A Learning-based Stochastic MPC Design for Cooperative Adaptive Cruise Control to Handle Interfering Vehicles

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Abstract—Vehicle to Vehicle (V2V) communication has a great potential to improve reaction accuracy of different driver assistance systems in critical driving situations. Cooperative Adaptive Cruise Control (CACC), which is an automated application, provides drivers with extra benefits such as traffic throughput maximization and collision avoidance. CACC systems must be designed in a way that are sufficiently robust against all special maneuvers such as cutting-into the CACC platoons by interfering vehicles or hard braking by leading cars. To address this problem, a Neural-Network (NN)-based cut-in detection and trajectory prediction scheme is proposed in the first part of this paper. Next, a probabilistic framework is developed in which the cut-in probability is calculated based on the output of the mentioned cut-in prediction block. Finally, a specific Stochastic Model Predictive Controller (SMPC) is designed which incorporates this cut-in probability to enhance its reaction against the detected dangerous cut-in maneuver. The overall system is implemented and its performance is evaluated using realistic driving scenarios from Safety Pilot Model Deployment (SPMD).

I. INTRODUCTION

DRIVERS are the most important and influential entities of non-autonomous vehicles in ground Intelligent Transportation System (ITS). A revolutionary age of modern driving has been initiated by the advent of safety and comfort driving applications that aim at assisting drivers in vehicle control. Forward collision warning [1]–[4], lane keep assistance [5]–[7], automatic braking [8], adaptive cruise control [9], [10], efficiency [11], [12], and pedestrian safety [13]–[15] are amongst the most important automated driving applications. The first generation of safety applications was designed by virtue of local sensors such as radars and cameras. Local sensors provide a mediocre level of safety due to their limited sensing range and data processing complexity. Moreover, they noticeably underperform in the presence of occluding obstacles. In order to handle these issues, some other sources of information are required to provide more accurate situational awareness within a broader neighboring area. Vehicle-to-Vehicle (V2V) communication has been proposed to remove this barrier through its omnidirectional and non-line of sight connectivity capabilities. Consequently, the performance of safety applications is expected to substantially improve by V2V communications. Currently, the most promising technology under consideration for V2V communication is the Dedicated Short Range Communication (DSRC) [16] technology.

Adaptive Cruise Control (ACC) is one of the most demanding automated driving applications. In comparison with its predecessor, i.e. conventional cruise control which had been solely designed to provide a fluctuation-free driver-specified velocity, ACC is also responsible for sustaining a certain level of safety by continuously tracking the vehicle longitudinal distance from its immediate leader and keeping this distance within a safe range. One step ahead in cruise technology would result in Cooperative Adaptive Cruise Control (CACC), which also leverages the V2V communication. This makes it more powerful to simultaneously preclude collision and maximize traffic throughput compared to ACC [17]. However, many CACC challenges still exist which need to be addressed. For instance, the CACC application should be robust against other vehicles’ maneuvers such as unforeseen lane changes [17]. Detection and appropriate reaction to these unexpected vehicle maneuvers are among the most challenging tasks, even in the normal driving situations and without CACC imposed constraints. These challenging tasks reveal the criticality and complexity of a well-behaved CACC design for these scenarios. Even though different theoretical and technical aspects of CACC have been investigated by researchers [18], handling interfering vehicles needs more elaborations.

In this paper, we specifically concentrate on cut-in maneuvers due to their imminent threat, as a vehicle in a stable CACC platoon has to perform a hard brake reaction when another vehicle makes a sudden lane change just in front of it. This hard brake reaction is extremely dangerous and can result in a severe crash [19], [20]. Thus, it is well-desired to predict cut-in intention of other drivers in advance. Moreover, cutting into the platoon deforms the platoon structure which should be compensated by a proper CACC design. Therefore, a meticulous CACC system should be able to both prevent possible crashes and maintain the normal platoon formation against entering vehicles from adjacent lanes.

Based on the above discussion, performance of CACC in...
that the controller which only acts based on the previous measurements without any predictive vision of driving scenario. The prevailing methods in the literature for driver behavior modeling, and some of the important proposed designs for adaptive cruise controller are mentioned in this section.

