Exact Aggregate Models for Optimal Management of Heterogeneous Fleets of Storage Devices

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Abstract—Future power grids will entail large fleets of storage devices capable of scheduling their charging/discharging profiles so as to achieve lower peak demand and reduce energy bills, by shifting absorption times in synchronous with the availability of renewable energy sources. Optimal management of such fleets entails large-scale optimization problems which can be dealt with in a hierarchical manner by clustering together individual devices into fleets. Based on recent results characterizing the set of aggregate demand profiles of a heterogeneous fleet of charging devices, an exact aggregate model is proposed to achieve optimality, in a unit commitment problem, by adopting a simplified formulation with a number of constraints for the fleet that scales linearly in the number of time-slots and is independent of the size of the fleet. This is remarkable, as it shows that under suitable conditions, a heterogeneous fleet of any size can effectively be treated as a single storage unit.

Index Terms—Aggregated demand response, energy storage systems, optimal control, unit commitment.

I. INTRODUCTION AND MOTIVATIONS

Recent years have witnessed considerable interest in the scientific community towards optimal management of distributed energy resources (DER). A survey has systematically classified a significant amount of literature dealing with optimal energy management for electric vehicles (EVs) alone [1]. While a fraction of such papers is concerned only with scheduling aspects related to a single EV and the benefits that this could enable for the household, many authors are also concerned with the issue of how solutions implemented at the level of the single household may impact the overall system or how to suitably manage (and suitably distribute) the resulting large scale optimization problem, e.g., energy cost minimization [2], peak demand reduction [3], and energy efficiency enhancement [4].

An emerging viewpoint, in such respect, is that a hierarchical approach, [5], may allow clustering multiple households, EVs or more generally prosumers, into groups which are regarded as a single entity by higher level grid abstraction layers, so that only their aggregate power absorption or generation profiles need to be considered, without mention of lower-level detailed dynamics and schedules. For instance, [6] considered an aggregated trajectory tracking problem to optimally control multiple battery sets in real time under the model predictive control (MPC) framework. In [7], an aggregator was proposed to provide predictions to the hierarchical MPC on the availability of EVs for load-frequency control. A hierarchical decomposition approach was presented in [8] to coordinate the charging/discharging behaviors of numerous EVs. The wisdom of the hierarchical setup is that this could provide an effective modular approach for tackling the inherent complexity of the optimization and scheduling task.

To make this proposition effective, however, it is important to characterise the degrees of freedom afforded by an aggregated fleet of flexible devices in a way that is accurate and still simple. For instance, Marzooghi et al. [9] introduces a heuristic aggregated model of flexible demand, to be used for the solution of unit commitment problems. Similarly, Calvillo et al. [10] modeled the aggregate behavior of inelastic flexible demand through heuristic constraints that limit the shifting ability afforded throughout the day. For computational tractability, distributed optimization is always employed by repeatedly solving multiple simpler local optimization problems while providing convergence claims [11], [12]. Nevertheless, the convergence of many distributed optimization algorithms is guaranteed only for convex problems. As an alternative approach to the hierarchical setup, approximate dynamic programming exhibits feasibility guarantees by polyhedral constraint for convex problems and inner approximations for nonconvex problems [13]. Furthermore, a comparative evaluation of several Minkowski sum approximation algorithms for demand-side flexibility aggregation is reviewed in [14]. However, none of the above approximation-based hierarchical methods can obtain the exact same optimal solution of the original individual agent-based optimization problem.

Demand dispatch for regulation of the power grid was considered in [15], based on randomized local control algorithms for homogeneous load in a mean-field control setting. Promising results, in this respect, are provided in [16], [17], [18], and [19] where optimal causal scheduling policies (affording maximal
flexibility) for heterogeneous fleets and arbitrary time-varying demand signals are provided and their intrinsic capabilities characterised graphically through so-called E-P transforms. Equivalent time-domain characterizations were independently explored in [20] and further expanded and interpreted in [21]. A related line of investigation, proposed the use of controlled invariant sets in order to characterise viable power profiles for aggregated fleets, so as to reduce their model complexity for the purpose of higher-level optimization [22]. In [23], invariant sets were also used to ensure the safe coordination of systems with both local and global constraints while a population of homogeneous air conditioners tracks a power trajectory.

More recently, the challenging task of designing optimal dispatch policies for devices with partial availability (as typical of EVs which are not always connected to the grid) was considered in [24]. In particular, Angeli et al. [24] proposed a scheduling algorithm for fleets with partial availability and, at the same time, characterises the set of aggregated demand profiles that heterogeneous fleets with partial flexibility induce when operated without cross-charging. By cross-charging we mean a situation where at least two distinct batteries are simultaneously charging and discharging so that energy can be thought of as flowing from one to the other. While allowing batteries to cross-charge may, in principle, expand the set of feasible schedules and, therefore, reduce overall costs, for realistic batteries with nonunity round-trip efficiency, it also necessarily introduces wasted energy. Therefore, our assumption appears to be meaningful whenever energy efficiency is prioritized. In our context, cross-charging is prevented by simply imposing a unidirectional power transfer constraint on individual batteries. It should be mentioned that we are not aware of the exact techniques able to replicate or extend these results to heterogeneous fleets in the presence of bidirectional power transfers.

For a fleet with $N$ storage devices and a horizon of $T$ time intervals, the results in [24] (suitably recast in a discrete-time setup with piecewise constant signals within the intersample time interval) allowed replacing a description involving $N \cdot T$ variables and $2 \cdot N \cdot T$ constraints with one that only entails $T$ variables (the aggregated demand signal) and $2^T$ constraints, thus drastically improving the scalability of the fleet model, and possibly of the underlying optimization problems, with respect to the size of the fleet. This article is dedicated to investigating equivalent aggregate models with fewer constraints, specifically, the main contributions are as follows.

1) In this article, we propose a way to achieve the exact optimality, in a unit commitment problem, by adopting a simplified formulation with a number of constraints for the fleet that scales linearly in the number $T$ of time slots considered and is also independent of the size of the fleet $N$. This is remarkable, as it shows that, under unidirectional power transfer, a heterogeneous fleet of any size can effectively be treated as a single storage unit.

2) The proposed exact aggregate model needs a combinatorial search of the $T$ active constraints for the fleet. To reduce the time used for constraint selection, two greedy nested descent algorithms are developed which are able to achieve near-optimal performance.

