Radiation from Side-Emitting Optical Fibers and Fiber Fabrics: Radiometric Model and Experimental Validation

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Side-emitting optical fibers are diffuse light sources that emit guided light through their cladding. Herein, two models to predict the generated radiation field are derived: one for the case of a cylinder source and one for a line source. The approach is based on the radiometric approximation and considers longitudinal and angle-dependent emission. Experimental validation is provided for the model parameters and the radiation field. It is shown that the longitudinal characteristic is relevant in proximity to the emitter’s surface and that the angular dependency determines the far field of emission. Comparison to the experiment shows that the cylinder source model allows for only slightly more accurate prediction at the cost of significantly higher computational effort. A combination of model and measurements is then used to predict the illumination performance of side-emitting fibers and fiber fabrics.

1. Introduction

Side-emitting optical fibers provide a means to illuminate spaces where common light sources face limitations, for example, in light penetration depth, thermal load, or volumetric homogeneity. These optical fibers emit a fraction of the guided light through their cladding, acting as a diffuse line source that is separated from the actual light emitter. This makes them easy to deploy, for example, in aquatic or highly humid environments. Additionally, they are thin, long, and flexible, which enables easy implementation even in tricky geometries. Different methods of fabrication and light coupling are established for side-emitting fiber made from inorganic glasses or plastics, e.g.,Typically, refractive index distortions (e.g., bubbles or particles) are introduced into the fiber to scatter light. Here, the standard case of homogeneously distributed scatterers results in an exponential decay in emission strength alongside the fiber, due to the Lambert–Beer law. In addition, the scattering process causes light emission, which is preferentially forward-directed. Both properties result in inhomogeneous illumination; their specific effect on the radiation field is largely unknown.

Previous efforts to model the radiation field assumed that the fiber light emission was constant over all angels (Lambertian emission) or used a stochastic Monte Carlo approach. Building on these methods, we now treat the side-emitting fiber as exhibiting angular scattering properties in between directional and diffuse, combined with a nonuniform longitudinal light emittance profile. Both of these characteristics are obtained from measurements. We will use this approach to discuss the generated radiation field in the proximity of the emitting surface and in the far field.

The primary aim of this work is to develop a parametric model of the radiation field surrounding the fiber. This will be shown in Section 2. Then, experimental methods of measuring the model parameters are presented in Section 3. The results of these measurements are presented in Section 4, and are subsequently compared to the calculated radiation field in Section 5 Discussion. Finally, in the same section, we will use the best-performing model to evaluate different strategies to create more uniform illumination from standard side-emitting fibers and fiber fabrics.

2. Theory

Side-emitting fibers are a light source with special properties: Their surface “emittance” $M(z)$ changes alongside the fiber, and the emitted radiation has an angular dependence, captured by the “phase function” $P(\Theta, \Phi)$. For example, the side-emitting poly(methyl methacrylate) optical fiber (PMMA fiber) used in this work exhibits an exponential decrease in emittance with length. In addition, the light is emitted preferentially at small angles to the fiber axis. Hereinafter, we will calculate the monochromatic radiation field surrounding the fiber in terms of its flux density, which is called “(spectral) irradiance” $E_s(\tau)$ in one arbitrary frequency interval $(\nu, \nu + d\nu)$. Vector quantities will be denoted in boldface. We will use this to calculate the radiation field of single fibers or fiber fabrics, consisting of many side-emitting fibers.

2.1. Radiance, Irradiance, and Radiation Transfer

We use the radiometric approximation to calculate the radiation field generated by the side-emitting fiber and assume no absorption or scattering in the surrounding medium. In this case, the
radiometric field quantity “radiance” $L_s(r, \hat{s}, \nu)$ is constant along a ray of light,\cite{15,16} basically representing its energy. The usual starting point for the radiometric derivation is to use the radiance to express the differential radiant flux $d\phi$ in the frequency interval $d\nu$ transported through an element of area $dA_1$ at an angle $\Theta_1$ and confined to an element of solid angle $d\Omega$ as

$$d\phi_s = L_s(r, \hat{s}, \nu) \cos \Theta_1 d\Omega dA_1 d\nu$$  \hspace{1cm} (1)

This defines the flux of a pencil of light rays as sketched in Figure 1. With the scalar product $\hat{s} \cdot \hat{n}_1 = \cos \Theta_1$ between the ray vector $\hat{s}$ and the surface normal $\hat{n}_1$ of $dA_1$, we can express the differential flux as the scalar product of the differential irradiance $dE_s = d\hat{E}_s \cdot \hat{n}_1$ and the surface element $dA_1 = \hat{n}_1 dA_1$ as

$$d\phi_s = L_s(r, \hat{s}, \nu) \hat{s} \cdot \hat{n}_1 d\Omega dA_1 d\nu = dE_s(r, \hat{s}, \nu) \cdot dA_1$$  \hspace{1cm} (2)

The vectorial irradiance describes both the spatial distribution of the flux density of the radiation field (light power per unit area) and the direction of the radiation. Therefore, we argue that it is the most useful quantity to describe the directional radiation surrounding a side-emitting fiber with angle-dependent emission. For example, if one were to shine the radiation from the fiber onto a piece of paper, the observable brightness distribution would be the projected irradiance distribution.

