Simple piecewise linearisation in time series for time-domain inversion to estimate physical parameters of nonlinear structures

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Summary

This study introduces simple piecewise linearisation in time series (SPLiTS), for time-domain inversion (TDI) to estimate the physical parameters of nonlinear structures. SPLiTS considers a nonlinear structure to be a set of piecewise linearised structures based on each half-cycle wave in the displacement response data. For the linearisation, SPLiTS intentionally minimises the effect of the central point shift that damaged structures commonly display. After theoretically introducing SPLiTS to the response data of single-/multi-degree-of-freedom (SDOF/MDOF) structures, parameter estimations are performed for data obtained from numerical simulations and experiments. The efficiency of consideration of the central-point shift is demonstrated in parameter estimations for numerical simulations where response data are obtained by a nonlinear SDOF structure having a trilinear hysteretic spring. SPLiTS is then applied to experimental data from shaking table tests, in which a specimen, imitating a one-storey structure, displayed yielding behaviour. TDI with SPLiTS reasonably estimates physical parameters that show increased damping and decreased stiffness in time series at the inelastic range. Next, SPLiTS is applied to experimental data from shaking table tests conducted for a full-scale, three-storey structure at E-Defense in 2013. The damping coefficient and stiffness of each storey are reasonably estimated for the specimen under different amplitudes. For these experimental data, estimated structural energy absorption calculated by damping in the time series corresponds well with the structural damage of the specimens. Thus, structural energy absorption is proven to be a useful index for indicating structural damage.

KEYWORDS

physical parameter estimation, shake table test, structural energy absorption, structural health monitoring, system identification, time-domain inversion
1 | INTRODUCTION

Structural health monitoring (SHM) is a key issue in various engineering fields, including aerospace, mechanical and civil engineering.\(^1\) In structural or civil engineering, monitoring techniques for identifying the existence, location and severity of structural damage are important for building and infrastructure maintenance.\(^2\) This is particularly important when natural disasters such as earthquakes occur because the structural condition assessment greatly affects the recovery plan.

Many researchers are developing new sensing devices and damage detection methods to quickly and accurately assess structural conditions. The advancement of microelectromechanical systems and battery energy-saving technology has contributed to the development of wireless sensors.\(^3\)–\(^5\) Using wireless sensors, it is now possible to acquire structural response data and has reduced human effort and installation expenses. More advanced devices are currently being developed for long-term usage, more accurate synchronisation between sensors and more efficient data transmission.\(^6\),\(^7\)

Visual inspection is the most straightforward scheme for detecting structural damages. However, its detection accuracy strongly relies on the capability of inspectors and accessibility to the damaged locations. Thus, a vibration-based scheme utilising vibration response data of structures is commonly employed to assess the conditions of the structures.\(^8\)–\(^9\) In the vibration-based scheme, there are two types of approaches. One is a model-based approach, which is applied to system identification of the structures when response outputs (e.g., displacements and accelerations) and inputs of the structures are available. Based on the input–output relation, this approach finds the best parameters for a model prepared by a priori information of the structure. Typical examples of this approach are the methods based on frequency response function (FRF)\(^2\),\(^10\)–\(^12\) and the prediction error, using autoregressive and other relevant models,\(^13\)–\(^18\) which are commonly employed to monitor buildings and civil infrastructure. The other is a data-based approach, which directly assesses the structural conditions based on the available data without requiring knowledge of the structures in advance. Currently, methods using various techniques, such as wavelet transform,\(^19\),\(^20\) machine learning\(^21\) and pattern recognition,\(^22\) have been developed for further improvement of this approach.

Although the data-based approach is useful for cases having a limited amount of data, the model-based approach is still powerful when enough data are available because established theories and methods support this approach. This is specifically applicable to assessing structural conditions within a laboratory environment because many experimenters try to make full use of their specimens and maximise the data acquisition of the structural responses. Thus, model-based approaches are still commonly employed in laboratory testing. These methods provide modal parameters, such as natural frequency, damping ratio and mode shapes, which are commonly used as indices of the structural conditions of specimens before and after excitation loading in laboratory testing. As an advanced technique, the detection of time-variant modal parameters\(^23\)–\(^25\) is an important research subject in SHM. However, modal parameters are insensitive to variations of the structural components because the modal parameters are affected by the condition of the entire structure.\(^26\) Thus, approaches to identify physical parameters, such as mass, stiffness and damping, are currently preferred for a more detailed assessment of structural conditions.\(^27\)–\(^28\)

Identification of hysteretic loops of nonlinear structures is a classical issue in earthquake and structural engineering, and various methods were contrived for the identification.\(^29\)–\(^31\) For this purpose, a method using wavelet\(^32\) was developed, and it was demonstrated that a wavelet transform is effective to estimate stiffness and damping of a nonlinear structure in a time series; however, its estimation accuracy is affected by the number of steps predetermined for the identification of the hysteretic loop. Currently, the wavelet transform is applied to various civil and structural engineering issues,\(^33\) and users still need to go through a learning curve to find the required steps to meet their purpose. Model updating based on a finite element (FE) model\(^34\),\(^35\) is also a powerful approach to detect nonlinear characteristics and even minute structural parameter changes in complicated structures. However, many engineers cannot easily access this approach because it requires expertise in FE modelling and analysis as well as data processing.

This study focusses on structural buildings that can be modelled by lumped mass models. We introduce a simple method that relies on basic signal processing techniques and enables to directly estimate physical parameters of nonlinear structures in the time domain, particularly in a rich-measurement environment, such as a laboratory. This method, referred to as simple piecewise linearisation in time series (SPLiTS), is an addition of data selection procedures as a preliminary step for the time-domain inversion (TDI), which is a common least-square method for parameter estimations.\(^29\)–\(^32\),\(^36\)–\(^38\) SPLiTS regards a nonlinear structure to be a set of piecewisely linearised structures, and this linearisation is performed on the base of each half-cyclic wave in the displacement data. This method minimises the effect of central-point shift (e.g., residual deformation of structures damaged) in the displacement response data. The
number of steps for each linearisation is automatically determined by the number of steps of each half-cyclic wave, which is an advantage of using SPLiTS.

In this study, we examine SPLiTS via structural conditions assessment using experimental data including tests of E-Defense, which has a three-dimensional shake table (size: 20 m $\times$ 15 m) and has been accumulating response data of near 100 full- or large-scale structures. As the proposed method can describe physical parameters of a structure in the time domain, the energy absorbed by the structure, which is referred to as structural energy absorption in this study, can also be calculated by the energy balance equation. Thus, this study examines the relationship between structural damage and structural energy absorption.

