Towards the Control over Electroweak Penguins in Nonleptonic $B$-Decays

ANDRZEJ J. BURAS
Technische Universität München, Physik Department
D–85748 Garching, Germany
Max-Planck-Institut für Physik
– Werner-Heisenberg-Institut –
Föhringer Ring 6, D–80805 München, Germany

ROBERT FLEISCHER
Institut für Theoretische Teilchenphysik
Universität Karlsruhe
D–76128 Karlsruhe, Germany

Abstract

We present strategies for determining electroweak penguins from experimental data. Using the CKM-angle $\gamma$ as one of our central inputs and making some reasonable approximations, we show that the $\bar{b} \to \bar{s}$ electroweak penguin amplitude can be determined in a two-step procedure involving i) $\text{BR}(B^+ \to \pi^0 K^+)$, $\text{BR}(B^- \to \pi^0 K^-)$, $\text{BR}(B^+ \to \pi^+ K^0)$ and ii) either $\text{BR}(B^0_d \to \pi^- K^+)$, $\text{BR}(\bar{B}^0 \to \pi^+ K^-)$ or $a_{CP}(t)$ of the mode $B_s \to K^+ K^-$. The determination employing the $B \to \pi K$ transitions is not affected by $SU(3)$-breaking effects. Relating the $\bar{b} \to \bar{s}$ electroweak penguin amplitude to the $\bar{b} \to \bar{d}$ case through $SU(3)$ symmetry arguments, we are in a position to estimate the electroweak penguin uncertainty affecting the extraction of the CKM-angle $\alpha$ by using isospin relations among $B \to \pi\pi$ decays. Our results allow in principle the determination of CKM-phases in a variety of $B$-decays.

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During the last two years there has been a considerable interest in the role of electroweak penguin contributions in non-leptonic $B$-decays. Since the Wilson coefficients of the corresponding local operators increase strongly with the top-quark mass, it has been found \cite{1, 2, 3} that the role of the electroweak penguins can be substantial in certain decays. This is for instance the case of the decay $B^{-} \to K^{-} \Phi$ \cite{1}, which exhibits sizable electroweak penguin effects. More interestingly, there are even some channels, such as $B^{-} \to \pi^{-} \Phi$ \cite{3} and $B_{s} \to \pi^{0} \Phi$ \cite{3}, which are dominated completely by electroweak penguin contributions and which should, thus, allow interesting insights into the physics of the corresponding operators. In this respect, the decay $B_{s} \to \pi^{0} \Phi$ (or similar transitions such as $B_{s} \to \rho^{0} \Phi$) is very promising due to its special isospin-, CKM- and colour-structure \cite{3}. As the branching ratio of this mode is expected to be of $\mathcal{O}(10^{-7})$, it will unfortunately be rather difficult to analyze this decay experimentally.

The electroweak penguin effects discussed in refs. \cite{1, 3} have been confirmed by other authors \cite{4}-\cite{6}.

In the foreseeable future the branching ratios of $\mathcal{O}(10^{-5})$ and possibly $\mathcal{O}(10^{-6})$ will be experimentally available and it is important to ask about the role of electroweak penguin effects in the corresponding channels. In particular, the question arises whether the usual strategies for the determination of the CKM-phases are affected by the presence of the electroweak penguin contributions.

It is evident that the pure tree diagram decays do not receive any contributions from electroweak penguins. Consequently, the very clean method for the determination of the phase $\gamma$ proposed by Gronau and Wyler \cite{7} involving charged $B$-decays of the type $B^{\pm} \to DK^{\pm}$ (see also ref. \cite{8}) remains unaffected by these new contributions. This applies also to the $\gamma$-determination proposed by Aleksan et al. \cite{9} which uses a measurement of the time-dependent decay rates of the transitions $B_{s} \to D_{s}^{\pm} K^{\mp}$. Similar comments apply to the “gold-plated” decay $B_{d} \to \psi K_{S}$ in which the electroweak penguins having the same phase as the leading tree contribution do not obscure a very clean determination of the phase $\beta$.

