Multivariate research in areas of phosphorus cast–iron brake shoes manufacturing using the statistical analysis and the multiple regression equations

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Abstract. The braking system is one of the most important and complex subsystems of railway vehicles, especially when it comes for safety. Therefore, installing efficient safe brakes on the modern railway vehicles is essential. Nowadays is devoted attention to solving problems connected with using high performance brake materials and its impact on thermal and mechanical loading of railway wheels. The main factor that influences the selection of a friction material for railway applications is the performance criterion, due to the interaction between the brake block and the wheel produce complex thermos–mechanical phenomena. In this work, the investigated subjects are the cast–iron brake shoes, which are still widely used on freight wagons. Therefore, the cast–iron brake shoes – with lamellar graphite and with a high content of phosphorus (0.8–1.1%) – need a special investigation. In order to establish the optimal condition for the cast–iron brake shoes we proposed a mathematical modelling study by using the statistical analysis and multiple regression equations. Multivariate research is important in areas of cast–iron brake shoes manufacturing, because many variables interact with each other simultaneously. Multivariate visualization comes to the fore when researchers have difficulties in comprehending many dimensions at one time. Technological data (hardness and chemical composition) obtained from cast–iron brake shoes were used for this purpose. In order to settle the multiple correlation between the hardness of the cast–iron brake shoes, and the chemical compositions elements several model of regression equation types has been proposed. Because a three–dimensional surface with variables on three axes is a common way to illustrate multivariate data, in which the maximum and minimum values are easily highlighted, we plotted graphical representation of the regression equations in order to explain interaction of the variables and locate the optimal level of each variable for maximal response. For the calculation of the regression coefficients, dispersion and correlation coefficients, the software Matlab was used.

1. Theoretical aspects

Brake shoes are made in a variety of materials (cast iron, composite, sintered), all of which have their own drawbacks (hard on the wheels, a low friction coefficient, cost etc.) [1–4]. Traditionally, freight wagons were fitted with brake systems using cast iron blocks. Passenger wagons still resist and are nowadays equipped with the original solution of cast iron brake blocks [5, 6].

Careful investigations on the parameters given by brake users, combined with the vast experience of the brake makers in the friction materials area, often lead to improved solutions for finding a shoes which gives the best possible technically compromise. In this way, the brake failure problems can be
solved by good co-operation [4–6]. Beside the mechanical properties of the brake shoes materials, the process technology and the parameters in the exploitation are decisive for the brake’s performance [5, 6]. Discussions are sometimes useful for and may help to create new ideas in area of the friction materials expertise and especially, in the brake shoes manufacturing (Figure 1 and Figure 2). However facts about brakes performance are most important for braking development [5–7].

In fact, the best way for brake makers to achieve better shoes is to ensure that better materials and improved manufacturing processes are used and that shoes users take account of friction conditions and improved braking processes. The main problem in shoe’s manufacturing processes is the optimal chemical composition assurance [5], [6], [8].

Therefore, the statistical analysis is useful for controlling the industrial production of braking shoes, due to the shoe’s chemical composition is not specified by railway customers. Due to industrial data analysis, the improving of the braking shoes technological process of manufacturing, according to the specifications, is needed. Statistical evaluations are helpful for brake shoe–makers in determining their actions to improve the shoe’s quality [5], [6]. Our studies on the multivariate analysis combine the statistical analysis techniques and the multivariate mapping for easier data interpretation.

Several interpretations of the correlations between the cast irons chemical components – Carbon (C), Silicon (Si), Manganese (Mn), Phosphorus (P) and Sulphur (S) – and the obtained brake shoes hardness (HB) was enounced in the recent literature related with the grey phosphorus cast–iron destined to the brake shoes, which proposed three kinds of correlations, using the Matlab area: [5], [6]

» statistical analysis related the general behavior of Carbon content in relation with the rest of chemical elements, which have influence on the hardness of brake shoes. As result, regression surfaces and correlation charts are revealed: [5], [6]

» statistical analyze related the combined behavior of all chemical elements of grey phosphorus cast iron on the hardness of brake shoes, in several correlations. As result, regression surfaces and correlation charts are revealed: [5], [6]

» statistical analyze related the equivalent carbon content value behavior on the hardness of cast iron brake shoes. As result, several regression surfaces and correlation charts are revealed [5], [6].

The current performed research upon a number of 100 charges of phosphorous cast iron brake shoes had in view to obtain multivariate three–dimensional correlations between the cast–iron brake shoes’ hardness and its chemical composition, defined by several representative elements – Manganese (Mn), Phosphorus (P) and Sulphur (S) –, which determine the microstructure.

