Covariant Matrix Model of Superparticle
in the Pure Spinor Formalism

Ichiro Oda

Edogawa University, 474 Komaki, Nagareyama City, Chiba 270-0198, JAPAN

Abstract

On the basis of the Berkovits pure spinor formalism of covariant quantization of supermembrane, we attempt to construct a M(atrix) theory which is covariant under $SO(1,10)$ Lorentz group. We first construct a bosonic M(atrix) theory by starting with the first-order formalism of bosonic membrane, which precisely gives us a bosonic sector of M(atrix) theory by BFSS. Next we generalize this method to the construction of M(atrix) theory of supermembranes. However, it seems to be difficult to obtain a covariant and supersymmetric M(atrix) theory from the Berkovits pure spinor formalism of supermembrane because of the matrix character of the BRST symmetry. Instead, in this paper, we construct a supersymmetric and covariant matrix model of 11D superparticle, which corresponds to a particle limit of covariant M(atrix) theory. By an explicit calculation, we show that the one-loop effective potential is trivial, thereby implying that this matrix model is a free theory at least at the one-loop level.

1E-mail address: ioda@edogawa-u.ac.jp
1 Introduction

Since the advent of M(atrix) theory by BFSS [1], there has been a strong desire to construct a manifestly Lorentz covariant M(atrix) theory, but no one has succeeded in constructing such a theory thus far.

Although M(atrix) theory has been derived from the low-energy effective action of D-particles which is obtained via the dimensional reduction from the maximally supersymmetric Yang-Mills theory in ten dimensions, this theory can be also interpreted as a regularized supermembrane theory in the light-cone gauge [2]. Then, it is natural to start with the supermembrane action in eleven dimensions and quantize it in a covariant manner in order to obtain the covariant M(atrix) theory. However, as is well known, the fermionic kappa symmetry, which is used to reduce the number of fermionic degrees of freedom by half, has given us a difficulty in covariant quantization of the supermembrane action.

Recently, there has been an interesting progress by Berkovits in the covariant quantization of the Green-Schwarz superstrings [3] using the pure spinors [4, 5, 6, 7, 8]. One of the key ingredients in the Berkovits approach is the existence of the BRST charge \( Q_{BRST} = \oint \lambda^\alpha d_\alpha \) where \( \lambda^\alpha \) are pure spinors satisfying the pure spinor equations \( \lambda^\alpha \Gamma^m_{\alpha\beta} \lambda^\beta = 0 \) and \( d_\alpha \approx 0 \) are the fermionic constraints associated with the kappa symmetry. It is remarkable that this approach provides us the same cohomology as the BRST charge of the Neveu-Schwarz-Ramond formalism [9] and the correct tree amplitudes of superstrings with keeping the Lorentz covariance of the theory. Afterwards, the Berkovits approach has been investigated from various different viewpoints [10, 11, 12, 13, 14, 15, 16]. In particular, more recently, the generalization of this approach to supermembrane has been done by Berkovits [17].

Combining the above-mentioned two observations, we are naturally led to think that we could make use of the Berkovits pure spinor formalism to construct a covariant M(atrix) theory since the covariant quantization of supermembrane has been made and the difficulty of the quantization associated with the kappa symmetry has been resolved in the pure spinor formalism. Actually, Berkovits has proposed such an interesting idea in the conference of Strings 2002 [18], but it is a pity that this work has not been completed so far as long as I know.

In this paper, we pursue this idea and attempt to construct a covariant M(atrix) theory by using the pure spinor formalism of supermembrane [17]. However, we will see that the construction of a covariant M(atrix) theory is rather difficult owing to the existence of the BRST invariance \( Q_{BRST} \) which is now promoted to a matrix symmetry (like a local gauge symmetry) in the matrix model. In this paper, we will explain in detail why it is difficult to apply the Berkovits formalism to the construction of the covariant M(atrix) model.

Thus, instead of constructing a covariant M(atrix) theory, we present how to construct a covariant matrix model of superparticle in eleven dimensions [19] which in some sense corresponds to a particle limit of a covariant M(atrix) theory.

This paper is organized as follows. In section 2, as a warmup, we construct a bosonic M(atrix) theory by starting with the first-order formalism of bosonic membrane. In section 3, we generalize the method to supermembrane and attempt to construct a covariant M(atrix)
theory from the pure spinor formalism of supermembrane by Berkovits. Here we find a difficulty of constructing M(atrix) theory which is invariant under the BRST symmetry. Hence, instead we turn to the construction of a covariant matrix model of superparticle invariant under both the supersymmetry and the BRST symmetry. Furthermore, in section 4, we calculate the one loop effective potential and show that our matrix model is a free theory owing to the lack of the potential term. The final section is devoted to the conclusion.

