A Model for Quantum Jumps in Magnetic Resonance Force Microscopy

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We propose a simple model which describes the statistical properties of quantum jumps in a single-spin measurement using the oscillating cantilever-driven adiabatic reversals technique in magnetic resonance force microscopy. Our computer simulations based on this model predict the average time interval between two consecutive quantum jumps and the correlation time to be proportional to the characteristic time of the magnetic noise and inversely proportional to the square of the magnetic noise amplitude.

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I. INTRODUCTION

Recent achievements in magnetic resonance force microscopy (MRFM) promise a single spin detection in the near future [1]. One may expect that single spin signal will represent a random sequence of quantum jumps. The important problem for the theory is modeling of quantum jumps in MRFM and the computation of their statistical characteristics. In this paper we propose a simple model which describes quantum jumps in MRFM single spin detection. We consider the oscillating cantilever-driven adiabatic reversals technique (OSCAR) which currently is the most promising approach for single spin detection [1]. In OSCAR the cantilever tip (CT) with a ferromagnetic particle oscillates, causing the periodic adiabatic reversals of the effective magnetic field on spin. The spin follows the effective magnetic field causing a tiny frequency shift of the CT vibrations which is measured.

In the second section we consider the Schrödinger dynamics of the CT-spin system which underlines our model of quantum jumps. In the third section we describe our model and present the results of the computer simulations. In Conclusion we discuss our results.

II. SCHRODINGER DYNAMICS OF THE CT-SPIN SYSTEM

We consider a vertical cantilever with a ferromagnetic particle attached to the CT and oscillating along the x-axis which is parallel to the surface of the sample. (See Fig. 1). The Hamiltonian of the CT-spin system in the system of coordinates which rotates with a circularly polarized rf field can be written as

$$H = \frac{1}{2}(p_x^2 + p_y^2) + \varepsilon S_x + 2\eta x_c S_z + \Delta(\tau) S_z.$$  (1)

Here $p_c$ is the effective momentum of the CT, $x_c$ is its coordinate, the first term describes the effective energy of the CT; the second term describes the interaction between the spin and the rf field; the third term describes the interaction of the spin and the effective magnetic field produced by the ferromagnetic particle at the interaction between the CT and the spin; and the last term describes the interaction between the spin and the random magnetic field which causes a deviation of spin from the effective magnetic field when the latter passes through the transverse $x$–$y$-plane [2, 3]. All quantities in (1) are dimensionless: $p_c$ and $x_c$ are expressed in terms of the “quantum units” $\hbar$ and $X_0$

$$X_0 = (\hbar \omega_c / k_c)^{1/2}, \quad P_0 = \hbar / X_0.$$  (2)

The dimensionless parameters $\varepsilon$, $\eta$, and $\Delta(\tau)$ are defined as

$$\varepsilon = \gamma B_1 / \omega_c, \quad \eta = \gamma(\hbar / k_c \omega_c)^{1/2} |\partial B_z(x) / \partial x| / 2, \quad \Delta(\tau) = \gamma \Delta B_z(\tau) / \omega_c, \quad \tau = \omega_c t.$$  (3)

Here $B_z$ is the z-component of the “regular magnetic field” which includes $\vec{B}_{ext}$ and the regular dipole-magnetic field produced by the ferromagnetic particle at

FIG. 1: Single spin OSCAR MRFM setup. $\vec{B}_{ext}$ is the external permanent magnetic field; $B_1$ is the rotating rf field; $\vec{m}$ is the magnetic moment of the ferromagnetic particle; $\vec{s}$ is a spin near the surface of the sample.
the location of the spin; $\Delta B_z(\tau)$ is the $z$-component of
the random field with zero average value; $\gamma$ is the mag-
nitude of the spin gyromagnetic ratio: $k_c$ and $\omega_c$ are the
effective spring constant and fundamental frequency of
the CT; and $\tau$ is the dimensionless time.

