The Effect of Randomness on the Mott State

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Introduction. Two non-trivial routes to insulating behavior in solids are connected with the names of Anderson and Mott. Non-interacting electrons in disordered solids are always localized in one and two space dimensions resulting from a combination of classical localization and quantum-mechanical interference. This Anderson insulator (AI) is characterized by both a finite ac conductivity at low frequencies and a finite compressibility. Interaction between electrons reduces the effect of disorder and may lead to metallic behavior in two dimensions. In Mott insulators, on the other hand, insulating behavior results from the blocking of sites by repulsive interaction between electrons and hence are dominated by correlation effects. The Mott-insulating (MI) phase is incompressible and has a finite gap in the conductivity. A natural question is then: what happens in systems when both disorder and interactions are non-negligible? Is there, depending on the strength of interaction or disorder, a single transition between these two phases or is the scenario more complex? A particularly interesting case to answer these questions is that of electrons in one space dimension: interactions are very strong, destroying Fermi liquid behavior, but interactions can be partially incorporated into the harmonic (bosonized) theory. Additional motivation to study this case comes from its physical realization in quantum wires and ultra-cold gases.

One-dimensional systems of this type have indeed been considered in a number of publications. Early work by Ma using real-space renormalization group (RG) for the disordered 1D Hubbard model suggests a direct transition from an AI to an MI phase. This result contrasts with more recent work (see e.g. and references therein) where a new type of order, different from the MI and AI phase, was found. In particular in the existence of a new Mott glass (MG) phase was postulated which is supposed to be incompressible but has no gap in the optical conductivity.

In this paper we reinvestigate this problem by relating both the compressibility and the low frequency conductivity to the energy of kinks in the displacement pattern of the bosonized electrons, resorting to ideas known from the discussion of the roughening transition. Using an instanton calculation of the conductivity we show that an incompressible phase has always a gap in the low frequency conductivity, excluding the possibility of a MG phase. A number of further consideration support this finding.

The model. Following Haldane we relate the mass density \( \rho(x) \) of the electrons to their displacement field \( \varphi(x) \): \( \rho(x) = \rho_0 + \frac{1}{2} \partial_x \varphi + \rho_0 \cos(2\varphi + 2k_F x) + \ldots \). Throughout the paper we will use \( e = \hbar = 1 \). The ground state energy of the system follows from \( E_0 = -\lim_{T \to 0} T \ln \int D\varphi e^{-S} \) where the action is

\[
S = \frac{1}{2\pi K} \int_0^L \int_0^\Lambda \lambda \varphi^2 - \frac{2\pi k \Lambda}{\Lambda} F \varphi - (\zeta(x)e^{-2\varphi} + h.c.) - w \cos 2\varphi \right) + S_{\text{diss}}.
\]

Here we introduced dimensionless space and imaginary time coordinates by the transformation \( \Lambda x \to x \) and \( \Lambda \tau \to \tau \) where \( v \) is the plasmon velocity and \( \Lambda \) a large momentum cut-off. \( \lambda \equiv v/T \) and \( L \) and denote the thermal de Broglie wavelength of the plasmons and the system size, respectively. \( w \) denotes the strength of the Umklapp scattering term (or the strength of a commensurate potential) and \( F \) the external driving force. \( \mu(x) \) and \( \zeta(x) \) result from the coupling of the random impurity potential to the long wave length and the periodic part of the density, respectively, with \( \langle \zeta(x) \zeta(x') \rangle = u^2 \delta(x-x'), \langle \mu(x) \mu(x') \rangle = \sigma \delta(x-x'), \) all other correlators vanish. Finally we add a dissipative action \( S_{\text{diss}} \) describing an Ohmic resistance which may e.g. result from the interaction with an external gate.

