A Multicore Tool for Constraint Solving

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Abstract

In Constraint Programming (CP), a portfolio solver uses a variety of different solvers for solving a given Constraint Satisfaction/Optimization Problem. In this paper we introduce sunny-cp2: the first parallel CP portfolio solver that enables a dynamic, cooperative, and simultaneous execution of its solvers in a multicore setting. It incorporates state-of-the-art solvers, providing also a usable and configurable framework. Empirical results are very promising. sunny-cp2 can even outperform the performance of the oracle solver which always selects the best solver of the portfolio for a given problem.

1 Introduction

The Constraint Programming (CP) paradigm enables to express complex relations in form of constraints to be satisfied. CP allows to model and solve Constraint Satisfaction Problems (CSPs) as well as Constraint Optimization Problems (COPs) [Rossi et al., 2006]. Solving a CSP means finding a solution that satisfies all the constraints of the problem, while a COP is a generalized CSP where the goal is to find a solution that minimises or maximises an objective function.

A fairly recent trend in CP is solving a problem by means of portfolio approaches [Gomes and Selman, 2001], which can be seen as instances of the more general Algorithm Selection problem [Rice, 1976]. A portfolio solver is a particular solver that exploits a collection of $m > 1$ constituent solvers $s_1, \ldots, s_m$ to get a globally better solver. When a new, unseen problem $p$ comes, the portfolio solver tries to predict the best constituent solver(s) $s_{i_1}, \ldots, s_{i_k}$ (with $1 \leq k \leq m$) for solving $p$ and then runs them.

Unfortunately —despite their proven effectiveness— portfolio solvers are scarcely used outside the walls of solvers competitions, and often confined to SAT solving. Regarding the CP field, the first portfolio solver (for solving CSPs only) was CPHydra [O’Mahony et al., 2008] that in 2008 won the International CSP Solver Competition. In 2014, the sequential portfolio solver sunny-cp [Amadini et al., 2013] attended the MiniZinc Challenge (MZC) [Stuckey et al., 2010] with respectable results (4th out of 18). To the best of our knowledge, no similar tool exist for solving both CSPs and COPs.

In this paper we take a major step forward by introducing sunny-cp2, a significant enhancement of sunny-cp. Improvements are manifold. Firstly, sunny-cp2 is parallel: it exploits multicore architectures for (possibly) running more constituent solvers simultaneously. Moreover, while most of the parallel portfolio solvers are static (i.e., they decide off-line the solvers to run, regardless of the problem to be solved), sunny-cp2 is instead dynamic: the solvers scheduling is predicted on-line according to a generalization of the SUNNY algorithm [Amadini et al., 2014c]. Furthermore, for COPs, the parallel execution is also cooperative: a running solver can exploit the current best bound found by another one through a configurable restarting mechanism. Finally, sunny-cp2 enriches sunny-cp by incorporating new, state-of-the-art solvers and by providing a framework which is more usable and configurable for the end user.

We validated sunny-cp2 on two benchmarks of CSPs and COPs (about 5000 instances each). Empirical evidences show very promising results: its performance is very close (and sometimes even better) to that of the Virtual Best Solver (VBS), i.e., the oracle solver which always selects the best solver of the portfolio for a given problem. Moreover, for COPs, sunny-cp2 using $c = 1, 2, 4, 8$ cores always outperforms the Virtual $c$-Parallel Solver (VPS), i.e., a static portfolio solver that runs in parallel a fixed selection of the best $c$ solvers of the portfolio.

Paper Structure. In Section 2 we generalize the SUNNY algorithm for a multicore setting. In Section 3 we describe the architecture and the novelties of sunny-cp2, while in Section 4 we discuss its empirical evaluation. In Section 5 we report the related work before concluding in Section 6.