The paper is organised as follows. Section 2 describes SPLiTS in detail. Section 3 examines parameter estimations with SPLiTS using the response data obtained by numerical simulations of a single-degree-of-freedom (SDOF) structure with or without nonlinear characteristics. Section 4 evaluates the parameter estimation based on experimental data of shaking table tests. Section 5 also examines the parameter estimation based on experimental data of shaking table tests at E-Defense. Section 6 summarises the conclusions of this study.

2 | SIMPLE PIECEWISE LINEARISATION IN TIME SERIES

SPLiTS is motivated by a comparison between linear SDOF structures with and without offset in their displacement responses; the offset in this study is rephrased as the central-point shift. When a linear SDOF structure does not have a shift, which corresponds to (I) in Figure 1a, the relationship between displacement and force associated with the spring is represented as follows:

$$f_k(t) = kx(t), \quad (1)$$

where $t$ is the time variable and $\{x, k, f_k\}$ is the set of displacement, stiffness and the restoring force of the stiffness in the linear SDOF structure, respectively. When the linear structure has a central-point shift, which corresponds to (II) in Figure 1a, the relation between the displacement and force becomes the following:

$$f_k(t) = kx(t) - f_{k0} = kx^*(t), \quad (2)$$

where $x^*(t) = x(t) - x_c$ and $x_c = f_{k0}/k$. According to Equation 2, by introducing the coordinate $x^*(t)$ considering the central-point shift, the linear SDOF structure having the shift can also be expressed by the same form of Equation 1. Note that nonlinear SDOF structures would show time varying shift $x_c(t)$ depending on its nonlinear characteristics, and Equation 2 can be applied to even such cases by introducing $x^*(t) = x(t) - x_c(t)$.

For a linear structure with/without the central-point shift, the displacement response of (I) and (II) in Figure 1b results in the hysteresis of (I) and (II) in Figure 1a, respectively. Note that the two responses in Figure 1b, consisting of $n$ sets of half-cyclic waves, are identical to each other apart from the shift $x_c$. Regarding the response, all amplitudes and durations in $n$ sets of half-cyclic waves would be different from each other: $A_1 \neq A_2 \neq \cdots \neq A_n$ and $T_1 \neq T_2 \neq \cdots \neq T_n$.

![Figure 1](image-url)  
**Figure 1** Response of linear single-degree-of-freedom structures without and with the central-point shift $x_c$ (I and II, respectively): (a) hysteresis and (b) time history
This difference is a minor issue when the response is derived from a linear structure. However, it becomes an important issue when the response is derived from a nonlinear structure because many of the nonlinear characteristics in structural buildings are closely related to the amplitude of the responses. In addition, the difference between the duration of each half-cyclic wave (i.e., $T_1 \neq T_2 \neq \cdots \neq T_n$) may be associated with variations in the stiffness of the structure. Thus, the set of amplitude and duration in half-cyclic waves is believed to contain essential information for the parameter estimation of nonlinear structures.

Based on the above study, we have developed SPLiTS to estimate physical properties of nonlinear structures in the time domain. The key concept of SPLiTS is piecewise linearisation of nonlinear structures based on half-cyclic waves in the displacement response data. This linearisation requires minimisation of the central-point shift observed in the displacement response data. To focus on the data associated with the vibrating behaviour of structures and exclude half-cyclic waves having a significantly small number of steps, SPLiTS employs thresholds for data selection. The following four procedures are used to estimate the physical parameters of a nonlinear structure:

P1. SPLiTS minimises the effect of the central-point shift in displacement response data. Residual deformation, which is commonly observed in structures that have been severely damaged by strong earthquake excitation, is a typical example of this shift.

P2. Following shift effect minimisation, cyclic waves in the data are divided into a set of half waves in positive and negative domains. Based on each half-cyclic wave, SPLiTS linearises the nonlinear structure. For example, when displacement response data contains 10 half-cyclic waves, SPLiTS handles the structure as a set of 10 different linear structures. This concept automatically determines the number of steps for the TDI.

P3. For enhancing the accuracy of estimation, SPLiTS should be applied to effective data in which the signal–noise (SN) ratio is not considerably small. The effective data are selected based on displacement and velocity response so that the selected data are associated with the vibrating behaviour of the structures. In other words, the effective data selected from the original data should satisfy the following data selection criteria: $|\dot{x}(t)| > \varepsilon_d$ or $|\ddot{x}(t)| > \varepsilon_v$, where $\varepsilon_d (>0)$ and $\varepsilon_v (>0)$ are thresholds for response data of displacement $\dot{x}(t)$ and velocity $\ddot{x}(t)$, respectively. Based on the time-history response data, these thresholds need to be determined to remove data that is not related to major vibrations of the structures.

P4. SPLiTS sets a threshold for the number of steps, $n_0$, to exclude waves with very few steps in the parameter estimation. When response data are measured by the sampling time interval $dt$, this threshold effectively ignores half-cyclic waves lasting for less than $n_0 dt$ s and is equivalent to ignoring frequency components over $1/(2n_0 dt)$ Hz; thus, the threshold can be roughly determined from the frequencies of interest.

In SPLiTS, these four procedures are applied to response data (acceleration, velocity and displacement) and external force applied to the structure as a preliminary step for parameter estimations of nonlinear structures.

In this section, the application of SPLiTS to response data is detailed in Section 2.1. Parameter estimations for SDOF and MDOF structures, based on response data processed by SPLiTS, are described in Sections 2.2 and 2.3, respectively. The velocity estimation, which is important for processing experimental data, is introduced in Section 2.4.

### 2.1 Data processing by SPLiTS

A nonlinear structure excited by external force $f(t)$ in Figure 2 is considered as a function producing response data, expressed by the following:

$$F(x(t),\dot{x}(t),\ddot{x}(t)) = f(t).$$

Its response data may be affected by the central-point shift. Following the concept P1, we minimise the effect on parameter estimation in advance. This is achieved by the following:

$$
\begin{align*}
\dot{x}'(t) &= x(t) - x_c(t) \\
\ddot{x}'(t) &= \dot{x}(t) - \dot{x}_c(t) \\
\dddot{x}'(t) &= \ddot{x}(t) - \ddot{x}_c(t) \\
\dddot{f}'(t) &= f(t)
\end{align*}
$$

In this section, the application of SPLiTS to response data is detailed in Section 2.1. Parameter estimations for SDOF and MDOF structures, based on response data processed by SPLiTS, are described in Sections 2.2 and 2.3, respectively. The velocity estimation, which is important for processing experimental data, is introduced in Section 2.4.
where $\dot{x}(t)$ is the displacement data minimising the effect of the central-point shift, $x(t)$. Equation 4 is the general form for both linear and nonlinear structures; $x(t)$ becomes zero in the case of linear structures. Note that the central-point shift $x(t)$ is mainly caused by very low frequency components, and its derivatives $\{\dot{x}(t), \ddot{x}(t)\}$ are expected to be near zero.

