A volume inequality for quantum Fisher information and the uncertainty principle. (English)

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Summary: Let $A_1, \ldots, A_N$ be complex self-adjoint matrices and let $\rho$ be a density matrix. The Robertson uncertainty principle

$$\det\left\{\text{Cov}(A_h, A_j)\right\} \geq \det\left\{-\frac{i}{2} \text{Tr}(\rho [A_h, A_j])\right\}$$

gives a bound for the quantum generalized covariance in terms of the commutators $[A_h, A_j]$. The right side matrix is antisymmetric and therefore the bound is trivial (equal to zero) in the odd case $N = 2m + 1$.

Let $f$ be an arbitrary normalized symmetric operator monotone function and let $\langle \cdot, \cdot \rangle_{\rho, f}$ be the associated quantum Fisher information. Based on previous results of several authors, we propose here as a conjecture the inequality

$$\det\left\{\text{Cov}(A_h, A_j)\right\} \geq \det\left\{\frac{f(0)}{2} \langle i[\rho, A_h], i[\rho, A_j] \rangle_{\rho, f}\right\}$$

whose validity would give a non-trivial bound for any $N \in \mathbb{N}$ using the commutators $i[\rho, A_h]$.

MSC:

81P68 Quantum computation
94A17 Measures of information, entropy

Keywords:

Generalized variance; Uncertainty principle; Operator monotone functions; Matrix means; Quantum Fisher information

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References:

