Generation of Orthogonal Sequence Sets for M-ary Spread Spectrum Communications with OFDM Modulation

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Abstract. The number of orthogonal sequences is a key point of M-ary spread spectrum (MSS) communication system since more sequences mean higher data rate. When orthogonal frequency division multiplexing (OFDM) modulation is used into MSS system, the additional low peak-to-average power ratio (PAPR) property is required due to high power efficiency. Based on the segmentation-shift operation, multiple orthogonal sequence sets with the same size and sequence length are generated in this paper. By combining these orthogonal sequence sets, the number of the generated orthogonal sequences is very large. For any two orthogonal sequence sets, there exists good cross-correlation performance between them at zero shift. In addition to large set size and good correlation property, the generated multiple orthogonal sequence sets possess low PAPR, which is very suitable for OFDM modulation. The generation algorithm of multiple orthogonal sequence sets is provided in this paper, and the corresponding simulation results show the effectiveness of the provide method.

1. Introduction

As an important role of spread spectrum communication system, the property of spread spectrum sequence will have a great impact on system performance. The sequence length will determine the processing gain of spread spectrum system while the auto-correlation function (ACF) and cross-correlation function (CCF) will affect the ability of the system to resist multipath interference and multiple access interference [1]. In order to increase data rate of spread spectrum communication system, M-ary spread spectrum (MSS) replaced traditional direct sequence spread spectrum (DSSS) [2-5]. For MSS communications, there generally exist two main spreading methods, namely time-domain spread spectrum [2, 4] and frequency-domain spread spectrum [3, 5]. Different from time-domain spread spectrum requiring good ACF and CCF at any shift, frequency-domain spread spectrum only focus on CCF at zero shift, namely in-phase cross-correlation (IPCC). When IPCC between any two sequences is equal to zero, these two sequences are orthogonal to each other [6]. Although lower requirement for correlation property, frequency-domain spread spectrum will lead to higher peak-to-average power ratio (PAPR) since orthogonal frequency division multiplexing (OFDM) modulation will be employed [7]. As a result, both of small IPCC and low PAPR are very important for MSS-OFDM communication system.

In addition to IPCC and PAPR, the number of spread spectrum sequences is another interesting parameter. For MSS-OFDM communication system with set size of $2^n$, each data group with $n$ bits will be mapped into one spread spectrum sequence [4], which means that more sequences will lead to higher data rate. It is well known that the set size of any orthogonal sequence set with sequence length
of $2^n$ is not larger than $2^n$ [6]. Then, the generation of more sequences with small IPCC is a key point. The main idea to expand set size of spread spectrum sequences is to generate more sequence sets with low inter-set CCF values [8]. In order to increase the number of sequence sets, several expansion-set methods based on initial orthogonal complementary sequence set were provided, such as shift-based method [9], shifted m-sequence method [10], permutation method [11] and so on. For these known methods, the number of orthogonal sequence sets is not larger than sequence length since good IPCC property should be maintained.

In this paper, a new generation method of orthogonal sequence sets for MSS-OFDM communication system is proposed. Since complementary sequence possesses 3dB-PAPR property [12], then the traditional orthogonal complementary sequence set in [13] is still considered as initial generation set for the proposed method. However, different the known methods, the proposed method can obtain more orthogonal sequence sets on the basis of segmentation-shift method. By segmenting a complementary sequence into multiple sub-sequences, the corresponding shift operation for sub-sequences will be used to design different orthogonal sequence sets. When the number of the segmented sub-sequences is equal to 1, the generated multiple orthogonal sequence sets will become those sets in [9]. Then, the shift-based method proposed in [9] can be seen as a special case of the method proposed in this paper.

This paper can be organized as follows. We first introduce MSS-OFDM communication system, and the corresponding requirements for spread spectrum sequences are given in Section 2. Then, the segmentation-shift method is presented, and the generation algorithm of multiple orthogonal sequence sets is proposed in Section 3. The performance of the generated sequences is analysed in Section 4, including the number of sequences, IPCC and PAPR. Finally, Section 5 summarizes the results.