One of the important research mainstreams in driver behavior modeling is based on utilizing classification methods, such as Support Vector Machine (SVM) and Neural Network (NN), to differentiate between distinguishable driver behaviors. The main idea behind another major class of driver behavior modeling schemes in the literature, such as Hidden Markov Models (HMMs) and Dynamic Bayesian Networks (DBNs), is developing a probabilistic causal framework which tries to find the next most likely driving maneuvers using available data sequences from the driving history and then chain these predicted consecutive maneuvers to construct the most probable future scenario.

Authors in [26] developed a hierarchical classifier for observed scenes of the host vehicle from remote vehicle’s lane change. These scenes were then assigned to the nodes of the hierarchy in the model to specify a pattern from the top nodes to the leaves. However, their overall scheme is not generalizable to other contexts, such as potential maneuver alternatives, since it is remarkably specialized.

SVM-based methods are proposed to classify lateral actions of drivers based on detection of preparatory behaviors, vehicle dynamics, and the environmental data prior to and during the maneuvers such as lane change [27]. A Relevance Vector Machine (RVM), was employed in [28] to distinguish between lane change and lane keeping maneuvers. In [29], feed forward artificial neural networks are used to predict the trajectory of the vehicle based on its movements history. The goal was to study the possibility of accurate movement prediction for a lane changing vehicle by an autonomous driving vehicle.

An Object-Oriented Bayesian Network (OObN) is utilized to recognize special highway driving maneuvers, such as lane change [30]. This approach models different driving maneuvers as vehicle-lane and vehicle-vehicle relations on four hierarchical levels which can tolerate uncertainties in both the model and the measurements. A finite set of driving behaviors are classified and future trajectories of the vehicle are predicted based on currently understood situational context using a DBN-based model [31].

Hidden Markov model (HMM) technique has been widely utilized to associate the observable time series of the vehicle to the unobserved driver intentions sequence during his maneuvers [24], [25], [32]–[37]. Some pioneer works in driver behavior modeling, [32], [33], proposed a decomposition of driver behaviors into small scale and large scale categories. Time sequence of unobserved large scale driver actions are assumed to have Markovian property and HMM is suggested as an acceptable method to model this sequence. This modeling approach accuracy was validated by its results of the lane change maneuver prediction. Sensor collected information was used as the observation set in the designed HMM predictor.

Using the data from V2V communication, two HMMs were utilized to discriminate different types of driver lane change
intent, namely dangerous and normal [25], [34]. A trajectory prediction stage and an MPC controller were mounted on top of the lane change prediction algorithm to manage reformation of a new CACC string after cut-in.

A controller for a CACC string which takes into account both V2V and non-V2V equipped vehicles was designed in [17]. This controller tries to handle cut-in and cut-out scenarios with a smooth reaction to the new condition of the host vehicles lane. No prediction is performed in this work to detect the cut-in or cut-out scenarios in advance. Another CACC design based on switched sampled-data model is presented in [18] which investigates the stability problem in the presence of sensor failures.

In our work, a learning-based driver behavior modeling method is combined with an MPC design in a probabilistic manner to improve the overall CACC performance.

III. System Description

In our framework, which is schematically depicted in Fig. 1, the vehicle inside the platoon, which is directly affected by the cut-in suspicious vehicle, is referred to as the host vehicle. The immediate vehicle in front of the host vehicle is known as the preceding vehicle, and the first vehicle of the platoon is the leading vehicle or leader. The dangerous area in front of the host vehicle is referred to as bad-set. This area and its dimensions will be discussed in details later.

Although, detection of cut-in by the vehicle itself is beneficial to some applications such as lane keep assist system (LKAS) and blind spot warning (BSW), CACC and platooning need the lane change maneuver to be detected remotely by the host vehicle. The remote lane change detection is required because the host vehicle should react in a timely manner to avoid hazardous situations.

