$B_{s,d}^0 \to l^+l^-$ in the minimal gauged $(B - L)$ supersymmetry

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Abstract

Complete expressions of effective Hamilton for $b \to s l^+l^- (l = \mu, \tau)$ are derived in the framework of minimal supersymmetric extension of the standard model with local $B - L$ gauge symmetry. With some assumptions on parameters of the model, a numerical analysis of the supersymmetric contributions to the branching ratios of $B_s^0 \to l^+l^- (l = \mu, \tau)$ is presented.

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I. INTRODUCTION

The study on rare B decays can detect new physics beyond the standard model (SM) since the theoretical evaluations on relevant physical quantities are not seriously affected by the uncertainties due to unperturbative QCD effects. The LHCb collaboration reports the observed branching ratios of $B^0_{s,d} \rightarrow \mu^+\mu^-$ as

$$BR(B^0_s \rightarrow \mu^+\mu^-)_{\text{EXP}} = (3.20^{+1.5}_{-1.2}) \times 10^{-9},$$

$$BR(B^0_d \rightarrow \mu^+\mu^-)_{\text{EXP}} < 9.4 \times 10^{-10}. \tag{1}$$

Now, Particle Data Group (PDG) gives the observed averages as

$$BR(B^0_s \rightarrow \mu^+\mu^-)_{\text{EXP}} = (3.1 \pm 0.7) \times 10^{-9},$$

$$BR(B^0_d \rightarrow \mu^+\mu^-)_{\text{EXP}} < 6.3 \times 10^{-10}, \tag{2}$$

where the experimental observable on branching ratio of $B^0_s \rightarrow \mu^+\mu^-$ is nicely consistent with the correspondingly SM prediction

$$BR(B^0_s \rightarrow \mu^+\mu^-)_{\text{SM}} = (3.23 \pm 0.27) \times 10^{-9}, \tag{3}$$

and the experimental precision on $B^0_d \rightarrow \mu^+\mu^-$ is already of the same order as the SM evaluation

$$BR(B^0_d \rightarrow \mu^+\mu^-)_{\text{SM}} = (1.07 \pm 0.10) \times 10^{-10}. \tag{4}$$

The precise measurements on the rare B-decay processes set more strict constraints on the new physics beyond SM. The main purpose of investigation of B-decays is to search for traces of new physics and determine its parameter space.

In all the extensions of SM, the supersymmetry is considered as one of the most plausible candidates. In the general supersymmetric extension of SM, new sources of flavor violation may appear in those soft breaking terms. If we believe that the SM is only an effective theory and the supersymmetry is more fundamental, study on rare B-processes will definitely enrich our knowledge in this field. But before we can really pin down any new physics effects, we need to carry out a thorough exploration in this field, not only in SM, but also in
supersymmetric models. Actually the analyses of constraints on parameters in the minimal supersymmetric extensions of the SM (MSSM) are extensively discussed in literature. The calculation of the rate of inclusive decay $B \rightarrow X_s\gamma$ is presented by authors of Refs. [5–7] in the two-Higgs doublet model (2HDM). The supersymmetric effect on $B \rightarrow X_s\gamma$ is discussed in Refs. [8–12] and the next-to-leading order (NLO) QCD corrections are given in Refs. [13]. The transition $b \rightarrow s\gamma\gamma$ in the supersymmetric extension of the standard model is computed in Ref. [14]. The hadronic B decays [15] and CP-violation in those processes [16] have been discussed also. The authors of Ref. [17] have discussed possibility of observing supersymmetric effects in rare decays $B \rightarrow X_s\gamma$ and $B \rightarrow X_s e^+ e^-$ at the B-factory. Studies on decays $B \rightarrow (K, K^*) \mu^+ \mu^-$ in the SM and supersymmetric model have been carried out in Refs. [18, 19]. The supersymmetric effects on these processes are very interesting and studies on them may shed some light on the general characteristics of the supersymmetric model. A relevant review can be found in Refs. [20, 21]. For oscillations of $B_0 - \bar{B}_0 (K_0 - \bar{K}_0)$, calculations have been done in the SM and 2HDM. As for the supersymmetric extension of SM, the calculation involving the gluino contributions should be re-studied carefully for gluino has a nonzero mass. At the NLO approximation, the QCD corrections to the $B_0 - \bar{B}_0$ mixing in the supersymmetry model have been discussed also. The authors of Refs. [22, 23] applied the mass-insertion method to estimate QCD corrections to the $B_0 - \bar{B}_0$ mixing. The calculations including the gluon-mediated QCD were given in Ref. [24], and later we have re-derived the formulation by including the contribution of gluinos [25].

The discovery of Higgs on the Large Hadron Collider (LHC) implies that we finish the spectrum of particles predicted by the standard model (SM) now [26, 27]. One main target of particle physics is testing the SM precisely and searching for the new physics (NP) beyond it. Experimentally the LHCb experiment can measure the quantities of exclusive hadronic, semi-leptonic, and leptonic $B$ and $B_s$ decays at a high sensitivity [28]. In addition the measurements on inclusive rare $B$ decay and decays with neutrino final states will be performed also in two next generation B factories in near future [29, 30].

In supersymmetry, R-parity is defined through $R = (-1)^{3(B-L)+2S}$, where $B$, $L$ and $S$ are baryon number, lepton number and spin respectively for a concerned field [31, 32]. In the MSSM with local $U(1)_{B-L}$ symmetry, R-parity is spontaneously broken when left- and right-
handed sneutrinos acquire nonzero vacuum expectation values (VEVs) \cite{33-36}. Meanwhile, the nonzero VEVs of left- and right-handed sneutrinos induce the mixing between neutralinos (charginos) and neutrinos (charged leptons). Furthermore, the MSSM with local $U(1)_{B-L}$ symmetry naturally predicates two sterile neutrinos \cite{37-39}, which are favored by the Big-bang nucleosynthesis (BBN) in cosmology \cite{40}. In other words, there are exotic sources to mediate flavor changing neutral current processes (FCNC) in this model.

