Simulation study of surfing acceleration in magnetized space plasmas

B Eliasson, M E Dieckmann and P K Shukla

Fakultät für Physik und Astronomie, Institut für Theoretische Physik IV, Ruhr-Universität Bochum, D-44780 Bochum, Germany
E-mail: bengt@tp4.rub.de

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Abstract. We present a numerical study of the surfing mechanism in which electrons are trapped in Bernstein–Greene–Kruskal (BGK) modes, and are accelerated across the magnetic field direction by the Lorentz force in magnetized space plasmas. The BGK modes are the product of an ion-beam Buneman instability that excites large-amplitude electrostatic upper-hybrid waves in the plasma. Our study, which is performed with particle-in-cell (PIC) and Vlasov codes, reveals the stability of the BGK mode as a function of the magnetic field strength and the ion beam speed. It is found that the surfing acceleration is more effective for a weaker magnetic field owing to the longer lifetime of the BGK modes. The importance of our investigation to electron acceleration in astrophysical environments has been emphasized.

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Acknowledgments

References

1 Author to whom any correspondence should be addressed.
2 On leave from the Department of Science and Technology, Linköping University, SE-601 74 Norrköping, Sweden.
1. Introduction

A natural saturation mechanism of large-amplitude electrostatic waves in plasmas is the formation of Bernstein–Greene–Kruskal (BGK) waves [1], where a population of electrons is trapped in the potential wells of the wave, and where the wave potential is reinforced by these trapped electrons. In such nonlinear waves, one thus has one population of trapped electrons which bounce in the potential well of the wave, and the other population of untrapped electrons which quiver in the passing wave potential. In their classic paper, Sagdeev and Shapiro [2] presented an investigation of the surfing acceleration and Landau damping in a non-relativistic magnetoplasma, where electrons that are transported across the magnetic field direction experience an acceleration perpendicular to both the magnetic field and the wave propagation directions by the Lorentz force. In the non-relativistic case, the particles will eventually escape the potential well [2, 3], while in the relativistic case, electrons can, in principle, be accelerated to unlimited speeds [4].

The surfing acceleration can also be mediated, for example, by large amplitude magnetosonic waves and shocks [5], which can accelerate ions to relativistic speeds. The surfing acceleration can be terminated if the driving electrostatic wave collapses and the trapped particles are released from the potential well. In an unmagnetized plasma, where the periodic simulation box is the same as the wavelength of the periodic BGK mode, the BGK modes are extremely long-lived [6, 7]. Long-lived BGK modes can also be considered as a possible candidate for an effective surfing acceleration in magnetized plasmas. However, when one considers the interaction of several BGK modes, one has recognized that they perform non-elastic collisions and have a tendency to merge [8]. A similar phenomenon occurs for periodic trains of electron holes, where the sideband instability makes adjacent holes merge, leading to a doubling of the wavelength of periodic structures [9]–[11], and eventually to a collapse of the BGK modes. A probable end product is that a few, well separated in space, electron holes survive and can live for a longer time, since they no longer experience the sideband instability. Observations by several spacecrafts in the Earth’s auroral zone and magnetosphere indeed reveal solitary potential structures, which are thought to be electron holes [12]–[15]. A numerical study [11] shows, however, that the sideband instability may be less effective than previously thought, so that surfing acceleration can have the time to take place. In multiple spatial dimensions, the phase space holes seem to be unstable and disintegrate, while computer simulations with strong magnetic fields indicate that standing electron holes can be stabilized and form phase space tubes perpendicular to the magnetic field direction [16], and where the stabilization of the electron holes occur if the electron gyrofrequency is larger than the bouncing frequency of the trapped electrons [17]. On the other hand, propagating electron holes can be destabilized by a magnetic field which is aligned perpendicular to the propagation direction [18], if the electron gyrofrequency is of the order of 10% of the electron plasma frequency, while PIC simulations where the gyrofrequency is 1% of the plasma frequency makes the electron hole more stable and an efficient surfing acceleration has the time to take place, which is needed for the Fermi acceleration at shocks [19].

