Electron/Nuclear spin domain walls in quantum Hall systems

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Motivated by recent all optical NMR experiments$^1$ on GaAs quantum wells, we propose new experiments that would involve creating spatially modulated nuclear spin profiles. Due to the hyperfine coupling these would appear as spatially modulated Overhauser fields for the electrons that could have an amplitude large enough to cancel or even reverse the external Zeeman field at some places. We discuss 2D electron gas transport in the quantum Hall regime at filling factor $\nu=1$, and demonstrate the existence of collective modes and topological excitations induced in the electron gas by various nuclear spin patterns. We calculate the $1/T_1$ relaxation rate of the nuclear spins due to coupling with these low lying collective modes and also discuss how transport and the low energy modes would be affected by a highly anisotropic g-tensor, which is special to a GaAs quantum well grown in the [110] direction.

I. INTRODUCTION

Quantum Hall systems that are realised in GaAs quantum wells and heterostructures show very interesting physics both in terms of electron charge and spin degrees of freedom$^{2}$. Spin orbit coupling in GaAs reduces the bulk g factor from 2 to -0.4 and together with a reduced electron effective mass ($m^* \sim 0.07 \, m_e$), causes the Zeeman splitting $g \mu_B B$ to be almost 70 times smaller than the cyclotron energy. Therefore it is possible to be in a temperature regime where the electrons are confined to the lowest Landau level (LLL), yet low energy spin fluctuations are not completely frozen out. Strong coulomb exchange interactions together with the dispersionless kinetic energy of the electrons make these quantum Hall systems ideal ferromagnets, while the low energy spin fluctuations are simply the goldstone modes of the ferromagnet$^3$.

Besides the electron coulomb energy and the Zeeman energy, a third energy scale, namely the hyperfine interaction of the electrons and the GaAs nuclei could play an important role in the physics. As a result of the hyperfine coupling, a net electron spin polarisation acts like an effective magnetic field $B_e$ for the nuclei and the corresponding energy shift is referred to as the Knight shift. Similarly, a net nuclear polarisation shows up as an effective additional magnetic field $B_n$ seen by the electrons, and the corresponding electron energy shift is referred to as the Overhauser shift. In a uniform quantum Hall ferromagnet the goldstone modes have a gap equal to the Zeeman splitting. This energy scale while small, is still several orders of magnitude larger than the nuclear spin precession frequency and therefore the goldstone modes do not affect the nuclear spin lattice relaxation rates.

The situation would be different however if one could make domain walls in the electron spins by creating regions where the effective electron Zeeman energy changes sign. We propose this be done by creating spatial patterns in the degree of nuclear polarization. Domain walls are characterised by electron spins non-colinear to the external applied magnetic field. Due to this, the domain walls can support gapless goldstone modes associated with the zero energy cost for rotating spins around the Zeeman axis. These zero energy modes can effectively couple to the nuclei causing shorter spin lattice relaxation times and can also significantly alter charge transport.

Spatial patterns in nuclear polarisation can in principle be created by magnetic resonance imaging (MRI) techniques. Nuclear spins can also be polarised by optical pumping techniques$^4$, and more recently ultrafast optical spectroscopy$^5$ has made it possible to create and detect spatially localised regions of polarised nuclear spins, the size of the localised pockets in present experiments being of the order of 70 microns. This is achieved by focussing the pump and probe beams by means of a lens. Near field scanning optical microscopy$^7$ could in principle be used to achieve spatial resolutions below the diffraction limit, but has practical difficulties due to loss of incident light intensity$^8$. If this difficulty is overcome, NSOM would be a very powerful and novel technique to “write” various nuclear spin patterns similar to the ones we discuss in this paper.

In this paper we discuss electron transport and low lying excitations in a quantum Hall system at Landau level filling factor $\nu=1$ formed in a GaAs [110] quantum well where the effective Zeeman field seen by the electrons, which is the sum of the external quantising magnetic field and the Overhauser field, has been spatially modulated. The spatial modulation may be produced either by near-field scanning probe methods or by allowing the pump beam to be a standing wave. The latter would create
a sinusoidal variation in the incident light intensity, and this would in turn create a commensurate sinusoidally varying nuclear polarisation. From recent experiments we may also assume that the nuclear polarisation amplitude is large enough to produce oscillations in the sign of the effective Zeeman field and create lines along which the effective magnetic field seen by the electrons is zero.

