Performance study of preloaded cryogenic bearings in liquid hydrogen pump

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Abstract. As a clean and efficient energy, liquid hydrogen has become an important solution for the energy restructuring in the future. The centrifugal liquid hydrogen pump has been applied in the storage, transportation and distribution of liquid hydrogen. As the support of pump rotor, the performance of cryogenic bearings are essential to the dynamic characteristics and design optimization of liquid hydrogen pump. In this paper, an improved quasi-static model for cryogenic bearing in liquid hydrogen pump was introduced, in which the effects of high-speed centrifugal expansion and shrinkage under cryogenic condition on bearing rings were considered. Based on this model, the effects of rotating speed and axial preload on bearing dynamic characteristics were studied. At the same time, the friction within the cryogenic bearing was analyzed and calculated. Finally, the load variation of bearing preload disc spring at cryogenic temperature was calculated and experimentally studied. The results of this paper will provide theoretical suggestions for further engineering design and applications of cryogenic bearings in liquid hydrogen pump.

1. Introduction

Liquid hydrogen pump is an important piece of equipment in the process of liquid hydrogen storage and transportation, in which the cryogenic bearing is the support of rotor-impeller system. Cryogenic bearings are mostly angular contact ball bearings, which can adapt to high speed and radial-axial combined load, and usually need preloading. In order to analyze the performance if cryogenic bearing, a dynamic model considering the effect of cryogenic shrinkage was established.

The quasi-static model proposed by Jones [1] is suitable for analyzing the dynamic characteristics of angular contact ball bearings, as it takes into account the centrifugal force and gyroscopic moment of the bearing ball. For the above reason, the quasi-static model has been applied in many previous studies [2, 3].

When the ball bearing rotates at high speed, the inner ring expands due to the centrifugal effect. At the same time, the low temperature causes the shrinkage of the bearing ring, shaft and bearing seat, which changes the interference amount and thus affects the contact state between the ball and the ring. Preloading can improve stiffness and reduce vibration during the running of bearing, while improper preload may lead to abnormal friction and heating of bearing, resulting in reduced bearing service life.
[2, 4]. In the liquid hydrogen pump involved in this paper, the cryogenic bearing is preloaded by a set of disc springs (also known as Belleville spring).

Therefore, in order to analyze the dynamic characteristics of cryogenic bearing and the influence of preload on bearing, a quasi-static model considering the effect of cryogenic shrinkage on bearing internal radial clearance was established. At the same time, the elastic modulus of the disc spring was tested at low temperature, and the corresponding preload was predicted based on the test results. Combined with the results above, the bearing frictional heat was calculated and the impact of preload on bearing heat generation was evaluated.

2. Quasi-static model

2.1. Cryogenic effects on bearing contact angle

The initial contact state of bearing ball and raceway under ambient and low temperature environment is shown in Figure 2, where $\alpha_0$ is the initial bearing contact angle, and $\alpha_0$ changes to $\alpha_C$ due to the bearing ring shrinkage induced by low temperature. $D_w$ is the diameter of the rolling ball, $d_{io}$ is the diameter of the contact point at inner raceway and outer raceway, $r_{io}$ is the curvature radius of the raceway, $D_b$ and $d_b$ are the outer and inner diameter of the bearing, respectively. The initial contact angle $\alpha_0$ can be expressed as follows:

$$\alpha_0 = \cos^{-1}\left(1 - \frac{P_0^b}{2A}\right)$$

where $P_0^b$ is the initial clearance of the bearing, $A$ is the distance between the curvature centers ($C_i$ and $C_o$) of the inner and outer raceways. The expressions of $P_0^b$ and $A$ are as follows:

$$P_0^b = d_i + d_o - 2D_w$$
$$A = r_i + r_o - D_w$$

Shrinkage due to cryogenic effects

Figure 2. Geometric relationship between rolling ball and raceways.
In the low temperature environment, the shrinkage of the ball, inner and outer ring causes the clearance \( P^0_d \) to change to \( P^C_d \), and the initial contact angle \( \alpha_0 \) also changes to \( \alpha_C \), which can be presented as follows:

\[
P^C_d = P^0_d + \Delta \delta_i + \Delta \delta_o + \delta_i
\]

\[
\alpha_C = \cos^{-1} \left( \frac{1 - \frac{P^C_d}{2A}}{d} \right)
\]

In the above formula, \( \delta_i \) represents the change of clearance due to the temperature drop:

\[
\delta_i = \alpha_i^{CTE} d_i - \alpha_o^{CTE} d_o + 2\alpha_w^{CTE} D_o
\]

where \( \alpha_i^{CTE} \) is the thermal expansion coefficient of the inner and outer ring, and \( \alpha_w^{CTE} \) is the thermal expansion coefficient of the ball.