3) We provide the relevant theory in support of our simplification algorithms and compare the different approaches on a large number of randomly generated examples and case studies. While these results might be useful in real-time operation, we believe that simplified models of this kind are essential in large-scale optimization and planning problems, where years and possibly decades of grid operation are modeled under uncertainty scenarios (potentially leading to a huge number of variables whose aggregated behavior can therefore be captured in a non-conservative way, by adopting our reduced models).

In this respect, it is worth pointing out that in real-time applications distributed approaches can also serve as an appealing alternative to hierarchical methods; this is instead less likely in planning problems where the large-scale decentralized nature of the problem is further hindered by the simultaneous consideration of a huge number of scheduling intervals.

The rest of this article is organized as follows. Section II presents an aggregate model of the storage fleet. Section III introduces the aggregated model and the associated optimization algorithms for storage devices. The advantages of the proposed aggregate model in the computation are addressed in Section IV. While the theoretical results only apply to single-area networks, Section V validates the methods in more general setups by applying the aggregation algorithms to a two-area system. Finally, Section VI concludes this article, and the Appendix contains the proofs of the technical results.

II. AGGREGATE FLEET MODEL FOR OPTIMIZATION

We consider a finite discrete time horizon $T = \{1, 2, \ldots, T\}$ and a storage fleet with $N$ batteries, which we denote as $N = \{1, 2, \ldots, N\}$. For each battery $j \in N$, its available time period of charging is denoted by the set $A_j \subseteq T$. Each battery is initially empty (without loss of generality) and needs to reach a certain target energy level $\bar{E}_j$ (available for later use) by charging only at times within its availability set $A_j$. Moreover, each battery has a maximum rated power $\bar{P}_j$ which is the maximum power it can absorb while charging. Notice that since power is measured externally (power absorbed from the grid rather than power transferred to the battery), the amount of absorbed energy $\bar{E}_j$ should factor in the round-trip efficiency $\eta_j$ and be computed according to $\bar{E}_j = \bar{E}_j / \eta_j$.

We consider the problem of minimizing generation costs for the task of charging the fleet while additionally meeting some assigned arbitrary nonnegative inflexible demand profile $D(t)$. It is assumed that each battery has a sufficient plug-in time duration to absorb the required energy, i.e., $\bar{E}_j \leq \sum_{t \in A_j} \bar{P}_j$. Moreover, as the time scale considered is that of a single charge cycle, we neglect battery degradation phenomena. For a given set of generation units $I$, the minimum and maximum power ratings are represented by $\bar{G}_i$ and $G_i$, respectively, $\forall i \in I$ and such that $0 \leq \bar{G}_i \leq \bar{G}_i$. In particular, the following optimization problem is of interest:

\[
\min_{g_j(t), \quad j \in N, \quad t \in T} \sum_{i \in I} \sum_{t \in T} C_i(g_i(t)) \quad (1a)
\]

s.t. \[
\sum_{i \in I} g_i(t) = D(t) + \sum_{j \in N} u_j(t) \quad \forall t \in T \quad (1b)
\]

\[
0 \leq u_j(t) \leq \bar{P}_j \quad \forall t \in T \quad \forall j \in N \quad (1c)
\]

\[
G_i \leq g_i(t) \leq \bar{G}_i \quad \forall t \in T \quad \forall i \in I \quad (1d)
\]
where the decision variables \( u_j(t) \) is the power of battery \( j \), \( g_i(\cdot) \) is the power of generation unit \( i \), and the objective functions \( C_i(\cdot) \) are convex increasing functions on their domain \([G_i, \bar{G}_i]\).

While \( D^i(t) \) is an arbitrary nonnegative signal, the feasibility of problem (1) implicitly entails upper and lower bounds to the value of \( D^i(t) \). Let us consider the set of possible aggregated demand signals of a fleet of batteries operating without cross-charging and within given availability windows

\[
\mathcal{D} := \left\{ d : \mathcal{T} \to \mathbb{R} : \exists u_j : \mathcal{T} \to [0, \bar{P}_j] : d(t) = \sum_{j \in \mathcal{N}} u_j(t), \quad u_j(t) = 0 \ \forall \ j \in \mathcal{N} \ \forall \ t \notin \mathcal{A}_j, \text{ and } \sum_{t \in \mathcal{T}} u_j(t) \leq \bar{E}_j \right\}.
\]

The set \( \mathcal{D} \) is a polyhedron in the \( T \)-dimensional real space and it is described implicitly by using \( 2NT + N \) inequality constraints and \( T + TN \) variables, with (typically) \( N \gg T \), (i.e., \( T = 24 \), while \( N \) could be in the thousands or even larger).

Alternatively, one may seek to describe the set of feasible aggregate power demands without explicit reference to individual battery profiles, viz., by only considering constraints on the aggregate demand. This is possible since the set \( \mathcal{D} \) is convex and essentially a projection of a polytope (of dimension \( NT + N \)) onto a \( T \)-dimensional polytope. It was shown in [24, Corollary 1] that \( \mathcal{D} \) can also be characterised as follows:

\[
\mathcal{D} = \{ d : \mathcal{T} \to \mathbb{R}_{\geq 0} : \sum_{t \in \mathcal{W}} d(t) \leq \sum_{j \in \mathcal{N}} \min\{\text{card}(\mathcal{A}_j \cap \mathcal{W}), \bar{E}_j/\bar{P}_j\}\bar{P}_j \ \forall \mathcal{W} \subseteq \mathcal{T} \}.
\]

Notice that \( \mathcal{D} \) is expressed by a family of linear constraints, indexed by \( \mathcal{W} \), that denotes any (nonempty) subset of the considered time horizon \( T \). Intuitively, to characterise \( \mathcal{D} \), one needs to specify for each possible selection of time intervals \( \mathcal{W} \), what is the maximum amount of energy the fleet is able to absorb within \( \mathcal{W} \). This is shown in Fig. 1 for the simple case of \( \mathcal{T} = \{1, 2, 3\} \), (notice the \( 2^3 - 1 = 7 \) active faces).

