Vertex-transitive graphs that have no Hamilton decomposition

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Abstract

It is shown that there are infinitely many connected vertex-transitive graphs that have no Hamilton decomposition, including infinitely many of valency 6, and including Cayley graphs of arbitrarily large valency.

1 Introduction

A famous question of Lovász concerns the existence of Hamilton paths in vertex-transitive graphs [27], and no example of a connected vertex transitive graph with no Hamilton path is known.

The related question of the existence of Hamilton cycles in vertex-transitive graphs is another interesting and well-studied problem in graph theory, see the survey [22]. Thomassen [32] (see [22]) has conjectured that there are only finitely many connected vertex-transitive graphs with no Hamilton cycle. On the other hand, Babai [5, 6] has conjectured that there are infinitely many such graphs. To date only five are known. These are the complete graph of order 2, the Petersen graph, the Coxeter graph, and the two graphs obtained from the Petersen and Coxeter graphs by replacing each vertex with a triangle.

For a regular graph of valency at least 4, a stronger property than the existence of a Hamilton cycle is the existence of a Hamilton decomposition. If $G$ is a $d$-regular graph, then a Hamilton decomposition of $G$ is a set of $\lfloor \frac{d}{2} \rfloor$ pairwise edge-disjoint Hamilton cycles in $G$. Given the small number of connected vertex-transitive graphs that are known to have no Hamilton cycle, and the uncertainty concerning the existence of others, it is natural to ask how many connected vertex-transitive graphs have no Hamilton decomposition. Mader [28] showed that a connected vertex-transitive graph of valency $d$ is $d$-edge-connected. So if $G$ is any connected vertex-transitive

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graph, then there is no obvious obstacle to the existence of a Hamilton decomposition of $G$. In this paper we show that there are in fact infinitely many connected vertex-transitive graphs that have no Hamilton decomposition, including infinitely many 6-regular graphs (see Theorem 7), and including Cayley graphs of arbitrarily large valency (see Theorems 3 and 4).

As far as we are aware, until now there were only six connected vertex-transitive graphs that were known to have no Hamilton decomposition. Firstly, there are the four non-Hamiltonian cubic graphs mentioned above. Secondly, Kotzig [18] has shown that a cubic graph has a Hamilton cycle if and only if its line graph has a Hamilton decomposition. Thus, the line graphs of these four graphs are 4-regular graphs with no Hamilton decomposition. However, of these, only the line graphs of the Petersen and Coxeter graphs are vertex-transitive. We have verified that every other connected vertex-transitive graph of order less than 28 has a Hamilton decomposition, using McKay and Royle’s list of vertex-transitive graphs that is available at http://staffhome.ecm.uwa.edu.au/~00013890/trans/.

Existence of Hamilton decompositions of vertex-transitive graphs has been established in many cases. It has been proved that all connected vertex-transitive graphs of order $p$ or $p^2$, where $p$ is prime, have Hamilton decompositions [3]. Every such graph is in fact a connected Cayley graph on an abelian group, and a long-standing conjecture of Alspach [1, 2] is that every connected Cayley graph on an abelian group has a Hamilton decomposition. This conjecture has been verified for graphs with valency at most 5 [4, 7, 8, 14], and in many cases for valency 6 [12, 13, 33, 34, 35]. Also, Liu [24, 25, 26] has proved strong results on the problem in cases where restrictions are placed on the connection set of the graph. We show that Alspach’s conjecture does not extend to Cayley graphs on non-Abelian groups by exhibiting two infinite families of connected Cayley graphs that have no Hamilton decomposition (see Theorems 3 and 4).

Hamilton decompositions of general graphs, not necessarily vertex transitive, have been studied extensively, see the survey [15]. Perhaps the most well-known problem on Hamilton decompositions is Nash-Williams’ conjecture [29], strengthened by Jackson [17], that every $d$-regular graph on at most $2d + 1$ vertices has a Hamilton decomposition. This conjecture has recently been proved by Kühn and Osthus et al for all sufficiently large $d$ [10, 11, 19, 20, 21]. Grünbaum and Malkevitch [16] have shown that there exist 4-regular 4-connected graphs that have no Hamilton decomposition, and moreover that there exist planar graphs with this property. One of the main ideas in their paper is a critical step in our constructions. We also mention two papers by Pike [30, 31] which
concern Hamilton decompositions, and in particular contain some questions on the existence of Hamilton decompositions of vertex transitive graphs.

