A “Model-on-Demand” Methodology For Energy Intake Estimation to Improve Gestational Weight Control Interventions

Penghong Guo*, Daniel E. Rivera†, Abigail M. Pauley‡, Krista S. Leonard‡, Jennifer S. Savage***, and Danielle S. Downs****

*School for Engineering of Matter, Transport, and Energy, Arizona State University, Tempe, AZ 85281 USA
†Exercise Psychology Laboratory, Department of Kinesiology, Penn State University, University Park, PA, USA
‡Center for Childhood Obesity Research and the Department of Nutritional Sciences, Penn State University, University Park, PA, USA.
****Department of Obstetrics and Gynecology, Penn State College of Medicine, Hershey, PA, USA.

Abstract

Energy intake underreporting is a frequent concern in weight control interventions. In prior work, a series of estimation approaches were developed to better understand the issue of underreporting of energy intake; among these is an approach based on semi-physical identification principles that adjusts energy intake self-reports by obtaining a functional relationship for the extent of underreporting. In this paper, this global modeling approach is extended, and for comparison purposes, a local modeling approach based on the concept of Model-on-Demand (MoD) is developed. The local approach displays comparable performance, but involves reduced engineering effort and demands less a priori information. Cross-validation is utilized to evaluate both approaches, which in practice serves as the basis for selecting parsimonious yet accurate models. The effectiveness of the enhanced global and MoD local estimation methods is evaluated with data obtained from Healthy Mom Zone, a novel gestational weight intervention study focused on the needs of obese and overweight women.

Keywords

Semi-physical Identification; Model-on-Demand; Estimation; Weight Interventions

1. INTRODUCTION

Maternal obesity and excessive gestational weight gain (GWG) are worldwide health concerns due to their high prevalence and associated adverse obstetric outcomes. Haugen et al. (2014) showed that 71% of overweight (OW) women and 61% of obese (OB) women gained weight in excess of the recommendations by the US Institute of Medicine (IOM) (Rasmussen and Yaktine, 2009). High GWG substantially elevates the risk of maternal
gestational diabetes mellitus, hypertension, emergency cesarean delivery, and postpartum weight retention in pregnant women, and it is also a strong predictor of macrosomia and early onset of obesity in the offspring (Rasmussen and Yaktine, 2009; Gilmore et al., 2015). Therefore, interventions are needed to effectively and efficiently promote healthy GWG, especially for overweight and obese women.

Motivated by these needs, Healthy Mom Zone (HMZ) (Downs et al., 2017a,b), a behavioral intervention study conducted at Pennsylvania State University, aims to develop and validate an individually-tailored and “intensively adaptive” approach to effectively manage GWG and to promote optimal maternal and infant health. From a modeling perspective, this novel intervention involves the first-principles energy balance model originally developed by Thomas et al. (2012) to predict GWG based on longitudinal measurements of maternal energy intake (EI), physical activity (PA) and resting metabolic rate (RMR). The discretized model is described as

\[ GWG(k + 1) = K_1 EI(k) + K_2 PA(k) + K_3 RMR(k) \]  

where the daily maternal weight change (GWG) is defined as \( GWG(k + 1) = W(k + 1) - W(k) \) (W: total weight). \( K_1 \) and \( K_2 \) are the system gains derived from Thomas et al. (2012).

More details of the model development are described in Guo et al. (2016).

In intervention practice, accurate prediction of maternal weight from the energy balance model in (1) can be compromised due to biased input measurements; this concern especially arises when self-reported energy intake from participants is used for model predictions. The literature documents that a large number of weight intervention participants under-report their energy intake by as much as 59% (Lichtman et al., 1992; Trabulsi and Schoeller, 2001). Misreporting of energy intake makes it difficult for clinicians to determine whether participants are meeting their caloric goals, and further prevents appropriate health counseling advice to be provided. More importantly, the significant bias observed in self-reported EI limits the use of the energy balance model for closed-loop controller design. To illustrate the biases resulting in model prediction from underreporting, simulation results using measured data from a representative HMZ participant are shown in Fig. 1. Here it can be seen that predicted weight (W) using self-reported EI lies below the measured W, with discrepancies accumulating along the intervention weeks. Similar bias in weight prediction is also observed with other participant data that have been collected so far.

