Selection of waterflooding systems for enhanced oil recovery by solving two-phase filtration problem

S P Rodionov\textsuperscript{1,2}, V P Kosyakov\textsuperscript{1,2} and E N Musakaev\textsuperscript{1,2}

\textsuperscript{1}UNI-CONCORD LTD, 625003, Tyumen, Russia
\textsuperscript{2}Tyumen Branch of the Khristianovich Institute of Theoretical and Applied Mechanics, Siberian Branch of RAS, 625026, Tyumen, Russia

E-mail: musakaev91@gmail.com

Abstract. On the basis of analytical solution of two-phase flow problem, an express method is proposed for calculation of advantageous system of oil reservoir development, function of production and injection wells under the given arrangement. The method allows real-time obtaining of one of advantageous variants of field development system with hundreds of wells. The computational efficiency of the proposed method is investigated on synthetic reservoir simulation models of a nine-spot development element.

Introduction

One of the most urgent tasks in oil reservoir management is to determine the most favorable development system at any time during the field's exploitation. Common waterflood-pattern configurations with symmetrically located injection and production wells (five-spot, in-line, seven-spot, nine-spot, etc.) designed for a homogeneous reservoir, when applied to a heterogeneous (real) reservoir may not be the best [1,2]. Due to the fact that information about the reservoir is specified as the oilfield is exploited, it becomes necessary to adjust the initial development system. The use of selective (non-regular) flooding systems is justified in heterogeneous formations, characterized by a high degree of zonal inhomogeneity [3].

The solution of optimization problems of oilfield development is usually made using the optimal control methods, based on the Lagrange-Pontryagin formalism and multiphase filtration equations [4-6]. To determine the favorable variants of the development system, in general, it is required to involve integer programming methods. The application of these methods leads to large computational costs, since the number of calculations rapidly increases with the increase in the number of variables (the number of wells). In particular, when solving the optimization problem of the development system including $N_w$ wells, by a brute force search method, the computational costs grow as $2^{N_w}$. Evolutionary methods in computational terms are also time-consuming.

In this paper, an effective method for calculating an optimal development system with a known well location is proposed, which allows real-time calculation. The speed of the method is due to the use of an analytical solution obtained by making some simplifying assumptions about the structure of the filtration flows in the formation. To study the computational efficiency of the proposed algorithm, synthetic reservoir simulation models of a nine-spot development element with various kinds of inhomogeneous inclusions on permeability were considered. The development systems obtained with
the help of the proposed algorithm were compared with the systems obtained as a result of a brute force search of all possible variants.

So now let us look more closely at the proposed method, which includes an analytical solution to the problem of oil displacement by water from a zone-inhomogeneous formation, the optimal control problem and the algorithm for its solution.

Analytical solution
In [7, 8], the analytical solution was obtained for the problem of the one-dimensional displacement of oil by water from a zone-inhomogeneous reservoir according to the Leibenson-Muskat scheme. In this case, liquids and rocks are considered incompressible. It is shown in [9] that the application of the obtained analytical solution for practical calculations on real reservoirs gives a positive result. A schematic representation of the problem is shown in Figure 1. Expressions for the full time of the oil displacement from a zone-based inhomogeneous reservoir for option 1 \((t_1)\), when the injection row of wells is located in Zone 1 and a production row - in zone 2, and for option 2 \((t_2)\), when there is a reverse order of placing the rows of wells, as follows [7, 8]:

\[
t_1 = \frac{\alpha_1}{\Delta p}, \quad t_2 = \frac{\alpha_2}{\Delta p},
\]

\[
\alpha_1 = \left(\frac{\mu_w V_1^2}{k_1} + \frac{\mu_o V_1^2}{k_2} + \frac{\mu_o V_1 V_2}{k_1} \right) + \left(\frac{\mu_w V_1 V_2}{k_1} + \frac{\mu_w V_2^2}{k_2} + \frac{\mu_o V_2^2}{k_2} \right),
\]

\[
\alpha_2 = \left(\frac{\mu_w V_2^2}{k_2} + \frac{\mu_o V_2^2}{k_1} + \frac{\mu_o V_1 V_2}{k_2} \right) + \left(\frac{\mu_w V_2 V_1}{k_2} + \frac{\mu_w V_1^2}{k_1} + \frac{\mu_o V_1^2}{k_1} \right),
\]

