Quantum theory of complete and incomplete fusion in collisions of weakly bound nuclei

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We propose a new quantum mechanical method to evaluate complete and incomplete fusion in collisions of weakly bound nuclei. The method is applied to the $^7\text{Li} + ^{209}\text{Bi}$ system and the results are compared to experimental data. The overall agreement between theory and experiment is very good, above and below the Coulomb barrier.

Nuclear reactions with weakly bound projectiles is one of the main research topics in low energy Nuclear Physics [1-6]. Owing to the low breakup threshold, all reaction channels are influenced by the continuum, and fusion is particularly affected. Besides direct complete fusion (DCF), where the whole projectile merges with the target, there are fusion processes following breakup. There is incomplete fusion (ICF), when only non of the breakup fragments fuses with the target, and sequential complete fusion (SCF), when all fragments are sequentially absorbed by the target. The sum DCF + SCF is called complete fusion (CF), and the sum of all fusion processes, that is DCF + SCF + ICF, is called total fusion (TF).

Determining CF and ICF cross sections has been a great challenge for both experimentalists and theorists. Most experiments determine only TF cross sections. However, individual CF and ICF measurements have been performed for a few particular projectile-target combinations. Dasgupta et al. measured CF and ICF cross sections in collisions of $^6,^7\text{Li}$ projectiles on $^{209}\text{Bi}$ [7, 8] and of $^9\text{Be}$ projectiles on $^{208}\text{Pb}$ [8, 9].

Similar experiments have been performed to study collisions of $^6,^7\text{Li}$ projectiles on $^{159}\text{Ta}$ [10, 12], $^{144,152}\text{Sm}$ [13, 15], $^{165}\text{Ho}$ [16], $^{198}\text{Pt}$ [17, 18], $^{154}\text{Sm}$ [19], $^{96}\text{Zr}$ [20], $^{124}\text{Sn}$ [21], and $^{197}\text{Au}$ [22] targets, and $^9\text{Be}$ projectile on $^{89}\text{Y}$ [23], $^{124}\text{Sn}$ [24], $^{144}\text{Sm}$ [25, 26], $^{186}\text{W}$ [27], $^{169}\text{Tm}$ [28], $^{187}\text{Re}$ [29], $^{181}\text{Ta}$ [29], and $^{209}\text{Bi}$ [30] targets. Very recently, Cook et al. [31] performed an experiment to investigate the origin of the suppression of CF in collisions of $^7\text{Li}$.

The first CF and ICF calculations in collisions of weakly bound nuclei were based on classical mechanics [7, 32, 33]. Although they were able to reproduce the main trends of the data, they missed important features of the fusion process, like tunnelling effects. Such effects were approximately taken into account in semi-classical [36, 37] models, which, however, were not fully satisfactory. More realistic quantum mechanical calculations, based on the continuum discretized coupled channel (CDCC) method, have also been developed. However, most calculations provided only the TF cross section [38-41]. The exceptions are the works of Hagino et al. [42] and Diaz-Torres and Thompson [43], which evaluated CF and TF in collisions of $^{11}\text{Be}$ ($=^{10}\text{Be} + n$) on $^{208}\text{Pb}$. In their method, the TF cross section was written as a sum of radial integral of the imaginary potential, in the subspace of bound channels and in the subspace of bins (continuum discretized states). The former and the latter were then associated with CF and ICF, respectively. Since their imaginary potential depended exclusively on the projectile-target distance, cross terms between bound channels and bins vanished identically. This method is nice but it can only be used for projectiles like $^{11}\text{Be}$, which break up into a heavy charged fragment and a light uncharged one. It relies on the assumption that the center of mass of the projectile is close to that of the heavy fragment, and far from the light one. Thus, it cannot be used for projectiles like $^7\text{Li}$, that breaks up into two fragments of comparable masses. In Ref. [44] V. V. Parkar et al. proposed an approximation for the calculation of CF, ICF, and TF. The method consisted of performing separate CDCC calculations including different fragment - target short-range and projectile-target potentials to determine the cross-sections. Although the method has some success in describing the individual experimental cross-sections for the $^6,^7\text{Li} + ^{209}\text{Bi}$, $^{198}\text{Pt}$ fusion reactions, it has the inconsistency that not all the fusion cross sections are determined from the same CDCC calculation. Very recently, Lei and Moro [45] implemented a quantum mechanical approach based on the spectator-participant inclusive breakup model of Ichimura, Austern and Vincent [46]. In their work, the CF cross section was indirectly determined subtracting from the total reaction cross section the contributions from inelastic scattering, elastic breakup and inclusive non-elastic breakup. The method was applied to the $^6,^7\text{Li} + ^{209}\text{Bi}$ systems and the results were shown to describe very well the experimental CF cross sections above the Coulomb barrier. Although the work of Lei and Moro are a significant ad-
vance on the existing theories, there is considerable room for improvements. One important point is that in this approach the ICF cross section is contained in the inclusive non-elastic breakup cross section. Thus, it cannot be evaluated individually. Boselli and Diaz-Torres [17] proposed a quantum-mechanical model to describe the time-evolution of wave packets. The method was used to study collisions of tightly bound systems [38, 39] and also to estimate CF and ICF cross sections for the weakly bound $^7$Li + $^{209}$Bi system. The method is promising but so far it has not been used in realistic calculations involving weakly bound projectiles.

