Analysis of Calculus Learning Beliefs and Students’ Understanding about Limit of Functions: A Case Study

Usman, RM Bambang, S, M. Hasbi, MZ Mardhiah
Syiah Kuala University, Jl. Teuku Nyak Arief Darussalam, Banda Aceh 23111, Indonesia

E-mail: usmanagani@unsyiah.ac.id

Abstract. Belief is an individual construction result based on learning experiences. This study aimed to describe students’ calculus learning beliefs and its relations with an understanding of solving limit of functions problems. This study used a qualitative descriptive approach. The subjects of this research were two Mathematics education students of Syiah Kuala University. The data collected by giving calculus learning belief questionnaire, the limit of functions understanding test, and interview guidelines. The data were analyzed by displaying data, analyzing, interpreting, and concluding. The result showed that both students who had high-beliefs in calculus learning were able to solve the limit of functions problems by using concepts, principles, and procedures.

1. Introduction
Beliefs have an essential role in the success of mathematics learning [23],[14]. Besides, students’ beliefs have an impact on the success of solving a problem [19]. Beliefs are also crucial for illustrating students’ success in interpreting learning, both directly and indirectly [11], [16]. So, it is essential to discuss students’ beliefs in mathematics learning.

Calculus is a required subject for mathematics education students. [18] in standard content formulation for teacher program stated ”candidates demonstrate a conceptual understanding of limit, continuity, differentiation, and integration and a thorough background in the techniques and application of the calculus.” Ti means that prospective mathematics teacher students must have abilities to demonstrate a conceptual understanding of limits, continuity, differential, and integral, and its applications. So, learning achievement of real functions of limits is an absolute requirement that must be mastered by mathematics education students.

Limit of function is one of the basic concepts for understanding other mathematics concepts such as calculus and real analysis. [21] Said that “many scholars hold the position that the concept of limit is fundamental to the study of calculus and analysis.” It is relevant with [10] who argued that “the concepts limit and continuity are central for further mathematics learning, i.e., analysis, differential equations, differential geometry, and advanced mathematical statistics.” Thus, limits of functions need to be understood by students so that the students can adequately understand other calculus or mathematics concepts.

The students generally challenging to understand the limit of function concepts. [27] said that in general, the difficulty is in the understanding limit of function definition. On the other hand, the students are generally able to solve the limit of functions problems by using the procedures, but unable to explain the reason for using those procedures [1]. Then, [2] argued that “students’ learning the definition of the limit concept could not interpret as this concept has learned because the students generally memorize the definition of the limit concept and processes that would facilitate finding a limit.”
That means the student is unable to interpret the limit of function concepts definition. Therefore, research about limits of functions understanding needs to discuss in mathematics education researcher [9],[4], [6].

Previous studies about limits of functions were discussed by [8], [26], [22], [21], and 24. [8] discussed exploring limits of functions meaning only at a point for 16-17 years old college students. The finding showed that most of the students said that limits of functions defined as “$f(x)$ approaches a number when $x$ approaches $a$.” Then, [22] found a study about the relations between mathematics beliefs and limits of functions understanding. The finding is as below.

“The perspective renders counterexample ineffectual and mathematical arguments unconvincing; students who hold these beliefs are left with incomplete or contradictory model of limit. These students often cannot give a coherent definition of the limit of function or explain why the formula and procedures that they use to solve limit problems are valid. Many hold misconceptions of limit as abound that cannot be crossed or as unreachable.”

Based on previous studies above, limits of functions researches only focused on limits of function understanding in the aspects of its meaning and definition. However, it had not focused on aspects of how the results of verbal representations and graphs of limits of functions definition. The results of function descriptions that have a limit of functions or do not have limits at a point, and how the results of proving the limits of function by applying the formal definition in the limit of function/theorem.

Belief is construction results of mental that describe someone experiences. Belief is one of the sub-dimensions of ideas from someone's decision from experience [14]. On the other hand, lecturers' and students' belief in learning materials related to motivation, performance, and success is an essential indication of beliefs [17]. [19] defined that “beliefs are mental constructions representing the codification of individual’s experiences, behaviours, understanding in the problem-solving process.” So, mathematics beliefs are construction results of mental related to motivation, performance, and success in mathematics learning.

[23] described the dimension of beliefs system as expressed by Green, namely a belief system that is independent of each other with a belief system, a belief system linked to Psychologist and a central point. For example, [20] researched the influence of students’ mathematical conceptions in interpreting problem assignments and problem-solving performance. [22] said that “student beliefs about content that logically underlies the limit concepts (beliefs about the real number, infinity, and functions) are termed contents beliefs, beliefs about how mathematical truth and validity are established are called sources of conviction.” So, a student's beliefs about mathematics included assumptions about acceptable evidence and ways of expressing validity. Students’ beliefs about mathematics included real numbers, infinite real numbers, and real functions.