V2V communication periodically provides the parameters of the cut-in suspicious vehicles via broadcasting basic safety messages (BSM) [16], [38]. In our model, we assume that the host vehicle, which is in a stable condition in the platoon, periodically receives the BSMs of its surrounding vehicles and continuously traces them prior to any probable cut-in maneuver. From BSM part one of the SAE J2735 standard, [38], we utilize the following parameters for our behavior modeling: latitude, longitude, elevation, speed, heading, steering wheel angle, 4-way acceleration set, and the vehicle size. The latitude, longitude, and elevation represent the location of the vehicles center of gravity in the WGS-84 coordinate system. The 4-way acceleration set consists of acceleration values in 3 orthogonal directions plus yaw rate, which are calculated based on the assumption that the front of the vehicle is toward the positive longitudinal axis, right side of it is the positive lateral axis, and clockwise rotation as the positive yaw rate.

In CACC platooning, a safe longitudinal gap must be continuously kept between every two consecutive vehicles. The deviation from the safe gap, which is known as spacing error, should remain as small as possible to reduce the risk of collision and take the advantages of platoon formation, such as lower fuel consumption and higher traffic throughput [39]. As mentioned, we define bad-set as the dangerous area in front of the vehicle in which the safe gap is violated. In other words, our bad-set is a rectangle aligned to the road surface in front of the host vehicle, while its longitudinal dimension, \( L_{bs} \), depends on the platoon speed and is equal to the desired longitudinal safe gap and its lateral dimension, \( W_{bs} \), is the lane width. The front bumper of the host vehicle is always located at the center of the bad-set rear lateral edge. These definitions are illustrated in Fig. 1.

The goal of our lane-change monitoring block is tracking and predicting the trajectory of all of the vehicles in the adjacent lanes of the host vehicle. The model should not only predict the immediate kinematics of the vehicles, but also the high-level driving maneuvers. Therefore, the position of neighboring vehicles should be predicted for multiple future steps based on their current and previous communicated information. The number of required prediction steps is determined by the duration of a complete high-level maneuver and denoted by \( S_m \). This multi-step prediction is then used to determine the probability of unsafe lane change which is passed to the SMPC for better estimation of the required inter-vehicle spacing gap.

A. Lane Change Monitoring Block Design

Each lane change maneuver consists of four separate phases: Intention phase, Preparation phase, Transition phase, and the Completion phase [40], [41]. It is worth mentioning that some more complicated maneuvers, such as overtaking, have also been investigated in the literature. For instance two-phase and five-phase overtake modeling frameworks are proposed in [42] and [43], respectively. The lateral acceleration and lateral speed in a lane change maneuver are bounded by the comfortable lateral acceleration threshold and the maximum tolerable lateral speed, respectively [44]. To safeguard a smooth transition of the vehicle between lanes, the acceleration is bounded by -0.2g and 0.2g [45]. Our model is designed to not only predict the immediate kinematics of the vehicles in the transition phase but also the complete four-phase lane change maneuvers. The trajectory of each remote vehicle is modeled as a time series. In our model, we separate the learning of lateral and longitudinal behaviors of the driver as they are influenced by different control inputs.

Artificial neural networks (ANNs) are one of the most famous tools for description and prediction of nonlinear systems [46], [47]. Neural networks with hidden units can principally predict any well-behaved function. In the case of time series, in order to handle the dependency of the prediction to a finite set of past values and time varying nature of the input signals, neural network topologies need to be equipped with a short
term memory mechanism which is called the feedback delay. In this work, we used feedback delay- based ANNs, namely nonlinear autoregressive (NAR), nonlinear autoregressive exogenous (NARX) and recurrent neural networks (RNN) toward driver behavior and lane change prediction.

NARX is a neural network with feedback delay that can be trained and used to predict a time series from its past values and an exogenous one, in spite of NAR which does not rely on any external inputs. We use NAR model to predict the future pattern of different system inputs, i.e. steering wheel angle, yaw rate, heading, speed, and longitudinal acceleration, based on their currently available values. A NARX model is employed to predict the longitudinal trajectory of the vehicle during the lane change using some of the previously estimated sequences of input signals as the exogenous input. The exogenous inputs in our framework are yaw rate, heading, speed, and longitudinal acceleration.