Here we investigate the FCNC processes with a $B_{s,d}^0 \rightarrow l^+ l^-$ ($l = \mu, \tau$) transition in the MSSM with local $U(1)_{B-L}$ symmetry, our presentation is organized as follows. In section II we briefly summarize the main ingredients of the MSSM with local $U(1)_{B-L}$ symmetry, then present effective Hamilton for $b \rightarrow s l^+ l^-$ in section III and the decay widths at hadronic scale in section IV respectively. The numerical analyses are given in section V and our conclusions are summarized in section VI.

### II. THE MSSM WITH LOCAL $U(1)_{B-L}$ SYMMETRY

When $U(1)_{B-L}$ is a local gauge symmetry, one can enlarge the local gauge group of the SM to $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_{(B-L)}$. In the model proposed in Refs. \cite{33-36}, the exotic superfields are three generation right-handed neutrinos $\hat{N}_i^c \sim (1, 1, 0, 1)$. Meanwhile, quantum numbers of the matter chiral superfields for quarks and leptons are given by

$$
\hat{Q}_i = \begin{pmatrix} \hat{U}_i \\ \hat{D}_i \end{pmatrix} \sim (3, 2, \frac{1}{3}, \frac{1}{3}) , \quad \hat{L}_i = \begin{pmatrix} \hat{\nu}_i \\ \hat{E}_i \end{pmatrix} \sim (1, 2, -1, -1) ,
$$

$$
\hat{U}_i^c \sim (3, 1, -\frac{4}{3}, -\frac{1}{3}) , \quad \hat{D}_i^c \sim (3, 1, \frac{2}{3}, -\frac{1}{3}) , \quad \hat{E}_i^c \sim (1, 1, 2, 1) ,
$$

with $I = 1, 2, 3$ denoting the index of generation. In addition, the quantum numbers of two Higgs doublets are assigned as

$$
\hat{H}_u = \begin{pmatrix} \hat{H}_u^+ \\ \hat{H}_u^0 \end{pmatrix} \sim (1, 2, 1, 0) , \quad \hat{H}_d = \begin{pmatrix} \hat{H}_d^0 \\ \hat{H}_d^- \end{pmatrix} \sim (1, 2, -1, 0) .
$$

The superpotential of the MSSM with local $U(1)_{B-L}$ symmetry is written as

$$
\mathcal{W} = \mathcal{W}_{MSSM} + \mathcal{W}_{(B-L)}^{(1)} .
$$
Here $\mathcal{W}_{\text{MSSM}}$ is superpotential of the MSSM, and

$$\mathcal{W}^{(1)}_{(B-L)} = \left(Y_N\right)_{IJ} \hat{H}_u^T i\sigma_2 \hat{L}_I \tilde{N}_J^c. \quad (8)$$

Correspondingly, the soft breaking terms for the MSSM with local $U(1)_{B-L}$ symmetry are generally given as

$$\mathcal{L}_{\text{soft}} = \mathcal{L}_{\text{soft}}^{\text{MSSM}} + \mathcal{L}^{(1)}_{\text{soft}}, \quad (9)$$

Here $\mathcal{L}_{\text{soft}}^{\text{MSSM}}$ is soft breaking terms of the MSSM, and

$$\mathcal{L}^{(1)}_{\text{soft}} = -(m_{\tilde{N}_c}^2)_{IJ} \tilde{N}_I^c \tilde{N}_J^c - (m_{BL} \lambda_{BL} \lambda_{BL} + h.c.) + \left\{ \left(A_N\right)_{IJ} H_u^T i\sigma_2 \hat{L}_I \tilde{N}_J^c + h.c. \right\}, \quad (10)$$

with $\lambda_{BL}$ denoting the gaugino of $U(1)_{B-L}$. After the $SU(2)_L$ doublets $H_u$, $H_d$, $\tilde{L}_I$ and $SU(2)_L$ singlets $\tilde{N}_I^c$ acquire the nonzero VEVs,

$$H_u = \begin{pmatrix} H_u^+ \\ \frac{1}{\sqrt{2}} \left(v_u + H_u^0 + iP_u\right) \end{pmatrix},$$

$$H_d = \begin{pmatrix} H_d^- \\ \frac{1}{\sqrt{2}} \left(v_d + H_d^0 + iP_d\right) \end{pmatrix},$$

$$\tilde{L}_I = \begin{pmatrix} \frac{1}{\sqrt{2}} \left(v_{L_I} + \bar{\nu}_{L_I} + iP_{L_I}\right) \\ \tilde{L}_I^- \end{pmatrix},$$

$$\tilde{N}_I^c = \frac{1}{\sqrt{2}} \left(v_{N_I} + \bar{\nu}_{R_I} + iP_{N_I}\right), \quad (11)$$

the R-parity is broken spontaneously, and the local gauge symmetry $SU(2)_L \otimes U(1)_Y \otimes U(1)_{(B-L)}$ is broken down to the electromagnetic symmetry $U(1)_e$, and the neutral and charged gauge bosons acquire the nonzero masses as

$$m_Z^2 = \frac{1}{4} (g_1^2 + g_2^2) v_{EW}^2,$$

$$m_W^2 = \frac{1}{4} g_2^2 v_{EW}^2,$$

$$m_{ZBL}^2 = g_{BL}^2 \left(v_N^2 + v_{EW}^2 - v_{SM}^2\right). \quad (12)$$

Where $v_{SM}^2 = v_u^2 + v_d^2$, $v_{EW}^2 = v_u^2 + v_d^2 + \sum_{\alpha=1}^3 v_{1\alpha}^2$, $v_N^2 = \sum_{\alpha=1}^3 v_{N\alpha}^2$, and $g_2$, $g_1$, $g_{BL}$ denote the gauge couplings of $SU(2)_L$, $U(1)_Y$ and $U(1)_{(B-L)}$, respectively.
To satisfy present electroweak precision observations, we assume the mass of neutral $U(1)_{(B-L)}$ gauge boson $m_{Z_{BL}} > 1$ TeV which implies $v_{\nu_{\mu}} > 1$ TeV when $g_{BL} < 1$, then we derive $\max((Y_N)_{ij}) \lesssim 10^{-6}$ and $\max(v_{L_L}) \lesssim 10^{-3}$ GeV to explain experimental data on neutrino oscillation. Considering the minimization conditions at one-loop level, we formulate the $3 \times 3$ mass-squared matrix for right-handed sneutrinos as

