The nonlinear anisotropic model of the Universe with the linear potential

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Abstract: In the Bianchi I cosmology some subclasses of the Horndeski theory allow for the non-standard anisotropy behavior. For example, the anisotropy is damped near the initial singularity instead of tending to infinity. In this article, we analyze the nonlinear anisotropic model with the linear potential. We have considered an example of such theory, for which the anisotropy is always finite. The anisotropy reaches its a maximum value at the initial moment. The anisotropy suppression occurs during the inflationary stage, and it approaches zero at later times. This cosmological model does not contain the singular point.

Keywords: Horndeski theory; Bianchi I cosmology; Dark energy

1. Introduction

It is usually stated that the state of the modern Universe is isotropic [1]. In his work [2] Mizner C. W speaks about this mysterious fact. Hawking and others formulated the so-called cosmic no-hair conjecture long ago [3, 4]. The states that the late Universe is homogeneous and isotropic, i.e., would obey the cosmological principle, regardless of initial states of the Universe. Moreover the initial states of the Universe may not obey this principle. There are only partial proofs for this hypothesis, for example [5, 6]. On the other hand in the work [7], based on SNe Ia datasets, the arguments are given for the anisotropic cosmic acceleration. Authors [8] examined the Pantheon sample of SNe Ia and five combinations among Pantheon. Authors conclude that the cosmic anisotropy is preferable for most observations. However the cosmic anisotropy found in Pantheon sample significantly relies on the inhomogeneous distribution of SNe Ia in the sky. More homogeneous distribution of SNe Ia is necessary for a clear answer. Thus, the absolute isotropy of the modern Universe raises questions.

Was there an anisotropic phase in the past? The isotropy of the Universe in the past does not follow from any general principles. One of the main arguments for the existence of the anisotropic phase of the Universe is the anisotropy at various scales of the microscopic wave background (CMB). The CMB contains information about the past of the Universe. There are anomalies in the CMB at the largest scales [9–16]. The Bianchi Universe can explain these anomalies of the CMB [17–19]. The Bianchi type-I (BI) space-time is a popular model. BI models have been studied from different perspectives [20–30]. For example, authors [25–27] consider constraints on the BI spacetime extension of the standard ΛCDM model.

An important criterion for the viability of any anisotropic model is the dynamic properties of anisotropic characteristics. In particular, it was argued in [28] that, from the point of view of the particle production, the isotropization of the Universe should occur quite early, no later than the beginning of primary nucleosynthesis (\( t \approx 1 \) s). The work [29] analyzed the effects caused by cosmic anisotropy on the primordial production of \(^4\)He. It was found that in the anisotropic Universe there is an overproduction of \(^4\)He with respect to the standard isotropic case. In order to agree with observational data, it is necessary to limit the anisotropy level at the time of freeze-out. There is an opposite effect [30]. Authors showed that the particle production provides the early isotropization. In General Relativity (GR) the process of isotropization occurs naturally. When the Universe expands the anisotropy terms decrease faster than the contribution of other forms of energy subject in the Einstein equations and the Universe rapidly approaches a locally isotropic state [31]. The anisotropic terms become dominant when approaching

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the beginning of the Universe, then they endure an infinite discontinuity. The state of affairs changes for the modified theories of gravity. The issue of isotropization in the modified theories of gravity is important and is considered by many researchers [32–39]. In the BI cosmology, some subclasses of the Horndeski gravity (HG) allow for the non-standard behavior of anisotropy. In the works [36, 40, 41], the HG theories were considered, in which the anisotropy has nonmonotonic dynamics.

