Co-Design to Enable User-Friendly Tools to Assess the Impact of Future Mobility Solutions

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Abstract—The design of future mobility solutions and the design of the mobility systems they enable are closely coupled. Indeed, knowledge about the intended service of novel mobility solutions would impact their design and deployment process, whilst insights about their technological development could significantly affect transportation management policies. This requires tools to study such a coupling and co-design mobility systems in terms of different objectives. We present a framework to address such co-design problems, leveraging a mathematical theory of co-design to frame and solve the problem of designing and deploying an intermodal mobility system, whereby autonomous vehicles service travel demands jointly with micromobility solutions and public transit, in terms of fleets sizing, vehicle characteristics, and public transit service frequency. Our framework is modular and compositional, allowing one to describe the design as the interconnection of simple components and to tackle it from a systemic perspective. Moreover, it requires general monotonicity assumptions and naturally handles multiple objectives, delivering rational, actionable solutions for policy makers. We showcase our methodology in a case study of Washington D.C., USA. Our work suggests the possibility to create user-friendly optimization tools to systematically assess costs and benefits of interventions, and to inform policy-making in the future.

Index Terms—Network optimization and control, Network resource allocation, Complex Networks, Cyber-Physical Network Co-Design and Analysis, Transportation Systems Analysis, Service network design, planning, and scheduling, Transportation infrastructure and investment, Emerging topics in transportation and logistics networks.

I. INTRODUCTION

CURRENT transportation systems are undergoing dramatic mutations, arising from the deployment of novel mobility solutions, such as autonomous vehicles (AVs) and μM systems. New mobility paradigms promise to drastically reduce negative externalities produced by the transportation system, such as emissions, travel time, parking spaces and, critically, fatalities (for a review on the subject, refer to [1]). However, industrial experience shows that the current design process for new mobility solutions often suffers from the lack of clear, specific requirements in terms of the service they will be providing [2]. Yet, knowledge about their intended service (e.g., last-mile versus point-to-point travel) might dramatically impact how vehicles are designed and significantly ease their development process. For instance, if for a given city we knew that for an effective on-demand mobility system AVs only need to drive up to 30 mph and only on relatively simple roads, their design would be greatly simplified and their deployment could be accelerated. Furthermore, from the system-level perspective of transportation management, knowledge about the trajectory of technology development for new mobility solutions would certainly impact decisions on future infrastructure investments and provisions of service. In other words, the design of future mobility solutions and the design of a mobility system leveraging them are intimately coupled. This calls for methods to reason about such a coupling, and in particular to co-design the individual mobility solutions and the associated mobility systems. A key requirement in this context is to be able to account for a range of heterogeneous objectives that are often not directly comparable (consider, for instance, travel time, public expense, and externalities), to formulate hierarchical design problems involving different disciplines, and to solve them in a computationally tractable manner.

Accordingly, the goal of this paper is to lay the foundations for a framework through which one can systematically co-design future mobility systems. Specifically, we show how to leverage a recently developed monotone theory of co-design [3], [4], [5], [6], which provides a general methodology to co-design complex systems in a modular and compositional fashion [7], [8]. This tool delivers the set of rational design solutions lying on the Pareto front, allowing one to reason about costs and benefits of the individual design options. The framework is instantiated in the setting of co-designing intermodal mobility systems [9], whereby fleets of self-driving vehicles provide on-demand mobility jointly with fleets of μMVs such as e-scooters (ESs), shared bikes (SBs), mopeds (Ms) and fuel-cell mopeds (FCMs), and public transit. Aspects that are subject to co-design include fleet sizes, vehicle-specific characteristics for AVs and μMVs, and service features, such as public transit service frequency, prices, and serviced networks.
A. Related Literature

Our work lies at the interface of the design of public transportation services and the design of novel mobility solutions. The first research stream is reviewed in [10], [11], [12] and comprises strategic long-term infrastructure modifications and operational short-term scheduling. The joint design of traffic network topology and control infrastructure has been presented in [13], [14]. Public transportation scheduling has been solved jointly with the design of the transit network by optimizing passengers’ and operators’ costs in [15], the satisfied demand in [16], and the energy consumptions of the system in [17]. However, these works mainly focus on a single infrastructure (road network or public transportation), and do not consider its joint design with new mobility solutions.

The research on novel mobility solutions mainly pertains to AVs, Autonomous Mobility-on-Demand (AMoD) systems, and μM. The research on design of AMoD systems is thoroughly reviewed in [1] and references therein, and mainly concerns their fleet sizing. In this regard, existing studies range from simulation-based approaches [18], [19], [20], [21], [22], [23] to analytical methods [24]. In [25], the fleet size and the charging infrastructure of an AMoD system are jointly designed, and the arising design problem is formulated as a mixed integer linear program. In [26], the fleet sizing problem is solved together with the vehicle allocation problem. Furthermore, [27] proposes a framework to jointly design the AMoD fleet size and its composition. More recently, the joint design of multi-modal transit networks and AMoD systems was formulated in [28] as a bilevel optimization problem and solved with heuristics, and coupled with infrastructure design in [29], using multi-objective linear optimization. Overall, the problem-specific structure of existing design methods for AMoD systems is often not amenable to a modular and compositional problem formulation. Furthermore, key AV characteristics, such as the achievable speed, are not considered. Research on the design and impact of μM solutions has been reviewed in [30], which focuses on the urban deployment of SBs and ESs. In particular, [31] presents a design framework for a multi-modal public transportation system, including various μM solutions and buses, optimizing user preferences and social costs. Fleet deployment models are analyzed in [14], [32], [33], [34]. The optimal allocation of SBs in a city is studied through mathematical programming models in [32], and solved through stochastic optimization in [14], [33]. Finally, [35] explores the impact of μM on urban planning and identifies strategies to increase μMVs utilization.

At a higher abstraction level, the problem we are trying to solve matches some of the principles of collaborative engineering [36], [37], which, while providing interesting insights, do not offer a mathematical theory and a scalable computational framework to deal with hierarchical, multi-objective design problems.

In conclusion, to the best of the authors’ knowledge, the existing design frameworks for mobility systems have a fixed problem-specific structure, and therefore do not permit to co-design the mobility infrastructure in a modular and compositional manner. Moreover, previous works neither capture important aspects of future mobility systems, such as the interactions among different transportation modes, nor specific design parameters of novel mobility solutions, as for instance the level of autonomy and the serviced network of AVs, in a computationally tractable and compositional way.

B. Statement of Contributions

In this paper, we lay the foundations for the systematic study of the design of future mobility systems. Specifically, we leverage a mathematical theory of co-design [3], [6] to devise a framework to study the design of intermodal mobility systems in terms of mobility solutions and fleet characteristics, enabling the computation of the rational solutions lying on the Pareto front of minimal travel time, transportation costs, and externalities. Our framework paves the way to structure the design problem in a modular way, in which each different transportation option can be “plugged in” in a larger model. Each model has minimal assumptions: Rather than properties such as linearity, continuity, or convexity, we ask for very general monotonicity assumptions, proven to be reasonable in the present paper and in previous works [5], [7], [8]. For example, we assume that the cost of automation of an AV does not decrease with the increase of the speed achievable by the AV. We are able to obtain the full Pareto front of rational solutions or, given policies, to weigh incomparable costs (such as travel time and emissions) and to present actionable information to the stakeholders of the mobility ecosystem. We consider the case study of Washington D.C., USA, to showcase our methodology. We illustrate how, given the model, we can easily formulate and answer several questions regarding the introduction of new technologies and investigate possible infrastructure interventions. This article significantly extends the preliminary material previously presented in [38], [39]. In particular, we first broaden and extend the formal presentation of the mathematical theory of co-design, and detail its application in this work by formalizing the introduced design problems (including proofs, new insights, and generalizability of the approach). Second, we extend the discussion of the literature, including recent research pertaining to the co-design of future mobility systems, with a focus on μM. Third, we showcase the modularity of our framework by including the design of μM solutions (both at the vehicle and at the fleet level) in the future mobility co-design problem, and evaluate their impact on the transportation system for the case study of Washington D.C., USA, whereby we leverage the network of the city, as well as demand datasets. Furthermore, we include a study of pricing strategies in the mobility co-design problem, highlighting the flexibility of the proposed framework. Finally, we extend our case studies with further scenarios and provide new managerial insights.

C. Organization of the Paper

The remainder of this paper is structured as follows: Section II reviews the mathematical background on which our framework is based. Section III presents models for future mobility systems and related co-design problems, by introducing the single design problems (DPs) and their interconnection.
II. MONOTONE CO-DESIGN THEORY

In this section, we present the main concepts related to the mathematical theory of co-design, presented in [3], [4] and extensively in [6]. We will recall basic concepts of order theory along the way. For more details, the interested reader is referred to [40].

The plot of the section is the following. We will introduce a theory of co-design whose atoms are MDPIs, which are (monotone) feasibility relations between functionalities and resources. We will then explain how one can compose MDPIs in various ways, define optimization problems related to the introduced structures, and hint at the solution techniques (with details reported in the Section A1 for convenience).

Resources in this theory are quantified via partially ordered sets (posets).

**Definition 2.1 (Poset):** A partially ordered set (poset) is a tuple \( P = \langle P, \leq_p \rangle \), where \( P \) is a set and \( \leq_p \) is a partial order, defined as a reflexive, transitive, and antisymmetric relation.

This structure allows one to describe standard engineering quantities (typically totally ordered sets), such as \( \langle R_{\geq 0}, \leq \rangle \) (e.g., energy, costs) and \( \langle N, \leq \rangle \) (e.g., number of vehicles in a fleet), but also more complex ones, which we will introduce later in this work.

Given a poset, we can consider its “reversed” version.

**Definition 2.2 (Opposite of a poset):** The opposite of a poset \( P = \langle P, \leq_p \rangle \) is the poset \( \langle P, \geq_p \rangle \), which has the same elements as \( P \), and the reverse ordering.

To be able to define MDPIs, we further need to introduce the notions of product poset and of monotone map.

