Possible Signature of First-Order Phase Transition in the Multi-messenger Data of Neutron Stars

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The equation of state (EoS) of the neutron star (NS) matter remains an enigma. In this work we perform the Bayesian parameter inference with the gravitational wave data (GW170817) and mass-radius observations of some NSs (PSR J0030+0451, PSR J0437-4715, and 4U 1702-429) using the phenomenologically constructed EoS models to search potential first-order phase transition. The whole data set, together with some additional/general conditions and the widely-adopted assumption that \( M_{\mathrm{TOV}} \) lies between 2.04 and 2.3 solar mass, yield a signature of first-order phase transition at the density of \( \sim 2.7 \rho_{\text{sat}} \), with a density jump of \( \sim 1.1 \rho_{\text{sat}} \), where \( \rho_{\text{sat}} \) is the nuclear saturation density. These parameters are in agreement with the current constraints and can be further tested with the new gravitational wave data as well as the upcoming NICER measurements of NSs in the near future.

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Introduction. Neutron stars (NSs) are natural laboratories to examine the unknown equation of state (EoS) of dense matter with the highest density in the Universe, which are mainly composed of hadrons or deconfined quark matter [1]. The EoS of the NS matter has been widely studied by theoretical calculations [2], phenomenological parameterizations [3, 4], and nonparametric methods [5]. Given the central conditions like the energy density in the core, each possible EoS can uniquely determine the global structures of nonrotating NSs including their masses, radii, and tidal deformabilities. Thus one of the informative constraints on the EoS is from the mass-radius (M-R) of NSs determined by the traditional spectroscopic measurements or the pulse profile modeling method (see Özel and Freire [6] for a recent review). The remarkable observations of the binary NS merger event GW170817 by LIGO/Virgo detectors [7] have also provided us a novel probe of the EoS [8–10]. The joint analyses of the M-R measurements, gravitational wave (GW) data, and the observed maximum mass of NSs, have set stringent constraints on the EoS [11–16].

The nature of matter in the core of NSs remains to be better understood. At sufficiently high energy density, fundamental theories like quantum chromodynamics (QCD) predict a deconfinement transition of hadronic nuclear matter into a new phase of quarks and gluons [17]. Depending on the possible compositions of the unknown matter, the compact stars can be either normal NSs [18, 19], or hybrid stars [20, 21], or strange quark stars [22, 23]. By evaluating the sound velocity in strongly interacting matter, Annala et al. [24] find that a sizable quark core in the massive NSs (> \( 2M_\odot \)) should present unless the conformal bound has been seriously violated. Many works have analyzed the feasibility of observing the presence of this exotic core (e.g., [25–27]). Among the massive NSs, the merger remnant of BNS, which is expected to have extremely high density after the collision, is promising for exhibiting strong phase transition [28, 29] that may leave imprints on the post-merger GW signals [30, 31]. While in the scenario that phase transition occurs at relatively low density, a particular family of NSs named “twin stars” whose M-R curve containing two or more branches, can present when the transition is sufficiently strong [32–34]. Therefore the M-R characteristics can be adopted to constrain the hadron-quark phase transition [35, 36]. Besides, the observed/inferred maximum mass of NSs (e.g., [37, 38]) may provide another ingredient to the research [39–42]. Benefiting from the M-R measurements and GW observations, numerous efforts have been made to answer these open questions (e.g., [43–49]).

Thanks to the excellent performance of NICER, the mass and radius of PSR J0030+0451 were measured with unprecedented precision [50, 51]. The radius measurement of PSR J0437-4715 has been updated in González-Canuel et al. [52], which will be directly tested by the dedicated NICER observations in the near future. Via the direct atmosphere-model fits to the time-evolving X-ray burst spectra [53], the M-R measurements of 4U 1702-429 were obtained with (significantly) smaller uncertainties compared to the sources measured in other indirect ways [24]. In this work, we implement phenomenological parameterization models that incorporate a phase transition, and simultaneously fit the GW data (GW170817) and M-R information of these three sources with Bayesian inference.
method. Our main finding is a signature of first-order phase transition at the density of $\sim 2.7 \rho_{\text{sat}}$ with a density jump of $\sim 1.1 \rho_{\text{sat}}$, where $\rho_{\text{sat}}$ is the nuclear saturation density.

