Successive Four-Dimensional Stokes-Space Direct Detection

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Abstract

We present a successive detection scheme for the fourth dimension in a four-dimensional Stokes-space direct detection receiver. At the expense of a small number of electrical-domain computations, the additional information rate can be substantial.

1 Introduction

Recently, the authors of [1] have presented the Stokes-vector direct detection transceiver as a promising candidate for noncoherent short-haul fiber-optic communication. Their method was improved by Morsy-Osman, et al. in [2] to exploit all four transmitted signal-space dimensions, but their proposed methods for detecting the fourth dimension all suffer various practical drawbacks. In this paper we propose a successive detection scheme with the same optical receiver structure as [2], but with some additional computations in the electrical domain. We show that the additional achievable information rate of the subchannel carried by the fourth dimension can be quite substantial.

2 Four-Dimensional Stokes-Space Direct Detection

Our method to detect three out of four used dimensions is the same as in [2], and is explained briefly here. For more details the reader is referred to [2]. At time index \( n \), the transmitter sends two complex numbers \( E_x[n] \) and \( E_y[n] \), chosen from a quadrature amplitude modulation constellation, over \( X \) and \( Y \) polarizations, respectively. The transmitted message is encoded in \( |E_x[n]|, |E_y[n]|, \arg(E_x[n]E^*_y[n]) \) and \( \arg(E_x[n]E^*_y[n-1]) \), where \( z^* \) denotes the complex conjugate of \( z \). We refer to these respective quantities as the first up to the fourth dimension.

In single mode fibers, the output of the channel can be modeled as [2]

\[
\begin{bmatrix}
F_x[n] \\
F_y[n]
\end{bmatrix} = \begin{bmatrix}
a & b \\
-b^* & a^*
\end{bmatrix} \begin{bmatrix}
E_x[n] \\
E_y[n]
\end{bmatrix} + \begin{bmatrix}
z_x[n] \\
z_y[n]
\end{bmatrix} = \begin{bmatrix}
K_x[n] \\
K_y[n]
\end{bmatrix} + \begin{bmatrix}
z_x[n] \\
z_y[n]
\end{bmatrix},
\]

where \( F_x[n] \) and \( F_y[n] \) are the noisy received light in the two polarizations, \( z_x \) and \( z_y \) are complex-valued circularly-symmetric zero-mean additive white Gaussian noise with variance \( 2\sigma^2 \) due to the amplified spontaneous emission of the preamplifier, \( K_x[n] \) and \( K_y[n] \) are the light components before contaminating with the preamplifier noise, and the channel matrix is random and unitary and is assumed to have a coherence time much larger than the symbol duration. The received light is passed through the receiver shown in Fig. I(a) to produce \( w_1 \) up to \( w_6 \). Without loss of generality, we assume that the responsivity of the photo-diodes is
fewer photo-diodes than the former. In this case respectively. The symbol-error rate performance of this scheme is shown in Section 3.

We can recover the intensities of $E_x[n]$, $E_y[n]$ and their phase difference from the equation

$$
\begin{bmatrix}
    w_1[n] \\
    w_2[n] \\
    w_3[n] \\
    w_4[n] \\
\end{bmatrix} = \begin{bmatrix}
    |a|^2 & |b|^2 & \Re(ab^*) & -\Im(ab^*) \\
    |b|^2 & |a|^2 & -\Re(ab^*) & \Im(ab^*) \\
    -2\Re(ab) & 2\Re(ab) & \Re(a^2) - \Re(b^2) & -\Im(a^2) + \Im(b^2) \\
    -2\Im(ab) & 2\Im(ab) & \Im(a^2) - \Re(b^2) & \Re(a^2) + \Re(b^2) \\
\end{bmatrix} \begin{bmatrix}
    |E_x[n]|^2 \\
    |E_y[n]|^2 \\
    2\Re(E_x[n]E_y[n]^{*}) \\
    2\Im(E_x[n]E_y[n]^{*}) \\
\end{bmatrix} + \begin{bmatrix}
    \hat{z}_1 \\
    \hat{z}_2 \\
    \hat{z}_3 \\
    \hat{z}_4 \\
\end{bmatrix}
$$