Effective data are obtained by applying the data selection criterion (i.e., concept P3) to the data set $\{x^+(t), \ddot{x}^+(t), \dddot{x}^+(t), f^+(t)\}$. The effective data can be rewritten as follows:

\[
\begin{align*}
(\lvert x^+(t) \rvert > e_d) \cup (\lvert \dot{x}^+(t) \rvert > e_v) & \Rightarrow x_+^+(t) = x^+(t), \ \dot{x}_+^+(t) = \dot{x}^+(t), \ \ddot{x}_+^+(t) = \ddot{x}^+(t), \ f_+^+(t) = f^+(t) \\
(\lvert x^+(t) \rvert \leq e_d) \cap (\lvert \dot{x}^+(t) \rvert \leq e_v) & \Rightarrow x_0^+(t) = 0, \ \dot{x}_0^+(t) = 0, \ \ddot{x}_0^+(t) = 0, \ f_0^+(t) = 0
\end{align*}
\]

where $\{x_+^+(t), \dot{x}_+^+(t), \ddot{x}_+^+(t), f_+^+(t)\}$ is the effective data set for displacement, velocity, acceleration and external force, respectively.

For distinguishing half-cyclic waves in positive and negative domains, the effective data need to be separated by the concept P2. The separation is performed based on the displacement data and is expressed as follows:

\[
\begin{align*}
x_+^+(t) = x_+^+(t)sgn^+(t), \ x_{-}^+(t) = -x_{-}^+(t)sgn^-(t) \\
\dot{x}_+^+(t) = \dot{x}_+^+(t)sgn^+(t), \ \dot{x}_{-}^+(t) = -\dot{x}_{-}^+(t)sgn^-(t) \\
\ddot{x}_+^+(t) = \ddot{x}_+^+(t)sgn^+(t), \ \ddot{x}_{-}^+(t) = -\ddot{x}_{-}^+(t)sgn^-(t) \\
f_+^+(t) = f_+^+(t)sgn^+(t), \ f_{-}^+(t) = -f_{-}^+(t)sgn^-(t)
\end{align*}
\]

where $sgn^+(t)$ ($sgn^-(t)$) is the positive (negative) values in $\text{sgn}(x_+^+(t))$ and is filled by zero values apart from the positive (negative) values, and $\text{sgn}(a) = \{1 \ (a > 0), 0 \ (a = 0), -1 \ (a < 0)\}$. In Equation 6, $\{x_+^+(t), \dot{x}_+^+(t), \ddot{x}_+^+(t), f_+^+(t)\}$ ($\{x_{-}^+(t), \dot{x}_{-}^+(t), \ddot{x}_{-}^+(t), f_{-}^+(t)\}$) is the set of effective data separated into the positive (negative) domain.

To exclude half-cyclic waves with fewer than $n_0$ steps, the concept P4 is projected to response data that have already been processed by concepts P1, P2 and P3. When the number of data of the $l$th half-cyclic wave, $n_l$ is less than the threshold (i.e., $n_l < n_0$), all data associated with the $l$th half-cyclic wave are modified to zero. For example, when this happens to the $l$th half-cyclic wave of the positive displacement data $x_{+}^+(t)$ ($t_l + dt \leq t \leq t_l + ndt$), all response data in the period are changed to $x_{+}^+(t) = \dot{x}_{+}^+(t) = \ddot{x}_{+}^+(t) = f_{+}^+(t) = 0$.

The following two subsections discuss parameter estimations of S/MDOF structures based on response data processed by SPLiTS.

2.2 | SPLiTS for SDOF structures

Here, SPLiTS is applied to parameter estimation of an SDOF structure in Figure 2. When this structure has time-variant damping and stiffness as well as constant mass, the equation of motion is expressed as follows:
\[ m\ddot{x}(t) + f_c(t) + f_k(t) = f(x), \]  

where \( f \) is the external force and \( f_c \) and \( f_k \) are restoring forces associated with damping and stiffness, respectively. In Equation 7, \( x \) is the relative displacement, equalling the inter-storey drift. Here, the damping coefficient \( c \) and stiffness \( k \) are unknown parameters that must be estimated from the known information of mass \( m \) and its response data \( \{x(t),\dot{x}(t),\ddot{x}(t),f(t)\} \).

As mentioned for the concept P2, SPLiTS regards the nonlinear structure as a set of linear structures with different constant parameters. These constant parameters vary depending on the half-cyclic waves. For instance, for the \( l \)th half-cyclic wave in the positive domain, the constant parameters have the following relation with the response data of \( \{x^*_+(t),\dot{x}^*_+(t),\ddot{x}^*_+(t),f^*_+(t)\} \):

\[
Q_lP_l = F_l, \tag{8}
\]

where

\[
Q_l(\in \mathbb{R}^{n_l\times 2}) = \begin{bmatrix}
\dot{x}^*_+(t_l + dt) & \dot{x}^*_+(t_l + dt) \\
\vdots & \vdots \\
\dot{x}^*_+(t_l + n_l dt) & \dot{x}^*_+(t_l + n_l dt)
\end{bmatrix}, \quad F_l(\in \mathbb{R}^{n_l\times 1}) = \begin{bmatrix}
f^*_+(t_l + dt) \\
\vdots \\
f^*_+(t_l + n_l dt)
\end{bmatrix}, \tag{9}
\]

\[
P_l(\in \mathbb{R}^{2\times 1}) = \begin{bmatrix}
c^*_{l+1} \\
k^*_{l+1}
\end{bmatrix}, \quad f^*_+(t_l + pdt) = f^*_+(t_l + pdt) - m\ddot{x}^*_+(t_l + pdt) \quad (p = 1 \cdots n_l)
\]

Note that \( \mathbb{R} \) is a real number, and its superscript describes the dimension of the matrix. In Equation 8, \( P_l \) containing \( c^*_{l+1} \) and \( k^*_{l+1} \) are vectors to be estimated from the known response data. When the number of steps is \( n_l > 2 \), which is for most cases, \( Q_l \) is not a regular matrix, and its direct inversion is infeasible. Thus, instead of the direct solution, the vector \( P_l \) is obtained by a pseudo inversion expressed by the following:

\[
P_l = (Q_l^TQ_l)^{-1}Q_l^TF_l. \tag{10}
\]

Note that the parameters estimated by Equation 10 represent stiffness and damping for the entire duration of the \( l \)th half-cyclic wave: \( c^*(t) = c^*_{l+1}, k^*(t) = k^*_{l+1} \) and \( t_l + dt \leq t \leq t_l + n_l dt \).

The parameter estimations in the negative domain \( c^*_-(t) \) and \( k^*_-(t) \) also can be achieved by applying the same procedure to the set \( \{x^*_-(t),\dot{x}^*_-(t),\ddot{x}^*_-(t),f^*_-(t)\} \). Then by combining the estimated parameters in both domains, the parameters of the nonlinear structure can be identified as \( c^*(t) = c^*_+(t) + c^*_-(t) \) and \( k^*(t) = k^*_+(t) + k^*_-(t) \).

