VELOCITY GRADIENTS AS A TRACER FOR MAGNETIC FIELDS

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ABSTRACT

Strong Alfvénic turbulence develops eddy-like motions perpendicular to the local direction of magnetic fields. This local alignment induces velocity gradients perpendicular to the local direction of the magnetic field. We use this fact to propose a new technique of studying the direction of magnetic fields from observations, which we call the velocity gradient technique. We test our idea by employing the synthetic observations obtained via 3D magnetohydrodynamical (MHD) numerical simulations for different sonic and Alfvén Mach numbers. We calculate the velocity gradient, $\Omega$, using the velocity centroids. We find that $\Omega$ traces the projected magnetic field best for the synthetic maps obtained with sub-Alfvénic simulations and provides good point-wise correspondence between the magnetic field direction and the direction of $\Omega$. The reported alignment is much better than the alignment between the density gradients and the magnetic field, and we demonstrate that it can be used to find the magnetic field strength with an analog of the Chandrasekhar–Fermi method. This new technique does not require dust polarimetry, and our study opens up a new way of studying magnetic fields using spectroscopic data.

Key words: ISM: kinematics and dynamics – ISM: magnetic fields – magnetohydrodynamics (MHD) – turbulence

1. INTRODUCTION

It is well established that the interstellar medium (ISM) is turbulent, which affects the dynamics of different astrophysical phenomena such as star formation, cosmic-ray acceleration, galaxy evolution, and feedback (Ferrière 2001; Elmegreen & Scalo 2004; de Avillez & Breitschwerdt 2005; McKee & Ostriker 2007; Falgarone et al. 2008). Widely used evidence of the turbulence in the ISM is seen in the so-called big power law in the sky (Armstrong et al. 1995; Chepurnov & Lazarian 2010), which reflects the Kolmogorov spectrum of electron density fluctuations in nonthermal Doppler broadening (see Draine 2011), in the power-law scalings of the fluctuations measured in position–position velocity (PPV) space (see Lazarian & Pogosyan 2000; Stanimirović & Lazarian 2001; Padoan et al. 2009; Chepurnov et al. 2010, 2015) (see Lazarian et al. 2009, for a review), and in velocity centroids (see Miesch et al. 1999; Miville-Deschênes et al. 2003).

The study of the direction and measurements of the intensity of magnetic fields present a challenging problem. Polarization arising from grains aligned with longer axes perpendicular to a magnetic field allows to estimate the direction of the magnetic field, but the existing uncertainties in grain alignment theory and the failure of grain alignment at large optical depths limit the ability of the technique to trace magnetic fields without ambiguities (see Lazarian 2007; Lazarian et al. 2015a, for a review). This also affects the Chandrasekhar–Fermi (C–F) technique, which is used to determine the intensity of magnetic fields when the variations in polarization direction and velocity dispersion are both known.

Other techniques that are used to study magnetic fields in molecular clouds, e.g., based on the Zeeman effect or on the Goldreich–Kylafis effect, have their own limitations (see Crutcher 2012). Faraday rotation is able to measure the intensity of the field along the line of sight (LOS) in ionized media (Padoan et al. 2001; Ostriker 2003, pp. 252–270; Crutcher 2012). The atomic/ionic alignment technique provides a promising approach for studies of the magnetic field, but the ability of the technique to trace magnetic field has not yet been demonstrated with observational data (see Yan & Lazarian 2015).

Therefore there is intense interest in developing new techniques. For instance, the anisotropy analysis of statistical properties of variations in the observable Doppler-shifted lines (Lazarian et al. 2002; Heyer & Brunt 2004; Esquivel & Lazarian 2005, 2009; Burkhart et al. 2015) proved to be a promising way of obtaining the mean magnetic field as well as the media magnetization (see also Esquivel et al. 2015; Kandel et al. 2016a, 2016b). The difference in the velocities of ions and neutrals that is caused by the difference in their damping in magnetohydrodynamical (MHD) turbulence has been suggested as another way of studying magnetic fields (Li & Houlde 2008). The quantitative treatment of the technique shows, however, that it has its limitations (see Xu et al. 2015). New approaches for studying magnetic fields using synchrotron intensity and polarization have been suggested in Lazarian & Pogosyan (2012, 2016). All in all, the search of new ways of studying magnetic fields is characterized by intensive research; all techniques have some shortcomings and limitations.

The Chandrasekhar & Fermi (1953, C–F) method can determine magnetic field strengths by estimating the variations in the directions of the magnetic field lines, assuming Alfvénic motions (Ostriker et al. 2001; Falceta-Gonçalves et al. 2008; Houde et al. 2009; Novak et al. 2009). The C–F method in its simplest from estimates the magnetic field strength as

$$B \approx \sqrt{4\pi \rho \delta v / \delta \phi},$$

where $\rho$ is the density, $\delta v$ is the velocity dispersion, and $\delta \phi$ is the polarization angle dispersion. An improved C–F technique in (Falceta-Gonçalves et al. 2008) was shown to be able to determine the intensity of the magnetic field with an uncertainty lower than a 20%. In the case when turbulence is injected on small scales, Cho & Yoo (2016) provide another modification of the C–F method.

In a strongly magnetized turbulent medium, the turbulent eddies rotate preferentially about the local direction of the
magnetic field (Goldreich & Sridhar 1995; Lazarian & Vishniac 1999; Cho & Vishniac 2000; Cho et al. 2002a). Thus the analysis of their rotation can determine the direction of the magnetic field. This is the basis for the new idea of studying the magnetic field that we advocate in this paper.

