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Modeling the dynamics of COVID-19 using fractal-fractional operator with a case study

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A B S T R A C T

This research study consists of a newly proposed Atangana–Baleanu derivative for transmission dynamics of the coronavirus (COVID-19) epidemic. Taking the advantage of non-local Atangana–Baleanu fractional-derivative approach, the dynamics of the well-known COVID-19 have been examined and analyzed with the induction of various infection phases and multiple routes of transmissions. For this purpose, an attempt is made to present a novel approach that initially formulates the proposed model using classical integer-order differential equations, followed by application of the fractal fractional derivative for obtaining the fractional COVID-19 model having arbitrary order \( \Psi \) and the fractal dimension \( \Xi \). With this motive, some basic properties of the model that include equilibria and reproduction number are presented as well. Then, the stability of the equilibrium points is examined. Furthermore, a novel numerical method is introduced based on Adams–Bashforth fractal-fractional approach for the derivation of an iterative scheme of the fractal-fractional ABC model. This in turns, has helped us to obtained detailed graphical representation for several values of fractional and fractal orders \( \Psi \) and \( \Xi \), respectively. In the end, graphical results and numerical simulation are presented for comprehending the impacts of the different model parameters and fractional order on the disease dynamics and the control. The outcomes of this research would provide strong theoretical insights for understanding mechanism of the infectious diseases and help the worldwide practitioners in adopting controlling strategies.

Introduction

At present, the accessibility to legitimate and successful treatment is scars for a COVID-19 infected individuals with the exception of certain medications as Remdesivir which are endorsed by certain countries including Australia and the European Union [4]. According to the literature, and industrial experts, there exists no influential and approved antibody for this novel contamination albeit not many nations have guaranteed it. The best avoidance methodologies utilized in certain regions for the complete control are the successive tests to decide the infected people, detachment and lockdown, social separating, utilization of severe Standard Operating Procedures (SOPs), and so forth until successful medicines and antibody become accessible. The preventive measures have been proved to be one of the effective tools in controlling the faster transmission of the contagious diseases. For this purpose, the academic researchers, and the pharmaceutical experts are putting their considerable efforts. Several approaches have been used to investigate the transmission mechanism of these infectious diseases.

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For understanding the mechanism and theoretical implementation, numerical modeling has proved to be an effective tool for these diseases. Numerous epidemic models have also been introduced to investigate the dynamic of the COVID-19 to present various controlling strategies around the globe. For example, a model of COVID-19 with Lockdown is proposed in [5], and the effect of undetected cases by means of a numerical model is investigated in [6]. The effect of some preventive measure on the reducing the COVID-19 in Pakistan by means of another numerical model is introduced in [7,8]. A transmission numerical model considering the ecological spread of the infection with a contextual analysis of Saudi Arabia is studied in [9].

A recent contribution to fractional calculus was made by Atangana and Baleanu, “who presented operators based on generalized Mittag-Leffler functions to solve fractional integrals and derivatives [10], as the Mittag-Leffler function is more suitable in expressing nature than power function. It can be recalled that the Mittag-Leffler function has been introduced to provide a response to the conventional question of complex analysis, in particular to portray the procedure of the analytic continuation of power-law series outside the disc of their convergence. Since 2016, the Atangana–Baleanu operators have inspired an explosion of new research in fractional calculus. This work is growing at a remarkable rate in the fields of mathematics, science, and engineering”. The Atangana–Baleanu derivative is a nonlocal fractional derivative with a nonsingular kernel that is connected with a variety of applications.

A powerful tools that described the real world situation in mathematical concept and terminology is known as mathematical modeling. “The different aspects for the majority of biological and general dynamics are well described via aforementioned techniques of mathematics. In this regards, the researchers use the tools of mathematical modeling to study the transmission and make further plan to prevent the mankind from the effects of mentioned infectious disease. In this regards, many researchers developed different mathematical models for the current COVID-19”, for detail see [11–14].

A large portion of the mathematical models of COVID-19 are formulated in terms of the integer order derivatives which have a few restrictions to portray the realistic aspects of a phenomena under consideration. To manage those constraints, non-integer order derivatives provide a practical mean to the sickness dynamic and beneficial results that need to comprehend the models. non-integer order models have memory appropriateness and give a superior situation to depict an epidemic model. Many mathematical models on the elements various illnesses in term of non-integer order derivatives were proposed see for occasion [15–24] and the literature referenced therein. Fractal fractional calculus is the generalization of classical calculus [25–29]. To get a better insight into a mathematical model and to deeply understand phenomena, non-integer order operators can be used.

For the ease of understanding this research is organized as follows: the mathematical model with fractal fractional-order derivative is formulated in Section “Fractional COVID-19 transmission models”. In Section “Equilibrium and basic reproductive number (R0)”, the equilibrium points and basic reproductive number R0 are presented. The local stability of the disease-free and endemic equilibria for the deterministic version model are presented in Section “Stability analysis”. Furthermore, the parameters estimation is shown in Section “Case study”. The existence and qualitative analysis with Hyers–Ulam Stability of fractional-order model in the sense of ABC presented in Section “Qualitative analysis of the COVID-19 model”. The numerical schemes and discussions are presented in Section “Simulation results & numerical schemes”, and in the last section we presented the concluding remarks.

### Fractional COVID-19 transmission models

A compartmental approach is used to develop the mathematical model for COVID-19 transmission dynamics. “The total population N is divided into six compartments named S, E, I, A, H, G, and R represent susceptible, exposed, symptomatically infected, asymptotically infected, isolated, or hospitalized, and Recovered/immune cases respectively. In the mathematical model developed in this study, humans get into the suspected group S at the rate of G and infected with Coronavirus as a result of contact with individuals in the group of A or I. The exposed group E gains population from infection induced by the Coronavirus. A proportion G1, (0 < G1 < 1) of the members of the group E advance to the asymptomatic group A and the remaining proportion 1 − G1 progresses to the symptomatic group I. People in the group I and A progress either to the Hospitalization group H or recovery group R at the rates indicated in Table 1. In the construction of the mathematical model, the exposed compartment E is included because people who are contracted with the virus do not get infectious immediately; there is an incubation period for the virus to get infectious. The groups I and A are included in the model, as people infected with Coronavirus are either symptomatic or asymptomatic. COVID-19 induced death rate G11 is also considered in the model. As a result, the authors are convinced that the model considered in this study named SEIAHR model incorporates all essential components of COVID-19 to study its transmission dynamics, in agreement with the definition of a mathematical model in” [30]. The mathematical model used in this study called SEIAHR model is shown in (1),

\[
\begin{align*}
S(t) &= G - \frac{G_1 (G_2 I(t) + A) S(t)}{N} - G_3 S(t), \\
E(t) &= \frac{G_1 (G_2 I(t) + A) S(t)}{N} - (G_4 + G_5) E(t), \\
I(t) &= (1 - G_1) G_3 E(t) - (G_5 + G_6 + G_7) I(t), \\
A(t) &= G_2 G_4 E(t) - (G_8 + G_9) A(t), \\
H(t) &= G_7 I(t) + G_9 A(t) - (G_{10} + G_8 + G_9) H(t), \\
R(t) &= G_2 I(t) + G_7 A(t) + (G_{11} + G_8) H(t) - G_5 R(t).
\end{align*}
\]

with initial condition

\[
S(0) \geq 0, \quad E(0) \geq 0, \quad I(0) \geq 0, \quad A(0) \geq 0, \quad H(0) \geq 0, \quad R(0) \geq 0.
\]

