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Capacitive micromachined ultrasonic transducer for ultra-low pressure measurement: Theoretical study

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Ultra-low pressure measurement is necessary in many areas, such as high-vacuum environment monitoring, process control and biomedical applications. This paper presents a novel approach for ultra-low pressure measurement where capacitive micromachined ultrasonic transducers (CMUTs) are used as the sensing elements. The working principle is based on the resonant frequency shift of the membrane under the applied pressure. The membranes of the biased CMUTs can produce a larger resonant frequency shift than the diaphragms with no DC bias in the state-of-the-art resonant pressure sensors, which contributes to pressure sensitivity improvement. The theoretical analysis and finite element method (FEM) simulation were employed to study the relationship between the resonant frequency and the pressure. The results demonstrated excellent capability of the CMUTs for ultra-low pressure measurement. It is shown that the resonant frequency of the CMUT varies linearly with the applied pressure. A sensitivity of more than 6.33 ppm/Pa (68 kHz/kPa) was obtained within a pressure range of 0 to 100 Pa when the CMUTs were biased at a DC voltage of 90% of the collapse voltage. It was also demonstrated that the pressure sensitivity can be adjusted by the DC bias voltage. In addition, the effects of air damping and ambient temperature on the resonant frequency were also studied. The effect of air damping is negligible for the pressures below 1000 Pa. To eliminate the temperature effect on the resonant frequency, a temperature compensating method was proposed.

I. INTRODUCTION

Ultra-low pressure measurement is of great importance in areas of high-vacuum environment monitoring, process control and biomedical applications, which is a challenging problem as the measured pressure is extremely small.¹–³ With recent advancement in MEMS technology, numerous investigations focused on silicon-based micromachined pressure sensors due to their small size, high sensitivity and low-energy consumption. Silicon-based micromachined pressure sensors can be mainly divided into three categories: piezoresistive, capacitive, and resonant pressure sensors.⁴–⁷ However, all three techniques suffer from intrinsic drawbacks in measuring the pressures below 1000 Pa. For piezoresistive sensors, the measurement limit of low pressure can be improved by reducing the piezoresistive diaphragm thickness, however, reduction of the diaphragm thickness dramatically increases the mechanical nonlinearity.⁴,⁸ The capacitive pressure sensors suffer inherent nonlinearity and small output capacitance, which are susceptible to parasitic effects and requires complex post-processing circuits to compensate for such negative effects.³ For resonant pressure sensors, the

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common complicated stress-relieving vacuum packaging, process and electrical feedthroughs to the outside make its design and fabrication a challenging problem, hindering its widespread application.\(^6\)

In this study, we used capacitive micromachined ultrasonic transducers (CMUTs) as an electrostatically actuated resonant sensor for ultra-low pressure measurement. Compared with the clamped diaphragm or diaphragm-beam resonant elements in the state-of-the-art resonant pressure sensors,\(^6\) CMUT membranes (thin plates in most cases) under DC bias voltages are able to achieve a larger resonant frequency shift for the same applied pressure. This is due to the fact that the pressure can generate larger changes in the deflection and stresses of the CMUT membrane under bias voltage than the diaphragm or diaphragm-beam elements without DC bias voltage. This fact can be analytically explained by the following two expressions:

\[
\Delta w_0(V_{\text{bias}}, P) = \frac{\Delta P}{64[D - Q_1 R^4/(42d_0)]}(R^2 - r^2)^2, \tag{1}
\]

and

\[
\Delta w_0(P) = \frac{\Delta P}{64D}(R^2 - r^2)^2, \tag{2}
\]