**Definition 2.3 (Product poset):** Let \( \langle P, \leq_p \rangle \) and \( \langle Q, \leq_q \rangle \) be posets. Then, \( P \times Q = \langle P \times Q, \leq \rangle \), with \( \langle p_1, q_1 \rangle \leq \langle p_2, q_2 \rangle \Leftrightarrow p_1 \leq_p p_2 \) and \( q_1 \leq_q q_2 \).

for all \( p_1, p_2 \in P, q_1, q_2 \in Q \), is the product poset of \( \langle P, \leq_p \rangle \) and \( \langle Q, \leq_q \rangle \).

**Definition 2.4 (Monotone map):** A map \( f : P \to Q \) between two posets \( \langle P, \leq_p \rangle \), \( \langle Q, \leq_q \rangle \) is monotone iff \( x \leq_p y \implies f(x) \leq_q f(y) \). Note that monotonicity is compositionnal.

We are now ready to define the main atom of the monotone co-design theory.

**Definition 2.5 (MDPI):** Given the posets \( F, R \), representing functionalities and resources, respectively, we define a monotone design problem with implementation (MDPI) as a tuple \( \langle I_d, \text{prov}, \text{reqs} \rangle \), where \( I_d \) is the set of implementations, and \( \text{prov}, \text{reqs} \) are functions from \( I_d \) to \( F \) and \( R \), respectively:

\[
\langle \text{prov}, \text{reqs} \rangle : I_d \to F \leftrightarrow R.
\]

(Maps \( \text{prov}, \text{reqs} \) are mnemonics for the fact that each implementation provides some functionality and requires some resources.) We compactly denote the MDPI as \( d : F \leftrightarrow R \).

forming a co-design problem (CDP). We showcase our methodology with several case studies for Washington D.C., USA, in Section IV-B. Section V concludes the paper with a discussion and an overview on future research directions. Nomenclature is available in the appendix.

Furthermore, to each MDPI we associate a monotone map \( d \), given by:

\[
d : F \leftrightarrow R \to \langle P(I_d), \leq \rangle
d(f^*, r) = \{i \in I_d : (\text{prov}(i) \geq f) \land (\text{reqs}(i) \leq R r)\},
\]

where \( \langle \cdot \rangle^op \) reverses the order of a poset. We represent a MDPI in diagrammatic form as in Fig. 1(a).

**Remark 1** (Intended semantics for MDPIs): The expression \( d(f^*, r) \) returns the set of implementations (design choices) \( S \subseteq I_d \) for which the functionalities \( f \) are feasible with resources \( r \). For instance, a battery provides energy, requires mass and has a cost. Different battery models represent different implementations.

**Remark 2** (Monotonicity of MDPIs): Consider an MDPI with \( d(f^*, r) = S \).

- Consider \( f' \leq f \). Then, \( d(f'^*, r) = S' \supseteq S \). In other words, decreasing the desired functionality cannot increase the required resources.
- Consider \( r' \geq r \). Then, \( d(f^*, r') = S'' \supseteq S \). In other words, increasing the available resources cannot decrease the provided functionalities.

For further related examples, refer to [6].

Individual MDPIs can be composed in several ways to form a co-design problem (Fig. 1), allowing one to decompose a large problem into smaller subproblems, and to interconnect them. We report technical details in the appendix, and give a practical intuition in the following. Series composition represents the case in which the functionality of a MDPI is required by another MDPI. For instance, the energy provided by a battery is required by an electric motor to produce torque. The posetal relation “\( \leq \)” in Fig. 1 represents a co-design constraint: The resource one component requires can be at most as much as the one provided by another component. Parallel composition formalizes decoupled processes happening together. Finally, loop composition describes feedback. The composition operations preserve monotonicity, meaning that the composition of two MDPIs is a MDPI. We call the composition of MDPIs a co-design problem with implementation (CDPI) [3], [5].

We can now formulate problems and describe solutions. Given a poset, we formalize the idea of “Pareto front” via anti-chains, which are useful to describe incomparable designs.

\[1\] It can be proved that the formalization of feedback makes the category of MDPIs a traced monoidal category [6], [41].
Definition 2.6 (Antichains): A subset $S \subseteq \mathcal{P}$ of a poset $\mathcal{P}$ is an antichain iff no elements are comparable: For $x, y \in S$, $x \leq_{\mathcal{P}} y$ implies $x = y$. We denote by $\mathcal{A}\mathcal{P}$ the set of all antichains in $\mathcal{P}$.

Definition 2.7 (Functionality to resources map): Given an MDPI $d$, one can define a monotone map $h_d : \mathcal{F} \rightarrow \mathcal{A}\mathcal{R}$, mapping a functionality to the minimum antichain of resources providing it. Dually, one can define $h'_d : \mathcal{R} \rightarrow \mathcal{A}\mathcal{F}$, mapping a resource to the maximum antichain of functionalities provided by it.

In the design of battery models, $h_d$ maps a particular desired energy to the antichain of masses and costs of (incomparable) battery models providing at least that energy.

We are now ready to state the problem.

Problem 1: We are given a CDPI (interconnection of MDPIs) with functionalities $\mathcal{F}$ and resources $\mathcal{R}$, and we can evaluate the map $h_d$ for each MDPI $d$ involved. Given a functionality $f \in \mathcal{F}$ of the CDPI, we wish to find the minimal resources in $\mathcal{R}$, for which there exists a feasible implementation vector which makes all sub-problems feasible simultaneously, and all co-design constraints satisfied; or, if none exist, we want to provide a certificate of infeasibility.

In other words, given the maps $h_d$ for the subproblems, we want to evaluate the map $h$ for the entire CDPI.

If $h$ is Scott continuous, and the posets are complete partial orders, one can rely on Kleene’s fixed point theorem to find the solution for the interconnected optimization problem [4]. Problem 1 as an instance of a multi-objective optimization problem, which does not require the objectives to be convex, differentiable, continuous, or even defined on continuous spaces. This class of problems can be solved recursively, and their complexity is linear in the number of design options, not combinatorial. These computational properties, and the ease of modeling within the proposed framework, make CDPIs a class of hierarchical multi-objective optimization problems that is computationally scalable, modular, and easily composable. For convenience, we report a detailed description of the solution techniques and their complexity in Section A1.

Remark 3 (A user-friendly framework): The theory presented in this section and in Section A1 might seem complex. However, it represents the developer-view, as opposed to the optimization framework’s user-view. As a user, one just needs to decompose the problem at hand in smaller problems, formulate them as MDPIs (via analytical relations, catalogues, and simulations), and formalize their interconnections. (A user can also focus on a single MDPI and interconnect it with other users’ ones.) We will show how, once the problem is formulated, a ready-to-use solver will smoothly provide the solutions to custom queries.

III. CO-DESIGN OF FUTURE MOBILITY SYSTEMS

In this section, we detail our co-design framework for future mobility systems, and instantiate it for the specific case of an intermodal transportation network.

A. Intermodal Mobility Framework

1) Multi-Commodity Flow Model: The transportation system and its different modes are modeled using the edge-labeled digraph $G = \langle V, A, c \rangle$, sketched in Fig. 2.

It is described through a set of nodes $V$ and a set of arcs $A \subseteq V \times V$, labeled with metrics $c : A \rightarrow S$. Specifically, $c_{ij} := c(i, j) \in S$ are the metrics associated to arc $(i, j) \in A$. Metrics of interest include edge length, travel time, energy consumption properties, and congestion models. $G$ is composed of four layers: The road network layer $G_R = \langle V_R, A_R, c_R \rangle$, consisting of an AVs layer $G_{R,V} = \langle V_{R,V}, A_{R,V, c_{R,V}} \rangle$ and a MVs layer $G_{R,M} = \langle V_{R,M}, A_{R,M}, c_{R,M} \rangle$, the public transportation layer $G_P = \langle V_P, A_P, c_P \rangle$, and a walking layer $G_W = \langle V_W, A_W, c_W \rangle$. The AVs and the MVs networks are characterized by intersections $i \in V_{R,V}$, $i \in V_{R,M}$ and road segments $(i, j) \in A_{R,V}$, $(i, j) \in A_{R,M}$, respectively. Similarly, public transportation lines are modeled through station nodes $i \in V_P$ and line segments $(i, j) \in A_P$. The walking network describes walkable streets $(i, j) \in A_W$, connecting intersections $i \in V_W$. Mode-switching arcs are modeled as

$$ A_C \subseteq V_{R,V} \times V_W \cup V_W \times V_{R,V} \cup V_{R,M} \times V_W \cup V_W \times V_{R,V} \cup V_{R,M} \times V_P \cup V_P \times V_W \times V_P, $$

connecting the AVs, the MVs, and the public transportation layers to the walking layer. To these arcs, we associate metrics $c_C$.

Consequently, $V = V_W \cup V_{R,V} \cup V_{R,M} \cup V_P$ and $A = A_W \cup A_{R,V} \cup A_{R,M} \cup A_P \cup A_C$. Consistently with structural properties of transportation networks in urban environments, we assume $G$ to be strongly connected.

We now characterize the partial order of edge labeled graphs, which will be instrumental when modeling the MDPI of the mobility system. To do so, we first need to define a partial order on functions.

Definition 3.1 (Poset of functions): Consider posets $A, B$, and consider the set of functions $A \rightarrow B$, denoted by $B^A$. 
Given any two functions \( f, g : A \to B \), we define

\[
f \preceq_B g \iff f(a) \leq_B g(a), \quad \forall a \in A.
\]

**Lemma 3.2:** Definition 3.1 defines a poset.
Definition 3.1 allows one to define a poset of edge-labeled multigraphs.

**Definition 3.3 (Poset of edge-labeled multigraphs):** Consider the set of edge-labeled multigraphs, denoted by \( G \). Given \( G_1, G_2 \in G \), with \( G_1 = (V_1, A_1, c_1) \), \( G_2 = (V_2, A_2, c_2) \), \( c_1 : A_1 \to C \), \( c_2 : A_2 \to C \) we define the order:

\[
G_1 \preceq G_2 \iff (V_1 \subseteq V_2) \land (A_1 \subseteq A_2) \land (c_1 \preceq c_2)_{A_1}.
\]

where \( c_2|_{A_1} \) is the restriction of \( c_2 \) onto the domain of \( c_1 \).

