Robust iterative learning control for uncertain continuous-time system with input delay and random iteration-varying uncertainties

Hamid Shokri-Ghaleh | Soheil Ganjefar | Alireza Mohammad Shahri

School of Electrical Engineering, Iran University of Science and Technology, Tehran 1684613114, Iran

Correspondence
Soheil Ganjefar, School of Electrical Engineering, Iran University of Science and Technology, Tehran 1684613114, Iran.
Email: s_ganjefar@iust.ac.ir

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Abstract
This study deals with the problem of robust iterative learning control (ILC) for linear continuous-time systems with input delay subject to uncertainties in input delay, plant dynamic, reference trajectory, initial conditions and disturbances. Using the internal model control (IMC) structure in the frequency domain, an ILC scheme is proposed in which the IMC structure is responsible for coping with uncertainties in both delay time and plant dynamic. Sufficient conditions are derived to ensure that the tracking error expectation is bounded and converges monotonically to a small neighbourhood of zero (in the \(L_2\)-norm sense) when uncertainties in reference trajectory, initial conditions and disturbances vary randomly from trial to trial. It is shown that the derived conditions are still valid to guarantee both boundedness and monotonic convergence of the tracking error variance (in the \(L_2\)-norm sense). Illustrative examples are provided to demonstrate the effectiveness of the proposed method.

1 | INTRODUCTION

Iterative learning control (ILC), which is categorized as a well-established intelligent control approach [1], is an effective method that is implemented over-and-over on a finite time interval to improve the tracking accuracy of dynamical systems. It updates the current control signal by incorporating the control signal from previous iterations and some correction terms. For more explanations of ILC and its application areas, for example, see [1–6], where various ILC algorithms have been designed in both discrete- and continuous-time domains using the time- or frequency domain approach. We also refer readers to [7] for a survey on the synthesis and convergence analysis of stochastic ILC.

Because uncertainties are inevitable, especially in engineering applications, several ILC methods have been presented in the area of uncertain systems, for example, see [8–11]. However, the ILC designs in [8–11] depend on the assumption that some factors such as reference trajectory, initial conditions and disturbances should be strictly iteration-invariant, which does not match the ‘practical nature’ of ILC [12]. Therefore, over the past years, many studies have been devoted to deal with the issue of iteration-varying (nonrepetitive) uncertainties in ILC designs, for example, see [12–20]. Despite these significant results, all of them have focused on systems without time-delay.

Note that the time-delay phenomenon in control systems may degrade the control performance and even give rise to instability [21]. Consequently, many results have been devoted to the stabilization problem of systems with input-delay [22, 23], state-delay [24–26] and a detailed review can be found in [27]. Due to that ILC has been found as an effective method to improve the tracking performance of uncertain systems; ILC techniques for uncertain time-delay systems have also drawn much attention over the past years (see, e.g. [28–31] for ILC designs under the assumption of iteration-invariant uncertainties, [6, 32–37] for overcoming the ill influence of iteration-varying uncertainties in initial conditions and [6, 21, 33, 34, 36, 38] for attenuating nonrepetitive disturbances).

For most existing control applications, the reference trajectory is precisely available a priori. However, there are also some practical applications in which we encounter uncertainty in the reference trajectory (see, e.g. [39, 40] for maximum power
extraction from a wind turbine/solar Photovoltaic (PV) system, [41] for consensus tracking of Multi-agent systems when only some agents (leaders/masters) can acquire the exact information of the reference trajectory. As argued in [16, 42, 43], if there is a change in the reference trajectory, no matter how small it might be, the traditional ILC control systems will have to start the learning from the very beginning. Nevertheless, unlike free delay systems, in the field of ILC methods for time-delay systems subject to iteration-varying uncertainties in the reference trajectory, only one result is available [44], which has been devoted to systems with state delay. To our knowledge, no results have been reported on this issue for systems with input delay yet. It is worth pointing out that input delay is often encountered in practical applications because of time-consuming information processing and transport phenomena [21, 23].

Internal Model Control (IMC) is well-known in industrial process control applications due to its ability to provide a suitable trade-off between the closed-loop system robust stability and control performance [45, 46]. Using the IMC structure in the frequency domain, two ILC schemes were proposed in [29, 30] for uncertain batch processes with input delay. However, the IMC-based ILC schemes studied in [29, 30] require that the reference trajectory, initial conditions and disturbances be strictly invariant for all iterations.

The main features of this study and its contribution relative to the related works are highlighted as follows:

1. This study is the first attempt in the ILC literature to extend the problem of robust ILC design for the systems with input delay subject to iteration-varying uncertainties in the reference trajectory.

2. Using the indirect-type ILC method [47], an ILC scheme is proposed based on IMC structure in the frequency domain, in which the IMC structure is responsible for ensuring the stability of the feedback part and coping with uncertainties in both input delay and plant dynamic. However, in comparison with [29, 30], we relax the restrictive assumptions of iteration-invariant reference trajectory, initial conditions and disturbances in our scheme and consider their variation from a random viewpoint. According to works [12, 48, 49], to reflect the effect of random iteration-varying uncertainties arising from random iteration-varying uncertainties in both initial conditions and disturbances, an additional term (a random iteration-varying exogenous signal) is considered at the system output described by frequency domain. Moreover, to provide a good compromise between zero tracking and robust stability at the interested frequency regions when uncertainties in the reference trajectory and exogenous signal vary randomly from trial to trial, a learning weighting function is considered in the ILC part.

3. The convergence analysis of expectation/variance of tracking error is performed. It is shown that the same convergence conditions can guarantee both boundedness and monotonic convergence of both the expectation and variance of the tracking error (in the $L_2$-norm sense).

For clarity, this manuscript is organised as follows. In Section 2, the problem formulation is discussed. The proposed ILC scheme is introduced in Section 3. Section 4 presents the convergence analysis of both the expectation and variance of tracking error. Illustrative examples are given in Section 5. Finally, we draw our conclusions in Section 6.

