Bound $q\bar{q}$ systems in the framework of two-body Dirac equations obtained from different versions of 3D-reductions of the Bethe-Salpeter equations

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Abstract

The two-body Dirac equations for the bound $q\bar{q}$ systems are obtained from the different (five) versions of the 3D-equations derived from Bethe-Salpeter equation with the instantaneous kernel in the momentum space using the additional approximations. There are formulated the normalization conditions for the wave functions satisfying the obtained two-body Dirac equations. The spin structure of the confining $qq$ interaction potential is taken in the form $x\gamma_1^0 \otimes \gamma_2^0 + (1 - x)I_1 \otimes I_2$, with $0 \leq x \leq 1$. It is shown that the two-body Dirac equations obtained from the Salpeter equation does not depend on $x$. As to other four versions such a dependence is left. For the systems $(u\bar{s})$, $(c\bar{u})$, $(c\bar{s})$ the dependence of the stable solutions of the Dirac equations obtained in the different version on the mixture parameter $x$ is investigated and results is compared with such dependence of 3D-equations derived from Bethe-Salpeter equations without the additional approximation and some new conclusions are obtained.

Keywords: Bethe-Salpeter Equation, Quasipotential Approach, $q\bar{q}$ systems, Two-Body Dirac equations, normalization condition

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1 Introduction

The Bethe-Salpeter (BS) equation provides a natural basis for the relativistic treatment of bound $q\bar{q}$ systems in the framework of the constituent quark model. But due to fact that the BS wave function (amplitude) has not probability interpretation, three-dimensional (3D) reduction is necessary. Review of investigations of bound $q\bar{q}$ systems (mesons) on basis of equations obtained in different versions of 3D-reduction of BS equation in the instantaneous (static) approximation for kernel of BS equation is given in Ref. [1]. In literature there are known five such versions formulated in Refs. [1]-[7], noted as SAL [2], GR [3], MW [4], CJ [5] and MNK[6], [7] versions. The last four 3D-equations have correct one-body limit (the Dirac equation) when the mass of one of the particles tends to infinity. As it is well-known the Salpeter has not such a limit. Note that Gross equation is obtained only for $m_1 \neq m_2$ case, while other versions work for the equal masses ($m_1 = m_2$) too.

Below we shall consider the problem how to get two-body Dirac equations for the bound $q\bar{q}$ systems from above mentioned 3D-relativistic equations, and how to formulate corresponding normalization conditions for the wave functions. Then, these equations will be used for the investigation of some aspects of the problem connected to the mass spectra for bound $q\bar{q}$ systems (mesons). Namely, the dependence of the existence of stable solutions of obtained equations and of the mass spectra on the Lorentz(spin) structure of confinement potential will be studied. Further, the comparison to the results obtained without additional approximations will be made.

2 The two-body Dirac equation for bound $q\bar{q}$ systems and normalization conditions for the corresponding wave functions

To derive such an equation note that all 3D-equations given in Ref. [1] can be written in the common form (c.m.f.)

$$[M - h_1(p) - h_2(-p)]\Phi_M(p) = \Pi(M; p)\gamma_1^0 \otimes \gamma_2^0 \int \frac{d^3p'}{(2\pi)^3} V(p, p')\Phi_M(p')$$

(1)

where

$$\Pi(M; p) = \frac{1}{2} \left( \frac{b_1}{\omega_1} + \frac{b_2}{\omega_2} \right), \quad (\text{SAL})$$

$$= \frac{1}{2} \left( \frac{1 + h_1}{\omega_1} \right), \quad (\text{GR})$$

$$= \frac{1}{2} \left( \frac{h_1}{\omega_1} + \frac{h_2}{\omega_2} \right) + \frac{M}{\omega_1 + \omega_2} \left( 1 - \frac{h_1}{\omega_1} \otimes \frac{h_2}{\omega_2} \right), \quad (\text{MW})$$

$$= \frac{1}{2} \left[ \frac{M + h_1 + h_2}{\omega_1 + \omega_2} \right], \quad (\text{CJ})$$

$$= \frac{1}{2} \left[ \frac{M(a - (p_0^+)^2) - p_0^+ M(b_1 - h_2) + B(h_1 - h_2)}{BR} \right], \quad (\text{MNK})$$

$$a = E_1^2 - E_2^2 = \frac{1}{4} \left( M^2 + b_0^2 - 2(\omega_1^2 + \omega_2^2) \right), \quad b_0 = E_1 - E_2, \quad E_{(1)} = \frac{M}{2} (1 + d_{12})$$

$$d_{12} = \frac{m_1^2 - m_2^2}{M^2}, \quad p_0^+ = \frac{R - b}{2y}, \quad y = \frac{m_1 - m_2}{m_1 + m_2}, \quad R = \sqrt{b^2 - 4y^2a}, \quad b = M + b_0y,$$

$$B = a + p_0^+ b_0 + (p_0^+)²$$

$$h_i = \bar{\alpha}_i \bar{\beta}_i + m_i \gamma_i^0, \quad \omega_i = \sqrt{m_i^2 + p_i^2}, \quad O_1 = O_1 \otimes I_2, \quad O_2 = I_1 \otimes O_2$$
Note that from the eq.(1) with the operators $\Pi$ (2) immediately follow the system of equations (3.61) in Ref.[1] with definition (3.61-63), if eq.(1) is multiplied from left by projection operator $\Lambda_{12}^{(\alpha_1\alpha_2)}$ and are used their properties:

$$
\Lambda_{12}^{(\alpha_1\alpha_2)} = \Lambda_1^{(\alpha_1)} \otimes \Lambda_2^{(\alpha_2)}, \quad \Lambda_i^{(\alpha_i)} = \frac{\omega_i + \alpha_i h_i}{2\omega_i}, \quad \Lambda_i^{(\alpha_i)} \Lambda_i^{(\beta_i)} = \delta_{\alpha_i\beta_i} \Lambda^{(\alpha_i)}
$$

(3)

$$
\Pi^{SAL} = \Lambda_{12}^{(+)} - \Lambda_{12}^{(-)} = \frac{1}{2} \left( \frac{h_1}{\omega_1} + \frac{h_2}{\omega_2} \right), \quad \Pi^{GR} = \Lambda_{12}^{(+)} + \Lambda_{12}^{(-)} = \frac{1}{2} \left( 1 + \frac{h_1}{\omega_1} \right)
$$

Now if in the operator $\Pi^{SAL}$ use the approximation

$$
\frac{h_i}{\omega_i} \rightarrow \frac{h_i}{\omega_i} |_{p \rightarrow 0} = \gamma^0_i
$$

(4)

then we obtain the two-body Dirac equation

$$
\begin{align*}
[M - h_1(p) - h_2(-p)]\Psi_M(p) &= \Pi_0^{SAL} \gamma^0_1 \otimes \gamma^0_2 \int \frac{d^3p'}{(2\pi)^3} V(p, p') \Psi_M(p') \\
\Pi_0^{SAL} &= \frac{1}{2} \left( \gamma^0_1 + \gamma^0_2 \right)
\end{align*}
$$

(5)

(6)

which already was used for bound $q\bar{q}$ systems in Refs.[8],[9].