The hyperfine interaction between the GaAs nuclei and the electron gas may be written as

$$ H_F = \frac{8\pi}{3} \gamma_e \gamma_N \hbar^2 \sum_{i,j} \mathbf{S}_i \cdot \mathbf{I}_j \delta(\mathbf{r}_i - \mathbf{r}_j) $$

(1)

Averaging the above expression with respect to the Slater determinant state describing a spin polarised 2DEG at $\nu = 1$, it is easy to see that the Knight shift energy and the Overhauser shift energy are related as follows when the nuclei carry spin $3/2$,

$$ E_O = \frac{3p \rho_n N}{n_e} $$

(2)

where $n_e$ and $n_N$ are the 3D electron and nuclear spin densities respectively, while $p$ is the extent of nuclear spin polarisation. GaAs has a zinc-blend structure with a cubic unit cell side of 5.65 Å, corresponding to a Ga nuclear density $n_N = 2.2 \times 10^{28} \text{m}^{-3}$. Moreover the size of Knight shifts for the Ga nuclei obtained by Barrett et al. is $\sim 20$kHz. Using this and the fact that Knight shifts are enhanced by narrower wells and higher 2D densities, we estimate the Overhauser shift for the samples used by Awschalom’s group to be around $E_O \sim 15 - 20$GHz from the Ga nuclei alone. Precise calculations predict that the nuclear fields due to polarised As nuclei would be almost twice as large as due to Ga, and therefore the maximum Overhauser shift can be as large as 39.5 GHz, which corresponds to nuclear magnetic fields (for $g^* \sim 0.053$) of 53 T. Thus by creating nuclear spins polarised in the appropriate direction one may have regions where the effective magnetic field seen by the electrons is zero or even negative.

**II. DOMAIN WALLS**

Falko et al. have described low energy excitations in a quantum Hall system where the Zeeman field abruptly changes sign, causing the formation of domain walls in the electron spin. They were looking at the effect of pressure inhomogeneities that would cause the $g$ factor to fluctuate about zero in a sample. We do a similar analysis to derive the low energy excitations for a linearly varying effective Zeeman field which however has been produced by a controlled spatial manipulation of nuclear spins. In addition, we also address the question of edge transport in a single domain wall system which may be a part of an array of domain walls separated roughly by the wavelength of the pump beam.

Fig. 1 is a schematic picture of the domain wall profile. We will assume that both the quantising magnetic field that tunes the 2DEG to be at $\nu = 1$ and the Overhauser field due to the polarised nuclei point in the $\hat{z}$ direction. Moreover the spatial variation of the effective Zeeman field is assumed to be linear and along the $\hat{x}$ direction. The energy density functional describing the 2DEG can therefore be written as

$$ H = \frac{\rho_s}{2} \left[ \left( \frac{\partial \theta(x)}{\partial x} \right)^2 + \sin^2 \theta(x) \left( \frac{\partial \phi(y)}{\partial y} \right)^2 \right] - \frac{e_z}{4\pi l^2} Q x \cos \theta(x) $$

(3)

The above energy functional is minimised by a domain wall solution where $\phi$ is uniform and arbitrary (reflecting the U(1) symmetry in the problem associated with rotations about the axis of the effective magnetic field), while $\theta(x)$ may be chosen to have the following variational form

$$ \cos \theta_0(x) = \text{sgn}(x) \left[ 1 - 2 \text{sech}(\beta |x|) + \ln(\sqrt{2} + 1) \right] $$

(4)

The constant term in the argument of sech ensures the boundary condition that for $\beta x = -(+)\infty$, $\theta = \pi(0)$ and at $x = 0$, $\theta = \frac{\pi}{2}$. We determine $\beta$ by requiring that

$$ \frac{\partial}{\partial x} \int dx H(x, \beta) = 0 $$

and find

$$ \beta = \frac{2 \sqrt{2} \pi}{\sqrt{2} + 1} \ln \left( \frac{2 \sqrt{2} \pi}{\sqrt{2} + 1} \right) \frac{e_z}{4\pi l^2 \rho_s} Q \frac{\pi}{2} $$

