Particle Acceleration Around 5-dimensional Kerr Black Hole

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On the lines of the 4-dimensional Kerr black hole we consider the particle acceleration near a 5-dimensional Kerr black hole which has the two rotation parameters. It turns out that the center of mass energy of the two equal mass colliding particles as expected diverges for the extremal black hole and there is a symmetry in the results for $\theta = 0, \pi/2$. Because of the two rotation parameters, $r = 0$ can be a horizon without being a curvature singularity. It is shown that the acceleration of particles to high energies near the 5-D extreme rotating black hole avoids fine-tuning of the angular momentum of particles.

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I. INTRODUCTION

In the paper \cite{1} authors have studied the center-of-mass (CM) energy of colliding two particles near the rotating black hole. They observed the divergence of the CM energy in the extreme rotating black hole case with the fine-tuning of the momentum of one of the particles (so called BSW process after the names of the authors - Banados, Silk, and West). The analysis of the CM energy of two colliding particles at the equatorial plane tends to extremely high energies for the extremal central black hole when it rotates with maximal speed and the maximal rotating black hole can be considered as high energy scale collider of normal and dark matter particles which can be detected by the observer at infinity. Since BSW process has been introduced the mechanism of particle acceleration near the black hole has been intensively studied by many of authors for the different space-time metric describing black hole. Authors of the Ref. \cite{2} have studied two particles acceleration, the multiple scattering and their CM energy in case of non extreme rotating black hole. The acceleration mechanism of the particles when they are in the stable circular orbits has been considered in \cite{3}.

Recent studies have shown that the naked singularities that are formed due to the gravitational collapse of massive stars provide a suitable environment where particles could get accelerated and collide at arbitrarily high center-of-mass energies \cite{4,7}.

Authors of Ref. \cite{8} studied the collision of two particles with the different rest masses moving in the equatorial plane in a Kerr-Taub-NUT spacetime and found that the CM energy depends not only on the rotation parameter, but also on the NUT charge. The collision of particles in the vicinity of a horizon of a weakly magnetized non-rotating black hole has been studied in \cite{8}. Acceleration of particles by black hole with gravitomagnetic charge immersed in magnetic field \cite{10}, by rotating black hole in a Randall-Sundrum brane with a cosmological constant \cite{11}, and by rotating black hole in Hořava-Lifshitz gravity \cite{12} have been studied in detail. Acceleration of electric current-carrying string loop near a Schwarzschild black hole embedded in an external magnetic field in the parallel direction to the axis of symmetry considered in \cite{13}.

Black holes are very interesting gravitational, as well as geometric, objects which may exist in multidimensional spacetimes. Other interesting axisymmetric object is the five dimensional supergravity black hole \cite{14}, which is an important solution of supergravity Einstein-Maxwell equation. Recently, a charged black hole solution in the limit of slow rotation was constructed in \cite{15} (also see \cite{16}). Also, charged rotating black hole solutions have been discussed in the context of supergravity and string theory \cite{17,19}. The solution obtained by Chong et. al. \cite{14} of minimal gauged supergravity theory comes closest to Kerr-Newman analogue. Energetics of a rotating charged black hole in 5-dimensional supergravity space-time has been studied in \cite{20} where energy extraction even for axial fall has been predicted.

In this paper, our main aim is to show particle acceleration for the axial collisions by studying the collision of two particles with the same rest masses in the background spacetime of the 5-D Kerr black hole and derive a general formula for the CM energy for the near-horizon collision of two particles on the equatorial plane and polar plane.

The outline of the paper is the following. In the Sect. \textsuperscript{11} we study the particle acceleration followed by the discussion of particle collisions and the CM energy extraction in the next Sect. \textsuperscript{111} We conclude with a discussion in the last Sect. \textsuperscript{11V} Throughout the manuscript we use $G = c = 1$. 