In approximation (4) from (2) follows

$$
\Pi_0^{GR} = \frac{1}{2} \left( 1 + \gamma^0_1 \right)
$$

(7)

As to MW, CJ and MNK versions for derivation of corresponding two-body Dirac equations the additional to (4) approximation is need, namely

$$
\Pi(M; 0) \rightarrow \Pi(m_1 + m_2; 0) = \Pi_0
$$

(8)

which is quite natural because it corresponds to zero approximation in iteration procedure for solving nonlinear over $M$ eq.(1) for the MW, CJ and MNK versions. As a result from (2) can be obtained

$$
\Pi_0^{MW} = \frac{1}{2} \left[ \gamma^0_1 + \gamma^0_2 + (1 - \gamma^0_1 \otimes \gamma^0_2) \right]
$$

(9)

$$
\Pi_0 = \frac{1}{2} \left[ 1 + \mu_1 \gamma^0_1 + \mu_2 \gamma^0_2 \right], \quad \mu_i = \frac{m_i}{m_1 + m_2} \quad \text{(CJ)}
$$

$$
\Pi_0 = \frac{1}{2} \left[ 1 + \mu_1 \gamma^0_1 + \mu_2 \gamma^0_2 \right], \quad \mu_i = \frac{m_i^2}{m_1^2 + m_2^2} \quad \text{(MNK)}
$$

(10)

Thus, we have the following two-body Dirac equations obtained from (1), (2)
\[ [M - h_1(p) - h_2(-p)]\Psi_M(p) = \Pi_0 \gamma_1^0 \otimes \gamma_2^0 \int \frac{d^3p'}{(2\pi)^3} V(p, p')\Psi_M(p') \]  

(11)

where the operator \(\Pi_0\) is given by the formulae (6,7,9,10).

Note that there is another approach for formulation the two-body Dirac equations, namely, generation of the one-body Dirac equation to two-body one, using constrain dynamics and relation to quantum field theory. Review of such an approach is given in Ref.[10].

Representing the wave function \(\Psi_M(p)\) as sum of “frequency” components

\[ \Psi_M(p) = \sum_{\alpha_1\alpha_2} \Lambda^{(\alpha_1\alpha_2)}_{12}(p)\Psi_M(p) = \sum_{\alpha_1\alpha_2} \Psi_M^{(\alpha_1\alpha_2)}(p) \]  

(12)

from the eq.(11) follows the system of the equation for the functions \(\Psi_M^{(\alpha_1\alpha_2)}\)

\[ [M - (\alpha_1\omega_1 + \alpha_2\omega_2)]\Psi_M^{(\alpha_1\alpha_2)}(p) = \Lambda^{(\alpha_1\alpha_2)}_{12} \Pi_0 \gamma_1^0 \otimes \gamma_2^0 \int \frac{d^3p'}{(2\pi)^3} V(p, p') \sum_{\alpha_1'\alpha_2'} \Psi_M^{(\alpha_1'\alpha_2')}(p'). \]  

(13)

Taking the \(q\bar{q}\) interaction operator \(V\) in the form [1] (combination of one-gluon exchange and confining part of potential)

\[ V = \gamma_1^0 \otimes \gamma_2^0 V_{OG} + \left[ x\gamma_1^0 \otimes \gamma_2^0 + (1 - x)I_1 \otimes I_2 \right] V_C \]  

(14)

and representing the function \(\Psi_M^{(\alpha_1\alpha_2)}\) as

\[ \Psi_M^{(\alpha_1\alpha_2)}(p) = N^{(\alpha_1\alpha_2)}_{12}(p) \left[ \left( \frac{1}{\omega_1 + \alpha_1m_1} \right) \otimes \left( \frac{1}{\omega_2 + \alpha_2m_2} \right) \right] \equiv f^{(\alpha_1\alpha_2)}_{12}(\vec{p}) \chi_M^{(\alpha_1\alpha_2)}(p) \]  

(15)

where

\[ N^{(\alpha_1\alpha_2)}_{12} = \sqrt{\frac{\omega_1 + \alpha_1 m_1}{2\omega_1}} \sqrt{\frac{\omega_2 + \alpha_2 m_2}{2\omega_2}}. \]  

(16)

then for the wave functions \(\chi_M^{(\alpha_1\alpha_2)}\) from (13) can be obtained the following system of equations

\[ [M - (\alpha_1\omega_1 + \alpha_2\omega_2)]\chi_M^{(\alpha_1\alpha_2)}(p) = \sum_{\alpha_1'\alpha_2'} \int \frac{d^3p'}{(2\pi)^3} V_{eff}^{(\alpha_1\alpha_2\alpha_1'\alpha_2')}(p, p')\chi_M^{(\alpha_1'\alpha_2')}(p') \]  

(17)

where

\[ V_{eff}^{(\alpha_1\alpha_2\alpha_1'\alpha_2')}(p, p') = N^{(\alpha_1\alpha_2)}_{12}(p)B^{(\alpha_1\alpha_2\alpha_1'\alpha_2')}(p, p')N^{(\alpha_1'\alpha_2')}_{12}(p') \]  

(18)
\[ B^{(\alpha_1 \alpha_2 \alpha'_1 \alpha'_2)}(\mathbf{p}, \mathbf{p'}) = \left[ 1 - \frac{\alpha_1 \alpha_2 \alpha'_1 \alpha'_2 (\bar{\sigma}_1 \bar{p})(\bar{\sigma}_2 \bar{p})'(\bar{\sigma}_1 \bar{p}')'(\bar{\sigma}_2 \bar{p}'')}{(\omega_1 + \alpha_1 m_1)(\omega_2 + \alpha_2 m_2)(\omega'_1 + \alpha'_1 m_1)(\omega'_2 + \alpha'_2 m_2)} \right] V_1(\bar{p}, \bar{p}') \quad \text{(SAL)} \tag{19} \]

\[ B^{(\alpha_1 \alpha_2 \alpha'_1 \alpha'_2)}(\mathbf{p}, \mathbf{p'}) = \left[ V_1(\bar{p}, \bar{p}') + \frac{\alpha_2 \alpha'_2 (\bar{\sigma}_2 \bar{p})(\bar{\sigma}_2 \bar{p})'}{(\omega_2 + \alpha_2 m_2)(\omega'_2 + \alpha'_2 m_2)} V_2(x; \bar{p}, \bar{p}') \right] \quad \text{(GR)} \tag{20} \]

