Axion condensate as a model for dark matter halos

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Localized solutions of an axion-like scalar model with a periodic self-interaction are analyzed as a model of dark matter halos. It is shown that such a cold Bose–Einstein type condensate can provide a substantial contribution to the observed rotation curves of galaxies, as well provide a soliton type interpretation of the dark matter ‘bullets’ observed via gravitational lensing in merging clusters.

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1. Introduction

The dominant nonvisible “dark fraction” of the total energy density or tension of the Universe is known to exist only from its gravitational effects. Since the dark matter (DM) part is distributed over rather large distances, its interaction, including a possible self-interaction [44], must be rather weak [6]. The most prominent candidates for such weakly interacting particles (WIMPs) are the (universal) axion or the lightest particle in hypothetical supersymmetric extensions of the standard model, as e.g. the neutralino, cf. Ref. [45].

Moreover, heterotic string theory provides a very light universal axion which may avoid [11] the strong CP problem in QCD [34]. The discovery of instanton solutions in non-Abelian gauge theories topologically classified by a Pontrjagin term of type \( \vec{E} \cdot \vec{B} \) has posed a problem in quantum chromodynamics (QCD): The experimental data for the electric dipole moment of the neutron lead to the bound \( \theta := \theta_{\text{QCD}} + \text{ArgDet} M < 10^{-10} \) on the effective vacuum angle, after diagonalizing the quark masses. A non-zero \( \theta \) would imply parity or even CP violation.

The Peccei–Quinn (PQ) solution [34] to the strong CP problem is to introduce a dynamical field, the axion \( a \), as a pseudo-Nambu–Goldstone boson associated with a new global \( U(1)_{\text{PQ}} \) symmetry, spontaneously broken at a scale \( f_a \). Non-perturbative effects of QCD induce a potential \( V(\phi) \) whose minimum at \( \phi = 0 \) cancels the \( \theta \) term and thus solve the strong CP problem.

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It is characteristic for the axion that it couples derivatively to spinor matter, it couples non-derivatively to two gluons and it has, via the Primakoff effect, an effective coupling to photons being used for its detection [3].

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2. Effective axion potential

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Standard QCD is based on the Yang–Mills Lagrangian

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\[ \hat{L}_{YM} := -\frac{1}{2} \text{Tr}(G \wedge^* G) + \frac{\theta}{2} dC, \]

amended by a coupling to the topological Pontrjagin term \( dC \) proportional to the so-called \( \theta \) angle.

After integrating out the fermion fields, cf. Ref. [19], the generating functional for QCD including the term \( \theta \) \( dC \) induces an effective axion potential
\[ V(\theta) = \frac{1}{2} U(\theta) = \Lambda_\theta(1 - \cos \theta) \]
\[ = m^2_a^2 \left( \frac{1}{2} - \frac{a^2}{12 f_a^2} + \frac{a^4}{360 f_a^4} - \cdots \right) \]
induced by QCD instantons. In terms of the \( \theta \)-angle, this potential displays a periodicity with a period of \( 2\pi \) and has a minimum at \( \theta = 0 \), as required. The resulting curvature of the potential at the minimum can be interpreted as an induced axion mass which for zero temperature is of the order \( m_a = \sqrt{\Lambda_{\theta}}/f_a \simeq (6 \times 10^{12} \text{ GeV})/f_a \mu\text{eV} \).

If the axion exist in its “invisible” form [19] and its energy scale \( f_a \) is not far from \( 10^{12} \text{ GeV} \), it may constitute a substantial fraction of the dark matter (DM) of the universe [8]. However, searches for the conversion an axion into a single photon via the inverse Primakoff effect have already excluded the \( \mu\text{eV}/c^2 \) mass range as a possible constituent of the local dark matter halo. On the other hand, in anthropic scenarios [14] or for a torsion-induced quintaxion [30], large \( f_a > 10^{13} \text{ GeV} \) are permitted for small misalignment angle \( \theta_t \).

For an ultralight axion of \( m_a < 10^{-11} \text{ eV} \) the absence of gamma rays from the supernova SN 1987A yields the bound of \( g_{\gamma\nu} < 0.3 \times 10^{-11} \text{ GeV}^{-1} \) on the coupling constant of a photon mixing with axion-like particles [43].

