Bounded Model Checking for Hyperproperties

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Abstract. This paper introduces the first bounded model checking (BMC) algorithm for hyperproperties expressed in HyperLTL. Just as the classic BMC technique for LTL primarily aiming at finding bugs, our approach also targets identifying counterexamples. LTL describes the property of individual traces and BMC for LTL is reduced to SAT solving. HyperLTL allows explicit and simultaneous quantification over traces and describes the property of multiple traces and, hence, our BMC approach naturally reduces to QBF solving. We report on successful and efficient model checking of a rich set of experiments on a variety of case studies, including security/privacy, concurrent data structures, and path planning in robotics applications.

1 Introduction

Hyperproperties \cite{9} have been shown to be a powerful framework for specifying and reasoning about important sets of requirements that were not possible with trace-based languages such as the classic temporal logics. Examples include information-flow security, consistency models in concurrent computing \cite{5}, and robustness models in cyber-physical systems \cite{30}. The temporal logic HyperLTL \cite{8} extends LTL by allowing explicit and simultaneous quantification over execution traces, describing the property of multiple traces. For example, observational determinism can be specified by the following HyperLTL formula:

\[ \forall \pi. \forall \pi'. (o\pi \leftrightarrow o'\pi) \text{W}(i\pi \leftrightarrow i'\pi) \]

stipulates that every pair of traces \( \pi \) and \( \pi' \) have to agree on the value of the (public) output \( o \) as long as they agree on the value of the (secret) input \( i \), where \( \text{W} \) denotes the weak until operator.

There has been a recent surge of model checking techniques for HyperLTL specifications \cite{8,11,20,22}. These approaches employ various techniques (e.g., alternating automata, model counting, strategy synthesis, etc) to verify hyperproperties. However, they generally fall short in proposing an effective method to deal with identifying bugs with respect to alternating HyperLTL formulas. Indeed, quantifier alternation has been shown to generally elevate the complexity class of model checking HyperLTL specifications in different shapes of Kripke structures.
For example, consider the simple Kripke structure $K$ in Fig. 1 and HyperLTL formulas $\varphi_1 = \forall \pi. \forall \pi'. [a_\pi \leftrightarrow a_{\pi'}]$ and $\varphi_2 = \exists \pi. \exists \pi'. [a_\pi \not\leftrightarrow a_{\pi'}]$. Verifying whether $K \models \varphi_1$ can be reduced to building the self-composition of $K$ (i.e., parallel composition of $K$ with itself) and applying standard LTL model checking, resulting in worst-case complexity $|K|^2$. In the size of the model. On the contrary, dealing with formula $\varphi_2$ is not as straightforward. In worst case, it requires a subset generation to encode the existential quantifier within the Kripke structure, resulting in $|K|^2$. In addition, the quantification is over traces rather than states, adding to the complexity of reasoning.

Following the great success of bounded model checking (BMC) for LTL specifications [7], in this paper, we propose the first BMC algorithm for HyperLTL. Just as BMC for LTL is reduced to SAT solving to search for a counterexample trace whose length is bounded by some integer $k$, we reduce BMC for HyperLTL to QBF solving to be able to deal with quantified counterexample traces in the input model. More formally, given a Kripke structure $K$ and HyperLTL formula (for example, of the form) $\varphi = \forall \pi. \exists \pi'. \psi$, the reduction involves three main components. First, the transition relation of $K$ is represented by a Boolean encoding $J_K$ with a separate copy for each quantifier in $\varphi$. Secondly, the inner LTL subformula $\psi$ is translated to a Boolean fixpoint representation $J_\psi$ in a similar fashion to the standard BMC technique for LTL. This way, the QBF encoding roughly appears as:

$$\boxed{[K, \neg \varphi]_k = \exists (x_0, x_1, \ldots, x_n). \forall (y_0, y_1, \ldots, y_n). [K_0]_k \land ([K_1]_k \rightarrow [\neg \psi]_k)}$$

where Boolean variables $x_0 \cdots x_n$ (respectively, $y_0 \cdots y_n$) encode $K$ with respect to the universal (respectively, existential) trace quantifier in $\varphi$, $[K_0]_k$ (respectively, $[K_1]_k$) is the unrolling of $K$ for the universal (respectively, existential) quantifier in $\varphi$, and $[\psi]$ is the fixpoint Boolean encoding of $\psi$. It is straightforward to extend QBF encoding (1) to any HyperLTL formula.

While this QBF encoding is a natural generalization of BMC for HyperLTL, there are subtle issues with respect to satisfiability of $[K, \neg \varphi]_k$ and soundness of our approach that needs to be addressed. For instance, consider a HyperLTL formula of the form $\varphi = \forall \pi. \exists \pi'. \psi$, where $\psi$ is a safety LTL subformula. This means that in $\neg \varphi$, $\neg \psi$ is a co-safety LTL formula. Thus, in our QBF encoding (1), if the solver finds a witness to the existential quantifier over $x_0 \cdots x_n$, it is straightforward to extend the witness to the universal quantifier over all states in $K$.
and concludes satisfiability of the formula, we have a counterexample. In this setting, unsatisfiability means up to bound $k$, there is no bug, but we cannot make a general verification conclusion. On the contrary, suppose $\psi$ is a co-safety LTL formula and the QBF solver reports unsatisfiability. This means that $K$ is indeed a model of $\varphi$. As can be seen satisfiability and unsatisfiability for different formulas have different meanings. In order to interpret the outcome the QBF solver and relate it to the original model checking decision problem, we propose three orthogonal semantics for BMC for HyperLTL. In the pessimistic semantics (which is the common for LTL BMC) pending eventualities are considered to be unfulfilled. The optimistic semantics considers the dual case, where pending eventualities are considered to be fulfilled. Bounded semantics use additional information about the system: if the system has reached termination, then the final state will repeated ad infinitum, so if all traces have terminated, then the verdict can be decided.

We have fully implemented our technique the the tool HyperQube. Our experimental evaluation includes a rich set of case studies, such information-flow security/privacy, concurrent data structures (in particular, linearizability), and path planning in robotics applications. Our evaluation shows that our technique is effective and efficient in finding bugs in several prominent examples. We also show that our approach can also be used as as tool for synthesis. Indeed, a witness to an existential quantifier in a HyperLTL formula is an execution path that satisfy the formula. Our experiments on path planning for robots show cases this feature of HyperCube.

2 Preliminaries

2.1 Kripke Structures

Let $\text{AP}$ be a finite set of atomic propositions and $\Sigma = 2^{\text{AP}}$ be the alphabet. A letter is an element of $\Sigma$. A trace $t \in \Sigma^\omega$ over alphabet $\Sigma$ is an infinite sequence of letters: $t = t(0)t(1)t(2)\ldots$

Definition 1. A Kripke structure is a tuple $K = \langle S, s_{\text{init}}, \delta, L \rangle$, where

- $S$ is a finite set of states;
- $s_{\text{init}} \in S$ is the initial state;
- $\delta \subseteq S \times S$ is a transition relation, and
- $L : S \rightarrow \Sigma$ is a labeling function on the states of $K$.

