\( \eta d \) scattering in the region of the \( S_{11} \) resonance

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Abstract

We have studied the reaction \( \eta d \rightarrow \eta d \) close to threshold within a nonrelativistic three-body formalism. We considered several \( \eta N \) and \( NN \) models, in particular potentials with separable form, fitted to the low-energy \( \eta N \) and \( NN \) data to represent the two-body interactions. We found that with realistic two-body interactions a quasibound state does not exist in this system, although there is an enhancement of the cross section by one order of magnitude, in the region near threshold, which is a genuine three-body effect not predicted within the impulse approximation.

14.40.Ag,25.40.Ve,25.80.Hp
I. INTRODUCTION

The elastic scattering $\eta d$ reaction has been studied recently by several authors \cite{1-4} in order to investigate the existence of a resonance or a quasibound state in this system, for which there are experimental indications \cite{5-8}. Some of these studies concluded that such an state would exist for certain values of the two-body $\eta N$ data. However, since they used in one form or another incomplete information on the two-body subsystems (in particular that corresponding to the $\eta N$ sector), we believe that a new calculation is required which takes into account all the information that is now available. In particular, we study the effect of the repulsion at short distances of the $NN$ interaction and take into account the $\eta N \rightarrow \eta N$ scattering amplitude that has been determined recently \cite{9-12}.

We will present the Faddeev formalism in section II. In section III we will first calculate the eta-deuteron scattering length to compare with the results of multiple-scattering theories as well as with separable potentials models and finally we will present the predictions of our model for $\eta d$ scattering. We will give our conclusions in section IV.

II. FADDEEV FORMALISM

Let us consider a system of three particles, where two of them are identical, interacting pairwise through separable potentials that act only in S-waves. In the case of the $\eta d$ system, S-wave means, for the eta-nucleon pair the $S_{11}$ channel, and for the nucleon-nucleon pair the $^3S_1$ channel. The two-body T-matrix of the pair $jk$ will be assumed of the separable form

$$t_{ij}(p_i, p'_i; E) = g_i(p_i) \tau_i(E) g_i(p'_i),$$

where $\tau_i(E)$ and $g_i(p_i)$ will be specified later. In the following we will identify particle 1 with the $\eta$ and the identical particles 2 and 3 with the two nucleons.

The Faddeev equations for the case of $\eta d$ scattering can be solved explicitely for the $\eta d \rightarrow N(N\eta)$ transition amplitude $T_2$, with nucleon 2 being the spectator particle in the final state. One obtains
\[ T_2(q_2; E) = 2K_{21}(q_2, q_{10}; E) + \int_0^\infty q_2^2 dq_2' M(q_2, q_2'; E)\tau_2(E - q_2^2/2\nu_2)T_2(q_2'; E), \]

where

\[ M(q_2, q_2'; E) = K_{23}(q_2, q_2'; E) + 2\int_0^\infty q_1^2 dq_1 K_{21}(q_2, q_1; E)\tau_1(E - q_1^2/2\nu_1)K_{12}(q_1, q_2'; E), \]

and the driving term for the transition from the amplitude with spectator j in the initial state to the one with spectator i in the final state is

\[ K_{ij}(q_i, q_j; E) = \frac{1}{2} \int_{-1}^{1} d\cos\theta \frac{g_i(p_i)g_j(p_j)}{E - p_i^2/2\mu_i - p_j^2/2\nu_i + i\epsilon}, \]

with

\[ p_i = \left( \frac{\mu_i^2}{m_i^2} q_i^2 + q_i^2 + 2\frac{\mu_i}{m_k} q_i q_j \cos\theta \right)^{1/2}, \]

\[ p_j = \left( \frac{\mu_j^2}{m_j^2} q_j^2 + q_j^2 + 2\frac{\mu_j}{m_k} q_i q_j \cos\theta \right)^{1/2}. \]

\( \mu_i \) and \( \nu_i \) are the reduced masses

\[ \mu_i = \frac{m_j m_k}{m_j + m_k}, \]

\[ \nu_i = \frac{m_i(m_j + m_k)}{m_i + m_j + m_k}, \]

and the \( \eta d \) on-shell relative momentum \( q_{10} \) is defined by the relation

\[ E = \frac{q_{10}^2}{2\nu_1} - B_d, \]

where \( B_d \) is the binding energy of the deuteron.

