Carbon mechanism on sustainable multi-objective solid transportation problem for waste management in Pythagorean hesitant fuzzy environment

Shyamali Ghosh · Karl-Heinz Küfer · Sankar Kumar Roy · Gerhard-Wilhelm Weber

Received: 12 May 2021 / Accepted: 3 February 2022 / Published online: 18 March 2022
© The Author(s) 2022

Abstract
Waste management involved in various fields of global ecosystem that provides several positive effects for green environment and sustainable development. We devise a multi-objective solid transportation model of waste management problem in agriculture field and forest department for urban or rural development. Starting to end point of the problem covered by considering the objective functions as transportation cost, job opportunity and carbon emission. Carbon emission is restricted by the combination of several policies of carbon mechanism (carbon tax, cap-and-trade and offset policy). Various critical stichs appear in such realistic process and uncertainty attached with related data. Here we prefer Pythagorean hesitant fuzzy environment to overcome deep uncertainty rather than single uncertainty. After that, we initiate a ranking approach to convert uncertain data into crisp data. To justify the appropriateness of the formulated model and to select the best policy of carbon mechanism, we study two industrial applications with various cases of such mechanism. To derive the Pareto-optimal solution of the problems, two fuzzy techniques, namely, fuzzy programming and Pythagorean hesitant fuzzy programming, are utilized here. Comparative study, model validation, sensitivity analysis, managerial insights and conclusions with future research scopes are outlined at last.

Keywords Multi-objective decision making · Waste management · Solid transportation problem · Carbon tax, cap, trade and offset policy · Sustainability · Pythagorean hesitant fuzzy programming

Introduction
In this section we focus on solid waste management (SWM), sustainability with sustainable development (SD), extension and activities of transportation problem (TP) in various environments, and application of carbon mechanism in several cases.

Solid waste management
Solid waste management is a new concept in sustainability and transportation. This management relates with solid wastes which are irregular combinations of rejected and unwanted items that created bad smell, attracted to virus and bacteria, and pollute environment directly or indirectly. A part of solid wastes (e.g., paper, wood, cotton, vegetables, cardboard, leather, ash, crop, spoil fruit, etc.) finished their activities in first user or first time, but these may rework second time by proper way of recycling. After recycling process the generated items are resold in market, and SWM provides
A sustainable transportation with an integrated AHP–DEA multi-objective optimization model in mining industry was proposed by Gupta et al. [17]. A robust optimization model with resilient closed loop supply chain was designed by Lotfi et al. [27] for sustainability. Vafaei et al. [50] provided a case study for designing a distribution network on sustainable multi-channel supply chain. A green closed-loop supply chain network was designed by Zhen et al. [61] in an uncertain situation for sustainability.

Sustainability and sustainable development

Sustainability has a major area due to cooperate on environment, social and economical domain. Definition of “Sustainability” is that “Development which meets the demands of the present without compromising the capability of the future generations to meet their own demands” (Brundtland Commission [8]). SD followed by sustainability and development which are concerned with three common parts as economical, social and environmental fact together. The three attributes of SD are fulfilled through transportation and SWM as follows.

Society: Social problem is created by the lack of job position when regional development is neglected. During SWM and transportation, social responsibility is completed by creating maximum number of job opportunities, satisfying public demand, human health, modern transportation planning, road management, pricing policies, improved vehicle technology, using clean fuels for minimum emissions, supplying recycled items, etc.

Economy: Economy and transportation are two sections of sustainable transportation that cannot be separated any time. Minimum transportation cost, carbon emission cost, and economical value from recycled items cause economic progression of sustainability.

Environment: Sustainable transportation has a responsibility on environment to reduce perilous carbon emissions and air pollution that can critically mutilate global health and disorder to urban ecosystem. It needs to promote a sustainable transportation system using green vehicles (e.g., electric vehicles, compressed air vehicles, hydrogen and fuel-cell vehicles, natural-gas vehicles, clean diesel vehicles, etc.). Minimum carbon emission from the recycle process of SWM can support towards an environment preservation.

Intersection of society, economy and environment: If optimum requirements of society, economy and environment factors are coincide, then SD and SWM provide green transportation by protecting environment, fulfilling social satisfaction and providing a highly growth economy.

A sustainable transportation with an integrated AHP–DEA multi-objective optimization model in mining industry was proposed by Gupta et al. [17]. A robust optimization model with resilient closed loop supply chain was designed by Lotfi et al. [27] for sustainability. Vafaei et al. [50] provided a case study for designing a distribution network on sustainable multi-channel supply chain. A green closed-loop supply chain network was designed by Zhen et al. [61] in an uncertain situation for sustainability.
Extension of TP with SWM and SD

TP is generated as linear programming problem initiated by Hitchcock [20] and it is renamed as Hitchcock–Koopmans TP. To transform the fresh items or waste items, closed type of vehicles are always used to prevent scattering or to control odour. The fresh items are transported from sources to destinations through several types of conveyances such as trucks, goods train, ships, etc. and dumper placer, hydraulic compactor vehicle, tripper, etc., are used for transporting waste items. Electric vehicles are suggested for sustainable transportation with economical and environmental improvement. Adding conveyance constraints on classical TP, the extended TP is called as solid transportation problem (STP), which was first defined by Haley [18]. Asim et al. [4] designed an integrated uncertain model on closed loop supply chain for transportation of production with cost reliability. An STP in fully intuitionistic fuzzy (IF) situation was initiated by Ghosh et al. [15], where the problem was expanded for multi-objective scenarios and fixed-charge.

Sustainable transportation on SWM includes transportation cost, job opportunity, carbon emission, etc., during shipment of items from distinct sources to several destinations by completing various steps. Traditional TP is not enough to control such situation that balance all the conflicting facts of sustainability. Therefore, a multi-objective transportation problem (MOTP) was initiated and extended as multi-objective solid transportation problem (MOSTP) by including conveyance constraint. TP with its several additions in various situations and different methodologies are incorporated from literature. Some related applications are selected here as: Dalman et al. [10] designed an MOSTP with multi-item and applied fuzzy approach in uncertain situation for finding a solution. An MOTP with truckload constraints was established by Ghosh and Roy [14] in fuzzy-rough environment. They analyzed fixed-charge by including product blending constraints and transfer station. Maity et al. [28] proposed an MOTP through time variation for SD with interval valued data. An MOTP with sustainability was introduced by Mehlawat et al. [30], where the problem was covered by three stages and fixed-charge. An MOSTP in IF environment was augmented by Midya et al. [31] for green supply chain with multiple stage and fixed-charge. Roy et al. [43] incorporated twofold uncertainty on MOSTP with fixed-charge that continued with multiple items. An MOTP with multiple items was discussed by Mardanya et al. [29] on the concept of just-in-time.

Carbon mechanism

Nowadays, global warming, environment contamination, health implications and green house gas (GHG) emissions become a big challenge from transportation or from industrial plant due to burning of fossil fuel or disposal of waste items. For global development and to augment ozone layer, sustainable transportation is needed and carbon mechanism is introduced to encourage the reduction of CO2 emission. To reduce such CO2 emission there exists an agreement such as Kyoto Protocol, such that rate of emissions from fossil fuel does not outstrip the assimilative capacity of the environment. Government as the third party helps to curb the emission by introducing carbon regulation. There exist several policies of carbon mechanism, such as “carbon tax, carbon cap, carbon cap-and-trade, carbon subsidy, carbon offset” policy. For such different policies, there exist various facilities and their combinations give a better profit which are, defined in some of the following ways:

Carbon tax with cap: For “carbon tax with cap” policy, the transportation system provides a tax to government for their regular carbon emissions and total emission is under a cap (i.e., free amount of maximum allowances). In this case the system does not give any penalty charge. For lower tax, the emission rate would not be reduced here.

Carbon tax with cap-and-trade: In this policy, the system gets an upper limit (i.e., cap) for emission and there exist two criteria. In the first criterion, if the transportation system emits more carbon than the cap unit, then the system would be penalized and will pay a penalty charge, and buy the shortage unit by paying an extra charge. In the second criterion, there exists a facility that if the system emits less carbon than the cap unit then the system can sell the extra unit in trading market and find a revenue. Therefore, for low emission, total carbon tax is decreased and for greater emission total tax is increased.

Carbon tax with offset: This policy consists of two agreements. First one is that, when total emission is less than the cap unit then the system cannot find any reward by selling their extra unit and total tax remains same. However, in second case, when total emission becomes the surplus of cap unit, then a penalty charge is to pay as an extra carbon tax and cannot buy the shortage carbon unit.

From literature we find various applications of carbon mechanism which are provided here as: Chiu et al. [9] launched carbon tax and trading theory on emissions for implementation of energy price. Das et al. [11] studied carbon emission tax, cap and offset policy on a green MOSTP in type-2 fuzzy environment and they found solution by fuzzy and non-fuzzy techniques with considering location and dwell time. Ghosh et al. [13] initiated carbon tax policy on a two-echelon supply chain and provided a collaborative model with uncertain demand. He et al. [19] represented cap-and-trade and carbon tax policy for a production with lot-sizing problem. Carbon subsidy policy for re-manufacturing closed-loop supply chain was defined by Li et al. [24]. In [25],...
the authors Li et al. analyzed single and multiple carbon policies for production and transportation in the supply chain by outsourcing the decisions. Trade-off between service time and carbon emissions on multi-echelon supply chains for safety stock was provided by Ni and Shu [36]. Palak et al. [37] applied the impacts of carbon mechanisms for bio-fuel supply chain with supplier and mode selection decision making problem. Cap-and-trade mechanism on production and pricing problems for supply chain was presented by Xu et al. [51]. Carbon emission constraint on global reverse supply chain was proposed by Xu et al. [52] for recycling waste items under uncertainties. Xu et al. [53] designed green supply chains and provided literature review of quantitative models based on carbon policies. A supply chain for energy saving and cost sharing was introduced by Yi and Li [55], where the emissions reduced by government subsidies and carbon tax policy. Yuyin and Jinxi [56] discussed the outcome of governmental policies as carbon taxes and energy-saving subsidies on a two-echelon supply chain to take enterprise decisions. Carbon pricing at variance with emissions trading on a supply chain was studied by Zakeri et al. [60]. Zhou et al. [62] showed the effects of carbon tariff by applying it on green supply chain design.

Environment selection

Sustainable MOSTP with SWM applied in realistic background, where all the data are not precisely defined for the presence of several hesitations, insufficient information, lack of evidence, competitive economic condition, fluctuations of financial market, etc. Some uncertain circumstances exist, where decision makers (DMs) cannot assign the precise value with their limited knowledge. These situations are evaluated by fuzzy, hesitant fuzzy, IF, neutrosophic, interval, rough, stochastic, randomness, PF, etc. Some uncontrolled realistic situations exist, where single uncertainty or traditional fuzzy set (FS) theory cannot sufficient to reflect the fact of decision making problem. In this state, we introduce a complex uncertainty as pythagorean hesitant fuzzy set (PHFS) which is the extension of PF set (PFS) [54] and hesitant fuzzy set (HFS) [49].

From literature we select some related works on HFS, PFS and PHFS as: Abbas et al. [2] presented cubic PFSs with unknown weight information and applied on multi-attribute decision making problem. Adhami and Ahmad [3] proposed an algorithm for solving MOTP in PHF uncertainty. Probabilistic HFSs for multiple criteria decision making problem was introduced by Farhadinia et al. [12]. Karaaslan and Ozlu [21] defined type-2 HFS with correlation coefficients and applied on clustering analysis. Pythagorean fuzzy approach applied by Kumar et al. [23] on TP. Peng and Selvachandran [39] discussed about the art and future directions of PFS. Nguyen et al. [35] explored PFS in decision making process by the application of exponential similarity measures. Qu et al. [40] implemented dual hesitant fuzzy linguistic uncertainty on stochastic multi-attribute method to measure the sustainability performance for supplier based and triple bottom line approach. Applications of correlation coefficients in PF situation was provided by Singh and Ganie [45]. Now, we place several research contributions in Table 1 which are connected with our work.

Research contributions of the present study

Table 1 highlights some research gaps and our current study provides some research contributions which are presented as follows:

- Comparing single objective TP form references [19, 25, 37, 42, 48], we fill the research gap by designing a mathematical model of MOSTP. Analyzing the references [3, 5, 33, 42, 43, 48, 52], we find that they were not concerned about three facts of sustainability and we augment such research by providing sustainable MOSTP that optimizes three factors of SD.

- References [3, 28, 30, 33, 42, 43, 48] are prepared without carbon emission or without carbon mechanism. We generate our present study that minimizes carbon emission by analyzing various policies of carbon mechanism. The attributes of carbon mechanism are focused through graphical presentation of results.

- The authors of the references [3, 17, 19, 25, 28, 30, 37, 43, 50, 60–62] did not include SWM in their articles. We apply the project of SWM in our present model that converts an undeveloped rural area into an modern and smart urban area.

- The existing literature [5, 19, 25, 30, 37, 42, 50, 60, 62] did not investigate about any type of uncertainty for realistic problem, but our designed model is implemented by considering a new environment as PHFS which is a combination of PFS and HFS. In this situation, pythagorean hesitant fuzzy programming (PHFP) and fuzzy programming (FP) validate the proposed model by finding the Pareto-optimal solution.

- We provide managerial insights of this study after displaying a sensitivity analysis and we illustrate the comprehensive discussions with conclusions.

Organization of the paper

The rest of this paper is outlined as follows: Motivation of our study are presented in the next section. The fundamental definitions of FS, HFS, PFS, PHFS with some basic properties are described in the following section. The following section depicts the notations with assumptions and the mathematical model of sustainable MOSTP with PHF uncertainty in con-
| Author(s) | SWM     | Environment | Multi-objective | Transportation mode | Cost  | Job opportunity | CO₂ emission | Carbon mechanism |
|-----------|---------|-------------|----------------|---------------------|-------|-----------------|--------------|-----------------|
| Adhami and Ahmad [3] | ✓       | PHF         | ✓              | ✓                   | ✓     |                 |              |                 |
| Asl et al. [5]       | ✓       | Crisp       | ✓              | ✓                   | ✓     |                 |              |                 |
| Gupta et al. [17]    | ✓       | Fuzzy       | ✓              | ✓                   | ✓     |                 |              |                 |
| He et al. [19]       | ✓       | Crisp       | ✓              |                      |       |                 |              |                 |
| Li et al. [25]       | ✓       | Crisp       | ✓              |                      |       |                 |              |                 |
| Maity et al. [28]    | ✓       | Interval    | ✓              |                      | ✓     |                 |              |                 |
| Mehlawat et al. [30] | ✓       | Crisp       | ✓              |                      |       |                 |              |                 |
| Muneeb et al. [33]   | ✓       | Uncertain   | ✓              |                      | ✓     |                 |              |                 |
| Palak et al. [37]    | ✓       | Crisp       | ✓              |                      |       |                 |              |                 |
| Rathore and Sarmah [42] | ✓     | Crisp       | ✓              |                      |       |                 |              |                 |
| Roy et al. [43]      | ✓       | Twofold uncertainty | ✓             |                      | ✓     |                 |              |                 |
| Tirkolaee et al. [48] | ✓      | Uncertain   | ✓              |                      |       |                 |              |                 |
| Vafaei et al. [50]   | ✓       | Crisp       | ✓              |                      | ✓     |                 |              |                 |
| Xu et al. [52]       | ✓       | Uncertain   | ✓              |                      |       |                 |              |                 |
| Zakeri et al. [60]   | ✓       | Crisp       | ✓              |                      | ✓     |                 |              |                 |
| Zhen et al. [61]     | ✓       | Uncertain   | ✓              |                      |       |                 |              |                 |
| Zhou et al. [62]     | ✓       | Crisp       | ✓              |                      |       |                 |              |                 |
| This investigation   | ✓       | PHF         | ✓              |                      | ✓     |                 |              |                 |
Table 2  Abbreviations with full names

| Abbreviations | Full name                      |
|---------------|--------------------------------|
| SWM           | Solid waste management         |
| SD            | Sustainable development        |
| TP            | Transportation problem         |
| PF            | Pythagorean fuzzy              |
| STP           | Solid transportation problem   |
| IF            | Intuitionistic fuzzy           |
| MOTP          | Multi-objective transportation problem |
| MOSTP         | Multi-objective solid transportation problem |
| GHG           | Green house gas                |
| DM            | Decision maker                 |
| FS            | Fuzzy set                      |
| PHFS          | Pythagorean hesitant fuzzy set |
| PFS           | Pythagorean fuzzy set          |
| HFS           | Hesitant fuzzy set             |
| TrPHFN        | Trapezoidal pythagorean hesitant fuzzy number |
| FP            | Fuzzy programming              |
| PHFP          | Pythagorean hesitant fuzzy programming |
| PIS           | Positive ideal solution        |
| NIS           | Negative ideal solution        |
| NFS           | No feasible solution           |

Abbreviations: Table 2 represents all the abbreviations which are used in this study.