We require that for each $s \in S$, there exists $s' \in S$, such that $(s, s') \in \delta$.

Figure 1 shows an example Kripke structure where $L(s_{\text{init}}) = \{a\}, L(s_3) = \{b\}, \text{etc.}$ The size of the Kripke structure is the number of its states. The directed graph $F = \langle S, \delta \rangle$ is called the Kripke frame of the Kripke structure $K$. A loop in $F$ is a finite sequence $s_0s_1\cdots s_n$, such that $(s_i, s_{i+1}) \in \delta$, for all $0 \leq i < n$, and $(s_n, s_0) \in \delta$. We call a Kripke frame acyclic, if the only loops are self-loops on
otherwise terminal states, i.e., on states that have no other outgoing transition. See Fig. 1 for an example. Since Definition 1 does not allow terminal states, we only consider acyclic Kripke structures with such added self-loops.

We call a Kripke frame tree-shaped, or, in short, a tree, if every state \( s \) has a unique state \( s' \) with \( (s', s) \in \delta \), except for the root node, which has no predecessor, and the leaf nodes, which, again because of Definition 1, additionally have a self-loop but no other outgoing transitions.

A path of a Kripke structure is an infinite sequence of states \( s(0)s(1)\cdots \in S^\omega \), such that:
1. \( s(0) = s_{\text{init}} \), and
2. \( (s(i), s(i + 1)) \in \delta \), for all \( i \geq 0 \).

A trace of a Kripke structure is a trace \( t(0)t(1)t(2)\cdots \in \Sigma^\omega \), such that there exists a path \( s(0)s(1)\cdots \in S^\omega \) with \( t(i) = L(s(i)) \) for all \( i \geq 0 \). We denote by \( \text{Traces}(\mathcal{K}, s) \) the set of all traces of \( \mathcal{K} \) with paths that start in state \( s \in S \).

In some cases, the system at hand is given as a tree-shaped or acyclic Kripke structure. Examples include session-based security protocols and space-efficient execution logs, because trees allow us to organize the traces according to common prefixes and acyclic graphs according to both common prefixes and common suffixes.

### 2.2 The Temporal Logic HyperLTL

HyperLTL [8] is an extension of linear-time temporal logic (LTL) for hyperproperties. The syntax of HyperLTL formulas is defined inductively by the following grammar:

\[
\varphi ::= \exists \pi. \varphi \mid \forall \pi. \varphi \mid \phi \\
\phi ::= \text{true} \mid a_\pi \mid \neg \phi \mid \phi \lor \phi \mid \phi \ U \phi \mid \lozenge \phi
\]

where \( a \in \text{AP} \) is an atomic proposition and \( \pi \) is a trace variable from an infinite supply of variables \( \mathcal{V} \). The Boolean connectives \( \lor \) and \( \land \) have the usual meaning, \( \ U \) is the temporal until operator and \( \lozenge \) is the temporal next operator. We also consider the usual derived Boolean connectives, such as \( \land \), \( \rightarrow \), and \( \leftrightarrow \), and the derived temporal operators eventually \( \Diamond \varphi \equiv \text{true} \ U \varphi \) and globally \( \Box \varphi \equiv \neg \Diamond \neg \varphi \). The quantified formulas \( \exists \pi \) and \( \forall \pi \) are read as ‘along some trace \( \pi \)’ and ‘along all traces \( \pi \)’, respectively.

**Standard Semantics.** The semantics of HyperLTL is defined with respect to a trace assignment, a partial mapping \( \Pi : \mathcal{V} \rightarrow \Sigma^\omega \). The assignment with empty domain is denoted by \( \Pi_\emptyset \). Given a trace assignment \( \Pi \), a trace variable \( \pi \), and a concrete trace \( t \in \Sigma^\omega \), we denote by \( \Pi[\pi \rightarrow t] \) the assignment that coincides with \( \Pi \) everywhere but at \( \pi \), which is mapped to trace \( t \). Furthermore, \( \Pi[j, \infty] \) denotes the assignment mapping each trace \( \pi \) in \( \Pi \)'s domain to \( \Pi(\pi)(j)\Pi(\pi)(j + 1)\Pi(\pi)(j + 2)\cdots \). The satisfaction of a HyperLTL formula \( \varphi \)
over a trace assignment \( \Pi \) and a set of traces \( T \subseteq \Sigma^\omega \), denoted by \( T, \Pi \models \varphi \), is defined as follows:

\[
\begin{align*}
T, \Pi \models \text{true} & \quad \text{iff} \quad \forall t \in T: T, \Pi \models \psi,
T, \Pi \models a_\pi & \quad \text{iff} \quad a \in \Pi(\pi)(0),
T, \Pi \models \neg \psi & \quad \text{iff} \quad T, \Pi \not\models \psi,
T, \Pi \models \psi_1 \lor \psi_2 & \quad \text{iff} \quad T, \Pi \models \psi_1 \text{ or } T, \Pi \models \psi_2,
T, \Pi \models \Box \psi & \quad \text{iff} \quad T, \Pi[1, \infty] \models \psi,
T, \Pi \models \psi_1 \lor \psi_2 & \quad \text{iff} \quad \exists i \geq 0: T, \Pi[i, \infty] \models \psi_2 \land \forall j \in [0, i): T, \Pi[j, \infty] \models \psi_1,
T, \Pi \models \exists \pi. \psi & \quad \text{iff} \quad \exists \pi \in T: T, \Pi[\pi \rightarrow t] \models \psi,
T, \Pi \models \forall \pi. \psi & \quad \text{iff} \quad \forall \pi \in T: T, \Pi[\pi \rightarrow t] \models \psi.
\end{align*}
\]

We say that a set \( T \) of traces satisfies a sentence \( \varphi \), denoted by \( T \models \phi \), if \( T, \Pi_0 \models \varphi \). If the set \( T \) is generated by a Kripke structure \( K \), we write \( K \models \varphi \).

