Uplink Performance Analysis of User-Centric Small Cell Aided Dense HC Nets With Uplink-Downlink Decoupling

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ABSTRACT The deployment of low power small base stations (SBSs) in high density user areas enhances the performance of heterogeneous cellular networks (HC Nets). Such user-centric SBS deployment (UCSD) is more appropriate as compared with the conventional uniformly deployed user equipment (UE) based HC Nets, because it captures the correlation between UEs and SBSs. In the literature, the downlink (DL) performance of UCSD based HC Net (UC-HCNet) has been analyzed under maximum received signal power (MRSP) association strategy. In this paper, we develop and analyze the uplink (UL) performance of UC-HC Nets with decoupled UL-DL association (DUDA) using stochastic geometry. We evaluate the total UL interference considering two special cases of Poisson cluster process (PCP), i.e., Matern cluster process (MCP) and Thomas cluster process (TCP), in UC-HCNet. The proposed DUDA scheme enhances the system performance gain by assuming dual connectivity of UE and BSs in the UL and DL directions. Furthermore, fractional power control (FPC) policy is considered to limit the UL power consumption. The system performance is evaluated in terms of outage probability and rate coverage. The numerical results are validated through simulations. Numerical results show that the performance improvement of DUDA in sparse scenario is higher as compared with dense scenario. Furthermore, it is investigated that the performance gain of DUDA is higher for larger UE variance and cluster radius in case of MCP and TCP, respectively.

INDEX TERMS Heterogeneous cellular network, Matern cluster process, Poisson cluster process, stochastic geometry, uplink-downlink decoupling, user-centric small cell deployment.
cluster of UEs is served by low power SBS. As a result, the UCSD is considered as one of the dominant themes in future cellular architectures [6].

In denser HCnets, the difference between the transmit power levels of different tier BSs in the downlink (DL) transmission increases, whereas, the transmit power of UEs in the uplink (UL) is generally same which leads to DL-UL power level asymmetry [7]. The association in which a UE connects with same BS both in UL and DL, also termed as coupled UL-DL association (CUDA), is not assumed as an optimal association strategy. The decoupled UL-DL association (DUDA) is considered as more promising association approach compared with conventional CUDA [8]. In this paper, we focus on the performance analysis of the capacity driven user centric HCnets (UC-HCnets), while assuming the DUDA scheme.

A. RELATED WORK

By using tools from stochastic geometry, the performance of HCnets has been analyzed in the literature with uniform deployment of UEs and BSs [9]–[13]. The authors in [14], used the conditional thinning property of PPP to model and analyze the non-uniformity of UEs in the HCnets. In [8]–[13], the conventional HCNet with uniformly distributed UEs and BSs over a finite area is analyzed while assuming DL association strategy.

The UL performance of multi-tier HCNet is analyzed in [15] and [16], where the authors studied the impact of power control and association biasing, respectively. To minimize the cross-tier interference in HCNet, the authors in [17]–[19] proved through system level simulation that efficient power control strategy leads to improved system throughput. The authors in [20] analyzed the coverage probability and spectral efficiency of dense cellular network while assuming line-of-sight (LOS) and non-line-of-sight (NLOS) transmissions with CUDA scheme between UEs and BSs. The authors in [21] highlighted the key benefits of DUDA in HCnets through system level simulations. The CUDA and DUDA in K-tier HCnets are studied in [22], [23] and significant performance gain in UL transmission is observed. Rate analysis of DUDA along with association biasing is studied in [24]. Moreover, the authors in [8], [25], [26] analyzed coverage rate and spectral efficiency in HCNet and it has been shown that the UL performance of HCNet improves with DUDA. The performance of decoupling is analyzed in [27] for coexisted WiFi and LTE in unlicensed band. The authors observed that almost double sum throughput is achieved by DUDA compared to CUDA. All these works on DUDA assume uniform UE distribution throughout the network. However, the assumption of uniform UEs and BSs in HCnets does not reflect the real HCnets scenario.

In capacity driven UCSD, the SBSs are assumed to be deployed in the areas with crowded UEs to enhance the performance of HCNet. The authors in [28] studied a two-tier HCnet with multi-channel hybrid access, by modeling the locations of the clustered femto cell according to Neyman-Scott process while the locations of the UEs are modeled using PPPP. Using clustered stochastic geometry, the authors in [29] and [30] used PCP to model the locations of SBSs while UEs are distributed according to HPPP throughout the network. However, the correlation between the locations of UEs and BSs is not assumed. Considering the correlation between UEs and BSs, the authors in [31] modeled the clustered UEs according to Thomas cluster process (TCP) around the MBS in a single-tier network. The authors in [32] analyzed the DL coverage probability in HCNet by modeling high density of UEs through PCP while the SBSs are deployed at the cluster center. Such a model couples the UEs and BSs, as already used by the third generation partnership project (3GPP). The authors in [33], [34] proved that UE deployment according to PCP around uniformly distributed BSs closely resembles 3GPP model. Moreover, several unified models are developed and analyzed to minimize the gap between realistic UE and BS deployments and the conventional PPP based HCNet.

The DL coverage probability in HCNet with non-uniformly deployed UEs and uniformly deployed BSs is analyzed in [35]. The authors in [36] developed HCnet model with UCSD by modeling the clustered UEs around SBS using Matern cluster process (MCP) and TCP. The authors analyzed DL coverage probability and proved that by considering the correlation between UE and BS, the performance of the network is improved, which reflects the realistic 3GPP inspired HCnets model. Moreover, the authors showed that the performance of a capacity driven UC-HCNet surpasses the performance of conventional HCnets. The authors in [37] extended the model in [36] for DL millimeter waves HCNet while assuming LOS and NLOS transmissions. In [38], the authors modeled the UE clusters and BSs according to PCP, however, the crowded UEs and BSs follow TCP around common cluster center. The numerical analysis showed far better performance of UC-HCnets than the uniformly deployed UEs and BSs with no correlation between them. The authors in [39], [40] extended the model in [38] and analyzed the impact of fading environment and load balancing.

All these prior works on capacity driven UC-HCNet analyzed DL performance with CUDA between UEs and BSs. To the best of our knowledge the work in [41] analyzed the UL performance of HCNet with conventional decoupling while considering MCP distributed UEs, while ignoring the significance of cluster center SBS and power control. In this paper, we analyze the performance of UC-HCnets with realistic DUDA along with fractional power control (FPC).

