A stochastic background of gravitational waves from hybrid preheating

Juan García-Bellido and Daniel G. Figueroa
Departamento de Física Teórica C-XI, Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain
(Dated: October 14, 2006)

The process of reheating the universe after hybrid inflation is extremely violent. It proceeds through the nucleation and subsequent collision of large concentrations of energy density in bubble-like structures, which generate a significant fraction of energy in the form of gravitational waves. We study the power spectrum of the stochastic background of gravitational waves produced at reheating after hybrid inflation. We find that the amplitude could be significant for high-scale models, although the typical frequencies are well beyond what could be reached by planned gravitational wave observatories like LIGO, LISA or BBO. On the other hand, low-scale models could still produce a detectable stochastic background at frequencies accessible to those detectors. The discovery of such a background would open a new window into the very early universe.

According to general relativity, the present universe should be permeated by a diffuse gravitational wave background (GWB) with a variety of origins, from unresolved point sources (gravitational collapse of supernovae, neutron star and black hole coalescence, etc.) to relic stochastic backgrounds from early universe phase transitions, inflation, turbulent plasmas, cosmic strings, etc. [1]. These backgrounds have very different spectral shapes and amplitudes that may, in the future, allow gravitational wave observatories like LIGO, LISA, BBO or DECIGO [1] to disentangle their origin.

There are already a series of constraints on some of these backgrounds, the most stringent one coming from the large-scale polarization anisotropies in the Cosmic Microwave Background (CMB), which may soon be measured by Planck, if the scale of inflation is sufficiently high [2]. There are also constraints coming from Big Bang nucleosynthesis [3], as well as from millisecond pulsar timing [4]. Furthermore, it has recently been proposed a new constraint on primordial GW coming from CMB anisotropies [5]. Most of these constraints come at very low frequencies (typically from $10^{-18}$ Hz to $10^{-8}$ Hz), while present GW detectors work at frequencies of order 1-100 Hz, and planned observatories will range from $10^{-3}$ Hz of LISA to $10^{3}$ Hz of Advanced-LIGO [1], which could detect GW associated with early universe phenomena like first-order phase transitions [6, 7], or cosmic turbulence [8], if these occur around the electroweak scale.

In this Letter we want to describe a new stochastic background of gravitational waves that may help open a window into the very early universe phenomena. Recent observations of the CMB anisotropies seem to suggest that something like inflation must have occurred in the early universe. The process by which the energy density driving inflation is converted into all the radiation and matter we observe today is called reheating, and corresponds to the true Big Bang of the Standard Cosmological Model. The first stage of conversion, preheating [11], is known to be explosive, and generates in less than a Hubble time the huge entropy measured today. In chaotic inflation, the coherent oscillations of the inflatons during preheating generates, via parametric resonance, a population of highly occupied modes that behave like waves of matter, which collide among themselves and whose scattering leads to homogeneization and local thermal equilibrium. These collisions occur in a highly relativistic and very asymmetric way, being responsible for the generation of a stochastic background of gravitational waves [12, 13] with a typical frequency today of the order of $10^{-7}$ to $10^{9}$ Hz, corresponding to the present size of the causal horizon at the end of high-scale inflation. There is at present no chance to detect such a background, not even by resonant superconducting microwave cavities [14].

However, there are models like hybrid inflation in which the end of inflation is sudden [15] and the conversion into radiation occurs almost instantaneously. Indeed, since the work of Ref. [16] we know that hybrid models preheat in an even more violent way than chaotic inflation models, via the spinodal instability of the symmetry breaking field that triggers the end of inflation, irrespective of the couplings that this field may have to the rest of matter. Such a process is known as tachyonic preheating [16, 17] and could be responsible for copious production of dark matter particles [18], lepto and baryogenesis [19], topological defects [16], primordial magnetic fields [20], etc. Moreover, it was speculated in Ref. [21] that in (low-scale) models of hybrid inflation it might be possible to generate a stochastic GWB in the frequency range accessible to present detectors, if the scale of inflation is as low as $H_{inf} \sim 1$ TeV. However, the amplitude was estimated using the parametric resonance formalism of chaotic preheating, which may not be applicable in this case. In Ref. [17] it was shown that the process of symmetry breaking proceeds via the nucleation of dense bubble-like structures moving at the speed of light, which collide and break up into smaller structures (see Figs. 7 and 8 of Ref. [17]). We conjectured at that time that such collisions would be a very strong source of gravitational waves, analogous to the gravity wave production associated with strongly first order phase transitions [6]. As we will show in this Letter, this is indeed the case during preheating in hybrid inflation.

