The Richness and Beauty of the Physics of Cosmological Recombination

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Abstract. The physical ingredients to describe the epoch of cosmological recombination are amazingly simple and well-understood. This fact allows us to take into account a very large variety of processes, still finding potentially measurable consequences. In this contribution we highlight some of the detailed physics that were recently studied in connection with cosmological hydrogen recombination. The impact of these considerations is two-fold: (i) the associated release of photons during this epoch leads to interesting and unique deviations of the Cosmic Microwave Background (CMB) energy spectrum from a perfect blackbody, which, in particular at decimeter wavelength, may become observable in the near future. Observing these distortions, in principle would provide an additional way to determine some of the key parameters of the Universe (e.g. the specific entropy, the CMB monopole temperature and the pre-stellar abundance of helium), not suffering from limitations set by cosmic variance. Also it permits us to confront our detailed understanding of the recombination process with direct observational evidence. In this contribution we illustrate how the theoretical spectral template for the cosmological recombination spectrum may be utilized for this purpose. (ii) with the advent of high precision CMB data, e.g. as will be available using the Planck Surveyor or Cmbpol, a very accurate theoretical understanding of the ionization history of the Universe becomes necessary for the interpretation of the CMB temperature and polarization anisotropies. Here we show that the uncertainty in the ionization history due to several processes that until now are not taken in to account in the standard recombination code Recfast exceed the level of 0.1\% to 0.5\% for each of them. However, it is indeed surprising how inert the cosmological recombination history is even at percent-level accuracy.

1. Introduction.

1.1. What is so rich and beautiful about cosmological recombination?

Within the cosmological concordance model the physical environment during the epoch of cosmological recombination (redshifts $500 \leq z \leq 2000$ for hydrogen, $1600 \leq z \leq 3500$ for He II$\rightarrow$He I and $5000 \leq z \leq 8000$ for He III$\rightarrow$He II recombination) is extremely simple: the Universe is homogeneous and isotropic, globally neutral and is expanding at a rate that can be computed knowing a small set of cosmological parameters. The baryonic matter component is dominated by hydrogen ($\sim 76\%$) and helium ($\sim 24\%$), with negligibly small traces of other light elements, such as deuterium and lithium, and it is continuously exposed to a bath of isotropic blackbody radiation, which contains roughly $1.6 \times 10^9$ photons per baryon. These initially simple and very unique
settings in principle allows us to predict the ionization history of the Universe and the cosmological recombination spectrum (see Sect. 2) with extremely high accuracy, where the limitations are mainly set by our understanding of the atomic processes and associated transition rates. It is this simplicity that offers us the possibility to enter a rich field of physical processes and to challenge our understanding of atomic physics, radiative transfer and cosmology, eventually leading to a beautiful variety of potentially observable effects.

1.2. What is so special about cosmological recombination?

The main reason for this simplicity is the extremely large specific entropy and the slow expansion of our Universe. Due to the huge number of CMB photons, the free electrons are tightly coupled to the radiation field due to tiny energy exchange during Compton scattering off thermal electrons until rather low redshifts, such that during recombination the thermodynamic temperature of electrons is equal to the CMB blackbody temperature with very high precision. In addition, the very fast Coulomb interaction and atom-ion collisions allows to maintain full thermodynamic equilibrium among the electrons, ions and neutral atoms down to \( z \approx 150 \) (Zeldovich et al. 1968). Also, processes in the baryonic sector cannot severely affect any of the radiation properties, down to redshift where the first stars and galaxies appear, and the atomic rates are largely dominated by radiative processes, including stimulated recombination, induced emission and absorption of photons. On the other hand, the slow expansion of the Universe allows us to consider the evolution of the atomic species along a sequence of quasi-stationary stages, where the populations of the levels are nearly in full equilibrium with the radiation field, but only subsequently and very slowly drop out of equilibrium, finally leading to recombination and the release of additional photons in uncompensated bound-bound and free-bound transitions.