The \( \eta d \) elastic-scattering amplitude is obtained from the solution of Eq. (2) by inserting into it a final \( \eta d \) state, as

\[ F_{\eta d}(q_{10}) = -\frac{\pi \nu_1}{N} \int_0^\infty q_2^2 dq_2 K_{12}(q_{10}, q_2; E)\tau_2(E - q_2^2/2\nu_2)T_2(q_2; E). \]

where \( N \) is the normalization of the deuteron wave function.
\[ N = \int_0^\infty p_1^2 dp_1 \left[ \frac{g_1(p_1)}{B_d + p_1^2/2\mu_1} \right]^2. \]  \hspace{1cm} (11)

The \( \eta d \) scattering length is given by

\[ A_{\eta d} = F_{\eta d}(0), \]  \hspace{1cm} (12)

while the integrated elastic cross section is given by

\[ \sigma_{\text{ELAS}} = 4\pi|F_{\eta d}(q_{10})|^2. \]  \hspace{1cm} (13)

Notice that Eqs. (2) - (4) and (10) do not include the \( \pi N \) channel explicitly, but only through the inelasticity of the \( \eta N \) channel. As for the \( \eta N \) inelasticity due to the \( \pi \pi N \) channel, its contribution is not yet included at this stage of the calculations.

### III. RESULTS

We started by calculating the \( \eta d \) scattering length (12), by solving the integral equation (2) with the method of matrix inversion after replacing the integration with a gaussian mesh. Notice that in this case Eqs. (2)-(4) are free of singularities (the singularity of \( \tau_1 \) at \( E = -B_d \) in eq. (3) occurs for \( q_1 = 0 \) and is regularized by the integration volume element).

Additionally, we also calculated the integrated elastic cross section of \( \eta d \) scattering. In order to solve the integral equation (2) above threshold, we used the method of contour rotation [13].

#### A. \( A_{\eta d} \) with non-dynamical separable models

We will calculate here the \( \eta d \) scattering length \( A_{\eta d} \) for the models proposed in Refs. [3,4]. The signal that a quasibound state exists for a given model is that the real part of \( A_{\eta d} \) becomes negative while the imaginary part gets large.

Notice that in Eq. (1) we have assumed a separable model for the two-body amplitudes \( t_i \). This form of the T-matrix is obtained if one assumes a separable potential between particles.
and $k$: a dynamical two-body equation determines the function $\tau_i(E)$ by the form factors $g_i(p_i)$, which carry information on the range of the potential, and the strength parameter of the potential. However, in the AGS formalism [14] used in Ref. [4] and in the multiple-scattering approach used in Ref. [3] the function $\tau_2(E)$ for the $\eta N$ subsystem, instead of being calculated, has been chosen independently of the form factor $g_2(p_2)$. Such an assumption violates the spirit of the Faddeev approach which requires a two-body interaction in order to relate the off-shell behavior of the T-matrix in the energy variable $E$ to the off-shell behavior in the momentum variable $p_2$. Nevertheless, it is instructive to repeat those calculations in order to check the accuracy of our numerical solution by comparing with the exact result of Ref. [4] as well as to test the convergence of the multiple-scattering scheme developed in [3] which is based in a partial summation of the multiple-scattering series.

