Discharge Coefficient for V-Notch Broad Crested Side Weir

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Abstract

In this paper the hydraulic characteristics of V-notch broad crested side weir have been experimentally studied. Sixteen V-notch broad crest weirs were constructed with six apex angles ($\theta=30^\circ, 45^\circ, 60^\circ, 90^\circ, 120^\circ, and 150^\circ$) and the apex height change three times ($p=5,10,15$cm) for the first forth angles and change two times for the last two angles ($p=10,15$cm), each model can be tested by changing flow in the main channel several times and from the analysis of experimental results it was found that the coefficient of discharge ($Cd$) for the V-notch broad crested weir is depend on the hydraulic and geometric characteristics of channel and weir, and from the hydraulic characteristics of the main channel it was found that the coefficient of discharge ($Cd$) directly proportional to the main channel Froude number before side channel , and a maximum value of ($Cd$) can be reached for a larger acute V-notch angle but a maximum value of ($Cd$) can be reached for the smaller obtuse V-notch angle. Also from the hydraulic and geometric effect for both channel and weir it is found that ($Cd$) inversely proportional with ($y_1/p$) and directly proportional with ($y_1/h, p/h$) and a maximum ($Cd$) can be reached maximum for both acute and obtuse V-notch angle. Suitable equations for discharge coefficient are obtained for different apex angle, then the predicted discharge coefficient from the equations were plotted against the calculated value with coefficient of determination ($R^2=0.9$) and it was found to be good.
Introduction:

A broad crested weir is an over flow structure with a horizontal crest above which the stream lines are practically straight and parallel. Broad crested weirs can have a variety of cross sections in control sections, depending on the requirements. The simplest type is a square edged weir with a rectangular cross section [7]. A side broad crested weir is a fixed structure installed at the side of the main channel and used as a standard discharge device to control the flow depth and discharge. Estimation of discharge over the side weir is still an important issue and an on-going problem in the area of water measurement. The normal side weirs may be of different shape (i.e., rectangular, triangular, trapezoidal etc.). Further side weirs may be made sharp or broad crested [6]. Most pervious research works for side weir carried out in channel with rectangular cross section, the sharp and broad crested rectangular side weirs have been studied extensively by many investigators [2,4,5,6,7,8] . It is obvious that almost all investigators have studied hydraulic characteristics of rectangular and triangular sharp crested side weir and less attention has been given to the behavior of flow over V-notch broad crested side weirs [6] . In this paper, experiments were carried on a V-notch broad crested side weir with apex angles of ($\theta=30^\circ$, $45^\circ$, $60^\circ$, $90^\circ$, $120^\circ$, and $150^\circ$) with different height of the apex for each angle. The experimental results were presented to validate a general expression for the estimation of discharge coefficient in terms of both hydraulic and geometrical parameters.

Experimental set up:

The experimental work was conducted at the Hydraulics Laboratory of Water Resource Engineering at the University of Mosul, Mosul, Iraq, using a long horizontal tilting main channel of (10)m long and a cross-section of (0.3)m width and (0.45)m height. The side channel was constructed perpendicular to the main channel at a distance (4.6)m upstream the main channel, the length of side channel is (2)m with cross section of (0.15)m width and (0.3)m height and zero slope. The main channel consisted of toughened glass walls, and the side channel consisted of plastic walls and both of a stainless steel floor. Three movable carriages with point gauges were mounted on brass rails at the top of the channel sides as shown in figure (1). Sixteen V-notch broad crested weirs models were made from wood of (0.3)m length and (0.2)m height. In these models, a V-notch of six different angles, with three different apex height, details of the models are shown in figure (2), and table (1). The V-notch broad crested weir was installed at the upstream end of the side channel. For discharge measurement, a full width thin plate sharp crested rectangular weir (10)cm height fixed at the tail end of the main channel section manufactured according to British standard 1965 [3], the head over the weir was measured two times, first by locking the inlet of the side channel and the second time by the opening the inlet of the side channel to determine the discharge in the main channel before and after side channel $Q_1$ & $Q_2$ respectively, the head over the weir was measured with a precision point gauge whose least count was (0.1)mm.

