FURTHER EXAMPLES OF STABLE BUNDLES OF RANK 2 WITH 4 SECTIONS

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Abstract. In this paper we construct new examples of stable bundles of rank 2 of small degree with 4 sections on a smooth irreducible curve of maximal Clifford index. The corresponding Brill-Noether loci have negative expected dimension of arbitrarily large absolute value.

1. Introduction

It has been apparent for some time that the classical Brill-Noether theory for line bundles on a smooth irreducible curve does not extend readily to bundles of higher rank. Some aspects of this have been clarified recently by the introduction of Clifford indices of higher rank [7]. An example of a stable rank-3 bundle with Clifford index less than the classical Clifford index on a general curve of genus 9 or 11 is given in [8], disproving a conjecture of Mercat [9]. Very recently, it was proved in [4] that there exist curves of any genus \( \geq 11 \) for which the rank-2 Clifford index is strictly smaller than the classical Clifford index. In this paper we use the methods of [4] to present further examples of this, showing in particular that the difference between the two Clifford indices can be arbitrarily large.

For any positive integer \( n \) the rank-\( n \) Clifford index \( \gamma'_n(C) \) of a smooth projective curve of genus \( g \geq 4 \) over an algebraically closed field of characteristic 0 is defined as follows. For any vector bundle \( E \) of rank \( n \) and degree \( d \) on \( C \) define

\[
\gamma(E) := \frac{1}{n}(d - 2(h^0(E) - n)).
\]
Then
\[ \gamma_n' = \gamma_n'(C) := \min \left\{ \gamma(E) \mid \begin{array}{l} E \text{ semistable of rank } n \text{ with } \\ d \leq n(g - 1) \text{ and } h^0(E) \geq 2n \end{array} \right\}. \]

Here \( \gamma_1 = \gamma_1' \) is the classical Clifford index of \( C \) and it is easy to see that \( \gamma_n' \leq \gamma_1 \) for all \( n \).

The gonality sequence \( (d_r)_{r \in \mathbb{N}} \) is defined by
\[ d_r := \min_{L \in \text{Pic}(C)} \{ \deg L \mid h^0(L) \geq r + 1 \}. \]

In classical terms \( d_r \) is the minimum number \( d \) for which a \( g_d^r \) exists.

In the case of a general curve we have for all \( r \),
\[ d_r = g + r - \left\lfloor \frac{g}{r + 1} \right\rfloor. \]

According to [9], [7] a version of Mercat’s conjecture states that
\[ \gamma_n' = \gamma_1 \quad \text{for all } n. \]

As mentioned above, counterexamples in rank 3 and rank 2 are now known. For the rest of the paper we concentrate on rank 2.

For \( \gamma_1 \leq 4 \) it is known that \( \gamma_2' = \gamma_1 \) (see [7, Proposition 3.8]). In any case, we have according to [7, Theorem 5.2]
\[ \gamma_2' \geq \min \left\{ \gamma_1, \frac{d_4}{2} - 2 \right\}. \]

For the general curve of genus 11 we have \( \gamma_1 = 5 \) and \( d_4 = 13 \). So in this case, \( \gamma_2' = 5 \) or \( \frac{g}{2} \). It is shown in [4, Theorem 3.6] that there exist curves \( C \) of genus 11 with \( \gamma_1 = 5 \) and \( \gamma_2' = \frac{g}{2} \), but this cannot happen on a general curve of genus 11 [4, Theorems 1.6 and 1.7]. Counterexamples to the conjecture in higher genus were also constructed in [4]. All examples \( E \) constructed in [4] have \( \gamma(E) = \gamma_1 - \frac{1}{2} \).

In this paper we use the methods of [4] to generalize these examples. Our main result is the following theorem.

