Spin pumping into a spin glass material

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Spin pumping is a recently established means for generating a pure spin current, whereby spins are pumped from a magnet into the adjacent target material under the ferromagnetic resonance condition. We theoretically investigate the spin pumping from an insulating ferromagnet into spin glass materials. Combining a dynamic theory of spin glasses with the linear-response formulation of the spin pumping, we calculate temperature dependence of the spin pumping near the spin glass transition. The analysis predicts that a characteristic peak appears in the spin pumping signal, reflecting that the spin fluctuations slow down upon the onset of spin freezing.

I. INTRODUCTION

Spin current is a flow of spin angular momentum [1]. Over the last two decades, great progress has been made in generating, manipulating, and detecting the spin current [2–3]. With regard to the spin current generation, as nicely reviewed in Ref. [4], the spin pumping is now established as a charge-free and versatile means [5–8]. In this method a pure spin current, which is unaccompanied by a charge current, is pumped from a ferromagnet into the adjacent spin sink material by a stimulus of microwaves satisfying the ferromagnetic resonance (FMR) condition. Thanks to the advent of the spin pumping technique, the spin current physics has so far been investigated in a variety of spin sink materials, ranging from nonmagnetic metals [9–12], semiconductors [13–16], magnetic metals [17–19], insulators [20], to more exotic systems such as graphene [21–23], transition metal dichalcogenides [24], organic materials [25–26], and strongly spin-orbit coupled materials [27–28].

Recently, the playground of the spin current physics has been extended to disordered magnets or the so-called spin glass (SG) materials [29–30]. The SGs are characterized by a freezing of random spins [31], and its nature has long been studied both experimentally and theoretically [32,33]. However, despite its long history of research, the interplay of spin current and the SG ordering has not yet been well examined. Thus, it is quite natural to ask what happens if we inject a pure spin current into a SG material by the spin pumping. Experimentally, the spin pumping into a SG material was reported in 2011 using a Ag$_{90}$Mn$_{10}$/Ni$_{81}$Fe$_{19}$ bilayer [34]. To the best of our knowledge, however, no theoretical work on the spin pumping into SG materials can be found in the literature. Therefore, developing a theory of spin pumping into the SG material is highly desirable.

In this paper, we theoretically investigate the spin pumping into SG materials. Although a metallic magnet Ni$_{81}$Fe$_{19}$ was used as the spin injecting magnet in the previous experiment [34], we consider here an insulating ferromagnet such as yttrium iron garnet for the spin injector, since it makes the spin pumping signal more visible. Our strategy to calculate the spin pumping into SG materials is as follows. First, we use a linear-response approach to the spin pumping [35–37]. The notion derived from the linear response approach, that the spin pumping is intimately related to the dynamic spin susceptibility of the spin sink layer, has successfully been applied to the spin pumping into a ferromagnet [38], antiferromagnets [39–40], and recently it was also applied to the spin pumping into superconductors [41–42]. Thus, we relate the spin pumping signal to the dynamic spin susceptibility of the SG layer. Next, we calculate the susceptibility of the SG layer by employing a dynamic theory [43–46], but also that the dynamic theory is more suitable to discuss the dynamic quantity such as the dynamic spin susceptibility [47].

In the literature, the dynamic spin susceptibility near the SG transition was calculated [47], but the result was limited to an extremely low-frequency regime less than 10 KHz, which is out of the FMR condition. In the present paper, we extend the susceptibility calculation of Ref. [47] to the GHz frequency regime that is relevant to spin pumping experiments, and calculate temperature dependence of the spin pumping into SG materials. With this, we show that a characteristic peak structure appears near the SG transition, which is a consequence of the slowing down of spin fluctuations that is concomitant with the spin freezing of the system.

The plan of this paper is as follows. In the next section, we introduce our microscopic model, and relate the spin pumping with the dynamic spin susceptibility of impurity spins. In Sec. III on the basis of the dynamic theory of SGs, we explain how to calculate the dynamic spin susceptibility of impurity spins. In Sec. IV the spin pumping signal into a SG material is calculated as a function of temperature. Finally, in Sec. V we discuss and summarize our results.

II. MODEL

We consider a bilayer composed of a ferromagnetic insulator (FI) and a SG material, as shown in Fig. 1. More
concretely, we may think of yttrium iron garnet (YIG) for the FI layer and Mn-doped Cu (Cu:Mn) for the SG layer. We assume that a static magnetic field $H_0 = H_0\hat{z}$ is applied to the FI/SG bilayer in the lateral direction, and that the anisotropy field is much smaller than $H_0$ such that it can be discarded.