2 Preliminaries

For any given graph $G$, the multigraph in which there are $m$ edges joining $u$ to $v$ when $u$ is joined to $v$ in $G$, and with zero edges joining $u$ to $v$ otherwise, is denoted by $mG$. Let $G$ be a loopless $d$-regular multigraph of order $n$. We define $K(G)$ to be the graph obtained from $G$ by replacing each vertex of $G$ with a complete graph of order $d$. Formally, $K(G)$ has vertex set $\{ (x, e) : x \in V(G), e \in E(G), x \in e \}$ (the vertices of $K(G)$ are the incidences in $G$ of vertices with edges), and the edge set of $K(G)$ is given by joining $(x, e)$ to $(y, f)$ if and only if $x = y$ or $e = f$ (two incidences are adjacent if and only if they share a common vertex or a common edge). Observe that $K(G)$ is a $d$-regular graph of order $dn$ (in particular, $K(G)$ has no edges of multiplicity greater than 1), and that $K(G)$ is connected if and only if $G$ is connected. The proof of Lemma 1 is routine.

**Lemma 1** The graph $K(G)$ is vertex-transitive if and only if $G$ is arc-transitive.

**Lemma 2** Let $G$ be a regular multigraph. The graph $K(G)$ has a Hamilton decomposition if and only if $G$ has a Hamilton decomposition.

**Proof** Let $d$ be the valency of $G$ and let $t = \lfloor \frac{d}{2} \rfloor$. First suppose $K(G)$ has a Hamilton decomposition $\{ H_1, H_2, \ldots, H_t \}$. For each $v \in V(G)$ let $S_v$ be the set of vertices of $K(G)$ that have first coordinate $v$, and let $E_v$ be the set of edges of $K(G)$ having exactly one endpoint in $S_v$. We have $|E_v| = d$ and $|E_v \cap E(H_i)|$ is positive and even for $v \in V$ and $i \in \{ 1, 2, \ldots, t \}$. Hence $|E_v \cap E(H_i)| = 2$ for each $v \in V$ and each $i \in \{ 1, 2, \ldots, t \}$. It follows that for each $i \in \{ 1, 2, \ldots, t \}$, $\{ e \in E(G) : \{ (u, e), (v, e) \} \in E(H_i) \}$ is the edge-set of a Hamilton cycle $C_i$ in $G$, and that $\{ C_i : i \in \{ 1, 2, \ldots, t \} \}$ is a Hamilton decomposition of $G$.

Now conversely suppose that $G$ has a Hamilton decomposition $\{ C_1, C_2, \ldots, C_t \}$. For each $i \in \{ 1, 2, \ldots, t \}$ let $F_i = \{ (u, e), (v, e) \} \in E(K(G)) : e \in E(C_i) \}$. It is clear that each $F_i$ can be completed to the edge set of a Hamilton cycle in $K(G)$ by adding, for each $v \in V(G)$, the edges of a Hamilton path in the complete graph (of order $d$) with vertex set $\{ (v, e) : e \in E(G), v \in e \}$. 

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Now, if $d$ is even, then the complete graph of order $d$ can be decomposed into $\frac{d}{2}$ pairwise edge-disjoint Hamilton paths, and in any such decomposition each vertex is an endpoint of exactly one of the Hamilton paths. Also, if $d$ is odd, then the complete graph of order $d$ can be decomposed into $\frac{d-1}{2}$ pairwise edge-disjoint Hamilton paths and a 1-regular graph of order $d - 1$. In any such decomposition each vertex of the 1-regular graph is an endpoint of exactly one of the Hamilton paths. Thus, both when $d$ is even and when $d$ is odd, $F_1, F_2, \ldots, F_t$ can be extended to a Hamilton decomposition of $K(G)$. \qed

We are interested in vertex-transitive graphs that have no Hamilton decomposition, and Lemmas 1 and 2 give us a method for constructing them. If $G$ is any arc-transitive multigraph that has no Hamilton decomposition, then $K(G)$ is a vertex-transitive graph with no Hamilton decomposition. Let $P$ denote the Petersen graph, let $C$ denote the Coxeter graph, and let $L(G)$ denote the line graph of an arbitrary graph $G$. We observe that since $L(P)$ and $L(C)$ are arc-transitive, it follows from Lemmas 1 and 2 that $K(L(P))$ and $K(L(C))$ are 4-regular vertex-transitive graphs that have no Hamilton decomposition.

Lemmas 1 and 2 also give us vertex-transitive graphs of arbitrarily large valency that have no Hamilton decomposition. Since $P$ is arc-transitive and has no Hamilton cycle, $mP$ is arc-transitive and has no Hamilton decomposition where $m$ is any positive integer. Thus, $K(mP)$ is a simple connected $3m$-regular vertex-transitive graph of order $30m$ that has no Hamilton decomposition. The same argument can be applied to the Coxeter graph because it is also arc-transitive and non-Hamiltonian. This argument gives us connected vertex-transitive graphs of arbitrarily large valency that have no Hamilton decomposition.

