**1/f Noise in Electron Glasses**

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We show that 1/f noise is produced in a 3D electron glass by charge fluctuations due to electrons hopping between isolated sites and a percolating network at low temperatures. The low frequency noise spectrum goes as $f^{-\alpha}$ with $\alpha$ slightly larger than 1. This result together with the temperature dependence of $\alpha$ and the noise amplitude are in agreement with the recent experiments. These results hold true both with a flat, noninteracting density of states and with a density of states that includes Coulomb interactions. In the latter case, the density of states has a Coulomb gap that fills in with increasing temperature. For a large Coulomb gap width, this density of states gives a dc conductivity with a hopping exponent of $\approx 0.75$ which has been observed in recent experiments. For a small Coulomb gap width, the hopping exponent $\approx 0.5$.

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**INTRODUCTION**

Low frequency 1/f noise [1, 2, 3] is found in a wide variety of conducting systems such as metals, semiconductors, tunnel junctions [4], and even superconducting SQUIDs [5, 6]. Yet the microscopic mechanisms are still not well understood. One example is an electron glass which is an insulator where electrons are localized by a strong random potential. A special case of this is a Coulomb glass in which the electrons interact with one another via a long range Coulomb potential. Doped semiconductors and strongly disordered metals provide examples of electron glasses. Experimental studies on doped silicon inversion layers have shown that low frequency 1/f noise is produced by hopping conduction [7]. Because the systems are glassy, electron hopping can occur on very long time scales which can produce low frequency noise. In this paper we show that the resulting noise spectrum goes as $f^{-\alpha}$ where $f$ is frequency and the temperature dependent exponent $\alpha > 1$.

Shklovskii has suggested that 1/f noise is caused by fluctuations in the number of electrons in an infinite percolating cluster [8]. These fluctuations are caused by the slow exchange of electrons between the infinite conducting cluster and small isolated donor clusters. Subsequently Kogan and Shklovskii combined a more rigorous calculation with numerical simulations and found a noise spectrum where $\alpha$ was considerably lower than 1 [9]. Furthermore, below a minimum frequency of order 1–100 Hz, the noise spectral density saturated and became a constant independent of frequency. Their calculations were valid only in the high temperature regime where the impurity band was assumed to be occupied uniformly and long-range Coulomb correlations were essentially neglected. Since then there have been attempts to include the effects of correlations.

In particular, Kozub suggested a model [10] in which electron hops within isolated pairs of impurities produce fluctuations in the potential seen by other hopping electrons that contribute to the current. While leading to 1/f–type noise within some frequency range, this model also shows low frequency noise saturation due to the exponentially small probability of finding an isolated pair of sites with a long tunneling time. Moreover, the noise magnitude is predicted to increase as the temperature $T \to 0$ in contradiction with the recent experimental findings of Massey and Lee [11]. This, in part, led Massey and Lee to the conclusion that the single particle picture is inconsistent with the observed noise behavior. A different approach was proposed by Kogan [12] who considered intervalley transitions as the source of the hopping conduction noise. Unfortunately this approach does not seem to be analytically tractable and is not easily generalizable.

In this paper we extend Kogan and Shklovskii’s approach [9] by including the energy dependence of the hopping as well as the effects of electron–electron interactions on the single particle density of states $g(\varepsilon)$. This is essentially a mean field approximation: we assume that charge is carried by electron-like quasiparticles whose interaction with the other charges is taken into account via the single particle density of states. Later we will present some justification for why we believe this approach works for low frequency noise. For comparison we also consider the case of noninteracting electrons with a flat density of states.

The paper is organized as follows. In section IIa, we describe our calculation of the noise spectral density. In section IIb, we present the density of states that includes the Coulomb gap and that models the decrease in the gap with increasing temperature. We show that this form of the density of states yields the usual value of the hopping exponent $\delta \approx 0.5$ for small values of the Coulomb gap width $E_g$. However, for large values of $E_g$, $\delta \approx 0.75$. Both values have been seen experimentally [11, 13, 14, 15, 16, 17]. In section III, we present our results.

