Supplementary data for:

**Quantification of dissipation and deformation in ambient atomic force microscopy**

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The equation of motion to be solved via numerical integration is

\[
\begin{aligned}
    m \frac{d^2 z}{dt^2} + \frac{m \omega^2}{Q} \frac{dz}{dt} + k z &= F_{ts} + F_0 \cos \omega t \\
\end{aligned}
\]  

(S1)

where \(k\) is the spring constant, \(Q\) is the Q factor due to dissipation with the medium, \(\omega=\omega_0\) are the drive and natural angular frequency of oscillation respectively, \(m=k/\omega^2\), \(F_{ts}\) is the instantaneous (net) tip-sample interaction, \(z\) is the instantaneous position of the tip relative to the cantilever and \(F_0\) is the drive force. For \(F_{ts}\) the tip-surface interaction (Fig. 2) we write[1, 2]

\[
F_s = -\frac{H_s R}{6d_s^2} \quad d>d_{\text{off}} \quad \text{(tip retraction)} \quad \text{and} \quad d>d_{\text{on}} \quad \text{(tip approach)}
\]  

(S2)
\[ F_{AD}(d) = \frac{F_{Doff} - F_{Don}}{d_{off} - d_{on}} + F_{Don} \quad \text{d}_{off} > d > d_{on} \text{ (tip retraction)} \]  

(S3)

where

\[ F_{Don} = -\frac{R_{tip}}{6a_0^2} \left[ \frac{H_w - H}{d_{off} - a_0} (d_{off} - a_0) + H \right] \]  

(S4)

and

\[ F_{Doff} = -C_{off} \frac{R H_w}{6a_0^2} \quad C_{off} \geq 0 \]  

(S5)

\[ F_{AD} = -\frac{H^* R}{6a_0^2} \quad a_0 < d < d_{on} \]  

(S6)

where

\[ H^* = \frac{H_w - H}{d_{off} - a_0} (d_{off} - a_0) + H \]  

(S7)

\[ F_{AD} = -\frac{HR}{6a_0^2} = -4aR \chi \quad d \leq a_0 \]  

(S8)

where \( d \) is the instantaneous tip-sample's surface distance, \( F_a \) is the attractive and conservative long range force, \( F_{AD} \) is the adhesion force, \( d_{off} \) is the distance at which the capillary force ruptures, \( d_{on} \) is the distance at which the capillary force breaks, \( H \) is the Hamaker constant for the tip and the sample, \( H_w \) is the Hamaker constant corresponding to the hydrated tip-hydrated
surface interaction, $d_w$ is an effective distance of interaction that takes into account the fact that in the presence of adsorbed water films the tip and the surface as $d_w = d - 2h$ where $h$ is the height of the water films on the tip and sample's surfaces [1-3], $a_0$ is an intermolecular distance that implies that matter interpenetration cannot occur, $\gamma$ is the surface energy and $C_{off}$ which is defined next.

The interpretation of (S3-S8) is that that when the water layers overlap as a consequence of capillary neck formation, the force is relatively constant for a distance $a_0 < d < d_{on}$ but significantly decays with separation at the larger distances $d_{on} < d < d_{off}$[1-3]. The decay in the adhesion force is controlled by $C_{off}$ where $C_{off} = 0.3$ has been used here throughout. When the capillary neck is formed the capillary force is also added[2, 4]

\[
F_{cap}(d) = -\frac{2 \pi \gamma_w R_{tip}}{1 + \frac{\pi d^3}{V_{men}}}
\]
provided capillary on and $d > a_0$  \hspace{1cm} (S9)

where $\gamma_w$ is the surface energy of water. Also, $H_w = 24\pi(a_0)^2\gamma_w$ and $H = 24\pi(a_0)^2\gamma$[5].

The distance $d_{on}$ has been taken to be $d_{on} = 3h[2]$. Additionally $d_{off}$ can be calculated numerically by solving the Laplace-Young equation to give[6, 7]

\[
d_{off} \approx V_{men}^{1/3} - \frac{1}{5R}V_{men}^{2/3}
\]
\hspace{1cm} (S10)
where $V_{men}$ is the volume of the meniscus or the volume of water forming the water bridge. $V_{men}$ is calculated using geometrical considerations[7]

$$V_{men} = 4 \pi R h^2 + \frac{4}{3} \pi h^3$$  \hspace{1cm} (S11)

From the above expressions, it is clear that for $d_{on}<d<d_{off}$ there is hysteresis due to capillarity. Thus energy is dissipated (green are in Fig. 2). The conservative short range repulsive force is written as[8, 9]

$$F_{DMT} = a \phi^{\frac{3}{2}} \hspace{1cm} d \leq a_0$$  \hspace{1cm} (S12)

where

$$a = \frac{4}{3} E^* \sqrt{R}$$  \hspace{1cm} (S13)

and where $E^*$ is the effective elastic modulus of the contacting bodies (tip $E_t$ and sample $E_s$). The dissipation in during sample deformation, i.e. $d<a_0$, is modeled by (3) and (4).

In the numerical simulations in the main text $k=40$N/m, $Q=500$, $\omega=2\pi f_0$ ($f_0=300$ kHz), $E_t=170$ GPa (Young modulus of the tip), $E_s=1$ GPa, $a_0=0.165$ nm, $R=7$ nm, $\gamma=7$ mJ, $\gamma_w=10$ mJ, $F_{AD} \approx 1$ nN (net), $\alpha=0.5$ and $\eta=500$ Pa·s. The equation of motion has been solved numerically with the use of a Runge Kutta algorithm and implemented in both C and Matlab[10]. Both implementations were equivalent while C was more than an order of magnitude faster.
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