The detail of the used unknown variables and parameters are given below in Tables 1 and 2 respectively:

Recently, it has been studied that the theory of fractional-calculus is rich for applications and researchers obtained more accurate results through fractional system rather than ordinary systems. Hence, we structured the above model (1) of COVID-19 infection in the framework of new fractal fractional derivative with a generalized Mittag-Leffler kernel as follows:

\[
\begin{align*}
\mathbb{D}^{\gamma}_{0+} S(t) &= \frac{G_1 (G_2 I(t) + A) S(t)}{N} - G_3 S(t), \\
\mathbb{D}^{\gamma}_{0+} E(t) &= \frac{G_1 (G_2 I(t) + A) S(t)}{N} - (G_4 + G_5) E(t), \\
\mathbb{D}^{\gamma}_{0+} I(t) &= (1 - G_1) G_3 E(t) - (G_5 + G_6 + G_7) I(t), \\
\mathbb{D}^{\gamma}_{0+} A(t) &= G_2 G_4 E(t) - (G_8 + G_9) A(t), \\
\mathbb{D}^{\gamma}_{0+} H(t) &= G_7 I(t) + G_9 A(t) - (G_{10} + G_8 + G_9) H(t), \\
\mathbb{D}^{\gamma}_{0+} R(t) &= G_2 I(t) + G_7 A(t) + (G_{11} + G_8) H(t) - G_5 R(t).
\end{align*}
\]

| Variables | Description |
|-----------|-------------|
| S         | The class of susceptible individuals |
| E         | The class of exposed individuals |
| I         | The class of symptomatic infected individuals |
| A         | The class of asymptomatic infected individuals |
| H         | The class of Hospitalized individuals |
| R         | The class of Recovered individuals |

| Variables | Description |
|-----------|-------------|
| \( S(t) \) | The class of susceptible individuals |
| \( E(t) \) | The class of exposed individuals |
| \( I(t) \) | The class of symptomatic infected individuals |
| \( A(t) \) | The class of asymptomatic infected individuals |
| \( H(t) \) | The class of Hospitalized individuals |
| \( R(t) \) | The class of Recovered individuals |

\[ S(0) \geq 0, \quad E(0) \geq 0, \quad I(0) \geq 0, \quad A(0) \geq 0, \quad H(0) \geq 0, \quad R(0) \geq 0. \]
With initial condition
\[ S(0) \geq 0, \quad E(0) \geq 0, \quad I(0) \geq 0, \quad A(0) \geq 0, \quad H(0) \geq 0, \quad R(0) \geq 0. \] (4)

Where the symbol \( \mathbb{F}^{\alpha, \beta} \) represents the fractal fractional order derivative with fractional order \( 0 < \beta \leq 1 \) and the fractal dimension \( \alpha > 0 \). Now, applying the AB fractional integral to both sides of (3), we obtained the following system
\[
\begin{align*}
S(t) - S(0) & = \frac{\mathbb{E}(1 - \beta)\mathbb{E}^{\alpha-1}}{M(\mathbb{E})} F_1(t, S, E, I, A, H, R) \\
E(t) - E(0) & = \frac{\mathbb{E}(1 - \beta)\mathbb{E}^{\alpha-1}}{M(\mathbb{E})} F_2(t, S, E, I, A, H, R) \\
I(t) - I(0) & = \frac{\mathbb{E}(1 - \beta)\mathbb{E}^{\alpha-1}}{M(\mathbb{E})} F_3(t, S, E, I, A, H, R) \\
A(t) - A(0) & = \frac{\mathbb{E}(1 - \beta)\mathbb{E}^{\alpha-1}}{M(\mathbb{E})} F_4(t, S, E, I, A, H, R) \\
H(t) - H(0) & = \frac{\mathbb{E}(1 - \beta)\mathbb{E}^{\alpha-1}}{M(\mathbb{E})} F_5(t, S, E, I, A, H, R) \\
R(t) - R(0) & = \frac{\mathbb{E}(1 - \beta)\mathbb{E}^{\alpha-1}}{M(\mathbb{E})} F_6(t, S, E, I, A, H, R)
\end{align*}
\] (5)

Equilibria and basic reproductive number \( R_0 \)

In order to proceed the dynamical behavior analysis, we firstly present some basic theoretical properties of the proposed model (1), including basic reproductive number, disease free and endemic equilibria. Additionally, an analytical expression for the important biological parameter termed as the basic reproductive number is provided. We obtained the following two equilibrium points for the proposed model (1):

Disease-free equilibrium point (DFE)

The proposed epidemiological model (1) of the COVID-19 is examined for the disease free equilibrium, for this purpose let \( N_0 \) is the disease free equilibrium of the proposed model (1), then for analyzing this point the population under consideration is assumed to be infection free. Thus the system reported by \( N_0 = \left( \frac{S_0}{Y_0}, 0, 0, 0, 0, 0 \right) \), where \( S_0 = \frac{G}{G_0} \).

Basic reproductive number \( R_0 \)

The endemic equilibrium (EE) of the COVID-19 vaccine model (1) denoted by \( T_E \) for the above model (1), we utilize the next generation matrix technique [1,31]. The Jacobian matrix around the DFE point \( N_0 \) is given by:
\[
J(EE) = \begin{pmatrix}
-G_1 & 0 & -G_2 & -G_3 & 0 & 0 & 0 \\
0 & -G_4 & G_1 & 0 & 0 & 0 & 0 \\
G_1 & 0 & 0 & -G_5 & G_4 & 0 & 0 \\
0 & G_2 & 0 & 0 & 0 & -G_6 & G_5 \\
G_3 & 0 & 0 & 0 & 0 & 0 & -G_7 \\
G_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
-G_2 & -G_3 & -G_4 & -G_5 & -G_6 & -G_7 & -G_8 & -G_9
\end{pmatrix}
\] (6)

where \( J_E = (1 - G_1) G_4 - (G_4 + G_6 + G_8) \).

