Electroweak Penguins, Final State Interaction Phases
and CP Violation in $B \to K\pi$ Decays

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Abstract

The recently observed $B^- \to K^-\pi^0$, $\bar{K}^0\pi^-$ and $\bar{B}^0 \to K^-\pi^+$ decay modes appear to have nearly equal branching ratios. This suggests that tree and electroweak penguins play an important role, and inclusion of the latter improves agreement between factorization calculation and experimental data. The value of $\gamma$ in the range $90^\circ - 130^\circ$ and $220^\circ - 260^\circ$ is favored, while the $B^0 \to K^0\pi^0$ rate is suppressed. Direct CP violation for $B \to K\pi$ modes can be large if final state interaction phases are large.

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Recently the CLEO Collaboration has measured the branching ratio \( (Br) \) of the \( B^- \rightarrow K^-\pi^0 \) mode for the first time, finding \( (1.5 \pm 0.4 \pm 0.3) \times 10^{-5} \) \[1\]. Two previously measured \( K\pi \) modes have also been remeasured with improved errors. The new \( B^0 \rightarrow K^-\pi^+ \) rate of \( (1.4 \pm 0.3 \pm 0.2) \times 10^{-5} \) remains almost the same as a year ago \[2\], but the central value for \( Br(B^- \rightarrow \bar{K}^0\pi^-) = (1.4 \pm 0.5 \pm 0.2) \times 10^{-5} \) has dropped considerably though still consistent within errors. The three rates now appear remarkably close to each other. We note that if these decay modes were dominated by the strong penguin interaction, then the \( B^- \rightarrow K^-\pi^0 \) rate is expected to be about half that of \( B^- \rightarrow \bar{K}^0\pi^- \). One of course has to wait for further experimental improvements, but if the present pattern persists, there are important implications for the interference between the strong penguin, the tree, and the electroweak penguin interactions \[3\], final state interaction (FSI) phases \[4–6\] and CP asymmetries in these decays. The isospin related \( B^0 \rightarrow \bar{K}^0\pi^0 \) decay rate can also be predicted once the other three are precisely determined. In the following we carry out an analysis in the Standard Model (SM) using the recent CLEO data. Our conclusions are suggestive and depend on the final experimental branching ratios being close to the present central values.

We parametrize the decay amplitudes according to final state isospin decompositions. This enables us to easily carry out analyses for FSI phases, and also compare tree, strong penguin and electroweak penguin contributions. For \( B \rightarrow K\pi \) decays, there are \( I = 1/2 \) and \( 3/2 \) amplitudes. In SM these amplitudes are generated by the \( \Delta I = 0 \) strong penguin \( (H^S_0) \), and \( \Delta I = 0, 1 \) tree \( (H^T_{0,1}) \) and electroweak penguin \( (H^W_{0,1}) \) effective Hamiltonians. We can write the amplitudes \( A_{K\pi} \) as

\[
A_{K^-\pi^0} = \frac{2}{3} b_3 e^{i\delta_3} + \sqrt{\frac{1}{3}} (a_1 + b_1) e^{i\delta_1}, \\
A_{\bar{K}^0\pi^-} = -\frac{\sqrt{2}}{3} b_3 e^{i\delta_3} + \sqrt{\frac{2}{3}} (a_1 + b_1) e^{i\delta_1}, \\
A_{K^-\pi^+} = \frac{\sqrt{2}}{3} b_3 e^{i\delta_3} + \sqrt{\frac{2}{3}} (a_1 - b_1) e^{i\delta_1}, \\
A_{\bar{K}^0\pi^0} = \frac{2}{3} b_3 e^{i\delta_3} - \sqrt{\frac{1}{3}} (a_1 - b_1) e^{i\delta_1},
\]

