Feedback channel designs for fair scheduling in MISO–OFDMA systems

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Abstract
In the next generation of wireless communication, adaptive resource allocation techniques will play an important role to improve quality of service and spectral efficiency. In order to employ adaptive fairness scheduling for multiple-input single-output (MISO) orthogonal frequency-division multiple access (OFDMA), the channel state information (CSI) belonging to all users is required at the base station. However, the amount of feedback increases with the number of users, transmit antennas and subchannels. Therefore, it is important to perform a user selection at the receiver side without cooperation among the users and to quantize the CSI. In this article, the reduced feedback channel designs are examined for MISO–OFDMA systems while providing fairness between the users. In order to reduce the feedback rate, we choose the users considering their norm and orthogonality properties as well as their location in the cell. In order to limit the feedback rate, channel direction information is quantized by designing a specific codebook thanks to the proposed criterion. We obtain an expression to determine the amount of required feedback information for MISO–OFDMA systems to support more than one beam per subchannel for a given number of users, subchannels and transmit antennas. The performance results of the reduced feedback channel designs are evaluated for fair scheduling in wireless channels.

Introduction
In future wireless radio networks, adaptive resource allocation can have a significant role to improve the spectral efficiency for multiuser orthogonal frequency division multiplexing (OFDM) systems with multiple transmit and/or receive antennas by exploiting a large number of degrees of freedom in space, frequency, and time. OFDM inherits its superiority of mitigating multipath fading to maximize throughput. Besides, in a multiple-antenna system with the help of precoding techniques, it is possible to increase the spectral efficiency by multiplexing multiple users on the same subchannel. For orthogonal frequency-division multiple access (OFDMA) systems, the sum rate can be maximized by assigning each subcarrier to the user having the best channel gain. However, for multiuser multiple-input single-output (MISO)–OFDM systems, the sum rate is maximized by choosing the optimal set of co-channel users for each subchannels [1,2]. It is also important to satisfy all users’ requirements even if the users are far from the base station (BS) by allocating the clusters and beams fairly. Therefore, not only the optimal set but also the fairness issue is considered by employing proportional fair scheduling algorithms (PFS) for MISO–OFDMA systems [3].

In order to achieve the gain of MISO–OFDMA systems, the channel state information (CSI) of all subchannels from all users for all antennas is required at the transmitter side. This causes a high feedback load and a sophisticated resource allocation algorithm at the BS. In order to design an efficient MISO–OFDMA system, the selection of users and the quantization of their CSI using efficient codebooks at the receiver side is needed to fed back the users’ CSI to the BS through a reduced rate feedback channel.

In order to design reduced feedback channels, user selection algorithms at the receiver side are performed by deciding the number of users that are fed back their quantized CSI by using codebooks known at both the transmitter and the receiver sides [4]. For multiuser MISO single-carrier systems with linear precoding, the semi-orthogonal user selection algorithm to reduce the feedback load by combining the classical norm criterion and
with a criterion based on the orthogonality between the co-channel users has been presented in [5,6]. The codebook design for narrowband single user communication system has been well studied in [7]. The extension to the OFDM case has been considered in [8,9].

For multiuser multiple-input multiple output (MIMO) with zero-forcing (ZF) beamforming, the codebook design and quantization algorithm have been proposed in [10] to enable high resolution quantization. In [11], an adaptive limited feedback linear precoding technique for temporally correlated MIMO channels has been presented by performing a differential feedback where a perturbation added to the previous precoder for adaptation to the time correlation structure. Limited feedback using a polar-cap differential codebook which utilizes the temporal correlation in MISO channels has been presented in [12].

A two-stage feedback scheme has been presented in [13] where in the first step a coarse estimates of all user channels are feedback to the BS then in the second step only $N_t$ users are selected to feed back more accurate channel quantization. In [14], it has been underscored the tradeoff between getting coarse channel feedback from large number of users and providing multuser diversity gain versus getting high-quality channel feedback from a lower number of users. In [15], two stages for scheduling process based on partial probing of the users has been presented, in order to reduce the feedback, at the second stage the probing process stops and only the remaining users are requested to feedback their channel quality.

A scheduler at the BS usually does not schedule users on their weakest clusters. Hence, the amount of feedback information can be reduced by letting each user feedback information only about its strongest clusters. Following this phenomenon, in order to achieve reduction on the feedback load without sacrificing performance too much, the so-called clustered S-best criterion has been proposed for OFDMA systems in [16]. This criterion is based on the clusters where adjacent subcarriers are grouped and only the CSI related to the strongest $S$ clusters of each user are fed back to the transmitter. In [17], a quantization method for OFDMA systems has been proposed to reduce the amount of feedback bits by determining subchannel block size and feedback periodicity according to users’ channel conditions. This scheme has been extended to MISO–OFDMA systems in [18] by using space-frequency matrix. For OFDMA systems, a feedback reduction algorithm that considers feedback efficiency as a feedback decision metric instead of the received signal-to-noise ratio (SNR) has been examined in [19] to improve the fairness between the users.