As a consequence, the previous optimization problem can be equivalently recast as

\[
\begin{align*}
\min_{\substack{d(t) \in \mathcal{T} \ \forall \ t \in \mathcal{T}, \ \forall i \in \mathcal{I} \ \forall t \in \mathcal{T} \ \forall i \in \mathcal{I}}} & \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} C_i(g_i(t)) \\
\text{s.t.} & \sum_{i \in \mathcal{I}} g_i(t) = D^i(t) + d(t) \ \forall t \in \mathcal{T} \\
& G_i \leq g_i(t) \leq \bar{G}_i \ \forall t \in \mathcal{T} \ \forall i \in \mathcal{I} \\
& \sum_{t \in \mathcal{T}} d(t) = \sum_{j \in \mathcal{N}} \bar{E}_j \\
& \sum_{t \in \mathcal{W}} d(t) \leq \sum_{j \in \mathcal{N}} \min\{\text{card}(\mathcal{A}_j \cap \mathcal{W}), \bar{E}_j/\bar{P}_j\}\bar{P}_j \\
& \forall \mathcal{W} \subseteq \mathcal{T}.
\end{align*}
\]

A systematic analysis and comparison between problems (2) and (1) from the numerical point of view are performed in Section IV. While the optimization model in (2) does not involve detailed descriptions of individual batteries (in that it only involves aggregated demand profiles), it still requires a potentially large number of inequality constraints due to (2e), \( 2^T - 1 \), in order to capture the flexibility afforded by the fleet. Therefore, its use is beneficial only when \( 2^T < 2NT + T \), viz., for huge fleets or for a relatively low number of considered time intervals.

In this context, we expect to reduce the number of constraints in (2e). The following section will develop equivalent aggregate models that can replace the \( 2^T - 1 \) constraints with \( T \) constraints and will discuss the relevant active constraint selection algorithms.

III. AGGREGATE MODELS WITH FEWER CONSTRAINTS

We start with the result for the simpler scenario of batteries with full availability, followed by the more challenging one of batteries with partial time availability windows. In a nutshell, the key technical achievement proves that retaining constraints of availability windows, which minimize the maximum average total demand for every given cardinality, is enough. This guarantees the same optimal solution of the full problem.

A. Simplified Aggregate Model for Full Availability

Note that some of the constraints in (2) will typically be redundant, their a priori elimination appears to be too computationally demanding as it entails the solution of a linear programming problem of comparable computational complexity as the problem itself. We propose next a way to replace the \( 2^T \) constraints with \( T \) inequality constraints which still afford computing the true optimal solution (or a very close approximation).

Let us consider the case of full availability which means all batteries can implement charging operation over the whole time horizon, viz. \( \mathcal{A}_j = \mathcal{T} \) for all \( j \in \mathcal{N} \). Under such a scenario, let \( t_1, t_2, \ldots, t_T \) be such that \( D^i(t_1) \leq D^i(t_2) \leq D^i(t_3) \leq \ldots \leq D^i(t_T) \).

Now, we present our first main result as follows.

**Theorem 1:** For any \( \mathcal{W} \subseteq \mathcal{T} \) of cardinality \( k \) we have \( \sum_{t \in \mathcal{W}} d^i(t) \leq \sum_{i=1}^k d^i(t_i) \), hence the same optimal generation...
cost and solution of problem (2) can be computed by solving
\[
\min_{g_i(t), t \in T} \sum_{i \in I} \sum_{t \in T} C_i(g_i(t)) \tag{3a}
\]
subject to
\[
\sum_{i \in I} g_i(t) = D^f(t) + d(t) \quad \forall t \in T \tag{3b}
\]
\[
G_i \leq g_i(t) \leq \bar{G}_i \quad \forall t \in T \forall i \in I \tag{3c}
\]
\[
\sum_{t \in T} d(t) = \sum_{j \in N} \bar{E}_j 
\]
\[
\sum_{i=1}^{k} d(t_i) \leq \sum_{j \in N} \min\{k, \bar{E}_j / \bar{P}_j\} \bar{P}_j 
\quad \forall k \in \{1, 2, ..., T\}. \tag{3e}
\]

**Proof:** Due to the permutation invariance of the problem and the monotonicity of the cost function, we expect that
\[
d^*(t_1) \geq d^*(t_2) \geq \cdots \geq d^*(T)
\]
where \(d^*\) is the solution of optimization problem (2).

To see this, assume by contradiction that for some \(t_k\) we have \(d^*(t_k) < d^*(t_{k+1})\). Then, \(D^f(t_k) + d^*(t_k) < D^f(t_{k+1}) + d^*(t_{k+1})\). Hence, reducing \(d(t_{k+1})\) by a small amount of power \(\Delta\) (equal for instance to \((d^*(t_{k+1}) - d^*(t_k)) / 2\)) and increasing \(d(t_{k})\) by the same amount, yields a strict price reduction. Let us denote \(\tilde{d}\) this new demand profile, which fulfills
\[
\tilde{d}(t) = \begin{cases} 
\frac{d^*(t)}{2} & \text{if } t \notin \{t_k, t_{k+1}\} \\
\frac{d^*(t) + d^*(t_{k+1})}{2} & \text{if } t \in \{t_k, t_{k+1}\}.
\end{cases}
\]

Feasibility is also preserved since any constraint involving both \(t_k\) and \(t_{k+1}\) is unaltered by the power swap. Any constraint involving only \(t_{k+1}\) is also fulfilled since \(0 \leq \tilde{d}(t_{k+1}) \leq d^*(t_{k+1})\). Every constraint involving only \(t_k\) is also fulfilled since the time \(t_k\) can be swapped with time \(t_{k+1}\)

\[
\sum_{t \in W} \tilde{d}(t) = \sum_{t \in W \setminus \{t_{k+1}\} \setminus \{t_k\}} \tilde{d}(t) \leq \sum_{t \in W \setminus \{t_{k+1}\} \setminus \{t_k\}} \frac{d^*(t)}{2} \leq \sum_{t \in W \setminus \{t_{k+1}\} \setminus \{t_k\}} \min\{\text{card}(W), \bar{E}_j / \bar{P}_j\} \bar{P}_j.
\]

Hence, by the convexity of the set \(D\), this contradicts the optimality of \(d^*(t)\) for all sufficiently small power swap among \(t_k\) and \(t_{k+1}\) in the direction of \(\tilde{d}\) considered above. \(\square\)

**B. Aggregate Models for Partial Availability**

Next, we aim to address the more general case of batteries with partial availability, viz., \(A_j \subset T\) with strict inclusion also allowed. This is of great practical relevance as it corresponds to batteries which are physically connected to the grid only during a limited amount of time, i.e., as typical of EVs.

Unlike the case of full availability, the expected ordering of total demand levels (and generation prices) cannot be inferred by looking at inflexible demand alone. This is because different time intervals typically exhibit different sets of available batteries, and this, in turn, impacts the ability and convenience to charge at any given time slot.

