Research Article

Applying Hybrid PSO to Optimize Directional Overcurrent Relay Coordination in Variable Network Topologies

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1. Introduction

The aim of relay coordination in power systems is to quickly isolate fault areas to preserve service throughout most of the power systems. Protection relays play a primary role in the coordination of power system protection, covering five basic facets: reliability, selectivity, speed of operation, simplicity, and economic feasibility. Engineers must consider these five aspects as a group to overcome the problem of power system protection. During the past 40 years, the technology used in the creation and use of relays for power system protection has seen tremendous progress. Directional overcurrent relays (DOCRs) have been applied to the design of economical alternatives for the primary or backup protection of power systems. Most work on protection coordination is geared toward determining the values of time dial setting (TDS) and plug setting (PS) of the DOCRs [1].

Many studies on optimization have addressed the coordination of DOCRs. Since Urdaneta et al. [2] first introduced a methodology based on optimization principles, these techniques have been widely investigated to deal with the problem of optimal coordination of DOCRs in interconnected power networks. Urdaneta et al. [2] applied an iterative approach, the Gauss-Seidel method, to solve steady-state fixed configurations DOCRs optimization problems, and decomposition technique and hierarchical coordination procedure were adopted to find DOCRs optimal settings under steady-state multiple configurations. This study applies linear programming technique to determine the DOCRs optimal settings. TDS are calculated given the PS; then new
Particle swarm optimization (PSO) theory was introduced by Eberhart and Kennedy in 1995 as a means to handle nonlinear optimization problems [9], which has now been expanded to include a wide range of applications related to optimization. Del Valle et al. [10] presented an extensive discussion on the basic concepts, variants, and applications of PSO in power systems. The PSO technique can be applied to power system problems such as reactive power and voltage control, economic dispatch, power system reliability and security, generation expansion problem, state estimation, load flow and optimal power flow, short-term load forecasting, and capacitor placement. Wu [11] proposed a hybrid real-coded genetic algorithm with a PSO algorithm and a hybrid artificial immune algorithm with a PSO algorithm to solve global optimization problems with 13 constraints. Kao et al. [12] proposed a new hybrid algorithm based on two main swarm intelligence approaches, the ant colony optimization (ACO) and the PSO to solve capacitated vehicle-routing combinatorial optimization problems. Zhang et al. [13] proposed an improved PSO algorithm to solve a bilevel multiobjective programming problem (BLMPP) in the upper level and the lower level interactively and repeatedly until the accurate Pareto optimal solutions of the original problem were found.

There are other literatures that applied the modified PSO approach for the DOCRs optimal coordination problem. Mansour et al. [14] proposed a repair algorithm which enabled the PSO algorithm to solve a constrained optimization coordination problem that has been solved as an LP problem. Zeineldin et al. [15] proposed a modified PSO algorithm which treated the DOCRs coordination problem as a mixed integer nonlinear programming problem and determined both the optimal discrete PS and continuous TDS. This proposal is far more superior to the original PSO algorithm and the general algebraic modeling system solvers in finding a near-optimal solution for the coordination problem.

This study is an attempt to supplement the findings of these earlier studies. It is similar to the previous studies discussed above, in that the focus is on the protection coordination of DOCRs using the PSO algorithm. However, this study differs from the previous works in its approach to assess nonlinear programming for the TDS and PS for DOCRs. Little research has been done on the simultaneous consideration of the TDS and PS for the coordination optimization of DOCRs. In addition to near-end faults, this study incorporates far-end faults into the objective function and takes into account a large number of coordination constraints in the optimal coordination problem. Due to the changeability of system topology, various network topologies were included in this study. As a result, the DOCRs coordination problem is a complex optimization problem with many nonlinear constraints, and a hybrid PSO method is applied to determine the optimal relay settings. This study shows that the proposed hybrid PSO method not only is efficient but also improves the convergence of conventional PSO.

