Searching for Signatures of Quantum Gravity in Quantum Gases.

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We propose a scheme for testing the quantum nature of the gravitational field with an ensemble of ultra-cold atoms. The use of many microscopic particles may circumvent some of the experimental obstacles encountered in recent proposals involving a pairs of particles with mesoscopic mass. We employ multi-parameter estimation techniques, including the quantum and classical Fisher information to provide a criteria for the observability of the quantum effects, and compare to other recently proposed schemes. Crucially, we find that by preparing the appropriate initial state, interactions mediated via a quantum-valued gravitational field provide a signature that is distinct from classical gravitational interactions. We find that a test with ultra-cold atoms would be challenging, but not implausible with moderate improvements of current experimental techniques.

While a full theoretical quantum treatment of quantum gravity remains elusive, it has been shown that at the low energy scales involved in laboratory scale experiments, gravity can be approximated by an effective quantum field theory by treating the quantum field as a perturbation from the classical solution $\Box \phi$. However, whether or not the the gravitational field displays quantum properties, that is, if a quantum treatment is required at all, is a question that demands experimental resolution [3].

There have been several recent proposals for laboratory scale experiments to test for signatures of a quantised gravitational field created by test-masses in quantum superposition states [3, 5–10], and experiments that rules out classes of semi-classical gravity [11, 12]. Recently, it has been argued that if the gravitational interaction between two quantum systems can create entanglement, then the gravitational field must act as a quantum mediator [3, 6, 9, 13]. This principle is the basis of two experimental proposals by Bose et al. [16] and Marletto and Vedral [17] for testing this phenomenon by searching for gravitationally induced entanglement between two micron sized test masses. While these proposals are feasible with a modest improvement in current experimental capabilities, there are still significant experimental challenges associated with them. As such, it is worth exploring if the differences that other systems provide can prove advantageous.

In this paper, we investigate the gravitational interaction amongst an ensemble of ultra-cold atoms. We find that if the gravitational interaction is mediated by a quantum-valued gravitational field, it presents different signatures to that of a classical-valued gravitational field. By carefully engineering the many-particle quantum state of the system, we can maximise the effect of the observability of these signatures. Other than the use of many particles, an important difference between this work and the work of Bose et al. [16] and Marletto and Vedral [17] is that instead of searching for an entanglement witness, we formulate the experiment as a multi-parameter estimation problem, which, in principle allows us to search for arbitrarily weak interactions by performing many repetitions of the experiment. Crucially, we show that classical theories of gravity, and decoherence effects, produce a signatures in the measured probability distribution that are distinct from what an interaction mediated by a quantum-valued gravitational field predicts.

I. GRAVITATIONAL INTERACTION FOR AN ENSEMBLE OF TWO-LEVEL ATOMS

We assume an ensemble of ultracold bosonic atoms with two electronic states $|a\rangle$ and $|b\rangle$. Following [16, 17], rather than attempting to present a quantum description of the gravitational field, we describe the gravitational interactions between the particles directly. In the limit of Newtonian gravity, the contribution to the Hamiltonian due to the gravitational interaction energy is given by

$$\hat{H}_G = -\frac{1}{2}Gm^2 \int \int :\hat{n}(\mathbf{r})\hat{n}(\mathbf{r}'):\frac{1}{|\mathbf{r} - \mathbf{r}'|} d^3 \mathbf{r} d^3 \mathbf{r}', \quad (1)$$

where $m$ is the mass of each individual particle, and $G$ is Newton's constant for universal gravitation. The operator $\hat{n}$ is given by

$$\hat{n}(\mathbf{r}) = \hat{\psi}_a^\dagger(\mathbf{r})\hat{\psi}_a(\mathbf{r}) + \hat{\psi}_b^\dagger(\mathbf{r})\hat{\psi}_b(\mathbf{r}), \quad (2)$$

and represents the total particle density, and $\hat{\psi}_j(\mathbf{r})$ is the standard bosonic field operator annihilating a particle from state $|j\rangle$ at point $\mathbf{r}$, obeying the standard commutation relations

$$[\hat{\psi}(\mathbf{r}), \hat{\psi}_j^\dagger(\mathbf{r}')] = \delta_{ij}\delta(\mathbf{r} - \mathbf{r}'), \quad (3)$$

