Complete $\mathcal{O}(\alpha)$ QED corrections
to the process $ep \rightarrow eX$ in mixed variables

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Abstract

The complete set of $\mathcal{O}(\alpha)$ QED corrections with soft photon exponentiation to the process $ep \rightarrow eX$ in mixed variables ($y = y_h, Q^2 = Q_l^2$) is calculated in the quark parton model, including the lepton-quark interference and the quarkonic corrections which were unknown so far. The interference corrections amount to few percent or less and become negligible at small $x$. The leading logarithmic terms proportional to $\ln(Q^2/m_q^2)$ from radiation off quarks are discussed and the non-logarithmic quarkonic corrections found to be negligible for almost all experimentally accessible $x$ and $y$.

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1 Introduction

Traditionally, deep inelastic $ep$ scattering,

$$e(k_1) + p(p_1) \rightarrow e(k_2) + X(p_2),$$

accompanied by radiative corrections from the reaction

$$e(k_1) + p(p_1) \rightarrow e(k_2) + X(p_2) + n\gamma(k),$$

was studied in terms of kinematic variables, which could be determined exclusively from the electron. The new detector generation at HERA gives additional access to the hadron kinematics. For the radiative process (2) the 'leptonic' variables differ from those, which are determined partly or exclusively with information about the hadronic kinematics. QED corrections from the diagrams of figure 1 (henceforth called leptonic corrections) with soft photon exponentiation have been determined in a model independent approach in several variables [1, 2]. The same has been undertaken in leading logarithmic approximation, including higher order corrections [2, 3], for a larger variety of variable sets [4]. For a complete $O(\alpha)$ calculation, one has to use the quark-parton model and include corrections from the diagrams of figure 2 and from their interferences with those of figure 1. In terms of leptonic variables, this has been done some time ago [5].

In this letter, we report on the corresponding corrections in terms of mixed variables. These variables are defined as follows:

$$y_m \equiv y_n = -\frac{2p_1Q_h}{S}, \quad Q_m^2 \equiv Q_l^2 = (k_1 - k_2)^2, \quad x_m \equiv \frac{Q_m^2}{y_m S}.$$  (3)

In the next section, we explain the structure of the corrections and give explicit expressions for them. Section 3 contains a discussion of photonic bremsstrahlung from quarks and section 4 numerical results. The technically involved calculations will be described elsewhere.

2 The bremsstrahlung corrections

The cross section of reaction (2) may be written as follows\(^1\):

$$\frac{d^2\sigma}{dxdy} = \frac{2\pi\alpha^2 S x}{Q^4} \sum_{Q,\bar{Q}} \left\{ B_0(y, 1) f_Q(x) \left[ 1 + \frac{\alpha}{\pi} \sum_{a=e,i,q} c_a S_a(x, y|m_a^2) \right] \\
+ \frac{\alpha}{\pi} \sum_{a=e,i,q} \left[ \int_1^{1/x} dz \left[ B_a^V(z; x, y) + p_e p_Q B_a^A(z; x, y) \right] \\
+ \int_y^1 dz \left[ B_a^V(z; x, y) + p_e p_Q B_a^A(z; x, y) \right] \right] \right\} + \frac{d^2\sigma^{box}}{dxdy},$$

(4)

with

$$B_0(y, z) = V_0 Y_+ \left( \frac{y}{z} \right) + p_e p_Q A_0 Y_- \left( \frac{y}{z} \right)$$

(5)

and $Y_\pm(y) = 1 \pm (1 - y)^2$.

The sum over $Q$ and $\bar{Q}$ extends over the quark content of the proton. The second sum with $a = e, q, i$ extends over the photonic corrections originating from the lepton legs, from the quark legs, and from their interference. Correspondingly, $c_e = Q_e^2, c_q = Q_Q^2, c_i = Q_e Q_Q$. We use the convention $Q_+ = -1$. The couplings $Q_1, v_f, a_f$ are collected in overall factors of the vector type, $V(B, p)$, and axial vector type, $A(B, p)$ and the running coupling $\alpha_{QED}(Q^2)$, the Fermi constant $G_F$, and the ratio of the photon and $Z$ propagators in effective coupling strengths $K(B, p)$:

\(^1\)If not stated differently, the variables $x, y, Q^2$ are mixed variables.
Since the interaction of leptons with quarks proceeds via photon or Z boson exchange, we have three types of cross section pieces labeled with $B = \gamma, Z, I$, where the latter denotes again the interference. The $[B_1B_2] = \gamma\gamma, Z\gamma, ZZ$ correspond to $B = \gamma, I, Z$. The $V(B,p)$ and $K(B,p)$ contain weak one loop corrections; they are defined in [6]. The axial couplings $A(a,b)$ are related to the couplings $A(B,p)$ of [5] in the same manner as is explained above for the vector couplings. The sign factor is $p = p_e p_Q = +1$ for $PP$, $AA$ scattering and $p = -1$ for $PA$ scattering, where we denote particles with $P$ and anti-particles with $A$.

