Algorithm for Vertical Bearing Capacity Calculation of Rock-Socketed Piles in Karst Area Based on Load Transfer Method

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Abstract. Due to the complex geological conditions and many influencing factors in karst areas, this paper conducted theoretical, experimental research and calculation analysis on the design and construction of engineering piles of bridge engineering in karst cave areas. An algorithm for vertical capacity calculation of rock-socketed piles in karst area based on load transfer method was developed. The results using this new method were analyzed and compared with the results from the static load test of three engineering piles in karst area, and the differences are relatively small. Through static load tests of pile foundation in karst area and study of the side resistance of rock-socketed pile in karst area, the friction resistance of rock-socketed pile and the effect of pile-end resistance, the bearing capacity characteristics of rock-socketed pile in karst area were obtained, achieving the load-sharing effect of cave roof in a rather scientific way. In view of the failure mechanism of the karst cave roof under the pile tip and the large proportion of lateral resistance of the rock-socketed pile, a calculation model for the stability of the karst cave roof under the combined action of the side resistance of the rock-socketed pile and the pile tip resistance is proposed.

1. Introduction
For karst areas, due to its complex geological conditions and numerous influencing factors, it brings great safety hazards to the design and construction of buildings and substructures. Currently in China, research work on the vertical bearing capacity of pile foundations in karst areas is still at the exploratory stage. Most of the bridge pile foundations are in the form of rock-socketed piles. Based on the load transfer method, this paper proposes a bearing capacity-displacement calculation method suitable for rock-socketed piles.

In terms of the theory on bearing capacity of rock-socketed piles, a large number of previous studies have shown that the factors affecting the bearing capacity of rock-socketed piles are not independent of the performance of the bearing capacity of rock-socketed piles, but a complex coupling effect. Pells and Turner [1] believed that the cause of the failure of the rock-socketed pile was the plastic failure of the rock mass at the bottom of the pile. They suggested that the ultimate bearing capacity of the pile end requires a large displacement, and the finite element method was used to give the average settlement reduction coefficient of the foundation, the ratio of elastic modulus of rock and pile, and the relationship between embedded depth and rock-socketed ratio. Williams [2] proposed that rock mass will reduce its normal strength due to the existence of joints, then further reduce the side
resistance around the pile. They also proposed a calculation method of pile side resistance taking into consideration of rock mass joints.

Through indoor model tests, it concluded that: 1) The roughness of the pile-rock interface is the main factor affecting the change of Q-S curve; 2) The cleanliness of the interface has a great influence on the lateral resistance of the pile; 3) For rock masses with a low degree of weathering, the relationship between the uniaxial compressive strength of the rock and the ultimate lateral resistance can be established. Horvath [3] et al. summarized the relationship between rock uniaxial compressive strength and pile side resistance through test data of more than 50 piles around the world, and perfected Pells’ theory using field tests instead of indoor model tests. Serrano and Olalla [4-6] used the H-P constitutive to calculate the ultimate end resistance based on the minimum rock-socketing depth, used the same method to derive the theoretical formula for the ultimate side resistance of the pile, compared with the empirical formula and concluded that the formula is suitable for rock-socketed long pile. However, comparing the average side friction resistance of the field tests, the result of the calculation formula was too conservative [6], and the effect was not ideal.

Before the 1990s, China’s standards often treated rock-socketed piles as end-bearing piles; therefore, end resistance of end-bearing piles was an important research direction during this period of time. As a large number of researchers continue to explore the bearing mechanism of pile foundations, many scholars have found that pile-rock side resistance is an important part of the bearing capacity of rock-socketed piles. Their mechanism is not completely similar to end-bearing piles. The bearing mode of rock-socketed piles may have the characteristics of friction piles or end-bearing friction piles. Sichuan Provincial Highway Planning, Survey and Design Institute [7] proposed that the condition for exploring the vertical-bearing mechanism of rock-socketed piles is that the rock mass at the bottom of the pile has sufficient rigidity, and the load transmitted to the bottom of the pile is caused by the combined force of vertical reaction of the rock mass at the bottom of the pile and the friction of the pile-rock interface, and the method of multiplying the safety factor by the sum of the pile tip resistance and the lateral resistance of the rock-socketed section should be adopted when designing the pile. Long Qiuliang [8] based on the load transfer method and considering the softening characteristics of soil on the side of pile, Longqiuliang proposed a simplified algorithm for predicting the settlement of single pile. Li Tao [9] established the theoretical expressions and empirical values of the corresponding lateral friction load transfer functions respectively, which could ideally simulate the bearing characteristics of single pile, but the stiffness of single pile was slightly higher than the measured results. Guan Jinping [10] found that the length-diameter ratio was between 19.38 and 20.13, and the load shared by the pile tip resistance was between 54.8% and 55.2%, showing the characteristics of the friction-end bearing pile.

