Neutrino clustering in growing neutrino quintessence

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A growing neutrino mass can stop the dynamical evolution of a dark energy scalar field, thus explaining the “why now” problem. We show that such models lead to a substantial neutrino clustering on the scales of superclusters. Nonlinear neutrino lumps form at redshift \( z \approx 1 \) and could partially drag the clustering of dark matter. If observed, large scale non-linear structures could be an indication for a new attractive “cosmon force” stronger than gravity.

Quintessence cosmologies could explain the observed order of magnitude of dark energy. This holds, in particular, for cosmic attractors or scaling solutions, where the dark energy density decays in time as \( \sim t^{-2} \). Such models generically predict the presence of an homogeneous dark energy fraction \( \Omega_m \), of similar size as the dark matter fraction \( \Omega_m \). Nevertheless, upper bounds at high redshift restrict early dark energy to be \( \Omega_m(z \geq 5) < 0.1 \), while at present dark energy dominates \( \Omega_m(t_0) \approx 3/4 \).

Realistic quintessence models need to explain the recent increase of \( \Omega_m(z) \) by a factor of around ten or more. It has been recently proposed that a growing mass of the neutrinos may play a key role in stopping the dynamical evolution of the dark energy scalar field, the cosmon. For a slow evolution of the cosmon, the scalar potential acts like a cosmological constant, such that the equation of state of dark energy is close to \( w = -1 \) and the expansion of the universe accelerates. In these models, the onset of accelerated expansion is triggered by neutrinos becoming non-relativistic. For late cosmology, \( z > 5 \), the overall cosmology is very similar to the usual \( \Lambda \)CDM concordance model with a cosmological constant.

An efficient stopping of the cosmon evolution by the relatively small energy density of neutrinos needs a cosmon-neutrino coupling that is somewhat larger than gravitational strength. This is similar to mass varying neutrino models \([8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]\), even though the coupling in those models is generically much larger. In turn, the enhanced attraction between neutrinos leads to an enhanced growth of neutrino fluctuations, once the neutrinos have become non-relativistic \([3, 9, 10, 20]\). In view of the small present neutrino mass, \( m_\nu(t_0) < 2.3 eV \), and the time dependence of \( m_\nu \) which makes the mass even smaller in the past, the time when neutrinos become non-relativistic is typically in the recent history of the universe, say \( z_R \approx 5 \). Neutrinos have been free streaming for \( z > z_R \), with a correspondingly large free streaming length. Fluctuations on length scales larger than the free streaming length are still present at \( z_R \), and they start growing for \( z < z_R \) with a large growth rate. This opens the possibility that neutrinos form nonlinear lumps \([3, 21]\) on supercluster scales, thus opening a window for observable effects of the growing neutrino scenario.

In this letter we show that neutrino perturbations indeed grow non-linear in these models. Non-linear neutrino structures form at redshift \( z \approx 1 \) on the scale of superclusters and beyond. One may assume that these structures later turn into bound neutrino lumps of the type discussed in \([21]\). The cosmon field within the neutrino lumps does not vanish, and we see in our investigation how the cosmon-fluctuations are dragged by the neutrino fluctuations. We compute the neutrino clustering as a function of scale and redshift, pointing out also how the growth of cold dark matter fluctuations is affected within these scenarios. Our investigation is limited, however, to linear perturbations. We can therefore provide a reliable estimate for the time when the first fluctuations become non-linear. For later times, it should only be used to give qualitative limits.

