Open- and hidden-charm tetra-quark scalar mesons

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Masses of open- and hidden-charm tetra-quark scalar mesons are studied in a new approach. As the result, newly estimated masses of hidden-charm sector are significantly lower than our earlier ones by a naive quark counting. In particular, our new result on the non-strange iso-triplet scalar mesons should be compared with the experiment that an indication of tiny $\eta\pi$ peak around 3.2 GeV has been observed, while no signal in the $\eta\pi$ channel.

The charm-strange scalar $D_{s0}^+(2317)$ meson was discovered in the $D_s^+\pi^0$ mass distribution [1], and its multi-quark interpretations have attracted general interests [2]. For example, it was assigned to a tetra-quark meson, and its open- and hidden-charm partners were expected to exist [3, 4]. On the other hand, an indication of tiny peak around 3.2 GeV in the $\eta\pi$ invariant mass distribution was observed in two photon collisions [5]. If it is a true signal, it can be a hidden-charm partner of $D_{s0}^+(2317)$, which was called as $\delta^c(3200)$ in [6]. However, no resonance peak around the same energy has been observed in the $\eta\pi$ channel [7]. It is strange, because the $\delta^c(3200) \to \eta\pi$ decay is allowed under the OZI-rule [8] and the mass of $\delta^c(3200)$ is sufficiently higher than the threshold, when its mass is truly 3200 MeV as measured [9] (or $\sim 3300$ MeV as predicted by using a naive quark counting [4]). This suggests to see that its true mass is lower than the $\eta\pi$ threshold. In this short note, therefore, we study mass spectra of open- and hidden-charm tetra-quark scalar mesons from a new approach which is much different from our naive quark counting and discuss why partners of $D_{s0}^+(2317)$ (except for $\delta^c(3200)$) have not been observed, and, in particular, why $\delta^c(3200)$ was indicated in the $\eta\pi$ channel but not in the $\eta\pi$ channel.

Before going to mass spectra of open- and hidden-charm scalar mesons, we review very briefly on our picture of tetra-quark mesons. They can be classified into the following four groups [9],

$$
\{qq\bar{q}\bar{q}\} = [qq][\bar{q}\bar{q}] \oplus (qq)(\bar{q}\bar{q}) \oplus \{qq\bar{q}\bar{q}\} \pm (qq)[\bar{q}\bar{q}]
$$

(1) according to different symmetry properties of their flavor wavefunctions, where parentheses and square brackets denote symmetry and anti-symmetry, respectively, under exchange of flavors between them. Here it should be noted that there are two ways to get a color-singlet tetra-quark $\{qq\bar{q}\bar{q}\}$ state, i.e., $\{qq\}$ (and $\{\bar{q}\bar{q}\}$) are (i) of $3c$ (and $3c$), and (ii) of $6c$ (and $6c$), respectively, of the color $SU_c(3)$. Therefore, each term in the right-hand side (r.h.s.) of Eq. (1) is again classified into two groups, i.e., $\{qq\}_3$, $\{qq\}_3$, and $\{qq\}_6$, $\{qq\}_6$. We here consider that the former is lower than the latter, because forces between two quarks (and between two antiquarks) are attractive when they are of $3c$ (and of $3c$), while repulsive when of $6c$ (and of $6c$) [10]. However, these $\{qq\}_3$, $\{\bar{q}\bar{q}\}_3$, and $\{qq\}_6$, $\{\bar{q}\bar{q}\}_6$ states can significantly mix with each other in the light sector ($q = u, d, s$) [8]. In contrast, such a mixing will be much smaller in heavy mesons, because the energy scale under consideration is much higher than the QCD scale ($\Lambda_{\text{QCD}}$).

