CounterExample Guided Neural Synthesis

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Abstract. Program synthesis is the generation of a program from a specification. Correct synthesis is difficult, and methods that provide formal guarantees suffer from scalability issues. On the other hand, neural networks are able to generate programs from examples quickly but are unable to guarantee that the program they output actually meets the logical specification. In this work we combine neural networks with formal reasoning: using the latter to convert a logical specification into a sequence of examples that guides the neural network towards a correct solution, and to guarantee that any solution returned satisfies the formal specification. We apply our technique to synthesising loop invariants and compare the performance to existing solvers that use SMT and existing techniques that use neural networks. Our results show that the formal reasoning based guidance improves the performance of the neural network substantially, nearly doubling the number of benchmarks it can solve.

1 Introduction

Program synthesis is the task of automatically generating a program from a given specification. In its most general form, program synthesis is complex, and scalable implementation is a challenge. Existing methods typically combine heuristic search of the space of programs together with an SMT solver that asserts formally that the candidate program satisfies the specification; a well-known instance is the CEGIS loop \cite{CEGIS}. The key idea of syntax-guided synthesis (SyGuS) is to ameliorate the scalability issue by imposing restrictions on the search space by means of a syntactic template provided by the user \cite{SyGuS}.

In parallel to these developments, machine learning is now known to be able to solve the program synthesis problem effectively for a particular special case: deep neural networks can be trained to produce string manipulation programs given a set of examples of inputs and the desired program outputs \cite{NN}. While these approaches are fast, fully specifying a non-trivial program using input-output examples only is hard \cite{ML}, and thus, effective search for a program that provably satisfies a given logical specification remains an open problem.

In this paper we present two new algorithms that combine formal reasoning using SAT/SMT solvers with a deep neural network in order to generate programs that provably satisfy a full logical specification. The first algorithm, Example-Guided Neural Synthesis, comprises of three steps: 1) we use SMT to convert the given logical specification into a set of input-output examples; the goal is to
generate a set that captures as much of the specification as possible; 2) we use the neural network to generate candidate programs from the set of input-output examples; 3) finally, we use SMT to verify whether any of the candidate programs returned by the network meets the specification. If the network does not return a correct program, we terminate with no answer.

We observe that the performance of the neural network is highly dependent on the quality of the input-output examples provided. We therefore hypothesise that the neural network might benefit from a refined set of input-output examples. Thus, the second algorithm, *CounterExample Guided Neural Synthesis* (CEGNS), integrates the first algorithm into a CEGIS loop, extracting counterexamples when the programs generated by the neural network fail to meet the specification, and using these counterexamples to generate input-output examples. The loop iterates until the neural network generates a correct program. The loop is not guaranteed to terminate.

We evaluate our algorithms using a neural network trained to synthesise predicates from input-output examples. It is potentially possible to extend our approach to functions returning bit vectors, or to other logics, though further network designs need to be explored. We evaluate our algorithms on invariant synthesis benchmarks from the synthesis competition. The key finding is that the counterexample guidance significantly increases the performance of the neural network: CEGNS nearly doubles the number of benchmarks that the neural network can solve on its own. Overall, CEGNS performs comparably to a conventional CEGIS implementation [1], but solving a different subset of the benchmarks.

The paper is structured as follows: Sections 3 and 4 introduce the architecture of the neural networks we build our algorithms around, and describes how we train them; Section 5 and 6 present Example Guided Neural Synthesis and CounterExample Guided Neural Synthesis; Section 7 describes the experimental setup, and presents the experimental results; Section 8 discusses related work.

## 2 Preliminaries

### 2.1 Program Synthesis

Program synthesis tools solve a second-order existential logic problem that can be formulated as follows: \( \exists P. \forall x. \sigma(P, x) \) where \( P \) ranges over functions, which are represented by means of programs, \( x \) is a ground term corresponding to the input arguments for the target program, and \( \sigma \) is a quantifier-free formula that represents the properties that must be satisfied by the program to meet the user’s specification.

