Study of $J/\psi \rightarrow D_{s,d}V$ decays with perturbative QCD approach

Yueling Yang,¹ Junfeng Sun,¹ Jie Gao,¹ Qin Chang,¹ Jinshu Huang,² and Gongru Lu¹

¹Institute of Particle and Nuclear Physics,
Henan Normal University, Xinxiang 453007, China
²College of Physics and Electronic Engineering,
Nanyang Normal University, Nanyang 473061, China

Abstract

Inspired by the recent measurements on two-body nonleptonic $J/\psi$ weak decay at BESIII, the charm-changing $J/\psi \rightarrow D_{s,d}V$ weak decays are studied with perturbative QCD approach, where $V$ denotes $\rho$ and $K^*$ vector mesons. It is found that branching ratio for $J/\psi \rightarrow D_s\rho$ decay can reach up to $O(10^{-9})$, which is within the potential measurement capability of the future high-luminosity experiments.
I. INTRODUCTION

The $J/\psi$ particle is bound state of $c\bar{c}$ pair with given quantum numbers $I^GJ^{PC} = 0^-1^{--}$. Since its discovery in 1974 [2, 3], the $J/\psi$ meson is always a hot and active topic for particle physicists. The $c\bar{c}$ pair of the $J/\psi$ meson annihilate mainly into gluons, which provides a valuable resource to explore the properties of the quark-gluon coupling and the invisible gluons, to search for various glueballs and possible exotic hadrons. There are two hierarchies in the $J/\psi$ meson and other heavy quarkonium, one is dynamical energy scales responsible for production and decay interactions of particles, and the other is relative velocity of $c$ quark\(^a\). The $J/\psi$ meson plays a prominent role in investigation of QCD dynamical.

A conspicuous property of the $J/\psi$ meson is its narrow decay width, only about 30 ppm\(^b\) of its mass. The $J/\psi$ meson lies below the kinematic $D\bar{D}$ threshold. Its hadronic decay into light hadrons violates the phenomenological Okubo-Zweig-Iizuka rules [7–9]. Besides the decay dominated by the strong and electromagnetic interactions, the $J/\psi$ can also decay via the weak interaction within the standard model. In this paper, we will study the $J/\psi \to D_{s,d}\rho, D_{s,d}K^*$ weak decays with perturbative QCD (pQCD) approach [10–12].

Experimentally, thanks to the good performance of CLEO-c, BES, LHCb, B-factories, and so on, plenty of $J/\psi$ data samples have been accumulated. Recently, the $J/\psi \to D_{s,d}\rho, D_{s,d}K^*$ weak decays have been searched for at BESIII using part of the available $J/\psi$ samples [13]. It is eagerly expected to have about $10^{10}$ $J/\psi$ samples at BESIII per year with the designed luminosity [14], and over $10^{10}$ prompt $J/\psi$ samples at LHCb per $fb^{-1}$ data [15], which offers opportunities to discover phenomena that have been previously overlooked because of statistical limitations. So a careful scrutiny of $J/\psi$ weak decays at high-luminosity dedicated experiments may be possible in the future. In particular, the “flavor tag” of a single charged $D$ meson from $J/\psi$ decay will precisely identify potential signal from massive background. In addition, an abnormal large production rate of single $D$ meson from $J/\psi$ decay would be a hint of new physics.

Theoretically, the $J/\psi \to D_qV$ decay is, in fact, induced by $c \to q + W^+$ transition at

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\(^a\) According to the power counting rules of nonrelativistic quantum chromodynamics (NRQCD) [4–6], there are several distinct energy scales in charmonium, for example, typical three-momentum $m_cv$ and kinetic energy $m_cv^2/2$, where $v \ll 1$ is the typical relative velocity of heavy quark. Those energy scales satisfy a hierarchy relation $m_c \gg m_cv \gg m_cv^2$.

\(^b\) ppm means percent per million, i.e. $10^{-6}$.
quark level, where \( q = s \) and \( d \), the virtual \( W^+ \) boson materializes into a pair of quarks which then hadronizes into a vector meson \( V = \rho \) and \( K^* \). As it is well known, there must be the participation of strong interaction in nonleptonic \( J/\psi \) weak decay, and \( c \) quark mass is between perturbative and nonperturbative domain. In recent years, some QCD-inspired methods, such as pQCD approach [10–12], QCD factorization approach [16–18], soft and collinear effective theory [19–22], have been fully formulated to explain nonleptonic \( B \) decays. The \( J/\psi \rightarrow D_{s,d}V \) decays have been investigated based on collinear approximation [23–26]. In this paper, the \( J/\psi \rightarrow D_{s,d}V \) decays will be restudied based on \( k_T \) factorization. It is expected to glean new insights into factorization mechanism, nonperturbative dynamics, final state interactions, and so on, from nonleptonic \( J/\psi \) weak decay.

This paper is organized as follows. The theoretical framework and amplitudes for \( J/\psi \rightarrow D_{s,d}V \) decays are given in section II, followed by numerical results and discussion in section III. Finally, we summarize in the last section.

II. THEORETICAL FRAMEWORK

A. The effective Hamiltonian

Theoretically, one usually uses the effective Hamiltonian to describe hadron weak decay, where hard contributions can be decently factorized based on operator product expansion and the renormalization group (RG) method. The effective Hamiltonian responsible for \( J/\psi \rightarrow D_{s,d}V \) decay could be written as [27],

\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q_1,q_2} V_{cq_1}V_{q_2}^* \left\{ C_1(\mu) Q_1(\mu) + C_2(\mu) Q_2(\mu) \right\} + \text{h.c.}, \tag{1}
\]

where \( G_F \approx 1.166 \times 10^{-5} \text{GeV}^{-2} \) is Fermi constant; \( q_{1,2} = d \) and \( s \).

