Kinematics and dynamics of an incomplete circular wheel drive of an agricultural tractor

Yu Kazakov*, V Medvedev, V Batmanov and V Pavlov

Department of Transport-Technological Machines and Complexes, Chuvash State Agrarian University, 29 K. Marx Street, Cheboksary, 428003, Russian Federation

*E-mail: info@academy21.ru  https://orcid.org/0000-0002-3189-3759

Abstract. The disadvantage of wheeled tractors is soil compaction, slipping due to limited traction, low tangential force. Experimental studies of a tractor with incomplete circular wheel mover on stubble, sand and virgin snow showed an increase in cross-country ability, a decrease in skidding, an increase in traction, and an increase in productivity. The purpose of the study is to develop a methodology for kinematic and dynamic analysis of incompletely rounded wheel propellers with a built-in differential. The equation of motion of the wheel is obtained on the basis of two-stage overcoming by the wheel of a single threshold obstacle taking into account the longitudinal and radial stiffness of the tire, its deformation, air resistance in the tire. The main influence is provided by translational speed, wheel radius and radial stiffness, the moment of inertia of the wheel and the shoulder of the application of mass. Planetary gearbox proposed in which the shaft of the driving satellite is a bearing, while the radius of the gear is an order of magnitude smaller than the radius of the wheel. The direction of improvement of wheel mover, increasing their traction properties is justified.

1. Introduction

The wheel mover was widely used on land transport and technological means [1]. The mechanical behavior of an agricultural tire is a matter of extreme interest as it is related to the adherence of agricultural machines, and to the compaction of agricultural soil [2]. The disadvantages of wheeled tractors include compaction of the soil, slipping of the propellers due to coupling restrictions, low tangential force [3]. It is proposed various technological and structural solutions to improve the results of their application: tire pressure regulation, ballast application, installation of removable devices on the wheel and half-track move, etc. [4]. The technical problem of predicting the performance of a specific vehicle terrain system in the design and operation stages, looking at vehicle specifications, terrain types and uses, and traction performance parameters such as pull and speed, presented in [5]. The experimental results indicate that the measured distributions of soil stresses under tractor tire could provide more real insight into the soil–wheel interactions [6]. Increased size of farm machinery has improved farm efficiency but at the risk of soil compaction. Subsoil compaction increased as total axle load increased and was independent of ground pressure. For the same tire configuration, radial tire caused less soil compaction than the cross-ply [7]. The interaction of the caterpillar link with the ground was studied in [8]. The compaction of wheeled tractors extends to the subsurface layer up to 40 cm, and on moist soils the degree of compaction is greater [9]. The use of paired wheels or three wheels on tractors for processing fields with a complex configuration and a small area is not justified. The results of the experiment presented in [6, 9] show the need to further study the distribution of stresses under tires. In
order to form a high tangential force by the driving wheel, an incomplete circular wheel mover (ICWM) is proposed. Single incompletely rounded wheels with a built-in differential provide increased tangential force, reduced slipping. Theoretical analysis of the interaction of an ICWM with a threshold obstacle is groundage on a graphic-analytical method, but it does not reflect the properties of the tire [10]. The article is devoted to the kinematics and dynamics of an ICWM, the change in the tangential force in the process of overcoming the soil shaft, represented as a threshold, from structural and operational factors affecting this process.

2. Overcoming an obstacle with an ICWM

The process of interaction of ICWM with road obstacles in the form of rigid obstacle thresholds is divided into two stages. At the first stage the wheel is located in a horizontal section, contacts the soil with a section of a full-circular tire, the wheel section in the form of a concave sector interacts with the edge of the threshold. The deformation of the wheel in the horizontal section is proportional to the load on the tire, and at the point of contact with the obstacle, it is proportional to the longitudinal pushing force applied to the axis of the wheel. The sum of the projections of forces and reactions on the coordinate axes (figure 1a):

\[
P_k + F_k - R_{II} \sin \alpha + R_f \cos \alpha = 0 \tag{1}
\]

\[
R_z - G_k + R_{II} \cos \alpha + R_f \sin \alpha = 0 \tag{2}
\]

At the second stage of interaction, the wheel detaches from the horizontal portion of the supporting surface. The deformation of the wheel on the supporting surface is reduced to zero, since the vertical reaction has become equal to zero. At the point of contact with the obstacle (with the edge of the threshold), the reaction formed by the longitudinal pushing force will become maximum. As a result, additional deformation of the tire segment occurs. The tangential force \( P_{K_{\text{max}}} \) is described by the equation (3):

\[
P_{K_{\text{max}}} = \phi R_z, \tag{3}
\]

and the tangential reaction of the obstacle \( R_{T_{\text{max}}} \):

\[
R_{T_{\text{max}}} = \phi_{II} R_{II}, \tag{4}
\]

at the same time coupling coefficients of the tire with the support surface \( \phi \) is less than the same indicator of interaction with the edge of the threshold \( \phi_{II} \).

