Decoherence due to telegraph and 1/f noise in Josephson qubits

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We study decoherence due to random telegraph and 1/f noise in Josephson qubits. We illustrate differences between gaussian and non-gaussian effects at different working points and for different protocols. Features of the intrinsically non-gaussian and non-Markovian low-frequency noise may explain the rich physics observed in the spectroscopy and the dynamics of charge based devices.

I. INTRODUCTION

Considerable progress has been recently achieved in implementing two-state systems using superconducting nanocircuits. Time-resolved coherent oscillations have been measured in Josephson qubits[1, 2, 3] and signatures of the entanglement of coupled charge qubits have been observed[4]. Limitations in the performances arise from various noise sources[5, 6, 7, 8]. In this work we will compare effects of gaussian and non-gaussian noise, and show results indicating that discrete noise models potentially explain the experimental features, the main open question being a characterization beyond phenomenological theories describing the environment as a suitable set of harmonic oscillators[5, 9, 10]. Since strongly dependent on the particular device and on details of the protocol[2, 14, 15], Explanation of this rich physics is beyond phenomenology of the physics of the noise sources[8].

BFs[6, 7, 8] have been observed, this phenomenology being common (to different extent) to all solid state implementations. A variety of behaviors, ranging from broadening due to a slow environment[1, 2] to relaxation limited decoherence[3] has been also observed, this phenomenology being strongly dependent on the particular device and on details of the protocol[2, 14, 15]. Explanation of this rich physics is beyond phenomenological theories describing the environment as a suitable set of harmonic oscillators[5, 9, 10]. Since the physical sources of noise are discrete in nature, attention has been devoted to environments made of collections of BF[6, 7, 8]. In this work we will compare effects of gaussian and non-gaussian noise, and show results indicating that discrete noise models potentially explain the experimental features, the main open question being a characterization beyond phenomenology of the physics of the noise sources[8].

Let us consider the Hamiltonian $H = H_Q - \frac{v}{2} \xi \sigma_z$, representing a qubit ($H_Q = -\frac{1}{2} \vec{\Omega} \cdot \vec{\sigma}$) anisotropically coupled to a noise source, described by $\xi$. Sensitivity to noise can be modulated by tuning the operating point, i.e. the angle $\theta$ between $\vec{z}$ and $\vec{\Omega}$, the qubit splitting $\Omega$ being also tunable. For classical noise $\xi = \xi(t)$ is a stochastic process, whereas for quantum noise $\xi$ is an operator of the environment. Decoherence results from the sensitivity of the qubit to the environment. For instance if coupling is weak, relaxation and dephasing rates are $T_1^{-1} = \sin^2 \theta S(0)/2$ and $T_2^{-1} = (2 T_1)^{-1} + T_2^{-1}$, where $T_2^{-1} = \cos^2 \theta S(0)/2$ is the adiabatic rate, responsible for secular broadening[10]. The power spectrum $S(\omega) = v^2 (\xi \sigma_z \omega)$ appears, therefore the qubit is able to “measure” statistical properties of the environment at the level of two point correlations. In this regime the qubit is not sensitive to other details, for instance whether a certain Lorentzian line shape is due to a bistable fluctuator (BF) giving rise to Random Telegraph Noise (RTN) or to a continuous Gaussian process with the same Lorentzian Noise (GLN) spectrum (see Fig. 1a). In solid state devices $T_2^{-1}$ may be very large thus invalidating the weak coupling theory. This is the case of a sufficiently slow environment, no matter how small is $v/\Omega$. Consider for instance the Lorentzian spectrum $S(\omega) = \frac{v^2}{2} \frac{\gamma}{(\gamma^2 + \omega^2)}$. Rates in Fig. 1b show that $T_2^{-1} \sim v^2/\gamma$, diverges for $\gamma \to 0$. The technical reason of this failure is that the weak coupling expansion parameter is $g \propto v/\gamma$. The high-frequency cutoff sets the characteristic time scale $\gamma^{-1}$ of the noise, thus problems are encountered for slow noise, $g > 1$, since the qubit becomes sensitive to details of the dynamics of the environment. For instance it will distinguish RTN and GLN having the same $S(\omega)$. Indeed suppose we average the signal from an ensemble of experiments identical, apart for the uncontrolled preparation of the environment. The effect of RTN due to a single BF is to determine two angular frequencies for the qubit, $\Omega$ and $\Omega' = \Omega \left[ (v/\Omega + \cos \theta)^2 + \sin^2 \theta \right]^{1/2}$, and the signal will show beats at the frequency $\Omega' - \Omega$. If the BF is very slow, switches between the two frequencies produce decay with $T_1, 2 \sim \gamma^{-1}$. In this regime we can identify $g = (\Omega' - \Omega)/\gamma$ from the condition for beats to be observable, $g > 1$. Instead for slow GLN[10], a standard model in NMR[16], decay is determined by uncertainty in the preparation of the environment, an effect analogous to the “rigid lattice line breadth”[14]. By averaging the phase over a static distribution of the effective bias $\epsilon_0 = v \xi_0$, we find the decay of the qubit coherences $\Gamma(t) = -\ln |\rho_{\perp}(t)| = \ln(e^{it(\Omega^2 + 2T_1 \cos \theta + \epsilon_0^2)^{1/2}})$. The distribution of $\epsilon_0$ is gaussian for GLN, the standard deviation being $\sigma^2 = \int \frac{d\epsilon}{2\epsilon} S(\omega) = \frac{\epsilon^2}{4}$. The decay depends on $\theta$ and it is slower for $\theta = \pi/2$ where for $v \ll \Omega$ a simple integration yields $\Gamma(t) = -\frac{\epsilon^2 t}{2} \ln (1 + i \sigma^2 t / 2 \Omega)$. For $\theta = 0$ the full dynamic problem can be
BF dynamics analytically in the full adiabatic regime \( \gamma < \Omega \). Relaxation is not so sensitive: for \( \text{coupling regime: the appearance of beats can be modulated with the external bias.} \)

