Integral constraints in spectroscopic surveys

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Abstract. Current clustering analyses of galaxy surveys rely on the knowledge of the survey selection function. Density fluctuations are estimated by comparing the galaxy density field to a random synthetic catalogue accounting for the survey density and geometry. However, this survey selection function is commonly partly inferred from the observed data itself, leading to so-called integral constraints. We present a new derivation of the global integral constraint effect, arising when the expected galaxy density is taken to be the measured one. We extend the formalism to the case where the full radial selection function is estimated from the data redshift distribution, as is often the case in the literature. We find that the radial integral constraint effect can be as significant as the window function correction at large scales. We model the radial integral constraint for a Redshift Space Distortions (RSD) analysis but we emphasise that it can be of paramount importance for large-scale studies of primordial non-Gaussianity. Its effect in configuration space cannot be safely ignored either. Finally, as a further application, we show that potential angular systematics can be mitigated by nulling the density contrast on a chosen angular scale. We model the subsequent loss of clustering by an angular integral constraint which can be combined with the radial one. We review the survey selection function normalisation, the shot noise contribution to the integral constraint corrections and wide-angle contributions.
Spectroscopic surveys measure redshift-space positions of galaxies (or any other tracer of the matter density field). These surveys are not exhaustive and are characterised by a selection function $W(r)$, giving the expected mean density of observed galaxies at any redshift-space position $r$ in the absence of clustering. In case one has a full knowledge of this selection function, $n$-point statistics can be fairly estimated from the density fluctuations:

$$F(r) = n_g(r) - W(r)$$  (1.1)
where \( n_g(\mathbf{r}) \) denotes the observed density of galaxies. In particular, the mean density constraint \( \int d^3 r F(\mathbf{r}) \) can be non-zero, due to large-scale clustering modes.

However, the true selection function is difficult to determine in practice. Its norm, i.e., the expected mean density of galaxies, may be unknown. It is commonly taken to be the actually observed mean data density. Thus, the integral of \( F(\mathbf{r}) \) over the whole survey footprint is fixed to zero, leading to a so-called integral constraint \([1, 2]\). We propose a new derivation of this global integral constraint in section 2.

In the same spirit, the radial survey selection function may be estimated from the data itself, as it is the case in the current clustering analyses of the Baryon Oscillation Spectroscopic Survey (BOSS) (e.g. \([3–5]\)) and its extended program eBOSS (e.g. \([6, 7]\)). We show that this technique can have a significant effect on clustering measurements and model this effect as a radial integral constraint in section 3, using the same formalism as developed for the global integral constraint.

The computation of window functions required for our treatment of integral constraints is discussed in section 4.

We show the effect of the radial integral constraint in a RSD analysis of realistic BOSS-like mocks in section 5. Details about the set of high fidelity and approximate mock catalogues used to estimate the covariance matrix required for cosmological fits are given in section 5.1.

As an application of our formalism, we suggest in section 6 to mitigate potential angular systematics by imposing an angular integral constraint, which can usually be combined with the radial one.

## 2 Global integral constraint

### 2.1 Problem statement

Predicting the mean galaxy density of a survey is complex in practice. For example, following \([8]\) the Yamamoto power spectrum estimator \([9]\) makes use of the FKP field:

\[
F(\mathbf{r}) = n_g(\mathbf{r}) - \alpha n_s(\mathbf{r})
\] (2.1)

where \( n_g(\mathbf{r}) \) and \( n_s(\mathbf{r}) \) denote the density of observed and random galaxies, respectively. Random galaxies come from a Poisson sampled, synthetic catalogue which is assumed to reproduce the survey selection function, to some normalisation factor \( \alpha \). Then, \( \alpha \) is fixed by \( \alpha = \sum_{i=1}^{N_g} w_{g,i} / \sum_{i=1}^{N_s} w_{s,i} \), with \( w_g \) and \( w_s \) the weights of observed and random galaxies, which may include e.g., corrections for systematics effects or a redshift weighting scheme (such as FKP weights, see appendix A). Thus, by construction,

\[
\int d^3 r F(\mathbf{r}) = \sum_{i=1}^{N_g} w_{g,i} - \alpha \sum_{i=1}^{N_s} w_{s,i} = 0.
\] (2.2)

In practice, this global integral constraint damps the power spectrum on scales approaching the survey size. Ref. \([2]\), following \([1]\), accounts for this effect by a constant term removed from the correlation function and multiplied by the window function resulting from the finite survey geometry. Thus, they express the observed window-convolved, integral-constraint-corrected (subscript cic) power spectrum as:

\[
P^{\text{cic}}(\mathbf{k}) = P^c(\mathbf{k}) - P^c(0) |W(\mathbf{k})|^2
\] (2.3)
with $P^c(k)$ the window-convolved (subscript $c$) power spectrum. $W(k)$ is the (Fourier transform of) the survey selection function, rescaled by imposing $|W(k)|^2 = 1$ when $k \ll 1 \, h/\text{Mpc}$, so that $P^c(k) \to 0$ as $k \to 0$. A new, exact, formulation of this global integral constraint is derived in the remainder of this section.

### 2.2 Density contrast

The observed galaxy density is $n_g(r) = W(r) \{1 + \delta(r)\}$, with $W(r)$ the survey selection function and $\delta(r)$ the density contrast. We assume the shape of the survey selection function is known, and sampled by the synthetic catalogue: $n_s(r) \propto W(r)$. The scaling $\alpha$ is given by:

$$\alpha = \frac{\sum_{i=1}^{N_g} w_{g,i} n_g(x)}{\sum_{i=1}^{N_s} w_{s,i} n_s(x)} = \int d^3 x n_g(x) \int d^3 x n_s(x).$$  \hspace{1cm} (2.4)

Then, the observed density fluctuations are:

$$\delta^c(r) = n_g(r) - \alpha n_s(r)$$

$$= W(r) \{1 + \delta(r)\} - W(r) \frac{\int d^3 x W(x) \{1 + \delta(x)\}}{\int d^3 x W(x)}$$

$$= W(r) \left\{ \delta(r) - \int d^3 x W_{\text{glob}}(x) \delta(x) \right\},$$  \hspace{1cm} (2.5)

with $W_{\text{glob}}(r) = \frac{W(r)}{\int d^3 x W(x)}$.

We find two terms, $W(r)\delta(r)$ corresponding to the density contrast weighted by the selection function, and the integral constraint term $\int d^3 r W_{\text{glob}}(r) \delta(r)$. The normalisation of $W_{\text{glob}}(r)$ ensures that $\int d^3 r \delta^c(r) = 0$ over the entire footprint: modes larger than the survey size are suppressed.

### 2.3 Correlation function in the local plane-parallel approximation

We are interested in the even multipoles of the window-convolved, integral-constraint-corrected power spectrum $P^c_{\ell}(k)$. We will focus on the correlation function multipoles $\xi^c_{\ell}(s)$ since power spectrum and correlation function multipoles form a Hankel transform pair (see e.g. [2]):

$$P^c_{\ell}(k) = 4\pi (-i)^\ell \int s^2 ds j_\ell(k s) \xi^c_{\ell}(s).$$  \hspace{1cm} (2.6)

We assume the local plane parallel approximation, namely that the pair separation $s$ between two galaxies is small compared to the distance $x$ of any of the two galaxies to the observer. Then, the cosine angle between $s$ and $x$ can be well approximated by $\hat{\eta}_s \cdot \hat{s}$, with $\eta_s = x + s/2$ the mid-point line-of-sight and $\hat{s}$ the unit $s$ vector. We report the reader to appendix B for a full derivation in the case of the end-point line-of-sight for the Yamamoto estimator, taking into account wide-angle corrections to the plane parallel approximation.