Spin wave functions of $\{qq\}_3$ and $\{qq\}_6$ should be anti-symmetric and symmetric in the flavor symmetry limit, because their wavefunctions should be totally anti-symmetric in the limit, i.e., they are of singlet $(1_s)$ and triplet $(3_s)$, respectively, of spin $SU(2)$. With regard to flavor symmetry breaking in meson wave functions, it is considered as follows. A matrix element of flavor charge which generates a flavor transformation is given by a form factor $f^a_+(0)$ in a relevant matrix element of vector current at zero momentum transfer squared, and therefore, a measure of the flavor symmetry breaking is given by a deviation of its value from unity [11]. By the way, values of various $f^a_+(0)$’s have been summarized in [12]. As the first example, the chiral perturbation theory has provided $f^\pi_+(0) = 0.961 \pm 0.008$ [13] which implies that the flavor $SU_f(3)$ symmetry works well in meson wave functions. In addition, it works well even in the world of charm mesons, as seen in $f^{D_{sD}}_+(0)/f^{K_{sD}}_+(0) = 1.00 \pm 0.11 \pm 0.02$ [14] and 0.99 $\pm 0.08$ [15]. Although the measured $f^{(K_{sD})}_+(0) = 0.74 \pm 0.03$ [12] suggests that the $SU_f(4)$ symmetry in wave functions is broken, its deviation from unity is not fatally large, and therefore, we take wave functions of open- and hidden-charm tetra-quark mesons in the flavor $SU_f(4)$ symmetry limit (as their dominant part) in this short note. In this case, the first term $\{qq\}[\bar{q}\bar{q}]$ in the r.h.s. of Eq. (1) has (dominantly) $J^P = 0^+$, where $J$ and $P$ denote its spin and parity, respectively.

Now it is known that the mass spectrum of the observed light scalar nonet, $f_0(500)$, $a_0(980)$, $f_0(980)$ [10] and $\kappa(800)$ [17] is well reproduced by the light tetra-quark $[qq][\bar{q}\bar{q}]$ states [8], and it is easy to extend this framework to open- and hidden-charm scalar mesons, $[cq][\bar{q}\bar{q}]$ and $[cq][\bar{q}\bar{q}]$, putting their detailed structure of color wavefunctions aside. In fact, just after the discovery of $D_{s0}^+(2317)$, it was assigned to the iso-singlet $\{[cn][\bar{s}\bar{n}]\}_{1=0}$, $(n = u, d)$ state [8] to understand its narrow width. If $D_{s0}^+(2317) \to \{[cn][\bar{s}\bar{n}]\}_{1=0} \to D_s^+\pi^0$ through the isospin nonconserving hadronic interactions were dominant in its decays, its rate would be small, because such interactions are much weaker than the isospin conserving strong ones, and therefore, its narrow width could be understood. In this case, however, its
dominant decay should be the radiative $D_{s0}^+(2317) \to D_s^+\gamma$ \[18\], because of the hierarchy of hadron interactions \[19\], unless there exists any specific mechanism to suppress strongly the $D_s^+\gamma$ decay and/or to enhance extremely the $D_s^+\pi^0$, where $\alpha$ is the fine structure constant. However, such a mechanism is not known in any tetra-quark picture of $D_{s0}^+(2317)$ yet. Therefore, it is strange that no signal of $D_{s0}^+(2317)$ has been observed in the $D_s^+\gamma$ channel. In addition, it is expected that its iso-triplet partners which decay dominantly into $D_s^+\pi$ final states exist. In this case, its dominant decay is expected to be the isospin conserving $D_{s0}^+(2317) \to D_s^+\pi^0$. Although it is a strong decay, its rate can be small, because of a small overlap of wavefunctions between the initial $D_{s0}^+(2317)$ and the final $D_s^+\pi^0$, as was discussed in \[15\] and will be seen later again. It seems to be nice, while no signal of its neutral and doubly charged partners, $D_{s0}^0(2317)$ and $D_{s0}^{\pm}(2317)$, has been observed in $B$ decays \[20\]. It might imply that their production rates are much lower than that for $D_{s0}^+(2317)$, against our expectation \[21\].