The situation concerning the $\alpha$-determination by means of the isospin relations among $B \to \pi \pi$ decays proposed by Gronau and London \cite{10} is more involved, however. As pointed out first by Deshpande and He \cite{11}, the impact of electroweak penguins on this determination could be sizable. A closer look \cite{12} shows, however, that this impact is rather small, at most a few %. On the other hand, it is now well accepted \cite{11, 12} that the electroweak penguins should have a considerable impact on the methods proposed last year by Gronau, Hermández, London and Rosner \cite{13}-\cite{18} to measure both weak and strong phases by using $SU(3)$ triangle relations among $B \to \{\pi \pi, \pi K, K \bar{K}\}$ decays and making certain plausible dynamical assumptions (e.g. neglect of annihilation topologies).

While this point has been shown explicitly in ref. \cite{11}, a systematic classification of
electroweak penguins in two-body $B$-decays has been presented in ref. \[12\]. Moreover, in this paper, Gronau et al. have constructed an amplitude quadrangle for $B \rightarrow \pi K$ decays that can be used – at least in principle – to extract the CKM-angle $\gamma$ irrespectively of the presence of electroweak penguins. Unfortunately, from the experimental point of view this approach is rather difficult, because one diagonal of the quadrangle corresponds to the amplitude of the electroweak penguin dominated $B_s$-decay $B_s \rightarrow \pi^0 \eta$ which is expected to have a very small branching ratio at the $\mathcal{O}(10^{-7})$ level. Another $SU(3)$-symmetry based method of extracting $\gamma$, where electroweak penguins are also eliminated, has been presented very recently by Deshpande and He \[19\]. Although this approach using the charged $B$-decays $B^- \rightarrow \{\pi^-, K^0, \pi^0K^-, \eta K^-\}$ and $B^- \rightarrow \pi^-\pi^0$ should be more promising for experimentalists, it is affected by $\eta-\eta'$-mixing and other $SU(3)$-breaking effects and therefore cannot be regarded as a clean measurement of $\gamma$.

In view of this situation, it would be useful to determine the electroweak penguin contributions experimentally. Once this has been achieved, their role in a variety of $B$-decays could be explicitly found. Although some thoughts on this issue have appeared in \[12\], no constructive quantitative method has been proposed there.

Here we would like to suggest a different “philosophy” of applying the $SU(3)$ amplitude relations. In contrast to Gronau et al., we think that these relations are more useful from the phenomenological point of view if one uses the phase $\gamma$ as one of the central inputs. As we have stated above, there are already methods on the market allowing a measurement of this phase in an absolutely clean way without any effect coming from the electroweak penguins. Although these methods (for a review see e.g. ref. \[20\]) are quite difficult from the experimental point of view as well, they should be easier for experimentalists than the quadrangle of ref. \[12\].

At first sight, this new philosophy might appear not useful because one of the goals of the GHLR strategy was precisely the determination of $\gamma$. Yet, as we have seen, this program is difficult to realize without further inputs. On the other hand, as we will show below, once the phase $\gamma$ is used as an input, the electroweak penguin contributions can be straightforwardly determined. This knowledge subsequently allows the determination of CKM-phases in a variety of $B$-decays \[21\]. Consequently, with this new strategy, the GHLR method is resurrected. Moreover, the impact of electroweak penguins on the $\alpha$-determination using $B(\bar{B}) \rightarrow \pi\pi$ decays can be quantitatively estimated.

The central point of this letter is a strategy for determining the $\bar{b} \rightarrow \bar{s}$ electroweak penguin amplitude from experimental data. Whereas electroweak $\bar{b} \rightarrow \bar{d}$ penguins are expected to be rather small in the case of $B$-decays into two-pion final states, the corresponding $\bar{b} \rightarrow \bar{s}$ electroweak penguins are expected to affect $B \rightarrow \pi K$ transitions significantly \[11, 12\]. Besides the knowledge of the CKM-angle $\gamma$ our approach involves certain approximations that will be discussed in a moment.

