A DYNAMIC STUDY
OF A
PILOT ECKEY HORIZONTAL FRACTIONATOR
BY
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ABSTRACT

In order that the automatic control of a process system be optimal, the dynamics of the equipment involved must be thoroughly understood. Distillation is a complex non-linear process and as such, many of the dynamic system parameters are difficult to predict accurately and must be determined experimentally. The purpose of this study was to develop a series of dynamic models which characterize the response of the liquid temperature on the plates of a pilot Eckey Horizontal Fractionator, to variations in liquid feed rate, liquid feed composition, and vapor feed rate. This was done by experimentally determining the frequency response of the column indirectly, using the pulse technique.

The eight inch, 32 plate column was run as a stripper using the binary methanol-water system. Liquid feeds were introduced at the top with raw steam being fed into the bottom of the column. The column was run at pressures of one atmosphere and about 200 mm Hg. Pulse-like variations were introduced into the liquid feed rate and composition and in the vapor feed rate and the time histories of these pulses and the column responses on several plates were recorded. From these data, values for the system frequency response were calculated numerically using the computerized TAFT routine. This data was then plotted on Bode diagrams. The
Bode plots were analyzed to determine the form of the dynamic models of the column and their parameters.

It was found that liquid rate and composition disturbances showed first order responses plus delays which increased with distance from the feed plate. Vapor rate responses also showed first order dynamics but without delays. The major first order time constants were relatively constant on all plates for the liquid and vapor rate responses but increased with distance from the feed plate for liquid composition responses. Of considerable interest was the presence of resonance peaks in the frequency response curves. These are thought to be the result of oscillating composition transients traveling down the column in the liquid and up in the vapor flow. A term to account for resonance was included in the models. It was found that the time delays and first order time constants could be related to the liquid residence time and scale-up equations are developed for applying the results of this study to other Eckey horizontal columns.
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INTRODUCTION

The optimal use of proven equipment and the development of new designs remain continuing challenges to the chemical process industry. The newly developed Horizontal Fractionator is a horizontal, vapor-liquid contact column recently developed for use in low pressure distillation, absorption, stripping, and evaporation operations. Although performing the same functions as conventional sieve-tray or bubble-cap distillation units, phase contact is accomplished by passing vapor countercurrent to streams of liquid droplets providing effective interfaces for mass and heat transfer. This results in efficient liquid-vapor contact for even highly viscous materials. Combined with an extremely low pressure drop per theoretical stage of the order .05 to .2 mm Hg, the column is ideal for performing separations involving high boiling, heat sensitive materials under high vacuum.

The horizontally mounted eight inch diameter column contains 32 phase contact stages. A section of the column is shown in Figure 1. Liquid flows from stage to stage, but only after it has been sprayed by the impellers located in each stage. The impellers, mounted on a rotating shaft extending lengthwise through the column below the axis, they are fabricated from a series of "sieve plates" and "spacer rings" and designed so that large volumes of liquid
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Figure 1 -- Internal views of Eckey Column
entering the inside periphery are sprayed out forming a pattern of uniform drop size and density. The spray then impinges on the shell of the column above the impeller and runs back down the wall, where it is divided. A portion returns to the sump containing the impeller to be resprayed, and the rest enters the downstream part of the stage and flows over a weir to the next stage. Wire mesh inflectors are located in the vapor space between each stage. These serve to prevent liquid entrainment in the vapor flow and to coalesce the droplets and direct the liquid flow down the column walls to its proper stage. Stages designed in this way are very compact and lead to multistage equipment much shorter in length than conventional distillation columns.

In order to insure efficient and economical design and high quality performance of process equipment, there must be a thorough understanding of the unit’s operating characteristics and behavior. This is especially true where adequate control of certain system parameters is to be achieved. In checking the performance of or pinpointing troubles in existing control systems, investigating the controllability of process equipment, or designing optimal control systems, it is necessary to have information concerning the dynamic behavior of the equipment involved.

This data is acquired by determining functional relationships between the independent and dependent variables of the system. A dynamic model may be rigorously derived if the physical laws describing the action and interaction of
these variables are completely understood. Often however, the system is of such complexity that only an approximate model can be constructed and experimental information is required to determine the parameters occurring in this model and to verify its accuracy and reliability. The experimental testing usually consists of subjecting an input (independent variable to some type of disturbance and measuring the response of some output (dependent) variable to that disturbance. Appropriate analysis of the forcing and the system response will then yield the desired dynamic information.

THE LITERATURE

Two main methods for determining dynamic properties, as discussed by Lees (58) are transient response testing and frequency response testing. Both of these methods yield sufficient information to construct or test a linearized model of the system. In transient response response testing, usually a ramp or step change in some system input is made and the time history of this and the system outputs of interest are recorded. These data are analyzed by standard methods to yield values for the time constants and other parameters occurring in a dynamic model. However for higher order more complex systems, interpretation of transient data becomes difficult. Also, a permanent change in the system variables is produced which in practical situations may not be desirable.
Frequency response testing involves the technique of applying a steady sinusoidal variation to a system input and measuring the output response. This is a technique that has been long used in electrical engineering applications. Theoretically the forcing can cover an infinite range of frequencies, but actually, data of interest usually lie in a more narrow range. By proper analysis of amplitude ratios and phase angle shifts between input forcing and the system's response over a range of input frequencies, it is possible to accurately determine values for the parameters occurring in a dynamic model of the system. From this information, one could then predict the response of the system to any disturbance in the range of linear operation. Frequency response data also yield information regarding stability of the system, reveal resonance effects, and are more useful for higher order systems than transient data.

The direct method of obtaining this data is to actually vary a system input sinusoidally and measure the response over a wide range of input frequencies. This however can be a very time consuming process and may require a large amount of expensive testing apparatus. Also if the equipment being tested is in use, the experimentation could cause dangerous side effects or result in the production of large quantities of off-specification product. Fortunately there are simpler indirect methods of obtaining frequency response data which are available. Gallier (37) and Lees (58) discuss methods of obtaining frequency response data by statistically analyzing
random variations in input and output variables using correlation techniques. However a more practical and widely applicable method is pulse testing.

It has been shown by Hougen (45, 47), Clements and Schnelle (22), and Draper et al (28) that by subjecting a system to a single pulse-like disturbance, the same frequency response information can be acquired as that determined by forcing the system sinusoidally over a range of frequencies. The experimental testing time then is much shorter than that required for direct frequency response testing. Since an exact sinusoidal variation in an input variable is not needed, the testing equipment can often be simpler. The pulse is made by varying the input from its steady-state value for some finite period of time and then returning it to this value. The time history of the input and the system response are recorded. The pulse must be large enough to generate a measurable response but should not be so large as to excite the system beyond linear operation. It may have a precise shape, such as triangular, rectangular, or half sine, but this is not a requirement.

Driefke (29) has made a detailed investigation into the effect of pulse height, width and shape on the accuracy of frequency response data derived from experimental pulse testing. This study involved analog testing of first, second, third, and higher order systems by forcing them with many different pulses. He concluded that several different pulses gave good results, and in general, longer pulses gave more
reliable low frequency data and shorter pulses more reliable high frequency data. It was found that half sine pulses gave better results than rectangular pulses and that the spectral content of the input pulse was very important. Accuracy was also affected by the data reduction process, confirming earlier work.

Some of the earliest applications of pulse testing were in the area of aircraft stability and control. Smith and Triplett (78) and Eggleston and Mathews (32) discuss several methods of obtaining frequency response data from pulse information gotten during flight tests. In recent years this technique has been applied in many areas of the chemical process industry. Hougen and Lees (46) pulse tested a condenser to determine the response of the outlet liquid temperature to changes in the inlet liquid flow rate. The frequency response was then calculated, compared to the sinusoidal forced response, and fitted to a dynamic model of the process. Hougen and Walsh (47) discuss the application of the pulse testing method to ten systems including servomechanisms, analog models, heat exchangers and mixing chambers. The frequency response data obtained were found to be in excellent agreement with those acquired by direct sinusoidal forcing.

There have been numerous studies made of the dynamics of fractionating equipment. A distillation column consists of a complex sequence of interacting heat and mass transfer stages. A series of non-linear differential equations
describing the distillation process can be written for each stage, but the simultaneous solution of these becomes quite difficult. Attempts to solve simplified versions of these have been made and the results compared with experimentally determined transient and frequency response data.

Williams (83) and Archer and Rothfus (4) have completed extensive literature surveys of recent work done in the area of distillation dynamics. Rosenbrock (64) discusses a digital computer routine used to numerically solve the theoretical equations.

A five plate benzene-carbon tetrachloride column was studied at total reflux by Armstrong and Wilkson (5, 6) to determine the transient response to step changes in vapor composition. An analytical solution of simplified modeling equations showed good comparison with the experimental data for the initial portion of the response. In further studies, Armstrong et al (7, 9) investigated the transient response of the plate liquid composition of a 22 plate four inch benzene-carbon tetrachloride column to step changes in feed composition and reflux ratio. Predicted responses showed good comparison with the experimental work and the authors suggested that the response could be characterized by a time delay followed by a first order lag.

Lamb and Pigford (57) made a theoretical study of transient behavior of five, 16, and 21 plate columns by solving the differential perturbation equations describing the column. An analog circuit was set up simulate changes in feed
rate and composition. An oscillating transient was noticed in the liquid composition on each plate and attributed to wavelike disturbances moving up in the vapor and down in the liquid. They seemed to originate at the point of upset and be reflected at the ends of the column. It was also felt that the responses at low frequency could be simply characterized by delays and first order lags.

In a series of papers, Gerster et al (10, 11, 36, 38) conducted experimental transient response studies on a pilot plant distillation column. Step changes in liquid feed rate, vapor rate, and reflux rate were made for five plate and ten plate, two foot diameter benzene-acetone columns. The results, when compared with analog solutions to the perturbation equations of Lamb and Pigford, showed good agreement. Approximate first order responses were noted, with disturbances in vapor rates being felt almost immediately throughout the column.

More complex equations were developed by Franke et al (34), and their solutions were compared with experimental tests performed on a twelve plate methanol-tertiary butyl alcohol column. The response to step changes in reflux rate showed approximately equal first order lags on each plate.

Wahl and Harriot (81), in analyzing a computer-simulated column, felt changes in vapor rates showed first order responses on any plate while responses to liquid rate and concentration changes were nth order at a distance of n plates from the point of upset.
Wood and Armstrong (8, 84) have developed and solved mathematical models of theoretical columns to study the frequency response to sinusoidal changes in liquid feed composition and reflux rate. The presence of possible oscillatory effects is noted.

The frequency response of a 20 plate gasoline splitting column was studied by Aikman (1) in order to provide a high quality control system. Sinusoidal variations in the vapor and reflux rates were made, with the outlet vapor temperature measured. The response was characterized by time delays and first order time constants.

Pell (68) compared a mathematical model with experimental data for the frequency response of a multiplate column. Direct sinusoidal forcing was used for changes in liquid feed rate, and the response to feed composition changes was determined by pulse methods. Good agreement was obtained, but the author noted some deviation between computed and experimental phase lags.

Marino and Stutzman (62, 63) have made frequency response tests using both sinusoidal and pulse forcings on a 24 plate, eight inch benzene-acetone column. Rectangular and cosine pulses were introduced in the reflux flow, with the plate liquid temperature at several points being recorded. The pulse technique is shown to give reliable data, with the response approximating first order plus a delay term. Recommendations are also given regarding maximum pulse width.
DISTILLATION DYNAMICS

During the actual operation of distillation columns, upsets are most likely to be introduced because of changes in liquid (or possibly vapor) feed rate and composition. Control of the rate or concentration of some product stream(s) is usually maintained by suitable manipulation of liquid reflux flow or vapor flow to the bottom of the column from the reboiler. It therefore becomes necessary to understand how variations in these parameters affect the operation of the column. In this study, the frequency response of the Eckey Horizontal Fractionator was studied using pulse techniques. The column was subjected to pulse-like disturbances in the liquid feed rate, liquid feed composition, and vapor rate. The time histories of the input pulse and of the responses of the liquid temperature on several plates were recorded. Methanol-water solutions were used as the working fluid and the column was run at both atmospheric and reduced pressure. The input and response data were processed to yield frequency response information, which was analyzed to determine the parameters occurring in the transfer functions showing how the liquid composition at several points in the column would respond to variations in vapor flow and feed liquid flow and concentration.

Effect of Liquid and Vapor Flow Disturbances

The propagation of flow and concentration disturbances
within a column is a very complicated process. A change in the liquid feed rate to one stage is eventually felt by each successive downstream stage as a new hydraulic equilibrium is approached. This results in a change in the liquid concentration in each stage, which in turn causes a change in the concentration of the vapor passing countercurrently through the liquid spray above each plate. These changes are propagated upstream in the vapor flow and act further to affect the liquid composition on the upstream plates. If the liquid's feed rate remains constant but its composition is changed, a disturbance is also propagated down the column by the liquid flow. As before, the compositions of the vapor above each plate starts to change, which in turn affects the plate liquid compositions as the vapor travels through the column.

A change in the flow rate of the vapor to a column is felt almost immediately at each stage. This is because the column is designed to give little resistance to vapor flow, hence the extremely low pressure drop across the column. As mixing occurs on each stage, the plate liquid compositions begin to change. The liquid composition on each plate is then also affected by other upstream plates as the liquid flows through the column.

Since there is a great deal of interaction between the flow streams, the response of the column to liquid and vapor perturbations becomes quite complex. Sinusoidal disturbances being propagated up and down the column in the liquid
and vapor flows also lead to the possibility of reinforce-
ment and inhibition of composition changes, resulting in
resonance effects. It would be extremely difficult to de-
velop a workable model which involved all of the mechanisms
present in the distillation process. It is possible however,
to construct simpler linearized models from frequency re-
sponse data which characterize well the response of the sys-
tem. This is common practice in dynamic analysis, (1, 38,
39, 46, 53, 59, 62, 65).

System Transfer Function

These models are often conveniently written in the form
of a transfer function, which is the ratio of the LaPlace
transforms of the system response and the input forcing.
Consistent with results from dynamic response tests perfo-
med on other distillation units, the modeling equation for
the Eckey Fractionator will be of the form:

\[ G(s) = \frac{\mathcal{L} \text{(OUTPUT)}}{\mathcal{L} \text{(INPUT)}} = \frac{-Ls}{(\gamma s + 1)} \frac{(1 - Ae)}{\gamma s + 1} \]

where \( G(s) \) defines the transfer function as the ratio of
LaPlace transforms and \( s \) is the LaPlacian operator. The re-
sponse of a system to any input forcing can be found simply
by multiplying the system transfer function by the LaPlace
transform of the forcing and taking the inverse transforma-
tion.
When a change is made in the liquid feed rate or concentration to a column, there is usually a period of time before a response is noted on the other plates in the column. This time $L$ is represented in the transfer function by the expression for delay, $e^{-Ls}$. It should be possible to relate $L$ to the time it takes the liquid to flow from the point of disturbance to the plate at which the response is measured.

The quantity $\tau$ represents the major time constant associated with a first order model of the response. If more accuracy were warranted, a second order model could be used. The major time constant is sometimes estimated from the liquid residence time in the whole column and a minor one from a single plate. These methods are often inaccurate though and useful only in estimating the order of magnitude of these parameters. The possibility of recurring resonance peaks in the frequency response is taken into account by including in the transfer function the term $(1 - Ae^{-Bs})$ where $A$ and $B$ are related to the magnitude and period of the peaks. Koppel (53), Kuemmel (56) and Lim (59) have found resonance effects present in both experimental and theoretical studies of several distributed systems. Process models which they have developed, when simplified, have shown this term to represent these resonance effects with good accuracy.