Notations: $\mathbb{N} = \{1, 2, 3, \ldots\}$; $\mathbb{Z} = \{0, 1, 2, \ldots\}$; $\ell[*]$ and $\ell^{-1}[*]$ are used for the Laplace operator and the inverse Laplace operator, respectively; The $2$-norm and $\infty$-norm of the transfer function $X$ are given by $\|X\|_2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$ and $\|X\|_{\infty} = \sup_{\omega} |X(j\omega)|$, respectively; $\deg(X)$ represents the numerator/denominator degree of the transfer function $X$; The $\mathcal{RH}_\infty$ space describes all proper and real rational stable transfer functions; $I_n$ denotes the identity matrix of size $n$; $\mathbb{E}[X_k(i)]$ and $\var[X_k(i)]$ represent the expectation and variance of signal $X(i)$ for a fixed $i$ over the independent identically distributed (i.i.d) signal samples at iteration $k$, respectively.

### 2 PROBLEM FORMULATION

Consider the single-input single-output (SISO) time-delay system shown in Figure 1, where $k \in \mathbb{Z}$ is the iteration index; $Y_k(i) = \ell[y_k(i)]$ is the system output; $U_k(i) = \ell[u_k(i)]$ is the control input; $\Psi_k(i) = \ell[\Psi_k(i)]$ is a random iteration-varying exogenous signal; and the plant $P(s)$ is composed of a rational transfer function $G(s)$ with uncertain real coefficients and an uncertain delay time $\Theta$

$$P(i) = G(i)e^{-\Theta i}. \quad (1)$$

Then in the frequency domain, the system output $Y_k(i)$ is given by

$$Y_k(i) = P(i)U_k(i) + \Psi_k(i), \forall k \in \mathbb{Z}. \quad (2)$$

Note that, in the frequency domain, the delay time $\Theta$ in (1) can represent the input delay or the output response delay [27, 29, 30].
Remark 1. The random iteration-varying exogenous signal \( \Psi_k(t) \) in (2) can reflect the effect of random iteration-varying initial conditions and random iteration-varying disturbances simultaneously during the learning process (see also [12, 48, 49]). For more clarity, let us consider a SISO continuous-time delay system with the following state-space (ss) representation:

\[
\begin{align*}
\dot{x}_k(t) &= A_{st} x_k(t) + B_{st} v_k(t) + w_k(t) , \\
y_k(t) &= C_{st} x_k(t) + r_k(t) , \\
x_k(0) &= x_{0k} ,
\end{align*}
\]

where \( t \in [0,T] \) is the continuous-time index running from \( t = 0 \) to \( t = T \); \( n_k(t) \in \mathbb{R}, y_k(t) \in \mathbb{R} \) and \( x_k(t) \in \mathbb{R}^n \) are the control input, system output and state vector, respectively; \( x_k(0) \in \mathbb{R}^n \) is the iteration-varying initial conditions; \( v_k(t) \in \mathbb{R} \) and \( w_k(t) \in \mathbb{R}^n \) are the iteration-varying load and measurement disturbances, respectively; \( A_{st}, B_{st} \) and \( C_{st} \) are uncertain constant matrices of appropriate dimensions; and \( \Theta \) is an uncertain constant input delay. It is clear that the system (3) is subject to uncertainties in both input delay and plant dynamic also iteration-varying uncertainties in initial conditions, load disturbance and measurement disturbance. Then by using the Laplace transform, the output of the system (3) in frequency-domain is given by (2) with

\[
P(s) = C_{st} (sI_n - A_{st})^{-1} B_{st} e^{-s\hat{\theta}} = G(s) e^{-s\hat{\theta}},
\]

where \( \Psi_k(s) = C_{st} (sI_n - A_{st})^{-1} [\mathcal{L}_k(s) + x_k(0)] + V_k(s) \), \( \forall k \in \mathbb{Z} \),

\[
\Psi_k(s) = C_{st} (sI_n - A_{st})^{-1} [\mathcal{W}_k(s) + x_k(0)] + V_k(s) , \forall k \in \mathbb{Z},
\]

where \( \mathcal{W}_k(s) \) and \( V_k(s) \) are the Laplace transform of \( w_k(t) \) and \( r_k(t) \), respectively. Consequently, random iteration-varying exogenous signal \( \Psi_k(t) \) includes random iteration-varying uncertainties in the load disturbance, measurement disturbance and initial conditions.

In the ILC literature, from an input-output viewpoint, the tracking error of an iteration-varying reference trajectory \( r_k(t) \) for a delay-free system is given by \( e_k(t) = r_k(t) - y_k(t) \), \( \forall k \in \mathbb{Z} \). The tracking error of a fixed reference trajectory \( r(t) \) for a time-delay system in the presence of uncertainty in input delay \( \Theta \) is described by \( e_k(t) = r(t - \hat{\Theta}) - y_k(t) \), \( \forall k \in \mathbb{Z} \), where \( \hat{\Theta} \) is an estimate of \( \Theta \) [28–30]. Now, according to this discussion, we introduce the following definition.

Definition 1. Let us define the tracking error of an iteration-varying reference trajectory \( r_k(t) \) for a time-delay system in the presence of uncertainty in input delay \( \Theta \) as follows:

\[
e_k(t) = r_k(t - \hat{\Theta}) - y_k(t) , \forall k \in \mathbb{Z}.
\]
The IMC-based ILC scheme shown in Figure 2, where the dotted block is the ILC part, and the rest is the IMC structure. In Figure 2, the dashed block represents the system described by (2); the dash-dotted block is the nominal model described by (10); \( C \) is the controller; \( W_p^\theta \) is an appropriate transfer function as the learning weighting function; and ‘Memory’ is a storage used for recording (providing) current (last) iteration information.

From Figure 2, the input update law can be derived as follows:

\[
U_k = V_k + C (H_k + R_k - A_k), \quad \forall k \in \mathbb{Z},
\]

where \( V_k = W_p^\theta U_{k-1} \), \( A_k = Y_k - \hat{Y}_k \), \( \hat{Y}_k = U_k \), \( \hat{\beta} = \hat{\beta} \), and \( H_k = A_k - U_k G - W_p^\theta A_{k-1} \hat{\beta} \). Note that \( A_{k-1} \hat{\beta} \) is realizable since it uses previous iteration operation data only. The proposed control scheme relies on the following assumption.

