The Dependence of Surface Diffuseness Parameter on N/Z Ratio of The Fusion of Neutron-Rich Colliding Nuclei

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Abstract Surface diffuseness parameter used in Woods-Saxon form of potential have been extracted from a large number of experimentally studied neutron-rich fusion cross sections at near barrier energies. The results of our systematic study reveals that the extracted diffuseness parameter depend linearly on the N/Z ratio of the fusing nuclei. Further, we demonstrated that the extracted values of surface diffuseness parameter lies within the range $a = 0.40$ to $0.73$ fm as compared to commonly accepted value form elastic scattering data i.e. $0.63$ fm.

Keywords: Fusion, Fusion barrier height, Potential, Fusion cross-section

1. INTRODUCTION

In recent years, lot of research work is being carried out in order to best describe the value of the measured fusion cross section by fitting the nucleus-nucleus interaction potential parameters [1–5]. The nucleus-nucleus interaction potential can be described with large variety of models based on microscopic / macroscopic concept [6–13]. The most often choice of the nucleus-nucleus interaction potential is Woods-Saxon form of the potential [1, 6] given as

$$V_N(r) = -V_0 / (1 + \exp((r - R_0) / a)) \text{ MeV}.$$  \hspace{1cm} (1)

Where $V_0$ is the depth, $R_0$ is radius and ‘$a$’ is the surface diffuseness parameter. These potential parameters are required in coupled channel calculations to
extract information about barrier. Several authors used these parameters to best fit their data and hence to extract barrier parameters [2]. In this potential, the value of diffuseness parameter play significant role in deciding the shape of the potential, height of the fusion barrier and ultimately the fusion cross section at near barrier energies. At the same time, in Ref. [1] a very large values of diffuseness parameter (= 0.75 to 1.5 fm) is employed to fit adequately the measured fusion cross section. On the other hand, relatively lesser value of diffuseness parameter is required to fit elastic scattering data [11]. The extracted value of surface diffuseness parameter and the possible explanation for its high value suggested in ref [1] is still not clear.

As is clear from literature, large numbers of nuclei including symmetric, asymmetric and neutron-rich target/projectile combinations have been studied experimentally to explore various structural effects and fusion dynamics in detail [14–22]. For example, collision of \(^{28}\text{Si}^{+}^{28,29,30}\text{Si} [14], \(^{12}\text{C}^{+}^{16,17,18}\text{O}, \(^{28,29,30}\text{Si}, \(^{46,48,50}\text{Ti} [15–17], \(^{16}\text{O}^{+}^{28,29,30}\text{Si}, \(^{70,72,73,74,76}\text{Ge}, \(^{144,146,148}\text{Sm} [18–20], \(^{27}\text{Al}^{+}^{70,72,73,74,76}\text{Ge} [21] \) and \(^{32,34,36}\text{S}^{+}^{58}\text{Ni} [22] \) with N/Z ratio as high as 1.43. Also, some studies indicate the strong role of neutron richness on the diffuseness parameter [1]. But no study is so far reported in the literature where a complete variation of diffuseness parameter on neutron richness is presented. Therefore, it is interesting to systematically study the extracted surface diffuseness parameter magnitude from measured fusion data for various neutron-rich colliding nuclei within the framework of Woods-Saxon form of potential.

So in this paper we try to focus on this problem in context to neutron rich colliding nuclei using standard potential based on Woods Saxon form parameterized within proximity formalism [13]. Section 1 describes the model in brief, section 2 depicts the results and summary is presented in section 3.

2. THE MODEL

The total ion-ion interaction potential is given by

\[
V_T(r) = V_N(r) + \frac{Z_P Z_T e^2}{r} \text{ MeV},
\]

where \(Z_P\) and \(Z_T\) are the charge numbers of projectile and target nuclei respectively and ‘r’ denotes the distance between the centre of mass of two spherical nuclei in fm. Here, \(V_N(r)\) represents the nuclear part of the interaction potential and can be calculated using Woods-Saxon parameterization due to Winther and collaborators [7, 12, 13, 23] as briefly explained below.
2.1 Christensen and Winther 1976 (CW 76)