Although this model is fairly simple in nature, it can, within limits, represent the complex non-linear distillation process with good accuracy.
THEORY OF PULSE TESTING

The frequency response of a system is a set of forced responses to sinusoidal variations in some input covering a wide range of frequencies. If the relationships between the dependent and independent variables of the system can be defined by a set of linear differential equations with constant coefficients, the system is said to be linear. When an input to such a system is varied sinusoidally at some frequency, the forced response of the system will also be a sinusoid and of the same frequency. Figure 2 shows the forced response, $y(t)$, of a typical linear system to a sinusoidal variation in input, $x(t)$. The frequency response information deduced from a set of plots like this are the phase shift, $\phi$, between the input and output, and the ratio of their amplitudes, $A_y/A_x$. A different set of values of the phase angle and amplitude ratio will be obtained for each value of the frequency of the input forcing.

A convenient and useful way of presenting frequency response data is through the use of Bode diagrams. In this method a log-log plot of the amplitude ratios versus frequency is made accompanied by a semi-log plot of the phase angles versus the log of the frequency. Often the plot of amplitude ratios is normalized with respect to the amplitude
Figure 2- Sinusoidal forced response of typical linear system
ratio at zero frequency, i.e. the steady-state gain. From these plots it is possible to determine the maximum gain for stable operation, if the system is put under feedback control, and the critical frequency of the system. This information is useful in determining optimum controller settings, judging the speed of response and comparing the controllability of the system under various proposed control schemes. When the frequency response of the components of a cascade of non-interacting subsystems are known it is a relatively simple matter to combine these to get the response of the entire system. Although frequency response analysis is theoretically accurate only when dealing with linear systems, it can provide information which is extremely useful when studying the dynamics of a system whose response can be assumed linear over a certain range of operation. By proper graphical analysis it is possible to determine values for the time constants occurring in linearized dynamic models of a system or validate the models. The presence of resonance effects can also be determined.

The Pulse Method

Frequency response information can be extracted from the time response of a system to a pulse-like disturbance in some system input. A periodic function such as a sequence of pulses can be written as a Fourier series of sinusoidal waves of different frequencies and amplitudes. Figure 3a shows a sequence of rectangular pulses of period $P$ together
Figure 3-- Frequency spectrum for rectangular pulse trains showing effect of decreasing rate of repetition of pulse.
with a line spectrum of the Fourier series coefficients associated with the pulse train. The ordinate represents the amplitude of the sine waves occurring in the series (the Fourier series coefficients) and the abcissa indicates frequency. If the pulse occurs at a frequency $\omega$, the frequencies of the sinusoids in the Fourier series will be integral multiples of $\omega$. As shown in Figure 3b, when the shape of the pulse remains the same, but its frequency of repetition decreases, more lines appear in the line spectrum. The limiting case of Figure 3c is of course when the pulse occurs only once. At this point the lines merge into a continuous spectrum. Thus, a single pulse contains sinusoidal components at all frequencies, except those at which the envelope crosses the abcissa and the harmonic content is zero. Points on this envelope are obtained by evaluating the amplitude of the Fourier transform of the pulse. This is determined from the Fourier Integral:

$$F[f(t)] = \int_{0}^{T} f(t)e^{-j\omega t} \, dt$$

where

- $F$ = Fourier transform
- $f(t)$ = function of time (pulse) for which Fourier transform is desired
- $j = \sqrt{-1}$
- $T$ = non-zero duration of time function (pulse)
- $\omega$ = frequency, radians/time, for which Fourier transform is desired

If the exact Fourier transform of the input pulse to a
system is known together with the exact value of the Fourier transform of the response of the system to that pulse, the frequency response of the system is by definition:

\[ G(j\omega) = \frac{F(Y)}{F(X)} = \frac{\int_0^{T_y} y(t)e^{-j\omega t} \, dt}{\int_0^{T_x} x(t)e^{-j\omega t} \, dt} \]

where \( x(t) \) and \( y(t) \) are the input pulse and output response time histories respectively and \( T_x \) and \( T_y \) are the duration of \( x(t) \) and \( y(t) \).

The complex variable \( G(j\omega) \) is a function of \( \omega \). The magnitudes and phase angles associated with this number can be determined over the range of frequencies of interest and the results displayed on a Bode plot.

The TAFT Routine

If precise mathematical expressions are known for the input pulse and output response, the evaluation of the Fourier integrals in the equation for the frequency response can be done by calculus. However, in most cases, evaluation must be done by operating mathematically on experimental pulse input-output time histories. Several methods for determining numerical Fourier transform approximations from experimental data are discussed in the literature. Draper, McKay and Lees (28) have developed a simple stepped-curve approximation and also discuss a type of trapezoidal method. Clements and Schnelle (22) discuss the use of Filon’s quad-
ature formula, where experimental time histories are approximated by parabolic line segments. In this study, a trapezoidal method, known as the TAFT (Trapezoidal Approximation to Fourier Transform) routine, was used to determine numerical approximations to the Fourier transforms of the input pulses and output responses of the system. This method has been used in recent works by Driefke (29), Marino (62) and Hougen (45).

In the development of the TAFT routine operating on a typical time function which differs from zero for a period of time $T$, (see Figure 4 for notation) it is first necessary to write the equation for the straight line connecting the ordinates $f_i$ and $f_{i+1}$ over the interval $\Delta T$:

$$f(t) = f_i(1 - \frac{t}{\Delta T}) + f_{i+1}(\frac{t}{\Delta T} + 1 - 1)$$

The transform of the function can now be evaluated in the $i$th interval from the definition.

$$F_i[f(t)] \approx TAFT_i(j\omega) = \int_{(i-1)\Delta T}^{i\Delta T} f(t)e^{-j\omega t} \, dt$$

$$= \frac{f_i}{\omega} \left[ \frac{1}{\Delta T} e^{-j\omega(i\Delta T)} - \left( \frac{1}{\omega \Delta T} + 1 \right) e^{-j\omega(i-1)\Delta T} \right]$$

$$+ \frac{f_{i+1}}{\omega} \left[ (1 - \frac{1}{\omega \Delta T}) e^{-j\omega(i\Delta T)} + \frac{1}{\omega \Delta T} e^{-j\omega(i-1)\Delta T} \right]$$

To determine the Fourier transform of the entire function, the integrals for the trapezoidal intervals must now
Figure 4 -- Notation for Trapezoidal Approximation to Fourier Transform (TAFT routine)
be summed over the time period from 0 to T (i.e. \( i = 1 \) to \( N \)).

\[
TAFT_T(j\omega) = \sum_{1}^{N} TAFT_i(j\omega)
\]

When this is done, the Trapezoidal Approximation to the Fourier Transform, separated into its real and imaginary parts reads:

\[
TAFT(j\omega) = \left\{ \begin{array}{l}
\Delta T f_1 \left[ \frac{1}{2} \left( \frac{\sin \frac{\omega AT}{2}}{\frac{\omega AT}{2}} \right)^2 \right] \\
+ \Delta T \left( \frac{\sin \frac{\omega AT}{2}}{\frac{\omega AT}{2}} \right)^2 \sum_{i=2}^{N} f_i \cos(\omega(1-i)\Delta T) \\
+ \Delta T \frac{\sin \frac{\omega AT}{2}}{\frac{\omega AT}{2}} \cos(\omega NAT) + \frac{1}{\omega AT} \left( \frac{1 - \sin \omega AT}{\omega AT} \right) \sin \omega NAT \\
+ f_1 \sin(\omega(1-1)\Delta T) \\
+ \Delta T f_{N+1} \left[ \frac{1}{2} \left( \frac{\sin \frac{\omega AT}{2}}{\frac{\omega AT}{2}} \right)^2 \sin(\omega NAT) + \frac{1}{\omega AT} \left( \frac{1 - \sin \omega AT}{\omega AT} \right) \cos \omega NAT \right] \\
\end{array} \right.
\]

From the above equation, the Fourier transform of the input pulse or of the output response can be numerically computed.
from the experimental time histories.

Because of the large amount of calculating involved, processing of the experimental pulse data was done using the University of Rhode Island IBM 360/50 digital computer. Data supplied to the computer were lists of numbers representing the time histories of the pulse forcing and the system response \((x, y, T_x, T_y)\), the time interval to be used in the TAFT approximation, and the values of \(\omega\) for which frequency response data were to be computed. To simplify data processing and interpretation, any apparent pure time delays in the responses were accounted for before data reduction by shifting the zero time of the output to the point of the first observable deviation from steady state.

The program used the TAFT routine to compute, for each value of \(\omega\), the real and imaginary parts of the Fourier transform of the pulse forcing and system response \((x\) and \(y)\) and the spectral content of the input pulse. The frequency response was then determined from the complex function \(F(y)/F(x)\), as values for each frequency; of the phase angle \((\arctan \frac{\text{Im}(F(y)/F(x))}{\text{Re}(F(y)/F(x))})\) and magnitude \((\sqrt{\text{square root} \left( \frac{\text{Im}(F(y)/F(x))}{\text{Re}(F(y)/F(x))} \right)^2 + \left( \frac{\text{Re}(F(y)/F(x))}{\text{Re}(F(y)/F(x))} \right)^2})\). The magnitude was then normalized by dividing each value by the magnitude at zero frequency. The frequency response data was then in a form suitable for plotting on a Bode diagram. A copy of the computer program used and simplified data processing flow sheet are found in Appendix B.
The overall equipment setup for the experimental work is shown in Figure 5. The 32 stage Eckey Fractionator was run as a stripping column with liquid feed, at room temperature, entering on plate 30, two plates from the "top". Using terminology consistent with vertical columns, vapor is considered to enter the "bottom" and exit at the "top" of this horizontal column. Binary methanol-water liquid feeds were used and flow control was maintained using constant head tanks. The liquid product exited from the bottom of the column into a storage tank. Pure steam, which stripped the alcohol from the incoming liquid, was fed into the bottom of the column. The methanol rich vapor product left the top of the column, was condensed, and stored in holding tanks. Provisions were made for sampling all feed and product streams. When desired, reduced pressure in the system was established by pulling a vacuum on the condenser. For economy, black iron pipe was used throughout the system. All liquid lines and the steam inlet line were ½ inch, sch-40, with 2 inch, sch-40 being used for the vapor exit line, to insure a low pressure drop between the column and condenser.
Figure 5 -- Overall equipment setup
The column was run as a stripper using methanol-water for several reasons. In a distillation system which includes reboiler and reflux setups, the response of the column to liquid and vapor upsets will be affected by the response of the reboiler and reflux systems, since they are an integral part of the overall system. Using a reflux setup would complicate the system greatly. The composition response of the vapor stream to pulses in feed liquid flow rate and composition or vapor rate would be carried up the column and out with the overhead vapor. Part of this vapor when condensed and fed back into the column as reflux would act as an additional input forcing. The column then would be responding to both the initial pulse forcing and the secondary forcing resulting from the reflux flow.

If a reboiler were used to generate vapor at the bottom of the column, vapor rate disturbances would be made by varying the heat flow to the reboiler. Because of the substantial capacity and resistance in the reboiler, a pulse in heat flow rate would result in a highly distorted, delayed pulse in vapor rate. Of much greater value is the column response to a distinct, direct, well controlled vapor rate pulse. In order to study the column alone, minimize the effect of auxiliaries, and reduce complexity, it was decided to run the column as a stripper without reflux and use a vapor source which was independent of the exit liquid. Steam, which is both cheap and plentiful, was used as the vapor, making water one component of the system.
In this study, the dynamic response of the column was studied by making pulse-like changes in an independent variable (liquid feed rate, liquid feed composition, vapor feed rate) and measuring the response of the various plate liquid temperatures to these changes. In order for this method to be feasible, the response must be measurable. When the column is pulsed, the resulting changes in plate liquid composition and saturated liquid temperature must be large enough to be accurately measured by recording devices. One criteria then of the binary system to be used was that there be a relatively large change in boiling point with composition. However in order to study the column response on several plates, the vapor-liquid equilibrium relationships between the two components must be such that complete separation of the components by distillation requires several equilibrium stages. The methanol-water system with a pure component boiling point difference of about 60°F satisfied these conditions. Also, methanol-water solutions are economical in cost, safe to handle, and relatively non-toxic. Copper-constantan thermocouples were used to measure plate liquid temperatures because of their chemical inertness to the methanol-water system and their relatively large change in emf with temperature (about 0.025 mv/°F).

For simplification, the equipment and instrumentation setups will be broken into sections for discussion. The major pieces of equipment in the accompanying figures are numbered with reference to Appendix A where detailed manu-
facturers specifications are found.

I. The Eckey Column

The eight inch, 32-stage Eckey fractionator is approximately eight feet long and was mounted, for optimum efficiency, at a 1.5 degree angle from the horizontal, as shown in Figure 6. This unit was capable of handling liquid loads up to 0.5 gpm. The motor-driven longitudinal shaft on which the impellers are mounted is supported by four main bearings and rotated at a speed of 1730 rpm. The shaft enters the column shell through an oil-cooled shaft seal (John Crane Chemlon). Motor oil was circulated by a small gear pump at 20 psig through the seal for lubrication and to maintain the temperature at a value less than 300°F. Sight glasses are present at the top and bottom of the column, and above the 13th and 26th plates. Thermocouples measured the inlet and outlet vapor temperatures, and a vacuum gauge located above plate 30 indicated column pressure. Copper-constantan bare wire thermocouples were mounted in the liquid pools, on plates 28, 26, 21, 16, 11, and 2, to measure the liquid temperatures. During pulse runs, time histories of these temperatures were recorded using two Honeywell ELECTRONIK 19 strip chart emf recorders. The sensitivity of the liquid temperature recording system was such that a one inch change in chart width represented about a 2°F temperature change.
II. Liquid Feed System

Liquid feed entered the column on plate 30 from either the main or auxiliary feed system. In both systems, the feed was pumped from stainless-steel storage tanks to ten gallon constant-head tanks mounted approximately ten feet above the column. To maintain a constant head, overflow pipes were mounted in the overhead tanks to carry excess feed back to storage. The feeds from both head tanks were filtered and passed through rotometers for flow rate measurement as shown in Figure 7. The capacities for the main and auxiliary feed systems were 100 and 60 gallons respectively. Flow was controlled in the auxiliary system by means of a hand operated needle valve, and switching between the feeds was accomplished using a 3-way solenoid valve.

Liquid flow rate disturbances were introduced into the system in the main feed line. A pulse in liquid rate resulted from a similar pulse in the air pressure signal to a Foxboro pneumatic control valve. The pulsed liquid flow rate was recorded using a Foxboro magnetic turbine flow transmitter. This instrument generates an oscillating voltage signal whose frequency is converted to an output voltage directly proportional to volumetric flow rate.

Rectangular pulses in feed liquid composition were made by switching from the main feed to the auxiliary feed of a different composition but the same flow rate for a fixed period of time. The duration of these pulses was recorded using an electric timer.
III. Vapor Feed and Bottoms Liquid Systems

Pure steam at 90-100 psig was available as vapor feed to the bottom of the column from the University high pressure steam line shown in Figure 8. With the Foxboro pneumatic control valve set to full open, the vapor flow rate during steady state operation was set using a Maisoneilon pressure regulating valve. A negative pulse-like change in the vapor feed was made by introducing a pulse in the air pressure signal to a Foxboro pneumatic control valve. This pulse acted to decrease the vapor rate for a period of time and then return it to the steady state value. The vapor rate was measured using a critical flow convergent nozzle designed so that the mass flow rate was directly proportional to the upstream steam pressure. This pressure was sensed by a Foxboro electronic pressure transmitter, to yield a recorded voltage which varied linearly with mass flow.

