\( \Lambda(1405) \) as a Resonance in the Baryon-Meson Scattering Coupled to the \( q^3 \) State in a Quark Model

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In order to describe \( \Lambda(1405) \) as a resonance in the baryon-meson scattering, we have investigated \( q^3-qq \) scattering system with the flavor-singlet \( q^3 (0s)^2(0p) \) state (the \( \Lambda^1 \) pole). The \( \Lambda^1 \) pole is treated as a bound state embedded in the continuum. We found that the peak appears below the \( NNK \) threshold in the spin \( \frac{1}{2} \), isospin \( 0 \) channel even if the mass of the \( \Lambda^1 \) pole is above the threshold. This peak disappears when the coupling to the \( \Lambda^1 \) pole is switched off. To use the observed hadron mass in the kinetic part of QCM is also found to be important to reproduce a peak just below the \( NNK \) threshold.

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I. INTRODUCTION

Constituent quark models are known to reproduce the observed low-energy features of baryons. The number of ground state baryons corresponds to the observed one. The masses and magnetic moments of the flavor-octet and decuplet baryons are also reproduced well. The picture that the baryon mass comes mainly from the masses of the three constituent quarks, which interact with each other by exchanging gluons and/or Goldstone bosons, seems appropriate.\[1, 2, 3, 4, 5, 6\].

There, however, are some exceptions. One of these is that such models cannot give the observed light mass of \( \Lambda(1405) \) nor the large mass difference between \( \Lambda(1405, \frac{1}{2}^-) \) and \( \Lambda(1520, \frac{3}{2}^-) \). In the conventional constituent quark model, each of these baryons is treated as a system of three quarks in the flavor-singlet \( (0s)^2(0p) \) state \[1\]. Then, the hyperfine interaction only gives a part (\( \sim 150 \text{ MeV} \)) of the observed mass difference between the flavor-singlet and the flavor-octet spin-\( \frac{1}{2} \) baryons, \( \sim 200 \text{ MeV} \). Moreover, in order to give the large mass difference between \( \Lambda(1405) \) and \( \Lambda(1520) \), one has to assume a strong spin-orbit force between quarks, which is absent in other negative-parity baryons.

The excited baryons are embedded in the baryon-meson scattering states. Therefore it is most appropriate to investigate \( \Lambda(1405) \) as a meson-baryon resonance. In ref. \[7\], the idea that \( \Lambda(1405) \) is a \( NNK \) bound state has been presented. A more systematic treatment may be found in the chiral unitary approach, where the lowest order baryon-meson vertices of the nonlinear chiral Lagrangian are employed as the baryon-meson interaction \[8, 9, 10, 11, 12\]. Because the model Lagrangian is an flavor-SU(3) extended version, this interaction is flavor-flavor type; the relative strength of the potentials among the SU(3) octet baryons and mesons is governed by the SU(3) relation. The interaction is very short-ranged and strongly attractive in both of the \( \Sigma \pi \) and \( \Sigma K (TJ) = (0\frac{1}{2}) \) channels. Furthermore, it is easily seen that this flavor-flavor type interaction gives the strongest attraction for the flavor-singlet state among the baryon-meson system. This is essentially the origin to produce the \( \Lambda(1405) \) resonance in this scheme.

There are also some attempts to express this baryon-meson picture by using quark models. To treat the negative-parity baryons as \( q^3q \) systems was proposed in ref. \[13\]. This idea, however, has been abandoned because it was found by the \( q^3q (0s)^5 \) calculations that some of the other flavor-octet negative-parity baryons are also found to have light mass. There is a report to examine the \( NNK \) scattering from a quark model \[14\]. Recent progress in the calculation method for few-body systems allows us to solve a \( q^3q \) system by taking the orbital correlation fully into account. Then, it is found that the one-gluon exchange potential (OGE) \[15\], which is strongly attractive in the \( \Sigma \pi \) channel,
causes a bound state below the $\Sigma \pi$ threshold in this $(TJ) = (0\frac{1}{2})$ channel \cite{16}. Since there is no strong attraction from color-magnetic interaction (CMI) in the $N\bar{K}$ channel, it seems unlikely to have a resonance close to but below the $N\bar{K}$ threshold in this approach. To explain $\Lambda(1405)$ by the baryon-meson picture directly using the quark models does not seem to work so far.

An example to combine the above two pictures, viz., the $q^3$ picture and the baryon-meson one, is found in ref. \cite{17}, where the meson-cloud effects on the flavor-singlet $q^3$ baryons were investigated. It was reported that the self-energy of the $q^3$ state makes the baryon mass considerably lower. It seems necessary to introduce such a $q^3$ state, in addition to the baryon-meson state, for the constituent quark models to express the negative-parity baryons. In this paper, in order to clarify the above situation of $\Lambda(1405)$, we perform the dynamical calculations of the $q^3$-$q\bar{q}$ scattering systems with a flavor-singlet $q^3 (0s)^2 0p$ state as an embedded pole in the continuum, which we call ‘the $\Lambda^1$ pole’ in the following. For this purpose, we employ the quark cluster model (QCM), which has succeeded in describing the baryon-baryon scattering quantitatively, and study the baryon-meson scattering in this paper \cite{18, 19, 20}. There are two points which are taken into account especially in this baryon-meson scattering problem. One of them, as was mentioned above, is a coupling to the $\Lambda^1$ pole in addition to the usual coupled-channel baryon-meson scattering calculation. The other one is a modification of the relative kinetic energy of QCM. In the nonrelativistic scheme of the quark models, the reduced mass of the baryon-meson system is $\mu = 6m_q/5$ with the constituent quark mass, $m_q$. When we study the $\Lambda(1405)$ in terms of the baryon-meson scattering, the most important channel is $\Sigma \pi$ channel. In reality, it has much smaller reduced mass, 124 MeV, compared with that of QCM, which is roughly 400 MeV. The kinetic energy is underestimated severely. To remove this, we replace the reduced mass in QCM by the real one; we have found that to take this effects into account is essential to discuss the baryon-meson scatterings by the quark model.