Motivation of the study

Due to rapid growth of urbanization or industrialization, different types of domestic and commercial wastes are created which called as municipal solid wastes, and these waste items generated by living community. Similarly, rural areas (agriculture field, forest department, cold storage) produce huge amount of waste items (e.g., vegetable waste, fruit waste, woody waste) in post-harvest, natural processes or in some commercial activities. India is successful in agriculture resources, where 30% of the vegetables or fruit items are wasted and thrown everyday, but these items are found by free of cost and convert into useful product (bio-fuel or compost) by proper treatment or recycle process. It is not so easy to collect, transport, recycle and reuse of these waste items in a systematic way with considering sustainability. Demand of petroleum products are increasing but these cannot be renewable, so bio-fuel generation from waste is an alternative renewable energy source that balancing between energy and pollution. Hence, an optimization strategy is required that provides an intellectual plan for converting waste to sustainable energy.

We observed that a simple uncertainty is presented by fuzzy data appears by the membership value that shows the amount of uncertainty of an event. The uncertainty of occurrence of such event is displayed by truth membership value and uncertainty of non-occurrence of that event is defined by the falsity membership value. The sum of membership value and non-membership value lies in [0, 1] and such uncertainty is called as IF uncertainty. If the sum of membership value and non-membership value greater than 1, then such uncertainty converts into PF uncertainty. A hesitant situation of membership and non-membership grades cannot be presented by a single value and this type of uncertainty is noted as PHF uncertainty.

Example 1 Suppose a certain type of items is to be reached in destinations within a required time. For some natural conditions (e.g., bad weather, sudden road strike, or road accident) the items cannot be reached in destination that DMs expect. A situation arises that it is supported for yes as 0.7 which is the membership grade. If the non-membership grade for supporting to no criteria is 0.4, then this state is only covered by PFS, not by FS nor by IFS. Let there exist three DMs and they assign the membership values as 0.2, 0.7 and 0.8; then hesitancy occurs on membership values for their partial knowledge. Similarly, they assign the non-membership grades as 0.1, 0.4 and 0.5. This uncertain situation handles by PHFS, not by PFS for the hesitancy. In addition, this is not controlled by HFS for the existence of membership and non-membership grades. Connecting about such situation, we design an effective model on MOSTP which is concerned with waste management and sustainability in PHF uncertainty.

Basic definitions and operations

To establish our present model, certain constructive definitions with their specific properties and fundamental operations based on FS, PFS, HFS and PHFS are defined here.

Definition 3.1 (Fuzzy set (FS), [57]) An FS $\tilde{A}$ in a universal set $X$ is specified by a membership function $\mu_{\tilde{A}}(x)$ that identifies each element $x$ in $X$ to a real number in the inter-
val [0, 1]. In FS, trapezoidal fuzzy number is a quadruplet defined as $A^n = (a, b, c, d)$, where $a \leq b \leq c \leq d$.

**Definition 3.2** (Pythagorean fuzzy set (PFS), [54]) Let $X$ be a universal set, then a PFS, $\tilde{A}^p$ in $X$ is explained as: $\tilde{A}^p = \{(x, \mu_{\tilde{A}^p}(x), \gamma_{\tilde{A}^p}(x)) : 0 \leq \mu_{\tilde{A}^p}(x) + \gamma_{\tilde{A}^p}(x) \leq 1, \mu_{\tilde{A}^p}(x), \gamma_{\tilde{A}^p}(x) \in [0, 1], x \in X\}$, where $\mu_{\tilde{A}^p}(x)$ and $\gamma_{\tilde{A}^p}(x)$ are the membership and non-membership function. The degree of indeterminacy of an element $x$ in the set $\tilde{A}^p$ is defined as the function $\pi_{\tilde{A}^p}(x) = \sqrt{1 - \mu_{\tilde{A}^p}^2(x) - \gamma_{\tilde{A}^p}^2(x)}$. It may be in the form $\mu_{\tilde{A}^p}(x) + \gamma_{\tilde{A}^p}(x) \geq 1$ or $\mu_{\tilde{A}^p}(x) + \gamma_{\tilde{A}^p}(x) \leq 1$.

**Definition 3.3** (Hesitant fuzzy set (HFS), [49]) Let $X$ be a universal set, then HFS, $\tilde{A}^h$ on $X$ is stated as $\tilde{A}^h = \{x, \mu_{\tilde{A}^h}(x) : x \in X\}$, where $\mu_{\tilde{A}^h}(x)$ is a set of some different values in $[0, 1]$, denoting the possible membership grades of the element $x$ in the set.

**Example** A set $\tilde{A}^h = \{x, 0.2, 0.5, 0.7, 0.9\}$ is a HFS with membership degree 0.2, 0.5, 0.7, and 0.9 of the element $x$ in the universal set.

**Definition 3.4** (Pythagorean hesitant fuzzy set (PHFS), [26]) To express the hesitant situation of PFS, HFS is introduced which extends PFS into PHFS. For a universal set $X$, PHFS $\tilde{A}$ is defined as: $\tilde{A} = \{(x, \mu_{\tilde{A}}(x), \gamma_{\tilde{A}}(x)) : x \in X\}$, where $\mu_{\tilde{A}}(x)$ and $\gamma_{\tilde{A}}(x)$ are the sets of some values in $[0, 1]$ denoting the possible Pythagorean hesitant membership and Pythagorean hesitant non-membership degree of the element $x \in X$ to the set $\tilde{A}$, respectively, with the conditions $0 \leq \theta, \eta \leq 1$ and $0 \leq \theta^2 + \eta^2 \leq 1$, where $\theta = \mu_{\tilde{A}}(x)$, $\eta = \gamma_{\tilde{A}}(x)$, $\forall x \in X$. Here, the membership and non-membership degrees are called hesitant fuzzy elements.

**Example** A set $\tilde{A} = \{(x, 0.4, 0.6), (0.2, 0.5, 0.7)\}$ is a PHFS of membership degree 0.4 and 0.6, and non-membership degree 0.2, 0.5 and 0.7 of the element $x$ in the set.

If a PHFS is convex and defined on a closed and bounded interval then the set is called as Pythagorean hesitant fuzzy number (PHFN).

**Trapezoidal Pythagorean hesitant fuzzy number (TrPHFN)**

Let $X$ be a universal set, then TrPHFN $\tilde{z}$ is defined as $\tilde{z} = \{(a, b, c, d), \mu_{\tilde{z}}(x), \gamma_{\tilde{z}}(x) : 0 \leq \mu_{\tilde{z}}(x) + \gamma_{\tilde{z}}(x) \leq 1, \mu_{\tilde{z}}(x), \gamma_{\tilde{z}}(x) \in [0, 1], x \in X\}$, where $\mu_{\tilde{z}}(x)$ and $\gamma_{\tilde{z}}(x)$ are the sets of some values in $[0, 1]$ denoting the possible Pythagorean hesitant membership and Pythagorean hesitant non-membership degrees of the element $x \in X$ with $a \leq b \leq c \leq d$. Furthermore, $\omega_1, \omega_2 \in [0, 1]$ are the set of hesitant values for membership and non-membership functions, respectively, and $0 \leq \omega_1^2 + \omega_2^2 \leq 1$.

The Pythagorean hesitant membership and non-membership function of $\tilde{A}$ defined as

$$
\mu_{\tilde{A}}(x) = \begin{cases}
\omega_1 \left(\frac{x-a}{b-a}\right), & \text{if } a \leq x < b, \\
\omega_1, & \text{if } b \leq x < c, \\
\omega_1 \left(\frac{d-x}{d-c}\right), & \text{if } c \leq x \leq d, \\
0, & \text{if } x < a \text{ or } x > d.
\end{cases}
$$

$$
\gamma_{\tilde{A}}(x) = \begin{cases}
\omega_2 \sqrt{1 - \left(\frac{x-a}{b-a}\right)^2}, & \text{if } a \leq x < b, \\
\omega_2, & \text{if } b \leq x < c, \\
\omega_2 \sqrt{1 - \left(\frac{d-x}{d-c}\right)^2}, & \text{if } c \leq x \leq d, \\
1, & \text{if } x < a \text{ or } x > d.
\end{cases}
$$

**Arithmetic operation of TrPHFNs**

Arithmetic operations of TrPHFNs are required for the calculations of several data taken as TrPHFNs. Several ranking approaches are chosen to convert TrPHFNs into crisp numbers by utilizing these operations which are extended into different forms and applied in multi-attribute decision making problem. Let two TrPHFNs $\tilde{A}_1 = ((a_1^1, a_1^2, a_1^3, a_1^4); \mu_{\tilde{A}_1}^1, \gamma_{\tilde{A}_1}^1)$ and $\tilde{A}_2 = ((a_2^1, a_2^2, a_2^3, a_2^4); \mu_{\tilde{A}_2}^1, \gamma_{\tilde{A}_2}^1)$ be given, where $a_1^1, a_2^1 \in [0, 1]$, $0 \leq a_1^2 + a_2^2 \leq 1$, $0 \leq a_1^3 + a_2^3 \leq 1$, and $0 \leq a_1^4 + a_2^4 \leq 1$. The arithmetic operations are expounded as follows:

**Addition:** $\tilde{A}_1 \oplus \tilde{A}_2 = ((a_1^1 + a_2^1, a_1^2 + a_2^2, a_1^3 + a_2^3, a_1^4 + a_2^4); \mu_{\tilde{A}_1}^1 \land \mu_{\tilde{A}_2}^1, \gamma_{\tilde{A}_1}^1 \lor \gamma_{\tilde{A}_2}^1)$, where $\theta_1 \in \mu_{\tilde{A}_1}^1 \land \mu_{\tilde{A}_2}^1, \theta_2 \in \gamma_{\tilde{A}_1}^1 \lor \gamma_{\tilde{A}_2}^1$ and $0 \leq \theta_1^2 + \theta_2^2 \leq 1$.

**Subtraction:** $\tilde{A}_1 \ominus \tilde{A}_2 = ((a_1^1 - a_2^1, a_1^2 - a_2^2, a_1^3 - a_2^3, a_1^4 - a_2^4); \mu_{\tilde{A}_1}^1 \land \mu_{\tilde{A}_2}^1, \gamma_{\tilde{A}_1}^1 \lor \gamma_{\tilde{A}_2}^1)$, where $0 \leq \theta_1^2 + \theta_2^2 \leq 1$.

**Multiplication:** $\tilde{A}_1 \odot \tilde{A}_2 = ((a_1^1 a_2^1, a_1^2 a_2^2, a_1^3 a_2^3, a_1^4 a_2^4); \mu_{\tilde{A}_1}^1 \land \mu_{\tilde{A}_2}^1, \gamma_{\tilde{A}_1}^1 \lor \gamma_{\tilde{A}_2}^1)$, where $0 \leq \theta_1^2 + \theta_2^2 \leq 1$.

**Scalar multiplication:** Scalar multiplication for any real $k$ is described as

$$
k \tilde{A}_1 = \begin{cases}
((ka_1^1, ka_1^2, ka_1^3, ka_1^4); \mu_{\tilde{A}_1}^1, \gamma_{\tilde{A}_1}^1); \omega_1, \omega_2, & \text{if } k \geq 0, \\
((ka_1^1, ka_1^2, ka_1^3, ka_1^4); \mu_{\tilde{A}_1}^1, \gamma_{\tilde{A}_1}^1); \omega_1, \omega_2, & \text{if } k < 0.
\end{cases}
$$

**Ranking function**

To convert TrPHFNs to crisp numbers and to compare with TrPHFNs, defuzzification plays an important role. Different defuzzification methods exist in the literature, such as $\alpha$-cut, possibility concept, integration method, linguistic approach,
etc. Here, we introduce an advanced and simple ranking approach through integration in terms of the TrPHFN that transforms TrPHFN into crisp number, which is applied in our proposed problem. Let \( \tilde{\mathbf{A}} \) be a TrPHFN then ranking index is defined by \( \mathfrak{R}(\tilde{\mathbf{A}}) \), which maps TrPHFNs to real line, i.e., \( \mathfrak{R} : \mathcal{F}(\tilde{\mathbf{A}}) \rightarrow \mathbb{R} \), where \( \mathcal{F}(\tilde{\mathbf{A}}) \) is the collection of all TrPHFNs and \( \mathbb{R} \) is the set of real numbers. Mathematically, the ranking index is defined for a TrPHFN \( \tilde{\mathbf{A}} = ((a, b, c, d); \mu_{\tilde{\mathbf{A}}}, \upsilon_{\tilde{\mathbf{A}}}; \omega_1, \omega_2) \), as follows:

\[
\mathfrak{R}(\tilde{\mathbf{A}}) = \frac{\int_a^b x\mu_{\tilde{\mathbf{A}}} dx + \int_a^b x\upsilon_{\tilde{\mathbf{A}}} dx}{\int_a^b \mu_{\tilde{\mathbf{A}}} dx + \int_a^b \upsilon_{\tilde{\mathbf{A}}} dx} = \frac{\omega_1^2(-a^2 - 3b^2 + 3c^2 + d^2 - 2ab + 2cd) + \omega_2^2(-5a^2 + 3b^2 - 3c^2 + 5d^2 + 2ab - 2cd)}{\omega_1^2(-a - 2b + 2c + d) + \omega_2^2(-2a + 2b - 2c + 2d)}.
\]  

(3.1)

Let \( \tilde{\mathbf{A}}_1 \) and \( \tilde{\mathbf{A}}_2 \) be TrPHFNs. Then, the ranking operation obeys the following comparisons as

**Case (i)** \( \mathfrak{R}(\tilde{\mathbf{A}}_1) > \mathfrak{R}(\tilde{\mathbf{A}}_2) \Rightarrow \tilde{\mathbf{A}}_1 > \tilde{\mathbf{A}}_2 \), i.e., \( \min[\tilde{\mathbf{A}}_1, \tilde{\mathbf{A}}_2] = \tilde{\mathbf{A}}_2 \).