2. Formulation of the DOCRs Coordination Issue

The principal aim of the coordination problem of DOCRs is to determine the TDS and PS of each relay. Then the optimal operating times of the primary relays can be minimized, for the protection coordination pairs of primary/backup relays coordination constraints to be satisfied. Therefore, the optimization objective function can be described as follows:

$$
\text{minObj} = \sum_{i=1}^{N_{near}} t_{near,primary}^i + \sum_{j=1}^{N_{far}} t_{far,primary}^j
$$

(1)

where $N_{near}$ and $N_{far}$ are the number of primary relays responding to near-end and far-end faults, respectively. The variables $t_{near,primary}^i$ and $t_{far,primary}^j$ indicate the operating time of primary relays for near-end and far-end faults.

2.1. Coordination Criteria. The coordination constraints between the primary and the backup relays are as follows:

$$
t_{near,backup}^i - t_{near,primary}^i \geq \text{CTI},
$$

$$
t_{far,backup}^i - t_{far,primary}^j \geq \text{CTI},
$$

(2)

where $t_{near,backup}^i$ and $t_{far,backup}^j$ reveal the operation time of the backup relay for the near-end and far-end faults, respectively. CTI is the minimum coordination time interval between backup and primary relay; the aim of the time interval is to ensure that circuit breakers clear the fault as directed. There are several factors influencing the operating time of relays, including CT error, the dc offset component of fault current, and relay overtravel. Therefore, typical CTI ranges from 0.2 to 0.5 s. In this study, a CTI of 0.2 s was selected.
2.2. Relay Characteristics. The nonlinear relay characteristics function based on standard IEEE C37.112-1996 can be stated as follows:

\[ t^i = \text{TDS}^i \times \left( \frac{28.2}{(p_{ct}^i/f_{ct}^i)^2} + 0.1217 \right). \]  

(3)

The constants 28.2, 0.1217, and 2 in (3) define the shape of the extremely inverse trip characteristics.

2.3. Bounds on the TDS and PS of Relays. The time dial setting TDS is the adjustable delay of the i\textsuperscript{th} relay. In this study, the TDS values range from 0.1000 to 1.1000:

\[ 0.1000 \leq \text{TDS} \leq 1.1000. \]  

(4)

\[ I_{p} \] is the pickup current setting of the i\textsuperscript{th} relay. As usual, the value of \( I_{p} \) can be determined by

\[ 2 \text{(maximum value of load current)} \leq I_{p} \leq \frac{1}{2} \text{(minimum value of fault current)}. \]  

(5)

In (5), the "2" of 2 times maximum value of load current represents safety factor, and the "2" of 1/2 times minimum value of fault current represents dependability. In this study, the \( I_{p} \) values range from 10 to 1000, for each relay. The PS is equal to \( I_{p} \) divided by the CT ratio. The current transformer (CT) ratio for each relay is 500 : 1.

Consider,

\[ \frac{I_{p}}{\text{CT ratio}} = \text{PS}, \]  

(6)

\[ 0.02 \leq \text{PS} \leq 2. \]

\( I_{sc} \) is the short circuit current passing through the i\textsuperscript{th} relay.

2.4. Variable Network Topology Constraints. When line(s) outage occur(s), loading change or equipment is maintained; modifying network topology can improve power quality, ensuring that power supply is not affected or the duration of power failure is shortened. Once the network topology changes, the original TDS and PS values of DOCRs settings in the system are no longer suitable for the protection coordination under the new network topology. Among the literature, only [4, 5] deal with the impact of changes in the network topology on protection coordination. Adopting the original parameter settings at this point leads to miscoordination of DOCRs. Therefore, it is necessary to investigate new sets of coordination constraints corresponding to various network topologies.

In consideration of changes in network topology, (2) are revised as

\[ \left( t_{\text{nearbackup}}^i \right)^s - \left( t_{\text{nearprimary}}^i \right)^s \geq \text{CTI}, \quad s \in S, \]  

\[ \left( t_{\text{farbackup}}^i \right)^s - \left( t_{\text{farprimary}}^i \right)^s \geq \text{CTI}, \quad s \in S, \]  

(7)

where S is the set of all topologies, when a single line outage occurs in the main topology.