In this article, we propose reduced feedback channel designs for fair scheduling where the users are uniformly distributed in the cell for MISO–OFDMA systems. Both user selection and quantization algorithms dedicated to the selected users are applied to solve the major problem which arises from the fact that the total feedback load increases with the number of users, subchannels and antennas. The clustered S-best and the combined user selection algorithms with a codebook design thanks to the semi-orthogonal criterion are presented by choosing adaptively the number of feedback clusters depending on the location of the users and/or the number of users in the cell. These algorithms allow a precoding scheme creating more than one beam to schedule more than one user for each cluster by providing fairness between the users. We also provide the amount of required feedback information to create more than one beam for a given number of users, subchannels and transmit antennas.

This article is organized as follows: In Section "System model", the system model for MISO–OFDMA over wireless channels is described. Then, the reduced feedback channel designs are presented for fair scheduling in Section “The proposed designs”. The performance results are demonstrated in Section “Performance results” considering perfect and quantized CSI in wireless channels. Section "Conclusion" draws the conclusion of the article.

**System model**

A MISO–OFDMA system with $N_t$ transmit antennas, $Q$ clusters and $K$ users is considered. A cluster structure where the correlation is high among the subcarriers is employed so that the feedback of only one value is sufficient. For this model, the frequency channel vector between the BS and the $k$th user for the $q$th cluster at frame $n$ is described by,

$$H_{k,q}^n = \left[ H_{k,q,1}^n \ H_{k,q,2}^n \ \cdots \ H_{k,q,K}^n \right]$$

(1)

where $H_{k,q,t}^n$ is the channel coefficient from the $t$th transmit antenna to the $k$th user for the $q$th cluster at frame $n$.

In order to prevent the outage, we have set the value of cluster as the CSI that has the minimum channel vector gain belonging to the subcarriers in the cluster. Then, the channel coefficient of a cluster is determined by,

$$\vec{H}_{k,m}^n = \hat{H}_{k,m}^n$$

(2)

where $\hat{H}_{k,m}^n$ is the $N_c \times 1$ channel vector associated to the $k$th user and the $m$th subcarrier and $m$ is:

$$m = (q-1)N_Q + \arg\min_{1 \leq i \leq N_Q} \{ ||H_{k,(q-1)N_Q+i}^n||^2 \}$$

(3)

where $N_Q$ is the number of subcarriers in one cluster and calculated as $N_Q = M/Q$ with $M$ is the total number of subcarriers in a OFDM symbol.

The time channel vector consists of multipath components for each antenna $t$ and each user $k$ is described as,
\[
\mathbf{h}_{n,k,t} = \begin{bmatrix}
  h_{n,k,t}^1 & h_{n,k,t}^2 & \cdots & h_{n,k,t}^L
\end{bmatrix}
\] (4)
where \( h_{n,k,t}^i \) is channel coefficient in time domain belonging to \( k\)th user, \( t\)th antenna and \( i\)th path and \( L \) is the number of multipath components.

The channel coefficients belonging to subcarriers for each user and each antenna are obtained by applying the Fourier Transform to the channel vector in Equation (4).

For a system where \( N_t \leq K \), let \( \mathcal{S}_q \) be the set of assigned users that scheduled to cluster \( q \):
\[
\mathcal{S}_q = \{ A_{n,k,q,b}^q | A_{n,k,q,b}^q = 1; \ \forall k, \forall b \}
\]
where \( A_{n,k,q,b}^q \) is a binary variable that indicates cluster \( q \) is allocated to user \( k \) for beam \( b \).

The total number of scheduled users for each cluster is \( N_t \) and \( \mathbf{H}^n(\mathcal{S}_q) \) denotes the matrix consisting of \( N_t \) channel vectors of these scheduled users for cluster \( q \). Then, the associated users’ data is transmitted after performing ZF precoding [20,21]. The ZF transmit beamforming matrix is calculated by,
\[
\mathbf{W}^n(\mathcal{S}_q) = \beta (\mathbf{H}^n(\mathcal{S}_q))^H (\mathbf{H}^n(\mathcal{S}_q)\mathbf{H}^n(\mathcal{S}_q))^H)^{-1} \tag{5}
\]
which includes \( N_t \) elements as \( \mathbf{W}^n(\mathcal{S}_q) = [\mathbf{W}^n_1(\mathcal{S}_q), \mathbf{W}^n_2(\mathcal{S}_q), \ldots, \mathbf{W}^n_{N_t}(\mathcal{S}_q)]^T \) where \( \mathbf{W}^n_q(\mathcal{S}_q) \) is the precoding vector for \( b\)th beam and \( q\)th cluster with the dimension of \( N_t \times 1 \).

