Constitutive models for FE analysis of pile founded buried arch bridge

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Abstract. The paper presents plane strain FE modeling of a pile founded bridge, made from reinforced concrete, by using plate elements for the structure and the soil body. The soil-structure contact is modeled by interface finite elements. The numerical analysis is performed by the Plaxis 2D software. The mechanical behavior of the bridge is approximated by the constitutive models of Hooke and Mohr-Coulomb. Two sets of values of the Mohr-Coulomb strength parameters (cohesion and internal friction angle) for the concrete are obtained - on the basis of the theoretical uniaxial compressive and tensile strengths and on the basis of relationships in the Eurocode 2. The advanced constitutive HSM model is used for the soil. The construction and service periods of the bridge are simulated by a sequence of representative states. The mechanical response of the bridge undergoing backfill and road loads is analysed.

1. Introduction

Finite element models for the interaction between the structure of a buried arch bridge and surrounding soil body are analysed. The modeling is carried out by the Plaxis 2D software. The purpose of the study is an assessment of the application of different constitutive models for the bridge structure and soil materials. The prototype of the analysed structure is the bridge Tri Voditsi with width of 15.29 m and height of 8.67 m which is built of prefabricated elements of concrete C50/60. It is the structure of the railway-road junction near the villages Tri Voditsi and Novo Selo in Bulgaria (Figure 1). The same bridge superstructure founded on strip footings is considered in previous analyses [1], [2]. In the present research the substructure is a piled-raft foundation of concrete C25/30. This type of foundation reduces significantly displacements of the bridge structure and it has been realized in the design project.

The usual procedure for the finite element modeling of a soil-structure interaction is to introduce one-dimensional (beam) elements for concrete structures within two-dimensional soil elements. For the purposes of the present analysis both the soil body and the concrete structure are discretized by a mesh of 2D plane strain plate elements. This approach allows to describe the complex geometry of the bridge structure more accurate. The stiffness parameters of the discrete piles are adapted to the plane strain model. This approximation gives more conservative results in comparison with a model of 3D discretization of the piled-raft foundation [3].

The mechanical behavior of the bridge structure is modeled by the linear elastic constitutive model of the Hooke and the Mohr-Coulomb elastic-plastic model. Two sets of values for the concrete Mohr-
Coulomb parameters (cohesion and friction angle) are defined - on the basis of the theoretical uniaxial compressive and tensile strengths and on the basis of relationships in the Eurocode 2.

The advanced constitutive Hardening Soil Model is used for modelling of the soil.

The loading conditions of the bridge structure during the construction and service periods are simulated by a sequence of representative states. The mechanical behavior of the bridge is analysed undergoing the backfill and road loads. Only static loadings of the bridge are considered. Time history dynamic analysis of the bridge using an accelerogram of the earthquake Pernik-2012 in Bulgaria [4], [5] is performed in reference [6].

2. Finite Element Modeling

2.1. Finite element mesh and boundary conditions

A 2D plane strain finite element model is created considering the symmetry of the geometry and the loading of the bridge structure (Figure 2). 15-node triangular elements are used for discretization of the soil body and the bridge structure and quartic interpolations of the displacements, horizontal, $U_x$, and vertical, $U_y$, are conducted. Interface finite elements, developed in the Plaxis software, are used for modeling of the soil-structure contact.

![Figure 2. Plane strain FE model of the soil body and the bridge structure](image)

The bridge structure is undergone the backfill self weight and the traffic loads on the road which is built 0.50 m above the top of the bridge. The traffic loads are simulated by a uniformly distributed load.
of value $q = 9$ kPa which corresponds to the load on the road lane number one according to the UDL system of the traffic model LM1 in the Eurocode 1 [7].

2.2. Constitutive model for the soils

The constitutive model (law) is a mathematical description of stress-strain behavior of the material in response to the applied loads. Here the elastic-plastic Hardening Soil Model (HSM), implemented in the Plaxis software [8], is used for modeling of the soil ground and backfill of the bridge. The HSM is a state-of-the-art scientific achievement which integrates many conceptions of the soil mechanics [9], [10]. It uses more than 10 material parameters and approximates the mechanical behaviour of the soil with a high degree of accuracy.