The Cabibbo-Kobayashi-Maskawa (CKM) factors are written as

\[
\begin{align*}
V_{cs}V_{ud}^* &= 1 - \lambda^2 - \frac{1}{2} \lambda^2 A^2 \lambda^4 + \mathcal{O}(\lambda^6), \quad \text{for } J/\psi \rightarrow D_s\rho \text{ decay} \\
V_{cs}V_{us}^* &= \lambda - \frac{1}{2} \lambda^3 - \frac{1}{8} \lambda^5 (1 + 4A^2) + \mathcal{O}(\lambda^6), \quad \text{for } J/\psi \rightarrow D_sK^* \text{ decay} \\
V_{cd}V_{ud}^* &= -V_{cs}V_{us}^* - A^2 \lambda^5 (\rho + i\eta) + \mathcal{O}(\lambda^6), \quad \text{for } J/\psi \rightarrow D_d\rho \text{ decay} \\
V_{cd}V_{us}^* &= -\lambda^2 + \mathcal{O}(\lambda^6), \quad \text{for } J/\psi \rightarrow D_dK^* \text{ decay}
\end{align*}
\tag{2}
\]

where \( A, \lambda, \rho, \eta \) are the Wolfenstein parameters; \( \lambda = \sin \theta_c \approx 0.2 \) and \( \theta_c \) is the Cabibbo angle. It is clearly seen that the \( J/\psi \rightarrow D_s\rho \) decay is favored by the CKM factor \( V_{cs}V_{ud}^* \).
The Wilson coefficients $C_{1,2}(\mu)$ summarize the physical contributions above the scales of $\mu$. They are calculated at scale of the $W$ boson mass $\mu \sim O(m_W)$ with perturbation theory, and then evolved to scale of the $c$ quark mass $\mu \sim O(m_c)$ with RG evolution function,

$$\vec{C}(\mu) = U_4(\mu, m_b)U_5(m_b, m_W)\vec{C}(m_W),$$

(3)

where $U_f(\mu_j, \mu_i)$ is RG evolution matrix $[27]$. The Wilson coefficients are independent of a particular process in the same role of universal gauge couplings. They have properly been evaluated to the next-to-leading order.

Generally, the penguin contributions induced by flavor changing neutral current transitions are proportional to small Wilson coefficients relative to tree contributions. Besides, for $c$ quark decay, the penguin contributions are also severely suppressed by the CKM factors $V_{cd}V_{ud}^* + V_{cs}V_{us}^* = -V_{cb}V_{ub}^* \sim O(\lambda^5)$. Hence, only the tree operators related to $W$ emission contributions are considered here. The expressions of tree operators are

$$Q_1 = [\bar{q}_{1,\alpha}\gamma_\mu(1 - \gamma_5)c_{\alpha}]\bar{u}_{\beta}\gamma^{\mu}(1 - \gamma_5)q_{2,\beta},$$

(4)

$$Q_2 = [\bar{q}_{1,\alpha}\gamma_\mu(1 - \gamma_5)c_{\beta}]\bar{u}_{\beta}\gamma^{\mu}(1 - \gamma_5)q_{2,\alpha},$$

(5)

where $\alpha$ and $\beta$ are color indices.

The physical contributions below scales of $\mu$ are included in hadronic matrix elements (HME). Because of the participation of the strong interaction, the entanglement perturbative and nonperturbative effects, the inadequate comprehension of hadronization mechanism and low energy QCD behavior, HME is the most complicated and intractable part. To get the amplitude, one has to face directly the HME calculation.

B. Hadronic matrix elements

Phenomenologically, the simplest approximation is that HME is parameterized into the production of transition form factors and decay constant based on naive factorization (NF) scheme $[28]$. The NF treatment on HME deprives any physical mechanism that could illustrate strong phases and rescattering among participating hadrons, and loses the $\mu$ dependence of HME which must exist to cancel that of Wilson coefficients. So the Lepage-Brodsky hard scattering approach $[29]$ is usually used, and HME is generally expressed as the convolution of hard scattering kernel with distribution amplitudes (DAs), where DAs reflect...
nonperturbative contributions but are universal. The hard part is, in principle, perturbatively calculable as a power of series of coupling $\alpha_s$. To suppress the soft contributions and avoid the problem of the endpoint singularity from collinear assumption \([16-18]\), the transverse momentum of quarks are retained explicitly and the Sudakov factors are introduced for each of meson wave functions in evaluation of potentially infrared contributions with pQCD approach \([10-12]\). Finally, a decay amplitude could be written as a convolution integral of three parts \([10-12]\): the hard effects enclosed by the Wilson coefficients $C_i$, the rescattering kernel amplitudes $H$, and process-independent wave functions $\Phi$,

$$\int dk \, C_i(t) \, H(t, k) \, \Phi(k) \, e^{-S},$$

where $k$ is the momentum of valence quarks, $t$ is a typical scale and $e^{-S}$ is a Sudakov factor.

