U(1) \(_Q\) invariance and SU(3)\(_C\) \(\otimes\) SU(3)\(_L\) \(\otimes\) U(1)\(_X\) models with \(\beta\) arbitrary

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Abstract

Using the U(1)\(_Q\) invariance, the photon eigenstate and matching gauge coupling constants in SU(3)\(_C\) \(\otimes\) SU(3)\(_L\) \(\otimes\) U(1)\(_X\) models with \(\beta\) arbitrary are given. The mass matrix of neutral gauge bosons is exactly diagonalized, and the photon eigenstate is independent on the symmetry breaking parameters - VEV’s of Higgs scalars. By obtaining the electromagnetic vertex, the model is embedded naturally into the standard model.

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1 Introduction

The detection of neutrino oscillation [1] experimentally indicates that neutrinos are massive particles and that flavour lepton number is not conserved. Since in the standard model (SM), neutrinos are massless and flavour lepton number is conserved. The neutrino oscillation experiments are a clear sign that the SM has to be extended.

A very common alternative to solve some of the problems of the SM consists of enlarging the group of gauge symmetry, where the larger group embeds properly the SM. For instance, the SU(5) grand unification model [2] can unify the interactions and predicts the electric charge quantization, while the group E\(_6\) can also unify the interactions and might explain the masses of the neutrinos [3], and etc. [4]. Nevertheless, such models cannot explain the generations number problem.

A very interesting alternative to explain the origin of generations comes from the cancelation of quiral anomalies [5]. In particular, the models with gauge group \(G_{331} = SU(3)\(_C\) \(\otimes\) SU(3)\(_L\) \(\otimes\) U(1)\(_X\)\), also called 3-3-1 models [6, 7, 8], arise as a possible solution to this puzzle, since some of such models require the three generations in order to cancel chiral anomalies completely. An additional motivation to study this kind of models comes from the fact that they can also predict the charge quantization [9].
In the literature about the 3-3-1 models, it is known that the matching of gauge coupling constants at the SU(3)$_L \otimes$ U(1)$_X$ breaking is dependent on the constraints among the VEV’s \cite{6}. In addition, the independence on the VEV’s of the photon eigenstate and mass has not been explained yet \cite{10}.

In this paper, we have pointed out that the photon eigenstate is independent on the VEV’s; and the matching of gauge coupling constants is not dependent on VEV’s structure.

The paper is organized as follows. In Sec.2 we recall some features of the 3-3-1 models with $\beta$ arbitrary and study the mass Lagrangian of the neutral gauge bosons, the photon eigenstate and mass. Matching gauge coupling constants and diagonalizing the neutral bosons gauge mass matrix are obtained in Sec.3. Finally, our conclusions are summarized in the last section.

2 Photon eigenstate

The 3-3-1 model with $\beta$ arbitrary \cite{11} has the electric charge operator in the following form

$$Q = T_3 + \beta T_8 + X, \quad (2.1)$$

where $T_3 = \lambda_3/2, T_8 = \lambda_8/2$ are the SU(3)$_L$ gauge charges, and $X$ is the U(1)$_X$ gauge charge.

Under the gauge symmetry $G_{331}$, the fermion representations are given in the triplets 3, antitriplets $3^*$, and singlets 1 (for the right-handed counterparts) of the SU(3)$_L$ group. In order to cancel anomalies, the same number of fermion triplets and antitriplets must be present \cite{11}.

A triplet of the SU(3)$_L$ group is composed from a doublet 2 and a singlet 1 of the SU(2)$_L$ group of the SM, therefore it is decomposed as follows

$$\begin{pmatrix} u, & d, & s \end{pmatrix}^T = \begin{pmatrix} u, & d \end{pmatrix}^T \oplus s, \quad (2.2)$$

or

$$\begin{pmatrix} 3, & X \end{pmatrix} = \begin{pmatrix} 2, & X \end{pmatrix} \oplus \begin{pmatrix} 1, & X \end{pmatrix}, \quad (2.3)$$

where $u, d,$ and $s$ denote the first, the second members of the doublets and of the singlets, respectively. In the case of an antitriplet $3^*$, it is also decomposed into a antidoublet $2^*$ and a singlet of the SU(2)$_L$ group

$$\begin{pmatrix} d, & -u, & s' \end{pmatrix}^T = \begin{pmatrix} d, & -u \end{pmatrix}^T \oplus s', \quad (2.4)$$

or

$$\begin{pmatrix} 3^*, & -X \end{pmatrix} = \begin{pmatrix} 2^*, & -X \end{pmatrix} \oplus \begin{pmatrix} 1, & -X \end{pmatrix}. \quad (2.5)$$

To find the hyper-charge $Y$ of the doublets and the singlets, we should use $Y = 2(\beta T_8 + X)$ which is obtained directly from (2.1).

