Problem-Based Learning for Critical Thinking Skills in Mathematics

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Abstract. This study aims to find out how to develop a student's critical thinking skills through problem-based learning. Single-case single-side case design was selected as the method of this study and a lecture was involved to carry out the classroom instruction. The data were collected through observations, interviews, and document analyses. The findings suggest that the development of mathematical critical thinking skills using problem-based learning was performed in two stages: (1) controversial issue and (2) active debate. Our study confirms that problem-based learning is an effective alternative to develop university student's critical thinking skills in mathematics.

1. Introduction
Critical thinking skills in mathematics are extremely important for every student [1]. Critical thinking has been a key factor in distinguishing students who are simply doing the math from those who truly understand what they are doing. Students apply critical thinking to find the best strategy out of many possible alternatives to obtain a solution. Critical thinking can be as much a part of a math class as learning concepts, computations, formulas, and theorems. Activities that stimulate critical thinking will also encourage students to think and speak in mathematical terms.

Critical thinking is a key component of the educational objectives of many countries [2]. Critical thinking skills in mathematics can assist students in problem-based learning. In addition, critical thinking skills in mathematics will gradually lead students from basic to advanced levels of critical thinking and encourage creativity and excitement about mathematics [3]. According to Diggs, a learning process that provides students with opportunities to actively interact through discussions, open-ended question-and-answer sessions, and well-constructed explanations, can stimulate the student's critical thinking skills.
Problem-based learning approach can shape the student’s thinking activities naturally and help them solve their learning problems [4-5]. Problem-based learning also proves to be more effective than conventional classroom instructions, as well as provides a positive impact on a student’s critical thinking skills [6]. As problem-solving through problem-based learning can help construct new knowledge [7], it is imperative that critical thinking skills in mathematics be continuously developed, particularly through problem-based learning.

The common problems among students today are that they understand a mathematical concept merely in its theoretical context and that their critical thinking skills are somewhat underdeveloped in learning basic mathematics, set operations, vindications of mathematical laws, Venn diagrams, and real-life applications of set operations [8-9]. These topics are the basics of subsequent learning in the following semesters, making critical thinking skills are indispensable in helping students identify problems they come across throughout the learning process [10]. A study [11] suggests that students typically struggle to adapt, manage discussions, communicate effectively, collaborate in a group, solve problems, respond to questions and feedback, and establish accurate conclusion [12-13]. Moreover, an apparent difference in critical thinking ability is observed between first-year students and their seniors. This stems from the fact that reasoning ability in solving mathematical problems basically varies among individuals [14].

Previous researches have investigated the development of critical thinking skills in mathematics and found that problem-based learning approach allows students to more actively explore and present their ideas in a group discussion to solve various mathematical problems [15-17]. Another research examined critical thinking development in terms of real analysis and the gradient of a straight-line equation [18]. Some studies also looked into the development of critical thinking in mathematics using several different methods, including problem-posing, multiple intelligence, and IMPROVE [19-21]. The way students think critically will be ultimately observable when they are trying to analyze problems related to the topics they are learning. Previous researches generally emphasize the importance of critical thinking skills in learning mathematics, and our current study discusses how these skills can be developed with the help of problem-based learning.

2. Methodology

Our study used a qualitative approach, in which a single-case single-side case design was selected as the research method [18]. A lecturer was involved to conduct classroom instruction [19] and a series of observations, interviews, and document analyses were performed to obtain the required data. These data were then analyzed qualitatively through reduction, presentation, and deduction [20]. Figure 2.1 below illustrates how the study was carried out.

![Figure 2.1 Illustrates The Study](image-url)
3. Results
The findings suggest that there are two stages in which mathematical critical thinking is exercised by the students: (1) controversial issue and (2) active debate. This confirms that problem-based learning is effective for developing critical thinking skills in mathematics among students, as described in Figure 4.1.

![Figure 3.1 Data triangulation (Creswell, 2013).](image)

| Acronym | Description |
|---------|-------------|
| Ov      | Observation result |
| AD      | Document analysis |
| MR      | Material review |
| EV      | Evaluation |
| RF      | Accurate references |
| KS      | Quiz |
| CI      | Controversial issue |
| PA      | Problem analysis |

“Controversial issue” (CI) in our study refers to the opportunities offered to the students to bring their own perspective and understanding to solve some controversial issue in mathematics within a group discussion. Figure 4.2 presents one example of how the students solve a problem of mathematical sets using their critical thinking skills.