Rigidities. We begin with a discussion of the generalized rigidities, which are related to the compressibility and the conductivity of the system. We first consider the application of a fixed strain \( \vartheta \) by imposing the boundary conditions \( \varphi(0, \tau) = 0 \) and \( \varphi(L, \tau) = \pi \vartheta L \). The boundary condition in the \( \tau \)-direction is assumed to be periodic. For \( \vartheta \ll 1 \) and \( L \to \infty \) the corresponding increase \( \Delta E_0(\vartheta, 0) = E_0(\vartheta, 0) - E_0(0, 0) \) of the ground state energy \( E_0(\vartheta, 0) \) is clearly an even but not necessarily an-
The r.h.s. of this relation has to be understood as follows: if $\Sigma_x = 0$ then the stiffness $\Upsilon_x = \Lambda^2/\kappa$ describes the response to the twisted boundary conditions, $\Upsilon_x^{-1}$ is the isothermal compressibility [13]. In this case the change of $\varphi$ is spread over the whole sample. If however $\Upsilon_x$ diverges, i.e. the system becomes incompressible, then the kink energy $\Sigma_x/\Lambda$ is non-zero. In this case the change of $\varphi$ from 0 to $\pi$ occurs in a narrow kink region of width $\xi$ much smaller than $L$. Creating a kink corresponds to adding (or removing) an electron at the kink position. A non-zero kink energy resembles the step free energy of a surface below the roughening transition [12, 27].

In a similar manner we can apply non-trivial boundary conditions in the $\tau$-direction by choosing $\varphi(x, 0) = 0$ and $\varphi(x, L_\tau) = \pi j L_\tau/v$. This corresponds to imposing an external current $j = (\partial_x \varphi)/\pi$ at $x = 0$ and $x = L$:

$$\frac{\Delta E_0(0, j)}{L} \bigg|_{L \to \infty} \cong \begin{cases} \Upsilon_\tau \varphi^2/2 & \text{if } \Sigma_\tau = 0, \\ \Sigma_\tau |j| & \text{if } \Upsilon_\tau^{-1} = 0. \end{cases}$$

(2)

(3)

$\Upsilon_\tau$ is related (at $T = 0$) to the charge stiffness $D = 1/\Upsilon_\tau = \kappa v^2$. D determines the Drude peak of the conductivity $\sigma(\omega) = D\delta(\omega) + \sigma_{\text{reg}}(\omega)$ [6, 17]. In Lorentz invariant systems (like the MI) $\Upsilon_x = (\Lambda v)^2 \Upsilon_\tau$ and $\Sigma_x = \Lambda v \Sigma_\tau$, provided they are finite.

So far we assumed that $\Upsilon_{x/\tau}$, $\Sigma_{x/\tau}$ are self-averaging if $L \to \infty$. In the same way we may introduce local rigidities by applying twisted boundary conditions over a large but finite interval $[x, x + L_x \Lambda]$, $\Lambda^{-1} \ll L_x \ll L$. The result for $\Upsilon_{x/\tau}$ and $\Sigma_{x/\tau}$ will then depend in general on the size $L_x$ of the interval, i.e. $\Sigma_{x/\tau} \to \Sigma_{x/\tau}(L_x)$.

Conductivity. In cases where $D$ vanishes, the field or frequency dependent conductivity $\sigma(F, \omega)$ may still be non-zero, provided $F$ or $\omega$ are finite. The non-linear dc-conductivity $\sigma(F, 0)$ follows from an instanton approach in which the current is calculated from the tunneling probability of electrons between consecutive metastable states (see Fig.11). The latter are the classical ground states in the absence of the driving force which follow from each other by a shift of a multiple of $\pi$. This approximate treatment is reliable for not too large $K$ and small $F$. The conductivity $\ln \sigma(F, 0) = -S_1(L_{x,s}, L_{\tau,s})$ is then related to the action $S_1$ of a critical droplet (the instanton) of the new metastable state on the background of the old one [13, 14]. Here $L_{x,s}, L_{\tau,s}$ are the saddle point values of the extension of the instanton which follow from

$$S_1 \approx \frac{2\Sigma_x(L_x)}{\Lambda v} L + L_x (2\Sigma_\tau + \eta \ln \frac{L_x}{\xi}) - \frac{1}{\pi v} FL_x L_\tau.$$  

(6)

For simplicity we assumed that the instanton is rectangular, a choice which is motivated by the fact that the disorder is correlated in time [13]. $\Sigma_x/(\Lambda v)$ and $\Sigma_\tau$ play the role of the surface tension of the instanton. The last term is the bulk contribution from the external field. The $\eta$-term results from the dissipation which allows relaxation in the next metastable state [13].