where, \(\Delta w_0\) is the deflection change of the clamped membrane, \(\Delta P\) is the pressure change, and \(Q_1\) indicates the effect of the applied bias voltage, \(V_{\text{bias}}\), (see Eq. (A1) for the relationship between \(Q_1\) and \(V_{\text{bias}}\)). The detailed definitions for expressions (1) and (2) were given in Ref. 9 and Ref. 10, respectively. Equation (1) represents the pressure-induced deflection change of a clamped circular membrane under a bias voltage, while the Eq. (2) denotes the deflection of the membrane caused only by the measured pressure. It can be seen that the membrane under a bias voltage has a larger deflection than the membrane with no bias with identical applied pressure, which finally leads to a larger frequency shift. Therefore, the DC voltage biased CMUTs are able to achieve an amplified frequency shift, thus, improved pressure sensitivity. Besides, the high resonant frequency (tens of megahertz), high quality factor \(Q\) (several hundred) and small membrane thickness of the CMUTs also contribute to sensitivity improvement.\(^11\) Additionally, CMUTs have the advantages of batch fabrication, ease of integration with circuitry and a robust structure allowing reliable operation in harsh environment. Therefore, CMUTs can be an attractive platform for pressure measurement. The performances (such as linearity and sensitivity) of the CMUTs for ultra-low pressure measurement were investigated by combining theoretical analyses and FEM simulations. The effects of air damping and temperature on the sensing performance were also studied.

II. WORKING PRINCIPLE

A. Fundamental theory

The structure schematic of a CMUT for pressure measurement is shown in Fig. 1(a). The silicon membrane has an effective radius of \(R\) and thickness of \(h\), and works as the top electrode and pressure-sensing element. The silicon substrate works as the bottom electrode of the CMUT. The electrode distance between the top and bottom electrodes is \(d_0\).

The resonant characteristics of the membrane are determined not only by its mechanical properties and geometry, but also by the DC bias voltage and the measured pressure. When a DC bias voltage, \(V_{\text{bias}}\), is applied across the top and bottom electrodes, the membrane is deflected towards the substrate and the well-known electrical spring softening phenomenon occurs, resulting in a shift in the resonant frequency of the membrane. If the measured pressure is superimposed on the membrane, the deflection change of the membrane caused by the pressure will lead to an increase in the electrostatic force, resulting in a further increase in the deflection of the membrane. As a result, the pressure applied to the membrane with the fixed bias voltage, \(V_{\text{bias}}\), induces a larger deflection than that caused by the pressure alone. By analogy with electrical spring softening, the additional electrostatic force change caused by the pressure can be interpreted as a pressure-induced electrical spring softening (PIESS). PIESSEffect also shifts the resonant frequency of the membrane. It has been demonstrated that the resonant frequency varies linearly with the pressure within a low range if a bias voltage
is applied across the electrodes.\(^9\) The relationship between the resonant frequency and the pressure acting on the membrane under the bias voltage is given as:

\[
f_P = f_0(a - bP),
\]

where \(f_P\) is the resonant frequency under the applied pressure, \(P\), \(f_0\) is the natural frequency of the membrane, and \(a\) and \(b\) are two coefficients determined by the mechanical properties and geometry of the membrane and the bias voltage, \(V_{\text{bias}}\). The \(a\) reflects the effect of the DC bias voltage on the resonant frequency, while \(b\) determines the resonant frequency shift due to the pressure, \(P\), under the bias voltage. The detailed expressions for \(f_0\), \(a\) and \(b\) are given in Ref. \(^9\) (see also Eqs. (A2) and (A3) in Appendix). The corresponding pressure sensitivity, \(S_P\), is defined as:

\[
S_P = \frac{1}{af_0} \frac{\partial f}{\partial P} = \frac{b}{a} \text{ (ppm/Pa)}.
\]

It can be seen from Eq. (4) that the sensitivity depends on not only the material and geometry parameters of the CMUT but also the applied bias voltage.

B. Resonant frequency detection

The circuit model shown in Fig. 1(b) is an equivalent circuit model of the CMUT mechanical parameters, which can be used for circuit design and resonant frequency analysis.\(^{12,13}\) The series capacitor \(C_m\) and inductor \(L_m\) represents the spring constant and the mass of the membrane, respectively. The negative capacitance \(-C_0\) accounts for the electrical spring softening effect due to the electromechanical interaction. The impedance \(Z_s\) indicates the motional resistance including the effects of radiation into the medium.\(^{13}\) The parallel capacitances \(C\) and \(C_{\text{ex}}\) are the clamped capacitance and external capacitance from the source such as interconnects and contact pads, respectively.\(^{12,13}\) The efficiency of this equivalent circuit model in modeling the impedance and frequency characteristics of the CMUT has been demonstrated in Ref. \(^{14}\).