$$m^2_{\tilde{N}} \simeq \begin{pmatrix}
\frac{\Lambda^2_{N_1}}{v_{N_1}} & \frac{\Lambda^2_{N_2}}{v_{N_2}} & \frac{-\nu_{N_1} \Lambda^2_{N_1}}{v_{N_1} v_{N_2}} \\
\frac{\nu_{N_1} \Lambda^2_{N_1}}{v_{N_1} v_{N_2}} & \frac{\Lambda^2_{N_2}}{v_{N_2}} & \frac{-\nu_{N_2} \Lambda^2_{N_2}}{v_{N_1} v_{N_2}} \\
\frac{-\nu_{N_1} \Lambda^2_{N_1}}{v_{N_1} v_{N_2}} & \frac{-\nu_{N_2} \Lambda^2_{N_2}}{v_{N_1} v_{N_2}} & \Lambda^2_{BL}
\end{pmatrix}$$

with $\Lambda^2_{BL} = m^2_{Z_{BL}}/2 + \Delta T_{\tilde{N}}$. Where $\Delta T_{\tilde{N}}$ denotes one-loop radiative corrections to the right-handed sneutrinos from top, bottom, tau and their supersymmetric partners.

### III. EFFECTIVE HAMILTON FOR $b \rightarrow s l^+ l^{-} (l = \mu, \tau)$

The transition $b \rightarrow s$ is attributed to the effective Hamilton at hadronic scale

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V^*_{ts} \left[ C_1 \mathcal{O}_1^c + C_2 \mathcal{O}_2^c + \sum_{i=3}^{6} C_i \mathcal{O}_i + \sum_{i=7}^{10} (C_i \mathcal{O}_i + C'_i \mathcal{O}'_i) \right] + \sum_{i=S,P} \left( C_i \mathcal{O}_i + C'_i \mathcal{O}'_i \right),$$

where $\mathcal{O}_i, (i = 1, 2, \cdots, 10, S, P)$ and $\mathcal{O}'_i, (i = 7, 8, \cdots, 10, S, P)$ are defined as

- $\mathcal{O}_1 = (\bar{s}_L \gamma_{\mu} T^a u_L)(\bar{u}_L \gamma^{\mu} T^a b_L)$, $\mathcal{O}_2 = (\bar{s}_L \gamma_{\mu} u_L)(\bar{u}_L \gamma^{\mu} b_L)$,
- $\mathcal{O}_3 = (\bar{s}_L \gamma_{\mu} b_L) \sum_q (\bar{q} \gamma^{\mu} q)$, $\mathcal{O}_4 = (\bar{s}_L \gamma_{\mu} T^a b_L) \sum_q (\bar{q} \gamma^{\mu} T^a q)$,
- $\mathcal{O}_5 = (\bar{s}_L \gamma_{\nu} \gamma_{\mu} b_L) \sum_q (\bar{q} \gamma^{\nu} \gamma^{\mu} \gamma^{\rho} q)$, $\mathcal{O}_6 = (\bar{s}_L \gamma_{\mu} \gamma_{\nu} T^a b_L) \sum_q (\bar{q} \gamma^{\mu} \gamma^{\nu} \gamma^{\rho} T^a q)$,
- $\mathcal{O}_7 = \frac{e}{g_s^2} m_b \langle \bar{s}_L \gamma_{\mu} b_R \rangle F^{\mu \nu}$, $\mathcal{O}'_7 = \frac{e}{g_s^2} m_b \langle \bar{s}_R \sigma_{\mu \nu} b_L \rangle F^{\mu \nu}$,
- $\mathcal{O}_8 = \frac{1}{g_s} m_b \langle \bar{s}_L \sigma_{\mu \nu} T^a b_R \rangle G^{a, \mu \nu}$, $\mathcal{O}'_8 = \frac{1}{g_s} m_b \langle \bar{s}_R \sigma_{\mu \nu} T^a b_L \rangle G^{a, \mu \nu}$,
- $\mathcal{O}_9 = \frac{e^2}{g_s^2} \langle \bar{s}_L \gamma_{\mu} b_L \rangle \bar{l} \gamma^{\mu} l$, $\mathcal{O}'_9 = \frac{e^2}{g_s^2} \langle \bar{s}_R \gamma_{\mu} b_R \rangle \bar{l} \gamma^{\mu} l$,
- $\mathcal{O}_{10} = \frac{e^2}{g_s^2} \langle \bar{s}_L \gamma_{\mu} b_L \rangle \bar{l} \gamma^{\mu} \gamma_5 l$, $\mathcal{O}'_{10} = \frac{e^2}{g_s^2} \langle \bar{s}_R \gamma_{\mu} b_R \rangle \bar{l} \gamma^{\mu} \gamma_5 l$.
\[ \begin{array}{cccc}
C_{\text{eff}7} & C_{\text{eff}8} & C_{\text{eff}9} - Y(q^2) & C_{\text{eff}10} \\
-0.304 & -0.167 & 4.211 & -4.103
\end{array} \]

| TABLE I: At hadronic scale \( \mu = m_b = 4.8 \text{GeV}, \) SM Wilson coefficients to NNLL accuracy. |
|-----|-----|-----|-----|
| \( O_S = \frac{e^2}{16\pi^2} m_b (\bar{s}_L b_R) \bar{l}l \), \( O'_S = \frac{e^2}{16\pi^2} m_b (\bar{s}_R b_L) \bar{l}l \), |
| \( O_p = \frac{e^2}{16\pi^2} m_b (\bar{s}_L b_R) l\gamma_5 l \), \( O'_p = \frac{e^2}{16\pi^2} m_b (\bar{s}_R b_L) l\gamma_5 l \). |

At the electroweak energy scale \( \mu_{\text{EW}} \), the Wilson coefficients \( C_{i,NP}(\mu_{\text{EW}}) \) from the new physics beyond SM can be found in Ref. [41] and elsewhere.

