Strength Calculation of Inclined Sections of Reinforced Concrete Elements under Transverse Bending

V B Filatov
Department of Building Constructions, Samara State Technical University, 194, Molodogvardeyskaya str., Samara 443001, Russia

E-mail: vb_filatov@mail.ru

Abstract. The authors propose a design model to determine the strength of inclined sections of bent reinforced concrete elements without shear reinforcement for the action of transverse force taking into account the aggregate interlock forces in the inclined crack. The calculated dependences to find out the components of forces acting in an inclined section are presented. The calculated dependences are obtained from the consideration of equilibrium conditions of the block over the inclined crack. A comparative analysis of the experimental values of the failure loads of the inclined section and the theoretical values obtained for the proposed dependencies and normative calculation methods is performed. It is shown that the proposed design model makes it possible to take into account the effect the longitudinal reinforcement percentage has on the inclined section strength, the element cross section height without the introduction of empirical coefficients which contributes to an increase in the structural safety of design solutions including the safety of high-strength concrete elements.

1. Introduction

The condition for the strength of inclined sections of bent elements in the domestic code for the calculation of reinforced concrete structures provides that the force from external loads transmitted to the cross section must not exceed the internal limiting force in the inclined section. The internal limiting force in calculating the strength of inclined sections on the action of transverse forces is added from the shear force in the compressed concrete zone and the total force in the transverse reinforcement that cross inclined crack.

The shear force in the compressed zone of concrete $Q_b$ in domestic and foreign design codes is determined by empirical relationships. In accordance SP 63.13330.2012 “Concrete and reinforced concrete. Design code” [1] shear force in the compressed zone of concrete $Q_b$ is determined on the basis of the empirical dependence obtained by the results of experimental studies on beams of medium strength concrete without transverse reinforcement [2]. The empirical dependence for the determination of $Q_b$ was proposed in [2], as a function of the concrete compressive strength and the projection length of an inclined crack. Subsequently, the compressive strength of concrete was replaced by its tensile strength, which improved the convergence of the calculated and experimental values. Experimental studies carried out in [3] have shown that in the inclined section the forces not taken into account in the design model are acting: the shear force in the longitudinal tensioned reinforcement (dowel action) and the aggregate interlock forces along the edges of the inclined crack. The method of calculating the strength of inclined sections reinforced concrete elements, taking into...
account the force of dowel action and the aggregate interlock forces along the edges of the inclined crack, was proposed in [4,5]. However, in the normative calculation methodology [6], these efforts continued to be considered indirectly - through the $Q_b$ value, due to the insufficiently representative base of the experimental data.

The development of the procedure for calculating the strength of inclined sections, taking into account the dowel action and the aggregate interlock forces in an inclined crack was proposed by the authors [7-9]. The force in the compressed concrete zone $Q_b$ is proposed to be determined by the refined dependence [7], which allows to avoid additional empirical constraints. The force $Q_b$ is not included in the strength condition and is expressed in terms of the remaining components of the stress state. Efforts to resist the shear of the inclined crack edges depend on the tensile concrete strength and the angle of inclination of the crack [8,9].

Analysis of the experimental studies results of the strength of inclined sections bending reinforced concrete elements [10,11] shows that the strength of bending elements to the action of shearing forces depends on the percentage of longitudinal reinforcement in the cross section and the height of cross section of the element (so-called "scale effect").

Further development of the theory of strength of reinforced concrete elements under the action of transverse forces occurs on the basis of a semi empirical approach, one of the directions of which is the modified theory of compression fields (MCT) proposed by the authors [12]. The technique proposed in [12] for calculating reinforced concrete elements under the action of shear forces makes it possible in particular to take into account the percentage of longitudinal reinforcement of the beam in determining the value of the shear force in the compressed concrete zone. The authors of [12] consider the work of reinforced concrete in transverse bending from the positions, in many respects similar to the theory of deformation of reinforced concrete with cracks, set forth in [13].

2. Methodology

Design model of the inclined section of a bending reinforced concrete element without shear reinforcement (figure 1) was proposed in [14].

