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The Study on D-stability of Dynamic Interval Systems Based on Robust State Observers

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Abstract. In this paper, we address the design problems of robust state observers for discrete-time dynamic interval systems with D-stability constrains. A method of designing the robust state observers is presented based on some matrix inequalities. It is not assumed that the system under consideration is necessarily regular and causal, the objective is to determine the observer gain matrix $K$ such that the desired Robust Performance and Time-domain performance can be satisfied in the whole observe procedure and the closed-loop poles are placed in a specified disk while the regularity, causality and stability of the closed-loop system can be guaranteed for all admissible uncertainties, furthermore, the analytical expression of desired observer gain matrix $K$ is also derived. Finally, a numerical example is given to demonstrate the application of the proposed method.

1. Introduction
Ever since Luenburger[1] introduced the concept of observer for dynamic systems, it has become one of the most active research fields with significant advances owing to its utility and its intimate connection with fundamental system concepts. It is not only widely used in on-line monitoring and control of dynamic systems, but also in finding out and recognizing the mistakes of dynamic systems[2]. Due to linearization approximation, measurement error or unmolded dynamics, it is inevitable that some uncertainties will occur in model used for analysis and design, therefore, this situation motivates the present investigation on designing robust state observers for discrete-time interval systems with admissible norm-bounded uncertainties. In this paper, we consider the problem of robust state observers for interval systems with D-stability. We are interested in designing state observers such that for all admissible uncertainties, the closed-loop system of observer error is regular, causal and D-stable.

In terms of a set of matrix inequalities, the solution to this problem is derived and the construction of the desired observer solving the problem is also resented.

2. Problem Statement
The following notation will be used throughout this paper, $\mathbb{R}^n$ denotes the $n$ dimensional Euclidean space, $\mathbb{R}^{n \times n}$ is the set of all $n \times n$ real matrices. We denote by $W^T$ and $W^{-1}$ the transpose and inverse of any matrix $W$, respectively. The notation $X \geq Y$ (respectively, $X > Y$) with $X$ and $Y$ being symmetric matrices, means that the matrix $X - Y$ is positive semi-definite (respectively, positive definite). $I$ is the identity matrix with appropriate dimension.

Let us consider the following dynamic interval system:
\[
\begin{align*}
E_{x_{k+1}} &= A(\theta)x_k + Bu_k \\
y_k &= C(\theta)x_k
\end{align*}
\] (1)

where \( x_k \in \mathbb{R}^n \) is the state, \( u_k \in \mathbb{R}^m \) is the control input, \( y_k \in \mathbb{R}^l \) is the controlled output. The derivative matrix \( E \in \mathbb{R}^{m \times n} \) may be singular, we shall assume that \( \text{rank} E = d \leq n \), \( A(\theta) = [a_{ij}^0] \in \mathbb{R}^{m \times n} \) and \( C(\theta) = [c_{ij}^0] \in \mathbb{R}^{l \times n} \) denote the system matrix and measurement matrix of (1), respectively, with \( A^m = [a_{ij}^m] \in \mathbb{R}^{m \times n}, \quad A^M = [a_{ij}^M] \in \mathbb{R}^{m \times n}, \quad C^m = [c_{ij}^m] \in \mathbb{R}^{l \times n}, \quad C^M = [c_{ij}^M] \in \mathbb{R}^{l \times n} \) satisfying \( a_{ij}^m \leq a_{ij}^0 \leq a_{ij}^M, \quad c_{ij}^m \leq c_{ij}^0 \leq c_{ij}^M \) for all \( 1 \leq i, j \leq n \), where \( A^m, \quad A^M, \quad C^M, \quad C^m \) are known real matrices.

Let

\[
A = [a_{ij}] = \frac{1}{2} (A^M + A^m), \quad \Delta A = [\Delta a_{ij}] = \frac{1}{2} (A^M - A^m),
\]

\[
C = [c_{ij}] = \frac{1}{2} (C^M + C^m), \quad \Delta C = [\Delta c_{ij}] = \frac{1}{2} (C^M - C^m)
\]

Then System (1) can be written as

\[
\begin{align*}
E_{x_{k+1}} &= [A + E_a]x_k + Bu_k \\
y_k &= [C + E_c]x_k
\end{align*}
\] (4)

where \( E_a = [f_{ij}], \quad |f_{ij}| \leq \Delta a_{ij} \) and \( E_c = [d_{ij}], \quad |d_{ij}| \leq \Delta c_{ij} \) for all \( 1 \leq i, j \leq n \).

Assumption 1:
The matrix is full row rank.

