Is there a paradox in $CP$ asymmetries of $\tau^\pm \rightarrow K_{L,S}\pi^\pm \nu$ decays?

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Based on the description of unstable $K_{L,S}$ particles in quantum field theory (QFT), we compute the time-dependent probabilities for transitions between asymptotic states in $\tau^\pm \rightarrow [\pi^+\pi^-]_K\pi^\pm \nu$ decays, where the pair $[\pi^+\pi^-]_K$ is the product of (intermediate state) neutral kaon decays. Then we propose a definition of $\tau$ decays into $K_L$ and $K_S$ states, which reflects into the cancellation between their $CP$ rate asymmetries, thus solving in a natural way the paradox pointed out in Ref. [1]. Since our definition of $K_{L,S}$ final states in $\tau$ decays is motivated on experimental grounds, our predictions for the integrated $CP$ rate asymmetries can be tested in a dedicated experiment.

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I. INTRODUCTION

In a recent paper, Bigi and Sanda [1] have pointed out that $\tau^\pm \rightarrow K_{L,S}\pi^\pm \nu$ decays exhibit a $CP$ asymmetry of the same size as the one measured in the charge asymmetry of semileptonic $K_L$ decays. The ‘known’ $CP$ rate asymmetries for $K_S$ and $K_L$ final states

\[
\frac{\Gamma(\tau^+ \rightarrow K_S\pi^+\bar{\nu}) - \Gamma(\tau^- \rightarrow K_S\pi^-\nu)}{\Gamma(\tau^+ \rightarrow K_S\pi^+\bar{\nu}) + \Gamma(\tau^- \rightarrow K_S\pi^-\nu)} = |p|^2 - |q|^2,
\]

(1)

\[
\frac{\Gamma(\tau^+ \rightarrow K_L\pi^+\bar{\nu}) - \Gamma(\tau^- \rightarrow K_L\pi^-\nu)}{\Gamma(\tau^+ \rightarrow K_L\pi^+\bar{\nu}) + \Gamma(\tau^- \rightarrow K_L\pi^-\nu)} = |p|^2 - |q|^2,
\]

(2)

turn out to be identical. If true, this would indicate a paradox because the total rates of $\tau^+$ and $\tau^-$ would be different, in contradiction with the CPT theorem.

Bigi and Sanda [1] proposed a solution to this contradiction by looking at the time evolution of the $K^0$ ($\bar{K}^0$) state produced in $\tau^+$ ($\tau^-$) decays at $t = 0$. They concluded that the sum of time integrated rates (from $t = 0$ to $\infty$) over all the final states that can be reached by neutral kaon decays is free from such $CP$ asymmetries, restoring the equality of the $\tau^\pm$ lifetimes. In their discussion of the problem [1], the interference of the $K_{S,L}$ states in the time-dependent rates plays an essential role in the cancellation of the contributions of pure $K_L$ and $K_S$ exponential decays.

In this paper we approach this problem from a different point of view. Based on the description of unstable $K_{L,S}$ particles in quantum field theory (QFT), we compute the time evolution of transition amplitudes between physical (in and out) states. Let us point out that, in the evaluation of the matrix elements giving rise to the rates that enter Eqs. (1,2), neutral kaons ($K_{L,S}$) are assumed to be asymptotic physical states (defined as outgoing states at $t \rightarrow +\infty$). Since unstable particles in QFT enter as intermediate states of the physical amplitudes, the paradox contained in Eqs. (1,2) does not appear. We compute the time-dependent probabilities for transitions between asymptotic states in $\tau^\pm \rightarrow [\pi^+\pi^-]_K\pi^\pm \nu$, where $[\pi^+\pi^-]_K$ is the pair produced from neutral kaon decays. Then we propose a definition of $\tau$ decays into $K_{L,S}$ states, which reflects into a natural cancellation between the $CP$ rate asymmetries defined in Eqs. (1) and (2).