**Table 1.** HSM parameters used in the FE model

| Parameter                                         | Symbol | Dimension | Backfill | Clayey sand |
|---------------------------------------------------|--------|-----------|----------|-------------|
| Unit weight of unsaturated soil                   | $\gamma_{\text{unsat}}$ | kN/m$^3$  | 18       | 18          |
| Unit weight of saturated soil                     | $\gamma_{\text{sat}}$  | kN/m$^3$  | -        | 20          |
| Reference stress for stiffnesses                  | $p_{\text{ref}}$      | MN/m$^2$  | 0,100    | 0,100       |
| Tangent stiffness for primary oedometer loading   | $E_{\text{oed,ref}}$  | MN/m$^2$  | 50       | 20          |
| Secant stiffness in standard drained triaxial test| $E_{\text{50,ref}}$   | MN/m$^2$  | 50       | 20          |
| Unloading/reloading stiffness                      | $E_{\text{ur,ref}}$   | MN/m$^2$  | 150      | 60          |
| Poisson’s ratio for unloading/reloading           | $\nu_{\text{ur}}$     | -         | 0,2      | 0,2         |
| Power for stress-level dependency of stiffness    | $m$      | -         | 0,5      | 0,6         |
| Coefficient of lateral earth pressure             | $K_0$   | -         | 0,384    | 0,515       |
| Failure ratio                                     | $R_f$   | -         | 0,9      | 0,9         |
| Friction angle                                    | $\phi$  | $^\text{o}$ | 38       | 29          |
| Cohesion                                          | $c$     | kPa       | 0        | 10          |

The Hooke model (law) of linear elasticity is the simplest constitutive law used for structure elements in the design practice. This model requires only two material parameters – Young modulus, $E$, and Poisson ratio, $\nu$, and assumes unlimited values of stresses. These features of the Hooke model are the reason for receiving of conservative results for stresses that are not relevant to the real mechanical behavior of the concrete.

The Mohr-Coulomb (MC) constitutive law describes elastic-perfectly plastic behavior and this model is appropriate for materials such as geomaterials, concrete, for which the compressive strength far exceeds the tensile strength. According to the MC model, the stresses are proportional to the strains until the yield (fail) point is reached. The stress state corresponding to the yield points is described by a set of six linear equations in effective principal stresses $\sigma_1'$, $\sigma_2'$, $\sigma_3'$, neglecting the effect of the intermediate principal stress. Assuming no order of the principal stresses the MC criterion is graphically represented as a hexagonal pyramid (Figure 3). When the stress state corresponds to the yield (fail), then the stress point lies on the pyramid surface. The stress points corresponding to the elastic behavior lie inside the surface.
Figure 3. Mohr-Coulomb yield function: a) Surface in 3D space of principal stresses; 
b) Cross section in deviator plane \((\sigma_1' + \sigma_2' + \sigma_3' = 0)\)

For the particular case \(\sigma_1' \geq \sigma_2' \geq \sigma_3'\) the MC criterion can be written as a following functions of the major \(\sigma_1'\) and the minor \(\sigma_3'\) principal stresses

\[
F(\sigma_1', \sigma_3') = (\sigma_1' - \sigma_3') - \sin \varphi \left(\sigma_1' + \sigma_3'\right) - 2c \cos \varphi,
\]

and normal \(\sigma\) and shear stresses \(\tau\) on the failure plane

\[
\tau = c + \sigma \tan \varphi.
\]

There are many experimental and theoretical researches on the application of the MC model to concrete. The most important point is the determination of the concrete strength parameters values. A review of the literature shows a very wide range values. In the reference [11] indirect tensile tests and numerical simulations are used and values \(c = 1.8\)\-\(2.7\)MPa and \(\varphi = 31^\circ\) are obtained. On the basis of equivalence of split test results and wedge theoretical solutions in [12] it is proved that the friction angle of the concrete is equal to \(\varphi = 55^\circ\)\-\(57.5^\circ\). Authors of reference [13] carried out direct shear tests and the cohesion and the friction angle are specified between 2.94MPa and 12.34MPa and between 29.8\(^\circ\) and 41.7\(^\circ\), respectively. Concrete samples are tested in a large shear box in [14] and values of \(c = 0.58\)\-\(2.27\)MPa and \(\varphi = 36^\circ\)\-\(52^\circ\) are summarised. Concrete Construction Engineering Handbook [15] recommends for the concrete value of \(c = 2.75\)MPa and \(\varphi = 30^\circ\)\-\(60^\circ\). The Plaxis Bulletin [16] presents three algorithms based on: Montoya’s considerations [17], the Spanish Structural Concrete Code EHE-98 and the Eurocode 2. According to these algorithms the following values of the strength parameters are determined: \(c = 0.387; 0.365; 0.712\)MPa for the concrete of compressive strength \(f_c = 15\)MPa and \(c = 0.5; 0.513; 1.186\)MPa for the concrete of \(f_c = 25\)MPa; \(\varphi = 9^\circ; 35^\circ; 55^\circ\) for the both concrete classes.

In the present research the theoretical grounds of three sets of material parameters for concrete, named Model I, Model II and Model III, are developed. They are applied to the bridge structure and a comparative analysis of the results is carried out.

- Model I – validity of the Hooke constitutive law for the concrete.
- Model II – validity of the MC constitutive law for the concrete with parameters \(c\) and \(\varphi\) calculated using the theoretical uniaxial compressive, \(f_c\), and tensile, \(f_t\), strengths.
In consequence of the MC relationships (Figure 4) the expressions are valid:

\[ f_c = 2c\cos \varphi / (1 - \sin \varphi), \]  
\[ f_t = 2c\cos \varphi / (1 + \sin \varphi). \]  

After the substitution of (3) and (4) in equation \( F(\sigma_1', \sigma_2') = 0 \), the following expressions are obtained for the strength parameters:

\[ \sin \varphi = (f_c - f_t) / (f_c + f_t), \]  
\[ c = f_c \cdot (1 - \sin \varphi) / (2\cos \varphi). \]  

The similar methodology is discussed in references [18], [19].

