Relationship between Hawking radiation
from black holes and spontaneous excitation of atoms

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Abstract

Using the formalism that separates the contributions of vacuum fluctuations and radiation reaction to the rate of change of the mean atomic energy, we show that a two-level atom in interaction with a quantum massless scalar field both in the Hartle-Hawking and Unruh vacuum in a 1+1 dimensional black hole background spontaneously excites as if there is thermal radiation at the Hawking temperature emanating from the black hole. Our calculation, therefore, ties the existence of Hawking radiation to the spontaneous excitation of a two-level atom placed in vacuum in the exterior of a black hole and shows pleasing consistence of two different physical phenomena, the Hawking radiation and the spontaneous excitation of atoms, which are quite prominent in their own right.
a. Introduction. Hawking radiation from black holes, as one of the most striking effects that arise from the combination of quantum theory and general relativity, has attracted widespread interest in physics community. Currently, several derivations of Hawking radiation have been proposed, including Hawking’s original one which calculates the Bogoliubov coefficients between the quantum scalar field modes of the in vacuum states and those of the out vacuum [1, 2], an elegant one based upon the Euclidean quantum gravity [3] which has been interpreted as a calculation of tunneling through classically forbidden trajectory [4], the approach based upon string theory [5, 6], and a recent proposal which ties its existence to the cancellation of gravitational anomalies at the horizon [7]. Here we discuss yet another approach which calculates the spontaneous excitation rate of a two-level atom interacting with massless quantum scalar fields in vacuum states in a black hole background. Our investigation ties the existence of Hawking radiation to the spontaneous excitation of a two-level atom placed in vacuum in the exterior of a black hole, thus revealing an interesting relationship between the existence of Hawking radiation from black holes and the spontaneous excitation of atoms in vacuum.

Spontaneous emission, on the other hand, is one of the most important features of atoms and so far mechanisms such as vacuum fluctuations [8, 9], radiation reaction [10], or a combination of them [11] have been put forward to explain why spontaneous emission occurs. The ambiguity in physical interpretation arises because of the freedom in the choice of ordering of commuting operators of the atom and field in a Heisenberg picture approach to the problem. The controversy was resolved when Dalibard, Dupont-Roc and Cohen-Tannoudji(DDC) [12, 13] proposed a formalism which distinctively separates the contributions of vacuum fluctuations and radiation reaction by demanding a symmetric operator ordering of atom and field variables. The DDC formalism has recently been generalized to study the spontaneous excitation of uniformly accelerated atoms in interaction with vacuum scalar and electromagnetic fields in a flat spacetime [14–16], and these studies show that when an atom is accelerated, the delicate balance between vacuum fluctuations and radiation reaction that ensures the ground state atom’s stability in vacuum is altered, making possible the transitions to excited states for ground-state atoms even in vacuum. In this paper, we apply this generalized DDC formalism to investigate the spontaneous excitation of an atom held static in the exterior region of a black hole and interacting with vacuum quantum massless scalar fields in two dimensions, and show that the atom spontaneously excites as if it were irradiated by or immersed in a thermal radiation at the Hawking temperature, depending on whether the scalar
field is in the Unruh or the Hartle-Hawking vacuum. In other words, atoms feel the Hawking radiation from black holes.

### b. Formalism

When vacuum fluctuations are concerned in a curved spacetime, a delicate issue then arises as to how the vacuum state of the massless scalar field is determined. Normally, the vacuum state is associated with non-occupation of positive frequency modes. However, the positive frequency of field modes are defined with respect to the time coordinate. Therefore, to define positive frequency, one has to first specify a definition of time. In a spherically symmetric black hole background, one definition is the Schwarzschild time and it is a natural definition of time in the exterior region. However, the vacuum state associated with this choice of time coordinate (Boulware vacuum) becomes problematic in the sense that the expectation value of the energy-momentum tensor, evaluated in a free falling frame, diverges at the horizon. Other possibilities that avoid this problem are the Unruh vacuum [17] and the Hartle-Hawking vacuum [18]. The Unruh vacuum is defined by taking modes that are incoming from $\mathcal{I}^-$ to be positive frequency with respect to the Schwarzschild time, while those that emanate from the past horizon are taken to be positive frequency with respect to the Kruskal coordinate $\bar{u}$, the canonical affine parameter on the past horizon. The Unruh vacuum is regarded as the vacuum state that best approximates the state that would obtain following the gravitational collapse of a massive body. The Hartle-Hawking vacuum, on the other hand, is defined by taking the incoming modes to be positive frequency with respect to $\bar{v}$, the canonical affine parameter on the future horizon, and outgoing modes to be positive frequency with respect to $\bar{u}$. Let us note that the Hartle-Hawking state does not correspond to our usual notion of a vacuum since it has thermal radiation incoming to the black hole from infinity and describes a black hole in equilibrium with a sea of thermal radiation.

