Many-Body Echo

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In this letter we propose a protocol to reverse a quantum many-body dynamical process. We name it “many-body echo” because the underlying physics is closely related to the spin echo effect in nuclear magnetic resonance systems. We consider a periodical modulation of the interaction strength in a weakly interacting Bose condensate, which resonantly excites quasi-particles from the condensate. A dramatic phenomenon is that, after pausing the interaction modulation for half a period and then continuing on with the same modulation, nearly all the excited quasi-particles in the resonance modes will be absorbed back into the condensate. During the intermediate half period, the free evolution introduces a \( \pi \) phase, which plays a role reminiscent of that played by the \( \pi \)-pulse in the spin echo. Comparing our protocol with another one implemented by the Chicago group in a recent experiment, we find that ours is more effective at reversing the many-body process. The difference between these two schemes manifests the physical effect of the micro-motion in the Floquet theory. Our scheme can be generalized to other periodically driven many-body systems.

How to reverse a quantum many-body dynamical process is a question of great interest, especially in recent discussions of quantum many-body chaos and quantum information scrambling [1–3]. Ultracold atomic gases provide a unique platform to address this kind of questions because of the following two reasons. Firstly, unlike other artificial quantum systems such as nuclear magnetic resonance (NMR) and trapped ions, where the number of qubits is currently limited to below a few hundreds, ultracold atomic gases are many-body systems containing a macroscopically large number of quantum particles. Secondly, in contrast to electronic systems in condensed matter materials where phonons are inevitably present and will cause decoherence and dissipation, ultracold atomic gases are isolated systems whose coherence times can be much longer than typical time scales of condensed matter systems.

One type of dynamics that has been widely explored in ultracold atoms is that under periodical driving [4]. For instance, the periodical modulation of optical lattices has been employed to create artificial magnetic fields [5,13] and topological bands [14,23] and to realise gauge field with dynamics [24,26]. Recently the Chicago group has explored the periodical modulation of the interacting strength between atoms in a weakly interacting Bose-Einstein condensate confined in a cylindrical box potential [27,28]. Such a modulation induces a parametric resonance and leads to an exponential growth of quasi-particles with energy close to half the modulation frequency [29]. To show that this many-body dynamics is indeed coherent, in a latest experiment they also attempted to reverse the many-body dynamics by inverting the time-dependence of the interaction modulation [30]. To be more precise, the following time-dependent interaction strength \( g(t) \) was considered

\[
 g(t) = \begin{cases} 
 g_0 \sin(\omega t) & 0 \leq t \leq nT \\
 g_0 \sin(\omega t - \pi) & nT \leq t \leq 2nT,
\end{cases}
\]  

where \( g_0 \) is the oscillation amplitude, \( \omega \) is the oscillation frequency, \( T = 2\pi/\omega \) is the period and \( n \) is an integer. This oscillation scheme is denoted by protocol (a) and shown in Fig. 1(a). During the first \( n \) periods of oscillations \( 0 \leq t \leq nT \), atoms are resonantly excited to states whose energy are in the vicinity of \( h\omega/2 \). In the second \( n \) periods of oscillations \( nT \leq t \leq 2nT \), however, a significant portion of those excitations are found to return to the condensate mode. This is a strong evidence that
a coherent many-body dynamical process can indeed be reversed.

In this letter we present a different oscillation scheme, denoted as the protocol (b) and shown in Fig. 1(b), which we show can reverse the many-body dynamics to a greater degree than the protocol (a). This scheme is mathematically described by

$$g(t) = \begin{cases} 
g_0 \sin(\omega t) & 0 \leq t \leq nT \\
0 & nT \leq t \leq (n + \frac{1}{2})T \\
g_0 \sin(\omega t - \pi) & (n + \frac{1}{2})T \leq t \leq (2n + \frac{1}{2})T. \end{cases}$$

(2)

In this scheme, the driving takes a half period break after the first $n$ periods of oscillation, whereby the system undergoes free evolution governed by the non-interacting Hamiltonian. The second $n$ periods of oscillation is a repetition of the first, which can be seen by letting $t' = t - T/2$ and writting $g(t') = g_0 \sin(\omega t')$ for $nT \leq t' \leq 2nT$. Without the half period pause inserted in between, we would simply have a single oscillation throughout the entire process and the quasi-particles will be continuously excited. Thus, the fact that our scheme can reverse the quasi-particle excitation process is quite counter-intuitive at first glance.

