Exploring the $\Sigma^+ p$ interaction by measurements of the correlation function

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Abstract

The spin-dependent components of the fundamental hyperon-nucleon interactions are largely unknown. We show that under reasonable assumptions, a measurement of the $\Sigma^+ p$ correlation function in high-energy proton-proton or heavy-ion collisions combined with cross section data allows to separate the singlet and the triplet $S$-wave contributions.

Introduction: There has been a renewal of interest in hyperon-nucleon interactions recently, triggered on the one side by the study of hypernuclei at facilities worldwide [1, 2, 3] and on the other side driven by studies of the equation of state of neutron matter. The latter is especially of relevance for understanding the properties of neutron stars, and in particular for the generation of gravitational waves in mergers of such compact objects, see, e.g., [4] and the recent reviews [5, 6]. In fact, the interactions of hyperons ($Y = \Lambda, \Sigma, \Xi$) with nucleons have been studied for many decades. However, so far only the bulk properties are experimentally established, and first of all those for the $\Lambda N$ system. More detailed information, specifically about the spin dependence of the $YN$ forces, is completely lacking. The latter is a consequence of the short-lived character of the hyperons so that no proper beams can be prepared in order to perform standard scattering experiments. An essential additional source of information on the $YN$ force is provided by studying their interaction in reactions where the two baryons emerge with low relative momentum in the final state [7, 8, 9, 10, 11, 12]. In such a case, even the scattering lengths can be extracted. Indeed, with that aim in mind the $\Lambda p$ final-state interaction in the reaction $pp \rightarrow K^* \Lambda p$ has been measured at the COSY accelerator in Jülich [13, 14]. A determination of the spin dependence, i.e. the separation of the spin-triplet and/or singlet amplitude, is possible in that reaction, but requires a single- or even double-polarization experiment [13, 15]. Unfortunately, efforts to determine the strength of the spin-triplet $\Lambda p$ interaction by the TOF collaboration [14] suffered from low statistics and, at the end, did not provide the anticipated conclusive results.

More recently, it was realized that information on the $YN$ scattering lengths can also be obtained from studying the corresponding two-body momentum correlation function as measured in heavy-ion collisions or high-energetic $pp$ collisions [17, 18]. With respect to $\Lambda p$ there are data from the STAR Collaboration [19] from a measurement in Au+Au collisions at $\sqrt{s} = 200$ GeV and by the ALICE collaboration in $pp$ collisions at $\sqrt{s} = 7$ TeV [20] and $\sqrt{s} = 13$ TeV [21], respectively. However, also here no detailed information on the spin dependence can be deduced. The singlet- and triplet contributions simply add up with the same statistical weight as in free scattering, as commonly assumed [17, 20].

Here, we point out that a separation of the spin states is feasible from measurements of the correlation function - in combination with cross section data - without any spin-dependent experiment. It is possible in specific cases, namely when the interaction in one of the spin states is attractive while that in the other one is repulsive. As will be discussed in more detail below, in such a situation the contributions of the spin states to the correlation function will partially cancel, because they always add up in case of the reaction cross section. This qualitatively different interplay allows one to disentangle the spin contributions, provided one has data on the correlation function and on the elastic cross section.

We exemplify this idea for the $\Sigma^+ p$ system, where cross section data at low energies have been available for a long time [22]. This channel has isospin $I = 3/2$ so...
that there is no coupling to the $\Lambda N$ system and the reaction is purely elastic, i.e. complications from open channels \([23, 24]\) are absent. At low energies, the observables are dominated by the (spin singlet) $^1S_0$ and (spin triplet) $^3S_1$ partial waves. There is strong evidence that the interaction in the $^3S_1$ partial wave is repulsive. First indications for its repulsive nature came from the analysis of level shifts and widths of $\Sigma^-$ atoms and from measurements of ($\pi^-, K^+$) inclusive spectra related to $\Sigma^-$ formation in heavy nuclei \([25]\). As discussed in detail in Ref. \([22]\), a repulsive $\Sigma$ potential in the medium can only be achieved when the interaction in the $^3S_1$ partial wave of the $\Sigma^+ p$ channel is repulsive. More recently, strong and more direct evidence for a repulsion has been provided by lattice QCD calculations \([27, 28]\). The first strong and more direct evidence for a repulsion has been described in detail in various publications.