Now, we decompose the above matrix in the form of \( T \) and \( V \) such that \( M = TV^{-1} \), where
\[
M = \begin{pmatrix}
-(G_4 + G_5) & G_1 & 0 \\
1 - G_4 & -G_5 & G_6 & 0 & 0 & 0 \\
0 & G_6 & G_7 & 0 & 0 & 0 \\
-(G_10 + G_16 + G_18) & 0 & & & & \\
0 & G_10 & -G_11 & -G_12 & -G_13
\end{pmatrix}
\] (7)

\[
T = \begin{pmatrix}
0 & G_1 & G_2 & G_3 \\
0 & G_2 & G_3 & G_4 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\] (8)

and
\[
V^{-1} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\] (9)

where, \( V_i = \frac{G_i (1 - G_i) G_4 + G_6 + G_8)}{(G_2 + G_3) (G_5 + G_6 + G_8)} \). The dominant eigenvalue of \( \rho(TV^{-1}) \) is called the basic reproductive number, and is given by
\[
R_0 = \frac{G_1 G_2 (1 - G_2) G_4}{(G_2 + G_3) (G_5 + G_6 + G_8)} + \frac{G_1 G_2 G_4}{(G_2 + G_3) (G_5 + G_6 + G_8)}
\] (10)

\( R_0 \) can be written as \( R_0 = R_1 + R_2 + R_3 = \frac{G_1 G_2 Y_2}{Y_1 Y_3} = \frac{G_1 G_2 Y_1}{Y_1 Y_4} \) where \( Y_i, i = 1 \ldots 6 \) are as defined above.
Endemic equilibrium point (EE)

Let \(N^*\) be the endemic equilibrium of model (1), then the solution of
the resultant algebraic equations will lead to the endemic equilibrium
which define, \(N^* = (S^*, E^*, I^*, A^*, H^*, R^*)\), where

\[
S^* = \frac{G}{G_4} + \frac{Y_1}{G_5}E^*,
\]

\[
I^* = -\frac{Y_2}{G_4}E^*,
\]

\[
A^* = -\frac{G_5G_4}{Y_4}E^*,
\]

\[
H^* = \frac{1}{G_4} \left( \left( G_5Y_3 + \frac{G_4G_5G_3}{Y_4} \right) - Y_1 \right) + \frac{Y_2}{G_4}E^*,
\]

\[
R^* = \frac{1}{G_5} \left( -G_5Y_2 - G_5G_4 + \frac{G_4G_5G_3}{Y_3} + \frac{G_4G_5G_3}{Y_4} \right) E^* - \frac{m_4}{m_1} = \frac{G_5G_5 - GR_0}{Y_1R_0},
\]

which is a solution of a quadratic equation

\[
m_1E^2 + m_2E + \frac{G_5G_5 - GR_0}{Y_1R_0} = 0,
\]

\[
r_1 = \frac{G_4G_2G_3}{Y_3Y_4},
\]

\[
r_2 = \frac{G_4G_3}{Y_3}Y_1 = \frac{G_4G_5G_3}{Y_3},
\]

\[
Y_1 = -(G_4 + G_6 + G_8),
\]

\[
Y_2 = -(G_4 + G_6 + G_8),
\]

\[
Y_3 = -(G_1 + G_9 + G_8),
\]

\[
Y_4 = -(G_1 + G_9 + G_8),
\]

(12)

Thus, for \(R_0 > 1\) a positive EE exists, with the assumption that \(G = G_1N^*\).

Stability analysis

The previous section presented the basic reproductive number, disease
free and endemic equilibria of the proposed model (1). This
analysis provides a clue for suggesting a better analysis of the dynamical
behavior of the model. Thus regarding the local as well as global
analysis of the proposed model we have the following stability results.

Stability analysis of DFE

Theorem 1. The proposed model (1) is locally asymptotically stable (LAS)
at DFE, if \(|\arg(\lambda)| > \frac{\pi}{2M}\) for all roots \(\lambda\) of the following associated equation,

\[
\det \left[ \Lambda (M + G_6)\right] = \sum_{i=0}^{10}(\Lambda + a_i)^2(\Lambda + b_i)^2\left( \begin{array}{l} Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \\ Y_6 \\ Y_7 \\ Y_8 \end{array} \right) = 0,
\]

where

\[
\begin{align*}
a_2 &= -(Y_1 + Y_2 + Y_4), \\
a_1 &= Y_1Y_3 + Y_1Y_4 + Y_2Y_3 - G_1G_2G_3 - G_1G_2Y_2, \\
a_0 &= -Y_1Y_3Y_3 + G_1G_2G_3Y_1 + G_1G_2Y_2Y_4,
\end{align*}
\]

where \(Y_i, j = 1 \ldots 6\) are as defined above in Eq. (12). From (14), we have \(a_2 > 0\) and \(a_1 = Y_1Y_3(1 - R_0) + Y_1Y_4(1 - R_0) + Y_3Y_4 > 0\) for \(R_0 < 1\) as \(R_0 < 1\) and \(R_0 > 1\) are positive and \(R_0 = 1\) is neutral. Besides, \(a_0 = -Y_1Y_2Y_3 + G_1G_2G_3Y_1 + G_1G_2Y_2Y_4 = Y_1Y_3(1 - R_0) > 0\) for \(R_0 < 1\). Further more, the eigenvalues of the equation

\[
\psi = (FM^M + a_2EM^M + a_1FM^M + a_0) = 0,
\]

if the Routh–Hurwitz stability condition \(a_0 - a_2a_1 > 0\) and \(a_0, a_2, a_1 > 0\)
are satisfied. That is,

\[
a_0 - a_2a_1 = Y_1Y_3Y_4(1 - R_0 - 1) - (Y_1Y_3 + Y_2Y_4)(Y_1Y_3(1 - R_0)) + Y_1Y_4(1 - R_0) + Y_3Y_4 > 0.
\]

(16)

For \(R_0 < 1\), \(R_2 < 1\), \(R_4 < 1\). The argument of the root of equations

\[
(F_{1}^M + G_6) = 0,
\]

\[
(F_{2}^M + G_6) = 0,
\]

\[
(F_{3}^M + G_6 + G_9 + G_11) = 0,
\]

are similar, that is:

\[
\left| \arg(\delta_i) \right| > \frac{\pi}{2M},
\]

where \(k = 0, 1, 2, 3, \ldots, (m - 1)\).

Similarly, we can find the arguments of the roots of the equation

\[
(F_{3}^M + a_2EM^M + a_1FM^M + a_0) = 0,
\]

are all greater than \(\frac{\pi}{2M}\) if \(R_0 < 1\), having an argument less than \(\frac{\pi}{2M}\) for \(R_0 > 1\). Thus, for \(R_0 < 1\) the DFE \(N^*\) is LAS.

Stability analysis of EE

Theorem 2. If \(R_0 > 1\), then the EE of model (1) LAS.

Proof. Since, we know that for \(R_0 > 1\) the EEP exists. Further the Jacobian matrix \(J\) at EEP is given by:

\[
J_{EEP} = \left( \begin{array}{cccccc} -Y_2 - G_8 & 0 & -G_2Y_2 & -Y_8 & 0 & 0 \\ Y_7 & Y_1 & G_2Y_3 & Y_3 & 0 & 0 \\ 0 & Y_2 & Y_3 & 0 & 0 & 0 \\ 0 & G_4 & 0 & Y_4 & 0 & 0 \\ 0 & 0 & G_5 & Y_5 & 0 & 0 \\ 0 & 0 & G_6 & G_7 & Y_6 & -G_8 \end{array} \right).
\]

(17)

where

\[
\begin{align*}
Y_1 &= -(G_4 + G_6), \\
Y_2 &= -(G_1 + G_9 + G_8), \\
Y_3 &= -(G_1 + G_9 + G_8), \\
Y_4 &= -(G_1 + G_9 + G_8), \\
Y_5 &= -(G_1 + G_9 + G_8), \\
Y_6 &= -(G_1 + G_9 + G_8), \\
Y_7 &= G_1Y_1. \\
\end{align*}
\]

