Numerical testing of the similarity conditions for the induction plasmatrons

S A Vasilevskii, A F Kolesnikov, A I Bryzgalov, S E Yakush

Ishlinsky Institute for Problems in Mechanics of the Russian Academy of Sciences (IPMech RAS), Prospekt Vernadskogo 101-1, Moscow, 119526, Russia

Corresponding author e-mail: vasil_ipmech@mail.ru

Abstract. Detailed comparison of numerical results obtained for IPG–4 plasmatron (base case), IPG–3H hypothetical plasmatron and IPG–3 real plasmatron is provided to estimate the accuracy of the similarity conditions and the applicability of that conditions for the pair of IPG–4 and IPG–3 plasmatrons. IPG–3H hypothetical plasmatron is geometrically similar to IPG–4 one and has twice larger dimensions. The operating regime of IPG–3H is obtained from the IPG–4 operating regime with use of the similarity conditions for the operating parameters. The operating regime of IPG–3 real plasmatron is the same as for IPG–3H. Vortical electric field generated by the inductor coils with high frequency current around the discharge channel is calculated with use of 2D model with far field boundary conditions. This calculation is integrated with the solver for Navier-Stokes equations for plasma flow. The comparison of IPG–4 and IPG–3H results shows about 4÷7% accuracy of the similarity conditions for main plasma parameters. The comparison of IPG–4 and IPG–3 results shows that application of the similarity conditions for this case provides a qualitative agreement but the quantitative difference in some parameters may be large, up to 35÷40%.

1. Introduction

Induction radio-frequency (RF) plasmatron is a powerful tool to test material thermochemical stability and determine its surface catalycity. Powerful induction plasmatrons are in operation in Russia (IPMech RAS [1], TsNII Mash [2], TsAGI [3]), in Belgium (VKI Plasmatron [4]), in Germany (Institut fur Raumfahrtsysteme of Stuttgart University IRS [5]). Two induction plasmatrons, IPG–4 and IPG–3, operate in IPMech RAS and are used intensively to study heat transfer and other physical processes in high-enthalpy jets and for material testing in such jets [1].

Induction plasmatron with a large discharge channel diameter provides capability to test relatively large material samples and even elements of heat protection system. On the other hand, large discharge channel leads to large operating cost and large cost of plasmatron design and manufacturing. Some institutes (e.g. IPMech RAS and VKI) start with design and manufacturing the small plasmatron to accumulate some experience in plasma flow investigation, and then proceed to design the large plasmatron. In that case it would be helpful to provide a technique to scale up the existing small plasmatron to the larger one. Such a technique would be useful in order to transfer the plasma flow results obtained in small diameter channel to large diameter channel; in comparison of the results obtained with the small and the large plasmatrons; in preparation the optimal test conditions for large plasmatron using the information obtained previously with the small plasmatron.
In the papers [6, 7] the required scaling laws have been proposed by means of conversion of both gasdynamic and electrodynamic governing equations to dimensionless form. That conversion revealed the 5 dimensionless numbers (criteria) $N_1$-$N_5$. These criteria fully determine the dimensionless solution of the system of governing equations under the assumptions that the flow is equilibrium and the necessary thermodynamic and transport gas/plasma properties (molar mass, viscosity, etc.) depend on flow temperature only and do not depend on pressure [6]. The second assumption is not accurate of course, but it is necessary to obtain the useful similarity conditions for the two flows - in the small and large discharge channels of the two plasmatrons.

2. Similarity conditions

Suppose there are the small (index "1") and the large (index "2") plasmatrons. We define $\kappa$ to be the ratio of the large discharge channel radius $R_2$ to the small one $R_1$: $\kappa = R_2 / R_1$. Then the similarity conditions [6, 7] for operating parameters of the two plasmatrons can be written as follows:

$\frac{f_2}{\kappa^2} = \frac{f_1}{\kappa^2}$ (1)

$\frac{P_2}{\kappa^2} = \frac{P_1}{\kappa^2}$ (2)

$N_{pl2} = \kappa \cdot N_{pl1}$ (3)

