Cavity mediated dissipative coupling of distant magnetic moments: theory and experiment

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We investigate long-range coherent and dissipative coupling between two spatially separated magnets while both are coupled to a microwave cavity. A careful examination of the system shows that the indirect interaction between two magnon modes is dependent on their individual mechanisms of direct coupling to the cavity. If both magnon modes share the same form of coupling to the cavity (either coherent or dissipative), then the indirect coupling between them will produce level repulsion. Conversely, if the magnon modes have different forms of coupling to the cavity (one coherent and one dissipative), then their indirect coupling will produce level attraction. We further demonstrate the cavity-mediated nature of the indirect interaction through investigating the dependence of the indirect coupling strength on the frequency detuning between the magnon and cavity modes. Our work theoretically and experimentally explores indirect cavity mediated interactions in systems exhibiting both coherent and dissipative coupling, which opens a new avenue for controlling and utilizing light-matter interactions.

I. INTRODUCTION

Light matter interactions in a hybrid system are of great interest in modern physics as a building block for coherent information processing. Ideally, two distant quantum systems can transfer information through their mutual coupling to a resonant photon system, which is highly desirable for any architecture of quantum information processing. As a newly discovered form of light-matter interaction, the strong coupling between microwave photons and magnons has been attracting increasing attention in recent years, since the interaction between electrodynamics and magnetization dynamics spawns the cavity magnon polariton (CMP). The CMP dispersion manifests as an elegant level repulsion, in which lies the profound physics of Rabi splitting. The high spin density and low room temperature damping rate of ferromagnetic insulators allows the coherent photon-spin interactions to enter the ultrastrong coupling regime. Consequently, the magnon photon coupling provides a perfect platform for studying and illustrating coupling related physics, specifically, in addition to being a promising candidate for coherent information processing.

Interestingly, when a cavity mode couples to two macroscopic magnetic moments simultaneously, the quantized magnetic field of the photons links the dynamics of distant magnons, thus inducing nonlocal interaction between the magnon modes. In turn, the coherent transport of magnons may survive over macroscopic distances. Recently, long range dispersive coupling between two macro-spin systems has been demonstrated experimentally by a model system with two ferrimagnets within a cavity. Combined with state of art spintronics technology, nonlocal spin current manipulation over several centimeters has also been achieved. In fact, photon mediated magnon coupling is so ubiquitous that it may even arise between ferromagnets and antiferromagnets. In this sense, cavity mediated coupling can combine ferromagnetic and antiferromagnetic spintronics within the frame of cavity spintronics. On the storage end, gradient memory architectures have been developed making use of the phase correlation and scalability of CMP systems.

Aside from coherent coupling, a form of dissipative coupling was revealed recently in CMP systems. In contrast to the level repulsion rising from coherent coupling between resonant modes, dissipative coupling shows an exotic level attraction. These two coupling mechanisms give us an unprecedented degree of freedom to control photon mediated interaction.

In this work, we revisit cavity mediated long range interaction between two magnets from the new perspective of controlling coupling mechanisms. By placing two yttrium iron garnet (YIG) spheres at different positions in a cavity, which correspond to coherent coupling or dissipative coupling, we experimentally show that the indirect coupling between the two magnon systems act as a XOR-like logic gate in the coupling phenomenon. Specifically, when both magnon modes share the same form of coupling to the cavity (either coherent or dissipative), then the indirect coupling between them will produce level repulsion. Conversely, if the magnon modes have different forms of coupling to the cavity (one coherent and one dissipative), then their indirect coupling will produce level attraction. Treating dissipative coupling on an equal footing with coherent coupling, we are now able to establish the correlation between local magnon-photon coupling and non-local magnon-magnon coupling. Our work reveals the cavity mediated dissipative coupling between distant magnetic moments, which opens a new avenue for controlling and utilizing light-matter interaction.

This paper is split into two main sections, which discuss the theoretical model and experimental results. In the theoretical model part, we first provide a brief com-
parison between coherent and dissipative magnon-photon coupling, particularly, the evolution of the eigenvector in the coupled system, which allows us to clearly distinguish the cases of level repulsion and attraction. Then we present the formula describing long-range coherent and dissipative indirect magnon-magnon interactions through their mutual coupling to a cavity photon mode. Finally, we present the implementation of our experimental set-up and quantitatively compare the experimental observations with the theoretical model.