1.3. Historical overview.

It was realized at the end of the 60's (Zeldovich et al. 1968; Peebles 1968), that during the epoch of cosmological hydrogen recombination (typical redshifts \( 800 \lesssim z \lesssim 1600 \)) any direct recombination of electrons to the ground state of hydrogen is immediately followed by the ionization of a neighboring neutral atom due to re-absorption of the newly released Lyman-continuum photon. In addition, because of the enormous difference in the \( 2p \leftrightarrow 1s \) dipole transition rate and the Hubble expansion rate, photons emitted close to the center of the Lyman-\( \alpha \) line scatter \( \sim 10^8 \) to \( 10^9 \) times before they can finally escape further interaction with the medium and thereby permit a successful settling of electrons in the 1s-level. It is due to these very peculiar circumstances that the \( 2s \leftrightarrow 1s \)-two-photon decay process, being \( \sim 10^8 \) orders of magnitude slower than the Lyman-\( \alpha \) resonance transition, is able to substantially control the dynamics of cosmological hydrogen recombination (Zeldovich et al. 1968; Peebles 1968), allowing about 57% of all hydrogen atoms in the Universe to recombine at redshift \( z \lesssim 1400 \) through this channel (Chluba & RS 2006b).

Shortly afterwards (RS & Zeldovich 1970; Peebles & Yu 1970), the importance of the ionization history as one of the key ingredients for the theoretical predictions of the Cosmic Microwave Background (CMB) temperature and polarization anisotropies became clear, and today these tiny directional variations of the CMB temperature (\( \Delta T / T_0 \sim 10^{-5} \)) around the mean value \( T_0 = 2.725 \pm 0.001 \) K (Fixsen & Mather 2002) have been observed for the whole sky using the COBE and WMAP satellites, beyond doubt with
great success. The high quality data coming from balloon-borne and ground-based CMB experiments (Boomerang, Maxima, Archeops, Cbi, Dasi and Vsa etc.) today certainly provides one of the mayor pillars for the cosmological concordance model (Bennett et al. 2003), and planned CMB mission like the Planck Surveyor will help to further establish the era of precision cosmology.

In September 1966, one of the authors (RS) was explaining during a seminar at the Shternberg Institute in Moscow how according to the Saha formula this recombination should occur. After the talk his friend (UV astronomer) Vladimir Kurt asked him: 'but where are all the redshifted Lyman-α photons that were released during recombination?' Indeed this was a great question, which was then addressed in detail by Zeldovich et al. (1968), leading to an understanding of the role of the 2s-two-photon decay, the delay of recombination as compared to the Saha-solution, the spectral distortions of the CMB due to two-photon continuum and Lyman-α emission, the frozen remnant of ionized atoms, and the radiation and matter temperature equality until \( z \sim 150 \).

All recombined electrons in hydrogen lead to the release of \( \sim 13.6 \text{ eV} \) in form of photons, but due to the large specific entropy of the Universe this will only add some fraction of \( \Delta \rho_\gamma / \rho_\gamma \sim 10^{-9} - 10^{-8} \) to the total energy density of the CMB spectrum, and hence the corresponding distortions are expected to be very small. However, all the photons connected with the Lyman-α transition and the 2s-two-photon continuum today appear in the Wien part of the CMB spectrum, where the number of photons in

![Figure 1. Ionization history of the Universe and the origin of CMB signals. The observed angular fluctuations in the CMB temperature are created close to the maximum of the Thomson visibility function around \( z \sim 10^{89} \), whereas the direct information carried by the photons in the cosmological hydrogen recombination spectrum is from earlier times.](image-url)
the CMB blackbody is dropping exponentially, and, as realized earlier (Zeldovich et al. 1968; Peebles 1968), these distortions are significant (see Sect. 2).

In 1975, Victor Dubrovich pointed out that the transitions among highly excited levels in hydrogen are producing additional photons, which after redshifting are reaching us in the cm-spectral band. This band is actually accessible from the ground. Later these early estimates were significantly refined by several groups (e.g. see Kholupenko et al. (2005) and Rubiño-Martín et al. (2006) for references), with the most recent calculation performed by Chluba & RS (2006b), also including the previously neglected free-bound component, and showing in detail that the relative distortions are becoming more significant in the decimeter Rayleigh-Jeans part of the CMB blackbody spectrum (Fig. 2). These kind of precise computations are becoming feasible today, because (i) our knowledge of atomic data (in particular for neutral helium) has significantly improved, and (ii) it is now possible to handle large systems of strongly coupled differential equations using modern computers.

