Bosonic enhancement of spontaneous emission near an interface

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Abstract

We show how the spontaneous emission rate of an excited two-level atom placed in a trapped Bose-Einstein condensate of ground-state atoms is enhanced by bosonic stimulation. This stimulation depends on the overlap of the excited matter-wave packet with the macroscopically occupied condensate wave function, and provides a probe of the spatial coherence of the Bose gas. The effect can be used to amplify the distance-dependent decay rate of an excited atom near an interface.

Introduction

Spontaneous emission from an excited atom can only take place when a vacuum mode is available to accommodate the emitted photon. This fact is employed in experiments in cavity quantum electrodynamics (cQED) to shift the spontaneous emission rate in small cavities where the structure of the electromagnetic (em) vacuum is modified [11, 12]. In particular, it is known that the lifetime of an excited atom near a plane surface (the simplest ‘cavity’ system) shows an oscillatory behavior for atom-surface distances comparable to the resonant photon wavelength. The effect is not easy to observe with ultracold atoms since distance control in the sub-micron range is required. A transient signal related to a change in decay rate was observed in the Amsterdam group [3], with cold atoms being probed spectroscopically in the vicinity of a surface using an evanescent light field. Quite analogous to the effect of a cavity on photon modes, the presence of a Bose-Einstein condensate (BEC) can alter the decay of bosonic atoms, as the macroscopic population of atomic modes simulates the transition into these [4, 5, 6]. This enhancement is significantly reduced, however, in a uniform system because of momentum conservation. For an excited atom initially at rest, the final state is shifted by the photon recoil momentum, and overlap with the condensate modes occurs only if repulsive interactions deplete the ground state [7], or alternatively, for a confined system where the BEC is spread over a finite width in momentum space.

In the present paper, we discuss the enhancement of spontaneous emission in a trapped BEC and show in particular that the small oscillations in the decay rate near an surface can be significantly amplified. The trap confinement and the temperature of the BEC are taken into account and provide only a moderate reduction compared to the scaling with the number of atoms in the BEC. We use a quantum field theory approach of atoms and photons to calculate complex self-energies (T-matrix elements) [8]. The spontaneous emission rate of an excited initial state is proportional to the phase of the term $\langle e | S^{(2)} | N,e \rangle = \text{Feynman diagram}$ (1)

$$= -\mu_{\alpha\beta} \int d^4x_1 d^4x_2 \Theta(t_2 - t_1) \langle T\{ E_{\alpha}(x_1) E_{\beta}(x_2) \} \rangle \times \langle e | \Psi_{e}(x_2) \Psi_{e}(x_1) | e \rangle |N | \Psi_{g}(x_2) \Psi_{g}(x_1) | N \rangle , \quad (2)$$

where $\mu_{\alpha}$ ($\alpha = x, y, z$) are the matrix elements of the transition dipole. The Feynman diagram represented above uses bold lines for the many-body system of ground state atoms, dashed lines for individual excited atoms and wavy lines for the photon propagator. The brackets $\langle \ldots \rangle$ in the first line of eqn. (2) denote an expectation value with respect to an equilibrium state of the em-field, and the symbol $T\{ \ldots \}$ denotes time-ordering. $\Psi_{e}$ and $\Psi_{g}$ are interaction-picture field operators for the atomic levels, with a time dependence governed by the unperturbed atomic Hamiltonian. In eqn. (2), the time dependent phase of the ground state correlation function $\langle N | \Psi_{g}(x_2) \Psi_{e}(x_1) | N \rangle$ is of the order of typical single particle energies in the trap and thus much smaller than the phase of the term $\langle e | \Psi_{e}(x_2) \Psi_{e}(x_1) | e \rangle$, which is proportional to $\exp[i \omega_{bg}(t_2 - t_1)]$ where $\omega_{bg}$ is the Bohr transition frequency. We will hence neglect the time dependence of the ground state correlation function in eqn. (2). (This is equivalent to summing the decay rate over the final ground state modes.) The time integrations in eqn. (2) can...
then be performed, yielding for the transition rate \((h = 1)\)

\[
\gamma_e(N) = 2[1 + \bar{n}(\omega_{eg})] \mu_\alpha \mu_\beta \int d^3x [\Phi(x)]^2 \text{Im} G_{\alpha \beta}(x, x, \omega_{eg})
\]

\[
+ \int d^3x_1 d^3x_2 \Phi(x_1) \Phi^*(x_2) \langle N | \Psi^\dagger_\alpha(x_1) \Psi_\beta(x_2) | N \rangle \times \text{Im} G_{\alpha \beta}(x_1, x_2, \omega_{eg})
\]

\(, \quad (3)\)

where the excited state is given by the normalized wave function \(\Phi(x)\). We denote \(\bar{n}(\omega_{eg})\) the average photon number at frequency \(\omega_{eg}\) in thermal equilibrium. For the sake of simplicity, we restrict ourselves for the rest of the paper to a

**frequency \(\omega\)**

as function of the atom-surface separation.