In both Refs. [3,4] the form factor $g_2(p_2)$ has been taken of the Yamaguchi form

$$g_2(p_2) = \frac{1}{\alpha_2^2 + p_2^2},$$ \hspace{1cm} (14)

with $\alpha_2 = 3.316 \text{ fm}^{-1}$. In Ref. [4], the function $\tau_2(E)$ has been parametrized as

$$\tau_2(E) = \frac{\lambda_\eta}{E - E_0 + i\Gamma/2},$$ \hspace{1cm} (15)

with $E_0 = 1535 \text{ MeV} - (m_N + m_\eta)$ and $\Gamma = 150 \text{ MeV}$. The parameter $\lambda_\eta$ in Eq. (15) was chosen to reproduce the complex $\eta N$ scattering length $a_{\eta N}$, by using the relation

$$t_2(0,0;0) = -\frac{a_{\eta N}}{\pi \mu_2},$$ \hspace{1cm} (16)

where $\mu_2$ is the $\eta N$ reduced mass. This leads to

$$\lambda_\eta = \frac{\alpha_2^4(E_0 - i\Gamma/2)}{\pi \mu_2} a_{\eta N}.$$ \hspace{1cm} (17)

As for the nucleon-nucleon separable T-matrix used in Ref. [4], it was generated from a Yamaguchi separable potential with an energy-dependent strength [4,15]. Using these parameters we calculated the $\eta d$ scattering length $A_{\eta d}$ with the formalism described in the previous section for a variety of values of the $\eta N$ scattering length $a_{\eta N}$ that have been proposed in the literature (see Refs. [3,4]). We show the results of this comparison in table
I. The results of Ref. [4] using the AGS formalism are given in column two and the ones obtained with the formalism of the previous section are given in column three. As one can see from table I there is very good agreement between the two calculations. The small discrepancies shown in some cases are of no significance since they occur for the values of $a_{\eta N}$ allowing the quasibound state to occur and thereby the solutions are highly unstable. In our case, in this situation we had to use a large number of mesh points in order to guarantee stability.

In the multiple-scattering approach of Ref. [3] an approximate formula was used which is based in a partial summation of the multiple-scattering series. The function $\tau_2(E)$ was taken to be constant

$$\tau_2(E) = \lambda_{\eta}.$$  \hspace{1cm} (18)

Using the relation (18) this gives

$$\lambda_{\eta} = -\frac{\alpha_4^2}{\pi \mu_2} a_{\eta N},$$ \hspace{1cm} (19)

which will be referred to as their model I. They used also a second model which will be referred to as model II for which instead of Eq. (19) they took

$$\lambda_{\eta} = -\frac{\alpha_4^2}{\pi \mu_2} \frac{a_{\eta N}}{1 - iq_0 a_{\eta N}},$$ \hspace{1cm} (20)

with $q_0 = 0.367$ fm$^{-1}$. For the nucleon-nucleon interaction they used a Yamaguchi separable potential with a range parameter $\alpha_1 = 1.41$ fm$^{-1}$.

We compare in table II the results of our exact calculations which we obtained using the parameters of Ref. [3] with their results using an approximate formula for the two models I and II. As it can be seen from this table, the approximate formula of the multiple scattering series works very well for small values of $a_{\eta N}$, as expected from convergence arguments. When $a_{\eta N}$ is large the multiple scattering series formula deviates more from the exact result, nevertheless, it is still qualitatively correct, since it predicts correctly the quasibound states in all the cases where they exist for both models.
B. \( \eta d \) with separable-potential models

In the previous subsection we have seen that the models of Refs. [3,4] predict a quasi-bound state if the real part of \( a_{\eta N} \) is of the order of 0.7-0.8 fm. However, since their \( \eta N \) T-matrix is not derived from a potential their function \( \tau_2(E) \) is not constrained by their form factor \( g_2(p_2) \). We will therefore construct separable potential models of the coupled \( \eta N - \pi N \) system that reproduce in one case just the complex scattering length \( a_{\eta N} \) for arbitrary values of the \( \eta N \) range-parameter \( \alpha_2 \) and in another case the full \( \eta N-\eta N \) scattering amplitude around the \( S_{11} \) resonance.