To calculate the irradiance at a point in space $r_1$, we only need to integrate the differential irradiance over the solid angle $\Omega$ surrounding it. Therefore the flux density of all rays intersecting at this point is summed up, which we write as the integral

$$E_s(r_1) = \int \frac{dE_s(r_1, \hat{s}, \nu)}{\Omega} = \int L_s(r_1, \hat{s}, \nu) \hat{s} d\Omega$$  \hspace{1cm} (3)

In the present case, the light only originates from the surface of the side-emitting fiber. Therefore, the irradiance at $r_1$ is calculated by integrating all the intersecting light rays coming from the fiber’s surface. The relationship between the solid angle element and a surface element of the side-emitting fiber is

$$d\Omega(\hat{s}) = \frac{\hat{s} \cdot \hat{n}_1 dA_2}{d^2} = \frac{\cos \Theta_1 dA_2}{d^2}$$  \hspace{1cm} (4)

This relation is also sketched in Figure 1: the pencil of rays crossing the observation point has its origin in the surface element $dA_2$ at the distance $d = r_1 - r_2$ on the fiber. This area spans the solid angle element $d\Omega$ when seen from $r_1$.\cite{17} Substituting Equation (4) and the ray vector $\hat{s} = d/d$ into Equation (3), we obtain the formula for calculating the irradiance in the volume surrounding the fiber by integration over its surface $A_F$.

$$E_s(r_1) = \int \frac{dE_s(r_1, \hat{s}, \nu)}{\Omega} = \int L_s(r_1, \hat{s}, \nu) \frac{\cos \Theta_1 dA_2}{d^2} \, dA_2$$  \hspace{1cm} (5)

The spectral (monochromatic) radiant quantity $E_s(r)$ can be turned into the radiometric quantity $E(r)$ by performing the integral over all involved frequencies $\int \frac{dE_s(r, \hat{s}, \nu)}{\Omega}$, One should keep in mind that the scattering process causing the light emission depends on the frequency and, therefore, the radiance’s angular distribution does too.

### 2.2. Radiation Field of a Cylinder Source

To perform the fiber surface integral in Equation (5), we model the fiber as a cylinder in the corresponding cylindrical coordinates $r = (\rho \cos \varphi, \rho \sin \varphi, z)$. The coordinate origin is the center of the start of the fiber; the light inside propagates in the positive $z$-direction. The fiber surface is defined by setting $\rho = R$, the radius of the fiber, which gives us the surface location vector $r_2 = (R \cos \varphi, R \sin \varphi, z)$. The radiance is measured at an arbitrary point of observation $O$. Because the fiber and its radiation field are rotationally symmetric, we chose $O(\varphi' = 0)$ as the point of observation with the location vector $r_1 = (\rho', \Theta, z')$ and $\rho' > R$. The distance vector $d$ from a surface element to the observation point is

$$d = r_1 - r_2 = \begin{pmatrix} \rho' - R \cos \varphi \\ -R \sin \varphi \\ z' - z \end{pmatrix}$$  \hspace{1cm} (6)

with the magnitude $d = |r_1 - r_2| = \sqrt{d \cdot d} = \sqrt{\rho^2 + R^2 - 2R\rho' \cos \varphi + (z' - z)^2}$. Consequently, the length and direction of this vector change from surface element to surface element.

Every surface element of the fiber radiates depending on $z$-position and emission angles $\Phi, \Theta$. The angles are defined in a local spherical coordinate system centered around the surface element, as shown in Figure 2. They have to be translated into the global cylindrical coordinate system for the integration: The surface vector $\hat{n}_2$ can be expressed as $\hat{n}_2 = \cos \varphi \hat{e}_\varphi + \sin \varphi \hat{e}_z$, as shown in Figure 2. Then, we can express the polar angle $\Theta$ in spherical coordinates with the aid of Equation (6) as

$$\cos \Theta = \hat{n}_2 \cdot \hat{s} = \frac{\hat{n}_2 \cdot d}{d} = \frac{\rho' \cos \varphi - R}{d}$$  \hspace{1cm} (7)
This result becomes zero when \( R = \rho' \cos \varphi \), then \( d \) is a tangent to the fiber’s surface. We will use this as the limit of the surface integration.