A crucial ingredient in this model is the dependence of the neutrino mass on the cosmon field \( \phi \), as encoded in the dimensionless cosmon-neutrino coupling \( \beta(\phi) \),

\[
\beta(\phi) \equiv -\frac{d \ln m_\nu}{d \phi}.
\]

For increasing \( \phi \) and \( \beta < 0 \) the neutrino mass increases with time

\[
m_\nu = \bar{m}_\nu e^{-\beta(\phi)\phi},
\]

where \( \bar{m}_\nu \) is a constant and \( \beta = \tilde{\beta} + \partial_\phi \tilde{\beta} / \partial \ln \phi \). The coupling \( \beta(\phi) \) can be either a constant \( \tilde{\beta} \) or, in general, a function of \( \phi \), as proposed in \([3]\) within a particle physics model. The cosmon field \( \phi \) is normalized in units of the reduced Planck mass \( M = (8\pi G_N)^{-1/2} \), and \( \beta \sim 1 \) corresponds to a cosmon mediated interaction for neutrinos with gravitational strength. For a given cosmological model with a given time dependence of \( \phi \), one can determine the time dependence of the neutrino mass \( m_\nu(t) \). For three degenerate neutrinos the present value of the neutrino mass \( m_\nu(t_0) \) can be related to the energy fraction in neutrinos

\[
\Omega_\nu(t_0) = \frac{3m_\nu(t_0)}{94 eV h^2}.
\]

The dynamics of the dark energy scalar field can be inferred from the Klein Gordon equation, now including an extra source due to the neutrino coupling,

\[
\phi'' + 2H \phi' + a^2 \frac{dU}{d\phi} = a^2 \beta(\phi)(\rho_\nu - 3p_\nu),
\]
with \( \rho_\nu \) and \( p_\nu = w_\nu \rho_\nu \) the energy density and pressure of the neutrinos. We choose an exponential potential \([1, 2, 3, 4]\):

\[
V(\phi) = M^2 U(\phi) = M^4 e^{-\alpha \phi},
\]

where the constant \( \alpha \) is one of the free parameters of our model.

The homogeneous energy density and pressure of the scalar field \( \phi \) are defined in the usual way as

\[
\rho_\phi = \frac{\phi'^2}{2a^2} + V(\phi) \quad \text{and} \quad p_\phi = \frac{\phi'^2}{2a^2} - V(\phi), \quad w_\phi = \frac{p_\phi}{\rho_\phi}.
\]

Finally, we can express the conservation equations for dark energy and growing matter in the form of neutrinos as follows \([22, 23]\):

\[
\rho_\phi' = -3H(1 + w_\phi)\rho_\phi + \beta(\phi)\phi'(1 - 3w_\nu)\rho_\nu, \quad \rho_\nu' = -3H(1 + w_\nu)\rho_\nu - \beta(\phi)\phi'(1 - 3w_\nu)\rho_\nu.
\]

The sum of the energy momentum tensors for neutrinos and the cosmon is conserved, but not the separate parts. We neglect a possible cosmon coupling to Cold Dark Matter (CDM), so that \( \rho_c' = -3H\rho_c \).

For a given potential \([5]\) the evolution equations for the different species can be numerically integrated, giving the background evolution shown in FIG. 11 (for constant \( \beta \)). The initial pattern is a typical early dark energy model, since neutrinos are still relativistic and almost massless. Radiation dominates until matter radiation equality, when CDM takes over. Dark energy is still subdominant and falls into the attractor provided by the exponential potential (see \([22, 23]\) for details).

As the mass of the neutrinos increases with time, the term \( \sim \beta \rho_\nu \) in the evolution equation for the cosmon \([11]\) (or \([11]\)) starts to play a more significant role, kicking \( \phi \) out of the attractor as soon as neutrinos become non-relativistic. This resembles the effect of the coupled dark matter component in \([24]\). Subsequently, small decaying oscillations characterize the \( \phi - \nu \) coupled fluid and the two components reach almost constant values.

The values of the energy densities today are in agreement with observations, once the precise crossing time for the end of the scaling solution has been fixed by an appropriate choice of the coupling \( \beta \). At present the neutrinos are still subdominant with respect to CDM, though in the future they will take the lead (see \([3]\) for details on the future attractor solution for constant \( \beta \)).

