Vibration dynamics and cutting process temperature at various stages of forming the wear of the cutting wedge

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Abstract. On today, there is no single and consistent mathematical model describing the complex connectivity of the metal cutting process. Therefore, the article proposes new approach based on the formation of feedback connecting subsystems describing the power, thermal and vibration reactions from the cutting process. The purpose of the work: By forming a consistent model of connections between subsystems describing the power, thermal and vibration reactions from the cutting process to the shaping movements of the tool, you can obtain a description of the mechanism of self-organization of the cutting process in the process of evolutionary changes of the tool. The resulting mechanism is needed to search for some mode of operation of the cutting system, in which further wear of the cutting wedge, cutting force, temperature in the cutting zone and vibration of the tool can be stabilized. Research methods: For processing and analysis of the obtained data, a Matlab package of mathematical programs was used, in which a subprogram was developed that allows a series of numerical experiments. The main conclusion on the work is the provision put forward by us on the self-organization of the cutting system, through the process of evolution of the instrument.

1. Introduction
In many ways, the modern base of metal cutting machines in its scientific and engineering complexity is not inferior to such an industry as astronautics. In metalworking today, the digital approach to the control and control of processing processes is most fully implemented, which is explained by the wide implementation of digital control systems (sensors) and data processing systems received from them. This approach makes it possible to use more complex models in the analysis of the cutting process on a particular machine than previously used. Here, a promising direction of quality control of the processing process is rapidly developing vibration monitoring and vibration diagnostics systems [1-5]. It is therefore an object to describe the relationship of vibrations measured during cutting with non-measurable but of engineering interest characteristics such as force reaction and temperature in the contact zone of the tool and the workpiece.

In the modern representation of vibrations arising during cutting, it is customary to divide them into three components: free vibrations, forced vibrations and self-excited oscillations [6]. Free oscillations are related to the quality of the cutting system and are a reaction to changes in the processing process, forced oscillations are due to external actions, such as, for example, beats in the bearings of the spindle assembly, vibrations of the machine body or beats in the IWP. To combat free and forced vibrations, many engineering methods have been developed today, but with self-excited oscillations that can extract energy from the interaction that arises in the cutting zone of unambiguous engineering solutions today. This makes the topic of self-excited oscillations in cutting very popular in modern scientific research [7-11]. However, in these works, the main emphasis is on assessing the effect on the oscillation of the tool, the so-called regenerative effect. For the first time, the regeneration of oscillations in metal
processing on metal cutting machines was investigated by Hahn R.S., Tobias S.A. and Merritt H.E. [12-14]. The works of these authors are the fundamental basis underlying the analysis of the dynamics of regenerative vibrations of the tool during cutting. Many works note the possibility of establishing the chaotic nature of instrument vibrations during vibration regeneration [15-18]. In these works, it is indicated that the main factor affecting the regenerative effect is the so-called time delay, which determines the dynamics of the process of regenerating the tool oscillations.

In the Russian scientific journalism devoted to self-excited oscillations (vibrations) of the instrument in metal cutting, the issues of estimation of the effect of cutting on the "trace" on the dynamics of vibrations of the instrument are considered indirectly, more attention is paid to the construction and analysis of models describing the interconnected dynamics of the processing process [19-21]. For example, in our previously published works [22-25], the analysis of the dynamics of the deformation vibrations of the tool is carried out on the basis of the connectivity, through the force reaction, of this deformation movement with the cutting elements of the NC system of the machine. In this case, the dynamic effects that occur during the simulation, in our opinion, more accurately reflect the nature of the interaction between the subsystems of the cutting control system in the contact area of the tool and the workpiece. In the fundamental works of leading Soviet and Russian scientists studying the vibrational dynamics of the cutting process [26-30], it is noted that in the cutting process, in addition to feedback on the cutting force, which takes into account the regeneration of vibrations during cutting along the "trace," thermodynamic feedback is formed, which is also associated with the vibration activity of the instrument and wear of the cutting wedge [26-30]. Thus, in the actual process of processing, there is a rather complex interaction involving the formation of a plurality of interconnected feedback.

2. Research methodology and simulation results

When presenting wear in the coordinates of the state, we rely on the connection of wear and work with the power of irreversible transformations of the supplied energy in the cutting zone. Fundamentally, the change in wear depending on irreversible transformations is disclosed in the works of A. L. Bershadsky, Kostetsky B.I., Ryzhkin A.A., etc. [30].

