Modification of Decay Constants of Superstring Axions: Effects of Flux Compactification and Axion Mixing

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We study possibilities for lowering the decay constants of superstring axions. In the heterotic Calabi-Yau compactification, a localized model-dependent axion can appear at a nearly collapsing 2-cycle. The effect of flux can be used for generating warp factor suppression of the axion decay constant. We also point out that the hidden sector instanton potential much higher than the QCD instanton potential picks up the larger effective axion decay constant as that of the QCD axion. We show that this can be converted by introducing many hidden-sector quarks so that the decay constant of the QCD axion turns out to be much smaller than the string scale.

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I. INTRODUCTION

One of the most puzzling problems of the standard model is the strong CP problem. There are a few appealing solutions on the strong CP problem [1], among which the most attractive and promising one is considered to be a spontaneously broken Peccei-Quinn(PQ) symmetry [2]. At present, the resulting invisible axion is phenomenologically allowed only within the window $10^9 \text{ GeV} \leq F_a \leq 10^{12} \text{ GeV}$, with the upper end plausible for the axion to be a candidate of dark matter. If so, it is better for the invisible axion to be realized in the most fundamental theory such as in superstring theory. Indeed, superstring theories provide such a candidate in the components of the antisymmetric tensor field $B_{MN} (M, N = 0, 1, \ldots, 9)$, in terms of the model-independent axion(MI-axion) [3] and/or the model-dependent axions(MD-axion) [4].

When we consider unbroken nonabelian groups, there appear corresponding $\theta$ parameters and as such the same number of axions is required to settle all the $\theta$ parameters. Our interest regarding the strong CP problem is the QCD axion for which the afore-mentioned window for the axion decay constant is applicable. For the MI-axion, the decay constant is of order $10^{16} \text{ GeV}$ which is harmful if it is interpreted as the QCD axion [5]. For the MD-axions, the decay constants take generic values of order the string scale since it is the only scale of the theory. If there exist several axions, axion masses are determined by the scales of the corresponding nonabelian gauge groups and also by the axion decay constants. If some plausible cancellation occurs such that one axion is almost a Goldstone boson [6], the decay constant of the corresponding axion can be much bigger than the Planck scale, which is desirable for it to be a quintessential axion. For some inflationary models, such a large decay constant is needed [6]. But generically, we expect that the decay constants are of order the string scale [6].

Therefore, it is desirable to have some smaller scales for decay constants of superstring axions. Recently, generating such hierarchical scales has been explored in the extra-dimensional context. Those mechanisms involve the localization of dynamical degrees of freedom at some location in compactified space. Especially, the localization of the gravity enables us to scale down the overall scales of physics of interest, introducing a warp factor [7].

The well-known warping from string models arises from the Klebanov-Strasser(KS) throat geometry [8]. In Type IIB string models, Giddings, Kachru and Polchinski(GKP) localized the $\langle F_{ijk}\rangle$ flux by the balancing act of the $\langle H_{ijk}\rangle$ flux [9] so that stringy realization of the Randall-Sundrum type model with stabilization mechanism has been successfully constructed. In heterotic strings, supersymmetric compactification with flux yields a warped product of 4D Minkowski spacetime and non-Kähler internal manifold, where the necessary conditions for a warp factor were given a long time ago by Strominger [10]. Recently Becker et al. worked on compactification of heterotic strings on the non-Kähler manifold with the use of fluxes of antisymmetric tensor field $B_{MN}$ [11]. Here, the interpretation of the warp factor in the closed string theory assumes some kind of localization of wave functions.

Our prime objective is to find out mechanisms of lowering axion decay constants. One easy method is an intermediate scale compactification [12]. In this paper, we look for a possibility of warping the effective axion decay constants in compactifications of string models. Previous discussions [2,13,14,15] on superstring axions did not consider the effects of warp factors and the localization of superstring axions. If the background is unwarped, it is unavoidable to have the axion decay constant at the string scale unless we deal with a large volume com-
pactification. To have a large hierarchy of axion decay constants, we emphasize that a warped background is effective for some localized MD-axions.

