The path-following control of the underactuated AUV with input saturation and multiple disturbances

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Abstract—Autonomous underwater vehicle (AUV) in marine resource surveys plays an important role. This paper proposes a new path-following control frame for the underactuated AUV with input saturation and multiple disturbances. The disturbances include external disturbances, model parameter uncertainties, unmodeled dynamics and other random disturbances. Compared to most of previously published literatures, which treat disturbances as lumped disturbances, a composite hierarchical anti-disturbance control (CHADC) strategy is adopted to achieve higher precision path following. A disturbance observer (DOB) is constructed to estimate and eliminate the disturbances with partial known information, while the $H_\infty$ theory is used to optimize the path-following controller to attenuate the other disturbances satisfying the $L_2$-norm bound condition and improve the robustness of system. Besides, Lyapunov direct method and back-stepping method are used to design the path-following controller, where the input saturation is considered, the extended state observer (ESO) is used to estimate the uncertainty of kinematic controller and the nonlinear tracking differentiator (NTD) is used to simplify the controller. Finally, simulations are given to demonstrate the effectiveness of the proposed control law.

1. Introduction

In recent years, the study of underactuated AUVs in theory and practice taking had preliminary in progress. Taking into account nonlinearity and external disturbances, [1] shown an adaptive nonlinear second-order sliding mode controller to achieve precise path following control for AUV. In [2], a Lyapunov-based model predictive control framework for AUV is designed, which can solve actual constraints and use computing resources to improve the trajectory tracking performance of AUV. [3] studied the multiple unmanned underwater vehicle cooperative control, in order to achieve synergy target tracking. [4] proposed a backstepping-based nonlinear controller for AUV, and [5] adopted a backstepping-based state feedback control method in the path following of underactuated AUV. However, the backstepping method often involves the problem of “explosion of complexity”, therefore, [6] adopted dynamic surface control (DSC) to solve the problem, [7] used NTD to avoid the problems. Uncertainties and external disturbances play an important role in AUV path-following control. In most studies, the disturbances to the aircraft are not considered, or the disturbances are assumed as lumped disturbances, which is not in line with reality. Therefore, [8] proposed that the external disturbances, unmodeled dynamics and the system uncertainties are regarded as equivalent. Uncertainty and
disturbance estimator (UDE) is used to estimate and provide feedforward compensation. Although this method can solve the interference problem, it is too conservative to achieve high-precision path tracking.

A composite anti-disturbances method was proposed in [9] to deal with external disturbances, then, the uncertainty of the exogenous system was further considered in [10], making this processing method more likely to be applied to the actual system.

In this paper, with the studies of previous advanced methods, we propose a novel path-following control frame for underactuated AUVs to address the problems of input saturation and multiple disturbances.

2. Problem statement

2.1. AUV Modeling

The schematic diagram of underactuated AUV path following system can be shown in Fig.1, the kinematics equation can be described as [4, 11]

\[\begin{align*}
\dot{x} &= u \cos(\psi_B) - v \sin(\psi_B) \\
\dot{y} &= u \sin(\psi_B) + v \cos(\psi_B) \\
\dot{\psi}_B &= r
\end{align*}\]

The dynamic equations can be described as [12]

\[\begin{align*}
\dot{u} &= -\frac{m_{22}}{m_{11}}v + \frac{X_\psi}{m_{11}}u - \frac{X_d}{m_{11}}u|u| + \tau_u + d_u \\
\dot{v} &= -\frac{m_{11}}{m_{22}}u + \frac{Y_\psi}{m_{22}}v - \frac{Y_d}{m_{22}}v|v| + d_v \\
\dot{r} &= \frac{m_{11} - m_{22}}{m_{33}}uv - \frac{N_\psi}{m_{33}}r - \frac{N_d}{m_{33}}r|v| + \tau_r + d_r
\end{align*}\]

where \(u, v, \) and \(r\) are the surge speed, sway speed and yaw speed of the underactuated AUV, \(\tau_u\) and \(\tau_r\) are the control input, \(m\) is the mass of AUV, \(I_Z\) is the moment of inertia, \(X_\psi, X_d, X_\psi, Y_\psi, Y_d, Y_\psi, Y_\psi\) are the nominal hydrodynamic parameters, which \(m_{11} = m - X_\psi, m_{22} = m - Y_\psi, m_{33} = I_Z - N_\psi\) represent the combined inertia and added mass terms. \(d_u, d_v, \) and \(d_r\) are the total disturbances, including environmental disturbance, unmodeled errors and system uncertainties.