Consider, in two dimensions, a two-level atom in interaction with a quantum real massless scalar field in a spherically symmetric black hole background, of which the metric is given by

$$ds^2 = \left(1 - \frac{2M}{r}\right) du dv - \frac{2M}{r} e^{-r/2M} d\bar{u} d\bar{v},$$

(1)

where

$$u = t - r^*, \quad v = t + r^*, \quad r^* = r + 2M \ln[(r/2M) - 1], \quad \bar{u} = -e^{-\kappa u}/\kappa, \quad \bar{v} = e^{-\kappa v}/\kappa.$$  

(2)

Here $\kappa = 1/4M$ is the surface gravity of the black hole. Without loss of generality, let us assume a pointlike two-level atom on a stationary space-time trajectory $x(\tau)$, where $\tau$ denotes the proper
time on the trajectory. The stationarity of the trajectory guarantees the existence of stationary atomic states, |+⟩ and |−⟩, with energies ±1/2ω₀ and a level spacing ω₀. The atom’s Hamiltonian which controls the time evolution with respect to τ is given, in Dicke’s notation [19], by

\[ H_\text{A}(\tau) = \omega_0 R_3(\tau), \]  

(3)

where \( R_3 = \frac{1}{2}|+\rangle\langle+| - \frac{1}{2}|−\rangle\langle−| \) is the pseudospin operator commonly used in the description of two-level atoms[19]. The free Hamiltonian of the quantum scalar field that governs its time evolution with respect to τ is

\[ H_\text{F}(\tau) = \int d^3k \omega_k \hat{a}_k^\dagger \hat{a}_k \frac{dt}{d\tau}. \]  

(4)

Here \( \hat{a}_k^\dagger, \hat{a}_k \) are the creation and annihilation operators with momentum \( \vec{k} \). Following Ref. [14], we assume that the interaction between the atom and the quantum field is described by a Hamiltonian

\[ H_\text{I}(\tau) = c R_2(\tau) \phi(x(\tau)) = \mu \omega_0 R_2(\tau) \phi(x(\tau)), \]  

(5)

where \( c \) is a coupling constant which we assume to be small, \( R_2 = \frac{1}{2}i(R_- - R_+) \), and \( R_+ = |+\rangle\langle−|, R_- = |−\rangle\langle+|. \) The coupling is effective only on the trajectory \( x(\tau) \) of the atom. Note that here we have defined a dimensionless parameter, \( \mu = c/\omega_0 \).

We can now write down the Heisenberg equations of motion for the atom and field observables. The field is always considered to be in its vacuum state \( |0⟩ \). We will separately discuss the two physical mechanisms that contribute to the rate of change of atomic observables: the contribution of vacuum fluctuations and that of radiation reaction. For this purpose, we can split the solution of field \( \phi \) of the Heisenberg equations into two parts: a free or vacuum part \( \phi^f \), which is present even in the absence of coupling, and a source part \( \phi^s \), which represents the field generated by the interaction between the atom and the field. Following DDC[12, 13], we choose a symmetric ordering between atom and field variables and consider the effects of \( \phi^f \) and \( \phi^s \) separately in the Heisenberg equations of an arbitrary atomic observable \( G \). Then, we obtain the individual contributions of vacuum fluctuations and radiation reaction to the rate of change of \( G \). Since we are interested in the spontaneous excitation of the atom, we will concentrate on the mean atomic excitation energy \( \langle H_\text{A}(\tau) \rangle \). The contributions of vacuum fluctuations(vf) and radiation reaction(rr) to the rate of change of \( \langle H_\text{A} \rangle \) can be written as ( cf. Ref.[12–16] )

\[ \left\langle \frac{dH_\text{A}(\tau)}{d\tau} \right\rangle_\text{vf} = 2i c^2 \int_{\tau_0}^{\tau} d\tau' \int C^F(x(\tau), x(\tau')) \frac{d}{d\tau} C^A(\tau, \tau') \frac{d}{d\tau}, \]  