The reason for evolutionary transformations is related to the power and operation of the cutting forces, that is, the energy of irreversible transformations in the processing zone. Thus, all the parameters characterizing the cutting process and the manufacturing quality of the part are not only interconnected, but also have one nature of change - this is the trajectory of the power of irreversible transformations in perfect work. To simulate evolutionary changes in this case, it is necessary to use Volterre integral operators of the second kind, having the following structure [25]:

$$h_3 = k \int_{0}^{A} w(t - \xi) \cdot N(\xi) d\xi$$  \hspace{1cm} (1)

where $w(t - \xi)$ is the core of the integral operator, is the phase trajectory of the power of irreversible transformations by perfect work, and $A$- the work of cutting forces. However, as follows from the structure of the operator, wear depends on the fluidity of irreversible transformations and its prehistory, which is taken into account by the core of the integral operator (see expression 1). The tool wear process is always bi-directional in nature, one of the directions of which is oriented towards the treatment of the cutting wedge to the cutting process, as a result of which we observe, the process of forming the tool contact area along the back face, through which the temperature field and the force reaction are stabilized, described by us in previous sections. The second direction is related to the growth of degrading features of the wear process, which subsequently lead to a significant change in cutting properties and properties. If $N(t)$ is not equal to $N_0 = \text{const}$, then we can assume that the measured or calculated discrete values of power $N_0, N_1, ..., N_n-1$, due to the smallness of $\Delta A = Ak-Ak-1$, are close to constant or constant, then we can obtain an approximating discrete sum describing the integral operator (2):
The expression (2) can be modeled using the Matlab/Simulink package of mathematical programs. When modeling the expression (2), we will use some of the workpieces we obtained earlier in the previous section, and consider the option of increasing the wear of the cutting wedge for the case of a low-oscillation processing process, when the secondary regeneration of oscillations is small and for the case of a high-oscillation cutting process, when the regenerative effect forms the vibrations of the tool along the entire path passed by it during cutting. To do this, as part of the model presented in the previous section, we will introduce an additional unit for calculating tool wear on the rear gras. The results of the simulation of the system taking into account this unit are shown in the figures below. Figure 1 shows the results of the simulation of the cutting force.

\[
h_3 = \frac{\beta_1}{\alpha_1} \left[ -N_0 e^{-\alpha_1 A} - (N_1 - N_0) e^{-\alpha_1 (A - A)} - (N_2 - N_1) e^{-\alpha_1 (A - 2A)} - \ldots - (N_{n-1} - N_{n-2}) e^{-\alpha_1 (A - (n-1)A)} + N_{n-1} \right] + \\
+ \frac{\beta_2}{\alpha_2} \left[ N_0 e^{\alpha_2 A} + (N_1 - N_0) e^{-\alpha_2 (A - A)} + (N_2 - N_1) e^{-\alpha_2 (A - 2A)} + \ldots + (N_{n-1} - N_{n-2}) e^{-\alpha_2 (A - (n-1)A)} - N_{n-1} \right]
\]

(2)

The power response from the processing process is quite large, this increase was made by us specifically in order to accelerate the process of wear growth, the results of which are shown in Figure 1.

As can be seen from Figure 2, the wear curve that we obtained for the specified case of simulating the cutting control system is similar to the one obtained experimentally, however, the area of stabilizing the wear characteristic is more pronounced here, as well as not less than the time of increasing critical wear. For subsequent modeling, it is of interest to consider separately the process of tapping described by the first part of the discretely equation (5) and the increase in the process of degradation of the cutting wedge of the in-tool, the second part of the discrete sum presented in the expression (5). Figure 2 shows the simulation of the curve reflecting the tool run-in.

![Wear Curve Plot](image)

**Figure 1. Wear Curve Plot**

As can be seen from Figure 2, the wear curve that we obtained for the specified case of simulating the cutting control system is similar to the one obtained experimentally, however, the area of stabilizing the wear characteristic is more pronounced here, as well as not less than the time of increasing critical wear. For subsequent modeling, it is of interest to consider separately the process of tapping described by the first part of the discretely equation (5) and the increase in the process of degradation of the cutting wedge of the in-tool, the second part of the discrete sum presented in the expression (5). Figure 2 shows the simulation of the curve reflecting the tool run-in.
As can be seen from Figure 2 b), the degradation of the tool begins to have a significant effect on the wear curve only after 9 minute of processing. The curve reflecting the process of degradation of the cutting wedge is different from zero and up to this minute, but its growth here is not very large, nor is its effect on the entire wear curve up to 10 seconds of the cutting process great.