(1)
where $a_i = \sum a_j^i$ with $j$ summed over $T$, $S$ and $W$, and likewise for $b_i$. The $I=1/2$ amplitudes $a_1^{T,S,W}$, $b_1^{T,S,W}$ and the $I=3/2$ amplitudes $b_3^{T,S,W}$ are generated by $H_{0(1)}^{T,S,W}$ and $H_1^{T,W}$, respectively. We have singled out the FSI phases $\delta_{1,3}$ of the corresponding amplitudes for clarity, but only the phase difference $\delta = \delta_3 - \delta_1$ is physically relevant for our purpose. There are additional phases in $a_1^i$ and $b_1^i$, such as the CP violating weak phases in KM matrix elements, and absorptive parts due to rescattering between different flavored intermediate states associated with the corresponding KM factors, such as charmless and charmed states, which can not be absorbed into $\delta_{1,3}$ \[3-5\]. The FSI phases and the additional phases due to flavored intermediate states are difficult to calculate. For the latter we will take the usual approximation and use the absorptive part obtained from quark level calculation \[7\]. Since we are summing over all intermediate states, to the extent that quark-hadron duality is valid, this approximation should represent the sizes and signs for the absorptive parts well. The FSI phase $\delta$ is more difficult to calculate as it comes from long distance effects. Theoretical calculations suggest that the phase $\delta$ is of order $10^\circ$ \[6\]. When necessary we will take $\delta = 10^\circ$ for illustration, but in general we will treat $\delta$ as a free parameter and try to obtain information about it from experimental data.

Let us now relate the above amplitudes to the Standard Model. In SM the amplitudes for B decays are generated by the following effective Hamiltonian \[8,9\]:

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} [V_{ub}V_{us}^*(c_1 O_1 + c_2 O_2) - \sum_{i=3}^{10} (V_{ub}V_{us}^* c_{i}^u + V_{cb}V_{cs}^* c_{i}^c + V_{tb}V_{ts}^* c_{i}^t) O_i] + H. C. ,$$  \(2\)

where the superscripts $u$, $c$, $t$ indicate the internal quarks. The operators $O_i$ and the Wilson coefficients $c^i_j$ are given explicitly in Ref. \[9\]. To obtain exclusive decay amplitudes, we need to calculate relevant hadronic matrix elements. We shall use factorization approximation to estimate the magnitudes, then insert the FSI phases $\delta_{1,3}$ after this is done as in Eg. (1). We have

$$a_1^T = i \frac{\sqrt{3}}{4} V_{ub}V_{us}^* \left[ c_1 \frac{N}{N} + c_2 \right] ,$$

$$b_1^T = i \frac{1}{2 \sqrt{3}} V_{ub}V_{us}^* \left[ - \frac{1}{2} \left( c_1 \frac{N}{N} + c_2 \right) + \left( c_1 + c_2 \frac{N}{N} \right) X \right] ,$$
absorptive parts depend on the Wilson coefficients, we use the values obtained in Ref. [9]. The associated quark level
MeV, \( F \) equally, then \( q \) and \( \bar{q} \).

In all later discussions. In Fig. 1, we show the branching ratios vs. \( FSI \) phase is small.

Although the branching ratios are not very sensitive to the specific value of \( m \), note that care also has to be taken to include absorptive parts from the gluon propagator \( \Gamma \).

Note that setting \( \delta_{1,3} = 0 \), we obtain the standard factorization amplitudes with the tree contribution to \( A_{F_{0,\pi}} \) equal to zero.

We use the following values [10] in our numerical calculations: \( f_{\pi} = 133 \text{ MeV}, f_{K} = 158 \text{ MeV}, F_{0}^{B_{\pi}}(0) = 0.36, F_{0}^{B_{K}}(0) = 0.41 \), and assume monopole \( k^2 \) dependence for \( F_{0}(k^2) \).

For the Wilson coefficients, we use the values obtained in Ref. [9]. The associated quark level absorptive parts [12] depend on the \( q^2 \) of the virtual gluon, which is not well determined. We note that care also has to be taken to include absorptive parts from the gluon propagator [1].

Although the branching ratios are not very sensitive to the specific value of \( q^2 \), the CP violating partial rate asymmetries are sensitive to \( q^2 \) when \( \delta \) is small. Two configurations need to be distinguished. When the pion comes off from the \( q\bar{q}' \) current in the penguin, \( q^2 \) should take the value of \( m_{\pi}^2 \). In case of Fierz transformed operators, the kaon contains the s quark and the \( q' \) quark. We assume that the two light quarks share the kaon momentum equally, then \( q^2 \) is approximately given by \( m_{\pi}^2/2 \), which is favorable for large CP asymmetries when FSI phase is small.

