Explicit incipient motion of cohesive and non-cohesive sediments using simple hydraulics

Befekadu Taddesse Woldegiorgis\textsuperscript{1} | Ann Van Griensven\textsuperscript{1,2} | Willy Bauwens\textsuperscript{1}

\textsuperscript{1}Department of Hydrology and Hydraulic Engineering, Vrije Universiteit Brussel (VUB), Brussels, Belgium
\textsuperscript{2}UNESCO-IHE Institute for Water Education, Delft, Netherlands

Abstract

Existing dimensionless expressions that represent the incipient motion of sediments are based on studies of non-cohesive sediments. Because of the complex behaviour of cohesive sediments, many simulators also assume non-cohesiveness when simulating the erosion of cohesive sediments. However, studies show that the critical shear force needed for entrainment is much higher for consolidated cohesive sediments than for similarly sized non-cohesive sediments. Treating cohesive sediments as non-cohesive sediments thus will introduce a significant error with regard to quantifying the eroded sediment mass. On the other hand, the existing expressions of non-cohesive sediments require relatively detailed hydraulic calculations to estimate the shear velocity or the bed shear stress and thus cannot be used with simplified simulators. Therefore, it is essential to have a versatile simple explicit method that estimates the incipient motion condition of both the consolidated cohesive and non-cohesive sediments whenever needed. In this paper, explicit analytical expressions are proposed that simulate the incipient motion of consolidated cohesive and non-cohesive sediments, based on the critical erosion curves of the Hjulström–Sundborg–Miedema diagram. The new method reproduces the latter diagram with high precision. It also mimics the critical incipient condition of non-cohesive sediments determined by a well-known analytical method for other experimental data sets and for the East Fork River without the need of an iterative solution. The new approach provides essential information for estimating the entrainment condition of pebbles or finer sediments. Besides, the use of the mean flow velocity and the flow depth as predictors of incipient condition allows for its easy and efficient implementation in conceptual simulators that do not perform detailed hydraulic calculations and for use by modelers that are not familiar with the hydrotechnical literature. It also reduces the computation time required for simulation.

KEYWORDS

consolidated cohesive sediment, erosion, Hjulström–Sundborg diagram, incipient motion, resuspension
INTRODUCTION

In this paper, a new expression for the calculation of incipient motion of cohesive and non-cohesive sediments is presented which can be incorporated in conceptual river models. Conceptual river models often only use mean flow and depth to represent the flow dynamics and prefer to use explicit expressions that can be solved without iteration, resulting in high calculation speed.

The study of incipient motion of sediments was first rationalised by Shields (1936). The famous Shields diagram is independent of the grain density and of the fluid properties. However, the shear velocity and grain size occur both in the Shields parameter and in the shear Reynolds number and therefore, his method uses an implicit graph (Cao, Pender, & Meng, 2006; Miedema, 2014; Paphitis, 2001; Simões, 2014; Zanke, 2003) that can be difficult to interpret (Yalin, 1972; Yang, 1973). The implicit graph developed by Shields was later extended by other researchers, to include finer and coarser particles. Detailed information on the extended experimental dataset can be found in Beheshti and Ataie-Ashtiani (2008) and Paphitis (2001). The combined experimental data are scattered and, consequently, it is difficult to fit analytical expressions to them (Simões, 2014). An alternative approach to the Shields method was proposed by Liu (1935). This method relates the movability number—the ratio of the shear velocity to the settling velocity—to the shear Reynolds number. Studies show that this approach narrows the scatter of the experimental data (Simões, 2014) and better represents the grain shape effects on the critical condition through calculation of the settling velocity (Paphitis, 2001). Since both the Shields method and the movability number approach are implicit in nature, several analytical expressions have been developed to simplify the application of the criteria of incipient motion in the field of sediment transport. However, they require hydrodynamic calculations. Because of this, these methods are not suitable for use in conceptual sediment transport simulators that rely on the hydraulics simulated by hydrologic concepts whereby the river is represented by lumped river reaches. In addition, none of the current methods can be used both for cohesive and non-cohesive sediments. Many simulators use the assumption of non-cohesive sediments to simulate the erosion behaviour of cohesive sediments. However, studies show that the critical shear force required to erode consolidated cohesive sediments is higher than that required to erode non-cohesive sediments of the same size. Treating cohesive sediments as non-cohesive sediments thus introduces a significant error with regard to quantifying the erosion of consolidated cohesive sediments from river beds.

The main objective of this research is to develop analytical expressions that enable a representation of the incipient motion condition of both the consolidated cohesive and non-cohesive sediments without executing detailed hydrodynamic calculations. These expressions allow improved calculation of incipient motion, especially for cohesive sediments. Also, the approach enables the application of incipient motion of sediments in simplified models that do not calculate the hydraulic variables in a detailed way. This outcome in turn allows simulation of size dependent processes (such as size-selective erosion, integrating sediment resuspension with the resuspension of sediment-bound pollutants with a higher affinity for fine sediments) in simple conceptual models. The aim here is to develop a unified dimensionless method that uses the flow velocity and the flow depth as predictors to determine the critical particle size of erosion of both the consolidated cohesive and the non-cohesive sediments without a need for detailed hydraulic calculations and iterative solutions.