(6)

\[ \left\langle \frac{dH_\text{A}(\tau)}{d\tau} \right\rangle_\text{rr} = 2i c^2 \int_{\tau_0}^{\tau} d\tau' \int C^F(x(\tau), x(\tau')) \frac{d}{d\tau} C^A(\tau, \tau') \frac{d}{d\tau}, \]  

(7)
with \(|\rangle = |a, 0\rangle\) representing the atom in the state \(|a\rangle\) and the field in the vacuum state \(|0\rangle\). Here the statistical functions of the atom, \(C^A(\tau, \tau')\) and \(\chi^A(\tau, \tau')\), are defined as

\[
C^A(\tau, \tau') = \frac{1}{2} \langle a|R^A_2(\tau), R^A_2(\tau')|a\rangle, \quad (8)
\]

\[
\chi^A(\tau, \tau') = \frac{1}{2} \langle a|R^A_2(\tau), R^A_2(\tau')|a\rangle, \quad (9)
\]

and those of the field are as

\[
C^F(x(\tau), x(\tau')) = \frac{1}{2} \langle 0|\phi'(x(\tau)), \phi'(x(\tau'))|0\rangle, \quad (10)
\]

\[
\chi^F(x(\tau), x(\tau')) = \frac{1}{2} \langle 0|\phi'(x(\tau)), \phi'(x(\tau'))|0\rangle. \quad (11)
\]

\(C^A\) is called the symmetric correlation function of the atom in the state \(|a\rangle\), \(\chi^A\) its linear susceptibility. \(C^F\) and \(\chi^F\) are the Hadamard function and Pauli-Jordan or Schwinger function of the field respectively. The explicit forms of the statistical functions of the atom are given by

\[
C^A(\tau, \tau') = \frac{1}{2} \sum_b \langle a|R^A_2(0)b\rangle^2 \left( e^{i\omega_{ab}(\tau-\tau')} + e^{-i\omega_{ab}(\tau-\tau')} \right), \quad (12)
\]

\[
\chi^A(\tau, \tau') = \frac{1}{2} \sum_b \langle a|R^A_2(0)b\rangle^2 \left( e^{i\omega_{ab}(\tau-\tau')} - e^{-i\omega_{ab}(\tau-\tau')} \right), \quad (13)
\]

where \(\omega_{ab} = \omega_a - \omega_b\) and the sum runs over a complete set of atomic states.

c. Spontaneous excitation of atoms. First let us apply the above formalism to the case of the Hartle-Hawking vacuum for the scalar field. Consider an atom held static at a radial distance \(R\) from the black hole. The Wightman function for massless scalar fields in the Hartle-Hawking vacuum in \((1+1)\) dimension is given by [20]

\[
D^+_H(x, x') = -\frac{1}{4\pi} \ln \frac{4e^{2\kappa R} \sinh^2\left(\kappa_R (\Delta t/2) - i\varepsilon\right)}{k^2}, \quad (14)
\]

where

\[
\Delta t = \Delta t \sqrt{g_{00}} = \Delta t \sqrt{1 - \frac{2M}{R}}, \quad \kappa_R = \frac{k}{\sqrt{1 - \frac{2M}{R}}}. \quad (15)
\]