The HG is determined by the following action density [42]:

\[ L_H = \sqrt{-g}(L_2 + L_3 + L_4 + L_5), \]  

with functions (in the parameterization of Ref. [43]):

\[ L_2 = G_2(\phi, X), \quad L_3 = -G_3(\phi, X)\Box \phi, \]
\[ L_4 = G_4(\phi, X)R + G_{4X}(\phi, X) \left( \Box \phi \right)^2 - (\nabla_\mu \nabla_\nu \phi)^2; \]
\[ L_5 = G_5(\phi, X)G_{\mu \nu} \nabla_\mu \nabla_\nu \phi \]
\[ - \frac{1}{6} G_{5X} \left( \Box \phi \right)^3 - 3 \Box \phi (\nabla_\mu \nabla_\nu \phi)^2 + 2 (\nabla_\mu \nabla_\nu \phi)^3, \]

respectively, where \( g \) is the determinant of a metric tensor \( g_{\mu \nu} \); \( R \) is the Ricci scalar and \( G_{\mu \nu} \) is the Einstein tensor; the factors \( G_i \) (for \( i = 2, 3, 4, 5 \)) are arbitrary functions of the scalar field \( \phi \) and the canonical kinetic term, \( X = -\frac{1}{2} \nabla_\nu \phi \nabla_\mu \phi. \) We consider the definitions
\[ G_{3X} \equiv \partial G_3/\partial X, \quad (\nabla_\mu \nabla_\nu \phi)^2 \equiv \nabla_\mu \nabla_\nu \phi \nabla^\nu \nabla_\mu \phi, \]
\[ (\nabla_\mu \nabla_\nu \phi)^3 \equiv \nabla_\mu \nabla_\nu \phi \nabla^\nu \nabla^\rho \nabla_\rho \nabla_\mu \phi \nabla_\nu \phi. \] The HG has a special place among the modified models. The field equations in GR are differential equations of the second order, thus, evading Ostrogradski instabilities arising [44, 45]. The HG is the most general variant of the scalar-tensor theory of gravitation with motion equations of the second order. The action density of HG contains several functions that provide a broad phenomenology. This makes it possible to solve important cosmological and astrophysical problems (screening of the cosmological constant, kinetic inflation, late de Sitter stage, hairy black holes, etc.) [46–54].

In this article, we review the HG within the framework of the BI cosmological model. In the case \( G_{5X} \neq 0 \) the gravitational equations give consequences containing the nonlinear anisotropic terms. Here we study the effects of nonlinear anisotropy with the function \( G_2(X, \phi) = -l \cdot \phi + A(X). \) Authors [55, 56] studied the linear potential \( l \cdot \phi \) for the isotropic Universe. In the work [40] the nonlinear anisotropy was studied for the model
\[ G_2 = X - \Lambda, \quad G_3 = 0, \quad G_4 = \text{const}, \quad G_5 = \text{const} + \xi \sqrt{2X}. \]

This model contains an early inflation and a late acceleration with suppressed anisotropy. The anisotropy shows a maximum at intermediate times. In the work [35] the nonlinearity leads to the effect of “anisotropization” in the later times, that is, the Universe evolves from an isotropic state to an anisotropic one. The HG is not the only theory with the nonlinear anisotropy. In the paper [32] the nonlinear growth of anisotropy in BI spacetime in metric \( f(R) \) cosmology was studied. Authors showed that any kind of anisotropy, if present initially, will be damped during the inflationary phase in quadratic gravity.

We aim to obtain and investigate the exact anisotropic cosmological solutions with certain properties. We want to find out what a model with the nonlinear anisotropic terms and the linear potential can offer. How interesting is this model for cosmology? What process will take place? Will there be a process of isotropization or “anisotropization”?

2. Bianchi I model

We consider the homogeneous and anisotropic Bianchi I metric:
\[ ds^2 = -dt^2 + a_1^2(t)dx^2 + a_2^2(t)dy^2 + a_3^2(t)dz^2, \]
with the three scale factors \( a_i \) and the scalar field \( \phi(t) \) depending only on \( t. \) Then the gravitational equations take the form [40]:

\[ G_0^'0 \left( G - 2G_{4X} \phi^2 - 2G_{4XX} \phi^4 + 2G_{5X} \phi^2 + G_{5XX} \phi^4 \right) \]
\[ = G_2 - G_2X \phi^2 - 3G_{3X} H \phi^3 + 3G_3 \phi^2 \]
\[ + 6G_{4X} \phi H + 6G_{4XX} \phi^3 H \]
\[ - 5G_{5X} H_1 H_2 H_3 \phi^3 - 5G_{3X} H_1 H_3 \phi^4, \]

\[ GG'_i - (H_j + H_k) \frac{dG}{dt} = G_2 - \frac{dG_1}{dt} + 2 \frac{d}{dt} \left( G_4 \phi \right) \]
\[ - \frac{d}{dt} \left( G_{5X} \phi^3 H_i H_k \right) - G_{5XX} \phi^3 H_i H_k(H_j + H_k). \]

