Braneworlds in six dimensions: new models with bulk scalars

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Abstract
Six-dimensional bulk spacetimes with 3- and 4-branes are constructed using certain non-conventional bulk scalars as sources. In particular, we investigate the consequences of having the phantom (negative kinetic energy) and the Brans–Dicke scalar in the bulk while obtaining such solutions. We find geometries with 4-branes with a compact on-brane dimension (hybrid compactification) which may be assumed to be small in order to realize a 3-brane world. On the other hand, we also construct, with similar sources, bulk spacetimes where a 3-brane is located at a conical singularity. Furthermore, we investigate the issue of localization of matter fields (scalar, fermion, graviton, vector) on these 3- and 4-branes and conclude with comments on our six-dimensional models.

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1. Introduction

The idea that our world could be visualized as a codimension 2-brane in six dimensions was first noticed by Akama [1] way back in 1982. Using the dynamics of the Nielson–Olesen vortex solution of the Abelian Higgs model in six dimensions, he was able to localize our spacetime within a 3-brane, with Einstein gravity being induced through the fluctuations of the brane. Subsequent to the work of Akama, Rubakov and Shaposhnikov [2] and Visser [3] pursued closely related ideas around the same time. These were, by and large, the early ideas on the notion of alternatives to the usual Kaluza–Klein compactification, which are being hotly pursued today following the work on large extra dimensions [4] and warped extra dimensions [5].
This paper is focused on braneworld models with codimension greater than 1. In particular, we shall be exclusively concerned with bulk spacetimes in six dimensions generically represented by the line element:

\[ dx^2 = \sigma(x^a) g_{\mu\nu} \, dx^\mu \, dx^\nu + \gamma_{ab}(x^a) \, dx^a \, dx^b, \]  

(1.1)

where \( \sigma(x^a) \) is a conformal factor depending on the extra coordinates and \( \gamma_{ab}(x^a) \) is line element representing the extra dimensional part (for a 6D bulk this is two dimensional). Certain specific solutions of the 6D Einstein equations with a positive bulk cosmological constant had been obtained in [2] where the extra dimensions are non-compact and assumed to be unobservable at low energies. Following the recent string inspired phenomenological brane world models proposed by Randall and Sundrum [5], a fair amount of activity has been generated involving possible extensions and generalizations, among which, codimension 2 models in six dimensions have been a topic of increasing interest.

Let us now briefly review the work done on codimension 2 models in six dimensions. Shortly after the work of Randall–Sundrum involving a warped geometry, several proposals came up which made use of two extra dimensions [6]. Most of these articles followed the original viewpoint (Akama, Rubakov and Shaposhnikov) with the four-dimensional world being a cosmic-string like topological defect [7] as opposed to the domain wall type defect in codimension 1 models. Chodos and Poppitz [6] first brought in the idea of warped braneworlds of codimension 2 where the presence of a brane appeared through the existence of a location with a conical deficit in the bulk. In particular, for some cases, the resolution of the hierarchy problem was also achieved and unlike Randall–Sundrum, there did not exist any fine-tuning between the brane tension and the bulk cosmological constant. Gravity was also found to be localizable on the defect (e.g. in [7]). A useful review on topological defects in higher dimensional models and its relation to braneworlds is available in [8].

Furthermore, Leblond et al obtained a set of consistency conditions for braneworld scenarios with a spatially periodic internal space in [9], from which one can see that the necessity of a negative tension brane appears in five dimensions and is absent in the higher dimensional constructions. It is also apparent from the multigravity scenario discussed in [10] that the radion stabilization problem and the presence of a negative tension brane is an artefact of five-dimensional spacetime. The non-trivial curvature of the internal space in the case of two or more extra dimensions provides the necessary bounce configuration of the warp factor without the need of any negative tension brane [11]. Generically, in six dimensions, two types of constructions are around. One of them involves 4-branes which localize gravity but one of their dimensions is compact, unwarped and of Planck length [10]. The other type of construction has conical singularities which support 3-branes (these can be of positive, negative and zero tension depending on the angle deficit, angle excess or no angle deficit respectively) [7].

From the above introduction, it is evident that in the context of factorizable (un-warped) as well as warped bulk spacetimes a fair amount of work has been carried out in the recent past. Apart from model construction, the question of solving the cosmological constant problem has been the primary issue addressed in several articles [12]. Other aspects such as cosmology, brane gravity etc have been discussed by numerous authors [13]. A list of some recent articles on codimension 2 models is provided in [14].

It is well known by now that in the braneworld scenario it is necessary to introduce dynamics which can determine the location of the branes in the bulk. Ever since Goldberger and Wise [15] added a bulk scalar field to fix the location of the branes in five dimensions, investigations with bulk fields became an active area of research. The consequences of different types of bulk scalars in the bulk spacetime geometry and their phenomenological
implications have been looked into in great detail over the last few years. For example, in the RS-II [5] set-up, it has been noted that spin 1/2 fields cannot be localized on the brane by the gravitational interaction only [16, 17]. Thus, it becomes necessary to introduce additional non-gravitational interactions (e.g. a fermion–scalar Yukawa coupling, say) to get spinor fields confined to the brane. A simple choice of such a non-gravitational field in the bulk is a scalar field coupled to gravity [17].

Motivated by the above-mentioned need of bulk scalars, we carry out our search for novel bulk spacetimes in six dimensions with such bulk scalars of various types as the sources in the six-dimensional Einstein field equations. The advantage of studying various types of models is related to the fact that it helps in revealing a wider spectrum of possibilities. In recent times, some interesting solutions of new brane models in 6D have been obtained. For example, a general regular warped solution with 4D Minkowski spacetime in six-dimensional gauged supergravity is obtained in [18]. A simple exact solution of six-dimensional braneworld which captures some essential features of warped flux compactification, including a warped geometry, compactification, a magnetic flux, and one or two 3-brane(s) is found in [18]. Higher dimensional fermions in a non-singular 6D brane background with an increasing warp factor has also been studied [18]. It has also been claimed in [19] that all the zero modes of the standard model fields can be localized on a single brane by means of only the gravitational interaction.