The evolution equations for linear perturbations (in Fourier space), in Newtonian gauge (in which the non diagonal metric perturbations are fixed to zero) \([22, 23]\), read for the growing neutrino scenario:

\[
\delta_i' = 3H(\rho_\phi - c_s^2)\delta_i
\]

\[
\begin{align*}
\rho_\phi' &= -3H(1 + w_\phi)\rho_\phi + \beta(\phi)\phi'(1 - 3w_\nu)\rho_\nu, \\
\rho_\nu' &= -3H(1 + w_\nu)\rho_\nu - \beta(\phi)\phi'(1 - 3w_\nu)\rho_\nu.
\end{align*}
\]

FIG. 1: Energy densities of neutrinos (dashed), cold dark matter (solid), dark energy (dotted) and photons (long dashed) are plotted vs redshift. For all plots we take a constant \( \beta = -52 \), with \( \alpha = 10 \) and large neutrino mass \( m_\nu = 2.11 \text{eV} \).

\[
\begin{align*}
\delta_i' &= 3\left[ H(1 - \beta(\phi)\phi') - c_s^2 \delta_i \right] \\
&= (1 + w_\nu)\rho_\nu - \beta(\phi)\phi'(1 - 3w_\nu)\delta_i
\end{align*}
\]

The equations for the density contrasts \( \delta_i(k) = \frac{1}{V} \int \delta_i(\mathbf{x})\exp(-i\mathbf{k} \cdot \mathbf{x})d^3x \) (defined as the Fourier transformation of the local density perturbation \( \delta_i(\mathbf{x}) = \delta \rho_i(x)/\rho_i(x) \) over a volume \( V \)) involve the velocity perturbations, which evolve according to

\[
\begin{align*}
v_{\phi}' &= -\frac{H(1 - 3w_\phi)v_\phi - \beta(\phi)\phi'(1 - 3w_\nu)v_\phi}{1 + w_\phi} \\
&= -\frac{w_\phi - \phi' - k\delta_i(1 + w_\phi)}{1 + w_\phi} + k\Psi \\
&= -\frac{2}{3}\frac{w_\phi - \phi' - k\delta_i(1 + w_\phi)}{1 + w_\phi} \frac{1 - 3w_\nu}{\rho_\phi} \\
v_{\nu}' &= \frac{(1 - 3w_\nu)(\beta(\phi)\phi' - H)v_\nu}{1 + w_\nu} - \frac{w_\nu}{1 + w_\nu}v_\nu.
\end{align*}
\]
\[ \Psi = -\Phi - \frac{a^2}{k^2 M^2} \sum_\alpha w_\alpha \rho_\alpha \pi_{T \alpha}, \]  
(13)

where \( \pi_{T \alpha} \) is the anisotropic stress for the species \( \alpha \) and the sound velocities are defined by \( c_i^2 \equiv \frac{\partial p_i}{\partial \rho_i}. \) The perturbed pressure for \( \phi \) is

\[ \delta p_\phi = \frac{\phi'}{a^2} \delta \phi' - \frac{\Psi}{a^2} \phi'^2 - \mathcal{U} \delta \phi \]  
(14)

and the anisotropic stress \( \pi_{T \alpha} = 0 \), as in uncoupled quintessence, since the coupling is treated as an external source in the Einstein equations. The linear perturbation of the cosmon, \( \delta \phi \), is related to \( v_\phi \) via

\[ \delta \phi = \phi' v_\phi/k \]  
(15)

\[ \delta \phi' = \frac{k}{\kappa} + \frac{1}{k} \left[ -2 \mathcal{H} \phi' - a^2 \frac{dU}{d\phi} + a^2 \beta(\phi)(\rho_\nu - 3p_\nu) \right] v_\phi . \]

Note that \( \delta \phi \) can equivalently be obtained as the solution of the perturbed Klein Gordon equation:

\[ \delta \phi'' + 2 \mathcal{H} \delta \phi' + \left( k^2 + a^2 \frac{d^2 U}{d\phi^2} \right) \delta \phi - \phi' (\Psi - 3 \Phi' - 3 \Phi) + 2 a^2 \frac{dU}{d\phi} \Psi = -a^2 \beta(\phi) \rho_\nu (1 - 3 c_\nu^2) \]  
(16)