Hybrid inflation models [15] arise in theories of particle physics with symmetry breaking fields ('Higgses') coupled to flat directions, and are present in many exten-
sions of the Standard Model, both in string theory and in supersymmetric theories [9]. Inflation occurs along the lifted flat direction, satisfying the slow-roll conditions thanks to a large vacuum energy \( \rho_0 \). Inflation ends when the inflaton \( \chi \) falls below a critical value and the symmetry breaking field \( \phi \) acquires a negative mass squared, which triggers the breaking of the symmetry and ends in the true vacuum, \( \phi = v \), within a Hubble time. These models do not require small couplings in order to generate the observed CMB anisotropies; e.g. a working model with GUT scale symmetry breaking, \( v = 10^{-3} M_P \), with a Higgs self-coupling \( \lambda \) and a Higgs-inflaton coupling \( g \) given by \( g = \sqrt{2 \lambda} = 0.05 \), satisfies all CMB constraints [10], and predicts a tiny tensor contribution to the CMB polarization. The main advantage of hybrid models is that, while most chaotic inflation models can only occur at high scales, with Planck scale values for the inflaton, and \( V_{\text{inf}}^{1/4} \sim 10^{16} \text{ GeV} \), one can choose the scale of inflation in hybrid models to range from GUT scales all the way down to TeV scales, while the Higgs v.e.v. can range from Planck scale, \( v = M_P \), to the Electroweak scale, \( v = 246 \text{ GeV} \), see Ref. [15, 16].

Reheating in hybrid inflation goes through four well-defined regimes: first, the exponential growth of long wave modes of the Higgs field via spinodal instability, which drives the explosive growth of all particles coupled to it, from scalars [16] to gauge fields [17] and fermions [18]; second, the nucleation and collision of high density contrast and highly relativistic bubble-like structures associated with the peaks of a Gaussian random field like the Higgs [17]; third, the turbulent regime that ensues after all these 'bubbles' have collided and the energy density in all fields cascades towards high momentum modes; finally, thermalization of all modes when local thermal and chemical equilibrium induces equipartition. The first three stages can be studied in detailed lattice simulations thanks to the semi-classical character of the process of preheating [22], while the last stage is intrinsically quantum and has never been studied in the lattice.

In this Letter we use lattice simulations to study the generation of gravitational waves during preheating in hybrid inflation and analyse the dependence of the shape and amplitude of the spectrum of gravity waves on the scale of hybrid inflation, and more specifically on the v.e.v. of the Higgs triggering the end of inflation. Gravitational waves are represented by a tensor metric perturbation, \( g_{\mu\nu} = h_{\mu\nu} + \eta_{\mu\nu} \), in the transverse traceless (or radiation) gauge. Its equation of motion in this gauge is \( \square h_{\mu\nu} = 16\pi G T_{\mu\nu} \), with the harmonic gauge condition \( \partial^\mu h_{\mu\nu} = 0 \) ensured by conservation of the energy-momentum tensor. In the radiation gauge we can fix \( h_00 = 0 \), and the resulting field is the usual tensor gauge-invariant metric perturbation \( h_{ij} \), which satisfies the evolution equation \( h''_{ij} - \nabla^2 h_{ij} = 16\pi G \Pi_{ij} \), with \( \Pi_{ij} \) the anisotropic stress tensor, sourced by both the inflaton and Higgs fields, \( \Pi_{ij} = \nabla_i \phi^a \nabla_j \phi^a + \nabla_i \chi \nabla_j \chi - 1/3 \delta_{ij} (\nabla \phi^a)^2 + (\nabla \chi)^2 \). We solve the evolution equations of the gravity waves \( h_{ij} \) together with those of the other coupled scalar fields in a discretized lattice, assuming initial quantum fluctuations for all fields and only a zero mode for the inflaton, following the prescription adopted in Ref. [17]. We also included the GW backreaction on the scalar fields' evolution via the gradient terms \( \partial^i \nabla_j \phi \partial_j \phi \), although for all practical purposes these are negligible throughout GW production. We then evaluate the mean field values, as well as the different energy components, see Fig. 1. For the energy in gravitational waves we use the expression \( (32\pi G) t_{ij} = \langle \partial_i h_{ij}^{\text{T}T} \partial_j h_{ij}^{\text{T}T} \rangle = \frac{2}{3} \langle \partial_i h_{ij} \partial_j h_{ij} \rangle \), where the expectation value is over a region sufficiently large to encompass enough physical curvature to have a gauge invariant measure of the GW energy [23], and we have expressed the average over the transverse traceless tensor \( h_{ij}^{\text{T}T} \) in terms of the average over \( h_{ij} \), the solution of the (traceless) tensor evolution equation. The fractional energy density in gravitational waves is then \( \Omega_{gw}/\rho_0 = 4t_{00}/\rho_0 m^2 \), which can be used to compute the corresponding density parameter today (with \( \Omega_{\text{rad}} h^2 \approx 3.5 \times 10^{-5} \)):

