The Okubo relation for the baryon magnetic moments and chiral perturbation theory

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Abstract

We analyze the Okubo $SU(3)$ relation among the hyperon magnetic moments in the usual scheme of chiral perturbation theory (ChPT). We classify the one-loop diagrams, including those with intermediate decuplet baryons, in a simple way according to whether or not they satisfy the Okubo relation. Contrary to the conventional wisdom, we find that one-loop contributions to the hyperon magnetic moments in general violate the Okubo relation if the physical masses are employed for the meson propagators in the loops.

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1 Introduction

The magnetic moments of the octet baryons were found to obey approximate $SU(3)$ symmetry a long time ago by Coleman and Glashow \cite{1}. The relations of Coleman and Glashow are satisfied by the observed magnetic moments up to about the 20\% level. Shortly thereafter Okubo \cite{2} derived a relation between the magnetic moments, based on the assumption that $SU(3)$ symmetry is broken linearly, which is satisfied to a great accuracy by the current high precision data on the magnetic moments \cite{3}.
Recently, many efforts have been made to study the baryon magnetic moments in chiral perturbation theory \[4, 5, 6, 7, 8, 9\]. Clearly, it is very relevant how the \(SU(3)\) breaking are treated in such study. In a scheme of chiral perturbation theory (ChPT)[6] that is popular in the literature the \(SU(3)\) breaking physical masses are used for strange mesons in the loop. This is the scheme that we shall discuss in this paper. (for an alternative scheme, see Ref.[9]). In such scheme, the hyperon magnetic moments receive contributions that are non-analytic in the strange quark mass \(m_s\), or equivalently, non-analytic in \(SU(3)\) breaking. Since the Okubo relation is a result of linear \(SU(3)\) breaking, one would therefore not expect that the moments calculated in ChPT will satisfy it. Nevertheless, it was shown \[6\] by an explicit calculation in ChPT that the \(\sqrt{m_s}\) corrections to the magnetic moments still satisfy the Okubo relation. In more recent applications of ChPT \[6, 7\] it was even claimed that both the \(\sqrt{m_s}\) and the \(m_s \ln m_s\) corrections satisfy the Okubo relation. Motivated by this apparent puzzle, we consider in this paper in more details the validity of the Okubo relation in this scheme of ChPT. We will identify the simple reason why the \(\sqrt{m_s}\) corrections to the hyperon magnetic moments in this scheme of ChPT satisfy the Okubo relation. On the other hand, while our calculational results are in agreement with that of Ref. \[6\], our conclusion for the \(m_s \ln m_s\) type non-analytic corrections is different. We will show that, contrary to the claim in the literature, \(m_s \ln m_s\) non-analytic corrections violate the Okubo relation.

This paper is organized as follows. In the next section we will introduce the Okubo relation. In Sec. 3 we will consider the one-loop diagrams in ChPT that contribute to the magnetic moments, and will show that they generally do not satisfy the Okubo relation. In Sec. 4 we will discuss our results and compare them with earlier calculations. In the Appendix, some calculational details are collected.

## 2 Okubo relation

The hyperon magnetic moments satisfy to great accuracy the Okubo relation \[2\]

\[
6\mu_\Lambda + \mu_\Sigma - 4\sqrt{3}\mu_{\Lambda \Sigma^0} - 4\mu_\pi + \mu_{\Sigma^0} - 4\mu_{\Xi^0} = 0, \tag{1}
\]

where \(\mu_{\Lambda \Sigma^0}\) is the \(\Sigma^0 \rightarrow \Lambda\) transition moment. This relation can be obtained \[4\] if one assumes that \(SU(3)\) breaking corrections to the moments are linear.
in the quark mass matrix $\sigma$, defined by

$$\sigma \equiv \text{diag}(0, 0, m_s).$$

(We neglect the masses of the $u$- and $d$-quark.) In leading order in the electromagnetic coupling and up to first order in $SU(3)$ symmetry breaking, the most general expression for the magnetic moments with this assumption is

$$\mu_{ab} = a_1 \langle \tilde{\lambda}_b^\dagger \{Q, \tilde{\lambda}_a\} \rangle + a_2 \langle \tilde{\lambda}_b^\dagger [Q, \tilde{\lambda}_a] \rangle + \alpha_1 \langle \tilde{\lambda}_b^\dagger [\sigma, [Q, \tilde{\lambda}_a]] \rangle + \alpha_2 \langle \tilde{\lambda}_b^\dagger [\sigma, [Q, \tilde{\lambda}_a]] \rangle + \alpha_3 \langle \tilde{\lambda}_b^\dagger [\sigma, [Q, \tilde{\lambda}_a]] \rangle + \alpha_4 \langle \tilde{\lambda}_b^\dagger [\sigma, [Q, \tilde{\lambda}_a]] \rangle + \alpha_5 \langle \tilde{\lambda}_b^\dagger \tilde{\lambda}_a \rangle \langle \sigma Q \rangle, $$

