Different applications such as laser plasma acceleration, colliders, and diffraction limited light sources require high gradient quadrupoles, with strength that can reach up to 200 T/m for a typical 10 mm bore diameter. We present here a permanent magnet based quadrupole (so-called QUAPEVA) composed of a Halbach ring and surrounded by four permanent magnet cylinders. Its design including magnetic simulation modeling enabling to reach 201 T/m with a gradient variability of 45%, and mechanical issues are reported. Magnetic measurements of seven systems of different lengths are presented and confirmed the theoretical expectations. The variation of the magnetic center while changing the gradient strength is $\pm 10 \mu m$.

A triplet of QUAPEVA magnets is used to efficiently focus a beam with large energy spread and high divergence that is generated by Laser Plasma Acceleration source for a free electron laser demonstration, and has enabled us to do beam based alignment and control the dispersion of the beam.

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I. INTRODUCTION

Accelerator physics and technology have recently seen tremendous developments. For example, colliders aim at beam focus at nanometer size scale for high energy physics applications, and thus require strong quadrupolar fields. The domain of synchrotron radiation is actively investigating low emittance storage rings (picometer scale) with multi-bend achromat optics for getting closer to the diffraction limit and providing a high degree of transverse coherence, for which high gradient quadrupoles with a small harmonic content is one of the issues. In addition, Laser Plasma Acceleration (LPA) can now generate a GeV beam within a very short accelerating distance (few centimeters), with high peak current $\sim 10 \text{ kA}$, but with high divergence (few mrad) and large energy spread (few percent). All these applications have the requirement for high gradient tunable quadrupoles, for example the electron beam produced by LPA needs quadrupoles with gradient as large as 200 T/m to handle its high divergence.

Permanent Magnet Quadrupoles (PMQs) achieve high gradient with compactness and with the absence of power supplies, letting them to be a solution for future sustainable green society. Several Halbach ring based PMQs with fixed gradient were designed and built: at Fermilab; at CESR; at Kyoto University / SLAC; at CORNELL; at the department für Physik; at ESRF. Various original designs were proposed and developed for the PMQ to provide a variable gradient, such as at SLAC / Fermilab collaboration (13 - 115 T/m gradient, 7 - 68.7 T integrated gradient with a bore radius of 6.5); at Kyoto U. / SLAC collaboration (17 - 120 T/m gradient with a bore radius of 10 mm and length of 230 mm);
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at STFC Daresbury Laboratory / CERN for the CLIC project (15 - 60.4 T/m gradient for a 241 mm length and 2.9 - 43.8 T/m for 194 mm length, with a bore radius of 13.6 mm); a "super hybrid quadrupole" concept (28 T/m gradient, 7 T integrated gradient with a bore radius of 27.5 mm and 30% variability); at SLAC for the next linear collider (up to 141 T/m gradient, 0.6-138 T integrated strength, and 324 - 972 mm effective lengths with a bore radius of 12.7 mm).

In this paper, a hybrid permanent magnet based quadrupole of variable strength (QUAPEVA) developed at Synchrotron SOLEIL is presented. It provides a high gradient strength (201 T/m), wide tunability (~45%), small magnetic center excursions (±10 µm), and compactness (bore radius of 5 mm). Simulation models and the mechanical design are presented. Seven systems of different integrated gradients have been built, two sets of triplets with magnetic lengths of "26 mm, 40.7 mm, 44.7 mm" and "47.1 mm, 66 mm, 81.1 mm", as well as a prototype of magnetic length 100 mm. Magnetic measurements using two different methods are shown. Finally, three QUAPEVAs have been used for the COXINEL project aiming at demonstrating LPA based Free Electron Laser amplification, making them one of the first tunable permanent magnet based quadrupoles installed in an accelerator line, and allowed us to control and shape the electron beam along an 8 m long transport line, and to actually observe undulator radiation.

II. QUAPEVA CONCEPT

Let's consider the local field \( B(x, y, s) \) in a quadrupole, with \( x \) (resp. \( y \)) the horizontal (resp. vertical) direction, and \( s \) the longitudinal axis. For an infinitely long magnet, the complex induction \( B(z) = B_y + iB_x \) with \( z = x + iy \), the vertical \( B_y \) and horizontal \( B_x \) components, can be expressed as:

\[
B(z) = \sum_{n=1}^{\infty} \left( B_n + iA_n \right) \frac{z^n}{r_0^n},
\]

with \( n \) the multipolar order, \( B_n \) and \( A_n \) are the normal and skew multipolar coefficients, \( r_0 \) the radius for which coefficients are computed or measured. "Normalized" components \( a_n = 10^4 A_n / B_2 \) and \( b_n = 10^4 B_n / B_2 \). For a perfect normal quadrupole (\( n=2, A_2=0 \)), the complex induction becomes:

\[
B_y + iB_x = B_2 \frac{x + iy}{r_0}.
\]

A realistic quadrupole contains higher order multipoles resulting from the structure, magnets, or mechanical assembly imperfections.