3. Anharmonic oscillatory axion density

In the following, we consider the periodic axion potential (2) with an arbitrary coupling constant \( \Lambda_\theta \). It will get absorbed by a rescaling of the radial coordinate.

Stationary solutions
\[ \theta = P_l(r) \exp(-i\omega\sqrt{\Lambda_{\theta}r}) Y_m(\theta, \varphi) \]
are the eigenstates of angular momentum [41] or axion-like field obey the nonlinear Klein–Gordon equation
\[ \left( \Box + \frac{dV}{d\theta} \right) \theta = 0 \]
in flat spacetime. After averaging out the angular dependence as in Ref. [26], the radial equation reads
\[ P'' + \frac{2}{x} P' - \frac{l(l+1)}{x^2} P + \omega^2 P - \sin P = 0, \]
\[ (4) \]
where \( \prime = d/dx \) is the derivative with respect to dimensionless radial coordinate \( x := \sqrt{\Lambda_{\theta}}r \). A real axion corresponds to the limiting case \( \omega = 0 \), cf. [23].

Then the energy density of the axionic solution is given by
\[ \rho_\theta = \frac{1}{2} \left[ -\dot{P}^2 + \left( \frac{dP}{dr} \right)^2 + U \right] \]
\[ = \frac{\Lambda_\theta}{2} \left( \omega^2 P^2 + (P')^2 + 2(1 - \cos P) \right), \]
\[ (5) \]
\[ (\text{since we adhere to the flat spacetime approximation. Scalar fields in a BEC type configuration, in contradistinction to cold dark matter (CDM), also exert the radial pressure}) \]
\[ p_\theta = \rho_\theta - U \]
\[ = \frac{\Lambda_\theta}{2} \left( \omega^2 P^2 + (P')^2 - 2(1 - \cos P) \right). \]
\[ (6) \]

3.1. Localized solutions of the linearized equations

As a first step, let us consider small axion amplitudes \( P \ll 1 \) such that \( \sin P \simeq P \) holds in this approximation. This holds, e.g. for a small initial misalignment \( \theta_t \ll 1 \). Then, the nonlinear differential equation (4) turns into the linear equation
\[ x^2 P'' + 2xP' + \left[ (\omega^2 - 1)x^2 - l(l+1) \right] P = 0. \]
\[ (7) \]

According to Ref. [22], its solutions are the so-called spherical Bessel functions
\[ P_l = \left( \frac{\pi}{2a} \right)^{1/2} \left[ J_{l+1/2}(x) + B Y_{l+1/2}(x) \right], \]
\[ (8) \]
where \( x := \sqrt{\omega^2 - 1} \) is for \( \omega^2 > 1 \) a real rescaled radial coordinate. For non-vanishing angular momentum \( l \gg 1 \), these solutions correspond to vortices in a BEC, see also Refs. [25,36]. Since the half-integer Bessel and Neumann functions are known to be elementary functions, we can rewrite our result in terms of parity eigenstates of angular momentum \( l \) as follows:
\[ P_l = (-1)^l x^l \left( \frac{d}{dx} \right)^l \frac{A \sin x - B \cos x}{x}. \]
\[ (9) \]

For the spherically symmetric case \( l = 0 \), a regular solution obeying the initial condition \( P(0) = \theta_t = \pi/2 \) of maximal initial misalignment\(^2\) reads
\[ P_0(x) = \frac{\pi}{2} \sin(x \sqrt{\omega^2 - 1}) \]
\[ x \sqrt{\omega^2 - 1}. \]
\[ (10) \]

It corresponds to the choice \( A = \pi/2 \) and \( B = 0 \), where the latter is a necessary condition for avoiding a singularity at the origin. Such a solution was first considered by Schunck [38] and recently rediscovered in Ref. [25].