Once the damping coefficient in the time series is given, the structural energy absorption for a nonlinear structure can be obtained by the following:

\[
E_c = \int_0^{T_a} c^*(t)\dot{x}^*(t)^2 dt, \tag{11}
\]

where \( E_c \) is the structural energy absorption and \( T_a \) is the final time stamp of the time series. \( E_c \) can become an index for SHM because the amount of structural energy absorption is closely related to structural damage.

## 2.3 SPLiTS for MDOF structures

SPLiTS is next applied to parameter estimation of an MDOF structure consisting of \( n \) storeys with masses, dampers and springs, as shown in Figure 3a. The equation of motion of the MDOF structure is commonly expressed by the relative displacement coordinate. This equation can be rewritten with the inter-storey drift coordinate shown in Figure 3b, which is more suitable for parameter estimations of the MDOF structure. The change to the coordinates is intensively discussed here.
The MDOF structure in Figure 3a has time-variant damping and stiffness as well as a constant mass on each storey. The structure is subject to the external force caused by ground motion excitation. Based on the relative displacement coordinate, its equation of motion can be expressed as follows:

$$M\ddot{x}(t) + C(t)\dot{x}(t) + K(t)x(t) = f(t),$$  \hspace{1cm} (12)

where

$$x(t)\in \mathbb{R}^{n\times1} = [x_1(t) \ x_2(t) \ ... \ x_n(t)]^T$$

$$f(t)\in \mathbb{R}^{n\times1} = [f_1(t) \ f_2(t) \ ... \ f_n(t)]^T, \quad M(\in \mathbb{R}^{n\times n}) = \text{diag}(m_1,m_2...m_n)$$

$$C(t)\in \mathbb{R}^{n\times n} = \begin{bmatrix} c_1(t) + c_2(t) & -c_2(t) & \cdots & 0 \\ -c_2(t) & c_2(t) + c_3(t) & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & -c_n(t) & c_n(t) \end{bmatrix}$$

$$K(t)\in \mathbb{R}^{n\times n} = \begin{bmatrix} k_1(t) + k_2(t) & -k_2(t) & \cdots & 0 \\ -k_2(t) & k_2(t) + k_3(t) & \cdots & \vdots \\ \vdots & \vdots & \ddots & -k_n(t) \\ 0 & \cdots & -k_n(t) & k_n(t) \end{bmatrix}$$

Here, $m_i, c_i, k_i, x_i$ and $f_i \ (i = 1 \ldots n)$ represent the mass, damping coefficient, stiffness, relative displacement and external force on the $i$th storey, respectively.

In Equation 13, the diagonal elements of the damping and stiffness matrices are coupled with values of the $i$th and $(i-1)$th storeys. This coupling makes parameter estimation of the MDOF structure difficult. The difficulty can be overcome by changing the coordinate to the inter-storey drift coordinate shown in Figure 3b. The new form can be written as follows:

**FIGURE 3** Nonlinear multi-degree-of-freedom structure: (a) under no excitations and (b) under excitations
\[
\ddot{\mathbf{x}}(t) + \mathbf{C}(t)\dot{\mathbf{x}}(t) + \mathbf{K}(t)\mathbf{x}(t) = \mathbf{f}(t), \tag{14}
\]

\[
\begin{bmatrix}
\dot{\mathbf{x}}(t)
\end{bmatrix} = \begin{bmatrix}
\ddot{x}_1(t) \\
\ddot{x}_2(t) \\
\vdots \\
\ddot{x}_n(t)
\end{bmatrix} = \Gamma\mathbf{x}(t)
\]

\[
\mathbf{f}(t) = \Gamma^{-T}\mathbf{f}(t)
\]

\[
\Gamma(\in \mathbb{R}^{n\times n}) = \begin{bmatrix}
1 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & -1
\end{bmatrix}
\]

\[
\mathbf{M}(t) = \Gamma^{-T}\mathbf{M}\Gamma^{-1} = \begin{bmatrix}
\alpha_1 & \alpha_2 & \cdots & \alpha_n \\
\alpha_2 & \alpha_2 & \cdots & \alpha_n \\
\vdots & \vdots & \ddots & \vdots \\
\alpha_n & \alpha_n & \cdots & \alpha_n
\end{bmatrix}, \quad \alpha_i = \sum_{q=1}^{n} m_q
\]

\[
\mathbf{C}(t) = \Gamma^{-T}\mathbf{C}(t)\Gamma^{-1} = \text{diag}(c_1(t), c_2(t), \ldots c_n(t))
\]

\[
\mathbf{K}(t) = \Gamma^{-T}\mathbf{K}(t)\Gamma^{-1} = \text{diag}(k_1(t), k_2(t), \ldots k_n(t))
\]

Note that \(\ddot{x}_i(t)\) is the \(i\)th inter-storey drift, which is expressed by \(x_i(t) - x_{i-1}(t)\) (n. \(x_0(t) = 0\)). In this coordinate, the damping and stiffness matrices can be expressed as diagonal matrices, although the mass matrix is more complicated than in Equation 13. In many cases, masses are constant, and weight can be directly measured or roughly estimated by the nominal design. Thus, the mass matrix is known in many cases, whereas damping and stiffness are unknown parameters to be estimated.

To estimate damping and stiffness from the response data and masses, Equation 14 can be rewritten as follows:

\[
\mathbf{\dot{C}}(t)\dot{\mathbf{x}}(t) + \mathbf{\dot{K}}(t)\dot{\mathbf{x}}(t) = \mathbf{f}'(t), \tag{16}
\]

where \(\mathbf{f}'(t) = \begin{bmatrix}
\mathbf{f}_1(t) \\
\mathbf{f}_2(t) \\
\vdots \\
\mathbf{f}_n(t)
\end{bmatrix} = \mathbf{f}(t) - \mathbf{M}\ddot{\mathbf{x}}(t)\). Because \(\mathbf{\dot{C}}(t)\) and \(\mathbf{\dot{K}}(t)\) are diagonal matrices in Equation 16, the parameters can be individually estimated for each storey with the approach described in Section 2.2 for the SDOF structure.