**Assumption 3.** The initial value of the ILC control law is set to be \( V_0 = H_0 = 0 \). Hence, the initial run \((k = 0)\) of the proposed scheme is an IMC structure and, thus, \( Y_0 \) and \( U_0 \) are the system output and control input of the IMC structure, respectively.

In the following, we obtain the system output \( Y_k \) for \( \forall k \in \mathbb{N} \). From (15) and Figure 2, it is straightforward to show that

\[
U_k = W_p^\theta U_{k-1} + C \left( A_k - U_k G - W_p^\theta A_{k-1} \hat{\beta} + R_k - A_k \right) \]

\[
= W_p^\theta U_{k-1} + C \left( -U_k G - W_p^\theta A_{k-1} \hat{\beta} + R_k \right) \]

\[
= W_p^\theta U_{k-1} + C \left( -U_k G - W_p^\theta (Y_{k-1} - \hat{Y}_{k-1}) \hat{\beta} + R_k \right) \]

\[
= W_p^\theta \left[ 1 - C \left( p \hat{\beta} - G \right) \right] U_{k-1} \]

\[
+ CR_k - C \hat{G} U_k - W_p^\theta C \Psi_{k-1} \hat{\beta}, \quad \forall k \in \mathbb{N}. \tag{16}
\]

From (16) and with the help of (9) and (10), it follows that

\[
U_k = \frac{C}{1 + CG} R_k \]

\[
+ \frac{W_p^\theta \left( 1 + CG - CP \hat{\beta} \right)}{1 + CG} U_{k-1} \]

\[
- \frac{W_p^\theta C \hat{\beta}}{1 + CG} \Psi_{k-1} = \frac{C}{1 + CG} R_k \]

\[
+ \frac{W_p^\theta (1 - C \hat{G} \Delta W)_{k-1}}{1 + CG} U_{k-1} \]

\[
- \frac{W_p^\theta C \hat{\beta}}{1 + CG} \Psi_{k-1}, \quad \forall k \in \mathbb{N}. \tag{17}
\]
From the above equation and using (2), we have
\[
\left( \frac{Y_k - \Psi_k}{p} \right) = \frac{C}{1 + C \hat{G}} R_k
\]
\[
W_k (1 - C \hat{G} \Delta W_s) \left( \frac{Y_{k-1} - \Psi_{k-1}}{p} \right)
\]
\[
+ \frac{W_k C \hat{G} \Delta \Psi_k}{1 + C \hat{G}}, \forall k \in \mathbb{N}, \quad \forall k \in \mathbb{N},
\]
which leads to
\[
Y_k = \frac{C (1 + \Delta W_s)}{1 + C \hat{G}} \hat{G} e^{-\hat{d}_k} R_k + \frac{W_k (1 - C \hat{G} \Delta W_s)}{1 + C \hat{G}} Y_{k-1}
\]
\[
+ \Psi_k \bar{W}_{k-1}, \forall k \in \mathbb{N}.
\]

4 | CONVERGENCE ANALYSIS

In this section, it is shown that both boundedness and monotonic convergence of both the expectation and variance of the tracking error can be guaranteed using the same convergence conditions.

4.1 | Tracking error expectation

To derive our results, we will need the following definition.

Definition 2. Let us define the remaining expected tracking error \(e_\theta(t)\) and functions \(Y_\theta\) and \(\Xi_\theta\) as follows:
\[
e_\theta(t) \equiv \ell^{-1} [E_\theta(t)]
\]
\[
= \ell^{-1} \left[ (W_k - 1) \left( \begin{array}{c}
(1 + C \hat{G}) \Psi_k - (1 - C \hat{G} \Delta W_s) R_k e^{-\hat{d}_k} \\
(1 + C \hat{G}) - W_k (1 - C \hat{G} \Delta W_s)
\end{array} \right) \right], \quad (21)
\]
\[
Y_\theta = W_k C \bar{P}, \quad \Xi_\theta = W_k \left( 1 - C \hat{G} \Delta W_s \right) \frac{1}{1 + C \hat{G}}. \quad (22)
\]

Now in the following Theorem, we can prove that tracking error expectation \(E[e_\theta(t)]\) is bounded for all iterations and converges to \(e_\theta(t)\) in the \(L_2\)-norm sense when \(k\) tends to infinity.

Theorem 1. Consider the H.C scheme shown in Figure 2, under Assumptions 1–3, and let the input update law (15) be applied. If there exist the controller \(C\) and the learning weighting function \(W_b\) such that the following conditions \(\mathcal{Q}_1, \mathcal{Q}_2, \mathcal{Q}_3\) are satisfied, then the tracking error expectation \(E[e_\theta(t)]\) is bounded for all \(k \in \mathbb{N}\) and convergent in the sense that
\[
\|E[e_\theta(t)] - e_\theta(t)\|_2 \to 0 \quad \text{uniformly as } k \to \infty.
\]
\(\mathcal{Q}_1\) \(W_0 \in RH_\infty\) and \((1 + C \hat{G})^{-1} \in RH_\infty\),
\(\mathcal{Q}_2\) \(C\) provides robust stability for the IMC structure; this implies \([45], C \in RH_\infty\) \(\|Y_\theta = W_k \bar{P}\|_\infty < 1\),
\(\mathcal{Q}_3\) \(\|\Xi_\theta = W_k (1 - C \hat{G} \Delta W_s)\|_\infty < 1\).

