Thermopower generation and thermoelectric cooling in a Kane-Mele normal-insulator-superconductor nano-junction

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Abstract – We have studied the thermoelectric effect of a Kane-Mele normal-insulator-superconductor (KMNIS) junction at low temperatures using a modified version of the Blonder-Tinkham-Klapwijk (BTK) theory. Since both the (electronic) charge and thermal current due to the carriers are sensitive to the strengths of the spin-orbit coupling (SOC) present in the Kane-Mele model, a tunability of this junction device with regard to its thermoelectric properties can be experimentally achieved by certain techniques that are used to manipulate the values of the spin-orbit couplings. We have computed the Seebeck coefficient, figure of merit, thermoelectric cooling, coefficient of performance of the KMNIS junction as a self-cooling device and investigated the role of the Rashba SOC (RSOC) and intrinsic SOC (ISOC) parameters therein. Our results on the thermoelectric cooling indicate the practical realizability and usefulness for efficient cooling detectors, sensors, and quantum devices and hence could be crucial to the experimental success of the thermoelectric applications of such junction devices. Further we have briefly touched upon the condition that possibly distinguishes the transmission through a topological insulator from an ordinary one.

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Introduction. – One of the most explored topics in the field of condensed-matter physics is the study of the graphene [1,2] which is a two-dimensional single layer of hexagonal lattice of carbon atoms. The conduction and the valance bands in graphene touch each other along the six Dirac points where the quasiparticles show a linear Dirac-like energy dispersion. The unique geometrical structure of graphene has generated tremendous interest in different fields, such as electronics [1–3], optoelectronics [4,5], and spintronics [6–8]. Further, some other interesting phenomena such as anomalous quantum Hall effect [9,10], chiral tunneling [11,12], Klein paradox [11,13] have been observed in graphene. Moreover, the superconducting features can be induced in graphene by possible intercalation with dopant molecules [14] or via proximity effects [15,16]. Such prospects provide newer scopes of fabricating devices based on graphene-based superconductors.

Recently, the thermal and thermoelectric properties of graphene structure have gained much attention because of the large Seebeck coefficient and high thermal conductivity [17–19]. Previously, due to the experimental limitations in accessing nano-scale devices, the charge and spin-dependent thermoelectric properties were often ignored [20,21]. But recently improved techniques in low-temperature measurement devices provide opportunities for experimental observation of thermoelectric physics. Very recently Zuev et al. [18] and Wei et al. [22] have performed theoretical and experimental investigations of the thermoelectric effects of graphene sheets.

Two types of spin-orbit couplings (SOC) are proposed in graphene, namely the Rashba SOC and the intrinsic SOC, where it has been predicted by Kane and Mele that the presence of both the spin-orbit terms may be the reason behind realizing a new type of topological state which is known as the quantum spin Hall (QSH) state [23]. However owing to the very small strength of the spin gap (typically ~ 0.01–0.05 meV) [24,25] in graphene, the QSH state is not achieved in experiments. But there are possible methods available to induce enhanced SOC strengths in graphene, such as via adatoms [26], using proximity effect of three-dimensional topological insulators [27,28], by functionalization with methyl groups [29], etc. It is experimentally obtained that in a graphene nano-sheet the
RSOC strength can be enhanced up to 17 meV by proximity effects. Further, it is observed that from gold (Au) intercalation at the graphene-Ni interface [30], the RSOC is enhanced up to approximately 100 meV. A large Rashba splitting of about 225 meV in epitaxial graphene layers on the surface of Ni [31] and a giant RSOC at the graphene-Ir surface from Pb intercalation have been observed [32]. Moreover, the tunability of the RSOC strength via an external gate voltage provides an additional impetus in the field of spintronics.

On a parallel front, quantum transport through the junction devices is gaining increased attention in the field of modern research for developing the nano-devices at atomic/molecular level. The junction devices have interesting applications in the fields of thermoelectric, thermometric, solid-state cooling etc. In the past a good number of studies have been performed [33–37] where the junction devices are found to be very useful in a wide range of applications [38–40]. The recent development in the field of thermoelectric physics in small-scale junction devices provides new directions for fabricating self-cooling devices, thermopower devices, etc.