$G_2 = \kappa \cdot G_1$ (4)

Here $f$, $P$, $N_{pl}$, $G$ are the frequency of the inductor current, operating static pressure, power input in plasma, mass flow rate. We assume that the number of current-carrying inductor coils is equal for the two plasmatrons and all geometry parameters (the diameter of inductor coil, the width of the gas inlet slot, etc.) are similar, e.g. the lengths of the channels obey the relation: $L_2 = L_1 \kappa$. As to the boundary conditions, we assume that the temperature of channel wall $T_{cw}$ and temperature of inflow flat face $T_{iw}$ are specified and $T_{cw1} = T_{cw2}$, $T_{iw1} = T_{iw2}$.

To formulate the similarity conditions for plasma parameters within the two discharge channels, we first take by definition that flow temperatures are equal for the two channels:

$T_2(z^*, r^*) = T_1(z^*, r^*)$ (5)

Here $z^*$ and $r^*$ are the dimensionless axial and radial coordinates: $z^* = z_1 / L_1 = z_2 / L_2$, $r^* = r_1 / R_1 = r_2 / R_2$. Then the similarity conditions for the plasma parameters can be written [6, 7]:

$H_{2z}(z^*, r^*) = H_{1z}(z^*, r^*)$ – magnetic field amplitude axial component (6)

$E_{2\theta}(z^*, r^*) = E_{1\theta}(z^*, r^*) / \kappa$ – electric field amplitude tangential component (7)

$U_2(z^*, r^*) = \kappa U_1(z^*, r^*)$ – axial velocity component (8)

$\frac{f_2(z^*, r^*)}{\kappa} = \frac{f_1(z^*, r^*)}{\kappa}$ – dimensionless stream function (9)

$\frac{\alpha_2(z^*, r^*)}{\kappa} = \frac{\alpha_1(z^*, r^*)}{\kappa}$ – flow enthalpy (10)

Additional similarity condition can be written for the scalar parameter:

$I_2 = \kappa I_1$ – amplitude of the inductor current (11)

The following part of the article is devoted to the comparison of numerical solutions obtained for the real plasmatron IPG–4 (base case) and for the hypothetical large plasmatron IPG–3H with the channel diameter and other dimensions twice as much as for the base one, in order to estimate the accuracy of the above similarity conditions (1) – (4), (5) – (11).

The inaccuracy of the similarity conditions (5) – (11) is due to the necessarily assumption that the thermodynamic and transport properties depend on flow temperature only and do not depend on the pressure [6]. The quantitative evaluation of this inaccuracy for the pare of plasmatrons IPG–4 and IPG–3H is the first objective of this work.
The second objective is to estimate the applicability of the similarity conditions to the pair of real operating plasmatrons IPG–4 and IPG–3. The real plasmatron IPG–3 is not exactly similar to IPG–4; the difference in dimensions of IPG–3 and IPG–3H is not large (about 10÷20%), but it is essential. Nevertheless, due to practical interest to the problem we provide the comparison of numerical solutions for the three plasmatrons: IPG–4, IPG–3H, IPG–3.

3. Numerical solution techniques

Numerical solution for the inductively coupled plasma flow in the discharge channel is based on Navier-Stokes gasdynamic equations written in the cylindrical coordinate system (z, r) coupled with the simplified 2D Maxwell equations for vortical electric field. We assume that the gasdynamic equations are time-averaged with respect to RF oscillations of the inductor current. Three velocity components U, V, W (axial, radial, tangential) are used to describe the flow in the channel with swirling in azimuthal direction. The flow is assumed to be axisymmetric, stationary, laminar and equilibrium. Radiative processes are supposed to be negligible. The gasdynamic equations include time-averaged source terms corresponding to RF electromagnetic field influence: axial and radial components of Lorentz force and Joule heat production [8].