2 Bosonic M(atrix) theory

In this section, we shall construct a bosonic M(atrix) theory since this construction gives us a good exercise in attempting to construct a M(atrix) theory of supermembrane based on the pure spinor formalism. In addition, we can clearly understand the difference of the construction of a matrix model between the bosonic theory and the supersymmetric one. A similar analysis has been thus far done from various different contexts [20, 21, 22, 23, 24].

We begin with the well-known Nambu-Goto action of the bosonic membrane in eleven dimensions in a flat space-time:

\[ S_{NG} = -T \int d^3 \sigma \sqrt{-g}, \]  

where \( T \) is the membrane tension with dimension \((\text{mass})^3\). And the induced metric and its determinant are respectively given by \( g_{ij} = \partial_i x^a \partial_j x^b \eta_{ab} \), and \( g = \det g_{ij} \). We take the Minkowskian metric signature \((-;++;\cdots;+)\). Moreover, the indices indicate \( i, j = 0, 1, 2 \) and \( a, b, c = 1, 2, \cdots, 11 \). We follow the notations and conventions of the Berkovits’ paper [17].

Let us perform the canonical quantization of the action (1). The canonical conjugate momenta of \( x^a \) are derived as

\[ P_a = \frac{\partial S_{NG}}{\partial \dot{x}^a} = -T \sqrt{-g} g^{0j} \partial_j x^a = -T \sqrt{-g} (g^{00} \partial_0 x^a + g^{0I} \partial_I x^a), \]  

where we have defined as \( \dot{x}^a = \partial_0 x^a \) and \( I, J = 1, 2 \). From this expression, we have the primary constraints which generate the world-volume reparametrization invariance as follows:

\[ \mathcal{H}_0 = \frac{1}{2T} P_a P^a + \frac{T}{2} h \approx 0, \]
\[ \mathcal{H}_I = P_a \partial_I x^a \approx 0, \]  

where \( h_{IJ} = \partial_I x^a \partial_J x^b \eta_{ab} \) and \( h = \det h_{IJ} \). Given the Poisson brackets

\[ \{ P_a(\sigma^0, \sigma'), x^b(\sigma^0, \sigma') \} = -\delta_a^b \delta^2(\sigma - \sigma'), \]  

where \( \sigma^0 = \sigma^0 + \delta^0 \) and \( \sigma' = \sigma' + \delta' \).
it is easy to show that the constraints constitute of the first-class constraints as required:

\[
\{ \mathcal{H}_0(\sigma^0, \vec{\sigma}), \mathcal{H}_0(\sigma^0, \vec{\sigma}') \} = \left[ H_I(\vec{\sigma})h(\vec{\sigma})h^{IJ}(\vec{\sigma}') + H_I(\vec{\sigma}')h(\vec{\sigma})h^{IJ}(\vec{\sigma}) \right] \partial_J \delta(\vec{\sigma} - \vec{\sigma}'), \\
\{ \mathcal{H}_0(\sigma^0, \vec{\sigma}), H_I(\sigma^0, \vec{\sigma}') \} = \left[ \mathcal{H}_0(\vec{\sigma}) + \mathcal{H}_0(\vec{\sigma}') \right] \partial_I \delta(\vec{\sigma} - \vec{\sigma}'), \\
\{ H_I(\sigma^0, \vec{\sigma}), H_J(\sigma^0, \vec{\sigma}') \} = H_J(\vec{\sigma}) \partial_I \delta(\vec{\sigma} - \vec{\sigma}') + H_I(\vec{\sigma}') \partial_J \delta(\vec{\sigma} - \vec{\sigma}').
\]

(5)

Since the Hamiltonian vanishes weakly, we can introduce the extended Hamiltonian which is purely proportional to the constraints

\[
H = \int d^2 \vec{\sigma} \left[ e^0 \mathcal{H}_0 + e^I H_I \right] \\
= \int d^2 \vec{\sigma} \left[ e^0 \left( \frac{1}{2T} P_a P_a + \frac{T}{2} h \right) + e^I P_a \partial_I x^a \right],
\]

(6)

where \( e^0 \) and \( e^I \) are the Lagrange multiplier fields. Via the Legendre transformation, we can obtain the first-order action:

\[
S_0 = \int d^3 \sigma P_a \partial_0 x^a - \int d\sigma^0 H \\
= \int d^3 \sigma \left[ P_a \partial_0 x^a - e^0 \left( \frac{1}{2T} P_a P_a + \frac{T}{2} h \right) - e^I P_a \partial_I x^a \right].
\]

(7)

Note that this action is very similar to the bosonic part of the Berkovits action of supermembrane [17] in that both the actions are in the first-order Hamiltonian form and invariant under only the world-volume reparametrizations as local symmetries, so it is worthwhile to construct a bosonic matrix model from this action. Actually, we will see that the construction of M(atrix) theory follows a very similar path to the present bosonic formalism.