As for the interaction between quarks, we follow the most common approach based on OGE and linear confinement. This model nicely reproduces the SU(3) octet and decuplet baryons. In order to describe the SU(3) octet and singlet meson spectra, we include the instanton induced interaction (III) \cite{21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34}. We also introduce a purely phenomenological potential to lower the pseudoscalar SU(3) octet mesons, which is considered to express the collective nature of the Goldstone bosons. Employing this model, we can reproduce the observed baryon and meson masses well after an assumption for the zero-point energy is introduced. We use this ‘OGE model’ rather than the ‘chiral model,’ in which the quarks interact with each other by exchanging the Goldstone bosons. The OGE model may not produce the long-range behavior of the system with a lack of meson-exchange between the baryon and meson. We, however, use this model as a first step, because it is very difficult for the chiral model to reproduce the meson spectrum, and because it is not clear how to remove a possible overcounting from the treatment which handles the scattering $q\bar{q}$ mesons and the exchanged mesons simultaneously if we use the chiral model for the baryon-meson scattering problem.

In the next section, we explain the model hamiltonian. Then in section 3, a brief summary of QCM for baryon-meson scattering will be given. There we explain how to modify the relative kinetic energy between baryon and meson in the quark cluster model, and how to take into account the $\Lambda^1$ pole in QCM. Numerical results for the coupled channel $\Sigma \pi$ and $N\bar{K}$ scattering and discussion are given in section 4. Summary is given in section 5.

II. HAMILTONIAN

We have employed a valence quark model. The hamiltonian is taken as:

\[ H_g = \sum_i \left( \frac{p_i^2}{2m_i} + V_0 \right) - K_G + \sum_{i<j} (V_{\text{OGE}ij} + V_{\text{Conf}ij}) + \sum_{i=1}^4 (V_{\text{OGE}i5}^{(q)} + V_{\text{coll}i5}) + V_{\text{INS}}. \]  

Here, $K_G$ is the kinetic energy from the center of mass motion. We take the zero-point energy, which is (the number of quarks) $\times V_0$. \hspace{1cm} (1)
The two-body potential term consists of the one-gluon-exchange potential, $V_{OGE}$ and $V_{\text{Conf}}$, which are defined as:

$$V_{OGEij} = \left( \frac{1}{4} \alpha_s \left( \frac{2m_i^2}{r_{ij}^2} + \frac{2m_j^2}{r_{ij}^2} + \frac{2\pi}{3m_i m_j} \delta(r_{ij}) \right) \right),$$

(2)

$$V_{\text{Conf}ij} = -\left( \frac{1}{4} \alpha_s \right) a_{\text{Conf}} r_{ij},$$

(3)

where $\lambda_i$, $\sigma_i$, $m_i$, and $r_{ij}$ are color SU(3) generator, Pauli spin operator, the mass for the $i$-th quark, and the distance between the $i$-th and $j$-th quarks, respectively. The $\alpha_s$ and $a_{\text{Conf}}$ are the strong coupling constant and the confinement strength. $V_{OGE55}$ is the term due to the pair annihilation, and $V_{\text{coll}}$ is introduced to act only on the color-singlet, pseudoscalar SU(3) octet mesons:

$$V_{OGE55}^{(4)} = \frac{1}{24} \left( \frac{16}{3} - \lambda_i \cdot \lambda_5 \right) \left( 3 + \sigma_i \cdot \sigma_5 \right) P_{i5} \pi \alpha_s \left( \frac{1}{4m_u} \delta(r_{i5}) \right),$$

(4)

$$V_{\text{coll}5} = \frac{1}{24} \left( \frac{16}{3} \right) f_i \cdot f_5 \left( \frac{1}{9} + \frac{\lambda_i \cdot \lambda_5}{6} \right) \delta(r_{ij}),$$

(5)

where $f_i$ is the flavor SU(3) generator for the $i$-th quark, and the antiquark is denoted as the 5-th quark. The flavor operator $P_{i5}$ has a matrix element only between a flavor singlet $q\bar{q}$ state when $\xi_5 = 1$. $V_{\text{coll}}$, which acts on a spin-0 flavor-octet color-singlet $q\bar{q}$ pair, stands for the collective nature of Goldstone bosons. The form of this term is determined by purely phenomenologically.

$V_{\text{INS}}$ is the instanton-induced interaction which stands for the short-ranged nonperturbative effects of the gluon field:

$$V_{\text{INS}} = V_{\text{INS}}^{(2)} + V_{\text{INS}}^{(2a)} + V_{\text{INS}}^{(3)} + V_{\text{INS}}^{(3a)},$$

(6)

Here, $V_{\text{INS}}^{(2)} + V_{\text{INS}}^{(2a)}$ and $V_{\text{INS}}^{(3)} + V_{\text{INS}}^{(3a)}$ are the two- and three-body parts of $V_{\text{INS}}$, respectively. $V_{\text{INS}}^{(2a)}$ corresponds to the two-body term due to the pair annihilation, which gives the $\eta' - \eta$ mass difference. $V_{\text{INS}}^{(3a)}$ corresponds to the three-body term also due to the pair annihilation. Their explicit forms are

$$V_{\text{INS}}^{(2)} = \frac{V_0^{(2)}}{2} \sum_{i<j} \xi_i \xi_j \mathcal{P}_{\text{INS}} U_{ij}^{(2)} \delta^3(r_{ij})$$

(7)

$$V_{\text{INS}}^{(2a)} = -\frac{V_0^{(2)}}{2} \sum_{i} \mathcal{P}_{\text{INS}} P_{i5} U_{ij}^{(2)} \delta^3(r_{i5})$$

(8)

$$V_{\text{INS}}^{(3)} = \frac{V_0^{(3)}}{4} \sum_{i<j<k} \mathcal{P}_{\text{INS}} U_{ijk}^{(3)} \delta^3(r_{ij}) \delta^3(r_{ik})$$

(9)

$$V_{\text{INS}}^{(3a)} = -\frac{V_0^{(3)}}{4} \sum_{i<j} \mathcal{P}_{\text{INS}} U_{ij5}^{(3)} \delta^3(r_{ij}) \delta^3(r_{i5})$$

(10)

with $\xi_i = m_u/m_i$. $V_0^{(2)}$ [$V_0^{(3)}$] is the effective strength of the two- [three-] body part of the $V_{\text{INS}}$ potential, which are related each other by the quark condensate:

$$V_0^{(2)} = \langle \langle \bar{u}u \rangle \rangle \frac{1}{2 \xi_s} V_0^{(3)}.$$

