A quark model analysis of the charge symmetry breaking in nuclear force

Takashi Nasu and Makoto Oka

Department of Physics, Tokyo Institute of Technology, Meguro, Tokyo 152-8551, Japan

Sachiko Takeuchi

Japan College of Social Work, Kiyose 204-8555, Japan

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Abstract

In order to investigate the charge symmetry breaking (CSB) in the short range part of the nuclear force, we calculate the difference of the masses of the neutron and the proton, $\Delta M$, the difference of the scattering lengths of the p-p and n-n scatterings, $\Delta a$, and the difference of the analyzing power of the proton and the neutron in the n-p scattering, $\Delta A(\theta)$, by a quark model. In the present model the sources of CSB are the mass difference of the up and down quarks and the electromagnetic interaction. We investigate how much each of them contributes to $\Delta M$, $\Delta a$ and $\Delta A(\theta)$. It is found that the contribution of CSB of the short range part in the nuclear force is large enough to explain the observed $\Delta A(\theta)$, while $\Delta a$ is rather underestimated.

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I. INTRODUCTION

The charge symmetry is the invariance under the charge-reflection, \(i.e.,\) the reflection about the 1-2 plane in the isospin space. If this were an exact symmetry, the masses of the proton and the neutron would be the same, as well as the binding energies of the mirror nuclei or the scattering lengths of the p-p and n-n scatterings. The charge symmetry holds only approximately in the real world. There are small but non-zero differences such as

\[
\Delta M = M_n - M_p = 1.29 \text{ [MeV]} \quad \text{and} \quad \Delta a = a_{pp} - a_{nn} = 1.5 \text{ [fm]}.
\]

These differences are manifestation of the charge symmetry breaking (CSB).

CSB appears also in spin-dependent observables. For example, the \(\vec{p}-\vec{n}\) system is the mirror of \(\vec{n}-\vec{p}\), where \(\vec{p}(\vec{n})\) is a polarized nucleon. There was found small difference in the analyzing powers of \(\vec{p}\) and \(\vec{n}\) in the medium energy scattering [1, 2],

\[
\Delta A(\theta) = A_n(\theta) - A_p(\theta).
\]

The study of \(\Delta A(\theta)\) is important because there is no Coulomb interaction between n and p.

It is important to understand CSB from the quantum chromodynamics (QCD) viewpoint [3]. From QCD we find that CSB has two origins: (i) the difference of the masses of the up and down quarks and (ii) the electromagnetic interaction. Thus the study of CSB phenomena can be a good probe to examine the behavior of the quarks and gluons in the low-energy region. The ultimate goal of the CSB study may be understanding their effects on hadron spectra and hadronic interactions directly from QCD, by, \(e.g.,\) lattice QCD simulation. As the direct approach is not available up to now, however, indirect approaches have been taken for the CSB study.

An often used approach to CSB is based on the meson exchange picture of the nuclear force. It was suggested that CSB of the nuclear force is generated by mixings of \(I = 0\) and \(I = 1\) mesons such as \(\rho-\omega\) mixing [4]. A model based on such a picture was reported to explain \(\Delta a\) well [5]. But it was also pointed out that the effect of the \(\rho-\omega\) mixing to CSB may be suppressed by the off-shell effect of the \(\rho-\omega\) mixing [6]. Thus, this problem is still open [7]. A class IV interaction [8] is also generated by the neutron-proton mass difference in the one-pion-exchange interaction [9]. It was pointed out that the effects of OPE and \(\rho-\omega\) mixing explain \(\Delta A(\theta)\) fairly well.
On the other hand, CSB appearing in the short-range part should be investigated by introducing subnucleonic degrees of freedom. One of the pioneering works to apply a quark model to CSB is found in Ref. [10], where the isovector mass shifts of isospin multiplets and the isospin-mixing matrix elements in 1s0d-shell nuclei are investigated by using the quark cluster model (QCM) [11, 12, 13, 14, 15]. It was concluded that the u-d quark constituent mass difference produces significant effects, which may explain the observed Okamoto-Nolen-Schiffer anomaly [16] well.

In the present work, we investigate CSB in $\Delta M$, $\Delta a$ and $\Delta A(\theta)$ by employing essentially the same model for all these three observables: a quark potential model for $\Delta M$ and QCM for $\Delta a$ and $\Delta A(\theta)$. The CSB sources are taken to be (a) the difference of the masses of the up and down constituent quarks and (b) the electromagnetic interaction between the constituent quarks. Our aim is to estimate the effect of CSB sources (a) and (b) on nuclear force by investigating the above three observables simultaneously.

Chentob and Yang [17] (CY) calculated $\Delta a$ using QCM, suggesting that the quark mass difference contributes to $\Delta a$ significantly. Later, Bräuer et al. [18, 19] studied $\Delta a$ and $\Delta A(\theta)$ using QCM and concluded that the effects of CSB sources (a) and (b) are too small to explain the observed value. However, their calculation of $\Delta A(\theta)$ suffers from a wrongly chosen factor, from omitting the symmetric spin-orbit term and from inconsistent use of the operators and wave functions (See sec IV).

In the present paper, we extend CY’s and Bräuer’s works in order to obtain more integrated knowledge on CSB. We investigate CSB in $\Delta M$, $\Delta a$ and $\Delta A(\theta)$ simultaneously. Also, we introduce the Instanton Induced Interaction (III) [20, 21, 22, 23, 24, 25, 26], which comes from the nonperturbative effects of QCD and explains the $\eta - \eta'$ mass splitting. Since III does not break the charge symmetry, its role in this study is mainly to make the effective strength of the one-gluon exchange interaction smaller. The strength becomes reasonably small, which fits to the picture that this term represents the perturbative effect of the gluons (See sec IV). Moreover, we include the symmetric spin-orbit term in the analysis of $\Delta A(\theta)$, whose effect is as large as the antisymmetric one. Furthermore, we solve QCM to obtain the relative wave function and use it to evaluate the matrix elements of $\Delta a$ and $\Delta A(\theta)$.

In section II, we show the Hamiltonian for quarks and the CSB sources. In section III, we explain the detail of the calculations of $\Delta M$, $\Delta a$ and $\Delta A(\theta)$. Results are discussed in section IV. Summary is given in section V.
II. HAMILTONIAN

We employ the constituent quark model with quark masses of order \( m \approx 300\,[\text{MeV}] \) in this study. The Hamiltonian is given by

\[
H = K + V
\]

(3)

\( K \) is the quark kinetic energy and considered as semirelativistic in calculation of \( \Delta M \) (See sec III A) and as non-relativistic in calculations of \( \Delta a \) and \( \Delta A(\theta) \) (See sec III B) in this study. The quark-quark interactions are represented by a static potential, which consists of the confinement (CF), the one-gluon-exchange (OGE) \,[27]\, the electromagnetic (EM) and the instanton induced (III) interactions.

\[
V = V_{\text{conf}} + V_{\text{OGE}} + V_{\text{EM}} + V_{\text{III}}
\]

(4)

\[
V_{\text{CF}} = \sum_{i<j} -a(\vec{\lambda}_i \cdot \vec{\lambda}_j)r_{ij}
\]