Similarly, we will consider two different models of the \( NN \) interaction; a simple Yamaguchi potential that does not have short-range repulsion and a PEST model which has the same half-off-shell behavior as the Paris potential so that it contains short-range repulsion.

If we use in the Lippmann-Schwinger equation of the coupled \( \eta N - \pi N \) system the separable model

\[
< p|V_{\eta\eta}|p' > = \lambda_{\eta}g_2(p)g_2(p'),
\]

(21)

\[
< p|V_{\pi\pi}|p' > = \lambda_{\pi}g_\pi(p)g_\pi(p'),
\]

(22)

\[
< p|V_{\eta\pi}|p' > = \pm \sqrt{\lambda_{\eta}\lambda_{\pi}}g_2(p)g_\pi(p'),
\]

(23)

the T-matrices are of the form

\[
< p|t_{\eta\eta}(E)|p' >= g_2(p)\tau_2(E)g_2(p'),
\]

(24)

\[
< p|t_{\pi\pi}(E)|p' > = \frac{\lambda_{\pi}}{\lambda_{\eta}}g_\pi(p)\tau_2(E)g_\pi(p'),
\]

(25)

\[
< p|t_{\eta\pi}(E)|p' > = \pm \sqrt{\frac{\lambda_{\pi}}{\lambda_{\eta}}}g_2(p)\tau_2(E)g_\pi(p'),
\]

(26)

with
\[ \frac{1}{\tau_2(E)} = \frac{1}{\lambda_\eta} - G_2(E) - \frac{\lambda_\pi}{\lambda_\eta} G_\pi(E), \quad (27) \]

\[ G_2(E) = \int_0^\infty p^2 dp \frac{g_2^2(p)}{E - p^2/2\mu_2 + i\epsilon}, \quad (28) \]

\[ G_\pi(E) = \int_0^\infty p^2 dp \frac{g_\pi^2(p)}{E + p_0^2/2\mu_\pi - p^2/2\mu_\pi + i\epsilon}. \quad (29) \]

\[ \mu_2 \text{ and } \mu_\pi \text{ are the } \eta N \text{ and } \pi N \text{ reduced masses respectively while } p_0 \text{ is the } \pi N \text{ relative momentum at the } \eta N \text{ threshold, i.e.,} \]

\[ p_0^2 = \frac{[s_0 - (m_\pi + m_N)^2][s_0 - (m_\pi - m_N)^2]}{4s_0}, \quad (30) \]

with

\[ s_0 = (m_\eta + m_N)^2. \quad (31) \]

If we use simple Yamaguchi form factors

\[ g_2(p) = \frac{1}{\alpha_2^2 + p^2}, \quad (32) \]

\[ g_\pi(p) = \frac{1}{\alpha_\pi^2 + p^2}, \quad (33) \]

we find that the strengths \( \lambda_\eta \) and \( \lambda_\pi \) can be obtained in terms of the real and imaginary parts of \( a_{\eta N} \) as

\[ \lambda_\eta^{-1} = -\frac{\pi \mu_2}{2\alpha_\eta^2} - \frac{\alpha_\eta^2}{2\alpha_\pi p_0} \pi \mu_2 \text{ Im } a_{\eta N} \alpha_\eta^4 \frac{\alpha_\eta^2}{\alpha_\pi^4} |a_{\eta N}|^2 - \frac{\pi \mu_2}{\alpha_2^4} \frac{\text{ Re } a_{\eta N}}{|a_{\eta N}|^2}, \quad (34) \]

\[ \lambda_\pi = \lambda_\eta \frac{\mu_2 (\alpha_\pi^2 + p_0^2)^2}{p_0 \alpha_\pi^4} \text{ Im } a_{\eta N} \frac{1}{|a_{\eta N}|^2}. \quad (35) \]