Formalism: The formalism for calculating the two-particle correlation function from a two-body interaction has been described in detail in various publications. Thus, in the following we provide only a summary of the employed formulae, where we follow very closely the paper of Ohnishi et al. \([36]\), see also \([18, 23, 37]\). The two-particle momentum correlation function $C(k)$ for two non-identical particles with interaction in a single $S$-wave state is given by

$$C(k) = 1 + \int_0^\infty 4\pi r^2 dr S_{12}(r) \left[ |\psi(k, r)|^2 - |j_0(kr)|^2 \right],$$

where $k$ is the center-of-mass momentum of the two-body system. $S_{12}$ is the so-called source function \([18]\) for which we adopt the usual static approximation and represent the source by a spherically symmetric Gaussian distribution, $S_{12}(r) = \exp(-r^2/4R^2)/(2\sqrt{\pi}R)^3$, so that it depends only on a single parameter, the source radius $R$. $\psi(k, r)$ is the scattering wave function that can be obtained by solving the Schrödinger or Lippmann-Schwinger equation for a given potential, and $j_0(kr)$ is the spherical Bessel function for $l = 0$. The wave function is normalized asymptotically to \([36]\)

$$\psi(kr) \rightarrow S^{-1} \left[ \frac{\sin(kr)}{kr} + \frac{f(k)}{r} \exp(ikr) \right]$$

where $f(k)$ is the scattering amplitude which is related to the $S$ matrix by $f(k) = (S - 1)/2ik$. For small momenta $k$, the amplitude can be written in terms of the effective range expansion, i.e. \(f(k) \approx 1/(-1/a_0 + \pi k^2/2 - ik)\), with $a_0$ and $r_0$ the scattering length and the effective range, respectively.

For illustrating how the sign of the scattering amplitude (scattering length) enters, let us take a look at the Lednicky and Lyuboshitz (LL) model \([38]\) where the actual wave function in Eq. \([1]\) is simply replaced by the asymptotic form Eq. \([2]\). Then the correlation function can be written completely in terms of the scattering amplitude,

$$\int_0^\infty 4\pi r^2 dr S_{12}(r) \left[ |\psi(k, r)|^2 - |j_0(kr)|^2 \right] \approx \frac{|f(k)|^2}{2\pi} F(r_0) + \frac{2\pi a_0}{3r_0^2} F_1(x) - \frac{\ln(a_0^2)}{r} F_2(x),$$

where $F_1(x) = \int_0^1 dt e^{-t^2 - x^2}/x$ and $F_2(x) = (1 - e^{-x^2}/x$, with $x = 2kr$. The factor $F(r_0) = 1 - r_0/(2\sqrt{\pi}R)$ is a correction that accounts for the deviation of the true wave function from the asymptotic form \([36]\). Obviously, there is a contribution proportional to Re$f(k)$ so that the correlation function is sensitive to its sign, i.e. to whether the interaction is attractive or repulsive. The term results from the interference between the incoming and outgoing waves when the wave function is squared, see Eq. \([2]\). Indeed, in general this term dominates the correlation function for small momenta, see, e.g., Ref. \([18]\) for a more detailed discussion. Thus, for a purely attractive interaction ($a_0 < 0$) the correlation function is enhanced, i.e. $C(k) \geq 1$, whereas for a purely repulsive interaction ($a_0 > 0$) the correlation function is suppressed, i.e. $C(k) \leq 1$.

As mentioned, at low energies the $\Sigma^+ p$ system will be in the spin singlet ($s$) and spin triplet ($t$) $S$-wave states. Then the cross section is given by $\sigma \sim (|f_s(k)|^2 + 3|f_t(k)|^2)/4$, considering the appropriate spin weights. Also for the correlation function \([1]\) an averaging over the spin has to be performed. Following the usual assumption that the weight is the same as for free scattering \([13, 18]\) leads to

$$C(k) = \frac{1}{4} C_s(k) + \frac{3}{4} C_t(k).$$

Considering the discussion above, $C_s$ and $C_t$ are expected to have opposite signs for the $\Sigma^+ p$ system so that
there will be a partial cancellation in the evaluation of the actual correlation function $C(k)$. Therefore, experimental information on the correlation function, together with data on the cross section, allow one to disentangle the $^1S_0$ and $^3S_1$ contributions, and possibly even to determine their scattering lengths, without performing any spin-dependent experiments.