In our first example here, we investigate the effect of having a phantom field in the six-dimensional bulk spacetime. Recently, in cosmology, the phantom scalar has been widely used [20] to explain dark energy and the accelerated expansion of the universe. The phantom is a hypothetical scalar field with a wrong-sign (negative) kinetic energy term in its Lagrangian. Even though questions of stability (unbounded negative energy) arise in such models, phenomenologically (e.g. in cosmology), they have been useful in explaining various scenarios. In order to justify the existence of the phantom, a model of phantom energy has been constructed in [21], using the graded super Lie algebra $SU(2/1)$, where the negative kinetic energy term seems to arise naturally. Furthermore, in our earlier work, we have seen that the presence of such a scalar field in the 5D bulk plays a crucial role in localizing massless as well as massive fermions on the brane [22]. Our investigation here is based on the exact solutions of the full 6D Einstein–phantom scalar equations. In the exact background geometry obtained from this setup it is possible to have the zero modes of all the standard model fields and gravity to be localized on 4-branes.

In our next example, we look for the consequence of introducing a Brans–Dicke scalar in the bulk (i.e. consider 6D Brans–Dicke gravity). Recently, the role of a Brans–Dicke scalar in five dimensions and the corresponding bulk solutions have been investigated in [23]. We obtain here, the bulk solution and the conditions for confinement of gravity as well as other matter fields on the brane.

In the above two models, the extra dimensional space is of finite volume with a negative Ricci scalar (hyperbolic two-dimensional geometries). Additionally, the models with a phantom or a Brans–Dicke bulk scalar both involve four brane constructions with an assumed, on-brane, compact, extra dimension (hybrid compactification).

Finally, we concentrate on models where a brane embedded in a six-dimensional bulk is realized via a conical deficit in the bulk spacetime (similar to the topological defect type braneworld models). We also investigate the issue of localization of fields for this class of models and comment on conditions under which localization of all fields is possible for a sufficiently broad class of warp and extra dimensional factors.

The organization of our paper is as follows. In section 2, we have obtained the exact solutions of the Einstein–scalar equations for the bulk phantom and the Brans–Dicke scalar.
We start with codimension 2 branes and finally generalize the results in higher codimensions in some particular cases. Section 3 discusses localization of gravity and other matter fields on the brane through the existence of normalizable zero modes on the brane. Section 4 deals with models with a conical deficit at the brane location and the issue of localization in such examples. In the last section, we conclude with discussions and open issues.

2. The ansatz, equations of motion and solutions for non-singular 4-branes

2.1. The models and the bulk solutions

Let us begin with the most general metric ansatz for a warped brane embedded in six dimensions obeying four-dimensional Poincaré invariance:

\[ ds^2 = g_{MN} dx^M dx^N = e^{2f(r)} \eta_{\mu\nu} dx^\mu dx^\nu + dr^2 + e^{2g(r)} L^2 d\theta^2, \]

where the radial coordinate \( r \) is infinitely extended \((0 < r < \infty)\) and the compact coordinate \( \theta \) ranges from \( 0 \leq \theta \leq 2\pi \). \( L \) is additional parameter characterizing the extra compact direction on the 4-brane. We also assume that the warp factors are functions of the extra dimensional radial coordinate, \( r \), only.

We will now focus on the time-independent solutions of the Einstein equations for two types of bulk scalar field sources: (i) a phantom scalar and (ii) a Brans–Dicke scalar. In this context an obvious question occurs—why do we choose such fields in the bulk? The answer lies in our ignorance about what could be there in the bulk. With vacuum, we do not find appropriate solutions and with a negative \( \Lambda \) we have the Randall–Sundrum type models. In search of further models with distinct characteristics we consider as a first simple choice: scalar fields in the bulk. The usual scalar field (with a potential) does not seem to provide a useful (and non-trivial) solution; hence we turn to non-standard scalars such as those mentioned above. Honestly speaking, there is no rationale about our choice of matter in the bulk. However, if consequences which result on the brane are physically relevant then we might call our choice as reasonable, with the model being capable of representing our usual four-dimensional world as a surface in the bulk.

2.1.1. Model-I: Phantom scalar field in the bulk. The six-dimensional action for a bulk scalar field (phantom: with a ‘wrong’ sign kinetic term) in a potential \( V(\phi) \), minimally coupled to gravity in the presence of a cosmological constant is given by

\[ S = \int \sqrt{-(6)g} \left[ (R - 2\Lambda) \frac{M^4}{2} + \frac{1}{2} g^{AB} \nabla_A \phi \nabla_B \phi - V(\phi) \right] d^6x, \]

where \( M \) corresponds to the six-dimensional fundamental mass scale. We assume henceforth that the scalar field is a function of the coordinate \( r \) only. Variation of the action (2.2) with respect to the metric and the scalar field leads to the following field equations for the Einstein-scalar system:

\[ 6 f'^2 + 3 f''' + 3 f' g' + g'' + g'^2 = \alpha(\phi'^2/2 - V) - \Lambda \]

\[ = 10 f'^2 + 4 f'' \]

\[ 6 f'^2 + 4 f' g' = \alpha(-\phi'^2/2 - V) - \Lambda \]

\[ (\phi'' + 4 f' \phi' + g' \phi') = -\frac{\partial V}{\partial \phi}, \]
where \( \alpha = \frac{1}{M^4} \) and the prime denotes a derivative with respect to \( r \). Note that the scalar field equation is not independent, it can be obtained from the other three equations. To obtain an exact analytical solution for the warp factors and the scalar field we first work with \( V(\phi) = 0 \) and also tune the bulk cosmological constant to be zero. Further, assuming the radius of the compact dimension to be of the order of Planck scale, we obtain the following solutions for the warp factors and the bulk scalar field as

\[
e^{2f(r)} = e^{\frac{kr}{2}} \quad \text{and} \quad e^{2g(r)} = e^{-2kr}
\]

(2.7)

\[
\phi(r) = \left( \frac{5k^2}{4\alpha} \right)^{\frac{1}{2}} r,
\]

(2.8)

where \( k \) is an arbitrary constant. Note the distinct nature of the warp factors—the brane part is a growing function of \( r \) and the other part is a decaying function. The geometry of the spacetime has a Ricci curvature which can be obtained from the formula

\[
(0)R = (-20f'' - 8f') + (-2g'' - 2g') - 8f'g'.
\]

(2.9)

With the warp factors for the case of the phantom scalar field one obtains \( (0)R = -\frac{5}{4}k^2 \). The two-dimensional extra dimensional space has \( (3)R = -2k^2 \). The volume of the two-dimensional piece is also finite and is given by \( \frac{2\pi L}{k} \). Thus, both the full space and the extra dimensional space (considered separately) are both of negative Ricci curvature (AdS) with the latter having a finite volume.