(3)

where $a_1, a_2, \alpha_1, \ldots, \alpha_5$ are arbitrary parameters, $Q$ is the quark charge matrix

$$Q = \frac{1}{3} \text{diag}(2, -1, -1),$$

(4)

and $\tilde{\lambda}_a$ are generators of $SU(3)$ in the physical basis, defined e.g. in Ref. [5, 9]. Including the transition moment, Eq. (3) expresses the 9 magnetic moments in terms of 7 parameters. Therefore, two relations can be derived between the moments, which hold for any values of the parameters. The first is the Okubo $SU(3)$ relation Eq. (1), and the second is a simple isospin relation [9].

Motivated by this success, it was argued in Ref. [9] that one should adopt a scheme of ChPT in which the chiral symmetry breaking appears linearly in $m_s$. However, a more popular scheme of ChPT, as represented in Refs. [6, 7, 8], is to use the $SU(3)$ breaking physical meson masses in the loop and as a result contributions nonanalytic in strange quark mass are obtained. The puzzle is why Okubo relation is still claimed to be valid within this scheme.

3 Okubo relation in ChPT

To analyze the Okubo relation for the magnetic moments in this scheme of ChPT, we follow the calculations of Refs. [6, 7, 8]. The hyperon magnetic moments receive contributions from the one-meson loop diagrams in Figs. (1) and (2). Following the scheme in Refs. [6, 7, 8], we will use an $SU(3)$ invariant mass for the baryon propagators inside the loops, while, for the meson propagators inside the loops, their physical masses are employed. All strong-interaction vertices in the loops originate from the leading-order
SU(3) invariant Lagrangian (see e.g. Ref. [3]). Therefore, the meson propagators and the electromagnetic vertices are the only sources of SU(3) symmetry breaking.

Starting with the loop-diagram in Fig. (1a), it gives the amplitude

\[ \Gamma^\mu_{ab} = C_0 \sum_{c,d,e=1}^8 \int \frac{d^4k}{(2\pi)^4} \frac{F_{bce} S_v \cdot (k + q)(G_{ed}(2k + q)^\mu)(H_{cad} S_v \cdot k)}{[(k + q)^2 - M_e^2 - i\epsilon][k^2 - M_d^2 - i\epsilon][v \cdot k - i\epsilon]}, \tag{5} \]

where \( M_d \) and \( M_e \) are the masses of mesons \( d \) and \( e \), respectively, \( q^\mu \) is the incoming photon four-momentum, and \( C_0 \) is a number irrelevant for the following discussion. The vertex factors \( F_{bce}, G_{ed}, \) and \( H_{cad} \) in Eq. (5) are given by

\[ F_{bce} = D\langle \bar{\lambda}_b^\dagger \bar{\lambda}_c \rangle + F\langle \bar{\lambda}_b^\dagger \bar{\lambda}_e \rangle, \tag{6} \]
\[ G_{ed} = \langle \bar{\lambda}_e^\dagger (Q, \bar{\lambda}_d) \rangle, \tag{7} \]
and

\[ H_{cad} = D\langle \bar{\lambda}_c^\dagger \bar{\lambda}_d \rangle + F\langle \bar{\lambda}_c^\dagger \bar{\lambda}_a \rangle, \tag{8} \]

respectively.

The meanings of the flavor indices \( a, \ldots, e \) are as illustrated in Fig. (1a). Since we took an SU(3) invariant baryon mass, the flavor index \( c \) does not appear in the baryon propagator. From Eq. (7) it is obvious that the electromagnetic vertex \( G_{ed} \) is diagonal in the flavor indices \( e \) and \( d \), and therefore \( M_e = M_d \) and the related loop integral depends only on one meson mass. Using this property it is straightforward to show that the amplitude \( \Gamma^\mu_{ab} \) contributes to the magnetic moment \( \mu_{ab} \) as

\[ \mu_{ab}^{(1a)} = \sum_{c,e,d=1}^8 F_{bce} G_{ed} H_{cad} I_e, \tag{9} \]

where the functions \( I_e \) represent the momentum integration part of the loop diagram. Assuming isospin symmetry these functions are readily seen to satisfy

\[ I_1 = I_2 = I_3 = I_4^{(1a)}(M_\pi^2), \quad I_4 = I_5 = I_6 = I_7 = I^{(1a)}(M_K^2), \quad I_8 = I^{(1a)}(M_\eta^2), \tag{10} \]