In order to keep the short term power constraint, we determine \( \beta \) as,
\[
\beta = \frac{1}{\sqrt{\text{tr}((\mathbf{H}^n(\mathcal{S}_q))^H(\mathbf{H}^n(\mathcal{S}_q)\mathbf{H}^n(\mathcal{S}_q))^H)^{-1}}}} \tag{6}
\]
For the \( k\)th user and the \( q\)th cluster at \( n\)th frame, the relation between the data vector \( \mathbf{Y}^n(\mathcal{S}_q) \) and the received vector can be written as:
\[
\mathbf{Y}^n(\mathcal{S}_q) = \mathbf{H}^n(\mathcal{S}_q)\mathbf{X}^n(\mathcal{S}_q) + \mathbf{N}^n(\mathcal{S}_q) = \mathbf{H}^n(\mathcal{S}_q)\mathbf{W}^n(\mathcal{S}_q)\mathbf{P}_q\mathbf{S}^n(\mathcal{S}_q) + \mathbf{N}^n(\mathcal{S}_q) \tag{7}
\]
where \( \mathbf{P}_q = \text{diag}(\sqrt{P_T/(QN_1)}, \ldots, \sqrt{P_T/(QN_L)}) \) denoting that the total transmit power \( P_T \) is equally shared between the clusters and beams and \( \mathbf{X}^n(\mathcal{S}_q) = \mathcal{C}^{N_t \times 1} \) is the transmitted vector from the BS.

Our objective is to maximize the average sum rate while keeping fairness between the users by optimizing both the cluster and beam allocation [3,22], \( \mathbf{A}^n = [\mathbf{A}^1_q, \ldots, \mathbf{A}^{N_t}_q] \). In order to construct \( \mathbf{A}^n_q \), all vectors of \( \mathbf{A}^n_{k,q} = [A_{n,k,q,1}^n, A_{n,k,q,2}^n, \ldots, A_{n,k,q,N_t}^n]^T \) are stacked column by column.

In order to perform the user allocation, the ZF precoding is performed under the total power constraint equally shared between the clusters and the beams considering a proportional fairness scheduling (PFS). Mathematically, the optimization problem is formulated as,
\[
(A^n)^* = \arg \max_{A^n} \sum_{k=1}^{K} R^n_k \sum_{q=1}^{Q} \frac{R^n_k}{(T^n_k)^{a}} \tag{8}
\]
subject to
\[
\sum_{k=1}^{k} A^n_{k,q,b} \leq 1; \ \forall q, \forall b \tag{9}
\]
\[
A^n_{k,q,b} \in \{0,1\}; \ \forall k, \forall q, \forall b \tag{10}
\]
The instantaneous data rate \( R^n_k \) is calculated as,
\[
R^n_k = \sum_{q=1}^{Q} \sum_{b=1}^{N_t} A^n_{k,q,b} \log_2(1 + \text{SINR}^n_{k,q,b}) \tag{11}
\]
where the signal-to-interference-plus-noise-ratio (SINR) is:
\[
\text{SINR}^n_{k,q,b} = \frac{P_{r} \mathbf{H}^n_{k,q,b} \mathbf{W}^n_{k,q,b}}{N_0 + P_{r} \sum_{j=1;j\neq b}^{Q} \mathbf{H}^n_{k,q,j} \mathbf{W}^n_{k,q,j}} \tag{12}
\]
where \( N_0 \) is the power spectral density of additive white Gaussian noise (AWGN) and \( B \) is the total available bandwidth.

The weighted average data rate in Equation (8) is calculated by,
\[
\overline{T^{n+1} k} = \left( 1 - \frac{1}{t_c} \right) \overline{T^n k} + \frac{1}{t_c} R^n_k \tag{13}
\]
where \( t_c \) is the average window size.
To simplify the optimization problem, it is possible to perform the user allocation at a cluster level as,
\[
(A^n)^* = \arg \max_{A^n_q} \sum_{k=1}^{K} R^n_{k,q} \sum_{b=1}^{L} \frac{R^n_{k,q,b}}{(T^n_{k,q,b})^{a}}, \ \forall q \tag{14}
\]
The instantaneous user rate for each cluster is:
\[
R^n_{k,q} = \sum_{b=1}^{N_t} A^n_{k,q,b} \log_2(1 + \text{SINR}^n_{k,q,b}) \tag{15}
\]
The weighted average rate of each user is updated after allocating each cluster as,
\[
\overline{T^{n+1} k} = \left( 1 - \frac{1}{t_c} \right) \overline{T^n k} + \frac{1}{t_c} R^n_{k,q} \tag{16}
\]

The proposed designs
In order to exploit the multiuser diversity, the users need to feedback their CSI to the BS. The feedback rate can be reduced by letting each user send their CSI associated only to a subset of clusters after applying quantization. We present four different algorithms to perform user selection at the receiver side and a quantization method thanks to the properties of the semi-orthogonal criterion.
P1: The clustered S-best criterion

For the clustered S-best criterion, each user selects independently a set of $S^\text{in}_k$ composed of the S clusters with the highest channel norm $\|\mathbf{H}_{k,q}^\text{in}\|$. Then, each user feeds back only its CSI associated to the selected clusters to the BS.