The error dynamics equations are

\[\begin{align*}
\dot{x}_e &= -\dot{s} \left(1 - c_e y_e\right) + v_t \cos(\psi_e) \\
\dot{y}_e &= -c_e \dot{x}_e + v_t \sin(\psi_e) \\
\dot{\psi}_e &= r + \beta - c_e \dot{s}
\end{align*}\]

where \(s\) is the abscissa of the virtual target point \(P\) on the desired path, \(Cc\) is the curvature of the desired path, \(\beta = \tan^{-1}(v/u)\) is the sideslip angle, \(v_t = \sqrt{u^2 + v^2}\) is the total speed. \((x_e, y_e, \psi_e)\) are the follow errors in the Serret-Frenet frame.

Taking into account the effect of the input saturation of the control inputs in the practical application, the actual control force \(\tau_u\) and \(\tau_r\) moment can be formulated as:

\[\begin{align*}
\tau_i &= \begin{cases}
\tau_{i,\text{max}}, & \tau_i > \tau_{i,\text{max}} \\
\tau_i, & \tau_{i,\text{min}} \leq \tau_i \leq \tau_{i,\text{max}}, \ i = u, r \\
\tau_{i,\text{min}}, & \tau_i < \tau_{i,\text{min}}
\end{cases}
\]

\[\text{(4)}\]
where \( \tau_i^* \) \((i = u, r)\) represents the control input without considering the problem of input saturation. \( \tau_{i\text{max}} \) and \( \tau_{i\text{min}} \) \((i = u, r)\) are the upper and lower bounds of the control input, respectively.

The dynamic model of the underactuated AUV can be expressed in matrix form:

\[
\dot{v} = Mv + F_0f_0(v) + M_1r + d
\]

the disturbances can be written as \( d = M_1d_0 + H_1d_1 \) by layering. Hence, the model (5) can be rewritten as

\[
\dot{v} = Mv + F_0f_0(v) + M_1[r + d_0] + H_1d_1
\]

2.2. Problem Statement

The control objectives of this paper are that design the controller \( u, r \) and \( s \) to make

\[
\begin{align*}
\sup_{t \in [t_0, \infty]} \| x_e \| & \leq \delta_1, \\
\sup_{t \in [t_0, \infty]} \| y_e \| & \leq \delta_2, \\
\sup_{t \in [t_0, \infty]} \| \psi_e - \psi_{\text{los}} \| & \leq \delta_3, \\
\sup_{t \in [t_0, \infty]} \| u - u_d \| & \leq \delta_4,
\end{align*}
\]

under the joint action of multiple disturbances (\( d_u, d_v \) with \( d_r \)) and input saturation.

3. Controller design

3.1. Path-following controller design

Using the line-of-sight (LOS) guidance algorithm described in [13], the LOS guidance law is selected as:

\[
\psi_{\text{LOS}} = \tan^{-1}\left(\frac{-\psi_e}{\Delta}\right)
\]

where \( \psi_{\text{LOS}} \) is the LOS angle, \( \Delta \) is the look-ahead distance.

To avoid long periods of full rudder, we chose to add constraints at the virtual of heading angle velocity, therefore, the control law can be described as follows:

\[
r = c_e(s)\dot{s} - \beta + \psi_{\text{los}} - k_1(\psi_e - \psi_{\text{los}}) + k_1\chi_1
\]

where \( \chi_1 \) is auxiliary system state which has been design as[14]:

![Fig.1 Frame and variable definitions of AUV path following](image-url)
among the function, \( f_1 = \frac{|\psi \Delta r| + 0.5 \gamma_1^2 \Delta^2}{\chi_1^2}, k_x > \frac{1}{2}, \gamma_1 > 0, \psi = \psi_v - \psi_{\text{LOS}}, \Delta r = r - r_d, \chi_1 > 0 \).