Finally, an RNN is adopted to model the lateral trajectory of the vehicle based on the predicted input signals. RNNs can use their internal memory to process arbitrary sequences of inputs. The input signals to our lateral position prediction RNN are steering wheel angle, yaw rate, and heading. Using the internal memory, the RNN can distinguish between different maneuvers with partially similar input signals. For example, a steering due to the road curvature might look partially similar to the one from lane change maneuver, but the RNN can learn to distinguish between these two maneuvers by looking at a longer history of the signals or other input signals, such as road curvature. In the former case, the RNN should be trained on other maneuvers which share the same input signal patterns in a portion of their lifetime.

All of our ANNs have a hidden layer with 20 nodes and required prediction steps are also set to 10 steps for all of the ANNs, \( S_m = 10 \), which means that the we are predicting the behavior of the driver for 1 second in the future, since the driver can change his decision and behavior beyond this time [48]. As mentioned before, the NAR is used to model the patterns of input signals to the system. The NARX and RNN are used to model and predict the longitudinal and lateral position of the vehicle, respectively. The longitudinal position of the vehicle is modeled based on the predicted values of heading, speed, and longitudinal acceleration as external inputs. On the other hand, the RNN should not only predict the future lateral position of the vehicle, but should also distinguish between different lateral maneuvers. Therefore, the lateral model is also trained with some curve road data to be able to differentiate between different lateral movements.

\[ A_i = \max_{1 \leq s \leq S_m} P_{c_i} \]

\[ P_{c_i} = \frac{A_i}{\text{intersection area}, \left( S_{l1}, S_{l2}, S_{l3} \right)} \]

To mitigate the effect of noise, small variations of input signals, based on the nature of the signal, are filtered to smooth the time series and mitigate the effect of noise. Variation smaller than 3 degrees, 0.1 rad, 0.1 m/s, and 0.1 m/s\(^2\) are removed from steering wheel angle, heading, speed, and longitudinal acceleration, respectively. The resulting input signals during one maneuver is shown in Fig. 2.

All of our ANNs have a hidden layer with 20 nodes and 15 step short term memory, which means that they are using the past information of 1.5 seconds for future prediction. The required prediction steps are also set to 10 steps for all of the ANNs, \( S_m = 10 \), which means that the we are predicting the behavior of the driver for 1 second in the future, since the driver can change his decision and behavior beyond this time [48]. As mentioned before, the NAR is used to model the patterns of input signals to the system. The NARX and RNN are used to model and predict the longitudinal and lateral position of the vehicle, respectively. The longitudinal position of the vehicle is modeled based on the predicted values of heading, speed, and longitudinal acceleration as external inputs. On the other hand, the RNN should not only predict the future lateral position of the vehicle, but should also distinguish between different lateral maneuvers. Therefore, the lateral model is also trained with some curve road data to be able to differentiate between different lateral movements.

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B. Cut-in Probability Calculation

The results of the proposed cut-in prediction scheme is now applied to find a single value between 0 and 1 which represents the overall cut-in probability. This probability, which is denoted by \( P_c \) from now on, will be fed to our SMPC as its input. SMPC design details are discussed in the following subsection. At each prediction cycle we have \( S_m \) predicted future values for each of the longitudinal and lateral relative positions of the suspicious cut-in vehicle. In our implementation, each of these \( 2 \times S_m \) predicted values comes with a specific 90 percent confidence level. Hence, we have \( S_m \) rectangular areas, each of them determines the predicted area for the position of the cut-in vehicle in the corresponding upcoming time step with 90 percent accuracy. We take the most conservative approach to define the cut-in probability, \( P_c \), as follows:

- Each of these \( S_m \) rectangles, \( A_i \) in Fig. 3, is intersected with the host vehicle’s bad-set at that moment and its intersection area, \( A_{i1} \) in Fig. 3, is calculated.
- The resultant intersection area, \( A_{i1} \), is normalized by dividing it by the corresponding predicted area value,
modeled as follows [45]:

\[ \dot{x}_i = x_{i-1} - x_i - h v_i - L_i - d_0 \]

for all \( i \in \{1, 2, \ldots, n\} \), where \( x_i \) and \( v_i \) are longitudinal position and velocity of the \( i^{th} \) following vehicle, respectively \((x_0) \) stands for the longitudinal position of the lead vehicle; \( h \) headway which introduces a speed dependent spacing policy in addition to \( d_0 \) which is a constant minimum desired distance between each vehicle and its preceding vehicle in the platoon, and \( L_i \) is the length of the \( i^{th} \) vehicle. Based on these definitions, longitudinal dimension of the bad-set, \( L_{bs} \), for \( i^{th} \) vehicle could be represented as:

\[ L_{bs} = hv_i + d_0 \]

Then, the linearized dynamics of the \( i^{th} \) following vehicle is modeled as follows [45]:

\[ \dot{\delta}_i = v_{i-1} - v_i - h \dot{v}_i \]

\[ \Delta \dot{v}_i = a_{i-1} - a_i \]

\[ \dot{a}_i = -\frac{a_i}{\xi} + \frac{u_i}{\xi} \]

where \( \xi \) is the engine time constant, \( a_i \) is the acceleration of the \( i^{th} \) vehicle, and \( u_i \) is an input signal which comes from an MPC controller. However, due to the communication delay each vehicle receives the delayed version of its preceding vehicle’s acceleration value. Denoting the communicated acceleration of \( i^{th} \) vehicle at receivers by \( \bar{a}_i(t) \), the state space equation of a vehicle in a CACC system could be represented as follows

\[ \dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + G_i \bar{a}_{i-1}(t) \]

with state vector \( x = [\delta_i \ \Delta v_i \ \bar{a}_i]^{T} \), and

\[ A_i = \begin{bmatrix} 0 & 1 & -h \\ 0 & 0 & -1 \\ 0 & 0 & -\frac{1}{\xi} \end{bmatrix} \quad B_i = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad G_i = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \]

However, delay of the communication network is not considered in this work, so \( \bar{a}_{i-1}(t) = a_{i-1}(t) \).

An MPC controller with three primary objectives is required to control the platoon system described by (6). The controller must compute the input signal \( u_i \) to minimize the spacing error, keep the velocity of the host vehicle as close as possible to its preceding vehicle velocity, and finally, respond appropriately to a cut-in vehicle based on our prediction of the driver behavior. To this end, the system dynamics (6) is discretized and an optimal control problem, which satisfies the aforementioned control goals, is defined. The continuous time dynamics of the system is discretized using the Euler forward method with a time step \( T_s \):

\[ \dot{x}_i[k+1] = A_i^k x_i[k] + B_i^k u_i[k] + G_i^k \bar{a}_{i-1}[k] \]

where

\[ A_i^k = \begin{bmatrix} 1 & T_s & -h T_s \\ 0 & 1 & -T_s \\ 0 & 0 & 1 - \frac{T_s}{\xi} \end{bmatrix} \quad B_i^k = \begin{bmatrix} 0 \\ 0 \\ T_s \end{bmatrix} \quad G_i^k = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \]

Hereinafter, for simplicity of notation, we use \( A, B, G, x, \) and \( u \) instead of \( A_i^k, B_i^k, G_i^k, x_i, \) and \( u_i \), respectively. The cost function of the optimal control problem is defined based on the primary objectives of the controller:

\[ J[k] = \sum_{i=0}^{N-1} c_\delta \delta^2[k+i] + c_v \Delta v^2[k+i] + c_a \Delta u^2[k+i] \]

\[ = \sum_{i=0}^{N-1} [x^T[k+i]Qx[k+i] + u^T[k+i]Ru[k+i]] \]

where \( N \) is the control horizon, \( c_\delta, c_v, \) and \( c_a \) are weighting coefficients reflecting the relative importance of each term and

\[ \Delta u[k+n] = u[k+n] - u[k+n-1] \]

which is added to the cost function as an extra term to bound the jerk and prevent fast variations of the input signal. This constraint could be interpreted as comfort ride. The MPC law finds the optimal input sequence \( u^*[k] \) which minimizes the predicted cost function (10) at each time instant:

\[ u^*[k] = \arg \min_u J[k] \]

subject to \( \begin{cases} x_{min} \leq x[k+i] & \leq x_{max} \\ u_{min} \leq u[k+i] & \leq u_{max} \end{cases} \quad i = 1, \ldots, N-1 \)

To solve this MPC problem, the future values of the preceding vehicle’s acceleration are required. These values are obtained from the aforementioned NAR neural network.