EXISTING METHODS

Explicit dimensionless methods developed for non-cohesive sediments

Methods that require detailed hydraulic information

A review of many empirical equations that have been proposed for determining the critical Shields parameter as a function of the dimensionless grain size was presented by Beheshti and Ataie-Ashtiani (2008). Among the diverse dimensionless fit equations developed for the Shields parameter are the well-known algebraic equations of Brownlie (1981) (Equation 1) and Soulsby and Whitehouse (1997) (Equation 2):

$$\theta = \frac{0.22}{D_9^{0.9}} + 0.06e^{-17.77D_9^{0.9}}$$  \hspace{0.5cm} (1)

$$\theta = \frac{0.22}{D_9^{0.9}} + 0.55[1 - e^{(-0.02D_9^s)}]$$  \hspace{0.5cm} (2)

with

$$D_9 = \left(\frac{(s - 1)g}{v^2}\right)^{1/3}$$  \hspace{0.5cm} (3)

where, $\theta$ is the Shields parameter $[-]$; $D_9$ is the dimensionless grain size $[-]$ as defined in Equation 3; $s$ is the ratio of the density of the sediment to the density of water $[-]$; $d$ is the particle size $[m]$; $g$ is the gravitational acceleration $[m/s^2]$; $v$ is the kinematic viscosity $[m^2/s]$ and $\epsilon$ is the base of natural logarithm.

The Equations 1 and 2 relate the Shields parameter to the dimensionless grain size for non-cohesive sediments. Among the empirical equations fitted to the alternative approach (where the movability parameter is expressed as a
function of the dimensionless grain size) are the works of Simões (2014), Beheshti and Ataie-Ashtiani (2008) and Paphitis (2001). Simões (2014) demonstrated that his empirical equation (Equation 4) is a better fit to previous experimental data when compared with prior approaches:

$$\Lambda_c = 0.215 + \frac{6.79}{D_{cr}^{1.70}} - 0.0750e^{-2.62 \times 10^{-3}D_{cr}}$$  \hspace{1cm} (4)$$

where, $$\Lambda_c$$ is the critical movability parameter.

All analytical expressions fitted to either the Shields parameter or the movability number express the critical parameter of incipient motion (the Shields parameter or the movability number) of granular sediments as a function of the dimensionless grain size or grain diameter. Despite their wide applicability, the formulation of these methods does not take the cohesive nature of sediments into account. The need for detailed hydraulic information—the computation of the shear velocity or the bed shear stress—hampers their application in simple sediment transport simulators that do not compute detailed hydraulic variables. Moreover, for some sediment simulators that use multiple sediment size classes (Shrestha, Leta, De Fraine, van Griensven, & Bauwens, 2013), the critical size of motion should be solved iteratively from Equations 1, 2 or 4. The iteration increases the computational cost.

### 2.1.2 Methods that require limited hydraulic information

Giménez-Curto and Corniero (2009) developed an explicit dimensionless method to determine the incipient motion condition of granular sediments in open channels requiring limited hydraulic calculations. This method expresses the critical dimensionless mean flow velocity as a function of the bed slope, the angle of repose and the specific gravity of the granular sediment (Equation 5):

$$\frac{U_c}{\sqrt{g d_{cr} \sin(S)}} = 0.78 \left[ 1 + \sqrt{1 + 0.48(s-1)(\cot S - \cot \phi_{rep})} \right]^{3/2}$$  \hspace{1cm} (5)$$

where, $$U_c$$ is the critical mean flow velocity/[ms] required for motion of particle size $$d_{cr}$$ [m], $$S$$ is the slope of the channel bed [--] and $$\phi_{rep}$$ [--] is the angle of repose of the sediment. One of the advantages of this method in simplified simulators is that it does not require the shear stress and the shear velocity to be calculated. Besides, it does not require an iterative solution. However, it cannot be applied to cohesive sediments.

The above analytical expressions used for determining the critical incipient motion condition, based on detailed or limited hydraulic calculations, are developed for granular sediments. Thus, their application to cohesive sediments ignores the impact of the cohesiveness on the erosion behaviour. Although it is not uncommon to ignore the cohesive behaviour of sediments for simplicity (Bertrand-Krajewski, 2006), it is generally acknowledged that there is a considerable difference in the tractive force needed to initiate motion of cohesive and non-cohesive sediments (Fang, Shang, Chen, & He, 2014; Miedema, 2014). It is, therefore, recommended to consider the cohesive nature of the sediment whenever possible. This method has another limitation: it does not consider the effect of the flow depth and hence, ignores the effect of the relative roughness (the ratio of bed-roughness scale to flow depth)—an important factor for the initiation of motion (Lamb, Dietrich, & Venditti, 2008; Recking, 2009; Shvidchenko & Pender, 2000)—on the incipient motion.

### 2.2 The explicit dimensional methods developed for cohesive sediments

The methods under this category predict the incipient motion of cohesive sediments based on the flow velocity and the flow depth. Among these methods are the dimensional approaches developed by Hjulström (1939), Sundborg (1956) and Xu, Bai, Ji, and Williams (2015). Unlike the methods based on the Shields parameter or the movability number, the advantage of these methods is that they account for the effect of cohesiveness. However, their application is limited to certain circumstances.

Hjulström (1939) developed the well-known dimensional diagram that relates the mean flow velocity to the particle size for incipient motion in the same period as the work of Shields. He conducted his study on a canal with a depth of 1 m and included grain sizes finer than those considered by Shields (Miedema, 2014). The Hjulström diagram is used extensively by sedimentary geologists (Southard, 2006).

However, its application is limited because a new diagram has to be constructed for each combination of fluid properties and flow depth. Sundborg (1956) indeed demonstrated the dependence of the critical flow velocity on the water depth, for non-cohesive sediments. Williams (1967) also emphasized the need to adjust critical conditions for the depth effect. In the Hjulström–Sundborg diagram Sundborg (1956) extended the Hjulström diagram by considering separate—explicit—curves of sediment erosion for four water depths, i.e. 0.01, 0.1, 1 and 10 m. Miedema (2014) later extended the Hjulström–Sundborg curves for cohesive sediments using the model of Zanke (2003). The Hjulström–Sundborg diagram modified by Miedema, hereafter, is referred to as the Hjulström–Sundborg–Miedema diagram.

