Stochastic Gravitational Radiation from Phase Transitions

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Abstract. A stochastic background of gravitational radiation from cosmological processes in the very early Universe is potentially detectable. We review the gravitational radiation which may arise from cosmological phase transitions, covering both bubble collisions and turbulence as sources. Prospects for detecting a direct signal from the electroweak phase transition or other cosmological sources with a space-based laser interferometer are discussed.

INTRODUCTION

Scientific and design goals for gravitational radiation detectors have been driven primarily by point sources of radiation like binary compact objects. While the distribution and properties of such sources will undoubtedly be of cosmological interest, the most interesting gravitational wave signals for cosmology are likely to be stochastic backgrounds produced in the early Universe. Since the Universe is opaque to electromagnetic radiation at redshifts greater than $z = 1300$ due to Thomson scattering off free electrons, gravitational radiation offers the only hope of “imaging” the earliest epochs of the Universe when it was dominated by high-energy fundamental physics processes.

Several possible stochastic sources have been considered. The best motivated are cosmic defects like strings or textures; quantum fluctuations in the fields driving inflation; and phase transitions. More speculative ideas include string-theory inspired scenarios for cosmic evolution at times prior to the apparent initial singularity (so-called “pre-big-bang” cosmology) or explosive preheating at the end of inflation. Here we review the stochastic background of gravitational radiation expected from first-order phase transitions. New results for gravitational radiation from turbulence, which could be produced in a first-order phase transition, are included. The final section considers the detectability of various stochastic sources, with an emphasis on the electroweak phase transition. A thorough review of stochastic gravitational radiation, with extensive material about detector phe-
nomenclature, overviews of a variety of sources, and a comprehensive bibliography, has recently appeared [1].

GENERAL CONSIDERATIONS

The time evolution of cosmological gravitational radiation follows directly from the linearized Einstein equations,

\[ \ddot{h}_{ij}(k, \eta) + 2\frac{\dot{a}}{a}\dot{h}_{ij}(k, \eta) + k^2 h_{ij}(k, \eta) = 8\pi G a^2 \Pi_{ij}(k, \eta), \tag{1} \]

where \( k \) is a comoving wave vector, \( \eta \) is conformal time, overdots are derivatives with respect to \( \eta \), \( a \) is the scale factor of the Universe, \( h_{ij} \) is the tensor metric perturbation, and \( \Pi_{ij} \) is the tensor piece of the stress-energy tensor. We will also make use of the Hubble parameter, \( H = (1/a) da/dt \) and its present value \( h_{100} \) in units of 100 km/s/Mpc, and throughout we use natural units with \( \hbar = c = k_B = 1 \).

The right side of Eq. (1) is the source term for generating gravitational radiation, while the left side describes the free propagation of the radiation, including damping due to the expansion of the Universe. Since each comoving \( k \)-mode is independent, the physical wavelength of a given gravitational wave increases linearly with the scale factor \( a \), just as with electromagnetic radiation.

Solutions to the homogeneous equation with \( \Pi = 0 \) are easily obtained. For a radiation-dominated universe, \( a \propto \eta \) and

\[ h_{ij} \propto j_0(k\eta), \quad y_0(k\eta) \quad (2a) \]

while for the matter-dominated era, \( a \propto \eta^2 \) and

\[ h_{ij} \propto j_1(k\eta)/k\eta, \quad y_1(k\eta)/k\eta, \quad (2b) \]

where \( j_l \) and \( y_l \) are the usual spherical Bessel functions. It is straightforward to show that in both cases, the amplitude of the gravitational waves scales as \( a^{-1} \). This result also follows from the stress-energy tensor for gravitational radiation, whose energy density is given by [2]

\[ \rho_{GW} = \frac{1}{32\pi G} \left\langle \frac{\partial h_{ij}}{\partial t} \frac{\partial h^{ij}}{\partial t} \right\rangle \tag{3} \]