Christensen and Winther [11] derived the nucleus-nucleus interaction potential by analyzing the heavy-ion elastic-scattering data, based on the semiclassical arguments and the recognition that optical-model analysis of elastic scattering determines the real part of the interaction potential only in the vicinity of a characteristic distance. The nuclear part of the empirical potential due to Christensen and Winther is written as

\[ V_{N}^{CW\,76}(r) = -50 \frac{R_{1}R_{2}}{R_{1} + R_{2}} \Phi(r - R_{1} - R_{2}) \text{ MeV}, \]

where \( \frac{R_{1}R_{2}}{R_{1} + R_{2}} \) represents the geometrical factor that depends on the colliding nuclei and \( \Phi(r - R_{1} - R_{2}) \) is the universal function independent of colliding nuclei. The radius parameter used in this potential is of form

\[ R_{i} = 1.233A_{i}^{1/3} - 0.978A_{i}^{-1/3} \text{ fm} \quad (i = 1, 2). \]

Where \( A_{i} \) represents the mass number of projectile/target nuclei. The universal function has the following form

\[ \Phi(s) = \exp(-\frac{r - R_{1} - R_{2}}{a}), \]

here ‘a’ is diffuseness parameter. The exact value of a used in this model is 0.63 fm taken from elastic scattering [11]. This model was tested for more than 60 reactions and we labeled it as CW 76.

2.2 Broglia and Winther 1991 (BW 91)

A refined version of the above potential was derived by Broglia and Winther [23–24], by taking Woods-Saxon parametrization with subsidiary condition of being compatible with the value of the maximum nuclear force predicted by the proximity potential [12]. This refined potential resulted in

\[ V_{N}^{BW\,91}(r) = -\frac{V_{0}}{1 + \exp\left(\frac{r - R_{0}}{a}\right)} \text{ MeV;} \]

where
With

\[ V_0 = 16\pi \left( \frac{R_R}{R_1 + R_2} \right) \gamma a. \]  

(7)

Here again the exact value of \( a = 0.63 \) fm and

\[ R_0 = R_1 + R_2 + 0.29. \]  

(8)

Here radius \( R_i \) has the form

\[ R_i = 1.233A_i^{1/3} - 0.98A_i^{-1/3} \text{fm} \quad (i = 1,2). \]  

(9)

The form of the surface energy coefficient \( \gamma \) is

\[ \gamma = \gamma_0 \left[ 1 - k_s \left( \frac{N_p - Z_p}{A_p} \right) \left( \frac{N_L - Z_L}{A_i} \right) \right], \]  

(10)

where \( N, Z \) being the total number of neutrons and protons. In the above formula, \( \gamma_0 \) is the surface energy constant and \( k_s \) is the surface asymmetry constant. Both constants were first parametrized by Myers and Swiatecki by fitting the experimental binding energies. The first set of these constants yielded values \( \gamma_0 = 1.01734 \) MeV/fm\(^2\) and \( k_s = 1.79 \). Later on these constants were revised to \( \gamma_0 = 0.9517 \) MeV/fm\(^2\) and \( k_s = 1.7826 \). In the present version, \( \gamma_0 \) and \( k_s \) were taken to be 0.95 MeV/fm\(^2\) and 1.8, respectively [24].

### 2.3 Aage Winther (AW 95)

Winther adjusted the parameters of the above potential through an extensive comparison with experimental data for heavy-ion elastic scattering. This refined adjustment led to new formula for surface diffuseness parameter ‘\( a \)’ and is given as [13]

\[ a = \left[ \frac{1}{1.17\left(1 + a_0\left( A_i^{-1/3} + A_2^{-1/3} \right) \right)} \right] \text{fm.} \]  

(11)

Where \( a_0 = 0.53 \) in the present version. Here, we treat this constant as a free parameter to change the value of \( a \). It is important to note that here surface
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Diffuseness parameter depend on the masses of the colliding nuclei and not merely a constant. In this model nuclear radius is of form

$$R_i = 1.20 A_i^{1/3} - 0.09 \text{ fm} \quad (i = 1, 2).$$

(12)

We labeled this potential as AW 95. The details of these potentials are presented in Ref. [7].

