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S-Wave $\pi$-$\pi$ Scattering Phase Shifts

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Using an absorptive one-pion exchange mechanism for the reaction $\pi^- p \to \pi^- n$, a fit is made to experimental data at 4 GeV/c in order to extract the $\pi$-$\pi$ scattering phase shifts. The analysis was carried out up to $D$ waves in the $\pi$-$\pi$ amplitude. The best results indicate a very broad $I=0$ S-wave resonance with $M_R = 820$ MeV and $\Gamma = 600$ MeV.

The determination of low-energy $\pi$-$\pi$ scattering phase shifts has been pursued over the last several years. Two recent works in this direction are those of Oh et al. and of Chan et al. We present an analysis similar to that of Ref. 1, but carried out on different experimental data. We study the data of Johnson et al. and use a peripheral model with absorptive corrections to determine the $\pi$-$\pi$ scattering parameters. These data have been previously analyzed and the improvement in the present analyses consists in the inclusion of an $I=0$ D-wave amplitude in $\pi$-$\pi$ scattering. Though our analysis has been restricted to $m_{\pi\pi}<900$ MeV, the tail of the $f^0$ resonance is seen to be important.

The process under study is the reaction $\pi^- p \to \pi^- n$ at 4 GeV/c incident momentum. The production mechanism was assumed to be due to a one-pion exchange modified by initial- and final-state absorption. The details of this calculation are presented in a subsequent paper. A cut in $t$, the momentum transfer between the proton and final neutron, was made and events with $|t|<3m^2$ were considered. It is hoped that with this assumption the peripheral hypothesis will be valid. Likewise, we feel that 4 GeV/c is a more desirable incident momentum for this kind of analysis than is 7 GeV/c, which was used in the analysis of Ref. 1.

At 7 GeV/c the exchanges of the $\omega$ and $A_2$ Regge

![Graphs](image_url)
trajectories may have to be taken into account. The $\pi-\pi$ scattering amplitude was parametrized in an energy-dependent way. The $P$ and $D$ waves were taken to be resonance-dominated by the $\rho$ and $f^0$ mesons, respectively. The positions and widths were not varied but were set at the accepted values. In the case of the $P$ wave, a radius of interaction was included and varied in the fit. For the $D$ wave no such radius was included as we were always on the low side of the resonance. The $S$ wave was parametrized by an effective-range expression,

$$\frac{(q/\sqrt{s})\cot\delta_\pi(s)}{\sqrt{s}} = a_1 + b_1 q^2,$$

FIG. 2. Treiman-Yang distribution. (Normalized to total number of events.)

FIG. 3. $\pi-\pi$ phase shifts. The solid line is our solution; the dashed line is the solution of Ref. 1.
with \( q \) and \( s \) the relative momentum and effective mass squared of the final \( \pi-\pi \) system.

Though all the distributions were fitted, the one most useful in determining the phase shifts was the \( \cos \theta \) distribution, where \( \theta \) is the scattering angle in the \( \pi-\pi \) center-of-mass system. At each \( \pi-\pi \) effective mass, the distribution was expanded in a power series in \( \cos \theta \),

\[
\frac{dN}{d\cos \theta} = \sum_{n=0}^{N} C_n \cos^n \theta.
\]  

In Fig. 1 we show the experimental values of the expansion coefficients as well as the best fits with and without a \( D \)-wave contribution. Whereas the previous analysis of these data\(^3\)\(^4\) had difficulty in obtaining a good fit to the magnitude of the forward-backward asymmetry, the inclusion of the \( D \) wave reduces the discrepancy. We likewise present, in Fig. 2, the results for the fit to the distribution in the Treiman-Yang angle \( \phi \). Comparisons with other distributions are presented in Ref. 5.

The best fit to the data was attained with an \( S \) wave \( l=0 \) dominated by a broad resonance centered at 820 MeV with a width \( \Gamma = 600 \) MeV. This is consistent with the findings of other groups.\(^5\) To recapitulate, the results we obtain for the \( \pi-\pi \) amplitudes are (the quantities in square brackets were not varied)

\[
\frac{q}{\sqrt{s}} \cot \delta_{00}(s) = \frac{m_s^2 - s}{m_s^2} \frac{q_s}{m_s^2}.
\]

\[
q \cot \delta_{20}(s) = -12.5 \mu + 0.25 q^2 / \mu,\]

\[
\frac{q^3}{\sqrt{s}} \cot \delta_{11}(s) = \frac{m_p^2 - s}{m_p^2} \frac{q_p^5}{m_p^2} \frac{1 + 0.193 \mu^2 q^2}{1 + 0.193 \mu^2 q_p^2},
\]

where \( \mu \) is pion mass in MeV, \( m_s = 820 \) MeV, \( \Gamma_s = 600 \) MeV, \( [m_p] = 765 \) MeV, \( [\Gamma_p] = 125 \) MeV, \( [m_o] = 1264 \) MeV, \( [\Gamma_o] = 151 \) MeV, and \( q_i \) for \( l=S, P, D \) is \( q \) at the resonance in \( l \)th partial wave.

A graphical presentation of these results is given in Fig. 3 together with the results of Ref. 1.

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\(^1\)B. Y. Oh, A. F. Garfinkel, R. Morse, W. D. Walker, J. D. Prentice, E. C. West, and T. S. Yoon, Phys. Rev. D 1, 2494 (1970).

\(^2\)L. Chan, V. Hagopian, and P. K. Williams, Phys. Rev. D 2, 583 (1970).

\(^3\)P. B. Johnson et al., Phys. Rev. 176, 1651 (1968).

\(^4\)M. Bander, G. L. Shaw, and J. R. Fulco, Phys. Rev. 168, 1679 (1968).

\(^5\)H. Kim, thesis, University of California, Irvine (unpublished).

\(^6\)Particle Data Group, Phys. Letters 33B, 1 (1970).