**Matter wave function overlap**

and an additional term from any macroscopic body. We use the free-space expression

\[
\text{To illustrate the physics in eqn. (3), we will assume that both}
\]

ground state atoms. The bosonic stimulation is thus a probe that it depends on the two-point correlation function of the

\[\text{distribution of the excited state wavepacket. (For a study of enhancement. The first term has a natural interpretation in}
\]

the Lamb-Dicke limit \(k_{eg} a_0 \ll 1\) (well-localized trap), the exponential in eqn. (5) can be approximated by unity, and the overlap is optimal when the two wavepackets are matched in width, \(\eta = 1\). The opposite case looks closer to a homogeneous system and is easier to analyze in Fourier space where the photon recoil provides a shift of the momentum distribution. This reduces the overlap and can be compensated for by making an excited-state wave packet wider in momentum space, i.e., \(\eta < 1\). At the optimum value, the width is of the order of the photon momentum and the shifted excited-state wave packet still has some overlap with the sharp zero-momentum component of the BEC.

The temperature dependence in Fig. 1 closely follows the occupation of the ground-state (condensate) mode. We have used Ref.[13] where the two-point correlation function \(\langle N | \Psi^\dagger_\alpha(x_1) \Psi_\beta(x_2) | N \rangle\) for the ideal Bose gas in a 3D trap is given in a simple form, involving only a single summation. This provides the Bose enhancement of \(\gamma_e(N)\) in a straightforward manner for atom temperatures \(T_A\) below and above the critical temperature \(T_c\) (see caption). We see that for any \(\eta\), the transition rate drops below the zero-temperature value, and for temperatures above \(T_c\), it becomes comparable to \(\gamma^{(0)}\) (horizontal dashed line). The temperature dependence is shown in Fig. 2 and compared to the condensate fraction \(N_0/N\) (dashed line). The bosonic enhancement closely follows the population of the condensate mode because the excited-state wave packet \(\Phi(x)\) has the largest overlap with the trap ground state. The thermal occupation of higher lying trap states hence diminishes the integrals eqn. (7). We note that this behaviour would change qualitatively in lower-dimensional systems where the Bose gas occupies excited states with a relatively larger weight.

The more realistic case of an interacting Bose gas is also shown in Fig. 1. We focus here on repulsive interactions (corresponding to positive s-wave scattering length \(a_s\)), and restrict ourselves to temperatures far below \(T_c\), where it is legitimate to approximate the field operator by the condensate mode only:

\[
\langle N | \Psi^\dagger_\alpha(x_1) \Psi_\beta(x_2) | N \rangle \approx N \bar{\psi}_0^\dagger(x_1) \bar{\psi}_0(x_2).
\]

(8)

Elementary excitations of the condensate can be included within Bogoliubov theory [16] [17] [18]. The condensate wave function \(\psi_0(x)\) is a solution of the Gross-Pitaevskii equation.
We have used the approximate variational solution \[ \psi_0(x) = \frac{e^0(\tau)}{N_0 R \sqrt{2}} \left( 1 - x^2/R^2 \right)^{(1+\tau)/2} \theta(R - x), \] (9) that interpolates between a Gaussian and the Thomas-Fermi limit as the parameter \( N_0 a_s/a_0 \) is changing from zero to infinity. The length \( R \) and the exponent \( \tau \) are fixed by minimizing the Gross-Pitaevskii energy functional, and \( e^0 \) is a normalization constant. The result for the decay rate \( \gamma_e(N) \) is shown by the dashed-dotted curves in fig. 1 as the interaction parameter \( N_0 a_s/a_0 \) is increased. Relative to the s-wave scattering length \( \pi_s \) of rubidium, we took \( a_s = 1, 5, 10 \pi_s \) (top to bottom) which can be achieved using a Feshbach resonance, for example. The interacting gas shows a flatter density profile in the trap, as is well known; this results in smaller values of the overlap integrals eqn. (7).

