Investigation on 3D Fractal Dimension as Complexity Parameter of Interconnected Pore in 3D Porous Media

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Abstract. 3D Fractal dimension as a complexity parameter of interconnected pore in 3D porous media is investigated. Four sets of 3D digital porous media with the size of 150x150x150 pixels are constructed by randomly depositing spherical grains with radius in the range of 10-20 pixels in an empty cube. Fractal dimension is obtained by means of the box counting method. Interconnected pore in 3D porous media are identified by applying cubical random walk in three different directions. The complexity of the interconnected pore is then described means of tortuosity, which is defined as the ratio between geodesic lengths of the interconnected pore to the sidelength of the medium. To observe the consistency of 3D fractal dimension as complexity parameter of 3D porous media structure, the four sets of samples of 3D porous media were generated with four different porosity. The correlation coefficient obtained from the samples are 0.1999, 0.3962, 0.9012 and 0.5682. Based on the results 3D fractal dimension shows a positive correlation with average tortuosity for each different porosity. It indicates that the higher 3D fractal dimension for 3D porous media, the interconnected pore of the porous media is more complex.

1. Introduction

To identify the complexity of fluid flow in porous rocks usually requires experiments, which are commonly complicated and costly. Digital image analysis can be used to characterize the pore structure of rocks [1-10] as an alternative to the experiment-based measurements. There are several important physical parameters of rock such as permeability, tortuosity and porosity. Due to its various advantages, tortuosity of rock can be used as parameters to describe the complexity as well as the anisotropy degree of rocks. Relationship of these physical parameters can be analyzed by using computer modeling. A fractal rock model has previously been developed by Feranie [4] and Tobing [5]. Several other studies on porous media modeling have also been done by Sun [6] and Teo [7]. Digital image analysis is conducted to calculate the fractal dimension and tortuosity of rock in x, y and z direction. Different rock models with the same porosity does not necessarily produce the same tortuosity, therefore porosity cannot be used as a parameter to determine the fluid flow complexity in porous rocks.

Several studies of the fractal dimension have been developed by Cowan [8] who examined fractal dimension as a petrophysical parameter. In medicine, fractal dimension has also been used by Hentschel [9] who examined fractal dimension as imaging parameters of CT scans in anorexia nervosa patients before and after therapy.
This study investigates the fractal dimension as a parameter to determine the complexity of interconnected pore in porous rocks. The fractal dimension and its flow in the geological rock pore structure had previously been investigated by Selly [10]. The present study examines the relationship of fractal dimension to the complexity of interconnected pore.

2. Sample and Method

The 3D porous rock model used in this study were with four different porosity. 3D models of porous rocks were generated by distributing 10-20 pixel-sized grains randomly on a 3D model of 150 × 150 × 150 until the desired porosity is reached. An example of 3D visual of the constructed model is shown in Figure 3. The complexity of the models was then analyzed using tortuosity and fractal dimension.

In order to analyze the complexity of the models, the interconnected pore needs to be identified and traced. The algorithm for tracing the pore is the cubical full random walk method, which was developed by Fauzi and Ariwibowo [11]. The cubical full random walk method is a method used for tracing the interconnected pore element using the nearest neighborhood connection. In this method, the neighborhood connection uses a 27 pixel element where the tracer has 26 walking direction as shown in Figure 2. The tracing path algorithm that used in this study is the flow-based priority, where the tracer is considered as a single element of fluid flowing through the interconnected pore.

In this study, we use the definition of tortuosity as the ratio of the interconnected path length ($L'$) with the sidelenath of the flow direction $L$, which can be calculated as follows:

$$\tau = \frac{L'}{L}.$$  (1)
Tortuosity is estimated directly from image and can be calculated by tracing the interconnected pore in a particular direction (x, y and z directions).

For the fractal dimension, the measurement of 3D fractal object is based on the notion of the dimension of the Hausdorff-Besicovitch fractal as follows:

\[ V = N(r) r^{D_{3D}}. \]  

(2)

where \( r \) is the side length of the cube, \( N(r) \) is the number of sub-cube that covers a fractal object and \( D_{3D} \) is the fractal dimension. The fractal dimension itself is the dimension of a fractal object that has non-integer dimension. Based on Equation (2), the fractal dimension can be calculated using Equation (3).

\[ D_{3D} = \frac{\Delta \ln N(r)}{\Delta \ln r} \]  

(3)

3. Results and Discussions

The visualization of the porous rock model and the trajectory of the interconnected pore as traced by the full cubical random walk method can be observed in Figure 3. It can be seen that the large tortuosity value indicates the complexity of interconnected pore. From the five sets of samples generated with different porosities, the tortuosity and fractal dimension for each models are shown in Table 1. The average tortuosity is the average from the tortuosity of the three axes in the x, y and z directions in each sample.