As $W(r)$ is uncorrelated to $\delta(r)$, the window-convolved, integral-constraint-corrected correlation function can be expressed from the density fluctuations of eq. (2.5), to some
global normalisation factor discussed in section 4.2:

\[
\xi^{\text{cic}}(s) = \int d^3x W(x) W(x + s) \xi(s) - \int d^3 \Delta \xi(\Delta) \int d^3x W(x) W(x + s) W_{\text{glo}}(x + \Delta) .
\]

(2.7a)

\[
- \int d^3 \Delta \xi(\Delta) \int d^3x W(x) W(x - s) W_{\text{glo}}(x + \Delta) .
\]

(2.7b)

\[
+ \int d^3 \Delta \xi(\Delta) \int d^3x W(x) W(x + s) \int d^3y W_{\text{glo}}(y) W_{\text{glo}}(y + \Delta) .
\]

(2.7c)

Term (2.7a) is the true correlation function \(\xi(s)\) multiplied by the window function \(W_{\delta,\delta}(s) = \int d^3x W(x) W(x + s)\). We account for the multipoles of this term using the formalism of [2]:

\[
\xi^\ell(\hat{s}) = \sum_{p \neq q} A^p_q \frac{2\ell + 1}{2q + 1} \xi_p(s) W_{\delta,\delta}^q(\hat{s})
\]

(2.8)

where we use \(\xi_p(s)\) the multipoles of \(\xi(s)\). The window function multipoles are given by:

\[
W_{\delta,\delta}^q(s) = \frac{2\ell + 1}{4\pi} \int d\Omega_s \int d^3x W(x) W(x + s) \mathcal{L}_\ell(\hat{n}_s \cdot \hat{s})
\]

(2.9)

and coefficient \(A^p_q\) is defined by:

\[
\mathcal{L}_\ell(\mu) L_p(\mu) = \sum_{q=0}^{\ell+p} A^p_q \mathcal{L}_q(\mu).
\]

(2.10)

Cross-terms (2.7b) and (2.7c) account for the correlation between the density field and the integral constraint term of eq. (2.5). We are only interested in even multipoles w.r.t. the mid-point line-of-sight, so term \(IC_{\ell}^{\delta,\text{glo}}\) (2.7c) is equal to \(IC_{\ell}^{\delta,\text{glo}}\) (2.7b). We are thus left with evaluating the multipoles of term (2.7b):

\[
IC_{\ell}^{\delta,\text{glo}}(s) = \frac{2\ell + 1}{4\pi} \int d\Omega_s \int d^3 \Delta \xi(\Delta) \int d^3x W(x) W(x + s) W_{\text{glo}}(x + \Delta) \mathcal{L}_\ell(\hat{n}_s \cdot \hat{s}) .
\]

(2.11)

The redshift-space correlation function is fully described by its Legendre multipoles. We thus develop:

\[
\xi(\Delta) = \sum_\ell \xi_\ell(\Delta) \mathcal{L}_\ell(\hat{n}_\Delta \cdot \hat{\Delta}),
\]

(2.12)

leading to:

\[
IC_{\ell}^{\delta,\text{glo}}(s) = \frac{2\ell + 1}{4\pi} \int d\Omega_s \int \Delta^2 \Delta \int d\Omega_{\Delta} \sum_p \xi_p(\Delta)
\]

\[
\int d^3x W(x) W(x + s) W_{\text{glo}}(x + \Delta) \mathcal{L}_\ell(\hat{n}_s \cdot \hat{s}) \mathcal{L}_p(\hat{n}_\Delta \cdot \hat{\Delta}).
\]

(2.13)

Writing the 3-point correlation function of the selection function:

\[
\mathcal{W}_{\ell p}^{\delta,\text{glo}}(s, \Delta) = \frac{(2\ell + 1)(2p + 1)}{(4\pi)^2} \int d\Omega_s \int d\Omega_{\Delta}
\]

\[
\int d^3x W(x) W(x + s) W_{\text{glo}}(x + \Delta) \mathcal{L}_\ell(\hat{n}_s \cdot \hat{s}) \mathcal{L}_p(\hat{n}_\Delta \cdot \hat{\Delta}),
\]

(2.14)
we obtain the simple formula:

$$IC_{\delta, glo}^{\delta, glo}(s) = \int \Delta^2 d\Delta \sum_{p} \frac{4\pi}{2p + 1} \xi_p(\Delta) W_{\delta, \delta}^{\delta, glo}(s, \Delta).$$  \hfill (2.15)

Finally, term \((2.7d)\) accounts for the auto-correlation of the integral term in eq. \((2.5)\), and its multipoles are simply equal to:

$$IC_{\delta, glo}^{\delta, glo}(s) = \int \Delta^2 d\Delta \xi_{c}(\Delta) W_{\delta, \delta}^{\delta, glo}(s),$$  \hfill (2.16)

with \(\xi_{c}^{\ell}(s)\) defined in eq. \((2.8)\). Then, the convolved, integral-constraint-corrected correlation function multipoles are:

$$\xi_{c, ic}^{\ell}(s) = \xi_{c}^{\ell}(s) - IC_{\delta, glo}^{\delta, glo}(s) - IC_{\delta, glo}^{\delta, glo}(s) + IC_{\delta, glo}^{\delta, glo}(s).$$  \hfill (2.17)

We detail the different terms contributing to the global integral constraint on the left panel of figure 1. The effect of the complete global integral constraint is negligible at the scales involved in a RSD analysis \((k \gtrsim 0.01 \ h/Mpc)\), but is significant at large scales, where the monopole reaches 0, as expected. After a Fourier transform, the auto-correlation term \(IC_{\delta, glo}^{\delta, glo}(2.7d)\) corresponds to the last term of eq. \((2.3)\) suggested in \([1]\) and \([2]\) to model the global integral constraint, with an opposite sign. As shown in figure 1 (left panel), taking only this term for the integral constraint correction appears to be a very legitimate approximation in the illustrated case.

3 Radial integral constraint

3.1 Problem statement

The true radial selection function of a spectroscopic survey is often a complex function of the luminosity function, sky lines, spectrograph efficiency and redshift determination algorithm \([10]\). Thus, in BOSS the radial distribution of the synthetic catalogue is directly inferred from the data. Various techniques exist: random redshifts can be picked from the whole data
redshift distribution (the so-called shuffled scheme [11, 12]), or, assuming the true radial selection function should be somewhat smooth, it can be fitted by a spline from which random redshifts are drawn [11]. A third possibility (binned scheme) is to weight an arbitrary initial random redshift distribution to match the data radial density in redshift or comoving distance bins.

As can be seen in figure 2, similar power spectrum measurements are obtained with the binned scheme (using radial bins of size 2 Mpc/h) and the shuffled scheme. They both result in a loss of power parallel to the line-of-sight, thus mostly affecting the power spectrum quadrupole and hexadecapole, compared to the baseline relying on the true survey selection function. In this example, effects of the binned and shuffled schemes are artificially enhanced by determining random redshifts in 6 small chunks of the whole footprint, as detailed in section 5.1.
3.2 Density contrast

Following the same formalism as eq. (2.5), density fluctuations are forced to be zero in any redshift slice:

$$\delta^{\text{cic}}(r) = W(r) \left\{ \delta(r) - \int d^3x W_{\text{rad}}(x) \delta(x) \epsilon_{\text{rad}}(r, x) \right\}$$  \hspace{1cm} (3.1)

with \( \epsilon_{\text{rad}}(r, x) = \delta^D(r - x) \), and:

$$W_{\text{rad}}(r) = \frac{W(r)}{\int d^3x W(x) \epsilon_{\text{rad}}(r, x)}.$$  \hspace{1cm} (3.2)

\( \int d^3x W_{\text{rad}}(x) \delta(x) \epsilon_{\text{rad}}(r, x) \) is the integrated density contrast multiplied by the radially-normalised selection function over a plane perpendicular to the line-of-sight at distance \( r \). Note that the global integral constraint is thus also automatically imposed.

3.3 Correlation function in the local plane-parallel approximation

The correlation function can be expressed from the density fluctuations of eq. (3.1) according to:

$$\xi^{\text{cic}}(s) = \int d^3x W(x) W(x + s) \xi(s)$$  \hspace{1cm} (3.3a)

$$- \int d^3x \Delta \xi(\Delta) \int d^3x W(x) W(x + s) W_{\text{rad}}(x + \Delta) \epsilon_{\text{rad}}(x + s, x + \Delta)$$  \hspace{1cm} (3.3b)

$$- \int d^3x \Delta \xi(\Delta) \int d^3x W(x) W(x - s) W_{\text{rad}}(x + \Delta) \epsilon_{\text{rad}}(x - s, x + \Delta)$$  \hspace{1cm} (3.3c)

$$+ \int d^3x \Delta \xi(\Delta) \int d^3x W(x) W(x + s) \epsilon_{\text{rad}}(x, y)$$  \hspace{1cm} (3.3d)

