Students’ metacognitive skills in solving word problem

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Abstract. Problem solving is the primary purpose of the mathematics curriculum. Metacognitive skills play an important role in problem solving. Solving mathematics word problem is a part of the problem solving. Students need appropriate steps and strategies to solve word problems and, in many cases, each student uses different strategy in solving word problem. This is because the difference of mathematical ability. This study aimed to describe the profile of metacognitive skills of students with high math ability and low math ability in solving the mathematics word problem, from the aspects of metacognitive skills of planning, monitoring and evaluating. The type of this research is qualitative descriptive. Subjects of the study are two students of Senior High School in Yogyakarta. Data collection techniques are observation, test, documentation and interviews. The validity of the data using a triangulation method. Data analysis techniques of this research are data reduction, data display, and conclusion/verification. The results of the study show that students who have high mathematical abilities meet the indicators of metacognitive skills in the planning, monitoring and evaluating stages. While students who have low mathematical abilities meet indicators of metacognitive skills in the planning stage but, have not fully met the indicators of metacognitive skills in the monitoring and evaluating stages in solving problems.

1. Introduction
Mathematics is one of the exact sciences and systematically organized which underlies other sciences in developing technology in various fields. The role of mathematics is very important as a servant of various sciences in interpreting an idea and conclusion logically. In Indonesia, mathematics is one of the subjects that must be taught at all levels of education, from basic education to higher education. One of the objectives of mathematical learning according to Indonesian Education Curriculum is learning to solve problem [1].

According to NCTM, "Problem solving means engaging in a task for which the solution method is not known in advance. In order to find a solution, students must draw on their knowledge, and through this process, they will often develop new mathematical understandings. Solving problems is not only a goal of learning mathematics, but also a major means of doing so" [2]. Thus, problem solving is a part of the mathematics curriculum that is very important because in the learning process, students are allowed to gain experience using the knowledge and skills they already have to apply to solving problems that are not routine [3]. Word problems are a mathematical question presented in the form of sentences related to daily life. Students need the ability of understanding the problem in solving word problem.

The research of Mc Loughlin and Hollingworth [3] shows that effective problem solving can be obtained by giving students the opportunity to implement their metacognitive strategies when solving problems. According to research conducted by Ardina & Setianingsih [4], the mathematical abilities of student influence the abilities of solving mathematics word problem. Based on the results of that
research can be stated that each student has a different thinking process or strategy in solving problems due to differences in their level of mathematical ability.

The term metacognition was introduced by Flavell [5]. According to Flavell (1979), metacognition is defined as "cognition about cognition" or "thinking about thinking." He also explained that students who manage cognitive activities properly, allowing can handle tasks and solve problems well too. Furthermore, Flavell (1976) states that "Metacognition is the knowledge and awareness of one's cognitive processes and the ability to monitor, regulate and evaluate one's thinking". In line with Flavell, Woolfolk (1998) explains that metacognition refers to a way to raise awareness about thinking and learning is done [6]. This awareness will materialize when one can start thinking with a plan (planning), monitoring (monitoring) and evaluation (evaluating) and the results of the cognitive activity.

| Planning | Monitoring | Evaluating |
|----------|------------|------------|
| 1. Write down what is known, which is asked | 1. Believing the right path | 1. Check the strengths and weaknesses that have been done |
| 2. Can determine goals | 2. Establish results | 2. Resolve in a different way |
| 3. Can get a settlement plan | 3. Take steps steadily | 3. Can apply this method to other questions |
| 4. Can find relationships with questions that have been resolved | 4. Check the correctness of the steps | 4. Pay attention to how you work yourself |
| 5. Know the reasons for choosing a particular plan | 5. Look at different ways | 5. Evaluate the achievement of goals |
|          | 6. Analyze the suitability of the plans made with the implementation |                        |

Table 1. Indicators of the metacognition process [7]

Seeing the role of metacognitive skill in problem solving, the researcher interested in conducting research to describe the metacognitive skill of high school student in solving word problem based on the level of mathematical ability. From the problem, hopefully the researcher can describe metacognitive skills of student.

2. Method
This research was a qualitative research. This research was conducted in one of senior high school in Yogyakarta in the even semester of academic year 2017/2018. Data collection techniques were observation, test, documentation and interviews. The validity of the data was using a triangulation method. The subjects of the study consisted of two students of X grade, one student with high mathematical ability and one student with low mathematical ability. The instruments used in this research consist of the main instrument that was researcher while the supporting instruments that were word problem test and interview guide. The material used in this research was trigonometry related to its application in daily life because trigonometry could be solved by some strategies. The mathematics problem given to students was: “A park in the middle of Surakarta is in the form of a regular hexagon. In the middle of the park and in every corner of the park there is a flagpole. The distance of the pole in the middle of the park to the pole at the hexagon’s corner is 100 meters. What is the area of the park?”. The aim of the test was to know the steps of students in solving word problem, while the aim of the interview was to know the involvement of metacognitive skills of student that cannot be known though the test. Data analysis techniques of this research were data reduction, data display, and conclusion / verification [8].