The QUAPEVA is made of two entangled quadrupoles. A central one following a Halbach hybrid structure constituted of four \( Nd_2Fe_{14}B \) PMs and four Iron-Cobalt alloy magnetic poles. An outer one composed of four PM cylinders with a radial magnetic moment orientation, each connected to a motor producing a variable gradient by the rotation around their axis. Four Iron-cobalt alloy plates are placed behind the PM cylinders in order to maintain the magnetic flux within the outer diameter to increase the gradient in the quadrupole aperture. The magnetic system is inserted into a dedicated Aluminum support frame in order to maintain the magnetic elements in their positions due to the strong generated magnetic force.

Fig. 1 presents three particular configurations of the tuning magnets: (a) maximum gradient: tuning magnets easy axis towards the central magnetic poles, (b) intermediate gradient: the tuning magnets are in the reference position, i.e. their easy axis is perpendicular to the central magnetic poles, (c) minimum gradient: tuning magnets easy axis away from the central magnetic poles.

III. QUAPEVA DESIGN

The QUAPEVA specifications have been defined according to LPA beam transport in the COXINEL case. QUAPEVAs should be compact (6 mm bore radius) and adequate to vacuum environment, have magnetic lengths from 26 mm up to 100 mm, good magnet quality to ensure high remanence and coercivity, and guaranty a high gradient \( G \geq 100 \) T/m with a large tunability \( \geq 30\% \), alongside small harmonic components \( b_6 / b_2 \leq 3\% \).
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FIG. 1. Scheme of the QUAPEVA: Permanent magnet blocks (Blue) and rotating cylinders (Red), Vanadium Premendur magnetic plates (Green) and poles (Orange), Aluminum support frame (Grey). (a) maximum, (b) intermediate, and (c) minimum gradient.

\[ b_{10}/b_2 \leq 1.5\% \] Motors also should handle the magnetic forces induced by the magnetic system.

Two numerical tools are used to optimize the geometry and magnetic parameters of the QUAPEVAs: RADIA\textsuperscript{21} a magnetostatic code based on boundary integral method (Fig. 2-a); TOSCA\textsuperscript{22} a finite element magnetostatic code (Fig. 2-b), using non-linear steel properties from induction versus magnetic field data. The tuning magnets magnetization angles are parameterized in order to simulate the gradient tuning and check the gradient range.

FIG. 2. (a) RADIA model, (b) TOSCA model.

FIG. 3. Simulations of the prototype gradient evolution versus tuning magnets angle at the longitudinal center of the QUAPEVA, with (△) TOSCA and (□) RADIA. (Line) sinus fit. Remanent field: 1.26 T, coercivity: 1830 kA/m, pole saturation: 2.35 T.

Fig. 3 shows the simulated gradient evolution computed at a 4 mm radius while rotating the tuning magnets from 0 to \(2\pi\) radians by the same angle from their reference position, where the gradient reaches a maximum and a minimum value for a complete rotation. The simulation results of the two models are in good agreement. The evolution is fitted with a sinus function \(G(\theta) = G_0 + G_t \sin(\theta)\), where \(G_0\) is the fixed gradient of the main magnets, \(G_t\) the gradient contribution of all the tuning magnets, and \(\theta\) their corresponding angle. The gradient variation from peak to peak is \(\sim 90\) T/m and the maximum gradient reaches \(\sim 201\) T/m with the prototype one (\(l= 100\) mm).
Table I gives the prototype simulated field gradient and multipole components in the intermediate case (Fig. 1-b), alongside measurement results (cf section IV).

|          | RADIA | TOSCA | RC  | SSW |
|----------|-------|-------|-----|-----|
| $G_0$ (T/m) | 164.5 | 164.4 |
| $\int B_2 dl$ (T.m) | 0.0658 | 0.06576 | 0.06324 | 0.0627 |
| $b_6$ | 202 | 199 | 237 | 247 |
| $b_{10}$ | -158 | -152 | -133 | -138 |

TABLE I. Normalized first order multipoles and gradient results computed and measured at 4 mm radius for the prototype (tuning magnets at their reference position). RC: Rotating Coil, SSW: Single Stretched Wire.

