Features of coronal heating by drift waves

J. Vranjes and S. Poedts
Center for Plasma Astrophysics,
and Leuven Mathematical Modeling and Computational Science Research Centre (LMCC),
Celestijnenlaan 200B, 3001 Leuven, Belgium.
E-mail: jvranjes@yahoo.com; Stefaan.Poedts@wis.kuleuven.be

Abstract. The stochastic heating, well known from laboratory plasma physics where it has been confirmed in numerous experiments, has been completely ignored in past studies of coronal heating. In the present study, and in our very recent works, it has been shown that the inhomogeneous coronal plasma is, in fact, a perfect environment for fast growing drift waves and the consequent stochastic heating. As a matter of fact, the large growth rates are typically of the same order as the plasma frequency. The heating rates may exceed the required values for a sustained coronal heating by several orders of magnitude. Some aspects of these phenomena are investigated here. In particular the analysis of the particle dynamics within the growing wave is compared with the corresponding fluid analysis.

1. Introduction
The drift wave is driven by gradients of the background plasma parameters (e.g., gradients of the plasma density, temperature, and magnetic field) that may act either separately or all together. In the literature, the drift wave is frequently called the universally unstable mode because it is growing both within the (two- or multi-component) fluid theory and the kinetic theory. In the former (fluid) description, it is either a collisional instability (discussed in [1] in the context of the solar atmosphere) that appears as a combined effect of the electron collisions, the ion inertia, and the density gradient, or a reactive one [2] in the presence of all three gradients mentioned above where now the ions play a major role. The latter (kinetic) theory, on the other hand, can also describe these two instabilities. However, the kinetic description yields an additional (purely kinetic) instability that is absent within the fluid description. In application to the solar corona, this additional kinetic instability is studied in [3]-[5]. This purely kinetic growth of the drift wave is entirely due to electron dynamics and it develops provided the wave frequencies are below the electron diamagnetic drift frequency.

2. Results
For the purpose of the present study, in order to quantitatively describe some details of the heating and acceleration of plasma particles, we shall use the kinetic, density gradient driven instability from [3, 4]. The frequency and growth rate of the kinetic drift wave are given by

$$\omega_r = \frac{\omega_{ei} \Lambda_0(b_i)}{1 - \Lambda_0(b_i) + T_i/T_e + k_g \lambda_d}, \quad \omega = \frac{\Omega_i}{\Omega_e},$$

(1)
\[
\gamma \sim \left( \frac{\pi}{2} \right)^{1/2} \frac{\omega_r^2}{\omega_{\|}} \left[ \frac{T_i}{T_e} \left( \frac{\omega_r - \omega_{se}}{|k_z|v_{Te}} \right) \exp[-\omega_r^2/(k_z^2v_{Te}^2)] \right. \\
+ \left. \frac{\omega_r - \omega_{si}}{|k_z|v_{Ti}} \exp[-\omega_r^2/(k_z^2v_{Ti}^2)] \right]. \tag{2}
\]

Here, the geometry is chosen as follows. The constant magnetic field is given by \( \mathbf{B}_0 = B_0 \mathbf{\hat{z}} \), the equilibrium density gradient in the electrically neutral plasma with \( n_0 = n_\| = n_\perp = 0 \) is given by \( \nabla_\perp n_0 = -\varepsilon_e n_\|' = -\varepsilon_e d n_0/dx \), and the perturbations are of the form \( f(x) \exp(-i\omega t + ik_y y + ik_z z) \), with \( |k_z| \ll |k_y| \) and an \( x \)-dependent wave amplitude \( f \). The local approximation will be used. Hence, it will be assumed that \( |d/dx| \ll |k_y| \). The notation in Eqs. (1) and (2) is standard, i.e., \( \omega_{se} = -k_y (v_{Te}^2/\Omega_e)(n_\|/n_\perp) \), \( \omega_{si} = -(T_i/T_e) \omega_{se} \), \( v_{Tj}^2 = \kappa T_j/m_j \), \( \Lambda_0(b_i) = I_0(b_i) \exp(-b_i) \), \( b_i = k_y^2 \rho_i^2 \), \( \lambda_{di} = v_{Ti}/\omega_{pi} \), \( I_0 \) is the modified Bessel function of the first kind and of the order 0, and \( c_s \) is the ion sound speed. From Eq. (2) it is seen that within the present instability, the sign of \( \gamma \) can be changed only by the electron term if \( |\omega_r| < |\omega_{se}| \). This is, however, only a necessary condition. A sufficient condition is obtained when the contribution of the rest of the expression (the ion damping terms) is also taken into account.