Let us begin our analysis by considering the $B$-meson decays $B^+ \rightarrow \pi^+K^0$, $B^+ \rightarrow$
\( \pi^0 K^+, B^0_d \rightarrow \pi^- K^+ \) and, moreover, the \( B_s \)-transition \( B_s \rightarrow K^+ K^- \). Applying the \( SU(3) \) flavour symmetry of strong interactions and using the same notation as Gronau, Hernández, London and Rosner in ref. \[12\], the corresponding decay amplitudes take the form

\[
\begin{align*}
A(B^+ \rightarrow \pi^+ K^0) &= P' + c_d P'_{\text{EW}}^C \\
A(B^+ \rightarrow \pi^0 K^+) &= -\frac{1}{\sqrt{2}} [P' + T' + (c_u - c_d)P'_{\text{EW}} + C' + c_u P'_{\text{EW}}^C] \\
A(B^0_d \rightarrow \pi^- K^+) &= -(P' + T' + c_u P'_{\text{EW}}) \\
A(B^0_s \rightarrow K^+ K^-) &= -(P' + T' + c_u P'_{\text{EW}}),
\end{align*}
\]

(1)

where \( T' \) and \( C' \) describe colour-allowed and colour-suppressed \( \bar{b} \rightarrow \bar{u}u\bar{s} \) tree-level amplitudes, respectively, \( P' \) denotes \( \bar{b} \rightarrow \bar{s} \) QCD penguins, \( P'_{\text{EW}} \) is related to colour-allowed \( \bar{b} \rightarrow \bar{s} \) electroweak penguins and \( P'_{\text{EW}}^C \) to colour-suppressed electroweak penguins. Following the plausible arguments of Gronau et al. outlined in refs. \[12, 18\], we expect the following hierarchy of the different topologies given in eq. (1):

\[
\begin{align*}
1 : |P'| \\
\mathcal{O}(\lambda) : |T'|, |P'_{\text{EW}}| \\
\mathcal{O}(\lambda^2) : |C'|, |P'_{\text{EW}}^C|.
\end{align*}
\]

(2)

Note that the parameter \( \bar{\lambda} = \mathcal{O}(0.2) \) appearing in these relations is not related to the usual Wolfenstein parameter \( \lambda \). It has been introduced by Gronau et al. just to keep track of the expected orders of magnitudes. In eq. (2), we have named this quantity \( \bar{\lambda} \) in order not to confuse it with Wolfenstein’s \( \lambda \).

Consequently, if we neglect the colour-suppressed electroweak penguin contributions \( P'_{\text{EW}}^C \), which will simplify our analysis considerably, the \( C' \) amplitudes have to be neglected as well since both topologies are expected to be of the same order in \( \bar{\lambda} \). Within this approximation, we obtain

\[
\begin{align*}
A(B^+ \rightarrow \pi^+ K^0) &= P' \\
A(B^+ \rightarrow \pi^0 K^+) &= -\frac{1}{\sqrt{2}} [P' + T' + (c_u - c_d)P'_{\text{EW}}] \\
A(B^0_d \rightarrow \pi^- K^+) &= -(P' + T') \\
A(B^0_s \rightarrow K^+ K^-) &= -(P' + T').
\end{align*}
\]

(3)

Note that exchange and annihilation-type topologies, which have not been written explicitly in eq. (1), have also to be neglected within this approximation since they are expected to be \( \sim \mathcal{O}(\lambda^2) \) \[12, 18\].

Due to the special CKM-structure of the \( \bar{b} \rightarrow \bar{s} \) penguins, we have \[22\]

\[
\begin{align*}
P' &= |P'| e^{i\delta} e^{i\pi} = \bar{P}' \\
P'_{\text{EW}} &= |P'_{\text{EW}}| e^{i\delta_{\text{EW}}} e^{i\pi} = \bar{P}'_{\text{EW}},
\end{align*}
\]

(4)
where the phases $\delta$ are CP-conserving strong final state interaction phases and $\pi$ represents the CP-violating weak phase.

