Checking Race Freedom of Clocked X10 Programs

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Abstract
One of many approaches to better take advantage of parallelism, which has now become mainstream, is the introduction of parallel programming languages. However, parallelism is by nature nondeterministic, and not all parallel bugs can be avoided by language design. This paper proposes a method for guaranteeing absence of data races in the polyhedral subset of clocked X10 programs.

Clocks in X10 are similar to barriers, but are more dynamic; the subset of processes that participate in the synchronization can dynamically change at runtime. We construct the happens-before relation for clocked X10 programs, and show that the problem of race detection is undecidable. However, in many practical cases, modern tools are able to find solutions or disprove their existence. We present a set of benchmarks for which the analysis is possible and has an acceptable running time.

1. Introduction
Driven by the limitations of current micro-architecture technologies, parallel computing has gone mainstream. Parallel programs are now required to utilize the massive amount of parallelism provided by multi-core and many-core architectures. Efficiently parallelizing programs by hand requires significant effort, and the programmers must “think parallel”. However, automatic parallelization is extremely difficult, and has seen limited success so far.

As an alternative approach, many parallel programming languages are being developed [8, 17, 25]. These languages aim for both high performance and high productivity by employing new programming models. Parallel programming is inherently more difficult than sequential programming, especially when it comes to debugging. This is due to the non-deterministic nature of parallelism that brings a new set of bugs that are not consistently reproducible.

There are a number of parallel debuggers (e.g., 1, 14, 15, 24) to help detecting parallel bugs, but their use is time consuming, and give no guarantees. We seek to complement dynamic debuggers by statically analyzing parallel programs and providing guarantees.

Specifically, we target the X10 parallel programming language [17]. X10 provides (logical) deadlock-free guarantee for programs written following a small set of rules. However, data race is another class of common parallel bugs that remains to be detected. Static analyses of X10 programs for data race detection have been proposed [1, 21, 26], but are currently limited to programs without clocks.

Clock is a synchronization mechanism in X10 that is similar to barriers. However, it is more dynamic, meaning that the processes participating in a synchronization can dynamically change. For example, consider the following:

//Process 1
sync;
S0;
sync;

//Process 2
sync;
S1;

The program above depicts an abstracted view of two parallel processes synchronizing through calls to sync statements. If the sync statement corresponds to MPI-style barrier, there is a deadlock, because the two processes do not call the barrier the same number of times. In X10, Process 2 will be removed from the set of processes participating in the synchronization when it reaches its termination, and therefore the above does not cause a deadlock. This dynamic behavior of X10 clocks invalidates static analyses developed for MPI-style barriers [21, 13, 27].

In this paper, we present static race detection for X10 programs that can provide race-free guarantees for regions of programs amenable to our analysis. Race-free guarantee of program sub-regions, and the ability to detect parallel bugs at compile-time both contribute to reduce the debugging effort, and hence leads to increased productivity. The main limitation of our approach is that it is polyhedral; loop bounds and array accesses must be affine. This class of programs can be represented using the polyhedral model, a mathematical framework for reasoning about program transformations [11]. Although the applicability of the model is limited, it has been proven to be effective in automatic parallelization.

Specifically, we extend the work by Yuki et al [26] by extending the subset to all of its core parallel constructs; async, finish, and clocks. We retain the main strength of their work that the analysis is statement instance-wise, and array element-wise. Instance-wise analysis provides information at the granularity of statement instances: the execution of a statement for a specific value of the loop iterators. Element-wise analysis distinguishes array accesses at the granularity of array elements, so that accesses are flagged as in conflict only when the same element is accessed.

Our key contributions are:

- Extension of the operational semantics to include clocks. The “happens-before” relation due to clocks is defined. Intuitively, the happens-before relation requires to count how many times a process has synchronized in its execution trace.
- Formulation of the happens-before relation with clocks in a polyhedral context. The number of times a process has synchronized at a given statement instance can be formulated as counting integer points in a union of polyhedra.
- Proof of undecidability of race detection for clocked X10 programs. In the general case, the synchronization counts produce
polynomials. Thus, comparing two statement instances with the extended happens-before relation turns out to be undecidable.

- Race-free guarantee of clocked X10 programs by disproving all possible races. Since the happens-before relation involves polynomials in the general case, Integer Linear Programming commonly used in the polyhedral context cannot be used. Instead, we formulate the race condition as a constraint satisfaction problem and resort to advanced SMT solvers to disprove the existence of potential races.

- Prototype implementation of the above.

The rest of the paper is organized as follows. We introduce the subset of X10 we use in this paper in Section 2. We review the work of Yuki et al. [26] that we extend upon in Section 3. We develop the operational semantics and define the happens-before relation of clocked X10 programs in Section 4. We establish the connection between the semantics and polyhedral analysis and present the data race detection formulation in Section 5. We demonstrate our approach with examples in Section 6 and present our implementation in Section 7. Related work is discussed in Section 8 and we conclude in Section 9.

2. The X10 Subset

The subset of X10 [17] is considered in a related work [26] that deals with X10 programs without clocks is the following:

- Sequence (S T): Composes two statements in sequence.
- Sequential for loop: All loops have an associated loop iterator. X10 loops may scan a multidimensional iteration space, but such loops are expanded.
- async: The body of the async is executed by an independent lightweight thread, called activity in X10.
- finish: The activity executing a finish waits for all activities spawned within the body of finish to terminate before proceeding further.

The details of the leaf statements are not relevant; the only required information is the array (elements) that are read and written.

Additionally, the program must fit the polyhedral model. The polyhedral model requires loop bounds, and array access to be affine expressions of the surrounding loop indices and program parameters (run-time constants).

2.1 X10 Clocks

Clocks are generalization of barriers for dynamically varying sets of activities (X10 threads). Activities can be registered to clocks, and may synchronize by calls to advance statements that blocks until all activities registered to the clock executes an advance. X10 allows for the set of registered activities to dynamically change.

One subtle but important difference with global barriers is that if all but one activity had reached an advance, and the last activity reaches its termination, the activity is de-registered from its clocks, allowing the remaining activities to proceed.