Let $T^\text{in}_q$ be the set of users that feedback their CSI associated to the cluster $q$ as,

$$T^\text{in}_q = \{k \in \{1,\ldots,K\} : q \in S^\text{in}_k\}$$

(16)

Then, for each cluster $q$, the BS selects the set $S^\epsilon_q$ from the set $T^n_q$ to maximize the sum rate in Equation (14).

Since the total feedback rate is proportional to $K\epsilon$, it is reasonable to adjust $S$ in function of $K$ according to a desired function $S = f(K)$.

The objective is defined in terms of a fraction of all clusters, $\eta$, which for $N_t$ users send their CSI to the BS, $\eta$ can be adjusted to guarantee $\eta Q$ clusters can allocate at least $N_t$ users with a probability higher than $1 - P_{\text{obj}}$ as follows.

$$\Pr \left( \sum_{q=1}^{Q} I^N_q \leq \eta Q, S, K \right) \leq P_{\text{obj}}$$

(17)

with

$$I^N_q = \begin{cases} 1 & \text{if} |T^\text{in}_q| \geq N_t \\ 0 & \text{else} \end{cases}$$

The case of $N_t = 1$ has been examined in [16] for OFDMA systems with single antenna. In this article, we extend it to the MISO–OFDMA systems considering the case of $N_t = 2$ to allocate more than one beam for each cluster.

In order to achieve $P_{\text{obj}}$ with the parameters $\nu = \eta Q$ and $V_K = \sum_{q=1}^{Q} I^N_q$, that is the number of clusters having CSI of more than one user, the probability that $\nu$ or less different clusters are fed back with less than two users is obtained by,

$$\Pr(V_K \leq \nu) = \sum_{i=0}^{(Q+1)(\nu+1)} P_K(i)$$

(18)

The calculation of $P_K(i)$ in Equation (18) is given in the Appendix.

P2: The combined criterion

The clusters can be selected based not only on channel norm/quality but also on channel direction information (CDI) [23]. Therefore, for each cluster, $N_t$ random orthonormal vectors $\phi_{b,q}$ ($N_t \times 1$), $b = 1,\ldots,N_t$ are generated.

The users measure the orthogonality between their channels and the random vectors $\phi_{b,q}$ for each cluster using the chordal distance as:

$$d^2 \left( \tilde{\mathbf{H}}_{k,q}^\epsilon, \phi_{b,q} \right) = 1 - |(\tilde{\mathbf{H}}_{k,q}^\epsilon)^H \phi_{b,q}|^2$$

(19)

where $\tilde{\mathbf{H}}_{k,q}^\epsilon = \mathbf{H}_{k,q}^\epsilon / \|\mathbf{H}_{k,q}^\epsilon\|$ is the normalized channel vector of the user $k$ and cluster $q$.

Let $O^N$ be the unit sphere lying in $C^N$ and centered at the origin. Using the chordal distance metric, for any $0 < \epsilon < 1$, we can define a spherical cap on $O^N$ with center $o$ and square radius $\epsilon$ as the open set:

$$B_{\epsilon}(o) = \{ \mathbf{g} \in O^N : d^2(\mathbf{g}, o) \leq \epsilon \}$$

(20)

Following the definition of spherical cap with the center $\phi_{b,q}$ and square radius $\epsilon$, the constructed open set corresponds to semi-orthogonal neighborhood set of any orthonormal vector. Then, it is possible to calculate that the normalized channel vector is in the open set or not according to chordal distance metric.

Then, the semi-orthogonal criterion is defined as:

$$T^\epsilon_q = \left\{ k \in \{1,\ldots,K\} : \tilde{\mathbf{H}}_{k,q}^\epsilon \in \bigcup_{b=1}^{N_t} B_{\epsilon}(\phi_{b,q}) \right\}$$

(21)

where $T^\epsilon_q$ is the set of semi-orthogonal users for cluster $q$.

The set of the selected clusters by user $k$ is given by,

$$S^\epsilon_q = \{ q \in \{1,\ldots,Q\} : k \in T^\epsilon_q \}$$

(22)

According to Mukkavilli et al. [24], the number of clusters which satisfy $\epsilon$ criterion for each user is calculated approximately as $N_t\epsilon^{-e-1}$. Therefore, the choice of $\epsilon$ is critical since it is directly relative to the total number of clusters per user. Consequently, it should be guaranteed that at least $S$ clusters which satisfy the semi-orthogonal criterion for each user are selected by adjusting the $\epsilon$ parameter properly.

Then, each user selects $S$ best clusters in terms of channel quality/norm from the set $T^\epsilon_k$ and constructs a set $S^\epsilon_k$. From this set, for each $q$, we can obtain the set $T^\epsilon_q$ using Equation (16).

