Axion Electrodynamics in Strong Magnetic Backgrounds

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Abstract. The overcritical regime of axion-electrodynamics (AED) is investigated. For magnetic fields larger than the characteristic scale linked to AED, quantum vacuum fluctuations due to axion-like fields can dominate over those associated with the electron-positron fields. This hypothetical regime of the dominance of AED over QED is predicted to induce strong birefringence and screening effects. We show that, if the magnetic field lines are curved, extraordinary photons could be canalized along the field direction. It is also shown that the running QED coupling depends on the magnetic field strength, and for certain energy regimes, it could be screened almost to zero, making the QED building blocks very weakly interacting between each other. The impact of this phenomenon on the radiation mechanism of pulsars is discussed.

1. Introduction
The Peccei-Quinn mechanism is among the most appealing solutions to the so-called strong CP problem. However, it predicts the existence of a pseudoscalar Nambu-Goldstone boson which has not been observed so far: the axion [1–3]. Constraints on the invisible axion [4–9]—or more general—axion-like particles are deduced from their potential astro-cosmological consequences which are not reflected accordingly by the current observational data of our universe [10–13]. A basic assumption underlying this line of argument is that the interplay between axion-like particles and the well established Standard Model sector—photons in first place—is extremely feeble. As a consequence, axion-like particles produced copiously in the core of stars via the Primakoff effect might escape from there almost freely, constituting a leak of energy that accelerates the cooling of the star and, thus, shortens its lifetime. Therefore, the number of red giants in the helium-burning phase in globular clusters should diminish considerably. That this fact does not take place—at least not significantly—constraints the axion-diphoton coupling $g$ to lie below $g \lesssim 10^{-10} \text{ GeV}^{-1}$ for axion masses $m$ below the keV scale [14–16].

Precisely on the surface of stellar objects identified as neutron stars [17–20] and magnetars, magnetic fields as large as $B \sim \mathcal{O}(10^{13} - 10^{15}) \text{ G}$ are predicted to exist. As these strengths are bigger than the characteristic QED scale $B_0 = 4.42 \times 10^{13} \text{ G}$ [21–23], such astrophysical scenarios are propitious for the realization of a variety of yet unobserved quantum processes, which are central in our current understanding of the nature and origin of the pulsar radiation. Notable among them are: the photon capture effect by the magnetic field owing to the resonant
behavior [24] of the vacuum polarization in QED [25,26] and the photon splitting effect [27–29]. Clearly, the strong-field environments provided by these compact objects can also be favorable for an axion-like particles phenomenology – as they are in QED – contrary to what is predicted relying on the weak coupling assumption. This occurs, because the aforementioned field strengths could compensate for the weakness of the coupling and significantly stimulate quantum vacuum fluctuations of axion-like fields.

Theoretically, the problem linked to field strengths overpassing the corresponding electric $E_c = m/g$ and magnetic $B_c = m/g$ scales of AED have not yet attracted much attention, save in [30, 31], where their respective relevance has been analyzed in connection with phenomenologies predicted to take place in black holes and neutron stars. While an electric background $E \gtrsim E_c$ induces an axionic instability, for magnetic fields $B \gtrsim B_c$ the theory is stable and – as in QED – the screening and refractive properties induced by quantum vacuum fluctuations of axion-like fields are predicted to be more pronounced.

2. Refraction properties in a constant magnetic field

When the Lagrangian density of AED is embedded in the QED action and a magnetic field background is considered, quantum vacuum fluctuations of the axion-like field introduce a contribution to the polarization tensor, as shown in Fig. 1.\(^1\) For magnetic fields larger than\(^2\) the characteristic scale $B_c = m/g$ linked to AED, the phenomenology of the quantum vacuum could be ruled by virtual ALPs instead of fluctuations of the electron-positron field. This seems to be particularly accessible because the vacuum polarization tensor $\Pi^{\mu\nu}(x, \tilde{x})$ in AED exhibits a quadratic growth in the external field while the corresponding QED polarization tensor grows linearly [33, 34], provided $B \gg B_0$ with $B_0 = m^2_e/e \approx 4.42 \times 10^{13}$ G referring to the characteristic scale in QED. The consequence of this conjecture of axion dominance is explored in this contribution. The starting point of our analysis is the equation of motion for a small-amplitude electromagnetic wave $a_\mu(x)$ with $\partial \mu a = 0$, modified by $\Pi^{\mu\nu}(x, \tilde{x})$

$$\Box a_\mu(x) + i \int d^4 \tilde{x} \Pi^{\nu}_{\mu}(x, \tilde{x}) a^\nu(\tilde{x}) = 0. \quad (1)$$