Generally, there are two types of resonances in the equivalent circuit model of the CMUT, namely, parallel and series resonances. At the parallel resonance, the electrical impedance reaches its maximum value and the corresponding frequency is influenced (shifted down) by the inherent parasitic capacitance included in \(C\) and the external parasitic capacitance \(C_{\text{ex}}\).\(^{12}\) However, the series resonance occurs at the minimum of electrical impedance, and the corresponding frequency reduction is just caused by the negative capacitance \(-C_0\), which is a function of the deflection of the membrane and thus, is a function of both electrostatic force and the applied pressure in this study. Therefore, the series resonant frequency reflects the electrical spring softening effect and the PIESSEffect on the resonant frequency of the membrane, which is free from the influence of the
inherent and external parasitic capacitances. Therefore, detecting the changes in the series resonant frequency is a better choice for pressure measurement. This method can successfully avoid the effect of the parasitic capacitance on the sensing performance encountered in the state-of-the-art capacitive pressure sensors based on capacitance changes.\textsuperscript{5}

The circuit schematic shown in Fig. 1(a) is used to measure the resonant frequency. The matching circuits are used to effectively detect the electrical parameters (mainly the system impedance $Z_s$) and the resonant frequencies of the circuit system. Currently, there are two methods to obtain the resonant frequency. The first method is to use an impedance analyzer to measure the electrical impedance and determine the resonant frequency. The other method is to design an oscillator circuit to track the resonant frequency of the device. The later method allows for accurate real-time measurements using a digital frequency counter connected to the output of the oscillator circuit.\textsuperscript{15}

### III. PERFORMANCE STUDY

#### A. Validation of the sensing performance

Theoretical analyses and FEM simulations using a commercially available FEM package (ANSYS\textsuperscript{12.0}, ANSYS Inc., Canonsburg, PA) were used to characterize the performance of the CMUT in pressure measurement. Owing to the axisymmetry of the CMUT structure and the applied load, a 2D-axisymmetrical model was constructed to reduce computational time, which has the same accuracy as a full 3D model.\textsuperscript{12,16} As shown in the 2D model (Fig. 2), the membrane of the CMUT was constructed using axisymmetric plane elements (PLANE42), and the edge of the membrane was fixed. The electrostatic effect between the top and bottom electrodes of the CMUT was modeled using electromechanical coupling elements (TRANS126). The electromechanical coupling elements can apply electrostatic attraction forces to the nodes to which they are attached. The top nodes of these elements were attached to the lower surface of the membrane, and their bottom nodes were simply fixed to represent the bottom electrode. Multi-Field Solver was employed, and static analysis and pre-stressed modal analysis were conducted consecutively to determine the fundamental resonant frequencies of the membrane under different loads, such as different bias voltages, pressures and temperatures. For the nonlinear electrostatic analysis, the substep was set to make the applied voltage ramping in steps of 2 V. The mapped meshing was used, and the mesh was optimized by increasing the mesh in one dimension in step of 10. The resonant frequency difference between two adjacent meshing is less than $1.7\times10^{-3}\%$ at the optimized mesh, where the relatively accurate results is obtained. The detailed parameters for the FEM simulations are shown in Table I.