In Guo et al. (2016), a series of estimation approaches were developed for energy intake estimation. These include an approach based on semi-physical identification principles to correct future self-reports by parametrizing the extent of underreporting. Compared with the other two estimation methods described in that paper (i.e., back-calculation and Kalman filtering), the semi-physical approach with estimated model parameters is capable of providing energy intake predictions from self-reports alone, without requiring additional participant measurements of weight and energy expenditure (i.e., PA or RMR).
In this paper, this global modeling approach with the use of fixed parameters from Guo et al. (2016) is extended and further analyzed. As a counterpoint for the global estimation approach, a local modeling technique based on the concept of Model-on-Demand is applied; as will be demonstrated, comparable performance can be achieved with reduced engineering effort and less a priori information required. Two intervention participants (one from the intervention group, one control) obtained through the HMZ Study are selected to evaluate the effectiveness of the approaches. Cross-validation procedures are applied to test the performance of both modeling methods, with the procedure favoring the most parsimonious yet accurate model structures. The proposed approaches for estimating energy intake are deemed to be helpful aids in conducting the intervention, and promoting successful weight control.

The paper is organized as follows. In Section 2, a semi-physical estimation approach, followed by the Model-on-Demand approach, to estimating the energy intake from the self-reports is developed, and the tested results against the data from actual HMZ participants are provided in Section 3. Section 4 gives a summary of our conclusions.

2. CORRECTION FOR UNDERREPORTING

The semi-physical identification approach described in Guo et al. (2016) was designed based on a single linear model to correct for participant self-reported energy intake which contains potential underreporting. The idea behind this approach is to seek the quantitative relationships between the actual energy intake \( EI_{\text{actual}} \) and the \( EI \) self-reports \( EI_{\text{rept}} \), or other input variables if necessary, such as participant weight \( W_{\text{actual}} \). Fig. 2 depicts such relationships for modeling purposes, where \( EI_{\text{actual}} \) is a function of \( EI_{\text{rept}} \) and \( W_{\text{actual}} \), denoted as

\[
EI_{\text{actual}} = f\left(EI_{\text{rept}}, W_{\text{actual}}\right) \quad (2)
\]

Here, the functional relationship \( f \) can be structured differently at the user’s request. In this paper, this estimation approach is expanded to obtain a wider set of estimators. For simplicity, we only focus on linear and quadratic relationships here. Once the model is identified using linear regression from past data, it can be used to adjust future \( EI \) measurements, without the need for further modeling or ancillary data collection.

2.1 Semi-Physical Identification

A variety of model structures can be proposed to predict \( EI_{\text{actual}} \) (model output) from \( EI_{\text{rept}} \) (model input), that is, to correct the self-reported \( EI \) from misreporting. For example, a linear formula as shown in (3) can be assumed to model such relationship:

\[
EI_{\text{actual}}(k) = \alpha \cdot EI_{\text{rept}}(k) + \gamma \quad (3)
\]
This linear relationship describes the deterministic portion of under-reporting, which tends to be systematically observed in one’s behaviors. For example, a person may mistake a 200 kcal bagel to be 150 kcal, or repeatedly forgets to report calories from snacks. This consistent behavioral pattern is the target that the proposed relationship tries to capture and model. A challenging aspect of under-reporting is associated with possible random variations in the EI self-reports. The effect of such variations can be treated as an input noise signal ($n_{EI_{rept}}$) added to $EI_{rept}$:

$$EI_{rept}(k) = \overline{EI}_{rept}(k) + n_{EI_{rept}}(k) \quad (4)$$

where $EI_{rept}$ is the measured EI, $n_{EI_{rept}} \sim \mathcal{N}(0, \sigma^2_{nEI_{rept}})$ and $\sigma^2_{nEI_{rept}}$ is the variance of the white noise $n_{EI_{rept}}$ in self-reports. To form the output of the regression problem, $EI_{actual}(k)$ can be approximated from the model-based back-calculation method or Kalman filtering approaches described in Guo et al. (2016). For simplicity, $EI_{actual}(k)$ values computed directly from the energy balance model in (3) are used here, leading to,

