Resource efficient white light interferometry algorithm for non-equidistant sampling steps

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Abstract. White light interferometry is a key technology in optical measurements and surface reconstruction. Depending on the amount of the raw data the acquisition and processing is a time-consuming process. Furthermore, the amount of raw data can easily exceed several gigabytes of memory. Particularly on embedded devices it is necessary to compress the raw data. This results in scattered data which must be processed. Depending on the compression algorithm arrangements must be made. Especially detecting the zero passages of the interferogram results in trouble. Common reconstruction algorithms fail in this case. Here a fast and scalable algorithm will be presented. Major improvements are resource efficiency and exact surface reconstruction. Additionally, optimizing the resulting bandwidth and processing time increases the user experience.

1. Introduction
White light interferometry is a time-consuming surface reconstruction technology in the nanometre down to sub-nanometre range. Several properties negatively impact the user experience and reproducibility. Scanning a wide range of piezo actuator positions to achieve the interferograms results in a large image stack, long time to achieve the data and noise from mechanical vibrations. Accelerating this is important to minimize these negative effects. Especially in embedded situations the memory bandwidth, photo sensor and mechanical properties of the piezo actuator limit this. Another huge impact is the processing speed of the algorithm. To optimize bandwidth and processing speed an algorithm able to process scattered data is to be preferred. In this paper, an algorithm feasible for scattered data and scalable to embedded platforms will be presented.

2. Theoretical background
Caused by the low coherence length of the white light source the absolute surface height can be reconstructed. Each pixel data must be processed for the complete image stack. Each pixel data results in an enveloped characteristic fringe. The formula for the pixel fringe is given in equation 1, assuming an Gaussian envelope [1].

\[ I(z) = I_0 + I_1 \cos(kz + \phi) \exp[-(z - z_m)^2(2w^2)^{-1}] \]  

(1)

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Where $z_m$ is the envelope peak position, $w$ the width of the envelope. The wavenumber $k$ and the phase shift $\phi$ are characteristics of the carrier frequency. The intensity of the modulation is given by $I_1$ while the offset intensity is $I_0$.

Most reconstruction algorithms are based on the principle of phase shifting [2]. In many cases, this is a good and well-tested method for accurate surface reconstruction. The majority of these algorithms depend on the fast Fourier transform (FFT). This numerical algorithm is widely used on hardware ranging from microcontrollers to supercomputers. In all major programming and scripting languages, the FFT can be used directly, or at least external libraries exist. This comes with various advantages and disadvantages. The most important disadvantages are the dependence on equidistant sampling rate and the amount of sampling points. The sampling has to be at least twice the highest frequency in the effective signal to satisfy the Nyquist–Shannon sampling theorem. With FFT it is also impossible to reconstruct the signal from zero crossing data points alone. If these sampling properties cannot be guaranteed other reconstruction algorithms have to be chosen.

2.1. Wavelet-transform

In contrast to the classical Fourier approach, the Wavelet transform does not use periodic sinusoidal functions. Instead a similar mother wavelet, compared to the useful signal, is used to find the maximum value of the envelope. The continuous Wavelet transform (CWT) can detect the position and the best matching frequency. Additionally, the signal is filtered by this fitted energy signal. A CWT using the Morlet wavelet on white light raw data is shown in figure 1. Here can be seen that only a small frequency range, in best case only a single frequency, depending on the bandwidth of the white light source is relevant for the reconstruction. The highest amplitude in the Wavelet-transform is at the position of the best matching frequency and best matching piezo actuator position.

![Figure 1. continuous wavelet transformation calculated from white light raw pixel data using the Morlet wavelet](image)

2.2 Compressive sensing

Compressive sensing is a relatively recent approach, which offers great abilities for the reconstruction of sparse or compressible signals. A signal can be represented by a set of orthonormal basis functions. This can be Fourier basis, Wavelet basis or other well-fitting bases depending on the application [3]. Information is compressible in the case only a few coefficients are necessary to reconstruct the signal using this orthonormal basis. Considering this a matrix containing the set of functions can be developed and the best matching coefficients for the signal must be found. This is possible by applying an discrete
II minimization to the sampled data [4]. Here the basis function is known therefore it is basically a minimization of residuum between the real valued function and sampled data at discrete points. Same is possible for the l2 minimization which is the square of the residua. An exact reconstruction is possible even in the case of undersampling in the sense of the Nyquist-Shannon sampling theorem.