(11)

Due to the projection operator in the flavor space, $\mathcal{P}_{\text{INS}}$, these terms do not vanish only if all the incoming quark lines have different flavor from each other: it allows $u\bar{u} \rightarrow d\bar{u}, s\bar{s}$, etc. for the two-body part, and $u\bar{u} \rightarrow u\bar{s}, etc.$ for the three-body part, but forbids $u\bar{u} \rightarrow u\bar{u}$ or $u\bar{u} \rightarrow u\bar{u}$ and so on.
We employ the gaussian form for these single baryon and meson wave functions: 

The meson wave function, $\phi$ where $\xi$ is the size of $V$ with $m$ where the leading term of OGE for this coupling. The term can be written as:

The single baryon wave function, $\phi$ where $\xi$ as follows:

We assume that the coupling between the scattering state and the $\Lambda$ is the quark mass in the annihilating $q$, is also given by a product of orbital, color (singlet), spin, and flavor parts:

The single baryon wave function, $\phi_B$, which is antisymmetrized in terms of quark exchanges is given by a product of orbital, color and spin-flavor parts:

The meson wave function, $\phi_M$, is also given by a product of orbital, color (singlet), spin, and flavor parts:

We employ the gaussian form for these single baryon and meson wave functions $\varphi(\xi)$.

where $\xi_B$ and $\xi_M$ are internal coordinates of baryon $B$ and meson $M$. The $\xi_B$ and $\xi_M$ are given by the $i$-th quark coordinates $r_i$ as follows:

$\xi_B = (\xi_1, \xi_2) = (r_1 - r_2, \frac{r_1 + r_2}{2} - r_3)$, 
$\xi_M = (r_4 - r_5)$. 

FIG. 1: Annihilation diagram of the $j$-th quark and the $\bar{k}$-th antiquark by OGE, where the gluon carries three-momentum $k = p_j + \bar{p}_\tau$ to the $i$-th quark.

![Diagram](image-url)
TABLE I: Model parameters.

| $m_n$ | $m_s$ | $a_{\text{Conf}}$ | $\alpha_s$ | $V_0^{(2)}$ | $v_0$ | $V_0$ | $b$ |
|-------|-------|-------------------|------------|-------------|------|------|-----|
| (MeV) | (MeV) | (MeV fm$^3$)      | (MeV fm$^3$) | (MeV) | (MeV) | (fm) |     |
| 313   | 507   | 163.33            | 0.5040     | -94.10     | 9.7  | -468.49 | 0.49 |

TABLE II: Masses of baryons and mesons given by the present model. All entries are given in MeV.

| N   | $\Sigma$ | $\Xi$ | $\Lambda$ | $\pi$ | $K$ | $\eta$ | $\eta'$ | $\eta_{ud}$ | $\eta_s$ |
|-----|----------|------|----------|------|----|--------|--------|-----------|--------|
| Model | 939      | 1188 | 1308      | 1113 | 139| 487    | 552    | 931       |        |
| Exp. [35] | 938.9 | 1190.5 | 1318.1 | 1115.7 | 138.0 | 495.0 | 547.8 | 957.8 | -        |

The $g(r, b)$ is a normalized gaussian wave function with a size parameter $b$ given by

$$g(r, b) = (\sqrt{\pi}b)^{-3/2} \exp(-\frac{r^2}{2b^2}).$$

The parameter used in this work are shown in Table I. The calculated baryon and meson masses employing the above hamiltonian and wave functions are given in Table II. We assume the ideal mixing for $\eta$ in the coupled channel QCM, because the channel $\Lambda\eta$ is not open in the concerning energy region, and the u and d components must be important. The masses of $\eta_{ud} = (u \bar{u} + d \bar{d})/\sqrt{2}$ and $\eta_s = s \bar{s}$ are also be shown in Table II.

### III. QUARK CLUSTER MODEL

Here, we briefly explain the quark cluster model. A baryon consists of three quarks and a meson consists of quark and an antiquark, whose effective mass is about 300 MeV. The baryon-meson system, which consists of 4 quarks and 1 antiquark, is written as a product of these single baryon and meson wave functions $\phi_B(\xi_B)$ and $\phi_M(\xi_M)$ and the relative wave function $\chi(R_{BM})$ between these two clusters:

$$\Psi(\xi_B, \xi_M, R_{BM}) = A[\phi_B(\xi_B)\phi_M(\xi_M)\chi(R_{BM})].$$

The $A$ is an antisymmetrization operator among 4 quarks,

$$A = 1 - 3P_{34}.$$  

Here we assume that the baryon wave function is antisymmetrized.

Employing the known baryon and meson wave functions, $\phi_B(\xi_B)$ and $\phi_M(\xi_M)$, we obtain the following equation (RGM equation) to determine the relative wave function $\chi(R_{BM})$:

$$\int \left[ H_{\text{RGM}}(\mathbf{R}, \mathbf{R}') - E N_{\text{RGM}}(\mathbf{R}, \mathbf{R}') \right] \chi(\mathbf{R}') d\mathbf{R}' = 0,$$

where the hamiltonian $H_{\text{RGM}}$ and normalization $N_{\text{RGM}}$ kernels are given by

$$H_{\text{RGM}}(\mathbf{R}, \mathbf{R}') = \int \phi_B^{\dagger}(\xi_B)\phi_M(\xi_M)\delta(\mathbf{R} - \mathbf{R}_{BM}) \left\{ \begin{array}{c} H \\ 1 \end{array} \right\} \times A[\phi_B(\xi_B)\phi_M(\xi_M)\delta(\mathbf{R}' - \mathbf{R}_{BM})] d\xi_B d\xi_M dR_{BM}.$$ 

Here, $H$ is the total hamiltonian of the system where the center of mass kinetic energy is subtraced.

In the following, we explain how to modify the relative kinetic energy for the baryon-meson scattering problem. The RGM kernel $H$ and $N$ in eq. (17) consist of direct and exchange terms. We take out the direct term of the kinetic...
energy operator $K_D$ in the RGM kernel $H$. By subtracting the internal kinetic energy with the direct norm kernel $N_D$, the RGM kernel of the relative kinetic energy $K_R(R, R)$ is given by

$$K_R(R, R) = K_D(R, R) - 3K_0 \times N_D(R, R),$$

where $K_0 = \frac{3}{4m_q b^2}$.

