LETTER TO THE EDITOR

Polar type density of states in non-unitary odd-parity superconducting states of gap with point nodes

K Miyake
Division of Materials Physics, Department of Physical Science, Graduate School of Engineering Science, Osaka University, Toyonaka, Osaka 560-8531, Japan
E-mail: miyake@mp.es.osaka-u.ac.jp

Abstract. It is shown that the density of states (DOS) proportional to the excitation energy, the so-called polar like DOS, can arise in the odd-parity states with the superconducting gap vanishing at points even if the spin-orbit interaction for Cooper pairing is strong enough. Such gap structures are realized in the non-unitary states, $F_{1u}(1, i, 0)$, $F_{1u}(1, \varepsilon, \varepsilon^2)$, and $F_{2u}(1, i, 0)$, classified by Volovik and Gorkov, Sov. Phys.—JETP 61 (1985) 843. This is due to the fact that the gap vanishes in quadratic manner around the point on the Fermi surface. It is also shown that the region of quadratic energy dependence of DOS, in the state $F_{2u}(1, \varepsilon, \varepsilon^2)$, is restricted in very small energy region making it difficult to distinguish from the polar-like DOS.

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In early stage of research of the heavy fermion superconductors, it has been important to infer the gap anisotropy from the power-law of the temperature dependence of a series of physical quantities [1, 2, 3, 4]. It was a sort of the golden rule there that point node(s) of the superconducting gap leads to the density of states (DOS), $N_s(\omega) \propto \omega^2$, while line node(s) leads to $N_s(\omega) \propto \omega$. It was also emphasized that all the odd-parity pairings would have only point node(s) if the spin-orbit coupling for the pairing interaction were so strong that the spin- and orbital degrees of freedom of the gap function cannot change independently [5, 6, 7, 8, 9]. However, it is not so self-evident whether the spin-orbit coupling for pairing is really so strong to quench technically the independent variation in spin- and orbital space [10, 11]. In any case, the classification scheme proposed by Volovik and Gorkov (VG) has been believed to rule out the polar-like DOS for the odd-parity states. The purpose of this letter is to point out that three of the non-unitary states in VG scheme has the polar-like DOS because the $k$-dependence of the gap around the point node is quadratic rather than linear.

In the odd-parity manifold, the quasiparticle energy is the matrix in the representation of spin eigen state as follows [12]:

$$\hat{E}_k = [\xi_k^2 \hat{1} + \hat{\Delta}_k^\dagger \hat{\Delta}_k]^{1/2},$$

(1)

where the superconducting gap is also the $2 \times 2$ matrix in the presentation of spin eigenstate, and is represented in terms of the $d$-vector as

$$\hat{\Delta}_k = i \sum_j \Delta(\sigma_j \sigma_y) d_j(k),$$

(2)

where $\sigma_j$ is the Pauli matrix of the $j$-the component, with $j=x$, $y$, and $z$. The eigenvalues of the magnitude of the gap matrix is given as [12]

$$\text{det}(\hat{\Delta}_k^\dagger \hat{\Delta}_k) = \Delta^2 [\text{det}(d(k) \cdot d^*(k)) \pm \text{det}(d(k) \times d^*(k))].$$

(3)

It is remarked that the time reversal symmetry is broken in the non-unitary state where $i d(k) \times d^*(k) \neq 0$ The DOS in the superconducting state $N_s(\omega)$ is expressed as follows:

$$\frac{N_s(\omega)}{N(0)} = \frac{\omega}{2} \sum_{\alpha=\pm} \int \frac{d\hat{k}}{4\pi} \frac{\theta(\omega - |\Delta_{\alpha}(\hat{k})|)}{\sqrt{\omega^2 - (\hat{\Delta}_{\alpha}^\dagger \hat{\Delta}_k)_{\alpha}}} ,$$

(4)

where $\theta(x)$ is the Heaviside function.

The $d$-vector of the state $F_{1u}(1, i, 0)$, the class of group theoretical representaion $D_4(E)$, is given as [6]

$$d(k) = \Delta \left(\frac{3}{4}\right)^{1/2} \left[\hat{k}_x \hat{e}_y - \hat{k}_y \hat{e}_x + i(\hat{k}_x \hat{e}_z - \hat{k}_z \hat{e}_x)\right] ,$$

(5)

where $\hat{k} \equiv k/|k|$. Then, the magnitude of the gap is calculated, leading to the expression

$$\text{det}(\hat{\Delta}_k^\dagger \hat{\Delta}_k) = \frac{3}{4}\Delta^2 (1 \pm |\hat{k}_z|)^2.$$

(6)

It is remarked that the amplitude of the smaller gap $[\text{det}(\hat{\Delta}_k^\dagger \hat{\Delta}_k)]^{1/2}$ has point nodes in the direction $|\hat{k}_z| = 1$, and has a quadratic dependence as $\propto (\hat{k}_x^2 + \hat{k}_y^2)$ around the node.
on the Fermi sphere. Therefore, the DOS is proportional to the excitation energy $\omega$. With the use of (6), the DOS is calculated numerically by means of the formula (4). The result is shown in Fig. 1. The shape of the DOS is similar to those for the polar state.