This study aimed to describe mathematics education students’ beliefs in learning calculus and limits of functions understanding. Specifically, the purpose of the research was to describe learning calculus beliefs and the ways that are understood by students in solving limits of functions and infinite limits problems-the students’ belief obtained by using calculus learning beliefs questionnaire. Meanwhile, understanding the limit of functions was obtained by using the task of understanding the function limit and interview.

2. Method
This study used a qualitative descriptive approach. The subjects were the second semester of 86 mathematics education students of Syiah Kuala University for 2018/2019 academic year. The subjects had learned integral and differential calculus. By the curriculum, it indicates that the subjects already
had a learning experience and followed calculus class. Data of students’ calculus beliefs were obtained by giving calculus learning confidence questionnaire to all subjects, then two subjects were chosen to interview. The subjects who had the characteristics of having a high calculus learning beliefs score that the understanding performance, and communicative coded S01 and S02.

The beliefs instrument consists of 8 questions or statements in the form of a Likert Scale adopted from the [22] instrument. The beliefs scores adopted the score developed by [16]. Data on understanding the limit of functions obtained by giving the task of understanding the limits of functions to 86 research subjects. The task of understanding the limits of functions (TULF) in the form of a description of 5 questions covering the limits of functions and infinite limit. The TULF instrument adopted from [22]. Data analysis of learning beliefs were analyzed in a descriptively, while the data understanding of the limits of functions were analyzed in a qualitative way, which displays the data written work and interviews, interpreting, analyzing, and concluding [15].

3. Result and Discussion
3.1 Students’ Beliefs in Learning Calculus
The data on students’ beliefs in learning calculus obtained by giving questionnaires for 86 subjects. The data was in the form of assumptions experienced by the subject related to the learning process and calculus learning. The results data on students’ beliefs presented as follows.

Table 1. Percentages of students based one Problem-Solving Strategies

| No | The Way Students Solved The Problems | Percentages of Students |
|----|-------------------------------------|-------------------------|
| 1  | Found out even though the answers were different from the answer keys in the book | 47%                     |
| 2  | Solved the problem by finding the right answer by double-checking both answers which one was the correct one | 43%                     |
| 3  | Continue with other questions and re-check the correctness of the answers | 7%                      |
| 4  | Solved the problems related the textbook | 12%                     |
| 5  | Completed the answers by themselves | 12%                     |

It found that 47% of the subjects solved the calculus problem from the questionnaire data. They found out even though the answers were different from the answer keys in the book. 43% of the subjects solved the problem by finding the right answer by double-checking both answers which one was the correct one, 7% of the subjects continue with other questions and re-check the correctness of the answers, 12% of subjects chose answers related the textbook, and 12% of subjects completed the answers by themselves. Thus showing that in general, the subjects solve the problem of calculus, they try to find solutions even though the solutions different from the keys in the textbook and try to check the truth of the answers in the textbook and the solutions that they solve themselves.
From the questionnaire data about the students’ calculus learning process experience, it found that 39.5% of subjects strongly agreed that in calculus learning, it was necessary to remember, memorize definitions, principles, theorems or formulas to solve the problems, 45.4% of subjects agreed, 15.1% of subjects voted maybe. No subject chose to disagree or strongly disagree. Furthermore, 8.1% of subjects strongly agreed that in learning calculus, everything related to logical consistent goes hand in hand with logical consistent and I can solved it by myself, 27.9% of subjects agreed, 50% of subjects chose maybe, 12.9% of subjects disagree, and 1.2% of subjects strongly disagree. Then in learning calculus obtained from questionnaires that 24.5% of subjects chose strongly agree that in learning calculus, they can solve problems when the lecturer gave examples and the solutions, 54.6% of subjects agreed, 20.9% of subjects chose maybe. Furthermore, 52.3% of subjects strongly agreed that in learning calculus, they could solve problems if given problems were similar to the example problems, 41.9% of subjects agreed, 9.8% of subjects chose maybe, 0% of subjects chose to disagree. It found that 34.9% of the subjects chose to agree strongly that in learning calculus from the questionnaire data. They needed to memorize the procedures for completing the sample problems that had been completed in order to solve the other problems, 53.5% of the subjects chose to agree, 9.3% of the subjects chose maybe, and 2.3% of subjects chose to disagree. Furthermore, 29.1% of subjects strongly agreed that in learning calculus, they had to pay attention to how to formulate definitions, principles or formula and prove it, 45.3% of subjects agreed, 22.1% of subjects choose maybe, 3.5% of subjects chose to disagree, and none of the subjects chose firmly disagreed. Then,
32.6% of the subjects strongly agreed that in learning calculus, they had to pay attention to how definitions, principles or theorems or formula applied in problem-solving, 46.5% agreed, 18.6% might, 2.3% disagreed.