Then the relative kinetic energy is modified in the following way.

$$K_R(R, R) \rightarrow K_R(R, R) \times \frac{6m_q}{5} \frac{1}{\mu},$$

where $\mu$ is the reduced mass for the baryon-meson system calculated by using the masses given in Table II.

The extension to the coupled-channel calculation is straightforward. The inclusion of the $\Lambda^1$ pole as a bound state embedded in the continuum is also straightforward. We add the pole at a certain energy to the baryon-meson wave function.

$$\Psi = \Psi(\xi_B, \xi_M, R_{BM}) + \Psi(q^3).$$

The coupling of the baryon-meson state with the $\Lambda^1$ pole is treated in the following way. First we calculate $\langle 0s^20p|V_{3q-5q}|0s^5 \rangle$ with $V_{3q-5q}$ given in eq.(15). This calculation can be done straightforwardly (see appendix A). Their values calculated by the present parameter set are listed in Table III. Note that the $|0s^5 \rangle$ corresponds to a state where the relative wave function $\chi(R_{BM})$ is a gaussian with the size parameter $B = \sqrt{5/6b}$ as follows:

$$\chi(R_{BM}) = g(R_{BM}, B = \sqrt{5/6b}).$$

The dependence on the relative distance $R_{BM}$ of the baryon-meson coupling is taken into account by multiplying an overlap with the above $|0s^5 \rangle$ state. This overlap is given by the norm kernel of the generator coordinate method (GCM), whose details are given in appendix B.

### IV. RESULTS AND DISCUSSION

In describing the $\Lambda(1405)$, we first consider the following SU(3) octet baryon and meson systems.

$$8_B \times 8_M = 1_{BM} + 8_{BM} + 8_{BM} + 10_{BM} + \overline{10}_{BM} + 27_{BM}.$$ 

The strangeness=$-1$ and isospin $T = 0$ state appears in the $1_{BM}, 8_{BM}$ and $27_{BM}$ states. These four states are given by a linear combination of the following four baryon-meson systems.

$$\Sigma \pi, \ \overline{N}K, \ \Lambda \eta, \ \Xi K.$$ 

For example, the flavor singlet state, $|1_{BM} \rangle$, is given by

$$|1_{BM} \rangle = \sqrt{\frac{3}{8}}|\Sigma \pi \rangle - \frac{1}{2}|\overline{N}K \rangle + \sqrt{\frac{1}{8}}|\Lambda \eta \rangle + \frac{1}{2}|\Xi K \rangle.$$
TABLE IV: Matrix elements of the color magnetic operator for $T = 0$.

|       | $\Sigma\pi$ | N$K$ | $\Lambda\eta$ | $\Xi K$ |
|-------|-------------|------|---------------|--------|
| $\Sigma\pi$ | $\frac{16}{3}$ | $\frac{116\sqrt{7}}{21}$ | $\frac{16\sqrt{105}}{105}$ | 0 |
| N$K$    | 0           | $\frac{28\sqrt{15}}{15}$ | 0          |
| $\Lambda\eta$ | 112        | $\frac{40\sqrt{70}}{21}$ |
| $\Xi K$ | $-\frac{160}{21}$ |

First we estimate the contributions from the color-spin part of the OGE, usually called as the color magnetic interaction (CMI), which plays the most important role in the constituent quark model. The matrix elements for the SU(3) octet baryon is

$$\langle -\sum_{i<j} \sigma_i \cdot \sigma_j \lambda_i \cdot \lambda_j \rangle = -8$$

for the octet baryons.

Because the matrix element for the SU(3) decuplet baryon is 8, this color-spin interaction gives the main contribution to the mass difference between the octet and decuplet baryons in the OGE model. For the SU(3) octet pseudoscalar mesons, the matrix element is given by

$$\langle \sigma_4 \cdot \sigma_5 \lambda_4 \cdot \lambda_5^* \rangle = -16$$

for the octet pseudoscalar mesons.

The matrix element for the SU(3) octet vector meson is $16/3$. Therefore the CMI again succeeds to give the larger contribution to the mass difference between pseudoscalar and vector mesons than that of the octet and decuplet baryons.

Now let us discuss the interaction due to the color magnetic term of OGE between the baryon and meson system given as

$$-\sum_{i<j} \sigma_i \cdot \sigma_j \lambda_i \cdot \lambda_j + \sum_{i=1}^4 \sigma_i \cdot \sigma_5 \lambda_i \cdot \lambda_5^* - (-18 - 16).$$

It is obvious that the direct term has no contribution to the baryon meson interaction because each of the baryon and meson is color-singlet. There is, however, a sizable contribution when we include the exchange term due to the antisymmetrization over the quarks. The results for the isospin $T = 0$ baryon-meson system are shown in Table IV.

In order to compare the features of this OGE model to those of the baryon meson picture, we also show the matrix elements which appear in the Weinberg-Tomozawa term in the chiral unitary model in Table IV they are proportional to the inner product of the flavor-SU(3) generator $F_i (\sum_{i<j} F_i \cdot F_j)$. As seen in these tables, the $\Sigma\pi$ channel has a strong attraction in both of the interactions. The N$K$ channel, however, has a strong attraction only in the above $(F \cdot F)$-type interaction; the contribution from the CMI to the diagonal N$K$ interaction vanishes. Therefore, as long as we restrict ourselves to the OGE model with the $q^4\bar{q}$ system, it is rather difficult to explain the $\Lambda(1405)$ peak as a N$K$ bound state appearing in the $\Sigma\pi$ scattering.

Now let us show the results of the dynamical calculation. Here we consider $\Sigma\pi$, N$K$ and $\Lambda\eta_{ud}$ channels for isospin $T = 0$ state. The effect of each channel couplings will also be discussed.

