Establishing the Spectral Turnover of Blazar PKS 2155-304 as an Outcome of Radiative Losses

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ABSTRACT

The broad band optical/UV and X-ray spectra of blazars have been often modeled as synchrotron component arising from a broken power-law distribution of electrons. A broken power-law distribution is expected, since the high energy electrons undergo radiative losses effectively. The change in the energy index should then be 1 and corresponds to a spectral index difference of 0.5. However, one of the long outstanding problems has been that the observed index change is significantly different. On the other hand, recent high quality observations of blazars suggest that their local spectra may not be a power-law, instead have a slight curvature and often represented by a log parabola model. Using XMM-Newton observations spanning over 12 years for the BL Lac PKS 2155-304, we show that the optical/UV and X-ray spectra can be well represented by a broken log parabola model. Further, we show that such a spectrum can indicate the energy dependence of the electron escape timescale from the main acceleration zone. This novel approach, besides addressing the observed difference in the photon spectral indices, also tries to explain the spectral turn over in far UV/soft X-rays as a result of the radiative losses.

Key words: galaxies: active – BL Lacertae objects: individual: PKS 2155-304 – acceleration of particles – diffusion – X-rays: galaxies

1 INTRODUCTION

The low energy synchrotron spectral component of blazars is reproduced by a broken power-law function suggesting the underlying electron distribution to be of broken power-law in shape (Sambruna et al. 1994). A power-law electron distribution can be achieved under Fermi acceleration process, where the electrons gain energy while being scattered by magnetic turbulent structures embedded in the jet or by crossing a shock front (Rieger et al. 2007). Subsequently, the synchrotron losses modify the accelerated electron distribution into a broken power-law with the indices differing by unity (Kardashev 1962; Heavens & Meisenheimer 1987). However, the observed difference between the low and high energy synchrotron spectral components of blazars cannot be perceived within the synchrotron cooling interpretation of a power-law electron distribution (Mankuzhiyil et al. 2012, 2010).

Around the peak of the synchrotron component, the blazar spectra deviate considerably from a power-law with the spectra showing smooth curvature. In many cases, the spectra at the peak are well reproduced by a log-parabola function suggesting the underlying electron distribution to be a log-parabola as well (Massaro et al. 2004). The log parabola function however, is successful in explaining only a narrow band of the spectrum falling around the peak but fails to explain the spectrum over a broad energy range e.g., optical–X-ray energy bands (Massaro et al. 2004). Alternatively, the broadband synchrotron component of blazars is often fitted with a smooth broken power-law function or a power-law with an exponential cut off (Sinha et al. 2017).

PKS 2155-304 is a BL Lac class of blazars located at a redshift \( z = 0.116 \). Its synchrotron component peaks at UV energies (Madejski et al. 2016) and the broadband spec-
trum reflects a smooth broken power-law function (Chiappetti et al. 1999; Aharonian et al. 2005). An extensive study of PKS 2155-304 was carried out by *XMM-Newton* at optical/UV (180-600 nm) and X-ray (0.15-12 keV) energy bands (Gaur et al. 2017; Bhagwan et al. 2014) on different epochs spanning more than a decade. The broadband spectrum obtained through joint analysis of *NuSTAR* and *XMM-Newton* observations, supplemented with *Fermi* observations at gamma-ray energies, could be reproduced satisfactorily by a synchrotron and synchrotron self Compton emission models due to a broken power-law electron distribution (Madejski et al. 2016). However, narrow band X-ray analysis showed significant deviation from a power-law spectrum (Gaur et al. 2017). Massaro et al. (2008) performed a detailed X-ray analysis of the source using *BeppoSAX*, *XMM-Newton* and *Swift* observations from 1996 to 2007. They showed that the X-ray spectrum is well reproduced by a log-parabola function with the peak of the spectral energy distribution indicating a positive correlation with the spectral curvature. Nevertheless, the log-parabola model did not succeed well in explaining the combined optical/UV and the X-ray spectrum by *XMM-Newton* (Bhagwan et al. 2014).

In this work, we perform a detailed examination of *XMM-Newton* observations of PKS 2155-304 at optical/UV and X-ray energies. The optical/UV and X-ray data from 19 November 2000 to 28 April 2012 are analysed and the source spectra are obtained. The composite spectrum is then fitted with a broken log-parabola function. The rationale behind this being, the radiative cooling of a log-parabola electron distribution eventually modify it into a broken log-parabola distribution. We also show that such an electron distribution can be achieved when the electron escape timescale from the main acceleration region is energy dependent. Further, the analytical model developed under this scenario can fit the observed optical/UV and X-ray well and in turn, can hint the particle diffusion processes in AGN jets.