(5)

\[
V_{\text{OGE}} = \sum_{i<j}(\vec{\lambda}_i \cdot \vec{\lambda}_j)\frac{\alpha_s}{4}\left\{ \frac{1}{r_{ij}} - \left( \frac{\pi}{2m_i^2} + \frac{\pi}{2m_j^2} + \frac{2\pi}{3m_im_j} - \vec{\sigma}_i \cdot \vec{\sigma}_j \right)\delta(\vec{r}_{ij}) - \left[ \frac{1}{2r_{ij}} \left( \frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4}{m_im_j} \right) \vec{L}_{ij} \cdot \vec{\sigma}_i + \vec{\sigma}_j \right] - \left[ \frac{1}{4r_{ij}^3} \left( \frac{1}{m_i^2} - \frac{1}{m_j^2} \right) \vec{L}_{ij} \cdot \vec{\sigma}_i - \vec{\sigma}_j \right] \right\}
\]

(6)

\[
V_{\text{EM}} = \sum_{i<j} e_ie_j\alpha_{em}\left\{ \frac{1}{r_{ij}} - \left( \frac{\pi}{2m_i^2} + \frac{\pi}{2m_j^2} + \frac{2\pi}{3m_im_j} - \vec{\sigma}_i \cdot \vec{\sigma}_j \right)\delta(\vec{r}_{ij}) - \left[ \frac{1}{2r_{ij}^3} \left( \frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4}{m_im_j} \right) \vec{L}_{ij} \cdot \vec{\sigma}_i + \vec{\sigma}_j \right] - \left[ \frac{1}{4r_{ij}^3} \left( \frac{1}{m_i^2} - \frac{1}{m_j^2} \right) \vec{L}_{ij} \cdot \vec{\sigma}_i - \vec{\sigma}_j \right] \right\}
\]

(7)

\[
V_{\text{III}} = V_{\text{I}}^{(2)} \sum_{i<j} \left( 1 + \frac{3}{32} \vec{\lambda}_i \cdot \vec{\lambda}_j + \frac{9}{32} \vec{\lambda}_i \cdot \vec{\lambda}_j \vec{\sigma}_i \cdot \vec{\sigma}_j \right)\delta(\vec{r}_{ij}) - \frac{1}{8} \left\{ \left( -1 + \frac{3}{16} \vec{\lambda}_i \cdot \vec{\lambda}_j \right) \frac{2}{m^2} + \frac{9}{8m^2} \vec{\lambda}_i \cdot \vec{\lambda}_j \right\} \frac{\delta(\vec{r}_{ij})}{r^2} \vec{L}_{ij} \cdot \vec{\sigma}_i + \vec{\sigma}_j
\]

(8)

\( \vec{\lambda}_i \) is the color SU(3) Gell-Mann matrix and \( e_i \) is the quark electric charge in units of the proton charge \( e \). In this study it is assumed that the confinement potential does not break the charge symmetry. This is a natural assumption based on the confining potential obtained, for instance, from lattice QCD calculation. Yet there may exist velocity dependent terms associated with confinement which break the charge symmetry. We do not consider such
terms in this study. Taking the Breit-Fermi interaction naively, non-Galilei invariant terms appear in the LS terms. But we consider only the Galilei invariant terms such as LS term in Eqs. \[6\text{ }7\]. It should be noted that the Instanton Induced Interaction (III) is effective only on the flavor singlet (iso-singlet) quark-quark state. In other words, it works only on a pair of up and down quarks. Thus III does not break the charge symmetry.

In this Hamiltonian the terms including the quark mass and the electric charge may break the charge symmetry. In order to show the CSB terms explicitly we rewrite the quark mass and the electric charge in terms of the isospin operator.

\[
m_i = \frac{m_d + m_u}{2} - \frac{m_d - m_u}{2} \tau_3^{(i)}
\]

\[
= \bar{m}(1 - \frac{\Delta m}{2\bar{m}} \tau_3^{(i)})
\]

\[
= \bar{m}(1 - \epsilon \tau_3^{(i)})
\]

\[
e_i = \frac{\tau_3^{(i)}}{2} + \frac{1}{6}
\]

(9)

(10)

where

\[
\bar{m} = \frac{m_d + m_u}{2} \quad \Delta m = m_d - m_u
\]

\[
\epsilon = \frac{\Delta m}{2\bar{m}}
\]

(11)

Using the typical constituent quark mass \(\bar{m} \simeq 300\) MeV and the up and down quark mass difference \(\Delta m \simeq 6\) MeV, \(\epsilon \simeq \frac{6}{2 \times 300} = \frac{1}{100}\) is as small as the electromagnetic coupling constant, \(\alpha_{e.m.} \simeq 1/137\). So we divide the Hamiltonian into the charge symmetric part \(\bar{H}\) and the charge symmetry breaking part \(\Delta H_{\text{CSB}}\), and treat \(\Delta H_{\text{CSB}}\) perturbatively.

The CSB part of the Hamiltonian is given to the leading order in \(\epsilon\) and \(\alpha_{e.m.}\) by

\[
\Delta V_{\text{CSB}} = \Delta V_{\text{CSB}}^{\text{OGE}} + \Delta V_{\text{CSB}}^{\text{EM}}
\]

\[
\Delta V_{\text{CSB}}^{\text{OGE}} = \sum_{i<j} (\vec{\lambda}_i \cdot \vec{\lambda}_j) \frac{\alpha_s}{4} \{ - \frac{\pi}{\bar{m}^2} (\tau_3^{(i)} + \tau_3^{(j)})(1 + \frac{2}{3} \vec{\sigma}_i \cdot \vec{\sigma}_j) \delta(\vec{r}_{ij})
\]

\[
- \frac{3\alpha_s}{4\bar{m}^2 r_{ij}^3} \vec{L}_{ij} \cdot (\vec{\sigma}_i + \vec{\sigma}_j)(\tau_3^{(i)} + \tau_3^{(j)})
\]

\[
- \frac{\alpha_s}{4\bar{m}^2 r_{ij}^3} \vec{L}_{ij} \cdot (\vec{\sigma}_i - \vec{\sigma}_j)(\tau_3^{(i)} - \tau_3^{(j)})
\}

(13)

\[
\Delta V_{\text{CSB}}^{\text{EM}} = \sum_{i<j} \frac{\tau_3^{(i)} + \tau_3^{(j)}}{12} \alpha_{e.m.} \{ \frac{1}{r_{ij}} - \frac{\pi}{\bar{m}^2} (1 + \frac{2}{3} \vec{\sigma}_i \cdot \vec{\sigma}_j) \delta(\vec{r}_{ij})
\]

\[
- \frac{3}{4\bar{m}^2 r_{ij}^3} \vec{L}_{ij} \cdot (\vec{\sigma}_i + \vec{\sigma}_j)
\}

(14)
We ignore the second order terms $O(\epsilon^2, \alpha_{m, e.m.}, \epsilon \alpha_{e.m.})$. The CSB terms from the tensor interaction are excluded because the tensor interactions between quarks are small. But we consider them when solving the charge symmetric equation for the unperturbed wave function.

The Hamiltonian has 5 parameters, $\alpha_s, \bar{m}, a, V_0^{(2)}$ and $\Delta m$. The parameters are determined so as to reproduce the single baryon properties and the results are shown in section [V].

III. CALCULATIONS

In this section we present the formulas of the neutron-proton mass difference, $\Delta M$, the difference of the scattering lengths of the p-p and n-n scattering, $\Delta a$, and the difference of the analyzing power of the neutron and the proton of the n-p scattering, $\Delta A(\theta)$.