**CALCULATION**

**Noise Spectral Density**

We start with a model of the Coulomb glass in which electrons occupy half of the impurity sites. Each site can have at most 1 electron due to a large onsite repulsion. The sites...
are randomly placed according to a uniform spatial distribution, and each has a random onsite energy $\phi_i$ chosen from a uniform distribution extending from $-W/2$ to $W/2$. Thus, $g_{\sigma}$, the density of states without interactions, is flat. At $T = 0$ such a system is a perfect insulator while at low but finite temperatures it will be able to conduct via variable range hopping [18, 19, 20]. In this picture the DC conductivity is dominated by particles hopping along the percolating network, which is constructed as follows. The resistance $R_{ij}$ associated with a transition between sites $i$ and $j$ grows exponentially with both their separation $r_{ij}$ and energy difference $\epsilon_{ij}$:

$$R_{ij} = R_0\exp (x_{ij})$$

where the prefactor $R_0 = kT/(e^2\tau_{ij})$ with $\tau_{ij}$ given by [18]

$$\tau_{ij} = \frac{D^2|\Delta|^2}{\pi\kappa^3h^4} \left[ \frac{2e^{2}}{3\kappa|x|} \right] ^{2} \frac{1 + \left( \frac{\Delta}{2kT} \right)^2}{\xi^2} \right]^{-4}$$

where $D$ is the deformation potential, $s$ is the speed of sound, $\rho$ is the mass density, $\xi$ is the localization length and $\kappa$ is the dielectric constant. $\Delta_{ij} = \epsilon_j - \epsilon_i - e^2/\kappa r_{ij}$ is the change in energy that results from hopping from $i$ to $j$ with $\epsilon_i = \phi_i + \sum_j e^2/\kappa r_{ij} n_j$ being a single site energy. In Eq. (1), the exponent is given by

$$x_{ij} = \frac{2r_{ij}}{\xi_0} + \frac{\epsilon_{ij}}{kT}$$

The exponent reflects the thermally activated hopping rate between $i$ and $j$ as well as the wavefunction overlap between the sites.

$$\epsilon_{ij} = \left\{ \begin{array}{ll} |\epsilon_j - \epsilon_i| - e^2/\kappa r_{ij}, & (\epsilon_j - \mu)(\epsilon_i - \mu) < 0 \\ \max[|\epsilon_j - \mu|,|\epsilon_i - \mu|], & (\epsilon_j - \mu)(\epsilon_i - \mu) > 0 \end{array} \right.$$ (4)

(In what follows we choose the Fermi level $\mu = 0$.)

A noninteracting picture of DC conduction is described in terms of electron hopping between sites in a cluster that spans the entire sample. In order to determine which sites are in a cluster, we introduce the “acceptance” parameter $x$ such that any two sites $i$ and $j$ are considered “connected” if $x_{ij} \leq x$ and disconnected otherwise. For small values of $x$ only rare pairs of sites are connected. As we increase $x$, more such pairs appear and small clusters start coalescing into bigger ones until an infinite cluster – the critical percolating network – is formed at some $x_c$. At this point we can neglect the contribution of the remaining impurity sites to the DC conductivity since it is exponentially small compared to that of the sites already in the percolating network (although the former sites are important for understanding both AC conductivity and noise). In the same spirit, the resistance of the critical percolating network is dominated by a few pairs with $x_{ij} = x_c$ – these are the pairs that bridge the gaps between large finite clusters enabling the formation of the infinite cluster. Hence, the resistance of the entire sample is well approximated by $R_{\text{tot}} \approx R_c \exp (x_c)$ where $R_c = kT/(e^2/\kappa)$ being the average value of $\tau_{ij}$ given by Eq. (4).

In the presence of Coulomb interactions, there is no exact mapping of transport onto a percolation picture. We nevertheless assume that upon diagonalizing the interacting Hamiltonian one finds that charge carrying excitations are of a local nature, and so they can be treated within the percolation picture as noninteracting quasiparticles. The Coulomb interactions renormalize the single-particle density of states of which acquires a soft gap. We will discuss this in more detail in the section on the density of states. However, we will mention here that this approach appears to work well for DC conduction and leads to a temperature dependence of the conductivity [18, 21, 22] which is distinctly different from the noninteracting case and which agrees with experiment (see for example ref. 23). However, the question about the validity of this approach is still far from being settled – see [24] for a different point of view.

