Tri-bimaximal neutrino mixing and neutrinoless double beta decay

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We present a tri-bimaximal lepton mixing scheme where the neutrinoless double beta decay rate has a lower bound which correlates with the ratio \( \alpha \equiv \Delta m_{\text{sol}}^2/\Delta m_{\text{atm}}^2 \) well determined by current data, as well as with the unknown Majorana CP phase \( \phi_{12} \) characterizing the solar neutrino sub-system. For the special value \( \phi_{12} = \frac{\pi}{2} \) (opposite CP-sign neutrinos) the \( \beta \beta \) rate vanishes at tree level when \( \Delta m_{\text{sol}}^2/\Delta m_{\text{atm}}^2 = 3/80 \), only allowed at 3σ. For all other cases the rate is nonzero, and lies within current and projected experimental sensitivities close to \( \phi_{12} = 0 \). We suggest two model realizations of this scheme in terms of an \( A_4 \times Z_2 \) and \( A_4 \times Z_4 \) flavour symmetries.

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Current neutrino oscillation data \([1, 2, 3, 4, 5, 6, 7]\) indicate a peculiar pattern \([8]\) of neutrino masses and mixings quite at variance with the structure of the Cabibbo-Kobayashi-Maskawa quark mixing matrix \([9]\). However, they do not yet fully determine the absolute scale of neutrino masses nor shed any light on the issue of leptonic CP violation, two demanding challenges left for future experiments.

Lacking a basic theory for the origin of mass one needs theoretical models restricting the pattern of fermion masses and mixings and providing guidance for future experimental searches. An attractive phenomenological ansatz for leptons is the Harrison-Perkins-Scott (HPS) mixing \([10]\)

\[
U_{\text{HPS}} = \begin{pmatrix}
\sqrt{2/3} & 1/\sqrt{3} & 0 \\
-1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\
-1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2}
\end{pmatrix}
\]

(1)

which predicts the following values for the lepton mixing angles: \( \tan^2 \theta_{\text{atm}} = 1 \), \( \sin^2 \theta_{\text{Chooz}} = 0 \) and \( \tan^2 \theta_{\text{sol}} = 0.5 \), providing a good first approximation to the values \([8]\) indicated by neutrino oscillation experiments \([1, 2, 3, 4, 5]\).

As noted earlier \([11]\), when the charged lepton mass matrix \( M_l \) obeys

\[
M_{\ell} M_{\ell}^\dagger = U_{\omega} M_{\text{diag}}^2 U_{\omega}^\dagger,
\]

where \( U_{\omega} \) is the “magic” unitary matrix

\[
U_{\omega} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix},
\]

and the neutrino mass matrix has the form

\[
M_\nu \sim \begin{pmatrix} A & 0 & 0 \\ 0 & B & C \\ 0 & C & B \end{pmatrix},
\]

the resulting lepton mixing matrix has exactly the tri-bimaximal structure given in Eq. (1).

Here we consider schemes where neutrinos get mass a la seesaw, defined by the following mass matrices,

\[
M_l \sim \begin{pmatrix} \alpha & \beta & \gamma \\ \gamma & \alpha & \beta \\ \beta & \gamma & \alpha \end{pmatrix} = U_{\omega} M_{\text{diag}}^l U_{\omega}^\dagger;
\]

\[
m_D \sim \begin{pmatrix} a & 0 & 0 \\ 0 & a & b \\ 0 & b & a \end{pmatrix}; M_R \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\]

This “texture” constitutes a new ansatz for the lepton sector that can be realized (see below) in the framework of \( A_4 \)-based flavour symmetry models. The assumed symmetry of the Dirac mass term holds in \( SO(10) \) models where it comes from a 16 16 10 Yukawa coupling. In contrast with other existing tri-bimaximal \( A_4 \) based schemes, the gauge singlet seesaw mass term characterizing the heavy right-handed neutrinos is also a flavour singlet, instead of the neutrino Dirac mass term. This makes the scheme extremely predictice, as it involves as free parameters only the two modulii and the relative phase between \( a \) and \( b \).

