Phonon-modified spontaneous emission from single quantum dots in a structured photonic medium

Kaushik Roy-Choudhury and Stephen Hughes

Department of Physics, Queen’s University, Kingston, Ontario, Canada, K7L 3N6
(Dated: July 9, 2014)

We describe how a structured photonic medium controls the spontaneous emission rate from an excited quantum dot in the presence of electron-phonon coupling. We analyze this problem using a polaron transformed master equation and we consider specific examples of a photonic crystal cavity and a coupled cavity waveguide. We find that when the relaxation times of the photon and phonon baths are comparable, phonons influence spontaneous emission in a non-trivial way. We demonstrate why and how the broadband frequency dependence of the local photon density of states determines the photon emission rate, manifesting in a complete breakdown of Fermi’s golden rule. For a single cavity resonance, we generalize Purcell’s formula to include the effects of electron-phonon coupling. For a waveguide, we show a suppression and a 200-fold enhancement of the photon emission rate.

PACS numbers: 42.50.-p, 42.50.Ct, 42.50.Nn, 78.67.Hc

Semiconductor quantum dots (QDs) coupled to structured photonic reservoirs such as photonic crystals, provide a promising platform for tailoring light-matter interaction in a solid-state environment and have applications for on-chip quantum information processing [1]. However, electron-phonon coupling in solid-state media has been shown recently to significantly modify the emission properties of a QD as compared to an isolated atom [2]. Studying the role of phonons in governing the emission properties of QDs has been an intense area of research, leading to a number of interesting effects beyond a simple pure-dephasing model [3–5]. For field-driven QD excitons yielding Rabi oscillations, phonon coupling manifests in damping and frequency shifts [6–8]. In QD-cavity systems, phonons cause intensity-dependent broadening of Mollow side-bands [9], off-resonant cavity feeding [10] and asymmetric vacuum Rabi doublets [11, 12]. Theoretical descriptions of electron-phonon scattering in QDs systems include the independent Boson model [5], perturbative master equations (MEs) [10, 13], polaron MEs [4, 14–17], variational MEs [18] and real-time path integrals [19].

One of the primary interests in coupling QDs to structured reservoirs is for modifying its spontaneous emission rate (SE), $\gamma$, via the Purcell effect [20, 21]. Photonic crystals are a paradigm example of a structured photonic reservoir, and both photonic crystal cavities (Fig.1(a)) and coupled-cavity optical waveguide (CROW, Fig.1(b)) structures have been investigated for modifying QD SE rates [21–24]. For an unstructured reservoir, $\gamma$ remains unchanged in the presence of phonons [25]. For structured reservoirs, previous theories have always assumed phonon processes to be much faster than all relevant system dynamics [3, 16], thus restricting them to structures with sharp variations of photonic local density of states (LDOS) (e.g. high Q cavity, photonic band edge). A primary example of a structured reservoir is a cavity and existing theories [4, 15, 16] treat the cavity mode as a system operator and find that phonons modify the QD-cavity coupling rate, $g \rightarrow \langle B \rangle g$ [4, 14], where $\langle B \rangle$ is the thermal average of the coherent phonon bath displacement operators $B_{\pm}$ [4]. Hence the Purcell factor is believed to scale as $g^2 \rightarrow \langle B \rangle^2 g^2$ [3]. However, such theories do not apply to large $\kappa$ cavities, where $\kappa$ is the cavity decay rate, and one would expect to recover the result that $\gamma$—and thus $g$—are not affected by phonons. Moreover, for an arbitrary photonic bath medium, it is not known how phonons affect the SE rates, yet clearly such an effect is of significant fundamental interest and also important for understanding emerging experiments.

