Numerical modelling for a simplified bridge pier model under high-speed train passage over the bridge

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Abstract. In this work, we simplify a moving high-speed train as combination of a series of the axe loads from the front and rear bogies of cars. We build a simplified bridge pier model, as a result, train source expressions with cars moving on land and bridges are given respectively. Considering that the piers are inserted into the ground at a depth of a few tens of meters to reach the bedrock. When the high-speed train passes the piers, the piers will appear oscillating motion which is subjugated by the ground, and this oscillating motion is relative to the microstructure interactions of subsoil and bedrock in the frame of the generalized continuum mechanics theory. Hence, we derive the elastic wave equations based on the modified couple stress theory to perform numerical modelling for a simplified bridge pier model under high-speed train passage over the bridge. Based on the wave equation and the train source expressions, a theoretical seismogram of a moving high-speed train with 16 cars on the bridge are obtained. Compared with the real data recorded when a high-speed train passed the piers in Dingxing County, some conclusions are drawn.

1. Introduction
Due to the repeatability, definiteness and economy of high-speed train moving seismic source, high-speed-train seismology has been extensively studied by many researchers [3,5,8,9,12]. In order to use the high-speed train source to achieve structural imaging and inversion of shallow surface geological structures, we need focus on the research of the characteristic of seismic wave induced by the high-speed train. Cao and Chen [3] discussed the solution of the three-dimensional elastic wave equation in a half space with a moving line source. Zhang et al. [12] presented a high-speed train seismic wavelet time function of P-wave and S-wave sources by a high-speed train. Wang et al. [8] introduced the synchro squeezing time-frequency analysis to the seismic wave induced by the high-speed train.

Several models have been proposed to predict the wave propagation in the ground system under high-speed train moving [1,2,4,6,7,11], it is important to note that the wave propagation induced by moving high-speed train vary as per the ground conditions, especially for the case of passing over the bridge. Consider that the piers are inserted into the ground at a depth of a few tens of meters to reach the bedrock. When the high-speed train passes over the piers, the piers will appear oscillating motion which is subjugated by the ground, and this oscillating motion is relative to the microstructure interactions of subsoil and bedrock in the frame of the generalized continuum mechanics theory. By means of conventional continuum mechanics theory, it is difficult to describe the more complex microscopic
interactions, especially for the modelling of the seismic response of a train passing over bridge piers, hence, we derive the elastic wave equations based on the modified couple stress theory to perform numerical modelling for a simplified bridge pier model under high-speed train passage over the bridge.

In this paper, we simplify the moving high-speed train as combination of a series of the axe loads from the front and rear bogies of $N$ cars, and built a simplified bridge pier model, as a result, train source expressions with $N$ cars moving on land and on the bridge are given, respectively. Based on the wave equation and the train source expressions, the theoretical seismograms of a moving high-speed train with 16 cars on the bridge are obtained. Compared with the real data recorded when high-speed train passed the piers in Dingxing county, the seismogram in this case exhibits similar waveform and amplitude-frequency response features. The bridge pier model and train load model we build basically accord with the physical facts.

2. Theory and Model

2.1. Elastic wave equations based on generalized continuum mechanics theory

In order to make the bridge more stable, the piers are inserted into the ground at a depth of a few tens of meters to reach the bedrock. When the high-speed train passes the piers, the piers will appear oscillating motion which is subjugated by the ground, and this oscillating motion will motivate the microstructure interactions whose macroscopic effect is rotational motion in the frame of the generalized continuum mechanics theory, hence, we derive the elastic wave equations based on the modified couple stress theory to get the theoretical seismograms for a simplified bridge pier model under high-speed train passage.

Strain energy density function $W$ can be respectively expressed as equation (1) based on the modified couple stress theory [10]:

$$ W = \frac{1}{2} \lambda (\varepsilon_{ij})^2 + \mu (\varepsilon_{ij} \varepsilon_{ij} + l^2 \chi_{ij} \chi_{ij}) $$

(1)

where, $\varepsilon_{ij}$ are the symmetric strain tensors under small deformation assumption, $\chi_{ij}$ are the symmetric curvature tensors, which are defined as $\chi_{ij} = \omega_{lj} = \frac{1}{2} \varepsilon_{ji} \delta_{li} \omega_{l}$. $\omega_{i}$ are the rotation vector, $\varepsilon_{ji}$ is the permutation symbol, $\lambda$, $\mu$ are Lame constants, $l$ are the characteristic length of media.