**Case (ii)** \( \mathfrak{R}(\tilde{\mathbf{A}}_1) < \mathfrak{R}(\tilde{\mathbf{A}}_2) \Rightarrow \tilde{\mathbf{A}}_1 < \tilde{\mathbf{A}}_2 \), i.e., \( \min[\tilde{\mathbf{A}}_1, \tilde{\mathbf{A}}_2] = \tilde{\mathbf{A}}_1 \).

**Case (iii)** \( \mathfrak{R}(\tilde{\mathbf{A}}_1) = \mathfrak{R}(\tilde{\mathbf{A}}_2) \Rightarrow \tilde{\mathbf{A}}_1 = \tilde{\mathbf{A}}_2 \), i.e., \( \min[\tilde{\mathbf{A}}_1, \tilde{\mathbf{A}}_2] = \tilde{\mathbf{A}}_1 \) or \( \tilde{\mathbf{A}}_2 \).

**Theorem 3.1** \( \mathfrak{R}(\alpha \cdot \tilde{\mathbf{A}}) = \alpha \cdot \mathfrak{R}(\tilde{\mathbf{A}}) \); here, \( \alpha \in \mathbb{R}, \alpha \geq 0 \).

**Proof** Let \( \tilde{\mathbf{A}} = ((a, b, c, d); \mu_{\tilde{\mathbf{A}}}, \upsilon_{\tilde{\mathbf{A}}}; \omega_1, \omega_2) \) be a TrPHFN then from definition of ranking index (cf. Eq. (3.1)), we have

\[
\alpha \cdot \mathfrak{R}(\tilde{\mathbf{A}}) = \frac{4}{\omega_1^2(-a - 2b + 2c + d) + \omega_2^2(-2a + 2b - 2c + 2d)} \left[ \frac{\omega_1^2(-a^2 - 3b^2 + 3c^2 + d^2 - 2ab + 2cd) + \omega_2^2(-5a^2 + 3b^2 - 3c^2 + 5d^2 + 2ab - 2cd)}{\omega_1^2(-a^2 - 3b^2 + 3c^2 + d^2 - 2ab + 2cd) + \omega_2^2(-5a^2 + 3b^2 - 3c^2 + 5d^2 + 2ab - 2cd)} \right].
\]

Now

\[
\mathfrak{R}(\alpha \cdot \tilde{\mathbf{A}}) = \frac{4}{\omega_1^2(-a - 2b + 2c + d) + \omega_2^2(-2a + 2b - 2c + 2d)} \left[ \frac{\omega_1^2(-a^2 - 3b^2 + 3c^2 + d^2 - 2ab + 2cd) + \omega_2^2(-5a^2 + 3b^2 - 3c^2 + 5d^2 + 2ab - 2cd)}{\omega_1^2(-a^2 - 3b^2 + 3c^2 + d^2 - 2ab + 2cd) + \omega_2^2(-5a^2 + 3b^2 - 3c^2 + 5d^2 + 2ab - 2cd)} \right] = \alpha \cdot \mathfrak{R}(\tilde{\mathbf{A}}).
\]

**Problem description and model formulation**

This section contains some subsections which illustrate problem background, needed notations with assumptions and the proposed integrated model.

**Problem background**

We are to select an SWM and TP for agriculture field and forest department, such that sustainability criteria are followed up. At first the fresh fruits or grains or vegetables or other items are transported into cold storage for preservation and to use in future marketing. Various agriculture wastes are produced after harvesting and forestry wastes are produced from forest department as residue of main items, and some wastes are generated from cold storage by the deterioration of fresh items. These waste items are transported into bio-fuel production plant and compost production plant according to their nature. Various recycle processes produced bio-fuel and compost, and these are shifted to bio-energy supply centre and compost selling market for reuse and to achieve some revenues. This SWM is completed by performing several steps such that sustainability criteria (economical condition, social impact and environment improvement) are always fixed. For economical improvement, we try to minimize transportation cost, and for social impact we want to maximize the number of job opportunities. For environment protection, we incorporate carbon mechanism for reduction of carbon emission.
Fig. 1 Network design for processing steps of MOSTP with SWM

Here, the object of carbon mechanism is to minimize total carbon emission based on “carbon tax with cap”, “carbon tax with cap-and-trade” and “carbon tax with offset” policy. Having these goals, we formulate an unpredicted mathematical model of MOSTP by including source, demand and conveyance constraints in the PHF environment for inconsistent and incomplete information. The performing stages of the formulated model are graphically described in Fig. 1.

Notations and assumptions of the proposed study

A list of notations with their nature, meaning, and required assumptions are selected to design the proposed model. These notations are illustrated in Tables 3 and 4.

Assumption

- \( \tilde{a}_i^1 > 0, \tilde{a}_j^2 > 0, \tilde{b}_k^1 > 0, \tilde{b}_l^1 > 0, \tilde{b}_m^1 > 0, \tilde{b}_n^1 > 0, \tilde{b}_l^2 > 0, \tilde{b}_l^3 > 0, \tilde{b}_l^4 > 0, \forall \, i, j, k, l, m, n, p, r. \) The positivity criteria is obeyed if every element of the quadruplet of TrPHFNs is positive.
- Only agriculture and forestry wastes with cold storage waste items are managed here.
- All the wastes should be distributed according to their nature.
- Cost of bio-fuel production and compost production are ignored.
- Total carbon emission during transportation of SWM:

\[
E = E(e_{CO_2}; x, y, w, z) = e_{CO_2} \left( \sum_{i=1}^{I} \sum_{p=1}^{P} \sum_{k=1}^{K} x_{1kp}d_{1kp}^1 + \sum_{l=1}^{L} x_{2lp}d_{2lp}^2 + \sum_{m=1}^{M} x_{3mp}d_{3mp}^3 \right) + \sum_{j=1}^{J} \sum_{p=1}^{P} \left( \sum_{k=1}^{K} y_{jkp}d_{jkp}^1 + \sum_{l=1}^{L} y_{jlp}d_{jlp}^2 + \sum_{m=1}^{M} y_{jmp}d_{jmp}^3 \right) + \sum_{p=1}^{P} \left( \sum_{k=1}^{K} w_{1kp}d_{1kp}^1 + \sum_{m=1}^{M} w_{2mp}d_{2mp}^2 \right) \]
Table 3  Usefu notations with their nature and descriptions

| Description | Type |
|-------------|------|
| **Index sets** | |
| $i$ | Index for source of forest department ($i = 1, 2, \ldots, I$) | Integer |
| $j$ | Index for source of agriculture field ($j = 1, 2, \ldots, J$) | Integer |
| $k$ | Index for bio-fuel production plant ($k = 1, 2, \ldots, K$) | Integer |
| $l$ | Index for cold storage ($l = 1, 2, \ldots, L$) | Integer |
| $m$ | Index for compost production plant ($m = 1, 2, \ldots, M$) | Integer |
| $n$ | Index for bio-fuel supply centre ($n = 1, 2, \ldots, N$) | Integer |
| $r$ | Index for compost selling market centre ($r = 1, 2, \ldots, R$) | Integer |
| $p$ | Index for conveyance type ($p = 1, 2, \ldots, P$) | Integer |

**Decision variables**

| Description | Type |
|-------------|------|
| $x_{1kp}$ | Amount of forestry waste items that to be transported from $i$th forest department to $k$th bio-fuel production plant through $p$th conveyance | Real |
| $x_{2lp}$ | Amount of fresh forestry items that to be transported from $i$th forest department to $l$th cold storage through $p$th conveyance | Real |
| $x_{3mp}$ | Amount of forestry waste items that to be transported from $i$th forest department to $m$th compost production plant through $p$th conveyance | Real |
| $y_{1jkp}$ | Amount of agriculture waste items that to be transported from $j$th agriculture field to $k$th bio-fuel production plant through $p$th conveyance | Real |
| $y_{2jp}$ | Amount of fresh agriculture items that to be transported from $j$th agriculture field to $l$th cold storage through $p$th conveyance | Real |
| $y_{3jp}$ | Amount of agriculture waste items that to be transported from $j$th agriculture field to $m$th compost production plant through $p$th conveyance | Real |
| $w_{1kp}$ | Amount of waste items transported from $l$th cold storage to $k$th bio-fuel production plant through $p$th conveyance | Real |
| $w_{2mp}$ | Amount of waste items transported from $l$th cold storage to $m$th compost production plant through $p$th conveyance | Real |
| $z_{1knp}$ | Amount of bio-fuel transported from $k$th bio-fuel production plant to $n$th bio-fuel supply centre through $p$th conveyance | Real |
| $z_{2mrp}$ | Amount of compost items transported from $m$th compost production plant to $r$th compost selling market centre through $p$th conveyance | Real |

**Parameters**

| Description | Type |
|-------------|------|
| $\tilde{a}_{ii}$ | PHF available amount of items (fresh and waste) at $i$th source | Fuzzy |
| $\tilde{a}_{jj}$ | PHF available amount of items(fresh and waste) at $j$th source | Fuzzy |
| $\tilde{b}_{k}$ | PHF capability of $k$th bio-fuel production plant | Fuzzy |
| $\tilde{b}_{l}$ | PHF capability of $l$th cold storage | Fuzzy |
| $\tilde{b}_{m}$ | PHF capability of $m$th compost production plant | Fuzzy |
| $\tilde{b}_{n}$ | PHF demand of $n$th bio-fuel supply centre | Fuzzy |
| $\tilde{b}_{r}$ | PHF demand of $r$th compost selling market centre | Fuzzy |
| $\tilde{e}_{p}$ | PHF capacity of $p$th type of conveyance | Fuzzy |
| $\alpha, \beta$ | Rate of waste generation from $l$th cold storage that transported to $k$th bio-fuel production plant and $m$th compost production plant, respectively | Real |
| $e_{CO_2}^{1}$ | Rate of carbon emission of the vehicle per unit item and per unit distance | Real |
| $e_{CO_2}^{2}$ | Rate of carbon emission per unit item from bio-fuel production plant | Real |
| $e_{CO_2}^{3}$ | Rate of carbon emission per unit item from compost production plant | Real |
| $\theta_{1}$ | Tax for carbon emission during transportation | Real |
| $\theta_{2}$ | Tax for carbon emission to bio-fuel production plant | Real |
| $\theta_{3}$ | Tax for carbon emission to compost production plant | Real |
| $\phi$ | Carbon trading (buying) cost per unit item | Real |
| $\psi$ | Carbon trading (selling) cost per unit item | Real |
Table 3  continued

| Description | Type |
|-------------|------|
| $C$         | Carbon cap (maximum allowance of carbon emission) | Real |
| $P$         | Penalty charge per unit item that emitted surplus carbon to cap | Real |
| $Z_s$       | Objective function, $(s = 1, 2, 3)$ | Real |

Table 4  Symbols with their nature and descriptions

| Parameters | Description | Type |
|------------|-------------|------|
| $c_{ikp}^{11}$ | Transportation cost per unit forestry waste item that transported from $i$th source to $k$th bio-fuel production plant through $p$th conveyance | Real |
| $c_{ilp}^{12}$ | Transportation cost per unit forestry fresh item that transported from $i$th source to $l$th cold storage through $p$th conveyance | Real |
| $c_{imp}^{13}$ | Transportation cost per unit forestry waste item that transported from $i$th source to $m$th compost production plant through $p$th conveyance | Real |
| $c_{jkp}^{21}$ | Transportation cost per unit agriculture waste item that transported from $j$th source to $k$th bio-fuel production plant through $p$th conveyance | Real |
| $c_{jlp}^{22}$ | Transportation cost per unit agriculture fresh item that transported from $j$th source to $l$th cold storage through $p$th conveyance | Real |
| $c_{jmp}^{23}$ | Transportation cost per unit agriculture waste item that transported from $j$th source to $m$th compost production plant through $p$th conveyance | Real |
| $c_{lkp}^{31}$ | Transportation cost per unit waste item that transported from $l$th cold storage to $k$th bio-fuel production plant through $p$th conveyance | Real |
| $c_{lmp}^{32}$ | Transportation cost per unit waste items that transported from $l$th cold storage to $m$th compost production plant through $p$th conveyance | Real |
| $c_{knp}^{41}$ | Transportation cost per unit bio-fuel that transported from $k$th bio-fuel production plant to $n$th bio-fuel supply centre through $p$th conveyance | Real |
| $c_{mrp}^{42}$ | Transportation cost per unit compost that transported from $m$th compost production plant to $r$th compost selling market centre through $p$th conveyance | Real |
| $d_{ikp}^{11}$ | Distance from $i$th source to $k$th destination through $p$th conveyance | Real |
| $d_{ilp}^{12}$ | Distance from $i$th source to $l$th destination through $p$th conveyance | Real |
| $d_{imp}^{13}$ | Distance from $i$th source to $m$th destination through $p$th conveyance | Real |
| $d_{jkp}^{21}$ | Distance from $j$th source to $k$th destination through $p$th conveyance | Real |
| $d_{jlp}^{22}$ | Distance from $j$th source to $l$th destination through $p$th conveyance | Real |
| $d_{jmp}^{23}$ | Distance from $j$th source to $m$th destination through $p$th conveyance | Real |
| $d_{lkp}^{31}$ | Distance from $l$th source to $k$th destination through $p$th conveyance | Real |
| $d_{lmp}^{32}$ | Distance from $l$th source to $m$th destination through $p$th conveyance | Real |
| $d_{knp}^{41}$ | Distance from $k$th source to $n$th destination through $p$th conveyance | Real |
| $d_{mrp}^{42}$ | Distance from $m$th source to $r$th destination through $p$th conveyance | Real |
| $\eta_i^1$ | Number of job opportunity at $i$th forest department | Real |
| $\eta_i^2$ | Number of job opportunity at $i$th agriculture field | Real |
| $\eta_k^3$ | Number of job opportunity at $k$th bio-fuel production plant | Real |
| $\eta_i^4$ | Number of job opportunity at $i$th cold storage | Real |
| $\eta_m^5$ | Number of job opportunity at $m$th compost production plant | Real |
| $\eta_n^6$ | Number of job opportunity at $n$th bio-fuel supply centre | Real |
| $\eta_r^7$ | Number of job opportunity at $r$th compost selling market centre | Real |
ever, source, demand and vehicle capacity are affected due to several unpredictable conditions that are not previously detected. In such situations we consider these parameters as PHFNs and the corresponding model becomes in a PHF environment. The integrated optimization model of MOSTP with PHF environment is implemented in Model 1 to tackle real-life hesitation or uncertainty about SWM with sustainability. We strive to minimize transportation cost, maximize job opportunity and minimize carbon emission cost.