Only one question of interest is whether tool vibrations will be taken into account in the in-run and degradation curves of the cutting wedge when modeling wear using the Volterra operator. To answer this question, we simulate the Volterra operator with various types of vibration signals, which we will relatively evaluate using the vibration signal value module. This is due to the fact that direct integration of the complex cutting speed signal, which has a periodic part in its composition, will lead to the fact
that the integral of the periodic component over the observation period will be zero. To numerically estimate the instrument vibration signal, we used the following integral index:

$$V_A = \sqrt{\int_0^T \left( \frac{dx(t)}{dt} \right)^2 dt}$$

where $x(t)$ - the vibration signal itself, and $T_v$ - the signal observation period.

To simplify our reasoning, we assume that we are talking about a scalar case of cutting, in which the cutting force is constant, and the speed of relative movement of the tool along the workpiece is the sum of the cutting speed and deformation vibrations of the tool. This allows us to describe the power of irreversible transformations as the product of the cutting force ($F$) on the sum of the cutting speed ($V$) and the speed of the deformation movements of the tool ($dx/dt$), or in the form of an expression:

$$M(t) = F(V - \frac{dx(t)}{dt})$$

The operation of the cutting force in this case is conveniently modeled using the following expression:

$$A(t) = F(Vt + VA)$$

When modelling, we will use the initial data obtained by us earlier in experiments on a lathe, where the cutting speed $V = 2620$ mm/s, the cutting force $F = 45$ N., the amplitude of the tool vibration speed in the cutting direction was 50-200 mm/s, the oscillation frequency was about 2000 Hz. Consider two extreme cases, in the first we take two extreme cases, the case with an amplitude of oscillations of 50 mm/s and the case of oscillations of 200 mm/s. As can be seen from Figure 5, the energy growth of the vibration signal reflecting the dynamics of the deformation movement of the tool was 4 times with 50 mm/s in Figure 4 a), up to 200 mm/s in Figure 4 b). Results of simulation of in-run and degradation curves taking into account accepted two versions of tool vibrations in cutting direction are given in Figure 3.
As can be seen from Figure 3, changes in the curve reflecting the wear of the in-tool appear only in part of the degradation characteristic (see Figure 3 (b)) and almost do not appear in the run-in curve (see Figure 3 (a)). Note that in both characteristics, red, a characteristic associated with a more amplitude version of the instrument vibrations is shown.

The force that determines the reaction on the back face will remain unchanged, and the decomposition coefficients will be made dependent on the degree of degradation of the instrument. According to our research presented in the second section, the greatest impact of the degradation of the cutting wedge has on the components of the cutting forces $P_x, P_y$. We take as a hypothesis the following mathematical description of the dependence of decomposition coefficients:

$$
\chi_1 = \chi_1^0 + a_1 h_3^d \\
\chi_2 = \chi_2^0 + a_2 h_3^d \\
\chi_3 = \sqrt{\chi_1^2 + \chi_2^2}
$$

wherein $a_1$ and $a_2$ are coefficients linking the restructuring of the power response to the shaping movements of the tool when the cutting wedge is degraded. According to our experiments, the components of the cutting forces when the wear on the back face changes from zero to about 0.42 mm, increase along the x coordinate by 125%, along the y coordinate by 65%, and along the z coordinate by 36%. This makes it possible to make the assumption of pre-ascentancy $a_1$ over $a_2$ almost twice, we accept in the future that $a_1 = 1$ over $a_2 = 0.5$. Given this, we assume that the model of the dynamics of the deformation movement of the tool will take the following form:
To analyze the effect of the restructuring of the power reaction associated with the degradation of the cutting wedge on the static of the processing process, we assume that the forces formed on the back face of the tool are zero, in this case the static equations take the form:

$$m \frac{d^2 x}{dt^2} + h_{11} \frac{dx}{dt} + h_{12} \frac{dy}{dt} + h_{13} \frac{dz}{dt} + c_{11} x + c_{12} y + c_{13} z = \chi_1 F + \Phi_1$$

$$m \frac{d^2 y}{dt^2} + h_{21} \frac{dx}{dt} + h_{22} \frac{dy}{dt} + h_{23} \frac{dz}{dt} + c_{21} x + c_{22} y + c_{23} z = \chi_2 F + \Phi_2$$

$$m \frac{d^2 z}{dt^2} + h_{31} \frac{dx}{dt} + h_{32} \frac{dy}{dt} + h_{33} \frac{dz}{dt} + c_{31} x + c_{32} y + c_{33} z = \chi_3 F + \Phi_3$$