$$EI_{actual}(k) = \frac{(GWG(k + 1) - Kz(PA(k) + RMR(k)))}{K_1} \quad (5)$$

In the HMZ Study, maternal $W$, $EI$, $PA$, and $RMR$ of 27 OW/OB pregnant women (age mean: 28.9, std. dev.: 5.1, and pre-pregnancy body-mass-index (BMI) mean: 29.6, std. dev.: 4.0) were measured for 25–28 weeks. For the measurement of $W$, the participants weighed themselves daily using Aria Wifi smart digital scales. Participant $EI$ was obtained from self-reported $EI$ using a dietary intake phone app (MyFitnessPal), while measurements of $PA$ were obtained using a wrist-worn commercial monitor (Jawbone UP) on a daily basis. The $RMR$ was measured and objectively assessed on a weekly basis over pregnancy, much less frequently compared to the signals of $W$, $EI$ or $PA$. Hence, the $RMR$ estimated with the quadratic regression formula proposed by Prof. D. Thomas (personal communication) was used to simplify the problem:

$$RMR(t) = 0.1976W(t)^2 - 13.424W(t) + 1457.6 \quad (6)$$

where $W(t)$ is the maternal weight expressed in kg. All measurements/estimates are subject to noise, but the noise in measured weight gain ($n_{GWG}$) is relatively more significant than others considering daily weight changes that result from the individuals’ varying hydration status. Hence, the noise in the output of the correction model cannot be neglected, leading to the model-based estimates of energy intake expressed as,
\[
EI_{\text{est}}(k) = \frac{GWG_{\text{meas}}(k + 1) - K_2(PA(k) + RMR(k))}{K_1}
\]
\[
= EI_{\text{actual}}(k) + n_{GWG}(k + 1)/K_1
\] (7)

where \(n_{GWG} \sim \mathcal{N}(0, \sigma_{n_{GWG}}^2)\). Since the magnitude of the noise term \(\frac{n_{GWG}(k)}{K_1}\) tends to be quite large compared to \(EI_{\text{est}}\), smoothing techniques can be used to reduce \(n_{GWG}\) for an accurate estimation.

With the constructed input and output of the correction model, the model parameters \(\alpha_1\) and \(\gamma\) in (3) can be estimated by solving a regression problem formulated based on measurements as shown below,

\[
Z = R \theta
\] (8)

\[
\begin{bmatrix}
EI_{\text{est}}(k_1) \\
EI_{\text{est}}(k_2) \\
\vdots \\
EI_{\text{est}}(k_N)
\end{bmatrix} =
\begin{bmatrix}
EI_{\text{rept}}(k_1) \\
EI_{\text{rept}}(k_2) \\
\vdots \\
EI_{\text{rept}}(k_N)
\end{bmatrix} \begin{bmatrix}
1 \\
1 \\
\vdots \\
1
\end{bmatrix}
\begin{bmatrix}
\alpha_1 \\
\gamma
\end{bmatrix}
\]

where \(Z \in \mathbb{R}^N\) is the output vector based on \(EI_{\text{est}}\) obtained from (3); \(R \in \mathbb{R}^{N \times 2}\) is the regressor that stores input measurements; \(\theta \in \mathbb{R}^2\) is the parameter vector that needs to be estimated; \(k_1, k_2, \ldots, k_N\) are the intermittent days at which the involved measurements are taken. Note that self-reported \(EI\) is not obtained daily in order to minimize participant burden during intervention. This will result in the measurements involved in the regressor not necessarily being consecutive in terms of gestational age (days).

To identify the parameter vector \(\theta\), a least squares cost function is considered:

\[
J(k) = \min_{\hat{\theta}} \left\{ \frac{1}{2} \|Z - R\hat{\theta}\|^2 \right\}
\] (9)

Identification of this model will give the estimates of \(\theta\), from which \(\alpha_1\) and \(\gamma\), the coefficients used to model the under-reported \(EI\) in (3) can be calculated. This allows us to estimate the actual \(EI\) from the self-reported \(EI_{\text{rept}}\) as shown below,
\[ \hat{EI}_{est}(k) = \hat{\alpha} EI_{rept}(k) + \hat{\gamma} \]  

(10)

where “hat” denotes the estimate. Besides the linear structure as shown in (3), other structures that directly relate the \( EI_{rept} \) with the output of the energy balance model \( EI_{est} \) can be proposed, involving different number of parameters or with different polynomial orders. Table 1 summarizes all the evaluated structures for this approach. For each model structure, the regressor \( R \) and the estimator \( \theta \) should change accordingly. As seen from this table, nonlinear aspects are incorporated by including quadratic terms with respect to self-reported \( EI \), while computation complexity is not elevated by maintaining a linear regression solution. It might be noted that, structure F and G use maternal weight as one of the system inputs. During pregnancy, intervention compliance might change as gestation advances to a later stage. Hence, a gestational time dependency or maternal weight dependency might be a potential factor to significantly enhance the prediction of women’s underreporting behaviors.