(a) Sample 1, porosity 30%, average tortuosity 7,0134 and fractal dimension 2,1229

(b) Sample 2, porosity 30%, average tortuosity 4,8696 and fractal dimension 2,1158
Figure 3. The 3D porous rock model (a) – (e) shows the trajectory of the interconnected pore in y direction.
Table 1. The tortuosity and fractal dimension in each model.

| Porosity | Sample | x-Direction | y-Direction | z-Direction | Average tortuosity | Fractal Dimension |
|----------|--------|-------------|-------------|-------------|--------------------|-------------------|
|          | Sample 1 | 10.0354     | 1.6568      | 24          | 11.9266             | 2.097             |
| 20%      | Sample 2 | 1.0912      | 2.0502      | 2.1561      | 1.7658              | 2.096             |
|          | Sample 3 | 18.3934     | 5.7991      | 16.8088     | 13.6671             | 2.1106            |
|          | Sample 4 | 2.6161      | 5.7026      | 1.3035      | 3.2074              | 2.1101            |
|          | Sample 5 | 4.1181      | 4.1396      | 1.4833      | 3.247               | 2.0987            |
|          | Sample 1 | 5.8972      | 8.858       | 8.0629      | 7.6060              | 2.0974            |
| 25%      | Sample 2 | 5.9581      | 7.7094      | 3.8213      | 5.8296              | 2.1004            |
|          | Sample 3 | 3.5628      | 6.7496      | 1.5748      | 3.9624              | 2.0989            |
|          | Sample 4 | 9.6663      | 4.1355      | 7.3126      | 7.0381              | 2.1135            |
|          | Sample 5 | 3.678       | 3.9643      | 6.5975      | 4.7466              | 2.0975            |
|          | Sample 1 | 10.6147     | 2.2511      | 8           | 7.0134              | 2.1229            |
| 30%      | Sample 2 | 3.805       | 6.5298      | 4.274       | 4.8696              | 2.1158            |
|          | Sample 3 | 3.4888      | 4.3102      | 5.3327      | 4.3772              | 2.1059            |
|          | Sample 4 | 3.8591      | 5.0163      | 3.2452      | 4.0402              | 2.0985            |
|          | Sample 5 | 4.181       | 2.052       | 1.8159      | 2.6829              | 2.0995            |
|          | Sample 1 | 4.2297      | 4.9836      | 1.7471      | 3.6534              | 2.1063            |
| 35%      | Sample 2 | 1.5609      | 4.7871      | 2.0816      | 2.8098              | 2.1               |
|          | Sample 3 | 12.2709     | 2.4344      | 4.8927      | 6.5326              | 2.0978            |
|          | Sample 4 | 4.7962      | 3.233       | 6.5313      | 4.8535              | 2.0983            |
|          | Sample 5 | 7.2788      | 7.9472      | 5.8929      | 7.0396              | 2.1522            |

The correlation between fractal dimension and tortuosity from each sample sets with different porosity were analyzed from the correlation coefficient which are listed in Table 2. Based on these results we can conclude that the correlation value showed a positive result. It can be identified that the fractal dimension of the porous rock model is directly proportional to the complexity of interconnected pore of the model, which is represented by the tortuosity.

Table 2. The correlation values from each type of sample with different porosity value.

| Porosity of sample | Correlation |
|--------------------|-------------|
| 20%                | 0.1999      |
| 25%                | 0.39626     |
| 30%                | 0.90121     |
| 35%                | 0.56821     |

4. Summary

The correlation values obtained from the samples with the porosity values are 20%, 25%, 30% and 35% respectively are 0.1999, 0.3962, 0.9012 and 0.5682. Based on the result 3D fractal dimension shows a positive correlation with average tortuosity in each porosity. It indicates that the higher 3D fractal dimension for 3D porous media, the interconnected pore of the porous media is more complex. Based on these result 3D fractal dimension can describe the complexity of interconnected pore in 3D porous media. Based on the results of the analysis and discussion can be concluded that the fractal dimension can be used as parameters of the complexity of interconnected pore in 3D porous rock model.
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