Since we do not include the pion channel explicitly but only through the function \( \tau_2(E) \) (see Eq. (27)), we will fix the range of the \( \pi N \) potential to the value \( \alpha_\pi = p_0 \), for which case the second term in the r.h.s. of Eq. (34) drops out and it becomes clear that in this case the strength of the \( \eta N \) potential is determined by the real part of the \( \eta N \) scattering length and
the strength of the \( \pi N \) potential is determined by the imaginary part of the \( \eta N \) scattering length for a given value of the range \( \alpha_2 \). Since this models are based in a Yamaguchi form factor for the \( \eta N \) potential we will refer to them as \( Y_{\eta N} \) models.

We constructed also more realistic separable models that reproduce not only the \( \eta N \) scattering length but also the most important features of the \( S_{11} \) resonance such as its position and width. For this we considered the \( S_{11} \) amplitudes obtained from the analyses of Refs. [10–12]. We found that with a simple Yamaguchi model of the \( \eta N \) form factor is not possible to generate a resonance in the \( \eta N S_{11} \) channel. We therefore changed the form factor \( g_2(p) \) instead of Eq. (32) to

\[
g_2(p) = \frac{A + p^2}{(\alpha_2^2 + p^2)^2},
\]

while keeping for the \( \pi N \) form factor the Yamaguchi form (33). We give in table III the parameters \( \alpha_{\pi}, \lambda_{\pi}, \alpha_2, A, \lambda_{\eta} \) of the coupled \( \eta N-\pi N \) separable potentials fitted to the \( S_{11} \) amplitudes of [10,11] as well as to the models A, B, C, and D of [12]. We show in Fig. 1, as an example, the \( \eta N-\eta N \) amplitude of Ref. [10] (dashed lines) compared with the ones of our separable-potential model. Similar results are obtained for the other models. Since these models generate a resonance in the \( \eta N \) channel we will refer to them as \( R_{\eta N} \) models.

In the case of the \( NN \) interaction we have considered two models; the simple Yamaguchi model used in Ref. [3] which has a range parameter \( \alpha_1 = 1.41 \text{ fm}^{-1} \) (which we will refer to as the \( Y_{NN} \) model) and the PEST potential constructed in Ref. [16] (which we will refer to as the \( P_{NN} \) model) that is of the form

\[
g_1(p_1) = \sum_{n=1}^{6} \frac{C_n}{p_1^2 + \beta_n},
\]

where the parameters \( C_n \) and \( \beta_n \) are given in Ref. [16]. The half-off-shell T-matrix of this separable potential has the same behavior as that of the Paris potential and therefore it takes into account the repulsion at short distances that is present in the nucleon-nucleon force.
We give in table IV the results of our separable-potential models for the $\eta d$ scattering length $A_{\eta d}$ where we have considered all four combinations of the $\eta N$ and $NN$ separable-potential models. In the case of the $\eta N$ Yamaguchi model $Y_{\eta N}$ we took the range parameter $\alpha_2 = 3.316$ fm$^{-1}$ which is the same as in Refs. [4,3]. Here however the corresponding T-matrix is calculated through the Lippmann-Schwinger equation. In the first column we give the reference for the $\eta N$ $S_{11}$ amplitude that we used to fit the $R_{\eta N}$ model and in the second column we give the $\eta N$ scattering length of that amplitude which has been used to construct the $Y_{\eta N}$ model. The third column gives $A_{\eta d}$ using simple Yamaguchi models for the $\eta N$ and $NN$ interactions and it shows that even with these simple separable models the quasibound state only appears when $Re\ a_{\eta N}$ is about 1.05-1.07 fm while in the multiple-scattering approaches of the previous section it appeared already with $Re\ a_{\eta N} \approx 0.6-0.9$ fm. The fourth column gives the results of the Yamaguchi model for the $\eta N$ amplitude and the PEST model for the $NN$ amplitude and it shows that the $NN$ short-range repulsion also works against quasibinding since it wipes out the quasibound state although the $Im\ A_{\eta d}$ remains large. The fifth and sixth columns contain the results of the resonant model of the $\eta N$ amplitude with Yamaguchi and PEST models for the $NN$ interaction respectively, and they show that the attraction of the system is greatly reduced when one takes into account the resonant nature of the $\eta N$ amplitude. Notice, however, that experimentally the effects will be similar whether there is a quasibound state or not since in both cases there will be an enhancement of the cross section at threshold.

