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SOFT-X-RAY TOMOGRAPHY ON ALCATOR C

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ABSTRACT. Using 80 miniature soft-X-ray detectors viewing along chords through a plasma cross-section at one toroidal location, tomographic reconstructions of emissivity have been obtained without the need for assuming any symmetry or rotation of the plasma. In one class of plasma discharges, it is found that a large $m = 1$ oscillation, which previously had been ascribed to the rotation of an MHD instability, actually is not rotating at all.

Soft-X-ray arrays are routinely used on tokamaks to look at MHD phenomena, such as sawtoothing, the $m = 1$ precursor oscillation, $m \geq 2$ instabilities, as well as plasma position, impurity radiation, etc. Detailed analyses of the X-ray signals using X-ray tomography techniques have been made by several experimenters [1–5] to obtain a two-dimensional reconstruction of the emissivity function, $g(r, \theta)$. Typically, however, the number and arrangement of detector views available do not provide sufficient information to yield a useful image unless some simplifying assumptions are made. One common technique is to assume that the X-ray emissivity has a specified functional form which includes one or more adjustable parameters [2]. In this case, reconstructing an image consists of finding the values of the free parameters which best reproduce the original signals. In situations where there is a very limited number of viewing chords, only very simple functional forms with one or two adjustable parameters can be used.

Another technique, which is widely used when studying MHD instabilities, is to assume that the oscillations typically seen on the X-ray signals are due to a perturbation in the emissivity which is rotating with a known time behaviour (not necessarily sinusoidal) [3–5]. This enables the ‘generation’ of extra signals from angles at which there are no detectors, since rotation provides a direct correlation between measured signals versus time at a given angle and unmeasured signals at any other angle. This assumption is valid if the emission pattern is rotating rigidly and not changing amplitude during one oscillation period. Although researchers worry about whether or not these conditions are met in actual measurements, there has been very little questioning of the fundamental assumption made here, namely that rotation causes the oscillations in the X-ray signals.

In order to generate tomographic reconstructions without any of the above assumptions, it is necessary to have signals from line chords at many different angles, $\phi$, and impact radii, $p$. Figure 1 shows the definition of $p$ and $\phi$ for one chord. These two parameters uniquely identify each detector view. Ideally, one would wish to have chords approximately equally spaced in both $p$ and $\phi$. The finite number of detectors, and therefore of $p$ and $\phi$ values, puts limits on the spatial resolution in the radial and angular directions, respectively. This is true regardless of the mathematical algorithm used to do the reconstruction. On Alcator C, we have installed an 80-channel array of miniature solid-state soft-X-ray detectors arranged in ten fans, each fan having eight chords, all at one toroidal location [6]. Because of the limited access available on this tokamak, the detector fans are not equally spaced in angle; rather they are bunched towards the top and bottom of the plasma cross-section, as shown in the schematic in Fig.2. The lack of views in the horizontal direction leaves angular gaps in $(p, \phi)$ space and therefore limits the poloidal resolution. Although the 80 detector locations are clearly not optimum, there is still enough information to make tomographic reconstructions of some detail without any of the above assumptions, even allowing for realistic errors in detector calibration and positioning.

![FIG. 1. Definition of the chord impact parameter, $p$, and the chord angle, $\phi$. Note that $p$ is usually normalized to the range 0–1. The emitting region must be confined to $p < 1$ for the algorithm described in this paper.](image-url)
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FIG. 2. Illustration of the 80 different viewing chords of the Alcator tomography experiment. The system is configured in 10 "fans", each fan having 8 individual detectors.

After careful evaluation of several different mathematical techniques for reconstructing the two-dimensional emission function, we found that an analytic solution due to Cormack [7, 8] yields the best results with our experimental setup. Given that no emission exists outside a certain radius (which in Alcator is about 10–11 cm, with a = 16.5 cm), the X-ray signals may be approximated as line integrals as long as the widths of the viewing chords are much narrower than the spatial variation of the emissivity:

\[ f(p, \phi) = \int_{L(p, \phi)} g(r, \theta) dL \]  

(1)

where \( f(p, \phi) \) is the X-ray brightness \((W \cdot cm^{-2} \cdot sr^{-1})\) measured by the detector viewing along the chord \( L \), defined by \( p \) and \( \phi \), and \( g(r, \theta) \) is the soft-X-ray emissivity \((W \cdot cm^{-3})\). This is true as long as the plasma is optically thin in the X-ray spectral region. Tomography consists of inverting Eq. (1) for \( g(r, \theta) \), which is a two-dimensional problem. As a first step, this can be transformed into a set of one-dimensional equations by expanding in Fourier harmonics in angle. Let

\[ g(r, \theta) = \sum_{m = 0}^{\infty} \left[ f_m^x(r) \cos m \theta + f_m^y(r) \sin m \theta \right] \]  

(2)

Equation (1) then becomes:

\[ f_m^x(p) = \sum_{m = 0}^{\infty} \left[ \int \frac{g_m^x(r) T_m(p/r)}{(r^2 - p^2)^{1/2}} dp \right] \]  

(4)

where \( T_m(x) = \cos(m \cos^{-1} x) \) is a Tschebycheff polynomial. This representation is quite useful in reconstructing the emission from a tokamak plasma because MHD theory predicts that only the lowest angular harmonics are involved in the equilibrium and principal instabilities.