Let us next rescale the transition amplitudes of the decays $B^+ \to \pi^+K^0$ and $B^+ \to \pi^0K^+$ by a factor $|P'|$. Taking furthermore into account the relation

$$T' = e^{-2i\gamma}T', \tag{5}$$

one can easily draw Fig. 1 representing the first two decay amplitudes given in eq. (3) and those of the corresponding CP-conjugate modes. Looking at this figure implies that the $\bar{b} \to \bar{s}$ electroweak penguin amplitude $(c_u - c_d)P_{EW}'$ can be constructed by measuring the rates of the decays $B^+ \to \pi^0K^+$, $B^- \to \pi^0K^-$ and $B^+ \to \pi^+K^0$, provided both the amplitude

$$z \equiv \frac{T'}{|P'|} \tag{6}$$

and the CKM-angle $\gamma$ are known. Note that the quantity $z$ is given in the $x'-y'$-frame defined in Fig. 1 by the expression

$$z = e^{-i\omega} \frac{|T'|}{|P'|}. \tag{7}$$

The phase $\delta_{P'}$ determining the orientation of this frame cannot be fixed. However, concerning our phenomenological applications this quantity is irrelevant.

In the following discussion we shall present two different approaches of determining $z$ making use of the decays $B^+_d \to \pi^-K^+$ ($B^+_d \to \pi^+K^-$) and $B_s \to K^+K^-$, respectively. Let us describe the method involving the $B_d$-modes first. Taking into account both eqs. (4) and (5) and the expression for the amplitude $A(B^+_d \to \pi^-K^+)$ given in eq. (3), we can easily construct the two triangles shown in Fig. 2. As can be seen from this figure, if the CKM-angle $\gamma$ is known, the amplitude $z = T'/|P'|$ can be determined by measuring the rates of the decays $B^+_d \to \pi^-K^+$, $B^+_d \to \pi^+K^-$ and $B^+ \to \pi^+K^0$ which fixes $|P'|$. Note that this method requires no time-dependent measurements and that all involved branching ratios should be of $\mathcal{O}(10^{-5})$.

Let us now describe another independent approach of determining this quantity which is more formal and requires a measurement of the time-dependent CP-violating asymmetry of the mode $B_s \to K^+K^-$. Since this transition is the decay of a neutral $B_s$-meson into a CP-eigenstate, the corresponding CP asymmetry is given by

$$a_{CP}(t) \equiv \frac{\Gamma(B^0_s(t) \to K^+K^-) - \Gamma(\bar{B}^0_s(t) \to K^+K^-)}{\Gamma(B^0_s(t) \to K^+K^-) + \Gamma(\bar{B}^0_s(t) \to K^+K^-)} =$$

$$A_{CP}^{dir}(B_s \to K^+K^-) \cos(\Delta M_s t) + A_{CP}^{mix-ind}(B_s \to K^+K^-) \sin(\Delta M_s t), \tag{8}$$

where we have separated the direct CP-violating contributions, which are proportional to

$$A_{CP}^{dir}(B_s \to K^+K^-) \equiv \frac{1 - |s_{K^+K^-}^{(s)}|^2}{1 + |s_{K^+K^-}^{(s)}|^2}, \tag{9}$$

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from those describing mixing-induced CP violation which are characterized by

\[ \mathcal{A}_{\text{CP}}^{\text{mix-ind}}(B_s \to K^+ K^-) \equiv \frac{2 \text{Im} \xi_{K^+ K^-}^{(s)}}{1 + \left| \xi_{K^+ K^-}^{(s)} \right|^2}. \]  

(10)