The Wilson coefficients in Eq. (14) are calculated at the matching scale \( \mu_{\text{EW}} \), then evolved down to hadronic scale \( \mu \sim m_b \) by the renormalization group equations. In order to obtain hadronic matrix elements conveniently, we define effective coefficients [19]

\[ C_{\text{eff}7} = \frac{4\pi}{\alpha_s} C_7 - \frac{1}{3} C_3 - \frac{4}{9} C_4 - \frac{20}{3} C_5 - \frac{80}{9} C_6, \]

\[ C_{\text{eff}8} = \frac{4\pi}{\alpha_s} C_8 + C_3 - \frac{1}{6} C_4 + 20 C_5 - \frac{10}{3} C_6, \]

\[ C_{\text{eff}9} = \frac{4\pi}{\alpha_s} C_9 + Y(q^2), \]

\[ C_{\text{eff}10} = \frac{4\pi}{\alpha_s} C_{10}, \quad C_{\text{eff}7,8,9,10} = \frac{4\pi}{\alpha_s} C_{7,8,9,10}, \]

where the concrete expression for \( Y(q^2) \) can also be found in Ref. [19]. In our numerical analyses, we evaluate the Wilson coefficients from the SM to next-to-next-to-logarithmic (NNLL) accuracy in Table I at hadronic energy scale. On the other hand, the corrections to the Wilson coefficients from new physics are only included to one-loop accuracy:

\[ \bar{C}_{i,NP}(\mu) = \bar{U}(\mu, \mu_0) \bar{C}_{i,NP}(\mu_0), \]

\[ \bar{C}_{i,NP}(\mu) = \bar{U'}(\mu, \mu_0) \bar{C}_{i,NP}(\mu_0) \]

with

\[ \bar{C}_{i,NP}^T = (C_{1,NP}, \ldots, C_{6,NP}, C_{7,NP}, C_{8,NP}^{\text{eff}}, C_{9,NP}^{\text{eff}} - Y_{NP}(q^2), C_{10,NP}^{\text{eff}}), \]

\[ \bar{C}_{i,NP}^{\prime T} = (C_{7,NP}^{\prime \text{eff}}, C_{8,NP}^{\prime \text{eff}}, C_{9,NP}^{\prime \text{eff}}, C_{10,NP}^{\prime \text{eff}}). \]
Correspondingly the evolving matrices are approached as
\[ \hat{U}(\mu, \mu_0) \simeq 1 - \left[ \frac{1}{2\beta_0} \ln \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right] \hat{\gamma}^{(0)T}, \]
\[ \hat{U}'(\mu, \mu_0) \simeq 1 - \left[ \frac{1}{2\beta_0} \ln \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right] \hat{\gamma}'^{(0)T}, \] (19)
where the anomalous dimension matrices can be read from Ref. [42] as
\[
\hat{\gamma}^{(0)} = \begin{pmatrix}
-4 & \frac{8}{3} & 0 & -\frac{2}{9} & 0 & 0 & -\frac{208}{243} & \frac{173}{102} & -\frac{2272}{729} & 0 \\
12 & 0 & 0 & \frac{4}{3} & 0 & 0 & \frac{416}{81} & \frac{70}{27} & \frac{1952}{243} & 0 \\
0 & 0 & 0 & -\frac{52}{3} & 0 & 2 & -\frac{176}{81} & \frac{14}{27} & -\frac{6752}{243} & 0 \\
0 & 0 & -\frac{40}{9} & -\frac{100}{9} & \frac{4}{9} & 5 & -\frac{152}{243} & \frac{587}{27} & -\frac{2192}{729} & 0 \\
0 & 0 & 0 & -\frac{256}{3} & 0 & 20 & -\frac{6272}{81} & \frac{6596}{27} & -\frac{84032}{243} & 0 \\
0 & 0 & -\frac{256}{9} & \frac{56}{9} & \frac{40}{9} & -\frac{2}{3} & \frac{4624}{243} & \frac{4772}{81} & -\frac{37856}{729} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{32}{3} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{32}{9} & \frac{28}{3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},
\]
\[
\hat{\gamma}'^{(0)} = \begin{pmatrix}
\frac{32}{3} & 0 & 0 & 0 \\
-\frac{32}{9} & \frac{28}{3} & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}. \] (20)

In addition, the operators \( O_{9,10}^{(s,p)} \) do not mix with other operators and their Wilson coefficients are given by the corresponding coefficients at matching scale.

IV. THE BRANCHING RATIOS OF \( B_{s,d}^0 \to \bar{\ell} \ell \) AT HADRONIC SCALE

In the effective Hamilton Eq. (14), the rare decays \( B_q^0 \to \bar{\ell} \ell \) (\( l = \mu, \tau \) and \( q = s, d \)) are induced by the operators \( O_{9,10}, O_{s,p}, O'_9, O'_{s,p} \) at hadronic scale. Correspondingly the hadronic matrix elements of axial vector and pseudoscalar currents are parametrized as
\[
\langle 0 | \bar{q} \gamma_\mu \gamma_5 b | B_q^0(p) \rangle = i p^\mu f_{B_q^0},
\]
\[ \langle 0 | q \bar{q} 0 \gamma_5 b | B^0_q (p) \rangle = -i \frac{M^2_{\nu_q^0}}{m_b} f_{\nu_q^0} \, , \]

where \( f_{\nu_q^0} \) denote the decay constants respectively:

\[ f_{\nu_s^0} = (227 \pm 8) \text{ MeV} \, , \quad f_{\nu_d^0} = (190 \pm 8) \text{ MeV} \, , \]

and \( M_{\nu_q^0} \) denote the masses of neutral mesons

\[ M_{\nu_s^0} = 5.36677 \text{ GeV} \, , \quad M_{\nu_d^0} = 5.27958 \text{ GeV} \, . \]