Cormack [7] proved that the solution to Eq. (4) is

\[ g_m^x(r) = \frac{1}{\pi} \left( \int_{r}^{\infty} \frac{f_m^x(p) T_m(p/r)}{(p^2 - r^2)^{1/2}} dp \right) \]  

(5)

For the \( m = 0 \) component, this reduces to the familiar Abel inversion. Unfortunately, the direct numerical application of Eq. (5) is noise sensitive because of the differentiation involved and is therefore of limited use with real data. To overcome this problem, Cormack [8] found a special pair of expansion polynomials which satisfy Eq. (5). If \( f_m(r) \) is expanded as follows:

\[ f_m(r) = \sum_{\xi = 0}^{\infty} a_{m\xi} \sin [(m + 2\xi + 1) \cos^{-1} p] \]  

(6)

then

\[ g_m(r) = \sum_{\xi = 0}^{\infty} \left( m + 2\xi + 1 \right) a_{m\xi} R_{m\xi}(r) \]  

(7)

where

\[ R_{m\xi}(r) = \sum_{s = 0}^{\xi} \frac{(-1)^s (m + 2\xi - s)! \Gamma^{m + 2\xi - 2s}}{s! (m + \xi - s)! (\xi - s)!} \]
are called the Zernicke polynomials. The coefficients, \( a_{m\ell} \), are found by doing a least-squares fit of Eq. (6) to the data and thus this method is relatively insensitive to noise. In addition, the special expansion functions for \( f_m(p) \) satisfy certain mathematical constraints having to do with the number of radial nodes in the projections, which further reduces the effects of noise in the data. In practice, we find that the detector configuration on Alcator C allows for reconstructions having angular harmonics \( m = 0 \) and \( 1 \), and radial harmonics \( \ell = 0 \) to 6. (The addition of side-arrays in the near future will permit the reconstructions to include up to \( m = 2 \).) The limits are determined by an error analysis, which takes account of the fact that the viewing geometry and detector calibrations have both random and systematic errors that will degrade the reconstructions. Starting with a reasonable test emission function, we calculate the flux to each detector, assuming a systematic error (\( \sigma = 0.5 \) mm) in detector position as well as additional random noise due to calibration inaccuracies (\( \sigma = 3\% \)). The tomographic reconstruction of these pseudo-signals is then compared with the test function. We find that the \( m \) and \( \ell \) harmonic numbers must be restricted to the previously specified values in order to reliably reproduce the input. From detailed numerical analysis and testing it is found that this diagnostic can resolve changes in plasma position of a few millimetres and can image perturbations having a spatial extent as small as 1 cm near the central region of the plasma column. This resolution degrades somewhat as one goes further out in radius.

Three examples of X-ray tomography of Alcator plasmas will be given. The first example is from a typical sawtoothing plasma (Fig.3). The reconstructions in Figs 4 and 5 show the change in emissivity occurring at the crash of a sawtooth. It should be emphasized...
FIG. 6. X-ray signal from a pellet-fuelled plasma, showing large sawteeth and \( m = 1 \) precursor oscillations. Several reconstructions, which span the indicated cycle of precursor oscillation, are presented in Fig. 7.

that since only a finite number of harmonics are used, there is a residual 'ripple' noise in the reconstructions which is of order \( \sim 5\% \). Therefore, the slight hollowing in the three-dimensional plot after the crash may or may not be real. However, the general flattening of the centre and the broadening of the outer regions is resolvable. Note also that the \( m = 1 \) component evident in Fig. 4 is due to an outward shift of the equilibrium with respect to the centre of the vacuum vessel and not to an \( m = 1/n = 1 \) instability. (The tomography algorithm makes no a priori assumption of plasma centre position.)

The second example is from a pellet-fuelled sawtoothing plasma which exhibits large \( m = 1 \) 'precursor' oscillations immediately before each sawtooth crash (Fig. 6). This instability is believed to be a resistive tearing mode which forms a magnetic island and grows until it eventually causes the sawtooth crash [9–11]. The reason for the oscillatory nature of the X-ray perturbations has always been ascribed to island rotation, either due to \( \omega \) diamagnetic effects (the so-called drift tearing mode [12]) or to radial electric fields giving rise to an \( \vec{E} \times \vec{B} \) drift.

Figure 7 shows the reconstructions from one oscillation cycle. By following the peak of the emission from frame to frame, it is seen that the perturbation does indeed exhibit an apparent rotation in the electron diamagnetic direction, although it is not exactly sinusoidal. In frames (c) and (e) there is a flattened region near the \( q = 1 \) surface which could conceivably be the centre of a magnetic island. However, this feature is not apparent in the other frames and we do not understand why this is so. It may be that the perturbation changes shape as it rotates from high-field regions to low-field regions.