In summary, the bearing mechanism of rock-socketed piles is mostly focused on traditional experience analysis and experimental research based on actual engineering. There is still a lack of relevant theoretical research on the load transfer mechanism of rock-socketed piles and the difference in bearing performance caused by the variation in the rock mass at the end of the pile.

2. Introduction to Theoretical Models and Calculations

2.1. Load Transfer Method Theory
After the pile body is loaded, the frictional resistance between the pile and soil drives the soil around the pile to move downwards, and within the annular range of the soil around the pile, shear strain and shear stress are generated, and the shear strain and shear stress spread outward in a ring shape as shown in the figure 1.
Therefore, the shear strain at a distance from the center of the pile is:

\[ \gamma = \frac{dW_r}{dr} \approx \frac{\Delta W_r}{\delta_r} = \frac{\tau_r}{G} \]  

(1)

The shear stress can also be obtained according to the sum of the shear force on the soil ring around the pile and the pile side resistance: \( 2\pi r \tau_r = \pi dq_s \), and the shear stress is:

\[ \tau_r = \frac{d}{2r} q_s \]  

(2)

The vertical shear deformation of each radius circle of the soil around the pile is added together to obtain the settlement \( W \) of the bottom surface of the pile:

\[ \int_{\frac{1}{2}}^{d} dW_r = \int_{\frac{1}{2}}^{d} \frac{\tau_r}{G} dr, \text{ obtain } W = \frac{d}{2G_s} q_s \ln(2n) \]  

(3)

If assume the pile side resistance limit value is \( q_{su} \), the corresponding pile bottom settlement is:

\[ W = \frac{d}{2G_s} q_{su} \ln(2n) \]  

(4)

The exertion of pile side resistance and pile tip resistance is the process of load transfer between the pile and the soil. After the pile top is subjected to the load, the pile body deforms and then moves downward, so that the pile circumference and the pile tip soil give upward reaction force to the pile. The basic differential equation process of the load transfer analysis process in the upper pile-soil system is obtained as shown in the figure 2 and 3:

\[ Q(z) + dq(z) \]

Figure 1. Schematic diagram of soil deformation on pile side.

Figure 2. Schematic diagram of force on the micro-section of pile body.
Figure 3. Schematic diagram of pile-soil load transfer.

If the pile top load is $Q_0$, the axial force of the pile body section at any depth is:

$$Q(z) = Q_0 - U \int_0^z q_s(z)dz$$ \hspace{1cm} (5)

According to the compression stiffness of the pile, the vertical displacement can be obtained as:

$$u(z) = S_0 - \frac{1}{E_pA} \int_0^z Q(z)dz$$ \hspace{1cm} (6)

From the vertical balance condition of the micro-section of the pile body with length $dz$, $q_s(z)$ can be obtained as:

$$q_s(z) = -\frac{1}{U} \frac{dQ(z)}{dz}$$ \hspace{1cm} (7)

Correspondingly, the compression amount $ds(z)$ of the pile body is:

$$du(z) = -\frac{Q(z)}{E_pA} dz, Q(z) = -E_pA \frac{dW(z)}{dz}$$ \hspace{1cm} (8)

Substitute (3.4) into (3.3) to get:

$$q_s(z) = \frac{E_pA}{U} \frac{d^2u(z)}{dz^2}$$ \hspace{1cm} (9)

Among them, $A$ -the area of the pile section; $E_p$ - the elastic modulus of the pile body concrete; $U$ - the perimeter of the pile section.

