On Branching Ratios of $B_s$ Decays and the Search for New Physics in $B_s^0 \rightarrow \mu^+\mu^-$

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Abstract

The LHCb experiment has recently established a sizable width difference between the mass eigenstates of the $B_s$-meson system. This phenomenon leads to a subtle difference at the 10% level between the experimental branching ratios of $B_s$ decays extracted from time-integrated, untagged data samples and their theoretical counterparts. Measuring the corresponding effective $B_s$-decay lifetimes, both branching ratio concepts can be converted into each other. The rare decay $B^0_s \rightarrow \mu^+\mu^-$ and the search for New Physics through this channel is also affected by this effect, which enhances the Standard-Model reference value of the branching ratio by $O(10\%)$, while the effective lifetime offers a new observable to search for physics beyond the Standard Model that is complementary to the branching ratio.

Keywords: $B_s$ decays, branching ratios, effective lifetimes, New Physics

1. Introduction

Weak decays of $B_s^0$ mesons encode valuable information about the quark-flavour sector of the Standard Model (SM) of particle physics. The conceptually simplest observables are branching ratios, which describe the probability for the considered decay to occur.

Measurements of $B_s$-decay branching ratios at hadron colliders would require precise knowledge of the $B_s$ production cross section, which is not available, and rely therefore on experimental control channels and the ratio of the $f_s/f_{s,d}$ fragmentation functions (for a detailed discussion, see Ref. [1]). At the $e^+e^-$ B factories operating at the $\Gamma(\Upsilon 5S)$ resonance, $B_s$-decay branching ratios can be extracted since the total number of produced $B_s$ mesons can be determined separately [2].

The neutral $B_s$ mesons exhibit $B_s^0-B_s^\pm$ mixing. Measuring the time-dependent angular distribution of the $B_s^0 \rightarrow J/\psi \phi$ decay products [3], LHCb has recently established a non-vanishing difference $\Delta\Gamma_s$ between the decay widths of the $B_s$ mass eigenstates [4]:

$$y_s \equiv \frac{\Delta\Gamma_s}{2\Gamma_s} \equiv \frac{\Gamma_L^{(s)} - \Gamma_H^{(s)}}{2\Gamma_s} = 0.088 \pm 0.014, \quad (1)$$

where $\Gamma_s$ is the inverse of the average $B_s$ lifetime $\tau_{B_s}$. Since a discrete ambiguity could also be resolved [5], we are left with the sign in (1), which agrees with the SM expectation. A sizable value of $\Delta\Gamma_s$ was theoretically expected since decades [6].

In view of the sizable $\Delta\Gamma_s$, special care has to be taken when dealing with the concept of a branching ratio, and the question of how to convert measured “experimental” $B_s$-decay branching ratios into “theoretical” $B_s$ branching ratios arises. This issue is the central topic of this writeup, summarizing the results of Refs. [7,8]. A special emphasis will be put on the rare decay $B^0_s \rightarrow \mu^+\mu^-$. 

2. Branching Ratios of $B_s$ Decays

2.1. Experimental vs. Theoretical Branching Ratios

The untagged rate of a $B_s$ decay, were no distinction between initially present $B^0_s$ or $\bar{B}^0_s$ mesons is made, is a sum of two exponentials:

$$\langle \Gamma(B_s(t) \rightarrow f) \rangle \equiv \Gamma(B^0_s(t) \rightarrow f) + \Gamma(\bar{B}^0_s(t) \rightarrow f) = R_I e^{-\tau_{B_s}(f)} + R_L e^{-\tau_{B_s}(f)}, \quad (2)$$

where $R_I$ and $R_L$ are the contributions of the initial $B^0_s$ and $\bar{B}^0_s$ mesons, respectively.

In the SM, the difference $\Delta\Gamma_s$ only affects the contribution $R_I$, which leads to a suppression of $\Delta\Gamma_s$ below the experimental uncertainty. This is an important aspect when discussing $\Delta\Gamma_s$ in the context of New Physics searches. New Physics contributions to $\Delta\Gamma_s$ are expected to be of similar size as the SM contributions to $\Delta\Gamma_s$. In addition, New Physics can be searched for in $\bar{B}^0_s$ decays, which are dominated by $R_L$ contributions.