Taking into account (3) and (4), equations (1) and (2) will appear as:

\[
R_z \phi + F_k - R_{II} \sin \alpha + \phi_{II} R_{II} \cos \alpha = 0 \tag{5}
\]

\[
R_z - G_k + R_{II} \cos \alpha + \phi_{II} R_{II} \sin \alpha = 0 \tag{6}
\]

The transformation of equations (5) and (6) leads to the equation (7):

\[
tg \alpha = \frac{\phi_{II} G_k + F_k + \left(\phi + \phi_{II}\right) R_z}{G_k - \phi_{II} F_k - \left(1 + \phi_{II}\right) R_z} \tag{7}
\]

From the design parameters of the wheel, the jumper and the obstacle threshold, it follows:

\[
tg \alpha = \sqrt{\frac{\left(r_0 - \Delta_k\right)^2}{\left(r_0 - h_{II} - h_z\right)^2}} - 1 \tag{8}
\]

It follows from (5) and (6) that the pushing force for overcoming an obstacle is equal to (9):

\[
F_k = \frac{\left(tg \alpha - \phi_{II}\right) G_k - \left[\left(\phi + \phi_{II}\right) + \left(1 + \phi_{II}\right)\right] R_z}{1 + \phi_{II} \left(tg \alpha\right)}. \tag{9}
\]
Figure 1. Incomplete round wheel drive on the first (a) and second (b) stages.

At the beginning of the second stage when \( l < l_k \) (figure 1b), \( R_z = 0 \), relative resistance:

\[
\frac{F_K}{G_K} = \frac{\lg \alpha - \varphi_\parallel}{1 + \varphi_\parallel \tan \alpha}
\] (10)

Taking into account the additional strain \( \Delta K \), equations (8) and (9) allow us to reflect the change in the relative resistance. By solving the inverse problem, you can set an increase in the relative height of the threshold to be overcome by an incompletely circular wheel with a constant pushing force (10).

Suppose a boom of a segment exceeds the wheel deformation by a factor of 5: for the ICWM, taking into account the basic inevitable deformation, \( \Delta K = 0.2r_0 \), and for a round wheel \( \Delta K_l = 0.05r_0 \). The horizontal deformation \( h_z \) is equal for both cases. The results are presented in figure 2.

Figure 2. Dependence of the relative pushing force on the relative height of the obstacle to be overcome and the relative size of the boom of the ICWM segment.
With an increase in the relative height from 0 to 0.8\(r_0\) and the relative value of the boom of the segment to 0.08\(r_0\), the relative pushing force decreased from 1.2 to 0.93 (by 22.5%).

Having solved the inverse problem, it is possible to establish an increase in the relative magnitude of the overcome threshold of the IC WM in comparison with a round wheel with the same relative magnitude of the longitudinal pushing force. Therefore, the developed mathematical model of the interaction of an IC WM with a rigid obstacle made it possible to identify factors that significantly affect the process under study: wheel design parameters, a boom of a segment, coupling coefficients of the tire with the support surface and with the edge of the threshold.

3. Kinematics and dynamics of an incompletely circular wheel drive

The behaviour of a loaded tyre during its deformation is complex, due to the combined contributions of the carcass components, the tread rubber and the air contained within it. The analysis of the kinematics and dynamics of the IC WM is carried out taking into account structural and technological factors, and to identify factors for improving the dynamics of the wheel.

The equation of motion of an IC WM with a tire taking into account its viscous and elastic properties is obtained on the basis of theoretical mechanics, as a body performing oscillatory movements.

At the moment of starting the rotation of the wheel with a radius \(r_d\), mass \(m\) and polar moment \(I_k\) around the rib, the vertical reaction is equal zero, the pressure on the obstacle rib increases, the tire sliding relative to this point will stop. In this case, the longitudinal pushing force reaches the value \(F_x\).

We compose the equation of motion of the tire, taking into account its elastic and viscous properties.

\[
x + 2n x + k^2 x = P_l,
\]

where \(2n = \frac{\beta_r}{m^2 + \frac{I_k}{r_d}}\) is the tire damping coefficient, \(k^2 = \frac{c_x}{m^2 + \frac{I_k}{r_d}}\) - the natural oscillation frequency, and \(p_l = \frac{P_k}{m^2 + \frac{I_k}{r_d}}\) - Specific excess longitudinal force.