![Figure 1. Design model of the inclined section.](image)

A feature of the proposed model is the allocation in the calculated dependencies of the aggregate interlock forces in the inclined crack as a separate component in the set of forces ensuring the strength of the inclined section when calculated on the action of the transverse forces [15,16]. The pattern of forces acting in the inclined section of the reinforced concrete element is adopted in accordance with [3].

It is assumed [17,18] that the transverse force from the applied load is perceived in the inclined section of the bending element without the transverse reinforcement by an internal force, which consists of the following components: shear force in the compressed concrete zone at the end of the inclined crack ($Q_{b1}$), shear force in the longitudinal tensioned reinforcement (dowel action $Q_s$) at the
beginning of the inclined crack, vertical component of the aggregate interlock force along the length of
the inclined crack \( Q_a \).

The dependencies for the calculation are compiled from the equilibrium conditions of the block
above the inclined crack. The equation of equilibrium of the projections of forces on the vertical axis:

\[ Q = Q_{b1} + Q_s + Q_a \]  \hspace{1cm} (1)

The equation of equilibrium of the projections of forces on the horizontal axis:

\[ N_{s2} = N_{b1} + N_a \]  \hspace{1cm} (2)

The equation of equilibrium of the moments of forces with respect to the point A:

\[ N_{b1} \times z_{b1} + T_a \times z_a + Q_s \times c = Q \times a \] \hspace{1cm} (3)

where \( N_{b1} \) and \( z_{b1} \) are normal force in the compressed concrete zone above the inclined crack and the
shoulder of this force, respectively; \( T_a \) and \( z_a \) are aggregate interlock force along the edges of the
inclined crack and the shoulder of this force, respectively; \( Q_s \) and \( c \) are shear force in the longitudinal
tensioned reinforcement and the shoulder of this force, respectively; \( Q \) and \( a \) are acting shear force
and the shoulder of this force, respectively.

Substituting (1) and (2) into (3), we obtain:

\[ Q_{b1} \times a + Q_a \times (a - z_a / \sin \theta) + (N_a - N_{s2}) \times z_{b1} + Q_s \times (a - c) = 0 \] \hspace{1cm} (4)

The force \( Q_s \) is determined by the equation (5):

\[ Q_s = N_{s2} \times c / (h_0 \times \gamma_v) = \sigma_{s2} \times A_s \times \cot \theta / 8 \] \hspace{1cm} (5)

To determine the aggregate interlock forces in a crack, we use the empirical dependence proposed
by the authors [11]

\[ \tau = \frac{0.18 \sqrt{f'_c}}{0.31 + \frac{24 a_{crc}}{a_g + 16}} \] \hspace{1cm} (6)

where \( f'_c \) is concrete compressive strength (MPa); \( a_{crc} \) is width crack opening (mm); \( a_g \) is maximum
size of coarse aggregate (mm).

Having done some modification of the dependence (6), we obtain an expression for determining the
vertical projection of the aggregate interlock force in an inclined crack:

\[ Q_a = \frac{0.4 R_{bh} h_0 (1 - \xi)}{n \times \left( 0.31 + \frac{24 a_{crc}}{a_g + 16} \right)} \] \hspace{1cm} (7)

where \( n = a / h_0 \).

The value of the relative height of the compressed concrete zone over the inclined crack \( \xi \) is found
from equation (8):

\[ \xi^2 + 2 \mu \alpha \xi - 2 \mu \varphi \alpha = 0 \] \hspace{1cm} (8)

where \( \mu = A_s / h_0; \alpha = E_s / E_b; \varphi = 1 - 0,7 / (100 \mu + 1) \).

