Low-lying continuum states of drip-line Oxygen isotopes

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Motivation

Emission from doorway-state

A bound state is shifted to continuum suddenly through a nuclear reaction

V.S.

Emission from single-particle resonance
Spin-orbit splitting

Eigenvalues of HO potential

Magic numbers
Mayer and Jensen (1949)

126
82
50
28
20
8
2

Mayer & Jensen

d_{3/2}
Neutron single-particle energies at $N=20$ for $Z=8$−$20$

solid line: full (central + tensor)
dashed line: central only

$d_{3/2}$
bound in F
unbound in O
due to interplay between tensor and three-body forces

TO, et al. PRL 104, 012501 (2010)
One of the Backgrounds
Why is the drip line of Oxygen so near?

next issue → oxygen anomaly and continuum
The clue: Fujita-Miyazawa 3N mechanism (Δ-hole excitation)

Δ particle
m = 1232 MeV
S = 3/2, I = 3/2

Miyazawa, 2007
Pauli blocking effect on the renormalization of single-particle energy

Renormalization of single particle energy due to $\Delta$-hole excitation

$\rightarrow$ more binding (attractive)

Another valence particle in state $m'$

Pauli Forbidden $\rightarrow$ The effect is suppressed

Single particle states
Most important message with Fujita-Miyazawa 3NF

- Renormalization of single particle energy
- Effective monopole repulsive interaction
- Pauli blocking

Monopole part of Fujita-Miyazawa 3-body force
(i) Δ-hole excitation in a conventional way

(a) G-matrix NN + 3N (Δ) forces

(b) $V_{\text{low } k}$ NN + 3N ($\Delta, N^2\text{LO}$) forces

(ii) EFT with Δ

(iii) EFT incl. contact terms ($N^2\text{LO}$)

Δ-hole dominant role in determining oxygen drip line

(c) 3-body interaction

(d) 3-body interaction with one more neutron added to (c)
Ground-state energies of oxygen isotopes

\[ \text{NN force + 3N-induced NN force} \]

(Fujita-Miyazawa force)

\[ ^{16}\text{O core} \]

(a) Energies calculated from phenomenological forces
(b) Energies calculated from G-matrix NN + 3N (\(\Delta\)) forces
(c) Energies calculated from \(V_{\text{low k}}\) NN + 3N (\(\Delta, N^2\text{LO}\)) forces

**Drip line**
(i) Δ-hole excitation in a conventional way

- **G-matrix NN + 3N (Δ) forces**
- **V_{low \kappa} NN + 3N (Δ, N^{2}LO) forces**

(ii) EFT with Δ

- **EFT incl. contact terms (N^{2}LO)**

(iii) EFT with contact terms (N^{2}LO)

Δ-hole dominant role in determining oxygen drip line
Continuum-coupled shell model (CCSM)

Hamiltonian:

$$H = H_0 + \hat{V} = \sum_j \tilde{\epsilon}_j n_j + \hat{V}$$

$$H_0 = T + U_{WS} + V_{wall} = \sum_j \tilde{\epsilon}_j n_j$$

basis state-vector (denoted by $j$):
bound states + discretized continuum states
wall very far (3000 fm, ~3000 basis states)

$V_{NN} + \overset{16}{\text{O core}}$

$V_{NN}$ included

approximated by Gaussian
(i) $\Delta$-hole excitation in a conventional way

(a) G-matrix $\text{NN} + 3\text{N} (\Delta)$ forces

(b) $V_{\text{low }k} \text{NN} + 3\text{N} (\Delta, \text{N}^2 \text{LO})$ forces

(ii) EFT with $\Delta$

$\Delta$-hole dominant role in determining oxygen drip line

phenomenological shell model

$\Delta$-hole excitation in a conventional way

(a) G-matrix $\text{NN} + 3\text{N} (\Delta)$ forces

(b) $V_{\text{low }k} \text{NN} + 3\text{N} (\Delta, \text{N}^2 \text{LO})$ forces

(c) 3-body interaction

(d) 3-body interaction with one more neutron added to (c)

$^{16}\text{O}$ core
\[ \hat{V}(r) = \sum_{i=1,2} g_i (1 + a_i \sigma \cdot \sigma) e^{-r^2 / d_i^2} \]

\[ d_{1,2} = 1.4, \ 0.7 \ \text{fm} \]

SDPF-M TBME = TBME of this \( V(r) \) for HO wave functions

\[ \langle 1s_{1/2} 0d_{3/2} | V | 1s_{1/2} 0d_{3/2} \rangle_{J=1,2} \]
\[ \langle 0d_{3/2} 0d_{3/2} | V | 0d_{3/2} 0d_{3/2} \rangle_{J=0,2} \]

under the assumption that 3-body force effect is included in SDPF-M interaction effectively

\( V(r) \) is fixed only by interaction
$^{24}O = ^{22}O + 2n$ in the space

**ground state:** $2n$ in $1s_{1/2}$

**excited states of $1^+$ and $2^+$:**

\[ |iJ^+\rangle = |1s_{1/2} \otimes id_{3/2}; J^+\rangle \]

**discretized continuum $id_{3/2}$ ($i = 1, 2, ...$)**

$1s_{1/2}$ : solution of Woods-Saxon potential with observed $S_n$

**diagonalize $H$**

**Eigenfunctions:**

\[ |J_k^+\rangle = \sum_i c_i^{(J,k)} |iJ^+\rangle \]
Reaction mechanism

\[ \rightarrow \text{Doorway state} \]
Removal of one proton and one neutron from $^{26}\text{F}$

**Knockout Reaction @ MSU (2009)**

- $^{9}\text{Be}(^{26}\text{F},^{24}\text{O})X$
  - C. Hoffman, M. Thoennessen et al.