3. Hybrid Nelder-Mead Method and Particle Swarm Optimization

The advantage of the PSO method [9] is that it belongs to the class of global search procedures; however, numerous particles are required, and the convergence can be very slow. To compensate for problems such as these, this study adopts PSO as the basic structure and uses the Nelder-Mead (NM) simplex search method [16] to form a hybrid Nelder-Mead simplex search method and particle swarm optimization. This method avoids falling easily into the local best solution with the added feature of the NM search method, which increases the rate of movement in the correct direction. This hybrid optimization method can lead to relatively rapid convergence to find a feasible and optimal solution. The following case study makes it even clearer that the proposed NM-PSO method is superior to conventional PSO methods. This is an important contribution of this study. A conventional PSO method is incapable of dealing with constrained optimization problems. Therefore, the proposed NM-PSO method must be combined with a gradient-based repair method [17, 18] to overcome NLP problems with constraints.

Next, we introduce the algorithm procedure of the proposed NM-PSO. Assuming that the question to be solved is in n dimensions, we first produce \( N \) particles (\( N > n + 1 \), according to the experimental results of the case study) to form a swarm. For every particle that violates the constraints, we use the gradient repair method to direct infeasible solutions toward feasible regions, and in most cases, the repair method is successful in doing this. Next we arrange the calculation results of the objective function in the order of good to bad and divide the \( N \) particles into three groups, that is, \( n \) particles, the \( (n + 1) \)th particle, and \( N - (n + 1) \) particles. We calculate the top \( n \) particles and the \( (n + 1) \)th particle using the NM simplex method. The \( (n + 1) \)th particle is quickly modified to move in the correct direction, which is the key to the proposed algorithm. Through the calculation of a simple NM algorithm, the probability of finding the preferable optimal solution is increased. Subsequently, we employ the PSO method to adjust all of the \( N \) particles, which (of course) include the best particles using the NM simplex method. This procedure for adjusting the \( N \) particles involves the selection of the global best particle, selection of the neighborhood best particle, and finally the velocity updates. Thus, the global best particle of the population is determined. The entire NM-PSO optimal process is repeated until the termination condition is fulfilled. The details of the theory of the proposed NM-PSO algorithm are shown in [18, 19].

4. Simulation Results

An IEEE 8-bus test system was used to test and demonstrate the proposed method for solving a new problem formulated for the optimal protection coordination of relays. The 8-bus
Table 1: Primary/backup relay pairs and the fault currents in a fixed network topology.

| P/B relay pairs | Near-end fault current (A) | Far-end fault current (A) |
|-----------------|----------------------------|---------------------------|
| Primary relay no. | Backup relay no. | Primary relay | Backup relay | Primary relay | Backup relay |
| 1 | 6 | 3230 | 3230 | 995 | 995 |
| 8 | 9 | 6080 | 1160 | 2990 | 553 |
| 8 | 7 | 6080 | 1880 | 2990 | 78* |
| 2 | 1 | 5910 | 993 | 3550 | 393 |
| 9 | 10 | 2480 | 2480 | 1160 | 1160 |
| 8 | 7 | 5910 | 1880 | 3550 | 746 |
| 3 | 2 | 5950 | 1880 | 2240 | 2240 |
| 10 | 11 | 3880 | 2340 | 2480 | 1140 |
| 6 | 5 | 6100 | 1200 | 3230 | 626 |
| 6 | 14 | 6100 | 1870 | 3230 | 78* |
| 13 | 8 | 2980 | 2980 | 986 | 986 |
| 14 | 9 | 5190 | 1160 | 1870 | 151 |
| 7 | 5 | 5210 | 1200 | 1890 | 181 |
| 14 | 1 | 5190 | 993 | 1870 | 986* |
| 7 | 13 | 5210 | 985 | 1890 | 994* |
| 4 | 3 | 3780 | 2240 | 2400 | 1050 |
| 11 | 12 | 3700 | 3700 | 2340 | 2340 |
| 5 | 4 | 2400 | 2400 | 1200 | 1200 |
| 12 | 13 | 5890 | 985 | 3700 | 433 |
| 12 | 14 | 5890 | 1870 | 3700 | 821 |

* Passing through DOCRs fault current flow reverse direction.