3. Result and Discussion
Based on the scores of mathematical report and consultation with a partner teacher, then the selected subjects of the research are presented in the following table.
Table 2. The Subjects

| No. | Name Code | Mathematical Ability |
|-----|-----------|----------------------|
| 1   | H         | High                 |
| 2   | L         | Low                  |

Here are the result of research and data analysis of students’ metacognitive skill in problem solving.

3.1. Subject with High Mathematical Ability (H)
3.1.1. Subject’s Problem Solving Result

![Figure 1. The answer from subject H for method I](image1)

![Figure 2. The answer from subject H for method II and III](image2)

3.1.2. Analysis of Subject’s Problem Solving Result. Subject H works in 3 methods (in figure 1 and figure 2). For the first method (method I), subject H looks for the area of a triangle that is known to its three sides. He began the process by drawing circle and hexagons. He wants to make sure that a
regular hexagon is made up of 6 equilateral triangles. He draws a circle so that it is easy to know the angle in the triangles and in the end he finds that the angle in the triangle at the center of the circle is $60^\circ$.

Then subject H takes a triangle which is part of the hexagon to find the unknown length of the side. He looks for the length of side $o$ with the rule of cosine and he obtains the side length $o = 100$ meters. Because it is known that the three sides are the same, then he looks for the area of the triangle with the formula $L = \sqrt{s(s-a)(s-b)(s-c)}$ with $s = \frac{1}{2}(a + b + c)$ so that the area is $2500\sqrt{3}$.

Since one of the area of the triangle is known, subject H then multiplies by 6 according to the number of triangles in the hexagon. Subject H found the park area was $15000\sqrt{3} m^2$.

For the second method, subject H immediately calculates the area of the park with the rule of sine because it has known two sides and one corner of the triangle in the hexagon namely $L = \frac{1}{2} \cdot a \cdot b \cdot \sin O$, so that the same answer is obtained in the method I is $15000\sqrt{3} m^2$.

As for the third method, from the triangles in the hexagon, subject H looks for the height of the triangle and he applies the Pythagoras formula to find the unknown side, the upright side which is the height of the triangle. The height of the triangle obtained is $50\sqrt{3} m$. After that, he determines the area of the triangle with the formula $L = \frac{\text{base} \times \text{height}}{2}$ and the same result is $15000\sqrt{3} m^2$. Subject H also has another idea or way of determining the height of a triangle, which is the ratio of cosine angles and obtaining the same height which is $50\sqrt{3} m$.

### 3.1.3. Metacognitive Skill Analysis of Subject H

Based on the results of interviews and the results of the subject’s work, subject metacognition skills were obtained in solving problems as follows.

#### 3.1.3.1 Planning aspect

Subject H did not write down what was known and asked from the question. But based on the results of the interview he can explain that what is known is the shape of the garden is a regular hexagon. He already understands some of the characteristics of a regular hexagon which has the same angle and length of the side.

Then what was asked was the garden area in the form of a regular hexagonal. Based on what he already knew, he would solve the problem. Subject H first make a circle to determine the angle of the triangle in the center of the circle. Subject H divides $360^\circ$ by 6 and gets $60^\circ$. He uses three methods to solve this problem.

Based on the results of the interview also, subject H understood the purpose of this problem, because he had studied material about calculating the area and he had also found a problem like this when he took part in the olympics at middle school. Subject H uses the area of a triangle as the basis for calculating the area of a hexagon so that it also uses symbols related to a triangle. He chose to calculate the area of a triangle because a regular hexagon is made up of triangles.

Based on the information above, subject H has implemented all planning indicators optimally.

#### 3.1.3.2 Monitoring aspect

Information about regular hexagons is very important for the subject. This information made him think of looking for the area of the hexagon by looking for the area of the triangles that make up the hexagon first. After he has made a midpoint on the hexagon, he draws a line towards the vertex at the hexagon and there are 6 triangles.

Subject H calculates carefully especially when calculating the square number and square root of a number. If something goes wrong, he will fix it. When he checked the correctness of the calculation on method I, he tried to work with method II and method III. Method II and III also use the area of a triangle but how to calculate the area of a triangle is different from the method I. If in the method II and method III will later give the same answer, the answer is correct. Based on the information above, subject H has implemented all monitoring indicators optimally.
3.1.3.3 Evaluating aspect. Based on the results of the interview and the results of the student's answer, subject H had checked when he had found the answer with method I, then he was sure again when he used method II and found the same result. He managed to solve the problem with three methods even he explained it could be more than that. He has tested the truth of the answer by checking every step he takes. He believes that between method I, method II and method III is the right answer because it gives the same result, namely $15000\sqrt{3} m^2$. According to subject H, the problem he had given him had answered well and correctly. He can also take advantage of the time provided. Subject H works smoothly without significant difficulties.