In this letter we introduce a self-consistent ME approach with both phonon and photon reservoirs included and we explore in detail the influence of a photon reservoir on the phonon-modified SE rate. We show how one can substantially tune the phonon modification by tailor-
ing the properties of the photonic LDOS. The frequency dependence of the LDOS is found to dictate how phonons modify the SE rates, causing a clear breakdown of Fermi’s golden rule which depends on the LDOS at the emitter’s frequency. Our theory can be applied to any general LDOS function and as specific examples, we consider cavities and coupled cavity waveguides—see Fig. 1 which also shows an energy level diagram of the QD system including both phonon and photon baths.

We model the QD as a two-level system interacting with an inhomogeneous semiconductor-based photonic reservoir and an acoustic phonon bath [4]. Assuming the QD of dipole moment $\mathbf{d} = \hat{d}\mathbf{n}_d$ at spatial position $\mathbf{r}_d$, the total Hamiltonian of the system in a frame rotating at the QD excitation frequency $\omega_x$, is [26]

$$
H = \hbar \int d\mathbf{r} \int_0^\infty d\mathbf{f}(\mathbf{r}, \omega) \mathbf{f}(\mathbf{r}, \omega) + \sum_q \hbar \omega_q \mathbf{b}_q^\dagger \mathbf{b}_q
- \frac{\sigma^+ e^{i\omega_x t}}{\hbar} \int_0^\infty d\omega \mathbf{d} \cdot \mathbf{E}(\mathbf{r}_d, \omega) + H.c
+ \sigma^+ \sigma^- \sum_q \hbar \lambda_q (\mathbf{b}_q^\dagger + \mathbf{b}_q),
$$

(1)

where $\sigma^+ / \sigma^-$ are the Pauli operators of the exciton (electron-hole pair), $\mathbf{b}_q^\dagger / \mathbf{b}_q$ are the annihilation and creation operators of the acoustic phonon reservoir and the exciton-phonon coupling strength $\lambda_q$ is assumed to be real. The operators $\mathbf{f} / \mathbf{f}^\dagger$ are the boson field operators of the photon reservoir and these satisfy the usual commutation rules for boson operators. The interaction between the QD and the photonic reservoir is written using the dipole and the rotating wave approximation. The electric-field operator $\mathbf{E}(\mathbf{r}, \omega)$ is given by $\mathbf{E}(\mathbf{r}, \omega) = i \int d\mathbf{r}' \mathbf{G}(\mathbf{r}, \mathbf{r}'; \omega) \sqrt{\frac{\hbar}{2\pi}} e^{i\mathbf{E}(\mathbf{r}', \omega) \mathbf{f}(\mathbf{r}', \omega)}$ [26], where $\mathbf{G}(\mathbf{r}, \mathbf{r}'; \omega)$ is the electric field Green’s function for the medium and $\mathbf{E}(\mathbf{r}, \omega)$ satisfies the Kramers-Kronig relations, with a complex dielectric constant $\varepsilon = \varepsilon_R + i\varepsilon_I$.

To include phonon interactions nonperturbatively, we perform the polaron transform on the Hamiltonian $\hat{H}$ given by $\hat{H}' = e^{\hat{P}} \hat{H} e^{-\hat{P}}$ where $P = \sigma^+ \sigma^- \sum_q \frac{\omega_q}{\varepsilon_R} (\mathbf{b}_q^\dagger - \mathbf{b}_q)$ [27]. Assuming weak coupling with the photon bath, a time-convolutionless [28] polaron ME for the QD reduced density operator $\rho$ is then derived using the Born approximation. The usual incoherent terms from the phonon reservoir can be written as $\frac{\partial \rho_\text{ph}}{\partial t}_\text{inc} = \mathcal{L}_\text{ph}(\rho)$. However, starting from Eq. (1) we find that the photonic reservoir is now modified by the phonon bath and is given by

$$
\mathcal{L}_\text{ph}(\rho) = \int_0^t dt' \int_0^\infty d\omega J_{\text{ph}}(\omega)(-C_{\text{ph}}(\tau)\sigma^+ \sigma^- e^{i\Delta_x \tau} \rho
+ C^*_{\text{ph}}(\tau)\sigma^- \rho \sigma^+ e^{-i\Delta_x \tau} + C_{\text{ph}}(\tau)\sigma^- \rho \sigma^+ e^{i\Delta_x \tau}
- C^*_{\text{ph}}(\tau)\rho \sigma^- \sigma^+ e^{-i\Delta_x \tau}),
$$