Above all three versions of potential have different strengths. In fact, the last two versions are purely based on Woods-Saxon form. As stated in the introduction, we are interested in the role of diffuseness parameter used in three potentials in the fusion of neutron-rich colliding nuclei. Here, the nuclear surface diffuseness parameter enter via universal function has taken to be arbitrary. The last version differ in their values. Therefore, it will certainly affect the fusion barriers and cross sections significantly.

Once the total ion–ion interaction potential is calculated [using equation (2)] one can extract the barrier height and barrier position using the following conditions:

$$\frac{dV_T(r)}{dr}\bigg|_{r=R_g} = 0; \quad \text{and} \quad \frac{d^2V_T(r)}{dr^2}\bigg|_{r=R_g} \leq 0; \quad \text{and} \quad \frac{d^2V_T(r)}{dr^2}\bigg|_{r=R_g} \leq 0;$$

(13)

The knowledge of the shape of the potential as well as barrier position and height, allows one to calculate the fusion cross section at a microscopic level. To study the fusion cross sections, we shall use the model given by Wong [25]. In this formalism, the cross section for complete fusion is given by

$$\sigma_{\text{ fus}}(E_{cm}) = \frac{\pi}{k^2} \sum_{l=0}^{l_{\text{max}}} (2l + 1) T_l(E_{cm}),$$

(14)

where $k = \sqrt{\frac{2\mu E_{cm}}{\hbar^2}}$ and here $\mu$ is the reduced mass. The centre-of-mass energy is denoted by $E_{cm}$. In the above formula, $l_{\text{max}}$ corresponds to the largest partial wave for which a pocket still exists in the interaction potential and $T_l(E_{cm})$ is the energy dependent barrier penetration factor and is given by,

$$T_l(E_{cm}) = \left\{1 + \exp\left[\frac{2\pi}{\hbar\omega_l} (V_{B_l} - E_{cm})\right]\right\}^{-1}.$$  

(15)
Where $\eta$ is the curvature of the inverted parabola. If we assume that the barrier position and width are independent of $l$, the fusion cross section reduces to

$$\sigma_{\text{fus}}(mb) = \frac{10R_B^2\hbar \eta}{2E_{cm}} \times \ln \left\{ 1 + \exp \left[ \frac{2\pi}{\hbar \eta} (E_{\text{c.m.}} - V_B) \right] \right\}. \quad (16)$$

For $E_{cm} \gg V_B$, the above formula reduces to well-known sharp cutoff formula

$$\sigma_{\text{fus}}(mb) = 10\pi R_B^2 \left( 1 - \frac{V_B}{E_{cm}} \right), \quad (17)$$

whereas for $E_{cm} \ll V_B$, the above formula reduces to

$$\sigma_{\text{fus}}(mb) = \frac{10R_B^2\hbar \eta}{2E_{cm}} \exp \left[ \frac{2\pi}{\hbar \eta} (E_{\text{c.m.}} - V_B) \right]. \quad (18)$$

We used Eq. (16) to calculate the fusion cross sections using parabolic approximation.

3. RESULTS AND DISCUSSION

The present study is conducted using the above stated three potentials based on Woods- Saxon form parameterized within proximity concept. We have systematically analyze the fusion of large number of colliding partners such as $^{28}\text{Si}^+, ^{28,29,30}\text{Si}$, $^{12}\text{C}^+, ^{16,17,18}\text{O}$, $^{28,29,30}\text{Si}$, $^{46,48,50}\text{Ti}$, $^{16}\text{O}^+, ^{28,29,30}\text{Si}$, $^{70,72,73,74,76}\text{Ge}$, $^{144,146,148}\text{Sm}$, $^{27}\text{Al}^+$, $^{70,72,73,74,76}\text{Ge}$ and $^{32,34,36}\text{S}^+{^{58}}\text{Ni}$ with $N/Z$ content as high as 1.43. The diffuseness parameter used in the above stated potentials is taken here to be a free parameter. We systematically vary the value of $a$ to best fit the available experimental data [14–22] on fusion cross sections for the above stated reactions.