We start from the following Hamiltonian:

$$\mathcal{H} = \mathcal{H}_{\text{SG}} + \mathcal{H}_{\text{FI}-\text{SG}},$$

(1)

where the first term,

$$\mathcal{H}_{\text{SG}} = \sum_p \xi_p c_p^\dagger c_p + J_{\text{ES}} \sum_{r_a} \sigma(r_a) \cdot S(r_a),$$

(2)

describes the SG layer \[48, 49\]. Here, the first term on the right-hand side describes the conduction-electron kinetic energy, and the second term the coupling between the conduction-electron spin and magnetic impurity at an impurity position $r_a$, where $J_{\text{ES}}$ is the sd-type exchange coupling. Here, $c_p^\dagger = (c_{p,\uparrow}^\dagger, c_{p,\downarrow}^\dagger)$ is the electron creation operator for spin projection $\uparrow$ and $\downarrow$, $S$ is an impurity spin, $\sigma(r) = c^\dagger(r)\hat{\sigma}c(r)$ is the spin density operator with $\hat{\sigma}$ being the Pauli matrices, and $c(r) = N_{\text{SG}}^{-1/2} \sum_p c_p e^{i p \cdot r}$ with $N_{\text{SG}}$ being the number of lattice sites at the SG layer.

The second term of Eq. (1),

$$\mathcal{H}_{\text{FI}-\text{SG}} = J_{\text{int}} \sum_{\text{int}} \sigma(\text{int}) \cdot \Omega(\text{int}),$$

(3)

describes the interaction between the FI and SG layers. Here, $J_{\text{int}}$ is the interfacial $ad$ coupling between the conduction-electron spins in the SG and the localized spins in the FI, where $\text{int}$ is a position at the FI/SG interface.

In order to investigate the spin pumping in the present system, we use the linear-response formulation of the spin pumping \[34, 35\]. We consider the situation where an external microwave with the angular frequency $\omega_{\text{ac}}$ is applied to the FI/SG bilayer that drives the FMR of the FI side. The linear-response formulation uses the following magnon language. In the absence of the adjacent SG layer, the uniform-mode (Kittel mode) magnon has an intrinsic damping rate $\alpha_0\omega_{\text{ac}}$, where $\alpha_0$ is the intrinsic Gilbert damping constant. In the presence of the SG layer, since the spin-relaxation rate due to the SG layer is additive and hence an additional spin dissipation channel opens, there arises an additional magnon damping rate. Therefore, the total Gilbert damping constant $\alpha$ for the bilayer is given by

$$\alpha = \alpha_0 + \delta \alpha,$$

(4)

where $\delta \alpha$ is the additional Gilbert damping constant.

According to the linear-response formulation \[34, 35\], the additional magnon damping rate can be calculated from the corresponding magnon self-energy $\Sigma(\omega)$ whose process involves the spin transfer across the interface (Fig. 2). In the present situation, up to the lowest order with respect to $J_{\text{int}}$, the self-energy is given by

$$\Sigma(\omega) = -\frac{J_{\text{int}}^2 N_{\text{int}}}{\hbar^2 N_{\text{SG}} N_{\text{FI}}} \sum_q \chi_{\text{q}}^{\text{S}}(\omega) J_{\text{ES}} \chi_{\text{q}}^{\text{S}}(\omega) J_{\text{ES}} \chi_{\text{q}}^{\text{S}}(\omega),$$

(5)

where $N_{\text{int}}$ is the number of the localized spins $\Omega$ at the FI/SG interface, and $N_{\text{FI}}$ is the number of lattice sites in the FI layer. In the above equation, $\chi_{\text{q}}^{\text{S}}(\omega)$ is the Fourier transform of the retarded susceptibility of the conduction-electron spin $\sigma$, i.e., $\chi_{\text{q}}^{\text{S}}(t - t') = i\Theta(t - t') \langle [\sigma_{\text{q}}(t), \sigma_{\text{q}}(t')] \rangle$, where $\Theta(t)$ is the step function and we defined $O^\pm = O^\pm \pm i O^0$ for a vector operator $O$. By contrast, $\chi_{\text{q}}^{\text{S}}(\omega)$ is the Fourier transform of the retarded susceptibility of the impurity spin $S$, i.e., $\chi_{\text{q}}^{\text{S}}(t - t') = i\Theta(t - t') \langle [S_{\text{q}}(t), S_{\text{q}}(t')] \rangle$.

Using the relation $\delta \alpha = -\omega_{\text{ac}}^{-1} \text{Im} \Sigma(\omega_{\text{ac}})$ \[34\], the additional Gilbert damping constant $\delta \alpha$ is expressed as

$$\delta \alpha \approx \frac{J_{\text{int}}^2 N_{\text{int}}}{\hbar^2 N_{\text{FI}}} \left( \chi_{\text{O}}^{(\text{S})} J_{\text{ES}} \right)^2 \frac{1}{\omega_{\text{ac}}} \text{Im} \chi_{\text{loc}}^{(\text{S})}(\omega_{\text{ac}}),$$

(6)
where we introduced a shorthand notation $\chi^{(s)}_0 = \chi^{(s)}_{q=0}(\omega = 0)$, and $\chi^{(S)}_{\text{loc}}(\omega) = N^{-1} \sum_q \chi^{(S)}_q(\omega)$ is the local susceptibility of impurity spins. In obtaining the above result, we made use of the fact that $\chi^{(S)}_{\text{loc}}(\omega)$ is defined in a small $q$ region $q \lesssim 2\pi/b$ where $b$ is the average distance of two magnetic impurities. In this small $q$ region, the conduction-electron spin susceptibility is approximated by the uniform and static component $\chi^{(s)}_0$, where this quantity is pure real.