Searches for $CP$ violation effects in a double kinematical distribution of $\tau^\pm \rightarrow K_S\pi^\pm \nu$ decays have been pursued recently by the CLEO Collaboration [2]. Prospects for improved experimental searches are interesting in the light...
Similarly, we can obtain the corresponding time-dependent amplitude for the \( \pi^+ \pi^- \) pair at time \( t = \tau \).

of the larger data samples of \( \tau \) pairs accumulated at \( B \)-factories [3]. These exclusive decays can be used to provide further tests on the violation of the CP symmetry [1, 2, 4, 5]. On the other hand, within the standard model, the CP rate asymmetry turns out to be negligibly small (of order \( 10^{-12} \)) in \( \tau^{\pm} \to K^{\pm} \pi^0 \nu_\tau \) decays [3], opening a large window to consider the effects of New Physics contributions. Indeed, virtual effects of supersymmetric particles may enhance this CP rate asymmetry to the level of \( 10^{-7} \sim 10^{-6} \) [6].

II. TIME-DEPENDENT AMPLITUDES IN \( \tau \) LEPTON DECAYS

In this section we focus on calculation of the time evolution amplitudes of \( \tau \) lepton decays into asymptotic physical states, as dictated by the S-matrix formalism of QFT.

Let us start by defining, at time \( t = 0 \), the \( \tau \) lepton decays into virtual strange states of definite flavor (\( \tau^{+} \to K^{0} \pi^+ \nu \) or \( \tau^{-} \to K^{0} \pi^- \bar{\nu} \)). The decay amplitudes of these two processes are the same owing to the CPT invariance of weak interactions, and will be denoted by \( A \). After being produced, initial kaon states of definite flavor evolve as a mixing of two orthogonal states of definite mass and width (\( K_L \) and \( K_S \)) and decay a later time \( t = \tau \) into a final state \( f \). For definiteness we will confine ourselves to \( f = \pi^+ \pi^- \) (see Fig. 1).

In order to introduce the CP violating effects, we change from the basis of definite flavor (\( K^0, \bar{K}^0 \)) to the basis of definite CP parity (\( K_1, K_2 \)) of neutral kaons:

\[
\begin{pmatrix}
K_1 \\
K_2
\end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix}
1 & 1 \\
1 & -1
\end{pmatrix} \begin{pmatrix}
K^0 \\
\bar{K}^0
\end{pmatrix}
\]

with the convention \( CP|K^0 = |\bar{K}^0 \rangle \).

In momentum space, the transition amplitudes for the \( K_1 - K_2 \) system propagating with momentum \( p \) from \( t = 0 \) to \( t = \tau \), are given by (see for example [7]):

\[
D_{K_1 K_2}^{K^0}(p^2) = \frac{1}{1 - \epsilon^2} \begin{pmatrix} 1 & \epsilon \\ \epsilon & 1 \end{pmatrix} \begin{pmatrix} d_s^{-1} & 0 \\ 0 & d_L^{-1} \end{pmatrix} \begin{pmatrix} 1 & -\epsilon \\ -\epsilon & 1 \end{pmatrix},
\]

where \( d_{S,L} \equiv p^2 - M_{S,L}^2 + i m_{S,L} \Gamma_{S,L} \). Equivalently, the matrix \( D(p^2) = \text{diag}(d_{S}^{-1}, d_{L}^{-1}) \) describes the propagation of the two independent physical modes of definite mass (\( M_{L,S} \)) and width (\( \Gamma_{L,S} \)).

Following the procedure discussed in Ref. [7], we compute now the time-dependent transition amplitude of the whole \( \tau^{-}(q) \to K^0(p_1 + p_2; t = 0) \pi^-(k)\nu(q') \to \pi^+(p_1)\pi^-(p_2)\pi^-\nu(q') \) decay process. The pair \( \pi^+(p_1)\pi^-(p_2) \) is the product of kaon decay at time \( t = \tau \), thus we define \( E = p_1^2 + p_2^2 \) as the total energy of the \( \pi^+ \pi^- \) pair and \( E_{S,L} = \sqrt{(p_1^2 + p_2^2)^2 + m_{S,L}^2} \). The time-dependent amplitude becomes:

\[
T_-(\tau) = \frac{1}{2} \delta^{(4)}(q - k - q' - p_1 - p_2) A \frac{1}{1 - \epsilon} \{ M(K_1 \to \pi^+ \pi^-) e^{iE \tau} \\
\times \left\{ \frac{1 + \chi + \epsilon}{2E_S} e^{-iE_{S\tau}} e^{-\frac{\Gamma_S}{2} \frac{m_{S}^2}{E_S}} - \frac{\epsilon + \chi - \epsilon}{2E_L} e^{-iE_{L\tau}} e^{-\frac{\Gamma_L}{2} \frac{m_{L}^2}{E_L}} \right\}. \]

