Two energy scales and close relationship between the pseudogap and superconductivity in underdoped cuprate superconductors

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By measuring the low temperature specific heat, the low energy quasi-particle excitation has been derived and analyzed in systematically doped La$_{2-x}$Sr$_x$CuO$_4$ single crystals. The Volovik’s relation predicted for a d-wave superconductor has been well demonstrated in wide doping regime, showing a robust evidence for the d-wave pairing symmetry. Furthermore the nodal gap slope $v_\Delta$ of the superconducting gap is derived and is found to follow the same doping dependence of the pseudogap obtained from ARPES and tunnelling measurement. This strongly suggests a close relationship between the pseudogap and superconductivity. Taking the entropy conservation into account, we argue that the ground state of the pseudogap phase should have Fermi arcs with finite density of states at zero K, and the transport data show that it behaves like an insulator due to probably weak localization. A nodal metal picture for the pseudogap phase cannot interpret the data. Based on the Fermi arc picture for the pseudogap phase it is found that the superconducting energy scale or $T_c$ in underdoped regime is governed by both the maximum gap and the spectral weight from the Fermi arcs. This suggests that there are two energy scales: superconducting energy scale and the pseudogap. The superconductivity may be formed by the condensation of Fermi arc quasiparticles through pairing by exchanging virtue bosons.

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Since the discovery of the cuprate superconductors, about 20 years have elapsed without a consensus about its mechanism. Many exotic features beyond the Bardeen-Cooper-Schrieffer theory have been observed. One of the core issues is about the origin of the pseudogap (PG) and its relationship with the superconductivity. One scenario assumes that the PG (with the energy scale $\Delta_p$) marks only a competing or coexisting order with the superconductivity and it has nothing to do with the pairing origin. However other pictures, typically the Anderson’s resonating-valence-bond (RVB) model predicts that the spin-singlet pairing in the RVB state (which causes the formation of the PG) may lend its pairing strength to the mobile electrons and make them naturally pair and then condense at $T_c$. According to this picture there should be a close relationship between the PG and the superconductivity.

In order to get a deeper insight about the relationship between the PG and superconductivity, we need to collect the information for the PG and the superconducting energy scale, especially their doping dependence. The PG values $\Delta_p$ or its corresponding temperature $k_B T^* (\propto \Delta_p)$ and its doping dependence have been measured through experiments. To determine the superconducting energy scale, we note that the normal state Fermi surface is formed by four small arcs near the nodal points. As temperature is lowered below $T_c$, a new gap opens on these arcs. To illustrate this point more clearly, in Fig. 1 we present a schematic plot for different gaps or energy scales. The dashed blue line represents the gap structure of the PG state, assuming the presence of Fermi arcs near the nodal points. The region of zero gap corresponds to the Fermi arc at zero K if the superconductivity would be suppressed completely. The red solid line represents the general quasiparticle (QP) gap on the Fermi arcs in the superconducting state. In superconducting state, the general QPs gap may construct a standard $\Delta$-wave gap. Based on this picture, we see that the nodal gap slope, which is defined as $v_\Delta = |d\Delta_s/d\theta|_{\text{node}} / h k_F$, can be used to determine the superconducting energy scale. The relationship between $v_\Delta$ and the maximum PG $\Delta_p$ remains to be a big puzzle. In particular, the two quantities may be suppressed completely.

FIG. 1: Schematic plot for the general quasiparticle gap, the PG energy (dashed blue line), superconducting energy scale (red solid curve) and nodal gap slope at $\theta = 45^\circ$. The angle $\theta$ counts from the $k$-space axis $k_z$, $p$ represents the hole concentration.
be independent of each other if the superconductivity is not induced by the formation of the PG. Therefore to measure the nodal gap slope \( v_\Delta \) near nodal point in the zero temperature limit becomes highly desired. When combined with the known results on the PG \( \Delta_p \), this will allow us to tell whether there is a relationship between the PG and the superconductivity. In this paper, we report the evidence of a proportionality between \( v_\Delta \) and the PG temperature \( T^* \). We also find that \( T_c \) is governed by both \( v_\Delta \) and the spectral weight from the Fermi arcs (\( k_{\text{arc}} \)).