The SM expectation for $\Delta\Gamma_s$ is obtained by

$$\Delta\Gamma_s = \frac{\Gamma_B^{(s)} - \Gamma_B^{(\bar{s})}}{\Gamma_B^{(s)} + \Gamma_B^{(\bar{s})}} \equiv \frac{\Gamma_L^{(s)} - \Gamma_H^{(s)}}{\Gamma_L^{(s)} + \Gamma_H^{(s)}} = 0.088 \pm 0.014, \quad (3)$$

where $\Gamma_B^{(s)}$ and $\Gamma_B^{(\bar{s})}$ are the hadronic decay widths of the $B_s$ and $\bar{B}_s$ mesons, respectively.

Since $\Delta\Gamma_s$ is defined as the difference between the hadronic decay widths of the $B_s$ and $\bar{B}_s$ mesons, it is natural to consider the ratio $\Gamma_L^{(s)}/\Gamma_H^{(s)}$ instead of $\Delta\Gamma_s$. This ratio is defined in the SM as

$$\frac{\Gamma_L^{(s)}}{\Gamma_H^{(s)}} = \frac{\Gamma^{(s)}_{LL} + \Gamma^{(s)}_{LL}}{\Gamma^{(s)}_{HH} + \Gamma^{(s)}_{HH}} = 0.088 \pm 0.014, \quad (4)$$

where $\Gamma^{(s)}_{LL}$ and $\Gamma^{(s)}_{HH}$ are the hadronic decay widths of the $B_s$ and $\bar{B}_s$ mesons, respectively.

The ratio $\Gamma_L^{(s)}/\Gamma_H^{(s)}$ is the inverse of the average $B_s$ lifetime $\tau_{B_s}$ and provides a strong constraint on the difference between $\Gamma_L^{(s)}$ and $\Gamma_H^{(s)}$. This ratio is expected to be close to one in the SM, and deviations from one can be used to search for New Physics.

In summary, the difference $\Delta\Gamma_s$ between the hadronic decay widths of the $B_s$ and $\bar{B}_s$ mesons can be measured in $B_s$ decays and provides a strong constraint on New Physics searches. New Physics contributions to $\Delta\Gamma_s$ can be searched for in $\bar{B}^0_s$ decays, which are dominated by $R_L$ contributions. The ratio $\Gamma_L^{(s)}/\Gamma_H^{(s)}$ is the inverse of the average $B_s$ lifetime $\tau_{B_s}$ and provides a strong constraint on the difference between $\Gamma_L^{(s)}$ and $\Gamma_H^{(s)}$. This ratio is expected to be close to one in the SM, and deviations from one can be used to search for New Physics.
which can be rewritten as

\[
\langle \Gamma(B_s(t) \to f) \rangle = \left( R_H^f + R_L^f \right) \exp \frac{-\Delta \gamma t}{2}
\times \left[ \cosh (y_s t/\tau_{B_s}) + \mathcal{A}^f_{\Delta \gamma} \sinh (y_s t/\tau_{B_s}) \right].
\] (3)

Here the parameter \( y_s \) was introduced in (1), and

\[
\mathcal{A}^f_{\Delta \gamma} \equiv \frac{R_H^f - R_L^f}{R_H^f + R_L^f}
\]

is an observable depending on the final state \( f \). The branching ratios given by experiments are extracted from total event yields, without taking time information into account, and can be defined as follows [7,9]:

\[
\text{BR}(B_s \to f)_{\text{exp}} \equiv \frac{1}{2} \int_0^{\infty} \langle \Gamma(B_s(t) \to f) \rangle \, dt
\]

(4)

(5)

\[
= \frac{1}{2} \left[ \frac{R_H^f}{\Gamma_{\text{tot}}^H} + \frac{R_L^f}{\Gamma_{\text{tot}}^L} \right] = \frac{\tau_{B_s}}{2} \left( R_H^f + R_L^f \right) \left[ 1 + \mathcal{A}^f_{\Delta \gamma} y_s \right].
\]

(6)

(7)

(8)

(9)

2.2. Effective \( B_s \) Decay Lifetimes

Once time information for the untagged \( B_s \) decay data sample becomes available, the theoretical input for determining \( \mathcal{A}^f_{\Delta \gamma} \) can be avoided in the extraction of the theoretical branching ratio [6,7].