(2)

where $\Delta_x = \omega_x - \omega$, and a trace over the phonon [4] and photon variables [29] has been performed where the phonon and reservoir operators are assumed to be in thermal equilibrium and statistically independent [30]. The photon-reservoir spectral function $J_{\text{ph}}(\omega) = \frac{d \text{Im} \{G_{\mathbf{r}_d, \mathbf{r}_d}(\omega)\}}{\pi \hbar \omega}$, and the phonon bath correlation function $C_{\text{ph}}(\tau)$ is defined as $C_{\text{ph}}(\tau) = e^{i\phi(\tau) - \phi(0)}$ where $\phi(t) = \int_0^\infty d\omega \frac{\omega}{\sinh(\omega/2k_B T)}\cos(\omega t) - i \sin(\omega t)$, and $J_{\text{ph}}(\omega)$ is the phonon spectral function [31]. For the calculations, we use the continuous form of the phonon spectral function $J_{\text{ph}}(\omega) = \alpha_{\text{ph}} \omega \exp[-\omega^2]$ for longitudinal acoustic (LA) phonon interaction, and use experimental numbers consistent with InAs QDs [32]; below we use a dipole moment $d = 50$ Debye, phonon cutoff frequency $\omega_b = 1$ meV, and exciton-phonon coupling strength $\alpha_{\text{ph}}/(2\pi)^2 = 0.06$ ps$^{-2}$ [31]. For simplicity, a polaron shift $\Delta_{\rho}$ is implicitly absorbed in the definition of $\omega_x$, and we also define a phonon correlation function that decays to zero, through $C_{\text{ph}}(\tau) = e^{i\phi(\tau) - 1}$. A simple expression for the phonon-modified SE decay rate can be derived from the real part of $\mathcal{L}_\text{ph}$, so that $\text{Re}(\mathcal{L}_\text{ph})$ reduces to a familiar Lindblad form $[\gamma(t)\sigma^-] \sigma^+$ for SE decay of a QD, with the SE decay rate given by

$$
\gamma(t) = 2 \int_0^t \text{Re}[C_{\text{ph}}(\tau) J_{\text{ph}}(\tau)] dt,
$$

(3)

where $J_{\text{ph}}(\tau) = \int_0^\infty d\omega J_{\text{ph}}(\omega) e^{i(\omega_x - \omega)\tau}$ is the photon bath correlation function, and $L[O] = \frac{1}{2}L_\text{ph}[O_{\rho} - \rho O^\dagger O]$.

Note that the imaginary part of $\mathcal{L}_\text{ph}$ yield Lamb shifts [31]. In the Markov limit ($t \to \infty$), Eq. (3) generalises Fermi’s golden rule for SE, since the LDOS at various frequencies can now contribute to the phonon-modified $\gamma$. In absence of the phonon coupling, the SE decay rate of the QD in a structured phonon reservoir reduces to $\gamma(t) = 2 \int_0^t \text{Re}[J_{\text{ph}}(\tau)] dt$, where $\gamma(t \to \infty) \propto \text{LDOS}(\omega_x)$. McCutcheon and Nazir [25] also use a similar approach to argue that $\dot{\gamma} \to \gamma$ for a free space medium.