2.2 Explicit and Implicit Clocks

X10 has two different syntaxes for clocks: explicit and implicit clocks. In the explicit syntax, clocks are represented as objects in the program. The programmer has fine grain control over when to synchronize on a particular clock, and can even de-register an activity from some clocks.

However, the use of the explicit syntax is not necessary in the majority of the cases. For instance, we were able to rewrite a significant set of benchmarks (see Section 2.2) using only implicit clocks. Explicit syntax can lead to deadlocks, although it can easily be avoided by following simple rules [17], and needs a difficult points-to analysis for its verification. Thus, programmers are strongly encouraged to use the alternative syntax: implicit clocks.

2.3 Implicit Clock Syntax

In implicit clock syntax both finish and async may be annotated with an optional keyword clocked. The clocked variants must satisfy the following:

- clocked async must be enclosed by clocked finish or clocked async
- clocked async must not be enclosed by unclocked async or unclocked finish
- advance must be enclosed by clocked finish
- advance must not be enclosed by unclocked async

and has the following semantics:

- clocked finish creates a clock and registers itself to it,
- clocked async registers itself to the clock created by its governing clocked finish,
- clocked finish de-registers itself from the clock it created when reaching the end of its enclosed statement,
- clocked async de-registers itself from its clock when reaching the end of its enclosed statement, and
- advance synchronizes with the clock created by its governing clocked finish.

where the governing clocked finish is the first clocked finish found when traversing the AST towards the root from a node.

2.4 Advance Counts

Informally, the clock may be considered to have as many associated counters as there are registered activities. When all participating activities execute an advance, synchronization takes place, and all counters are incremented. The statements that may-happen-in-parallel are restricted to those that are executed when the value of the counters match. This notion is formalized in Section 4.2.

2.5 Example

Figure 1 illustrates simple uses X10 clocks using implicit syntax. Note the dynamic changes in the activities participating to the clock. In Figure 1a all activities blocks until the primary activity reaches the end of finish. The number of participants gradually decreases, since they do not have the same trip count. Figure 1e is an example of the opposite behavior, where the number of participants gradually increases.

3. Array Dataflow Analysis for Polyhedral X10 Programs

Our work is an extension to the work by Yuki et al. [26], which only handles async and finish. These authors present a concise (and affine) expression of the happens-before relation for X10 programs with async and finish. The happens-before relation is then used to extend array dataflow analysis [10] that finds precise (statement instance-wise and array element-wise) dependence information. We briefly describe the key ideas that we reuse in this paper.

3.1 Paths and Iteration Vectors

The statement instances are identified by a vector of integers, called an iteration vector. This vector is computed (symbolically) as a path from the root to nodes in the AST. As the AST is traversed, values are appended to the vector based on the following rules:
3.2 Happens-Before Relation

The happens-before relation for X10 programs with \texttt{finish} and \texttt{async} is formulated as an incomplete lexicographic order. For sequential programs, the full lexicographical order denotes the execution order. In the presence of parallelism, the execution order is no longer total. Yuki et al. [26] show that for the \texttt{finish/async} subset of X10 programs, the happens-before relationship can be expressed as an incomplete lexicographic order.

The iteration vector for a statement instance is obtained by instantiating the loop iterators to integer values. This is similar to the conventional iteration vectors used in the polyhedral literature, with the addition of \texttt{a} and \texttt{f}. Due to structural constraints, when two iteration vectors are compared, \texttt{a} and \texttt{f} are never compared to anything but themselves, and thus their order is irrelevant.

\begin{align*}
\text{clocked finish} \\
\text{for (i=1:N)} \\
\text{clocked async} \{ \\
\quad \text{for (j=i:N)} \\
\quad \quad \text{advance}; \\
\quad \}
\end{align*}

(a) Barrier-like synchronization with implicit clocks

\begin{align*}
\text{clocked finish} \\
\text{for (i=1:N)} \\
\text{clocked async} \{ \\
\quad \text{for (j=i:N)} \\
\quad \quad \text{advance}; \\
\quad \}
\end{align*}

(c) Slightly different synchronization pattern

\begin{align*}
\text{clocked finish} \\
\text{for (i=1:N)} \\
\text{clocked async} \{ \\
\quad \text{for (j=i:N)} \\
\quad \quad \text{advance}; \\
\quad \}
\end{align*}

(d) Parallel iterations

Figure 1: Examples of X10 clock usage. Figure 1a illustrates barrier-like synchronization where \(N\) activities spawned by the \(i\) loop synchronize every step of the \(j\) loop. Every spawned activity immediately reaches an advance, and blocks until the primary activity reaches the end of the \texttt{finish} block, de-registering itself from the clock. Figure 1b is a slight variation, where the primary activity executes an advance after each spawn. Thus, the spawned activities may proceed before all \(N\) activities are spawned, leading to a different parallel execution.

\begin{itemize}
\item \textbf{Sequence}: Integer \(x\) when taking the \(x\)-th branch of a sequence.
\item \textbf{For}: Loop iterator \(i\).
\item \textbf{Async}: \(a\)
\item \textbf{Finish}: \(f\)
\end{itemize}

The iteration vector for a statement instance is obtained by instantiating the loop iterators to integer values. This is similar to the conventional iteration vectors used in the polyhedral literature, with the addition of \(a\) and \(f\). Due to structural constraints, when two iteration vectors are compared, \(a\) and \(f\) are never compared to anything but themselves, and thus their order is irrelevant.

\begin{align*}
\text{for} \quad (i = 1: N) \quad \text{clocked finish} \\
\quad \text{clocked async} \{ \\
\quad \quad \text{for} \quad (j = i: N) \\
\quad \quad \quad \text{advance}; \\
\quad \}
\end{align*}

(b) Parallel iterations of 1a

\begin{align*}
\text{for} \quad (j = i: N) \quad \text{advance}; \\
\end{align*}

(2)

3.3 Race Detection through Dataflow Analysis

Using the iteration vectors and happens-before ordering, the authors develop an extension to the array dataflow analysis [10] for X10 programs. Array dataflow analysis finds the statement instance that produced the value used by an instance of a read. Given reader and writer statements \(R, W\), and memory access functions \(f_R, f_W\), the set of potential sources is defined by:

\begin{align*}
\forall r \in D_R, \quad w \in D_W, \\
f_W(w) = f_R(r), \\
\neg(r < w) \land r \neq w
\end{align*}

where

\begin{itemize}
\item Constraints [3] and [5] restrict the statement instances to its domain (the set of legal iterations).
\item Constraint [6] restricts to those that access the same array element, and
\item Constraint [7] excludes writers that happen after reads, and writes by the same statement instance.
\end{itemize}

In a sequential program, \(<\) is total, hence \(\neg(v < w) \land v \neq w\), which is the usual formulation [10].