\[ B^{(\alpha_1 \alpha_2 \alpha'_1 \alpha'_2)}(\mathbf{p}, \mathbf{p'}) = \left[ 1 - \frac{\alpha_1 \alpha_2 \alpha'_1 \alpha'_2 (\bar{\sigma}_1 \bar{p})(\bar{\sigma}_2 \bar{p})'(\bar{\sigma}_1 \bar{p}')'(\bar{\sigma}_2 \bar{p}'')}{(\omega_1 + \alpha_1 m_1)(\omega_2 + \alpha_2 m_2)(\omega'_1 + \alpha'_1 m_1)(\omega'_2 + \alpha'_2 m_2)} \right] V_1(\bar{p}, \bar{p}') 
+ \left[ \frac{\alpha_1 \alpha'_1 (\bar{\sigma}_1 \bar{p})(\bar{\sigma}_1 \bar{p}')(\bar{\sigma}_1 \bar{p}')}{(\omega_1 + \alpha_1 m_1)(\omega'_1 + \alpha'_1 m_1)} \right] V_2(x; \bar{p}, \bar{p}') \quad \text{(MW)} \tag{21} \]

\[ B^{(\alpha_1 \alpha_2 \alpha'_1 \alpha'_2)}(\mathbf{p}, \mathbf{p'}) = V_1(\bar{p}, \bar{p}') + \left[ \frac{\alpha_1 \alpha'_1 (\bar{\sigma}_1 \bar{p})(\bar{\sigma}_1 \bar{p}')(\bar{\sigma}_1 \bar{p}')}{(\omega_1 + \alpha_1 m_1)(\omega'_1 + \alpha'_1 m_1)} \mu_2 + \frac{\alpha_2 \alpha'_2 (\bar{\sigma}_2 \bar{p})(\bar{\sigma}_2 \bar{p}')}{(\omega_2 + \alpha_2 m_2)(\omega'_2 + \alpha'_2 m_2)} \mu_1 \right] V_2(x; \bar{p}, \bar{p}') \quad \text{(CJ, MNK)} \tag{22} \]

\[ \omega'_i = \sqrt{m_i^2 + \mathbf{p}^2}, \quad V_1(\mathbf{p}, \mathbf{p'}) = V_{OG}(\mathbf{p}, \mathbf{p'}) + V_C(\mathbf{p}, \mathbf{p'}), \]
\[ V_2(x; \mathbf{p}, \mathbf{p'}) = V_{OG}(\mathbf{p}, \mathbf{p'}) + (2x - 1)V_C(\mathbf{p}, \mathbf{p'}) \tag{23} \]

It is very important that the two-body Dirac equation (17) with effective potential (18) with (19) obtained from the equation (1), corresponding to SAL version (2) does not depend on parameter \(x\) interned in the interaction operator (14), which means that from this equation can not be obtained any information on the Lorentz (spin) structure of the confining \(q \bar{q}\) interaction potential (14). Second interesting result is that the wave functions satisfying the two-body Dirac equation (17) with effective potentials (18) with expression (19, 20) obtained for SAL and GR versions (2) of the 3D-relativistic equations have all nonzero "frequency components" whereas two components of the wave functions satisfying the equation (1) with projection operators (2), are zero, namely:

\[ \tilde{\Phi}_M^{(+\mp)} = \Lambda_{12}^{(+\mp)} \tilde{\Phi}_M = 0 \quad \text{(SAL)} \]
\[ \tilde{\Phi}_M^{(-\pm)} = \Lambda_{12}^{(-\pm)} \tilde{\Phi}_M = 0 \quad \text{(GR)} \tag{24} \]

which directly follows (and is well known) from the eq.(1) if it is multiplied (from left) by the operators \(\Lambda_{12}^{(+\mp)}\), \(\Lambda_{12}^{(-\pm)}\) and used the formulae (3).

For formulation normalization condition for the wave function (12) which satisfies the equation (11), we note that normalization condition for Salpeter wave function obtained in Ref.[1] (see relation (3.14)) can be written in the form

\[ < \tilde{\Phi}_M | \Pi^{SAL} | \tilde{\Phi}_M > = 2M \tag{25} \]

The analogous condition can be derived for wave function satisfying Gross equation (1, 2) if we use equation for full Green operator corresponding to the equation (1)

\[ \tilde{G} = g_0 \Pi G_0 + g_0 \tilde{U} \tilde{G}, \quad g_0 = [M - h_1 - h_2]^{-1}, \quad \Gamma_0 = \gamma_1^0 \otimes \gamma_2^0, \quad \tilde{U} = \Pi G_0 V \tag{26} \]

Assuming that the operator \(\tilde{G}^{-1}\) exists (being natural at any rate in the bound states, we need) from eq.(26) after some transformations can be obtained the following relation

\[ \tilde{G} \Gamma_0 \Pi \left[ g_0^{-1} - \tilde{U} \right] \tilde{G} \Gamma_0 = \tilde{G} \Gamma_0 \Pi \Pi. \tag{27} \]
Noting that $\Pi_{GR} \Pi_{GR} = \Pi_{GR}$ from (27) we have

$$\tilde{G}\Gamma_0 \Pi^{GR} \left[ g_0^{-1} - \tilde{U} \right] \tilde{G}\Gamma_0 \Pi^{GR} = \tilde{G}\Gamma_0 \Pi^{GR}. \quad (28)$$

Now using the spectral representation of Green operator $\tilde{G}$

$$\tilde{G}(p) = \sum_B \frac{\tilde{\Phi}_P |B| \tilde{\Phi}_P |B\rangle}{P^2 - M^2_B} + \tilde{R}(P), \quad <\tilde{\Phi}_P |B| \tilde{\Phi}_P |B\rangle = <\tilde{\Phi}_P |B| \Gamma_0 \quad (29)$$

from (28) can be obtained the relation

$$<\tilde{\Phi}_M |\Pi^{GR} |\Phi_M > \Pi^{GR} = 2M \Pi^{GR} \quad (30)$$

It means that the normalization condition analogous to (25)

$$<\tilde{\Phi}_M |\Pi^{GR} |\Phi_M >= 2M \quad (31)$$

holds only in corresponding subspace of the Gilbert space. Note that the condition (31) can be obtained from the formula (3.28) of ref.[1], which was not derived, but supposed with an analogy to (3.14).