As we will explain in detail later, the underlying principle by which the protocol (b) reverses the dynamics is reminiscent of the spin echo \[31, 32\]. Spin echo in a NMR system is a scheme to refocus the magnetisation against the dephasing due to the inhomogeneous magnetic field. There, the magnetic field under which the spins process does not change, similar to the fact that our protocol (b) involves exactly the same modulation $g(t)$ in the first and the second $n$ periods of driving. The key of the spin echo effect is a $\pi$ pulse during the spin procession that inverts the spin orientation. In our protocol (b), the analogy of the $\pi$ pulse is the free evolution that introduces a phase to the wave function. Because of this close analogy with the spin echo and the many-body nature of our problem, we refer to the dynamics under our protocol as the “many-body echo”. Although we introduce the concept of many-body echo using a weakly interacting Bose condensate as an example, the idea can be generally applied to other many-body systems.

**Bogoliubov Theory.** We consider a Bose gas with a periodically modulated interaction, described by the Hamiltonian

$$\hat{H} = \int d\mathbf{r} \hat{\psi}^\dagger(\mathbf{r}) \left[ -\frac{\hbar^2 \nabla^2}{2m} + V_{tr}(\mathbf{r}) \right] \hat{\psi}(\mathbf{r}) + \frac{g(t)}{2} \int d\mathbf{r} \hat{\psi}^\dagger(\mathbf{r}) \hat{\psi}(\mathbf{r}) \hat{\psi}(\mathbf{r}) \hat{\psi}(\mathbf{r}),$$

(3)

where $\hat{\psi}(\mathbf{r})$ is the bosonic field, $V_{tr}(\mathbf{r})$ is the trapping potential, $m$ is the atom mass and $g(t)$ is the interaction strength. At $t = 0$, the system is non-interacting and all the particles are condensed in the ground state $\varphi_0(\mathbf{r})$ of the single particle Hamiltonian $\hat{h}(\mathbf{r}) = -\hbar^2 \nabla^2/(2m) + V_{tr}(\mathbf{r})$. After the interaction modulation is turned on, we monitor the dynamics by calculating the population of resonant excitations as a function of time from both protocol (a) (blue) and protocol (b) (red) depicted in Fig. 1. The dashed lines are the populations at $\epsilon_k = \hbar \omega/2$ and the solid lines are those at $\epsilon_k = (1/2 + 3\gamma^2/8)\hbar \omega$. Calculations are done for two values of modulation strength, i.e., $\gamma = 0.06$ in the upper panel and $\gamma = 0.04$ in the lower panel.

![FIG. 2. Normalised population of resonant excitations as a function of time from both protocol (a) (blue) and protocol (b) (red) depicted in Fig. 1. The dashed lines are the populations at $\epsilon_k = \hbar \omega/2$ and the solid lines are those at $\epsilon_k = (1/2 + 3\gamma^2/8)\hbar \omega$. Calculations are done for two values of modulation strength, i.e., $\gamma = 0.06$ in the upper panel and $\gamma = 0.04$ in the lower panel.](image-url)
opposite momentum collide and absorb one quanta of energy $\hbar \omega$. Shown in Fig. [2] are the population of atoms excited to the resonant energy $\epsilon_k = \hbar \omega/2$ and to a slightly modified resonant energy $\epsilon_k = (1/2 + 3\gamma^2/8)\hbar \omega$ (the significance of this modification will be explained later), calculated for the interaction modulations depicted in both protocol (a) and (b). Here $\gamma = g_0 n_0/(\hbar \omega)$ is a relatively small, dimensionless parameter that characterises the strength of the modulation. As we can see, for both schemes, the atoms are excited during the first stage of interaction modulation, but most of them are absorbed back to condensate after the second stage. It is also clear that the protocol (b) reverses the many-body process much better than the protocol (a), particularly for larger modulation strengths.