**Parameterizing EoS.** We use a combination of piecewise polytrope [3, 54] and constant-speed-of-sound (CSS) parameterization [21] methods to describe the EOS with first-order transition between the hadronic and the quark phases (see also Refs.[44, 55]). The adiabatic indices used to construct the EoS are expressed as

$$\Gamma(\epsilon, p, h) = \begin{cases} \frac{\log p_{1}}{\log p_{0}} / \frac{\log p_{1}}{\log p_{0}} & \text{if } \rho \leq \rho_{1}, \\ \frac{\rho_{tr}}{p} / \frac{\rho_{tr}}{p} & \text{else if } \rho \leq \rho_{tr}, \\ \frac{\rho_{tr}}{p} / \frac{\rho_{tr}}{p} & \text{else if } \rho \leq \rho_{QCD}, \\ (e + p)/p & \text{else.} \end{cases}$$

where $\epsilon$, $p$, and $h$ respectively denote the internal energy, the total pressure, and the pseudo enthalpy, and the rest mass density is calculated by $\rho = (e + p)/\exp(h)$. We take $\Gamma_{tr} = 0.03$ as the Maxwell construction since the mixed phase is quite uncertain [56]. Benefiting from the nuclear theories/experiments [57, 58], the pressure at nuclear saturation density $\rho_{sat}$ is well constrained, so we fix $\rho_{1}$ to $2.7 \times 10^{14}$ g cm$^{-3}$ and $\rho_{tr}$ to $3.91 \times 10^{33}$ dyn cm$^{-2}$. And $\rho_{0}$ is the pressure at $\rho_{0} = 0.33 \rho_{sat}$, which is determined by the EoS SLy [59] that interpolates the low-density range of our models. While in the high-density region, the EoS is phenomenologically parameterized by the phase transition onset density $\rho_{tr}$ and the corresponding pressure $p_{tr}$ as well as the density discontinuity $\Delta \rho$. Above the density $\rho_{tr} + \Delta \rho$, we use two models to describe the quark phase. Model A (i.e., Maxwell CSS) is parameterized by the speed of sound $v_{s}$ with a constant $c$ ($c/\sqrt{3}$) below (above) the density $\rho_{QCD}$ corresponding to the onset of the perturbative QCD limit. While Model B (i.e., Maxwell polytrope) uses a constant adiabatic index $\Gamma_{q}$ to construct the quark core until achieving the causality constraint, and is extended to higher densities with $v_{s} = c$. The representative EoSs constructed from each model are shown in Fig.1. We choose reasonable ranges for parameters $\theta_{EoS}$ with $\rho_{tr} \in [0.8, 15] \rho_{sat}$ ($P_{1.85\rho_{sat}}^{1} = 1.21 \times 10^{34}$ dyn cm$^{-2}$ [60]), $\rho_{sa} \in [1.5, 4.4] \rho_{sat}$, $\Delta \rho \in [0.2, 3.0] \rho_{sat}$, $\rho_{QCD} \in [8, 10.5] \rho_{sat}$, and $\Gamma_{q} \in [0.5, 8]$, 

![FIG. 1. Sketch of the EoS models in the form of pressure versus rest-mass density. The green line represents the low density region of the EoS SLy.](image-url)
which can encompass a wide variety of candidate EoSs [42, 44, 55]. Additionally, all of the parameterized EoSs satisfy the following conditions: 1) causality constraint $v_s \in [0, c]$, 2) thermal stability $de/dp > 0$, and 3) maximum mass limits $M_{\text{TTOV}} \in [2.04, 2.3] M_\odot$. The left boundary is the 68.3% lower limit of PSR J0740+6620’s mass measurement [37], and the upper bound is chosen based on the constraints from the multimessenger analysis of GW170817/GRB170817A/AT2017gfo [61–64]. We implement both models with the enthalpy-based formulae of Lindblom and Indik [4] to solve the Tolman-Oppenhimer-Volkoff and Regge-Wheeler equations (see also Refs.[11, 65]). Thus we can map the mass or central enthalpy to other bulk properties, e.g., radius $R$ and tidal deformability $\Lambda$.