Here the dot denotes the \( t \)-derivative (\( \phi \equiv \frac{d\phi}{dt} \)), one has \( H_i = \dot{a}_i/a_i \), and the average Hubble parameter is
\[ H = \frac{1}{3} \sum_{i=1}^3 H_i \equiv \dot{a}/a \quad \text{with} \quad a = (a_1 a_2 a_3)^{1/3}. \] The Einstein tensor components are
\[ G_0^' = -(H_i H_2 + H_2 H_3 + H_3 H_1), \]
\[ G'_i = - \left( H_j + H_k + H_j^2 + H_k^2 + H_j H_k \right), \]
where the triples of indices \( \{i, j, k\} \) take values \( \{1, 2, 3\}, \{2, 3, 1\}, \text{or} \{3, 1, 2\}. \) In addition, we define
\( \mathcal{G} = 2G_4 - 2G_{4X} \phi^2 + G_{5\phi} \phi^2. \)  
\( \text{(9)} \)

Varying the action (1) with respect to \( \phi \) yields the equation
\[
\frac{1}{a^3} \frac{d}{dt} (a^3 \mathcal{J}) = \mathcal{P},
\]
\( \text{(10)} \)

with
\[
\mathcal{J} = \dot{\phi} \left[ G_{2X} - 2G_{3\phi} + 3H \dot{\phi} (G_{3X} - 2G_{4X}) \right. \\
+ G_0^0 (-2G_{4X} - 2\phi^2 G_{4XX} + 2G_{5\phi} + G_{5X} \dot{\phi}^2) \\
+ H_1 H_2 H_3 (3G_{5X} \dot{\phi} + G_{5XX} \dot{\phi}^3) \],
\]
\( \text{(11)} \)
\[
\mathcal{P} = \frac{3}{8\pi} (H^2 - \sigma^2) = l \dot{\phi} - A + \dot{\phi}^2 A_X + 3G_{3X} H \dot{\phi}^3 \\
+ \dot{\phi}^3 (5G_{5X} + G_{5XX} \dot{\phi}^2) (H - 2\dot{\beta}_+) \left[ (H + \dot{\beta}_+) - 2\dot{\beta}_+ \right].
\]
\( \text{(17)} \)

For convenience let’s consider the following parametrization of three scalar factors:
\[
a_1 = ae^{\beta_+ + \sqrt{3} \beta_-}, \quad a_2 = ae^{\beta_+ - \sqrt{3} \beta_-}, \quad a_3 = ae^{-2\beta_+},
\]
\( \text{(13)} \)

This parametrization explicitly separates the isotropic and anisotropic parts. The functions \( e^{\beta_+ + \sqrt{3} \beta_-}, e^{\beta_+ - \sqrt{3} \beta_-} \) and \( e^{-2\beta_+} \) are the deviation from isotropy, and \( a(t) \) is the isotropic part. The rate of expansion in the direction of \( x, y \) and \( z \) are given by
\[
H_1 = H + \dot{\beta}_+ + \sqrt{3} \dot{\beta}_-, H_2 = H + \dot{\beta}_+ - \sqrt{3} \dot{\beta}_-, H_3 = H - 2\dot{\beta}_+.
\]
\( \text{(15)} \)

The anisotropies are determined by \( \dot{\beta}_\pm \), and if they vanish, then \( H_1 = H_2 = H_3 = H \) and the Universe is isotropic. This parametrization simplifies the process of integrating field equations.