where the average is over many wavelengths. So the energy density is proportional to \( h_c^2 f^2 \), where \( h_c \) is the characteristic amplitude of the metric perturbation \( h_{ij} \) and \( f \) is the (physical) frequency of the wave. The energy density scales like \( a^{-4} \) with the expansion of the Universe, while the frequency scales like \( a^{-1} \). It follows that \( h_c \propto a^{-1} \) as derived above from the evolution equation. Note this is different from the amplitude scaling of the electric and magnetic fields in an electromagnetic wave, which drop off like \( a^{-2} \).
In practice, only very large stress-energy sources contribute significantly to the stochastic gravitational wave amplitude. If a particular source acts for a period of time shorter than the Hubble time $H^{-1}$, then the expansion of the Universe can be neglected during the time the source is active and the resulting gravitational wave amplitude can be conveniently calculated in flat space. Then the entire subsequent evolution of the gravitational waves is simply a reduction of amplitude and frequency by the factor

$$\frac{a_s}{a_0} = 8.0 \times 10^{-14} \left(\frac{100}{g_*}\right)^{1/3} \left(\frac{1 \text{ GeV}}{T_*}\right),$$

where $a_s$ and $T_s$ are the scale factor and temperature at the time the source is active, $g_*$ is the total number of relativistic degrees of freedom at the time of the source, and $a_0$ is the scale factor today. This expression is valid as long as the evolution of the Universe since the source has been adiabatic. If a source exists for a time interval long compared to the Hubble time, then the expansion of the Universe may impact the source evolution and the full evolution equation, Eq. (1), must be used.

A stochastic background of gravitational waves can be characterized by $\Omega_{GW}(f)$, its energy density per frequency octave in units of the critical density $\rho_c = 3H^2/8\pi G$. From this, we follow Thorne [3] in defining a convenient characteristic amplitude at frequency $f$ as

$$h_c(f) \equiv 1.3 \times 10^{-18} \left[\Omega_{GW}(f)h_{100}^2\right]^{1/2} \left(\frac{1 \text{ Hz}}{f}\right).$$

We will calculate characteristic energy densities in gravitational wave backgrounds and convert to characteristic amplitudes with this relation.

**MODEL PHASE TRANSITIONS**

The most interesting potential source of a cosmological gravitational wave background is an early-Universe phase transition. Phase transitions very likely occurred in the early Universe. The standard model of particle physics provides two: the electroweak phase transition at $T \simeq 100$ GeV, at which the electroweak symmetry was spontaneously broken, and the QCD phase transition at $T \simeq 150$ MeV, at which chiral symmetry was broken. If the standard model is actually the result of the breaking of a larger gauge symmetry (i.e. a Grand Unified Theory), then a GUT phase transition at an energy scale of $T \simeq 10^{16}$ GeV is also likely. Other more speculative phase transitions involving the breaking of hypothesized additional symmetries have also been discussed.

The details of a particular phase transition, in particular its order, latent heat, and dynamics, are determined by the effective potential driving the phase transition. In a given particle theory, calculation of an effective potential is in general
a difficult, non-perturbative problem of finite-temperature field theory. For purposes of gravitational wave signals, we consider a generic, model first-order phase transition described by several physical parameters; a specific phase transition corresponds to some set of model parameters.

As the Universe cools due to its expansion, phase transitions can occur if a new state with lower energy density becomes physically possible. Assume this occurs at some characteristic temperature scale $T_\ast$. The energy density difference between the two phases is roughly the vacuum energy density, $\rho_{\text{vac}}$. If the two phases are separated by a significant potential energy barrier (greater than the characteristic thermal energy density at temperature $T_\ast$), then the transition must proceed via the nucleation of new-phase bubbles via quantum or thermal processes. We define $\alpha \equiv \rho_{\text{vac}}/a_R T_\ast^4$, the ratio of vacuum energy density to thermal energy density ($a_R$ is the radiation constant). Once a bubble of new phase is nucleated, the potential energy difference between the two phases exerts an outward force on the walls of the bubble, causing it to expand. Hydrodynamic forces will tend to resist the bubble expansion, which will quickly attain some equilibrium velocity $v$.

We neglect possible instabilities which might distort the bubble’s spherical shape and redistribute energy. As the bubbles expand, some fraction $\kappa$ of the available vacuum energy is converted to the kinetic energy of the expanding bubble walls, while the rest goes into heat.

