The Non-perturbative Effect on $R = \sigma_L/\sigma_T$

from QCD Vacuum

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Abstract

We investigate the non-perturbative effects on the ratio $R = \sigma_L/\sigma_T$ in lepton-nucleon deep inelastic scattering by taking into account the lowest dimensional condensate contributions from the QCD vacuum. By combining conventional perturbative QCD corrections and the Georgi-Politzer target-mass effects with the non-perturbative effects from the QCD vacuum, we give a good description of the $Q^2$ and $x$ dependences of $R$ in comparison with the recent experimental data.

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1 Introduction

The process of lepton-nucleon deep inelastic scattering has proven to be an effective tool in probing the structure of nucleons. The quantity $R = \sigma_L/\sigma_T$, the ratio of longitudinal ($\sigma_L$) to transverse ($\sigma_T$) virtual photon absorption cross sections, is one of the sensitive quantities that can yield information about the quark-gluon structure of the nucleon, thus can provide an additional consistency check of various theory predictions. Within the naive parton model with spin-1/2 partons, $R$ is expected to be small, and to decrease rapidly with increasing momentum transfer $Q^2$, whereas with spin-0 partons, $R$ should be large and increase with $Q^2$. The earlier measurements of $R$ indicated the domination of scattering from spin-1/2 constituents and confirmed the identification of partons with quarks.

In the naive quark-parton model at finite $Q^2$, one can take into account the target-mass effect by the Callan-Gross relation, i.e. $2xF_1 = F_2$, and easily find that

$$R(x, Q^2) = \frac{4M^2x^2}{Q^2}.$$  \hspace{1cm} (1)

In the perturbative QCD, $R$ is expected to decrease logarithmically with increasing $Q^2$. However, at finite $Q^2$ the less known non-perturbative effects and higher twist effects et al. may also have significant consequences on $R$. Therefore accurate measurements of $R$ with $Q^2$ and $x$ dependence might be vital for revealing the non-perturbative effects and higher twist effects by the difference between the data and the predictions with conventional pQCD corrections and target-mass effects et al. taken into account. There have been some precision measurements of the quantity $R(x, Q^2)$ in recent years [1, 2, 3], and it has been observed that the data are not completely in compatible with the predictions with pQCD corrections and target-mass effects included [4]. This indicates the necessary to take into account the
non-perturbative effects and the higher twist effects et al..

The purpose of this paper is to investigate the possible non-perturbative effects in $R(x, Q^2)$. We will first review the conventional treatment with pQCD corrections and the the Georgi-Politzer (GP) target-mass effects included. Then we will study the non-perturbative effects on $R(x, Q^2)$ by taking into account the lowest dimensional condensate contributions from the QCD vacuum. It will be shown that a good description of the $Q^2$ and $x$ dependences of the data $R$ can be obtained by combining the pQCD corrections and the Georgi-Politzer (GP) target-mass effects with the non-perturbative effects from the QCD vacuum.

2 The pQCD corrections and the Georgi-Politzer mass effects

We now briefly review the conventional treatment with the pQCD corrections and the Georgi-Politzer (GP) target-mass effects taken into account. The quantity $R$ is defined as the ratio $\sigma_L/\sigma_T$ and is related to the structure functions $F_1(x, Q^2)$ and $F_2(x, Q^2)$ by

$$R(x, Q^2) = \frac{\sigma_L}{\sigma_T} = \frac{F_2}{2xF_1}(1 + \frac{4M^2x^2}{Q^2}) - 1.$$  \hfill (2)

In the framework of pQCD, logarithmic scaling violations occur due to quark-gluon interactions. To the order $\alpha_s$, hard gluon bremsstrahlung from quarks, and photon-gluon interaction effects yield contributions to lepton-production. The QCD structure functions are given by