Term (3.3a) is equal to (2.7a) and is treated in the same manner. We recall that we use the plane-parallel approximation and we are interested in even multipoles only, so term \( IC_{\ell}^{\text{rad},\delta} \) (3.3c) is equal to \( IC_{\ell}^{\text{rad},\epsilon} \) (3.3b). Similarly to section 2.3, multipoles of terms (3.3b) and (3.3d) become:

$$IC_{\ell}^{\text{rad},i,j}(s) = \int \Delta d\Delta \sum_p \frac{4\pi}{2p + 1} \xi_p(\Delta) W_{\ell p}^{i,j}(s, \Delta) \quad (i, j) \in \{(\delta, \text{rad}), (\text{rad}, \text{rad})\},$$  \hspace{1cm} (3.4)

where we use the window function multipoles:

$$W_{\ell p}^{\delta,\text{rad}}(s, \Delta) = \frac{(2\ell + 1)(2p + 1)}{(4\pi)^2} \int d\Omega_s \int d\Omega_{\Delta}$$  \hspace{1cm} (3.5)

$$\int d^3x W(x) W(x + s) W_{\text{rad}}(x + \Delta) L_\ell(\hat{n}_s \cdot \hat{s}) L_p(\hat{n}_\Delta \cdot \hat{x}) \epsilon_{\text{rad}}(x + s, x + \Delta)$$

and

$$W_{\ell p}^{\epsilon,\text{rad}}(s, \Delta) = \frac{(2\ell + 1)(2p + 1)}{(4\pi)^2} \int d\Omega_s \int d\Omega_{\Delta} \int d^3x W(x) W(x + s)$$  \hspace{1cm} (3.6)

$$\int d^3y W_{\text{rad}}(y) W_{\text{rad}}(y + \Delta) L_\ell(\hat{n}_s \cdot \hat{s}) L_p(\hat{n}_\Delta \cdot \hat{\Delta}) \epsilon_{\text{rad}}(x, y) \epsilon_{\text{rad}}(x + s, y + \Delta).$$

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The expressions of $IC_{\ell}^{s,\text{glo}}(s)$ (eq. (2.15)) and $IC_{\ell}^{s,\text{glo},\text{glo}}(s)$ (eq. (2.16)) for the global integral constraint are recovered by setting $\epsilon_{\text{rad}}$ to 1. We use a similar equation to (2.17) to compute the window-convolved, integral-constraint-corrected correlation function multipoles:

$$\xi_{\ell}^{\text{IC}}(s) = \xi_{\ell}(s) - IC_{\ell}^{s,\text{rad}}(s) - IC_{\ell}^{s,\text{rad},\delta}(s) + IC_{\ell}^{s,\text{rad},\text{rad}}(s).$$

(3.7)

The right panel of figure 1 displays the different contributions to the radial integral constraint. Compared to the global integral constraint, the total radial integral constraint has a larger effect in the quadrupole and hexadecapole, where differences of the full integral constraint correction to $-IC_{\ell}^{s,\text{rad},\text{rad}}$ are also slightly more significant.

4 Computing window functions

This section is devoted to the practical computation of window functions $W_\ell^{s,\delta}(s)$ and $W_\ell^{s,j}(s,\Delta)$ required in sections 2.3 and 3.3. We argue that a reasonable approximation for the true selection function can be inferred from data in section 4.1. We then discuss the normalisation of the window functions, and detail their response to a Poisson shot noise (spike at $s, \Delta = 0$) in sections 4.2 and 4.3. Algorithms required to compute these window functions from a synthetic catalogue sampling the survey selection function are given in section 4.4.

4.1 Approaching the true survey selection function

Our derivation of the integral constraint corrections makes use of the true, underlying, survey selection function $W(r)$. However, as we mentioned previously, one has only access to a synthetic catalogue whose radial density $n_s(r)$ is tuned to match the actual data. Thus, clustering modes imprinted in the radial data distribution are propagated to this synthetic catalogue. Interestingly enough, the normalised $W_{\text{rad}}(r)$ calculated using $n_s(r)$, whose radial part $n_s(r)$ is tuned on the observed data, is equal to the true one. However, if that radially-tuned $n_s(r)$ is taken for $W(r)$, the calculated window effect statistically differs from the truth.

Small discrepancies, at the level of the uncertainties on the mean of the mocks, have been noticed between predictions using the true survey selection function $W(r)$ and that based on the selection function $n_s(r)$ calculated with a synthetic catalogue tuned to match the radial distribution (using the binned scheme) of one realisation of mock data. We checked that these differences are almost completely absorbed by nuisance parameters when fitting mock data and that no bias is induced on cosmological parameters. We therefore estimate the radial selection function from data throughout the paper, as would be done in an actual data analysis.

We can nevertheless estimate first order corrections to this approximation. Let us recall that:

$$n_s(r) = W(r) \left\{ 1 + \int d^3x W_{\text{rad}}(x) \delta(x) \epsilon_{\text{rad}}(r, x) \right\},$$

(4.1)

where $W_{\text{rad}}(r)$ is well described by $n_{s,\text{rad}}(r) = \frac{n_s(r)}{\int d^3x n_s(x) \epsilon_{\text{rad}}(r, x)}$. Then, the multipoles of the 2-point window function estimated from $n_s(r)$ can be written:

$$S_\ell^{s,\delta}(s) = W_\ell^{s,\delta}(s) + IC_{\ell}^{s,\text{rad},\text{rad}}(s).$$

(4.2)

Taking $n_s(r)$ for $W(r)$ in $IC_{\ell}^{s,\text{rad},\text{rad}}(s)$, eq. (4.2) provides an estimate for $W_\ell^{s,\delta}(s)$ at first order in the radial integral constraint corrections. We checked that using this estimate for $W_\ell^{s,\delta}(s)$ in the model almost completely removes the small discrepancies mentioned hereabove.
4.2 Normalisation

Ref. [2] suggested to normalise the 2-point window function in the limit $s \to 0$. However, this limit may be ill-defined in practice, since its estimation requires a very fine sampling of the window function. In current Fourier space analyses, power spectrum measurements are normalised by [4]:

$$A = \alpha^2 \int d^3 r n_s^2(r) = \alpha^2 \sum_{i=1}^{N_s} w_{s,i} n_{s,i},$$  \tag{4.3}

using the notations of eq. (2.1). This corresponds to the limit $\mathcal{W}^{\delta,\delta}(0) = \int d^3 x W^2(x)$ if $W$ is represented by the same synthetic catalogue as in eq. (4.3).

Besides, the estimation of $n_s(r)$ in eq. (4.3) is in practice non-trivial when accounting for various survey selection effects. For example, $n_s(r)$ is commonly computed in redshift slices only [12], while it can also be a function of the angular position on the sky.

We thus decide to also use term (4.3) in the normalisation of window functions $\mathcal{W}^{\delta,\delta}_\ell$ and $\mathcal{W}^{i,j}_\ell$, so that $A$ terms divide both the power spectrum measurements and model, and compensate. Therefore, the estimation of $A$ does not impact the estimation of cosmological parameters. Integral constraint terms are properly normalised using e.g. eq. (3.2).

4.3 Shot noise

In principle, formulae given in section 2.3 and section 3.3 fully describe global and radial integral constraint corrections. However, they are derived in real space, where accounting for a shot noise term (a spike at $s = 0$) is numerically challenging.

The Poisson shot noise term is a Dirac at $s = 0$ in the monopole of the true (unconvolved) correlation function. The survey selection function being fully isotropic as the separation $s \to 0$, the shot noise contribution to the convolved correlation function only shows up in the monopole:

$$\xi^\delta_0(s) \ni \mathcal{W}^{\delta,\delta}_0(0) \delta^D(s) N,$$  \tag{4.4}

which, in Fourier space, is a simple offset in the monopole. However, the shot noise contribution to the integral constraint correction leaks on all scales into all multipoles, e.g. in the case of the radial integral constraint:

$$IC^{i,j}_\ell(s) \ni \mathcal{W}^{i,j}_0(0) N \quad (i, j) \in \{ (\delta, \text{rad}), (\text{rad}, \delta), (\text{rad}, \text{rad}) \},$$  \tag{4.5}

with:

$$\mathcal{W}^{\text{rad},\delta}_0(s, 0) = \frac{2\ell + 1}{4\pi} \int d\Omega_s \int d^3 x W(x) W_{\text{rad}}(x) W(x + s) L_\ell(\hat{n}_s \cdot \hat{s}) \epsilon_{\text{rad}}(x, x + s)$$  \tag{4.6}

and

$$\mathcal{W}^{\text{rad},\text{rad}}_0(s, 0) = \frac{2\ell + 1}{4\pi} \int d\Omega_s \int d^3 x W(x) W(x + s) \int d^3 y W^2_{\text{rad}}(y) L_\ell(\hat{n}_s \cdot \hat{s}) \epsilon_{\text{rad}}(x, y) \epsilon_{\text{rad}}(x + s, y)$$  \tag{4.7}

At first sight, an accurate estimate of the ill-defined limit $\mathcal{W}^{\delta,\delta}(0) = \int d^3 x W^2(x)$ would be required. However, $N$ can be rescaled according to $N \to \mathcal{W}^{\delta,\delta}_0(0) N$. Since the shot noise contribution to $\xi^\delta_0(s)$ and $IC^{i,j}_\ell(s)$ should match on large scales, the correct normalisation for
Figure 3. Normalised shot noise contributions from the global and radial integral constraints (blue: monopole, red: quadrupole, green: hexadecapole). As expected, the Poisson shot noise in $P_c(k)$ is cancelled by the integral constraint contribution at large scales.