### 2. MSS-OFDM Communication System

As a kind of spread spectrum communication simultaneously employing multiple sequences, MSS communication can map multiple data bits into one sequence. Given a spread spectrum sequence set $S = \{\overline{S}_i, 0 \leq i \leq 2^n - 1\}$ with $\overline{S}_i = \{S_i(0), S_i(1), \ldots, S_i(2^n - 1)\}$, it is obvious that both of its set size and sequence length are equal to $2^n$. Then, $n$ bits may be mapped into one sequence, namely [4]

$$ (b_1b_2 \cdots b_n) \leftrightarrow \overline{S}_i $$

(1)

where $i = 2^{n-1}b_1 + 2^{n-2}b_2 + \cdots + 2b_{n-1} + b_n$, and each bit satisfies $b_1, b_2, \ldots, b_n \in \{0, 1\}$.

From (1), the number of the mapped bits is determined by the size of set $S$. In order to obtain higher data rate, more spread spectrum sequences are required. Also, the performance of demapping operation greatly depends on CCF property of $S$ when a correlator at receiver is employed. Then, good correlation property is important to generate spread spectrum sequence set.

It can be seen that MSS method is more suitable for high data rate communication since data rate of MSS is $n$ times that of traditional DSSS. When frequency-domain MSS is considered, OFDM modulation is generally combined with MSS method and the combination can be called MSS-OFDM communication [3]. Let $\{X(k), 0 \leq k \leq N - 1\}$ be a frequency-domain symbol of MSS-OFDM communication system, and then the relationship between spread spectrum sequence $\overline{S}_i = \{S_i(0), S_i(1), \ldots, S_i(2^n - 1)\}$ and frequency-domain symbol $\{X(k), 0 \leq k \leq N - 1\}$ can be given by

$$ \{X(k), 0 \leq k \leq N - 1\} = \left\{0, S_i(2^n - 1), S_i(2^n - 1 + 1), \ldots, S_i(2^n - 1), 0, \ldots, 0, S_i(0), S_i(1), \ldots, S_i(2^n - 1 - 1)\right\} $$

(2)

where $X(k) = 0$ corresponds to direct-current subcarrier, and $N$ denotes the number of fast Fourier transform (FFT) points.
Each MSS-OFDM frequency-domain symbol can be realized on the basis of FFT, which will reduce the complexity of system implementation. The MSS-OFDM time-domain symbol \( \{x(m), 0 \leq m \leq N-1\} \) can be calculated as follows [3],

\[
x(m) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k)e^{-j\frac{2\pi mk}{N}}
\]

(3)

As a multiple-carrier communication technique, MSS-OFDM will have to face the impact of high PAPR, which can be seen from (3). So we hope reduce PAPR by generating suitable spread spectrum sequences in this paper.

3. Generation Algorithm

In this section, a new generation method of multiple orthogonal sequence sets will be provided. In essence, the proposed method can be considered as a kind of permutation-based generation method since the orthogonality of each sequence set will have to be maintained. Different the known results, the permutation sequences on the basis of segmentation-shift operation are proposed in this paper.

3.1. Permutation-based Generation Method

For a given binary row orthogonal sequence set \( C \) with set size \( I \) and sequence length \( L \), we describe it in the form of matrix as follows,

\[
C = \begin{bmatrix}
\bar{C}_0 \\
\bar{C}_1 \\
\vdots \\
\bar{C}_{L-1}
\end{bmatrix} = \begin{bmatrix}
C_0(0), C_0(1), \ldots, C_0(L-1) \\
C_1(0), C_1(1), \ldots, C_1(L-1) \\
\vdots \\
C_{L-1}(0), C_{L-1}(1), \ldots, C_{L-1}(L-1)
\end{bmatrix} = [C_0, C_1, \ldots, C_{L-1}]
\]

(4)

where \( \bar{C}_i \) denotes a row sequence with sequence length \( L \) and \( C_j \) denotes column sequence with sequence length \( I \), \( 0 \leq i \leq I-1 \) and \( 0 \leq l \leq L-1 \). Any two row sequences in \( C \) are orthogonal to each other, that is, their IPCC is equal to 0.

From (4), it is obvious that a new set \( C' \) is still an orthogonal matrix when reordering column sequences. For example, \( C' = [C_1, C_0, C_3, C_2] \), \( C' = [C_3, C_2, C_1, C_0] \) and so on, they are also orthogonal sequence sets for the initial set \( C = [C_0, C_1, C_2, C_3] \).