1) Conventional MPC Design: In this section, MPC design problem without incorporating the calculated cut-in probability, \( P_c \), is investigated. We referred to this MPC design as conventional design in this paper. The values of \( a_{i-1}[k] \) could be considered as a measured disturbance when its model is available to the MPC controller. Then, the system equations could be rewritten in the standard form as

\[ \dot{x}_i[k+1] = A \dot{x}_i[k] + B u[k] \]

where \( \dot{x}_i[k] = [x[k], v[k]]^{T} \) is the augmented state vector and the measured disturbance state vector \( v[k] = [\dot{a}_i[k], \Delta v_i[k]]^{T} \) is the measured disturbance state vector.
Therefore, selecting $Q$ be feasible at all time system rely on the assumption that tail conditions, the stability and convergence of the closed loop feasible for all should be satisfied at each time instant is met, provided it is feasible at $k$.

It is shown that the largest possible $\Omega$ is derived by

$$\Omega = \{ x : \ u_{\min} \leq K(A + BK)^i x \leq u_{\max}, \ x_{\min} \leq (A + BK)^i x \leq x_{\max}, \ i = 0, 1, \ldots \}$$

To show that $\Omega$ is invariant over the infinite horizon of the second mode of (15), constraint satisfaction should be checked over a long enough finite horizon ($N_c$). Here, $N_c$ is the smallest number which satisfies the following equations

$$\{ u_{\min} \leq K(A + BK)^{N_c+1} x, \quad \pi = \max_x K(A + BK)^{N_c+1} x \}$$

such that

$$u_{\min} \leq u \leq u_{\max}$$

Having $N_c$ found, adding the following constraints to the MPC problem guarantees the feasibility of the controller:

$$u_{\min} \leq K(A + BK)^i x[k + N][k] \leq u_{\max}, \quad x_{\min} \leq (A + BK)^i x[k + N][k] \leq x_{\max}, \quad i = 0, 1, \ldots, N_c$$

2) **MPC design: Incorporating cut-in probability**: The designed MPC controller in the previous section satisfies our first two primary goals, namely spacing error and velocity error minimization. However, the controller should be able to react appropriately if a cut-in suspicious vehicle enters the platoon unexpectedly and pushes the host vehicle to decelerate to reestablish the safe distance.

Hereafter, the probability of the suspicious vehicle’s cut-in trajectory intersection with the host vehicle’s bad-set has been determined. Based on this, we propose a new stochastic definition for the spacing error:

$$\delta_i = \frac{x_{i-1} - x_i}{2 - e^{-\alpha P_c}} - hv_i - L_i - d_0$$

where $P_c$ is the probability of the cut-in vehicle being in the bad-set of the host vehicle, and is a constant control parameter which adjusts the reaction sensitivity of the MPC controller to the cut-in probability. Clearly, when the probability is one, assuming $\alpha$ has been set to a sufficiently large number, the controller starts doubling the distance from its current preceding vehicle by halving the numerator of the first term in the proposed equation for spacing error, (24). Consequently, the cut-in vehicle has enough safe gap to enter the CACC platoon. On the contrary, when the probability is zero, the suspicious vehicle is not expected to cut in or it has the safe distance from the host vehicle for its maneuver. This zero probability sets the denominator of (24) to one, which means the host vehicle keeps the normal safe distance, $hv_i + d_0$, from its preceding vehicle.

Although the stability and feasibility of the controller is already guaranteed for the MPC design with deterministic
spacing error, it should be proved under the new circumstances due to the stochastic spacing error definition.