Using the parameters presented in [1]

$$\omega_c/2\pi = 6.6kHz, \; k_c = 6 \times 10^{-4}N/m,$$  

$$B_1 = 0.3mT, \; |\partial B_z(x)/\partial x| \approx 4.3 \times 10^5T/m,$$  

we can estimate all parameters in (1) except of $\Delta(\tau)$:

$$X_0 = 85fm, \; P_0 = 1.2 \times 10^{-21}Ns, \; \eta = 0.078, \; \varepsilon = 1270.$$  

To simplify computer simulations we considered the func-
tion $\Delta(\tau)$ to be a random telegraph signal with two
values $\pm \Delta$. The time interval between two consecutive
“kicks” of $\Delta(\tau)$ was taken randomly from the interval
$(\tau_0 - \delta\tau, \tau_0 + \delta\tau)$, with the average time interval, $\tau_0$, close
to the Rabi period $\tau_R$

$$\tau_R = \omega_c(2\pi/\gamma B_1) = 2\pi/\varepsilon = 4.95 \times 10^{-3}.$$

We choose the initial state of the CT to be a coherent
quasiclassical state, and the average spin to be pointed
along the “regular” effective magnetic field with the compo-
ents ($\varepsilon, 0, 2\eta x_c$).

Below we describe briefly the results of our computer
simulations. Our simulations reveal the formation of a
Schrödinger cat state for the CT: the probability distri-
bution function

$$Schr¨ odinger \; cat \; state \; for \; the \; CT: \; the \; probability \; dis-
tri-
bution \; of \; the \; spin \; jumps.$$  

We assume that every “kick” provided by the function
$\Delta(\tau)$ is followed by the collapse of the wave function.
Before the kick, the spin points in (or opposite to) the
direction of the effective magnetic field. After the kick
there appears the finite angle ($\Delta \Theta$) between the new
direction of the effective field and the average spin. The
probability for the spin to “accept” the “before-kick” di-
rection relative to the new effective field is $\cos^2(\Delta \Theta/2)$.
The probability to “accept” the opposite direction, i.e.
the probability of a quantum jump is $\sin^2(\Delta \Theta/2)$.
(It is clear that a significant probability of a quantum jump
appears only when the effective field passes the transvers-
al $x - y$-plane. In the transversal plane, the effective
field has the minimum value. Thus, after every kick of
the random field our computer code decides the “fate”
of the spin in accordance with the probabilities of two
events: to restore the previous direction relatively to the
effective field, or to experience a quantum jump. In our
model the CT experiences harmonic oscillations

$$x_c(\pm)(\tau) = x_m \cos(1 \pm \delta\omega)\tau,$$  

where $(\pm)$ correspond to two CT trajectories with the spin
pointing in the direction of (or opposite to) the corre-
responding effective field, and $\delta\omega$ is the CT frequency
shift. The components of the effective field are given by

$$(\varepsilon, 0, 2\eta x_c(\tau) + \Delta(\tau)).$$  

From the experimental data [1] for the CT amplitude
$X_m = 10$ nm we obtain $x_m = 1.2 \times 10^5$. The frequency
shift, $\delta\omega$, can be estimated as [5]

$$\delta\omega = \Delta \omega/\omega_c = \frac{2G\mu_B}{\pi X_m k_c} = 4.2 \times 10^{-7}.$$  

III. SIMPLE MODEL OF QUANTUM JUMPS

FIG. 2: (a) Distribution function of time intervals between
two consecutive quantum jumps for $\Delta = 100, \tau_0 = 0.01$ and
$10^3$ kicks; the solid line is a fit with $\tau_d = 32$; (b) Enlargement of
(a)
Note that our model contains two important simplifications: first, we assume that the wave function collapse occurs immediately after the “kick” of the random field. Thus, we ignore the finite time when the spin-CT system is in an entangled state. Second, in a real situation the deviation of the spin from the effective field is a “quasi–resonance” process caused by the cantilever modes whose frequencies are close to the Rabi frequency. In our model this deviation appears as a result of the “kick” of the random field.