The spontaneous symmetry breaking from the $G_{331}$ to the $G_{SM}$ group of the SM \cite{12} will allow the singlet member separated from the triplets or antitriplets and get mass. This is
archived by a Higgs scalar triplet $\chi$ with the VEV as follows

$$
\langle \chi \rangle^T = \left( 0, 0, \frac{v_s}{\sqrt{2}} \right).
$$

Then the neutral gauge bosons of the theory get mass from

$$
\mathcal{L}_{mass}^\chi = (D^H_\mu \langle \chi \rangle) + (D^{H*}_\mu \langle \chi \rangle),
$$

where subscripts $H$ denotes diagonal part of the covariant derivative

$$
D^H_\mu = \partial_\mu + igT_3W^3_\mu + igT_8W^8_\mu + igX_T9X_B\mu.
$$

Here $g$ and $g_X$ are the gauge coupling constants of the SU(3)$_L$ and U(1)$_X$ groups, respectively. $X_\chi$ is the U(1)$_X$ charge of the $\chi$ Higgs scalar. $T_9 = diag(1,1,1)/\sqrt{6}$ is chosen such that $Tr(T_aT_b) = \delta_{ab}/2$; $a,b = 1,2,\ldots,9$. Substituting the charge $X_\chi$ from (2.1) into (2.8), we get

$$
D^H_\mu = \partial_\mu + igT_3W^3_\mu + igT_8W^8_\mu + igX/\sqrt{6}B_\mu (Q - T_3 - \beta T_8).
$$

The $U(1)_Q$ invariance requires $Q\langle \chi \rangle = 0$, therefore we get

$$
D^H_\mu \langle \chi \rangle = \frac{igv_s}{2\sqrt{2}} \left( -\frac{2}{\sqrt{3}}W^8_\mu + \frac{2t}{\sqrt{6}}B_\mu (Q - T_3 - \beta T_8) \right)
$$

$$
= \frac{igv_s}{2\sqrt{2}} \Delta^3_\mu,
$$

where the following notations are used

$$
\Delta^3_\mu \equiv \left( -\frac{2}{\sqrt{3}}W^8_\mu + \frac{2t}{\sqrt{6}}B_\mu \beta \right),
$$

$$
t \equiv g_X/g.
$$

Hence

$$
\mathcal{L}_{mass}^\chi = \frac{g^2v^2_s}{8} \Delta^2_3,
$$

where

$$
\Delta^2_3 = \Delta^3_\mu \Delta^\mu_3.
$$

In the second step of symmetry breaking [5, 6, 12], the $G_{SM}$ group must be decomposed into the SU(3)$_C \otimes U(1)_Q$ group, two $\eta, \rho$ Higgs triplets are introduced with following VEV’s

$$
\langle \eta \rangle^T = \left( \frac{v_H}{\sqrt{2}}, 0, 0 \right),
$$

$$
\langle \rho \rangle^T = \left( 0, \frac{v_H}{\sqrt{2}}, 0 \right).
$$
The neutral gauge bosons also gain mass from two Lagrangians given by

\[ \mathcal{L}_{\text{mass}}^\eta = (D_H^\mu(\eta))^+(D_H^\mu(\eta)), \]
\[ \mathcal{L}_{\text{mass}}^\rho = (D_H^\mu(\rho))^+(D_H^\mu(\rho)). \]

(2.15)

(2.16)

Noting that \( Q(\eta) = Q(\rho) = 0 \), we get

\[ D_H^\mu(\eta) = \frac{igv_u}{2\sqrt{2}} \left[ W_3^\mu + \frac{1}{\sqrt{3}} W_8^\mu + \frac{2t}{\sqrt{6}} B_\mu \left( -\frac{1}{2} - \frac{\beta}{2\sqrt{3}} \right) \right] \]
\[ = \frac{igv_u}{2\sqrt{2}} \Delta_{1\mu}, \]

(2.17)

\[ D_H^\mu(\rho) = \frac{igv_d}{2\sqrt{2}} \left[ -W_3^\mu + \frac{1}{\sqrt{3}} W_8^\mu + \frac{2t}{\sqrt{6}} B_\mu \left( \frac{1}{2} - \frac{\beta}{2\sqrt{3}} \right) \right] \]
\[ = \frac{igv_d}{2\sqrt{2}} \Delta_{2\mu}, \]