To this end, utilizing a Stochastic Hybrid System (SHS) design is beneficial. A Time-Triggered Stochastic Impulsive System (TTSIS) SHS model, [21], incorporating the probability of an upcoming cut, is utilized to this end as shown in Fig. (4).

In this model, $p \in [0,1]$ is a random variable which represents the cut-in probability, $G(p)$ is a guard condition that must be hold for the discrete transition (time trigger), and $R(x,p)$ is a reset function which describes the changes in the continuous states after the transition. The stochastic nature of our system comes from the dependency of the guard on the random variable $p$.

**Proposition 1:** The TTSIS of Fig (4) is stable under the designed MPC controller if the following condition is satisfied:

- There is a finite invariant region $S_R$ in which
  \[
  x[k] \in S_R \quad \forall p \in [0,1], \quad x_{min} \leq x[k] \leq x_{max}
  \]  
  (25)

The region $S_R$ is a subset of the region of attraction for the MPC law, denoted by $S_\Omega (S_R \subset S_\Omega)$. Here, $S_\Omega$ is defined as the set of all initial states from which a sequence of inputs exists that forces the state predictions to reach the terminal constraint set in the first mode of control law (15), i.e.

\[
S_\Omega = \left\{ x[0] : \exists \mathbf{u}[0], \mathbf{x}[N|0] \in \Omega, \right. \\
\left. \quad \text{s.t.} \begin{align*}
    x_{min} \leq x[i] \leq x_{max}, \quad \forall i \leq N \\
    u_{min} \leq u[i] \leq u_{max}, \quad \forall i \leq N - 1
  \end{align*} \right\}
\]  
(26)

Therefore, to guarantee the stability of the system over a larger set of conditions, the constraints on the system states should be relaxed as much as the safety is not violated. To this end, we set the constraint on the spacing error to $[\delta_{min}, \delta_{max}] = (-h v_i - L_i - d_0 + \delta_s, +\infty)$ for the case of no cut-in detected, where $\delta_s$ is the minimum desired safe gap between the vehicles. Clearly, after each discrete transition, the value of spacing error can only jump with a value between $-h v_i - L_i - d_0$ and $h v_i + L_i + d_0$. However, for the case of a positive cut-in probability, we should choose a more conservative constraint, $[\delta_{min}, \delta_{max}] = (-h v_i - L_i - d_0 + (x_{i-1} - x_{TV}) + \delta_s, +\infty)$ to assure the collision avoidance where $x_{TV}$ is the position of the cut-in suspicious vehicle. Finally, if the is no constraint found which can guarantee the feasibility, the driving situation is considered as an unsafe or a harsh maneuver and the controller temporarily is overwritten with $u_i$ set to the maximum possible deceleration (usually up to $-10 m/s^2$ [51], to prevent the collision) or the maximum predefined acceleration of the vehicle till the feasibility of the controller can be guaranteed again.

### IV. Evaluation

In this section overall performance of the proposed system framework is evaluated using realistic driving scenarios from Safety Pilot Model Deployment (SPMD) dataset [49]. First, the practicality of our cut-in trajectory prediction method is shown by its performance comparison versus the kinematic-based trajectory prediction as a ground truth. Next, noticeable better behavior of the proposed SMPC controller versus the conventional MPC is discussed.

#### A. Cut-in Trajectory Prediction Performance Evaluation

To evaluate the performance of our method, we extracted 90 lane change maneuvers from the BSMs generated by participating vehicles in SPMD dataset in Ann Arbor, Michigan. BSM broadcast rate had been set to 10Hz in this dataset. Therefore, we have all of the time series recorded in this rate. The signals from all 90 maneuvers are concatenated to create a long univariate time series for each input signal. Finally, a 70-15-15 percent training, cross-validation, and testing data selection is used for ANNs training and performance evaluation.