where $\hat{z}_1$ up to $\hat{z}_4$ are correlated signal-dependent noise. The maximum-likelihood detector would select $|E_x[n]|$, $|E_y[n]|$ and $\arg(E_x[n]E_y[n]^{*})$ to maximize $f(w_1[n], w_2[n], w_3[n], w_4[n] | |E_x[n]|, |E_y[n]|, \arg(E_x[n]E_y[n]^{*}))$, where $f(\cdot, \cdot)$ denotes the conditional probability density function (PDF). As finding the exact conditional PDF is cumbersome, we approximate it with a Gaussian with the same mean vector and covariance matrix. After estimation of the channel matrix, $(|E_x[n]|, |E_y[n]|, \arg(E_x[n]E_y[n]^{*}))$ and $(|K_x[n]|, |K_y[n]|, \arg(K_x[n]K_y[n]^{*}))$ are in one-to-one correspondence, so we deal with the latter quantities. If $\delta = \arg(K_x[n]K_y[n]^{*})$, the covariance matrix and the mean vector are given as

$$
\begin{bmatrix}
    4\sigma^4 + 4\sigma^2|K_x|^2 & 0 & 4\sigma^2|K_x||K_y|\cos(\delta) & 4\sigma^2|K_x||K_y|\sin(\delta) \\
    0 & 4\sigma^4 + 4\sigma^2|K_y|^2 & 4\sigma^2|K_x||K_y|\cos(\delta) & 4\sigma^2|K_x||K_y|\sin(\delta) \\
    4\sigma^2|K_x||K_y|\cos(\delta) & 4\sigma^2|K_x||K_y|\cos(\delta) & 8\sigma^4 + 4\sigma^2(|K_x|^2 + |K_y|^2) & 0 \\
    4\sigma^2|K_x||K_y|\sin(\delta) & 4\sigma^2|K_x||K_y|\sin(\delta) & 0 & 8\sigma^4 + 4\sigma^2(|K_x|^2 + |K_y|^2) \\
\end{bmatrix}
= \begin{bmatrix}
    2\sigma^2 + |K_x|^2 \\
    2\sigma^2 + |K_y|^2 \\
    2|K_x||K_y|\cos(\delta) \\
    2|K_x||K_y|\sin(\delta) \\
\end{bmatrix}
$$

respectively. The symbol-error rate performance of this scheme is shown in Section 3.

Instead of the receiver shown in Fig. 1(a), we can use the one shown in Fig. 1(b) which has a structure similar to the Fig. 1b of [1]. As each balanced photo-detector has two photo-diodes, the latter uses four fewer photo-diodes than the former. In this case $w'_3[n] = w_1[n] + w_2[n] + \frac{1}{2}w_3[n]$ and we can recover $w_3$ in the electrical domain, thereby reducing the number of optical/electro-optical devices. The same argument applies for $w'_4, w'_5$ and $w'_6$. In this paper, the calculations are based on $w_1$ up to $w_6$, which can be obtained from the receiver shown in Fig. 1(b) with appropriate post-processing.

Until this point, we have detected the first three dimensions. To detect the last dimension, $w_5$ and $w_6$ are used. These are related to their transmitted counterparts by

$$
F_x[n]F_y[n]^{*} = \ell^T \begin{bmatrix}
    E_x[n]E_y^{*}[n-1] \\
    E_y[n]E_x^{*}[n-1] \\
    E_x[n]E_y^{*}[n-1] \\
    E_y[n]E_x^{*}[n-1] \\
\end{bmatrix} + \hat{z}_5, \quad \ell = \begin{bmatrix}
    a^2 \\
    -b^2 \\
    -ab \\
    ab \\
\end{bmatrix},
$$

and where $\hat{z}_5$ is a signal dependent complex noise with correlated components. As is apparent from (1), the received quantity is not just a function of $E_x[n]E_y^{*}[n-1]$, but a function of three more beating terms. In [2], three suggestions for resolving this issue are given: (i) If $b = 0$, then the channel has no rotation and in [1], $F_x[n]F_y^{*}[n-1]$ is only a function of $E_x[n]E_y^{*}[n-1]$. This model is too simple and is not realistic. (ii) In [1] we have four unknowns, but just one equation. So in order to solve a system of linear equations, we need three more equations that can be obtained by measuring $F_y[n]F_y^{*}[n-1], F_x[n]F_x^{*}[n-1]$ and $F_y[n]F_x^{*}[n-1]$ as well. By using the same method as in Fig. 1(a), it needs three more optical hybrids and six more balanced photo-detectors which makes it quite complex. (iii) By feeding back the channel rotation matrix to the transmitter, we can pre-rotate the signal before transmission; however, this requires a feedback channel.
Here we present our method to detect the fourth dimension. If we examine (1) more carefully, we see that upon more examination we can extract $E_x[n]E_y^*[n-1]$. If we denote the phase difference of $E_x[n]$ and $E_y[n]$ by $\theta[n]$ and the phase difference of $E_x[n]$ and $E_y[n-1]$ by $\eta[n]$, by using Fig. 1(c) we can rewrite (1) as

$$F_x[n]F_y^*[n-1] = \ell^T \begin{bmatrix} |E_y[n]| \cdot |E_y[n-1]| \cdot \exp(i\eta[n]) \\ |E_y[n]| \cdot |E_y[n-1]| \cdot \exp(i(\theta[n] + \eta[n] + \theta[n-1])) \\ |E_y[n]| \cdot |E_y[n-1]| \cdot \exp(i(\eta[n] + \theta[n] + \theta[n-1])) \end{bmatrix} + \hat{z}_5,$$