Now, noting that the two-body Dirac equations (11) for the SAL and GR versions of the 3D-relativistic equation (1) were obtained in the approximation (4) for the projection operators $\Pi^{SAL}$ and $\Pi^{GR}$, the corresponding condition for wave function can be obtained from (25, 31) by replacement $\Pi^{SAL} \Rightarrow \Pi^{SAL}_0 (6)$ and $\Pi^{GR} \Rightarrow \Pi^{GR}_0 (7)$. Thus we have

$$<\Psi_M |\Pi^{SAL,GR}_0 |\Psi_M >= 2M \quad (32)$$

where $\Pi^{SAL}_0$ and $\Pi^{GR}_0$ are given by formulae (6), (7).

Further, noting that the projection operator $\Pi^{MW}_0 (9)$ satisfies condition $\Pi^{MW}_0 \Pi^{MW}_0 = 1$, from relation (27) can be obtained the normalization condition analogous to (32) i.e.

$$<\Psi_M |\Pi^{MW}_0 |\Psi_M >= 2M. \quad (33)$$

As to normalization conditions for wave functions satisfying the two-body Dirac equation (11), corresponding to the CJ and MNK versions, they can not be derived analogously because the corresponding projection operators $\Pi_0 (10)$does not satisfy the conditions $\Pi_0 \Pi_0 = \Pi_0$ or $\Pi_0 \Pi_0 = 1$. But bellow we assume (suppose) that the condition analogous to (33) can be written in common form

$$<\Psi_M |\Pi_0 |\Psi_M >= 2M. \quad (34)$$

where operator $\Pi_0$ is given by the formulae (6, 7, 9, 10) for all versions. As a result with an account of the formulae (12, 15, 16) the normalization condition for the components of the wave functions $\chi^{(\alpha_1\alpha_2)}_M$ takes the form
\[
\sum_{\alpha_1\alpha_2\beta_1\beta_2} < \chi_M^{(\alpha_1\alpha_2)}|N_{12}^{(\alpha_1\alpha_2)} f_{12}^{(\alpha_1\alpha_2)} + \Pi_0 f_{12}^{(\beta_1\beta_2)} N_{12}^{(\beta_1\beta_2)} |\chi_M^{(\beta_1\beta_2)}> = 2M \tag{35}
\]

from which follows

\[
\int \frac{d^3p}{(2\pi)^3} \sum_{\alpha_1\alpha_2\beta_1\beta_2} \frac{1}{4} \left[ E_{12}^{(\alpha_1\alpha_2\beta_1\beta_2)} \left( \begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \\ \end{array} \right) + \alpha_1\beta_1 E_{12}^{(-\alpha_1\alpha_2-\beta_1\beta_2)} \left( \begin{array}{c} 0 \\ 0 \\ 1 \\ \mu_2 \\ \end{array} \right) + \alpha_2\beta_2 E_{12}^{(\alpha_1\alpha_2\beta_1\beta_2)} \left( \begin{array}{c} 0 \\ 1 \\ 1 \\ \mu_1 \\ \end{array} \right) - \\
- \alpha_1\alpha_2\beta_1\beta_2 E_{12}^{(-\alpha_1\alpha_2-\beta_1\beta_2)} \left( \begin{array}{c} 1 \\ 0 \\ 1 \\ 0 \\ \end{array} \right) \right] \chi_M^{(\alpha_1\alpha_2)}(p) \chi_M^{(\beta_1\beta_2)}(p) = 2M, \tag{36}
\]

where

\[
E_{12}^{(\alpha_1\alpha_2\beta_1\beta_2)} = \sqrt{\left(1 + \alpha_1 \frac{m_1}{\omega_1}\right) \left(1 + \alpha_2 \frac{m_2}{\omega_2}\right) \left(1 + \beta_1 \frac{m_1}{\omega_1}\right) \left(1 + \beta_2 \frac{m_2}{\omega_2}\right)} \tag{37}
\]

Now we use the partial-wave expansion for the function \(\chi_M^{(\alpha_1\alpha_2)}(p)\) [1]

\[
\chi_M^{(\alpha_1\alpha_2)}(p) = \sum_{LSJM_J} < n|LSJM_J > R_{LSJ}^{(\alpha_1\alpha_2)}(p) \equiv \sum_{JM_J} \chi_{JM_J}^{(\alpha_1\alpha_2)}(p), \quad (n = \frac{p}{p}) \tag{38}
\]

where \(R_{LSJ}^{(\alpha_1\alpha_2)}(p)\) are corresponding radial wave functions. And the potential functions \(V_{OG}(p,p')\), \(V_C(p,p')\) are represented in form (local potentials)

\[
V(p-p') = (2\pi)^3 \sum_{LSJM_J} V^L(p,p') < n|LSJM_J > < L\bar{S}\bar{J}\bar{M}_J | n' > \tag{39}
\]

where

\[
V^L(p,p') = \frac{2}{\pi} \int_0^\infty j_L(pr)V(r)j_L(p'r)r^2dr \tag{40}
\]

\(j_L(x)\) being the spherical Bessel function. Then from the system of equations (17), the effective potentials of which is defined by the formulae (18-23) we obtain the following system of equations for the radial functions \(R_{LSJ}^{(\alpha_1\alpha_2)}(p)\)

**SAL version**

\[
[M - (\alpha_1\omega_1 + \alpha_2\omega_2)] R_{j_{i,j,J}^{(\alpha_1\alpha_2)}}(p) = \sum_{\alpha'_1\alpha'_2} \int_0^\infty p'^2 dp' \left[ \left( N_{12}^{(\alpha_1\alpha_2)}(p) N_{12}^{(\alpha'_1\alpha'_2)}(p') \right) - \\
- \alpha_1\alpha_2\alpha'_1\alpha'_2 N_{12}(\alpha_1\alpha_2)(p) N_{12}(\alpha'_1\alpha'_2)(p') V_J^{(\pm 1)}(p,p') \right] R_{j_{i,j,J}^{(\alpha'_1\alpha'_2)}}(p') \tag{41}
\]