and ‘::’ denotes normal ordering. It's important to note that in writing Eq. (1), we have implicitly assumed that the gravitational field can be quantum valued. Assuming that the interaction between the atoms is mediated by a gravitational field (i.e., not a direct non-local action at a distance), this field must be quantum valued.
Specifically, Eq. (11) can be re-written as
\[ \hat{H}_G = \frac{m}{2} \int : \hat{n}(\mathbf{r}) \hat{\Phi}_G(\mathbf{r}) : d^3 \mathbf{r}, \]  
(4)
where
\[ \hat{\Phi}_G(\mathbf{r}) = -G m \int \frac{\hat{n}(\mathbf{r}')} {\lvert \mathbf{r} - \mathbf{r}' \rvert} d^3 \mathbf{r}', \]  
(5)
is the (operator valued) gravitational potential due to source masses at location \( \mathbf{r}' \), and the factor of \( \frac{1}{2} \) removes double-counting. That is, the gravitational potential is quantum valued as a result of the quantum uncertainty in the location of the source masses.

Consider, in contrast, a classical-valued gravitational potential \( \hat{\Phi}_C(\mathbf{r}) \), originating from some classical source. Placing our quantum test-particles in this field results in the gravitational potential
\[ \hat{H}_C = m \int \hat{n}(\mathbf{r}) \hat{\Phi}_C(\mathbf{r}) d^3 \mathbf{r}. \]  
(6)
Clearly, this gravitational interaction energy can also be quantum valued, but only due to the quantum-ness of the location of the particles, rather than the quantum-valued nature of the gravitational source (and therefore field) as in Eq. (11). A theory of gravity that does not allow for a quantum valued field would then be described by a Hamiltonian such as Eq. (5), with the value of \( \hat{\Phi}_C(\mathbf{r}) \) depending on how a classical valued field arises from a quantum distribution of source masses. For example, in a semi-classical theory of gravity \( \hat{\Phi}_C(\mathbf{r}) = \langle \hat{\Phi}_C(\mathbf{r}) \rangle \). 

However, we do not restrict ourself to this case, and simply consider any classical gravitational field, without making any assumptions on how the value of \( \hat{\Phi}_C(\mathbf{r}) \) is determined from the quantum distribution of source masses. Our task is then to devise an experiment that can distinguish between the effects of Eqs. (1) and (6).

II. SCHEME

Our scheme is based on a standard Mach-Zehnder interferometer, and is illustrated in Fig. 1. Each atom is initially prepared in state \( |a\rangle \), before being placed in an equal coherent superposition of \( |a\rangle \) and \( |b\rangle \) at time \( t = 0 \) via a \( \pi/2 \) pulse implemented via a radio-frequency transition. The two components are then spatially separated via a state dependent potential. Alternatively, if a two-photon Raman transition is used instead of a radio-frequency transition, the need for a potential is removed as the two components will separate due to the momentum transfer of the Raman transition. After evolving under the effects of the gravitational interaction for time \( t \), the two components are then brought back together and recombined by a second \( \pi/2 \) pulse, and the number in each component is measured. Quantitatively, we describe the evolution as follows: The state \( |\Psi_i\rangle = \hat{U}_0 |\Psi_0\rangle \) is prepared by splitting the initial state into two components. We also allow for the possibility that \( \hat{U}_0 \) may contribute additional operations during this step, such as creating entanglement between the particles. The atoms interact gravitationally, via the interaction \( \hat{I}_G |\Psi_i\rangle \), which may be a unitary mapping, or perhaps some operation that causes decoherence, before a second unitary operation \( \hat{U}_2 \) is applied, and a measurement \( \hat{M} \) is made.