In this paper, we present results of simulation studies of electron surfing acceleration that are carried out by means of PIC [20] and Vlasov [21, 22] codes. The simulation studies are performed in a periodic box with a length which equals the width of a single electron hole, thereby excluding all but one unstable sideband mode [23]. We shall investigate the stability of electron holes and the effectiveness of the surfing electron acceleration for the magnetic field strength that is relevant for the solar wind at the Earth’s bow shock, and for the weaker magnetic field strength that is relevant at supernova remnant shocks. The paper is organized in the following
fashion. We describe the numerical setup and physical parameters in section 2. In section 3 we discuss numerical results from the Vlasov and PIC simulations. Conclusions and applications of our results are discussed in section 4.

2. Physical model and numerical setup

Collisionless shocks exist both at the bow shock of the Earth, where the surrounding solar wind streams with supersonic speed and collides with the geomagnetic field, and in the vicinity of supernovae where the plasma is ejected and propagates with high speed into the interstellar medium. Ions can be reflected either by the electrostatic potential of the shock or by the compressed magnetic field downstream the shock, to form a beam of ions in front of the shock. The beam may then be rotated in the upstream magnetic field and returned into the shock as a counterpropagating beam [24]. The ion beams can interact with electron waves and give rise to a Buneman instability where waves that have the same phase speed as the beam grow exponentially. The nonlinear saturation of the electrostatic wave occurs when electrons get trapped at the potential maxima of the wave potential, and subsequently BGK modes are formed.

In our numerical model, we use as an initial condition, two counterpropagating ion beams, which are propagating relative to the bulk plasma with the speeds $\pm v_b$. The electrons are assumed initially to form a homogeneous Maxwell-distributed background with a temperature $T_e$ and a zero mean velocity. The ion beam propagating in the positive $x$ direction with speed $v_b$ (relative to the mean speed of the electrons) is assumed to have a shifted Maxwellian particle distribution with the temperature $10T_e$, while the counterpropagating ion beam (with speed $-v_b$) has the temperature $100T_e$. The bulk ions have a temperature of $10T_e$. We assume that both the bulk and beam ions consist of protons with mass $m_i = 1836m_e$, where $m_e$ is the electron mass. The particle number densities of the two ion beams are each taken to be $n_b = n_0/6$, where $n_0$ denotes the unperturbed electron number density, while the bulk protons have the density $n_{\text{bulk}} = 2n_0/3$ so that the unperturbed plasma is charge neutral. In our simulations, we have set spatial box length $L = 2\pi v_b/\omega_{pe}$, which corresponds approximately to the wavelength of the most unstable Buneman mode. Here, $\omega_{pe} = (n_0e^2/\epsilon_0m_e)^{1/2}$ is the electron plasma frequency, $e$ the magnitude of the electron charge and $\epsilon_0$ the electric permittivity in vacuum. The electrostatic and non-relativistic Vlasov simulations solve the Vlasov–Poisson system

$$\frac{\partial f_j}{\partial t} + v_x \frac{\partial f_j}{\partial x} + \frac{q_j}{m_j} \left[ -\frac{\partial \phi}{\partial x} \hat{x} + (v_x \hat{x} + v_y \hat{y}) \times B_0 \hat{z} \right] \cdot \frac{\partial f_j}{\partial \mathbf{v}} = 0,$$

where $B_0$ is the constant external magnetic field, and

$$-\frac{\partial^2 \phi}{\partial x^2} = \frac{\rho}{\epsilon_0},$$

where the charge density is $\rho = e(n_i - n_e)$ and the particle number densities

$$n_j = \int_{-\infty}^{\infty} f_j \, dv$$

are obtained from the continuous particle distribution functions $f_j(x, v_x, v_y, t)$. Here, $j$ equals $e$ for electrons and $i$ for ions, $q_e = -e$ and $q_i = e$, where $e$ is the magnitude of the electron
charge, and $\hat{x}$, $\hat{y}$ and $\hat{z}$ denote the unit vectors in the $x$, $y$ and $z$ directions, respectively. The values of $f_j$ are represented on a fixed grid, and the numerical method is based on a Fourier method in velocity space, a pseudo-spectral method in space and a fourth-order Runge–Kutta scheme in time [21, 22]. The PIC simulation code represents the particle distribution function with a large number of discrete superparticles, where each superparticle represents a large number of real electrons or ions. The electromagnetic field is represented on a fixed equidistant grid in space, and the value of the electric and magnetic fields are interpolated to the position of each particle. Then the momentum and position of particle $k$ is advanced via the equations of motion

$$\frac{dp_j^{(k)}}{dt} = Q_j \left[ E_j(x_j^{(k)})\hat{x} + \frac{p_j^{(k)}}{\gamma_j^{(k)} M_j} \times B(x_j^{(k)})\hat{z} \right],$$