Adaptive P1: Adaptive clustered S-best criterion

In order to further reduce the feedback rate, the number of feedback clusters, $|S^\epsilon_k| = S^\epsilon_k$, is adjusted adaptively according to the location of the users in the cell. Since the users that are far from BS have lowest rate per cluster, they require more clusters for fair scheduling. Therefore, the purpose of the adaptive feedback algorithm is to give more scheduling opportunities to the users far from the BS by providing more feedback clusters.

Let’s define a variable

$$P^\nu_k = \left[ \frac{Z^\nu_k}{Q} \right]_1$$

(23)

where $\lfloor . \rfloor_1$ denotes the orthogonal projection onto interval $[0,1]$, $e$ is a fixed spectral efficiency parameter that adjust to rate of the feedback channel and $Z^\nu_k$ is the instantaneous number of allocated clusters for the $k$th user.
\[ Z^n_k = \sum_{b=1}^{N_t} \sum_{q=1}^{Q} A^n_{k,q,b} \]  

\(^n\) is proportionally changed according to the location of the users where the farthest user will required more clusters than the nearest user for proportionally fair scheduling. Since \( p^n_k \) is proportionally to \( Z^n_k \), the farthest user will feedback more clusters than the nearest user.

After that, each user generates a binomial random variable with parameters \( Q \) and \( p^n_k \) to determine the number of feedback clusters, \( S^n_k \). Then, as described in \textit{P1 algorithm}, the set \( S^n_k \) which includes the best \( S^n_k \) clusters of user \( k \) is constructed.

\textbf{Adaptive P2: adaptive combined criterion}

The adaptive combined criterion is described as follows: For each user \( k \):

- As described in \textit{Adaptive P1 algorithm}, the required number of feedback clusters, \( S^n_k \), is calculated depending on the instantaneous number of allocated clusters.
- Choose the corresponding epsilon value, \( \epsilon^n_k \) as well as the number of feedback cluster, \( S^n_k \).
  - The set of the predefined number of clusters and the corresponding epsilon values are given, respectively:
    \[ C = \{C_1, C_2, \ldots, C_T \} \]
    \[ E = \{e_1, e_2, \ldots, e_T \} \]
    where \( T \) is the number of codebooks for different epsilon values.
  - Find the index of the closest value:
    \[ t^* = \arg \min_{t=1,2,\ldots,T} |C_t - S^n_k|, \quad t = 1, 2, \ldots, T \]
    \[ \epsilon^n_k = \epsilon_{t^*} \]  

- According to these parameters, the semi-orthogonal criterion described in \textit{P2 algorithm} is performed.

The \textbf{codebook design}

In order to transmit the CSI of the selected users to the BS, a classical solution is to quantize the CDI and the channel quality information (CQI) before transmission over the finite rate feedback link. In contrast to the CDI which is identically independent distributed (i.i.d) in \( \mathbb{O}^{N_t} \) isotropically, since we are employing the semi-orthogonal criterion in the combined user selection algorithms, the clusters are selected according to a spherical cap region [6]. In order to reduce the error quantization, it is important to build a codebook using a quantization of the localized region or local packing [8]. A local Grassmannian packing with parameters \( N_t, \epsilon \) is a set of \( N_t \) vectors, where \( N_t \) is the codebook size, \( g_i \), \( i = 1, \ldots, N_t \), constrained to a spherical cap \( \mathcal{B}_t(\epsilon) \) in \( \mathbb{O}^{N_t} \) such that

\[ \min_{1 \leq i < j \leq N_t} d^2(g_i, g_j) \]

is maximized. As in the i.i.d., case for CDI, we use a practical vector quantization scheme, namely the generalized Lloyd-Max algorithm to design these local packings.

For the semi-orthogonal criterion, the codebook must be adapted according to the orthogonal vectors \( \phi_{b,q} \). From the local packing associated to the spherical cap \( \mathcal{B}_t(\epsilon) \), it is possible to compute the local packing, \( \mathcal{B}_t(\phi_{b,q}) \), using the rotation matrix as,

\[ \phi_{b,q} = U_{\text{rot}} \cdot \mathbf{o} \]  

where \( U_{\text{rot}} \) is the unitary rotation matrix.

The users selected by the combined criterion feed back \( \log_2(N_c) \) bits corresponding to the codebook index. In addition to that, it is necessary to feedback \( \log_2(N_t) \) bits for the index of the vector \( \phi_{b,q} \). Consequently, for a codebook size of \( N_c \), \( \log_2(N_c \times N_t) \) bits are necessary to quantify the CDI for P2 algorithm while \( \log_2(N_t) \) bits are required for P1 algorithm for each cluster. In terms of complexity, predefined tables can be used for generation of \( N_t \) random orthogonal vectors in practical systems.