If the atoms in state \( |j\rangle \) remain localised in spatial mode \( u_j(\mathbf{r}, t) \), then its reasonable to make the approximation
\[ \hat{\psi}_j(\mathbf{r}) = j u_j(\mathbf{r}, t) \]  
(8)
with
\[ [\hat{j}, \hat{k}] = 0, \]  
(9)
Inserting this into Eq. (11) gives
\[ \hat{H} = \kappa_{aa}(t) \hat{a}^\dagger \hat{a}^t \hat{a} + \kappa_{bb}(t) \hat{b}^\dagger \hat{b}^t \hat{b} + 2 \kappa_{ab}(t) \hat{a}^\dagger \hat{b} \]  
(10)
where
\[ \kappa_{ij} = -\frac{1}{2} G m^2 \int \int \lvert u_i(\mathbf{r}) \rvert^2 \lvert u_j(\mathbf{r}') \rvert^2 \frac{1} {\lvert \mathbf{r} - \mathbf{r}' \rvert} d^3 \mathbf{r} d^3 \mathbf{r}'. \]  
(11)
Meanwhile, making the same substitution in Eq. (5) gives
\[ \hat{H}_C = \nu_a \hat{a}^\dagger \hat{a} + \nu_b \hat{b}^\dagger \hat{b} \]  
(12)
where
\[ \nu_j = m \int \lvert u_j(\mathbf{r}) \rvert^2 \hat{\Phi}_C(\mathbf{r}) d^3 \mathbf{r}. \]  
(13)
If for the majority of the evolution the wavepackets are spatially separated by significantly more than their spatial extent, \( \kappa_{ab} \) is much less than \( \kappa_{aa}, \kappa_{bb} \). This means
that the gravitational potential energy of the system will be larger when all of the atoms are in one mode compared to the case when they are evenly distributed across both modes (and therefore occupying a larger volume). Setting $\kappa_{aa} = \kappa_{bb} = \kappa_{a}$, $\kappa_{ab} = 0$, the unitary operator for the evolution given by Eq. (10) is

$$\hat{U}_Q = \exp \left( i \alpha \hat{A} \right).$$

(14)

where $\alpha = \kappa_0 t / h$, and $\hat{A} = \hat{a} \hat{a}^\dagger + \hat{b} \hat{b}^\dagger$. In this case, the gravitational interaction is simply described by $I_G |\Psi_i\rangle = U_Q |\Psi_i\rangle$.

Meanwhile, the unitary evolution operator resulting from Eq. (10) is

$$\hat{U}_C = \exp \left[ i (\beta \hat{J}_z + \gamma \hat{N}_+) \right].$$

(15)

where $\beta = (v_b - v_a) t$, $\gamma = \frac{1}{2} (v_a + v_b) t$, $\hat{N}_+ = \hat{a} \hat{a}^\dagger + \hat{b} \hat{b}^\dagger$, and $\hat{J}_z = \frac{1}{2} (\hat{a} \hat{a}^\dagger - \hat{b} \hat{b}^\dagger)$, and $I_G |\Psi_i\rangle = U_C |\Psi_i\rangle$. If we assume a mean-field gravity model, then $\Phi(r) = \langle \hat{\Phi}(r) \rangle$, and we can estimate the expected values of $v_a$ and $v_b$. However, we do not restrict ourself to this case, and treat these as unknown parameters to allow for the possibility of other models of classical gravity. Our task of demonstrating the quantum valued nature of the gravitational field can now be summarised as inferring a non-zero value of $\alpha$, while distinguishing this from the effects of a possibly non-zero, unknown value of $\beta$ and $\gamma$. Fortunately, we can ignore $\gamma$, as for a fixed total number of particles, this parameter results only in a global phase shift to the state, which is un-detectable. We therefore estimate the smallest value of $\alpha$ that can be detected in the presence of the unknown nuisance parameter $\beta$. We can place a bound on our ability to determine $\alpha$ by introducing the quantum Fisher covariance matrix [19], with elements

$$F_{i,j} = 2 \left( \langle \partial_i \Phi | \partial_j \Phi \rangle + \langle \partial_j \Phi | \partial_i \Phi \rangle - 2 \langle \Phi | \partial_i \Phi \rangle \langle \partial_j \Phi | \Phi \rangle \right).$$

(16)

for $\{i, j\} = \{\alpha, \beta\}$. In general, the quantum Fisher covariance matrix resulting from the state

$$|\Psi\rangle = \hat{U}_Q \hat{U}_C |\Psi_i\rangle,$$

(17)

contains off-diagonal terms, and therefore our ability to distinguish the presence of $\alpha$ from the possible presence of $\beta$ is hindered. However, by carefully choosing our initial state, we can find a diagonal quantum Fisher covariance matrix with large diagonal terms. Introducing the collective spin operators $\hat{J}_k = \frac{1}{2} \hat{a}^\dagger \sigma_k \hat{a}$, where $\alpha = (\hat{a}, \hat{b})^T$ and $\sigma_k$ is the $k$th Pauli matrix, and denoting $|m_k\rangle$ as the eigenstate of $\hat{J}_k$ with eigenvalue $m_k$, we find that the state

$$|\Psi_i\rangle = \frac{1}{\sqrt{2}} \left[ |0_z\rangle + \frac{1}{\sqrt{2}} \left( |(N/2)_z\rangle + |(-N/2)_z\rangle \right) \right],$$