Miedema (2014) fitted an empirical model to the Hjulström diagram (Equation 6). His equation defines the
critical velocity required for initiation of motion as a function of the grain size for a water depth of 1 m:

\[ U_c = 1.5 \left( \frac{v}{d} \right)^{0.8} + 0.85 \left( \frac{v}{d} \right)^{0.35} + 9.5 \frac{(s - 1)gd}{(1 + 2.25(s - 1)gd)} \]

(6)

where, \( U_c \) is the critical mean flow velocity/[m/s] required for motion of particle size \( d \) [m].

Superimposing the curve reconstructed by Miedema’s equation on the Hjulström–Sundborg–Miedema diagram shows that his equation does not fit the curves that are developed for water depths other than 1 m. So far, there is no unified explicit equation to determine the critical particle sizes that will erode in cohesive sediments under an arbitrary flow depth. This is an important research gap that needs to be addressed to enable the application of the concept of incipient motion of consolidated cohesive sediments in sediment simulators that have been simulating cohesive sediments using the concepts of non-cohesive sediments. Also, this research enables the application of the concept of incipient motion in simplified sediment simulators without the need to calculate the hydraulics in detail.

Besides the methods based on the Hjulström diagram, the method developed by Xu et al. (2015) is a dimensional method that does not require detailed hydraulic calculations. Xu et al. (2015) investigated the influence of the mud density on the incipient motion of coastal mud. They developed an explicit function, expressing the critical mean flow velocity of incipient motion in turbulent open channel flow as a function of the mud density and the water depth (Equation 7):

\[ U_c = \sqrt{\frac{C_1 (\rho_m - \rho) C_2}{\kappa^2 \rho}} \ln \left( \frac{9Y}{v} \sqrt{\frac{C_1 (\rho_m - \rho) C_2}{\rho}} \right) - \ln(Y) - 1 \]

(7)

where, \( C_1 = 1.59 \) E-08 [Nm/kg], \( C_2 = 3.06 \), \( \kappa \) is the von Karman constant (approximated as 0.4), \( Y \) is the water depth [m], \( \rho_m \) is the mud density [kg/m³], \( \rho \) is the density of water [kg/m³] and \( v \) is the kinematic viscosity of water [m²/s].

The equation is applicable where the incipient motion does not occur for individual sediment particles. It is not applicable for non-cohesive sediments because all of the fluid forces responsible for the initiation of motion in such cases are related to the particle size (Xu et al., 2015). Therefore, this method cannot be used to determine the critical particle size of erosion and hence its application for the quantification of the eroded sediment mass in grain size-based approaches is problematic. Therefore, it is not suited to fill the gap raised in this research.

2.3 | Overview of the current methods

In summary, the application of the current dimensionless methods developed for estimating the incipient motion of non-cohesive sediments—except the method of Giménez-Curto and Corniero (2009)—presents a problem in simplified models that do not calculate the shear stress or the shear velocity. The method of Giménez-Curto and Corniero (2009) is limited as it does not account for the effect of flow depth, an important factor affecting incipient motion. Besides, the use of these methods for simulating the erosion behaviour of the consolidated cohesive sediments in any sediment transport simulator induces a significant error with regard to quantifying the eroded sediment mass. The dimensional methods developed for estimating the critical motion condition of consolidated cohesive sediments are either developed for a specific water depth (e.g. the equations of Miedema) or cannot be integrated to sediment transport modelling for quantifying the eroded mass of sediment (e.g. the method of Xu et al. (2015)). Literature shows that depth-averaged flow velocity and flow depth are important factors that influence incipient motion (Lamb et al., 2008). Consequently, the two hydraulic variables can be used as predictors of incipient motion.

3 | THE NEW METHOD

3.1 | The conceptual framework

In order to develop the new equation, the Hjulström–Sundborg–Miedema diagram was used as a reference as it relies only on the mean flow velocity and flow depth and therefore does not require detailed hydraulics calculations.

The four curves of the Hjulström–Sundborg–Miedema diagram can be collapsed to a unified dimensionless curve as shown in Figure 1. It is an indication that a unique flow depth-independent expression can be developed to estimate the critical particle size of incipient motion. The dimensionless parameter of the unified curve was determined after experimenting with different combinations of dimensionless parameters that explain the variability of incipient conditions. The unified curve relates a dimensionless parameter (\( \zeta_1 \)) defined in Equation 8 to the dimensionless grain size (\( D_s \)):

\[ \zeta_1 = \log_{10} \left[ \left( \frac{U_Y}{Y} \right) \left( \frac{Y_{10}}{Y} \right)^{1.05 + 0.045 \log_{10}(D_s + 10)} \right] \]

(8)

where, \( \zeta_1 \) is a dimensionless parameter; \( U_Y \) is the surface velocity/[m/s]; \( Y \) is the flow depth [m]; \( Y_{10} = 10 \) m; \( Y_{10}/Y \) is the relative depth of the flow compared to the 10 m flow depth [-].

The new dimensionless parameter, \( \zeta_1 \), follows the same pattern as the Shields parameter for the consolidated
cohesive sediment range. However, unlike the Shields parameter, it has a strictly increasing slope for the coarse sediment range. Consequently, no clear relationship is found between \( \zeta_1 \) and the Shields parameter. The dimensionless parameter, \( \zeta_1 \), increases where greater bed shear stress is required to initiate motion of the sediment particles and vice versa (see Figure 1). The high Nash-Sutcliffe Efficiency (NSE) and per cent bias (PBIAS) (Equation 9) goodness of fit indices (Table 1) show that the dimensionless parameter \( \zeta_1 \) generated using the experimental data collapses to a nearly single curve with insignificant scatter. This fact is an indication that the unified dimensionless curve represents all the experimental incipient conditions provided in the Hjulström–Sundborg–Miedema diagram with high accuracy and hence can be projected for other water depths.