2 Generalizing the SUNNY Algorithm

SUNNY is the algorithm that underpins sunny-cp. Fixed a solving timeout $T$ and a portfolio $P$, SUNNY exploits instances similarity to produce a sequential schedule $\sigma = [(s_1, t_1), \ldots, (s_n, t_n)]$ where solver $s_i$ has to be run for $t_i$ seconds and $\sum_{i=1}^{n} t_i = T$. For any input problem $p$, SUNNY uses a $k$-Nearest Neighbours ($k$-NN) algorithm to select from a training set of known instances the subset $N(p, k)$ of the $k$ instances closer to $p$. According to the $N(p, k)$ instances, SUNNY relies on three heuristics: $h_{sel}$, for selecting the most
promising solvers to run; \(h_{alt}\), for allocating to each solver a certain runtime (the more a solver is promising, the more time is allocated); and \(h_{sch}\), for scheduling the sequential execution of the solvers according to their presumed speed. These heuristics depend on the application domain. For example, for CSPs \(h_{sel}\) selects the smallest sub-portfolio \(S \subseteq \Pi\) that solves the most instances in \(N(p, k)\), by using the solving time for breaking ties. \(h_{alt}\) allocates to each \(s_i \in S\) a time \(t_i\), proportional to the instances that \(S\) can solve in \(N(p, k)\), while \(h_{sch}\) sorts the solvers by increasing solving time in \(N(p, k)\). For COPs the approach is analogous, but different performance metrics are used [Amadini et al., 2014b].

As an example, let \(p\) be a CSP, \(\Pi = \{s_1, s_2, s_3, s_4\}\) a portfolio of 4 solvers, \(T = 1800\) seconds the solving timeout, \(N(p, k) = \{p_1, ..., p_4\}\) the \(k = 4\) neighbours of \(p\), and the runtimes of solver \(s_1\) on problem \(p_j\), defined as in Table 1. In this case, the smallest sub-portfolios that solve the most instances are \(\{s_1, s_2, s_3\}\), \(\{s_1, s_2, s_4\}\), and \(\{s_3, s_3, s_4\}\). The heuristic \(h_{alt}\) selects \(S = \{s_1, s_2, s_4\}\) because these solvers are faster in solving the instances in the neighbourhood. Since \(s_1\) and \(s_4\) solve 2 instances and \(s_2\) solves 1 instance, \(h_{alt}\) splits the solving time window \([0, T]\) into 5 slots: 2 allocated to \(s_1\) and \(s_4\), and 1 to \(s_2\). Finally, \(h_{sch}\) sorts the solvers by increasing solving time. The final schedule produced by SUNNY is therefore \(\sigma = [(s_4, 720), (s_1, 720), (s_2, 360)]\).

\[
\begin{array}{|c|c|c|c|c|}
\hline
& P_1 & P_2 & P_3 & P_4 \\
\hline
s_1 & T & 5 & 278 & \\
\hline
s_2 & 593 & T & T & \\
\hline
s_3 & T & 36 & 1452 & T \\
\hline
s_4 & T & 122 & 60 & \\
\hline
\end{array}
\]

Table 1: Runtimes (in seconds). \(T\) means the solver timeout.

For a detailed description and evaluation of SUNNY we refer the reader to [Amadini et al., 2014c] [Amadini et al., 2014b]. Here we focus instead on how we generalized SUNNY for a multicore setting where \(c \geq 1\) cores are available and, typically, we have more solvers than cores. sunny-cp2 uses a dynamic approach: the schedule is decided on-line according to the instance to be solved. The approach used is rather simple: starting from the sequential schedule \(\sigma\) produced by SUNNY on a given problem, we first detect the \(c-1\) solvers of \(\sigma\) having the largest allocated time, and we assign a different core to each of them. The other solvers of \(\sigma\) (if any) are scheduled on the last core, by preserving their original order in \(\sigma\) and by widening their allocated times to cover the entire time window \([0, T]\).