Although the constructions described in the preceding two paragraphs use the Petersen and Coxeter graphs, and rely on the fact that each is arc-transitive and has no Hamilton cycle, Lemmas 1 and 2 do not require the existence of such a graph, but only the existence of an arc-transitive graph $G$ such that $mG$ has no Hamilton decomposition for some positive integer $m$. Interestingly, it turns out that there are many such graphs, and the first example we discuss is the graph of the 3-cube, which is denoted by $Q_3$. 
3 Cayley graphs with no Hamilton decomposition

Firstly, observe that $Q_3$ contains only six distinct Hamilton cycles. Let these Hamilton cycles be $H_1, H_2, \ldots, H_6$. Also, for $i = 1, 2, \ldots, 6$, let $n_i$ be the number of Hamilton cycles which traverse the vertices of $mQ_3$ in the same order as $H_i$ in a putative Hamilton decomposition of $mQ_3$. If $u$ and $v$ are adjacent vertices in $mQ_3$, then it follows that the equation $\sum_{i=1}^{6} \delta_i n_i = m$ holds, where $\delta_i = 1$ if $H_i$ has an edge with endpoints $u$ and $v$, and $\delta_i = 0$ otherwise. The twelve edges of $Q_3$ thus give us twelve equations in the variables $n_1, n_2, \ldots, n_6$, and it is routine to check that these have no integral solution when $m \equiv 2 \pmod{4}$. It follows that for each positive integer $m \equiv 2 \pmod{4}$, $K(mQ_3)$ is a connected vertex-transitive graph that has no Hamilton decomposition.

We now observe that $K(mQ_3)$ is a Cayley graph. It is easy to check that

$$K(Q_3) \cong \text{Cay}(\text{Sym}(4); \{(1 2), (2 3 4), (2 4 3)\})$$

where $\text{Sym}(4)$ is the symmetric group acting on $\{1, 2, 3, 4\}$. Let $e$ denote the identity of $\text{Sym}(4)$, let $\mathbb{Z}_m = \{1, r, r^2, \ldots, r^{m-1}\}$ be a cyclic group of order $m$ with generator $r$, and let $H$ be the subgroup of $\text{Sym}(4) \times \mathbb{Z}_m$ generated by $((2 3 4), 1)$ and $(e, r)$. Since $K(Q_3) \cong \text{Cay}(\text{Sym}(4); \{(1 2), (2 3 4), (2 4 3)\})$, it is easy to see that for each positive integer $m$,

$$K(mQ_3) \cong \text{Cay}(\text{Sym}(4) \times \mathbb{Z}_m; \{(1 2, 1)\} \cup H \setminus \{(e, 1)\}).$$

Thus, we have the following result.

**Theorem 3** For each positive integer $m \equiv 2 \pmod{4}$, the connected Cayley graph

$$\text{Cay}(\text{Sym}(4) \times \mathbb{Z}_m; \{(1 2, 1)\} \cup H \setminus \{(e, 1)\})$$

is isomorphic to $K(mQ_3)$ and has no Hamilton decomposition.

Using a similar argument to that which was used to show $mQ_3$ has no Hamilton decomposition when $m \equiv 2 \pmod{4}$, it can be shown that $mF_{16}$ also has no Hamilton decomposition when $m \equiv 2 \pmod{4}$. Here $F_{16}$ is the Möbius-Kantor graph, which is a connected cubic vertex-transitive graph of order 16. It thus follows that for each positive integer $m \equiv 2 \pmod{4}$, $K(mF_{16})$ is a connected vertex-transitive graph that has no Hamilton decomposition.

Now, it can be easily checked that if $GL(2, 3)$ denotes the general linear group of invertible 2 by 2 matrices over the field with three elements, then

$$K(F_{16}) \cong \text{Cay}(GL(2, 3); \{A, B, B^{-1}\})$$

where $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. 

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Similarly to above, from here it is easy to see that for each positive integer \( m \),

\[
K(mF_{16}) \cong \text{Cay}(GL(2, 3) \times \mathbb{Z}_m; \{(A, 1)\} \cup H \setminus \{(I, 1)\}),
\]

where \( I \) is the identity of \( GL(2, 3) \), \( \mathbb{Z}_m = \{1, r, r^2, \ldots, r^{m-1}\} \) is a cyclic group of order \( m \), and \( H \) is the subgroup of \( GL(2, 3) \times \mathbb{Z}_m \) generated by \( (B, 1) \) and \( (I, r) \). This gives us another infinite family of connected Cayley graphs that have no Hamilton decomposition.

**Theorem 4** For each positive integer \( m \equiv 2 \pmod{4} \), the connected Cayley graph

\[
\text{Cay}(GL(2, 3) \times \mathbb{Z}_m; \{(A, 1)\} \cup H \setminus \{(I, 1)\})
\]

is isomorphic to \( K(mF_{16}) \) and has no Hamilton decomposition.