**Implementation of uncertain model**

**Model 1**

minimize $Z_1(x, y, z, w) =$

\[
\sum_{i=1}^{L} \sum_{j=1}^{P} \left( \sum_{k=1}^{K} x_{ikp}^1 \right) + \sum_{j=1}^{P} \left( \sum_{m=1}^{M} x_{imp}^3 \right) + \sum_{j=1}^{P} \left( \sum_{p=1}^{P} \sum_{k=1}^{K} x_{lmp}^1 \right)
\]

subject to

\[
\sum_{p=1}^{P} \left( \sum_{k=1}^{K} x_{ikp}^1 + \sum_{l=1}^{L} x_{imp}^2 \right)
\]

maximize $Z_2(x, y, z, w) =$

\[
\sum_{i=1}^{L} \sum_{j=1}^{P} \left( \sum_{k=1}^{K} x_{ikp}^1 \right) + \sum_{m=1}^{M} x_{imp}^3 \]

subject to

\[
\sum_{p=1}^{P} \left( \sum_{k=1}^{K} x_{ikp}^1 + \sum_{l=1}^{L} x_{imp}^2 \right)
\]
The feasibility conditions of this TP are set as:

\( \sum_{i=1}^{l} \sum_{k=1}^{K} x_{ikp}^1 \leq \tilde{e}_p \ (p = 1, 2, \ldots, P), \) (4.13)

\( \sum_{i=1}^{l} \sum_{l=1}^{L} x_{ilp}^2 \leq \tilde{e}_p \ (p = 1, 2, \ldots, P), \) (4.14)

\( \sum_{i=1}^{l} \sum_{m=1}^{M} x_{imp}^3 \leq \tilde{e}_p \ (p = 1, 2, \ldots, P), \) (4.15)

\( \sum_{j=1}^{L} k \ y_{1kp}^1 \leq \tilde{e}_p \ (p = 1, 2, \ldots, P), \) (4.16)

\( \sum_{j=1}^{L} l \ y_{1lp}^2 \leq \tilde{e}_p \ (p = 1, 2, \ldots, P), \) (4.17)

\( \sum_{j=1}^{L} M \ y_{1mp}^3 \leq \tilde{e}_p \ (p = 1, 2, \ldots, P), \) (4.18)

\( \sum_{k=1}^{K} \sum_{l=1}^{L} w_{lkp}^1 \leq \tilde{e}_p \ (p = 1, 2, \ldots, P), \) (4.19)

\( \sum_{l=1}^{L} \sum_{m=1}^{M} w_{imp}^2 \leq \tilde{e}_p \ (p = 1, 2, \ldots, P), \) (4.20)

\( \sum_{k=1}^{K} \sum_{n=1}^{N} z_{knp}^1 \leq \tilde{e}_p \ (p = 1, 2, \ldots, P), \) (4.21)

\( \sum_{m=1}^{M} \sum_{r=1}^{R} z_{mrdp}^2 \leq \tilde{e}_p \ (p = 1, 2, \ldots, P). \) (4.22)

\( x_{ikp}^1 \geq 0, \ x_{ilp}^2 \geq 0, \ x_{imp}^3 \geq 0, \ y_{1kp}^1 \geq 0, \)

\( y_{1lp}^2 \geq 0, \ y_{1mp}^3 \geq 0, \ w_{lkp}^1 \geq 0, \ w_{imp}^2 \geq 0, \ z_{knp}^1 \geq 0, \)

\( z_{mrdp}^2 \geq 0, \ \forall \ i, j, k, l, m, n, p, r. \) (4.23)

The feasibility conditions of this TP are set as: \( \sum_{l=1}^{L} \tilde{a}_l^1 \geq \sum_{k=1}^{K} \tilde{b}_k^{11} + \sum_{l=1}^{L} \tilde{a}_l^2 \geq \sum_{j=1}^{J} \tilde{a}_j^3 \geq \sum_{l=1}^{L} \tilde{a}_l^4 \geq \sum_{m=1}^{M} \tilde{b}_m^{13}; \)

\( \sum_{j=1}^{J} \tilde{a}_j^5 \geq \sum_{k=1}^{K} \tilde{b}_k^{14} + \sum_{j=1}^{J} \tilde{a}_j^6 \geq \sum_{l=1}^{L} \tilde{a}_l^7 \geq \sum_{m=1}^{M} \tilde{b}_m^{15}; \)

\( \sum_{m=1}^{M} \tilde{b}_m^{16} \sum_{k=1}^{K} \tilde{b}_k^{17} \geq \sum_{n=1}^{N} \tilde{b}_n^{21} + \sum_{m=1}^{M} \tilde{b}_m^{22} \geq \sum_{r=1}^{R} \tilde{b}_r^{23} \geq \sum_{m=1}^{M} \tilde{b}_m^{24} + \sum_{l=1}^{L} \tilde{b}_l^{25} \geq \sum_{r=1}^{R} \tilde{b}_r^{26}. \)

**Elementary information about the model**

In Model 1, three objective functions are illustrated by Eqs. (4.1)–(4.3). The 1st objective function calculates the total transportation cost that to be minimized for transporting fresh items, waste items and recycled items. The 1st part consists of three parts which state the total transportation cost from \( i \)th forest department to \( k \)th bio-fuel production plant, to \( l \)th cold storage and to \( m \)th compost production plant through \( p \)th conveyance, respectively. The 2nd part contains three parts that describe the total transportation cost from \( j \)th agriculture field to \( k \)th bio-fuel production plant, \( l \)th cold storage and \( m \)th compost production plant by \( p \)th mode of vehicle, respectively. The 3rd part includes two parts which indicate the total transportation cost from \( l \)th cold storage to \( k \)th and \( m \)th destination, respectively, with \( p \)th set of conveyance. The 4th part composes two parts such as total transportation cost from \( k \)th bio-fuel production plant to \( n \)th bio-fuel supply centre and from \( m \)th compost production plant to \( r \)th compost selling market centre using \( p \)th type of conveyance.

The 2nd objective function is the total number of job opportunity which to be maximized for transporting the items and for other works (e.g., loading, unloading, treatment, marketing, managing, etc.) through this SWM problem. The 1st, 2nd and 3rd parts are the total number of job opportunity from \( i \)th source to \( k \)th bio-fuel production plant, \( l \)th cold storage and \( m \)th compost production plant, respectively, and transportation of items completed through \( p \)th conveyance. The 4th, 5th and 6th parts are the entire number of job opportunity from \( j \)th source to \( k \)th bio-fuel production plant, \( l \)th cold storage and \( m \)th compost production plant, respectively, with \( p \)th model of conveyance. The 9th part is the total number of job opportunity from \( k \)th bio-fuel production plants to \( n \)th bio-fuel supply centre and items transported using \( p \)th kind of conveyance. The last part denotes the total number of job opportunity from \( m \)th compost production plant to \( r \)th compost selling market centre, where transportation of items covered by \( p \)th nature of conveyance.

The 3rd objective function is the total carbon emission cost which is to be minimized. The 1st part is the total carbon emission cost as tax during all transportation from each source to each destination. Here, \( E \) is the total emission which is defined in assumption (Section “Notations and assumptions of the proposed study”) to reduce the complexity of mathematical model. The 2nd part is the total penalty charge for carbon emission at \( k \)th bio-fuel production plant during bio-energy creation from waste items which are collected from \( i \)th, \( j \)th and \( l \)th sources by transporting \( p \)th class of conveyance. The 3rd part is the total penalty charge of \( m \)th compost production plant that emits carbon during compost production and the required waste items are supplied from \( i \)th, \( j \)th and \( l \)th origins, and transporting with \( p \)th mode of conveyance.

Constraints (4.4) and (4.5) describe the total source availability condition of \( i \)th and \( j \)th source. Demand capability of \( k \)th, \( l \)th and \( n \)th destinations ensured by the constraints (4.6), (4.7) and (4.8). Two types of waste items are produced from \( l \)th cold storage and the total rate of waste productions defined by constraints (4.9) and (4.10). First type of waste
is transported into bio-fuel production plant and second type of waste is again transported into compost production plant. Constraints (4.11) and (4.12) indicate the demand constraint of second stage, i.e., demand aptitude of nth bio-fuel supply centre and rth compost selling market centre. Constraints (4.13)–(4.22) employ to show the conveyance capacity for pth set of vehicles. Constraints (4.23) define the non negativity restriction of the variables.

Identical deterministic model

The formulated model is in PHF environment for existence of uncertainty in source, demand and conveyance parameters. The proposed model cannot be easily evaluated, and therefore, defuzzification ranking index (from Eq. (3.1)) is introduced in the model that converts Model 1 into a deterministic form which is presented in Model 2 as follows:

Model 2

\[
\begin{align*}
\text{minimize } & Z_1(x, y, z, w) \\
\text{maximize } & Z_2(x, y, z, w) \\
\text{minimize } & Z_3(x, y, z, w) \\
\text{subject to } & \sum_{p=1}^{P} \left( \sum_{k=1}^{K} x_{ikp} + \sum_{l=1}^{L} y_{ilp} + \sum_{m=1}^{M} x_{imp} \right) \\
& \leq \mathcal{R}(\tilde{A}_i) \quad (i = 1, 2, \ldots, I), \\
& \sum_{p=1}^{P} \left( \sum_{k=1}^{K} x_{ikp} + \sum_{j=1}^{J} y_{jkp} + \sum_{l=1}^{L} w_{lkp} \right) \\
& \geq \mathcal{R}(\tilde{B}_k) \quad (k = 1, 2, \ldots, I), \\
& \sum_{p=1}^{P} \left( \sum_{i=1}^{I} x_{ikp} + \sum_{j=1}^{J} y_{ijp} + \sum_{l=1}^{L} w_{ljp} \right) \\
& \geq \mathcal{R}(\tilde{B}_j) \quad (j = 1, 2, \ldots, J), \\
& \sum_{p=1}^{P} \left( \sum_{i=1}^{I} x_{imp} + \sum_{j=1}^{J} y_{jmp} + \sum_{l=1}^{L} w_{lmp} \right) \\
& \geq \mathcal{R}(\tilde{B}_m) \quad (m = 1, 2, \ldots, M), \\
\end{align*}
\]

constraints (4.9)–(4.10).

Extended model for carbon mechanism

In the proposed model (Model 1 or Model 2), the third objective function is focused on minimum carbon emission. Due to existence of carbon mechanism with different combined policies as “carbon tax with cap”, “carbon tax with cap-and-trade” and “carbon tax with offset” policy, the proposed...
model generates several feasible regions of solutions. In such situation the deterministic model is extended and designed into the following three Models 3–5:

**Model 3 (Tax with cap):**

minimize $Z_1(x, y, z, w)$  
maximize $Z_2(x, y, z, w)$  
minimize $Z_3(x, y, z, w)$  
subject to the constraints (4.9)–(4.10),  
the constraints (4.24)–(4.41),  
$E \leq C$.

This model is same as of Model 2, but an extra constraint $E \leq C$ is added as the restriction of cap policy.

**Model 4 (Tax with cap-and-trade policy):**

minimize $Z_1(x, y, z, w)$  
maximize $Z_2(x, y, z, w)$  
minimize $Z_3^3(x, y, z, w)$  
subject to the constraints (4.9)–(4.10),  
the constraints (4.24)–(4.41).

Model 5 (Tax with offset policy):

minimize $Z_1(x, y, z, w)$  
maximize $Z_2(x, y, z, w)$  
minimize $Z_3^{\text{offset}}(x, y, z, w) = \theta_1 E + P[E - C]^+$  
subject to the constraints (4.9)–(4.10),  
the constraints (4.24)–(4.41).
into two cases and the first is defined when cap is greater than the total carbon emission of transportation and second case is that when cap is less than the total carbon emission of transportation. First case is Model 5A, where no reward found for less emission and total emission cost is same as of Model 3, i.e., the problem becomes carbon emission with tax and cap. The second case is Model 5B, where penalty charge \( P \) is multiplied for extra emission \([E - C]\) and the system cannot buy their short permission with any cost. Therefore, total emission cost is increased by \( P[E - C] \). For such two different cases, the models are elaborated as follows.

**Model 5A**

\[
\begin{align*}
\text{minimize} & \quad Z_1(x, y, z, w) \\
\text{maximize} & \quad Z_2(x, y, z, w) \\
\text{minimize} & \quad Z^{**}_3(x, y, w, z) = \theta_1 E + \theta_2 e^{C}_{CO_2} \sum_{k=1}^{K} \sum_{p=1}^{P} \left( \sum_{i=1}^{I} x_{ikp}^1 + \sum_{j=1}^{J} y_{jkp}^1 \right) + \theta_3 e^{3}_{CO_2} \sum_{m=1}^{M} \sum_{p=1}^{P} \left( \sum_{i=1}^{I} x_{imp}^3 \right) + \sum_{j=1}^{J} y_{jmp}^3 + \sum_{l=1}^{L} w_{lmp}^2 \\
\text{subject to the constraints} & \quad (4.9)-(4.10), \\
\text{the constraints} & \quad (4.24)-(4.41), \\
\text{the constraints} & \quad (4.42).
\end{align*}
\]

**Model 5B**

\[
\begin{align*}
\text{minimize} & \quad Z_1(x, y, z, w) \\
\text{maximize} & \quad Z_2(x, y, z, w) \\
\text{minimize} & \quad Z^{**}_3(x, y, w, z) = \theta_1 E + P[E - C] + \theta_2 e^{C}_{CO_2} \sum_{k=1}^{K} \sum_{p=1}^{P} \left( \sum_{i=1}^{I} x_{ikp}^1 + \sum_{j=1}^{J} y_{jkp}^1 \right) + \sum_{l=1}^{L} w_{lkp}^1 + \theta_3 e^{3}_{CO_2} \sum_{m=1}^{M} \sum_{p=1}^{P} \left( \sum_{i=1}^{I} x_{imp}^3 \right) + \sum_{j=1}^{J} y_{jmp}^3 + \sum_{l=1}^{L} w_{lmp}^2 \\
\text{subject to the constraints} & \quad (4.9)-(4.10), \\
\text{the constraints} & \quad (4.24)-(4.41), \\
\text{the constraints} & \quad (4.43).
\end{align*}
\]

The feasibility conditions of all the above models are same as of Model 2.

Proposition 4.1 If we find the solutions of Model 4A and Model 4B then we compare them to select the final optimal solution of Model 4. However, if one of Model 4A or Model 4B provides a feasible solution and another cannot, then the feasible solution is the final optimal solution of Model 4. Similarly this holds for Model 5, Model 5A and Model 5B.

Proposition 4.2 \( Z^*_3 \leq Z^{**}_3 \leq Z_3 \).

Proof The feasible region of “carbon tax with offset” policy is a strict subset of the feasible region of “carbon tax with cap-and-trade” policy. The feasible solution for carbon tax with offset policy has same objective value in both offset and cap-and-trade model. The first inequality is followed by this condition. Feasible region of carbon tax with cap policy is a strict subset of the feasible region of tax with offset policy. The emission cost for any solution that becomes feasible region of the carbon tax with cap policy has the same objective function in both tax with cap and tax with offset models. Thus, second inequality is fulfilled. This is the complete proof of the Proposition.

Definition 4.1 (Pareto-optimal solution) A solution \((x', y', z', w') \in F\) (where \( F \) is the feasible region) is said to be a Pareto-optimal solution (non-dominated solution) of Model 3 or Model 4A/Model 4B or Model 5A/Model 5B if and only if there is no other \((x, y, z, w) \in F\), such that \(Z_s(x, y, z, w) \leq Z_s(x', y', z', w'), \ s = 1, 3\), and \(Z_2(x, y, z, w) \geq Z_2(x', y', z', w')\) for at least one inequality holds with strict inequality.