B. APPROACH AND CONTRIBUTIONS

The main focus of this research work is to investigate the performance of DUDA in capacity driven UC-HCNet along with FPC. To enable capacity driven SBS deployment, the UEs are assumed to be clustered around low power SBSs and are served by different BSs in UL and in DL. The main contributions of this paper are highlighted as follow.
1) We derive expressions for the UL outage probability in UC-HCNet while assuming user distributions according to MCP and TCP.

2) We present the impact of DUDA on the performance of UC-HCNet. The UL power control is taken into account to derive accurate expressions for UL outage probability and rate coverage. The UL and DL association probability and interference expressions are presented. Furthermore the performance gain of DUDA over CUDA is highlighted for MCP and TCP.

3) Our results show that the UL performance of UC-HCNet improves with DUDA. Furthermore, large number of UEs are associated with SBSs in the UL while assuming DUDA. The performance gain of DUDA in UC-HCNet is higher in case of larger cluster while negligible performance gain is observed for closely packed clusters. Unlike the conventional HCNet, the performance gain of DUDA in UC-HCNet for sparse scenario is higher as compared with the dense scenario.

The remainder of this paper is organized as follows. The system model comprising SBS and UE distributions is discussed in Section II. Analyses of outage probability are carried out in Sections III and IV, respectively. Numerical results are discussed in Section V and the paper is concluded in Section VI. The notations used in the paper are summarized in Table 1.

### TABLE 1. Notation summary.

| Notation   | Description                                                                 |
|------------|-----------------------------------------------------------------------------|
| 0th-tier   | Consists of single BS located at representative cluster                      |
| \( K_k \)  | Number of BS tiers excluding 0th tier SBS                                    |
| \( N_{BS} \) | Number of BS tiers including 0th tier SBS                                    |
| \( \Phi_k \) | Distribution process of kth tier BSs                                        |
| \( \lambda_k \) | Density of kth tier BSs                                                      |
| \( \Phi_U \) | Distribution process of UEs                                                  |
| \( \lambda_U \) | Density of UEs                                                              |
| \( n_k \) | Number of UEs per cluster                                                   |
| \( \eta_k \) | Number of active UEs per cluster                                            |
| \( R \) | Cluster radius in MCP                                                        |
| \( \sigma^2 \) | UE variance in TCP                                                          |
| \( \tau_B \) | SINR threshold in B link direction                                           |
| \( B_k \) | UL, DL                                                                      |
| \( h_k \) | Channel fading gain                                                          |
| \( P_{DL,k} \) | DL transmit power of kth tier                                                 |
| \( P_{UL,k} \) | UL transmit power of UEs                                                     |
| \( P_{UL} \) | Initial uplink transmit power                                                 |
| \( B_k \) | Biasing power of kth tier                                                    |
| \( y_0 \) | Location of 0th tier BS                                                      |
| \( y_k \) | Location of kth tier BS                                                      |
| \( \rho \) | Fractional power control factor                                              |
| \( N_{path,noise} \) | System added noise                                                          |
| \( \alpha_k \) | Path loss exponent                                                          |
| \( \bar{A}_{R,k} \) | Association probability of kth tier in B link                               |
| \( \bar{C}_{R,k} \) | Outage probability of kth tier in B link                                     |
| \( T_{DL,k} \) | Interference from kth tier in open access BSs in downlink                   |
| \( T_{DL,0} \) | Interference from open access BSs located at the cluster center             |
| \( T_{DL,k} \) | Interference from kth tier in closed access BSs in downlink                 |
| \( T_{UL,k} \) | Inter-cluster interference                                                  |
| \( T_{UL,0} \) | Intra-cluster interference                                                  |
| \( \nu \) | Percentage of decoupled UEs                                                 |
| \( \nu_{CUDA} \) | Represents CUDA, DUDA schemes                                                |

### II. SYSTEM MODEL

A two-tier UC-HCNet model composed of MBSs and SBSs is considered, where the MBS coverage area is overlaid by low power SBSs as shown in Fig. 1. We have assumed that the location of the cluster center is HPPP distributed and the SBSs are assumed to be deployed at the center of high density UE areas or UE clusters. Therefore, SBSs act as the parent points of the clusters. The locations of the deployed BSs in each tier are represented by tier indices \( K = \{1, 2\} \) and are modeled according to independent HPPPs, \( \Phi_k^B \) and \( \Phi_k^B \), respectively.

The deployment density of kth-tier HPPP is denoted by \( \lambda_k^B \), \( \forall k \in K \). The BSs in each tier are assumed to have the same signal-to-interference-plus-noise ratio (SINR) but differ in terms of transmit power. We further assume that the BSs in each tier are the superposition of open access and closed access BSs represented by two sub HPPP \( \Phi_k^{(B,o)} \) and \( \Phi_k^{(B,c)} \) with deployment densities \( \lambda_k^B \) and \( \lambda_k^C \), respectively, such that the total BS density in the network is \( \lambda_k^B + \lambda_k^C \). The transmit power level of kth-tier BS in the DL is denoted by \( P_{DL,k} \), while based on FPC strategy for UL transmit power compensation, the UL power is given as \( P_{UL,k}^\psi = P_{UL,k}^\psi \gamma^\alpha \), where \( \alpha \) denotes the path loss, \( \epsilon \) is the power control fraction which can take value between 0 and 1, and \( y \) is the distance between UE and its associated BS.

### A. UE DISTRIBUTION

We focus on the UCSD in UC-HCNet which assumes correlation between UEs and BSs with DUDA. In UCSD, it is more likely that the UE is located in the closed proximity of BSs. The SBSs are deployed in the area with high UE density. This is more suitable for HCNet in terms of performance improvement and quality-of-service (QoS). The high density UE area is considered as UE cluster or hotspot. The locations of the clustered UEs are modeled according to Poisson cluster.
process (PCP), $\Phi_U$, with density $\lambda_U$. The HPPP distributed SBSs are considered as a base for the PCP which are referred to as the parent points while the UEs are assumed to be the daughter points scattered around the parent points. In our proposed system model, the parent points are represented by $\Phi^{B,0}_U$ process. The daughter points, also termed as cluster members, are assumed to be independent of the parent points. Let $\bar{n}$ be the mean number of UEs per cluster then the UE density throughout the network is $\bar{n}\lambda_2$. The locations of UEs in the cluster can be either modeled according to TCP or MCP. The difference between the two processes is the UE distribution around an SBS within the cluster. That is, in TCP, UEs follow Gaussian distribution around the parent points with variance $\sigma^2$, while MCP considers uniformly distributed UEs around the parent point within circular area having radius $R$.