The scalar product \( d \cdot \hat{e}_z = d \sin \Theta \cos \Phi = z' - z \) between the connection vector \( d \) and \( \hat{e}_z \), expressed in cylindrical and spherical coordinates gives the relation for \( \Phi \), and in combination with \( \cos^2 \Theta + \sin^2 \Theta = 1 \) we obtain

\[
\cos \Phi = \frac{z' - z}{d \sqrt{1 - \cos^2 \Theta}} = \frac{z' - z}{\sqrt{d^2 - (\rho' \cos \varphi - R)^2}}
\]

(8)

Now all properties of the local spherical coordinate system are expressed in the global cylindrical coordinate system. We insert Equation (7) in Equation (5), with \( \Theta_1 = \Theta \), replace the surface element by its representation in cylindrical coordinates \( dA_1 = R d\varphi dz \), and obtain the final equation for the irradiance vector-field of the cylinder source

\[
E_\nu (\rho', z') = \frac{1}{4\pi} \int_{\rho_0}^{\rho_1} \int_{\varphi_0}^{\varphi_1} \frac{M_z (z, \nu)}{d^2} \left( \frac{\rho' \cos \varphi - R}{d^2} \right) \cdot \left( \begin{array}{c} \rho' - R \cos \varphi \\ -R \sin \varphi \\ z' - z \end{array} \right) R d\varphi dz.
\]

(9)

From Equation (7), we obtain the limits of integration \( \varphi_0 = -\arccos(R/\rho) \) and \( \varphi_1 = +\arccos(R/\rho) \); \( l \) is the length of the fiber.

Two additional remarks to Equation (9): First, the distance to the surface has to be bigger than zero \( d > 0 \rightarrow \rho' > 0 \). Second, the symmetry of the formula would cause the second entry of \( E_\nu \) always to integrate to zero because the irradiance passing through this surface element is equal from both sides. This is only true for a virtual surface and not for a real one, which would block radiation from one side of the fiber.

2.3. Radiance, Emittance, and the Phase Function

The radiant emittance of a surface element is distributed on the hemisphere above it. This distribution results from the heterogeneous scattering process inside the fiber and the refraction and secondary scattering on the fiber surface. To account for this, we separate the radiance into the product of the emittance \( M \) of the fiber surface element with its affiliated phase function \( P \), which contains the normalized angular information of the emitted radiation.

\[
L_\nu (z, \Theta, \Phi, \nu) = M_\nu (z, \nu) P_\nu (z, \Theta, \Phi, \nu) \approx M_\nu (z, \nu) P_\nu (\Theta, \Phi, \nu)
\]

(10)

We assume homogeneous scattering throughout the fiber, so only the emittance \( M \) depends on \( z \). Then, we can separate the radiance into the directional \( P \) and the positional contribution \( M \), which allows us to determine them independently with different experiments.

2.4. Line Source Approximation

The calculation can be greatly simplified by using a line source approximation. Here, the light is only emitted radially, therefore its phase function has no angular component everywhere except for \( \Phi = 0 \). This means that we can replace the \( \Phi \)-dependency of the phase function with the delta function

\[
P_\nu (\Theta, \Phi, \varphi) = P_\nu (\Theta, \varphi) \delta(\varphi)
\]

(11)

Inserting this into Equation (9) and performing the \( \varphi \)-integration lead to the radiant flux density of the line source with an angular-dependent emission in \( \Theta \). Additionally, as a line has no radial extend, we set all resulting \( (\rho' - R) = \rho' \). The remaining \( R \) from the surface element \( dA = dR \varphi dz \) is combined with the surface emissivity to yield the radial flux emission \( M_\nu (z, \nu) R = L_\nu (z, \nu) \). The resulting equation for the line source is

\[
E_\nu (\rho', z') = \frac{1}{4\pi} \int_{\rho_0}^{\rho_1} \int_{\varphi_0}^{\varphi_1} L_\nu (z, \nu) P_\nu (\Theta, \varphi) \frac{\rho' \cos \varphi}{d^2} \left( \frac{\rho' - R \cos \varphi}{d^2} \right) \left( \begin{array}{c} \rho' - R \cos \varphi \\ -R \sin \varphi \\ z' - z \end{array} \right) d\varphi dz
\]

(12)

with \( d^2 = (\rho')^2 + (z' - z)^2 \); the irradiance can now be calculated by integration over \( z \).

3. Experimental Section

The radiation field of a rotationally symmetric side-emitting fiber can be determined if the radiant emittance \( M(z) \) and the phase function \( P(\Phi, \Theta) \) are known. These parameters are determined experimentally, with two different setups: The “side emission
measurement” will determine the emittance, and the microscopy-based “angular measurement” will measure the phase function. Except for the calibration of the angular measurement, both methods are also described elsewhere in more detail.[18,19] Additionally, we show how to measure the resulting irradiance field of a fiber band with a scattering screen.

All light measurements are subject to unknown systematic attenuation due to light decoupling or transmission losses. To account for this, all measurements are normalized to the maximum measured value. This does not affect the calculated distributions according to Section 2. If absolute values are required, it is sufficient to measure the maximum brightness with a calibrated device and multiply the calculated distribution by this to obtain the absolute value of the irradiance distribution.