Using the effective \( B_s \) decay lifetime

\[
\tau_{B_s} \equiv \frac{\int_0^{\infty} \langle \Gamma(B_s(t) \to f) \rangle \, dt}{\int_0^{\infty} (\Gamma(B_s(t) \to f) ) \, dt}
\]

(9)

(10)

we obtain

\[
\text{BR}(B_s \to f)_{\text{theo}} = 2 - \left( 1 - y_s^2 \right) \frac{\tau_{B_s}}{\tau_{B_s}},
\]

(11)

where only measurable quantities appear on the right-hand side. The measurement of effective \( B_s \) decay lifetimes is hence an integral part of the extraction of the theoretical branching ratios from the data and not only an interesting topic to constrain the \( B_s^0 - \bar{B}_s^0 \) mixing parameters [10]. The use of the theoretically clean relation in (9) is advocated for the compilation of \( B_s \) decay properties in particle listings.

For a discussion of experimental subtleties related to the measurement of \( B_s \) decay branching ratios and effective lifetimes, the reader is referred to Ref. [7].

2.3. \( B_s \to VV \) Decays

The branching ratio measurements of \( B_s \to VV \) decays into two vector mesons, such as \( B_s \to J/\psi \phi \), \( B_s \to K^{*0}R^{0} \) and \( B_s \to D_s^{*+}D_s^{-} \), are also affected by the sizable width difference \( \Delta \gamma \) [13,14]. Here an angular analysis of the decay products of the vector mesons has to be performed in order to disentangle the CP-even \((0,0)\) and CP-odd \((\pm \pm)\) final states, with

\[
f_{VV,k}^{\text{exp}} \equiv \text{BR}_{VV,k}^{\text{exp}} / \text{BR}_{VV}^{\text{exp}}.
\]

The experimental branching ratios can then be converted into the theoretical branching ratios through

\[
\text{BR}_{\text{theo}}^{VV} = \left( 1 - y_s^2 \right) \sum_{k=0,\pm \pm} \frac{f_{VV,k}^{\text{exp}}}{1 + y_s \mathcal{A}^{VV}_{\Delta \gamma}} \text{BR}_{\text{exp}}^{VV}
\]

(11)

(12)

with the help of theoretical information on the \( \mathcal{A}^{VV}_{\Delta \gamma} \) observables, or by means of the relation

\[
\text{BR}_{\text{theo}}^{VV} = \frac{\text{BR}_{\text{exp}}^{VV}}{2} \left( 1 - y_s^2 \right) \frac{\tau_{B_s}}{\tau_{B_s}} f_{VV,k}^{\text{exp}}
\]

using effective lifetime measurements [7].
3. $B_s^0 \rightarrow \mu^+\mu^-$ and a New Window for New Physics

3.1. Setting the Stage

A key probe of New Physics (NP) is the rare decay $B^0_s \rightarrow \mu^+\mu^-$, which receives only loop contributions from box and penguin topologies in the SM. Here the (theoretical) branching ratio is predicted as follows [15]:

$$\text{BR}(B_s \rightarrow \mu^+\mu^-)_{\text{SM}} = (3.2 \pm 0.2) \times 10^{-9}. \quad (13)$$

In the presence of NP, the branching ratio may be affected by new particles in the loops or new contributions at the tree level [16]. The error of (13) is dominated by lattice QCD input for non-perturbative physics [17].

The limiting factor for the $B^0_s \rightarrow \mu^+\mu^-$ branching ratio measurement at hadron colliders is the ratio $f_s/f_d$, where the fragmentation functions $f_{s(d)}$ describe the probability that a $b$ quark fragments in a $\bar{B}^0_{(s)}$ meson. A new method for determining $f_s/f_d$ using nonleptonic $\bar{B}^0 \rightarrow D^{+}\pi^{-}$, $\bar{B}^0 \rightarrow D^{0}K^{-}$, $\bar{B}^0 \rightarrow D^{*}\pi^{-}$ decays [18] was recently implemented at LHCb [19], with a result in good agreement with measurements using semileptonic decays [20]. The $SU(3)$-breaking form-factor ratio entering the non-leptonic method has recently been calculated with lattice QCD [17, 21].