(18)

leads to a quantum Fisher covariance matrix with elements $F_{\alpha, \alpha} = N^4 / 4$, $F_{\beta, \beta} = 2N^2$, and $F_{\alpha, \beta} = F_{\beta, \alpha} = 0$. This state is shown in Fig. 2. The lack off-diagonal terms means that the matrix is trivial to invert, and we should be able to infer the presence of a non-zero $\alpha$ as long as

$$\alpha \lesssim \frac{1}{\sqrt{k F_{\alpha, \alpha}}} = \frac{2}{\sqrt{k N^2}},$$

(19)

where $k$ is the number of repetitions of the experiment. We note that as this state is a superposition of maximal and minimal eigenstates of $\hat{A}$, it is the state that maximises $F_{\alpha, \alpha}$ for $N$ particles.

The question remains how to engineer this state, and to find a measurement scheme that saturates the Quantum Cramer-Rao bound (QCRB) [19–22]. While standard atom-optics techniques cannot easily produce Eq. (18), we can create a state that performs nearly as well via the well-studied one-axis twisting (OAT) technique [23,30]. In particular, we envisage a controllable one-axis twisting interaction via an optically induced nonlinearity as demonstrated in [24,31,33]. Starting with all the particles in mode $|a\rangle$, i.e., $|\Psi_0\rangle = (\hat{a})^N \sqrt{\frac{1}{N!}} |0\rangle = |(N/2)_z\rangle$, and applying $\pi / 2$ rotation about the $\hat{J}_x$ axis, followed by a one-axis twisting interaction for period $\tau$, followed by a second $\pi / 2$ rotation results in the state preparation unitary

$$\hat{U}_0 = \exp(i \hat{J}_x \frac{\pi}{2}) \exp(i \chi \tau \hat{J}_z^2) \exp(i \hat{J}_y \frac{\pi}{2}).$$

(20)

Setting $\chi \tau = \frac{\pi}{4}$ gives the state shown in Fig. 2 b, which is

$$|\Psi_i\rangle = \hat{U}_0 |\Psi_0\rangle = N e^{\frac{\chi \tau}{4}} \left( |\eta_x\rangle + e^{-i \frac{\chi \tau}{4}} |\eta_x\rangle \right),$$

(21)

where

$$|\eta_x\rangle = \frac{1}{\sqrt{2}} \left( |(N/2)_z\rangle + |(-N/2)_z\rangle \right),$$

(22a)

and

$$|\eta_x\rangle = \frac{1}{\sqrt{2}} \left( |(N/2)_z\rangle - e^{i \frac{\chi \tau}{4}} |(-N/2)_z\rangle \right),$$

(22b)

and the normalisation factor $N \approx \frac{1}{\kappa_a}$ as a result of $|\langle \eta_x | \eta_x \rangle| \ll 1$ for $N \gg 1$. This state has $F_{\alpha, \alpha} = N^4 / 4 - O(N^3)$, $F_{\beta, \beta} = 2(N^2 + N)$, and $F_{\alpha, \beta} = 0$. However, the task remains to find a measurement that saturates the QCRB. That is, the obtainable sensitivity from a particular measurement is given by the diagonal terms of the covariance matrix, which can be calculated from the inverse of the classical Fisher information matrix

$$\Delta \alpha^2 = \left[ F^{-1} \right]_{\alpha, \alpha},$$

(23)

where $F$ is the classical Fisher information matrix with elements

$$F_{ij} = \sum_m \frac{\partial_i P_m \partial_j P_m}{P_m}.$$

(24)

Following [34,35], we can saturate the QCRB by applying the unitary operator $\hat{U}_2 = \hat{U}_0$ after the interrogation
FIG. 2. The Husimi-Q function $Q(\theta, \phi)$ for the state given in Eq. (18) (a) and Eq. (21) (b). The Husimi-Q function is defined as $Q(\theta, \phi) = |\langle \alpha(\theta, \phi) | \Psi \rangle|^2$, where $|\alpha(\theta, \phi)\rangle = \exp(i\phi J_z) \exp(i\theta J_x)(N/2)\rangle$ is a coherent spin state. The projection into the $J_x$, $J_y$ and $J_z$ basis are also shown for Eq. (18) (c, e, g) and Eq. (21) (d, f, h) respectively.

time, and then measuring in a basis that projects into $|\Psi_0\rangle$, which can be achieved by measuring in the $J_z$ basis. This results in a classical Fisher covariance matrix with elements $F_{ij} = F_{ji}$, and therefore we can achieve the sensitivity given by Eq. (19). To illustrate how the effect of two potential models of gravity can be distinguished, figure 3 shows $\partial P_{mz}$ for $k = \alpha, \beta$. A small, non-zero value of $\alpha$ results in transfer of probability from $P_{N/2}$ to $P_{-N/2}$, while a small, non-zero value of $\beta$ results in transfer to a distribution of values around $m_z \approx 0$.