The evolution of neutrinos requires solving the Boltzmann equation in the case in which an interaction between neutrinos and the cosmon is present [29]. The first order Boltzmann equation written in Newtonian gauge reads [28]

\[ \frac{\partial \Psi_{ps}}{\partial \tau} + \frac{q}{\epsilon} (k \cdot n) \Psi_{ps} + \frac{d \ln f_0}{d \ln q} \left[ -\Phi' - \frac{\epsilon}{q} (k \cdot n) \Psi \right] = 0 \]  
(17)

where \( \Psi_{ps} \) is defined as the perturbed term in the phase space distribution [28, 29].

\[ f(x', \tau, q, n_j) = f_0(q) \left[ 1 + \Psi_{ps}(x', \tau, q, n_j) \right] \]  
(18)

\( \Psi \) and \( \Phi \) are the metric perturbations, \( x' \) the spatial coordinates, \( \tau \) is the conformal time, \( q = a \mathbf{p} = q \mathbf{n} \) is the comoving 3-momentum, \( \epsilon = \epsilon(\phi) = \sqrt{q^2 + m^2(\phi)^2 a^2} \), \( f \) is the phase space distribution and \( f_0 \) its zeroth-order term (Fermi-Dirac distribution).

The Boltzmann hierarchy for neutrinos, obtained expanding the perturbation \( \Psi_{ps} \) in a Legendre series can be written in Newtonian gauge as

\[ \Psi_{ps, 0} = - \frac{q k}{\epsilon} \Psi_{ps, 1} + \Phi \frac{d \ln f_0}{d \ln q} \]  
(19)

\[ \Psi_{ps, 1} = \frac{q k}{3 \epsilon} (\Psi_{ps, 0} - 2 \Psi_{ps, 2}) - \frac{e k}{3 q} \Psi \frac{d \ln f_0}{d \ln q} + \kappa \]  
(20)

\[ \Psi_{ps, l} = \frac{q k}{(2l + 1) \epsilon} \left[ l \Psi_{ps, l-1} - (l + 1) \Psi_{ps, l+1} \right] \]  
(21)
where \[28, 29\]
\[
\kappa = -\frac{1}{3} \frac{q^2 m_\nu^2}{q^2} \frac{\partial}{\partial \phi} \ln m_\nu \frac{d \ln f_0}{d \ln q} \phi .
\]

This allows us to calculate the perturbed energy and pressure as well as the shear for neutrinos:
\[
\delta \rho_\nu = a^{-4} \int q^2 f_0(q) \left[ \epsilon(\phi) \Psi_{ps,0} + \epsilon(\phi) \frac{\partial \epsilon}{\partial \phi} \right] dq d\Omega ,
\]
\[
\delta p_\nu = a^{-4} \int \frac{q^4}{3} f_0(q) \left[ \epsilon(\phi) \Psi_{ps,0} - \epsilon(\phi) \frac{\partial \epsilon}{\partial \phi} \right] dq d\Omega ,
\]
\[
(\rho_\nu + p_\nu) \sigma_\nu = \frac{8\pi}{3} a^{-4} \int q^2 dq q^2 f_0(q) \Psi_{ps,2} .
\]

The anisotropic stress is related to the shear via \(\pi_T = \frac{3}{2\rho_\nu}(\rho_\nu + p_\nu)\sigma\) and in our case
\[
\frac{\partial \epsilon}{\partial \phi} = \frac{a^2 m_\nu^2}{\epsilon} \frac{\partial}{\partial \phi} \ln m_\nu = -\beta(\phi) \frac{a^2 m_\nu^2(\phi)}{\epsilon(\phi)} .
\]