**Multi-model Semantics.** In this paper we use a slightly different semantics for HyperLTL formulas to allow comparing traces from different models. We still use trace mapping \( \Pi : \mathcal{V} \rightarrow \Sigma^\omega \) as before. But we now consider different sets of traces \( T(\pi) \), with one set of traces for each trace variable \( \pi \). Formally, \( T \) is a total function \( T : \mathcal{V} \rightarrow 2^{\Sigma^\omega} \). We call \( T \) a multi-interpretation. As before, given a trace assignment \( \Pi \), a trace variable \( \pi \), and a concrete trace \( t \in \Sigma^\omega \), we denote by \( \Pi[\pi \rightarrow t] \) the assignment that coincides with \( \Pi \) everywhere but at \( \pi \), which is mapped to trace \( t \). Furthermore, \( \Pi[j, \infty] \) denotes the assignment mapping each trace \( \pi \) in \( \Pi \)'s domain to \( \Pi[\pi](j) \Pi[\pi](j+1) \Pi[\pi](j+2) \cdots \). The satisfaction of a HyperLTL formula \( \varphi \) over a trace assignment \( \Pi \) and a multi-interpretation \( T : \mathcal{V} \rightarrow 2^{\Sigma^\omega} \), denoted by \( T, \Pi \models \varphi \), is defined as follows:

\[
\begin{align*}
T, \Pi \models \text{true} & \quad \text{iff} \quad \forall t \in T: T, \Pi \models \psi,
T, \Pi \models a_\pi & \quad \text{iff} \quad a \in \Pi(\pi)(0),
T, \Pi \models \neg \psi & \quad \text{iff} \quad T, \Pi \not\models \psi,
T, \Pi \models \psi_1 \lor \psi_2 & \quad \text{iff} \quad T, \Pi \models \psi_1 \text{ or } T, \Pi \models \psi_2,
T, \Pi \models \Box \psi & \quad \text{iff} \quad T, \Pi[1, \infty] \models \psi,
T, \Pi \models \psi_1 \lor \psi_2 & \quad \text{iff} \quad \exists i \geq 0: T, \Pi[i, \infty] \models \psi_2 \land \forall j \in [0, i): T, \Pi[j, \infty] \models \psi_1,
T, \Pi \models \exists \pi. \psi & \quad \text{iff} \quad \exists \pi \in T(\pi): T, \Pi[\pi \rightarrow t] \models \psi,
T, \Pi \models \forall \pi. \psi & \quad \text{iff} \quad \forall \pi \in T(\pi): T, \Pi[\pi \rightarrow t] \models \psi.
\end{align*}
\]

We say that a multi-interpretation \( T \) satisfies a sentence \( \varphi \), denoted by \( T \models \phi \), if \( T, \Pi_0 \models \varphi \), and we say that the multi-interpretation \( T \) is a multi-model of \( \phi \). If each set \( T(\pi) \) is generated by a family of Kripke structures with one \( K(\pi) \) per variable \( \pi \), we write \( K \models \varphi \). Note that by choosing the same set of traces for each variable (i.e. \( T(\pi) = T(\pi') \) for each \( \pi, \pi' \)) we recover the standard semantics of HyperLTL.

### 2.3 Quantified Boolean Formula Satisfiability

The *quantified Boolean formula* (QBF) satisfiability problem [23] is the following:
Given is a set of Boolean variables, \( \{x_1, x_2, \ldots, x_n\} \), and a quantified Boolean formula
\[
y = Q_1 x_1, Q_2 x_2, \ldots, Q_{n-1} x_{n-1}, Q_n x_n, \psi
\]
where each \( Q_i \in \{\forall, \exists\} \) (\( i \in [1, n] \)) and \( \psi \) is an arbitrary Boolean formula over variables \( \{x_1, \ldots, x_n\} \). Is \( y \) true?

Figure 2 shows a satisfying model for formula:
\[
y = \exists x_1. \forall x_2. \exists x_3. \exists x_4. \forall x_5. (x_1 \lor \neg x_2 \lor x_3) \land (\neg x_1 \lor x_2 \lor \neg x_4) \land (\neg x_3 \lor x_4 \lor \neg x_5) \land (x_1 \lor x_4 \lor x_5).
\]

3 Bounded Semantics of HyperLTL

In this section we present the bounded semantics of HyperLTL

3.1 Bounded Semantics

We assume the formula is closed and has \( n \) quantifiers \( Q_1 \ldots Q_n \), and it has been converted into negation-normal form (NNF) so that the negation symbol only occurs in front of atomic symbols \( \neg a_\pi \). We describe in this section how different QBF queries can be generated, depending on the structure of the formula, and what can be inferred in each case from the outcome of the QBF solver about the model-checking problem.

The main idea of bounded model-checking is to perform incremental exploration of the state space of the systems by unrolling the systems and the formula up-to a bound.

Let \( k \geq 0 \) be the unrolling bound and let \( \langle T_1 \ldots T_n \rangle \) be a collection of finite sets of finite traces, one per path variable. We use \( T(\pi_l) \) to refer to \( T_l \), the set of traces that \( \pi_l \) can range over. Our intention is to define satisfaction relation
$\models_k$ between models $(T, \Pi, i)$ and formulas for a bounded exploration $k$. We will defined two different finite satisfaction relations for general models: $\models_k^{\text{pes}}$ (pessimistic) and $\models_k^{\text{opt}}$ (optimistic) and their variants for models that inform about the termination of traces $\models_k^{\text{pes}}$ and $\models_k^{\text{opt}}$. These semantics differ in how to interpret the possible unseen future events after the bound of observation $k$. All semantics coincide in the interpretations quantifiers and Boolean connectives at all points, and in the interpretations of the temporal operators up-to instant $k - 1$:

**Quantifiers.** For every $i \leq k$:

1. $(T, \Pi, 0) \models_k \exists \pi. \psi$ iff there is a $t \in T(\pi) : (T, \Pi[\pi \to t], 0) \models_k \psi$, \hspace{1cm} (1)
2. $(T, \Pi, 0) \models_k \forall \pi. \psi$ iff for all $t \in T(\pi) : (T, \Pi[\pi \to t], 0) \models_k \psi$. \hspace{1cm} (2)

**Boolean operators.** For every $i \leq k$:

1. $(T, \Pi, i) \models_k \top$ iff always holds \hspace{1cm} (3)
2. $(T, \Pi, i) \models_k a_\pi$ iff $a \in \Pi(\pi)(i)$, \hspace{1cm} (4)
3. $(T, \Pi, i) \models_k \neg a_\pi$ iff $a \notin \Pi(\pi)(i)$, \hspace{1cm} (5)
4. $(T, \Pi, i) \models_k \psi_1 \lor \psi_2$ iff $(T, \Pi, i) \models_k \psi_1$ or $(T, \Pi, i) \models_k \psi_2$, \hspace{1cm} (6)
5. $(T, \Pi, i) \models_k \psi_1 \land \psi_2$ iff $(T, \Pi, i) \models_k \psi_1$ and $(T, \Pi, i) \models_k \psi_2$ \hspace{1cm} (7)

**Temporal connectives** for $(i < k)$.

1. $(T, \Pi, i) \models_k \Diamond \psi$ iff $(T, \Pi, i + 1) \models_k \psi$ \hspace{1cm} (8)
2. $(T, \Pi, i) \models_k \psi_1 \U \psi_2$ iff $(T, \Pi, i) \models_k \psi_2$, or $(T, \Pi, i) \models_k \psi_1$ and $(T, \Pi, i + 1) \models_k \psi_1 \U \psi_2$ \hspace{1cm} (9)
3. $(T, \Pi, i) \models_k \psi_1 \R \psi_2$ iff $(T, \Pi, i) \models_k \psi_2$, and $(T, \Pi, i) \models_k \psi_1$ or $(T, \Pi, i + 1) \models_k \psi_1 \R \psi_2$ \hspace{1cm} (10)

