Efficiency enhancement of THz radiation from an electron bunch in a waveguide due to low-frequency stabilization

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Abstract. Simultaneous generation of pulses of high-frequency Super-Radiance and low-frequency Coherent Spontaneous Radiation from an ultra-relativistic electron bunch moving in a waveguide placed in a spatially periodic and / or uniform longitudinal magnetic field is studied. The low-frequency radiation can weaken expansion of the bunch and decrease a velocity spread. This significantly improves electron micro-bunching and increases energy of high-frequency pulses. Such effect makes possible fairly efficient high-frequency cyclotron radiation of the bunch near an autoresonance regime, when the wave phase velocity is very close to speed of light.

1. Introduction

Photo-injectors allow the formation of picosecond and sub-picosecond bunches of ultrarelativistic electrons with a large charge [1-4]. Such bunches can be used to generate powerful electromagnetic pulses in various frequency ranges, in particular in the terahertz range [5-18]. One of obstacles to creating efficient sources of coherent radiation in this way is the longitudinal Coulomb repulsion of particles, leading to rapid expansion of the bunch length and a decrease in radiation power. In works [19] and [20], mechanisms of radiative self-compression and phase size stabilization of bunches were described for the cases of undulator and cyclotron Coherent Spontaneous Radiation, when the phase size of the bunch is less than $2\pi$. These effects are mostly significant at the group synchronism, when the group velocity of radiation is close to the longitudinal velocity of particles, and a bunch duration is of the order of half the emitted wavelength. In this paper, it is proposed to use this method to obtain coherent undulator radiation of bunches in a waveguide in the regime of simultaneous excitation of low- and high-frequency waves that greatly differ in frequency, so that the low-frequency wave is emitted in the Coherent Spontaneous Radiation regime and stabilizes a bunch size that accelerates self-micro-bunching of particles and leads to a significant increase in the efficiency of high-frequency Super-Radiance.
2. Stabilization in two-frequency regime

Consider the particle motion and radiation in a cylindrical waveguide immersed in a periodic transverse undulator field and guiding uniform magnetic field or only in axial magnetic field (Fig. 1 a). The condition of undulator or cyclotron resonances between a particle oscillating in such fields and a wave radiated by it in a waveguide is as follows:

$$\omega - k_z v_z = s\Omega,$$  

(1)

here, $\omega$ and $k_z$ are the wave frequency and the longitudinal wavenumber, $v_z$ is the particle longitudinal velocity, $s$ is the number of harmonic of the oscillation frequency $\Omega$; for the undulator field with the period $d$, $\Omega = 2\pi v_z/d$ is the undulator frequency and for the uniform axial field $H_0 z_0$, $\Omega = eH_0/m\gamma c$ is the cyclotron frequency, $e$, $m$ and $\gamma$ are the electron charge, mass, and Lorentz factor. There are two characteristic radiation regimes for electrons in a waveguide depending on the value of $\Omega$ (Fig. 1 b): 1) tangency of dispersion characteristics (group synchronism, G), and 2) intersection of the dispersion characteristics of the bunch and wave, at which high-frequency (H) and low-frequency (L) waves are simultaneously excited.

![Figure 1. (a) A bunch of electrons moving along helical trajectories in a cylindrical waveguide; (b) regimes of grazing (G) and two-frequency intersection (H and L) of the dispersion characteristics of the wave and bunch.](image)

If an initial bunch length is less than the low-frequency wavelength, $l < \lambda_L$ the low-frequency pulse is generated in the regime of Coherent Spontaneous Radiation. At the same time, the bunch can be long and initially non-micro-bunched on the scale of a high-frequency wave, $l >> \lambda_H$. The low-frequency generation of an oscillated bunch begins immediately. The high-frequency generation starts from incoherent spontaneous emission of electrons, which slowly (over a considerable length) causes a corresponding, at first very weak, high-frequency seed micro-bunching. The bunch “overtakes” the maximum of low-frequency pulse emitted by it, so the efficiency of the low-frequency generation is significantly lower, than at the group synchronism when the longitudinal velocity of particles is close to the group velocity of the radiated wave, $v_z = v_g$. At the same time, the group velocity of the high-frequency pulse is much closer to the longitudinal velocity of the bunch and its interaction with particles may be more efficient.