In this paper we introduce a new technique, the velocity gradient technique (VGT), and demonstrate its ability to trace the direction of the magnetic field. For our studies we use synthetic maps obtained with 3D MHD simulations and use gradients of the first-order velocity centroids.\(^1\) To test the ability of the VGS to represent magnetic field, we rotate these gradients by 90\(^\circ\) and compare their direction with the direction of the magnetic field averaged along the LOS.

In what follows, in Section 2, we explore the theoretical approach to the use of velocity gradients; in Section 3, we describe the numerical code and setup for the simulations; in Section 4, we present our method of the VGT and the alignment with the magnetic field; in Section 5, we present observational properties of the velocity gradients; 6, we present the use of the C–F method on the VGT; in Section 7, we discuss our technique; and in Section 9, we give our conclusions.

2. THEORETICAL CONSIDERATIONS

The VGT is based on the modern theory of MHD turbulence (see Brandenburg & Lazarian 2013, for a review). In strong MHD turbulence, the Alfvénic turbulence motions are eddy-like, as described by Goldreich & Sridhar (1995, henceforth GS95). However, unlike the hydrodynamical case, the eddy motions have a preferential direction of motion that is set by the magnetic field direction. The eddies mix magnetic fields and matter perpendicular to the local direction of the magnetic field. The local system of reference, which was not a part of the original GS95 picture, is currently one of the main pillars of our understanding of MHD turbulence (Lazarian & Vishniac 1999; Cho & Vishniac 2000; Maron & Goldreich 2001; Cho et al. 2002b). In this local frame, the relation between parallel and perpendicular scales of the eddies is set by a “critical balance”:

\[
l_{\parallel}^{-1}V_{\parallel} \approx l_{\perp}^{-1}u_{\perp},
\]

with \(V_{\parallel}\) the Alfvén speed, \(u_{\parallel}\) the eddy velocity, and \(l_{\parallel}\) and \(l_{\perp}\) the eddy scales parallel and perpendicular to the local direction of the magnetic field. The Alfvén speed \(V_{\parallel}\) is \(\langle |B| \rangle / \sqrt{4\pi \rho_{\parallel}}\), where \(\langle \cdot \rangle\) is the average over the entire data set. This critical balance determines the eddy size by the distance an Alfvénic perturbation can propagate during an eddy turnover (for a review see Lazarian et al. 2012). For the sub-Alfvénic regime, the eddy velocity and scales can be written in terms of the injection velocity \(V_{\parallel}\) as (Lazarian & Vishniac 1999)

\[
l_{\parallel} \approx L \left( \frac{l_{\perp}}{L} \right)^{2/3} M_{A}^{-4/3},
\]

\[
u_{\parallel} \approx V_{\parallel} \left( \frac{l_{\perp}}{L} \right)^{1/3} M_{A}^{1/3},
\]

where \(M_{A} = \langle |v|/V_{\parallel} \rangle\) is the Alfvénic Mach number, \(|v|\) is the local magnitude of the velocity field, and \(L\) is the injection scale with injection velocity \(V_{\parallel}\). These intrinsic properties of the eddies imprinted by the Alfvénic turbulence imply not only the condition of a preferential direction along the local magnetic field, but also that the eddy velocity depends the size of the eddy.

The effects of the anisotropy of the velocity fluctuations on the turbulent medium have been described by analyzing intensity anisotropies of spectral line cube velocity channels (Lazarian et al. 2002; Burkhardt et al. 2014; Esquivel et al. 2015; Kandel et al. 2016b), correlations of velocity centroids (Esquivel & Lazarian 2005; Federrath et al. 2010; Kandel et al. 2016a), the bispectrum (Burkhart et al. 2009), and higher order statistical moments (Kowel et al. 2007), as well as using principal component analysis (PCA) (Heyer et al. 2008). All these techniques require, however, that the statistical samples limit their ability to trace magnetic fields over sufficiently small patches of the sky.

It is also important to mention that the local system of reference cannot be studied in observations where the averaging along the LOS is performed. The projection effects inevitably mask the actual direction of the magnetic field within individual eddies along the LOS. As a result, the scale-dependent anisotropy predicted in the GS95 model is not valid for the observer measuring parallel and perpendicular scales of projected and averaged (along the LOS) eddies. The anisotropy of eddies becomes scale independent, and the degree of anisotropy is determined by the anisotropy of the largest eddies for which projections are mapped (Cho & Lazarian 2002; Esquivel & Lazarian 2005). The projections of eddies for sub-Alfvénic turbulence are aligned along the magnetic field. A more recent theoretical study by A. Lazarian 2016 (in preparation) shows that the alignment of the velocity gradients with magnetic field is also expected in superAlfvénic turbulence.

The elongated eddies have the largest velocity gradient perpendicular to the their longest axes. Thus we expect the direction of the maximum velocity gradient to be perpendicular to the local magnetic field. In this way, the velocity gradients can trace the directions of the local magnetic field, while one can expect that the observed gradients of the Doppler-shifted spectral lines will trace the plane-of-sky variations in the magnetic fields. The measurements do not require determining the correlations and therefore can be made more local than the anisotropies of correlations that we have discussed above.

3. MAGNETOHYDRODYNAMIC SIMULATIONS

We used two MHD codes to simulate the data, AMUN for the sub-Alfvénic regime (Kowel et al. 2007, 2009), and a code developed by Cho & Lazarian (2002) for the super-Alfvénic regime. Both codes solve the ideal MHD equations with periodic boundary conditions,

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,
\]

\[
\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot \left( \rho \mathbf{v} \mathbf{v} + \left( p + \frac{B^2}{8\pi} \right) \mathbf{I} - \frac{\mathbf{B} \mathbf{B}}{4\pi} \right) = \mathbf{f},
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}),
\]

\[
p = c_s^2 \rho,
\]

\[
\nabla \cdot \mathbf{B} = 0,
\]

\[
\mathbf{v} = 0,
\]

\[
\mathbf{B} = 0.
\]
were \( \rho \) is the density, \( p \) is the pressure, \( c_s \) is the isothermal sound speed, \( B \) is the magnetic field, \( v \) is the velocity, and \( f \) is the external force (in this case, the turbulence injection force).