There are many problems that can be expressed as synthesis problems of the form above. As an exemplar, we consider the problem of synthesising an invariant for a given loop. Let \( x \) denote a program state, \( I \) denote the predicate for the initial condition, \( A \) an assertion, and \( T \) a transition relation. Using the criteria of the invariant track in the SyGuS competition, a predicate \( P \) is an invariant if
it is a model for the following formula:

\[ \exists P \forall x, x'. (I(x) \Rightarrow P(x)) \land (P(x) \land T(x, x') \Rightarrow P(x')) \land (P(x) \Rightarrow A(x)) \].

2.2 CounterExample Guided Inductive Synthesis

CounterExample-Guided Inductive Synthesis (CEGIS), illustrated in Fig. 1, is an iterative process consisting of two phases: a synthesis phase and a verification phase. Given the specification \( \sigma \) of the desired program, the inductive synthesis procedure generates a candidate program \( P^* \) that satisfies \( \sigma(P^*, x) \) for a subset \( x_{inputs} \) of all possible inputs. The candidate program \( P^* \) is passed to the verification phase, which checks whether it satisfies the specification \( \sigma(P^*, x) \) for all possible inputs. This is done by checking whether \( \neg \sigma(P^*, x) \) is unsatisfiable. If so, then \( \forall x. \sigma(P^*, x) \) is valid, and we have successfully synthesised a solution and the algorithm terminates. Otherwise, the verifier extracts a counterexample \( c \) from the satisfying assignment, which is then added to the set of inputs passed to the synthesiser, and the loop reiterates.

The verification phase is typically implemented using a SAT or a SMT solver. There is a wide range of options for the inductive synthesis component; a popular choice is heuristic enumeration in combination with an SMT solver \cite{5}; another is synthesis using a SAT solver \cite{31}. A well-known limitation of enumeration-, SAT- and SMT-based techniques is their inability to synthesise constants that are not given as part of the problem description, which can potentially result in an inefficient enumeration of the search space.

3 Neural Networks for Synthesising Programs

A neural network consists of a set of neurons that aggregate input values using a weighted linear combination and a non-linear activation function to return a single output. The output of a neuron with \( k \) inputs is

\[ f(\Sigma_{i=1}^{k}(w_i \cdot x_i) + b) \]

where \( w_i \) denotes the weight for the \( i \)th input, \( x_i \) denotes the value of the \( i \)th input, \( b \) denotes the bias and \( f \) is the non-linear activation function.
The input to the neural network in our algorithms is a set of multiple I/O examples. An I/O example is an assignment to the input parameters of the function to be synthesised and a corresponding correct output. The input parameters are bitvectors of width 32, and we feed the network both the binary representation of the bitvector and the decimal representation of the value normalised to be between $-1$ and $1$. The output assignment in each example is a Boolean value. Discrete values are represented using an embedding that maps them to real numbers. The use of such embedding functions is standard.

The output of the neural network is a candidate program, represented as a sequence of program tokens. The network has a vocabulary of 50 possible program tokens, 16 of which are used to represent constants, and the remaining 34 represent program instructions, parentheses, and indicators for “start of program” and “end of program”.

We empirically explored a variety of neural network architectures. We describe a simple network to illustrate our ideas, and a more complex network with similar architecture to that used in neural program synthesis for string manipulation programs [13]. The simple network fails to solve any but the most trivial benchmarks (e.g., where true is a sufficiently strong invariant), and so our experimental work uses the complex network.

3.1 Network 1 – A Simple Network

Network 1 is a simple feed-forward neural network. In a feed-forward network, neurons are organised into layers, where all neurons in a layer have access to the same inputs; however, each neuron has its own set of weights. Information in a feed-forward network flows forward through these layers, such that the output of a layer is the input for the next. No recurrence is used, and thus no neuron is used more than once in the computation of the network’s output. Owing to this structure, feed-forward architectures can only processes data sequences of
bounded length. Data given to feed-forward networks must be preprocessed to
fixed length.