Due to the complexity of the calculation of \( \dot{\beta} \), choose ESO to estimate the uncertainty \( d_\psi \) which represent \( \dot{\beta} \) [12].

\[
\begin{align*}
\dot{\xi}_1 &= -\alpha_1 \xi_1 - \alpha_2 \psi_v - \alpha_3 (r - c_v \dot{s}) \\
d_\psi &= \alpha_4 \psi_v + \xi_1, \alpha_4 > 0
\end{align*}
\]

As a result, the control law of virtual yaw speed becomes:

\[
\tau_d = c_v(s) \dot{s} - \hat{d}_\psi + \psi_{\text{los}} - k_1 (\psi_v - \psi_{\text{los}}) + k_1 \xi_1
\]

(10)

The virtual target movement control law is designed as

\[
\ddot{s} = k_2 \chi_v + \ddot{v}_e \cos \psi_v
\]

(11)

The control input \( \tau_u \) can be designed as follow, notice the saturation of \( \tau_u \).

\[
\tau_u = m_{11} \left[ \dot{u}_d - k_5 (u - u_d) - \frac{m_{22}}{m_{11}} \ddot{v} + \frac{X_u}{m_{11}} u + \frac{X_d}{m_{11}} |u| - d_u + k_5 \chi_2 \right]
\]

(12)

where \( \chi_2 \) is the is auxiliary system state which has been designed as \( \chi_1 \), \( k_5 > 0 \) is the control gain.

Design control input \( \tau_r \) as follow:

\[
\tau_r = m_{33} \left[ \ddot{r}_d - k_3 (r - r_d) - k_4 (\psi_v - \psi_{\text{los}}) - \frac{m_{11} - m_{22}}{m_{33}} \dot{u}v + \frac{N_r}{m_{33}} r - \frac{N_r}{m_{33}} |r| - d_r \right]
\]

(13)

where \( k_3 > 0 \) and \( k_4 > 0 \) are the control gains. Due to the form \( \dot{r}_d \) is too complicated, using NTD can simplify the above design controller (16) as follows[12]:

\[
\tau_r = m_{33} \left[ \ddot{r}_v - k_3 (r - r_v) - k_4 (\psi_v - \psi_{\text{los}}) - \frac{m_{11} - m_{22}}{m_{33}} \dot{u}v + \frac{N_r}{m_{33}} r - \frac{N_r}{m_{33}} |r| - d_r \right]
\]

(14)

3.2. Composite controller design:

First, design the disturbance observer:

\[
\begin{align*}
\ddot{d}_0 &= V\omega \\
\dot{\omega} &= \dot{\xi}_2 - L\dot{v} \\
\dot{\xi}_2 &= (W + LM_1 V)\omega + L(Mv + F_{01}f_{01} + M_1 \tau)
\end{align*}
\]

(15)

where \( \dot{d}_0 \) is the estimation of disturbances \( d_0 \), \( \dot{\omega} \) is the estimation of \( \omega \), \( \dot{\xi}_2 \) is auxiliary state of the observer. The estimation error is \( \dot{e}_{\omega} = \omega - \dot{\omega} \), then

\[
\dot{\hat{e}}_{\omega} = (W + LMV)e_{\omega} + LH_1 d_1 + H_2 \delta
\]

(16)

Then, make \( \tau = -d_0 + \tau_0 \), where \( \tau_0 \) is the control input when the disturbances are absent, where \( \dot{\hat{\psi}}_v = \psi_v - \psi_{\text{LOS}} \). Substituting \( \tau \) into (5) yields

\[
\dot{v} = Mv + F_{01}f_{01} + M_1 (d_0 - d_0 + \tau_0) + H_1 d_1
\]

(17)

make \( \dot{\hat{v}} = v - v_d \)

\[
\dot{\hat{v}} = (G_0 + K)\ddot{v} + F_{01}f_{01} + M_1 V e_{\omega} + H_1 d_1 + H_3 d_2
\]