**Solution methodology**

In the proposed multi-objective optimization problem, each objective contradicts to each other objective, and there does not always exist a strategy for optimal solution which satisfies all the objective functions. That is the solution will be the best for one objective function and that may be worst for another objective function. Only Pareto-optimal solution exists on the border between feasible and non-feasible solutions, which is achieved by multi-objective optimization technique. From literature survey, we find various fuzzy and non-fuzzy techniques to provide Pareto-optimal solution of multi-objective decision making problem. Most of fuzzy techniques are FP, intuitionistic fuzzy programming, neutrosophic linear programming, PHFP, etc. From these methods, we choose two approved methods as FP and PHFP which generate Pareto-optimal solution through simple technical process. Established models are solved individually by utilizing these two methods that find Pareto-optimal solutions. The solution of each method for all models are analyzed. Thereafter we choose a preferable model for a better policy that provides a final Pareto-optimal solution.
FP

FP was initiated by Zimmermann [63] for solving any multi-objective linear programming problem. This is one of the simple fuzzy technique that provides Pareto-optimal solution in short elapsed time and simple way. To find the best Pareto-optimal solution, we utilize the facility of FP to solve our proposed MOSTP. The required steps are illustrated as follows:

- **Step 1:** Convert the PHF model to deterministic model and reformulate several crisp models for different policies of carbon mechanism.
- **Step 2:** Solve each problem individually with subject to all constraints.
- **Step 3:** Determine the positive ideal solution (PIS) and negative ideal solution (NIS) and formulate the membership function \( \mu_s(x) \) corresponding to each objective function \( Z_s(x) \) by setting their tolerance. The membership function is defined as

\[
\mu_s(Z_s(x)) = \begin{cases} 
1, & \text{if } Z_s \leq L_s^T, \\
\frac{U_s^T - Z_s}{U_s^T - L_s^T}, & \text{if } L_s^T \leq Z_s \leq U_s^T, \\
0, & \text{if } Z_s \geq U_s^T,
\end{cases}
\]

and

\[
\mu_s(Z_s(x)) = \begin{cases} 
1, & \text{if } Z_s \geq U_s^T, \\
\frac{Z_s - L_s^T}{U_s^T - L_s^T}, & \text{if } L_s^T \leq Z_s \leq U_s^T, \\
0, & \text{if } Z_s \leq L_s^T,
\end{cases}
\]

Here, PIS = \( L_s^T = \min\{Z_{s1}, Z_{s2}, Z_{s3}\} \) and NIS = \( U_s^T = \max\{Z_{s1}, Z_{s2}, Z_{s3}\} \), for \( s = 1, 3 \).

Again for \( s = 2 \), PIS = \( U_s^T = \max\{Z_{s1}, Z_{s2}, Z_{s3}\} \) and NIS = \( L_s^T = \min\{Z_{s1}, Z_{s2}, Z_{s3}\} \), where \( Z_{sr} = Z_s(x', y') \) (\( r = 1, 2, 3 \)).

- **Step 4:** Maximize the degree of acceptance of each objective function and setting the degree of acceptance as \( \theta \), then Model 6 is developed with the help of FP as follows:

Model 6

maximize \( \theta \)

subject to \( \mu_s(Z_s(x)) \geq \theta \) (\( s = 1, 2, 3 \)),

\( \theta \in [0, 1] \),

the constraints (4.9)–(4.10),

the constraints (4.24)–(4.41),

the constraints (4.42)/(4.43).

Third objective is selected from Model 3, Model 4A/Model 4B and Model 5A/Model 5B, and added the constraints (4.42)/(4.43) with condition of cap.

- **Step 5:** Solve Model 6 by LINGO iterative scheme with finding maximum value of parameter \( \theta \) and achieve the solution.

**Theorem 5.1** If \((x', y', z', w', \theta')\) is an optimal solution of Model 6 then it is also Pareto-optimal (non-dominated) solution of Model 3 or Model 4A/Model 4B or Model 5A/Model 5B or both.

**Proof** Assuming that it is not true that \((x', y', z', w')\) is a Pareto-optimal (non-dominated) solution of Model 3 or Model 4A/Model 4B or Model 5A/Model 5B or both. From Definition (4.1), we find that there exists at least one \( x \), one \( y \), one \( z \) and one \( w \), such that \( Z_s(x, y, z, w) \leq Z_s(x', y', z', w') \) for \( s = 1, 3 \) and \( Z_2(x, y, z, w) \geq Z_2(x', y', z', w') \) with at least one inequality holds for strict inequality. Membership function \( \mu_s(Z_s(x, y, z, w)) \) is strictly decreasing with respect to the corresponding objective function \( Z_s \) in [0, 1]. Hence, \( \mu_s(Z_s(x, y, z, w)) \geq \mu_s(Z_s(x', y', z', w')) \) \( \forall s \) and \( \mu_s(Z_s(x, y, z, w)) > \mu_s(Z_s(x', y', z', w')) \) for at least one \( s \). Now, \( \theta = \min \mu_s(Z_s(x, y, z, w)) \geq \min \mu_s(Z_s(x', y', z', w')) = \theta' \) which contradicts that \((x', y', z', w')\) is an optimal solution of Model 3 or Model 4A/Model 4B or Model 5A/Model 5B or both. Here, \( \theta' \) is the value of \( \theta \) at \((x', y', z', w')\). This completes the proof of the theorem. \( \square \)

**PHFP**

For finding Pareto-optimal solution of the proposed MOSTP, a new modified programming approach PHFP based on PHFS is introduced here. PHFP maximizes Pythagorean hesitant membership grade and minimizes Pythagorean hesitant non-membership grade. The decision making concept through fuzzy set was first initialized by Bellman and Zadeh [6]. The fuzzy decision set \((D)\) on a decision making problem can be defined by the intersection of fuzzy objective function \((\tilde{O})\) and fuzzy constraint \((\tilde{C})\). Pythagorean hesitant fuzzy decision \((\tilde{D})\) is an extension of fuzzy decision that can be described as \( \tilde{D} = \tilde{O} \cap \tilde{C} = \{x, y, z, w\} \), where \( \{\mu_{\tilde{D}}(x) \in (\mu_{\tilde{O}}(x) \cap \mu_{\tilde{C}}(x)) : \mu_{\tilde{D}}(x) \leq \min(\max(\mu_{\tilde{O}}(x) \cap \mu_{\tilde{C}}(x)))\} \) and \( \{v_{\tilde{D}}(x) \in (v_{\tilde{O}}(x) \cap v_{\tilde{C}}(x)) : v_{\tilde{D}}(x) \geq \max(\min(v_{\tilde{O}}(x) \cap v_{\tilde{C}}(x)))\} \). \( \mu_{\tilde{D}}(x) \) and \( v_{\tilde{D}}(x) \) are the sets of membership value of acceptance and rejection of PHF solution under PHF decision set, respectively, and \( \tilde{O}, \tilde{C} \) are the hesitant fuzzy objective function and hesitant fuzzy constraint, respectively.

Using such PHF solution criteria we represent the stepwise procedure for solving the proposed model(s) as follows:

- **Step 1:** Converting the PHF model into crisp model and solve each crisp objective problem independently with subject to all constraints for finding the solution.
Step 2: Substituting the obtained solution of Step 1 in each objective function and determine PIS and NIS as of Step 3 (from PHFP).

Step 3: Utilizing PIS and NIS, formulate the membership and non-membership function corresponding to each objective function in PHF environment as:

For $s = 1, 3$:

$$
\mu^P_h(Z_s(x)) = \begin{cases} 
1, & \text{if } Z_s(x) \leq L_s, \\
\alpha_s(\frac{U_s - Z_s(x)}{U_s - L_s}), & \text{if } L_s \leq Z_s(x) \leq U_s, \\
0, & \text{if } Z_s(x) \geq U_s,
\end{cases}
$$

$$
v^P_h(Z_s(x)) = \begin{cases} 
0, & \text{if } Z_s(x) \leq L_s, \\
\beta_s(\frac{Z_s(x) - L_s}{U_s - L_s}), & \text{if } L_s \leq Z_s(x) \leq U_s, \\
1, & \text{if } Z_s(x) \geq U_s.
\end{cases}
$$

For $s = 2$:

$$
\mu^P_h(Z_2(x)) = \begin{cases} 
0, & \text{if } Z_2(x) \leq L_s, \\
\alpha_s(\frac{Z_2(x) - L_s}{U_s - L_s}), & \text{if } L_s \leq Z_2(x) \leq U_s, \\
1, & \text{if } Z_2(x) \geq U_s,
\end{cases}
$$

$$
v^P_h(Z_2(x)) = \begin{cases} 
0, & \text{if } Z_2(x) \leq L_s, \\
\beta_s(\frac{U_s - Z_2(x)}{U_s - L_s}), & \text{if } L_s \leq Z_2(x) \leq U_s, \\
1, & \text{if } Z_2(x) \geq U_s.
\end{cases}
$$

The parameters $\alpha_s, \beta_s \in [0, 1]$ are the sets of hesitant values corresponding to membership and non-membership functions, respectively, and selected by the DM’s own choice in PHF environment.

Step 4: To find the highest degree for satisfaction and lowest degree for rejection, the PHFP model (modified from [3]) corresponding to MOSTP can be newly clarified in the following model:

Model 7

Maximize $\xi^2 - \eta^2$

Subject to:

$$
\begin{align*}
\mu^P_h(Z_s(x))^2 &\geq \xi^2 
\text{ (s = 1, 2, 3),} \\
v^P_h(Z_s(x))^2 &\leq \eta^2 
\text{ (s = 1, 2, 3),} \\
\xi^2 &\geq \eta^2, \\
\xi^2 + \eta^2 &\in [0, 1], \\
\xi^2 &\in [0, 1], \\
\text{the constraints (4.9)–(4.10),} \\
\text{the constraints (4.24)–(4.41),} \\
\text{the constraints (4.42)–(4.43).}
\end{align*}
$$

Here, $\xi$ and $\eta$ are the grades of membership and non-membership of each objective function.

Step 5: Solving Model 7 by any mathematical programming with parameters $\xi$ and $\eta$, and obtain Pareto-optimal solution of proposed model(s).

Theorem 5.2 If $(x', y', z', w')$ is an optimal solution of Model 7 then it becomes a Pareto-optimal (non-dominated) solution of Model 3 or Model 4A/Model 4B or Model 5A/Model 5B or both.

Proof Let $(x', y', z', w')$ is not a Pareto-optimal (non-dominated) solution of Model 3 or Model 4A/Model 4B or Model 5A/Model 5B or both. From Definition (4.1), we consider that there exist at least one $x$, one $y$, one $z$ and one $w$, such that $Z_s(x, y, z, w) \leq Z_s(x', y', z', w')$ for $s = 1, 3$ and $Z_2(x, y, z, w) \geq Z_2(x', y', z', w')$ with at least one inequality holds for strict inequality. The membership function $\mu_s(Z_s(x, y, z, w))$ is strictly decreasing with respect to the corresponding objective function $Z_s$ in $[0, 1]$. $\mu_s(Z_s(x, y, z, w)) \geq \mu_s(Z_s(x', y', z', w')) \forall s$ and $\mu_s(Z_s(x, y, z, w)) > \mu_s(Z_s(x', y', z', w'))$ for at least one $s$. The non-membership function $v_s(Z_s(x, y, z, w))$ is strictly increasing with respect to the corresponding objective function $Z_s$ in $[0, 1]$. Hence, $v_s(Z_s(x, y, z, w)) \leq v_s(Z_s(x', y', z', w')) \forall s$ and $v_s(Z_s(x, y, z, w)) < v_s(Z_s(x', y', z', w'))$ for at least one $s$. $\xi = \min_{s} \mu_s(Z_s(x, y, z, w)) \geq \min_{s} \mu_s(Z_s(x', y', z', w')) = \xi'$ and $\eta = \max_{s} v_s(Z_s(x, y, z, w)) \leq \max_{s} v_s(Z_s(x', y', z', w')) = \eta'$. Thus, $\xi^2 - \eta^2 \geq \xi'^2 - \eta'^2$ contradicts that $(x', y', z', w')$ is an optimal solution of Model 3 or Model 4A/Model 4B or Model 5A/Model 5B or both. $\xi'$ and $\eta'$ are the values of $\xi$ and $\eta$ at $(x', y', z', w')$, respectively. This completes the proof of the theorem. □

Remark 1 If the consequential part of the Pareto sets is non-convex then Pascoletti–Serafini runs (Pascoletti and Serafini [38]) are required to approximate the part inside the vital accuracy. Combination of Sandwich and Hyperboxing algorithm finds the samples in both convex and non-convex regions. Sandwich approximation is suited to approximate the convex part, whereas non-convex part is efficiently computed by hyperboxing scheme. More details can be seen from Küfner et al. [22]. The DMs navigate (see the Appendix B of Bortz et al. [7]) on the whole Pareto set to find a trade-off between best compromises, and reach a final decision by comparing several alternatives with respect to distinct objectives.

Advantages and limitations

Some advantages with limitations are provided in this section.

- We have generated a model of MOSTP by the link of SWM and SD. The contribution of this model is that the three sections of sustainability are optimized in the sense of urban or rural development.
- Three types of combined policy of carbon mechanism are analyzed to minimize carbon emission by providing some facilities or relaxations to the user and to government or a third party. This is the supporting fact to reduce GHG
emission and the study provides an opportunity to choose an appropriate choice among several policies which are suitable for the user.

- Extending PFS and HFS, we select a new environment as PHFS in our study. The facility is that this study is prepared to challenge any difficult uncertainty.
- A new programming approach PHFP is implemented to find the Pareto-optimal solution of the proposed model. The advantage is that this method is always capable to find the solution of any multi-objective decision making problem.
- One of the limitations is that here we did not introduce any type of vehicle that emits zero carbon. We are not considered the carbon emission from deterioration, but nowadays this is a common issue of transportation when transporting perishable items. Here, we did not choose the deterioration rate, but some perishable items are transported here, and we did not impose any type of preservation technology that may help to prevent deterioration.
- Our selected environment (PHF) is differ from stochastic environment, as the models may be solved by several methods and provided more space for uncertain factors. However, stochastic system always focuses on data. PHF environment measures the degree of uncertainty of event that may occur or may not occurs. In contrast for stochastic environment, the randomness always presents the uncertainty of the event of occurrence.
- The proposed methods (FP and PHFP) have advantages as the arithmetic mean and standard deviation are not necessary, but in stochastic programming these are required. The disadvantage of FP and PHFP is that in these programming all the parameters must be in deterministic form, but in stochastic programming some or each parameter(s) are in uncertain nature. Stochastic programming [59] not only optimizes the criteria of decision maker, but also measures the uncertainty of parameters in approximate value, whereas this measure cannot be found in FP and PHFP.

Real-life experiments

In this section, we study two real-life problems to examine the effectiveness of our proposed model. Connecting with industrial organizations and for rural/urban development we start with two types of sources as \( I(=2) \) and \( J(=3) \) for agriculture field and forest department, respectively. Mainly SWM with transportation system controls such problem by transporting the fresh items and waste items into three types \((K, L, M)\) of destinations for future storage and for recycl. The recycle plants are as \( K(=2) \) for bio-fuel production and \( M(=2) \) for compost production from waste items. Here, \( L(=2) \) is considered for cold storage that stores fresh items for future marketing with high profit. After producing bio-fuel and compost, these items are transported into market for resell and we select \( N(=3) \) for bio-fuel supply centre and \( R(=3) \) for compost selling market centre. For transporting the items, \( P(=2) \) types of vehicles are considered here. All the required data are presented in Tables 5, 6, 7, 8 and 9. Source, demand and conveyance are considered here as \( \text{TrPHFN} \) and we use ranking index for defuzzification of the parameters. To find most simplified ranking value, we select the set of two hesitant values (corresponding to Pythagorean hesitant membership and non-membership function) as minimum of membership grades and maximum of non-membership grades, respectively. For SD, three objective functions are optimized by minimizing transportation cost, maximizing job opportunity and minimizing carbon emission with carbon mechanism. For “carbon tax with cap-and-trade” policy and for “carbon tax with offset” policy, the designed problem is divided into two cases separately. For “carbon tax with cap” policy, the problem becomes another case which is different from the other two policies.