In order to construct a matrix model, we first perform the integration over \( P_a \) whose result is given by

\[
S_0 = \frac{T}{2} \int d^3 \sigma \left[ \frac{1}{e^0} (\partial_0 x^a - e^I \partial_I x^a)^2 - e^0 h \right] \\
= \frac{T}{2} \int d^3 \sigma \left[ \frac{1}{e^0} (\partial_0 x^a - e^I \partial_I x^a)^2 - \frac{1}{2} e^0 \{ x^a, x^b \}^2 \right],
\]

(8)

where in the second equation we have introduced the Lie bracket defined as

\[
\{ X, Y \} = e^{IJ} \partial_I X \partial_J Y.
\]

(9)

Here let us try to understand the geometrical meaning of the Lagrange multiplier fields, which can be done by comparing the above action with the Polyakov action (which is at least classically equivalent to the Nambu-Goto action (1))

\[
S_P = T \int d^3 \sigma \left( - \frac{1}{2} \sqrt{-g} g^{ij} \partial_i x^a \partial_j x^b \eta_{ab} + \frac{1}{2} \sqrt{-g} \right).
\]

(10)
Then we can express the metric tensor in terms of the Lagrange multiplier fields

\[
g_{ij} = \begin{pmatrix}
e^i e^j h_{IJ} - (e^0)^2 h & h_{IK} e^K \\
h_{IL} e^L & h_{IJ}
\end{pmatrix},
\]

\[
g^{ij} = \begin{pmatrix}
-\frac{(e^0)^2 h}{e^i} & \frac{e^j}{(e^0)^2 h} \\
\frac{e^i}{(e^0)^2 h} & h^{IJ} - \frac{e^i e^j}{(e^0)^2 h}\n\end{pmatrix} \tag{11}
\]

Next we will fix the reparametrization invariance by two gauge conditions \(2\). The first choice of the gauge conditions is given by

\[
e^0 = \frac{1}{\sqrt{h}}, \quad e^i = 0, \tag{12}\]

or equivalently, from (11),

\[
g_{ij} = \begin{pmatrix}
-1 & 0 \\
0 & h_{IJ}
\end{pmatrix} \tag{13}
\]

With the gauge conditions (12), the action (8) reduces to

\[
S_0 = \frac{T}{2} \int d\sigma^0 \int d^2 \sigma \sqrt{h} \left[ (\partial_0 x^a)^2 - \frac{1}{2h} \{x^a, x^b\}^2 \right]. \tag{14}\]

Finally, we make the following replacements

\[
\int d^2 \sigma \sqrt{h} \rightarrow Tr,
\]

\[
\frac{1}{\sqrt{h}} \{x^a, x^b\} \rightarrow i[x^a, x^b]. \tag{15}\]

Consequently, we arrive at a matrix model of the bosonic membrane

\[
S_0 = \int d\tau Tr \left\{ \frac{1}{2} (\partial_\tau x^a)^2 + \frac{1}{4} [x^a, x^b]^2 \right\}, \tag{16}\]

where we have set \(T = 1\) and \(\sigma^0 = \tau\). This matrix model describes a matrix model of the bosonic membrane.

We can also select another form of the gauge conditions \(e^0 = \frac{1}{\sqrt{h}}\) and \(e^i = \frac{1}{\sqrt{h}} e^I J \partial J A_0\). Then, the action (8) takes the form

\[
S_0 = \frac{T}{2} \int d\sigma^0 \int d^2 \sigma \sqrt{h} \left[ (\partial_0 x^a + \frac{1}{\sqrt{h}} \{A_0, x^a\})^2 - \frac{1}{2h} \{x^a, x^b\}^2 \right]. \tag{17}\]

\(^2\)For comparison with the case of supermembrane in the next section, we will not take the light-cone gauge explicitly in what follows.
With the replacements (15), we have a matrix model

$$S_0 = \int d\tau Tr\left\{ \frac{1}{2}(D_\tau x^a)^2 + \frac{1}{4}[x^a, x^b]^2 \right\}$$