Liquid product leaving the bottom of the column passed into a small holding tank. After specific gravity measurements showed this to be essentially alcohol free, it was discarded.

IV. Condensate Collection System

Figure 9 illustrates the condensate collection and vacuum control systems. The methanol rich vapor product leaving the top of the column was condensed in a horizontally mounted, stainless-steel, 47 sq. ft., 1-1, shell and tube,
heat exchanger. Cooling water passed through the tubes and vapor entered the shell. Provisions were made for withdrawing samples from the condensate effluent line for composition analysis. The distillate was collected in two forty gallon receivers, which were vented during atmospheric pressure operation.

When required, reduced pressure was maintained in the system by pulling a partial vacuum on the vapor space above the liquid in the receivers, using an NCR rotary gas-ballast pump. A small condenser in the vacuum line prevented volatiles from entering the pump. System pressure was measured using a column mounted vacuum gauge.

V. Pulse Generating System

The purpose of the pulse generating circuit was to produce an air pressure signal pulsed in the range 3 to 15 psig for a finite period of time. This signal was used to actuate pneumatic control valves producing similar pulse-like changes in either liquid or vapor feed rate. The principal piece of equipment used was a Hewlett-Packard 202A low frequency function generator. This instrument, in normal operation, is capable of producing sinusoidal, triangular, and rectangular waves over a wide range of frequencies and amplitudes. But it can be modified to produce single pulses of triangular, rectangular, or half-sine shape. This is done by externally driving the 202A with a pulse activator powered by a 300 volt D.C. source. This activator produces
a fast rising trigger pulse of sufficient height and duration to initiate one cycle of operation of the function generator, which, in turn produces a single pulse of chosen shape, amplitude and duration.

It was possible to produce pulses up to 150 seconds in length. The pulsed voltage signal was received by a Fisher electro-pneumatic transducer where it was converted to a pulsed pressure signal used to actuate the pneumatic control valves. This circuit is shown in Figure 10.
Figure 6 -- Eckey column
Figure 7 -- Liquid feed system
Figure 8 -- Vapor feed and bottoms liquid systems
Figure 9 -- Distillate collection and vacuum systems
Figure 10 -- Pulse generating circuit
EXPERIMENTAL PROCEDURE

The experimental work covered the period from February to October 1970 and took place in the following sequence:

1. Tests at atmospheric pressure
   (a) Steady state runs
   (b) Vapor pulse run
   (c) Liquid pulse run
   (d) Concentration pulse run

2. Tests at reduced pressure (200±20 mm Hg)
   (a) Steady state runs
   (b) Liquid pulse run
   (c) Vapor pulse run
   (d) Concentration pulse run

In order for dynamic testing to be successful, a change in some system input must produce a measurable change in some system output. Prior to the pulse runs a series of steady state runs at various liquid and vapor feed rates and liquid feed compositions were made at both atmospheric and reduced pressure. This permitted optimum operating conditions for the dynamic tests to be determined and insured that pulse-like changes in vapor and liquid flows would produce measurable changes in the plate liquid temperatures in the column.

It was decided to make atmospheric steady state runs using feed liquid concentrations of about 30, 40 and 50 mole
percent methanol. It was felt that the optimum feed liquid concentration for the pulse runs to follow would fall somewhere in this range. From inspection of the methanol-water vapor liquid equilibrium diagram, it can be seen that several equilibrium stages will be needed to strip the alcohol from liquid feeds of the above compositions. Using feeds of lower methanol concentrations would result in methanol removal occurring in only a few stages at the top of the column. Using a highly concentrated feed would not appreciably increase the number of equilibrium stages required for stripping. Highly concentrated solutions could also be extremely dangerous to handle and would require large amounts of the limited supply of methanol available. Actual liquid feed composition used during these runs were 29.9, 42.1 and 50.3 mole percent methanol. During each run, several liquid to vapor ratios were used.

To conserve feed, the column was first heated until a thermal equilibrium had been reached by feeding steam in the bottom and using pure water as liquid feed. Equilibrium was reached when the plate liquid reached the saturation temperature at column pressure. Following this, the methanol-water feed solution was introduced into the column. Liquid bottoms product and condensate samples, together with plate liquid temperature readings, were taken at various time intervals until steady state conditions were reached. At this point the liquid to vapor ratio was changed and product samples and temperature readings were again taken until the
next steady state was reached. This procedure was followed for each run using a different liquid feed composition.

Another series of steady state runs was made, prior to the reduced pressure pulse tests, at a column pressure of about 200 mm Hg. Liquid feed concentrations used were 39.5 and 47.5 mole percent methanol. For each feed, various liquid to vapor ratios were tried with the sampling procedure identical to that for atmospheric steady state runs.

The compositions of the distillate and bottoms products were determined from specific gravity measurements made on a Christian Becker balance. The plate liquid compositions were found from their liquid temperatures assuming saturation at column pressure. This is justified on the basis that when pure water was used as the liquid feed, the plate liquids did indeed reach and maintain their saturation temperatures. Using these measurements, plate liquid temperature and composition profiles throughout the column were constructed. The operating conditions and results of the steady state runs are shown in Table 1. From these results, the approximate optimum operating conditions for the dynamic pulse tests to follow were determined. The actual operating conditions for the pulse runs are shown in Table 2. Dynamic testing is primarily concerned with changes in variables - changes in output caused by changes in input. Extremely accurate measurements of the absolute values of system variables is often not important. Values of the liquid feed rate, vapor rate, and liquid feed composition were chosen
| Steady State Run No. | Column Pressure Atm. | Feed Liq. Comp. % MeOH | Feed Liq. Rate GPM | Feed Vapor Rate LB/HR | Liquid to Vapor Ratio | Dist. Comp. Mole % MeOH | Bot. Comp. Mole % MeOH |
|---------------------|----------------------|------------------------|-------------------|------------------------|----------------------|-------------------------|------------------------|
| 1                   | 1.0                  | 29.9                   | .33               | 84.0                   | 1.7                  | 56.0                    | 0.0                    |
| 2                   | 1.0                  | 29.9                   | .19               | 56.0                   | 1.3                  | 41.7                    | 0.0                    |
| 3                   | 1.0                  | 29.9                   | .25               | 84.0                   | 1.3                  | 35.9                    | 0.0                    |
| 4                   | 1.0                  | 29.9                   | .20               | 55.0                   | 1.1                  | 34.4                    | 0.0                    |
| 5                   | 1.0                  | 42.1                   | .35               | 84.0                   | 1.7                  | 75.6                    | 0.0                    |
| 6                   | 1.0                  | 42.1                   | .28               | 83.0                   | 1.3                  | 55.5                    | 0.0                    |
| 7                   | 1.0                  | 42.1                   | .22               | 82.0                   | .99                  | 41.2                    | 0.0                    |
| 8                   | 1.0                  | 50.3                   | .34               | 83.0                   | 1.5                  | 78.6                    | 0.0                    |
| 9                   | 1.0                  | 50.3                   | .29               | 83.0                   | 1.2                  | 60.6                    | 0.0                    |
| 10                  | .26                  | 50.3                   | .23               | 83.0                   | .96                  | 47.8                    | 0.0                    |
| 11                  | .26                  | 39.5                   | .26               | 47.0                   | 1.9                  | 77.0                    | 0.0                    |
| 12                  | .26                  | 39.5                   | .26               | 59.0                   | 1.5                  | 61.9                    | 0.0                    |
| 13                  | .26                  | 39.5                   | .25               | 88.0                   | 1.0                  | 39.8                    | 0.0                    |
| 14                  | .28                  | 47.5                   | .27               | 44.0                   | 2.0                  | 81.1                    | 0.0                    |
| 15                  | .27                  | 47.5                   | .27               | 60.0                   | 1.4                  | 67.0                    | 0.0                    |
| 16                  | .26                  | 47.5                   | .27               | 75.0                   | 1.2                  | 55.0                    | 0.0                    |
**TABLE 2**

**STEADY STATE OPERATING CONDITIONS FOR PULSE RUNS**

| Pulse Type | Column Pressure Atm. | Feed Liq. Comp. % MeOH | Feed Liq. Rate GPM | Feed Vapor Rate LB/HR | Liquid to Vapor Ratio | Dist. Comp. Mole % MeOH | Bot. Comp. Mole % MeOH |
|------------|----------------------|-------------------------|--------------------|------------------------|-----------------------|-------------------------|------------------------|
| Vapor rate | 1.0                  | 44.3                    | .29                | 92.                    | 1.1                   | 48.0                    | 0.0                    |
| Liquid rate| 1.0                  | 45.5                    | .30                | 78.0                   | 1.3                   | 60.0                    | 0.0                    |
| Liquid conc.| 1.0                  | 49.9                    | .30                | 63.                    | 1.5                   | 79.0                    | 0.0                    |
| Vapor rate | .26                  | 42.0                    | .27                | 73.                    | 1.2                   | 51.0                    | 0.0                    |
| Liquid rate| .24                  | 40.6                    | .25                | 73.                    | 1.2                   | 48.6                    | 0.0                    |
| Liquid conc.| .26                  | 45.5                    | .25                | 48.                    | 1.6                   | 75.5                    | 0.0                    |
such that pulse-like changes in these values would produce measurable changes in plate liquid temperatures in the column. Measurement of the actual value of the liquid and vapor rate to more than two figure accuracy was not necessary. It was also possible to estimate the maximum column response which might be expected when changes were made in vapor rate and liquid rate and composition.

It was found that the feed liquid was stripped of its alcohol in the top two thirds of the column and was essentially pure water at this point, regardless of the feed used. Therefore no column response would be felt by the thermocouples in the liquid on plates 11 and 2. During the pulse runs the liquid temperatures on these plates were not recorded.

The dynamic testing was done during six experimental sessions, one each for liquid rate, liquid composition, and vapor rate pulses at atmospheric pressure and again for low pressure column operation. The total liquid feed storage capacity was 100 gallons, and liquid rates were kept between .25 and .3 gpm allowing 5-6 hours of operation for each pulse session. This was ample time to permit the use of pulses of different shapes, amplitudes, and duration.

Liquid and Vapor Rate Pulses

Pulses in liquid feed rate were made by applying a pulsed pressure signal to the pneumatic control valve in the liquid feed line. The column was first brought to a steady
state. A pulse-like change was then made in liquid rate with the pulse and the responses of the liquid temperatures on plates 28, 26, 21 and 16 being recorded. After the column had returned to steady state again, additional pulses could be made.

The procedure for pulsing the vapor flow was similar to that for liquid flow, except that the air pressure signal was applied to the pneumatic operated valve in the steam line. After the column had reached steady state, a pulse in the vapor rate was made and the column was allowed to return to steady state with the input and responses being recorded.

A variety of triangular, half-sine and rectangular pulses were tried during both the high and low pressure runs. Because of non-linearities inherent in the pulse circuit, transducer and control valve, the resulting pulses in liquid rate became somewhat deformed (see Figure 11) and the triangular and half-sine pulses appeared quite similar in shape. This was also true for the pulses made in the vapor flow rate to the bottom of the column. The effect is to partially smooth out any discontinuities in the pulse (e.g. to round off the point of the triangular pulses). It was possible to make more vapor pulses than liquid pulses during equal testing periods because the column responded more rapidly to vapor disturbances.

Liquid Composition Pulses
Figure 11 -- Illustration of theoretical vapor and liquid flow rate pulses and actual pulses produced showing distortions caused by system non-linearities.
To determine the response of the column to pulses in liquid feed composition, the unit was first brought to steady state using the main feed system. The column was then switched to the auxiliary feed of a different composition for a short period of time and back to the main feed. The pulse duration was measured using an electric timer accurate to .1 seconds. During this period, the volumetric flow rate was maintained constant. The response of the column to this disturbance was recorded until it had returned to steady state. Identical procedures were followed for both atmospheric and low pressure runs with pulses of several different amplitudes (compositions) being used. Pulses made in this way would be considered rectangular in shape.

Response Data

The thermocouple emf recordings representing the plate liquid responses to liquid and vapor pulses were hand smoothed prior to data processing. The pulses, to three significant figures, were read off the smoothed response curves for time intervals of eight seconds. From the unsmoothed input pulse curves values were taken for two second intervals. These intervals were decided upon after some experimentation indicated that using smaller time intervals (more data points), while requiring more processing time, would not improve the accuracy of the results. Using much larger time intervals would have unduly distorted the functions being approximated. When pure time delays were apparent in the
response curves, these were recorded, and the zero point was shifted to the point of first deviation from the steady state. The rectangular concentration pulses were all assigned an amplitude of unity for their duration. The input–response data were then processed by the TAFT routine which calculates the input and output Fourier transforms and from these, standard frequency response information. While producing a printed output, the program also utilized the computer plotting system to automatically construct and plot the results as Bode diagrams.
RESULTS

A system subjected to a pulse forcing will be excited at all frequencies except those at which the frequency content (the amplitude of the Fourier transform) of the input pulse becomes zero. At these points the expression for the amplitude and phase angle of the frequency response are reduced to the indeterminate form 0/0. In practical computations however, small inaccuracies arising from point reading and roundoff errors inherent in numerical approximations will result in highly erroneous values if calculation is attempted.

As might be expected, there is often noticeable scatter in the computed frequency response curves as these "zero frequencies" are approached. However the experimental curves usually remain smooth and scattering does not become apparent until after the onset of error. It has been widely shown experimentally that the computed data deviate from the true system frequency response well before a frequency value equal to the first zero of the frequency content of the input pulse.

Hougen and Walsh (47), as a result of pulse testing many experimental systems, have observed that the calculated frequency response information can be considered reliable up to 70-80 percent of the first zero frequency. Although it
was sometimes possible to recover useful information between zeros, this was not recommended. Driefke (29) in his study on the effect of pulse height and width reached similar conclusions. After experience with many experimental and analytically computed pulse tests, Clements and Schnelle (22) concluded that deviation from true response sets in when frequency content of the input pulse, normalized at \( \omega = 0 \), drops to .2-.3.

Consistent with these findings, the experimentally determined frequency response data in this study were calculated and considered reliable below frequencies at which the normalized input pulse frequency content equaled .25. These data expressed as the amplitude ratio and phase angle are plotted on Bode diagrams, and found in Appendix C. These plots were analyzed to determine values for the parameters occurring in the transfer functions showing the response of liquid temperature on plates 16, 21, 26 and 28 to changes in vapor rate, liquid feed rate and liquid composition.

Determination of First Order Time Constants

Although it was possible to characterize the column responses by first, second or higher order approximations, after careful examination of the experimental Bode diagrams, it was decided that determination of more than a major first order time constant would be unwarranted. There were several reasons for this. The response of distillation units to
changes in liquid flows (rate and composition) is often estimated to be first order on each plate in the column, making the total response nth order at a distance n plates from the point of disturbance. It is common practice in cases such as this for the resulting response to be estimated by a time delay and a first order time constant (1, 7, 9, 10, 36, 57). In this study, as previously mentioned, time delays were extracted from the experimental time histories of the responses to liquid rate and composition pulses before data processing by the TAFT routine.