3. Results
The algorithm was tested on simulated test data both with and without added noise and on data acquired by our white light interferometer targeting an inclined micrometre step. The result of the algorithm is the position of the envelope maximum. In Figure 2 the reconstructed height along a column and a row is shown. The coarse curves are the maximum values of the image stack raw data per pixel position using a step size of 20nm for the piezo actuator. The achievable resolution corresponds to the piezo actuator step size. Additionally, in the case of a phase shift the maximum of the envelope and the maximum of the light intensity differ. The acquired data is noisy, and the step size is usually not fine enough therefore it is necessary to reconstruct the raw data. The fine curves show the proposed algorithm reconstructs the surface as it. The aforementioned problems are compensated and the geometry shows a good fit.

![acquired row height of a µm step](image1.png)

![acquired column height of a inclined plane](image2.png)

**Figure 2.** reconstructed height of a µm step across a row and column using the maximum per pixel and the reconstruction using 20nm piezo step size

Mechanical vibrations negatively impact the quality of the raw data. Additionally, the time and memory consumption increase with smaller step sizes. For that reasons, a short acquisition time and big step sizes are preferable. The distance between the zeroth and the first maximum of the light intensity is half the central wavelength. To reconstruct the light intensity, it is necessary to sample at least two points of the curve between neighbouring maxima. In Table 1 simulated data of a white light interferometer with a white light source of 600nm and an envelope centre at \( z_m = 50000 \text{nm} \) was used to achieve the values. Different step sizes were used in the simulation. Equally distributed noise was added to the signal. The envelope amplitude was set to a value of 50 without an offset and the added noise amplitude varied from zero to ten. Comparing the reconstructed position of the envelope maximum with the theoretical position there is a constant offset of 2nm. Obviously, the noise negatively inflicts the quality of the reconstructed signal. This is perfectly true for higher step sizes. While the noiseless signal could be reconstructed even
at sub Nyquist step sizes real world data will have normal distributed noise. To achieve a good balance between reconstruction quality and acquisition time a step size of 100nm seems a good compromise.

**Table 1.** Reconstructed height from $l_1$ minimization (upper half) and $l_2$ minimization (lower half) with a pseudo random noise added and phase

| step size (nm) | $\phi=0$, Noise=0 | $\phi=-90$, Noise=0 | $\phi=0$, Noise=5 | $\phi=0$, Noise=10 |
|---------------|-------------------|---------------------|-------------------|-------------------|
| $l_1$         |                   |                     |                   |                   |
| 1             | 5002              | 5002                | 5000              | 5001              |
| 20            | 5002              | 5002                | 5003              | 5004              |
| 100           | 5002              | 5002                | 5007              | 5019              |
| 200           | 5002              | 5002                | 5015              | 5031              |
| 500           | 5002              | 5002                | 4995              | 5049              |

| $l_2$         |                   |                     |                   |                   |
| 1             | 5002              | 5002                | 5002              | 5001              |
| 20            | 5002              | 5002                | 5003              | 5009              |
| 100           | 5002              | 5002                | 5003              | 5013              |
| 200           | 5002              | 5002                | 5009              | 4992              |
| 500           | 5002              | 5002                | 4996              | 5043              |

4. Conclusions and future work
The presented approach is a promising combination of the well-known Wavelet transform and the $l_1$ minimization from compressive sensing acquired on white light interferometry fringes. Undersampled data could be reconstructed in many cases. Additionally, this approach can be accessed on non-equidistant data and even on data consisting only on zero crossings. Implementing this algorithm scales easily depending on the necessary accuracy in terms of execution time. On parallel hardware this solution benefits from the pixel independence of the white light interferometry technology. While the raw data of white light interferometry can easily exceed several gigabytes, only a small fraction of that must be held in memory at a time. Therefore, an inline implementation would improve the amount of data in temporary memory.

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