where the functional form of \( I \) will be discussed later.
Making similar analysis for the other diagrams in Figs. (1) and (2), one finds that each one-meson loop diagrams will contribute to the moments $\mu_{ab}$ as

$$\mu_{ab}(K) = \sum_{e, e' = 1}^{8} T_{e e'}^{K} I_{e}^{K} \delta_{ee'},$$  \hspace{1cm} (11)$$

where, in addition to the meson mass dependence, we add the $K$ index to the integral $I$ to label different diagrams. For every $K$, the functions $I_{e}^{K}$ obey relations as in Eq. (10). For Fig. (1a) considered above one obviously has

$$T_{abee'}^{(1a)} = \sum_{c, d}^{8} F_{bc}^{e'} G_{ed} H_{cd}.$$  \hspace{1cm} (12)$$

For diagrams in Fig. 2, each diagram contains only one internal meson line. Take Fig. (2e), which contains an intermediate decuplet propagator, for example, one has

$$T_{abee'}^{(2e)} = \delta_{ed} \epsilon_{ijk} (\bar{\lambda}_{b} \lambda_{e'})_{m} \Lambda^{kln} \Lambda_{e' j' k'}^{(Q)} \Lambda^{l' j' m'} (\bar{\lambda}_{e'} \lambda_{a})_{k''} = 16 \delta_{ij} \delta_{k l} \delta_{m'} - 2 \delta_{ij} \delta_{k l} \delta_{m'},$$  \hspace{1cm} (13)$$

where $\Lambda_{ijk} = \bar{\Lambda}^{ijk}$ is a basis for the decuplet, given by [11]

$$\Lambda_{ijk} = 1, \hspace{1cm} i = j = k, \hspace{1cm} \frac{1}{3}, \hspace{1cm} i = j \neq k, \hspace{1cm} \frac{1}{6}, \hspace{1cm} i, j, k \text{ distinct.}$$  \hspace{1cm} (14)$$

The internal octet and decuplet indices in $T_{abee'}$ can be easily summed over using

$$\sum_{a=1}^{8} (\bar{\lambda}_{a})_{\beta}^{\alpha} (\lambda_{a})_{\gamma}^{\rho} = 2 \delta_{\alpha}^{\rho} \delta_{\gamma}^{\beta} - \frac{2}{3} \delta_{\alpha}^{\rho} \delta_{\gamma}^{\beta},$$  \hspace{1cm} (15)$$

and

$$\Lambda^{kln} \Lambda_{k' l' m'} = \frac{1}{6} (\delta_{k}^{m} \delta_{l}^{n} \delta_{m' k'} + \delta_{k}^{m} \delta_{l}^{n} \delta_{m' k'} + \delta_{k}^{m} \delta_{l}^{n} \delta_{m' k'} + \delta_{k}^{m} \delta_{l}^{n} \delta_{m' k'} + \delta_{k}^{m} \delta_{l}^{n} \delta_{m' k'} + \delta_{k}^{m} \delta_{l}^{n} \delta_{m' k'}).$$  \hspace{1cm} (16)$$

The general result after these summations is that $T_{abee'}$ in Eq. (11) is a flavor structure constructed out the remaining matrices $\lambda_{a}$, $\bar{\lambda}_{a}$, $\bar{\lambda}_{b}$, $\lambda_{e'}$, $\bar{\lambda}_{e'}$, and $Q$, in which each of these matrices appears exactly one time. In fact, $T_{abee'}$ would
be a $SU(3)$ tensor if not for the $SU(3)$ breaking matrix $Q$. The fact that all the one-meson loops in Figs. (1) and (2) can be written in the form of Eq. (11) makes it possible to examine their $SU(3)$ symmetry breaking pattern in a simple way.

To analyze the $SU(3)$ symmetry structure of the magnetic moments in Eq. (11), we first note that, for each $K$, $I_e^K$ can be rewritten as

$$ I_e^K = \hat{I} + \sum_{e,e'} T_{ab}^{K} \lambda_{e'} \{ \hat{I}, \lambda_e \} = \delta_{ee'} \langle \hat{\lambda}_{e'} \{ \hat{I}, \lambda_e \} \rangle + \frac{1}{3} I(M_{\pi}^2) \delta_{ee'} \delta_{es}, \quad (17) $$

where $\hat{I}$ is the following $3 \times 3$ matrix

$$ \hat{I} = \frac{1}{2} I(M_{\eta}^2) + \frac{1}{m_s} [I(M_{K}^2) - I(M_{\pi}^2)] \sigma, \quad (18) $$