The two eigenvalues \(i_2 = -G_8\) and \(i_3 = Y_3 = -(G_1 + G_9 + G_8)\) of the
matrix (17) are negative. Further more, for the remaining eigenvalues
we can utilize the following equation

\[
f(\lambda) = \lambda^4 + B_1\lambda^3 + B_2\lambda^2 + B_3\lambda + B_0,
\]

where

\[
\begin{align*}
B_3 &= Y_1 - Y_3 - Y_4 + Y_7, \\
B_2 &= Y_5 + Y_1Y_4 + Y_1Y_3 + Y_2Y_3 + Y_2Y_4 - Y_3Y_2 - Y_3Y_7 - G_8Y_1 \\
&- G_8Y_5 - G_8Y_4 - G_8G_5Y_3 - G_6Y_2Y_4, \\
B_1 &= G_9Y_3 + G_5Y_3 + G_5Y_4 + G_5Y_2Y_4 - Y_1Y_3Y_4 + Y_1Y_3Y_7 \\
&+ Y_2Y_4Y_2 + Y_3Y_2Y_5 + G_1G_2G_3Y_8 - G_2G_6Y_2Y_8, \\
&+ G_6G_7Y_1Y_1 + G_4Y_2Y_8, \\
B_0 &= G_7G_2G_3Y_8 - G_8Y_3Y_4 - Y_1Y_3Y_4Y_7 + G_2G_6Y_2Y_4Y_8.
\end{align*}
\]

(19)
The coefficient $B_3$ can easily be shown to be positive and $B_2, B_1, B_0$ are also positive as shown below:

$$B_2 = \left( (Y_1 Y_2 R_1 + Y_1 Y_3 R_2) / R_0 \right) + Y_3 Y_4 - Y_3 Y_7 - Y_4 Y_8 - G_y Y_1 - G_g Y_1 - G_g Y_4 > 0.$$  

$$B_1 = G_y Y_1 Y_3 (R_2 / R_0) + G_y Y_2 Y_8 (R_1 / R_0) + G_y Y_2 Y_4 - 2 Y_1 Y_2 Y_3 + Y_1 Y_2 Y_7 + Y_2 Y_3 Y_7 > 0.$$  

$$B_0 = G_y G_2 G_3 Y_3 Y_8 - G_y Y_1 Y_4 + Y_1 Y_1 Y_7 + G_y G_2 Y_2 Y_4 Y_7 = - Y_1 Y_1 Y_7 Y_7 > 0.$$  

Since it is not hard to show that $B_0 B_1 B_2^2 - B_1 B_3 B_0 < 0$, the Routh–Hurwitz stability conditions for Eq. (18) are satisfied. Thus all the eigenvalues of the Eq. (18) have a negative real part. Accordingly, the EEP $N^*$ is LAS for $R_0 > 1$.

**Case study**

The parameters used in the system (1) are estimated depend on the total number of confirmed incidents, and deaths data in Khyber Pukhtunkhwa Pakistan. The ordinary Least Square Solution (OLS) is utilized to reduce the error terms for the daily reports, and the related relative error is used in the goodness of fit.

$$\min \left( \sum_{i=1}^{n} \left( I_i - \hat{I}_i \right)^2 \right)^{1/2}$$  

(21)

where $I_i$ is the reported total number of infected, and $\hat{I}_i$ is the simulated total number of infected. The simulated cumulative number of infected are calculated by summing the individuals transit from the infected compartment to the recovered compartment for each day. The Fig. 1 shows the fit of model to the data. Estimated values of parameters are shown in Table 3.

**Qualitative analysis of the COVID-19 model**

In the present section, we are going prove the uniqueness, existence, Ulam–Hyers stability of the solution for the proposed model with help of fixed point approaches. Before that, we rewrite the model (3) as

$$\mathbb{F} F_t S(t) = F_1(t, S, E, I, A, H, R),$$  

$$\mathbb{F} F_t E(t) = F_2(t, S, E, I, A, H, R),$$  

$$\mathbb{F} F_t I(t) = F_3(t, S, E, I, A, H, R),$$  

$$\mathbb{F} F_t A(t) = F_4(t, S, E, I, A, H, R),$$  

$$\mathbb{F} F_t H(t) = F_5(t, S, E, I, A, H, R),$$  

$$\mathbb{F} F_t R(t) = F_6(t, S, E, I, A, H, R).$$  

(22)

The system (22) can be turned to the following formula,

$$\mathbb{W}(t) = \mathbb{W}_0 + \frac{\Theta(1 - \Psi)^{\epsilon^{-1}} \mathbb{S}(t, \mathbb{W}(t))}{M(\Psi)}$$  

(26)

Next, for the analysis, the below assumptions $H_1$ and $H_2$ should be fulfilled:

- $H_1 : \sigma : J \times I \mapsto R$ is continuous and there exists two constants $\tau_0, \eta_0 > 0$ such that $|\sigma(t, \mathbb{W}(t))| \leq \tau_0 + |\mathbb{W}(t)| \eta_0$ for $t \in J$ and $\mathbb{W} \in F$.

- $H_2 :$ there should be exists constant $L_0 > 0$ such that $|\sigma(t, \mathbb{W}_1(t)) - \sigma(t, \mathbb{W}_2(t))| \leq L_0 |\mathbb{W}_1(t) - \mathbb{W}_2(t)|$, for $t \in J$ and $\mathbb{W} \in F$.
**Theorem 3.** Assume that \( H_1 \) and \( H_2 \) holds. Then Eq. (24) identical to the system (22) has a solution, provided that

\[
\frac{\varepsilon (1 - \Psi) T^{z-1}}{M(\Psi)} I_{\text{wo}} < 1
\]

(27)

and

\[
\Delta_1 = |W_0| + \left[ \frac{\varepsilon (1 - \Psi) T^{z-1}}{M(\Psi)} + \frac{\tilde{\varepsilon} \Psi B(\Psi, \Xi T^{z+p-1})}{M(\Psi)} \right] \eta_\omega
\]

(28)

**Proof.** We turn the given system (22) into a fixed point problems, i.e

\( W = \Phi W, W \in F \) Where the operator \( \Phi : F \rightarrow F \) defined by

\[
(\Phi W) (t) = W_0 + \frac{\varepsilon (1 - \Psi) T^{z-1}}{M(\Psi)} \sigma (t, W(t)) + \frac{\tilde{\varepsilon} \Psi B(\Psi, \Xi T^{z+p-1})}{M(\Psi)} \int_0^t \sigma (\theta, W(\theta)) d\theta.
\]

(29)

Let

\[ \Pi_\zeta = \{ W \in F : \| W \| \leq \zeta \} \]

(30)

be close, convex, bounded subset with

\[ \zeta \geq \frac{\Delta_1}{1 - \Delta_2} \]

(31)

where

\[
\Delta_2 = \left[ \frac{\varepsilon (1 - \Psi) T^{z-1}}{M(\Psi)} + \frac{\tilde{\varepsilon} \Psi B(\Psi, \Xi T^{z+p-1})}{M(\Psi)} \right] \eta_\omega
\]

(32)