2 OBSERVATIONS AND DATA REDUCTION

We selected twenty XMM Newton archival data of PKS 2155-304, starting from 19 November 2000 to 28 April 2012, such that they have at least one simultaneous Optical/UV exposures with X-ray. For X-ray data, we used only the European Photon Imaging Camera (EPIC)-pn data and EPIC MOS data were avoided due to their low sensitivity, quantum efficiency and chances of pile up. This data were reduced using *XMM-Newton* Science Analysis System (SAS Version 14.0) following standard procedures. The calibrated photon event files for the pn data processing were produced using the command *epchain*. For pn data processing, both single and double events (PATTERN ≤ 4) of good quality (FLAG = 0) in the energy range 0.2-10 keV were considered. In the 10-12 keV energy range, a “Good Time Interval (GTI)” event list was produced by studying the light curve and fixing a threshold rate to omit background particle flaring.

The source spectrum was obtained using a circular region of size 40″ around the source. The background was estimated using two circular regions of similar size located away from the source on the same source CCD chip. For nine observations, *epatplot* indicated significant pile up and the source spectrum was extracted using an annular ring of inner radius 10″ and an outer radius between 38″ and 40″, within the CCD chip.

The Optical Monitor (OM) observations during the selected epochs were reprocessed with SAS pipeline *omichain*. The optical/UV data contain a significant galactic contamination which can manifest as large systematic error. To investigate this, we fitted the optical/UV data by a simple power-law function with addition of appropriate systematic error on data, required for better fit statistics. Four observations were discarded as they contained less than three optical/UV filter exposures. Similarly, two other observations demand a large systematic error (>30%) for a better statistics and hence they were also omitted in the present study. On an average, we found adding 3% systematic error to the rest of 14 observations can result in better fit statistics. The details of these observations are given in Table 1.

3 BROKEN LOG-PARABOLA MODEL

The optical/UV and the X-ray data of PKS 2155-304 for the selected epochs were fitted with X-ray Spectral Fitting Package (XSPEC) using user-defined (local) and the inbuilt models (Arnaud 1996). The X-ray absorption due to Galactic neutral hydrogen in the direction of PKS 2155-304 was estimated by fixing the hydrogen column density to 1.71 \times 10^{20} cm^{-2} (Bhagwan et al. 2014). The optical/UV data were corrected for galactic reddening using the parameter $E_{B-V} = 0.019$ (Seaton 1979; Schlafly & Finkbeiner 2011). Similar to earlier works, the X-ray spectra were found to significantly deviate from a power-law and were better represented by a log parabola, but failed to explain the optical/UV data (Gaur et al. 2017; Bhagwan et al. 2014).

In order to develop a consistent model capable of fitting both the optical/UV and X-ray data, we considered a scenario where a log parabola electron distribution is losing its energy under a synchrotron emission process (Massaro et al. 2004). For a small curvature, the radiative losses will steepen the index by ∼ 1 (Appendix A). If the escape of electrons from the main emission region is also considered, then the electron distribution will transform into a broken log parabola distribution with break occurring at an energy where the electron cooling timescale is equal to the escape timescale. The synchrotron spectrum resulting from such an electron distribution will again be a broken log parabola with the index differing by ∼ 0.5 (Sahayanathan et al. 2018).

To diagnose whether this interpretation is capable of explaining the broadband distribution of PKS 2155-304, we performed a joint fitting of optical/UV and X-ray (0.6-10 keV) data using a broken log parabola function defined by

$$ F(\epsilon) \propto \begin{cases} \left( \frac{\epsilon}{\epsilon_0} \right)^{-\alpha} \times \log(\epsilon/\epsilon_0), & \text{for } \frac{\epsilon}{\epsilon_0} \leq 1 \\ \left( \frac{\epsilon}{\epsilon_0} \right)^{-\alpha-\Delta} \times \log(\epsilon/\epsilon_0), & \text{for } \frac{\epsilon}{\epsilon_0} > 1 \end{cases} $$

(1)

Here, $\epsilon$ is the photon energy, $\epsilon_0$ is the break energy\(^{1}\), $\alpha$ is the index, $\Delta$ is the difference between the indices and $\beta$