When a structure is shaken by ground motion acceleration \(\ddot{x}_g(t)\) such as that caused by an earthquake, the external force in Equation 12 becomes \(\mathbf{f}(t) = -\mathbf{M}\ddot{x}_g(t)\) where \(\eta = (\in \mathbb{R}^{n\times 1}) = [1 \cdots 1]^T\). Then the force in Equation 14 becomes \(\mathbf{f}(t) = -\Gamma^{-T}\mathbf{M}\eta\ddot{x}_g(t)\) and Equation 16 can be rewritten as follows:

\[
\mathbf{f}'(t) = -\Gamma^{-T}\mathbf{M}\eta\ddot{x}_g(t) - \mathbf{M}\ddot{\mathbf{x}}(t) = -\Gamma^{-T}\mathbf{M}(\ddot{x}_g(t)\eta + \ddot{\mathbf{x}}(t)) = -m_s\mathbf{M}_s(\ddot{x}_g(t)\eta + \ddot{\mathbf{x}}(t)), \tag{17}
\]

where \(\Gamma^{-T}\mathbf{M}(\in \mathbb{R}^{n\times n}) = \begin{bmatrix}
m_1 & m_2 & \cdots & m_n \\
0 & m_2 & \cdots & m_n \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & m_n
\end{bmatrix}, \quad \mathbf{M}_s(\in \mathbb{R}^{n\times 1}) = \begin{bmatrix}
m_1 & m_2 & \cdots & m_n \\
m_s & m_s & \cdots & m_s \\
0 & m_s & \cdots & m_s \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & m_s
\end{bmatrix}
\]

and \(m_s\) is the reference mass for standardisation.

According to Equation 17, even when actual masses are unknown, and only the mass ratio is known, parameters of the structure can be obtained by the following:
\[ \dot{C}_s(t) \ddot{x}(t) + K_s(t) \dot{x}(t) = -M_s(\ddot{x}_g(t) + \dot{x}(t)), \]  

(18)

where \( \dot{C}_s = \text{diag} \left( \frac{c_1}{m_1}, \ldots, \frac{c_n}{m_n} \right) \) and \( K_s = \text{diag} \left( \frac{k_1}{m_1}, \ldots, \frac{k_n}{m_n} \right) \). Note that \( \dot{C}_s \) and \( K_s \) are damping and stiffness, respectively, standardised by the reference mass. Equation 18 indicates that stiffness and damping standardised by the reference mass can be estimated based on mass ratio, even without knowing accurate weight for each storey.

Structural energy absorption for MDOF structures is calculated by Equation 11, with the inter-storey velocity and damping coefficient estimated for each storey. Then based on the structural energy absorption of each storey, it is possible to identify which storey absorbs the most energy. This information is very useful in visual inspection because it allows us to roughly decide the inspection priority of each storey.

2.4 | Estimation of velocity response data

In many experiments, data acquisition for earthquake/structural engineering purposes is focused on acceleration and displacement response data, whereas velocity data are a lower priority. Many experiments, therefore, lack velocity response data, limiting applications of SPLiTS. Accurate estimation of velocity is required to make SPLiTS feasible for such experiments, as well as for the structural energy absorption calculations described by Equation 11.

Directly estimating velocity by the differentiation of displacement (integration of acceleration) tends to generate unexpectedly high (low) frequency. To compensate for the deficiencies of these two simple approaches, we estimate the velocity response data by using a composite filtering technique.\(^{40}\) In the Laplace domain, the first-order filtering technique is described by the following:

\[
\dot{x}(s) \left( \frac{s}{s + \omega_c} \right) = F_a(s) \ddot{x}(s) + F_d(s) x(s),
\]

(19)

where \( s \) is the Laplace operator, \( F_a(s) = 1/(s + \omega_c) \), \( F_d(s) = s\omega_c/(s + \omega_c) \) and \( \omega_c \) is the switching frequency for \( F_a \) and \( F_d \). The switching frequency determines how much acceleration and displacement contribute to the velocity. This composite filtering technique is essential for experimental data, but it is not for data obtained in numerical simulations, which can easily provide all response data required for SPLiTS.

3 | NUMERICAL SIMULATIONS

Parameter estimation with SPLiTS is examined by numerical simulations of nonlinear/linear SDOF structures that are excited by the external force caused by ground motion acceleration. In this case, the external force in Equation 7 is described by \( f(t) = -m \ddot{x}_g(t) \). The damping coefficient and stiffness are parameters estimated from known mass \( m \) and response data \( \{x(t), \dot{x}(t), \ddot{x}(t)\} \), as well as from ground motion acceleration \( \ddot{x}_g(t) \).

The linear SDOF structure has properties \( m = 200 \text{ kg}, c = 377.0 \text{ Ns/m and } k = 71.1 \text{ kN/m} \), which result from the natural frequency of \( 3.0 \text{ Hz} \) and the damping ratio of \( 0.05 \). When the structure is treated as a nonlinear SDOF, the spring in the linear structure is replaced by the trilinear hysteretic spring in Figure 4 with the following properties:

- \( r_1 = 0.5 \), \( r_2 = 0.1 \), \( \delta_1 = 10 \text{ mm} \) and \( \delta_2 = 30 \text{ mm} \).

Numerical simulations here are performed by the sampling time interval \( dt = 0.001 \text{ s} \). These simulations employ the ground motion acceleration recorded by the Japan Meteorological Agency (JMA) at Hyogo-ken Nanbu/Kobe Earthquake 1995, and this motion is referred to as JMA Kobe motion in this study. The motion does not contain frequency components lower than \( 0.1 \times 2\pi \text{ rad/s} \), and such diminished frequency in the response data indicates central-point shift. Thus, the shift is estimated by the second-order Butterworth high-pass filter with cut-off frequency \( 0.1 \times 2\pi \text{ rad/s} \). To focus on the data associated with the vibration behaviour of the structure, we set the data selection criteria as \( e_d = 2.0 \text{ mm}, e_v = 20.0 \text{ mm/s and } n_0 = 100 \). The thresholds of \( e_d \) and \( e_v \) are determined based on the assumption that measurement noises in displacement, velocity and acceleration data are approximately \( 1 \text{ mm}, 10 \text{ mm/s} \) and \( 100 \text{ mm/s}^2 \), respectively. The threshold of the steps \( n_0 \) indicates that SPLiTS employs only half-cyclic waves that last for more than \( 0.1 \text{ s} \) for the parameter estimation. Structural energy absorption, \( E_{Ec} \), is calculated by Equation 11 with estimated inter-storey velocity and damping coefficient.
The parameter estimations for an SDOF structure with a linear (nonlinear) spring are performed in Section 3.1 (3.2). The effect of the central-point shift in parameter estimation is studied using SPLiTS with/without considering the central-point shift. SPLiTS without consideration shift is equivalent to Equation 4, with \( x_c(t) = \dot{x}_c(t) = \ddot{x}_c(t) = 0 \). In addition, the effect of the thresholds in SPLiTS is examined together with response data containing noises in Section 3.3. In these examinations, the basic TDI, which is equivalent to the linearisation of the entire time series responses, is additionally considered for comparison.