We will now proceed to derive an effective action describing the spin wave modes that arise when $\theta$ and $\phi$ fluctuate about $\theta_0$ and $\phi_0 = \text{constant}$ respectively. The leading order term in the effective energy functional for the spin waves has two parts. One part is obtained by simply integrating out the the domain wall profile in the $\hat{x}$ direction in the second term in Eq 3. This gives the following term in the action

$$ U \approx \frac{\rho_s}{2} \frac{1.72}{\beta} \int dy \left( \frac{\partial \phi}{\partial y} \right)^2 $$

(5)

The second term in the spin-wave energy functional is a measure of the Zeeman energy cost for deviations from $\theta_0$ and is obtained variationally by evaluating the change in Zeeman energy for a domain wall profile whose center is shifted from $x = 0$ to $x = X_0$. This gives rise to a net magnetisation density along the transverse $\hat{y}$ direction $m_\perp^{\text{eff}} = \frac{X_0}{\pi e}$ and the cost in Zeeman energy turns out to be
\[ E_z = \int dy \frac{\pi e Q^2}{4} \left( m_z^{1D}(y) \right)^2 \] (6)

The action may be derived by starting with the Berry’s phase term for a quantum Hall ferromagnet \[ S_n \int dx \int dy A^a \cdot \bar{m}_a \], where \( \bar{A} = \bar{\nabla}_m \times \hat{m} \). For the domain wall region, the spin is almost completely in the x-y plane and this would mean \( \bar{A} \approx -m_z \hat{y} \). Using this and integrating out the \( \hat{x} \) direction, the Berry phase term for the 1D action is \( \frac{i}{\hbar} \int dy \partial_y m_z^{1D} \). Combining all these terms, the total action describing the domain wall is

\[ S = \int dy \int dx \frac{i}{2} \frac{\partial \phi}{\partial t} m_z^{1D} - \frac{\Gamma}{8} (m_z^{1D})^2 - \frac{\rho_s^{1D}}{2} \left( \frac{\partial \phi}{\partial y} \right)^2 \] (7)

where \( \Gamma = 2\pi Q_0 \) and \( \rho_s^{1D} = \frac{\pi}{2} \rho_s \). Next, one can integrate out the massive \( m_z^{1D} \) fluctuations from the above action and arrive at \( S = \int dy \int dx \frac{\rho_s^{1D}}{2} \left( \frac{\partial \phi}{\partial y} \right)^2 + \frac{1}{8\pi} \left( \frac{\partial \phi}{\partial x} \right)^2 \).

The quantised Hall conductance implies a direct relation between the charge and spin fluctuations. Hence the domain wall profile which minimises the energy functional, over the \( \hat{x} \) direction gives the following relation for 1D charge deviations along the \( \hat{y} \) direction \( \rho(y) = \frac{1}{\pi v_g} \frac{\partial \phi}{\partial y} \). Thus the above action for spin waves along the domain wall can be mapped onto a Luttinger liquid with interaction parameter \( g = \frac{1}{\pi v_g} \) and collective mode velocity \( c = \sqrt{\frac{1}{\pi v_g}} \).

Assuming that the nuclear spin profile is established optically by either using a standing light wave of wavelength \( \lambda \) or by a diffraction limited focussed beam, the scale of \( Q \) will be set by \( A = \frac{\pi}{\lambda} \sim 100 \ell \). The above estimate for \( Q \), along with the fact that \( e_z \sim 3K \) and using \( \rho_s = \frac{1}{16v_g2\pi} \) gives a domain wall width, \( \frac{\lambda}{Q} \sim 6 \ell \). For this the collective mode velocity along the domain wall is estimated to be \( \sim 10^3 \) m/s.