To this end, we define, for each \(W \subset T\), the maximum average total demand as follows:
\[
\bar{D}(W) := \sum_{t \in W} D^f(t) + \sum_{t \in T \setminus W} \sum_{j \in N} \min\{\text{card}(A_j \cap W), \bar{E}_j / \bar{P}_j\} \bar{P}_j / \text{card}(W)
\]

Notice that this is affected both by the inflexible demand and by the availability windows of individual agents and is computed assuming that within \(W\) and compatibly with individual availability windows, batteries are drawing energy at their maximal rates (until completion of the charging task).

Then, let us define the set \(W^*_k\) of cardinality \(k\), minimizing the maximum average total demand, as follows:
\[
W^*_k := \arg \min_{W \subset T, \text{card}(W) = k} \bar{D}(W) \tag{4}
\]

which can be computed on the basis of a priori available information. Based on this set \(W^*_k\), the main result of this article can be stated below:

**Theorem 2:** Let \(d^*(t)\) be any optimal profile of problem (2) and \(W^*_k\) be defined as in (4). Then, the minimum of (2) is equal to the minimum of
\[
\min_{g_i(t), t \in T} \sum_{i \in I} \sum_{t \in T} C_i(g_i(t)) \tag{5a}
\]
subject to
\[
\sum_{i \in I} g_i(t) = D^f(t) + \tilde{d}(t) \quad \forall t \in T \tag{5b}
\]
\[
G_i \leq g_i(t) \leq \bar{G}_i \quad \forall t \in T \forall i \in I \tag{5c}
\]
\[
\sum_{t \in T} \tilde{d}(t) = \sum_{j \in N} \bar{E}_j 
\]
\[
\sum_{t \in W} \tilde{d}(t) \leq \sum_{j \in N} \min\{\text{card}(A_j \cap W^*_k), \bar{E}_j / \bar{P}_j\} \bar{P}_j 
\quad \forall k \in \{1, 2, ..., T\}. \tag{5e}
\]

**Remark 1:** Theorem 2 shows that any fleet of batteries operating with unidirectional power transfers (i.e., charging only, which also implies no cross-charging) can be treated within the optimization problem considered as equivalent to a “single” big battery provided its charging profile is constrained according to the \(T\) upper-bounds computed in (5d). In particular, the approach allows dealing with individual batteries of arbitrary power rating, availability window, round-trip efficiency and/or values of initial and target energy levels. This, together with the total inflexible demand, is the information needed to derive a simplified aggregate model. It is conceivable that such information might become available in future smart grids, whereby aggregators directly demand it to participating batteries and/or try to infer it from historical data. It is worth pointing out that the current formulation only allows dealing with deterministic target energy levels and electricity prices. A possible interesting future direction of investigation is to look at the situation where some or this parametric information is in fact uncertain (for instance stochastic in nature). While this is outside the scope of this note, it is interesting to remark that as long as variations on inelastic demand or target energy levels are sufficiently small, they do...
not affect the sets $W_k^*$ defined in (4), thus confirming a degree of robustness of the performed constraints selection.

To prove Theorem 2, several key lemmas need to be clarified. First, Lemma 1 plays an important role in gaining an a priori understanding of optimal power profiles.

Lemma 1: Let $u_j^*$ be optimal dispatch policies for problem (1) corresponding to the optimal aggregate demand profile $d^*(t)$ for problem (2), where

$$\sum_{j \in \mathcal{N}} u_j^*(t) = d^*(t) \quad \forall t \in \mathcal{T}. \quad (6)$$

Then, for all $j \in \mathcal{N}$, there exists $d_j > 0$ such that

$$u_j^*(t) = \begin{cases} \bar{P}_j & \text{if } t \in A_j \text{ and } D^I(t) + d^*(t) < d_j \\ 0 & \text{if } t \in A_j \text{ and } D^I(t) + d^*(t) > d_j. \end{cases} \quad (7)$$

Proof: See Appendix A. □

Our aim is to devise an efficient algorithmic procedure to a priori select a subset of constraints in (2d), of cardinality $T$, so as to retain the same optimal solution. The set $W \subseteq T$ is said to correspond to an active constraint for $d^*(t)$ if it holds $\sum_{t \in W} d^*(t) = \sum_{j \in \mathcal{N}} \min\{\text{card}(A_j \cap W), \bar{E}_j / \bar{P}_j\} \bar{P}_j$. The following lemma provides an important insight into some of the active constraints.

Lemma 2: Let $d^*(t)$ be an optimal aggregated demand profile for the problem (2). Consider for any $d > 0$, the following sets:

$$W_d := \{t \in \mathcal{T} : D^I(t) + d^*(t) < d\}. \quad (8)$$

Any such set corresponds to an active constraint, viz.,

$$\sum_{t \in W_d} d^*(t) = \sum_{j \in \mathcal{N}} \min\{\text{card}(A_j \cap W_d), \bar{E}_j / \bar{P}_j\} \bar{P}_j. \quad (9)$$

Proof: See Appendix B. □

While Lemma 2 identifies an important set of active constraints, it does not provide any indication of how to compute these without solving the optimization problem (2). In fact, the definition of $W_d$ explicitly requires the optimal aggregate power profile $d^*(t)$. The next result gives some important hints on how to look for active constraints by only using a priori available information.

Lemma 3: Let $d^*(t)$ be the optimal aggregated demand for the problem (2) and $d \geq 0$ be arbitrary. Define $W_d$ according to (7). Then, the following holds:

$$\bar{D}(W_d) = \min_{W \subseteq T : \text{card}(W) = \text{card}(W_d)} \bar{D}(W). \quad (10)$$

Moreover, $W_d$ is the unique minimizer in (8).

Proof: See Appendix C. □

Lemma 3 states that $W_d$ achieves the minimum value of maximum average available demand $\bar{D}(W)$ over sets of its cardinality. While the $W_d$ sets defined in (7) are not the only active constraints that one may find by solving (2), it turns out that they are enough to fully characterise the optimal solution $d^*(t)$, as clarified in the following Lemma 4.