3.1 | Linear SDOF

The response of a linear SDOF structure under JMA Kobe motion is displayed in Figure 5a,b. In this case, the displacement response exhibits no central-point shift characteristics. Based on the response data and the input motion, \( \{ x(t), \dot{x}(t), \ddot{x}(t), \dddot{x}(t) \} \), the parameters of the SDOF structure are estimated by SPLiTS with/without considering the shift.

According to Figure 5c,d, parameters are accurately estimated in both cases. The average values of damping coefficient and restoring force obtained by SPLiTS with (without) considering the shift are 375.2 Ns/m and 71.0 kN (377.0 Ns/m and 71.1 kN), respectively, nearly identical to the assigned values.

When the estimated values are accurate, the force \( f(x) - mx \dot{x} \) used for the estimation should be accurately reproduced by the estimated values. In this examination, all reproductions are very close to the true values, as shown in Figure 5e,f.

Based on the damping estimation of SPLiTS with (without) the central-point shift, the structural energy absorption is calculated to be \( E_c = 340.5 \) (\( E_c = 341.3 \)) Nm. These values are close to the true structural energy absorption 342.9 Nm, which is calculated by the true damping coefficient assigned.

3.2 | Nonlinear SDOF

The nonlinear SDOF structure under the JMA Kobe motion displays the response in Figure 6a,b. The hysteretic loop obtained in Figure 6a shows the characteristics of the trilinear hysteretic spring. Unlike the linear structure result, the inter-storey drift shows a central-point shift in Figure 6b. Based on the response data and the input motion \( \{ x(t), \dot{x}(t), \ddot{x}(t), \dddot{x}(t) \} \), SPLiTS estimates the parameters of the nonlinear SDOF structure with/without considering the central-point shift.

As shown in Figure 6c, SPLiTS without shift consideration does not produce estimations over 15 s because the response data over the time are not selected as effective data, due to the shift shown in Figure 6b. As shown in Figure 6d, when the shift is considered, SPLiTS produces more reasonable estimations under the same conditions. In this case, the estimations are more closely matched to the characteristics of a trilinear spring; the stiffness at a small deformation returns to the initial stiffness even after a large deformation. The estimated damping coefficient sharply rise when the inter-storey drift reaches its maximum at 5–15 s and returns to its initial value at small deformation. This
A rapid increase in damping coefficient clarifies that SPLiTS regards the energy dissipation caused by the trilinear hysteretic loop as an additional damping effect within the structure. Note the estimates $k$ and $c$ of the basic TDI have become $15.2 \text{ kN/m}$ and $1.0 \text{ kN.s/m}$, respectively.

Reproductions of the force used for the estimation $f(x) - m\ddot{x}(t)$ are illustrated in Figure 6e,f. As shown in Figure 6e, the reproduction for the basic TDI differs from the true force and SPLiTS without the shift consideration shows better reproduction than the basic TDI. However, as shown in Figure 6f, SPLiTS with the consideration has produced the closest result to the true force. The comparison confirms the estimation accuracy of SPLiTS with shift consideration.

Structural energy absorption based on the damping coefficients by SPLiTS with (without) the central-point shift becomes $E_c = 370.9 \ (359.4) \text{ Nm}$. The difference in structural energy absorption is mainly caused by the difference in the number of effective data for the two cases. Thus, structural energy absorption is also found to be affected by the central-point shift.
3.3 Efficiency of thresholds to response data containing noise

Here, we examine the effect of thresholds on response data containing noise. The responses having noise are described as follows:

\[ x(t) + w_d(t) \]
\[ \dot{x}(t) + w_v(t) \]
\[ \ddot{x}(t) + w_a(t) \]  

where \( \{w_d, w_v, w_a\} \) is the set of noises contaminating the displacement, velocity and acceleration data, respectively.

In this study, noises are created by applying the first-order Butterworth low-pass filter with the cut-off frequency of 100.0 Hz to random numbers that are generated so as not to have the correlations with each other. The noises of \( w_d \), \( w_v \), and \( w_a \) are standardised to make those maximum values become 1.0 mm, 10.0 mm/s and 100.0 m/s\(^2\), respectively.
These noises are added to responses obtained by the linear structure in Section 3.1 and nonlinear structure in Section 3.2.

The set of the thresholds employed in the previous simulations (i.e., \( \varepsilon_d = 2.0 \text{ mm} \), \( \varepsilon_v = 20.0 \text{ mm/s} \) and \( n_0 = 100 \)) are handled as the standard set for the parameter estimations. The sets of \([\varepsilon_d, \varepsilon_v, n_0/10]\) and \([\varepsilon_d/10, \varepsilon_v/10, n_0/10]\), referred to as sets 1 and 2, respectively, are employed to examine the effect of the three thresholds. The effective data in the three sets will be increased in the order of standard set, set 1 and set 2, because effective data become larger, as the thresholds become smaller.

Based on these sets, parameter estimations are performed for the responses of the linear and nonlinear structures, and the results are illustrated in Figures 7 and 8, respectively. As shown in Figures 7a and 8a, the standard set provides very similar results with the results shown in Figures 5d and 6d, which were obtained by the simulations without the noises. However, the estimations based on set 1 in Figures 7b and 8b have more noise than the ones obtained by the standard set. This is particularly significant in the estimated damping coefficients. As seen in Figures 7c and 8c, set 2 has the worst performance, clarifying the efficiency of the standard set.

4 | SHAKE TABLE TESTS FOR AN SDOF STRUCTURE

For evaluating more realistic situations, SPLiTS is applied to parameter estimation of a physical model used as the specimen in shake table experiments\(^4^1\) to examine the performance of nonlinear signal-based control.\(^4^2\),\(^4^3\) As shown in Figure 9a, the specimen was shaken by a single-axis electrodynamic shake table with the following specifications: table...
weight 200 kg, table size 1.2 m × 1.2 m, loading capacity 500 kg, maximum stroke ±300 mm, maximum velocity 1.0 m/s and available frequency range 0.5–15.0 Hz. The specimen had a single-storey (i.e., SDOF) structure and measured 0.9 m × 0.9 m with a height of 0.6 m and weight of 200.0 kg. A set of steel plates connected the table, and the physical model was installed to demonstrate the flexibility of the structure as well as to display nonlinear characteristics at deformation over the elastic range. Eight plates (four plates on each side) were installed to make the natural frequency of the specimen become 3.0 Hz.

Two accelerometers were placed on the table and two on the specimen for the experiment. Two-wire displacement transducers were attached near the top of the protection stands on the table to measure inter-storey drift. The sampling time interval for the data acquisition was 0.001 s. High-frequency noise was eliminated by postprocessing the experimental data using a second-order Butterworth low-pass filter with a cut-off frequency of 30 Hz. Hysteresis loops were then acquired from the acceleration and inter-storey drift of the specimen.

Basic dynamic properties of the specimen are investigated by the identification system discussed in Section 4.1. Parameter estimations are described in Section 4.2 for the specimen subjected to an earthquake excitation with different amplitudes.

