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CLASSICAL AND NEW PLUMBED HOMOLOGY SPHERES 
BOUNDING CONTRACTIBLE MANIFOLDS

OĞUZ ŞAVK

Abstract. A central problem in low-dimensional topology asks which homology 3-spheres bound contractible 4-manifolds. In this paper, we address this question for plumbed 3-manifolds and we present two new infinite families. We consider most of the classical examples around the nineteen-eighties by reproving that they all bound Mazur manifolds. Also, we show that several well-known families bound possibly different types of contractible 4-manifolds. To unify classical and new results in a fairly simple way, we modify Mazur’s famous argument and introduce generalized Mazur manifolds.

1. Introduction

Freedman’s breakthrough work \cite{Fre82} expresses that every integral homology 3-sphere bounds a topological contractible 4-manifold. However, the smooth analogue of this implication produces an unresolved problem in low-dimensional topology.

\textbf{Problem 1.1} (Problem 4.2, \cite{Kir78b}). Which integral homology 3-spheres bound smooth contractible 4-manifolds or smooth integral homology 4-balls?\footnote{In this paper, we always work with integral homology 3-spheres, smooth contractible 4-manifolds, and smooth integral homology 4-balls, hence we drop the \textit{integral}, \textit{smooth}, 3- and 4- prefixes.}

It is mysterious to give a complete answer to this problem. However, in the past four decades, a considerable amount of progress has been achieved by restricting to plumbed 3-manifolds, and especially Seifert fibered homology spheres with three singular fibers. In general, Seifert fibered homology spheres can be uniquely realized as the boundaries of negative-definite, unimodular plumbing graphs of 4-manifolds. These graphs have one node with adjacent branches and the number of branches corresponds to the number of singular fibers.

Several infinite families of Seifert fibered homology spheres with three singular fibers were known to be bound contractible manifolds. After Kirby’s celebrated work \cite{Kir78a}, the classical articles appeared subsequently: Akbulut and Kirby \cite{AK79}, Casson and Harer \cite{CH81}, Stern \cite{Ste78}, Fintushel and Stern \cite{FS81}, Maruyama \cite{Mar81, Mar82}, and Fickle \cite{Fic84}. In addition, some of these results were found independently of Kirby calculus, see Fukuhara \cite{Fuk78} and Martin \cite{Mar79}. One can weaken Problem 1.1 to ask which homology spheres bound rational homology balls. Surprisingly, this slight version of the problem still remains hard. For affirmative answers, see \cite{FS84, AL18, Şav20} and \cite{Sim21}.

When the number of singular fibers increases, there is a strong conjecture for Seifert fibered homology spheres bounding homology balls first indicated by Fintushel and Stern \cite{FS87}, and explicitly stated by Kollár \cite{Kol08}. Thus, the above plenty number of examples may seem sporadic.

\textbf{Conjecture 1.2} (Three Fibers Conjecture\footnote{The proposed name for the conjecture comes from an e-mail correspondence with Ronald Fintushel.}). A Seifert fibered homology sphere with more than three singular fibers cannot bound a homology ball.
The main obstruction comes from Fintushel-Stern \( R \)-invariant \[\text{FS85}\]. Together with the short-cut of Neumann and Zagier \[\text{NZ85}\], we know that Seifert fibered homology spheres with the node different than minus one weight do not bound homology balls. This condition is even enough for the obstruction from bounding rational homology balls \[\text{IM20}\].

From the perspective of 4-dimensional handlebodies, the simplest contractible manifolds after the 4-ball \(B^4\) are Mazur manifolds \[\text{Maz61}\]. They are contractible manifolds obtained by attaching a single 1- and 2-handle to \(B^4\). In this paper, we provide a modification of Mazur’s argument by increasing the number of 1- and 2-handles through the eyes of knot theory. In the original construction, we change the role of the unknot with any ribbon knot; therefore, we call the resulting spaces generalized Mazur manifolds, see Section 2 for details.

We first notice that Conjecture 1.2 cannot be generalized for plumbed homology spheres that are not Seifert fibered. The first examples were provided by Maruyama \[\text{Mar82}\], independently refound by Akbulut and Karakurt \[\text{AK14}\]. These manifolds are not Seifert fibered unless \(n = 1\) and they can be realized as the boundary of a plumbing graph with two nodes and five branches, and are reproven via our approach.

