Statistical investigation and experimental verification of frequency stirring reverberation chamber

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Abstract. A statistically uniform E-field can be created in a reverberation chamber by frequency stirring. The statistical properties of frequency stirring reverberation chamber in over-moded are analyzed, and the ideal probability density functions (PDF) of E-field and field power are presented. In order to verify the statistical model, the S21 parameter which is the normalized power is measured between transmitting and receiving antenna. The result of the examination and processed data shows that the histogram of S21 can be good agreement with the theoretical model when the number of sample is enough.

1. Introduction
Reverberation chambers are becoming more and more popular for electromagnetic compatibility and immunity testing [1]. Such chambers can generate high field strengths using modest power sources. The goal of reverberation chambers is to create a statistically uniform field that eliminates the need to rotate the test object [2]. A mechanical mode stirrer (paddle wheel) is usually used to vary the chamber boundary conditions in an attempt to obtain statistical field uniformity. Mechanical stirring can be quite effective, but it is fairly slow [3]. It has been pointed out that the continuous changing of the mechanical tuner position has some equivalent effect with frequency modulation of the source and experimental work has been done by Loughry in 1991 to show the possibility of using frequency stirring in a reverberation chamber [4]. A computational investigation on frequency stirring in an ideal two-dimensional rectangular cavity has been done by Hill [5] and the good field uniformity results imply that frequency stirring could be an alternative to mechanical stirring in reverberation chambers. In this paper we are going to investigate the statistics of the field in the frequency stirring reverberation chamber and the organization of this paper is as follows: In section 2, the statistics of frequency stirring reverberation chamber is analyzed, and the theoretical E-field PDF model is presented. In Section 3 the experimental setup used in this paper is discussed. Section 4 presents experimental and theoretical results to illustrate the validity of the E-field PDF model. Section 5 thanks the funded projects and Section 6 summarizes the research presented here briefly and discusses future work.

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2. Statistical characteristics of frequency stirring reverberation chamber

The theory of reverberation chamber derived from wave-guide resonance cavity. In reverberation chamber, a mode stirrer is commonly used to change the boundary conditions for the electromagnetic field. Each location of the stirrer creates an independent field distribution. Similarly, independent electromagnetic environment can be provided by the frequency stirring process, because mechanical stirring leads to the physical dimension alter, meanwhile, the frequency-stirred mode makes the electrical dimension change.

The E-field at a location in the frequency stirring reverberation chamber varies greatly depending on the electrical dimension alter by changing the operating frequency. When the dimensions of the chamber are large compared to the wavelength at the operating frequency, that is, when a sufficient number of resonant modes in the chamber exists, the E-field vector at a location in the chamber is the sum of multipath plane waves with random phases. The number of resonant modes (N) is given approximately by using Weyl's formula [6].

\[ N \approx \frac{8\pi abc}{3\lambda^3} \]  

where \( a, b, \) and \( c \) are the dimensions of the chamber and \( \lambda \) is the wavelength.

The E-field \( E \) as the vector sum of three rectangular components, each of them consists of a real and imaginary part, that is, as six parameters in total, are represented by

\[ E = E_x + E_y + E_z \]  
\[ E_{x,y,z} = \text{Re}(E_{x,y,z}) + i \text{Im}(E_{x,y,z}) \]

As the large number of multipath waves arrives at a location in the chamber by moving the stirrers, each real and imaginary part of three rectangular components is normally distributed by the central limit theorem. Since these real and imaginary parts are not correlated, they are independent random variables. Therefore, six parameters of the E-field are independent random variables and each parameter is normally distributed with zero mean and variance \( \sigma^2 \)

\[ f(E_{r}) = \frac{1}{\sqrt{2\pi}} e^{-\frac{E_r^2}{2\sigma^2}} \]

The other parameters \( ExI, EyR, EyI, EzR, EzI \) also obey the law of normal distribution.

\[ E_x^2 = E_{xR}^2 + E_{xI}^2, \ E_x^2 \]  
is the chi-square distributed with two degrees. The PDF of \( E_x^2 \) is

\[ f(E_x^2) = \frac{1}{2\sigma^2} e^{-\frac{E_x^2}{2\sigma^2}} \]