After the seesaw mechanism, one obtains the effective light neutrino mass matrix \( M_\nu \) given as

\[
M_\nu = m_D \frac{1}{M_R} m_R^T \sim \begin{pmatrix} a^2 & 0 & 0 \\ 0 & a^2 + b^2 & 2ab \\ 0 & 2ab & a^2 + b^2 \end{pmatrix}.
\]
Rewriting the effective light neutrino mass matrix in the basis where charged leptons are diagonal one finds
\[
\mathcal{M}_\nu \equiv \begin{pmatrix}
  a^2 + \frac{4ab}{3} + \frac{2b^2}{3} & -\frac{1}{3}b(2a + b) & -\frac{1}{3}b(2a + b) \\
  -\frac{1}{3}b(2a + b) & \frac{2}{3}b(4a - b) & a^2 - \frac{2ab}{3} + \frac{2b^2}{3} \\
  -\frac{1}{3}b(2a + b) & a^2 - \frac{2ab}{3} + \frac{2b^2}{3} & \frac{2}{3}b(4a - b)
\end{pmatrix}.
\]

This matrix is fully determined by two complex parameters \(a\) and \(b\), which imply three physical real parameters, namely two moduli and a relative phase, which is the only source of leptonic CP violation in the scheme.

We note that \(\mathcal{M}_\nu\) is \(\mu \leftrightarrow \tau\) invariant, so it gives \(\theta_{13} = 0\) and \(\sin^2 \theta_{23} = 1/2\) as predictions. The state \((1, 1, 1)^t\) is an eigenstate of \(\mathcal{M}_\nu\) with eigenvalue \(a^2\), so the neutrino mass matrix \(\mathcal{M}_\nu\) is diagonalized by the tri-bimaximal mixing matrix, leading then to \(\tan^2 \theta_{12} = 0.5\). The three neutrino mass eigenvalues are
\[
\{m_1, m_2, m_3\} = \{(a + b)^2, a^2, -(a - b)^2\}.
\]

Data from neutrino oscillation experiments [1, 2, 3, 4, 5] determine pretty well two of the three parameters on the left-hand side \([8]\), namely the solar and atmospheric mass-square splittings. The remaining observable is precisely the neutrino-exchange amplitude for neutrinoless double beta decay, given by
\[
\langle m_\nu \rangle \equiv |m_{ee}| = |a^2 + \frac{4ab}{3} + \frac{2b^2}{3}|.
\]

This parameter can be given as a function of the three independent model parameters, which we choose to express in terms of the observables \(\Delta m^2_{\text{sol}}/\Delta m^2_{\text{atm}}\), \(\alpha\) and the relative phase between \(a\) and \(b\). The latter is directly related to the Majorana CP phase \([12, 13, 14, 15]\) characterizing the solar neutrino sub-system, \(\phi_{12}\), in a symmetric parametrization of the lepton mixing matrix where all phases appear attached to the corresponding mixing angle \([12, 13]\).

First we note that our scheme is compatible with negligible neutrinoless double beta decay, vanishing at the tree level i.e. \(m_{ee} = 0\). This happens only when CP is conserved with opposite CP parities \([16, 17]\) between \(\nu_1\) and \(\nu_2\) and for
\[
\alpha = \frac{\Delta m^2_{\text{sol}}}{\Delta m^2_{\text{atm}}} = \frac{3}{80} = 0.0375,
\]
as seen in Fig. [1] which is currently allowed at 3 \(\sigma\). For all other values of the CP phase the model gives a lower bound on the neutrinoless double beta decay which we display in Fig. [2] which we call the “Niemeyer” plot \([32]\). This plot exhibits two dips characterized by very small