Figure 4. Left: the window function multipoles $W^{\delta,\delta}_\ell(s)$. Right: the window function multipoles $W^{\mathrm{rad,rad}}_\ell(s,\Delta)$. $W^{\mathrm{rad,\delta}}_\ell(s,0)$ and $W^{\mathrm{rad,rad}}_\ell(s,0)$ is their integral over $s$. The same result holds for the global integral constraint, with $\epsilon_{\mathrm{rad}} \rightarrow 1$.

Shot noise contributions to the global and radial integral constraints are shown in figure 3. By definition, they reach 1 as $k \rightarrow 0$ in the monopole. Their impact on the quadrupole and hexadecapole cannot be ignored in the case of the radial integral constraint.

4.4 Calculation from a synthetic catalogue

The survey selection function is randomly sampled by a synthetic catalogue. We use a classic pair-count algorithm to compute the anisotropic 2-point correlation and implement the algorithm from [13] to compute the anisotropic 3-point correlation of the synthetic catalogue. We define the lines-of-sight according to our power spectrum estimator, as detailed in appendix B. We show $W^{\delta,\delta}_\ell(s)$ and $W^{\mathrm{rad,rad}}_\ell(s,\Delta)$ in figure 4.

\[1\]In the case $W$ is sampled by a synthetic catalogue, the normalisation for $W^{\mathrm{rad,\delta}}_\ell(s,0)$ and $W^{\mathrm{rad,rad}}_\ell(s,0)$ is the (weighted) number of correlated pairs.
In order to compute (3.2), we slice the synthetic catalogue in comoving distance bins of size $\delta r$, and normalise density in each radial bin. Thus, we formally replace $\epsilon_{\text{rad}}(x, y) = \delta^D(y-x)$ by a top-hat function centered on 0 of width $\delta r$. This reformulation is an exact derivation in the case redshifts of the random catalogue are forced to match the data redshift distribution in the same bins of size $\delta r$, which is the case of the binned scheme, with $\delta r = 2$ Mpc/$h$. In general, the bin size $\delta r$ should represent the radial scale where the radial integral constraint is imposed.

All window functions (for the global and radial integral constraints) could be accurately computed in $\sim 1500$ CPU hours.

5 Radial integral constraint in RSD analyses

5.1 Mock catalogues

We work on a set of 84 high fidelity N-series mocks used for the LRG clustering mock challenge [14] of the BOSS Data Release 12 (DR12) [5]. These mocks were built from the 3 projections of 7 independent, periodic box realisations of side 2600 Mpc/$h$ at redshift $z = 0$. The simulated cosmology is a flat $\Lambda$CDM model with $\Omega_m = 0.286$, $\Omega_\Lambda = 0.714$, $\sigma_8 = 0.82$, $n_s = 0.96$ and $h = 0.7$. Simulations were run with the GADGET2 code [15], and a HOD modelling was used to populate dark matter halos with galaxies, mimicking the observed clustering in the data.

The precision matrix required for cosmological fits is built from 2048 Multidark Patchy mocks provided by the BOSS collaboration. These approximate mocks were calibrated on BigMultiDark simulations [16]; halo abundance matching was applied to reproduce the 2 and 3-point clustering measurements and mocks at different redshifts were combined into light cones [17]. We use V6C catalogues, which were adjusted to reproduce the data clustering measurements. These mocks are also used to compute the standard deviation (blue shaded area) shown in figures 2, 8 and 12.

Both N-series and Multidark Patchy mocks were trimmed following the DR12 CMASS NGC selection function using the make_survey software [18]. To enhance the radial integral constraint effect coming from the binned or shuffled schemes, we divide the CMASS footprint ($\approx 7420$ deg$^2$) in 6 chunks (from $\approx 980$ deg$^2$ to $\approx 1570$ deg$^2$; see figure 5). The global and radial integral constraints are applied to the 6 chunks separately, before they are combined for the power spectrum measurement. This procedure, though not representative of the real BOSS DR12 analysis, allows us to test our modelling of the radial integral constraint in stringent conditions. Note that the radial selection function of future spectroscopic surveys like DESI [19] and Euclid [20] may vary on the sky; modelling this effect may indeed consist in combining small patches with locally constant radial selection functions.

We account for the chunk-splitting by adding the condition that $r$ and $x$ should belong to the same chunk for $\epsilon_{\text{rad}}(r, x)$ to be non-zero (similarly for the global integral constraint). We impose integral constraints consistently on N-series and Multidark Patchy mocks in each series of cosmological fits.

We apply FKP weights [8] $w_{\text{FKP}} = \frac{1}{1+n(z)P_0}$ to both mock data and randoms using $P_0 = 20000$ (Mpc/$h$)$^3$. To obtain the baseline power spectrum measurements, we compute the density $n(z)$ in bins $\Delta z = 0.005$ (as in [12]) on the full random catalogue accounting for the true survey selection function. As would be done on real data, the radial density $n(z)$ is computed for each CMASS mock data realisation when we impose the radial integral constraint (binned or shuffled scheme). As we divide the CMASS footprint into 6 chunks, $n(z)$
should in principle be measured in the 6 chunks separately. However, we show in appendix A that doing so significantly biases clustering measurements as FKP weights, using local $n(z)$, smooth out clustering. We therefore choose to measure $n(z)$ from the whole CMASS sample, making the bias due to FKP weights negligible.

5.2 Analysis methods

We use the Python toolkit \texttt{nbodykit} \cite{Munoz09} to compute mock data power spectra. We take a large box size of 4000 Mpc$/h$ to reduce sampling effects in low $k$-bins. The FKP field is interpolated on a $512^3$ mesh following the Triangular Shaped Cloud (TSC) scheme. The Nyquist frequency is thus $k \approx 0.4$ h$/\text{Mpc}$, more than twice larger than the maximum wavenumber used in our analysis. We employ the interlacing technique \cite{McCloskey10} to mitigate aliasing effects. Power spectrum multipoles are measured in bins of $\Delta k = 0.01$ h$/\text{Mpc}$, from $k = 0$ h$/\text{Mpc}$.

Power spectrum multipoles are calculated on a discrete $k$-space grid, making the angular modes distribution irregular at large scales. We correct for this effect using the technique employed in \cite{Kazin2014}. The correction is very small, given the large box size used to measure power spectra. Hankel transforms between power spectrum and correlation functions multipoles (e.g. (2.6)) are performed using the \texttt{FFTLog} \cite{Eisenstein02} software. As in \cite{Kazin2014} we only consider correlation function multipoles up to $\ell = 4$ in our calculations. We checked that adding $\xi_6(s)$ has a completely negligible impact on the model prediction.

Mock data are fitted by the RSD TNS \cite{Diemer14} model with the bias prescription described in \cite{Kazin2014,Diemer15}. A common practice of Fourier space clustering analyses is to remove the shot noise contribution from the power spectrum monopole measurement. Instead, we fit the power spectrum monopole with its shot noise and add up in the model the full shot noise term measured from the data, including contributions from integral constraints as shown in section 4.3. As in \cite{Kazin2014}, we marginalise over the constant galaxy stochastic term (see \cite{Diemer15}).

Power spectrum monopole, quadrupole and hexadecapole are fitted from $0.01$ h$/\text{Mpc}$ to $0.15$ h$/\text{Mpc}$. Fitted cosmological parameters are the growth rate of structure $f$ (times the power spectrum normalisation $\sigma_8$) and the Alcock-Paczynski $\alpha_\parallel$ and $\alpha_\perp$. We consider 4 nuisance parameters: the linear and second order biases $b_1$ and $b_2$, the velocity dispersion $\sigma_v$ and $A_g = N_g/N$, with $N_g$ the constant galaxy stochastic term and $N$ the
measured Poisson shot noise. Minimisations are performed using the algorithm Minuit \cite{minuit, minuit2}, taking large variation intervals for all parameters. We checked that the fitted parameters do not reach the input boundaries.