The low frequency ac-conductivity $\sigma(0, \omega)$ results from the spontaneous tunneling processes between metastable states and their instanton configurations (1 and 2 in Fig. 11). Tunneling leads to a level splitting of the order

$$\delta E \approx \sqrt{4\Sigma_x^2(L_x)\Lambda^{-2} + C(v\Lambda \xi K)^2 e^{-4L_x\Sigma_\tau}},$$

(7)

which has to match the energy $\omega$ of the external field [14]. $1/\Sigma_x \sim K \xi$ plays the role of the tunneling length, $C$ is a numerical factor. This mechanism was first considered for non-interacting electrons by Mott [8] and extended to the interacting case via instantons in [14]. Thus, to get a non-zero $\sigma(0, \omega)$ for arbitrary low frequency $\omega$, $2\Sigma_x(L_x) < \omega \Lambda \to 0$ which requires $2\Sigma_x(L_x) \to 0$ for a finite density of kink positions, which implies a finite compressibility. The distance $L_x$ between the sites involved in the tunneling is then $L_x(\omega) \approx \xi K \ln(1/(\kappa \xi \omega))$.

Fixed points and phases. We come now to the discussion of the possible phases of model (11) by attributing them to their RG fixed points (denoted by superscript *). Bare values will get a subscript 0. For small $u$ and $w$ the lowest order RG-equations read [18]:

$$\frac{dK}{dl} = -K(au^2 + bw^2), \quad \frac{d\sigma}{dl} = \sigma(1 - cw^2)$$

(8)

$$\frac{dw}{dl} = w(2 - K - \frac{2}{\pi} \sigma)$$

(9)

$$\frac{du^2}{dl} = u^2(3 - 2K) + \frac{1}{\pi} \sigma w^2$$

(10)

$$\frac{ds}{dl} = -\kappa(z - 1 + \frac{c}{2} w^2)$$

(11)

where $l$ is the logarithm of the length scale and $a, b, c$ are positive non-universal constants. $z$ denotes the dynamical critical exponent which has to be determined from the fixed point condition.
Luttinger liquid (LL). The LL phase is characterized by $u_A^* = w_A^* = 0$ and hence $\Sigma_\gamma = \Sigma_\sigma = 0$. $z = 1$, $K^*_F > 0$ and $\kappa^*_L = K^*_F / (\pi \xi^*_L) > 0$. The fixed point is reached for sufficiently large values of $K$ and $\sigma$. The long time and large scale behavior of the system is that of a clean LL characterized both by a finite compressibility $\kappa^*_L$ and a finite charge stiffness $D_L = \kappa^*_L \nu^2$. The dynamical conductivity is given by $\sigma_{reg} = iD_L / (\pi \omega)$. The presence of the forward scattering term $\sim \mu(x) \partial_x \varphi$ does not change these results since it can be always removed by the transformation $\varphi = -f(x) \partial x \mu(x)$.

Mott insulator (MI). Here $K^*_M = \kappa^*_M = u^*_M = \sigma^*_M = 0$ but $w^*_M \gg 1$. Clearly the fixed point $w^*_M$ is outside the applicability range of the $[3] - [11]$ but nevertheless some general properties of this phase can be concluded. The system is in the universality class of the 2-dimensional classical sine Gordon model which describes inter alia the MI to LL transition and the roughening transition of a 2-dimensional classical crystalline surface $[12]$. In the MI phase $\Sigma_A$ and $\Sigma_y$ diverge. The classical ground state is given by $\varphi(x) = n\pi$ with $n$ integer. The system is characterized by a finite kink energy $\Sigma_x \sim A(\kappa_0 \xi_M)^{-1}$ where $\xi_M$ denotes the correlation length of the MI-phase $[12]$. From (6) we get $\Sigma(u, 0) \sim \exp(-F_M / (\kappa F))$ where $F_M = 4\pi \sum I_\gamma / \Lambda / 1 / (\kappa_0 \xi_M)$ is a characteristic depinning field $[22]$. According to (7) the ac conductivity vanishes for $\omega \lesssim 2\Sigma_x / \Lambda$.

