Mathematical Modeling of Sun and Solar Drying Kinetics of Fermented Cocoa Beans

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Abstract— In this study, thin layer drying experiments were conducted to compute drying characteristics of fermented cocoa beans in open sun and indirect natural convection solar dryer. The drying experiments were conducted at the same time for comparison. Three different thin layers of the fermented beans were examined under field conditions for Akure, Nigeria. The drying process took place only in the falling rate period. The drying curves obtained from the experimental data were fitted to thirteen (13) different thin layer mathematical models. All the models were compared according to three evaluation parameters. These include coefficient of determination ($R^2$), Root mean square error (RMSE) and Chi-square ($X^2$). The results showed that increasing drying air temperature resulted to shorter drying times. The Vermal et al. model was found to be the most suitable for describing the drying curve of the convective indirect solar drying process of cocoa beans with $R^2 = 0.9562$, $X^2=0.0069$ and RMSE=0.0067; while, the Midilli and Kucuk model, best described the drying curve of fermented cocoa beans under open sun with $R^2 = 0.9866$, $X^2=0.0024$ and RMSE=0.0023.

Keywords— Thin-layer drying, moisture content, modelling, cut test, pH, Cocoa beans.

I. INTRODUCTION

Drying is one of the oldest methods of food preservation (Doymaz, 2007). Agricultural and other products have been dried by sun and wind in the open air for thousands of years. The purpose is either to preserve them for later use, as in the case with food; or as an integral part of the production process as with tobacco and cocoa beans. It is necessary that the traditional techniques be replaced with industrial drying methods. (Ertekin and Yaldiz, 2004). Mathematical modelling and simulation of drying curves under different conditions is important to obtain an overall improvement of the quality of the final product. Simulation models of the drying process are used for developing new designs, improving existing drying systems, predicting the airflow over the product and for the control of the process. (Aghbashlo, et al., 2008). Thin layer drying equations are used to estimate drying kinetics for several products and also to generalize drying curves.

A critically important aspect of drying technology is mathematical modelling of the drying process. Modelling of drying process and kinetics is a tool for process control and necessary to choose suitable method of drying for specific product. Modelling is also essential for engineers to choose the most suitable climatic conditions in order to design appropriate drying equipment for perishable crops. The aim of this work is to study the drying process and select the most suitable model (in terms of fitting ability) to describe the thin-layer drying of cocoa beans. Although much information has been reported about modelling of thin layer drying (Togrul&Pehlivan, 2002) there is no information about modelling of thin layer drying of cocoa beans in Nigeria.

II. MATERIALS AND METHODS

2.1 Drying experiments

In this study, fresh healthy cocoa pods (Amenlonado variety) were procured from Oda village, Akure South Local Government, Akure Ondo State. The drying experiments were carried out using mobile solar dryer in the Department of Agricultural Engineering, Federal University of Technology, Akure. Plate 1 shows the schematic diagram of the solar dryer used for the experimental work which consists of a solar collector and a drying chamber. The samples were weighed using a digital balance with 0.01g sensitivity every 60 minutes throughout the drying process.

Three different thin samples of wet fermented cocoa beans were spread evenly into the solar dryer and in the open sun drier for the dehydration test. The experiment was replicated thrice and the mean value was used. Thermal drying method was used in the determination of moisture content of the samples.100g of sample were placed in oven at 105± 3°C and allowed to dry to a constant weight for 24 hours (Lagha-Benamrouche, S. and Madani, K., 2013). The moisture content (MC) was calculated by expressing the weight loss upon drying a fraction of the initial weight of sample used. The moisture content of the seeds was determined by gravimetric method which determines the mass loss from the sample.
by drying to constant weight (ASABE STANDARDS, 1993 and AOAC, 2000).

\[ DM(\%) = \frac{W_3 - W_0}{W_1 - W_0} \times 100 \]  

(1)

\[ \%MC_{db} = 100 - DM\% \]  

(2)

Where \( W_0 \) is weight of empty crucible, \( W_1 \) is weight of crucible plus sample before drying, \( W_3 \) is weight of crucible plus sample after drying, DM is dry matter and MC\(_{db}\) is the cocoa beans moisture content (g water/g dry base, d.b).

