Trajectory based interpretation of the laser light diffraction on a sharp edge

Milena D Davidović · Miloš D Davidović · Angel S Sanz · Mirjana Božić · Darko Vasiljević

Abstract In a diffraction pattern of a laser beam on a sharp edge of a half-plane two characteristic regions are noticeable. In the central region, one can notice the diffraction of laser light in the region of geometric shadow, while intensity oscillations are observed in the non-obstructed area. There are also, on both sides of the edge, very long light traces along the normal to the edge of the obstacle. The theoretical explanation is based on the Fresnel-Kirchhoff diffraction theory applied to the Gaussian beam propagation behind the obstacle.

Keywords Diffraction · Bohmian mechanics · Classical Electromagnetism

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1 Introduction

Bohmian mechanics enables visualization and interpretation of quantum mechanical behavior of massive particles through trajectories associated with the probability current density [1]. Electromagnetic field also admits hydrodynamic formulation when the existence of suitably defined photon wave function

M. Davidović
Faculty of Civil Engineering, University of Belgrade, Serbia
E-mail: milena@grf.bg.ac.rs

M. Davidović
Vinča Institute of Nuclear Sciences, University of Belgrade, Serbia

A. S. Sanz
Department of Optics, Universidad Complutense de Madrid, Spain

M. Božić
Institute of Physics, University of Belgrade, Serbia

D. Vasiljević
Institute of Physics, University of Belgrade, Serbia
is assumed [2]. This formulation gives possibility to interpret the optical phenomena in a picturesque way through photon trajectories which describe the evolution of the electromagnetic energy density behind an obstacle.

This approach, based on the trajectories, was used in the analysis of Youngs double slit diffraction [3], in the context of the Arago-Fresnel laws [4], as well as in the analysis of the modes in the optical and microwave waveguides [5]. A group of scientists from the University of Toronto under the guidance of professor Steinberg, has been able to experimentally determine the mean paths of single photons in the Youngs experiment [6]. The measured trajectories show good agreement with theoretically anticipated trajectories presented in [2]-[4]. The achievement of Steinbergs group was selected by the Physics World as the top breakthrough in physics for the year 2011, as the discovery that is shifting the moral of quantum measurement [7].

Theoretical solution for the diffraction of plane wave by the edge of the perfectly conducting plane was given by Sommerfeld in 1896 [8], and this solution became the starting point in solving the diffraction problems for various two dimensional obstacles [9]-[10]. Diffraction of Gaussian beam by the edge was studied since the sixties of the last century [11] but more attention was given to the central part of the diffraction image, while the less pronounced side trails were analyzed much later [12]. In this paper we use photon trajectories approach to analyze the diffraction pattern obtained on the screen put behind the laser beam partially covered by a sharp edge, such as a razor blade.

2 Propagation of the electromagnetic wave

Electric and magnetic fields in vacuum obey the following wave equations:

\[
\nabla^2 \vec{E} (\vec{r}, t) - \frac{1}{c^2} \frac{\partial^2 \vec{E} (\vec{r}, t)}{\partial t^2} = 0, \tag{1}
\]

\[
\nabla^2 \vec{H} (\vec{r}, t) - \frac{1}{c^2} \frac{\partial^2 \vec{H} (\vec{r}, t)}{\partial t^2} = 0, \tag{2}
\]

where \(c\) is the speed of light in vacuum, \(\vec{r}\) is the position vector and \(t\) is time. For a monochromatic EM wave the electric and magnetic fields are given by \(\vec{E}(\vec{r}, t) = \vec{E}(\vec{r}) \cdot e^{-i\omega t}\) and \(\vec{H}(\vec{r}, t) = \vec{H}(\vec{r}) \cdot \exp^{-i\omega t}\), so from (1) and (2) it follows that the complex amplitudes \(\vec{E}\) and \(\vec{H}\) satisfy the Helmholtz equation

\[
\nabla^2 \vec{E} (\vec{r}) + k^2 \vec{E} (\vec{r}) = 0, \tag{3}
\]

\[
\nabla^2 \vec{H} (\vec{r}) + k^2 \vec{H} (\vec{r}) = 0, \tag{4}
\]

where \(k = \frac{\omega}{c} = \frac{2\pi}{\lambda}\). The electromagnetic energy flow lines are determined using the energy flux vector, given by the real part of the complex Poynting vector

\[
\vec{S} (\vec{r}) = \frac{1}{2} \text{Re}[\vec{E}(\vec{r}) \times \vec{H}^*(\vec{r})], \tag{5}
\]
from the equation
\[ \frac{d\vec{r}}{ds} = \frac{\overrightarrow{S}}{S} \] (6)
where \( ds \) is elementary arc length along the EME flow line.