![Figure 3.2 Solving the problem of set vindication.](image)
On the other hand, “active debate” provides the students to elaborate their answers by giving arguments and critical and logical explanation, which are mathematically acceptable, to seek for solutions to mathematical problems, as shown in Figures 4.3 and 4.4 below.

![Figure 3.3 Active debate in group discussion.](image)

![Figure 3.4 Active debate in group discussion.](image)

4. Discussion
During classroom instruction, the lecture provided the students with opportunities to solve mathematical problems. The students responded with logical arguments to deal with the problems. This would show how the students were thinking critically to take on serious problems. Active debate served as a means to evoke a controversial issue. The discussion among the students was conducted to settle differences and disagreement concerning mathematical problems as a result of concept mapping. The following is a problem of mathematical sets which could be used for a controversial issue and active debate sessions:

> “Among 15 students, 8 of them like to play tennis, 9 like to play chess, and 5 like to play both tennis and chess.”

To analyze the problem, the students could use a series of structured steps: (i) understanding the problem by paraphrasing the expression; (ii) outlining a plan to find the solution; (iii) finding the solution according to the plan; (iv) reviewing the steps that have been carried out; and (v) drawing conclusion, followed by comparing and examining the “controversial issue”, that is which solution is more efficient.

According to Peter (2012), learning mathematics trains our logical reasoning and critical thinking to solve problems. In the Quran (Al-Baqarah: 75), which says “… and then they distort it after they had understood it while they were knowing?”, the word ‘aql is interpreted as “having practical intelligence that is translated to problem-solving skills” [23]. The interpretation entails the ability to deal with problems by understanding, analyzing and logical reasoning. The Quran suggests that thinking is a good deed, and Islam honored those who use their reasoning ability to understand things and observe the truths. The history of human reasoning in understanding the world’s problems can be found in the book “Revelation and Reason” [24]. Today, educators must be able to analyze and promote the development of critical thinking skills among their students during classroom instruction.

The critical thinking skills developed using a problem-based learning approach include orientation, organization, guidance, elaboration and presentation, analysis and evaluation. There are five indicators in the development of mathematical critical thinking skills [25]: (1) elementary clarification, (2) basic support, (3) inference, (4) advanced clarification, and (5) strategies and tactics.
Critical thinking skills in mathematics depend on self-confidence in dealing with problems (Leonard & Amanah, 2014), which was observed in the interaction among the students during their group discussions. Individuals naturally vary in many things, including in the way they think; some are good at critical thinking, some struggle to do so (Novtiar & Aripin, 2017; Hidayat, Tresnawati, & Euis Eti Rohaeti, 2017; Rochmad, 2013). It is the great task of educators to help students to fully develop their own skills, character, and potentials, as well as to become individuals who are God-fearing, honorable, knowledgeable, creative, and responsible (Syafri, Mohd.Ishak, & Erlina, 2017).

Critical thinking skills require self-assurance, thus to avoid doubt and worry (Rosita, 2017; Hidayat, 2017). Teachers and lecturers alike have to be creative and empathetic in motivating their students [29]. Motivation is linked to necessity (Sardiman, 2005); therefore, teachers should make their students aware that critical thinking skills are required to tackle a wide range of problems, including mathematical ones. Critical thinking skills will enable them to arrive at solutions, provide convincing logical reasoning, retain various concepts, paraphrase knowledge, interpret and apply concepts according to cognitive structure [34].

The application of problem-based learning is a solution to problem-solving, as suggested by Sianturi, Sipayung, & Argareta (2018) who claim that problem-based learning helps develop specific skills, including the ability to think critically; to analyze and solve complex, real-world problems; to find, evaluate, and use appropriate learning resources and intellectual skills; and to become continual learners. In problem-based learning, the focus is not merely on the problem, but also on understanding any relevant concepts [36]. Innovation in problem-based learning is expected to facilitate optimal development of critical thinking skills in mathematics through systematic group discussions. Today’s classroom instruction to teach mathematics is commonly teacher-centered (Jumaisyaroh & Hasratuddin, 2015), which can be greatly improved with the application of problem-based learning.