The results are different in nature from those obtained by considering only a cosmological constant in the bulk [10] and for a smooth local defect in the bulk [7]. The energy–momentum tensor for the bulk field which gives rise to the above solution has components given by

\[
\rho = -p_{x,y,z} = -\frac{5}{8}k^2
\]

(2.10)

\[
p_r = -\frac{5}{8}k^2 \quad \text{and} \quad p_\theta = \frac{5}{8}k^2.
\]

(2.11)

It is clear from the above expressions that the matter stress energy which gives rise to the background geometry (2.7) violates all the energy conditions (namely WEC, SEC, NEC) [24]. However, the energy density and pressures are constant and therefore bounded (and, obviously less problematic than an infinitely unbounded negative energy density). Thus, our solution, as far as the bulk source is concerned, is not very different from the usual Randall–Sundrum solution in five dimensions, where the bulk has only a negative cosmological constant. In the phantom case, however, we have \( p_\theta = -\rho \), \( p_r = \rho \) (unlike the cosmological constant which would have required \( p_i = -\rho \) for all \( i \)).

We can generalize the results for situations where the total spacetime dimensions are more than six. The extra dimensional space is constructed with one non-compact dimension and \( (p-1) \) compact dimensions where \( p \) is the number of extra dimensions. The warp factors and the scalar field are assumed to be a function of only the radial dimension \( r \). The exact analytical solution of the Einstein–scalar equations in \( (4 + p) \) dimensions follows turns out to be:

\[
ds^2 = e^{(p-1)kr} \eta_{\mu\nu} \, dx^\mu \, dx^\nu + \, dr^2 + e^{-2kr} \eta_{mn} \, dy^m \, dy^n,
\]

(2.12)

where \( m, n \) runs from 1 to \( p \). The nature of the warp factors remain same as those obtained in six dimensions. In this case also the scalar field is a monotonically increasing function of \( r \) as given in equation (2.8).
2.1.2. Model-II: Brans–Dicke scalar field in the bulk. We now introduce another example of a bulk scalar coupling with gravity (namely the Brans–Dicke coupling) in a six-dimensional spacetime. The scalar–tensor Brans–Dicke theory [25] is known to be an alternative theory to Einstein’s general relativity. It is similar to GR, except the reciprocal of the gravitational constant is itself a one-component field, the scalar field \( \phi \), which is generated by matter through an additional equation (the scalar field equation). Thus \( \phi \) as well as usual matter both play their roles in determining the metric via a modified version of Einstein’s equations. In fact, Brans–Dicke theory is distinguishable from general relativity only by the value of its single dimensionless parameter \( \omega \) which determines the effectiveness of matter in producing \( \phi \). The larger \( \omega \), the closer the Brans–Dicke theory predictions are to those for general relativity. Initially, a popular alternative to general relativity, the Brans–Dicke theory lost favour as it became clear that \( \omega \) must be very large—an artificial requirement. Nevertheless, the theory has remained a paradigm for the introduction of scalar fields into gravitational theory, and as such has enjoyed a revival in connection with low energy effective theories derivable from quantum string theory. The so-called dilaton gravity can be identified as a \( \omega = -1 \) Brans–Dicke theory.

In our work here we examine the consequences of having a Brans–Dicke scalar in the bulk. Apart from obtaining the line elements satisfying the equations of motion we also study the localization of fields on the brane in these models.

For Brans–Dicke theory the gravitational and scalar field equations are given as

\[
G_{AB} = \frac{8\pi}{\phi} T_{MAB} + \frac{\omega}{\phi^2} \left( \nabla_A \phi \nabla_B \phi - \frac{1}{2} g_{AB} \nabla^C \phi \nabla_C \phi \right) + \frac{1}{\phi} \left( \nabla_A \nabla_B \phi - g_{AB} \Box^2 \phi \right) \tag{2.13}
\]

\[
\Box^2 \phi = \frac{8\pi}{3 + 2\omega} T_{M^A}. \tag{2.14}
\]

We choose, the energy–momentum tensor for matter to be zero i.e. \( T^A_{\mu} = 0 \). With this choice we shall now find exact analytical solutions for the background geometry first in six dimensions then in arbitrary dimensions.

In the background geometry defined by the line element in equation (2.1), the above equations reduce to

\[
6 f'^2 + 3 f'' + 3 f' g' + g'' + g'^2 = \left[ -\frac{\omega \phi'^2}{2 \phi^2} - \frac{1}{\phi} (\phi'' + 3 f' \phi' + g' \phi') \right] \tag{2.15}
\]

\[
6 f'^2 + 4 f' g' = \left[ \frac{\omega \phi'^2}{2 \phi^2} - \frac{1}{\phi} (4 f' \phi' + g' \phi') \right] \tag{2.16}
\]

\[
10 f'^2 + 4 f'' = - \left[ \frac{\omega \phi'^2}{2 \phi^2} + \frac{1}{\phi} (\phi'' + 4 f' \phi') \right] \tag{2.17}
\]

\[
\phi'' + 4 f' \phi' + g' \phi' = 0. \tag{2.18}
\]

