The Q.Q Interaction and Variations of Single Particle Energies

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March 31, 2020

1 Abstract

In this work we make further studies of the quadruple quadruple interaction used in shell model calculations. Whereas in a previous work we adjusted the single particle energies so as to obtain the rotational spectrum of the Elliott model, we here vary the single particle energies and examine the various spectral shapes that evolve.

2 Introduction

In a previous work \cite{1} we used a Q.Q interaction to study the spectrum of \textsuperscript{20}Ne in the context of a shell model calculation. To get the much studied Elliott model results \cite{2-7}, namely of a \(J(J+1)\) spectrum, one has to choose the single particle energies with care.

The Elliott formula \cite{2} for the energies is

\[
E(SU(3)) = \chi'[\lambda^2 + \mu^2 + \lambda \mu + 3(\lambda + \mu)] + 3\chi' L(L + 1) \tag{1}
\]

where \(\chi' = 5b^4/(32\pi) \chi\).

To get these SU(3) results in the shell model one has to introduce a single particle energy splitting \cite{8, 9}

\[
E(L_2) - E(L_1) = 3\chi' [L_2(L_2 + 1) - L_1(L_1 + 1)] \tag{2}
\]

We can now use the simple Q.Q interaction without the momentum terms. For completeness we also list the rotational model formulas of Bohr and Mottelson \cite{10}.

\[
B(E_2(I_I K \rightarrow I_f K)) = (5/16\pi)Q_0^2(I_I K 2 0|I_f K)^2. \tag{3}
\]
\[ Q(I, K) = (3K^2 - I(I + 1))/(I + 1)(2I + 3))Q_0. \] (4)

It should be noted that although both the Elliott model and the rotational model yield \( J(J + 1) \) spectra, they do not agree on the \( B(E2) \)'s. For a \( J \) to \( (J - 2) \) transition, the Elliott model shows a decreased \( B(E2) \) with increasing \( J \) and band termination whereas with Bohr and Mottelson, the corresponding \( B(E2) \)s increase with increasing \( J \).

What is new in this work is that we will change the single particle energies. We will then of course no longer get rotational spectra but it will be of interest to see what new spectral shapes emerge.

3 The 0p shell.

If we make the \( 0p_{3/2} \) and \( 0p_{1/2} \) single particle energies degenerate, then with the Q.Q interaction we get “rotational band” energies for the yrast \( J = 0, 2, 4 \) states — \( E(J) = CJ(J + 1) \). In Table 1 we show the \( p \) shell yrast spectrum as we introduce a single particle splitting \( e(p_{1/2}) - e(p_{3/2}) = \Delta \). The results are also shown in Fig 1.

| \( J/\Delta \) | 0   | 1   | 5   | 10  | 100 |
|----------------|-----|-----|-----|-----|-----|
| 0              | 0   | 0   | 0   | 0   | 0   |
| 2              | 0.8953 | 0.9122 | 0.8753 | 0.8416 | 0.7965 |
| 4              | 2.9844 | 2.8967 | 2.2777 | 2.1340 | 1.9912 |
| \( E(4)/E(2) \) | 3.3333 | 3.1777 | 2.5930 | 2.5362 | 2.5 |

Table 1: Energy Levels for the Configuration \((p^2)_\pi \) and \((p^2)_\mu \) as a Function of \( \Delta = e(p_{1/2}) - e(p_{3/2}) \). The Q.Q Interaction Is Used

![Figure 1: P Shell Yrast Spectrum](image-url)
Note that the ratio $E_4/E_2$ starts at 3.3333 and ends at 2.5. The beginning is of course the rotational limit, as was shown by Elliott. The ratio 2.5 corresponds to the single $j$ shell case — pure $p_{3/2}$. In this limit the quadruple moment of the $2^+$ state is zero because we are at midshell.

Amusingly, in a recent publication by Sharon et al. [11] a similar journey was described from a strong prolate deformation to a gamma soft rotor. At the latter $O(6)$ limit [13, 14], the quadruple moment was also zero and the ratio of energies $E_4/E_2$ was equal to 2.5.

4 The 1s 0d shell.

As noted by Kingan et al. [1] if, with the Q.Q interaction, we choose the single particle energies in the 1s−0d shell such that $e(0d_3/2) = e(0d_5/2) = 0.8952$ and $e(1s_{1/2}) = 0$, we get a rotational band. Starting from there we now move the 1s above the degenerate $d$ pair by an amount $\Delta$ and explore what interesting spaces evolve. We show numerical results in Table 2 and visual results in Fig. 2. As shown in figure 2, an interesting behavior emerges near $\Delta = 9$. One gets

Table 2: Separating 1s From 0d by an Amount $\Delta$ for 2 Protons and 2 Neutrons in the 1s−0d Shell. The Q.Q Interaction Is Used.

| $J/\Delta$ | Rot | 3   | 8   | 9   | 10  | 20  | 100 |
|------------|-----|-----|-----|-----|-----|-----|-----|
| 0          | 0.8952 | 0.6469 | 1.1499 | 1.2420 | 1.3199 | 1.2075 | 0.7264 |
| 2          | 2.9840 | 2.0062 | 1.3372 | 1.2999 | 1.2701 | 1.1362 | 1.0247 |
| 4          | 6.2664 | 4.2063 | 3.2534 | 3.2486 | 3.2532 | 3.3453 | 2.9701 |
| 6          | 10.7424 | 6.8472 | 3.7202 | 3.5202 | 3.3587 | 2.6402 | 2.1115 |
| $E(4)/E(2)$ | 3.3333 | 3.1013 | 1.1629 | 1.0466 | 0.9623 | 0.9410 | 1.4107 |

Figure 2: Separating 1s From 0d by an Amount $\Delta$ for 2 Protons and 2 Neutrons in the 1s−0d Shell
2 sets of near doublets with the $J = 2^+$ and $4^+$ nearly degenerate and likewise the $6^+$ and $8^+$. Note also that from $\Delta=10$ to $20$, the $2^+$ state is at a higher energy than the $4^+$ state. However for very large $\Delta$, the usual order recovers with $J = 4^+$ higher than $J = 2^+$. However the $J = 8^+$ state comes below the $J = 6^+$ state causing $J = 8^+$ to be isomeric. Such behavior is not unusual.

![Figure 3: The $1s\frac{1}{2}$ and $0d\frac{3}{2}$ Are Set Equal and Moved an Amount $\Delta$ Above $0d\frac{5}{2}$](image)

The trend in Fig 3, where both the $1s\frac{1}{2}$ and $0d\frac{3}{2}$ single particle energies are raised relative to $0d\frac{5}{2}$ is simpler than in Fig.2. Note that the ratio $E4/E2$, which in the rotational limit is 10/3, has a value of 3.298 for $\Delta=1$ whereas when $\Delta$ is very large the value reduces to 2.417. The latter is not too far off from the value of 2.5 in the p shell.

## 5 Added Remarks

We have had a long standing interest in results emanating from schematic interactions and in particular those from the Q,Q interaction. For example, with Esduderos [15,17] interesting degeneracies were found which involved isospin $T = 0$ and $T = 2$ "doublets". With Harper [16] it was found that the wave functions of a single $j$ shell for a system of 2 protons and 2 neutrons could be well approximated by unitary 9$j$ coefficients. Generally speaking, schematic interactions can give simple physical insights to more complex behaviors. The calculations in this work were carried out using The Shell Model Code NUSHELLX@MSU[17].
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