**Temporal connectives** for $(i = k)$. In this case we distinguish the different semantics. In the pessimistic semantics the eventualities (including $\Diamond$) are assumed to never be fulfilled:

1. $(T, \Pi, k) \models_k^{\text{pes}} \Diamond \psi$ iff never happens \hspace{1cm} (P1)
2. $(T, \Pi, k) \models_k^{\text{pes}} \psi_1 \U \psi_2$ iff never happens \hspace{1cm} (P2)
3. $(T, \Pi, k) \models_k^{\text{pes}} \psi_1 \R \psi_2$ iff never happens \hspace{1cm} (P3)

On the other hand, in the optimistic semantics the eventualities are assumed to be fulfilled in the future:

1. $(T, \Pi, k) \models_k^{\text{opt}} \Diamond \psi$ iff always happens \hspace{1cm} (O1)
2. $(T, \Pi, k) \models_k^{\text{opt}} \psi_1 \U \psi_2$ iff always happens \hspace{1cm} (O2)
3. $(T, \Pi, k) \models_k^{\text{opt}} \psi_1 \R \psi_2$ iff always happens \hspace{1cm} (O3)

In order to capture the halting semantics, we assume that the Kripke structure is equipped with a predicate $\text{halt}$ that is true if the state corresponds to a
halting state, and define the auxiliary predicate \( \text{halted} \) \( \equiv \land_i \text{halt}_i \). Introduce \( \text{halt}_i \) before Then, the bounded semantics of the temporal case for \( i = k \) in the pessimistic case consider the halting case to infer the actual value of the temporal operators on the (now fully known) trace:

\[
(T, \Pi, k) \models^{\text{hpes}} \psi \quad \text{iff} \quad (T, \Pi, k) \models^k \text{halted} \text{ and } (T, \Pi, k) \models^{\text{hpes}} \psi (BP_1)
\]

\[
(T, \Pi, k) \models^k \psi_1 U \psi_2 \quad \text{iff} \quad (T, \Pi, k) \models^k \text{halted} \text{ and } (T, \Pi, k) \models^{\text{hpes}} \psi_2 (BP_2)
\]

\[
(T, \Pi, k) \models^{\text{hpes}} \psi_1 R \psi_2 \quad \text{iff} \quad (T, \Pi, k) \models^k \text{halted} \text{ and } (T, \Pi, k) \models^{\text{hpes}} \psi_1 (BP_3)
\]

Dually, in the halting optimistic case:

\[
(T, \Pi, k) \models^{\text{hopt}} \psi \quad \text{iff} \quad (T, \Pi, k) \not\models^k \text{halted} \text{ or } (T, \Pi, k) \models^{\text{hopt}} \psi (BO_1)
\]

\[
(T, \Pi, k) \models^{\text{hopt}} \psi_1 U \psi_2 \quad \text{iff} \quad (T, \Pi, k) \not\models^k \text{halted} \text{ or } (T, \Pi, k) \models^{\text{hopt}} \psi_2 (BO_2)
\]

\[
(T, \Pi, k) \models^{\text{hopt}} \psi_1 R \psi_2 \quad \text{iff} \quad (T, \Pi, k) \not\models^k \text{halted} \text{ or } (T, \Pi, k) \models^{\text{hopt}} \psi_1 (BO_3)
\]

Now we are ready to define the four semantics:

- \( \models^k_{\text{pes}} \): the pessimistic semantics use (1)-(10) and \((P_1)-(P_3)\).
- \( \models^k_{\text{opt}} \): the optimistic semantics use (1)-(10) and \((O_1)-(O_3)\).
- \( \models^{\text{hpes}}_k \): the bounded pessimistic semantics use (1)-(10) and \((BP_1)-(BP_3)\).
- \( \models^{\text{hopt}}_k \): the bounded optimistic semantics use (1)-(10) and \((BO_1)-(BO_3)\).

The pessimistic semantics is the semantics in the traditional LTL BMC, where pending eventualities are considered to be unfulfilled. In the pessimistic semantics a formula is declared false unless it is witnessed to be true within the bound explored. In other words, formulas can only get “truer” with more information obtained by a longer unrolling. Introduce example with \( G \varphi \) (which is false) and \( F \varphi \) (which is equivalent to a big disjunction). Dually, the optimistic semantics considers a formula true unless there is evidence within the bounded exploration on the contrary. Therefore, formulas only get “false” with further unrolling. The following lemma formalizes this intuition.

**Lemma 1.** Let \( k \leq j \). Then,

1. If \((T, \Pi, 0) \models^{\text{pes}}_k \varphi \) then \((T, \Pi, 0) \models^j \varphi \).
2. If \((T, \Pi, 0) \not\models^{\text{opt}}_k \varphi \) then \((T, \Pi, 0) \not\models^j \varphi \).
3. If \((T, \Pi, 0) \models^{\text{hpes}}_k \varphi \) then \((T, \Pi, 0) \models^j \varphi \).
4. If \((T, \Pi, 0) \not\models^{\text{hopt}}_k \varphi \) then \((T, \Pi, 0) \not\models^j \varphi \).

In turn, the verdict obtained from the exploration up-to \( k \) can (in some cases) be used to infer the verdict of the model checking problem. As in classical BMC, if the pessimistic semantics find a model, then it is a model. Similarly, if the optimistic semantics fail to find a model, then there is no model. The next lemma formally captures this intuition.

**Lemma 2 (Infinite inference).** The following hold for every \( k \),

1. If \((T, \Pi, 0) \models^{\text{pes}}_k \varphi \) then \((T, \Pi, 0) \models \varphi \).
2. If \((T, \Pi, 0) \not\models^{\text{opt}}_k \varphi \) then \((T, \Pi, 0) \not\models \varphi \).
3. If \((T, \Pi, 0) \models^{\text{hpes}}_k \varphi \) then \((T, \Pi, 0) \models \varphi \).
4. If $(T,Π,0)\not\models_k^{\text{opt}} \varphi$ then $(T,Π,0)\not\models \varphi$.

3.2 Examples

4 Reducing BMC to QBF

Given a Kripke structure $K = (S, s_{\text{init}}, \delta, L)$, a HyperLTL formula $\varphi = Q_1 \pi_1 . Q_2 \pi_2 . \cdots . Q_n \pi_n . \psi$, and a bound $k \geq 0$, we will construct a quantified Boolean formula $\mathcal{K}_k,\varphi_k$. The unrolling of the transition relation of the Kripke structure up to bound $k$ is the following:

$$[[K]]_k = I(s_{\text{init}}) \land \bigwedge_{i=0}^{k-1} \delta(s_i, s_{i+1})$$

This is done in the same fashion as in classic BMC for LTL.