Floquet Hamiltonian. We first present our understanding of the above phenomenon in terms of the Floquet Hamiltonian, which governs the stroboscopic evolution of the system. We begin with the Bogoliubov Hamiltonian for the uniform condensate $\hat{H}_{\text{Bg}}(t) = g(t) N^2/(2V) + \sum_k \hat{H}_{\text{Bg}}^k(t)$, where $V$ is the volume and

$$\hat{H}_{\text{Bg}}^k(t) = [\epsilon_k + g(t)n_0] \hat{a}_k^\dagger \hat{a}_k + \frac{n_0 g(t)}{2} (\hat{a}_k^\dagger \hat{a}_{-k}^\dagger + h.c.).$$

(6)

To derive the Floquet Hamiltonian using the high frequency expansion, it is necessary to first apply the rotating frame transformation

$$\hat{R}(t) = \exp\left(\frac{i \omega t}{2} \sum_{k \neq 0} \hat{a}_k^\dagger \hat{a}_k\right).$$

(7)

to eliminate the resonance energy term. In doing so, the Hamiltonian in the rotating frame is given by $\hat{H}_R(t) = \hat{R}(t)[\hat{H}_{\text{Bg}}(t) - i\hbar \partial_t] \hat{R}^\dagger(t)$, which yields $\hat{H}_R(t) = g(t) N^2/(2V) + \sum_k \hat{H}_R^k(t)$ with

$$\hat{H}_R^k(t) = [\epsilon_k - \frac{\hbar \omega}{2} + g(t)n_0] \hat{a}_k^\dagger \hat{a}_k$$

$$\quad + \frac{g(t)n_0}{2} (e^{i \omega t} \hat{a}_k^\dagger \hat{a}_{-k}^\dagger + e^{-i \omega t} \hat{a}_k \hat{a}_{-k}).$$

(8)

For an interaction strength $g(t)$ periodic in $T$, an effective Floquet Hamiltonian $\hat{H}_{\text{eff}}$ capturing the evolution at integer periods of oscillation can be introduced by

$$T \exp\left(-\frac{i}{\hbar} \int_{0}^{T} \hat{H}_R(t)dt\right) = \exp\left(-\frac{i}{\hbar} \hat{H}_{\text{eff}} T\right),$$

(9)

where $T$ is the time-ordering operator and $\alpha$ specifies the initial reference time. By Fourier transforming $\hat{H}_R(t) = \sum_p \hat{H}_p e^{ip\omega t}$ and using the $1/\omega$ expansion, we obtain

$$\hat{H}_{\text{eff}} \approx \hat{H}_0^+$$

$$\sum_{p>0} \left(\frac{\hat{H}_p, \hat{H}_{-p}}{\hbar \omega} + \frac{\hat{H}_p, \hat{H}_0}{\hbar \omega e^{-i2\omega \alpha}} + \frac{\hat{H}_{-p}, \hat{H}_0}{\hbar \omega e^{i2\omega \alpha}}\right).$$

(10)

The effective Floquet Hamiltonian we define here is different from the conventional one [35, 36], in which the information of the initial state is absorbed in the kick operator.

For $0 \leq t \leq nT$, the time dependence of $g(t)$ is the same for protocol (a) and (b). Writing $\hat{H}_{\text{eff}} = \sum_k \hat{H}_{\text{eff}}^k$ and following Eq. [10] we obtain for this duration

$$\frac{\dot{\hat{H}}^k_{\text{eff}}}{\hbar \omega} = -\frac{1}{2} \gamma \hat{A}_y^k - \gamma^2 \hat{A}_x^k + \Delta_k \left(2 \hat{A}_z^k - 1\right),$$

(11)

where $\Delta_k \equiv \epsilon_k/\hbar \omega - 1/2 - 3\gamma^2/8$ and

$$\hat{A}_x^k = \frac{1}{2i} \left(\hat{a}_k^\dagger \hat{a}_{-k} - \hat{a}_k \hat{a}_{-k}\right),$$

$$\hat{A}_y^k = \frac{1}{2i} \left(\hat{a}_k^\dagger \hat{a}_{-k} + \hat{a}_k \hat{a}_{-k}\right),$$

$$\hat{A}_z^k = \frac{1}{2} \left(\hat{a}_k^\dagger \hat{a}_k + \hat{a}_{-k}^\dagger \hat{a}_{-k}\right).$$

These three operators form the group of pseudo-rotations, SU(1,1), obeying the commutation relations $\left[\hat{A}_x^k, \hat{A}_y^k\right] = -i \hat{A}_z^k$, $\left[\hat{A}_y^k, \hat{A}_z^k\right] = i \hat{A}_x^k$, and $\left[\hat{A}_x^k, \hat{A}_x^k\right] = i \hat{A}_x^k$.