\[ MR = \frac{M - M_e}{M_0 - M_e} \]  

(3)

The values of \( M_e \) are relatively small compared to those of \( M \) or \( M_0 \) hence error involved in its simplification as negligible. (Aghbashlo, Kianmerhrk & Samini-Akhljahani, 2008), hence moisture ratio is calculated

\[ MR = \frac{M}{M_0} \text{ or } \frac{M}{M_e} \]  

(4)

For drying model selection, drying data were fitted into thirteen well known thin layer drying models which are given in Table 1.

| S/N | Model name              | Model equation                  | References                  |
|-----|-------------------------|---------------------------------|----------------------------|
| 1   | Newton                  | \( MR = \exp(-kt) \)             | Upadhyay et al., 2008      |
| 2   | Page                    | \( MR = \exp(-kt^n) \)           | Saeed et al., (2006)       |
| 3   | Modified page           | \( MR = \exp[-(kt)^n] \)         | Ceylan et al., (2007)      |
| 4   | Henderson and Pabis     | \( MR = a \exp(-kt) \)           | Kashaninejad and Tabil (2004) |
| 5   | Logarithmic             | \( MR = a \exp(-kt) + c \)       | Wang et al., (2007)        |
| 6   | Two-term                | \( MR = a \exp(-kt) + b \exp(-nt) \) | Wang et al., (2007)        |
| 7   | Two-term exponential    | \( MR = a \exp(-kt) \exp(-kt) \) | Tarigart et al., (2007)    |
| 8   | Wang and Singh          | \( MR = 1 + at + [bt]^2 \)      | Wang and Singh (1978)      |
| 9   | Diffusion approach      | \( MR = a \exp(-kt) + (1-a) \exp(-kt) \) | Wang et al., (2007); |
| 10  | Modified Henderson and Pabis | \( MR = a \exp(-kt) + b \exp(-kt) + c \exp(-ht) \) | Kaya et al., (2007b) |
| 11  | Verma et al             | \( MR = a \exp(-kt(n)) + bt \)  | Doymaz, (2005b)            |
| 12  | Midilli and Kucuk       | \( MR = a \exp(-kt) + bt \)     | Midilli et al., (2002)     |
| 13  | Thomson                 | \( t = a \ln MR + b (\ln MR)^2 \) | Thomson et al., (1968)     |

Moisture ratio (MR) = dependent variable, Drying constant constant (k) = independent variable.

The goodness of fit was determined using three parameters; coefficient of determination \( R^2 \), reduced chi-square \( x^2 \) and the root mean square error (RMSE) using equations (4) - (6) as in Sacilik and Elicin (2008). The statistical analyses were carried out using SPSS 13.0 software and non-regression technique.

Plate 1: Mobile Solar dryer
a. Coefficient of determination ($R^2$)

$$R^2 = \frac{\sum_{i=1}^{n}(MR_i - MR_{pre,i})^2}{\sum_{i=1}^{n}(MR_i - MR_{exp,i})^2} \times \frac{\sum_{i=1}^{n}(MR_i - MR_{pre,i})}{\sum_{i=1}^{n}(MR_i - MR_{exp,i})} \times \frac{\sqrt{\sum_{i=1}^{n}(MR_i - MR_{pre,i})}}{\sqrt{\sum_{i=1}^{n}(MR_i - MR_{exp,i})}}$$

b. Chi-square ($x^2$)

$$x^2 = \frac{\sum_{i=1}^{N}(MR_{exp,i} - MR_{pre,i})^2}{N - n}$$

Where $MR_{exp,i}$ is the $i$th experimentally observed moisture ratio, $MR_{pre,i}$ is the $i$th predicted moisture ratio, $N$ is the number of observation, $n$ is the number of model constants.

### III. RESULTS AND DISCUSSIONS

3.1 Drying Kinetics of Fermented cocoa beans

From the experimental data, the moisture content (%wb) of fermented cocoa beans for the solar dryer and open sun drying at any time are represented in Figures 1-3. It was clearly evident from these curves that the drying rate of fermented cocoa beans in the solar dryer was faster than that of the open sun drying. The moisture content of the fermented cocoa beans reached 6.5% dry basis in 32 hours of drying in the solar dryer, whereas the final moisture of the same product dried by open sun drying was only 9.87% dry basis thus moisture content was not enough for safe storage. When it was dried under open sun drying, the duration of dry was about two (2) sunshine days to bring it to the same moisture level.