(18)

where $D_\tau x^a = \partial_\tau x^a + i[A_\tau, x^a]$ and $A_\tau \equiv A_0$. This matrix model is obviously invariant under the $SU(N)$ gauge symmetry

$$x^a \rightarrow x'^a = U^{-1}x^aU,$$

$$A_\tau \rightarrow A'^\tau = U^{-1}A_\tau U - iU^{-1}\partial_\tau U.$$  (19)

With the gauge condition $A_\tau = 0$, this matrix model reduces to the previous matrix model (16). Note that if we selected the light-cone gauge, the matrix model (18) would become equivalent to the bosonic part of M(atrix) theory by BFSS except irrelevant dimensional factors and numerical constants [1] ³. In this way, we can obtain the bosonic M(atrix) theory by starting with the bosonic membrane action and utilizing the first-order Hamiltonian formalism.

### 3 A covariant matrix model of 11D superparticle

We now turn our attention to an attempt of the construction of a covariant M(atrix) theory of supermembrane in the pure spinor formalism and point out a difficulty of it. Then we construct a new matrix model of superparticle in the pure spinor formalism.

Before doing so, let us start by reviewing the pure spinor formalism of supermembrane [17]. From now on, we consider only the flat membrane such as toroidal membrane where the scalar density $\sqrt{h}$ can be set to unity.

The first-order Hamiltonian action of supermembrane reads

$$S = \int d^3\sigma\left[ P_i \Pi^i_0 + L_{WZ} + e^0 \left( P_c P^c + \det(\Pi^c_i \Pi^c_j) \right) + e^I P^i \Pi^i_I \right]$$

(20)

where $\Pi^c_i = \partial_i x^c + \frac{i}{2} \theta \Gamma^c \partial_i \theta$ and $L_{WZ}$ denotes the Wess-Zumino term whose concrete expression takes the form

$$L_{WZ} = \frac{i}{4} \varepsilon^{ijk} \theta \Gamma_{cd} \partial_i \theta \left( \Pi^c_j \Pi^d_k - \frac{i}{2} \Pi^c_j \theta \Gamma^d \partial_k \theta - \frac{1}{12} \theta \Gamma^c \partial_j \theta \theta \Gamma^d \partial_k \theta \right),$$

(21)

where we define as $\varepsilon_{012} = -\varepsilon^{012} = +1$ and $\varepsilon^{01J} = -\varepsilon^{IJ}$. This action is invariant under the kappa symmetry and the global space-time supersymmetry as well as the world-volume reparametrizations. The primary constraints consisting of 16 first-class and 16 second-class

³M(atrix) theory manifestly depends on the background flat metric, so it is not a background independent formalism. See [25] for the pioneering works of the background independent matrix models.
constraints appear when we evaluate the canonical conjugate momenta \( p_\alpha \) of the spinor fields \( \theta^\alpha \), which are given by

\[
d_\alpha \equiv p_\alpha - \frac{\partial R S}{\partial \dot{\theta}^\alpha} = p_\alpha - \frac{i}{2} P^c (\Gamma_c \theta)_\alpha + \frac{i}{4} \varepsilon^{IJ} (\Gamma_{cd} \theta)_\alpha \left( \Pi^d_I \Pi^d_J - \frac{i}{2} \Pi^d_I \theta^d \partial_d - \frac{1}{12} \theta^c \partial_t \theta^d \partial_d \theta \right) + \frac{1}{8} \varepsilon^{IJ} \Gamma_{cd} \partial_d \theta \left( \Pi^d_I - \frac{i}{6} \theta^d \partial_d \theta \right) (\Gamma^c \theta)_\alpha \approx 0,
\]

where the superscript \( R \) on \( \frac{\partial R S}{\partial \dot{\theta}^\alpha} \) denotes the right differentiation. These constraints satisfy the following Poisson bracket

\[
\{ d_\alpha (\sigma^0, \theta), d_\beta (\sigma^0, \theta') \} = \left[ -i P_c \Gamma_{\alpha \beta} + \frac{i}{2} \varepsilon^{IJ} \Pi_c \Pi_{Jd} \Gamma_{\alpha \beta} \right] \delta^2 (\theta - \theta').
\]