Searches for the $B^0_s \rightarrow \mu^+\mu^-$ decay were performed by the CDF [22], D0 [23], ATLAS [24], CMS [25] and LHCb collaborations. The latter experiment has recently reported the currently most stringent upper bound on the branching ratio, which corresponds to

$$\text{BR}(B_s \rightarrow \mu^+\mu^-) < 4.5 \times 10^{-9}. \quad (14)$$

at the 95% confidence level [26], and is approaching the SM prediction [13]. For a recent review of the experimental studies, see Ref. [22].

In the analyses of the $\bar{B}^0_s \rightarrow \mu^+\mu^-$ decay, the impact of $\Delta \Gamma_s$ was not taken into account. In view of the discussion in the previous section, the question arises how the sizable value of $\Delta \Gamma_s$ affects the theoretical interpretation of the $\bar{B}^0_s \rightarrow \mu^+\mu^-$ data and whether we can actually take advantage of this decay width difference [18].

3.2. Low-Energy Effective Hamiltonian

In order to address this issue, we use the general low-energy effective Hamiltonian describing the decay $\bar{B}^0_s \rightarrow \mu^+\mu^-$ as the starting point. Using a notation similar to Ref. [23], where a model-independent NP analysis was performed, it can be written as

$$\mathcal{H}_{\text{eff}} = - \frac{G_F}{\sqrt{2} \pi} \alpha V_{ts} V_{tb} \left[ C_{10} O_{10} + C_5 O_5 + C_P O_P + C_{10}' O_{10}' + C_5' O_5' + C_P' O_P' \right]. \quad (15)$$

where $G_F$ and $\alpha$ are the Fermi and QED fine-structure constants, respectively, and the $V_{iq}$ are elements of the Cabibbo–Kobayashi–Maskawa (CKM) matrix. The short-distance physics is encoded in the Wilson coefficients $C_i, C_i'$ of the four-fermion operators

$$O_{10} = (\bar{s} \gamma_\mu P_L b)(\bar{\ell} \gamma^\nu \gamma_\ell \ell) \quad O_S = m_b (3 P_R b)(\bar{\ell} \ell) \quad O_F = m_b (3 P_R b)(\bar{\ell} \gamma_\ell \ell), \quad (16)$$

where $m_b$ denotes the $b$ quark mass, $P_{LR} \equiv (1 \mp \gamma_5)/2$, and the $O'_i$ are obtained from the $O_i$ through the replacements $P_L \leftrightarrow P_R$. It should be noted that only operators with non-vanishing contributions to $\bar{B}^0_s \rightarrow \mu^+\mu^-$ are included in (15); in particular the matrix elements of operators involving the $\bar{\ell} \gamma_\ell \ell$ vector current vanish.

The hadronic structure of the leptonic $\bar{B}^0_s \rightarrow \mu^+\mu^-$ decay is very simple and can be expressed in terms of a single, non-perturbative parameter, which is the $B_s$-meson decay constant $f_b$ [15].

In the SM, only the $O_{10}$ operator contributes with a real Wilson coefficient $C_{10}^{\text{SM}}$, leading to the prediction in [13]. The sensitivity to (pseudo-)scalar lepton densities entering the $O_{10}^{PS}$ and $O_{10}^{PS'}$ operators is an outstanding feature of the $\bar{B}^0_s \rightarrow \mu^+\mu^-$ channel. The corresponding Wilson coefficients are still largely unconstrained, thereby leaving ample space for NP [28].

3.3. Observables

For the calculation of the $B_s \rightarrow \mu^+\mu^-$ observables, it is convenient to go to the rest frame of the decaying $\bar{B}^0_s$ meson and to use the notation $\mu_1^+ \mu_2^-$ to distinguish between the left-handed ($\lambda = L$) and right-handed ($\lambda = R$) muon helicity configurations. In this setting, the $\mu_1^+ \mu_L^-$ and $\mu_2^+ \mu_R^-$ states are simply related to each other through CP transformations.