The side-emitting optical fibers (multi-mode, PMMA, diameter 500 µm) and the textile fiber band, containing 19 corresponding fibers oriented parallel to each other with an average distance of 2.6 mm (see Figure 5c), were provided by F.J.RAMMER GmbH. For all experiments, we used a 100 mW 520 nm green laser diode which is butt-coupled (direct contact without focusing optics) to the fibers of the band. For micrographs and phase function measurement, we used a JenaPol Interphako microscope. The radiation field was imaged with a Canon EOS 650D camera and an EF 18–55 objective focused on a frosted glass plane as a scattering screen.

### 3.1. Side Emission Measurement

A custom-made integrating sphere (see Figure 3a) was used to measure the fiber emittance. It consists of two fiber guides: a baffle to protect the detector port from direct irradiation and an optical fiber to connect the sphere to a spectrometer (Ocean Optics: Maya2000 Pro). The side-emitting fiber was threaded through the sphere with the help of two hollow fiber guides, leaving only a small segment of the length Δz exposed to the interior of the sphere. The emitted flux of the fiber segment Δθ(z), which is related to the emissivity by Δθ(z) = 2πRΔz M(z), is distributed homogeneously by multiple diffuse reflections on the sphere walls. Therefore, the measured irradiance E_m(z) is proportional to the flux collected by the sphere Δθ(z).[6,19] By measuring the flux at different positions along the fiber, we captured the z-dependence of the emittance of the side-emitting fiber. Here, fiber coupling, light transmission, and absorption in the integrating sphere (walls and air) represent systematic sources of error. To take this into account, the measurement is normalized to the maximum value.

### 3.2. Angular Measurement

The angular light distribution on the hemisphere in Figure 2 was captured with a large numerical aperture (NA) objective: In its back focal plane, the light is decomposed into its angular components.[18,20] The relation between emission angle Θ and back focal plane radial distance h, for infinity-corrected objectives, is sketched in Figure 3b and given by the Abbe sine condition[21]

\[
\sin \Theta = \frac{h}{f}
\]  

(13)

The unknown focal length f can be replaced by \( f = n_0 h_{\text{max}} / \text{NA} \) by using the NA and Equation (13): \( NA = n_0 \sin \Theta_{\text{max}} = n_0 h_{\text{max}} / f \). Here, \( h_{\text{max}} \) is the radius of the circular back focal plane image.

All real-world objectives with high NA have angle and polarization-dependent transmission losses.[22] For correction, we use a Lambertian scattering standard (provided by QSI GmbH Quarzschmelze Ilmenau), which should have an ideal flat irradiance profile in the back focal plane. The correction is performed by dividing the measurement image pixel-wise by the image of the scattering standard. To show this correction property, we derive the transfer of a lossless objective with an ideal scattering standard: As shown in Figure 3b, the flux Φ emerges from the focal point in a cone around the observed ray and is transformed into a non-divergent pencil of rays on the reference sphere while conserving its energy. The flux passing through \( dA_2 \) is equal to the flux passing through \( dA_1 \cos \Theta dA_1 \), so the irradiance has to vary accordingly \( \Phi = E_1 dA_1 = E_2 dA_2 = E_2 dA_1 \cos \Theta \).

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**Figure 3.** a) Emittance measurement: Light emitted by the fiber segment Δz (limited by the aperture) is homogeneously distributed on the sphere wall by multiple diffuse reflections. The irradiance on the detector port is proportional to the emitted flux \( E \propto \Phi \). b) Phase function measurement: Rays spanning an angle Θ with the optical axis (dashed) are transformed in parallel rays with a distance h by refraction on the reference-sphere (blue, radius f) according to the Abbe sine condition Equation (13).
Therefore, the irradiance in an ideal objective is amplified according to \( E_2(\Theta) = E_1(\Theta)/\cos \Theta \) for increasing \( \Theta \).

Suppose we insert the irradiance of a Lambertian diffuser \( E_1(\Theta) = E_1 \cos \Theta \) into this formula. In that case, the cosines cancel, and we obtain \( E_2 = E_2(\Theta) = \text{const.} \): a Lambertian diffuser should have an ideally flat intensity profile in the back focal plane of an objective. This correction was performed separately for both polarization states, and then both states were averaged to obtain the corrected unpolarized back focal plane image.