### III. EFFECTS OF DE-COHERENCE

However, another potential effect that our measurement must be able to discriminate is the possibility that the gravitational field causes decoherence \cite{5, 6, 12, 37, 38}. That is, that superposition of different gravitational source configurations suffer de-phasing. We must also allow for the possibility of other unaccounted sources of decoherence, as this could cause a signature in the detection signal that is falsely attributed to a non-zero value of $\alpha$. We account for the possibility of decoherence (due to gravitational or other effects) phenomenologically by introducing an additional parameter $\delta$, such that the effect of the de-phasing interaction on our initial state is

$$I_G[|\Psi_i\rangle] = \rho,$$

where

$$\rho = \sum_n \sum_m \exp(-\delta |\lambda_n - \lambda_m|) |n\rangle \langle m| \rho_i |m\rangle \langle n|,$$

\hspace{1cm} (25)

$$\rho_i = |\Psi_i\rangle \langle \Psi_i|,$$ and $\hat{\Gamma}|n\rangle = \lambda_n|n\rangle$ \cite{39, 40}. By setting $\hat{\Gamma} = \hat{A}$, the eigenstates represent states with well-defined gravitational interaction energy. That is, off-diagonal terms representing superpositions of states with a large difference in their gravitational interaction energy will suffer greater decoherence. Another possibility is that it is just superpositions of the source potential, rather than the total interaction energy that decohere, which could be accounted for by setting $\hat{\Gamma} = \hat{J}_z$. This choice of $\hat{\Gamma}$ would also describe decoherence due to the presence of a fluctuating field interacting with a non-vanishing difference in the magnetic dipole moment of the two atomic species.

While it is challenging to compute the full quantum Fisher covariance matrix including $\delta_A$ and $\delta_J_z$, it is straightforward to numerically compute the classical
Fisher covariance matrix. Unfortunately, the state preparation and measurement procedure presented in the previous section ($\hat{U}_2 = \hat{U}_0^1$, followed by measurement in the $J_z$ basis) leads to a non-diagonal Fisher covariance matrix for the parameters $\alpha$, $\beta$, $\delta_A$ and $\delta_J$. In fact, setting $\hat{\Gamma} = \hat{A}$, results in $\partial_\alpha P(J_z)$ with nearly identical profile to $\partial_\alpha P(J_z)$. Fortunately, an alternate choice of $\hat{U}_2$ remedies the situation. Specifically, choosing $\hat{U}_2 = \hat{U}_0$ (rather than $\hat{U}_0^1$) and computing the Fisher covariance matrix numerically results in values of $F_{\alpha,k} \ll F_{\alpha,\alpha}$ for $k \neq \alpha$, and as such can be inverted to find $[F]_{\alpha,\alpha}^{-1} \approx F_{\alpha,\alpha}$. In figure 3 we show $P(J_z)$ and $\partial_\beta P(J_z)$ for $k = \alpha$, $\beta$, $\delta_A$, and $\delta_J$. Again, we see that $\beta$, $\delta_A$, and $\delta_J$ give signatures that are easy to distinguish from $U_Q$. An additional benefit of using $\hat{U}_2 = \hat{U}_0$ (rather than $\hat{U}_2 = \hat{U}_0^1$) is that the final state has a maximally bi-modal probability distribution and is therefore optimally robust to detection noise such that our inference of $\alpha$ saturates the noisy quantum Cramer-Rao bound, as introduced in [41].

IV. EXPERIMENTAL CONSIDERATIONS

We can estimate the magnitude of $\alpha$ by assuming $u_{a(b)}(r)$ are Gaussian wave-packets of the form

$$|u_{a(b)}(r)|^2 = \frac{1}{\sigma^3 \pi^{3/2}} \exp \left[ -\frac{(r - (+)x_0)^2}{\sigma^2} \right].$$  

(26)

Inserting this into Eq. (11) gives

$$\alpha = \frac{tGm^2}{\hbar \sigma \sqrt{\pi}},$$

(27)

where we have assumed that $x_0 \gg \sigma$ such that we can ignore the contribution from $\kappa_{ab}$. Using Eq. (19), the minimum number of particles required to observe signatures of quantum gravity is then

$$N = \frac{2\hbar \sigma \sqrt{\pi}}{\sqrt{kGm^2 t}}.$$  

(28)