We determine the properties of the nodal quasiparticles by measuring low temperature electronic specific heat on systematically doped \( \text{La}_{2-x}\text{Sr}_x\text{CuO}_4 \) single crystals (\( p = 0.063, 0.069, 0.075, 0.09, 0.11, 0.15, 0.22 \)). Details about the sample characterization, the specific heat measurement, the residual linear term and extensive analysis were reported in our recent papers [4, 5]. Here we present a further analysis of these data. The full squares in Fig. 4 represent the transition temperatures of our samples. In all measurements the magnetic field was applied parallel to \( c \)-axis.

One of the important discovery in our measurement is that the low temperature specific heat coefficient \( \gamma = C_v/T \) always increases with the magnetic field. An example for the very underdoped sample (\( p = 0.069 \)) is shown in Fig. 2. This behavior is in sharp contrast with the low temperature thermal conductivity data [4, 7] which shows an unchanged or even decline of the thermal conductivity \( \kappa_0/T \). Our data clearly show that the field has induced new quasiparticles, although they are localized leading to the decrease of the thermal conductivity and the diverging of the resistivity. In this sense the ground state when the superconductivity is suppressed completely has finite density of states at the Fermi level although it is insulating with localized quasiparticles.

Next we show the robust evidence of \( d \)-wave pairing symmetry in regimes from very underdoped to very overdoped. It has been widely perceived that the pairing symmetry in the hole doped cuprate superconductors is of \( d \)-wave with line nodes in the gap function. In the mixed state, due to the presence of vortices, Volovik [8] pointed out that supercurrents around a vortex core lead to a Doppler shift to the QP excitation spectrum. This will dominate the low energy QP excitation and the specific heat (per mol) behaves as \( C_{\text{vol}}/T = A H^{1/2} \) with \( A \propto 1/v_\Delta \). This square-root relation has been verified by many measurements which were taken as evidence for \( d \)-wave symmetry. In this way one can determine the nodal gap slope \( v_\Delta \). Since the phonon part of the specific heat is independent on magnetic field, this allows to remove the phonon contribution by subtracting the \( C/T \) at a certain field with that at zero field, one has \( \Delta \gamma = \Delta C/T = |C(H) - C(0)|/T \). For a \( d \)-wave superconductor, in the zero temperature limit \( \Delta \gamma = \Delta C/T = C_{\text{vol}}/T = A H^{1/2} \) is anticipated.

In order to get \( \Delta \gamma \) in the zero temperature limit, we extrapolate the low temperature data of \( C/T \) vs. \( T^2 \) (between 2 K to 4 K) to zero K. The data taken in this way and normalized at 12 T are presented in our recent papers [4, 5]. It is found that the Volovik’s \( H^{1/2} \) relation describes the data rather well for all doping concentrations. Furthermore we can determine the prefactor \( A \) in \( \Delta \gamma = \Delta C/T = C_{\text{vol}}/T = A H^{1/2} \) and \( v_\Delta \). Fig. 3(a) shows the doping dependence of the pre-factor \( A \). The error bar is obtained by fitting the extracted zero temperature data to \( \Delta \gamma = A H^{1/2} \). For a typical \( d \)-wave superconductor, by calculating the Dirac fermion excitation spectrum near the nodes, it was shown that [4]

\[
A = \alpha_p 4k_F^2 \sqrt{\frac{\pi}{3\hbar c}} \frac{nV_{\text{mol}}}{\Phi_0} \frac{1}{v_\Delta} \tag{1}
\]