As before, the scalar field is assumed to be a function of fifth coordinate \( r \) only and the prime denotes the derivative with respect to this coordinate. An exact solution of the above set of equations is given as

\[
f(r) = k_1 r \quad \text{and} \quad g(r) = k_2 r \tag{2.19}
\]

\[
\phi(r) = e^{k_3 r}. \tag{2.20}
\]
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Figure 1. The ratio $k_1/k_2$ as a function of $|\omega|$. The upper (lower) graphs are the ones for the $-$(+) sign before the square root.

where the constants are constrained by the following relations:

$$4k_1 + k_2 = -k_3$$
$$\frac{\omega}{2} = -k_2 k_3 - 10k_1^2 = 6k_1^2 + 4k_1 k_2 - k_3^2 = -6k_1^2 - 3k_1 k_2 - k_3^2 + k_1 k_3. \quad (2.21)$$

Using the first constraint in the second one arrives at a equation involving $k_1$ and $k_2$ which can be solved to give

$$\frac{k_1}{k_2} = \left[ \frac{1 - |\omega|}{5 - 4|\omega|} \pm \frac{\sqrt{-6 + 5|\omega|}}{2(5 - 4|\omega|)} \right], \quad (2.22)$$

where $\omega = -|\omega|$. Also $|\omega| > \frac{5}{4}$ and $\omega$ is necessarily negative. One can further show that

$$\omega = -\frac{4k_1^2 + k_2^2 + k_3^2}{k_3^2}. \quad (2.23)$$

Using the above constraints one may construct a typical example. If $k_1 = k(>0)$ and $k_2 = -k$ then $k_3 = -3k$ and $\omega = -\frac{14}{9}$. Thus the brane has growing warp factor, the scalar field decays for larger values of the extra dimension and the extra dimensional geometry (two dimensional) has a decaying exponential (anti-de Sitter in two dimensions). On the other hand, we may also have $k_1 = -k$ and $k_2 = k(>0)$ for which $k_3 = 3k$ and $\omega = -\frac{14}{9}$. This gives a decaying brane warp factor, a growing scalar field and a de Sitter extra dimensional space. Many other possibilities exist with varying values of $k_1$, $k_2$, $k_3$ and $\omega$. A plot of $k_1/k_2$ as a function of $|\omega|$ (figure 1) illustrates the allowed range of solutions.

For a generalization of the solutions to codimension $p$ branes we derive the scalar–gravity equations in $(4+p)$ dimensions. An exact static solution of the spacetime geometry is obtained for relations

$$4k_1 + (p - 1)k_2 + k_3 = 0 \quad (2.24)$$
$$\omega = -\frac{4k_1^2 + (p - 1)k_2^2 + k_3^2}{k_3^2}. \quad (2.25)$$

The scalar field is the same exponential function of $r$. The constant $k_3$ is rescaled by the parameter $p$. The whole six-dimensional spacetime has, as before, a negative curvature scalar.
2.1.3. Placing the single branes. In all the above models one needs to place branes in order to have a braneworld. Following standard methods, we now take into account the effects of placing branes in the above bulk geometries. In particular, we evaluate the brane tension as a function of the parameters which appear in the expression for the bulk fields.

To include a single 4-brane here, we notice that we need to modify the bulk action with a 4-brane contribution (world volume action). Correspondingly, the Einstein field equations change and the effect is seen in the $G_{00}$ and $G_{\mu\nu}$ ($\mu = 1, 2, 3$) terms through the presence of a Dirac delta function term $\lambda \delta(r)$ (where $\lambda$ is the brane tension). To achieve a delta function in the RHS of the Einstein equation we modify the warp factors by extending the domain of $r$ to $-\infty < r < \infty$ and replacing $r$ by $|r|$. The $G_{00}$ Einstein equation finally yields

$$\lambda = 3k_1 + k_2 = -\frac{k}{4}. \quad (2.26)$$

2.2. Localization of gravity and other matter fields

An important issue related to the viability of a braneworld model is the question of localization of gravity and other matter fields on the brane [16, 17]. To address this point, we now consider localization of different types of matter fields in the context of the models discussed above.

In order to see whether the fields are confined or not we first employ the simplest test, originally outlined in [16]. For fields of different spins in six dimensions we assume, at the outset, that they are independent of the extra coordinates. The consistency check is then done by showing that the effective coupling constants emerging after dimensional reduction are non-vanishing and finite. In a sense, this approach assumes localization as a starting point. On the other hand, one may consider the fields to be dependent on the extra dimensional coordinates and then solve the relevant equations to see whether the behaviour of the fields conforms with localization. To have localized modes one requires the extra dimensional part of the field to peak around the brane and the full solution to be normalizable and finite everywhere.

2.2.1. Gravitational field (spin 2). We analyse the spectrum of linearized tensor fluctuations to see whether gravity is localized on the brane so that the model remains consistent with the results of the usual Newtonian and 4D GR experiments. For a fluctuation of the full 6D metric $g_{MN} \rightarrow g_{MN} + h_{MN}$ one has a variety of polarizations, or graviton modes [11]. In six dimensions there are three kinds of modes: (i) transverse traceless modes which are polarized along the Lorentz invariant hypersurface represented by $x^\mu$, (ii) vector modes polarized along the circle and in the flat piece of the brane, (iii) scalar polarization which are not traceless and related to radion field. We focus only on the transverse, traceless (TT) graviton which is represented by the metric perturbation around the classical solution of the 4D metric on the brane:

$$ds^2 = e^{2f(r)}(\eta_{\mu\nu} + H_{\mu\nu})\, dx^\mu \, dx^\nu + dr^2 + e^{2g(r)} L^2 \, d\theta^2. \quad (2.27)$$