The turbulence is driven solenoidally in Fourier space at a scale 2.5 times smaller than the simulation box size, i.e., 2.5 \( l_{\text{inj}} = l_{\text{box}} \). This scale defines the injection scale in our modes in Fourier space, and therefore the largest scale of the eddies. Density structures in turbulence can be associated with the effects of slow and fast modes (Beresnyak et al. 2005; Kowal et al. 2007). The code units of length are defined in terms of the box size \( L \), and the time as the eddy turnover time \( (L/v) \). For this simulation the velocity and density fields were set to unity in code units, while the pressure was changed to obtain the different sonic Mach numbers. The sonic Mach number is defined as \( M_s = (v/c_s) \). The simulations are isothermal scale-free, so they can be scaled for any parameters of the observed media studied (see Burkhart et al. 2009) provided that the cooling in the media is efficient to keep it isothermal. The properties of different phases of the ISM can be found in Draine & Lazarian (1998). To reach saturation and stability, the simulations run for 5–7 turnover times of the largest eddy.

The magnetic field has a uniform component \( (B_0) \) and a fluctuating field \( \langle \delta B \rangle \), i.e., \( B = B_0 + \delta B \). Initially, \( \langle \delta B \rangle = 0 \), and for all times \( \langle \delta B \rangle = 0 \). The fluctuating field is produced by the turbulence injection. The mean field is in the \( x \) direction. The database consists of 10 numerical simulations with a resolution of 512\(^3\) and 256\(^3\) for the super-Alfvénic and 192\(^3\) and 256\(^3\) for the sub-Alfvénic regime, where the mean magnetic field has values of 10, 1, and 0.1 in units of the driving velocity (see Table 1). Therefore a magnetic field \( B = 10 \) corresponds to the Alfvén Mach number \( M_{A} = V_A/v = 0.1 \), where \( V_A \) is the velocity at the scale \( L \) of turbulent injection and \( V_A \) is the Alfvén velocity, while the trans-Alfvénic turbulence corresponds to \( B = 1 \). The supersonic simulations have a resolution of 512\(^3\) and 256\(^3\). For most of the analysis we use the first three models.

### Table 1

| Model | \( B_0 \) | \( M_A \) | \( M_s \) | Resolution |
|-------|-----------|----------|-----------|------------|
| 1     | 10        | 0.1      | 0.7       | 192\(^3\)  |
| 2     | 1         | 0.7      | 0.7       | 256\(^3\)  |
| 3     | 0.1       | 2.7      | 0.7       | 512\(^3\)  |
| 4     | 1         | 0.7      | 1         | 512\(^3\)  |
| 5     | 1         | 0.7      | 1.5       | 256\(^3\)  |
| 6     | 1         | 0.7      | 3         | 512\(^3\)  |
| 7     | 1         | 0.7      | 4.5       | 512\(^3\)  |
| 8     | 1         | 0.7      | 5.5       | 256\(^3\)  |
| 9     | 1         | 0.7      | 7         | 512\(^3\)  |
| 10    | 0.1       | 2.7      | 2         | 512\(^3\)  |

4. ALIGNMENT OF VELOCITY GRADIENTS AND MAGNETIC FIELD

#### 4.1. Velocity Centroids

Observational information, such as velocity, density, intensity, and magnetic field, correspond to the projected information of the 3D medium. The projection is made along the LOS generating a 2D the plane-of-sky field.

A common technique used to study Doppler-shifted spectral lines is based on the analysis of velocity centroids. The most popular of them are the first moments of spectral line (see Münch & Wheelon 1958; Kleiner & Dickman 1985; O’dell & Castaneda 1987; Miesch et al. 1999). Velocity centroids were also suggested as a means of measuring the anisotropy of turbulence using velocity correlations (Esquivel & Lazarian 2005, 2009; Burkhart et al. 2014). A theoretical elaboration of the latter technique using the analytical description of PPV (Lazarian & Pogosyan 2000, 2004, 2008) was obtained in Kandel et al. (2016a). In the VGT we do not use correlations of centroids, but their gradients.

Velocity centroids \( C(x) \) and \( S(x) \) (normalized and un-normalized, respectively), give information on the velocity field of the medium. In addition to velocity centroids, we use the intensity of the total emission. For our model we assume that the intensity, \( I(x) \), is proportional to the column density, just like the case of optically thin H\(1 \) emission:

\[
C(x) = \int v_z(x, z) \rho(x, z) dz, \\
S(x) = \int v_z(x, z) \rho(x, z) dz, \\
I(x) = \int \rho(x, z) dz, 
\tag{5}
\]

where \( \rho \) is the density, \( v_z \) is the LOS component of the velocity, \( x \) is the position of the plane of the sky, and \( z \) is the position along the LOS. For velocity centroids of higher order, \( v_z \) can enter with different power. For instance, quadratic centroids can have \( v_z^2 \), cubic \( v_z^3 \), etc. The use of higher moments increases the velocity contribution to the measure, but it can also enhance the noise in the signal. In this work we only demonstrate the technique’s abilities using the first-order centroids. Centroids or arbitrary moments can be contracted from spectral line observations.

Using Equation (5), we construct three 2D maps with the mean magnetic field, \( B_0 \), perpendicular to the LOS: one map for the intensity (density), and two for the centroids (velocity). While velocities directly trace turbulence, our earlier studies showed that density is a much more distorted tracer, especially at high Mach numbers (Beresnyak et al. 2005; Kowal et al. 2007). The differences between the velocity and density gradient directions is a very important measure whose relevance will be studied elsewhere. Combining the two, we cannot only obtain information on the Mach number from the difference in gradient directions, but provide a much better idea of at which regions the velocity tracing may be distorted by shocks and other motions unrelated to MHD turbulence.