here \( l_c = 13.28 \) Å is the \( c \)-axis lattice constant, \( V_{\text{mol}} = 58 \) cm\(^3\) (the volume per mol), \( \alpha_p \) a dimensionless constant taking 0.5 (0.465) for a square (triangle) vortex lattice, \( n = 2 \) (the number of Cu-O plane in one unit cell), \( \Phi_0 \) the flux quanta. The \( v_\Delta \) has then been calculated without any adjusting parameter (taking \( \alpha_p = 0.465 \)) and shown in Fig. 3(b). It is remarkable that \( v_\Delta \) has a very similar doping dependence as the PG temperature \( T^* \), indicating that \( v_\Delta \propto T^* \propto \Delta_p \). If converting the data \( v_\Delta \) into the virtual maximum quasiparticle gap (\( \Delta_q \)) via \( v_\Delta = 2\Delta_q/k_F \), here \( k_F = \pi/\sqrt{2a} \) is the Fermi vector of the nodal point with \( a = 3.8 \) Å (the in-plane lattice constant), surprisingly the resultant \( \Delta_q \) value [shown by the filled squares in Fig. 3(b)] is related to \( T^* \) in a simple way (\( \Delta_q \approx 0.46k_BT^* \)). It is important to emphasize that this result is obtained without any adjusting parameters. Counting the uncertainties in determining \( T^* \)
and the value of $\alpha_p$, this relation is remarkable since $\Delta_q$ and $T^*$ are determined in totally different experiments. Because $v_\Delta$ (or $\Delta_q$) reflect mainly the information near nodes which is predominantly contributed by the superconductivity, above discovery, i.e., $v_\Delta \propto T^* \propto \Delta_p$ (or $\Delta_q \approx 0.46k_BT^*$) strongly suggests a close relationship between the PG and superconductivity.

In above discussion, we see the consistency between our low temperature specific heat data and the Volovik’s relation $\Delta \gamma = A \sqrt{H}$. This seems surprising since the temperature range considered here is about several Kelvin. At such an energy scale, the impurity scattering will strongly alter the DOS in the low energy region. However, by applying a magnetic field, the Doppler shift of the quasiparticle excitation spectrum will contribute a new part to DOS. As argued in our recent paper, this energy shift can be described by $\Delta \varepsilon = 3.67\alpha_{FL}\sqrt{B/|H|}$ meV. For example, taking the maximum field (12 T) in our experiment, we get $\Delta \varepsilon = 12.2\alpha_{FL}$ meV which is actually a relatively large energy scale compared to the temperature since $\alpha_{FL} \approx 1.3$. This may explain why the Volovik’s simple relation $\Delta \gamma = A \sqrt{H}$ can be easily observed in our single crystals with inevitable certain amount of impurities.

In the following we will investigate what governs $T_c$. Baring the doping dependence of $v_\Delta$ in mind, it is easy to understand that $v_\Delta$ should not be a good estimate of the superconducting energy scale for the under-doped samples since the $T_c$ and $v_\Delta$ have opposite doping dependence. The basic reason is that the normal-state Fermi surface contains small arcs of length $k_{arc}$ near the nodal points. The superconducting transition occurs by forming extra gaps on these Fermi arcs. So the effective superconducting energy scale should be estimated as $E_s \approx 1/2v_\Delta k_{arc}$. From the normal state electronic specific heat $C_{el} = \gamma_n T$, we have $\gamma_n = 4\pi k_B^2 k_{arc} V_{mol}/hv_{F el}$. Assuming $E_s \approx k_BT_c$ we find

$$T_c = \frac{\hbar v_{F el c} \gamma_n v_\Delta}{\alpha_{s} 8\pi k_B^2 V_{mol}} = \beta \gamma_n v_\Delta \quad (2)$$

where $\alpha_s$ is a dimensionless constant in the order of unity, $v_{F el}$ is the nodal Fermi velocity normal the Fermi surface. The value of $\gamma_n(0)$ can be estimated from specific heat [11]. Here we take the values for $\gamma_n(0)$ summarized by Matsuzaki et al. [11] and fit it (in unit of mJ/mol K$^2$) with a formula $\gamma_n = \zeta(p - p_c)^\eta$ yielding $\zeta = 182.6, p_c = 0.03, \eta = 1.54$. In the inset of Fig. 4 we present the doping dependence of the zero-temperature specific heat coefficient $\gamma_n(0)$ from which one can calculate $k_{arc}$. In the