We use the fiducial BOSS DR12 cosmology in our analysis:

\[
H_0 = 0.676, \quad \Omega_m = 0.31, \quad \Omega_\Lambda = 0.69, \quad \Omega_b h^2 = 0.022, \quad \sigma_8 = 0.80.
\] (5.1)

In all figures showing the power spectrum model (1, 6), we use \(f = 0.7, b_1 = 2, b_2 = 1, \sigma_v = 5\).

Except for baseline cosmological fits, which rely on the true selection function, to make our tests more realistic we estimate the radial survey selection function from one realisation of the mocks only, using the binned scheme. As already discussed in section 4.1, we have checked that differences in the model prediction based on the true selection function and that inferred from one mock data realisation are of the order of the uncertainty on the mean of the N-series mocks, and are mostly absorbed by nuisance parameters in the cosmological fits.

### 5.3 Cosmological fits with the radial integral constraint

Figure 6 displays the model for the global and radial integral constraint corrections. By construction, the power spectrum monopoles converge to zero at large scales. We stress again that window functions are normalised according to section 4.2, without using any low-\(k\) (nor low-\(s\)) limit. The effect of the radial integral constraint is shown to be large on the window-convolved correlation in both Fourier and configuration space, though the latter prediction cannot be directly compared to measurements using e.g. the Landy-Szalay estimator \cite{landy} from which the window function effect is already removed. The effects on the quadrupole and hexadecapole are not negligible in the \(k\)-range of interest for our analysis. We thus expect that neglecting the radial integral constraint will lead to significant bias in the fitted cosmological parameters.

Figure 7 and table 1 present the cosmological fit results obtained in three different cases. The baseline cosmological fits (column 1 in table 1) are obtained with the power spectrum
Figure 7. Left: distributions of the cosmological parameters $f\sigma_8$, $\alpha_\parallel$, $\alpha_\perp$ measured on the 84 N-series mocks. The baseline (blue) uses the true selection function of the whole CMASS footprint. In orange, the binned scheme is applied to the mocks. In green, the radial integral constraint is added to the model. Continuous lines give the mean of the 84 best fits; the size of the cross is the standard deviation of the best fits divided by $\sqrt{84}$. Dashed lines show the expected values from the mock cosmology. Right: the corresponding $\chi^2$ distributions. The vertical dashed line shows the number of degrees of freedom ($42 - 7 = 35$).

| Parameter | Baseline global IC | Binned global IC | Binned radial IC | Expected |
|-----------|--------------------|------------------|------------------|----------|
| $\alpha_\parallel$ | 0.991 ± 0.031 | 0.982 ± 0.032 | 0.990 ± 0.031 | 0.989 |
| $\alpha_\perp$ | 0.980 ± 0.019 | 0.986 ± 0.019 | 0.980 ± 0.019 | 0.979 |
| $f\sigma_8$ | 0.465 ± 0.036 | 0.452 ± 0.036 | 0.463 ± 0.036 | 0.470 |

Table 1. Mean and standard deviation of the cosmological parameters fitted on the 84 N-series mocks, corresponding to figure 7. Error bars should be divided by $\sqrt{84} \sim 10$ to obtain errors on the mean of the mocks.

measurements using the true selection function of the whole CMASS footprint (see baseline in figure 2). The global integral constraint is applied in the model. The expected parameter values (column 4 in table 1), predicted from the mock fiducial cosmology, are recovered to the statistical uncertainty on the mean of the mocks.

Applying the binned scheme to the mocks (see binned in figure 2) while modelling the global integral constraint only results in a large bias on all cosmological parameters of roughly 30% of the statistical error bars on one realisation (column 2 in table 1). The goodness-of-fit (probed by the $\chi^2$ distribution, figure 7) is significantly degraded ($\Delta\chi^2 \simeq 5$).

The modelling of the radial integral constraint successfully removes the bias to better than the statistical uncertainty on the mean of the mocks (column 3 in table 1), and the goodness-of-fit is well recovered. We obtained similar results with the shuffled scheme. No increase on cosmological parameter errors is detected.
6 Angular integral constraint to mitigate angular systematics

6.1 Problem statement

As the radial selection function, the angular selection function of a spectroscopic survey can be difficult to evaluate because of residual photometric calibration errors or other potential photometric systematics. As a case study, we inject photometric systematics into our mocks as a function of right ascension (R.A.) and declination (Dec), using a weight:

\[ w_{\text{sys}} = 1 + 0.2 \sin \left( \frac{2\pi}{10(\text{deg})} \text{R.A.} (\text{deg}) \right) \sin \left( \frac{2\pi}{5(\text{deg})} \text{Dec} (\text{deg}) \right) . \]  (6.1)

The amplitude of the systematics (±20%) is very large compared to the typical requirements of a target selection (e.g. ±7.5% in [30]). \( w_{\text{sys}} \) has a drastic impact on power spectrum measurements, as can be seen on the left-hand plots of figure 8. A standard RSD analysis, using scales 0.01 \( h/\text{Mpc} < k < 0.15 \ h/\text{Mpc} \) would be impossible. We will see that these systematics can be drastically reduced by using a similar procedure as in section 3.

6.2 Angular integral constraint

We suggest to remove the contaminated modes by weighting randoms from the synthetic catalogue by \( \sum_{\text{pixel}} w_{g,i} / \sum_{\text{pixel}} w_{s,i} \) in pixels (the pixelated scheme), thus nulling the density contrast in each pixel. We use a HEALPix\(^2\) map with \( n_{\text{side}} = 64 \) (pixel area of \( \simeq 0.84 \deg^2 \)), for which the contaminated and uncontaminated mocks look similar (see figure 8, left). The pixelated scheme completely mitigates angular systematics in the quadrupole and hexadecapole. However, a difference close to a scale factor and statistically significant at small scales remains in the monopole.

We model the pixelated scheme by an angular integral constraint. A similar idea is developed in [31] to mitigate the impact of the DESI [19] fiber assignment. Formulae are directly deduced from the radial integral constraint (section 3) by changing \( \epsilon_{\text{rad}}(\mathbf{x}, \mathbf{y}) \) into \( \epsilon_{\text{ang}}(\mathbf{x}, \mathbf{y}) \), being non-zero if \( \mathbf{x} \) and \( \mathbf{y} \) lie within the same pixel (and within the same chunk). The shot noise contribution to the angular integral constraint is very large in the quadrupole and hexadecapole, as can be seen in figure 9.

Applying the pixelated scheme to uncontaminated mocks, cosmological parameters are well recovered when modelling the angular integral constraint: as shown in figure 10 and table 2, differences with the baseline (obtained with the true survey selection function of full CMASS, as in section 5) are below the uncertainty on the mean of the mocks (column 2 in table 2). The error bar on \( f\sigma_8 \) increases by 17%.

The magnitude of angular systematics makes it impossible to perform any relevant standard cosmological fit. However, cosmological parameters can be measured when applying the pixelated scheme to the contaminated mocks and the angular integral constraint in the model. Indeed, Alcock-Paczynski parameters are recovered well within the statistical uncertainty on the mean of the mocks (column 3 in table 2). Though a bias below 20% of the error on a single realisation can be seen on \( f\sigma_8 \), the cosmological analysis remains possible. A lower bias would be expected with more realistic angular systematics.

\(^2\)http://healpix.jpl.nasa.gov/
Figure 8. Left column, top panels: power spectrum multipoles (top: monopole, middle: quadrupole, bottom: hexadecapole) measured from the 84 N-series mocks. In orange, angular systematics (eq. (6.1)) are injected into the mocks. In the pixelated scheme (green), random objects are weighted to match the mock data density in pixels of area $\approx 0.84 \text{deg}^2$. In red, the pixelated scheme is applied to the contaminated mocks. The blue shaded area represents the standard deviation of the mocks. Left column, bottom panels: difference of the pixelated scheme with and without angular systematics, with the standard deviation of the difference given by the error bars, normalised by the standard deviation of the mocks. Right column, top panels: in orange, the pixelated scheme is recalled (same as the green curve in the left column). In green, the binned (see figure 2) and pixelated schemes are both applied to the uncontaminated mocks. In red, angular systematics are added onto the mocks. Right column, bottom panels: normalised difference of the binned and pixelated scheme with and without angular systematics.
Figure 9. Normalised shot noise contributions to the radial, angular, and combined (radial x angular) integral constraints (blue: monopole, red: quadrupole, green: hexadecapole).