Note also that the unperturbed neutrino density and pressure read
\[
\rho_\nu = a^{-4} \int q^2 dq d\Omega \epsilon(\phi) f_0(q) ,
\]
\[
p_\nu = \frac{1}{3} a^{-4} \int q^2 dq d\Omega \frac{q^2}{\epsilon} \epsilon(\phi) f_0(q) .
\]

We numerically compute the linear density perturbations both using a modified version of CMBEASY \[31\] and, independently, a modified version of CAMB \[34\], written in synchronous gauge. The initial conditions are chosen such that \(\delta_\nu = \delta_\nu = \sqrt{A}\) at redshift \(z_{14} \sim 1100\), with \(A\) the primordial power spectrum amplitude as determined from the CMB anisotropies \[30\]. We plot the density fluctuations \(\delta_\nu\) as a function of redshift for a fixed \(k\) in FIG.2. The neutrino equation of state is also shown, starting from 1/3 when neutrinos are relativistic and then decreasing to its present value when neutrinos become nonrelativistic. The turning point marks the time at which neutrino perturbations start to increase. At the scale of \(k = 0.1 h/Mpc\) (corresponding to superclusters scales) shown in FIG.2, neutrino perturbations eventually overtake CDM perturbations and even force \(\phi\) perturbations to increase as well, in analogy with dark energy clustering expected in \[32, 33\] within scalar tensor theories. Notice, however, that the scale at which neutrinos form nonlinear clumps depends on the model parameters, in particular the coupling \(\beta\), the potential parameter \(\alpha\) and the present days neutrino mass. Those are related to the neutrino free-streaming length, the range of the cosmon field and its mass. A detailed investigation of the parameter space will be performed in future work, but we mention that the model \[3\] with varying \(\beta\) gives qualitatively similar results.

We emphasize again that the linear approximation loses its quantitative reliability once one of the \(\delta_\nu\) reaches one - for the wavelength shown in FIG.2 this first concerns neutrinos. Strong neutrino clumping could produce a gravitational potential that, in turn, drags the CDM fluctuations, as seen in the linear approximation in FIG.2. However, once the neutrinos form strong nonlinearities - neutrino lumps - one expects that nonlinear effects substantially slow down the increase of \(\delta_\nu\) and even stop it. The magnitude of the CDM-dragging by neutrinos is therefore not shown - it might be much smaller than visible in FIG.2. These remarks concern the quantitative interpretation of all the following figures, which are always computed in the linear approximation.

Nevertheless, the linear approximation demonstrates well the mechanisms at work. As can be seen from FIG.3, the gravitational potential shows a strong growth at the moment when neutrinos go nonlinear. Despite the small neutrino fraction \(\Omega_\nu\), strong neutrino fluctuations can become its main source. In turn, the gravitational potential will source the growth of CDM, whereas the cosmon field inhomogeneities are dragged by the even stronger interaction with the neutrinos. In the nonlinear regime, one expects that the neutrino structures decouple from the expansion and form stable lumps of the type described in \[21\]. Typically, the gravitational potential in such static loops is much smaller than the huge values obtained in the linear approximation in FIG.3. This demonstrates again that a quantitative understanding of
the gravitational potential and the CDM dragging has to wait for a better understanding of the nonlinear evolution of neutrino fluctuations. This will also be crucial for an estimate of the role of neutrino clumping on the CMB anisotropies via the Integrated Sachs-Wolfe effect.

In FIG. 4 we plot the redshift \( z_{\text{nl}} \) at which CDM, neutrinos and \( \phi \) become nonlinear as a function of the wavenumber \( k \). The case of CDM in the concordance \( \Lambda \text{CDM} \) model with the same present value of \( \Omega_\phi \) and for massless neutrinos is also shown for reference (dotted line). The redshift \( z_{\text{nl}} \) roughly measures when nonlinearities first appear by evaluating the time at which \( \delta(z_{\text{nl}}) = 1 \) for each species. The curves in FIG. 4 are obtained in the linear approximation, such that only the highest curves are quantitatively reliable. This concerns CDM for large \( k \) and neutrinos for small \( k \). The subleading components are influenced by dragging effects and may be, in reality, substantially lower.