$V_0 = V(1,1),$ $V(a,b) = \sum_{B=\gamma,I,Z} K(a,b;B,p_e p_Q) V(B,p_e p_Q),$ $K(a,b;B,p) = \chi_{B_1}(a) \chi_{B_2}(b) K(B,p),$ $\chi_B(a) = \frac{Q^2}{aQ^2 + M_B^2}.$

Figure 1: Leptonic bremsstrahlung.

Figure 2: Bremsstrahlung from quarks.

\[ V_0 = V(1,1), \]
\[ V(a,b) = \sum_{B=\gamma,I,Z} K(a,b;B,p_e p_Q) V(B,p_e p_Q), \quad (6) \]
\[ K(a,b;B,p) = \chi_{B_1}(a) \chi_{B_2}(b) K(B,p), \]
\[ \chi_B(a) = \frac{Q^2}{aQ^2 + M_B^2}. \]
The functions $S_e, S_q$ contain factorizing soft photon corrections and the corresponding vertex corrections:

$$S_e(x, y|m_e^2) = \left(\ln \frac{Q^2}{m_e^2} - 1\right) \ln \left[\frac{(1-y)(1-x)}{y} \right] + \frac{3}{2} \ln \frac{Q^2}{m_e^2} - 2 - \ln(1-y) \ln(xy)$$

$$+ 2 \text{Li}_2(1) - \text{Li}_2(xy) + \text{Li}_2 \left[-\frac{(1-x)y}{1-y}\right],$$

$$S_q(x, y|m_Q^2) = S_e(x, y|m_Q^2) - \left[\text{Li}_2(1-y) + \text{Li}_2 \left(-\frac{1-x}{x}\right)\right].$$

(7)

(8)

With the exception of the last two terms in (8) the expressions for $S_e(x, y|m_e^2)$ and $S_q(x, y|m_Q^2)$ agree.

The soft photon interference term $S_i$ has to be combined with the $\gamma\gamma$ and $\gamma Z$ boxes in order to get an infrared finite answer; the latter are added as an extra piece after the divergences are canceled, and it is:

$$S_i(x, y) = \ln(1-y) \ln \left(\frac{(1-y)^3(1-x)^2}{y^4}\right) - 6 \text{Li}_2(y) + 2 \text{Li}_2(xy) - 2 \text{Li}_2 \left[-\frac{(1-x)y}{1-y}\right].$$

(9)

The corrections $B^{V,A}$ and $\bar{B}^{V,A}$ in (11) are due to real photonic bremsstrahlung. Although we leave out all details of the involved calculations, a remark on the phase space parameterization should be made. The double-differential bremsstrahlung corrections are the result of a threefold integration, which has been organized such that two of the integrals may be performed analytically. At the parton level, there are two nontrivial integrals to be performed whose result has to be convoluted with the parton momentum distribution:

$$d\sigma^{\text{brem}} = \frac{\alpha^3}{(2\pi)} \cdot \sum_{Q, Q, f} f_Q(x_h)(dx_h/x_h) dQ_h^2 dQ_f^2 dy_l d\Gamma_k |\mathcal{M}_{eq}^{\text{brem}}|^2.$$  We chose the following parameterization of the phase space:

$$d^2\sigma^{\text{brem}} = \frac{\alpha^3}{2\pi} Q_i^2 \sum_{Q, Q} \int dq Q(zx) \int dq_l d\Gamma_k |\mathcal{M}_{eq}^{\text{brem}}|^2.$$  (10)

Here, it is

$$z = \frac{x_h}{x_m} = \frac{Q_h^2}{Q_l^2},$$

with $Q_h^2 = (p_2 - p_1)^2$. The integration variable $z$ may be larger or smaller than 1. As may be seen in figure 39 of (11), for these two cases the physical regions for the integration over $y_l$ (at fixed values of $Q_h^2$ or, equivalently, of $z$) are different, which in turn leads to different integrands in (11) for the subsequent integration over $z$. The integral over $y_l$ and that over the photon phase space, indicated by $d\Gamma_k$, are performed analytically. In fact the latter one is transformed into an integral over $dz_2/(2\sqrt{F_z})$, where $z_2 = -2kk_2$ is the denominator of the final state electron propagator before an emission of a photon (11).