Equation of motion of the tire taking into account its elastic and viscous properties (figure 3a):

\[
x = \frac{v_k}{k} e^{-nt} \sin kt + \frac{p_l}{c}(1 - e^{-nt} \cos kt).
\]

The elastic force is equal to:

\[
F_x = \frac{c v_k}{k} e^{-nt} \sin kt + (P - p_l)(1 - e^{-nt} \cos kt).
\]

By the end of the first phase the force \(F_x\) reaches the highest value and \(F_x = P_k\), there will be wheel slippage relative to the rolling surface. The duration of the first phase is equal to:

\[
t_1 = \frac{1}{k} \sin^{-1} \left( \frac{P_k}{c_o v_k} \right).
\]

The longitudinal component of the reaction fins on the tire wheel will be:

\[
F_x = C_x x + \beta_r \dot{x},
\]

where \(C_x, x, \dot{x}\) are longitudinal stiffness and horizontal deformation of the tire and the speed of it, \(\beta_r\) - coefficient of viscous friction of the tire, and \(C_x = C_x\sin^2 \propto k\), where \(C_t\) - radial stiffness.

At the initial moment of meeting an obstacle, the forward speed of the wheel decreases, and the wheel can roll back from the obstacle. But under the action of the pushing force, the wheel will re-enter into contact with the obstacle and begin its rotation relative to the edge of the obstacle.

Taking into account the tire slipping relative to the edge, the instantaneous center of rotation will occupy a position corresponding to a certain radius \(r_\delta = r_d(1 - \delta)\), where \(\delta\) - is the wheel slipping coefficient relative to the obstacle edge. When \(\delta = 1\), raising the wheel to the threshold is impossible.

Assumptions when considering the behavior of the wheel at the second stage of overcoming obstacles are: 1) the second stage is short-lived, the longitudinal component of the speed during the second stage
remains constant, and we take it equal to the value of the speed at the end of the first stage; and 2) radial deformation of the tire \( \delta_k \) remains constant during the second stage.

Figure 3. Diagram of ICWM with applied forces and reactions: a) the tractor’s gravity \( G_t \) is applied to the center of the wheel; b) the tractor’s gravity \( G_t \) is applied to the center of the bearing drive satellite of the wheel reducer.

The angular velocity \( \phi \) of the wheel turning around the edge, during which the angle of contact \( \alpha_k \) with the edge of the obstacle will decrease by the value of the angle \( \varphi \):

\[
\phi = \frac{x}{r_A \cos(\alpha_K - \varphi)}. \tag{16}
\]

The speed of lifting the wheel on the obstacle:

\[
\dot{z} = \dot{x} \tan(\alpha_k - \varphi) + \delta_z. \tag{17}
\]

Where \( \delta_z \) – is the vertical strain rate, \( \delta_z = \delta_t \cos(\alpha_k - \varphi) \) – is the vertical component of the radial strain. The deformation \( h_z \) depends on the properties of the tire, the height \( h_n \) of the obstacle threshold, the contact angle \( \alpha_k \), and the rotation angle \( \varphi \).

Contact angle

\[
\alpha_k \approx \arctan \left( \frac{(2r_d h_n - h_n^2)^{0.5}}{(r_d - h_n)} \right). \tag{18}
\]

The vertical acceleration of \( \ddot{z} \) is comparable to the acceleration of free fall of \( g \).

\[
\ddot{z} = - \frac{v_A^2}{r_A \cos^2(\alpha_k - \varphi)} + \delta_t \ddot{\varphi} \sin(\alpha_k - \varphi) - \frac{\delta_t v_A^2}{r_A^2} \cos(\alpha_k - \varphi). \tag{19}
\]

At the end of the climb we have equality of angles \( \varphi = \alpha_k \), thus

\[
\ddot{z} = - \frac{v_A^2}{r_A} \left( 1 + \frac{\delta_k}{r_A} \right). \tag{20}
\]

Under the condition \( G_K < m \ddot{z} \), the wheel is detached from the surface, since

\[
g < \frac{v_A^2}{r_A} \left( 1 + \frac{\delta_k}{r_A} \right). \tag{21}
\]

The use of a spring with a force of \( F_{zp} \) helps to stabilize the behavior of the wheel at the time of lifting. The use of a wheel reducer with a satellite bearing drive shaft reduces the moment of inertia of the wheel and the turning radius (figure 3b). In this case, the load from the gravity of the tractor \( G_t \) will be applied to the point \( O_1 \), the gravity of the wheel \( G_k \) will be applied to the point \( O \). The longitudinal pushing force \( F_k \) will also be applied to the point \( O_1 \). The position of the point \( O_1 \), depending on the height of the obstacle to be overcome, will be variable, both vertically and horizontally. Consequently,
a continuous change in the gear ratio of the wheel reducer, longitudinal pushing force, clearance will be achieved [10]. This helps to increase the tangential force of the wheel, reduce the slipping of a tractor equipped with upgraded wheels, and increase the efficiency of its use.