### 3.3. Measurement of the Irradiance on a Scattering Screen

The radiation field of a band of several side-emitting fibers is measured with a simple setup shown in Figure 4. The idea is that the screen makes the radiation field in its plane visible by secondary scattering. The irradiance \( E_0(x, y) = n(x)E(x) \), which is intersected by the screen at a certain point, is turned into the emissivity on the other side \( M(x, y) \propto E_0(x, y) \) by transmission through the screen. We assume that the screen acts as an ideal Lambertian diffuser: the irradiance which is observed by the camera on the other side of the screen is

\[
E(x, y, \alpha) = M(x, y) \frac{\cos \beta}{d^2} \propto E_0(x, y) \frac{\cos \beta}{d^2} \tag{14}
\]

This allows us to measure the light field at the scattering screen just by taking a picture of it and correcting for the angular and distance attenuation: We define the position of the pixel relative to the center of the screen as shown in Figure 4, so \( d = \sqrt{x^2 + y^2} \). Therefore, the geometric attenuation is

\[
\frac{\cos \beta}{d^2} = \frac{l}{d^2} = \frac{l}{(p^2 + x^2 + y^2)^{1/2}} \tag{15}
\]

which is used to correct the measurement for geometric angle and distance attenuation. Because the scattering behavior of the screen is expected to follow the cos-dependence only approximately, especially for large angles, it is best to choose a large distance \( l \) between the screen and the camera. Also, the camera response has to be checked for linearity.

### 4. Results

The commercial side-emitting fiber in Figure 5a appears transparent and featureless to the naked eye and under the microscope. When light is butt-coupled in the fiber, as in Figure 5b, it lights up and appears self-luminous. This luminosity is not equally distributed across the fiber, but it is brighter at the fringes and dimmer in the center when observed under the microscope. Additionally, attached particles and small scratches on the fiber surface become visible. The fiber emits strongest close to the coupling, and then the emission decreases quickly toward the fiber end.

The band of equivalent plastic side-emitting fibers in Figure 5c shows a similar decline in emission, but additionally, the overall brightness varies from one fiber to the other because of difficulties that arise from distributing light equally from one large-diameter fiber to many small ones via butt-coupling.

To quantify the loss in fiber brightness with length, a single fiber is clamped to the side emission setup from Section 3.1, and

![Figure 4](image1.png)

**Figure 4.** Scattering camera measurement: A CCD camera is focused \((l = 900\text{mm})\) on a scattering screen with the surface area \( A_2 = 650\text{mm} \times 450\text{mm}\). The fiber band is mounted to a straight holder, which is clamped to an optical bench. A laser is coupled to the side-emitting fibers, and the room is darkened. The fibers are moved to different positions \( D (10–800\text{mm}) \) relative to the glass plate, and a picture is taken of the scattering glass plate for each position without changing the focus of the camera.

![Figure 5](image2.png)

**Figure 5.** Micrographs of the side-emitting fiber under bright field microscope illumination a) and dark field self-illumination b). c) Nineteen fibers woven in a fiber band in self-illumination. d) \( z \)-dependence of the side-emitting fiber emittance \( M \) with increasing distance to the light coupling, the data is normalized by the maximum emittance. A biexponential function with the two decay constants \( \alpha_1 = 0.0021 \text{mm}^{-1} \) and \( \alpha_2 = 0.0080 \text{mm}^{-1} \) and the corresponding amplitudes \( A_1 = 0.58 \) and \( A_2 = 0.42 \) has been fitted to the measurement data. Panel (b) reproduced from[1] under CC-BY 4.0 Licence. Copyright 2019, The authors. Published by Springer Nature.
the relative change in surface emittance is measured. The data was normalized by the maximum value, and the plot in Figure 5d shows a monotonous decaying curve that decreases to 2% of the maximum value at the end of the fiber. This decrease can be fitted with a biexponential decay function.

4.1. Phase Function Measurement

The fiber’s light emission depends on the emission angle. This can easily be verified by observing the fiber (band) from different positions. This angular emission behavior was measured with Fourier microscopy in two orthogonal polarization directions, shown in Figure 6a,b. The radiation is concentrated on the right side of the circle in a half-moon shape, so light scatters preferentially forward under low angles. The remainder of the back focal plane image is dark, which means that comparatively little radiation is scattered in these directions.

We determine the phenomenological, polarization-independent phase function $P(\Theta, \Phi)$ from these measurements by imaging 50 pictures on different positions in each polarization direction, correcting them according to Section 3.2, and averaging all of them. The resulting phase function in Figure 6c is still brighter to the right and is now almost rotationally symmetric with respect to $\hat{e}_z$. The half-moon shape brightness, which is caused by the large angle objective amplification, is gone due to the correction procedure. Additionally, the phase function was normalized so that the integral over the hemisphere is equal to one.

To better illustrate the forward scattering of the fiber surface, we present it in Figure 7 as a polar plot in the $\hat{n}_1 \cdot \hat{e}_z$-plane from Figure 6. This data will also serve as the phenomenological phase function $P(\Theta)$ of the line source. Here, we see an isotropic Lambertian surface compared to the $\hat{n}_1 \cdot \hat{e}_z$-cross section of the phase function and the average scattering. The average was calculated by integrating over the surface of the hemisphere in a rotation around the $\hat{e}_z$ vector and then dividing by the arc length.