**Theorem 1.1.** Suppose \( d = g - s \) with an integer \( s \geq -1 \) and
\[ g \geq \max\{4s + 14, 12\}. \]

Suppose further that the quadratic form
\[ 3m^2 + dmn + (g - 1)n^2 \]
cannot take the value \(-1\) for any integers \( m, n \in \mathbb{Z} \). Then there exists a curve \( C \) of genus \( g \) having \( \gamma_1(C) = \left\lfloor \frac{g - 1}{2} \right\rfloor \) and a stable bundle \( E \) of rank 2 on \( C \) with \( \gamma(E) = \frac{g - s}{2} - 2 \) and hence
\[ \gamma_1 - \gamma_2' \geq \left\lfloor \frac{g - 1}{2} \right\rfloor - \frac{g - s}{2} + 2 > 0. \]

In particular the difference \( \gamma_1 - \gamma_2' \) can be arbitrarily large.
This statement can also be written in terms of the Brill-Noether loci $B(2, d, 4)$ which are defined as follows. Let $M(2, d)$ denote the moduli space of stable bundles of rank 2 and degree $d$ on $C$. Then

$$B(2, d, 4) := \{ E \in M(2, d) \mid h^0(E) \geq 4 \}.$$ 

Theorem 1.1 says that under the given hypotheses $B(2, g - s, 4)$ is non-empty. It may be noted that the expected dimension of $B(2, g - s, 4)$ is $-4s - 11 < 0$.

The key point in proving this theorem is the construction of the curves $C$, which all lie on K3-surfaces and are therefore not general, although they do have maximal Clifford index.

**Theorem 1.2.** Suppose $d = g - s$ with an integer $s \geq -1$ and

$$g \geq \max\{ 4s + 14, 12 \}.$$ 

Then there exists a smooth K3-surface $S$ of type $(2, 3)$ in $\mathbb{P}^4$ containing a smooth curve $C$ of genus $g$ and degree $d$ with

$$\text{Pic}(S) = H\mathbb{Z} \oplus C\mathbb{Z},$$

where $H$ is the polarization, such that $S$ contains no divisor $D$ with $D^2 = 0$. Moreover, if $S$ does not contain a $(-2)$-curve, then $C$ is of maximal Clifford index $\lceil \frac{g - 1}{2} \rceil$.

The proof of Theorem 1.2, which uses the methods of [3] and [4], is given in Section 2. This is followed in Section 3 by the proof of Theorem 1.1.

2. Proof of Theorem 1.2

**Lemma 2.1.** Let $d = g - s$ with $g \geq 4s + 14$ and $s \geq -1$. Then $d^2 - 6(2g - 2)$ is not a perfect square.

**Proof.** If $d^2 - 6(2g - 2) = g^2 - (2s + 12)g + s^2 + 12 = m^2$ for some non-negative integer $m$, then the discriminant

$$(s + 6)^2 - (s^2 + 12 - m^2) = 12s + 24 + m^2$$

is a perfect square of the form $(m+b)^2$ with $b \geq 2$. Solving the equation $g^2 - (2s + 12)g + (s^2 + 12 - m^2) = 0$ for $g$, we get

$$g = s + 6 \pm (m + b).$$

Now, since $b \geq 2$, we have $(m + b - 2)^2 \geq m^2$ and hence

$$4(m + b) - 4 = (m + b)^2 - (m + b - 2)^2 \leq 12s + 24$$

which gives $m + b \leq 3s + 7$. So (2.1) implies $g \leq 4s + 13$, which contradicts the hypothesis. \qed
Proposition 2.2. Let \( g \geq 4s + 14 \) with \( s \geq -1 \). Then there exists a smooth K3-surface \( S \) of type \((2, 3)\) in \( \mathbb{P}^4 \) containing a smooth curve \( C \) of genus \( g \) and degree \( d = g - s \) with

\[
\text{Pic}(S) = H\mathbb{Z} \oplus C\mathbb{Z},
\]

where \( H \) is the polarization, such that \( S \) contains no divisor \( D \) with \( D^2 = 0 \).