In this letter, we propose a new quantum mechanical method to derive CF and ICF (and hence TF) cross sections in collisions of any weakly bound projectile. The method is applied to the $^7$Li + $^{209}$Bi system and the results are compared to the data of Dasgupta et al. [78].

We consider the collision of a weakly bound projectile formed by two fragments, $c_1$ and $c_2$, on a spherical target. The projectile-target relative vector and the vector between the two fragments of the projectile are denoted by $R$ and $r$, respectively. The fragments interact with the target through the complex potentials, $\Psi^{(i)}(r_i) = U^{(i)}(r_i) - iW^{(i)}(r_i)$, with $i = 1, 2$, where $r_i$ is the distance from the centers of fragment $c_i$ and the target. As in Refs. [42, 43], we start from the expression for the TF cross section within the CDCC approach,

$$\sigma_{\text{TF}} = \frac{1}{|A|^2} \frac{k}{E} \langle \Psi^{(+)} | W | \Psi^{(+)} \rangle \equiv \frac{1}{|A|^2} \frac{k}{E} \sum_{\alpha,\alpha'} \langle \psi_{\alpha} | W_{\alpha,\alpha'} | \psi_{\alpha'} \rangle,$$

where the indices $\alpha$ and $\alpha'$ run over bound channels and bins. Above, $\psi_{\alpha}(R)$ is the projectile-target relative wave function in channel $\alpha$, and $W_{\alpha,\alpha'}(R)$ is the matrix-elements of the imaginary potential between channels $\alpha$ and $\alpha'$.

Refs. [42, 43] adopted a short-range imaginary potential, $W(R)$, that acts on the center of mass of the projectile, independently of it being bound or unbound. Since it does not depend on $r$, its matrix-elements are diagonal in channel space. In this way, the TF cross section of Eq. (1) reduces to a sum of independent contributions from the channel. Hagino et al. [12] then assigned the contributions from bound channels to CF, and those of unbound channels to ICF. However, their choice of the imaginary potential does not allow for the individual absorption of one of the fragments. To avoid this problem, we adopt the imaginary potential $W(R,r) = W^{(1)}(r_1) + W^{(2)}(r_2)$, which is not diagonal in channel space. Then we keep off-diagonal matrix-elements within the same subspace (bound (B) or continuum (C)), but neglect off-diagonal matrix-elements between channels in different subspaces. The sum of Eq. (1) then splits as, $\sigma_{\text{TF}} = \sigma_{\text{TF}}^B + \sigma_{\text{TF}}^C$. As in Refs. [42, 43], we make the assumption:

$$\sigma_{\text{DCF}} = \sigma_{\text{TF}}^B.$$

The physical meaning of $\sigma_{\text{TF}}^B$ requires further discussion. Performing proper angular momentum expansions of the wave functions and the imaginary potentials, $\sigma_{\text{TF}}^B$ can be put in the form

$$\sigma_{\text{TF}}^B = \frac{\pi}{k^2} \sum_J (2J+1) \left[ \mathcal{P}^{(1)}(J) + \mathcal{P}^{(2)}(J) \right].$$

Above, $\mathcal{P}^{(i)}(J)$ is the probability of absorption of fragment $c_i$ in a collision with total angular momentum $J$. It corresponds to the contribution of the potential $W^{(i)}$ to the $J$-expanded version of Eq. (1), with $\alpha$ and $\alpha'$ constrained to channel in the continuum.