(4)

where $\gamma_j^{(k)} = \sqrt{1 + [p_j^{(k)}]^2/M_j^2 c^2}$ and

$$\frac{dx_j^{(k)}}{dt} = \frac{p_j^{(k)}}{\gamma_j^{(k)} M_j},$$

(5)

where $Q_j$ and $M_j$ are the charge and mass, respectively, of the superparticles. From the positions and velocities of each particle, the charge and current densities $\rho$ and $j$ are calculated via an interpolating scheme and entered into Maxwell’s system of equations

$$\frac{\partial B}{\partial t} = -\nabla \times E$$

(6)

and

$$\frac{\partial E}{\partial t} = -\frac{j}{\epsilon_0} + c^2 \nabla \times B,$$

(7)

which is advanced in time with the initial conditions $B = B_0 \hat{z}$ and $\nabla \cdot E = \rho/\epsilon_0$ for the magnetic and electric fields. In our one-dimensional case, we have $\nabla = \hat{x} \partial / \partial x$. The PIC code is based on a Lagrangian scheme [20] which has a relatively low numerical noise level. In the physical problem, the box length and particle velocities are relatively small so that relativistic and electromagnetic effects will play only a minor role, thus making the two codes comparable. The Vlasov code uses a maximum representation of velocity space of $v_{\text{max}} = 20\pi v_{\text{Te}} \approx 63 v_{\text{Te}}$, which is resolved with 400 intervals, where $v_{\text{Te}} = (k_B T_e/m_e)^{1/2}$ is the electron thermal speed and $k_B$ is the Boltzmann constant. The space is resolved with 50 intervals, and the time step is adjusted dynamically to maintain numerical stability. The PIC simulations use 180 intervals to resolve the space, 2.7 million particles for the electrons and 432 000 particles for each proton species (the bulk protons and the two beams).

3. Simulation results

We first study the linear growth rates of the exponentially growing $\propto \exp(\gamma t)$ amplitude of the Buneman instability. The results are presented in figure 1, where one can see that the Buneman instability is strongly dependent on the beam velocity. The growth rate was not large enough to
be measurable in the numerical experiments for beam speeds below \( v_b \approx 7v_{Te} \), while for larger speeds the growth rate approached the theoretical value for a cold ion beam in a cold electron plasma, as given by [25]

\[
\frac{\gamma_{\text{cold}}}{\omega_{pe}} = \left( \frac{3\sqrt{3} \omega^2_{pb}}{16 \omega^2_{pe}} \right)^{1/3},
\]

where \( \omega_{pb} = (n_b e^2/\epsilon_0 m_b)^{1/2} \) denotes the plasma frequency of the beam. Here, we have \( \omega^2_{pb}/\omega^2_{pe} = (n_b/n_0)(m_e/m_b) \), where \( n_b/n_0 = 1/6 \) and \( m_e/m_b = 1/1836 \), yielding the value \( \gamma_{\text{cold}}/\omega_{pe} \approx 0.03 \), indicated by the dashed line in figure 1. We observe that the linear growth rates obtained from the Vlasov and PIC simulations are in excellent agreement.

We next turn to the nonlinear saturation of the Buneman instability. When the electrostatic wave reaches a large enough amplitude, electrons get trapped in the potential maxima of the wave, and at this time the transfer of energy from the ion beam to the wave becomes less effective, and the growth of the wave amplitude gets reduced or halted. In figure 2, we have compared the amplitude of the electric field as a function of time for different beam speeds and magnetic fields, latter represented by different values of the electron gyrofrequency \( eB_0/m_e \). We clearly see the initial exponential growth phase of the instability, followed by the saturation where the electric field reaches a maximum value, and then a more or less rapid decrease of the electric field. In general, the growth of the wave starts to saturate at a somewhat earlier stage (at a smaller amplitude) in the Vlasov simulation than in the PIC simulation. This is most clearly seen in the lower panels for the weak magnetic field case where the electric field in the Vlasov
The peak electric field as a function of time, obtained from the Vlasov (left panels) and PIC simulations (right panels). Parameters are: $v_b = 15v_{Te}$ and $\omega_{pe} = 30\omega_{ce}$ (upper panels), $v_b = 10v_{Te}$ and $\omega_{pe} = 30\omega_{ce}$ (middle panels) and $v_b = 15v_{Te}$ and $\omega_{pe} = 100\omega_{ce}$ (lower panels).

