Quantum Teleportation in a Solid State System

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Abstract

We propose a practical solid-state system capable of demonstrating quantum teleportation. The set-up exploits recent advances in the optical control of excitons in coupled quantum dots, in order to produce maximally-entangled Bell and Greenberger-Horne-Zeilinger (GHZ) states. Only two unitary transformations are then required: a quantum Controlled-Not gate and a Hadamard gate. The laser pulses necessary to generate the maximally-entangled states, and the corresponding unitary transformations, are given explicitly.
Since the original idea of quantum teleportation considered in 1993 by Bennett et al. [1], great efforts have been made to realize the physical implementation of teleportation devices [2]. The general scheme of teleportation [1], which is based on Einstein-Podolsky-Rosen (EPR) pairs [3] and Bell measurements [4] using classical and purely nonclassical correlations, enables the transportation of an arbitrary quantum state from one location to another without knowledge or movement of the state itself through space. This process has been explored from various points of view [2]; however none of the experimental set-ups to date have considered a solid-state approach, despite the recent advances in semiconductor nanostructure fabrication and measurement [5–8]. Reference [6], for example, demonstrates the remarkable degree of control which is now possible over quantum states of individual quantum dots (QDs) using ultra-fast spectroscopy. The possibility therefore exists to use optically-driven QDs as “quantum memory” elements in quantum computation operations, via a precise and controlled excitation of the system. In this Letter we propose a practical scheme for quantum teleportation which exploits currently available ultrafast spectroscopy techniques in order to prepare and manipulate entangled states of excitons in coupled QDs. To our knowledge, this is the first practical proposal for demonstrating quantum teleportation in a solid state system.

In order to implement the quantum operations for the description of the teleportation scheme proposed here, we employ two elements: the quantum controlled-not gate (CN gate), and the Hadamard transformation. In the orthonormal computation basis of single qubits \{|0\rangle, |1\rangle\}, the CN gate acts on two qubits \(|\varphi_i\rangle\) and \(|\varphi_j\rangle\) simultaneously as follows: 
\[
CN_{ij}(|\varphi_i\rangle |\varphi_j\rangle) \leftrightarrow |\varphi_i\rangle |\varphi_i \oplus \varphi_j\rangle. 
\]
Here \(\oplus\) denotes addition modulo 2. The indices \(i\) and \(j\) refer to the control bit and the target bit respectively. The Hadamard transformation \(H\) acts only on single qubits by performing the rotations 
\[
H(|0\rangle) \mapsto \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \quad \text{and} \quad H(|1\rangle) \mapsto \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle). 
\]
We also introduce a pure state \(|\Psi\rangle\) in this Hilbert space given by 
\[
|\Psi\rangle = \alpha |0\rangle + \beta |1\rangle \quad \text{with} \quad |\alpha|^2 + |\beta|^2 = 1, \quad \text{where} \quad \alpha \text{ and } \beta \text{ are complex numbers. As discussed later, } |0\rangle \text{ represents the vacuum state for excitons while } |1\rangle \text{ represents a single exciton.}
\]

Figure 1 shows our general computational approach which is inspired by the work of...
Brassard et al. [9]. As usual, we refer to two parties, Alice and Bob. Alice wants to teleport an arbitrary, unknown qubit state $|\Psi\rangle$ to Bob. Figure 2 shows the specific realization we are proposing using optically controlled quantum dots with QD $a$ initially containing $|\Psi\rangle$.

Alice prepares two qubits (QDs $b$ and $c$) in the state $|0\rangle$ and then gives the state $|\Psi_{00}\rangle$ as the input to the system. By performing the series of transformations shown in Fig 1(a), Bob receives as the output of the circuit the entangled state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_a \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_b |\Psi\rangle_c$ (Fig. 2c). In Fig. 1(b) we extend the analysis of the teleportation process to the case of a four qubit quantum circuit, which can be realized by four coupled QDs. As before, Alice wants to teleport the state $|\Psi\rangle_a$ to Bob. She prepares three qubits in the state $|0\rangle$ (QDs $b, c$ and $d$) and gives the state $|\Psi_{000}\rangle$ as the input to the system. From Fig. 1(b) it is clear that the function of the first three operations performed by Alice is to obtain the maximally entangled GHZ state $\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$. The next two operations realized by Alice (before the arrow in Fig 1(b)) leave the system in the state

$$\frac{1}{2}\{|000\rangle(\alpha|0\rangle + \beta|1\rangle) + |011\rangle(\beta|0\rangle + \alpha|1\rangle) + |100\rangle(\alpha|0\rangle - \beta|1\rangle) + |111\rangle(-\beta|0\rangle + \alpha|1\rangle)\}. \quad (1)$$