In this section, we consider the heterotic string theory compactified with flux. In compactification of string models, it is required for the vacuum to possess a four dimensional (4D) \( N = 1 \) supersymmetry after compactification, which is studied from the \( D = 10 \) supersymmetry transformation of Fermi fields: gravitino \( \psi_M \), dilatino \( \lambda \) and gaugino \( \xi \) which transform by

\[
\delta_{\epsilon} \psi_M = \nabla_M \epsilon + \frac{i}{2} H_M \epsilon, \\
\delta_{\epsilon} \lambda = (\partial \phi) \epsilon + \frac{i}{2} \chi \epsilon, \\
\delta_{\epsilon} \xi = 2 \Phi \epsilon, \tag{1}
\]

where \( \phi \) is dilaton, \( F_{MN} \) is the field strengths of gauge fields and 3-form flux \( H_{MNP} \) is defined in terms of NS 2-form potential \( B_{MN} \) and the Yang-Mills Chern-Simons term \( \omega_Y \) and Lorentz Chern-Simons term \( \omega_L \) according to

\[
H = dB + \frac{i}{4} \left( \frac{1}{30} \omega_Y - \omega_L \right) \tag{2}
\]

Here, the bold faced ones are the field strengths contracted with appropriate 10D \( \gamma \) matrices, for example \( H_M = H_{MNP} \gamma^N P \).

The pioneering work of Candelas et al. searched for vacua having non-zero covariantly constant supersymmetry parameter \( \epsilon \) in case fluxes vanish, \( H_M = 0 \) and \( \Phi = 0 \), and obtained the background manifold with Kähler geometry. For the case of 6D internal space, it is the Calabi-Yau space which has \( SU(3) \) holonomy.

More general conditions required for 4D \( N = 1 \) supersymmetry with non-vanishing fluxes have been obtained in Ref. \[10\]. The recent work \[11\] obtained non-singular solutions satisfying Strominger’s four conditions. With non-vanishing fluxes, the internal manifold turns out to be non-Kähler.

The geometry of compactification and the dynamics of low energy fields in the string theory are described by 10D low energy supergravity whose bosonic part is

\[
S = \frac{1}{2 \kappa_{10}^2} \int d^{10} x \sqrt{-g} e^{-2 \phi} \times \\
\left( R + 4|\partial \phi|^2 - \frac{1}{2} |H|^2 + \frac{\alpha'}{4} \text{tr}|F|^2 + 2 \mathcal{L}_{GS} \right) \tag{3}
\]

where \( \kappa_{10} \) is the inverse of reduced 10D Planck mass, and \( \mathcal{L}_{GS} \) is the Green-Schwarz term.

The MI-axion is the antisymmetric tensor field in the tangent 4D space. Its equation of motion is determined universally from the Bianchi identity:

\[
dH = R \wedge R - \frac{1}{30} F \wedge F, \tag{4}
\]

i.e. the \( H_{\mu \nu \rho} \) coupling to the gluon field strength is

\[
\frac{1}{30} \epsilon_{\mu \nu \rho \sigma} \partial_\mu H_{\nu \rho \sigma} = -\frac{1}{30} \left( \frac{1}{2} \right)^2 \epsilon_{\mu \nu \rho \sigma} F_{\mu \nu} F_{\rho \sigma}. \tag{5}
\]

Defining \( H_{\mu \nu \rho} = M_{\mu \nu \rho} \theta^a a_M \) in terms of the properly normalized MI-axion \( a_M \), basically \( M \) turns out to be proportional to the MI-axion decay constant. \( M \) is determined by 4D Planck scale and the scale of the coefficient of \( |H|^2 \) term in 4D effective action. From the ratio of warp factors of the Einstein-Hilbert term and the \( |H|^2 \) term in \( \mathcal{L}_{GS} \), we find that the MI-axion does not have a warp factor dependent decay constant. This is because the RHS of Eq. \( \mathcal{L}_{GS} \) is a topological term.

For MD-axions, \( B_{ij} \), \( i, j \) are internal space indices. For these MD-axions, there can be warp factor dependence. The MD-axions couple to field strengths via the Green-Schwarz term \[18\],

\[
\int d^{10} x \mathcal{L}_{GS} = c \int (-3B X_8 + 2X_3^2 X_7^0) \tag{6}
\]

for which the couplings of MD-axions are schematically given as \( \mathcal{L}_{GS} \),

\[
\int B \wedge F \wedge F \wedge F \wedge F + \cdots. \tag{7}
\]

Depending on the way \( B_{ij} \) is embedded in the internal space, MD-axions can be localized. The Hodge-Betti number \( b_{(1,1)} \) corresponds to the number of MD-axions. In CY spaces, it is known that \( b_{(1,1)} \geq 1 \); thus there exists at least one MD-axion. The MD-axion in the simplest case of \( b_{(1,1)} = 1 \) corresponds to the breathing mode of the internal space. Even with fluxes, this breathing mode couples to the \( E_8 \) and \( E_8 \) anomalies without exponentially small decay constant since the breathing mode is not localized.