Generally the matrix element \( \mathcal{M} \) is expressed as:

\[ \mathcal{M}_q = i G_F V_{tb} V_{ts}^* \left\{ f_S^q \bar{l}l + f_P^q \bar{l} \gamma_5 l + f_V^q p_\mu \bar{l} \gamma_\mu l + f_A^q p_\mu \bar{l} \gamma_\mu \gamma_5 l \right\} \, , \]

where the form factors \( F^q_S, F^q_P, F^q_V, F^q_A \) of the scalar, pseudoscalar, vector and axial-vector currents are given

\[ F^q_S = \frac{\alpha_{EW}(\mu_b)}{8\pi} \frac{m_b M^2_{\nu_q^0}}{m_b + m_q} f_{\nu_q^0} (C^q_S - C'^q_S) \, , \]
\[ F^q_P = \frac{\alpha_{EW}(\mu_b)}{8\pi} \frac{m_b M^2_{\nu_q^0}}{m_b + m_q} f_{\nu_q^0} (C^q_P - C'^q_P) \, , \]
\[ F^q_V = \frac{\alpha_{EW}(\mu_b)}{8\pi} f_{\nu_q^0} \left[ C^{eff}_{9q} (\mu_b) - C'^{eff}_{9q} (\mu_b) \right] \, , \]
\[ F^q_A = \frac{\alpha_{EW}(\mu_b)}{8\pi} f_{\nu_q^0} \left[ C^{eff}_{10q} (\mu_b) - C'^{eff}_{10q} (\mu_b) \right] . \]

Correspondingly the squared amplitude is

\[ |\mathcal{M}_q|^2 = 16 G_F^2 |V_{tb} V_{ts}^*|^2 M^2_{\nu_q^0} \left\{ |F^q_S|^2 + |F^q_P + 2m_l F^q_A|^2 \right\} . \]

The branching ratio is then given by

\[ BR(B^0_q \to \bar{\ell} \ell) = \frac{\tau_{\nu_q^0}}{16\pi} \frac{|\mathcal{M}_q|^2}{M^2_{\nu_q^0}} \sqrt{1 - \frac{4m^2_l}{M^2_{\nu_q^0}}} \]

with \( \tau_{\nu_s^0} = 1.466(31) \text{ ps} \, , \quad \tau_{\nu_d^0} = 1.519(7) \text{ ps} \) denoting the life time of mesons.

In generic new physics, the branching ratio \( BR(B^0_q \to \bar{\ell} \ell)_{NP} \) is sensitive to the operators \( C^{(l)}_{10q} \) and \( O^{(l)}_{s,p} \):

\[ \frac{BR(B^0_q \to \bar{\ell} \ell)_{NP}}{BR(B^0_q \to \bar{\ell} \ell)_{SM}} = |S|^2 \left( 1 - \frac{4m^2_l}{M^2_{\nu_q^0}} \right) + |P|^2 \]
| Input                  | Input                  |
|-----------------------|-----------------------|
| $m_B = 5.280$ GeV     | $m_{K^*} = 0.896$ GeV |
| $m_{B_s} = 5.367$ GeV | $m_{\mu} = 0.106$ GeV |
| $m_w = 80.40$ GeV     | $m_Z = 91.19$ GeV     |
| $\tau_B = 2.307 \times 10^{12}$ GeV | $f_B = 0.190 \pm 0.004$ |
| $\alpha_s(m_Z) = 0.118 \pm 0.002$ | $\alpha_s(m_Z) = 1/128.9$ |
| $m_c(m_c) = 1.27 \pm 0.11$ GeV | $m_b(m_b) = 4.18 \pm 0.17$ GeV |
| $m_t^\text{pole} = 173.1 \pm 1.3$ GeV | $m_b(m_b) = 4.18 \pm 0.17$ GeV |
| $\lambda_{CKM} = 0.225 \pm 0.001$ | $A_{CKM} = 0.811 \pm 0.022$ |
| $\bar{\rho} = 0.131 \pm 0.026$ | $\bar{\eta} = 0.345 \pm 0.014$ |

TABLE II: Input parameters $^2$ used in the numerical analysis

with

$$S \simeq \frac{M^2_{B^0}}{2m_t} \cdot \frac{C_S - C'_S}{|C_{10,SM}^{\text{eff}}(\mu_b)|},$$

$$P \simeq \frac{M^2_{B^0}}{2m_t} \cdot \frac{C_P - C'_P}{|C_{10,SM}^{\text{eff}}(\mu_b)|} + \frac{C_{10}^{\text{eff}}(\mu_b) - C_{10}^{\text{eff}}(\mu_b)}{|C_{10,SM}^{\text{eff}}(\mu_b)|}. \quad (29)$$

V. NUMERICAL ANALYSES

For the experimental observations in $\bar{B} \to X_s \gamma$ and $B^0_s \to l^+l^-$, the relevant SM inputs are presented in table III. The supersymmetric parameters involved here are soft breaking masses of the 2nd and 3rd generation squarks, $m_{\tilde{Q}_{2,3}}^2$, $m_{\tilde{U}_{2,3}}^2$, $m_{\tilde{D}_{2,3}}^2$, neutralino and chargino masses $m_{\tilde{\chi}_\alpha^0}$, $m_{\tilde{\chi}_\beta^\pm}$, ($\alpha = 1, \cdots, 4$, $\beta = 1, 2$) and their mixing matrices. Additionally the free parameters also include $B - L$ gaugino/right-handed neutrino masses and mixing which are mainly determined from the nonzero VEVs of right-handed sneutrinos, the local $B - L$ gauge coupling $g_{BL}$ and the soft gaugino mass $m_{BL}$. The flavor conservation mix-
ing between left- and right-handed squarks \((\delta_{LR})_{33} = m_{\tilde{t}_3}^2/\Lambda_{NP}^2\), \((\delta_{LR})_{33} = m_{\tilde{b}_3}^2/\Lambda_{NP}^2\) are chosen to give the lightest Higgs mass in the range \(124–126\) GeV, where \(\Lambda_{NP}\) represents the energy scale of supersymmetry and the concrete expressions of \(m_{\tilde{t}_3}^2\), \(m_{\tilde{b}_3}^2\) are presented in appendix A. The \(b \to s\) transitions are mediated by those flavor changing insertions

\[
(\delta_{LL})_{23} = \frac{\delta_{LL}}{\Lambda_{NP}^2}, \quad (\delta_{LR})_{23} = \frac{\delta_{LR}}{\Lambda_{NP}^2}, \quad (\delta_{RR})_{23} = \frac{\delta_{RR}}{\Lambda_{NP}^2},
\]

which are originated from flavour-violating scalar mass terms and trilinear scalar couplings in soft breaking terms.