In deriving this equation, we need to use the eleven dimensional Fierz identity \( \Gamma_{(\alpha \beta} \Gamma^{\gamma \delta)} \eta_{bc} = 0 \) and the Poisson brackets

\[
\{ \tilde{P}_c (\sigma^0, \theta), x^d (\sigma^0, \theta') \} = -\delta_c^d \delta^2 (\theta - \theta'),
\]

\[
\{ p_\alpha (\sigma^0, \theta), \theta^\beta (\sigma^0, \theta') \} = \delta^\beta_\alpha \delta^2 (\theta - \theta'),
\]

where \( \tilde{P}_c \), the conjugate momenta of \( x^c \), are defined as

\[
\tilde{P}_c \equiv \frac{\partial S}{\partial \dot{x}^c} = P_c + \frac{i}{2} \varepsilon^{IJ} \theta \Gamma_{cd} \partial_d \theta \left( \Pi^d_I - \frac{i}{4} \theta^d \partial_d \theta \right).
\]

Since we cannot quantize the action (20) covariantly owing to the kappa symmetry, Berkovits has proposed a pure spinor action, which is of form

\[
S = \int d^9 \sigma \left[ P_c \Pi^c_c + L_{WZ} + d_\alpha \partial_\alpha \lambda^\alpha + w_\alpha \partial_\alpha \lambda^\alpha - \frac{1}{2} \left( P_c P^c + \text{det}(\Pi^c_I \Pi^c_J) \right) \right. \\
+ \left. (d \Gamma_c \partial_\alpha \theta) \Pi^c_c \varepsilon^{IJ} + (w \Gamma_c \partial_\alpha \lambda) \Pi^c_c \varepsilon^{IJ} - i \varepsilon^{IJ} (w \Gamma_c \partial_\alpha \partial_t \theta) (\lambda \Gamma^c \partial_d \theta) + i \varepsilon^{IJ} (w_\alpha \partial_\alpha \partial_t \theta^\beta) (\lambda \partial_d \theta^\beta) \right.
\]

\[
+ \left. e^I (P_c \Pi^c_I + d_\alpha \partial_\alpha \theta^\alpha + w_\alpha \partial_\alpha \lambda^\alpha) \right],
\]

where \( d_\alpha \) is defined as in (22). In this action, the kappa symmetry has been already gauge-fixed covariantly, whereas the shift symmetries of the world-volume reparametrizations are still remained. (The lapse symmetry is gauge-fixed to \( e^0 = -\frac{1}{2} \).) This action is invariant under the BRST transformation \( Q_B = \int d^9 \sigma \lambda^\alpha d_\alpha \). As a peculiar feature of supermembrane, additional constraints

\[
\lambda \Gamma^c \lambda = 0, \ (\lambda \Gamma^{cd} \lambda) \Pi^c_J = 0, \ \lambda_\alpha \partial_J \lambda^\alpha = 0
\]
are required to guarantee the BRST invariance of the action and the nilpotence of the BRST transformation. Note that the constraints (27) break the covariance on the world-volume explicitly.

Note that the bosonic part in the pure spinor action (26) of supermembrane shares the same form as in the bosonic membrane argued in the previous section, so as in the bosonic membrane, let us proceed to integrate over $P_c$ and choose the gauge conditions $e^I = -\varepsilon^{IJ} \partial_J A_0$.

As a result, the action (26) reduces to the form

$$S = \int d^3\sigma \left\{ \frac{1}{2} (D_0 x^c + \frac{i}{2} \theta \Gamma^c D_0 \theta)^2 + L_{WZ} + d_{\alpha} D_0 \theta^\alpha + w_{\alpha} D_0 \lambda^\alpha - \frac{1}{2} \det(\Pi^I_J \Pi_{Jc}) ight. \right.$$  

$$+ (d \Gamma_c \partial_I \theta) \Pi^c_J \varepsilon^{IJ} + (w \Gamma_c \partial_I \lambda) \Pi^c_J \varepsilon^{IJ} - i \varepsilon^{IJ} (w \Gamma_c \partial_I \theta) (\lambda \Gamma^c \partial_J \theta)$$  

$$+ i \varepsilon^{IJ} (w_{\alpha} \partial_I \theta^\alpha) (\lambda_{\beta} \partial_J \theta^\beta) \right\},$$  

where we have defined as $D_0 = \partial_0 - \varepsilon^{IJ} \partial_J A_0 \partial_I$.