We set up Network 1 such that it synthesises functions with up to three input
parameters: any missing input parameters are filled with padding data. The
padding data is a learnable variable of the network: it can thus learn a value that
best encodes the non-existence of the corresponding parameter. Network 1, given
in Fig. 2 consists of

1. a feed-forward network for encoding;
2. a max-pooling operation aggregating the encodings for each input-output
element;
3. a feed-forward network for decoding.

All feed-forward network layer neurons use a ReLU activation function, which is
\( \max\{0, x\} \). This activation function is commonplace \[17\]. The encoder processes
an I/O example as follows: it concatenates the inputs and the output into a
single vector. This vector is fed into a feed-forward network with two neuron
layers; a first is a hidden layer and the second is the output layer. The output
of this network is a vector of floating-point values called the *encoding*, which
summarises the I/O example the network is given.

To process multiple I/O examples, the encoder network is applied separately
to each example, and so produces an encoding for each example. These encodings
are aggregated using the *max-pooling operation*, which combines the encodings
by taking the maximum value from each element of the vector. The output
of the max pooling operation applied to \( k \) encodings is computed as follows:

\[
O[j] = \max_{1 \leq i \leq k} Enc_i[j]
\]

where \( Enc_i[j] \) is the \( j \)th dimension of the \( i \)th encoding vector. The max pooling
operator is chosen based on its successful use in \[13, 27\]. Using pooling, the
network can process an arbitrary number of I/O examples by computing the
encoding using the encoder network and consolidating all encodings via the
pooling operation.

The decoder is another feed-forward network with two neuron layers. The
decoder accepts the aggregated encoding as input, and maps this to a sequence of
program tokens. The final layer of this network contains 5000 neurons, which can
represent programs of up to 100 tokens long, using our vocabulary of 50 tokens.
Each block of 50 neurons generates a single token. We use the softmax function
to convert these floating-point outputs into a discrete probability distribution
over the vocabulary for each token. The final program is computed as the most
likely token obtained from each block of 50 neurons, with the exception of the
first token, which yields the “end of program” symbol.

This neural network architecture can scale to an arbitrary number of I/O
examples, while also being very fast to run and train due to its feed-forward
structure. However, it lacks representational power. For instance, the decoder
network does not explicitly enforce a dependence between distinct program tokens
dictated by its output, whereas this dependency is evident in programs, e.g., the
token “bvgt” is often followed by a constant. Furthermore, the network does
not explicitly account for the order of input parameters, which is critical for
non-commutative program synthesis.

3.2 Network 2 – A Complex Network

Recurrent Neural Networks (RNNs) are designed to recognise patterns in sequences
of data, and have a temporal dimension. Information is cycled in a loop through
the network, and the decision a recurrent network reached at time step $t - 1$
affects the decision it will reach at the next time step. This memory allows the
network to find correlations between events that are separated by many moments
in time, and means that they can consider sequences of variable length. This
ability to consider a sequence of arbitrary length enables us to consider programs
with arbitrary numbers of input parameters. Network 2 is based on a recurrent
neural network.

Long-Short Term Memory (LSTM) cells allow RNNs to learn patterns
from sequential data more efficiently, in particular with respect to long-term
dependencies. LSTM cells compute a state representation at every time step of
an input sequence which describes the information it has processed thus far. This
state is represented using two vectors: the cell state and the hidden state. The cell
state serves as memory to retain knowledge over time, while the hidden state is
primarily used to produce the cell’s output. LSTM cell states are updated using
a combination of linear operations (e.g., multiplication and summation), which
are easily differentiable and whose gradient does not decay over time, unlike
most non-linear activations. This helps the network retain information over a
larger number of time steps. An LSTM can be conditioned on the output of a
previous cell by using the final cell state and hidden state of the previous cell as
the current cell’s initial hidden state and cell state. Network 2, shown in Fig. 3,
consists of, for each input-output example:

1. an input encoder LSTM recurrent network, which receives the input pa-
parameter values from the input-output pair, fed in sequentially;
2. an output encoder LSTM cell, which receives the Boolean output embed-
ding value from the input-output pair and is conditioned on the final cell and
hidden states of the input encoder network;
3. a decoder LSTM recurrent network, which is trained on the target pro-
gram and conditioned on the final output encoder state.