Lemma 4: Let $d^*(t)$ be any optimal profile of problem (2) and $W_d$ be defined as in (7). Then, the minimum and minimizers of (2) are equal to the minimum and minimizers of

$$\min_{d^*(t) \in \mathcal{T}} \sum_{i \in I} \sum_{t \in T} C_i(g_i(t)) \quad (11)$$

subject to $g_i(t) = D^I(t) + \tilde{d}(t) \quad \forall t \in \mathcal{T} \quad (12)$

$$G_i \leq g_i(t) \leq \bar{G}_i \quad \forall t \in \mathcal{T} \quad (13)$$

$$\sum_{t \in \mathcal{T}} \tilde{d}(t) = \sum_{j \in \mathcal{N}} \bar{E}_j \quad (14)$$

$$\sum_{t \in W_d} \min\{\text{card}(A_j \cap W_d), \bar{E}_j / \bar{P}_j\} \bar{P}_j \leq \sum_{j \in \mathcal{N}} \sum_{t \in W_d} \min\{\text{card}(A_j \cap W_d), \bar{E}_j / \bar{P}_j\} \bar{P}_j \quad \forall d \geq 0. \quad (15)$$

Proof: See Appendix D. □

It is worth pointing out that as $d$ ranges in $[0, +\infty)$, there are at most $T$ distinct sets of $W_d$. This is because the $W_d$ are nested with each other.

Let $d^*(t)$ be any optimal profile of problem (2) and $W_d$ be defined as in (7). We already remarked that there are at most $T$ distinct sets $W_d$ (as $d$ ranges in $(0, +\infty)$) and that they are nested with one another so that we may rename them and denote them in ascending order as

$$W_1 \subseteq W_2 \subseteq W_3 \subseteq \ldots \subseteq W_Q.$$}

Moreover, it is clear by their definition that the following implication holds:

$$\sum_{t \in W_k} D^I(t) + d^*(t) \leq \sum_{t \in W_{k+1}} D^I(t) + d^*(t) / \text{card}(W_k).$$

As a consequence, it holds $\bar{D}(W_k) < \bar{D}(W_{k+1})$, for $k = 1, \ldots, Q - 1$, with $Q \leq T$.

The discussion so far clearly shows that the procedure of choosing $W_k^*$ in (4) can be adopted to precompute $T$ constraints to be included in a simplified version of (2).

Notice that the sets $W_k^*$, unlike $W_d$ previously discussed, can be computed on the basis of a priori available information. Furthermore, by virtue of Lemma 3, the following inclusion holds:

$$\{W_d : d \geq 0\} \subseteq \{W_k^* : k \in \{1, 2, \ldots, T\}\} \subset 2^T. \quad (16)$$

Hence, we may replace the constraints of support $W_d$ in (9) with similar constraints of support $W_k^*$, without affecting the cost of the optimal solution. At this point, one is ready to prove Theorem 2 which is shown in Appendix E.

C. Algorithms for Active Constraints Selection

Application of Theorem 2 requires prior selection of the set $W_k^*$ as shown in (4). This constraint selection is described in Algorithm 1. As shown in (5e), $T$ constraints are required and the cardinality of set $W_k^*$ ranges from 1 to $T$. Hence, in Algorithm 1, given each cardinality $k$, the corresponding sets $W_k^*$ are selected by solving the problem (4).

However, one drawback of (4) is that, for every assigned value of $k$, it is a combinatorial optimization problem that entails generation of $\binom{T}{k}$ sets and computation of the corresponding maximum available power. While this would typically be faster than a linear programming solver dealing with $2^T$ constraints, it might still be computationally inefficient, and lead to scalability issues.

Therefore, we propose the following two greedy algorithms (of limited computational complexity) for the precomputation...
Algorithm 1: Combinatorial Optimization Algorithm.

1. **Initialization:**
   \( k = T \)

2. **Update procedure:**
   while \( k > 0 \) do
     4. Find \( W_k^* \) by solving (4)
     5. Select the inequality corresponding to \( W_k^* \) as an active constraint, decrease the cardinality by \( k = k - 1 \)

6. **Final results:** The set of selected active constraints are returned.

Algorithm 2: Greedy Nested Descent Algorithm.

1. **Initialization:**
   \( k = T; \hat{W}_T = T; S = 1 \)

2. **Update procedure:**
   while \( k > 1 \) do
     4. Calculate \( \hat{W}_{k-1}^S := \arg \min_{W \subset \hat{W}_k, |W| = k-S} D(W) \)
     5. if \( \hat{W}_{k-1}^S \) is not a singleton or \( D(\hat{W}_{k-1}^S) \geq D(\hat{W}_k) \) then
     6. \( S = S + 1 \)
     7. else
        8. Select the inequality corresponding to \( \hat{W}_{k-1}^S \) as an active constraint, decrease the cardinality by \( k = k - S \), and reset \( S = 1 \)

9. **Final results:** The set of selected \( \leq T \) active constraints are returned.

Algorithm 3: Greedy Step-1 Nested Descent Algorithm.

1. **Initialization:**
   \( k := T; \hat{W}_T := T \)

2. **Update procedure:**
   while \( k > 1 \) do
     4. Calculate \( \hat{W}_{k-1}^{(m)} := \arg \min_{W \subset \hat{W}_k, |W| = k-1} D(W), \forall \hat{W}_{k-1}^{(m)} \in \hat{W}_k = \{ \hat{W}_k^{(1)}, \hat{W}_k^{(2)}, \ldots, \hat{W}_k^{(M)} \} \)
     5. Let \( \hat{W}_{k-1}^{(m)} = \arg \min_{\hat{W}_{k-1}^{(m)} \in \{ \hat{W}_k^{(1)}, \hat{W}_k^{(2)}, \ldots, \hat{W}_k^{(M)} \}} \bar{D}(\hat{W}_{k-1}^{(m)}) \)
     6. Select the inequality corresponding to \( \hat{W}_{k-1}^{(m)} \) as an active constraint
     7. Decrease the cardinality by \( k = k - 1 \)

8. **Final results:** The set of selected active constraints are returned.

The advantage of Algorithm 3 compared to Algorithm 2 is that the number of candidate \( W \) at the \( k \)th iteration is \( (k-1) = k \) which avoids the combinatorial explosion and terminates in a number of steps of order \( T^2 \). However, this fast greedy step-1 algorithm might lose optimality for certain examples because the minimizer \( \hat{W}_{k-1} \) is restricted to be selected from the subset of \( \hat{W}_k \) which disables the active constraint selection for \( S > 1 \) in Algorithm 2.

IV. ADVANTAGES IN COMPUTATION

This section is devoted to applying the proposed aggregate demand model-based optimization problem and the active constraints selection algorithms to some numerical case studies. All the tested optimization problems were solved by the YALMIP toolbox and Gurobi optimizer in MATLAB R2019a, on a computer with a two-core 3.50 GHz Intel(R) Xeon(R) E5-1650 processor and 32 GB RAM.

The computational advantages of using optimization problem (2) and the greedy constraint selection algorithms in a large population of storage devices are evaluated. In particular, the following methods for exploring the optimal dispatch of storage fleets are considered throughout the rest of this section.