Evaluation of discharge:

The basic equation of V-notch broad crested weir used to evaluate the discharge over the side weir is:

$$Q_{the} = Cd \left( \frac{8}{15} \sqrt{2g} \tan \left( \frac{\theta}{2} \right) h^{2.5} \right)$$

In which: $Q_{the}$= theoretical discharge over side weir $m^3/sec$, $Cd$=coefficient of discharge, $h$=head of water upstream broad crested weir m, $g$=acceleration due to gravity $m/sec^2$, $\theta$=apex V-notch angle in degree.
The actual discharge over the side weir computed by the following equations:

\[ Q_1 \quad Q_2 \]

**Fig. (2) Definition Sketch**

**Fig. (1) Channel Cross Section**
Table 1: Details of weir model investigated

| Model No. | Run No. | Apex Angle (Degree) | Apex Height (cm) |
|-----------|---------|---------------------|------------------|
| 1         | 1-5     | 30                  | 5                |
| 2         | 6-11    | 30                  | 10               |
| 3         | 12-17   | 30                  | 15               |
| 4         | 18-23   | 45                  | 5                |
| 5         | 24-29   | 45                  | 10               |
| 6         | 30-35   | 45                  | 15               |
| 7         | 36-41   | 60                  | 5                |
| 8         | 42-47   | 60                  | 10               |
| 9         | 48-53   | 60                  | 15               |
| 10        | 54-59   | 90                  | 5                |
| 11        | 60-65   | 90                  | 10               |
| 12        | 66-71   | 90                  | 15               |
| 13        | 72-77   | 120                 | 10               |
| 14        | 78-83   | 120                 | 15               |
| 15        | 84-89   | 150                 | 10               |
| 16        | 90-95   | 150                 | 15               |

\[ Q_b = (Q_1 - Q_2) \]  
\[ Q_1 = 0.57988 \ h_1^{1.5} \]  
\[ Q_2 = 0.57988 \ h_2^{1.5} \]

Dimensional analysis:
For flow over triangular broad crested weir, the variables involved are

\[ Cd = f(\mu, \beta, \rho, g, p, \theta, h, y_1, Q_1) \]
\[ Cd = f (F_1, Re, We, p/h, y_1/h, y_1/p, \theta) \]  
\[ \text{------(6)} \]

In which: \( F_1 = \frac{V_1}{\sqrt{g y_1}} \) = Froude number in the main channel, \( V_1 = \) velocity of flow before side channel m/sec, \( Re = \) Reynolds number, \( We = \) Weber number.

By assuming that the crest of the weir is smooth, Reynolds number is large enough to neglect viscous effect and Weber number is large enough to neglect surface tension effects [2]. Therefore equation (6) can be reduced to:

\[ Cd = f (F_1, p/h, y_1/h, y_1/p, \theta) \]  
\[ \text{------(7)} \]

**Analysis of Results and discussion:**

The discharge coefficient for V-notch broad crested side weir was calculated for six apex angle (\( \theta = 30^\circ, 45^\circ, 60^\circ, 90^\circ, 120^\circ, \) and \( 150^\circ \)). It is found that (\( F_1, p/h, y_1/h, \) and \( y_1/p \)) has a significant effect on (\( Cd \)) as follows:

1- The relations between discharge coefficient (\( Cd \)) and Froude number \( F_1 \) is plotted in figures (3 and 4), and from these figures it is found that for each V-notch angle, \( (Cd) \) increases as \( F_1 \) increase, and from figure (3) when V-notch is acute a maximum value of \( (Cd) \) can be reached for acute angle (90\(^\circ\)) and decrease gradually as acute angle decrease (60\(^\circ\), 45\(^\circ\), 30\(^\circ\)). While from figure (4) when V-notch is obtuse a maximum \( (Cd) \) can be reached for an obtuse angle (150\(^\circ\)) and decrease as obtuse angle decrease to (120\(^\circ\)).