We define the four arbitrary functions \( G_i (i = 2, 3, 4, 5) \) as follows
\[
G_2 = -l \cdot \phi + A(X), \quad G_3 = G_3(X), \quad G_4 = \frac{1}{16\pi}, \quad G_5 = G_5(X).
\]
\( \text{(16)} \)

In the future, the expression \( l \cdot \phi \) will provide the dynamic solution to \( \dot{\phi}(t), \dot{\beta}_\pm(t) \). The theory with \( G_5(X) \) gives a nontrivial behavior of anisotropy. Taking into account (14), (15) and (16) from Eqs. (5), (6) and (10) we obtain the consequences

\[
\frac{1}{8\pi} (2H + 3H^2 + 3\sigma^2) = l \dot{\phi} - A + G_{3X} \dot{\phi}^2 \phi
\]
\[
+ \frac{d}{dt} \left[ G_{5X} \dot{\phi}^3 (H^2 - \sigma^2) + 2G_{5XX} \dot{\phi}^3 \left( H^3 + \dot{\beta}_+ - 3\dot{\beta}_+ \right) \right],
\]
\( \text{(18)} \)

\[
\frac{3}{8\pi} (H^2 - \sigma^2) = l \dot{\phi} - A + \dot{\phi}^2 A_X + 3G_{3X} H \dot{\phi}^3
\]
\[
+ \dot{\phi}^3 (5G_{5X} + G_{5XX} \dot{\phi}^2) (H - 2\dot{\beta}_+) \left[ (H + \dot{\beta}_+) - 2\dot{\beta}_+ \right].
\]
\( \text{(17)} \)

The theory with \( G_{5X} \neq 0 \) gives the system of nonlinear Eqs. (19), (20) for \( \dot{\beta}_\pm \). From this point of view, we consider the nonlinear anisotropic model.

Further, we put
\[
C_\phi = C_- = C_+ = 0.
\]
\( \text{(23)} \)

Then the simplest solution is the isotropic one,
\[
\dot{\beta}_\pm = 0.
\]
\( \text{(24)} \)

In addition, since the equations are nonlinear, there are also solutions with \( \dot{\beta}_\pm \neq 0 \):
\[
\dot{\beta}_+ = \frac{1}{2} \left( H - \frac{1}{8\pi \cdot G_{5X} \dot{\phi}^3} \right),
\]
\( \text{(25)} \)
\[
\dot{\beta}_- = \pm \frac{\sqrt{3}}{2} \left( H - \frac{1}{8\pi \cdot G_{5X} \dot{\phi}^3} \right),
\]
\( \text{(26)} \)

In the solution (25), the signs “+” and “−” of \( \dot{\beta}_- \) correspond to
Thus, the assumption \( C_– = C_+ = 0 \) gives the locally rotationally symmetric (LRS) BI model. We will consider the model (27).

In view of (25) and (26), from Eqs. (17) and (21) we obtain

\[
3H \left\{ G_{3X} \phi^3 + \frac{1}{(8\pi^2) \cdot G_{5X} \phi^3} \left[ 3 + \frac{G_{XX} \cdot \phi^2}{G_{5X}} \right] \right\} = -l \phi + A - \phi^2 A_X
\]

\[
+ \frac{1}{(8\pi^2) \cdot (G_{5X} \phi^3)^2} \left[ 7 + \frac{2G_{XX} \cdot \phi^2}{G_{5X}} \right],
\]

\[
3H \left\{ G_{3X} \phi^3 + \frac{1}{(8\pi^2) \cdot G_{5X} \phi^3} \left[ 3 + \frac{G_{XX} \cdot \phi^2}{G_{5X}} \right] \right\} = -\phi^2 A_X - \frac{1}{a^3} \int a^3 dt
\]

\[
+ \frac{2}{(8\pi^2) \cdot (G_{5X} \phi^3)^2} \left[ 3 + \frac{G_{XX} \cdot \phi^2}{G_{5X}} \right].
\]

Eq. (18) can be ignored, since it is automatically fulfilled by virtue of the Bianchi identities. The combination of (29) and (30) gives the equation:

\[
\frac{1}{(8\pi^2) \cdot (G_{5X} \phi^3)^2} - l \phi + A = -\frac{1}{a^3} \int a^3 dt.
\]

Let us note an important property of the presented model. If \( l = 0 \) then the system (25), (29), (30) can only have the stationary solution \( \dot{\phi} = 0, H, \phi = \text{const} \). A necessary condition for the existence of the dynamic solution \( \dot{\phi} \cdot (t), \phi(t) \) is the presence of the term \( l \cdot \phi \neq 0 \). In the work [40] the dynamics of the solution was provided by nonzero charges \( C_\phi, C_\pm \).