The linearized gravitational fluctuation equation for the TT modes can be written as

$$\frac{1}{\sqrt{-g}} \partial_M (\sqrt{-g} g^{MN} \partial_N h_{\mu\nu}) = 0, \quad (2.28)$$

where $M, N$ denote the bulk spacetime indices. For the above equation we can obtain solutions in the following form $H_{\mu\nu}(x^\mu, r, \theta) = h_{\mu\nu}(x^\mu) \sum \phi_m(r) e^{i \ell \theta}$ where $h_{\mu\nu}$ satisfy the four-dimensional field equation $\Box h_{\mu\nu} = m_0^2 h_{\mu\nu}$. $m_0$ represents the mass of the corresponding
modes. With these ansätze and in the background of the metric (2.7) equation (2.28) reduces to the following form:

$$\varphi''_m + \left( \frac{m_0^2}{e^{2r}} - \frac{l^2}{L^2 e^{-2kr}} \right) \varphi_m = 0, \quad (2.29)$$

where the modes $\varphi_m$ satisfy the orthonormality condition

$$2\pi \int_0^{\infty} dr L e^{2fr} \varphi_m \varphi_n = \delta_{mn}. \quad (2.30)$$

There exists a zero mass ($m_0 = 0$) and s-wave ($l = 0$) solution of the above equation given by $\varphi_0 = \text{constant}$. So the normalized zero mode wavefunction can be written as

$$\psi_0 = \sqrt{\frac{k}{2L}} e^{-\frac{k}{2}r}, \quad (2.31)$$

which shows that the zero mode is localized near the origin $r = 0$. The modes for $m \neq 0$ and $l \neq 0$ may be obtained by solving the equation for $\varphi_m$ mentioned above. In $(4+p)$ dimensions the modes will be localized for $p < 3$.

In the case of the background metric with a bulk Brans–Dicke scalar, note that in the Brans–Dicke gauge, the equations for the gravitational and Brans–Dicke scalar field fluctuations are

$$\Box H_{\mu\nu} = 0 \quad \text{and} \quad \Box \xi = 0 \quad (\xi \text{ being the BD scalar fluctuation}) \quad (26).$$

Thus the BD scalar zero modes and the graviton zero modes will be localized if the condition $(2k_1 + k_2) < 0$ is obeyed. The fluctuation equations and hence the condition for localization, will however change provided we have other matter fields in the bulk.

2.2.2. Scalar field (spin 0). We now turn towards discussing the localization of a spin 0, scalar field in either of the fixed bulk background line elements discussed in the previous subsection. The action for a massless scalar field coupled to gravity in $D$ dimensions is given by

$$S_0 = -\frac{1}{2} \int d^D x \sqrt{-g} g^{AB} \partial_A \Phi \partial_B \Phi, \quad (2.32)$$

where $A, B$ denote bulk spacetime indices. The localization condition is equivalent to the normalizability of the ground state wavefunction around the brane [17]. The criterion will not be different from the case of the graviton discussed in the previous section. In a nutshell, one needs the integrals over the extra coordinates in the action to be finite so that the four-dimensional part reduces to the usual 4D Klein–Gordon equation. In the background geometry given by the metric in (2.1) the above action can be recast in the following form:

$$S_0 = -\pi L \int_0^{\infty} e^{2f(r) + g(r)} dr \int n_{\mu\nu} \partial_{\mu}\phi \partial_{\nu}\phi d^4 x. \quad (2.33)$$

For the line element with the bulk phantom field, as given in equation (2.7) we see that the integral over $r$ is finite. So the scalar field zero modes will be localized on the brane. For $p$ extra dimensions one is restricted to $p < 3$ for the localization of the zero mode. In the case of bulk Brans–Dicke scalar the normalization condition will be satisfied only if $(2k_1 + k_2) < 0$. The same condition holds for six $(4+p)$ dimensions as well.

2.2.3. Vector field (spin 1). It is known that in five dimensions the spin 1 vector (Maxwell) fields are not localized on the brane with increasing/decreasing warp factors. This is an inherent problem with the five-dimensional models. As we show below, in six dimensions, the spin 1 fields can be localized on the brane. The action for a $U(1)$ vector field reduces to the following form (with the choice of a vector potential with no functional dependence on the
extra dimensional coordinates):

\[ S_1 = -\frac{1}{4} \int d^Dx \sqrt{-g} g^{AB} g^{MN} F_{AM} F_{BN} \]

\[ = -\frac{\pi}{2} L \int dr e^{g(r)} \int d^4x \sqrt{-\eta} \eta^{\mu\nu} \eta^{\rho\sigma} F_{\mu\rho} F_{\nu\sigma}. \tag{2.33} \]

If the integration over the extra coordinates is finite then the above action will reduce to the standard Maxwell action in four dimensions. For the bulk phantom model \( g(r) \) is a decaying function in six dimensions as well as in \((4 + p)\)—as a result the integration over \( r \) is finite and we can achieve localized \( U(1) \) gauge field around the brane. In case of a bulk Brans–Dicke scalar the condition for the confinement of vector field on the brane is \( k_2 < 0 \).

2.2.4. Spinor field (spin \( \frac{1}{2} \)). For spinor fields we need to look at the Dirac action in a \( D \)-dimensional warped spacetime. The Dirac equation is given by

\[ \Gamma^A V^M_A (\partial_M - \Omega_M) \Psi(x^A) = 0, \tag{2.34} \]

where, \( V^M_A \) is the extension of the usual vierbein (tetrad) to six dimensions, \( \Omega_M = \frac{1}{4} \Omega_M^{AB} \Sigma^{AB} \) the spin connection and \( \Sigma^{AB} = \frac{1}{2} [\Gamma^A, \Gamma^B] \), \( \Gamma^A \) are the curved space gamma matrices. We look for the solutions of the form \( \Psi(x^A) = \psi(x^\mu) U(r) \) where \( \Gamma^\mu D_\mu = 0 \) and \( \mu \) stands for brane coordinate index. The assumption that the wavefunction does not depend on \( \theta \) leads to the solution \( U(r) = u_0 e^{-(2f+g/2)/2} \). Substituting this result into the action of the spinor fields in curved spacetime we obtain

\[ S_{\text{Dirac}} = u_0^2 \int e^{-f(r)} dr \int i\sqrt{-\eta} \psi D_\mu \psi d^4x. \tag{2.35} \]

The condition of trapping of spin \( \frac{1}{2} \) fields on the brane now becomes equivalent to having the integral over the extra coordinates as finite. In model-I \( f(r) \) is a growing function, so the integral is finite and non-vanishing, which, in turn, guarantees the localization of spin \( 1/2 \) fermions on the brane. In codimension \( p \) branes \( f(r) \) will remain an increasing function of \( r \) only for \( p > 1 \). To have localized fermions in model-II one needs the condition \( k_1 > 0 \) to be satisfied for both the codimension 2 and codimension \( p \) branes. This is easily satisfied by our models in both the cases.