### C. Kinematic variables

In the center-of-mass frame of $J/\psi$ meson, the light-cone kinematic variables are defined as follows.

\begin{align*}
p_{J/\psi} &= p_1 = \frac{m_1}{\sqrt{2}} (1, 1, 0), \\
p_D &= p_2 = (p_2^+, p_2^-, 0), \\
p_V &= p_3 = (p_3^-, p_3^+, 0), \\
k_i &= x_i p_i + (0, 0, \vec{k}_iT), \\
e_i^\parallel &= \frac{p_i}{m_i} - \frac{m_i}{p_i \cdot n_+} n_+, \\
n_+ &= (1, 0, 0), \\
n_- &= (0, 1, 0), \\
p_i^\pm &= (E_i \pm p)/\sqrt{2}, \\
s &= 2 \, p_2 \cdot p_3, \\
t &= 2 \, p_1 \cdot p_2 = 2 \, m_1 \, E_2, \\
u &= 2 \, p_1 \cdot p_3 = 2 \, m_1 \, E_3, \\
p &= \frac{\sqrt{m_1^2 - (m_2 + m_3)^2} \, [m_1^2 - (m_2 - m_3)^2]}{2 \, m_1},
\end{align*}
where the subscript \( i = 1, 2, 3 \) on variables, including polarization vector \( \epsilon_i \), four dimensional momentum \( p_i \), energy \( E_i \) and mass \( m_i \), correspond to initial \( J/\psi \) meson, recoiled \( D \) meson, emitted vector meson \( V = \rho \) and \( K^* \), respectively; \( x_i \) and \( k_i (\vec{k}_{iT}) \) denote the longitudinal momentum fraction and (transverse) momentum of valence quark \( s \), respectively; \( n_+ \) is the plus null vector; \( s, t \) and \( u \) are Lorentz transformation scalars; \( p \) is the common momentum of final states. The kinematic variables are displayed in Fig. 2(a).

### D. Wave functions

Taking the convention of Ref. [30, 31], HME of the diquark operators squeezed between the vacuum and meson state is defined as below.

\[
\langle 0|c_i(z)\bar{c}_j(0)|\psi(p_1,\epsilon_1^\parallel)\rangle = \frac{f_\psi}{4} \int d^4k_1 e^{-ik_1\cdot z} \left\{ \epsilon_1^\parallel \left[ m_1 \phi_\psi^V(k_1) - \not{p}_1 \phi_\psi^t(k_1) \right] \right\}_{ji},
\]

(19)

\[
\langle 0|c_i(z)\bar{c}_j(0)|\psi(p_1,\epsilon_1^\perp)\rangle = \frac{f_\psi}{4} \int d^4k_1 e^{-ik_1\cdot z} \left\{ \epsilon_1^\perp \left[ m_1 \phi_\psi^V(k_1) - \not{p}_1 \phi_\psi^T(k_1) \right] \right\}_{ji},
\]

(20)

\[
\langle D_q(p_2)|\bar{q}_i(z)c_j(0)|0\rangle = \frac{if_{D_q}}{4} \int d^4k_2 e^{ik_2\cdot z} \left\{ \gamma_5 \left[ \not{p}_2 \Phi_3^D(k_2) + m_2 \Phi_3^p(k_2) \right] \right\}_{ji},
\]

(21)

\[
\langle V(p_3,\epsilon_3^\parallel)|u_i(z)\bar{q}_j(0)|0\rangle = \frac{1}{4} \int_0^1 dk_3 e^{ik_3\cdot z} \left\{ \epsilon_3^\parallel m_3 \Phi_V^v(k_3) + \epsilon_3^\parallel \not{p}_3 \Phi_V^t(k_3) - m_3 \Phi_V^s(k_3) \right\}_{ji},
\]

(22)

\[
\langle V(p_3,\epsilon_3^\perp)|u_i(z)\bar{q}_j(0)|0\rangle = \frac{1}{4} \int_0^1 dk_3 e^{ik_3\cdot z} \left\{ \epsilon_3^\perp m_3 \Phi_V^v(k_3) + \not{p}_3 \Phi_V^t(k_3) + \frac{i m_3}{p_3 \cdot n_+} \varepsilon_{\alpha \beta \mu \nu} \gamma_5 \gamma^\mu \gamma_3 \varepsilon_3^\parallel n_+^\beta \Phi_V^A(k_3) \right\}_{ji},
\]

(23)

where \( f_\psi \) and \( f_{D_q} \) are decay constants; wave functions \( \Phi_{\psi,V}^v,T \) and \( \Phi_{D}^p \) are twist-2; wave functions \( \Phi_{\psi,V}^{t,s,V,A} \) and \( \Phi_{D}^p \) are twist-3. For wave functions of light vector meson, only \( \Phi_V^v \) and \( \Phi_V^{V,A} \) are involved in decay amplitudes (see Appendix A). Their expressions are [30, 31]:

\[
\phi_V^v(x) = 6x(1 + \sum_{i=1}^3 a_i^V \left| c_i^V \right|^{3/2})
\]

(24)

\[
\phi_V^T(x) = \frac{3}{4}(1 + t^2)
\]

(25)

\[
\phi_V^A(x) = \frac{3}{2}(-t)
\]

(26)
where $\bar{x} = 1 - x$ and $t = \bar{x} - x$; $a_i^V$ is nonperturbative Gegenbauer moment and corresponds to Gegenbauer polynomial $C^{3/2}_i(t)$.

$$C^{3/2}_1(t) = 3t, \quad C^{3/2}_2(t) = \frac{3}{2}(5t^2 - 1), \quad \cdots$$

(27)

With the relation of $m_{J/\psi} \simeq 2m_c$ and $m_{D_q} \simeq m_c + m_q$, it is suspected that the motion of valence quarks in $J/\psi$ and $D_q$ mesons is nearly nonrelativistic. So their spectrum can be described with time-independent Schrödinger equation. Suppose the interaction between valence quarks is an isotropic harmonic oscillator potential, the ground state eigenfunction with quantum numbers $nL = 1S$ is expressed as:

$$\phi_{1S}(\vec{k}) \sim e^{-\vec{k}^2/2\omega^2},$$

(28)

where parameter $\omega$ determines the average transverse momentum, $\langle 1S|k_T^2|1S \rangle = \omega^2$. By using the transformation [32],

$$\bar{k}^2 \rightarrow \frac{1}{4}\left(\frac{\bar{k}_T^2 + m_q^2}{x_1} + \frac{\bar{k}_T^2 + m_q^2}{x_2}\right),$$

