Early Time Evolution of High Energy Heavy Ion Collisions

Rainer J Fries
Cyclotron Institute and Department of Physics, Texas A&M University, College Station, TX 77843, USA
RIKEN/BNL Research Center, Brookhaven National Laboratory, Upton NY 11973
E-mail: rjfries@comp.tamu.edu

Abstract. We solve the Yang-Mills equations in the framework of the McLerran-Venugopalan model for small times $\tau$ after a collision of two nuclei. An analytic expansion around $\tau = 0$ leads to explicit results for the field strength and the energy momentum tensor of the gluon field at early times. We then discuss constraints for the energy density, pressure and flow of the plasma phase that emerges after thermalization of the gluon field.

PACS numbers: 12.38.Mh,25.75.-q,24.85.+p,25.75.Nq

Submitted to: J. Phys. G: Nucl. Phys.

The Color Glass Condensate (CGC) model \cite{1,2,3,4,5} has provided valuable tools to understand the interaction of hadrons and nuclei at very high energies. The idea that the rapid growth of the gluon distribution at high energy is tamed by gluon recombination leads to a saturated gluon density described by a saturation scale $Q_s$. Due to the high occupation numbers the gluons can be approximated by a classical field \cite{1}. In high energy nuclear collisions the gluon density in each nucleus is enhanced by a factor $\sim A^{1/3}$ and it has been argued that the CGC can describe the initial particle production at the Relativistic Heavy Ion Collider (RHIC) \cite{6}.

Here, we discuss what can be learned about the energy and momentum deposited in the space between the two receding nuclei immediately after the collision. This is an important question because there is convincing evidence that relativistic hydrodynamics governs the evolution of the fireball starting at rather early times $\tau_0 \approx 0.5 \ldots 1.0$ fm/c. The plasma phase, given by an energy momentum tensor $T_{\text{pl}}^{\mu\nu} = (e + p)u^\mu u^\nu - pg^{\mu\nu}$ emerges through a rather rapid and not yet completely understood thermalization process from the initial gluon field after overlap, described by an energy momentum tensor $T_I^{\mu\nu}$. Our goal here is to constrain the energy density $e$, pressure $p$ and flow velocity $v$ (with $u^\mu = \gamma(1,v)$) at the beginning of the plasma phase, using the CGC model in its version first conceived by McLerran and Venugopalan (MV) \cite{1,2}.

We have to solve the Yang-Mills equations $[D_\mu, F^{\mu\nu}] = J^\nu$ for a current $J^\nu$ created by infinitely thin color charge distributions $\rho_1(x_\perp)$ and $\rho_2(x_\perp)$ propagating on the plus
and minus light cone, respectively, and overlapping at time $t = 0$. In light cone gauge the field of each nucleus before the collision, $A_1^i(x_\perp)$ and $A_2^i(x_\perp)$ respectively, $(i = 1, 2)$, is transverse. In the forward light cone, i.e. after the collision, the field has light cone components $A^\pm = \pm x^\pm A$ and transverse components $A^i$. $A$ and $A^i$ are functions of the proper time $\tau = \sqrt{t^2 - z^2}$ and the transverse position $x_\perp$, but not of the space-time rapidity $\eta$.

For small times $\tau$, immediately after the collision, the Yang-Mills equations in the forward light cone can be rewritten using an expansion in powers of $\tau$, $A^\mu = \sum_n \tau^n A_{(n)}^\mu$. The resulting infinite tower of differential equations can be solved recursively to arbitrary order in $\tau$ [7]. Explicit solutions for $A^\mu$ and the field strength $F_{\mu\nu}$ up to order $\tau^3$ have been discussed. The lowest order in $\tau$ ($O(\tau^0)$) coincides with the familiar boundary conditions for the field on the light cone. They imply that at the earliest time, for $\tau \to 0$, strong longitudinal electric and magnetic fields between the nuclei, [8, 7]

$$E_z = ig[A_1^i, A_2^i], \quad B_z = -ig\epsilon^{ij}[A_1^i, A_2^i],$$

(1)

dominate. This was also observed, e.g., in [9, 10]. The next order in $\tau$ corresponds to a linear build-up of transverse fields.