To provide some details of the particle acceleration and heating by the growing drift wave in the context of the solar corona, we need specific data. Hence, as an example we here use a set of parameters from [3, 4] that are known to yield the instability: \( B_0 = 10^{-2} \text{T} \), \( n_0 = 10^{15} \text{m}^{-3} \), \( l_n = [(d n_0/dx)/n_0]^{-1} = s \cdot 100 \text{ m} \), \( \lambda_y = 0.5 \text{ m} \), and \( \lambda_z = s \cdot 10^4 \text{ m} \). These are just representative parameters with no particular significance. As discussed in [4], the instability will in fact develop even when the density is varied by several orders of magnitude around the given value, and the same holds for the magnetic field and the temperature. The parameter \( s \) is introduced in [3, 4] for convenience only, because it was realized that the ratio \( \gamma/\omega_r \) remains exactly the same in case the ratio \( \lambda_z/l_n \) is fixed. Here, \( l_n \) is the characteristic scale-length for the inhomogeneous equilibrium density. This further implies that we may go to very different scales, e.g., by taking \( s \) in the range \( 0.1 - 10^{-4} \). As long as \( \lambda_y \) is kept constant, the ratio \( \gamma/\omega_r \) will remain the same, although the actual values for these two quantities will certainly change. Hence, for the given parameters, from Eqs. (1) and (2) one finds \( \gamma/\omega_r = 0.26 \). In the case \( s = 1 \), this in fact implies \( \omega_r = 254 \text{ Hz} \) and \( \gamma = 66 \text{ Hz} \). While in the case \( s = 10^4 \), this would give a mode with \( \omega_r = 0.254 \text{ Hz} \) and \( \gamma = 0.66 \text{ Hz} \). Going to such small values of \( l_n \) (tens of meters) in fact makes sense because the perpendicular diffusion, that naturally develops in such a plasma with density gradients, is indeed very small, with the diffusion coefficient \( D_1 \) of the order of \( 0.01 \text{ m}^2/\text{s} \) and the corresponding diffusion velocity \( D_1 \nabla_\perp n/n \) of just a few millimeters per second. Short values of \( l_n \) in the same time imply very high frequency drift waves and fast growing instabilities, so that the same time heating will develop at time scales that are orders of magnitude shorter than the plasma diffusion. Note that the parameters \( \lambda_z \) and \( l_n \) can of course also be varied independently of each other.

A direct numerical integration for plasma particles is required for a full description of the heating mechanism. The numerical simulation has to take into account the actual values of the electric field in the position of a specific gyrating plasma particle. For that purpose, taking the drift-wave electric field with a time-varying amplitude, the appropriate set of equations for a single particle reads

\[
y''(t) + y(t) - a(t) \sin \left[ y(t) + \frac{k_z}{k_y} z(t) - bt \right] = 0, \tag{3}
\]

\[
z''(t) - a(t) \frac{k_z}{k_y} \sin \left[ y(t) + \frac{k_z}{k_y} z(t) - bt \right] = 0, \quad x'(t) - y(t) = 0. \tag{4}
\]

Here, the prime denotes a derivative in time, and the spatial and time coordinates are normalized as \( x, y, z \rightarrow k_y x, k_y y, k_y z, t \rightarrow \Omega_i t, b = \omega_r/\Omega_i \).
The set of ordinary differential equations (3)-(4) is solved numerically. We do this first for a very small wave amplitude in order to demonstrate some basic features of the particle motion due to the wave electric field. The result is shown in Fig. 1, where we give the projection of the proton positions in the perpendicular plane, \( x(t), y(t) \), in a time-varying potential \( \phi_1(t) \) due to the above demonstrated drift wave instability. Restricting the time interval to relatively short values (below the onset time of the stochastic heating), yields a spiral trajectory in the \( x, y \)-plane. The potential is given by 

\[
\phi_1(t) = \hat{\phi} \exp(\gamma t/\Omega_i), \quad \gamma = 0.26\omega, \quad \omega = 254 \text{ Hz}, \quad \hat{\phi} = \phi_1(0).
\]

The maximum time in physical units here is 0.04 s, and in this moment the potential has reached the value of 17 V. Here and further in the text we take \( \lambda_y = 0.5 \text{ m}, \quad \lambda_z = 2 \cdot 10^4 \text{ m}, \) which consequently from Eq. (1) yields the wave frequency \( \omega = 254 \text{ Hz} \). The normalization is the same as above, viz. \((x, y) \equiv (k_y x, k_y y)\).

**Figure 1.** Normalized particle positions in the \( x, y \)-plane after it starts from the point A with \((x, y, z) = (0, 0, 0)\), for a growing electric field potential \( \phi_1(t) = \hat{\phi} \exp(\gamma t/\Omega_i) \), and \( \hat{\phi} = 0.86 \text{ V} \).

