The $t \to cH$ decay width in the standard model.

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Abstract

The $t \to cH$ decay width has been computed in the standard model with a light Higgs boson. The corresponding branching fraction has been found to be in the range $B(t \to cH) \simeq 10^{-13} \div 10^{-14}$ for $M_Z \lesssim m_H \lesssim 2M_W$. Our results correct the numerical evaluation usually quoted in the literature.

The one-loop flavor-changing transitions, $t \to cg$, $t \to c\gamma$, $t \to cZ$ and $t \to cH$, are particularly interesting, among the top quark rare decays. Indeed, new physics, such as supersymmetry, an extended Higgs sector and heavier-fermion families could conspicuously affect the rates for this decays. In the standard model (SM), these processes are in general quite suppressed due to the Glashow-Iliopoulos-Maiani (GIM) mechanism, controlled by the light masses of the $b, s, d$ quarks circulating in the loop. The corresponding branching fractions $B_i = \Gamma_i/\Gamma_T$ are further decreased by the large total decay width $\Gamma_T$ of the top quark. The complete calculations of the one-loop flavour-changing top decays have been performed, before the top quark experimental observation, in the paper by Eilam, Hewett and Soni [1] (also based on Eilam, Haeri and Soni [2]). Assuming $m_t = 175$GeV, the value of the total width $\Gamma_T \simeq \Gamma(t \to bW)$ is $\Gamma_T \simeq 1.55$ GeV, and one gets from ref. [1]

\begin{equation}
B(t \to cg) \simeq 4 \cdot 10^{-11}, \quad B(t \to c\gamma) \simeq 5 \cdot 10^{-13}, \quad B(t \to cZ) \simeq 1.3 \cdot 10^{-13}.
\end{equation}

In the same ref. [1], a much larger branching fraction for the decay $t \to cH$ is presented as function of the top and Higgs masses (in Fig. 1 the relevant Feynman graphs for this channel are shown). For $m_t \simeq 175$ GeV and $40$ GeV $\lesssim m_H \lesssim 2M_W$, the value

\begin{equation}
B(t \to cH) \simeq 10^{-7} \div 10^{-8}
\end{equation}
is obtained, by means of the analytical formulae presented in ref. [2] for the fourth-
genration quark decay $b' \rightarrow bH$, in a theoretical framework assuming four flavour families. Such relatively large values for $B(t \rightarrow cH)$ look surprising, since the topology of the Feynman graphs for the different one-loop channels is similar, and a GIM suppression, governed by the down-type quark masses, is acting in all the decays.

In order to clarify the situation, we recomputed from scratch the complete analytical decay width for $t \rightarrow cH$, as described in [3]. The corresponding numerical results for $B(t \rightarrow cH)$, when $m_t = 175\text{GeV}$ and $\Gamma(t \rightarrow bW) \simeq 1.55 \text{GeV}$, are reported in Table 1. We used $M_W = 80.3\text{GeV}$, $m_b = 5\text{GeV}$, $m_s = 0.2\text{GeV}$, and for the Kobayashi-Maskawa matrix elements $|V_{tb}^*V_{cb}| = 0.04$. Furthermore, we assumed $|V_{ts}^*V_{cs}| = |V_{tb}^*V_{cb}|$. As a consequence, the $m_d$ dependence in the amplitude drops out.

Our results are several orders of magnitude smaller than the ones reported in the literature. In particular, for $m_H \simeq M_Z$ we obtain

$$B_{\text{new}}(t \rightarrow cH) \simeq 1.2 \cdot 10^{-13} \quad (3)$$

to be compared with the corresponding value presented in ref. [1]

$$B_{\text{old}}(t \rightarrow cH) \simeq 6 \cdot 10^{-8}. \quad (4)$$

In order to trace back the source of this inconsistency, we performed a thorough study of the analytical formula in eq. (3) of ref. [3], for the decay width of the fourth-family down-type quark $b' \rightarrow bH$, that is the basis for the numerical evaluation of $B(t \rightarrow cH)$.
presented in ref. [1]. The result of this study was that we agreed with the analytical computation in [2], but we disagreed with the numerical evaluation of $B(t \to cH)$ in [1].

The explanation for this situation can be ascribed to some error in the computer code used by the authors of ref. [1] to work out their Fig. 3. This explanation has been confirmed to us by one of the authors of ref. [1] (J.L.H.), and by the erratum appeared consequently [4], whose evaluation we now completely agree with.