II. THEORETICAL MODEL

For a quantitative understanding of the long-range magnon-magnon interaction, we first study a general theoretical model involving coherent and dissipative coupling in a two-mode system. From this we clearly see the distinguishing features of level repulsion and level attraction in the dispersion, as well as the eigenvector of the coupled system. Then we consider the long-range indirect interaction between two magnon modes, which are coupled to a common microwave cavity. We rewrite the eigenfrequencies and eigenvectors of the system in a more explicit form using the dispersive approximation, where the frequency detuning between the magnon mode and the cavity mode is assumed to be large compared to the coupling strength.

A. Eigenfrequency and eigenvector in a strongly coupled magnon-photon system

We start with a general theoretical model of a two-mode system involving both coherent and dissipative coupling, which can be described by an equivalent non-Hermitian Hamiltonian as

$$H = \hbar \omega_c a^\dagger a + \hbar \omega_m m^\dagger m + \hbar g (a^\dagger m + e^{i\Phi} m a^\dagger),$$  

(1)

where $\omega_c$ ($\omega_m$) is the frequency of the cavity (magnon) mode, $a^\dagger$ ($a$) and $m^\dagger$ ($m$) are the creation (annihilation) operators for the cavity and magnon mode respectively. The coupling rate $g$ is chosen to be a real positive number. The coupling phase $\Phi$ describes the competing coherent and dissipative couplings: $\Phi = 0$ for level repulsion and $\Phi = \pi$ for level attraction.

In the Heisenberg picture, this leads to the equation of motion

$$\frac{d}{dt} \begin{pmatrix} a \\ m \end{pmatrix} = \begin{pmatrix} \omega_c & g \\ e^{i\Phi} g & \omega_m \end{pmatrix} \begin{pmatrix} a \\ m \end{pmatrix}. \quad (2)$$

Following the $e^{-i\omega t}$ convention, the hybridized eigenvalues of the system are found by diagonalizing the matrix in Eq. (2), and have eigenfrequencies

$$\omega = \frac{1}{2} \left( \omega_m + \omega_c \pm \sqrt{(\omega_m - \omega_c)^2 + 4e^{i\Phi} g^2} \right),$$  

(3)

and eigenvectors

$$\begin{pmatrix} a \\ m \end{pmatrix} = \begin{pmatrix} \Delta_H/2 \pm \sqrt{\Delta_H^2/4 + e^{i\Phi} g^2} \\ \Delta_H \end{pmatrix},$$

with $\Delta_H = \omega_m - \omega_c$.  

(4)

In the frequency domain, two coupled modes (indicated by coloured lines) repel with each other for $\Phi = 0$ and attract each other for $\Phi = \pi$ as clearly shown in Figs. 1(a) and (b) respectively. As a result, for level attraction, the two hybridized modes coalesce and have an identical eigenvector at a condition of $\Delta_H = \omega_m - \omega_c = \pm 2g$, resembling an exceptional point through linear dynamics in the absence of damping.

The phase correlation, $\phi_m - \phi_c$, of magnetization ($m$) and electrodynamics ($a$) calculated by Eq. (4) is shown in Figs. 1(c) and (d) for level repulsion and attraction, respectively. In level repulsion case, the phase correlation between $m$ and $a$ is quite simple: $\phi_m - \phi_c = 0$.

![Image](https://via.placeholder.com/150)
(corresponding to in-phase $a - m$ motion) for $\omega_+$, and $\phi_m - \phi_c = -\pi$ (corresponding to out-of-phase $a - m$ motion) for $\omega_-$. For level attraction, it can be found that the phase correlation is completely different: for both $\omega_\pm$, $\phi_m - \phi_c = 0$ when $\Delta \mu < -2g$ and $\phi_m - \phi_c = \pi$ when $\Delta \mu > 2g$; in between those $\Delta \mu$ values the $m$ phase for $\omega_-$ rotates anti-clockwise from 0 to $\pi$ with respect to $a$ while for $\omega_-$ the $m$ phase rotates clockwise from 0 to $-\pi$. Although the two hybridized magnon-photon modes follow an identical dispersion over a wide range for $|\Delta \mu| < 2g$ \( \text{Re}(\omega_\pm) = (\omega_c + \omega_m)/2 \) and furthermore both consist of half magnon and half photon, the two states are independent because their correlation phases have opposite signs.