$$F_2^{QCD}(x, Q^2) = \sum_i e_i^2x[q_i(x, Q^2) + q_i(x, Q^2)],$$  \hfill (3)
We use the formula of Ref. [4] for the longitudinal structure function \( F_L \) in next to leading order QCD,
\[
F_{QCD}^{L}(x,Q^2) = \frac{\alpha_s(Q^2)}{2\pi} x^2 \int_x^1 \frac{du}{u^3} \left[ \frac{8}{3} F_2^{QCD}(u, Q^2) + 4 \sum_i e_i^2 u G(u, Q^2)(1 - x/u) \right],
\]
(4)
\[
2xF_1^{QCD}(x, Q^2) = F_2^{QCD} - F_L^{QCD}
\]
(5)
and
\[
R^{QCD}(x, Q^2) = \frac{F_L^{QCD}}{2xF_1^{QCD}}
\]
(6)
where
\[
\alpha_s(Q^2) = \frac{12\pi}{(33 - n_f)\ln[Q^2/\Lambda^2(n_f)]},
\]
(7)
and \( n_f \) is the flavor number of quarks. The values of \( R^{QCD} \), which are shown as dotted curves in Fig. 1, decrease with increasing \( Q^2 \). In addition, the kinematic effects due to target mass, which dominate at small \( Q^2 \) and large \( x \), should be further considered. These effects were first calculated in the framework of operator product expansion and moment analysis by Georgi and Politzer (GP) [3]. The structure functions including these GP target-mass effects are given by
\[
2F_1^{QT M}(x, Q^2) = \frac{x^2 F_1^{QCD}(\xi, Q^2)}{\xi} + \frac{2M^2 x^2}{Q^2} \frac{I_1}{k^2} + \frac{4M^4 x^3}{Q^4} \frac{I_2}{k^3}
\]
(8)
\[
F_2^{QT M}(x, Q^2) = \frac{x^2 F_2^{QCD}(\xi, Q^2)}{\xi^2} + \frac{6M^2 x^3}{Q^2} \frac{I_1}{k^4} + \frac{12M^4 x^4}{Q^4} \frac{I_2}{k^5}
\]
(9)
and
\[
R^{QT M}(x, Q^2) = \frac{F_2^{QT M}}{2xF_1^{QT M}} k^2 - 1,
\]
(10)
where
\[ k = (1 + \frac{4x^2 M^2}{Q^2})^{1/2}, \]  
\[ \xi = \frac{2x}{1 + k}, \]  
\[ I_1 = \int_{\xi}^{1} \frac{F_{QCD}^2(u, Q^2)}{u^2}, \]  
and
\[ I_2 = \int_{\xi}^{1} \int \frac{F_{QCD}^2(v, Q^2)}{v^2}. \]  

The predicted results of \( R \) including pQCD corrections and GP target-mass effects are also shown as dashed curves in Fig. 1. It has been observed that the results are still significantly lower than the data from recent precision measurement \([1]\). This indicates the necessity to take into account other contributions such as the non-perturbative effects.

3 The non-perturbative effects from the QCD vacuum

We now study the \( \langle \bar{q}q \rangle \)-corrected quark propagator by taking into account the lowest dimensional condensate contributions from the QCD vacuum. Using the obtained non-perturbative quark propagator, we describe the non-perturbative effects on the quantity \( R \).

Let us start by writing the free quark propagator
\[ i[S_F(x-y)]_{\alpha\beta}^{ab} = \langle 0 | T q_a^\alpha (x) q_b^\beta (y) | 0 \rangle, \]  
where \( a, b \) are color indices and \( \alpha, \beta \) are spinor indices. The free quark propagator without any modification of condensation can be expressed in momentum space as
\[ S_F^{-1}(p) = \not{p} - m_c \]  

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with the perturbative (current-) quark mass $m_c$ which can be neglected in large momentum transfer process. But in medium energy region, the quark propagator should be modified by taking into account non-perturbative effects \cite{7, 8, 9}. In this paper we consider the non-perturbative effects from the Feynman diagrams as shown in Fig. 2 to the quark propagator. To derive the effects of dimension-3 quark condensate contribution to the quark propagator, we use the non-perturbative vacuum expectation value (VEV) of two quark fields

$$\langle 0| \bar{q}_a^\alpha(x) q_b^\beta(y) |0\rangle_{NP} = \frac{1}{12} \delta_{ab} (1 + \frac{im\gamma_\mu (x-y)\mu}{4})_{\alpha\beta} \langle \bar{q}q \rangle + \cdots . \quad (17)$$

For the dimension-4 gluon condensate contribution to the quark propagator, we take the non-perturbative VEV of two gluon fields

$$\langle 0| A^a_\mu(x) A^b_\nu(y) |0\rangle_{NP} = \frac{1}{4} x^\lambda y^\rho \langle 0| G^a_{\lambda\mu} G^b_{\rho\nu} |0\rangle + \cdots \quad (18)$$

$$= \frac{1}{4} \frac{1}{96} x^\lambda y^\rho (g_{\lambda\rho} g_{\mu\nu} - g_{\lambda\nu} g_{\mu\rho}) \langle GG \rangle + \cdots$$

in the fixed point gauge $x^\mu A_\mu(x) = 0$, where

$$\langle GG \rangle = \langle 0| G^a_{\lambda\mu}(0) G^b_{\lambda\nu}(0) |0\rangle . \quad (19)$$