(29)

then integrating out transverse momentum $k_T$ and combining with their asymptotic forms, finally, DAs for $J/\psi$ and $D$ mesons are written as

$$\phi^v(x) = \phi^T(x) = A x \bar{x} \exp\left\{-\frac{m_c^2}{8\omega_1^2 x \bar{x}}\right\},$$

(30)

$$\phi^v(x) = B t^2 \exp\left\{-\frac{m_c^2}{8\omega_1^2 x \bar{x}}\right\},$$

(31)

$$\phi^V(x) = C (1 + t^2) \exp\left\{-\frac{m_c^2}{8\omega_1^2 x \bar{x}}\right\},$$

(32)

$$\phi^a_D(x) = D x \bar{x} \exp\left\{-\frac{\bar{x} m_q^2 + x m_c^2}{8\omega_2^2 x \bar{x}}\right\},$$

(33)

$$\phi^p_D(x) = E \exp\left\{-\frac{\bar{x} m_q^2 + x m_c^2}{8\omega_2^2 x \bar{x}}\right\},$$

(34)

where $\omega_i = m_i \alpha_s$ according to the NRQCD power counting rules [4], coefficients of $A$, $B$, $C$, $D$, $E$ are determined by the normalization conditions,

$$\int_0^1 dx \phi^v_{\psi}^{v,t,V,T}(x) = 1, \quad \int_0^1 dx \phi^a_D(x) = 1, \quad \int_0^1 dx \phi^p_D(x) = 1.$$
Here, it should be pointed out that there are many wave function models for $D$ meson, for example, Eq.(30) in Ref. [33]. The preferred one in Ref. [33] is:

$$\phi_D(x, b) = 6 x \bar{x} \left\{1 + C_D (1 - 2x)\right\} \exp\left\{-\frac{1}{2} w^2 b^2\right\},$$  \hspace{1cm} (37)$$

where $C_D = 0.4$ and $w = 0.2$ GeV for $D_s$ meson; $C_D = 0.5$ and $w = 0.1$ GeV for $D_d$ meson. In addition, the same form of Eq.(37), without a distinction between twist-2 and twist-3, is used in many practical calculation.

The shape lines of DAs for $J/\psi$ and $D_{s,d}$ mesons are displayed in Fig.1. It is clearly seen that (1) DAs for $J/\psi$ meson is symmetric versus $x$, and a broad peak of $\phi^{v,t,V}_D(x)$ appears at $x < 0.5$ regions, which is basically in line with the picture that momentum fraction is proportional to valence quark mass. (2) under the influence of exponential functions, DAs of Eqs.(30—34) fall quickly down to zero at endpoint $x, \bar{x} \to 0$, which is bound to suppress soft contributions. (3) The flavor symmetry breaking effects between $D_d$ and $D_s$ mesons, and difference between twist-2 and twist-3 are obvious in Eqs.(33,34) rather than Eq.(37). In this paper, we will use DAs of Eqs.(33,34) for $D$ meson.

E. Decay amplitudes

The Feynman diagrams for $J/\psi \to D_s \rho$ decay are shown in Fig.2 including factorizable emission topologies (a) and (b) where gluon connects $J/\psi$ with $D_s$ meson, and nonfactorizable emission topologies (c) and (d) where gluon couples the spectator quark with emitted
The amplitude for $J/\psi \rightarrow D_q V$ decay is written as [34]

$$A(J/\psi \rightarrow D_q V) = A_L(e_1^\parallel, e_3^\parallel) + A_N(e_1^\perp \cdot e_3^\perp) + i A_T \varepsilon_{\mu \nu \alpha \beta} e_1^\mu e_3^\nu p_1^\alpha p_3^\beta,$$  

which is conventionally written as helicity amplitudes [34],

$$A_0 = -F \sum_i A_{i,L}(e_1^\parallel, e_3^\parallel),$$  

$$A_\parallel = \sqrt{2} F \sum_i A_{i,N},$$  

$$A_\perp = \sqrt{2} F m_1 p \sum_i A_{i,T},$$

$$F = \frac{i}{\sqrt{2}} \frac{G_F}{N_c} \frac{C_F}{N_c} \pi f_\psi f_{D_q} f_V V_{eq1} V_{uq2},$$  

where the color number $N_c = 3$ and color factor $C_F = (N_c^2 - 1)/2N_c$; the subscript $i$ on $A_{i,j}$ corresponds to indices of Fig 2. The expressions of building blocks $A_{i,j}$ can be found in Appendix A. Our results show that (1) factorizable contributions [Fig 2 (a) and (b)] are color-favored, i.e., $a_1$-dominated; (2) nonfactorizable contributions [Fig 2 (c) and (d)] are proportional to small Wilson coefficient $C_2$ and suppressed by color factor $1/N_c$.