Models A to G in Table 1 involve increasing number of model parameters, as well as requiring additional pieces of information relevant to the energy balance model. For example, Model G contains four parameters to be identified, and once estimated, participant weight measurement is required in addition to the self-reported \( EI \). In comparison, Model A with one parameter to be identified needs no measurements. The comparison of how different structures perform in terms of their predictive ability will be discussed in the next section.

### 2.2 Model-on-Demand Approach

The semi-physical identification approach described previously falls in the category of global parametric modeling methods, where all the available data points are used for batch estimation, leading to a single estimator for every operating condition. The model obtained with this global approach is assumed to be valid over the entire regressor space. Considering the dynamical changes in both physiological and psychological status during gestation, it may not be sensible to use a fixed model to describe the maternal energy intake behaviors by averaging the data collected in different gestational stages. Hence, the usefulness of this approach is limited.

Alternatively, local modeling techniques such as the Model-on-Demand (MoD) predictor use only portions of the data, relevant to the region of interest, to determine a model as needed (Braun et al., 2001). In MoD estimation all observations are stored on a database, and a local regression is performed using an “on demand” linear or quadratic model to estimate the system output at each time step. Hence, a model/prediction is obtained “on demand” and the data used for every iteration is selected independently, making this estimator capable of coping with nonlinearities presented in the model. This data-centric, nonlinear estimation method substantially enhances the classical local modeling problem. Since the MoD technique is data-driven, the user can dedicate less time making decisions regarding model structure; the requirement, however, is an informative database.

Consider a SISO system with nonlinear ARX structure
\[ y(t) = m(\varphi(t)) + e(t) \quad (11) \]

where \( m(\cdot) \) is an unknown nonlinear mapping and \( e \) is an error term modelled as i.i.d. random variables with zero mean. The regressor space \( \varphi(t) \) is of the same form as an ARX model:

\[ \varphi(t) = \left[ y(t - 1), \ldots, y(t - n_a), u(t - n_k), \ldots, u(t - n_b - n_k) \right]^T \quad (12) \]

where \( n_a, n_b \) and \( n_k \) denote the number of previous outputs, inputs and delays in the model. The MoD algorithm is designed to provide an estimate of \( \hat{y}(t) \) based on local neighborhood of \( \varphi(t) \), denoted as \( \varphi(k) \). Considering computation complexity and efficiency, a local linear or quadratic relationship is proposed to approximate \( m(\cdot) \) for further optimization. For example, a local linear structure with respect to estimator \( \beta = [\beta_0, \beta_1] \) can be assumed as below,

\[ \hat{m}(\varphi(k), \hat{\beta}) = \hat{\beta}_0 + \hat{\beta}_1^T \varphi(k) - \varphi(t) \quad (13) \]

The estimates of \( \beta \) is computed from the following objective function:

\[ \hat{\beta} = \min_{\beta} \sum_{k=1}^{N} \ell(\hat{\varphi}(k) - \hat{m}(\varphi(k), \hat{\beta}))W(\frac{\|\varphi(k) - \varphi(t)\|_M}{h}) \]

where \( \ell(\cdot) \) is the function for computing quadratic norm; \( \|d\|_M \) is a scaled distance function, defined as \( \sqrt{d^T M d} \). The bandwidth parameter \( h \) is used to control the size of the local neighborhood and is computed adaptively for each prediction with a localized version of the Akaike Information Criterion (AIC) method. The selection of the bandwidth reflects the trade-off between the bias and variance of the estimate errors. \( W(\cdot) \) is the kernel function to assign weights to every data point within the selected neighborhood window. The weight for each point is determined based on its distance from the operating condition with the goal of minimizing the point-wise mean square error of the estimate. Specifically, a tricube window function is used in lieu of the typical bell-shaped function, to improve computational tractability (Braun et al., 2001).