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II. PARTICLE MOTION AROUND A ROTATING BLACK HOLE

The Ricci flat metric for the 5-dimensional Kerr black hole in the Boyer-Lindquist coordinates \((t, r, \theta, \varphi, \psi)\) has the following form [21]:

\[ ds^2 = -\frac{\Delta}{\rho^2}dT^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2d\theta^2 + \rho^2\sin^2\theta d\varphi^2 + \rho^2 \frac{\Delta}{r^2}(b\sin^2\theta d\Phi + a \cos^2\theta d\Psi)^2, \]  

where

\[
\begin{align*}
    dT &= dt - a\sin^2\theta d\Phi - b\cos^2\theta d\Psi, \\
    d\nu &= b\sin^2\theta d\Phi + a \cos^2\theta d\Psi, \\
    \rho^2 d\Phi &= (r^2 + a^2) d\varphi, \\
    \rho^2 d\Psi &= b dt - (r^2 + b^2) d\psi, \\
    \Delta &= (r^2 + a^2)(r^2 + b^2) - 2M, \\
    \rho^2 &= r^2 + a^2 \cos^2\theta + b^2 \sin^2\theta.
\end{align*}
\]

Here \(a\) and \(b\) are the rotational parameters related to the specific angular momenta of black hole with the total mass \(M\) corresponding to the coordinates \(\varphi\) and \(\psi\), respectively. The angular coordinates range over, \(\theta \in [0, \pi/2]\) and \(\varphi, \psi \in [0, 2\pi]\).

Using the equation \(\Delta = 0\) one can easily find the black hole horizon (higher positive root) in the form

\[ r_+ = \left[ \left( M - \frac{a^2 + b^2}{2} \right) + \sqrt{\left( M - \frac{a^2 + b^2}{2} \right)^2 - a^2 b^2} \right]^{1/2}. \]

The horizon exists if \(a^2 + b^2 + 2|ab| \leq 2M\).

The motion of particles and light in a space-time of a five-dimensional rotating black hole has been studied in [22]. Complete integrability of geodesic motion in higher-dimensional rotating black-hole spacetimes has been studied in [23]. Here we will study the equation of motion for a test particle with mass \(m_0\) in the field of a 5-dimensional rotating black hole. The Lagrangian reads as

\[
\mathcal{L} = \frac{1}{2} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu \]  

which readily leads to the conserved energy and angular momenta:

\[
\begin{align*}
    -E &= g_{tt} \dot{t} + g_{t\varphi} \dot{\varphi} + g_{t\psi} \dot{\psi}, \\
    l_\varphi &= g_{\varphi t} \dot{t} + g_{\varphi\varphi} \dot{\varphi} + g_{\varphi\psi} \dot{\psi}, \\
    l_\psi &= g_{\psi t} \dot{t} + g_{\psi\varphi} \dot{\varphi} + g_{\psi\psi} \dot{\psi}.
\end{align*}
\]

Solving the equations (4) - (6), one can write

\[
\begin{align*}
    \frac{dt}{ds} &= -\Upsilon^{-1} \left[ E(g_{t\varphi}^2 - g_{t\psi} g_{\psi\varphi}) - l_\varphi g_{t\psi} g_{\varphi\psi} \right] + (l_\varphi g_{t\varphi} + l_\psi g_{t\psi}) g_{t\psi} - l_\varphi g_{t\psi} M, \\
    \frac{d\varphi}{ds} &= -\Upsilon^{-1} \left[ E(g_{t\varphi} g_{\varphi\psi} - g_{t\psi} g_{\varphi\psi}) + (l_\varphi g_{t\psi} - l_\psi g_{t\varphi}) g_{t\varphi} \right] - (l_\varphi g_{t\varphi} - l_\psi g_{t\psi}) M, \\
    \frac{d\psi}{ds} &= -\Upsilon^{-1} \left[ E(g_{t\psi} g_{\psi\varphi} - g_{t\varphi} g_{\psi\psi}) - (l_\varphi g_{t\varphi} - l_\psi g_{t\psi}) g_{t\varphi} \right] + (l_\psi g_{t\psi} - l_\varphi g_{t\varphi}) M.
\end{align*}
\]

where

\[
\Upsilon = (g_{t\varphi} g_{\varphi\psi} - 2g_{t\psi} g_{\varphi\varphi} + g_{\varphi\varphi} g_{\psi\psi} + g_{\varphi\psi} g_{\psi\varphi} + 2 g_{t\varphi} g_{t\psi} - g_{\varphi\varphi} g_{t\psi} - g_{\psi\psi} g_{t\varphi}).
\]