Experimental data design

**Example 2** In this example we input carbon cap \( C = 25000 \) gm, and carbon tax, penalty charge, buying or selling permission cost for surplus or slack carbon emission are provided. For two types of recycle plants, there exist two types of carbon tax. Here, all types of cost is in $, job opportunity in job, distance in kilometers (km), unit of items in ton are defined.

**Example 3** Here, we reduce carbon cap as \( C = 18000 \) gm to check the effect of carbon mechanism. Except cap value, all the other parameters remain identical as of Example 2.

Now, we solve two examples with considering three policies of carbon mechanism by keeping fixed the carbon cap.

Results and comparison analysis

This section includes the results of two numerical examples, provides graphical presentation for comparison analysis, and reports the model validation.

Allocation from case study

Here, we find the optimal allocations by evaluating two examples by regarding three policies and by the help of PHFP and FP. In Example 2, cap \( C = 25000 \) gm provides feasible solutions of three policies with all cases and solutions are presented in Table 10. For Example 3, when cap \( C = 18000 \) gm then only second case of second policy (i.e., tax with
Table 5  Supply, demand of 1st stage, demand of 2nd stage, conveyance capacity in TrPHFNs and their ranking values

| Case | Demand of 1st Stage | Demand of 2nd Stage | Conveyance Capacity |
|------|---------------------|---------------------|---------------------|
| 1    | 150, 250, 350, 450 | 325, 375, 400       | 250, 300, 400       |
| 2    | 120, 150, 180, 210 | 133, 150, 170, 205 | 165, 195, 210, 240 |
| 3    | 115, 130, 162, 195 | 60, 113, 166, 219  | 70, 80, 100, 120   |
| 4    | 68, 80, 92, 104   | 35, 45, 55, 65     | 100, 200, 300, 400 |


Table 6  Transportation cost

| Case | Cost of 1st Stage | Cost of 2nd Stage | Conveyance Cost |
|------|------------------|------------------|-----------------|
| 1    | 7, 8, 9, 10      | 6, 12, 16, 16    | 10              |
| 2    | 5.5, 6.5, 8.5    | 12, 16, 16       | 14              |
| 3    | 7.5, 8, 9, 10    | 6, 12, 16, 16    | 10              |
| 4    | 7.5, 8, 9, 10    | 6, 12, 16, 16    | 10              |

Table 7  Job opportunity of whole process

| Case | Opportunity of 1st Stage | Opportunity of 2nd Stage | Carbon Cap |
|------|--------------------------|--------------------------|-----------|
| 1    | 3, 4                     | 5, 6                     | 10        |
| 2    | 5, 6                     | 7, 8                     | 10        |
| 3    | 7, 8                     | 9, 10                    | 10        |
| 4    | 7, 8                     | 9, 10                    | 10        |
| 5    | 2, 3                     | 4, 5                     | 10        |
| 6    | 2, 4                     | 3, 5                     | 10        |

For the third case, the objective values for the considered methods, we pick an appropriate solution to specify the final Pareto-optimal solution of both examples and star mark uses to select bet-

---

The cap-and-trade policy and second case of third policy (i.e., tax with offset policy) provide the feasible solutions defined in Table 11. If total emission is less than carbon cap then this example provides no feasible solution (NFS). The objective values in both examples with two methods are summarized in Table 12 and the preferable method, PHFP is highlighted by bold face for suitable comparison. Observing all over the objective values for the considered methods, we pick an appropriate solution to select a bet-
ter policy among three policies of carbon mechanism which declares as final result.

**Discussion with graphical presentation**

Tables 10 and 11 provide the Pareto-optimal solution of two numerical examples, respectively. From these tables, we find the objective values which define the main differences between the solutions. Table 10 shows that here the cap value is greater, and therefore, all the policies provide feasible solution in two methods. In Table 11, where cap value is lower and hence only Model 4B and Model 5B give solution. That is the two tables does not display the same type of solutions of two numerical examples, respectively.

From Table 12, we highlight the different optimal solutions in FP and PHFP for several reasons. FP supports only on the membership grades of the objective functions, but PHFP considers both the membership and non-membership grades of the objectives. FP maximizes the membership grade but PHFP maximizes the degree of satisfaction by finding highest value of Pythagorean hesitant membership grade and minimizes the degree of rejection by providing lowest value of Pythagorean hesitant non-membership grade. The hesitancy of membership and non-membership is tackled in PHFP, whereas FP cannot face to any hesitancy. PHFP overcomes different complexities, and displays more efficient Pareto-optimal solutions than FP as DMs have a privilege to choose the efficient solution in a technical way by setting the hesitant values for each membership and non-membership function.

Between FP and PHFP, we select PHFP as a better method which provides final Pareto-optimal solution of both examples. From Table 12, we note that Example 2 provides feasible solution in two methods with all of three policies. However, for Example 3, only Model 4B and Model 5B provide objective values, whereas the other cases have NFS in both methods. We compare three policies of both examples in two methods and notice that “carbon tax with cap-and-trade” policy (i.e., Model 4A and Model 4B) provides appropriate result than other two policies. In this policy we analyze that when total emission is less than carbon cap (i.e., Model 4A) then the organization can sell extra unused carbon units and find a profit that reduced emission cost 7214.13$. If total emission is greater than carbon cap (i.e., Model 4B) then organization

| Table 8 | Distance in km for transportation |
|---------|----------------------------------|
| $d_{1p}^{1}$ | 65 | $d_{1p}^{1}$ | 75 |
| $d_{2p}^{1}$ | 52 | $d_{2p}^{1}$ | 45 |
| $d_{3p}^{1}$ | 67 | $d_{3p}^{1}$ | 58 |
| $d_{4p}^{1}$ | 37 | $d_{4p}^{1}$ | 30 |
| $d_{1p}^{2}$ | 33 | $d_{1p}^{2}$ | 58 |
| $d_{2p}^{2}$ | 47 | $d_{2p}^{2}$ | 28 |
| $d_{3p}^{2}$ | 40 | $d_{3p}^{2}$ | 25 |
| $d_{4p}^{2}$ | 47 | $d_{4p}^{2}$ | 68 |

| Table 9 | Carbon emission rate, tax, carbon trading cost, penalty charge, rate of waste creation from cold storage |
|---------|--------------------------------------------------|
| $e_{CO_{2}}^{1} = 0.60$ | $e_{CO_{2}}^{2} = 0.40$ | $e_{CO_{2}}^{3} = 0.50$ | $\theta_{1} = 0.30$, $\theta_{2} = 0.50$, $\theta_{3} = 0.40$, $\phi = 0.5$, $\psi = 0.3$, $P = 0.6$, $\alpha = 10\%$, $\beta = 15\%$ |

![Graphical presentation of three objective values for two examples](image-url)

**Fig. 2** Graphical presentation of three objective values for two examples.
### Table 10  Solution of Example 2

| FP | PHFP |
|-----------------|-----------------|
| **Model 3**     | **Model 4A**    |
| $\theta = 0.4055$; $x_{122}^1 = 141.36$; $y_{111}^1 = 45.14$, $y_{1312}^1 = 66.11$, $y_{112}^2 = 187.5$, $y_{212}^2 = 236.39$, $y_{312}^2 = 136.99$, $y_{212}^3 = 101.65$; $w_{1111}^1 = 18.75$, $w_{222}^1 = 23.64$, $w_{112}^2 = 28.12$, $w_{211}^2 = 5.21$, $w_{311}^2 = 16.48$; $z_{112}^1 = 58$, $z_{122}^2 = 47.5$, $z_{132}^2 = 77.5$, $z_{112}^1 = 33.75$, $z_{212}^2 = 86$, $z_{231}^2 = 50$; the other variables are zero | $\xi = 0.6504179$, $\eta = 0.3489311 \times 10^{-3}$; $x_{122}^1 = 145.88$, $y_{111}^1 = 109.93$, $y_{12}^1 = 1.32$, $y_{112}^2 = 187.5$, $y_{212}^2 = 191.25$, $y_{1111}^2 = 27.57$, $y_{112}^2 = 111.93$, $y_{212}^3 = 103.78$, $w_{111}^1 = 18.75$, $w_{222}^1 = 19.12$, $w_{212}^2 = 28.12$, $w_{211}^2 = 14.34$; $z_{112}^1 = 58$, $z_{122}^2 = 47.5$, $z_{132}^2 = 77.5$, $z_{1111}^2 = 33.75$, $z_{212}^2 = 86$, $z_{231}^2 = 50$; the other variables are zero |
| **Model 4B**    | **Model 5A**    |
| $\theta = 0.5593774$; $x_{121}^1 = 250$, $x_{122}^1 = 362.5$, $y_{111}^1 = 66.11$, $y_{12}^1 = 628.12$, $y_{111}^2 = 153.12$, $y_{12}^2 = 9.38$, $y_{212}^2 = 6.81$, $y_{1111}^2 = 34.38$, $y_{1111}^3 = 76.88$, $y_{212}^3 = 173.12$, $w_{111} = 18.75$, $w_{212}^2 = 362.5$, $w_{312}^2 = 9.56$, $w_{111}^2 = 28.12$, $w_{112}^2 = 18.68$, $w_{211}^2 = 28.69$, $z_{112}^2 = 47.5$, $z_{122}^1 = 237.5$, $z_{132}^2 = 77.5$, $z_{112}^2 = 33.75$, $z_{212}^3 = 13.87$, $z_{231}^2 = 72.12$, $z_{231}^2 = 144.12$; other variables are zero. | $\xi = 0.9642012$, $\eta = 0.357892 \times 10^{-6}$; $x_{122}^1 = 155.44$, $y_{111}^1 = 111.25$, $y_{12}^1 = 58.75$, $y_{212}^2 = 128.75$, $y_{111}^2 = 128.75$, $y_{1111}^2 = 62.5$, $y_{1111}^3 = 87.69$, $y_{212}^3 = 141.25$, $w_{111} = 18.75$, $w_{212}^2 = 9.56$, $w_{111}^2 = 23.12$, $w_{112}^2 = 8$, $w_{211}^2 = 28.69$; $z_{112}^2 = 58$, $z_{122}^1 = 47.5$, $z_{132}^2 = 77.5$, $z_{1111}^2 = 33.75$, $z_{212}^3 = 62.29$, $z_{212}^3 = 23.71$, $z_{231}^2 = 50$; the other variables are zero. |
| **Model 5B**    | **Model 5B**    |
| $\theta = 0.560517$; $x_{121}^1 = 250$, $x_{122}^1 = 362.5$, $y_{111}^1 = 66.11$, $y_{12}^1 = 628.12$, $y_{111}^2 = 153.12$, $y_{12}^2 = 9.38$, $y_{212}^2 = 6.81$, $y_{1111}^2 = 34.38$, $y_{1111}^3 = 76.88$, $y_{212}^3 = 173.12$, $w_{111} = 18.75$, $w_{212}^2 = 362.5$, $w_{312}^2 = 9.56$, $w_{111}^2 = 28.12$, $w_{112}^2 = 18.68$, $w_{211}^2 = 28.69$, $z_{112}^2 = 47.5$, $z_{122}^1 = 237.5$, $z_{132}^2 = 77.5$, $z_{112}^2 = 33.75$, $z_{212}^3 = 13.87$, $z_{231}^2 = 72.12$, $z_{231}^2 = 144.12$; other variables are zero. | $\xi = 0.698533$, $\eta = 0.3137012 \times 10^{-5}$; $x_{122}^1 = 155.44$, $y_{111}^1 = 111.25$, $y_{12}^1 = 80.94$, $y_{111}^2 = 22.19$, $y_{212}^2 = 123.75$, $y_{1111}^2 = 106.56$, $y_{1111}^3 = 45.31$, $y_{212}^3 = 82.69$, $y_{212}^3 = 146.25$, $w_{111} = 18.75$, $w_{212}^2 = 9.56$, $w_{111}^2 = 28.12$, $w_{112}^2 = 28.69$; $z_{112}^2 = 58$, $z_{122}^1 = 47.5$, $z_{132}^2 = 77.5$, $z_{1111}^2 = 33.75$, $z_{212}^3 = 86$, $z_{231}^2 = 50$; the other variables are zero. |
Table 11  

| PHFP | FP | NFS | FP | NFS | FP | NFS | FP | NFS |
|------|----|-----|----|-----|----|-----|----|-----|
| Model 3 | N | N | N | N | N | N | N | N |
| Model 4A | N | N | N | N | N | N | N | N |
| Model 4B | N | N | N | N | N | N | N | N |
| Model 5A | N | N | N | N | N | N | N | N |
| Model 5B | N | N | N | N | N | N | N | N |

Each Hesitant Parameter generates a random number in $[0, 1]$. The optimal values of objective functions are resulted on three objective functions (Model 4B and Example 2). We validate the proposed PHFP model by selecting 9 random achievements of the parameters under 10 violations. Each hesitant parameter generates a random number in $[0, 1]$. The optimal values of objective functions are resulted by PHFP model. Tables 13 and 14 display the objective val-

purchases the deficit carbon unit and pays a penalty charge for extra emission. In such case total carbon emission cost is increased and it is 9283.47. We find that carbon emission cost 7390.37 (Model 3 and Model 5A) and 10527.32 (Model 5B) are obtained for “carbon tax with cap” and “car- bon tax with offset” policy, which indicate that emission cost is increased that of the previous policy. In this policy, when total emission is greater than carbon cap (i.e., Model 5B) then the organization pays a penalty charge but could not purchase their deficit carbon units. Whenever total emission is less than carbon cap (i.e., Model 3 and Model 5A) then the system cannot sell their surplus carbon unit and emission cost is not decreased. Therefore, Model 4A is better than Model 5A and Model 3, and Model 4B is better than Model 5B. Model 5A has a disadvantage that for low emission the system does not find any reward, whereas Model 4A finds such reward by selling their extra permission. Between Model 4A and Model 4B, we fix Model 4B of Example 2 and also Model 4B of Example 3, to provide final Pareto-optimal solution, as this model minimizes transportation cost and at the same time maximizes job opportunity. However, Model 4A only provides less emission cost than of Model 4B. For better comparison, we define three objective values of both example in Fig. 2. This graph defines the effect of carbon mechanism by the third objective value which varies for several cap values, i.e., these show that total emission cost of Example 2 is less than that of Example 3. Carbon market has made “carbon tax with cap-and-trade” mechanism that becomes a larger policy and attractive in many countries for reduction of carbon emission, and this policy measures the limit index of carbon emission for a sustainable trans-