To coincide with updated experimental data on supersymmetric particle searching from LHC etc. \cite{2}, we choose \(m_{\tilde{Q}_2} = m_{\tilde{Q}_3} = m_{\tilde{u}_2} = m_{\tilde{u}_3} = 2\) TeV, \(m_{\tilde{b}_1} = 1\) TeV, \(\Lambda_{NP} = A_t = A_b = 1\) TeV. For those parameters in Higgsino and gaugino sectors of the MSSM, we set \(m_1 = 200\) GeV, \(m_2 = 400\) GeV, \(m_3 = 2\) TeV, \(\mu = 600\) GeV. For the gauge coupling of local \(B - L\) symmetry and relevant gaugino mass, we take \(g_{BL} = 0.7\), \(m_{BL} = 0.5\) TeV, \(v_N = (0, 0, 3)\) TeV here. Similar to scenarios of the MSSM, the \(b \to s\gamma\) transition can be evoked by the insertions \((\delta_{LL})_{23}, (\delta_{LR})_{23}, (\delta_{RR})_{23}\) through one loop diagrams composed by virtual charginos and up-type scalar quarks, which are extensively discussed in literature before. In order to simplify our analyses here, we choose \((\delta_{LL})_{23} = (\delta_{LR})_{23} = (\delta_{RR})_{23} = 0\) unless a particular specification being made. Actually the numerical results of \(BR(B_s^0 \to l^+l^-), (l = \mu, \tau)\) depend on the insertion \((\delta_{LR})_{23}\) and CP phase \(\theta_{BL}\) mildly with this choice on the parameter space. Because of the reason above, we set \((\delta_{LR})_{23} = \theta_{BL} = 0\) and mass of the lightest CP-odd Higgs as \(m_{a_0} = 1\) TeV. With those assumptions on parameters of the model considered here, one obtains theoretical prediction on the lightest CP-even Higgs mass around the value \(125\) GeV as \(\tan \beta = 40\) partnering with \(A_t = 0.5\) TeV, \(\tan \beta = 20\) partnering with \(A_t = 0.6\) TeV, or \(\tan \beta = 10\) partnering with \(A_t = 1\) TeV respectively, which coincides with the experimental data from LHC.

It is well known that the experimental observation on \(BR(\bar{B} \to X_s\gamma)\) constrains the relevant parameters strongly, the average experimental data on the branching ratio of the inclusive \(\bar{B} \to X_s\gamma\) reads \cite{2}

\[
BR(\bar{B} \to X_s\gamma)_{EXP} = (3.40 \pm 0.21) \times 10^{-4},
\]

(30)
FIG. 1: Taking $\theta_s = 0$, $(\delta^{RR})_{23} = 0$, we plot $BR(\bar{B} \to X_s \gamma)$, $R(\mu)$ and $R(\tau)$ varying with the insertion $(\delta^{LL})_{23}$ in (a), (b) and (c), respectively. Where the solid line represents $\tan\beta = 40$, $A_t = 0.5$ TeV, the dashed line represents $\tan\beta = 20$, $A_t = 0.6$ TeV, and the dotted line represents $\tan\beta = 10$, $A_t = 1$ TeV. In addition, the gray regions represent the experimental results within 3$\sigma$ permission.

which is consistent with the correspondingly SM prediction at NNLO order \[44, 45\]

$$BR(\bar{B} \to X_s \gamma)_{SM} = (3.36 \pm 0.23) \times 10^{-4}.$$ (31)

Through scanning the parameter space, we find that theoretical predictions on the branching ratio of $\bar{B} \to X_s \gamma$ depends on the insertions $(\delta^{LL})_{23}, (\delta^{RR})_{23}$ weakly in the model considered here.

Under our assumptions on the relevant parameter space, the supersymmetric correc-
FIG. 2: Taking $\theta_9 = 0$, $(\delta_{D}^{LR})_{23} = 0$, we plot $BR(\bar{B} \rightarrow X_s \gamma)$, $R(\mu)$ and $R(\tau)$ varying with the insertion $(\delta_{D}^{RR})_{23}$ in (a), (b) and (c), respectively. Where the solid line represents $\tan \beta = 40$, $A_t = 0.5$ TeV, the dashed line represents $\tan \beta = 20$, $A_t = 0.6$ TeV, and the dotted line represents $\tan \beta = 10$, $A_t = 1$ TeV. In addition, the gray regions represent the experimental results within $3\sigma$ permission.

Tions to the $b \rightarrow s l^+l^-$ transition are mainly originated from the insertions $(\delta_{D}^{LL})_{23}$, $(\delta_{D}^{RR})_{23}$ through one loop diagrams composed by $U(1)_{B-L}$ gaugino/gluino and down type squarks of the 2nd and 3rd generations. Assuming CP phase $\theta_9 = 0$, $(\delta_{D}^{RR})_{23} = 0$, we plot $BR(\bar{B} \rightarrow X_s \gamma)$, $R(\mu) = BR(B_s^0 \rightarrow \mu^- \mu^+)_{NP}/BR(B_s^0 \rightarrow \mu^- \mu^+)_{SM}$ and $R(\tau) = BR(B_s^0 \rightarrow \tau^- \tau^+)_{NP}/BR(B_s^0 \rightarrow \tau^- \tau^+)_{SM}$ varying with $(\delta_{D}^{LL})_{23}$ in Fig. 2 where the gray regions denote the experimental data within 3 standard deviations. The new physics corrections to $BR(\bar{B} \rightarrow X_s \gamma)$ mainly originate from the insertion $(\delta_{D}^{LR})_{23}$ through Feynman diagrams
FIG. 3: Taking $(\delta_{LL}^{UL})_{23} = (\delta_{LR}^{UL})_{23} = (\delta_{RR}^{UL})_{23} = 0.02$ and $(\delta_{LL}^{UR})_{23} = (\delta_{RR}^{UR})_{23} = -0.04$, we plot $BR(\bar{B} \rightarrow X \gamma)$, $R(\mu)$ and $R(\tau)$ varying with the CP phase $\theta_g$ in (a), (b) and (c), respectively. Where the solid line represents $\tan \beta = 40, \ A_t = 0.5 \ TeV$, the dashed line represents $\tan \beta = 20, \ A_t = 0.6 \ TeV$, and the dotted line represents $\tan \beta = 10, \ A_t = 1 \ TeV$. In addition, the gray regions represent the experimental results within $3\sigma$ permission.