Here \(\mu_w\) and \(\mu_o\) are the viscosities of the oil and water, respectively; \(V_1\) and \(V_2\) are mobile pore volumes of zones; \(L_1\) and \(L_2\) are the lengths of zones; \(\omega_1\), \(\omega_2\) are the cross-sectional areas of zones; \(\phi_1\), \(\phi_2\) are the value of shares of the movable pore space of zones, \(p_{w,1}\), \(p_{w,2}\) are the pressures in the wells located in zones 1 and 2, respectively; \(\phi\) is the porosity. \(S_{wc}\) and \(S_{or}\) show critical saturations of water and oil, respectively.

![Figure 1](image-url)

**Figure 1.** Schematic representation of one-dimensional problem of oil displacement by water from a zone-heterogeneous reservoir with different placement of injection and production wells.
Forming drainage areas

The reservoir area is divided into cells—the control volumes. These control volumes are divided into internal and boundary ones. In the internal control volumes there are real wells, and in the boundary volumes—dummy wells that mimic the contour of the reservoir (figure 2). Thus, each well is only connected to the wells that are in opposite corners of the associated control volumes.

Let us consider two arbitrary adjacent wells with numbers $i$ and $j$. The control volume relating to these wells consists of triangular shaped areas 1 and 2 with appropriate volumes ($V_1$, $V_2$), thicknesses ($h_1$, $h_2$), and lengths ($L_1$, $L_2$). In that case, the length $L_1$ equals the distance between the $i$-th well and the border of the cell toward the $j$-th well, and $L_2$ is the distance between the $j$-th well and the border of the cell towards the $i$-th well.

![Figure 2. Diagram of bonds between the $i$-th well and adjacent wells. Control volumes within which the oil is displaced by water are highlighted with a filling.](image)

![Figure 3. Building the drainage volume (highlighted with the filling) between the wells with numbers $i$ and $j$.](image)

An algorithm for building the control volumes:

1. Build the wells bonds “each to every”
2. Introduce limitations on the distance between the wells and the angles at the vertices of which the wells are located.
3. The control volumes are built in a form of quadrilaterals in which the oil is displaced by water (figure 2).
4. In order to apply the analytical solution, each of these control volumes is presented in the form of effective rectangles.

For example, let us consider a rectangle CEAF (figure 3). It is built as follows. Straight lines CE and AE are built based on the condition that they form an angle defined by the relations:

$$L_{AD}^2 \phi_{DAE} = L_{DA}^2 \phi_{EAC}, \ \phi_{DAC} = \phi_{DAE} + \phi_{EDC}.$$

The constructed lines intersect at point $E$. Similarly we define point $F$. The points $E$ and $F$ are connected by segment $EF$.

The triangles $CEF$ and $EAF$ form Zones 1 and 2 to apply the analytical solution there to (1). These triangles are recalculated to effective rectangles as follows. If areas 1 and 2 were in the form of rectangles, then they would have a cross-sectional area defined by the following equations:

$$V_1 = \phi_1 h_1 \frac{1}{2} L_1 d = \phi_1 \omega_1 L_1, \quad V_2 = \phi_2 h_2 \frac{1}{2} L_2 d = \phi_2 \omega_2 L_2,$$
where \( d \) is the length of the common boundary between these adjacent \( i \)-th and \( j \)-th cells. Hence we have \( \omega_1 = \frac{h_1 d}{2} \), \( \omega_2 = \frac{h_2 d}{2} \).

We assume that the pressure in the internal control volumes equals the BHP in wells, and in the boundary cells—the pressure on the reservoir contour. Furthermore, it is assumed that the fluid flow in the \( ij \)-th control volume \( V_{ij}^0 = V_i + V_j \) does not depend on time.