**Model validation**

In this subsection, we inspect the effect of hesitant parameters $\alpha_t$ and $\beta_t$ on three objective functions (Model 4B and Example 2). We validate the proposed PHFP model by selecting 9 random achievements of the parameters under 10 violations. Each hesitant parameter generates a random number in $[0, 1]$. The optimal values of objective functions are resulted by PHFP model. Tables 13 and 14 display the objective val-
Table 12  Objective values of two examples

| Models   | Methods | Example 2 (Cap C=25000) | Example 3 (Cap C=18000) |
|----------|---------|--------------------------|--------------------------|
| Model 3  | FP      | (11784.86, 17419.2, 7501.48) | NFS                      |
| Model 3  | PHFP    | (11342.77, 16666.05, 7390.37) | NFS                      |
| Model 4A | FP      | (11787.48, 17417.45, 7383.03) | NFS                      |
| Model 4A | PHFP    | (11273.95, 16677.52, 7214.13) | NFS                      |
| Model 4B | FP      | (18052.4, 34824.3, 34775.09) | (18045.27, 34811.69, 36853.44) |
| Model 4B | PHFP    | (9572.09, 16935.56, 9283.47)* | (9572.27, 16861.18, 11379.91)* |
| Model 5A | FP      | (11784.86, 17419.2, 7501.48) | NFS                      |
| Model 5A | PHFP    | (11342.77, 16666.05, 7390.37) | NFS                      |
| Model 5B | FP      | (18028.67, 34883.92, 48659.99) | (18043.87, 34821.83, 52445.68) |
| Model 5B | PHFP    | (9479.52, 16856.8, 10527.32)  | (9572.87, 16861.18, 14311.95) |

Bold form highlight that the results obtained in Pythagorean hesitant fuzzy programming (PHFP) are better than in fuzzy programming (FP)

Table 13  Performance of proposed model under several hesitant values corresponding membership and non-membership function

| Cases | Fixed hesitant values | Adjusted hesitant values | Objective values |
|-------|-----------------------|--------------------------|------------------|
| 1     | $\alpha_2 = 0.1 = \beta_2; \alpha_3 = 0.1 = \beta_3$ | $\alpha_1 = 0.1, \beta_1 = 0.1$ | (9572.093, 16686.06, 9283.474) |
|       |                       | $\alpha_1 = 0.2, \beta_1 = 0.2$ | (10672.28, 16526.62, 7647.084) |
|       |                       | $\alpha_1 = 0.3, \beta_1 = 0.3$ | (9910.767, 16609.66, 8591.418) |
|       |                       | $\alpha_1 = 0.4, \beta_1 = 0.4$ | (9750.938, 16662.94, 8821.579) |
|       |                       | $\alpha_1 = 0.5−0.6, \beta_1 = 0.5−0.6$ | (9512.812, 16639.81, 9437.127) |
|       |                       | $\alpha_1 = 0.7, \beta_1 = 0.7$ | (9411.562, 16426.06, 9801.627) |
|       |                       | $\alpha_1 = 0.8, \beta_1 = 0.8$ | (9883.438, 16472.94, 9926.502) |
|       |                       | $\alpha_1 = 0.9, \beta_1 = 0.9$ | (9275.425, 16386.53, 10533.10) |
|       |                       | $\alpha_1 = 1.0, \beta_1 = 1.0$ | (9246.297, 16300.09, 10698.43) |
| 2     | $\alpha_2 = 0.5 = \beta_2; \alpha_3 = 0.5 = \beta_3$ | $\alpha_1 = 0.1, \beta_1 = 0.1$ | (9572.093, 16686.06, 9437.127) |
|       |                       | $\alpha_1 = 0.2, \beta_1 = 0.2$ | (9225.203, 16328.22, 10845.24) |
|       |                       | $\alpha_1 = 0.3, \beta_1 = 0.3$ | (8993.953, 16483.22, 12963.84) |
|       |                       | $\alpha_1 = 0.4, \beta_1 = 0.4$ | (8974.306, 16542.16, 13338.71) |
|       |                       | $\alpha_1 = 0.5, \beta_1 = 0.5$ | (9572.093, 16686.06, 9283.474) |
|       |                       | $\alpha_1 = 0.6−0.8, \beta_1 = 0.6−0.8$ | (10714.74, 16521.38, 7616.15) |
|       |                       | $\alpha_1 = 0.9−1.0, \beta_1 = 0.9−1.0$ | (10672.28, 16526.62, 7647.084) |
| 3     | $\alpha_2 = 1.0 = \beta_2; \alpha_3 = 1.0 = \beta_3$ | $\alpha_1 = 0.1, \beta_1 = 0.1$ | (10087, 16551.17, 8337.829) |
|       |                       | $\alpha_1 = 0.2, \beta_1 = 0.2$ | (9512.812, 16686.06, 9437.127) |
|       |                       | $\alpha_1 = 0.3, \beta_1 = 0.3$ | (9372.92, 16464.52, 9985.586) |
|       |                       | $\alpha_1 = 0.4, \beta_1 = 0.4$ | (9225.203, 16328.22, 10845.24) |
|       |                       | $\alpha_1 = 0.5, \beta_1 = 0.5$ | (9088.718, 16420.20, 12025.55) |
|       |                       | $\alpha_1 = 0.6−0.7, \beta_1 = 0.6−0.7$ | (8993.953, 16483.22, 12963.84) |
|       |                       | $\alpha_1 = 0.8, \beta_1 = 0.8$ | (8974.305, 16542.16, 13338.73) |
|       |                       | $\alpha_1 = 0.9, \beta_1 = 0.9$ | (8955.874, 16597.45, 13690.39) |
|       |                       | $\alpha_1 = 1.0, \beta_1 = 1.0$ | (9572.093, 16639.81, 9283.474) |
| 4     | $\alpha_1 = 0.1 = \beta_1; \alpha_3 = 0.1 = \beta_3$ | $\alpha_2 = 0.1−0.3, \beta_2 = 0.1−0.3$ | (9572.093, 16686.06, 9283.474) |
|       |                       | $\alpha_2 = 0.4, \beta_2 = 0.4$ | (9572.093, 16640.31, 9283.474) |
|       |                       | $\alpha_2 = 0.5, \beta_2 = 0.5$ | (9572.093, 16686.06, 9283.474) |
|       |                       | $\alpha_2 = 0.6, \beta_2 = 0.6$ | (9572.093, 16683.81, 9283.474) |
|       |                       | $\alpha_2 = 0.7, \beta_2 = 0.7$ | (9572.093, 16639.81, 9283.474) |
|       |                       | $\alpha_2 = 0.8, \beta_2 = 0.8$ | (9572.477, 16364.12, 9284.63) |
|       |                       | $\alpha_2 = 0.9, \beta_2 = 0.9$ | (9573.584, 16617.71, 9287.96) |
|       |                       | $\alpha_2 = 1.0, \beta_2 = 1.0$ | (9574.473, 16604.56, 9290.632) |
Table 13 continued

| Cases | Fixed hesitant values | Adjusted hesitant values | Objective values |
|-------|-----------------------|--------------------------|------------------|
| 5     | \(\alpha_1 = 0.5 = \beta_1; \alpha_3 = 0.5 = \beta_3\) | \(\alpha_2 = 0.1, \beta_2 = 0.1\) | (9793.588, 15955.21, 9949.655) |
|       | \(\alpha_2 = 0.2, \beta_2 = 0.2\)                     | (10520.44, 15009.56, 12135.77) |
|       | \(\alpha_2 = 0.3-0.4, \beta_2 = 0.3-0.4\)             | (10768.55, 14889.66, 12881.99) |
|       | \(\alpha_2 = 0.5, \beta_2 = 0.5\)                     | (9572.093, 16686.06, 9283.474) |
|       | \(\alpha_2 = 0.6-0.9, \beta_2 = 0.6-0.9\)             | (9572.093, 16639.81, 9283.474) |
|       | \(\alpha_2 = 1.0, \beta_2 = 1.0\)                     | (9572.093, 16642.43, 9283.474) |
| 6     | \(\alpha_1 = 1.0 = \beta_1; \alpha_3 = 1.0 = \beta_3\) | \(\alpha_2 = 0.1, \beta_2 = 0.1\) | (9593.572, 16367.28, 9348.096) |
|       | \(\alpha_2 = 0.2, \beta_2 = 0.2\)                     | (9804.995, 15933.99, 9983.962) |
|       | \(\alpha_2 = 0.3, \beta_2 = 0.3\)                     | (10208.24, 15285.88, 11196.77) |
|       | \(\alpha_2 = 0.4, \beta_2 = 0.4\)                     | (10520.44, 15009.56, 12135.77) |
|       | \(\alpha_2 = 0.5, \beta_2 = 0.5\)                     | (10549.95, 14988.44, 12224.52) |
|       | \(\alpha_2 = 0.6-0.8, \beta_2 = 0.6-0.8\)             | (10768.55, 14889.66, 12881.99) |
|       | \(\alpha_2 = 0.9, \beta_2 = 0.9\)                     | (10228.61, 15265.66, 11258.04) |
|       | \(\alpha_2 = 1.0, \beta_2 = 1.0\)                     | (9572.093, 16639.81, 9283.474) |

Table 14 Performance of proposed model under several hesitant values corresponding membership and non-membership function

| Cases | Fixed hesitant values | Adjusted hesitant values | Objective values |
|-------|-----------------------|--------------------------|------------------|
| 7     | \(\alpha_1 = 0.1 = \beta_1; \alpha_2 = 0.1 = \beta_2\) | \(\alpha_3 = 0.1, \beta_3 = 0.1\) | (9572.093, 16686.06, 9283.474) |
|       | \(\alpha_3 = 0.2, \beta_3 = 0.2\)                     | (9088.715, 16420.20, 12025.57) |
|       | \(\alpha_3 = 0.3, \beta_3 = 0.3\)                     | (9258.438, 16372.94, 10628.5) |
|       | \(\alpha_3 = 0.4, \beta_3 = 0.4\)                     | (9411.562, 16426.06, 9801.627) |
|       | \(\alpha_3 = 0.5, \beta_3 = 0.5\)                     | (9512.812, 16639.81, 9437.127) |
|       | \(\alpha_3 = 0.6, \beta_3 = 0.6\)                     | (9691.042, 16639.81, 8975.156) |
|       | \(\alpha_3 = 0.7-0.8, \beta_3 = 0.7-0.8\)             | (9750.938, 16662.94, 8821.572) |
|       | \(\alpha_3 = 0.9, \beta_3 = 0.9\)                     | (9927.365, 16604.13, 8567.517) |
|       | \(\alpha_3 = 1.0, \beta_3 = 1.0\)                     | (10087.32, 16551.10, 8337.394) |
| 8     | \(\alpha_1 = 0.5 = \beta_1; \alpha_2 = 0.5 = \beta_2\) | \(\alpha_3 = 0.1, \beta_3 = 0.1\) | (9512.812, 16639.81, 9437.127) |
|       | \(\alpha_3 = 0.2, \beta_3 = 0.2\)                     | (10230.38, 16553.31, 8144.367) |
|       | \(\alpha_3 = 0.3, \beta_3 = 0.3\)                     | (10683.28, 16526.62, 7638.804) |
|       | \(\alpha_3 = 0.4, \beta_3 = 0.4\)                     | (10714.74, 16521.38, 7616.15) |
|       | \(\alpha_3 = 0.5, \beta_3 = 0.5\)                     | (9572.093, 16686.06, 9283.474) |
|       | \(\alpha_3 = 0.6, \beta_3 = 0.6\)                     | (8945.678, 16628.04, 13884.93) |
|       | \(\alpha_3 = 0.7-0.8, \beta_3 = 0.7-0.8\)             | (8993.953, 16483.22, 12963.84) |
|       | \(\alpha_3 = 0.9, \beta_3 = 0.9\)                     | (9031.061, 16483.22, 12576.44) |
|       | \(\alpha_3 = 1.0, \beta_3 = 1.0\)                     | (9088.717, 16420.20, 12025.55) |
| 9     | \(\alpha_1 = 1.0 = \beta_1; \alpha_2 = 1.0 = \beta_2\) | \(\alpha_3 = 0.1, \beta_3 = 0.1\) | (9246.297, 16300.09, 10698.43) |
|       | \(\alpha_3 = 0.2, \beta_3 = 0.2\)                     | (9512.812, 16662.45, 9437.127) |
|       | \(\alpha_3 = 0.3, \beta_3 = 0.3\)                     | (9750.937, 16662.94, 8821.572) |
|       | \(\alpha_3 = 0.4, \beta_3 = 0.4\)                     | (10230.38, 16553.31, 8144.367) |
|       | \(\alpha_3 = 0.5, \beta_3 = 0.5\)                     | (10672.28, 16526.62, 7647.084) |
|       | \(\alpha_3 = 0.6, \beta_3 = 0.6\)                     | (10683.28, 16526.62, 7638.804) |
|       | \(\alpha_3 = 0.7-0.9, \beta_3 = 0.7-0.9\)             | (10714.74, 16521.38, 7616.15) |
|       | \(\alpha_3 = 1.0, \beta_3 = 1.0\)                     | (9572.093, 16639.81, 9283.474) |
ues whenever four values of $\alpha$ and $\beta$ are fixed at 0.1, 0.5 and 1.0, whereas other two values of $\alpha$ and $\beta$ are violated at 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 1.0. All these cases are highlighted in Figs. 3, 4 and 5. According to the validation of the model, we find that the objective values are violated in the realization model for the cause of hesitant values and most selected optimal values appear in any middle or end or starting situation, where objectives are not increased or decreased corresponding to the increased or decreased valued of $\alpha$ and $\beta$. From this perspective, it can be decided that the proposed PHFP model offers an opportunity to the DMs for selecting a better decision in an uncertain situation. Important novelty of this approach is that the obtained solution of feasible region provides a spectrum of uncertainty which is different from fuzzy and IF.

Sensitivity analysis on carbon emission

This section analyzes the impact of cap parameter of MOSTP. Today, carbon policy is a challenging task to keep green world and a largest flexible range of this parameter is required for future study. Table 15 represents the ranges by the variation of cap parameter $C$ with carbon emission tax is attached. We study the sensitivity analysis based on our proposed method PHFP and this investigation is allowable only when one parameter is changed at a time and other parameters have fixed with their initial values. Cap parameter is analyzed with three policies in different cases and for both examples. The analysis is performed by changing the parameter $C$ into $C^\ast$. We utilize an easy method [15], such that basic variables remain stable, but their values may have changed. The processing steps are defined as follows:

- **Step 1:** Include the basic variables of the optimal allocation for MOSTP which are derived by PHFP.
- **Step 2:** Oscillate the value of the parameter by keeping rigid to the other parameters at that time and solve by LINGO 13 iterative scheme.
- **Step 3:** Go to Step 2, till NFS occurs or the basic variable replaces in optimal solution.
- **Step 4:** Collect the range of parameters obtained in Step 3.