A. The proton-neutron mass difference $\Delta M$

The differences of the mass of the isodoublet hadrons were evaluated in the constituent quark model by Isgur [28]. We also evaluate the neutron-proton mass difference in order to determine the mass difference of the up and down constituent quarks. Our approach is different in the following two points. First, we consider the semi-relativistic kinetic energy term,

$$K = \sum_{i}^{3} \sqrt{\bar{m}_{i}^2 + p_{i}^2}$$

(15)

Eq. (15) can be divided into the charge symmetric part and the charge symmetry breaking part,

$$K = \bar{K} + \Delta K_{CSB}$$

(16)

$$\bar{K} = \sum_{i}^{3} \sqrt{\bar{m}_{i}^2 + p_{i}^2}$$

(17)

$$\Delta K_{CSB} = -\sum_{i}^{3} \frac{\bar{m}_{i}^2}{\sqrt{\bar{m}_{i}^2 + p_{i}^2}} \epsilon \tau_{3}^{(i)}$$

(18)

Eq. (15) contains the kinetic energy of the center of mass coordinate, which must be subtracted in order to calculate the baryon mass. For the semirelativistic kinematics, the
center-of-mass energy can not be treated exactly. Therefore we use the following approximation,

\[ M_N = \langle \sqrt{H^2 - P_G^2} \rangle \]
\[ \approx \langle H \rangle - \frac{\langle P_G^2 \rangle}{2\langle H \rangle} \]  \hfill (19)

The relativistic effect is partially included as the convergence of the expansion in \( \langle P_G^2 \rangle \langle H \rangle \) is better than that in \( \langle p_{\text{im}} \rangle \). Then the nucleon mass can be written in terms of \( \bar{H} \) and \( \Delta H_{CSB} \) as

\[ M_N = \langle \bar{H} \rangle - \frac{\langle P_G^2 \rangle}{2\langle H \rangle} + \langle \Delta H_{CSB} \rangle \left( 1 + \frac{\langle P_G^2 \rangle}{2\langle H \rangle^2} \right) \]  \hfill (20)

where

\[ H = \bar{H} + \Delta H_{CSB} \]  \hfill (21)
\[ \bar{H} = \bar{K} + \bar{V} \]  \hfill (22)
\[ \Delta H_{CSB} = \Delta K_{CSB} + \Delta V_{CSB} \]  \hfill (23)

\( \bar{H} \) is the charge symmetric part of the Hamiltonian and \( \Delta H_{CSB} \) contains Eqs. (18) and (12). The first two terms of Eq. (20) give the average mass of the nucleon and the third term contributes to \( \Delta M \). The up-down quark mass difference \( \Delta m \) is determined so as to reproduce \( \Delta M \) by using Eq. (20).

The second difference from the Isgur’s work is that the Instanton Induced Interaction (III) is considered in this study. III has the contact spin-spin interaction and contributes to the difference of the masses of the Nucleon and \( \Delta (1232) \) just like the color magnetic interaction. We choose the coupling constant of the OGE, \( \alpha_s \), and the III, \( V_0^{(2)} \), so as to reproduce the nucleon-\( \Delta \) mass difference in total. So \( \alpha_s \) becomes smaller effectively by considering III.

**B. CSB in the N-N scattering**

In the calculation of the scattering lengths and analyzing powers, we employ the quark cluster model (QCM) which describes two-nucleon systems in terms of their quark coordinates. The scattering wave functions, which are used as the unperturbed states, are calculated by solving the resonating group method (RGM) equation. By mainly
technical reasons the kinetic energy term is treated purely in the non-relativistic way, i.e. the semirelativistic kinematics is not taken into account contrary to the case of single baryon mass. This approximation can be justified because the relativistic effect on the kinetic energy term is smaller for the motion of the two baryons. Then the kinetic energy is given as

\[ K = \sum_{i}^{6} K_{i} - K_{G} \]  

(24)

\[ K_{i} = \left( m_{i} + \frac{p_{i}^{2}}{2m_{i}} \right) \]  

(25)

\[ K_{G} = \frac{P_{G}^{2}}{2M_{G}} \]  

(26)

where

\[ M_{G} = \sum_{i}^{6} m_{i} \quad P_{G} = \sum_{i}^{6} p_{i} \]  

(27)

The RGM equation for the baryon A and baryon B is as follows,

\[ \int \phi_{A}(\xi_{A})\phi_{B}(\xi_{B})(H - E)A[\phi_{A}(\xi_{A})\phi_{B}(\xi_{B})\chi(R_{AB})]d\xi_{A}d\xi_{B} = 0 \]  

(28)

\[ \phi_{A}(\xi_{A}) = \frac{1}{2\pi}\frac{1}{b^{3}}\frac{3}{(2\pi b^{2})^{\frac{3}{4}}} \exp\left( -\frac{\xi_{A1}^{2}}{4b^{2}} - \frac{\xi_{A2}^{2}}{3b^{2}} \right) \]  

(29)

\( \phi_{A(B)} \) and \( \xi_{A(B)} \) is the internal wave function and coordinates of the baryon A(B). \( R_{AB} \) is the relative coordinates of the baryon A and B. The parameter \( b \) is the gaussian size parameter, which represents a nucleon size. \( A\) is the antisymmetrization operator for six quarks and is written as follows.

\[ A = 1 - A' = 1 - \sum_{i\in A,j\in B} P_{ij} \]  

(30)

In the end, the following equation is obtained

\[ \left[ \frac{P_{AB}^{2}}{2\mu_{AB}} + V_{\text{rel}}^{(D)}(R) - \frac{k^{2}}{2\mu_{AB}} \right] \chi(R) - \int dR'\left( K^{(EX)}(R, R') + V^{(EX)}(R, R') \right) \]  

\[ - E\chi^{(EX)}(R) = 0 \]  

(31)

where \( P_{AB} \) is the momentum operator of the relative motion of the baryons A and B, and

\[ E = \tilde{M}_{A} + \tilde{M}_{B} + \frac{k^{2}}{2\mu_{AB}} \]  

(32)

\[ \frac{1}{\mu_{AB}} = \frac{1}{M_{A}} + \frac{1}{M_{B}} \]  

(33)
\[ M_{A(B)} = \sum_{i \in A(B)} m_i \]  
(34)

\[ \frac{1}{\tilde{\mu}_{AB}} = \frac{1}{M_A} + \frac{1}{M_B} \]  
(35)

\[ \tilde{M}_{A(B)} : \text{observed mass of the baryon } A(B) \]  
(36)

It should be noted here that \( M_{A(B)} \) and \( \tilde{M}_{A(B)} \) may not agree with each other completely. We take \( m_i = 313 \text{ [MeV]} \) in our calculation so that the difference is small, but for the charge symmetry breaking we assume that \( \mu_{AB} = \tilde{\mu}_{AB} \). The observed masses of the proton and neutron are given by

\[ \tilde{M}_A = \tilde{M}(1 - \epsilon_N \tau_3^{(A)}) \]  
(37)

\[ \epsilon_N = \frac{\Delta \tilde{M}}{2 \tilde{M}} \]  
(38)

\[ \tilde{M} = \frac{\tilde{M}_p + \tilde{M}_n}{2} = 939 \text{ MeV} \]  
(39)

\[ \Delta \tilde{M} = \tilde{M}_n - \tilde{M}_p = 1.29 \text{ MeV} \]  
(40)