Intuitively, a labeled multigraph dominates another if it includes its nodes and edges, and if the labels are dominating.

**Lemma 3.4:** Definition 3.3 defines a poset.

We represent customer movements by means of travel requests. A travel request refers to a customer flow starting its trip at a node \( o \in V \) and ending it at a node \( d \in V \).

**Definition 3.5 (Travel demand):** A travel demand \( q \) is a triple \((o, d, \alpha) \in V \times V \times R_+\), described by an origin node \( o \in V \), a destination node \( d \in V \), and the request rate \( \alpha > 0 \) (i.e., the number of customers who want to travel from \( o \) to \( d \) per unit time).

Without loss of generality, we can assume that in a set of requests origin-destination pairs are not repeated, and denote the set of all possible sets of requests \( Q \subseteq \mathcal{P}(V \times V \times R_{\geq 0}) \). This set can be ordered as follows.

**Definition 3.6 (Poset of travel demands):** Consider the set of sets of travel requests \( Q \). Given any \( Q_1, Q_2 \in Q \), one has:

\[
Q_1 \preceq Q_2 \iff \forall \{(o_1^i, d_1^i, \alpha_1^i)\}_{i=1}^{M_1} \subseteq \mathcal{Q} \quad \forall \{(o_2^i, d_2^i, \alpha_2^i)\}_{i=1}^{M_2} \subseteq Q_2
\]

iff for all \( \langle o_1^i, d_1^i, \alpha_1^i \rangle \in Q_1 \), there is some \( \langle o_2^i, d_2^i, \alpha_2^i \rangle \in Q_2 \) with \( o_1^i = o_2^i, d_1^i = d_2^i, \) and \( \alpha_2^i > \alpha_1^i \). In other words, \( Q_1 \preceq Q_2 \) if every travel request in \( Q_1 \) is in \( Q_2 \) as well.

**Lemma 3.7:** Definition 3.6 defines a poset.

To ensure that a customer is not biased to use a given transportation mode, we assume all requests to appear on the walking digraph, i.e., \( o_m, d_m \in V_W \) for all \( m \in M := \{1, \ldots, M\} \). The flow \( i \) of customers per unit time traveling on arc \( \langle i, j \rangle \in A \) and satisfying a travel request \( m \). Furthermore, \( f_{o,V}(i, j) \geq 0 \) and \( f_{o,M}(i, j) \geq 0 \) denote the flow of empty AVs and \( \mu \) MVs on AVs arcs \( \langle i, j \rangle \in A_{R,V} \) and \( \mu \) MVs arcs \( \langle i, j \rangle \in A_{R,M} \), respectively. This accounts for rebalancing flows of AVs and \( \mu \) MVs between a customer’s drop-off and the next customer’s pick-up. Assuming AVs and \( \mu \) MVs to carry one customer at a time, the flows satisfy

\[
\sum_{i \in A} f_m(i, j) + 1_{j=\alpha_m} \cdot \alpha_m = \sum_{k \in A} f_m(j, k) + 1_{j=\alpha_m} \cdot \alpha_m \quad \forall m \in M, j \in \mathcal{V}
\]

\[
\sum_{i \in A_{R,V}} f_{tot,V}(i, j) = \sum_{k \in A_{R,V}} f_{tot,V}(j, k) \quad \forall j \in A_{R,V}
\]

\[
\sum_{i \in A_{R,M}} f_{tot,M}(i, j) = \sum_{k \in A_{R,M}} f_{tot,M}(j, k) \quad \forall j \in A_{R,M}
\]

where \( 1_{j=\alpha} \) denotes the boolean indicator function, \( f_{tot,V}(i, j) = f_{o,V}(i, j) + \sum_{m \in M} f_{o,M}(i, j), \) and \( f_{tot,M}(i, j) = f_{o,M}(i, j) + \sum_{m \in M} f_{o,M}(i, j) \). Specifically, (1a) guarantees flows conservation for every transportation demand, (1b) preserves flow conservation for AVs on every road node, and (1c) preserves flow conservation for \( \mu \) MVs on every road node. Combining conservation of customers (1a) with the conservation of AVs (1b) and \( \mu \) MVs (1c) guarantees rebalancing AVs and \( \mu \) MVs to match the demand.

**B. Labeling Graphs with Relevant Attributes**

In the following, we specify how to label the graphs composing the full mobility network. Specifically, edge-labeling maps will be of the form \( c : A \to S \), where \( S = \mathbb{R}_{\geq 0} \) represents link length, time needed to traverse it, its speed limit, related emissions, and capacity.

1) **Walking Arcs:** We infer arc lengths \( s_{ij} \) from geographical data and, assuming constant walking speed \( v_W \), travel time results from \( t_{ij} = s_{ij}/v_W \). As speeds limits, congestion, and energy consumption do not apply to walking graphs, we set \( v_{L,ij} = \infty, k_{ij} = \infty, e_{ij} = 0 \). Accordingly,

\[
c_W : A_W \to \mathcal{S}
\]

\[
(i, j) \mapsto (s_{ij}, t_{ij}, v_{L,ij}, e_{ij}, k_{ij}).
\]

2) **Public Transit Arcs:** We infer arc lengths \( s_{ij} \) from public transit network data. Furthermore, assuming that the public transportation system at node \( j \) operates with the frequency \( q_j \), travel time results from \( t_{ij} = t_{mon} + t_{WS} + 1/2(q_j) \), where \( t_{mon} \) is the in-vehicle travel time (inferred from public transit schedules) and \( t_{WS} \) is a constant sidewalk-to-station travel time. We ignore capacity and speed limits, so that \( k_{ij} = v_{L,ij} = \infty \). For the public transportation system we assume a constant energy consumption per unit time. This approximation is reasonable in urban environments, where the operation of the public transportation system is independent from the number of customer serviced, and its energy consumption is therefore invariant. Therefore, we write \( e_{ij} = k \epsilon_{ij}, k > 0 \). Accordingly,

\[
c_P : A_P \to \mathcal{S}
\]

\[
(i, j) \mapsto (s_{ij}, t_{ij}, v_{L,ij}, e_{ij}, k_{ij}).
\]

3) **Road Arcs for AVs:** Each road arc is characterized by a length \( s_{ij} \), a speed limit \( v_{L,ij} \), and a capacity \( k_{ij} \), all derived from road network data. We consider AVs driving at speed \( v_W \), so that travel time reads
We compute the energy consumption of AVs via an urban driving cycle. In particular, the cycle is scaled so that its average speed \( v_{\text{avg, cycle}} \) matches the free-flow speed on the link. The energy consumption of road link \((i, j)\) is scaled as
\[
e_{ij} = e_{\text{cycle}} \frac{s_{ij}}{s_{\text{cycle}}}.
\]

Collectively,
\[
c_{R,V} : \mathcal{A}_{R,V} \to \mathcal{S}
\]
\[
(i,j) \mapsto (s_{ij}, t_{ij}, v_{L,ij}, e_{ij}, k_{ij}).
\]

### 4) Road Arcs for \( \mu \text{MVs} \):
Each road arc is characterized by a length \( s_{ij} \) and a speed limit \( v_{L,ij} \), derived from road network data, while we neglect arc capacity (i.e., \( k_{ij} = \infty \)). Assuming \( \mu \text{MVs} \) driving at speed \( v_M \), travel time reads
\[
t_{ij} = \frac{s_{ij}}{\min\{v_M, v_{L,ij}\}}.
\]

For \( \mu \text{MVs} \) we consider a distance-based energy consumption, i.e. \( e_{ij} = ts_{ij} \), with \( t > 0 \). Overall,
\[
c_{R,M} : \mathcal{A}_{R,M} \to \mathcal{S}
\]
\[
(i,j) \mapsto (s_{ij}, t_{ij}, v_{L,ij}, e_{ij}, k_{ij}).
\]

### 5) Transfer Arcs:
We define travel time \( t_{ij} \) as follows: we assume that the average waiting time for AVs is \( t_{W,V} \); the average time needed to reach a \( \mu \text{MV} \) is \( t_{W,M} \), and switching from the AVs graph, the \( \mu \text{MVs} \) graph, and the public transit graph to the pedestrian graph takes the transfer times \( t_{V,W}, t_{M,W}, \) and \( t_{W,V} \), respectively. For each arc, we set length and energy consumption to zero (i.e., \( s_{ij} = e_{ij} = 0 \)) and ignore capacity and speed limit (i.e., \( k_{ij} = v_{L,ij} = \infty \)). Overall,
\[
c_{C} : \mathcal{A}_{C} \to \mathcal{S}
\]
\[
(i,j) \mapsto (s_{ij}, t_{ij}, v_{L,ij}, e_{ij}, k_{ij}).
\]

### C. Road Congestion
We assume that road arcs are subject to a normalized capacity \( k_{ij} \), which could arise from the difference of the nominal road capacity and the exogenous road usage:
\[
f_{\text{tot, } V}(i,j) \leq k_{ij}.
\]

We assume that the central authority operates the AMoD fleet such that vehicles travel at free-flow speed throughout the road network of the city, meaning that the total flow on each road link must be below the link’s capacity. Therefore, we capture congestion effects with the threshold model. Finally, we assume \( \mu \text{M} \) to not significantly contribute to congestion [42].

### D. Discussion
First, the demand is assumed to be time-invariant and flows are allowed to have fractional values. This assumption is in line with the mesoscopic and system-level planning perspective of our study. Second, we model congestion effects using a threshold model. This approach can be interpreted as a municipality preventing mobility solutions to exceed the critical flow density on road arcs. AVs and \( \mu \text{MVs} \) can therefore be assumed to travel at free flow speed [43]. This assumption is realistic for an initial low penetration of new mobility systems in the transportation market, especially when the AV and \( \mu \text{MV} \) fleets are limited in size. Finally, we allow AVs and \( \mu \text{MVs} \) to transport one customer at a time [44].