This leads to the following statistical functions of the scalar field

\[
C^F(x(\tau), x(\tau')) = -\frac{1}{8\pi} \left[ 2 \ln \frac{4e^{2\kappa R}}{k^4} + \ln \sinh^2\left(\frac{\kappa_R}{2} \Delta t - i\varepsilon\right) + \ln \sinh^2\left(\frac{\kappa_R}{2} \Delta t + i\varepsilon\right) \right], \quad (16)
\]

\[
\chi^F(x(\tau), x(\tau')) = -\frac{1}{8\pi} \left[ \ln \sinh^2\left(\frac{\kappa_R}{2} \Delta t - i\varepsilon\right) - \ln \sinh^2\left(\frac{\kappa_R}{2} \Delta t + i\varepsilon\right) \right]. \quad (17)
\]
Plugging the above expressions into Eq. (6) and Eq. (7) and performing the integration yields the contribution of the vacuum fluctuations to the rate of change of the mean atomic energy for the atom held static at a distance $R$ from the black hole

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{\text{vf}} = -\mu^2 \left[ \sum_{\omega_a > \omega_b} \omega_0^2 |\langle a| R_2^f(0)|b\rangle|^2 \left( \frac{1}{2} + \frac{1}{e^{\omega_{ab}(2\pi/\kappa R)} - 1} \right) \right. - \sum_{\omega_a < \omega_b} \omega_0^2 |\langle a| R_2^f(0)|b\rangle|^2 \left( \frac{1}{2} + \frac{1}{e^{\omega_{ab}(2\pi/\kappa R)} - 1} \right) \left. \right], \quad (18)$$

and that of radiation reaction

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{\text{rr}} = -\mu^2 \left[ \sum_{\omega_a > \omega_b} \frac{\omega_0^2}{2} |\langle a| R_2^f(0)|b\rangle|^2 + \sum_{\omega_a < \omega_b} \frac{\omega_0^2}{2} |\langle a| R_2^f(0)|b\rangle|^2 \right]. \quad (19)$$

Here we have extended the integration range to infinity for sufficiently long times $\tau - \tau_0$. Adding up two contributions, we obtain the total rate of change of the mean atomic energy

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{\text{tot}} = -\mu^2 \left[ \sum_{\omega_a > \omega_b} \omega_0^2 |\langle a| R_2^f(0)|b\rangle|^2 \left( 1 + \frac{1}{e^{\omega_{ab}(2\pi/\kappa R)} - 1} \right) \right. - \sum_{\omega_a < \omega_b} \omega_0^2 |\langle a| R_2^f(0)|b\rangle|^2 \left( \frac{1}{e^{\omega_{ab}(2\pi/\kappa R)} - 1} \right) \left. \right]. \quad (20)$$

One can see that, for a ground state atom held static at a radial distance $R$ from the black hole, only the second term ($\omega_a < \omega_b$) contributes and this contribution is positive, revealing that the atom spontaneously excites and thus transitions from ground state to the excited states occur spontaneously in the exterior region of the black hole. The most striking feature is that the spontaneous excitation rate is what one would obtain if the atom were immersed in a thermal bath of radiation at the temperature

$$T = \frac{\kappa}{2\pi} \frac{1}{\sqrt{1 - \frac{2M}{R}}} = (g_{00})^{-1/2} T_H, \quad (21)$$

where $T_H = \kappa/2\pi$ is just the Hawking temperature of the black hole. In fact, the above result is the well-known Tolman relation [21] which gives the proper temperature as measured by a local observer. While for an atom at spatial infinity ($R \to \infty$), the temperature as felt by the atom, $T$, approaches $T_H$. Therefore, an atom infinitely far away from the black hole (in the asymptotic region) would spontaneously excite as if in a thermal bath of radiation at the Hawking temperature. However, as the atom approaches the horizon ($R \to 2M$), the temperature $T$ diverges. This can be understood as the fact that the atom must be in acceleration relative to the local free-falling
frame, which blows up at the horizon, to maintain at a fixed distance from the black hole, and this acceleration gives rise to additional thermal effect [17].