The above gives a set of writer instances \(w\) that may be a producer for a read instance \(r\) for a single writer statement \(W\). The proposed analysis proceeds by finding the most recent \(w\) among all statements that write to the same array. Since the happens-before relation is not total, the most recent \(w\) may not be unique, and there is a race when a producer cannot be uniquely identified. There are two kinds of races:

\begin{itemize}
\item \textbf{Read-Write Race}: When an instance of reader \((r)\) and an instance of writer \((w)\) are not ordered. The write may or may not happen before the read, and thus the result is not deterministic.
\item \textbf{Write-Write Race}: When two writer instances, not necessarily of the same statement, is not ordered among themselves, but both of them happen-before the reader instance.
\end{itemize}

Our proposed approach consists of disproving all races found by using additional happens-before order deduced from clocks.

4. Semantics of Clocks

In this section, we define the operational semantics, and the happens-before relation for the subset of X10 we consider.

4.1 Operational Semantics

We provide a simple, concise structural operational semantics (SOS) for the fragment of X10 considered in this paper. The semantics is based on the same ideas as [26], but extends it in two ways. It provides a treatment of full sequential composition (permitting, e.g., \({\{s_1; s_2; s_3\}}\)) – [26] permits only \({\{s_1; s_2; s_3\}}\). More importantly, it provides a formal treatment for clocks that is considerably simpler than [18].

We assume that a set of (typed) locations \(\text{Loc}\), and a set of values, \(\text{Val}\), is given. \(\text{Loc}\) typically includes the set of variables in the program under consideration. With every \(d\)-dimensional \(N_1 \times \ldots \times \ldots \times N_d\)
\[ \cdots \times N_d \text{ array-valued variable } a \text{ of type array are associated a set of distinct locations, designated } a(0), \ldots, a(N_d-1) \] The set of values includes integers and arrays.

A **heap** is a partial (finite) mapping from Loc to Val. For \( h \) a heap, \( t \) a location and \( v \) a value by \( h[|v|] \) we mean the value \( e \) to which \( h \) maps \( l \). We let \( H \) denote the space of all heaps.

**Definition 4.1 (Statements).** The statements are defined by the productions:

\[
\text{Statements} \quad s \quad ::= \\
\quad b; \quad \text{Basic statements.}
\]

\[
\quad \text{advance;} \quad \text{Clock advance statement.}
\]

\[
\quad \{t\}; \quad \text{Execute } s \text{ for } x \text{ in } c1 \ldots c2.
\]

\[
\quad \text{async } s; \quad \text{Spawn } s.
\]

\[
\quad \text{finish } s; \quad \text{Execute } s \text{ and wait for termination.}
\]

We will assume that the set of basic statements includes \textit{skip}, a statement that immediately terminates with an unchanged heap.

We let \( S \) denote the space of all statements.

Procedures (methods) can be defined in X10 in a manner familiar from object-oriented programming languages like Java. Unlike Cilk, concurrency constructs in X10 can cross procedure boundaries. For example, the body \( s \) of a \textit{finish} \( s \) or an \textit{async} \( s \) could contain a call to a method whose body contains an \textit{async} \( t \), a \textit{finish} \( t \), or an \textit{advance}. For the purposes of this paper we do not formally model procedure calls, leaving it for future work. The main implication is that, in general, static concurrency analysis of X10 programs involves inter-procedural reasoning.

**Execution relation.** As is conventional in SOS, we shall take a configuration to be a pair \( \langle s, h \rangle \) (representing a state in which \( s \) has to be executed in the heap \( h \)) or \( h \) (representing a terminated computation.) Formally, the space of configurations \( K \) is given by \( K = (S \times H) + H \), where \( \times \) represents the cross product of two sets and \( + \) their disjoint union.

The operational execution relation \( \rightarrow \) is defined as a binary relation on configurations. We use the "matrix" convention for presenting rules compactly. A rule such as:

\[
\begin{array}{c}
\gamma \rightarrow \gamma_0 | \ldots | \gamma_{n-1} \\
(d_0) \gamma_0 \rightarrow \delta_0 | \ldots | \delta_{n-1} \\
(d_{m-1}) \gamma_{m-1} \rightarrow \delta_{m-1}^m | \ldots | \delta_{n-1}^m
\end{array}
\] (with \( p \geq 0, m > 0, n > 0 \) is taken as shorthand for \( m \times n \) rules: infer \( \gamma_i \rightarrow \delta_i \) from \( c, d_i, \gamma \rightarrow \gamma_j \), for \( i < m, j < n \). Here, \( c \) and \( (d_i) \) are sequences of conditions, omitted when the sequence is empty.

To define the axioms and rules of inference we need two auxiliary structural predicates on statements. They define what it means for a statement to be asynchronous and synchronous. A statement is asynchronous if it is an \textit{async} \( s \), or a sequential composition of asynchronous statements.

\[
\vdash \text{isasync } s \\
\vdash \text{isasync } t \\
\vdash \text{isasync } \{s, t\} \\
\vdash \text{isasync } \{s\} \\
\vdash \text{isasync } \{s, t\}
\]

A statement is synchronous if it is not asynchronous. We can give a positive definition thus:

\[
\vdash \text{issync } b; \\
\vdash \text{issync } \{s\} \\
\vdash \text{issync } \{s, t\} \\
\vdash \text{issync } \{s\} \\
\vdash \text{issync } \{{s'}\} \\
\vdash \text{issync } \{s \in e_1 \ldots e_2\} s
\]

The following proposition is established by structural induction.

**Proposition 4.1.** For any statement \( s \), either \( \vdash \text{isasync } s \) or \( \vdash \text{issync } s \).

Now we turn to the axiomatization of the transition relation. This is the same as \textcolor{red}{[26]} except that we permit the first statement of a sequential composition to be arbitrarily nested.