### III. NUMERICAL RESULTS AND DISCUSSION

In the rest frame of $J/\psi$ meson, branching ratio is defined as

$$B_r = \frac{1}{12\pi} \frac{p}{m_\psi^2 \Gamma_\psi} \left\{|A_0|^2 + |A_\parallel|^2 + |A_\perp|^2\right\}.$$  

FIG. 2: Feynman diagrams for $J/\psi \rightarrow D_s \rho$ decay, including factorizable diagrams (a) and (b), and nonfactorizable diagrams (c) and (d).
TABLE I: The numerical values of input parameters.

| CKM parameters \[1\] |
|----------------------|
| \(A = 0.814^{+0.023}_{-0.024}\), \(\lambda = 0.22537\pm0.00061\), \(\varrho = 0.117\pm0.021\), \(\bar{\eta} = 0.353\pm0.013\), |
| mass and decay constants |
| \(m_{\psi} = 3096.916\pm0.011\text{ MeV} \[1\]\), \(m_{D_s} = 1968.30\pm0.11\text{ MeV} \[1\]\), \(m_{D_d} = 1869.61\pm0.10\text{ MeV} \[1\]\), |
| \(f_{\psi} = 395.1\pm5.0\text{ MeV}^d\), \(f_{D_s} = 257.5\pm4.6\text{ MeV} \[1\]\), \(f_{D_d} = 204.6\pm5.0\text{ MeV} \[1\]\), |
| \(m_{K^*} = 891.66\pm0.26\text{ MeV} \[1\]\), \(m_{\rho} = 775.26\pm0.25\text{ MeV} \[1\]\), \(m_{c} = 1.67\pm0.07\text{ GeV} \[1\]\), |
| \(f_{K^*} = 220\pm5\text{ MeV} \[31\]\), \(f_{\rho} = 216\pm3\text{ MeV} \[31\]\), \(\Gamma_{\psi} = 92.9\pm2.8\text{ keV} \[1\]\), |
| \(m_s \approx 510\text{ MeV} \[35\]\), \(m_d \approx 310\text{ MeV} \[35\]\), |

Gegenbauer moments at \(\mu = 1\text{ GeV} \[31\]\)

| \(a_1^{K^*} = -0.03\pm0.02\), \(a_2^{K^*} = 0.11\pm0.09\), \(a_0^d = 0\), \(a_2^\rho = 0.15\pm0.07\). |

TABLE II: Branching ratios for \(J/\psi \to DV\) decays, where uncertainties of our results come from scale \((1\pm0.1)t_i\), quark mass \(m_c\), hadronic parameters and CKM parameters, respectively.

| Reference | \[24\]^e | \[25\] | \[26\] | \[36\] | this work |
|-----------|-------------|-------------|-------------|-------------|--------------|
| \(10^9 \times \text{Br}(J/\psi \to D_s \rho)\) | 2.54 | 5.1 | 2.2 | 1.3 | \(3.33^{+0.07+0.47+0.17+0.002}_{-0.42-0.51-0.17-0.002}\) |
| \(10^{10} \times \text{Br}(J/\psi \to D_s K^*)\) | 1.48 | 2.8 | 1.2 | 0.8 | \(1.86^{+0.57+0.28+0.12+0.010}_{-0.24-0.35-0.12-0.010}\) |
| \(10^{10} \times \text{Br}(J/\psi \to D_d \rho)\) | 1.54 | 2.2 | 1.1 | 0.4 | \(1.32^{+0.37+0.14+0.08+0.007}_{-0.16-0.19-0.08-0.007}\) |
| \(10^{11} \times \text{Br}(J/\psi \to D_d K^*)\) | ... | 1.3 | 0.6 | ... | \(0.80^{+0.23+0.05+0.06+0.009}_{-0.10-0.12-0.06-0.009}\) |

The values of input parameters are listed in Table I, where if it is not specified explicitly, their central values will be taken as the default inputs. Our numerical results are presented.

e The relations between CKM parameters \((\varrho, \eta)\) and \((\varrho', \eta')\) are \[1\]: \((\varrho + i\eta) = \frac{\sqrt{1 - A^2 \lambda^4 (\varrho' + i\eta')}}{\sqrt{1 - \lambda^2 [1 - A^2 \lambda^4 (\varrho' + i\eta')]}}\).

d The decay constant \(f_{\psi}\) can be obtained from experimental branching ratios for electromagnetic \(J/\psi\) decay into charged lepton pairs through the formula

\[
\text{Br}(J/\psi \to \ell^+ \ell^-) = \frac{16\pi}{27} \frac{f_{\psi}^2 \alpha_{\text{QED}}^2}{m_{\psi} \Gamma_{\psi}} \sqrt{1 - 4 \frac{m_{\ell}^2}{m_{\psi}^2} \{1 + 2 \frac{m_{\ell}^2}{m_{\psi}^2}\}},
\]

where \(\alpha_{\text{QED}}\) is the fine-structure constant, \(\ell = e\) and \(\mu\). One can get \[1\] \(f_{\psi} = 395.4\pm7.0\text{ MeV with} \text{Br}(J/\psi \to e^+ e^-) = (5.971\pm0.032)\%\), and \(f_{\psi} = 394.8\pm7.1\text{ MeV with} \text{Br}(J/\psi \to \mu^+ \mu^-) = (5.961\pm0.033)\%\), respectively, where the errors arise from mass \(m_{\psi}\), decay width \(\Gamma_{\psi}\) and branching ratios. The weighted average is \(f_{\psi} = 395.1\pm5.0\text{ MeV} \[23\].

The updated results are listed in Table 4 of Ref. \[23\].
in Table II, where the first uncertainty comes from the choice of the typical scale \((1\pm0.1)t_i\), and expression of \(t_i\) is given in Eq.(A24) and Eq.(A25); the second uncertainty is from quark mass \(m_c\); the third uncertainty is from hadronic parameters including decay constants and Gegenbauer moments; and the fourth uncertainty of branching ratio comes from CKM parameters. The following are some comments.