The Construction of the inner LTL formula is analogous to standard BMC as well. In particular, we introduce the following inductive construction.

- Inductive Case: for all $i \leq k$:
  
  $$[[p]]_k^i := p_i^k$$
  $$[[\neg p]]_k^i := \neg p_i^k$$
  $$[[\psi_1 \lor \psi_2]]_k^i := [[\psi_1]]_k^i \lor [[\psi_2]]_k^i$$
  $$[[\psi_1 \land \psi_2]]_k^i := [[\psi_1]]_k^i \land [[\psi_2]]_k^i$$
  $$[[\psi_1 U \psi_2]]_k^i := [[\psi_2]]_k^i \lor \left([[\psi_1]]_k^i \land [[\psi_1 U \psi_2]]_k^{i+1}\right)$$
  $$[[\psi_1 R \psi_2]]_k^i := [[\psi_2]]_k^i \lor \left([[\psi_1]]_k^i \land [[\psi_1 R \psi_2]]_k^{i+1}\right)$$
  $$[[\psi_1 \psi_2]]_k^i := [[\psi]]_k^{i+1}$$

- Base case:
  
  $$[[\psi]]_{k+1} := \text{false}$$

Combining all components, the encoding of the HyperLTL BMC problem in QBF is the following:

$$[[K, \varphi]]_k = Q_1 . Q_2 . \cdots . Q_n . \left([[K]]_k^0 . [[K]]_k^1 . \cdots . [[K]]_k^{n-1} . [[K]]_k^n . [[\psi]]_k^0\right)$$

where

$$\alpha_i = \begin{cases} \land & \text{if } Q_i = \exists \\ \to & \text{if } Q_i = \forall \end{cases}$$

For example, if $\varphi$ is $\forall \pi . \exists \pi'. \psi$. Let $\pi = \{p_i^k | 0 \leq i \leq k, \ p \in AP\}$ and $\overline{\pi} = \{p_i^k | 0 \leq i \leq k, \ p \in AP\}$.

$$[[K, \varphi]]_k = \forall \pi . \exists \pi'. \left(([K]]_k^1 . \cdots . (K]]_k^0)\right)$$

Similarly, take $\varphi$ is $\exists \pi . \forall \pi'. \psi$. Let $\pi = \{p_i^k | 0 \leq i \leq k, \ p \in AP\}$ and $\overline{\pi} = \{p_i^k | 0 \leq i \leq k, \ p \in AP\}$.
\[ [K, \varphi]_k = \exists x. \forall y. ([K_1]_k \land ([K_2]_k \rightarrow [\psi]_k^0)) \]

The following holds for the last formula \( \varphi : \exists x \forall y. \psi \). If \([K, \varphi]_k\) holds for a given \( k \) then there is a finite trace \( \sigma_1 \) of length \( k \) of \( K_1 \) such that for all traces \( \sigma_2 \) of length \( k \) of \( K_2 \), \( (\sigma_1, \sigma_2) \models_k \psi \).

5 Evaluation and Case Studies

In this section, we evaluate our technique using a rich set of case studies including verification of symmetry, linearizability, non-interference, and non-repudiation. We also used our technique in path planning in robotics as well as test case generation for mutation testing. Our tool HyperCube works as follows. Given a transition relation, we automatically unfold it up to some bound \( k \geq 0 \) using a home-grown tool written in Ocaml. The unfolded transition relation along with the QBF encoding of input HyperLTL formula will form a complete QBF instance which will then be fed to the QBF solver Quabs [25]. All experiments in this section are run on an iMac desktop with XXX CPU and XXX RAM.

5.1 Case study 1: Symmetry in the Bakery Algorithm

We first investigate the symmetry property in Lamport’s Bakery algorithm for enforcing mutual exclusion in a concurrent program. The algorithm works as follows. When a process intends to enter the critical section, a ticket with a special number will be drawn by the process. When more than one process requests to enter the critical section, the process with the smallest ticket number enters first, while other processes wait. In a concurrent program, it is also possible that two or more processes hold tickets with same number if they drew tickets at the same time. To solve this, while ticket comparison is a tie, we let the process with smaller process ID enter the critical section first and keep others in waiting.

We now consider the symmetry property, where if there is no specific process in a program that owns special privilege. We use atomic proposition \( \text{select} \) to represent the process selected to proceed in the next state, and \( \text{pause} \) to indicate if the process is not moving. Each process \( P_n \) has a program counter denoted by \( pc(P_n) \). The symmetry property of the bakery algorithm is the following. For all traces \( \pi \), there exists a trace \( \pi' \), such that if both traces pause together and both select the next process to execute symmetrically, then the program counter of each process would be completely symmetric as well. For example, for two processes \( P_0 \) and \( P_1 \), trace \( \pi \) selects \( P_0 \) iff trace \( \pi' \) selects \( P_1 \), and \( \pi \) selects \( P_1 \) iff \( \pi' \) selects \( P_0 \). Such a scenario is presented as \( \text{sym}(\text{select}_\pi, \text{select}_{\pi'}) \). We now describe the specification as the following HyperLTL formula:

| Symmetry | \( \varphi_{\text{sym}} = \forall \pi. \exists \pi'. \Box \left( \text{sym}(\text{select}_\pi, \text{select}_{\pi'}) \land (\text{pause}_\pi = \text{pause}_{\pi'}) \land (pc(P_0) = pc(P_1) \land (pc(P_1) = pc(P_0))) \right) \) |
Now, observe that the negation of the above formula is:

\[ \neg \text{Symmetry} \]

\[ \neg \phi_{\text{sym}} = \exists \pi. \forall \pi'. \left( \neg \text{sym}(\text{select}_\pi, \text{select}_{\pi'}) \lor (\text{pause}_{\pi} \neq \text{pause}_{\pi'}) \lor (\text{pc}(P_0)_{\pi} \neq \text{pc}(P_1)_{\pi'}) \lor (\text{pc}(P_1)_{\pi} \neq \text{pc}(P_0)_{\pi'}) \right) \]

In this case study, HyperQube returns SAT, which indicates that there exists a trace that satisfies \( \neg \phi_{\text{sym}} \). The returned trace represents a witness in the Bakery algorithm that violates symmetry and thus falsifies the original formula \( \phi_{\text{sym}} \). That is, the Bakery algorithm does not satisfy symmetry.

5.2 Case study 2: Linearizability in SNARK Algorithm

Next, we investigate linearizability property in SNARK algorithm. Linearizability is a correctness property for concurrent program. In a concurrent system, a history is a collection of method invocations and responses executed by different threads. We say that a history is linearizable [26], if there exists a sequential order of invocations and responses, such that the responses are returned with atomic executions of the whole method. A concurrent program satisfies linearizability if all possible histories are linearizable. In [5], the authors show that linearizability is a hyperproperty of the form \( \forall \exists \), where the domain of the universal quantifier is over all possible executions of the concurrent data structure and the domain of the existential quantifier is over all possible executions of the sequential implementation of the data structure. Thus, reasoning about linearizability requires our multi-model semantics introduced in Section 2.