The complementary cumulative distribution function (CCDF) and probability density function (PDF) of UE being in TCP [36], are given, respectively, as

CCDF: $\tilde{F}_{y_0}(y_0) = \exp \left( -\frac{y_0^2}{2\sigma^2} \right), \quad y_0 \geq 0,$ \hspace{1cm} (1a)

PDF: $f_{y_0}(y_0) = \frac{y_0}{\sigma^2} \exp \left( -\frac{y_0^2}{2\sigma^2} \right), \quad y_0 \geq 0,$ \hspace{1cm} (1b)

where $\sigma^2$ is the variance and $y_0$ is the random variable representing the separation distance between UE and cluster center.

Similarly, for $\Phi_U$ in case of MCP [36], CCDF and PDF can be written, respectively, as

CCDF: $\tilde{F}_{y_0}(y_0) = 1 - \frac{y_0^2}{R^2}, \quad 0 < y_0 \leq R,$ \hspace{1cm} (2a)

PDF: $f_{y_0}(y_0) = \frac{2y_0}{R^2}, \quad 0 < y_0 \leq R,$ \hspace{1cm} (2b)

where $R$ represents the cluster radius.

Similar to [36], by splitting the SBS tier, we enrich the BS tiers with 0th tier consisting of SBS located at the cluster center. The set of SBSs located outside the clusters are treated as separate tier. The tier indices set can now be written as $K = \{0, K\}$. For the sake of simplicity, the analysis is performed for a randomly selected UE, called typical UE (TUE), from a randomly selected cluster in DL; and randomly selected BS, called typical BS (TBS), in the UL scenario. Based on Slivnyak Theorem [2], the location of the TUE is shifted to the origin without affecting the statistics of stationary HPPP.

Being the parent point of PCP, the locations of all the BSs outside the representative cluster follow HPPP, therefore the distance separation between UE and $k$-th tier BS is represented by random variable $\{Y_k\}_{k=1,2}$ [9], [10]. The CCDF and PDF of $Y_k \forall k \in \{1, 2\}$ can be written, respectively, as

$$f_{y_k}(y) = 2\pi \lambda_k y_k \exp \left( -\pi \lambda_k y_k^2 \right), \quad y_k \geq 0,$$ \hspace{1cm} (3a)

$$\tilde{F}_{y_k}(y) = \exp \left( -\pi \lambda_k y_k^2 \right), \quad y_k \geq 0.$$ \hspace{1cm} (3b)

B. UPLINK/DOWNLINK TRANSMIT POWER AND CHANNEL MODEL

In the DL scenario, we assume constant BS transmit power $P_{DL,k}^B$ in the $k$-th tier, such that MBS transmit power is higher than SBS transmit power. The DL received power at the TUE, from the $k$-th tier BS $\forall k \in \{0, 1, 2\}$, is given as $P_{DL,k}^B = P_{DL,k}^B h_k \|y_k\|^{-a_k}$ where $h_k$ denotes Rayleigh fading gain, $a_k$ is the path loss exponent, and $y_k$ is the distance separation between the TUE and $k$-th tier BS. The serving BS in DL is selected based on the maximum biased average received power (BARP) scheme. The location of the serving BS under BARP scheme can be written as

$$y_{DL,k}^* = \arg \max_{y_k \in \cup_{k \in K}} B_k P_{DL,k}^B h_k \|y_k\|^{-a_k}, \quad \forall k \in \{0, 1, 2\},$$ \hspace{1cm} (4)

where $B_k$ is the association bias of the $k$-th tier BS in the DL transmission.

In the UL transmission, FPC is considered where the dynamic UE transmit power associated with $k$-th tier BS is denoted by

$$P_{UL,k}^U(y_k) = P^U_k \|y_k\|^{\epsilon_k},$$

where $P^U_k$ is the constant transmit power of UE, $\epsilon_k \in [0, 1]$ is the power control factor for path loss compensation. The value of $\epsilon_k$ can be set to enhance network performance and decrease the overall interference power by limiting the UE transmit power when the serving BS is located in the close proximity. The minimum value of power control factor, i.e., $\epsilon_k = 0$, refers to no path loss compensation or static power while $\epsilon_k = 1$ refers to full path loss compensation.

Based on BARP, the location of the BS associated with the TUE in UL is given as

$$y_{UL,k}^* = \arg \max_{y_k \in \cup_{k \in K}} B_k^* P_{UL,k}^B h_k \|y_k\|^{\epsilon_k a_k - a_k}, \quad \forall k \in \{0, 1, 2\}.$$ \hspace{1cm} (5)

The serving distances $y_{DL,k}^*$ and $y_{UL,k}^*$ are equal in CUDA and different in DUDA.

The received DL and UL SINR given that the TUE is connected with $k$-th tier BS can be expressed, respectively, as

$$\text{SINR}_{DL}(\|y_{DL,k}^*\|) = \frac{B_k P_{DL,k}^B h_k \|y_{DL,k}^*\|^{-a_k}}{N_k + I_{DL,k}},$$ \hspace{1cm} (6)

$$\text{SINR}_{UL}(\|y_{UL,k}^*\|) = \frac{B_k^* P_{UL,k}^B h_k \|y_{UL,k}^*\|^{\epsilon_k a_k - a_k}}{N_k + I_{UL,k}}.$$ \hspace{1cm} (7)

where $I_{DL,k}$ and $I_{UL,k}$ denote the interference power levels in the UL and DL directions, respectively, and $N_k$ is the system thermal noise. As $\Phi_U$ is the superposition of open access and closed access BSs so the total DL interference can be interpreted as the sum of interference power from open access and closed access BSs. Similarly, in the UL scenario, the total
interference is the sum of interference power received at the TBS from the intra-cluster and inter-cluster UEs. We interpret the intra-cluster UEs as the set of UEs located inside the representative cluster and inter-cluster UEs as the set of UEs located outside the representative cluster. Based on the association scheme, let a random variable $Y_j$ represent the serving distance between the TUE connected with the BS from $\Phi_k$ in the DL and TBS connected with UE from $\Phi_U$ in the UL. The generic expression for total interference power from all the BSs $\in \Phi_k$ and UEs $\in \Phi_U$ in $B$ link directions, $\forall B \in \{DL, UL\}$, can be written as (8), shown at the bottom of the page. In (8), $y_0$ is the distance between UE in the representative cluster and TBS located at the center of the cluster, and $y_k$ is the distance between TBS and UE located outside the representative cluster.