4.1 | System identification

System identification of the specimen was performed by a band-limited white noise excitation containing frequency components of 0.1–50.0 Hz. In an identification test in which the table displacement remained within 5.0 mm, the dynamics of the physical model were identified, yielding natural frequency $\omega = 2.22 \times 2\pi$ rad/s and damping ratio $\zeta = 0.011$. In Figure 9b, FRF for the input and response acceleration of the physical model was found to be precisely modelled by $G(s) = (2\zeta\omega + \omega^2)/(s^2 + 2\zeta\omega + \omega^2)$. Based on this modelling and the weight of the specimen $m = 200$ kg, the damping coefficient and stiffness were found to be $c = 61.4$ Ns/m and $k = 38.9$ kN/m, respectively.

4.2 | Parameter estimations

The specimen was excited by the JMA Kobe motion at three different amplitudes of 25%, 50% and 80%; the experiment with 100% amplitude was not implemented because of its safety management. In these experiments, acceleration and displacement of the table and specimen were directly measured, whereas the velocity response was estimated using the composite filtering technique of Equation 19 with $\omega_c = 1.0 \times 2\pi$ rad/s. Hereafter, estimated velocity response is mentioned as response data in the experiments.

For parameter estimation in each experiment, the central-point shift is calculated by applying the second-order Butterworth high-pass filter with a cut-off frequency of $0.1 \times 2\pi$ rad/s to the inter-storey response data (i.e., drift, velocity and acceleration). Data selection criteria are set to be $\varepsilon_d = 2.0$ mm, $\varepsilon_v = 20.0$ mm/s and $n_0 = 100$. The structural energy absorption $E_c$ is calculated by Equation 11 with estimations of both inter-storey velocity and damping coefficient.

**FIGURE 9** Physical model for shaking table tests: (a) photograph and (b) frequency response function
4.2.1 | JMA Kobe 25%

Figure 10a,b shows the inter-storey response data in the JMA Kobe 25% experiment. The central-point shift was not observed in this experiment because the specimen did not exceed its elastic range during the excitation, as observed in Figure 10a.

Figure 10c shows the estimated parameters and $E_c$. The average values of the estimated damping and stiffness are $c = 69.0$ Ns/m and $k = 38.3$ kN/m, respectively, which are similar to values obtained at system identification. In the time series shown in Figure 10c, the estimated stiffness does not show a large variation.

The structural energy absorption in this experiment is thus $E_c = 51.2$ Nm. According to Figure 10c, nearly 90% of the energy is absorbed within 10–14 s.

4.2.2 | JMA Kobe 50%

Figure 11a,b shows the inter-storey response data in the JMA Kobe 50% experiment. The hysteresis loop in Figure 11a and the central-point shift observed in Figure 11b indicate yielding in the specimen.

Figure 11c shows the estimated parameters and $E_c$. The maximum damping coefficient in the estimation is 685 Ns/m, which is 9.9 times the average damping coefficient obtained at JMA Kobe 25%. This increased damping coefficient is caused by the yielding because energy dissipation by the yielding behaviour tends to be evaluated as additional damping. The estimated stiffness also shows the variation in the value at around 10–15 s, although its variation is less significant than that of the damping coefficient. The estimated stiffness past 15 s returns to the initially estimated stiffness before 10 s. This characteristic corresponds well with the inclination toward slightly damaged structures.

The structural energy in this experiment is $E_c = 203.5$ Nm, nearly four times that of the absorption at JMA Kobe 25%. This result proves that the additional damping effect by the yielding can be properly evaluated and quantified by Equation 11.

4.2.3 | JMA Kobe 80%

Figure 12a,b shows the inter-storey response data from the JMA Kobe 80% experiment. The central-point shift, observed in Figure 12b, is larger than in the experiment of JMA Kobe 50%. The fat hysteresis loop in Figure 12a nearly confirms the occurrence of yielding, because the loop describes some large energy dissipation. The steel plates showed traces of yielding when the physical model was visually inspected after this excitation.
Figure 12c shows the estimated parameters and $E_c$. The maximum damping coefficient is estimated to be 1,120 Ns/m, which is 16.2 times the average value at JMA Kobe 25% and 1.6 times the maximum value at JMA Kobe 50%. The increased damping coefficient is caused by the fat hysteresis loop in Figure 12a. Figure 12c shows the variation of stiffness in the duration of 5–20 s. The decreased stiffness is understandable for the yielding behaviour, whereas the increased stiffness opposes the nature of the behaviour. The estimation of the central-point shift may have caused the increase because overestimation can result in higher stiffness than the actual value. The estimation accuracy of the stiffness is also found to be affected by the estimation of the central-point shift effect.

The structural energy absorption in this experiment is $E_c = 491.9$ Nm, which is 9.6 (2.4) times the absorption at JMA Kobe 25% (50%), respectively. This result indicates that the structural energy absorption properly evaluates the effect of yielding.

5 | SHAKE TABLE TESTS FOR A THREE-STOREY STEEL BUILDING

For evaluating SPLiTS with more realistic buildings, parameter estimations were performed for a three-storey steel structural frame that was shaken by E-Defense in October 2013, as shown in Figure 13. The aim of this experiment was
to investigate the structural damage caused by the 1995 Kobe earthquake. For understanding both the situation and the structural damage caused by this earthquake, a specimen was designed to duplicate a common steel structure at the time of the earthquake. The size of the specimen was 18 m × 12 m with a height of 13.5 m. The mass of the first, second and third storeys were measured to be 44.9, 43.9 and 40.6 t, respectively.

Five accelerometer units consisting of three accelerometers for each of the three axes were allocated to the four corners and the centre of each storey. Additionally, four laser displacement transducers were placed at the four corners of each storey to measure inter-storey drift in the longitudinal and transverse directions. All experiments were performed by uniaxial excitation to the longitudinal direction with a sampling time interval of 0.001 s. Similar to the SDOF structure, experimental data were postprocessed by the second-order Butterworth low-pass filter with a 30-Hz cut-off frequency, to eliminate higher frequency components. The hysteretic loop on each storey was acquired from acceleration and inter-storey drift of the specimen.

In these experiments, the inter-storey velocity for each storey is estimated by the composite filtering technique with $\omega_c = 1.0 \times 2\pi$ rad/s in Equation 19 as well as by the inter-storey acceleration and displacement. Hereafter, the estimated velocity is referred to as response data in the experiments. Structural energy absorption for $i$th storey $E_{ci}$ ($i = 1, 2, 3$) is calculated by Equation 11 with estimated $i$th inter-storey velocity and damping coefficients.

Basic dynamic properties of the three-storey steel structural frame are investigated by the system identification in Section 5.1. Parameter estimations of the frame subjected to an earthquake excitation are discussed in Section 5.2.