The SNARK algorithm [12] is a concurrent implementation of a double-ended queue data structure. It uses double-compare-and-swap (DCAS) with doubly linked-list that stores values in nodes while each node is connected to its two neighbors, \( L \) and \( R \). When a modification of data happens, e.g., by invoking \text{pushRight()} or \text{popLeft()}, SNARK conducts DCAS by comparing two memory locations to decide if such modification is appropriate.

We define the hyperproperty of linearizability using two different models. Let \( \pi_m \) denote the trace variable over the traces of the model (in this case SNARK). This model allows multiple threads to execute each method with interleavings. Let \( \pi_s \) represents the trace variable over traces of the sequential implementation of a double-ended queue (i.e., the specification), where only atomic invocations are allowed. The HyperLTL formula specifying a linearizable program is as follows:

\[ \text{Linearizability} \quad \phi_{\text{linearizability}} = \forall \pi_m. \exists \pi_s. \Box (\text{history}_{\pi_m} \leftrightarrow \text{history}_{\pi_s}) \]

The negation of the above formula is:

\[ \neg \text{Linearizability} \quad \neg \phi_{\text{linearizability}} = \exists \pi_m. \forall \pi_s. \Diamond (\text{history}_{\pi_m} \not\leftrightarrow \text{history}_{\pi_s}) \]

In this case study, HyperQube returns SAT, indicating that a witness of linearizability violation has been found in SNARK algorithm. The returned trace is a history that cannot be performed when only atomic executions are allowed. The bug we identified by using HyperQube is the same as the one reported in [12].
5.3 Case study 3: Non-interference in Typed Multi-threaded Programs

We also investigate non-interference in a multi-threaded program with type system. As a security policy, non-interference guarantees that low-security variables are independent from the high-security variables, thus, preserving secure information flow. For a concurrent program, a type system classifies each variable as either high or low security, labeled as high-variable and low-variable. Non-interference requires that all information about a high-variable cannot be inferred by observing any the value of a low-variable. In this case study, we look at a concurrent system example in [28], which contains three threads $\alpha$, $\beta$, and $\omega$. The variables are assigned with different security level as follows. PIN, trigger0, and trigger1 are classified as high-variable, and maintrigger, mask, and result are low-variables. In a multi-threaded setting, assuming that thread scheduling is fair, the program satisfies non-interference, if for all executions, there exists another one such that they start from different high-inputs (i.e., the values of PIN are not equal) and at termination point, they are in low-equivalent states (i.e., the values of Result are equal).

Furthermore, in order to search for a witness of non-interference violation in bounded time, we also consider the termination bound introduced in Section 3. In this particular program, the execution terminates when the low-variable MASK contains value zero. The corresponding HyperLTL formula is as follows:

$$\varphi_{NI} = \forall \pi. \exists \pi'. (PIN_\pi \neq PIN_{\pi'}) \land \left( (\neg terminate_\pi \lor \neg terminate_{\pi'}) \land (\text{Result}_\pi = \text{Result}_{\pi'}) \right)$$

where atomic proposition terminate denotes the terminating state (MASK contains a zero bit) and by abuse of notation $PIN_\pi$ (respectively, $Result_\pi$) denotes the value of PIN (respectively, Result) in trace $\pi$. The negated formula for BMC is then:

$$\neg \varphi_{NI} = \exists \pi. \forall \pi'. (PIN_\pi \neq PIN_{\pi'}) \rightarrow \left( (\neg terminate_\pi \lor \neg terminate_{\pi'}) \lor (\text{Result}_\pi \neq \text{Result}_{\pi'}) \right)$$

In this case study, HyperQube returns SAT, indicating that there is a trace we can detect the difference of high-variable by observing low variable, that is, violating non-interference by 27 unrollings. Moreover, by taking terminating bound into consideration, further unrollings of transition relation will not affect the result of original model checking problem.

5.4 Case study 4: Fairness in Non-repudiation Protocols

A non-repudiation protocol consists of three parties: a message sender ($A$), a message receiver ($B$), and a trusted third party $T$. In a message exchange event, the message receiver should obtain a receipt from the sender, named non-repudiation
of origin (NRO), and the message sender should end up having an evidence named non-repudiation of receipt (NRR). A fair non-repudiation protocol guarantees that two parties can exchange messages fairly without any party being able to deny sending out evidence while having received an evidence. The three participants can take the following actions:

\[ Act_A = \{A \to B : m, A \to T : m, A \to B : NRO, A \to T : NRO, A : \text{skip}\} \]
\[ Act_B = \{B \to A : NRR, B \to T : NRR, B : \text{skip}\} \]
\[ Act_T = \{T \to A : NRR, T \to B : NRO, T : \text{skip}\} \]

We say that a trace is effective if message, NRR, and NRO are all received. Assuming that each party will take turns and take different actions, the fairness of non-repudiation protocol can be presented as hyperproperty as follows. There exists an effective trace \( \pi \), such that for all other traces \( \pi' \), if \( A \) in both traces always take the same action while \( B \) behave arbitrarily, or both \( B \) take the same action and \( A \) behave arbitrarily, then for \( \pi' \), eventually NRR gets received by \( A \) if and only if NRO gets received by \( B \). The complete specification for non-repudiation is the following:

\[
\varphi_{\text{fair}} = \exists \pi. \forall \pi'. (\Box m_\pi) \land (\Diamond \text{NRR}_\pi) \land (\Diamond \text{NRO}_\pi) \land (\square \land_{a \in \text{Act}_A} a_\pi \leftrightarrow a_{\pi'}) \leftrightarrow ((\Diamond \text{NRR}_{\pi'}) \leftrightarrow (\Diamond \text{NRO}_{\pi'})) \land (\square \land_{a \in \text{Act}_B} a_\pi \leftrightarrow a_{\pi'}) \leftrightarrow ((\Diamond \text{NRR}_{\pi'} \leftrightarrow (\Diamond \text{NRO}_{\pi'}))
\]

Observe that trace \( \pi \) expresses effectiveness (i.e., an honest behavior of all parties), while trace \( \pi' \) is a trace that behaves similarly to trace \( \pi \) as far as the actions of \( A \) or \( B \) are concerned while ensuring fair receipt of NRR and NRO. The negated formula for BMC is then:

\[
\neg \varphi_{\text{fair}} = \forall \pi. \exists \pi'. \neg((\Box m_\pi) \land (\Diamond \text{NRR}_\pi) \land (\Diamond \text{NRO}_\pi)) \lor (\square \land_{a \in \text{Act}_A} a_\pi \leftrightarrow a_{\pi'}) \land \neg(((\Diamond \text{NRR}_{\pi'}) \leftrightarrow (\Diamond \text{NRO}_{\pi'})) \lor (\square \land_{a \in \text{Act}_B} a_\pi \leftrightarrow a_{\pi'}) \land \neg(((\Diamond \text{NRR}_{\pi'}) \leftrightarrow (\Diamond \text{NRO}_{\pi'})))
\]