Then the dependence of oil volume on the time remaining in the control volume at the time point \( t \) can be expressed as follows:

\[
V_j(t) = V_j^0 \left( 1 - \frac{t}{\tau_j} \right)^\eta \left( 1 - \frac{t}{\tau_j} \right) \quad (i \neq j, \; V_j^0 = V_j^0, \; V_j \neq V_j),
\]

where \( \eta(x) \) - a Heaviside unit function, \( V_j^0 \) - a movable pore volume of the cell at the initial point of time, and \( \tau \) can be equal to \( t_1 \) or \( t_2 \) (see (1)) depending on the flow direction. The direction of flow is determined by the sign of the difference of BHP values in wells. If the flow is directed from the \( i \)-th to the \( j \)-th well, then \( \tau_j = t_1 \), and when the flow is directed from well \( j \) to well \( i \), then \( \tau_j = t_2 \).

Initially, the formation can not be completely oil-saturated. If the initial oil saturation is \( S_o \), then for an approximate account of the factor of a partial filling of the reservoir in formulas (1) instead of \( (1 - S_{wc} - S_{or}) \) it is substituted \( (S_o - S_{or}) \).

**Optimal control problem**

The components of the control vector \( u \) are BHP on wells. The objective functional \( J = J(u) \) is the volume of oil remaining in the reservoir at time \( T \). Thus, the optimality criterion is:

\[
J(u) = V(u, T) \rightarrow \min ,
\]

where the volume \( V \) is equal to the sum of all volumes \( V_{ij} \)

\[
V(u, T) = \sum V_{ij} \quad (3)
\]

Dependency of \( V_{ij} \) from controls based on the flow direction under conditions

\[
V_{ij} \neq V_{ji}, \quad V_{ij}^0 = V_{ji}^0 \quad i \neq j,
\]

can be represented as follows:

\[
V_{ij} = V_{ij}(u_i, u_j, T) = V_{ij}^0 \left( 1 - \frac{T}{\tau_{ij}} \right)^\eta \left( 1 - \frac{T}{\tau_{ij}} \right),
\]

\[
\tau_{ij} = \alpha_{ij} / |u_i - u_j|, \quad \alpha_{ij} = \alpha_{1,ij} \eta(u_i - u_j) + \alpha_{2,ij} (1 - \eta(u_i - u_j)) ,
\]

where \( \alpha_i \) and \( \alpha_2 \) are determined individually for each of the \( ij \)-th control volume according to (1).

Limitations on the control parameters are:

\[
u_{\min,i} \leq u \leq u_{\max,i} \quad \forall \; i=1,2,...,N_w
\]

where \( u_{\max,i} \) corresponds to BHP at the \( i \)-th injection well, and \( u_{\min,i} \) at the production well. \( N_w \) denotes the number of wells.

When the control is changed at the \( i \)-th well from \( u = u_{\max,i} \) to \( u = u_{\min,i} \), the objective functional changes by the value:

\[
\Delta J_i = J(u_i, ..., u_i = u_{max,i}, ..., u_{N_w}) - J(u_i, ..., u_i = u_{min,i}, ..., u_{N_w}) \quad \forall \; i=1,2,...,N_w.
\]

(5)
The value $\Delta J_i$ can be calculated using (3) and (4). However, the calculations can be shortened if one observes that according to (4) control parameter $u_i$ enters the expression only for the volumes of cells with numbers $j$ adjacent to the $i$-th well. Substituting (3) into (4), one can see that the terms that do not relate to the control volumes adjacent to the $i$-th well are mutually reduced. Then the expression for the "increment of the objective functional" will take the following form:

$$
\Delta J_i = \sum_j (V_j(u_{\text{max}}, u_j, T) - V_j(u_{\text{min}}, u_j, T)) \quad \forall \; i=1,2,\ldots,N_w.
$$

(6)

For a three-dimensional formation, assuming a layered flow structure, the value $\Delta J_i$ will be determined by summing expressions (6) across all layers.