Our results are given in Figs. 1 to 6. Since the branching ratios measured so far are averaged over \( B \) and anti-\( B \) decays, we will present the results for averaged branching ratios in all later discussions. In Fig. 1, we show the branching ratios vs. \( \delta \), taking \( \gamma = 64^\circ \) which
is the present best fit value \[13\]. We see that the agreement with data is not so good, but can be made to within one sigma level by scaling up \(F_0^{B\pi}(0)\) by 20%. A non-zero \(\delta\) tends to split \(B^- \to \bar{K}^0\pi^-\) from the other two measured modes, and the separation reaches maximum at \(\delta = 180^\circ\). Clearly the data prefers smaller values of \(\delta\) such as \(\delta \sim 0\) or \(10^\circ\), but larger values are not ruled out within the errors at present. For small \(\delta\), the branching ratio for \(\bar{B}^0 \to \bar{K}^0\pi^0\) is about two to three times smaller than other three branching ratios.

It is clear that the branching ratios in Fig. 1 are widely separated. Can they be brought closer to each other? The branching ratios depend strongly on \(\gamma\) and for certain values of \(\gamma\), the branching ratios indeed can be much closer to each other. We show this in Fig. 2 for \(\delta = 0\). We see that the preferred value for \(\gamma\) is in the range of \(90^\circ - 130^\circ\) and \(220^\circ - 260^\circ\), in which the three branching ratios are within one standard deviation of the experimental central values. The corresponding branching ratio for \(\bar{B}^0 \to \bar{K}^0\pi^0\) is typically three times smaller. We note that \(B^- \to K^-\pi^0\) is never larger than \(B^0 \to K^-\pi^+\).

As pointed out some time ago, electroweak penguins are important in some \(B \to K\pi\) decay modes \[3\]. To see this in light of the present experimental data, we set electroweak penguin effects to zero and give the results in Fig. 3. It is clear that in this case the strong penguin effects dominate, resulting in the expected ratio of \(Br(B^- \to K^-\pi^0)/Br(B^- \to \bar{K}^0\pi^-)\) and \(Br(\bar{B}^0 \to \bar{K}^0\pi^0)/Br(\bar{B}^0 \to K^-\pi^+)\sim 1/2\) for small \(\delta\). This is a general feature for the branching ratios without electroweak penguins which was pointed out in Ref. \[12\]. For large FSI phase, the branching ratios for \(B^- \to \bar{K}^0\pi^-\) and \(\bar{B}^0 \to K^-\pi^+\) approach each other, which is very different from Fig. 1 when electroweak penguins are included.

We consider dependence on \(\gamma\) in Fig. 4 when electroweak penguin effects are neglected and with \(\delta = 0\). We see that without electroweak penguins, the branching ratios are more widely separated and there is no consistent solution where all three observed branching ratios are close to each other. This again shows the importance of the electroweak penguins. A direct way of measuring the strength of the electroweak penguin is to measure the branching ratios for \(\bar{B}_s \to \pi(\eta, \phi)\) \[11\]. However, these rates are expected to be small. The study of \(B \to K\pi\) modes thus provide a more practical method of probing electroweak penguin
effects through interference. The present data also has implications for the method proposed to constrain the phase angle $\gamma$ from $\text{Br}(\bar{B}^0 \to K^-\pi^+)/\text{Br}(B^- \to \bar{K}^0\pi^-)$ \[^{[14]}\]. The ratio is now almost one and gives no constraint on $\gamma$. However, from the near equality of three observed branching ratios, $\gamma$ is favored to be in the range of $90^\circ - 130^\circ$ and $220^\circ - 260^\circ$.

Direct CP violating partial rate asymmetries in these decays depend on both $\gamma$ and $\delta$. In Fig. 5 we show the $\gamma$ dependence with $\delta = 0$. The rate asymmetries can be as large as 13% for $\bar{B}^0 \to K^-\pi^+$ and 8% for $B^- \to K^-\pi^0$, but they are small for both $B^- \to \bar{K}^0\pi^-$ and $\bar{B}^0 \to \bar{K}^0\pi^0$ because the tree contribution to the former is zero and the latter is color suppressed. In Fig. 6 we show the dependence of the asymmetries on $\delta$ for $\gamma = 64^\circ$. For $\delta$ around $10^\circ$ as suggested by theoretical calculations, the asymmetries are 3%, 3%, 10% and 8% for $\bar{B} \to K^-\pi^0$, $\bar{K}^0\pi^-$, $K^-\pi^+$ and $\bar{K}^0\pi^0$ respectively. If the phase $\delta$ turns out to be large, direct CP violation can be large too. For example, with $\delta \approx 100^\circ$, the asymmetries for the above modes can become as large as -27%, 19%, -10% and 40%, respectively.