To study how phonons modify the SE rates in a structured photonic reservoir, we first consider a Lorentzian cavity similar to that shown in Fig. 1(a). This LDOS will likely sit on top of some background photonic reservoir, which can also be included in our theory. For a single cavity mode, in a dielectric cavity with $\varepsilon = n_g^2$, then

$$
J_{\text{ph}}(\omega) = g^2 \frac{1}{\pi} \frac{\omega}{(\omega - \omega_c)^2 + (\frac{\delta}{2})^2},
$$

(4)

where $g = \sqrt{\frac{d^2 \omega}{2\hbar n_0}} V_{\text{eff}}^{1/2}$ is the QD-cavity coupling rate, and we assume that the QD has its dipole aligned with the cavity mode polarization and is positioned at the field antinode. This allows us to characterize the coupling strength in terms of the effective mode volume, $V_{\text{eff}}$. Defining the long-time SE rate as $\gamma \equiv \gamma(t \to \infty)$, then from Eq. (3) and Eq. (4), we obtain

$$
\dot{\gamma} = 2g^2 \langle B \rangle^2 \text{Re} \left[ \int_0^\infty e^{i\phi(\tau) - i\Delta_{\rho} \tau - \kappa \tau/2} d\tau \right],
$$

(5)
where $\langle B \rangle = \exp[-\frac{1}{2}\phi(0)]$ [14, 27] and $\Delta_{xc} = \omega_c - \omega_0$. We now generalize Purcell’s formula [PF] for the enhanced SE rate of a QD in a semiconductor cavity:

$$PF_{QD} = \left[ \frac{3}{4\pi^2} \left( \frac{\lambda_0}{\hbar_0} \right)^3 \frac{Q}{\Gamma_{\text{eff}}} \left( \Delta_{\text{eff}}^2 + \kappa_n^2 \right) \right] \chi(T),$$

where $\lambda_0 = \omega_c/(2\pi c)$ and $Q = \omega_c/\kappa$ is the quality factor. We have included the explicit detuning dependence from the cavity and introduced the phonon-modification factor, $\chi \equiv \tilde{\chi}/\chi_0 = (\frac{1}{2} + \frac{1}{2} \Delta_{\text{eff}}^2) (B^2 g_0^2 \int_0^\infty d\tau \exp(-\Delta_{xc} + \kappa \tau/2), which also depends on temperature through $\langle B \rangle$ and $\phi$. In the limit $\kappa \to \infty$, then $\chi = 1$ [25].

It is interesting to compare Eq. (5) with previous polaron ME approaches that treat the cavity mode operator at the level of a system operator, with $\kappa$ and $\gamma_0$ introduced phenomenologically, where $\gamma_0$ does not include cavity coupling; here one also finds that $g \to \langle B \rangle g$, and for weak fields, the analytical phonon-mediated cavity scattering rates in [4] are written in terms of cavity feeding ($\Gamma^{\sigma^+ \sigma^-}$) and exciton feeding ($\Gamma^{\sigma+ \sigma^+}$), defined as $\Gamma^{\sigma^+ \sigma^-} \equiv 2 \langle B \rangle^2 g e^{\phi(\tau)} - \Delta_{xc} + \kappa \tau/2), [31], with corresponding Lindblad terms $L[a^\dagger \sigma^-]$ and $L[\sigma^\dagger a]$. Using the cavity-QED equations [31] in the weak driving limit, we derive the SE rate analytically, yielding

$$\tilde{\chi}_{\text{WEA}} = \Gamma^{\sigma^+} + 2g^2 \langle B \rangle^2 \frac{\kappa + \Gamma^{\sigma^+ \sigma^-}}{\Delta_{xc}^2 + (\kappa + \Gamma^{\sigma^+ \sigma^-})^2}. \quad (7)$$

For a cavity with $\kappa \gg \Gamma^{\sigma^+ \sigma^-}$, $\tilde{\chi}_{\text{WEA}}$ agrees with $\tilde{\gamma}$ in Eq. (5) in the limit $\kappa^{-1} \gg \tau_{pn}$ (where $\tau_{pn}$ is the phonon relaxation time). Our theory thus not only recovers previous cavity-QED results [4, 31] in the appropriate limit, but also reveals a fundamental limitation of these approaches for sufficiently large $\kappa$ cavities. Specifically, when the cavity relaxation time becomes comparable to the phonon relaxation time, or smaller, these formalisms break down. More importantly, our current formalism can be applied to any LDOS medium and is therefore not limited to simple Lorentzian cavity structures.