Formally, given \(c\) cores and the sequential schedule \(\sigma = [(s_1, t_1), ..., (s_n, t_n)]\) computed by SUNNY, we define a total order \(\prec_{\sigma}\) such that \(s_i \prec_{\sigma} s_j \iff t_i > t_j \land 1 < j\), and a ranking function \(\text{rank}_{\sigma}\) such that \(\text{rank}_{\sigma}(s) = i\) if and only if \(s\) is the \(i\)-th solver of \(\sigma\) according to \(\prec_{\sigma}\). We define the parallelisation of \(\sigma\) on \(c\) cores as a function \(P_{\sigma,c}\) which associates to each core \(i \in \{1, ..., c\}\) a sequential schedule \(\Pi_{\sigma,c}(i)\) such that:

\[
\Pi_{\sigma,c}(i) = \begin{cases} \emptyset & \text{if } i > n \\ [(s, T)] & \text{if } i = \text{rank}_{\sigma}(s) \land 1 \leq i < c \\ [(s, T_c)] & \text{if } i = c \end{cases}
\]

where \(T_c = \sum \{t \mid (s, t) \in \sigma \land \text{rank}_{\sigma}(s) \geq c\}\).

Let us consider the schedule of the previous example: \(\sigma = [(s_4, 720), (s_1, 720), (s_2, 360)]\). With \(c = 2\) cores, the parallelisation of \(\sigma\) is defined as \(P_{\sigma,2}(1) = [(s_4, 1800)]\) and \(P_{\sigma,2}(2) = [(s_1, 1800/1080 \times 720), (s_2, 1800/1080 \times 360)]\).

The rationale behind the schedule parallelisation \(P_{\sigma,c}\) is that SUNNY aims to allocate more time to the most promising solvers, scheduling them according to their speed. Assuming a good starting schedule \(\sigma\), its parallelisation \(P_{\sigma,c}\) is rather straightforward and it is easy to prove that, assuming an independent execution of the solvers without synchronization and memory contention issues, \(P_{\sigma,c}\) can never solve less problems than \(\sigma\).

Obviously, different ways to parallelise the sequential schedule are possible. Here we focus on what we think to be one of the most simple yet promising ones. An empirical comparison of other parallelisation methods is outside the scope of this paper.

3 SUNNY-CP 2

sunny-cp2 solves both CSPs and COPs encoded in the MiniZinc language [Nethercote et al., 2007], nowadays the de-facto standard for modelling CP problems. By default, sunny-cp2 uses a portfolio \(\Pi\) of 12 solvers disparate in their nature. In addition to the 8 solvers of sunny-cp (viz., Chuffed, CPX, G12/CBC, G12/FD, G12/LazyFD, G12/Gurobi, Gecode, and MinisatID) it includes new, state-of-the-art solvers able to win four gold medals in the MZC 2014, namely: Choco, HaifaCSP, iZplus, and OR-Tools.

Figure 1 summarizes the execution flow of sunny-cp2. It takes as input a problem \(p\) to be solved and a list of input parameters specified by the user (e.g., the timeout \(T\) used by SUNNY, the number of cores \(c\) to use, etc.). If a parameter is not specified, a corresponding default value is used.
The solving process then relies on two sequential steps, later detailed: (i) the pre-solving phase, where a static schedule of solvers may be run and the instance neighbourhood is computed; (ii) the solving phase, which runs the dynamic schedule on different cores.

If \( p \) is a CSP, the output of sunny-cp2 can be either SAT (a solution exists for \( p \)), UNS (\( p \) has no solutions), or UNB (sunny-cp2 can not say anything about \( p \)). In addition, if \( p \) is a COP, two more answers are possible: OPT (sunny-cp2 proves the optimality of a solution), or UNB (\( p \) is unbounded).

### 3.1 Pre-solving

The pre-solving step consists in the simultaneous execution on \( c \) cores of two distinct tasks: the execution of a (possibly empty) static schedule, and the neighbourhood detection.

Running for a short time a static schedule has proven to be effective. It enables to solve those instances that some solvers can solve very quickly (e.g., see [Kadioglu et al., 2011]) and, for COPs, to quickly find good sub-optimal solutions [Amadini and Stuckey, 2014].