III. CELL ASSOCIATION PROBABILITY AND DISTRIBUTION OF SERVING DISTANCE

This section focuses on the derivation of cell association probability and distribution of the serving distance between UE and BS in UCSD based HCNet model presented in Section II. The association probability and distribution of serving distances are presented in Subsections II-A and II-B, respectively. These are later used in the derivation of outage probability and rate coverage.

A. CELL ASSOCIATION PROBABILITY

The cell association probability can be interpreted as the success probability of UE association with open access BS and vice versa in the DL and UL, respectively. In the conventional HCNet setup, a UE associates with a BS from the $k$th tier in UL as well as in DL, based on the strongest received signal power. Each UE association is also referred as CUDA strategy and mostly depends upon $k$th-tier BS density. In UCSD, the SBSs are assumed to be located at the cluster center so the distances between the UE and SBSs are relatively small compared with distance between UE and MBS. Hence, the chance that the UE associates with MBS in UL direction is less. In such case, when the UE associates with BS from different tiers in the UL and DL, it is dual connectivity or DUDA association as shown in Fig. 2.

For the derivation of association probability, we define an association event $N_{B,k} = n$ as the TUE association with the $k$th-tier BS in the $B$ link direction $\forall B \in \{DL, UL\}$, and can be written as

$$\begin{align*}
N_{B,k} = n
\end{align*}$$

be written as

$$\begin{align*}
& (N_{B,k} = n) \\
& = \begin{cases} \\
B_k P_{l,B,k}^{j} Y_k^{\alpha_k} > \bigcup_{j \in K \setminus k} B_j P_{l,B,j}^{j} Y_j^{\alpha_j}, & \text{for } B = DL, \\
B_k' P_{u,B,k}^{j} Y_k^{\alpha_k} > \bigcup_{j \in K \setminus k} B_j' P_{u,B,j}^{j} Y_j^{\alpha_j}, & \text{for } B = UL.
\end{cases}
\end{align*}$$

(9)

It is worth mentioning that the the 0th tier consists of a single SBS located at the cluster center while all other SBSs located in the network are collectively referred to as Tier 2. The association probability of the TUE in the $B$ link is given in Lemma 1.

Lemma 1: The association probability of the TUE such that it associates with $k$-th-tier BS in $B$ link, $A_{B,k}$, is given as

$$\begin{align*}
& A_{B,k} \\
& = \begin{cases} \\
\int f_{Y_0}(y_0) \prod_{j=0}^{K} F_{Y_j} \left[ \Psi_{B,k,j}(y) \right] dy, & \text{if } k = 0, \\
2\pi \lambda_k \int \sum_{y_0 \in \Phi_k \setminus \Psi_{B,k,j}(y)} f_{Y_0}(y_0) \prod_{j=1}^{K} F_{Y_j} \left[ \Psi_{B,k,j}(y) \right] dy, & \text{if } k \in \{1, 2\},
\end{cases}
\end{align*}$$

(10)
where $\mathcal{B} \in \{\text{DL, UL}\}$. The PDF and CCDF in case of $k = 0$ are given in (1a), (1b) and (2a), (2b) for TCP and MCP, respectively. The $\Psi_{\text{DL},k,j}(y)$ and $\Psi_{\text{UL},k,j}(y)$, can be written, respectively, as

$$
\Psi_{\text{DL},k,j}(y) = \left( \frac{B_1 P_{\text{DL},j}}{B_k P_{\text{DL},k}} \right)^{\frac{1}{k}} \frac{\alpha_k}{y^{\beta_k}}, \quad (11)
$$

$$
\Psi_{\text{UL},k,j}(y) = \left( \frac{B'_1 P_{\text{UL},j}}{B'_k P_{\text{UL},k}} \right)^{\frac{1}{k}} \frac{\alpha_k}{y^{\beta_k}}, \quad (12)
$$

Here, $B_k$ denotes association bias given that the UE associates with $k$-th tier BS. $\alpha_k$ is the path loss exponent and $\epsilon_k$ is the power control factor.

**Proof:** See Appendix A.

### B. DISTRIBUTION OF SERVING DISTANCE

Now we derive the expression for the distribution of the serving distance, $Y_{\mathcal{B},k}$, between the TUE and the $k$-th tier BS, when event $\mathcal{K}_{\mathcal{B},k} = k$ has occurred, where $\mathcal{B} \in \{\text{DL, UL}\}$. The serving distance is the distance between the TUE and the nearest BS in $\Phi_k \forall \mathcal{K} \in \{0, 1, 2\}$. Let $Y_{\mathcal{B},k}$ is a random variable representing the distance between the TUE and the nearest BS in $\Phi_k$, then the distribution of $Y_{\mathcal{B},k} = (Y_{\mathcal{B},k} | \mathcal{K}_{\mathcal{B},k} = k)$ in the UL and DL scenario is given in Lemma 2.

**Lemma 2:** The conditional PDF of the serving distance, $f_{\mathcal{Y}_{\mathcal{B},k}}(y)$, given that the TUE associates with $k$-th tier BS in $\mathcal{B}$ link $\forall \mathcal{B} \in \{\text{DL, UL}\}$, can be written as

$$
f_{\mathcal{Y}_{\mathcal{B},k}}(y) = \frac{1}{A_{\mathcal{B},0}} f_{\mathcal{Y}_{\mathcal{B}}}(y_0) \prod_{j=0 \backslash k}^{K} \bar{F}_{Y_j} \left[ \Psi_{\mathcal{B},0,j}(y) \right], \quad \text{if } k = 0,
$$

$$
f_{\mathcal{Y}_{\mathcal{B},k}}(y) = \frac{2\pi \lambda_k}{A_{\mathcal{B},k}} f_{\mathcal{Y}_{\mathcal{B}}}(y) \prod_{j=1 \backslash k}^{K} \bar{F}_{Y_j} \left[ \Psi_{\mathcal{B},k,j}(y) \right] dy, \quad \text{if } k \in \{1, 2\},
$$

where $\Psi_{\mathcal{B},k,j} \forall \mathcal{B} \in \{\text{DL, UL}\}$ is given in (11) and (12).