Figure 3 shows the resonant frequency as a function of the applied pressure within the range of 0 Pa to 100 Pa under a bias voltage of 65% of the collapse voltage, $V_{\text{colla}}$. The theoretical values were calculated using Eq. (3) by setting $f_0$ equal to the simulated value and eliminating the frequency difference between theoretical analysis and FEM simulation only under DC bias voltage.\textsuperscript{9} The simulation

**FIG. 2.** An axisymmetric 2D model for the CMUT. Y-axis is the symmetry axis of the 2D model. J and I nodes are referred to as the top and bottom nodes of the TRANS126 element, respectively. The potential of the bottom nodes of the TRANS126 element are set to be zero, and different potentials are applied to the top nodes to simulate the electrostatic force. It should be noted that the dimensions of this model are not as the same as those given in Table I for a clear view of the model.
TABLE I. Material and geometry parameters for simulations.

| Parameters                  | Value        | Parameters                  | Value                      |
|-----------------------------|--------------|-----------------------------|----------------------------|
| Radius (R)                  | 10 μm        | Density (ρm)                | 2332 kg/m³                 |
| Thickness (h)               | 0.4 μm       | Poisson’s ratio (ν)         | 0.29                       |
| Cavity gap (d₀)             | 0.12 μm      | Dielectric constant (ε₀)    | 8.854×10⁻⁶ pF/μm           |
| Young’s modulus (E)         | 169×10³ MPa  | Thermal expansion coefficient (α) | 2.6×10⁻⁶/°C |

results are consistent with those from the theoretical analysis. Both results show that the resonant frequencies vary linearly with the applied pressure. The pressure sensitivities from the simulation and theoretical analysis are -0.409 ppm/Pa and -0.391 ppm/Pa, respectively. The relative difference between them is 4.4%. Figure 4 shows a linear relationship between the resonant frequency and the applied pressure under a bias voltage of 90% of the $V_{colla}$. The corresponding pressure sensitivity is -6.33 ppm/Pa (68 kHz/kPa), which is two orders of magnitude larger than the sensitivities obtained by the state-of-the-art resonant pressure sensors with diaphragm-beam structures. Furthermore, Fig. 4 shows higher pressure sensitivity than that shown in Fig. 3. This indicates that the pressure sensitivity is improved by increasing the bias voltage. For better understanding this point, the pressure sensitivities under different bias voltages were calculated through FEM simulations and theoretical analysis using Eq. (4), the results of which are shown in Fig. 5. Both simulated and calculated results show that pressure sensitivity increases with the bias voltage.

As mentioned above, an increased bias voltage can enable improved sensitivity. However, as CMUTs work in its traditional mode (un-collapsed mode), the increased bias voltage will reduce the maximum pressure that the CMUTs can detect in its un-collapsed operation mode. As a result, there is a trade-off between the pressure measurement range and the sensitivity for a certain CMUT-based resonant pressure sensor.

B. Effect of air damping

When the membrane vibrates in air, air damping also influences the resonant frequency of CMUTs. Therefore, the effect of air damping is necessary to be considered. For the CMUTs with a vacuum cavity, there is no squeeze-film damping between the membrane and substrate. The main energy dissipation is the acoustic radiation into its surrounding medium from one side of the membrane, which can be equivalent to a virtual added mass on the membrane. The resonant frequency with air damping effects can be given as:

$$f_d = f_p / \sqrt{1 + \beta}, \quad \beta = 0.6689 \rho R / (\rho m h),$$

(5)

FIG. 3. Variation of the resonant frequency with respect to the applied pressure under a DC bias voltage of 16 V, 65% of the $V_{colla}$. 

FIG. 3. Variation of the resonant frequency with respect to the applied pressure under a DC bias voltage of 16 V, 65% of the $V_{colla}$. 

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where, $\beta$ is known as the added virtual mass factor,$^{22}$ $\rho_f$ is density of the fluid (the air in this study); $R$, $h$ and $\rho_m$ are the parameters of the CMUT membrane. As the $\beta<1$ in the air,$^{22,23}$ Eq. (5) can be linearized using Maclaurin’s theorem as:

$$f_d = f_p(1 - 0.5\beta),$$

(6)

where $f_d$ is the resonant frequency with air damping. The density of air at a temperature of 293 K can be calculated as the following:$^{22,24}$

$$\rho_f = 1.18 \times 10^{-5}P/P_0,$$

(7)

where, $P_0 = 1$ Pa. Using the parameters shown in Table I, the added virtual mass factor is obtained as $\beta = 8.46 \times 10^{-8}P/P_0$. Substituting the expression for $\beta$ into Eq. (6), Eq. (6) can be rewritten as:

$$f_d = f_p(1 - 4.23 \times 10^{-8}P/P_0).$$

(8)