As mentioned before, the spin waves along the domain wall are gapless and therefore can couple to the GaAs nuclei causing a finite spin-lattice relaxation time \( T_1 \) given by

\[ \frac{1}{T_1} = \lim_{\omega \to 0} \frac{2\pi}{\hbar} \mu \left( \frac{A}{2} \right)^2 < 3 \left| I_+ \right| \left| I_- \right| > \frac{2}{1 - e^{-\rho_s \mu}} \text{Im} \chi^+_- (\omega) \] (8)

where \( \text{Im} \chi^+_- (\omega) \) is the dissipative part of the spin susceptibility. Note that \( A \) is related to the Knight Shift as \( E_K = A < S_z > \). In our formalism, the spin raising operator is defined as \( \hat{S}_+(x, y, \tau) = \frac{\Delta}{2} \sin \theta_0 (x) e^{i\theta(x, y, \tau)} \) where \( \theta \) is a coordinate along the domain wall, while \( x \) is the coordinate in the transverse direction. \( \text{Im} \chi^+_- \) is obtained by evaluating the following correlation function in imaginary time, \( C(\tau) = < T_+ \hat{S}_+(x, y = 0, \tau) \hat{S}_-(x, y = 0, 0) > \), followed by simultaneously doing the fourier transform and the analytic continuation using the identity \( \text{Im} \chi^+_- (0, \omega) = \frac{\Delta}{2\pi} \int dt e^{-i\omega t} \chi (\beta/2 - it) \). Due to the gaussian action of the Luttinger liquid, the above correlation function can be analytically evaluated and we obtain

\[ \frac{1}{T_1} = \frac{\sin^2 \theta_0 (x) 2\pi}{\hbar (1/4)^2} < 3 \left| I_+ \right| < \frac{2}{1 - e^{-\rho_s \mu}} \text{Im} \chi^+_- (\omega) \]

The above expression yields the the Korringa law \( T_1 \) \( T_2 \) constant in the non-interacting limit \( g = 1 \). Note that \( \Delta v_F \) is a short time cutoff that arises in evaluating the correlation functions. While we can only give a heuristic estimate for what this cut-off ought to be, we find from the variational treatment of the domain wall above that \( g \sim 0.02 \). For such tiny values of \( g \) the size of \( T_1 \) is not very sensitive to our choice of cut-off. The reason why \( g \) is much smaller than is typical for 1D interacting electron gases is that for our system \( g \) scales as \( g \sim (QI)^{1/2} \), where \( Q \) is the wave-vector of the pump beam. If we assume the momentum cutoff to be set by inverse of the domain wall width \( (\ell \sim \frac{1}{Q}) \) and if we take \( v_F \sim 10^3 \text{m/s} \), which yields an energy cutoff \( h \Delta v_F \) of 1K, we find \( T_1 \sim 0.1 \text{s} \). This time scale indicates that the domain wall will stay intact long enough to carry out electron transport measurements. This time scale is however shorter by at least 4 orders of magnitude from typical nuclear relaxation times observed in quantum Hall samples in uniform Zeeman fields close to \( \nu = 1 \), and 2 orders of magnitude smaller than nuclear relaxation times observed in \( \nu = 0.88 \) quantum Hall samples that contain skyrmions.

It is interesting to notice that for times longer than \( T_1 \), the nuclei at \( x = 0 \) would depolarise significantly. But since the nuclear spins far from the domain wall center are still polarised parallel/anti-parallel to the external Zeeman field, the domain wall in electron spins and the gapless excitations characterised by them will remain intact. However the details of the domain wall profile (length, spin wave velocity etc.) would get modified and the center of the domain wall will also shift along the direction of the nuclear field gradient.

We now address the question of transport in the above geometry. Figure 2 shows what we have in mind. We imagine feeding current into the sample through the edges that are adiabatically connected to the reservoirs at the two ends, while towards the center of the sample the spin degree of freedom associated with the edges rotates to follow the domain wall profile calculated above. We would like to know the transmission probability across this domain wall. Perfect transmission would mean that voltage probes V1 and V2 are at the same potential (i.e., \( \rho_{xx} = 0 \)) and the \( \nu = 1 \) quantum Hall state is restored. Perfect reflection on the other hand would correspond to destruction of the quantum Hall state.
In order to analyse this we note that the domain wall is equivalent to two interacting chiral Luttinger liquids of opposite spins. This is made clear by doing the following change of variables in the domain wall action (Eq. 8), \( \phi = \phi_L + \phi_R \) and \( m_z = \frac{1}{4\lambda} (\frac{\partial \phi_L}{\partial x} \frac{\partial \phi_R}{\partial x}) \). This change of variables preserves the canonical commutation relations between \( \phi \) and \( m_z \), provided \( \phi_L \) and \( \phi_R \) obey the Kac-Moody algebra. In terms of these new variables the action may be written as \( S = S_L + S_R + S_{\text{int}} \) where