\textbf{PFS algorithm}

Since it is assumed that the users are distributed heterogeneously, the proportional fairness (\( \alpha \) PF) algorithm is used to allocate users by adding a set of nonlinear constraints. This scheduler brings a compromise between the sum rate and the fairness. The fairness index (FI) is calculated by using the Jain index [25] given as:

\[ \text{FI} = \frac{\left( \frac{\sum_{n=1}^{N} \sum_{k=1}^{K} R^n_k}{K} \right)^2}{\frac{\sum_{n=1}^{N} \sum_{k=1}^{K} (R^n_k)^2}{K}} \]

The FI ranges between 0 (no fairness) and 1 (perfect fairness) in which all users would achieve the same data rate.

After applying user selection algorithms and quantization at the receiver side as presented in the previous sections, the CSI of the selected users is feedback through the feedback channel. Then, the users’ CDI and CQI are available at the BS to perform PFS by choosing the user set for each cluster. Since the search space becomes \( 2^{|T_q|} \) instead of \( 2^K \) with the advantage of the user selection
algorithms, the complexity of the resource allocation is significantly reduced at the BS.

- For each cluster $q$:
  - Set $A_{k,q,b}^{n} = 0$ for $k = 1, 2, \ldots, K$ and $b = 1, 2, \ldots, N_t$.
  - Construct $\left( \frac{|T_q|}{N_t} \right)$ sets as $S_{q}^{b_i}; b = 1, 2, \ldots, N_t$ from the set of $T_q$.
  For $i = 1, 2, \ldots |T_q|:
  - Calculate the user rate for $k' \in S_{q}^{b_i}; k' = 1, 2, \ldots, K; b = 1, 2, \ldots, N_t$:
    \[ R_{k,q,b}^{n} = \log_2(1 + \text{SINR}_{k,q,b}^{n}) \]  
  - Choose the allocation pattern that maximizes the sum weighted rate as:
    \[ i^* = \arg\max_{i} \sum_{b=1}^{N_t} \frac{R_{k,q,b}^{n}}{L_{t,q}} \]  
  - Update $A_{k',q,b'}^{n} = 1$ for $k' \in S_{q}^{b_i}; k' = 1, 2, \ldots, N_t$.
  - Update the weighted average data rate according to Equation (15).

- End.

**Performance results**

We obtain the performance results to illustrate the benefits of the reduced rate feedback channel in a single-cell MISO–OFDMA system with two transmit and one receive antennas through wireless channels. The users are uniformly distributed in a cell with a diameter of 750 m. The transmitted power and the noise density power are set at 43.10 dBm and $-174$ dBm/Hz, respectively. The path loss model is $L_P = 128.1 + 37.6 \log_{10}(d(\text{km}))$ dB and the wireless channel is modeled using 3GPP-TU. The bandwidth, the carrier frequency and the number of clusters are selected 10 MHz, 2.4 GHz and 48 with a velocity of 30 km/h. The clusters are grouped into 18 subcarriers. Assuming the slot duration is 100 ms, the feedback information is provided every 1 ms. The parameter $\alpha$ is chosen as 2 with $t_c = 100$ for the PFS algorithm.

Firstly, we choose the reduced rate feedback channel parameters of $S$ and $\epsilon$ as a function of $K, P_{\text{obj}}$ and $N_t$ with the help of calculation in Appendix. In order to increase the multiuser diversity, the $\epsilon$ values are adjusted to have 1.35 clusters after performing the semi-orthogonal criterion and the proper spectral efficiency parameter to achieve approximately the same sum capacity, $\epsilon$, is chosen for adaptive schemes. In Table 1, the parameters are listed for a given target according to the number of users in the cell. The spectral efficiency parameter in adaptive schemes is also adaptively changed according to the number of users in the cell as indicated in Table 1.

![Figure 1: Fairness performance of the user selection algorithms.](image-url)
we focus on the quantization error on CDI assuming the CQI is perfectly available at the transmitter. In order to show the performance of the proposed codebook design, the CDI of the selected users and clusters is quantized by using different codebooks. The simulation results for different reduced feedback algorithms and different codebook sizes are illustrated in Figures 4 and 5. It is observed that the adaptive schemes give almost the same performance as the fixed algorithms. When the number of feedback bits for CDI is increased, the performance of the sum rate is improved significantly. As illustrated in Figure 6, the combined user selection algorithms outperform the clustered S-best criterion by reducing the quantization error of the CDI with the usage of the proposed codebook design. The reason is that since the MISO–OFDMA systems are more sensitive to interference, it is important to minimize the CDI quantization error. Consequently, the sum rate performance of the proposed combined algorithms is much higher than the clustered S-best algorithm especially when the number of CDI quantization bits is quite limited.