The boundary conditions for gasdynamic equations are as follows. Zero values for velocity components are specified at all rigid surfaces - wall of the channel and the gas injector interface at the channel inlet section. Temperature value \( T_w = 300 \text{ K} \) is applied also at all rigid surfaces. Symmetry conditions are applied at the channel axis. The inlet flow parameters corresponding to the room temperature air with the specified mass flow rate are specified at all rigid surfaces. The boundary conditions for gasdynamic equations are as follows. Zero values for velocity components are specified at all rigid surfaces - wall of the channel and the gas injector interface at the channel inlet section. Temperature value \( T_w = 300 \text{ K} \) is applied also at all rigid surfaces. Symmetry conditions are applied at the channel axis. The inlet flow parameters corresponding to the room temperature air with the specified mass flow rate are specified at all rigid surfaces.

The result of calculation electric field amplitude tangential component \( E(z, r) \) is solved on far-field mesh, extended beyond the channel, to allow the use of the simple far-field boundary conditions:

\[
E_0(z, r=\pm \infty)=0; \quad E_0(z=\pm \infty, r) = 0
\]  

In fact, this condition is changed to less hard condition for \( E_{p0} \) [9, 10]:

\[
E_{p0}(z, r=\pm \infty)=0; \quad E_{p0}(z=\pm \infty, r) = 0
\]

here \( E_{p0} \) is the plasma-induced part of \( E_0 \), so that \( E_0 = E_{p0} + E_{j0} \). \( E_{j0} \) is induced by the outer inductor rings in vacuum and is calculated as the sum of analytical expressions for each ring [11]. The comparison of the two results obtained for electric field with use of far-field meshes extended to \( 3R_c \) and to \( 5R_c \) in radial direction revealed negligible difference in plasma and electric field parameters within the discharge channel, so the \( 3R_c \) radial grid extension was used in further calculations.

Symmetry condition \( E_0(z, r=0) = 0 \) is used at the channel axis. Numerical method for the solution of 2D equation (12) is described in our work [10]. The numerical method for the solution of Navier-Stokes equations is based on the control volumes approach by Patankar and Spalding and is described in [8].

The result of calculation electric field amplitude tangential component \( E_0 \) and its plasma-induced part \( E_{p0} \) on the far-field mesh is shown in figures 1, 2 for IPG–3 plasmatron for its operating
conditions: \( f = 0.44 \text{ MHz}, \ P = 25 \text{ mbar}, \ G = 4.8 \text{ g/s}, \ N_{pl} = 58 \text{ kW} \); the diameter of IPG–3 discharge channel is \( D_c = 192 \text{ mm} \), working gas is air. Black rectangular in these figures is the IPG–3 channel.

**Figure 1.** Contours of \( E_\theta = E_{V\theta} + E_{P\theta} \) calculated on the extended far-field mesh for IPG–3 channel.

**Figure 2.** Contours of \( E_{P\theta} \) calculated on the extended far-field mesh for IPG–3 channel.

4. **Estimation for the accuracy of similarity conditions by comparison of numerical solutions for IPG–4 and IPG–3H plasmatrons.**

Estimation of applicability of similarity conditions for IPG–4 and IPG–3 plasmatrons

To estimate the accuracy of similarity conditions, we choose IPG–4 plasmatron with the air working gas as the base case (subscript "1"). Its discharge channel diameter \( D_{c1} = 80 \text{ mm} \), the channel length \( L_{c1} = 400 \text{ mm} \); the inductor current frequency \( f_1 = 1.76 \text{ MHz} \), the number of inductor coils \( n = 5 \). The base operating regime for IPG–4 is as follows: \( P_1 = 100 \text{ mbar}, \ G_1 = 2.4 \text{ g/s}, \ N_{pl1} = 29 \text{ kW} \).

To compare with the IPG–4 base case we use hypothetical plasmatron IPG–3H (not the real one working in IPMech RAS) with the channel dimensions twice as much as the base one. IPG–3H is geometrically similar to IPG–4, it has the same number of inductor coils \( n = 5 \), its channel diameter \( D_{c2} = 160 \text{ mm} \) and channel length \( L_{c2} = 800 \text{ mm} \). According to the conditions (1) – (4) we choose the operating parameters for IPG–3H: \( f_2 = 0.44 \text{ MHz}, \ P_2 = 25 \text{ mbar}, \ N_{pl2} = 58 \text{ kW}, \ G_2 = 4.8 \text{ g/s} \). Of course the working gas of IPG–3H is air too.