\[
[M - (\alpha_1\omega_1 + \alpha_2\omega_2)] R_{j_{j_{i,j,J}^{(\alpha_1\alpha_2)}}}(p) = \sum_{\alpha'_1\alpha'_2} \int_0^\infty p'^2 dp' \left[ \left( N_{12}^{(\alpha_1\alpha_2)}(p) N_{12}^{(\alpha'_1\alpha'_2)}(p') V_J^{(\pm 1)}(p,p') \right) - \\
- \alpha_1\alpha_2\alpha'_1\alpha'_2 N_{12}(\alpha_1\alpha_2)(p) N_{12}(\alpha'_1\alpha'_2)(p') V_J^{(\pm 1)}(p,p') \right] R_{j_{j_{i,j,J}^{(\alpha'_1\alpha'_2)}}}(p') \tag{42}
\]
\[ [M - (\alpha_1 \omega_1 + \alpha_2 \omega_2)] R^{(\alpha_1 \alpha_2)}_{J_1 J_2}(p) = \sum_{\alpha_1' \alpha_2'} \int_0^\infty p^2 \, dp' \left\{ \left[ N^{(\alpha_1 \alpha_2)}_{12}(p) N^{(\alpha_1' \alpha_2')}_{12}(p') \right] V^J_1(p, p') + \right. \\
+ \alpha_2 \alpha_2' N^{(\alpha_1 - \alpha_2)}_{12}(p) N^{(\alpha_1' - \alpha_2')}_{12}(p') R^{(\alpha_1' \alpha_2')}_{J_1 J_2}(p') \left\} - \right. \\
\left[ \alpha_2 \alpha_2' N^{(\alpha_1 - \alpha_2)}_{12}(p) N^{(\alpha_1' - \alpha_2')}_{12}(p') V^{(\alpha_1 \alpha_2)}_{J_1 J_2}(x, p, p') \right] R^{(\alpha_1' \alpha_2')}_{J_1 J_2}(p') \right) \]  

\[ [M - (\alpha_1 \omega_1 + \alpha_2 \omega_2)] R^{(\alpha_1 \alpha_2)}_{J_{\pm 1} J}(p) = \sum_{\alpha_1' \alpha_2'} \int_0^\infty p^2 \, dp' \left\{ \left[ N^{(\alpha_1 \alpha_2)}_{12}(p) \right] N^{(\alpha_1' \alpha_2')}_{12}(p') \right\} \left[ \begin{array}{c} 1 \\ J^1 \end{array} \right] + \\
+ \alpha_2 \alpha_2' N^{(\alpha_1 - \alpha_2)}_{12}(p) N^{(\alpha_1' - \alpha_2')}_{12}(p') \left[ \begin{array}{c} 1 \\ J^1 \end{array} \right] \right\} R^{(\alpha_1' \alpha_2')}_{J_{\pm 1} J}(p') \]  

\[ [M - (\alpha_1 \omega_1 + \alpha_2 \omega_2)] R^{(\alpha_1 \alpha_2)}_{J_{\pm 1} J}(p) = \sum_{\alpha_1' \alpha_2'} \int_0^\infty p^2 \, dp' \left\{ \left[ N^{(\alpha_1 \alpha_2)}_{12}(p) \right] N^{(\alpha_1' \alpha_2')}_{12}(p') \right\} \left[ \begin{array}{c} 1 \\ J^1 \end{array} \right] - \\
\left[ \alpha_2 \alpha_2' N^{(\alpha_1 - \alpha_2)}_{12}(p) N^{(\alpha_1' - \alpha_2')}_{12}(p') \right] \left[ \begin{array}{c} 1 \\ J^1 \end{array} \right] \right\} R^{(\alpha_1' \alpha_2')}_{J_{\pm 1} J}(p') \]  

Maxwell, Cattaneo-Johnson and MNK versions

\[ [M - (\alpha_1 \omega_1 + \alpha_2 \omega_2)] R^{(\alpha_1 \alpha_2)}_{J_{\pm 1} J}(p) = \sum_{\alpha_1' \alpha_2'} \int_0^\infty p^2 \, dp' \left\{ \left[ N^{(\alpha_1 \alpha_2)}_{12}(p) \right] N^{(\alpha_1' \alpha_2')}_{12}(p') \right\} \left[ \begin{array}{c} 1 \\ J^1 \end{array} \right] + \\
\left[ \alpha_2 \alpha_2' N^{(\alpha_1 - \alpha_2)}_{12}(p) N^{(\alpha_1' - \alpha_2')}_{12}(p') \right] \left[ \begin{array}{c} 1 \\ J^1 \end{array} \right] \right\} R^{(\alpha_1' \alpha_2')}_{J_{\pm 1} J}(p') \]  

\[ V^{(\alpha_1 \alpha_2)}_{n \oplus J}(p) = \frac{1}{2J + 1} \left[ (J^1) V^{(J + 1)}_{n J} + (J^1) V^{(J - 1)}_{n J} \right] \]
\[ V_{n\otimes J} = \sqrt{\frac{J(J+1)}{2J+1}} [V_{n}^{J+1} - V_{n}^{J-1}], \quad n = 1, 2 \]  
\[ V_{n(J\pm 1)} = \frac{1}{(2J+1)^2} \left[ V_{n}^{J\pm 1} + 4J(J+1)V_{n}^{J+1} \right] \]

It is interesting to compare the system of equations (41-46) with the system of equations obtained from (1) without the approximation (4, 8) (see eqs. (4.16, 17) in [1], neglecting the terms corresponding to t’Hooft interaction)

\[
[M - (\alpha_1 \omega_1 + \alpha_2 \omega_2)] R_{JL_S J}^{(\alpha_1 \alpha_2)}(p) = A^{(\alpha_1 \alpha_2)}(M; p) \sum_{\alpha_1' \alpha_2'} \int_0^\infty p'^2 \, dp' \left\{ \left[ (N_{12}^{(\alpha_1 \alpha_2)}(p) N_{12}^{(\alpha_1' \alpha_2')}(p') + \alpha_1 \alpha_2 \alpha_1' \alpha_2' N_{12}^{(-\alpha_1 - \alpha_2)}(p) N_{12}^{(-\alpha_1' - \alpha_2')}(p')) V_1^{J}(p, p') + \right. \right.
\left. + \left( \alpha_1 \alpha_1' N_{12}^{(-\alpha_1 \alpha_2)}(p) N_{12}^{(-\alpha_1' \alpha_2')}(p') + \alpha_2 \alpha_2' N_{12}^{(-\alpha_1 - \alpha_2)}(p) N_{12}^{(-\alpha_1' - \alpha_2')}(p')) V_2^{(i)}(x; p, p') \right] R_{JL_S J}^{(\alpha_1' \alpha_2')}(p') \right. \\
\left. - \left[ (\alpha_1' \alpha_1 N_{12}^{(-\alpha_1 \alpha_2)}(p) N_{12}^{(-\alpha_1' \alpha_2')}(p') - \alpha_2 \alpha_2' N_{12}^{(-\alpha_1 - \alpha_2)}(p) N_{12}^{(-\alpha_1' - \alpha_2')}(p')) V_2^{\otimes J}(x; p, p') \right] \right\} R_{JL_S J}^{(\alpha_1' \alpha_2')}(p') \right\} \right]\]