For phonon bath temperatures of 4 K and 40 K, Figs. 2(a) and 2(b) show the phonon correlation function versus time and frequency, respectively. The time evolution of the real part of the phonon correlation function shows that typical phonon correlation times are very fast ($\tau_{pn} \leq 5$ ps). For comparison, we also show a photon correlation function in Figs. 2(c) and 2(d), for several different values of $\kappa$; the cavity bath correlation functions $\langle p_h(t) \rangle$ are oscillatory functions damped at the cavity decay rate, with an oscillation frequency is determined by the QD-cavity detuning $\Delta_{xc}$. For the case of zero detuning, we show results for cavities with $\kappa = 2.4$ meV (thick light line), 0.6 meV (thin dark line), and 0.06 meV (thin light line). From Eq. (5) and Fig. 2, we expect that (i) phonons should not influence the SE rate $\gamma$ in a strongly damped cavity and (ii) phonons will reduce the SE rate to its mean-field limit [3] ($\langle B \rangle^2$) only for a weakly damped cavity (e.g., with $\kappa = 0.06$ meV, which for a cavity frequency of $\omega_c/2\pi = 1440$ meV yields a $Q \approx 24,000$). Significant deviations from these two limits occur when the damping times of the photon and phonon correlation functions are comparable. To test this hypothesis, we next investigate the influence of phonons on the frequency-dependent SE rates.

In Fig. 3 we plot the PF (left panels) and phonon-modified SE factor $\chi$ (right panels), for the three $\kappa$ values. For each cavity, we investigate two different bath temperatures, and the dashed lines on the left panels represent PFs without phonon modification. The asymmetry in $\chi$ about the LDOS peak arises due to the fact that phonon emission is more probable than absorption [31] at low temperatures. The results can be explained by writing $\tilde{\gamma} = (B^2 \gamma + \tilde{\gamma}')$ where $\langle B \rangle^2 \gamma$ is the coherently renormalized bare SE rate and arises due to local ($\omega_s$) sampling of photonic LDOS and $\tilde{\gamma}' = \text{Re}[2 \int_0^\infty d\tau \exp(-\chi(\tau) - 1)]J_{\phi}(\tau)d\tau$, accounts for the non-local contribution. When $\kappa$ is small, $\tilde{\gamma}' \to \Gamma^{\sigma^+ \sigma^-}$. Due to the non-local component, the reduction of the SE rate is always $\geq \langle B \rangle^2$, at zero detuning. At large detunings, the non-local component dominates leading to an overall enhancement of spontaneous emission. Figures 3(b, d, f) shows that $\chi$ varies significantly over several meV. The dashed lines on right panels represent $\chi_{\text{WEA}} = \tilde{\gamma}_{\text{WEA}}/\gamma$, which evidently differs from our full calculations in the limit $\kappa \approx \tau_{pn}^{-1}$ (Figs. 3(d, f)). The structure of the photon bath (Figs. 2 (d)) is properly accounted in the full calculations (c.f. Eq. (5)) and is not approximated as a high-Q cavity, with respect to
the phonon bath (Figs. 2(b)). We highlight that low $Q$ (several hundred) cavities are commonly employed for measuring the vertical emission from QDs in planar cavities [2, 32], and for modifying the SE rates in simple photonic crystal cavities [21], and intermediate $Q$ ($\approx 3000$) cavities are used for all optical switching [33] and efficient single photon emission [34]. These cavity findings are also important as we can now anticipate a much more general result for a complex structured photonic reservoir.

