Average power of Laguerre-Gaussian laser beams along the atmospheric path

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Abstract. Numerical simulation of various mode propagation of a Laguerre-Gaussian laser beam reflected by a flat mirror at different intensities of optical turbulence along the atmospheric path are carried out. It is found that the strength of optical turbulence increases, the average power of the received signal at the end of a location path exceeds ones at the end of a propagation path double distance in the forward direction. The greater the mode order of the Laguerre-Gaussian beam, the stronger the effect.

1. Introduction
A large number of scientific publications have devoted to problems an analysis of influence of an orbital angular moment value on characteristics of a laser beam propagating in a turbulent atmosphere along direct paths [1]. Vortex beams have attracted considerable research interest due to their prospects for the practical use of such beams, in particular, to increase the efficiency of atmospheric optical systems for transmitting energy and information through the atmosphere.

In this work, based on numerical simulations, we investigate the average power of backscattered laser radiation of Laguerre-Gaussian beam modes detected by a receiver in a general scheme of monostatic location along the atmospheric path.

2. Theoretical background
Let us consider the problem of propagation of vortex laser beams in a location path along a coordinate \( x \geq 0 \) in the turbulent atmosphere. We assume that the laser radiation propagates to a reflector in a form of a planar infinite mirror at a distance \( x = x_L \) and vice versa. The reflected radiation is recorded by the receiver, combined in space with the source of the laser radiation in the plane \( x = x_0 \); \( \mathbf{p}_L = \{y_L, z_L\} \), \( \mathbf{p} = \{y, z\} \) and \( \mathbf{p}_0 = \{y_0, z_0\} \) are radius vectors in the plane transverse to the propagation direction, coinciding with the planes of the reflector, receiver and laser radiation source, respectively.

We introduce the spectral amplitude of the electric field intensity:

\[
E(x, \mathbf{p}) = E_{nm}^0(x, \mathbf{p}) \exp \left\{ jk < n > x \right\},
\]

where \( j = \sqrt{-1} \); \( k = 2\pi/\lambda \) is the wave number; \( \lambda \) is the wavelength;

\[
<n> = 1 + 10^{-6} \frac{P_a}{<T>} \left[ 77.6 + \frac{0.584}{\lambda^2} \right]
\]
is the average refractive index; \( P_a \) is the atmospheric pressure; \( < T > \) is the temperature averaged over the ensemble of realizations.

We set the initial distribution \( E_{nm}^0(x_0, \rho_0) \) in Eq. \( (1) \) \( (at \ x=x_0=0 \ and \ \rho=\rho_0) \) in the form of Laguerre-Gaussian beam modes \[2\]

\[
E_{nm}^0(x_0, \rho_0) = (-j)^m E_0 \left( -\frac{\rho_0}{a_0} \right)^m \exp \left\{ -\frac{\rho_0^2}{2a_0^2} + j\psi_0 + j m\theta \right\} L_m^m \left( -\frac{\rho_0^2}{a_0^2} \right), \tag{3}
\]

where the parameter \( a \) determines the boundedness of the Laguerre mode in the space; \( \theta \) is an angular coordinate; \( L_m^m(x) \) is the Laguerre polynomial; \( n \) and \( m \) are the radial and azimuthal order of the Laguerre-Gaussian beam mode \( E_{nm}^0 \) respectively; \( \psi_0 \) is a wave phase; \( E_0 \) is a beam amplitude on its axis.

The wave parabolic equation describing the propagation of the laser radiation (Eq. \( (1) \)) in the turbulent atmosphere for the complex spectral amplitude \( E_{nm}^0(x, \rho) \) has the form \[3\]

\[
j2k \frac{\partial E_{nm}^0(x, \rho)}{\partial x} + \Delta_\perp E_{nm}^0(x, \rho) + 2k^2 n'(x, \rho) E_{nm}^0(x, \rho) = 0, \tag{4}
\]

\( \Delta_\perp = \partial^2 / \partial y^2 + \partial^2 / \partial z^2 \) is the transverse Laplace operator;

\[
n'(x, \rho) = [1 - < n(f) >] \frac{T'(x, \rho)}{< T >} \tag{5}
\]

is fluctuations of the refractive index caused by turbulent variations of air temperature; \( T'(x, \rho) \) is the turbulent fluctuations of air temperature, with the boundary condition, according to Eqs. \( (1) \) and \( (3) \),

\[
E(x = 0, \rho) = E_{nm}^0(x_0, \rho_0). \tag{6}
\]

We denote by

\[
\langle I(x, \rho) \rangle = \left\langle \left| E_{nm}^0(-L, \rho_0) \right|^2 \right\rangle \tag{7}
\]

is the distribution of the intensity ensemble-averaged random realizations of the laser radiation scattered from a point \( (L, \rho_L) \) in backward direction to a point \( (0, \rho) \), where

\[
E_{nm}^0(x=-x_L, \rho) = \frac{jV_0}{\lambda x_L} E_{nm}^0(L, \rho_L) \exp \left\{ -\frac{jk(\rho-\rho_L)}{2x_L} \right\} \exp \left\{ -j\Psi(\rho) \right\} = \frac{V_0}{(\lambda x_L)} \exp \left\{ j\Psi(\rho_L) \right\} \exp \left\{ -j\Psi(\rho) \right\} \times \int_{-\infty}^{\infty} d^2 \rho_0 \rho_{nm}(\rho_0) \exp \left\{ -\frac{jk(\rho_0-\rho_L)^2}{2x_L} \right\} \int_{-\infty}^{\infty} d^2 \rho_L \rho_{nm}(L, \rho) \exp \left\{ -\frac{jk(\rho-L)^2}{2x_L} \right\}. \tag{8}
\]

is the complex amplitude of the field scattered by the flat mirror.