We compare the directions of projected velocity centroid gradients with the direction of projected magnetic fields. The magnetic fields were also projected along the LOS by

\[
B_x(x) = \int B_x(x, z) dz/\Delta z, 
\tag{6}
\]

2 The traditionally used centroids are normalized, but the study in Esquivel & Lazarian (2005) showed that for practical purposes, the normalization does not give much, but significantly complicates the statistical study. Thus we introduced un-normalized centroids, which were used for many subsequent studies (see Kandel et al. 2016a).
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64, in powers of $2^n$. The results are independent of the lag for lags larger than $\sim 8$. Below this size, the circular neighborhood presents the effects of a square grid, slightly increasing the error on the measurement. In other words, a larger lag results in greater precision in the determined direction (angle) of the maximum gradient. For this pilot study we do not use an interpolation. In the case of $r = 1$, there are accordingly only four angle options, while for $r = 10$ there are over 50. In practice, the calculation of the gradients should be performed on the scales larger that the scale at which numerical effects become important. For our simulations this scale corresponds to $r \sim 8$.

The velocity gradient field, $\Omega(x)$, which marks the direction of the maximum gradient, is defined as

$$\Omega^U(x) = x'. \quad (9)$$

$\Omega$ is therefore constructed such that in Alfvénic turbulence its direction is preferentially perpendicular to the local magnetic field. Figure 1 (third row) shows the projected magnetic field and the rotated 90° velocity gradient vector. The rotation accounts for the fact, as we discussed above, that the velocity gradients tend to be perpendicular to the magnetic field. We observe a fair alignment of the magnetic field and the rotated velocity gradients obtained with velocity centroids (see Figure 1).

We treat the velocity gradient field in the way analogous to the polarization from aligned grains (see Lazarian 2007), i.e., we do not care which way gradients are pointed, but care that they trace the direction of magnetic fields. To compare the alignment between the two vector fields, namely, $\Omega$ and $B$, one can use the angle between the two vectors, $\phi^\Omega$. We do not care which way gradients are pointed, but care that the correlation of the magnetic field direction and the gradient direction is best for high degrees of media magnetization, i.e., corresponding to $B = 10$, and it deteriorates for super-Alfvénic turbulence with $B = 0.1$. The magnetization we used in our earlier papers (see Esquivel & Lazarian 2005; Burkhart et al. 2014; Esquivel et al. 2015) was measured with Alfvén Mach number $M_A = V_L/V_A$, where $V_L$ is the injection velocity and $V_A$ is the Alfvén velocity. Therefore $M_A \sim 1/B$ for the units adopted in this paper, and the exact values can be found in Table 1.

Figure 2 shows the cumulative distribution and the histogram for the angle measurements of the velocity centroids and intensity. The histograms are normalized to have the maximum value set to one to better compare between different resolutions. Table 2 gives the standard deviation, $\sigma_{\phi^\Omega}$, of the angle distribution for both $C$ and $S$. Since $C$ and $S$ have similar angle distributions, future analysis will center on the un-normalized velocity centroid (Section 5). Figure 2 quantitatively illustrates the correlation of the magnetic field with the velocity gradients measured by velocity centroids, as well as the variation of this correlation with the level of magnetization.

The different panels of Figure 2 present the results using different styles. The first column shows the histogram or

$$B_i(x) = \int B_i(x, z) dz/\Delta z, \quad (7)$$

where $B_i$ and $B_\phi$ are the components of the magnetic field perpendicular to the LOS, and $\Delta z$ is the length of the LOS. The map of the projected magnetic field is shown in the first row of Figure 1.

4.2. The Velocity Gradient Technique

The velocity gradient information is available in observations using velocity centroids, which are the measures integrated along the LOS. Hence the gradients of velocity centroids are affected by the projection effects. The maximum gradient that we use for the analysis is defined as

$$\nabla U(x) = \max \left\{ \frac{|U(x) - U(x + x')|}{|x'|} \right\}, \quad (8)$$

where $U(x)$ is the projected information ($C(x)$ or $S(x)$), and $x'$ is defined in a circular punctured neighborhood around the point $x$ of radius $r$. For our calculations we use $r = 10$ cells. To estimate the lag effects on the velocity gradient, we modify the lag for the estimation of the velocity gradient, ranging form 2 to

Figure 1. First row: projected magnetic field. $B$. Second row: gradient field for the un-normalized centroids, $\Omega^U$. Third row: gradient field for the un-normalized centroids rotated 90° in black, and the $B$ in red for comparing, and fourth row: angle between the two vector fields, $\phi^\Omega$. From left to right the different models decrease in magnetic field strength. All plots correspond to a subregion of 100° cells of the simulation. The figure shows the technique of the velocity gradient applied to a subsection of the simulation.

$\Omega^U(x) = x'$. \quad (9)
3.1. Alignment of Density Gradients and Magnetic Fields

The VGT traces the intrinsic velocity gradient present in a turbulent medium. The turbulence also has its imprint on the density distribution (see Beresnyak et al. 2005). A similar analysis to the one used on the velocity centroids is applied to the density (intensity), $I(x)$. As shown in Figure 2, the density gradient is not well correlated with the direction of the magnetic field, giving much larger error estimates for the direction of the magnetic field (Table 2).