The relationship between the axial force and lateral resistance of the pile shaft and the change of the pile shaft can be obtained through equations (5) and (7) as shown in the figure 4.

Figure 4. Spring load transfer model.
A similar method can also be used to analyze the load transfer law of the pile and the settlement calculation of the pile. According to the static balance equation of any element on the pile body, it can be obtained:

\[
\frac{dP(z)}{dz} = -U \tau(z)
\]

(10)

Pile unit compression is:

\[
du = -\frac{P(z)dz}{A_p E_p} \quad \text{or} \quad \frac{du}{dz} = -\frac{1}{A_p E_p} P(z)
\]

(11)

Taking the derivative and substitute, obtain:

\[
\frac{d^2u}{dz^2} = \frac{U}{A_p E_p} \tau(z) \quad \text{which is} \quad \frac{d^2u}{dz^2} = \frac{ku}{A_p E_p}
\]

(12)

2.2. Calculation Method of Rock-Socketed Pile Bearing Capacity Based on Load Transfer Method

This study assumes that the pile-soil load transfer is an ideal elastoplastic function, and the pile concrete is a linear elastic material during the vertical compression of the pile foundation. According to the above load transfer method, the process is derived:

\[
\frac{d^2u}{dz^2} = \frac{ku}{A_p E_p}
\]

(13)

According to the preliminary results of the test, among the components of the bearing capacity of the foundation pile, the pile side resistance comes into play before the pile tip resistance. This method makes the following assumptions:

When the load level is low, each layer of soil around the pile is in an elastic condition, and the rock mass at the end of the pile is also in an elastic condition; as the load level increases, the soil around the pile gradually transforms from elastic deformation to plastic deformation; finally as the load level continues to increase, the soil layers around the pile are transformed into plastic deformation.

Because the development of fissures in the rock portion of the pile tip cannot be known accurately, the ultimate load that the bedrock could bear cannot be accurately determined. Therefore, it is assumed that the pile-end rock mass under normal working conditions is in the elastic stage, and the pile-end rock mass will fail if plastic deformation occurs.

Due to low load level, there is lateral resistance along the length of the pile, and the soil around the pile and the rock at the end have not deformed.

According to:

\[
\frac{d^2S(z)}{dz^2} = \frac{U}{E_p A_p} \cdot \tau(z)
\]

(14)

\[
\tau(z) = \lambda_s S(z)
\]

(15)

Can obtain:

\[
\frac{d^2S(z)}{dz^2} - \frac{U}{E_p A_p} \cdot \lambda_s S(z) = 0
\]

(16)
If let $\alpha_1 = \sqrt{\frac{\lambda U}{E_pA_p}}$, then the above equation can be simplified to:

$$\frac{d^2 S(z)}{dz^2} - \alpha_1^2 S(z) = 0$$

(17)

According to the pile-end section axial force and the pile-end reaction force are interaction forces, and the displacement function value is the length of the pile length, the value of the pile-end displacement can be obtained by solving the above differential equation:

$$S(z) = A_1 \cdot sh(\alpha_1 z) + A_2 \cdot ch(\alpha_1 z)$$

(18)

Further according to:

$$Q(z) = -E_pA_p \cdot \frac{dS(z)}{dz}$$

(19)

Can obtain:

$$Q(z) = -E_pA_p\left[A_1 \cdot sh(\alpha_1 z) + A_2 \cdot ch(\alpha_1 z)\right]$$

(20)

And can solve the above equations $A_1$ and $A_2$:

$$A_1 = \frac{Q_b}{E_pA_p\alpha_1} \cdot ch(\alpha_1 L) - S_b \cdot sh(\alpha_1 L) \quad A_2 = \frac{Q_b}{E_pA_p\alpha_1} \cdot sh(\alpha_1 L) - S_b \cdot ch(\alpha_1 L)$$

(21)

The displacement and load expressions of the pile top ($z = 0$) of the rock-socketed pile are solved by the above differential equation:

$$S_0 = \frac{Q_b}{E_pA_p\alpha_1} \cdot sh(\alpha_1 L) + S_b \cdot ch(\alpha_1 L)$$

(22)

$$Q_0 = Q_b ch(\alpha_1 L) + E_pA_p\alpha_1 S_b sh(\alpha_1 L)$$

(23)