Its solutions in the Fourier space are characterized by a massive mode and two massless states. Among the massless excitations, there is an ordinary mode with dispersion relation $q_{0,o} = |q|$ and an extraordinary mode, the dispersion law of which reads [see Fig. 2]

$$q^2_{0,e} = q^2 + \frac{1}{2} m^2_* \left[ 1 - \sqrt{1 + \frac{4 b^2}{1 + b^2} \frac{q^2}{m^2_*}} \right], \quad (2)$$

where $m_* = m \sqrt{1 + b^2}$ is the ALP mass dressed by the magnetic field parameter $b = B/B_c$. Here $q_\perp (q_\parallel)$ is the momentum perpendicular (parallel) to the field direction and $|q|^2 = q^2_\parallel + q^2_\perp$.

1 Also a one-loop contribution is involved [32]. However, in a strong field $B \gg B_c$ it can be ignored.
2 We use units with the Planck constant $\hbar$, the speed of light $c$ and the vacuum permittivity $\varepsilon_0$ taken to be unity [$\hbar = c = \varepsilon_0 = 1$].
In contrast to the ordinary mode, the dispersion relation of the extraordinary one depends on the magnetic field strength which reveals the axion-modified birefringent feature of the vacuum. When calculating the components of the group velocity \( v_{\perp,\|} = \partial q_0 / \partial q_{\perp,\|} \) linked to the extraordinary branch, we find \( q_\perp \ll m_\ast \) 

\[
v_{\perp,\|} \approx \frac{q_{\|}}{q_{0,\|}} \quad \text{and} \quad v_{\perp,\|} \approx \frac{1}{1 + b^2 q_{0,\|}} \left[ 1 + \frac{2b^4 q_{\perp}^2}{1 + b^2 m_\ast^2} \right].
\]  

(3)

As a consequence, the angle between the direction of propagation of the electromagnetic energy and the external magnetic field \( \theta = \arctan( v_{\perp} / v_{\|}) \) does not coincide with the one between the photon momentum \( q \) and \( B \), i.e. \( \theta = \arctan(q_{\perp} / q_{\|}) \). Furthermore, we observe that it tends to vanish when \( q_{\perp} \ll m_\ast \approx m_B \) the faster, the stronger the field is, since one has \( v_{\perp,\|} \to 0 \) and \( v_{\perp,\|} \to 1 \). This implies that the group velocity of extraordinary excitations tends to be parallel to the magnetic field for hard, as well as for soft photons with \( q_{\perp} \ll m_\ast \). Beyond this limit, i.e. for \( q_{\perp} \gg m_\ast \), the components of the group velocity approach \( v_{\perp,\|} \approx q_{\perp} / q_{0,\perp} \), \( v_{\perp,\|} = q_{\|} / q_{0,\|} \), and the angle \( \theta \) between the direction of propagation of the electromagnetic energy and \( B \) does not deviate substantially from the one formed by the momentum of the small-amplitude wave and the external field \( [\theta \approx \theta] \).

### 3. Running QED coupling and modified Coulomb potential

The problem of determining how the quantum vacuum fluctuation of a pseudoscalar axion field modifies the Coulomb potential \( a_C(x) = q / (4\pi|x|) \) reduces, to a large extent, to determine the explicit expression for the photon Green function including the vacuum polarization effect. This can be reached by inverting the operator which defines the equation of motion [see Eq. (1)]. As a consequence, the following expression for the axion-modified Coulomb potential was found [for details we refer the reader to Ref. [31]]:

\[
a_0(x) = \frac{q}{(2\pi)^3} \frac{e^{-iq\cdot x}}{q^2} \frac{1}{1 + \frac{e^2B^2q^2}{q^2(q^2 + m^2)}} \quad \text{for} \quad |q| \to 0 \quad \text{and} \quad |q| \to 0 \quad [21, 22].
\]  

(4)

where \( q \) is the charge of a point-like source.