The most interesting aspect of this radiation is that it has a very peculiar but well-defined, quasi-periodic spectral dependence, where the photons are coming from redshifts \( z \sim 1300 – 1400 \), i.e. before the time of the formation of the CMB angular fluctuation close to the maximum of the Thomson visibility function (see Fig. 1). Therefore, measuring these distortions of the CMB spectrum would provide a way to confront our understanding of the recombination epoch with direct experimental evidence, and in principle may open another independent way to determine some of the key parameters of the Universe, in particular the value of the CMB monopole temperature, \( T_0 \), the number density of baryons, \( \propto \Omega_b h^2 \), or alternatively the specific entropy, and the primordial helium abundance (e.g. see Chluba & RS (2007c) and references therein).

2. The hydrogen recombination spectrum.

Within the picture described above it is possible to compute the cosmological hydrogen recombination spectrum with high accuracy. In Figure 2 we give the results of our computations for frequencies from 100 MHz up to 3000 GHz. The free-bound and bound-bound atomic transitions among 5050 atomic levels had to be taken into account in these computations. At high frequencies one can clearly see the features connected with the Lyman-\( \alpha \) line, and the Balmer-, Paschen- and Brackett-series, whereas below \( \nu \sim 1 \) GHz the lines coming from transitions between highly excited level start to merge to a continuum. Also the features due to the Balmer and the 2s-1s two-photon continuum are visible. In total \( \sim 5 \) photons per hydrogen atom are released in the full hydrogen recombination spectrum.

One can also see from Figure 2 that both in the Wien and the Rayleigh-Jeans region of the CMB blackbody spectrum the relative distortion is growing. In the vicinity of the Lyman-\( \alpha \) line the relative distortion exceed unity by several orders of magnitude, but unfortunately at these frequencies the cosmic infra-red background due to submillimeter, dusty galaxies renders a direct measurement impossible. Similarly, around the maximum of the CMB blackbody at \( \sim 150 \) GHz it will be hard to measure these distortions with current technology, although there the spectral variability of the recombination radiation is largest. However, at low frequencies the relative distortion exceeds the level of \( \Delta I/I \sim 10^{-7} \) at frequency \( \nu \sim 2 \) GHz but still has variability with well-defined frequency dependence at a level of several percent.
Figure 2. The full hydrogen recombination spectrum including the free-bound emission. The results of the computation for 100 shells were used. The contribution due to the 2s two-photon decay is also accounted for. The dashed lines indicate the expected level of emission when including more shells. In the upper panel we also show the free-bound continuum spectrum for different values of $n_{\text{max}}$ (dashed-dotted). The inlay gives the free-bound emission for $n = 30, 40,$ and $50$. The lower panel shows the distortion relative to the CMB blackbody spectrum, and the inlay illustrates the modulation of the total emission spectrum for $1 \text{GHz} \leq \nu \leq 30 \text{GHz}$ in convenient coordinates. The figure is from Chluba & RS (2006b).
Figure 3. The bound-bound hydrogen recombination spectrum for $n_{\text{max}} = 50$. The upper panel illustrates the dependence on $\Omega_b h^2$, and the lower the dependence on the value of $T_0$. The figure is from Chluba & RS (2007b).
2.1. Dependence the recombination spectrum on cosmological parameters.

In this Section we want to illustrate the impact of different cosmological parameters on the hydrogen recombination spectrum. We restrict ourselves to the bound-bound emission spectrum and included only 50 shells for the hydrogen atom into our computations. A more rigorous investigation is in preparation, however, the principle conclusions should not be affected.