Motivated by the above, we have performed an extensive study of the thermoelectric effect of a Kane-Mele normal-insulator-superconductor (KMNIS) nano-junction by employing a modified Blonder-Tinkham-Klapwijk (BTK) theory that describes the low-energy transmission characteristics of nano/mesoscopic junctions. Physically the scenario corresponds to adatom decorated graphene NIS junction to account for finite strengths of the spin-orbit couplings. Thus, a KMNIS is used interchangeably with the insulating layer extending from $x \geq 0$ to $x = 0$ to $x = d$. It is considered that the $x \geq d$ region of the junction system has been produced by proximity effect by an external superconductor.

The effective Hamiltonian for graphene with both the spin-orbit couplings is given by

$$ H = -t_1 \sum_{(ij)} a_i^\dagger b_j + i \lambda_I \sum_{(ij)} V_{ij} (a_i^\dagger \sigma_z a_j$$

$$+ b_i^\dagger \sigma_z b_j) + i \lambda_R \sum_{(ij)} a_i^\dagger (\hat{\sigma} \times \hat{d}_{ij}) \cdot \hat{n} b_j + \lambda_v \sum_i a_i^\dagger a_i$$

$$- \lambda_v \sum_i b_i^\dagger b_i + \text{h.c.} \quad (1)$$

The first term denotes the nearest-neighbour (NN) hopping with a strength $t_1$, the second term denotes the intrinsic spin-orbit coupling (ISOC) given by the next-nearest-neighbours (NNN) hopping with the ISOC strength $\lambda_I$. $\langle \langle ij \rangle \rangle$ represents the summation over NNN, $V_{ij} = + (-)$ if the hopping is clock (anti-clock) wise. The third term indicates the Rashba term given by the nearest-neighbour (NN) hopping with the Rashba spin-orbit coupling (RSOC) strength $\lambda_R$, where $d_{ij}$ signifies the unit vector from site $i$ to $j$, $\sigma$ denotes the Pauli matrices and $\hat{n} = \hat{x}$ is unit vector along the interface normal.

An on-site energy term is given by $\lambda_v$, which is different for the two sites within a unit cell. In particular we have considered “+” energy for A sites and “−” energy for B sites. The operator $a_i^\dagger (b_i^\dagger)$ denotes creation and $a_i (b_i)$ denotes annihilation of an electron at the $R_i$ site of the A (B) sublattice. Various details related for the simplification of the Hamiltonian and the BTK formalism applied to the Kane-Mele normal-insulator-superconductor nano-junctions appear elsewhere [41].

It is important to note that, though in the presence of the RSOC and ISOC there are openings of gaps in the electronic dispersion (insulating regime) at the valley points ($K$ and $K'$), the incident particles will still be able to pass through the insulating gap under certain condition. This condition can be described as in the following. The values of the intrinsic and the Rashba coupling terms should be such that $(\lambda'_{1\sigma}^N + \lambda_R^2) < (E_F^N \pm E)^2$ (see eq. (10) and eq. (11) in ref. [41])$^1$, where $\lambda'_{1\sigma} = \lambda_v - \sigma \sqrt{3} \lambda_I$, $\lambda_R = 3 \lambda_R / 2$, $E_F^N$ is the Fermi energy and $E$ is the energy of the carriers. If this condition is violated, then there will be no transmission (see footnote $^1$). In a sense, this condition may be contemplated to distinguish a graphene-based quantum spin Hall insulator from that of an ordinary insulator. In this regard, the inset of fig. 1 of ref. [42] may be seen where a diamond-shaped region in the $\lambda_R$ vs. $\lambda_v$ plane (both scaled by $\lambda_I$) is identified as the topological phase where the transport is possible only via the edge states, while the bulk remains insulating. In a similar sense, the above condition yields a range of values for $\lambda_R$ and $\lambda'_{1\sigma}$ that are conducive to transmission.