2- The relation between discharge coefficient (\( Cd \)) and the parameter \( (y_1/p) \) for all V-notch angles, is plotted in figures (5 and 6), the figures shows that \( (Cd) \) decreases as \( (y_1/p) \) increase, and maximum \( (Cd) \) can be reached at a maximum V-notch angle, and \( (Cd) \) decrease when the angle decrease for both acute and obtuse V-notch.

3- Figures (7) to (10) show the relation between discharge coefficient (\( Cd \)) with the parameters \( (y_1/h, \text{ and } p/h) \) for all V-notch angles, the figures show that \( (Cd) \) increases as \( (y_1/h, \text{ and } p/h) \) increase, a maximum \( (Cd) \) can be reached at the maximum V-notch angle, and \( (Cd) \) decrease as the angle decrease for both acute and obtuse V-notch.

4- From the analysis of experimental results, table (2) shows the range of values of discharge coefficient (\( Cd \)) for each degree of V-notch of broad crested side weirs that be studied.

A mathematical models is developed by multiple nonlinear regression analysis using the experimental data for various values of apex angle which is used to correlate the value of discharge coefficient (\( Cd \)) to both hydraulic and geometrical parameters (\( F_1, p/h, y_1/h, \) and \( y_1/p \)). The following equations fits the data for each apex angle with the coefficient of determination \( (R^2 = 0.88, 0.81, 0.68, 0.52, 0.98, 0.999) \) for the following angles respectively:

\[ Cd = 1 - 0.2576 (F_1) - 0.09 (p/h) - 0.413 (y_1/h) - 0.9185 (y_1/p) \]  
\[ \text{for } \theta = 30^\circ \]  
\[ \text{------(8)} \]

\[ Cd = 1 - 0.3718 (F_1) - 0.2682 (p/h) - 0.285 (y_1/h) - 0.8351 (y_1/p) \]  
\[ \text{for } \theta = 45^\circ \]  
\[ \text{------(9)} \]

\[ Cd = 1 - 0.1042 (F_1) + 0.9414 (p/h) - 0.9522 (y_1/h) - 0.6361 (y_1/p) \]  
\[ \text{for } \theta = 60^\circ \]  
\[ \text{------(10)} \]

\[ Cd = 1 - 0.5842 (F_1) - 0.2091 (p/h) + 0.1482 (y_1/h) - 0.9318 (y_1/p) \]  
\[ \text{for } \theta = 90^\circ \]  
\[ \text{------(11)} \]

\[ Cd = 1 + 0.456 (F_1) + 0.6448 (p/h) - 0.6896 (y_1/h) - 0.832 (y_1/p) \]  
\[ \text{for } \theta = 120^\circ \]  
\[ \text{------(12)} \]
Cd = 1 + 0.7275 (F1) + 0.7247 (p/h) – 0.7493 (y1/h) - 0.9133 (y1/p) for θ = 150° ---- (13)
Table 2: The discharge coefficient range for each degree of V-notch apex angle

| V-Notch Apex Angle (Degree) | Discharge Coefficient (Cd)       |
|-----------------------------|----------------------------------|
| 30                          | 0.49-0.79                        |
| 45                          | 0.41-0.75                        |
| 60                          | 0.50-0.91                        |
| 90                          | 0.53-0.92                        |
| 120                         | 0.24-0.79                        |
| 150                         | 0.20-0.91                        |

Figure 11 shows the relation between the predicated values of (Cd) from equations (8 to 13) and the calculated values of (Cd) with a coefficient of determination ($R^2 = 0.9$). The figure indicates an obvious a good agreement between the calculated and predicated values of (Cd).
Conclusions:
The discharge coefficient (Cd) for broad crested V-notch side weir is studied as a function of both hydraulic and geometrical parameters (F1, p/h, y1/h, and y1/p) and from analysis of experimental results a mathematical models were developed for different apex angles which enable to determine the side weir discharge coefficient (Cd). The previously developed models give a good estimation when compared with predicated and calculated values of discharge coefficient.

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