Since the rock at the end of the pile is in elastic stage, then:

$$\frac{Q_0}{S_0} = E_pA_p\alpha_1 \frac{k_1 + E_pA_p\alpha_1 th(\alpha_1 L)}{E_pA_p\alpha_1 + k_1 th(\alpha_1 L)}$$

(24)

During the continued loading process, the soil layers around the pile gradually transition from elastic stage to plastic stage. According to the test results, it can be seen that in the process of continuous load application, the soil on the upper part of the pile will give priority to the increased load, and then develop from top to bottom to the pile tip. Assuming that all the soil round the pile perimeter from $L_t$ above enters the plastic stage, and the soil layer at a certain elevation ($z = L_t$) is at the plastic stage, the expression of the load transfer function of the soil around the pile is:

$$\tau(z) = \lambda_S u$$

(25)

Then

$$\frac{d^2 S(z)}{dz^2} = \frac{U}{E_pA_p} \cdot \lambda_S u$$

(26)
If let $\alpha_2 = \frac{\lambda_2 U}{E_p A_p}$, based on the axial force at the assumed elevation positon ($z = L_x$) as displacement condition, solve the differential equation to achieve:

$$S(z) = A_3 \cdot sh(\alpha_2 z) + A_4 \cdot ch(\alpha_2 z) - \lambda_2 S_u$$

(27)

$$Q(z) = -E_p A_p A_2 \left[ A_3 \cdot ch(\alpha_2 z) + A_4 \cdot sh(\alpha_2 z) \right]$$

(28)

Because the elevation ($z = L_x$) has already entered the plastic stage, hence $S_x = S_u$. Then since the pile section below this elevation is still at the elastic stage, the interface axial force can be calculated:

$$Q_x = E_p A_p \alpha_1 \left[ k_1 + E_p A_p \alpha_1 \cdot \alpha_i \cdot th[\alpha_i (L - L_x)] \right] \cdot S_u$$

(29)

And solve the above equation:

$$A_3 = -\frac{Q_x}{E_p A_p \alpha_2} \cdot ch(\alpha_2 L_x) + S_x \cdot sh(\alpha_2 L_x) - \lambda_2 S_u \cdot sh(\alpha_2 L_x)$$

(30)

$$A_4 = -\frac{Q_x}{E_p A_p \alpha_2} \cdot sh(\alpha_2 L_x) + S_x \cdot ch(\alpha_2 L_x) + \lambda_2 S_u \cdot ch(\alpha_2 L_x)$$

(31)

If let $\alpha = \frac{\alpha_1}{\alpha_2}$, $K_x = \frac{Q_x}{S_x}$, $\beta = \frac{S_x}{S_u}$. Solve according to the L'Hospital's Rule:

$$Q_0 = (\alpha_i^2 L_x E_p A_p + K_x) \cdot S_u$$

(32)

$$S_0 = (1 + \frac{K_x L_x}{E_p A_p} + \frac{1}{2} \alpha_i^2 L_x^2) \cdot S_u$$

(33)

In this calculation method, if the load continues to be applied, since it is assumed that the soil around the pile is an ideal elastoplastic body, after the soil around the pile enters the plastic stage, all the additional load is borne by the rock at the end of the pile, so:

$$\Delta Q_0 = \Delta Q_o$$

(34)

Since the rock at the end of the pile is in the elastic stage, then:

$$\Delta Q_0 = k_i \Delta S_p$$

(35)

Assuming that the vertical compression of the pile body concrete occurs only after the end bearing force is exerted, the pile-top displacement can be expressed as:

$$\Delta S_0 = \Delta S_p + \Delta S_p$$

(36)

According to the above assumptions and derivation process, we have:

$$\Delta Q_0 = \frac{k_i}{1 + k_i L / E_p A_p} \Delta S_0$$

(37)
It is only necessary to substitute the calculation results of the above-mentioned layers of soil on the
side of the pile to the full pile side and substitute it into the layers of soil on the side of the full pile to
enter the plastic stage \( z = L \), and then solve the expression:

\[
S_0 = (1 + \frac{k_L}{E_p A_p}) \cdot S_{sw} + \frac{1}{2} \alpha_s^2 L^2 S_u \tag{38}
\]

The above derivation process conforms to the basic principles of the load transfer method, but due
to the diversity of actual engineering and the complexity of geotechnical materials, the derivation
result cannot be accurately used as the analytical solution of the Q-S curve obtained from the single
pile static load test. Therefore, this derivation process is only used to analyze the current stress state of
the test pile and properly predict the limit load range of the test pile.