\[
[M - (\alpha_1 \omega_1 + \alpha_2 \omega_2)] R_{JL_S J}^{(\alpha_1 \alpha_2)}(p) = A^{(\alpha_1 \alpha_2)}(M; p) \sum_{\alpha_1' \alpha_2'} \int_0^\infty p'^2 \, dp' \times \\
\times \left\{ \left[ (N_{12}^{(\alpha_1 \alpha_2)}(p) N_{12}^{(\alpha_1' \alpha_2')}(p') V_1^{J\pm 1}(p, p') + \alpha_1 \alpha_2 \alpha_1' \alpha_2' N_{12}^{(-\alpha_1 - \alpha_2)}(p) N_{12}^{(-\alpha_1' - \alpha_2')}(p')) V_1^{J\pm 1}(p, p') + \right. \right.
\left. + \left( \alpha_1 \alpha_1' N_{12}^{(-\alpha_1 \alpha_2)}(p) N_{12}^{(-\alpha_1' \alpha_2')}(p') + \alpha_2 \alpha_2' N_{12}^{(-\alpha_1 - \alpha_2)}(p) N_{12}^{(-\alpha_1' - \alpha_2')}(p')) V_2^{J}(x; p, p') \right) R_{JL_S J}^{(\alpha_1' \alpha_2')}(p') \right. \\
\left. + \left[ \alpha_1 \alpha_2 \alpha_1' \alpha_2' N_{12}^{(-\alpha_1 - \alpha_2)}(p) N_{12}^{(-\alpha_1' - \alpha_2')}(p') \right] \frac{2}{2J+1} V_1^{\otimes J}(x; p, p') \right\} R_{JL_S J}^{(\alpha_1' \alpha_2')}(p') \right\} \right]\]

where

\[ A^{(\pm \pm)} = \pm 1, A^{(\pm \mp)} = 0, \text{ (SAL)}; \quad A^{(\pm \pm)} = \pm 1, A^{(- \pm)} = 0, \text{ (GR)}; \]
\[ A^{(+ \pm)} = \pm 1, A^{(+ \mp)} = M \frac{1}{\omega_1 + \omega_2}, \text{ (MW)}; \quad A^{(\alpha_1 \alpha_2)} = \frac{M + (\alpha_1 \omega_1 + \alpha_2 \omega_2)}{2(\omega_1 + \omega_2)}, \text{ (CJ)}; \]
\[ A^{(\alpha_1 \alpha_2)} = \frac{1}{2BR} \left[ M(a - (p_0^+)^2) - p_0^+ M(\alpha_1 \omega_1 - \alpha_2 \omega_2) + B(\alpha_1 \omega_1 + \alpha_2 \omega_2) \right], \text{ (MNK)} \]

Note that the last expression in (50) is obtained from (3.62) in [1] after some transformation.

Main difference between the system of equations (41-46) and (48, 49) with the expression (50) is following: 1) In the wave functions \( R_{L_S J}^{(\alpha_1 \alpha_2)} \) satisfying the system of equations (48, 49) for (SAL) and (GR) versions the nonzero functions are only \( R_{L_S J}^{(\pm \pm)} \) and \( R_{L_S J}^{(+ \pm)} \) respectively (about this fact was mentioned above), whereas in the corresponding system of equations (41), (42), (43), (44) all components of wave functions \( R_{L_S J}^{(\alpha_1 \alpha_2)} \) are nonzero; 2) The system of equations (48, 49) for the MW, CJ and MNK versions are nonlinear over \( M \), whereas the system of equations (45, 46) are linear one; 3) Dirac equations (41,42) obtained from the Salpeter, equation do not depend on \( x \).
3 Procedure for solving the obtained equations

For solving bound-state equations (41-46) or (48,49), we need to specify the interquark interaction potentials $V_{OG}$ and $V_C$ (14). Below for $V_C(r)$ we use the following form [1], [11]

$$V_C(r) = \frac{4}{3} \alpha_S(m_{12}^2) \left( \frac{\mu_{12}\omega_0^2 r^2}{2\sqrt{1 + A_0 m_1 m_2 r^2}} - V_0 \right)$$ (51)

$$\alpha_S(Q^2) = \frac{12\pi}{33 - 2n_f} \left[ \ln \frac{Q^2}{\Lambda^2} \right]^{-1}, \quad m_{12} = m_1 + m_2, \quad \mu_{12} = \frac{m_1 m_2}{m_{12}}$$ (52)

where $Q^2$ is the momentum transferred and the $\frac{4}{3}$ comes from the color-dependent part of the $q\bar{q}$ interaction, $n_f$ is the number of flavors ($n_f = 3$ for $u,d,s$ quarks; $n_f = 4$ for $u,d,s,c$; $n_f = 5$ for $u,d,s,c,b$). $\omega_0$, $A_0$, $V_0$ and $\Lambda$ are considered to be the free parameters of the model. The potential given by expression (51) effectively reduces to the harmonic oscillator potential for the light quarks $u,d,s$ and to the linear potential to the heavy $c,b$ quarks if the dimensionless parameter $A_0$ is chosen small enough. Moreover, asymptotically, for a large $r$ it is linear and almost flavor-independent. The one-gluon exchange potential is given by standard expression [1], [11]

$$V_{OG}(r) = -\frac{4}{3} \frac{\alpha_S(m_{12}^2)}{r}$$ (53)

Now we have to specify the numerical procedure for solution of the systems of radial equations (41-45),(48),(49). A possible algorithm looks as follows: we choose the known basis functions denoted by $R_{nL}(p)$. The unknown radial wave functions are expanded in the linear combination of the basis functions

$$R^{(\alpha_1\alpha_2)}_{LSJ}(p) = \sqrt{2M(2\pi)^3} \sum_{n=0}^{\infty} C_{nLSJ}^{(\alpha_1\alpha_2)} R_{nL}(p)$$ (54)

where $C_{nLSJ}^{(\alpha_1\alpha_2)}$ are the coefficients of the expansion. The integral equation for the radial wave functions is then transformed into the system of linear equations for these coefficients. If the transaction is carried out the finite system of equations is obtained that can be solved by using conventional numerical methods. The convergence of the whole procedure, with more terms taken into account in the expansion (54) depend on the successful choice of the basis. In case of the confining potential of form (51) it is natural to take as a basis the functions corresponding to oscillator potential, which is obtained from (51) at $A_0 = 0$, in non-relativistic limit of the system of equations obtained from from (41-46),(48),(49). The radial wave functions in this case have the form [1](the formula (4.52)).