Anderson insulator (AI). Here $w^*_A = K^*_A = 0$ but $u^*_A \gg 1$. $\kappa^*_L \approx \kappa_0$ is finite which is the result of the so-called statistical tilt symmetry $[22]$ corresponding to $z = 1$. The fixed point Hamiltonian is in the universality class of the 1 + 1 dimensional sine Gordon model with a random phase correlated in the $\gamma$-direction. The transition to the LL phase occurs at $K = K^*_A(u) \geq 3/2$.

Next we look at $\Sigma_A(x)$ finite length scale $\xi_A \ll L_x \ll L$ such that the parameters are close to their fixed point values. $\xi_A$ is the correlation length which diverges at the Beresinskii-Kosterlitz-Thouless (BKT) transition to the LL phase. To find the classical ground state of the system under periodic boundary conditions we have to choose $2\varphi_i + \alpha_i = 2\pi n_i$ with $n_i$ integer, and secondly the elastic term has to be minimized with respect to the $n_i$. The subscript $i$ refers to the sites of the lattice with spacing $\xi_A$ and $\xi^*_i = (\xi_A e^{i\alpha_i}$). The solution is $n_i = \sum_{j} \delta \xi_i (|\xi_j - \alpha_i| - 1) / 2\pi G$ $[13]$. $[\xi_i]$ denotes the closest integer to $x$. The ground state is uniquely determined by the $\alpha_j$ apart from the pairs of sites (of measure zero) at which $\alpha_j - \alpha_{j-1} = \pm \pi$. At such pairs the ground state bifurcates since two solutions are possible. In the case of non-interacting electrons ($K = 1$) bifurcation sites are states at the Fermi energy. For pairs at which $\alpha_j - \alpha_{j-1} = \pm \pi + \epsilon$, $|\epsilon| \ll 1$ we can go over to an excited state by creating a kink which costs at most the energy $\Sigma_x \approx cA / (\kappa \xi_A)$ $[13]$. Those ’almost’ bifurcating sites correspond to states close to the Fermi energy. The smallest $\epsilon$ found with probability of order one in a sample of length $L_x$ is of the order $\xi / L_x$ and hence $\Sigma_x \approx A / (L_x K_0)$, i.e. the kink energy vanishes for $L_x \rightarrow \infty$ $[13]$ and hence the system is compressible.

Twisted boundary conditions in the $y$-direction give $\Sigma_y \approx (K_0 \xi_A)^{-1}$ $[13]$. The non-linear conductivity is given by $\sigma(F, \omega = 0) \sim \exp(-F_A / (K F^2))$ where $F_A \sim 1 / (\kappa \xi_A^4)$ $[13]$. This result can be understood as electron tunneling between sites at which $\Sigma_x \sim 1 / L_x$. As explained already a vanishing $\Sigma_x$ is also crucial for the existence of the low frequency conductivity $\sigma(0, \omega) \approx \xi_M K (\omega \Sigma_0 K^*_M(\ln(\kappa_0 \xi_M \omega))^2$ as has been discussed in detail in $[14]$. This result can be understood in terms of tunneling processes between rare positions at which the kink energies $\Sigma_x(x)$ are much smaller than $1 / (\kappa \xi L_x)$.