Via the replacements (15) (recall that we have set $h = 1$) from the continuum theory to the matrix model, we obtain a covariant matrix model corresponding to the pure spinor action of supermembrane:

$$S = \int d\tau \text{Tr} \left\{ \frac{1}{2} (D_{\tau} x^c + \frac{i}{4} (\theta \Gamma^c D_\tau \theta - D_\tau \theta \Gamma^c \theta))^2 + d_{\alpha} D_\tau \theta^\alpha + w_{\alpha} D_\tau \lambda^\alpha 
$$

$$- (\Gamma_c d)_{\alpha} \left( i [x^c, \theta^\alpha] - \frac{1}{2} (\Gamma^c \theta)_{\beta} \{ \theta^\alpha, \theta^\beta \} \right) - (\Gamma_c w)_{\alpha} \left( i [x^c, \lambda^\alpha] + \frac{1}{2} (\Gamma^c \theta)_{\beta} [\lambda^\alpha, \theta^\beta] \right)$$

$$- (w_{\alpha} \lambda_{\beta} - (\Gamma_c w)_{\alpha} (\Gamma^c \lambda)_{\beta}) \{ \theta^\alpha, \theta^\beta \} + L_{WZ} + L_{\det \pi^2} \right\},$$  

where we have defined the covariant derivative as $D_{\tau} x^c = \partial_{\tau} x^c + i [A_{\tau}, x^c]$ as before, and the curly bracket $\{ , , \}$ denotes the anti-commutator whereas the square bracket $[ , , ]$ denotes the commutator. The last two terms $L_{WZ}$ and $L_{\det \pi^2}$ come from the Wess-Zumino term and $\frac{1}{2} \det(\Pi^I_J \Pi_{Jc})$, respectively, and involve the complicated expression. Note that in moving the continuum theory to the matrix theory we must pay attention to how to order various terms (in particular, in $L_{WZ}$ and $L_{\det \pi^2}$). Our guiding principle is to order the terms in order to keep symmetries of the theory as much as possible.

At this stage, compared with the bosonic membrane in the previous section, we further have to impose the requirements of the supersymmetry and the BRST invariance on the matrix model (29). First, let us consider the supersymmetry. This symmetry is a global symmetry, so the matrix extension can be given by

$$\delta x^c = \frac{i}{2} \theta \Gamma^c \epsilon, \quad \delta \theta^\alpha = \epsilon^\alpha, \quad \delta \lambda^\alpha = \epsilon^\alpha,$$  

where the parameter $\epsilon$ is not a matrix but a mere number. We have checked that under this supersymmetry the matrix model (29) is invariant except the Wess-Zumino term $L_{WZ}$. To

\footnote{Comparing (7) and (26), we notice that the definition of $e^0$ and $e^I$ in supermembrane differs from that in the bosonic membrane by the minus sign.}
Thus, using Eq. (15) the matrix extension must take the form of the BRST symmetry. The Berkovits’ BRST symmetry is simply given by

$$Q = \delta_{\alpha} T_{\alpha} + \frac{i}{4} (P.c(\Gamma^c \theta)_{\alpha} + (\Gamma^c \theta)_{\alpha} P.c).$$

(32)

Then we have the following BRST transformation

$$Q_B \theta^\alpha = \lambda^\alpha,$$

$$Q_B x^c = \frac{i}{4} (\theta \Gamma^c \lambda + \lambda \Gamma^c \theta),$$

$$Q_B d_{\alpha} = -\frac{i}{2} (P.c(\Gamma^c \lambda)_{\alpha} + (\Gamma^c \lambda)_{\alpha} P.c),$$

$$Q_B w_{\alpha} = d_{\alpha}.$$ (33)

Although this BRST symmetry has a rather simple form (essentially is of the same form as in superparticle), this symmetry constrains the form of the action severely since all the fields are now promoted to matrices. Actually we can check that the BRST-invariant matrix model must take the form

$$S = \int d\tau Tr \left\{ \frac{1}{2} \left( D_\tau x^c + \frac{i}{4} (\theta \Gamma^c D_\tau \theta - D_\tau \theta \Gamma^c \theta) \right)^2 + d_{\alpha} D_\tau \theta^\alpha + w_{\alpha} D_\tau \lambda^\alpha \right\}. \quad (34)$$

It is also worthwhile to notice that the BRST transformation is nilpotent up to the ‘gauge’ transformations

$$\delta_G d_{\alpha} = \frac{1}{4} [(D_\tau \theta \Gamma^c \lambda + \lambda \Gamma^c D_\tau \theta)(\Gamma^c \lambda)_{\alpha} + (\Gamma^c \lambda)_{\alpha} (D_\tau \theta \Gamma^c \lambda + \lambda \Gamma^c D_\tau \theta)],$$

$$\delta_G w_{\alpha} = -\frac{i}{2} [P.c(\Gamma^c \lambda)_{\alpha} + (\Gamma^c \lambda)_{\alpha} P.c], \quad (35)$$