3. Calculation and Test Analysis of Rock-Socketed Pile Bearing Capacity Based on the
Calculation Method

3.1. Project Overview
In the engineering project area, the Carboniferous Shidengzi Formation (C1s) has developed soluble
limestone, and the degree of karst development in the limestone is uneven. Among the 392 geological
boreholes, 111 boreholes have exposed karst and soil caves, with a cave-in rate of 28.32%, and the
degree of karst development is moderate. Among the 111 boreholes with exposing karst caves, 44 of
them are medium and large karst caves with a height of 3-10m, and large karst caves are greater than
or equal to 10m). The proportion of medium to large-scale karst caves is about 40%. Most of the caves
are single-layer caves, and the height of a single cave is 0.3~12.8m. Dividing the height of the caves
into three classes, below 3 meters, 3-10m, and above 10m, the ratio of the three is about 50%:48%:2%
respectively. The filling in the cave is mainly non-filled or semi-filled.

Geological profile of Y0-C# pile: the first layer is debris fill, about 3.5 meters thick; the second
layer is silty clay, about 2.1 meters thick; the third layer is silt, about 2.3 meters thick; the fourth layer
is silty clay with a thickness of about 5.6 meters; the fifth layer is silty sand with a thickness of about
2.5 meters; the sixth layer is gravel sand with a thickness of about 4.5 meters; the seventh layer is silty
clay with a thickness of about 2.0 meters; The eighth layer is a soil hole without filling, and the
thickness is about 2.5 meters; the ninth layer is silty clay with a thickness of about 1.1 meters; the
tenth layer is slightly weathered limestone, the core is columnar and a few blocks, and the joint length
is 5-50cm, and the rock is hard. The thickness is about 10.3 meters.

The specific test method is to weld steel stress gauges and strain sensors in the pile foundation
reinforcement cage in advance during the construction of the pile foundation, and perform a static load
test on the pile foundation after the pile foundation reaches a suitable age. During the static load
process, the pile body stress and deformation are continuously read and recorded to obtain the pile
side friction resistance value, and the total load is recorded. The total load minus the side friction value is
the pile-end resistance value.

3.2. Sensor Layout and Static Load Test
In this test, a steel bar stress gauge was embedded in the reinforcement cage along the depth direction,
and an earth pressure box was welded to the reinforcement cage at the bottom of the pile. Under
various loads, the pile side resistance and pile tip resistance of each pile section were obtained based
on the above sensor data analysis.

(1) Installation position: At each section, the reinforcement cage of the pile body is arranged
symmetrically along the axis of the pile

(2) Installation method: Before pouring the concrete, connect it with the corresponding structural
steel bar by butt welding. This installation form does not change the overall strength of the original
structure design because the pile body reinforcement cage is not cut.
The static load test conditions: Since the design load of the engineering pile in this test is 3000kN, the test load is 6000kN. According to the total applied load, it is divided into 10 grades evenly, with a total of 8 loads (the first and second loads are each applied with 2 grades). The applied sequential load sizes are: 1200kN, 2400kN, 3000kN, 3600kN, 4200kN, 4800kN, 5400kN, 6000kN.

Use jacks, oil pumps, oil pipes, pressure sensors, displacement sensors, reference beams, magnetic stand, and data acquisition instruments as shown in the figure 5 and 6.