Putting this problem aside for a while, we now study masses of open- and hidden-charm scalar tetra-quark mesons. Before going to the main subject, we study masses of ordinary mesons. An ideally mixed ordinary meson which is considered as a color singlet quark-antiquark $\{q\bar{q}\}_c$ system has a mass which is given by a sum of spin averaged mass $M(\{q\bar{q}\}_c)$ and hyperfine splitting of the $\{q\bar{q}\}_c$ pair \[22\],

$$M(\{\text{meson}\}_{ij}) = M(\{q\bar{q}\}_{1c}) + a_{ij} \frac{\sigma_i \cdot \sigma_j}{m_i m_j},$$

where $m_i$ is the mass of quark $q_i = u, d, s,$ or $c$ and its spin is given by $\frac{1}{2} \sigma_i$. In this case, masses of ideally mixed mesons are given by

$$M(J/\psi) = M(\{cc\}_1) + 2a_{cc}/(m_c)^2, \quad M(\eta_c) = M(\{cc\}_1) - 6a_{cc}/(m_c)^2,$$

$$M(D^*_s) = M(\{cs\}_1) + 2a_{cs}/(m_c m_s), \quad M(D_s) = M(\{cs\}_1) - 6a_{cs}/(m_c m_s),$$

$$M(D^+) = M(\{c\bar{n}\}_1) + 2a_{cn}/(m_c m_n), \quad M(D) = M(\{c\bar{n}\}_1) - 6a_{cn}/(m_c m_n),$$

$$M(\bar{K}^*)_c = M(\{n\bar{s}\}_1) + 2a_{sn}/(m_s m_n), \quad M(\bar{K}) = M(\{n\bar{s}\}_1) - 6a_{sn}/(m_s m_n),$$

$$M(\rho) = M(\omega) = M(\{n\bar{n}\}_1) + 2a_{nn}/(m_n)^2, \quad M(\pi) = M(\{n\bar{n}\}_1) - 6a_{nn}/(m_n)^2,$$

where $\{ss\}$ mesons have been excluded in the above, because the $\eta\eta'$ mixing is far from the ideal one. In this way, the spin averaged masses are determined as

$$\{M(\{cc\}_1) = [3M(J/\psi) + M(\eta_c)]/4 = 3068.5 \text{ MeV}, \quad M(\{cs\}_1) = [3M(D^*_s) + M(D_s)]/4 = 2076.4 \text{ MeV},$$

$$M(\{c\bar{n}\}_1) = [3M(D^+) + M(D)]/4 = 1973.3 \text{ MeV}, \quad M(\{n\bar{s}\}_1) = [3M(\bar{K}^*)_c + M(\bar{K})]/4 = 794.5 \text{ MeV},$$

$$M(\{n\bar{n}\}_1) = [3M(\omega) + M(\rho)]/2 + M(\pi)/4 = 618.4 \text{ MeV},$$

by taking the measured mass values of mesons in \[23\], where their small errors have been neglected.

The spin averaged mass $M(\{q\bar{q}\}_c)$ is given by a sum of masses of quarks which construct the system and their binding energy, and the so-called constituent quark masses have been taken as those of the constituents in \[22\]. In this short note, however, their running masses are taken and, at the same time, a confinement effect (including possible gluon contributions) in the ordinary meson is parametrized by $G$,

$$M(\{q\bar{q}\}_c) = m_i + m_j + G + B(\{q\bar{q}\}_c),$$

where $G$ is assumed to be flavor independent. To estimate the binding energy $B(\{q\bar{q}\}_c)$, we take the following values of running quark masses: $m_u(\mu = 2 \text{ GeV}) = (3.5^{+0.3}_{-0.2}) \text{ GeV}$, $m_s(\mu = 2 \text{ GeV}) = (96^{+6}_{-5}) \text{ MeV}$, $m_c(\mu = m_c) = (1280\pm30) \text{ MeV}$ \[23\] and $m_c(\mu = 3 \text{ GeV}) = (993\pm8) \text{ MeV}$ \[24\]. We here assume, for simplicity, that $m_u(\mu)$ and $m_c(\mu)$ are rather stable (do not so rapidly run) in the energy scale of open- and hidden-charm meson masses under consideration, because their contribution is not very large and, as the result, this assumption does not make serious errors in the present work. In addition, it is assumed that $m_c(\mu \sim 2 \text{ GeV})$ is not very much different from $m_c(\mu = m_c)$. By taking the above values of quark masses, spin averaged binding energies $B(\{q\bar{q}\}_c)$'s of color singlet $\{q\bar{q}\}_c$ states at the energy scale ($\mu \sim 2$ or 3 GeV) under consideration can be crudely obtained as