(18)
where \( G_0 = \text{diag}([0 -Y_v/m_{22} 0]) \), \( f_{01} = (0 f_v 0)^T \), \( d_2 = [k_5 \chi_2 -k_4 \ddot{\psi}_d (-\dot{Y}_v/m_{22})\nu_d -\dot{\nu}_d]^T \),
\[
K = \begin{bmatrix} k_5 & 0 & 0 \\ 0 & 0 & k_3 \end{bmatrix}^T .
\]
Combine (16) and (18), we can gain that
\[
\begin{bmatrix} \dot{v} \\ \ddot{\omega} \end{bmatrix} = Gv + Ff_{01} + Hd
\]
where
\[
-v = \begin{bmatrix} \dot{v} \\ \dot{\omega} \end{bmatrix},
G = \begin{bmatrix} G_0 + K & M_1V \\ 0 & W + LM_1V \end{bmatrix},
F = \begin{bmatrix} F_{01} \\ 0 \end{bmatrix},
H = \begin{bmatrix} H_1 & 0 & H_3 \\ LH_1 & H_2 & 0 \end{bmatrix},
d = [d_1 \delta \ddot{d}_2]^T ,
C = [C_1 \ 0] .
\]
It is noted that we denote by a matrix \( M < 0 \) the \( M \) is a negative definite matrix[10].

Therefore, for given parameters \( \lambda > 0, \gamma > 0 \), if there are real symmetric matrices \( Q_1 > 0, \ P_2 > 0, \ R_1 \), \( R_2 \), such that
\[
\begin{bmatrix} \Pi_1 & F & H_1 & 0 & H_3 & Q_1U_1^T & M_1V \\ * & -1/\lambda^2 I & 0 & 0 & 0 & 0 & 0 \\ * & * & -\gamma^2 I & 0 & 0 & H_1^T R_2^T & 0 \\ * & * & * & -\gamma^2 I & 0 & H_2^T P_2^T & 0 \\ * & * & * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & * & * & * & \Pi_2 \end{bmatrix} < 0
\]
(20)
where * represents the corresponding elements in the symmetric matrix, \( R_1 = KQ_1 \), \( R_2 = P_2L \),
\[
\Pi_1 = G_0Q_1 + Q_1^T G_0^T + KQ_1 + Q_1^T K^T , \quad \Pi_2 = P_2W + W^T P_2^T + P_2LM_1V + V^T M_1^T L^T P_2 ,
\]
then the nonlinear system represented by equation (19) is robust and asymptotically stable when the disturbance \( d \) is absent, and satisfies \( \|\ddot{d}\|_2 < \gamma \|d\|_2 \). If formula (20) is true, it needs to be satisfied \( \Pi_1 < 0 \), \( \Pi_2 < 0 \), and \( K \) and \( L \) can be obtained by linear matrix inequality (LMI).

4. Numerical Simulation

Table 1 shows the nominal parameters of the AUV model. The initial conditions of the AUV, the designed parameters for the proposed control law and the bounds of control forces and virtual yaw speed are set as Table 2.

| Table 1 The nominal parameters of the underactuated AUV [11] |
|-------------------------------------------------------------|
| \( m = 185 \text{kg} \), \( I_z = 50 \text{kgm}^2 \), \( X_u = -30 \text{kg} \), \( Y_v = -80 \text{kg} \), \( N_r = -30 \text{kgm}^2 \), \( X_u = 70 \text{kg} / \text{s} \), \( X_u' = 100 \text{kg} / \text{m} \), \( Y_v' = 100 \text{kg} / \text{m} \), \( Y_v' = 200 \text{kg} / \text{m} \), \( N_r = 50 \text{kgm}^2 / \text{s} \), \( N_r' = 100 \text{kgm}^2 \) |