In our treatment we will focus on the noise caused by quasiparticle hopping between isolated clusters and the percolating network, producing fluctuations of charge in the latter [8, 9]. Let $N_{\alpha}$ be the average number of such particles in the critical percolating network and $\Delta N_{\alpha}(t)$ be its time-dependent fluctuation. Assuming that only stationary processes are involved (i.e. $\langle \Delta N_{\alpha}(t)\Delta N_{\alpha}(t') \rangle = f(t_2 - t_1)$), we can use the Wiener–Khintchine theorem [3] to relate the noise spectral density $S_i(\omega)$ to current fluctuations to the Fourier transform of the autocorrelation function:

$$S_i(\omega) = \frac{2(\Delta N_{\alpha}(t)\Delta N_{\alpha}(t'))_0}{N_{\alpha}^2} \frac{2kT}{e^2} \sum_{\alpha \neq 0} \frac{\tau_{\alpha}}{1 + \omega^2 \tau_{\alpha}^2} \sum_{\mu \neq \alpha} C_i \psi_{\alpha}(i) \psi_{\mu}(j)$$ (6)

Here $C_i \equiv (e^2/kT)f_i(1 - f_i)$ is the “capacitance” of site $i$ (with $f_i = [\exp(\epsilon_i/kT) + 1]^{-1}$ being its equilibrium occupancy) while $\tau_{\alpha}^{-1}$ and $\psi_{\alpha}(i)$ are the $\alpha$-th eigenvalue and eigenvector of the following system of linear equations:

$$\sum_{j} R_{ij}^{-1} \left| \psi_{\alpha}(i) - \psi_{\alpha}(j) \right| = \tau_{\alpha}^{-1} C_i \psi_{\alpha}(j)$$ (7)

with $R_{ij}$ being the inter-site resistances given by Eq. (1). Since $R_{ij}^{-1}$ is proportional to the hopping rate $\tau_{ij}^{-1} = \tau_{ij}^0 \exp (-x_{ij})$ from site $i$ to site $j$, Eq. (4) relates $\tau_{ij}^{-1}$ to the relaxation rates $\tau_{\alpha}^{-1}$ of the entire percolating network. The sum over sites $i$ in Eq. (6) runs only over those sites that belong to the critical percolating network (CN) since only their occupancies affect
the current through the sample. The physical meaning of the quantity \( C_\lambda \psi_\alpha(i) \) is that it is proportional to the fluctuation \( \delta f_i \) of the occupation of site \( i \) and decays exponentially with the associated time constant \( \tau_\alpha \). The eigenvectors satisfy the following conditions:

\[
\sum_\alpha C_\lambda \psi_\alpha(i) \psi_\beta^\ast(j) = \delta_{ij} \quad (8)
\]

\[
\sum_\alpha C_\lambda \psi_\alpha(i) \psi_\alpha(j) = \delta_{ij} \quad (9)
\]

\[
\sum_\alpha C_\lambda \psi_\alpha(i) = 0 \quad \forall \alpha \neq 0 \quad (10)
\]

The first condition states that the eigenfunctions are orthonormal; the second states that the functions form a complete set. One of the eigenfunctions is a constant which we take to be the one corresponding to \( \alpha = 0 \). This has the eigenvalue \( \tau_0^{-1} = 0 \). Eq. (10) is the orthonormalization condition between this eigenstate and the others. It represents the fact that the fluctuations in occupation represented by the \( \alpha \neq 0 \) modes do not affect the total number of electrons on the impurity sites. Thus the last equation is just the statement of overall charge conservation. We remark here that Eqs. (7) are linear only within the assumption made earlier of noninteracting quasiparticles. Otherwise the \( R_{ij} \) are not constant coefficients; they depend on the on-site energies, which in turn depend on the occupancies of other sites.

Since we are interested in the modes that affect the charge in the conducting network, we can replace the sum over \( \alpha \) by a sum over all finite clusters that coalesce with the infinite cluster as the acceptance parameter increases above \( x_c \). In particular we can replace the sum over \( \alpha \) by an integral over \( x \) and a sum over all finite clusters merging with the infinite cluster at a given value of \( x \). With this in mind, we can evaluate Eq. (5) using Eqs. (8) and (10). For a single mode \( \alpha \) the sum over sites \( i \) can be split into a sum over finite clusters (FC) and a sum over the infinite cluster (IC). So we can write the normalization condition Eq. (8) and the charge conservation condition Eq. (10) as

\[
\sum_{m \in \text{FC}} C_m \psi_\alpha^2(m) + \sum_{n \in \text{IC}} C_n \psi_\alpha^2(n) = 1 \quad (11)
\]

\[
\sum_{m \in \text{FC}} C_m \psi_\alpha(m) + \sum_{n \in \text{IC}} C_n \psi_\alpha(n) = 0 \quad (12)
\]