$$\{B(\{cc\}_1) = 1082.5 \text{ MeV} - G, \quad B(\{cs\}_1) = 700.4\pm30 \text{ MeV} - G,$$

$$B(\{c\bar{n}\}_1) = 689.8\pm30 \text{ MeV} - G, \quad B(\{n\bar{s}\}_1) = 695.0 \text{ MeV} - G,$$

$$B(\{n\bar{n}\}_1) = 611.4 \text{ MeV} - G.$$
under the isospin $SU(2)$ symmetry, where errors from $m_u(\mu = 2\text{GeV})$ and $m_s(\mu = 2\text{GeV})$ have been neglected, because they do not make serious uncertainties in the above results.

Next, we study masses of open- and hidden-charm tetra-quark scalar mesons in a new picture similar to a QCD-string-junction model [22]. In this picture, a mass of tetra-quark $\{|q\bar{q}I_{3c}\} \rightarrow |q\bar{q}I_{3c}\} \rightarrow \{q\bar{q}I_{3c}\}$ meson is given in the form

$$M(\{q\bar{q}I_{3c}\} \rightarrow |q\bar{q}I_{3c}\} \rightarrow \{q\bar{q}I_{3c}\} \rightarrow \{q\bar{q}I_{3c}\}) = 2\hat{S} + (m_i + m_j + m_k + m_e) + \hat{G} + B(\{q\bar{q}I_{3c}\} \rightarrow |q\bar{q}I_{3c}\} \rightarrow \{q\bar{q}I_{3c}\} \rightarrow \{q\bar{q}I_{3c}\}),$$

(7)

where $\hat{S}$ and $\hat{G}$ denote a QCD-string-junction contribution and a confinement effect in the tetra-quark system. In the above equation, we have neglected a contribution of spin-spin force between the so-called diquark $|q\bar{q}I_{3c}\} \rightarrow \text{antidiquark } |\bar{q}\bar{q}I_{3c}\} \rightarrow \text{because their spins are vanishing in the present case, in contrast to [22].}$ To compute the binding energies $B(\{q\bar{q}I_{3c}\} \rightarrow |q\bar{q}I_{3c}\} \rightarrow \{q\bar{q}I_{3c}\} \rightarrow \{q\bar{q}I_{3c}\})$ of tetra-quark scalar mesons, we first decompose each of tetra-quark states into a sum of products of $\{|q\bar{q}\} \rightarrow \text{pairs, as in [26]. (Our notations } \hat{F}_i, \hat{F}_0, \hat{D}_i, (\hat{D}_s), \hat{E}^q, \hat{k}^c, \hat{d}^c, (\hat{\sigma}^c) \text{ and } \hat{c}^c \text{ of the open- and hidden-charm tetra-quark scalar mesons have been provided in [3] and [6], respectively, where } \hat{D}_s \text{ and } \hat{\sigma}^c \text{ are not considered in this short note, because each of them contains an } \{s\bar{s}\} \text{ pair.) To save space, we here list only the result on } \hat{F}_i^+ \text{ which is assigned to } D_{s0}^+(2317),

$$|\hat{F}_i^+\rangle = \frac{1}{\sqrt{2}} [\bar{c}u_{I_{1c}} [\bar{s}\bar{u}]_{I_{1c}} - [\bar{c}d_{I_{1c}} [\bar{s}\bar{d}]_{I_{1c}}]_{I_{1c}}$$

$$= \frac{1}{\sqrt{2}} \left\{ \frac{3}{12} \left[ (\bar{c}s)_{I_{1c}} \right]_{I_{1c}} \left[ (\bar{u}\bar{u})_{I_{1c}} \right]_{I_{1c}} + \frac{1}{\sqrt{2}} \left[ (\bar{c}s)_{I_{1c}} \right]_{I_{1c}} \left[ (\bar{u}\bar{u})_{I_{1c}} \right]_{I_{1c}} + \frac{1}{\sqrt{2}} \left[ (\bar{c}s)_{I_{1c}} \right]_{I_{1c}} \left[ (\bar{u}\bar{u})_{I_{1c}} \right]_{I_{1c}} \right\},$$