Note that in case of the $\Sigma^+p$ system there is a caveat, namely the presence of the Coulomb force. Due to the repulsive Coulomb interaction between the $\Sigma^+$ and the proton eventually, for momenta $k \to 0$, $C_i(k)$ as well as $C_i(k)$ have to show the characteristics of a repulsive interaction. It will be interesting to see in how far the Coulomb interaction affects the overall results. In the presence of the Coulomb interaction modifications of Eqs. (1) and (2) are required, see Ref. [57]. Specifically, the wave functions $\psi(k,r)$ and $j_0(kr)$ have to be replaced by the corresponding solutions including the Coulomb interaction, $\psi^C(k,r)$ and $F_0(kr) / (kr)$, where $F_0(kr)$ is the regular Coulomb wave function for $l = 0$.

Finally, let us emphasize that the results presented below are all based on Eq. (1), i.e. from calculations with the full scattering wave functions. When the scattering length is small and the effective range large, as is the case for the $\Sigma^+p$ $^1S_1$ partial wave, see Table 1 the LL approximation [4] is not reliable [12, 58].

**Results:** For $\Sigma^+p$ scattering at low energies cross sections but also angular distributions have been available for a long time [22]. The data are shown in Fig. 1 together with results from $YN$ interactions derived within chiral effective field theory (EFT) at next-to-leading order (NLO) (specifically, we use the NLO19 version) [31] (red bands) or based on the traditional meson-exchange picture [32] (dashed line). One can see that the data are fairly well reproduced by these potentials, which actually aimed at a consistent description of all low-energy $\Lambda N$ and $\Sigma N$ scattering data under the assumption of an (approximate) SU(3) flavor symmetry. A more specific analysis of the $\Sigma^+p$ data has been presented by Nagels et al. [39]. A dedicated phase shift analysis at $p_{lab} = 170$ MeV/c has been attempted in [40].

Selected results for the $\Sigma^+p$ scattering lengths from the literature are summarized in Table 1. One can see that there are several predictions with a $^1S_0$ scattering length of $a_i \approx -4$ fm so that the $^1S_0$ contribution alone practically saturates the measured cross section [35]. Then the $^3S_1$ contribution has to be very small and the scattering length amounts to only $a_i \approx 0.5$ fm. On the other hand, in the analysis in [39] the singlet scattering length is significantly smaller and, accordingly, the one in the triplet state noticeably larger. The latter situation is also predicted by a $YN$ potential that has been derived within the constituent-quark model [34].

In order to quantify the impact of such differences on $\Sigma^+p$ observables and specifically on the correlation function we constructed a $\Sigma N$ interaction within chiral EFT that simulates the latter situation. To be concrete, we adopted the NLO19 potential with cutoff $\Lambda = 600$ MeV from [31] and re-adjusted the low-energy constants (LECs) corresponding to the SU(3) [27] and [10] irreps [40] to get values in the range suggested by Refs. [34, 39], see the entry denoted as NLO(sim) in Table 1. No attempt was made to reproduce one or the other result of these works exactly. However, we made sure that the total $\chi^2$ (based on the cross section) for NLO(sim) and for NLO19(600) are identical (i.e. $\chi^2 = 0.4$ in both cases). The pertinent results for $\Sigma^+p$ are indicated by the dash-dotted lines in Fig. 1. One can see that the total cross section for the smaller $a_i$ (larger $a_s$) scenario exhibits a slightly weaker momentum dependence, see also [39, 44], while the angular depen-

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**Table 1:** $\Sigma^+p$ Coulomb modified scattering lengths and effective range parameters (in fm) in the $^1S_0$ $(s)$ and $^3S_1$ $(t)$ partial waves.