The static schedule \( \sigma = [(s_1,t_1), \ldots, (s_n,t_n)] \) is a prefixed schedule of \( n \geq 0 \) solvers decided off-line, regardless of the input problem \( p \). To execute it, sunny-cp2 applies a “First-Come, First-Served” policy by following the schedule order. Formally, the first \( m \) solvers \( s_1, \ldots, s_m \) with \( m = \min(c,n) \) are launched on different cores. Then, for \( i = m + 1, \ldots, n \), the solver \( s_i \) is started as soon as a solver \( s_j \) (with \( 1 \leq j \leq i \)) terminates its execution without solving \( p \). If a solver \( s_i \) fails prematurely before \( t_i \) (e.g., due to memory overflows or unsupported constraints) then \( s_i \) is discarded from the portfolio for avoiding to run it again later. If instead \( s_i \) reached its timeout, then it is just suspended: if it has to run again later, it will be resumed instead of restarted from scratch. The user has the possibility of setting the static schedule as an input parameter. For simplicity, the static schedule of sunny-cp2 is empty by default.

When solving COPs, the timeout defined by the user may be overruled by sunny-cp2 that examines the solvers behaviour and may decide to delay the interruption of a solver. Indeed, a significant novelty of sunny-cp2 is the use of a waiting threshold \( T_w \). A solver \( s \) scheduled for \( t \) seconds is never interrupted if it has found a new solution in the last \( T_w \) seconds, regardless of whether the timeout \( t \) is expired or not. The underlying logic of \( T_w \) is to stop a solver which is actively producing solutions. Thus, if \( T_w > 0 \) a solver may be executed for more than its allocated time. Setting \( T_w \) to a high value might therefore delay, and thus hinder, the execution of the other solvers.

For COPs, sunny-cp2 also enables the bounds communication between solvers: the sub-optimal solutions found by a solver are used to narrow the search space of the other scheduled solvers. In Amadini and Stuckey, 2014 this technique is successfully applied in a sequential setting, where the best objective bound found by a solver is exploited by the next scheduled ones. Things get more complicated in a multicore setting, where solvers are run simultaneously as black boxes and there is no support for communicating bounds to another running solver without interrupting its solving process. We decided to overcome this problem by using a restarting threshold \( T_r \). If a running solver \( s \) finds no solution in the last \( T_r \) seconds, and its current best bound \( v \) is obsolete w.r.t. the overall best bound \( v' \) found by another solver, then \( s \) is restarted with the new bound \( v' \). The reason behind \( T_r \) is that restarting an active solver is risky and potentially harmful even when its current best bound is actually obsolete. However, also ignoring the solutions found by other solvers could mean to neglect valuable information. The choice of \( T_r \) is hence critical: a too small value might cause too many restarts, while a big threshold inhibits the bound communication. Care should be taken also because restarting a solver means to lose all the knowledge it gained during the search.

After performing some empirical investigations, we found reasonable to set the default values of \( T_w \) and \( T_r \) to 2 and 5 seconds respectively. The user can however configure such parameters as she wishes, even by defining a customized setting for each different solver of the portfolio.

The neighbourhood detection phase begins in parallel with the static schedule \( \sigma \) execution only after all the solvers of \( \sigma \) has been started. This phase has lower priority than \( \sigma \), since its purpose is not quickly finding solutions, but detecting the neighbourhood \( N(p,k) \) later on used to compute the dynamic schedule. For this reason, the computation of the neighbourhood starts as soon as one core is free (i.e., the number of running solvers is smaller than \( c \)). The first step of pre-scheduling is the extraction of the feature vector of \( p \), i.e., a collection of numerical attributes (e.g., number of variables, constraints, etc.) that characterizes the instance \( p \). The feature vector is then used for detecting the set \( N(p,k) \) of the \( k \) nearest neighbours of \( p \) within a dataset of known instances.

The default dataset \( \Delta \) of sunny-cp2 is the union of a set \( \Delta_{\text{CSP}} \) of 5527 CSPs and a set \( \Delta_{\text{COP}} \) of 4988 COPs, retrieved from the instances of the MiniZinc 1.6 benchmarks, the MZCs 2012–2014, and the International CSP Solver Competitions 2008/09. sunny-cp2 provides a default knowledge base that associates to every instance \( p_i \in \Delta \) its feature vector and the runtime information of each solver of the portfolio on \( p_i \) (e.g., solving time, anytime performance, etc.). In particular, the feature vectors are used for computing \( N(p,k) \) while the runtime information are later used for computing the dynamic schedule \( \sigma \) by means of SUNNY algorithm.