Based on the information above, subject H has implemented all evaluating indicators optimally.

3.2. Subject with Low Mathematical Ability (L)

3.2.1. Subject’s Problem Solving Result

![Method I](image1)

![Method II](image2)

**Figure 3.** The answer method I and II from subject L

3.2.2. Analysis of Subject’s Problem Solving Result. In the figure 3, the first step subject L answers the question is by drawing a hexagon and writing things that are known in the hexagon. Then, subject L calculates the angles in the triangle. He found that the angles in the triangle were all the same at 60°.
After that, he could conclude that the triangles were equal, or he wants to make sure that a regular hexagon is made up of 6 equilateral triangles. The step taken by the subject in the method I is to find the height of the triangle first then look for the area of the triangle and end multiplying it by 6. But the answers obtained are wrong because the subject is not careful. He counts $3 \times 100 \times 50\sqrt{3}$ and divides it by 2 which produces $7500\sqrt{3}$.

For method II, subject L uses a triangle area formula known for the length of the three sides. In that way, the steps taken by subject L are just right, but at the end, he does an incorrect calculation. Subject L calculates the area of hexagon by $2500\sqrt{3} \times 6$ and divides it by 2 which gives the wrong answer.

3.2.3. Analysis Metacognitive Skill of Subject L. Based on the results of interviews and the results of the subject's work, subject metacognition skills were obtained in solving problems as follows.

3.2.3.1. Planning aspect. Based on the results of interviews and the work of the subject, subject L has been able to explain what is known and asked, know the information implied and he can determine the purpose of giving this matter.

Subject L makes a small circle first so that he can determine the angle of the triangle at the center of the circle. He calculates by dividing $360^\circ$ by 6 and obtaining $60^\circ$. He already understands some of the characteristics of a regular hexagon which has the same angle and length of the side. Subject L uses the area of a triangle as the basis for calculating the area of the hexagon. So he uses symbols related to triangles.

Subject L had already found a problem with this type (topic: finding area) when he looked for the area of an octagonal area. He realized that the previous knowledge he had taught and that he had experienced helped him greatly in answering this question. Subject L predicts how long he will solve the problem but turns out to be far from his prediction.

From the aspect of planning, subject L has implemented an optimal indicator where the subject is able to explain well what is known and asked, both explicit and implicit, and understand well the relationship between the two and related concepts or formulas.

3.2.3.2. Monitoring aspect. Subject L has solved the problem according to his plan and coherently. When working on the problem, subject L is not aware of any calculation errors that he did in method I and subject L does not test the solution for completion with method I. Subject L considers the answer in method I to be correct and becomes the reference for working on method II. The impact of subject L is only on the answer $7500\sqrt{3}$ which turns out to provide an incorrect solution.

In terms of monitoring, subject L has not implemented the monitoring indicators optimally because subject L did not check the correctness of the steps he had taken since he solved the problem with method I.

3.2.3.3. Evaluating aspect. At the stage of evaluating, subject L did not check when he had found the answer by method I. Subject L immediately used method II and found the same result. Subject L did not consider every step he worked on from the start.

In terms of evaluating, subject L has not implemented the evaluating indicator optimally. Subject L has not checked the strengths and weaknesses he has done.

4. Conclusion

Based on the results of the analysis, the results of the study show that students who have high mathematical abilities meet the indicators of metacognitive skills in the planning, monitoring and evaluating stages. While students who have low mathematical abilities meet indicators of metacognitive skills in the planning stage but, have not fully met the indicators of metacognitive skills in the monitoring and evaluating stages in solving problems.

References

[1] Kementerian Menteri Pendidikan dan Kebudayaan Republik Indonesia 2014 *Permendikbud No.*
59 Tentang Kurikulum SMA/MA/SMK/MAK (Jakarta: Kemendikbud)

[2] NCTM 2000 Principles and Standards for School Mathematics (Virginia: NCTM)

[3] Dwiani 2015 Proses metakognisi dalam pemecahan masalah matematika pada siswa kelas XI di SMA negeri Banyumas Jurnal Elektronik Pembelajaran Matematika 3 1021-34

[4] Ardina A R and Setianingsih R 2017 Metacognitive skills of senior high school students’ in solving word problems based on the level of mathematical ability Jurnal Ilmiah Pendidikan Matematika 3 127-31

[5] Flavell J H 1979 Metacognition and Cognitive Monitoring: A New Area of Cognitive-Developmental Inquiry American Psychologist 34 906-11

[6] Woolfolk A E 1998 Educational Psychology (Seventh Edition) (Boston: Allyn and Bacon)

[7] Wardani G A K and Yunianta T N H 2017 Analisis metakognisi siswa dalam memecahkan masalah matematika materi spldv ditinjau dari perbedaan gender Jurnal Mitra Pendidikan 1 1031-45

[8] Moleong L J 2007 Metode Penelitian Kualitatif (Bandung: Remaja Rosdakarya)