In eq. (8), \( \Delta M_s \) denotes the mass splitting of the physical \( B_s^0 - \bar{B}_s^0 \)-mixing eigenstates. The quantity \( \xi_{K^+ K^-}^{(s)} \) containing essentially all the information needed to evaluate the asymmetries (9) and (10) is given by

\[ \xi_{K^+ K^-}^{(s)} = -e^{-i0} \frac{A(\bar{B}_s^0 \to K^+ K^-)}{A(B_s^0 \to K^+ K^-)}, \]  

(11)

where the factor \( -e^{-i0} \) is related to \( B_s^0 - \bar{B}_s^0 \)-mixing. Using eqs. (3), (4) and writing the colour-allowed tree-amplitude \( T' \) in the form

\[ T' = |T'| e^{i\delta_{T'}} e^{i\gamma}, \]  

(12)

where \( \delta_{T'} \) is a strong phase shift and \( \gamma \) is the usual CKM-angle, we obtain

\[
A(B_s^0 \to K^+ K^-) = -|P'| e^{i\delta_{T'}} \left[ e^{i\pi} + \frac{|T'|}{|P'|} e^{-i\omega} \right],
\]

\[
A(\bar{B}_s^0 \to K^+ K^-) = -|P'| e^{i\delta_{T'}} \left[ e^{i\pi} + \left( \frac{|T'|}{|P'|} e^{-i\omega} \right) e^{-2i\gamma} \right],
\]

(13)

where \( \omega \) is given by

\[ \omega = \delta_{pr} - \delta_{T'} - \gamma. \]  

(14)

Consequently, the quantity \( |T'|/|P'| e^{-i\omega} \), which describes the amplitude \( z = T'/|P'| \) in the \( x'-y' \)-frame specified in Fig. 1, is related to \( \xi_{K^+ K^-}^{(s)} \) through the expression

\[ \frac{|T'|}{|P'|} e^{-i\omega} = \frac{1 + \xi_{K^+ K^-}^{(s)}}{e^{-2i\gamma} + \xi_{K^+ K^-}^{(s)}}. \]  

(15)

If one measures the time-dependent CP asymmetry of the decay \( B_s \to K^+ K^- \), which is probably a rather difficult task for experimentalists due to the large \( B_s^0 - \bar{B}_s^0 \)-mixing parameter \( x_s \equiv \tau_{B_s} \Delta M_s \gtrsim 10 \), the quantity \( \xi_{K^+ K^-}^{(s)} \) can be determined by using eqs. (8), (4) and (14) up to a two-fold ambiguity. This ambiguity can be resolved in principle, if one takes into account the life-time splitting of the neutral \( B_s \)-meson system which has been neglected in eqs. (3)-(14) (for a discussion of this point see e.g. ref. [9]). Inserting \( \xi_{K^+ K^-}^{(s)} \) extracted this way into the expression (15), the quantity \( z \) appearing in Fig. 1 can be determined provided the CKM-angle \( \gamma \) is known, for example, by applying the approach proposed by Gronau and Wyler [7]. In contrast to the method shown in Fig. 2, the approach using eq. (15) to determine \( z \) suffers from \( SU(3) \)-breaking corrections that
are related to the spectator $s$-quark of the decaying $B_s$-meson. A reliable theoretical treatment of these corrections is unfortunately not possible at present.

Let us note that one can extract in principle both $\gamma$ and the amplitude $z$ simultaneously by combining Fig. 2 with eq. (15). This approach requires both time-independent measurements of the branching ratios $BR(B_d^0 \to \pi^- K^+)$, $BR(\bar{B}_d^0 \to \pi^+ K^-)$, $BR(B^+ \to \pi^+ K^0) = BR(B^- \to \pi^- K^0)$ and a time-dependent measurement of the CP asymmetry $a_{CP}(t)$ of the decay $B_s \to K^+ K^-$ that has been defined by eq. (8). From the experimental point of view this simultaneous approach seems, however, to be quite difficult.