Proof—Step 1: By using (8) and (20), the tracking error \(E_k\) for all \(k \in \mathbb{N}\) is obtained as follows:
\[
E_k = \frac{W_k (1 - C \hat{G} \Delta W_s)}{1 + C \hat{G}} E_{k-1} + \frac{(1 - C \hat{G} \Delta W_s) e^{-\hat{d}_k}}{1 + C \hat{G}} R_k
\]
\[
- \frac{W_k (1 - C \hat{G} \Delta W_s)}{1 + C \hat{G}} \Psi_k + \bar{W}_k \bar{W}_{k-1}, \quad \forall k \in \mathbb{N}.
\]

Implementing the expectation operator \(E\) on both sides of (23) and then using (12), one can obtain
\[
E (E_k) = \frac{W_k (1 - C \hat{G} \Delta W_s)}{1 + C \hat{G}} E (E_{k-1})
\]
\[
+ (W_k - 1) \left( \Psi_k - \frac{1 - C \hat{G} \Delta W_s}{1 + C \hat{G}} R_k \right),
\]
\[
\forall k \in \mathbb{N}.
\]

From the above equation and Definition 2, we get
\[
E (E_k) = \Xi_\theta E (E_{k-1}) + E_\theta [1 - \Xi_\theta], \forall k \in \mathbb{N},
\]
which results in
\[
[E (E_k) - E_\theta] = \Xi_\theta [E (E_{k-1}) - E_\theta], \forall k \in \mathbb{N}.
\]

With repetitive substituting into (26), it can be expressed in terms of \(E (E_\theta) - E_\theta\) as
\[
[E (E_k) - E_\theta] = \Xi_\theta^k [E (E_\theta) - E_\theta], \forall k \in \mathbb{N}.
\]
Taking the norm on both sides of (27), gives the following inequality
\[
\| E( E_k) - E_s \|_2 \leq \| E( E_k) - E_s \|_2 \\
\leq \| E( E_k) - E_s \|_2 \\
\leq \| E( E_k) - E_s \|_2, \forall k \in \mathbb{N}.
\]

Step 2: From the condition \( \varphi_3 \), it follows that \( \| W_b(1 - C\hat{G}\Delta W_d) \|_{\infty} < 1 + C\hat{G} \|_{\infty} \). From this and condition \( \varphi_1( W_b \in RH_{\infty}) \) together with the condition \( \varphi_2( C \in RH_{\infty}) \) and Assumption 1 \( \{ \hat{G}, \Delta, W_d \} \in RH_{\infty} \), it can be concluded that \( E_s \) is well defined.

Step 3: Assumption 3, together with the condition \( \varphi_2 \), ensure that \( E_0 \) is bounded and so is \( E( E_0) \).

Step 4: Based on the condition \( \varphi_1 \), condition \( \varphi_2( C \in RH_{\infty}) \) and Assumption 1 \( \{ \hat{G}, \Delta, W_d \} \in RH_{\infty} \), it is clear that \( \Xi_{\infty} \in RH_{\infty} \).

Step 5: From (28) and the results of steps 2–4 together with the condition \( \varphi_3 \), we deduce \( E( E_k) \) is bounded for all \( k \in \mathbb{Z} \) and \( \| E( E_k) - E_s \|_2 \) converges toward zero as \( k \to \infty \).

Now, from the well-known Parseval’s theorem and (28), one can conclude that
\[
\| E( \varepsilon_k(t)) - \varepsilon_s(t) \|_2 = \| E( E_k) - E_s \|_2 \\
\leq \left( \| \Xi_{\infty} \|_k \| E( E_k) - E_s \|_2 \right) \\
= \left( \| \Xi_\infty \|_k \| E( \varepsilon_k(t)) - \varepsilon_s(t) \|_2 \right), \forall k \in \mathbb{N}.
\]

This, together with the result of step 5, completes the proof.

It is worth noting that both conditions \( \varphi_1 \) and \( \varphi_2 \) are reasonable for practical applications. However, the satisfaction of condition \( \varphi_3 \) depends on the exact knowledge of \( \Delta \), which is not available in practice. Therefore, in the following theorem, we attempt to solve this difficulty.

**Theorem 2.** A sufficient condition for satisfying condition \( \varphi_3 \) is
\[
\varphi_4:\| W_b \|_{\infty} (\| S \| + \| W_d \| T) \|_{\infty} < 1,
\]
where
\[
S = \frac{1}{1 + C\hat{G}},\quad T = \frac{C\hat{G}}{1 + C\hat{G}}.
\]

**Proof.** Assume that condition \( \varphi_4 \) holds. From there, it concludes
\[
1 > | W_b | (| S | + | W_d | T), \forall \omega \in [0, \infty).
\]

From the above equation and Assumption 1 \( (| \Delta | \leq 1) \), one can conclude that
\[
\| W_b | (| S | + | W_d | T) \|_{\infty} \geq (| W_b | (| S | + | W_d | T) ) \\
\geq \left( \frac{| W_b | (| S | - | W_d | T) }{1 + C\hat{G}} \right), \forall \omega \in [0, \infty).
\]

The condition \( \varphi_4 \) follows as an immediate consequence of (31) and (32). The proof is completed.

Theorem 1, together with Theorem 2, provides sufficient and practical conditions \( \varphi_1, \varphi_2 \) and \( \varphi_4 \) under which \( \lim_{k \to \infty} E( \varepsilon_k(t)) = \varepsilon_s(t) \) in the \( L_2 \)-norm sense. Referring to (21), we can see that if \( W_b = 1 \), then \( \varepsilon_k(t) = 0 \) and, thus, the tracking error expectation converges toward zero as \( k \to \infty \). However, the use of \( W_b = 1 \) has some disadvantages, as stated in the following.

**Remark 3.** By choosing an appropriate \( W_b \), one can design the controller \( C \) satisfying convergence conditions \( \varphi_1, \varphi_2 \) and \( \varphi_4 \) using, for example, the \( \mu \) -synthesis approach [53] or population-based methods [54]. However, since the proposed ILC scheme in this study is based on IMC structure, similar to other related works reported in [29, 30], we are also interested in using a unified controller derived from IMC theory. To this end, we consider the controller given in [29] as follows:
\[
C(\omega) = \frac{K_D(\omega)b}{\hat{K}_D(\omega) + \omega^{n}b}\Xi_\infty, \quad \Xi_\infty = \Xi, \quad \Xi_\infty = \Xi,
\]
where \( K_D(\omega) \) and \( \hat{K}_D(\omega) \) are adjustable constant parameters for controller tuning; \( N_\Xi(\omega) \) represents the complex conjugate of \( \Xi_\infty(\omega) \); and \( n \) is chosen such that \( C \) can be realized (Numerator degree \( \leq \) Denominator degree). Note that using (33), conditions \( \varphi_1 \) and \( \varphi_2 \) are easy to achieve. However, if we choose \( W_b = 1 \), then the unified controller (33) cannot be used to satisfy condition \( \varphi_4 \). The reason for this behaviour is that if \( W_b = 1 \), then substituting (33) into \( \varepsilon_k(t) \), it follows that \( \| Y_\delta(\omega) \| = \| S(\omega) \| + | W_d(\omega) T(\omega) | \) is larger than one at other frequency regions. Consequently, to overcome this problem, we choose \( W_b \) such that \( | W_b | < 1 \) at the frequency regions that \( | S(\omega) | + | W_d(\omega) T(\omega) | \geq 1 \).