![Counterforce device of ballast platform](image)

**Figure 5.** Schematic diagram of counterforce device of ballast platform.

![Load-displacement curve](image)

**Figure 6.** Measured curve of Y0-C pile.

According to the analysis of the test data, the maximum settlement of the pile in the static load test is 2.94 mm, and the load-displacement curve basically changes linearly, and then the upper soil compaction leads to an increase in lateral resistance and a slight decrease in slope. When the last two loads are applied, the slope of the curve increases, but the degree of change is low. After unloading, the pile rebounds obviously, the maximum rebound is 2.62 mm, and the rebound rate is 89.1%. The settlement caused by each level of load does not change much, but at 3600 kN, the settlement of the pile body increases slowly with time, and it is not easy to stabilize. Judge based on the load-displacement curve basically showing a linear change, the pile side resistance does not cause great deformation to the pile side rock and soil, and the slope is basically a certain value, the soil and rock layers around the pile are in the elastic stage. In summary, it is inferred that the bearing capacity of the test pile is much greater than 6000 kN as shown in the tables 1 and 2.
Table 1. Summary table of Y0-C# pile static load test results.

| Serial number | Load (kN) | Current level Duration (min) | Cumulative duration (min) | Current level settlement (mm) | Cumulative settlement (mm) |
|---------------|-----------|------------------------------|---------------------------|------------------------------|---------------------------|
| 0             | 0         | 0                            | 0                         | 0                            | 0                         |
| 1             | 1200      | 60                           | 60                        | 0.44                         | 0.44                      |
| 2             | 2400      | 90                           | 150                       | 0.54                         | 0.98                      |
| 3             | 3000      | 60                           | 210                       | 0.2                          | 1.18                      |
| 4             | 3600      | 135                          | 345                       | 0.24                         | 1.42                      |
| 5             | 4200      | 105                          | 450                       | 0.26                         | 1.68                      |
| 6             | 4800      | 60                           | 510                       | 0.35                         | 2.03                      |
| 7             | 5400      | 75                           | 585                       | 0.41                         | 2.44                      |
| 8             | 6000      | 60                           | 645                       | 0.5                          | 2.94                      |
| 9             | 4800      | 15                           | 660                       | -0.23                        | 2.71                      |
| 10            | 3600      | 15                           | 675                       | -0.53                        | 2.18                      |
| 11            | 2400      | 15                           | 690                       | -0.54                        | 1.64                      |
| 12            | 1200      | 15                           | 705                       | -0.42                        | 1.22                      |
| 13            | 0         | 60                           | 765                       | -0.9                         | 0.32                      |

Table 2. Relationship between Y0-C pile shaft axial force and depth.

| Pile Top Load Value (kN) | Depth (m) | 0 | 6.8 | 9.1 | 14.7 | 17.2 | 23.7 | 27.3 | 19.8 | 32 |
|-------------------------|-----------|---|-----|-----|------|------|------|------|------|----|
| 1200kN                  | 1200      | 1200 | 831 | 906 | 610 | 436 | 353 | 216 | 108 | 0 |
| 2400kN                  | 2400      | 2400 | 1820 | 1953 | 1180 | 1267 | 789 | 482 | 307 | 0 |
| 3000kN                  | 3000      | 3000 | 2360 | 2476 | 1541 | 1587 | 1043 | 573 | 407 | 0 |
| 3600kN                  | 3600      | 3600 | 2984 | 3104 | 1982 | 2086 | 1321 | 822 | 502 | 0 |
| 4200kN                  | 4200      | 4200 | 3507 | 3727 | 2418 | 2593 | 1600 | 1039 | 635 | 0 |
| 4800kN                  | 4800      | 4800 | 4106 | 4363 | 2896 | 3166 | 1915 | 1255 | 773 | 0 |
| 5400kN                  | 5400      | 5400 | 4638 | 4953 | 3403 | 3669 | 2314 | 1479 | 881 | 0 |
| 6000kN                  | 6000      | 6000 | 5677 | 4056 | 4413 | 2560 | 1155 | 1795 | 1122 | 0 |

As shown in the figure 7 the side resistance of Y0-C pile is basically positively correlated with the depth of the section, but there are obvious side frictional resistances in the 6~9 m and 15~16 m sections. The reason is that the crossing layers of the two pile sections are saturated silt of quartz and feldspar, with poor gradation and high compressibility, makes the soil layer and super-high pile deform under the action of high level load. The pile body enters the slightly weathered granite at a depth of about 26m, and the side resistance of the rock-socketed section accounts for a relatively large proportion, about 29-42.8%.
3.3. Calculation and Analysis of Load-Displacement Curve Based on Load Transfer Method

0-D pile (there is karst cave at the depth of 20~26.8m, the depth through the upper part of the karst cave is 5.3m, and the depth of the lower part of the karst cave is 2m)

The calculation parameters that need to be determined are: \( \lambda_1 \) slope of the first section of the pile side transfer function; \( \lambda_2 \) slope of the second section of the pile side transfer function.