Next, we use the reconstruction method. Let’s make an assumption for the average scale factor

\[
\frac{1}{a^3} \int a^3 dt = \mu = \text{const} > 0.
\]

This assumption gives the exact solution, and in this case the Universe is expanding with acceleration:

\[
H = \frac{l}{3\mu} = \text{const}, a(t) = a_e \exp \left( \frac{l \cdot t}{3\mu} \right), l > 0.
\]

In other words, we consider the isotropization process of the Universe, which is approaching de Sitter’s world. The proposed model can describe the primary inflation of the early Universe. The type of the scale factor (33) is substantiated and used by many authors to describe the cosmic inflation in anisotropic models [32, 33, 60, 61]. The model (33) is interesting in the context of inflationary mechanism for suppressing the anisotropy. We choose the function \( A(X) \) as follows

\[
A(X) = -\mu \phi = -\mu \sqrt{2X}, \dot{\phi} \geq 0.
\]

The model with \( G_2 \propto \sqrt{X} \) corresponds to Cuscuton scenarios [57–59]. From Eq. (31) it follows

\[
\frac{1}{(8\pi^2) \cdot (G_{5X} \phi^3)^2} - l \cdot \phi = 0 \Rightarrow \frac{1}{8\pi \cdot G_{5X} \phi^3} = \pm \sqrt{8\pi l \cdot \phi}.
\]

We choose the “+” sign. In this case the Universe expands in all directions, \( H_1 > 0 \) (see (27), (28)).

We want to get a model that is isotropic in later times \( t \rightarrow +\infty \). Therefore, it must be fulfilled

\[
\frac{\dot{\beta}_\pm}{H} \rightarrow 0 \text{ as } t \rightarrow +\infty,
\]

i.e.

\[
\frac{1}{8\pi \cdot G_{5X} \phi^3} \rightarrow H \text{ as } t \rightarrow +\infty.
\]

The dynamics of \( \dot{\phi}(t) \) allows one to construct solution (25) with the isotropization. To satisfy the isotropization condition (37), we put

\[
\frac{1}{8\pi \cdot G_{5X} \phi^3} = H + \gamma \phi^{1/3}, \gamma = \text{const}.
\]

Then Eq. (35) is transformed to the form

\[
H + \gamma \phi^{1/3} = \sqrt{8\pi l \cdot \phi}.
\]

This equation has a nonsingular solution

\[
\dot{\phi} = \frac{H^2}{8\pi l} \cdot \left( \frac{1 + \frac{8\pi l \cdot t}{|\gamma|}}{1 + \frac{8\pi l \cdot t}{|\gamma|} - \frac{|\gamma|^3}{8\pi l H}} \right), \gamma < 0, t \geq t_* \equiv -\frac{|\gamma|^3}{8\pi l H}.
\]

The scalar field \( \phi \) is the bounded function, \( 0 \leq \phi < H^2 / (8\pi l) \). As seen in Fig. 1, the function \( \phi \) tends monotonically to a constant value over time and there is no infinite discontinuity. With time the linear potential \( V = l \cdot \phi \) begins to behave like the cosmological constant.

Considering (25), (38) and (40) we get
As seen in Fig. 2, the functions $\beta_{\pm}$ tend monotonically to zero over time, that is, the Universe becomes isotropic at later times. The functions $\beta_{\pm}$ are finite at the beginning ($t_s$) of the Universe. This is a non-standard behavior of the Universe anisotropy.

The Universe is expanding along the $x$, $y$ and $z$ axes:

\begin{align*}
H_1 &= H \cdot \frac{3 + \sqrt{1 + \frac{t}{|t_s|}}}{1 + \sqrt{1 + \frac{t}{|t_s|}}} > 0, \\
H_2 &= H \cdot \frac{1}{1 + \sqrt{1 + \frac{t}{|t_s|}}} > 0.
\end{align*}

As seen in Fig. 3, the functions $H_i$ tend monotonically to a constant value over time, $H_i \to H = \text{const}$, $t/t_s \gg 1$. There is no infinite discontinuity. The Universe is approaching the de Sitter’s world.