Define the operator \( \Phi_1, \Phi_2 \) such that \( \Phi = \Phi_1 + \Phi_2 \)

\[
\Phi_1 W(t) = W_0 + \frac{\varepsilon (1 - \Psi) T^{z-1}}{M(\Psi)} \sigma (t, W(t))
\]

\[
\Phi_2 W(t) = \frac{\tilde{\varepsilon} \Psi B(\Psi, \Xi T^{z+p-1})}{M(\Psi)} \int_0^t \sigma (\theta, W(\theta)) d\theta.
\]

(33)

Now we split the proof in the following steps as:

Step (1): \( \Phi_1 W(t) + \Phi_2 W(t) \in \Pi_\zeta \) for all \( W, W^* \in \Pi_\zeta \). Indeed, we have

\[
\| \Phi_1 W + \Phi_2 W \| = \max_{\omega \in J} |W_0| + \frac{\varepsilon (1 - \Psi) T^{z-1}}{M(\Psi)} \| \sigma (t, W(t)) \| + \frac{\tilde{\varepsilon} \Psi B(\Psi, \Xi T^{z+p-1})}{M(\Psi)} \| \int_0^t \sigma (\theta, W(\theta)) d\theta \|
\]

\[
\leq |W_0| + \frac{\varepsilon (1 - \Psi) T^{z-1}}{M(\Psi)} \| \sigma (t, W(t)) \| + \frac{\tilde{\varepsilon} \Psi B(\Psi, \Xi T^{z+p-1})}{M(\Psi)} \| \int_0^t \sigma (\theta, W(\theta)) d\theta \|
\]

\[
= |W_0| + \frac{\varepsilon (1 - \Psi) T^{z-1}}{M(\Psi)} \| \sigma (t, W(t)) \| + \frac{\tilde{\varepsilon} \Psi B(\Psi, \Xi T^{z+p-1})}{M(\Psi)} \| \int_0^t \sigma (\theta, W(\theta)) d\theta \|
\]

\[
\leq |W_0| + \frac{\varepsilon (1 - \Psi) T^{z-1}}{M(\Psi)} \| \sigma (t, W(t)) \| + \frac{\tilde{\varepsilon} \Psi B(\Psi, \Xi T^{z+p-1})}{M(\Psi)} \eta_\omega \zeta
\]

\[
\leq \Delta_1 + \Delta_2 \zeta \leq \zeta.
\]

This proves that

\[
\Phi_1 W(t) + \Phi_2 W(t) \in \Pi_\zeta.
\]

(35)

Step (3): \( \Phi_1 \) is contraction. Let \( W_1, W_2 \in \Phi_1 \). Then via \( (H_2) \), we get

\[
|\Phi_1 W_1 - \Phi_1 W_2| = \max_{\omega \in J} \frac{\varepsilon (1 - \Psi) T^{z-1}}{M(\Psi)} \| \sigma (t, W_1(t)) - \sigma (t, W_2(t)) \|
\]

\[
\leq \max_{\omega \in J} \frac{\varepsilon (1 - \Psi) T^{z-1}}{M(\Psi)} \| W_1(t) - W_2(t) \|
\]

\[
\leq \frac{\varepsilon (1 - \Psi) T^{z-1}}{M(\Psi)} \| W_1 - W_2 \|.
\]

(36)

Step (3): \( \Phi_2 \) is relatively compact.

Case 1: \( \Phi_2 \) is continuous. Due to \( W(t) \) is continuous, then \( \Phi_2 W(t) \) is continuous too.

Case 2: \( \Phi_2 \) is uniformly bounded on \( \Pi_\zeta \). Let \( W(t) \in \Pi_\zeta \). Then, we have

\[
\| \Phi_1 W \| = \max_{\omega \in J} \frac{\varepsilon \Psi B(\Psi, \Xi T^{z+p-1})}{M(\Psi)} \int_0^t \| \sigma (\theta, W(\theta)) \| d\theta
\]

\[
\leq \frac{\varepsilon \Psi B(\Psi, \Xi T^{z+p-1})}{M(\Psi)} \eta_\omega \zeta.
\]

Hence \( \Phi_2 \) is uniformly bounded on \( \Pi_\zeta \).

Case 3: \( \Phi_3 \) is equicontinuous. Let \( W \in \Pi_\zeta \) and \( 0 < t_1 < t_2 < T \). Then

\[
\| \Phi_2 W(t_2) - \Phi_2 W(t_1) \| = \max_{\omega \in J} \frac{\varepsilon \Psi B(\Psi, \Xi T^{z+p-1})}{M(\Psi)} \int_0^t \| \sigma (\theta, W(\theta)) \| d\theta
\]

\[
\leq \frac{\varepsilon \Psi B(\Psi, \Xi T^{z+p-1})}{M(\Psi)} \eta_\omega \zeta \| W(t_2) - W(t_1) \|.
\]

(38)

It follows that

\[ \| \Phi_2 W(t_2) - \Phi_2 W(t_1) \| \rightarrow 0, \text{ as } t_1 \rightarrow t_2. \]

Thus, by Arzela–Ascoli theorem, we deduce that \( \Phi_2 \) is completely continuous. The Eq. (26) has at least one solution, so the proposed model has unique solution.

**Theorem 4.** Assume that \( (H_2) \) holds if

\[
\Delta_1 = \left[ \frac{\varepsilon (1 - \Psi) T^{z-1}}{M(\Psi)} + \frac{\tilde{\varepsilon} \Psi B(\Psi, \Xi T^{z+p-1})}{M(\Psi)} \right] \| \omega \| > 1
\]

(39)

then the integral Eq. (24) has a unique solution which implies that the model (3) has a unique solution.

**Proof.** Taking the operator \( \Phi : F \rightarrow F \) defined by (26). Let \( W_1, W_2 \in F \) and \( t \in J \). Then

\[
|\Phi W_1 - \Phi W_2| \leq \max_{\omega \in J} \frac{\varepsilon (1 - \Psi) T^{z-1}}{M(\Psi)} \| \sigma (t, W_1(t)) - \sigma (t, W_2(t)) \|
\]

\[
+ max_{\omega \in J} \frac{\varepsilon \Psi B(\Psi, \Xi T^{z+p-1})}{M(\Psi)} \eta_\omega \| W_1(t) - W_2(t) \|
\]

\[
\leq A \| W_1 - W_2 \|
\]

due to, \( \Phi \) is contraction. Thus (26) has a unique solution, which yield that the model (3) has a unique solution.

