Higher Order Spin Resonances in a 2.1 GeV/c Polarized Proton Beam

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Spin resonances can depolarize or spin-flip a polarized beam. We studied 1st and higher order spin resonances with stored 2.1 GeV/c vertically polarized protons. The 1st order vertical (νy) resonance caused almost full spin-flip, while some higher order νy resonances caused partial depolarization. The 1st order horizontal (νx) resonance caused almost full depolarization, while some higher order νx resonances again caused partial depolarization. Moreover, a 2nd order νy resonance is about as strong as some 3rd order νx resonances, while some 3rd order νy resonances are much stronger than a 2nd order νx resonance. One thought that νy spin resonances are far stronger than νx, and that lower order resonances are stronger than higher order; the data do not support this.

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To study the strong interaction’s spin dependence with polarized proton beams, one must preserve and control the polarization during acceleration and storage. This can be difficult due to many 1st and higher order depolarizing (spin) resonances. For vertically polarized beams in flat accelerators, it was thought that vertical spin resonances should be stronger than horizontal resonances, and lower order resonances should be stronger than higher order resonances. There were several theoretical attempts to calculate the strengths of higher order spin resonances. Some 2nd order and synchrotron-sideband resonances were seen in electron rings and proton rings. Moreover, a 2nd order proton resonance was studied in detail at IUCF. We used 2.1 GeV/c polarized protons stored in the COSY synchrotron for a detailed experimental study of higher order spin resonances. Our preliminary νy data was presented at SPIN 2004, but both the νy data and the never-presented νx data needed significant reanalysis. The properly reanalyzed data presented here suggest that many higher order spin resonances, both νy and νx, must be overcome to accelerate polarized protons to high energies.

In flat circular rings, a beam proton’s spin precesses around the vertical fields of the ring’s dipole magnets. The spin tune νs = Gγ is the number of spin precessions during one turn around the ring, where G = (g − 2)/2 is the proton’s gyromagnetic anomaly and γ is its Lorentz energy factor. Horizontal magnetic fields can perturb the proton’s stable vertical polarization creating a spin resonance. Spin resonances occur when

νs = kνx + lνy + m,  \hspace{1cm} (1)

where k, l and m are integers; νx and νy are the horizontal and vertical betatron tunes, respectively. Imperfection spin resonances occur when k = l = 0. Intrinsic spin resonances occur when either k ≠ 0 or l ≠ 0, or both; the sum |k| + |l| defines each resonance’s order.

The experiment’s apparatus, including the COSY storage ring, EDDA detector, electron cooler, low energy polarimeter (LEP), injector cyclotron, and polarized ion source, were shown in Fig. 1 of Ref. 29. The beam from the polarized H− ion source was accelerated by the cyclotron to 45 MeV and then strip-injected into COSY.

Before this injection, the LEP measured the H− beam’s polarization to monitor its stability. The cylindrical EDDA detector measured the beam’s polarization in COSY after crossing the resonances. We reduced its systematic errors by cycling the polarized source between the up and down vertical polarization states. The measured flat-top polarization, before crossing any resonances, was typically about 75%.

In the COSY ring, the protons’ average circulation frequency fc was 1.491 85 MHz at 2.1 GeV/c, where their Lorentz energy factor was γ = 2.4514. For these parameters, the spin tune νs = Gγ was 4.395. During injection, acceleration and at the beginning of the flat-top the betatron tunes νx and νy were kept fixed at 3.575 and 3.525, respectively. This kept both betatron tunes away from any 1st, 2nd, or 3rd order spin resonances on flat-top. After reaching the flat-top, we varied the ring quadrupoles’ currents to vary either νy or νx, while keeping the other tune fixed; then we measured the polarization.

Figure 1 shows the betatron tunes’ behavior in a typical COSY cycle, during the higher order vertical (νy) spin resonance study; we first ramped νy rapidly from 3.525 to some value between 3.51 and 3.71 during 0.5 s. Next we slowly ramped νy through a very small tune range of about 0.002 during 2 s, with νx fixed at 3.575; then we measured the polarization. The rapid ramp reduced the effects of the resonances between the injection tune

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of 3.525 and the start of the slow ramp, while each slow \( \nu_y \) ramp enhanced the effect of any spin resonance in its very small \( \nu_y \) range.

The Low Energy Polarimeter (LEP) monitored the beam polarization before injection into COSY. The measured LEP asymmetries indicated that the initial polarization changed during the experiment by about 10%. Thus, we normalized each final COSY polarization measured by EDDA to the measured LEP asymmetry for that data-run. The typical duration of each EDDA data-run was 25 min; thus, the LEP data bin sizes were typically 60 min (±30 min) to include one high count-rate LEP run before each data-run, and one after. When needed, the LEP bin size was increased to include a high count-rate LEP run both before and after each data run.

The measured polarizations for the higher order vertical (\( \nu_y \)) spin resonance study are plotted against the measured \( \nu_y \) values in Fig. 2. The measured slow betatron tune ramps of about 0.002 are shown as a horizontal bar for each data-point. We used Eq. (1) to calculate the positions of 1\(^{\text{st}}\), 2\(^{\text{nd}}\) and 3\(^{\text{rd}}\) order resonances that could be studied between the half-integer 3.5 and quarter-integer 3.75 beam blow-up resonances.