Therefore, the relative difference between the resonant frequencies with and without air damping can be written as:

$$e = (f_p - f_d)/f_p = (4.23 \times 10^{-6}P/P_0) \%.$$  

(9)
It can be shown from Eq. (9) that variation caused by the air damping is negligible within the studied pressure range of 0 to 100 Pa, even for the pressure up to 1000 Pa. Therefore, the effect of the air damping on the resonant frequency can be neglected.

C. Effect of ambient temperature

Another factor that can affect the resonant frequency of CMUTs is the ambient temperature, which is a common problem for all pressure sensors. Assuming the temperature distribution is uniform for micro scale structures, the radial thermal stress of the membrane with the same thermal expansion coefficient (TEC) as the substrate can be given by:

$$\sigma_r = \frac{E\alpha}{1-\nu}(\Delta T_\text{m} - \Delta T_\text{s}) = \frac{E\alpha}{1-\nu}\Delta T, \quad (\Delta T_\text{m} - \Delta T_\text{s} = \Delta T),$$  

(10)

where, $\Delta T_\text{m}$ and $\Delta T_\text{s}$ are the temperature variations of the membrane and substrate, respectively, and $\Delta T$ is the temperature difference between them. The temperature difference causes thermal stress in the membrane, which finally leads to a resonant frequency shift of the membrane. To eliminate the temperature effect, two identical CMUTs subjected to different bias voltages can be used to achieve different pressure and temperature sensitivities. Figure 6 shows a study of the temperature effect on the resonant frequency of the example CMUT membrane under different pressures with a fixed bias voltage, in which the reference temperature is $20^\circ\text{C}$. As shown in Fig. 6, the resonant frequency shifts caused by the same temperature variation are identical under the different applied pressures. It indicates that the applied pressure has no cross-impact on the thermal-induced frequency shift. Figures 6 also show a linear relationship between the resonant frequency and temperature under different pressures. Therefore, the resonant frequencies of the two CMUTs with both effects of temperature and pressure considered can be given as:

$$f_1 = f_\text{V1} - s_1P - k_1\Delta T, \quad (f_\text{V1} = f_\text{0a1}_1, \quad s_1 = f_\text{0b1}_1)$$

$$f_2 = f_\text{V2} - s_2P - k_2\Delta T, \quad (f_\text{V2} = f_\text{0a2}_2, \quad s_2 = f_\text{0b2}_2),$$  

(11)

where, the subscripts 1 and 2 represent the working and compensating CMUTs, respectively; $f_\text{V1}$, $f_\text{V2}$ are the resonant frequencies of the working and compensating CMUTs under the bias voltages of $V_\text{bias1}$ and $V_\text{bias2}$, respectively; $s_1$ and $s_2$ are the coefficients determining the pressure-induced frequency shift; $k_1$ and $k_2$ are the coefficients reflecting the temperature-induced frequency shift; $f_1$ and $f_2$ are the resonant frequencies under the co-action of pressure, temperature and bias voltages. In order to validate Eq. (11), a comparison of the resonant frequencies extracted from FEM simulations and Eq. (11) under a pressure of 40 Pa and a bias voltage of 16 V are shown in Fig. 7, which shows a maximum difference of 0.057% between the simulated and calculated results. Therefore, Eq. (11) can

![FIG. 6. The effect of temperature on the resonant frequency under pressures of 0 Pa and 60 Pa with a constant bias voltage of 16 V.](image-url)
FIG. 7. Comparison of the resonant frequencies obtained by simulations and analytical calculations, the coefficients $f_v$, $s$ and $k$ were determined using FEM simulations firstly.