Figure 10. Left: distributions of the cosmological parameters $f\sigma_8$, $\alpha_\parallel$, $\alpha_\perp$ measured on the 84 N-series mocks. The baseline (blue) uses the true selection function of the whole CMASS footprint. In orange, the pixelated scheme is applied on the mocks and the angular integral constraint is used in the model. In green, angular systematics (eq. (6.1)) are added onto the mocks. Right: the corresponding $\chi^2$ distributions.

| Parameter | Baseline global IC | Pixelated angular IC | Systematics, pixelated angular IC | Expected |
|-----------|--------------------|----------------------|----------------------------------|----------|
| $\alpha_\parallel$ | 0.991 ± 0.031 | 0.992 ± 0.032 | 0.994 ± 0.032 | 0.989 |
| $\alpha_\perp$ | 0.980 ± 0.019 | 0.980 ± 0.020 | 0.979 ± 0.020 | 0.979 |
| $f\sigma_8$ | 0.465 ± 0.036 | 0.466 ± 0.043 | 0.458 ± 0.042 | 0.470 |

Table 2. Mean and standard deviation of the cosmological parameters fitted on the 84 N-series mocks, corresponding to figure 10. Error bars should be divided by $\sqrt{84} \sim 10$ to obtain errors on the mean of the mocks.
6.3 Combining radial and angular integral constraints

Yet, one would probably like to combine the radial and angular integral constraints, to account for both unknown radial and angular selection functions. Here we need to assume that the radial and angular selection functions are independent, i.e. the redshift distribution does not depend on the angular position on the sky in each chunk of the survey. This assumption is usually satisfied, in particular if the survey is divided into small patches of locally constant radial selection function.

Right-hand plots of figure 8 show the effects of the combined binned and pixelated schemes on N-series mocks. They add up in the hexadecapole, and partially cancel in the quadrupole. This is expected as \( L_2 \) is negative around \( \mu = 0 \) (where the angular integral constraint removes signal) and positive around \( \mu = 1 \) (where the radial integral constraint plays up). As with the pixelated scheme alone (left-hand plots of figure 8), angular systematics are mitigated in the quadrupole and hexadecapole, but a multiplicative effect remains in the monopole.

Let us model the radial and angular integral constraints in a row:

\[
\delta^{\text{cic}}(r) = W(r) \left\{ \delta(r) - \int d^3x W_{\text{rad}}(x) \epsilon_{\text{rad}}(r, x) \delta(x) - \int d^3x W_{\text{ang}}(x) \epsilon_{\text{ang}}(r, x) \delta(x) + \int d^3x W_{\text{ang}}(x) \epsilon_{\text{ang}}(r, x) \int d^3y W_{\text{rad}}(y) \epsilon_{\text{rad}}(x, y) \delta(y) \right\}. \tag{6.2}
\]

Since the radial and angular parts of \( W \) are independent, the last term is just the integral of the density contrast over the whole (chunk) footprint, i.e. the global integral constraint. Thus, the two integral constraints commute. Then, building up the window-convolved, integral-constraint-corrected correlation function, we find 16 terms:

\[
\xi^{\text{cic}}(s) = \xi^{\text{c}}(s) - IC_{\ell}^{\delta,\text{rad}}(s) - IC_{\ell}^{\delta,\text{ang}}(s) + IC_{\ell}^{\text{rad},\text{rad}}(s) + IC_{\ell}^{\text{rad},\text{ang}}(s) + IC_{\ell}^{\text{ang},\text{ang}}(s)
\]

\[
- IC_{\ell}^{\delta,\text{ang}}(s) - IC_{\ell}^{\text{ang},\text{rad}}(s) + IC_{\ell}^{\text{ang},\text{ang}}(s)
\]

\[
+ IC_{\ell}^{\delta,\text{glo}}(s) + IC_{\ell}^{\text{glo},\text{ang}}(s) + IC_{\ell}^{\text{glo},\text{glo}}(s)
\]

\[
- IC_{\ell}^{\text{glo},\text{rad}}(s) - IC_{\ell}^{\text{rad},\text{glo}}(s) - IC_{\ell}^{\text{glo},\text{ang}}(s) - IC_{\ell}^{\text{ang},\text{glo}}(s)
\]

\[
+ IC_{\ell}^{\text{rad},\text{ang}}(s) + IC_{\ell}^{\text{ang},\text{rad}}(s). \tag{6.3d}
\]

Terms (6.3a), (6.3b) and (6.3c) correspond to the radial, angular and global integral constraints, while terms (6.3d) are the cross-integral constraints, given by a formula similar to eq. (3.4), with:

\[
W_{\ell p}^{i,j}(s, \Delta) = \frac{(2\ell + 1)(2p + 1)}{(4\pi)^2} \int d\Omega_s \int d\Omega_\Delta \int d^3x W(x) W(x + s)
\]

\[
\int d^3y W_r(y) W_j(y + \Delta) L_\ell(\hat{\eta}_s \cdot \hat{s}) L_\ell(\hat{\eta}_\Delta \cdot \hat{\Delta}) \epsilon_i(x, y) \epsilon_j(x + s, y + \Delta). \tag{6.4}
\]

As shown in figure 11 and table 3, the combined (radial x angular) integral constraint accounts well for the binned and pixelated schemes in the mocks: cosmological parameters are recovered well within the uncertainty on the mean of the mocks (column 2 in table 3). Cosmological fits of contaminated mocks are not further degraded by adding the radial integral constraint on top of the angular one. Results obtained with the combined integral constraint are extremely close to those obtained with the angular constraint alone (column 3 in table 2).
Table 3. Mean and standard deviation of the cosmological parameters fitted on the 84 N-series mocks, corresponding to figure 11. Error bars should be divided by $\sqrt{84} \sim 10$ to obtain errors on the mean of the mocks. Results are extremely close to those obtained in table 2.

| Parameter | Baseline global IC | Binned, pixelated radial x angular IC | Systematics, binned, pixelated radial x angular IC | Expected |
|-----------|--------------------|--------------------------------------|-------------------------------------------------|----------|
| $\alpha_\parallel$ | 0.991 ± 0.031 | 0.992 ± 0.032 | 0.993 ± 0.033 | 0.989 |
| $\alpha_\perp$ | 0.980 ± 0.019 | 0.981 ± 0.020 | 0.980 ± 0.020 | 0.979 |
| $f_\sigma_8$ | 0.465 ± 0.036 | 0.466 ± 0.042 | 0.458 ± 0.041 | 0.470 |

6.4 Caveat: multiplicative systematics

We emphasise that the angular (pixel) integral constraint can only mitigate the additive part of angular systematics. Let us call $c(\mathbf{r})$ the contamination signal, constant over a pixel. First, let us suppose $c$ to be purely additive. Applying the pixelated scheme, we would measure the power spectrum of the density contrast:

$$
\delta^{\text{pic}}(\mathbf{r}) = W(\mathbf{r}) \{1 + \delta(\mathbf{r}) + c(\mathbf{r})\} - W(\mathbf{r}) \int d^3x W(x) \{1 + \delta(x) + c(x)\} \epsilon_{\text{ang}}(\mathbf{r}, x) \int d^3x W(x) \epsilon_{\text{ang}}(\mathbf{r}, x)
$$

(6.5)

Then, as $c$ is constant over a pixel, $c(x)\epsilon_{\text{ang}}(\mathbf{r}, x) = c(\mathbf{r})\epsilon_{\text{ang}}(\mathbf{r}, x)$ and:

$$
\delta^{\text{pic}}(\mathbf{r}) = W(\mathbf{r}) \left\{ \delta(\mathbf{r}) - \int d^3x W_{\text{ang}}(x) \delta(x) \epsilon_{\text{ang}}(\mathbf{r}, x) \right\},
$$

(6.6)

as expected: $c$ disappears from the analysis.

Now, let us consider $c$ to be multiplicative (as implemented in our contamination model (6.1)):

$$
\delta^{\text{pic}}(\mathbf{r}) = W(\mathbf{r}) c(\mathbf{r}) \{1 + \delta(\mathbf{r})\} - W(\mathbf{r}) \int d^3x W(x) c(x) \{1 + \delta(x)\} \epsilon_{\text{ang}}(\mathbf{r}, x) \int d^3x W(x) \epsilon_{\text{ang}}(\mathbf{r}, x)
$$

(6.7a)

$$
= W(\mathbf{r}) c(\mathbf{r}) \left\{ \delta(\mathbf{r}) - \int d^3x W_{\text{ang}}(x) \delta(x) \epsilon_{\text{ang}}(\mathbf{r}, x) \right\},
$$

(6.7b)
i.e. \( c \) multiplies the selection function \( W \). However, by definition, \( c \) is unknown and therefore cannot be taken into account in \( W \). The resulting multiplicative systematics can explain the observed bias on \( f\sigma_8 \) (compensating for a higher \( b_1\sigma_8 \)). A possible way to alleviate this effect may consist in estimating the angular survey selection function (including \( c \)) with the *pixelated* scheme. This would induce a bias which can be estimated using a method similar to that of section 4.1. We report the reader to e.g. [32] for a fully coherent treatment of unknown multiplicative systematics.