Let us consider the electrostatic energy \( U(x) = -ea_0(x) \) of an electron. The insertion of Eq. (4) into this formula with \( q \to e \) determines how quantum vacuum fluctuations of axion-like fields in the presence of a strong magnetic background \( [b \gg 1] \) modify the running QED coupling

\[
a_{EM}(q_{\perp}^2, q_{\|}^2) \approx \frac{\alpha}{1 + \frac{e^2B^2q_{\perp}^2}{q^2(q^2 + m^2)}} \quad \text{for} \quad q_{\perp}^2, q_{\|}^2 \ll m^2,
\]  

(5)

with \( \alpha = e^2/(4\pi) \approx 137^{-1} \) defined at \( b \to 0 \) and \( |q| \to 0 \) [21,22]. Notably, if the dominance of AED over QED takes place, \( a_{EM}(q) \) is free of the characteristic Landau pole arising in QED that rises the coupling constant to an infinite value for large enough momentum. Furthermore, its dependence on the magnetic field leads to an anisotropic behavior along and transverse to \( B \). This dependence decreases the value of the running QED coupling as the magnetic field \( B \) grows while keeping the momentum fixed. Indeed, for \( q_{\perp}^2 \ll q_{\|}^2 \ll m^2 \), one finds
that $\alpha_{\text{EM}}(q_\perp^2, q_\parallel^2) \sim \alpha/b^2 \ll 1$ tends to be screened to zero. This phenomenon makes QED an almost "trivial" theory in the aforementioned regime, meaning that the corresponding charged matter sector and the small-amplitude electromagnetic waves turn out to be very weakly interacting between each other. This situation becomes particularly dramatic if the axion mass turns out to be larger than the first pair creation threshold, i.e. $m > 2m_e$, where $m_e$ refers to the electron mass. Indeed, under such a restriction, radiation channels such as the recombination of pairs, cyclotron radiation and the photon splitting effect along with the main pair production mechanisms in highly-magnetized pulsars would be suppressed. This sort of quantum triviality [35–37] persists in domains other than the one discussed before [31]. Conversely, for $q_\perp^2/m^2 \ll 1 \ll b^2 \ll q_\parallel^2/m^2$ or $1 \ll b^2 \ll q_\perp^2/q_\parallel^2 \ll m^2/q_\parallel^2$, the screening effects are negligibly small and the running QED coupling approximates to its standard value $\alpha_{\text{EM}}(q_\perp^2, q_\parallel^2) \sim \alpha$.

The described properties of the running electromagnetic coupling have direct implications on the Coulomb potential [see Eq. (4)]. For strong magnetic fields $[b \gg 1]$ and at distances from the source much smaller than the Compton wavelength of the axion $2\pi m^{-1} \gg |x_\perp|$, the Coulomb potential of a point-like charge turns out to be of Yukawa-type in the direction perpendicular to the magnetic field while along $B$ it follows approximately half of the Coulomb law:

$$a_0(x_\perp, 0) \approx \frac{q}{4\pi|x_\perp|}e^{-\frac{1}{2}m_e|x_\perp|}, \quad a_0(0, x_\parallel) \approx \frac{1}{2} \frac{q}{4\pi|x_\parallel|}$$

Figure 3 exhibits the evolution of the electrostatic energy of an electron $\mathcal{U}(x) = -e a_0(x)$ as the magnetic field grows gradually. This evaluation was carried out from Eq. (4). While the panel most to the left has been obtained by considering the standard Coulomb potential $a_C(x) = q/(4\pi|x|)$, the remaining ones were generated by taking the field parameter $b = 1$,
b = 10, b = 50 and b → ∞, respectively. These results show that, as the field strength increases, the static field of an electric charge is anisotropic and tends to be squeezed towards the axis parallel to B. This behavior is a clear manifestation of a dimensional reduction linked to the corresponding gauge sector, and it still appears at distances for which |x_⊥| ≫ m^−1, where the Coulomb's law acquires the form

\[ a_0(x) = \frac{q}{4\pi \sqrt{|x_\perp|^2(1 + b^2) + |x_\parallel|^2}}. \]  

Unless |x_⊥| = 0, this formula tends to vanish as the external field grows and |x_⊥| ≫ b^−1|x_∥|. Conversely, for b ≫ 1 and |x_⊥| ≫ b|x_∥|, the Coulomb potential tends to be one-dimensional

\[ a_0(x) \approx \frac{q}{(4\pi |x_\parallel|)}. \]

We remark that the expression above resembles closely the long-range behavior found within a pure QED context, when the external magnetic field exceeds the characteristic scale associated with this framework \( B_0 = m_e^2/e \approx 4.42 \times 10^{13} \text{ G} \) [38–40].