![FIG. 3: (a) Doping dependence of the pre-factor $A$ determined in present work (circles). Here the point at $p = 0.19$ was adopted from the work by Nohara et al. on a single crystal [10]. (b) Doping dependence of the PG temperature $T^*$ (open symbols) summarized in Ref. 1 (see Fig. 26 there) and our data $v_\Delta$ (solid line). The full squares represent the calculated virtual maximum quasi-particle gap $\Delta_q$ derived from $v_\Delta$ without any adjusting parameters. Surprisingly both sets of data are correlated through a simple relation $\Delta_q \approx 0.46k_BT^*$ although they are determined in totally different experiments. This result implies a close relationship between the PG $\Delta_p$ and the nodal gap slope $v_\Delta$.](image)

![FIG. 4: Doping dependence of the measured $T_c$ (full squares) and that calculated one by $T_c = \beta v_\Delta \gamma_n(0)$ (open squares) with $\beta = 0.7445$ K$^3$ mol s/3 m. The solid line represents the empirical relation $T_c = T_c^{max} = 1 - 82.6(p - 0.16)^2$ with $T_c^{max} = 98$ K. The inset shows the residual value of $\gamma_n(0)$ of the PG state, the solid line is a fit to $\gamma_n = \zeta(p - p_c)^\eta$ with $\zeta = 182.6, p_c = 0.03$, and $\eta = 1.54$.](image)
FIG. 5: Schematic plot for temperature dependence of the electronic specific heat coefficient $C_e/T$ for (a) “arc metal” with a finite DOS at $T = 0$ and (b) “nodal metal” for the PG phase. By applying a magnetic field, in the superconducting state at $T = 0$, it is found that the DOS increases, which is not consistent with the expectation of the nodal metal ground state for the PG phase. The two short dashed lines in the zero temperature limit illustrate how does $C_e/T$ change with increasing magnetic field (from bottom to up) if the PG ground state is (a) an “arc metal” or (b) a “nodal metal”.

main frame of Fig. 4 we present the doping dependence of the measured $T_c$ (filled squares) and the calculated value (open squares) by eq. (2) with $\beta = 0.7445$ K$^3$ mol s/J m. In underdoped region, the measured and calculated $T_c$ values coincide rather well implying the validity of eq. (2). So the energy scale of the superconductivity is not given by $v_\Delta \hbar k_F$, but by $E_s = 1/2v_\Delta \hbar k_{arc}$ or more precisely by eq. (2) in the underdoped region. In overdoped region, $\gamma_n (0)$ will gradually become doping independent, therefore one expects $T_c \propto \Delta_q \propto v_\Delta$.

Our discussion here is totally based on the assumption of the Fermi arc ground state for the PG phase. Although some preliminary evidence for the existence of Fermi arcs has been found by ARPES in, in the following we argue that the Fermi “arc metal” instead of the “nodal metal” is the ground state of the PG phase. From the specific heat data of Matsuzaki et al.\textsuperscript{[11]} and Loram et al.\textsuperscript{[12]} one can see that the PG phase (when the superconductivity is completely suppressed) has actually a finite DOS at zero K based on the entropy conservation consideration, as shown by Fig. 5(a). This finite DOS at zero K of the PG phase can be interpreted as due to two possible reasons: either induced by the impurity scattering of the nodal quasiparticles of a $d$-wave PG (if it would be a “nodal metal”), or given by the zero-temperature Fermi arcs of the PG. For a “nodal metal” ground state, no extra quasiparticles can be generated by increasing the magnetic field in the zero temperature limit (as shown in Fig. 5(b)). However as shown in Fig. 2, our specific heat data clearly show that there are extra DOS generated by applying the magnetic field which is just the case shown by Fig. 5(a). Therefore from experiments, it is evident that the specific heat behaves in the way as Fig. 5(a) for an “arc metal” instead of Fig. 5(b) for a “nodal metal”. It is thus tempting to conclude that there are Fermi arcs near nodes in the PG phase in the zero temperature limit.

In summary, a close relationship between the PG and superconductivity has been found. Based on the Fermi arc picture of the PG phase, it is found that the superconducting energy scale in underdoped regime is governed by both the maximum gap and the spectral weight from the Fermi arcs. This suggests that there are two energy scales and the superconductivity may be formed by the Fermi arc quasiparticles through pairing mediated by exchanging virtue bosons, such as the spin interaction or phonons.

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