**Theorem 1.3** (Maruyama, Akbulut-Karakurt). Let \(X(n)\) be Maruyama plumbed 4-manifold in the left-hand side of Figure 1. Then for each \(n \geq 1\), its boundary \(\partial X(n)\) is a homology sphere which bounds a Mazur manifold with one 0-handle, one 1-handle and one 2-handle.

![Figure 1. Maruyama plumbed 4-manifold \(X(n)\) and its companion \(X'(n)\).](image)

Next, we present a new infinite family with the same number of nodes and branches, which are not Seifert fibered\(^3\) again unless \(n = 1\).

**Theorem 1.4.** Let \(X'(n)\) be the companion of Maruyama plumbed 4-manifold in the right-hand side of Figure 1. Then for each \(n \geq 1\), its boundary \(\partial X'(n)\) is a homology sphere which bounds a Mazur manifold with one 0-handle, one 1-handle and one 2-handle.

We exhibit one more new infinite family by increasing the complexity of the graph whose plumbing has three nodes and seven branches.

**Theorem 1.5.** Let \(W(n)\) be Ramanujam plumbed 4-manifold in Figure 2. Then for each \(n \geq 1\), its boundary \(\partial W(n)\) is a homology sphere which bounds a generalized Mazur manifold with one 0-handle, two 1-handles and two 2-handles.

The case \(n = 1\) of our theorem interestingly appeared as a counter-example in the influential article of Ramanujam \[\text{Ram71}\] before the Kirby calculus was introduced.

\(^3\) Note that \(\partial X(1) = \Sigma(2, 5, 7)\) and \(\partial X'(1) = \Sigma(3, 4, 5)\), compare with \[\text{AK79}, \text{CH81}, \text{and Sav20}\].
This was the first example of a complex affine surface that is contractible and not analytically isomorphic to $\mathbb{C}^2$. This also provided the exhibition of the first exotic algebraic structure on $\mathbb{C}^3$ since $W(1) \times \mathbb{C}^2$ is not diffeomorphic to $\mathbb{C}^3$. For further directions about this subject, see papers [Man80], [Zai01] and [SS05]. To the author’s knowledge, this was also the first example of plumbed homology spheres bounding contractible manifolds.

We reprove most of the classical results about Seifert fibered homology spheres bounding contractible manifolds. Such spheres coincide with Brieskorn homology spheres $\Sigma(p,q,r)$ which are the links of singularities $x^p + y^q + z^r = 0$. The following families were from the lists of Casson and Harer [CH81]. The special cases of their list also independently exhibited in the papers of Martin [Mar79] and Maruyama [Mar81].

**Theorem 1.6** (Fukuhara, Maruyama, Martin, Casson-Harer). Let $p$ and $s$ be integers such that $p$ is odd. Then each Seifert fibered homology sphere $\Sigma(p,ps+1,ps+2)$ and $\Sigma(p,ps-2,ps-1)$ bounds a Mazur manifold with one 0-handle, one 1-handle and one 2-handle.

The Seifert fibered homology spheres $\Sigma(2,3,13)$ and $\Sigma(2,3,25)$ were known to be bound Mazur manifolds, see [AK79] and [Pic84]. Fickle proved that Stern’s examples [Ste78] bound Mazur manifolds as well. Our current perspective indicates that they also bound generalized Mazur manifolds.

**Theorem 1.7.** The following Seifert fibered homology spheres bound generalized Mazur manifolds with one 0-handle, two 1-handles and two 2-handles: $\Sigma(2,3,13)$, $\Sigma(2,3,25)$, $\Sigma(2,4n+1,20n+7)$, $\Sigma(3,3n+1,21n+8)$, $\Sigma(2,4n+3,20n+13)$, and $\Sigma(3,3n+2,21n+13)$ for each $n \geq 1$.

Therein, our approach simply unifies the proofs of classical and new examples of plumbed homology spheres bounding contractible manifolds. All these examples start with the only ribbon twist knots [CG86], namely the unknot and the stevedore knot, except new manifolds presented in the paper. Their initial ribbon knot is the square knot. Note that the proofs of $\Sigma(2,3,13)$ and $\Sigma(2,3,25)$ varied from those found in [Sav20]. Here, we cannot use the surgery description of manifolds $\Sigma(2,3,6n+1)$ in terms of the twist knots. Using the similar additional handle attachments, one can reprove that $\Sigma(2,3,7)$ and $\Sigma(2,3,19)$ bound rational homology balls, compare with [AL18, Proposition 3]. Since the proofs are usually addressed to the dual approach, new contractible manifolds may or may not be diffeomorphic to classical ones. This is currently unknown to the author but is highly unlikely to be the case.
For homology spheres, bounding homology balls is equivalent to being homology cobordant to the 3-sphere $S^3$. Therefore, they represent the trivial element in the homology cobordism group $\Theta_3^Z$. On the other hand, the algebraic structure of $\Theta_3^Z$ is very complicated. The pioneering work of Dai, Hom, Stoffregen and, Truong refers that $\Theta_3^Z$ admits a $\mathbb{Z}^\infty$ summand [DHST18], for more examples see [KS22].