(1) As it is aforementioned, the \(J/\psi\) decay modes considered here are dominated by the color-favored factorizable contributions and insensitive to nonfactorizable contributions. So, generally, branching ratio for a given \(J/\psi \to D_{s,d}V\) decay has the same order of magnitude even with different phenomenological models.

(2) There is a clear hierarchical pattern among branching ratios, mainly resulting from the hierarchical structure of CKM factors in Eq.(2), i.e.,

\[
\text{Br}(J/\psi \to D_s\rho) \gg \text{Br}(J/\psi \to D_sK^*) \sim \text{Br}(J/\psi \to D_d\rho) \gg \text{Br}(J/\psi \to D_dK^*). \tag{45}
\]

In addition, because nonfactorizable contributions are suppressed by both small \(C_2\) and color factor \(1/N_c\), there is an approximate relationship,

\[
\frac{\text{Br}(J/\psi \to D_sK^*)}{\text{Br}(J/\psi \to D_s\rho)} \approx \frac{\text{Br}(J/\psi \to D_dK^*)}{\text{Br}(J/\psi \to D_d\rho)} \approx \lambda^2 \frac{f_{K^*}^2}{f_\rho^2}. \tag{46}
\]

Above all, the Cabibbo- and color-favored \(J/\psi \to D_s\rho\) decay has branching ratio \(\sim \mathcal{O}(10^{-9})\), which is well within the measurement capability of the future high-luminosity experiments, such as super tau-charm factory, LHC and SuperKEKB.

![FIG. 3: Contributions to branching ratio \(\text{Br}(J/\psi \to D_s\rho)\) versus \(\alpha_s/\pi\), where the numbers over histogram denote the percentage of the corresponding contributions.](image-url)
Here, one might question the practicability of pQCD approach and the feasibility of perturbative calculation because c quark mass seems to be not large enough. To clear this issue up or to check what percentage of contributions come from perturbative domain, contributions to branching ratio $Br(J/\psi \to D_s\rho)$ from different $\alpha_s/\pi$ region are displayed in Fig. 3. It is easily seen that about 80% contributions come from $\alpha_s/\pi \leq 0.4$ regions, which implies that the calculation with pQCD approach is valid. One of crucial reasons for the small percentage in the region $\alpha_s/\pi \leq 0.1$ is that the absolute values of Wilson coefficients $C_{1,2}$, $a_1$ and coupling $\alpha_s$ decrease along with the increase of renormalization scale $\mu$. Of course, a perturbative calculation with pQCD approach is influenced by many factors, such as Sudakov factors, the choice of scale $t$, models of wave functions, etc., which deserve much attention but beyond the scope of this paper.

There are many uncertainties on branching ratios. The first uncertainty from scale $t$ could be reduced by the inclusion of higher order corrections to HME and an improved control on nonperturbative contributions. The second uncertainty from wave function models or parameter $m_c$ will be greatly lessened with the relative rate of branching ratios, for example, Eq. (46). The third uncertainty is dominated by decay constants whose effects will be weakened with the increasing precision of experimental measurements and/or theoretical calculation using nonperturbative methods (such as lattice QCD and so on). The uncertainty from CKM factor is small. Moreover, other factors, such as the final state interactions which is important and necessary for $c$ quark decay, are not properly considered here, but deserve massive dedicated study. Our results just provide an order of magnitude estimation on branching ratio.

IV. SUMMARY

Within the standard model, the $J/\psi$ meson can decay via the weak interaction, besides the strong and electromagnetic interactions. With anticipation of copious $J/\psi$ data samples at the future high-luminosity experiments and gradual improvement of particle identification techniques, we investigated the charm-changing $J/\psi \to D_{s,d}\rho$, $D_{s,d}K^*$ weak decays with pQCD approach. It is found that the estimated branching ratio for the color- and CKM-favored $J/\psi \to D_s\rho$ decay can be up to $\mathcal{O}(10^{-9})$, which is very likely to be measured in the future.
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Appendix A: Building blocks of decay amplitudes

The expressions of building blocks $A_{i,j}$ are listed as follows, where subscript $i$ corresponds to indices of Fig.2 and $j$ corresponds to helicity amplitudes.

$$A_{a,L} = \int_0^1 dx_1 \int_0^1 dx_2 \int_0^\infty b_1 db_1 \int_0^\infty b_2 db_2 \phi_\psi^a(x_1) E_a(t_a)$$

$$\times H_a(\alpha, \beta_a, b_1, b_2) \alpha_s(t_a) a_1(t_a) \left\{ \phi_D^p(x_2) m_2 m_c u + \phi_D^a(x_2) \left[ m_1^2 s - (4 m_1^2 p^2 + m_2^2 u) \bar{x}_2 \right] \right\}. \quad (A1)$$

$$A_{a,N} = m_1 m_3 \int_0^1 dx_1 \int_0^1 dx_2 \int_0^\infty b_1 db_1 \int_0^\infty b_2 db_2 \phi_\psi^v(x_1)$$

$$\times E_a(t_a) H_a(\alpha, \beta_a, b_1, b_2) \left\{ -2 m_2 m_c \phi_D^p(x_2) + \phi_D^a(x_2) \left[ 2 m_2^2 \bar{x}_2 - t \right] \right\} \alpha_s(t_a) a_1(t_a). \quad (A2)$$

$$A_{a,T} = 2 m_1 m_3 \int_0^1 dx_1 \int_0^1 dx_2 \int_0^\infty b_1 db_1 \int_0^\infty b_2 db_2 \phi_\psi^v(x_1)$$

$$\times \phi_D^a(x_2) E_a(t_a) H_a(\alpha, \beta_a, b_1, b_2) \alpha_s(t_a) a_1(t_a), \quad (A3)$$