| Interaction          | $a_s^c$ | $r_s^c$ | $a_t^c$ | $r_t^c$ |
|----------------------|---------|---------|---------|---------|
| NLO13(600) [30]      | $-3.56$ | $3.54$  | $0.49$  | $-5.08$ |
| NLO19(600) [31]      | $-3.62$ | $3.50$  | $0.47$  | $-5.77$ |
| Jülich '04 [32]      | $-3.60$ | $3.24$  | $0.31$  | $-12.2$ |
| ESC16 [33]           | $-4.30$ | $3.25$  | $0.57$  | $-3.11$ |
| Nagels '73 [39]      | $-2.42 \pm 0.30$ | $3.41 \pm 0.30$ | $0.71$  | $-0.78$ |
| fss2 [34]            | $-2.28$ | $4.68$  | $0.83$  | $-1.52$ |
| NLO(sim)             | $-2.39$ | $4.61$  | $0.80$  | $-1.25$ |
Figure 1: Total and differential $\Sigma^+ p$ cross section. The bands represent results from the chiral EFT potential NLO19 (red-dark) [31], while the dashed lines are those of the Jülich '04 meson-exchange potential [32]. The dash-dotted line corresponds to an interaction that simulates effective-range parameters as suggested in Refs. [39, 34], see Table 1. The data are from Ref. [22].

dence is slightly more pronounced. This result suggests that, in principle, a very accurate measurement of the $\Sigma^+ p$ cross section itself could already allow to disentangle the singlet- and triplet contributions. However, as argued above, since the contributions add up there is less sensitivity. Furthermore, and more decisive, measuring $\Sigma^+ p$ elastic scattering so close to threshold is practically impossible.

Let us now look at the predictions for the correlation function. A calculation without inclusion of the Coulomb interaction is presented in Fig. 2, which essentially corresponds to the $\Sigma^- n$ case. Results are shown for a source radius of $R = 1.2$ fm, utilizing NLO19(600) (solid line) and NLO(sim) (dash-dotted line), respectively. The contributions of the $^1S_0$ and $^3S_1$ partial waves alone (cf. upper and lower curves) are indicated by dashed and dotted lines, respectively. One can clearly see that there is a strong interplay between the two $S$-wave contributions. Moreover, and as expected, there is a sizable difference in the overall magnitude of $C(k)$ for the two potentials.

Actual results for $\Sigma^+ p$, i.e. with the Coulomb interaction included, are presented in Fig. 3 for a source radius of $R = 1.2$ fm (left) and of $R = 2.5$ fm (right), respectively. Note that the former value corresponds to what
one expects in high-energetic \( pp \) collisions as those studied by the ALICE Collaboration \[20, 21\] whereas the latter is more characteristic for heavy-ion collisions as performed by the STAR Collaboration \[19\]. As expected, the Coulomb repulsion between \( \Sigma^+ \) and the proton suppresses the correlation function at very small momenta and, thus, also the differences in the \( ^1S_0 \) and the \( ^3S_1 \) amplitudes. However, for \( R = 1.2 \) fm there is definitely a range of momenta, say from 25 to 75 MeV/c, with a pronounced difference and where pertinent measurements could allow one to discriminate between the interactions and, thus, facilitate the determination of the singlet- and the triplet \( S \)-wave amplitudes in combination with the measured \( \Sigma^- p \) cross section.

There is a visible signal for \( R = 2.5 \) fm, too, see Fig. 3 (right). However, it is overall smaller so that data with rather high precision are needed for distinguishing between the differences in the singlet- and triplet amplitudes.

**Conclusions:** In the present paper we have shown that a determination of the spin dependence of two-body interactions like \( \Sigma^+ p \) can be achieved by measuring the pertinent momentum correlation function. It is feasible under the premises that there is a sign difference in the scattering amplitudes of the two spin states involved and that the integrated reaction cross section is known. The latter is fulfilled for the \( \Sigma^+ p \) system, while for the former there is strong evidence not least from lattice QCD simulations.

With the \( \Sigma^+ p \) interaction determined, also the one in the \( \Sigma^- n \) system is practically fixed, since effects of charge symmetry breaking are expected to be small. This would have consequences for the already mentioned ongoing discussion on the properties of neutron starts and the role played by hyperons for the equation of state. At present the relevance of the \( \Sigma^- \) is still unsettled \[41\], and in some scenarios the \( \Sigma^- \) appears at similar density in neutron stars as the \( \Lambda \) \[42\]. Solid constraints on the strength of the \( \Sigma^- n \) interaction that can be deduced from measuring and analyzing the \( \Sigma^+ p \) correlation function would provide here a definite answer.

Be aware that measurements of the \( \Sigma^+ p \) correlation function are not easily done. But, of course, this is likewise true for conventional scattering experiments involving the \( \Sigma^+ \) \[43\].