We now depart from the simple cavity structure, and consider the case of a CROW (cf. Fig. 1(b)). Photonic crystal waveguides (Fig. 1(b)) are useful for slow-light propagation [35–37] and for manipulating the emission properties of embedded QDs for on-chip single photon emission [38–41], with a number of recent experiments emerging. As a representative photonic crystal waveguide we adopt a model LDOS for a CROW [42]. An analytical tight-binding technique is employed to calculate the CROW band structure [43] and it allows one to obtain the Green function and the LDOS. The photon reservoir spectral function is

$$J_{\text{ph}}(\omega) = \frac{-d^2\omega}{2\hbar\epsilon_0 n_f^2 V_{\text{eff}}} \left[ \frac{1}{\pi} \text{Im} \left( \frac{1}{\sqrt{(\omega - \omega_u)(\omega - \omega_l)}} \right) \right],$$

(8)

where $\omega_{u,l} = \omega_{u,l} \pm i\kappa_{u,l}$ [43], $\omega_{u,l}$ is the mode-edge frequencies of the waveguide (see Fig. 4(a)), $\kappa_{u,l}$ is the effective damping, and $V_{\text{eff}}$ is the mode volume of a single cavity in the CROW. In this case the photonic LDOS has a rich non-trivial structure compared to a smooth Lorentzian cavity, within the band width of the phonon bath (cf. Fig. 2(b)). For our calculations, we use parameters that closely represent a CROW made up of a local width modulation of a line-defect photonic crystal cavity [44] which yields a band structure [45] consistent with experiments [46]. In Fig. 4(a), we show the computed PF with (solid) and without (dashed) phonons. We see that phonons strongly influence the SE rates, causing a reduction at the mode edges and an enhancement inside and outside the waveguide band. Figure 4(b) shows $\chi$ at $T = 40$ K, and the slight asymmetry in phonon modification is due to unequal phonon emission and absorption rates.

The phonon modification in a waveguide can be qualitatively understood by treating $J_{\text{ph}}(\omega)$ as a sum of two Lorentzians located at the mode edges ($\omega_{u,l}$). The corresponding $J_{\text{ph}}(t) = e^{-(\omega_u + \lambda)t} + e^{-(\omega_l + \lambda)t}$ is a sum of two exponentially damped oscillatory functions, where $2\lambda$ is the bandwidth of the waveguide mode edge LDOS. At the sharp mode-edge, the contribution from local ($\omega_x$) photonic LDOS dominates and $\chi \approx \langle B^2 \rangle / 3$. Away from the mode-edge (Fig. 4(b) and 4(b) inset), SE rate is enhanced due to non-local effects. This simple model is however approximate as in reality the mode-edge LDOS is non-Lorentzian. For a symmetric Lorentzian with the same bandwidth, $\lambda^{-1} \approx 50$ ps. $J_{\text{ph}}(t)$ damps much faster than $\lambda^{-1}$ initially and damps very slowly thereafter (Fig. 4(a), inset). The long time decay rate is set by the linewidth of the sharper side of the mode-edge LDOS ($\approx 0.1$ ns). This non-Lorentzian mode-edge in turn leads to a very strong enhancement of the PF ($\times 200$) outside the waveguide band (Fig. 4(b)), compared to a symmetric Lorentzian line-shape (see Fig. 3(b)).
In conclusion, we have demonstrated how the frequency dependence of the LDOS of a photonic reservoir determines the extent to which phonons modify the SE of a coupled QD. The relative dynamics between the phonon and the photon bath correlation functions is found to play a fundamentally important role; specifically, when the relaxation times are comparable, phonons strongly modify the emission spectra leading to non-Lorentzian cavity lineshapes and even enhanced SE. These effects are not obtained using the usual Fermi's golden rule. Our formalism is important for understanding related experiments with QD-cavity systems and is broadly applicable to various photonic reservoirs. We have exemplified our theory by studying the modified SE rate from both simple cavity structures and CROWs.

This work was supported by the Natural Sciences and Engineering Research Council of Canada.