The approximate first order time constants were determined by superimposing on the amplitude ratio curves of each Bode diagram, transparencies on which were plotted similar curves for known first order systems. Thus an observable best fit was obtained which would be considered accurate enough for engineering applications. The experimental amplitude ratio curves showed good agreement with first order dynamics at low frequencies but the true higher order nature of the responses became evident because of the more rapid attenuation of the curves at high frequencies. The phase lags also showed lower values at the corner frequencies (reciprocal of the first order time constants) than the -45° predicted from first order dynamics. It was felt however that any attempt to calculate a second time constant would produce values highly inaccurate and of little significance.

Because of the extremely low pressure drop in the column, a vapor pulse is felt almost immediately on each plate.
The response then contains no time delay. In this case a first order approximation to the response is more viable, although again at high frequencies, the higher order nature of the response is evident.

Characterization of Resonance Peaks

Perhaps the most distinct characteristic of the experimentally determined frequency response curves is presence of resonance peaks. Lamb and Pigford (57) and Wood (84) have both noted the possibility of this occurrence. The Eckey column is basically a countercurrent phase contact unit. Sinusoidal composition transients in the liquid moving down the column interact with the vapor on each plate causing oscillating variations in the composition of the vapor moving up the column. The reverse is also true. Because of the column's design for low pressure drop, the vapor composition oscillations are unlikely to be broken up by passing through the plates, as in conventional columns. When the sinusoidal liquid and vapor composition transients are in phase, reinforcement of the waves would occur, producing a peak in the amplitude ratio and phase plots. Inhibition occurring when the disturbances are 180° out of phase would produce a corresponding trough.

In theory, these peaks should occur on all plates at the same frequencies. The experimental curves bear this out. As expected, the resonance frequencies were not noticeably affected by the shape, height or duration of the input pulse.
In addition, resonance peaks were evident in the frequency response curves for all liquid and vapor forcings at both atmospheric and reduced pressure.

The quantity B in the experimental transfer functions was calculated from the period of the resonance peaks occurring in the amplitude ratio curves of the frequency response plots. The quantity A is related to the amplitude of the resonance peaks and was calculated, where possible from the phase angle curves. Details of the methods of calculation of these parameters are shown below with reference to Figure 12.

Determination of Parameters A and B

Let the transfer function for a system which shows resonance peaks in the frequency response be of the form:

(1) \( G(s) = D(s) R(s) \)

where \( G(s) = \) total system transfer function

(2) \( R(s) = (1 - Ae^{-Bs}) \) - component accounting for resonance

\( D(s) = \) remaining part of \( G(s) \)

To obtain the frequency response, let \( s = j\omega \), where \( j = \sqrt{-1} \)

\( \omega = \) frequency, rad/sec

\( G(j\omega) = D(j\omega)(1 - Ae^{-j\omega B}) \)

\( = D(j\omega)(1 - A\cos B\omega + jA\sin B\omega) \)

(3) \( = D(j\omega) \left[ (1 - A\cos B\omega) + j(A\sin B\omega) \right] \)

Calculation of B

Obtain magnitude of \( G(j\omega) \)

(4) \( \text{Mag}(G) = \text{Mag}(D)(1 - 2A\cos B\omega + A^2 \cos^2 B\omega + A^2 \sin^2 B\omega)^{\frac{1}{2}} \)
\[ = \text{Mag}(D)(1 - 2A \cos B \omega + A^2)^{1/2} \]

Obtain amplitude ratio by division by \( \text{Mag}(G) \) at \( \omega = 0 \)

\[ (5) \quad \text{AR}(G) = \frac{\text{Mag}(G)}{\text{Mag}(G(0))} = \frac{\text{Mag}(D)}{\text{Mag}(D(0))} \frac{1}{1 - (1 - 2A \cos B \omega + A^2)^{1/2}} \]

Oscillations in a plot of \( \text{AR}(G) \) versus \( \log \omega \) as shown in Figure 12 are caused by the term, \(-2A \cos B \omega\). This oscillates with a "period", \( P = \frac{2\pi}{B} \). From Figure 12, the "period" is the difference between successive peaks or troughs. Therefore \( B \) is determined from the relation:

\[ (6) \quad B = \frac{2\pi}{P} = \frac{2\pi}{\omega_2 - \omega_1} \]

**Calculation of \( A \)**

\[ (7) \quad \angle R(j\omega) = \angle G(j\omega) - \angle D(j\omega) \]

from equation (3)

\[ (8) \quad \angle R(j\omega) = \angle \left[ (1 - A \cos B \omega) + j(A \sin B \omega) \right] \]

\[ = \tan^{-1} \frac{A \sin B \omega}{1 - A \cos B \omega} \]

This term is oscillatory as seen from the table below.

| \( B \omega \) | \( \angle R \) |
|----------------|--------------|
| 0              | 0            |
| \( \pi/2 \)    | \( \tan^{-1} A \) |
| \( \pi \)      | 0            |
| \( \pi/2 \)    | \( \tan^{-1} - A \) |
| \( 2\pi \)     | 0            |

Table I.

From Figure 12, \( \angle R = \phi_1 - \phi_2 \).

From Table I, \( (\phi_1 - \phi_2) \) max = \( \tan^{-1} A \) or \(-\tan^{-1} A \)

Therefore,

\[ A = \tan \left| (\phi_1 - \phi_2) \right| \text{max} \]
Bode Plot for Typical System Showing Resonance Peaks

Figure 12 -- Illustration of notation used for calculation of transfer function parameters $A$ and $B$. 
Summary of Calculated Transfer Function Parameters

Appendix C contains the experimentally determined Bode Diagrams and is divided into six sections according to the nature of the forcing and the system pressure. The values for the parameters in the transfer function model were calculated from the Bode plot for each pulse and are found in Tables 3-8. Each table is divided into parts a, b, and c. Part (a) shows the characteristics of the input pulses used.

The pulses are characterized by:

- **Type**: T - triangular, S - half-sine, R - rectangular
- **Amp**: Maximum amplitude of pulse expressed as percentage change in steady state value
- **Dur**: Duration of pulses - seconds

Parts (b) and (c) show the calculated transfer function parameters for each pulse response on plates 16, 21, 26 and 28.

The following transfer function parameters are listed.

- **τ** - major first order time constant - minutes
- **L** - time delay - minutes
- **B** - "period" of resonance - minutes
- **A** - dimensionless parameter related to magnitude of resonance peaks

From these tables, the values of the parameters which best describe the response on plates 16, 21, 26 and 28 of the Eckey column to variations in vapor rate, liquid rate, and liquid composition at both atmospheric and reduced pressure were determined and are shown in Table 9.
### TABLE 3a

**VAPOR RATE PULSE**

**COLUMN PRESSURE - 1 ATM.**

| Pulse No. | Type | Amp | Dur |
|-----------|------|-----|-----|
| 1         | T    | 19. | 48. |
| 2         | T    | 9.8 | 52. |
| 3         | T    | 27. | 62. |
| 4         | T    | 25. | 75. |
| 5         | T    | 30. | 76. |
| 6         | T    | 24. | 118.|
| 7         | T    | 65. | 60. |
| 8         | T    | 58. | 48. |
| 9         | T    | 55. | 28. |
| 10        | T    | 58. | 82. |
| 11        | T    | 27. | 80. |
| 12        | T    | 14. | 124.|
| 13        | T    | 17. | 74. |
| 14        | S    | 45. | 48. |
| 15        | S    | 22. | 72. |
| 16        | S    | 40. | 66. |
| 17        | S    | 56. | 30. |
| 18        | S    | 58. | 56. |
| 19        | R    | 12. | 70. |
| 20        | R    | 38. | 46. |
| 21        | R    | 46. | 54. |
| 22        | R    | 56. | 62. |
| 23        | R    | 56. | 26. |
| 24        | R    | 57. | 30. |
| Pulse No | Plate 16 | | | Plate 21 | | |
|----------|---------|---|---|---------|---|---|
|          | L  | τ | B | A | L  | τ | B | A |
| 1        | 0.0 | 0.7 | 1.5 | -  | 0.0 | 2.1 | 4.5 | -  |
| 2        | 0.0 | 1.5 | 3.7 | -  | 0.0 | 1.2 | 3.2 | -  |
| 3        | 0.0 | 1.8 | 4.4 | -  | 0.0 | 2.1 | 5.0 | -  |
| 4        | 0.0 | 1.9 | 4.8 | -  | 0.0 | 0.8 | 4.4 | -  |
| 5        | 0.0 | 1.4 | 4.8 | -  | 0.0 | 1.9 | 4.4 | -  |
| 6        | 0.0 | 1.7 | 4.5 | 42  | 0.0 | 1.4 | 4.4 | -  |
| 7        | 0.0 | 1.8 | 4.5 | -  | 0.0 | 1.7 | 4.8 | 18  |
| 8        | 0.0 | 1.8 | 4.5 | -  | 0.0 | 1.7 | 5.0 | -  |
| 9        | 0.0 | 1.8 | 4.5 | -  | 0.0 | 1.7 | 4.4 | -  |
| 10       | 0.0 | 1.6 | 4.6 | -  | 0.0 | 1.6 | 4.0 | 18  |
| 11       | 0.0 | 1.5 | 4.5 | -  | 0.0 | 1.5 | 4.5 | 32  |
| 12       | 0.0 | 2.0 | 4.2 | -  | 0.0 | 1.9 | 4.5 | 32  |
| 13       | 0.0 | 2.0 | 4.2 | -  | 0.0 | 1.9 | 4.8 | -  |
| 14       | 0.0 | 1.9 | 4.8 | 25  | 0.0 | 1.9 | 4.8 | 18  |
| 15       | 0.0 | 2.1 | 4.4 | -  | 0.0 | 2.4 | 5.0 | 27  |
| 16       | 0.0 | 2.2 | 4.4 | -  | 0.0 | 1.8 | 4.0 | -  |
| 17       | 0.0 | 2.2 | 4.4 | -  | 0.0 | 2.2 | 4.5 | 58  |
| 18       | 0.0 | 2.2 | 4.4 | -  | 0.0 | 2.2 | 5.0 | 14  |
| 19       | 0.0 | 2.4 | 4.8 | 84  | 0.0 | 2.1 | 4.8 | 21  |
| 20       | 0.0 | 2.2 | 5.0 | 29  | 0.0 | 1.8 | 4.5 | -  |
| 21       | 0.0 | 2.4 | 4.8 | 23  | 0.0 | 1.4 | 3.0 | -  |
| 22       | 0.0 | 1.4 | 4.4 | -  | 0.0 | -   | -   | -   |
| 23       | 0.0 | 1.8 | 5.0 | -  | 0.0 | -   | -   | -   |
| 24       | 0.0 | -   | -   | -  | 0.0 | -   | -   | -   |
TABLE 3c

TRANSFER FUNCTION PARAMETERS FROM BODE PLOTS

| Pulse No | Plate 26 | Plate 28 |
|----------|----------|----------|
|          | L  | T | B  | A   | L  | T | B  | A   |
| 1        | 0.0 | 2.1 | 4.4 | 0.32 | 0.0 | 2.1 | 4.5 | 0.58 |
| 2        | 0.0 | 0.6 | 3.2 | -    | 0.0 | 0.7 | -   | -    |
| 3        | 0.0 | 2.1 | 4.2 | -    | 0.0 | 1.0 | -   | -    |
| 4        | 0.0 | 0.6 | -   | -    | 0.0 | 1.4 | -   | -    |
| 5        | 0.0 | 1.9 | 3.9 | -    | 0.0 | 1.4 | 5.2 | -    |
| 6        | 0.0 | 1.3 | 4.4 | -    | 0.0 | 1.6 | 5.0 | -    |
| 7        | 0.0 | 1.5 | 4.5 | -    | 0.0 | 1.6 | 4.8 | -    |
| 8        | 0.0 | 1.8 | 4.5 | 0.19 | 0.0 | 1.5 | 4.4 | 0.18 |
| 9        | 0.0 | 1.8 | 4.5 | 0.27 | 0.0 | 1.8 | 4.4 | -    |
| 10       | 0.0 | 1.3 | 3.6 | -    | 0.0 | 1.8 | 4.5 | -    |
| 11       | 0.0 | 1.6 | 3.9 | -    | 0.0 | 1.8 | 4.4 | -    |
| 12       | 0.0 | 1.5 | 3.9 | -    | 0.0 | 1.8 | -   | -    |
| 13       | 0.0 | 1.7 | -   | -    | 0.0 | 2.1 | 4.5 | 0.58 |
| 14       | 0.0 | 2.1 | 4.4 | 0.32 | 0.0 | 2.1 | 4.5 | 0.58 |
| 15       | 0.0 | 0.6 | 3.2 | -    | 0.0 | 0.7 | -   | -    |
| 16       | 0.0 | 2.1 | 4.2 | -    | 0.0 | 1.0 | -   | -    |
| 17       | 0.0 | 1.9 | 3.9 | -    | 0.0 | 1.4 | 5.2 | -    |
| 18       | 0.0 | 1.3 | 4.4 | 0.19 | 0.0 | 1.5 | 4.4 | 0.19 |
| 19       | 0.0 | 1.6 | 3.6 | -    | 0.0 | 1.8 | 3.4 | -    |
| 20       | 0.0 | 1.5 | 4.5 | -    | 0.0 | 1.8 | 3.4 | -    |
| 21       | 0.0 | 2.1 | 4.4 | -    | 0.0 | 1.8 | -   | -    |
| 22       | 0.0 | 1.0 | -   | -    | 0.0 | 1.8 | -   | -    |
| 23       | 0.0 | 1.8 | -   | -    | 0.0 | 1.8 | -   | -    |
| 24       | 0.0 | 0.9 | 4.5 | -    | 0.0 | 0.7 | -   | -    |
### TABLE 4a