**Basic statements.** We assume that basic statements come with a definition of the inference relation on configurations. For instance, if assignment were included as a basic statement, it would be formalized thus:

\[
(10) \quad l = h(a), v = h(e) \\
\vdash a = c, h, h \rightarrow h[|v|]
\]

Similarly, \textit{skip} is formalized thus:

\[
(11) \quad \langle \text{skip}, h \rangle \rightarrow h
\]

**Sequencing, finish, async.**

\[
(12) \quad \langle s, h \rangle \rightarrow \langle s', h' \rangle | h'
\]

\[
(13) \quad \langle \text{finish } s, h \rangle \rightarrow \langle \text{finish } s', h' \rangle | h'
\]

\[
(14) \quad \langle \text{for } x \in e_0 \ldots e_n, s \rangle, h \rightarrow \langle \{T' \text{ for } x \in m \ldots u \} s, h' \rangle | \langle \text{for } x \in m \ldots u, s, h' \rangle
\]
statement is an advance.\footnote{We use the matrix notation defined above for transition relations, adapting it for stuck s judgments.} 
\[ \begin{align*}  
\vdash & \text{stuck advance;} \\
\vdash & \text{stuck s} \\
\vdash & \text{stuck async s} \\
\vdash & \text{issync s \vdash \text{stuck } \{ t \} } \\
\vdash & \text{issync s \vdash \text{stuck } \{ s \} t} \\
\vdash & \text{stuck \{ s \} t} 
\end{align*} \] (15)

**Proposition 4.2** (Stuck configurations are stuck). Let \( \rightarrow' \) be a transition relation on configurations defined by the rules introduced so far, i.e. Rules (10-12). Then \( \vdash \text{stuck s} \) iff for no \( t, h, h' \) is it the case that \( (s, h) \rightarrow' (t, h') \) or \( (s, h) \rightarrow' h' \).

Next we define a relation on statements. We write \( s \Rightarrow t \) (and say “\( s \) yields \( t \) after a clock step”). 
\[ \begin{align*}  
\text{advance;} & \Rightarrow \text{skip;} \\
b; & \Rightarrow b; \\
\vdash & \text{async s} \\
\vdash & \text{sync t} \\
\vdash & \text{issync s \{ s t \} u} \\
\vdash & \text{issync s \{ s' t \} s t} \\
\vdash & \text{stuck \{ s t \} s t} 
\end{align*} \] (16)

Now we can define the transition relation. If \( s \) is stuck, and \( s \) yields \( t \) after a clock step, then \( \text{finish } s \) can transition to \( \text{finish } t \), and leave the heap unchanged:
\[ \begin{align*}  
\vdash & \text{stuck s} \\
(\text{finish } s, h) & \Rightarrow (\text{finish } t, h)  
\end{align*} \] (17)

The Rules (10-12) and (17) complete the definition of the transition relation \( \rightarrow \).

**Proposition 4.3.** The only terminal configurations for the transition system \( (K, \rightarrow) \) are of the form \( \{ h \} \in H \).

**Semantics.** We now define appropriate semantical notions.

**Definition 4.2** (Semantics). Let \( \Rightarrow \) represent the reflexive, transitive closure of \( \rightarrow \). The operational semantics, \( \mathcal{O}[s] \), of a statement \( s \) is the relation
\[ \mathcal{O}[s] \overset{\text{def}}{=} \{(h, h') \mid (s, h) \Rightarrow h'\} \]

Sometimes a set of observable variables is defined by the programmer, and the notion of semantics appropriately refined:
\[ \mathcal{O}[s, V] \overset{\text{def}}{=} \{(h, h') \mid (s, h) \Rightarrow h'\} \]

where for a function \( f : D \rightarrow R \) and \( V \subseteq D \) by \( f|_V \) we mean the function \( f \) restricted to the domain \( V \).

### 4.2 The CLOCKED Happens-Before-Relation

The semantics of Section 4.1 is operational: it can be considered as a blueprint for a rudimentary X10 interpreter. Concurrency is represented as non determinism. An interpretation of a program \( s \) is a linear succession of reductions according to the rules of Section 4.1. Each reduction is associated to the execution of a basic statement or to the crossing of a barrier. Due to the fact that a sequence \( \{ s \} t \) can be reduced in two ways if \( \text{issync s} \) is true, a program can have many interpretations, all of which are interleaves of the same set of basic operations.

In contrast, in building the happens-before realisation, our aim is to specify the X10 semantics as a partial order: operation \( u \) happens before \( v \) if it occurs to the left of \( v \) in every legal interpretation.

To simplify the presentation, we will assume that basic statements have distinct names. For polyhedral X10 programs, we can use paths as defined in Section 4.1 for that purpose.

Polyhedral X10 programs have static control: the set of operations and their execution order are fixed as soon as a few size parameters are known. Hence, we can simplify the semantics of Section 4.1 by dispensing with the heap. A convenient way of achieving this simplification is to assume that all basic statements are skips. Under this assumption, all configurations have the same heap as the initial configuration of the program.

As a first effort, we will ignore loops in what follows. Note that when size parameters are given, loops can be statically expanded into nests of sequence constructs.

Each step in the interpretation of a program is a deduction according to the rules of inference of Section 4.1; each such deduction must start with one of the axioms (11) or (12). Each reduction is associated either to a basic statement or to the set of advances which are transformed into skips by rule (14). No reduction can use more than one axiom, since there is no transitivity rule for \( \rightarrow \). To such an interpretation we can associate the trace obtained by successively appending either the name of the reduced skip or the set of transformed advances. Observe that at each step, either the number of advances or the number of basic statements is reduced, hence, all reductions terminate. This is an indirect proof that the fragment of X10 we consider has no deadlock.

**Proposition 4.4** (Normal Forms). Let \( r \) be a program which is not stuck. Either \( r \) does not contain advances, or there is a unique stuck \( r' \) such that every reduction path from \( r \) which does not use (12) terminates in \( r' \).

For reasons to be made clear presently, \( r' \) will be called the normal form of \( r, N(r) \). If \( r \) does not contain advances, \( N(r) \) is a terminal configuration: a configuration without a continuation.