For the crucial role of $\Theta_3^Z$ in topology, one may consult Saveliev’s book [Sav02] and Manolescu’s article [Man18].

Organization. The structure of the paper is as follows. In Section 2, we give the preliminaries about the essential tools for the proof of theorems. In Section 3, we describe the blow down procedures for surgery diagrams of plumbed homology spheres. We first present the proof for Theorem 1.5. Just after, the proofs of Theorem 1.3 and Theorem 1.4 are given together. Based on the previous work [Şav20], we finally prove Theorem 1.6 and Theorem 1.7 jointly.

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2. Plumbing, Slice and Ribbon Knots, and Contractible Manifolds

2.1. Plumbings. A plumbing graph $G$ is a weighted tree such that each vertex $v_i$ is decorated by an integer $e_i$. A plumbing graph $G$ gives rise to a simply connected smooth 4-manifold with boundary $X(G) = X$ which is obtained by plumbing together a collection of $D^2$-bundles over $S^2$. Thus, the Euler number of $D^2$-bundle corresponding to the vertex $v_i$ is given by $e_i$, see the recipe in [Sav02 Section 1.1.9].

Let $Y(G) = Y$ be the 3-manifold which is the boundary of $X$. Then $Y$ is said to be a plumbed 3-manifold.

The straight lines between any two vertices resulted in the plumbing process form edges and a linear plumbing graph having at least one edge is called a branch. Vertices with at least three adjacent branches are called nodes. Plumbing graphs are generally characterized by the number of nodes and branches respectively.

Let $|G|$ denote the number of vertices of a plumbing graph $G$. The second homology group $H_2(X, \mathbb{Z})$ of the plumbed 4-manifold $X$ is generated by vertices of $G$ and the intersection form on $H_2(X, \mathbb{Z})$ is given by the adjacency matrix

$$I = (a_{ij})_{i,j \in \{1,\ldots,|G|\}} \text{ where } a_{ij} = \begin{cases} e_i, & \text{if } v_i = v_j, \\ 1, & \text{if } v_i \text{ and } v_j \text{ are connected by one edge,} \\ 0, & \text{otherwise.} \end{cases}$$

The plumbed 3-manifold $Y$ is called a plumbed homology sphere if and only if the determinant of the intersection matrix $I$ is $\pm 1$. In this case, the corresponding plumbing graph is said to be unimodular. When the signature of $I$ is equal to the minus of number of vertices, a plumbing graph is called negative-definite.

2.2. Slice and Ribbon Knots, and Contractible Manifolds. A knot $K \subset S^3$ is said to be a slice knot if $K$ bounds a slice disk, i.e. smoothly properly embedded disk $D \subset B^4$. A slice disk having the only index 0 and 1 critical points with respect to the Morse function inherited from the radial function on $B^4$ is called a ribbon
disk and its boundary is said to be a ribbon knot. Equivalently, a knot $K \subset S^3$ is a ribbon knot if it can be built by attaching bands amongst components of an unlink. The minimal number of bands to produce a ribbon disk for a ribbon knot is called fusion number, see [Miy86].

Now we highlight Mazur’s famous argument [Maz61] in detail. Attaching a single 1-handle $B^1 \times B^3$ to a 0-handle $B^4$ gives $S^1 \times B^3$. If we attach a 2-handle $B^2 \times B^2$ to $S^1 \times B^3$ along a knot $J$ in $S^1 \times S^2$ that normally generates the fundamental group of $S^1 \times B^3$, then we produce a Mazur manifold - a contractible 4-manifold $W$ with one 0-handle, one 1-handle and one 2-handle. We have the following three observations in the original construction:

1. The boundary of $S^1 \times B^3$ is clearly $S^1 \times S^2$, which is also 0-surgery on the unknot in $S^3$,
2. The unknot in $S^3$ bounds a smooth disk $D$ embedded in $B^4$ and the tubular neighborhood of $D$, $\nu(D)$, is diffeomorphic to $D \times B^2$,
3. The 4-manifold $S^1 \times B^3$ is also the disk exterior $B^4 - \nu(D)$.