The expression for the charge current through the KMNIS junction using the BTK theory can be found to have the following form [43]:

$$ I_{NS,\sigma} (E_F^N, T^N, E_F^{TN}, T^S) = e A_r v_N^S \int \tau_{\sigma} (E, \theta_{N1}) \times (f^N (E_F^N, T^N) - f^S (E_F^{TN}, T^S)) N(E) dE \cos \theta_{N1} d\theta_{N1}, \quad (2)$$

$^1$The radicand in the expression for the momentum in eq. (10) of [41] becomes negative if the stated condition is violated resulting in no transmission.
where \( N(E) \) denotes the density of states, \( v^N \) is the Fermi velocity, \( A_r \) is the area of contact, and \( f^N, f^S \) are the Fermi distribution functions for the normal and the superconducting leads, respectively. \( \tau_{\sigma}(E, \theta_{N1}) \) is the transmission probability where \( \tau_{\sigma}(E, \theta_{N1}) = 1 - |b_{\sigma}(E, \theta_{N1})|^2 + |a_{\sigma}(E, \theta_{N1})|^2 \), \( a_{\sigma} \) is the amplitude of the Andreev reflection, \( b_{\sigma} \) is the amplitude of the normal reflection, \( \theta_{N1} \) is the angle of incidence for electrons. The Fermi energy variation across the junction system is assumed to be of the form, \( \Delta E = E_F^N \Theta(-x) + E_F^D \Theta(d-x) + E_F^S \Theta(x-d) \), where \( E_F^N \) and \( E_F^S \) are the Fermi energies of the normal and the superconducting leads. \( E_F^P \) is the Fermi energy of the insulating barrier which is defined by, \( E_F^P = E_F^N + V_0 \). It should be noted that the Fermi energy of the insulating layer is ramped by \( V_0 \) via an arbitrary gate voltage applied across the barrier region. As the normal state resistance, \( R_N \) is given by \( R_N = \frac{2\pi e^2 N_0}{\hbar A_r} \) (the factor 2 comes due to the spin degeneracy, \( N_0 \) denotes the density of states at Fermi level), the electrical charge current takes the following form:

\[
I_{NS}(E_F^N, T^N, E_F^S, T^S) = \frac{1}{2e R_N N_0} \int \int \tau_{\sigma}(E, \theta_{N1}) \times |f^N(E_F^N, T^N) - f^S(E_F^S, T^S)| N(E) dE \cos \theta_{N1} \theta_{N1}, \tag{3}
\]

where the energy-dependent quantity, \( N(E) = \frac{|E_F^N + E_F^D|}{2e \hbar} \) is the number of transverse modes in a graphene sheet of width \( W \) [44].

**Seebeck coefficient.** Here we present the theory to calculate the Seebeck coefficient. A temperature difference between two dissimilar materials produces a voltage difference and this phenomenon is known as Seebeck effect. The Seebeck coefficient is a measurement of the amount of potential induced in the device divided by the temperature difference and is defined by \( S = \frac{\delta V}{\delta T} \).

We consider the left and right electrodes to serve as independent temperature reservoirs where the left electrode is fixed at temperature, \( T^N = T - \delta T/2 \) and the right electrode is fixed at temperature \( T^S = T + \delta T/2 \). The population of electrons in the left and the right leads are described by the Fermi-Dirac distribution functions, \( f^N \) and \( f^S \), respectively, where \( E_F^N = E_F^S \) at zero external bias.