In contrast to the global semi-physical identification approach which leads to a fixed estimator from batch estimation, the MoD algorithms re-perform the optimization problem for each operating condition with newly computed weighting values. These iterations provide dynamically changing estimators/predictions corresponding to the local data characteristics. In a user-friendly MoD software package\(^1\), the only user decision required is

\(^1\)http://csel.asu.edu/MoDMPCtoolbox

Proc IFAC World Congress. Author manuscript; available in PMC 2018 November 24.
the choice of regressor vector parameters $n_a$, $n_b$ and $n_k$, local model order (linear or quadratic), and a lower bound on the number of data the bandwidth selector can use (readily determined by validation in a few iterations). This greatly enhances ease of use and acceptance of the technique.

From our experience from the global semi-physical approach, the output, $y$, of the MoD predictor corresponds to $EI_{\text{actual}}$ in the semi-physical model while $\hat{y}$ corresponds to $EI_{\text{est}}$.

The input to the predictor $u$ can be combinations of the signals $EI_{\text{rept}}$, $EI_{\text{rept}}^2$, and $W_{\text{meas}}$ (per Table 1). To construct the regressor, $n_a = 0$, $n_b = 1$, and $n_k = 0$.

3. CASE STUDY

In this section, the two estimation approaches described in Section 2 are evaluated against participant data from the HMZ Study, and cross-validation techniques are used to test their performance. Cross-validation is a common test procedure in system identification to examine how accurately this predictive model performs on an independent data set. Considering the characteristics of the data collected during gestation intervention, an interspersed way of data partitioning is applied by choosing one data point for estimation and every other point for validation. In this manner, the estimation and validation data sets each occupy 50% of the entire data set respectively, but are spread out uniformly over the intervention span.

For each model structure, estimation based on the assigned data set is performed followed by the validation on the remaining independent data set. To evaluate the performance of model prediction, multiple criteria are examined. Specifically, the mean and standard deviation of $EI_{\text{est}}$ as well as the computed root mean square (RMS) of the residuals from regression is used for analysis.

Based on these evaluation criteria, the best model can be selected while maintaining a parsimonious structure with minimum inputs. In Fig. 3, the results of $EI_{\text{est}}$ calculated for the two representative study participants are presented based on the model structure C and MoD approach. For Participant A, the residuals remain random and stationary while an increasing drift is observed in Participant B. This is caused by the significant increase in $EI_{\text{est}}$ towards late pregnancy due to the increasing rate of maternal weight change, while the self-reports are not reflecting such increase but remaining stationary. Such observations in maternal weight gains can lead to a nonlinear relationship between $EI_{\text{rept}}$ and $EI_{\text{est}}$. For participants with such characteristics, a time-dependent input such as gestational age or maternal weight (as shown in Model Structure F and G) can improve the estimates significantly, while for some other participants, this additional input might not change the results as much. On the other hand, MoD approach can successfully remove the drift in the residuals by capturing the unmodeled dynamics without requiring any extra piece of time-dependent information.

The estimated results for the two participants using different model structures are tabulated in Table 2 and 3, and the parallel comparison for the two approaches can be found in Table 4. As can be seen from Table 4, MoD approach can achieve better prediction by requiring less pieces of information/measurements from participants by comparing the RMS for each
estimator. This advantage from MoD approach can reduce participant burdens and contribute to the success of future intervention.

In summary, when there is no significant increase in $EI_{\text{rept}}$, the 1st order model with two parameters and the 2nd order model with three parameters provide the best (and comparable) estimation results. When significant increases in self-reports are observed due to dramatic weight gain (e.g., Participant B), the dynamics in the energy intake cannot be fully captured with correction models that are only dependent on $EI_{\text{rept}}$ (which are usually being stationary signals). Augmentation of a time-dependent variable in the models, e.g., gestational age or weight, will improve the predictive performance as shown in the residual analysis. It is important to note that the input noise poses a challenge for both estimation methods due to the problem of errors-in-variables, as shown in Fig. 2. This issue is part of current research. The MoD approach, on the other hand, can achieve comparable and even better prediction performance, while requiring less information of the energy balance system from participants, as well as involving reduced engineering effort.

4. SUMMARY & CONCLUSIONS

Both past literature and the experience with the Healthy Mom Zone Study point to the fact that misreporting is a common problem in self-reported measures, and especially significant in self-reported energy intake. To address this concern, we have developed two estimation approaches based on system identification principles to estimate energy intake and adjust biased self-reported measurements of $EI$ in the future. The estimated models are useful in determining the portion of energy intake that is systematically underreported, enabling health providers to deliver informative health guidance for participants, and allowing energy balance models to be more accurately used in intervention settings.