B. Two distanced magnons coupled with a common cavity mode

Based on the key features of level repulsion and attraction shown in Sec. IIA, now we study a three-mode coupled magnon-photon system, where two spatially separated magnon modes, $m_1$ and $m_2$, couple with a cavity mode ($a$). The schematic diagram of this three-mode coupled system is illustrated in Fig. 2(a). The microwave current drives or impedes the dynamics of magnetization through the competition of Ampere’s law and the cavity Lenz effect (indicated by blue arrow)\(^23\) Meanwhile, due to the effects of Faraday’s law, the magnetization precession also creates a back action effect onto the cavity field (indicated by red and green arrows). Thus the two magnon modes are coupled to the cavity mode with a coupling strength of $g_{1,2}$. By exchanging virtual photons, the two magnon modes are strongly coupled with an exchange coupling rate of $J$ (grey dashed arrow).

The equivalent non-Hermitian Hamiltonian of this three-mode system can be written as

\[
H = \hbar \omega_1 a^\dagger a + \hbar \omega_1 m_1^\dagger m_1 + \hbar g_1 (a^\dagger m_1 + e^{i\Phi_1} a m_1^\dagger) \quad (5) \\
\hbar \omega_2 m_2^\dagger m_2 + \hbar g_2 (a^\dagger m_2 + e^{i\Phi_2} a m_2^\dagger),
\]

where $\omega_{1,2}$, $g_{1,2}$ and $\Phi_{1,2}$ are the frequency, coupling strength to the cavity mode and coupling phase for the magnon mode $m_{1,2}$, respectively. Assuming sufficient spatial separation between the two magnetic samples, we neglect direct interactions between $m_1$ and $m_2$\(^23\). The hybridized eigenmodes of the system can be solved from the equation of motion

\[
\frac{d}{dt} \begin{pmatrix} a \\ m_1 \\ m_2 \end{pmatrix} = i \begin{pmatrix} \omega_c & g_1 & g_2 \\ e^{i\Phi_1} g_1 & \omega_{m1} & 0 \\ e^{i\Phi_2} g_2 & 0 & \omega_{m2} \end{pmatrix} \begin{pmatrix} a \\ m_1 \\ m_2 \end{pmatrix}. \quad (6)
\]

For this three-mode coupled system with two identical YIG spheres placed within a cavity, a global magnetic field $H$ is applied to tune the frequency of the magnon modes according to $\omega_{m1,m2}(H) = \gamma (H + H_{A1,A2})$, where

\[
\gamma = 2\pi \times 27.6 \text{ GHz/T} \text{ is gyromagnetic ratio, and } H_{A1,A2} \text{ is the anisotropy field for each individual YIG sample.}
\]

In the meantime, the field at each sphere can be locally adjusted by $\pm \delta H/2$ via a small coil. In the following discussion, we operate the system in the dispersive limit,
where both magnons are significantly detuned from the cavity (\(|\Delta_{1,2}|=\omega_{m1,m2} - \omega_c| \gg g_{1,2}\)).

Figures 2(b-e) show the calculated dispersions of the hybridized modes for four cases based on the coupling state of \((\Phi_1, \Phi_2)\), describing either coherent or dissipative coupling between the individual magnon modes and the cavity mode. Focusing on the region highlighted by a dotted box, where the two magnon modes (red and green dashed lines) cross each other, level repulsion is observed for \((0,0)\) and \((\pi, \pi)\) states, while level attraction is observed for \((0, \pi)\) and \((\pi, 0)\) states. In order to give a more detailed explanation for this striking feature, we analytically solved Eq. (6) in the dispersive limit and rewrite the frequencies of hybridized modes in an explicit form, which is similar to Eq. (3) for directly coupled \(a-m\) system, as

$$
\omega_{m\pm} = \frac{1}{2} \left[ \omega_{m1} + \omega_{m2} \pm \sqrt{(\omega_{m1} - \omega_{m2})^2 + 4e^{i(\Phi_1 + \Phi_2)}J^2} \right],
$$

(7)

where \(\omega_{m1,m2} = \omega_{m1} + e^{i\Phi_1}g_{1,2}^2/\Delta_{1,2}\) includes a finite Lamb shift in the energy level, which can be either blue or red shift dependent on not only the sign of detuning \(\Delta_{1,2}\) but also the nature of the coupling between the magnon and cavity. \(J = \frac{g_1g_2}{\Delta_{1,2}}\) is the effective coupling strength between \(m_1\) and \(m_2\).