In this approximation, it can be regarded as a standing wave of an axion field coherently oscillating in its potential \( U \), cf. Ref. [20]. Since in our approximation \( U(\theta) \simeq \Lambda_\theta P^2 \) holds, we find for the energy density and pressure
\[ \rho_\theta \simeq \frac{\Lambda_\theta}{2} \left[ (\omega^2 + 1)P^2 + (\omega^2 - 1)(dP/d\theta)^2 \right], \]
\[ \rho_\theta \simeq \frac{\Lambda_\theta}{2} (\omega^2 - 1) \left[ P^2 + (dP/d\theta)^2 \right] = \rho_\theta - \Lambda_\theta P^2, \]
\[ (11, 12) \]
respectively.

For a real axion with \( \omega = 0 \) the pressure will always be negative. Due to the recurrence relations for Bessel functions, the first derivative in Eqs. (11) and (12) may as well be expressed via
\[ \frac{d}{dx} P_l = P_{l-1} - \frac{l+1}{x} P_l, \]
\[ (13) \]

3.2. Numerical solutions

In order to see the effect of the nonlinear potential, let us consider a spherically symmetric solution for which the axion is maximally ‘misaligned’ at the origin, i.e. that satisfies the initial condition \( P_0(0) = \pi/2 \) and \( P_0'(0) = 0 \). Such a configuration will be localized at the origin, provided the change of the slope remains initially negative, i.e. \( P'' < 0 \), which, according to (4), holds for \( \omega > \sqrt{2/\pi} \). For this initial conditions, we have solved numerically the nonlinear differential equation

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\(^1\) For \( l = 0 \) and \( \omega = 1 \), Eq. (7) admits the singular solution \( P = A + B/x \) for which the radial pressure (12) vanishes.

\(^2\) The related average \( \langle \theta \rangle = \pi^2/3 \) is considered in Ref. [14].
oscillating around its linearly rising growth in an axionic DM halo.

4. Rotation curve with wiggles

The circular velocities of stars or HI gas in galaxies are bounded by $v_{\phi}/c \lesssim 10^{-3}$ and thus are non-relativistic. Nevertheless, we depart from the general relativistic formula [33]

$$v_{\phi}^2 := \frac{r}{2} \frac{d\nu}{dr} = \frac{\kappa}{2} \left[ \frac{M(r)}{4\pi r} - \kappa M(r) \right]$$

$$\simeq \frac{\kappa}{2} \left[ \frac{M(r)}{4\pi r} + P_{\text{tot}}^2 \right]$$

(15)

for the tangential velocity squared, in order to keep track of the effect of the radial pressure. Here $\kappa = 8\pi G$ is the gravitational coupling constant and, for a spherically symmetric metric, $\nu = 2\ln N$ is related to be the lapse function $N$ in general relativity. Even the approximation above for a small mass function goes beyond the usual Kepler law $v_{\phi,\text{Newton}} \simeq GM(r)/r$ inasmuch as it takes the effect of the radial pressure $P_{\phi}$ into account.

The total mass enclosed in a halo of radius $r$ is given by the familiar Newtonian mass function

$$M(r) := 4\pi \int_0^r \rho_0 \xi^2 d\xi$$

(16)

shown in Fig. 5. For a round halo, i.e. for $l = 0$, the solution (10) of the linearized equations yields

$$M(r) \simeq \frac{\pi^3 \sqrt{\Lambda_{\theta}}}{2\sqrt{\omega^2 - 1}} \left[ \omega^2 - \frac{\sin(2\hat{x})}{2\hat{x}} - \frac{(\omega^2 - 1) \sin^2 \hat{x}}{\hat{x}^2} \right] \hat{x}^3$$

(17)

Likewise, an $l$-dependent DM halo profile can be obtained by integrating (11) with the aid of MATHEMATICA and further simplification [22].