Using the amplitude $z$ determined by applying either the approach shown in Fig. 2 or the time-dependent CP asymmetry of the decay $B_s \to K^+ K^-$, the $\bar{b} \to \bar{s}$ electroweak penguin amplitude $(c_u - c_d) P_{EW}'$ can be extracted with the help of Fig. 1. If one follows Fig. 2 to determine $z$ and defines $(c_u - c_d) P_{EW}'$ as the electroweak penguin contribution to the decays $B^\pm \to \pi^0 K^\pm$, $SU(3)$-breaking does not affect the determination of this amplitude, since we have only to deal with $B_{u,d}$ decays into $\pi K$ final states. Consequently, besides the corrections related to the neglect of the $C'$ and $P'_{EW}$ topologies (see eq. (2)), there are only isospin-breaking corrections present in this approach.

Since electroweak penguins are dominated to a good approximation by internal top-quark exchanges – in contrast to the situation concerning QCD penguins – the $\bar{b} \to \bar{d}$ electroweak penguin amplitude $(c_u - c_d) P_{EW}$ is related in the limit of an exact $SU(3)$ flavour symmetry of strong interactions to the corresponding $\bar{b} \to \bar{s}$ amplitude through the relation

$$(c_u - c_d) P_{EW} = -\lambda R_t e^{-i\beta} (c_u - c_d) P_{EW}'. \quad (16)$$

Here, $\lambda$ is the usual Wolfenstein parameter (in contrast to the parameter $\bar{\lambda}$ in eq. (2)) and $R_t$ represents the side of the unitarity triangle that is related to $B_d^0 - \bar{B}_d^0$ mixing. It is given by the CKM-combination

$$R_t \equiv \frac{1}{\lambda} \frac{|V_{td}|}{|V_{cb}|}. \quad (17)$$

From present experimental data, we expect $R_t$ being of $O(1)$.

The $\bar{b} \to \bar{d}$ electroweak penguin amplitude being $O(\lambda^2)$ is mainly interesting in connection with a clean determination of the CKM-angle $\alpha$ by using isospin relations among $B \to \pi \pi$ decays. As we have pointed out already, although electroweak penguins are expected to lead to small effects in this case it is an interesting and important question to control the corresponding corrections quantitatively.

In Fig. 3 we have drawn the $B(\bar{B}) \to \pi \pi$ isospin triangles in a way which differs from the one given in ref. in order to illustrate the electroweak penguin corrections more clearly. In particular we have rotated the $A(\bar{B} \to \pi \pi)$ amplitudes by the phase factor $e^{-2i\beta}$, which allows to rotate $\bar{P}_{EW}^{(C)}$ back to $P_{EW}^{(C)}$:

$$e^{-2i\beta} \bar{P}_{EW}^{(C)} = P_{EW}^{(C)}. \quad (18)$$
This equation expresses the fact that the electroweak penguins are dominated by internal top-quark exchanges. The angle $\phi$ appearing in Fig. 3 fixing the relative orientation of the $B \to \pi\pi$ and $\bar{B} \to \pi\pi$ triangles is measured directly by the mixing-induced CP asymmetry of the decay $B_d \to \pi^+\pi^-$ given by

$$A_{CP}^{\text{mix-ind}}(B_d \to \pi^+\pi^-) = -\frac{2|A(\bar{B}_d^0 \to \pi^+\pi^-)||A(B^0_d \to \pi^+\pi^-)|}{|A(\bar{B}_d^0 \to \pi^+\pi^-)|^2 + |A(B^0_d \to \pi^+\pi^-)|^2} \sin \phi$$

which enters a formula for the corresponding time-dependent CP asymmetry in an analogous way as in eq. (8). Note that we would have $A_{CP}^{\text{mix-ind}}(B_d \to \pi^+\pi^-) = -\sin 2\alpha$ and, thus, $\phi = 2\alpha$, if we neglected the penguin contributions to the decay $B_d \to \pi^+\pi^-$ completely.