**LIQUID RATE PULSE**  
**COLUMN PRESSURE—1 ATM.**

| Pulse No. | Type | Amp  | Dur  |
|-----------|------|------|------|
| 1         | T    | 29.  | 42.  |
| 2         | T    | 38.  | 80.  |
| 3         | T    | 95.  | 58.  |
| 4         | T    | 88.  | 34.  |
| 5         | S    | 61.  | 34.  |
| 6         | S    | 62.  | 62.  |
| 7         | S    | 28.  | 66.  |
| 8         | S    | 88.  | 72.  |
| 9         | R    | 30.  | 44.  |
| 10        | R    | 29.  | 40.  |
| 11        | R    | 59.  | 44.  |
| 12        | R    | 62.  | 86.  |
| 13        | T    | 81.  | 32.  |
| 14        | T    | 72.  | 104. |
| 15        | T    | 68.  | 20.  |
| 16        | T    | 64.  | 14.  |
| 17        | T    | 60.  | 62.  |
| Pulse No | L  | T  | B  | A  | L  | T  | B  | A  |
|---------|----|----|----|----|----|----|----|----|
| 1       | 1.6| 1.2| 3.9| -.36|    |    |    |    |
| 2       | 1.0| .9 | 3.7| -.36|    |    |    |    |
| 3       | .93|.7 | 3.7| -.36|    |    |    |    |
| 4       | .93| 1.4| 4.8| -.36|    |    |    |    |
| 5       |    |    |    |    |    |    |    |    |
| 6       |    |    |    |    |    |    |    |    |
| 7       |    |    |    |    |    |    |    |    |
| 8       |    |    |    |    |    |    |    |    |
| 9       |    |    |    |    |    |    |    |    |
| 10      |    |    |    |    |    |    |    |    |
| 11      | .97| 1.6| 4.5| .27 |    |    |    |    |
| 12      |    |    |    |    |    |    |    |    |
| 13      | .83| 1.3| 5.0| .25 |    |    |    |    |
| 14      | 1.7|    |    | .25 |    |    |    |    |
| 15      |    |    |    |    |    |    |    |    |
| 16      |    |    |    |    |    |    |    |    |
| 17      |    |    |    |    |    |    |    |    |
| Pulse No | L  | $\tau$ | B  | A  | L  | $\tau$ | B  | A  |
|----------|----|--------|----|----|----|--------|----|----|
| 1        | -  | -      | -  | -  | -  | -      | -  | -  |
| 2        | .6 | 1.7    | 4.8| .30|    |        |    |    |
| 3        | .3 | 1.5    | 5.0| -  |    |        |    |    |
| 4        | .3 | 1.5    | 4.5| .18|    |        |    |    |
| 5        | .23| 1.8    | 4.5| .36|    |        |    |    |
| 6        | .4 | 1.4    | 5.0| -  | .37| 1.9    | 5.5|    |
| 7        | .43| 1.0    | 4.2| -  | .33| 1.6    | 4.5|    |
| 8        | .43| 1.6    | 5.0| -  | .27| 1.0    |    |    |
| 9        | .4 | 1.4    | 4.8| .27|    |        |    |    |
| 10       | .37| 1.3    | 3.7| -  |    |        |    |    |
| 11       | .23| 1.7    | 5.2| -  |    |        |    |    |
| 12       | .2 | 1.9    | 4.8| -  |    |        |    |    |
| 13       | .4 | 1.3    | -  | -  |    |        |    |    |
| 14       | .47| 1.9    | 4.5| .34|    |        |    |    |
| 15       | .23| 1.7    | 4.4| -  | .07| 1.5    | 4.8| .14|
| 16       | .23| 1.0    | 4.4| -  | .07| 1.3    | 4.5| .18|
| 17       | .33| 1.5    | 5.0| .21| .1 | 1.5    | 4.8| -  |
### TABLE 5a

**LIQUID CONCENTRATION PULSE COLUMN PRESSURE - 1 ATM.**

| Pulse No. | Type | Amp  | Dur |
|-----------|------|------|-----|
| 1         | R    | 30.9 | 42. |
| 2         | R    | 30.9 | 64. |
| 3         | R    | 30.9 | 113.|
| 4         | R    | 61.1 | 43. |
| 5         | R    | 61.1 | 20. |
| 6         | R    | 61.1 | 80. |
| 7         | R    | 100.0| 20. |
| 8         | R    | 100.0| 10. |
| 9         | R    | 100.0| 4.9 |
| 10        | R    | 100.0| 49. |
| Pulse No | Plate 16 | Plate 21 |
|----------|----------|----------|
|          | L    |   | L    |   | |
| 1        | .63  | 1.0 | .58  | 1.5 | 4.0 | .58 |
| 2        | .60  | 1.2 | 3.7  | 1.5 | 3.9 | .36 |
| 3        | .67  | 1.2 | .83  | 1.4 | 4.4 | .47 |
| 4        | 1.0  | 1.5 | 4.0  | 1.2 | 4.4 | .36 |
| 5        | .77  | 1.7 | 5.2  | 1.0 |    |    |
| 6        | .53  | 1.7 | 3.3  | 1.5 | 3.7 | .40 |
| 7        | .6   | 1.6 | 4.2  | 1.6 | 4.4 |    |
| 8        | -    |   | -    | 1.3 | 4.0 |    |
| 9        | -    |   | -    |    |    |    |
| 10       | .67  | 1.6 | .47  | .23 |    | .27 |
| Pulse No | L  | τ  | B  | A  | Plate 26 | L  | τ  | B  | A  | Plate 28 |
|----------|----|----|----|----|----------|----|----|----|----|----------|
| 1        | .37| 1.3| 4.5| .14|          |    |    |    |    |          |
| 2        | .33| 1.0| 4.3| .18|          | .13| .8 |    |    | .05      |
| 3        | .33| 1.2| 4.4| .09|          | .17| .8 |    |    | .05      |
| 4        | .33| 1.0| 4.8| .09|          | .13| .7 | 5.0| .05|          |
| 5        | .27| 1.0| 3.9| .14|          | .13| .7 | 4.2| .05|          |
| 6        | .27| 1.2| 4.8| .14|          | .10| .8 | 4.2| .05|          |
| 7        | .27| .7 |    |    |          | .13| .5 |    |    |          |
| 8        | .27| .7 |    |    |          | .10| .5 |    |    |          |
| 9        | .23| .7 |    |    |          | .13| .7 | 5.0|    |          |
| 10       | .27| 1.0| 4.5| .14|          | .13| .7 | 5.0|    |          |
TABLE 6a

VAPOUr RATE PULSE
COLUMN PRESSURE- .26 ATM.

| Pulse No. | Type | Amp | Dur |
|-----------|------|-----|-----|
| 1         | T    | 54. | 76. |
| 2         | T    | 52. | 112.|
| 3         | T    | 64. | 48. |
| 4         | T    | 48. | 90. |
| 5         | T    | 38. | 144.|
| 6         | T    | 61. | 40. |
| 7         | S    | 54. | 94. |
| 8         | S    | 60. | 26. |
| 9         | S    | 60. | 90. |
| 10        | S    | 47. | 42. |
| 11        | S    | 64. | 28. |
| 12        | R    | 42. | 78. |
| 13        | R    | 59. | 82. |
| 14        | R    | 58. | 32. |
| 15        | R    | 41. | 56. |
| 16        | R    | 49. | 82. |
| 17        | R    | 61. | 32. |
| 18        | T    | 63. | 136.|
| 19        | T    | 64. | 36. |
| 20        | S    | 68. | 94. |
| 21        | R    | 64. | 82. |
| Pulse No | L  | $\tau$ | B  | A  | L  | $\tau$ | B  | A  |
|----------|----|--------|----|----|----|--------|----|----|
| 1        | 0.0| 2.1    | 4.0| .47| 0.0| 2.1    | 4.2| .36|
| 2        | 0.0| 1.8    | 3.9|   | 0.0| 1.6    | 4.0|   |
| 3        | 0.0| 1.7    | 3.7|   | 0.0| 1.7    | 4.8|   |
| 4        | 0.0| 1.6    | 4.4|   | 0.0| 2.0    | 4.0|   |
| 5        | 0.0| 1.6    | 4.4|   | 0.0| 1.8    | 4.0|   |
| 6        | 0.0| 1.6    | 4.4|   | 0.0| 2.0    | 4.5|   |
| 7        | 0.0| 1.7    | 4.0|   | 0.0| 1.7    | 4.0|   |
| 8        | 0.0| 1.6    | 4.4|   | 0.0| 1.2    | 4.0|   |
| 9        | 0.0| 1.8    | 4.0|   | 0.0| 1.6    | 4.8|   |
| 10       | 0.0| 1.6    | 4.4|   | 0.0| 1.8    | 4.8|   |
| 11       | 0.0| 1.6    | 4.4|   | 0.0| 1.6    | 4.4|   |
| 12       | 0.0| 1.6    | 4.4|   | 0.0| 1.6    | 4.4|   |
| 13       | 0.0| 1.9    | 4.0|   | 0.0| 1.9    | 4.8|   |
| 14       | 0.0| 1.5    | 3.7|  .27 | 0.0| 2.0    | 4.5|  .23 |
| 15       | 0.0| 1.5    | 3.7|  .27 | 0.0| 1.7    | 4.4|  .21 |
| 16       | 0.0| 1.5    | 3.7|  .27 | 0.0| 1.2    | 5.8|  .21 |
| 17       | 0.0| 1.7    | 5.2|  .27 | 0.0| 1.2    | 5.8|  .21 |
| 18       | 0.0| 1.3    | 5.2|  .27 | 0.0| 1.3    | 5.2|  .21 |
| 19       | 0.0| 1.4    | 5.2|  .27 | 0.0| 1.6    | 5.2|  .21 |
| 20       | 0.0| 1.2    | 4.4|  .27 | 0.0| 1.7    | 4.0|  .21 |
| 21       | 0.0| 1.4    | 4.5|  .27 | 0.0| 1.8    | 4.8|  .21 |
| Pulse No | L  | γ  | B  | A  | L  | γ  | B  | A  |
|---------|----|----|----|----|----|----|----|----|
| 1       | 0.0| 1.4| 4.4| .25|     |     |     |    |
| 2       | 0.0| 1.3| 3.4| .21|     |     |     |    |
| 3       | 0.0| 1.5| 4.5| .23|     |     |     |    |
| 4       | 0.0| 1.3| 3.9| .21|     |     |     |    |
| 5       | 0.0| 1.3| 3.6| .27|     |     |     |    |
| 6       | 0.0| 1.4| 3.5| .25|     |     |     |    |
| 7       | 0.0| 1.3| 3.7| .25|     |     |     |    |
| 8       | 0.0| 1.1| 4.2|     |     |     |     |    |
| 9       | 0.0| 1.2| 4.2|     |     |     |     |    |
| 10      | 0.0| 1.2|     |     |     |     |     |    |
| 11      | 0.0| 1.3| 3.5| .18|     |     |     |    |
| 12      | 0.0| 1.4| 4.8|     |     |     |     |    |
| 13      | 0.0| 1.4| 3.7|     |     |     |     |    |
| 14      | 0.0| 1.0| 4.4| .23|     |     |     |    |
| 15      | 0.0| 1.7| 4.8|     |     |     |     |    |
| 16      | 0.0| 1.4| 4.0|     |     |     |     |    |
| 17      | 0.0| 1.6|     |     |     |     |     |    |
| 18      | 0.0| 1.3| 2.9|     |     |     |     |    |
| 19      | 0.0| 1.2|     |     |     |     |     |    |
| 20      | 0.0| 1.2| 4.4|     |     |     |     |    |
| 21      | 0.0| 1.3| 4.8|     |     |     |     |    |
### TABLE 7a

**LIQUID RATE PULSE**  
COLUMN PRESSURE= .24 ATM.

| Pulse No. | Type | Amp | Dur |
|-----------|------|-----|-----|
| 1         | -    | -   | -   |
| 2         | T    | 67. | 76. |
| 3         | T    | 85. | 114.|
| 4         | T    | 82. | 36. |
| 5         | T    | 57. | 118.|
| 6         | R    | 63. | 96. |
| 7         | R    | 70. | 42. |
| 8         | R    | 59. | 40. |
| 9         | R    | 46. | 42. |
| 10        | S    | 63. | 94. |
| 11        | S    | 74. | 52. |
| 12        | -    | -   | -   |
| 13        | S    | 74. | 70. |
| 14        | T    | 59. | 88. |
| Pulse No | L  |  \( \gamma \) | B  | A  | L  |  \( \gamma \) | B  | A  |
|----------|----|------------|----|----|----|------------|----|----|
| 1        |    |            |    |    |    |            |    |    |
| 2        |    |            |    |    |    |            |    |    |
| 3        | .8 | 2.1        | 5.0|    |    |            |    |    |
| 4        |    |            |    |    |    |            |    |    |
| 5        | .93| 1.0        | 5.8|    |    |            |    |    |
| 6        | .53| 1.5        | 3.9|    |    |            |    |    |
| 7        | .53| 1.7        | 5.0|    |    |            |    |    |
| 8        | .53| 1.7        | 4.2|    |    |            |    |    |
| 9        | .63| 1.5        | 4.8|    |    |            |    |    |
| 10       | .57| 1.5        | 5.2|    |    |            |    |    |
| 11       |    |            |    |    |    |            |    |    |
| 12       |    |            |    |    |    |            |    |    |
| 13       | .73| 1.5        | 3.9|    |    |            |    |    |
| 14       |    |            |    |    |    |            |    |    |

**Table 7b**

 Transfer Function Parameters from Bode Plots
### TABLE 7c

**TRANSFER FUNCTION PARAMETERS FROM BODE PLOTS**

| Pulse No | L   | \( \tau \) | B   | A   | Plate 26 | L   | \( \tau \) | B   | A   | Plate 28 | L   | \( \tau \) | B   | A   |
|----------|-----|----------|-----|-----|----------|-----|----------|-----|-----|----------|-----|----------|-----|-----|
| 1        | -   | -        | -   | -   | -        | -   | -        | -   | -   | -        | -   | -        | -   | -   |
| 2        | .33 | .9       | 5.0 | -   | -        | -   | -        | -   | -   | -        | -   | -        | -   | -   |
| 3        | .47 | 1.5      | 5.2 | -   | -        | -   | -        | -   | -   | -        | -   | -        | -   | -   |
| 4        | .33 | 1.7      | 5.0 | .42 | -        | -   | -        | -   | -   | -        | -   | -        | -   | -   |
| 5        | .57 | 1.7      | 4.5 | .25 | -        | -   | -        | -   | -   | -        | -   | -        | -   | -   |
| 6        | .33 | 1.2      | 3.7 | .32 | -        | -   | -        | -   | -   | -        | -   | -        | -   | -   |
| 7        | .37 | 1.8      | 4.8 | .34 | -        | -   | -        | -   | -   | -        | -   | -        | -   | -   |
| 8        | .3  | 1.9      | 4.8 | -   | .27      | 1.9 | 4.4      | 5.8 | .32 | -        | -   | -        | -   | -   |
| 9        | .4  | 1.3      | 4.8 | -   | .3       | 1.3 | 5.2      | -   | -   | -        | -   | -        | -   | -   |
| 10       | .47 | 1.7      | 3.6 | -   | .3       | 1.5 | 4.8      | -   | -   | -        | -   | -        | -   | -   |
| 11       | .5  | 1.1      | -   | -   | .33      | 1.1 | 5.2      | -   | -   | -        | -   | -        | -   | -   |
| 12       | -   | -        | -   | -   | -        | -   | -        | -   | -   | -        | -   | -        | -   | -   |
| 13       | -   | -        | -   | -   | -        | -   | -        | -   | -   | -        | -   | -        | -   | -   |
| 14       | .63 | 1.7      | 4.4 | -   | -        | -   | -        | -   | -   | -        | -   | -        | -   | -   |
| Pulse No. | Type | Amp  | Dur |
|-----------|------|------|-----|
| 1         | R    | 66.8 | 40  |
| 2         | R    | 66.8 | 64  |
| 3         | R    | 66.8 | 20  |
| 4         | R    | 66.8 | 80  |
| 5         | R    | 66.8 | 60  |
| 6         | R    | 66.8 | 100 |
| 7         | R    | 100.0| 20  |
| 8         | R    | 100.0| 40  |
| 9         | R    | 100.0| 60  |
| 10        | R    | 100.0| 10  |
| Pulse No | Plate 16 | | | | Plate 21 | | | |
|---|---|---|---|---|---|---|---|---|
| | L | τ | B | A | L | τ | B | A |
| 1 | - | - | - | - | - | - | - | - |
| 2 | - | - | - | - | 1.0 | 1.2 | 5.8 | - |
| 3 | 0.0 | 1.3 | 3.9 | - | .87 | 1.7 | 3.9 | - |
| 4 | - | - | - | - | .9 | 1.4 | 4.5 | - |
| 5 | - | - | - | - | .63 | 1.0 | 4.2 | .40 |
| 6 | - | - | - | - | .47 | 1.6 | 4.2 | - |
| 7 | - | - | - | - | .47 | 1.0 | 4.2 | - |
| 8 | 0.0 | 1.8 | 4.2 | - | .06 | - | - | - |
| 9 | - | - | - | - | - | - | - | - |
| 10 | 0.0 | 1.8 | 4.2 | - | - | - | - | - |
| Pulse No | Plate 26 | Plate 28 |
|---------|---------|---------|
|         | L       | τ       | B   | A   | L   | τ       | B   | A   |
| 1       | .43     | 1.3     | 4.8 | .14 | .1   | 9       | 5.2 | -   |
| 2       | .33     | 1.3     | 5.0 | .18 | .13  | .8      | 4.8 | .05 |
| 3       | .33     | .6      | -   | -   | .1   | 1       | .5  | -   |
| 4       | .3      | 1.5     | 4.2 | .21 | 1.1   | 1.2     | 3.9 | .09 |
| 5       | .33     | 1.1     | 4.8 | -   | 1.1   | .8      | 4.5 | -   |
| 6       | .3      | 1.3     | -   | -   | -     | -       | -   | -   |
| 7       | .33     | 1.4     | 4.4 | .21 | .07  | .5      | -   | -   |
| 8       | .33     | 1.0     | 4.5 | -   | .07  | .7      | -   | -   |
| 9       | .27     | 1.1     | 4.2 | -   | .07  | .7      | 4.5 | -   |
| 10      | .27     | .7      | -   | -   | .07  | .5      | -   | -   |
TABLE 9a