Our post-Newtonian approximation reveals that the radial pressure term $p_{\phi} \xi^2 \simeq \pi^2 (\hat{x}^2 - \hat{x} \sin(2\hat{x}) + \sin^2 \hat{x})/8\hat{x}^2$ for $l = 0$ of a scalar ‘fluid’ contributes to the shape of the configuration as well as to the rotation curve. For a round halo the “Keplerian” term $M(r)/4\pi r \simeq \pi^2 (2\omega^2 \hat{x}^2 - \hat{x} \sin(2\hat{x}) - 2(\omega^2 - 1) \sin^2 \hat{x})/16(\omega^2 - 1)\hat{x}^2$ is partially dominating over the pressure term, with the result that
self-interacting due to the periodic potential (2) provides a similar halo substructure. When ‘superposed’ with its ‘vortex solution’ the axionic rotation curve remains. Likewise, for the nonlinear Emden model, the dynamics of galaxies in the Coma cluster provided the first dynamical simulation of the baryonic and DM components. In fact, the dynamics of galaxies in the Coma model exhibits an intriguing bifurcating into several nearly Einsteinian branches\[29,40\]. Ultimately, this may indicate that soliton models of DM halos are equivalent to the purely gravitational geons\[1\], similarly to those of boson stars\[41\].

Asymptotically, the deviation increases due to the almost linearly rising mass (17) of the axionic halo. Then, modifications of the rotation curve due to self-gravity are mandatory, cf. Ref.\[27\] for related attempts in a general-relativistic setting.

5. Discussion

For a BEC below its critical temperature

\[ T_c = \frac{2\pi^2}{mk_B} \left[ \frac{N}{\xi(3/2)V} \right]^{2/3} \]  

(20)

where \( \xi(z) \) is the Riemann zeta function, the \( N = nV \) scalar particles in a volume \( V \) are in a coherent state and thus do not collide. Since the axion is very weakly interacting, the reheating phase after initial inflation cannot destroy coherence\[2\]. Thus an axion-like DM halo can be in a non-relativistic BEC state even until now.

It is interesting to note that both, the critical temperature \( T_c \) as well as the Kaup limit\[31\]

\[ M_{\text{Kaup}} \leq \frac{2}{\pi} \frac{M_{\text{Planck}}^2}{m} \]  

(21)

for the stable branch of spherical localized boson configurations are inversely proportional to the constituent mass \( m \). Thus an almost vanishing mass is mandatory in order to extrapolate a BEC to a galactic scale.\(^3\)

Consequently, in order that a BEC can account for DM halos, the Compton wave length of DM particles has to be of the order of the size of a galaxy, i.e. \( \lambda_{\text{Compton}} = h/mc \sim 10 \) kpc. This naive estimate would lead to an ultra light mass of \( m \sim 10^{-26} \text{ eV} \) which is twenty orders of magnitude below the usual mass range \( m_a \sim \mu \text{eV} \) of axions. If we identify the average radius \( \langle r \rangle = \int_0^\infty \rho_0 r dr / \int_0^\infty \rho_0 dr \propto \frac{1}{m_a f_a} \) of our axion condensate with the size of a galaxy, we obtain a much higher constituent mass, depending on the axion coupling \( f_a \).

Recently, the so-called ‘bullet cluster’\[1\] (1E0657-558) provided indirect evidence\[9\] for the existence of DM, see also Ref.\[24\] for a dynamical simulation of the baryonic and DM components. In fact, the dynamics of galaxies in the Coma model provided the first clue for Zwicky in postulating ‘missing matter’. Clusters provide an excellent laboratory for constraining DM parameters such as their lifetime\[35\]. During a collision or merging of galaxy clusters, the DM halos are displaced from the stars and apparently pass each other in a soliton-like way. There will be no annihilation, if the halos are described by two coherent states of nonlinearly interacting scalar fields\[4,21\]. Although the observations in the bullet cluster\[9\] can be given a rather conventional interpretation\[24\], axionic BECs offer the possibility of lump-like DM structures\[1\], similarly to those of boson stars\[41\].

On the other hand, the real part of a scalar field can be regarded as a conformal mode of the spacetime metric which, via a Legendre transformation, induces a nonlinear modification of general relativity, exhibiting an intriguing bifurcating into several nearly Einsteinian branches\[29,40\]. Ultimately, this may indicate that soliton models of DM halos are equivalent to the purely gravitational geons\[16\] of Wheeler.

\(^3\) For a small critical temperature \( T_c \simeq 3 \) K, the resulting equivalent bound \( M_{\text{Kaup}} \leq 4\pi T_c (\xi(3/2)/n)^{2/3} M_{\text{Planck}}^2 / m^2 \) would require a tiny number density \( n \), independent of the constituent mass \( m \).
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