Let us now consider an extra infinitesimal current induced by an additional voltage, \( \delta V \) and the temperature difference, \( \delta T \) across the junction in an open circuit. The currents induced by \( \delta T \) and \( \delta V \) are given by \( (dI)_T = I(E_F^N, T^N, E_F^P) - I(E_F^N, T^N, E_F^P) = T^N + \delta T) \) and \( (dI)_V = I(E_F^N, T^N, E_F^P) - I(E_F^N, T^N, E_F^P) = T^N \). Suppose that the current cannot flow in an open circuit, thus \( (dI)_T \) counterbalances \( (dI)_V \). It allows us to write

\[
(dI)_V = (dI)_T + (dI)_V = 0 \tag{4}
\]

where the expressions for the \( (dI)_T \) and \( (dI)_V \) can be obtained from eq. (3). Now the first-order expansion of the Fermi-Dirac distribution function in \( (dI)_T \) and \( (dI)_V \) yields the expression for the spin-dependent Seebeck coefficient as

\[
S_{\sigma} = \frac{\delta V}{\delta T} = \frac{1}{\epsilon T} \int dE d\theta_{N1} \cos \theta_{N1} E(E_F^N + E\tau_{\sigma}(E, \theta_{N1}) \frac{\partial f}{\partial E} \tag{5}
\]

where the energy \( E \) is shifted by the Fermi energy, \( E_F^N \).

Now the charge and spin Seebeck coefficients are usually defined by [45]

\[
S_{ch} = \frac{1}{2}(S_{up} + S_{down}), \quad S_{sp} = \frac{1}{2}|S_{up} - S_{down}| \tag{6}
\]

which can be computed from eq. (5) for \( \sigma = \text{up/down} \). The Seebeck coefficient, \( S \) is dimensionless (since \( e = 1 \) and \( k_B = 1 \)).

The efficiency of the device depends upon a quantity called as “figure of merit” (FM). To get a clear idea of the efficiency, one should compute the spin-dependent FM which is given by

\[
Z_{\sigma}T = \frac{S_{\sigma}G_{\sigma}T}{K_{\sigma}}, \tag{7}
\]

where \( S_{\sigma} \) is the Seebeck coefficient, \( G_{\sigma} \) is the electrical conductance, \( K_{\sigma} \) is the thermal conductance, and \( T \) is the absolute temperature. \( G_{\sigma} \) can be calculated from the relation \( G_{\sigma} = \frac{dL_{\sigma}}{dV} \) and is given by the form

\[
G_{\sigma} = \frac{1}{2e R_N E_F^P} \int \int \tau_{\sigma}(E, \theta_{N1}) \left( \frac{\partial f}{\partial E} \right) \times (E + E_F^N) dE \cos \theta_{N1} d\theta_{N1}. \tag{8}
\]

The thermal conductance, \( K_{\sigma} \) can be calculated from the relationship, \( K_{\sigma} = \frac{dL_{NS}}{dV} \), where \( L_{NS} \) is the thermal current flowing from the normal to the superconducting region. In the next subsection we shall discuss how the thermal current and the thermal conductance can be calculated.

Additionally, the charge FM (\( Z_{ch}T \)) and spin FM (\( Z_{sp}T \)) are defined as [45-47]

\[
Z_{ch}T = \frac{S_{ch}^2 (G_{up} + G_{down}) T}{K_{up} + K_{down}}, \quad Z_{sp}T = \frac{S_{sp}^2 |G_{up} - G_{down}| T}{K_{up} + K_{down}}. \tag{9}
\]

**Thermoelectric cooling.** The flow of electrons can also transport thermal energy through the junction which is responsible for the thermal current. The thermal current is defined as the rate at which the thermal energy flows from the left lead to the right lead. As said earlier, the left electrode, that is the normal lead, serves as the cold reservoir and the right one serves as hot reservoir. The junction device is connected to an external bias voltage \( V_B = (E_F^N - E_F^S)/e \) which drives the electrons to flow
from the normal lead to the superconducting lead. Thus, the electron removes the heat energy from the normal lead and transfers it to the superconducting lead which further makes the cold reservoir (normal) cool. The energy conservation allows us to write

\[
J_{NS_a}(E_F^N, T^N; E_F^S, T^S) + I_{NS_a}(E_F^N, T^N; E_F^S, T^S)V_B = J_{SN_a}(E_F^N, T^N; E_F^S, T^S).
\]

(10)