In Fig. 3 we illustrate the dependence of the hydrogen recombination spectrum on the value of the CMB monopole temperature, $T_0$. The value of $T_0$ mainly defines the time of recombination, and consequently when most of the emission in each transition appears. This leads to a dependence of the line positions on $T_0$, but the total intensity in each transition (especially at frequencies $\nu \lesssim 30$ GHz) remains practically the same. We found that the fractional shift of the low frequency spectral features along the frequency axis scales roughly like $\Delta \nu/\nu \sim \Delta T/T_0$. Since the maxima and minima of the line features due to the large duration of recombination are rather broad ($\sim 10 - 20\%$), it is probably better to look for these shifts close to the steep parts of the lines, where the derivatives of the spectral distortion due to hydrogen recombination are largest. It is also important to mention that the hydrogen recombination spectrum is shifted as a whole, allowing to increase the significance of a measurement by considering many spectral features at several frequencies.

We showed in Chluba & RS (2007b) that the cosmological hydrogen recombination spectrum is practically independent of the value of $h$. Only the features due to the Lyman, Balmer, Paschen and Brackett series are slightly modified. This is connected to the fact, that $h$ affects the ratio of the atomic time-scales to the expansion time. Therefore changing $h$ affects the escape rate of photons in the Lyman-$\alpha$ transition and the relative importance of the 2s-1s transition. For transitions among highly excited states it is not crucial via which channel the electrons finally reach the ground state of hydrogen and hence the modifications of the recombination spectrum at low frequencies due to changes of $h$ are small. Changes of $\Omega_m h^2$ should affect the recombination spectrum for the same reason.

The lower panel in Fig. 3 illustrates the dependence of the hydrogen recombination spectrum on $\Omega_b h^2$. It was shown that the total number of photons released during hydrogen recombination is directly related to the total number of hydrogen nuclei (e.g. Chluba & RS 2007c). Therefore one expects that the overall normalization of the recombination spectrum depends on the total number of baryons, $N_b \propto \Omega_b h^2$, and the helium to hydrogen abundance ratio, $Y_p$. Varying $\Omega_b h^2$ indeed leads to a change in the overall amplitude $\propto \Delta(\Omega_b h^2)/(\Omega_b h^2)$. Similarly, changes of $Y_p$ should affect the normalization of the hydrogen recombination spectrum, but here it is important to also take the helium recombination spectrum into account.

2.2. A possible observing strategy.

In order to measure these distortions one should scan the CMB spectrum along the frequency axis including several spectral bands (for illustration see Fig. 4). Because the CMB spectrum is the same in all directions, one can collect the flux of large regions on the sky, particularly choosing patches that a the least contaminated by other astrophysical foregrounds. No absolute measurement is necessary, but one only has to look for a modulated signal at the $\sim \mu$K level, with typical amplitude of $\sim 30$ nK and $\Delta \nu/\nu \sim 0.1$, where this signal can be predicted with high accuracy, yielding a spectral template for the full cosmological recombination spectrum, which should also include the contribu-
Figure 4. Comparison of observing strategies: top panel – observations of the CMB angular fluctuations. Here one is scanning the sky at fixed frequency in different directions. lower panel – proposed strategy for the signal from cosmological recombination. For this one may fix the observing direction, choosing a large, least contaminated part of the sky, and scan along the frequency axis instead.
tions from helium. For observations of the CMB angular fluctuations a sensitivity level of 10 nK in principle can be already achieved (Readhead 2007).

3. Previously neglected physical processes during hydrogen recombination.

With the improvement of available CMB data also refinements of the computations regarding the ionization history became necessary, leading to the development of the widely used Recfast code (Seager et al. 1999, 2000). However, the prospects with the Planck Surveyor have motivated several groups to re-examine the problem of cosmological hydrogen and helium recombination, with the aim to identify previously neglected physical processes that could affect the ionization history of the Universe at the level of $\gtrsim 0.1\%$. Such accuracy becomes necessary to achieve the promised precision for the estimation of cosmological parameters using the observation of the CMB angular fluctuations and acoustic peaks. Here we wish to provide an overview of the most important additions in this context and to highlight some of the previously neglected physical processes in particular. Most of them are also important during the epoch of helium recombination (e.g. Switzer & Hirata 2007), but here we focus our discussion on hydrogen only. The superposition of all effects listed below lead to an ambiguity in the ionization history during the epoch cosmological hydrogen recombination that exceeds the level of 0.1% to 0.5% each, and therefore should be taken into account in the detailed analysis of future CMB data. Still this shows that the simple picture, as explained in Sect. [1], is amazingly stable.