We can find $\beta_{\pm}$ from the formula (41):

$$
\beta_{\pm} = 2s_{\pm}^* H[t_s]\left[\sqrt{1 + \frac{t}{|t_s|}} - \ln\left(1 + \sqrt{1 + \frac{t}{|t_s|}}\right)\right] + \text{const}_{\pm}.
$$

On integration (42) we get the scale factors

\begin{align*}
a_1(t) &= a_1^* \exp\left\{H|t_s|\left[1 + \frac{t}{|t_s|} + 4\sqrt{1 + \frac{t}{|t_s|}}\right.ight. \\
&\left.- 4\ln\left(1 + \sqrt{1 + \frac{t}{|t_s|}}\right)\right]\}, \\
a_2,3(t) &= a_2,3^* \exp\left\{H|t_s|\left[1 + \frac{t}{|t_s|} - 2\sqrt{1 + \frac{t}{|t_s|}}\right.ight. \\
&\left.+ 2\ln\left(1 + \sqrt{1 + \frac{t}{|t_s|}}\right)\right]\}, t \geq t_s,
\end{align*}

where $a_1^*$ and $a_{2,3}^*$ are integration constants, $a_i^* = a_i(t_s)$.

This cosmological model does not contain a singular point. The Ricci scalar (invariant)
From the assumption (38), we restore the function $G_3(X)$:

$$G_3 = \frac{\gamma}{48\sqrt{2}\pi} \left[-X^{-1/3} + \frac{4\sqrt{2}\gamma}{HX^{1/6}}\right] + \text{const.} \quad (47)$$

From the assumption (38), we restore the function $G_5(X)$:

$$G_5 = -\frac{3|\gamma|^2}{8 \cdot 2^{1/6} \pi H^2 X^{1/6}} - \frac{3|\gamma|}{16 \cdot 2^{1/3} \pi H^2 X^{1/3}}$$

$$- \frac{3|\gamma|^3}{8 \pi H^4} \ln \left[ \frac{2^{1/6} |\gamma| X^{1/6}}{2^{1/6} |\gamma| X^{1/6} - H} \right] + \text{const.} \quad (48)$$

Thus, we have presented the theory that allows the isotropization process of the Universe with the finite anisotropy.

3. Conclusions

We have studied the anisotropic BI cosmology within the HG with functions: $G_2(X, \phi) = -l \cdot \phi + A(X)$, $G_3(X) \neq 0$, $G_4 = \text{const}$ and $G_5(X) \neq 0$. Our aim was to see what the nonlinear anisotropy with the linear potential could produce. We have studied the isolated influence of the linear potential on the dynamics of the field functions, assuming that the scalar charge and the anisotropic charges are equal to zero, $C_{\phi} = C_{\pm} = 0$. The value of anisotropic charges affects the symmetry of space-time. The assumption $C_{\pm} = 0$ gives the locally rotationally symmetric Bianchi I model: $a_1 \neq a_2 = a_3$.

We have presented the exact cosmological solution. It has the following properties. First, the anisotropic terms are always finite and they reach their a maximum value at the initial moment. This is the non-standard behavior of the Universe anisotropy. Secondly, the anisotropy suppression occurs during the inflationary stage, and it approaches zero at later times. When the model is isotropized over time, we get the de Sitter's world. Third, the cosmological model does not contain a singular point.

To find the exact solution, we applied the reconstruction method. This method is often used in the modified theories of gravity [39, 51, 52, 55, 56]. The reconstruction method is interesting and optimal for assessing the viability of the modified theory of gravity under study. The Lagrangian functions $G_2$, $G_4$ are entered manually. A priori, the average Hubble parameter was set constant, $H = \text{const}$. Modeling the process of isotropization, we have chosen the function $G_5$. Such assumptions allow you to restore the desired functions $G_3$, $G_5$.

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