\[
S_{L/R} = \int dx \int dt \left( \frac{1}{4\pi} \frac{\partial \phi}{\partial x} \right)^2 (\pm \frac{\partial \phi}{\partial t} - \nu_0 \frac{\partial \phi}{\partial x})
\]

and \( \lambda = \rho^{1D} - \frac{e}{16\pi} \) and \( \nu = 2\pi \rho^{1D} + \frac{e}{16\pi} \). \( S_L \) and \( S_R \) are actions for left-moving and right-moving non-interacting chiral Luttinger liquids while the third term shows that these chiral modes interact with each other through the parameter \( \lambda \). This also explains why the spin raising operator \( S_+ \propto e^{i\phi} \). This is because the spin raising operator is equivalent to \( \Psi_L \Psi_R \propto e^{i\phi} e^{i(\phi_L + \phi_R)} \). Note that for our problem \( k_F \) is zero since microscopically the domain wall profile is taken by a linear combination of up-spin and down-spin single electron states at the same momentum (or guiding center index). For the usual Luttinger liquid obtained from bosonizing spinful fermions, the spin raising operator has a more complicated form since unlike our case, a spin flip can occur along more than one channel (such as spin flips between unidirectional and counterpropagating modes).

We can now describe transport via the edge modes in Fig. 2 as the same as transport along a 1D chain that has 3 parts. The 1st and 3rd parts consist of Luttinger liquids with interaction parameter \( g = 1 \). This represents the chiral non-interacting edge-modes that feed in/out of the reservoirs. The middle part of the chain however is an interacting Luttinger liquid and represents the domain wall. Transport in such coupled Luttinger liquids have been studied by various authors.\(^{13} \)

The central result of Safi et al.\(^{13} \) is that as long as there is no disorder, the central wire acts as a Fabry Perot interferometer and there is always perfect transmission along the wire in the dc limit. In the present problem perfect transmission occurs because of conservation of the \( z \) component of the electron spin and due to the fact that the two chiral modes carry opposite spins, which makes backscattering impossible. Perfect transmission along the 1D chain implies that a wave incident along one of the edges in Fig. 2 travels along the domain wall and gets perfectly reflected back into the same reservoir. In our language this corresponds to perfect reflection at the domain wall and this would give rise to a finite voltage drop between probes V1 and V2 and hence a destruction of the \( \nu = 1 \) quantum Hall state.

One could get finite transmission across the domain wall if the spin waves along the domain wall are gapped. When this happens a low energy mode incident from one of the reservoirs would not have any propagating state to scatter to along the domain wall. Thus it would travel along the domain wall as an evanescent wave, and for a long enough domain wall, may completely decay before reaching the other end. This situation would correspond to complete transmission across the domain wall. The physics of this has been analysed in detail in a different context involving two \( \nu = 1 \) 2DEGs separated by a narrow but high barrier. In the next section we discuss how such a gapping can arise.

### III. ANISOTROPIC G-TENSOR

In our present set-up the spin waves can be gapped by introducing spin-orbit interactions which destroys the U(1) symmetry for rotations about the Zeeman axis. The effect of spin-orbit interactions in the regime of vanishing Zeeman energy has been studied in detail by Falko et al.\(^{13} \) They explicitly show that a Rashba spin-orbit interaction gives rise to a small additional term in the spin wave action proportional to \( \cos \phi \), so that the total action looks like the integrable Sine-Gordon model.

We find that for a 2DEG formed in a GaAs [110] heterostructure the spin waves can be gapped even in the absence of explicit spin-orbit coupling terms. This is due to the anisotropic g tensor (implicitly due to spin-orbit effects) and may be understood as follows. The crystal symmetry in GaAs is such that the principal axes of the g-tensor coincide with the [110],[1-10] and [001] directions. For orientations of the external B field that do not coincide with the principal axes, the electrons spins would want to align along an axis \( \Omega^\mu = g^{\mu\nu}B^\nu \), non-collinear to \( B \). Unlike the electrons, the polarised nuclear spins would continue to precess about \( B \) and therefore the time-averaged nuclear spin points along \( B \) and may be written as \( < I > = \hat{I}(r)\hat{B} \). As the nuclear polarisation \( I(x) \) varies spatially, the total effective magnetic field now rotates in the \( \Omega - B \) plane. This is a more complicated situation than before where the net B field was always along the same axis. If we denote \( \phi \) to be the angle that the electron spin makes with the \( \Omega - B \) plane, and \( \theta \) the angle it makes with \( \Omega \), then the Zeeman energy density has the following form

\[
E_z = s_{||} \left( \Omega + E_O p(x) \Omega \cdot \hat{B} \right) + s_{\perp} E_O p(x) |\Omega \times \hat{B}| \cos \phi
\]