Proof. The conditions of [6, Theorem 6.1,2.] are fulfilled to give the existence of \( S \) and \( C \). Let

\[
D = mH + nC \quad \text{with} \quad m, n \in \mathbb{Z}.
\]

We want to show that the equation \( D^2 = 0 \) does not have an integer solution. Now

\[
D^2 = 6m^2 + 2dmn + (2g - 2)n^2.
\]

For an integer solution we must have that the discriminant \( d^2 - 6(2g - 2) \) is a perfect square and this contradicts Lemma 2.1. \( \Box \)

Lemma 2.3. Under the hypotheses of Proposition 2.2, the curve \( C \) is an ample divisor on \( S \).

Proof. We show that \( C \cdot D > 0 \) for any effective divisor on \( S \) which we may assume to be irreducible. So let \( D \sim mH + nC \) be an irreducible curve on \( S \). So

\[
C \cdot D = m(g - s) + n(2g - 2).
\]

Note first that, since \( H \) is a hyperplane, we have

\[
(2.2) \quad D \cdot H = 6m + (g - s)n > 0.
\]

If \( m, n \geq 0 \), then one of them has to be positive and then clearly \( C \cdot D > 0 \). The case \( m, n \leq 0 \) contradicts (2.2).

Suppose \( m > 0 \) and \( n < 0 \). Then, using (2.2) we have

\[
C \cdot D = m(g - s) + n(2g - 2) > -n \left( \frac{(g - s)^2}{6} - (2g - 2) \right).
\]

So \( C \cdot D > 0 \) for \( g > s + 6 + 2\sqrt{3s + 6} \), which holds, since \( g \geq 4s + 14 \).

Finally, suppose \( m < 0 \) and \( n > 0 \). Then, since we assumed \( D \) irreducible,

\[
nC \cdot D = -mD \cdot H + D^2 \geq -mD \cdot H - 2 \geq -m - 2.
\]

If \( m \leq -3 \), then \( nC \cdot D > 0 \). If \( m = -1 \), we have

\[
C \cdot D = -(g - s) + n(2g - 2) \geq g + s - 2 > 0.
\]

The same argument works for \( m = -2 \), \( n \geq 2 \). Finally, if \( m = -2 \) and \( n = 1 \), we still get \( C \cdot D > 0 \) unless \( D \cdot H = 1 \) and \( D^2 = -2 \). Solving these equations gives \( s = 1, g = 14 \), contradicting the hypotheses. \( \Box \)
Theorem 2.4. Let the situation be as above with \( d = g - s, \) \( s \geq -1 \) and
\[
g \geq \max\{4s + 14, 12\}.
\]
If \( S \) does not contain a \((-2)\)-curve, then \( C \) is of maximal Clifford index \( \left\lfloor \frac{g - 1}{2} \right\rfloor \).

Note that a stronger form of this has been proved for \( s = -2 \) and \( g \) odd in [4, Theorem 3.6] and for \( s = -1 \) and \( g \) even in [4, Theorem 3.7]. The proof follows closely that of [3, Theorem 3.3], but, since some of the estimates are delicate and our hypotheses differ, we give full details.

Proof. Since \( C \) is ample by Lemma 2.3, it follows from [1, Proposition 3.3] that \( C \) is of Clifford dimension 1.

Suppose that \( \gamma_1(C) < \left\lfloor \frac{g - 1}{2} \right\rfloor \). According to [2] there is an effective divisor \( D \) on \( S \) such that \( D|_C \) computes \( \gamma_1(C) \) and satisfying
\[
h^0(S, D) \geq 2, \quad h^0(S, C - D) \geq 2 \quad \text{and} \quad \deg(D|_C) \leq g - 1.
\]
We consider the exact cohomology sequence
\[
0 \to H^0(S, D - C) \to H^0(S, D) \to H^0(C, D|_C) \to H^0(S, D - C).
\]
Since \( C - D \) is effective, and not equivalent to zero, we get
\[
H^0(S, D - C) = 0.
\]
By assumption \( S \) does not contain \((-2)\)-curves, so \( |D - C| \) has no fixed components. According to Proposition 2.2 the equation \((C - D)^2 = 0\) has no solutions, therefore \((C - D)^2 > 0\) and the general element of \( |C - D| \) is smooth and irreducible. It follows that
\[
H^1(S, D - C) = H^1(S, C - D)^* = 0
\]
and
\[
\gamma_1(C) = \gamma(D|_C) = D \cdot C - 2 \dim |D| = D \cdot C - D^2 - 2
\]
by Riemann-Roch. We shall prove that
\[
D \cdot C - D^2 - 2 \geq \left\lfloor \frac{g - 1}{2} \right\rfloor,
\]
a contradiction.