Therefore we may rewrite the kinetic energy terms as

\[ \frac{P^2_{AB}}{2\tilde{\mu}_{AB}} - \frac{k^2}{2\tilde{\mu}_{AB}} = \frac{P^2_{AB} - k^2}{2\tilde{\mu}} \left( 1 + \frac{\tau_3^{(A)} + \tau_3^{(B)}}{2} \epsilon_N \right) \]  
(41)

and the energy in Eq. (31) as

\[ E = 2\tilde{M}(1 - \frac{\tau_3^{(A)} + \tau_3^{(B)}}{2} \epsilon_N) + \frac{k^2}{2\tilde{\mu}} \left( 1 + \frac{\tau_3^{(A)} + \tau_3^{(B)}}{2} \epsilon_N \right) \]  
(42)

\[ = 2\tilde{M} + \frac{k^2}{2\tilde{\mu}} + (-2\tilde{M} + \frac{k^2}{2\tilde{\mu}}) \left( \frac{\tau_3^{(A)} + \tau_3^{(B)}}{2} \epsilon_N \right) \]  
(43)

because

\[ \frac{1}{2\tilde{\mu}_{AB}} = \frac{1}{2\tilde{\mu}} \left( 1 + \frac{\tau_3^{(A)} + \tau_3^{(B)}}{2} \epsilon_N \right) \]  
(44)

\[ \tilde{\mu} = \frac{\tilde{M}}{2} \]  
(45)

The RGM kernels \( V_{rel}^{(D)}, N^{(EX)}, K^{(EX)}, V^{(EX)} \) are defined by

\[ V_{rel}^{(D)}(R) = \int d\xi_A d\xi_B dR_{AB} \phi_A(\xi_A) \phi_B(\xi_B) \]
\[
\sum_{i \in A, j \in B} V_{ij} \delta(R - R_{AB}) \phi_A(\xi_A) \phi_B(\xi_B)
\] (46)

\[
\begin{pmatrix}
N^{(EX)}(R', R) \\
K^{(EX)}(R', R) \\
V^{(EX)}(R', R)
\end{pmatrix} = \int d\xi_A d\xi_B d\tilde{R}_{AB} \phi_A(\xi_A) \phi_B(\xi_B) \delta(R' - R_{AB})
\begin{pmatrix}
1 \\
K \\
V
\end{pmatrix}
\]

\[
A'[\delta(R - R_{AB}) \phi_A(\xi_A) \phi_B(\xi_B)]
= \begin{pmatrix}
\tilde{N}^{(EX)}(R', R) \\
\tilde{K}^{(EX)}(R', R) + \Delta K_{CSB}(R', R) \\
\tilde{V}^{(EX)}(R', R) + \Delta V_{CSB}(R', R)
\end{pmatrix}
\] (47)

K and V are given by Eqs. (24) and (4) and can be divided into the charge symmetric part \(\tilde{K}, \tilde{V}\) and the charge symmetry breaking part \(\Delta K_{CSB}, \Delta V_{CSB}\). Therefore RGM kernels are divided into the charge symmetric part \(\tilde{K}^{(EX)}, \tilde{V}^{(EX)}\) and the charge symmetry breaking part \(\Delta K_{CSB}^{(EX)}, \Delta V_{CSB}^{(EX)}\).

In order to treat the CSB part perturbatively, we employ the distorted wave Born approximation (DWBA) in this study. we solve the following equation to obtain the distorted wave.

\[
\left(\frac{P_{AB}^2}{2\mu} - \frac{k^2}{2\mu}\right) \chi_{\text{dist}}(R) - \int dR' (\tilde{K}^{(EX)}(R, R') + \tilde{V}^{(EX)}(R, R'))
- \tilde{E} \tilde{N}^{(EX)}(R, R') \chi_{\text{dist}}(R') = 0
\] (48)

The direct kernel \(V_{rel}^{(D)}(R)\) comes from the electromagnetic interaction of quarks and corresponds to the electromagnetic interaction of baryons. We are interested in effects of CSB at the quark level, not at the hadron level. So we ignore the direct kernel. But we consider the exchange kernel of the electromagnetic interaction of quarks. Using the distorted wave \(\chi_{\text{dist}}(R)\), we estimate the following CSB parts.

\[
\text{(CSB part)} = \frac{P_{AB}^2}{2\mu} - \frac{k^2}{2\mu} \left(\frac{\tau_3^{(A)} + \tau_3^{(B)}}{2} \epsilon_N\right) \chi_{\text{dist}}(R)
- \int dR' (\Delta K_{CSB}^{(EX)}(R, R') + \Delta V_{CSB}^{(EX)}(R, R'))
- \Delta E_{CSB} \tilde{N}^{(EX)}(R, R') \chi_{\text{dist}}(R')
\] (49)
C. CSB in the analyzing power

There is a special CSB interaction in the neutron-proton system, which is called the class IV interaction, according to the classification by Henley and Miller [8].

\[
V_{IV} \propto (\tau_3^A - \tau_3^B)(\vec{\sigma}_A - \vec{\sigma}_B) \tag{50}
\]

or

\[
(\vec{\tau}_A \times \vec{\tau}_B)_z (\vec{\sigma}_A \times \vec{\sigma}_B) \tag{51}
\]

one sees that the class IV interaction mixes spin-singlet states and spin-triplet states. The spin singlet-triplet mixing induces asymmetries of spin polarization observables such as the analyzing power. At the level of the quark-quark interaction, CSB in the spin-orbit interactions is given as [See Eqs. (6-7)]

\[
V_{CSB}^{LS} = V_{qSLS}^{OGE} + V_{qALS}^{OGE} + V_{qSLS}^{EM} \tag{52}
\]

\[
V_{qSLS}^{OGE} = -\sum_{i<j} (\vec{\lambda}_i \cdot \vec{\lambda}_j) \frac{3\alpha_s}{16\bar{m}^2} \frac{L_{ij}}{r_{ij}^3} [((\vec{\sigma}_i + \vec{\sigma}_j)(\tau_3^{(i)} + \tau_3^{(j)})] \tag{53}
\]

\[
V_{qALS}^{OGE} = -\sum_{i<j} (\vec{\lambda}_i \cdot \vec{\lambda}_j) \frac{\alpha_s}{16\bar{m}^2} \frac{L_{ij}}{r_{ij}^3} [((\vec{\sigma}_i - \vec{\sigma}_j)(\tau_3^{(i)} - \tau_3^{(j)})] \tag{54}
\]

\[
V_{qSLS}^{EM} = -\sum_{i<j} \frac{\alpha_{em}}{16\bar{m}^2} \frac{L_{ij}}{r_{ij}^3} [((\vec{\sigma}_i + \vec{\sigma}_j)(\tau_3^{(i)} + \tau_3^{(j)})] \tag{55}
\]

The first two terms of Eq. (52) come from the one-gluon-exchange interaction and the third term from the electromagnetic interaction of quarks. It should be noted that the symmetric spin-orbit interaction of quarks (qSLS) induces the class IV interaction of baryons as well as the antisymmetric one (qALS). Bräuer et al. calculated \(\Delta A(\theta)\) using a similar model without including the qSLS terms [19]. They concluded that the contribution of quarks to \(\Delta A(\theta)\) is very small. But we will see that the contribution of quark spin-orbit interactions, Eq. (52), to \(\Delta A(\theta)\) is large enough to reproduce the observed \(\Delta A(\theta)\).