**Proof:** See Appendix B.

For a large cluster size, UEs located at the edge of cluster experience severe interference from MBS and SBSs located outside the cluster. In such case, it is more probable that UE connects with the BS located outside the cluster in the UL. This type of association is defined earlier as decoupled association. The total number of UEs in the network is the superposition of UEs associated via CUDA and DUDA. The UEs connected via CUDA are associated with the single BS in the UL and well as in DL, while the UEs connected via DUDA associate with BSs from different tiers in UL and DL.

The portion of the total UEs in the proposed network is either coupled or decoupled with the $k$-th tier BS in UL and DL. Therefore, the percentage of the UEs that are connected with $k$-th tier BS in the UL and $j$-th tier BS in the DL is presented in the next preposition.

**Proposition 1 (Percentage of DUDA-Associated UEs):** The percentage of total DUDA-associated UEs in the network can be written as

$$
N_{\text{D}}^{\mathcal{B}} = 1 - \sum_{k=K}^{\nu} \mathbb{P} \left( N_{\text{DL},k} = n, N_{\text{UL},k} = n \right).
$$

**Proof:** (a) follows due to the fact that the processes $\mathcal{K}_k$ are independent of each other, while Step (b) follows the expectation of probability distribution.

### IV. PERFORMANCE EVALUATION

This section focuses on the performance evaluation of UCSD based HCNet in terms of outage probability, rate coverage, and energy efficiency coverage. Using the association probability and PDF of the serving distance from Section III, we derive expressions for outage probability, rate coverage and energy efficiency in $\mathcal{B}$ link direction for the proposed UC-HCNet.
where $\gamma$ is the interference from the open access BS, closed access BS, and cluster center SBS are split into two cases for DL scenario and is given in (18), as shown at the top of the page, where the interference from the cluster center SBS is also included when $k = \{1, 2\}$. Outage probability for the UL scenario can be written as (19), shown at the top of the page.

Proof: See Appendix C

**B. LAPLACE TRANSFORM OF INTERFERENCE POWER**

We now focus on the derivation of Laplace transform of interference in B link association $\forall B \in \{\text{DL, UL}\}$. The Laplace transform of the interferences from open access BS, closed access BS, and cluster center SBS are the key requirements for the derivation of outage probability expression in (18).

**Lemma 3:** The Laplace transform of the interference from HPPP distributed open access and closed access BSs, $L_{\text{DL,k}}^{\text{oa}}(\cdot)$ and $L_{\text{DL,k}}^{\text{ca}}(\cdot)$, are given, respectively, as

$$L_{\text{DL,k}}^{\text{oa}}(\cdot) = \int_0^\infty \exp \left( -\frac{\tau_{\text{DL}} Y_{\text{DL}}^k}{P_{\text{DL,k}}} \right) L_{\text{DL,k}}^{\text{oa}} \left( \frac{\tau_{\text{DL}} Y_{\text{DL}}^k}{P_{\text{DL,k}}} \right) f_{Y_0}(y)dy,$$

for $k = \{1, 2\}$, (18)

$$L_{\text{DL,k}}^{\text{ca}}(\cdot) = \int_0^\infty \exp \left( -\frac{\tau_{\text{DL}} Y_{\text{DL}}^k}{P_{\text{DL,k}}} \right) L_{\text{DL,k}}^{\text{ca}} \left( \frac{\tau_{\text{DL}} Y_{\text{DL}}^k}{P_{\text{DL,k}}} \right) f_{Y_0}(y)dy,$$

and $\gamma_{\text{DL}}(\alpha, \tau_{\text{DL}}) = \frac{2m_2}{\alpha^2} \frac{2sF_1}{a} \left[ 1, 1 - \frac{2}{\alpha}; 2 - \frac{2}{\alpha}, -\tau_{\text{DL}} \right]$, where $F_1[\alpha, b, c, t] = \frac{\Gamma(c) - \frac{\Gamma(a)}{\Gamma(b - 1)} \int_0^t 1/(1-x)^{b-1} dx}{\Gamma(c)}$ represents hypergeometric function. The Laplace transform from the cluster center SBS can be upper bounded as

$$L_{\text{DL,k}}^{\text{ca}} \left( \frac{\tau_{\text{DL}} Y_{\text{DL}}^k}{P_{\text{DL,k}}} \right) \leq \frac{1}{1 + sP_i \nu_0 Y_0 - \nu_0 \bar{P}_i \nu_0},$$

where $\bar{P}_i$ represents the beta function and $\bar{n}_a$ is the number of active UEs per cluster.

Proof: See Appendix D

The interference in the UL when the UEs are distributed according to PCC, is the sum of interference from representative cluster UEs and the interference from UEs located outside representative cluster. The total interference in UL direction can be written as

$$L_{\text{UL,k}}^{\text{ca}} \left( \frac{\tau_{\text{UL}} Y_{\text{UL}}^k}{P_{\text{UL,k}}} \right) = L_{\text{inter}}^{\text{ca}} \left( \frac{\tau_{\text{UL}} Y_{\text{UL}}^k}{P_{\text{UL,k}}} \right),$$

Based on the assumption that the number of UEs located within the cluster share orthogonal resources, the intra-cluster interference diminishes and the total interference in the UL is caused by the inter-cluster UEs only. The inter-cluster interference is presented in Lemma 4.

**Lemma 4:** The laplace transform of inter-cluster interference in PCC can be written as

$$L_{\text{inter}}^{\text{ca}} \left( \frac{\tau_{\text{UL}} Y_{\text{UL}}^k}{P_{\text{UL,k}}} \right) \leq \exp \left( -\pi \lambda_m (sP_i)^2 \bar{n}_a B \left[ 1 - \frac{2}{\alpha}; \bar{n}_a + \frac{2}{\alpha} \right] \right),$$

where $B[\cdot]$ represents the beta function and $\bar{n}_a$ is the number of active UEs per cluster.