The hard bremsstrahlung functions in (11) are:

$$B_a^{V,A}(z; x, y) = \left\{ \frac{V_a}{A_a} \right\} \left\{ \sum_n \text{Reg} \left[ \frac{(-)^{V,A}}{F_{\alpha n} (z, y)} \int f_Q(zx) \right] (-)^{V,A} \right\} L_{\alpha n} + (-)^{V,A} (z, y) f_Q(zx).$$  (12)
The bremsstrahlung vector couplings are:

\[ V_e = c_e V(z, z), \quad V_i = c_i V(1, z), \quad V_q = c_q V(1, 1). \]  

(13)

The bremsstrahlung axial couplings are defined analogously. The different arguments of \( V(a, b) \) reflect the different momentum flows in the exchanged boson propagators in figure \( \| \) (\( Q_e^2 \)) and figure \( \| \) (\( Q_q^2 \)).

For \( a = e, i, q \), the \( n \) runs from 1 to 2, 1, 2. The functions \( F_{an} \) are multiplied by a factor \( 1/(1 - z) \) which becomes singular at \( z = 1 \) due to the infrared singularity of the corrections in this phase space region. This singularity is regularized by a subtraction:

\[ \text{Reg} \left[ \frac{(-)^{V,A} F_{an}}{F_{an}} (z, y) f_Q(zx) \right] = \frac{y}{(1 - z)} \left[ \frac{(-)^{V,A} F_{an}}{F_{an}} (z, y) f_Q(zx) - \frac{(-)^{V,A} F_{an}}{F_{an}} (1, y) f_Q(x) \right]. \]  

(14)

The quarkonic bremsstrahlung functions are:

\[ F_{q1}^{V,A} (z, y) = \frac{1}{2} \left( 1 + z^2 \right) \frac{z}{y} Y_+ \left( \frac{y}{z} \right), \]  

(15)

\[ F_{q2}^{V,A} (z, y) = \frac{1}{2} \left( 1 + z^2 \right) \frac{1}{y} Y_+ (y), \]  

(16)

\[ U_q^V = \frac{1}{2} \left( 2 + 3z + 2z^2 \right) Y_+ \left( \frac{y}{z} \right) + \frac{1}{z} \left( 1 - \frac{y}{z} \right) - \frac{y^2}{2z^2} \left( 1 + z^2 \right) + y(1 + z) - \frac{1}{2} + \frac{y^2}{z} \ln \frac{z}{y}, \]  

(17)

\[ U_q^A = \frac{1}{2} \left( 1 + 3z + 2z^2 \right) Y_- \left( \frac{y}{z} \right) - \frac{y^2}{2z^2} \left( 1 + z^2 \right) - y(1 + z), \]  

(18)

\[ L_{q1} = L_1 \left( z, y \left| \frac{m_q^2}{z} \right) \right., \]  

(19)

\[ L_1(z, y|m^2) = \ln \frac{Q^2}{m^2} - 1 + \ln \frac{z - y}{z(1 - z)}, \]  

(20)

\[ L_{q2}(z, y) = \ln \frac{1 - z}{1 - y}. \]  

(21)

The leptonic corrections differ by a simple factor:

\[ \frac{(-)^{V,A} F_{ea}}{L_{e1}} = z \frac{(-)^{V,A} F_{qa}}{L_{e2}}, \quad a = 1, 2, \quad \frac{(-)^{V,A} U_e}{L_{e2}} = z \frac{(-)^{V,A} U_q}{L_{e2}}. \]  

(22)

The lepton quark interference corrections have no mass singularities:

\[ \frac{(-)^{V,A} F_{i}}{L_i} = \frac{z}{y} \left[ Y_+ \left( \frac{y}{z} \right) + Y_+ (y) \right], \quad \frac{(-)^{V,A} U_i}{L_i} = y U_i, \]  

(23)

\[ L_i = \ln(1 - y), \]  

(24)

\[ U_i^V = 2 \left[ (2 - y) \ln(1 - y) - \left( 4 - y - \frac{y}{z} \right) \ln(1 - z) - y \ln \frac{z}{y} \right] - 4(z - y), \]  