Using DWBA, we calculate the following matrix elements for J=\(L\leq 3\),

\[
\Delta T_{CSB} = \langle {}^3L_J|V_{CSB}^{LS}|{}^1L_J \rangle \tag{56}
\]

Then the total T-matrix is given as follows,

\[
T = \tilde{T}_{CS} + \Delta T_{CSB} \tag{57}
\]
\( T_{CS} \) is obtained by solving the RGM equation. \( T \) is regarded as a matrix based on the spin states and the analyzing power is given by

\[
A_N(\theta) = \frac{\text{Tr}[T^\dagger \sigma_N T]}{\text{Tr}[T^\dagger T]} \quad (58)
\]

Then \( \Delta A(\theta) \) is given in terms of \( T_{CS} \) and \( \Delta T_{CSB} \)

\[
\Delta A(\theta) = A_n(\theta) - A_p(\theta) = \frac{2 \text{Re} \text{Tr}[T_{CS}^\dagger (\sigma_n - \sigma_p) \Delta T_{CSB}]}{\text{Tr}[T_{CS}^\dagger T_{CS}]} \quad (59)
\]

We show the explicit forms of the T-matrix and of \( \Delta A(\theta) \) in Appendix A.

**IV. RESULTS**

The parameters in our calculation are determined so as to reproduce the single nucleon property. In order to show explicitly how much the contribution of the Instanton Induced Interaction (III) to the Nucleon-\( \Delta \) splitting is, we introduce a new parameter \( P_{III} \), which denotes the ratio of the contribution of III to the whole Nucleon-\( \Delta \) splitting. For example, when \( P_{III} = 0.4 \) the contribution of III to the Nucleon-\( \Delta \) splitting is 40\% of the whole one. \( V_0^{(2)} \) is determined so as to reproduce the \( \eta \) and \( \eta' \) mass splitting. Our analysis shows that \( P_{III} \sim 0.4 \text{--} 0.5 \) gives the right \( \eta \text{-} \eta' \) splitting. Here we try two values \( P_{III} = 0.4 \) and 0.5. Using the nucleon mass formula Eq. (20), we obtain \( \Delta m \) for each \( P_{III} \). The results are given in Table I. The parameter \( b \) is the gaussian size parameter for the internal wave function of the nucleon, which represents the nucleon size.

| TABLE I: Parameters |
|---------------------|
| \( P_{III} \) | \( \Delta m \) | \( \bar{m} \) [MeV] | \( b \) [fm] | \( \alpha_s \) | \( a \) [MeV/fm] | \( V_0^{(2)} \) [MeV fm\(^3\)] |
| A         | 0.4 | 7.3 | 313 | 0.6 | 0.91 | 44.29 | -177.2 |
| B         | 0.5 | 5.2 | 313 | 0.6 | 0.76 | 40.34 | -221.5 |

Another possible source of the N-\( \Delta \) splitting is contribution of pion cloud around the baryon. For instance, the cloudy bag model predicts the N-\( \Delta \) splitting of about 100 MeV [29]. This effect may reduce the roles of OGE and III, but it is not taken into account in this approach.
By increasing $P_{III}$, we reduce $\alpha_s$ accordingly so that the N-$\Delta$ mass difference is fixed. For $P_{III} = 0.4$, $\alpha_s$ becomes 0.91, while $\alpha_s = 1.52$ is necessary to reproduce the N-$\Delta$ mass difference only by OGE. In order to show the effect of the Instanton Induced Interaction to $\Delta M$, we show contribution of each term to $\Delta M$ in Table II for various $P_{III}$. The Kin,OGE and EM represent the contributions of the kinetic energy, the one-gluon exchange interaction and the electromagnetic interaction to $\Delta M$. It should be noted that when $P_{III} = 0$ we cannot reproduce the $\Delta M$ because OGE gives large contribution, which goes to the opposite direction. This shows the essential role of the III, which reduces the OGE strength.

| $P_{III}$ | Kin | OGE | EM | $M_n - M_p$ |
|----------|-----|-----|----|------------|
| 0        | 4.72 | -5.54 | -0.41 | -1.23     |
| 0.1      | 4.72 | -5.54 | -0.41 | -0.67     |
| 0.2      | 4.72 | -4.99 | -0.41 | -0.12     |
| 0.3      | 4.72 | -4.43 | -0.41 | 0.44      |
| 0.4      | 4.72 | -3.88 | -0.41 | 0.99      |
| 0.5      | 4.72 | -2.77 | -0.41 | 1.54      |

It is also found that the calculation of “Strong hyperfine” for “p-n” in Table I of Ref. 28 is different from our calculation even if we use the same potential. This is because Isgur considers distortion of the quark wave function from the u-d quark mass difference. However, to be consistent the distortion of the wave function should not contribute to the energy in the first order of the perturbation theory. The contribution of the “Strong hyperfine” to $\Delta M$ should be $\frac{1}{3} \delta \frac{\Delta m}{m}$ instead of $\frac{1}{24} \delta \frac{\Delta m}{m}$ in Ref. 28, where $\delta$ is the nucleon-$\Delta$ mass splitting.

Next we calculate $\Delta a$ using the parameters in Table I. The results are shown in Table III $\bar{a}(\bar{r})$ and $\Delta a(\Delta r)$ is the average and the difference of the scattering lengths (effective ranges) of the p-p and n-n scatterings.

$$\bar{a} = \frac{a_{pp} + a_{nn}}{2} \quad \Delta a = a_{pp} - a_{nn}$$

$$\bar{r} = \frac{r_{pp} + r_{nn}}{2} \quad \Delta r = r_{pp} - r_{nn}$$

1 Chemtob and Yang also point out the mismatch with Isgur in their paper 17.
Our results, $\Delta a = 0.79$ and 0.52 [fm] for $P_{III} = 0.4$ and 0.5, are somewhat smaller than the observed value $\sim 1.5$ [fm]. We, however, point out that $\Delta a$ is sensitive to the parameters because it is given by a cancellation of positive and negative terms.

### TABLE III: Scattering length

|       | $P_{III}$ | $\Delta m$ [MeV] | $\bar{a}$ [fm] | $\Delta a$[fm] | $\bar{r}$[fm] | $\Delta r$[fm] |
|-------|-----------|------------------|----------------|----------------|--------------|---------------|
| A     | 0.4       | 7.3              | -17.9          | 0.79           | 2.42         | -0.39         |
| B     | 0.5       | 5.2              | -17.9          | 0.52           | 2.46         | -0.25         |
| Exp[3]|           |                  | -18.1±0.5      | 1.5±0.5        | 2.80±0.12    | 0.10±0.12     |
| B [18]|           |                  | 5.0            | 20.07          | 0.46         |               |
| CY [17]|         |                  | 6.0            | 2~3.5          |             |               |

In Table IV we show each contribution of CSB terms Eq. (49) to $\Delta a$. NMD, Kin, OGE and EM are contributions of the first term of Eq. (49), the quark kinetic energy (including the $\Delta E_{CSB}$ term), the one-gluon exchange interaction and the electromagnetic interaction, respectively.