For realistic experimental parameters, ($\sigma = 1\mu m$, $t = 1s$, $k = 10^5$) and considering $^{87}$Rb, the required number of particles is $N \approx 10^9$, which is 2 orders of magnitude more than are used in current state-of-the-art precision sensing experiments. If we consider more massive atoms such as Ytterbium, and the possibility of employing di-atomic molecules, then the required number of particles is reduced further. Additionally, increasing the interrogation time ($t$), or the number of repetitions ($k$) are also not inconceivable. $k = 10^5$ equates to approximately 1 day of...
continuous operation (with \( t = 1 \text{s} \)), or 10 days including 10s of state preparation time. Although the preparation of exotic quantum states such as Eq. (21) has not been demonstrated in large ensembles of particles, the field is currently in a state of rapid progress [42–44].

Obviously, when searching for such minuscule signals, it is important to consider sources of systematic error. Linear shifts in energy such as Zeeman and Stark shifts due to fluctuating electric and magnetic fields, as well as fluctuations in any confining potentials could potentially be orders of magnitude larger than the gravitational signature. However, these shifts are all single-particle effects, and as our multi-parameter analysis reveals, with the appropriate choice of quantum state, these effects de-couple from the gravitational signature that we are searching for. That is, these effects would couple into the value of \( \beta \) or \( \delta J_+ \), which cause a change in the measured probability distribution that is entirely distinguishable from \( \alpha \). A potentially much more serious issue is imperfect knowledge of the inter-particle interactions due to short-range van der Waals interaction. In the single-mode approximation, these interactions contribute to a term in the Hamiltonian of the same form as the gravitational interaction. That is, they contribute to the coefficient of \( \tilde{A} \) and therefore are indistinguishable from a change in \( \alpha \). However, this coefficient has different scaling with \( \sigma \) compared to \( \alpha \). That is, as gravity is a long-range force, \( \alpha \propto \sigma^{-1} \). However, the s-wave interactions that generally dominate ultra-cold atomic gases scale as \( \sigma^{-3} \), so by varying the value of \( \sigma \) is may be possible to isolate the two contributions. Alternatively, very precise measurements of these interactions could also remove this source of systematic error.

V. DISCUSSION

It is worth discussing how a scheme involving the use of many particles differs from the schemes proposed by Bose et al. [16], and Marletto and Vedral [17], and how these differences might prove advantageous. Firstly, the use of many particles allows for a far greater space of possible quantum states, and therefore a much greater capacity to tailor the specific state to the particular requirements. Formulating the problem in the language of multi-parameter estimation, rather than searching for an entanglement witness, means that, in principle, arbitrary small signals can be found by performing more repetitions of the experiment to increase precision, assuming long-term stability can be achieved. Additionally, the freedom in the choice of the quantum state means that we can engineer our quantum state such that it is maximally sensitive to the parameter of interest (in our case, the gravitational interaction), and ensuring that this parameter provides a signature that is distinct from other effects. It is interesting to consider the scaling of the scheme with the number of particles, \( N \). Increasing \( N \) not only increases the magnitude of gravitational interaction, it also increases the ability of the quantum state to perceive a small change. Although we are not searching for entanglement, and therefore not directly demonstrating entanglement mediated through the gravitational field, a Hamiltonian of the form Eq. (10) can create entanglement, so proving that the gravitational field obeys a Hamiltonian of this form is evidence that the gravitational field can transmit entanglement. It was recently argued by Hall and Reginatto that, depending on how quantum-classical interactions are modelled, the generation of entanglement through a gravitational interaction is not definitive proof of non-classical gravity [43, 44]. It’s possible that the scheme considered here could perhaps differentiate between a broader class of classical gravitational models than the schemes presented in Bose et al. [16] and Marletto and Vedral [17], however, further investigation is required.

In summary, we have investigated a scheme to search for quantum signatures of gravity in ultra-cold quantum gases. Although it appears that the observation of quantum signatures of gravitation in ultra-cold quantum gases is beyond the ability of current experiments, improvements in particle number, interrogation time, long-term stability, and knowledge of the residual electro-magnetic forces between the atoms may enable the detection of effects of quantum gravity. The use of many microscopic particles and multi-parameter estimation may provide additional advantages over the previously proposed schemes.

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