(2)

All the terms at time index $n - 1$ are decoded and known. In addition, as explained in Section 2, we have detected the first three dimensions and as a result $|E_x[n]|, |E_y[n]|$ and $\theta[n]$ are also known. So the only unknown (besides noise) of the right-hand side of (2) is $\eta[n]$, which is the fourth dimension. By rearranging the terms, it can be shown that

$$\exp(i\eta[n]) = \frac{F_x[n]F_y^*[n-1] - \hat{z}_5}{\ell^T v}, \quad v = \begin{bmatrix} |E_{x,t}[n]| \cdot |E_{y,t}[n-1]| \\ |E_{x,t}[n]| \cdot |E_{x,t}[n-1]| \cdot \exp(i(\theta[n] + \eta[n] + \theta[n-1])) \\ |E_{x,t}[n]| \cdot |E_{y,t}[n-1]| \cdot \exp(i(\theta[n] - 1)) \end{bmatrix}$$

where $v$ is a known vector.

In summary, we perform successive detection. First we decide on $|E_x[n]|, |E_y[n]|$ and $\theta[n]$. Then we proceed to decide on $\eta[n]$ based on $w_5, w_6$ and the decoded three dimensions by approximately maximizing $f(w_5, w_6, |E_x[n]|, |E_y[n]|, \theta[n] \mid \eta[n])$ over all valid values for $\eta[n]$, which is equivalent to maximizing $f(w_5, w_6, \eta[n], |E_x[n]|, |E_y[n]|, \theta[n])$. Again, after estimating the channel matrix and decoding $E_x[n], E_y[n]$ and $\theta[n]$, there is a one-to-one correspondence between $\eta[n]$ and $\arg(K_x[n]K_y^*[n-1])$. So if $\arg(K_x[n]K_y^*[n-1]) = \alpha'$, then the covariance matrix and mean vector for $w_5$ and $w_6$ are given, respectively, as $(8\sigma^4 + 8\sigma^4(K_x[n])^2 + |K_y[n-1]|^2))I_{2 \times 2}$ and $[2|K_x[n]| ||K_y[n-1]| \cos(\alpha'), 2|K_x[n]||K_y[n-1]| \sin(\alpha')]^T$, where $I_{2 \times 2}$ is the identity matrix.

We use an $N_r$-ring/$N_p$-ary PSK modulation, with equally spaced squared radii, as in [2]. The fiber loss is compensated by a preamplifier right before the PBS at the receiver (see Fig. 1(a)). The amplifier is the main source of noise. The noisy waveform then enters the receiver, and at the output we perform successive detection, using a Gaussian approximation of the conditional PDF as previously described. The resulting symbol error rate is shown in Fig. 2(a).

In Fig. 2(b) the average achievable rate of the fourth dimension for several constellations is depicted. To obtain this figure, we estimate mutual information by a discretization of the complex plane. Fig. 2(b) shows that the achievable rate of the fourth dimension can be substantial. For example, in the 2-ring/4-ary PSK
scheme at an OSNR near 20 dB, the achievable rate of the first three dimensions is < 4 bits/channel-use, while the fourth dimension nearly provides an additional 2 bits/channel-use.

Figure 2: (a): SER of different dimensions for 2-rings/4-ary PSK constellation by approximating the likelihood function as a Gaussian function. (b): Achievable rate of the fourth dimension for \((N_r, N_p)\) constellations \((N_r\text{-ring}/N_p\text{-ary PSK})\).

References

[1] Di Che, An Li, Xi Chen, Qian Hu, Yifei Wang, and William Shieh, "Stokes vector direct detection for short-reach optical communication," Opt. Lett. 39, 3110-3113 (2014).

[2] M. Morsy-Osman, M. Chagnon and D. V. Plant, “Four-Dimensional Modulation and Stokes Direct Detection of Polarization Division Multiplexed Intensities, Inter Polarization Phase and Inter Polarization Differential Phase,” J. of Lightwave Technology, 34, 1585-1592, (Apr. 1, 2016).