**Hyers–Ulam Stability**

**Definition 1 ([32]).** The fractal fractional integral system given by Eqs. (5) is said to be Hyers–Ulam stable if exist constants \( \Delta_i > 0, i \in N^0 \) satisfying: For every \( \gamma_i > 0, i \in N^0 \), for
there exist \( \bar{S}, \bar{E}, \bar{I}, \bar{A}, \bar{H}, \bar{R} \) which are satisfying

\[
\begin{align*}
\dot{\bar{S}}(t) &= \frac{1}{B(\psi)} \bar{\xi}_1(\bar{\psi}, t, \bar{S}(t)) + \frac{\psi}{B(\psi)} I(\psi) \int_0^t (t - \delta)^{\psi - 1} \bar{\xi}_1(\psi, \delta, \bar{S}(\delta)) d\delta, \\
\dot{\bar{E}}(t) &= \frac{1}{B(\psi)} \bar{\xi}_2(\bar{\psi}, t, \bar{E}(t)) + \frac{\psi}{B(\psi)} I(\psi) \int_0^t (t - \delta)^{\psi - 1} \bar{\xi}_2(\psi, \delta, \bar{E}(\delta)) d\delta, \\
\dot{\bar{I}}(t) &= \frac{1}{B(\psi)} \bar{\xi}_3(\bar{\psi}, t, \bar{I}(t)) + \frac{\psi}{B(\psi)} I(\psi) \int_0^t (t - \delta)^{\psi - 1} \bar{\xi}_3(\psi, \delta, \bar{I}(\delta)) d\delta, \\
\dot{\bar{A}}(t) &= \frac{1}{B(\psi)} \bar{\xi}_4(\bar{\psi}, t, \bar{A}(t)) + \frac{\psi}{B(\psi)} I(\psi) \int_0^t (t - \delta)^{\psi - 1} \bar{\xi}_4(\psi, \delta, \bar{A}(\delta)) d\delta, \\
\dot{\bar{H}}(t) &= \frac{1}{B(\psi)} \bar{\xi}_5(\bar{\psi}, t, \bar{H}(t)) + \frac{\psi}{B(\psi)} I(\psi) \int_0^t (t - \delta)^{\psi - 1} \bar{\xi}_5(\psi, \delta, \bar{H}(\delta)) d\delta, \\
\dot{\bar{R}}(t) &= \frac{1}{B(\psi)} \bar{\xi}_6(\bar{\psi}, t, \bar{R}(t)) + \frac{\psi}{B(\psi)} I(\psi) \int_0^t (t - \delta)^{\psi - 1} \bar{\xi}_6(\psi, \delta, \bar{R}(\delta)) d\delta.
\end{align*}
\]

Taking, \( \gamma_1 = \psi, \Delta t = \frac{1}{\Gamma(\psi)} + \frac{\psi}{B(\psi)} I(\psi), \) this implies

\[
\begin{align*}
\|S - \bar{S}\| &\leq \gamma_2 \delta_1, \\
\|E - \bar{E}\| &\leq \gamma_2 \delta_2, \\
\|I - \bar{I}\| &\leq \gamma_2 \delta_3, \\
\|A - \bar{A}\| &\leq \gamma_2 \delta_4, \\
\|H - \bar{H}\| &\leq \gamma_2 \delta_5, \\
\|R - \bar{R}\| &\leq \gamma_2 \delta_6.
\end{align*}
\]

Theorem 5. \textbf{Presume that the assumption of Theorem 4 are satisfied. Then the model (3) will be UH stable.}

\textbf{Proof.} The fractal fractional model (5) has at least one solution \((S, E, I, A, H, R)\) satisfying equations of system (5). Then, we have

\[
\begin{align*}
\|S - \bar{S}\| &\leq \frac{1}{B(\psi)} ||\bar{\xi}_1(\bar{\psi}, t, S) - \bar{\xi}_1(\bar{\psi}, t, \bar{S})|| \\
&+ \frac{\psi}{B(\psi) I(\psi)} \int_0^t (t - \delta)^{\psi - 1} ||\bar{\xi}_1(\psi, \delta, \bar{S}(\delta))|| d\delta \\
&\leq \left[ \frac{1}{B(\psi)} + \frac{\psi}{B(\psi) I(\psi)} \right] \psi \|S - \bar{S}\| \\
\|E - \bar{E}\| &\leq \frac{1}{B(\psi)} ||\bar{\xi}_2(\bar{\psi}, t, E) - \bar{\xi}_2(\bar{\psi}, t, \bar{E})|| \\
&+ \frac{\psi}{B(\psi) I(\psi)} \int_0^t (t - \delta)^{\psi - 1} ||\bar{\xi}_2(\psi, \delta, \bar{E}(\delta))|| d\delta \\
&\leq \left[ \frac{1}{B(\psi)} + \frac{\psi}{B(\psi) I(\psi)} \right] \psi \|E - \bar{E}\|
\end{align*}
\]

\textbf{Simulation results & numerical schemes}

With the help of the numerical scheme as presented in the above sections, the models are simulated under various fractional orders for model (5). This is very important to show the feasibility of the reported work and investigate the validity of the analytical work using large-scale numerical simulation. It is important to point out that, unlike traditional numerical analysis, there are not as many options to choose schemes for the numerical analysis of the fractional order epidemiological models simulations. For the numerical solution of the fractal fractional model (5), we utilized the initial conditions, i.e, \( S(0) = 150, E(0) = 100, I(0) = 50, A(0) = 80, H(0) = 80, R(0) = 10, \) and the parameters value taken from Table 2. We can clear see from Figs. 2–4, the Adams–Bashforth method is faster to capture the solution of the nonlinear fractal fractional model as compared to the Newton polynomial method.

\textbf{Solution by Newton polynomial}

The numerical scheme for Newton polynomial we can see in Fig. 2 the initial days the susceptible class is decreasing at different fractional order. Consequently, the exposed class first increases which also
increase the infected class for initial few days. In same line the recovered class is raising which indicate that the infection is considerably reducing or they are going to die due to infection. Also, the virus class first increase and after the due to reduction in infection this class is also went on reducing. First, we can express model (1) as follows:

\[
\text{FFM} \begin{align*}
\mathbf{r}_G^s &= G - G_{G1} (G_2 I + A^*) S^r - G_6 S^r, \\
\mathbf{r}_G^e &= G - G_{G1} (G_2 I + A^*) s S^r - G_4 (G_4 + G_6) E^r, \\
\mathbf{r}_G^p &= (1 - G_1) G_2 E^r - (G_4 + G_6) I^r, \\
\mathbf{r}_G^a &= G_5 G_6 E^r - (G_1 + G_6) A^r.
\end{align*}
\]

(51)

Now, we can rewrite the above system as:

\[
\text{FFM} \begin{align*}
\mathbf{r}_G^s &= S^r (t, s, E, I, A, H, R), \\
\mathbf{r}_G^e &= E^r (t, s, E, I, A, H, R), \\
\mathbf{r}_G^p &= I^r (t, s, E, I, A, H, R), \\
\mathbf{r}_G^a &= A^r (t, s, E, I, A, H, R), \\
\mathbf{r}_G^h &= H^r (t, s, E, I, A, H, R), \\
\mathbf{r}_G^r &= R^r (t, s, E, I, A, H, R).
\end{align*}
\]

(52)