**C. $\eta d$ scattering**

We calculated the integrated elastic cross section of $\eta d$ scattering in the region of the $S_{11}$ resonance for the six resonant models of the $\eta N$ interaction given in table III and the realistic PEST potential for the $NN$ interaction. We show in Fig. 2 the results of the three-body model (solid lines) and of the impulse approximation (dashed lines). At threshold, the results of all the three-body models are about one order of magnitude larger than those
of the impulse approximation while at higher energies they are 2 or 3 times smaller. The behavior of the cross section at threshold indicates that even though the quasibound state is not present, the interaction in this region is very strong since it enhances the impulse approximation result by about one order of magnitude. Thus, a signal of this behavior may appear also in other processes where there is an $\eta N$ final state like the $np \rightarrow \eta d$ reaction where a large enhancement in the cross section has been observed in the region near threshold.

In order to illustrate the effect of the strong $\eta d$ interaction in the reaction $np \rightarrow \eta d$ we will estimate the enhancement of the $np \rightarrow \eta d$ cross section at threshold due to the $\eta d$ rescattering. We write the amplitude of the process $np \rightarrow \eta d$ as

$$A = B + BG_0 T_{\eta d},$$

where $B$ is the amplitude of the $np \rightarrow \eta d$ process without $\eta d$ rescattering, $G_0$ is the two-body Lippmann-Schwinger propagator of the intermediate $\eta d$ state, and $T_{\eta d}$ is the half-off-shell $T$-matrix of the elastic $\eta d$ process. If we introduce the $\eta d$ elastic-scattering amplitude $F_{\eta d} = -\frac{\pi}{\mu_{\eta d}} T_{\eta d}$, where $\mu_{\eta d}$ is the $\eta d$ reduced mass, then at threshold, Eq. (38) is written explicitly as

$$A = B(0)[1 + \frac{2}{\pi} \int_0^\infty dq_1 \frac{B(q_1)}{B(0)} F_{\eta d}(q_1)],$$

where $F_{\eta d}(q_1)$ is given by Eq. (10) with $q_{10}$ replaced by $q_1$. Therefore, the enhancement factor of the $np \rightarrow \eta d$ cross section due to $\eta d$ rescattering is

$$f = |1 + \frac{2}{\pi} \int_0^\infty dq_1 \frac{B(q_1)}{B(0)} F_{\eta d}(q_1)|^2.$$

The amplitude $B$ of the $np \rightarrow \eta d$ process without $\eta d$ rescattering is presumably given by meson exchanges such as $\pi$, $\rho$, and $\eta$ followed by the excitation and decay of the $S_{11}$ resonance. The explicit form of the production operator without $\eta d$ rescattering is not so important since in our estimate of the enhancement factor given by Eq. (40) only the ratio $B(q_1)/B(0)$ enters. Therefore, we take for it the $\eta$-exchange amplitude generated by our three-body model, i.e.,
\[ B(q_1) = \int_0^\infty q_2^2 dq_2 K_{12}(q_1, q_2; E) \tau_2(E - q_2^2/2\nu_2)K_{23}(q_2, q_3; E), \]  

where \( E = -B_d \) and \( q_3 = \sqrt{(m_\eta + m_d)^2 - m_N^2} \) is the momentum corresponding to an initial \( NN \) state. Using the models 1-6 of Fig. 2, taken from references [10–12] and included in Table IV, we obtained the enhancement factors \( f = 2.5, 2.7, 3.1, 3.3, 4.7, \) and \( 5.1 \) respectively. These estimates are quite comparable to the enhancement factors observed in Ref. [7], in special for the 4 first models.