Conclusion
In this article, we have examined efficient algorithms to design reduced rate feedback channel for MISO–OFDMA systems. We have presented fixed and adaptive cluster S-best and the combined semi-orthogonal criterion by employing more than one user for each cluster. According to the number of active users in cell and a given target, we have obtained the calculations of the number of feedback clusters for OFDMA with multiple antennas. The quantization error on the channel direction has been reduced since the CDI codebooks are designed using a local packing by taking into account the users’ direction. It has been illustrated that the proposed algorithms in a quantized reduced feedback link improve sum capacity significantly while providing fairness among the users in wireless channels. It has been also shown that the adaptive algorithm adjusts the number of selected clusters according to the location of the users and achieve the same fairness performance while further reducing the feedback link load. The proposed reduced feedback designs will be extent to multicell MISO–OFDMA networks as a future work.

Appendix
We define respectively $U_k$ and $V_k$ as the number of clusters having CSI of only one user and more than one user when $k$ users are fed back their CSI to the BS. For the case $N_t = 2$, we compute the probability that $v$ different clusters are sending back less than $N_t$ users. It is assumed that each user feds back $S$ clusters among the $Q$ clusters.

The fraction of clusters that have at least $N_t$ users’ CSI is $\bar{U} = (U' / Q)$ where $U' = \lvert \bigcap_{q=1}^{Q} S_q \rvert$.

Following [16], the probability $\Pr(U_k = A; V_k = C)$ can be stacked in a column vector $P_k$ of size $(Q + 1)^2$ and computed recursively:

$$ P_k = \begin{pmatrix} \Pr(U_k = 0; V_k = 0) \\ \Pr(U_k = 0; V_k = 1) \\ \vdots \\ \Pr(U_k = Q; V_k = Q - 1) \\ \Pr(U_k = Q; V_k = Q) \end{pmatrix} = BP_{k-1} \quad (31) $$

![Figure 2 Sum capacity performance for the user selection algorithms without quantization.](image1)

![Figure 3 Sum capacity performance of PFS for the P1 and P2 algorithms with quantized CSI.](image2)

| $K$  | 10 | 20 | 30 | 40 | 50 |
|------|----|----|----|----|----|
| Full | 10 | 20 | 30 | 40 | 50 |
| Fixed| 6.25 | 6.25 | 6.25 | 6.67 | 6.25 |
| Adaptive | 4.15 | 4.48 | 4.74 | 5.12 | 5.51 |
where

\[
B = \begin{pmatrix}
\Pr(U_k = 0; V_k = 0 | U_{k-1} = 0; V_{k-1} = 0) & \ldots & \Pr(U_k = 0; V_k = 0 | U_{k-1} = Q; V_{k-1} = Q) \\
\Pr(U_k = 0; V_k = 1 | U_{k-1} = 0; V_{k-1} = 0) & \ldots & \Pr(U_k = 0; V_k = 1 | U_{k-1} = Q; V_{k-1} = Q) \\
\Pr(U_k = Q; V_k = 0 | U_{k-1} = 0; V_{k-1} = 0) & \ldots & \Pr(U_k = Q; V_k = 0 | U_{k-1} = Q; V_{k-1} = Q) \\
\Pr(U_k = Q; V_k = Q | U_{k-1} = 0; V_{k-1} = 0) & \ldots & \Pr(U_k = Q; V_k = Q | U_{k-1} = Q; V_{k-1} = Q)
\end{pmatrix}
\]

If the following constraints:

\[
D \geq S - (A - B) - 2(C - D) \\
S - (A - B) - 2(C - D) \geq 0 \\
B \geq C - D \\
C - D \geq 0 \\
Q - B - D \geq C + A - D - B \\
C + A - D - B \geq 0 \\
Q - B - D \geq 0
\]

are satisfied, the elements of \( B \) are calculated as follows:

\[
\Pr(U_k = A; V_k = C | U_{k-1} = B; V_{k-1} = D) = \frac{D}{(S - 2C + 2D + B - A)} \left( \begin{array}{c}
B \\
(C - D) \\
(C + A - D - B)
\end{array} \right) \left( \begin{array}{c}
Q - B - D \\
C - D \\
B - D
\end{array} \right)
\]

(32)

Otherwise, \( \Pr(U_k = A; V_k = C | U_{k-1} = B; V_{k-1} = D) = 0. \)

Since the first user \((k = 1)\) picks \( S \) clusters, all the elements of \( P_1 \) are zero except \( \Pr(U_1 = S; V_1 = 0) = 1. \) Then, \( P_K \) can be computed as follows:

\[
P_K = B^K P_1
\]

(33)

The probability that \( v \) or less different clusters are fed back with less than 2 users is obtained by,

\[
\Pr(V_K \leq v) = \sum_{i=0}^{(Q+1)(v+1)} P_K(i)
\]

(34)

where \( P_K(i) \) denotes the \( i \)th element of \( P_K. \)
Competing interests
The authors declare that they have no competing interests.