### E. Co-Design Framework
We integrate the intermodal framework presented in Section III-A in the co-design formalism, allowing the decoupling of the CDPI of a complex system in the MDPIs of its individual components in a modular, compositional, and systematic fashion. To achieve this, we decouple the CDPI in the MDPIs of the individual AV (Section III-E1), the AVs fleet (Sections III-E4 to III-E6), the individual \( \mu \text{MV} \) (Section III-E2), the \( \mu \text{MVs} \) fleet (Sections III-E4 to III-E6), and the public transportation system (Section III-E3). Their interconnection is presented in Sections III-E7 to III-E9, where we propose multiple model versions, showcasing the flexibility of the developed framework. We aim at computing the antichain of resources, quantified in terms of costs, average travel time per trip, and emissions required to provide the mobility service to a set of customers. For each model, we provide descriptions and formal proofs of integration in the co-design framework.

1) The AV MDPI: The AV MDPI selects the labeled graph on which the AV provider wants to operate. The selection happens via the choice of the achievable speed of the AVs as follows. AVs safety protocols impose a maximum achievable velocity \( v_V \). Furthermore, in order to prevent too slow and therefore dangerous driving behaviors, we only consider AVs arcs through which the AVs can drive at least at a fraction \( \beta \) of the speed limit. Specifically, AVs can drive on arc \((i,j)\) if and only if
\[
v_V \geq \beta \cdot \pi_{\text{in}} c_{R,V}(i,j),
\]
where \( \beta \in (0, 1] \), and \( \pi_{\text{in}} \) projects the part of \( c_{R,V}(i,j) \) related to \( v_V \). The elimination of forbidden arcs given an achievable speed can be achieved through the following map (mnemonics for reduction):
\[
70, 0, 0_{R,V} : \mathbb{R}_{\geq 0} \to \langle \mathbb{G}, \leq \mathbb{G} \rangle
\]
\[
v_V \mapsto \langle \mathbb{G}_V, \mathbb{A}, c \rangle,
\]

where
\[
\mathbb{A} = \{ a \in \mathcal{A}_{R,V} : (3) \text{ holds} \},
\]
\[
c = \langle \pi_{\text{in}}, c_{R,V} : \min\{v_V, \pi_{\text{in}} c_{R,V} \}, \pi_{\text{in}} c_{R,V}, \pi_{\text{in}} c_{R,V}, \pi_{\text{in}} c_{R,V} \rangle.
\]

Lemma 3.8: The map 70, 0, 0_{R,V} is monotone.

Under the rationale that driving safely at higher speed requires more advanced sensing and algorithmic capabilities [7], we model the achievable speed of the AVs \( v_V \) as a monotone function of the vehicle fixed costs \( C_{V, f} \) (resulting from the cost of the vehicle \( C_{V, c} \) and the cost of its automation \( C_{V, a} \)) and the mileage-dependent operational costs \( C_{V, o} \)
MDPI Definition: The AV MDPI, denoted $d_{AV}$, provides the functionality $G_{AV} \in G$ (i.e., the functionality of servicing a specific network with a specific performance) and requires the resources $C_{V_{AV}}, C_{V_{V}} \in \mathbb{R}_{\geq 0}$. The implementations space $I_{AV}$ consists of models of the AVs. Formally: $d_{AV} : G \rightarrow \mathbb{R}_{\geq 0}^2$.

Lemma 3.9: $d_{AV}$ is a well-defined MDPI.

2) The $\mu$MV MDPI: The $\mu$M MDPI comprises the selection of the labeled graph on which to operate, again resumed in the maximal speed achievable by $\mu$MVs. Given an achievable speed $v_{M}$, one obtains the resulting graph as follows:

$$70, 0, 0_{RM} : R_{\geq 0} \rightarrow \langle G, \leq G \rangle$$

where $\leq = (\pi_{s,CR_{M}}, \pi_{s,CR_{M}})$, $\pi_{s,CR_{M}}$ is the functionality of servicing a specific network with a specific performance and requires the resources $C_{M_{CR_{M}}}, C_{M_{CR_{M}}} \in \mathbb{R}_{\geq 0}$. The implementations space $I_{M}$ consists of instances of the $\mu$MVs. Formally: $d_{MM} : G \rightarrow \mathbb{R}_{\geq 0}^2$.

Lemma 3.10: $d_{MM}$ is a well-defined MDPI.

3) The Subway MDPI: The public transit MDPI comprises the selection of the labeled network on which to operate, now resumed in the choice of fleet size for the subway system. Specifically, we assume the service frequency $f_{avg}$ to scale monotonically with the size of the train fleet $n_{S}$. In the linear case, one has:

$$\frac{\varphi_{j}}{\varphi_{j,base}} = \frac{n_{S}}{n_{S,base}},$$

where $\varphi_{j,base}$ and $n_{S,base}$ are respective existing baselines. Given a train fleet size, one obtains the resulting network as follows:

$$70, 0, 0_{P} : N \rightarrow \langle G, \leq G \rangle$$

where $n_{S} = \varphi_{j} + \frac{n_{S,base}}{2}$

$$c = \langle \pi_{s,CP}, l_{WS} + \frac{n_{S,base}}{2} \pi_{s,CP} = \varphi_{j,base}, \pi_{s,CP}, \pi_{s,CP} \rangle.$$

Lemma 3.12: The map $70, 0, 0_{P}$ is monotone.

We relate a train fleet of size $n_{S}$ to the fixed costs $C_{S_{f}}$ (accounting for train and infrastructural costs) and to the operational costs $C_{S, o}$ (accounting for energy consumption, vehicles depreciation, and train operators’ wages). Given the passengers-independent public transit operation in today’s cities, we assume the operational costs $C_{S, o}$ to be mileage independent and to only vary with the size of the fleet. Assuming an average train’s life of $l_{S}$, and a baseline subway fleet of $n_{S, baseline}$ trains, costs are

$$C_{S} = \frac{C_{S_{f}}}{l_{S}} \cdot n_{S,base} + C_{S, o}.$$

Moreover, operating a fleet of trains entails the CO$_2$ emissions

$$m_{CO_{2},tot} = m_{CO_{2},S} \cdot n_{S}.$$

MDPI Definition: The public transit MDPI, denoted $d_{P}$, provides the functionality $G_{P} \in G$ (i.e., the functionality of servicing a specific network with a specific performance) and requires the resources $C_{S} \in \mathbb{R}_{\geq 0}$ and $m_{CO_{2},S, tot} \in \mathbb{R}_{\geq 0}$. The implementations space $I_{P}$ consists of different train acquisition choices. Formally: $d_{P} : G \rightarrow \mathbb{R}_{\geq 0}^2$.

Lemma 3.13: $d_{P}$ is a well-defined MDPI.

4) The Intermodal Mobility System MDPI (Version 1): The first version of the intermodal mobility system MDPI considers demand satisfaction as a functionality.

To successfully satisfy a given set of travel requests, we require the following resources:

- the mobility network resulting from the design of AVs, $AVs G_{AV} = \langle V_{AV}, A_{AV}, c_{AV} \rangle$,
- the mobility network resulting from the design of public transit $G_{P} = \langle V_{P}, A_{P}, c_{P} \rangle$,
- the number of available AVs per fleet $n_{V, max}$,
- the average travel time of a trip $t_{avg} := \frac{1}{\alpha_{tot}} \sum_{(i,j) \in A_{AV}, \forall i, \forall j} \pi_{s,AV}(i, j) \cdot f_{tot}(i, j)$,

where $\alpha_{tot} := \sum_{m \in M} \alpha_{m}$.

- the total distance driven by the AVs per unit time $s_{V, tot} := \sum_{(i, j) \in A_{AV}} \pi_{s,AV}(i, j) \cdot f_{tot, V}(i, j)$,
- the total AVs CO$_2$ emissions per unit time $m_{CO_{2}, tot} := \gamma \cdot \sum_{(i, j) \in A_{AV}} \pi_{s,AV}(i, j) \cdot f_{tot, V}(i, j)$.

We assume that AVs are routed to maximize the customers’ welfare, defined without loss of generality as the average travel time $t_{avg}$. Hence, we link functionality and resources of the mobility system MDPI through the optimization problem:

$$\min_{t_{avg} \leq t_{lim}} \frac{1}{h_{V}}$$

s.t. Eq.(1),

$$\forall (i, j) \in A_{AV},$$

$$\sum_{(i, j) \in A_{AV}} f_{tot, V}(i, j) \cdot \pi_{s,AV}(i, j) \leq n_{V, max},$$
where we express the number of vehicles on arc \((i,j)\) as the multiplication of the total vehicles flow on the arc and its travel time.

**MDPI Definition:** The intermodal mobility system MDPI has as functionality the satisfied requests \(Q \subseteq \mathcal{Q}\) and the mentioned resources. Furthermore, \(\mathcal{I}_0\) consists of specific intermodal scenarios. Formally: \(d_{\text{IAMOD}} : \mathcal{Q} \rightarrow \mathbb{G}^2 \times \mathbb{N} \times \mathbb{R}^3_0\).