In the following figures, we show the phase shift, $\delta$, of the $\Sigma\pi$ channel for the angular momentum $L = 0$. We also show the $\Sigma\pi$ mass spectrum given by

$$|1 - \eta e^{2i\delta}|^2 / k,$$
TABLE V: Matrix elements of \((F \cdot F)\) operator for \(T = 0\).

| \(\Sigma \pi\) | \(N\bar{K}\) | \(\Lambda\eta\) | \(\Xi K\) |
|----------------|--------------|----------------|----------|
| \(-8\) \(\sqrt{6}\) | \(-6\) \(3\sqrt{2}\) | \(-6\) \(3\sqrt{2}\) | \(-6\) |

FIG. 2: Phase shift and the mass spectrum of the \(\Sigma \pi\) channel for the angular momentum \(L = 0\) given by the QCM calculation without the \(\Lambda^1\) pole with the constituent-quark reduced mass, \(6m_q/5\).

where scattering matrix \(S\) is denoted as \(S = \eta e^{2i\delta}\) and \(k\) is the relative wave number for the \(\Sigma \pi\) scattering.

First we show the results of the pure QCM calculation without the \(\Lambda^1\) pole in Fig. 2 where the reduced mass for the relative kinetic energy is \(6m_q/5\). As seen in the figure, there is no structure in the mass spectrum when the coupling to the \(\Lambda^1\) pole is not included.

Next we show the result of the calculation including a coupling with the \(\Lambda^1\) pole in the baryon-meson scattering in Fig. 3. Before the coupling to the baryon-meson is switched on, the energy of the pole, \(E^{(0)}_{\text{pole}}\), is assumed to be 160 MeV above the \(\Sigma \pi\) threshold. As seen in the figure, there appears a bound state with the binding energy 1.7 MeV. The probability of the \(\Lambda^1\) pole in this bound state is 0.20, while those of \(\Sigma \pi\), \(N\bar{K}\), and \(\Lambda\eta_{ud}\) are 0.77, 0.04, and 0.004, respectively. This state is essentially a \(\Sigma \pi\) state bound by the strongly attractive CMI with the help of the \(\Lambda^1\) pole coupling.

The phase shift at the low energy region for the \(\Sigma \pi\) channel becomes negative, because there is a bound state. The phase shift, however, goes eventually to zero, rather than \(-\pi\), because we put an extra closed state, the \(\Lambda^1\) pole, in the system. When we move the original energy of the pole higher, e.g. 200 MeV above the \(\Sigma \pi\) threshold, the bound state disappears and becomes a resonance just above the \(\Sigma \pi\) threshold. Model ambiguity allows us to change \(E^{(0)}_{\text{pole}}\) higher, or to make the coupling to the pole smaller. Then, however, the resonance just vanishes without moving to the higher energy: namely, no resonance appears around \(E = 1405\) MeV by this model. It is because the attraction in the \(\Sigma \pi\) channel is much stronger than that of the \(N\bar{K}\) channel.
We must note, however, no peak appears around the observed energy region either even if we employ the \((F \cdot F)\)-type interaction, so long as the the rather heavy reduced mass, \(6m_q/5 = 376\) MeV, is used for the \(\Sigma \pi\) channel. There appears a \(\Sigma \pi\) bound state due to the heavy reduced mass when the attraction for \(\Sigma \pi\) and \(N \overline{K}\) channels is strong enough. It is, therefore, important to use the observed value, 124 MeV for \(\Sigma \pi\) and 324 MeV for \(N \overline{K}\), which add a repulsive effect especially to the \(\Sigma \pi\) channel. Therefore, in the following calculations, we employ the realistic reduced masses, calculated from the masses given by the present quark model (Table II).

Now we show the results of the coupled channels calculation employing the realistic reduced masses including the \(\Lambda^1\) pole (Fig. 4). Here it is also assumed that \(E_{\text{pole}}^{(0)}\) is 160 MeV above the \(\Sigma \pi\) threshold. As seen in the figure, the bound state found in the previous calculation vanishes. Instead, a resonance appears around 75 MeV above the \(\Sigma \pi\) threshold, which corresponds to 1404 MeV. The width of this peak is found to be 55 MeV. Both of the results agree with the observed mass and width of \(\Lambda(1405): 1406\pm4\) MeV and 50\(\pm2\) MeV, respectively [35].

The wave functions at the resonance (\(k = 0.70\) fm\(^{-1}\)) are also shown in Fig. 4. The ratio of the mixing probabilities of the \(\Lambda^1\) pole and \(N \overline{K}\) state is roughly 2.8, which indicates that the resonance is essentially the \(\Lambda^1\) pole. The coupling to the baryon-meson channels, however, is very important to reproduce the resonance at such a low energy: it causes the energy shift of 85 MeV, which lowers the pole position from 160 MeV down to 75 MeV.

The diagonal parts of the S-matrix for the coupled channel baryon-meson scattering are shown in Fig. 5. Above the \(N \overline{K}\) threshold, the elasticity decreases sharply. The \(N \overline{K}\) phase shift goes to negative: the scattering length is \(-0.75 + i0.38\) fm, which, as a simple model, agrees with the experimental value, \((-1.70 \pm 0.07) + i(0.68 \pm 0.04)\) fm [36], reasonably well.

Next we discuss the effect of the channel couplings. The mass spectra given by the single channel \(\Sigma \pi\) calculation, the coupled channel calculations with two channels \(\Sigma \pi + N \overline{K}\) and with three channels \(\Sigma \pi + N \overline{K} + \Lambda \eta_{ud}\) are shown in Fig. 6. As seen in the figure, in the single baryon-meson channel calculation, there appears a resonance around 122 MeV above the \(\Sigma \pi\) threshold, which means that the resonance is shifted by 38 MeV by the coupling. Then the peak is shifted lower down to 84 MeV above the \(\Sigma \pi\) threshold by adding the \(N \overline{K}\) channel. The inclusion of the \(\Lambda \eta_{ud}\) channel...
FIG. 4: Phase shift and the mass spectrum of the Σπ channel for the angular momentum \( L = 0 \) and the wave function at the resonance. The QCM calculation is performed with the \( \Lambda^1 \) pole at \( E_{pole}^{(0)} = 160 \text{ MeV} \) and the realistic reduced mass.