3 Experimental setup and diffraction picture

In Fig. 1(a) the experimental setup consisting of the optical bench holding a green laser pointer with the wavelength \( \lambda = 532 \text{nm} \), an opaque barrier with a razor glued along its vertical edge and the observation screen is shown. If the laser beam is propagating freely there is a circular bright spot on the screen as in Fig. 1(b), and if a half of the beam is blocked in the diffraction pattern shown in Figures 1(c) and 1(d) the central part of high intensity and the long horizontal light line of smaller intensity are clearly visible. The theoretical solution of the diffraction problem is obtained by solving the Helmholtz equation behind the obstacle so that the boundary conditions at the obstacle are satisfied. The solution can be written in the form of the Fresnel-Kirchhoff integral [10]. Let us consider the incident beam travelling along the axis coming to the opaque obstacle located at xOz plane, with the edge along the z axis as shown in Fig. 2. For simplicity we will assume that the incident wave does not depend on the z-coordinate, and it is Gaussian along the x axis. In that case the Fresnel-Kirchhoff integral reads:

\[ \Psi(x, y) = \sqrt{\frac{k}{2\pi y}} e^{-i\pi/4} e^{iky} \int \Psi_0(x', 0^+) e^{ik(x-x')^2/2y} dx' \] (7)

\[ \Psi_0(x', 0^+) = \begin{cases} 0, & x' > 0, \\ Ae^{-x'^2/4\sigma^2}, & x' \leq 0 \end{cases} \] (8)

The incident wave is the superposition of two waves \( H \) polarized with the magnetic field along the z (A components) and \( E \) polarized with the electric field along the z (B components), with the phase shift \( \phi \) between them. As shown by Born and Wolf [10] and in [2, 3] the solution behind the obstacle is given by:

\[ \overrightarrow{H} = -ik^{-1} Be^{i\phi} \partial \Psi \overrightarrow{e}_x + ik^{-1} Be^{i\phi} \partial \Psi \overrightarrow{e}_y + A \Psi \overrightarrow{e}_z, \] (9)

\[ \overrightarrow{E} = \frac{iA}{\epsilon_0 \omega} \partial \Psi \overrightarrow{e}_x - \frac{iA}{\epsilon_0 \omega} \partial \Psi \overrightarrow{e}_y + \frac{kB}{\epsilon_0 \omega} e^{i\phi} \Psi \overrightarrow{e}_z, \] (10)

so that the components of the Poynting vector can be expressed as:

\[ S_x = \frac{i}{4\epsilon_0 \omega} (A^2 + B^2)(\Psi \partial \Psi^* \overrightarrow{e}_x - \Psi^* \partial \Psi \overrightarrow{e}_x), \] (11)
Fig. 1  Experimental setup (a) a green pointer laser spot (b) and diffraction patterns at distances y=0.6 m (c) and y=3 m (d) behind a razor covering approximately half of the laser beam

\[ S_y = \frac{i}{4\epsilon_0\omega}(A^2 + B^2)(\psi \frac{\partial\psi^*}{\partial y} - \psi^* \frac{\partial\psi}{\partial y}) , \]  \hspace{1cm} (12)

\[ S_z = \frac{i}{2\epsilon_0\omega k} AB \sin \phi (\frac{\partial\psi}{\partial x} \frac{\partial\psi^*}{\partial y} - \frac{\partial\psi^*}{\partial x} \frac{\partial\psi}{\partial y}) . \]  \hspace{1cm} (13)

The differential equations

\[ \frac{dx}{dy} = \frac{S_z}{S_y} \]  \hspace{1cm} (14)
Trajectory based interpretation of the laser light diffraction on a sharp edge

Fig. 2 Beam of light incident on a half plane

\[ \frac{dz}{dy} = \frac{S_z}{S_y}, \]

(15)

determine the photon paths, which are obtained by numerical integration and shown at Fig. [4] The histogram of the number of trajectories ending at various points along the x-axis at the chosen distance from the half plane shows very good agreement with the corresponding intensity curve.

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References

1. Bohm D., Hiley B.J., The Undivided Universe, New York: Routledge (1993).
2. Davidović M.D., Sanz A. S., Arsenović D., Božić M., Miret-Artes S., Electromagnetic energy flow lines as possible paths of photons, Phys. Scr. T135, 014009 (2009)
3. Sanz A. S., Davidović M. D., Božić M., Miret-Artes S., Understanding interference experiments with polarized light through photon trajectories, Ann. Phys. 325, 4, 763 (2010).
4. Davidović M. D., Sanz A. S., Božić M., Arsenović D., Dimić D., Trajectory-based interpretation of Youngs experiment, the Arago-Fresnel laws and the Poisson-Arago spot for photons and massive particles, Phys.Scr. T153, 014015(2013).
5. Davidović Miloš D., Davidović Milena D., Mode analysis of the optical and the microwave waveguides using electromagnetic energy flow lines, Acta Phys. Pol. A 116, 4, 672 (2009).
6. Kocsis S., Braverman B., Ravets S., Stevens M. J., Mirin R. P., Shalm L. K., Steinberg, A. M., Observing the average trajectories of single photons in a two-slit interferometer, Science 332, 1170 (2011).
7. http://physicsworld.com/cws/article/news/2011/dec/16/ physics-world-reveals-its-top-10-breakthroughs-for-2011
8. Sommerfeld, A.,Mathematische Theorie der Diffraction, Mathematische Annalen 47, 134 (1896).
9. Sommerfeld, A.,Optics, New York: Academic Press, 249-266 (1969).
10. Born M., Wolf E., Principles of Optics, New York: Pergamon Press, 428-435 (1986).
Fig. 3  Photon paths and intensity distribution at distances $y=5$ cm (a), $y=60$ cm (b) and photon paths in the vicinity of the half plane edge (c)

11. Pearson J. E., McGill T. C., Kurtin S., Yariv, A., Diffraction of Gaussian laser beams by a semi-infinite plane, J. Opt. Soc. Am. 59, 1440 (1969).

12. Anakhov S. P., Lymarenko R. A., Khizhnyak A. E., Wide-angle diffraction of the laser beam by a sharp edge, Radiophys. Quant. El. 47, 926 (2004).
Fig. 4  Histogram of the number of trajectories ending at various points along the x axis and the corresponding intensity curves (solid line) for distance y=0.6 m