**Lemma 3.14:** \(d_{\text{IAMOD,1}}\) is a well-defined MDPI.

5) The Intermodal Mobility System MDPI (Version 2): The second version of the intermodal mobility system MDPI still considers demand satisfaction as a functionality, now including \(\mu M\) options. To successfully satisfy a given set of travel requests, we require the following resources:

- \(G_{AV} = (\mathcal{V}_{AV}, \mathcal{A}_{AV}, c_{AV})\) as in Section III-E4,
- \(G_{P} = (\mathcal{V}_{P}, \mathcal{A}_{P}, c_{P})\) as in Section III-E4,
- the mobility network resulting from the design of \(\mu Ms\), \(\mu Ms G_{MM} = (\mathcal{V}_{MM}, \mathcal{A}_{MM}, c_{MM})\),
- \(n_{V,max}\) as in Section III-E4,
- the number of available \(\mu M\)Vs per fleet \(n_{MV,max}\),
- the (adapted) average travel time of a trip

\[
\bar{t}_{avg} := \frac{1}{\alpha_{tot}} \sum_{(i,j) \in \mathcal{A}_{AV}} \pi_t c_{AV}(i,j) \cdot f_{m}(i,j),
\]

with \(\alpha_{tot}\) as in (5),

- \(s_{V,tot}\) as in (6),
- the total distance driven by the \(\mu M\)Vs per unit time

\[
s_{M,tot} := \sum_{(i,j) \in \mathcal{A}_{MM}} \pi_t c_{MM}(i,j) \cdot f_{tot,M}(i,j),
\]

- \(m_{CO2,V,tot}\) as in (7),
- the total \(\mu M\)Vs CO2 emissions per unit time

\[
m_{CO2,M,tot} := \gamma \sum_{(i,j) \in \mathcal{A}_{MM}} \pi_t c_{MM}(i,j) \cdot f_{tot,M}(i,j),
\]

where \(\gamma\) relates energy consumption and CO2 emissions. We assume that AVs and \(\mu M\)Vs are routed to maximize the customers’ welfare, defined without loss of generality as the average travel time \(\bar{t}_{avg}\). Hence, we link functionality and resources of the mobility system MDPI through the following optimization problem, extending (8):

\[
\begin{align*}
\min_{\{h\}_m,h_{AV},h_{MM}} & t_{avg} \\
\text{s.t.} \quad & \text{Eq. (1),} \\
& \text{Eq. (2) \quad \forall (i,j) \in \mathcal{A}_{AV},} \\
& \sum_{(i,j) \in \mathcal{A}_{AV}} f_{tot,V}(i,j) \cdot \pi_t c_{AV}(i,j) \leq n_{V,max}, \\
& \sum_{(i,j) \in \mathcal{A}_{MM}} f_{tot,M}(i,j) \cdot \pi_t c_{MM}(i,j) \leq n_{M,max},
\end{align*}
\]

where we express the number of vehicles on arc \((i,j)\) as the multiplication of the total vehicles flow on the arc and its travel time.

**MDPI Definition:** The intermodal mobility system MDPI has as functionality \(Q \subseteq \mathcal{Q}\) and the mentioned resources. Furthermore, \(\mathcal{I}_0\) consists of specific intermodal scenarios. Formally: \(d_{\text{IAMOD,2}} : \mathcal{Q} \rightarrow \mathbb{G}^3 \times \mathbb{N} \times \mathbb{R}^5_{\geq 0}\).

**Lemma 3.15:** \(d_{\text{IAMOD,2}}\) is a well-defined MDPI.

6) The Intermodal Mobility System MDPI (Version 3): We extend the setting presented in Section III-E4 by including a new functionality. Specifically, the intermodal mobility system MDPI not only provides demand satisfaction as a functionality, but also provides the revenue \(\rho\) arising from the mobility offer, which reads:

\[
\rho = p_{AV}s_{V,tot} + p_{P} \sum_{(i,j) \in \mathcal{A}_{AV}} f_{m}(i,j),
\]

where \(p_{AV}\) is a distance-based price to use AVs and \(p_{P}\) is a fixed entry price for the subway system. Accordingly, we modify the optimization problem to account for both average travel time and average cost of fare:

\[
\begin{align*}
\min_{\{h\}_m,h_{AV},h_{MM}} & V_{T}t_{avg} + \frac{1}{\alpha_{tot}} \rho \\
\text{s.t.} \quad & \text{Eq. (1),} \\
& \text{Eq. (2) \quad \forall (i,j) \in \mathcal{A}_{AV},} \\
& \sum_{(i,j) \in \mathcal{A}_{AV}} f_{tot,V}(i,j) \cdot \pi_t c_{AV}(i,j) \leq n_{V,max},
\end{align*}
\]

where \(V_{T}\) is the value of time.

**MDPI Definition:** This new version of the intermodal mobility system MDPI has as functionality the satisfied requests \(Q \subseteq \mathcal{Q}\) and the total revenue \(\rho \in \mathbb{R}^3_{\geq 0}\) and the mentioned resources. Furthermore, \(\mathcal{I}_0\) consists of specific intermodal scenarios (including specific price choices). Formally: \(d_{\text{IAMOD,3}} : \mathcal{Q} \rightarrow \mathbb{G}^3 \times \mathbb{N} \times \mathbb{R}^5_{\geq 0}\).

**Lemma 3.16:** \(d_{\text{IAMOD,3}}\) is a well-defined MDPI.

7) The Mobility MDPI (Version 1): The functionality of the system is to satisfy the customers’ demand. Formally, the functionality provided by the CDP is the set of travel requests and coincides with the functionalities of \(d_{l_V}\). To provide the mobility service, three resources are required. First, on the customers’ side, we require the average travel time defined in Section III-E4. Second, on the side of the central authority, the resource is the total transportation cost of the intermodal mobility system. Assuming an average AV’s life of \(l_{AV}\), an average \(\mu M\)V’s life of \(l_{MM}\), we express the total costs as

\[
C_{tot} = C_{V} + C_{S},
\]

where \(C_{V}\) is the AVs-related cost

\[
C_{V} = \frac{C_{V,f}}{l_{AV}} \cdot n_{V,max} + C_{V,o} \cdot s_{V,tot},
\]

and \(C_{S}\) is the public transit-related cost. Third, on the environmental side, we consider the total CO2 emissions

\[
m_{CO2,tot} = m_{CO2,V,tot} + m_{CO2,S,tot}.
\]
Formally:

The MDPI of the total costs, i.e., $C_{\text{tot}}$, we get

$$C_{\text{tot}} = C_V + C_M + C_S,$$

where $C_M$ is the $\mu$MV-related cost

$$C_M = \frac{C_{Mf}}{t_M} \cdot n_{M,\text{max}} + C_{M_o} \cdot s_{M,\text{tot}},$$

Third, we add the $\mu$M-related emissions to the ones computed in Section III-E7:

$$m_{CO_2,\text{tot}} = m_{CO_2,V,\text{tot}} + m_{CO_2,M,\text{tot}} + m_{CO_2,S,\text{tot}}.$$
from OpenStreetMap [50], whilst the public transit network together with its schedules are extracted from GTFS [51]. Original demand data is obtained by merging origin-destination pairs of the morning peak of 1st May, 2017, provided by taxi companies [52] and the Washington Metropolitan Area Transit Authority (WMATA) [53]. On the public transportation side, we focus our studies on the MetroRail system and its design. To take account of the recently increased presence of ride-hailing companies, the taxi demand rate is scaled by a factor of 5 [54]. The complete demand dataset includes 16430 distinct origin-destination pairs, describing travel requests. To account for congestion effects, the nominal road capacity is computed as in [55] and an average baseline usage of 93 % is assumed, in line with [56]. We assume an AV fleet composed of battery electric BEV-250 mile AVs [57]. We summarize the main parameters characterizing our case studies together with their bibliographic sources in Table I. In the remainder of this section, we solve the co-design problem presented in Section III 2.

The diagrams we reported represent the “skeleton” of the design hierarchy. In order to fill the blocks, one needs feasibility relations, which have been described in previous sections. Note that the proposed approach is extremely flexible, since it allows one to specify feasibility relations via catalogues (e.g., for vehicle models), formulas (e.g., for the cost structures), and simulation/optimization problems (e.g., for the intermodal mobility system). Once one identifies the MDPIs, one can directly use the PyMCDP solver [58]. The solver provides the full set of optimal solutions. If it converges to an empty set, the solution corresponds to a certificate of infeasibility. Beside our basic setting (S1), we evaluate the sensitivity of the design strategies to different models of automation costs of AVs (S2–S4), assess the impact of emerging m solutions, showing how one can easily include new modes of transportation in the framework (S5), and investigate pricing strategies in (S6). We summarize the considered mobility solutions and their complementarity in Table II.

### Table I

| Parameter                     | Variable         | Value | Units | Source |
|-------------------------------|------------------|-------|-------|--------|
| Road usage                    | \( n_{ij} \)     | 93    | %     | [58]   |
| AVs operational cost          | \( CV_{av} \)    | 0.084 |      |        |
| Vehicle cost                  | \( CV_{v} \)     | 32    | 0.084 |        |
| 20 mph                        | 15 20 3.7 500 20 3.7 | 0.084 |        |
| 25 mph                        | 15 30 4.4 500 0 4.4 |      |        |
| 30 mph                        | 15 55 6.2 500 0 6.2 |      |        |
| AV automation cost            | \( CV_{av} \)    | 32    | 0.084 |        |
| 26 mph                        | 15 90 8.7 500 90 8.7 | 0.084 |        |
| 40 mph                        | 15 115 9.8 500 0 9.8 |      |        |
| 45 mph                        | 15 130 12 500 0 12 |      |        |
| 50 mph                        | 15 150 15 500 0 15 |      |        |
| AV life                       | \( i_{v} \)      | 5     | 5     | [9]    |
| CO2 per Joule                 | \( \gamma \)     | 0.14  | 0.14  |        |
| Time \( t_{AV} \) to \( t_{AV} \) | \( t_{NV} \)     | 300   | 300   | s      |
| Time \( t_{AV} \) to \( t_{AV} \) | \( t_{NV} \)     | 300   | 300   | s      |
| Time from \( t_{AV} \) to \( t_{AV} \) | \( t_{AV} \)     | 60    | 60    | s      |
| Speed limit fraction \( \beta \) | 1/1.3             | 1/1.3 | 1/1.3 | 1/1.3 |