NSE and PBIAS statistical performance indicators (Equation 9) are used to measure the efficiency of the new expression. The performance is evaluated based on the reproducibility of the critical velocities for given dimensionless grain sizes and vice versa. The dimensionless grain size varies up to six orders of magnitudes and hence a log-transformation is required to reduce the heteroscedasticity of the residuals before evaluating the efficiencies:

\[
\begin{align*}
\text{NSE} & = 1 - \frac{\sum (Q_{\text{new},i} - Q_{\text{exp}})^2}{\sum (Q_{\text{exp}} - Q_{\text{exp}})^2} \\
\text{PBIAS} & = 100 \times \frac{\sum (Q_{\text{new},i} - Q_{\text{exp}})}{\sum (Q_{\text{new}})}
\end{align*}
\]  

(9)

where, \( Q_{\text{exp},i} \) is the \( i \)th experimental value; \( Q_{\text{new},i} \) is the \( i \)th simulated value and, \( Q_{\text{exp}} \) is the average of the experimental values, \( n \) is the number of experimental data points.

The dimensionless unified curve is implicit and hence cannot be implemented without iteration. To avoid the need for iteration and enable an explicit formulation, an equivalent explicit dimensionless unified analytical expression was developed that defines the dimensionless flow depth \( \left( Y/d_{cr} \right) \) as a function of dimensionless variables (Equation 10) for consolidated cohesive and coarse granular sediments. Hereby, a dimensionless analytical expression is chosen as (a) a dimensionless expression appears to provide a better match to all the curves of the Hjulström–Sundborg–Miedema diagram than the dimensional expressions that were tested and (b) the dimensionless approach offers the advantage of avoiding scale dependency of the expression:

\[
Y = Y_{10} \times \left\{ \begin{array}{ll} 
5.16118 \times 10^{-9} \times \left[ \frac{U_Y}{v} \times \left( \frac{d_{cr}}{Y} \right) \right]^{1.08} & \text{if } D_s < 5.5, \text{ consolidated cohesive} \\
0.0894511 \times \left[ \frac{U_Y}{v} \times \left( \frac{d_{cr}}{Y} \right) \right]^{1.08} & - 360904.32 & \text{if } d_{cr} \end{array} \right. 
\]

\[
D_s > 5.5, \text{ non-choesive}
\]

(10)

where, \( d_{cr} \) is the critical particle size [m]. The expression is valid both for wide and narrow channels. For wide channels, the expression \( U_Y/v \) becomes the Reynolds number and hence the dimensionless flow depth \( \left( Y/d_{cr} \right) \) becomes a function of the Reynolds number. The first equation of Equation 10 is only applicable to consolidated cohesive sediments because it is developed using consolidated cohesive sediment data.
For the four entrainment curves of unconsolidated fine sediments constructed by Miedema (2014) using the Soulsby and Whitehouse (1997) equations, a unified explicit curve was constructed where the four curves collapse (see Figure 2). This collapse again indicates that a unified explicit expression can be developed to reproduce the incipient condition of unconsolidated fine sediments.

The unified explicit dimensionless expression developed here for the incipient condition of unconsolidated fine sediments is provided in Equation 11. It reproduces the unified incipient condition curve of the unconsolidated fine sediments with high accuracy (Figure 3, and see Table 2):

\[
D_0 = 10^{1 - \sqrt{\left(77.21845 - 11.57541 \log 10 \left(\frac{U_Y}{\nu} \frac{Y}{C_3 \nu} \frac{1}{C_3} \right) \right)}}, \quad D_0 \leq 5.5
\]  

(11)

### 3.2 | Which flow velocity?

The velocity used to derive the curves on the Hjulstrom-Sundborg-Miedema diagram corresponds to the surface velocity. Accordingly, the 10 m depth curve uses the velocity measured at 10 m above the bed, and the 1 m depth curve uses the velocity measured at 1 m above the bed (Sundborg, 1956). However, the surface velocity mostly differs from the mean velocity, depending on the velocity profile, the effect of the bank roughness—dependent on the width of the river—and the wind action. The ratio of the mean velocity to the surface velocity becomes 0.85 if the velocity follows the log-low-of-the-wall (Cheng & Gartner, 2003). Studies carried out on natural rivers show that the ratio of the mean velocity to the surface velocity is quite close to the theoretical value of 0.85 (Rantz (1982) showed that this ratio varies between 0.85 and 0.86 for natural rivers and Cheng and Gartner (2003) showed that this ratio varies between 0.8 and 0.93 with a mean value of 0.88). For smooth artificial channels, the ratio is 0.9 (Rantz, 1982). Therefore, if only the mean velocity is available, it has to be divided by 0.85 and 0.9, respectively, for natural river and flume experiments before being used in the expressions presented here. The procedure for determining the mean velocity, if the surface velocity is not provided, is available in Rantz (1982).