Thanks to $\bar{B}^0_s - \bar{B}^0_s$ mixing, we get interference effects between the $\bar{B}^0_s \rightarrow \mu_1^+ \mu_2^-$ and $\bar{B}^0_s \rightarrow \mu_2^+ \mu_1^-$ decay processes that are described by the observable

$$\xi_3 \equiv -e^{-i \phi_3} \left[ \frac{\epsilon_{\text{CP}}(B_s)}{A(B_s \rightarrow \mu_1^+ \mu_2^-)} \right] \frac{A(B_s \rightarrow \mu_2^+ \mu_1^-)}{A(B_s \rightarrow \mu_1^+ \mu_2^-)} \quad (17)$$

Here $\phi_3$ is the $\bar{B}^0_s - \bar{B}^0_s$ mixing phase, whereas $\epsilon_{\text{CP}}(B_s)$ denotes a convention-dependent phase which is associated with CP transformations [29]. Expressing the $\bar{B}^0_s \rightarrow \mu_1^+ \mu_1^-$ decay amplitude as

$$A(B_s \rightarrow \mu_1^+ \mu_1^-) = (\mu_1^+ \mu_1^-) |\mathcal{H}_{\text{eff}}(B_s)^{\mu_1^+ \mu_1^-}| \quad (18)$$

and performing appropriate CP transformation results eventually in the following expression [8]:

$$\xi_3 = \frac{\eta_3 P + S}{-\eta_3 P^* + S^*}. \quad (19)$$
where all convention-dependent quantities (such as the $\phi_{CP}(B_s)$ phases) cancel, $\eta_{f(R)} = \langle -1 \rangle$, and

$$P \equiv \frac{C_{10} - C'_{10}}{C_{SM}^{10}} + \frac{m_0}{C_{SM}^{10}} + \frac{m_0}{C_{SM}^{10}} \left( C - C'_{SM} \right)$$

and

$$S \equiv \sqrt{1 - 4 \frac{m_0^2 + |m_0|^2}{M_{B_s}^2} \left( \frac{m_0}{C_{SM}^{10}} \right)^2} \left( C - C'_{SM} \right).$$

These combinations of Wilson coefficient functions have been introduced in such a way that we simply have $P = 1$ and $S = 0$ in the SM, whereas $P \equiv |P|e^{i\phi}$ and $S \equiv |S|e^{i\phi_s}$ carry, in general, non-trivial CP-violating phases $\phi_P$ and $\phi_S$ (see also Ref. [28]). It should be noted that the non-perturbative $B_s$-meson decay constant $f_{B_s}$, which arises in the parametrization of (18) and affects the SM prediction (13), cancels in the observable (19).

Before having a closer look at the branching ratio, it is interesting to consider the following time-dependent rate asymmetries, which require tagging information and knowledge of the muon helicity $\lambda$:

$$\Gamma(B_s^0(t) \to \mu^+\mu^-) - \Gamma(B_s^0(t) \to \mu^+\mu^-)$$

$$\Gamma(B_s^0(t) \to \mu^+\mu^-) + \Gamma(B_s^0(t) \to \mu^+\mu^-)$$

$$= \frac{C_4}{y_s^*} \sin(\Delta M_t) + S_4 \sin(\Delta M_t) \cosh(y_s^* \tau_{B_s}) + \mathcal{A}_4 \sinh(y_s^* \tau_{B_s}).$$

Here $\Delta M_t$ denotes the mass difference between the heavy and light $B_s$ mass eigenstates while $y_s$ is given in (1). Neglecting the impact of $\Delta \Gamma_s$, such CP asymmetries were considered for $B_{sL} \to \ell^+\ell^-$ decays within various NP scenarios in the previous literature [30][31][32].