It is worth pointing out that multiplicative uncertain form is a commonly used uncertainty model in the literature because the effect of uncertainty in the model can be quickly assessed by
choosing the uncertainty weighting function \( W_u \) (see Remark 2). Therefore, \( \| W_u(\omega) \|_{\infty} \) determine the amount of uncertainty in the model. The small value of \( \| W_u(\omega) \|_{\infty} \) indicates small uncertainty in the model which does not match the ‘practical nature’ of engineering applications. In fact, in most practical applications \( \| W_u(\omega) \|_{\infty} \) is larger than one \( (\| W_u(\omega) \|_{\infty} > 1) \).

Now, if in a practical application \( \| W_u(\omega) \|_{\infty} > 1 \) and we choose \( W_u = 1 \), then satisfaction of condition \( \mathcal{G}_4 \) cannot be guaranteed. The reason for this behaviour is proved in the following Theorem.

**Theorem 3.** If \( W_u = 1 \), then a necessary condition for the condition \( \mathcal{G}_4 \) is

\[
\| W_u(\omega) \|_{\infty} \leq 1.
\]

**Proof.** To prove Theorem 3, we use reductio ad absurdum. First of all, from \( W_u = 1 \) and condition \( \mathcal{G}_4 \), we have

\[
|S| + |W_u^T| < 1, \quad \forall \omega \in [0, \infty).
\]

From (30), we get

\[
1 = |S + T| \leq |S| + |T|, \quad \forall \omega \in [0, \infty).
\]

Now assume that (34) is not satisfied, that is, \( \| W_u(\omega) \|_{\infty} > 1 \). As a consequence, there exists a frequency \( \omega \) such that \( |W_u(\omega)| > 1 \). This, together with (36), leads to

\[
1 \leq |S| + |T| < |S| + |W_u^T|, \quad \text{for } \omega = \omega_0,
\]

which is in contradiction to (35). The proof is completed.

**Remark 4.** From Theorem 3, it is clear that the choice \( W_u = 1 \) is critical. Nevertheless, returning to (21), we can conclude that if we choose \( W_u \) such that \( |W_u - 1| \approx 0 \), then \( \epsilon_u(t) \approx 0 \), and consequently, \( \lim_{k \to \infty} E[e_k(t)] \approx 0 \). Therefore, the performance degradation caused by \( W_u \neq 1 \) can be negligible in practice. Now, from this and Remark 3, it follows that we should choose \( W_u \) such that \( |W_u - 1| \approx 0 \) at the frequency regions that \( |S(\omega)| + |W_u(\omega)^T(\omega)| < 1 \).

### 4.2 Tracking error variance

To demonstrate that the tracking error variance is both bounded and convergent, let us introduce the following definition.

**Definition 3.** For convenience, let us define functions \( \Theta, \Xi_0, \Xi_1, \Xi_2, \Xi_3, \Xi_4, \Xi_f, \Xi_g \) and the remaining tracking error variance \( \epsilon_{uk}(t) \) as follows:

\[
\Theta = \frac{\left( 1 - C \tilde{G} \Delta W_s \right)}{1 + C \tilde{G}}, \quad \Xi_0 = \frac{\Delta}{1 + C \tilde{G}}, \quad \Xi_1 = -W_s \Theta \epsilon^\Delta,
\]

\[
\Xi_2 = -1, \quad \Xi_3 = W_s^\Delta, \quad \Xi_4 = \Xi_f^\Delta, \quad \Xi_5 = \Xi_g^\Delta.
\]

Now, we are in a position to show that the conditions \( \mathcal{G}_1, \mathcal{G}_2 \) and \( \mathcal{G}_4 \) are still valid to ensure that the tracking error variance \( \text{var}[e_k(t)] \) is bounded for all \( k \in \mathbb{Z} \) and convergent in such a way that \( \lim_{k \to \infty} \text{var}[e_k(t)] = \epsilon_{uk}(t) \) in the \( L_2 \)-norm sense. In other words, the same convergence conditions can guarantee both boundedness and monotonic convergence of both the expectation and variance of the tracking error (in the \( L_2 \)-norm sense). To this end, the following theorem is given.

**Theorem 4.** Consider the ILC scheme shown in Figure 2 under Assumptions 1–3, and let the input update law (15) be applied. If there exist the controller \( C \) and the learning weighting function \( W_s \) such that the conditions \( \mathcal{G}_1, \mathcal{G}_2, \) and \( \mathcal{G}_4 \) are satisfied, then the tracking error variance \( \text{var}[e_k(t)] \) is bounded for all \( k \in \mathbb{Z} \) and convergent in the sense that \( \| \text{var}[e_k(t)] - \epsilon_{uk}(t) \|_2 \to 0 \) uniformly as \( k \to \infty \).