$$A_{b,L} = \int_0^1 dx_1 \int_0^1 dx_2 \int_0^\infty b_1 db_1 \int_0^\infty b_2 db_2 H_b(\alpha, \beta_b, b_2, b_1)$$

$$\times \phi_\psi^v(x_1) \phi_D^a(x_2) \left[ m_1^2 (s - 4 p^2) \bar{x}_1 - m_2^2 u \right]$$

$$+ 2 m_1 m_2 \phi_\psi^v(x_1) \phi_D^p(x_2) (s - u \bar{x}_1) \alpha_s(t_b) a_1(t_b). \quad (A4)$$

$$A_{b,N} = \int_0^1 dx_1 \int_0^1 dx_2 \int_0^\infty b_1 db_1 \int_0^\infty b_2 db_2 H_b(\alpha, \beta_b, b_2, b_1)$$

$$\times E_b(t_b) \alpha_s(t_b) \left\{ m_1 m_3 \phi_\psi^v(x_1) \phi_D^a(x_2) (2 m_2^2 - t \bar{x}_1) + 2 m_2 m_3 \phi_\psi^v(x_1) \phi_D^p(x_2) (2 m_1^2 \bar{x}_1 - t) \right\} a_1(t_b), \quad (A5)$$

$$A_{b,T} = 2 m_3 \int_0^1 dx_1 \int_0^1 dx_2 \int_0^\infty b_1 db_1 \int_0^\infty b_2 db_2 H_b(\alpha, \beta_b, b_2, b_1) E_b(t_b)$$

$$\times \alpha_s(t_b) a_1(t_b) \left\{ 2 m_2 \phi_\psi^v(x_1) \phi_D^p(x_2) - m_1 \phi_\psi^V(x_1) \phi_D^0(x_2) \bar{x}_1 \right\}, \quad (A6)$$
\[ A_{c,L} = \frac{1}{N_c} \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx_3 \int_0^\infty db_1 \int_0^\infty b_2 db_2 \int_0^\infty b_3 db_3 \]  
\[ \times \phi^\nu_\rho(x_3) E_c(t_c) H_c(\alpha, \beta_c, b_2, b_3) \alpha_s(t_c) \delta(b_1 - b_2) \]  
\[ \times C_2(t_c) \left\{ \phi^\nu_\psi(x_1) \phi^\nu_\psi(x_2) u \left[ t(\bar{x}_2 - \bar{x}_1) + s(x_3 - \bar{x}_2) \right] \right. \]  
\[ \left. + \phi^\nu_\psi(x_1) \phi^\nu_\psi(x_2) m_1 m_2 \left[ u(x_3 - \bar{x}_1) + s(x_3 - \bar{x}_2) \right] \right\}, \quad (A7) \]

\[ A_{c,N} = \frac{m_3}{N_c} \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx_3 \int_0^\infty db_1 \int_0^\infty b_2 db_2 \int_0^\infty b_3 db_3 \]  
\[ \times E_c(t_c) H_c(\alpha, \beta_c, b_2, b_3) \alpha_s(t_c) C_2(t_c) \delta(b_1 - b_2) \]  
\[ \times \left\{ \phi^\nu_\psi(x_1) \phi^\nu_\psi(x_2) \phi^\nu_\psi(x_3) 2 m_1 \left[ s(x_2 - x_3) + t(\bar{x}_1 - \bar{x}_2) \right] \right. \]  
\[ \left. + \phi^\nu_\psi(x_1) \phi^\nu_\psi(x_2) \phi^\nu_\psi(x_3) m_2 \left[ u(x_3 - \bar{x}_1) + t(\bar{x}_2 - \bar{x}_1) \right] \right. \]  
\[ \left. + \phi^\nu_\psi(x_1) \phi^\nu_\psi(x_2) \phi^\nu_\psi(x_3) 2 m_1 m_2 p(x_3 - \bar{x}_2) \right\}, \quad (A8) \]

\[ A_{c,T} = \frac{1}{N_c} \frac{m_3}{m_1 p} \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx_3 \int_0^\infty db_1 \int_0^\infty b_2 db_2 \int_0^\infty b_3 db_3 \]  
\[ \times E_c(t_c) H_c(\alpha, \beta_c, b_2, b_3) \alpha_s(t_c) C_2(t_c) \delta(b_1 - b_2) \]  
\[ \times \left\{ \phi^\nu_\psi(x_1) \phi^\nu_\psi(x_2) \phi^\nu_\psi(x_3) 2 m_1 \left[ s(x_2 - x_3) + t(\bar{x}_1 - \bar{x}_2) \right] \right. \]  
\[ \left. + \phi^\nu_\psi(x_1) \phi^\nu_\psi(x_2) \phi^\nu_\psi(x_3) m_2 \left[ u(x_3 - \bar{x}_1) + t(\bar{x}_2 - \bar{x}_1) \right] \right. \]  
\[ \left. + \phi^\nu_\psi(x_1) \phi^\nu_\psi(x_2) \phi^\nu_\psi(x_3) 2 m_1 m_2 p(x_3 - \bar{x}_2) \right\}, \quad (A9) \]

\[ A_{d,L} = \frac{1}{N_c} \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx_3 \int_0^\infty db_1 \int_0^\infty b_2 db_2 \int_0^\infty b_3 db_3 \]  
\[ \times \phi^\nu_\rho(x_3) E_d(t_d) H_d(\alpha, \beta_d, b_2, b_3) \alpha_s(t_d) \delta(b_1 - b_2) \]  
\[ \times \left\{ \phi^\nu_\psi(x_1) \phi^\nu_\psi(x_2) m_1 m_2 \left[ u(x_3 - x_1) + s(x_2 - x_3) \right] \right. \]  
\[ \left. + \phi^\nu_\psi(x_1) \phi^\nu_\psi(x_2) 4 m_1^2 p^2 (x_3 - x_2) \right\} C_2(t_d), \quad (A10) \]

