Optimization for cantilever piezoelectric energy harvester with a cavity

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Abstract. The structure optimization of piezoelectric energy harvester aims at frequency reduction, broadening bandwidth and increasing output mainly. This paper employs a cantilever with cavity modelled by the finite element software COMSOL Multiphysics and the effect of the cavity on transducer is studied. The first order characteristic frequency is obtained by modal analysis module and the frequency response curve is solved by frequency domain analysis module. Comparison results show that the introducing of cavity improves output voltage of energy harvester and reducing device nature frequency. Moreover, effect of the cavity size parameters such as the thickness, length and position on frequency, output and conversion rate of the harvester is studied, after that the energy harvester structure is optimized.

1. Introduction

With the continuous development of wireless sensor, the requirements for energy supply are increasing. Vibration energy harvester can absorb ambient vibration and convert into electric energy which has become an aroused general interest in recent years as its long-life, low-cost and environment friendly compared with traditional chemical batteries [1-5]. PEH (piezoelectric energy harvester) has high conversion rate and good compatibility with MEMS technology, making them more suitable for practical engineering applications [6,7]. The cantilever structure has a better characteristic of energy capture as it can generate large strain in piezoelectric layer when it vibrates [8,9], and most of current energy harvesters adopt cantilever structure.

The main goals of PEH optimization are frequency reduction, broadening bandwidth and increasing output. The voltage across the resistance reflects the amount of electrical energy converted per unit time and performance of the converter depends on output voltage. Introducing a cavity in the cantilever structure not only reduce the system nature frequency but also improve the output voltage, which is a simple and effective optimization method. Wang et al [10] designed a PZT layer-air-PZT layer composite cantilever structure, and found that the output voltage is significantly improved. Reddy et al [11] designed a PEH with a rectangular cavity and showed that an improved output voltage can be obtained compared to the beam without a cavity. Raju et al [12] developed and demonstrated a PEH with multiple rectangle cavities to harvest high power. It was found that the device with two cavities captured the highest voltage compared with the others. Ramalingam et al [13] designed a variable-
section beam with a trapezoidal cavity and then added a low-frequency beam as an optimization, reducing the nature frequency and widening the working band.

Raju et al [14] introduced cavities in two different variable width cantilever PEH and compared their conversion efficiency.

Most of the existing researches studied the influence of the presence or absence of cavity on device output from perspective of theoretical, there is few researches are about effect of cavity size parameters on output of the PEH. In this paper, a rectangle cavity is constructed in the cantilever beam modelled by finite element software COMOL Multiphysics. On the premise of ensuring the stability of the structure, the influence of cavity size parameters on output voltage, power and conversion rate is studied. This work finds that not all cases with cavity play a role in improving PEH performance and the size of cavity has a great impact on the results, and the optimal parameters are finally given.

2. Modeling

The structure of the PEH is shown in figure 1, which consists of base beam, piezoelectric layer and circuit part. The piezoelectric layer is attached to upper surface of the cantilever beam and a cavity is introduced in the base beam, where $l_b, l_p, l_c$ are length of beam, piezoelectric layer and cavity respectively; $w$ is width of structure; $h_b, h_p, h_c$ are thickness of beam, piezoelectric layer and cavity respectively; $t$ is the distance between the upper surface of the cavity and base beam; $R$ is the resistance of the external circuit; $V$ is the voltage across the resistor.

![Figure 1. Schematic for cantilever-type PEH with cavity.](image)

### Table 1. Dimension and properties of device.

| Description          | Symbol | Value | Units  |
|----------------------|--------|-------|--------|
| Length               | $l_b$  | 150   | mm     |
|                      | $l_p$  | 75    | mm     |
| Width                | $w$    | 20    | mm     |
| Thickness            | $h_b$  | 6     | mm     |
|                      | $h_p$  | 0.5   | mm     |
| Young’s modulus      | $E_b$  | 70    | Gpa    |
|                      | $E_p$  | 56    | Gpa    |
| Density              | $\rho_b$ | 2700 | Kg·m$^{-3}$ |
|                      | $\rho_p$ | 7500 | Kg·m$^{-3}$ |
| Piezoelectric stress constant | $e_{31}$ | -6.62 | C·m$^{-2}$ |
| Dielectric constant  | $\varepsilon_{33}$ | 1470 |       |
| Load                 | $F_A$  | 100   | N·m$^{-2}$ |
| Resistance           | $R$    | 5e4   | $\Omega$ |
| Damping coefficient  | $\eta_b$ | 0.015 |        |
|                      | $\eta_p$ | 0.025 |        |
The PEH is modelled by the finite element software COMSOL Multiphysics as shown in figure 2. The material of the base beam is selected as aluminium and the material of the piezoelectric layer is selected as PZT-5H. The parameters and geometric dimensions are shown in table 1. The cantilever beam structure is modelled by two-dimensional solid module and tetrahedral element is used for discretization. The complete grid contains about 5000 domain cells and 832 boundary elements and the freedom degree of solution is 10366. Considering the piezoelectric effect, the force field is coupled with the electric field and the finite element model is established, modal analysis and domain analysis are carried out then. In order to study the effect of the cavity size parameters, parametric scanning for $h_c$, $t$ and $l_c$ is performed.