This is a typical architecture for sequence-to-sequence networks. The final program is obtained by aggregating the outputs of the decoders using pooling. We then use either greedy decoding on the output of the pooling, similar to Network 1, or Beam Search, which is a compromise between performing a time-consuming complete search to find the optimal output sequence and performing a speedy but sub-optimal greedy decoding. Beam search returns $k$
output sequences, where $k$ is referred to as the beam size. Beam search computes
the set of $k$ likely candidate sequences at each time step until all $k$ candidates have reached an end-of-sequence token, or are longer than a pre-defined limit.

Though pooling the outputs at this later point incurs extra computational cost owing to the additional decoders, this approach delivers better results in practice [13]. Furthermore, the architecture lends itself to extensions, in particular with regards to incorporating attention [11]. We choose a recurrent synthesis decoder to overcome the limitations of MLPs, namely the lack of explicit time-dependence between the decoders’ tokens and the structural upper bound on output length, albeit at the expense of computational efficiency during training.

To train Network 2, the output of the decoder is compared with the target sequence using the cross-entropy loss function, which is computed as follows:

$$L = -\sum_{i=1}^{k} y_i \cdot \log(o_i)$$

where $o$ and $y$ are $k$-dimensional vectors denoting the output and ground-truth value respectively, and $o_i$ and $y_i$ denote the $i^{th}$ dimension value of $o$ and $y$.

To train our model, we use teacher forcing, where the target sequence is fed into the decoder directly during training and used to compute the next output. We give the full details of the training approach in the next section.

4 Training the Neural Networks

4.1 Sourcing the Training Data

The performance of a neural network is dependent on the quality and quantity of the data it is trained on [34]. Neural networks typically require millions of
training examples [13, 27]. Given we do not have millions of program synthesis benchmarks available, in order to obtain a large training data set, we randomly generate predicates constructed as syntactically correct combinations of bit-vector instructions. In an ideal world, we would randomly generate benchmarks in the same format as the SyGuS benchmarks. There are two reasons we elected not to do this: first, randomly generating logical specifications that are in some way similar to the problem we want to tackle is a hard problem; and second, in order to train the network using supervised learning we need to have both the logical specification and the solution to the logical specification, and finding the solutions to invariant generation benchmarks is hard.

Our neural network synthesises programs from I/O examples, where an I/O example is an assignment to all input parameters of the program to be synthesised, and a corresponding output that satisfies the specification. Our training data is a set of candidate programs, each accompanied by a corresponding set of input-output examples. We have experimentally identified two features of “bad training data”: 1) multiple programs with different syntax but equivalent semantics, which we term equivalent mutants and 2) input-output examples that do not sufficiently differentiate between programs. To avoid bad training data, we supplement our program generator with a set of syntactic rules and an SMT-based procedure for both reducing redundancy and generating informative I/O examples.

**Eliminating equivalent mutants** We enforce several rules in our random program generator to reduce the number of equivalent mutants generated. As an example, shifting a bit vector by any number greater than the bit vector width always produces the same result as shifting the bit vector by the width. Thus, we introduce a rule such that no generated program contains any shift operation with the second operand greater than the width of the first.