Let \( D \sim mH + nC \) with \( m, n \in \mathbb{Z} \). Since \( D \) is effective and \( S \) contains no \((-2)\)-curves, we have \( D^2 > 0 \) and \( D \cdot H > 2 \). Since \( C - D \) is also effective, we have \((C - D) \cdot H > 2\), i.e. \( D \cdot H < d - 2 \). These inequalities and \( \deg(D|_C) \leq g - 1 \) translate to the following inequalities
\[
\begin{align*}
(2.3) & \quad 3m^2 + mnd + n^2(g - 1) > 0, \\
(2.4) & \quad 2 < 6m + nd < d - 2, \\
(2.5) & \quad md + (2n - 1)(g - 1) \leq 0.
\end{align*}
\]
Consider the function
\[ f(m, n) := D \cdot C - D^2 - 2 = -6m^2 + (1 - 2n)dm + (n - n^2)(2g - 2) - 2, \]
and denote by
\[ a := \frac{1}{6} (d + \sqrt{d^2 - 12(g - 1)}) \quad \text{and} \quad b := \frac{1}{6} (d - \sqrt{d^2 - 12(g - 1)}) \]
the solutions of the equation \( 6x^2 - 2dx + 2g - 2 = 0 \). Note that \( d^2 > 12(g - 1) \). So \( a \) and \( b \) are positive real numbers.

Suppose first that \( n < 0 \). From (2.3) we have either \( m < -bn \) or \( m > -an \). If \( m < -bn \), then (2.4) implies that \( 2 < n \frac{d}{d - 6b} < 0 \), because \( n < 0 \) and \( d - 6b = \sqrt{d^2 - 12(g - 1)} > 0 \), which gives a contradiction.

If \( n < 0 \) and \( m > -an \), from (2.5) we get
\[ -an < m \leq \frac{(g - 1)(1 - 2n)}{d} < \frac{(1 - 2n)d}{12}, \]
since \( d^2 > 12(g - 1) \). For a fixed \( n \), \( f(m, n) \) is increasing as a function of \( m \) for \( m \leq \frac{(1 - 2n)d}{12} \) and therefore
\[ f(m, n) > f(-an, n) \]
\[ = \frac{d^2 - 12(g - 1) + d \sqrt{d^2 - 12(g - 1)}}{6} \cdot (-n) - 2 \]
\[ \geq \frac{d^2 - 12(g - 1) + d \sqrt{d^2 - 12(g - 1)}}{6} - 2 \]
\[ \geq \frac{g - 1}{2}, \]
which gives a contradiction. Here the last inequality reduces to
\[ d \sqrt{d^2 - 12(g - 1)} \geq 15g - 3 - d^2 \]
which certainly holds if \( d^2 \geq 15g - 3 \). This is true under our hypotheses on \( g \) if \( s \geq 1 \). The inequality can be checked directly in the cases \( s = 0 \) and \( s = -1 \).

Now suppose \( n > 0 \). From (2.3) we get that either \( m < -an \) or \( m > -bn \). If \( m < -an \), we get from (2.4), \( 2 < n(-6a + d) < 0 \), a contradiction.