The compression in the undulator field is highest at the group synchronism and a bunch length close to half the emitted wavelength [19]. In the case of a large mismatch from the group synchronism, the length of the bunch weakly decreases under the influence of the wave, but if the bunch length is even close to the wavelength, some stabilization of the bunch longitudinal size is still ensured. In the input part of the waveguide, the amplitude of the low-frequency wave is small. As the amplitude grows, the maximum of the decelerated phase of the low-frequency wave shifts, following the centre of the electron bunch towards the bunch “tail”, particles of the “tail” are gradually concentrated in a stable “zero” phase of the wave. Accelerating particles in the front fall into another stable zero phase, where they are also held by the wave field. As a result, the bulk of charge concentrates between neighboring stable neutral phases of the low-frequency wave, the bunch does not further expand (length stabilization), which obviously contributes to the excitation of the high-frequency wave and further self-consistent micro-
bunching of particles in its field (Fig. 2a). Such stabilization significantly improves the conditions for the generation of a high-frequency pulse.

In the case of cyclotron radiation, if the longitudinal velocity of particles exceeds the group velocity of the low-frequency wave, decrease in electrons energies due to the interaction with the wave leads to an increase in longitudinal velocities, and vice versa. Initially, the bunch length increases because of Coulomb repulsion. The center of the bunch accelerated by the field and the maximum of the so called “accelerating” phase follows the center of the bunch, and charge density increases at the front. The bunch splits into two bunches, centres of which concentrate near the neutral phases of the low-frequency wave (Fig. 2b) and the regions of increased density is in the front. Sufficiently compression of the formed bunch (up to order of the short wavelength) makes generation at the high-frequency possible, but a part of electrons is not included in this process.

The considered dynamics of bunch particles indicates that the optimal bunch length for the high-frequency undulator radiation is slightly less than the low-frequency wavelength. However, for the high-frequency cyclotron radiation, a more preferred is the bunch length close to the half the low-frequency wavelength or shorter, when the loss of particles is smaller.

3. Undulator radiation

In the simulations, we will be guided by the main expected parameters of the photo-injector of the Schlesinger Family Center of Ariel University (Israel), that is designed to generate terahertz pulses [12]. In accordance with the capabilities of the setup, the bunch energy of 6 MeV is supposed in the simulations, the bunch duration $T_e = 4$ ps, radius $R_e = 0.5$ mm, and charge 1 nC. For simplicity, we assume that bunches obtained in the photo-injector and formation section have a cylindrical shape at the entrance to the radiation section with a charge density independent of the radial and azimuthal coordinates, but gradually decreasing to the ends according to the law $\rho/[1 + \exp\{t - T_e/21/t\}]$ with the smoothing parameter $\tau = 0.2$ ps characterizing the bunch fronts. To realize the single-mode regime of simultaneous excitation of two electromagnetic pulses at the fundamental undulator harmonic and to obtain a high radiation frequency, we consider an operating waveguide with a small radius of 1 mm, placed in a micro-undulator with a period of 15 mm with a strong transverse undulator field $H_u = 0.5 T$
and guiding field \( H_0 = 1.5 \, T \), correspondingly, and the normalized transverse velocity \( \beta_u = V_u/c \approx 1/\gamma \). The low and high frequencies corresponding to the initial electron parameters are \( f_L = 210 \, GHz \) and \( f_H = 3.2 \, THz \), respectively. We divide the bunch in the longitudinal direction into N disks with charges determined by the distribution function \( Q(\vartheta) \), where \( \vartheta \) is the disk phase, to describe Coulomb interaction inside a bunch [18]. The change in the energies of the particles located in the considered disk is determined by their interaction with the excited low-frequency and high-frequency pulses, as well as by the Coulomb interaction with all other disks:

\[
\frac{dy}{d(\omega_H t)} = \frac{1}{\gamma_z^2} \frac{\partial y}{\partial \tau} = -f_{w,l} - f_{w,h} - f_c. \tag{2}
\]