Equation (7) indicates that the indirect coupling features of the long-range magnon-magnon interaction are solely determined by the phase between them \((\Phi_1 + \Phi_2)\): level repulsion for \(\cos(\Phi_1 + \Phi_2) = 1\) and level attraction of \(\cos(\Phi_1 + \Phi_2) = -1\). These relations well explains the coupling signature highlighted in Fig. 2(b-e).

Fig. 3(a) and (b) show a zoomed-in view of the boxed areas in Fig. 2(b) and (d). A careful examination shows that the hybridization modes always cross \(\omega_{m1}\) at the point where \(\omega_{m1} = \omega_{m2}\), which is different from the observations in directly coupled system [Figs. 1(a) and (b)]. Mathematically, this point results from the Lamb shift, which causes a shift in the frequency detuning \(\Delta_\phi\) with a magnitude of \(J\) for level repulsion and a shift in field detuning \(\Delta_H\) with a magnitude of \(2J\) for level attraction. Physically, this point is related to “dark” states of coupled system \((\Phi_1, \Phi_2)\). The hybridization of two magnon modes precess out of phase with an identical amplitude, and as a consequence, their interactions are decoupled from the cavity mode.

We can understand this effect by deducing the eigenvector of the \(m_1-m_2\) subsystem

$$
\begin{pmatrix}
m_1 \\
m_2
\end{pmatrix} = \left(\frac{\delta}{2} \pm \sqrt{\frac{\delta^2}{4} + e^{i(\Phi_1 + \Phi_2)}J^2}\right),
$$

(8)

with \(\delta = \omega_{m1} - \omega_{m2}\).

Following the similarity between Eqs. (8) and (4), the phase correlation \(\phi_{m2} - \phi_{m1}\) can be determined exactly the same way as \(\phi_m - \phi_a\) for the directly coupled \(a-m\) system shown in Fig. 1(c) and (d). Using the approximation of \(\Delta_1 \approx \Delta_2 \approx \Delta\) in the dispersive limit, the photon part of the eigenstate can be deduced as

$$
a = \frac{g_1m_1 + g_2m_2}{\Delta}.
$$

(9)

Combining Eqs. (8) and (9), we can determine that the dark state \((a = 0)\) appears at

$$
\delta = \frac{e^{i\Phi_1}g_1^2 - e^{i\Phi_2}g_2^2}{\Delta}.
$$

(10)

As an example, the calculated \(|a|\) (normalized by a factor \(g/\sqrt{|m_1|^2 + |m_2|^2}\) for clarity) is plotted in Figs. 3(c) and (d) for \(g_1 = g_2\) and \(\Delta > 0\), where the dark state clearly appears at \(\delta = 0\) on the \(\omega_{m-}\) branch for \(\Phi_1 = \Phi_2 = 0\) (level repulsion), and at \(\delta = 2J\) on the right exceptional point for \(\Phi_1 = 0\) and \(\Phi_2 = \pi\) (level attraction). Since \(\phi_{m2} - \phi_{m1}\) for \(\omega_{m+}\) and \(\omega_{m-}\) only differ in sign for the level attraction case [similar to \(\phi_m - \phi_a\) in Fig. 1(d)], it does not affect the amplitude of \(g_1m_1 + g_2m_2\) and hence \(|a|\). As a result, \(|a|\) is identical for both \(\omega_{m+}\) and \(\omega_{m-}\) branches as shown in Fig. 3(d).

In previous studies, the dark state always occurs at the hybridized mode closer to the cavity mode in frequency. Here, we find that the dark state of long-range
coherent coupling may also reside in the outer branch of
the hybridized modes if both magnon modes are dissipa-
tively coupled to the cavity mode (Φ$_1$ = Φ$_2$ = π). Our
model indicates that the dark state can be adjusted by
phases Φ$_1$ and Φ$_2$ in addition to the sign of Δ.