(2.18)

Here

\[ \Delta_{1\mu} \equiv \left[ W_3^\mu + \frac{1}{\sqrt{3}} W_8^\mu + \frac{2t}{\sqrt{6}} B_\mu \left( -\frac{1}{2} - \frac{\beta}{2\sqrt{3}} \right) \right], \]

(2.19)

\[ \Delta_{2\mu} \equiv \left[ -W_3^\mu + \frac{1}{\sqrt{3}} W_8^\mu + \frac{2t}{\sqrt{6}} B_\mu \left( \frac{1}{2} - \frac{\beta}{2\sqrt{3}} \right) \right]. \]

(2.20)

Hence

\[ \mathcal{L}_{\text{mass}}^\eta = \frac{g^2v_u^2}{8} \Delta_{1\mu}^2, \]
\[ \mathcal{L}_{\text{mass}}^\rho = \frac{g^2v_d^2}{8} \Delta_{2\mu}^2. \]

(2.21)

Finally, the mass Lagrangian of the neutral gauge bosons is given by

\[ \mathcal{L}_{\text{mass}}^{\text{NCC}} = \mathcal{L}_{\text{mass}}^\eta + \mathcal{L}_{\text{mass}}^\rho + \mathcal{L}_{\text{mass}}^\chi, \]
\[ = \frac{g^2}{8} \left( v_u^2 \Delta_{1}^2 + v_d^2 \Delta_{2}^2 + v_s^2 \Delta_{3}^2 \right). \]

(2.22)

In general, any 3-3-1 model need to have three Higgs triplets for breaking the \( G_{331} \) group into the \( SU(3)_C \otimes U(1)_Q \) group. However, some 3-3-1 models need less than three Higgs [13]. For our purpose in obtaining the mass matrix of the neutral gauge bosons, this corresponds to vanishing of \( v_u \) or \( v_d \). In the case with more than three Higgs triplets, one just makes the following appropriate replaces

\[ v_u^2 \rightarrow v_u^2 + v_{u1}^2 + v_{u2}^2 + ..., \]
\[ v_d^2 \rightarrow v_d^2 + v_{d1}^2 + v_{d2}^2 + ..., \]
\[ v_s^2 \rightarrow v_s^2 + v_{s1}^2 + v_{s2}^2 + ... \]

(2.23)

where \( v_{ui}, v_{dj}, v_{sk} \) are the VEV’s of the additional Higgs triplets. They belong to the up, down, and singlet members, respectively. This also remains correctly for the cases, if a Higgs
triplet has two neutral members with the non-zero VEV’s \([13]\), and for models with Higgs antitriplets.

In some models, for example the minimal 3-3-1 model \([6]\), to give mass to all leptons, we have to introduce a Higgs sextet. Let us denote the Higgs sextet by \(\Gamma_{ij}\). Then, the mass Lagrangian will get an addition

\[
\mathcal{L}_{\text{mass}}^\Gamma = (D_H^\mu \langle \Gamma \rangle_{ij})^+ (D_H^\mu \langle \Gamma \rangle_{ij}),
\]  

(2.24)

where

\[
D_H^\mu \langle \Gamma \rangle_{ij} = ig \left[ (W_3^\mu T_3 + W_8^\mu T_8)^k \langle \Gamma \rangle_{kj} + (W_3^\mu T_3 + W_8^\mu T_8)^k \langle \Gamma \rangle_{ki} \right] + \frac{igX}{\sqrt{6}} B_\mu \langle \Gamma \rangle_{ij}
\]  

\[
= ig \left[ \left( W_3^\mu T_3 + W_8^\mu T_8 + \frac{t}{\sqrt{6}}(Q - T_3 - \beta T_8)B_\mu \right)_{ij} \right]_{ij} + \frac{igX}{\sqrt{6}} B_\mu \langle \Gamma \rangle_{ij}
\]  

\[
= ig \left( \left\langle \Gamma \right\rangle_{12}(\Delta_{1\mu} + \Delta_{2\mu}) \left\langle \Gamma \right\rangle_{13}(\Delta_{1\mu} + \Delta_{3\mu}) \right)
\]  

(2.25)

It is easy to verify that

\[
\Delta_{1\mu} + \Delta_{2\mu} + \Delta_{3\mu} = 0.
\]  