$$R_{nL}(p) = p_0^{-3/2} R_{nL}(z), \quad p_0 = \sqrt{\mu_{12}\omega_0\sqrt{\frac{3}{4}} \alpha_S(m_{12}^2)}, \quad z = \frac{p}{p_0}$$

$$R_{nL}(z) = c_{nL} z^L \exp(-\frac{z^2}{2}) \frac{1}{1F1}(-n, L + \frac{3}{2}, z^2), \quad c_{nL} = \sqrt{\frac{2\Gamma(n+L+\frac{1}{2})}{\Gamma(n+1)}} \frac{1}{\Gamma(L+\frac{3}{2})}$$ (55)

where $1F1$ denotes the confluent hypergeometric function.
Now, satisfying the expression (54) into the system of equations (41-46),(48),(49), the following algebraic equations for the coefficients \( C_{\alpha_1 \alpha_2}^{(\alpha_{1 \alpha 2})} \) can be obtained:

\[
MC_{nLSJ}^{(\alpha_1 \alpha_2)} = \sum_{\alpha'_1 \alpha'_2} \sum_{n' L'S' J} H_{n' L'S' J}(M) C_{n' L'S' J}^{(\alpha'_1 \alpha'_2)}
\]

(56)

It is necessary to note that the matrix \( H_{\alpha \beta}(M) \) depends on meson mass \( M \) only for MW, CJ and MNK versions as it can be seen from equations (56) for \( M \) is not linear one and therefore should be solved, e.g. by iteration. As to the system of Dirac equations (41-46) such a problem does not exist.

4 The numerical results and discussions

The main problem we have investigated at first stage is dependence of the existence of stable solutions of the eq. (56) i.e. the equations (41-46),(48), (49) on Lorentz (spin) structure of the confining \( q\bar{q} \) interaction potential, i.e. on the parameter \( x \). This will be done taking as examples the \( u\bar{s} \), \( c\bar{u} \) and \( c\bar{s} \), bound states with constituent quark masses \( m_u = m_d = 280 \text{MeV}, m_s = 400 \text{MeV}, m_c = 1470 \text{MeV} \) and the free parameters of the confining potential (51,52) - \( \omega_0 = 710 \text{MeV}, V_0 = 525 \text{MeV}, A_0 = 0.0270, \Lambda = 120 \text{MeV} \).

Note, that in [11] only the SAL version of 3D-reduction of Bethe-Salpeter equation was considered as to MW, CJ and MNK without additional approximation (4) with oscillator like potential (\( A_0 = 0 \) in (51)) were considered in Refs. [7], [12].

The results of the calculations are given for states \(^{2S+1}L_J\) (note, that for cases \(^3S_1, ^3P_2, ^3P_1\) are neglected additional corresponding terms \(^3D_1, ^3F_2, ^1P_1\), because they give small contribution in the calculated mass).

The additional conclusions to pure theoretical results formulated at the end of section 2, which follow from the tables, are the following:

- The area of dependence on parameter \( x \) existence of stable solutions of corresponding equations is a little extended for corresponding Dirac equation.
- The results for CJ and MNK versions for the corresponding Dirac equations are almost the same which can be seen from formulae (10). Further, it can be shown exactly, that Dirac equations in the CJ and MNK versions are equivalent for the states \(^1S_0, ^3S_1\) when mixture of states \(^3S_1\) and \(^3D_1\) is neglected.
- For \( u\bar{c} \) and \( s\bar{c} \) bound systems Gross version works better what is related to the large difference of constituent masses.
- The area of the existence of the stable solutions is enlarged with increasing of the constituent masses which is theoretically understandable.
- Masses of the bound \( q\bar{q} \) systems obtained from solutions of Dirac equations are bigger then masses corresponding to 3D-equations obtained from BS equation for all versions except GR version case.

Note, that for \( x = 0.5 \) the stable solutions always exist.

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| $x$ | 0.0 | 0.1 | 0.3 | 0.5 | 0.7 | 0.9 | 1.0 |
|-----|-----|-----|-----|-----|-----|-----|-----|
| $u\bar{s}\,^1S_0\ (494)$ |   |   |   |   |  |  |  |
| SAL | * | * | 853 | 892 | 923 | 948 | 958 |
| SALD | 1000 |   |   |   |  |  |  |
| GR | 819 | 838 | 873 | 906 | * | * | * |
| GRD | 957 | 956 | 954 | 953 | * | * | * |
| MW | 847 | 858 | 883 | 909 | * | * | * |
| MWD | 908 | 917 | 938 | 964 | 998 | * | * |
| CJ | 865 | 878 | 905 | 935 | 968 | * | * |
| CJD | 924 | 930 | 942 | 955 | 972 | * | * |
| MNK | 866 | 793 | 819 | 844 | * | * | * |
| MNKD | 923 | 929 | 941 | 955 | 972 | * | * |
| $u\bar{s}\,^1S_1\ (892)$ |   |   |   |   |  |  |  |
| SAL | * | 812 | 870 | 914 | 950 | 980 | 993 |
| SALD | 979 |   |   |   |  |  |  |
| GR | 839 | 859 | 897 | 934 | 967 | * | * |
| GRD | 944 | 947 | 954 | 962 | 975 | * | * |
| MW | 863 | 877 | 907 | 943 | 983 | * | * |
| MWD | 879 | 887 | 905 | 928 | 957 | * | * |
| CJ | 878 | 893 | 924 | 959 | 998 | * | * |
| CJD | 924 | 930 | 942 | 955 | 972 | * | * |
| MNK | 814 | 830 | 861 | 891 | * | * | * |
| MNKD | 923 | 929 | 941 | 955 | 972 | * | * |
| $u\bar{s}\,^3P_0\ (1350)$ |   |   |   |   |  |  |  |
| SAL | 1189 | 1204 | 1213 | 1210 | 1202 | 1189 | 1182 |
| SALD | 1349 |   |   |   |  |  |  |
| GR | 1233 | 1232 | 1229 | 1223 | 1218 | * | * |
| GRD | 1304 | 1302 | 1300 | 1298 | 1298 | * | * |
| MW | 1255 | 1253 | 1249 | 1250 | * | * | * |
| MWD | 1278 | 1274 | 1267 | 1260 | 1257 | * | * |
| CJ | 1268 | 1267 | 1263 | 1260 | 1264 | * | * |
| CJD | 1296 | 1294 | 1290 | 1287 | 1284 | 1285 | * |
| MNK | 1217 | 1212 | 1202 | 1190 | * | * | * |
| MNKD | 1295 | 1293 | 1289 | 1286 | 1284 | * | * |
| $u\bar{s}\,^3P_2\ (1430)$ |   |   |   |   |  |  |  |
| SAL | * | * | 1189 | 1289 | 1367 | 1430 | 1458 |
| SALD | 1318 |   |   |   |  |  |  |
| GR | 1119 | 1159 | 1237 | 1310 | 1381 | * | * |
| GRD | 1278 | 1284 | 1297 | 1314 | 1336 | * | * |
| MW | 1184 | 1209 | 1262 | 1326 | 1384 | * | * |
| MWD | 1185 | 1200 | 1234 | 1275 | 1326 | * | * |
| CJ | 1181 | 1211 | 1276 | 1345 | 1421 | * | * |
| CJD | 1254 | 1264 | 1286 | 1310 | 1337 | 1369 | 1388 |
| MNK | 1137 | 1165 | 1223 | 1282 | 1344 | 1408 | * |
| MNKD | 1254 | 1264 | 1285 | 1309 | 1388 | 1372 | * |