| Phase | $\kappa^*$ | $K^*$ | $D$ | $\Sigma_\gamma$ | $\Sigma_\sigma$ | $\sigma(\omega \ll \Sigma_x)$ |
|-------|-------------|---------|-----|-----------------|-----------------|--------------------------|
| LL    | $\kappa_L > 0$ | $K_L > 0$ | $\kappa_L \xi^*_L$ | 0 | 0 | $iD_L / (\pi \omega)$ |
| AI    | $\kappa_A > 0$ | 0 | 0 | 0 | $\sim \xi^*_A$ | $\sim \omega^2 \ln^2 \omega$ |
| MI    | 0 | 0 | 0 | $\sim \xi^*_M$ | $\sim \xi^*_M$ | 0 |

Mott-glass (MG). This new hypothetical phase was proposed in $[11]$ to be characterized by a vanishing compressibility, $\kappa^*_G = 0$, but a non-zero optical conductivity at low frequencies $[11]$. Since the phase is considered to be glassy, both fixed point values $w^*_G, u^*_G \gg 1$. Similarly to the AI, the ground state can be found by minimizing first the two backward scattering terms followed by minimization of the elastic energy. Although the ground state solution is now more involved than for the MI and AI case, for $F = 0$ it is clearly periodic with period $\pi$. As before kinks (or anti-kinks) with $\delta \varphi = \pm \pi \pm \pi$ allow the accommodation of twisted boundary conditions and the formation of instantons. A vanishing compressibility corresponds to a finite kink energy $\Sigma_x \geq C > 0$ which, according to (7), leads to a gap in the ac-conductivity. Here we make the reasonable assumption that it is the instanton mechanism which dominates the low frequency response $[20]$. Thus, in a system with a non-zero $\sigma(\omega)$ for small $\omega$ also the compressibility has to be non-zero, contrary to the claims in $[11]$.

Phase diagram. We now come to the discussion of the phase diagram of our model $[11]$. From (10) follows that the random backward scattering term is generated by forward scattering and the commensurate pinning potential. Since $\sigma(l) = \sigma_0 e^l$ the two Eigenvalues $\lambda_1 = 3 - 2K$ and $\lambda_2 = 4 - 2K - 4\sigma / \pi$ describing the RG-flow of $u^2$ and $w^2$ around the the LL fixed point $u^* = w^* = 0$ have opposite sign: $u(l)$ increases whereas $w(l)$ decreases. Thus the hypothetical MG phase, if it existed, cannot reach up to the point $u = w = 0$, in contrast to the findings in $[11]$. From this we conclude that for not too large values of $w_0$ the AI phase is stable, as shown in Fig. $[11]$. To find the phase boundary to the MI phase we consider the stability of the MI phase with respect to the formation of a kink.
by the disorder. To lowest order in the disorder we get for the kink energy in the MI phase

\[ \Sigma_x \sim \frac{\sqrt{w_0}}{\kappa_0} \left[ 1 - \frac{1}{2} \left( \frac{\sigma_0^2}{w_0} \right)^{\frac{1}{4}} - u_0 / \left( \pi^2 w_0^{3/4} \right) \right] \]  

(12)

which gives for the phase boundary between the MI and the AI phase as depicted in Fig. 1. A similar result follows from the self-consistent harmonic approximation.

Thus, to conclude we only find the three phases LL, MI and AI.

Finally a word on the variational approach and replica symmetry breaking (RSB) which has been used in [11] where the Mott glass phase has been found. In the variational approach the full Hamiltonian is replaced by a functional approach the full Hamiltonian is replaced by a functional approach which leads to the decoupling of Fourier components. The functional RG (which takes the coupling of different Fourier modes into account) gives however only two phases: the flat and the rough phase, but at the transition a logarithmically diverging interface width \[ \kappa_0 \] . A similar situation may exist also in the present case although we are not able to calculate the properties of the AI to MI transition.

To conclude we have shown for a one-dimensional disordered Mott insulator [1], tracing back both the compressibility and the ac-conductivity to the kink energy \( \Sigma_x \) of the electronic displacement field that an incompressible system has also a vanishing optical conductiv-

ity. Thus we exclude the possibility of the existence of a Mott-glass phase.

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