Applying the fractal fractional integral and plugging Newton polynomials into these equations, we can get;

\[
S^{r+1} = 1 - \frac{Ψ}{AB(Ψ)} + \sum_{p=2}^{n} S(t_p, S^p, E^p, I^p, A^p, H^p, R^p) + \frac{Ψ(Δt)^p}{AB(Ψ)I(Ψ + 1)} \sum_{p=2}^{n} \sum_{k=2}^{m} S(t_{p-k}, S^{p-k}, E^{p-k}, I^{p-k}, A^{p-k}, H^{p-k}, R^{p-k}) II
\]

\[
E^{r+1} = 1 - \frac{Ψ}{AB(Ψ)} + \sum_{p=2}^{n} E(t_p, S^p, E^p, I^p, A^p, H^p, R^p) + \frac{Ψ(Δt)^p}{AB(Ψ)I(Ψ + 1)} \sum_{p=2}^{n} \sum_{k=2}^{m} E(t_{p-k}, S^{p-k}, E^{p-k}, I^{p-k}, A^{p-k}, H^{p-k}, R^{p-k}) II
\]

\[
H^{r+1} = 1 - \frac{Ψ}{AB(Ψ)} + \sum_{p=2}^{n} H(t_p, S^p, E^p, I^p, A^p, H^p, R^p) + \frac{Ψ(Δt)^p}{AB(Ψ)I(Ψ + 1)} \sum_{p=2}^{n} \sum_{k=2}^{m} H(t_{p-k}, S^{p-k}, E^{p-k}, I^{p-k}, A^{p-k}, H^{p-k}, R^{p-k}) II
\]

\[
R^{r+1} = 1 - \frac{Ψ}{AB(Ψ)} + \sum_{p=2}^{n} R(t_p, S^p, E^p, I^p, A^p, H^p, R^p) + \frac{Ψ(Δt)^p}{AB(Ψ)I(Ψ + 1)} \sum_{p=2}^{n} \sum_{k=2}^{m} R(t_{p-k}, S^{p-k}, E^{p-k}, I^{p-k}, A^{p-k}, H^{p-k}, R^{p-k}) II
\]
where
\[
\Delta = \begin{bmatrix}
(a - \mu + 1)^2 & 2(a - \mu)^2 + (5\Psi + 10)(a - \mu) \\
0 & 2(a - \mu)^2 + 9\Psi + 12 \\
(a - \mu)^2 & 2(a - \mu)^2 + (5\Psi + 10)(a - \mu) \\
0 & 2(a - \mu)^2 + 18\Psi + 12
\end{bmatrix},
\]
\[
\Sigma = \begin{bmatrix}
(a - \mu + 1)^2 & 2(a - \mu)^2 + (5\Psi + 10)(a - \mu) \\
0 & 2(a - \mu)^2 + 9\Psi + 12 \\
(a - \mu)^2 & 2(a - \mu)^2 + (5\Psi + 10)(a - \mu) \\
0 & 2(a - \mu)^2 + 18\Psi + 12
\end{bmatrix},
\]
\[
\Pi = [(a - \mu + 1)^2 - (a - \mu)^2].
\]

Numerical scheme by Adams Bashforth method

The numerical scheme for Adams Bashforth method we can see in Figs. 3–4 the initial days the susceptible class is decreasing at different fractional order. Consequently, the exposed class first increases which also increase the infected class for initial few days. In some line the recovered class is raising which indicate that the infection is considerably reducing or they are going to die due to infection. Also, the virus class first increase and after the due to reduction in infection this class is also went on reducing. First, we can express model (1) as follows:

\[
\begin{align*}
\frac{d^\Psi}{dt^\Psi} S(t) &= \Xi \Xi^{-1} F_1(t, S, E, I, A, H, R) \\
\frac{d^\Psi}{dt^\Psi} D(t) &= \Xi \Xi^{-1} F_2(t, S, E, I, A, H, R) \\
\frac{d^\Psi}{dt^\Psi} D(t) &= \Xi \Xi^{-1} F_3(t, S, E, I, A, H, R) \\
\frac{d^\Psi}{dt^\Psi} I(t) &= \Xi \Xi^{-1} F_4(t, S, E, I, A, H, R) \\
\frac{d^\Psi}{dt^\Psi} H(t) &= \Xi \Xi^{-1} F_5(t, S, E, I, A, H, R) \\
\frac{d^\Psi}{dt^\Psi} R(t) &= \Xi \Xi^{-1} F_6(t, S, E, I, A, H, R).
\end{align*}
\]

(53)

Now, applying the fractal fractional integral to both sides of (53), we obtained the following system

\[
\begin{align*}
S(t) - S(0) &= \frac{\Xi \Xi^{-1}}{\mathcal{M}(\Psi)} F_1(t, S, E, I, A, H, R) \\
+ \frac{\Xi \Xi^{-1}}{\mathcal{M}(\Psi)} \int_0^t (t - s)^{\Psi-1} \Xi^{-1} F_1(s, S, E, I, A, H, R) \, ds \\
E(t) - E(0) &= \frac{\Xi \Xi^{-1}}{\mathcal{M}(\Psi)} F_2(t, S, E, I, A, H, R) \\
+ \frac{\Xi \Xi^{-1}}{\mathcal{M}(\Psi)} \int_0^t (t - s)^{\Psi-1} \Xi^{-1} F_2(s, S, E, I, A, H, R) \, ds \\
I(t) - I(0) &= \frac{\Xi \Xi^{-1}}{\mathcal{M}(\Psi)} F_3(t, S, E, I, A, H, R) \\
+ \frac{\Xi \Xi^{-1}}{\mathcal{M}(\Psi)} \int_0^t (t - s)^{\Psi-1} \Xi^{-1} F_3(s, S, E, I, A, H, R) \, ds \\
H(t) - H(0) &= \frac{\Xi \Xi^{-1}}{\mathcal{M}(\Psi)} F_4(t, S, E, I, A, H, R) \\
+ \frac{\Xi \Xi^{-1}}{\mathcal{M}(\Psi)} \int_0^t (t - s)^{\Psi-1} \Xi^{-1} F_4(s, S, E, I, A, H, R) \, ds \\
A(t) - A(0) &= \frac{\Xi \Xi^{-1}}{\mathcal{M}(\Psi)} F_5(t, S, E, I, A, H, R) \\
+ \frac{\Xi \Xi^{-1}}{\mathcal{M}(\Psi)} \int_0^t (t - s)^{\Psi-1} \Xi^{-1} F_5(s, S, E, I, A, H, R) \, ds \\
R(t) - R(0) &= \frac{\Xi \Xi^{-1}}{\mathcal{M}(\Psi)} F_6(t, S, E, I, A, H, R) \\
+ \frac{\Xi \Xi^{-1}}{\mathcal{M}(\Psi)} \int_0^t (t - s)^{\Psi-1} \Xi^{-1} F_6(s, S, E, I, A, H, R) \, ds.
\end{align*}

(54)

