Numerical Investigation of Chaotic Mixing in Gas Stirred Steel Ladles

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1. Introduction

Fluid mixing has assumed an important role in a number of different processes in industries. The chemical efficiencies of typical processing operations carried out in modern days industries are intrinsically related to their hydrodynam- ic performance. In steel manufacturing, gas injection is often a preferred route for mixing. For high quality steelmaking, gas-stirring of steel in ladles has been widely applied to almost every steelmaking shop. The gas injected from the bottom of the ladles rises through the liquid steel and effects mixing, promotes chemical reactions, helps in achieving compositional and thermal homogeneity. In addition, gas injection also aids in agglomeration and float-out. In all of the above operations, mixing is very important and proper mixing can enhance the quality of steel being manufactured. In the last three decades a great attention has been paid to the studies on fluid flow1–3) and mixing phenomena4–6) in gas stirred ladle systems. Various physical and mathematical model studies on the subject have been carried out and reported in the literature. Most often, the concept of a mixing time, tm, has been applied to represent the state of agitation in the reactor vessels. The experimental work of Nakanishi et al.8) for the first time, proposed a functional relationship (tm = e−0.4t) between the mixing time and specific energy input rates for a range of metal processing operations. Since then, many empirical relationships similar to this type have been reported.9,10) In all these, influence of different operating variables (e.g., gas flow rates, vessel geometries, nozzle configurations) on mixing were studied and expressed by a suitable correlation. Mathematical modelling of mixing phenomena in a gas stirred melt primarily involves prediction of mixing time through numerical solution of the species conservation equation.5,9) An alternative calculation procedure was proposed by Sano and Mori11) and is commonly known as the “circulation time model”. The method assumes that circulation time is proportional to the mixing time and calculates circulation time in terms of operating variables through an energy balance method. Krishnamurthy et al.12) adopted a similar procedure to investigate mixing phenomena in gas stirred baths.

It can be noted here that most of the above-mentioned studies have typically investigated the hydrodynamics of gas stirred system in terms of velocity field existing in the bath and tried to quantify mixing through proper representation of the mixing time. Researchers in the past have however largely neglected an important aspect of mixing, namely chaotic advection which governs the transport behavior to a large extent.13) Demonstration of chaos in various physical systems have shown that chaotic systems due to their ergodic nature can lead to very efficient mixing. Enhancement of chemical reactions and heat transfer rate has also been reported in chaotic fluid systems.13) Pioneering work in the field of chaotic dynamics14) has shown that flow in several industrial mixing processes is characteristic of chaotic advection and can be examined by applying chaotic analysis to the trajectories (e.g. Poincare sections) and analysing chaotic quantitative measures (e.g. Lyapunov exponents). This realization has broad implications for many fields of science and technology, and it is only within the past decade or so that the field has undergone explosive growth. With the availability of high performance computing systems, several studies14,15) dealing with the optimization of mixing process in the chemical industries have been reported, however, a systematic study addressing this significant issue of chaotic mixing in a gas stirred steel vessel is yet to be found in the literature.

The present work is a preliminary attempt to study the mixing process and investigate the possible presence of chaotic mixing in an industrial gas stirred steel ladle. This is accomplished by numerical simulation of transient fluid flow in the ladle via a modified version of the previously reported two-dimensional Computational fluid dynamics (CFD) model pertinent to this process.16) The evolution of mixing pattern is studied by analyzing the composition distribution of tracer particle (which is an outcome of the numerical solution of species conservation equation5–9) and flow trajectories of large numbers of particles in a Lagrangian frame, using the transient Eulerian velocity field. The combined model, for the first time, throws light on the presence of chaotic advection in the flow field of a gas stirred steel ladle and is expected to provide more fundamental insight into the nature of the mixing process thereby facilitating the subsequent optimization of the overall process.