Once multiple bubbles have been nucleated in some region of the Universe, they will expand until eventually their walls meet and they merge together, converting the entire region to the lower-energy phase. The energy stored up in the bubble walls is eventually dissipated into heat via hydrodynamical turbulence. If $\alpha$ is large enough, the kinetic energy of the bubble walls will represent a significant fraction of the entire energy density of the Universe, and will have $v$ a significant fraction of $c$. Note that for $T_\ast > 10^5$ K, the energy density of the Universe is dominated by radiation, so the sound speed will be $c_s = c/\sqrt{3}$, the relativistic limit. As soon as the spherical symmetry of an individual bubble is broken due to collisions with other bubbles, gravitational radiation will be produced; the high energy densities and velocities involved make bubble collisions a potentially potent source [4–6].

The characteristic frequency of the gravitational radiation is determined by the duration of the phase transition, corresponding to the percolation time of the biggest bubbles with the largest energies. A simple bubble nucleation model takes an exponential bubble nucleation rate per unit volume $\Gamma = \Gamma_0 \exp(\beta t)$ [7]. This form is motivated by the general observation that $\Gamma$ will typically be the exponential of some nucleation action, and the time dependence of the nucleation action can be approximated as linear at the time of the phase transition. For such a nucleation rate, the duration of the phase transition is roughly $\tau \simeq \beta^{-1}$. The characteristic length scale is just $\beta^{-1} v$, the size of the largest bubbles at the end of the phase transition. In general, $\beta \simeq 4 \ln(m_{\text{Pl}}/T_\ast) H_\ast \simeq 100 H_\ast$, where $H_\ast$ is the Hubble parameter at the time of the phase transition [7]. Thus the characteristic wavelength of gravitational radiation at the time of the phase transition is perhaps a percent of the Hubble radius at that time.
A simple model of a first-order phase transition is comprised of the quantities defined above: the characteristic time scale $\beta^{-1}$; the vacuum energy $\rho_{\text{vac}}$; the bubble expansion velocity $v$; the temperature of the phase transition $T_*$ or equivalently the ratio of vacuum energy to thermal energy $\alpha$; and the efficiency factor $\kappa$.

The gravitational radiation resulting from the violent collision of bubbles in a first-order phase transition has been calculated in detail [8–10]. It was demonstrated that the radiation source is primarily the bubble walls, which for can be treated in the thin-wall approximation if the bubble walls propagate as detonation fronts. The traceless portion of the stress-energy tensor is nonzero only at the position of the spherically expanding bubble walls, and the amplitude of the stress-energy tensor follows simply from the amount of vacuum energy liberated by a particular bubble’s expansion. Then the radiation from many randomly nucleated bubbles in a sample volume can be computed numerically. The result is a stochastic background of radiation with energy density

$$\Omega_{\text{GW}}(f)h^2 \simeq 1.1 \times 10^{-6} \kappa^2 \frac{H_*}{\beta} \left( \frac{\alpha}{1 + \alpha} \right)^2 \left( \frac{v^3}{0.24 + v^3} \right) \left( \frac{100}{g_*} \right)^{1/3}, \quad (6)$$

peaking at a characteristic frequency

$$f_{\text{max}} \simeq 5.2 \times 10^{-8} \text{Hz} \left( \frac{\beta}{H_*} \right) \left( \frac{T_*}{1 \text{ GeV}} \right) \left( \frac{g_*}{100} \right)^{1/6}, \quad (7)$$

with characteristic amplitude

$$h_c(f_{\text{max}}) \simeq 1.8 \times 10^{-14} \kappa \left( \frac{\alpha}{\alpha + 1} \right) \left( \frac{H_*}{\beta} \right)^2 \left( \frac{1 \text{ GeV}}{T_*} \right) \left( \frac{v^3}{0.24 + v^3} \right)^{1/2} \left( \frac{100}{g_*} \right)^{1/3}. \quad (8)$$

At frequencies lower than the characteristic peak frequency, the energy density per frequency octave increases like $f^{2.5}$ while above this frequency it drops off more slowly [9]. For very strong phase transitions, the energy spectrum of gravitational radiation has a long tail at high frequencies, due to the presence of numerous small bubbles with significant energy densities. These results hold for a strongly first-order phase transition where the bubbles expand supersonically as detonation fronts. For weaker transitions with deflagration (subsonic) bubble walls, the dynamics are more complex, but the resulting radiation is expected to be much smaller.