| Table 2 Kinds of parameters of the underactuated AUV |
|-----------------------------------------------------|
| Initial position and posture of the underactuated AUV |
| \( x(0) = 15 \text{m} \), \( y(0) = -15 \text{m} \), \( u(0) = 0.1 \text{m/s} \), \( v(0) = 0 \text{m/s} \), \( r(0) = 0 \text{rad/s} \), \( \psi(0) = \pi / 2 \text{rad} \), \( s(0) = 0 \text{m} \), \( u_d = 1 \text{m/s} \) |
| The bounds of control force and virtual yaw speed |
| \( \tau_{u\max} = 200 \text{N} \), \( \tau_{u\min} = 0 \text{N} \), \( r_{d\max} = 0.6(\text{rad} / \text{s}) \), \( r_{d\min} = -0.6(\text{rad} / \text{s}) \) |
The proposed controller \( k_1 = 10, \ k_2 = 5, \ k_4 = 2, \ k_{x_1} = k_{x_2} = 1.6, \ h = 0.1 \text{s}^{-1}, \ R = 100 \text{m/s}^2 \)

\[ T = 0.01 \text{s}^{-1} \]

The underactuated AUV is required to follow a desired path described by (21) with the parameters shown in Table 3.

\[
x_F(\mu) = \sum_{i=0}^{4} a_i \mu^i, \quad y_F(\mu) = \sum_{i=0}^{4} b_i \mu^i
\]

Table 3 Desired path parameters

| Parameter | \( i=0 \) | \( i=1 \) | \( i=2 \) | \( i=3 \) | \( i=4 \) |
|-----------|----------|----------|----------|----------|----------|
| \( a_i \) | 0        | 0.87     | -0.02    | 10^{-5}  | 1.5 \times 10^{-6} |
| \( b_i \) | 0        | 0.5      | -5 \times 10^{-4} | 10^{-5} | 10^{-7} |

Select \( W = \begin{bmatrix} 0 & 0.5 \\ -0.5 & 0 \end{bmatrix}, \ V = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.5 \end{bmatrix} \) to make \( d_0 \) can be used to describe the harmonic disturbances, and the \( L \) and \( K \) can be calculated by LMI.

\[
L = \begin{bmatrix} -134.375 & 0 & -0.2261 \\ 0.3799 & 0 & -80 \end{bmatrix}, \quad K = \begin{bmatrix} -k_5 & 0 & 0 \\ 0 & 0 & -k_3 \end{bmatrix} = \begin{bmatrix} -0.5 & 0 & 0 \\ 0 & 0 & -0.5 \end{bmatrix}
\]

The rest disturbances \( d_1 \) can be set as

\[
\begin{align*}
d_{iu} &= 0.01 \sin(0.1t) + 0.01 \\
d_{iv} &= 0.01 \cos(0.1t) - 0.01 \\
d_{ir} &= 0.01 \sin(0.2t) + 0.01
\end{align*}
\]

The simulation results are shown in Fig. 2(a)-(d). It can be seen that the novel controller can make the AUV follow the desired path with high precision in Fig. 2(a). The Fig. 2(b) shows that the path following error asymptotically converges to neighborhood of zero. In Fig. 2(c), the surge speed \( u \) converges to \( u_d \) with great speed, and the sway speed and yaw speed are bounded. The Fig. 2(d) illustrate that the control input \( r_\tau \) can be limited to an appropriate range by placing restrictions on the virtual of virtual yaw speed \( r_d \), and the control input \( \tau_u \) also in an appropriate range.
5. Conclusions
This paper proposes a novel path following control scheme for the underactuated AUV with input saturation and multiple disturbances. By considering the characteristic of disturbances, which include external disturbances, model parameter uncertainties, unmodeled dynamics and other random disturbances, we assumed that the disturbances were divided into two types. The first type with partial known information was estimated by the disturbance observer and the other satisfied the $L_2$-norm bounded conditions was minimized by an $H_{\infty}$ based path-following controller. Besides, when the path-following controller was designed, the input saturation was considered, and the $\dot{\beta}$ was regarded as uncertainty which was on-line estimated by ESO, which reduced the reliance on models and improved the practicability of the AUV. Numerical simulations validate the effectiveness of the proposed controller.

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