**Proof.** Step 1: By using (22) and (38), we can rewrite (23) as follows:

\[
E_k = \Xi_0 E_{k-1} + \Xi_1 R_k + \Xi_2 R_{k-1} + \Xi_3 \Psi_k + \Xi_4 \Psi_{k-1}, \quad \forall k \in \mathbb{N}.
\]

With repetitive substituting into (40), it can be expressed as

\[ E_k = \Xi_0^k E_0 + \sum_{i=0}^{k-1} \Xi_0^i R_{k-i-1} + \sum_{i=0}^{k-2} \Xi_0^i \Psi_{k-i-1} + \Xi_0^i \Psi_k, \quad \forall k \geq 2, \]

and

\[ E_1 = \Xi_0 E_0 + \Xi_1 R_0 + \Xi_2 R_1 + \Xi_3 \Psi_0 + \Xi_4 \Psi_1. \]
And also, from (8), Assumption 3 and (38), we can drive

\[
E_{i0} = \left( e^{-\hat{\theta}I} + \Xi_j \right) R_0 + \Xi_\xi \Psi_0. \tag{43}
\]

Substituting (43) into both (41) and (42), we have

\[
E_{i_k} = \Xi_\xi^k \left[ \left( e^{-\hat{\theta}I} + \Xi_j \right) R_0 + \Xi_\xi \Psi_0 \right]
+ \Xi_\xi^{k-1} \Xi_\xi R_0
+ (\Xi_\xi \Xi_\xi + \Xi_\xi) \sum_{i=0}^{k-2} \Xi_\xi^i R_{k-i-1}
+ \Xi_\xi R_k
+ \Xi_\xi^{k-1} \Xi_\xi \Psi_0
+ (\Xi_\xi \Xi_\xi + \Xi_\xi) \sum_{i=0}^{k-2} \Xi_\xi^i \Psi_{k-i-1}
+ \Xi_\xi \Xi_\xi \Psi_k
\]

\[
= \Xi_\xi^{k-1} \left[ \Xi_\xi + \Xi_\xi \left( e^{-\hat{\theta}I} + \Xi_j \right) \right] R_0
+ (\Xi_\xi \Xi_\xi + \Xi_\xi) \sum_{i=0}^{k-2} \Xi_\xi^i R_{k-i-1}
+ \Xi_\xi R_k
+ \Xi_\xi^{k-1} \Xi_\xi \Psi_0
+ (\Xi_\xi \Xi_\xi + \Xi_\xi) \sum_{i=0}^{k-2} \Xi_\xi^i \Psi_{k-i-1}
+ \Xi_\xi \Xi_\xi \Psi_k,
\forall k \geq 2. \tag{44}
\]

Step 2: Based on Assumption 2, it is clear that all terms \((\bullet) \times R_i \) and \((\bullet) \times \Psi_j \) on the right sides of both (44) and (45) are uncorrelated \(\forall i, j \in \mathbb{Z} \). Now, implementing the variance operator \(\text{var} \) on both sides of (44) and then using (13), yields

\[
\text{var} (E_{i_k}) = \text{var} \left( \Xi_\xi^{k-1} \left[ \Xi_\xi + \Xi_\xi \left( e^{-\hat{\theta}I} + \Xi_j \right) \right] R_0 
+ (\Xi_\xi \Xi_\xi + \Xi_\xi) \sum_{i=0}^{k-2} \Xi_\xi^i R_{k-i-1}
+ \Xi_\xi R_k
+ \Xi_\xi^{k-1} \Xi_\xi \Psi_0
+ (\Xi_\xi \Xi_\xi + \Xi_\xi) \sum_{i=0}^{k-2} \Xi_\xi^i \Psi_{k-i-1}
+ \Xi_\xi \Xi_\xi \Psi_k \right)
\forall k \geq 2. \tag{49}
\]

By using the properties of the geometric progression, (46) can be rewritten as

\[
\text{var} (E_{i_k}) = \left[ \Xi_\xi + \Xi_\xi \left( e^{-\hat{\theta}I} + \Xi_j \right) \right]^2 R_0
+ (\Xi_\xi \Xi_\xi + \Xi_\xi) \frac{1 - \Xi_\xi^{2k-2}}{1 - \Xi_\xi^2} R_k
+ \Xi_\xi \Xi_\xi \Psi_k \forall k \geq 2. \tag{48}
\]

From (39), (47) and (48), it follows that

\[
\text{var} (E_{i_k}) = \Xi_\xi^{2k-2} \left[ \text{var} (E_{i_1}) - E_{i_k} \right], \forall k \geq 2. \tag{49}
\]
Taking the norm on both sides of (49), gives the following inequality

$$\|\text{var} (E_k) - E_{ss}\|_2 = \|\Xi_2^{2k-2} \text{var} (E_1) - E_{ss}\|_2$$

$$\leq \|\Xi_2^{2k-2}\|_\infty \|\text{var} (E_1) - E_{ss}\|_2$$

$$\leq \|\Xi_1\|_\infty^{2k-2} \|\text{var} (E_1) - E_{ss}\|_2, \forall k \geq 2. \tag{50}$$

Step 3: From condition $\mathcal{G}_1$ and condition $\mathcal{G}_2 (C \in RH_{\infty})$, together with Assumption 1 ($\{\hat{C}, \Delta, W_0\} \in RH_{\infty}$), it follows that $\{\Xi_1, \Theta\} \in RH_{\infty}$.

Step 4: Based on Theorem 2, the satisfaction of condition $\mathcal{G}_3$ results in $\|\Xi_\infty\|_\infty < 1$, and consequently, $\|\Xi_\infty\|_\infty < 1$. Hence, it follows that $\|W'_a (1 - C\hat{C} \Delta W'_0)\|_2^2 < \|1 + C\hat{C}\|_2^2$. From this and the result of step 3, it can be concluded that $E_{ss}$ is well defined.

The remaining steps in the proof procedure can be established in a way similar to the proof steps of Theorem 1, which are omitted here.

5 | SIMULATION RESULTS

In the following, two examples are provided to illustrate the effectiveness of the proposed scheme. Throughout this simulation, the MATLAB command “rand” is employed to generate all random variables, and the following notations are used:

- the zero-order hold method with the sample time 0.01 (s) is used to convert the discrete-time signals to the continuous-time.
- $\varpi_X (k, \ell)$ represents an APRBS (amplitude modulated pseudo random binary signal), which has 100 steps per second and random amplitudes in the range of $-0.002$ to $0.002$. It is worth highlighting that a PRBS is a periodical signal that closely imitates white noise [55, 56] using a deterministic algorithm, and an APRBS modifies PRBS by assigning different amplitudes in the specific range [55, 57]. For instance, a portion of the signal $\varpi_X (k, \ell)$ for three different iterations $\{a, b, c\} \in Z$ is plotted in Figure 3.