Note that the frequency range of the proposed LISA space-based interferometer, $10^{-5}$ to $10^{-1}$ Hz, is particularly well-suited to probing the electroweak phase transition at an energy scale of 100 GeV: $\beta/H_*$ is generically around 100, putting the frequency for the peak of the gravity wave power spectrum somewhere between $10^{-3}$ and $10^{-4}$ Hz. The strength of the electroweak phase transition is an open question; this issue is discussed below in the concluding section.
MODEL TURBULENCE

Substantial gravitational radiation can also be produced by a period of turbulence in the early Universe [10]. The following analysis [11] is completely general, without reference to a particular source of turbulence, although the most likely source is a first-order phase transition: once the bubbles of low-temperature phase begin to collide with each other, they stir up the primordial plasma on a characteristic length scale corresponding to the size of the largest bubbles in the transition. The gravitational radiation produced by the turbulence will be in addition to that from the bubbles given above.

We construct a simple model of isotropic cosmological turbulence as follows: at some particular temperature $T_*$ and corresponding enthalpy $w_* = \rho_* + P_*$ the turbulence commences, with energy input into the Universe on a largest length scale $L_S$. In general, the energy input will be a complicated process; we make the simple idealization that the turbulence will last for a time $\tau$ and that the turbulent energy will be distributed as a stationary Kolmogoroff spectrum with

$$E(k) \equiv \frac{1}{w} \frac{d\rho_{\text{turb}}}{dk} \simeq \varepsilon^{2/3} k^{-5/3},$$

where $\varepsilon$ is the energy density dissipation rate per unit enthalpy [12]. The turbulent energy injected at the scale $L_S$ will be written as $\varepsilon \rho_{\text{vac}}$, with $\varepsilon$ an efficiency factor. This energy cascades to smaller scales until it is dissipated by viscous damping at a scale determined by the kinematic viscosity $\nu$ of the fluid. The damping scale $L_D$ can be obtained from the Reynolds number, which can be approximated as

$$\text{Re} = \left( \frac{L_S}{L_D} \right)^{4/3} \simeq \frac{2}{3} \left( \frac{L_S}{2\pi} \right)^{4/3} \left( \frac{\varepsilon \rho_{\text{vac}}}{\nu^3 T} \right)^{1/3},$$

if the turbulent source lasts for a long time compared to the dynamical time scale of the turbulence on the scale $L_S$. The critical Reynolds number for the development of a Kolmogoroff spectrum of turbulence is around 2000; generally this number is easily exceeded for early-Universe phase transitions. If the source lasts for a time short compared to the dynamical time at the scale $L_S$ (as is likely with the electroweak phase transition [10]), then fully developed turbulence never appears, but we argue on physical grounds that, as far as gravitational wave production is concerned, this case can be treated like fully-developed turbulence which lasts for one dynamical time on the scale $L_S$ [11].

In analogy with the first-order phase transition calculation, the model turbulence is completely described by the physical quantities defined above: the characteristic time scale $\tau$, which we write as $\beta H_*^{-1}$; the energy density which goes into turbulent motions, $\varepsilon \rho_{\text{vac}}$; the characteristic length scale $L_S$, which we write as $\gamma H_*^{-1}$; the temperature of the phase transition $T_*$, which determines the enthalpy density $w_*$; and the kinematic viscosity $\nu$, which fixes the scale $L_D$ at which the turbulence is dissipated into heat.
Given this model, the turbulent stress-energy tensor can be constructed. The starting point is the Fourier-space expression

\[ T_{ij}(k) = \frac{w}{(2\pi)^3} \int dq u_i(q) u_j(k - q) \]  