The second $n$ periods of oscillation in the protocol (a) and (b) are governed by different Floquet Hamiltonians. For protocol (a), we find the following effective Hamiltonian

$$\frac{\hat{H}_{\text{eff,a}}^k}{\hbar \omega} = \frac{1}{2} \gamma \hat{A}_y^k - \gamma^2 \hat{A}_x^k + \Delta_k \left(2 \hat{A}_z^k - 1\right)$$

(12)

for $nT \leq t \leq 2nT$. If we consider the resonant modes $\epsilon_k \sim \hbar \omega/2$ such that $\Delta_k \sim 3\gamma^2/8$, it is clear that $\hat{H}_{\text{eff,a}}$ inverts $\hat{H}_{\text{eff}}^k$ up to the leading order of $\gamma$, but not to the second order of $\gamma^2$.

Now consider the protocol (b). During the half period of free evolution $nT \leq t \leq (n + 1/2)T$, the Hamiltonian in the rotating frame vanishes for the resonant modes with $\epsilon_k \sim \hbar \omega/2$. For $nT + T/2 \leq t \leq 2nT + T/2$, even though the functional form of $g(t)$ is the same as that in the second stage of protocol (a), the initial reference time characterized by parameter $\alpha$ in Eq. [9] is different. More specifically, we have $\alpha = 0$ for the protocol (a) and $\alpha = 1/2$ for the protocol (b). In the Floquet theory, this results in a difference in the so-called micro-motion term in the effective Hamiltonian [35]. Thus we find the effective Hamiltonian of the protocol (b) as

$$\frac{\dot{\hat{H}}_{\text{eff,b}}^k}{\hbar \omega} = \frac{1}{2} \gamma \hat{A}_y^k + \gamma^2 \hat{A}_x^k + \Delta_k \left(2 \hat{A}_z^k - 1\right)$$

(13)

for $nT + T/2 \leq t \leq 2nT + T/2$. Now we can see that both the first and the second term in $\hat{H}_{\text{eff,b}}^k$ are opposite to those in $\hat{H}_{\text{eff}}^k$. If we further consider the resonance modes specified by $\Delta_k = 0$, i.e., $\epsilon_k = (1/2 + 3\gamma^2/8)\hbar \omega$, $\hat{H}_{\text{eff,b}}^k$ completely inverts $\hat{H}_{\text{eff}}^k$ for all contributions up to
the order of $\gamma^2$. This explains why the protocol (b) reverses the many-body dynamical process better than the protocol (a). Since the difference between the two protocols lies in the second order terms of $\gamma$ in the effective Hamiltonian, it also explains why the difference is more significant for larger $\gamma$, as shown in Fig. 2. Finally, it can be shown that all the excitations with $\Delta_k \ll \gamma$ will be well reversed by our protocol.

**Many-Body Echo.** Now we discuss the connection between the underlying physics of the protocol (b) and the spin echo. For this purpose we introduce an alternative approach to understand the reversal of dynamics. As mentioned earlier, the first and second $n$ periods of oscillation in the protocol (b) are identical, which can be seen by writing $t' = t - T/2$, such that $g(t') = g_0 \sin(\omega t')$ for $nT \leq t' \leq 2nT$. However, they become inequivalent when viewed from the single rotating frame of reference introduced, leading to different Floquet Hamiltonians obtained earlier. Such an equivalence can be restored if we apply the unitary rotation $\hat{R}(t)$ for $0 \leq t \leq nT$ and another $\hat{R}(t')$ for $nT \leq t' \leq 2nT$ (i.e. $nT + T/2 \leq t \leq 2nT + T/2$). In this approach, the free evolution for the intermediate half period $nT \leq t \leq nT + T/2$ is according to the original Bogoliubov Hamiltonian in Eq. (6) but the system will be governed by the same effective Hamiltonian $\hat{H}_{\text{eff}}$ in Eq. (11) during both sections of the driving. Hence, the total evolution operator from $t = 0$ to $t = 2nT + T/2$ is given by

$$
\hat{U} = \hat{R}((2n+1)T)e^{-\frac{i}{\hbar}\hat{H}_{\text{eff}}nT}\hat{R}(nT) \times e^{-\frac{i}{\hbar}\sum_k \epsilon_k \hat{a}_k \hat{a}_k^\dagger R^k(nT)}e^{-\frac{i}{\hbar}\hat{H}_{\text{eff}}nT}\hat{R}(0).
$$