In the following lemma, we change the role of the unknot in Mazur’s argument with any ribbon knot. We call the resulting contractible 4-manifold bounded by a homology sphere a generalized Mazur manifold.

**Lemma 2.1.** Let $Y$ be the 3-manifold obtained by 0-surgery on a ribbon knot in $S^3$ with the fusion number $n$. Then any homology sphere obtained by an integral surgery on a knot in $Y$ bounds a generalized Mazur manifold with one 0-handle, $n + 1$ 1-handles, and $n + 1$ 2-handles.

**Proof.** Let $K \subset S^3$ be a ribbon knot with the fusion number $n$ that bounds a ribbon disk $D \subset B^4$. Consider the ribbon disk exterior $X = B^4 - \nu(D)$. Since $\nu(D)$ is diffeomorphic to $D \times B^2$, we clearly have $\partial X = Y$. Further, the ribbon disk exterior $X$ has a handle decomposition with one 0-handle, $n + 1$ 1-handles, and $n + 1$ 2-handles, see [GS99, Section 6.2].

Now let $J \subset Y$ be an arbitrary knot. An integral surgery on $J \subset Y$ corresponds to attaching a 2-handle $B^2 \times B^2$ to $X$ along $J$, and produces a 4-manifold, say $W$. Here, we can place $J$ in $S^1 \times D^2 \subset Y$, so the framing of the additional 2-handle is fixed due to Akbulut’s carving, see [Akb16, Section 1.4]. Then $W$ must be simply-connected if the resulting 3-manifold is a homology sphere. In this case, the knot $J$ normally generates $\pi_1(X)$ by van Kampen’s theorem and vice versa. Note that $W$ is also a homology ball, see for example [Sav20, Lemma 3.1]. Therefore, we conclude that $W$ is a contractible 4-manifold built with one 0-handle, $n + 1$ 1-handles, and $n + 1$ 2-handles by using the classical theorems of Hurewicz and Whitehead. \qed

**Remark 2.2.** Presumably, there would be handle cancellations in the handle decomposition of a ribbon disk exterior. Therefore, we may address contractible manifolds having fewer numbers of 1- and 2-handles. This is not the case for any ribbon knot in the Rolfsen’s table with fewer than 11 crossings, see Kawauchi’s exposition [Kaw96, Appendix F]. However, it happens for the generalized square knots $T_{p,q}^\# - (T_{p,q})$ where $T_{p,q}$ is the $(p,q)$-torus knot [MZ19, Proposition 5.3] or for the $(p,1)$-cable of a fusion number one ribbon knot or even its iterated cables, see [HKP20, Theorem 1.1].

Our next ingredient is the following trick of Akbulut and Larson coming from the proof of their main theorem [AL18, Theorem 1].

**Definition 2.3.** The Akbulut-Larson trick is an observation about describing the iterative procedure for passing from the surgery diagram of a homology sphere to a consecutive one by using a single blow-up with an isotopy, see Figure 3.
In this section, we prove our all theorems using the background in Section 2. We shall start with the proof of Theorem 1.5.

Proof of Theorem 1.5. Due to the combinatorial approach in [EN85, Chapter 5.21], it can be easily checked that the determinant of intersection matrix associated to the plumbing graph for $W(n)$ is $\pm 1$ according to the parity of $n$, so $\partial W(n)$ is a homology sphere for each $n \geq 1$. Also since the plumbing graph for $W(n)$ has three nodes and seven branches, it is not a Seifert fibered homology sphere, see [EN85, Chapter 2.7].

Here, we use the dual approach by giving integral surgeries from its plumbing graph displayed in Figure 2. We actually show that $\partial W(n)$ is obtained by $(-1)$-surgery on a knot in $Y$ where $Y$ is $0$-surgery on the square knot. Its surgery diagram corresponding to the first element of its plumbing graph appears in Figure 3. The dark black $(-1)$-framed component gives the necessary integral surgery to $Y$ after applying blow downs several times. Then the general family is obtained by applying the Akbulut-Larson trick successively. Therefore, we finish the proof by using Lemma 2.1 since the blow down operation does not change the boundary 3-manifold. $\square$

Our next proof is about the boundary of Maruyama plumbed 4-manifold as well as its companion introduced in the paper.