FINAL COLUMN TRANSFER FUNCTION MODEL PARAMETERS

| Forcing   | Location (Plate) | Column Pressure Atm. | Plate 16 | Plate 21 |
|-----------|------------------|----------------------|----------|----------|
|           |                  |                      | L | \( \gamma \) | B | A     | L | \( \gamma \) | B | A     |
| Vapor rate | 0                | 1.0                  | 0.0 | 1.9   | 4.5  | .3   | .00 | 1.9    | 4.6  | .3   |
| Vapor rate | 0                | .26                  | 0.0 | 1.7   | 4.5  | .3   | .00 | 1.8    | 4.5  | .3   |
| Liquid rate | 30               | 1.0                  | 1.1 | 1.3   | 4.2  | .3   | .64 | 1.5    | 4.6  | .3   |
| Liquid rate | 30               | .24                  | 1.3 | 1.5   | 4.4  | .3   | .66 | 1.6    | 4.8  | .3   |
| Liquid conc. | 30               | 1.0                  | .68 | 1.6   | 4.0  | .6   | .42 | 1.5    | 4.0  | .4   |
| Liquid conc. | 30               | .26                  | -   | 1.5   | 4.0  | .6   | .79 | 1.4    | 4.2  | .4   |
### TABLE 9b

**FINAL COLUMN TRANSFER FUNCTION MODEL PARAMETERS**

| Forcing    | Location (Plate) | Column Pressure (Atm.) | Plate 26 | Plate 28 |
|------------|------------------|-------------------------|----------|----------|
|            |                  |                         | L  | \( \gamma \) | B | A | L  | \( \gamma \) | B | A |
| Vapor rate | 0                | 1.0                      | .00 | 1.8   | 4.4 | .3 | .00 | 1.7 | 4.5 | .3 |
| Vapor rate | 0                | .26                      | .00 | 1.4   | 4.2 | .3 | .00 | 1.3 | 4.3 | .3 |
| Liquid rate | 30               | 1.0                      | .36 | 1.5   | 4.7 | .3 | .20 | 1.5 | 4.7 | .3 |
| Liquid rate | 30               | .24                      | .43 | 1.6   | 4.8 | .3 | .30 | 1.4 | 4.8 | .3 |
| Liquid conc. | 30               | 1.0                      | .29 | 1.1   | 4.7 | .15 | .13 | .7  | 5.0 | .05 |
| Liquid conc. | 30               | .26                      | .32 | 1.2   | 4.5 | .15 | .09 | .8  | 4.6 | .05 |
DISCUSSION

It was desired to determine the effect of pulse shape, height and width on the values of the model parameters determined from the experimental Bode plots. In theory, closed pulses of sufficient strength to excite the system, producing measurable responses, will yield identical frequency response information. In practice however, if the system is excited to saturation or far beyond linear operation, the results will vary with the forcing. From examination of the Bode Diagrams in Appendix C, it was found that the amplitude, duration and pulse shape had little effect on the calculated model parameters.

In this study, it was intended that pulsed input forcings in vapor and liquid rate of three shapes be used - rectangular, half-sine and triangular. However, non-linearities in the electronic and pneumatic systems produced flow pulse distortions such that the half-sine and triangular pulses strongly resembled each other and the rectangular pulses were rounded at the corners. This affects the reliability of the results only in that the frequency content of the pulse is dependent on its shape.

The response of the column to liquid rate changes at the feed plate is characterized by first order dynamics and a pure time delay, which as expected, is greater on plates
farther from the origin of the disturbance. The values of
the first order time constants for the different plates were
very close, with any differences probably attributable to
experimental errors. It would seem that once the distur­
bance is felt, the response of the liquid composition and
temperature is the same on all plates.

A similar situation is encountered with the response to
vapor rate changes, except there was no delay noted because
of the rapid propagation of vapor disturbances through the
column. Again the major first order time constants were
about the same on each plate, and are very close to those
characterizing the response to liquid flow changes. The
time constants in the responses for both forcings, shown in
Table 9, range from 1.3 to 1.9 minutes, with an average of
about 1.7 minutes. By changing the liquid or vapor rate on
a particular plate, the contact time between the liquid and
vapor phases is changed. This in turn affects the mass
transfer between the phases and the composition and boiling
point of the liquid pools on each plate. Thus, liquid and
vapor rate disturbance which change this contact time should
effect a similar response from the column.

A feed liquid composition pulse is a more powerful
forcing since it directly causes a change in the plate liq­
uid composition. Because of this, the response differed
from that of the liquid and vapor rate pulses. Time delays
were noted which increased with distance from the feed plate.
However, as seen in Table 9, they were shorter than those
which occurred in the responses to liquid rate pulses. The major time constant varied with position in the column from about .75 minute on plate 28 to 1.6 minutes on plate 16. The column also seemed to respond with slightly smaller delays and time constants as the amplitude of the composition pulse increased. The response, at reduced pressure, of plate 16 to composition pulses was very small in magnitude and produced results which are probably not reliable.

The time delays found in the liquid rate and composition pulse responses should be related to the average time required for the liquid to pass from the feed plate through the column — the liquid residence time. This is determined by dividing the liquid holdup below the feed plate by the volumetric liquid flow rate. For the pilot eight inch column used in this study, the holdup was about two gallons. Figures 13 and 14 show plots of the ratio of delay to total residence time versus plate number for liquid rate and liquid composition pulses respectively. This data was taken from the experimental response time histories and is tabulated in Tables 4, 5, 7 and 8. Although some scatter is noted, it was found that the time delays varied linearly with distance from the feed plate and could be approximated with reasonable accuracy from the equation:

\[ L_i = K(RT)(\frac{NP_i}{TP}) \]

where \( i \) = plate on which response is being determined

\( L_i \) = time delay
$RT = \text{total residence time of liquid below feed plate}$

$K = \text{empirical constant}$

- .35 for responses to liquid rate forcings
- .3 for responses to liquid concentration forcings

$TP = \text{total number of plates below feed plate}$

$NP_i = \text{number of plates between feed plate and plate } i$

The empirical constant being less than 1. indicates that the disturbances are being carried down the column in the liquid flow faster than might be predicted using an average flow rate.

The quantity $B$, which was calculated from the period of the resonance peaks in the amplitude ratio curves, varied from 4.0 to 5.0 minutes with an average of about 4.5 minutes. Values in this range were consistently found regardless of the type of input forcing used. The dimensionless quantity $A$ is related to the magnitude of the resonance peaks and was calculated, where possible, from the phase angle plots of the experimental Bode diagrams. Although there was little variation in the period of resonance, the magnitudes were often inconsistent and erratic, making calculation of meaningful values of $A$ extremely difficult. This was especially true at higher frequencies. It is felt that the technique of extracting frequency response information from transient pulse responses, which involves considerable data processing and numerical approximations, may not be accurate enough to produce reliable information regarding the magnitude of
Figure 13 -- Plot of the ratio of delay to total residence time (L/RT) versus plate number for liquid rate pulses.
Figure 14 -- Plot of the ratio of delay to total residence time (L/RT) versus plate number for liquid composition pulses.
these resonance peaks. For the response to liquid and vapor rate forcings, A seemed to vary from .15 to .4 with an average of about .3 on all plates. For the liquid concentration responses, A varied from about .05 on plate 28 to .6 on plate 16.

Reliability and Accuracy of Results

The reliability and hence the usefulness of the dynamic response models is a function of the accuracy of the experimental input pulse - output response time histories, the point reading process, the computerized TAFT routine used to calculate and plot the frequency response data, and the interpretation of the Bode diagram to determine model parameters.

The output response time histories were hand smoothed before point reading because of noise caused by variations in thermocouple readings. These small fluctuations in the plate liquid temperature were probably caused by liquid flow turbulence on the plates, and pressure variations during low pressure runs which changed the vapor-liquid equilibrium relationships. This resulted in a maximum margin of error in the amplitude of the points read of about ±.4°F.

It is often convenient to truncate the output pulse responses to facilitate data processing. This is done when the response decays at a very slow rate or does not return to zero, as was often the case when very strong input forcings were used. When this is done, the low frequency part
of the amplitude ratio curve is flattened while the high frequency part is attenuated too sharply. However Driefke (29) found that moderate truncation (up to 25-50 percent of the time history) had negligible detrimental effect.

It has been shown that calculation of frequency response data numerically, using the TAFT routine, is extremely accurate as long as a sufficient number of points are used (22, 29, 47). Hougen (47) recommends at least ten data points to approximate the input pulse while Clements and Schnelle (22) feel it is rarely necessary to approximate the output by more than 50 points. In this study it was found that a maximum 5 minutes of response time, with points taken at 8 second intervals, sufficiently described the output response. It is felt that the data processing routine contributes little in the way of errors to the results up to frequencies where the normalized frequency content of the input pulse equals .25.

Perhaps the best estimate of accuracy can be found by examining the reproducibility of the results. Calculated values of B showed a maximum variation of about 15 percent for all runs. The determination of the first order time constant from any single amplitude ratio curve by comparison with standard curves will naturally be subject to the experience and bias of the interpreter. However it is felt that the value determined would be within 10 percent of that of the author's. For the response on one plate, the variation in the values of τ, calculated from the amplitude ratio
curves, was about 25 percent. The time delays, \( L \), for liquid rate and concentration pulses were taken directly from the pulse response time histories. Variations in these values for a particular plate ranged from 15 percent to 35 percent, with the data being less consistent on plates farther from the feed plate. It was very difficult to determine values of \( A \). As previously mentioned, it is doubtful that frequency response data extracted from transient pulse data can give very accurate information regarding the amplitude of the resonance peaks. Those values of \( A \) stated can be assumed to have a margin of error of about 50 percent.
CONCLUSIONS

1. The pulse technique was shown to be an excellent method of experimentally determining the frequency response of the eight inch pilot Eckey Fractionator to upsets in liquid rate, liquid composition and vapor rate. Using properly executed pulses, this information can be found in a fraction of the time required for direct sinusoidal forcings.

2. The data, plotted on Bode diagrams, was analyzed to determine the parameters occurring in dynamic models of the system. Approximate first order behavior on all plates was noted with time delays evident in the responses to liquid rate and composition forcings. Resonance peaks were found in all responses. Good consistency in the calculated model parameters was obtained, while many pulses of different amplitude, duration and shape were used as input forcings. The results appear to be of sufficient accuracy to be useful in most engineering applications and in the design of feedback control systems.

3. The column operating pressure had no noticeable effect on the model parameters. Varying the pressure, changes only the vapor rate and density which would in turn change the vapor holdup and residence time. However, since the vapor holdup is very small relative to the liquid holdup,
little effect on the column response is expected.

4. It is felt that the results of this study can be applied to other Eckey columns of different sizes. Values of time delays for different liquid rates and column holdups can be estimated from equation (1), page 79. The first order time constants can be considered to be proportional to the total liquid residence time - total liquid holdup divided by liquid flow rate. From the results of this study, the estimate, $\tau = 0.25(RT)$, for responses to liquid and vapor rate forcings seems reasonable. For response to liquid composition disturbances, $\tau$ varied from $0.1(RT)$ on plate 28 to $0.2(RT)$ on plate 16. Until the resonance effects are more clearly understood there appears to be no simple method of scaling the parameters A and B for other units.

5. Further work can be suggested in several areas:

The effect on the column dynamics of (1) using different binary or multicomponent systems, (2) running at high vacuum, (3) using reboiler and reflux setups should be investigated,

Further investigation into the resonance effects using direct sinusoidal forcings or carefully selected pulses would be interesting.

An attempt should be made to mathematically derive and solve a set of modeling equations for the col-
umn. The validity of these could then be checked using the experimental results.

Additional investigation into the applications and limitations of pulse testing for dynamic analysis are needed. This would include study of (1) reliability, (2) accuracy, (3) onset of errors, (4) data processing routines, and (5) input pulse selection based on frequency content.
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Equipment Specifications

1. Mayney Pilot Horizontal Fractionator, 8 foot length, 6 inch diameter, 32 stage, stainless steel, Vulcan Manufacturing Company, serial #10542.

2. Foxboro Pneumatic Control Valve, #225435, 3/8 inch needle valve, reversible actuator.

3. Foxboro 1/2 inch transmitter, model 1/2-2-41251, 0-15 psi.

4. Foxboro 100 mesh wire screen filter.

5. Foxboro Frequency to D.C. Converter, model 70-305-8-2, Input: 10-3700 Hz, Output: 0-10 VDC.

6. Artistic and Roaring Rotometer, serial #53-9-304-7, 0-375 gpm, 0-250 psig.

7. Artistic and Roaring Rotometer, serial #53-9-304-7, 0-15 gpm, 0-250 psig.

8. Automatic Switch Company, 3-way solenoid valve, model 3B320412, serial #562935.

9. Daniel 1/2 inch, #300, raised face threaded orifice flange, model /30-RT.

   Critical Flow Orifice Plates:
   Plate 1: 6-65 psi, 0-65 lb/hr steam
   Plate 2: 6-65 psi, 0-140 lb/hr steam

10. Foxboro Electronic Pressure Transmitter, model #6-110-CH, serial #1435906, Input 0-100 psig, Output 10-50 mA.

11. Foxboro Power Supply, part #M12112, Output: 65 volts DC at 70 mA.

12. Foxboro Pneumatic Control Valve, #3944093, 1/4 inch needle valve, reversible motor.

13. Manostat Steam Pressure Reducing Valve, model #27-1, serial #279, 55-125 psi.

14. Stainless Steel, 1-1 shell and tube condenser, 47 sq. ft. Ferry Products Company.

APPENDIX A

Equipment Specifications
Equipment Specifications

1. Eckey Pilot Horizontal Fractionator, 8 foot length, 8 inch diameter, 32 stages, stainless steel, Vulcan Manufacturing Company, Serial #10542.