The metric functions have the following form:

\[
\begin{align*}
    g_{tt} &= 1 - \frac{2M}{\rho^2}, \\
    g_{t\varphi} &= -\frac{2aM}{\rho^2} \sin^2\theta, \\
    g_{t\psi} &= -\frac{2bM}{\rho^2} \cos^2\theta, \\
    g_{\varphi\varphi} &= (r^2 + a^2) \sin^2\theta + \frac{2aM}{\rho^2} a \sin^4\theta, \\
    g_{\psi\psi} &= (r^2 + b^2) \cos^2\theta + \frac{2bM}{\rho^2} b \cos^4\theta, \\
    g_{\varphi\psi} &= \frac{2abM}{\rho^2} \sin^2\theta \cos^2\theta, \\
    g_{rr} &= \frac{\rho^2}{\Delta}, \quad g_{\theta\theta} = \rho^2.
\end{align*}
\]

Now for the motion in the polar plane \(\theta = 0\), we have \(l_\varphi = 0\), \(\rho_\varphi^2 = r^2 + a^2\) and \(\theta = 0\),

\[
\begin{align*}
    \frac{dt}{d\tau} &= \frac{E}{\rho_\varphi^2 - 2M} \left[ \rho_\varphi^2 - \frac{4M^2 \rho_\varphi^2}{b^2 \rho_\varphi^2 + r^2 (\rho_\varphi^2 - 2M)} \right] \\
    &\quad - L_\psi \frac{2Mb}{b^2 \rho_\varphi^2 + r^2 (\rho_\varphi^2 - 2M)}, \\
    \frac{d\psi}{d\tau} &= \frac{(\rho_\varphi^2 - 2M) L_\varphi + 2bM E}{b^2 \rho_\varphi^2 + r^2 (\rho_\varphi^2 - 2M)}, \\
    \left( \frac{dr}{d\tau} \right)^2 &= \frac{\Delta}{\rho_\varphi^2} \left\{ \frac{E^2}{\rho_\varphi^2 - 2M} - \frac{1}{\rho_\varphi^2} \left[ (\rho_\varphi^2 - 2M) L_\varphi + 2bM E \right]^2 \right\}.
\end{align*}
\]

Note that the motion in the equatorial plane, \(\theta = \pi/2\), will be given by letting \(\psi \to \phi\), \(a \to b\), \(b \to a\).

Now one can easily write the radial equation of motion for the test massive particle in the equatorial plane and polar plane in the following form

\[
\frac{1}{2} \dot{r}^2 + V_{eff}(r) = 0,
\]
where the quantity
\[
V_{eff}(r) = \frac{1}{2} \Delta \rho_a^2 \left[ 1 - \frac{E^2}{\rho_a^2 - 2M} \right] + \left[ (\rho_a^2 - 2M) L_\psi + 2bM E \right]^2 / (\rho_a^2 - 2M) [b^2 \rho_a^2 + r^2 (\rho_a^2 - 2M)] \right] , \tag{14}
\]
can be interpreted as an effective potential for the polar plane \( \theta = 0 \) and similar form for the effective potential at equatorial plane \( (\theta = \pi/2) \) with transformations \( a \rightarrow b \) and \( b \rightarrow a \).