where we have used the notation like $\langle \Gamma_{\alpha \beta} \rangle_{\alpha} \langle \Gamma_{\beta \gamma} \rangle_{\beta} \equiv \frac{1}{2} (\langle \Gamma_{\alpha \beta} \rangle_{\alpha} \langle \Gamma_{\beta \gamma} \rangle_{\beta} - \langle \Gamma_{\beta \gamma} \rangle_{\alpha} \langle \Gamma_{\alpha \beta} \rangle_{\beta}).$ The reason why the Wess-Zumino term is not invariant under the supersymmetry might be related to the fact that this term breaks the $SU(N)$ gauge symmetry since it includes not the covariant derivative $D_\tau$ but the ordinary derivative $\partial_\tau.$ Since the supersymmetry is an essential ingredient of our formalism, we stick to keep this symmetry and drop the Wess-Zumino term $L_{WZ}$ from the matrix theory.

Furthermore, in case of supermembrane in the pure spinor formalism, we must respect the BRST symmetry. The Berkovits’ BRST symmetry is simply given by $Q_B = \int d\sigma \lambda^\alpha d_{\alpha}.$ Thus, using Eq. (15) the matrix extension must take the form of $Q_B = Tr \lambda^\alpha d_{\alpha}.$ Since we have dropped the Wess-Zumino term, $d_{\alpha}$ is now simply given by

$$d_{\alpha} = p_{\alpha} - \frac{i}{4} (P.c(\Gamma^c \theta)_{\alpha} + (\Gamma^c \theta)_{\alpha} P.c).$$

(32)
which are indeed symmetry of the matrix model (34). Let us notice that this matrix model is nothing but the matrix model which can be obtained from the pure spinor formalism of the 11D superparticle [19] by generalizing all the local fields to matrices.

We shall finally make comments on some features of matrix model (34). First of all, this matrix model is not only invariant under the space-time supersymmetry and the Berkovits’ BRST transformation but also manifestly covariant under SO(1,10) Lorentz group, which is the most appealing point of the model at hand. However, the matrix model does not have the potential term given by $([x^a,x^b])^2$ (which exists in $L_{\text{det} \pi^2}$) as in the BFSS M(atrix) model so the physical properties of the both models are quite different as shown in the next section.

4 The one loop effective potential

In this section, we wish to clarify the physical properties of our new matrix model of 11D superparticle. It is well known that the superparticle action in the continuum theory [19] is the zero-slope limit of the superstring theory [3], so it might hopefully shed some light on the underlying structure of space-time. However, as shown below by evaluating the one-loop effective potential the matrix theory of superparticle in the pure spinor formalism is a free theory, so scattering amplitudes should be calculated by determining the vertex operators and inserting them in the path integral.

In order to evaluate the one-loop effective potential, we take the gauge condition $A_\tau = 0$ and introduce the FP ghosts $(\bar{C}, C)$.

After integrating over $A_\tau$, we obtain the gauge-fixed, BRST-invariant action

$$S = \int d\tau \text{Tr} \left\{ \frac{1}{2} \left( \partial_\tau x^c + \frac{i}{4} (\theta \Gamma^c \partial_\tau \theta - \partial_\tau \theta \Gamma^c \theta) \right)^2 + d_\alpha \partial_\tau \theta^\alpha + w_\alpha \partial_\tau \lambda^\alpha - \bar{C} \partial_\tau C \right\}. \quad (36)$$

As a background, we select a non-trivial classical solution

$$x^1_{(0)} = \frac{1}{2} \begin{pmatrix} v \tau & 0 \\ 0 & -v \tau \end{pmatrix}, \quad x^2_{(0)} = \frac{1}{2} \begin{pmatrix} b & 0 \\ 0 & -b \end{pmatrix}, \quad (37)$$

which describes two particles moving with velocities $v/2$ and $-v/2$ and separated by the distance $b$ along the $x^2$-th axis. Around this background, we expand $x^c$ by $x^c = x^c_{(0)} + y^c$ where the fluctuation $y^c$ takes the off-diagonal form

$$y^c = \begin{pmatrix} 0 & y^c \\ y^{c\dagger} & 0 \end{pmatrix}. \quad (38)$$

Similarly, $C, \bar{C}, w_\alpha, \lambda^\alpha, p_\alpha$ and $\theta^\alpha$ are expanded in the off-diagonal form like $y^c$. (For convenience, we have used the same letters as the original fields for expressing the off-diagonal matrix elements.)