To test the data’s reproducibility, we tried to measure polarizations at the same \( \nu_y \) settings several times. However, when we precisely measured the \( \nu_y \) values after each setting, we found that the slow ramps were often not exactly identical at the ±0.0002 level. Thus, Fig. 2 has many partly-overlapping points, which obscure the polarization’s behavior near each resonance. We tried to clarify Fig. 2 by combining points with nearby \( \nu_y \) values, except in the regions where the polarization changed very rapidly (between \( \nu_y \) values of 3.586 to 3.620). We first combined all pairs of points that had \( \nu_y \) values within \( \delta \nu_y = 0.1 \times 10^{-4} \). To help ensure that this did not bias the results, we combined the data in both the increasing (Left-to-Right) and decreasing (Right-to-Left) \( \nu_y \) directions; the two results were identical. We then sequentially increased the \( \delta \nu_y \) intervals in steps of \( 0.1 \times 10^{-4} \); the input data for each step were the output data from the previous step. The error and position of each newly combined point after each step were the properly weighted averages of the errors and positions of the two combined points; each new horizontal bar encompassed the slow ramps of both combined points.

Figure 3 plots polarization vs. \( \nu_y \) for the combination interval of \( \delta \nu_y = 7.6 \times 10^{-4} \). The 76 combination steps reduced the number of data points from 131 to 95. The plot shows clear resonance behavior around several 3\(^{\text{rd}}\) order resonances, but the behavior around the 2\(^{\text{nd}}\) order resonance is still unclear. When we further increased the combination interval size, the polarization’s behavior around the narrow resonances was broadened excessively, as expected.

We observed full spin-flip when the 1\(^{\text{st}}\) order vertical (\( \nu_y \)) spin resonance was crossed; we also found partial depolarization near several 3\(^{\text{rd}}\) order resonances and possibly near a 2\(^{\text{nd}}\) order resonance. The 3\(^{\text{rd}}\) order \( \nu_s = 8 + \nu_x - 2 \nu_y \) resonance and the partly overlapping 3\(^{\text{rd}}\) order 15 - 3\( \nu_y \) and 8 - 2\( \nu_x + \nu_y \) resonances appear significantly stronger than the 2\(^{\text{nd}}\) order 2\( \nu_y - 3 \) resonance. This suggests that many significant 3\(^{\text{rd}}\) and possibly higher order spin resonances must be overcome to accelerate and store polarized protons above 100 GeV.

We also studied the higher order horizontal (\( \nu_z \)) spin resonances by using \( \nu_z \) ramps similar to the \( \nu_y \) ramps in Fig. 1, with \( \nu_y \) fixed at 3.525. We first rapidly ramped \( \nu_x \) from 3.575 to a value between 3.525 - 3.74 in 0.5 s; we next slowly ramped \( \nu_x \) through a range of about 0.002 in 2 s; then we measured the polarization. The rapid ramp
again reduced the effects of the resonances between the injection tune of 3.575 and the start of the slow ramp, while each slow tune ramp enhanced the effect of the resonance in that small νx range.

The polarizations are plotted in Fig. 4 against νx. The 5 pairs of overlapping points were combined, as earlier described for Fig. 3. Figure 4 shows almost full depolarization at the 1st order spin resonance. Above this resonance, the polarization increased steadily probably because this fairly strong resonance was crossed at increasing Δνy/Δt rates, which decreased the depolarization [10]. Δνx/Δt increased because the ramp time Δt was fixed at 0.5 s, while the ramp range Δνx was increased. Thus, we found partial depolarization near a 2nd order νx resonance and near several 3rd order νx resonances; these νx resonances all seem about equally strong. Recall that some 3rd order νy resonances seem significantly stronger than the 2nd order νy resonance.

Also note that the polarization increased significantly at the two νx beam-blow-up resonances probably because they removed mostly those beam particles with larger betatron amplitudes, as supported by the sharp decrease in the precisely measured count rates in EDDA at each blow-up resonance. These outside particles were probably more depolarized [31] when crossing the strong 1st order νx spin resonance; thus, removing them increased the beam’s polarization while decreasing its intensity.

The measured strengths of the 11 resonances, for which we had adequate data, are listed in Table 1. We first obtained the very strong 1st order νy resonance's P1 and Pf, respectively from the left and right horizontal dashed line fits in Fig. 3. We then obtained its strength ε using the measured Pf/P1 and the fast ramp’s time Δt of 0.5 s and Δν of 0.105 in the Froissart-Stora equation [10]:

$$P_f/P_1 = 2 \exp \left(-\left(\frac{\pi \varepsilon}{f_x}\right)^2 \frac{\Delta \nu}{\Delta t}\right) - 1.$$  \hspace{1cm} (2)

We could only set a lower limit on ε of $240 \times 10^{-6}$ because the 1st order νy resonance was so strong that the spin was fully flipped for our fixed Δt of 0.5 s. For the strong 1st order νy resonance, the blue dashed curve in Fig. 4 is the fit of Eq. (2) to the 8 data points just after crossing it, using Δt of 0.5 s and Δν equal to each point’s Δνx from the νx value at injection.