# Passing through DOCRs fault current fall below their pickup current.

The system comprises 7 lines, 2 transformers, and 14 DOCRs, as shown in Figure 1. The system parameters are stated in [4]. At bus 4, there is a link to another network modeled by a short circuit capacity of 400 MVA. However, the network topology is usually assumed to be fixed. In practice, the system may be operated in different topologies due to transmission lines outage. In order to take all the various network topologies into account, a large number of coordination constraints must be met. This study considers a variable network topology under steady-state rather than under transient-state as discussed in [20]. Additional constraints, such as the miscoordination caused by large transient currents in the backup relays of protection relay pairs resulting from the tripping of nearby relays, are not considered in this study.

It is assumed that all of the DOCRs have IEEE standard inverse-time characteristics as mentioned above in (3). When the values of TDS and PS of DOCRs need to be solved simultaneously, the relay coordination problem can be formulated as a nonlinear programming problem. 20 particles are used to solve an n-dimensional problem in this study. Therefore, the population size in the NM-PSO method is 20 × n + 1, and the population size in the PSO method is 20 × n, where n = 28 is the number of the PS and TDS for the 14 DOCRs.

A Matlab 7 program is developed to implement the NM-PSO algorithm and a conventional PSO algorithm and is executed on a PC with an Intel Core i5 3.1 GHz CPU and 4 GB RAM.

Table 2: Results of a conventional PSO and the proposed NM-PSO in a fixed network topology.

| Relay no. | PSO | NM-PSO |
|-----------|-----|--------|
| PS | TDS | PS | TDS |
| 1 | 0.4644 | 0.2171 | 0.4911 | 0.1854 |
| 2 | 1.0728 | 0.5065 | 1.1621 | 0.4096 |
| 3 | 0.5681 | 0.6440 | 0.7506 | 0.3810 |
| 4 | 0.4553 | 0.7807 | 0.6156 | 0.5056 |
| 5 | 0.4861 | 0.2293 | 0.6632 | 0.1491 |
| 6 | 1.1828 | 0.2348 | 0.4056 | 1.0842 |
| 7 | 0.5468 | 0.5244 | 0.7574 | 0.2783 |
| 8 | 0.5518 | 0.7422 | 0.7954 | 0.3902 |
| 9 | 0.6187 | 0.1462 | 0.4075 | 0.2891 |
| 10 | 0.3648 | 1.0287 | 1.0075 | 0.2176 |
| 11 | 0.4392 | 1.0460 | 0.7999 | 0.3419 |
| 12 | 0.7757 | 0.9921 | 1.1674 | 0.4226 |
| 13 | 0.4754 | 0.2358 | 0.6585 | 0.1000 |
| 14 | 0.5058 | 0.6784 | 0.5940 | 0.4299 |

Obj-Fun | 6.1328 | 5.5514 |

4.1. Fixed Network Topology. In this study, the minimization of the objective function and coordination constraints are based on the consideration of simultaneous near-end fault
Table 3: Operation times of primary/backup relay pairs in a fixed network topology (with near-end faults and far-end faults) using the proposed NM-PSO method.