In the graph on the left in Figure 7, we see the Lambertian phase function, which is constant over all angles ($P(\Theta) = 1/\pi$). The cross section of the phase function has its maximum value before it drops rapidly for angles greater than 71°. This cutoff is due to the limited opening angle of the objective. The average function shows a similar behavior, but decreases after its maximum at 60° before the cutoff angle.

When we multiply the phase function with the $\cos \Theta$ projection factor (Equation (5)), we see how much radiation is really scattered in a given direction from a surface element: the isotropic Lambertian surface is now turned into a circle with its maximum emission at 0°. The average curve and the cross-section curve become more similar in shape, with their respective maxima now at 53° and 58°. The influence of the cutoff angle is strongly diminished by the $\cos \Theta$ factor. This shows that the phase function can be satisfactorily determined even with a

![Figure 6](image-url)  
**Figure 6.** a,b) Typical back focal plane images for two orthogonal polarization directions (pol.) and the average corrected image c) from 100 images, 50 in each polarization direction: the phase function $P$. The coordinate system is analogous to Figure 2, when the hemisphere is viewed from the top.

![Figure 7](image-url)  
**Figure 7.** Polar plot of the normalized phase function $P$ (left) and the projected phase function with $\cos \Theta$ apodization (right). Lambertian $P$ is the isotropic case. The cross section is the data from the $\hat{n}_1 \cdot \hat{e}_z$-plane. The average is calculated by integrating along a rotation around $\hat{e}_z$ and dividing by arc length.
limited aperture because the projection factor dampens the missing large-angle phase function components.

4.2. Making the Radiation Field Visible

A scattering screen intercepts the radiation field of the fiber band at different distances $D$ and makes it visible. Black and white camera images of the screen are shown in Figure 8 and display the behavior of the radiation field: At close distance, we can almost distinguish single fibers when the band is closest to the screen. Analogous to a single fiber, this distribution is bright at the start and then decays rapidly toward the fiber end.

When the distance between the screen and band increases, as shown in Figure 8 from left to right, the radiation fields of the single fibers overlap and form a continuous enveloping distribution. Here, we observe a distinct maximum of brightness close, but not at the very start. Increasing the distance further leads to a broadening of the light distribution. This strong initial decay is visually unpleasing and can lead to overexposure in technical applications. A costly solution would be to optimize the optical coupling to the fiber core; a simpler solution is to cover this part of the fiber with an absorber (but at the expense of emission efficiency). One could also choose a fiber with a smaller scattering coefficient to stretch the exponent and obtain a more uniform illumination. However, this would also be at the expense of efficiency as more light would pass through the fiber unused without being scattered and emitted.

5. Discussion

The fiber in Figure 5d shows a biexponential decay in emitted radiation with increasing distance from the light entry point. This corresponds to a fiber where some light is guided in the core, and some are guided in the cladding,[19] each with its respective scattering coefficients $\sigma_1$ and $\sigma_2$ and amplitudes $\phi_1$ and $\phi_2$. This is the result of the butt-coupling, which excites core and cladding modes simultaneously. In the present case, we expect the cladding modes to experience stronger dampening because they interact with the fiber surface.

$$\phi(z) = \phi_1 e^{-\sigma_1 z} + \phi_2 e^{-\sigma_2 z} \quad \leftrightarrow \quad -\frac{d\phi}{dz} = \phi_1 \sigma_1 e^{-\sigma_1 z} + \phi_2 \sigma_2 e^{-\sigma_2 z}$$  \hspace{1cm} (16)

If we neglect absorption loss, we see how this leads to exponentially decaying emission (assuming flux conservation): any loss in transmitted flux is turned into out-scattered radiation $M(z) \propto -d\phi/dz$. So, the amplitudes extracted from the fit in Figure 5 are $A_1 = \phi_1 \sigma_1$ and $A_2 = \phi_2 \sigma_2$. They give us a ratio of cladding to core flux of $\phi_2/\phi_1 = 0.2$ at the start of the measurement.

Under the microscope, no bubbles, particles, or other sources of light scattering can be seen inside the fiber in Figure 5. Measurement of the angular distribution of the radiation emitted from a fiber surface element in Figure 6 and 7 shows a clear preference for forward scattering. This points to the presence of long-period refractive index distortions: Generally, all deviations from the ideal core-cladding structure in a step-index optical fiber cause light scattering.[23] These deviations can be thought of as refractive index fluctuations, which can be decomposed into a spectrum of mechanical waves.[18,24] Each wavelength is responsible for light scattering under a certain angle: longer wavelengths than the guided light causes forward scattering and vice versa. Therefore, the dominant wavelengths of the fluctuations are much longer than the wavelength of the green laser diode.