In summary, assuming that the standard model fields (e.g. scalar, vector or fermion) are independent of the extra dimensional coordinates, we find that the criteria for existence of localized zero modes are related to the finiteness of the following integrals:

\[ \int_0^\infty e^{2f+g} dr, \quad (\text{scalar}); \quad \int_0^\infty e^g dr, \quad (\text{vector}); \quad \int_0^\infty e^{-f} dr, \quad (\text{fermion}). \tag{2.36} \]

In addition, the localization criterion for the zero mode graviton is identical to that for the massless scalar.

3. Brane models with conical singularity

Till now, the models under consideration have been essentially 4-branes with a compact on-brane extra dimension. These models, therefore represent a hybrid between the usual Kaluza–Klein idea and the braneworld perspective where compactification is replaced by the notion of localization. In this section, we consider the possibility of having 3-branes as conical defects in the six-dimensional spacetime. The metric for a general six-dimensional spacetime containing
a warped codimension 2 brane and obeying the four-dimensional Poincaré invariance is given by
\[ ds^2 = g_{MN} dx^M dx^N = e^{2f(r)} \eta_{\mu \nu} dx^\mu dx^\nu + dr^2 + e^{2g(r)} d\theta^2. \] (3.1)

The radial dimension is semi-infinite i.e. \( 0 \leq r \leq \infty \) and the coordinate \( \theta \) is cyclic \( [\theta : 0 \rightarrow 2\pi] \). For the conical singularity at the location of the we impose the boundary condition
\[ e^{2g(r)} |_{r=0} = 0. \] (3.2)

Note that if \( e^{2g(r)} \) has multiple zeros (at different values of \( r \)), it will be possible to place branes at those points in the extra dimensional space.

The Einstein–scalar equations for a bulk phantom field with potential \( V(\phi) \) and a nonzero bulk cosmological constant \( \Lambda \) lead to the following solutions for the warp factors and the bulk scalar field
\[ e^{2f(r)} = \text{sech}^2(kr) \quad e^{2g(r)} = \frac{\sinh^2(kr)}{\cosh^2(kr)} \] (3.3)
\[ \phi(r) = \sqrt{\frac{4}{5\alpha}} \ln \cosh(kr) \quad V(\phi) = -\frac{\Lambda}{\alpha} \tanh^2(kr) = \frac{\Lambda}{\alpha} (e^{-\sqrt{5} \phi} - 1), \] (3.4)

where \( \alpha = \frac{1}{M^4} \) and \( k \) is an arbitrary constant. The bulk cosmological constant is related to \( k \) as \( \Lambda = \frac{4\alpha}{5} \). From the above relations, it is clear that the brane warp factor \( e^{2f} \) decays as we go away from the brane location (at \( r = 0 \)). The extra dimensional part of the line element has the factor \( e^{2g} \) which, as required, has a zero at \( r = 0 \) and therefore results in a conical deficit. The potential for the bulk scalar field is negative definite for a positive bulk cosmological constant. Another solution (with a growing warp factor) in vacuum has been obtained in [27].

A natural question, following our previous discussion of 4-branes, is—what happens if we have a Brans–Dicke scalar in the bulk? To this end, we assume the following forms for \( f(r) \), \( g(r) \) and \( \ln \phi \):
\[ f(r) = \alpha \ln \cosh kr + \beta \ln \sinh kr \] (3.5)
\[ g(r) = \gamma \ln \cosh kr + \eta \ln \sinh kr \] (3.6)
\[ \ln \phi = \mu \ln \cosh kr + \nu \ln \sinh kr. \] (3.7)

Substituting the above ansatz in the Brans–Dicke equations mentioned earlier and some manipulations, we end up with the following constraints:
\[ \alpha + \beta = \gamma + \eta = 0; \quad \mu = -(4\alpha + \gamma - 1); \quad \nu = -(4\beta + \eta - 1). \] (3.8)

An additional equation also exist which relates \( \alpha, \gamma \) and \( \omega \). These constraints show that the possible solutions for \( f(r) \) and \( g(r) \) are necessarily singular (by virtue of the fact that the Ricci scalar will have a divergence through the term proportional to \( \coth^2(kr) \)).

Following a similar argument as in the case of 4-branes discussed earlier, and using the result \( \delta(\vec{r}) = \frac{1}{2\pi^2} \delta(r) \) we find that for the model with only a cosmological constant we get the brane tension as \( \lambda = -5\pi k \) whereas for the model we have derived with a phantom bulk in a potential \( \lambda = -2\pi k \).

Let us now provide a somewhat general method of constructing the warp factors \( f \) and the extra dimensional factor \( g(r) \) through the specification of a single function: the determinant of the metric. We first note that, for a minimally coupled bulk scalar or a bulk phantom field
Table 1. \( R(r) = \cosh^2(\kappa r) \), \( S(r) = \sinh^2(\kappa r/2) \). The solutions for the warp factors are given for four different choices of the determinant of the bulk metric \( \bar{g} = \tanh(\kappa r) \), \( \sinh(\kappa r) \cosh^4(\kappa r) \), \( \text{sech}(\kappa r) \tanh(\kappa r) \) and \( e^{-\kappa r} \tanh(\kappa r) \), respectively.