This can be explained that the main factor influencing drying rate was the drying air temperature. Compared to open sun drying, solar dryer can generate higher air temperature and affected the significant increasing of evaporation rate of water and then result in lower final moisture content of drying samples. These results indicated that solar dryer was effective than open sun drying.

![Fig.1: Variation of moisture content with drying time for fermented cocoa beans for 3.97g/cm²](image1.jpg)

![Fig.2: Variation of moisture content with drying time for fermented cocoa beans for 3.21g/cm²](image2.jpg)
3.3 Mathematical modelling

The moisture content data at different experimental modes were converted to the more useful moisture ratio expression, and curve fitting computations with drying time were performed with the thirteen (13) drying models presented by previous workers (Table 1). The results of the statistical analyses undertaken on these models for the natural convention solar drying and the natural sun drying are given in Table 2 and 3, respectively. The models were evaluated based on Coefficient of determination ($R^2$), Chi-square ($\chi^2$), and Root Mean Square Error (RMSE). All equations gave consistently high ($R$) values in the range of 0.91 - 0.98. This indicates that all equations could satisfactorily describe the solar drying rates of fermented cocoa beans. RMSE ranged from 0.0067-1.6465, chi-square ranged from 0.0069-1.1000 and for solar drying while RMSE ranges from 0.0023-1.4672, and chi-square ranges from 0.0023-1.2700 for open sun drying. The result shows that for all thin layer drying models and conditions of solar drying, the Verma et al. (1985) model gave the best fit with $R^2 = 0.9562$, $\chi^2 = 0.0069$, and RMSE = 0.0067 for solar drying. The Midilli model gave the best fit with $R^2 = 0.9866$, $\chi^2 = 0.0024$, and RMSE = 0.0023 for open sun drying. The drying constants ($k$ and $l$) and coefficients ($a$ and $n$) values as well as the statistical parameters $R^2$, $\chi^2$, and RMSE are shown in Tables 2 and 3 for both solar drying and open sun drying. Validation of the Verma et al and Midilli and Kucuk models were made by comparing the predicted moisture ratio with the experimented moisture ratio values from all the tests. The performance of the Verma et al model for the thin solar drying and the Midilli and Kucuk model for natural sun drying was illustrated in Figures 4 and 5. The predicted data is banded around the straight line which showed the suitability of the Verma et al and Midilli and Kucuk models in describing the drying behaviour of fermented cocoa beans in solar and open drying respectively.

![Graph showing the variation of moisture content with drying time for fermented cocoa beans for 2.97g/cm²](image)

**Fig. 3: Variation of moisture content with drying time for fermented cocoa beans for 2.97g/cm²**

| MODELS          | COEFFICIENT       | $R^2$ | $\chi^2$ | RMSE   |
|-----------------|-------------------|-------|----------|--------|
| NEWTON          | $k = 0.0186$      | 0.9311| 0.0134   | 0.013  |
| PAGE            | $k = 0.0383$      | 0.9351| 0.0108   | 0.0105 |
| MODIFIED PAGE   | $k = 0.0192$      | 0.9351| 0.0675   | 0.0654 |
| HENDERSON & PABIS | $k = 0.0160$      | 0.9506| 0.0085   | 0.0082 |
| LOGARITHMIC     | $k = 0.0057$      | 0.9662| 0.0088   | 0.0082 |
|                 | $a = 0.8748$      |       |          |        |
|                 | $b = 0.4523$      |       |          |        |
| TWO TERM        | $k_0 = 0.0162$    | 0.9506| 0.0087   | 0.0081 |
|                 | $k_1 = 0.0160$    |       |          |        |
|                 | $b = 0.4226$      |       |          |        |
| TWO TERM MOD.   | $k = 0.2539$      | 0.9398| 0.0114   | 0.0107 |
| WANG & SINGH    | $a = -0.0139$     | 0.9234| 1.1      | 106465 |
|                 | $b = 5.0149$      |       |          |        |
| APPRO OF DIF    | $k = 1.1408$      | 0.9562| 0.2088   | 0.2021 |
| MOD HENDER      | $k = 0.0165$      | 0.9506| 0.009    | 0.0082 |
|                 | $a = 0.2746$      |       |          |        |
|                 | $b = 0.2954$      |       |          |        |
|                 | $c = 0.3049$      |       |          |        |
|                 | $g = 0.0160$      |       |          |        |
|                 | $h = 0.0158$      |       |          |        |
VERMA ET AL  
k=1.1444  
a=0.1705  
g=0.0152  
0.9562  
0.0069  
0.0067  
a=1.0050.