3.2 The Limits of Functions Understanding
Limits of functions data obtained by giving the task of understanding the limits of functions (TULF) for the subjects. Then two subjects who had high mathematical beliefs scores and communicative were chosen. The written data on completion of the TULF and interviews with two subjects S01 and S02 obtained as follows.

Problem 1: determine \( \lim_{x \to 2} 3x + 8 \). Explain it!

From S01’s written answers and interviews data obtained that S01 determined the limit value of the function by substituting the values of x approaches two into the form of the function 3x + 8, then explained that the limit of the function when x approaches two is equal to 14. Likewise, S02 determined the limit value by substituting the value of x approaches two into 3x + 8, then explain the substitution method. Thus, understanding related to determining the limit value of the function was carried out using the substitution method and its explanation.

Problem 2: determine \( \lim_{x \to \infty} \sin x \). Explain it!

The written answers data of S01 and S02 and interviews data, both obtained that S01 determined the limit value at the infinite point by substituting the values of x approaches to a more significant number. The function value will approach to a larger number when explained through interviews, S01 involved a graph of the sin x function in explaining x approaches a more significant number. Likewise, S02 determined the limit value at an infinite point by substituting a value of x approaches an infinite number, but S02 did not involve a graph of functions to explain x approaches an infinite number. Thus, both of S01 and S02 can be concluded that the subjects’ understanding related to the determination of the limit value at the infinite point did by using the method of substitution and explanation involving graphical representation.

Problem 3: determine \( \lim_{x \to \infty} \frac{\sqrt{x}}{(3x)^x} \). Explain it!

The data shown that S01 determined the limit value at an infinite point by substituting values of x approaches a more significant number, the function value approaches to a larger number. Likewise, S02 determined the limit value at an infinite point by substituting a value approaches infinity. Thus, both of S01 and S02 can conclude that the subjects understanding related to determining the limit value at the infinite point did by using the substitution method.

Problem 4: determine \( \lim_{t \to 3} \frac{t^2 - 9}{x^2 - 9} \). Explain it!

The data showed S01 and S02 determined the limit value by factoring the numerator, then crossing out the same factor. S01 explains that the limit value obtained is equal to 6 when t approaches 3. S02 also determines the limit value by factoring, simplify by crossing out, and substituting t approaches three but not equal to 3. Thus, it can be concluded that the understanding of the subjects in solving the limit problem by factoring, crossing out, and substituting.
Problem 5: determine \( \lim_{x \to 0} x + 1 + \frac{1}{10^x} \). Explain it!

The data showed that S01 determined the limit by using the L’Hopitals rule and then substituted the value of \( x \). Whereas, S02 determined the limit value by using the limit theorem of the sum of two functions, then substitutes the value. Thus, the understanding of the two subjects in determining the limit values using different methods.

The subjects understanding in solving limits of functions at a points’ problem did by using substitution method. Both subjects with high-calculus learning beliefs determined the limit value by involving the substitution method and explain that "the limit function when \( x \) approaches \( c \) is \( L \). The subjects believed that the truth of the limit value is obtained by explaining that by using substitution and involving the meaning of the limit. It is relevant with the findings of [22], [21],13 that explained by representing verbal and graphic form [7]. Likewise, the understanding of an infinite limit was determined by subjects who had high-calculus learning beliefs scores using the substitution method and explaining by involving verbal and graphical representations.

**Conclusion**

The finding of this research is a small part of the research of understanding and belief limits of the first year mathematics students at Syiah Kuala University. The finding shown that both subjects who had high-calculus learning beliefs understood and were able to solve limits of functions and infinite limits by doing the substitution method and explaining representing verbal and graphic form.

**References**

[1] Beynon, K.N & Zollman, A. (2015). Lacking a formal concept of limit: Advanced non-mathematics students’ personal concept definitions. *Investigations in Mathematics Learning, The Research Council on Mathematics Learning, 3*(1).

[2] Cetin, N. (2009). The performance of undergraduate students in the limit concepts. *International Journal of Mathematical Education in Science and Technology, 40*(3), 323-340.

[3] Cobb, P. (1986). Contexts, goal, beliefs, and learning mathematics. *For the Learning of Mathematics, 6*(2), 2-9

[4] Cory, B., L., & Garofalo, J. (2011). Using dynamic sketches to enhance preservice secondary mathematics teachers’ understanding of limit of sequences. *Journal for Research in Mathematics Education, 42*(1), 65-95

[5] Cottrill, J., Dubinsky, E., Nielchols, D., Schwingendorf, K., Thomas, K., & Vidakovic, D. (1996). Understanding the limit concepts: Beginning with a coordinated process schema. *Journal of Mathematics Behavior, 15* 92), 167-192.