Furthermore, the refractive index contrast of the distortion enhances its scattering power.[18] In summary, this means that the cause of the scattering inside the fiber is a long periodic fluctuation with low refractive index contrast or a disturbance of the core-cladding boundary with low amplitude, so it cannot be detected with the microscope. The visible scratches and particles on the fiber surface, on the other hand, are signs of wear and have high refractive index contrast. They are responsible for the attenuation of the light guided in the cladding, which

![Figure 8](image)

Figure 8. Black and white images of the scattering screen reveal the radiation distribution of a side-emitting fiber band for several distances (increasing from left to right), according to Figure 4. The light is coupled into the fiber from the top and loses its emissivity, due to the constant out-scattering of light, toward the end. Increasing the distance $D$ between the band and screen leads to a broadening of the light distribution.
explains the larger scattering coefficient $\sigma_2$ for the cladding modes.

5.2. Calculated Radiation Field

Comparing the numerically calculated fields in Figure 9 close to the fiber for the two fiber models, namely cylinder source and line source, together with the two different phase functions, namely Lambertian and the measured phenomenological phase function, reveals two properties: first, the irradiance in all four variants shows a reciprocal dependency on distance ($\rho^{-1}$). Second, the phase functions result in different magnitudes of irradiance close to the fiber: the two Lambertian models are congruent but result in a slightly higher irradiance.

These numerical results are plotted in Figure 9. The cylinder source was calculated by numerical integrating Equation (9) with the parameters from the fit to the measured values in Figure 5. The Lambertian phase function for the cylinder had the constant value $P(\Theta, \Phi) = (2\pi)^{-1}$ and for the line $P(\Theta) = \pi^{-1}$. The data for the phenomenological phase function was taken either from Figure 6 for the cylinder or from Figure 7 (average) for the line.

The discretization of the surface in the $\rho$ was done by dividing the angular interval $[\rho_0, \rho_1]$ into 20 equal pieces. For $z$, we converted interval $[0, l]$ for every $z$-position into the angular interval [arctan$(z/(\rho_0 - R))$, arctan$(l - z)/(\rho_0 - R))$] divided it into 100 equal angles and converted the angles back into $z$-coordinates. This improves numerical stability for small distances.

The deviation between phenomenological and Lambertian models close to the fiber is contradictory to the expected behavior in that all models should converge to the emissivity $M(z)$ of the fiber surface when $\rho \rightarrow R$. Three features of our phenomenological phase function are probably responsible for this: First, the phase function is only known for $\Theta < 71^\circ$ due to the NA limitation. Second, the normalization of the phase functions can only be performed up to a certain numerical accuracy using our present approach (float 64bit). Third, the discretization of the phase function leads to angular intervals with constant scattering.

Interestingly, the line source and the cylinder source give the same result in the Lambertian case, which indicates a property known for the Lambertian sphere, whose irradiance shows the same behavior ($\propto \rho^{-2}$) as an ideal point source. Thus, the conclusion is that the Lambertian cylinder’s irradiance behaves like that of a line source, although this remains to be proven mathematically.

We conclude that the Lambertian approximation is adequate to describe the irradiance close to the fiber, justifying the approach of Endruweit et al.[11] to calculate the field of a fiber from a Lambertian cylinder. However, it is even sufficient to solely use the line source. This is, of course, just possible in the absence of absorption and scattering in the surrounding medium. In the latter case, the irradiance of the line source and the cylinder source would deviate.

5.3. Comparison to the Measured Radiation Field

Section 5.2 concluded that the line source and the cylinder source differ only for different phase functions. Now, we compare the calculations for all four models to the measured field of a fiber band. We find that, in principle, the cylinder source with the phenomenological phase function performs best in the observed measurement range, although only slightly better than the phenomenological line source. Lambertian fiber models, which performed adequately in proximity to the fiber, perform worse for larger distances.

We use the same procedure as in Section 5.2 to numerically integrate the four models and obtain the radiation field for the half-space next to the fiber, which corresponds to the volume spanned by the scattering screen measurement in Figure 8. Then, we used the principle of superposition to calculate the radiation field of the fiber band from a single fiber: we made 19 duplicates of the calculated field, moved their $x$-coordinates to the respective positions of fibers on the band in Figure 5, and added them up. Additionally, we accounted for different coupling efficiencies by weighting the fields. This was done for all observed

![Figure 9](image_url) Figure 9. Comparison of the irradiance of the line source and the cylinder source close to the fiber for the phenomenological and the Lambertian phase function. At $z = 100$ mm, in reciprocal distance (left) or in distance (middle). Irradiance in a line parallel to the fiber in a distance of $\rho = 0.5$ mm.
distances. Exemplary results of these calculations of the cylinder source with the phenomenological phase function are shown in Figure 10 next to the measured values.

The calculated irradiance of the fiber band with the phenomenological phase function in Figure 10 shows two properties that match the measurement: First, the field in proximity to the fiber is dominated by the exponential decay of the radiant emission of the fiber surface. Second, the forward scattering property of the fiber causes a downward movement (away from the coupling) of the maximum of irradiance with increasing distance \( D \). That the forward scattering property of \( P \) causes this can be inferred by comparison with a Lambertian fiber, which does not show a movement of the maximum (not shown).