Similarly, we can obtain the corresponding time-dependent amplitude for the \( \tau^+(q) \to K^0(p_1 + p_2; t = 0) \pi^+(k)\bar{\nu}(q') \to \pi^+(p_1)\pi^-\pi^+(p_2)\pi^+\bar{\nu}(q') \) decay:

\[
T_+(\tau) = \frac{1}{2} \delta^{(4)}(q - k - q' - p_1 - p_2) A \frac{1}{1 + \epsilon} \{ M(K_1 \to \pi^+ \pi^-) e^{iE \tau} \\
\times \left\{ \frac{1 + \chi - \epsilon}{2E_S} e^{-iE_{S\tau}} e^{-\frac{\Gamma_S}{2} \frac{m_{S}^2}{E_S}} + \frac{\epsilon + \chi + \epsilon}{2E_L} e^{-iE_{L\tau}} e^{-\frac{\Gamma_L}{2} \frac{m_{L}^2}{E_L}} \right\}. \]
In the above expressions we have defined the direct CP violation parameter

$$\chi_{+-} \equiv \frac{\mathcal{M}(K_2 \rightarrow \pi^+\pi^-)}{\mathcal{M}(K_1 \rightarrow \pi^+\pi^-)},$$

(7)

and $\mathcal{M}(K_{1,2} \rightarrow \pi^+\pi^-)$ denote the ‘instantaneous’ decay amplitudes (at $t = \tau$) of the CP eigenstates into a $\pi^+\pi^-$ pair. Note that in Eqs. (5,6) we do not need to symmetrize the decay amplitudes because identical pions are produced at different space-time locations.

For further use in the following discussion we introduce the usual CP violation parameter

$$\eta_{+-} = \frac{\epsilon + \chi_{+-}}{1 + \chi_{+-}} e^{-i\phi_{+-}}.$$  

(8)

The time-dependent amplitudes given in Eqs. (5,6) are defined in an arbitrary reference frame [7]. Now, we choose the center of mass frame of the pion pair produced at $t = \tau$, which means $E_{S,L} = M_{S,L}$. The expressions for the time-dependent probabilities (up to the second order in the CP violation parameter and neglecting direct CP violation, i.e. $\chi_{+-} = 0$) are given by:

$$|T_-(\tau)|^2 \simeq \frac{B(1 + 2Re[\epsilon])}{4M_S^2}[e^{-\Gamma_S\tau} + |\epsilon|^2e^{-\Gamma_L\tau} - 2|\epsilon|e^{-\frac{1}{2}(\Gamma_S+\Gamma_L)\tau}\cos(\Delta m\tau - \phi_{+-})]$$

(9)

$$|T_+(\tau)|^2 \simeq \frac{B(1 - 2Re[\epsilon])}{4M_S^2}[e^{-\Gamma_S\tau} + |\epsilon|^2e^{-\Gamma_L\tau} + 2|\epsilon|e^{-\frac{1}{2}(\Gamma_S+\Gamma_L)\tau}\cos(\Delta m\tau - \phi_{+-})],$$

(10)

where we have defined the common factor $B = (2\pi)^4 g_0(q + q' - k - k' - p_1 - p_2) |\mathcal{A}(K_1 \rightarrow \pi^+\pi^-) e^{iE_\tau}|^2$ and $\Delta m = M_L - M_S$. Such expressions for the time-dependent probabilities are similar to the ones obtained in the framework of the usual Lee-Oehme-Yang formalism [8] and used in the data analysis of the CPLEAR experiment [9].