Equation (8) means that the additional Gilbert damping constant $\delta\alpha$ due to the spin pumping is proportional to the imaginary part of the dynamic spin susceptibility $\Im[\chi^{(S)}_{\text{loc}}(\omega)]$. In this expression, the strongest temperature dependence upon the SG transition results from $\Im[\chi^{(S)}_{\text{loc}}(\omega)]$. This means that, as long as the temperature dependence is concerned, the part other than $\Im[\chi^{(S)}_{\text{loc}}(\omega)]$ can be regarded as being temperature independent, and the temperature dependence is dominated by that of $\Im[\chi^{(S)}_{\text{loc}}(\omega)]$. We adopt this approximation in the numerical calculation in Sec. [LV].

The quantity $\chi^{(S)}_{\text{loc}}(\omega)$ is a correlation function between two impurity spins, and hence it can be evaluated using our knowledge on SGs. In the next section, we evaluate $\chi^{(S)}_{\text{loc}}(\omega)$ using a dynamic theory of SGs [42].

III. DYNAMIC SPIN SUSCEPTIBILITY OF IMPURITY SPINS

In this section, by employing a dynamic theory of SGs [42], we sketch our procedure for calculating the dynamic spin susceptibility $\chi^{(S)}_{\text{loc}}(\omega)$ appearing in the spin pumping signal [Eq. (6)]. In order to obtain a practical result for the present problem, we make the following simplifications. First, a vector spin $S(r_i)$ in Eq. (2) is replaced by Ising spin $S_S(r_i)$. This is justified because the spin precession in the spin sink layer is not important in the conventional spin pumping process [34]. Next, the transverse spin susceptibility $\chi^{(S)}_{\text{loc}}(\omega)$ appearing in Eq. (6) is approximated by the longitudinal one. We think this approximation does not bring any problems, because it is argued that the behavior of the transverse spin susceptibility is very similar to the longitudinal one (see the last paragraph of Ref. [50]).

With these assumptions in mind, after integrating out the conduction-electron degrees of freedom, the Hamiltonian for the SG layer $\mathcal{H}_{\text{SG}}$ is transformed into the following form [18,19]:

$$\mathcal{H}_{\text{SG}} = \frac{1}{2} \sum_{i \neq j} J_{ij} S(r_i) \cdot S(r_j),$$

(7)

where $J_{ij}$ is nominally the RKKY interaction of the form $J_{ij} = J_{\text{ES}} \cos(2\pi r_{ij}/r_0^3)$ with $r_{ij} = |r_i - r_j|$. However, following the standard approach to the SG problem [42], we regards $J_{ij}$ as Gaussian random variables with zero mean and variance $[J_{ij}]_\text{av} = J^2/N_S$, where $[\cdots]_\text{av}$ means the random average over the distribution of $J_{ij}$, and $N_S$ is the number of impurity spins.

Hamiltonian $\mathcal{H}_{\text{SG}}$ in Eq. (7) is the same as the Sherrington-Kirkpatrick (SK) model [41], so that we employ the established dynamical approach. Following Sompolinsky and Zippelius [42], we first replace $\mathcal{H}_{\text{SG}}$ with its soft-spin version:

$$\beta\tilde{\mathcal{H}}_{\text{SG}} = \frac{1}{2} \sum_{ij} (r_0 \delta_{ij} - \beta J_{ij}) S_i S_j - \sum_i h_i S_i,$$

(8)

where $\beta = 1/k_B T$ is the inverse temperature, and the length of the soft spin $S_i$ varies $-\infty < S_i < \infty$. Here, we consider a paramagnet/SG transition by ignoring any tendency to ferromagnetic order, such that $r_0$ in Eq. (8) is chosen to be a positive constant. Next, we introduce the Langevin dynamics

$$\Gamma_0^{-1} \partial_t S_i = -\frac{\partial (\beta\tilde{\mathcal{H}}_{\text{SG}})}{\partial S_i} + \xi_i(t),$$

(9)

where $\Gamma_0$ is the relaxation rate. In the above equation, $\xi_i(t)$ is a thermal noise represented by a Gaussian random variable with zero mean and variance $\langle \xi_i(t)\xi_j(t') \rangle_\xi = 2\Gamma_0^{-1} \delta_{ij} \delta(t-t')$,

(10)

where $\langle \cdots \rangle_\xi$ means the average over $\xi_i$. In the following calculation, the response function

$$G_{ij}(t-t') = \left[ \frac{\partial \langle S_i(t) \rangle_\xi}{\partial h_j(t')} \right]_{\text{av}}$$

plays an important role. This is because the spin susceptibility can be calculated by the relation

$$\chi_{ij}(\omega) = \beta G_{ij}(\omega),$$

(12)

where $G_{ij}(\omega)$ is the Fourier transform of $G_{ij}(t-t')$. Besides, we need to define the correlation function

$$C_{ij}(t-t') = \langle [S_i(t)S_j(t')] \rangle_{\text{av}}.$$  

(13)

Note that, as stated at the beginning of this section, we use the following relation:

$$\chi^{(S)}_{\text{loc}}(\omega) = \chi_{ii}(\omega).$$

(14)