As a result, new orthogonal sequence set can be generated from a permutation sequence. Let \( P = \{P_k, 0 \leq k \leq K-1\} \) with \( \bar{P}_k = (P_k(0), P_k(1), \ldots, P_k(L-1)) \) denote a permutation sequence set, and then we can obtain \( K \) orthogonal sequence sets \( \{C^k, 0 \leq k \leq K-1\} \) with sequence length \( L \), where

\[
C^k = [C_{P_k(0)}, C_{P_k(1)}, \ldots, C_{P_k(L-1)}]
\]

(5)

Especially, we have \( \bar{P}_0 = (0, 1, \ldots, L-1) \) when \( C^0 = C \).

3.2. Segmentation-shift Operation

Let the initial set \( C \) be a complementary orthogonal sequence set with \( L = I = 2^n \), then the sequence length of permutation sequence \( \bar{P}_k \) is equal to \( 2^n \). For \( \bar{P}_0 = (0, 1, \ldots, 2^n - 1) \), we segment it into \( g \) sub-sequences with the same length \( 2^n / g \), where \( g \) is the density of 2, namely \( g = 1, 2, 4, 8, \ldots, 2^{n-1} \). Then, by rotating all sub-sequences, we can obtain \( 2^n / g \) permutation sequences for a fixed segmentation value \( g \).
In order to show how the generation algorithm works, we will give a simple example for $L = 32$, where $P^{(g)} = \{\overrightarrow{P}_k^{(g)}, 0 \leq k \leq K - 1\}$ denotes the $g$-segmentation permutation sequence set.

- $g = 1$
  
  $P^{(1)}$
  
  $= \{\overrightarrow{P}_k^{(1)}, 0 \leq k \leq 31\}$
  
  $\overrightarrow{P}_0^{(1)} = (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31)$
  
  $\overrightarrow{P}_1^{(1)} = (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 0)$
  
  $= \overrightarrow{\overrightarrow{P}}_2^{(1)} = (2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 0, 1)$
  
  $\vdots$
  
  $\overrightarrow{P}_{31}^{(1)} = (31, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30)$ (6)

- $g = 2$

  $P^{(2)}$
  
  $= \{\overrightarrow{P}_k^{(2)}, 0 \leq k \leq 15\}$
  
  $\overrightarrow{P}_0^{(2)} = (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31)$
  
  $\overrightarrow{P}_1^{(2)} = (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 0, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 16)$
  
  $= \overrightarrow{\overrightarrow{P}}_2^{(2)} = (2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 0, 1, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 16, 17) (7)$
  
  $\vdots$
  
  $\overrightarrow{P}_{15}^{(2)} = (15, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 31, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30)$

- $g = 4$

  $P^{(4)}$
  
  $= \{\overrightarrow{P}_k^{(4)}, 0 \leq k \leq 7\}$
  
  $\overrightarrow{P}_0^{(4)} = (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31)$
  
  $\overrightarrow{P}_1^{(4)} = (1, 2, 3, 4, 5, 6, 7, 0, 9, 10, 11, 12, 13, 14, 15, 8, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 24)$
  
  $= \overrightarrow{\overrightarrow{P}}_2^{(4)} = (2, 3, 4, 5, 6, 7, 0, 1, 10, 11, 12, 13, 14, 15, 8, 9, 18, 19, 20, 21, 22, 23, 24, 16, 25, 26, 27, 28, 29, 30, 31, 24, 25) (8)$

  $\vdots$
  
  $\overrightarrow{P}_{7}^{(4)} = (7, 0, 1, 2, 3, 4, 5, 6, 15, 8, 9, 10, 11, 12, 13, 14, 16, 23, 16, 17, 18, 19, 20, 21, 22, 31, 24, 25, 26, 27, 28, 29, 30)$

For the other cases of $g = 8$ and $g = 16$, the similar results can be generated and is omitted. According to (5), multiple orthogonal sequence sets can be obtained on the basis of initial complementary orthogonal sequence set and permutation sequence sets. For the permutation sequence
the corresponding new orthogonal sequence set can be denoted as $C^{[g,g']}$, where $g = 1, 2, 4, 8, \cdots, 2^{n-1}$ and $g' = 0, 1, 2, \cdots, 2^n / g - 1$. It should be noted that the IPCCs between these generated new orthogonal sequence sets will be tested and only those sequence sets with good IPCC performance can be selected by the actual MSS-OFDM communication system.