To analyze the effect of the restructuring of the power reaction associated with the degradation of the cutting wedge on the static of the processing process, we assume that the forces formed on the back face of the tool are zero, in this case the static equations take the form:

$$c_{11} x + (c_{12} + \chi_{11} \rho) y + c_{13} z = \chi_1 F$$

$$c_{21} x + (c_{22} + \chi_{22} \rho) y + c_{23} z = \chi_2 F$$

$$c_{31} x + (c_{32} + \chi_{32} \rho) y + c_{33} z = \chi_3 F$$

As can be seen from the expression (11), the stiffness matrix of the tool under cutting conditions becomes skew-symmetric, which suggests that the presence of circulating forces will be observed in the system. However, this effect will be present in the cutting system regardless of the wear of the cutting wedge, it is of interest that it will occur during the restructuring of the power reaction during the degradation of the cutting wedge. To analyze the effect of cutting wedge degradation, consider three variants $h_d^3$, the first $h_d^3 = 0$, the second $h_d^3 = 0.1$, the third $h_d^3 = 0.2$, values of matrix coefficients

$$c = \begin{bmatrix} 1390 & 190 & 165 \\ 190 & 795 & 150 \\ 165 & 150 & 970 \end{bmatrix} \text{ kg/mm}. \text{ Orientation factors: } \chi_1 = 0.3369, \chi_2 = 0.48, \chi_3 = 0.81 . \text{ Process modes: depth } t = 2 \text{ mm}, \text{ feed } S = 0.1 \text{ mm}, \text{ spindle } n = 1000 \text{ rpm, } \rho = 400 \text{ kg/mm}^2, \text{ radius of machined part } R = 50 \text{ mm}. \text{ That is, consider the case given in the previous section, when the total cutting force was approximately equal to 50 H. Based on these data, the values of the forces that prevent the shaping movements of the tool for the case } h_d^3=0 \text{ will be:}$$

$$F_{x_0} = 16$$

$$F_{y_0} = 24$$

$$F_{z_0} = 40$$

The stationary values of the strain coordinates for this case are shown below:

$$x_0 = 0.0047$$

$$y_0 = 0.022$$

$$z_0 = 0.0376$$

For the case of the non-zero value of the degradation of the cutting wedge, that is, when $h_d^3=0.1$, the values of the forces that prevent the forming movements of the tool are given below:

$$F_{x_0} = 21.85$$

$$F_{y_0} = 26.5$$

$$F_{z_0} = 36.34$$

$$h_{31} \frac{dx}{dt} + h_{32} \frac{dy}{dt} + h_{33} \frac{dz}{dt} + c_{31} x + c_{32} y + c_{33} z = \chi_3 F + \Phi_3$$
As can be seen from expression 11, at this level of degradation of the cutting wedge, the force component directed along the feed axis has increased most, and the cutting force itself has even decreased slightly. In the real case of cutting, the cutting force will not decrease, but even increase due to the effect of the growth of friction forces on the back face. The values of the orientation coefficients of the resistance forces for this case of wear were:

\[
\begin{align*}
\chi_1 &= 0.437 \\
\chi_2 &= 0.53 \\
\chi_3 &= 0.7267
\end{align*}
\] (12)

As seen in comparison with the previous version of the orientation forces, there is a decrease in the effect of the coefficient associated with the main motion direction. The equilibrium values of the coordinates of the state of the de-formation movement system of the tool are given below:

\[
\begin{align*}
x_0 &= 0.0085 \\
y_0 &= 0.0253 \\
z_0 &= 0.0321
\end{align*}
\] (13)

The next version of wear of the tool will be the value of the degradation component in \( h^d_3 = 0.2 \), the results of modeling the distribution of the cutting force along the deformation axes are given in the system (17).

\[
\begin{align*}
Fx_0 &= 26.85 \\
Fy_0 &= 29 \\
Fz_0 &= 30.63
\end{align*}
\] (14)

As can be seen from the expression (17), the trend we noted above is also executed here. The orientation coefficient values are as follows:

\[
\begin{align*}
\chi_1 &= 0.537 \\
\chi_2 &= 0.58 \\
\chi_3 &= 0.6185
\end{align*}
\] (15)

The corresponding equilibrium values of the deformation coordinates for this case are given in the system (19):

\[
\begin{align*}
x_0 &= 0.0126 \\
y_0 &= 0.0288 \\
z_0 &= 0.0250
\end{align*}
\] (16)

Further degradation of the cutting wedge leads to a critical restructuring of the cutting system in the space of deformation movements of the tool, and, for example, with \( h^d_3 = 0.39 \), the equilibrium values of the deformation coordinates will be:

\[
\begin{align*}
x_0 &= 0.0213 \\
y_0 &= 0.0379 \\
z_0 &= -0.003
\end{align*}
\] (17)

As can be seen from the expression (17), with such wear of the tool, the cutting wedge is literally drawn into the cutting zone, which is unacceptable from the point of view of processing. A similar
situation is also possible with improper sharpening of the cutting wedge. In this case, cutting becomes impossible and the tool must break.