\(^{1}\) Here, $\epsilon_0$ is the energy at which the two log parabola functions assume the same value and not the peak of the log parabola.
is the curvature parameter. Motivated by the synchrotron cooling interpretation of a log parabola electron distribution, we fixed $\Delta$ at 0.5 and obtained the best fit parameters. The fit results are given in Table 2. We also provide fit statistics, assuming a power-law model with a log parabola tail (PLLT) following (Bhagwan et al. 2014), for comparison. In Fig. 1 (left) we show the spectral fit for the observation ID 0411780501. The best fit X-ray, UV and optical fluxes are also given in Table 1. The X-ray spectra corresponding to the observation IDs 0158961401, 0411780101 and 0411780201 show significant negative curvature suggesting a plausible contribution of Compton component. Hence, for these observations the model parameters cannot be constrained effectively. Presence of Compton contamination was evident in most of the observations with the high energy data deviating considerably from the best fit model (Fig. 1).

A Spearman rank correlation analysis between the flux and the fitted parameters was performed to investigate their dependence. We found no significant correlation between these quantities. However, a weak negative correlation between the curvature parameter $\beta$ and the optical/UV flux can be seen with the lowest null hypothesis probability $P_{\gamma} = 0.03$ and corresponding rank correlation coefficient $r_\beta = -0.62$. This correlation is not very significant and hence we speculate that the spectral shape does not have any implication on the observed flux level rather may depend on dynamics which are unrelated to the flux variation. A plausible scenario can be the spectral curvature being dependent on the electron diffusive processes which may not have a direct association with the flux variation.

### 4 ELECTRON ESCAPE TIME-SCALE AND SPECTRAL CURVATURE

To interpret the spectral curvature and the broken log parabola representation of the observed optical/UV and X-ray spectra of PKS 2155-304, we considered a scenario where the electrons are accelerated at a shock front and escape into the downstream region where they lose their energy through radiative processes. Consistently, we label the region around the shock front as Acceleration Region (AR) and the downstream region as Emission Region (ER). In general, the steady state non-thermal electron distribution $n(\gamma)$ generated by an acceleration process which is balanced by the escape rate can be expressed as (e.g. Kardashev 1962)

$$\frac{d}{d\gamma} \left[ \left( \frac{\gamma - \zeta \gamma^2}{\tau_a} \right) n(\gamma) \right] + \frac{n(\gamma)}{\tau_e} = Q_\delta (\gamma - \gamma_0)$$

Here, we assume a mono energetic injection into AR at energy $\gamma_0$, $\zeta$ being the dimensionless electron energy and, $\tau_a$ and $\tau_e$ define the characteristic acceleration and escape timescales, respectively. The radiative loss rate encountered by the electrons in AR is determined by $\zeta\gamma^2$. Expressing $\zeta$ in terms of maximum Lorentz factor $\gamma_{\text{max}}$ attained by the electron, $\zeta = (\gamma_{\text{max}}/\tau_a)^{-1}$, equation (2) can be reduced for