The ICF and the SCF cross sections can be determined using the probabilities $\mathcal{P}^{(1)}(J)$ and $\mathcal{P}^{(2)}(J)$. We follow the procedure of the semiclassical calculations of Ref. [57], writing for the ICF probabilities for fragments $c_1$ (ICF1) and $c_2$ (ICF2) as,

$$\mathcal{P}^{\text{ICF1}}(J) = \mathcal{P}^{(1)}(J) \times [1 - \mathcal{P}^{(2)}(J)],$$

$$\mathcal{P}^{\text{ICF2}}(J) = \mathcal{P}^{(2)}(J) \times [1 - \mathcal{P}^{(1)}(J)].$$

The ICF cross sections of fragment $c_i$ is then given by

$$\sigma_{\text{ICFi}} = \frac{\pi}{k^2} \sum_J (2J+1) \left[ 2 \mathcal{P}^{(1)}(J) \times \mathcal{P}^{(2)}(J) \right],$$

and the total ICF cross section is: $\sigma_{\text{ICF}} = \sigma_{\text{ICF1}} + \sigma_{\text{ICF2}}$.

Finally, the SCF cross section can be obtained subtracting $\sigma_{\text{ICF}}$ from $\sigma_{\text{TF}}^B$. One gets

$$\sigma_{\text{SCF}} = \frac{\pi}{k^2} \sum_J (2J+1) \left[ 2 \mathcal{P}^{(1)}(J) \times \mathcal{P}^{(2)}(J) \right].$$

Then, the CF cross section is $\sigma_{\text{CF}} = \sigma_{\text{DCF}} + \sigma_{\text{SCF}}$, with the DCF and SCF cross sections given respectively by Eqs. (2) and (7).

Our method was used to evaluate CF and TF cross sections for the $^7$Li + $^{209}$Bi system, which have been measured by the ANL group [78] at near-barrier energies. For this purpose, we wrote the CF-ICF computer code (unpublished), based on the angular momentum projected version of the above equations, using intrinsic and radial wave functions extracted from the CDCC version of the FRESCO code [53]. $^7$Li is a weakly bound projectile, which breaks up into $^3$H and $^4$He, with the breakup threshold of 2.47 MeV.

For the real parts of the interaction between the fragments and the target, $\Psi^{(1)}(r_1)$ and $\Psi^{(2)}(r_2)$, we used the São Paulo potential [54, 55] (SPP). The
FIG. 1. (Color on line) Calculated TF and CF cross sections for the $^7$Li + $^{209}$Bi system at near-barrier energies, in comparison with the data of Refs. [7, 8] (blue solid circles and green triangles). The results are shown in logarithmic (panel (a)) and linear (panel (b)) scales.

Coulomb barrier, given by the maximum of the potential $V(R) = \langle \phi_0 | V(1) + V(2) | \phi_0 \rangle$, is $V_B = 28.2$ MeV. Owing to the low binding energy of the fragments in $^7$Li, the barrier is 1.2 MeV lower than the one of the SPP for the $^7$Li + $^{209}$Bi system, where this property is ignored.

For the imaginary parts, we adopted typical short-range strong absorption potentials. We took Woods-Saxon functions with the depths, radii and diffusenesses: $W_0 = 50$ MeV, $R_w = 1.0 \left[ A_i^{1/3} + A_T^{1/3} \right]$ fm, where $A_i$ and $A_T$ are respectively the mass numbers of $c_i$ and the target, and $a_w = 0.2$ fm. The continuum discretization was carried out with bins of orbital angular momenta up to $l_{\text{max}} = 4 \hbar$ and energies up to $\varepsilon_{\text{max}} = 8$ MeV. For the $l = 0, 1, 2$ and $4\hbar$ we used bins of constant width, with density of 1 bin/MeV. At $l = 3\hbar$, we modified the mesh to account for the resonances at 2.2 and 4.2 MeV. Around these energies the bins were much sharper, with densities up to 10 bins/MeV. We made sure that the continuum discretization with these parameters guaranteed good convergence in our calculations.

Fig. 1 shows the TF and the CF cross sections predicted by our method in comparison with the data of Refs. [7, 8]. At sub-barrier energies and above-barrier energies up to $E_{\text{c.m.}} \sim 35$ MeV, the agreement between the calculated CF cross section and the data is excellent. Above 35 MeV, the agreement is slightly poorer. The calculations underestimate the experimental CF cross section by $\sim 10\%$.