To proceed further, we precisely follow the procedure of Ref. [42], which involves a lots of technical algebra. Since reviewing the details of Ref. [42] is beyond our scope, we leave it to the original paper and a famous textbook [41], and we briefly sketch the derivation of the self-consistent dynamical equation in the mean-field limit. First, we rewrite Eq. (9) in terms of a generating functional, and introduce an auxiliary field $\tilde{S}_i$ as was done by Martin, Sigia, and Rose [52]. Next, this generating functional is averaged over $J_{ij}$ without using replicas [53].
which generates temporally nonlocal quartic interactions among $S_i$ and $\tilde{S}_i$. Then, we introduce new auxiliary fields $Q_1, Q_2, Q_3$ and $Q_4$ to decouple the quartic terms, and we evaluate the functional integral by using the saddle-point approximation.

Following the above procedure, we obtain the new equation of motion for $S_i$ containing the local self-interaction, which in the frequency space is written as

$$-rac{i\omega}{\Gamma_0}S(\omega) = [-r_0 + \beta h(\omega) + (\beta J)^2G(\omega)]S(\omega) + \phi(\omega),$$

where $G(\omega)$ is the local response function $G_{ii}(\omega)$. Here and hereafter, the site index $i$ is discarded. In the above equation, $\phi$ is a new noise field satisfying

$$\langle \phi(\omega)\phi(\omega') \rangle = 2\pi\delta(\omega + \omega')[2\Gamma_0^{-1} + (\beta J)^2C(\omega)],$$

where $C(\omega)$ is the Fourier transform of the local correlation function $C_\theta(t-t')$, and $\langle \cdots \rangle_\phi$ is the average over $\phi(\omega)$. In the approach by Sompolinsky and Zippelius [42], the local correlation function is separated as $C(t) = q + \Delta C(t)$, where $q = C(t)|_{t\to\infty}$. In parallel with this separation, the noise field $\phi$ is divided into two parts,

$$\phi(\omega) = f(\omega) + z(\omega),$$

where the first term, $f(\omega)$, satisfies

$$\langle f(\omega)f(\omega') \rangle = 2\pi\delta(\omega + \omega') \left[2\Gamma_0^{-1} + (\beta J)^2\Delta C(\omega)\right]$$

with $\langle \cdots \rangle_f$ being the average over $f(\omega)$, whereas the second term, $z(\omega)$, satisfies

$$[z(\omega)z(\omega')] = (2\pi)^2\delta(\omega + \omega')\delta(\omega)(\beta J)^2q,$$

where $[\cdots]$ is the average over $z(\omega)$.

From Eqs. (18) and (19), we find that $f(\omega)$ is a usual thermal fluctuation, whereas $z(\omega)$ represents a frozen random field that breaks the ergodicity. Since $z$ acts as a static random field, Sompolinsky and Zippelius introduced the “unaveraged” response function over $z$:

$$g(\omega, z) = \frac{\partial(S(\omega))_f}{\partial h(\omega)}$$

where $g(\omega, z)$ is related to $G(\omega)$ through

$$G(\omega) = [g(\omega, z)]_z,$$

and, from Eq. (19), the average of a quantity $Q(z)$ over $z$ is given by

$$[Q(z)]_z = \frac{1}{\sqrt{2\pi(\beta J)^2q}} \int_{-\infty}^{\infty} dz e^{-\frac{z^2}{2(\beta J)^2q}}Q(z).$$

The Dyson equation for $g(\omega, z)$, which results from Eq. (15) with static random field $z$, is given by

$$g(\omega, z)^{-1} = G_0(\omega)^{-1} - \Sigma(\omega, z),$$

where the bare propagator is given by $G_0^{-1}(\omega) = r_0 - i\omega/(\Gamma_0 - (\beta J)^2G(\omega))$, and $\Sigma(\omega, z)$ is the self-energy coming from the frozen $z$-field. Since we are interested in the dynamic behavior of $G(\omega)$, we solve the Dyson equation by perturbation with respect to $\Delta G(\omega) = G(\omega) - G(0)$. We define $\eta(\omega, z) = i\omega/\Gamma_0 + (\beta J)^2\Delta G(\omega) - \Delta \Sigma(\omega, z)$, where $\Delta \Sigma(\omega, z) = \Sigma(\omega, z) - \Sigma(0, z)$, and rewrite the Dyson equation as $g(\omega, z)^{-1} = g(0, z)^{-1} - \eta(\omega, z)$. Then, after averaging over $z$, we obtain a quadratic equation for $\Delta G(\omega)$:

$$\Delta G(\omega) (1 - (\beta J)^2[g^2]_z) = \left(\frac{i\omega}{\Gamma_0} + (\beta J)^2\Delta G(\omega)\right)^2 [g^3]_z + \frac{i\omega}{\Gamma_0} [g^2]_z,$$

where we introduced the shorthand notation $[g^n]_z = [g(0, z)^n]_z$, and $\Delta \Sigma(\omega, z)$ was discarded as it brings only a small change [47]. Equation (24) can be solved for $\Delta G(\omega)$. After using the relation in Eq. (12), we obtain

$$\chi^{(S)}_{\text{loc}}(\omega) = \chi^{(S)}_{\text{loc}}(0) + \frac{\tilde{T}}{2[g^3]_z} \left(D - \sqrt{D^2 - \frac{4\omega}{\Gamma_0}[g^2]_z[g^3]_z}\right),$$

where $D = \tilde{T}^2 - [g^2]_z - 2i\omega[g^2]_z/\Gamma_0$, $\tilde{T} = T/T_g$, and $T_g = J/k_B$ is the SG transition temperature.