Since the fast modes equilibrate the occupations of sites within each cluster, the eigenfunctions do not depend on their site indices within each cluster, i.e., \( \psi_\alpha(m) = \psi_\alpha, \text{FC}, \forall m \in \text{FC} \) and \( \psi_\alpha(n) = \psi_\alpha, \text{IC}, \forall n \in \text{IC} \). As a result the we can take \( \psi_\alpha, \text{FC} \) and \( \psi_\alpha, \text{IC} \) out of the sums in Eqs. (11) and (12). The sum over capacitances in the finite clusters will be much smaller than the sum over the infinite cluster which implies that \( \psi_\alpha, \text{IC} \) is negligible in Eq. (11). This leads to

\[
\psi_\alpha, \text{FC} = \left( \sum_{m \in \text{FC}} C_m \right)^{-1/2}
\]

Plugging this into Eq. (12) yields

\[
\psi_\alpha, \text{IC} \sum_{m \in \text{IC}} C_m = - \left( \sum_{m \in \text{FC}} C_m \right)^{1/2}
\]

We can use these results to evaluate the sum over sites in Eq. (6) by noting that all the sites in the critical network are also in the infinite cluster by definition. Thus

\[
\sum_{i \in \text{FC}} C_\lambda \psi_\alpha(i) = \frac{N_\text{IC}(x)}{N_{\text{IC}(x)}} \sum_{i \in \text{IC}} C_\lambda \psi_\alpha, \text{IC}
\]

\[
= - \frac{N_\text{IC}(x)}{N_{\text{IC}(x)}} \left( \sum_{i \in \text{FC}} C_i \right)^{1/2}
\]

where \( N_{\text{IC}(x)} \) is the number of sites in the infinite cluster at a given value of \( x \).

In evaluating Eq. (6), we make the following approximation for \( \tau_\alpha \). Since we are interested in the modes \( \alpha \) that affect the charge of the percolating network, we only consider particle exchange between the isolated clusters and the infinite cluster. This involves hopping times that are longer than those within the percolating network itself by definition. Due to the exponentially wide distribution of hopping times \( \tau_j \) such exchange is likely to be dominated by the single closest pair of sites of which one belongs to the finite, and the other to the infinite cluster. The relaxation times within each cluster are much faster, and therefore the above mentioned pair serves as a “bottleneck” for intercluster relaxation. A simple diagonalization of the system of equations (7) for two clusters \( A_1 \) and \( A_2 \), with the “bottleneck” hopping resistance \( R = \min(R_{ij} \mid i \in A_1, j \in A_2) \) between them (and with the assumption that all other intercluster resistances are much higher and all intracluster resistances are much lower than \( R ) leads to the following expression for the intercluster relaxation time:

\[
\tau = R \left( \sum_{j \in A_1} C_j \right)^{-1} \left( \sum_{j \in A_2} C_j \right)^{-1}
\]

where we are interested only in the situation when one of the clusters is infinite, this simplifies Eq. (16): \( \tau = R \sum_{j \in \mathcal{A}} C_j \), where \( \mathcal{A} \) is the finite cluster.

We can substitute this value of \( \tau \) into Eq. (5) by replacing the sum over all modes \( \alpha \) by a sum over all finite clusters that coalesce with the infinite cluster as the acceptance parameter \( x \) is increased above \( x_c \). Each such finite cluster contributes one new term to the sum over \( \alpha \) in Eq. (6) with the corresponding \( \tau_\alpha \) given by (16).

We then can write the spectral density of the noise as follows:

\[
S_j(\omega) = \frac{1}{T^2} \frac{16kT}{e^2} \int_{x_c}^\infty dx \sum_{\mathcal{A}} N_\text{IC}^2(x) R(x) \sum C_j^2 \left( \sum_{i \in \mathcal{A}} C_i \right)^2
\]

where \( \sum' \) stands for the sum over all finite clusters that coalesce with the infinite cluster as \( x \) increases by \( dx \). The parameter \( \lambda > 1 \) sets the distance in \( x \) space from the percolation threshold.