| Warp factors | \( e^{2f(r)} \) | \( e^{2g(r)} \) |
|--------------|-----------------|-----------------|
| \( \text{sech}^2(\kappa r) \) | \( \sinh^2(\kappa r) \cosh^{-2}(\kappa r/2) \) | \( e^{\kappa \text{sech}(\kappa r)} \cosh^{-2}(\kappa r/2) \) |
| \( e^{-\kappa \text{sech}(\kappa r)} R(r) \cosh^2(\kappa r/2) \) | \( e^{\kappa \text{sech}(\kappa r)} \cosh^{-2}(\kappa r/2) \) | \( e^{\kappa \text{sech}(\kappa r)} \cosh^{-2}(\kappa r/2) \) |
| \( e^{-\kappa \cosh(\kappa r)} \tanh^2(\kappa r/2) \cosh^{-2}(\kappa r) \) | \( e^{-\kappa \cosh(\kappa r)} \tanh^2(\kappa r/2) \cosh^{-2}(\kappa r) \) | \( e^{-\kappa \cosh(\kappa r)} \tanh^2(\kappa r/2) \cosh^{-2}(\kappa r) \) |

dependent only on the transverse radial coordinate \( r \), the Einstein tensor components \( G_{00} \) and \( G_{66} \) are related by the equation, \( G_{00} = -G_{66} \). This, in turn, leads to the opportunity to explore the spectrum of possibilities of having various solutions of the warp factors for different choices of the scalar potential. The general solutions for \( f(r) \) and \( g(r) \) can be shown to functionally depend on the determinant \( (\bar{g}) \) of the bulk metric in the following way:

\[
\begin{align*}
f(r) &= \frac{1}{5} \ln \sqrt{-\bar{g}} - \frac{C}{5} \int \frac{dr'}{\sqrt{-\bar{g}}} + C_1, \\
g(r) &= \frac{1}{5} \ln \sqrt{-\bar{g}} + 4 \frac{C}{5} \int \frac{dr'}{\sqrt{-\bar{g}}} + C_1,
\end{align*}
\]

where \( C \) and \( C_1 \) are integration constants to be fixed by the other equations. In the following table 1 we give examples of several toy models constructed by specifying a functional form for the determinant. Note that to have a brane at a conical singularity one must choose the determinant to have a zero at some specific point(s) in the extra dimensional space.

The full solution of the scalar field equation has been already discussed for the first example but it is not easy to obtain the exact solutions for the scalar field in the other three cases. It can be shown, however, from a graphical analysis of the variation of \( \phi' \) w.r.t. radial coordinate \( r \) that the scalar field is real everywhere. The potential can be represented as a function of \( r \) in all the three cases though its representation as a function of \( \phi \) depends on the analytical solvability of the scalar field equation. It must be admitted that the above models are all pretty complicated and contrived in nature. We do not intend to discuss these models further here.

3.1. Localization of gravity and matter fields on the brane with a conical singularity

Let us now discuss the localization scenario for the 3-brane models discussed above. The transverse traceless modes of the linearized gravity fluctuation polarized along Lorentz invariant hypersurface satisfies the equation given in (2.28). For an infinitely extended transverse radial dimension we do not find any normalizable zero mode for the models given in the table 1 because the normalization integral \( \int_0^\infty e^{2f(r)+g(r)} dr \) is not finite in any of the cases. As a remedy, normalized zero modes may be achieved by truncating the radial direction at a certain value of \( r \) and placing a 4-brane (with a compact on-brane extra dimension) at that point [10]. The picture then will be like the RS-I model and the gravity localization problem
will be similar to the corresponding Schrödinger problem. Along with the localized massless mode one will also find discrete Kaluza–Klein modes. As we have discussed earlier, the same conditions are also applicable for the confinement of a scalar field. In the background geometry described by the warp factors in the second and fourth rows of table 1 we find localized spin \(1/2\) zero modes on the 3-brane. The integration over the transverse radial coordinate in equation (2.35) is finite in these two cases. It turns out that the spin-1 vector fields also do not have any massless mode localized on the 3-brane.

One may ask, what could be the functional forms of \(f(r)\) and \(g(r)\) in order to have localization of all the fields? Let us go back to the ansatz we had made earlier:

\[
f(r) = \alpha \ln \cosh kr + \beta \ln \sinh kr
\]

\[
g(r) = \gamma \ln \cosh kr + \eta \ln \sinh kr.
\]

Obviously, we do not want the warp factor \((e^{2f})\) to go to either zero or infinity at the location of the brane. This implies we must have \(\beta = 0\). Also, to have a zero in \(e^{2g(r)}\) at the location of the brane, we need \(\eta > 0\). With these choices, we can now go back to the requirement that the three integrals \(\int e^{2f} dr\), \(\int e^{-f} dr\) and \(\int e^{g} dr\) should give finite answers. The divergence of these integrals will stem from large values of \(r\) (i.e. \(r \to \infty\)). Taking the asymptotic forms of \(\cosh kr\) and \(\sinh kr\) as \(r \to \infty\), which, essentially implies including only the exponentially growing pieces in each of these, we find that the finiteness of the integrals imply that the following holds:

\[
2\alpha + \gamma + \eta < 0; \quad \alpha > 0; \quad \gamma + \eta < 0.
\]

(3.13)

Using \(\alpha > 0\), we find we must have \(\gamma < -2\alpha - \eta\). It can be shown easily that the two solutions we have quoted earlier do not satisfy these constraints. If we can have a solution of the six-dimensional field equations with the coefficients \(\alpha\), \(\gamma\) and \(\eta\) satisfying the above-mentioned constraints we will have a warped codimension 2 braneworld located at a conical singularity in the 6D bulk, where all the above three fields will be localized.

### 3.2. Localizing all fields: geometries and energy conditions

From the above discussion, we can write down, in an *ad hoc* manner, warped geometries with a conical singularity for which the zero modes of all fields will be localized. In this brief section, we provide some examples and then, investigate, the nature of matter through an analysis of some of the energy conditions [24]. We do not attempt here to arrive at solutions to the Einstein equations in the presence of specified matter sources (such as a scalar field etc). Our aim is to see whether we can have line elements which are non-singular and with properties enabling it to represent a warped braneworld in six dimensions where the brane is located at a conical singularity in the bulk.

Recall the forms of \(f, g\) mentioned in the previous section. We assume \(\eta = 1\) – this avoids the presence of a singularity at \(r = 0\). We also keep in mind the various constraints on \(\alpha\) and \(\gamma\).