\[\Delta \delta_o\] refers to the change of clearance caused by the change of the interference after the shrinkage caused by the low temperature.

\[
\Delta \delta_i = \frac{2 \left[ I - \left( \alpha_i^{CTE} d_i - \alpha_o^{CTE} D_o \right) \right] \left( \frac{d}{d_o} \right)}{\left[ \left( \frac{d}{d_o} \right)^2 - 1 \right]}
\]

\[
\Delta \delta_o = \frac{2 \left[ I - \left( \alpha_i^{CTE} d_i - \alpha_o^{CTE} D_o \right) \right] \left( \frac{D}{d_o} \right)}{\left[ \left( \frac{D}{d_o} \right)^2 - 1 \right]}
\]

where \( I_j \) is the initial interference between inner ring and shaft, \( I_o \) is the initial interference between outer ring and bearing seat. \( D_s \) is the outer diameter of the spindle, \( d_h \) is the inner diameter of the bearing seat. \( \alpha_i^{CTE} \) and \( \alpha_o^{CTE} \) are the thermal expansion coefficient of the spindle and bearing seat, respectively. \( E_s \) is the elastic modulus of the bearing ring, \( E_i \) and \( E_h \) are the elastic modulus of the spindle and bearing seat materials, respectively. \( v_s \) refers to the Poisson’s ratio of bearing ring, while \( v_i \) and \( v_h \) represents the Poisson’s ratio of shaft and bearing seat.

2.2. Balance of rolling element force and bearing internal-external load

When the bearing is at static state, the contact angle at inner raceway and outer raceway are equal. As the bearing rotates at high speed, the two contact angles become inconsistent, because of the effect of centrifugal force of the moving ball. The contact relationship between the ball and the raceway is presented in Figure 3, where \( F_a \) and \( F_r \) are the axial and radial load, \( \alpha_i(\omega_0) \) is the contact angle between the ball and inner/outer raceway, \( j \) represents the number of the rolling ball. \( C_i \) and \( C_o \) are the curvature center of the inner/outer raceway after bearing is loaded by \( F_a \) and \( F_r \).
In order to further illustrate the contact deformation between the rolling ball and the raceway, the relative position between the center of the \( j \)th rolling ball and the curvature center of the inner/outer raceway is shown in Figure 4. It is assumed that the position of the outer raceway curvature center \( C_o \) is fixed, while the inner raceway curvature center \( C_i \) moves due to the loading \( C_i' \). The ball center \( O \) also moves after the bearing is loaded \( O' \). The position angle is represented by \( \varphi_j \), and the relative axial and radial displacement of the inner ring to the outer ring are represented by \( \delta_a \) and \( \delta_r \), while \( \theta \) is the angular displacement.

\[ \Delta_{ij} = (f_i - 0.5)D_u + \delta_y - \delta_b \]  
\[ \Delta_{oj} = (f_o - 0.5)D_u + \delta_y - \delta_b \]  
among which, \( \delta_{ij} \) is the contact deformation between the ball and the raceway, \( f_{io} \) is the curvature radius coefficient of inner/outer raceway, \( \delta_b \) is the ball shrinkage at low temperature, which is obtained by the following formula:

\[ \delta_b = \frac{1}{2} \alpha_{CTE} D_u \]  

\( A_x \), \( A_y \) are the axial and radial components of the distance between the curvature center of the inner and outer raceway, they are calculated as follows:

\[ A_y = A \sin \alpha_x + \delta_y + R \theta \sin \varphi \]  
\[ A_x = A \cos \alpha_x + \delta_y \cos \varphi + u_{cent} \]  
where \( R \) is the distance from inner raceway curvature center to the spindle axis, and \( u_{cent} \) represents the inner ring expansion due to the centrifugal effect as bearing rotated at high speed. These values can be obtained by following formulas:

\[ R = \frac{1}{2} d_m + (f_i - 0.5)D_u \cos \alpha_p \]  
\[ u_{cent} = \rho \omega^2 \frac{d_i}{32E_r} [d_i^2 (3 + v_c) + d_i^2 (1 - v_c)] \]  
where \( \rho \) is the material density of bearing inner ring and \( \omega \) is the spindle rotating speed.