We have also investigated how the results change with other parameters. A smaller $N$ lowers the branching ratios but brings them closer to each other. A smaller $m_s$ enhances the strong penguin contribution and tends to widen the separation between the branching ratios \[^{[12]}\]. At present $1/N$ between $1/2$ to zero and $m_s$ between 100 MeV to 200 MeV are allowed by data. Improvements on the experimental branching ratios will further restrict the allowed ranges for the parameters involved.

The asymmetries discussed above depend strongly on the FSI phase $\delta$. If the rate differences for all the $B \to K\pi$ modes are measured, one can observe CP violation involving only $I = 1/2$ amplitudes which is free of $\delta$. This is in contrast with the kaon system where, in order to have rate asymmetry, there must be at least two isospin amplitudes \[^{[15]}\]. Defining

$$\Delta = \Delta_{K^-\pi^0} + \Delta_{\bar{K}^0\pi^-} - \Delta_{K^-\pi^+} - \Delta_{\bar{K}^0\pi^0}$$

$$\sim |a_1 + b_1|^2 - |\bar{a}_1 + \bar{b}_1|^2 + |a_1 - b_1|^2 - |\bar{a}_1 - \bar{b}_1|^2,$$ \hspace{1cm} (4)

where $\Delta_{ij}$ is the rate difference of $B$ and anti-$B$ decays to $ij$ final states. The barred amplitudes refer to anti-particle decays. Normalizing to the experimental branching ratio...
for $B^0 \rightarrow K^- \pi^+$, we find that the quark level calculation gives $\Delta/Br(B^0 \rightarrow K^- \pi^+)$ around 5% for $\gamma = 64^\circ$, which can be tested at B factories.

In conclusion, the present data on $B \rightarrow K\pi$ decay modes suggest that the electroweak penguins are important. Without these effects, the $B^- \rightarrow \bar{K}^0\pi^-$ or $K^-\pi^+$ rates would lie considerably higher than the $K^-\pi^0$ final state. This makes the study of $B \rightarrow K\pi$ modes a more practical way to probe the electroweak penguin effects than the study of $B_s \rightarrow \pi(\eta, \phi)$. The branching ratio for $\bar{B}^0 \rightarrow \bar{K}^0\pi^0$ is predicted to be a factor of two to three smaller if the other three decay modes have approximately equal rates. The present experimental data still allow sizable FSI phase $\delta$. CP violation in $\bar{B} \rightarrow K^-\pi^0, \bar{K}^0\pi^-, K^-\pi^+$ and $\bar{K}^0\pi^0$ modes can be as large as 33%, 19%, 27% and 45%, respectively, if $\delta$ is large.

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FIG. 1. $B \to K\pi$ branching ratios with electroweak penguin contributions as a function of $\delta$ for $\gamma = 64^\circ$. In all the figures we use $N = 3$ and $m_s = 200$ MeV for illustration, and solid, dot-dashed, dashed and dotted lines are for $B^- \to K^-\pi^0$, $\bar{K}^0\pi^-$ and $\bar{B}^0 \to K^-\pi^+$, $\bar{K}^0\pi^0$, respectively.

FIG. 2. $B \to K\pi$ branching ratios with electroweak penguin contributions vs. $\gamma$ for $\delta = 0$.

FIG. 3. $B \to K\pi$ branching ratios without electroweak penguin contributions vs. $\delta$ for $\gamma = 64^\circ$. 
FIG. 4. $B \rightarrow K\pi$ branching ratios without electroweak penguin contributions vs. $\gamma$ for $\delta = 0$.

FIG. 5. $B \rightarrow K\pi$ partial rate asymmetries with electroweak penguin contributions vs. $\gamma$ for $\delta = 0$.

FIG. 6. $B \rightarrow K\pi$ partial rate asymmetries with electroweak penguin contributions vs. $\delta$ for $\gamma = 64^\circ$. 