Mathematical critical thinking emphasizes problem-solving, concept mastery, and topic controversy in mathematics learning (Maulana, 2008; Sungur, 2014). Developing mathematical critical thinking involves not only mental state, hence should be stressed on basic reasoning skills. However, no learning approach is perfect. Problem-based learning has its own strengths and weaknesses and demands continuous development and evaluation as current information and communication technology progresses. Moreover, implementation of problem-based learning approach to establishing critical thinking skills is still at its early age of development [39-40].

5. Conclusion
The development of critical thinking skills in mathematics through problem-based learning centers on two stages: (1) controversial issue and (2) active debate. With these two stages, problem-based learning is an excellent alternative for students to develop their critical thinking skills in mathematics. Continuous implementation of this approach should be considered in designing classroom instruction to see how it works and develops in varied groups of students.

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References

[1] N. Ratnasari dkk 2018 Project Based Learning Model on the Mathematical Representation Ability J. tadiris Kegur. dan Ilmu Pendidik., 3 149
[2] David Hitchcock 2014 Russel and Critical Thinking J. Betrand Russel Stud
[3] C. S. Schumacher & M. J. Siegel 2015 Curriculum Guide to Majors in The Mathematical Sciences (America: The Mathematical Assosiation of America)
[4] H. Baharun & R. Ummah 2018 Strengthening Student’s Character in Akhlaq Subject Through Problem Based Learning Model Tadiris J. Kegur. dan Ilmu Tarb. 3 1 21–30
[5] B. Rosy & T. Pahlevi 2015 Penerapan Problem Based Learning Untuk Meningkatkan Kemampuan Berfikir Kritis dan Keterampilan Memecahkan Masalah Pros. Semin. Nas. UNS, 169
[6] A. H. Asyhar 2015 Aplikasi Metode Nilai Eigen Dalam Analytical Hierarchy Process Untuk Memilih Tempat Kerja J. MANTIK Pendidik. Mat. 1 1 7
[7] Padmavathy & Mareesh 2013 Evvectiveness of Problem Based Learning In Mathematics,” J. Educ. 2 1 49
[8] Netriwati 2018 Matematika Dasar, Ke-3 (Lampung: Permata Net)
[9] Jong Jek Siang 2009 Matematika Diskrit dan Aplikasinya Pada Ilmu Komputer (Yogyakarta: Andi OFFCET)
[10] U. D. Suiiati, M. Firdaus & Hodiyanto 2017 Analisis Kesulitan Belajar Mahasiswa Program Studi Matematika J. SAP. 1 3 230
[11] Z. K. Fadhilah 2016 Pendekatan Problem Based Learning Untuk Mengembangkan Kemampuan Berfikir Kritis Mahasiswa J. Pedagog. 5 2 281
[12] F. G. Putra 2017 Ekspertimetakan Pendekatan Kontekstual Berbantuan Hands On Activity (HoA) Terhadap Kemampuan Pemecahan Masalah Matematik Al-Jabar J. Pendidik. Mat. 8 1 73–80
[13] N. N. Akmalia, H. Pujiasutii & Y. Setiani 2016 Identifikasi Tahap Berpikir Kreatif Matematis Melalui Penerapan Model Problem Based Learning J. JPPM, 9 2 184
[14] Yanti & Syazali 2016 Analisis Proses Berfikir Siswa dalam Memecahkan Masalah Matematika Berdasarkan Langkah-langkah Bransford dan Stein Ditinjau dari Advertisy Question J. Al-Jabar Pendidik. Mat. 7 1 65
[15] L. Widayanti & P. Subekti 2017 Pendekatan Problem Based Learning untuk Meningkatkan Pemahaman Konsep Mahasiswa Prodi Teknik Informatika SINATIKA, 4 154
[16] R. Rustina 2016 Efektititas Penggunaan Model Pembelajaran Problem Posing Terhadap Peningkatan Kemampuan Berpikir Kritis Mahasiswa JP3M (Jurnal Penelit. Pendidik. dan Pengajaran Mat., 2 1 43–44
[17] Mujib & Mardiyah 2017 Kemampuan Berfikir Kritis Matematis Berdasarkan Multiple Intelligences Al-JabarJurnal Pendidik. Mat. 8 2 188
[18] Robbert Yin 2013 Qualitative Research Methodology J. Am. 3 1
[19] John W. Creswell The Case Study Approach BMC Med. Res. Methodol. 11 1 105
[20] Miles & Huberman 2007 Qualitative Data Analysis: A Sourcebook Of New Methods (California: Sage Publications, Inc)
[21] Bogdan & Biklen 2007 *Qualitative Research for Education: An Introduction to theories and Methods*, 5th ed (Boston: allyn bacon)