It would be impractical to check whether every program generated is equivalent to any previous program generated using an SMT solver. However, we do use an SMT solver to identify a further cause of equivalent mutants: An if-then-else statement that always returns one of its conditional outputs is semantically equivalent to the same program without the if-then-else statement, e.g., $(z = z) ? x : y$ is equivalent to $x$. We identify this equivalence by recursively identifying leaf operators (i.e., operators without nested `ite` statements) and checking their branch satisfiability with an SMT solver. If a branch is infeasible, we replace the if-then-else statement with the other branch and apply the recursive function again. We continue until no leaf statements are redundant.

Further to this, we use an SMT solver to remove certain trivial programs, namely programs that always return a single constant value, or always return a single one of the input arguments.

**Input-Output Example Generation** The simplest way to generate input-output examples from a known target program is to uniformly generate input

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3 This term is borrowed from the literature on mutation testing.
values and to execute the program in order to obtain the corresponding outputs. This may not differentiate between programs that do not represent a continuous function mapping the input to output, i.e., they contain features that produce a discontinuous output such as conditional statements. We generate input-output examples for each random program such that the examples are *informative* with respect to this program, i.e., the input-output covers as many conditional branches in a program as possible, using a combination of randomly generated inputs and SMT solving. In order to obtain a distribution of inputs over the input space when using an SMT solver, we use Z3 version 4.5.1 [25] with the phase selection set to random.

### 4.2 Training the network

To train the network, we randomly generate 32 million programs distributed over programs of up to three input parameters, and of length up to give operations, with up to one arbitrary constant, in addition to a zero and one constant.

Of these 32 million programs, redundancy checks discard 2,613,768, leaving a final training set of 29,386,232 programs. For each program, we generate ten input-output examples. Training for the networks was done on an AWS p3.xlarge instance with a Tesla Z100 GPU, and took 75 hours for the complex network. The network was trained once over the training data (i.e., for one epoch) using the Adam optimiser [23] and masked cross-entropy loss [24].

### 5 Example Guided Neural Synthesis

A limitation using neural networks to synthesise programs that meet logical specifications is that neural networks are best suited to recognising patterns in input-output examples rather than logical specifications, and, given a specification and an input, it is not necessarily possible to determine a single correct output for that input. In Example Guided Neural Synthesis, we use a combination of SMT solving and heuristics to generate input-output examples that guide the network to the candidate solution, and rely on the network’s resilience to noise to overcome this limitation. The EGNS algorithm, illustrated in Fig. 4, consists of three components: An SMT-solver used to generate a sequence of input-output examples from the invariant specification; a neural network as described in Section 3.2 and an SMT solver that checks each solution generated by the network against the logical specification.

![Fig. 4. Example Guided Neural Synthesis](image)
5.1 Generating Examples

We use a combination of two approaches to generate input-output examples from a logical specification: In general the approach we take is to randomly generate possible input values across the full bitvector range and use an SMT solver to determine a possible correct output from the program to be synthesised. If the number of possible correct outputs for a given input is large, we may generate input-output examples that when combined preclude viable candidate functions, and we rely on the statistical nature of the neural network to address this problem.

In the particular case of invariants we are able to further reduce such inconsistencies. Recall that, given a predicate $P$ is an invariant iff $\forall x, x'$

$$(I(x) \Rightarrow P(x)) \land (P(x) \land T(x, x') \Rightarrow P(x')) \land (P(x) \Rightarrow A(x))$$

For a given $x$, the result of $A(x)$, $T(x, x')$ and $I(x)$ are computable. We thus look for solutions to the following formula, where $P$ and $P'$ are Boolean variables representing a correct output from $P(x)$ and $P(x')$ respectively:

$$\exists P, P', x'. (I(x) \Rightarrow P) \land (P \land T(x, x') \Rightarrow P') \land (P \Rightarrow A(x))$$ (1)

