Missile Threat Assessment System Using Naive Bayes Classifier Approach

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MISSILE THREAT ASSESSMENT SYSTEM USING NAIVE BAYES CLASSIFIER APPROACH

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Abstract. In the modern era, missiles have been used to serve both defence and offense purposes. Numerous research projects have been conducted to improve various aspects of missile systems. One area of main interest regarding the development of missiles in general is automation, particularly surrounding a feature known as guidance system (Siouris, 2003). This feature had been proven to relieve a missile user of several workloads, particularly during aim and launch phase. One key component of a guidance system is known as threat assessment system. Such system is assigned to assess the threat situation that a missile’s engaged in. In this research paper, the developed guidance system uses an approach that is based on the naive Bayes classifier concept. The idea is to establish a system that first describes relationship between a missile and its’ target during mission in the form of a set of combat parameters. These combat parameters are then treated as evidence that is used by the Bayesian algorithm to update an assigned prior belief, yielding an updated posterior belief (Virtanen et al, 2006). The belief comes in a form of probability distribution with each probability representing a predefined threat situation class.

Keywords: missile, threat assessment, combat parameters, naive Bayes, probability distribution

1. Introduction
This paper introduces a missile’s threat assessment system that is developed based on naive Bayes classifier approach. The developed threat assessment system translates inertial geometric properties of the missile and its’ target to relative geometry between the missile and the target. Quantitative result of this translation procedure is defined as combat parameters and the physical situation depicted by these combat parameters is defined as combat geometry. The previously obtained combat parameters are then treated as a bag of evidence and each parameter is assumed to be independent of the rest (Koçkesen et al, 2007). Each combat parameter is then set to become input for a corresponding likelihood function. Such likelihood function is usually obtained through the process of training a machine learning program (Mithcell, 1997). In this paper, the utilized likelihood functions are assumed to be already obtained in order to preserve simplicity. After the system obtained each combat parameter’s likelihood value, these values are processed by the naive Bayes classifier. This process updates an assigned prior belief using the collected evidence, yielding a posterior belief (Ummels, 2010). These beliefs are presented in the
form of probability distribution and each probability in the distribution represents a threat situation outcome which acts as a class in a Bayesian classification approach. In this paper, the simulated scenarios involve two missiles and a target. However, the demonstrated threat assessment program is limited to interaction between the involved missiles with both missiles being featured with identical threat assessment system to preserve simplicity. As additional information, for comparison, another probabilistic approach using similar concept that is known as Probability of Kill (PK) was used by (Sugiyanto, 2014). The developed PK threat assessment program was developed for one-on-one air combat scenarios involving two fighter aircrafts and takes guns or armaments into account.

2. Combat Parameters

Statistical computation performed using any variation of Bayesian approach requires data, acting as set of evidence, as the inputs. In this case, these features can be obtained by defining a set of combat parameters. The established formulas of combat parameters are set to perform translation from independent geometry of two separate agents to expression of one agent’s state relative to the other. From here on, the involved agents will be labeled as B (blue missile), R (red missile), and T (target).

Since combat parameters implies the interaction between two agents, B’s state with respect to expression of one agent’s state relative to B’s perspective of evidence of the following two auxiliary

\[ \sigma_B = \cos \left( \frac{\dot{r}_B (\overrightarrow{V}_B, \overrightarrow{V}_R)}{|r_B||V_B|} \right) \]  \hspace{1cm} (1)

\[ \epsilon_B = \cos \left( \frac{\dot{r}_B (\overrightarrow{V}_B, \overrightarrow{V}_R)}{|V_B||V_R|} \right) \]  \hspace{1cm} (2)

\[ r_B = \frac{|\overrightarrow{pos}_R - \overrightarrow{pos}_B|}{|V_B||V_R||tB| \Delta t} \]  \hspace{1cm} (3)

Mathematical formulation of all five combat parameters above in order are as follows:

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\[ r_B = \frac{|\overrightarrow{pos}_R - \overrightarrow{pos}_B|}{|V_B||V_R||tB| \Delta t} \]  \hspace{1cm} (4)

\[ \dot{r}_B = \frac{-\left( |\overrightarrow{pos}_R - \overrightarrow{pos}_B + (\overrightarrow{V}_R - \overrightarrow{V}_B) \Delta t - r_B \right)}{(|V_B| + |V_R|) \Delta t} \]  \hspace{1cm} (5)

and those five combat parameters can be calculated with the assistance of the following two auxiliary parameters, position vector and range vector, which in order are formulated as follows:

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\[ \overrightarrow{pos}_B = \{x_B, y_B, z_B\}^T ; \overrightarrow{pos}_R = \{x_R, y_R, z_R\}^T \]  \hspace{1cm} (6)

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|combat parameters obtained by solving equations (1) through (5) are next treated as a bag of evidence. According to naive Bayes principal, the order of presentation of the evidence is not important. This
implies that the order in which the combat parameters are presented by equations (1) through (5) is arbitrarily chosen. All five combat parameters are also independent from one another primarily to comply with one of naive Bayes algorithm’s requirements, which is independence between one component of evidence and the other. Secondary advantage of establishing the combat parameters in such manner is isolability, resulting in enhanced convenience during analysis of each parameter’s influence to the threat assessment outcomes. In addition, the following figures are visualization of the previously defined and formulated combat parameters. Note that these figures assume missile B’s perspective against missile R.

3. Threat Situation Outcomes

In this paper, there are 6 classes that are defined as possible threat situation outcomes. These 6 possible threat assessment outcomes are expansions of the idea of using 4 classes as demonstrated by (Bergdahl, 2013). Definition of those 6 possible outcomes are as follows:

a. **Miss (M)** situation. When engaged in this situation, it is likely for \(\sigma, \varepsilon, r\) to have high values while \(\zeta\) value is likely to be low. The value of \(\dot{r}\) is likely negative.

b. **Chase (C)** situation. When engaged in this situation, it is likely for \(\sigma, \varepsilon, \zeta\) and \(r\) to have small values. The value of \(\dot{r}\) is likely positive.

c. **Undershoot (U)** situation. When engaged in this situation, it is likely for \(\sigma, \varepsilon, \zeta\), and \(r\) to have small values. The value of \(\dot{r}\) is likely negative.

d. **Lead (L)** situation. When engaged in this situation, it is likely for \(\sigma\) and \(\varepsilon\) to have high values while \(\zeta\) and \(r\) are likely to be low. The value of \(\dot{r}\) is likely positive.
e. **Overshoot (O)** situation. When engaged in this situation, it is likely for $\sigma$ and $\varepsilon$ to have high values while $\zeta$ and $r$ are likely to be low. The value of $\dot{r}$ is likely negative.

f. **Frontal (F)** situation. When engaged in this situation, it is likely for $\sigma$ and $r$ to have low values while $\zeta$ and $\varepsilon$ are likely to be high. The value of $\dot{r}$ is likely positive.

This expansion, particularly involving derivation of **Chase**, **Undershoot**, **Lead** and **Overshoot** from **Advantage** and **Disadvantage**, was made possible by introducing $\dot{r}$ as one of the utilized combat parameters. Provided below is the figure depicting all 6 possible threat assessment outcomes.

![Diagram of threat assessment outcomes](image)