2. Foxboro Pneumatic Control Valve, #238485, 3/8 inch needle valve, reversible motor.

3. Foxboro 1/2 inch turbine flow transmitter, model 1/2-2-81F501, 0-.5 gpm.

4. Number 100 mesh wire screen filter.

5. Foxboro Frequency to DC Converter, model FR-305-5-2, Input: 10-2700 HZ., Output: 0-5 volts DC.

6. Schutte and Koerting Rotometer, serial #63-01-627-M, 0-.575 gpm., 0-250 mm.

7. Schutte and Koerting Rotometer, serial #53-9-204-T, 0-.5 gpm., 0-250 mm.

8. Automatic Switch Company, 3-way solenoid valve, model #8320All, serial #682935.

9. Daniel 1/2 inch, #300, raised face threaded orifice flange, model #30-RT.
   Critical Flow Orifice Plates:
   Plate 1: 0-85 psia, 0-85 lb/hr steam
   Plate 2: 0-85 psia, 0-140 lb/hr steam

10. Foxboro Electronic Pressure Transmitter, model #6-11-GM, serial #1435208, Input 0-100 psig, Output 10-50 ma.

11. Foxboro Power Supply, part #N121AT, Output: 65 volts DC at 70 ma.

12. Foxboro Pneumatic Control Valve, #2944093, 1/4 inch needle valve, reversible motor.

13. Masoneilan Steam Pressure Reducing Valve, model #17-1, serial #279, 25-125 psi.

14. Stainless Steel, 1-1 shell and tube condenser, 47 sq. ft. Perry Products Company.
15. N.C.R. Rotary Gas Ballast Pump, type 15-D, serial #41402.

16. Honeywell Electronik 19 Multirange Lab/Test Wide Chart Recorder, model #194 112-002-002-001-01, serial #I8245974001.

17. Honeywell Electronik 19 Multirange Lab/Test Narrow Chart Recorder, model #19302-11-01, serial #E6757710001.

18. Everedy Mini-Max 300 volt battery, no.493.

19. Hewlett-Packard Low Frequency Function Generator, model 202A, range .008-1,200 Hz., 0-30 volts.

20. Hewlett-Packard Harrison 6204B DC Power Supply, 0-40 volts, .3amps.

21. Fisher Electro-Pneumatic Transducer, serial #4362320, Input: 1-9 volts DC, Output: 3-15 psig., Fisher Governor Company.

22. Pulse Activator, circuit constructed as per Hewlett-Packard Application Note 31: Externally Driving the 202A Low Frequency Function Generator.
Computer Program

This appendix contains the computer program used to calculate frequency response information from pulsed input and output response time histories, and a general flow diagram of the logic involved. The values of the pulse forcing, sampling interval and number of data points were input to the program together with values for the response, response sampling interval and number of points. Frequency values at which the frequency response was calculated were also specified. At a particular frequency, the program utilized the FFT routine to calculate the Fourier transforms of the input pulse and output response. The output transform was then divided by the input transform to give the real and imaginary parts of the frequency response. The magnitude and phase angle of this complex number was then found and the magnitude was normalized by the magnitude at zero frequency. The program printed out the frequency specified, the real and imaginary parts of the Fourier transforms of the input pulse and system response, the real and imaginary parts of the frequency response, and the normalized amplitude and phase angle.

Important nomenclature is shown below.

**NF** - number of pulse data points

**NT** - sampling interval or pulse forcing
Computer Program

This Appendix contains the computer program used to calculate frequency response information from pulsed input and output response time histories, and a general flow diagram of the logic involved. The values of the pulse forcing, sampling interval and number of data points were input to the program together with values for the response, response sampling interval and number of data points. Frequency values at which the frequency response was calculated were also specified. At a particular frequency, the program utilized the TAFT routine to calculate the Fourier transforms of the input pulse and output response. The output transform was then divided by the input transform to give the real and imaginary parts of the frequency response. The magnitude and phase angle of this complex number was then found and the magnitude was normalized by the magnitude at zero frequency. The program printed out the frequency specified, the real and imaginary parts of the Fourier transforms of the input pulse and system response, the real and imaginary parts of the frequency response, and the normalized amplitude and phase angle.

Important nomenclature is shown below.

NP - number of pulse data points
DTI - sampling interval of pulse forcing
DTO - sampling interval of response
G - values of response
F - values of pulse
W - frequency specified
FTRI - real part of Fourier transform of input pulse
FTMI - imaginary part of Fourier transform of input pulse
FTRO - real part of Fourier transform of output response
FTMO - imaginary part of Fourier transform of output response
FRR - real part of frequency response
FRM - imaginary part of frequency response
ARG - normalized magnitude (amplitude ratio) of frequency response
ANGLE - phase angle of frequency response
DIMENSION W(100)
DIMENSION F(100)
DIMENSION G(100)
1 FORMAT(1X,E11.4,2X,E11.4,4X,E11.4,2X,E11.4,2X,E11.4,2X,E11.4,2X,E11.4,2X,E11.4,2X)
20 FORMAT(T3,'FOURIER TRANSFORM OF',T31,'FOURIER TRANSFORM OF',T8
10,'FREQUENCY PESP:INSE',/)
30 FORMAT(T7,'INPUT PULSE',T38,'OUTPUT',//)
40 FORMAT(T5,'REAL',T16,'IMAGINARY',T33,'REAL',T44,'IMAGINARY',T59,'F
1REQUENCY',T74,'REAL',T85,'IMAGINARY',T99,'ARGUMENT',T113,'PHASE')
50 FORMAT(T5,'PART',T18,'PART',T33,'PART',T47,'PART',T74,'PART',T87
1,'PART',T113,'ANGLE',/)
100 FORMAT (5E15.4)
110 FORMAT (5E15.4)
120 FORMAT(1C)
130 FORMAT(2E15.4)
    WRITE(6,20)
    WRITE(6,30)
    WRITE(6,40)
    WRITE(6,50)
    READ(5,120)KK
    READ(5,100) (W(K),K=1,KK)
    READ(5,130)DTI
    READ(5,130)DT0
    READ(5,120)NP
    READ(5,110) (F(I),I=1,NP)
    READ(5,120)MP
    READ(5,110) (G(I),I=1,MP)
    NI=NP-1
    MI=MP-1
    A1=0.0
    DO 700 K=1,XX
       SD21 = ([SIN(W(K)*DTI/2.))/((W(K)*DTI/2.))]*2
       CSI = SIN(W(K)*(NI*DTI+A1))
       SD1 = (SIN(W(K)*DTI))/((W(K)*DTI))
       CALCULATE REAL PART OF INPUT FOURIER TRANSFORM
       SUM=0.0
       DO 380 I=2,N1
          DSM =F(I)*COS(W(K)*((I-1)*DTI+A1))
          SUM=SUM+DSM
       CONTINUE
    380     FTRI=DTI*F(1)* (SD21*COS(W(K)*A1))/2.*(1.-SD1)/(W(K)*DTI)*
           1SIN(W(K)*A1))
    7+DTI*SD21*SUM
    3+DTI*F(NP)* (SD21/2.*CSI+(1.-SD1)/(W(K)*DTI)*SS1)
CALCULATE IMAGINARY PART OF INPUT FOURIER TRANSFORM
SUM=0.0
DO 480 I=2,NI
DSUM=F(I)*(-SIN(W(K)*((I-1)*DTI+AI)))/2
SUM=SUM+DSUM
480 CONTINUE
490 FMI=DTI*F(I) + (SD2I/2.*SIN(W(K)*A1)-{(1.-SDI)/(W(K)*DTI))*
1COS(W(K)*A1))/2+DTI*SD2I*SUM +
3DTI*F(NP)* (-SD2I/2.*SSI+(1.-SDI)/(W(K)*DTI)*CSI)
SD2O = ((SIN(W(K)*DTO/2.))/(W(K)*DTO/2.))^2
SSO = SIN(W(K)*((M1*DTO+A1))
C50 = COS(W(K)*((M1*DTO+A1))

SD0 = (SIN(W(K)*DTO))/(W(K)*DTO)
CALCULATE REAL PART OF OUTPUT FOURIER TRANSFORM
SUM=0.0
DO 580 I=2,MI
DSUM =G(I)*COS(W(K)*((I-1)*DTO+A1))/2
SUM=SUM+DSUM
580 CONTINUE
590 FTR0=DTO*G(I) + (SD20*(COS(W(K)*A1)))/2-{(1.-SD0)/(W(K)*DTO))*
1SIN(W(K)*A1))/2+DTO*SD20*SUM +
3*DTO*G(NP) - (SD20/2.*CS0+(1.-SD0)/(W(K)*DTO)*SS0)
CALCULATE IMAGINARY PART OF OUTPUT FOURIER TRANSFORM
SUM=0.0
DO 680 I=2,MI
DSUM=G(I)*(-SIN(W(K)*((I-1)*DTO+A1)))/2
SUM=SUM+DSUM
680 CONTINUE
690 FTRM=DTO*G(I) + (SD20/2.*SIN(W(K)*A1)-{(1.-SD0)/(W(K)*DTO))*
1COS(W(K)*A1))/2+DTO*SD20*SUM +
3DTO*G(NP) - (SD20/2.*SS0+(1.-SD0)/(W(K)*DTO)*CS0)
FRR=(FTR1*FTR0+FTM*FTRM)/IFTRI**2+FTM**2)
FRM=(FTR1*FTR0-FTM*FTRM)/IFTRI**2+FTM**2)
ARG=(FRR**2+FRM**2)**.5
ANGLE=ATAN(FRM/FRR)
WRITE(6,100) (W(K),K=1,KK)
WRITE(6,110) (F(I),I=1,NP)
WRITE(6,110) (G(I),I=1,MP)
CALL EXIT
END
Flow sheet of computer program logic for calculation of frequency response from pulse input and output response data

1. Read in input pulse and output response data, sampling interval, and number of sampling points for each.

2. Specify range of frequencies, $\omega$, at which it is desired to calculate frequency response and let $\omega = \omega_{\text{initial}}$.

3. Calculate real and imaginary parts of Fourier transform of input pulse using TAFT routine.

4. Calculate real and imaginary parts of Fourier transform of output response using TAFT routine.

5. Divide output Fourier transform by input Fourier transform to get real and imaginary parts of frequency response.

6. Calculate magnitude and phase angle of frequency response and amplitude ratio by dividing magnitude by magnitude at $\omega = 0$. 
Print out real and imaginary parts of input and output Fourier transforms and frequency response, amplitude ratio, and phase angle and specified frequency.

Call plotting routine to plot amplitude ratio and phase angle versus frequency on Bode diagram.

Choose new $\omega$ at which to calculate frequency response until $\omega = \omega_{\text{final}}$.

end
This Appendix contains the experimentally determined frequency response plots (Bode diagrams) calculated from the input pulse - output response data by means of the computerized FFT routine. It is divided into six sections according to input pulse forcing and column pressure:

Vapor rate responses - 1.0 atm.
Liquid rates - 1.0 atm.
Liquid concentration responses - 1.0 atm.
Vapor rate - .25 atm.
Liquid - .25 atm.
Liquid concentration responses - .25 atm.

APPENDIX C
Experimental Frequency Response Plots
This Appendix contains the experimentally determined frequency response plots (Bode diagrams) calculated from the input pulse - output response data by means of the computerized TAFT routine. It is divided into six sections according to input pulse forcing and column pressure:

- Vapor rate responses - 1.0 atm.
- Liquid rate responses - 1.0 atm.
- Liquid concentration responses - 1.0 atm.
- Vapor rate responses - .26 atm.
- Liquid rate responses - .24 atm.
- Liquid concentration responses - .26 atm.
Bode Plots

Vapor Rate Responses

Column Pressure - 1.0 atm.
BODE PLOT

VAPOR PULSE 1 AT COLUMN PRESSURE OF 1.000 ATM.

AMPLITUDE RATIO

FREQUENCY - RAD/NS PER SECOND
BODE PLOT

VAPOR PULSE 2 AT COLUMN PRESSURE OF 1.000 ATM.
PHASE ANGLE

FREQUENCY - RADIANS PER SECOND

* - PLATE 16  0 - PLATE 26
X - PLATE 21  + - PLATE 28
BODE PLOT

VAPOUR PULSE 3 AT COLUMN PRESSURE OF 1.000 ATM.

AMPLITUDE RATIO

FREQUENCY - RADIANS PER SECOND
FREQUENCY - RADIANS PER SECOND

- PLATE 16
0 - PLATE 26
X - PLATE 21
+ - PLATE 28
BODE PLOT

VAPOUR PULSE AT COLUMN PRESSURE OF 1.000 ATM.
BODE PLOT

VAPOUR PULSE 5 AT COLUMN PRESSURE OF 1.000 ATM.

AMPLITUDE RATIO

FREQUENCY - RADIANS PER SECOND
BODE PLOT

VAPOR PULSE 6 AT COLUMN PRESSURE OF 1.000 ATM.
FREQUENCY - RADIANS PER SECOND

- PLATE 16
0 - PLATE 26
X - PLATE 21
+ - PLATE 28
BODE PLOT

VAPOR PULSE ? AT COLUMN PRESSURE OF 1.000 ATM.

AMPLITUDE RATIO

FREQUENCY - RADIANS PER SECOND
BODE PLOT

VAPOR PULSE 8 AT COLUMN PRESSURE OF 1.000 ATM.
FREQUENCY - RADIANS PER SECOND

- PLATE 18
O - PLATE 26
X - PLATE 21
+ - PLATE 28
BODE PLOT

VAPOR PULSE 5 AT COLUMN PRESSURE OF 1.000 ATM.
BODE PLOT

VAPOR PULSE 10 AT COLUMN PRESSURE OF 1.000 ATM.

AMPLITUDE RATIO

FREQUENCY - RADIANS PER SECOND
BODE PLOT

VAPOR PULSE II: AT COLUMN PRESSURE OF 1.000 ATM.

AMPLITUDE RATIO

FREQUENCY - RADIANS PER SECOND
BODE PLOT

VAPOR PULSE 12 AT COLUMN PRESSURE OF 1.000 ATM.
BODE PLOT

VAPOR PULSE 13 AT COLUMN PRESSURE OF 1.000 ATM.

AMPLITUDE RATIO

FREQUENCY - RADIANS PER SECOND
BODE PLOT

VAPOR PULSE 14 AT COLUMN PRESSURE OF 1.000 ATM.
BODE PLOT

VAPOR PULSE 15 AT COLUMN PRESSURE OF 1.000 ATM.
BODE PLOT

VAPOR PULSE 16 AT COLUMN PRESSURE OF 1.000 ATM.
FREQUENCY - RADIANS PER SECOND

- PLATE 16
O - PLATE 26
X - PLATE 21
+ - PLATE 28
BODE PLOT

VAPOR PULSE 17 AT COLUMN PRESSURE OF 1.000 ATM.
FREQUENCY - RADIANS PER SECOND

* - PLATE 16  0 - PLATE 26
X - PLATE 21  + - PLATE 28
BODE PLOT

VAPOR PULSE 18 AT COLUMN PRESSURE OF 1.000 ATM.
Phase Angle vs Frequency (Radian Per Second) with markers for Plate 16, Plate 26, Plate 21, and Plate 28.
BODE PLOT

VAPOR PULSE 19 AT COLUMN PRESSURE OF 1,000 ATM.

AMPLITUDE RATIO

FREQUENCY - RADIANS PER SECOND
FREQUENCY - RADIANS PER SECOND

* - PLATE 16  0 - PLATE 26
X - PLATE 21  + - PLATE 29
BODE PLOT

VAPOR PULSE 20 AT COLUMN PRESSURE OF 1.000 ATM.
FREQUENCY - RADIANS PER SECOND

- PLATE 16  O - PLATE 26
X - PLATE 21  + - PLATE 28
BODE PLOT

VAPOR PULSE 21 AT COLUMN PRESSURE OF 1.000 ATM.
BODE PLOT

VAPOR PULSE 22 AT COLUMN PRESSURE OF 1.000 ATM.