This equation is difficult to evaluate mathematically. Fortunately, however, we can extract the low frequency asymptotic behavior of Eq. (17) where the above approximations are well
Thus and with the onsite energy \( \epsilon \), the probability that no neighbors nearer than \( x \) have energy \( \epsilon \) is given by

\[
P_1(x, \epsilon) = \exp \left( - \int_0^x p(x', \epsilon) dx' \right)
\]  

(18)

We can write this expression for \( P_1(x, \epsilon) dx \) as the product of \( P_1(x, \epsilon) \), the probability of no neighbors within \( x \), multiplied by the probability of having a neighbor between \( x \) and \( x + dx \):

\[
P_1(x, \epsilon) dx = P_1(x, \epsilon) \left[ 1 - e^{-p(x, \epsilon) dx} \right] = -\left[ \frac{\partial}{\partial x} P_1(x, \epsilon) \right] dx
\]

(19)

(20)

Thus \(-p(x, \epsilon)/\partial x\) is the probability density for a site to have its nearest neighbor between \( x \) and \( x + dx \). We can now write the spectral density of current fluctuations as

\[
\frac{S_I(\omega)}{T^2} = \frac{16 k T V}{e^2 N^2 c} \int_{-\infty}^{\infty} dx \int_{-W/2}^{W/2} d\epsilon \ g(\epsilon, T) \left( -\frac{\partial P_1(x, \epsilon)}{\partial x} \right) \times \frac{R(x) C^2(\epsilon)}{1 + \omega^2 R^2(x) C^2(\epsilon)}
\]

(21)

where \( V \) is the volume, \( W \) is the bandwidth, and \( f(\epsilon) \) is the Fermi occupation number. To obtain an expression for \( P_1(x, \epsilon) \), we note that the average number \( dN \) of impurity sites found in a phase volume element \( d\Omega = d^d r d\epsilon \) within a distance \( x \) of a site with energy \( \epsilon \) is given by

\[
dN = g(\epsilon) \theta \left( x - 2r \frac{r}{\xi} - \frac{|\epsilon + |\epsilon'| + |\epsilon - \epsilon'|}{2kT} \right) d\epsilon' d r
\]

(22)

The probability that no sites are in \( d\Omega \) is given by

\[
\lim_{N \to \infty} \left[ 1 - \frac{dN}{N} \right]^N = e^{-dN}
\]

(23)

Thus the probability \( P_1(x, \epsilon) \) that a given site with the onsite energy \( \epsilon \) has no neighbors nearer than \( x \) is given by

\[
P_1(x, \epsilon) = \exp \left\{ - \int d^d r \int_{-W/2}^{W/2} d\epsilon' g(\epsilon', T) \times \theta \left( x - 2r \frac{r}{\xi} - \frac{|\epsilon + |\epsilon'| + |\epsilon - \epsilon'|}{2kT} \right) \right\}
\]

(24)

Notice the absence of the Coulomb energy in the argument of the \( \theta \) function in Eq. (24), in accordance with our quasiparticle picture. Our quasiparticle picture is likely to work best for hops between isolated sites and the infinite cluster. Although one such hop may result in a sequence of other hops, these will mostly happen within the infinite cluster on a much shorter time-scale, effectively renormalizing the properties of the “slow” particle. As was mentioned earlier, these renormalizations can be included in the single particle density of states \( g(\epsilon, T) \).

To facilitate evaluating the integral in Eq. (21) numerically for the case where we include a Coulomb gap in the density of states, we define the dimensionless variables \( \tilde{\epsilon} = \epsilon/E_g, \tilde{\omega} = \omega/\sqrt{2} T, \tilde{T} = kT/E_g, \tilde{x} = \sqrt{2} R(x) C(\epsilon) f(\epsilon)(1 - f(\epsilon))\epsilon' \), and \( \tilde{g}(\tilde{\epsilon}, \tilde{T}) = g(\epsilon, T)/g_o \). \( E_g \) is the noninteracting density of states and \( \tilde{E}_g = e^3 \sqrt{\pi g_o/3k^3} \) is the characteristic width of the Coulomb gap. Evaluating the integral over \( x \) in Eq. (21) leads us to define

\[
\tilde{x} = 2r + \frac{|\tilde{\epsilon}| + |\tilde{\epsilon}'| + |\tilde{\epsilon} - \tilde{\epsilon}'|}{2\tilde{T}}
\]

(25)