**IV. CONCLUSIONS**

Recently, the experimental band for the \( \eta N \) scattering length \( a_{\eta N} \) has been pushed towards larger values for its real part [11,12]. For those larger values, the \( \eta N \) models not generated directly from an integral equation, which would fix naturally their off-energy-shell behavior needed in three-body calculations, predict a quasi-bound \( \eta NN \) state. If instead, the new data is used to generate \( \eta N \) t-matrices calculated from a potential and an integral equation, our results indicate that a realistic NN interaction, like the Paris potential, through its short-range correlations, prevents the existence of the bound-state, independently of the \( \eta N \) models, provided they have been built dynamically. We confirmed then that the predictions of a \( \eta NN \) eta-mesic nucleus are crucially affected by the off-shell behavior of the underlying \( \eta -N \) models. Importantly, however, is that even for an inexisting quasi-bound state, an exact three-body calculation for the multi-scattering series in the final state predicts a severe enhancement of the elastic \( \eta d \) cross-section in the narrow region from threshold to 5-10 MeV above threshold. This result is independent of the \( \eta N \) two-body models. Very likely, the enhancement predicted by the exact three body calculations is related to the one observed in the reactions \( np \rightarrow \eta d \) [7] and \( \gamma d \rightarrow \eta d \) [8,13]. We actually made an estimate of the enhancement of the cross section of \( np \rightarrow \eta d \) due to the \( \eta d \) final state interaction. Within the three-body model of our work, this enhancement is in the ballpark of the empirical findings of [4].
ACKNOWLEDGMENTS

This work was supported in part by COFAA-IPN (México) and by Fundação para a Ciência e a Tecnologia, MCT (Portugal) under contracts PRAXIS XXI/BCC/18975/98 and PRAXIS/P/FIS/10031/1998.
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FIGURES

FIG. 1. Real and imaginary parts of the $\eta N \to \eta N$ $S_{11}$ amplitude. The solid and dashed lines are the results of our resonant $R_{\eta N}$ separable potential model 1; the symbols give the amplitudes of Ref. [10].

FIG. 2. Integrated elastic cross sections of the three-body model (solid lines) and of the impulse approximation (dashed lines) for the six resonant $\eta N$ models of table III using for the $NN$ interaction the PEST model, as a function of the c.m. $\eta d$ kinetic energy.
TABLES

TABLE I. $\eta d$ scattering length $A_{\eta d}$ (in fm) as calculated in Ref. [4] with the AGS formalism and as calculated here with the Faddeev formalism for different values of the $\eta N$ scattering length $a_{\eta N}$ (in fm).

| $a_{\eta N}$ | AGS [4]   | Faddeev  |
|-------------|-----------|----------|
| 0.25+i0.16  | 0.73+i0.56| 0.73+i0.56|
| 0.30+i0.30  | 0.61+i1.22| 0.61+i1.22|
| 0.46+i0.29  | 1.31+i1.99| 1.33+i1.99|
| 0.579+i0.399| 0.34+i3.31| 0.34+i3.33|
| 0.876+i0.274| -8.81+i4.30| -9.02+i4.15|
| 0.98+i0.37  | -4.69+i1.59| -4.74+i1.53|

TABLE II. $\eta d$ scattering length (in fm) for models I and II calculated in Ref. [3] using a multiple-scattering theory (MST) and as calculated here using the Faddeev formalism for different values of the $\eta N$ scattering length $a_{\eta N}$ (in fm).