Proof of Theorem 1.3 and Theorem 1.4. Following the recipe in [EN85, Chapter 5.21] again, it is straightforward to check that the determinant of intersection matrix associated to the plumbing graph for $X'(n)$ is $-1$ for each $n \geq 1$, so $\partial X'(n)$ is a homology sphere.

Again we use the dual approach for describing the additional integral surgeries from their plumbing graphs shown in Figure 1. For the case $n = 1$, see the analogous proof of [Sav20, Theorem 1.2]. Assume that $n \geq 2$. In particular, we prove that $\partial X'(n)$ is obtained by $(-1)$-surgery on a knot in $Y$ where $Y$ is $0$-surgery on the
unknot. In Figure 5, we draw the blow down sequences explicitly and we eventually reach the 3-manifold $Y$. Hence, the rest of proof is using Lemma 2.1 and the fact that blow down operation keeps the homeomorphism type of boundary 3-manifold same. The proof for $\partial X(n)$ is quite similar to the previous one and it is clearly displayed in Figure 6. For $n = 1$, one can again consult the proof of [Şav20, Theorem 1.2].

The discussion in the following remark is suggested by Marco Golla.

**Remark 3.1.** One may recognize that the upper part of the plumbing graph of $X'(n)$ behaves freely in the proof, so one might think that we obtain a two-parameter family of homology spheres, let say $X'(a, b)$ displayed in Figure 7. Then the determinant of its intersection matrix is $(-1)^{a-b-1}(a-b-1)^2$. Thus, its boundary $\partial X'(a, b)$ is a homology sphere if and only if $a = b$ or $a = b + 2$. However, in this case, $X'(b, b)$ and $X'(b + 2, b)$ are essentially same as our initial 4-manifold $X'(n)$.

We finally reshow the classical results about Seifert fibered homology spheres.
Proof of Theorem 1.6 and Theorem 1.7. Using the procedure in [Sav02, Example 1.17], it can be simply shown that the $\Sigma(2, 3, 13)$ and $\Sigma(2, 3, 25)$ are the boundary of the negative-definite unimodular plumbing graphs shown in Figure 8, respectively.

In Figure 9, we draw the additional $(-2)$-framed components to their surgery diagrams. Blowing down $(-1)$-framed dark black components 5- and 7-times respectively, we ultimately reach the 3-manifold $Y$ where $Y$ is 0-surgery on the stevedore knot. Applying Lemma 2.1, we finish the proofs of these cases. The explicit procedures for blow down sequences are left to readers as exercises.

The proof for Stern’s families listed in Theorem 1.7 are identical within [Sav20]. One can consult the handle diagrams appeared in [Sav20, Theorem 1.2]. The outcome of the theorem is different due to Lemma 2.1, compare with [Sav20, Lemma 3.1].

Next, we consider families of Casson and Harer. For odd values of the integer $p$, recall that $\Sigma(p, ps + 1, ps + 2)$ and $\Sigma(p, ps - 2, ps - 1)$ are the boundaries of the negative-definite unimodular plumbing graphs displayed in Figure 10 from the left to the right [CH81]. To complete the proof by applying Lemma 2.1, we similarly address the dual approach by giving integral surgeries from their plumbing graphs.
In fact, we show that \( \Sigma(p, ps + 1, ps + 2) \) and \( \Sigma(p, ps - 2, ps - 1) \) are both obtained by \((-1)\)-surgery on a knot in \( Y \) where \( Y \) is 0-surgery on the unknot.

Assume \( p = 3 \) for both families, so that we consider \( \Sigma(3, 3s + 1, 3s + 2) \) and \( \Sigma(3, 3s - 2, 3s - 1) \). The case \( s = 1 \) for the second one results \( \Sigma(3, 1, 2) = S^3 \).
The remaining elements are essentially same with the former family, and the proof of $\Sigma(3, 3s + 1, 3s + 2)$ is already in [Şav20] with the same technique. Henceforth, we suppose that $p \geq 5$ odd and $s \geq 1$ for the former family, and $p \geq 5$ odd and $s \geq 2$ for the latter one. The surgery diagrams corresponding to their first elements of plumbing graphs show in Figure 11 and Figure 12 respectively. Again the extra dark black $(-1)$-framed components indicate the required surgeries to $Y$ by following the blow down procedures. Then the whole families are obtained by applying the Akbulut-Larson trick successively.

\[\square\]
Figure 11. $(-1)$-surgery from $\Sigma(p, p+1, p+2)$ to $Y$. 
Figure 12. \((-1)\)-surgery from \(\Sigma(p, 2p - 2, 2p - 1)\) to \(Y\).
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