| No. | Backup relay Operating time (sec) | Near-end fault | Constraint value (sec) | No. | Backup relay Operating time (sec) | Far-end fault | Constraint value (sec) |
|-----|----------------------------------|----------------|-----------------------|-----|----------------------------------|--------------|-----------------------|
| 6   | 0.2583                           | 1              | 0.0583                | 6   | 3.6456                           | 1             | 3.2664                |
| 9   | 0.3337                           | 8              | 0.1335                | 9   | 1.8970                           | 8             | 1.6270                |
| 7   | 0.3799                           | 8              | 0.2464                | ×   | ×                                | ×             | ×                     |
| 1   | 0.3800                           | 2              | 0.2000                | 1   | 3.3115                           | 2             | 2.9161                |
| 10  | 0.2830                           | 9              | 0.2000                | 10  | 0.8607                           | 9             | 0.5270                |
| 7   | 0.3799                           | 2              | 0.2000                | 7   | 2.3585                           | 2             | 1.9631                |
| 2   | 0.3954                           | 3              | 0.1954                | ×   | ×                                | ×             | ×                     |
| 11  | 0.3895                           | 10             | 0.1895                | 11  | 1.2643                           | 10            | 0.9813                |
| 5   | 0.3046                           | 6              | 0.0914                | 5   | 1.1760                           | 6             | 0.9177                |
| 14  | 0.4390                           | 6              | 0.0914                | 14  | 1.8682                           | 13            | 1.4291                |
| 8   | 0.2713                           | 13             | 0.0713                | 8   | 1.3476                           | ×             | ×                     |
| 9   | 0.3337                           | 14             | 0.1045                | 9   | 0.2056                           | ×             | ×                     |
| 5   | 0.3046                           | 7              | 0.1045                | 5   | 0.2056                           | ×             | ×                     |
| 1   | 0.3807                           | 14             | 0.1045                | 1   | 0.2056                           | ×             | ×                     |
| 13  | 0.4391                           | 7              | 0.1045                | 13  | 0.2056                           | ×             | ×                     |
| 3   | 0.3752                           | 4              | 0.1728                | 3   | 1.5123                           | 4             | 1.2174                |
| 12  | 0.4316                           | 11             | 0.2316                | 12  | 0.9111                           | 11            | 0.5217                |
| 4   | 0.2949                           | 5              | 0.0949                | 4   | 0.9170                           | 5             | 0.6124                |
| 13  | 0.4391                           | 12             | 0.2389                | 13  | 2.8975                           | 12            | 2.4659                |
| 14  | 0.4390                           | 12             | 0.2389                | 14  | 2.0881                           | 12            | 1.6565                |

*: both the primary protection and the backup protection are unavailable because the current flowing through the DOCRs is either too small or in the wrong direction.

Table 4: Primary/backup relay pairs and the fault currents after the removal of Line 65.

| P/B relay pairs | Near-end fault current (A) | Far-end fault current (A) |
|-----------------|---------------------------|---------------------------|
| Primary relay no. | Backup relay no. | Primary relay | Backup relay | Primary relay | Backup relay |
| 1               | 6                        | 2990                    | 2990         | 940           | 940          |
| 8               | 9                        | 6130                    | 1320         | 3130          | 1080         |
| 8               | 7                        | 6130                    | 1780         | 3130          | 433          |
| 2               | 1                        | 5760                    | 939          | 4320          | 705          |
| 9               | 10                       | 1430                    | 1430         | 1320          | 1320         |
| 2               | 7                        | 5760                    | 1780         | 4320          | 1340         |
| 3               | 2                        | 4320                    | 4320         | 3520          | 3520         |
| 10              | 11                       | 1540                    | 0            | 1430          | 0            |
| 6               | 14                       | 5480                    | 2440         | 2990          | 433          |
| 13              | 8                        | 3130                    | 3130         | 1290          | 1290         |
| 14              | 9                        | 5290                    | 1320         | 2450          | 1130         |
| 14              | 1                        | 5290                    | 939          | 2450          | 1290*        |
| 7               | 13                       | 4320                    | 1290         | 1780          | 939*         |
| 4               | 3                        | 5060                    | 3520         | 4000          | 2790         |

*: Passing through DOCRs fault current flow reverse direction.
Table 5: Operation times of primary/backup relay pairs after the removal of Line 65 (near-end faults).