In general it is not possible to determine the correct values of $P$ and $P'$ (Figure 5). We know $P(x)$ must be true if $x$ is part of the initial conditions and does not violate the property, and that $P(x)$ must be false if $x$ violates the property, and that $P(x)'$ must be false if $x'$ violates the property. There is a region of the state space where a valid $P$ should be true because the state is reachable but we cannot determine that from a single input value $x$, and a region of the state space where a satisfying invariant could be either true or false because the states are unreachable but still satisfy the property. We test three heuristics: in the first we only generate example pairs that are definitively true or false; in the second we over-approximate the invariant, and assume that it returns true for every case where $x$ and $x'$ satisfy $A(x)$; and in the third we under-approximate the invariant and assume it returns false for every case where $x$ and $x'$ do not satisfy $I(x)$. Note that due to the statistical nature of the neural network, the use of incorrect input-output examples does not necessarily prevent the neural network from giving a correct answer. We find that the best results are obtained from a combination of all three heuristics, i.e., when we cannot determine the output, we randomly choose true or false.

If there exists an $x$ that is both in the initial conditions and does not satisfy the property, Formula 1 is not satisfiable and there no possible invariant. There are no benchmarks in the set that meet these criteria.

A limitation of randomly generating inputs across the full range of the bit vectors is that we may never hit an input for which the invariant is known to definitely be true or false. In the second approach, we guarantee that we generate inputs for which the invariant is both true and false by using a SAT solver to find values for $x$ while asserting that either the initial conditions are true, or that the property is false and the transition function is true.
6 CounterExample Guided Neural Synthesis

A natural extension to Example Guided Neural Synthesis is CounterExample Guided Neural Synthesis, where the network is integrated into an iterative process similar to CEGIS and the next set of examples are chosen based on the previous set of incorrect programs that were generated by the network, in order to try to guide the network to a program that meets the specification. The use of a counterexample-guided loop allows us to fix any incorrect outputs gradually.

An overview of this architecture is given in Fig. 6. The loop is initialised by generating a set of random input-output examples as described in Section 5.1. We over-approximate the invariant when generating the output examples. This approximation is corrected by counterexamples in subsequent iterations, inspired by IC3 [9].

6.1 CounterExample choice

It is known that counterexample choice is key for performance in CEGIS [21]. This is especially true with neural network synthesisers, as these generalise best when deployed on inputs that are similar to its training inputs. In this paper, the network trains for synthesis using very well-formed input-output sets.
which, by construction, cover all execution branches and are restricted to the program’s working input range, and we endeavour to replicate these informative input-output sets as far as possible inside the CEGNS loop.

**Probable candidate programs** The first heuristic we introduce for counterexample selection takes advantage of the complex neural network’s ability to produce not just one candidate program, but the \(k\) most probable candidate programs. Given \(k\) candidate programs and a set of input-output examples, we introduce a pre-verification step which checks each candidate starting with the most probable candidate, against the set of input-output examples. It continues checking candidates until it has found \(n\) candidates that satisfy all the input-output examples, where \(n\) is a heuristic parameter given by the user, or until it has checked the \(k^{th}\) program. It returns these programs that satisfied all input-output examples to the verifier. In the event that no candidates satisfy all input-output examples, it return the candidate which satisfies the most examples.

**Distribution of counterexamples** In order to obtain a distribution of counterexamples over the possible values that is as similar as possible to the training data, we use Z3 as a verifier with the same random phase-selection used for generating the training data. Though this does not guarantee similarity with training input-output, owing to Z3 solving a different and more constrained problem when generating counterexamples, it does increase the variance in counterexamples.

**Generation of output examples** Each counterexample consists of values assigned to the inputs \(x\) for which the invariant returned the wrong answer. To convert this to an input-output example we take the output produced by the counterexample inputs and negate it. In the case where we receive a counterexample due to states that violate the inductiveness of the candidate invariant (i.e., violate \(P(x) \land T(x, x') \Rightarrow P(x')\)), conventional model checking algorithms must make a choice whether to remove \(x\) from the invariant or to add \(x'\). Typically they act in a monotonic way, doing either one or the other. Since we initialise the loop with an over-approximation of the invariant, we opt for providing the example that guides the network towards removing \(x\) from the invariant. However, due to the statistical nature of the neural network, and the fact that the sequence of candidate invariants it generates will not be monotonic, this is not guaranteed to be the correct answer. As a measure to compensate for potentially generating input-output examples that are not compatible with a single invariant, we maintain a finite number of counterexamples, discarding older counterexamples as new ones are obtained. We also discard duplicate counterexamples.