The simulation shows a clear decrease of the growth rate at $\omega_{pe}t \approx 400$, while in the PIC simulation the electric field rapidly reaches its peak value after which it decreases. We have previously identified this effect as a difference between the Vlasov and PIC codes in the representation of the electron distribution function, where the Vlasov code can represent the phase density tail of the Maxwellian distribution to lower values than the PIC code [11]. This leads to an earlier trapping of electrons and the formation of electron phase space holes in the Vlasov simulations compared to the PIC simulations, and to an earlier saturation in the Vlasov simulations than in the PIC simulations. The electric field reaches the largest amplitude and also shows the most rapid decrease for the largest values of the beam speed and the magnetic field, displayed in the upper panel.
panels of figure 2. Compared to this case, the peak electric field is almost halved for the case with somewhat slower beam speed in the middle panels. Here the electric field amplitude does not decrease as rapidly as in the higher beam speed case, but shows a complicated oscillatory behaviour. In the weak magnetic field case, displayed in the lower panels of figure 2, we observe that after the peak of the electric field, the electric field amplitude decreases relatively slowly compared to the case with the stronger magnetic field in the upper panels, and for the Vlasov simulation (left panels), there is a more distinct peak of the electric field in the strong magnetic field case than in the weak magnetic field case. This can be explained in that the fastest electrons, which are trapped early in the wave potential, start to experience surfing acceleration and are accelerated in the \(v_x\) direction, after which they are decelerated and detrapped from the wave, so that they do not contribute to the saturation of the growing wave. In this way, the electric field can grow faster for a stronger magnetic field than in the weaker magnetic field, where the trapped electrons contribute to the saturation of the wave, clearly seen in the lower left panel of figure 2.

In order to investigate the influence of the magnetic field strength on the surfing mechanism, we have plotted the distribution function for electrons at different times for the case \(v_b = 15v_{Te}\) in figures 3–6. In these plots, we have made a projection of the three-dimensional distribution function in \((x, p_x, p_y)\) space down to two dimensions by integrating over the third dimension. Accordingly, we display the function \(F(x, v_x) = \int f \, dv_y\) in the left panels and the function \(F(v_x, v_y) = \int_{L_0}^{L} f \, dx\) in the right panels of figures 3–6. Figures 3 and 4 show the distribution functions at different times, corresponding to the electric field in the upper left and right panels of figure 2, respectively. The upper panels of figures 3 and 4 exhibit an electron distribution function at times shortly before the electric field has reached its peak value. Here (in the upper left panels), we see that unmistakable BGK modes have been formed in both the Vlasov and PIC simulations. In the second right panel from the top of figures 3 and 4, we can see that electrons are being accelerated in the \(v_y\) direction also by the magnetic field. In the lower panels of figures 3 and 4, we see that the BGK mode has disappeared (left panels) and that the distribution of electrons has reached a more or less isothermal distribution rotating in the \((v_x, v_y)\) space. The fastest electrons here have reached speeds of around 30\(v_{Te}\), which corresponds to the fastest particles initially trapped in the BGK mode seen in the upper panels. Thus, there has not been any efficient surfing acceleration in this case.