1) M1: Each storage unit is modeled as an individual agent and the optimization problem (1) is solved.
2) M2: Aggregate demand model-based optimization by solving problem (2) with \( 2T - 1 \) inequality constraints
3) M3: Aggregate demand model-based optimization by solving problem (5) with \( T \) inequality constraints defined in (4).
4) M4: Aggregate demand model-based optimization by solving problem (5) with \( \leq T \) constraints chosen by Algorithm 2.
5) M5: Aggregate demand model-based optimization by solving problem (5) with active constraints chosen by Algorithm 3.

To compare the performance of these methods, a series of case studies are designed considering various lengths of the full-time window and different numbers of storage devices in a...
fleets, i.e., $T_1 = 8\ h$, $T_2 = 16\ h$, $T_3 = 24\ h$; $N_1 = 10$, $N_2 = 100$, $N_3 = 1000$, and $N_4 = 10000$. For each case study, the constraint selection time (CS-T) and optimization time (OP-T) in the unit of seconds (s) are recorded to represent the computational performance.

The data are sampled every hour. Then, given a time window $T_i = \{1, 2, \ldots, T_i\}$ where $i \in \{1, 2, 3\}$, the availability window of each device is uniformly generated as a subset of $T_i$. For each battery, the rated power is set to be $P_j = 1\ kW$ and the target power dispatch algorithm for $j$ follows a uniform distribution on the continuous interval $[0, P_j \cdot \text{card}(A_j)]$. Based on these randomly generated examples, the following results are observed.

The computational performances of M1–M5 for $T_i = 8\ h$ and $T_3 = 24\ h$ are shown in Figs. 2 and 3. From the figures, it emerges that the time for solving optimization problem (1) is proportional to the number of agents, so M1 is inefficient for problems of large numbers of agents. In Fig. 2, when the considered time window is of small length, approaches M2–M4 which are based on the aggregate demand model are effective regardless of the number of devices. In particular, these methods are much faster than M1 for $N_4 = 10000$. However, the aggregate demand-based methods may be exposed to high computational burden if the window is large as shown in Fig. 3, i.e., $T_3 = 24\ h$. Specifically, M2 can provide the optimal aggregate demand profile, but it is computationally expensive due to $O^{24} - 1 \approx 1.6 \cdot 10^7$ constraints. Since the active constraint selection in M3–M5 is determined by the maximum average demand, a larger number of batteries can also increase the time spent on constraint selection, however, the solution of optimization (5) is extremely fast and unaffected by the agents’ number. Moreover, as in Section III-B, combinatorial explosion increases computational burden in searching for the optimal $W^*_k$ in M3 and possibly $\hat{W}_{k-s}$ in M4, the time on active constraint selection is much higher than that of using the greedy step-1 nested descent algorithm in M5.

Figs. 4 and 5 present the computational time of each optimal power dispatch algorithm for $T_i - T_3$ with $N_2$ and $N_4$ batteries. The results of M1 verify that the computational complexity for solving optimization problem (1) is linear with respect to $N \cdot T$. For the methods M3 and M4 which are based on the aggregate demand model, the performance is mainly affected by the length of the horizon and example parameters. Particularly, comparing the selection time of M4 at $T_3 = 24\ h$ in both figures, we can conclude that less combinatorial explosion is encountered for $N_4 = 10000$.

Notice that the computational performance of M5 using greedy step-1 nested descent algorithm for active constraints selection is the best for problems with a large number of batteries and long horizon interval. However, this improvement in computation is achieved at the expense of optimality for some examples. Although the methods M1–M3 are suffering from computational challenges, the results are optimal. To quantify the performance in obtaining the optimal solution using M4 and M5, $10^4$ scenarios with $N_1 = 10$ batteries and $T_4 = 24\ h$ time horizon are randomly generated. Finally, the success rate to reach the optimal solution for M4 and M5 are 95.6% and 93.6%, respectively (with small optimality gaps in the case of missed optimal solutions).

### V. Two-Area System

Next, we consider the application of the aggregate demand model on a two-area system $I = \{1, 2\}$ over a time horizon $T = \{1, 2, \ldots, 24\}$ h and an hourly discretization step. Each area is equipped with local dispatchable generators and numerous flexible storage devices. In this case study, to simplify the analysis, we assume that the size of a single generator is significantly smaller than the total installed power output.
capacity, i.e., the dispatchable power generation at area $i \in I$ can be approximated by one generator whose power is a continuous variable between $G_i = 0$ GW and $G_i = 60$ GW. The generation cost functions are expressed as

$$C_i(g_i(t)) = a_i \cdot (g_i(t))^2 + b_i \cdot g_i(t) \quad \forall i \in I$$

where $a_i = 1 \times 10^4$ £/GW²h, $b_i = 1.5 \times 10^4$ £/GWh, $a_2 = 2 \times 10^5$ £/GW²h, $b_2 = 1.4 \times 10^4$ £/GWh.

Moreover, the inflexible demands of these two areas $D_i^1$ (Area 1) and $D_i^2$ (Area 2) are assumed to be equal to historic data.

The optimal power flow $p(t)$ between the two areas is shown in Fig. 6(b). Negative values indicate that the power is flowing from area 1 to area 2. When the transmission capacity is no more than 5 GW, the line is congested over 24 hours for energy arbitrage. It is noted that the additional required transmission capacity to avoid congestion is around 15 GW.

The optimal costs using approaches MM1–MM4 are collected in Table I. Since the two methods MM1 and MM2 are equivalent, only MM1 is displayed. From Table I, we can observe that the increase in transmission capacity reduces the operational cost of the two-area system. Moreover, it is interesting to note that MM3 (MM4, respectively) achieve the same optimal cost of MM1 for values of $\bar{\sigma}$ such that transmission constraints are always active (respectively never active) over the considered time window. When the transmission line is partially congested during the day, both MM3 and MM4 have small gaps with respect to the optimal cost.