(25)

\[ U_i^A = 2 \frac{y}{z} \left[ \ln(1 - y) - (1 + z) \ln(1 - z) - z \ln \frac{z}{y} \right]. \]  

(26)

The integrands in the two integration regions in \( \| \) are related by a symmetry relation:

\[ \tilde{X} \left( z, y, \frac{y}{z}, \ldots \right) = X \left( \frac{1}{z}, \frac{y}{z}, y, \ldots \right), \quad X = L_e, L_i, U_i, F_q, L_q, U_q. \]  

(27)
Finally, we have to discuss the last term in (34), \(d^2\sigma_{\text{box}}/dx dy\). This is the finite part of the contribution from the interference of the \(\gamma\gamma\) and \(\gamma Z\) box diagrams with the Born diagrams. It contributes to the factorizing part of the lepton quark interference. Its variables are defined from the Born kinematics. For this reason, we may refer to [5], eqs. (2.1), (3.1), (3.4) and (C.11), (C.14). The latter two formulae contain the corrections from the box diagrams: \(S_i(B; a, b)_{\text{box}} = S_i(B; a, b) - S_{\text{soft}}\). \(B = \gamma, Z\), where it is \(S_{\text{soft}} = 4a^2 \ln(b/a) \ln[(1 - x)/x]\).

### 3 Photonic bremsstrahlung from quarks

In this section, we discuss the connection of our complete \(\mathcal{O}(\alpha)\) results with expressions to be expected from a LLA treatment. For this purpose, we rewrite the terms proportional to \(\ln(Q^2/m_Q^2)\) in the first of the integrals in (34) and perform a change of integration variable, \(z \to 1/z\):

\[
\frac{d^2\sigma_{\text{LLA}, q}}{dx dy} = \frac{2\pi\alpha^2 Sx}{Q^4} \sum_{q, Q} \frac{\alpha Q^2}{m_Q^2} B_0(y, 1) \int_x^1 \frac{dz}{Q^2} \frac{1 + z^2}{1 - z} \frac{1}{2z} f_Q \left(\frac{x}{z}\right) - \frac{1}{z(1 - z)} f_Q(x) \bigg[ 1 + z^2 - 1 \bigg] B_0(y, z) - \frac{1}{1 - z} f_Q(x) B_0(y, 1) .
\]

(28)

In the second integral, one has to collect terms correspondingly (here without change of integration variable):

\[
\frac{d^2\sigma_{\text{LLA}, q}}{dx dy} = \frac{2\pi\alpha^2 Sx}{Q^4} \sum_{q, Q} \frac{\alpha Q^2}{m_Q^2} \int_y^1 \frac{dz}{Q^2} \frac{1 + z^2}{1 - z} \frac{z f_Q(z) B_0(y, z)}{2z} - \frac{1}{1 - z} f_Q(x) B_0(y, 1) .
\]

(29)

Further, the soft photon correction (34) has to be rewritten:

\[
S_q(x, y|m_Q^2) \equiv S_q^{\text{ini}}(x|m_Q^2) + S_q^{\text{fin}}(y|m_Q^2) + S_q^{\text{finite}}(x, y) ,
\]

(30)

where

\[
S_q^{\text{ini}}(x|m_Q^2) = \ln \frac{Q^2}{m_Q^2} \left[ -\frac{1}{2} \int_0^1 \frac{dz}{1 - z} \left( \frac{1 + z^2}{1 - z} + \int_x^1 \frac{dz}{z(1 - z)} \right) \right] ,
\]

(31)

\[
S_q^{\text{fin}}(y|m_Q^2) = \ln \frac{Q^2}{m_Q^2} \left[ -\frac{1}{2} \int_0^1 \frac{dz}{1 - z} \left( \frac{1 + z^2}{1 - z} + \int_y^1 \frac{dz}{z(1 - z)} \right) \right] ,
\]

(32)

\[
S_q^{\text{finite}}(x, y) = - \ln \left[ (1 - y) \frac{1 - x}{x} \right] - 2 - \ln(1 - y) \ln(xy) + 2 \text{Li}_2(1) - \text{Li}_2(xy)
\]

\[+ \text{Li}_2 \left( \frac{-1}{y} \right) - \left[ \text{Li}_2(1 - y) + \text{Li}_2 \left( \frac{-1 - x}{x} \right) \right] .
\]

(33)