To consider the circular orbits one should use the following conditions:
\[
V_{eff}(r) = 0, \quad \frac{dV_{eff}(r)}{dr} = 0 , \tag{15}
\]
This leads to a limitation on the possible values of the angular momentum for collision of two particles and after some straightforward calculation, one can obtain the range of angular momenta of particles for the special cases when \( a = b \):
\[
-\frac{a - \sqrt{a^2 - 2a^4}}{a^2 - 1} \leq l \leq \frac{3a + \sqrt{a^2 - 1}}{2} , \tag{16}
\]
and when \( a = -b \):
\[
-\frac{3a + \sqrt{a^2 - 1}}{2} \leq l \leq \frac{a + \sqrt{a^2 + 2a^4}}{a^2 - 1} . \tag{17}
\]
One should have the satisfied condition in the polar plane
\[
E \left( \rho_a^4 (r^2 + b^2) + b^2 f_a \right) \geq L b f_a , \tag{18}
\]
in order to have \( dt/dr \geq 0 \).

In the limiting case when \( r \rightarrow r_+ \) for the massive test particle, one can gets
\[
E \geq \frac{b f_a}{\rho_a^4 (r^2 + b^2) + b^2 f_a} L = \omega_H L .
\]

**III. CENTER-OF-MASS ENERGY FOR A ROTATING BLACK HOLE IN 5 DIMENSIONAL SPACETIME**

This section is devoted to study the CM energy of the accelerating particles near the rotating black hole in 5-dimensional spacetime. Hereafter we assume that the motion of particles occurs both in the equatorial plane and the polar of a rotating black hole. For simplicity one may consider that two colliding particles has the same rest mass \( m_0 \) and they are at rest at infinity \( (E = m_0) \), then they approach the rotating black hole and collide at some radius \( r \). We assume that two particles 1 and 2 are at the same spacetime position and have angular momenta \( l_1 \) and \( l_2 \), respectively. Here, our aim is to compute the energy in the CM frame for this collision according to the calculation method developed in [1]. The momentum of the particle \( i \) \( (i = 1, 2) \) is given by
\[
P_i^\mu = m_0 u_i^\mu ,
\]
where \( u_i^\mu \) is the velocity of particles \( i \). One can construct the total momentum of two particles in the form
\[
P_i^\mu = P_1^\mu + P_2^\mu .
\]
It is easy to calculate the CM energy \( E_{c.m.} \) of accelerating particles using the standard formulae
\[
E_{c.m.} = \sqrt{2m_0 \sqrt{1 - g_{\mu\nu} u^\mu_{(1)} u^\nu_{(2)}}} . \tag{19}
\]

Here, we consider two particles coming from infinity with \( E_1/m_0 = E_2/m_0 = 1 \) for simplicity. Inserting equations (11)–(12) and equations of motion at the equatorial plane into the expression (19), one can easily calculate CM energies of the accelerating particles near the rotating black hole in the two cases of the 5 dimensional spacetime.

In the first case, specializing to motion along \( \theta = 0 \), we have \( L_\varphi = 0 \) and \( g_a = \rho_a^4 - f_a \). CM energy of the two particles is calculated as
and similar form for the expression for CM energy of the two particles at equatorial plane (\( \theta = \pi/2 \)) with transformations \( a \to b \) and \( b \to a \).

### A. Classification of center of mass energy of two colliding particles near rotating 5 dimensional black hole

Below we will analyze the expression for CM energy of two particles \(^{(20)}\). In all cases mass of BH is taken to be \( M = 1 \).

- If both rotational parameters are vanishing: \( a = b = 0 \), then event horizon is located at \( r_+ = \sqrt{2} \).
- Center of mass energy is finite in this case and has the following limit:

\[
\frac{E_{\text{c.m.}}^2}{2m^2} = (l_1 - l_2)^2 + 8 \tag{21}
\]

The radial dependence of the center of mass energy for the different values of the angular momentum of the particles are shown in the Fig. 2

- Rotational parameter \( b \) is vanishing: \( b = 0 \). If condition \( a = \pm \sqrt{2} \) will be satisfied, then BH is extremal. Center of energy diverges in any values of angular momentums of the particles in the range: \(-\sqrt{2} \leq l_1 \leq \sqrt{2}, -\sqrt{2} \leq l_2 \leq \sqrt{2}\):

\[
\frac{E_{\text{c.m.}}^2}{2m^2} = 2 + \frac{2 - l_1 l_2 - \sqrt{(2 - l_1^2)(2 - l_2^2)}}{r^2} \tag{22}
\]