**Figure 6 Depiction of the established possible threat situation outcomes**

4. **Likelihood Functions**

In the designed threat assessment program, the likelihood of a situation is computed based on the likelihoods of the combat parameters having their current values while being engaged in that situation. Therefore, formulation of a set of likelihood functions involving all utilized combat parameters is necessary. To preserve each combat parameter’s independency of the rest, these likelihood functions are set to be single-attributed, meaning that each function only contain one combat parameter. Last, these likelihood functions affect threat assessment outcome but not the other way around. This results in implementation of likelihood functions that are time-invariant being sufficient. The likelihood functions are set to use exponential distribution instead of linear distribution which was used by (Bergdahl, 2013). Formulation of the set of single-attribute time-invariant likelihood functions being used for depiction of a missile’s status against the other missile are as shown in **Table 1**. The superscript $i$ represents interaction of an agent with one other agent. Parameter $a_i$ acts as a constant that affects a function’s gradient and maximum likelihood value. Similarly, the constant parameter $k_i$ is assigned to manipulate influence level given by $r$ and $k_i\dot{r}$ is assigned to regulate influence level given by $\dot{r}$. In this paper, the value of $a_i$ set to be 1. The value of $k_i$ and $k_i\dot{r}$ are set to be 2. In addition, the value of $a_i$ should be equal or greater than 1 to avoid any occurrence of negative likelihood value. Regardless, the values of the constants can be arbitrarily chosen (Suseno, 2017).
Table 1 List of utilized single-attribute and time-invariant likelihood functions

| Threat Situation | Likelihood Function | $i = BR, RB$ |
|------------------|---------------------|-------------|
| $M, L, O$        | $L^i_\sigma = \exp \left\{ a^i \left( \frac{\sigma^i}{\pi} \right) \right\} - 1$ | (8)         |
| $C, U, F$        | $L^i_\sigma = \exp \left\{ a^i \left( 1 - \frac{\sigma^i}{\pi} \right) \right\} - 1$ | (9)         |
| $L, O, F$        | $L^i_\xi = \exp \left\{ a^i \left( \frac{\xi^i}{\pi} \right) \right\} - 1$ | (10)        |
| $M, C, U$        | $L^i_\xi = \exp \left\{ a^i \left( 1 - \frac{\xi^i}{\pi} \right) \right\} - 1$ | (11)        |
| $M, F$           | $L^i_\xi = \exp \left\{ a^i \left( \frac{\xi^i}{\pi} \right) \right\} - 1$ | (12)        |
| $C, U, L, O$     | $L^i_\xi = \exp \left\{ a^i \left( 1 - \frac{\xi^i}{\pi} \right) \right\} - 1$ | (13)        |
| $M$              | $L^i_\rho = \exp \left\{ k^i_\rho a^i \left( \frac{r^i}{r^i_{\text{max}}} \right) \right\} - 1$ | (14)        |
| $C, U, L, O, F$  | $L^i_\rho = \exp \left\{ k^i_\rho a^i \left( 1 - \frac{r^i}{r^i_{\text{max}}} \right) \right\} - 1$ | (15)        |
| $C, L, F$        | $L^i_\rho = \exp \left\{ k^i_\rho a^i \left( \dot{r}^i \right) \right\}$ | (16)        |
| $M, U, O$        | $L^i_\rho = \exp \left\{ k^i_\rho a^i \left( -\dot{r}^i \right) \right\}$ | (17)        |

5. Naive Bayes Algorithm

Following the referred naive Bayes algorithm, the outputs of prior likelihood $L$ functions are then processed in the way that is expressed by Equation (18) below.

$$L(c^i_k|\Gamma^i_k = j) = L(\sigma^i_k|\Gamma^i_k = j)L(\xi^i_k|\Gamma^i_k = j)L(\epsilon^i_k|\Gamma^i_k = j)L(r^i_k|\Gamma^i_k = j)L(\dot{r}^i_k|\Gamma^i_k = j)$$ (18)

The subscript $i$ corresponds to either B or R missile, the subscript $j$ corresponds to a possible threat situation outcome, and the subscript $k$ denotes corresponding stage at which the computation is performed. Equation (18) yields the likelihood of all possible outcomes given the obtained evidence. The probability $P$ of each outcome can be obtained by evaluation formula shown in Equation (19) below.

$$P(c^i_k|\Gamma^i_k = j) = \frac{L(c^i_k|\Gamma^i_k = j)}{\sum_{Q=1}^{6} L(c^i_k|\Gamma^i_k = Q)}$$ (19)

The idea of Bayesian approach in general is that update of assumption or belief is bound to happen given the accounted evidence (Berrar, 2018). This implies that the calculation process requires the existence of prior belief or assumption, which is formally defined as prior probabilities. Calculation process yields updated belief or assumption, formally defined as posterior probabilities. This procedure requires
initialization of belief. In this paper, the initial belief vector represented by $P$ is set to be $[1/6 \ 1/6 \ 1/6 \ 1/6 \ 1/6 \ 1/6]$ for both missiles.