\( s_{||/\perp} \) is the component of the electron spin density along/perpendicular to \( \Omega \) and \( E_O \) is the amplitude of the Overhauser shift and \( p(x) \) is the spatially varying net nuclear spin polarisation along \( B \).

For small deviations of the applied magnetic field from the crystal symmetry axis, one can still assume that the domain wall profile is given by Eq. 4. By looking at fluctuations about the static domain wall solution one now obtains the integrable Sine-Gordon model,
characterised by a Luttinger liquid part and an additional term proportional to \( \cos \phi \). The coefficient of \( \cos \phi \) term reflects the fact that the spin waves are gapped if there is a net nuclear polarisation in addition to an anisotropic g-tensor. The spin wave gap keeping only the leading order term in \( \mathbf{\Omega} \times \mathbf{B} \) is given by
\[
\Delta_s = \sqrt{\frac{\Gamma_{\text{opt}}}{\pi}} \left( f \frac{\Omega}{\sin \theta_0(x)} \right) \mathbf{\Omega} \times \mathbf{B} \approx 0.66 \sqrt{\mathbf{\Omega} \times \mathbf{B}} \text{ K}
\]
for a 10 T field slightly misaligned with the [110] axis. Transport measurements are sensitive to the charge gap \( \Delta_c \) which is twice the energy required to create a soliton in the field \( \phi \), and the classical expression for this to leading order in the tilt angle is given by
\[
\Delta_c = \frac{\Delta}{2} \approx 26.3 \sqrt{\mathbf{\Omega} \times \mathbf{B}} \text{ K}.
\]
This estimate for the charge gap is in principle reduced by quantum fluctuations which may be calculated exactly, but in the limit \( g \to 0 \), the classical estimate is more and more exact.

Therefore for the purposes here \( g \approx 0.02 \) we use the classical expression and find that at temperatures of 1K, for the domain wall excitations to appear gapless, the external B field has to be aligned along [110] as precisely as 0.08 deg. The evanescent wave decay length \( \frac{1}{\kappa} = \frac{\lambda_0}{\hbar c} \) at this angle is only 77Å which also sets the upper limit on the domain wall length for which one would observe perfect reflection at the domain wall. Due to these two reasons, namely the precision with which the magnetic field has to be aligned along the [110] crystal symmetry axis and the difficulty in creating domain walls of length \( < \ell \), experimentally it seems one would always measure a finite charge gap.

IV. SKYRMIONS

It is also interesting to study the nature of the collective modes in a geometry where a tiny circular patch of region has spin-reversed nuclei, yielding an effective B field which is along the negative \( \hat{z} \) direction, while everywhere outside the patch the B field is along the positive \( \hat{z} \) direction. One would now expect the domain wall of spins to be circular (see Fig. 3). Let us suppose the radius of the patch is \( R \). Then the excitations along the domain wall will be of two kinds. The first will be neutral excitations that do not carry any topological charge, and have an energy given by
\[
\omega_n = \frac{c k_n}{\sqrt{\Gamma_{\text{opt}}}} \left( \frac{\pi}{2} \right),
\]
where \( n \) is any integer. The second kind of excitation along the domain wall are charge carrying excitations where the field \( \phi \) winds by \( 2\pi n \) along the circumference and therefore carries a net charge of \( me \). The energy of these modes which is a sum of the exchange energy and the electrostatic hartree energy of a ring of charge of radius \( R \), in the limit of \( r_0 \ll R \), is given by
\[
\omega_m = \frac{m^2}{\kappa} \left( \frac{\pi}{2} \rho_s e^2 \ln \frac{r_0}{\hbar c} \right),
\]
where \( m \) now labels the topological charge, and \( r_0 \sim \ell \) is an ultraviolet cutoff associated with the finite thickness of the ring of charge.