The radial dependence of the center of mass energy for the different values of the angular momentum of the particles are shown in the Fig. 3 when a) \( a = 1, r_+ = 1 \) and b) \( a = 0.5, r_+ = \sqrt{7}/4 \). Note that in particular case when \( a = 1 \) the center of mass
The radial dependence of the center of mass energy of accelerating particles around rotating five dimensional black hole for the different values of the angular momentum of the particles in the cases when a) $b = 0, a = 1$ and b) $b = 0, a = 0.5$ which are relevant to nonextremal rotating black hole. The vertical lines correspond to the event horizon.

The radial dependence of the center of mass energy of the particles in nonextremal case of $a = 0, b = \sqrt{2}$ which is corresponding to extremal rotating black hole. The value of the event horizon radius is $r_+ = 0$.

The radial dependence of the center of mass energy of the particles for the different values of the angular momentum of the particle are shown in the Fig. 5.

The radial dependence of the center of mass energy of the particles for the different values of the angular momentum of the particle are shown in the Fig. 6.

The following condition should be satisfied to be extremal BH: (i) $1 - (a + b)^2/2 = 0$ and (ii) $1 - (a - b)^2/2 = 0$. Let us consider extra condition: $r_+ = 0 \Rightarrow a^2 + b^2 = 2$ then one can find the solution for $a$ and $b$ as: $a = 0, b = \pm \sqrt{2}$ and as: $b = 0, a = \pm \sqrt{2}$. In all cases the center of mass energy diverges.

Consider the extremal rotating 5-D black hole with nonvanishing $r_+$. This implies the conditions i) $a = \sqrt{2} - b$ and ii) $a = b - \sqrt{2}$. In Fig. 7 the radial dependence of the center of mass energy of the particles for the different values of the angular momentum of the particle are shown. The upper and lower plots correspond to the condition (i) and (ii), respectively. From this dependence one may conclude that the centre of mass energy of the particles diverge near the event horizon when the central object is the 5-D extreme rotating black hole. However, one can see that the fine-tuning for the energy has the following form:

$$\frac{E_{\text{c.m.}}^2}{m^2} = \frac{1}{2} [ (l_1 - l_2)^2 + 4] . \quad (23)$$
angular momentum of the particles is not required for 5-D rotating black hole. Note that for 4 dimensional rotating black hole one needs significant fine-tuning to get sensible cross sections for particles.

**IV. CONCLUSION**

The study described in this manuscript devoted to particles acceleration mechanism at the equatorial plane and polar region of a rotating black hole in the 5 dimensional spacetime. We have derived a general formula for the CM energy and made analyse it for the different cases. It was pointed out by the Banados, Silk, and West [1] that a rotating black hole in 4 dimensional spacetime can, in principle, accelerate the particles falling to the central black hole to arbitrarily high energies if one of the particles has angular momentum $\ell = 2$. We have derived a general formula for the CM energy near the horizon on the equatorial plane and polar plane.

We have found that particles will collide near-extremal singularity and the center of mass energy for collision of the two particles can be unlimited near-extremal singularity of the 5 dimensional spacetime. Our result shows that arbitrarily high CM energy appears near-extremal singularity even for the axial collision which is a significant difference from other black holes. In particular, energy could be extracted even in the polar region through $aq$ coupling producing a rotation. This is similar to the energy extraction by Penrose process discussed in the paper [21] by one of the authors of this paper.

The frame-dragging effects in a pure Kerr black hole spacetime can accelerate particles and some needs significant fine-tuning to get sensible cross sections for particles (at least one of particles has to have critical angular momentum). Recently it was shown that the acceleration process near the 4 dimensional naked singularity avoids fine-tuning of the parameters of the particle geodesics for the unbound center of mass energy of collisions [3, 4]. Here we show that the CM energy diverge for any values of particles falling to central objects. The unbound CM energy can be observed for any particles coming inward to 5 dimensional extreme rotating black hole.

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