**Bayesian Inference.** Assuming that compact stars share the same EoS, we take the likelihood $L = L_{\text{GW}}(d | \hat{\theta}_{\text{GW}}) \prod P_i(M(\hat{\theta}_{\text{EOS}}, h_i), R(\hat{\theta}_{\text{EOS}}, h_i))$ to constrain the parameters $\hat{\theta}_{\text{EOS}}$ that characterize the ultra dense matter EoS [65], by performing Bayesian inference with BILBY [66] and PYMULTINEST [67] packages. For the $M$-$R$ observations of PSR J0030+0451 by NICER [50, 51] and 4U 1702-429 [53], we use the posterior samples $(\mathcal{S})$ to construct the

**FIG. 2.** Posteriors of $\hat{\theta}_{\text{EOS}}$ and some bulk properties obtained from different models and data sets.
kernel density estimate (KDE) as $P_{i}(M, R) = \text{KDE}(M, R \mid S)$ \cite{11}. While for PSR J0437-4715, we approximate the $M-R$ measurements by the products of two KDEs, i.e., $P_{i}(M, R) = \text{KDE}(M \mid S_{M}) \times \text{KDE}(R \mid S_{R})$, where $S_{M}$ and $S_{R}$ are posterior samples of mass and radius \cite{52, 68}. Each pair of $(M, R)$ are calculated by varying the central enthalpy $h_{i}$ in the range of $[0.05, 1]$. The contribution of GW to the likelihood is determined by its strain data and power spectral densities (detailed processing follows \cite{65}), waveform models (e.g., IMRPhenomD,NRTidalv2, \cite{69}) as well as the corresponding parameters $\vec{\theta}_{\text{GW}}$. We fix the source location of GW170817 to the known position (R.A.=197.450374°, decl.=−23.381495°, $z=0.0099$) as determined by electromagnetic (EM) observations \cite{70, 71}. To break the degeneracy between component masses and improve the efficiency in nest sampling, the chirp mass $M_{c}$ and mass ratio $q$ are sampled instead of $m_{1}, m_{2}$. Thus the GW parameters of the marginalized-phase likelihood are $\vec{\theta}_{\text{GW}} = \{\Lambda_{1}(m_{1}^{\text{src}}, \vec{\theta}_{\text{EOS}}), \Lambda_{2}(m_{2}^{\text{src}}, \vec{\theta}_{\text{EOS}})\} \cup \{M_{c}, q, \chi_{1z}, \chi_{2z}, \theta_{\text{JN}}, t_{c}, \Psi\}$, where $\Lambda_{1,2}$ are dimensionless tidal deformabilities that mapped from source frame masses. The priors of $M_{c}, q$ are given by $P(M_{c}, q) \propto M_{c}(1 + q)^{2/5}q^{-6/5}$ ($M_{c} \in [0.87, 1.74]M_{\odot}$, $q \in [0.5, 1]$) and the additional constraints $m_{1,2} \in [1, 2]M_{\odot}$, which yield uniform distribution in $m_{1}-m_{2}$ plane. An aligned low-spin prior is assigned to $\chi_{1z}$ and $\chi_{2z}$, while $\sin(\theta_{\text{JN}})$ and other parameters (e.g., $\vec{\theta}_{\text{EOS}}, h_{i}, \Psi$) are uniformly distributed in their domains.