This model assumes that the pile side soil is an ideal elastoplastic material, so take 0; \( k_1 \) the slope of the first section of rock mass end resistance at the pile tip; \( k_2 \) the slope of the second section of rock mass end resistance at the pile tip, generally taken \( k_1/15 \); \( \tau_u \) the weighted ultimate side resistance with consideration of the side resistance of the rock-socketed section and the side resistance of the upper soil body; \( S_u \) lateral displacement of pile under \( \tau_u \); \( Q_u \) the ultimate end resistance embedded in the rock mass; \( S_{u,m} \) the boundary displacement between the elastic and plastic stages of the pile end.

Slope of pile side resistance transfer function:

\[
\lambda_1 = \frac{G_s}{r_0 \cdot \ln \frac{r_m}{r_0}} = 70.43\text{kpa/mm}, \quad \lambda_2 = 0\text{kpa/mm}
\]

(39)

Considering the rock-socketed part, according to 1/20 of the uniaxial compressive strength of the pile-rock bond strength zone, the weighted average side resistance can be obtained:

\[
\tau_u = \sum_{i=1}^{n} \frac{L_i}{L} \cdot \tau_i
\]

\[
= \frac{3.0}{28.8} \times 30 + \frac{5.5}{28.8} \times 50 + \frac{2.5}{28.8} \times 30 + \frac{3.7}{28.8} \times 60 + \frac{5.3}{28.8} \times 250 + \frac{6.8}{28.8} \times 0 + \frac{2.0}{28.8} \times 250
\]

\[
= 86\text{kpa}
\]

(40)

Then, the pile-soil side resistance and pile-rock side resistance are average weighted, and the pile side displacement under the limit:

\[
S_u = \frac{\tau_u}{\lambda_1} = \frac{86\text{kpa}}{70.43\text{kpa/mm}} = 1.22\text{mm}
\]

(41)

The slope of the end resistance transfer function of the pile end embedded in the rock mass:
\[ G_s = \frac{E}{2(1 + \nu)} = \frac{2.0 \times 10^4 \text{Mpa}}{2 \times (1 + 0.3)} = 7.7 \times 10^6 \text{kpa} \]  

\[ k_1 = \frac{4G_r r_0}{\alpha(1 - \nu)} = \frac{4 \times 7.7 \times 10^6 \text{kpa} \times 0.6 \text{m}}{0.78 \times 0.7} = 3.38 \times 10^{11} \text{kN/mm} \]  

\[ k_2 = \frac{k_1}{15} = 2.26 \times 10^{10} \text{kN/mm} \]

According to the uniaxial saturated compressive strength of the rock, the ultimate end resistance of the rock mass is:

\[ Q_{bu} = 4000 \times 3.14 \times 0.6^2 = 4521.6 \text{kN} \]

\[ S_{bu} = \frac{Q_{bu}}{k_1} = \frac{4521.6 \text{kN}}{3.38 \times 10^{11} \text{kN/m}} = 1.34 \text{mm} \]

According to the application of the first-level load, the pile top displacement is 0.4 mm,

\[ Q_0 = \sqrt{\frac{\lambda_1 U E_p A_p \cdot S_0}{10 \text{mm}}} = 3000 \text{kN/mm} \cdot S_0, \quad \text{get} \quad Q_0 = 1200.28 \text{kN} \]

\[ \alpha_1 = \sqrt{\frac{\lambda_1 U}{E_p A_p}} = 0.09 \]

\[ K_2 = \alpha_1 E_p A_p \cdot \frac{k_1 + \alpha_1 E_p A_p \cdot \text{th}(\alpha_1 L)}{\alpha_1 E_p A_p + k_1 \cdot \text{th}(\alpha_1 L)} = 3052 \text{kN/mm} \cdot \frac{k_1 + \alpha_1 E_p A_p \cdot \text{th}(\alpha_1 L)}{\alpha_1 E_p A_p + k_1 \cdot \text{th}(\alpha_1 L)} = 3083 \text{kN/mm} \]