composed by virtual chargino-stop particles, and theoretical evaluations for $BR(\bar{B} \rightarrow X \gamma)$ depends on $(\delta_{LL}^{UL})_{23}$ mildly. Meanwhile the experimental data on $BR(\bar{B}_s^0 \rightarrow \mu^- \mu^+)$ favor the insertion $(\delta_{LL}^{UL})_{23}$ lying in the range $-0.1 \leq (\delta_{LL}^{UL})_{23} \leq 0.1$. In limit of large $\tan \beta$, dominating corrections to the Wilson coefficients $C_{S,P}^{(t)}$ from new physics are proportional to the mass of lepton in final states $m_l$. Nevertheless the dependence on $m_l$ is compensated by $m_l$ from denominator in the first terms of $S, P$ respectively in Eq. (29). Because of the reason, the theoretical evaluations on $R(\tau)$ are not differ from that on $R(\mu)$ obviously.
In Fig. 2, we plot $BR(\bar{B} \to X_s\gamma)$, $R(\mu)$ and $R(\tau)$ varying with $(\delta^R_D)_{23}$, as $\theta_g = 0$, $(\delta^L_L)_{23} = 0$. The theoretical evaluations of $BR(\bar{B} \to X_s\gamma)$ depends on $(\delta^R_R)_{23}$ mildly, too. Meanwhile the experimental data on $BR(B^0_s \to \mu^-\mu^+)$ favor the insertion $(\delta^R_R)_{23}$ lying in the ranges $-0.1 \leq (\delta^R_R)_{23} \leq 0.05$ as $\tan \beta = 40$, $A_t = 0.5$ TeV, $-0.25 \leq (\delta^R_R)_{23} \leq 0.1$ as $\tan \beta = 20$, $A_t = 0.6$ TeV, and $-0.6 \leq (\delta^R_R)_{23} \leq 0.2$ as $\tan \beta = 10$, $A_t = 1$ TeV, respectively. Because of the reason mentioned above, the theoretical evaluations on $R(\tau)$ are not differ from that on $R(\mu)$ obviously.

Taking $(\delta^L_L)_{23} = (\delta^R_R)_{23} = (\delta^L_L)_{23} = (\delta^R_R)_{23} = 0.02$ and $(\delta^L_R)_{23} = (\delta^R_R)_{23} = -0.04$, we plot $BR(\bar{B} \to X_s\gamma)$, $R(\mu)$ and $R(\tau)$ varying with the CP phase $\theta_g$ in Fig. 3. In Fig. 3(a,b) the gray regions represents the experimental data on $BR(\bar{B} \to X_s\gamma)$ and $R(\mu)$ within 3 standard deviations, respectively. Adopting our assumptions on relevant parameter space, one finds that those theoretical evaluations on $R(\mu)$ and $R(\tau)$ depend on the CP phase $\theta_g$ acutely as $\tan \beta = 40$. Along with decreasing of $\tan \beta$, those numerical evaluations on $R(\mu)$ and $R(\tau)$ vary with the CP phase $\theta_g$ mildly.

VI. SUMMARY

Considering the constraint from the observed Higgs signal at the LHC, we study the supersymmetric corrections to the branching ratios $BR(\bar{B} \to X_s\gamma)$, $BR(B^0_s \to l^+l^-)$ ($l = \mu, \tau$) in the MSSM with local $U(1)_{B-L}$ symmetry with nonuniversal soft breaking terms. Under our assumptions on parameters of the considered model, the numerical analyses indicate that the insertions $(\delta^L_L)_{23}$, $(\delta^R_R)_{23}$ affects the theoretical predictions on $BR(B^0_q \to l^+l^-)$ ($l = \mu, \tau$) strongly when the numerical evaluations of $BR(\bar{B} \to X_s\gamma)$ are coincide with corresponding experimental observations. In addition, the CP phase $\theta_g$ also affects the numerical results acutely when the neutral gauginos $m_\tilde{g} \sim m_{BL} \geq 1$ TeV and the squarks acquire the masses around several TeVs in large $\tan \beta$ scenarios.
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Appendix A: The mass squared matrices for squarks

With the minimal flavor violation assumption, the $2 \times 2$ mass squared matrix for scalar tops is given as

$$
Z_t^† \begin{pmatrix}
m^2_{\tilde{t}_L} & m^2_{\tilde{t}_X} \\
m^2_{\tilde{t}_X} & m^2_{\tilde{t}_R}
\end{pmatrix} Z_t = \text{diag}(m^2_{\tilde{t}_1}, m^2_{\tilde{t}_2}),
$$

(A1)

with

$$
m^2_{\tilde{t}_L} = \frac{(g_1^2 + g_2^2)v_{\text{EW}}^2}{24} \left(1 - 2 \cos^2 \beta \right) \left(1 - 4c_w^2 \right) + \frac{g_2^2}{6} \left(v_N^2 - v_{\text{EW}}^2 + v_{\text{SM}}^2 \right) + m_t^2 + m_{Q_3}^2,
$$

$$
m^2_{\tilde{t}_R} = -\frac{g_1^2 v_{\text{EW}}}{6} \left(1 - 2 \cos^2 \beta \right) - \frac{g_2^2}{6} \left(v_N^2 - v_{\text{EW}}^2 + v_{\text{SM}}^2 \right) + m_t^2 + m_{\bar{U}_3}^2,
$$

$$
m^2_{\tilde{t}_X} = -\frac{v_d}{\sqrt{2}} A_t Y_t + \frac{\mu Y_t}{\sqrt{2}} Y_t.
$$