In order to see what happens for larger amplitudes of the electrostatic drift wave when the stochastic heating is supposed to be in action, Eqs. (3)-(4) are solved by allowing a slightly larger time range. One of the results is shown in Fig. 2 for the kinetic energy of a particle with a unit mass 

\[
E(t)/m = \left( v_x(t)^2 + v_y(t)^2 + v_z(t)^2 \right)/2,
\]

with normalized velocities as above. The other parameters are the same as in the previous text. Here, the stochastic heating takes place after around 0.078 s, in the moment when the growing wave amplitude reaches the value of around 150 V. Note that this is by about a factor 2.5 larger than the value that can be obtained from the fluid theory [6] and which reads

\[
a \equiv k_y^2 \phi_1(t)/(B_0\Omega_i) = k_y^2 \rho_i^2 e\phi_1(t)/(\kappa T_i) \geq 1.
\]

**Figure 2.** Change in time of the normalized kinetic energy of a particle with a unit mass 

\[
E(t) = (v_x^2 + v_y^2 + v_z^2)/2.
\]
drift in this limit becomes inaccurate. Hence, the difference between the two descriptions is expected. Yet, Eq. (5) describes the general trend for the polarization drift effect: a fast and significant acceleration and heating of the plasma particles will definitely take place, as experimentally verified in [7, 8].

The corresponding plot of the displacement in the direction of the wave \( y(t) \), which is in fact also equal to the velocity \( v_x(t) \) is shown in Fig. 3.

![Figure 3. Normalized displacement in the direction of the polarization drift \( y(t) \), and the normalized perpendicular velocity component \( y(t) \equiv v_x(t) \).](image)

![Figure 4. Normalized perturbed velocity in the direction parallel to the magnetic field.](image)

The plot of the parallel velocity \( v_z(t) \), presented in Fig. 4, shows that along the magnetic field the particle dynamics remains non-stochastic, as should be expected in view of the analysis presented in the text above. The velocity magnitude is below 1, thus still smaller than the perpendicular velocity which is the consequence of the stochastic heating.

3. Summary
In this work, the trajectories of individual plasma particles have been investigated within the framework of the drift wave instability and the associated stochastic heating, with the aim to describe some fine details of the particle dynamics during the heating process. The calculation of the actual growth rates of the mode is not repeated here because it has been given in detail in our recent studies. From the present study and from our recent works it follows that the general trend of the ion behavior in the wave field with a growing amplitude can be described analytically from the fluid point of view. However, a more detailed picture is obtained by following a single particle which is moving in the wave field. For example, this is seen from the critical threshold for the onset of the heating.

It is also interesting to note that, although the individual particle dynamics becomes stochastic for relatively large wave amplitudes, the overall picture of the macroscopic motion of
the plasma, caused by the wave, actually remains well described by the fluid description. This has been directly experimentally verified by [9]. The parameter $a$ [c.f. Eq. (5)] in their experiment was around 2, and thus the stochastic heating was in action, yet the observed electrostatic potential and density profiles nicely matched the theoretical contour plots corresponding to a wave described within the fluid theory and drift approximation. This is in agreement with the nature of the heating; the increased effective temperature $T_{\text{max}}$ follows from the stochastic velocity $v_{\text{max}}$ which in fact is not random, yet in practical terms it is indistinguishable from standard collisional thermalization and the broadening of the distribution function [6].

The model used here implies a Maxwellian distribution function for plasma species, that is the basis for both the kinetic and the fluid description of the instability. However, the stochastic heating with all its features [2, 4] in fact implies that such a starting distribution function will necessarily evolve. The perpendicular ion temperature will grow and eventually become larger than the parallel one. Hence, the particle distribution may pass through several stages, going through the Oort’s distribution with the temperature anisotropy and to the more general loss-cone distribution [10], or to something else. This opens an interesting possibility for eventual numerical simulations because such fine details are beyond the scope of an analytical study.

Acknowledgments
The results presented here are obtained in the framework of the projects G.0304.07 (FWO-Vlaanderen), C 90347 (Prodex), GOA/2009-009 (K.U.Leuven). Financial support by the European Commission through the SOLAIRE Network (MTRN-CT-2006-035484) is gratefully acknowledged.

[1] Vranjes J and Poedts S 2006 Astron. Astrophys. 458 635
[2] Vranjes J and Poedts S 2009 MNRAS 400 2147
[3] Vranjes J and Poedts S 2009 EPL 86 39001
[4] Vranjes J and Poedts S 2009 MNRAS 398 918
[5] Vranjes J and Poedts S 2009 Phys. Plasmas 16 092902
[6] Bellan P M 2006 Fundamentals of Plasma Physics (Cambridge: Cambridge Univ. Press) p 106
[7] McChesney J M, Stern R A and Bellan P M 1987 Phys. Rev. Lett. 59 1436
[8] Sanders S J, Bellan P M and Stern R A 1998 Phys. Plasmas 5 716
[9] Bailey A D, Stern R A and Bellan P M 1993 Phys. Rev. Lett. 71 3123
[10] Baronia A and Tiwari M S 1999 Planet. Space Sci. 47 1111