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\langle \sigma_y \rangle = \frac{\gamma}{t} \approx \gamma \ln(T/\gamma) \quad \text{and} \quad \Gamma(t) = (\nu/2\gamma)(\gamma t - 1 + e^{-\gamma t}) \approx (\nu t)^2/8. \]

In both cases the signal amplitude is strongly reduced already at times \( t \sim \nu^{-1} \ll \gamma^{-1} \) giving an apparent decay time \( T_2^* \) independent on \( \gamma \) and finite. Differences between slow RTN and GLN are also apparent in the line-shapes (see Fig. 1(c)).

II. QUBIT DYNAMICS IN THE PRESENCE OF IMPURITIES

Decoherence due to discrete noise sources has been studied in Ref. 6, 7, where the fluctuator has been modeled by a quantum impurity described by the Fano Anderson model. Rather than study higher orders \[9\] of the perturbative expansion in \( v \), a different strategy has been used, namely one may write a master equation (ME) for a four-level system system including the impurity level, which is traced out at the end of the calculation. Thus fast components of the environment are effectively treated at the ME level, whereas slow fluctuations are accounted for nonperturbatively.

In practice one has at most to diagonalize a simple \( 8 \times 8 \) Lindblad map which implies that the qubit dynamics is characterized by combining exponentials showing two relaxation times \( T_{1 \pm} \) and two dephasing times \( T_{2 \pm} \) (Fig. 2(a)). If \( g \ll 1 \) the dominant rates for \( t \gg \gamma^{-1} \) coincide with the weak coupling ones (and the classical GLN ones) and describe homogeneous broadening. Instead for impurities with \( g \gg 1 \), both \( T_{2 \pm} \) are important since dephasing reflects the bistable nature of the slow environment. Relaxation is not so sensitive: for \( \nu \ll \Omega \), the dominant rate is the weak coupling one the other rate being \( T_{1 \pm} \approx \gamma \). Rates have the form \( T_{2 \pm}^{-1} + i\delta \Gamma/2 = \gamma(1 \pm \alpha)/2 \) and \( T_{1 \pm}^{-1} = \gamma(1 + \alpha_r)/2 \), \( \delta \Omega \) being the splitting between the two spectroscopic peaks induced by the bistable impurity. They can be found analytically in the full adiabatic regime \( \gamma < \Omega \), which includes the crossover region. In the stochastic limit for the BF dynamics \( \alpha = [\delta p - ig]^2 + \cos^2(\delta \theta)(1 - \delta p^2)]^{1/2} \) and \( \alpha_r = [1 - \sin^2(\delta \theta)(1 - \delta p^2)]^{1/2} \). Features of the nature of the BF appear if \( \delta \Omega > \gamma \), i.e. \( 3\alpha > 1 \), this criterion for “strong coupling” depending on both \( v/\gamma \) and the operating point (see Fig. 2(b)). Strongly coupled impurities are non-gaussian, and determine non-exponential decay. Moreover their effect depends on their preparation, so they are non-stationary and non-Markovian, determining memory effects. As a consequence decoherence depends on details of the protocols, a critical feature for \( 1/f \) noise.
III. 1/F NOISE AT OPTIMAL WORKING POINT