AMPLITUDE RATIO

FREQUENCY - RADIANS PER SECOND
FREQUENCY - RADIANS PER SECOND

- PLATE 16
- PLATE 21
- PLATE 23
+ PLATE 26
BODE PLOT

VAPOR PULSE 23 AT COLUMN PRESSURE OF 1.000 ATM.
BODE PLOT

VAPOR PULSE 24 AT COLUMN PRESSURE OF 1.000 ATM.

AMPLITUDE RATIO

FREQUENCY - RADIANS PER SECOND
FREQUENCY - RADIANs PER SECOND

* - PLATE 16  O - PLATE 26
X - PLATE 21  + - PLATE 28
Bode Plots

Liquid Rate Responses

Column Pressure - 1.0 atm.
BODE PLOT

LIQUID PULSE 1 AT COLUMN PRESSURE OF 1.000 ATM.

AMPLITUDE RATIO

FREQUENCY - RADIANS PER SECOND
FREQUENCY - RADIANS PER SECOND

- PLATE 16

0 - PLATE 26

X - PLATE 21

+ - PLATE 28
BODE PLOT

LIQUID PULSE 2 AT COLUMN PRESSURE OF 1.000 ATM.

AMPLITUDE RATIO

FREQUENCY - RADIANS PER SECOND
BODE PLOT

LIQUID PULSE 3 AT COLUMN PRESSURE OF 1.000 ATM.
BODE PLOT

LIQUID PULSE AT COLUMN PRESSURE OF 1.000 ATM.
FREQUENCY - RADIANs PER SECOND

- PLATE 16
0 - PLATE 26
X - PLATE 21
+ - PLATE 28
LIQUIDPULSE 5 AT COLUMN PRESSURE OF 1.000 ATM.
BODE PLOT

LIQUID PULSE at COLUMN PRESSURE OF 1.000 ATM.

AMPLITUDE RATIO

FREQUENCY - RADIANS PER SECOND
# FREQUENCY vs. RADIANS PER SECOND

- **#** - PLATE 16
- **O** - PLATE 26
- **X** - PLATE 21
- **+** - PLATE 28
BODE PLOT

LIQUID PULSE AT COLUMN PRESSURE OF 1.000 ATM.

FREQUENCY - RADIANS PER SECOND

AMPLITUDE RATIO
FREQUENCY - RADIANS PER SECOND

- PLATE 18
- PLATE 26
- PLATE 21
- PLATE 28
BODE PLOT

LIQUID PULSE 9 AT COLUMN PRESSURE OF 1.000 ATM.

| AMPLITUDE RATIO | FREQUENCY - RADIANS PER SECOND |
|-----------------|--------------------------------|
| 0.1             |                               |
| 0.2             |                               |
| 0.3             |                               |
| 0.4             |                               |
| 0.5             |                               |
| 0.6             |                               |
| 0.7             |                               |
| 0.8             |                               |
| 0.9             |                               |
| 1.0             |                               |
| 1.1             |                               |
| 1.2             |                               |
| 1.3             |                               |
| 1.4             |                               |
| 1.5             |                               |
| 1.6             |                               |
| 1.7             |                               |
| 1.8             |                               |
| 1.9             |                               |
| 2.0             |                               |
FREQUENCY - RADIANS PER SECOND

- PLATE 16
0 - PLATE 26
X - PLATE 21
+ - PLATE 28
BOEDE PLOT

LIQUID PULSE 10 AT COLUMN PRESSURE OF 1.000 ATM.

AMPLITUDE RATIO

FREQUENCY - RADIANS PER SECOND
BODE PLOT

LIQUID PULSE II AT COLUMN PRESSURE OF 1.000 ATM.
BODE PLOT

LIQUID PULSE 12 AT COLUMN PRESSURE OF 1.000 ATM.

AMPLITUDE RATIO

FREQUENCY - RADIAN PER SECOND
PLATE 16
0 - PLATE 26
X - PLATE 21
+ - PLATE 28

FREQUENCY - RADIANS PER SECOND
BODE PLOT

LIQUID PULSE 13 AT COLUMN PRESSURE OF 1.000 ATM.

AMPLITUDE RATIO

FREQUENCY - RADIANS PER SECOND
BODE PLOT

LIQUID PULSE AT COLUMN PRESSURE OF 1.000 ATM.
FREQUENCY - RADIANS PER SECOND

- PLATE 16
- PLATE 21
- PLATE 26
+ - PLATE 28
BODE PLOT

LIQUID PULSE 15 AT COLUMN PRESSURE OF 1.000 ATM.

AMPLITUDE RATIO

FREQUENCY - RADIANS PER SECOND
BODE PLOT

LIQUID PULSE 16 AT COLUMN PRESSURE OF 1.000 ATM.
BODE PLOT

LIQUID PULSE 17 AT COLUMN PRESSURE OF 1.000 ATM.

FREQUENCY - RADIANS PER SECOND

AMPLITUDE RATIO
Bode Plots

Liquid Concentration Responses

Column Pressure - 1.0 atm.
BODE PLOT

CONCENTRATION PULSE 1 AT COLUMN PRESSURE OF 1.000 ATM.
BODE PLOT

CONCENTRATION PULSE 2 AT COLUMN PRESSURE OF 1.000 ATM.

FREQUENCY - RADIANS PER SECOND

AMPLITUDE RATIO
FREQUENCY - RADIANS PER SECOND

* - PLATE 16  O - PLATE 26
X - PLATE 21  + - PLATE 28
BODE PLOT

CONCENTRATION PULSE 3 AT COLUMN PRESSURE OF 1.000 ATM.
Bode Plot

Concentration Pulse at Column Pressure of 1.000 atm.

Amplitude Ratio

Frequency - Radians per Second
FREQUENCY - RADIANS PER SECOND

- PLATE 16
O - PLATE 26
X - PLATE 21
+ - PLATE 28
BODE PLOT

CONCENTRATION PULSE 5 AT COLUMN PRESSURE OF 1.000 ATM.

AMPLITUDE RATIO

FREQUENCY - RADIANS PER SECOND
FREQUENCY - RADIANS PER SECOND

- PLATE 16
- PLATE 21
O - PLATE 26
+ - PLATE 28
BODE PLOT

CONCENTRATION PULSE 6 AT COLUMN PRESSURE OF 1.000 ATM.
BODE PLOT

CONCENTRATION PULSE at COLUMN PRESSURE OF 1.000 ATM.

AMPLITUDE RATIO

FREQUENCY - RADIAN PER SECOND
BODE PLOT

CONCENTRATION PULSE 8 AT COLUMN PRESSURE OF 1.000 ATM.
FREQUENCY - RADIAN'S PER SECOND

* - PLATE 16
0 - PLATE 26
X - PLATE 21
+ - PLATE 29
BODE PLOT

CONCENTRATION PULSE 9 AT COLUMN PRESSURE OF 1.000 ATM.
FREQUENCY - RADIANS PER SECOND

- PLATE 16
- PLATE 21
0 - PLATE 26
+ - PLATE 28
BODE PLOT

CONCENTRATION PULSE 10 AT COLUMN PRESSURE OF 1.000 ATM.
FREQUENCY - RADIIANS PER SECOND

- PLATE 16
- PLATE 26
- PLATE 21
- PLATE 29
Bode Plots

Vapor Rate Responses

Column Pressure - .26 atm.
BODE PLOT

VAPOUR PULSE 1 AT COLUMN PRESSURE OF 0.263 ATM.
FREQUENCY - RADIANS PER SECOND

- PLATE 18
0 - PLATE 26
X - PLATE 21
+ - PLATE 28
BODE PLOT

VAPOR PULSE 2 AT COLUMN PRESSURE OF 0.263 ATM.

AMPLITUDE RATIO

FREQUENCY - RADIANS PER SECOND
BODE PLOT

VAPOR PULSE 3 AT COLUMN PRESSURE OF 0.263 ATM.

AMPLITUDE RATIO

FREQUENCY - RADIANS PER SECOND
BODE PLOT

VAPOR PULSE AT COLUMN PRESSURE OF 0.263 ATM.
BODE PLOT

VAPOR PULSE 5 AT COLUMN PRESSURE OF 0.263 ATM.

AMPLITUDE RATIO

FREQUENCY - RADIANS PER SECOND
BODE PLOT

VAPOR PULSE 6 AT COLUMN PRESSURE OF 0.263 ATM.
BODE PLOT

VAPOR PULSE at column pressure of 0.263 ATM.
BODE PLOT

VAPOR PULSE AT COLUMN PRESSURE OF 0.263 ATM.
FREQUENCY - RADIANS PER SECOND

- PLATE 16  0 - PLATE 26
- PLATE 21  + - PLATE 28
BODE PLOT

VAPOR PULSE 9 AT COLUMN PRESSURE OF 0.263 ATM.
FREQUENCY - RADIANS PER SECOND

- PLATE 16
- PLATE 21
- PLATE 28
BODE PLOT

VAPOR PULSE 10 AT COLUMN PRESSURE OF 0.263 ATM.

AMPLITUDE RATIO

FREQUENCY - RADIANS PER SECOND
FREQUENCY - RADIANS PER SECOND

* - PLATE 16
X - PLATE 21
O - PLATE 26
+ - PLATE 28
BODE PLOT

VAPOR PULSE 11 AT COLUMN PRESSURE OF 0.263 ATM.

AMPLITUDE RATIO

FREQUENCY - RADIANS PER SECOND
BODE PLOT

VAPOR PULSE 12 AT COLUMN PRESSURE OF 0.263 ATM.
BODE PLOT

VAPOR PULSE 13 AT COLUMN PRESSURE OF 0.263 ATM.
FREQUENCY - RADIANS PER SECOND

* - PLATE 16  0 - PLATE 26
X - PLATE 21  + - PLATE 29
BODE PLOT

VAPOR PULSE 14 AT COLUMN PRESSURE OF 0.263 ATM.
FREQUENCY - RADIANS PER SECOND

- PLATE 16  O - PLATE 26
X - PLATE 21  + - PLATE 29
BODE PLOT

VAPOR PULSE 15 AT COLUMN PRESSURE OF 0.263 ATM.
FREQUENCY - RADIANS PER SECOND

- PLATE 16
0 - PLATE 26
X - PLATE 21
+ - PLATE 28
BODE PLOT

VAPOR PULSE 16 AT COLUMN PRESSURE OF 0.263 ATM.

AMPLITUDE RATIO

FREQUENCY - RADIANS PER SECOND
BODE PLOT

VAPOR PULSE 17 AT COLUMN PRESSURE OF 0.263 ATM.
BODE PLOT

VAPOR PULSE 13 AT COLUMN PRESSURE OF 0.263 ATM.
FREQUENCY - RADIANs PER SECOND

* - PLATE 16  O - PLATE 26
X - PLATE 21  + - PLATE 28
BODE PLOT

VAPOR PULSE 19 AT COLUMN PRESSURE OF 0.263 ATM.
FREQUENCY - RADIANS PER SECOND

* - PLATE 16  
O - PLATE 26  
X - PLATE 21  
+ - PLATE 28
BODE PLOT

VAPOR PULSE 20 AT COLUMN PRESSURE OF 0.263 ATM.

AMPLITUDE RATIO

FREQUENCY - RADIANS PER SECOND
BODE PLOT

VAPOR PULSE 21 AT COLUMN PRESSURE OF 0.263 ATM.
Bode Plots

Liquid Rate Responses

Column Pressure - .24 atm.
BODE PLOT

LIQUID PULSE 2 AT COLUMN PRESSURE OF 0.243 ATM.

FREQUENCY - RADIANS PER SECOND
BODE PLOT

LIQUID PULSE 3 AT COLUMN PRESSURE OF 0.243 ATM.
BODE PLOT

LIQUID PULSE AT COLUMN PRESSURE OF 0.243 ATM.
FREQUENCY - RADIANS PER SECOND

- PLATE 16
- PLATE 21
- PLATE 26
- PLATE 28
BODE PLOT

LIQUID PULSE 5 AT COLUMN PRESSURE OF 0.243 ATM.

AMPLITUDE RATIO

FREQUENCY - RADIANS PER SECOND
FREQUENCY - RADIANS PER SECOND

- PLATE 16
- PLATE 26
X - PLATE 21
+ - PLATE 28
BODE PLOT

LIQUID PULSE 6 AT COLUMN PRESSURE OF 0.243 ATM.

FREQUENCY - RADIANS PER SECOND

AMPLITUDE RATIO
BODE PLOT

LIQUID PULSE 7 AT COLUMN PRESSURE OF 0.242 ATM.
BODE PLOT

LIQUID PULSE 8 AT COLUMN PRESSURE OF 0.293 ATM.

FREQUENCY - RADIANS PER SECOND

AMPLITUDE RATIO
BODE PLOT

LIQUID PULSE AT COLUMN PRESSURE OF 0.243 ATM.
BODE PLOT

LIQUID PULSE 10 AT COLUMN PRESSURE OF 0.243 ATM.
BODE PLOT

LIQUID PULSE 11 AT COLUMN PRESSURE OF 0.243 ATM.

AMPLITUDE RATIO

FREQUENCY - RADIANS PER SECOND
BODE PLOT

LIQUID PULSE 13 AT COLUMN PRESSURE OF 0.243 ATM.
FREQUENCY - RADIANS PER SECOND

- PLATE 16
- PLATE 26
X - PLATE 21
+ - PLATE 28
BODE PLOT

LIQUID PULSE 14 AT COLUMN PRESSURE OF 0.243 ATM.
FREQUENCY - RADIANS PER SECOND

- PLATE 16  0 - PLATE 26

X - PLATE 21  + - PLATE 28
Bode Plots

Liquid Concentration Responses

Column Pressure - .26 atm.
BODE PLOT

CONCENTRATION PULSE 1 AT COLUMN PRESSURE OF 0.263 ATM.
BODE PLOT

CONCENTRATION PULSE 2 AT COLUMN PRESSURE OF 0.263 ATM.
CONCENTRATION PULSE 3 AT COLUMN PRESSURE OF 0.263 ATM.

FREQUENCY - RADIANS PER SECOND
FREQUENCY - RADIANS PER SECOND

* - PLATE 16  0 - PLATE 26
X - PLATE 21  + - PLATE 28
BODE PLOT

CONCENTRATION PULSE AT COLUMN PRESSURE OF 0.263 ATM.
BODE PLOT

CONCENTRATION PULSE AT COLUMN PRESSURE OF 0.263 ATM.
BODE PLOT

CONCENTRATION PULSE AT COLUMN PRESSURE OF 0.263 ATM.
BODE PLOT

CONCENTRATION PULSE at column pressure of 0.263 atm.

FREQUENCY - RADIANS PER SECOND

AMPLITUDE RATIO
BODE PLOT

CONCENTRATION PULSE 8 AT COLUMN PRESSURE OF 0.253 ATM.

FREQUENCY - RADIANS PER SECOND

AMPLITUDE RATIO
FREQUENCY - RADIANS PER SECOND

* - PLATE 16
0 - PLATE 26
X - PLATE 21
+ - PLATE 29
BODE PLOT

CONCENTRATION PULSE AT COLUMN PRESSURE OF 0.263 ATM.
BODE PLOT

CONCENTRATION PULSE 10 AT COLUMN PRESSURE OF 0.263 ATM.

FREQUENCY - RADIANS PER SECOND

AMPLITUDE RATIO
FREQUENCY - RADIANS PER SECOND

# - PLATE 16     O - PLATE 26
X - PLATE 21     + - PLATE 29