When \( m > -bn \), first suppose \( n = 1 \). Then (2.5) gives
\[ (2.6) \quad -b < m \leq -\frac{g - 1}{d}. \]
We claim that
\[ (2.7) \quad 1 < b < \frac{4}{3}. \]
In terms of $s$ we have
\[
6b = g - s - \sqrt{(g - s)^2 - 12(g - 1)} = g - s - \sqrt{(g - (s + 6))^2 - 12s - 24} > g - s - (g - (s + 6)) = 6,
\]
since $s \geq -1$. This gives $1 < b$. For the second inequality note that $b = \frac{4}{3}$ gives $s = \frac{g - 13}{4}$ and $b$ is a strictly increasing function of $s$ in the interval $[-1, \frac{g - 13}{4}]$. Since certainly $s < \frac{g - 13}{4}$, we obtain $b < \frac{4}{3}$.

So there are no solutions of (2.6) unless $d \geq g - 1$, i.e. $s = 1, 0$ or $-1$. For these values of $s$ we must have $m = -1$ and
\[
f(m, n) = f(-1, 1) = d - 8.
\]

So $f(-1, 1) \geq \left[\frac{g - 1}{2}\right]$ if and only if $g \geq 2s + 14$.

Now suppose $m > -bn$ and $n \geq 2$. Then (2.5) gives
\[
f(m, n) \geq \min \left\{ f \left(-\frac{(g - 1)(2n - 1)}{d}, n\right), f(-bn, n)\right\}.
\]

We have
\[
f\left(-\frac{(g - 1)(2n - 1)}{d}, n\right) = \frac{g - 1}{2} \left(\frac{(2n - 1)}{2} \left(1 - \frac{12(g - 1)}{d^2}\right) + 1\right) - 2.
\]

It is easy to see that $f \left(-\frac{(g - 1)(2n - 1)}{d}, n\right) \geq \frac{g - 1}{2}$ for $n \geq 2$. Moreover,
\[
f(-bn, n) = -b\cdot n(2g - 2) - 2 = n(2g - 2 - bd) - 2.
\]

Note that
\[
2g - 2 - bd = \frac{\sqrt{d^2 - 12(g - 1)}}{6}(d - \sqrt{d^2 - 12(g - 1)}) > 0.
\]

So $f(-bn, n)$ is a strictly increasing function of $n$. Hence it suffices to show that $f(-2b, 2) \geq \frac{g - 1}{2}$ or equivalently
\[
7(g - 1) - 4bd - 4 \geq 0.
\]

According to (2.7) we have $b < \frac{4}{3}$. So, since $d \leq g + 1$, we have
\[
7(g - 1) - 4bd - 4 \geq 7(g - 1) - \frac{16}{3}d - 4 \geq 7g - 7 - \frac{16}{3}g - \frac{16}{3} - 4 = \frac{1}{3}(5g - 49) > 0.
\]

This completes the argument for $m > -bn, n > 0$.

Finally, suppose $n = 0$. Then
\[
f(m, 0) = -6m^2 + dm - 2.
\]

As a function of $m$ this takes its maximum value at $\frac{d}{12}$. By (2.5), $m \leq \frac{g - 1}{d} \leq \frac{d}{12}$. So $f(m, 0)$ takes its minimal value in the allowable range at $m = 1$. Since $f(1, 0) = d - 8$, we require $d - 8 \geq \left[\frac{g - 1}{2}\right]$ or equivalently
\[
g \geq 2s + 14,
\]
which is valid by hypothesis.

This completes the proof of Theorem 1.2.

**Remark 2.5.** For \( s = 0 \) or \(-1\) the assumptions of the theorem are best possible, since in these cases \( \gamma(H|_C) = \gamma((C - H)|_C) = d - 8 \) would otherwise be less than \( \left\lfloor \frac{g-1}{2} \right\rfloor \). For \( s \geq 1 \) the conditions can be relaxed. For example, if \( s \geq 1 \) and \( g = 4s + 12 \), the only places where the argument can fail are in the proofs of Lemma 2.1 and formula (2.7). In the first case, one can show directly that \( d^2 - 6(2g - 2) \) is not a perfect square; in the second, one can show that \( b < \frac{3}{2} \), which is sufficient.