Recently, Becker et al. obtained an interesting non-Kähler geometry with heterotic flux compactification \[11\]. In the string frame, the background is a direct product of 4D spacetime and an internal six-dimensional manifold with the 10D metric

\[
ds^2 = \eta_{\mu \nu} dx^\mu dx^\nu + ds^2_{(6)}. \tag{8}
\]
The internal space geometry is a $T^2$ bundle over the base K3 manifold

$$ds^2_{K3} = e^{2\phi} ds^2_{K3} + (dy_5 + a_5)^2 + (dy_6 + a_6)^2$$

where $ds^2_{K3}$ is the metric for the four dimensional base manifold K3, with coordinates $y_1, \cdots, y_4$. Here, $\phi$ is dilaton and 1-forms $a_5$ and $a_6$ depend only on the K3 coordinates. To obtain 4D $\mathcal{N} = 1$ SUSY, the torus must be fibered nontrivially, which leads to the complication of the solution. However, such twisting has only global effects.

To discuss the property of warped geometry arising near 2-cycles of the above solution, it is enough to consider the trivially fibered case. In the orbifold limit of K3-manifold [21], $ds^2_{K3}$ describes a flat 4-torus $T^4$ and

$$\Delta^2 = c_0 + A z^1 + B z^2 + c.c., \quad \Delta = e^\phi,$$

$$H = Adz^1 \wedge dz^2 \wedge dz^3 \wedge dz^4 + Bdz^1 \wedge dz^2 \wedge dz^3 + c.c.$$  \hspace{1cm} (10)

where $z^1$ and $z^2$ are the coordinate of $T^4$. The 10D metric in the Einstein frame is given by

$$ds^2_E = \Delta^{-1/2} (\eta_{\mu\nu} dx^\mu dx^\nu)$$

$$+ \Delta^{-1/2} ((dx + \alpha_1)^2 + (dy + \alpha_2)^2) + \Delta^{3/2} ds^2_{K3}.$$  \hspace{1cm} (11)

One can easily see that the Becker et al. solution gives a power-law warp factor for $c_0 \ll A z^1 + B z^2$. Even though we have not obtained an exponential warp factor here, exponential warp factors may arise in some other flux compactifications.

The warp factor effect is significant for localized fields [7]. Strings on fixed points (in the twisted sector) in the orbifold compactification are considered to be localized, and hence can have warp factors if the orbifold is a limit of some manifold with fluxes. These localized strings cannot be moved to the bulk because of the closed string condition, satisfying the modding group, around the fixed point. For some CY spaces, there exist the corresponding orbifolds from which the CY space can be made by blowing up the fixed points as shown for $T^4/Z_2$ orbifold in Fig. [1].

Let us call the corresponding region in the CY space ‘squeezed throat’. Around the fixed point of the orbifold and hence around the cut line of the corresponding CY space, topological invariants must be the same. Namely, the curve around the fixed point (solid and dashed line ring of Fig. [1(a)]) and the closed ring in the bulk (the dashed line ring of Fig. [1(a)]) have different topological properties and hence from the topological argument the solid-dash ring cannot be moved to the dashed ring without obstruction of topology. Even though there is no singularity in the corresponding CY space as shown in Fig. [1(b)], the solid-dash ring of Fig. [1(b)] cannot be moved to the dashed ring of Fig. [1(b)] without obstruction of topology, because the topological number must change in the CY space also for this to happen.

Now from physical argument, we can guess which MD-axions can have warp factors. The MD-axion corresponding to the breathing mode must belong to the cycle encompassing the whole internal space, and hence it does not belong to a localized string. So the breathing mode MD-axion would not have a warp factor. The remaining MD-axions belonging to some squeezed throat of the CY space and can have warp factors. The maximum number of MD-axions having the warp factors is

$$b_{(1,1)} - 1.$$  \hspace{1cm} (12)

Even if a MD-axion is localized at a fixed point, it does not necessarily mean that it has a warp factor. The additional condition is to have a stable squeezed throat à la KS and GKP.