The feature extractor used by sunny-cp2 is the same as mzn2feat1. The neighbourhood size is set by default to \( k = 70 \); following Dada et al., 2000 we chose a value of \( k \) close to \( \sqrt{n} \), where \( n \) is the number of training samples. Note that during the pre-solving phase only \( N(p,k) \) is computed. This is because SUNNY requires a timeout \( T \) for computing the schedule \( \sigma \). The total time taken by the pre-solving phase (\( C \) in Figure 1) must therefore be subtracted from the initial timeout \( T \) (by default, \( T = 1800 \) seconds as in sunny-cp).

This can be done only when the pre-solving ends, since \( C \) is not predictable in advance.

The pre-solving phase terminates when either: (i) a solver of the static schedule \( \sigma \) solves \( p \), or (ii) the neighbourhood

1 The feature extractor mzn2feat is available at [https://github.com/CP-Unibo/mzn2feat](https://github.com/CP-Unibo/mzn2feat) For further information please see Amadini et al., 2014a and Amadini et al., 2015.
computation and all the solvers of $\sigma$ have finished their execution. In the first case sunny-cp2 outputs the solving outcome and stops its execution, skipping the solving phase since the instance is already solved.

3.2 Solving

The solving phase receives in input the time $C$ taken by pre-solving and the neighbourhood $N(p, k)$. These parameters are used by SUNNY to dynamically compute the sequential schedule $\sigma$ with a reduced timeout $T - C$. Then, $\sigma$ is parallelised according to the $P_{\sigma,c}$ operator defined in Section 3.1.

Once computed $P_{\sigma,c}$, each scheduled solver is run on $p$ according to the input parameters. If a solver of $P_{\sigma,c}$ had already been run in the static schedule $\sigma$, then its execution is resumed instead of restarted. If $p$ is a COP, the solvers execution follows the approach explained in Section 3.1 according to $T_w$ and $T_r$ thresholds. As soon as a solver of $\sigma$ solves $p$, the execution of sunny-cp2 is interrupted and the solving outcome is outputted.

Note that we decided to make sunny-cp2 an anytime solver, i.e., a solver that can run indefinitely until a solution is found. For this reason we let the solvers run indefinitely even after the timeout $T$ until a solution is found. Moreover, at any time we try to have exactly $c$ running solvers. In particular, if $c$ is greater or equal than the portfolio size no prediction is needed and we simply run a solver per core for indefinite time. When a scheduled solver fails during its execution, the overall solving process is not affected since sunny-cp2 will simply run the next scheduled solver, if any, or another one in its portfolio otherwise.

Apart from the scheduling parallelisation, sunny-cp2 definitely improves sunny-cp also from the engineering point of view. One of the main purposes of sunny-cp2 is indeed to provide a framework which is easy to use and configure by the end user. First of all, sunny-cp2 provides utilities and procedures for adding new constituent solvers to $H$ and for customizing their settings. Even though the feature extractor of sunny-cp2 is the same as sunny-cp, the user can now define and use her own tool for feature processing. The user can also define her own knowledge base, starting from raw comma-separated value files, by means of suitable utility scripts. Furthermore, sunny-cp2 is much more parametrisable than sunny-cp. Apart from the aforementioned $T_w$ and $T_r$ parameters, sunny-cp2 provides many more options allowing, e.g., to set the number $c$ of cores to use, limit the memory usage, ignore the search annotations, and specify customized options for each constituent solver.

The source code of sunny-cp2 is entirely written in Python and publicly available at [https://github.com/CP-Unibo/sunny-cp](https://github.com/CP-Unibo/sunny-cp).

4 Validation

We validated the performance of sunny-cp2 by running it with its default settings on every instance of $\Delta_{CSP}$ (5527 CSPs) and $\Delta_{COP}$ (4988 COPs). Following the standard practices, we used a 10-fold cross-validation: we partitioned each dataset in 10 disjoint folds, treating in turn one fold as test set and the union of the remaining folds as the training set.