### 7 Conclusions

This paper revisited the notion of integral constraints which we showed to be useful in clustering analyses to account for biases related to calibrating the survey selection function partly on data.

We first made an exact derivation of the global integral constraint mentioned in [1] and [2].

We noticed that the common practice of drawing random redshifts based on data to account for the survey selection function may significantly bias large-scale clustering measurements. This radial integral constraint effect can be particularly large compared to the window function effect if the survey is composed of several patches whose radial selection functions must be treated separately. In particular, its impact would be potentially very large in analyses focusing on large scales, e.g. devoted to primordial non-Gaussianity [33, 34].

Extending the derivations for the global integral constraint, our modelling of the radial integral constraint successfully reduces the systematic bias on cosmological parameters in the case of a RSD analysis. We discussed the window function normalisation and the shot noise contribution to the integral constraints. We also applied to our calculations the framework recently developed by [35] to account for wide-angle effects. Though we led our analyses in Fourier space, base calculations were made in configuration space, where the effect of the radial integral constraint may not be safely ignored either.

As a further application, we showed that we could similarly apply an angular integral constraint to help mitigating angular systematics. This angular integral constraint can be combined to the radial one, as required if the radial selection function is also estimated from the data. Though any additive systematics can be fully accounted for, our scheme performs well enough with multiplicative systematics on a BOSS CMASS-like survey.

Further developments may concern the use of a fully analytic selection function, as it can often be split into a radial and angular components. This would enable a faster estimation of the window functions required in our analysis.

We noted that a potential bias can emerge when the selection function used in the window function calculations for integral constraint corrections is estimated from the data itself. We suggested a way to estimate this bias, which remained very subdominant in our analysis. A workaround for future analyses may consist in predicting the radial selection function from first principles (e.g. from the luminosity function), without relying on the observed data, while marginalising on possible unknowns in the cosmological fits.

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A FKP weights

We apply FKP weights [8] to both mock data and randoms. They are calculated according to:

\[ w_{\text{FKP}} = \frac{1}{1 + n(z)P_0}, \]  

(A.1)

using \( P_0 = 20000 \) (Mpc/h)^3, which should be representative of the value of the power spectrum at the scales of interest for the clustering analysis. These weights require an estimation of the true radial density \( n(z) \) in absence of clustering. If \( n(z) \) is computed from the data itself, using narrow bins in \( z \), clustering overdensities may be smoothed out along the line-of-sight.

As in [12], we compute the density \( n(z) \) in bins \( \Delta z = 0.005 \). Figure 12 shows power spectrum measurements obtained with the binned scheme, with different \( n(z) \) estimations. Measuring \( n(z) \) from the full CMASS sample induces a negligible bias w.r.t. taking the true \( n(z) \). However, measuring \( n(z) \) separately in each of the 6 chunks dividing the CMASS footprint, as would be natural to do, leads to an additional bias on all scales of all multipoles. Though the density \( n(z) \) also enters the power spectrum normalisation (see eq. (4.3)), the main effect comes from the smoothing of the clustering along the line-of-sight, as can be seen from the loss of power in the quadrupole and hexadecapole. This effect is multiplicative and would be difficult to take into account.

A simple way to prevent FKP weights from biasing clustering measurements would be to fit a simple spline to the redshift distribution in wide redshift bins to reduce the correlation between \( w_{\text{FKP}} \) and the density field. As stated in section 5.1, we choose for simplicity to estimate \( n(z) \) from each data realisation, using the full CMASS sample (instead of the 6 different chunks), making the bias from \( w_{\text{FKP}} \) almost invisible.

B Modelling wide-angle effects

Wide-angle effects arise when a single line-of-sight must be chosen for a galaxy pair [36]. They can be accounted for by expanding both correlation and survey window functions in powers of \( x = s/d \), with \( s \) the separation of a pair of galaxies and \( d \) its comoving distance to the observer. To allow a considerable reduction of computation time, the Yamamoto power spectrum estimator [9] uses one galaxy of a pair as line-of-sight: the so-called end-point line-of-sight (see figure 13). As shown in [35], this choice leads to detectable wide-angle effects, mainly in odd multipoles. The correlation function in the end-point (ep) line-of-sight can be written [35]:

\[ \xi^\text{ep}(s) = \sum_{p,q,n} A_p^q \frac{2 \ell + 1}{2q + 1} s^n \xi^\text{ep,}(n)(s) W_{\ell}^{\delta,\delta,}(n)(s) \]  

(B.1)

where:

\[ W_{\ell}^{\delta,\delta,}(n)(s) = \frac{2 \ell + 1}{4 \pi} \int d\Omega_s \int d^3 x x^{-n} W(x) W(x - s) \mathcal{L}_{\ell}(\hat{x} \cdot \hat{s}). \]  

(B.2)

\( \xi^\text{ep,}(n) \) can be predicted within linear perturbation theory [35, 36].

\(^3\)Note that we now use the same triangle definition as in [36] (see figure 13) \( s = r_1 - r_2 \), which explains the minus signs in the selection functions.
Figure 12. Top panels: power spectrum multipoles (upper left: monopole, upper right: quadrupole, bottom: hexadecapole) obtained using the binned scheme and different \( n(z) \) estimations. First, we use the true density \( n(z) \) from the radial selection function (orange). In green, \( n(z) \) is estimated from each full CMASS mock realisation. In red, \( n(z) \) is measured from each mock realisation, in the 6 chunks separately. We recall the baseline in blue, obtained with the true selection function. The blue shaded area represents the standard deviation of the mocks. Bottom panels: difference of the binned scheme with the different (true, full CMASS, chunk) \( n(z) \) estimations to the baseline, with the standard deviation of the difference given by the error bars, normalised by the standard deviation of the mocks.

\[
\mathbf{d} = \mathbf{r}_1
\]

\[
\mathbf{s} = \mathbf{r}_1 - \mathbf{r}_2
\]

Figure 13. The assumed geometry and angles. The two galaxies lie at \( \mathbf{r}_1 \) and \( \mathbf{r}_2 \), with the separation vector \( \mathbf{s} = \mathbf{r}_1 - \mathbf{r}_2 \). The end-point line-of-sight is used: \( \mathbf{d} = \mathbf{r}_1 \).