To set up the dynamical scale of axion fields by warp factor, it is crucial to understand how the wavefunction of a 4D massless field is localized in the compactified internal manifold $X$. The zero modes are determined by harmonic $p$-forms $\omega$ satisfying $\Delta \omega = 0$ where the Laplacian is defined by $\Delta = d^* d + dd^* = (d + d^*)^2$. Here, the adjoint exterior derivative operator on $p$-form $\omega = \omega_1 \cdots \omega_p dx^{i_1} \wedge \cdots \wedge dx^{i_p}$ is

$$d^* \omega = \frac{1}{p!} \nabla^{\mu} \omega_{\mu i_2 \cdots i_p} dx^{i_2} \wedge \cdots \wedge dx^{i_p}.$$  \hspace{1cm} (13)

The Laplacian acted on the $p$-form $\omega$ is written explicitly by

$$\Delta d \omega = \frac{1}{p!} \left( \nabla_{[\mu_1} \nabla^{\nu} \omega_{\nu i_2 \cdots i_p]} - \nabla^\nu \nabla_{[\nu \omega_{\mu_2 \cdots i_p}]} \right) dx^{i_1} \wedge \cdots \wedge dx^{i_p}.$$  \hspace{1cm} (14)

Harmonic $p$-forms belong to the cohomology group $H^p(X)$ on $X$. If we have a set of $p$-cycles $\{C_j\}$ on $X$, we find the basis $\{\omega_j\}$ for the harmonic $p$-forms such that the period satisfies

$$\int_{C_i} \omega_j = \delta_{ij}$$  \hspace{1cm} (15)

due to de Rham’s theorem. The harmonic $p$-forms are the zero mode wave functions. The period integrated
over the same homology class where the integration is performed has the same value. This fact indicates that the wave function has the largest value on the smallest cycle in the same homology class. If a cycle can be deformed to a nearly vanishing size, the zero mode wave function is localized like a delta-function at that point, which is the case for the twisted mode in the orbifold compactification.

The zero mode wave function of $E_8 \times E_8$ gauge fields are determined by harmonic zero forms which cannot have the localization effect discussed in the preceding paragraph. However, the zero mode wave function of MD-axions are from harmonic 2-forms, and thus it is important to examine the 2-cycles which can shrink to a point where the effect of the warp factor is significantly large.

In summary, some MD-axion(s) in the heterotic string can be localized and its scale can have a warp factor suppression. But, for MD-axions, there may appear dangerously large superpotential terms as pointed out by Wen and Witten [21]. However, the size of these terms depend on details of compactification.

III. AXION MIXING

If there exist more than one axion, one crucial question is what is the QCD axion and its decay constant. Even if the scale of one MD-axion is warped to give an intermediate scale, it is not automatic that the scale of the QCD axion is at the intermediate scale because the scale of MI-axion is of order $10^{16}$ GeV. A naive guess leads to a $10^{16}$ GeV for the scale of the QCD axion.

In fact, the above pessimistic comment is related to our another method of lowering the axion decay constant.

Let us consider two nonabelian gauge groups, the hidden sector group, say $SU(N)_h$, confining at the intermediate scale and QCD $SU(3)_c$, and two axions $a_1$ and $a_2$ with decay constants $F_1$ and $F_2$ with a hierarchy $F_1 \gg F_2$. Considering two axions is equivalent to introducing two global symmetries. If mass eigenstates are nontrivial mixtures of $a_1$ and $a_2$, then the higher instanton potential corresponds to the lower decay constant and the lower instanton potential corresponds to the higher decay constant [14]. On the other hand, remember that if there exists a massless quark then the instanton potential vanishes, which corresponds to a flat axion potential. Therefore, for a light quark mass $m_q$ the instanton potential is proportional to $m_q^2$ [22]. For $n$ light quarks, one might naively expect a power law of $m_q^n$, which however is not warranted. This is because there is only one $\theta$ parameter for a nonabelian group and only one current out of $n$ quark currents contributes to the axion potential. This is the reason that the QCD axion potential has only one light quark mass factor [1]. Here also, this property of one power of current mass is not changed.

Naively, it is guessed that the hidden sector instanton potential is much higher than the QCD instanton potential in which case the QCD axion corresponds to the larger axion decay constant $F_1$, which is harmful. But, if almost massless hidden-sector quarks exist, then one can obtain a much lower hidden-sector instanton potential but the suppression is only with one mass factor of hidden-sector quarks. But the following study shows that one must introduce sufficiently many hidden-sector quarks.