### TABLE IV: The contributions to $\Delta a$ of CSB terms [fm]

|       | NMD | Kin | OGE | EM |
|-------|-----|-----|-----|----|
| A     | 0.3 | -2.6| 2.9 | 0.2|
| B     | 0.3 | -1.7| 1.7 | 0.2|

We estimate $\Delta a$ in our formulation for the parameters of Ref. 18 (B) and 17 (CY). The contributions of Kin and OGE should be given by

$$\Delta a_{\text{Kin}} \propto \frac{\Delta m}{m^2b^2} \equiv \Delta b_{\text{Kin}} \quad (62)$$

$$\Delta a_{\text{OGE}} \propto \frac{\alpha_s \Delta m}{m^3b^3} \equiv \Delta b_{\text{OGE}} \quad (63)$$

In Table V we show $\Delta b_{\text{Kin}}$ and $\Delta b_{\text{OGE}}$ for the parameters of B and CY. Using the values of Table IV and V we find

$$\Delta a_{\text{Kin}} + \Delta a_{\text{OGE}}|_B = -2.6 \times \frac{4.6}{8.0} + 2.9 \times \frac{5.2}{7.7} \approx -1.5 + 2.0 = 0.5 \quad (64)$$
\[
\Delta a_{\text{Kin}} + \Delta a_{\text{OGE}}|_C = -2.6 \times \frac{5.6}{8.0} + 2.9 \times \frac{8.9}{7.7} = -1.8 + 3.4 = 1.6
\] (65)

These estimates suggest that our results may become larger by the changing the parameters. As \(\Delta b_{\text{Kin}}\) is larger than \(\Delta b_{\text{OGE}}\) in our parameter choice, the cancellation of \(\Delta a_{\text{Kin}}\) and \(\Delta a_{\text{OGE}}\) is stronger than the other cases. On the other hand \(\Delta r\) is too large and has the wrong sign. More investigation should be done for \(\Delta r\), which reflects not only the strength of the interaction but also its radial dependence.

| \(\Delta b_{\text{Kin}} \) [MeV] | \(\Delta b_{\text{OGE}} \) [Mev] |
|------------------|------------------|
| A 8.0            | 7.7              |
| B 4.6            | 5.2              |
| CY  5.6          | 8.9              |

Finally we calculated \(\Delta A(\theta)\) at two energy points, taking \(P_{\text{III}} = 0.4\). The results at \(E_n = 183\) and 477 [MeV] are shown in Fig. 1 and 2. The results at \(E_n = 183\) and 477 MeV are large enough to reproduce the data [1, 2], which disagree with the conclusion of Br"auer et al. [19]. The difference mainly comes from two points. The first point is that they consider only the antisymmetric spin-orbit interaction of quarks (qALS) not the symmetric spin-orbit interaction of quarks (sSLS). The factor of qSLS is three times as large as that of qALS (See Eqs. (53-54)). The remaining discrepancy might be attributed to their erroneous choice of the unit of \(\gamma_1\) in the formula Eq. (3.6) in their paper [19]. We convert their value of \(\gamma_1\) in radian into that in degrees and obtain \(\Delta A(\theta = 96^\circ) = 5.4 \times 10^{-4}\), which is of the same order as our estimate. Our result at \(E_n = 477\) [MeV] is too large. It is not surprising since we fit the phase shift of the N-N scattering up to \(E_n = 400\) [MeV] and we may not apply QCM at higher energy and we need higher partial waves.

Fig. 3 and 4 show the contributions of \(\langle 1P_1|\hat{T}|3P_1 \rangle\), \(\langle 1D_2|\hat{T}|3D_2 \rangle\) and \(\langle 1F_3|\hat{T}|3F_3 \rangle\) to \(\Delta A(\theta)\). It is found that the contribution of \(\langle 1P_1|\hat{T}|3P_1 \rangle\) is dominant in the observed \(\theta\) region. But the other mixings of partial wave become important for the other \(\theta\) region.

We also investigate each contribution of the one-gluon exchange interaction and the electromagnetic interaction. (Fig. 5 and 6) It is found that the contribution of OGE depends on
the incident energy much strongly than that of the electromagnetic interaction does. This is because the dominant contribution of the EM interaction is the direct interaction while OGE interaction contributes as the exchange interaction. Therefore their energy dependences are different from each other, which may be studied by future experiment at various energy points.
FIG. 3: The contribution of each partial wave mixing at $E_n = 183$ MeV.

FIG. 4: The contribution of each partial wave mixing at $E_n = 477$ MeV.

V. CONCLUSION

We have calculated the difference of the masses of the neutron and the proton, $\Delta M$, the difference of the scattering lengths of the p-p and n-n scatterings, $\Delta a$, and the difference of the analyzing power of the proton and the neutron in the n-p scattering, $\Delta A(\theta)$, using the quark cluster model. In the calculation of $\Delta M$, we treated the kinetic energy in the semirelativistic way and introduce the Instanton Induced Interaction (III). We have found that the contribution of the one-gluon-exchange interaction (OGE) is suppressed by the
introduction of the III and have determined the up-down quark mass difference, $\Delta m = 7.3$ and 5.2 [MeV] for $P_{III} = 0.4$ and 0.5.

We have calculated $\Delta a$ for the CSB parameters fixed by $\Delta M$. Our results are $\Delta a = 0.8$ and 0.5 [fm] for $P_{III} = 0.4$ and 0.5, which are smaller than the observed value. It is found that the contribution of the u-d mass difference to $\Delta a$ is comparable with that from EM interaction because the contributions of OGE and the quark kinetic energy cancel out each other. It is pointed out that $\Delta a$ is sensitive to the choice of the quark model parameters because of this cancellation.
The P-wave CSB observable, $\Delta A(\theta)$, is calculated for $P_{III} = 0.4$. It is found that CSB of the short range part in nuclear force is large enough to explain $\Delta A(\theta)$. This result is different from the conclusion of Bräuer et al. We have found that this discrepancy is attributed to the introduction of the quark symmetric spin-orbit interaction and the erroneous choice of the $\gamma_1$ in their paper. We also have investigated the importance of individual mixing matrix element, $\langle \hat{T} | P_1 \rangle$, $\langle \hat{T} | D_2 \rangle$, and $\langle \hat{T} | F_3 \rangle$ and also the relative importance of the OGE and EM interaction. It is found that the contributions of $\langle \hat{T} | P_1 \rangle$ and OGE are dominant in the observed $\theta$ region. Future experiments for other angles as well as different energies may give us further information of the mixings of other partial waves and properties the spin-orbit parts of the OGE and EM interactions. In fact, we have observed that at $E_n = 477$ [MeV] the contributions of the higher partial waves become more important than at $E_n = 183$ [MeV]. The present quark model description is found to account for the short-range part of CSB. We would like to stress that the CSB for the single nucleon as well as the central and spin-orbit parts of the nuclear force are consistently described. There is a possible remaining short-range contribution introduced by Goldman et al. in Ref. 30 (GMS), which comes from interference between the QCD and QED effects. GMS pointed out that such an interference is necessary to explain the mass difference of the neutral and charged pions. Its effect on the NN scattering was studied by Kao and Yang. Because this effect has much ambiguity, we have not included its effect in the present study in order to see how the current data can be accounted without such complex effects.

Effects of longer range CSB may require further analysis. Approaches based on the chiral effective theory were performed in Refs. 32. Although the applicability of the chiral perturbation theory at high energy NN scattering phenomena is not established, its extension to the spin-orbit interaction might be interesting to pursue, which is a subject for future works.