As is indicated by the choice of indices, the selected corrections may be described by the following generic expression for the collinear radiation of photons from the initial or final state quark:

\[
\frac{d^2\sigma}{dx dy} = \frac{d^2\sigma_{\text{LLA}, q}}{dx dy} + \sum_{q, Q} \frac{2\alpha^3 Sx}{Q^4} B_0(y, 1) f_Q(x) c_q S_q^a(x, y|m_Q^2)
\]

(34)

\[
= \sum_{q=Q, Q} \frac{\alpha}{2\pi} \frac{Q^2}{m_Q^2} \int_0^1 \frac{dz}{1 - z} \left( \theta(z - z_0^a) \mathcal{J}_a(x, y, Q^2) \frac{d^2\sigma_q^a}{dx dy} \right)_{x=x, y=y, s=S} - \frac{d^2\sigma_q^a}{dx dy} .
\]
where $a = \text{ini, fin}$ and $m_Q$ is a constant quark mass. The appropriate choices of initial and final state scalings are

$$
\hat{p}_1^\text{ini} = z p_1, \quad \hat{p}_2^\text{fin} = \frac{1}{z} p_2,
$$

and the corresponding Jacobian is

$$
J(x, y, Q^2) = \left| \frac{\partial (\hat{x}, \hat{y})}{\partial (x, y)} \right|.
$$

Further details about scaling of kinematic variables may be found in tables 1 and 2. These tables contain, in addition, the scaling properties for other experimentally interesting sets of kinematical variables.

We now may address the following question: Is it possible, as has been proposed for the case of leptonic variables [7, 8, 9], to absorb the quark mass dependences completely by a redefinition of the quark distribution functions? For initial state radiation, this is evidently the case. The LLA cross section in mixed variables has the same structure as that in leptonic variables [5, 8] and may be treated in the same way. Otherwise, the contribution would be substantially overestimated (see [9]). This holds also for the other variables in table 1 with the notable exception of the hadronic ones.

In leptonic variables, there is in accordance with the KLN theorem no LLA contribution from final state radiation [10, 5]. For mixed variables (and other ones which are also not totally inclusive in the hadronic final state (see table 2)), this is different: There is a LLA contribution and its Born function $B_0(y, z)$ in (29) does not factorize, preventing the framework as developed in [9] to be operative here.

| Initial state radiation | $\hat{S}$ | $\hat{Q}^2$ | $\hat{y}$ | $z_0$ |
|-------------------------|-----------|------------|----------|------|
| leptonic variables      | $S z$     | $Q_i^2$    | $y_l$    | $x_l$|
| hadronic variables      | $S z$     | $z Q_h^2$  | $y_h$    | 0    |

Table 1: The definition of scaling variables for the leading logarithmic corrections due to photon emission from the initial quark leg. Those variable sets of table 2, which are not contained here transform as the leptonic variables do. The definitions of variables are explained in detail in [3,4].
|                  | $\hat{S}$ | $\hat{Q}^2$ | $\hat{y}$ | $z_0$ |
|------------------|-----------|-------------|-----------|-------|
| **leptonic variables** | $S$       | $Q_l^2$     | $y_l$     | $0$   |
| **mixed variables**     | $S$       | $Q_l^2$     | $\frac{y_h}{z}$ | $y_h$ |
| **hadronic variables**  | $S$       | $\frac{Q_h^2}{z}$ | $\frac{y_h}{z}$ | $y_h$ |
| **JB variables**        | $S$       | $Q_{JB}^2 \frac{1-y_h}{z(z-y_h)}$ | $\frac{y_h}{z}$ | $y_h + x_{JB}(1-y_h)$ |
| **double angle method** | $S$       | $Q_{DA}^2$  | $y_{DA}$  | $0$   |
| $\theta_1, y_h$        | $S$       | $Q_{\theta y}^2 \frac{z-y_h}{z(1-y_h)}$ | $\frac{y_h}{z}$ | $\frac{1-y_h(1-x_{\theta y})}{x_{\theta y}}$ |
| **$\Sigma$ method**    | $S$       | $Q_{\Sigma}^2 \frac{[y_{\Sigma}+z(1-y_{\Sigma})]}{z}$ | $\frac{y_{\Sigma}}{y_{\Sigma}+z(1-y_{\Sigma})}$ | $(\star)$ |
| **$e\Sigma$ method**   | $S$       | $Q_l^2 \frac{y_{e\Sigma}z}{[y_{e\Sigma}+z(1-y_{e\Sigma})]^2}$ | $(\star)$ | |

Table 2: The definition of scaling variables for the leading logarithmic corrections due to photon emission from the final state quark leg. Variable sets without rescaling have no final state LLA radiation (leptonic and double angle kinematics). The definitions of variables are explained in detail in [3,4].