\[ A_{d,N} = \frac{m_2 m_3}{N_c} \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx_3 \int_0^\infty db_1 \int_0^\infty b_2 db_2 \int_0^\infty b_3 db_3 \]  
\[ \times \phi^\nu_\rho(x_3) E_d(t_d) H_d(\alpha, \beta_d, b_2, b_3) \alpha_s(t_d) C_2(t_d) \delta(b_1 - b_2) \]  
\[ \times \left\{ \phi^\nu_\psi(x_1) \left[ 2 m_1^2 x_1 - t x_2 - u x_3 \right] + 2 m_1 p \phi^\nu_\psi(x_3) (x_2 - x_3) \right\}, \quad (A11) \]

\[ A_{d,T} = \frac{1}{N_c} \frac{m_2 m_3}{m_1 p} \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx_3 \int_0^\infty db_1 \int_0^\infty b_2 db_2 \int_0^\infty b_3 db_3 \]  
\[ \times \phi^\nu_\rho(x_3) E_d(t_d) H_d(\alpha, \beta_d, b_2, b_3) \alpha_s(t_d) C_2(t_d) \delta(b_1 - b_2) \]  
\[ \times \left\{ \phi^\nu_\psi(x_1) \left[ 2 m_1^2 x_1 - t x_2 - u x_3 \right] + 2 m_1 p \phi^\nu_\psi(x_3) (x_2 - x_3) \right\}, \quad (A12) \]
where $b_i$ is the conjugate variable of the transverse momentum $k_{iT}$; $\alpha_s$ is the QCD running coupling; $a_1 = C_1 + C_2/N_c$; $C_{1,2}$ are the Wilson coefficients.

The hard scattering function $H_i$ and Sudakov factor $E_i$ are defined as follows.

$$ H_{a(b)}(\alpha, \beta, b_i, b_j) = K_0(b_i\sqrt{-\alpha})\left\{\theta(b_i - b_j)K_0(b_j\sqrt{-\beta})I_0(b_j\sqrt{-\beta}) + (b_i \leftrightarrow b_j)\right\}, \quad (A13) $$

$$ H_{c(d)}(\alpha, \beta, b_2, b_3) = \left\{\theta(-\beta)K_0(b_3\sqrt{\beta}) + \frac{\pi}{2}\theta(\beta)\left[iJ_0(b_3\sqrt{\beta}) - Y_0(b_3\sqrt{\beta})\right]\right\} $$

$$ \times \left\{\theta(b_2 - b_3)K_0(b_2\sqrt{-\alpha})I_0(b_3\sqrt{-\alpha}) + (b_2 \leftrightarrow b_3)\right\}, \quad (A14) $$

$$ E_i(t) = \begin{cases} \exp\{-S_\psi(t) - S_D(t)\}, & \text{for } i = a, b \\ \exp\{-S_\psi(t) - S_D(t) - S_V(t)\}, & \text{for } i = c, d \end{cases} \quad (A15) $$

$$ S_\psi(t) = s(x_1, p_1^\perp, 1/b_1) + 2\int_{1/b_1}^t \frac{d\mu}{\mu} \gamma_q, \quad (A16) $$

$$ S_D(t) = s(x_2, p_2^\perp, 1/b_2) + 2\int_{1/b_2}^t \frac{d\mu}{\mu} \gamma_q, \quad (A17) $$

$$ S_V(t) = s(x_3, p_3^\perp, 1/b_3) + s(\bar{x}_3, p_3^\perp, 1/b_3) + 2\int_{1/b_3}^t \frac{d\mu}{\mu} \gamma_q, \quad (A18) $$

where $I_0$, $J_0$, $K_0$, $Y_0$ are Bessel functions; the expression of $s(x, Q, 1/b)$ can be found in Ref.\[10]; $\gamma_q = -\alpha_s/\pi$ is the quark anomalous dimension; $\alpha$ and $\beta_i$ are gluon and quark virtuality, respectively, where subscript $i$ on $\beta_i$ corresponds to indices of Fig.2.

$$ \alpha = \bar{x}_1^2 m_1^2 + \bar{x}_2^2 m_2^2 - \bar{x}_1 \bar{x}_2 t, \quad (A19) $$

$$ \beta_a = m_1^2 - m_c^2 + \bar{x}_2^2 m_2^2 - \bar{x}_2 t, \quad (A20) $$

$$ \beta_b = m_2^2 + \bar{x}_1^2 m_1^2 - \bar{x}_1 t, \quad (A21) $$

$$ \beta_c = \bar{x}_1^2 m_1^2 + \bar{x}_2^2 m_2^2 + \bar{x}_3^2 m_3^2 $$

$$ - \bar{x}_1 \bar{x}_2 t - \bar{x}_1 x_3 u + \bar{x}_2 x_3 s, \quad (A22) $$

$$ \beta_d = x_1^2 m_1^2 + x_2^2 m_2^2 + x_3^2 m_3^2 $$

$$ - x_1 x_2 t - x_1 x_3 u + x_2 x_3 s, \quad (A23) $$

$$ t_{a(b)} = \max(\sqrt{-\alpha}, \sqrt{-\beta_{a(b)}}, 1/b_1, 1/b_2), \quad (A24) $$

$$ t_{c(d)} = \max(\sqrt{-\alpha}, \sqrt{\beta_{c(d)}}, 1/b_2, 1/b_3). \quad (A25) $$

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