By performing partial derivative operations on the strain energy density function seen as equation (1), the constitutive relations of the modified couple stress theory [10] are as below:

$$ \sigma_{ij} = \lambda \varepsilon_{ij} \delta_{li} + 2\mu \varepsilon_{ij} $$

(2)

$$ \mu_{ij} = 2\eta \chi_{ij} $$

(3)

where, $\sigma_{ij}$ are the symmetric stress tensors, $\mu_{ij}$ are the deviatoric couple stress tensors, $\eta$ is the medium parameter, $\eta = l^2 \mu$.

Applying the law of conservation of momentum and the principle of conservation of moment of momentum to unit volume elements with surface stress and surface couples (without consideration of body couples), we obtain:

$$ \sigma_{ij,j} + \frac{1}{2} \varepsilon_{ij} \mu_{ij} + \rho f_i = \rho \ddot{u}_i $$

(4)

where, $f_i$ are the body force.

Substituting equation (2) and (3) into equation (4) to derive elastic wave equations based on the modified couple stress theory [10] (regardless of body force couple) as below:
\[(\lambda + \mu)u_{j,i} + \mu u_{j,j} + \frac{1}{2}\eta e^{\frac{1}{2}e_{e,ijkl} + \frac{1}{2}e_{e,ijkl}} + \rho f_i = \rho u_j\]  

(5)

2.2. High-speed train load model

High-speed train load model is shown in Figure 1, we consider a train consisting of \(N\) numbers of cars, and move along the positive x-axis with a constant velocity \(c\). The train load is comprised of the axe loads from the front and rear bogies.

The external loading can be described in time domain as:

\[f(x, y, z, t) = \sum_{i=1}^{N} \left[ G_{n1}\delta(x - ct + \sum_{j=0}^{n-1} L_i + a) + G_{n2}\delta(x - ct + \sum_{j=0}^{n-1} L_i + a + b) + G_{s1}\delta(x - ct + \sum_{j=0}^{n-1} L_i + 2a + b) \right] g(t)\delta(y)\delta(z) \]

(6)

where, \(G_{n1}\) and \(G_{n2}\) are respectively the axe loads from the front and rear bogies, \(G_{n1} = G_{n2} = 170\ kN\), \(L_i\) represent the length of each car, \(L_i = 28\ m\), and, \(a\), \(b\) are the distances between axles, \(a = 2.7\ m\), \(b = 17.5\ m\), \(\omega_0\) is the natural vibration angular frequency of the train, \(g(t)\) is Ricker wavelet, \(g(t) = \left[1 - 2(\pi f_s t)^2\right]e^{-\pi f_s t^2}\).

\[\text{Figure 1. High-speed train load model.}\]

We consider a moving train which move along the positive x-axis with a constant velocity \(c\), rewrite equation (6), we obtain:

\[f(x, y, z, t) = \sum_{n=1}^{N} \left[ G_{n1}g\left(t - \frac{L}{c}(n-1)\right) + G_{n2}g\left(t - \frac{L}{c}(n-1) - \frac{a}{c}\right) + G_{s1}g\left(t - \frac{L}{c}(n-1) - \frac{a+b}{c}\right) + G_{s2}g\left(t - \frac{L}{c}(n-1) - \frac{2a+b}{c}\right) \right] \delta(x)\delta(y)\delta(z) \]

(7)

Equation (7) can be considered as a train source with \(N\) cars moving on land.