The calculation based on the \( q^4\pi(0s)^5 \) configuration shows that some of the flavor-octet negative-parity baryons, e.g. \( \Sigma^*(\frac{1}{2}^-) \), also gains strong attraction. In the present dynamical model, however, no bound state nor resonance is found without the pole in the \( \Lambda\pi-\Sigma\pi-N\bar{K} \) \((TJ)=(1\frac{1}{2})\) scattering; the situation is similar to the the above \((TJ)=(0\frac{1}{2})\) case. The \( q^4(0s)^20p \) state also exists in this channel, but has much heavier mass: 128 MeV above the \( \Lambda^1 \) mass if the mass difference is calculated by the conventional quark model. Also, the matrix elements between the baryon-meson states and the \( \Sigma^*(\frac{1}{2}^-) \) \((0s)^20p \) pole are much smaller (Table III). We perform the coupled-channel QCM calculation also for \( T = 1 \) channel with this pole at 360 MeV above the \( \Lambda\pi \) threshold, which corresponds to 125 MeV above the \( \Lambda^1 \) pole used for the \( T=0 \) channel. A resonance appears in the \( N\bar{K} \) channel when the coupling to this pole is switched on. The energy of the resonance, however, is shifted only by 12 MeV: the peak appears at about the same energy as that of the pole. Namely, in the present model scheme, no resonance is found in this channel at an energy as low as \( \Lambda(1405) \).

As we mentioned before, we assume that \( E_{pole}^{(0)} = 160 \text{ MeV} \) above the \( \Sigma\pi \) threshold, which corresponds to 1489 MeV. We choose this value so that the system produces the resonance around 1405 MeV in the present model scheme. The value, 1489 MeV, is not unreasonable because the predicted value in ref. [1] is 1490 MeV. In this scheme, however, it is somewhat lower than the energy where the pole is considered to exist. For example, the mass of the flavor-singlet \( q^3(0s)^20p \) state calculated with the present parameter set is 1551 MeV. Moreover, \( \Lambda(1520) \) should be closer to the mass of the \( \Lambda^1 \) pole because it couples to the scattering states of the ground state baryons and mesons only by the non-central part of the interaction. Thus, the pole probably exist at around 1520-1550 MeV in this scheme. Nevertheless, we consider it very interesting that the present simple model with the assumption that OGE induces the coupling only between the \( q^4\pi(0s)^5 \) and \( q^3(0s)^20p \) state can give such a peak. We argue that the mechanism of the \( \Lambda(1405) \) resonance can be the baryon-meson scattering coupled to the three-quark state.
V. SUMMARY

We have investigated the negative-parity $\Lambda(1405)$ state in terms of the baryon-meson $S$-wave scattering by employing a quark cluster model. The model hamiltonian has $qq$ and $q\bar{q}$ hyperfine interactions coming from the one-gluon-exchange (OGE) as well as the instanton-induced interaction ($V_{INS}$). The parameters are taken so that all the masses of the octet baryons and mesons are reproduced quite well.

We perform the $\Sigma\pi$-$N\bar{K}$-$\Lambda\eta_{ud}$ coupled channel QCM calculation with and without the coupling to the $\Lambda^1$ pole by OGE. The results show that (1) there is a strong attraction in the $\Sigma\pi$ channel but not in the $N\bar{K}$ channel, (2) no peak is found in the $\Sigma\pi$-$N\bar{K}$-$\Lambda\eta_{ud}$ coupled channel QCM calculation if we employ the realistic reduced mass for the kinetic energy, and (3) a reasonable peak appears if the $\Lambda^1$ pole is included above the $N\bar{K}$ threshold.

In the baryon-meson picture where the flavor-flavor type interaction, $(F\cdot F)$, is employed, the peak appears because of the attraction in the $N\bar{K}$ channel$^8$. In the present scheme of the color-spin interaction, there is no such an attraction in the $N\bar{K}$ channel, which requires the introducing the $\Lambda^1$ pole to reproduce the resonance. The spin-independent $(F\cdot F)$-type interaction appears when we introduce the vector meson exchange potential between quarks in addition to the Goldstone boson exchange or to the gluonic interactions. To include such an interaction may be interesting but beyond the scope of the present work. Here we would like to emphasize that a quark model with the one-gluon exchange and the instanton-induced interaction can reproduce the bulk feature of the $\Lambda(1405)$ with the help of the $\Lambda^1$ pole.

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**APPENDIX A: COUPLING BETWEEN THE $\Lambda^1$-POLE AND THE BARYON-MESON STATE**

We assume that OGE induces the coupling between the spin-1/2 $(0s)^2(0p)$ state, and $q^4\overline{q}(0s)^5$ state.

First let us calculate the matrix element between $u$ and $us\bar{s}$ states (See Fig. 1). Each of these states can be written as

$$\langle u(\xi') | = (0|a_u(\xi') \rangle \quad (A1)$$

$$|u(\xi)s(\xi_j)\rangle = a^j_u(\xi) a^j_s(\xi_j)|0\rangle.$$  

Then the matrix element is given by

$$M = \alpha_s \frac{1}{\sqrt{2\epsilon_1 2\epsilon_i 2\epsilon_j 2\epsilon_k}} \langle \bar{u}(\xi') | \gamma^\mu \gamma^2 u(\xi) \rangle \langle \bar{s}(\xi) | (\pi_{-\xi} \gamma^\nu s_{\xi}) e^{i(p'_{-\xi} - p_{-\xi} - p_k)x},$$

where $u_{\xi}$ $|s_{\xi}\rangle$ is a spinor of the u-quark $|s$-quark$\rangle$ with the quantum number $\xi$, and $\pi_{-\xi}$ is a spinor of the s-antiquark with quantum number $\xi$. Since the vertex and the lowest-order term of the propagator of the gluon are given by

$$O^a_\mu = g \frac{\lambda^a \gamma_\mu}{2}$$

$$D_{\mu\nu} = \frac{\pi}{m^2} g_{\mu\nu},$$

the matrix element becomes

$$M = \alpha_s \frac{\pi}{m^2} \frac{1}{\sqrt{2\epsilon_1 2\epsilon_i 2\epsilon_j 2\epsilon_k}} e^{-i(p'_{-\xi} - p_{-\xi} - p_k)x} \{ \left( \frac{\lambda^a \gamma^0}{2} u_{\xi} \right) \langle \pi_{-\xi} \gamma^\nu s_{\xi} \rangle - \left( \frac{\lambda^a \gamma^0}{2} u_{\xi} \right) \langle \pi_{-\xi} \gamma^\nu s_{\xi} \rangle \}.$$  