where \( u \) is the relativistic velocity of the fluid at a given point and \( w \) is the fluid enthalpy, assumed to be independent of position. The source for gravitational radiation can be obtained via a projection tensor [13],

\[ \Pi_{ij}(k) = \left(P_{il} P_{jm} - \frac{1}{2} P_{ij} P_{lm}\right) T_{lm}(k) \]  

where \( P_{ij} \equiv \delta_{ij} - \hat{k}_i \hat{k}_j \) is a projector onto the transverse plane. The connection to isotropic turbulence is obtained via the two-point correlation function

\[ \langle u_i(k) u_j^*(k') \rangle = (2\pi)^3 P_{ij} P(k) \delta(k - k') \]  

which holds for any statistically isotropic and homogeneous velocity field. For Kolmogoroff turbulence, the power spectrum is

\[ P(k) \simeq \pi^2 \varepsilon^{2/3} k^{-11/3}. \]  

After a lengthy calculation which combines these pieces, the resulting gravitational radiation spectrum can be estimated as [11]

\[ h_c(f) \simeq 7 \times 10^{-14} \gamma^{5/3} \beta^{1/3} \left(\frac{\rho_{\text{vac}}}{\rho_\ast}\right)^{2/3} \left(\frac{100}{g_\ast}\right)^{1/3} \left(\frac{1 \text{ GeV}}{T_\ast}\right) \left(\frac{f_S}{f}\right)^{4/3}, \]  

with the lowest frequency \( f_S \) corresponding to the comoving scale \( L_S \),

\[ f_S = 1.6 \times 10^{-7} \text{ Hz} \gamma^{-1} \left(\frac{T_\ast}{1 \text{ GeV}}\right) \left(\frac{g_\ast}{100}\right)^{1/6}, \]  

using Eq. (4). The spectrum of the gravitational radiation extends to the highest frequency corresponding to the dissipation length scale. This can be approximated, but it suffices to note that the Reynolds number is large in the early Universe, so the range of frequencies will span at least a factor of several hundred. Turbulence can be just as efficient at producing gravitational radiation as the first-order phase transition that spawned it. In cases where the phase transition is only weakly first-order, the turbulence may be the dominant source.

Turbulence has the possibility of producing gravitational radiation via another mechanism. If any small seed magnetic field is present at the onset of turbulence, a turbulent dynamo will amplify the field exponentially until it saturates at equipartition with the turbulent kinetic energy. In Kolmogoroff turbulence, the dynamical time on a given scale is shorter at smaller scales, so the diffusion scale magnetic
fields will saturate first. Typically, the smallest scale dynamical time is at least 100 times shorter than the largest scale dynamical time, so the magnetic seed field on the smallest scales will be amplified by at least 100 e-foldings. We can crudely approximate the resulting magnetic field as isotropic with a power spectrum that has the Kolmogoroff form from the smallest scale up to some saturation scale, and then an exponential drop up to the largest scale.

The magnetic fields generated by turbulence are potentially interesting because they also generate gravitational radiation. Roughly, the equipartition magnetic fields are as efficient as the turbulence at making gravitational waves, with the crucial difference that the magnetic fields last much longer. The turbulence damps out on the turbulence time scale, while the magnetic fields remain until matter-radiation equality. The resulting gravitational radiation energy density can be approximated as that from the turbulence times an additional factor of $\ln(z_*/z_{\text{eq}})$, where $z_*$ is the redshift of the turbulence and $z_{\text{eq}}$ is the redshift of matter-radiation equality. For the electroweak phase transition, this factor is around 25, giving a large increase in the gravitational wave amplitude. This mechanism would yield a distinctive power spectrum of gravitational waves: it peaks at a frequency corresponding to the largest scale on which the dynamo saturates, drops off like a power law at smaller scales, and drops off exponentially at larger scales. The major question about this mechanism is whether an adequate seed field exists. The plasma in the very early Universe is strongly in the MHD regime (collision time short compared to dynamical time) and has a very high conductivity, making it difficult for any seed field generation mechanism to work. It has been suggested that the expanding bubbles in the electroweak phase transition will produce adequate seed fields [14], while thermal seed fields have also been advocated [15]. We are currently investigating gravitational radiation from turbulent-dynamo magnetic fields in detail.