The observables entering (22) are governed by $\xi_{\lambda}$ in (19) and take the following expressions [8]:

$$C_4 = \frac{1 - |\xi_{\lambda}|^2}{1 + |\xi_{\lambda}|^2}, \quad S_4 = \frac{2|\xi_{\lambda}|^2}{1 + |\xi_{\lambda}|^2}$$

$$\mathcal{A}_4 = \frac{2 \text{Re} \xi_{\lambda}}{1 + |\xi_{\lambda}|^2},$$

which are theoretically clean. Note that $S_{CP} \equiv S_4$ and $\mathcal{A}_{SM} \equiv \mathcal{A}_{SM}$ do not depend on the muon helicity $\lambda$.

In the discussion given above, it was assumed that NP enters only through the Wilson coefficients governing the $B_s^0 \to \mu^+\mu^-$ decay and that the $B_s^0 - \bar{B}_s^0$ mixing phase $\phi_{SM} + \phi_{NP}$ takes its SM value $\phi_{SM} \equiv 2\arg(V_{cb}V_{cb}^*)$, which is cancelled in (17) through the CKM factors of the ratio of decay amplitudes. In (24) and (25), NP in $B_s^0 - \bar{B}_s^0$ mixing can straightforwardly be included through the replacements $2e_P \rightarrow 2e_P + \phi_{NP}$. The LHCb data for CP violation in $B_s \rightarrow J/\psi\phi$, $J/\psi f_0(980)$ decays already constrain $\phi_{NP}$ to the few-degree level [3]. Consequently, this effect is negligible from the practical point of view for the following considerations.

It is difficult to measure the muon helicity. In order to circumvent this problem, we consider the

$$\Gamma(B_s^0(t) \to \mu^+\mu^-) = \sum_{\lambda = L,R} \Gamma(B_s^0(t) \to \mu^+\mu^-)$$

rate and its counterpart for initially present $B_s^0$ mesons, which can be combined into the CP asymmetry

$$\Gamma(B_s^0(t) \to \mu^+\mu^-) - \Gamma(B_s^0(t) \to \mu^+\mu^-)$$

$$\Gamma(B_s^0(t) \to \mu^+\mu^-) + \Gamma(B_s^0(t) \to \mu^+\mu^-)$$

$$= \mathcal{S}_{CP} \sin(\Delta M_t/2) \cosh(y_s^* \tau_{B_s}) + \mathcal{A}_{SM} \sinh(y_s^* \tau_{B_s}).$$

Since a non-zero value would immediately signal new CP-violating phases, it would be most interesting to measure this asymmetry. This feature was recently highlighted in Ref. [33] for minimal $U(2)$ models [34]. Unfortunately, despite the independence on the muon helicity, this is still challenging from the practical point of view as tagging and time information are required. An analogous expression holds for the rare $B_d \rightarrow \mu^+\mu^-$ decays, where $y_d$ is negligibly small.

### 3.4. Closer Look at the Branching Ratios

From the practical point of view, the branching ratio extracted from untagged data samples, ignoring the decay-time information, is the first measure:

$$\text{BR}(B_s \rightarrow \mu^+\mu^-)_{\text{exp}}$$

$$= \int_0^\infty \Gamma(B_s(t) \rightarrow \mu^+\mu^-)) dt.$$ (28)

Here the untagged $(\Gamma(B_s(t) \rightarrow \mu^+\mu^-))$ rate is given in general terms in (2) and (3).

Since $\mathcal{A}_{SM}$ in (25) does actually not depend on the muon helicity, i.e. $\mathcal{A}_{SM} \equiv \mathcal{A}_{SM}$, we can apply (7) to extract the theoretical branching ratio from the experimental branching ratio (28):

$$\text{BR}(B_s \rightarrow \mu^+\mu^-)_{\text{theo}}$$

$$= \left[ 1 - \frac{v^2}{1 + \mathcal{A}_{SM}} \right] \text{BR}(B_s \rightarrow \mu^+\mu^-)_{\text{exp}}.$$ (29)

The former is considered and calculated by the theoretical community (see, e.g., Refs. [15][28]), and satisfies

$$\frac{\text{BR}(B_s \rightarrow \mu^+\mu^-)_{\text{theo}}}{\text{BR}(B_s \rightarrow \mu^+\mu^-)_{\text{SM}}} = \left| p^2 + |S|^2 \right|^2.$$ (30)
The $y_s$ terms in (29) had not been taken into account in the comparison between theory and experiment [8].