The real plasmatron IPG–3 operating in IPMech RAS is not exactly geometrically similar to IPG–4; its length and diameter are \( L_c = 870 \text{ mm}, \ D_c = 192 \text{ mm} \). The difference in IPG–3 and IPG–3H dimensions is not large (about \( 10\%\text{–}20\% \)), but it is essential. Inductor current frequency of IPG–3 \( f = 0.44 \text{ MHz} \) is the same as for IPG–3H, but the number of inductor coils in IPG–3 is \( n = 4 \). So, the difference in IPG–4 plasma parameters recalculated according to the similarity conditions and
IPG–3 parameters, as well as the difference in calculated plasma parameters for IPG–3H and IPG–3 plasmatrons, is due to the difference in their geometry and should not be associated with inaccuracy of the similarity conditions. Nevertheless the estimation of applicability of the similarity conditions for the pair of IPG–4 and IPG–3 plasmatrons has a significant practical meaning because they are the two operating facilities in IPMech RAS. So, we add the plasma parameters calculated for IPG–3 plasmatron to the comparison of plasma parameters of the pair of IPG–4 and IPG–3H. In calculations we specify operating parameters of IPG–3 to be the same as for IPG–3H. In comparison of the parameters $U, E_0$ calculated in IPG–3 channel with ones calculated in IPG–4, the same scaling parameter $\kappa=2$ will be used.

Results of numerical solutions for plasma and electric field parameters within the discharge channels of IPG–4, IPG–3H and IPG–3 are presented below in figures 3–15.

Figure 3. Contours of dimensionless stream function (a) and isotherms (b) in IPG–4 channel.

Figure 4. Contours of dimensionless stream function (a) and isotherms (b) in IPG–3H channel.
Figure 5. Contours of dimensionless stream function (a) and isotherms (b) in IPG–3 channel.

Figures 3–5 show the isolines of dimensionless stream function and isotherms within IPG–4, IPG–3H and IPG–3 channels. The corresponding contours for IPG–4 and IPG–3H are similar, though the maximum temperature within the IPG–4 channel is higher than that for IPG–3H one: $T_{\text{max}}=10733$ K (IPG–4), $T_{\text{max}}=10215$ K (IPG–3H). Value of maximum temperature within IPG–3 channel is 9592 K.

Figures 6–8 show the isolines of electric field amplitude tangential component $E_{\theta}$ and magnetic field amplitude axial component $H_z$ within IPG–4, IPG–3H, and IPG–3 channels.

Figure 6. Contours of electric field amplitude tangential component $E_{\theta}$ and magnetic field amplitude axial component $H_z$ within IPG–4 channel.
Figure 7. Contours of electric field amplitude tangential component $E_\theta$ and magnetic field amplitude axial component $H_z$ within IPG–3H channel.

Figure 8. Contours of electric field amplitude tangential component $E_\theta$ and magnetic field amplitude axial component $H_z$ within IPG–3 channel.

Values of $E_\theta$ for IPG–3H and IPG–3 channel are to be twice less than that for IPG–4 according to (7). With this in mind, the corresponding contours are very similar for IPG–4 and IPG–3H, maximum of $E_\theta$ within IPG–3H channel is practically equal to $0.5E_\theta$ for IPG–4 one: $E_{\theta\text{max}}$=1072 V/m for IPG–3H, $E_{\theta\text{max}}$=2146 V/m for IPG–4. The calculated contours of $H_z$ are also very similar for IPG–4 and IPG–3H channels; the maximum values are 9159 A/m (IPG–4) and 9162 A/m (IPG–3H). For
IPG–3 the maximum value of $H_r$ is $12422$ A/m near the wall in front of the inductor current turn, and $E_{0\max}=1147$ V/m. Large values of $H_{r\max}$ and $E_{0\max}$ for IPG–3 are due to the smaller diameter of the inductor coil compared with IPG–3H: the inductor current turns in IPG–3 are close to the channel wall.