| Backup relay No. | Operating time (sec) | Operating time (sec) | Constraint value (sec) |
|------------------|---------------------|---------------------|------------------------|
| 6                | 0.2982              | 1                   | 0.0636                 |
| 9                | 0.2576              | 8                   | 0.1328                 |
| 7                | 0.4212              | 8                   | 0.1328                 |
| 1                | 0.4254              | 2                   | 0.1866                 |
| 10               | 0.6050              | 9                   | 0.2204                 |
| 7                | 0.4212              | 2                   | 0.1866                 |
| 2                | 0.2853              | 3                   | 0.1572                 |
| 14               | 0.2903              | 6                   | 0.1066                 |
| 8                | 0.2542              | 13                  | 0.0673                 |
| 9                | 0.2576              | 14                  | 0.1264                 |
| 1                | 0.4254              | 14                  | 0.1264                 |
| 13               | 0.2625              | 7                   | 0.1233                 |
| 3                | 0.1974              | 4                   | 0.1397                 |

Table 6: Results of a conventional PSO and the proposed NM-PSO in variable network topologies.

| Relay no. | PSO TDS | NM-PSO TDS |
|-----------|---------|-----------|
| 1         | 0.7999  | 0.4297    |
| 2         | 0.8892  | 1.0540    |
| 3         | 0.7498  | 0.7342    |
| 4         | 1.1999  | 0.2423    |
| 5         | 0.7921  | 0.2679    |
| 6         | 0.6497  | 1.0998    |
| 7         | 0.7674  | 0.6715    |
| 8         | 0.8868  | 0.7033    |
| 9         | 0.4900  | 0.6088    |
| 10        | 1.0138  | 0.3585    |
| 11        | 0.6267  | 1.0189    |
| 12        | 1.1804  | 0.7422    |
| 13        | 0.5661  | 0.8237    |
| 14        | 0.7781  | 0.6595    |

Table 7: 10 independent runs of a conventional PSO and the proposed NM-PSO in variable network topologies.

| 10 independent runs | PSO       | NM-PSO    |
|---------------------|-----------|-----------|
| 1                   | 65.0723   | 64.6398   |
| 2                   | 66.6168   | 65.6285   |
| 3                   | 65.2207   | 65.6045   |
| 4                   | 67.0376   | 65.6308   |
| 5                   | 66.0238   | 65.5951   |
| 6                   | 67.0316   | 64.7014   |
| 7                   | 66.9528   | 65.5822   |
| 8                   | 66.7907   | 64.8591   |
| 9                   | 66.7058   | 65.3773   |
| 10                  | 66.1551   | 65.0092   |

Mean: 66.3607, Standard deviations: 0.72601, 0.41393

In approximately 52 iterations, whereas a conventional PSO method converges to the optimal object function value in 6.1328 s. Due to the added feature of the proposed NM search method which increases the rate of movement in the correct direction, a globally optimal solution is more likely reached than a locally optimal solution. The result of an example case of 20 particles shows that the output of the proposed NM-PSO method converges rapidly to a feasible and optimal solution.

The optimal values of the PS and TDS for both the proposed NM-PSO method and a conventional PSO method, considering fixed network topology are presented in Table 2. The operating times of primary/backup relay pairs for near-end and far-end faults are shown in Table 3. The constraint values are larger or equal to 0.2 in the column of Table 3. This means that all the coordination constraints have satisfied the coordination criteria.

4.2 Variable Network Topologies. When modifications are required in the network topology due to changes in load demand or emergency conditions, continued use of the TDS and PS values of DOCRs as system settings planned according to fixed network topology will lead to miscoordination. For example, in Figure 2, when Line 65 was removed, the network topology changed in response. The new relationship of primary/backup relay pairs and new fault current at this point is shown in Table 4. Under this condition, Table 5 shows the operating time of primary/backup relay pairs and constraint values between them. In this table, there are some constraint values that do not satisfy the coordination criteria.