The analogue between the electronic charge current and the electronic thermal current allows us to write the outbound energy flow rate from the normal lead to the superconducting lead as

\[
J_{NS_a} = \frac{1}{2e^2R_NE_F^N} \int \int (E - eV_B)(E + E_F^N)\tau'_a(E, \theta_{N1}) \times |f^N(E - eV_B, T^N) - f^S(E, T^S)|dE \cos \theta_{N1}d\theta_{N1}.
\]

(11)

Similarly, the reverse, that is the rate at which the superconducting lead receives the thermal energy, is written as

\[
J_{SN_a} = \frac{1}{2e^2R_NE_F^N} \int \int E(E + E_F^N)\tau'_a(E, \theta_{N1}) \times |f^N(E - eV_B, T^N) - f^S(E, T^S)|dE \cos \theta_{N1}d\theta_{N1}.
\]

(12)

where again the energies are shifted by the Fermi energy and \( \tau'_a \) is given by the form

\[
\tau'_a(E, \theta_{N1}) = 1 - |b_a(E, \theta_{N1})|^2 - |a_a(E, \theta_{N1})|^2.
\]

(13)

The thermal conductance, \( K_\sigma \) can be calculated from the temperature derivative of \( J_{NS_a} \), that is, \( \frac{dJ_{NS_a}}{dT} \). This normal-insulator-superconductor (NIS) junction can be regarded as the electronic cooling device only when \( J_{NS_a} > 0 \), which implies that it is capable to remove the heat from the cold reservoir, thereby making it cooler.

The performance of this junction as a self-cooling device can be measured by the coefficient of performance (COP), where COP is defined as the ratio of the heat removed from the cold reservoir to the electrical power needed for driving the system. The COP for the electronic thermal current, namely, COP\(_\sigma\), is given by

\[
\text{COP}_\sigma = \frac{J_{NS_a}}{I_{NS_a}V_B} = \frac{J_{NS_a}}{J_{NS_a} - J_{SN_a}}.
\]

(14)

Results and discussions. – We have investigated the thermoelectric effect of a graphene-based normal-insulator-superconductor (NIS) junction device in the presence of Rashba and intrinsic spin-orbit couplings assumingly induced by the transition metal adatoms, where the effects of the spin-orbit couplings in graphene are mimicked by the Kane-Mele model [23]. When adatoms are adsorbed by graphene, the electrons of the outer most shell of adatoms are distributed among the carbon atoms of graphene. This causes a net positive charge in the vicinity of the adatoms and to screen this charge, electrons start to accumulate surrounding the adatom. This electron cloud results in an inhomogeneous electric field which cause enhanced spin-orbit couplings.

Here we include a note on the values of various parameters used for our numerical computation. To put things in perspective, we have considered some reasonable values of \( \Delta_0 \), for example, \( \Delta_0 \sim (10^{-3} - 10^{-2}) \text{ eV} \). \( E_F^N \) has been considered as \( 50\Delta_0 \). The Fermi velocity and the hopping parameter \( t_1 \) can be calculated from the Fermi energy through the relation, \( E_F = hV_Fk_F \), with \( V_F = \frac{3\sqrt{3}a}{2\pi} \). The strength of the NN hopping, \( \lambda_N \), and the NN hopping, \( \lambda_t \), are varied in the range \([0 : 0.165t_1]\) and \([0 : 0.02t_1]\) respectively. Physically, it implies decorating the graphene nano-ribbon by adatoms which induce the SOC couplings. From first-principle calculation it is obtained that Au-decorated graphene yields \( \lambda_N = 0.007t_1 \) and \( \lambda_R = 0.0165t_1 \) [26]. In our computation the value of the RSOC parameter is varied up to one order greater magnitude compared to that of the Au-decorated graphene and the strength of the ISOC is varied up to a slightly higher value. It is worth mentioning that, using different techniques, such as using an external gate voltage or organic solvents, etc., it is possible to enhance the SOC strengths up to a couple of orders of magnitude. Finally, the staggered term is taken as \( \lambda_0 = 0.1t_1 \). The temperature difference, \( \delta T \) that exists across the junction is taken as \( \delta T = 0.02\Delta_0 \), ensuring that the difference is indeed small.