Ideally quantum protocols assume we can measure individual members of an ensemble of identical (meaning that preparation is controlled) time evolutions of the qubit. The environment limits control of the preparation. At most one can recalibrate, for each individual member, the collective variable ξ. With this feedback scheme error would be due solely to decoherence during time evolution. In actual experiments lack of control on the environment determines additional defocusing of the signal, analogous to inhomogeneous broadening in NMR, which depends on the statistics of the environment at appropriate low-frequencies.

We model 1/f noise with a set of impurities switching at rates γi. For charge based devices they are coupled with the qubit via the total polarizing charge, νeff = ∑i νi ni, where ni = 0, 1. The standard assumption of a distribution of γi with P(γ) ∝ 1/γ for γ ∈ [γm, γM] leads to the spectrum S(ω) = ∑ i 1 2 νi2 (1 − δp2) γi(ω + ωi), which is 1/f at frequencies 2πf ∈ [γm, γM]. In other words, impurities with both large and small g are present. Decoherence for this model has been studied for θ = 0, where exact solutions are available even for non-linear environments.

The feedback protocol results not to be sensitive to very slow impurities, having γi < γ* ∼ π/10, which sets an effective intrinsic low-frequency cutoff γ*. This effect is due to the nongaussianity of the environment, and to the fact that BF with g ∼ 1 determine strong decay. Instead BF with g < 1 behave as an environment of quantum harmonic oscillators, being sensitive only to the amplitude of noise ∝ nBFg. In absence of recalibration, inhomogeneous broadening corresponds to a proper averaging of the initial conditions of the BF and one may prove that γ* moves to γ* ∼ min{π/10, 1/m}, where m is the overall measurement time of the experiment and a large number of repetitions is assumed.

The most effective implemented strategy α for defeating 1/f is to tune optimally the working point, which in our case means setting θ = π/2. Here the qubit splitting is less sensitive to bias fluctuations, or equivalently the lowest order adiabatic rate T 2 2−1 vanishes. In other words, operating with θ modifies the parameters gi and the quantitative characterization of the spectrum in terms of γ*, the main effect being that smaller gi means smaller effectiveness of noise. At first sight one expects that differences between a BF-1/f environment and a set of oscillators with 1/f spectrum (G-1/f) decrease by operating at θ = π/2, although they do not necessarily disappear. Decoherence at the optimal point for a G-1/f environment has been recently studied by Makhlin and Shnirman, combining the adiabatic approximation and diagrammatic perturbation theory. Instead we study a BF-1/f environment by solving numerically the stochastic Schrödinger equation. We checked that very slow BF are ineffective, so we choose γm = 10^5 Hz. We consider first an adiabatic environment, γM ≪ Ω (Fig. 3a) studying relaxation via ⟨σx(t)⟩. The resulting rate reproduces the weak coupling result, in both protocols with and without recalibration of the environment. Instead dephasing is much faster and recalibration is able to reduce defocusing effects. Inhomogeneous broadening can be estimated with the rigid lattice line breadth formula. If one assumes NFL to be large enough that the initial ε0 = ∑ NBF i=1 νi ni(0) is gaussian distributed, one finds σ2 = vNBF/4 = 16AC ln (γM/γm), where physically γm ∼ γ* = 1/m. This simple result accounts for the initial reduction of the signal amplitude, and coincides with the short-time behavior of the diagrammatic theory. Adding faster BF to the environment increases relaxation more than dephasing. In our example (Fig. 3b) decoherence is limited by relaxation T2 ≈ T1, this latter being due to the fast part of the spectrum ω ∼ Ω. Inhomogeneous broadening does not reduce drastically the amplitude and,
FIG. 4: (a) $\langle \sigma_y \rangle$ at $\theta = \pi/2$, $\Omega = 10^{10}$ Hz. The effect of weak adiabatic $1/f$ noise (light gray line) ($\gamma \in [10^5, 10^9]$ Hz, uniform $v = 0.002 \Omega$, $n_d = 250$) is strongly enhanced by adding a single slow ($\gamma/\Omega = 0.01$) more strongly coupled ($n_0/\Omega = 0.2$) impurity (black line), which alone would give rise to beats (gray and dashed line). (b) The latter two cases display a characteristic behavior of the Fourier transform, shown for the single impurity alone (dashed line) and for $1/f$ noise plus impurity (solid line).