The correlation of the density gradients and magnetic field was first noted by Soler et al. (2013). Their gradients were calculated using the square neighborhood around the cell, with a kernel of $3 \times 3$. In Figure 2 we present the correlation of column density gradients with the magnetic field. Our earlier studies (see Beresnyak et al. 2005; Kowal et al. 2007) suggest that velocity traces magnetic fields better, and therefore velocity gradients should provide a more accurate direction of the magnetic field. Note that simulations in Soler et al. (2013) included self-gravity, which explains one of the differences in the correlation of their and our density gradients. For the simulations without self-gravity, the problematic nature of using density gradients is expected to increase for high Mach number turbulence when the density fluctuations lose a clear correlation with the magnetic field. Low-contrast density fluctuations according to Beresnyak et al. (2005) follow the picture of GS95, but high-contrast fluctuations may be perpendicular or not well aligned (see Cho & Lazarian 2003, Kowal et al. 2007).

5. PROPERTIES OF THE VELOCITY GRADIENT

5.1. Reduction Factor

In what follows, we introduce the reduction factor (RF) that measures the correspondence between two velocity gradients and the magnetic field. This RF is analogous to the Rayleigh RF in dust alignment theory suggested by Greenberg (1968). Our RF is defined as

$$R = 2 \left( \cos(\phi)^2 - \frac{1}{2} \right),$$

where $\phi$ is the angle between the velocity gradients and the magnetic field. The difference with the Rayleigh RF is that we introduce our RF for the 2D distribution instead of for the 3D distribution. $R$ is 0 for no alignment, and it is 1 when the gradients are perpendicular to the projected magnetic field.

MHD turbulence theory supports the presence of a velocity gradient perpendicular to the magnetic field. Since RF is a squared quantity of the cosine of the angle, it does not distinguish the direction of the vectors (that form the angle), making it advantageous to characterize the accuracy of the VGT.

We use the RF to measure the correspondence between the velocity centroid gradients, the intensity gradient, and the magnetic field. The values of the RF for all models are presented in Figure 3. The negative value of $R$ for the velocity centroids and column density gradients means that they tend to be aligned. The positive value of $R$ between the velocity centroid gradient and the magnetic field means that the velocity gradients tend to be perpendicular to the magnetic field, as is expected in theory.
The eddy velocity depends on the Mach number $M_S$ of the medium. A higher $M_S$ is expected to weaken the correspondence between the velocity gradient and the magnetic field because in the case of supersonic turbulence, the presence of shocks changes the properties of the velocity gradient.

To analyze the effects of the $M_S$ on the velocity centroid, we measure the spread of their distribution ($\sigma$). The spread measurements only use the velocity gradient measurements. This spread, $\sigma$, most not be confused with the spread of the angle distribution ($\sigma_B$), which requires both velocity gradient measurements and magnetic field measurements. While both measurements are affected by $M_S$ and $M_A$, $\sigma$ is an intrinsic property of the velocity gradient. With only the measurement of $\sigma$, both Mach numbers in the media are determined. We suggest that this standard deviation is used to measure the strength of the magnetic field (see Section 6). Figure 4 shows $\sigma$ as a function of $M_S$ for different $M_A$. We do not see the theoretically expected increase in the dispersion with $M_S$, which we attribute to the insufficient accuracy of our measurements. A more detailed study of the effect will be presented elsewhere.

5.2. Effects of the $M_S$ on the VGT

The eddy velocity depends on the Mach number $M_S$ of the medium. A higher $M_S$ is expected to weaken the correspondence between the velocity gradient and the magnetic field because in the case of supersonic turbulence, the presence of shocks changes the properties of the velocity gradient.

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5.3. Interferometric Studies Using Velocity Gradients

Gradients can be measured using interferometers, and their advantage is that it is not necessary to first restore the images. For instance, in Gaensler et al. (2011) and Burkhart et al. (2012), synchrotron polarization gradients were used to study turbulence for data that lacked single-dish observations.

The velocity centroids can be calculated using raw interferometric data (see Kandel et al. 2016b). Gradients can be obtained from these data and can be used to trace magnetic fields in distant objects, such as other galaxies. Examples of this application will be presented in future work.

5.3.1. The Effects of the Base Line on the Gradient Calculation

Spectroscopic observations in PPV format permit measuring the velocity properties of the medium, which in turn allows measuring the turbulent velocity field. This is fundamental since density measurements do not always trace the turbulent medium and its properties (Chepurnov et al. 2008). To understand the limitations and scales that are needed in spectroscopic observations when the VGT is to be used, we transform the simulated data and remove the long and short baselines. This is done using a Fourier transformation on the PPV data cube and filtering different scales. We remove information at 10%, 5%, and 1% of the inner cells (small $k$, long $l$) and outer cells (big $k$, short $l$), where $k$ is the wave number of the resolution Fourier plane and $l$ is the telescope baseline. Once the information is removed, we reconstruct the plane of the sky for the velocity and the intensity, and then apply the VGT. Therefore removing long baseline $l$ decreases the resolution at smaller scales in the velocity and intensity maps. The removal of the short $l$ does not affect the technique; it gives the same $\sigma$ value for all cases, but the opposite does change the value of $\sigma$. It is therefore important to have a high resolution at small scales.

6. OBTAINING THE MAGNETIC FIELD INTENSITY: ANALOG OF THE CHANDRASEKHAR-FERMI TECHNIQUE

The C–F technique and its later modifications have been used to determine the intensity of the magnetic field on the plane of the sky using the velocity dispersion and dust polarization measurements (see Falceta-Gonçalves et al. 2008, and ref therein). Here we use the fact that velocity gradients similar to aligned grains tend to align perpendicular to the magnetic field. Therefore here we propose a new technique that is analogous to the C–F, but the gradients of velocity centroids are used instead of polarization vectors. Equation (1) can be rewritten in terms of the velocity centroid for the velocity dispersion, and the angle dispersion from the velocity gradient ($\sigma^\theta$):

$$B = \gamma \sqrt{4 \pi \rho} \frac{\delta v}{\sigma^\theta},$$

where $\gamma$ is the correction factor that can potentially depend on $M_S$ and other parameters, e.g., the self-gravity effects. In the classical C–F method there is also an analog of our $\gamma$. For instance, the factor $\sim 0.5$ was suggested using numerical simulations in Ostriker et al. (2001). More accurate expressions have also been suggested (see Falceta-Gonçalves et al. 2008).
However, for the sake of simplicity, in this study we use Equation (11).