| $a_{\eta N}$ | MST I [3] | Faddeev | MST II [3] | Faddeev |
|-------------|-----------|---------|------------|---------|
| 0.25+i0.16  | 0.66+i0.71| 0.64+i0.71| 0.66+i0.58| 0.65+i0.58|
| 0.30+i0.30  | 0.39+i1.28| 0.38+i1.25| 0.58+i1.11| 0.56+i1.09|
| 0.46+i0.29  | 0.72+i2.04| 0.62+i1.95| 1.11+i1.54| 1.04+i1.54|
| 0.579+i0.399| -0.13+i2.64| -0.08+i2.31| 0.93+i2.41| 0.74+i2.28|
| 0.876+i0.274| -2.76+i4.24| -1.54+i2.55| 2.42+i5.55| 0.46+i5.21|
| 0.98+i0.37  | -2.75+i2.77| -1.16+i2.05| -0.06+i6.20| -1.12+i4.21|
TABLE III. Parameters of the $\eta N$-$\pi N$ separable potential models fitted to the $S_{11}$ resonant amplitudes given in Refs. [10-12].

| Model | Ref. | $a_{\eta N}$ | $\alpha_\pi$ | $\lambda_\pi$ | $\alpha_2$ | $A$ | $\lambda_\eta$ |
|-------|------|-------------|-------------|-------------|-------------|-----|-------------|
| 1     | [6]  | 0.72+i0.26  | 1.2         | -0.028013   | 15.5        | 21.394872 | -8817.102932 |
| 2     | [7]  | 0.75 + i0.27| 1.2         | -0.028013   | 14.5        | 18.657140 | -7222.670614 |
| 3     | [8](D)| 0.83 + i0.27| 1.2         | -0.028013   | 13.68       | 17.7754   | -5970.573595 |
| 4     | [8](A)| 0.87+i0.27  | 1.2         | -0.028013   | 13.1        | 16.646837 | -5215.563508 |
| 5     | [8](B)| 1.05 + i0.27| 1.2         | -0.028013   | 11.1        | 13.357879 | -3080.397824 |
| 6     | [8](C)| 1.07+i0.26  | 1.2         | -0.028013   | 10.47       | 11.952401 | -2579.494411 |

TABLE IV. $\eta d$ scattering length (in fm) for various models of the $\eta N$ and $NN$ interactions where the respective t-matrices were calculated through a dynamical equation. $Y_{\eta N}$ stands for the Yamaguchi model of the $\eta N$ interaction with a range parameter $\alpha_2 = 3.316$ fm$^{-1}$, $R_{\eta N}$ stands for the resonant model of the $\eta N$ interaction, $Y_{NN}$ stands for the Yamaguchi model of the $NN$ interaction, and $P_{NN}$ stands for the PEST model of the $NN$ interaction. The first column indicates the reference on which the resonant $\eta N$ model is based and the second column indicates the $\eta N$ scattering length (in fm) of that model.

| Ref. | $a_{\eta N}$ | $Y_{\eta N}Y_{NN}$ | $Y_{\eta N}P_{NN}$ | $R_{\eta N}Y_{NN}$ | $R_{\eta N}P_{NN}$ |
|------|-------------|------------------|------------------|------------------|------------------|
| [6]  | 0.72+i0.26  | 2.38+i3.04       | 2.59+i2.37       | 2.43+i1.60       | 2.46+i1.62       |
| [7]  | 0.75+i0.27  | 2.38+i3.41       | 2.70+i2.64       | 2.56+i1.65       | 2.61+i1.72       |
| [8](D)| 0.83+i0.27  | 2.57+i4.51       | 3.21+i3.34       | 3.00+i1.87       | 3.10+i2.03       |
| [8](A)| 0.87+i0.27  | 2.54+i5.20       | 3.47+i3.78       | 3.20+i1.94       | 3.36+i2.19       |
| [8](B)| 1.05+i0.27  | 0.06+i8.66       | 4.27+i7.05       | 4.20+i2.24       | 4.81+i3.19       |
| [8](C)| 1.07+i0.26  | -0.42+i9.21      | 4.53+i7.60       | 4.21+i2.04       | 5.02+i3.14       |