The $d$-vector of the state $F_{2u}(1, i, 0)$, the class of group theoretical representaion $D_4(E)$, is given as (6)

$$d(k) = \Delta \left( -\frac{3}{4} \right)^{1/2} \left[ \hat{k}_z \hat{e}_y + \hat{k}_y \hat{e}_z + i(\hat{k}_x \hat{e}_z + \hat{k}_z \hat{e}_x) \right].$$

The magnitude of the gap is calculated, leading to the same expression as (6). Therefore, the DOS $N_d(\omega)$ is the same as shown in Fig. 1 the polar-like one.

The $d$-vector of the state $F_{1u}(1, \varepsilon, \varepsilon^2)$, the class of group theoretical representaion $D_3(E)$, is given as (6)

$$d(k) = \Delta \left( -\frac{1}{2} \right)^{1/2} \left[ \hat{k}_z \hat{e}_y - \hat{k}_y \hat{e}_z + \varepsilon(\hat{k}_x \hat{e}_z - \hat{k}_z \hat{e}_x) + \varepsilon^2(\hat{k}_y \hat{e}_x - \hat{k}_x \hat{e}_y) \right],$$

where $\varepsilon \equiv e^{i2\pi/3}$. Then, the magnitude of the gap is calculated, leading to the expression

$$\langle \hat{\Delta}_k^\dagger \hat{\Delta}_k \rangle_{\pm} = \frac{\Delta^2}{4} \left( \sqrt{3} \pm |\hat{k}_x + \hat{k}_y + \hat{k}_z| \right)^2.$$  

The smaller gap $[(\hat{\Delta}_k^\dagger \hat{\Delta}_k)_{-}]^{1/2}$ has point nodes in the direction $\hat{k} = (\pm 1/\sqrt{3}, \pm 1/\sqrt{3}, \pm 1/\sqrt{3})$, and has also a quadratic dependence as (6). So, the DOS is proportional to the excitation energy $\omega$, and its explicit dependence is calculated numerically by means of the formula (4). The result is shown in Fig. 1. The DOS is the same as those for the polar state $\Delta_k = \sqrt{3} \Delta k_z$.

**Figure 1.** $N_s(\omega)/N_F$ in the states $F_{1x}(1, i, 0)$ and $F_{2u}(1, i, 0)$, $N_F$ being the DOS in the normal state at the Fermi level. The dashed curve is for the DOS of the polar state $\Delta_{k} = \sqrt{3} \Delta k_z$. 

The $d$-vector of the state $F_{2u}(1, \varepsilon, \varepsilon^2)$, the class of group theoretical representation $D_3(E)$, is given as \[ (d(k)) = \Delta \left( \frac{1}{2} \right) \frac{1}{2} \left[ \hat{k}_x \hat{e}_y + \hat{k}_y \hat{e}_z \right] + \varepsilon(\hat{k}_x \hat{e}_z + \hat{k}_z \hat{e}_x) + \varepsilon^2(\hat{k}_y \hat{e}_x + \hat{k}_x \hat{e}_y) \]. (10)

The magnitude of the gap is calculated as \[ (\Delta_k^+ \Delta_k^-) = \Delta \left( \frac{1}{2} \right) \left[ 2 - (\hat{k}_x \hat{k}_y + \hat{k}_y \hat{k}_z + \hat{k}_z \hat{k}_x) \right] \pm \sqrt{3} \left[ 1 - 2(\hat{k}_x \hat{k}_y + \hat{k}_y \hat{k}_z + \hat{k}_z \hat{k}_x) + 4\hat{k}_x \hat{k}_y \hat{k}_z (\hat{k}_x + \hat{k}_y + \hat{k}_z) \right]^{1/2} \]. (11)

The smaller gap $[(\Delta_k^+ \Delta_k^-)]^{1/2}$ has point nodes in the direction $\hat{k} = (\pm 1/\sqrt{3}, \pm 1/\sqrt{3}, \pm 1/\sqrt{3})$. The gap vanishes linearly around the nodes so that the DOS is proportional to the square of the energy $\omega$. However, the region of $\omega$ where $N_s(\omega) \propto \omega^2$ holds is very restricted, i.e., $\omega < \Delta/10$, as shown in Fig. 3. Therefore, the temperature dependence of physical quantities is hard to distinguish from those of the polar-like state.

Other example, in which the point node(s) give the polar-like DOS, is the so-called planar state with $E_{2u}$ symmetry which is unitary state and was proposed as a candidate of that of UPt$_3$ [13, 14]. Such a state gives a magnitude of the gap as

\[ |\Delta_k| \propto \hat{k}_z [(\hat{k}_x^2 - \hat{k}_y^2)^2 + 4\hat{k}_x^2 \hat{k}_y^2]^{1/2}. \] (12)

This gap has point nodes at $|\hat{k}_z| = 1$ and shows the quardatic behaviour around the node on the Fermi surface. So, the quasiparticles around the point nodes should also give the polar-like DOS if there exists the Fermi surface around the nodes.

In conclusion, we have pointed out by explicit calculations that the superconducting gap with point nodes in the non-unitary states, $F_{1u}(1, i, 0)$, $F_{1u}(1, \varepsilon, \varepsilon^2)$, and $F_{2u}(1, i, 0)$,
classified by Volovik and Gorkov, exhibits the polar-like DOS which is proportional to the excitation energy itself rather than its square. This results arises from the fact that the $k$-dependence of the gap around the point nodes is quadratic rather than linear as expected in general.

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