Here \( \gamma_z^2 = 1/(1 - \beta_z^2) \) is the longitudinal Lorentz-factor, \( \beta_z = V_z/c \) – the normalized longitudinal electron velocity, \( \tau = \omega_H(t - V_{z,0}z/c^2) \); \( f_{w,l} = \frac{\beta_u}{2} \text{Re}[a_H \exp(i\vartheta_H)] \), \( f_{w,l} = \frac{\omega_L}{\omega_H} \frac{\beta_u}{2} \text{Re}[a_L \exp(i\vartheta_L)] \), \( a_{H,L} = eA/mc\omega_{H,L} \) are normalized amplitudes of high- and low-frequency waves, \( f_c(\vartheta) = \frac{2i\epsilon^2\tau^2}{l_A^2 \gamma_z^2} \sum Q(\vartheta)(\text{sign}\Delta \vartheta)D(\Delta \vartheta) \), \( I \) – bunch current, \( I_A = 17 \, kA \), \( D = \left\{ 1 - \left[1 + \left( \frac{k_H^2 R_L^2}{\gamma_z \Delta \vartheta_H} \right)^2 \right]^{\frac{1}{2}} \right\} \) is the normalized Coulomb field of the disk at its axis. Equation for slowly varying normalized amplitude of the high-frequency wave has a form

\[
2i\epsilon \frac{\partial a_H}{\partial \tau} - 2i\epsilon_H \frac{\partial a_H}{\partial \zeta} - \frac{\partial^2 a_H}{\partial \zeta^2} = iG_H \beta_u \rho L \frac{F(\zeta)}{\omega_H T_e}, \tag{3}
\]

\( \zeta = k_{z,H}(z - V_{z,0}t), s = (1 - \beta_z \beta_{g,H}), \epsilon_H = (\beta_z - \beta_{g,H}) \gamma_z^2 \) is the parameter of slipping of the electron bunch relative to the emitted pulse.

The radiated low-frequency pulse duration significantly exceeds a bunch duration, that allows use of approximate simple integral solution

\[
a_L(\zeta = 0, \tau) = \frac{\omega_L G_L}{\omega_H} \left[ \frac{l}{2\pi} \int_0^\tau \frac{\beta_u \rho L}{2\sqrt{\tau - \tau'}} \exp \left( - \frac{i\omega_L \epsilon_L^2 (\tau - \tau')}{2\omega_H} \right) d\tau' \right]. \tag{4}
\]

For the case of a helical undulator with a guiding homogeneous field, changes in particle phases relative to the emitted waves are described by equations:

\[
\frac{1}{\gamma_z^2} \frac{\partial \vartheta_H}{\partial \tau} = -\mu \Delta \gamma, \quad \frac{1}{\gamma_z^2} \frac{\partial \vartheta_L}{\partial \tau} = -\frac{\omega_L}{\omega_H} \mu \Delta \gamma, \tag{5}
\]

where, the parameter of the phase bunching is determined by the approximate formula \( \mu = \frac{1}{\gamma^3} \left( 1 - \frac{K}{\Delta} \right) \), \( K = \frac{e\mu_0 d}{\pi mc^2} \) and \( \Delta = (1 - \frac{\omega_d}{\omega_q}) \) are the undulator parameter and the resonance mismatch between forced undulator and free cyclotron oscillations.

The low-frequency Coherent Spontaneous Radiation begins immediately (Fig. 3a), but due to sufficiently large mismatch of the group synchronism the saturated wave intensity and generation efficiency are relatively low, since the maximum of the low-frequency pulse lags the bunch. The field of the low-frequency radiation holds the main charge in the bunch centre (Fig. 3b) (Fig. 3b). Such stabilization of the bunch length drastically improves the particles’ micro-bunching in the field of the simultaneously generated high-frequency pulse and increases the efficiency of its generation (compare Fig. 3 a with Fig. 3 d). In the case of only high-frequency radiation, the maximum efficiency is 3 times lower, and Coulomb bunch expansion prevents the bunching process (Fig. 3 c), the saturation reaches slower (Fig. 3 d). In the regime of two-frequency generation, the efficiency at the high frequency exceeds 2%, that is sufficiently high for such frequencies.
Figure 3. (a): Efficiency of high-frequency (solid purple curve) and low-frequency (dashed blue curve) generation from a bunch with a charge of 1 nC and an initial duration of 4 ps as a function of the longitudinal coordinate; (b): longitudinal positions of particles during two-waves generation. (c): Efficiency of high-frequency generation as a function of the longitudinal coordinate, with (dashed) and without (solid line) the simultaneous excitation of a low-frequency pulse; (d): longitudinal positions of particles during generation of only the high-frequency pulse.