Proof: See Appendix E.
TABLE 2. Simulation parameters.

| Parameter | Value |
|-----------|-------|
| $\lambda_1$ | $2/\text{km}^2$ |
| $\lambda_2$ | $20\lambda_1$ |
| $P_{UL,1}^t$ | 53 dBm |
| $P_{UL,2}^t$ | 33 dBm |
| $P_{UL}^t$ | 23 dBm |
| $\gamma_1$ | 1.032 |
| $\bar{a}$ | 3.5 |
| $\bar{Y}$ | 10 |
| $B_1$ | 0 dB |
| $B_a$ | 10 |
| $\beta$ | 0.2 |
| $N_t$ | -174 dBm |

V. NUMERICAL RESULTS

In this section we present the numerical results of the proposed model for the association probability and outage probability. The simulation parameters and their values are presented in Table 2.

Fig. 3 presents the UL and DL outage performances of the proposed UC-HCNet. The outage probability versus SINR threshold for different values of cluster radius in case of MCP based UC-HCNet is shown in Fig. 3 (a). It can be observed that the DL and UL outage probabilities of MCP based UC-HCNet increase for clusters with large radius. Similarly the outage probability versus SINR threshold for different value of UE variance in case of TCP based UC-HCNet is presented in Fig. 3 (b). In case of TCP based UC-HCNet the cluster size is a function of UE variance and, hence, the outage probability for a fixed value of SINR threshold increases with increase in UE variance. This is because the distance between UEs and cluster center SBS increases with increase in the cluster radius and UE variance per cluster in case of MCP and TCP, respectively. In both cases, the DL performance is better than the UL performance due to lower transmit power in UL. Moreover, the Monte Carlo simulation results for both UL as well as DL are also plotted to further validate the accuracy of the proposed UC-HCNet model.

A. IMPACT OF DECOUPLING ON ASSOCIATION PROBABILITY

The association probability with cluster center radius and UE variance for MCP and TCP based UC-HCNet are shown in Fig. 4 (a) and Fig. 4 (b), respectively. The dashed lines represent the UL association probability and solid lines represent the DL association probability. The per tier UL association probability with cluster center SBS and the SBSs located outside the cluster is higher than DL due to DUDA and smaller interference received by the UEs. Similarly, the UL associations with MBS in both cases are almost zero because of the large transmission distance between UEs and MBS. As shown in Fig. 4 (a) and Fig. 4 (b), the UL association probability of UEs with cluster center SBS decreases while the UL association with SBS located outside representative cluster enhances with increase in cluster radius and UE variance, respectively. This is due to the fact that the UE distance from the cluster center SBS depends upon cluster radius and UE variance in case of MCP and TCP, respectively. For higher values of these cluster parameters, (i.e., cluster radius and UE variance) the distance between the cluster center SBS and UE is larger and hence the UE is closer to the serving SBSs located outside the cluster.

$$
L_{UL,k} (\frac{\tau_{UL,y_k^a_k}}{P_{UL,k}}) (c) \left[ \exp \left( -s \sum_{x \in \Phi_y \setminus \Phi_0} \sum_{y \in \chi^2} P_{UL,k}^t h_y \|x + y\|^{-\alpha_k} \right) \right]_{1 + \frac{\tau_{UL,y_k^a_k}}{P_{UL,k}}},
$$

$$
= \prod_{y \in \Phi_y \setminus \Phi_0} \left( \frac{1}{1 + s P_{UL,k}^t \|x + y\|^{-\alpha_k}} \right) \bar{Y} \bar{a},
$$

$$
= \exp \left( -\lambda_p \int_{\mathbb{R}^2} \left( 1 - \left( \frac{1}{1 + s P_{UL,k}^t \|x + y\|^{-\alpha_k}} \right) \bar{Y} \bar{a} \right) \ dy \right),
$$

$$
= \exp \left( -\lambda_p \int_{\mathbb{R}^2} \left( 1 - \left( \int_{\mathbb{R}^2} \left( \frac{1}{1 + s P_{UL,k}^t \|x + y\|^{-\alpha_k}} \right) f_Y(y) \ dy \right) \bar{Y} \bar{a} \right) \ dy \right),
$$

$$
= \exp \left( -\lambda_p \int_{\mathbb{R}^2} \left[ 1 - \left( \frac{1}{1 + s P_{UL,k}^t \|x + y\|^{-\alpha_k}} \right) \bar{Y} \bar{a} \right] f_Y(y) \ dy \right),
$$

$$
= \exp \left( -\pi \lambda_p (s P_{UL,k}^t)^\frac{\alpha_k}{2} \bar{Y} \bar{a} \left( 1 - \frac{2}{\alpha_k} \bar{Y} \bar{a} + \frac{2}{\alpha_k} \right) \right), \quad (25)
$$
FIGURE 3. UL and DL outage probabilities versus SINR threshold (dB): (a) for different values of $R$ in MCP based UC-HCNet, and (b) for different values of $\sigma$ in TCP based UC-HCNet with $\epsilon = 0$, $\lambda_1 = 1/km^2$, $\lambda_2 = 100/km^2$ and $B_k = 0$ dB.

FIGURE 4. UL and DL association probabilities (a) versus cluster radius in MCP based UC-HCNet, and (b) versus UE variance in TCP based UC-HCNet with $\epsilon = 0$, $\lambda_1 = 1/km^2$, $\lambda_2 = 100/km^2$ and fixed SINR threshold $\tau_{UL} = 0$ dB.

B. IMPACT OF SBS DENSITY ON ASSOCIATION PROBABILITY

In Fig. 5 (a) and Fig. 5 (b) the association probabilities with number of clusters per km$^2$ in case of MCP and TCP are presented, respectively. The association of cluster center SBS decreases and the UL association of UEs with SBS located outside the cluster center increases with increase in the number of clusters per km$^2$. Similarly, the UL association of MBS decreases with increasing number of clusters. Because for larger number of clusters, the UE density is high and the clusters are more compact and overlapping between the clusters increases. Hence, UEs are more likely to associate with SBSs located outside the representative cluster.