A. Analogy to old $U(1)$ problem, instanton potential and $\eta'$-like particle masses

Our objective is to lower the hidden-sector instanton potential below that of the QCD instanton potential. In analogy with the resolution [23] of the old $U(1)$ problem, we determine the height of the hidden-sector axion potential. The relevant Lagrangian with the hidden-sector SUSY $SU(N)_h$ gauge group with one hidden-sector quark $q$ is

$$\mathcal{L}_\chi = -M\lambda\lambda - m\bar{q}q + \mathcal{L}_{\text{inst}} + \text{h.c.}$$

where $\lambda$ is the hidden-sector gaugino, $q$ is the hidden-sector quark, and $\mathcal{L}_{\text{inst}}$ is the ’t Hooft determinantial interaction,

$$\mathcal{L}_{\text{inst}} = e^{-\frac{\lambda\lambda}{\alpha} - i\theta(\lambda\lambda)(\bar{q}q)}$$

which shows that a chiral rotation by angle $\alpha$ on the hidden-sector quark is equivalent to changing the $\theta$ parameter by $\alpha$, and a chiral rotation by angle $\alpha$ on the hidden-sector gaugino is equivalent to changing the $\theta$ parameter by $N\alpha$. The resolution of the old $U(1)$ problem comes from the instanton term, which will be manifest in the following discussion also.

The condensations of hidden-sector gluino and hidden-sector quark produce composite pseudoscalar particles which are denoted as $\eta_\lambda \equiv F_\lambda\theta_\lambda$ and $\eta_\theta \equiv F_\theta\theta_\theta$, respectively, where $\langle \eta_\lambda \rangle \simeq -e^{\lambda\lambda}e^{\theta_\lambda}$ and $\langle \bar{q}q \rangle \simeq -e^{\bar{q}q}e^{\theta_\theta}$. Then, in the chiral perturbation theory language, we have the following terms from [16] and [17]

$$-M\lambda^3 \cos\theta_\lambda - m\nu^3 \cos\theta_\theta + \langle \mathcal{L}_{\text{inst}} \rangle.$$

The problem is to calculate the expectation value of $\mathcal{L}_{\text{inst}}$. For this purpose, we insert the identity,

$$1 = |0\rangle\langle 0| + |\eta_\lambda\rangle\langle \eta_\lambda| + |\eta_\theta\rangle\langle \eta_\theta| + \cdots + (|\eta_\lambda\rangle\langle \eta_\lambda|)^N|\eta_\theta\rangle\langle \eta_\theta| + \cdots$$

where momenta sum is understood. It can be schematically shown as Fig. [2] The uncondensed fermion lines are connected with the current masses.

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2 In Ref. [6] the power was not correctly given.

3 The instanton potential for $\eta'$ does not have a quark mass.
Thus, we obtain an effective interaction Hamiltonian of the form,
\[ \mathcal{H}_\chi = -MA^3 \cos \theta_\chi - m v^3 \cos \theta_q \\
- f_0 \Lambda^3 v^3 \cos \left[ N \theta_\chi + \theta_q + (N + 1) \theta \right] \\
- N f_1 M \Lambda^3 (N-1) v^3 \cos \left[ N \theta_\chi + \theta_q + (N + 1) \theta + \alpha_\chi \right] \\
- f_2 m \Lambda^3 \cos \left[ N \theta_\chi + \theta_q + (N + 1) \theta + \alpha_q \right] + \cdots \]
where \( f_0, f_1 \) and \( f_2 \) are constants. Let us choose \( M \) and \( m \) real, i.e. take \( \alpha_\chi = \alpha_q = 0 \). For two condensates and one axion, the \( 3 \times 3 \) mass matrix is
\[
M^2 = \begin{pmatrix}
\frac{(N^2 \tilde{\Lambda}^4 + \Lambda^4)}{F^2_q} & \frac{N^2 \tilde{\Lambda}^4}{F^2_q} & \frac{N(N+1) \tilde{\Lambda}^4}{F^2_q} \\
\frac{N^2 \tilde{\Lambda}^4}{F^2_q} & \frac{\tilde{\Lambda}^4 + m v^3}{F^2_q} & \frac{N \tilde{\Lambda}^4}{F^2_q} \\
\frac{N(N+1) \tilde{\Lambda}^4}{F^2_q} & \frac{N \tilde{\Lambda}^4}{F^2_q} & \frac{(N+1) \tilde{\Lambda}^4}{F^2_q}
\end{pmatrix}
\]
so
\[
M^2 = \begin{pmatrix}
\frac{(N^2 \tilde{\Lambda}^4 + \Lambda^4)}{F^2_q} & \frac{N^2 \tilde{\Lambda}^4}{F^2_q} & \frac{N(N+1) \tilde{\Lambda}^4}{F^2_q} \\
\frac{N^2 \tilde{\Lambda}^4}{F^2_q} & \frac{\tilde{\Lambda}^4 + m v^3}{F^2_q} & \frac{N \tilde{\Lambda}^4}{F^2_q} \\
\frac{N(N+1) \tilde{\Lambda}^4}{F^2_q} & \frac{N \tilde{\Lambda}^4}{F^2_q} & \frac{(N+1) \tilde{\Lambda}^4}{F^2_q}
\end{pmatrix}
\]
where ellipses denote higher power terms of \( m \) and \( M \).
In the limit of \( \Lambda \gg M, m \), we obtain three masses as
\[
\begin{align*}
\eta_\chi^2 &= (a_\chi^2 + a_\chi^2 + \alpha_\chi^2) \tilde{\Lambda}^2 \\
\eta_q^2 &= (a_q^2 + a_q^2) M^3 \Lambda^3 / F^2_q + (a_\chi^2 + a_\chi^2) m v^3 / F^2_q \\
m_a^2 &= \frac{m v^3}{(1 + \alpha_q) (1 + \alpha_\chi) Z \tilde{\Lambda} F_a}
\end{align*}
\]
where
\[
Z = m / M, \quad \alpha_q = a_q^2 / a_\chi^2, \quad \alpha_\chi = a_\chi^2 / a_q^2, \quad \beta = v^3 / \Lambda^3, \\
\alpha_\chi = N \tilde{\Lambda} / F_a, \quad a_q = \tilde{\Lambda} / F_q, \quad a_\chi = (N + 1) \tilde{\Lambda} / F_a.
\]
From Eq. (19), with a large \( m \) there is no pseudoscalar which is almost massless. It is equivalent to saying that the QCD instanton potential is lower than the hidden-sector instanton potential. Then the decay constant of the QCD axion is at the string scale even if we lowered some axion decay constant to a lower scale.