7 Experimental Results

7.1 Experimental Setup
We use a 12-core 2.40 GHz Intel Xeon E5-2440 with 96 GB of RAM running Linux. We implement the counterexample generation, the example generation and the
program verification in C++ and the neural network in Python. We evaluate Example Guided Neural Synthesis and CounterExample Guided Neural Synthesis using a two sets of benchmarks: 1) problems from the loop invariant category taken from the Syntax Guided Synthesis Competition where we have replaced unbounded integer types by bit-vector types; 2) benchmarks that correspond to safety invariants for C programs [12]. We apply a timeout of 600s. Our code, benchmarks and the scripts in order to run the experiments are available to download\footnote{https://drive.google.com/open?id=1VkCyy7Sbipy7m3553R4gOhquXRjz4d}. 

We compare with CVC4\cite{CVC4} version 1.7-prerelease [git master dd9246f3] (CVC4 1.5 was the clear winner of the general track of the 2018 competition), our implementation of a standard CEGIS loop \cite{12} with an SMT solver as the verifier and CEGIS(T)\cite{1}.

We compare the two best configurations of CEGNS and EGNS and the solver-based tools. There were two tools that we would have liked to compare to but were unable to; the first is LoopInvGen, which narrowly beats CVC4 in the invariant track of the SyGuS competition [30]. LoopInvGen\cite{26} is limited to arithmetic over unbounded integers; consequently, we ran it on equivalent benchmarks translated to use linear integer arithmetic, of which it solved 67 in \(\sim 10\) s per benchmark. However, owing to the semantic gap, some of the benchmarks have different solutions, and it is unclear whether the performance observed is achievable on bit vectors. The second tool is Code2Inv \cite{30}, a tool based on neural networks for synthesising loop invariants for C code. Even though Code2Inv takes C files as input, it treats C integers as unbounded integers. Unfortunately, it was unable to parse our benchmarks.

7.2 Results

We empirically sample across the following parameters controlling how counterexamples are generated:

1. 1. beam: the number of programs produced by the network at once, e.g., a beam of size of \(k\) means that the network generates the \(k\) most likely programs.

2. progs: the maximum number of programs returned by the network to the verifier, as explained in Section 6.

We find that storing the maximum number of programs possible after each network call allows us to solve a few more benchmarks than only storing the first one that satisfies all the input-output examples, although it is slower. A beam size of 100 solves more benchmarks than a beam size of 10. We opt for maximising the number of benchmarks solved, in our comparison in our comparison with other solvers shown in Table 1 and use a beam size of 100, and maximally store up to 100 programs per network call.

Example Guided Neural Synthesis solves only 12 benchmarks with beam size 100 although the runtime is quick. CEGNS solves nearly twice as many benchmarks as EGNS, demonstrating that the counterexample guidance improves
Table 1. Comparison of CEGNS with the state of the art

|                        | CEGNS | EGNS | CEGIS | CEGIS(T) | CVC4 |
|------------------------|-------|------|-------|----------|------|
| Benchmarks solved      | 21    | 12   | 23    | 26       | 53   |
| Average time (s)       | 46.3 s| 13.7 s| 1.5 s | 74.3 s   | 5.5 s|

the performance of the neural network, and given a more performant network CEGNS could be a powerful tool. CEGNS solves a similar number of benchmarks to our traditional solver-based CEGIS implementation, and solves five benchmarks that CEGIS fails to solve, and two that CEGIS(T) fails to solve.