| ES                            | SB                            | M | FCM |
|-------------------------------|-------------------------------|---|-----|
| \( \mu MV \) operational cost | \( CM_{av} \)                 | 0.79 | 1.58 | 2.05 | 1.20 | USD/mile | [67]–[69] |
| \( \mu MV \) cost             | \( CM_{v} \)                  | 0.79 | 1.58 | 2.05 | 1.20 | USD/mile | [67]–[69] |
| \( \mu MV \) achievable speed | \( CM_{ij} \)                 | 15  | 15   | 15   | 15   | mph      |        |
| \( \mu MV \) life             | \( t_{MT} \)                  | 0.085 | 7.0  | 10.0 | 10.0 | year     | [68]–[70] |
| \( \mu MV \) emissions        | \( m_{CO2,M_{av}} \)          | 0.101 | 0.033 | 0.158 | 0.033 | kg/mile | [68], [71]–[73] |
| Time from \( t_{AV} \) to \( t_{AV} \) | \( t_{AV} \)     | 60    | 60    | 60    | 60    | s        |        |
| Time from \( t_{AV} \) to \( t_{AV} \) | \( t_{AV} \)     | 60    | 60    | 60    | 60    | s        |        |
| 100 %                         | 148,000,000                 | 148,000,000 | USD/year | [74] |
| 150 %                         | 222,000,000                 | 222,000,000 | USD/year | [74] |
| 200 %                         | 295,000,000                 | 295,000,000 | USD/year | [74] |
| \( CS_{v} \)                  | 14,500,000                 | 14,500,000 | USD/train | [75] |
| \( CS_{f} \)                  | 14,500,000                 | 14,500,000 | USD/train | [75] |
| Train life                    | \( i_{v} \)                 | 100 % | 100   | 100   | 100   | year     | [73]   |
| Subway fixed cost            | \( CS_{f} \)                 | 14,500,000 | USD/train | [75] |
| Subway emissions per train    | \( t_{AV} \)                | 140,000 | USD/year | [76] |
| Train fleet baseline          | \( m_{AV} \)                | 112  | 112   | 112   | 112   | train    | [75]   |
| Subway service frequency      | \( \psi_{f} \)              | 1/6  | 1/6   | 1/6   | 1/6   | s        |        |
| Time \( t_{AV} \) to \( t_{AV} \) | \( t_{AV} \)     | 60    | 60    | 60    | 60    | s        |        |

2 The solution techniques for this kind of optimization problems and their complexity are described in [3, Proposition 5], in the appendix, and in our talk at https://bit.ly/3eliO6f. We are writing books on the subject, and teaching classes; see https://applied-compositional-thinking.engineering.
structure, detailed in Table I. The large variance in sensing technologies available on the market and their performances suggests that AV costs are, in fact, performance-dependent [7], [76]. Indeed, the technology currently required to safely operate an autonomous vehicle at 50 mph is substantially more sophisticated, and therefore more expensive, than the one needed at 20 mph. Furthermore, the frenetic evolution of automation techniques will inevitably reduce automation costs: Experts forecast a massive automation cost reduction (up to 90%) in the next decade, principally due to mass-production of AVs sensing technology [77], [78]. Therefore, we perform our studies with current (2022) automation costs as well as with their projections for the upcoming years (2025) [57], [75], [78].

**S3 - High automation costs:** We assess the impact of high automation costs. In particular, we assume a performance-independent automation cost of 0.5 MilUSD/car, capturing the extremely high research and development costs that AVs companies are facing today [79], as well as insurance costs and infrastructural investments. The latter, often referred to as “autonomy-enabling infrastructure,” would allow high driving speeds, and could consist of dedicated roads, equipped with sensors and cloud computing capabilities, enhancing the performance of AVs.

**S4 - MoD setting:** We analyze the current Mobility-on-Demand (MoD) case. The cost structure of MoD systems is characterized by lower vehicle costs (due to lack of automation) and higher operation costs, mainly due to drivers’ salaries.

**S5 - Impact of new transportation modes:** We show the modularity of our framework by evaluating the impact of μM solutions on urban mobility (Section III-E8). We consider ESs (e.g., Lime in DC), SBs (e.g., Capital Bikeshare in DC), Ms (e.g., Revel in DC), and FCMs. In addition to the design parameters introduced in the basic setting, we design the specific μM solution $M \in \{ES, SB, M, FCM\}$ and the μM fleet size $n_{M, max} \in \{500, 1000, \ldots, 5000\}$ vehicles (see Lemma 3.10). We study the joint deployment of μM solutions and AVs, and therefore consider the extended settings of 2022 and 2025.

**S6 - Pricing:** We show another extension of our framework to capture pricing strategies and infrastructure-contributing revenues (Section III-E9) in the 2022 setting. We consider AMoD service providers that choose from an exemplary set of prices $\{0.8, 1.2, 1.6, 2.4, 3.2\}$ (expressed in USD/mile) and public transit authorities choosing fare prices from the set $\{1.0, 2.0, 4.0, 6.0\}$ (expressed in USD per ride). Furthermore, we consider a municipality willing to cover 50% (just a particular choice) of the investment cost through the revenues of mobility services. (i.e., the revenue gained from travelers paying for the trips should at least be enough to cover 50% of the investment costs).

### B. Results

1) **Basic Setting:** Fig. 4(a) reports the solution of the co-design problem through the antichain consisting of the total $CO_2$ emissions, average travel time, and total transportation cost. The design solutions are rational (and not comparable), since there exists no instance which simultaneously yields lower emissions, average travel time, and cost.

In the interest of clarity, we prefer a two-dimensional antichain representation, where emissions are included in the costs via a conversion factor of $40/USD/kg$ [80]. Note that this transformation preserves the monotonicity of the CDPI and therefore integrates in our framework. The two-dimensional antichain and the corresponding central authority’s decisions are reported in Fig. 4(b). In general, as the municipality budget increases, the average travel time per trip required to satisfy the given demand decreases, reaching a minimum of about 20.7 min, with a monthly public expense of around 36 Mil USD/month. This configuration corresponds to a fleet of 4000 AVs able to drive at 50 mph, and to the doubling of the current MetroRail train fleet. Furthermore, the smallest rational investment of 13 Mil USD/month leads to a 22% higher average travel time, corresponding to the current situation, i.e., to a non-existent AVs fleet, and an unchanged subway infrastructure. Notably, an expense of 18 Mil USD/month (50% lower than the highest rational investment) only increases the minimal required travel time by 8%, requiring a fleet of 3000 AVs able to drive at 45 mph and no acquisition of trains. Conversely, an expense of 15 Mil USD/month (just 2 Mil USD/month higher than the minimal rational investment) provides a 2 min shorter travel time. Finally, it is rational to improve the subway system starting from a budget of 23 Mil USD/month, leading to a travel improvement of just 8%. This trend can be explained with the high train acquisition cost and increased operation costs, related to the reinforcement of the subway system. This phenomenon is expected to be even more marked for other cities, considering the moderate operation costs of the MetroRail subway system, due to its automation and related benefits [81].

2) **Speed-Dependent Automation Costs:**

2022: We report the results in Fig. 5(a). A comparison with our basic setting (cf. Fig. 4) confirms the trends concerning public expense. Indeed, a public expense of 26 Mil USD/month (43% lower than the highest rational expense) only increases
the average travel time by 5%, requiring a fleet of 2000 AVs able to reach 30 mph and a subway reinforcement of 50%. Nevertheless, our comparison shows two substantial differences. First, the budget required for an average travel time of 13 min is 25% higher compared to S1. Second, the higher AV costs result in an average AVs fleet growth of 9%, an average velocity reduction of 15%, and an average train fleet growth of 14%. The latter suggests a shift towards poorer AVs performance in favor of fleets reinforcements.

2025: The maximal rational budget is 23% lower than in the case of immediate deployment (Fig. 5(b)). Further, the reduction in autonomy costs incentivizes the acquisition of more performant AVs, increasing the average vehicle speed by 14%. Hence, AVs and train fleets are 10% and 13% smaller.

3) High Automation Costs Analysis: Fig. 6 shows the results for high automation costs. First, we observe a substantial shift towards larger train fleet sizes (65% larger than in S1) and smaller AVs fleets (55% smaller than in S1). Second, minimizing the average travel time entails an expense of approximately 68 Mil USD/month, basically doubling the investments observed in the basic setting.
4) MoD Setting: We summarize the results for the MoD scenario in Fig. 7. In particular, by comparing the MoD case with the 2025 setting, we can notice the game-changing properties that AVs introduce in the mobility ecosystem. In particular, the average train fleet size and the average vehicle fleet sizes increase by $130\%$ and $66\%$, suggesting a clear transition in investments from public transit to AVs, and testifies to the interest in AMoD systems developed in the past years.

5) Impact of New Transportation Modes: To assess the impact of $\mu M$ solutions, we compare the arising design solutions, reported in Fig. 8, with their counterpart in S2 (cf. Fig. 5).

2022: Fig. 8(a), together with Fig. 5(a), demonstrates an overall benefit from $\mu M$ solutions. For instance, the most time-efficient solution in S2 yields an average travel time of $20.7$ min at an expense of $45$ Mil USD/month. The deployment of $\mu M$ solutions lowers the average travel time achievable with the same expense by $10\%$ ($18.8$ min) and allows for even lower average travel times, with a time-efficient solution of $17.6$ min at an investment plan of $84$ Mil USD/month. Overall, the average AVs fleet size and the average train fleet size are $35\%$ and $6\%$ smaller, in favor of an average $\mu M$ fleet of 2280 $\mu$MVs.

2025: Fig. 8(b), together with Fig. 5(b), shows that the benefit of $\mu M$ solutions is less marked than in 2022. For instance, an expense of $35$ Mil USD/month (same as the maximal expense in Fig. 5(b)) results in an average travel time of $19.5$ min, i.e., only $6\%$ lower than in the case without $\mu M$. Furthermore, we observe an average AVs fleet size enlargement of $17\%$, and an average train fleet size reduction of $27\%$. Finally, the comparison with the 2022 case highlights a $\mu M$Vs fleet reduction of $23\%$, which suggests the comparative advantage of AVs in the future. Indeed, the stronger the reduction of the cost of automation, the more investments in AVs are rational. The benefits of employing $\mu M$ solutions could therefore just be temporary, and gradually vanish as the costs of automation of AVs decrease.