Both approaches described in this paper exhibit pros and cons. The global semi-physical approach can generate good estimates with low computational effort, but requires a priori knowledge to make sensible decisions of model structures. The identified model remains fixed over the entire span of interest. In contrast, the Model-on-Demand approach can obtain time-varying models that capture nonlinearities observed in the data. While this local method requires reduced engineering effort from users and can be implemented via user-friendly software, it still involves higher complexity in the computation. When applying these illustrated approaches in diverse settings, the features of each method need to be considered and examined carefully based on the requirements of the specific application.

Acknowledgments

Support for this work has been provided by the National Heart, Lung, and Blood Institute (NHLBI) through grant R01 HL119245. The opinions expressed in this article are the authors’ own and do not necessarily reflect the views of NHLBI.

REFERENCES

Braun MW, Rivera DE, and Stenman A (2001). A ‘Model-on-Demand’ identification methodology for nonlinear process systems. International Journal of Control, 74(18), 1708–1717.
Downs D et al. (2017a). Individually-tailored, adaptive intervention to manage gestational weight gain: protocol for a randomized controlled trial in women with over-weight and obesity. JMIR Res Protoc (In Press).

Downs DS, Savage JS, Rivera DE, Leonard KS, Pauley AM, Hohman EE, and Guo P (2017b). Influence of a feasibility study on the design of an individually-tailored, adaptive intervention to manage weight in overweight and obese pregnant women. Annals of Behavioral Medicine, 51, S1451–S1452.

Gilmore LA, Klempel-Donchenko M, and Redman LM (2015). Pregnancy as a window to future health: Excessive gestational weight gain and obesity. In Seminars in Perinatology, volume 39, 296–303. [PubMed: 26096078]

Guo P, Rivera DE, Downs DS, and Savage JS (2016). Semi-physical identification and state estimation of energy intake for interventions to manage gestational weight gain. In Proceedings of 2016 American Control Conference (ACC), 1271–1276.

Haugen M et al. (2014). Associations of pre-pregnancy body mass index and gestational weight gain with pregnancy outcome and postpartum weight retention: a prospective observational cohort study. BMC Pregnancy and Childbirth, 14(1), 201. [PubMed: 24917037]

Lichtman SW et al. (1992). Discrepancy between self-reported and actual caloric intake and exercise in obese subjects. New England Journal of Medicine, 327(27), 1893–1898. [PubMed: 1454084]

Rasmussen KM and Yaktine AL (eds.) (2009). Weight Gain During Pregnancy: Reexamining The Guidelines. National Academies Press.

Thomas DM, Navarro-Barrientos JE, Rivera DE, et al. (2012). Dynamic energy balance model predicting gestational weight gain. The American Journal of Clinical Nutrition, 95(1), 115–122. [PubMed: 22170365]

Trabulsi J and Schoeller DA (2001). Evaluation of dietary assessment instruments against doubly labeled water, a biomarker of habitual energy intake. American Journal of Physiology-Endocrinology And Metabolism, 281(5)(5), E891–E899. [PubMed: 11595643]
Fig. 1.
Weight predictions from the energy balance model per (1) using a representative participant from the Healthy Mom Zone Study shows evidence of significant underreporting of energy intake.
Fig. 2.
Block diagram of the correction model.
Fig. 3.
Estimate comparison on validation data set with the two approaches for Participant A (an intervention participant) and B (a control participant) from HMZ Study. Model structure C was used for semi-physical approach, while one input case for MoD approach.
Table 1.
Summary of all the proposed structures to correct self-reported energy intake. Each structure is characterized by the number of model parameters and the number of pieces of required information (longitudinal measurements of energy balance variables). The estimator corresponding to each regression model is a subset of \([a_1, \gamma, a_2, \beta]\)