Now let us briefly discuss what happens if the Hartle-Hawking vacuum is replaced by the Unruh vacuum. Then it is easy to show that the statistical functions of the scalar field become

\[ C^F (x(\tau), x(\tau')) = -\frac{1}{8\pi} \left\{ 2 \ln \frac{2e^{KR}}{k\sqrt{1 - \frac{2M}{R}}} + \ln \left[ \sinh \left( \frac{a}{2}\Delta \tau - \frac{i\epsilon}{2} \right) \left( \Delta \tau - \frac{i\epsilon}{2} \right) \right] + \ln \left[ \sinh \left( \frac{a}{2}\Delta \tau + \frac{i\epsilon}{2} \right) \left( \Delta \tau + \frac{i\epsilon}{2} \right) \right] - a(\tau + \tau') \right\} , \]

\[ (22) \]

\[ \chi^F (x(\tau), x(\tau')) = -\frac{1}{8\pi} \left\{ \ln \left[ \sinh \left( \frac{a}{2}\Delta \tau - \frac{i\epsilon}{2} \right) \left( \Delta \tau - \frac{i\epsilon}{2} \right) \right] - \ln \left[ \sinh \left( \frac{a}{2}\Delta \tau + \frac{i\epsilon}{2} \right) \left( \Delta \tau + \frac{i\epsilon}{2} \right) \right] \right\} , \]

\[ (23) \]

and the contribution of the vacuum fluctuations to the rate of change of the mean atomic energy for the atom held static at a distance \( R \) from the black hole is given by

\[ \left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{vf} = -\mu^2 \left[ \sum_{\omega_a > \omega_b} \omega_0^2 |\langle a|R^f_2(0)|b\rangle|^2 \left( \frac{1}{2} + \frac{1}{2} e^{\omega_{ab}(2\pi/\kappa_R)} - 1 \right) \right. \]

\[ - \left. \sum_{\omega_a < \omega_b} \omega_0^2 |\langle a|R^f_2(0)|b\rangle|^2 \left( \frac{1}{2} + \frac{1}{2} e^{\omega_{ab}(2\pi/\kappa_R)} - 1 \right) \right] , \]

\[ (24) \]

and that of radiation reaction by

\[ \left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{rr} = -\mu^2 \left[ \sum_{\omega_a > \omega_b} \omega_0^2 |\langle a|R^f_2(0)|b\rangle|^2 + \sum_{\omega_a < \omega_b} \omega_0^2 |\langle a|R^f_2(0)|b\rangle|^2 \right] . \]

\[ (25) \]

Consequently, we obtain, by adding up two contributions, the total rate of change of the mean atomic energy

\[ \left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{tot} = -\mu^2 \left[ \sum_{\omega_a > \omega_b} \omega_0^2 |\langle a|R^f_2(0)|b\rangle|^2 \left( 1 + \frac{1}{2} e^{\omega_{ab}(2\pi/\kappa_R)} - 1 \right) \right. \]

\[ - \left. \sum_{\omega_a < \omega_b} \omega_0^2 |\langle a|R^f_2(0)|b\rangle|^2 \frac{1}{2} e^{\omega_{ab}(2\pi/\kappa_R)} - 1 \right] . \]

\[ (26) \]

It is interesting to note that the term of the thermal like contribution is half of that in the Hartle-Hawking case (refer to Eq. (20)). This is consistent with our understanding that the Unruh vacuum corresponds to the state following the collapsing of a massive body to form a black hole, and as a result, the atom, held static at a radial distance \( R \), spontaneously excites as if it were irradiated by a beam of outgoing thermal radiation at the temperature \( T = \kappa_R/2\pi \), in other words, atoms feel the Hawking radiation. While the Hartle-Hawking vacuum is the state that includes a thermal radiation at the Hawking temperature incoming from infinity and describes an eternal black hole in thermal equilibrium with the incoming thermal radiation. Therefore the spontaneous excitation rate doubles.
d. Summary By evaluating the rate of change of the mean atomic energy for a two-level atom in interaction with a massless scalar field in both the Hartle-Hawking and the Unruh vacuum in a 1+1 dimensional black hole background, we have demonstrated that an inertial atom far away from the black hole (in the asymptotic region) would spontaneously excite as if there is thermal radiation at the Hawking temperature emanating from the black hole, or in other words, atoms feel the Hawking radiation from black holes. Therefore, our discussion can be considered as providing another approach to the derivation of the Hawking radiation. Our result also reveals an interesting relationship between the existence of Hawking radiation from black holes and the spontaneous excitation of a two-level atom in vacuum in the exterior of a black hole, and shows pleasing consistency of two different physical phenomena, the Hawking radiation and the spontaneous excitation of atoms, which are quite prominent in their own right.

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