2. The phosphorus cast iron

The properties of grey iron are primarily dependent on its composition, being influenced by the elements present in irons. The lower strength grades can be consistently produced by simply selecting the proper melting stock, but the higher strength grades require close control of their processing as well as their composition. Within a class of grey iron, hardness is a good indicator of its engineering properties and many of these properties are directly related to its hardness. The hardness is affected by the processing of the grey iron as well as the composition because these factors influence the microstructure [5], [6], [9–11].
Along with Carbon (C), Silicon (Si), Manganese (Mn) and Sulphur (S), Phosphorus (P) is traditionally regarded as one of the five elements normally present in grey cast iron. In the absence of carbide–forming components and with low cooling rates (cast in sand mould), Phosphorus form a low melting point phase in grey iron that is commonly referred as steadite (Fe₃P) [12].

The minor elements in grey iron are Phosphorus and the two interrelated elements: Manganese and Sulphur [12]. Sulphur is a very significant element because it exerts marked effects on the solidification behavior of iron. The influence of Sulphur needs to be considered relative to its reaction with Manganese in iron. For this reason, the Sulphur content in iron is usually controlled within limits and with a selected ratio of the Manganese content. Sulphur combines chemically with Manganese to form manganese sulphide (MnS), and with Iron (Fe) will form iron sulphide (FeS) and segregates on grain boundaries during freezing and precipitates during final stage of freezing [12].

Table 1. Chemical compositions of cast iron [5], [6]

| Main components   | Proportion, [%] |
|-------------------|-----------------|
| Carbon, [C]       | 3.0–3.5         |
| Phosphorus, [P]   | 1.3–1.5         |
| Silicon, [Si]     | 1.5–2.0         |
| Sulphur, [S]      | 0.1–0.15        |
| Manganese, [Mn]   | 0.5–0.8         |

Typically the alloy has a composition corresponding to conventional grey cast iron, except for the high phosphorus content. Phosphorus (P) is used as an alloying element in irons for special applications, the classical example being the brake shoes destined to trains. Due to its high wear resistance, high phosphorus grey cast iron is the best material selection for brake shoes [5], [6], [8–11]. Preferably, the Phosphorus is added to cast iron in the form of Ferro–phosphorus, which may be incorporated into cast iron in the proportions necessary to provide an alloy with the desired Phosphorus content [8]. Increasing phosphorus content, the hardness will increase. In fact, increasing steadite increases the hardness of cast iron overall [8–11].
The single effects of the main elements on the hardness of phosphorous grey cast iron are shown in Figures 3–6. Two–dimensional rendering represents only the tendency of an influence of the chemical composition, by decreasing or increasing the analyzed parameter (hardness), according to one of component or the other. The polynomial functions appreciate only on the whole influence of the chemical elements upon the hardness, at different points of the shoes, indicating only the limits of variation. For this reason, it requires an analysis of the influence of a number of cumulative items on the shoe’s hardness. In this sense, the chemical composition is an important cumulative parameter.

3. Methodology
The mathematical modeling establishes a methodology for the determination of the technological parameters values, for which a mechanical characteristic (the hardness) has the desirable values. Because is disposed of real data, the optimization model is based on industrial data, obtained from cast–iron brake shoes [5], [6].

Multiple linear regression is one of the fundamental models in statistics used to represent the relationship between a dependent variable and several independent variables and, finally, for predict (or estimate) the value of a variable by knowing the value of other variables. This article focuses on expressing the multiple linear regression model related to the hardness assurance by the chemical composition of the phosphorous cast irons destined to the brake shoes, having in view that the regression coefficients will illustrate the unrelated contributions of each independent variable towards predicting the dependent variable. The main goal of a multiple regression analysis is to see whether two or more variables co vary, and to quantify the strength of the relationship between these, whereas regression expresses the relationship in the form of a regression equation [5], [6].

In fact, multiple regression allows us to ask (and hopefully answer) the general question: What is the best predictor from the chemical composition of the phosphorous cast irons for a proper hardness of the brake shoes? In the current statistical experiments we use the correlation and regression for the following scopes:

» the interest related the cause–and–effect relationships, commonly used for investigating the relationship between two or more quantitative variables. In this case, we try to determine if the variation of the chemical composition of the phosphorous cast irons causes variation in hardness value, using the regression equations.

» the interest related the associated variables (if exist), without necessarily inferring a cause–and–effect relationship, knowing that a strong association between two or more components results in an increase in the accuracy of the prediction of proper hardness.