Under the chain approximation, one can obtain the complete quark propagator in momentum space \cite{3}

$$S_F^{-1}(p) = \not{p} \left[ 1 + \frac{g^2 \langle \bar{q}q \rangle (1-\xi)m}{9p^2} + \frac{g^2 \langle GG \rangle m_c^2}{12(p^2-m_c^2)^2} \right]$$

$$- \left[ m_c + \frac{g^2 \langle \bar{q}q \rangle (4-\xi)}{9p^2} + \frac{g^2 \langle GG \rangle m_c p^2}{12(p^2-m_c^2)^2} \right], \quad (20)$$

where $m$ in $S_F^{-1}(p)$ arises from incorporating the QCD equation of motion

$$(i\not{D} - m)\psi = 0. \quad (21)$$

It is necessary to emphasize that $m$, which includes the effects of the condensates of non-perturbative QCD, is different from the purely perturbative
(current-) quark mass $m_c$. Note also that $S_F^{-1}(p)$ is gauge parameter $\xi$ dependent due to the internal gluon line appearing in Fig. 2 (b). In common sense, the current quark mass $m_c$ is small, and it can be neglected in large momentum transfer process, which is equivalent to neglecting the gluon condensate term in $S_F^{-1}(p)$. Therefore $S_F^{-1}(p)$ can be rewritten as

$$S_F^{-1}(p) = \not{p} - M(p)$$

with

$$M(p) = \frac{g^2}{9p^2} \left[ (4 - \xi) - \frac{(1 - \xi) \not{p} m}{p^2} \right].$$

We require the pole of the $\langle \bar{q}q \rangle$-corrected propagator corresponding with $m$ in equation of motion (8), i.e.,

$$M(p)|_{\not{p}=m} = \frac{g^2}{3m^2} \langle \bar{q}q \rangle = m. \quad (24)$$

From this equation, one can obtain the solution of $m$ which is independent of gauge parameter $\xi$

$$m = M(p)|_{\not{p}=m} = \left( \frac{4\pi a_s(Q^2)}{3} \right)^{1/3}. \quad (25)$$

Thus the $\langle \bar{q}q \rangle$-corrected quark propagator can be written as

$$S_F^{-1}(p) = \not{p} - \left( \frac{4\pi a_s(Q^2)}{3} \langle \bar{q}q \rangle \right)^{1/3}, \quad (26)$$

We now discuss the effects of the $\langle \bar{q}q \rangle$ condensate in the nucleon structure function by means of the non-perturbative quark propagator Eq. (26). Consider the inclusive lepton-nucleon scattering

$$l + N \rightarrow l + X \quad (27)$$

where the hadronic structure is entirely contained in the tensor $W_{\mu\nu}$

$$W_{\mu\nu} = (2\pi)^3 \sum_X \langle P | J_\mu | X \rangle \langle X | J_\nu | P \rangle \delta^4(P_X - P - q)$$

$$= (-g_{\mu\nu} + \frac{gq_{\mu}q_{\nu}}{q^2}) W_1 + \frac{1}{M^2}(P_\mu - \frac{Pq}{q^2}q_\mu)(P_\nu - \frac{Pq}{q^2}q_\nu) W_2, \quad (28)$$
here $M$ is the mass of the nucleon. If $W_{\mu \nu}$ is given, one can extract $W_1$ and $W_2$ through the following formulas:

$$W_1 = \frac{1}{2} [c_2 - (1 - \frac{\nu^2}{q^2})c_1](1 - \frac{\nu^2}{q^2})^{-1};$$  \hspace{1cm} (29)$$

$$W_2 = \frac{1}{2} [3c_2 - (1 - \frac{\nu^2}{q^2})c_1](1 - \frac{\nu^2}{q^2})^{-2};$$  \hspace{1cm} (30)$$

with $c_1 = W_{\mu}^{\mu}$ and $c_2 \equiv \frac{P_{\mu}^{\nu}P_{\nu}^{\mu}}{M^2}W_{\mu \nu}$ \([10]\). All non-perturbative effects are entirely contained in $W_{\mu \nu}$. In this paper, we try to study the non-perturbative effects in the nucleon structure function from the quarks in the QCD physical vacuum. We suppose that the proton is made up of bound partons that appear as “free” Dirac particles but with the non-perturbative propagator because of being in the QCD vacuum. With the incoherence assumption, one parton contribution to $W_{\mu \nu}$ is