The performance of two trajectory prediction methods, vehicle kinematic model, and our trained RNN, for a lane changing vehicle in terms of lateral confidence levels at each time step ahead, averaged on all 90 scenarios, are shown in Fig. 5(a).
The classic car model could be represented by kinematic-based differential equations as follows:

\[
\dot{x}_i = v_i \cos \theta_i, \quad \dot{y}_i = v_i \sin \theta_i, \quad \dot{\theta}_i = \frac{v_i}{L_i} \tan \phi_i
\]  

(27)

where \(x_i, y_i, \) and \(v_i\) are longitudinal position, lateral position, and velocity of the \(i^{th}\) vehicle, respectively. Also, \(\phi_i\) denotes the steering angle, \(\theta_i\) stands for the angle between the vehicle’s instantaneous heading and the road direction, and \(L_i = 5\) is the length of the vehicle [52].

Fig. 5(b) shows the same comparison for the longitudinal position prediction.

B. Designed SMPC Performance Evaluation

In this section, superiority of designed SMPC versus conventional MPC is investigated. To this end, reactions of these two different designs to real cut-in maneuvers should be compared. A general cut-in maneuver duration is between 3.5-6.5 seconds for urban scenarios with the mean of 5 seconds and 3.5-8.5 seconds for highway scenarios with the mean of 5.8 seconds [53]. For the sake of comparison fairness, two types of cut-in maneuvers, i.e. the harshest type with 3.5 seconds duration, and the average class with 5.5 seconds duration, have been selected and the outputs of two aforementioned controllers are compared in each case. In both cases, the platoon velocity is assumed 27 m/s or 60 mph.

Cut-in probabilities, calculated based on the discussed method in section III-B, for average and harsh maneuvers are depicted in Fig. 6. As mentioned before, these probabilities are fed into our SMPC controller at each prediction cycle. The vertical dashed lines in both figures stand for the moments at which the cut-in vehicles cross the road line between two adjacent lanes. It is clear that our cut-in detection starts around 1.25 seconds and 2 seconds ahead of this moment for harsh and average maneuvers, respectively, which provides a noticeable extra reaction time for the controller.

Fig. 7, illustrates the changes in spacing error, velocity and acceleration of the host vehicle produced by two different controllers in response to the average cut-in maneuver. It is noteworthy that our controller starts its reaction to compensate the situation notably sooner than the conventional one which results in a noticeable smoother reaction. In addition, it highly increases the reaction safety as a consequence of its considerable lower maximum spacing error which is evident by comparing the spacing error in Figs. 7(a) and 7(b). For instance, as it is clear in Fig. 7(a), the worst SMPC spacing error reaches 10 meters, while its counterpart in conventional MPC system is around 17 meters. Moreover, the spacing error of the SMPC controller is around 6 meters when the suspicious vehicle entered the CACC lane while in conventional MPC it is on its maximum value of 17 meters. Finally, the SMPC cut-in detection starts around 2 seconds from the beginning of the scenario, which gives the controller about 2 seconds additional reaction time in comparison with the conventional system.

The same plots for harsh maneuver, which are depicted in Figs. 7(b), 7(d), and 7(f), demonstrate the dominance of the proposed SMPC performance in terms of sooner and safer reaction.

V. Conclusion

In this paper, a probabilistic framework for handling cut-in maneuvers into a CACC platoon is proposed and its better performance compared to the conventional controller design is demonstrated. At the first step of the designed procedure, a cut-in maneuver of an interfering vehicle is detected and its trajectory is predicted using a novel three-layer neural network-based approach. The high accuracy of this method is demonstrated by comparing its results against the state of the art Kinematic-based deterministic models. Afterwards, the output of this phase is utilized to calculate the probability of cut-in predicted trajectory overlap with the host vehicles bad-set area. This probability, which is referred to as cut-in probability, specifies the severity level of the dangerous situation caused by a sudden cut-in into the stable CACC platoon. Obviously, higher values of this probability need more urgent reactions from the host vehicles controller to prevent the possible collision with a smooth and safe reaction. This goal is achieved by giving this probability to a new stochastic MPC controller, designed based on the emerging SHS concept. The overall performance of the designed system is evaluated and its effectiveness for better regulation of the host vehicles reaction to dangerous cut-in situations is discussed using realistic cut-in driving scenarios from SPMD dataset.

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