Comparison of the key parameters calculated for IPG–4, IPG–3H, IPG–3 channels is shown in the table 1. Here $T_c$, $h_c$ and $U_c$ are the temperature, enthalpy and velocity at a channel exit section at the flow axis; $I_0$ is the inductor current amplitude; $T_{r\max}$, $E_{0\max}$ and $H_{r\max}$ are the maximum temperature, electric and magnetic field amplitudes within a channel.

**Table 1.** Comparison of flow and electric field parameters for IPG–4, IPG–3H, IPG–3 channels.

|       | $T_c$  | $h_c$  | $U_c$  | $I_0$ | $T_{r\max}$ | $E_{0\max}$ | $H_{r\max}$ |
|-------|--------|--------|--------|-------|-------------|-------------|-------------|
| ВГУ–4 | 7223   | 37.8   | 157    | 320   | 10733       | 2146        | 9159        |
| ВГУ–3H| 7401   | 40.3   | 153.2  | 319.2 | 10215       | 2144/2      | 9162        |
| ВГУ–3 | 6769   | 36.5   | 102.2  | 314.2 | 9592        | 2294/2      | 12422       |

In the lines of table 1 for IPG–3H, IPG–3 the $U_c$, $I_0$ and $E_{0\max}$ parameters are presented as $U_{cmp}\cdot2$, $I_{0cmp}/2$, $E_{0maxcmp}/2$ to account for scaling rules (8), (11), (7), so that $U_c$ for IPG–4 should be compared with the scaled value $U_{cmp}$ for IPG–3H, IPG–3, and so on. The percent deviations of the scaled parameters calculated for IPG–3H, IPG–3 channels from that of IPG–4 are shown in the table below the corresponding parameters.

For more detailed comparison of plasma and electromagnetic field parameters calculated in IPG–4, IPG–3H, and IPG–3 channels, we provide the plots of radial profiles of parameters (figures 9–13) and plots of axial distribution of plasma temperature and velocity along the channel axis (figures 14–15). In that plots the blue curves with circles correspond to IPG–4, black curves with squares - IPG–3H, magenta curves with rhomb - IPG–3. Note that only flow temperature, enthalpy and magnetic field amplitude axial component $H_r$ can be compared directly. As to the flow velocity $U$ and $E_0$, we should compare the values obtained in IPG–4 channel $U_{\kappa}(z^*, r^*)$, $E_0(\ z^*, r^*)$ with the scaled values of IPG–3H and IPG–3 channels: $U_{\kappa}(z^*, r^*) = (1/\kappa) U_{\kappa}(z^*, r^*)$, $E_0(\ z^*, r^*) = \kappa E_{02}(\ z^*, r^*)$. We use for IPG–3 the same scaling parameter $\kappa=2$ as for IPG–3H. So, in figures 10,15 the presented values are $U_{cmp}=U$ – for IPG–4 and $U_{cmp}=(1/2)U$ - for IPG–3H and IPG–3; in figure 12 the presented values are $E_{0cmp}=E_0$ – for IPG–4 and $E_{0cmp}=2E_0$ - for IPG–3H and IPG–3. The parameters are compared in the corresponding points with equal values of $z/Z_c$ and $r/R_c$.

Figures 9–11 show the comparison of radial profiles of flow temperature, axial velocity and enthalpy at the exit sections of the channels. Deviation of the results calculated for IPG–4 and IPG–3H (with account for scaling factor $\kappa$ for $U$) is about $4\text{–}7\%$. The results calculated in IPG–3 channel are close to ones in IPG–3H channel for $T$ and $h$, but they are very different for $U$.

Figures 12,13 show the comparison of radial profiles of $E_0$ and $H_r$ at the sections of the channels in the middle of inductor coil. Note that $E_0$ and $H_r$ in general are nearly the same for IPG–4 and IPG–3H (with account for scaling factor $\kappa=2$). For IPG–3 the calculated values of $E_0$ and $H_r$ are essentially lower than that for IPG–3H. Probably it is due to the difference in the number of inductor turns and the position of the middle of inductor coil: for IPG–4 and IPG–3H the middle of inductor coil is in front of the medial turn; for IPG–3 it is not so.
Figures 14, 15 show the comparison of axial distributions of plasma parameters $T$ and $U_{cmp}$ along the symmetry axis for the channels of IPG–4, IPG–3H and IPG–3. Again, the results for IPG–4 and IPG–3H in general are similar. For IPG–3 the calculated temperature is not very different from the one for IPG–3H, but $U_{cmp}$ is essentially different from the one calculated for IPG–3H.