6) Pricing: We report the results in Fig. 9. In particular, we report the Pareto front between system performance (average travel time) and produced externalities.

C. Discussion

First, the presented case studies showcase the ability of our framework to extract the set of rational design strategies for a future mobility system, including AVs, $\mu M$Vs, and public transit. This way, stakeholders such as mobility providers, transportation authorities, and policy makers can get transparent and interpretable insights on the impact of future interventions, inducing further reflection on this complex socio-technical problem. Note that this kind of results is only one of the many factors affecting negotiations when interacting with stakeholders. Second, we perform a sensitivity analysis through the variation of autonomy cost structures, and show the capacity of our framework to capture various models. On the one hand, this reveals a clear transition from small fleets of fast AVs (in the case of low autonomy costs) to large fleets of slow AVs (in the case of high
V. CONCLUSION

This paper leverages the mathematical theory of co-design to propose a co-design framework for future mobility systems. The nature of our framework offers a different viewpoint on the future mobility problem, enabling the modular and compositional interconnection of the design problems of different mobility options and their optimization, given multiple objectives. Starting from the multi-commodity flow model of an intermodal mobility system, we designed AVs, μMVs, and public transit both from a vehicle-centric and fleet-level perspective. Specifically, we studied the problem of deploying a fleet of self-driving vehicles providing on-demand mobility in cooperation with μM solutions and public transit, adapting the speed achievable by AVs and μMVs, their fleet sizes, and the service frequency of the subway lines. Our framework allows stakeholders involved in the mobility ecosystem, from vehicle developers all the way to mobility-as-a-service companies and central authorities, to characterize rational trajectories for technology and investment development. We showcased both the developer and the user views of the framework, explaining how practitioners can easily use their models within it. The proposed methodology is showcased in a case study based on data for Washington D.C., USA. Notably, we highlighted how our problem formulation allows for a systematic analysis of incomparable objectives, such as public expense, average travel time, and emissions, providing stakeholders with analytical insights for the socio-technical design of future mobility systems. This work urges the following future research streams:

**Modeling:** First, we would like to capture heterogeneous fleets of AVs, with different autonomy pipelines, propulsion systems, and passenger capacity. For instance, the modular nature of the framework allows one to easily include complex autonomy models in the design problem of the AV fleet [7], [8]. Second, we would like to investigate variable demand models. Third, we would like to analyze the interactions between multiple stakeholders in the mobility ecosystem, characterized by conflicting interests and different action spaces. It is advantageous to formulate this as a game, and to characterize potentially arising equilibria [82], [83], possibly leveraging recent results in posetal games [84]. This might bring realism and effectiveness in the actionable information proposed to the mobility stakeholders. Finally, we would like to explicitly include (and not just via costs) more elements of urban design in the co-design model, to account for more realistic scenarios [85]. These include parking spaces and autonomy-enabling infrastructure.

**Algorithms:** We are interested in tailoring general co-design algorithmic frameworks to the particular case of transportation design problems, leveraging their specific structure, and characterizing their solutions. In particular, we would like to study adaptive approaches to cleverly simulate mobility systems.

**Appendix A**

**Nomenclature**

**AMoD-related symbols**

- $s_{ij}$ = Length of arc $(i, j)$
- $c$ = Edge-coloring maps.
- $A$ = Set of arcs of a digraph.
- $G$ = Digraph.
- $V$ = Set of vertices of a digraph.
- $\rho$ = Revenue.
- $q$ = Travel demand.
- $\sigma_{cycle}$ = Length of cycle.
- $v_{AV}$ = Speed autonomous vehicle (AV).
- $v_M$ = Speed micromobility ($\mu$M).
- $\beta$ = Threshold for minimal speed.
- $\alpha$ = Demand rate.
- $\gamma$ = Energy consumption to CO$_2$ emissions.
- $C_{V,t}$ = Vehicle fix cost.
- $n_{CO_2,V, tot}$ = Emissions micromobility vehicle ($\mu$M).
- $n_{CO_2,S, tot}$ = Emissions subway.
- $n_{CO_2,V, tot}$ = Emissions AV.
- $\varphi_j$ = Subway service frequency.
- $b_{base}$ = Subway baseline service frequency.
- $l_M$ = Lifetime μM.
- $l_{AV}$ = Lifetime subway train.
- $l_{V}$ = Lifetime AV.
- $\text{red}$ = Graph reduction maps.
- $n_{M, max}$ = Fleet size μMVs.
- $n_S$ = Fleet size subway.
- $n_{AV, tot}$ = Used μMVs.
- $n_{V, tot}$ = Used AVs.
- $n_{V, max}$ = Fleet size AVs.
- $\pi$ = Projection maps.
- $G$ = Set of graphs.
- $C_{M, tot}$ = μM fix cost.
- $C_{S, tot}$ = Subway fix cost.
- $C_{M, tot}$ = μM operational cost.
- $C_{S, tot}$ = Subway operational cost.
- $C_{AV}$ = AV operational cost.
- $C_{tot}$ = Total cost.
- $s_{M, tot}$ = Total distance μM.
- $s_{V, tot}$ = Total distance AV.
- $t_{ij}$ = Travel time of arc $(i, j)$
- $t_{avg}$ = Average travel time.
- $f_{c}(i, j)$ = Flow of customers per unit time on arc $(i, j)$
- $f_{OV}(i, j)$ = Flow of empty AVs on arc $(i, j)$
- $f_{OM}(i, j)$ = Flow of empty μMVs on arc $(i, j)$
- $f_{tot}(i, j)$ = Total flow of μMVs on arc $(i, j)$
- $f_{tot,AV}(i, j)$ = Total flow of AVs on arc $(i, j)$
- $v_W$ = Walking speed.
- $u_{ij}$ = Baseline usage for arc $(i, j)$
- $e_{ij}$ = Energy consumption for arc $(i, j)$
- $k_{ij}$ = Normal capacity of arc $(i, j)$
- $v_{ij}$ = Speed limit for arc $(i, j)$
- $v_{MO}$ = Speed of μMVs on arc $(i, j)$
- $v_{VJ}$ = Speed of AVs on arc $(i, j)$

**Related Symbols**

- $M$ = Set of arc sets.
- $V$ = Set of vertices.
- $C$ = Set of costs.
- $G$ = Digraph.
- $S$ = Set of edges.
- $A$ = Set of arcs.
- $\gamma$ = Fraction of the revenue to cover costs.
- $\kappa$ = Cost of the arc.
- $\pi$ = Projection maps.
- $\rho$ = Revenue.
- $T$ = Arrival time.
- $D$ = Departure time.
- $\delta$ = Fixed cost.
- $\Delta$ = Distance.
- $\alpha$ = Demand rate.
- $\beta$ = Threshold for minimal speed.
- $\gamma$ = Energy consumption to CO$_2$ emissions.
- $C_{V,t}$ = Vehicle fix cost.
- $n_{M, max}$ = Fleet size μMVs.
- $n_S$ = Fleet size subway.
- $n_{AV, tot}$ = Used μMVs.
Co-design-related symbols

\( \perp, \top \) = Bottom and top of a poset.
\( \mathcal{P} \) = Poset and powerset.
\( \mathcal{F} \) = Functionalities.
\( \mathcal{R} \) = Resources.
\( d \) = Design problem.
\( \mathcal{AP} \) = Set of antichains for poset \( \mathcal{P} \)
\( \text{prov} \) = Implementation-to-functionality map.
\( \text{reqs} \) = Implementation-to-resources map.

Background on Solution of Co-Design Problems

For the convenience of the reader, in the following we report technical results from [3], [6].

1) Solution of CDPI

We first recall concepts related to fixed points.

Definition 1.1 (Least fixed point): A least fixed point of \( f : \mathcal{P} \rightarrow \mathcal{P} \) is the minimum (if it exists) of the set of fixed points of \( f \):

\[
\text{lfp}(f) = \min \{ x \in \mathcal{P} : f(x) = x \}.
\]

A least fixed point might not exist. Monotonicity of the map \( f \) and completeness of the partial orders is sufficient to ensure existence.

Definition 1.2 (Completeness): A poset is a directed complete partial order (DCPO) if each of its directed subsets has a supremum (least of upper bounds). It is a complete partial order (CPO) if it also has a bottom.

Example 1.3: Consider \( \mathbb{R}_{\geq} = \{ x \in \mathbb{R} \mid x \geq 0 \} \), which has a bottom \( \perp = 0 \). One can make \( \langle \mathbb{R}_{\geq}, \leq \rangle \) a CPO by adding an artificial top element \( \top \), by defining \( \mathbb{R}_{\geq} := \mathbb{R}_{\geq} \cup \{ \top \} \), and extending the partial order \( \leq \) such that \( a \leq \top \) for all \( a \in \mathbb{R}_{\geq} \).

Lemma 1.4 (Lemma 3 in [3]): If \( \mathcal{P} \) is a CPO and \( f : \mathcal{P} \rightarrow \mathcal{P} \) is monotone, then lfp(\( f \)) exists.

Assuming Scott continuity of \( f \), Kleene’s algorithm is a systematic procedure to find the least fixed point.

Definition 1.5 (Scott continuity): A map \( f : \mathcal{P} \rightarrow \mathcal{Q} \) between DCPOs is Scott continuous if and only if, for each directed subset \( D \subseteq \mathcal{P} \), the image \( f(D) \) is directed, and \( f(\text{sup} D) = \text{sup} f(D) \).

Lemma 1.6 (Lemma 4 in [3]): Assume \( \mathcal{P} \) is a CPO and \( f : \mathcal{P} \rightarrow \mathcal{P} \) is Scott continuous. Then, the least fixed point of \( f \) is the supremum of the Kleene ascent chain

\[
\perp \leq f(\perp) \leq f(f(\perp)) \leq \ldots \leq f^n(\perp) \leq \ldots.
\]

Note that a sufficient condition is to assume all posets to be finite.