If $g_1 \neq g_2$, the dark state appears away from these
symmetric points on the hybridized magnon-magnon dis-

erption. However, by substituting the lamb shift into Eq.
(1), one can find a general relation for the dark state where
ω$_{m1} = $ ω$_{m2}$, regardless of the detailed coupling feature
between the individual magnons and the microwave cav-
ity. Here, the dark mode is induced by the hybridization
of the two magnon modes when they precess out of phase,

$$
\begin{pmatrix}
    n_1 \\
    n_2
\end{pmatrix} = \begin{pmatrix}
    -e^{i\Phi_1}J \\
    e^{i\Phi_2}\frac{g_1^2}{\Delta}
\end{pmatrix},
$$

and their coupling effects on the cavity mode cancel each other, resulting a vanishing total
response of the magnon dynamics to the cavity mode.

III. EXPERIMENT RESULTS AND
DISCUSSION

The experimental setup of our measurement system is
schematically shown in Fig. 4 (a). The microwave cav-
ity used in this work is a Fabry-Perot-like cavity based
on the Ku band (12-18 GHz) assembled waveguide appa-
ratus, where circular waveguides are connected through


![FIG. 4. (a) Experimental setup, with a VNA measuring the
microwave transmission through a waveguide loaded with two
YIG spheres. The simulated h field amplitude for the cavity
mode at the middle plane. YIG1 (white) is fixed at either a
node or antinode of the cavity mode system. Using a vector network analyzer (VNA)
coupling regime.](image)

We first calibrate the coupling effects between the cav-
ity mode and a single YIG sphere. YIG1 is fixed at a
position with an angle either θ = 180° (h— antinode)
or $\theta = -90^\circ$ (h− node). The S-parameter measurement for this single YIG allows us to determine $g_1/2\pi = 55$ MHz and $\Phi_1 = 0$ for $\theta = 180^\circ$ and $g_1/2\pi = 19$ MHz and $\Phi_1 = \pi$ for $\theta = -90^\circ$. Separately, YIG2 is rotated over an angle range of $-22^\circ \leq \theta \leq 112^\circ$ covering two distinct coupling regimes of coherent coupling and dissipative coupling. The deduced coupling strength, $g_2$, as a function of the angular position $\theta$ for YIG2 are summarized in Fig. 2(b), from which we found that the critical angle where the coupling regime switches is $70.5^\circ$.

By placing YIG1 at $\theta = 180^\circ$ corresponding to the coherent coupling region of $g_1$ and rotating YIG2 within the cavity, we measure the long-range coupling between magnon modes mediated by the cavity photon. Figure 4(c) shows the results for a fixed static magnetic field $\mu_0H = 488$ mT, where $\omega_c/2\pi = 12.76$ GHz and $\omega_{m1}/2\pi = 12.85$ GHz indicated by dashed lines are the uncoupled cavity mode and YIG1 magnon mode frequencies. By rotating YIG2, the interaction between the two magnon modes is clearly seen in Fig. 4(c) when $\omega_{m2}$ approaches $\omega_{m1}$, indicated by arrows label as $A_{1-3}$. At conditions $A_1$ and $A_2$, where both $\omega_{m1}$ and $\omega_{m2}$ are coherently coupled with $\omega_c$, the long-range interaction between the two magnon modes shows a characteristic feature of the avoided level crossing. As we rotate YIG2 clockwise across $70.5^\circ$, the two spheres enter different coupling regions. When their frequencies again meet at the condition $A_3$, level attraction between the indirectly coupled magnon modes is experimentally demonstrated for the first time.

Next we place YIG1 at $\theta = -90^\circ$ corresponding to the dissipative coupling region of $g_1$ and repeat the above measurement. The results are summarized in Fig. 4(d). Despite the same rotation trajectory of YIG2, the behavior of the long-range interaction between the magnon modes shows a different pattern. At conditions $B_1$ and $B_2$, where the two YIG spheres have different coupling mechanisms to the cavity mode, a characteristic feature of level attraction occurs. Meanwhile, level repulsion is observed when both YIG spheres are dissipatively coupled with the cavity.