VI. CONCLUSION

The article develops new methods for optimal management of heterogeneous storage fleets subject to availability constraints, by representing their aggregate behavior through a small number of linear inequality constraints. This can be successfully applied to unit commitment problems where the goal is to schedule the fleet’s power absorption in order to minimize generation costs. The novel formulation involves a number of constraints for the fleet that scales linearly in the length $T$ of the considered time horizon and is independent of the size $N$ of the fleet. This is remarkable, as it shows that, under suitable technical
conditions, a heterogeneous fleet of any size can effectively be treated as a single storage unit. The approach is compared with formulations involving a detailed description of the fleet (involving roughly 2NT constraints) or aggregated descriptions involving approximately 2T constraints. Numerical experiments confirm the theoretical results, showing that optimal performance is achieved with a drastic reduction in the cost of the optimization time. Since constraint selection is of exponential complexity in T, we additionally propose two greedy selection procedures which are shown, through extensive simulations, to perform the correct constraint selection in most considered cases. When the choice of constraints is not optimal, this leads to optimal power schedules which are unfeasible for the original fleet (due to the reduced number of constraints). However, numerical experiments have shown in all such cases a small optimality gap. The method is tested on a two-area system, where each area has its own fleet and ability to schedule it in a coordinated fashion. Transmission capacity limits are assumed among the two areas, so that marginal price differentials may be experienced at times when capacity constraints are active. While the theoretical analysis cannot be applied to this case, we show how the method yields promising results. The constraint reduction is performed by either considering the two fleets as a single fleet (i.e., ignoring capacity constraints) or, ignoring the existence of a transmission line, by carrying out the reduction of each fleet individually.

APPENDIX

A. Proof of Lemma 1

Let \( D_0 = \{ D^I(t) + d^*(t) : t \in T \} \). Of course, \( D_0 \) is a finite set due to finiteness of \( T \). For any \( j \in \mathcal{N} \) we claim that \( d_j \in D_0 \) exists such that (6) holds. Let us define \( d_j := \sup \{ d \in D_0 : u_j^*(t) = \bar{P}_j \forall t : D^I(t) + d^*(t) \leq d \} \), where \( d_j = -\infty \) if the supremum is taken over an empty set. Similarly, we define \( d_j := \inf \{ d \in D_0 : u_j^*(t) = 0 \forall t : D^I(t) + d^*(t) \geq d \} \), where \( d_j = +\infty \) if the infimum is taken over an empty set. By construction \( d_j < d \). We need to show that there exists at most a single element \( d \in D_0 \) with the property that \( d_j < d < d_j \). If the claim is true, indeed, (6) holds by choosing \( d_j = d \). By contradiction, should this not be the case, there would exist \( t_1 \) and \( t_2 \) in \( T \), with \( D^I(t_1) + d^*(t_1) < D^I(t_2) + d^*(t_2) \) such that \( u_j^*(t_1) = \bar{P}_j > d \) and \( u_j^*(t_2) = \bar{P}_j > d \). This, however, contradicts optimality of \( u_j^*(t) \) as for any sufficiently small \( d > 0 \) the policy obtained by taking \( u_j^*(t) = u_j^*(t_1) + \delta > d_j \) and \( u_j^*(t) = u_j^*(t_2) - \delta \) is feasible and of strictly lower cost.

B. Proof of Lemma 2

Let \( u_j^*(t) \), for \( j \in \mathcal{N} \) be part of an arbitrary optimal solution of (1). As remarked in the Lemma statement this fulfills \( \sum_{j \in \mathcal{N}} u_j^*(t) = d^*(t) \). Denote for any \( j \in \mathcal{N} \), \( d_j \) quantities identified in Lemma 1. By virtue of Lemma 1, \( u_j^*(t) = 0 \) for all \( t \) such that \( d^*(t) + D^I(t) > d_j \). In particular then, if \( d > d_j \), the same is true for all \( t \) such that \( d^*(t) + D^I(t) \geq d \), i.e., for all \( t \in W_0 \). Hence, \( E_j = \sum_{t \in \mathcal{N}} u_j^*(t) = \sum_{t \in W_0} u_j^*(t) \).

On the other hand, \( u_j^*(t) = \bar{P}_j \) for all \( t \in \mathcal{N} \) such that \( d^*(t) + D^I(t) < d_j \). In particular then, if \( d \leq d_j \), the same is true for all \( t \) such that \( d^*(t) + D^I(t) < d \), i.e., for all \( t \in W_d \). As a consequence, \( \sum_{t \in W_d} u_j^*(t) = \text{card}(A_j \cap W_d) \bar{P}_j \). For any \( j \in \mathcal{N} \) we see

\[
\sum_{t \in W_d} d^*(t) = \sum_{t \in \mathcal{N}} \sum_{j \in \mathcal{N}} u_j^*(t) = \sum_{t \in \mathcal{N}} \sum_{j \in \mathcal{N}} u_j^*(t) = \sum_{j \not\in \mathcal{N}} \sum_{t \in \mathcal{N}} u_j^*(t) = \sum_{j \not\in \mathcal{N}} \sum_{t \in \mathcal{N}} u_j^*(t) = \sum_{t \in \mathcal{N}} \text{card}(A_j \cap W_d) \bar{P}_j + \sum_{j \not\in \mathcal{N}} \bar{E}_j \geq \sum_{j \not\in \mathcal{N}} \min \{ \text{card}(A_j \cap W_d), \bar{E}_j / \bar{P}_j \} \bar{P}_j.
\]

Since \( d^*(t) \) is feasible, the latter inequality holds with the equal sign and \( W_d \) corresponds to an active constraint.

C. Proof of Lemma 3

Let \( d^*(t) \) be the optimal aggregated demand for the problem (2). Then, by Lemma 2 the following holds for any \( W \subset T \) of the same cardinality as \( W_d \):

\[
\sum_{t \in \mathcal{N}} \sum_{j \in \mathcal{N}} \text{card}(A_j \cap W_d) \bar{P}_j / \text{card}(W) \leq \sum_{t \in \mathcal{N}} \sum_{j \in \mathcal{N}} \text{card}(A_j \cap W_d) \bar{P}_j / \text{card}(W) = \sum_{t \in \mathcal{N}} \sum_{j \not\in \mathcal{N}} \text{card}(A_j \cap W_d) \bar{P}_j / \text{card}(W).
\]

Hence, it holds \( D(W_d) = \min_{W \subset T \text{card}(W) = \text{card}(W_d)} \bar{D}(W) \). The final claim follows by remarking that inequality:

\[
\sum_{t \in \mathcal{N}} \sum_{j \not\in \mathcal{N}} \text{card}(A_j \cap W_d) \bar{P}_j / \text{card}(W) \leq \sum_{t \in \mathcal{N}} \sum_{j \not\in \mathcal{N}} \text{card}(A_j \cap W_d) \bar{P}_j / \text{card}(W)
\]

is strict whenever \( W \neq W^d \) (this is because \( W^d \) must include at least a time instant with strictly greater \( D^I(t) + d^*(t) \) than the corresponding values for \( W_d \), by definition of \( W_d \)).