[6] Dunbel, D. G. (2014). Students’ misconceptions of the limit concept in a first calculus course. *Journal of Education and Practice, ISSN 2222-288X* (online), 5. 34. 24-40.

[7] Fernandez-Plaza, J.A., Ruiz-Hidalgo, J. F, & Romero, L. L. (2012). The concept offinite limit of a functionat one point as explained by students of non-compulsory secondary education. *Proceeding of the 36th Conference of the International Group for the Psychology of Mathematics Education, (2), 235-242. Taipei Taiwan*

[8] Fernandez-Plaza, J.A., Rico, L, & Ruiz-Hidalgo, J. F. (2013). Meaning of the concepts of finite limit of a function at one point as demonstrated by students in bachillerato.
Proceedings of 12th International Congress on Mathematical Education, pp 2715-2721, Seoul, Korea

[9] Kabael, T. 2014. Students’ formalising process of the limit concept. *Australian Senior Mathematics Journal*, 28 (2), 23-37.

[10] Karatas, I., Guven, B., & Cekmez, E. (2011). A cross-age study of students’ understanding of limit and continuity concepts. *Boletim de Educação Matemática*, 24(38), 245-264.

[11] Kloosterman, P. & Caogan, M.C. (1994). Students’ beliefs about learning school mathematics. *Elementary School Journal*, 94(4), 375-388

[12] Kula, Semih & Guzel, E., B. (2014). Misconceptions emerging in mathematics student teachers’ limit instruction and their reflections. Quasi Quant (2014) 48-3355-3372. DOI 10.1007/s11135-013-9961-y, Published online: 4 Desember 2013 © Springer Science+Business Media Dordrecht 2013.

[13] Martorides, E., & Zachariaides, T. (2004). Secondary mathematics teachers’ knowledge concerning the concept of limit and continuity. *Proceeding of the 28th Conference of the International Group for the Psychology of Mathematics Education*, 4, 481-488

[14] Memnun, D., S., Hart, C., L., Akkaya, R. (2012). A research on the mathematical problem solving beliefs of mathematics, science and elementary pre-service teachers in Turkey in terms of different variables. *International Journal of Humanitis and Social Science*, 2(24), 172-184

[15] Merriam, S. B. (1998). *Qualitative research and case study application in education*. San Fransisco: Josey Bass Publishers.

[16] Mkomange, W., C., & Ajagbe, M., A. (2012). Prospective secondary teachers’ beliefs about mathematical problem solving. *International Journal of Research in Management & Technology*, 2(2), 2249-9563

[17] National Council of Teacher of Mathematics. (1989). Principle and Standards for School Mathematics. Diumduhtanggal 10 Juli 2018 dari http://eric.ed.gov/follH/Ejjo54301.pdf.

[18] National Council of Teacher of Mathematics. (2000). Principle and Standards for School Mathematics. Diumduhtanggal 10 Juli 2018 dari http://eric.ed.gov/follH/Ejjo54301.pdf.

[19] Ozturk, T & Guven, B. (2016). Evaluating students’ beliefs in problem solving process: A case study. *Eurasia Journal of Mathematics, Sciences, & Technology Education*, 12 (2), 411-429.

[20] Schoenfeld, A.H. (1992). Learning to think mathematically: problem solving, metacognition and sense making in mathematics. In D.A. Groups (Ed). *Handbook of research on mathematics teaching and learning*. New York, NY: Macmillan

[21] Swinyard, C & Larsen, S. (2012). Coming to understand the formal definition of limit: Insights gained from engaging students in reinvention. *Journal for Research in Mathematical Education*, 43 (4), 465-493.

[22] Szydlik, J., E. (2000). Mathematical beliefs and conceptual understanding of the limit a function. *Journal for Research in Mathematics Education*, NCTM, 31 (3), 258-276.

[23] Thomspoon, A.G. (1992). Teachers’ beliefs and conceptions: A synthesis of the research. In D.A. Grouws (Ed), *Handbook of research on mathematics teaching and learning*, 127-146. New York, NY: Macmillan

[24] Usman, Juniati, D, & Siswono, T., Y., E. (2017a). Exploring the Conception of Prospective Students Teacher that Have Mathematical Ability about Limit of Function. *Conference Proceedings The 4th International Conference on Research, Implementation, and Education of Mathematics and Science* (4th ICRIEMS),
Yogjakarta, Indonesia. Volume 1869. Melville, New York, 2017 AIP Conference Proceedings

[25] Yazier, E, &Ertekin, R. (2010). Gender differences of elementary prospective teachers in mathematical belief and mathematics teaching anxiety. International Journal of Social, Education, and Management Engineering, 4(2), 327-330

[26] Williams, S. R. (1991). Models of limit held by college calculus students. *Journal for Research in Mathematics Education*, 22(3), 219-236.