As can be seen in (25), $\mathcal{A}_{\Delta \Gamma}$ depends sensitively on NP entering through the Wilson coefficients which govern the $B_s^0 \rightarrow \mu^+\mu^-$ channel. Consequently, this observable is currently unknown. Varying $\mathcal{A}_{\Delta \Gamma} \in [-1,+1]$ yields

$$\Delta \text{BR}(B_s \rightarrow \mu^+\mu^-)|_{y_s} = \pm y_s \text{BR}(B_s \rightarrow \mu^+\mu^-)_{\text{exp}}, \quad (31)$$

which has to be added to the experimental error of (28).

On the other hand, within the SM, we have the theoretically clean prediction $\mathcal{A}_{\Delta \Gamma}^{\text{SM}} = +1$. If we rescale the theoretical SM branching ratio in (13) correspondingly by a factor of $(1/y_s)$ and use (1), we obtain

$$\text{BR}(B_s \rightarrow \mu^+\mu^-)_{\text{SM}}|_{y_s} = (3.5 \pm 0.2) \times 10^{-9}. \quad (32)$$

This is the SM branching ratio reference value for the comparison with the experimental branching ratio (29).

### 3.5. Effective Lifetime

Once the $B_s^0 \rightarrow \mu^+\mu^-$ decay has been observed and more experimental data become available, it is possible to include also the decay time information in the analysis so that the effective $B_s^0 \rightarrow \mu^+\mu^-$ lifetime $\tau_{\mu^+\mu^-}$, which is defined as in [8], can be measured. Since $\mathcal{A}_{\Delta \Gamma}$ in (25) does not depend on the muon helicity, this observable can be extracted from the effective lifetime with the help of the relation

$$\mathcal{A}_{\Delta \Gamma} = \frac{1}{y_s} \left[ \left( 1-y_s^2 \right) \tau_{\mu^+\mu^-} - \left( 1+y_s^2 \right) \tau_{B_s} \right], \quad (33)$$

and results in

$$\text{BR} \left( B_s \rightarrow \mu^+\mu^- \right)_{\text{theo}} = \left[ 2 - \left( 1-y_s^2 \right) \frac{\tau_{\mu^+\mu^-}}{\tau_{B_s}} \right] \text{BR} \left( B_s \rightarrow \mu^+\mu^- \right)_{\text{exp}}. \quad (34)$$

This expression takes the same form as (9) and allows the conversion of the experimental $B_s \rightarrow \mu^+\mu^-$ branching ratio into its theoretical counterpart, irrespective of whether there are NP contributions present or not. Consequently, the error in (31) can then be eliminated.

The effective $B_s^0 \rightarrow \mu^+\mu^-$ lifetime and the extraction of $\mathcal{A}_{\Delta \Gamma}$ from untagged data samples is an important new measurement for the high-luminosity upgrade of the LHC. Extrapolating from the currently available analyses of the effective $B_s^0 \rightarrow J/\psi f_0(980)$ and $B_s^0 \rightarrow K^+K^-$ lifetimes performed by the CDF and LHCb collaborations, a precision of 5% or better appears feasible [8]. Detailed experimental studies of this exciting new feature of the $B_s^0 \rightarrow \mu^+\mu^-$ channel are strongly encouraged.

### 3.6. Constraints on New Physics

Looking at the expression for $\mathcal{A}_{\Delta \Gamma}$ in (25), it is obvious that this observable and the effective lifetime $\tau_{\mu^+\mu^-}$ may well be affected by NP. The $\Delta \Gamma_s$ effects propagate also into the constraints on NP parameters that can be obtained from the comparison of the experimental information on the $B_s \rightarrow \mu^+\mu^-$ branching ratio with the SM branching ratio, where it is useful to introduce [8]

$$R \equiv \frac{\text{BR}(B_s \rightarrow \mu^+\mu^-)_{\text{exp}}}{\text{BR}(B_s \rightarrow \mu^+\mu^-)_{\text{SM}}}.$$  