Fixed a solving timeout $T$, different metrics can be adopted to measure the effectiveness of a solver $s$ on a given problem $p$. If $p$ is a CSP we typically evaluate whether, and how quickly, $s$ can solve $p$. For this reason we use two metrics, namely proven and time, to measure the solved instances and the solving time. Formally, if $s$ solves $p$ in $t < T$ seconds, then proven($s, p$) = 1 and time($s, p$) = $t$; otherwise, proven($s, p$) = 0 and time($s, p$) = $T$. A straightforward generalization of these two metrics for COPs can be obtained by setting proven($s, p$) = 1 and time($s, p$) = $t$ if $s$ proves in $t < T$ seconds the optimality of a solution for $p$, the unsatisfiability of $p$ or its unboundedness. Otherwise, proven($s, p$) = 0 and time($s, p$) = $T$. However, a major drawback of such a generalization is that the solution quality is not addressed. To overcome this limitation, we use in addition the score and area metrics introduced in [Amadini and Stuckey, 2014].

The score metric measures the quality of the best solution found by a solver $s$ at the stroke of the timeout $T$. More specifically, score($s, p$) is a value in $[0,25,0,75]$ linearly proportional to the distance between the best solution that $s$ finds and the best known solution for $p$. An additional reward (score = 1) is given if $s$ is able to prove optimality, while a punishment (score = 0) is given if $s$ does not provide any answer. Differently from score, the area metric evaluates the behaviour of a solver in the whole solving time window $[0, T]$ and not only at the time edge $T$. It considers the area under the curve that associates to each time instant $t_i \in [0, T]$ the best objective value $v_i$ found by $s$ in $t_i$ seconds. The area value is properly scaled in the range $[0, T]$ so that the more a solver is slow in finding good solutions, the more its area is high (in particular, area = $T$ if and only if score = 0). The area metric folds in a number of measures the quality of the best solution found, how quickly any solution is found, whether optimality is proven, and how quickly good solutions are found. For a formal definition of area and score we refer the reader to [Amadini and Stuckey, 2014].

We ran sunny-cp2 on all the instances of $\Delta_{CSP}$ and $\Delta_{COP}$ within a timeout of $T = 1800$ seconds by varying the number $c$ of cores in $\{1, 2, 4, 8\}$. We compared sunny-cp2 against the very same version of sunny-cp submitted to MZC 2011 and the Virtual Best Solver (VBS), i.e., the oracle portfolio solver that —for a given problem and performance metric— always selects the best solver to run. Moreover, for each $c \in \{1, 2, 4, 8\}$ we consider as additional baseline the Virtual $c$-Parallel Solver (VPS), i.e., a static portfolio solver that always runs in parallel a fixed selection of the best $c$ solvers of the portfolio $H$. The VPS is a static portfolio since its solvers are selected in advance, regardless of the problem $p$ to be solved. Obviously, the portfolio composition of VPS$_c$ depends on a given evaluation metric. For each $\mu \in \{\text{proven, time, score, area}\}$ we therefore defined a corresponding VPS$_c$ by selecting the best $c$ solvers of $H$ according to the average value of $\mu$ over the $\Delta_{CSP}$ or $\Delta_{COP}$ instances. Note that VPS$_c$ is an “ideal” solver, since
it is an upper bound of the real performance achievable by actually running in parallel all its solvers. An approach similar to VPS, has been successfully used by the SAT portfolio solver ppfolio [Roussel, 2011]. The VPS is sometimes also called Single Best Solver (SBS) while VPS[H] is exactly equivalent to the Virtual Best Solver (VBS).

In the following, we will indicate with sunny-cp2[c] the version of sunny-cp2 exploiting c cores. Empirical results on CSPs and COPs are summarized in Table 2 and 3 reporting the average values of all the aforementioned evaluation metrics. On average, sunny-cp2 remarkably outperforms all its constituent solvers as well as sunny-cp for both CSPs and COPs. Concerning CSPs, a major boost is given by the introduction of HaifaCSP solver in the portfolio. HaifaCSP solves almost 90% of ΔCSP problems. However, as can be seen from the Tables 2 and 3, sunny-cp2[H] solves more than 95% of the instances and, for c > 1, sunny-cp2[c] solves nearly all the problems. This means that just few cores are enough to almost reach the VBS performance. The best proven performance is achieved by sunny-cp2[8]. Nevertheless, the average time of sunny-cp2[8] is slightly slower. However, difference are minimal: sunny-cp2[4] and sunny-cp2[8] are virtually equivalent. Note that sunny-cp2[c] is always better than VPS, in terms of proven and, for c < 8, also in terms of time. Furthermore, sunny-cp2[c] solves more instances than VPS4. In other words, running sequentially the solvers of sunny-cp2 on a single core allows to solve more instances than running independently the four solvers of the portfolio that solve the most number of problems of ΔCSP. Analogously, sunny-cp2[4] solves more instances than VPS4. This witnesses that in a multicore setting — where there are typically more available solvers than cores— sunny-cp2 can be very effective.