- $\chi_X (t_X, \omega_X, t)$ represents a sinusoidal signal with amplitude 0.002, delay time $t_X$, and frequency $\omega_X$, which is described by

$$\chi_X (t_X, \omega_X, t) = \begin{cases} 0 & 0 \leq t < t_X \\ 0.002 \sin (\omega_X t) & t \geq t_X \end{cases}. \tag{51}$$

- Variable $\beta_X (k)$ is an iteration-varying random number over the interval $[0, 1]$.

Example 1. In this example, a one-degree-of-freedom (1-DOF) slave robot with the dynamic equation of $(J_s \ddot{\theta} + B_s \dot{\theta} + M_g L_s) \alpha_s = f_i$ is used (as studied in [6]); where $M_s$, $J_s$, and $L_s$ are the mass, the moment of inertia and the length of the manipulator link, respectively; $g = 9.8 \, (m/s^2)$ is the gravity acceleration; $B_s$ represents the viscous friction coefficient; $\alpha_s$ is the rotational angle and $f_i$ is the input. For this robot, we notice multiplicative uncertainty (9) with the nominal values of $M_s = 1.5 \, (kg)$, $J_s = 0.85 \, (kg \cdot m^2)$, $L_s = 0.25 \, (m)$, $B_s = 4.5 \, (Nm/rad/s)$, $\dot{\theta} = 2.96 \, (rad)$ and the uncertainty weighting function

$$W'_a = \frac{5r + 6}{5r + 500}. \tag{52}$$

To address Remarks 3 and 4 and satisfy the convergence conditions $\mathcal{G}_1$, $\mathcal{G}_2$, and $\mathcal{G}_3$, the learning weighting function $W'_b$ and controller $C$ are considered as follows:

$$W'_b = \frac{100}{1.7r + 100}, \tag{53}$$

$$C = \frac{0.085r^2 + 0.45r + 0.3675}{0.01r^2 + 0.2r + 1} \quad (K = 0.1, \lambda = 0.1, n = 2), \tag{54}$$

which leads to

$$W'_b \in RH_{\infty} \text{ and } (1 + C\hat{C})^{-1} \in RH_{\infty} \text{ (condition } \mathcal{G}_1),$$

$$\|Y_a\|_{\infty} = 0.000512 < 1 \text{ (condition } \mathcal{G}_2),$$

$$\|Y_b\|_{\infty} = 0.991200 < 1 \text{ (condition } \mathcal{G}_3).$$

Figure 4 shows the magnitude plot of $Y_a$ and $Y_b$ with respect to the frequency.

Now, the boundedness and convergence of the tracking error are guaranteed with increasing iteration in the sense of expectation and variance.

To perform this simulation, we consider the continues-time form of reference trajectory given in [50] as follows:

$$r_k (t) = 10 \left(\frac{t}{50}\right)^2 \left(1 - \frac{t}{50}\right) + 10 \varpi_A (k, \ell), \quad t \in [0, 50]. \tag{55}$$
Case 1: It is assumed that we only intend to track (55) in the absence of $\psi_k(t)$. Note that the controller (54) can also satisfy the convergence condition presented in [29]. To show this, the convergence condition given in [29] is plotted in Figure 5 (see (1)-(4), (35), (43), (46) in [29]), which demonstrates that $|1 + \frac{K_{N_k}(\omega)}{N_1^*(\omega)(\lambda + 1)}| > |1 - \frac{K_{N_k}(\omega)\omega^s(\omega)}{N_1^*(\omega)(\lambda + 1)}|, \forall \omega \leq (\omega_k = 0.228).$ Figure 6 shows the tracking error evaluated by $\|e_k(t)\|_2$ for the first 2500 iterations. It is clear that using the IMC-based ILC scheme [29] the tracking error decreases in the first iterations and then starts to grow, while our proposed ILC scheme remains convergent. The reason for the instability of the scheme given in [29] is that it cannot guarantee convergence of ILC at high-frequency regions (see Section 4 of [29]). Consequently, if there exist rather sharp corners in the reference trajectory due to iteration-varying uncertainties or some other factors, then the tracking error will contain high-frequency components and may lead to the phenomenon of apparent convergence followed by divergence [59].

Case 2: In this case, we are still interested in the tracking of (55) in the presence of the iteration-varying exogenous signal $\psi_k(t) = 5\omega B_k(t) + (50 + 5\beta A_k)\chi(t)$. Figures 7 and 8 illustrate the simulation test results for the first 2500 iterations. From Figure 7, it can be seen that, similar to Case 1, the ILC scheme [29] is divergent while using the proposed method in this study the tracking error reduces to iteration $k = 100$ and then varies along the iteration axis within a specific bound. Figure 8 shows the reference trajectory $r_k(t)$ and output $y_k(t)$ obtained at a random iteration number between $k = 100$ to $k = 2500$ (here, the random iteration number is $k = 1465$ and due to the instability of the ILC scheme given in [29], its output is not plotted). It is clear from Figures 7 and 8 that despite random iteration-varying uncertainties, the learning
process evolves and the system output can be enabled to track the reference trajectory nicely.