Definition 1:
① The pair is said to be regular if is not identically zero.
② The pair is said to be causal if it is regular and .
③ The singular system or simply the pair is said to be D-stable if it is regular and all the roots of lie inside a specified disk with center at and radius , i.e., , where

\[
\text{Theorem 1}[4]:
\]
The pair is regular, causal and stable if and only if there exists an invertible symmetric matrix such that the following inequalities hold:

\[
(5)
\]

Assuming the dynamic interval system (1) is completely observable, we can construct the state observer as following:

\[
(6)
\]

where is the state estimate vector, is an observer gain matrix with appropriate dimensions to be determined.

Let estimate error vector , then

\[
(7)
\]

The objective of this paper is to determine the observer gain matrix such that for all admissible uncertainties and, the following requirements are simultaneously achieved:
① The closed-loop system (7) is regular and causal.
② All finite poles of the closed-loop system (7) are placed in the disk , that is . Assumption 1:
The matrix $\Delta C = [\Delta c_{ij}] = \frac{1}{2}(C^h - C^m)$ is full row rank.

**Definition 1:**

1. The pair $(E, A)$ is said to be regular if $\det(sE - A)$ is not identically zero.
2. The pair $(E, A)$ is said to be causal if it is regular and $\deg[\det(sE - A)] = \text{rank}E$.
3. The singular system $Ex_{k+1} = Ax_k$ or simply the pair $(E, A)$ is said to be D-stable if it is regular and all the roots of $\det(sE - A) = 0$ lie inside a specified disk $D(\alpha, \gamma)$ with center at $\alpha + j\gamma$ and radius $\gamma$, i.e. $\sigma(E, A) \subset D(\alpha, \gamma)$, where

   $\sigma(E, A) = \{s | \det(sE - A) = 0 | s \in C, |\alpha| + \gamma < 1\}$.

**Theorem 1 [4]:**

The pair $(E, A)$ is regular, causal and stable if and only if there exists an invertible symmetric matrix $P \in R^{n \times n}$ such that the following inequalities hold:

\[
\begin{aligned}
& EPE^T \geq 0 \\
& APA^T - EPE^T < 0
\end{aligned}
\]

Assuming the dynamic interval system (1) is completely observable, we can construct the state observer as following:

\[
Ez_{k+1} = [A + E_a]z_k + Bu_k - K[y_k - [C + E_c]z_k]
\]

where $z_k \in R^n$ is the state estimate vector, $K$ is an observer gain matrix with appropriate dimensions to be determined.

Let estimate error vector $e_k = x_k - z_k$, then

\[
Ee_{k+1} = ([A + E_a] + [C + E_c])e_k = A_e e_k
\]

The objective of this paper is to determine the observer gain matrix $K$ such that for all admissible uncertainties $E_a$ and $E_c$, the following requirements are simultaneously achieved:

1. The closed-loop system (7) is regular and causal.
2. All finite poles of the closed-loop system (7) are placed in the disk $D(\alpha, \gamma)$, that is

   $\sigma(E, A_k) \subset D(\alpha, \gamma)$.

3. **Main Results**

   Firstly, we present the following facts [3] and lemma which will be used in the proof of our main results.

   **Face 1:**

   Let $X = [x_{ij}] \in R^{n \times n}$, $Y = [y_{ij}] \in R^{n \times n}$ be real matrices with satisfying $|x_{ij}| \leq |y_{ij}|$ for all $1 \leq i, j \leq n$, then

   \[
   XX^T \leq YY^T
   \]

   **Fact 2:**

   Let $X, Y$ be real matrices of suitable dimensions. Then, for any scalar $\lambda > 0$,

   \[
   XY + Y^TX^T \leq \lambda XX^T + \frac{1}{\lambda} Y^TY
   \]

   **Face 3:**

   For any real nonsingular matrices $X_1, X_3$ and real matrices $X_2, X_4$ with appropriate dimensions, it follows that

   \[
   (X_1 + X_2X_3X_4)^{-1} = X_1^{-1} - X_1^{-1}X_2X_3^{-1} + X_4X_1^{-1}X_2^{-1}X_1
   \]

   **Fact 4:**
Let $X$ be a real positive definite symmetric matrix, then there exists a real lower triangular matrix $B$, it follows that

$$X = B^T B$$  \hspace{1cm} (11)

If its entries are positive in diagonal position, then its factorization is unique.

**Lemma 1:**

If there exists an invertible symmetric matrix $P \in \mathbb{R}^{m \times n}$ such that

$$EPE^T \geq 0$$

$$APA^T - \alpha EAP^T - \alpha APE^T + (\alpha^2 - \gamma^2)EPE^T < 0$$  \hspace{1cm} (12)

then the pair $(E, A)$ is regular, causal and $\sigma(E, A) \subseteq D(\alpha, \gamma)$.