Then we can rewrite Eq. (21) as

\[
\frac{S_I(\omega)}{T^2} = A \int_{-W/2}^{W/2} d\tilde{\epsilon} \tilde{g}(\tilde{\epsilon}, \tilde{T}) \int_{-W/2}^{W/2} d\tilde{\epsilon}' \tilde{g}(\tilde{\epsilon}', \tilde{T}) \int_0^{R_v} r^2 d\tilde{r} \times \theta(\tilde{x} - \lambda x_c) P_1(\tilde{x}, \tilde{\epsilon}) \tilde{g}(\tilde{\epsilon}, \tilde{T}) \left[ 1 - f(\tilde{\epsilon}) \right] \frac{R(x) C^2(\epsilon)}{1 + \omega^2 R^2(x) C^2(\epsilon)}
\]

(26)

where

\[
A = 64 \pi g_o^2 E_g^2 V_x^3/(N^2 \sqrt{2}) \ , \ R_v = (3V/4\pi)^{1/3}/x, \ W = W/E_g, \ \eta = 4\pi g_o E_g x^3, \ \text{and}
\]

\[
P_1(\tilde{x}, \tilde{\epsilon}) = \exp \left\{ -\eta \int_0^{R_v} r^2 d\tilde{r} \int_{-W/2}^{W/2} d\tilde{\epsilon}' \tilde{g}(\tilde{\epsilon}', \tilde{T}) \times \theta \left( \tilde{x} - 2\tilde{r} - \frac{|\tilde{\epsilon}| + |\tilde{\epsilon}'| + |\tilde{\epsilon} - \tilde{\epsilon}'|}{2\tilde{T}} \right) \right\}
\]

(27)

For comparison we also consider the case with no Coulomb gap by setting \( g(\epsilon, T) = g_o \) in Eqs. (21) and (24). Since there is no natural energy scale, we do not rescale the energies. However, we can define \( \tilde{r}, \tilde{x}, \tilde{\omega} \) as before. As a result, the definition of \( \tilde{x} \) in Eq. (25) becomes \( \tilde{x} = 2\tilde{r} + (|\tilde{\epsilon}| + |\tilde{\epsilon}'| + |\tilde{\epsilon} - \tilde{\epsilon}'|)/2\tilde{T} \). In Eq. (26), \( A \) is replaced by \( A_o = 64 \pi V g_o^2 E_g^2 V_x^3/3k^3 \) and \( W \) is replaced by simply \( W \). In Eq. (27) \( \eta \) is replaced by \( \eta_o = 4\pi^2 g_o \).

**DENSITY OF STATES**

At zero temperature, long-range interactions produce a Coulomb gap centered at the Fermi energy in \( g(\epsilon, T) \) [18, 21]
This gap arises because the stability of the ground state with respect to single electron hopping from an occupied site $i$ to an unoccupied site $j$ requires that the energy difference $\Delta_i^j > 0$. At finite temperatures the Coulomb gap is partially filled and the density of states no longer vanishes at the Fermi energy. The exact form of $g(\epsilon, T)$ is not known, but some have argued that its low temperature asymptotic behavior is described by $g(\epsilon = 0, T) \sim T^{-d-1}$. We have done Monte Carlo simulations of a three dimensional Coulomb glass with off-diagonal disorder and we find that $g(\epsilon = 0, T)$ cannot be described by a simple power law. The results of such simulations do not produce a density of states that is suitable for use in our noise integrals due to finite size effects. In particular $g(\epsilon, T)$ goes to zero at energies far away from the Fermi energy because of the finite size of the system.

Another way to approximate the density of states is to use the Bethe–Peierls–Weiss (BPW) approximation. The idea is to treat the interactions between one “central” site and all other sites (boundary sites) exactly, but to include the interactions between these boundary sites by means of effective fields. The density of states can then be written as a convolution

$$g(\epsilon, T) = \int_{-W_o/2}^{W_o/2} d\epsilon' g(\epsilon - \epsilon') \frac{1}{kT} h\left(\frac{\epsilon'}{kT}\right)$$

where $g(\epsilon)$ is the zero temperature density of states and $W_o$ is the bandwidth. The function $h(\epsilon/kT)$ takes into account thermal fluctuations in the occupation of the central site and the boundary sites. At low temperatures it has a sharp peak with a width of the order $kT$ at $\epsilon = 0$. We can make the approximation $(1/kT)h(\epsilon/kT) \approx -f'(\epsilon)$ where $f'(\epsilon)$ is the derivative of the Fermi function. The zero temperature density of states can be determined numerically by solving a self-consistent equation based on the ground state stability condition that a single electron hopping from an occupied site $i$ to an unoccupied site $j$ requires $\Delta_i^j > 0$. The result of evaluating Eq. (28) is shown in Fig. 1.