Performing the operations shown after the arrow in Fig. 1(b), Bob gets as the output of the circuit the entangled state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_a \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_b, c |\Psi\rangle_d$. Hence the state $|\Psi\rangle$ was teleported from dot $a$ to dot $d$ in the system. In order to describe in detail how this circuit may be implemented, we need to perform the following steps: Alice prepares three qubits in the state $|0\rangle$, and then sends the first two of them through the two first gates between QDs $b$ and $c$, as shown in Fig. 1(b). She keeps the information stored in QD $c$, namely $\gamma$, in her quantum memory and sends qubit $\beta$ to Bob. In the next step she pushes the other qubit which is in state $|0\rangle$ together with $\gamma$ to the third gate; after this operation, she keeps the last qubit of the system, $\delta$. Alice then receives from QD $a$ the qubit $|\Psi\rangle$ which she wants to teleport to Bob. To achieve this, she removes the $\delta$ qubit from her quantum memory and sends this, together with qubit $|\Psi\rangle$, to the next two gates of the circuit. She then performs a measurement of the output between QDs $a$ and $c$ [12] (at the arrow in Fig. 1(b)) in order
to turn the result into two classical bits \( \Lambda \) and \( \Gamma \) respectively. To finish the teleportation process, Alice needs to communicate \( \Lambda \) and \( \Gamma \) to Bob via a classical communication channel. Hence after the arrow, Bob receives the classical information and creates the quantum states \( |\Lambda\rangle \) and \( |\Gamma\rangle \). Next, he removes the qubits \( \beta \) and \( \delta \) from his quantum memory and sends the four qubits to his part of the circuit. At this point teleportation is complete since Bob receives at his output the state \( |\Psi\rangle \) on dot \( d \). Hence our quantum teleportation circuit (QTC) transforms the input \( |\Psi\rangle_a |0\rangle_b |0\rangle_c |0\rangle_d \) into the output \( \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_{a} |\text{Bell}\rangle_{b,c} |\Psi\rangle_{d} \).

Interestingly, the above teleportation process can be extended to an \( n \)–QTC using the Schrödinger’s cat state, as shown in Fig. 1(c). Again, the goal is to teleport the state \( |\Psi\rangle_a \) from Alice to Bob. She prepares \( n - 1 \) qubits in the state \( |0\rangle \) (QDs \( b, ..., n \)) and hence gives the state \( |\Psi_{00...0}\rangle \) as the input. After the first \( n - 1 \) operations (Fig 1(c)) she obtains the Schrödinger’s cat state \( \frac{1}{\sqrt{2}}(|00...0\rangle + |11...1\rangle)_{b,...,n} \) which, followed by the last two exclusive-or operations before the arrow, leaves the system in the following state of \( n \) qubits

\[
\frac{1}{2} \{|00...0\rangle (\alpha |0\rangle + \beta |1\rangle) + |011...1\rangle (\beta |0\rangle + \alpha |1\rangle) + |100...0\rangle (\alpha |0\rangle - \beta |1\rangle) + |11...1\rangle (-\beta |0\rangle + \alpha |1\rangle)\}.
\]