In order to determine the \( \gamma \) parameter for our technique, we calculate the values for Equation (11) for the first three models (same \( M_S \), but different \( M_A \)). \( \delta v \) is the dispersion of the velocity field measured by the normalized velocity centroid, and \( \sigma^{fi} \) is the dispersion of the velocity gradient (see Figure 4 for the different values). \( \gamma \) stands for the intensity \( (I) \), and both velocity centroids have \( \sim 1.29 \). The reason behind a larger parameter for the modified C–F method is a larger dispersion of the velocity gradient than the dispersion found in the traditional CF technique. Figure 5 shows the relative error estimation on the projected magnetic field. The value of \( \gamma \) is an average of the values obtained for the different gradients. Our results should be treated only as a demonstration of the potential applicability of the technique. For instance, in Figure 5 the errors for supersonic turbulence seem smaller than those of subsonic turbulence; this is a pure effect of the \( \gamma \) choice and not a condition on the turbulence properties. We expect that the errors of the technique can be significantly reduced by using a more sophisticated expression for the magnetic field strength, as well as an improved procedure of calculating velocity gradients.

To account for different degrees of compressibility in the medium, we use all data cubes (Table 1). We know that the velocity gradient dispersion is highly correlated with \( M_A \) and loosely correlated with \( M_S \) (Section 5.2), hence some dependence is expected. Since the C–F method also incorporates the spread of the velocity field, the overall measurements depend weakly on \( M_S \) and do not require extra parametrization.

7. ADDITIONAL EFFECTS

Further analyses of the properties of the velocity gradient are made with the correlation functions and statistical moments (see Appendix A). They all show that the velocity gradient is susceptible to different degrees of magnetization and that the velocity gradient aligns perpendicularly to the magnetic field.

7.1. Telescope Resolution Effects

Finite telescope resolution introduces additional uncertainties. In order to account for data averaging within the telescope beam, we use two different smoothing kernels on the velocity centroids—a square, and a Gaussian kernel. For the square kernel, each point in the velocity centroid is replaced with the average of the points in its vicinity. In this case, the vicinity was defined as square boxes of lag \( r = 2^n \), with \( n \) from 0 to 6. The velocity gradient is then estimated using the smooth velocity centroids and the unsmoothed magnetic field. This process reduces or increases the different values, as seen in Table 3. The process of smoothing is thus not a technique to enhance the results in all cases.

The Gaussian smoothing kernel was used to simulate more realistic observational data. Namely, observational data do not have a pencil-thin beam resolution, but more of a smooth beam resolution. We use four values for the FWHM of 2, 4, 8, and 16 (see Table 4).

7.2. Noise

Realistic observational data have intrinsic noise. We add artificial noise to the simulations to understand the changes in the VGT. We introduce Poisson noise to the projected data (the plane of the sky) with signal-to-noise ratios (S/Ns) of 10, 50, 100, and 400. The noise is added independently of both the intensity and velocity maps. Using the same procedure as described in Section 4.2, we obtain the gradient for the velocity gradient.
centroids and the intensity for all five models. We then estimate the standard deviation for the different noise levels. For most cases, the changes in the standard deviation are smaller than $0.5^\circ$ compared to the noiseless data. Table 5 shows the change in the standard deviation between the case with an S/N of 10 to the noiseless data.

### 7.3. Column Density Effects

Measurements of magnetic fields strengths are affected by column density effects (Evans 1999; Clark et al. 2014; Ntormousi et al. 2016; Planck Collaboration et al. 2016b). To understand the effects of column density in the VGT, we divided the data sample into low, medium, and high column density for models 1 and 2 (Table 1). The gradient for the intensity and the un-normalized velocity centroid as calculated in Section 4 are distributed into three groups using the column density criterion. Using only the low and high column density data, the cumulative distribution is obtained as shown in Figure 6. For both degrees of magnetization and both gradients, $\Omega^l$ and $\Omega^h$, the low column density has the smallest errors, followed by the global calculation, the high column density (Table 6). It is important to note that these simulations are performed in the subsonic regime where there are no shocks in the medium. In the presence of shocks, the velocity gradient can align with the magnetic field, which changes the properties of the technique (see Section 5.2).

#### 7.4. Sub-block Analysis

The VGT can assess the direction of the magnetic field with good point-wise correspondence. The values reported previously use the full simulated domain, but it is important to understand the limitations of smaller data samples. To analyze this effect, we subdivided the simulation into individual regions with different numbers of cells. At each region, we apply the VGT and obtain the velocity gradient. With the velocity gradient we obtain the standard deviation of the angle distribution ($\sigma_\theta$). For most regions the standard deviation, $\sigma_\theta$, was smaller than the deviation found in the full simulated box. To understand how the different standard deviations vary as a function of the number of cells, we estimate the spread ($\sigma_\theta$) and the mean ($\mu_\theta$) of the different $\sigma_\theta$ along the different regions. Figure 7 shows the mean, $\mu_\theta$, as a function of the number of cells, and the error bars correspond to the standard deviation, $\sigma_\theta$. It is clear that our technique is effective: even with small data samples, the technique works. If the observed or simulated media have shocks, a sub-grid analysis that removes (or properly analyzes) these regions would give a better overall performance of our technique.