Consequently, measuring both the $B(\bar{B}) \to \pi\pi$ rates and the asymmetry $A_{CP}^{\text{mix-ind}}(B_d \to \pi^+\pi^-)$, the solid and dashed triangles shown in Fig. 3 can be constructed and the angle $\tilde{\alpha}$ can be determined. This approach differs from the original proposal of Gronau and London [10] (see also ref. [12]). Applying elementary trigonometry, we find that the CKM-angle $\alpha$ is related to $\tilde{\alpha}$ through

$$\alpha = \tilde{\alpha} + \Delta \alpha,$$

where $\Delta \alpha$ is given by

$$\Delta \alpha = r \sin \alpha \cos(\rho - \alpha) + O(r^2)$$

with

$$r \equiv \frac{|(c_u - c_d)(P_{EW} + P_{EW}^C)|}{|T + C|}.$$  

The phase $\rho$ is defined by

$$(c_u - c_d)(P_{EW} + P_{EW}^C) \equiv e^{i\rho} r(T + C).$$

While it has been shown in ref. [12] that $\Delta \alpha = O(r)$, we have calculated this correction quantitatively in eq. (21).

Taking into account that $P_{EW}^C/P_{EW}, C/T = O(\lambda)$ [12, 18] and employing both eq. (16) and the $SU(3)$-relation $T' = r_u T$ with $r_u \equiv V_{us}/V_{ud} \approx \lambda$ [13-18], we find

$$r \approx \lambda r_u R_u \frac{|(c_u - c_d)P_{EW}'|}{|T'|}$$

$$\rho' \approx \rho' - \beta + \pi,$$

where $\rho'$ is a phase that is related to the $\bar{b} \to \bar{s}$ electroweak penguin amplitude and that is defined in analogy to eq. (23) through

$$(c_u - c_d)P_{EW}' \equiv e^{i\rho'} \frac{|(c_u - c_d)P_{EW}'|}{|T'|} T' = e^{i\rho'} \frac{|(c_u - c_d)P_{EW}'|}{|z|} z.$$
Strategies for the determination of the quantity $z$ have been discussed above (see Fig. 2 or eq. (15)).

Consequently, inserting (24) and (25) into (21) and using the relation $\gamma = \pi - \alpha - \beta$, we obtain

$$\Delta \alpha \approx \lambda r_u R_t \frac{|(c_u - c_d)P_{EW}^\prime|}{|T'|} \sin \tilde{\alpha} \cos(\rho + \gamma).$$

(27)

Note that replacing $\sin \alpha$ appearing in eq. (21) by $\sin \tilde{\alpha}$ leads to corrections of $\mathcal{O}(r^2)$ which have been neglected in eq. (27). The nice feature of this equation is related to the fact that it includes only quantities that can be determined by using Figs. 1–3 ($\gamma$ is one of our inputs). Therefore, using this expression we are in a position to estimate the electroweak penguin contribution to the value of $\tilde{\alpha}$ in a quantitative way and consequently we can extract the CKM-angle $\alpha$ with the help of eq. (20).

At this point a discussion of $SU(3)$-breaking effects seems to be in order. Whereas factorizable $SU(3)$-breaking affecting the relation between $T'$ and $T$ can be included straightforwardly by setting $r_u = \lambda f_K/f_\pi$ [13]–[18], such corrections on eq. (14) are more difficult to estimate as they involve not only meson decay constants but also hadronic form factors. Approximately, factorizable $SU(3)$-breaking can be taken into account in this equation by multiplying its r.h.s. by the factor $F_{B\pi}(0;0^+)/F_{BK}(0;0^+)$, where $F_{B\pi}(0;0^+)$ and $F_{BK}(0;0^+)$ are form factors parametrizing the hadronic quark-current matrix elements $\langle \pi^+ |(\bar{b}d)_{V-A}|B^+ \rangle$ and $\langle K^+ |(\bar{b}s)_{V-A}|B^+ \rangle$, respectively [24]. Combining these considerations, we obtain the following expression for $\Delta \alpha$:

$$\Delta \alpha \approx \left[ \frac{f_K}{f_\pi} \frac{F_{B\pi}(0;0^+)}{F_{BK}(0;0^+)} \right] \left[ \lambda^2 R_t \frac{|(c_u - c_d)P_{EW}^\prime|}{|T'|} \sin \tilde{\alpha} \cos(\rho + \gamma) \right],$$