Restricting ourselves to resonance modes with $\epsilon_k \sim \hbar \omega/2$ and using $\hat{R}(nT) = (-1)^n$, we obtain $\hat{U} = \prod_k \hat{U}_k$ where

$$
\hat{U}_k = e^{-\frac{i}{\hbar}\hat{H}_{\text{eff}}k^2 nT}e^{-i\epsilon_k}e^{-\frac{i}{\hbar}\hat{A}_k nT}.
$$

The operator $e^{-i\epsilon_k}$ is reminiscent of the $\pi$-pulse inserted in the spin echo experiment. More precisely, this operator acts on the Bogoliubov-type many-body ground state as

$$
e^{-i\epsilon_k} = e^{-i\epsilon_k(\chi_{k\Lambda}^\dagger e^{-\frac{i}{\hbar}\hat{A}_k nT} - \chi_{k\Lambda}^\dagger e^{-\frac{i}{\hbar}\hat{A}_k nT})}.  \]$$

We note that this operator adds a $\pi$ phase shift to the wave function of excitations, which plays a key role in reversing the many-body dynamics.

**Harmonic Trapped Case.** For the uniform system, the resonance of excitations due to the interaction modulation has a typical width of the order of $\gamma \hbar \omega$, while our protocol only reverses those satisfying $|\epsilon_k - \hbar \omega| \ll \gamma \hbar \omega$. In order to achieve a complete reversal of all excitations, we turn to a quasi-one-dimensional Bose condensate in a harmonic trap with the frequency $\omega_z$. The advantage of this setup is that the single particle eigen-energy $\epsilon_j = (j + 1/2)\hbar \omega_z$ is discrete, such that only pairs of particles with $(j_1, j_2) \omega = \omega$ can be excited if the level separation $\hbar \omega_z$ is large than or comparable with the amplitude of interaction energy modulation. In such a case, almost all the excitations can be reversed by our protocol.

To demonstrate this, we consider a condensate with a strong transverse confinement $\omega_\perp = 2\pi \times 430$ Hz such that the modulation will only excite the axial modes. The condensate thus behaves like a one-dimensional system with an effective interaction modulation amplitude $g_0 = g_0/(2\pi l_z^2)$, where $l_z = \sqrt{\hbar/m \omega_z}$. The modulation frequency is chosen to be $\omega = 20\omega_z$, where the axial trapping frequency is $\omega_z = 2\pi \times 200$ Hz. We numerically solve the number-conserving BdG equations described earlier for this system with a total atom number $N = 730$ and a modulation strength $\tilde{\gamma} = \frac{\hbar g_0}{\hbar \sqrt{\hbar/m \omega_z}} = 0.12$, where $\tilde{\gamma} = N/l_z$ with $l_z = \sqrt{\hbar/m \omega_z}$. Shown in Fig. 3 are the total number of excitations $N_{\text{tot}}(t) = \sum_j N_j(t)$ from both the protocol (a) and (b). As shown in the inset of Fig. 3, the occupied modes indeed mostly satisfy the resonance condition $j_1 + j_2 = \omega/\omega_z$. We see that our protocol, again much more effective than the protocol (a), achieves an almost perfect reversal of all the excitations.

**Outlook.** In summary, we have developed an analogy of the spin echo in a Floquet quantum many-body system, which we refer to as the many-body echo. Although we demonstrate our protocol to realise the many-body echo in a weakly interacting Bose gas, we believe this method can be generalised to other quantum many-body systems under periodical driving and will find broad applications in future research of Floquet quantum matter. One application, for instance, could be facilitating the experimental measurement of the out-of-time-ordered correlation function by reversing the many-body dynamics. Another application draws inspiration from the spin echo effect, where the imperfect refocusing can be used to detect decoherence time due to spin-spin interactions. Similarly, in a many-body system with significant quasiparticles interactions, the degree to which our protocol...
does not reverse the dynamics can then be attributed to the quasi-particle interactions. Finally, concerning a generic issue of Floquet driving, the system will eventually be heated to infinite temperature as it keeps absorbing energy from the driving [40]. Thus one has to reply on effects such as the many-body localization to prevent heating and allows the formation of interesting phases in the Floquet system [41]. Our protocol opens up an alternative route for preventing heating whereby novel Floquet physics may become accessible.

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