In this case study, we evaluate two different models of trusted third party from [27]. First, we pick an incorrect implementation in [27], named \( T_{\text{incorrect}} \), which \( B \) can choose not to send out NRR after receiving NRO. We obtain a SAT result from HyperQube. The challenge with a SAT result with a \( \forall \exists \) formula is that the solver understandably does not return an witness. Thus, a SAT result can be either due to non-existence of an effective trace or the fact that all conforming traces are unfair. To clarify this, one can verify the protocol with respect to formula \( \exists \pi. (\Box m_\pi \land \Diamond \text{NRR}_\pi \land \Diamond \text{NRO}_\pi) \). This step was successful, meaning that an effective trace exists. This means that the original SAT result indicates that the protocol includes an unfair trace.

On the contrary, the implementation named \( T_{\text{correct}} \) in [27], where \( T \) always guarantees the message exchange event is fair between the two parties, HyperQube returns UNSAT. This result indicates that all traces in the correct system satisfies fairness in non-repudiation.
5.5 Case study 5: Privacy-Preserving Path Planning for Robots

In addition to model checking problems, inspired by the work in [31], we also explore other applications that involve hyperproperties with quantifier alternation. One such application is searching the optimal solution for robotic planning. For example, given a map with an initial state and a goal state, the shortest path from initial state to goal state is a traces \( \pi \), such that \( \pi \) can reach the goal state and for all other traces \( \pi' \), \( \pi' \) should not reach the goal state until \( \pi \) reached it. In other words, the shortest path is a path on the map that reaches the goal state before all other paths did. We express this specification as the following hyperproperty:

\[
\varphi_{sp} = \exists \pi. \forall \pi'. (\neg \text{goal}_{\pi'} \cup \text{goal}_{\pi})
\]

where the atomic proposition \( \text{goal} \) denotes that the path has reached the goal state.

By enforcing the above formula directly with a map model, HyperQube returns SAT. The returned path represents a path that can reach the goal state from the initial state with least steps compared to all other paths on the same map. To further analyze the result, we also consider the terminating bound with the formula. An optimal path searching should terminate when the shortest path is found because when a shortest path has been discovered on the map, any further exploration will not affect the returning value.

Besides optimal solution searching, HyperLTL also allows us to specify the robustness of paths that are derived by uncertainty in robotic planning. For example, instead of one single initial state, we now consider a map with a set of initial states. We are interested in a strategy that can help all traces to reach the goal state regardless which initial state the path start from. A robustness strategy searching problem can be presented as follows. There exists a robust path \( \pi \), such that for all paths \( \pi' \) starting from any initial state, \( \pi' \) is able to reach the goal state using the same strategy as \( \pi \). Let the notation of \( \text{strategy} \) represent a sequential of movements the path took (i.e. move up, down, left, or right.). We write the formula as follows:

\[
\varphi_{rb} = \exists \pi. \forall \pi'. (\text{strategy}_{\pi'} \leftrightarrow \text{strategy}_{\pi}) \rightarrow (\text{goal}_{\pi} \land \text{goal}_{\pi'})
\]

By enforcing the above formula directly with the map model, HyperQube returns SAT, which indicates that a robustness path has been found. The returned path represents a strong strategy that all paths starting from different initial state can guarantee of goal reachability using the same strategy.

5.6 Case study 6: Generate Mutants in Mutation Testing

Another application of hyperproperty with quantifier alternation is for efficiently generating test suite for mutation testing [13]. We look at the beverage machine...
model in this paper. The beverage machine has three possible inputs: request, fill, or none. Based on the input, the machine may output coffee, tea, or none. We also use an atomic proposition mut to mark mutated traces, and ¬mut for non-mutated traces. In this non-deterministic model, a potentially killable mutants for mutation testing is a mutated trace π such that, for all other non-mutated π′ who have same inputs as π, the outputs will eventually diverge.

\[ \exists \pi \forall \pi' (mut_\pi \land \neg mut_{\pi'}) \land ((input_\pi \leftrightarrow input_{\pi'}) \lor (output_\pi \neq output_{\pi'})) \]

HyperQube returns SAT with the above formula. The returned path represents a valid mutant for mutation testing. Our experiment result shows that HyperQube is able to output a mutant with given formula in a very short amount of time, which provides an efficient solution for test suite generation of mutation testing.

5.7 Summary Table

6 Related Work

There has been a lot of recent progress in automatically verifying [11, 20–22] and monitoring [1, 5, 6, 18, 19, 24, 29] HyperLTL specifications. HyperLTL is also supported by a growing set of tools, including the model checker MCHyper [11, 22], the satisfiability checkers EAHyper [17] and MGHyper [15], and the runtime monitoring tool RVHyper [18].

The complexity of the model checking for HyperLTL for tree-shaped, acyclic, and general graphs was rigorously investigated in [2]. The first algorithms for model checking HyperLTL and HyperCTL* using alternating automata were introduced in [22]. These technique, however, were not able to deal with alternating HyperLTL formulas in a fully automated fashion. These algorithms were then extended to deal with hyperliveness and alternating formulas in [11] by finding a winning strategy in ∀∃ games. In this paper, we take an alternative approach by reducing the model checking problem to QBF solving, which is arguably more effective for finding bugs.
Table 1: Case studies results of BMC for hyperproperties using HyperQube

| Model               | Original $\varphi$ | #unroll | semantics | QBF result genqbf [s] | QuAbS [s] |
|---------------------|--------------------|---------|-----------|------------------------|------------|
| Bakery 3 processes  | $\forall (sym1)$  | 49      | PES       | SAT                    | 6.25       | 0.64       |
| Bakery 3 processes  | $\forall (sym2)$  | 144     | PES       | SAT                    | 71.47      | 14.40      |
| Bakery 3 processes  | $\forall (sym3)$  | 50      | OPT       | UNSAT                  | 7.76       | 23.66      |
| Bakery 3 processes  | $\varphi_{sym}$   | 50      | PES       | SAT                    | 6.20       | 0.57       |
| Bakery 5 processes  | $\varphi_{sym}$   | 50      | PES       | SAT                    | 139.64     | 27.06      |
| SNARK bug1          | $\varphi_{linearizability}$ | 26 | PES       | SAT                    | 88.42      | 383.60     |
| SNARK bug2          | $\varphi_{linearizability}$ | 40 | PES       | SAT                    | 718.09     | 779.76     |
| 3-Thread program    | $\varphi_{NI}$    | 57      | terminating-PES | SAT | 8.67       | 79.36      |
| Non-repudiation Protocol ($T_{incorrect}$) | $\varphi_{fair}$ | 15 | terminating-PES | SAT | 0.10       | 0.27       |
| Non-repudiation Protocol ($T_{correct}$) | $\varphi_{fair}$ | 15 | terminating-OPT | UNSAT | 0.08       | 0.12       |

The satisfiability problem for HyperLTL is shown to be undecidable in general but decidable for the $\exists^w \forall^w$ fragment and for any fragment that includes a $\forall \exists$ quantifier alternation [14]. The hierarchy of hyperlogics beyond HyperLTL has been studied in [10]. The synthesis problem for HyperLTL has been studied in problem in [3] in the form of program repair, in [4] in the form of controller synthesis, and in [16] for the general case.