(2.26)

Finally, the mass term for neutral gauge bosons from sextet is given by

\[
\mathcal{L}_{\text{mass}}^\Gamma = \frac{g}{\sqrt{6}} \left\{ [2\langle \Gamma \rangle_{11}^2 + \langle \Gamma \rangle_{23}^2] \Delta_1^2 + [2\langle \Gamma \rangle_{22}^2 + \langle \Gamma \rangle_{13}^2] \Delta_2^2 + [2\langle \Gamma \rangle_{33}^2 + \langle \Gamma \rangle_{12}^2] \Delta_3^2 \right\}.
\]  

(2.27)

Note that, only neutral members in sextet can have non-zero VEV’s. From (2.27) we see that the general form of mass Lagrangian (2.22) is not changed by adding \(\mathcal{L}_{\text{mass}}^\Gamma\).

In order to generate the fermion masses, the Higgs bosons should lie in either the triplet, antitriplet, sextet, or singlet representation of \(SU(3)_L\) \([12]\). In later case, the singlet must be neutral, and it does not give mass to gauge bosons. So, we conclude that for any 3-3-1 model, the mass matrix of the neutral gauge bosons always has the form given in (2.22).

The mass Lagrangian (2.22) can be rewritten

\[
\mathcal{L}_{\text{mass}} = \frac{1}{2} V^T M^2 V,
\]  

(2.28)

where \(V^T = (W^3, W^8, B)\), and

\[
M^2 = \frac{1}{4} g^2 \left( \begin{array}{ccc} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{array} \right),
\]  

(2.29)
with
\begin{align*}
m_{11} &= v_u^2 + v_d^2, \\
m_{12} &= \frac{1}{\sqrt{3}}(v_u^2 - v_d^2), \\
m_{13} &= \frac{t}{\sqrt{6}} \left[ v_u^2 \left( -1 - \beta \frac{1}{\sqrt{3}} \right) - v_d^2 \left( 1 - \frac{\beta}{\sqrt{3}} \right) \right], \\
m_{22} &= \frac{1}{3} \left( v_u^2 + v_d^2 + 4v_s^2 \right), \\
m_{23} &= \frac{t}{3\sqrt{2}} \left[ v_u^2 \left( -1 - \beta \frac{1}{\sqrt{3}} \right) + v_d^2 \left( 1 - \frac{\beta}{\sqrt{3}} \right) - v_s^2 \frac{4\beta}{\sqrt{3}} \right], \\
m_{33} &= \frac{t^2}{6} \left[ v_u^2 \left( -1 - \beta \frac{1}{\sqrt{3}} \right)^2 + v_d^2 \left( 1 - \frac{\beta}{\sqrt{3}} \right)^2 + v_s^2 \left( \frac{2\beta}{\sqrt{3}} \right)^2 \right]. \quad (2.30)
\end{align*}

It can be checked that the matrix $M^2$ has a non degenerate zero eigenvalue for any breaking parameters in any 3-3-1 model. Therefore, the zero eigenvalue is identified with the photon mass, $M^2_\gamma = 0$.

The eigenstate with the zero eigenvalue can be obtained directly from the following equation
\begin{equation}
M^2 \begin{pmatrix} A_{\gamma 1} \\ A_{\gamma 2} \\ A_{\gamma 3} \end{pmatrix} = 0. \quad (2.31)
\end{equation}
We get then
\begin{equation}
A_{\gamma} = \begin{pmatrix} t \\ \beta \frac{t}{\sqrt{6}} \end{pmatrix} \frac{1}{\sqrt{6 + (1 + \beta^2)t^2}}. \quad (2.32)
\end{equation}
So, the physical photon field $A_{\mu}$ is given by
\begin{equation}
A_{\mu} = \frac{t}{\sqrt{6 + (1 + \beta^2)t^2}} W^3_{\mu} + \frac{\beta t}{\sqrt{6 + (1 + \beta^2)t^2}} W^8_{\mu} + \frac{\sqrt{6}}{\sqrt{6 + (1 + \beta^2)t^2}} B_{\mu}. \quad (2.33)
\end{equation}

For any 3-3-1 model, we see that the photon eigenstate and mass are not dependent on the VEV’s $(v_u, v_d, v_s)$. These are a natural consequence of the $U(1)_Q$ invariance - the conservation of the electric charge.