\[FIG. 1: \text{Lower bound on the } \beta\beta_{0\nu} \text{ amplitude parameter } m_{ee} \text{ as function of } \alpha \equiv \Delta m^2_{\text{sol}}/\Delta m^2_{\text{atm}} \text{ for different values of the Majorana CP phase } \phi_{12} = -\pi/2 + t \text{ where } t = 0 \text{ (dark brown), } 0.001 \text{ (brown), } 0.004 \text{ (red), } 0.011 \text{ (dark orange), } 0.029 \text{ (orange), } 0.089 \text{ (yellow). The 1, 2 and 3\(\sigma\) ranges for } \alpha \text{ are also shown.}\]

\[FIG. 2: \text{Lower bound on the neutrinoless double beta decay amplitude parameter } m_{ee} \text{ as function of the Majorana CP phase } \phi_{12} \text{ for } \alpha \text{ within the 1\(\sigma\) (yellow) and 2\(\sigma\) (blue) ranges.} \]

\(\beta\beta_{0\nu}\) amplitudes, which correspond to almost full destructive interference between opposite CP sign neutrinos \(\nu_1\) and \(\nu_2\).

Notice that in the central region around the other CP conserving point \(\phi_{12} = 0\) the \(\beta\beta_{0\nu}\) amplitude is sizeable, and depends very sensitively on the Majorana phase \(\phi_{12}\), as displayed in Fig. [3].
It is a non-trivial task to produce a consistent flavour symmetry leading to a structure of the effective neutrino mass matrix $M_\nu$ that has, at least as a first approximation, the desired predictive pattern.

Here we suggest two possible realizations based on an $A_4$ flavour symmetry for the neutrino mass matrix. The discrete group $A_4$ is a relatively small and simple flavour group consisting of the 12 even permutations among four objects. It has a three-dimensional irreducible representation appropriate to describe the three generations observed. Originally, $A_4$ was proposed 18, 19] for understanding degenerate neutrino spectrum with nearly maximal atmospheric neutrino mixing angle. More recently, predictions for the solar neutrino mixing angle have also been incorporated within the so-called tri-bimaximal neutrino mixing schemes 20, 21, 22, 23, 24, 25, 26.

In our phenomenological $A_4$-based flavor symmetry schemes the neutrino mass comes from type-I seesaw mechanism with right-handed Majorana mass matrix proportional to the identity matrix. In both models leptons transform as $A_4$-triplets, while the standard Higgs is a flavour singlet 23. The lepton and scalar content of the models are specified in Tables I and II.

The $A_4 \times Z_2$ invariant Lagrangian characterizing the first model is renormalizable, and given by

$$\mathcal{L} = \lambda_0(L^cL)h + \lambda(L^cH) + \lambda'(L^c\varphi) + \lambda_R(\nu^c\nu^c)\xi,$$

where the first term involves an $A_4$-invariant coupling $\lambda_0$ that provides $\alpha$ in $M^l$, while the second involves a tensor $\lambda_{ij}$ with components $\beta$ and $\gamma$, and similarly for the next two terms. Note that in this case there are additional SU(2) $\otimes$ U(1) doublet Higgs scalar bosons $H_i, \Phi_i, \varphi$ transforming non-trivially under the flavour symmetry. We assume that these develop non-zero vacuum expectation values (vevs), with the structure

$$\langle H_i \rangle \sim (1, 1, 1); \quad \langle \Phi_i \rangle \sim (0, 0, 1)$$

Similar vev alignment condition has been used in Ref. 27. Note that the two zeros in $m_D$ follow from the alignment condition $\langle \Phi_1 \rangle = \langle \Phi_2 \rangle = 0$.