With the above expressions we can calculate the time-dependent CP rate asymmetry for the processes under consideration. We get the result:

$$A_{+-}(\tau) = \frac{|T_-(\tau)|^2 - |T_+(\tau)|^2}{|T_-(\tau)|^2 + |T_+(\tau)|^2} \simeq 2Re[\epsilon] \left[ \frac{\cos\phi_{+-} e^{-1/2(\Gamma_S+\Gamma_L)\tau}\cos(\Delta m\tau - \phi_{+-}) + e^{-\Gamma_S\tau} + |\epsilon|^2e^{-\Gamma_L\tau}}{e^{-\Gamma_S\tau} + |\epsilon|^2e^{-\Gamma_L\tau}} \right].$$

(11)

Note that this CP asymmetry, defined for an arbitrary decay time $\tau$ of neutral kaons, does not vanish.

III. A DEFINITION FOR THE $K_{L,S}$ STATES

Strictly speaking, within the S-matrix formalism we can not define physical amplitudes for final states containing unstable particles (they are not asymptotic states). Instead, unstable particles appear as intermediate states of the S-matrix amplitude connecting the production and decay processes. However, we can resort to a definition of $\tau^\pm \rightarrow K_{L,S}\pi^\pm\nu$ decay rates by introducing a time scale $T$ which allows to separate neutral kaon decays that occur at short and long decay times.

Thus, in order to define the CP asymmetry for $\tau$ decays into an specific $K$ meson final state, one has first to adopt a definition of the amplitude with $K_{L,S}$ states. Due to the large difference between $K_L$ ($t_L$) and $K_S$ ($t_S$) lifetimes, it is possible to adopt a procedure to separate ‘$K_L$’ and ‘$K_S$’ events. We introduce a time scale $T$ such that $t_S \ll T \ll t_L$. In this way, decays of neutral kaons that occur for $t \sim O(t_S)$ can be identified as $K_S$ events, while those taking place for $t \sim O(t_L)$ would be identified as $K_L$ events. As is usually done, to talk about a beam of $K_L$ mesons we should look at $\pi^+\pi^-$ pairs produced after a time $t \geq T$. Inversely, at short decay times ($t \leq T$), our $\pi^+\pi^-$ pairs will originate mainly from decays of the $K_S$ component.

Thus, in analogy with Ref. [1], we can define the CP rate asymmetry of $\tau$ leptons into $K_{S,L}\pi$ final states from our Eq. (11). The CP rate asymmetries for $\tau$ decays into $K_S$ (respectively $K_L$) become:
\[ A_{CP}^S \approx \int_0^T \frac{(-2|\epsilon|e^{-1/2(\Gamma_S+\Gamma_L)\tau})\cos(\Delta m\tau + \phi_{\pm}) + 2Re[\epsilon]e^{-\Gamma_S\tau}}{e^{-\Gamma_S\tau}} d\tau, \]  
\[ A_{CP}^L \approx \int_T^\infty \frac{2Re[\epsilon]e^{-\Gamma_L\tau}}{e^{-\Gamma_L\tau}} d\tau, \]

where we have used the approximations \( \Gamma_S \gg \Gamma_L \) and \( ts \ll T \ll tl \). A straightforward evaluation of the CP asymmetries from Eqs. (11,12) gives:

\[ A_{CP}^L \approx -A_{CP}^S \approx 2Re[\epsilon]. \]  

In other words, the CP asymmetry integrated over all decay times for a given physical final states does not exhibit the paradox discussed in the Introduction. Furthermore, it comes out that the ‘experimentally motivated’ definitions of the CP asymmetries for \( \tau^\pm \) decays into \( K_L \) and \( K_S \) proposed above cancel each other.

IV. CONCLUSIONS

In the framework of the S-matrix formalism of QFT, we compute the time evolution amplitudes for \( \tau^\pm \rightarrow [\pi^+\pi^-]_K\pi^\pm\nu_\tau \) decays, where \([\pi^+\pi^-]_K\) denote the decay products of a unstable neutral kaon. By introducing a definition of decay rates for short and long decay times of neutral unstable kaons, we show that the integrated CP rate asymmetries of these decays cancels each other in a natural way. Therefore, in the description of unstable states based on the S-matrix formalism there is not a paradox like the one described in Ref. [1]. Since our definition of \( K_{L,S} \) final states in \( \tau \) decays is motivated on experimental grounds, our predictions for the integrated CP rate asymmetries (Eqs. (14)) can be tested in a dedicated experiment.

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