Slow dynamos and decay of monopole plasma magnetic fields in the Early Universe

by

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Abstract

Previously Liao and Shuryak [Phys. Rev C (2008)] have investigated electrical flux tubes in monopole plasmas, where magnetic fields are non-solenoidal in quark-QCD plasmas. In this paper slow dynamos in diffusive plasma [Phys. Plasmas 15 (2008)] filaments (thin tubes) are obtained in the case of monopole plasmas. In the absence of diffusion the magnetic field decays in the Early Universe. The torsion is highly chaotic in dissipative large scale dynamos in the presence of magnetic monopoles. The magnetic field is given by the Heaviside step function in order to represent the non-uniform stretching of the dynamo filament. These results are obtained outside the junction condition. Stringent limits to the monopole flux were found by Lewis et al [Phys Rev D (2000)] by using the dispute between the dynamo action and monopole flux. Since magnetic monopoles flow dispute the dynamo action, it seems reasonable that their presence leads to a slow dynamo action in the best hypothesis or a decay of the magnetic field. Hindmarsh et al have computed the magnetic energy decay in the early universe as $E_M \approx t^{-0.5}$, while in our slow dynamo case linearization of the growth rate leads to a variation od magnetic energy of $\delta E_M \approx t$, due to the presence of magnetic monopoles. Da Rios equations of vortex filaments are used to place constraints on the geometry of monopole plasma filaments. PACS numbers: Plasma dynamos,47.65.Md, Magnetic monopoles 14.80.Hv.
I Introduction

As early as 1983 Arons and Blanford [1] have shown that dynamo growth of galactic field can still occur in the scales of $10^7 - 10^8 \text{yr}$ in the presence of a monopole halo. Resonant character of magnetic field damping by moving monopoles, allows the field to survive indefinitely when monopole plasma frequency is large enough. More recently Liao and Shuryak [2] have investigated electric flux tubes in monopole plasmas, in order to better understand the strong quark-gluon plasma (QCD). Earlier, Brandenberg and Zhang [3] have linked galactic dynamo mechanism to QCD scale, by using string (thin flux tubes) dynamics to explain galactic magnetic fields by dynamo action [4]. Earlier Lewis et al [5] have investigate this dispute between monopole dissipation and dynamo action in protogalactic ambient. They found more stringent limits to the monopole flux than the ones found by Parker. In this report it is shown that by considering a Riemannian model [6] of curved stretch-twist and fold [7] magnetic dynamo flux tube [8], the monopole current is created at the junction condition of the flux tube. These Riemannian tubes used here are similar to the Moebius strip, considered by Shukurov et al [9] to model experiments such as the Perm torus dynamo [10]. Earlier, Hindmarsh et al [11] were able to show that the magnetic field decay energy is given by $E_M \approx t^{-0.5}$ [12] in the early universe. The model we shall be discussing in this session is a slow dynamo model in the presence of dissipation given by the monopole flux. Here, we show that the magnetic energy decay is given by $E_M \approx t$, which is higher, due to the presence of the magnetic monopoles. This paper is organized as follow: In section 2 the dynamo filament model of non-uniform stretching as thin channels, for the monopole plasmas is presented and in section 3 the solution of self-induction equation is given and the physical properties of the dynamo action in monopole plasma universe is obtained. In this section, the Da Rios equation are shown to place constraints in the torsion and curvature of plasma monopole filaments. In Section 4, finally addresses conclusions.
II Monopole plasma dynamo flow

Let us start this section, by defining the main equations of the self-induction and the monopole ansatz magnetic field. By defining the magnetic field as

\[ B = B_0(t)H_0(s - s_0)t \]  

where \( H_0 \) is the Heaviside step function which physically means that along the filament coordinate \( s \), the junction condition establish the point where the magnetic field is turned on. The stationary flow speed is defined by

\[ v := v_0\delta(s - s_0)t \]

where \( v_0 \) is constant and \( \delta(s - s_0) \) is the delta Dirac distribution. When \( s_0 = nT \) where \( n \in \mathbb{Z} \) is an integer, this flow is a chaotic flow [13], with period \( T \). Here \( t \) is the tangent vector to the plasma filament, based on the Frenet frame \((t, n, b)\), which describes the time evolution. The self-induction equation

\[ \partial_t B = \nabla \times (v \times B) + \eta \nabla^2 B \]  