\[
\Omega_{gw} h^2 = \Omega_{\text{rad}} h^2 \frac{1}{8\pi G v^2 m^2} \left\langle \partial_i h_{ij}^{\text{T}T} \partial_j h_{ij}^{\text{T}T} \right\rangle ,
\]

where we have assumed that all the vacuum energy \( \rho_0 \) gets converted into radiation, an approximation which is always valid in generic hybrid inflation models with \( v \ll M_P \), and thus \( H \ll m = \sqrt{\chi v} \). We have shown in Fig. 1 the evolution in time of the fraction of energy density in GW. The first (tachyonic) stage is clearly visible, with a slope twice that of the anisotropic tensor \( \Pi_{ij} \). Then there is a small plateau corresponding to the production of GW from bubble collisions; and finally there is the linear growth due to turbulence. Note that in the case that \( H \ll m \), the maximal production of GW occurs in less than a Hubble time, soon after symmetry breaking, while turbulence lasts several decades in time units of \( m^{-1} \). Therefore, we can safely ignore the dilution due to the
Hubble expansion, until the universe finally reheat and the energy in gravitational waves redshifts like radiation thereafter.

We then compute the power spectrum per logarithmic interval in GW by performing a Fourier transform of the energy density, \( \Omega_{gw} = \int df \rho_{gw}(f) \), as a function of the frequency \( f \), where \( \Omega_{gw}(k) = k^3 \rho_{gw}(k)/2\pi^2 \rho_c \), with \( \rho_c \) the critical density today. Since gravitational waves below Planck scale remain decoupled from the plasma immediately after production, we can evaluate the power spectrum today from that obtained at preheating by simply converting the wavenumber \( k \) into frequency \( f \),

\[
f = 6 \times 10^{10} \text{ Hz} \frac{k}{\sqrt{H M_P^2}} = 5 \times 10^{10} \text{ Hz} \frac{k}{m} \lambda^{1/4}.
\]

We have shown in Fig. 2 the power spectrum of gravitational waves as a function of wavenumber \( k/m \). We have used different lattices in order to have lattice artifacts under control, specially at late times and high wavenumbers. We have checked that the power spectrum of GW follows (turbulent) scaling after \( mt \sim 40 \), and we can thus estimate the subsequent growth in energy density beyond our simulations. We have left for a future publication [24] the detailed analysis of turbulence in this system.