4. Captures of γ quanta: implications

Let us suppose a magnetic field with its lines of force curved as it is adopted in models describing the magnetosphere of pulsars. The fields of these highly magnetized objects rotate with certain angular velocity Ω. However, as long as both its curvature radius and its rotation period 2πΩ^−1 exceed considerably the photon wavelength and the Compton wavelength of ALPs, the analysis developed above can still be applied. Indeed, whenever the previous requirements are fulfilled a geometrical optical and an adiabatic approximation can be used simultaneously to investigate the propagation of those photons subject to the aforementioned limitations. To accomplish them, we shall refer hereafter to the so-called curvature γ-radiation (q_0 > 2m_e) which is emitted tangentially to the magnetic field lines. At the moment of its emission this electromagnetic wave decomposes into ordinary and extraordinary modes and both of them are localized at the origin of the diagram in Fig. 2. As the wave propagates, the momentum components q_⊥ and q_∥ of these modes change, while their energies q_0 are kept constant approximately. Thereby the dispersion relation of the ordinary and extraordinary photons evolve from the moment of their emission following the corresponding curves displayed in Fig. 2.

Since the curvature radiation appears well below the first pair creation threshold, the main process taking place in this regime is the photon splitting: \( γ + B \rightarrow γ' + γ'' + B \). This inelastic scattering becomes feasible as long as the photon wave vector q and the magnetic field B form a nontrivial angle [θ ≠ 0]. Under such a circumstance the ordinary mode is expected to decay into two extraordinary photons before reaching the first pair creation threshold, provided B \( \gtrsim 0.2 B_0 \) [26]. Mainly because the electromagnetic sector linked to this type of degree of freedom is unaffected by ALPs, and therefore, it is not subject to the aforementioned dominance. As a result of the decay of the ordinary mode, the curvature radiation acquires a high degree of polarization in the plane spanned by q and B before the extraordinary modes reach the first pair creation threshold.

3 Only virtual modes with the polarization of the extraordinary photons carry the electrostatic interaction.
The axion-diphoton interaction opens a new channel for the splitting of a photon [see Fig. 4]. Observe that, owing to the parity invariance, only ordinary $\gamma$ photons can decay via the AED channel, and that the result of the reaction involves photons with different nature. We remark that, in the equation of motion, the term linked to this photon splitting turns out to be sub-leading as compared to the term involving the axion-mediated polarization tensor. Mainly because it depends linearly on the external field.

Now, for $q_\perp \ll m_\gamma$ the dispersion law of an extraordinary mode tends to stick to the horizontal axis as $b$ grows. This is precisely the region in which the group velocity across the magnetic field disappears $[v_{\perp,e} \rightarrow 0]$. Hence, this $\gamma$ quantum would be canalized along the magnetic line of force immediately after its emission. It is worth remarking that, if this photon capture occurs, a single sufficiently hard – belonging to the $\gamma$-range – photon might have no chance to produce an electron-positron pair before it has propagated far enough to leave the region of the strong dipole field of the pulsar where, however, the one-photon pair creation is strongly suppressed. This is guaranteed if the distance traversed by the extraordinary photon is such that the local magnetic field satisfies the relation $[b = B/B_0]$

$$b \ll 1 \ll b$$

and, up to this point the lowest pair creation threshold is not reached i.e. $q_{\perp,e}^2 - q_{\perp}^2 < 4m_\gamma^2$. While the former relation introduces the condition $B_e \ll B_0$ along with $b \gg 1$, the latter provides a characteristic value for $q_\perp$ [31]:

$$(q_{\perp,\text{thr}})^2 = 4m_\gamma^2 + \frac{1}{2}m^2 \left( \sqrt{1 + 16b^2 \frac{m_\gamma^2}{m^2}} - 1 \right) > 4m_\gamma^2.$$  \tag{9}$$

We remark that the inequality above is valid for any value of $b$ and $m$. Observe that, for a dipole magnetic field $B(r) = B_s (r_s/r)^3$ with $B_s$ the field at the star surface and $r_s$ the star’s radius, the condition (8) translates into

$$b_s^{1/3} \gg \frac{r}{r_s} \gg b_s^{1/3},$$

where the respective field parameters at the pulsar surface are $b_s = B_s/B_0$ and $b_s = B_s/B_0$. We note, that the emission of curvature $\gamma$ quanta takes place in a vicinity of the pulsar surface $[r \gtrsim r_s]$ where the local field can exceed the characteristic scale of QED $[b_s \gtrsim 1]$. It is worth remarking that, for pulsars holding $b_s^{1/3} < 1$, the inequality in the right hand side is bounded by unity. Hence, instead of the relation above we should consider

$$b_s^{1/3} \gg \frac{r}{r_s} > \max \{1, b_s^{1/3} \}.$$  \tag{10}$$

For ALPs satisfying the relations $m_b < 2m_e$ and $b \gg 1$ at the moment when $b \ll 1^4$ the characteristic value of $q_\perp$ approximates to $q_{\perp,\text{thr}} \approx 2m_e[1 + mb/(2m_e)]$. In a dipole field, the condition $m_b < 2m_e$ introduces an additional restriction in Eq. (10):

$$b_s^{1/3} \gg \frac{r}{r_s} > \max \left\{ 1, b_s^{1/3}, \left( \frac{mb}{2m_e} \right)^{1/3} \right\}.$$  \tag{11}$$