Seebeck coefficient and figure of merit. Initially we show the results for the Seebeck coefficient for a pristine graphene (\( \lambda_R = \lambda_I = 0 \)). The variation of the Seebeck coefficient, \( S \), as a function of temperature (in units of superconducting gap, \( \Delta_0 \)) is shown in fig. 2(a). \( S \) increases from a low value at small \( T \) till \( T \sim 0.5\Delta_0 \), beyond which it saturates. To get an idea about the role of spin-orbit couplings on the behavior of thermopower, in fig. 2(b) we present the variation of the spin-resolved Seebeck coefficient, \( S \), as function of temperature for realistic values, that is, for Au-decorated graphene (see the values above). A comparison between the two does not yield any significant change in the thermopower profile and neither one gets spin-resolved signal. Thus, by some means, if

Fig. 2: The variation of the Seebeck coefficient \( S \) as a function of temperature, \( T \) (scaled by the superconducting order parameter, \( \Delta_0 \)) (a) for pristine graphene, (b) for Au-decorated graphene.
Fig. 3: The variation of the Seebeck coefficient $S$ as a function of temperature, $T$ (scaled by the superconducting order parameter, $\Delta_0$) for a larger RSOC parameter by one order greater magnitude compared to that of the Au-decorated graphene.

we are able to enhance the SOCs by one order of magnitude compared to the values available in the Au-decorated graphene, there could be noticeable effects of SOC. Thus, in fig. 3 we have shown the thermopower profile with graphene, there could be noticeable effects of SOC. Thus, in the subsequent discussions we shall use these values. In this context we would like to mention that, in the case of a standard NS junction (electrons with parabolic energy dispersion) there is no spin-resolved thermopower in the presence of RSOC. But a graphene-based junction in the presence of spin-orbit couplings yields a spin-resolved thermopower and hence should be relevant for spintronic applications. In the case of a graphene-based NS junction, it is observed that

$$a_S(E, \pm \theta N_1) \neq a_S(E, \mp \theta N_1),$$

$$b_S(E, \pm \theta N_1) \neq b_S(E, \mp \theta N_1),$$

where $a_S$ ($b_S$) is the Andreev (normal) amplitude. Thus, the integrals in eq. (5) yield different values for up- and down-spins, which eventually causes the spin-resolved thermopower. But in the case of a standard NS junction, the above inequalities are replaced by equalities, thereby yielding no spin-resolved thermopower.

To get an idea of how the spin-resolved Seebeck coefficient varies with both the spin-orbit couplings, and also to get an operating regime in the parameter space, we have plotted the spin-resolved Seebeck coefficient as a function of $\lambda_R$ and $\lambda_I$ in fig. 4(a) and fig. 4(b) at a specific value of the temperature, namely, $T = 0.5\Delta_0$. The color plots yield the information of the Seebeck coefficient for different values of the RSOC and the ISOC parameters. For certain values of the RSOC parameter ($> 0.1t_1$), irrespective of the ISOC strength, both spins show higher values of thermopower. Further, we have shown results for the charge and spin Seebeck coefficients in fig. 4(c) and fig. 4(d). The charge Seebeck coefficient shows higher values for larger strengths of RSOC, and for certain values of the SOC parameters, the spin Seebeck coefficient vanishes. The last result indicates the vanishing of the difference between the up and down Seebeck coefficients, that is $S_{up}$ becomes the same as $S_{down}$. This map aids in identifying an operating regime of the magnitude of the Seebeck coefficient corresponding to a variety of choices of $\lambda_R$ and $\lambda_I$.