**Results.** We carry out the Bayesian inference based on two models (i.e., Model A and Model B) with four data sets,
Our results show that adopting $M$-$R$ measurements of PSR J0030+0451 from Riley et al. [51] or Miller et al. [50] yield rather similar posterior distributions (see Fig.2). The inferred bulk properties of NSs (e.g., $\Lambda$, $R_{1.4}$, $\Lambda_{1.4}$), which are mainly dependent of the pressure at around $2\rho_{\text{sat}}$ [72], are strongly correlated with each other [73]. These bulk properties and the phase transition onset pressure $p_{\text{tr}}$ are enhanced in the cases of $D_3, D_4$, while these properties have minor difference between the model A and model B. As shown in Fig.3, the central densities $\rho_c$ of these sources may have exceeded the transition onset density $\rho_{\text{tr}}$ except the lighter component of GW170817 and PSR J0030+0451, i.e., the sources may undergo first-order phase transition. For the results obtained with joint analysis of GW170817 and PSR J0030+0451, the behavior of density discontinuity $\Delta\rho$ converging to the small density jump values (left boundary of the prior) makes it indistinguishable for the presence of a strong phase transition, which is consistent with Pang et al. [55] where GW170817 and GW190425 were analyzed. However, by the inclusion of the additional sources 4U 1702-429 and PSR J0437-4715, the striking difference appears in the distribution of density discontinuity.
$\Delta \rho$ which peaks at $\sim 1.1 \rho_{\text{sat}}$ (a log-uniform prior of $\Delta \rho$ yields similar phenomenon). This result indicates that above the 90% lower limit of $\rho_{\text{tr}}$ ($\sim 2.1 \rho_{\text{sat}}$), the first-order phase transition is plausible, especially for the heavy source 4U 1702-429. Our finding is not in tension with Christian and Schaffner-Bielich [36] where strong phase transition below $1.7 \rho_{\text{sat}}$ was found to be ruled out. The reconstructed $M$-$R$ posterior distributions (presented in Fig.4) show that each sources are well fitted, and the uncertainties of $M$-$R$ curve below $\sim 1.4 M_\odot$ are relatively narrow thanks to the informative sources. While the heavier object 4U 1702-429, though with a relatively larger measurement error, has contribution on constraining the twin-star branches and the phase transition parameters like $\Delta \rho$.

**Discussion and Summary.** We have performed the Bayesian parameter inference with the GW data (GW170817) and $M$-$R$ observations (PSR J0030+0451, PSR J0437-4715, and 4U 1702-429) using the phenomenologically constructed EoS models to search potential first-order phase transition. For the data sets $D_{1,2}$, the bulk properties of NSs are constrained to $R_{1,4} \sim 11.6 \text{ km} (\Lambda_{1,4} \sim 250)$. While for $D_{3,4}$, the corresponding results are $R_{1,4} \sim 12.8 \text{ km} (\Lambda_{1,4} \sim 550)$. An inspiring phenomenon appears in the analysis of data sets $D_{3,4}$, where the density discontinuities peak at relatively large values. This means that the heavy sources whose central densities exceed the transition onset density $\rho_{\text{tr}}$ are promising for undergoing first-order phase transition. Anyhow, our models are limited to the Maxwell-like phase transition, i.e., sharp density discontinuities are assumed. In the case of Gibbs-like phase transition, a mixed phase may smear out an observable EOS feature [55]. Alford et al. [20] found that a masquerade problem will appear for such EoSs, making their macroscopic structure properties ($M$-$R$ or $M$-$\Lambda$ relations) hard to be distinguished from purely nucleonic EoSs. Another model dependence is that without a tight bound on the maximum mass, the distinguishable peaks (in the cases of $D_{3,4}$) disappear in the distribution of $\Delta \rho$, this is understandable because the presence of a quark core will be disfavored by a very high $M_{\text{TOV}}$. In addition, the $M$-$R$ measurements of the sources (PSR J0437-4715 and 4U 1702-429) may still suffer from some systematic uncertainties [6, 50]. Benefiting from the dedicated observations by NICER, unprecedentedly precise $M$-$R$ measurements for massive NSs (e.g., PSR J0740+6620 and PSR J1614-2230 [74]) as well as PSR J0437-4715 will be available in the future. Hence the existence of such transition will be reliably probed, and then shed valuable lights on the dense matter in the core of NSs.

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