This means that the fraction of extraordinary photons with $q_\perp \ll 2m_e$, having traversed a radial distance $r$ which satisfies (11) will escape from the pulsar. This effect could explain the observed hard $\gamma$-rays emission $\gtrsim 10^3$ GeV from neutron stars, and might even provide new bounds on the parameter space of ALPs by requiring that the energy loss due to this mechanism does not exceed the corresponding luminosity of the pulsar.

\footnote{These two conditions imply that locally $m/(2m_e) \ll b^{-1} \ll 1$. Hence, we are basically referring to $m \ll 1$ MeV.}
Figure 5. Sketch of the dispersion relation for an extraordinary mode in an inhomogeneous magnetic field (red). The curve describes the scenario in which the $\gamma$ quantum reaches $q_{\perp} > 2m_e$ while $b, b > 1$. The green curve corresponds to the pure QED case, whereas the dashed blue line is linked to pure AED. As $q_{\perp}^2 \rightarrow 4m_e^2$, the combined scenario of QED+AED leads to a dispersion law which tends to be parallel to the dotted line that demarcates the first pair creation threshold. Once the field is such that $b < 1$ but $b > 1$, both the green and red curves are expected to cross the threshold. On the other hand, the fraction of extraordinary photons with $q_{\perp} > 2m_e[1 + mb/(2m_e)]$ that has traversed a distance $r$ according to (11), could decay into electron-positron pairs because their dispersion curves could cross the first threshold. Clearly, until fulfilling the aforementioned condition, these quanta could undergo a magnetic field strength characterized by $b > 1$. If during the time interval in which the previous condition applies both $q_{\perp} > 2m_e[1 + mb/(2m_e)]$ and $q_{\perp}^2 \approx 4m_e^2$ are met, the squared-root singularity of the QED polarization tensor becomes relevant [41]. Under such circumstances, QED restores the control of the phenomenology for a while. This means that an extraordinary photon which has been canalized initially due to the strong axion-induced refraction, i.e. having $q_{\perp} < 2m_e$, could be recaptured later on owing to the strong birefringent property of the vacuum driven by the quantum vacuum fluctuations of the electron-positron field. As the field decreases gradually, the capture due to the aforementioned singularity is expected to end, and eventually, the dispersion laws of these photons will cross the first pair creation threshold.

It is worth remarking that, under such a circumstance, the capture undergone by this part of the wave would induce an effective growing of the mean free path of the curvature radiation as compared to the case in which the vacuum polarization effects are considered perturbatively. This fact constitutes a striking effect that puts under scrutiny pulsar models in which the curvature radiation is supposed to propagate straightforwardly and to supply the electron-positron plasma [42–44]. In these models the plasma serves to limit a required polar gap the height of which – relative to the star surface – cannot exceed considerably the mean free path that a curvature $\gamma$–quantum traverses before creating an electron-positron pair [43]. We emphasize that the extension of the polar gap due to the axion-mediated capture of the extraordinary mode would modify the so far accepted expression for the pulsar luminosity which is proportional to the square of the gap height.

5. Discussion and outlook
Magnetic fields stronger than the characteristic scale of AED can induce nontrivial modifications of the QED phenomenology. In this contribution we have explored the scenario in which these modifications prevail hypothetically over the QED. It has been shown that, in super strong magnetic fields [$b \gg 1$], electromagnetic processes characterized by perpendicular momentum transfers much smaller than the axion mass tend to be suppressed due to the dimensional reduction that the gauge sector responsible for the electrostatic interaction undergoes. This effect represents a manifestation of the anisotropy of the running electromagnetic coupling that the axion-mediated vacuum polarization causes.

In the second part we evaluated some implications on the $\gamma$ emission mechanism of pulsars. This preliminary analysis indicates that, in a strong field regime – characterized by the
condition $B \gg B_c$ with $B_c \ll B_0$ – ALPs could enable the escape of certain part of the curvature radiation. The escaping of these photons could explain the observed hard $\gamma$-rays emission from pulsars. Hence, establishing how the gap height and the corresponding pulsar luminosity are modified by the axion-diphoton interaction would open an enticing window to probe the existence of axion-like particles. This will be the subject of a forthcoming investigation.

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