CVC4’s performance based on single invocation in formidable, in terms of both speed and benchmarks solved. However, in cases where our network and CVC4 both find a solution, the solution given by the network is typically shorter and more human readable. This is likely due to the style of the training data, and is synthesis biased towards human readable answers is an area worth exploring.

7.3 Threats to validity

Benchmark selection: We report an assessment of our approach on a diverse selection of benchmarks. Nevertheless, the set of benchmarks is limited within the scope of this paper, and the performance may not generalise to other problems. Optimality of neural network architecture: We evaluate CEGNS and EGNS on an empirically chosen neural network architecture; other architectures may be more performant. The architecture was chosen on the basis of existing literature. Choice of theories: CEGNS requires a neural network that is trained for the relevant theory, e.g., linear integer arithmetic, bit vectors etc.; our evaluation is limited to the theory of bit vectors, and results for other theories may be worse.

8 Related Work

Research applying neural networks to program synthesis typically focuses on synthesising programs from I/O examples. A main approach for this is supervised learning.

For instance, Parisotto et al. introduce the NSPS (Neuro-Symbolic Program Synthesis) system [27], which synthesise string manipulation programs from I/O examples using a Recursive-Reverse-Recursive Neural Network that syntheses programs by incrementally expanding partial programs. Neural GPUs [22] are capable of learning to perform discrete algorithmic tasks such as reversing a sequence but the program itself cannot be extracted. RobustFill also synthesises string manipulation program [13]. The architecture of the neural network of RobustFill is similar to that we use in CEGNS. Supervised learning methods require a large set of programs and I/O examples to train, which can be difficult to generate or acquire, but are very fast to query once trained.
Reinforcement learning used to train neural networks for program synthesis [2][10] does not require I/O examples to generate programs, and uses a solution checker to generate the reward needed for reinforcement learning; however, the programs synthesised are typically in restrictive languages with 10 or fewer operators, so it is not clear whether this approach would scale to synthesising invariants for bitvector programs.

Neural networks have also been used to supplement, rather than replace, conventional program synthesis techniques. For example, Balog et al. [6] develop DeepCoder, a program synthesis engine that uses a neural network to guide and augment conventional search techniques.

Code2Inv is a neural network based tool specifically for synthesising loop invariants for C programs [30]. The network is designed to mimic a human synthesising an invariant by working in a compositional fashion. The reported results are promising, but the tool will not work for loops in C programs where overflow semantics are critical to the safety of the loop, and in testing, we were unable to run the tool on any of our benchmarks.

SMT solvers are frequently used as oracles in formal program synthesis, including in the CEGIS algorithm first presented for program sketching [31][32]. Component-based approaches to program synthesis [3][14][16][18][19][29] are of interest to us as we feel neural networks would be equally capable of learning how to assemble programs from a database of components rather than from a set of instructions. Component based synthesis typically makes use of techniques from counterexample-guided synthesis [18] to type-directed search with lightweight SMT-based deduction and partial evaluation [15] and Petri nets [16].

There exist many algorithms specifically for invariant generation, and, although our algorithm has potential to be extended to more general program synthesis, it is worth mentioning the similarities. Our counterexample generation resembles IC3 [9] in the way it refines invariants. LoopInvGen [26] uses program synthesis based techniques to synthesise loop invariants, and is similarly data-driven. constraints.

9 Conclusions

We have introduced an Example Guided Neural Synthesis algorithm and a CounterExample Guided Neural Synthesis Algorithm. Both allow application of neural synthesis to logical specifications. CEGNS is a promising step towards guaranteed correct neural-network based program synthesis, and performs comparably to a purely SMT based CEGIS implementation, despite having a poorly performing network as a core component. We look forward to exploring use of counterexample guidance with more sophisticated network designs. CEGNS has potential to be extended to more general program synthesis problems, particularly if used in combination with a technique such as CEGIS(T), where the counterexamples returned provide more information than a single counterexample value.
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