A final example of tomography on Alcator C is from a lower-current pellet-fuelled plasma that is not sawtoothing but which does exhibit large \( m = 1 \) oscillations on the soft-X-ray signals (Fig. 8). As in the previous example, this behaviour seems to indicate mode rotation. However, the X-ray reconstructions over one cycle,

FIG. 7. Contour plots of X-ray emissivity, showing a rotating perturbation at a radius of approximately 4 cm.
FIG. 8. X-ray signal from another pellet-fuelled plasma, showing large oscillations but no sawteeth. Several reconstructions, which span the indicated single cycle of oscillation, are presented in Fig. 9.

FIG. 9. Contour plots of X-ray emissivity, showing a growing perturbation (location indicated by arrows) which appears to compress and push the plasma, but which does not rotate.

which are displayed in Fig. 9, show that there is no rotation associated with these oscillations. Instead, an instability seems to grow and to compress the plasma on the inner major-radius side. Then, within 100 μs (frames (e) to (f)), the mode disappears and the X-ray emissivity returns to its normal circular shape. This process repeats itself over and over again, resulting in the oscillating pattern on the individual soft-X-ray signals. We have made computer movie films of thousands of frames of the tomography data; they help to elucidate this motion and clearly show the difference between this type of discharge and a sawtoothing discharge.

What causes the difference in the rotation behaviour of these two types of discharges? There is no drastic difference in the pressure gradient and therefore any rotation due to $\omega_e$ effects would remain approximately unchanged. This leads one to postulate that a radial electric field may be contributing to the mode rotation and the substantial differences in $E_r$ may lead to the different rotation behaviour. This change in electric field would be expected to affect the radial impurity transport, among other things, and this is indeed seen. In Alcator C, non-sawtoothing plasmas have a relatively higher soft-X-ray flux than sawtoothing discharges, presumably due to enhanced impurity radiation. In addition, there are indications that the impurity confinement time in Alcator C, which is normally finite, may approach infinity in non-sawtoothing plasmas [13]. In other words, impurities may actually accumulate on axis.

In summary, using 80 soft-X-ray detectors at one toroidal location, we have obtained tomographic reconstructions of the X-ray emissivity without assuming any symmetry or rotation of the plasma. We find both rotating and non-rotating instabilities — the non-rotating oscillations are caused by the rapid and repetitive growth and decay (reconnection?) of an $m = 1$ mode rather than by the rotation of a saturated, steady-state island. This leads us to postulate that there is a change in the radial electric field, which in turn affects the nature of impurity confinement and possibly other plasma properties as well. Future work includes the addition of several side-arrays to enable higher Fourier poloidal harmonics to be included in the reconstructions. In addition, by injecting various impurities into the plasma, we plan to accurately measure the differences in impurity confinement between rotating and non-rotating discharges.

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GENERATION OF SUPRATHERMAL ELECTRONS DURING PLASMA CURRENT STARTUP BY LOWER HYBRID WAVES IN A TOKAMAK

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ABSTRACT. Suprathermal electrons which carry a seed current are generated by non-resonant parametric decay instability during the initial phase of lower-hybrid current startup in the JIPP T-IIU tokamak. From the numerical analysis it is found that parametrically excited lower-hybrid waves at the lower side-band can bridge the spectral gap between the thermal-velocity region and the low-velocity end of the pump power spectrum.

Plasma current startup by RF and steady-state operation of a tokamak have attracted considerable attention. This mode of operation would make possible a more practical tokamak design since the Ohmic heating coil system would not be needed. Recently, the WT-2 [1], JIPP T-IIU [2] and PLT [3] groups have succeeded in plasma current startup by lower-hybrid (LH) waves of up to 5 kA, 20 kA and 100 kA, respectively. As discussed in papers on LH current startup [1—3] and sustainment [4], the mechanisms of creating an initial seed current without an Ohmic field and filling up the velocity gap between the bulk electron thermal-velocity region and the low-velocity end of the pump-wave power spectrum have become of interest. In WT-2 [1], suprathermal electrons as seed-current carriers are provided by electron cyclotron heating (ECH). In PLT [3], energetic electrons of more than 20 keV can be produced by an initial small pulse of the vertical field. For JIPP T-IIU, it was shown that the plasma current could be started up by LH power, even in the absence of pre-ionization by ECH [2]. In this letter we report that suprathermal electrons resulting from non-resonant parametric decay instability generate the seed current in the early phase of the discharge and that LH waves parametrically excited at the lower side-band (LSB) with respect to the pump frequency can bridge the spectral gap.

The experiment was carried out in the JIPP T-IIU tokamak [2], with minor radius $a = 0.25$ m and major radius $R = 0.93$ m. When 35.5 GHz of microwave power is injected in the ordinary mode [5] at electron cyclotron resonance (ECR), an initial hydrogen plasma with a central electron temperature $T_{e0}$ of about 20 eV and a density $n_{e0}$ of about $4 \times 10^{13} \text{cm}^{-3}$ is produced in a magnetic field $B_t$ of 12.7 kG. Pulse-height analysis of soft X-rays shows that the ECR plasma has no suprathermal electrons, this being in striking contrast to the enhanced soft-X-ray emission from ECR plasma in WT-2 [1]. The RF power at 800 MHz is launched into the ECR plasma

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