#### 8. COMPARISON WITH OTHER TECHNIQUES

Many other techniques proposed to study magnetic fields have limitations. The measurements of polarization arising from aligned dust are made in the optical/near-IR for stars and in the far-IR/submillimeter for dust. The interpretation of these measurements requires an understanding of the dust alignment.
and modeling its failure for high optical depths. In spite of the significant progress of grain alignment theory (see Lazarian & Hoang 2007, 2008; Hoang & Lazarian 2008, 2009, 2016), the magnetic field studies employing dust polarimetry are not straightforward. In some cases it is necessary to provide detailed modeling of the radiation field as if the radiative field were insufficient, the grain alignment fails and does not reveal magnetic fields (see reviews by Andersson et al. 2015; Lazarian et al. 2015b). In addition, high-resolution high-sensitivity polarization data are not readily available. For instance, a great dust polarization map has been obtained by the Planck Collaboration et al. (2016a, see references therein). To obtain a map with a better resolution, we need to wait years for the next analogous mission. Tracing magnetic fields with higher resolution using the VGTI may be much easier.

In addition, the classical C–F method requires both polarization and spectroscopic velocity measurements to determine the intensity of the magnetic field. The technique that we suggested using velocity centroid gradients only requires the velocity measurements, which is a simplification.

The direction of magnetic fields can also be obtained using statistical techniques that use the predicted anisotropies of MHD turbulence (Lazarian et al. 2002; Esquivel & Lazarian 2005). These variations of this techniques using the PCA was used and showed that the magnetic fields are in agreement with the measurements using dust polarization (Heyer et al. 2008). However, being statistical in nature, the techniques are only able to provide average magnetic field directions. In comparison, the technique of tracing magnetic fields with the velocity gradients that we introduced in this paper provides a more detailed information of the magnetic field. The synergy of all these techniques is to be revealed in future publications.

We would like to stress the exploratory nature of our present study. We did not seek to provide detailed prescriptions for a better tracing of magnetic fields and for obtaining the magnetic field intensity from spectroscopic observations. This is the goal of further studies. Instead, we introduced a new way of tracing magnetic fields and showed its practical applicability using synthetic observations. In this way, we obtained encouraging results that stimulate further in-depth studies.

We would also like to emphasize that velocity gradients should not be treated as mere proxies of the magnetic field direction or only an alternative technique for tracing the magnetic field without polarimetric measurements. Velocity gradients are the measures of the interstellar physics. For instance, they are expected to respond to shocks and self-gravity in different ways compared to magnetic fields. Therefore the misalignment of velocity centroid gradients and dust polarization may be very informative. Similarly, the studies of the relative alignment of the velocity centroid gradients and the column density gradients opens up a new avenue for exploring interstellar physics. Thus we expect that the three measures of velocity gradients, density gradients, and dust polarization will be used simultaneously whenever possible.

The synergy of the VGT and other techniques giving magnetic field direction is still to be explored. Some advantages are obvious even now. For instance, the Goldreich–Kylafis technique as well as the technique based on atomic/ionic alignment (see Yan & Lazarian 2015) can provide a magnetic field with an ambiguity of 90°, which may be confusing. The VGT may be used to remove the ambiguity. We do not expect the qualitative nature of the VGT technique to be changed in the presence of self-absorption (see Lazarian & Pogosyan 2006, for more details). In the conditions where the infall induced by self-gravity is important, the alignment of velocity gradients and magnetic field can change. This issue is studied elsewhere.

Our work shows the advantages of using a theory-motivated approach to develop techniques for studying magnetic fields and turbulence from observations. The prediction that velocity gradients are expected to be perpendicular to the magnetic field follows from the MHD turbulence theory provided that the Alfvénic modes dominate the contribution of the fast modes (see Cho & Lazarian 2002, 2003). In this paper we proved that this provides a new way to study magnetic fields observationally.

In general, the synergy of different techniques is probably best suited for tracing magnetic fields, and our new technique, namely, the VGT in combination with other techniques, can be useful for studying magnetic fields in the diffuse ISM and molecular clouds.

9. CONCLUSIONS

This work presented a new technique, the VGT, to trace the magnetic field and to estimate its magnitude using only spectroscopic data. The method is based on the fact that the eddies align with the local 3D magnetic field and that this creates eddy velocity gradients perpendicular to the direction of the field. To test the technique, we used synthetic observations constructed with 3D MHD simulations. Further analysis with a larger set of initial conditions should be explored to fully understand the implications and limitations of this technique. A summary of the work is given as follows:

1. For observational studies, the velocity gradients can be represented by the velocity centroid gradients $\Omega$ that we showed trace the direction of the projected magnetic field reliably.

2. We proposed and successfully tested a new technique for estimating the level of magnetization of the media given by the Alfvén Mach number $M_A$ and the magnetic field intensity. This new technique only requires spectroscopic velocity data and does not require any polarimetry measurements.

3. We showed that the VTG can work in the presence of averaging arising from finite telescope resolution. The VTG can also employ interferometry data when some of the baselines are lacking. This opens up prospects of using the VTG for a wide variety of objects, including extragalactic magnetic field studies.

4. Our work suggests a synergy may be found in a simultaneous use of the VTG, polarimetry data, and density gradients in combination to study magnetic fields in order to explore the star formation processes in turbulent magnetized ISM.

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APPENDIX A
EXTRA PROPERTIES OF THE VELOCITY GRADIENTS

The velocity gradients are affected by noise in the data, telescope beam effects, and column density effects that change their observational properties, but they also present effects related to the level of the magnetization. Here we detail how the properties of the velocity gradient change. The change is measured with correlation functions and statistical moments.