**Figure 9.** Comparison of temperature at the channel exit sections for IPG–4, IPG–3H, IPG–3.

**Figure 10.** Comparison of velocity at the channel exit sections. Due to similarity conditions, the presented values $U_{cmp}$ are: $U$ – for IPG–4, $U/2$ – for IPG–3H, IPG–3.
Figure 11. Comparison of enthalpy at the channel exit sections for IPG–4, IPG–3H, IPG–3.

Figure 12. Comparison of $E_\theta$ at the middle of inductor section for the plasmatron channels. Due to similarity conditions, the presented values $E_{\text{cmp}}$ are: $E_0$ – for IPG–4, $E_0^2$ – for IPG–3H, IPG–3.
Figure 13. Comparison of $H_z$ at the middle of inductor section for IPG–4, IPG–3H, IPG–3 channels.

Figure 14. Comparison of $T(r=0, z/Z_c)$ along the symmetry axis for IPG–4, IPG–3H, IPG–3 channels.
Figure 15. Comparison of $U_{\text{cmp}}(r=0, z)$ along the symmetry axis for IPG$-4$, IPG$-3H$, IPG$-3$ channels. Due to similarity conditions, the presented values $U_{\text{cmp}}$ are: $U$ – for IPG$-4$, $(1/2)U$ – for IPG$-3H$ and IPG$-3$.

5. Conclusion
Detailed comparison is provided for numerical solutions of plasma flow and electromagnetic field within the discharge channels of plasmatrons IPG$-4$, IPG$-3H$, IPG$-3$. In the comparison the scaling factor was accounted for flow velocity $U$, electric field amplitude tangential component $E_\theta$ and amplitude of the inductor current $I_c$.

The channels of two plasmatrons IPG$-4$ and IPG$-3H$ are geometrically similar with scaling factor $\kappa=2$. Maximum deviation of plasma parameters for this pair of channels gives the estimation of the accuracy of similarity conditions (5)–(11): about 4÷7% for the temperature $T_c(r/R_c, Z_c)$, enthalpy $h_c(r/R_c, Z_c)$, velocity $U_c(r/R_c, Z_c)$ at the channel exit section; about 5÷8% for the electromagnetic field parameters $E_\theta(r/R_c, Z_{\text{min}})$, $H_z(r/R_c, Z_{\text{min}})$ in the section at the middle of inductor coil; the amplitude of inductor current $I_c$, the maximum values of $T$, $E_\theta$, $H_z$ within the channel are practically identical for IPG$-4$ and IPG$-3H$. So, the similarity conditions are reasonably accurate for geometrically similar plasmatrons IPG$-4$ and IPG$-3H$.

The channel of the real IPG$-3$ plasmatron is not geometrically similar to IPG$-4$ one. By this reason, the application of similarity conditions (5)–(11) with scaling factor $\kappa=2$ to the pair of IPG$-4$ and IPG$-3$ plasmatron channels provides only a qualitative agreement. The quantitative agreement here is satisfactory for $T$ and $h$, but is not satisfactory for $U$ and $H_z$. Namely, the maximum difference in calculated parameters (with account for scaling factor) for IPG$-4$ and IPG$-3$ channels is: for flow velocity $U$ – about 35%, for $H_z$ – about 40%. However, the differences in other parameters are not so large, e.g. the maximum differences in $h$ and $T$ are about 3 and 7%, the difference in $I_c$ is 2%. So, the similarity conditions for the pair of IPG$-4$ and IPG$-3$ plasmatrons can be used in general for qualitative estimations, and for $h$ and $T$ can be used also for quantitative characterization.
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