We will now compare our numerical results with analytical estimates. The tachyonic growth is dominated by the faster than exponential growth of the Higgs modes towards the true vacuum [17]. The (traceless) anisotropic stress tensor \( \Pi_{ij} \) grows rapidly to a value of order \( k^3 |\phi|^2 \sim 10^{-3} m^2 v^2 \), which gives a tensor perturbation \( |h_{TT}^{(T)}|^2 \sim 16 \pi G v^2 (m \Delta t)^2 10^{-3} \) and an energy density in GW, \( \rho_{gw}/\rho_0 \sim 64 \pi G v^2 (m \Delta t)^2 10^{-6} \sim G v^2 \), for \( m \Delta t \sim 16 \). In the case at hand, with \( v = 10^{-3} M_P \), we find \( \rho_{gw}/\rho_0 \sim 10^{-6} \) at symmetry breaking, which coincides with the numerical simulations at that time, see Fig. 1. The production of gravitational waves in the next stage proceeds through bubble collisions. Assuming the bubble walls contain most of the energy density, and since they travel close to the speed of light [17], it is expected that the asymmetric collisions will copiously produce GW, like those of a strongly first order phase transition. In that case, a quick estimate suggests that the fraction of energy density is given by \[
\rho_{gw}/\rho_0 \sim 1/(20(RH)^2) \sim 8 \pi/60 (Rm)^2 G v^2 \sim 2 G v^2,
\]
of the same order or slightly larger than the previous stage, for the typical size of bubbles, \( R \sim 3 m^{-1} \), upon collision [17], which again corresponds to what is observed in the numerical simulations, see Fig. 1. The subsequent turbulent stage [20, 23] is expected to further produce GW with a spectrum that scales with time in a well defined manner, see also [24],

\[
\frac{k^3}{2\pi^2} \frac{\rho_{gw}(k)}{\rho_0} = 0.2 G v^2 \tau^{1/0} k^2 \exp(-0.25 k^2 \tau^{-2} r),
\]

where \( \tau = mt \) and \( p = 1/7 \) is the corresponding turbulent exponent [20, 23]. This spectrum has a maximum at \( k/m \sim 1 \), and falls as \( k^2 \) for small \( k \) until it reaches the maximum wavelength, \( k \sim H \), corresponding to the minimum frequency today, \( f_{\text{min}} \sim 5 \times 10^{10} \text{ Hz} \lambda^{1/4} v/M_P \). For the case we were considering in our numerical simulations, with \( v = 10^{-3} M_P \) and \( \lambda \sim g^2 \sim 0.1 \), we find the power spectrum of Fig. 2.
corresponding to a rate of expansion $H \sim 100$ GeV. The high-scale hybrid model produces typically as much gravitational waves form preheating as the chaotic inflation models. The advantage of low-scale hybrid models of inflation is that the background produced is within reach of future GW detectors like BBO [20].

To summarize, we have shown that hybrid models are very efficient generators of gravity waves at preheating, in three well defined stages, first via the tachyonic growth of Higgs modes, which act as sources of gravity waves; then via the collisions of highly relativistic bubble-like structures with large amounts of energy density, and finally via the turbulent regime that drives the system towards thermalization. These waves remain decoupled since the moment of their production, and thus the predicted amplitude and shape of the gravitational wave spectrum today can be used as a probe of the reheating period in the very early universe. The characteristic spectrum can be used to distinguish between this stochastic background and others, like those arising from NS-NS and BH-BH coalescence, which are decreasing with frequency, or those arising from inflation, that are flat [27].

For a high-scale model of inflation, we may never see the predicted GW background coming from preheating, in spite of its large amplitude, because it appears at very high frequencies, much beyond present experiments’ sensitivities, where no detector has yet shown to be sensitive. On the other hand, if inflation occurred at low scales, even though we will never have a chance to detect the GW produced during inflation in the polarization anisotropies of the CMB, we do expect gravitational waves from preheating to contribute with an important background in sensitive detectors like BBO. The detection and characterization of such a GW background, coming from the complicated and mostly unknown epoch of reheating of the universe, may open a new window into the very early universe, while providing a new test on inflation.

Acknowledgments: We wish to thank Margarita García Pérez and Alfonso Sastre for useful comments on the numerical simulations. This work is supported in part by CICYT projects FPA2003-03801 and FPA2003-04597. D.G.F. also acknowledges support from a FPU-Fellowship from the Spanish M.E.C.