(A6)

After the nonrelativistic reduction, the lowest-order term of the $(p/m)$ expansion becomes

$$M = (\lambda_i \cdot \lambda_j) \frac{\alpha_s}{4} \frac{\pi}{m^2} e^{-i(p'_{-\xi} - p_{-\xi} - p_k)x} \left\{ \left( w^*_u \xi' \right) \left( w^*_u \xi \right) \left( w^*_s \xi \right) \left( \frac{1}{2m_s} \sigma \cdot (p_j + p_k) w^*_u \xi \right) \right.$$

$$- \left( w^*_u \xi' \right) \left( p_i^f + p_i + i \sigma \times (p_i^f - p_i) \right) \left( w^*_u \xi \right) \left( w^*_s \xi \sigma \cdot w^*_s \xi \right) \right\} + O((p/m)^2).$$  

(A7)
where $w_u\xi$ is 2-component spinor of the u-quark, and $w_{-\xi}$ is 2-component spinor of the $\bar{s}$-antiquark. The SU(3) generator of the color space, $\lambda_i/2$, is to be evaluated between the initial and final states of the $i$-th quark. The operator denoted by $\lambda_{ij}/2$ should be evaluated between the $k$-th antiquark and $j$-th quark, which corresponds to the $q\bar{q}$ pair annihilating at the vertex creating the gluon.

Using the momentum conservation, $p'_i = p_i + p_j + p_k$, with the notation $k = p_j + p_k$, we obtain the potential as:

$$V_{ij\bar{k}} = \lambda_i \cdot \lambda_{\bar{k}j} \frac{\alpha_s}{4} \pi \frac{\pi}{m_a} \left\{ \left( \frac{k + p_i + i\sigma \times k}{2m_i} \right) \cdot \sigma_{\bar{k}j} \right\} \delta^f_{kj},$$

where $\sigma_{\bar{k}j}$ is the spin operator which operates between the $k$-th antiquark and $j$-th quark, and $\delta^f_{kj}$ stands for that the flavor of the quark of the annihilating pair is equal to that of the antiquark.

Now let us evaluate the matrix element between the $q^3$-pole and the baryon-meson state. The $q^3$-pole is denoted as $|\Lambda^1(123)\rangle$, being assumed as the flavor-singlet spin-1/2 (0s)$^2(0p)$ state of the three quarks numbered from 1 to 3. The baryon in the scattering state is denoted as $|B(123)\rangle$, which is antisymmetrized among three quarks with appropriate quantum numbers. The meson is denoted as $|M(4\bar{5})\rangle$, the $q\bar{q}$ state with 4th quark and the antiquark numbered as $\bar{5}$.

Then the transfer matrix element can be expressed as

$$\langle \Lambda^1(123) | \sum_{i<j}^4 V_{ij\bar{k}} A_4 | B(123) M(4\bar{5}) \rangle = 6 \langle \Lambda^1(123) | V_{345\bar{5}} | B(123) M(4\bar{5}) \rangle,$$

where $A_4$ is the antisymmetrizing operator over the four quarks, and $P_{ij}$ is the exchange operator between the $i$-th and $j$-th quarks. The operator is evaluated in each color, flavor, spin and orbital space. The direct term vanishes because we assume that the one-gluon exchange induces the annihilation of the color singlet $q\bar{q}$ pair. Also, we assume the total momentum of the system is equal to zero. Since it is enough to calculate the operator $V_{345\bar{5}}$, only the mixed symmetric (MS) term in the orbital space is relevant for the $q^3$ state. So the transfer matrix element becomes

$$\langle \Lambda^1(123) | V_{345\bar{5}} P_{ij} | B(123) M(4\bar{5}) \rangle = -\sqrt{3} \sqrt{5} \sum_{\alpha} \frac{1}{\sqrt{2}} \langle (0s)^2(0p)MS; L_z = 1 | O_{\alpha}^{orb} | (0s)^3 \rangle \langle \Lambda^1 MS; m_z = \frac{1}{2} | O_{\alpha}^{orb} P_{ij}^c | BM \rangle \langle \lambda_i \cdot \lambda_{\bar{k}j} P_{ij}^c \rangle$$

with

$$V_0^{tr} = \frac{\pi \alpha_s}{8m_u^3}, \quad O_{1}^{orb} = k; \quad O_{2}^{orb} = -(p_3 + p_4); \quad O_{3}^{orb} = k,$$

$$O_{1}^{orb} = \xi_3 \sigma_{\bar{k}j} \delta^f_{\bar{k}j}, \quad O_{2}^{orb} = \xi_3 \sigma_{\bar{k}j} \delta^f_{\bar{k}j}, \quad O_{3}^{orb} = \xi_3 \sigma_{\bar{k}j} \delta^f_{\bar{k}j}, \quad \xi_3 = \frac{m_u}{m_3}, \quad \text{and} \quad \xi_3 = \frac{m_u}{m_3}.$$

As for the orbital part, since we assume only from the $(0s)^5$ component of the baryon-meson wave function couples to the $q^3$-pole, we omit the exchange operator from the above equation. Then,

$$\langle O_{1}^{orb} \rangle = \langle O_{2}^{orb} \rangle = 2A, \quad \langle O_{3}^{orb} \rangle = -4A, \quad \text{with} \quad A = -\frac{15/4}{\sqrt{3\pi}} \frac{1}{7 b^4}.$$

The matrix elements can be calculated straightforwardly. Color and flavor-spin parts of the matrix elements are shown in Table VI.

**APPENDIX B: GCM APPROACH TO THE BARYON MESON SCATTERING**

In order to solve the RGM equation, we employ the generator coordinate method (GCM). First we expand the relative wave function in terms of locally peaked gaussians centered at $R_i$ with the size parameter $B = \sqrt{5/6}b$ as follows:

$$\chi(R_{BM}) = \sum_{i=1}^n C_i g(R_{BM} - R_i, B = \sqrt{5/6}b).$$

(B1)
Because the relative wave function is expanded in terms of the locally peaked gaussian, this method can be applied to the bound state problem. The modification necessary for treating the scattering problem will be explained later.