One can now compute the energy momentum tensor of the gluon field. The leading terms $O(\tau^0)$ at small time are the diagonal elements

$$\varepsilon \equiv T^0_{0f} = T^{ii}_{if} = -T^{33}_{if}$$

(2)

$(i = 1, 2)$. Here $\varepsilon = (E_z^2 + B_z^2)/2$ denotes the energy density at $\tau = 0$. It turns out that $\varepsilon$ in the McLerran-Venugopalan model suffers from a ultraviolet divergence [7, 9]. In lattice calculations this divergence is regularized by the finite lattice spacing [9]. We can argue that this divergence comes from the fact that the ultraviolet sector of the initial particle production with large momenta $p_\perp >> Q_s$ is not perfectly described in the MV model which works best for $p_\perp \approx Q_s$. Instead, we can choose a cutoff $Q_0$ and replace the classical field above $Q_0$ with a quantum but weak coupling description known to be well-behaved in this regime, i.e. perturbative QCD.

At order $O(\tau^1)$ the components describing transverse flow of energy receive their leading contributions

$$T^0_{if} = \nu^i \cosh \eta, \quad T^{3i}_{if} = \nu^i \sinh \eta,$$

(3)

with the transverse flow vector

$$\nu^i = -\frac{\tau}{4} \nabla^i (E_z^2 + B_z^2)$$

(4)

$(i = 1, 2)$. We will see below that this translates directly to the existence of transverse flow in the plasma phase immediately after thermalization.

Further corrections to the components of $T^\mu_{\nu f}$ up to order $O(\tau^3)$ can be easily computed. This gives a rather accurate picture of the energy momentum tensor at times $\tau << 1/Q_s$. At $\tau \approx 1/Q_s$ the expansion might fail, however the asymptotic behavior for $\tau \to \infty$ is known from a weak coupling expansion [2] and an interpolation can be
used for intermediate values of $\tau$. This reproduces the time dependence of numerical solutions of the Yang-Mills equations, see e.g. \cite{9}.

In any case, the classical field approximation is not expected to hold for very large times anyway. Instead, the field is expected to decay on a time scale $\sim 1/Q_s$, maybe through instabilities in the color fields \cite{11} or particle production in the background gluon field \cite{12}. This is widely assumed, though not proved, to lead to a thermalized plasma within a rather short time.

One can, however, use energy and momentum conservation to estimate the energy momentum tensor of the plasma. Suppose the thermalization is sufficiently rapid around a time $\tau_0$ so that the system can be approximately described by $T^{\mu\nu}_f$ for $\tau \lesssim \tau_0$, and by $T^{\mu\nu}_{pl}$ for $\tau \gtrsim \tau_0$. Then one can show that

$$ e + p = \epsilon \left( 1 - \left( \frac{\nu}{\epsilon + p} \right)^2 \right), \quad (5) $$

$$ v^i = \frac{1}{\cosh \eta} \frac{\nu^i}{\epsilon + p}, \quad v^3 = \tanh \eta \quad (6) $$

(i = 1, 2) using the first two terms in the $\tau$ expansion. These four equations relate the five parameters $e$, $p$ and $v$ in the plasma phase with components of the energy momentum tensor of the field. The system of equations can be closed by providing an additional constraint, e.g., an equation of state. This result can be used as an initial condition for a hydrodynamic evolution of the system.

Let us briefly discuss this result. First we notice that we recover boost-invariance for the longitudinal flow velocity $v^3$. Secondly, as mentioned before, there is initial radial flow $v^i$ in the plasma phase which is directly proportional to the energy flow $\nu^i$ of the field. For collisions with finite impact parameter this also implies the existence of elliptic flow.