**B. Squark condensation**

To reduce the hidden sector instanton potential drastically, we may introduce a massless quark. Then from Eq. (19), the axion mass is vanishing. But the question is whether this masslessness of the hidden-sector quark is supported or not in our scenario with supersymmetry. The chiral symmetry is present when we introduced a massless hidden-sector quark. This chiral symmetry can be broken by a squark condensation, not breaking supersymmetry. So the hidden-sector quark is expected to obtain a mass of order \( \langle \tilde{q} \tilde{q} \rangle / (8 \pi^2) \Lambda \). In Fig. 2, the condensation of the hidden-sector quark cannot appear but only the effective mass can be present as shown in Fig. 3.

Thus, with \( n \) hidden-sector quarks, we expect the height of the hidden-sector instanton potential is
\[
MA^3 \left( \frac{\langle \tilde{q} \tilde{q} \rangle}{8 \pi^2 \Lambda^2} \right)^n.
\]
Here we inserted one power of the hidden-sector gaugino mass. Without the gaugino mass, i.e. without the gravitational effect \( M \sim \Lambda^3 / M_{Pl}^2 \), all insertions in the ‘t Hooft determinental interaction involve condensation and cannot contribute to the axion mass. It contributes to the \( \eta' \) mass in Eq. (19). At this level, \( \langle \tilde{q} \tilde{q} \rangle \) is not determined. There is no constraint on the scale of the hidden-sector squark condensation which can be very small. If we take an arbitrary value of order \( 10^{-3} \Lambda \), the suppression factor is of order \( 10^{-5n} \) which must bring down the instanton potential below the QCD scale for \( n > 8 \).

**IV. CONCLUSION**

In this paper, we discussed two plausible mechanisms toward lowering the decay constants of superstring axions. For the MD-axions winding a nearly vanishing 2-cycle background in the warped region, the localization is a plausible mechanism for lowering the axion decay constant. Here, the Wen-Witten world-sheet instanton problem must be evaded so that a large superpotential is not generated. Two axions are needed if we require the hidden sector confining force. In this case, the axion mixing must be studied to pinpoint what is the decay constant corresponding to the QCD axion. To lower the decay constant of the QCD axion much below the string scale, it is pointed out that it is required to make the hidden-sector instanton potential be lowered below the QCD instanton potential. Even if we introduced massless hidden-sector quarks, this problem is nontrivial because.
of the chiral symmetry breaking via the hidden-sector squark condensation. It was pointed out that it is possible with sufficiently many hidden-sector quarks with a low value of the hidden-sector squark condensation.

[Note added]: During this work, a work on the related topic appeared by P. Svrcek and E. Witten, hep-th/0605206

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