(8)

where $\{|u\bar{u}\} \rightarrow \{c\bar{s}\} \rightarrow \{u\bar{u}\} \rightarrow \{c\bar{s}\} \rightarrow \{u\bar{u}\}$ etc. have been replaced by $K^+, D^+_s, K^{*+}, D^{*+}$ etc., respectively, and the ellipsis denotes neglected contributions of products of color octet $\{q\bar{q}\}_{I_{1c}}$ pairs in the last equality. We here assume that the above decomposition is stable (not easily reshuffled by exchanging gluons), because the energy scale under consideration is much higher than $\Lambda_{QCD}$. In this way, it is seen that a wavefunction overlap between the initial $D_{s0}^+(2317) = \hat{F}_i^+$ and the final $D_s^+ \pi^0$ is small, so that a rate for the main decay $D_{s0}^+(2317) \rightarrow D_s^+ \pi^0$ can be small, and as the result, $D_{s0}^+(2317)$ can be narrow, as discussed before. In addition, a square of coefficient of each $\{|q\bar{q}\}$ state provides its share in the tetra-quark state under consideration. By noting the above decomposition of $\hat{F}_i^+$ (and those of its partners) and by taking the binding energies $B(\{q\bar{q}I_{3c}\} \rightarrow |q\bar{q}I_{3c}\} \rightarrow \{q\bar{q}I_{3c}\} \rightarrow \{q\bar{q}I_{3c}\}$'s in Eq. (3), $B(\{q\bar{q}I_{3c}\} \rightarrow |q\bar{q}I_{3c}\} \rightarrow \{q\bar{q}I_{3c}\} \rightarrow \{q\bar{q}I_{3c}\})$ which are equated to $-\frac{1}{8} B(\{q\bar{q}I_{3c}\} \rightarrow |q\bar{q}I_{3c}\} \rightarrow \{q\bar{q}I_{3c}\}) \rightarrow \{q\bar{q}I_{3c}\}$'s (because the force between $q$ and $\bar{q}$ with the color $8c$ is $-\frac{1}{8}$ of that with $1c$ (10) and the binding
energy $B_{s,s_c}$ between two $\{q\bar{q}\}_{s_c}$ pairs, the total binding energies $B([q\bar{q}]_3^3_{\Sigma_c}([q\bar{q}]_{3}^1_{\Sigma_c})$'s can be obtained as

$$
\begin{align*}
B(\hat{F}_I^+) &= (337.1 \pm 5.3) \text{ MeV} - \frac{G}{2} + \frac{2}{3} B_{s,s_c}, \\
B(\hat{D}) &= (325.3 \pm 7.5) \text{ MeV} - \frac{G}{2} + \frac{2}{3} B_{s,s_c}, \\
B(\hat{E}^0) &= (346.2 \pm 7.5) \text{ MeV} - \frac{G}{2} + \frac{2}{3} B_{s,s_c}, \\
B(\sigma^c) &= (384.2 \pm 7.5) \text{ MeV} - \frac{G}{2} + \frac{2}{3} B_{s,s_c}, \\
B(\kappa^c) &= (396.0 \pm 5.3) \text{ MeV} - \frac{G}{2} + \frac{2}{3} B_{s,s_c},
\end{align*}
$$