\[ Q_0 = 1.22 \text{mm} \times 3083 \text{kN/mm} = 3761 \text{kN} \]

\[ Q_e = K_2 \cdot S_e = \alpha_1 E_p A_p \cdot \frac{k_1 + \alpha_1 E_p A_p \cdot \text{th}(\alpha_1 L)}{\alpha_1 E_p A_p + k_1 \cdot \text{th}(\alpha_1 L)} \cdot S_u, \quad S_e = S_u. \]

\[ K_e = \alpha_1 E_p A_p \cdot \frac{k_1 + \alpha_1 E_p A_p \cdot \text{th}(\alpha_1 L)}{\alpha_1 E_p A_p + k_1 \cdot \text{th}(\alpha_1 L)} = 3082 \text{kN} \]

According to equation: \[ Q_0 = (\alpha_1^2 L_1 E_p A_p + K_e) \cdot S_u, \quad S_0 = \left(1 + \frac{K_2 E_p A_p}{1 + \frac{1}{2} \alpha_1^2 L_1^2} \right) S_u \text{ equation(45)} \]

Through calculation, the result is as table 3 below:

**Table 3.** Load-displacement relationship of 0-D pile surrounding soil under different plastic depth.

| Plastic section depth (m) | Pile top load (kN) | Pile top displacement (mm) |
|--------------------------|-------------------|---------------------------|
| 5.7                      | 6228              | 2.20                      |
| 11.4                     | 8326              | 3.42                      |
| 17.1                     | 10424             | 4.99                      |
| 22.8                     | 12522             | 6.91                      |
| 28.8                     | 14731             | 9.31                      |
From $\Delta Q_0 = \frac{k_1}{1+k_iL/E_pA_p} \cdot \Delta S_0$, $K = \frac{k_1}{1+k_iL/E_pA_p} = 1177kN/mm$ and substitute the points obtained above into (14731, 9.31) as shown in the figure 8 and 9.

$$Q_0 = 1177kN/mm \times S_0 - 1520.11$$

**Figure 8.** Comparison of theory and experiment within the test load range.

**Figure 9.** Comparison of theory and experiment within the calculated load range.

Within the load range of the test, the QS curve calculated this time has a slightly larger error than the actual measurement result, and the error is positively correlated with the increase of the load level. When the maximum load of 6000 kN is applied in the test, the maximum error is 0.37 mm, and the percentage error is 14.5%. The calculated settlement is lower than the actual value, and the vertical bearing capacity of the obtained foundation pile is higher. Therefore, the vertical bearing capacity of the foundation pile calculated according to the theoretical Q-S curve is about 19000 kN, which is only for reference for construction and design.

4. Conclusion and Prospect

Based on the study of the mechanism of the load transfer method, this paper used the basic differential equations of the load transfer method to establish a theoretical method for analyzing the Q-S curve of a single pile static load test. Because the static load test shows that the vertical ultimate bearing capacity of the Q-S curve is the closest to the actual working condition of the engineering project, the obtained results are relatively close to the ultimate bearing capacity of the design following the specification, and the transfer function used for analyzing the load transfer pattern of the pile and the displacement calculation is feasible. The calculation method based on the load transfer rule has relatively high accuracy at predicting the bearing capacity of foundation piles running through the cave, providing
reference for design and construction. Constrained by the level of applied load, the end bearing capacity of the project pile is basically not exerted. It is concluded that the bearing capacity of the rock-socketed pile is not necessarily the main end bearing capacity during the service process of the designed bearing capacity.

The basic idea of the derivation process in this paper is the load transfer method. Although the result of analyzing the vertical static load test of a single pile using this basic theory as the guiding ideology is within an acceptable range of errors, it is restricted by its basic assumptions. In terms of theoretical research, it violates the objective fact of soil continuity. Further theoretical methods are needed for follow-up research work. For the sake of simple calculation and easy selection of parameters, the pile-soil transfer function used in this paper is an ideal elastoplastic model, which does not completely conform to the hyperbola under real conditions. Therefore, it causes some errors in theoretical analysis, and further research is needed.

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