**APPENDIX A: THE DECOMPOSITION OF THE T-MATRICES**

The representations of the T-matrices in the basis of the nucleon spins are shown explicitly in Appendix A. First we expand the wave function of the two nucleons as

$$|\vec{p}, s_z^a, s_z^b\rangle = \sqrt{4\pi} \sum_{L,S,J} \sum_{L_a+S_a=J_a} \langle L, L_z, S, S_z|J, J_z\rangle Y_{L,L_a}(\hat{p}) |s_z^a, s_z^b\rangle$$  (A1)
Using the wave function Eq. (A1), we calculate the T-matrix. For example, the T-matrix of the $^3\text{P}_0 \to ^3\text{P}_0$ scattering is given by

$$\mathcal{T}_{^3\text{P}_0 \to ^3\text{P}_0} = 4\pi \sum_{m,s_z} \langle 1, m, 1, s_z | 0, 0 \rangle^* \langle 1, 0, 1, 0 | 0, 0 \rangle Y_{1,m}(\hat{k})^* Y_{1,0}(\hat{p})$$

$$\langle ^3\text{P}_0 | T | ^3\text{P}_0 \rangle \langle s_z^c, s_z^d | s_z^a, s_z^b \rangle | s_z^a + s_z^d = 0, s_z^c + s_z^d = s_z \rangle = \frac{1}{2} T_{^3\text{P}_0} \begin{pmatrix} 0 & -s e^{-i\phi} & -s e^{-i\phi} \\ c & c \\ c & c \\ s e^{i\phi} & s e^{i\phi} \end{pmatrix}$$

(A2)

where $\hat{p}$ is the unit vector along the initial momentum $\vec{p}$ and we take it along the z-axis. We show the the T-matrix of each partial in terms of $s \equiv \sin \theta, c \equiv \cos \theta$ and $\phi$, where $(\theta, \phi)$ is the scattering angle in the center of mass system.

$$\mathcal{T}_{^1\text{S}_0 \to ^1\text{S}_0} = \frac{1}{2} T_{^1\text{S}_0} \begin{pmatrix} 0 \\ 1 & -1 \\ -1 & 1 \\ 0 \end{pmatrix}$$

(A3)

$$\mathcal{T}_{^3\text{S}_1 \to ^3\text{S}_1} = \frac{1}{2} T_{^3\text{S}_1} \begin{pmatrix} 2 \\ 1 & 1 \\ 1 & 1 \\ 2 \end{pmatrix}$$

(A4)

$$\mathcal{T}_{^1\text{P}_1 \to ^1\text{P}_1} = \frac{3}{2} T_{^1\text{P}_1} c \begin{pmatrix} 0 \\ 1 & -1 \\ -1 & 1 \\ 0 \end{pmatrix}$$

(A5)

$$\mathcal{T}_{^3\text{P}_0 \to ^3\text{P}_0} = \frac{1}{2} T_{^3\text{P}_0} \begin{pmatrix} 0 & -s e^{-i\phi} & -s e^{-i\phi} \\ c & c \\ c & c \\ s e^{i\phi} & s e^{i\phi} \end{pmatrix}$$

(A6)
\[ T_{3p_1 \rightarrow 3p_1} = \frac{3}{4} T_{3p_1} \begin{pmatrix} 2c & 0 & 0 & 0 \\ s e^{i\phi} & 0 & 0 & -s e^{-i\phi} \\ s e^{i\phi} & 0 & 0 & -s e^{-i\phi} \\ 0 & 0 & 0 & 2c \end{pmatrix} \]  
(A7)

\[ T_{3p_2 \rightarrow 3p_2} = \frac{1}{4} T_{3p_2} \begin{pmatrix} 6c & 2s e^{-i\phi} & 2s e^{-i\phi} & 0 \\ -3s e^{i\phi} & 4c & 4c & 3s e^{-i\phi} \\ -3s e^{i\phi} & 4c & 4c & 3s e^{-i\phi} \\ 0 & -2s e^{i\phi} & -2s e^{i\phi} & 6c \end{pmatrix} \]  
(A8)

\[ T_{1D_2 \rightarrow 1D_2} = \frac{5}{4} T_{1D_2} (3c^2 - 1) \begin{pmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 1 & 0 \end{pmatrix} \]  
(A9)

\[ T_{3D_1 \rightarrow 3D_1} = \frac{1}{4} T_{3D_1} \begin{pmatrix} (3c^2 - 1) & -6sc e^{-i\phi} & -6sc e^{-i\phi} & 3s^2 e^{-i2\phi} \\ 3sc e^{i\phi} & 2(3c^2 - 1) & 2(3c^2 - 1) & -3sc e^{-i\phi} \\ 3sc e^{i\phi} & 2(3c^2 - 1) & 2(3c^2 - 1) & -3sc e^{-i\phi} \\ 3s^2 e^{2i\phi} & 6sc e^{i\phi} & 6sc e^{i\phi} & (3c^2 - 1) \end{pmatrix} \]  
(A10)

\[ T_{1D_2 \rightarrow 1D_2} = \frac{5}{4} T_{1D_2} \begin{pmatrix} (3c^2 - 1) & -s^2 e^{-i2\phi} \\ sc e^{i\phi} & 0 & 0 & -sc e^{-i\phi} \\ sc e^{i\phi} & 0 & 0 & -sc e^{-i\phi} \\ -s^2 e^{2i\phi} & (3c^2 - 1) \end{pmatrix} \]  
(A11)

\[ T_{3D_3 \rightarrow 3D_3} = \frac{1}{4} T_{3D_3} \begin{pmatrix} 4(3c^2 - 1) & 6sc e^{-i\phi} & 6sc e^{-i\phi} & 2s^2 e^{-i2\phi} \\ -8sc e^{i\phi} & 3(3c^2 - 1) & 3(3c^2 - 1) & 8sc e^{-i\phi} \\ -8sc e^{i\phi} & 3(3c^2 - 1) & 3(3c^2 - 1) & 8sc e^{-i\phi} \\ 2s^2 e^{2i\phi} & -6sc e^{i\phi} & -6sc e^{i\phi} & 4(3c^2 - 1) \end{pmatrix} \]  
(A12)

\[ T_{1F_3 \rightarrow 1F_3} = \frac{7}{4} T_{1F_3} (5c^3 - 3c) \begin{pmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \]  
(A13)
\[
T_{3F_2 \rightarrow 3F_2} = \frac{1}{2} T_{3F_2} \begin{pmatrix}
3c(5c^2 - 3) & -\frac{3}{2} s(5c^2 - 1) e^{-i\phi} & -\frac{3}{2} s(5c^2 - 1) e^{-i\phi} & 5s^2 c e^{-2i\phi} \\
s(5c^2 - 1) e^{i\phi} & \frac{3}{2} c(5c^2 - 3) & \frac{3}{2} c(5c^2 - 3) & -s(5c^2 - 1) e^{-i\phi} \\
s(5c^2 - 1) e^{i\phi} & \frac{3}{2} c(5c^2 - 3) & \frac{3}{2} c(5c^2 - 3) & -s(5c^2 - 1) e^{-i\phi} \\
5s^2 c e^{2i\phi} & \frac{3}{2} s(5c^2 - 1) e^{i\phi} & \frac{3}{2} s(5c^2 - 1) e^{i\phi} & c(5c^2 - 3)
\end{pmatrix}
\]

(A14)

\[
T_{3F_3 \rightarrow 3F_3} = \frac{1}{2} T_{3F_3} \begin{pmatrix}
\frac{7}{2} c(5c^2 - 3) & -\frac{35}{4} s^2 c e^{-2i\phi} \\
\frac{7}{8} s(5c^2 - 1) e^{i\phi} & 0 & 0 & -\frac{7}{8} s(5c^2 - 1) e^{-i\phi} \\
\frac{7}{8} s(5c^2 - 1) e^{i\phi} & 0 & 0 & -\frac{7}{8} s(5c^2 - 1) e^{-i\phi} \\
-\frac{35}{4} s^2 c e^{2i\phi} & \frac{7}{2} c(5c^2 - 3)
\end{pmatrix}
\]