which is a simple linear combination of the unit matrix and the quark mass matrix. Note that each integration factor, $I$, in Eqs.(17) and (18) have diagramatic, $K$, dependence as well which we suppressed in notation. Using Eq. (17) one can then split the magnetic moments $\mu_{ab}$ in two parts as

$$ \mu_{ab} = \mu_{ab}^I + \mu_{ab}^II, \quad (19) $$

where $\mu_{ab}^I$ is defined by

$$ \mu_{ab}^I = \sum_K \sum_{e,e'} T_{ab}^{K} \langle \hat{\lambda}_{e'} \{ \hat{I}, \lambda_e \} \rangle, \quad (20) $$

and $\mu_{ab}^II$ is defined by

$$ \mu_{ab}^II = \sum_K \mu_{ab}^{II}(K) = \sum_K \left[ (I^K(M_{\eta}^2) - \frac{4}{3} I^K(M_{K}^2) + \frac{1}{3} I^K(M_{\pi}^2) \right] T_{ab}^{K}. \quad (21) $$

Note that because the $\eta$ meson is charge neutral, the electromagnetic vertex $G_{ed}$ is zero for $\eta$ loop in Figs.(1a,b). As a result, Figs.(1a,b) contribute only to $\mu_{ab}^I$ because $T_{ab}^{(1a,b)} = 0$. Figs. (2) can contribute to both $\mu_{ab}^I$.

The symmetry properties of $\mu_{ab}^I$ can be easily analyzed. For the summations over $e$ and $e'$ in Eq. (20) one can again use the completeness relation Eq. (15). Since the matrix $\hat{I}$ is a linear combination of the unit matrix and the quark mass matrix $\sigma$, and given the properties of $T_{ab}^{(1a,b)}$ as discussed after
Eq. (16), one finds that \( \mu_{1ab} \) is a tensor formed out of the matrices \( \tilde{\lambda}_a, \tilde{\lambda}_b^\dagger, Q, \) and \( \sigma \), in which \( \tilde{\lambda}_a, \tilde{\lambda}_b^\dagger, \) and \( Q \) each appears exactly one time, and the matrix \( \sigma \) appears at most one time. The general form of such a tensor is given by Eq. (3). As was discussed after Eq. (3), the \( \mu_{1ab} \) component of the magnetic moments therefore satisfy the Okubo relation Eq. (1) no matter what kind of meson mass dependences result from the integration part of the contribution.

Whether or not the magnetic moments \( \mu_{ab} \), Eq. (11), satisfy the Okubo relation then depends on whether or not the moments \( \mu_{1ab} \), Eq. (21), satisfy the Okubo relation. As a result we only have to worry about the contributions of diagrams in Figs(2). For any given one-loop diagram, the contribution to the magnetic moment from only the \( \eta \)-loop, denoted here by \( \mu_{ab}^\eta \), is, according to Eqs. (11) and (10), given by

\[
\mu_{ab}^\eta(K) = I^K(M_\eta^2)T_{ab}^{\delta88} \tag{22}
\]

Using Eq. (22) we can relate \( \mu_{ab}^{11} \) to the \( \eta \)-loop as

\[
\mu_{ab}^{11} = \sum_K \frac{\Delta^K}{3I^K(M_\eta^2)}\mu_{ab}^\eta(K), \tag{23}
\]

where we have defined

\[
\Delta^K = 3I^K(M_\eta^2) - 4I^K(M_K^2) + I^K(M_\pi^2). \tag{24}
\]

Since the meson masses are non-degenerate, \( \Delta^K \) in Eq. (23) is in general nonzero. The magnitude of \( \Delta^K \) depends on the function \( I^K \) in Eq. (24). By closer inspection one finds that the generic form of \( I \) for the diagrams in Fig. (1) is given by

\[
I^K(X^2) = A_1^K + A_2^K \sqrt{X^2}, \tag{25}
\]

while the generic form of \( I^K \) for the diagrams in Fig. (2) is given by

\[
I^K(X^2) = B_1^K + B_2^K X^2 \ln \left( X^2/\mu^2 \right), \tag{26}
\]

where the coefficients \( A_1^K, A_2^K, B_1^K, \) and \( B_2^K \) depend on the diagrams but are independent of \( X \), and \( \mu \) is the renormalization scale. Note that in terms of the quark masses \( m_q \), Eq. (23) leads to corrections of the form \( \sqrt{m_q} \), and Eq. (26) leads to corrections of the form \( m_q \ln m_q \). We conclude immediately
here that the nonanalytic contributions of the form $\sqrt{m_q}$ satisfy the Okubo relation. This part, we agree with Refs. [6, 7].