Observation and managerial insights

Three objective functions connecting with SD are minimum transportation cost, maximum job opportunity and minimum carbon emission cost which are included in the proposed model, that should be relevant to any industrial problem with transportation system. The objective functions are chosen for
Table 15  Sensitivity analysis for carbon cap of both examples

| Real value of $C$       | Range of $C^*$            |
|-------------------------|---------------------------|
| Example 2               |                           |
| $C = 25000$ (Model 3)   | $24236.76 \leq C^* < \infty$ |
| $C = 25000$ (Model 4A)  | $24236.76 \leq C^* < \infty$ |
| $C = 25000$ (Model 4B)  | $21667.32 \leq C^* \leq 43572.15$ |
| $C = 25000$ (Model 5A)  | $24236.76 \leq C^* < \infty$ |
| $C = 25000$ (Model 5B)  | $21141.30 \leq C^* \leq 38225.40$ |
| Example 3               |                           |
| $C = 18000$ (Model 4B)  | $17884.30 \leq C^* \leq 18683.98$ |
| $C = 18000$ (Model 5B)  | $17505.54 \leq C^* \leq 18550.86$ |

the focus of SWM, such that the proposed model provides a plan to optimize all the objectives by considering source, demand and conveyance constraints. The following managerial insights are traced out from this study as follows:

- Introduction of SWM in the proposed model has several benefits with different sides that can be managed in vast areas, advices that urban or rural areas are developed comfortably and provided green or smart city certainly.
- Sustainability criteria are developed by regarding three main objectives as minimum transportation cost for economical opportunity, maximum job opportunity for social impact and minimum carbon emission for safety of environment. At the time when these factors meet, then the system becomes more adaptable.
- Considering the analysis of carbon mechanism with different combined policies, the organization can select a suitable policy for predicting the positive impact of it which drops down the carbon emission. Utilizing appropriate policy, industrial organisation can also reduce the carbon emission from the production plant to prevent global warming.
- The suggested model is established on the basis of TP, waste management, SD and in the presence of PHF environment. As DM tackles such complicated uncertainty with multiple criteria, therefore, he/she is prepared to challenge any type of difficult uncertain situation with several criteria.
- From Pareto-optimal solution, it is decided that all the objectives provide a satisfactory result which can help to formulate any network design related with transportation, waste management or SD.

Concluding remarks and outlook to future research

In this research, for the first time in MOSTP with SWM has been rigorously initiated for SD under PHF uncertain situation. The proposed new model and solution methodology becomes highly applicable in real-life decision making problems as three conflicting objective functions have been concentrated on minimum transportation cost for economical aspect, maximum job opportunity for social impact and low carbon emission for environment effect. Three different types of scenarios (models) have been presented by considering three policies of carbon mechanism and all these objective functions are optimized in the presence of such carbon mechanism, and adding with all required constraints. The problem has been divided into different stages for waste management and that controlled by considering multiple variables. Source, demand and conveyance are taken as TrPHFNs to overcome deep uncertainty by extending two uncertainty as hesitant fuzzy and PF. An advance ranking index has been utilized here to defuzzify the uncertainty in the problem. Proposed models have been solved technically by utilizing two fuzzy techniques as FP and PHFP. Two realistic examples have been experimented that show the advantages of the proposed model based on the improved methods. A vast discussion has been elaborated in view of carbon mechanism for choosing a better policy among other different policies which mitigate CO2 emission of sustainable transportation with SWM. Model validation is investigated based on the preferable policy.

Our existing study has some limitations and modifying all the drawbacks, and extending our proposed study, researchers can provide various opportunities for future research scope. Preservation technology may be included for transporting perishable items and effect of carbon emission during preservation can be included in our model. One may improve our model in new environment, such as polygonal fuzzy, type-2 neutrosophic, fuzzy-rough [14], type-2 zigzag uncertainty, interval PF, soft set, Python gorean probabilistic hesitant fuzzy, etc., with different ranking approaches on non-linear membership functions (e.g., hyperbolic, exponential, quadratic, etc.). Our model can be remodelled using green supply chain management with stochastic data, fixed-charge, robust ranking, supplier selection, portfolio optimization, etc. Another scope is that a large industrial problem concerning with probabilistic industrial waste, mining waste, other several combination of carbon emission policy, etc., may be added in our model. Some sophisticated methods, such as ant colony optimization [47], genetic algorithm, particle swarm optimization, robust optimization model [48], etc., can be introduced in our model as well. Such directions of investigation can be pursued for future research.

Declarations

Conflict of interest The authors announce that there is no conflict of interest.
Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article’s Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article’s Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4.0/.

References

1. Abdullah L, Ghosh P (2019) Decision making method based on Pythagorean fuzzy set and its application to solid waste management. Complex Intell Syst 5:185–198
2. Abbas SZ, Khan MSA, Abdullah S, Sun H, Hussain F (2019) Cubic Pythagorean fuzzy sets and their application to multi-attribute decision making with unknown weight information. J Intell Fuzzy Syst 37:1529–1544
3. Adhami AY, Ahmad F (2020) Interactive Pythagorean hesitant fuzzy computational algorithm for multi-objective transportation problem under uncertainty. Int J Manag 15(4):1–10
4. Asim Z, Jali SA, Javaid S (2019) An uncertain model for integrated production-transportation closed-loop supply chain network with cost reliability. Sustain Prod Consum 17:298–310
5. Asl HF, Makui A, Jabbarzadeh A, Barzinpour F (2018) Solving a multi-objective sustainable waste collection problem considering a new collection network. Oper Res 20:1977–2015
6. Bellman RE, Zadeh LA (1970) Decision making in a fuzzy environment. Manag Sci 17(4):141–164
7. Bortz M, Burger J, Asprion N, Scheithauer A, Welke R, Küfer KH, Hasse H (2014) Multi-criteria optimization in chemical process design and decision support by navigation on Pareto sets. Comput Chem Eng 60:354–363
8. Brundtland G, Khalid M, Agnelli S, Al-Athel S, Chidzero B, Fadika L, Hauff V, Lang I, Shijun M, de Botero MM, Singh M (1987) Our common future. Brundtland Report
9. Chiu FP, Kuo HI, Chen CC, Hsu CS (2015) The energy price equivalence of carbon taxes and emissions trading-theory and evidence. Appl Energy 160:164–171
10. Dalman H, Güzel N, Sivri M (2016) A fuzzy set based approach to multi-objective multi-item solid transportation problem under uncertainty. Int J Fuzzy Syst 18:73–79
11. Das SK, Roy SK, Weber GW (2020) Application of type-2 fuzzy logic to a multi-objective green solid transportation-location problem with dwell time under carbon emission tax, cap and offset policy: fuzzy vs. non-fuzzy techniques. IEEE Trans Fuzzy Syst 28(11):2711–2725
12. Farhadinia B, Aickelin U, Khorshidi HA (2020) Uncertainty measures for probabilistic hesitant fuzzy sets in multiple criteria decision making. Int J Intell Syst 35(11):1646–1679
13. Ghosh A, Sarmah SP, Jha JK (2018) Collaborative model for a two-echelon supply chain with uncertain demand under carbon tax policy. Sadhana 43(9):144
14. Ghosh S, Roy SK (2021) Fuzzy-rough multi-objective product blending fixed-charge transportation problem with truck load constraints through transfer station. RAIRO Oper Res 55:S2923–S2952
15. Ghosh S, Roy SK, Ebrahimnejad A, Verdegay JL (2021) Multi-objective fully intuitionistic fuzzy fixed–charged solid transportation problem. Complex Intell Syst 7(2):1009–1023
16. Gosavi P, Chaudhary Y, Gupta AD (2017) Production of bio-fuel from fruits and vegetable wastes. Eur J Biotechnol Biosci 5(3):69–73
17. Gupta P, Mehlawat MK, Aggarwal U, Charles V (2018) An integrated AHP-DEA multi-objective optimization model for sustainable transportation in mining industry. Resour Policy. https://doi.org/10.1016/j.resourpol.2018.04.007
18. Haley KB (1962) The solid transportation problem. Oper Res 10:448–463
19. He P, Zhang W, Xu X, Bian Y (2015) Production lot sizing and carbon emissions under cap-and-trade and carbon tax regulations. J Clean Prod 103(15):241–248
20. Hitchcock FL (1941) The distribution of a product from several sources to numerous localities. J Math Phys 20:224–230
21. Karaaslan F, Olu S (2020) Correlation coefficients of dual type-2 hesitant fuzzy sets and their applications in clustering analysis. Int J Intell Syst 35(7):1200–1229
22. Küfer KH, Monz M, Scherrer A, Süß P, Alonso F, Sultan ASA, Bortfeld T, Thieke C (2009) Multi-criteria optimization in intensity modulated radiotherapy planning. In: Handbook of optimization in medicine. Springer, pp 123–168
23. Kumar R, Edalatpanah SA, Jha S, Singh R (2019) A Pythagorean fuzzy approach to the transportation problem. Complex Intell Syst 5(3):255–263
24. Li J, Du W, Yang F, Hua G (2014) The carbon subsidy analysis in remanufacturing closed-loop supply chain. Sustainability 6:3861–3877
25. Li J, Su Q, Ma L (2017) Production and transportation outsourcing decisions in the supply chain under single and multiple carbon policies. J Clean Prod 141:1109–1122
26. Liang D, Xu Z (2017) The new extension of TOPSIS method for multiple criteria decision making with hesitant Pythagorean fuzzy sets. Appl Soft Comput 60:167–179
27. Lotfi R, Mehrjerdi YZ, Pishvaa MS, Sadeghieh A, Weber GW (2021) A robust optimization model for sustainable and resilient closed-loop supply chain network design considering conditional value at risk. Numer Algebra Control Optim 11(2):221–253
28. Maiti G, Roy SK, Verdegay JL (2019) Time variant multi-objective interval valued transportation problem in sustainable development. Sustainability 11(21):6161
29. Mardana D, Maiti G, Roy SK (2021) The multi-objective multi-item just-in-time transportation problem. Optimization. https://doi.org/10.1080/02331934.2021.1963246
30. Mehlawat MK, Kannan D, Gupta P, Aggarwal U (2019) Sustainable transportation planning for a three stage fixed-charge multi-objective transportation problem. Ann Oper Res. https://doi.org/10.1007/s10479-019-03451-4
31. Midya S, Roy SK, Yu VF (2021) Intuitionistic fuzzy multi-stage multi-objective fixed-charge solid transportation problem in a green supply chain. Int J Mach Learn Cybern 12(4):699–717
32. Mingaleva Z, Vukovic N, Volkova I, Salimova T (2020) Waste management in green and smart cities: a case study of Russia. Sustainability 12:94
33. Muneeb SM, Adhami AY, Jalil SA, Asim Z (2018) Decentralized bi-level decision planning model for municipal solid waste recycling and management with cost reliability under uncertain environment. Sustain Prod Consum 16:33–44
34. Nagarajan G, Rajakumar S, Ayyasamy PM (2014) Vegetable wastes: an alternative resource for bio-gas and bio-compost production through lab scale process. Int J Curr Microbiol 3(10):379–387
35. Nguyen XT, Nguyen VN, Nguyen VH, Garg H (2019) Exponential similarity measures for Pythagorean fuzzy sets and their applica-
tions to pattern recognition and decision making process. Complex Intell Syst 5(2):217–228
36. Ni W, Shu J (2015) Trade-off between service time and carbon emissions for safety stock placement in multi-echelon supply chains. Int J Prod Res 53:6701–6718
37. Palak G, Eksioglu SD, Geunes J (2014) Analyzing the impacts of carbon regulatory mechanisms on supplier and mode selection decisions: an application to a bio-fuel supply chain. Int J Prod Econ 154:198–216
38. Pascoletti A, Serafini P (1984) Scalarizing vector optimization problems. J Optim Theory Appl 42:499–524
39. Peng X, Selvachandran G (2019) Pythagorean fuzzy set: state of the art and future directions. Artif Intell Rev 52:1873–1927
40. Qu G, Xue R, Li T; Qu W, Xu Z (2020) A stochastic multi-attribute method for measuring sustainability performance of a supplier based on a triple bottom line approach in a dual hesitant fuzzy linguistic environment. Int J Environ Res Public Health 17(6):2138
41. Rabbani M, Heidari R, Farrokhi AH, Rahimi N (2018) Using meta-heuristic algorithms to solve a multi-objective industrial hazardous waste location-routing problem considering incompatible waste types. J Clean Prod 170:227–241
42. Rathore P, Sarmah SP (2019) Modeling transfer station locations considering source separation of solid waste in urban centers: a case study of Bilaspur city, India. J Clean Prod 211:44–60
43. Roy SK, Midya S, Weber GW (2019) Multi-objective multi-item fixed-charge solid transportation problem under two-fold uncertainty. Neural Comput Appl 31:8593–8613
44. Singh A, Kulia A, Adak S, Bishai M, Banerjee R (2012) Utilization of vegetable wastes for bioenergy generation. Agric Res 1(3):213–222
45. Singh S, Ganie AH (2020) On some correlation coefficients in Pythagorean fuzzy environment with applications. Int J Intell Syst 35(4):682–717
46. Tirkolaee EB, Goli A, Pahlevan M, Kordestanizadeh RM (2019) A robust bi-objective multi-trip periodic capacitated arc routing problem for urban waste collection using a multi-objective invasive weed optimization. Waste Manag Res 37(11):1089–1101
47. Tirkolaee EB, Mahdavi I, Esfahani MMS, Weber GW (2019) A hybrid augmented ant colony optimization for the multi-trip capacitated arc routing problem under fuzzy demands for urban solid waste management. Waste Manag Res 38:156–172
48. Tirkolaee EB, Mahdavi I, Esfahani MMS, Weber GW (2020) A robust green location-allocation inventory problem to design an urban waste management system under uncertainty. Waste Manag 102:340–350
49. Torra V, Narukawa Y (2009) On hesitant fuzzy sets and decision. In: IEEE Int Conf Fuzzy Syst, pp 1378–1382. https://doi.org/10.1109/FUZZY.2009.5276884 (ISSN 10987584)
50. Vafaei A, Yaghoubi S, Tajik J, Barzinpour F (2020) Designing a sustainable multi-channel supply chain distribution network: a case study. J Clean Prod 251:119628
51. Xu X, Zhang W, He P, Xu X (2017) Production and pricing problems in make to order supply chain with cap and trade regulation. Int J Manag Sci 66:248–257
52. Xu Z, Elomri A, Pokharel S, Zhang Q, Ming XG, Liu W (2017) Global reverse supply chain design for solid waste recycling under uncertainties and carbon emission constraint. Waste Manag 64:358–370
53. Xu Z, Elomri A, Pokharel S, Mutlu F (2019) The design of green supply chains under carbon policies: a literature review of quantitative models. Sustainability 11(11):3094
54. Yager RR, Abbasov AM (2013) Pythagorean membership grades, complex numbers, and decision making. Int J Intell Syst 28(5):436–452
55. Yi Y, Li J (2018) Cost sharing contracts for energy saving and emissions reduction of a supply chain under the conditions of government subsidies and a carbon tax. Sustainability 10(3):895
56. Yuyin Y, Jinxi L (2018) The effect of governmental policies of carbon taxes and energy-saving subsidies on enterprise decisions in a two-echelon supply chain. J Clean Prod 181:675–691
57. Zadeh LA (1965) Fuzzy sets. Inf. Control 8(3):338–353
58. Zakeri A, Dehghanian F, Fahimnia B, Sarkis J (2015) Carbon pricing versus emissions trading: a supply chain planning perspective. J Int Prod Econ 146:197–205
59. Zaeimi MB, Rassafi AA (2021) Designing an integrated municipal solid waste management system using a fuzzy chance-constrained programming model considering economic and environmental aspects under uncertainty. Waste Manag 125:268–279
60. Zaeimi MB, Rassafi AA (2021) Optimization model for integrated municipal solid waste system using stochastic chance-constrained programming under uncertainty: a case study in Qazvin, Iran. J Adv Transp Article ID 9994853:16
61. Zakeri A, Dehghanian F, Fahimnia B, Sarkis J (2015) Carbon pricing versus emissions trading: a supply chain planning perspective. J Int Prod Econ 146:197–205
62. Zhou Y, Gong DC, Huang B, Peters BA (2015) The impacts of carbon tariff on green supply chain design. IEEE Trans Autom Sci Eng 14(3):1–14
63. Zimmermann HJ (1978) Fuzzy programming and linear programming with several objective functions. Fuzzy Sets Syst 1(1):45–55

Publisher’s Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.