7 Conclusion and Future Work

In this paper, we introduced the first bounded model checking (BMC) technique for verification of hyperproperties expressed in HyperLTL. To this end, we proposed three different semantics that jointly ensure the soundness of our approach. To handle trace quantification in HyperLTL, we reduced the BMC problem to checking satisfiability of quantified Boolean formulas (QBF). This is analogous
Table 2: Case studies results of hyperproperties for robotic planning using HyperCube, in comparison with the experimental results in [11]

| title of Table | # unroll | gen [s] | Z3 [s] | Total [s] | genqbf [s] | QuAbS [s] | Total [s] |
|----------------|----------|---------|--------|-----------|------------|-----------|----------|
| Shortest path (map size : 10^2) | 20 | 8.31 | 0.33 | **8.64** | 1.30 | 0.57 | **1.87** |
| Shortest path (map size : 20^2) | 40 | 124.66 | 6.41 | **131.06** | 4.53 | 12.16 | **16.69** |
| Shortest path (map size : 40^2) | 80 | 1093.12 | 72.99 | **1166.11** | 36.04 | 35.75 | **71.79** |
| Shortest path (map size : 60^2) | 120 | 4360.75 | 532.11 | **4892.86** | 105.82 | 120.84 | **226.66** |
| Initial state robustness (map size : 10^2) | 20 | 11.14 | 0.45 | **11.59** | 1.40 | 0.35 | **1.75** |
| Initial state robustness (map size : 20^2) | 40 | 49.59 | 2.67 | **52.26** | 15.92 | 15.32 | **31.14** |
| Initial state robustness (map size : 40^2) | 80 | 216.16 | 19.81 | **235.97** | 63.16 | 20.13 | **83.29** |

As for future work, our first step is to solve the loop condition problem. This is necessary to establish completeness conditions for BMC. The application of QBF-based techniques in the framework of abstraction/refinement is another unexplored area. Success of BMC for hyperproperties inherently depends on effectiveness of QBF solvers and since QBF solving is arguably not as mature as SAT/SMT solving techniques, it is imperative to conduct more research to enhance the performance of QBF solvers.

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A Pseudo-code of Case Studies

A.1 Bakery Algorithm

Algorithm 1: Bakery

1  init(MAX/ \text{P}_1.ticket... \text{P}_n.ticket/ \text{P}_1.status... \text{P}_n.status):= 0/ 0...0/ noncrit...noncrit ;
2  while true do
3    foreach k in 1...n do
4      if select(\text{P}_k) then
5        \text{P}_k.ticket = MAX + 1;
6        \text{P}_k.status = wait;
7      else if \text{P}_k.status = wait then
8        if \text{P}_k.ticket = \text{min}(\text{P}_1.ticket... \text{P}_n.ticket) then
9          \text{P}_k.status = crit ;
10         else
11          \text{P}_k.status = wait ;
12        end
13      end
14  end
A.2 SNARK Algorithm

Algorithm 2: SNARK

1. popRight()
2. while true do
3.   rh = RightHat;
4.   lh = LeftHat;
5.   if rh→R = rh then
6.     return "empty";
7.   end
8.   if rh = lh then
9.     if DCAS(&RightHat, &LeftHat, rh, lh, Dummy, Dummy) then
10.    return rh→V;
11.   end
12. else
13.   rhL = rh→L;
14.   if DCAS(&RightHat, &rh→L, rh, rhL, rhL, rh) then
15.     result = rh→V;
16.     rh→R = Dummy;
17.     return result;
18.   end
19. end
20. popLeft()
21. while true do
22.   lh = LeftHat;
23.   rh = RightHat;
24.   if lh→L = lh then
25.     return "empty";
26. end
27. if lh = rh then
28.   if DCAS(&LeftHat, &RightHat, lh, rh, Dummy, Dummy) then
29.     return lh→V;
30.   end
31. else
32.   lhR = lh→R;
33.   if DCAS(&LeftHat, &lh→R, lh, lhR, lhR, lh) then
34.     result = lh→V;
35.     lh→L = Dummy;
36.     return result;
37.   end
38. end
pushRight()
nd = new Node();
if nd = null then
    return "full";
nd→R = Dummy;
nd→V = v;
while true do
    rh = RightHat;
    rhR = rh→R;
    if rhR = rh then
        nd→L = Dummy;
        lh = LeftHat; if DCAS(&RightHat, &LeftHat, rh, lh, nd, Dummy) then
            return success;
    else
        nd→L = rh;
        if DCAS(&RightHat, &lh→R, rh, rhR, nd, nd) then
            return success;
Algorithm 3: Typed Multi-threaded Program

1 Thread $\alpha$:
2 while $\text{mask} \neq 0$ do
3     while $\text{trigger}0 = 0$ do
4         no-op;
5     end
6     $\text{result} = \text{result} \parallel \text{mask}$; // bitwise 'or'
7     $\text{trigger}0 = 0$;
8     $\text{maintrigger} = \text{matintrigger} + 1$
9     if $\text{maintrigger} = 1$ then
10        $\text{trigger}1 = 1$;
11     end
12 end
13 Thread $\beta$:
14 while $\text{mask} \neq 0$ do
15     while $\text{trigger}1 = 0$ do
16         no-op;
17     end
18     $\text{result} = \text{result} \& \text{!mask}$; // bitwise 'and'
19     $\text{trigger}1 = 0$;
20     $\text{maintrigger} = \text{matintrigger} + 1$
21     if $\text{maintrigger} = 1$ then
22        $\text{trigger}0 = 1$;
23     end
24 end
25 Thread $\gamma$:
26 while $\text{mask} \neq 0$ do
27     $\text{maintrigger} = 0$
28     if $\text{PIN} \& \text{mask} = 0$ then
29         $\text{trigger}0 = 1$;
30     else
31        $\text{trigger}1 = 1$;
32     end
33     while $\text{maintrigger} \neq 2$ do
34         no-op;
35     end
36     $\text{mask} = \text{mask}/2$;
37 end
38 $\text{trigger}0 = 1$;
39 $\text{trigger}1 = 1$;
Algorithm 4: Non-repudiation Protocol

\begin{algorithm}
\begin{algorithmic}[1]
\Function{}\end{algorithmic}
\end{algorithm}