Fig. 5 (a) shows that the difference between the UL and DL association probabilities of cluster center SBS is higher for larger cluster radius due to DUDA and diminishes with increasing density of the clusters. For higher number of clusters, the UEs are more likely to experience CUDA due to cluster overlapping. Similarly, association of UEs with cluster center BS is higher while the association probability of SBS outside representative cluster and MBS is lower for smaller cluster radius. Similarly, Fig. 5 (b) shows the impact of cluster density on per tier association probability in case of MCP. The association probability of cluster center SBS and MBS decreases while association of SBSs located outside the cluster decreases with increase in the number of clusters.
FIGURE 5. UL and DL association versus number of clusters per km$^2$ in case of (a) TCP and (b) MCP, with $\epsilon = 0$, $\lambda_1 = 1$/km$^2$ and fixed SINR threshold $\tau_B = 0$ dB.

C. IMPACT OF DECOUPLING ON UL SINR OUTAGE PROBABILITY

The UL SINR outage probability for TCP based UC-HCNet for different values of UE variance in case of sparse and dense scenarios is presented in Fig. 6 (a) and Fig. 6 (b), respectively. Solid lines represent the UL SINR outage probability for DUDA and dashed lines represent SINR outage probability for CUDA. It is evident from the figures that the
UL performance for DUDA case is better when UE variance is larger and becomes approximately equal to that of CUDA for smaller UE variance. This is due to the fact that for smaller UE variance, UEs and SBSs are located closer to each other and, hence, the nearest BS in both DUDA and CUDA cases is the cluster center SBS. Due to smaller transmission distance between UEs and cluster center SBS in case of smaller cluster, UEs experience lower outage.

The UL SINR outage probability for MCP based UC-HCNet for different values of cluster radius in sparse and dense scenarios is shown in Fig. 6 (c) and Fig. 6 (d), respectively. Due to large transmission distances between UE and SBSs, higher performance gain is observed for larger cluster radius as compared with smaller cluster radius.

Moreover, in contrast to conventional HCNets, for both TCP and MCP based UC-HCNet, higher UL performance gain in sparse case is observed as compared with dense scenario.

**D. IMPACT OF SMALL CELL DENSITY ON UL SINR OUTAGE PROBABILITY**

The impact of number of clusters per simulation area on the UL outage probability for TCP and MCP based UC-HCNet is plotted in Fig. 7 (a) and Fig. 7 (b), respectively. Dashed lines show outage probability for CUDA and solid lines represent DUDA. The outage probability is less in case of DUDA as compared with CUDA. The difference between the outage
probability in case of DUDA and CUDA is interpreted as the decoupling gain. It can be seen that negligible decoupling gain is observed for smaller values of $\sigma$ and $R$ in case (a) and case (b), respectively. Similarly, for larger values of $\sigma$ and $R$, the decoupling gain is higher and becomes zero when the number of cluster exceeds 100.

Fig. 8 (a) and Fig. 8 (b) show the impact of active number of UEs per cluster for TCP and MCP based UC-HCNet, respectively. It can be observed from the figures that in both cases the UL outage probability enhances with increasing number of active UEs per cluster. A single active UE per cluster leads to better network performance but represents unrealistic scenario. Increasing number of active UEs per cluster results in severe UL interference and, hence, the outage probability increases.

VI. CONCLUSION

In this paper we have analyzed the performance of UC-HCNet and investigated the significance of DUDA over CUDA. UC-HCNet is modeled by assuming TCP and MCP based clustered UEs. The impact of cluster size and number of clusters on the association probability and outage probability are investigated for the proposed UC-HCNet model. The results show that the performance of DUDA is better than CUDA when the cluster size is larger, and a negligible performance gain is observed for the smaller cluster size. In contrast with the conventional HCNet model, the performance of UC-HCNet is better in case of sparse network scenario compared with the dense scenario. Moreover, the DUDA gain decreases with increase in the number of clusters throughout the network.

APPENDIX A

PROOF OF LEMMA 1

The TUE associates with $k$-th tier BS in DL, if the association event in (9) has occurred. Hence, the DL association probability of TUE with $k$-th tier BS can be written as

$$\mathcal{A}_{DL,k} = \mathbb{P} \left[ B_k P_{DL,k}^{(k)} \gamma_{DL,k} < \bigcup_{j \in K \setminus k} B_j P_{DL,j}^{(j)} \gamma_{DL,j} \right]$$

$$= \int_{0}^{\infty} \prod_{j \in K \setminus k} \mathbb{P} \left[ Y_j > \frac{B_j P_{DL,j}^{(j)} \gamma_{DL,j}}{B_k P_{DL,k}^{(k)} \gamma_{DL,k}} \right] f_{Y_k}(y) dy.$$

(26)

From (26), the association probability of TUE with 0th-tier (cluster center) SBS in $B$ link direction is deduced as

$$\mathcal{A}_{DL,0} = \int_{0}^{\infty} \prod_{j \in K \setminus 0} \tilde{F}_{Y_0} \left[ \Psi_{DL,j}(y) \right] f_{Y_0}(y_0) dy_0.$$

(27)

The association probability of TUE with $k$-tier BS, $\forall k \in K \setminus k$, located outside the representative cluster in $B$ link direction is given as

$$\mathcal{A}_{DL,k} = \int_{0}^{\infty} \prod_{j \in K \setminus k} \tilde{F}_{Y_j} \left[ \Psi_{DL,j}(y) \right] f_{Y_k}(y_k) dy_k.$$

$$= 2\pi \lambda_k \int_{0}^{\infty} \prod_{j \in K} \tilde{F}_{Y_j} \left[ \Psi_{DL,j}(y) \right] f_{Y_0}(y_0) dy_0.$$

(28)

Similarly, based on the association event in (12), the UL association probability of TUE with $k$-th tier BS can be written as

$$\mathcal{A}_{UL,k} = \mathbb{P} \left[ B_k P_{UL,k}^{(k)} \gamma_{UL,k} \right. $$

$$\left. \chi_{UL,k} > \bigcup_{j \in K \setminus k} B_j P_{UL,j}^{(j)} \gamma_{UL,j} \right]$$

$$= \int_{0}^{\infty} \prod_{j \in K \setminus k} \mathbb{P} \left[ Y_j > \frac{B_j P_{UL,j}^{(j)} \gamma_{UL,j}}{B_k P_{UL,k}^{(k)} \gamma_{UL,k}} \right] f_{Y_0}(y) dy.$$

(29)

The UL association probability of TUE with 0th-tier BS and $k$-th tier BS is obtained by replacing $\Psi_{DL,k}(y)$ with $\Psi_{UL,k}(y)$ in (27) and (28), respectively.