(A15)

\[
T_{3S_1 \rightarrow 3D_1} = \frac{\sqrt{7}}{4} T_{3S_1 \rightarrow 3D_1} \begin{pmatrix}
3c^2 - 1 & 3sc e^{-i\phi} & 3sc e^{-i\phi} & 3s^2 e^{-i\phi} \\
3sc e^{i\phi} & -(3c^2 - 1) & -(3c^2 - 1) & -3sc e^{i\phi} \\
3sc e^{i\phi} & -(3c^2 - 1) & -(3c^2 - 1) & -3sc e^{i\phi} \\
3s^2 e^{2i\phi} & -3sc e^{i\phi} & -3sc e^{i\phi} & 3c^2 - 1
\end{pmatrix}
\]

(A16)

\[
T_{3D_1 \rightarrow 3S_1} = \frac{\sqrt{7}}{2} T_{3D_1 \rightarrow 3S_1} \begin{pmatrix}
1 & -1 & -1 & 0 \\
-1 & -1 & -1 & 0 \\
1 & 1 & 1 & 1
\end{pmatrix}
\]

(A17)

\[
T_{3P_2 \rightarrow 3F_2} = \frac{\sqrt{6}}{4} T_{3P_2 \rightarrow 3F_2} \begin{pmatrix}
c(5c^2 - 3) & s(5c^2 - 1) e^{-i\phi} & s(5c^2 - 1) e^{-i\phi} & 5s^2 c e^{-2i\phi} \\
-s(5c^2 - 1) e^{i\phi} & -c(5c^2 - 3) & -c(5c^2 - 3) & -s(5c^2 - 1) e^{-i\phi} \\
-s(5c^2 - 1) e^{i\phi} & -c(5c^2 - 3) & -c(5c^2 - 3) & -s(5c^2 - 1) e^{-i\phi} \\
5s^2 c e^{2i\phi} & -s(5c^2 - 1) e^{i\phi} & -s(5c^2 - 1) e^{i\phi} & c(5c^2 - 3)
\end{pmatrix}
\]

(A18)

\[
T_{3F_2 \rightarrow 3P_2} = \frac{\sqrt{6}}{4} T_{3F_2 \rightarrow 3P_2} \begin{pmatrix}
2c & -s e^{-i\phi} & -s e^{-i\phi} & 0 \\
-s e^{i\phi} & -2c & -2c & s e^{-i\phi} \\
-s e^{i\phi} & -2d & -2c & s e^{-i\phi} \\
0 & s e^{i\phi} & s e^{i\phi} & 2c
\end{pmatrix}
\]

(A19)

\[
T_{3P_1 \rightarrow 3P_1} = \frac{3\sqrt{6}}{4} T_{3P_1 \rightarrow 3P_1} \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]

(A20)
Substituting the above T-matrices into the denominator and the numerator of Eq. (59), we obtain

\[
\text{Tr}[\bar{T}^\dagger T] = \frac{1}{8} - 2T_{1s_0} + 2T_{3s_1} - 2\sqrt{2}T_{3s_1 \rightarrow 3d_1} + 2c(-3T_{1p_1} + T_{3p_0} + 2T_{3p_2} - \sqrt{6}T_{3p_2 \rightarrow 3f_2}) + (3c^2 - 1)(-5T_{1d_2} + 2T_{3d_1} + 3T_{3d_3} - \sqrt{2}T_{3s_1 \rightarrow 3d_1}) + c(5c^2 - 3)(-7T_{1f_3} + 3T_{3f_2} - \sqrt{6}T_{3p_2 \rightarrow 3f_2}) + \frac{1}{8}2T_{1s_0} + 2T_{3s_1} - 2\sqrt{2}T_{3s_1 \rightarrow 3d_1} + 2c(3T_{1p_1} + T_{3p_0} + 2T_{3p_2} - \sqrt{6}T_{3p_2 \rightarrow 3f_2}) + (3c^2 - 1)(5T_{1d_2} + 2T_{3d_1} + 3T_{3d_3} - \sqrt{2}T_{3s_1 \rightarrow 3d_1})
\]
+ c(5c^2 - 3)(7T_{1F_3} + 3T_{3P_2} - \sqrt{6}T_{3P_2 \rightarrow 3F_2})^2
+ \frac{1}{8}[4T_{3S_1} + 2\sqrt{2}T_{3S_1 \rightarrow 3D_1} + 2c(3T_{3P_1} + 3T_{3P_2} + \sqrt{6}T_{3P_2 \rightarrow 3F_2})
+ (3c^2 - 1)(T_{3D_1} + 5T_{3D_2} + 4T_{3D_3} + \sqrt{2}T_{3S_1 \rightarrow 3D_1})
+ c(5c^2 - 3)(2T_{3P_2} + 7T_{3F_3} + \sqrt{6}T_{3P_2 \rightarrow 3F_2})^2
+ \frac{1}{8}s^4[3T_{3D_1} - 5T_{3D_2} + 2T_{3D_3} + 3\sqrt{2}T_{3S_1 \rightarrow 3D_1}
+ c(10T_{3P_2} - \frac{35}{2}T_{3P_3} + 5\sqrt{6}T_{3P_2 \rightarrow 3F_2})^2
+ \frac{1}{4}s^2|2T_{3P_0} - 2T_{3P_2} + \sqrt{6}T_{3P_2 \rightarrow 3F_2}
+ 3c(2T_{3D_1} - 2T_{3D_3} - \sqrt{2}T_{3S_1 \rightarrow 3D_1})
+ (5c^2 - 1)(3T_{3F_2} - \sqrt{6}T_{3P_2 \rightarrow 3F_2})^2
+ \frac{1}{4}s^2|3T_{3P_1} - 3T_{3P_2} - \sqrt{6}T_{3P_2 \rightarrow 3F_2}
+ c(3T_{3D_1} + 5T_{3D_2} - 8T_{3D_3} + 3\sqrt{2}T_{3S_1 \rightarrow 3D_1})
+ (5c^2 - 1)(2T_{3F_2} + \frac{7}{4}T_{3F_3} + \sqrt{6}T_{3P_2 \rightarrow 3F_2})^2 (A26)

\text{Tr}[\mathcal{T}^i(\sigma_n - \sigma_p)\Delta T_{CSB}]
= \frac{1}{4}i(3\sqrt{6}sT_{3P_1 \rightarrow 3P_1} + 5\sqrt{6}scT_{3D_2 \rightarrow 3D_2} + \frac{7\sqrt{3}}{2}s(5c^2 - 1)T_{3F_3 \rightarrow 3F_3})
\{4T_{1S_0} + 4T_{3S_1} + 2\sqrt{2}T_{3S_1 \rightarrow 3D_1}
+ 2c(3T_{3P_1} + 3T_{3P_2} + \sqrt{6}T_{3P_2 \rightarrow 3F_2})
+ (3c^2 - 1)(10T_{1D_2} + T_{3D_1} + 5T_{3D_2} + 4T_{3D_3} + 3\sqrt{2}T_{3S_1 \rightarrow 3D_1}
+ (5c^2 - 3)c(14T_{1F_3} + 2T_{3F_2} + 7T_{3F_3} + \sqrt{6}T_{3P_2 \rightarrow 3F_2})
+ s^2(3T_{3D_1} - 5T_{3D_2} + 2T_{3D_3} + 3\sqrt{2}T_{3S_1 \rightarrow 3D_1})
+ s^2c(10T_{3F_2} - \frac{35}{2}T_{3F_3} + 5\sqrt{6}T_{3P_2 \rightarrow 3F_2})\} (A27)

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