As we know that it is very difficult to reproduce the experimental data in all energy region using one-dimensional model due to Wong [25]. Therefore, we first define the energy range within which the exact fusion cross section best fit the experimental data. We fit the experimental cross sections of large number of colliding nuclei within the energy region $V_B \pm 5 \text{ MeV}$ i.e., near
barrier region with different potential models having Woods-Saxon form by treating the surface diffuseness as an adjustable parameter. To this end we first calculate the ion-ion interaction potential, fusion barrier parameters and then calculate the fusion cross section using parabolic approximation.

As a first step, we calculated the ion-ion interaction potential using Eq. (2). In Fig. (1), we display the total ion-ion interaction potential along with nuclear

| Figure 1: The total ion-ion interaction potential $V_T$ (MeV) along with nuclear and Coulomb part as a function of inter-nuclear distance ‘r’ for reaction of $^{12}\text{C} + ^{28}\text{Si}$, $^{12}\text{C} + ^{29}\text{Si}$ and $^{12}\text{C} + ^{30}\text{Si}$ using three different potential. Only three different value of diffuseness parameter are shown to maintain the clarity of the figure.
and Coulomb part as a function of inter-nuclear distance ‘r’ for the reactions $^{12}\text{C} + ^{28}\text{Si}$, $^{12}\text{C} + ^{29}\text{Si}$ and $^{12}\text{C} + ^{30}\text{Si}$ for potentials CW 76 (in the upper panel), BW 91 (in the middle panel) and AW 95 (in the lower panel), respectively for different values of diffuseness parameter. In this figure we have shown only three different values of surface diffuseness parameter to maintain the clarity of the figure. The first value represented with solid line correspond to the standard value used in particular potential. We have systematically vary diffuseness parameter to get the best fit with the measured fusion excitation function in the near barrier energy region. The best fit so obtained with experimental data is represented with dotted line. The dashed line represents almost middle of the extreme values. It is also clear from the figure that the nuclear potential $V_N(r)$ is more attractive for smaller value of diffuseness parameter compared to other values. This leads to deeper pocket and hence has layer probabilities of fusion. Since fusion is a surface phenomenon, therefore value of diffuseness at surface can have drastic effects on fusion process. As a result, the height as well as shape of the potential changes drastically with the increase in the value of diffuseness parameter. The systematically decrease in the height of the barrier for neutron -rich nuclei with different values of diffuseness parameter is also noticed. These effects will certainly affect the near barrier fusion probabilities.

In Fig. (2), we display the total interaction potential $V_T(r)$ (solid line) as a function of distance $r$ (in fm) for the colliding nuclei $^{12}\text{C} + ^{28}\text{Si}$, $^{12}\text{C} + ^{29}\text{Si}$ and $^{12}\text{C} + ^{30}\text{Si}$ at the maximum value. The total interaction potential fitted with inverted parabolic approximation is also presented with solid dots. It is clear from the figure that the inverted parabolic approximation nicely reproduce the extract values near the maximum of the barrier. This gives us the information about barrier curvature ($\hbar \omega_l$) that helps us to determine the fusion probability by using Eq. (16).

In Fig. (3), we present the fusion cross section $\sigma_{\text{fus}}$ (mb) as a function of centre of mass energy $E_{\text{c.m.}}$ for the reactions of $^{12}\text{C} + ^{28}\text{Si}$, $^{12}\text{C} + ^{29}\text{Si}$ and $^{12}\text{C} + ^{30}\text{Si}$ using three different potentials and for two extreme value of diffuseness parameter. The first value represented with solid line showed the standard value used in these potentials. Its value is 0.63 fm for CW 76 and BW 91 potentials, whereas in case of AW 95 its value depend on the mass number of colliding nuclei. To vary the value of diffuseness parameter in case of AW 95 potential, we vary the value of parameter ‘$a_0$’ [Eq. (11)] so that the outcome is closer to experimental findings. It is very far from the measured fusion cross section. As a result, we find the suitable values of ‘$a$’ so that fusion cross section can be nicely reproduce in the barrier region. Surprisingly, its value is quite less than the value extracted from elastic scattering data. It is clear from the figure that the original value of diffuseness parameter is very large and it
overestimate the experimental data whereas, the smaller value of diffuseness parameter is needed to reproduce the data.