For each isolated 2nd and 3rd order resonance, we obtained its dip’s depth or polarization loss (Pf/P1) by using a χ² minimization fit of a 2nd order Lorentzian to that resonances data with a baseline obtained from its nearby points. The (Pf/P1) values of the two overlapping 3rd order νy resonances in Fig. 3 were obtained by a fit using two overlapping Lorentzians and the baseline shown by the horizontal dashed blue line. We simultaneously fit the stronger ($8 - 2\nu_x + \nu_y$) resonance to a 1st order Lorentzian, with its frequency a variable in the fit, and the weaker ($15 - 3\nu_y$) resonance to a 2nd order Lorentzian.
Lorentzian with its frequency held fixed at the calculated value shown by its dashed green line. The fits to all 2nd and 3rd order resonances are shown by the solid red curves in Figs. 3 and 4. Three 2nd and 3rd order resonances had no observable dip at their calculated \( \nu_y \) or \( \nu_y \) value; therefore, the lower limits on their \( P_f/P_i \) were taken to be 95\%, which was 4 times the average error on straight line fits to the data points near these apparently weak resonances. We then phenomenologically used \( (P_f/P_i) \) in Eq. (2) with our fixed experimental \( \Delta t \) of 2 s and \( \Delta \nu \) of 0.002 to obtain \( \varepsilon \) for each 2nd and 3rd order resonance.

| Type | Order | Resonance | \( P_f/P_i(\%) \) | \( \varepsilon \times 10^{-6} \) |
|------|-------|-----------|-----------------|-----------------|
| \( \nu_y \) | 1st | \( 8 - \nu_y \) | -97.2 ± 1.4 | > 240 |
| \( \nu_y \) | 2nd | \( 2\nu_y - 3 \) | > 95 | < 1.3 |
| \( \nu_y \) | 3rd | \( 15 - \nu_x - 2\nu_y \) | > 95 | < 1.3 |
| \( \nu_y \) | 2nd | \( 15 - 3\nu_y \) | 92.5 ± 0.6 | 1.6 ± 0.4 |
| \( \nu_y \) | 3rd | \( 8 - 2\nu_x + \nu_y \) | 80.1 ± 0.7 | 2.7 ± 0.2 |
| \( \nu_y \) | 3rd | \( 8 + \nu_x - 2\nu_y \) | 76.9 ± 4.5 | 2.9 ± 0.4 |
| \( \nu_x \) | 1st | \( 8 - \nu_x \) | F-S eq. fit | 41 ± 2 |
| \( \nu_x \) | 2nd | \( 2\nu_x - 3 \) | 77.7 ± 0.2 | 2.8 ± 0.2 |
| \( \nu_x \) | 3rd | \( 15 - 3\nu_x \) | inadequate data |
| \( \nu_x \) | 3rd | \( 15 - 2\nu_x - \nu_y \) | inadequate data |
| \( \nu_x \) | 3rd | \( 15 - \nu_x - 2\nu_y \) | > 95 | < 1.3 |
| \( \nu_x \) | 3rd | \( 8 - 2\nu_x + \nu_y \) | 77.7 ± 0.2 | 2.8 ± 0.2 |
| \( \nu_x \) | 3rd | \( 1 - \nu_x + 2\nu_y \) | 85.8 ± 4.7 | 2.2 ± 0.5 |

There were several theoretical attempts\(^{[6][11]}\) to calculate the strengths of higher order spin resonances; one\(^{[9]}\) suggests that odd order resonances may be stronger than even order resonances for rings with Siberian snakes. It is not yet clear if these theoretical approaches allow one to explain our experimental results.

In summary, we used 2.1 GeV/c polarized protons stored in the COSY synchrotron to study 1st and higher order spin resonances. We observed almost full spin-flip when the 1st order \( 8 - \nu_y \) spin resonance was crossed and partial depolarization near the 2nd and 3rd order spin resonances. We also observed almost full depolarization near the 1st order \( 8 - \nu_x \) spin resonance and partial depolarization near the 2nd and 3rd order spin resonances. It was thought that, for vertically polarized protons in flat accelerators, vertical spin resonances are stronger than horizontal resonances, and lower order resonances are stronger than higher order resonances. The data suggest that many higher order spin resonances, both horizontal and vertical, must be overcome to accelerate polarized protons to high energies; these data may help RHIC to better overcome its snake resonances between 100 and 250 GeV/c.

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NOTE: The below figure was not included in the PRL due to space limitation, but was included in a March 2012 CERN Courier.

FIG. 5: The measured spin resonance strengths obtained from Fig. 3 and Fig. 4 are plotted against the resonance order. The up arrow indicates a lower limit on the resonance strength, while the down arrows each indicate an upper limit. The paper was submitted to PRL on 25 April 2011 and published on 13 February 2012 PRL 108, 074801 (2012) without Fig. 5 due to space limitation.