In the following calculation, we take the Ising limit as was done in Ref. [47]. In this case, $[g^2]_z$ is calculated from $[g^2]_z = [(1 - \langle S^2 \rangle^2)]$, where the quantity $\langle S^2 \rangle$ is given by

$$\langle S^2 \rangle = \frac{1}{\sqrt{2\pi(\beta J)^2q}} \int_{-\infty}^{\infty} dz e^{-\frac{z^2}{2(\beta J)^2q}} \tanh^{2s}(z).$$

At the SG transition, the Almeida-Thouless condition [54] holds, which in the present notation takes the form

$$[g^2]_z = 1 - \langle S^2 \rangle_4 = \tilde{T}^2.$$

Then, following Sompolinsky and Zippelius [42] we assume that Eq. (27) holds not only at $T_g$ but also below $T_g$, meaning that the SG phase is characterized by the marginal instability condition.

IV. RESULTS FOR SPIN PUMPING INTO SPIN GLASS MATERIALS

In this section, using the formalism developed in the previous two sections, we calculate temperature dependence of the spin pumping signal. The key equation in the present argument is Eq. (6), which relates the spin pumping with the dynamic spin susceptibility of SG materials.

Before presenting our results for the spin pumping that is intimately related to the dynamic spin susceptibility $\chi^{(S)}_{\text{loc}}(\omega)$, it is instructive to examine the static spin susceptibility $\chi^{(S)}_{\text{loc}}(0)$. This quantity can be calculated by
the Fischer relation \[54\]:
\[
\chi_{\text{loc}}^{(S)}(0) = \frac{1}{T}(1 - q).
\]

Figure 3 shows the static spin susceptibility \(\chi_{\text{loc}}^{(S)}(0)\) as a function of temperature, calculated from Eq. \(28\). The result reproduces the cusp structure at \(T_g\), but since we calculate the SG order parameter \(q\) by the marginal instability condition [Eq. \(27\)] instead of the Sherrington-Kirkpatrick equation \(q_{\text{SK}} = (2\pi)^{-1/2} \int_{-\infty}^{\infty} dz e^{-z^2/2} \tanh(\sqrt{q_{\text{SK}}/\beta J_z})\) \[44\], the decrease of \(\chi_{\text{loc}}^{(S)}(0)\) below \(T_g\) is slightly stronger than that of \[44\].

Now we discuss the spin pumping into a SG material. In discussing the spin pumping signal in the present case, the important parameter is the ratio of the microwave angular frequency \(\omega_{ac}\) to the relaxation rate of localized spins \(\Gamma_0\), i.e., \(\tilde{\omega}_{ac} \equiv \omega_{ac}/\Gamma_0\). Since the magnitude of \(\omega_{ac}\) under the FMR condition is of the order of 60 GHz, the parameter \(\tilde{\omega}_{ac}\) is determined by a material parameter \(\Gamma_0\). The spin relaxation time in a prototypical SG material Cu:Mn is reported to be of the order of picosecond (corresponding to \(\Gamma_0 \sim 10^3\) GHz) \[56\], and hence the parameter \(\tilde{\omega}_{ac}\) under the FMR condition is estimated to be \(\tilde{\omega}_{ac} \sim 0.1\). Note that the previous calculation of the dynamic spin susceptibility \[44\] was done for very low frequencies \(\omega_{ac} \lesssim 10^{-4}\), which is far out of the FMR condition.

Figure 4 shows temperature dependence of the spin pumping signal, calculated from Eqs. \(6\) and \(25\). First, we see that a clear peak structure appears at \(T_g\). Second, upon the increase of the parameter \(\tilde{\omega}_{ac}\), the height of the peak is reduced. This is because the peak structure originates from the critical slowing down of spins that develops on the verge of the spin freezing \[51\], so that the effects of the slowing down are more prominent when the paramagnetic state has a more rapid dynamics (i.e., larger \(\Gamma_0\)) in comparison to the spin frozen state.

As shown in Fig. 4, the peak structure is visible for a parameter region \(\tilde{\omega}_{ac} \lesssim 0.1\). Since the parameter \(\tilde{\omega}_{ac}\) for a prototypical SG material Cu:Mn is estimated about \(\tilde{\omega}_{ac} \sim 0.1\) \[56\] as mentioned in the last paragraph, we expect that we can observe a peak structure in the spin pumping signal near \(T_g\). Therefore, we propose a spin pumping experiment for a Cu:Mn/YIG system in order to test our theoretical prediction. Frequency \(\omega_{ac}\) dependence of the peak mentioned above, namely, the lower the frequency \(\omega_{ac}\), the higher the peak, is a key to identify the predicted signal.