4. Performance of the Generated Orthogonal Sequence Sets

For the multiple orthogonal sequence sets proposed in Section 3, we will analyse the corresponding properties of set size, IPCC and PAPR in this section.

4.1. The Number of the Generated Orthogonal Sequences

It is obvious that a large number of orthogonal sequence sets have been generated according to construction algorithm in Section 3. For a given initial complementary orthogonal sequence set with both of sequence length and set size being equal to $2^n$, the number $M$ of the expanded orthogonal sequence sets can be calculated by

$$M = \sum_{u=0}^{n-1} (2^{n-u} - 1)$$

According to the real requirements of MSS-OFDM communication system, the set optimization from $M$ orthogonal sequence sets should be operated in terms of IPCC and PAPR.

4.2. IPCC between Different Orthogonal Sequence Sets

The generated orthogonal sequence sets have good IPCC performance. Here we use a length-32 complementary orthogonal sequence set in [6] as the initial set of the proposed generation algorithm. According to (5)-(8), a large number of orthogonal sequence sets can be obtained. In order to show their IPCC performance, we provide figure 1 where the normalized IPCC distribution between different orthogonal sequence subsets are shown.

For the sequence index from 1 to 64 in figure 1-(a), the former 32 sequences come from $C^{[4,1]}$ and the latter 32 sequences come from $C^{[4,3]}$. From figure 1-(a), two generated sequence sets possess orthogonality due to their zero-IPCC property. For inter-IPCC between $C^{[4,1]}$ and $C^{[4,3]}$, it is obvious that the orthogonality is destroyed since the number of sequences is larger than sequence length.
However, the normalized IPCC can be controlled into several fixed values \( \{0, \pm 0.5\} \). The similar results can be seen in figure 1-(b), the main difference from figure 1-(a) is that their IPCC distribution is in the range of \( \{0, \pm 0.125, \pm 0.25, \pm 0.5\} \).

4.3. PAPR Distribution
When power efficiency is required in MSS-OFDM communication system, the problem of PAPR has to be solved. For the initial complementary orthogonal sequence set, its PAPR can be limited to 3dB due to its ideal aperiodic ACF property. However, the expanded orthogonal sequence sets generally possess higher PAPR.

For \( g = 4 \), the PAPR distribution of the former 4 orthogonal sequence sets, namely \( \{C^{(4,g')}, g' = 0,1,2,3\} \), is shown in figure 2. It is obvious that the initial set \( C^{(4,0)} \) has best PAPR property and the corresponding PAPR values of its 32 sequences are not larger than 3dB. The case of \( C^{(4,2)} \) is inferior to \( C^{(4,0)} \) and its PAPR is smaller than or equal to 5dB. The worst case comes from two sets of \( C^{(4,1)} \) and \( C^{(4,3)} \), and their maximum PAPR is larger than 6dB.

![Figure 2. PAPR distribution of the generated orthogonal sequence sets \( \{C^{(4,g')}, g' = 0,1,2,3\} \).](image)

5. Conclusion
This paper focuses on generating more sets of orthogonal sequences for MSS-OFDM communication system. By the means of segmentation-shift operation, a large number of orthogonal sequence sets can be constructed from an initial complementary orthogonal sequence set. In addition to large set size, the performance of IPCC and PAPR should be considered to suppress interference and improve power efficiency. The simulation results show that their IPCCs can be limited into several fixed values and the corresponding PAPRs are not larger than 6.5dB. For the actual MSS-OFDM communication application, the suitable number of orthogonal sequence sets should be selected and tested so as to improve the overall performance. It should be note that not all of IPCCs between two sequence sets can be controlled to low values although there exist several fixed IPCCs, which can be seen as the cost of increasing set size. In order to obtain low IPCCs, the corresponding optimization algorithm has to be researched. This will be our future work.

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