(A2)
Here $Y_t$, $A_t$ denote Yukawa coupling and trilinear soft-breaking parameters in top quark sector, respectively. In a similar way, the mass-squared matrix for scalar bottoms is

$$Z_b^\dagger \begin{pmatrix} m_{b_L}^2 & m_{b_X}^2 \\ m_{b_X}^2 & m_{b_R}^2 \end{pmatrix} Z_b = \text{diag}(m_{b_1}^2, m_{b_2}^2), \quad (A3)$$

with

$$m_{b_L}^2 = \frac{(g_1^2 + g_2^2)v_{\text{EW}}^2}{24} \left(1 - 2\cos^2\beta \right) \left(1 + 2c_W^2 \right) + \frac{g_{BL}^2}{6} \left(v_N^2 - v_{\text{EW}}^2 + v_{\text{SM}}^2 \right) + m_b^2 + m_{\tilde{u}_3}^2,$$

$$m_{b_R}^2 = \frac{g_1^2 v_{\text{EW}}^2}{12} \left(1 - 2\cos^2\beta \right) - \frac{g_{BL}^2}{6} \left(v_N^2 - v_{\text{EW}}^2 + v_{\text{SM}}^2 \right) + m_b^2 + m_{\tilde{d}_3}^2,$$

$$m_{b_X}^2 = \frac{v_d}{\sqrt{2}} A_b Y_b - \frac{\mu\nu_u}{\sqrt{2}} Y_b, \quad (A4)$$

Here $Y_b$, $A_b$ denote Yukawa couplings and trilinear soft-breaking parameters in b quark sector, respectively.

[1] R. Aaij et al. (LHCb collaboration), Phys. Rev. Lett. 110(2013)021801.
[2] K. A. Olive et al. (Particle Data Group), Chin. Phys. C38(2014)090001.
[3] A. J. Buras, J. Girrbach, G. Isidori, Eur. Phys. J. C72(2012)2172.
[4] E. Gabrielli et al., Phys. Lett. B374(1996)80.
[5] M. Ciuchini, G. Degrassi, P. Gambino and G. F. Giudice, Nucl. Phys. B527(1998)21.
[6] P. Ciafaloni, A. Romanino and A. Strumia, Nucl. Phys. B524(1998)361.
[7] F. Borzumati and C. Greub, Phys. Rev. D58(1998)074004.
[8] S. Bertolini, F. Borzumati, A. Masiero and G. Ridolfi, Nucl. Phys. B353(1991)591.
[9] R. Barbieri and G. F. Giudice, Phys. Lett. B309(1993)86.
[10] F. Borzumati and C. Greub, T. Hurth and D. Wyler, Phys. Rev. D62(2000)075005.
[11] M. Causse and J. Orloff, Eur. Phys. J. C23(2002)749.
[12] S. Prelovsek and D. Wyler, Phys. Lett. B500 (2001) 304.
[13] M. Ciuchini, G. Degrassi, P. Gambino and G. F. Giudice, Nucl. Phys. B534 (1998) 3.
[14] S. Bertolini and J. Matias, Phys. Rev. D57 (1998) 4197.
[15] W. N. Cottingham, H. Mehrban and I. B. Whittingham, Phys. Rev. D60 (1999) 114029.
[16] G. Barenboim and M. Raidal, Phys. Lett. B457 (1999) 109.
[17] J. L. Hewett and D. Wells, Phys. Rev. D55 (1997) 5549.
[18] A. Ali, P. Ball, L. T. Handoko and G. Hiller, Phys. Rev. D61 (2000) 074024.
[19] W. Altmannshofer, P. Ball, A. Bharucha, A. J. Buras, D. M. Straub, and M. Wick, JHEP 0901 (2009) 019, arXiv:0811.1214.
[20] A. Masiero and L. Silvestrini, Honolulu 1997, B physics and CP violation, p.172, hep-ph/9709244.
[21] A. Masiero and L. Silvestrini, Erice 1997, Highlights of subnuclear physics, p.404, hep-ph/9711401.
[22] M. Ciuchini et al., JHEP 9810 (1998) 008, arXiv:hep-ph/9808328.
[23] R. Contion, I. Scimemi, Eur. Phys. J. C10 (1999) 347.
[24] F. Krauss, G. Soff, Nucl. Phys. B633 (2002) 237.
[25] Tai-Fu Feng, Xue-Qian Li, Wen-Gan Ma and Feng Zhang, Phys. Rev. D63 (2001) 015013.
[26] CMS Collaboration, Phys. Lett. B716 (2012) 30.
[27] ATLAS Collaboration, Phys. Lett. B716 (2012) 1.
[28] B. Adeva et al. (LHCb Collaboration), Roadmap for selected key measurements of LHCb, arXiv:0912.4179.
[29] T. Aushev et al., Physics at Super B Factory, arXiv:1002.5012.
[30] B. O’Leary et al. (SuperB Collaboration), SuperB Progress Reports, arXiv:1008.1541.
[31] R. Barbier et al., Phys. Rep. 420 (2005) 1.
[32] C.-H. Chang, T.-F. Feng, Eur. Phys. J. C12 (2000) 137.
[33] P. Fileviez Perez and S. Spinner, Phys. Lett. B673 (2009) 251.
[34] V. Barger, P. Fileviez Perez, and S. Spinner, Phys. Rev. Lett. 102 (2009) 181802.
[35] P. Fileviez Perez and S. Spinner, Phys. Rev. D80 (2009) 015004.
[36] P. Fileviez Perez and S. Spinner, JHEP 1204 (2012) 118, arXiv:1201.5923.
[37] V. Barger, P. Fileviez Perez and S. Spinner, Phys. Lett. B696(2011)509.
[38] D. K. Ghosh, G. Senjanovic, Y. Zhang, Phys. Lett. B698(2011)420.
[39] C.-H. Chang, T.-F. Feng, Y.-L. Yan, H.-B. Zhang, S.-M. Zhao, Phys. Rev. D90(2014)035013.
[40] J. Hamann, S. Hannestad, G. Raffelt, I. Tamborra, and Y. Y. Y. Wong, Phys. Rev. Lett.105(2010)181301.
[41] T.-F. Feng, Y.-L. Yan, H.-B. Zhang, S.-M. Zhao, Phys. Rev. D92(2015)055024.
[42] P. Gambino, M. Gorbahn and U. Haisch, Nucl. Phys. B673(2003)238.
[43] A. Dedes, J. Rosiek and P. Tanedo, Phys. Rev. D79(2009)055006.
[44] M. Misiak et al., Phys. Rev. Lett.114(2015)221801, arXiv:1503.01789.
[45] M. Czakon et al., arXiv:1503.01791.