By substituting the functional forms Eq. (26) into Eq. (24), the diagrams in Figs. (12) all yield $\Delta^K \neq 0$. Therefore, by Eq. (23), $\mu_{ab}^{\eta}$ is proportional to the $\eta$-loop contribution to the magnetic moments. For the one-loop diagrams contributing to the hyperon magnetic moments we then conclude: A diagram satisfies the Okubo relation if and only if its $\eta$ loop contribution, or equivalently its $T_{a88}$ term, satisfies the Okubo relation. This observation greatly simplifies the analysis of the validity of the Okubo relation of hyperon magnetic moments calculated in current scheme of ChPT.

Finally we note that $\mu_{ab}^{\eta}$ is explicitly renormalization scale independent. This can be seen by substituting Eq. (26) in Eq. (24), we find that

$$\Delta^K = -B_2(K) \left[ 3M_\eta^2 - 4M_K^2 + M_\pi^2 \right] \ln \mu^2 + \text{(renormalization independent terms)},$$  

(27)

The dependence on $\mu$ in Eq. (27) vanishes upon using the Gell-Mann-Okubo relation for the meson masses. On the other hand, $\mu_{ab}^{\eta}$ is renormalization scale dependent, however, since they are of the form as in Eq. (3), they can be removed by introducing renormalization counterterms. We can now discuss the consequence of the above analysis.

4 Discussions and summary

From the discussion in the previous section it follows that contributions from diagrams which have a vanishing $\eta$-loop, will trivially satisfy the Okubo relation. This is the case for the diagrams in Fig. (1). This gives a simple explanation why the contributions of the diagrams in Fig. (1), which give rise to the $\sqrt{m_q}$ corrections, satisfy the Okubo relation consistent with the observations in Refs. [4, 5].

For the same reason, the diagram Fig. (2b,2c) yields magnetic moments that satisfy the Okubo relation. For Fig. (2b), it is because the $\eta$ meson is charge neutral. For Fig. (2c), it is purely due to accidental cancellation as shown in the Appendix.

On the other hand, as shown explicitly in the Appendix, the $\eta$ loop contribution to the magnetic moments from each of the diagrams Figs. (2a), (2d), (2e), (2f), (2g), (2h) and (2i) is non-vanishing and violates the Okubo
relation. They all give contributions to the hyperon moments of the form $m_q \ln m_q$. We also show that their sum also remains nonzero. While our detail analytic result, as demonstrated in the Appendix, agrees with Refs. [1], our conclusion is in conflict with that in Refs. [3, 4] which claimed that corrections of the form $m_q \ln m_q$ satisfy the Okubo relation. The magnitude of the deviation from the Okubo relation will in general depend on the parameters from the ChPT Lagrangian and the meson masses. There is no reason, based upon $SU(3)$ symmetry arguments alone, a priori that this deviation should vanish or should be small.

One should finally also note that in the modified scheme of $SU(3)$ ChPT proposed in Ref. [9, 10], loop diagrams give rise to linear $SU(3)$ symmetry breaking automatically in the lowest nontrivial order in $SU(3)$ breaking. In that case one can easily find that $\Delta^K = 0$ automatically in Eq. (23), and the magnetic moments satisfy the Okubo symmetry relation.

The hyperon magnetic moments satisfy to a high precision the Okubo relation. This relation is based on the assumption that $SU(3)$ is broken linearly in the strange quark mass $m_s$. In this paper we have studied the validity of the Okubo relation if the moments are calculated in a scheme of $SU(3)$ chiral perturbation theory (ChPT) at the one-loop level.

We have found a simple way to classify the one-loop diagrams of this ChPT contributing to the magnetic moments according to whether or not they generate magnetic moments that satisfy the Okubo $SU(3)$ relation. A diagram satisfies the Okubo relation if and only if its $\eta$ loop satisfies the Okubo relation. Using this observation, we have shown that the $\sqrt{m_s}$ corrections satisfy the Okubo relation. However, some of the $m_s \ln m_s$ corrections violate the Okubo relations as opposed to what was claimed in recent literature. Note that the above analysis could in principle be extended to the scheme in which that $SU(3)$ breaking masses are allowed even in the baryon propagators.

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Appendix

In this appendix we give the deviation from the Okubo relation among the hyperon magnetic moments for each one-loop diagrams in Figs. (1) and (2) and the total deviation.