4. Results of modelling
Starting from the above–mentioned affirmations related to the values of Manganese (Mn), Phosphorus (P) and Sulphur (S) in the cast–irons, two new statistical experiments are effectuated. The equations of regression hiperplanes, which describe the mathematical dependency between the above–mentioned elements and the hardness, are determined. Therefore, is searched a mathematical solution which can determine the optimum chemical composition for the hardness desirable values. Thus the optimal additions can be determined in these elements to assure the proper hardness [5], [6].

In the first statistical experiment, the cumulative effect of Carbon (C), Manganese (Mn) and Sulphur (S) is analyzed. The optimal form of modeling in the case of HB = f([C], [Mn], [S]) is given by the regression equation (1), where the correlation coefficient is \( r_{f(1)} = 0.5773 \).

In the second statistical experiment, the cumulative effect of Carbon (C), Phosphorous (P) and Sulphur (S) is analyzed. The optimal form of modeling in the case of HB = f([C], [S], [P]) is given by the regression equation (2), where the correlation coefficient is \( r_{f(2)} = 0.6301 \).

\[
\text{HB} = a_1 [C]^2 + a_2 [Mn]^2 + a_3 [S]^2 + a_4 [C][Mn] + a_5 [Mn][S] + a_6 [S][C] \\
+ a_7 [C] + a_8 [Mn] + a_9 [S] + a_{10}
\]

where: \( a_1 = -553.9699; a_2 = -114.4673; a_3 = 2598.4966; a_4 = -73.6059; a_5 = 4435.1292; \)
\( a_6 = -2868.1394; a_7 = 3325.7384; a_8 = 592.9469; a_9 = -13164.8786; a_{10} = -4884.8396 \)
HB = a_1[C]^2 + a_2[S]^2 + a_3[P]^2 + a_4[C][S] + a_5[S][P] + a_6[P][C] (2)

where: a_1 = 580.2331; a_2 = 11760.6092; a_3 = -65.3734; a_4 = 3184.6546; a_5 = -38.2403; a_6 = -1017.4488; a_7 = -3894.4233; a_8 = -11324.9699; a_9 = 317.2420; a_{10} = 6703.7864

Because these surfaces, described by the equation (1) and (2), cannot be represented in the three–dimensional space, the independent variables ([C], [Mn], [S] and [P]) were successively replaced with their average values ([C]_{med}, [Mn]_{med}, [S]_{med} and [P]_{med}). Therefore, in the case of correlation HB = f([C], [Mn], [S]), the equations (3)–(5) were obtained, in which the correlation coefficient are rf_{(3)} = 0.4046, rf_{(4)} = 0.4572 and rf_{(5)} = 0.4218.

\[ HB \left[ C \right]_{med} = b_1[Mn]^2 + b_2[S]^2 + b_3[Mn][S] + b_4[Mn] + b_5[S] + b_6 \] (3)

\[ HB \left[ Mn \right]_{med} = c_1[C]^2 + c_2[S]^2 + c_3[C][S] + c_4[C] + c_5[S] + c_6 \] (4)

\[ HB \left[ S \right]_{med} = d_1[C]^2 + d_2[Mn]^2 + d_3[C][Mn] + d_4[C] + d_5[Mn] + d_6 \] (5)

In the case of correlation HB = f([C], [S], [P]), the equations (6)–(8) were obtained, in which the correlation coefficient are rf_{(6)} = 0.4902, rf_{(7)} = 0.4473 and rf_{(8)} = 0.4618.

\[ HB \left[ C \right]_{med} = b_1[S]^2 + b_2[P]^2 + b_3[S][P] + b_4[S] + b_5[P] + b_6 \] (6)

\[ HB \left[ S \right]_{med} = c_1[P]^2 + c_2[C]^2 + c_3[P][C] + c_4[P] + c_5[C] + c_6 \] (7)

\[ HB \left[ P \right]_{med} = d_1[C]^2 + d_2[S]^2 + d_3[C][S] + d_4[C] + d_5[S] + d_6 \] (8)

5. Generating the regression surfaces and the correlation charts

These regression surfaces, belonging to the three–dimensional space, can be represented graphically, resulting several variation domains, which can be interpreted as correlation charts. In this way, the correlation of the values of the two independent variables can be made, so that the dependent variables (HB) can be obtained between the requested limits.