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References
1. YJ Zhang, KB Letaif, An efficient resource allocation scheme for spatial multiuser access in MIMO/OFDM system. IEEE Trans. Commun. 53, 107–116 (2005)
2. TF Maciel, A Klein, A resource allocation strategy for SDMA/OFDMA systems. in Proc. of 16th IST Mobile and Wireless Communications Summit, (2007)
3. TF Maciel, A Klein, On the performance, complexity and fairness of suboptimal resource allocation for multiuser MIMO-OFDMA systems. IEEE Trans. Veh. Technol. 59, 406–419 (2010)
4. DJ Love, RW Heath, VKN Lau, D Gesbert, BD Rao, M Andrews, An overview of limited feedback in wireless communication systems. IEEE J. Sel. Areas Commun. (JSAC). 26, 1341–1365 (2008)
5. C Swannack, E Uysal-Biyikoglu, GW Wornell, Finding NEMO: near mutually orthogonal sets and applications to MIMO broadcast scheduling. in Proc. of Int. Conf. Wireless Networks, Communications, Mobile Computing (WIRELESSCOM), (Maui, HI, 2005)
6. D Le Ruyet, H Khanfir, B Özbek, Performance of semi-orthogonal user selection for multiuser MISO systems. in Proc. of IEEE International Workshop on Signal Processing Advances in Wireless Communications (SPAWC), (Brazil, 2008)
7. S Zhou, Z Wang, G Giannakis, Performance analysis for transmit-beamforming with finite-rate feedback. in Proc. of 38th Conf. on Info. Sciences and Systems, (2004)
8. J Choi, RW Heath, Interpolation based transmit beamforming for MIMO-OFDM with limited feedback. IEEE Trans. Signal Process. 53, 4125–4135 (2005)
9. T Pande, DJ Love, JV Krogmeier, Reduced feedback MIMO–OFDM precoding and antenna selection. IEEE Trans. Signal Process. 55, 2284–2293 (2007)
10. RW Heath, T Wu, ACK Soong, Progressive refinement of beamforming vectors for high-resolution limited feedback. EURASIP J. Adv. Signal Process. 2009, 13 (Article ID 463823) (2009)
11. T Kim, DJ Love, B Clerckx, MIMO systems with limited rate differential feedback in slowly varying channels. IEEE Trans. Commun. 59, 1175–1189 (2011)
12. J Choi, B Clerckx, N Lee, G Kim, A new design of polar-cap differential codebook for temporally/spatially correlated MISO channels. IEEE Trans. Commun. 11, 703–711 (2012)
13. R Zakhour, D Gesbert, A two-stage approach to feedback design in MU-MIMO channels with limited channel state information. in Proc. IEEE Personal, Indoor and Mobile Radio Commun. Symp. (PIMRC), (France, 2008)
14. N Ravindran, N Jindal, Multi-user diversity vs. accurate channel feedback for MIMO broadcast channels. in Proc. IEEE Personal, Indoor and Mobile Radio Commun. Symp. (PIMRC), (France, 2008)
15. Y Al-Harthi, Opportunistic multiuser scheduling with reduced feedback load. Eur. Trans. Telecommun. 21, 299–311 (2010)
16. P Svedman, Sk Wilson, LJ Cimini, B Ottersten, Opportunistic beamforming and scheduling for OFDMA systems. IEEE Trans. Commun. 55, 941–952 (2007)
17. V Majigi, R Agarwal, J Cioffi, User profiling: a method for limited feedback in OFDMA systems. in Proc. of IEEE International Communications Conference (ICC), (Dresden, Germany, 2009)
18. M Trivellato, S Toma, N Benvenuto, On channel quantization and feedback strategies for multiuser MIMO-OFDM downlink systems. IEEE Trans. Commun. 57, 2645–2654 (2009)
19. J Jeon, K Son, HW Lee, S Chong, IEEE Trans. Veh. Technol. 59, 160–169 (2010)
20. QH Spencer, AL Swindlehurst, M Haardt, Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels. IEEE Trans. Signal Process. 52, 461–471 (2004)
21. T Yoo, A Goldsmith, On the optimality of multiantenna broadcast scheduling using zero-forcing beamforming. IEEE J. Sel. Areas Commun. (JSAC). 24, 528–541 (2006)
22. YD Papouatis, IG Fraimis, SA Kotropoulos, Fairness-aware user selection and resource allocation in MISO-OFDMA. Eur. Trans. Telecommun. 21, 568–574 (2010)
23. B Özbek, D LeRuyet, Reduced feedback designs for SDMA-OFDMA systems. in Proc. of IEEE International Communication Conference (ICC), (Dresden, Germany, 2010)
24. KK Mukkavilli, A Sabharwal, E Erkip, B Azazhang, On beamforming with finite rate feedback in multiple antenna systems. IEEE Trans. Inf. Theory. 49, 2562–2579 (2003)
25. R Jain, DM Chiou, WR Haeve, A quantitative measure of fairness and discrimination for resource allocation shared computer systems. in Digital Equipment Corporation technical report TR-307, (1984)

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