APPENDIX B

PROOF OF LEMMA 2

Using the conditional probability property, the PDF is obtained by taking derivative of conditional CCDF. The PDF of serving distance given that the TUE is associated with $k$-th tier BS in the $B$ link direction is given as

$$f_{Y_{B,k}}(y) = \frac{d}{dy} \left[ \mathbb{P} \left[ Y_k \leq y | N_{B,k} = n \right] \right]$$

$$= \frac{d}{dy} \left[ \mathbb{P} \left[ Y_k \leq y | N_{B,k} = n \right] \right]$$

$$= \frac{d}{dy} \left[ \frac{1}{A_{B,k}} \int_{0}^{\infty} \prod_{j \in K \setminus k} \tilde{F}_{Y_j} \left[ \Psi_{B,k,j}(y) \right] f_{Y_k}(y) dy \right].$$

(30)

From (30), the PDF of serving distance with 0th-tier (cluster center) SBS in the $B$ link direction can be written as

$$f_{Y_{B,0}}(y) = \frac{1}{A_{B,0}} \prod_{j \in K \setminus k} \tilde{F}_{Y_j} \left[ \Psi_{B,0,j}(y) \right] f_{Y_0}(y_0).$$

(31)
Similarly, the PDF of serving distance with k-tier BS, ∀
k ∈ \mathcal{K}_k, located out side the representative cluster, in B link direction ∀ B \in \{UL, DL\}, can be expressed as

\[ f_{N_{B,k}}(y) = \frac{1}{A_{B,k}} \prod_{j \in \mathcal{K}_k} \tilde{F}_{N_{B,j}}[\Psi_{B,0,j}(y)] \tilde{F}_{N_{B,k}}[\Psi_{B,k,j}(y)] f_{Y_k}(y_k). \]  

(32)

**APPENDIX C**

**PROOF OF THEOREM 1**

For the per tier outage probability conditioned that the TUE is associated with kth-tier BS in B link direction, (15) can be rewritten as

\[ O_{B,k}^v(\tau_B) = 1 - P\left( \text{SINR} \left( Y_{B,k} \right) > \tau_B | A_{B,k} \right). \]

By substituting (7), \( O_{B,k}^v(\tau_B) \), can be expressed as

\[ O_{B,k}^v(\tau_B) = 1 - P \left( \frac{B_k' P_{B_k} h_{y_k} \| Y_{B,k} \|^2_B}{A_{B,k}} > \tau_B \right) \]

\[ = 1 - P \left( h_{y_k} > \frac{\tau_B \left( N_{B,k} + I_{B,k} \right)}{B_k' P_{B_k} \| Y_{B,k} \|^2_B} \right) \]

\[ \overset{(l)}{=} 1 - E_{y_k^*} I_{B,k} \left( \exp \left( \frac{\tau_B \left( N_{B,k} + I_{B,k} \right)}{B_k' P_{B_k} \| Y_{B,k} \|^2_B} \right) \right) \]

\[ \overset{(m)}{=} 1 - \int_0^\infty \exp \left( - \frac{\tau_B N_{B,k} y_k^2}{P_{B,k}^2} \right) L_{I_{B,k}} \left( \frac{\tau B y_k^2}{P_{B,k}^2} \right) f_{Y_k}(y_k) \text{dy}. \]  

(33)

In (33), Step (l) follows from the assumption of Rayleigh fading channel and Step (m) is obtained from the definition of Laplace transform following by taking the expectation over random variable.

Putting the Laplace transform of UL interference from (23) in (33), we get

\[ O_{B,k}^v(\tau_B) = 1 - \int_0^\infty \exp \left( - \frac{\tau_{UL} N_{y_k^2} y_k^2}{P_{UL,k}^2} \right) L_{I_{UL,k}} \left( \frac{\tau_{UL} Y_k^2}{P_{UL,k}^2} \right) \]

\[ \times L_{I_{UL,k}} \left( \frac{\tau_{UL} Y_k^2}{P_{UL,k}^2} \right) f_{Y_k}(y_k) \text{dy}. \]  

(34)

The total outage probability in (17) is obtained by combining (34) and (16). After splitting for the 0-tier and k-th tier similar to (27) and (28), we obtain (19).

**APPENDIX D**

The proofs of (20) and (21) are well known results and follow the same derivation as given in [36, Lemma 3, and Lemma 5]. The Expression (22) is derived by ignoring the interference from cluster center SBS located farther from randomly selected UE compared with the serving kth-tier BS. The interference from the cluster center SBS is assumed to be negligible compared with the total interference in the network.

**APPENDIX E**

From (8), the UL interference from the inter-cluster UE is given as

\[ I_{UL,k} = \sum_{y_k \in \mathcal{B}_k} B_k' P_{UL} h_{y_k} \| y_k \|^{\kappa_{UL,k} - \alpha_k}. \]

From the definition of Laplace transform, following the same procedure as in [42], the Laplace transform of inter-cluster interference can be derived in (25), as shown at the bottom of the page 8, where Step (c) is derived by following the definition of Laplace transform, and Step (d) is obtained by applying Laplace transform on Rayleigh fading and exploiting the property of exponential function. Here, x being the distance between the clustered UE and the TBS located at the center of representative cluster, and Step (e) follow the probability generating functional (PGFL) of HPPP. Since the location of the cluster center follows HPPP, hence, (e) is obtained by using the PGFL of HPPP. Step (f) follows the expansion of expectation in Step (e). The upper bound on the inter-cluster interference in PCP is obtained in Step (g) by applying Jensen inequality stated as (\( E_{\mathcal{B}}[C^k] \)) ≤ \( E_{\mathcal{B}}[C^n] \), where \( \lambda_0 \) denotes the density of the parent points in PCP, located at the cluster center. Step (h) is obtained by exploiting beta function.

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