Now we show the results on the “figure of merit” (FM) which defines the efficiency of this system as a thermopower device. The variations of the spin-dependent FM, $Z_{sp}T$, as a function of the spin-orbit couplings ($\lambda_R$ and $\lambda_I$) are presented in fig. 5(a) and fig. 5(b). Interestingly, the down-spin shows more efficiency compared to that of the up-spin and hence it contradicts the behaviour of the Seebeck coefficient. Further, we have shown the results for charge and spin FM in fig. 5(c) and fig. 5(d), where it is observed that for higher values of ISOC the charge FM becomes larger. The spin FM becomes zero for

\begin{align}
Z_{sp} &= \frac{S_{sp}T}{R_{sp}}, \\
Z_{ch} &= \frac{S_{ch}T}{R_{ch}}, \\
Z_T &= \frac{S_T}{R_T}.
\end{align}
the lower values of the RSOC parameters irrespective of the ISOC strength, thereby implying $G_{up} = G_{down}$. Such regions, along with others, are shown by black patches in fig. 5(d). Thus, these maps aid in deciding on the values of the parameters that may be used for maximizing the gain of these KMNIS junction devices.

**Thermoelectric cooling and coefficient of performance.**

Here we show the results of the thermoelectric cooling of the KMNIS junction. To get an idea of the effect of spin-orbit couplings on the thermal current, in fig. 6(a) we present the variation of a dimensionless quantity, $2J_{NS}e^2R_N/\Delta_0^2$ as a function of the biasing voltage where the temperature is fixed at $T = 0.5\Delta_0$. It is evident that at zero biasing voltage, the rate of the thermal current extracted from the cold (normal) reservoir is negative. Thus, to achieve cooling effects, a lower threshold voltage of the battery, namely, $V_{lower}$, is needed. That is, the thermoelectric cooling does not work when $V_B < V_{lower}$. Also beyond an upper threshold voltage, $V_{upper}$, the refrigeration effect ceases to exist. From the plots it is visible that for a very small range of the biasing voltage (having values in the vicinity of the superconducting order parameter), the thermoelectric cooling process is efficient and the maximum refrigeration occurs at around, $V_B \sim 0.9\Delta_0$. Moreover, we have checked that a lower threshold of the voltage, $V_{lower}$ required to trigger the cooling effect does not depend on the choice of the SOC parameters.

Next we have shown the spin-resolved thermal current as a function of both the spin-orbit couplings in fig. 6(b) and fig. 6(c) which provides an idea of the desired (for maximum thermoelectric cooling) range of values of the RSOC, the ISOC parameters and the corresponding thermal current. The biasing voltage is fixed at $V_B = 0.5\Delta_0$. For higher values of the RSOC parameter, the thermoelectric cooling for both spins becomes smaller. But at small SOC values, the variation is negligible. The reason behind this can be explained as follows. In the expression of the thermal current (see (11) and eq. (13)), the contributions of both the Andreev reflection (AR) and the normal reflection (NR) are negative. It is noticed that AR and NR always show opposite behaviour as functions of both the SOCs, in a sense that if AR increases with SOCs, then NR decreases and vice versa. As both AR and NR contributions are negative, the net change in the thermal current is very small.

Here we show the results of the performance of the KMNIS junction as a self-cooling device. To recapitulate, a measure of the performance and the efficiency of the KMNIS junction as a thermoelectric nano-refrigerator is defined by the coefficient of performance (COP). Figure 7(a) reveals that for a narrow range of the biasing voltage, the nano-refrigeration works (namely, $\sim 0.063\Delta_0$). Since $\Delta_0$ is in meV range for a conventional superconductor, the operating voltage is low. Further, it is understood that when the driving voltage exceeds the value of the superconducting order parameter, $\Delta_0$, the refrigeration vanishes, irrespective of the strengths of SOC.

Finally to complete our enumeration of the tunability of a KMNIS junction, the COP is plotted as a function of the RSOC and ISOC strengths in fig. 7(b) and fig. 7(c). The color plots provide the idea of the range of values of the RSOC, the ISOC and the corresponding coefficient of performance. The biasing voltage is fixed at $V_B = 0.125\Delta_0$ where the COP shows a large value (see fig. 7(a)). It is clear that the RSOC helps to enhance the COP for both the spins. The color plots provide a helpful hint on the tunable coefficient of performance.

**Conclusion.** – In summary, we have investigated the thermoelectric properties of a KMNIS junction in the
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