### 3.3 | Application to sand-mud mixtures

Mixtures of cohesive and non-cohesive sediments can be modelled in one of two ways (Manning & Schoellhamer, 2013): (1) the segregated approach that treats the cohesive and non-cohesive parts separately, and (2) the approach that allows the cohesive and non-cohesive sediments to interact with each other so that the mixture exhibits different characteristics. Literature (Manning, Baugh, Spearman, Pidduck, & Whitehouse, 2011; Manning & Dyer, 2007) shows that sand and cohesive mud coexist as a single mixture. The erosion threshold of the sand-mud mixture is affected by the cohesiveness (Jacobs, Hir, Van Kesteren, Cann, & Kesteren, 2011). Also, cohesive material forms either small compact micro flocs or larger porous macro flocs (Eisma, 1986; Manning et al., 2013). The floc formation changes the terminal settling velocity, particle effective density, and the effective particle size from their compositional base (Manning et al., 2013). Therefore, sand-mud mixtures exhibit different erosion and settlement characteristics compared to purely cohesive and purely non-cohesive sediment. River sediments often are a mixture of different particle sizes and mixtures can be expected to have different characteristics.
than the individual constituents (Whitehouse, Soulsby, Roberts, & Mitchener, 2000).

The expressions used here are derived from experimental curves generated for either purely cohesive or purely non-cohesive sediments. Consequently, they have limitations when used to simulate the complex behaviour of coexisting sand and cohesive mud. This method is thus primarily designed for purely cohesive or purely non-cohesive sediments. Nevertheless, it may be applied to a mixture of sand and cohesive mud using the threshold fraction of cohesive sediment suggested in the literature to categorize the sediment as purely cohesive or purely non-cohesive. Accordingly, a sediment mixture with a clay fraction of greater than 10% may be treated as a purely cohesive sediment as the cohesive property becomes dominant for this condition (Mitchener & Torfs, 1996; van Rijn, 1993) and vice versa.

### 3.3.1 | The evaluation strategy

#### Comparison with the Hjulström–Sundborg–Miedema diagram

The unified dimensionless expressions are first calibrated on the 10 m and 0.1 m flow depth experiments of the critical curves of the Hjulström–Sundborg–Miedema diagram and then validated on the 1 m and 0.01 m flow depth experiments of the latter diagram both for the consolidated cohesive and for non-cohesive sediments.

Comparison with the method of Soulsby and Whitehouse (1997) for flume experimental data

The other evaluation strategy used in this paper compares the critical particle size of incipient motion of the new method with that determined using the method of Soulsby and Whitehouse (1997) provided earlier. Included in this comparison are data from three flume experiments (Sun & Donahue, 2000; Wilcock & McArdell, 1997; Wu & Yang, 2004) and the field data of the East Fork River (Leopold & Emmett, 1976) for non-cohesive sediments. The Shields parameter was determined for use in the method of Soulsby and Whitehouse (1997) using Equation 12. The average bed shear stress determined using Equation 13 is corrected for the effect of smooth sided flume walls using the method of Vanoni and Brooks (1957). The critical particle size of incipient motion for the method of Soulsby and Whitehouse (1997) is iteratively determined by combining Equations 2 and 3:

\[
\theta = \frac{\tau_0}{(\rho_s - \rho)gd} \tag{12}
\]

where, \( \tau_0 \) is the bed shear stress/[Nm²] determined using Equation 13; \( \rho_s \) is the density of the sediment [kg/m³]; \( \rho \) the density of the fluid [kg/m³]; \( g \) is the acceleration due to gravity/[ms²] and \( d \) is the particle size [m]:

\[
\tau_0 = \rho_s g R s_w \tag{13}
\]

where, \( R \) is the hydraulic radius [m]; \( s_w \) is water surface slope approximated as the angle of the channel bed with the horizontal [-].

The mean velocities of the flume experimental data are divided by 0.9 and that of the East Fork River is divided by 0.85 to approximate the surface velocity for use in the new method.

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### Table 2

| Flow depth (m) | NSE   | PBIAS |
|---------------|-------|-------|
| 10 m          | 0.996 | 0.67  |
| 1 m           | 0.992 | 0.30  |
| 0.1 m         | 0.982 | 0.40  |
| 0.01 m        | 0.973 | 0.30  |

The mean velocities of the flume experimental data are divided by 0.9 and that of the East Fork River is divided by 0.85 to approximate the surface velocity for use in the new method.
4 | RESULTS AND DISCUSSION

4.1 | Reconstructing the Hjulström–Sundborg–Miedema diagram

The calibration results of the new method for the experiments with 10 m and 0.1 m flow depth are depicted in Figure 4. The new method reproduces both the critical velocities and the critical dimensionless grain sizes of the calibration experiments accurately (Table 3). It reproduces the critical velocities of the calibration experiments with a minimum NSE value of 0.99 and a maximum PBIAS of 4.0%. The dimensionless grain sizes are simulated with a minimum NSE of 0.92 and a maximum PBIAS of 14%. The relatively large PBIAS in the latter case is due to deviations of the simulated dimensionless grain sizes corresponding to the 10 m experiment towards the high \( D^* \) values (Figure 4). The heteroscedasticity of the residuals leads to a large total bias. The NSE of the new method with regard to simulating the dimensionless grain sizes of both calibration experiments increases to 0.99 when the \( D^* \) values are log-transformed (reduced heteroscedasticity of residuals). The PBIAS of the 10 m experiment also reduces to −0.9% after the log-transformation (Table 3).

The critical velocities simulated by the new method provide very high NSE values of for both the 0.01 m and the 1 m water depth validation experiments. The PBIAS of the new method is less than 2% for both validation experiments. With regard to reproducing the validation dimensionless grain sizes, the new method provides high NSE and low PBIAS for the 0.01 m water depth. The relatively low NSE and high PBIAS of the equation for the 1 m depth validation experiment is explained by the fact that the new method underestimates the critical dimensionless grain sizes when \( D^* \) is greater than 1000 (corresponding to very coarse gravel) (see Figure 5). This interpretation is confirmed by the increase of the NSE to 0.99 and the decrease of the PBIAS to −3.9% when the curve of 1 m depth is ignored when \( D^* \) is greater than 1000. For log-transformed dimensionless grain sizes, the residual heteroscedasticity decreases and hence the NSE of the new method becomes very high for both validation experiments over the full range of the \( D^* \) values (Table 3).