Using (25) and (29), the ratio $R$ takes the form

$$R = \frac{1+\mathcal{A}_{\Delta \Gamma} y_s}{1-y_s^2} \left( |P|^2 + |S|^2 \right)$$

$$= \frac{1+y_s \cos 2\varphi_P}{1-y_s^2} \left| P \right|^2 + \frac{1-y_s \cos 2\varphi_S}{1-y_s^2} \left| S \right|^2. \quad (36)$$

Combining (13) and (14) yields the bound $R < 1.4$, where the theoretical uncertainty of the SM prediction of the $B_s^0 \rightarrow \mu^+\mu^-$ branching ratio was neglected.

The ratio $R$ would fix a circle in the $|P|\rightarrow|S|$ plane for $y_s = 0$, i.e. for a vanishing $B_s$ decay width difference. On the other hand, for non-zero values of $y_s$, $R$ can be converted into ellipses which depend on the CP-violating phases $\varphi_{P,S}$. As the latter quantities are in general unknown, $R$ results in a circular band, with the upper bounds $|P|,|S| \leq \sqrt{(1+y_s)R}$. Since the $S$ and $P$ contributions cannot be separated through experimental information on $R$, as can be seen in (36), there may still significant NP contributions be present in $B_s^0 \rightarrow \mu^+\mu^-$, even if the branching ratio should eventually be measured close to the SM expectation.

As was pointed out in Ref. [8], the measurement of the effective lifetime $\tau_{\mu^+\mu^-}$ and the associated untagged $\mathcal{A}_{\Delta \Gamma}$ observable allows a resolution of this situation. The point is that

$$|S| = \left| \sqrt{\frac{\cos 2\varphi_P - \mathcal{A}_{\Delta \Gamma}}{\cos 2\varphi_S + \mathcal{A}_{\Delta \Gamma}}} \right| \quad (37)$$

fixes a straight line through the origin in the $|P|\rightarrow|S|$ plane. For illustrations, the reader is referred to the figures shown in Ref. [8].

In the most recent analyses of the constraints on NP parameter space that are implied by the experimental upper bound on the $B_s \rightarrow \mu^+\mu^-$ branching ratio for various extensions of the SM, authors have now started to take the effect of $\Delta \Gamma_s$ into account (see, for instance, the papers listed in Refs. [33, 35]).
4. Conclusions

The non-vanishing width difference of the $B_{s}$-meson system, which has recently been established by LHCb, leads to subtleties in the extraction of $B_{s}$ branching ratio information from the data but offers also new observables. The differences between the experimental and theoretical branching ratios can be as large as 10%, depending on the final state. Both branching ratios can be converted into each other either through theoretical considerations or through the measurement of the effective $B_{s} \rightarrow f$ decay lifetimes. As the latter involves only experimental data, it is generally the preferred avenue.

The rare decay $B_{s}^{0} \rightarrow \mu^{+}\mu^{-}$ is also affected by $\Delta \Gamma_{s}$, where the theoretical branching ratio in $[13]$ has to be rescaled by $1/(1 - y_{s})$ for the comparison with the experimental branching ratio, resulting in the SM reference value of $(3.5 \pm 0.2) \times 10^{-9}$. Thanks to $\Delta \Gamma_{s}$, the $B_{s}^{0} \rightarrow \mu^{+}\mu^{-}$ decay offers a new observable, which is the effective lifetime $\tau_{\mu^{+}\mu^{-}}$. It allows the inclusion of the $\Delta \Gamma_{s}$ effects in the conversion of the experimental into the theoretical branching ratio. Moreover, it offers also a new, theoretically clean NP probe that may still show large NP effects, in particular those originating from the (pseudo-)scalar $\ell^{\pm}\ell^{-}$ densities entering the four-fermion operators. This observable may even show NP should the $B_{s}^{0} \rightarrow \mu^{+}\mu^{-}$ branching ratio be found close to the SM prediction. The measurement of $\tau_{\mu^{+}\mu^{-}}$ and the associated $A_{\mu^{+}\mu^{-}}$ observable is an exciting new topic for the high-luminosity upgrade of the LHC. Detailed feasibility studies are strongly encouraged.

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