According to the discussion in Sec.3, the deviation from the Okubo relation for each one-loop diagram only depends on the second term $\mu_{ab}^{\Pi}$ in Eq.(19) and can be expressed as

$$\Omega(K) = 6\mu_{\Lambda}^{\Pi}(K) + \mu_{\Sigma+}^{\Pi}(K) - 4\sqrt{3}\mu_{\Lambda\Sigma}^{\Pi}(K) - 4\mu_{n}^{\Pi}(K)
+ \mu_{\Sigma+}^{\Pi}(K) - 4\mu_{\Xi0}^{\Pi}(K),$$

(28)

where $K$ label different diagrams. Using Eqs.(21) and (24), the $\Omega(K)$ can be denoted as

$$\Omega(K) = \frac{1}{3} \Delta^K T(K)$$

(29)

where $\Delta(K)$ is defined in Eq.(24) and

$$T(K) = 6T^K_{8888} + T^K_{2288} - 4\sqrt{3}T^K_{8388} - 4T^K_{6688}
+ T^K_{1188} - 4T^K_{7788}.$$ 

(30)

The $T_{ab8}$ is expected to be nonzero only for the diagonal components in the flavor indices $(a,b)$ i.e. $a=b$, and for $(a,b)=(3,8)$ or $(8,3)$, which is related to the transition magnetic moment $\mu_{\Lambda\Sigma}$. In the following, we give the $T_{ab88}$ for each diagram, and, at the second line of each equation, we list explicitly only the values of diagonal components in indices $(a,b)$ and component $(a,b)=(8,3)$ in the order $(a,b)=\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6),(7,7),(8,8),(8,3)\}$, corresponding to the magnetic moments in the order $(\mu_{\Sigma+}, \mu_{\Sigma-}, \mu_{\Lambda\Sigma}, \mu_{p}, \mu_{\Xi-}, \mu_{n}, \mu_{\Xi0}, \mu_{\Lambda}, \mu_{\Lambda\Sigma})$.

$$T_{ab88}^{(1a)} = \sum_{c} F_{bc8} G_{88} H_{ca8},$$

$$= \{0, 0, 0, 0, 0, 0, 0, 0\}$$

(31)

$$T_{ab88}^{(1b)} = \sum_{c} T_{bc8} G_{88} R_{ca8},$$

$$= \{0, 0, 0, 0, 0, 0, 0, 0\}$$

(32)
\[ T^{(2a)}_{a88} = \sum_{c,d} F_{bc8} V_{cd} H_{da8}, \]
\[ = \left\{ \frac{2}{9}D^2(\mu_D + 3\mu_F), \frac{2}{9}D^2(\mu_D - 3\mu_F), \frac{2}{9}D^2\mu_D, \right. \]
\[ \frac{1}{18}(D - 3F)^2(\mu_D + 3\mu_F), \frac{1}{18}(D + 3F)^2(\mu_D - 3\mu_F), \]
\[ -\frac{1}{9}(D - 3F)^2\mu_D, -\frac{1}{9}(D + 3F)^2\mu_D, -\frac{2}{9}D^2\mu_D, \]
\[ -\frac{2}{3\sqrt{3}}D^2\mu_D \right\} \] (33)

\[ T^{(2b)}_{a88} = \langle \tilde{\lambda}_8^\dagger [\tilde{\lambda}_8, [\tilde{\lambda}_8^\dagger, Q]], \tilde{\lambda}_a \rangle, \]
\[ = \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\} \] (34)

\[ T^{(2c)}_{a88} = \sum_{c,d} T_{bc8} U_{cd} R_{da8}, \]
\[ = \left\{ 6\mathcal{C}^2\mu_C, -6\mathcal{C}^2\mu_C, 0, 6\mathcal{C}^2\mu_C, 0, 0, 0, 0, 0 \right\} \] (35)

\[ T^{(2d)}_{a88} = \sum_{c,d} T_{bc8} X_{cd} H_{da8}, \]
\[ = \left\{ -2\mathcal{C}\mu_T, 0, -\mathcal{C}\mu_T, 0, 0, 0, (D + 3F)\mathcal{C}\mu_T, \right. \]
\[ 0, -\sqrt{3}\mathcal{C}\mu_T \right\} \] (36)

\[ T^{(2e)}_{a88} = \sum_{c,d} T_{bc8} X_{cd} H_{da8}, \]
\[ = \left\{ -2\mathcal{C}\mu_T, 0, -\mathcal{C}\mu_T, 0, 0, 0, (D + 3F)\mathcal{C}\mu_T, \right. \]
\[ 0, 0 \right\} \] (37)

\[ T^{(2f)}_{a88} = \sum_{c} F_{bc8} H_{c88} V_{ba}, \]
\[ = \left\{ \frac{2}{9}D^2(\mu_D + 3\mu_F), \frac{2}{9}D^2(\mu_D - 3\mu_F), \frac{2}{9}D^2\mu_D, \right. \]
\[
\frac{1}{18}(D - 3F)^2(\mu_D + 3\mu_F), \frac{1}{18}(D + 3F)^2(\mu_D - 3\mu_F),
\]
\[
-\frac{1}{9}(D - 3F)^2\mu_D, -\frac{1}{9}(D + 3F)^2\mu_D, -\frac{2}{9}D^2\mu_D,
\]
\[
\frac{2}{3\sqrt{3}}D^2\mu_D \}
\]