**Proof** It is clear that the only case in which several reductions are possible is if \( r = \{ s \} t \) and \( \vdash \text{issync s} \). One can either reduce \( s \), giving \( \{ s' t \} \), or \( t \), giving \( \{ s t' \} \). This last term can be reduced further, giving \( \{ s' t' \} \). It easy to convince oneself that \( s' \) is either terminal or \( \vdash \text{issync s'} \). In the first case, both terms reduce to \( r' \). In the second case, \( \{ s' t' \} \) reduces to \( \{ s t' \} \). In both cases, the reduction system has the weak diamond property, and since reductions terminate, the unique normal form is Church-Rosser [3, Chapter 3] property, Q.E.D.

This result is the key to the analysis of clocks in polyhedral X10 programs.

### 4.3 The Case of a Single Clock Program

Consider first a one clock program, i.e. a program of the form \( \text{finish } r \) where \( r \) does not contain any clocked finish. This program has a unique normal form \( \text{finish } s \) where \( s \) is stuck. The reduction can only progress by applying rule (17), giving a new program \( \text{finish } t \) which may be further reduced. The elaboration of the initial program therefore proceeds in phases, where only rules (11-12) are applied, separated by applications of (17). One convenient way of expressing these observations is to assign a number in sequence to each application of rule (17), and to assign a phase number \( \phi(u) \) to each operation \( u \) which is executed between applications \( \phi(u) \) and \( \phi(u) + 1 \) of (17). It is then obvious that if \( \phi(u) < \phi(v) \) then \( u \ll v \). The fragment of code which constitutes a phase fits into the model of [26], and hence has the same happens-before relation. The clocked happens-before relation is therefore:
\[ u \ll v \equiv \phi(u) < \phi(v) \lor u \ll v. \]
4.4 Multiple Clocks

Let us now consider two operation $u$ and $v$ in a multiple clock program. There exists an innermost finish $F$ which contains both $u$ and $v$. In $F$, there exists an outermost finish $f_u$ (resp. $f_v$) which contains $u$ (resp. $v$). Either $f_u$ or $f_v$ or both may be the same as $F$. If $f_u = f_v = F$, we are back to a single clock program and the conclusion of the preceding section stands.

Suppose now that neither $f_u$ nor $f_v$ are equal to $F$. In the text of the program, replace $f_u$ (resp. $f_v$) by a fictitious basic statement $U$ (resp. $V$) and evaluate $U \ll V$, again by the method of the preceding section. We claim that $u \ll v \equiv U \ll V$. Assume first that $U \ll V$ is true. In all reductions of the transformed $F$, $U$ is replaced before $V$. We can construct a reduction of the original $F$ by replacing the reduction of $U$ by the reduction of $f_u$, and, by the semantics of finish, no operation of $f_v$ will execute until $f_u$ has terminated. Hence, $u \ll v$ is true. The case $V \ll U$ is symmetric. If neither $U \ll V$ nor $V \ll U$ are true, then there is a reduction in which $U$ and hence $u$ occurs first, and another one in which $V$ hence $v$ occurs first, hence neither $u \ll v$ nor $v \ll u$ are true. The remaining two cases can be handled in the same way.

4.5 Loops

The analysis of loops poses both a practical and a theoretical problem. On the practical side, when loop bounds are known numbers, loops can be eliminated in favor of sequences by repeated application of rules (4.3). However, the resulting program may be so large as to make counting phases unpractical. But in the polytope model, loop bounds may depend on unknown symbolic parameters, hence the application of rule (4.3) may never terminate. The trick here is to observe that a program which depends on symbolic constants is a shorthand for a possibly infinite family of programs, which are obtained by giving every admissible value to the parameters. For each program in the family, the conclusions of the preceding section still stand. All that is needed is to find a closed form for the advance counters $\phi$ and for the unlocked happens-before relation, $\prec$. For the later, the authors of [26] have given a closed formula as an incomplete lexicographic order. It remains to find a symbolic way of computing $\phi$.

To this aim, assume that advances are temporarilily considered as ordinary basic statements, to which $\prec$ applies. One way of interpreting the fact that a configuration is stuck is to say that no reduction can be done unless one advance at least is reduced, which can be done only by using rule (4.7). The initial advances of a stuck $s$ are the advances which are replaced by skips before application of (4.7).

**Proposition 4.5.** If $s$ is stuck, then for every elementary statement $x$ in $s$, either $s = x$ is an initial advance, or there exists a unique initial advance $a$ such that $a \prec x$.

**Proof** The proof is by induction on the $\bot$ stuck $s$ inference. There is nothing to prove if $s$ is an advance. If $s = u \prec a \prec t$, then $t$ is stuck and $x$ is in $t$, and the result follows. If $s = \{t \cup u\}$ and $t$ is asynchronous, then both $t$ and $u$ are stuck and $x$ is in either $t$ or $u$. If $t$ is synchronous, then $t$ is stuck. If $x$ is in $t$, then the induction hypothesis applies. If $x$ is in $u$, let $a$ be the initial adavance of $t$, which exists by the induction hypothesis. By the semantics of sequential composition, $a \prec x$. Q.E.D.

Let $A$ be the set of advance instances in the program under study. From the above result follows that, for each statement $u$:

$$\phi(x) = \text{Card}\{a \in A | a \prec x\}.$$  

For polyhedral programs, we will show in Section 5.4 how to compute closed forms for $\phi$.

5. Clocks and Races

The important observation is that clocks only add additional synchronizations among activities, and hence strictly decreases the set of may-happen-in-parallel iterations. Therefore, we propose to guarantee race-freedom by disproving all races found with out taking clocks into account [26].

5.1 Computing the $\phi$ Function

The first step is to automatically compute the $\phi$ functions that define the happens-before relation with respect to clocks. What we are interested in is a function that gives the number of advances, associated with a clock, an activity has executed before executing an iteration of a statement. Thus, the computed function must be parametric to the statement instance in question, as well as the program parameters.

This is achieved by constructing the following union of parametric polytopes, and computing the number of integer points it contains:

$$x \in D_S,$$  

$$a \in D_A,$$  

$$a \prec x$$

where

- $D_S$ is the domain of a statement $S$,
- $D_A$ is the union of domains of advance statements that operates with the clock in question, and
- Constraint (20) restricts the advance statement instances to those that happens-before an instance $x$ of $S$.