V. DISCUSSION AND CONCLUSION

The main result of this paper is the theoretical prediction that the spin pumping into a SG material, whose signal is proportional to the additional Gilbert damping constant \(\delta\alpha\), can be enhanced near the SG transition. The physics behind this enhancement is explained in the following way. First, a SG material exhibits the critical slowing down upon the spin freezing \[51\], meaning that the spin relaxation rate of the SG material is reduced. Then, recalling that the spin pumping represents an additional damping of magnons in the spin injecting magnet, this type of enhancement in the spin pumping signal can be interpreted as a kind of the inverse of the motional narrowing \[57, 58\] as discussed in Ref. \[59\] (see Sec. VI therein). It means that a reduction of the spin relaxation rate in the spin sink material results in a broadening of the magnon damping in the adjacent magnet, leading to the enhancement of the spin pumping.

The spin pumping has an advantage that it can measure the dynamic spin susceptibility of a thin film sample.
So far, this fact has been applied to the spin pumping into ferromagnets [34, 36], antiferromagnets [37, 38], and superconductors [39–41]. Extending the same idea to SGs within the linear-response approach, we formulated the spin pumping into a SG material, and shown that the signal is expressed by using the local spin susceptibility of the SG material [Eq. (3)]. Moreover, the spin pumping into a SG material is predicted to exhibit a characteristic peak around $T_g$ (Fig. 1).

The height of the predicted peak in the spin pumping signal is controlled by the ratio of the microwave angular frequency to the relaxation rate of impurity spins, i.e., $\omega_{ac} = \omega_{ac}/\Gamma_0$. That is, a smaller $\omega_{ac}$ is better for an experimental detection of the peak. Conversely, it means that if the parameter $\omega_{ac}$ is too large, the peak is not visible. To test our theoretical prediction, we hope to experimentally detect such a spin pumping signal more visible owing to the smallness of the intrinsic Gilbert damping term $\alpha_0$.

Before conclusion, we briefly comment on the previous experiment of the spin pumping into a SG material using $\text{Ag}_{90}\text{Mn}_{10}/\text{Ni}_{81}\text{Fe}_{19}$ [33]. In that experiment, the SG transition temperature $T_g = 25$ K was measured around $g = 25$ K from the cusp in the susceptibility data of a thick $\text{Ag}_{90}\text{Mn}_{10}$ film. Note that this thick film is different from the thin film used for the spin pumping experiment. In the spin pumping experiment, a weak temperature dependence of the signal was measured around $T_g$, but no pronounced peak expected from the theoretical calculation (Fig. 3) can be seen. The reason could be either i) the important parameter $\omega_{ac} = \omega_{ac}/\Gamma_0$ is too large in $\text{Ag}_{90}\text{Mn}_{10}$ for the enhanced spin pumping to be detected (see Fig. 1), or ii) the SG transition temperature of the thin film sample is much lower than $25$ K, since the thick film may have lower $T_g$ than the thick film used to determine $T_g = 25$ K.

To conclude, we have theoretically examined the spin pumping into a SG material. We have shown that the temperature dependence exhibits a characteristic peak near the SG transition, whose height is controlled by the dimensionless angular frequency $\omega_{ac} = \omega_{ac}/\Gamma_0$. This is a consequence of the critical slowing down of spin fluctuations upon the spin freezing. Since the spin pumping has an advantage of being able to measure the spin dynamics of a thin film sample [39], we hope that the present theory stimulates further experiments of the spin pumping into SG materials.