Working with the general case of the radial integral constraint (see eq. (3.4)), similar expansions can be derived for \( IC_{\ell}^{i,j}(s) \), \( (i,j) \in \{(\delta, \text{rad}), (\text{rad}, \delta), (\text{rad}, \text{rad})\} \). The first cross-
term \((\delta, \text{rad})\) reads:
\[
IC_{\ell}^{\delta, \text{rad}}(k) = \frac{2\ell + 1}{4\pi} \int d\Omega_k \int d^3r_1 \int d^3r_2 \int d^3r_3 e^{-ik(r_1 - r_2)} \langle \delta(r_1)\delta(r_3) \rangle
\]
\[
W(r_1)W(r_2)W_{\text{rad}}(r_3)\mathcal{L}_\ell(\hat{k} \cdot \hat{r}_1)\epsilon_{\text{rad}}(r_2, r_3)
\]
with:
\[
\langle \delta(r_1)\delta(r_3) \rangle = \sum_{n,p} \left( \frac{\Delta}{r_1} \right)^n \xi_{p}^{\text{ep},(n)}(\Delta) \mathcal{L}_p(\hat{r}_1 \cdot \hat{\Delta}), \quad \Delta = r_1 - r_3.
\]
Using the Rayleigh plane wave expansion:
\[
e^{-ik\cdot s} = \sum_{q=0}^{\infty} (-i)^q (2q + 1) j_q(ks) \mathcal{L}_q(\hat{k} \cdot \hat{s})
\]
and
\[
\int \frac{d\Omega_k}{4\pi} \mathcal{L}_\ell(\hat{k} \cdot \hat{r}_1) \mathcal{L}_q(\hat{k} \cdot \hat{s}) = \frac{\delta_{\ell q}}{2\ell + 1} \mathcal{L}_\ell(\hat{r}_1 \cdot \hat{s}),
\]
one gets:
\[
IC_{\ell}^{\delta, \text{rad}}(k) = (2\ell + 1)(-i)^\ell \int d^3s j_\ell(ks) \sum_{p,n} \int d^3s d^3s \delta_{\ell q} \xi_{p}^{\text{ep},(n)}(\Delta) \mathcal{L}_n(\hat{s}_1 \cdot \hat{s}_2) \mathcal{L}_p(\hat{s}_3 \cdot \hat{s}_4)\epsilon_{\text{rad}}(r_1 - s, r_1 - \Delta)
\]
with \(s = r_1 - r_2\). As in eq. (3.4), one just needs to take the Hankel transform of:
\[
IC_{\ell}^{i,j}(s) = \int \Delta^2 d\Delta \sum_{p,n} \frac{4\pi}{2p + 1} \Delta^n \xi_{p}^{\text{ep},(n)}(\Delta) W_{\ell p}^{i,j,(n)}(s, \Delta)
\]
with \((i, j) = (\delta, \text{rad})\), if we define:
\[
W_{\ell p}^{i,j,(n)}(s, \Delta) = \frac{(2\ell + 1)(2p + 1)}{(4\pi)^2} \int d\Omega_s \int d\Omega_\Delta \mathcal{L}_\ell(\hat{s} \cdot \hat{\Delta}) \mathcal{L}_n(\hat{s} \cdot \hat{\Delta}) \mathcal{L}_p(\hat{s} \cdot \hat{\Delta}) \epsilon_{\text{rad}}(x - s, x - \Delta).
\]
The second cross-term \((\text{rad}, \delta)\) is not trivially equal to the first one for even multipoles. Indeed:
\[
IC_{\ell}^{\text{rad}, \delta}(k) = \frac{2\ell + 1}{4\pi} \int d\Omega_k \int d^3r_1 \int d^3r_2 \int d^3r_3 e^{-ik(r_1 - r_2)} \langle \delta(r_2)\delta(r_3) \rangle
\]
\[
W(r_1)W(r_2)W_{\text{rad}}(r_3)\mathcal{L}_\ell(\hat{k} \cdot \hat{r}_1)\epsilon_{\text{rad}}(r_1, r_3)
\]
gives:
\[
IC_{\ell}^{\text{rad}, \delta}(k) = (2\ell + 1)(-i)^\ell \int d^3s j_\ell(ks) \sum_{p,n} \int d^3s d^3s \delta_{\ell q} \xi_{p}^{\text{ep},(n)}(\Delta) \mathcal{L}_n(\hat{s}_1 \cdot \hat{s}_2) \mathcal{L}_p(\hat{s}_3 \cdot \hat{s}_4)\epsilon_{\text{rad}}(s + r_2, s + r_2 - \Delta)
\]
with \( s = r_1 - r_2, \Delta = r_3 - r_4 \). Taking the opposite \( s \rightarrow -s \) and defining:

\[
W^{\delta,\delta(n)}_{\ell p}(s, \Delta) = \frac{(2\ell + 1)(2p + 1)}{(4\pi)^2} \int d\Omega_s \int d\Omega_{\Delta} \int d^3x x^{-n} W(x) W(x - s) W_{\text{rad}}(x - \Delta) \mathcal{L}_\ell \mathcal{L}_p (\hat{x} \cdot \hat{\Delta}) \varepsilon_{\text{rad}}(x - s, x - \Delta)
\]

we obtain the integral constraint correction eq. (B.8), with \((i, j) = (\delta, \delta)\). Let us move to the last term \((\text{rad, rad})\):

\[
IC^{\text{rad, rad}}_\ell (k) = \frac{2\ell + 1}{4\pi} \int d\Omega_k \int d^3r_1 \int d^3r_2 \int d^3r_3 \int d^3r_4 e^{-ik(r_1 - r_2)} \langle \delta(r_3) \delta(r_4) \rangle
\]

\[
W(r_1) W(r_2) W_{\text{rad}}(r_3) W_{\text{rad}}(r_4) \mathcal{L}_\ell (\hat{k} \cdot \hat{r}_1) \varepsilon_{\text{rad}}(r_1, r_3) \varepsilon_{\text{rad}}(r_2, r_4)
\]

(B.13)

gives:

\[
IC^{\text{rad, rad}}_\ell (k) = (2\ell + 1)(-i)^\ell \int d^3s j_\ell (ks) \sum_{p,n} d^3s d^3\Delta \Delta^\text{exp}(n)(\Delta)(2\ell + 1) \int d^3r_1 W(r_1) W(r_1 - s) \int d^3r_3 W_{\text{rad}}(r_3) W_{\text{rad}}(r_3 - \Delta) \mathcal{L}_\ell (\hat{r}_3 \cdot \hat{s}) \mathcal{L}_p (\hat{r}_3 \cdot \hat{\Delta}) \varepsilon_{\text{rad}}(r_1, r_3) \varepsilon_{\text{rad}}(r_1 - s, r_3 - \Delta)
\]

(B.14)

with \( s = r_1 - r_2, \Delta = r_3 - r_4 \). This is eq. (B.8) with \((i, j) = (\text{rad, rad})\) if we define:

\[
W^{\delta,\delta(n)}_{\ell p}(s, \Delta) = \frac{(2\ell + 1)(2p + 1)}{(4\pi)^2} \int d\Omega_s \int d\Omega_{\Delta} \int d^3y W(y) W(y - s) \int d^3x x^{-n} W_{\text{rad}}(x) W_{\text{rad}}(x - \Delta) \mathcal{L}_\ell (\hat{y} \cdot \hat{s}) \mathcal{L}_p (\hat{y} \cdot \hat{\Delta}) \varepsilon_{\text{rad}}(y, x) \varepsilon_{\text{rad}}(y - s, x - \Delta).
\]

(B.15)

Results for the global integral constraint are obtained by replacing \( \text{rad} \rightarrow \text{glo} \) and \( \varepsilon_{\text{rad}} \rightarrow 1 \). In particular, eq. (2.16) is recovered at any order \( n \) if one takes \( W^{\delta,\delta}(0) \) as \( W^{\delta,\delta}_0 \) (as stressed out by [35]).

Contrary to the line-of-sight definition in \( W^{\delta,\delta,\text{exp}(n)}_{\ell p}(s, \Delta) \) (and the first line-of-sight in \( W^{\delta,\delta,\text{exp}(n)}_{\ell p}(s, \Delta) \)), which should be the same as in the power spectrum (or correlation function) estimator, the line-of-sight connecting \( \xi^{\text{exp}(n)}_p(\Delta) \) to \( W^{\delta,\delta,\text{exp}(n)}_{\ell p}(s, \Delta) \) is a purely practical choice. Our calculations use the end-point line-of-sight, but the derivation with any line-of-sight \( d \) is straightforward by replacing \( x^{-n} \) in eq. (B.9), (B.12) and (B.15) by \( d^{-n} \), changing the arguments of Legendre polynomials accordingly and taking the corresponding \( \xi^{d,\delta(n)}_p(\Delta) \) in eq. (B.8). In particular, taking the mid-point line-of-sight makes first-order \((n = 1)\) wide-angle corrections vanish.

For clarity, we have used the end-point line-of-sight as second line-of-sight in \( W^{\delta,\delta,\text{exp}(n)}_{\ell p}(s, \Delta) \) throughout the paper. With this choice, by definition, the integral constraint corrections completely cancel with the convolved power spectrum monopole \( P^\text{glo}_0(k) \) on large scales. This would only be asymptotically true (as the wide-angle correction order \( n \rightarrow \infty \)) if a different line-of-sight definition was used in \( W^{\delta,\delta,\text{exp}(n)}_{\ell p} \) and \( W^{\delta,\delta,\text{exp}(n)}_{\ell p} \). We show wide-angle contributions to the convolved power spectrum multipoles and the radial integral constraint in figure 14. They are
Figure 14. Ratio of wide-angle corrections to the convolved power spectrum multipoles $P_c^\ell(k)$ (upper left: monopole, upper right: quadrupole, bottom: hexadecapole) at zeroth order. In blue are shown wide-angle contributions to the convolved power spectrum monopole up to order $n = 1, 2$ (dashed and dotted lines respectively). Corrections for the radial integral constraint up to order $n = 0, 1, 2$ (continuous, dashed and dotted lines respectively) are plotted in orange.

significant for $k \lesssim 10^{-2} \, h/\text{Mpc}$. However, they remain small compared to the radial integral constraint correction. Then, for simplicity, we do not include these wide-angle corrections ($n \geq 1$) in our analysis. However, wide-angle contributions may dominate over radial integral constraint corrections for a large survey with a constant radial selection function.