For a quantitative comparison of the models with the measurement, we calculated the “relative residual” as the absolute difference between the measured \( E_m \) and the calculated \( E_c \) irradiance divided by the measured irradiance \( |E_m - E_c|/E_m \) for every pixel. This gives the pictures on the right in the subfigures in Figure 10. For a more comprehensive depiction, we calculated the average residual and the standard deviation of the relative residual for each plane of observation, which is shown in Figure 11. Here, we see that all models start with the highest residual, but only those with a phenomenological phase function surpass an error of 10% while the Lambertian level off at around 30%.

The cause of the high residual for small distances between screen and band is shown in Figure 10: we see that the residual directly above the band is small, but next to it is large. In this plane, the fibers will block the light from each other because fibers and the screen are approximately situated in the same plane. Additionally, a possible interaction between the scattering screen and the fiber band makes the measured scattering more diffuse than in the calculation: some light is scattered back and forth between band and screen, causing additional diffuse irradiation. Also, the screen has no real Lambertian transmission for large incident angles. We conclude that the scattering screen measurement is unfit to measure the irradiance in proximity to the fiber.

With increasing distance \( D \) between screen and band, the aforementioned effects weaken, so the calculation and the measurement converge. Still, the line source shows slightly higher residuals and standard deviation. This shows that the cylinder source is the slightly more precise way of calculating the...
Spigulis et al. [10] proposed two methods to mediate the exponential decay in emittance and a visually unpleasing appearance. The phenomenon hinders the application of side-emitting fibers almost equally well to create a much more homogeneous irradiance field.

From the four proposed schemes we do explore, three perform better than single-sided coupling, leading to uneven illumination than single-sided coupling, but shows the same disadvantages of having a strong difference in irradiance from start to end.

5.4. Testing Alternative Fiber Coupling Schemes by Superposition

At last, we explore alternative light coupling schemes with the best performing fiber model, the cylinder source with the phenomenological phase function, to see if they create a more homogeneous illumination from a fiber band than basic single side coupling. We use the relative standard deviation of irradiance for each calculated plane as a quantitative comparison. From the four proposed schemes we do explore, three perform almost equally well to create a much more homogeneous irradiance field.

The exponential decay in emittance and the forward scattering property hinder the application of side-emitting fibers, leading to uneven illumination and a visually unpleasing appearance. Spigulis et al. [10] proposed two methods to mediate the exponential decay without having to resort to fibers with self-compensating scattering coefficient \( \sigma(z) \): First, double coupling, where light is coupled in both ends of the fiber, and second, a fiber end face mirror, to reuse the transmitted light by reflecting it back into the fiber. We additionally propose two more schemes: alternating, where light is coupled alternating from one side or the other in neighboring fibers, and a combination of alternating with end mirror.

We can now easily test these schemes for a fiber band with the calculated irradiance field of one fiber and the principle of superposition: For the double coupling, we use the result from Section 5.3, duplicate it, rotate the duplicate by 180°, and add it to the original calculated field. For the end mirror, we proceed in the same manner, but weigh the duplicate with the appropriate attenuation caused by the fiber transmission loss. Alternating these methods for every single fiber gives the other two schemes. The relative standard deviation is obtained by dividing the standard deviation of the irradiance in each plane of observation by the average irradiance in that plane.

In Figure 12 (left), we show an example for every scheme in two distances (30 and 100 mm) and the relative standard deviation (right). As it turns out, three of the four schemes lead to comparable uniform irradiance fields: both alternating and alternating with mirror provide the most homogeneous illumination with the same amount of couplings as the basic one-sided version. Double coupling is third but requires more couplings. Just using an end mirror on one side results in a more homogeneous illumination than single-sided coupling, but shows the same disadvantages of having a strong difference in irradiance from start to end.

6. Conclusion

We considered two methods to calculate the emitted light field of a side-emitting fiber in the radiometric approximation: A cylinder source and a line source. We validated experimentally that a standard side-emitting fiber possesses a position and angle-dependent radiance, which is properly represented in these models. The two contributions influence the emitted light field in two ways: The z-position dependence of the emittance is dominant close to the side-emitting fiber, and the angular dependence influences the field in the distance. We showed this by comparing calculated data from both approaches with real-world measurements of the radiation field. Both models are in good agreement with the measurement of the light field for distances larger than 80 mm from the emitter. Here, the cylinder source possesses slightly better predictive capability compared to the line source at the expense of higher computational effort. From the presented models, the radiation field of complicated arrangements of side-emitting fibers can be calculated by superposition. This was used to show that alternating the side of the light coupling between neighboring, parallel fibers in a fiber fabric can greatly improve the homogeneity of the generated radiation field.

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Conflict of Interest
The authors declare no conflict of interest.

Data Availability Statement
The data that support the findings of this study are available from the corresponding author upon reasonable request.

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line source, optical fiber fabrics, optical fiber illumination, radiometry, side-emitting optical fiber

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