The width of inclined crack opening \( a_{crc} \) in equation (6) is expressed in terms of the stresses in
longitudinal reinforcement at the beginning of inclined crack \( \sigma_{s2} = N_{s2} / A_s \):

\[ a_{crc} = \sigma_{s2} \times l_{crc} / E_s = \sigma_{s2} \times h_0 (1 - \xi) \cot \theta / E_s \] \hspace{1cm} (9)

The shear force at the compressed concrete zone above the inclined crack \( Q_{b1} \) is proposed to
determine by equation (10):
\[
Q_{bl} = \frac{R_{th}bh_0 \tan \theta}{16(1-\xi)}
\]  
(10)

Substituting expressions (5), (7), (9), (10) in (4) we obtain a quadratic equation with respect to \(\sigma_{z2}\). Having from (4) determined value of \(\sigma_{z2}\), using the formulas (5), (7) and (10), we find the values of \(Q_s\), \(Q_a\) and \(Q_{bl}\), respectively.

3. Results and Discussion

The results of calculations for proposed dependencies (\(Q_{cal}\)), as well as their comparison with the results of calculations according to ACI-318 (\(Q_{ACI}\)), EUROCODE 2 (\(Q_{EC}\)) and SP 63.13330.2012 (\(Q_{SP}\)) are given in tables 1, 2. For the comparative analysis the tests results of samples, published in [10, 11] were used. The strength of inclined sections according to SP 63.13330.2012 was determined from \(Q_b = 0.5R_{th}bh_0\). The results of determining the strength of inclined sections according to ACI-318 are taken from [10].

**Table 1.** Results of comparison experimental and calculated values of the ultimate shear forces (at different concrete strength and the height section of the element).

| b, mm | h0, mm | \(f'_c\), MPa | \(\mu\), % | \(a/h_0\) | Qexp, kN | QSP, kN | QAC, kN | QEC, kN | Qcal, / Qexp | QSP / Qexp | QAC / Qexp | QEC / Qexp | Qcal / Qexp |
|-------|--------|---------------|---------|----------|---------|--------|--------|--------|-----------|----------|-----------|-----------|-----------|
| BN12  | 300    | 110           | 37,2    | 0,91     | 3,07    | 40     | 46     | 34     | 38        | 45       | 1,14      | 0,85      | 0,95      | 1,11      |
| BN25  | 300    | 225           | 37,2    | 0,89     | 3,00    | 73     | 94     | 69     | 76        | 75       | 1,28      | 0,95      | 1,04      | 1,03      |
| BN50  | 300    | 450           | 37,2    | 0,81     | 3,00    | 132    | 187    | 137    | 126       | 119      | 1,42      | 1,04      | 0,95      | 0,90      |
| BN100 | 300    | 925           | 37,2    | 0,76     | 2,92    | 192    | 385    | 282    | 223       | 190      | 2,04      | 1,47      | 1,16      | 0,99      |
| B100  | 300    | 925           | 36,0    | 1,01     | 2,92    | 225    | 378    | 278    | 242       | 212      | 1,68      | 1,24      | 1,08      | 0,94      |
| B100L | 300    | 925           | 39,0    | 1,01     | 2,92    | 223    | 394    | 289    | 249       | 216      | 1,77      | 1,30      | 1,12      | 0,97      |
| BH25  | 300    | 225           | 98,8    | 0,89     | 3,00    | 85     | 127    | 93     | 93        | 81       | 1,50      | 1,09      | 1,09      | 0,95      |
| BH50  | 300    | 450           | 98,8    | 0,81     | 3,00    | 132    | 255    | 187    | 155       | 125      | 1,93      | 1,42      | 1,17      | 0,95      |
| BH100 | 300    | 925           | 98,8    | 0,76     | 2,92    | 193    | 523    | 384    | 274       | 200      | 2,71      | 1,99      | 1,42      | 1,03      |
| BRL100| 300    | 925           | 94,0    | 0,50     | 2,92    | 163    | 523    | 384    | 238       | 167      | 3,21      | 2,36      | 1,46      | 1,02      |
| B100H | 300    | 925           | 98,0    | 1,01     | 2,92    | 193    | 523    | 384    | 301       | 227      | 2,71      | 1,99      | 1,56      | 1,17      |

Average value 1,94 1,43 1,18 1,01
Standard deviation 0,64 0,47 0,21 0,08
Coefficient of variation, % 32,7 32,6 17,5 7,5