Below we describe the results of our simulations. Fig. 3 demonstrates a typical probability distribution of time intervals, \( \tau_{\text{jump}} \), between two consecutive quantum jumps. The probability distribution is a sequence of sharp peaks at \( \tau_{\text{jump}} = \tau_n = n\pi \) with the Poisson-like amplitude

\[
P(\tau_n) \sim \exp(-\tau_n/\tau_d). \tag{10}
\]

(Certainly, \( P(\tau_{\text{jump}}) = 0 \) at \( \tau < \tau_0 - \delta \tau \).) The sharp peak appears as the probability of the quantum jump is significant when the spin passes through the transversal plane, i.e. every half-period of the CT oscillation which is equal to \( \pi \). The average value of the time interval \( \langle \tau_{\text{jump}} \rangle \) was found to be

\[
\langle \tau_{\text{jump}} \rangle \approx \tau_d, \tag{11}
\]

with an error less than 3%. The standard deviation is equal to \( \tau_d \) with the same accuracy

\[
(\langle \tau_{\text{jump}}^2 \rangle - \langle \tau_{\text{jump}} \rangle^2)^{1/2} \approx \tau_d. \tag{12}
\]

We studied the dependence of the average value \( \langle \tau_{\text{jump}} \rangle \) on the parameters of our model. We have found that \( \langle \tau_{\text{jump}} \rangle \) does not depend on \( \delta \tau \) or \( \delta \omega \). (We varied \( \delta \tau \) from 0 to \( \tau_0 \) and changed \( \delta \omega \) up to one order of magnitude.) At a fixed value of the amplitude \( x_m \) the value of \( \langle \tau_{\text{jump}} \rangle \) is approximately proportional to \( \tau_0/\Delta^2 \). Fig. 3 demonstrates this dependence.

The best fit for the numerical points in Fig. 3 is given by

\[
\ln\langle \tau_{\text{jump}} \rangle = p + q \ln(\tau_0/\Delta^2). \tag{13}
\]

For \( x_m = 1.2 \times 10^5 \) we have \( p = 17.9, q = 0.993 \). For the six fold value \( x_m = 7.2 \times 10^5 \) we obtained the same value of \( q \) and \( p = 19.743 \). If we estimate the amplitude of the random CT vibrations near the Rabi frequency as \( 1 \) pm, then \( \Delta = 1.8 \). Putting \( \tau_0 = \tau_R \) and the experimental value for \( \tau_R \) (6) we obtain \( \omega_c \langle \tau_{\text{jump}} \rangle = 2.3 \) s for \( x_m = 100\mu m \) and \( \omega_c \langle \tau_{\text{jump}} \rangle = 14.5 \) s for \( x_m = 60\mu m \). These values are close to the experimental characteristic times of 3 s and 20 s reported in [1].

Next we computed the correlation function for the CT frequency shift

\[
C(\tau) = \frac{\langle (\delta \omega(t)) (\delta \omega(t + \tau)) \rangle - \langle (\delta \omega(t)) \rangle \langle (\delta \omega(t + \tau)) \rangle}{\langle (\delta \omega(t))^2 \rangle - \langle (\delta \omega)^2 \rangle}, \tag{14}
\]

where \( \overline{\delta \omega} = \langle \delta \omega(t) \rangle = 0 \), and \( \langle ... \rangle \) indicates an average over time.

In Fig. 4 we show the correlation function \( C(\tau) \) for three different values of parameters as indicated in the legend. As one can see, the general behavior is well described by the exponential function (indicated by dashed lines in Fig. 4) \( \exp(-\tau/\tau_c) \). The relation between the correlation time \( \tau_c \) and \( \langle \tau_{\text{jump}} \rangle \) was found to be \( \langle \tau_{\text{jump}} \rangle \approx 2.5\tau_c \).
IV. CONCLUSION

We developed a simple model which describes quantum jumps of spins in the OSCAR MRFM technique and which allowed us to compute the statistical characteristics of quantum jumps. In our model the average time interval $\langle \tau_{\text{jump}} \rangle$ between two consecutive jumps and the correlation time $\tau_c$ are proportional to the characteristic time of the magnetic noise, and inversely proportional to the square of the magnetic noise amplitude. Using experimental parameters $[1]$ and a reasonable value for the CT noisy amplitude we obtained the value of $\langle \tau_{\text{jump}} \rangle$ which is close to the experimental value of the characteristic time of OSCAR MRFM signal.

Acknowledgments

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