| Obs.ID | Exposure (s) | X-Ray | UVW2 | UVM2 | UVW1 | U  | B  | V  |
|-------|--------------|-------|------|------|------|----|----|----|
| 0124930501 | 104868       | 7.534±0.036 | 0.782±0.039 | -    | -    | 2.17±0.110 | 1.004±0.081 | 4.419±0.291 |
| 0124930601 | 111475       | 5.161±0.011 | 0.733±0.015 | 1.844±0.002 | -0.036 | 2.565±0.062 | 1.989±0.101 | 9.555±0.048 | 3.607±0.198 |
| 0158961001 | 27159        | 4.497±0.004 | 2.663±0.016 | -0.135 | 1.841±0.091 | 2.909±0.148 | -    | -    |
| 0158960901 | 28919        | 4.849±0.050 | -    | -    | 2.981±0.152 | 1.38±0.070 | -0.089 | -0.357 |
| 0158961001 | 40419        | 6.714±0.007 | 1.059±0.062 | 2.599±0.129 | -0.180 | 1.851±0.092 | -    | 1.341±0.067 | -    |
| 0158961301 | 60415        | 10.617±0.013 | 2.032±0.102 | 5.603±0.280 | -0.281 | 3.776±0.187 | -0.188 | 2.865±0.144 | 14.695±0.906 |
| 0158961401 | 68814        | 4.506±0.023 | 1.277±0.064 | 3.561±0.180 | 2.622±0.131 | 4.296±0.251 | 2.12±0.108 | 10.726±0.722 |
| 0411780101 | 101102       | 6.04±0.015 | 1.855±0.093 | 5.05±0.248 | 3.399±0.167 | 5.65±0.279 | 2.749±0.136 | 14.685±0.738 |
| 0411780201 | 67911        | 13.178±0.036 | 2.143±0.107 | 5.715±0.284 | 4.049±0.201 | 6.872±0.342 | 3.123±0.156 | 15.842±0.873 |
| 0411780301 | 61216        | 16.125±0.042 | 1.888±0.097 | 4.169±0.205 | -0.205 | 2.799±0.138 | 4.138±0.207 | 1.947±0.097 | 7.901±0.426 |
| 0411780401 | 64820        | 8.811±0.221 | 1.739±0.087 | 4.589±0.229 | 3.181±0.158 | 5.162±0.259 | 2.377±0.119 | 10.209±0.588 |
| 0411780501 | 74298        | 5.254±0.019 | 0.959±0.046 | 2.464±0.124 | -0.124 | 1.644±0.083 | 2.493±0.128 | 1.812±0.081 | 5.109±0.313 |
| 0411780601 | 63818        | 7.523±0.028 | 1.022±0.050 | 2.543±0.127 | 1.795±0.089 | 2.844±0.143 | 1.354±0.088 | 5.551±0.318 |
| 0411780701 | 68735        | 1.466±0.007 | 0.452±0.023 | 1.254±0.064 | 0.849±0.042 | 1.369±0.071 | 0.686±0.038 | 3.414±0.199 |

Table 1. Observation details of PKS 2155-304 with XMM-Newton and the best fit X-ray and optical/UV fluxes. Pile up were noticed for the observation IDs with *.

Figure 1. Unfolded optical/UV and X-ray spectrum along with best fit models for Observation ID 0411780501. The models are broken log-parabola (left) and synchrotron emission with energy dependent electron escape (right).

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\[ \gamma_0 < \gamma < \gamma_{\text{max}}, \quad \text{to} \]
\[ \frac{d}{dy} \frac{\gamma}{\tau_0} = \frac{n(y)}{\tau_e} \tag{3} \]
This can be rewritten for an energy independent \(\tau_a\) as
\[ \frac{d \ln n(y)}{d \ln y} = -\xi \tag{4} \]
where, \(S(y) = \gamma n(y)\) is the energy density and \(\xi = \tau_0/\tau_e\). If we assume \(\tau_e\) also to be energy independent, then the resulting electron distribution will be a powerlaw \(n(y) \propto y^{-p}\) with index \(p = \xi + 1\). On the other hand, if we consider the escape timescale to be energy dependent such that \(\xi\) takes a form \(\xi = \xi_0 y^{\beta'},\) then equation (4) will have a solution
\[ S(y) \propto \exp \left( -\frac{\xi_0}{\beta'} y^{\beta'} \right) \tag{5} \]
Similarly, the steady state non-thermal electron distribution \(N(y)\) in ER can be expressed as
\[ \frac{d}{dy} \left[ y^2 N(y) \right] + \frac{N(y)}{\tau_e} = \frac{n(y)}{\tau_0(y)} \tag{6} \]
where, \(\eta_2(y)\) is the radiative loss rate in ER and \(\tau_e\) is the characteristic escape timescale from ER. For energy independent \(\tau_e\), solution to equation (6) will be of the form
\[ N(y) \propto \begin{cases} y^{\beta'-1} \exp \left( -\frac{\xi_0}{\beta'} y^{\beta'} \right), & \text{for } \gamma < \gamma_b \\ y^{\gamma_2} \exp \left( -\frac{\xi_0}{\beta'} y^{\beta'} \right), & \text{for } \gamma \gg \gamma_b \end{cases} \tag{7} \]
where, \(\gamma_b\) corresponds to the energy for which the cooling time scale equals to \(\tau_e\). The synchrotron photon flux due to this electron distribution will be (Sahayyanathan et al. 2018)
\[ F_{\nu y}(\epsilon) \propto \begin{cases} e^{-1+\beta'/2} \exp \left( -\psi \epsilon^{\beta'/2} \right), & \text{for } \xi_0 \ll 1 \\ e^{-3/2} \exp \left( -\psi \epsilon^{\beta'/2} \right), & \text{for } \xi_0 \gg 1 \end{cases} \tag{8} \]
where, \(\psi\) is a parameter relating the observed photon frequency with electron energy and \(\epsilon'\) is the emitted photon corresponding to electron energy \(\gamma_b\). For \(\beta' \ll 1\), equation (8) will be equivalent to equation (1) with the parameters related as
\[ \beta' = \frac{3.5 \beta}{2a - 3}, \quad \psi = \left( \frac{2a}{3} \right)^{3.5} \frac{2a - 3}{3.5} \beta \tag{9} \]
The observed optical/UV and X-ray data of PKS 2155-304 for the selected epochs were fitted using the synchrotron spectrum from ER given by (8). Similar to the broken log parabola function, we found that this physical model can also fit observations well and in Table 2, we provide the fit results. In Fig. 1 (right) we show the spectral fit corresponding to this model for the observation ID 0411780501. This study thereby suggests the observed curvature in the X-ray spectrum of blazars may indicate energy dependence of the escape time scale. This in turn will depend on the electron diffusion in the jet medium and hence can provide information regarding the magnetic field structure of blazar jets.