Since using the BPW approximation to evaluate Eqs. (21) and (24) is rather awkward, we model the finite temperature density of states by

$$g(\epsilon, T) = g_o \frac{\epsilon^2 + (kT)^2}{E_o^2 + \epsilon^2 + (kT)^2}.$$  

Notice that for $T = 0$, $g(\epsilon, T = 0) \sim \epsilon^2$ for $\epsilon \ll E_o$ as is expected for a Coulomb gap in three dimensions. For large energies ($\epsilon \gg E_o$ and $\epsilon \gg kT$), $g(\epsilon, T)$ approaches the noninteracting value $g_o$. A comparison of Eq. (29) with the BPW approximation at various temperatures is shown in Fig. 1. Eq. (29) is the expression we use for the density of states of a Coulomb glass in Eqs. (21) and (24).

We can calculate the DC conductivity resulting from this density of states by following Mott’s argument for variable range hopping. Mott pointed out that hopping conduction at low temperatures comes from states near the Fermi energy. If we consider states within $\epsilon_o$ of the Fermi energy ($E_F = 0$), then the concentration of states in this band is

$$N(\epsilon_o, T) = \int_{-\epsilon_o}^{\epsilon_o} g(\epsilon, T) d\epsilon$$

where $g(\epsilon, T)$ is given by Eq. (29). So the typical separation between sites is $R_o = [N(\epsilon_o, T)]^{-1/3}$. To estimate the resistance corresponding to hopping between two typical states in the band, we replace $r_{ij}$ with $R_o$ and $\epsilon_{ij}$ with $\epsilon_o$ in Eq. (3). We find that at low temperatures ($T \ll E_o$)

$$\sigma(T) = \sigma_o \exp\left[-\left(\frac{T}{T_o}\right)^\delta\right]$$

where $\delta$ is the hopping exponent. The value of $\delta$ depends on $E_o$. For large values of the Coulomb gap ($E_o \gtrsim 50$ K) $\delta \approx 0.75$ while for small values of the Coulomb gap ($E_o \lesssim 1$ K) $\delta \approx 0.5$. When we tried intermediate values of $T = 8, 10, \text{and} 20$ K, we found that $\ln|\epsilon(\epsilon_o)|$ versus $\ln(T)$ had a break in slope with $\delta \approx 0.5$ at low temperatures and with $\delta \approx 0.72 - 0.75$ at high temperatures. Examples are shown in Fig. 3. $\delta = 0.75$ is higher than the Mott value of $\delta = 0.25$ associated with a flat density of states and the value of $\delta = 0.5$ derived by Efros and Shklovskii for the zero temperature Coulomb gap. However, experiments on materials such as ultrathin metal films find values for $\delta = 0.75 \pm 0.05$. In agreement with our value of $\delta$ for large $E_o$, the mechanism behind this exponent has been a puzzle. Here we see that a possible simple explanation for the experimental observation of an anomalous hopping exponent is that the
For example, changing \( T \) with increasing temperature. If one takes this into account in the variable range hopping calculations, then the observed exponent of 0.75 can be obtained naturally. However, we should caution that our calculation applies to three dimensions while a two dimensional calculation may be more appropriate for ultrathin films. In fact we find that the analogous two dimensional calculation with a density of states \( g(\varepsilon, T) = g_o(|\varepsilon| + kT) / (E_g + |\varepsilon| + kT) \) yields \( \delta \approx 0.5 \).

**Results**

We evaluate Eqs. (26) and (27) numerically and display the results in Figs. 3[21] and 4[21]. In Fig. 3, we show the spectral density of the noise as a function of frequency. We find that for a wide range of parameters the noise spectral density is given by \( S(\omega) \sim \omega^{-\alpha} \) with the spectral exponent \( \alpha \) between 1.07 and 1.16 (see Figs. 3[21] and 4[21]) which is “1/f” noise. For comparison we show in Fig. 4 the noise spectrum in the absence of a Coulomb gap with \( g(\varepsilon, T) = g_o \) in Eqs. (21) and (24). The slope of a line through the open squares is \(-1.12\) which is very close to the values obtained with a Coulomb gap. Notice that the presence of a Coulomb gap reduces the noise amplitude at low temperatures.