Insertion of Eq. (9) into Eq. (10) leads to the following masses of open- and hidden-charm tetra-quark scalars,

$$
\begin{align*}
M(\hat{F}_I) &= 2\hat{S} + (\hat{G} + m_c + m_s + 2m_n) + B(\hat{F}_I) = M(\hat{F}_0^+) \\
&= (1720.1 \pm 30.5) \text{ MeV} + [2\hat{S} + \hat{G} - \frac{1}{3}G + \frac{2}{3} B_{s,s_c}], \\
M(\hat{D}) &= 2\hat{S} + (\hat{G} + m_c + 3m_n) + B(\hat{D}) \\
&= (1615.8 \pm 30.9) \text{ MeV} + [2\hat{S} + \hat{G} - \frac{1}{3}G + \frac{2}{3} B_{s,s_c}], \\
M(\hat{E}^0) &= 2\hat{S} + (\hat{G} + m_c + m_s + 2m_n) + B(\hat{E}^0) \\
&= (1729.2 \pm 30.9) \text{ MeV} + [2\hat{S} + \hat{G} - \frac{1}{3}G + \frac{2}{3} B_{s,s_c}], \\
M(\sigma^c) &= 2\hat{S} + (\hat{G} + 2m_c + 2m_n) + B(\sigma^c) = M(\sigma^c) \\
&= (2383.2 \pm 7.5) \text{ MeV} + [2\hat{S} + \hat{G} - \frac{1}{3}G + \frac{2}{3} B_{s,s_c}], \\
M(\kappa^c) &= 2\hat{S} + (\hat{G} + 2m_c + m_s + m_n) + B(\kappa^c) \\
&= (2481.5 \pm 5.3) \text{ MeV} + [2\hat{S} + \hat{G} - \frac{1}{3}G + \frac{2}{3} B_{s,s_c}].
\end{align*}
$$

Because of $B(\hat{F}_I) = B(\hat{F}_0^+)$, masses of $\hat{F}_I$ and $\hat{F}_0$ are degenerate in the present approach, as in our earlier work [4]. The above masses of open- and hidden-charm tetra-quark scalar mesons include a combination of unknown paramters, $2\hat{S} + \hat{G} - \frac{1}{3}G + \frac{2}{3} B_{s,s_c}$, where it has been assumed that the combination does not (significantly) run within the region of energy scale under consideration. By taking the measured $M(D_{s0}^*(2317))_{\exp} = 2317.8 \pm 0.6$ MeV [23] as the input data, it can be determined as

$$
2\hat{S} + \hat{G} - \frac{1}{2}G + \frac{2}{3} B_{s,s_c} = (597.7 \pm 30.5) \text{ MeV}.
$$

Inserting the above result into Eq. (10), we can obtain mass values of open- and hidden-charm tetra-quark scalars as listed in Table I. As seen in the table, the present results on masses of open-charm sector are not so much different from the previous ones in [4], while newly estimated masses of hidden-charm mesons are considerably lower than our earlier ones in [4].

The iso-triplet $\hat{F}_I$ can decay dominantly into $D^+_s \pi$ through isospin conserving strong interactions, while the iso-singlet $\hat{F}_0^+$ should decay dominantly into $D^+_s \gamma$, because of the hierarchy of hadron interactions, as discussed before.

Table I. Estimated masses of open- and hidden-charm tetra-quark scalar mesons, where $\hat{D}^*$ and $\hat{\sigma}^c$ are not included because each of them contains an $\{s\bar{s}\}$ pair. The measured $M(D_{s0}^*(2317))_{\exp} = 2317.8 \pm 0.6$ MeV [23] and the running quark mass values which are listed in the text are taken as the input data. The listed errors are mainly from $m_c(\mu = m_c)$ in [23].

| $S$ | $I = 1$ | $I = 1/2$ | $I = 0$ | Predicted mass (MeV) | Lowest OZI allowed mode | Threshold (MeV) |
|-----|---------|---------|--------|----------------------|-------------------------|-----------------|
| 0   | $\hat{F}_I$ | 2317.8 ± 0.6 (‡) | $D^+_s \pi$ | 2106.5 |
| 0   | $\hat{F}_0^+$ | 2317.8 ± 0.6 | $D^+_s \eta$ | 2516.1 |
| -1  | $\hat{E}^0$ | 2213.5 ± 43.4 | $D \pi$ | 2005.2 |
| 1   | $\hat{\sigma}^c$ | 3079.2 ± 30.5 | $\eta_c K$ | 3479.4 |
| 0   | $\hat{\delta}^c$ | 2980.6 ± 31.4 | $\eta_c \pi$ | 3121.4 |
|     | $\hat{\kappa}^c$ | 2980.6 ± 31.4 | $\eta_c \eta$ | 3531.3 |