![Feynman graphs for the decay $t \to bW H$ ($t \to bW Z$).](image)

Figure 2: Feynman graphs for the decay $t \to bW H$ ($t \to bW Z$).

In the following we give some heuristic considerations useful in order to understand the correct order of magnitude of the rate for the decay $t \to cH$. The comparison between the rates for $t \to cZ$ and $t \to cH$ and the corresponding rates for the tree-level decays $t \to bWZ$ and $t \to bWH$, when $m_H \approx M_Z$ can give some hint on this order of magnitude. In fact, the latter channels can be considered a sort of lower-order parent processes for the one-loop decays, as can be seen in Fig. 2, where the relevant Feynman graphs are shown. Indeed, the Feynman graphs for $t \to cZ$ and $t \to cH$ can be obtained by recombining the final $b$ quark and $W$ into a $c$ quark in the three-body decays $t \to bWZ$ and $t \to bWH$, respectively, and by adding analogous contributions where the $b$ quark is replaced by the $s$ and $d$ quarks. Then, the depletion of the $t \to cH$ rate with respect to the parent $t \to bWH$ rate is expected to be of the same order of magnitude of the depletion of $t \to cZ$ with respect to $t \to bWZ$, for $m_H \approx M_Z$. In fact, the GIM mechanism acts in a similar way in the one-loop decays into $H$ and $Z$.

The $t \to bWZ$ and $t \to bWH$ decay rates have been computed, taking into account crucial $W$ and $Z$ finite-width effects, in ref. [5]. For $m_H \approx M_Z$, the two widths are
Table 1: Branching ratio for the decay $t \rightarrow cH$ versus $m_H$. We assume $m_t = 175\text{GeV}$ and $m_c = 1.5\text{GeV}$.

| $m_H$ (GeV) | $B(t \rightarrow cH)$ |
|------------|----------------------|
| 80         | $0.1532 \cdot 10^{-12}$ |
| 90         | $0.1169 \cdot 10^{-12}$ |
| 100        | $0.8777 \cdot 10^{-13}$ |
| 110        | $0.6452 \cdot 10^{-13}$ |
| 120        | $0.4605 \cdot 10^{-13}$ |
| 130        | $0.3146 \cdot 10^{-13}$ |
| 140        | $0.1998 \cdot 10^{-13}$ |
| 150        | $0.1105 \cdot 10^{-13}$ |
| 160        | $0.4410 \cdot 10^{-14}$ |
comparable. In particular, for $m_t \simeq 175\text{GeV}$, one has [3]

$$B(t \to bWZ) \simeq 6 \cdot 10^{-7} \quad B(t \to bWH) \simeq 3 \cdot 10^{-7}. \tag{5}$$

From [1], $B(t \to cH) \simeq 6 \cdot 10^{-8}$ for $m_H \simeq M_Z$. Accordingly, the ratio of the one-loop and tree-level decay rates is

$$r_H \equiv \frac{B(t \to cH)}{B(t \to bWH)} \sim 0.2 \tag{6}$$

to be confronted with

$$r_Z \equiv \frac{B(t \to cZ)}{B(t \to bWZ)} \sim 2 \cdot 10^{-7}. \tag{7}$$

On the other hand, $r_H$ and $r_Z$ are related to the quantity

$$\left( \frac{g}{\sqrt{2}} |V_{tb}V_{cb}| \frac{m_b}{M_W^2} \right)^2 \sim 10^{-8} \tag{8}$$

(where $V_{ij}$ are the Kobayashi-Maskawa matrix elements) arising from the higher-order in the weak coupling and the GIM suppression mechanism of the one-loop decay width. The large discrepancy between the value of the ratio $r_H$ in eq. (6) and what was expected from the factor in eq. (8), which, on the other hand, is supported by the value of $r_Z$, was a further indication that the values for $B(t \to cH)$ reported in eq. (2) could be incorrect.

Indeed, the new value of $B(t \to cH)$ in eq. (3) gives $r_H \sim 4 \cdot 10^{-7}$.

In conclusion, we have pointed out that one of the numerical results of ref. [1] establishing a relatively large branching ratio for the decay $t \to cH$ in the SM has been overestimated. The correct numerical estimates are shown in Table 1. We find $B(t \to cH) \simeq 1 \cdot 10^{-13} \div 4 \cdot 10^{-15}$ for $M_Z \lesssim m_H \lesssim 2M_W$. Such a small rate will not be measurable even at the highest luminosity accelerators that are presently conceivable. An eventual experimental signal in the rare $t$ decays will definitely have to be ascribed to some new physics effect.

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