Even better results are reached by sunny-cp2 on the instances of ΔCOP, where there is not a clear dominant solver like HaifaCSP for ΔCSP. sunny-cp2 outperforms sunny-cp in all the considered metrics, and the gain of performance in this case is not only due to the introduction of new solvers. Indeed, the best solver according to time and proven is Chuffed, which was already present in sunny-cp. For each c ∈ {1, 2, 4, 8} and for each performance metric, sunny-cp2[c] is always better than the corresponding VPS[c]. Moreover, as for CSPs, sunny-cp2 has very good performances even with few cores. For example, by considering the proven, time, and score results, sunny-cp2[H] is always better than VPS and its score even outperforms that of VPS[H]. This clearly indicates that the dynamic selection of solvers, together with the bounds communication and the waiting/restarting policies implemented by sunny-cp2, makes a remarkable performance difference.

Results also show a nice monotonicity: for all the considered metrics, if c > c′ then sunny-cp2[c] is better than sunny-cp2[c′]. In particular, it is somehow impressive to see that on average sunny-cp2 is able to outperform the VBS in terms of time, proven, and area, and that for score the performance difference is negligible (0.01%). This is better shown in Figure 2 depicting for each performance metric the number of times that sunny-cp2 is able to overcome the performance of the VBS with c = 1, 2, 4, 8 cores. As c increases, the number of times the VBS is beaten gets bigger. In particular, the time and area results show that sunny-cp2 can be very effective to reduce

| Metric       | sunny-cp | sunny-cp2 | VPS     | VBS     |
|--------------|----------|-----------|---------|---------|
| | 1 core | 2 cores | 4 cores | 8 cores | 1 core | 2 cores | 4 cores | 8 cores |
| time (s)     | 95.26    | 99.11     | 99.38   | 99.35   | 89.04  | 93.51    | 95.19   | 99.24   | 100    | 54.30 |
| area (s)     | 223.21   | 136.04    | 112.60  | 112.32  | 297.30 | 211.13   | 176.81  | 68.52   | 132.28 |
| score * 100  | 504.10   | 112.32    | 112.32  | 112.32  | 112.32 | 112.32   | 112.32  | 112.32  | 112.32  |

Table 2: Experimental results over CSPs.

| Metric       | sunny-cp | sunny-cp2 | VPS     | VBS     |
|--------------|----------|-----------|---------|---------|
| | 1 core | 2 cores | 4 cores | 8 cores | 1 core | 2 cores | 4 cores | 8 cores |
| time (s)     | 71.55    | 74.40     | 75.68   | 76.34   | 61.33  | 63.45    | 65.60   | 75.86   | 76.30  | 457.00 |
| score * 100  | 594.79   | 501.35    | 482.95  | 469.74  | 545.54 | 718.86   | 682.35  | 645.68  | 463.63 |
| area (s)     | 90.50    | 92.26     | 93.00   | 93.45   | 93.62  | 82.80    | 85.99   | 89.53   | 90.79  | 93.63  |