$$w_{\mu \nu} = (2\pi)^{\frac{3}{2}} \sum_{s,s'} \langle \vec{p}, s| J_\mu | \vec{p}', s' \rangle \langle \vec{p}', s'| J_\nu | \vec{p}, s \rangle \delta^4 (p' - p - q)$$

$$= e_i^2 \int \frac{d^3p'}{2p_0'} \delta^4 (p' - p - q) \frac{1}{2} Tr [\gamma_\mu (p' + m) \gamma_\nu (p + m)].$$  \hspace{1cm} (31)$$

Neglecting the difference among light quark masses, the quark mass $m$ in the above equation is actually a c-number multiplied by a unit matrix. According to the trace theorem that the trace of an odd number of $\gamma$’s vanishes, $w_{\mu \nu}$ can also be equivalently expressed as

$$w_{\mu \nu} = e_i^2 \int \frac{d^3p'}{2p_0'} \delta^4 (p' - p - q) \frac{1}{2} Tr [\gamma_\mu (p' + m) \gamma_\nu (p + m)];$$  \hspace{1cm} (32)$$

i.e.,

$$w_{\mu \nu} = e_i^2 \int \frac{d^3p'}{2p_0'} \delta^4 (p' - p - q) \frac{1}{2} Tr [\gamma_\mu S^{-1}_F (p') \gamma_\nu S^{-1}_F (p)],$$  \hspace{1cm} (33)$$

where $iS_F(p)$ is the quark propagator and $e_i$ is the charge of quark in unit of $e$. Generally, one should take the complete non-perturbative quark propagator including the corrections due to $\langle \bar{q}q \rangle$, $\langle GG \rangle$ and higher dimensional
condensate. However, as a simple qualitative analysis, we take only the $\langle \bar{q}q \rangle$-corrected quark propagator given by Eq. (26). For the sake of simplicity, we adopt the parton picture in which the parton 4-momentum is expressed as $p^\mu = y P^\mu$ ($0 \leq y \leq 1$) with the nucleon 4-momentum $P^\mu$; i.e., we assume that all transverse momenta are negligible and that no parton moves oppositely to the nucleon. Using Eqs. (29) and (30), we can extract one quark contribution of type $i$ quark to $W_2$, and the corresponding contribution to the nucleon structure function $F_{NP}^i(x)$ is

$$F_{NP}^{i}(y) = 2M^2 \epsilon_i^2 \delta(y - x) r_{2NP}^i(x, Q^2),$$

with

$$x = \frac{Q^2}{2M\nu},$$

$$r_{2NP}^i(x, Q^2) = 1 + \frac{(3 - 4\kappa)\nu}{2MQ^2\kappa x} \frac{4\pi\alpha_s \langle \bar{q}q \rangle}{3} \frac{2}{3},$$

and

$$\kappa = 1 + \frac{Q^2}{4M^2x^2}. $$

Suppose that the nucleon state contains $f_i(y)dy$ parton states of the type $i$ in the interval $dy$, then

$$F_{NP}^i = \sum_i \int_0^1 dy f_i(y) F_{NP}^{i}(y).$$

We adopt the convention of Ref. [10] in which a parton state has $2p_0$ partons per unit volume, while a nucleon state has $P_0/M$ nucleons per unit volume. Therefore, in one nucleon, the number of partons of type $i$, in the interval $dy$ is $f_i(y)$ multiplied by $\frac{2p_0}{(P_0/M)} = 2My$, i.e., $q_i(y)dy = 2My f_i(y)dy$, where $q_i(y)$ is the quark parton distribution with constraint of parton flavor number conservation. Summing all contributions of all quarks in the nucleon, we
obtain the structure function of the nucleon

\begin{align*}
F_2^{NP}(x, Q^2) &= \sum_i q_i(x, Q^2) x e_i^2 r_2^{NP}(x, Q^2) \\
&= F_2(x, Q^2) r_2^{NP}(x, Q^2) = \sum_i \tilde{q}_i(x, Q^2) x e_i^2,
\end{align*}

where

\begin{equation}
\tilde{q}_i(x, Q^2) = q_i(x, Q^2) r_2^{NP}(x, Q^2),
\end{equation}