(‡): Input data
Therefore, the present approach expects observations of peaks around 2317 MeV not only in the \( D_s^+ \pi^0 \) channel but also in the \( D_s^{++} \gamma \) (arising from \( E_s^+ \) and \( F_s^+ \), respectively), because of \( M(E_s^+) = M(F_s^+) \). Nevertheless, experiments observed no signal of the \( D_s^{++} \gamma \) peak. Even if \( D_s^{\ast 0}(2317) \) were identified to the ordinary \( D_s^{\ast 0} \) as the \( ^2P_0 \{c\bar{s}\} \), it also should decay dominantly into \( D_s^{\ast +} \gamma \) for the same reason as the above \[18\]. No signal of \( D_s^{\ast +}(2317) \) in the \( D_s^{\ast +} \gamma \) channel and no indication of \( D_s^{0}(2317) \) and \( D_s^{\ast 0}(2317) \) in \( B \) decays are problems in our tetra-quark interpretations of \( D_s^{\ast 0}(2317) \), as discussed before.

In regard to the non-strange \( \hat{D} \sim \{[c\bar{u}]_{\frac{1}{2}}[\bar{u}d]_{\frac{1}{2}}\} \), it can decay into \( D\pi \) final states through isospin coserving strong interactions. However, it will be narrow for the same reason as the narrow width of \( D_s^{\ast +}(2317) \) \[18\]. Therefore, the observed broad \( D_s^{\ast 0}(2400) \) with a mass \( M(D_s^{\ast 0})_{\text{exp}} = 2318 \pm 29 \text{ MeV} \) \[23\] will be the ordinary \(^3P_0 \{c\bar{s}\} \), meson. This implies that the tetra-quark \( \hat{D} \) with the predicted mass \( M(D_s) = 2213.5 \pm 43.4 \text{ MeV} \) in Table I will be observed as a tiny peak on the lower tail of the observed broad \( D\pi \) enhancement, \( D_s^{\ast 0} \), because production rate of \( \hat{D} \) will be much lower than that of \( D_s^{\ast 0} \).

The predicted mass of the truly exotic \( \hat{E}^0 = \{[c\bar{s}]_{\frac{1}{2}}[\bar{u}d]_{\frac{1}{2}}\} \) is lower than the \( D\bar{K} \) threshold, (at most, a little bit higher than the threshold, even if the upper bound of the estimated large errors is taken), in the present approach. Therefore, its strong decay would be forbidden (or kinematically suppressed), and, in addition, its radiative decay is not allowed. This suggests that its search in inclusive \( e^+e^- \) annihilation might not be very easy. If its strong decay is strictly forbidden, it might be detected, for example, in the successive weak decays, \( B \to \hat{D}\hat{E}^0 \to \hat{D}(\overline{K}K\pi)^0 \) \[24\] and \( B \to D\hat{E}^0 \to D(\overline{K}K\ell\nu) \), where \( \ell = \mu \) or \( e \).

The hidden-charm non-strange \( \delta^c \) and \( \sigma^c \) have a degenerate mass \( M(\delta^c) = M(\sigma^c) \sim 3.0 \text{ GeV} \), which is lower than the \( \eta\pi \) threshold, in the present approach and therefore, their searches are expected to be done in OZI-suppressed channels and radiative ones. Here it should be noted that the above result is much lower than those from the other approaches, for example, 3723 MeV in the present work as the mass of the lowest hidden-charm non-strange scalar meson from the diquark-antidiquark model \[27\] and around 3700 MeV from a unitarized chiral model \[28\]. Therefore, confirmation of existence of \( \delta^c(3200) \) and \( \sigma^c(3200) \) will be useful to select a realistic model of multi-quark mesons.