To summarize the data of Fig.1, we find a relatively strong enhancement of the spontaneous decay rate of an excited atom embedded in a Bose condensate. This happens despite the non-perfect overlap that encodes the constraints of momentum conservation and photon recoil. The optimum conditions correspond to a well-localized excited-state wavepacket (on the scale of the transition wavelength) and a strong condensate fraction \( T \lesssim 0.5 T_c \).

3 Bose enhancement near a surface

In this section, we calculate the transition rate eqn. (3) near a surface and demonstrate its enhancement in a Bose condensate of oblate shape. This scenario can be realized with an optical lattice, by retro-reflecting an off-resonant laser beam at the surface [20], or in a bichromatic evanescent wave [21]. We take the surface in the \( xy \)-plane and the trapped atoms centered at a distance \( d \) from the surface in the positive \( z \)-direction. Concerning the surface material, we use the idealized model of a perfectly reflecting mirror for the sake of simplicity; but with the appropriate choice of (frequency-dependent) reflection coefficients that appear in the photonic Green function \( G_{\alpha\beta}(x_1, x_2, \omega) \), a wide range of surface materials can be treated in the same manner [22, 13].

As we have measurements in mind where the control over the distance \( d \) is essential, we take an oblate Bose condensate and assume for simplicity a single gaussian mode with widths \( a_0 \) (in the \( xy \)-plane) and \( a_0/\sqrt{\lambda} \ll d \) (along the \( z \)-axis). The depletion of the condensate and its broadening due to repulsive interactions could be incorporated as in Sec. 2. For the excited state, we adopt again a gaussian wave packet localized in the cloud center, with widths \( \eta a_0 \) and \( \eta_{\parallel} a_0/\sqrt{\lambda} \), respectively. The actual values of the trap frequency are given in the caption of fig. 3. As the very narrow confinement in the \( z \)-direction describes a quasi-2D scenario, the temperature has to be lower than \( T_{2D}^c(\omega) = \omega_{\parallel}(N/\zeta[2]^{1/2}) \) to ensure a strong condensate occupation.

Fig. 3 illustrates the decay rate \( \gamma_e(N) \) obtained from eqn. (3) as a function of the distance \( d \). As is well known, the rate depends on the orientation of the dipole moment (parallel or perpendicular to the surface, represented in red and blue, respectively). The full curves show the Bose-enhanced contribution \( \gamma_e^{\parallel}(N, d) \), while the dashed curves give the
single-atom part $\gamma_c^{(0)}(d)$, re-scaled such that the asymptotic value for large distances $d$ coincides with $\gamma_c^{\text{BEC}}(N, d \to \infty)$. The numbers given in fig. 3 are the result of a compromise between a tight confinement in the vertical (z-) direction and a localized wave packet in the excited state. The atomic wave packets must be confined below the wavelength in the z-direction, otherwise the oscillations in $\gamma_c$ vs. distance are averaged out. In this limit, the optimal Bose enhancement is found for an excited wave packet that is matched to the condensate ($\eta_z = 1$). For the size parameter in the $xy$-plane, we find an optimum at $\eta = 0.07$. The asymptotic values $\gamma_e(N, d \to \infty)$ are enhanced by factors of 390 and 150 compared to $\gamma_e^{(0)}(d \to \infty)$ for the parallel and perpendicular dipole, respectively. The difference between these two numbers and the relative phase shift of the oscillation pattern in $\gamma_e^{\text{BEC}}(d)$ are due to the radiation pattern of the dipole emission, combined with the shape of the ground state mode that modulates the Bose enhancement in $k$-space.

Fig. 3 thus demonstrates a significant amplification of the decay rate above the surface, with the oscillation amplitude receiving an additional enhancement relative to the asymptotic free-space component. It suggests that even at a distance of a few microns (several transition wavelengths), Bose enhancement can bring the tiny interference structure of the decay rate into an experimentally detectable regime.

4 Virtual excited atoms produced by laser absorption

The calculation above assumed the presence of an excited atom prepared in a gaussian wave packet, and one may ask whether this is a realistic description. Indeed, the preparation of such a state would typically proceed by illuminating the system. We therefore describe in this section a calculation of a typical absorption spectrum. We find that the results of the previous section are qualitatively unchanged. The method also illustrates the relevance of two- and four-point correlation functions of the Bose gas. For the sake of simplicity, we restrict this analysis to the ideal Bose gas.

The calculation proceeds by keeping a continuum of modes for the excited state field operator $\Psi_e(x)$ and by identifying the absorption spectrum of a weak laser field with a suitable $T$-matrix element (self-energy). We take the laser field to be described by a coherent state $|\beta\rangle$ in a given plane-wave mode.