The chosen motors (HARMONIC DRIVE, FHA-C mini motors) have sufficient torque to counteract the magnetic forces and are very compact (48.5 x 50 x 50 mm$^3$). Each tuning magnet is connected to one motor allowing a precise positioning of each magnet that minimizes the magnetic center shift at different gradients. The magnetic system is mounted on an Aluminum frame. A non-magnetic belt transmits the rotation movement from the motor to the cylindrical magnets. The quadrupole is supported by a translation table (horizontal and vertical displacement) used to compensate any residual magnetic axis shift when varying the gradient. Fig. 4 shows the resulting mechanical design (left), and an assembled QUAPEVA on the translation table (right).

IV. MAGNETIC MEASUREMENTS OF QUAPEVA

Two different magnetic measurements are performed to characterize the quadrupoles. A dedicated radial rotating coil was built for the SOLEIL magnet characterization bench$^{23}$, and a stretched-wire bench developed at Laboratoire de l’Accélérateur Linéaire (LAL).

A linear model is built taking into account the contribution of the inner and outer quadrupoles, and considering that the saturated steel behaves as permanent magnets. The main multipole $B_2$ becomes: $B_2 = B_2^{eq} + \sum_{k=1}^{4} B_2^k \sin(\theta^k + \phi B_2^k)$ where $B_2^{eq}$ is the main magnet contribution, $B_2^k$ the contribution of the $k^{th}$ tuning magnet number, $\theta^k$ its angle and $\phi B_2^k$ the multipolar phase shift. Not considering the harmonic dependence with the tuning angles of the cylinders, the tuning magnet angle for a given gradient can be computed using: $\theta_k = \sin^{-1}\left(\frac{(B_2^{eq} - B_2^k)}{4B_2^k}\right) - \phi B_2^k$ where $B_2^{eq}$ is the required normal quadrupolar term. Applying this modeling, one can then measure the gradient change of one QUAPEVA while the different cylindrical magnets are rotated simultaneously, as shown in Fig. 5-a ($G = \frac{\int B_2 dl}{R}$), where $R$ is the radius of the measured field region. Measurements with rotating coil and stretched wire are in good agreement and correspond to the expectations from the RADIA and TOSCA models (see Table I). Fig. 5-b, c shows however that the gradient variation leads to a harmonic excursion about 20% of the average value.
A crucial aspect for the operation of the QUAPEVA for practical use is to maintain the evolution of the magnetic center as small as possible. The mechanical design anticipated a residual evolution to be compensated by applying proper feed-forward tables deduced from the magnetic measurements on the horizontal and vertical position of the translation stages. Different measurements of the magnetic center versus gradient were carried out. Starting from a first reference position of the maximum gradient case and by rotating the PM cylinders in the opposite direction to maintain the symmetry, the magnetic center excursions (horizontal and vertical) versus gradient are measured (Fig. 6) using a rotating coil and a single stretched wire. The change of the magnetic axis is kept within typically \(\pm 10 \, \mu m\).

Fig. 7 shows the systematic 2\(^{nd}\), 6\(^{th}\), and 10\(^{th}\) multipole terms computed by the models and calculated by the measurements. Indeed they present very good agreement with a difference less than 1%.

V. APPLICATION TO COXINEL

The QUAPEVAs are the first tunable permanent magnet based quadrupoles that have been installed and commissioned in a beam transport line. A first triplet (26 mm, 40.7 mm, 44.7 mm magnetic length) is installed in-vacuum, right after the gas jet where the electrons are produced, for focusing the electron beam produced by the LPA source at Laboratoire d’Optique Appliquée (LOA). The pulsed wire measurement have been used for QUAPEVA alignment. A beam observation on the first screen is shown in Fig. 8, (a) without and (b) with the first QUAPEVA installed 5 cm away from the electron source. The large divergence of the electron beam (\(~5\) mrad) is properly controlled and focused, leading to a beam size reduction from \(\sigma_x(\sigma_z) = 4.6 (2.8)\) mm to \(\sigma_x(\sigma_z) = 1.8 (1.1)\) mm RMS and enabling transport through the beam line.
VI. CONCLUSION

The design of a permanent magnet based quadrupole of high gradient strength ($\sim 201$ T/m) with a wide tuning range ($\sim 90$ T/m) have been presented. These results are confirmed by different magnetic measurements. The residual excursion of the magnetic center has been limited to a $\pm 10$ $\mu$m range thanks to an appropriate choice of the reference position and rotation direction of the cylindrical magnets. Three QUAPEVAs have been installed successively at COXINEL beam line, and are able to achieve good focusing with a highly divergent large energy spread beam. The gradient could be enhanced $\sim 30\%$ by integrating a cooling system\textsuperscript{25} at liquid nitrogen temperature and using $Pr_2Fe_{14}B$ PMs\textsuperscript{26}. Besides, a design with a hyperbolic shape\textsuperscript{27} would enable to reduce the multipole content.
in compromising on the gradient variability, is of great interest for low emittance storage rings.

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