$(\star)$ $z_0 = \frac{[1-2xy(1-y)-(1-4xy(1-y))^{1/2}]}{2x(1-y)^2}$
4 Numerical results

The QED corrections will be described by the following correction factor:

\[
\delta_{\text{mix}} = \frac{d^2\sigma - d^2\sigma_{\text{Born}}}{d^2\sigma_{\text{Born}}} \times 100\% \equiv \delta_e + \delta_i + \delta_q.
\]  

(37)

QED corrections in mixed variables are quite different from those in leptonic variables. Numerical results have been obtained with HECTOR [2], a Fortran program for the calculation of QED, electroweak, and QCD corrections to deep inelastic lepton nucleon scattering in different variables and different formal approaches: LLA – leading logarithmic approximation, with possible inclusion of higher order corrections; MI – complete \(O(\alpha)\), plus (or without) higher order LLA, [quark-parton] model-independent; QPM – complete \(O(\alpha)\), plus (or without) higher order LLA, in the quark-parton model. The latter approach is advocated here.

As a matter of fact we mention that the leptonic corrections \(\delta_e\) in mixed variables determined here in the QPM agree numerically exactly with those determined in the MI (when using the same structure functions). The latter rely on completely different expressions and are described in [1], where also the general features of QED corrections in mixed variables are explained.

On top of the leptonic corrections \(\delta_e\), which are the numerically largest ones, the effect of the interference corrections \(\delta_i\) is shown in figure 3. They are positive at intermediate values of \(y\) and become negative at \(y \geq 0.7\), nowhere exceeding in magnitude 1% except very large values of \(x\) and \(y\). The additional influence of the quarkonic corrections is shown in figures 4 and 5. In figure 4 we present the effect of the quarkonic corrections, calculated for definiteness with the assumption \(m_Q = x_h M_p\). Their effect is quite sizeable due to the unphysical quark mass singularities. Excluding both the initial and final state mass singularities from the prediction (and assuming now the case of constant quark masses, which chooses the sign discussed in footnote 4), we get figure 3. In this case the quarkonic corrections are rather small and the situation as shown in figure 3 is re-established.

The exclusion of initial state mass singularities is proposed in [9]. There it is shown that the photonic initial state LLA corrections, although modifying the QCD evolution equations, give rise to negligible numerical effects in the kinematical domain of HERA. The exclusion of final state mass singularities might be motivated by experimental reasons. It is scarcely possible to distinguish between bremsstrahlung photons and those from \(\pi^0\) decay, say. For an adequate treatment one has to redefine the kinematical variables by adding the energy of a final state photon to the hadronic energy. In such variables, the final state LLA corrections would vanish in accordance with KLN theorem.

To summarize, the QED corrections in mixed variables have a substantially different behavior compared to those measured in leptonic variables. The bulk of the corrections comes from the leptons. If the data will reach an accuracy of the order of a percent one has to take into account also the interference bremsstrahlung corrections which, together with the quarkonic ones, have been calculated here for the first time. The major part of the latter is associated with quark mass singularities and may be excluded from the corrections.

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5 The parton distributions CTEQ2L [11] and the HECTOR flag ITERAD=1 are chosen. This configuration uses the correct dependence of the structure functions on \(Q^2\). We also include higher order LLA corrections and the photonic vacuum polarization in the numerics. The latter gives a sizeable contribution of 5 \(\div\) 15%.

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Figure 3: Comparison of the radiative corrections $\delta_{\text{mix}}^{e+i} = \delta_e + \delta_i$ (broken line) with the leptonic corrections $\delta_e$ (solid line) at HERA in mixed variables.

Figure 4: Comparison of $\delta_{\text{mix}}^{e+i} = \delta_e + \delta_i$ (broken line) with $\delta_{\text{mix}} = \delta_e + \delta_i + \delta_q$ (solid line) at HERA.
Figure 5: Comparison of $\delta_{\text{mix}}^{e+i} = \delta_e + \delta_i$ (broken line) with $\delta_{\text{mix}} = \delta_e + \delta_i + \delta_{q^{\text{non-LLA}}}$ (solid line) at HERA.
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