Set \( t = t_{m+1} \) for \( m = 0, 1, 2, \ldots \), it follows that

\[
S(t_{m+1}) - S(0) = \frac{\Xi \Xi^{-1}}{\mathcal{M}(\Psi)} F_1(t_{m+1}, S, E, I, A, H, R) \\
+ \frac{\Xi \Xi^{-1}}{\mathcal{M}(\Psi)} \int_{t_m}^{t_{m+1}} (t_{m+1} - s)^{\Psi-1} \Xi^{-1} F_1(s, S, E, I, A, H, R) \, ds \\
E(t_{m+1}) - E(0) = \frac{\Xi \Xi^{-1}}{\mathcal{M}(\Psi)} F_2(t_{m+1}, S, E, I, A, H, R) \\
+ \frac{\Xi \Xi^{-1}}{\mathcal{M}(\Psi)} \int_{t_m}^{t_{m+1}} (t_{m+1} - s)^{\Psi-1} \Xi^{-1} F_2(s, S, E, I, A, H, R) \, ds \\
I(t_{m+1}) - I(0) = \frac{\Xi \Xi^{-1}}{\mathcal{M}(\Psi)} F_3(t_{m+1}, S, E, I, A, H, R) \\
+ \frac{\Xi \Xi^{-1}}{\mathcal{M}(\Psi)} \int_{t_m}^{t_{m+1}} (t_{m+1} - s)^{\Psi-1} \Xi^{-1} F_3(s, S, E, I, A, H, R) \, ds \\
H(t_{m+1}) - H(0) = \frac{\Xi \Xi^{-1}}{\mathcal{M}(\Psi)} F_4(t_{m+1}, S, E, I, A, H, R) \\
+ \frac{\Xi \Xi^{-1}}{\mathcal{M}(\Psi)} \int_{t_m}^{t_{m+1}} (t_{m+1} - s)^{\Psi-1} \Xi^{-1} F_4(s, S, E, I, A, H, R) \, ds \\
A(t_{m+1}) - A(0) = \frac{\Xi \Xi^{-1}}{\mathcal{M}(\Psi)} F_5(t_{m+1}, S, E, I, A, H, R) \\
+ \frac{\Xi \Xi^{-1}}{\mathcal{M}(\Psi)} \int_{t_m}^{t_{m+1}} (t_{m+1} - s)^{\Psi-1} \Xi^{-1} F_5(s, S, E, I, A, H, R) \, ds \\
R(t_{m+1}) - R(0) = \frac{\Xi \Xi^{-1}}{\mathcal{M}(\Psi)} F_6(t_{m+1}, S, E, I, A, H, R) \\
+ \frac{\Xi \Xi^{-1}}{\mathcal{M}(\Psi)} \int_{t_m}^{t_{m+1}} (t_{m+1} - s)^{\Psi-1} \Xi^{-1} F_6(s, S, E, I, A, H, R) \, ds.
\]

Here, we approximate the functions \( (t_{m+1} - s) \) by the interpolation polynomial with \( h = t_{m+1} - t_m \) as follows

\[
x_i(t) = \left( t - t_{m+1} \right) \Xi^{-1} F_i(t_{m+1}, S, E, I, A, H, R) \]

\[
\Xi^{-1} \Xi^{-1} \left( \frac{t - t_{m+1}}{h} \right) \Xi^{-1} F_i(t_m, S(t_m), E(t_m), I(t_m), A(t_m), H(t_m)) \]

(55)

\[
\Xi^{-1} \Xi^{-1} \left( \frac{t - t_{m+1}}{h} \right) \Xi^{-1} F_i(t_m, S(t_m), E(t_m), I(t_m), A(t_m), H(t_m)) \]

(56)

By the approximate the functions \( x_i(s) \), (56) becomes

\[
S(t_{m+1}) = \left[ S(0) + \frac{\Xi \Xi^{-1}}{\mathcal{M}(\Psi)} F_1(t_{m+1}, S, E, I, A, H, R) \\
+ \frac{\Xi \Xi^{-1}}{\mathcal{M}(\Psi)} \sum_{i=1}^{N_1} \Xi^{-1} F_i(t, S(t_m), E(t_m), I(t_m), A(t_m), H(t_m)) \right] N_1 \\
- \frac{\Xi \Xi^{-1}}{\mathcal{M}(\Psi)} \sum_{i=1}^{N_2} \Xi^{-1} F_i(t, S(t_m), E(t_m), I(t_m), A(t_m), H(t_m)) \right] N_2
\]

(57)
Fig. 2. Simulation results for the proposed model (5) via Newton polynomial for the different values fractal dimension $\mathcal{E}$ and fractional order $\Psi$. 

(a) Simulation result of susceptible group
(b) Simulation result of exposed group
(c) Simulation result of symptomatic Infected group
(d) Simulation result of asymptomatic Infected group
(e) Simulation result of hospitalized group
(f) Simulation result of recovered group
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where

\[
(1 + \Psi + 1) \frac{1}{\Psi} \left[ (t_{n+1} - t_n) \Psi^{n+1} - (t_{n+1} - t_n) \Psi^{n+1} \right],
\]

d and

\[
2 \Psi \sum_{n=1}^\infty \frac{\Psi^{n+1}}{(1 + \Psi + 1)} \left[ (t_{n+1} - t_n) \Psi^{n+1} - (t_{n+1} - t_n) \Psi^{n+1} \right].
\]

put \( t_n = nh \), we get

\[
N_1 = \frac{\Psi^{n+1}}{\Psi(1 + \Psi + 1)} \left[ (m + 1 - n)(m - n + 2 + \Psi) - (m - n + 2 + 2\Psi) \right],
\]

and

\[
N_2 = \frac{\Psi^{n+1}}{\Psi(1 + \Psi + 1)} \left[ (m + 1 - n)^2 - (m - n)^2 (m - n + 1 + \Psi) \right].
\]

Substituting (63) and (64) into Eqs. (57)–(62), we get

\[
E(t_{n+1}) = \sum_{n=1}^\infty \left[ \frac{\Psi^{n+1}}{\Psi(1 + \Psi + 1)} \left[ (m + 1 - n)(m - n + 2 + \Psi) - (m - n + 2 + 2\Psi) \right] - \frac{\Psi^{n+1}}{\Psi} \right]
\]

Conclusions

This study presents a novel approach for understanding the dynamics of the mathematical modeling approach that provides strong conclusions on the transmission mechanism of the newly but deeply investigated COVID-19 pandemic driven infections. To define the proposed model a COVID-19, the infected people are divided into two classes, namely, detected and undetected classes. The Fractal fractional order derivative with fractal dimension \( \Psi \) and fractional order \( \Psi \) in ABC sense is used to more readily investigate the infection dynamics. After the model definition, at first, we introduced the fundamental and essential numerical provisions of the fractal fractional COVID-19 pandemic model. We make use of the fractional order stability in...
Fig. 3. Simulation results for the proposed model (5) via Adams–Bashforth method for the different values fractal dimension $\beta$ and fractional order $\psi$. 

(a) Simulation result of susceptible group.

(b) Simulation result of exposed group.

(c) Simulation result of symptomatic Infected group.

(d) Simulation result of asymptomatic Infected group.

(e) Simulation result of hospitalized group.

(f) Simulation result of recovered group.
Fig. 4. Simulation results for the proposed model (5) via Adams-Bashforth method for another set of initial condition at different values fractal dimension $\gamma$ and fractional order $\varphi$. 

(a) Simulation result of susceptible group.

(b) Simulation result of exposed group.

(c) Simulation result of symptomatic Infected group.

(d) Simulation result of asymptomatic Infected group.

(e) Simulation result of hospitalized group.

(f) Simulation result of recovered group.
approach for the local stability of both endemic as well as the disease-free equilibrium points. The fractal and fractional order mathematical model in the ABC sense are solved numerically via Newton polynomial and Adams–Bashforth techniques. We believe that the attempt made in this work will provide fruitful insights for adopting strategies in reducing the continuous COVID-19 pandemic.

CRediT authorship contribution statement

Jian-Cun Zhou: Conceptualization, Data curation, Methodology,
Writing - original draft. Soheil Salahshour: Software, Validation, Formal analysis, Review editing. Ali Ahmadian: Supervision, Project administration, Funding acquisition. Norazak Senu: Visualization, Software, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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