4.2 | The new method versus the Soulsby and Whitehouse (1997) method for non-cohesive sediments

The performance of the new method is further validated by comparing with the method of Soulsby and Whitehouse (1997) using the data from the three different flume experiments and the field measurement data of the East Fork River (Leopold & Emmett, 1976). The new simplified method provides critical particle sizes of incipient motion generally comparable to the method of Soulsby and Whitehouse (1997) (Figure 6). The field data are simulated with high efficiency (Table 4). The agreement with the experimental data is also plausible, given the simplicity of the new method and the complexity of the processes. The maximum PBIAS after log transformation of the data is 2.9%, which is low. It is worth mentioning that the performance of the new method improved significantly—especially for the field data—when the surface velocity is used instead of the mean velocity. Given the fact that the new simplified method does not rely on detailed hydrodynamic calculations and iterations, simulating the critical particle sizes of non-cohesive sediments with the given efficiencies—especially for the real case—shows that the new method can be used to estimate the incipient conditions of non-cohesive sediments in natural rivers and artificial channels.

**FIGURE 4** The calibration of the analytical expression reproduces the experimental data of Hjulström–Sundborg–Miedema for flow depths of 10 m and 0.1 m.

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4.3 Limitations of the new method

Although the new method is easily applicable—as compared with the other methods that are dependent on hydrodynamic calculations—for simulating both non-cohesive and consolidated cohesive sediments just using the surface velocity and the flow depth, it does have limitations. Miedema (2014) used the model of Zanke (2003) to extend the four experimental curves of Sundborg (1956) to the cohesive part of the sediment—the range for which Sundborg

| Flow depth (m) | NSE, $U_{cr}$ | PBIAS, $U_{cr}$ | NSE, $D_*$ | PBIAS, $D_*$ | NSE, log10($D_*$) | PBIAS, log10($D_*$) |
|---------------|---------------|-----------------|-------------|--------------|--------------------|--------------------|
| Calibration   | 10 m          | 0.99            | 0.4         | 0.95         | -14.10             | 1.0                | -0.9               |
|               | 0.1 m         | 0.99            | -4.0        | 0.92         | -5.0               | 0.99               | 12.1               |
| Validation    | 1 m           | 0.99            | -2.0        | 0.77         | -31.3              | 0.99               | 3.8                |
|               | 0.01 m        | 0.99            | -1.6        | 0.99         | -4.4               | 0.99               | -120               |

**TABLE 3** The efficiency of the new analytical expression with regards to reconstructing the Hjulström–Sundborg–Miedema diagram

**FIGURE 5** The validation of the analytical expression shows a good match with the experimental data of Hjulström–Sundborg–Miedema for 1 m and 0.1 m flow depths

**FIGURE 6** The comparison of the critical particle size simulated by the new method vs. the method based on Soulsby and Whitehouse (1997) using the three experimental dataset and one field data generally provides a good agreement
(1956) did not provide definite curves due to the large uncertainty of the critical condition in the cohesive part (Southard, 2006; Sundborg, 1956). Because the formulations used here for the consolidated cohesive sediments are based on the simulation of Miedema (2014), any uncertainty in the Zanke model would propagate to the consolidated cohesive formulations of the new method as well. Studies (Fang et al., 2014) show that the critical bed shear stress required for the motion of cohesive sediments could be two orders of magnitude higher than that required for non-cohesive sediments of the same particle size. Therefore, the uncertainty of the Zanke model seems less important than the magnitude of error that could be introduced by considering the critical condition determined for non-cohesive sediments as a critical condition of consolidated cohesive sediments.

Similar to other incipient motion experiments, Sundborg (1956) conducted his experiments on materials with uniform grain size because of the complexity of experimenting on mixed size sediments where some particles are exposed and some are hidden. The finer particles are exposed to less than the average bed shear stress, whereas the coarser particles are exposed to more than the average bed shear stress (Wu & Yang, 2004). Accordingly, the Hjulström–Sundborg–Miedema diagram understimates the incipient condition of coarser particles of mixed size sediment but overestimates the incipient condition of finer particles in mixed size sediments. Consequently, the method utilized here inherits this limitation. This limitation actually exists for other explicit methods based on Shield’s diagram, as the Shields diagram is also based on uniformly sized sediments. Therefore, the critical particle sizes determined based on an assumed uniform sediment dataset should be carefully used for mixed size sediments.

### TABLE 4

The efficiency of the new analytical expression with regards to reproducing the critical particle sizes of incipient motion (log-transformed) determined using the Soulsby and Whitehouse (1997) method for the flume and field data from the literature.

| Data                        | NSE  | PBIAS | Remark                     |
|-----------------------------|------|-------|----------------------------|
| Leopold and Emmett (1976)   | 0.97 | 0.5   | Field data of East Fork River |
| Wilcock and McArvell (1997) | 0.49 | −2.9  | Flume experimental data    |
| Sun and Donahue (2000)      | 0.62 | 0.2   | Flume experimental data    |
| Wu and Yang (2004)          | 0.63 | −1.9  | Flume experimental data    |