The contact angle between the ball and the inner/outer raceway can be expressed as follows:

\[ \sin \alpha_y = \frac{A_y - X_{ij}}{(f_i - 0.5)D_u + \delta_y} \]  
\[ \sin \alpha_y = \frac{X_{oj}}{(f_o - 0.5)D_u + \delta_y} \]
The geometric relationship between the ball and the raceways can be described as a set deformation coordination equations, which can be illustrated by Figure 4:

\[
(A_{ij} - X_{ij})^2 + (A_{ij} - X_{ij})^2 - [(f_{ij} - 0.5)D_w + \delta_{ij}]^2 = 0
\]

(18)

\[
X_{ij}^2 + X_{ij}^2 - [(f_{ij} - 0.5)D_w + \delta_{ij}]^2 = 0
\]

(19)

The relationship between the centrifugal force caused by the ball revolution, the gyroscopic moment caused by the ball rotation and the contact load at the inner/outer raceway is shown in Figure 5, among which \( M_{ij} \) is the gyroscopic moment, \( F_{ij} \) is the centrifugal force and \( Q_{i\in(j)} \) is the contact load between the ball and the inner/outer raceway.

According to Figure 5, each ball of the bearing has a group of local force balance equations as follows:

\[
Q_{ij} \sin \alpha_{ij} - Q_{ij} \sin \alpha_{ij} - \frac{M_{ij}}{D_w} (\lambda_{ij} \cos \alpha_{ij} - \lambda_{ij} \cos \alpha_{ij}) = 0
\]

\[
Q_{ij} \cos \alpha_{ij} - Q_{ij} \cos \alpha_{ij} + \frac{M_{ij}}{D_w} (\lambda_{ij} \cos \alpha_{ij} - \lambda_{ij} \cos \alpha_{ij}) + F_{ij} = 0
\]

(20)

The outer raceway control assumption is adopted for high speed rolling bearing in which \( \lambda_{ij} = 0 \), \( \lambda_{ij} = 2 [5] \). The centrifugal force and gyroscopic moment should be calculated by the following formulas, respectively:

\[
F_{ij} = \frac{1}{2} m \omega_m^2
\]

\[
M_{ij} = J \omega_b \omega_m \sin \varphi_j
\]

(21)

(22)

where \( \omega_m \) is the common angular velocity of the rolling ball, \( \omega_b \) is the angular velocity of the self-rotation of the ball, and \( J \) represents the moment of inertia of the rolling ball.

Assuming that the outer ring of the bearing is fixed, the external load acts on the rolling ball through the inner ring of the bearing, and the global balance equations of the external load, the contact load of the ball and the inertia are as follows:

\[
F_a - \sum_{j=1}^{N} (Q_{ij} \sin \alpha_{ij} - \frac{M_{ij}}{D_w} \lambda_{ij} \cos \alpha_{ij}) = 0
\]

\[
F_a - \sum_{j=1}^{N} (Q_{ij} \cos \alpha_{ij} + \frac{M_{ij}}{D_w} \lambda_{ij} \sin \alpha_{ij}) \cos \varphi_j = 0
\]

(23)

Combined with the above analysis, it can be considered that the quasi-static model of cryogenic bearing consists of two parts (20) and (23), which can be solved by the iteration of Newton-Raphson method.
3. Friction power loss model
In the operation of the liquid hydrogen pump, the frictional heat generation from bearing is one of the main heat sources. According to Palmgren’s theory [6], the frictional torque of cryogenic bearing ($M$) is mostly related to the load distribution inside bearing ($M_l$) and the lubricant viscosity ($M_v$), which can be described as follows:

\[
\begin{align*}
M &= M_l + M_v \\
M_l &= f_P d_m \\
M_v &= \begin{cases} 
10^{-7} f_0 (v_n)^{2/3} d_m^3 & \text{if } v_n \geq 2000 \\
160 \times 10^{-7} f_0 d_m^3 & \text{if } v_n < 2000
\end{cases}
\] (24)
\]

for $M_l$, $P_1$ is dynamical equivalent load, $f_1$ is a coefficient related to bearing type and bearing load, $d_m$ is bearing pitch diameter. For $M_v$, $v$ is the kinematic viscosity of the fluid, $f_0$ is the coefficient related to bearing type, $n$ is bearing rotation speed in r/min. Because the viscosity of cryogenic fluids (LN2, LH2, LHe, etc.) is very low, the torque value is small.