**Remark 2.6.** The condition that \( S \) does not contain a \((-2)\)-curve certainly holds if \( 3m^2 + dmn + (g - 1)n^2 = -1 \) has no solutions. We do not know precisely when this is true, but it certainly holds if both \( g - 1 \) and \( g - s \) are divisible by 3. So the conclusion of Theorem 2.4 holds for \( s \equiv 1 \mod 3 \), if \( g \geq 4s + 14 \) and \( g \equiv 1 \mod 3 \). The conclusion also holds, for example, for \( g = 16 \) and \( s = 1 \) (see Remark 2.5).

### 3. Proof of Theorem 1.1

**Lemma 3.1.** Let \( C \) and \( H \) be as in Proposition 2.2 with \( d = g - s \), \( s \geq -1 \) and suppose that \( S \) has no \((-2)\)-curves. Then \( H|_C \) is a generated line bundle on \( C \) with \( h^0(\mathcal{O}_C(H|_C)) = 5 \) and

\[
S^2H^0(\mathcal{O}_C(H|_C)) \to H^0(\mathcal{O}_C(H^2|_C))
\]

is not injective.

**Proof.** Consider the exact sequence

\[
0 \to \mathcal{O}_S(H - C) \to \mathcal{O}_S(H) \to \mathcal{O}_C(H|_C) \to 0.
\]

\( H - C \) is not effective, since \( (H - C) \cdot H = 6 - d < 0 \). So we have

\[
0 \to H^0(\mathcal{O}_S(H)) \to H^0(\mathcal{O}_C(H|_C)) \to H^1(\mathcal{O}_S(H - C)) \to 0.
\]

Now

\[
(C - H)^2 = 2g - 2 - 2d + 6 = 2s + 4 \geq 2
\]

and

\[
H^2(\mathcal{O}_S(C - H)) = H^0(\mathcal{O}_S(H - C))^* = 0.
\]

So by Riemann-Roch \( h^0(\mathcal{O}_S(C - H)) \geq 3 \). Since \( S \) has no \((-2)\)-curves, it follows that the linear system \(|C - H|\) has no fixed components and hence its general element is smooth and irreducible (see [10]). Hence \( h^1(\mathcal{O}_S(H - C)) = 0 \) and therefore \( h^0(\mathcal{O}_C(H|_C)) = h^0(\mathcal{O}_S(H)) = 5 \). The last assertion follows from the fact that \( S \) is contained in a quadric. \( \Box \)

**Remark 3.2.** Lemma 3.1 implies that \( H|_C \) belongs to \( W^4_{g-s} \). So \( g - s \geq d_4 \). Since the generic value of \( d_4 \) is \( g + 4 - \left\lfloor \frac{g}{2} \right\rfloor \), it follows that \( C \) has non-generic \( d_4 \) if \( g < 5s + 20 \).
Lemma 3.3. Let $C$ be a smooth irreducible curve and $M$ a generated line bundle on $C$ of degree $d < 2d_1$ with $h^0(M) = 5$ and such that $S^2H^0(M) \rightarrow H^0(M^2)$ is not injective. Then $B(2, d, 4) \neq \emptyset$.

The proof is identical with that of [5] Theorem 3.2 (ii)]. □

Theorem 3.4. Let $C$ be as in Theorem 2.4. Then

(i) $B(2, g - s, 4) \neq \emptyset$;
(ii) $\gamma_2(C) \leq \frac{g-s}{2} - 2 < \gamma_1(C)$.

Proof. This follows from Theorem 2.4 and Lemmas 3.1 and 3.3. □

This completes the proof of Theorem 1.1 where the last assertion follows from Remark 2.6.

Corollary 3.5. $\gamma_{2n}(C) < \gamma_1(C)$ for every positive integer $n$.

Proof. This follows from Theorem 3.4 and [7] Lemma 2.2]. □

Remark 3.6. Under the conditions of Theorem 1.1, for any stable bundle $E$ of rank 2 and degree $g - s$ on $C$ with $h^0(E) = 4$, it follows from [5] Proposition 5.1] that the coherent system $(E, H^0(E))$ is $\alpha$-stable for all $\alpha > 0$. So the corresponding moduli spaces of coherent systems are non-empty.

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