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[1] S. Maekawa, H. Adachi, K. Uchida, J. Ieda, and E. Saitoh, Spin Current: Experimental and Theoretical Aspects, J. Phys. Soc. Jpn 82, 102002 (2013).
[2] I. ˘Zuti´c, J. Fabian, and S. D. Sarma, Spintronics: Fundamentals and applications Rev. Mod. Phys. 76, 323 (2004).
[3] Spin Current, edited by S. Maekawa, S. O. Valenzuela, E. Saitoh, and T. Kimura (Oxford University Press, Oxford, 2012).
[4] I. ˘Zuti´c and H. Dery, Taming spin currents, Nat. Mater. 10, 647 (2011).
[5] R. Urban, G. Woltersdorf, and B. Heinrich, Gilbert Damping in Single and Multilayer Ultrathin Films: Role of Interfaces in Nonlocal Spin Dynamics, Phys. Rev. Lett. 87, 217204 (2001).
[6] S. Mizukami, Y. Ando, and T. Miyazaki, Effect of spin diffusion on Gilbert damping for a very thin permalloy layer in Cu/permalloy/Cu/Pt films, Phys. Rev. B 66, 104413 (2002).
[7] Y. Tserkovnyak, A. Brataas, and G. E. W. Bauer, Spin pumping and magnetization dynamics in metallic multilayers, Phys. Rev. B 66, 224403 (2002).
[8] E. Saitoh, M. Ueda, H. Miyajima, and G. Tatara, Conversion of spin current into charge current at room temperature: Inverse spin-Hall effect, Appl. Phys. Lett. 88, 182509 (2006).
[9] O. Mosendz, J. E. Pearson, F. Y. Fradin, G. E. W. Bauer, S. D. Bader, and A. Hoffmann, Quantifying Spin Hall Angles from Spin Pumping: Experiments and Theory, Phys. Rev. Lett. 104, 046601 (2010).
[10] C. W. Sandweg, Y. Kajiwara, A. V. Chumak, A. A. Serga, V. I. Vasyuchika, M. B. Jungfleisch, E. Saitoh, and B. Hillebrands, Spin Pumping by Parametrically Excited Exchange Magnons, Phys. Rev. Lett. 106, 216601 (2011).
[11] F. D. Czeschka, L. Dreher, M. S. Brandt, M. Weiler, M. Althammer, I.-M. Imort, G. Reiss, A. Thomas, W. Schoch, W. Limmer, H. Huebl, R. Gross, and S. T. B. Goennenwein, Scaling Behavior of the Spin Pumping Effect in Ferromagnet-Platinum Bilayers, Phys. Rev. Lett. 107, 046601 (2011).
[12] V. Castel, N. Vlietstra, J. Ben Youssef, and B. J. van Wees, Platinum thickness dependence of the inverse spin-Hall voltage from spin pumping in a hybrid yttrium iron garnet/platinum system, Appl. Phys. Lett. 101, 132414 (2012).
[13] C. Hahn, G. de Loubens, O. Klein, M. Viret, V. V. Naletov, and J. Ben Youssef, Comparative measurements of inverse spin Hall effects and magnetoresistance in YIG/Pt and YIG/Ta, Phys. Rev. B 87, 174417 (2013).
[14] K. Ando, S. Takahashi, J. Ieda, H. Kurebayashi, T. Trypiniotis, C. H. Barnes, S. Maekawa, and E. Saitoh, Electrically tunable spin injector free from the impedance mismatch problem, Nat. Mater. 10, 655 (2011).
[15] J.-C. Rojas-Sanchez, M. Cubukcu, A. Jain, C. Vergnaud, C. Portemont, C. Ducruet, A. Barski, A. Marty, L. Vila, J.-F. Attane, E. Augendre, G. Desfonds, S. Gambarelli, H. Jaffres, J.-M. George, and M. Jamet, Spin pumping...
and inverse spin Hall effect in germanium, Phys. Rev. B 88, 064403 (2013).

[16] E. Shikoh, K. Ando, K. Kubo, E. Saitoh, T. Shinjo, and M. Shiraishi, Spin-pump-induced spin transport in p-type Si at room temperature, Phys. Rev. Lett. 110, 127201 (2013).

[17] P. Hyde, L. Bai, D. M. J. Kumar, B. W. Southern, C.-M. Hu, S. Y. Huang, B. F. Miao, and C. L. Chien, Electrical detection of direct and alternating spin current injected from a ferromagnetic insulator into a ferromagnetic metal, Phys. Rev. B 89, 184404(R) (2014).

[18] J. B. S. Mendes, R. O. Cunha, O. Alves Santos, P. R. T. Ribeiro, F. L. A. Machado, R. L. Rodriguez-Suárez, A. Azevedo, and S. M. Rezende, Large inverse spin Hall effect in the antiferromagnetic metal Ir$_2$Mn$_{80}$, Phys. Rev. B 89, 140406(R) (2014).

[19] H. Saglam, W. Zhang, M. B. Jungfleisch, J. Sklenar, J. E. Pearson, J. B. Ketterson, and A. Hoffmann, Spin transport through the metallic antiferromagnet FeMn, Phys. Rev. B 94, 140412(R) (2016).

[20] H. Wang, C. Du, P. C. Hammel, and F. Y. Yang, Anti-ferromagnetic Spin Transport from $\text{Y}_3\text{Fe}_5\text{O}_{12}$ to NiO, Phys. Rev. Lett. 113, 097202 (2014).

[21] A. K. Patra, S. Singh, B. Barin, Y. Lee, J.-H. Ahn, E. del Barco, E. R. Mucciolo, and B. Özyilmaz, Dynamic spin injection into chemical vapor deposited graphene, Appl. Phys. Lett. 101, 162407 (2012).

[22] Z. Tang, E. Shikoh, H. Ago, K. Kawahara, Y. Ando, T. Shinjo, and M. Shiraishi, Dynamically generated pure spin current in single-layer graphene, Phys. Rev. B 87, 140401(R) (2013).

[23] D. Khodadadi, J. B. Mohammadi, C. Mewes, T. Mewes, M. Manno, C. Leighton, and C. W. Miller, Enhanced spin pumping near a magnetic ordering transition, Phys. Rev. B 96, 054436 (2017).

[24] M. Inoue, M. Ichioka, and H. Adachi, Spin pumping into superconductors: A new probe of spin dynamics in a superconducting thin film, Phys. Rev. B 96, 024414 (2017).

[25] Y. Yao, Q. Song, Y. Takamura, J. P. Cascales, W. Yuan, K. Uchida, K. Sato, S. Okamoto, Y. Tserkovnyak, Z. Qiu, and E. Saitoh, Spin-current probe for phase transition in an insulator, Nat. Commun. 7, 12670 (2016).