Based on the calculation above, the total frictional heat generation of bearing in Watts is obtained as follows:

\[
H = M \cdot \omega
\] (25)

where $\omega$ represent the angular speed of the bearing inner ring, the relationship between $\omega$ and the rotational speed $n$ is expressed as follows:

\[
\omega = \frac{2 \pi n}{60}
\] (26)

4. Preload of disc spring at low temperature
The cryogenic bearing in this paper is preloaded by disc spring. The disc spring is made of alloy for cryogenic application. For the preload of disc spring, the elastic modulus is the key parameter affected by temperature. In order to calculate the preload of the disc spring at low temperature, the samples of the disc spring were tested at temperatures of LN2 (77 K), LH2 (20 K), LHe (4.2 K) and room temperature (296 K).

The test samples and test platform are shown in Figure 6, and the test result of elastic modulus is shown in Figure 7.

![Figure 6. The test platform and test samples.](image)

According to the reference [7], the preload of disc spring with the above test result is shown in Figure 8. The inner cone height (compressibility) of disc spring is 1.1mm, and when the compressibility changes in the range of 0.1–1mm, the preload at different temperatures first increases and then decreases, from which it can be concluded that there is a maximum preload for disc spring within the compression range. Another point that should be noted is the preloads with the same compression are different as the
temperature variates, and it can be seen that the disc spring produces the max preload at the temperature of LHe, which can be attributed to the rise of the elastic modulus as the temperature drops.

![Figure 8. Variation of preload.](image)

5. Example and calculation results
Combined with the model described above, the performance analysis and frictional heat calculation were carried out. The bearing type involved in this study is 7206C.

At the constant temperature (77 K), the contact angle between the ball and the inner raceway ($\alpha_i$) is significantly different from that between the ball and the outer raceway ($\alpha_o$), as shown in Figure 9. With the increase of the axial force, the difference between $\alpha_i$ and $\alpha_o$ reduces and the two contact angles are getting close to a constant value. At the same time, $\alpha_i$ increases with the rise of speed, while $\alpha_o$ changes along the opposite tendency. Figure 10 shows the contact load at inner raceway ($Q_i$) and outer raceway ($Q_o$) increase linearly with the increase of axial force, while $Q_i$ and $Q_o$ moves in different direction as the speed rises.

In summary, the result shows that the decrease of centrifugal force entails the two contact state to approach to the static state, which indicates the centrifugal force is the key factor for the dynamics of bearing running state.

![Figure 9. Variations of contact angle $\alpha_i$ and $\alpha_o$.](image)

![Figure 10. Variations of contact load $Q_i$ and $Q_o$.](image)

The influence of low temperature shrinkage on the contact state of bearing was considered in the model. Under the condition of constant axial load and rotating speed, the influence of environment temperature on the contact state of ball-raceway is shown in Figure 11. With the decrease of temperature, the contact load between ball-inner raceway and ball-outer raceway gradually increases, which indicates that low temperature has great impact on the contact state of ball-raceway.

Finally, based on the prediction of the preload, the influence of disc spring on the frictional heat generation of the cryogenic bearing was analyzed. In the liquid hydrogen pump, the cryogenic bearing is usually preloaded by several disc springs. In this paper, the frictional heat of the cryogenic bearing preloaded by four disc springs was calculated, as shown in Figure 12. Compared with the room
temperature, it can be found that the heat generation increase at low temperature, and the greater the preload becomes, the more obviously the heat generation grows.

Under a certain preload, the heat generation changes slightly in the temperature range from LN2 to LHe, which indicates the change of physical properties of the cryogenic disc spring described in this paper has no obvious effect on the cryogenic bearing.

Figure 11. Variation of contact load with temperature.

Figure 12. The effect of the preload on bearing frictional heat generation (n=18000r/min).

6. Conclusion
To conclude, based on the consideration above, the dynamic characteristics of cryogenic bearing was modelled, and the performance of bearing was analyzed. The conclusions are as follows:

The impact of preload and rotational speed on contact state is significant, the centrifugal force varying with the rotational speed is the reason for the difference between the inner and outer contact state. Meanwhile, the drop of temperature leads to a significant change of contact state between the ball and raceway, which can be attributed to the decrease of initial contact angle caused by low temperature.

In addition, the elastic modulus of the disc spring material at low temperature was measured by a customized platform. It can be concluded that the compression of the disc spring should be controlled in a proper range when used for cryogenic purposes, and the effect of low temperature on the preload should be considered at the same time.

Through the calculation of frictional heat generation, it was found that the change of preload of disc spring at low temperature will lead to the increase of friction heating of bearing, but the effect is not obvious. Therefore, the influence of low temperature on preload and bearing frictional heat generation can be ignored in the case of small preload, but in the case of high preload, the influence of temperature on bearing frictional heat still needs to be considered.

7. Reference
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