\footnote{See [26, 27, 28] for calculations of the effective action in M(atrix) theory.}
After inserting these equations into the action (34) and taking the quadratic terms with respect to the fluctuations, we obtain the following action:

\[
S_2 = \int dt Tr \left( -y^i \partial^2 y^c + p_\alpha \partial_\tau \theta^\alpha + p^i_\alpha \partial_\tau \theta^\alpha + w_\alpha \partial_\tau \lambda^\alpha + w^i_\alpha \partial_\tau \lambda^\alpha \\
- \bar{C} \partial_\tau C - \bar{C} ^i \partial_\tau C \right). \tag{39}
\]

In deriving this quadratic action, we have used the fact that in the one-loop approximation, we can put \( P^c = \partial_\tau x^c(0) \). Then the partition function is given by

\[
Z = \int D\bar{X} e^{-S_2} = (\det \partial^2_\tau)^{-11}(\det \partial_\tau)^{-46}(\det \partial_\tau)^6(\det \partial_\tau)^2(\det \partial_\tau)^2 \\
= (\det \partial_\tau)^{-22-46+64+2+2} \\
= 1, \tag{40}
\]

where we have symbolically denoted the integration measure by \( D\bar{X} \) and taken account of the contribution from the missing ghosts \((b, c)\) [17] in the pure spinor formalism. The result shows that at least in the one-loop level the theory is trivial, in other words, two particles do not interact with each other. Recall that in M(atrix) theory by BFSS the similar calculation leads to the phase shift of D-particles in the eikonal approximation [1, 26, 27, 28]. Our matrix theory therefore seems to be a free theory owing to the lack of the potential term \(([x^a, x^b])^2\). Thus, in order to have non-trivial physical scattering amplitudes we must evaluate the expectation values of the vertex operators even in the matrix theory.

Finally, let us ask ourselves why we have obtained the matrix model (34) which is quite different from the BFSS matrix model. First, we should notice that the transformation law of the supersymmetry is completely different in both the formalisms. That is, our law (30) is purely from supermembrane whereas their law is from super Yang-Mills theory [1]

\[
\delta X^i = -2\epsilon^T \gamma^i \theta, \\
\delta \theta = \frac{1}{2} \left( D_i X^j \gamma_i + \gamma_- + \frac{1}{2}[X^i, X^j] \gamma_{ij} \right) \epsilon + \epsilon' \\
\delta A_0 = -2\epsilon^T \theta, \tag{41}
\]

where \( \epsilon \) and \( \epsilon' \) are two independent 16 component constant parameters. (Note that \( A_0 \) is needed to make the algebra of supersymmetry close.) Second, the Berkovits’ BRST invariance plays a role similar to a local symmetry in the matrix model, thereby strongly restricting the

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\(^{6}\)One subtle point of the above calculation is that we have taken the ‘axial’ gauge \( A_\tau = 0 \). It is known that the effective potential in general depends on the gauge conditions whereas the S-matrix does not depend on the gauge. To have the gauge-invariant effective action, we usually take the background field-dependent gauge as in [26, 27, 28], which guarantees the gauge invariance at all the stage of calculations. Our result obtained above, however, is manifestly gauge-invariant so it is free from the problem of the gauge dependence.
form of the action of the matrix model. In particular, the non-trivial potential \( ([x^a, x^b])^2 \) in M(atrix) theory, which is also present in the \( L_{\text{det} \pi^2} \) in Eq. (31), is not allowed to satisfy the matrix version of the Berkovits’ BRST symmetry. In any case, since the symmetries in both the present matrix model and the BFSS M(atrix) theory are different so that the two theories belong to different universality classes, it is natural to obtain the different theories in the both approaches.

5 Conclusion

In this article, we have investigated the possibility of making use of the Berkovits pure spinor formalism in order to make a Lorentz covariant M(atrix) theory. We have clarified that the naive expectation of it does not work well since symmetries in the Berkovits pure spinor formalism and the BFSS M(atrix) theory are different. Moreover, we have pointed out that the Berkovits’ BRST symmetry excludes the presence of the potential \( ([x^a, x^b])^2 \) which not only leads to an interesting interpretation of space-time relevant to the non-commutative geometry but also produces the non-trivial interaction of 11D supergravitons in M(atrix) theory. Instead, we have constructed a matrix model of 11D superparticle which is in a sense a particle limit of M(atrix) theory.

Obviously we have many remaining future works to be investigated. For instance, we have not constructed the vertex operators which should be also invariant under the Berkovits’ BRST transformation. Related to this work, there is a computation of scattering amplitude using superparticle in the continuum theory [29] where the amplitude leads to a divergent result and the coefficient is fixed by using the duality of superstring theory. We think that once the vertex operators are constructed in the present formalism, the scattering amplitude can be calculated and gives rise to a finite result. This study is under investigation and we wish to report the results in future publication.

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