The results presented so far are functions of the single nucleus fields $A_{1,2}^i$ and have to be evaluated by putting the correct expressions for those fields. They have been discussed in the literature, see e.g. \cite{3}. In \cite{7} estimates were presented in an abelianized approximation in which the non-abelian effects were mimicked by color screening at a typical distance $R_c = 1/Q_s$. To be precise, the screening radius $R_c$ at a given point in the transverse plane was chosen to be $R_c^{-2} = 4\alpha_s \sigma/3$ where $\sigma$ is the number density of color charges in the nucleus \cite{7}. This approximation reproduces the gluon two-point function $\langle A_i^j(x) A^i_j(y) \rangle$ \cite{3} quite well. Thus we expect this approximation to give a good estimate of the energy momentum tensor, since it has been shown that $\epsilon$ depends on $A_{1,2}^i$ solely through this two-point function \cite{9}. Using this scheme one finds very simple formulae for the center of two very large nuclei colliding head-on,

$$ \epsilon = \frac{2\pi \alpha_s^2}{N_c} \sigma_1 \sigma_2 \ln(1 + c\zeta^2), \quad (7) $$

$$ \nu^i = -\tau \frac{\pi \alpha_s^2}{2N_c} \nabla^i(\sigma_1 \sigma_2) \ln(1 + c\zeta^2), \quad (8) $$

where $c \approx 0.42$ is a numerical constant and $\zeta = R_c Q_0$. $\sigma_{1,2}$ are the number densities of
Early Time Evolution of High Energy Heavy Ion Collisions

Figure 1. Initial energy density $\epsilon_0$ of the gluon field at $\tau \to 0$ and saturation scale $Q_s = 1/R_c$ as a function of the cutoff $Q_0$.

charges in the two nuclei, which lead to the $SU(3)$ color densities $\rho_{1,2}$.

Fig. 1 shows the dependence of the results on the (unphysical) cutoff $Q_0$. The (physical) saturation scale $Q_s = R_c^{-1}$ is independent of $Q_0$ as it should be, while the initial energy density $\epsilon$ ($\epsilon_0$ in the plot) grows with $Q_0$. For a rather reasonable value $Q_0 = 2.5$ GeV the estimated initial energy density is roughly $\epsilon = 260$ GeV/fm$^3$.

Acknowledgments

The author would like to thank his collaborators, J I Kapusta and Y Li, and the organizers of QM 2006 for a memorable conference. This work was supported in part by DOE grants DE-FG02-87ER40328, DE-AC02-98CH10886, RIKEN/BNL and the Texas A&M College of Science.

References

[1] McLerran L D and Venugopalan R 1994 Phys. Rev. D 49, 2233
  McLerran L D and Venugopalan R 1994 Phys. Rev. D 49, 3352
[2] Kovner A, McLerran L D and Weigert H 1995 Phys. Rev. D 52 6231
  Kovner A, McLerran L D and Weigert H 1995 Phys. Rev. D 52 3809
[3] Jalilian-Marian J, Kovner A, McLerran L D and Weigert H 1997 Phys. Rev. D 55 5414
[4] Kovchegov Y V 1996 Phys. Rev. D 66 5463
  Kovchegov Y V and Mueller A H 1998 Nucl. Phys. B 529 451
[5] Gelis F 2007 These Proceedings, Preprint hep-ph/0701225
[6] Kharzeev D and Nardi M 2001 Phys. Lett. B 507 121
[7] Fries R J, Kapusta J I and Li Y 2006 Preprint nucl-th/0604054
[8] Fries R J, Kapusta J I and Li Y 2006 Nucl. Phys. A 774 861
[9] Lappi T 2006 Phys. Lett. B 643 11
[10] Lappi T and McLerran L 2006 Nucl. Phys. A 772 200
[11] Strickland M 2007 These Proceedings, Preprint hep-ph/0701238
  Romatschke P and Venugopalan R 2006 Phys. Rev. Lett. 96 062302
[12] Kharzeev D and Tuchin K 2005 *Nucl. Phys. A* **753** 316

Gelis F, Kajantie K and Lappi T 2006 *Phys. Rev. Lett.* **96** 032304