Note that this corresponds to the definition of the $\phi$ function in Section 4.5. By treating $x$ as parameters of the polytope, we obtain a parametric expression of the number of advance statements that happens-before $x$. We compute such expression for each pair of statement and clock in the program.

5.2 Counting Integer Points in Polytopes

For polyhedral iteration spaces, the question of counting advances can be cast as counting the number of integer points in polyhedra. Ehrhart [9] showed that the number of integer points in a polytope can be expressed as periodic polynomials. We use an algorithm proposed by Verdoolaege et al. [23] for computing such polynomials, which handles parametric polytopes.

5.3 Disproving Races

We may refine the dataflow analysis formulation for X10 programs without clocks (overviewed in Section 5.1) using the new happens-before relation for clocked programs. The only change required is to replace $\prec$ with $\prec$. However, the problem stems from the $\phi$ functions not being affine in general. Parametric integer linear programming [9] can no longer be used, and there is no known alternative that can handle polynomial expressions.

Therefore, our proposed solution is to first detect races without clocks taken into consideration, and then later use constraint solvers to verify if the statement instances involved in a race can take the same value of $\phi$.

5.4 Problem Formulation

The races detected have precise information regarding which statement instances are involved in a race. Recall that we have two main kinds of races (Section 3.3), Read-Write and Write-Write. For each detected race, we have the following:

- $r \in D_R$: Read instances involved in the race.
...w ∈ D^*_w(r): Write instances involved in the race, parametric to r. For Write-Write races, we obtain two of such sets, as we have two writers in conflict.

Given a set of clocks C in the program, and φ functions φ_c for each c ∈ C. Recall (Section 4.4) that among the set of clocks, only one clock is relevant for each pair of statements, and thus the case with multiple clocks was reduced to single clock case. Let us define reduce(C, S1, S2) to be a function that gives a single clock, c*, from the set of clocks C that is relevant when defining the happens-before relation between instances of statements S1 and S2.

The problem is to simply check if there exists a pair of instances that are involved in a race, and take the same value of φ_c* for each potential race.

The constraints for Read-Write race to occur, and respectively for Write-Write race to occur, are the following:

\[
\begin{align*}
\& r \in D^*_r \\
\& w \in D^*_w(r) \\
\& c^* = \text{reduce}(C, R, W) \\
\phi_{c^*}(r) &= \phi_{c^*}(w)
\end{align*}
\]

5.5 Undecidability

To prove that the race problem for clocked X10 is undecidable, we need the following construction:

Given an arbitrary polynomial \( P(x) \) in \( n \) variables \( x_1, \ldots, x_n \) with integer coefficients, build an X10 program that has a race if and only if \( P(x) \) has an integral root.

Since deciding if \( P(x) \) has a root is undecidable (Hilbert 10th problem), it follows that the race problem is undecidable.

As we will see later, we may have to build not one X10 program but a finite number of programs (in fact, \( 2^n \) programs) such that \( P(x) \) has a root iff one of those programs has a race, but this does not change the conclusion.

5.5.1 The Shape of the Test Program

Let us write \( P(x) = P_1(x) - P_2(x) \), and consider the following:

```c
for(x in D){
  clocked finish{
    clocked async{
      L1;
      u = f(); //U
    }
    clocked async{
      L2;
      g(u); //G
    }
  }
}
```

where \( x \) is in fact a vector of dimension \( n \) which scans the domain \( D \) to be defined later, and \( L_1 \) (resp. \( L_2 \)) is a loop nest which executes exactly \( P_1(x) \) (resp. \( P_2(x) \)) advances. It is clear that this program will have an MHP race iff \( P(x) \) has a root in \( D \).

To understand the behaviour of this program, assume first, without loss of generality, that for a given value of \( x \), \( P_1(x) < P_2(x) \). Then the loops \( L_1 \) and \( L_2 \) will execute in lockstep, \( L_1 \) will terminates first and \( U \) will be executed. The first activity will terminate and de-register itself, and then \( L_2 \) will execute its remaining iterations, and then execute \( G \). Contrarywise, if \( P_1(x) = P_2(x) \), \( L_1 \) and \( L_2 \) will terminate at the same (logical) time, and the execution order of \( U \) and \( G \) will be undefined.

However, we must first insure that the program is realistic. Observe that the number of iterations of a loop nest can never be negative; hence, we must insure that \( P_1(x) \) and \( P_2(x) \) are non-negative for \( x \in D \). This can be guaranteed if \( D \) is the positive orthant and if \( P_1 \) and \( P_2 \) have positive coefficients. In this way, we will test only the existence of a positive integral root of \( P \). To be complete, we must apply the above construction to the \( 2^n \) polynomials \( P(e_1 x_1 + \cdots + e_n x_n) \), where the \( e_s \) take all combinations of the values \( \{ +1, -1 \} \).

The fact that \( D \) is unbounded is irrelevant, since we do not intend to run the above program to find races. We just have to be careful in writing the \( x \) loop, but this is a well know problem (see for instance the classical proof that \( \text{Card } N = \text{Card } N^2 \)).

5.5.2 Constructing Counting Loop Nest

Our aim now is, given a polynomial \( Q(x) \) with positive integer coefficients, to construct a loop nest which compute \( Q(x) \) only using increments. It will be enough then to replace each incrementation by an advance to prove the theorem. In the following, we accept more general forms of increments \( \text{phi ++$d; \text{where $d$ is a positive integer, representing $d$ consecutive advances.} \)

Let us select one variable, say \( x_1 \) and let us write \( Q(x) = Q(x_1, x_r) \) where the vector of variables \( x_r \) may be empty. Let \( m \) be the degree of \( Q \) in \( x_1 \). The first difference of \( Q \) is:

\[
Q^{(1)}(x_1, x_r) = Q(x_1 + 1, x_r) - Q(x_1, x_r)
\]

and it is clear that the program

```c
phi = Q(0, x_r);
for(i=0; i<x; i++)
  phi += Q1(i, x_r);
compute Q(x). The degree of \( Q^{(1)}(i, x_r) \) is \( m - 1 \) in \( i \), hence we can iterate this construction to obtain:
```

\[
\begin{align*}
&\text{phi} = Q(0, x_r); \\
&\text{for}(i=0; i<x; i++) \\
&  \text{phi} += Q1(i, x_r); \\
&  \text{for}(j=0; j<i; j++) \\
&    \text{phi} += Q2(j, x_r);
\end{align*}
\]

where \( Q^{(2)} \) is the second difference of \( Q \) with respect to \( x_1 \).