2. Mathematical Modelling

2.1. Chaotic Advection

Chaos refers to the strange, non-periodic responses exhibited by deterministic non-linear systems. Chaotic systems are inherently unpredictable—tiny errors in measurements are amplified rapidly leading to incorrect forecasts. Chaos theory is the theory of dynamical systems which exhibit a myriad non-linear behavior including chaos. Although chaotic behavior signifies an end of predictability of even simple dynamical systems, such systems are characterized by some universal qualitative and quantitative features. These universal features are independent of the details of the particular system. The Navier–Stokes equation is inherently non-linear and hence such systems are capable of displaying chaotic behavior. Research in the recent past14,15) has shown that flow field in different industrial mixing processes is characteristic of chaotic advection. Numerical computation of the flow field along with the quantitative analysis of the fluid motion has pointed to the fact that mixing can be chaotic in nature.

2.2. The Physical Problem

The problem domain considered here is a cylindrical ladle of radius R (R=0.3 m), containing molten steel to a depth H (H=0.6 m). Argon gas is purged from a centrally placed nozzle at the bottom of the steel ladle. The gas, while rising as a plume to the free surface, induces recirculatory fluid flow in the vessel. It is assumed that axial symmetry exists about the centreline. The velocity fields are calculated as a function of time. It can be noted here that the details of the governing equations and the associated methodology adopted to describe the hydrodynamics of bottom gas stirring are described in Ganguly et al.18) and are not mentioned here for the purpose of brevity.

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2.3. Numerical Solution Procedure

The present approach to the study of mixing in the gas stirred flow field applies Computational fluid dynamics theory and Lagrangian description of fluid flow. The Lagrangian description uses coordinates that move with a particle. The equations which governs the fluid motion in the gas stirred liquid bath are first solved using a two-dimensional CFD model to obtain the velocity field \( u(x,t) \), the composition distribution of tracer particle in the flow domain is obtained by numerically solving the species-conservation equation. The dynamics of the transport and mixing of a fluid particle is studied by the trajectories of:

\[
\dot{x} = u(x,t) \quad \text{.......................... (1)}
\]

where a trajectory is the path the fluid particle takes through the fluid. This is obtained by numerically integrating the velocity field. The amount of fluid mixing is analysed by calculating Lyapunov exponent which is a measure of the chaotic mixing in the flow field.

2.3.1. Calculation of Lyapunov Exponent

Lyapunov exponent is generally used to provide a quantitative measure of the chaotic mixing. The dynamical analogy to the mixing of fluids is the stretching and folding of a cluster of initial conditions associated with the chaotic motion in state space, which causes the cluster to be spread throughout a significant region in state space. Lyapunov exponent is a measure of the rate of divergence of state-space trajectories in the system. It gives an estimate of whether trajectories starting from nearby points would converge, diverge or remain equidistant.

For calculating Lyapunov exponent, two close points are taken in the state space and are allowed to evolve for some time, say \( t \). If two points in the state space had a separation \( d_1 \), initially, and a separation \( d_2 \) after time \( t \) then the Lyapunov exponent is given by:

\[
d_2 = d_1 e^{\lambda t} \quad \text{.......................... (2)}
\]

where \( \lambda \) is the Lyapunov exponent. A positive Lyapunov exponent indicates chaos while quasi-periodicity is characterized by zero Lyapunov exponent.

3. Results and Discussions

The hydrodynamic theory of two dimensional fluid flows can be specified in terms of the streamfunction \( \psi \) for an isochoric flow. The streamfunction plays the role of the Hamiltonian function for Hamiltonian Systems. If the fluid flow is steady, then the fluid system is equivalent to a Hamiltonian system with one degree of freedom. Here there is no possibility of chaotic behaviour. If the streamfunction is periodic in time, the fluid flow system is equivalent to a Hamiltonian system with “one-and-a-half degrees of freedom” and chaotic behaviour is possible. Accordingly, in the present study, transient simulation of the fluid flow was carried out to capture the symptoms of chaotic mixing in the flow field. Simulations were carried out with Argon flow rate varying from \( 1.0 \times 10^{-8} \) to \( 1.0 \times 10^{-7} \) m/s and evolution of the flow field and mixing pattern were studied. This values of the Argon flow rate are consistent with the much smaller cylindrical ladle chosen for the purpose of present simulation as compared to the actual steelmaking ladle.