4. Conclusion
The analysis of kinematics and dynamics of ICWM is carried out. As a result of the analysis, factors contributing to an increase in tangential force, a decrease in slipping, and the prevention of soil over-compaction through the use of ICWM with a built-in differential on tractors have been established. A time-stretched process of acceleration of the machine-tractor unit at working speed with buried working bodies is achieved, preventing overload of the wheeled tractor engine. All other things being equal, it is important to choose the speed of translational motion rationally for an ICWM. As the speed of movement increases, the radius of the ICWM decreases, the probability of its separation from the support surface increases. With an increase in the radial stiffness of the tire, its incompletely circular section, the probability of wheel bounce from the support surface decreases. A promising direction for improving wheel drives is to reduce the moment of inertia of the wheel and the shoulder of the tractor's gravity, the point of application of which should be the bearing drive shaft—the gear of the wheel torque transformer. An increase in the traction capabilities of the wheel is achieved while reducing soil compaction. Field tests of a wheeled tractor of traction class 14 kN equipped with ICWM confirmed an increase in productivity by 5%, a decrease in slipping from 18% to 4% with the same hook load compared to a tractor with serial wheels [10]. Thus, the direction of improvement of wheel mover, increasing their traction properties is justified. It became possible to develop soil-sparing wheel propellers that form a large tangential force without the use of ballast. Such propellers prevent the breakdown of soil particles in the contact spot due to a smooth change in the driving torque on the wheel. This helps to increase productivity, reduce per-hectare fuel consumption, and preserve the soil structure.

References
[1] Bekker M 1960 Off-the-Road Lokomotion: Research and Development in Terramechanics (Ann Arbor: University of Michigan Press) p 259
[2] Anifantis A S, Cutini M and Bietresato M 2020 An Experimental – numerical approach for modelling the mechanical behaviour of a pneumatic tire for agricultural machines. Appl. Sci. 103481 https://doi.org/10.3390/app10103481
[3] Mudarisov S, Gainullin I, Gabitov I, Hasanov E and Farhutdinov I 2020 Soil compaction management: reduce soil compaction using a chain-track tractor. J. Terramechanics, 89 1 http://dx.doi.org/10.1016/j.jterra.2020.02.002
[4] Muro T and O’Brien J 2004 Terramechanics: Land Locomotion Mechanics (London: Taylor & Francis Group) p 180
[5] Arvidsson J and Keller T 2007 Soil stress as affect by wheel load and tyre inflation pressure. Soil Till. Res. 96(1-2) 284 http://dx.doi.org/10.1016/j.still.2007.06.012
[6] Nguyen V, Matsuo T, Inaba S and Kuomoto T 2008 Experimental analysis of vertical soil reaction and soil stress distribution under off-road tires. J. Terramechanics 45(1-2) 25 http://dx.doi.org/10.1016/j.jterra.2008.03.005
[7] Botta G, et al. 2008 Soil compaction produced by tractor with radialand cross-ply tires in two tillage regimes. Soil Till. Res. 101(1-2) 44 http://dx.doi.org/10.1016/j.still.2008.06.001
[8] Ma Z.-D and Perkins N 2002 A track-wheel-terrain interaction model for dynamic simulation of tracked vehicles. Vehicle Syst. Dyn. 6401 http://dx.doi.org/10.1076/vesd.37.6.401.3522
[9] Jamali H, Nachimuthu G, Palmer B, Hodgson D, Hundtc A, Nunnb C and Braunack M 2021 Soil compaction in a new light: Know the cost of doing nothing – A cotton case study. Soil Till. Res. 213(4) 105158 https://doi.org/10.1016/j.still.2021.105158
[10] Medvedev V, Akimov A and Batmanov V 2005 Efficiency of incomplete tractor pneumatic tires on surfaces with low load capacity and unstable micro relief. Tractors and Agricultural Machines 5 32 [in Russian]