We can continue in this way until we reach the \( m \)-th difference, which is independent of its first variable. At this point, all the increments in the program depend only on \( x_r \). We can select another variable and apply the same construction, until all increments are constant. This terminates the proof.

5.5.3 An Example

Let us construct the counting nest for \( Q(x, y) = x^2 + xy + y^2 \). The first difference is \( 2x + y + 1 \), hence the first program is:

```c
phi = yy;
for(i1 = 0; i1<x; i++)
  phi += 2*i1+y+1;
```

The second difference is simply 2, hence the second program:

```c
phi = yy;
for(i1 = 0; i1<x; i++)
  phi += y+1
for(i2 = 0; i2<i1; i++)
  phi += 2;
```

At this point, the increments depend only on \( y \). Applying the same algorithm to \( y^2 \) and \( y + 1 \) yields the final program:
phi = 0;
for (i3 = 0; i3<y; i3++) {
    phi += 1;
    for (i4 = 0; i4<i3; i4++)
        phi += 2;
}
for (i1 = 0; i1<x; i1++) {
    phi += 1;
    for (i5 = 0; i5<y; i5++)
        phi += 2;
    phi += 2;
}

Note that a new induction variable is needed for each loop.

6. Examples

In this section, we illustrate different types of synchronizations that can be analyzed by our proposed approach through examples.

6.1 Barrier-like Synchronization

The following is a simplified implementation of 1D Jacobi-style stencil computation using clocks.

clocked finish
for (i=1:N-1)
clocked async
for (t=0:T)
    B[i] = S0(A[i-1], A[i], A[i+1]);
    advance;
    A[i] = S1(B[i-1], B[i], B[i+1]);
    advance;

The above use of clocks are similar to barriers; the synchronization does not rely on an activity being de-registered from a clock, aside from the primary activity.

Without taking the additional happens-before relation due to clocks into account, there are four read-write races in the program.

• Read $A[i-1]$ by $S0(i, t)$ is in read-write race with $S1(i-1, t')$ when $1 < i < N$ and $0 \leq t' \leq T$.
• Read $A[i+1]$ by $S0(i, t)$ is in read-write race with $S1(i+1, t')$ when $1 \leq i < N - 1$ and $0 \leq t' \leq T$.
• Read $B[i-1]$ by $S1(i, t)$ is in read-write race with $S0(i-1, t')$ when $1 < i < N$ and $0 \leq t' \leq T$.
• Read $B[i+1]$ by $S1(i, t)$ is in read-write race with $S0(i+1, t')$ when $1 \leq i < N - 1$ and $0 \leq t' \leq T$.

Note that $t'$ refers to all possible values that $t$ can take. Without the clock synchronization, all writes to the same element of the array at different time steps are in conflict.

The $\phi$ functions for $S0$ and $S1$ are $\phi_{S0} = 2t$, and $\phi_{S1} = 2t+1$. The four races can trivially be disproved by using the $\phi$ functions, since it guarantees $S0$ and $S1$ never execute in parallel. In this case, the $\phi$ functions are actually affine, and hence it can directly be incorporated into array dataflow analysis.

6.2 Activity Specific $\phi$ Functions

The following is a parallelization of Gauss-Seidel style stencil computation. The difference is that the reference $A[i - 1]$ refers to a value computed at the same $t$, rather than $t - 1$ in the case of Jacobi style stencils.


clocked finish
for (i=1:N-1)

The four races can trivially be disproved by using the $\phi$ functions, since it guarantees $S0$ and $S1$ never execute in parallel. In this case, the $\phi$ functions are actually affine, and hence it can directly be incorporated into array dataflow analysis.

The following is a parallelization of Gauss-Seidel style stencil computation using clocks.

clocked finish
for (i=1:N-1)
clocked async
for (t=0:T)
    B[i] = S0(A[i-1], A[i], A[i+1]);
    advance;
    A[i] = S1(B[i-1], B[i], B[i+1]);
    advance;

The parallelization using clocks is illustrated in Figure 2.

There are two read-write races in the program before clocks are taken into consideration.

• Read $A[i-1]$ by $S0(i, t)$ is in read-write race with $S0(i-1, t')$ when $1 < i < N$ and $0 \leq t' \leq T$.
• Read $A[i+1]$ by $S0(i, t)$ is in read-write race with $S0(i+1, t')$ when $1 \leq i < N - 1$ and $0 \leq t' \leq T$.

The $\phi$ function for $S0$ is $\phi_{S0} = 2t + i$.

Using the $\phi$ function, constraint solvers can easily disprove these races. The problem reduces to the existence of values of $t$ and $t'$ within its domain that satisfies:

• $2t + i = 2t' + i - 1$, or
• $2t + i = 2t' + i + 1$.

It can easily be found that the LHS is limited to even numbers and the RHS is limited to odd numbers.

Note that the $\phi$ function found involves $i$ that takes different values in each activity. This is because the primary thread also executes an advance statement after spawning each thread, allowing earlier activities to proceed. The number of activities participating in a synchronization dynamically changes as the program executes, which is different from how barrier synchronization typically work.
6.3 Polynomial $\phi$ Functions

The following is a possible parallelization of QR decomposition (via CORDIC) using clocks. The advance statement is surrounded by two loops and an affine if-guard. The statement domains are no longer rectangular like other examples, leading to more complicated $\phi$ functions.

```c
clocked finish
for (j=0:N-1)
clocked async
for (k=0:N-2)
for (i=0:N-2-k)
if (j>=k) {
    M[N-i-1][j] = S0(M[N-i-1][j], M[N-i-2][j], M[N-i-1][k], M[N-i-2][k]);
    M[N-i-2][j] = S1(M[N-i-1][j], M[N-i-2][j], M[N-i-1][k], M[N-i-2][k]);
    advance;
}
```

There are a total of eight Read-Write races in the above program. We do not enumerate them as they are quite similar. Writes to $M$ by both statements are in race with reads where $k$ is used in the access.

