Solar cycle variation in GONG and MDI data: 1995-2002

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Abstract. Both GONG and MDI projects have measured $p$-mode frequencies of the Sun for more than 7 years. Here we review what we have learnt from the temporal variation of the oscillation frequencies and splitting coefficients.

1. Introduction

Helioseismology has now been accepted as a powerful diagnostic tool to carry information from inside the sun at different depths. The basic data which enables to disseminate this information are the global oscillation modes which are characterised by the eigenfrequencies. During the solar cycle 21, it was observed that mode frequencies change with the evolution of the solar cycle (Woodard and Noyce, 1985; Libbrecht and Woodard, 1989; Elsworth et al., 1990) which suggest that these changes are manifestation of the variation of the magnetic field with solar cycle. During the early stages of the current solar cycle 23, the frequency variation has also been shown to be strongly correlated with many other activity indices at the solar surface (e.g. Bhatnagar, Jain and Tripathy, 1999). With nearly 7 years of co-temporal helioseismic data from two different instruments from two different platforms, we are now in a stronger position to study the time variation of $p$-mode frequencies with changes in the levels of solar activity.

The solar oscillations are described as spherical waves which sense the spherical geometry of the sun. The angular frequencies are represented by spherical harmonics $Y_{\ell}^m$, where the degree $\ell$ is the total number of nodal lines on the spherical surface while $m$ is the number of nodes along the equator and is restricted between $-\ell \leq m \leq \ell$; each mode with a given $\ell$ has $2\ell + 1$ values of $m$ associated with it. The cyclic frequencies $\omega_{n\ell m}$ depends on the mode which is lebeled by the value of $\ell$, $m$, and $n$; $n$ being the radial order which signifies the number of nodes in the radial direction (cf. Christensen-Dalsgaard, 1996).

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1998). The frequencies of modes of oscillations are typically represented as a mean frequency $\nu_{nt} \left( \omega_{ntm} / 2\pi \right)$ and frequency splittings $\nu_{ntm} - \nu_{nt}$ between modes in the same multiplet. The frequency splittings of the order $m$ are normally parametrised according to the formula

$$\nu_{ntm} = \nu_{nt} + \sum_{j=1}^{j_{\text{max}}} a_j(n, \ell) p_j^\ell \quad (1)$$

where the basis functions are polynomials related to the Clebsch-Gordan coefficients (Ritzwoller and Lavely, 1991). The coefficients $a_j$ are normally referred as the $a$ coefficients. The odd $a$ coefficients express the difference between prograde and retrograde mode frequencies caused by the solar rotation and can be used to calculate the solar rotation. The even $a$ coefficients sense longitudinally symmetric but spherically asymmetric properties—local variation in the sound speed or the asphericity or the magnetic field strength or a combination of the two.

2. Analysis and results

For the present study, we consider 68 overlapping GONG data sets (GONG month 2-69) derived from 108 day long time series but spaced in an interval of 36 days. These data sets cover a period from May 7, 1995 to March 30, 2002 and were produced through the standard GONG pipeline (Hill et al., 1996). Each set yields about 60,000 useful frequencies for individual $n$, $\ell$, $m$ modes for $\ell = 0$ to 150 in about 1600 multiplets. Central frequencies $\nu_{nt}$ and $a$ coefficients up to $a_9$ are derived from these multiplets. The MDI data consists of 31 non-overlapping data sets starting at May 1, 1996 and ending on November 1, 2002 with interruptions between June 16 and October 22, 1998 due to the loss of contact with SOHO. All these data sets are 72 days long except those immediately before and after the break, which are shorter. Each data set consist of centroid $p$-mode frequencies up to $\ell \approx 200$ and $a$-coefficients up to $a_{18}$ (Schou, 1999).

2.1 Frequency shifts

The mean shift $\delta \nu$ for a given $\ell$ and $n$ is calculated from the relation

$$\delta \nu(t) = \sum_{n,\ell} \frac{\delta \nu_{n,\ell}(t)}{\sigma_{n,\ell}^2} / \sum_{n,\ell} \frac{1}{\sigma_{n,\ell}^2} \quad (2)$$

where $\delta \nu_{n,\ell}(t)$ is the change in measured frequency for degree $\ell$ and radial order $n$ and $\sigma_{n,\ell}$ is the error in the observed frequency. Since the solar activity changes over the cycle, we have defined the change $\delta \nu_{n,\ell}(t) = \nu_{n,\ell}(t) - < \nu_{n,\ell} >$ where the average is taken over all the available data sets instead of a particular reference set.

The frequency shift is well known to have a strong dependence on frequency and for
Figure 1. The change in mean frequency for the period 1995-2002. In panel (a), the triangles represent the shifts calculated from non-overlapping GONG data sets while the stars represent the shifts from MDI data. Panel (b) shows the frequency shifts of the overlapping GONG data sets along with the scaled activity level represented by 10.7 cm radio flux (dashed line).