[1] H. Kim, R. Bose, T. Shen, G. Solomon, and E. Waks, Nature Photonics 7, 373 (2013).
[2] S. Weiler, A. Ulhaq, S. M. Ulrich, D. Richter, M. Jetter, P. Michler, C. Roy, and S. Hughes, Phys. Rev. B 86, 241304 (2012).
[3] D. Hoang, Johannes Beetz, Leonardo Midolo, Matthias Skacel, Matthias Lermer, Martin Kamp, Sven Höfling, Laurent Balet, Nicolas Chauvin, and Andrea Fiore, Appl. Phys. Lett. 100, 061122 (2012).
[4] G. Lecamp, P. Lalanne, and J. P. Hugonin, Phys. Rev. Lett. 99, 023902 (2007).
[5] D. P. S. McCutcheon, A. Nazir, Phys. Rev. Lett. 110, 217401 (2013).
[6] S. Scheel, L. Knöll, and D.-G. Welsch, Phys. Rev. A 60, 4094 (1999).
[7] G. D. Mahan, Many-Particle Physics, Plenum, New York, 1990.
[8] H.-P. Breuer and F. Petruccione, The Theory of Open Quantum Systems, Oxford University Press, 2002.
[9] A. Kowalski-Kula and R. Tanaś, J. Mod. Opt. 48, 347 (2001).
[10] H. J. Carmichael, Statistical Methods in Quantum Optics 1: Master Equations and Fokker-Planck Equations, Springer, 2003.
[11] A. Nazir, Phys. Rev. B 78, 153309 (2008).
[12] Y. Ota, S. Iwamoto, N. Kumagai, Y. Arakawa, Y. Yamamoto, and J. Vučković Phys. Rev. Lett. 95, 013904 (2005).
[13] Thang Ba Hoang, Johannes Beetz, Leonardo Midolo, Matthias Skacel, Matthias Lermer, Martin Kamp, Sven Höfling, Laurent Balet, Nicolas Chauvin, and Andrea Fiore, Appl. Phys. Lett. 100, 061122 (2012).
[14] G. Lecamp, P. Lalanne, and J. P. Hugonin, Phys. Rev. Lett. 99, 023902 (2007).
[15] D. P. S. McCutcheon, A. Nazir, Phys. Rev. Lett. 110, 217401 (2013).
[16] S. Scheel, L. Knöll, and D.-G. Welsch, Phys. Rev. A 60, 4094 (1999).
[17] G. D. Mahan, Many-Particle Physics, Plenum, New York, 1990.
[18] H.-P. Breuer and F. Petruccione, The Theory of Open Quantum Systems, Oxford University Press, 2002.
[19] A. Kowalski-Kula and R. Tanaś, J. Mod. Opt. 48, 347 (2001).
[20] H. J. Carmichael, Statistical Methods in Quantum Optics 1: Master Equations and Fokker-Planck Equations, Springer, 2003.
[21] A. Nazir, Phys. Rev. B 78, 153309 (2008).
[22] Y. Ota, S. Iwamoto, N. Kumagai, Y. Arakawa, Y. Yamamoto, and J. Vučković Phys. Rev. Lett. 95, 013904 (2005).
[23] Thang Ba Hoang, Johannes Beetz, Leonardo Midolo, Matthias Skacel, Matthias Lermer, Martin Kamp, Sven Höfling, Laurent Balet, Nicolas Chauvin, and Andrea Fiore, Appl. Phys. Lett. 100, 061122 (2012).
[24] G. Lecamp, P. Lalanne, and J. P. Hugonin, Phys. Rev. Lett. 99, 023902 (2007).
[25] D. P. S. McCutcheon, A. Nazir, Phys. Rev. Lett. 110, 217401 (2013).
[26] S. Scheel, L. Knöll, and D.-G. Welsch, Phys. Rev. A 60, 4094 (1999).
[27] G. D. Mahan, Many-Particle Physics, Plenum, New York, 1990.
[28] H.-P. Breuer and F. Petruccione, The Theory of Open Quantum Systems, Oxford University Press, 2002.
[29] A. Kowalski-Kula and R. Tanaś, J. Mod. Opt. 48, 347 (2001).