## 4 6-regular graphs with no Hamilton decomposition

We now proceed to construct an infinite family of 6-regular vertex-transitive graphs that have no Hamilton decomposition. The following lemma shows that if \( G \) is cubic, then the existence of a Hamilton decomposition of \( 2G \) is equivalent to the existence of a perfect 1-factorisation of \( G \). A **perfect 1-factorisation** of a \( d \)-regular graph is a set of \( d \) pairwise edge-disjoint 1-factors (spanning 1-regular subgraphs) such that the union of any two of these 1-factors is a Hamilton cycle.

**Lemma 5** If \( G \) is a cubic graph, then \( G \) has a perfect 1-factorisation if and only if \( 2G \) has a Hamilton decomposition.

**Proof** If \( \{F_1, F_2, F_3\} \) is a perfect 1-factorisation of \( G \), then \( \{F_1 \cup F_2, F_1 \cup F_3, F_2 \cup F_3\} \) is a Hamilton decomposition of \( 2G \). Conversely, if \( \{H_1, H_2, H_3\} \) is a Hamilton decomposition of \( 2G \), and we let \( F_1 \) contain those edges of \( G \) such that the corresponding two edges of \( 2G \) are in \( H_1 \) and \( H_2 \), let \( F_2 \) contain those edges of \( G \) such that the corresponding two edges of \( 2G \) are in \( H_1 \) and \( H_3 \), and let \( F_3 \) contain those edges of \( G \) such that the corresponding two edges of \( 2G \) are in \( H_2 \) and \( H_3 \), then \( \{F_1, F_2, F_3\} \) is a perfect 1-factorisation of \( G \).

An immediate corollary of Lemma 5 (combined with Lemmas 1 and 2) is that if \( G \) is a cubic arc-transitive graph that has no perfect 1-factorisation, then \( K(2G) \) is a 6-regular vertex-transitive graph that has no Hamilton decomposition. The following result, which Laufer [23] attributes to Kotzig [18], is thus important for us.
Theorem 6 (Kotzig, [18]) If $G$ is a regular bipartite graph with order congruent to 0 (mod 4) and valency at least 3, then $G$ has no perfect 1-factorisation.

Combining Lemma 5 with Theorem 6 we have that if $G$ is any connected bipartite arc-transitive cubic graph of order congruent to 0 (mod 4), then $2G$ is a connected vertex-transitive multigraph that has no Hamilton decomposition. It then follows by Lemmas 1 and 2 that $K(2G)$ is a 6-regular connected vertex-transitive graph that has no Hamilton decomposition. It is known that there are infinitely many connected bipartite arc-transitive cubic graphs of order congruent to 0 (mod 4), see [9] for example, and so we have the following theorem.

Theorem 7 If $G$ is a connected bipartite arc-transitive cubic graph of order congruent to 0 (mod 4), then $K(2G)$ is a 6-regular connected vertex-transitive graph that has no Hamilton decomposition. Thus, there are infinitely many 6-regular connected vertex-transitive graphs that have no Hamilton decomposition.

In Theorem 7 we can actually take $G$ to be any connected arc-transitive cubic graph for which $2G$ has no Hamilton decomposition (we do not require that $G$ has order congruent to 0 (mod 4) except to prove that $2G$ has no Hamilton decomposition). For example, it is easily verified that $2F_{18}$ has no Hamilton decomposition, where $F_{18}$ is the Pappus Graph that has order 18. It follows that $K(2F_{18})$ is another connected 6-regular vertex-transitive graph that has no Hamilton decomposition.

5 Concluding remarks and open problems

It is a well-known conjecture that all connected Cayley graphs have a Hamilton cycle (except the complete graph of order 2). Obviously, if true, this means that all connected Cayley graphs of valency 3 have a Hamilton decomposition. We have shown that there are at least two Cayley graphs of valency 6 that have no Hamilton decomposition, namely $K(2Q_3)$ and $K(2F_{18})$, and we have constructed Cayley graphs of arbitrarily large valency that have no Hamilton decomposition. An obvious question then, is whether there exist Cayley graphs of valency 4 (or 5) that have no Hamilton decomposition.

The graph $K(2Q_3)$ is a connected Cayley graph of valency 6 and order 48 that has no Hamilton decomposition. It would be interesting to know if there exist any smaller connected Cayley graphs
that have no Hamilton decomposition. Any such graph has order at least 28. It would also be interesting to know whether $K(2Q_3)$ is the smallest connected vertex-transitive graph of valency 6 that has no Hamilton decomposition. Another open question is whether there are any connected Cayley graphs of odd order which have no Hamilton decomposition. At present, $L(P)$ is the only connected vertex-transitive graph of odd order that is known to have no Hamilton decomposition.

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