2.3. Bridge Pier Model
As shown in Figure 2, a simplified bridge pier model is built. Seismic waves propagate in a range of size $400 \times 100$, and the grid intervals are $dx = 4 \, m$, $dz = 1 \, m$. The velocity model is divided into low velocity layer and high velocity layer, the first layer is low-speed layer with the thickness of 10 m which corresponds to the soil, and the density, P-wave velocity are respectively $400 \, kg/m^3$ and $0.5 \, km/s$. The second layer is high-speed layer with the thickness of $90 \, m$ which corresponds to the rock, and the density, P-wave velocity are respectively $1400 \, kg/m^3$ and $1.6 \, km/s$, respectively. The S-wave velocity is a scaled version of the P-wave velocity with $v_s / v_p = 1.7$.

The simplified bridge pier model contains five piers which are inserted into the ground at a depth of $50 \, m$ to reach the bedrock, and each source is set every $10 \, m$ depth. It is assumed that the bridge piers are tightly coupled with the soil and bedrock. When the train passes the piers with a constant velocity $c = 300 \, km/h$, we can derive the high-speed source for the simplified bridge pier model:

$$f(x, y, z, t) = \sum_{i=1}^{M} \sum_{n=0}^{N} \left[ G_n g \left( t - \frac{L}{c} (n-1) - \frac{d}{c} (i-1) \right) 
+ G_{n+1} g \left( t - \frac{L}{c} (n-1) - \frac{d}{c} (i-1) - \frac{a}{c} \right) \delta(x-x_0-d(i-1)) \delta(y) \delta(z) \right]$$

where, $d$ is the distance between each two piers, $M$ is the number of piers, $x_0$ is the position of first pier.

3. Numerical modelling
In this section, we perform numerical modelling for the simplified bridge pier model as shown in Figure 2. The number of cars of train and bridge piers are 16 and 5, respectively. The dominant frequency of moving sources is $20 \, Hz$, and the record length is $10 \, s$ with a time interval of $0.0005 \, s$.

The simplified bridge pier model contains five piers which are inserted into the ground at a depth of $50 \, m$ to reach the bedrock, and each source is set every $10 \, m$ depth. It is assumed that the P-wave velocity in pier is $4.0 \, km/s$, hence, the time delay of each source in piers is $10/4000 \, s$. The seismogram can be considered as the superposition of that for moving cars of train and bridge piers, the high-speed source for the simplified bridge pier model can be obtain:
Where, $D$ is the distance of every two sources in piers, $K$ is the number of source set in each pier.

Based on equation (5) and equation (9), we perform the numerical modelling for the simplified bridge pier model. Figure 3 shows the seismogram ($z$ component) for this bridge case. Compared with the real data recorded when high-speed train passed the piers in Dingxing county, the seismogram in this case exhibits similar waveform features. When the high-speed rail has passed the piers at a speed of 300 kilometers per hour, the excited seismic wave field received at the piers is mainly dominated by low-frequency energy around 10Hz, which matches the real data well. The numerical modelling proves that the bridge pier model we build basically accords with the physical facts.
4. Conclusions
In this work, we simplify the moving high-speed train as combination of a series of the axe loads from the front and rear bogies of \( N \) cars, and built a simplified bridge pier model, as a result, train source expressions with \( N \) cars moving on land and on the bridge are given, respectively. Consider that the piers are inserted into the ground at a depth of a few tens of meters to reach the bedrock. When the high-speed train passes the piers, the piers will appear oscillating motion which is subjugated by the ground, and this oscillating motion is relative to the microstructure interactions of subsoil and bedrock in the frame of the generalized continuum mechanics theory. By means of conventional continuum mechanics theory, it is difficult to describe the more complex microscopic interactions, especially for the modelling of the seismic response of a train passing over bridge piers, hence, we derive the elastic wave equations based on the modified couple stress theory to perform numerical modelling for a simplified bridge pier model under high-speed train passage over the bridge. Based on the wave equation and the train source expressions, the theoretical seismograms of a moving high-speed train with 16 cars on the bridge are obtained. Compared with the real data recorded when high-speed train passed the piers in Dingxing county, the seismogram in this case exhibits similar waveform features, and is mainly dominated by low-frequency energy around 10 Hz, which matches the real data well. The numerical modelling proves that the bridge pier model we build basically accords with the physical facts.

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