In Eq. \( (6) \) \( V_0 \) is a reflector amplitude multiplier; \( L = const \) is a propagation path length; the multiplier \( \exp \left\{ j\Psi(\rho_L) \right\} \), where

\[
\Psi(\rho_L) = \int_{0}^{L} dx' \psi(x_L + x_0, \rho_L), \tag{9}
\]

takes into account the phase distortion of the wave by turbulent inhomogeneities of the air refractive index;
is the complex amplitude of the field incident on the scattering surface.

Using Eqs. (1) and (7), the expression for the average power \( P(L, -L) \) of the received scattered radiation in the turbulent atmosphere when the reflected radiation is completely intercepted has the form

\[
P(L, -L) = \int_{-\infty}^{\infty} d^2 \mathbf{p} \langle I(-L, \mathbf{p}) \rangle.
\]

(11)

Also, the average power of the wave passed the double distance \( 2L \) was calculated

\[
P(0, 2L) = \int_{-\infty}^{\infty} d^2 \mathbf{p} \langle I(2L, \mathbf{p}) \rangle
\]

(12)

for various parameter values [4]

\[
\beta_0 = \sqrt{1.23C_n^2k^{7/6}\lambda^{1/6}},
\]

(13)

characterizing the intensity of the optical turbulence on the propagation path, where \( C_n^2 \) is the structural constant of the air refractive index.

For calculations using Eqs. (8) and (10), we use numerical simulation of the forward and backward propagation of the laser radiation in the turbulent atmosphere, which is described by Eq. (4) by the method of splitting into physical factors [5] and the algorithm for modeling random phase screens in Eq. (9).

3. Results of numerical simulations

For numerical simulations of random distributions of complex amplitudes \( E_{nm}(x_L, \mathbf{p}_L) \) (Eq. (10)) and \( E_{nm}(-x_L, \mathbf{p}) \) (Eq. (8)), the following parameters are taken: \( a_0 = 2.5 \text{ cm}; \lambda = 0.63 \mu\text{m} ; V_0 = 1; E_0 = 1; L = 1 \text{ km} \) on a grid dimension \( M \times M = 512 \times 512 \) with grid spacing \( h = 2 \text{ mm} \); \( n = 0 \) and \( m = 3 \) and 6. The parameter \( 0.12 \leq \beta_0^2 \leq 56 \), which corresponds to the regime of weak (\( \beta_0^2 < 1 \)) and strong (\( \beta_0^2 \gg 1 \)) optical turbulence, was calculated for values \( 1.23 \times 10^{-15} \leq C_n^2 \leq 2.77 \times 10^{-13} \text{ m}^{-2/3} \), respectively. Averaging was carried out over 7000 independent random implementations of 2D distributions of wave intensity in the transverse plane.

In Figure 1 shows the results of calculating the ratio of average powers \( P(L, -L) / P(0, 2L) \) reflected from the plane mirror to the double-distance direct propagation, calculated by formulas (11) and (12) respectively, of the laser radiation of modes \( E_{03}^0 \) (Figure 1a) and \( E_{06}^0 \) (Figure 1b) of the Laguerre-Gaussian beam at \( \beta_0 = 0.3; 0.7; 1.5; 2–6 \) and 7.5 depending on the radius of the receiving aperture \( a \).
4. Results

Figure 1 clearly demonstrates how the average power of the backscattered radiation of the Laguerre-Gaussian beam mode increases due to the correlation of counterpropagating waves compared with the average power of the laser radiation detected at the end of the double-length direct path [6]. The optical turbulence increase (the parameter $\beta_0 \geq 1.5$), the power of the received signal of the Laguerre-Gaussian beam mode $E_{06}^0$ at the end of the location path exceeds the power of the corresponding mode at the end of the direct path of doubled length by almost two times (Figure 1b).

At $\beta_0 \geq 2$ (Eq. (11)), the difference in the magnitude of the maximum between the curves $\langle I(-L, \rho) \rangle$ and $\langle I(2L, \rho) \rangle$ decreases and, consequently, the ratio of integral powers over the area of the receiving
aperture $a$ decreases. The smaller the mode order $m$, the smaller this effect (Figure 1a).

In this case, regardless of the order of the Laguerre-Gaussian beam mode and the size of the receiving aperture $a$, the opposite effect is observed - a weakening of the average power of the reflected radiation. This effect is explained by a decrease in the beam energy in the receiving plane and, accordingly, in the intensity in the direction strictly backward in comparison with corresponding values for direct propagation at the double distance due to the displacement of the beam as a whole [6]. The smaller the mode order $m$, the smaller the parameter $\beta_0$ this effect is manifested.

5. Conclusion

In this work, based on the numerical solution of the parabolic wave equation for the complex amplitude of the wave field by splitting into physical factors method and modeling phase screens, the average power of the backscattered by a flat mirror laser radiation of Laguerre-Gaussian beam modes in the turbulent atmosphere is analyzed.

From a comparison of the results of numerical simulations, it follows that with an increase in the intensity of atmospheric turbulence (parameter $\beta_0$), the average power distribution in the location path exceeds the corresponding distributions for the same Laguerre-Gaussian beam mode, but passed double distance in the forward direction, by almost two times. The larger the order of the mode $m$, the larger the parameter $\beta_0$ this effect is manifested.

With strong optical turbulence, the opposite effect is observed, a weakening of the average power of the reflected radiation. This effect does not depend on the order of the Laguerre-Gaussian beam mode and the size of the receiving aperture.

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