A.1. Statistical Moments of the Velocity Gradient

The different levels of magnetization produced in the medium modify the distribution of the velocity gradient, \( \Omega \). These differences in the distribution are quantified by the statistical moments. We use the L-moments to understand its properties and relate them to the Alfvénic Mach number. L-moments, introduced by Hosking (1990), measure the properties of the distribution like regular statistical moments. We use the L-moments to understand its statistical moments. We use the L-moments to understand its properties and relate them to the Alfvénic Mach number.

The L-moments used are as defined in Wang (1996), and the L-moment ratios are \( l_3 = l_1/l_2 \) and \( l_4 = l_2/l_1 \):

\[
\begin{align*}
l_1 &= \frac{1}{C_{1}^{n}} \sum_{i=1}^{n} x_i, \\
l_2 &= \frac{1}{2} \frac{1}{C_{2}^{n}} \sum_{i=1}^{n} (C_{1}^{i-1} - C_{1}^{n-1}) x_i, \\
l_3 &= \frac{1}{3} \frac{1}{C_{3}^{n}} \sum_{i=1}^{n} (C_{2}^{i-1} - 2C_{1}^{i-1} C_{1}^{n-1} - C_{2}^{n-1}) x_i, \\
l_4 &= \frac{1}{4} \frac{1}{C_{4}^{n}} \sum_{i=1}^{n} (C_{3}^{i-1} - 3C_{2}^{i-1} C_{1}^{n-1} - 3C_{1}^{i-1} C_{2}^{n-1} - C_{3}^{n-1}) x_i, \\
&\quad + 3C_{1}^{i-1} C_{2}^{n-1} - C_{3}^{n-1}) x_i,
\end{align*}
\]

where \( x_i \) is the data sample, and \( C_{m}^{n} \) is the number of combination of \( m \) items from \( n \) defined as

\[
C_{m}^{n} = \frac{m!}{k!(m-k)!}.
\]

Because L-moments are a linear function of the data, they are less susceptible to sampling variability (such as outliers in the data) than conventional moments. The L-moment ratios such as L-skew \( (l_3) \) and L-kurtosis \( (l_4) \) have the property \( |l_3| < 1 \).

The two components of the velocity gradient (parallel and perpendicular to the mean magnetic field) and the magnitude of the gradient are analyzed separately using the L-moments for the full angle span (0°–180°). \( \Omega_{x}^{\parallel} \) is the component of the velocity gradient that is parallel to the mean magnetic field, while \( \Omega_{z}^{\perp} \) is in the perpendicular direction. L-skew and L-mean are close to zero for all the cases of the velocity gradient components, giving no information on the intensity of the magnetic field. The L-mean and L-skew for \( \Omega_{x}^{\parallel} \) is always constant (as a function of \( M_{A} \)) since its distribution always has a peak around 90° (perpendicular to the magnetic field), for \( \Omega_{x}^{\perp} \) the lack of changes is due to no preferential direction of motion set by the magnetic field and therefore a more homogenous medium. The L-kurtosis and L-mean are shown as a function of the Alfvénic Mach number in Figure 8. The moments are only a function of \( M_{A} \) for \( \Omega_{x}^{\parallel} \) since this component is susceptible to the changes in the intensity of the field. The decrease in the L-kurtosis and increase in the L-scale for \( \Omega_{x}^{\parallel} \) reflects that \( \Omega_{x}^{\parallel} \) is mostly perpendicular to the magnetic field with most of its components around zero. This implies that given a distribution of the velocity gradient, one can estimate the intensity of the magnetic field and its global direction by measuring the L-moments for both components. To estimate the mean field direction, it is necessary that the components of the velocity gradient match those presented here.

A.2. Anisotropy

Correlation functions are a two-point statistical tool. In a turbulent medium they can be used to measure the power spectrum of the energy cascade and to analyze the anisotropy of the medium (Esquivel et al. 2003; Esquivel & Lazarian 2005):

\[
\text{CF}(r) = \langle (f(x) \cdot f(x + r) \rangle,
\]

where CF is the correlation function, \( r \) is the “lag,” \( \langle \cdot \rangle \) denotes the average over all points, and \( f \) denotes the desired function—in this case the velocity centroids, intensity, and velocity gradient. The power and energy spectrum can be estimated by a correlation function of the velocity centroid.

The anisotropies of the medium are measured using the correlation functions by making the lag a function of the angle to the global mean magnetic field, \( r(\theta) \). The angle \( \theta \) has a span of 90°, going from \( r(0°) = r_{\parallel} \) \( (r_{\parallel} \times B = 0) \) to \( r(90°) = r_{\perp} \) \( (r_{\perp} \cdot B = 0) \). In an isotropic medium the values of the correlation function should be independent of the direction of the lag, i.e., \( \text{CF}(r_{\parallel}) = \text{CF}(r_{\perp}) \). In the case of an anisotropic medium, such as in Alfvénic turbulence, the correlation presents a preferential direction. The preferential direction is set by the mean magnetic field—in other words, the correlation function changes depending on the direction of the mean magnetic field. The intensity of the magnetic field determines the level of the anisotropy and hence the elongation of the isocontours of the velocity centroids (Figure 9). The mean direction of the magnetic field sets the elongation direction of
the isocountours. Even if the CF can trace the mean magnetic field, as several other techniques, it is important to understand new ways to trace them such as the VGT. Moreover, the CF requires many sample data to roughly estimate the direction, and so is important to find techniques that can map the magnetic field on a smaller scale.

Applying the same method of correlation functions to the velocity gradient $\Omega$, one can see that just as the velocity centroid is affected by the intensity of the magnetic field, so is the velocity gradient. Hence the correlation function of the velocity gradient can determine the level of magnetization of the medium and the direction of the mean field to determine if the VGT can be used.

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