D. Proof of Lemma 4

Let \( d^*(t) \) be the optimal solution of (9). We claim that for any \( k \in \{1, \ldots, Q - 1\} \) the signal \( D^I(t) + d^*(t) \) is constant on \( W_{k+1} \setminus W_k \). This claim is trivial if \( W_{k+1} \setminus W_k \) has cardinality 1. If the cardinality is bigger than one we see arguing by contradiction that a sufficiently small power swap from the time instant where the maximum is achieved towards the time instant where the minimum is achieved would still fulfill all constraints and imply a net cost reduction, thus violating optimality of \( d^*(t) \). Moreover, \( d^*(t) = d^*(t) \) for all \( t \). We prove the result \( W_k \) by induction showing that, \( \sum_{t \in \mathcal{N}} d^*(t) = \sum_{t \in \mathcal{N}} d^*(t) \), viz. the constraint of support \( W_k \) is active. For \( k = 1 \) notice that \( \sum_{t \in \mathcal{N}} d^*(t) < \sum_{t \in \mathcal{N}} d^*(t) \) implies that \( D^I(t) + d^*(t) > D^I(t) + d^*(t) \) for some \( t \notin W_1 \). The latter value is in turn bigger than \( D^I(t) + d^*(t) \) for all \( t \in W_1 \). Hence, a power swap from such \( t \) into all time instances in \( W_1 \) (equally distributed) would still preserve constraints and yield an overall cost reduction, violating the optimality of \( d^*(t) \). In particular, then \( d^*(t) = d^*(t) \) for \( t \in W_1 \). By induction, a similar argument shows that \( \sum_{t \in \mathcal{N}} d^*(t) = \sum_{t \in \mathcal{N}} d^*(t) \).
\[ \sum_{i \in W_k} d_i^t(t). \] Assume by the hypothesis that the result is true for \( k - 1 \). If, by contradiction \( \sum_{i \in W_k} d_i^t(t) < \sum_{i \in W_k} d_i^t(t) \), then 
\( \sum_{i \in W_k \setminus W_{k-1}} d_i^t(t) < \sum_{i \in W_k \setminus W_{k-1}} d_i^t(t) \) implies that 
\( D^t(t) + d^t(t) > D^t(t) + d^t(t) \) for some \( t \not\in W_k \). The latter value is in turn bigger than \( D^t(t) + d^t(t) \) for \( t \in W_k \). Hence, a power swap from such \( t \) into all time instants in \( W_k \setminus W_{k-1} \) (equally distributed) would still preserve constraints and yield an overall cost reduction, violating optimality of \( d^t(t) \). As a consequence \( d^t(t) = d^t(t) \), for all \( t \in W_k \).

**E. Proof of Theorem 2**

Thanks to the inclusion (10), we see that the optimal cost of (9) is less or equal to the optimal cost of (5) (because (5) is minimized over a subset of the feasible region of (9)). On the other hand, again by virtue of (10), the optimal cost of (5) is less or equal to the optimal cost of (2), since (2) is minimized over a subset of the feasible region of (5). Hence, the claim follows by Lemma 4.

**REFERENCES**

[1] T. Weitzel and C. H. Glock, “Energy management for stationary electric energy storage systems: A systematic literature review,” Eur. J. Oper. Res., vol. 264, no. 2, pp. 582–606, 2018.

[2] G. Bianchini, M. Casini, D. Pepe, A. Vicino, and G. G. Zanvettor, “An integrated model predictive control approach for optimal HVAC and energy storage operation in large-scale buildings,” Appl. Energy, vol. 240, pp. 327–340, 2019.

[3] J. Ma, J. Qin, T. Salsbury, and P. Xu, “Demand reduction in building energy systems based on economic model predictive control,” Chem. Eng. Sci., vol. 67, no. 1, pp. 92–100, 2012.

[4] G. Serale, M. Fiorentini, A. Capozzoli, D. Bernardini, and A. Bemporad, “Model predictive control (MPC) for enhancing building and HVAC system energy efficiency: Problem formulation, applications and opportunities,” Energies, vol. 11, no. 3, 2018, Art. no. 631.

[5] A. Bernstein, L. Reyes-Chamorro, J.-Y. Le Boudec, and M. Paolone, “A compositional method for real-time control of active distribution networks with explicit power setpoints. Part I: Framework,” Electric Power Syst. Res., vol. 125, pp. 254–264, 2015.

[6] P. Fortenbacher, G. Andersson, and J. L. Mathieu, “Optimal real-time control of multiple battery sets for power system applications,” in Proc. IEEE Eindhoven PowerTech, 2015, pp. 1–6.

[7] F. Kennel, D. Görges, and S. Liu, “Energy management for smart grids with electric vehicles based on hierarchical MPC,” IEEE Trans. Ind. Informat., vol. 9, no. 3, pp. 1528–1537, Aug. 2013.

[8] W. Yao, J. Zhao, F. Wen, Y. Xue, and G. Ledwich, “A hierarchical decomposition approach for coordinated dispatch of plug-in electric vehicles,” IEEE Trans. Power Syst., vol. 28, no. 3, pp. 2768–2778, Aug. 2013.

[9] H. Marzooghi, G. Verbič, and D. J. Hill, “Aggregated demand response modelling for future grid scenarios,” Sustain. Energy, Grids Netw., vol. 5, pp. 94–104, 2016.

[10] C. Calvillo, A. Sánchez-Miralles, J. Villar, and F. Martín, “Optimal planning and operation of aggregated distributed energy resources with market participation,” Appl. Energy, vol. 182, pp. 340–357, 2016.

[11] D. K. Molzahn et al., “A survey of distributed optimization and control algorithms for electric power systems,” IEEE Trans. Smart Grid, vol. 8, no. 6, pp. 2941–2962, Nov. 2017.

[12] E. Dall’Anese, S. S. Guggilam, A. Simonetto, Y. C. Chen, and S. V. Dhople, “Optimal regulation of virtual power plants,” IEEE Trans. Power Syst., vol. 33, no. 2, pp. 1868–1881, Mar. 2017.

[13] A. Engelmann, M. B. Bandeira, and T. Faulwasser, “Approximate dynamic programming with feasibility guarantees,” 2023, arXiv:2306.06201.

[14] E. Özertürk, K. Rheinberger, T. Faulwasser, K. Worthmann, and M. Preißinger, “Aggregation of demand-side flexibilities: A comparative study of approximation algorithms,” Energies, vol. 15, no. 7, 2022, Art. no. 2501.