Theorem 1.7: The map \( h \) for a CDPI has an explicit expression in terms of the maps \( h_d \) of its subproblems.

We report the specific definition of these maps.

Definition 1.8 (Series): For two maps \( h_1 : \mathcal{F}_1 \rightarrow \mathcal{AR}_1, \ h_2 : \mathcal{F}_2 \rightarrow \mathcal{AR}_2 \), if \( \mathcal{R}_1 = \mathcal{F}_2 \), define

\[
h_1; h_2 : \mathcal{F}_1 \rightarrow \mathcal{AR}_2, \quad h_1; h_2(f) = \min \{ h_2(s) \mid s \in h_1(f) \}.
\]

Definition 1.9 (Parallel): For two maps \( h_1 : \mathcal{F}_1 \rightarrow \mathcal{AR}_1, \ h_2 : \mathcal{F}_2 \rightarrow \mathcal{AR}_2 \), define

\[
h_1 \otimes h_2 : \mathcal{F}_1 \times \mathcal{F}_2 \rightarrow \mathcal{AR}_1 \times \mathcal{AR}_2, \quad (f_1, f_2) \mapsto h_1(f_1) \times h_2(f_2).
\]

Definition 1.10 (Loop): For \( h : \mathcal{F} \times \mathcal{R} \rightarrow \mathcal{AR} \) define

\[
h^l : \mathcal{F} \rightarrow \mathcal{AR}, \quad f \mapsto \text{lfp}(\Psi^l_f),
\]

where

\[
\Psi^l_f : \mathcal{AR} \rightarrow \mathcal{AR}, \quad \Psi^l_f = \min \bigcup_{r \in \mathcal{R}} h(f, r) \cap \uparrow \{ r \},
\]

where \( \uparrow \) represents the upper closure operator.

Definition 1.11 (Coprod): For \( h_1, h_2 : \mathcal{F} \rightarrow \mathcal{AR} \), define

\[
h_1 \lor h_2 : \mathcal{F} \rightarrow \mathcal{AR}, \quad f \mapsto \min \{ h_1(f) \cup h_2(f) \}.
\]

Algorithmic sketch

Given the aforementioned discovery, the algorithmic procedure to solve co-design problems is the following. 1) Take an arbitrary CDPI (in connection of MDPIs). 2) Flatten it to a graph. 3) Re-write the graph in form of a series of series-parallel and feedback graphs. 4) Write the graph as a tree of composition operations. 5) Run Kleene’s iteration recursively on the graph.

2) Complexity of the Solution

The results for complexity are described in [3]. Consider a CDPI. The space of the solution is bounded by the width of \( \mathcal{R} \). At each iteration, the number of evaluations of each component is linear in the number of options. The number of execution steps depends on the height of the poset of antichains of \( \mathcal{R} \).

B. PROOFS

Proof of Lemma 3.2: Consider partial orders \( A, B, C \) and maps \( f, g, h : A \rightarrow B \). Clearly \( f \leq g \rightarrow f \). Furthermore, if \( f \preceq g \) and \( g \preceq h \) (i.e., \( f(a) \preceq g(a) \) and \( g(a) \preceq h(a) \), \( \forall a \in A \)), then \( f(a) \preceq g h(a) \Rightarrow a \in A \), implying \( f \preceq g \). Finally, if \( f \preceq g \) and \( g \preceq f \) one has \( f = g \).

Proof of Lemma 3.4: Consider \( G_1, G_2, G_3 \in \mathbb{G} \) with

\[
G_1 = \langle V_1, A_1, c_1 \rangle, \quad G_2 = \langle V_2, A_2, c_2 \rangle, \quad G_3 = \langle V_3, A_3, c_3 \rangle,
\]

and \( c_1 : A_1 \rightarrow C \). Clearly \( G_1 \preceq G_1 \), since \( V_1 \subseteq V_1, A_1 \subseteq A_1, \) and \( c_1 \preceq c_1 \). Furthermore, \( G_1 \preceq G_2 \) and \( G_2 \preceq G_3 \) (i.e., \( V_1 \subseteq V_2 \subseteq V_3, A_1 \subseteq A_2 \subseteq A_3, \) \( c_1 \preceq c_1 \), \( c_2 \preceq c_3 \)), one has \( V_1 \subseteq V_3, A_1 \subseteq A_3, \) and \( c_1 \preceq c_2 \), \( c_2 \preceq c_3 \), implying \( G_1 \preceq G_2 \). Finally, it is easy to see that \( G_1 \preceq G_2 \) and \( G_2 \preceq G_3 \) implies \( G_1 \preceq G_3 \).

Proof of Lemma 3.7: Consider \( Q_1, Q_2, Q_3 \in \mathbb{Q} \). Clearly \( Q_1 \preceq Q_1 \). Let \( Q_1 \preceq Q_2 \) and \( Q_2 \preceq Q_3 \), and let \( \langle \alpha^1, d^1, \alpha^1 \rangle \in Q_1 \). Since \( Q_1 \preceq Q_2 \), there is \( \langle \alpha^2, d^2, \alpha^2 \rangle \in Q_2 \) such that \( \alpha^1 \preceq \alpha^2, d^1 \preceq d^2, \) and \( \alpha^2 \preceq \alpha^2 \). Since \( Q_2 \preceq Q_3 \), there is \( \langle \alpha^3, d^3, \alpha^3 \rangle \in Q_3 \) such that \( \alpha^2 \preceq \alpha^3, d^2 \preceq d^3, \) and \( \alpha^3 \preceq \alpha^3 \). So, \( \alpha^1 \preceq \alpha^3, d^1 \preceq d^3, \alpha^3 \preceq \alpha^3 \), proving that \( Q_1 \preceq Q_3 \). Finally,
$Q_1 \succeq Q_2$ and $Q_2 \succeq Q_1$ implies $Q_1 = Q_2$ (given that origin-destination pairs are not repeated).

**Proof of Lemma 3.8:** We need to prove that for $v_1, v_2 \in \mathbb{R}_{\geq 0}$ one has: $v_1 \leq v_2 \Rightarrow 70, 0, 0_{R,V}(v_1) \leq 70, 0, 0_{R,V}(v_2)$. Following the definition, 70, 0, $0_{R,V}(v_1)$ and 70, 0, $0_{R,V}(v_2)$ will share the same set of vertices (satisfying the vertex condition). Furthermore, $v_1 \leq v_2$ implies that the arcs $A_1$ of 70, 0, $0_{R,V}(v_1)$ and 70, 0, $0_{R,V}(v_2)$ will be a subset of the set of arcs $A_2$ of 70, 0, $0_{R,V}(v_2)$. Finally, the edge colors remain unchanged, except for speed-related one. Let $c_1, c_2$ the colors associated to 70, 0, $0_{R,V}(v_1)$ and 70, 0, $0_{R,V}(v_2)$, respectively. Clearly $\min\{v_1, x\} \leq \min\{v_2, x\}$ for any $x \in \mathbb{R}_{\geq 0}$. This, together with (4), gives $c_1 \succeq c_2$, proving monotonicity.

**Proof of Lemma 3.9:** $C_{V,1}, C_{V,0}$ are monotone functions of the AV’s achievable speed. Leveraging Lemma 3.8, we know that the serviced network is a monotone function of the speed.

**Proof of Lemma 3.10:** We need to prove that given $v_1, v_2 \in \mathbb{R}_{\geq 0}$, one has: $v_1 \leq v_2 \Rightarrow 70, 0, 0_{R_M}(v_1) \leq 70, 0, 0_{R_M}(v_2)$. First, notice that sets of vertices and arcs are preserved by 70, 0, $0_{R,M}$ Second, the argument for the edge attributes is analogous to the one in the proof of Lemma 3.8. The two facts together prove monotonicity.

**Proof of Lemma 3.11:** $C_{M1}, C_{M0}$ are monotone functions of the $\mu$MV’s achievable speed. Leveraging Lemma 3.10, we know that the serviced network is a monotone function of the speed.

**Proof of Lemma 3.12:** We need to prove that given $n_1, n_2 \in \mathbb{R}_{\geq 0}$, one has: $n_1 \leq n_2 \Rightarrow 70, 0, 0_p(n_1) \leq 70, 0, 0_p(n_2)$. Again, notice that the set of vertices and arcs are preserved by 70, 0, $0_p$. Furthermore, $n_1 \leq n_2$ implies that $t_{WS} + \frac{n_{NS \_base}}{2 n_{Q \_base}} \geq t_{WS} + \frac{n_{NS \_base}}{2 n_{Q \_base}}$, proving monotonicity.

**Proof of Lemma 3.13:** First, notice that $C_S$ and $m_{CO_2 \_S \_tot}$ are monotone functions of $n_S$. Furthermore, leveraging Lemma 3.12, we know that the serviced network relates monotonically to $n_S$.

**Proof of Lemma 3.14:** Let $r \preceq \mathbf{g}^2 \times \mathbb{N} \times \mathbb{R}_{\geq 0}$ and $r'$. Since all feasible solutions of (8) with $r$ remain feasible with $r'$, $\hat{d}_{\text{DIAMOD}}(q', r') \subseteq \hat{d}_{\text{DIAMOD}}(q', r)$ for all $q \in \mathbb{Q}$. Similarly, let $q' \preceq q$. Since all feasible solutions of (8) remain feasible (possibly by replacing demand with empty vehicles and by artificially adding loops to the graph), $\hat{d}_{\text{DIAMOD}}(q', r) \subseteq \hat{d}_{\text{DIAMOD}}(q, r)$ for all $r \in \mathbf{g}^2 \times \mathbb{N} \times \mathbb{R}_{\geq 0}$. This proves monotonicity.

**Proof of Lemmas 3.15 and 3.16:** The proofs parallel the proof of Lemma 3.14.

**Proof of Lemmas 3.17, 3.18, and 3.19:** The MDPIs are monotone, since they consist of the valid composition of monotone MDPIs [3].

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