Table 1: The dependence of the $q\bar{q}$ system mass for light constituent quarks on the mixing parameter $x$ in the different 3D-reductions of Bethe-Salpeter equations and corresponding Dirac equations. "*" denotes the absence of the stable solutions. Masses are given in $MeV$. 
Table 2: The dependence of the $q\bar{q}$ system mass for heavy constituent quarks on the mixing parameter $x$ in the different 3D-reductions of Bethe-Salpeter equations and corresponding Dirac equations. "*" denotes the absence of the stable solutions. Masses are given in MeV.

| $x$  | 0.0  | 0.1  | 0.3  | 0.5  | 0.7  | 0.9  | 1.0  |
|------|------|------|------|------|------|------|------|
|      | $u\bar{c}^1S_0$ (1863) |      |      |      |      |      |      |
| SAL  | 1881 | 1895 | 1920 | 1943 | 1965 | 1985 | 1994 |
| SALD |      |    1983 |      |      |      |      |      |
| GR   | 1883 | 1896 | 1921 | 1944 | 1966 | 1986 | 1995 |
| GRD  | 1979 | 1979 | 1978 | 1978 | 1978 | 1978 | 1979 |
| MW   | 1915 | 1922 | 1935 | 1951 | 1972 |    * |    * |
| MWD  | 1924 | 1929 | 1942 | 1958 | 1979 |    * |    * |
| CJ   | 1921 | 1928 | 1943 | 1960 | 1982 |    * |    * |
| CJD  | 1932 | 1937 | 1948 | 1961 | 1978 |    * | 2003 |
| MNK  | 1928 | 1934 | 1946 | 1961 | 1978 |    * |    * |
| MNKD | 1927 | 1932 | 1944 | 1958 | 1977 |    * |    * |
|      | $u\bar{c}^3S_1$ (2010) |      |      |      |      |      |      |
| SAL  | 1883 | 1897 | 1922 | 1946 | 1968 | 1988 | 1998 |
| SALD |      |    1981 |      |      |      |      |      |
| GR   | 1886 | 1899 | 1924 | 1947 | 1969 | 1989 | 1999 |
| GRD  | 1977 | 1977 | 1978 | 1979 | 1981 | 1982 | 1983 |
| MW   | 1918 | 1924 | 1938 | 1955 | 1977 |    * |    * |
| MWD  | 1921 | 1926 | 1939 | 1955 | 1975 |    * |    * |
| CJ   | 1923 | 1930 | 1948 | 1963 | 1981 |    * |    * |
| CJD  | 1932 | 1937 | 1948 | 1961 | 1978 |    * | 2003 |
| MNK  | 1930 | 1935 | 1948 | 1963 | 1981 |    * |    * |
| MNKD | 1927 | 1932 | 1944 | 1958 | 1977 |    * |    * |
|      | $s\bar{c}^1S_0$ (1971) |      |      |      |      |      |      |
| SAL  | 2020 | 2031 | 2055 | 2070 | 2088 | 2105 | 2113 |
| SALD |      |    2106 |      |      |      |      |      |
| GR   | 2023 | 2033 | 2052 | 2071 | 2089 | 2106 | 2114 |
| GRD  | 2106 | 2100 | 2100 | 2100 | 2100 | 2100 | 2100 |
| MW   | 2044 | 2050 | 2062 | 2077 | 2094 | 2118 |    * |
| MWD  | 2052 | 2058 | 2070 | 2084 | 2101 | 2126 |    * |
| CJ   | 2051 | 2057 | 2071 | 2087 | 2105 | 2126 |    * |
| CJD  | 2063 | 2067 | 2077 | 2087 | 2100 | 2116 | 2127 |
| MNK  | 2052 | 2057 | 2069 | 2082 | 2097 | 2116 |    * |
| MNKD | 2059 | 2063 | 2073 | 2085 | 2100 | 2120 |    * |
|      | $s\bar{c}^3S_1$ (2107) |      |      |      |      |      |      |
| SAL  | 2023 | 2033 | 2054 | 2073 | 2091 | 2108 | 2116 |
| SALD |      |    2104 |      |      |      |      |      |
| GR   | 2025 | 2035 | 2055 | 2074 | 2092 | 2110 | 2118 |
| GRD  | 2098 | 2099 | 2100 | 2102 | 2103 | 2105 | 2106 |
| MW   | 2047 | 2053 | 2065 | 2080 | 2098 | 2124 |    * |
| MWD  | 2049 | 2054 | 2066 | 2080 | 2097 | 2121 |    * |
| CJ   | 2053 | 2060 | 2074 | 2089 | 2108 |    * |    * |
| CJD  | 2063 | 2067 | 2077 | 2087 | 2100 | 2116 | 2127 |
| MNK  | 2054 | 2059 | 2071 | 2084 | 2100 | 2119 |    * |
| MNKD | 2059 | 2063 | 2073 | 2085 | 2100 | 2120 |    * |
References

[1] T.Kopaleishvili, Phys. Part. Nucl. 32 (2001) 560-588
[2] E.E.Salpeter, Phys. Rev. 87 (1952) 328
[3] F.Gross, Phys. Rev. 186 (1969) 1448; C26 (1982) 2203, 2226
[4] V.B.Mandelzweig, S.J.Wallace, Phys. Lett. B197 (1987) 469
[5] E.D.Cooper, B.K.Jennings, Nucl.Phys. A500 (1989), 551
[6] K.M.Moung, J.W.Norbury, D.E.Kahana, J.Phys. G. Nucl. Part. Phys. 22 (1996) 315
[7] T.Babutsidze, T.Kopaleishvili, A.Rusetsky, Phys. Lett. B426 (1998) 139
[8] Z.K.Silagadze, A.A.Khelashvili, Theor.Math.Phys. 61 (1984) 431
[9] C.Semay, R.Cenleneer, Phys. Rev. D48 (1993)4361
[10] H.W.Crater, P.Van Alstine invited paper presented at a conference on September 12th, 1997 at the University of Georgia in honour of Professor Donald Robson on his 60th birthday.
[11] A.Archvadze, M.Chachkhunashvili, T.Kopaleishvili, A.Rusetsky, Nucl. Phys. A581 (1995) 460
[12] T.Babutsidze, T.Kopaleishvili, A.Rusetsky Phys. Rev. C59 (1999) 976