(38)

\[
T_{ab88}^{(2g)} = \sum_c V_{ba} F_{ac8} H_{ca8},
\]
\[
= \{ \frac{2}{9}D^2(\mu_D + 3\mu_F), \frac{2}{9}D^2(\mu_D - 3\mu_F), \frac{2}{9}D^2\mu_D,
\]
\[
\frac{1}{18}(D - 3F)^2(\mu_D + 3\mu_F), \frac{1}{18}(D + 3F)^2(\mu_D - 3\mu_F),
\]
\[
-\frac{1}{9}(D - 3F)^2\mu_D, -\frac{1}{9}(D + 3F)^2\mu_D, -\frac{2}{9}D^2\mu_D,
\]
\[
\frac{2}{3\sqrt{3}}D^2\mu_D \}
\]

(39)

\[
T_{ab88}^{(2h)} = \sum_c T_{bc8} R_{cb8} V_{ba},
\]
\[
= \{ 2\mathcal{C}^2(\mu_D + 3\mu_F), 2\mathcal{C}^2(\mu_D - 3\mu_F), 2\mathcal{C}^2\mu_D,
\]
\[
0, 2\mathcal{C}^2(\mu_D - 3\mu_F), 0, -4\mathcal{C}^2\mu_D, 0, 0 \}
\]

(40)

\[
T_{ab88}^{(2i)} = \sum_c V_{ba} T_{ac8} R_{ca8},
\]
\[
= \{ 2\mathcal{C}^2(\mu_D + 3\mu_F), 2\mathcal{C}^2(\mu_D - 3\mu_F), 2\mathcal{C}^2\mu_D,
\]
\[
0, 2\mathcal{C}^2(\mu_D - 3\mu_F), 0, -4\mathcal{C}^2\mu_D, 0, 2\sqrt{3}\mathcal{C}^2\mu_D \}
\]

(41)

where

\[
T_{bac} = \mathcal{C}\varepsilon_{ijk}(\tilde{\lambda}_b^i)^i_j^i_m^i \Lambda_{klm}
\]

(42)

\[
R_{bac} = \mathcal{C}\varepsilon_{ijk} \tilde{\lambda}_{klm}(\tilde{\lambda}_c^m)^i_j^i_l^i m
\]

(43)

\[
V_{ba} = \mu_D \tilde{\lambda}_b^i \{ Q, \tilde{\lambda}_a \} + \mu_F \tilde{\lambda}_b^i \{ Q, \tilde{\lambda}_a \},
\]

(44)

\[
U_{ba} = \mu_C \tilde{\lambda}_{ijk}(Q)^i_j^i_l^i \Lambda_{ijkl}
\]

(45)
Table 1: The deviation $T(K)$ from the Okubo relation for the octet baryon magnetic moments in each label $K$ diagram. All moments are given in nuclear magnetons

| $K$     | $T(K)$                                 |
|---------|----------------------------------------|
| Fig.(1a) | 0                                      |
| Fig.(1b) | 0                                      |
| Fig.(2a) | $\frac{2}{3}\mu_D(D^2 + 3F^2)$        |
| Fig.(2b) | 0                                      |
| Fig.(2c) | 0                                      |
| Fig.(2d) | $\mu_T C(D - 2F)$                     |
| Fig.(2e) | $-\mu_T C(D + 2F)$                    |
| Fig.(2f) | $-\frac{2}{3}\mu_D(D^2 - 3F^2)$      |
| Fig.(2g) | $-\frac{2}{3}\mu_D(D^2 - 3F^2)$      |
| Fig.(2h) | $-\frac{4}{3}\mu_D C^2$              |
| Fig.(2i) | $-\frac{4}{3}\mu_D C^2$              |

$$X_{ba} = \mu_T \epsilon^{ijk} Q_i^l \bar{A}_{k}^{lm} (\tilde{\Lambda}_a)_j^m$$  \hspace{1cm} (46) $$Y_{ba} = \mu_T \epsilon_{ijk} Q_i^l (\tilde{\Lambda}_b)_j^m A^{klm}$$ \hspace{1cm} (47) with the coupling constants $D$, $F$, $C$, $\mu_D$, $\mu_F$, $\mu_C$, and $\mu_T$ defined in Ref. [6]. The vertex factors $F_{bac}$, $G_{ba}$, and $H_{bac}$ are given in Eqs. (6), (7), and (8), respectively.