$^{26}\text{F}$

$^{16}\text{O}$

$^{16}\text{O}$

$^{16}\text{O}$

$^{16}\text{O}$

**Ground State**

- $1s_{1/2}$ is bound.
  - Kanungo et al. (2009)

**Doorway State**

$|1s_{1/2}0d_{3/2}; J_k^+\rangle$

**Continuum**

**Excited States in $^{24}\text{O}$**

$$H_{CCSM}^k |J_k^+\rangle = E_k |J_k^+\rangle$$

- Less probable
- $$\leq$$ large $s_{1/2}$-$d_{3/2}$ neutron gap
Neutron single-particle energies at N=20 for Z=8~20

- Solid line: full (central + tensor)
- Dashed line: central only

$d_{3/2}$
- Bound in F
- Unbound in O

Due to interplay between tensor and three-body forces

TO, et al. PRL 104, 012501 (2010)
Removal of one proron and one neutron from $^{26}$F

Before the removal, neutron $d_{3/2}$ is well-bound in $^{26}$F and can be described by a HO wave function.

Sudden removal $\rightarrow$ doorway state with HO $d_{3/2}$

Decay of neutron from this $d_{3/2}$ through overlap with continuum states:

$$\zeta^{(J)}_k \equiv \langle J^+_k | 1s_{1/2}0d_{3/2}; J^+ \rangle = \sum_i c^{(J,k)}_i \langle id_{3/2} | 0d_{3/2} \rangle$$

$$p^{(J)}_k \equiv |\zeta^{(J)}_k|^2 \quad \rightarrow \quad \text{Spectrum of emitted neutron}$$
Low-lying Continuum Spectra in $^{24}$O

Doorway state $\rightarrow$ continuum states in $^{24}$O

$p_k^J = |\langle J_k^+ | \Phi_{\text{doorway}} \rangle|^2 = | \sum_i C_i^{(k)} \langle id_{3/2} | 0d_{3/2} \rangle |^2$

- **bound approximation:**
  Normal shell model with the same Hamiltonian: NO continuum effect

- **CCSM:** With continuum effect
  incl. residual interaction

- **no int.:** With continuum effect but no residual interaction.

- Continuum effect is about 1 MeV
- No bound excited state.
- $1^+-2^+$ splitting by 2-body interaction
- $1^+-2^+$ splitting is in good agreement with experiments.
Notable difference between $1^+$ and $2^+$ states.

The peak states in CCSM reproduce the behavior of "resonance wave" at far distance (phase shift of $\pi/2$).
Peak Energies of neutron emission

Continuum spectra are consistent with the shell evolution.
The results do not change so much if $L$ is taken to be sufficiently large.

Even usual values of $L \sim 50$ fm are not stable.
Comparison to single-particle resonance
Effective phase shift and one-body reduction

Can many-body resonance be described by effective one-body problem?

CCSM: continuum spectra are obtained by taking the overlap between the doorway state and CCSM eigenstates in continuum.

Effective phase shift

We define effective phase shift by introducing 1-body reduction of CCSM wave function.

\[
|J_{k}^{+}\rangle = \sum_{i} c_{i}^{(J,k)} |1s_{1/2} \otimes id_{3/2}; J\rangle
\]

\[
=: |1s_{1/2} \otimes \tilde{d}_{3/2}; J, k; J\rangle
\]

\[
|\tilde{d}_{3/2}; J, k; J\rangle = \sum_{i} c_{i}^{(J,k)} |id_{3/2}\rangle
\]

One can then obtain phase shift, and can use it for calculating the cross section.

\[
\sigma_{J} = \frac{4\pi}{k^{2}} (2l + 1) \sin^{2} \delta_{J}
\]
Effective phase shift and one-body reduction

- CCSM (doorway state approach) and effective phase shift approach give very similar results for peak positions.

Notable difference appears for the width of $2^+$ in $^{24}\text{O}$.
- Doorway state decays faster.

| Unit          | $^{23}\text{O}$ | $^{24}\text{O}$ |
|---------------|------------------|------------------|
| states        | $3/2^+$          | $1^+$            | $2^+$            |
| CCSM E        | 0.92             | 1.35             | 0.61             |
| CCSM $\Gamma$ | 0.11             | 0.28             | **0.06**         |
| Phase shift E | 0.92             | 1.36             | 0.61             |
| Phase shift $\Gamma$ | 0.11     | 0.28             | **0.04**         |

50% longer life time
Phase shift

\[ \tan \delta_J \]

\[ E \text{ [MeV]} \]

\[ \Delta \, {}^{23}\text{O} \, 3/2^+ \]

\[ {}^{24}\text{O} \, 1^+ \]

\[ {}^{24}\text{O} \, 2^+ \]
Although resonance state and doorway state are different, continuum spectra are similar.

What is the meaning of single-particle resonance states in complex dynamical processes such as multi-nucleon transfer heavy-ion reactions???

Time scale of the heavy-ion reaction may be shorter than the resonance life time.

Coupling to continuum lowers the (peak) energies by more than 1 MeV for oxygen isotopes.