In the leading order of perturbation theory, the absorption by the atom cloud of a photon out of the coherent state $|\beta\rangle$ and re-emitting it into the same state,

$$\langle N, \beta | T^{(2)} | N, \beta \rangle \approx \delta(\omega_{\text{eg}} - \omega_L - \epsilon) ,$$

results in a (complex) energy shift of the laser plus atom system that is described by the $T$-matrix element

$$\langle N, \beta | T^{(2)} | N, \beta \rangle = \frac{\omega_L}{2} \frac{\mu_{\alpha_0} \mu_{\beta} \delta(\omega_{\text{eg}} - \omega_L - \epsilon)}{\omega_{\text{eg}} - \omega_L - \epsilon} .$$

In eqn. (11), $|\beta|^2$ is the number of photons in the coherent state, $\omega_L$ and $k_L$ denote the frequency and wave vector of the absorbed photons, the unit vectors $\hat{e}(k_L)$ denote axes of (linear) photon polarization, and the infinitesimal $\epsilon \gg 0$ ensures the adiabatic switching-on of the laser field. At this order of perturbation theory, the absorption of photons by the atom cloud is proportional to $\text{Im} \langle N, \beta | T^{(2)} | N, \beta \rangle \propto \delta(\omega_{\text{eg}} - \omega_L)$.

The next order in perturbation theory brings about the diagram

$$\langle N, \beta | T^{(4)} | N, \beta \rangle = \frac{\omega_L}{2} \frac{\mu_{\alpha_0} \mu_{\beta} \delta(\omega_{\text{eg}} - \omega_L - \epsilon)}{\omega_{\text{eg}} - \omega_L - \epsilon} \times \int d^3x_1 \int d^3x_2 \langle \Psi_{\gamma_1}^{(1)}(x_1) \Psi_{\gamma_2}^{(1)}(x_2) \rangle \langle \Psi_{\gamma_1}^{(1)}(x_1) \Psi_{\gamma_2}^{(1)}(x_2) \rangle \times \mu_{\gamma_1} \mu_{\gamma_2} G_{\gamma_1\gamma_2} e^{-i k_L \cdot (x_1 - x_2)} .$$

Let us introduce the density correlation function of the Bose gas as

$$C(x_2, x_1) = \langle \Psi_{\gamma_1}^{(1)}(x_2) \Psi_{\gamma_2}^{(1)}(x_1) \Psi_{\gamma_1}^{(1)}(x_1) \rangle - n(x_2)n(x_1)$$

where $n(x_1) = \langle \Psi_{\gamma_1}^{(1)}(x) \Psi_{\gamma_1}^{(1)}(x) \rangle$ is the average density. This splits eqn. (13) in two parts: $T^{(4)} = T^{(4)}_{\text{scat}} + T^{(4)}_{\text{abs}}$. The former contains only densities and can be identified with the

Figure 3: Bose-enhanced decay rate $\gamma_c(N)$ near an interface. The BEC contains $3.4 \times 10^7$ Rb atoms in an oblate wave function at an average distance $d$ from a perfectly reflecting surface. The size parameters are $\alpha_0 = 3.4 \mu m$ parallel and $\alpha_0/\sqrt{A} = 0.01 \alpha_0$ perpendicular to the surface, corresponding to trapping frequencies of $\omega_j/2\pi = 10$ Hz and $\omega_\perp = 10^4 \omega_0$ and a critical temperature $T_{\text{c}}^{(2D)} = 118$ nK. The excited wave packet (resonance frequency as in fig. 1) is spatially centered in the BEC, with size parameters $\eta = 0.07$ and $\eta_z = 1$ relative to $\alpha_0$. Full red curve: Bose-enhanced decay rate $\gamma_{\text{BEC}}(N)$ given by eqn. (3), for an excited atom with its dipole moment oriented parallel to the surface. We normalize to the free-space decay rate $\gamma_c^{(0)}(d \to \infty)$. Full blue curve: $\gamma_{\text{BEC}}(N)$ for an excited atom with perpendicular dipole moment. Horizontal dashed lines: asymptotic values of $\gamma_{\text{BEC}}(N, d \to \infty)$ at large separation. Dashed red (blue) curve: single-atom decay rate $\gamma_c^{(0)}(d \to \infty)$.
Eqs. (17, 18) can now be compared to the decay rate \( \gamma \) done analytically, at the low temperatures derived under the integral. At zero temperature, the summations can be \( \gamma \) wavepacket is hence no longer pure. This makes the temperature profile matched to the condensate density. The second laser spectroscopy effectively prepares an excited state density. The first term in both expressions is very similar, and we see that the equivalence \( 2 \) is appropriate for the ideal Bose gas:

\[
\langle T^{(2)} \rangle + \langle T^{(4)} \rangle_{\text{abs}} = \frac{\beta^2 \omega_L}{2} \delta_{\alpha \beta}(k_L) \delta_{\beta}(k_L)
\]

\[
\times \left[ \alpha_{\alpha \beta}^{\text{res}}(\omega_L) + \delta \omega_{eg} \frac{\partial \alpha_{\alpha \beta}^{\text{res}}(\omega_L)}{\partial \delta \omega_{eg}} \right].
\]

By identifying eqn. (15) and eqn. (16), we can read off the frequency shift \( \delta \omega_{eg} \) whose imaginary part yields the atomic line width (the inverse lifetime of the virtual state involving an excited atom)

\[
\gamma_{\text{abs}}(N) = -2 \text{Im} \delta \omega_{eg}
\]

\[
= \frac{2}{N} \mu_{\alpha \beta} \int d^3x n(x) \text{Im} G_{\alpha \beta}(x, x, \omega_L)
\]

\[
+ \int d^3x_1 d^3x_2 C(x_1, x_2) \text{Im} G_{\alpha \beta}(x_1, x_2, \omega_L) e^{-i k_L \cdot (x_1 - x_2)}. \tag{17}
\]

This function depends weakly on the laser frequency, and we evaluate it at \( \omega_L = \omega_{eg} \) for simplicity. We shall use below Wick’s theorem to evaluate the density correlation function (eqn. (3)), as appropriate for the ideal Bose gas:

\[
C(x_1, x_2) = \left| \langle \Psi^+_\alpha(x_2) \Psi_\beta(x_1) \rangle \right|^2 \tag{18}
\]

Eqs. (17) (18) can now be compared to the decay rate \( \gamma(N) \) of an excited-state wave packet (eqn. (3)). The first term in both expressions is very similar, and we see that the laser spectroscopy effectively prepares an excited state density profile matched to the condensate density. The second term differs because the laser wave-vector appears explicitly. Also the one-body density matrix for the excited state, \( \langle \epsilon | \Psi^+_{\alpha}(x_2) \Psi_\beta(x_1) | \epsilon \rangle \) in eqn. (2), is replaced by its ground-state equivalent \( \langle N | \Psi^+_{\alpha}(x_2) \Psi_\beta(x_1) | N \rangle \) in eqn. (17). The prepared wavepacket is hence no longer pure. This makes the temperature dependence of \( \gamma_{\text{abs}}(N) \) stronger, as can be seen in Fig. 2 (compare the blue and red curves). The calculation of \( \gamma_{\text{abs}}(N) \) involves, because of the squared correlation function, a double summation over single-particle trap states (15) under the integral. At zero temperature, the summations can be done analytically, at the low temperatures \( T_A = 0.2, 0.3 T_c \), the double sum could be evaluated numerically (circles in fig. 2), while for \( T_A/T_c \geq 0.5 \), the summations can be accurately replaced by integrations that evaluate faster (denoted by squares). The size parameter \( \eta \) was set to the optimal value obtained from Fig. 1. Although the line width \( \gamma_{\text{abs}}(N) \) is for these parameters around 30% smaller than the optimized decay rate \( \gamma(N) \), the strong Bose enhancement is still working in a qualitatively similar way for both types of processes. We expect a similar result to hold for an absorption experiment near a surface, using for example evanescent fields as discussed in Refs. 26–28.

### 5 Summary

To summarize, the presence of a trapped BEC can significantly enhance the decay of an excited atom by bosonic stimulation. The magnitude of the effect depends on the overlap between the atomic wave functions and the wavevector of the photon involved in the decay. More precisely, our calculations based on a quantum field theory of the atom-photon interaction illustrate the importance of two- and four-point correlation functions of the ground-state field for the Bose enhancement. For an excited atom prepared in a Gaussian wavepacket, the transition rate to the ground state can be increased under optimum conditions by a factor \( N/10 \) where \( N \) is the atom number in the BEC. This effect also amplifies the small oscillations of the decay rate near an interface. We have provided an alternative calculation based on the absorption of a laser beam that qualitatively confirms the simper wave packet picture. The main difference is that absorption from the laser field prepares a non-pure excited state which matches the one-body density matrix of the Bose gas.

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