**Proof:** see[4]

Now we are in a position to present the result of this paper based on matrix inequalities.

**Theorem 1:**

Consider the closed-loop system(7). Given a positive definite symmetric matrix $Q \in \mathbb{R}^{n \times n}$ and $I \succeq Q$, let $A_0 = A + KC - \alpha E$, if there exists an invertible symmetric matrix $P \in \mathbb{R}^{m \times n}$, such that the following inequalities hold:

$$EPE^T \geq 0$$

$$Q - P > 0$$

$$A_0(P^{-1} - Q^{-1})^{-1}A_0^T + 2\Delta \Delta \Delta A^T + 2K \Delta \Delta \Delta C^T K^T - \gamma^2 EPE^T < 0$$  \hspace{1cm} (13)

then, the closed-loop system(7) is regular, causal and all finite poles of the closed-loop system(7) are placed in the disk $D(\alpha, \gamma)$, that is $\sigma(E, A_0) \subseteq D(\alpha, \gamma)$.

**Proof:**

Suppose that (13) has an invertible solution $P$. Note that since $Q - P > 0$, by Fact 4, there exists an invertible lower triangular matrix $G$ with appropriate dimensions such that

$$Q - P = G^T G$$  \hspace{1cm} (14)

Let $D = A_0 PG^{-1} - (E_a + KE_c)G^T$, then $DD^T > 0$. Next, we will show that the following inequality holds.

$$DD^T = [A_0PG^{-1} - (E_a + KE_c)G^T][A_0PG^{-1} - (E_a + KE_c)G^T]^T$$

$$= A_0PG^{-1}PA_0^T - (E_a + KE_c)G^T(G^T)^{-1}PA_0^T - A_0PG^{-1}G(E_a + KE_c)^T + (E_a + KE_c)G^T(G^T)^{-1}PA_0^T$$

$$= A_0P(Q - P)^{-1}PA_0^T - (E_a + KE_c)PA_0^T - A_0P(E_a + KE_c)^T$$

$$+ (E_a + KE_c)(Q - P)(E_a + KE_c)^T$$

$$= A_0[P + P(Q - P)^{-1}]A_0^T - (A_c - \alpha E)P(A_c - \alpha E)^T$$

$$+ (E_a + KE_c)Q(E_a + KE_c)^T$$  \hspace{1cm} (15)

i.e.

$$(A_c - \alpha E)P(A_c - \alpha E)^T$$

$$\leq A_0[P + P(Q - P)^{-1}]A_0^T + (E_a + KE_c)Q(E_a + KE_c)^T$$

$$\leq A_0[P + P(Q - P)^{-1}]A_0^T + (E_a + KE_c)(E_a + KE_c)^T$$  \hspace{1cm} (16)

Using Fact 2 and Fact 1, (16) can be rewritten as

$$(A_c - \alpha E)P(A_c - \alpha E)^T$$
\[ \leq A_0 [P + P(Q - P)^{-1} P] A_0^T + 2E_0E_{0}^T + 2(KE_0)(KE_0)^T \]
\[ \leq A_0 [P + P(Q - P)^{-1} P] A_0^T + 2\Delta A\Delta A^T + 2(KA)(KA)^T \]  
\[ = (A_0 - \alpha E)(A_0 - \alpha E)^T - \gamma^2 P^T \]
\[ \leq A_0 [P + P(Q - P)^{-1} P] A_0^T + 2\Delta A\Delta A^T + 2(KA)(KA)^T + (\alpha^2 - \gamma^2)EPE^T \]  
\[ = A_0 [P^{-1} - Q^{-1}]^{-1} A_0^T + 2\Delta A\Delta A^T + 2(KA)(KA)^T - \gamma^2 EPE^T < 0 \]  
\[ \text{Hence, by Lemma 1, it is easy to see that the pair } (E, A_0) \text{ is regular, causal and} \]
\[ \sigma(E, A_0) \subset D(\alpha, \gamma). \]

**Theorem 2:**

The closed-loop system (7) is regular, causal and all finite poles of the closed-loop system (7) are placed in the disk \( D(\alpha, \gamma) \), if there exists a positive definite symmetric matrix \( Q \in R^{n \times n} \) and \( I \geq Q \), such that the following inequalities

\[
\begin{align*}
EPE^T &\geq 0 \\
Q - P &\geq 0 \\
(A - \alpha E)[\psi^{-1} + \frac{1}{2}C^T(\Delta C\Delta C^T)^{-1}C]^{-1}(A - \alpha E)^T + 2\Delta A\Delta A^T - \gamma^2 EPE^T < 0
\end{align*}
\]

have an invertible symmetric matrix solution \( P \). And in this case, a suitable state observer can be chosen as:

\[ K = -(A - \alpha E)\psi C^T [C\psi C^T + 2\Delta C\Delta C^T]^{-1} \]  
\[ \psi = [P^{-1} - Q^{-1}]^{-1} \]  