For a meaningful analysis, we have considered only those \( p \)-modes which are present in all GONG and MDI data sets in the frequency range of 1500–3500 \( \mu \text{Hz} \). Figure 1(a) shows the temporal variation of mean frequency shift for MDI and non-overlapping GONG data sets and we clearly note that there is a systematic offset between the two shifts. This may have been caused either due to different analysis technique to derive the mode frequencies (Schou et al., 2002) or because of the different time series lengths over which averages are taken to compute the frequency shifts (also see Jain and Bhatnagar, 2003). The mean frequency shift is correlated with various activity indices representing photospheric, chromospheric and coronal activities. In panel (b), we show the frequency shifts calculated from the overlapping GONG data sets along with the scaled activity index represented by 10.7 cm radio flux. It is evident that the change in frequency follows the change in solar activity very closely, the proximity being marginally higher during the ascending phase of the solar cycle. A similar result for the low-\( \ell \) \( p \)-mode frequencies were reported by Chaplin et al. (2001). For a detailed investigation, we calculate the mean frequency shifts corresponding to four different frequency bins of 500 \( \mu \text{Hz} \) (Figure 2). The offset between GONG and MDI data sets is quite apparent in lower frequency bands and slowly decreases for higher frequency ranges. For the highest frequency band (3000-3500 \( \mu \text{Hz} \)), the offset is seen only when the activity level is high. In low frequency bands, deviations from the simple activity dependence is observed during the activity minimum period.

Since both GONG and MDI frequencies are obtained from time series spanning over a
long period, these can be used to study the temporal evolution of a single \((n,\ell)\) multiplet. The variation of the central frequency of two multiplets for \(n = 6\) and \(n = 9\) corresponding to \(\ell = 60\), along with the scaled activity index is shown in Figure 3. It is remarkable that even the frequency of a single mode of oscillation closely follows the changes in activity level. As mentioned earlier, a small offset between the absolute values for GONG and MDI frequencies at lower \(n\) values corresponding to lower frequency range is clearly seen. However, the sensitivity to the activity level appears independent of the data sets used.

2.2 Variation in solar rotation rate

Early helioseismic results have confirmed that the surface differential rotation detected with Doppler Observations persists throughout the convection zone (e.g. Brown et al. 1989). The rotation rate in the convection zone along different latitudes is nearly constant, while at the base of the convection zone, a shear layer (the tachocline) separates the radiative interior which rotates almost like a solid body. In solar dynamo theories, it is generally believed that the rotation stretches the poloidal field lines near the tachocline and creates the toroidal fields, hence it is important to look for possible variations in rotation rate with time. There are two approaches, the first one is an analytical approach (Morrow, 1988; Jain, Tripathy and Bhatnagar, 2000) which calculates the surface rotation rates at different latitudes by combination of the odd \(a\) coefficients. In the second ap-
Figure 3. Temporal variation of centroid frequency for two different values of $n$ corresponding to same value of $\ell$ for GONG (triangles) and MDI (stars) data sets. The 1 $\sigma$ errors are also shown. The continuous curves represent 10.7 cm radio flux, scaled by the best fit to GONG (solid line) and MDI (dashed line) data.

Figure 4. Variation of average solar rotation rate as a function of latitude as derived from different measurements. The solid line represents the Doppler measurements from Snodgrass (1984), the dashed line represents the non-overlapping GONG data and the dash-dot line represent the MDI data. The 1 $\sigma$ uncertainties are shown by the dotted lines for the GONG data and as error bars for the MDI data.

proach, one uses the inversion techniques which provides information both with latitude and depth.

Figure 4 shows the rotation rate as a function of latitude from different measurements and clearly displays the differential nature of the solar rotation, the maximum rate being at the equator and the minimum at the poles. It also shows that the rotation rate derived
from the linear combination of the averaged GONG and MDI odd a’s are in good agreement with each other and also with those from surface Doppler measurements (Snodgrass, 1984). The residual in rotation rate contains the temporally varying component of the rotation and are best studied through inversion techniques. Inversions show that at low latitudes the bands of faster and slower regions (zonal flows) move towards the equator with time (Schou, 1999; Antia and Basu, 2000). These are analogous to the torsional oscillations seen at the solar surface (Howard and LaBonte, 1980) which are believed to arise from the nonlinear interactions between magnetic fields and differential rotation. At high latitudes, the bands seem to move toward the poles (Antia and Basu 2001; Ulrich 2001). The changing pattern of the zonal flows implies that the maximum and minimum velocities for each latitude occur at different times (Antia and Basu, 2003). However, its implication for solar dynamo theory is not well understood.

3. Conclusions

We have analysed the p-mode oscillation frequencies obtained from GONG and MDI instruments covering a period of seven years which includes the descending phase of solar cycle 22 and ascending phase of cycle 23. The frequencies show an increase with solar activity and changes are found to be well correlated with activity indices. There appears to be a small offset between the frequency shifts derived from the GONG and MDI data sets. A marginally higher correlation is seen for the higher frequencies in the ascending phase of the solar cycle. We also note that the residual rotation rate behaves differently at low and high latitudes.

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