The power is in proportion to the square values of E-field, so each rectangular components of the power also obey the chi distributed with two degrees. The PDF of \( P_x \) is

\[ f(P_x) = \frac{1}{2\sigma^2} e^{-\frac{P_x}{2\sigma^2}} \]

The square values of E-field is

\[ E^2 = E_x^2 + E_y^2 + E_z^2 = E_{xR}^2 + E_{xI}^2 + E_{yR}^2 + E_{yI}^2 + E_{zR}^2 + E_{zI}^2 \]

The sum or the square values of the six parameters, i.e. the squared magnitude of the resultant field, is the chi-square distributed with six degrees of freedom. Similarly, the root value of the chi-square
distribution, i.e. the magnitude of the resultant field, is chi distributed with six degrees of freedom. The probability density functions of these distributions are as follows:

\[
f(E^2) = \frac{E^5}{16\sigma^2} e^{-\frac{E^2}{2\sigma^2}}
\]

\[
f(E) = \frac{E^5}{8\sigma^2} e^{-\frac{E^2}{2\sigma^2}}
\]

The PDF of the power is

\[
f(P) = \frac{P^2}{16\sigma^2} e^{-\frac{P}{2\sigma^2}}
\]

3. Experiment method and setup

In order to verify the statistical properties of frequency stirring reverberation chamber, we test the power at a location by sweeping the S21 measurement between the transmitting antenna and the receiving antenna in the test region.

The measurement setup is shown in figure 1. A two-port vector network analyzer (VNA) is used to measure the S21 parameters. Port 1 connected to a transmitting antenna, port 2 connected to a receiving antenna and both of the two horns are Log-periodic antenna.

![Figure 1. Diagram of the testing system.](image1)

![Figure 2. Configuration of the experiment.](image2)
The configuration of the experiment is shown in figure 2. In the test, the dimension of the reverberation chamber is 10.5 m by 8 m by 4.3 m and the lowest useable frequency is 80 MHz. The operating frequency of VNA is from 300 KHz to 1.5 GHz and the number of frequency points set on the NVA is always chosen to be the maximum 1601. We take the example of the swept frequency region from 500 MHz to 1 GHz, so the gap between each frequency point is about 0.625 MHz.

4. Experimental Results and analysis

Figure 3 shows the measured S21 parameter fluctuates with the operating frequency changes, and the result approves that frequency stirring is an alternative way to mechanical stirring to achieve statistical field uniformity in an over-moded chamber.

![Figure 3. The S21 curve at a location.](image)

![Figure 4. Comparison of the histogram and PDF when the number of sample is 50.](image)

![Figure 5. Comparison of the histogram and PDF when the number of sample is 100.](image)

When the operating frequency is 1 GHz, the reverberation chamber is in over-moded status. Figure 4–7 shows the measured S21 histogram of 50, 100, 200 and 300 samples and is compared with the theoretical PDF which is presented in formula 10. When the number of samples is 50, the characteristics of the measured distribution is not agreement with the theoretical value, because the independent sample is not enough, and when the number of samples increases to 200, the PDF of the S21 distribution consists with the theoretical curve well, even more better when the number reach to 300. The processed data proves that the statistical characteristics of frequency stirring reverberation
chamber can be satisfied with the theoretical model, therefore statistically uniform spatial E-field can be created efficiently by frequency-stirred mode.

![Graph](image1)

**Figure 6.** Comparison of the histogram and PDF when the number of sample is 200.

![Graph](image2)

**Figure 7.** Comparison of the histogram and PDF when the number of sample is 300.

5. Conclusion
In the paper, the statistical property of a frequency stirring reverberation chamber is analyzed, and in order to verify the theoretical PDF model, the S21 parameter (normalized power) is measured by a two-port vector network analyzer in the paper. The processed data shows that the histogram of S21 can be good agree with theoretical model when the number of sample is more than 200, and this proves that frequency stirring can achieve spatial uniformity efficiently. Meanwhile we should see that the effect of the nonzero bandwidth of frequency stirring in performing immunity measurements, particularly for test objects with sharp resonances and the frequency stirring method used in practical application needs further theoretical and experimental studies.

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