since both flow speed and magnetic field are along the magnetic flow as in the London equation \( J = \lambda B \) [14]. Magnetic monopoles are characterized by the violation of the solenoidal equation, which is replaced by the monopole equation

\[ \nabla \cdot B := \rho_m \]

where \( \rho_m \) is the magnetic monopole density. By Substitution of equation (II.4) into the equation (II.3) one obtains the scalar equation

\[ \partial_t \rho_m = \eta \nabla^2 \rho_m \]  

where the first term on the RHS of equation vanishes which shows by simple inspection that the monopole density decays in time, physically showing why the monopoles are so difficult to find in the present universe. Nevertheless as show by Arnon and Blanford,
monopole density can be very high in galactic scales where dynamo growth can recover this loss. The model we shall be discussing in this session is a slow dynamo model in the presence of dissipation given by the monopole flux. Now let us discussed the model based on plasma monopole filaments where the dynamo action, is given by stretching the filament, which in turn produces a non-uniform stretching which is fundamental for a strong magnetic field. The presence of magnetic monopole flux seems to explain the reason why a fast dynamo action does not take place and in its place a slow dynamo shows up. At the junction condition the plasma flow is compressible, in the sense that the divergence of the flow is

\[ \nabla \cdot \mathbf{v} = v_0 \frac{\delta(s - s_0)}{s - s_0} \]  

which diverges at the junction \( s = s_0 \), note however that, the integration of this equation yields

\[ \int (\nabla \cdot \mathbf{v}) dV = \frac{v_0 \pi r^2}{s - s_0} \]  

which shows that the integral does not necessarily diverge at the junction, since the radius of the filament \( r \to 0 \), and so is the magnetic field away from the junction condition, namely the magnetic monopoles are absent away from the junction condition of plasma filament. Note that off the junction this integral vanishes and the flow is incompressible away from the junction. Therefore there is a flow of monopoles at the junction. Monopole density as high as \( 10^{18}\frac{GeV}{c^2} \) in the galactic halo. In our case monopole density diverges and need to be normalized by a quantum theory. The density \( \rho_m \) is given by

\[ B_0 \delta(s - s_0) := \rho_m \]  

By assuming that \( B_0 = e^{\gamma t} \), where \( \gamma \) is the growth rate of the magnetic field, substitution of the magnetic field ansatz into the self-induction equation yields

\[ [\gamma + \delta(s - s_0)B - \kappa(s, t)B_0[\tau(s, t) + v_0\delta(s - s_0)]H_0 \mathbf{n} + B_0\kappa_\mathbf{s}H_0 \mathbf{b} = \eta[B_0\kappa_\mathbf{s}H_0 \mathbf{n} + 2B_0\kappa_\mathbf{n} + \mathbf{d}']\frac{1}{H_0}\mathbf{B}] \]  

where one has used the following Frenet generalized equations [14]

\[ \frac{dt}{dt} = -\kappa(s, t)\tau(s, t)\mathbf{n} + \kappa_\mathbf{s} \mathbf{b} \]  

(II.10)
\[
\frac{dt}{ds} = \kappa(s,t)n \\
\frac{dn}{ds} = -\kappa(s,t)n + \tau b
\] (II.11)

and

\[
\frac{db}{ds} = -\tau(s,t)n
\] (II.12)

Where here \(\kappa\) and \(\tau\) represents respectively Frenet curvature and torsion, of the plasma filaments. Expression (II.9) can be further simplified to

\[
\gamma_{\text{eff}} B = -\kappa(s,t)B[s \cdot \delta(s - s_0)]H_0 n + B_0 \kappa_s H_0 b = \eta[ B_0 (\kappa_s H_0 + 2\kappa \delta)] n + B_0 \kappa_s H_0 b
\] (II.14)

where the effective growth rate now is given by

\[
\gamma_{\text{eff}} = \gamma + \delta - \kappa^2
\] (II.15)

which shows that curvature of filaments appears non-linearly in the dynamo equation for the monopole dynamo filamentation. One notes therefore that the curvature of plasma filamentation contributes to weaken dynamo effects while the presence of Dirac delta function acts as to enhance the dynamo action.