### 5 Discussion

We show that the combined optical/UV and X-ray observations of PKS 2155-304 using XMM-Newton, spanning over a period of 12 years, are better represented by a broken log parabola function with minimal curvature. The index differences are consistent with the spectral turnover introduced by synchrotron loss of a parent log parabola electron distribution. This study, thereby resolves the inadequacy of the broken power-law representation of blazar spectra to explain the index difference under the synchrotron loss phenomena. Further, we show that the curvature of the fitted function can indicate the energy dependence of the electron escape rate from the main acceleration region. The simplistic physical model developed under this scenario can fit the observed optical/UV and X-ray data very well.

The spectral and temporal properties of PKS 2155-304 were also studied by Kapanadze et al. (2014) using Swift observations during 2005-2012. The X-ray spectra showed significant curvature and were reproduced better by a log parabola function. The observed anti-correlation between the spectral index at 1 keV and 0.3-10 keV flux suggested
that the spectra hardens at high flux states. The non-availability of information at low energy prevents the authors from precise estimation of the peak energy. A log parabola X-ray spectral shape of PKS 2155-304 was already identified in XMM-Newton observations during a period overlapping with the one considered here (Gaur et al. 2017). However, the main attempt of these works was to highlight the curved X-ray spectra and their behaviour during different flux states. On the other hand, here we perform a joint spectral fitting of optical–X-ray spectra. The inadequacy of a log parabola function to explain the combined optical–X-ray spectra was initially shown by Massaro et al. (2004), for the case of MKN 421. In case of PKS 2155-304, a satisfactory fit of optical–X-ray spectra can be obtained by PLLP model (Bhagwan et al. 2014). A plausible scenario under which such a spectrum can be obtained is when the electron escape timescale in ER is energy dependent; whereas, the acceleration and escape timescales in AR are energy independent (Sinha et al. 2017). The resultant particle distribution in AR will then be a power-law. On subsequent injection into ER, it will develop a smooth curvature at high energy, imitating a log parabola. On the other hand, here we show that a broken log parabola can provide a better fit without invoking additional number of parameters. This asserts the energy dependence of the escape timescale in AR rather than the ER.

Through the present work, we demonstrate that the slight curvature observed in addition to the power-law component in the blazar spectra can be translated into energy dependence of the escape timescale from the main acceleration region. This identification in turn, can provide clues on the electron diffusion processes in blazar jets. For instance, if the electron diffusion is mainly governed through pitch angle scattering, then this result can be helpful in understanding the magneto hydrodynamic nature of the blazar jets (Summerlin & Baring 2012). Alternatively, the information about the energy dependence of the escape timescale can indicate the magnetic field alignment in blazar emission zone (Achterberg & Ball 1994). This energy dependence can be coupled with cross-field and/or align-field diffusion coefficients and supplemented with the polarisation information, this can be used to draw a global picture regarding the magnetic field structure in blazars.

This research is based on the observations obtained with XMM-Newton satellite, an ESA science mission with instruments and contributions directly funded by ESA member states and NASA. This work has made use of the NASA/IPAC Extra Galactic Database operated by Jet Propulsion Laboratory, California Institute of Technology and the High Energy Astrophysics Science Archive Research Center (HEASARC) provided by NASA’s Goddard Space Flight Center. JKS thanks Jithesh V and Savithri H Ezhiokode for useful discussions. JKS thanks the University Grants Commission for the financial support through the RGNF scheme. Authors thank the anonymous referee for the comments that improved the quality of the manuscript.