(28)

which includes factorizable $SU(3)$-breaking in an approximate way. At present there is no reliable theoretical technique available to calculate non-factorizable $SU(3)$-breaking corrections to this expression.
Let us summarize briefly the main results of this letter:

- Using the CKM-phase $\gamma$ as an input and making some reasonable approximations, we have shown that the $\bar{b} \to \bar{s}$ electroweak penguin amplitude $(c_u - c_d)P'_{\text{EW}}$ can be straightforwardly determined.

- To this end, one has to measure the three branching ratios $\text{BR}(B^+ \to \pi^0 K^+)$, $\text{BR}(B^- \to \pi^0 K^-)$, $\text{BR}(B^+ \to \pi^+ K^0) = \text{BR}(B^- \to \pi^- \bar{K}^0) \propto |P'|^2$ and has, moreover, to determine the amplitude $z \equiv T'/|P'|$.

- We have presented two different strategies for extracting the quantity $z$:
  - A geometrical construction using the branching ratios $\text{BR}(B^0_d \to \pi^- K^+)$ and $\text{BR}(B^0_d \to \pi^+ K^-)$.
  - A more formal method using the time-dependent CP asymmetry of the mode $B_s \to K^+ K^-$. Whereas the latter approach suffers from $SU(3)$-breaking corrections that are related to the spectator $s$-quark of the decaying $B_s$-meson, there is no $SU(3)$-breaking present in the former one and in the corresponding determination of $(c_u - c_d)P'_{\text{EW}}$; if one defines this amplitude as the electroweak penguin contribution to the decays $B^\pm \to \pi^0 K^\pm$.

- Note that all branching ratios involved are expected to be of $\mathcal{O}(10^{-5})$ and should be available in the foreseeable future. A measurement of the time-evolution of the decay $B_s \to K^+ K^-$ will, however, be rather difficult.

- As electroweak penguins are dominated by internal top-quark exchanges, we obtain a simple $SU(3)$-relation between the $\bar{b} \to \bar{s}$ and $\bar{b} \to \bar{d}$ electroweak penguin amplitudes.

- Using this relation and the experimentally determined amplitude $(c_u - c_d)P'_{\text{EW}}$ we are in a position to estimate the electroweak penguin contribution $\Delta \alpha$ to the angle $\tilde{\alpha} = \alpha - \Delta \alpha$. This angle can be determined following the approach of Gronau and London [10] by measuring the branching ratios of the decays $B(\bar{B}) \to \pi \pi$ and the CP asymmetry $A_{\text{CP}}^{\text{mix-ind}}(B_d \to \pi^+ \pi^-)$. Therefore, the CKM-angle $\alpha$ can be extracted and the electroweak penguin corrections are – at least in principle – under control.

The possibility of determining $P'_{\text{EW}}$ and $P_{\text{EW}}$ experimentally as suggested here opens the door for a quantitative study of electroweak penguin effects in other $B$-decays. We will return to this in a separate publication [21].
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Figure Captions

Fig. 1: A geometrical strategy for determining the $\bar{b} \rightarrow \bar{s}$ electroweak penguin amplitude $(c_u - c_d)P'_{\text{EW}}$.

Fig. 2: The determination of the amplitude $z$ by using the modes $B_d \rightarrow \pi^- K^+$, $B_d^0 \rightarrow \pi^+ K^-$ and $B^+ \rightarrow \pi^+ K^0$ to fix $|P'|$.

Fig. 3: The determination of the angle $\tilde{\alpha}$ by using $B(\bar{B}) \rightarrow \pi \pi$ decays and its relation to the CKM-angle $\alpha$. 
Figure 1:
Figure 2:
Figure 3:
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