3 Matching gauge coupling constants

To embed the 3-3-1 model into the SM, we will work with electromagnetic vertex. Let us denote two remain massive fields by $Z^1_{\mu}, Z^2_{\mu}$. We change basis by unitary matrix
\begin{equation}
(A_{\mu}, Z^1_{\mu}, Z^2_{\mu}) = (W^3_{\mu}, W^8_{\mu}, B_{\mu})U, \quad (3.1)
\end{equation}
where $U$ has the following form

$$
U = \begin{pmatrix}
t & U_{12} & U_{13} \\
\frac{\sqrt{6+(1+\beta^2)t^2}}{\beta t} & U_{22} & U_{23} \\
\frac{\sqrt{6+(1+\beta^2)t^2}}{\sqrt{6}} & U_{32} & U_{33}
\end{pmatrix}.
$$

Here elements in the second and third columns are not necessary to determine. From (3.1) and (3.2), we get

$$
W_3^\mu = \frac{t}{\sqrt{6+(1+\beta^2)t^2}} A_\mu + U_{12} Z_1^\mu + U_{13} Z_2^\mu,
$$

$$
W_8^\mu = \frac{\beta t}{\sqrt{6+(1+\beta^2)t^2}} A_\mu + U_{22} Z_1^\mu + U_{23} Z_2^\mu,
$$

$$
B^\mu = \frac{\sqrt{6}}{\sqrt{6+(1+\beta^2)t^2}} A_\mu + U_{32} Z_1^\mu + U_{33} Z_2^\mu.
$$

The interactions among the gauge bosons and fermions are given by

$$
L_F = \bar{R} i \gamma^\mu \left( \partial_\mu + i \frac{g_X}{\sqrt{6}} X B_\mu \right) R + \bar{L} i \gamma^\mu \left( \partial_\mu + i g W^a_\mu \lambda^a \frac{1}{2} + i \frac{g_X}{\sqrt{6}} X B_\mu \right) L,
$$

where $R$ represents any right-handed singlet and $L$-any left-handed triplet or antitriplet. Substituting $W^3, W^8, B$ from (3.3) into (3.4), for $R = e_R$ with $X e_R = -1$ and $L = (\nu_e, e_L, E_L)^T$ with $X_L = -1/2 - \beta/2\sqrt{3}$, we get

$$
\mathcal{L}^{\text{int}}_{\bar{e}e\gamma} = -\bar{e}_R i \gamma^\mu \left[ \frac{i g_X}{\sqrt{6}} \frac{\sqrt{6}}{\sqrt{6+(1+\beta^2)t^2}} \right] A_\mu e_R + \bar{\nu}_L i \gamma^\mu \left[ \frac{2}{\sqrt{6+(1+\beta^2)t^2}} \frac{t}{\sqrt{6}} + \frac{\beta t}{2\sqrt{3}} \right] A_\mu e_L - \frac{i g_X}{\sqrt{6}} \left( \frac{1}{2} + \frac{\beta}{2\sqrt{3}} \right) \frac{\sqrt{6}}{\sqrt{6+(1+\beta^2)t^2}} A_\mu e_L
$$

$$
= \frac{g_X}{\sqrt{6+(1+\beta^2)t^2}} \bar{e} \gamma^\mu e A_\mu.
$$

The coefficient of the $\bar{e}e\gamma$ vertex in (3.5) is identified with the electromagnetic coupling constant