Using an IEEE 8-bus test system, consideration of single line outage leads to seven different network topologies. The near-end faults and far-end faults in these seven circumstances result in a total of 136 primary/backup relay pairs that must satisfy coordination constraints. To find the optimal values of the PS and TDS for both the proposed NM-PSO method and a conventional PSO method, the program is run 10 times independently for each algorithm with 300 iterations in each run, and the best result of each run is recorded in Table 6. Figure 3 shows the convergence characteristics in
Table 8: Operation times of primary/backup relay pairs after the removal of Line 65 (near-end faults) using the proposed NM-PSO method.

| Backup relay No. | Operating time (sec) | Primary relay No. | Operating time (sec) | Constraint value (sec) |
|------------------|----------------------|-------------------|----------------------|------------------------|
| 6                | 0.5452               | 1                 | 0.2968               | 0.2484                 |
| 9                | 0.7377               | 8                 | 0.2034               | 0.5343                 |
| 7                | 0.9302               | 8                 | 0.2034               | 0.7267                 |
| 1                | 2.2755               | 2                 | 0.2873               | 1.9883                 |
| 10               | 1.3815               | 9                 | 0.6249               | 0.7566                 |
| 7                | 0.9302               | 2                 | 0.2873               | 0.6429                 |
| 2                | 0.4457               | 3                 | 0.2457               | 0.2000                 |
| 14               | 0.5666               | 6                 | 0.2445               | 0.3221                 |
| 8                | 0.4834               | 13                | 0.2834               | 0.2000                 |
| 9                | 0.7377               | 14                | 0.2140               | 0.5237                 |
| 1                | 2.2755               | 14                | 0.2140               | 2.0616                 |
| 13               | 1.3070               | 7                 | 0.2441               | 1.0630                 |
| 3                | 0.3261               | 4                 | 0.1261               | 0.2000                 |

Finding the best values. As stated previously, the proposed NM-PSO method converges to an optimal solution faster than a conventional PSO method because the proposed NM-PSO method has a higher rate of movement in the correct direction and converges to a globally optimal solution more likely than a locally optimal solution. Although faster convergence and better optimal values can be obtained by increasing the number of particles, the complexity of the optimization process will also increase. The statistical results for 10 independent runs are shown in Table 7. Together Tables 6 and 7 demonstrate the superiority of NM-PSO. The operating times of primary/backup relay pairs with constraint values between them are examined, and the results are listed in Table 8. The fact that the results conform to requirements demonstrates the robustness of the proposed NM-PSO algorithm in dealing with large numbers of coordination constraints.

5. Conclusions

In this paper, the coordination optimization problem is represented by a nonlinear relay characteristic function and
a range of nonlinear coordination constraints. A detailed formulation of the optimal DOCRs coordination problem in variable network topologies is presented. This study proposes an NM-PSO method to find an optimal solution for the complex optimization problem such that a large number of coordination constraints corresponding to various network topologies are satisfied simultaneously. The proposed method has the advantages of NM and PSO methods but none of their individual drawbacks and finds the optimal solution more accurately and efficiently. It has been shown that the proposed NM-PSO method outperforms a conventional PSO method.

Abbreviations

| Symbol | Description |
|--------|-------------|
| \( N_{\text{near}} \) | The number of primary relays responding to near-end faults |
| \( N_{\text{far}} \) | The number of primary relays responding to far-end faults |
| \( t_{\text{near}}^{i} \) | The operating time of primary relays for near-end faults |
| \( t_{\text{far}}^{i} \) | The operating time of primary relays for far-end faults |
| \( t_{\text{nearbackup}}^{i} \) | The operation time of the backup relay for the near-end faults |
| \( t_{\text{farbackup}}^{i} \) | The operation time of the backup relay for the far-end faults |
| CTI | The coordination time interval |
| TDS | The time dial setting of the \( i \)th relay |
| PS | The plug setting |
| CT ratio | The current transformer ratio |
| \( I_{sc}^{i} \) | The short circuit current passing through the \( i \)th relay |

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