**Proof:** Suppose that the inequalities (19) have an invertible symmetric matrix solution \( P \). Considering the state observer (6), by some simple algebraic manipulations, it follows that

\[ A_0(P^{-1} - Q^{-1})^{-1} A_0^T + 2\Delta A\Delta A^T + 2(KA)(KA)^T - \gamma^2 EPE^T \]
\[ = (A - \alpha E + KC)\psi(A - \alpha E + KC)^T + 2\Delta A\Delta A^T + 2(KA)(KA)^T - \gamma^2 EPE^T \]
\[ = (A - \alpha E)\psi(A - \alpha E)^T + (A - \alpha E)\psi C^T K^T + KC\psi(A - \alpha E)^T + 2\Delta A\Delta A^T + 2(KA)(KA)^T - \gamma^2 EPE^T \]
\[ = (A - \alpha E)[\psi - \psi C^T(C\psi C^T + 2\Delta C\Delta C^T)^{-1}C\psi](A - \alpha E)^T + 2\Delta A\Delta A^T - \gamma^2 EPE^T \]  
\[ \text{Note that Assumption 1 holds, it follows that } \Delta C\Delta C^T \text{ is invertible. Hence, by Fact 3, the} \]
expression (19) can be rewritten as

\[ (A - \alpha E)[\psi^{-1} + \frac{1}{2}C^T(\Delta C\Delta C^T)^{-1}C]^{-1}(A - \alpha E)^T + 2\Delta A\Delta A^T - \gamma^2 EPE^T \]
\[ = (A - \alpha E)[\psi - \psi C^T[C\psi C^T + 2\Delta C\Delta C^T]^{-1}C\psi](A - \alpha E)^T + 2\Delta A\Delta A^T - \gamma^2 EPE^T \]  
Thus, from (22), we have

\[ A_0 [P^{-1} - Q^{-1}]^{-1} A_0^T + 2\Delta A\Delta A^T + 2(KA)(KA)^T - \gamma^2 EPE^T < 0 \]  
and, hence, by Theorem 1, the desired results follows immediately.

**Remark 1.**
Theorem 2 shows that when there exists a positive definite symmetric matrix $Q \in \mathbb{R}^{n \times n}$ and $I \geq Q$ and an invertible symmetric matrix $P \in \mathbb{R}^{n \times n}$ satisfying matrix inequalities (19), the problem of robust state observer for dynamic interval systems with D-stability is solvable, and in this case a desired state observer can be obtained in terms of a solution to (23). The design procedure becomes relatively simple by using of the powerful LMI toolbox such as Matlab’s LMI Toolbox.

4. Numerical Example
In this section, we give an example to illustrate the effectiveness of the proposed method. Consider the dynamic interval system (1) with parameters as follows:

\[
E = \begin{bmatrix} 6 & 0.2 \\ 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 0.1 & 0 \\ 0.2 & 0.5 \end{bmatrix}, \quad \Delta A = \begin{bmatrix} 0.06 & 0.01 \\ 0.03 & 0.005 \end{bmatrix}, \quad C = \begin{bmatrix} 0.2 & 0.3 \\ 1 & 0 \end{bmatrix},
\]

\[
\Delta C = \begin{bmatrix} 0.45 & 0.075 \\ 0.78 & 0.13 \end{bmatrix}.
\]

Let $\alpha = 0.009$, $\gamma = 0.78$, $Q = \begin{bmatrix} 0.5 & 0.1 \\ 0.1 & 0.5 \end{bmatrix}$, by Theorem 3, a feasible solution of LMI (21) is as follows:

\[
P = \begin{bmatrix} 0.4066 & 0.1612 \\ 0.1612 & 1.7629 \end{bmatrix}, \quad K = \begin{bmatrix} 0.2712 & -0.1326 \\ 3.3526 & -1.4837 \end{bmatrix}.
\]

5. Conclusions
This note has studied the robust state observer problem for the dynamic interval systems with pole placement in a specified disk which is not assumed to be necessarily regular. A sufficient condition, which ensures not only regularity and causality of the closed-loop system, but also the closed-loop poles constraint in a specified disk for all admissible uncertainties, has been proposed. The desired robust state observer can be constructed by solving a set of matrix inequalities. The proposed method is demonstrated with an example. Extension of the results in this paper to the pure delay dynamic interval systems case is a topic for further research.

6. References
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