Figure 1 shows the computed streamfunction contours and concentration contours for a low value of bottom gas flow rate \( \sim 1.0 \times 10^{-7} \) m/s during the initial period of blowing. Due to axis-symmetry, only a half of the flow field are shown in the figure. The streamlines (Fig. 1(a)) reveals the formation of recirculatory flow pattern which is the characteristic of an axisymmetric, gas bubble driven system. Figure 1(b) shows the concentration field in the gas stirred flow field. The composition distribution values shown in the Fig. 1(b) indicates the fraction of concentration at a particular location, thereby qualitatively depicting the amount of fluid mixing.

Figure 2 illustrates the streamfunction contours and concentration contours for a comparatively high value of bottom gas flow rate \( \sim 1.0 \times 10^{-6} \) m/s. The recirculatory vortex is still visible in the main bulk of the liquid. Figure 2(a) shows possible existence of elliptic points (‘A’ and ‘B’) and hyperbolic point (‘C’). Elliptic points and hyperbolic points, analogous to those for a Hamiltonian system organize the fluid flow and is an indication of a possible chaotic behaviour. A comparison with Fig. 1 shows that for lower gas-injection values the mixing is poor (Fig. 1(b)) in contrast to the mixing for high gas-injection values (Fig. 2(b)). The good mixing is attributed to the presence of chaotic patterns in the flow field. To investigate the matter further, Lyapunov exponent has been calculated at an initial point near the hyperbolic point (‘C’ in Fig. 2(a)). Figure 3(a) shows the variation of finite time Lyapunov exponent with time for a gas flow rate of \( 1.0 \times 10^{-7} \) m/s. A positive Lyapunov exponent confirms chaotic behavior of the mixing pattern. Figure 3(b) compares the behavior of Lyapunov Exponent for three different gas flow rate \( \sim 1.0 \times 10^{-5}, 1.0 \times 10^{-4}, \) and \( 1.0 \times 10^{-8} \) m/s. The Lyapunov Exponent has been calculated about a point located in the region C (in Fig. 2(a)). The figure clearly shows that for higher values of gas flow rate \( \sim 1.0 \times 10^{-4} \) m/s, the Lyapunov Exponent increases and remains positive, thereby confirming the presence of chaotic behavior in the flow. However, for lower gas flow rates \( \sim 1.0 \times 10^{-7} \) and \( 1.0 \times 10^{-8} \) m/s, negative Lyapunov Exponent values indicates absence of chaos in the flow field.

Figure 4(a) shows the variation of streamfunction at a particular point (located in the region C in Fig. 2(a)) with time. It demonstrates the periodic behaviour of streamfunction, and as it has been mentioned earlier, the periodicity is an indication of possible presence of chaos in the system. It can also be seen from the figure that the amplitude of variation gradually decreases with time and finally streamfunction plot flattens. It suggests that chaos does not exist forever in this system and present only in the initial transient period. This observation is further reinforced by Fig. 4(b) which plots the trajectory of a simulated fluid particle in the flow field. The fluid particle undergoes recirculatory motion thus forming a loop and intersects itself a number of times before finally falling into the basin of attraction of stable attracting region. This shows that the chaotic orbit is unstable and co-exists with a stable attracting region which finally ‘pulls’ the particle inwards.

4. Conclusions

A numerical modelling exercise has been carried out to investigate the possible presence of chaotic mixing in the flow field of a gas stirred ladle system. This is accomplished by numerical simulation of the transient fluid flow in the ladle along with the study of composition distribution of tracer particles and the trajectory of fluid particles in the flow domain. Different aspects of chaotic advection pertaining to an industrial mixing process have been discussed. As a quantitative measure of chaos, Lyapunov exponent is calculated. The results, for the first time indicates the chaotic behaviour of mixing pattern in gas stirred ladle system.

Nomenclature

- \( H \): Liquid steel height
- \( Q \): Volumetric gas flow rate
- \( r \): Radial coordinate
- \( R \): Ladle radius
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