In Fig. 5 we use the transport value of \( E_g \approx 0.4K \), not the tunneling one \( \sim 8K \); the two were found to be different by an order of magnitude[22][23]. We find that increasing \( E_g \) by a factor of 20 does not produce a noticeable change of the results at low temperatures \( T = 0.1 E_g \), but at high temperatures \( T = 10 E_g \) it does lead to saturation of the noise power at low frequencies. This is shown in Figure 6 which also shows that saturation occurs in the absence of a Coulomb gap when \( \eta_o \) is increased by a factor of 20. This saturation of the noise power occurs because the probability \( P_1(x, \varepsilon) \) of finding a site with no neighbors closer than \( x \) (see Eq. (24)) decreases exponentially with increasing temperature and with increasing \( \eta \) or \( \eta_o \). In addition \( P_1(x, \varepsilon) \) becomes exponentially small as \( x \) becomes large, and it is the large values of \( x \) that contribute to the low frequency noise. Finally we note that decreasing \( E_g \) by a factor of 10 does not produce a noticeable change of the results for either low temperatures \( T = 0.1 E_g \) or high temperatures \( T = 10 E_g \). We plot the spectral exponent \( \alpha \) in Fig. 5 versus the cases with and without a Coulomb gap in the density of states. In both cases we see that it decreases slightly with increasing temperature and eventually saturates in qualitatively agreement with experiment[11]. Fig. 6 shows that the noise amplitude \( \sqrt{S} \) grows with temperature and eventually saturates, both in good qualitative agreement with the...
FIG. 4: The noise power spectrum as a function of frequency at $T = 10 E_g$ for various values of $\eta = 4\pi \alpha E_g^2 / g_0$. The rest of the parameters are the same as in Fig. 3. Notice the saturation at low frequencies for large $\eta$. For comparison we show the case with no Coulomb gap at $T = 10$ with a large value of $\eta_0 = 4\pi \xi_0^3 / g_0$. Large values of $\eta_0$ lead to saturation but small values do not.

experimental results of Massey and Lee [11]. The data of Massey and Lee span 2 decades in frequency while our calculations are able to cover a much broader range. Again we see from Fig. 5 that the presence of a Coulomb gap reduces the noise amplitude at low temperatures. We obtain qualitatively the same results both with and without a Coulomb gap in the density of states which implies that the behavior of the noise spectral density with respect to temperature and frequency is not strongly tied to the hopping exponent $\delta$ or to the particular form of the density of states.

We will now discuss some of the physical reasons behind our results. The fact that we obtain $1/f$ noise is perhaps to be expected since weighted sums over Lorentzians (see Eq. 26) often result in $1/f$ noise [1]. The subtlety lies in the temperature dependence of the noise amplitude. For simplicity let us consider the case of a density of states with no Coulomb gap which gives qualitatively the same results as the case with a Coulomb gap. The decrease in the noise amplitude $\sqrt{S}$ with decreasing temperature is due to the presence of activated hopping processes which decrease with decreasing temperature. However, this is not at all obvious from Eq. 26. The integral for the noise power at low frequencies is dominated by large $\bar{\epsilon}$ which corresponds to long relaxation times $\tilde{\tau} \sim \exp(\bar{\epsilon})$. In this case the factor of $f(\bar{\epsilon})|1 - f(\bar{\epsilon})|$ cancels between the numerator and denominator leaving the temperature dependence of the integrand dominated by $P_\uparrow(x, \epsilon) \exp(-\tilde{\tau})$. $P_\uparrow(x, \epsilon)$ increases while $\exp(-\tilde{\tau})$ decreases with decreasing temperature. The fact that our calculations yield an increase in the noise amplitude with decreasing temperature implies that the activated hopping processes associated with $\exp(-\tilde{\tau})$ dominate. We should mention that experimentally the noise power does not always decrease with decreasing temperature. In some cases it increases with decreasing temperature [38, 39] but we do not know the differences in the samples which can account for this difference in behavior.

To summarize, recent experiments on $1/f$ noise [1] are consistent with a quasiparticle percolation picture of transport in electron glasses, though this does not exclude multi-particle correlations.

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