All the one-loop diagrams in Fig. (2) contribute the $m_q \ln m_q$ corrections to the hyperon magnetic moments, but only the contributions of the diagrams in Figs. (2b) and (2c) explicitly satisfy the Okubo relation.

For the diagrams in Figs. (2a), (2d), (2e), (2f), (2g), (2h) and (2i) the fact that $T(K) \neq 0$ individually means that each diagram violates the Okubo relation. However, one must check explicitly whether or not there is any cancellation when these contributions from different diagrams add up. According to their coupling constant factors, the Okubo relation violating one-loop diagrams can be classified into three groups: (I) diagrams (2a), (2f), (2g) with factor $\mu_D D^2$ or $\mu_D F^2$; (II) diagrams (2d), (2e) with factor $\mu_T CD$ or $\mu_T CF$; and (III) diagrams (2h), (2i) with factor $\mu_D \cdot C^2$. 

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The Feynman integral $I$ in Eq. (21) for diagram Fig. (2a) is

$$I^{(2a)}(M^2) = -\frac{1}{32\pi^2 f^2} M^2 \ln\left(\frac{M^2}{\mu^2}\right), \quad (48)$$

where $\mu$ is an arbitrary renormalization scale and $M$ is the physical mass of the intermediate meson state. The contributions from Figs. (2f) and (2g) correspond to wave function renormalization with intermediate octet baryon. The integrals $I$ for both diagrams turn out to be the same and equal to

$$I^{(2f)}(M^2) = -\frac{3}{2} \cdot \frac{1}{32\pi^2 f^2} M^2 \ln\left(\frac{M^2}{\mu^2}\right). \quad (49)$$

Summing over the contributions from Figs. (2a), (2f), and (2g), we get

$$\sum_{K=\text{Fig.}(2a),(2f),(2g)} \Omega(K) = \frac{1}{32\pi^2 f^2} \left[3 M_\eta^2 \ln\left(\frac{M_\eta^2}{\mu^2}\right) - 4 M_K^2 \ln\left(\frac{M_K^2}{\mu^2}\right) + M_\pi^2 \ln\left(\frac{M_\pi^2}{\mu^2}\right)\right] \left[\frac{16}{9} \mu_D(D^2 - 6 F^2)\right]. \quad (50)$$

For the diagrams in Figs. (2d) and (2e), using the SU(3) invariant mass values for intermediate octet and decuplet baryons, the one-loop integrals $I(M^2)$ for both diagrams can be easily shown to be equal and

$$I^{(2d)}(M^2) = \frac{2}{3} \cdot \frac{1}{32\pi^2 f^2} M^2 \ln\left(\frac{M^2}{\mu^2}\right). \quad (51)$$

Figs. (2h) and (2i) are related to the wave function renormalization with the intermediate decuplet baryon. The integration function $I(M^2)$ for Fig. (2h) or (2i) is

$$I^{(2h)}(M^2) = -\frac{1}{32\pi^2 f^2} M^2 \ln\left(\frac{M^2}{\mu^2}\right). \quad (52)$$

Summing over all the contributions in Fig. (2), the deviation from the Okubo relation is

$$\Omega = \sum_{K} \Omega(K) = \frac{1}{32\pi^2 f^2} \left[3 M_\eta^2 \ln\left(\frac{M_\eta^2}{\mu^2}\right) - 4 M_K^2 \ln\left(\frac{M_K^2}{\mu^2}\right) + M_\pi^2 \ln\left(\frac{M_\pi^2}{\mu^2}\right)\right] \left[\frac{4}{9} \mu_D(D^2 - 24 F^2 - C^2) + \frac{8}{9} \mu_T C F\right]. \quad (53)$$
If we add up the contributions listed in Ref. [6] from various diagrams, the result actually consistent with our sum above. However, unless there is a very special relation among $D$, $F$, $C$, $\mu_D$ and $\mu_T$, which is not the case, the Okubo relation is indeed violated by the logarithmic corrections in $m_q$ contrary to the claim in Ref. [3].

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Figure captions

**Fig. 1:** One-loop diagrams that give rise to non-analytic $\sqrt{m_q}$ corrections to the hyperon magnetic moments. The dashed lines denote the pions, the single solid lines denote octet hyperons, and the double solid lines denote decuplet hyperons.

**Fig. 2:** One-loop diagrams that give rise to non-analytic $m_q \ln m_q$ corrections to the hyperon magnetic moments. See Fig. (1) for the meaning of the lines.
Fig. 1
Fig. 2