The agreement between the theoretical TF cross section and the data below 35 MeV is also excellent. Above this energy, the theoretical cross section exceeds the experimental results by up to $\sim 15\%$. In fact, the agreement in this case may be better. The authors of the experiment pointed out that their ICF data above 36 MeV should be considered a lower bound to the actual cross section. The reason is that the contribution from $^{209}$Po, which is in the decay chain of the compound nuclei produced in both ICF processes ($^{213}$At and $^{212}$Po), cannot be measured owing to its long half life ($t_{1/2} = 102$ y), and estimates made with the code PACE [56] indicate that this channel becomes important above $\sim 36$ MeV. Since $\sigma_{\text{TF}}$ contains $\sigma_{\text{ICF}}$, the calculated TF cross section is consistent with the data. It is interesting to compare also the ICF cross section predicted by our method and the data. This is done in Fig. 2. Clearly, the agreement between theory and experiment is excellent up to $\sim 33 - 34$ MeV, and at higher energies the calculated cross section is larger than the data.

The above comparison between theory and experiment indicates that the data of Refs. [7, 8] can be very well described by our CDCC-based model, which considers only inelastic channels and breakup. At first sight, it seems to disagree with Ref. [31], which concluded that the main process contributing to the ICF data of Refs. [7, 8] is direct triton stripping. But, in fact,
we do not think there is any contradiction. We made coupled reaction channel calculations to estimate the triton-stripping cross section to bound states in $^{212}$Po and obtained cross sections orders of magnitude lower than the data. Thus, the stripping processes considered in Ref. [31] must be mainly to the continuum of $^{212}$Po and, as has been pointed out in Ref. [52], breakup can be though of as transfer to the continuum of the target.

The influence of breakup couplings on fusion can be better understood comparing the CF, ICF and TF cross sections of our CF-ICF calculation with the one obtained with a one-channel calculation, where all couplings are switched off. This is done in Fig. 3. The behavior of the cross sections above the Coulomb barrier can be observed more clearly in the linear plot of panel (b). The CF cross section is strongly suppressed with respect to the fusion cross section of the one-channel calculation. On the other hand, the TF cross section can hardly be distinguished from the cross section of the one-channel calculation. This indicates that breakup couplings split the one-channel cross section into CF and ICF components, keeping their sum practically unchanged. The behavior of the cross sections at sub-barrier energies can be observed in the logarithmic plot of panel (a). There is still some suppression of CF, but it decreases with the collision energy. On the other hand, the TF cross section below the Coulomb barrier is strongly enhanced. One sees that in this energy range the TF cross section is dominated by the ICF process. This is not surprising since at low energies the transmission coefficients for the lighter fragments must be much larger than that for the whole projectile.

We have proposed a new method to evaluate CF and ICF cross sections in collisions of weakly bound projectiles. Our method has the advantages of being based on full quantum mechanics and being applicable to any weakly bound projectile that breaks up into two fragments. As an example, the method was used to calculate CF, ICF and TF cross sections in the $^7$Li + $^{209}$Bi collision, and the results were compared with experimental data. Considering that our calculations are based on standard heavy ion potentials and that it contains no free parameter, the overall agreement between theory and experiment is extremely good, above and below the Coulomb barrier.

Although in its present stage our method is restricted to projectiles composed of two fragments, it can be generalized to projectiles that break up into three fragments, like $^{9}$Be.

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[1] L. F. Canto, P. R. S. Gomes, R. Donangelo, and M. S. Hussein, Phys. Rep. 424, 1 (2006).

[2] L. F. Canto, P. R. S. Gomes, R. Donangelo, J. Lubian, and M. S. Hussein, Phys. Rep. 596, 1 (2015).
[49] A. Diaz-Torres and M. Wiescher, Phys. Rev. C 97, 055802 (2018).
[50] L. F. Canto and M. S. Hussein, *Scattering Theory of Molecules, Atoms and Nuclei* (World Scientific Publishing Co. Pte. Ltd., 2013).
[51] G. R. Satchler, M. A. Nagarajan, J. S. Liley, and I. J. Thompson, Ann. Phys. (NY) 178, 110 (1987).
[52] G. Potel, F. M. Nunes, and I. J. Thompson, Phys. Rev. C 92, 034611 (2015).
[53] I. J. Thompson, Comput. Phys. Rep. 7, 167 (1988).
[54] L. C. Chamon, D. Pereira, M. S. Hussein, M. A. Cândido Ribeiro, and D. Galetti, Phys. Rev. Lett. 79, 5218 (1997).
[55] L. C. Chamon, B. V. Carlson, L. R. Gasques, D. Pereira, C. De Conti, M. A. G. Alvarez, M. S. Hussein, M. A. Cândido Ribeiro, E. S. Rossi Jr., and C. P. Silva, Phys. Rev. C 66, 014610 (2002).
[56] A. Gavron, Phys. Rev. C21, 230 (1980).