**DETECTABILITY OF STOCHASTIC SOURCES**

Phase transitions are almost sure to have occurred in the early Universe; in particular, electroweak symmetry breaking at $T \approx 100$ GeV and the QCD phase transition at $T \approx 1$ GeV follow from our current understanding of the standard model of particle physics. However, only phase transitions which are first order are likely to have caused large enough stress-energy fluctuations to leave behind a significant gravitational wave background. In the standard model, both the electroweak and QCD phase transitions appear to be second-order. On the other hand, no high-energy theorist believes that the standard model is the final story. Once additional symmetries and fields are allowed, as with supersymmetry, the strength of the electroweak phase transition can be greatly enhanced. Belief in a strongly first-order electroweak phase transition is motivated by electroweak baryogenesis [16], which requires significant departure from thermodynamic equilibrium and thus a strongly first-order phase transition (but see [17]). Baryogenesis at the electroweak
energy scale is a natural possibility, since at higher energies, sphaleron processes which violate baryon number occur rapidly enough to erase any net baryon number. The very existence of matter in the Universe may be connected to a detectable gravitational wave background; we are currently studying this issue.

Inflationary gravitational radiation may be detectable with LISA, although it is difficult to come up with sensible models which give signals much above the anticipated LISA sensitivity [19]. The amplitude of the gravitational radiation at the present horizon scale is limited by the COBE measurement of large angular scale fluctuations in the microwave background temperature, and general considerations of slow-roll inflation argue that the power spectrum of gravitational radiation must drop as the scale decreases. Detection of such a background would be extremely important, because it would reflect the energy scale at which inflation occurred.

Cosmic defect sources, while well-motivated, offer less hopeful direct detection prospects. At this point, measurements of large-scale structure and the microwave background likely rule out any cosmic defects as the primary mechanism for structure formation in the Universe [20]. On the other hand, defects are a generic product of symmetry breaking. The gauge group of the standard model of particle physics is \( \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y \), which has a \( \text{U}(1) \) subgroup. A topological theorem states that the breaking of any larger group to a subgroup containing a \( \text{U}(1) \) factor will result in one-dimensional defects, i.e. cosmic strings. Thus the production of cosmic strings is a generic feature of any cosmology incorporating breaking of a grand-unified (GUT) symmetry to the standard model. (although if the GUT transition is inflationary, the number density of cosmic strings will be exponentially suppressed, making them completely irrelevant). Even if strings or other defects do not drive structure formation, they could conceivably produce detectable amounts of stochastic gravitational radiation [18].

Other cosmological sources of gravitational radiation, such as an early string phase of cosmic evolution [21] or complex field dynamics during reheating after inflation [22], offer detection possibilities, but these sources are based on more speculative underlying physics. Until further theoretical advances put them on more secure footing, they are perhaps best considered examples of the flavor of theories which might give an unanticipated signal.

In summary, potential cosmological sources of a stochastic background of gravitational radiation are more speculative than the compact binary point sources towards which LIGO is largely oriented. But several sources are well motivated, and the potential scientific payoff is spectacular. In particular, a direct probe of electroweak symmetry breaking with the LISA space-based interferometer is a distinct possibility, as is the direct detection of gravitational waves from inflation. All conjectured gravitational wave sources in the early Universe are intimately connected with fundamental physics at energy scales between 100 GeV and \( 10^{16} \) GeV. For all but the lowest portion of this range, gravitational radiation represents the only direct probe of physics available with currently envisioned technology. It also seems likely that stochastic backgrounds offer a greater chance at serendipitous discovery than point sources, since the evolution of the very early Universe is deter-
mined by physics which is partly unknown, while point sources are probably known astrophysical objects. We strongly encourage designers of gravitational radiation detectors to make stochastic background sensitivity a primary design consideration.

This work has been supported by NASA’s Astrophysics Theory Program. T.K. has been partially supported by the National Research Council’s COBASE program. A.K. is a Cotrell Scholar of the Research Corporation.

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