MIDILLI & KUCUK  
k=0.2021  
n=0.1734  
b=0.0049  
0.9787  
0.4659  
0.4358  
Thompson  
a=-64.7773  
b=-8.0810  
0.9655  
0.8923  
0.9568

**Fig. 4:** Comparison of experimental and predicted dimensionless moisture ratio for solar drying

**Table 3:** Modelling the drying process of fermented cocoa beans using open sun drying.

| MODELS        | COEFFICIENTS | R²     | X²     | RMSE    |
|---------------|--------------|--------|--------|---------|
| NEWTON        | k=0.0199     | 0.9423 | 0.0089 | 0.0086  |
| PAGE          | k=0.0710     | 0.9617 | 0.0063 | 0.0061  |
| MODIFIED PAGE | k=0.0217     | 0.9617 | 0.1376 | 0.1339  |
| HENDERSON     | k=0.0160     | 0.9741 | 0.0044 | 0.0042  |
| &PABIS        | a=0.8334     |        |        |         |
| LOGARITHMIC   | k=0.0125     | 0.977  | 0.0038 | 0.0037  |
|               | a=0.9083     |        |        |         |
|               | c=-0.0955    |        |        |         |
| TWO TERM      | k=0.0160     | 0.9741 | 0.0045 | 0.0042  |
| &PAB          | a=0.1071     |        |        |         |
| TWO TERM MOD. | k=0.1656     | 0.9572 | 0.007  | 0.0066  |
| WANG &SINGH   | a=-0.0144    | 0.9149 | 1.27   | 1.1197  |
|              | b=5.3256     |        |        |         |
| APPRO OF DIF  | k=0.8487     | 0.9827 | 0.0032 | 0.0031  |
| MOD HENDER     | k=0.0159     | 0.9741 | 0.0048 | 0.0042  |
|              | a=0.2610     |        |        |         |
|              | b=0.2814     |        |        |         |
|              | c=0.2909     |        |        |         |
|              | g=0.0161     |        |        |         |
|              | h=0.0161     |        |        |         |
| VERMA ET AL   | k=0.1921     | 0.9866 | 0.0024 | 0.0023  |
|              | a=0.3235     |        |        |         |
|              | b=-0.0023    |        |        |         |
| MIDILLI &     | k=0.0192     | 0.9827 | 0.3246 | 1.4672  |
| KUCUK         | n=0.0040     |        |        |         |
|              | a=0.0040     |        |        |         |
|              | b=-0.0023    |        |        |         |
| THOMPSON      | a=-59.2342   |        |        |         |
|              | b=-4.0169    |        |        |         |
IV. CONCLUSION
The solar dryer proves useful for local farmer as it ensures high quality, good colour and flavour and reduces the drying time. It does not pollute the environment, requires minimal maintenance once it is installed and with good quality. In order to explain the drying behaviour and the mathematical models of fermented cocoa beans, thirteen models were applied to thin layer convective indirect solar dryer and open drying processes. The result showed that the Verma et al model was found to be the most suitable model for describing the drying curve of the convective indirect solar drying process of cocoa beans with $R^2 = 0.9562$, $x^2=0.0068$, MBE=0.0383 and RMSE=0.0067 while, the Midilli and Kucuk model, best described the drying curve of fermented cocoa beans under open sun with $R^2 = 0.9866$, $x^2=0.0024$, MBE=0.0078 and RMSE=0.0023.

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