**Detailed evolution of the populations in the angular momentum sub-states.** The numerical solution of the hydrogen recombination history and the associated spectral distortions of the CMB requires the integration of a stiff system of coupled ordinary differential equations, describing the evolution of the populations of the different hydrogen levels, with extremely high accuracy. Until now this task was only completed using additional simplifying assumptions. Among these the most important simplification is to assume full statistical equilibrium. However, as was shown in Rubiño-Martín et al. (2006) and Chluba et al. (2007), this leads to an overestimation of the hydrogen recombination rate at low redshift by up to $\sim 10\%$. This is mainly because during hydrogen recombination collision are so much weaker than radiative processes, so that the populations within a given atomic shell depart from SE. It was also shown that for the highly excited level stimulated emission and recombination (see Fig. 5) is important.

**Induced two-photon decay of the hydrogen 2s-level.** In the transition of electrons from the 2s-level to the ground state two photons are emitted in a broad continuum (see Fig. 5). Due to the presence of a large number of CMB photons at low frequencies, stimulated emission becomes important when one of the photons is emitted close to the Lyman-α transition frequency, and, as demonstrated in Chluba & RS (2006a), leads to an increase in the effective two-photon transition rate during hydrogen recombination by more than 1%. This speeds up the rate of hydrogen recombination, leading to a maximal change in the ionization history of $\Delta N_e/N_e \sim -1.3\%$ at $z \sim 1050$.

**Re-absorption of Lyman-α photons.** The strongest distortion of the CMB blackbody spectrum is associated with the Lyman-α transition and 2s-1s continuum emission. Due to redshifting these access photons can affect energetically lower transitions. The huge
Figure 5. **Upper panel** – $l$-dependence of the recombination coefficient, $\alpha_{nl}$, at $z = 1300$ for different shells. The curves have been re-scaled by the total recombination coefficient, $\alpha_{n,\text{tot}} = \sum_l \alpha_{nl}$, and multiplied by $l_{\text{max}}^{-1}$ such that the 'integral' over $\xi = l/l_{\text{max}}$ becomes unity. Also the results obtained within the Kramers’ approximation, i.e. $\alpha_{nl}^K = \text{const} \times (2l + 1)$, and without the inclusion of stimulated recombination for $n = 100$ are presented. **Lower panel** – Two-photon decay profile for the 2s-level in hydrogen: the solid line shows the broad two-photon continuum assuming that there is no ambient radiation field. In contrast, the dashed line includes the effects of induced emission due to the presence of CMB photons at $z = 1500$. The figures are from Chluba & RS (2006a) and Chluba et al. (2007).
excess of photons in the Wien-tail of the CMB slightly increases the $1s \rightarrow 2s$ twophoton absorption rate, resulting in percent-level corrections to the ionization history during hydrogen recombination (Kholupenko & Ivanchik 2006).

**Feedback within the Lyman-series.** Due to redshifting, all the Lyman-series photons emitted in the transition of electrons from levels with $n > 2$ have to pass through the next lower-lying Lyman-transition, leading to additional feedback corrections like in the case of Lyman-α absorption in the $2s-1s$ two-photon continuum. However, here the photons connected with $\text{Ly}$ are completely absorbed by the $\text{Ly}(n-1)$ resonance and eventually all $\text{Ly}$ photons are converted into Lyman-α or $2s-1s$ two-photon decay quanta. As shown in Chluba & RS (2007a), this process leads to a maximal correction to the ionization history of $\Delta N_e/N_e \sim 0.22\%$ at $z \sim 1050$.