Novel and easily applicable expressions are proposed here that simulate the incipient motion of both consolidated cohesive sediments and non-cohesive sediments, based on the Hjulström–Sundborg–Miedema diagram. The four curves of the Hjulström–Sundborg–Miedema diagram turned out to collapse to a unified curve, relating a dimensionless parameter to the dimensionless grain size. The existence of such a unified curve implies the possibility of developing a unified expression that can reproduce the Hjulström–Sundborg–Miedema diagram. Consequently, a unified dimensionless expression was developed that simulates the critical particle size of incipient motion with high efficiency based only on mean flow velocity and water depth without the need of an iterative solution. The validation of the new unified expression using three other experimental data and field data of the East Fork River showed a good agreement with the method of Soulsby and Whitehouse (1997). Contrary to most of the current analytical expressions that require detailed hydrodynamic calculations to estimate the bed shear stress or shear velocity, the use of the mean flow velocity and the flow depth as predictors of the incipient condition in the new method allows for applications even in simplified models. For instance, it provides useful information on the critical particle sizes of incipient motion required as input for the conceptual probabilistic sediment mobility simulator developed by Woldegiorgis, Wu, van Griensven, and Bawens (2017). It can also be used by some hydrodynamic sediment transport simulators such as the multiple class-size sediment transport simulator developed by Shrestha et al. (2013).

### ORCID

Befekadu Taddease Woldegiorgis [http://orcid.org/0000-0002-0576-4312](http://orcid.org/0000-0002-0576-4312)

### REFERENCES

Beheshiti, A. A., & Ataie-Ashtiani, B. (2008). Analysis of threshold and incipient conditions for sediment movement. *Coastal Engineering*, 55(5), 423–430. https://doi.org/10.1016/j.coastaleng.2008.01.003

Bertrand-Krajewski, J.-L. (2006). Modelling of sewer solids production and transport. In J.-L. Bertrand-Krajewski (Ed.), *Cours de DEA “Hydrologie Urbaine”* (pp. 1–39). Lyon: INSA de Lyon.

Brownlie, W. R. (1981). *Compilation of alluvial channel data: Laboratory and field*. Technical Report KH-R-43B. Pasadena, CA: California Institute of Technology.

Cao, Z., Pender, G., & Meng, J. (2006). Explicit formulation of the shields diagram for incipient motion of sediment. *Journal of Hydraulic Engineering*, 9429(JUNE 2006), 553–562. https://doi.org/10.1061/(ASCE)0733-9429(2006)132
Cheng, R. T., & Gartner, J. W. (2003). Complete velocity distribution in river cross-sections measured by acoustic instruments. *Proceedings of the IEEE/OES Seventh Working Conference on Current Measurement Technology*, 2003, 21–26. https://doi.org/10.1109/CCM.2003.1194276

Eisma, D. (1986). Flocculation and de-flocculation of suspended matter in estuaries. *Netherlands Journal of Sea Research*, 20(2/3), 183–199.

Fang, H., Shang, Q., Chen, M., & He, G. (2014). Changes in the critical erosion velocity for sediment colonization by biofilm. *Sedimentology*, 61(3), 648–659. https://doi.org/10.1111/sed.12065

Giménez-Curto, L. A., & Corniero, M. A. (2009). Entrainment threshold of cohesionless sediment grains under steady flow of air and water. *Sedimentology*, 56(2), 493–509. https://doi.org/10.1111/j.1365-3091.2008.00981.x

Hjulstrom, F. (1939). Transportation of detritus by moving water: Part 1. Transportation. In P. D. Trask (Ed.), *Recent marine sediments. A Symposium: Tulsa, Oklahoma* (pp. 5–31). Tulsa, OK: AAPG.

Jacobs, W., Hir, P. Le., Van Kesteren, W., Cann, P., & Kesteren, V. (2011). Erosion threshold of sand–mud mixtures. *Continental Shelf Research*, 31(10), S15–S25.

Lamb, M. P., Dietrich, W. E., & Venditti, J. G. (2008). Is the critical shields stress for incipient sediment motion dependent on channel bed slope? *Journal of Geophysical Research: Earth Surface*, 113(2), 1–20. https://doi.org/10.1029/2007JF000831

Leopold, L. B., & Emmett, W. W. (1976). Bedload measurements. East Fork River, Wyoming. *Proceedings of the National Academy of Sciences of the United States of America*, 73, 1000–1004.

Liu, T.-Y. (1935). *Transportation of the Bottom Load in an Open Channel (Thesis)*. Iowa: University of Iowa.

Manning, A. J., Baugh, J. V., Spearman, J. R., Pidduck, E. L., & Whitehouse, R. J. S. (2011). The settling dynamics of flocculating mud–sand mixtures: Part 1-Empirical algorithm development. *Ocean Dynamics*, 61(2–3), 311–350. https://doi.org/10.1007/s10236-011-0394-7

Manning, A. J., & Dyer, K. R. (2007). Mass settling flux of fine sediments in Northern European estuaries: Measurements and predictions. *Marine Geology*, 245, 107–122. https://doi.org/10.1016/j.margeo.2007.07.005

Manning, A. J., & Schoellhamer, D. H. (2013). Factors controlling floc settling velocity along a longitudinal estuarine transect. *Marine Geology*, 345, 266–280. https://doi.org/10.1016/j.margeo.2013.06.018

Manning, A. J., Spearman, J. R., Whitehouse, R. J. S., Pidduck, E. L., Baugh, J. V., & Spencer, K. L. (2013). Flocculation dynamics of mud: Sand mixed suspensions. In A. J. Manning (Ed.), *Sediment transport processes and their modelling applications* (pp. 119–164). Rijeka, Croatia: InTech. https://doi.org/10.5772/3401

Miedema, S. A. (2014). Constructing the Shields curve, a new theoretical approach and its applications. In R. H. Curran (Ed.), 9th *World Dredging Congress (WODCON XIX)*: Beijing, China, 9–14 September 2010; [technical proceedings] (pp. 732–750). New York: Eastern Dredging Association.