[1] Čencov, N.N.: Statistical Decision Rules and Optimal Inference. American Mathematical Society, Providence (1982). Translation from the Russian edited by Lev J. Leifman
[2] Daoud, M.: Representations and properties of generalized A r statistics, coherent states and Robertson uncertainty relations. J. Phys. A: Math. Gen. 39, 889–901 (2006) · Zbl 1087.81030 · doi:10.1088/0305-4470/39/4/010
[3] Dodonov, A.V., Dodonov, V.V., Mizrahi, S.S.: Separability dynamics of two-mode Gaussian states in parametric conversion and amplification. J. Phys. A: Math. Gen. 38, 683–696 (2005) · Zbl 1063.81026 · doi:10.1088/0305-4470/38/3/008
[4] Gibilisco, P., Isola, T.: A characterization of Wigner-Yanase skew information among statistically monotone metrics. Infin. Dimens. Anal. Quantum Probab. 4(4), 553–557 (2001) · Zbl 1041.81011 · doi:10.1142/S0219025701000644
[5] Gibilisco, P., Isola, T.: Wigner-Yanase skew information on quantum state space: the geometric approach. J. Math. Phys. 44(9), 3752–3762 (2003) · Zbl 1062.81019 · doi:10.1063/1.1598279
[6] Gibilisco, P., Isola, T.: On the characterization of paired monotone metrics. Ann. Inst. Stat. Math. 56, 369–381 (2004) · Zbl 1071.81021 · doi:10.1007/BF02530551
[7] Gibilisco, P., Isola, T.: On the monotonicity of scalar curvature in classical and quantum information geometry. J. Math. Phys. 46(2), 023501–14 (2005) · Zbl 1076.81006 · doi:10.1063/1.1834693
[8] Gibilisco, P., Isola, T.: Uncertainty principle and quantum Fisher information. Ann. Inst. Stat. Math. 59, 147–159 (2007) · Zbl 1146.81012 · doi:10.1007/s10463-006-0103-3
[9] Gibilisco, P., Imparato, D., Isola, T.: Uncertainty principle and quantum Fisher information II. J. Math. Phys. 48, 072109 (2007) · Zbl 1144.81349 · doi:10.1063/1.2748210
[10] Gibilisco, P., Imparato, D., Isola, T.: Inequality for quantum Fisher information (2007). arXiv:math-ph/0702058 · Zbl 1144.81349
[11] Hansen, F.: Extension of Lieb’s concavity theorem. J. Stat. Phys. 124(1), 87–101 (2006) · Zbl 1157.47305 · doi:10.1007/s10955-
[12] Hansen, F.: Metric adjusted skew information (2006). arXiv:math-ph/0607049v3 · Zbl 1205.94058
[13] Heisenberg, W.: Über den Anschaulichen Inhalt der Quantentheoretischen Kinematik und Mechanik. Z. Phys. 43, 172–198 (1927) · Zbl 53.0853.05 · doi:10.1007/BF01397280
[14] Jarvis, P.D., Morgan, S.O.: Born reciprocity and the granularity of spacetime. Found. Phys. Lett. 19, 501 (2006) · Zbl 1111.81077 · doi:10.1007/s10702-006-1006-5
[15] Kennard, E.H.: Zur Quantenmechanik einfacher Bewegungstypen. Z. Phys. 44, 326–352 (1927) · Zbl 53.0853.02 · doi:10.1007/BF01391200
[16] Kosaki, H.: Matrix trace inequality related to uncertainty principle. Int. J. Math. 16(6), 629–645 (2005) · Zbl 1083.15033 · doi:10.1142/S0129167X0500303X
[17] Kubo, F., Ando, T.: Means of positive linear operators. Math. Ann. 246(3), 205–224 (1979/80) · Zbl 0421.47011 · doi:10.1007/BF01371042
[18] Luo, S.: Quantum Fisher information and uncertainty relations. Lett. Math. Phys. 53, 243–251 (2000) · Zbl 1083.15033 · doi:10.1023/A:1011080128419
[19] Luo, S.: Wigner-Yanase skew information and uncertainty relations. Phys. Rev. Lett. 91, 180403 (2003) · doi:10.1103/PhysRevLett.91.180403
[20] Luo, S., Luo, Y.: Correlation and entanglement. Acta Math. Appl. Sin. 19(4), 581–598 (2003) · Zbl 1073.81539
[21] Luo, S., Zhang, Z.: An informational characterization of Schrödinger’s uncertainty relations. J. Stat. Phys. 114(5–6), 1557–1576 (2004) · Zbl 1059.81024 · doi:10.1023/B:JOSS.0000013971.75667.c8
[22] Petz, D.: Monotone metrics on matrix spaces. Linear Algebra Appl. 244, 81–96 (1996) · Zbl 0856.15023 · doi:10.1016/0024-3795(94)00211-8
[23] Petz, D.: Geometry of quantum states. J. Math. Phys. 37, 2662–2673 (1996) · Zbl 0868.60098 · doi:10.1063/1.531535
[24] Petz, D., Tenesi, R.: Means of positive numbers and matrices. SIAM J. Matrix Anal. Appl. 27(3), 712–720 (2005) (electronic) · Zbl 1108.47020 · doi:10.1137/050621906
[25] Robertson, H.P.: The uncertainty principle. Phys. Rev. 34, 573–574 (1929) · doi:10.1103/PhysRev.34.163
[26] Robertson, H.P.: An indeterminacy relation for several observables and its classical interpretation. Phys. Rev. 46, 794–801 (1934) · Zbl 1009.81501 · doi:10.1103/PhysRev.46.794
[27] Schrödinger, E.: About Heisenberg uncertainty relation (original annotation by Angelow, A. and Batoni, M.C.). Bulg. J. Phys. 26(5–6), 193–203 (1999). Translation of Proc. Prussian Acad. Sci. Phys. Math. Sect. 19, 296–303 (1930)
[28] Trifonov, D.A.: Generalized intelligent states and squeezing. J. Math. Phys. 35(5), 2297–2308 (1994) · Zbl 0824.47056 · doi:10.1063/1.530553
[29] Trifonov, D.A.: Generalized intelligent states and squeezing. J. Math. Phys. 35(5), 2297–2308 (1994) · Zbl 0824.47056 · doi:10.1063/1.530553
[30] Trifonov, D.A.: State extended uncertainty relations. J. Phys. A: Math. Gen. 33, 299–304 (2000) · Zbl 1009.81501 · doi:10.1088/0305-4470/33/32/102
[31] Trifonov, D.A.: Generalizations of Heisenberg uncertainty relation. Eur. Phys. J. B 29, 349–353 (2002) · doi:10.1140/epjb/e2002-00315-6
[32] Yanagi, K., Furuichi, S., Kuriyama, K.: A generalized skew information and uncertainty relation. IEEE Trans. Inf. Theory 51(12), 4401–4404 (2005) · Zbl 1171.94330 · doi:10.1109/TIT.2005.858871

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