Express algorithm for calculating the waterflooding system

Using the expression for $\Delta J_i$, we have the following simple algorithm for calculating the selective waterflooding system, which is classified as "greedy":

**Step 1.** Set $u^0; \; \text{iter}=0; \; U(\text{iter})=u^0; \; T$ ;

**Step 2.** Calculate $\Delta J_i \forall i=1,2,\ldots,N_w \; ; \; \Delta J_k = \min_i \Delta J_i$ ;

**Step 3.** if $\Delta J_k > 0$ and $u_k = u_{\text{min},k}$ then $u_k = u_{\text{max},k}$ ;

**Step 4.** if $\Delta J_k < 0$ and $u_k = u_{\text{max},k}$ then $u_k = u_{\text{min},k}$ ;

**Step 5.** if $u^0 \not\in U$ then $U(\text{iter})=u^0; \; \text{iter}:=\text{iter}+1; \; \text{go to Step 2 else end.}$

The convergence of this algorithm is very high. Usually it converges for 4-5 iterations. Iteration is understood as a single execution of steps 2 and 3 by the algorithm. Due to the fact that $\Delta J_i$ is determined with the help of the analytical solution, it becomes possible to calculate the development system in the fields containing hundreds of wells in real time (about one minute).

In this method it is easy to take into account additional constraints. For example, such a restriction as the ban on the transfer of specific injectors into producers or producers into injectors. These specific wells can be selected according to certain criteria. Also, as a constraint, it is possible to use the condition that the ratio of the number of producing and injection wells should not be greater than or less than a certain predetermined value.

Results

Investigation of the computational efficiency of the proposed algorithm was carried out on the problems of hydrodynamic modeling of oil reservoirs waterflooding. Synthetic reservoir simulation models of a nine-spot development element were considered. The development period was chosen in such a way that the average water cut of the produced liquid was in range of 90 to 95%. The development systems obtained with the help of the proposed algorithm were compared with the brute force search result - the system obtained as a result of a complete search of all possible variants. The task parameters are given in Table 1.

**Figure 4.** Four cases of inhomogeneities are presented. - K1, - K2 (K1 < K2)
Table 1. Parameters

| Parameter                                      | Value          |
|------------------------------------------------|----------------|
| bottomhole pressure on production wells        | 30 (atm)       |
| bottom pressure on injection wells             | 70 (atm)       |
| porosity                                       | 0.2            |
| thickness                                      | 10 (m)         |
| water and oil viscosity                        | 1 cP, 10 cP    |
| critical saturation of water and oil           | 0.25, 0.6      |
| initial water saturation                       | 0.25           |
| number of Voronoi cells                        | ~200           |

In the proposed algorithm, all possible options for assigning wells (overall 512 variants) were submitted as initial values. In the case of a low-permeability inclusion of the "island" type (figure 4A), the algorithm produces 14 unique development systems, the range of the ORF values of which are close to the maximum (figure 5A). One of the distributions of control parameters is a regular five-spot waterflood-pattern configuration.

With a highly permeable inclusion (figure 4B), the express algorithm offers 38 unique well assignments, but the ORF range (figure 5B) is favorable. Among the proposed layouts there is a regular nine-spot configuration.

With a non-uniform permeability field of the "river" type (figure 4C, figure 4D), the express algorithm finds 28 and 52 unique well types assignments, respectively. In both cases, the ORF range is at the top of the graphs (figure 5C and figure 5D). Suggested direct line drive well spot configurations by brute force search were also proposed by the express algorithm.

Figure 5. Ranking of the options for assigning wells using ORF value for a 9-spot development element with a different permeability fields'

![Graph A](image1.png)  ![Graph B](image2.png)

![Graph C](image3.png)  ![Graph D](image4.png)
It is worth noting that the express algorithm calculates all 512 variants in about 13 seconds, while brute force search takes about an hour.

**Conclusion**

The proposed algorithm does not depend on the method of calculating the increment of the objective functional $\Delta J_i$. Therefore, formulas (5) and (6) can also be used for other calculation methods, for example, using the streamline method [10]. But then a much longer time will be required for calculation than on the basis of the analytical solution (4). The optimal control, determined on the basis of the proposed method, can be used in further calculations using algorithms [1, 2] as the "initial approximation". As can be seen from results of the method on synthetic reservoir simulation models of a nine-spot development element, the number of possible variants of well types is significantly reduced. In this case, all variants proposed by the method are favorable from the point of view of the objective functional.

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