**REFERENCES**

Achterberg A., Ball L., 1994, A&A, 285, 687

Aharonian F., et al., 2005, A&A, 442, 895

Arnaud K. A., 1996, in Jacoby G. H., Barnes J., eds, Astronomical Society of the Pacific Conference Series Vol. 101, Astronomical Data Analysis Software and Systems V. p. 17

Bhagwan J., Gupta A. C., Papadakis I. E., Wiita P. J., 2014, MNRAS, 444, 3647

Chiappetti L., et al., 1999, ApJ, 521, 552

Gaur H., Chen L., Misra R., Sahayanathan S., Gu M. F., Kushwaha P., Dewangan G. C., 2017, ApJ, 800, 209

Heavens A. F., Meisenheimer K., 1987, MNRAS, 225, 335

Kapanadze B., Romano P., Vercellone S., Kapanadze S., 2014, MNRAS, 444, 1077

Kardashev N. S., 1962, Soviet Ast., 6, 317

Madejski G. M., et al., 2016, ApJ, 831, 142

Mankuzhiyil N., Persic M., Tavecchio F., 2010, ApJ, 715, L16

Mankuzhiyil N., Ansoldi S., Persic M., Rivers E., Rothschild R., Tavecchio F., 2012, ApJ, 753, 154

Massaro E., Perri M., Giommi P., Nesci R., 2004, A&A, 413, 489

Massaro F., Tramacere A., Cavaliere A., Perri M., Giommi P., 2008, A&A, 478, 395

Rieger F. M., Bosch-Ramon V., Duffy P., 2007, Ap&SS, 309, 119

Sahayanathan S., Misra R., 2018, A&A, 18, 35

Sambruna R. M., Barr P., Giommi P., Maraschi L., Tagliaferri G., Treves A., 1994, ApJS, 95, 371

Schlafly E. F., Finkbeiner D. P., 2011, ApJ, 737, 103

Seaton M. J., 1979, MNRAS, 187, 73P

Sinha A., Sahayanathan S., Acharya B. S., Anupama G. C., Chitnis V. R., Singh B. B., 2017, ApJ, 836, 83

Summerlin E. J., Baring M. G., 2012, ApJ, 745, 63

APPENDIX A: LOG PARABOLA ELECTRON DISTRIBUTION UNDERGOING RADIATIVE LOSSES

A log parabola electron distribution, \( u(\gamma) \), undergoing synchrotron loss can be described by

\[
-d \frac{d}{dy} \left( \gamma^2 u(\gamma) \right) = Q_0 \gamma^{-a-b \ln \gamma} \tag{A1}
\]

Here, \( \gamma^2 \) is the radiative energy loss rate. Equation (A1) can be rewritten as

\[
\frac{d \ln W(\gamma)}{d \ln \gamma} = - \frac{Q_0}{\gamma W(\gamma)} \gamma^{-a+1-1-b \ln \gamma} \tag{A2}
\]

where, \( W(\gamma) = \gamma^2 u(\gamma) \). We assume a priori that the solution \( W(\gamma) \) is of a log-parabola type:

\[
W(\gamma) = W_0 \gamma^{-c-d \ln \gamma} \tag{A3}
\]

and recast equation (A2) to

\[
\frac{d \ln W(\gamma)}{d \ln \gamma} = - \frac{Q_0}{\gamma W_0} \gamma^{-c-1} - \frac{Q_0}{\gamma W_0} (1 + k \ln \gamma) \tag{A4}
\]

where, \( k = -a + c + 1 - (b-d) \ln \gamma \ll 1 \). Using equation (A3)

\[
\frac{d \ln W(\gamma)}{d \ln \gamma} = -c - 2d \ln \gamma \tag{A5}
\]

Comparing equations (A4) and (A5), we note that for consistency \( \kappa \) is required to be energy independent and hence, \( d = b \). Moreover, \( c = c - a + 1 = 2b \) and for \( b \ll 1 \),

\[
c \approx (a - 1) \left[ 1 + \frac{2b}{(a - 1)^2} \right]
\]

The resultant electron distribution will be

\[
u(\gamma) \propto \gamma^{-(a+1)-b \ln \gamma-\frac{2b}{a-1}} \tag{A6}
\]
Hence, the synchrotron loss process steepens the injected log parabola distribution by $1 + \frac{2b}{\sigma^2-1}$.

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