We can identify four regimes: i) At very big scales (larger than superclusters) the universe is homogeneous and perturbations are still linear today. ii) The range of length scales going from 14.5 Mpc to about \( 4.4 \times 10^3 \) Mpc appears to be highly affected by the neutrino coupling in growing matter scenarios: neutrino perturbations are the first ones to go nonlinear and neutrinos seem to form clumps in which then both the scalar field and CDM could fall into. Note that the effect of the neutrino fluctuations on the gravitational potential induces CDM to cluster earlier with respect to the concordance \( \Lambda \text{CDM} \) model, where CDM is still linear at scales above \( \sim 87 \) Mpc. iii) For lengths included in the range between 0.9 Mpc and 14.5 Mpc, CDM takes over. That is in fact expected since neutrinos start to approach the free streaming scale. In this regime CDM drags neutrinos, and this effect may be overestimated in the linear approximation. Notice that in our model CDM clusters later than it would do in \( \Lambda \text{CDM} \). There are two reasons for this effect. At early times, the presence of a homogeneous component of early dark energy, \( \Omega_\phi \sim 3/\alpha^2 \), implies that \( \Omega_m \) is somewhat smaller than one and therefore clustering is slower [35]. At later times, \( \Omega_m \) is smaller than in the \( \Lambda \text{CDM} \) model since for the same \( \Omega_\phi \), part of \( 1 - \Omega_\phi = \Omega_m + \Omega_\nu \) is now attributed to neutrinos. In consequence massive neutrinos reduce structure at smaller scales when they do not contribute to the clumping. The second effect is reduced for a smaller present day neutrino mass. iv) Finally, at very small scales (below clusters), CDM becomes highly non linear and neutrinos enter the free streaming regime, their perturbations do not growth and remain inside the linear regime.

The longitudinal density perturbation for CDM and for neutrinos is shown in FIG. 5 as a function of \( k \), for two values of redshift \( z \). For the present epoch (\( z = 0 \)) the total size of the density contrast is presumably strongly overestimated in the linear approximation. Nevertheless, this figure visualizes the range of scales for which strong neutrino clumping is expected. In this range also the dragging effect on CDM is maximal.
Maximum neutrino clustering occurs on supercluster scales and one may ask about observable consequences. First of all, the neutrino clusters could have an imprint on the CMB-fluctuations. Taking the linear approximation at face value, the ISW-effect of the particular model presented here would be huge and strongly ruled out by observations. However, non-linear effects will substantially reduce the neutrino-generated gravitational potential and the ISW-effect. Further reduction is expected for smaller values of $\beta$ (accompanied by smaller $\alpha$). It is well conceivable that realistic models for the growing neutrino scenario lead to an ISW-effect in a range interesting for observations. Particular features are possible consequences of the oscillations in the neutrino sector. This typically leads to an ISW effect which can show structures as a function of the angular momentum $l$.

A second possibility concerns the detection of nonlinear structures at very large length scales. Such structures can be found via their gravitational potential, independently of the question if neutrinos or CDM source the gravitational field. Very large nonlinear structures are extremely unlikely in the $\Lambda$CDM concordance model. An establishment of a population of such structures, and their possible direct correlation with the CMB-map [36, 37, 38, 39, 40], could therefore give a clear hint for “cosmological actors” beyond the $\Lambda$CDM model. For any flat primordial spectrum the gravitational force will be insufficient to produce large scale clumping, whatever the ingredients are. Large scale clumping could thus be an indication for a new attractive force stronger than gravity - in our model mediated by the cosmon.

DFM and VP acknowledge the Alexander von Humbold Foundation. GR acknowledges support by the Deutsche Forschungsgemeinschaft, grant TRR33 “The Dark Universe”.

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