Example 2. Consider an uncertain continuous time-delay system with the state-space representation (3) as follows:

\[ A_n = \begin{bmatrix} a & b & c & d \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B_n = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad C_n = \begin{bmatrix} 4 & e & f & g \end{bmatrix} \]

\[ x_k(0) = \begin{bmatrix} -0.03 + 0.06\beta \lambda_{A_n}, -0.03 + 0.06\beta \lambda_{B_n}, \\ -0.03 + 0.06\beta \lambda_{C_n}, -0.03 + 0.06\beta \lambda_{D_n} \end{bmatrix}^T \]  

(56)

together with

\[ w_k(t) = \begin{bmatrix} 5\pi_C (k, t) + (5 + 2.5\beta \lambda_{E_n})\chi_B (10, 1, t) \\ 5\pi_D (k, t) + (5 + 2.5\beta \lambda_{E_n})\chi_C (12, 1, t) \\ 5\pi_E (k, t) + (2.5 + 5\beta \lambda_{F_n})\chi_D (30, 0, 5, t) \\ 5\pi_F (k, t) + (2.5 + 5\beta \lambda_{F_n})\chi_F (8, 0, 5, t) \end{bmatrix}, \]  

(57)

and

\[ v_k(t) = 25\pi_G (k, t) + (25 + 2.5\beta \lambda_{E_n})\chi_f (60, 1, t). \]  

(58)

All parameters \(a, b, c, d, e, f, g\) and \(\Theta\) represent the uncertain parameters of the system, which are unknown but belong to \(-3.85 \leq a \leq -2.75, -4.13 \leq b \leq -3.81, -2.215 \leq c \leq -1.895, -0.425 \leq d \leq -0.345, 12.54 \leq e \leq 13.30, 10.004 \leq f \leq 10.924, 0.2024 \leq a \leq 0.4024\) and \(4.24 \leq \Theta \leq 4.30\) with the nominal values of \(\dot{a} = -3.30, \dot{b} = -3.97, \dot{c} = -2.055, \dot{d} = -0.385, \dot{e} = 12.92, \dot{f} = 10.464, \dot{g} = 0.3042, \dot{\Theta} = 4.30\) and \(\dot{x}_k(0) = [0, 0, 0, 0]^T\). Hence, for the above system, a nominal model is described by

\[ \dot{\hat{A}}_n = \begin{bmatrix} \hat{a} & \hat{b} & \hat{c} & \hat{d} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \dot{\hat{B}}_n = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \dot{\hat{C}}_n = \begin{bmatrix} 1 & \hat{f} & \hat{g} & \hat{h} \end{bmatrix}. \]  

(59)

Therefore, following [52, 60] (see also Remark 2), to model the above system in a multiplicative uncertain form (9), \(W'_n\) should be chosen such that

\[ |G(\omega) = \frac{G(\omega) e^{-\lambda_0 \text{ad}} - \hat{G}(\omega) e^{-\lambda_0 \text{ad}}}{\hat{G}(\omega) e^{-\lambda_0 \text{ad}}}| \leq |W'_n(\omega)| \]  

is satisfied for all \(\omega, a, b, c, d, e, f, g, h, \Theta\), where \(G(\omega) = C_n (j\omega - \hat{A}_n)^{-1}B_n\) and \(\hat{G}(\omega) = \hat{C}_n (j\omega - \hat{\hat{A}}_n)^{-1}\hat{B}_n\). By using the Bode magnitude plot, a suitable uncertainty weighting function \(W'_n\) is

\[ W'_n = \frac{2.7 + 28.13}{s + 38.13}. \]  

(60)

FIGURE 9 The Bode magnitude plots of \(|W'_n(\omega)|\) (black dashed line) and \(|G(\omega) e^{-\lambda_0 \text{ad}} - \hat{G}(\omega) e^{-\lambda_0 \text{ad}}|\) (blue solid lines) for 1000 random samples of parameters uncertainties.

FIGURE 10 The magnitude plot of \(Y_a\) and \(Y_b\) with respect to the frequency.

Figure 9 shows the Bode magnitude plots of \(|W'_n(\omega)|\) and \(|G(\omega) e^{-\lambda_0 \text{ad}} - \hat{G}(\omega) e^{-\lambda_0 \text{ad}}|\) for 1000 random samples of parameters uncertainties. As can be seen from Figure 9, the chosen \(W'_n\) is suitable.

To address Remarks 3 and 4 and satisfy the convergence conditions \(\phi_1, \phi_2\) and \(\phi_3\), we adopt \(W'_n\) and \(C\) as follows:

\[ W'_n = \frac{10}{0.1s + 10}, \]  

(61)

\[ C = \frac{0.06s^4 + 0.198s^3 + 0.238s^2 + 0.123s + 0.0231}{0.4s^4 + 7.052s^3 + 19.65s^2 + 15.1s + 0.4355} \]  

(62)

which leads to

\[ W'_n \in RH_\infty \text{ and } (1 + C \hat{G})^{-1} \in RH_\infty \text{ (condition } \phi_1), \]  

\[ ||Y_n||_\infty < 0.032311 < 1 \text{ (condition } \phi_2), \]  

\[ ||Y_n||_\infty < 0.999897 < 1 \text{ (condition } \phi_3). \]  

Figure 10 shows the magnitude plot of \(Y_a\) and \(Y_b\) with respect to the frequency.

Now, both boundedness and convergence of both expectation and variance of the tracking error are guaranteed as \(k \to \infty\).
initial conditions and disturbances are relaxed. By using the scheme proposed in this paper, it was shown that both boundedness and monotonic convergence of both the expectation and variance of the tracking error can be guaranteed using the same convergence conditions when uncertainties in reference trajectory, initial conditions and disturbances vary randomly from trial to trial. For more convenience in achieving convergence conditions, a unified controller derived from the IMC theory was considered. Two examples were given to verify the effectiveness of the proposed method. In our future works, we will focus on the extension of our method to the uncertain continuous-time systems subject to random iteration-varying uncertainties in both plant dynamic and input delay as well as consensus tracking of heterogeneous multi-agent systems with the virtual leader.

6 | CONCLUSION

This study has been devoted to the IMC-based ILC design for uncertain continuous systems with input-delay, for which the assumptions of strictly iteration-invariant reference trajectory, initial conditions and disturbances are relaxed. By using the scheme proposed in this paper, it was shown that both boundedness and monotonic convergence of both the expectation and variance of the tracking error can be guaranteed using the same convergence conditions when uncertainties in reference trajectory, initial conditions and disturbances vary randomly from trial to trial. For more convenience in achieving convergence conditions, a unified controller derived from the IMC theory was considered. Two examples were given to verify the effectiveness of the proposed method. In our future works, we will focus on the extension of our method to the uncertain continuous-time systems subject to random iteration-varying uncertainties in both plant dynamic and input delay as well as consensus tracking of heterogeneous multi-agent systems with the virtual leader.

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