6. Results and Discussions

Two scenarios are presented in this section. Both scenarios were given similar initial setup. The difference between the two are the maneuvers assigned to both missiles. The target is assigned with maneuvers too, but results involving the target are not recorded aside from its trajectory since this paper focuses only on the interactions between the involved missiles. Initial setup of the scenarios is as shown in Table 2. Simulations are performed numerically using MATLAB as the numerical platform.

| Initial Parameter | Parameter Description | Blue Missile (B) | Red Missile (R) | Target (T) |
|-------------------|-----------------------|------------------|-----------------|------------|
| $x_c (m)$         | position in x axis    | 3000             | 2000            | 5000       |
| $y_c (m)$         | position in y axis    | 4000             | 4000            | 1000       |
| $h_c (m)$         | altitude              | 3000             | 2000            | 0          |
| $\gamma$ (deg)   | pitch angle           | 0                | 0               | 0          |
| $\chi$ (deg)     | heading angle         | 0                | 0               | 0          |
| $V$ (m/s)         | velocity              | 250              | 280             | 100        |

6.1. Scenario 1

In this simulation, missile B is assigned to engage a flight path with constant radius and altitude. Missile R and target T are assigned to maintain their respective directions, velocities, and altitudes. The performed simulation consists of 40 stages with $\Delta t$ between stages value being 1 second. The visualization of the scenario’s result is as follows:

![Scenario 1 trajectory visualization](image)

The obtained probability distribution plots are as follows:
From Figure 7, missile B experienced a change in situation with respect to missile R and vice versa. At early stages of the simulation, missile B happened to engage in *Chase* situation as missile R engaged a *Lead* situation. At latter stages of the simulation, both missiles ended up in a *Miss* situation. This interpretation is supported by Figure 8. Interestingly, Figure 8 captures the threat assessment system’s interpretation of a phase of missile B’s transition from *Lead* to *Miss* as an *Overshoot* situation. Figure 9 indicates that a similar case happened to missile R with the only difference being missile R’s transitional phase was interpreted as a case of *Undershoot*. Both recorded interpretation of the transition phase was due to the exponential likelihood functions being used.

### 6.2. Scenario 2

In this simulation, missile B is assigned to engage a flight path with varying curvature and decreasing altitude, target T is assigned to engage a path with constant radius and altitude, and missile R is assigned to engage a flight path with varying curvature and increasing velocity. The performed simulation consists of 40 stages with $\Delta t$ between stages value being 1 second. The visualization of the scenario’s result is as follows:

The obtained probability distribution plots are as follows:
At early stages of the simulation, missile B happened to engage in Lead situation as missile R engaged a Chase situation. This also happened in Scenario 1. However, at latter stages of Scenario 2’s simulation, missile B ended up in an Undershoot situation while missile R ended up in an Overshoot situation. This interpretation is supported by Figure 11. Interestingly, unlike Scenario 1, no specific threat status visibly dominating the rest is present in Figure 11. Figure 12 indicates that a similar case happened to missile R. However, the cause of the overlapping probabilities during the transition is the same as in Scenario 1, which is the utilization of exponential likelihood functions.

7. Conclusions
In this paper, the threat assessment system is developed based on naive Bayes classifier approach. Five combat parameters in the form of line of sight, aspect angle, heading-crossing angle, range, and closure are defined to act in unison as a bag of evidence to be processed by the predefined exponential, single-attribute, and time-invariant likelihood functions. The obtained set of single-attribute likelihoods are then utilized by the naive Bayes algorithm to transform a missile’s probability distribution from an arbitrarily assumed distribution to a distribution given the evidence the system collected. Both scenarios provide threat assessment results that are aligned with the established possible threat status class provided by Chapter 4. However, due to the usage of exponential likelihood functions, interesting distribution are recorded during phases that are considered transitional according to Chapter 6. Regardless, the developed threat assessment system works properly.

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