Table 3: Experimental results over COPs.
the solving time both for completing the search and, especially, for quickly finding sub-optimal solutions. For example, for 414 problems sunny-cp2[9] has a lower time than VBS, while for 977 instances it has a lower area. Table [4] reports a clear example of the sunny-cp2 potential on an instance of the Resource-Constrained Project Scheduling Problem (RCRSP) [Bruker et al., 1999] taken from the MiniZinc-1.6 suite. Firstly, note that just half of the solvers of II can find at least a solution. Second, these solvers find their best bound \( b_{\text{best}} \) very quickly (i.e., in a time \( t_{\text{best}} \) of between 1.46 and 4.05 seconds) but none of them is able to prove the optimality of the best bound \( v^* = 958 \) within a timeout of \( T = 1800 \) seconds. Conversely, sunny-cp2 finds \( v^* \) and proves its optimality in less than 11 seconds. Exploiting the fact that CPX finds \( v^* \) in a short time (for CPX \( t_{\text{best}} = 2.42 \) seconds, while sunny-cp2 takes 5 seconds due to the neighbourhood detection and scheduling computation), after \( T_s = 5 \) seconds any other scheduled solver is restarted with the new bound of \( v^* \). Now, Gecode can prove almost instantaneously that \( v^* \) is optimal, while without this help it can not even find it in half an hour of computation.

### Table 4: Benefits of sunny-cp2 on a RCPSP instance. \( T = 1800 \) indicates the solvers timeout.

| Solver   | Sunny-cp2 | Choco | Chuffed | CPX | G12/FD | Gecode | OR-Tools | Other solvers |
|----------|-----------|-------|---------|-----|--------|--------|----------|--------------|
| \( t_{\text{best}} \) (s) | 2.42 | 4.05 | 2.42 | 2.87 | 3.31 | 1.46 | N/A | 5.00 |
| \( v_{\text{best}} \) | 958 | 959 | 2.87 | 959 | 959 | 959 | N/A | 10.56 |
| time     | T | T | T | T | T | T | T | 508 |

Apart from the aforementioned CPHydra and sunny-cp2 solvers, another portfolio approach for CSPs is Proteus [Hutter et al., 2014]. However, with the exception of a preliminary investigation about a CPHydra parallelisation [Yun and Epstein, 2012], all these solvers are sequential and except for sunny-cp2 they solve just CSPs. Hence, to the best of our knowledge, sunny-cp2 is today the only parallel and dynamic CP portfolio solver able to deal with also optimization problems.

The parallelisation of portfolio solvers is a hot topic which is drawing some attention in the community. For instance, parallel extensions of well-known sequential portfolio approaches are studied in [Hoos et al., 2015b]. In [Hoos et al., 2015a] ASP techniques are used for computing a static schedule of solvers which can even be executed in parallel, while [Cire et al., 2014] considers the problem of parallelising restarted backtrack search for CSPs.

### 6 Conclusions

In this paper we introduced sunny-cp2, the first parallel CP portfolio solver able to dynamically schedule the solvers to run and to solve both CSPs and COPs encoded in the MiniZinc language. It incorporates state-of-the-art solvers, providing also a usable and configurable framework.

The performance of sunny-cp2, validated on heterogeneous and large benchmarks, is promising. Indeed, sunny-cp2 greatly outperforms all its constituent solvers and its earlier version sunny-cp. It can be far better than a ppfolio-like approach [Roussel, 2011] which statically determine a fixed selection of the best solvers to run. For CSPs, sunny-cp2 almost reaches the performance of the Virtual Best Solver, while for COPs sunny-cp2 is even able to outperform it.

We hope that sunny-cp2 can stimulate the adoption and the dissemination of CP portfolio solvers. Indeed, sunny-cp was the only portfolio entrant of MiniZinc Challenge 2014. We are interested in submitting sunny-cp2 to the 2015 edition in order to compare it with other possibly parallel portfolio solvers.

There are many lines of research that can be explored, both from the scientific and engineering perspective. As a future work we would like to extend the sunny-cp2 by adding new, possibly parallel solvers. Moreover, different parallelisations, distance metrics, and neighbourhood sizes can be evaluated.

Given the variety of parameters provided by sunny-cp2, it could be also interesting to exploit Algorithm Configuration techniques [Hutter et al., 2011] [Kadioglu et al., 2010] for the automatic tuning of the sunny-cp2 parameters, as well as the parameters of its constituent solvers.

4The model is rcpsp.mzn while the data is in la10.x2.dzn
Finally, we are also interested in making sunny-cp2 more usable and portable, e.g., by pre-installing it on virtual machines or multi-container technologies.

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