[22] Ebindele Ebosele Peter 2012 *Critical Thinking: Essesnce for Teaching Mathematics and Mathematics Problem Solving Skills* *African J. Math. Comput. Sci. Res.* **5** 3 39

[23] T. Izutsu 1996 *Etika Beragama Dalam Qur’an* (Jakarta: Pustaka Firdaus)

[24] A. Alberry 2007 *Revelation and Reason in Islam* (Inggris: Routledge)

[25] Leonard & Amanah 2014 *Pengaruh Adversity Question dan Kemampuan Berfikir Kritis Terhadap Prestasi Belajar Matematika* *J. Perspekt. Ilmu Pendidik.* **28** 1 55–64

[26] C. Novtiar & U. Aripin 2017 *Meningkatkan Kemampuan Berfikir Kritis Matematis dan Kepercayaan Diri Siswa SMP Melalui Pendekatan Open-Ended* *J. Prism.* **4** 2 119–131

[27] W. Hidayat 2017 *Tresnawati & Euis Eti Rohaeti Kemampuan Berpikir Kritis Matematis dan Kepercayaan Diri Siswa* *J. Res. Math. Learn. Educ.* **2** 116–122

[28] Rochmad 2013 *Keterampilan Berfikir Kritis & Kreatif Dalam Pembelajaran Matematika*,” *Semin. Pres. UNY*

[29] I. H. Batubara 2017 *Peningkatan Kemampuan Berfikir Kritis Matematis Melalui Model Pembelajaran Berbasis Masalah Berbantuan Autograph dan Geogebra di SMA Freemethodist Medan* *J. Wahana Inov.* **6** 1 98

[30] Syafrimen, N. Mohd.Ishak & N. Erlina 2017 *Emotional Intelligence Profile of Prospective Teacher* *J. Eng. Appl. Sci.* **12** 7 1677–1680

[31] Rosita 2017 *Implementasi Pembelajaran Matematika dengan Pendekatan Open-Ended Terhadap Sikap Siswa* *Pas. J. Res. Math. Learn. Educ.* **1** 1 13

[32] Hidayat 2017 *Adversity Question and Penalaran Kreatif Matematis Siswa SMA Dalam Pembelajaran Argument Driven Inquiry pada Materi Turunan Fungsi* *J. Pendidik. Mat.* **2** 19

[33] Syafrimen, N. Mohd.Ishak & N. Erlina 2017 *Six Ways to Develop Empathy of Educators,”* *J. Eng. Appl. Sci.* **12** 7 1689

[34] Safery 2006 *Overview of Problem Based Learning: Definitions and Distictions,”* *Interdiscip. J. Probl. Based Learn.* **1** 1 10–20

[35] A. Sianturi, T. N. Sipayung & M. Argareta 2018 *Pengaruh Model Problem Based Learning (PBL) Terhadap Kemampuan Berpikir Kritis Matematis Siswa SMPN 5 Sumbul,”* *J. UNION Pendidik. Mat.* **6** 1 29–42

[36] Rusman 2014 *Model-Model Pembelajaran (Mengembangkan Profesionalisme Guru)* (Jakarta: Rajawali Pers)

[37] Maulana 2008 *Pendekatan Metakognitive Sebagai Alternative Pembelajaran Matematika Untuk Meningkatkan Kemampuan Berfikir Kritis Mahasiswa* *J. Pendidik. Dasar.* **10** 39–46

[38] Sumarmo 2006 *Berpikir Matematik Tingkat Tinggi : Apa,Mengapa,Bagaimana dikembangkan pada Siswa Sekolah Menengah dan Mahasiswa Calon Guru* *Pres. Semin. Nas. FMIPA Univ. Padjajaran,* **22**

[39] S. Sungur 2014 *Improving Achievement Through Problem-Based Learning* *J. Autumn,* **40** 4 156

[40] Agustini, R & Suyatna, A 2018 *Developing Inquiry-Based Practice Equipment of Heat Conductivity to Foster the Students’ Critical Thinking Ability* *Jurnal Ilmiah Pendidikan Fisika Al-BiRuNi,* **7** 1 49–57