[1] M. Maggiore, Phys. Rept. 331, 283 (2000); C. J. Hogan, arXiv:astro-ph/0608567
[2] M. Kamionkowski, A. Kosowsky and A. Stebbins, Phys. Rev. Lett. 78, 2058 (1997); U. Seljak and M. Zaldarriaga, Phys. Rev. Lett. 78, 2054 (1997).
[3] B. Allen, gr-qc/9604033
[4] I. H. Stairs, Living Rev. Rel. 6, 5 (2003).
[5] T. L. Smith, E. Pierpaoli and M. Kamionkowski, Phys. Rev. Lett. 97, 021301 (2006).
[6] A. Kosowsky, M. S. Turner and R. Watkins, Phys. Rev. Lett. 69, 2026 (1992); A. Kosowsky and M. S. Turner, Phys. Rev. D 47, 4372 (1993); M. Kamionkowski, A. Kosowsky and M. S. Turner, Phys. Rev. D 49, 2837 (1994).
[7] A. Nicolis, Class. Quant. Grav. 21, L27 (2004); C. Grojean and G. Servant, arXiv:hep-ph/0607107
[8] A. Kosowsky, A. Mack and T. Kainshavili, Phys. Rev. D 66, 024030 (2002); A. D. Dolgov, D. Grasso and A. Nicolis, Phys. Rev. D 66, 103505 (2002).
[9] D. H. Lyth and A. Riotto, Phys. Rept. 314, 1 (1999).
[10] D. N. Spergel et al., astro-ph/0003449; M. Tegmark et al., astro-ph/0608632; W. H. Kinney, E. W. Kolb, A. Melchiorri and A. Riotto, Phys. Rev. D 74, 023502 (2006). H. Peiris and R. Easther, astro-ph/0609003
[11] L. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Rev. Lett. 73, 3195 (1994); L. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Rev. D 56, 3258 (1997).
[12] S. Y. Khlebnikov and I. I. Tkachev, Phys. Rev. D 56, 653 (1997).
[13] R. Easther and E. A. Lim, JCAP 0604, 010 (2006).
[14] F. Pegoraro, L. A. Radicati, P. Bernard and E. Picasso, Phys. Lett. A 68, 165 (1978); G. Gemme, A. Chincarini, R. Parodi, P. Bernard and E. Picasso, gr-qc/0112021; R. Ballantini et al., gr-qc/0502054.
[15] A. D. Linde, Phys. Rev. D 49, 748 (1994); J. García-Bellido and A. D. Linde, Phys. Rev. D 57, 6075 (1998).
[16] G. N. Felder, J. Garcia-Bellido, P. B. Greene, L. Kofman, A. D. Linde and I. Tkachev, Phys. Rev. Lett. 87, 011601 (2001); G. N. Felder, L. Kofman and A. D. Linde, Phys. Rev. D 64, 123517 (2001).
[17] J. Garcia-Bellido, M. Garcia Perez and A. Gonzalez-Arroyo, Phys. Rev. D 67, 103501 (2003).
[18] J. Garcia-Bellido and E. Ruiz Morales, Phys. Lett. B 536, 193 (2002).
[19] J. Garcia-Bellido, D. Y. Grigoriev, A. Kusenko and M. E. Shaposhnikov, Phys. Rev. D 60, 123504 (1999). J. Garcia-Bellido, M. Garcia-Perez and A. Gonzalez-Arroyo, Phys. Rev. D 69, 023504 (2004); A. Tranberg and J. Smit, JHEP 0311, 016 (2003).
[20] A. Diaz-Gil, J. Garcia-Bellido, M. Garcia Perez and A. Gonzalez-Arroyo, PoS LAT2005, 242 (2006).
[21] J. Garcia-Bellido, hep-ph/9804205.
[22] S. Y. Khlebnikov and I. I. Tkachev, Phys. Rev. Lett. 77, 219 (1996); 79, 1607 (1997); T. Prokopec and T. G. Roos, Phys. Rev. D 55, 3768 (1997).
[23] S. Carroll, “Spacetime and Geometry: An introduction to General Relativity,” Addison Wesley (2003).
[24] J. Garcia-Bellido, D. G. Figueroa and A. Sastre, in preparation.
[25] R. Micha and I. I. Tkachev, Phys. Rev. Lett. 90, 121301 (2003); Phys. Rev. D 70, 043538 (2004).
[26] V. Corbin and N. J. Cornish, Class. Quant. Grav. 23, 2435 (2006); G. M. Harry, P. Fritschel, D. A. Shaddock, W. Folkner and E. S. Phinney, Class. Quant. Grav. 23, 4887 (2006).
[27] T. L. Smith, M. Kamionkowski and A. Coroy, Phys. Rev. D 73, 023504 (2006).
[28] A. J. Farmer and E. S. Phinney, Mon. Not. Roy. Astron. Soc. 346, 1197 (2003).