### III Slow dynamos and constraints in plasma filamentation curvature and torsion

In this section we shall solve the self-induction equation and apply the results to found when dynamos or decay or magnetic fields takes place in the early universe under monopole structures. Separation of the equation (II.14) along the Frenet frame yields the following three equations

\[
\gamma = -\delta + \eta[\kappa^2 + \frac{\delta}{[s - s_0]H_0}] \\
\kappa \tau = \frac{\nu_0}{2} \delta(s - s_0)
\] (III.16)

and

\[
\kappa_s = \eta \tau \kappa
\] (III.18)
From the first equation (III.16) in this group it is clear that in the diffusion free limit 
\( \eta \to 0 \), the dynamo growth \( \gamma \) vanishes in the absence of monopoles, or away from the 
junction of plasma filaments, where the Dirac function vanishes upon integration. Thus 
this actually implies that in the absence of monopoles the dynamo is slow, while in the 
presence of monopoles the dynamo effect is absent in the diffusion or dissipative case, 
since then

\[
\gamma = -\delta(s - s_0) \tag{III.19}
\]

which represents a strong and very fast decay of monopole magnetic fields in the junction 
of plasma filaments. The remaining equations can be used along with the Da Rios vortex 
filaments equations

\[
\kappa_t = -2\kappa_s \tau - \tau_s \kappa \tag{III.20}
\]

and

\[
\tau_t = [-\frac{\kappa_{ss}}{\kappa} + \frac{\kappa^2}{2} - \tau^2]_s \tag{III.21}
\]

By assuming that the plasma filaments are stationary or \( \kappa_t = \tau_t = 0 \), yields

\[
\tau(s) = -\frac{1}{\eta s} \tag{III.22}
\]

which shows that the dissipation brought by monopole current yields a damping on the 
torsion of the filamentation process. The Frenet curvature is finally given by

\[
\kappa^2 = \frac{1}{4\eta s^2} \tag{III.23}
\]

The magnetic field is expressed as

\[
\mathbf{B} = e^{\gamma t}H_0(s - s_0)\mathbf{t} \tag{III.24}
\]

while the repeated process that leads to the dynamo chaotic structure can be obtained 
by simply substitution of \( s_0 = nT \). Computation of the magnetic energy integral

\[
E_M = \frac{1}{8\pi} \int \mathbf{B}^2 dV = \frac{1}{8\pi} \int e^{\gamma t}H_0(s - s_0)\mathbf{t} dV \tag{III.25}
\]

which yields \( \delta E_M \approx \eta \kappa^2 t \) obtained by linearizing the exponential stretching. By considering equations (III.17), (III.22) and (III.23) yields

\[
\frac{1}{4\eta^2 s^2} = \frac{v_0}{2}\delta(s - s_0) \tag{III.26}
\]
which by integration and applying the Dirac constraint
\[ \int \delta(s - s_0) ds = 1 \]  \hspace{1cm} (III.27)

at the junction condition \( s = s_0 \), one obtains
\[ \int \frac{1}{4\eta^2 s^2} ds = \frac{v_0}{2} \int \delta(s - s_0) ds \]  \hspace{1cm} (III.28)
or
\[ v_0 = -\frac{1}{4\eta^2 s_0} \]  \hspace{1cm} (III.29)

which completely determines the constant velocity of the plasma dynamo flow in terms of the dissipation. Since the diffusion coefficient \( \eta \) is inversely proportional to the magnetic Reynolds number \( Re_m \) one obtains finally that
\[ v_0 = -\frac{Re_m^2}{4s_0} \]  \hspace{1cm} (III.30)

which shows that the plasma filamentary flow is damped in small scale dynamos where \( R_m \) are small and is enhance for large scale astrophysical and cosmic dynamos. This is physically interesting since in astrophysical dynamo flows the speed of plasma monopoles should be faster than in non-monopole plasmas in laboratory.
IV Conclusions

Plasma filamentation which is used in this brief report in the context of the monopole plasma in cosmology, may also be used in tokamak and plasma instabilities [15]. Earlier Uby et al [?] have used also, Frenet frame plasma filamentation dynamics to investigate the effects on the superconductivity type II which makes explicit use of London equations. The results obtained here reinforce the existence of dynamo action versus monopole dissipation in the early universe, giving rise to an energy which is proportional to $\delta E_M \approx t$ where the proportionally coefficient is given by the product of dissipative coefficient $\eta$ and the curvature of plasma filaments.

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