accurately predict the temperature influence on the resonant frequency. Eliminating $\Delta T$ in Eq. (11), the measured pressure, $P$, after compensation can be written as:

$$P = \frac{k_2(f_1 - f_{V1}) - k_1(f_2 - f_{V2})}{s_2k_1 - s_1k_2}. \tag{12}$$

Such method can reduce the temperature effect on pressure measurement if the temperature effect is accurately determined. This method has been experimentally demonstrated in Ref. 26 and was proven to be practical. Besides, since the thermal-induced resonant frequency shift depends on the temperature difference, $\Delta T$, and the TEC of the membrane, making $\Delta T$ close to zero and using materials with small TEC may be the most effective approach to eliminate the temperature effects. It should be noted that Fig. 6 shows extremely large temperature difference between the membrane and substrate of the CMUTs in order to better demonstrate the temperature effects on the resonant frequency of the CMUT membrane. However, in practice, temperature differences between different parts of a micro scale device, like CMUTs, would not come close to such high numbers. In addition, the temperature gradient in CMUTs approaches zero as the micro device is placed in ambient environment.

IV. CONCLUSION

In this study, we proposed a CMUT-based resonant pressure sensor for ultra-low pressure measurement. The fundamental theory and detection method, using frequency shift, were illustrated in detail. Theoretical analysis and FEM simulations were carried out to validate the performance of the CMUT for pressure measurement. The results for the studied case demonstrated a high pressure sensitivity of 68 kHz/kPa which is almost two order of magnitude higher than the state-of-the-art diaphragm-beam based resonant pressure sensors. It was also demonstrated that the sensitivity can be adjusted by the applied bias voltage. Furthermore, the effect of air damping on the resonant frequency was theoretically studied. The analytical results showed that the air damping effect is negligible for the pressures below 1000 Pa. Finally, a method was provided to eliminate the effect of ambient temperature on the resonant frequency. Overall, CMUTs show excellent performance for ultra-low pressure measurement, and are promising to be used as resonant pressure sensors.

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APPENDIX

Equation (1) shows the pressure-induced deflection change of a CMUT membrane under a bias voltage, $V_{bias}$, where the mathematic relationship between variables $Q_1$ and $V_{bias}$, is given as:

$$Q_1 = \frac{\varepsilon_0 V_{bias}^2}{2d_0^2}, \quad (A1)$$

where, $\varepsilon_0$ is the dielectric constant of the space between the top and bottom electrodes and $d_0$ is the effective electrode distance or cavity gap of CMUTs.

The expressions for variables $a$ and $b$ have been derived in Ref. 9. In this paper, $a$ and $b$ in Eqs. (3) and (4) can be rewritten as:

$$a = \left\{ 1 - 0.0958 \times \frac{\varepsilon_0 V_{bias}^2}{D R^6} \left[ \frac{5NQ_1 R^6 - 3d_0 R^2}{16N^2 Q_1^2 (d_0 - N Q_1 R^2)^2} + \frac{3}{32} \frac{\ln \sqrt{d_0 + \sqrt{NQ_1 R^2}}}{\ln \sqrt{d_0 - \sqrt{NQ_1 R^2}}} \right] \right\}^{\frac{1}{2}}, \quad (A2)$$

and

$$b = 0.0479 \times \frac{\varepsilon_0 V_{bias}^2}{a D N^2 Q_1^3} \left[ \frac{15d_0^2}{32R^4 (d_0 - N Q_1 R^2)^3} - \frac{15}{64R^8 \sqrt{d_0 NQ_1}} \frac{\ln \sqrt{d_0 + \sqrt{NQ_1 R^2}}}{\ln \sqrt{d_0 - \sqrt{NQ_1 R^2}}} \right], \quad (A3)$$

where, $N$ is given as

$$N = \frac{1}{64[D - Q_1 R^2/(42 d_0)]^3}, \quad (A4)$$

where, $D$ is the flexible stiffness of CMUT membrane (thin plate in most cases). $D = Eh^3/[12(1 - v^2)]$, where $E$ is the Young’s modulus; $v$ is the Poisson ratio.

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