The binding energy \( E \) and the expansion coefficients \( C_i \) are given by the eigenvalues and eigenvectors of the following GCM equation:

\[
\sum_{j=1}^{n} H_{ij} C_j = E \sum_{j=1}^{n} N_{ij} C_j, \tag{B2}
\]

where \( n \) is a dimension of the GCM kernels whose matrix elements are given by

\[
\begin{align*}
\left\{ \begin{array}{c}
H_{ij} \\
N_{ij}
\end{array} \right\} & = \int \phi_{BM}(R_i) \left\{ \begin{array}{c}
H \\
1
\end{array} \right\} A \phi_{BM}(R_j) \prod_{k=1}^{5} dr_k. \tag{B3}
\end{align*}
\]

Here the \( \phi_{BM}(R_i) \) is the five quark (4 quarks and 1 antiquark) wave function whose orbital part \( \varphi_{BM}(R_i) \) is given by the following product:

\[
\varphi_{BM}(R_i) = \varphi_B(\xi_B) \varphi_M(\xi_M) g(R_B - R_i), B = \sqrt{\frac{7}{6}} b, g(R_G, B = \sqrt{\frac{7}{5}} b).
\]

When we rewrite the integrals over the internal and relative coordinates in terms of single quark coordinates in eq. (B3), we have employed the following equation for the center of mass coordinate \( R_G \):

\[
\int g(R_G, B = \sqrt{\frac{7}{5}} b) g(R_G, B = \sqrt{\frac{7}{5}} b) dR_G = 1.
\]

Employing the gaussian form for the internal wave function, the orbital part \( \varphi_{BM}(R_i) \) is written as a product of single quark wave functions with size parameter \( b \), centered at \( 2R_i/5 \) and \( -3R_i/5 \):

\[
\varphi_{BM}(R_i) = \prod_{k=1}^{3} g(r_k - \frac{2R_i}{5}, b) \prod_{k=4}^{5} g(r_k + \frac{3R_i}{5}, b). \tag{B4}
\]
Note that it is quite easy to perform the antisymmetrization of the four quarks when the wave function is given in terms of single-quark coordinates as in eq. (B3).

Now let us discuss the renormalization of the wave function. Employing the following equation,

\[ H\chi = EN\chi \rightarrow \frac{1}{\sqrt{N}}H\frac{1}{\sqrt{N}}\sqrt{N}\chi = E\sqrt{N}\chi, \]

we call \( \sqrt{N}\chi \) the renormalized wave function which satisfies the Schrödinger equation. Using the coefficients \( C_i \) given by solving eq. (B2) which are normalized as

\[ \sum_{ij} C_i N_{ij} C_j = 1, \]

the wave function \( \sqrt{N}\chi \) is properly orthonormalized.

The modification of the relative kinetic energy for the baryon-meson scattering problem explained in section III is rewritten in terms of the GCM kernel as follows. We take out the direct term of the kinetic energy operator \( K_{Dij} \) in the GCM kernel \( H_{ij} \). By subtracting the internal kinetic energy with the direct norm kernel \( N_{Dij} \), the GCM kernel of the relative kinetic energy is given by

\[ K_{Rij} = K_{Dij} - 3K_0 \times N_{Dij}, \quad \text{where} \quad K_0 = \frac{3}{4m_q b^2}. \]

Then the relative kinetic energy \( K_{Rij} \) is modified in the following way.

\[ K_{Rij} \rightarrow K_{Rij} \times \frac{6m_q}{5} \frac{1}{\mu}, \]

where \( \mu \) is the reduced mass for the baryon-meson system calculated by using the masses given in Table II. The GCM kernel of the coupling of the baryon-meson state with the 3q state is taken as

\[ \langle 3q|V_{3q-5q}|BM, R_j \rangle = \langle 0s^20p|V_{3q-5q}|0s^5 \rangle N_{ij} \quad \text{with} \quad R_i = 0. \]

Now we will explain how to modify the GCM in order to treat the scattering problem. Because practical calculations of the bound state and scattering problem are done in terms of partial waves, we first explain the partial wave expansion. The relative wave function in eq. (B1) is expanded in terms of locally peaked wave functions with a definite angular momentum \( l,m \):

\[ \chi(R_{BM}) = \sum_{i=1}^{n} C_i \chi_i^{(l)}(R_{BM}) Y_{lm}(\hat{R}_{BM}). \]  

Here, \( \chi_i^{(l)}(R) \) is given by an expansion of locally peaked gaussian wave functions:

\[ g(R - R_i, B) = (\sqrt{\pi}B)^{-3/2} \exp(-\frac{(R - R_i)^2}{2B^2}) = \sum_{lm} \chi_i^{(l)}(R) Y_{lm}(\hat{R}) Y_{lm}^*(\hat{R}_i). \]

The explicit form of \( \chi \) is given by

\[ \chi_i^{(l)}(R) = 4\pi(\sqrt{\pi}B)^{-3/2} \exp(-\frac{R^2 + R_i^2}{2B^2}) i_l(R R_i B), \]

where \( i_l \) is the modified spherical Bessel function. When we treat the scattering problem, we use the following modified wave functions for the expansion

\[ \tilde{\chi}_i^{(l)}(R) = \alpha_l \chi_i^{(l)}(R), \quad (R < R_c) \]

\[ \tilde{\chi}_i^{(l)}(R) = h_i^{(2)}(kR) + s_i h_i^{(1)}(kR), \quad (R \geq R_c). \]
Here, $k$ is a wave number and $h^{(1)}(l)$ and $h^{(2)}(l)$ are spherical Hankel functions. The coefficients $\alpha_i$ and $s_i$ are determined by a continuity condition of $\tilde{\chi}$ at $R = R_c$. The relative wave function is expanded in terms of $\tilde{\chi}$ as

$$\chi^{(l)}(R_{BM}) = \sum_{i=1}^{n} C_i \tilde{\chi}^{(l)}(R_{BM}) \quad \text{with} \quad \sum_{i=1}^{n} C_i = 1.$$ 

Then the $S$-matrix is given in terms of the coefficients $C_i$ as:

$$S = \sum_{i=1}^{n} C_i s_i.$$

The details how to determine the expansion coefficients can be found in Oka and Yazaki [18].

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