4. Cyclotron radiation

In our works [18, 21-24], the possibility of low-frequency wave stabilization in the case of motion of the bunch in uniform magnetic field was already mentioned. In this case, Eqs. (2) – (5) should be supplemented by the equation for the longitudinal electron momentum $p_z = \beta_z \gamma$:

$$\frac{1}{\gamma_{z,0}^2} \frac{\partial p_z}{\partial \tau} = -\beta_{g,l} f_{w,l} - \beta_{g,h} f_{w,h} - \frac{f_c}{\beta_z}.$$  

Besides, equations for changes in particle phases relative to excited waves in approximation of inertial electron bunching (5) are written as follows:

$$\frac{1}{\gamma_{z,0}^2} \frac{\partial \theta_{H/L}}{\partial \tau} = \frac{\omega_{H/L}}{\omega_H} \left \{ 1 - \beta_z \beta_{g,H/L} - \gamma_0 \frac{\Delta \gamma_c}{\omega_{H/L}} \right \}.$$  

(7)

There is the two-dimensional bunching since the cyclotron frequency of a relativistic particle depends on its energy. In the case of the only one radiated high-frequency pulse Eq. (7) is as follows [18]:

$$\frac{1}{\gamma_{z,0}^2} \frac{\partial \theta_H}{\partial \tau} = \left \{ 1 - \beta_{g,H} \right \} \left \{ \beta_{g,H} f_{w,H} \frac{\Delta \gamma_c}{\gamma_0} + (1 - \beta_{g,H}^2) \Delta \gamma_w \right \},$$  

(8)

where, $\Delta \gamma_c$ and $\Delta \gamma_w$ are the energy changes caused by the Coulomb repulsion and high-frequency pulse, respectively. The group velocity of the high-frequency pulse is close to the initial particle longitudinal velocity and to the speed of light ($\beta_{g,H} \approx 1$), when the bunching process is very weak [25, 26], and the efficiency of radiation must be small. The simulations confirm it. In the simulations for the cyclotron case, we use some parameters different from those in the undulator case, namely, the guiding magnetic field 8.7 T; the bunch duration and charge are 2 ps and 0.5 nC. The efficiency of high-frequency radiation without simultaneous low-frequency radiation (Fig. 4 a) is significantly lower, than in the regime of two-frequency generation. The short dense bunch is formed by the low-frequency pulse excited in regime of Coherent Spontaneous Radiation (Fig. 4b) that makes possible a fairly efficient high-frequency Super-Radiance even very close to the conditions of exact autoresonance.
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Figure 4. (a): Efficiency of high- and low-frequency cyclotron generation from a bunch with a charge of 0.5 nC and an initial duration of 2 ps as a function of the longitudinal coordinate for the cases of two waves (solid purple curve) and one wave (dashed purple curve) excitation; (b): longitudinal positions of particles during two-wave generation.

5. Conclusion
The low-frequency Coherent Spontaneous Radiation of a short dense electron bunch in a waveguide can stabilize the length of the bunch and thereby help accelerate the high-frequency micro-bunching of particles and make much more efficient their high-frequency Super-Radiance for the both studied undulator and cyclotron mechanisms. In the cyclotron case, it is possible even in the regime of very close to the autoresonance. The efficiency of high-frequency generation exceeds 2% and its saturation is reached after several tens cm. For an additional validation, the described assumption was also examined and confirmed with a more accurate 3D, space–frequency approach using WB3D numerical code [23,27].

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