![Figure 2. Finite element grid of PEH.](image)

3. Result and analysis

3.1. Effect of cavity thickness

The fundamental frequency, output voltage and power of the harvester without cavity are 254.2 Hz, 18.5 V and 3.4 mW under resonance. In order to investigate the influence of the cavity, different thicknesses of cavity are selected and set $t = \frac{h_b - h_c}{2}$. The cavity middle plane is in coincident with base layer middle plane. Perform modal analysis firstly, the nature frequency is computed, then apply uniform load to base beam and conduct frequency domain analysis, the voltage-frequency curve for different $h_c$ and nature frequency/maximum output voltage-$h_c$ curve are obtained.

It can be seen from figures 3 and 4 that the thickness of the cavity has a significant influence on the characteristic frequency and voltage: the voltage is positively correlated with the thickness of the cavity, while the natural frequency is negatively. Compared with the case without cavity, the introducing of the cavity not only reduces the characteristic frequency of the device, but also increases the output voltage during resonance, especially when $h_c$ is 4 mm, the frequency is reduced by 58%, and the voltage is increased by 35%, which verifies the necessity of cavity introducing.

![Figure 3. Voltage-frequency response curve at different $h_c$.](image)

![Figure 4. Effect of thickness of cavity $h_c$.](image)

3.2. Effect of cavity position

The parameter $t$ is the distance between the upper surface of base beam and cavity. Once $t$ has changed,
the neutral axis is obviously shifted. The expression of the strain is: \[ \varepsilon_{xx} = \frac{\partial^2 w}{\partial x^2} \], where \( z \) is the distance of point to neutral axis. Change in the position of neutral axis leads to variation of \( z \), while the strain in piezoelectric layer also changes which has an effect on the output voltage according to the force-electric coupling equation.

The thickness \( h_c \) of 1 mm, 2 mm, 3 mm, 4 mm are selected to study the effect of the \( t \) comparatively. From figure 5, with the change of the position of the cavity, it can be seen that the natural frequency decreases first and then increases. The frequency is the lowest when the cavity is at the coincident position where the central axis of cavity and beam coincide, and the bigger \( h_c \) is, the more obvious variation trend is. The same in figure 6, when the \( h_c \) is 1 mm and 2 mm, the voltage is almost constant while \( h_c \) is 3 mm and 4 mm, the variation of output voltage reaches 4 V and 6 V. Choosing \( h_c \) and \( t \) are 4 mm, 0.5 mm respectively, output voltage at resonance reaches 30.3 V, which is 26.2% higher than the case at coincident position and 63.8% higher than that without cavity. From the data it indicates that the position of the cavity has a great influence on the voltage.

As shown in figures 7-9, the conversion rate is changed with \( t \) and reaches lowest when cavity is at middle position. From the curve in figure 9, the conversion rate is only improved when it is in rectangle area at the upper left corner compared to the case without cavity. Compared the results in figures 6 and 9, it can be seen that the output voltage is highest at \( h_c = 4 \) mm, the conversion rate is lowest even much lower than the case without cavity. Therefore, considering the effect of the cavity on the output voltage and the conversion efficiency comprehensively, the thickness is selected to be 3 mm, and the size of \( t \) is
selected to be 0.5 mm.

![Figure 9](image1.png)  ![Figure 10](image2.png)

**Figure 9.** Effect of cavity position on conversion rate.  **Figure 10.** Effect of length on conversion rate and voltage.

3.3. **Effect of cavity length**

After selecting of $h_c$ and $t$, the influence of the length of the cavity $l_c$ on the energy harvester is further studied. Under the same external excitation, the calculated maximum voltage and conversion rate are shown in figure 10.

The length of the piezoelectric layer is half of the base layer. The output voltage in figure 10 increases almost linearly when the length ratio $l_c/l_b$ less than 50%, while voltage drops first and then slowly increases when the ratio is greater than 50%. For conversion rate, the conversion efficiency increases slowly and then decrease, when the ratio is greater than 50%, the conversion efficiency drops rapidly. It can be seen that the output voltage growth rate is much lower than the conversion rate reduction rate when the ratio is greater than 50%, and the optimal length is determined as 75 mm finally.

4. **Conclusion**

This paper established a model of PEH with a cavity by COMSOL Multiphysics. Comparative studies show that the introduction of cavity obviously improves the capability of the device, especially when $h_c$ is 4 mm, output voltage is increased by 35% compared with the case without cavity. The thickness, length and position have obvious impact on the output voltage and conversion rate; at last choosing the length, thickness and $t$ are 75 mm, 3 mm, 0.5 mm, corresponding nature frequency is 164 Hz that is reduced by 35.4% and the output voltage, conversion rate reach 25.6 V, 41.3%, which are much higher than the case without cavity.

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