The various components of the energy momentum tensor turn out to be:

\[
\rho = \kappa [(-6\alpha^2 + 3\alpha - 3\alpha\gamma + \gamma - \gamma^2) \tanh^2 kr + (-6\alpha - 3\gamma - 1)]
\]

(3.14)

\[
p_{r,\gamma,\zeta} = \kappa [(6\alpha^2 - 3\alpha + 3\alpha\gamma - \gamma + \gamma^2) \tanh^2 kr + (6\alpha + 3\gamma + 1)]
\]

(3.15)

\[
p_r = \kappa [(6\alpha^2 + 4\alpha\gamma) \tanh^2 kr + 4\alpha]
\]

(3.16)

\[
p_\theta = \kappa [(10\alpha^2 - 4\alpha) \tanh^2 kr + 4\alpha].
\]

(3.17)
One can consider looking at the inequalities that need to hold if the null energy condition \((\rho + p_i \geq 0)\) or the weak energy condition \((\rho \geq 0, \rho + p_i \geq 0)\) have to be satisfied. It is easy to note that \(\rho + p_{x,y,z} = 0\) but the other inequalities need to be checked. It turns out that the requirement \(2\alpha + \gamma + 1 \leq 0\) is not compatible with what one needs in order to satisfy the energy conditions. In other words, violation seems to be a necessity in order to achieve localization.

As an example, we choose \(\alpha = \frac{1}{2}, \gamma = -3\) and write down the various components for this choice.

\[
\rho = -\kappa \left(\frac{15}{4} \tanh^2 kr - 5\right) = -p_{x,y,z} \quad \text{(3.18)}
\]

\[
p_r = \kappa \left(-\frac{9}{2} \tanh^2 kr + 2\right), \quad p_\theta = \kappa \left(\frac{1}{2} \tanh^2 kr + 2\right) \quad \text{(3.19)}
\]

where \(k^2\) has been absorbed in the definition of \(\kappa\).

Note, even though it is negative the energy density is bounded. So, are the pressures. The line element is

\[
ds^2 = \cosh kr (\eta_{\mu\nu} dx^\mu dx^\nu) + dr^2 + \frac{\sinh^2 kr}{\cosh kr} d\theta^2. \quad \text{(3.20)}
\]

The above is an example of a warped six-dimensional line element for which all fields will be localized on the brane and the brane will be located at a conical singularity in the bulk line element. This, of course is not the only possibility, there are infinitely many such line elements. It will be nice to find bulk matter sources which can generate them.

4. Summary and conclusions

Let us now summarize the results obtained in this paper and discuss the open issues.

Exact bulk solutions for some new, six-dimensional braneworld models are obtained for the bulk phantom scalar field and the Brans–Dicke scalar field respectively. We have shown that the phantom field and the BD scalar can provide the adequate source terms (in the Einstein equations) which enable the existence of various types of solutions. In particular, for the BD scalar we find that there are solutions which can have a decaying warp factor and a growing extra dimensional factor and vice versa. The above-mentioned class of models are essentially 4-brane models with an on-brane, compact (angular) extra dimension. Further, in these models, we address the issue of localization of gravity as well as other fields in this context. A unique feature of the models is the localization of massless spin fields ranging from 0 to 2 on a single brane by means of gravity only. In particular, the sixth dimension seems to facilitate the localization of vector fields, a result which does not exist in five dimensions. We have also generalized the results obtained for codimension 2 branes to that for codimension \(p\) branes with \((p - 1)\) compact \((S^1)\) and one non-compact extra dimension. Subsequently, we have studied the genuine 3-brane models where the brane is located at a conical singularity in the bulk. In the presence of a bulk phantom scalar we have constructed a viable model with a 3-brane. We then discuss a general method of constructing the warp factor and the extra dimensional factor with the determinant of the bulk metric as the only input. Finally, we address the localization scenario using the generic criteria obtained in the previous section. Apart from localized fermions, none of the other fields can be localized within this class of models with the brane as a conical defect. We point out (with examples) the conditions under which, for a sufficiently broad class of warp and extra dimensional factors, we can construct a model with a conical singularity where all the massless modes of the standard model fields can be localized.
The energy–momentum tensor corresponding to the bulk phantom scalar violates all the energy conditions and the bulk spacetime obtained in this setup is probably not dynamically stable. However, in the bulk, the brane is geometrically stable against small normal deformations (analysed via the Jacobi equations [28]) in a fixed bulk. In this regard, we mention that the bulk in the RS model [5] also violates the energy conditions but the brane is not geometrically stable under small normal deformations in a fixed bulk [28]. In order to investigate the full dynamical stability of the bulk, we need to look at the full gravitational as well as scalar field fluctuations (perturbing both sides of the Einstein equations) and obtain the resulting criteria. Note that there are possible subtleties which may arise because of the presence of the two functions $f$ and $g$ in the bulk metric, which can have growing/decaying characteristics. Merely discarding these models as unstable because of the phantom’s presence is possibly not the right thing to do [29]. We intend to give this issue a more careful look later.

A further point concerns localization issues for the bulk solution with a Brans–Dicke field. We have stated that the criteria for gravity localization is the same as for the phantom (modulo the nature and coefficients in the warp factor). This happens because the equations for the perturbations of the bulk metric and the Brans–Dicke scalar in the so-called Brans–Dicke gauge remain the same as that for the bulk phantom. If one places extra matter sources on the brane or in the bulk, things will surely change (because the perturbation equations change). It will be interesting to probe, what kind of gravity a Brans–Dicke bulk can induce on the brane. Recall that in the work of Garriga–Tanaka [30], the gravity induced on the positive and negative tension branes were of Brans–Dicke type (with different Brans–Dicke parameters).

Additionally, we need to reconsider issues like solution of hierarchy problem, construction of Friedman–Robertson–Walker (FRW) branes in a six-dimensional bulk, the stability of such FRW branes under fluctuations and graviton massive mode localization in the background geometries describing 3- and 4-branes. It also remains an open issue to find a good model with a conical singularity for a given matter source (e.g. scalar fields with some potential) and with all the known matter fields localized.

We hope to make progress with the above aspects in our forthcoming articles.

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