Table 2. The results of comparison of the experimental and calculated values of the ultimate shear forces (at different percentages of longitudinal reinforcement).

| b, mm | h0, mm | \(f'_c\), MPa | \(\mu\), % | \(a/h_0\) | Qexp, kN | QSP, kN | QAC, kN | QEC, kN | Qcal, / Qexp | QSP / Qexp | QAC / Qexp | QEC / Qexp | Qcal / Qexp |
|-------|--------|---------------|---------|----------|---------|--------|--------|--------|-----------|----------|-----------|-----------|-----------|
| BN100 | 300    | 925           | 37,2    | 0,76     | 2,92    | 192    | 385    | 34     | 223       | 190      | 2,00      | 1,47      | 1,16      | 0,99      |
| B100  | 300    | 925           | 36,0    | 1,01     | 2,92    | 225    | 378    | 69     | 242       | 212      | 1,68      | 1,24      | 1,08      | 0,94      |
| B100L | 300    | 925           | 39,0    | 1,01     | 2,92    | 223    | 394    | 137    | 249       | 216      | 1,77      | 1,30      | 1,12      | 0,97      |
| B4    | 240    | 1200          | 25,2    | 1,26     | 3       | 177    | 329    | 282    | 231       | 229      | 1,86      | 1,36      | 1,31      | 1,29      |
| A1    | 400    | 930           | 28,7    | 1,35     | 3       | 358    | 453    | 278    | 332       | 354      | 1,27      | 0,93      | 0,93      | 0,99      |
| A2    | 400    | 930           | 22,6    | 1,35     | 3       | 328    | 402    | 289    | 306       | 290      | 1,23      | 0,90      | 0,93      | 0,88      |
| DB230 | 300    | 895           | 32,0    | 2,09     | 3,02    | 257    | 345    | 93     | 289       | 264      | 1,34      | 0,98      | 1,12      | 1,03      |
| 3043  | 154    | 1092          | 27,0    | 2,71     | 3       | 165    | 199    | 187    | 181       | 170      | 1,21      | 0,88      | 1,1      | 1,03      |

Average value 1,55 1,13 1,09 1,02
Standard deviation 0,32 0,24 0,12 0,12
Coefficient of variation, % 20,5 20,8 11,3 12,0

Analysis of the results given in the tables shows that with increasing sample heights and strength of concrete (table 1), as well as with decreasing percentage of longitudinal reinforcement of the element
(table 2) the calculation technique adopted in SP 63.13330.2012 significantly overestimates the strength of inclined cross-sections of bending elements without shear reinforcement. The same trend is typical for ACI-318 [19], despite the existing limitations on the absolute height of the section and strength of concrete. The best conformity of calculated and experimental values among the normative methods considered is provided by the EUROCODE 2 [20]. The obvious reason for this is the consideration in the calculated dependence of two factors: the percentage of longitudinal reinforcement of the section and the coefficient $\xi$, taking into account absolute height of the element cross section. However, both factors are taken into account on a purely empirical basis with the introduction of restrictions, so, for example, the percentage of longitudinal reinforcement is allowed to take no more than two.

4. Conclusion

The proposed technique allows one to take into account the influence of the sectional height and the percentage of longitudinal reinforcement on the strength of inclined section from considering equilibrium conditions of the block over inclined crack, without resorting to empirical constraints. In addition, the dependence (7) makes it possible to determine the value of the aggregate interlock forces in inclined crack, taking into account the width of its opening and the size of coarse aggregate. The technique also allows you to evaluate ponderability of each component on the right side of equation (1) and, if necessary, perform a targeted adjustment to ensure the strength of inclined sections.

The proposed design model of the inclined section of a reinforced concrete bending element, taking into account the aggregate interlock forces in inclined crack in calculating the strength for the action of shear force, allows eliminating the observed (under certain conditions) systematic excess of design values over the experimental values of failure loads and increasing the reliability of design solutions, including for high-strength concrete elements.

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