The $\phi$ functions for two statements are the following:

- $\phi_{S0} = \phi_{S1} = Nk + i - \frac{k^2 + k}{2}$

We illustrate the problem formulation with one of the eight races. Give a writer instance $S0(j, k, i)$, and a read access $M[N-i-1][k]$ of instance $S1(j', k', i')$, there is a race when $j > k$ and $i = i'$ and $k = k'$ and $k >= k'$ and $i <= N - 2 - k'$.

To disprove this race, we must ensure that there is no pair of distinct instances that satisfies:

- $i = i'$, $k = k'$, $k >= k'$, $j > k$, and $Nk + i - \frac{k^2 + k}{2} = Nk' + i' - \frac{k'^2 + k'}{2}$.

With constraint solvers that can handle polynomials over integers, it can be verified that the above cannot be satisfied.

7. Implementation

We have implemented our analysis for the subset of X10 we handle. We take a simplified representation of the program only concerning the access to variables, disregarding the specifics of the operations performed in statements. This simplified representation can easily be extracted from the full X10 AST.

Significant amount of effort has been made towards identifying polyhedral regions of loop programs [12, 16]. However, integrating such effort to X10 is beyond the scope of this paper.

We use the Integer Set Library [23] for polyhedral operations and parametric integer programming, and the Barvinok library [23] for counting of integer points. Other parts of the analysis are implemented in Java, and we use native bindings for library calls.

We use the Z3 SMT solver [7] for disproving races involving polynomials. The constraints are given to Z3 using the SMT-LIB format [4], and many other solvers that support the same standard can also be used. We interface with Z3 through the command line.

Table 1: Performance (in seconds) of our implementation on JGF benchmarks and examples in Section 6. All races found were disproved by the SMT solver. Our experiments were conducted with 4 core Intel i7 (2.4 GHz) and 4GB of memory. We used Java 1.7, ISL 0.11, Barvinok 0.36, and Z3 4.3.1.

| Benchmark   | Races Found | DFA Time | Counting Time | SMT Time | Total Time |
|-------------|-------------|----------|---------------|----------|------------|
| SOR         | 4           | 3.02     | 0.54          | 0.18     | 3.57       |
| MOLDYN      | 16          | 5.35     | 6.55          | 0.33     | 11.95      |
| LUFAC1      | 5           | 0.71     | 0.25          | 0.15     | 0.97       |
| JACOBI      | 4           | 0.32     | 0.23          | 0.09     | 0.56       |
| GAUSS-SEIDEL| 2           | 0.19     | 0.25          | 0.07     | 0.44       |
| QR          | 8           | 1.35     | 0.22          | 0.29     | 1.85       |

8. Related Work

In this section, we place our work in context of the previous work on analysis of barrier synchronization.

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2 Our implementation is open source, and will be made available when the paper is published.
8.1 Analysis of Clocks

There are very little work in the literature that deals with clocks in X10 (or similar parallel constructs). Vasudevan et al. [20] have presented static analysis for verifying syntactic correctness of clocks to avoid runtime exceptions. They also propose a few optimizations for certain patterns of clock usage, e.g., when some activities are registered but do not participate in synchronization.

We distinguish our work by providing race free guarantee to the polyhedral subset of X10 programs. We are not aware of any other work that verify absence of races in the presence of clocks.

8.2 Analysis of Barriers

Barrier synchronization in conventional parallel programming models (e.g., MPI, OpenMP) has similar semantics as X10 clocks. There are tools for dynamic analysis of these languages [12, 13, 23]. Our work complements these tools by providing race-free guarantee for program regions amenable to our static analysis.

There are static analysis techniques developed for analyzing barrier synchronization of SPMD-style programs [2, 13, 23]. SPMD (Single Program Multiple Data) is a common parallel programming model where the same code is executed on multiple processes. Typically, each process works on different data by referring to its process ID and synchronize/communicate accordingly.

Aiken and Gay [2] introduced single-valued expressions, expressions that can be proved to evaluate to the same value in all processes, to ensure that all processes execute the same number of barriers. Kamil and Yelick [13] extend the work of Aiken and Gay in the context of Titanium parallel programming language [25] to perform May-Happen-in-Parallel analysis.

Titanium requires all the barriers to be textually aligned, i.e., all processes must execute the same barrier in the same order. Zhang and Duesterwald [23] present a method for more general SPMD-style programs where barriers are not necessarily textually aligned.

X10 is not SPMD; activities are dynamically created executing its own piece of code. Furthermore, the participants of barriers can dynamically change, complicating static analysis.

8.3 Handling of at and places

X10 uses Partitioned Global Address Space (PGAS) programming model, where the address space is sparated into multiple places. The at construct allows the programmer to specify the place where operations are performed.

Agarwal et al. presented an algorithm to find the set of iterations that may happen in parallel for X10 programs [1]. Although they do not handle clocks, their algorithm handles at and places. They assume that places are identified as some function of the loop indices. Two statement instances execute at the same place only if the expressions evaluate to the same value.

They also handle atomic blocks in X10, which are similar to atomic blocks in other languages, but only allow concurrent execution of critical sections if two processes are in different places.

Both of these may be used in combination with our work to further extend the applicability of our analysis.

9. Conclusions and Future Work

We have presented a method for guaranteeing race freedom of polyhedral X10 programs with clocks. We show that the problem is undecidable and resort to constraint solvers for providing the guarantee. The idea is not limited to X10 programs, and can easily be adapted to handle its dialect, Habanero-Java, and possibly other languages with less dynamic synchronizations. When combined with the work by Agarwal et al. [1], we may now analyze all of the basic parallel constructs in X10.

There has been little work on static analysis of parallel programming languages. The application of the main ideas in our work is not limited to data race detection. The formalization and happens-before relation opens many opportunities such as scheduling, memory allocation, program transformations, and so on.

Another direction of future work is to relax the polyhedral requirement. This is a limitation shared among any polyhedral analysis, and our work will also benefit from progress in this direction.

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