which is different from \(q_i(x)\) since \(q_i(x)\) represents the probability distribution of quarks of type \(i\) and satisfies the parton flavor number sum rule, but \(\tilde{q}_i(x)\) does not. From Eq. (39), we can also extract \(F_1^{NP}\),

\begin{equation}
F_1^{NP}(x, Q^2) = F_1(x, Q^2) r_1^{NP}(x, Q^2),
\end{equation}

with

\begin{equation}
r_1^{NP}(x, Q^2) = 1 + \left(1 - 4\kappa\right) \left(\frac{4\pi\alpha_s(Q^2) \langle \bar{q}q \rangle}{3}\right)^{2/3}. \tag{42}
\end{equation}

The difference between \(r_1^{NP}(x, Q^2)\) and \(r_2^{NP}(x, Q^2)\), which is due to the non-perturbative effects from QCD vacuum, causes the violation of the Callan-Gross relation between \(F_1\) and \(F_2\). In Eqs. (39) and (41), \(F_1\) and \(F_2\) are in common sense nucleon structure functions in terms of realistic quark distributions, which are taken as those including pQCD corrections and GP target-mass effects, i.e., \(F_{1,2} = F_{1,2}^{QTM}\). Therefore the ratio \(R\) including the pQCD corrections, the GP target-mass effects and the non-perturbative effects from QCD vacuum can be written as

\begin{equation}
R(x, Q^2) = \frac{F_2^{QTM}(x, Q^2) r_2^{NP}(x, Q^2)}{2x F_1^{QTM}(x, Q^2) r_1^{NP}(x, Q^2)} k^2 - 1. \tag{43}
\end{equation}

The calculated results of the ratio \(R\) based on Eq. (43) are presented in Fig. 1. In the estimates of \(r_1^{NP}(x, Q^2)\) and \(r_2^{NP}(x, Q^2)\), we take the standard phenomenological value of the quark condensate \(\langle \bar{q}q \rangle = (250\text{MeV})^3\) obtained
from QCD sum rule [6]. We find from Fig. 1 (a) and (b) that the $Q^2$-
dependence of $R$ is much improved as compared with the results in which
only conventional pQCD corrections and target-mass effects are taken into
account. The results shown in Fig. 1 (c) and (d) indicate that the $x$-
dependence of $R$ is also much better than that with only pQCD corrections
and target-mass effects. This reflects the importance of taking into account
the non-perturbative effects, besides the pQCD corrections and the target-
mass effects, for a better description of the realistic behaviors of $R$.

In principle the non-perturbative effects from the QCD vacuum should
also exist in other quantities related to quark distributions, such as parton
sum rules. The non-perturbative effects on the Gottfried sum rule have been
studied in a separate work [11] and a non-trivial $Q^2$ dependence beyond the
perturbative QCD is predicted. Unlike the case of the quantity $R$ considered
in this paper, the non-perturbative effects from QCD vacuum are small in
that case and do not seem to be the dominant source for the violation of the
Gottfried sum rule.

4 Summary

We investigated the non-perturbative effects on $R$ by taking into account the
lowest dimensional condensate contributions from the QCD vacuum in the
quark propagator. We found a non-trivial modification of the conventional
quark parton model formula of the nucleon structure functions at finite
$Q^2$ and provided a better description of the $Q^2$ and $x$ dependences of the
recent precision data of $R$ by combining conventional perturbative QCD
corrections and target-mass effects with the non-perturbative effects from
the QCD vacuum.
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Figure Captions

Fig. 1. The value of $R$ at different $Q^2$ versus $x$: (a) $Q^2 = 2.5 GeV^2$; (b) $Q^2 = 5.0 GeV^2$. The value of $R$ at different $x$ versus $Q^2$: (c) $x=0.35$; (d) $x=0.5$. The dotted curves are the predictions including only pQCD corrections; the dashed are the results including the pQCD corrections and target-mass effects; and the solid curves are the results including also the non-perturbative effects from QCD vacuum, besides conventional pQCD corrections and target-mass effects. The experimental data are taken from Ref. [1]. The calculations are performed by using the GRV parameterization [5] of quark-gluon distribution functions.

Fig. 2. The non-perturbative quark propagator including:
(a). The perturbative free quark propagator;
(b). Lowest-order correction due to the nonvanishing value of $\langle \bar{q}q \rangle$;
(c). Lowest-order correction due to the nonvanishing value of $\langle GG \rangle$. 

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Fig. 2