**Figure 7.** The three–dimensional regression area HB = f([S], [Mn], [C]), when [C] = [C]_{med}

**Figure 8.** The correlation chart HB = f([S], [Mn], [C]), when [C] = [C]_{med}
Figure 9. The three-dimensional regression area \(HB = f([S], [Mn], [C])\), when \([Mn] = [Mn]_{med}\).

Figure 10. The correlation chart \(HB = f([S], [Mn], [C])\), when \([Mn] = [Mn]_{med}\).

Figure 11. The three-dimensional regression area \(HB = f([S], [Mn], [C])\), when \([S] = [S]_{med}\).

Figure 12. The correlation chart \(HB = f([S], [Mn], [C])\), when \([S] = [S]_{med}\).

Figure 13. The three-dimensional regression area \(HB = f([C], [S], [P])\), when \([C] = [C]_{med}\).

Figure 14. The correlation chart \(HB = f([C], [S], [P])\), when \([C] = [C]_{med}\).
6. Discussions

The mathematical model quantifies the influences of the independent variables ([C], [Mn] and [S], respectively [C], [S] and [P]) on the dependent variable ([HB]), and it was established by a statistical analysis. The real values of the process variables were chosen statistically, their limits of variation being given by the industrial data from the phosphorous cast–iron brake shoes manufacturing.

In order to settle the correlation between the [HB], and the above–mentioned parameters (independent variables, i.e. [C], [Mn], [S] and [P]) the following model of polynomial equation type has been proposed:

\[
Y = a_1 [X_1]^2 + a_2 [X_2]^2 + a_3 [X_3]^2 + a_4 [X_1] [X_2] + a_5 [X_1] [X_3] + a_6 [X_2] [X_3] + a_7 [X_1] + a_8 [X_2] + a_9 [X_3] + a_{10}
\]

where \( a_1, \ldots, a_{10} \) are the regression coefficients.

For visualization of the regression surfaces and for the calculation correlation coefficients the software Matlab was used. The spatial representations of the response surfaces [HB] for constant values of variables reveal that all data are located inside the considered surface.
In order to estimate the independent variables overall influence on hardness [ HB], the multiple correlation coefficients were calculated. The obtained values are: \( rf_1 = 0.5773 \) and \( rf_2 = 0.6301 \), respectively. Analysing the values of regression coefficients one can conclude that the process performance it is noticeably influenced by all three parameters, in both statistical experiments. From the value of the determination coefficient resulted that the proposed mathematical model is adequate for a confidence interval of 57–63%. In other words, in the chosen intervals the independent variables influence the hardness in an extent of approximately 60%. The rest of approximately 40% can be attributed to the effect of other chemical elements or manufacturing factors ignored during these statistical experiments.

The three–dimensional regression surface is a graphical representation of the regression equation. It is plotted to explain interaction of the variables and locate the optimal level of each variable for maximal response. Each regression surface represents the different combinations of two variables at one time while maintaining the other variable at the average value. These regression surface (Figures 7, 9, 11, 13, 15 and 17) and its respective correlation charts Figures 8, 10, 12, 14, 16 and 18) provide a visual interpretation of the interaction between two factors (when the third factor is constant) and facilitate the determination of optimum experimental conditions. The point for which the response is optimized is the point called the stationary point. The stationary point may be a point of maximum response, minimum response or a saddle point (point of inflexion).

7. Conclusions
The regression analysis provides an opportunity to specify hypotheses concerning the quality assurance of brake shoes, as well as explanatory factors in the manufacturing practice. When the analysis is correctly executed, having on base the statistically valid adjustment, this can produce quantitative effects on the quality assurance process. In the practice of the phosphorous cast–iron brake shoes manufacturing many variables interact with each other simultaneously. Multivariate research and visualization comes to the fore when the engineers have difficulties in comprehending many dimensions at one time. In order to establish the optimal condition for hardness assurance in the phosphorous cast–iron brake shoes manufacturing we proposed a mathematical modelling study by using the statistical analysis and regression equation.

Based on the experiments, on the results obtained from data processing and on the technical analysis of these data, we concluded that realization of the optimum chemical compositions of the phosphorus cast–iron can constitute a technical efficient way to assure the exploitation properties (like hardness), the material from which the brake shoes are manufactured having an important role in this sense. We believe that a chemical composition within acceptable limits, correlated with the general requirements on brake shoes manufacturing process, can give the required or proper hardness. A three–dimensional regression surfaces with variables on three axes is a common way to illustrate multivariate data, which can be applied to the brake shoes manufacturing. The advantage of this approach is that the maximum and minimum values are easily highlighted.

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