To analyze the effect of cutting wedge degradation on the dynamics of the cutting process, consider this system in the version of equations (17), but under the condition of zero values of the forces formed on the back face of the tool (0).

The simulation results for case $h_d^d=0.1$ and $n = 880$ are shown in Figure 4.

As can be seen from Figure 5, in comparison with Figure 3, as a result of the resulting degradation of the cutting wedge, the cutting system lost stability, given the above stated conditions of the experiment, this loss of stability is associated precisely with the restructuring of the power reaction. Results of simulation of cutting system with degradation value $h_d^d = 0.2$ and $n = 880$ are given in Figure 5.

![Figure 4. Simulation results - tool strain coordinates at $h_d^d = 0.1$](image1)

![Figure 5. Simulation results - tool strain coordinates at $h_d^d = 0.2$](image2)

As can be seen from Figure 4, the system is also not stable, as in the previous case, but the process of fluctuation divergence is much faster than in the previous case.

Consider the dynamics of the vibrational motion of the cutting wedge for the case of stable system behavior, that is, at a processing speed of 1772 revolutions per minute. A variant of such treatment, in conditions of zero level of cutting wedge degradation is shown in Figure 5.
As shown in Figure 6 in comparison with Figure 5, there is no loss of stability in the cutting process. The vibration vibrations of the tool are resistant to this level of degradation of the cutting wedge, however, the vibrations in the feed direction are non-extinguishing, which distinguishes them quite strongly from the vibrations shown in Figure 7. The results of the modeling of the cutting system, for the case of the critical level of degradation of the cutting wedge with $h_3^d = 0.2$, are shown in Figure 8.
As shown in Figure 8, despite the fact that at a spindle speed of 1772 revolutions per minute, the regenerative effect is minimal, we observe a loss of stability by the cutting system, which is caused by the degradation of the cutting wedge.

The research allows us to form the following position: during the evolution of the tool during cutting, the formation of the contact area of the tool with the machined part, serves the purpose of self-organization of the cutting system through the formation of additional thermodynamic feedback, the stabilization of which is ensured by a certain combination of tool wear and limited vibration cutting mode. After this, the interconnectedness of the thermodynamic, power and vibration subsystems already ensures the stabilization of the wear intensity of the wired tool. In other words, during the cutting process, the cutting system tends to work the cutting wedge of the tool to reach a certain level of wear, which reduces the level of vibration activity of the tool and provides the function of thermodynamic feedback, which allows, through stabilization of the power reaction, to achieve the maximum possible decrease in the further wear intensity of the cutting wedge.

3. Discussion and conclusions

The validity of our position, in addition to the results of modeling the integral operator we obtained, is confirmed by the provision widely known from the works of A.D. Makarov on the existence of the optimal cutting mode from the point of view of ensuring the maximum resistance of the tool [28]. In his reasoning, A.D. Makarov relies on the principle of V. Reichel, which can be reduced to the statement that a certain period of resistance for a given tool-part pair corresponds to the same temperature in the contact zone of the tool and the workpiece. Based on this principle, A.D. Makarov introduces the concept...
of optimal cutting temperature, which provides optimal tool resistance. However, the temperature in the contact area of the tool and the workpiece cannot be stationary, since the processing process itself is not substantially stationary. Therefore, in order to ensure that the optimum temperature is constant over a long period of the cutting process, it is necessary to provide, if not a steady state of the entire processing process, at least some quasi-stationary of this process. It is this quasi-stationary that is formed during the tool run-in and the formation of the primary wear of the cutting wedge, the stabilization of this quasi-stationary state of the cutting process occurs due to the stabilization of the cutting force, which in turn is stabilized by the temperature, which depends on the vibration activity of the tool, depending on the cutting force.

4. Acknowledgments
The study was carried out with the financial support of RFFI grant No. 19-08-00022.

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Conflict of interest
The authors declare that there is no conflict of interest.