5.1 | System identification

A system identification test was performed using band-limited white noise excitation. During this test, no inter-storey drift exceeded 3.0 mm. Figure 14 shows the FRF obtained using the curve-fitting method. The first, second and third natural frequencies (and corresponding damping ratios) of the specimen were 1.37, 4.46 and 8.21 Hz (0.03, 0.03 and 0.02), respectively.

For the parameter estimations, $\epsilon_d = 0.5$ mm, $\epsilon_v = 5.0$ m/s and $n_0 = 100$ are employed as data selection criteria, and parameters are estimated from inter-storey response data processed by SPLiTS. Although the number of steps satisfying the criteria are few because of the low amplitude excitation, the estimations in Figure 15 show rough values of the damping coefficient and stiffness for each storey. The average values of the damping coefficient (stiffness) for the first, second and third storeys are 226, 175 and 127 kNs/m (17,481, 15,338 and 12,132 kN/m), respectively.

In this experiment, the structural energy absorption for the first, second and third storeys is $\{E_{c1}, E_{c2}, E_{c3}\} = \{1.3, 0.7, 0.4\}$ kNm, respectively. These values will be found low compared with those obtained in the following experiments.
Parameter estimations

A series of experiments were performed for the steel frame with the ground motion acceleration recorded at the Takatori station during the 1995 Kobe earthquake, referred to as Takatori motion in this study. Four tests were performed with amplitudes of 40%, 60%, 80% and 100%. At 100% excitation, the inter-storey drift in the first storey exceeded the measurable range of the laser displacement transducer. Thus, excluding the amplitude of 100%, response data for the smallest and largest amplitudes (i.e., 40% and 80%) are selected as parameter estimations in this study. Following the estimations in Section 4.2, $\varepsilon_d = 2.0$ mm, $\varepsilon_v = 20.0$ mm/s and $n_0 = 100$ are employed as the data selection criteria.

5.2.1 | Takatori motion 40%

Figure 16a,b shows the response data in the experiment with 40% Takatori motion. In Figure 16a1,b1, the first storey displayed the largest response, and each subsequent ascending storey had a smaller response. Because the first storey did not show a fat hysteresis loop in Figure 16a1, the structural damage and energy absorption caused by this excitation are anticipated to be minor.

Figure 16c shows the parameters estimated and $E_c$. In addition to the first storey, which had the largest response, the second storey also shows a sharp rise in the damping coefficient at 15 s, as shown in Figure 16c1,c2. Then the maximum damping coefficient of the first (second) storey becomes 330 (295) kNsm/m, which is 1.5 (1.7) times the average values of the identification test. Because the increase in damping is small, the corresponding structural damage is expected to be insignificant. The structural energy absorption in this experiment is $\{E_{c1}, E_{c2}, E_{c3}\} = \{23.9, 10.5, 4.1\}$ kNm. According to this structural energy absorption, structural damage would likely be found in the order of first, second and third storeys. According to the detailed report containing the visual inspection of the specimen, many minute visible damages such as cracks were concentrated on the first storey.
5.2.2 Takatori motion 80%

Figure 17a,b shows the response data from the 80% Takatori motion experiment. Like the 40% experiment, the first storey again displayed the largest response with a fat hysteresis loop in Figure 17a1. Because this hysteretic loop indicates the occurrence of some yielding, its structural damage and energy absorption are expected to be significant.

Figure 17c shows the estimated parameters and $E_c$. As seen in Figure 17c, the estimated stiffness for the first and second storeys show some reduction within 10–30 s, but the variations are less obvious than those of the damping coefficient. Further, even after experiencing large deformation, the stiffnesses of the first storey returns to values similar to
those obtained at system identification. This indicates that postexperiment stiffness cannot precisely project the severity of structural damage.

The maximum damping on the first, second and third storeys are 568, 686 and 416 kNs/m, which are 2.5, 3.9 and 3.3 times the average values obtained at system identification, respectively. The second storey has the largest increase rate, whereas the first storey, which experienced the largest response, is the smallest. This is the opposite of the energy dissipation anticipated by the hysteresis loops in Figure 17a. However, the structural energy absorption is \( E_{c1}, E_{c2}, E_{c3} \) = \{174.9, 77.1, 20.3\} kNm, which matches the anticipation toward the hysteretic loop. The structural energy absorption of the first storey is the largest in this excitation and 7.3 times the absorption at Takatori motion 40%, indicating severe structural damage. According to the detailed experimental report, cracks and local buckling on the beam of the

FIGURE 17 Parameter estimations for the specimen under Takatori motion 80%: (a) inter-storey drift versus shear force, (b) response data of displacement and velocity and (c) parameter estimations and structural energy absorption. The subscript number in the captions correspond to the number of storeys
first storey were found during the on-site visual inspection of the specimen. Based on this result, structural damage is
more appropriately evaluated by structural energy absorption than by variation of the damping coefficient.

6 | CONCLUSIONS

This study introduced SPLiTS for physical parameter estimations of nonlinear structures. For an effective response data
selected by the criteria, SPLiTS minimises the effect of central-point shifts for piecewise linearisation, which are
performed based on each half-cyclic wave. Parameter estimations using SPLiTS were examined via data obtained in experi-
ments and numerical simulations.

First, SPLiTS was applied to data obtained from numerical simulations for a nonlinear SDOF structure. The results
of parameter estimations were affected by the central-point shift, which appeared as the nature of the nonlinear hys-
teric spring in the structure. SPLiTS provided more reasonable results when the shift was considered. Then SPLiTS
was applied to experimental data obtained from shaking table tests where the specimen was a simple physical model
imitating a one-storey structure. TDI with SPLiTS reasonably estimated physical parameters for the specimen under
different amplitudes. When the specimen went into the inelastic range, the estimation showed increased damping and
decreased stiffness, which are the typical characteristics of the inelastic behaviour. Finally, the parameter estimations
were applied to data, which was obtained by shaking table tests of a full-scale three-storey structure at E-Defense. The
damping coefficient and stiffness for each storey were estimated reasonably for the specimen under different amplitudes. Structural energy absorption is found to be more appropriate for indicating structural damage than the
damping coefficient.

The current SPLiTS requires both acceleration and displacement data directly measured by measurement devices.
Although the advancement of wireless measurement sensors enables ease of access to acceleration response data of
urban structural buildings than before, the acquisition of those displacement data remains a challenge. For the application
of SPLiTS to urban structures, it needs to be improved to realise the same performance even without using dis-
placement data. Additionally, we will consider its application to sparse measurement issues. The current SPLiTS is
limited to simple buildings, which can be modelled by the lumped mass model, with many measurement devices. Thus,
to enhance its identification accuracy, we will develop other forms of SPLiTS that are suitable to more elaborate models
e.g., fishbone model45).

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