In summary the present results on masses of open-charm tetra-quark scalar mesons are not very much different from our previous estimates by using a naive quark counting, while those of hidden-charm sector are considerably lower than the earlier ones, so that expected decay property of the latter is now drastically changed. In particular, the predicted mass of \( \delta^c \) in the present work is much lower than our previous result, so that it now cannot decay into \( \eta\pi \), in contrast to our previous work. As the result, its dominant decay will be the OZI-suppressed \( \delta^c \to e^+e^- \). This should be compared with the existing result from two photon collisions that a tiny \( \eta\pi \) peak around 3.2 GeV was indicated but no signal in the \( \eta\pi \) channel. (If its mass is truly 3.2 GeV or higher, the experiment should have observed a peak at the same energy in the \( \eta\pi \) channel.) In addition, to search for hidden-charm tetra-quark scalar mesons, their OZI-suppressed and radiative decay channels would be important, because their OZI-allowed hadronic decays are kinematically suppressed or not allowed. However, detailed studies of these decays are left intact as our future subjects.

With regard to the charm-strange scalar \( D_{s0}^{\ast 0}(2317) \), no indication of \( D_{s0}^{\ast 0}(2317) \to D_s^{\ast +}\gamma \) and no signal of its neutral and doubly charged partners in \( B \) decays would be a serious dilemma in its tetra-quark interpretations, unless production rates for its neutral and doubly charged partners in addition to its iso-singlet one are unexpectedly suppressed. In contrast, there exist arguments that hadronic loop contributions might induce a strong suppression of the \( D_s^{\ast +}\gamma \) decay and/or an extraordinary enhancement of the isospin nonconserving \( D_s^{\ast +}\pi^0 \) decay of iso-singlet \( DK \) molecule \[29\,30\]. Therefore, one might consider that the \( DK \) molecular picture of \( D_{s0}^{\ast 0}(2317) \) is favored by experiments. Nevertheless, these analyses contain various adjustable parameters and exponential tunings of their values are needed to solve the above puzzle. In \[29\], for example, a size parameter \( \Lambda_{D_{s0}^{\ast 0}(2317)} \) describing the size of \( D_{s0}^{\ast 0}(2317) \) has been introduced, and it has been discussed that the above puzzle can be solved for \( \Lambda_{D_{s0}^{\ast 0}(2317)} > 1 \text{ GeV} \). However, this condition implies that the size of the molecular \( DK \sim D_{s0}^{\ast 0}(2317) \) is much compact than the constituent \( K \) meson with its charge radius \( \sqrt{\langle r^2 \rangle} \approx 0.58 \text{ fm} \) \[31\], i.e., the constituent \( K \), which is treated as a point particle in the analysis, has a size larger than the molecular \( D_{s0}^{\ast 0}(2317) \). On the other hand, in a more recent analysis \[30\] in which more restricted loop contributions have been taken into account, it has been argued that the isospin nonconserving \( D_s^{\ast +}\pi^0 \) decay of \( D_{s0}^{\ast 0}(2317) \) is sensitive to tuning of adjustable parameters involved, and the resulting ratio of rates \( R = \Gamma(D_{s0}^{\ast 0}(2317) \to D_s^{\ast +}\gamma)/\Gamma(D_{s0}^{\ast 0}(2317) \to D_s^{\ast +}\pi^0) \) with a makeshift tuning of their values can be considerably lower than unity, while it cannot completely satisfy its measured restriction \( R_{\text{exp}} < 0.059 \) \[23\]. In this case, it might not be easy to understand why no indication of \( D_{s0}^{\ast 0}(2317) \) has been observed in the \( D_s^{\ast +}\gamma \) channel. Thus, it seems to be still unclear at the present stage if the iso-singlet \( DK \) molecular picture of \( D_{s0}^{\ast 0}(2317) \) is acceptable.

As seen above, physics of \( D_{s0}^{\ast 0}(2317) \) and its partners seems to be still confusing. To determine their structure, more experimental and theoretical investigations will be needed.
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