In contrast the second model contains only one SU(2) $\otimes$ U(1) doublet Higgs boson and its $A_4 \times Z_4$ symmetric leading-order Lagrangian is written as

$$\mathcal{L} = \lambda_0(L^cL^c)h \xi_1 + \lambda(L^cH)h + \lambda'(L^c\varphi^c)h \xi_2 + \lambda_R(\nu^c\nu^c)\xi_3,$$

where $\lambda_R$ is dimensionless while the others scale as inverse mass. Note the appearance of gauge singlet scalars $\phi, \phi'$ and $\xi_i$, transforming non-trivially under the flavour symmetry and coupling non-renormalizably to the lepton doublets. Their only renormalizable is the one giving rise to the large Majorana mass term. We assume that these “flavon” fields develop non-zero vacuum expectation values (vevs), with the structure

$$\langle \phi \rangle \sim (1, 1, 1); \quad \langle \phi' \rangle \sim (0, 0, 1)$$

Note that either way we obtain the desired predictive charged lepton and neutrino mass matrices discussed above.

| fields | $L_i$ | $\ell_i$ | $\nu_i^c$ | $h$ | $\phi$ | $\phi'$ | $\xi_1$ | $\xi_2$ | $\xi_3$ |
|--------|------|---------|--------|----|------|-------|-------|-------|-------|
| $SU(2)_L$ | 2 | 1 | 1 | 2 | 1 | 1 | 1 | 1 | 1 |
| $A_4$ | 3 | 3 | 3 | 1 | 3 | 1 | 1 | 1 | 1 |
| $Z_2$ | 1 | $\omega^3$ | $\omega$ | 1 | $\omega^3 \omega^3 \omega^3 \omega^3$ |

TABLE I: Lepton multiplet structure of model I

In summary, here we have proposed two $A_4$-based flavour symmetries leading to tri-bimaximal lepton mixing, namely $\tan^2 \theta_{\text{atm}} = 1$, $\sin^2 \theta_{\text{Chooz}} = 0$ and $\tan^2 \theta_{\text{sol}} = \ldots$
Although this implies a boring scenario for upcoming long baseline oscillation experiments \cite{28, 29} aiming to probe $\theta_{13}$ and leptonic CP violation in oscillations, we have analysed its implications for neutrinoless double beta decay. We have seen how the $\beta\beta$ amplitude parameter $m_{ee}$ has a lower bound which correlates with the ratio $\alpha \equiv \Delta m^2_{\text{sol}}/\Delta m^2_{\text{atm}}$, well determined by current neutrino oscillation data, as well as with the Majorana CP violating phase $\phi_{12}$. Accelerator neutrino oscillation experiments like MINOS, T2K and NOvA are expected to improve the determination of $\alpha$ in the not-too-distant future.

For the special value $\phi_{12} = \frac{\pi}{2}$ (opposite CP-sign neutrinos) one finds that $\beta\beta_{\nu}$ vanishes at tree level when $\Delta m^2_{\text{sol}}/\Delta m^2_{\text{atm}} \approx 3/80$. However this is only allowed at $3\sigma$, as seen from Fig. 11 at 1 $\sigma$ we currently have a lower bound $|m_{ee}| \gtrsim \text{few } \times 10^{-4}$ eV. For all other cases one has a nonzero $\beta\beta_{\nu}$ decay rate, with CP conservation with same CP-sign neutrinos already excluded. We have also presented in Fig. 13 the lower bound in the region close to $\phi_{12} = 0$, corresponding to the case of same CP-parity neutrinos, where neutrinoless double beta decay could soon be observed.

All our considerations refer to an effective low-energy model which assumes the vev alignment conditions, and the symmetry of the neutrino Dirac mass matrix, a relation which holds in the framework of $\text{SO}(10)$ unification \cite{30, 31}. A more complete picture formulated at the unified level is outside the scope of this letter. In principle the structure presented here can be lifted to the $\text{SO}(10)$ level, though fitting the flavour structure of quarks will require additional fields and/or symmetries. In such more complete scenario exact tri-bimaximality would be just a first approximation, corrections leading to calculable deviations from the predictions reported here. These issues will be taken up elsewhere.

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