$$
g_X \frac{1}{\sqrt{6+(1+\beta^2)t^2}} \equiv e.
$$
In the SM, we have
\[ \frac{g_2 g_Y}{\sqrt{g_2^2 + g_Y^2}} = e, \] (3.7)
where \( g_2, g_Y \) are coupling constants of \( SU(2)_L \) and \( U(1)_Y \) gauge group, respectively. Using continuation of gauge coupling constant of \( SU(3)_L \) group at the spontaneous symmetry breaking point,
\[ g = g_2 = g, \] (3.8)
from (3.6) and (3.7), we get
\[ \frac{1}{g^2_Y} = \frac{\beta^2}{g^2} + \frac{6}{g^2_X}. \] (3.9)
From (3.6), we obtain
\[ \frac{t}{\sqrt{6 + (1 + \beta^2)t^2}} = \frac{e}{g}. \] (3.10)
As in the SM, we put
\[ \frac{t}{\sqrt{6 + (1 + \beta^2)t^2}} = \sin \theta_W. \] (3.11)
The (3.9) yields
\[ t = \frac{\sqrt{6} t_W}{\sqrt{1 - \beta^2 t_W^2}}. \] (3.12)
Hence, the photon eigenstate can be rewritten in the form
\[ A_\mu = s_W W^3_\mu + c_W \left( \beta t_W W^8_\mu + \sqrt{1 - \beta^2 t_W^2} B_\mu \right). \] (3.13)
From orthogonal condition of the photon eigenstate to two remain gauge vectors, we can write
\[ Z_\mu = c_W W^3_\mu - s_W \left( \beta t_W W^8_\mu + \sqrt{1 - \beta^2 t_W^2} B_\mu \right), \] (3.14)
\[ Z'_\mu = \sqrt{1 - \beta^2 t_W^2} W^8_\mu - \beta t_W B_\mu. \] (3.15)
Therefore, in this basis, the mass matrix \( M^2 \to M'^2 \) has the form as follows
\[ M'^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & M_Z^2 & M_{ZZ'}^2 \\ 0 & M_{ZZ'}^2 & M_Z'^2 \end{pmatrix}, \] (3.16)
where
\[ M_Z^2 = \frac{g^2}{4c_W^2} (v_u^2 + v_d^2), \]
\[ M_{ZZ'}^2 = \frac{g^2}{4\sqrt{3}c_W \sqrt{1 - (1 + \beta^2)s_W^2}} \left\{ [\sqrt{3}\beta - 1)s_W^2 + 1]v_u^2 + ((\sqrt{3}\beta + 1) s_W^2 - 1)v_d^2 \right\}, \]
\[ M_Z'^2 = \frac{g^2}{12(1 - \beta^2 t_W^2)} \left\{ (1 + \sqrt{3}\beta t_W^2)^2 v_u^2 + (1 - \sqrt{3}\beta t_W^2)^2 v_d^2 + 4v_s^2 \right\}. \] (3.17)
The matrix, $M^2'$, gives mixtures between $Z_\mu$ and $Z'_\mu$, by rotating an angle $\phi$ in the plane $(Z_\mu, Z'_\mu) \rightarrow (Z^1_\mu, Z^2_\mu)$, the mass eigenvectors are

$$
Z^1_\mu = Z_\mu \cos \phi - Z'_\mu \sin \phi,
Z^2_\mu = Z_\mu \sin \phi + Z'_\mu \cos \phi,
$$

(3.18)

where $\phi$ is defined by

$$
\tan^2 \phi = \frac{M^2_Z - M^2_{Z^1}}{M^2_{Z^2} - M^2_Z},
$$

(3.19)

and the physical mass eigenvalues:

$$
M^2_{Z^1} = \frac{1}{2} \left[ M^2_Z + M^2_{Z'} - \sqrt{(M^2_Z - M^2_{Z'})^2 + 4(M^2_{ZZ'})^2} \right],
$$

(3.20)

$$
M^2_{Z^2} = \frac{1}{2} \left[ M^2_Z + M^2_{Z'} + \sqrt{(M^2_Z - M^2_{Z'})^2 + 4(M^2_{ZZ'})^2} \right].
$$

(3.21)

From the mixing mass matrix of $Z$ and $Z'$ we see that $\phi = 0$ if $v_s \gg v_u, v_d$ or

$$
v^2_u = \frac{1 - (\sqrt{3} \beta + 1)s^2_W}{1 + (\sqrt{3} \beta - 1)s^2_W} v^2_d
$$

(3.22)

Here $A, Z^1$ correspond to the neutral gauge bosons the SM ($\gamma, Z$), and $Z^2$ is a new neutral gauge boson.

To finish this section, we note that the matching condition of the coupling constants (3.6) at the $SU(3)_L \otimes U(1)_X$ breaking is very obvious as the matching in the SM. It is not dependent on the constraint $v_s \gg v_u, v_d$ as in the literature [6]. After the matching, we rewrote the photon field with the coefficients in the Weinberg mixing angle, and then taking exact diagonalization of the mass matrix for the neutral gauge bosons.

4 Conclusion

In this paper, the photon eigenstate and the matching of coupling constants in 3-3-1 models are obtained in general form containing Higgs triplets, antitriplets as well as sextet. We emphasized that the matching of coupling constants is not dependent on condition that vacuum expectation value of Higgs boson of the first step of breaking symmetry must be much larger than those of the second step namely $\langle s \rangle \gg \langle u \rangle, \langle d \rangle$.

This technique can be extended for electroweak models which are based on the larger gauge groups such as $SU(3)_C \otimes SU(4)_L \otimes U(1)_X$ [14].

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