Skyrmions are charged topological spin excitations that arise in quantum Hall ferromagnets and their energy and size is determined by a competition between coulomb interactions and the Zeeman energy. Detailed Hartree-Fock calculations have been done to estimate the size of a skyrmion in GaAs to be a few magnetic lengths so that the skyrmions contain about 3-4 overturned spins. NMR Knight shift experiments also support this estimate. However skyrmions can be much larger at high pressure where the g factor is reduced and at \( \nu = 1 \) can have arbitrarily many flipped spins in the limit \( g^* \to 0 \). Here we point out that instead of pressure tuning, one could vary the degree of nuclear polarization to obtain small effective Zeeman fields and hence large scale size skyrmions. In addition, by spatially modulating the degree of nuclear polarization one could produce effective potential wells that could trap the skyrmions and/or modify their transport. For example, if the regions inside and outside the circular patch are characterised by an effective Zeeman field of the same strength, but opposite orientations, then skyrmions with K spin flips formed outside the patch (say) would be attracted to the patch which would appear as a potential well of depth \( 2g^* \mu_B BK \sim 2.2 \) Kelvin for a 10 T field and \( g^* \approx 0.04 \). This situation is not stable however and for time scales longer than \( T_1 \) the skyrmion would increase in size by acquiring additional spin flips. Eventually the skyrmion would turn into the topological excitation described above (Fig. 3) whose charge lies on a circular ring domain wall and the electron spins in the interior of the ring would be all reversed.

V. BILAYER DOMAIN WALLS

Analogous to the derivation done above for the domain wall collective modes in a \( \nu = 1 \) quantum Hall sample, we can do a similar analysis for a quantum Hall double layer, where the spins are replaced by pseudospins. The pseudospin labels which layer the electron is in and the spatially varying effective Zeeman field is replaced by a spatially varying external bias potential which unbalances the charge density in the two layers. With appropriate split gates one could arrange the external bias potential to be large and positive for \( x < 0 \) say, and large and negative for \( x > 0 \), so that the pseudospin is “up” on the left and “down” on the right, i.e., the electrons like to sit completely in the upper/lower layer on the left/right.

In the intermediate region around \( x = 0 \) we would however expect a pseudospin domain wall where the pseudospin gradually tilts from +1/2 to -1/2 and in this region the electrons may be regarded to be in a coherent superposition of both layers. We do a similar analysis as before on the full energy functional for the double layer given by
\[
E[\mathbf{m}] = \int d^2 r \beta (m_z - V_B(x))^2 + C[\mathbf{m}] + \frac{\rho_A}{2} (\nabla m_z)^2 + \frac{\rho E_0}{2} [ (\nabla m_x)^2 + (\nabla m_y)^2 ]
\] (13)
$V_B(x)$ is the external bias potential which we assume varies linearly and passes through zero at $x = 0$. The effective action for the domain wall we derive also looks like that of a Luttinger liquid, where the scalar field $\phi$ is now related to the X-Y orientation of the pseudospin along the domain wall. For $\beta = 0.005\frac{e^2}{\hbar c} \rho$ and $\rho_E = 0.012\frac{e^2}{\hbar c}$, the velocity of the mode turns out to be $v \sim 0.17c^2 = 3 \times 10^4$ m/s, slightly larger than the bulk collective mode velocity in a balanced double layer system.\[4\]

VI. CONCLUSION

The method of all optical NMR allows the nuclear polarisation to be spatially modulated. Moreover tiny and anisotropic $g$-tensors can give rise to large Overhauser fields that can cancel the external Zeeman field seen by the electrons. In this paper we have discussed electron transport and the nature of low-lying excitations for a variety of spatial patterns of nuclear spin polarisations that we suggest can now be achieved experimentally.

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FIG. 1. Electron spin domain wall profile in the vicinity of vanishing effective Zeeman field.

FIG. 2. Top view of sample. Central portion is the domain wall characterised by electron spins with an in-plane component. In the absence of spin-orbit interactions, $S_z$ is conserved. Hence edge modes are completely reflected at the domain wall.
FIG. 3. Electron spin texture associated with creating circular pockets of nuclear spins polarised anti-parallel to the external applied field. Here the charge associated with the circular domain wall is exactly 1e.