| Model Structure | Par # | Info # |
|------------------|-------|--------|
| A \(EI_{est}(k) = \gamma\) | 1     | 0      |
| B \(EI_{est}(k) = a_1 EI_{rept}(k)\) | 1     | 1      |
| C \(EI_{est}(k) = \gamma + a_1 EI_{rept}(k)\) | 2     | 1      |
| D \(EI_{est}(k) = a_1 EI_{rept}(k) + a_2 EI_{rept}(k)^2\) | 2     | 1      |
| E \(EI_{est}(k) = \gamma + a_1 EI_{rept}(k) + a_2 EI_{rept}(k)^2\) | 3     | 1      |
| F \(EI_{est}(k) = \gamma + a_1 EI_{rept}(k) + \beta W_{meas}(k)\) | 3     | 2      |
| G \(EI_{est}(k) = \gamma + a_1 EI_{rept}(k) + \beta W_{meas}(k) + a_2 EI_{rept}(k)^2\) | 3     | 2      |
Table 2.
Estimation Results for Participants A and B using the global semi-physical approach for all the proposed model structures (A to G in Table 1).

| Model Structure | Participant A | | Participant B | | | | | | |
|-----------------|--------------|---|--------------|---|---|---|---|---|---|
|                 | Estimation   | Validation | Estimation   | Validation | | | | | |
|                 | $\hat{E}_{est}$ | RMS | $\hat{E}_{est}$ | RMS | $\hat{E}_{est}$ | RMS | $\hat{E}_{est}$ | RMS |
| A               | 2455±699     | 8188 | 2257±771     | 9354 | 2591±5002     | 408 | 2628±410     | 3606 |
| B               | 2665±3E-12   | 5315 | 2665±3E-12   | 5443 | 2671±1E-12    | 3182 | 2671±1E-12   | 3019 |
| C               | 2665±39      | 5305 | 2676±43      | 5428 | 2671±90      | 3131 | 2677±74      | 2982 |
| D               | 2594±457     | 6250 | 2540±536     | 7561 | 2646±289     | 3471 | 2680±98      | 2937 |
| E               | 2665±91      | 5263 | 2677±69      | 5541 | 2671±90      | 3131 | 2677±79      | 2985 |
| F               | 2665±50      | 5299 | 2674±50      | 5361 | 2671±198     | 2926 | 2674±183     | 2552 |
| G               | 2665±95      | 5258 | 2675±74      | 5480 | 2671±203     | 2912 | 2678±189     | 2534 |
Table 3.
Estimation Results for Participants A and B using Model-on-Demand (MoD).

| Inputs | Participant A | | Participant B | |
|-----------------|-----------------|-----------------|-----------------|-----------------|
|                | Estimation | Validation | Estimation | Validation |
| \(E_{lent}\) | 2675±94 | 5271 | 2670±99 | 5273 |
| \(E_{lent}^2\) | 2791±152 | 2498 | 2713±159 | 2471 |
| \(E_{lent}^2\) \(\times W\) | 2499±95 | 5315 | 2671±95 | 5495 |
| \(E_{lent}^2\) \(\times W\) | 2781±154 | 2512 | 2712±155 | 2502 |
| \(E_{lent}^2\) \(\times W\) | 2665±109 | 5105 | 2676±102 | 5100 |
| \(E_{lent}^2\) \(\times W\) | 2761±147 | 3131 | 2553±153 | 2537 |
| \(E_{lent}^2\) \(\times W\) | 2632±97 | 5380 | 2682±89 | 5385 |
| \(E_{lent}^2\) \(\times W\) | 2706±144 | 3471 | 2211±159 | 2268 |
Table 4.
Comparison of estimation results for the two approaches for Participants A and B.

| Participant A | Semi-physical Approach | Participant B |
|---------------|------------------------|---------------|
| MoD Method | Model Structure | MoD Method | Inputs |
| | | | |
| | | | |
| Inputs | RMS | Model Structure | RMS | Inputs |
| | | | |
| $[EI_{rep}]$ | 5273 | 5443 | B | 3019 | $[EI_{rep}]$ |
| | 5428 | 3019 | C | 2982 |
| $[EI_{app}, EI_{rep}^2]$ | 5495 | 7561 | D | 2502 | $[EI_{app}, EI_{rep}^2]$ |
| | 5541 | 2937 | E | 2985 |
| $[EI_{app}, W_{meas}]$ | 5100 | 5361 | F | 2537 | $[EI_{app}, W_{meas}]$ |
| | 5385 | 2552 | G | 2268 | $[EI_{app}, EI_{rep}^2, W_{meas}]$ |

Note: model structure A of the semi-physical approach is not included in the comparison.