**Two-photon transitions from higher levels.** One of the most promising additional corrections to the ionization history is due to the two-photon transition of highly excited hydrogen states to the ground state as proposed by Dubrovich & Grachev (2005). The estimated correction was anticipated to be as large as $\sim 5\%$ very close to the maximum of the Thomson visibility function, and therefore should have had a large impact on the theoretical predictions for the CMB power spectra. It is true that in the extremely low density plasmas the cascade of permitted transitions (for example the chain $3s \rightarrow 2p \rightarrow 1s$) goes unperturbed and might be considered as two photon process with two resonances (Göppert-Mayer 1931). In addition there is a continuum analogous to $2s-1s$ decay spectrum and interference term between resonances and this weak continuum (see Fig. 6 and Chluba & RS (2007b)). However, the estimates of Dubrovich & Grachev (2005) only included the contribution to the two-photon decay rate coming from the two-photon continuum due to virtual transitions, and as shown in Chluba & RS (2007b) in particular the interference between the resonance and non-resonance continuum plays an important role in addition. It is the departures of the two-photon line profile from the Lorentzian shape in the very distant red wing of the Lyman-α line that matters, and the overall corrections to the ionization history likely do not reach the percent-level.

4. Additional sources of uncertainty during hydrogen recombination.

**Feedback due to helium lines.** The feedback of high frequency photons released during helium recombination should also affect the dynamics of hydrogen recombination. Here it is interesting that most of the photons from $\text{He} \, \text{iii} \rightarrow \text{He} \, \text{ii}$ will be already reprocessed by neutral helium before they can directly affect hydrogen. However, those photons emitted by neutral helium can directly feedback on hydrogen, but in order to take this feedback into account more detailed computations of the helium recombination spectrum are required. This process should affect the hydrogen recombination history on a level exceeding $0.1\%$.

**Uncertainty in the CMB monopole temperature.** The recombination history depends exponentially on the exact value of the CMB monopole. As shown in Chluba & RS (2007c) even for an error $\sim 1\,\text{mK}$ this leads to percent-level corrections in the ionization history. However, the effect on the CMB power spectra is only slightly larger than $0.1\%$. At this level also the uncertainties in the helium abundance ratio, $Y_p$, and the
Figure 6. Two-photon emission profile for the $3s \rightarrow 1s$ and $3d \rightarrow 1s$ transition. The non-resonant, cascade and combined spectra are shown as labeled. Also we give the analytic approximation as given in Chluba & RS (2007b) and show the usual Lorentzian corresponding to the Lyman-$\alpha$ line (long dashed). The figure is from Chluba & RS (2007b).
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effective number of neutrinos, $N_\nu$, should also be considered. In particular the value of $Y_p$ has a rather strong impact, because it directly affects the peak of the Thomson visibility function due to the dependence of the number of hydrogen nuclei, $N_H$, for a fixed number of baryons on the helium abundance ratio ($N_H \propto [1 - Y_p]$).

5. Conclusions.

It took several decades until measurements of the CMB temperature fluctuations became a reality. After COBE the progress in experimental technology has accelerated by orders of magnitude. Today CMB scientists are even able to measure $E$-mode polarization, and the future will likely allow to access the $B$-mode component of the CMB in addition. Similarly, one may hope that the development of new technologies will render the consequences of the discussed physical processes observable. Therefore, also the photons emerging during the epoch of cosmological (hydrogen) recombination could open another way to refine our understanding of the Universe.

As we illustrated in this contribution, by observing the CMB spectral distortions from the epoch of cosmological recombination we can in principle directly measure cosmological parameters like the value of the CMB monopole temperature, the specific entropy, and the pre-stellar helium abundance, not suffering from limitations set by cosmic variance. Furthermore, we could directly test our detailed understanding of the recombination process using observational data. It is also remarkable that the discussed CMB signal is coming from redshifts $z \sim 1300 - 1400$, i.e. before bulk of the CMB angular fluctuations were actually formed. To achieve this task, no absolute measurement is necessary, but one only has to look for a modulated signal at the $\sim \mu$K level, with typical amplitude of $\sim 30$ nK and $\Delta \nu/\nu \sim 0.1$, where this signal can be predicted with high accuracy, yielding a spectral template for the full cosmological recombination spectrum, also including the contributions from helium.

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