In general, it has been observed while fitting the fusion cross section that the original value of surface diffuseness parameter used in all potentials is not sufficient to reproduce the experimental data within near energy region. Whereas,
slight change in the value of diffuseness parameter (either increase or decrease) nicely fit the experimental data within the near barrier energy region.

Interestingly, if we consider a neutron rich colliding nuclei than the higher value of diffuseness parameter is needed to reproduce the fusion cross section data. Of course, these calculations do not account any channel coupling effects.

Figure 3: The fusion cross sections $\sigma_{\text{fus}}$ (mb) as a function of centre of mass energy $E_{\text{c.m.}}$ (MeV) for the reaction of $^{12}\text{C} + ^{28}\text{Si}$, $^{12}\text{C} + ^{29}\text{Si}$ and $^{12}\text{C} + ^{30}\text{Si}$ using three different potentials and for two extreme values of diffuseness parameter. The experimental values are taken from Ref. [15].
However it is sufficient for our present work, as we are interested only in the study of role of diffuseness parameter in neutron-rich colliding nuclei.  

The results so obtained in our systematic study for the collision of $^{12}\text{C} + ^{16,17,18}\text{O}$, $^{12}\text{C} + ^{28,29,30}\text{Si}$, $^{12}\text{C} + ^{46,48,50}\text{Ti}$, $^{16}\text{O} + ^{28,29,30}\text{Si}$, $^{16}\text{O} + ^{70,72,73,74,76}\text{Ge}$, $^{16}\text{O} + ^{144,148,154}\text{Sm}$, $^{28}\text{Si} + ^{28,29,30}\text{Si}$, $^{27}\text{Al} + ^{70,72,73,74,76}\text{Ge}$ and $^{32,34,36}\text{S} + ^{58}\text{Ni}$ is plotted as a function of N/Z ratio for different potentials in Figs. (4), (5) and (6). In all these figures the straight line fit best explain the extracted values.

In Figs. (4), (5) and (6), solid line represents the straight line least square fit over the data points and is represented as

**Figure 4:** The variation of surface diffuseness parameter ‘$a$’ with N/Z ratio of colliding nuclei namely $^{12}\text{C} + ^{16,17,18}\text{O}$, $^{12}\text{C} + ^{28,29,30}\text{Si}$ and $^{12}\text{C} + ^{46,48,50}\text{Ti}$ for three different potentials. The solid line represents the straight line least square fit over the data points.
Figure 5: Same as Fig. (4), but for colliding nuclei $^{16}O + ^{28,29,30}Si$, $^{16}O + ^{70,72,73,74,76}Ge$ and $^{16}O + ^{144,148,154}Sm$.

$$a = \alpha \left( \frac{N}{Z} \right) + \beta$$

(19)

where $\alpha$ and $\beta$ are constants that vary from potential to potential. The value of constants $\alpha$ and $\beta$ are shown in Figs. (4), (5) and (6). It is clear from these figures that the extracted values of surface diffuseness shows a linear variation with N/Z ratio of the colliding partners [26]. This indicates the fact that a
neutron number increases the potential is more extended and hence shows the larger value of diffuseness parameter. All reaction series stated above follow the similar pattern and the extracted value of diffuseness parameter vary from 0.40 fm (for $^{12}$C + $^{28,29,30}$Si) to 0.73 fm (for $^{16}$O + $^{28,29,30}$Si). It has been observed that the extracted values of surface diffuseness from measured fusion cross sections are significantly different from the commonly accepted value $a = 0.63$ fm from elastic scattering data.

**Figure 6:** Same as Fig. (4), but for colliding nuclei $^{28}$Si + $^{28,29,30}$Si, $^{27}$Al + $^{70,72,73,74,76}$Ge and $^{32,34,36}$S + $^{58}$Ni.
CONCLUSIONS

Our systematic study over large number of colliding nuclei reveals that the surface diffuseness parameter extracted from measured fusion cross sections follows a linear trend with N/Z content of the colliding nuclei. Also, the extracted value of diffuseness parameter varies between 0.40 to 0.73 fm that is not consistent with commonly accepted value from elastic scattering i.e. $a = 0.63$ fm.

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