Can the idea proposed here be exploited also in studies of other interactions? In principle, it should be applicable to any system with only two \( S \)-wave states, i.e. for the scattering of two octet baryons or of an octet and a decuplet baryon. There are several candidates in the strangeness \( S = -2 \) to \(-4 \) sectors where chiral EFT and/or lattice QCD, but also phenomenological models, predict opposite signs for the \( ^1S_0 \) and \( ^3S_1 \) scattering lengths. For example, it is the case for \( \Xi^0 p \) \[34, 44, 45\], for the \( S = -3 \) system \( \Xi^- \Lambda \) \[34\], and also for \( \Xi^- \Xi^- \) \[34, 44\].

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References

[1] H. Ohnishi, F. Sakuma and T. Takahashi, Prog. Part. Nucl. Phys. 113, 103773 (2020).
[2] F. Garibaldi et al. [Jefferson Lab Hall A], Phys. Rev. C 99, 054309 (2019).
[3] K. Schöning, Springer Proc. Phys. 238, 931 (2020).
[4] A. Anna, T. Gordan, A. Kurkela and A. Vuorinen, Phys. Rev. Lett. 120, no.17, 172703 (2018).
[5] D. Chatterjee and I. Vida, Phys. Rev. C 52, 29 (2016).
[6] J. Haidenbauer and U.-G. Meißner, Prog. Part. Nucl. Phys. 112, 103770 (2020).
[7] D. T. Tan, Phys. Rev. D 7, 600 (1973).
[8] F. Hinterberger and A. Sibirtsev, Eur. Phys. J. A 21, 313 (2004).
[9] A. Budzanowski et al., Phys. Lett. B 487, 31 (2010).
[10] M. Abdel-Bary et al. [TOF, Eur. Phys. J. A 48, 23 (2012).
[11] R. Münzer et al., Phys. Lett. B 785, 574 (2018).
[12] C. Wilkin, Eur. Phys. J. A 53, 114 (2017).
[13] M. Rother et al. [COSY-TOF], Eur. Phys. J. A 49, 157 (2013).
[14] F. Haukta et al. [COSY-TOF Collaboration], Phys. Rev. C 95, 034001 (2017).
[15] A. Gasparyan, J. Haidenbauer, C. Hanhart and J. Speth, Phys. Rev. C 69, 034006 (2004).
[16] A. Gasparyan, J. Haidenbauer and C. Hanhart, Phys. Rev. C 72, 034006 (2005).
[17] V. M. Shapoval, B. Erazmus, R. Lednicky and Y. M. Sinyukov, Phys. Rev. C 92, 034010 (2015).
[18] S. Cho et al. [ExHIC Collaboration], Prog. Part. Nucl. Phys. 95, 279 (2017).
[19] J. Adams et al. [STAR Collaboration], Phys. Rev. C 74, 064906 (2006).
[20] S. Acharya et al. [ALICE Collaboration], Phys. Rev. C 99, 024001 (2019).
[21] S. Acharya et al. [ALICE], [arXiv:2104.04427] [nucl-ex].
[22] F. Eisele, H. Filthuth, W. Föllisch, V. Hepp, G. Zech, Phys. Lett. 37B, 204 (1971).
[23] J. Haidenbauer, Nucl. Phys. A 981, 1 (2019).
[24] Y. Kamiya, K. Sasaki, T. Fukui, T. Hyodo, K. Morita, K. Ogata, A. Ohnishi and T. Hatsuda, Phys. Rev. C 105, 014915 (2022).
[25] A. Gal, E.V. Hungerford, D.J. Millener, Rev. Mod. Phys. 88, 035004 (2016).
[26] J. Haidenbauer and U.-G. Meißner, Nucl. Phys. A 936, 29 (2015).
[27] S. R. Beane et al., Phys. Rev. Lett. 109, 172001 (2012).
[28] H. Nemura et al., EPJ Web Conf. 175, 05030 (2018).
[29] H. Polinder, J. Haidenbauer and U.-G. Meißner, Nucl. Phys. A 779, 244 (2006).
[30] J. Haidenbauer, S.Petschauer, N. Kaiser, U.-G. Meißner, A. Nogga and W. Weise, Nucl. Phys. A 915, 24 (2013).
[31] J. Haidenbauer, U.-G. Meißner and A. Nogga, Eur. Phys. J. A 56, 91 (2020).
[32] J. Haidenbauer, U.-G. Meißner, Phys. Rev. C 72, 044005 (2005).