This experiment unambiguously validates our model, demonstrating long-range indirect coherent and dissipative interactions between two spatially separated magnons. The characteristic features of the long-range interaction are solely determined by the relative phase between $\Phi_1$ and $\Phi_2$. Furthermore, the dark state is clearly seen at the condition $A_1$, where $g_1 \simeq g_2$. At conditions $A_2$, $A_3$, $B_1$ and $B_2$, the dark state predicted by Eq. (10) appears far away from the strongly coupling regime due to the significant difference between $g_1^2$ and $g_2^2$ and is thus not well resolved in current experiment.

To quantitatively explain the experimental observation of long-range interactions between two magnon modes, we calculated the dispersion by solving the determinant of Eq. (6). We focus on the three conditions $A_1$, $B_1$, and $B_3$, which correspond to three typical cases of long-range magnon-magnon interaction: (a) both magnon modes coherently coupled with the cavity, (b) one magnon mode coherently and the other dissipatively coupled with the cavity, and (c) both magnon modes dissipatively coupled with the cavity. For the calculation, $g_1$ and $g_2$ are determined by experimental measurements. For simplicity, we assume $\omega_{m2}$ follows a relation as $\omega_{m2} - \omega_{m1} \propto \sin(\theta - \theta_0) + C$ within a $20^\circ$ range where $\theta_0$ and $C$ are constants. As shown in Fig. 5, the comparison between simulation and experiment illustrates a quantitative agreement.

Thanks to the high sensitivity of our experimental implementation, we can study the dependence of the indirect coupling strength $J$ on detuning $\Delta$ by varying the static magnetic field $H$. As clearly seen in Fig. 6(a), the amplitude of the hybridized magnon modes decreases with increasing $\Delta$, and furthermore the gap between the hybridized magnon modes shrinks. Although amplitude of the hybridized magnon modes gradually decrease, the dark state is well resolved.

For the case of indirect coherent coupling, the coupling strength $J$ can be directly determined from the polariton gap ($=2J$) of the dispersion as indicated in Fig. 3(a). The measured amplitude of $J$ is plotted as black squares in Fig 6(b). To compare with our model, we first calculate the dispersion [dashed lines in Fig. 6(a)] using Eq. (6), which is in agreement with experimental results. During the calculation one set of parameters ($g_1/2\pi = 55$ MHz, $g_2/2\pi = 52$ MHz, and $\Phi_1 = \Phi_2 = 0$) was used. The $J$ deduced from our calculations (solid line) is in agree-
FIG. 6. (a) For long-range coherent coupling, transmission spectra at various external field demonstrate the decay of the coupling strength $J$ when increasing $\Delta$. (b) $J$ as a function of $\Delta$ at $A_1$ when both magnon modes are coherently coupled with the cavity mode. Black squares are determined by measured data in (a), while the red line represents calculation result based on Eq.(6) and the blue dashed line follows the $g_1 g_2/\Delta$ dependence in the dispersive limit.

FIG. 7. (a) For long-range dissipative coupling, transmission spectra at various external field demonstrate the decay of the coupling strength $J$ with increased $\Delta$. (b) $J$ as a function of $\Delta$ at $B_1$ with one magnon coherently and one dissipatively coupled with the cavity mode. Black squares are determined by measured data in (a), while the red line represents the calculation results based on Eq.(6) and the blue dashed line follows $g_1 g_2/\Delta$ dependence in the dispersive limit.

IV. CONCLUSIONS

We have presented a systematic study of the effect of indirect coupling between two magnon modes mediated by a cavity mode. A theoretical model based on a phenomenological approach was developed to describe the dispersions and phase information of the system in the dispersive limit. The characteristic properties of XOR-like coupling relations and magnon dark states are revealed both theoretically and experimentally. Putting magnets on a similar basis to qubits and atoms in cavities, our work provides a new method for studying cavity mediated coupling in the framework of cQED. Furthermore, in a general context, our model system demonstrates the transition rule of the coupling state where two sub-systems interact with each other through a mediating oscillator, which can act as a building block to further understand long range light-matter interactions.
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Note added.-During the manuscript preparation, we found a preprint theoretical work on dissipative long-range magnon-magnon interactions in a coupled magnon-photon system [Ref. 40], which presents an alternative consistent theoretical picture for our experiment.

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