[26] M. Umeh, Y. Shiomi, T. Kikkawa, T. Nizeki, J. Lustikova, S. Takahashi, and E. Saitoh, Spin-current coherence peak in superconductor/magnet junctions, Appl. Phys. Lett. 112, 232601 (2018).

[27] Y. Shimo, K. Nomura, Y. Kajiwara, K. Eto, M. Novak, K. Segawa, Y. Ando, and E. Saitoh, Spin-Electricity Conversion Induced by Spin Injection into Topological Insulators, Phys. Rev. Lett. 113, 196601 (2014).

[28] J. C. Rojas Sánchez, L. Vila, G. Desfonds, S. Gambarelli, J. P. Attané, J. M. De Teresa, C. Magén, and A. Fert, Spin-to-charge conversion using Rashba coupling at the interface between non-magnetic materials, Nat. Commun. 4, 2944 (2013).

[29] Y. Niimi, M. Kimata, Y. Omori, B. Gu, T. Ziman, S. Maekawa, A. Fert, and Y. Otani, Strong Suppression of the Spin Hall Effect in the Spin Glass State, Phys. Rev. Lett. 115, 196602 (2015).

[30] D. Wesenberg, T. Liu, D. Balzar, M. Wu, B. L. Zink, Long-distance spin transport in a disordered magnetic insulator, Nat. Phys. 13, 987 (2017).

[31] P. W. Anderson, in Ill-Condensed Matter, edited by R. Balian, R. Maynard, and G. Toulouse (North-Holland, Amsterdam, 1979).

[32] K. Binder and A. P. Young, Spin glasses: Experimental facts, theoretical concepts, and open questions, Rev. Mod. Phys. 58, 801 (1986).

[33] R. Iguchi, K. Ando, E. Saitoh, and T. Sato, Spin current study of spin glass AgMn using spin pumping effect, J. Phys.: Conf. Ser. 266, 012089 (2011).

[34] Y. Ohnuma, H. Adachi, E. Saitoh, and S. Maekawa, Enhanced dc spin pumping into a fluctuating ferromagnet near $T_C$, Phys. Rev. B 89, 174417 (2014).

[35] Y. Ohnuma, H. Adachi, E. Saitoh, and S. Maekawa, Magnon instability driven by heat current in magnetic bilayers, Phys. Rev. B 92, 224404 (2015).

[36] B. Khodadadi, J. B. Mohammadi, C. Mewes, T. Mewes, M. Manno, C. Leighton, and C. W. Miller, Enhanced spin pumping near a magnetic ordering transition, Phys. Rev. B 96, 054436 (2017).

[37] L. Frangou, S. Oyarzún, S. Auﬀert, L. Vila, S. Gambarelli, and V. Baltz, Enhanced Spin Pumping Efficiency in Antiferromagnetic IrMn Thin Films around the Magnetic Phase Transition, Phys. Rev. Lett. 116, 077203 (2016).

[38] Z. Qiu, J. Li, D. Hou, E. Arenholz, A. T. N’Diaye, A. Tan, K. Uchida, K. Sato, S. Okamoto, Y. Tserkovnyak, Z. Qiu, and E. Saitoh, Spin-current probe for phase transition in an insulator, Nat. Commun. 7, 12670 (2016).

[39] M. Inoue, M. Ichioka, and H. Adachi, Spin pumping into superconductors: A new probe of spin dynamics in a superconducting thin film, Phys. Rev. B 96, 024414 (2017).

[40] Y. Yao, Q. Song, Y. Takamura, J. P. Cascales, W. Yuan, Y. Ma, Y. Yun, X. C. Xie, J. S. Moodera, and W. Han, Probe of spin dynamics in superconducting NbN thin films via spin pumping, Phys. Rev. B 97, 224414 (2018).

[41] M. Umeh, Y. Shiomi, T. Kikkawa, T. Nizeki, J. Lustikova, S. Takahashi, and E. Saitoh, Spin-current coherence peak in superconductor/magnet junctions, Appl. Phys. Lett. 112, 232601 (2018).
[52] P. C. Martin, E. D. Siggia, and H. A. Rose, Statistical Dynamics of Classical Systems, Phys. Rev. A 8, 423 (1973).

[53] C. De Dominicis, Dynamics as a substitute for replicas in systems with quenched random impurities, Phys. Rev. B 18, 4913 (1978).

[54] K. H. Fischer, Theory of spin glasses: Susceptibility and specific heat in a magnetic field, Solid State Commun. 18, 1515 (1976).

[55] J. R. L. de Almeida and D. J. Thouless, Stability of the Sherrington-Kirkpatrick solution of a spin glass model, J. Phys. A 11, 983 (1978).

[56] F. Mezei, Neutron spin echo study of spin glass dynamics (invited), J. Appl. Phys. 53, 7654 (1982).

[57] R. Kubo, M. Toda, and N. Hashitsume, Statistical Physics II: Nonequilibrium Statistical Mechanics (Springer, 1991).

[58] C. P. Slichter, Principles of magnetic resonance (Springer, Berlin, 2010).

[59] T. Taira, M. Ichioka, S. Takei, and H. Adachi, Spin diffusion equation in superconductors in the vicinity of $T_C$ Phys. Rev. B 98, 214437 (2018).