The values of the strength parameters calculated according to the Model II for the concrete classes used in the bridge structure are given in Table 2.

**Table 2. Values of \( c \) and \( \varphi \) for the concrete according to the Model II**

| Concrete class | \( c \) (MPa) | \( \varphi^\circ \) |
|----------------|---------------|-------------------|
| C25/30         | 3.36          | 60                |
| C50/60         | 6.04          | 63                |

- Model III – validity of the MC law for the concrete with parameters \( c \) and \( \varphi \) determined according to the Eurocode 2.

The Eurocode 2 suggests the following formula for the design shear resistance of concrete elements without shear reinforcement

\[ \tau_{rd,c} = \nu_{\text{min}} + k_1 \sigma_{cp}, \]  

where:

\( \nu_{\text{min}} = 0.035 \cdot k^{3/2} \cdot f_{ck}^{1/2} \), where \( f_{ck} \) is the characteristic compression strength in MPa;

\( k = 1 + (200/d)^{1/2} \leq 2 \), where \( d \) is the effective height of a cross section in mm;

\( k_1 = 0.15 \) – empirical coefficient;

\( \sigma_{cp} \) – design normal stress.

The form of the formula (7) is identical to the expression (2) of the MC criterion. Using the substitutions

\[ c = \nu_{\text{min}} \quad \text{and} \quad \tan \varphi = k_1, \]  

the design values of the strength parameters \( c \) and \( \varphi \) can be calculated.
The values of the strength parameters calculated according to the Model III for the concrete classes used in the bridge structure are given in Table 3.

| Concrete class | c (MPa) | φ(°) |
|----------------|---------|------|
| C25/30         | 0,495   | 9    |
| C50/60         | 0,700   | 9    |

2.4. *Simulation of the construction and service periods of the bridge*
Six loading states in the period of backfilling (State 1 – State 6) and one loading state in the service period (State 7) of the bridge are numerically analysed (Figure 5).

![Figure 5. States in construction and service periods of the bridge](image)

3. Results from FE analyses

3.1. *Initial stress conditions*

The initial stresses (“in situ” stresses) in the ground exist as a result of action of the soil and ground water self-weight. Diagrams of the “in situ” stresses are shown in Figure 6, where the following symbols are used:

- $\sigma_{y,0}$ – normal vertical effective (in the soil skeleton) stress;
- $\sigma_{x,0}$ – normal horizontal effective (in the soil skeleton) stress;
- $p_w$ – hydrostatic pressure.

The initial ground displacements are equal to zero.
3.2. Service period analysis
The mechanical response of the bridge, undergoing backfill and road loads (State 7 in Figure 5) is analysed. Results from FE solutions using defined above constitutive models for the bridge structure: Model I, Model II and Model III, are presented in Figures 7-9.

**Figure 7.** State 7: Diagrams of the normal vertical effective stresses $\sigma_{y,0}''$

**Figure 8.** State 7: Diagrams of the normal horizontal effective stresses $\sigma_{x,0}''$
4. Conclusion

The comparative analysis of the results confirms the following conclusions:

- The FE analyses of Model I, Model II and Model III give similar distribution of areas with extreme values of stresses;
- The solution using the Hooke constitutive model for the bridge structure gives the most conservative results for the stresses;
- The values of the Mohr-Coulomb strength parameters – cohesion and angle of internal friction – have significant influence on the results of the stresses in the bridge structure. The Model II uses strength parameters about 7 times larger than the strength parameters in the Model III. And the Model II gives larger values of max stresses in the bridge in comparison with the Model III: 3 times in the vertical normal stress; 2.6 times in the horizontal normal stress; 1.3 times in the shear stresses;
- The rather high values of the strength parameters in the elastic-plastic Model II are the reason for results of this model closed to the results of the elastic Model I: the differences are about 15% and 4% in the normal and the shear stresses, respectively.

The present study shows that the constitutive modeling of concrete structures according to the Mohr-Coulomb law requires precise analysis of the strength parameters – cohesion and internal friction angle. The parameters influence significantly on the results of the mechanical behavior of concrete structures. The literature sources include different experimental and theoretical methods for determination of the concrete strength parameters and the obtained values are in very large ranges. Two methods are applied in the present paper for determination of the strength parameters. The first method is based on the theoretical relationships between uniaxial compressive and tensile strengths of the concrete. The second method is based on the analogy between the equation of the Mohr-Coulomb criterion and the formula for the design shear resistance of concrete elements without shear reinforcement in the Eurocode 2. The application of these methods for a bridge structure shows big differences in the results of the structure mechanical response.
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