We now turn to the weaker magnetic field case, displayed in figures 5 and 6. The initial stage with the trapping of electrons is here similar to the stronger magnetic field case, as depicted in figures 3 and 4; therefore in the following, we present the phase-space distribution at somewhat later times. We see in the left panels of figures 5 and 6 that the BGK modes survive for a much longer time than in the stronger magnetic field case, displayed in figures 3 and 5. In the right panels of figures 5 and 6, we see that the acceleration cone in the \(v_y\) direction continues to grow, and in the final stage, we see in the lower right panel of figure 6 that the fastest electrons have attained speeds of approximately 50\(v_{Te}\), which well exceeds the speed of the initially trapped electrons. At this stage, a population of electrons get detrapped from the BGK wave potential and start to rotate counterclockwise in the \((v_x, v_y)\) space. The surfing acceleration mechanism has thus been more efficient in the weak magnetic field case than in the strong magnetic field case, owing to the stability of the BGK modes.
Figure 3. The electron phase-space function (10-logarithmic colour scale) obtained from the Vlasov simulation, integrated over $v_y$ as a function of $v_x$ and $x$ (left panels) and integrated over $x$ as a function of $v_x$ and $v_y$ (right panels) at different times, for the strong magnetic field case $\omega_{pe}/\omega_{ce} = 30$. The beam speed is $v_b = 15v_{Te}$.
Figure 4. The electron phase-space function (10-logarithmic colour scale) obtained from the PIC simulation, integrated over $v_y$ as a function of $v_x$ and $x$ (left panels) and integrated over $x$ as a function of $v_x$ and $v_y$ (right panels) at different times, for the strong magnetic field case $\omega_{pe}/\omega_{ce} = 30$. The beam speed is $v_b = 15v_{Te}$. 

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Figure 5. The electron phase-space function (10-logarithmic colour scale) obtained from the Vlasov simulation, integrated over $v_y$ as a function of $v_x$ and $x$ (left panels) and integrated over $x$ as a function of $v_x$ and $v_y$ (right panels) at different times, for the weak magnetic field case $\omega_{pe}/\omega_{ce} = 100$. The beam speed is $v_b = 15v_{Te}$. 

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Figure 6. The electron phase-space function (10-logarithmic colour scale) obtained from the PIC simulation, integrated over $v_y$ as a function of $v_x$ and $x$ (left panels) and integrated over $x$ as a function of $v_x$ and $v_y$ (right panels) at different times, for the weak magnetic field case $\omega_{pe}/\omega_{ce} = 100$. The beam speed is $v_b = 15v_{Te}$.
4. Conclusions and discussion

To conclude, we have performed numerical simulation studies of the surfing mechanism in magnetized space plasmas, appropriate for a stronger magnetic field in the solar wind and for a weaker magnetic field relevant for supernova remnant shocks. We find that in plasmas, the stronger magnetic field, namely $\omega_{\text{pe}}/\omega_{\text{ce}} = 30$, waves grow to a maximum amplitude, followed by a rapid collapse of the BGK modes. In this case, the electrons suffer only a minor surfing acceleration perpendicular to the wave direction of motion, where the speed does not exceed the speed of the electrons initially trapped in the BGK modes. On the other hand, in the weaker magnetic field case, namely $\omega_{\text{pe}}/\omega_{\text{ce}} = 100$, the waves grow to a maximum amplitude, after which the amplitude slowly decreases. In this case, there is also a clear surfing acceleration of the electrons, where the electrons reach speeds approximately twice the speed of the initially trapped electrons. By examining the distribution of electrons in phase space at different times, we find that in the stronger magnetic field, the trapped particles first experience an acceleration perpendicular to the magnetic field, followed by a rapid detrapping of the particles and a collapse of the BGK mode. In the weaker magnetic field, however, the electrons have the time to bounce many times in the potential well of the BGK mode, while also being accelerated perpendicularly to the wave direction. In this case, the electrons are only slowly being detrapped, so that the BGK mode remains stable for a longer time. A population of electrons then have the time to experience surfing acceleration, leading to speeds approximately twice of those in the stronger magnetic field case. We thus conclude that surfing acceleration may not be an effective electron surfing mechanism by propagating electron holes in the solar interplanetary medium, while it can be more effective in the weaker magnetic field of supernovae remnant shocks and other astrophysical objects. We further remark that relativistic effects may play an important role to stabilize the electron holes both against the detrapping mechanism studied here and against the sideband instability. Some astrophysical observations seem to indicate that there exist a threshold for the speed of quasi-perpendicular shocks (between 1 and 3% of the light speed) above which the efficiency of the electron injection mechanism increases by orders of magnitude [24]. A numerical investigation of the impact of relativistic effects on the surfing mechanism is the subject of forthcoming studies.

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