Mitchener, H., & Torfs, H. (1996). Erosion of mud/sand mixtures. *Coastal Engineering*, 29(1–2), 1–25. https://doi.org/10.1016/S0378-3839(96)00002-6

Paphitis, D. (2001). Sediment movement under unidirectional flows: An assessment of empirical threshold curves. *Coastal Engineering*, 43(3–4), 227–245. https://doi.org/10.1016/S0378-3839(01)00015-1

Rantz, S. E. (1982). *Measurement and computation of streamflow: Volume 1*, Measurement of stage and discharge. Measurement of Stage and Discharge, USGS Water Supply Paper 2175. Washington, DC: U.S. G.P.O. Retrieved from https://doi.org/10.1029/wr017i001p00131

Recking, A. (2009). Theoretical development on the effects of changing flow hydraulics on incipient bed load motion. *Water Resources Research*, 45(4), 1–16. https://doi.org/10.1029/2008WR006826

Shields, A. (1936). *Anwendung der Aehnlichkeitsmechanik und der Turbulenzforschung auf die Geschiebewegung. Mitt. Preuss. Versuchsanstalt Wasserbau Schiffbau* (Vol. 26). Heft 26, Berlin. Belin: Mittelung der Preussischen Versuchsanstalt fur Wasserbau und Schiffbau.

Shrestha, N. K., Leta, O. T., De Frainne, B., van Griendsen, A., & Bauwens, W. (2013). OpenMI-based integrated sediment transport modelling of the river Zenne, Belgium. *Environmental Modelling and Software*, 47, 193–206. https://doi.org/10.1016/j.envsoft.2013.05.004

Shvidchenko, A. B., & Pender, G. (2000). Flume study of the effect of relative depth on the incipient motion of coarse uniform sediments. *Water Resources Research*, 36(2), 619–628.

Simões, F. J. M. (2014). Shear velocity criterion for incipient motion of sediment. *Water Science and Engineering*, 7(2), 183–193. https://doi.org/10.3882/j.issn.1674-2370.2014.02.006

Soulby, R., & Whitehouse, R. (1997). *Threshold of sediment motion in coastal environments. Pacific Coasts and Ports’ 97*: Proceedings of the 13th Australasian Coastal and Ocean Engineering Conference and the 6th Australasian Port and Harbour Conference; Volume 1 (p. 145). Centre for Advanced Engineering, University of Canterbury.

Southard, J. (2006). Threshold of movement. In J. Southard (Ed.), *Special topics: An introduction to fluid motions, sediment transport, and current-generated sedimentary structures* (pp. 260–284). Cambridge, MA: MIT OpenCourseWare. Retrieved from http://ocw.mit.edu/courses/earth-atmospheric-and-planetary-sciences/12-090-special-topics-an-introduction-to-fluid-motions-sediment-transport-and-current-generated-sedimentary-structures-fall-2006

Sun, Z., & Donahue, J. (2000). Statistically derived bedload formula for any fraction of nonuniform sediment. *Journal of Hydraulic Engineering*, 126(2), 105–111.

Sundborg, A. (1956). The River Klarälven: A study of fluvial processes. *Geografiska Annaler*, 38(2–3), 238–316.

van Rijn, L. C. (1993). Simple general formulae for sand transport in rivers, estuaries and coastal waters. Retrieved from www.leovanrijn-sediment.com

Vanoni, V. A., & Brooks, N. H. (1957). *Laboratory studies of the roughness and suspended load of alluvial streams. California Institute of Technology (Vol. 112)*. Pasadena, CA: California Institute of Technology.

Whitehouse, R. J. S., Soulsby, R. L., Roberts, W., & Mitchener, H. J. (2000). Dynamics of estuarine muds: a manual for practical applications. Report SR 527. Retrieved from http://www.scopus.com/inward/record.url?eid=2-s2.0-0033506599&partnerID=40&md5=58152c0925f1888745e5f663acc71ce4

Wilcock, P. R., & McArdell, B. W. (1997). Partial transport of a sand/gravel sediment flume. *Water Resources Research*, 33(1), 235–245. https://doi.org/10.1029/96WR02672
Williams, G. P. (1967). *Flume experiments on the transport of a coarse sand. Sediment transport in alluvial channels* (Vol. 562–B). US Government Printing Office.

Woldegiorgis, B. T., Wu, F.-C., van Griensven, A., & Bauwens, W. (2017). A heuristic probabilistic approach to estimating size-dependent mobility of nonuniform sediment. *Stochastic Environmental Research and Risk Assessment*, in press. https://doi.org/10.1007/s00477-017-1471-3

Wu, F.-C., & Yang, K.-H. (2004). A stochastic partial transport model for mixed-size sediment: Application to assessment of fractional mobility. *Water Resources Research, 40*(4), 1–18. https://doi.org/10.1029/2003WR002256

Xu, D., Bai, Y., Ji, C., & Williams, J. (2015). Experimental study of the density influence on the incipient motion and erosion modes of muds in unidirectional flows: The case of Huangmaohai Estuary. *Ocean Dynamics, 65*(2), 187–201. https://doi.org/10.1007/s10236-014-0803-9

Yalin, M. S. (1972). *Mechanics of sediment transport* (M. S. Yalin, Ed.) (1st ed.). New York: Pergamon Press.

Yang, C. T. (1973). Incipient motion and sediment transport. *Journal of the Hydraulics Division, 99*(10), 1697–1704.

Zanke, U. C. E. (2003). On the influence of turbulence on the initiation of sediment motion. *International Journal of Sediment Research, 18*(1), 17–31. Retrieved from http://www.waser.cn/journal/fulltext/2003-1/02.pdf

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