as expected, dephasing is underestimated by adiabatic approaches [9].

Finally, we show that even a single impurity on a $1/f$ background may cause a substantial reduction of the amplitude. This strongly poses the problem of reliability of charge based devices. Effects of realistic BF distributions were pointed out by Galperin et al.[8]. Using reasonable parameters, we obtain that indeed an additional BF has a substantial effect (see Fig. 4), whose signature is an asymmetric Fourier transform double-peak, which is similar to recent spectroscopy observations [11].

[1] Y. Nakamura et al., Nature 398, 786 (1999); Y. Yu et al., Science 296, 889 (2002); J. Martinis et al., Phys. Rev. Lett. 89, 117901 (2002); I. Chiorescu et al., Science, 299, 1869, (2003); T. Yamamoto et al., Nature 425, 941 (2003).
[2] D. Vion et al., Science 296, 886 (2002).
[3] T. Duty et al., Phys. Rev. B, 69, 140503 (2004); O. Astafiev et al., preprint 2004.
[4] Yu. A. Pashkin et al., Nature 421, 823 (2003).
[5] Y. Makhlin, G. Schön and A. Shnirman, Rev. Mod. Phys. 73, 357 (2001); L. Tian et al. in Quantum Mesoscopic Phenomena and Mesoscopic Devices in Microelectronics, I.O. Kulik and R. Ellatioglu Eds., Kluwer Pub. 2000, pg. 429.
[6] E. Paladino, L. Faoro, G. Falci, R. Fazio, Phys. Rev. Lett., 88, 228304 (2002).
[7] E. Paladino, L. Faoro, G. Falci, Adv. Sol. State Phys., 43, 747 (2003).
[8] Y. M. Galperin, B. L. Altshuler, D. V. Shantsev, cond-mat/0312200.
[9] Y. Makhlin, A. Shnirman, Phys. Rev. Lett. 92, 178301 (2004), cond-mat/0308297.
[10] K. Rabenstein, V. A. Sverdlov, D. V. Averin, cond-mat/0401519.
[11] Chalmers Group (Göteborg), Quantronics Group (Saclay), NTT Group (Atsugi), private communications.
[12] M.B. Weissman, Rev. Mod. Phys. 60, 537 (1988).
[13] A.B. Zorin et al., Phys. Rev. B 53, 13682 (1996). M. Covington et al., Phys. Rev. Lett. 84, 5192 (2000).
[14] Y. Nakamura et al., Phys. Rev. Lett. 88, 047901 (2002).
[15] G. Falci, E. Paladino, R. Fazio, in Quantum Phenomena of Mesoscopic Systems, B. L. Altshuler and V. Tognetti Eds., IOS Press (2003), cond-mat/0308297.
[16] C. P. Slichter, Principles of Magnetic Resonance, Springer Verlag (1996)
[17] C. Cohen-Tannoudji et al., Atom-Photocon Interactions, Wiley-Interscience (1993).
[18] Notice that in the regime $v \ll \Omega$, $g$ is approximated by $g \approx \cos \theta v/\gamma + \sin^2 \theta v^2/(2\gamma\Omega)$. Thus at $\theta = \pi/2$ it can be translated in $g \approx S(0)/\Omega$ and the condition $g < 1$ simply means that broadening as given by the weak coupling theory should be smaller than the qubit splitting.
[19] The result gives the short time behavior found in Refs. [9, 10] with a much more complete analysis.