The procedure to realize the circuit of Fig. 1(c) follows directly from the description given for Fig. 1(b). In the case of Fig 1(c), the measurement performed by Alice at the end of her part of the circuit, must be realized between the \( a \)–th and the \( (n - 1) \)th QDs \[12\] in order to turn the result into two classical bits \( \Theta \) and \( \Upsilon \) respectively. Hence Alice communicates these bits to Bob via a classical channel and, after the arrow, Bob receives the classical information and creates the quantum states \( |\Theta\rangle \) and \( |\Upsilon\rangle \). Next, he removes from his quantum memory the other \( n - 2 \) qubits, thereby ultimately obtaining \( |\Psi\rangle \) on the \( n \)th–dot. In this way, the QTC presented here transforms the input state \( |\Psi\rangle_a |0\rangle_b |0\rangle_c ... |0\rangle_n \) into the output \( \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_{a} \frac{1}{\sqrt{2}}(|00...0\rangle + |11...1\rangle)_{b,...,n-1} |\Psi\rangle_{n} \). From Figs. 1(b) and 1(c) we note that the final stage of the QTC may be used as a subroutine in larger quantum computations or for quantum communication; specifically this is because we are recovering at the output a 2, 3, ..., or \( (n - 2) \)–maximally entangled state \[13\]. We also note that the structure of Bob’s part of the circuit is the same for all the circuits in Fig. 1. This is because Bob’s function in the QTC is
to realize the “appropriate rotations” over the general state given in Eq. (2). It is interesting to note that if Bob, instead of performing the operations after the arrow, chooses one of such appropriate unitary transformations [15] to apply to the $n$–th qubit after receiving the classical bits from Alice’s measurement, then he does not need to perform his part of the circuit. For this reason only two unitary exclusive-ors transformations are needed in order to teleport the state $|\Psi\rangle$. However, from the point of view of our implementation and, more generally, for quantum computer algorithms, it is better to undertake the complete process shown in the QTC than to choose such a special rotation. Although our goal is the practical realization using what is at the limit of current optoelectronics technology, we note that the circuits of Fig. 1 are not restricted to QD systems: they can be applied to any system where the task of entangled-state preparation has been achieved.

In order to describe the physical implementation of the quantum circuits using coupled quantum dots, we exploit the recent experimental results involving coherent control of excitons in single quantum dots on the nanometer and femtosecond scales [3,4]. Consider a system of $N$ identical and equispaced QDs containing no net charge which are radiated by long-wavelength classical light, as illustrated schematically in Fig. 2(b) for the case $N = 3$. The formation of single excitons within the individual QDs and their inter-dot hopping can be described by the Hamiltonian [16,10]

$$H(t) = \epsilon J_z + W (J^2 - J^2_z) + \xi(t) J_+ + \xi^*(t) J_-,$$

(3)

where $J_+ = \sum_{p=0}^N c^\dagger_p h^\dagger_p$, $J_- = \sum_{p=0}^N h_p c_p$, and $J_z = \frac{1}{2} \sum_{p=0}^N \left( c^\dagger_p c_p - h^\dagger_p h_p \right)$. Here $c^\dagger_p$ ($h^\dagger_p$) is the electron (hole) creation operator in the $p$th QD; $\epsilon$ represents the QD band gap, $W$ is the interdot interaction parameter, $\xi(t)$ the laser pulse shape, while the quasi-spin $J$–operators satisfy the usual commutation relations $[J_z, J_\pm] = \pm \hbar J_\pm$, $[J_+, J_-] = 2\hbar J_z$ and $[J^2, J_+] = [J^2, J_-] = [J^2, J_z] = 0$. By solving the eigenvalue problem associated with the time-dependent Hamiltonian (3), we have shown [10] for several different values of the phase $\phi$, that Bell and GHZ states of the form $\frac{1}{\sqrt{2}} \left( |00\rangle + e^{i\phi} |11\rangle \right)$ and $\frac{1}{\sqrt{2}} \left( |000\rangle + e^{i\phi} |111\rangle \right)$ can be prepared in systems comprising two and three coupled quantum dots, respectively. The practical requirements
are realizable in present experiments employing both ultrafast \cite{6,7} and near-field optical spectroscopy \cite{8} of quantum dots. In Figure 3 we present the generation of $\phi-$pulses which lead to the implementation of our QTC. As mentioned previously, $|0\rangle$ represents the vacuum for excitons while $|1\rangle$ denotes a single-exciton state. For the practical proposal of Fig. 1(a), we require 3 equidistant QDs which initially must be prepared in the state $|\Psi\rangle_a |0\rangle_b |0\rangle_c$. As shown in Fig. 2(a), one of these (QD $a$) contains the quantum state $|\Psi\rangle_a$ that we wish to teleport, while the other two (QDs $b$ and $c$) are initialized in the state $|00\rangle_{bc}$ – this latter state is easy to achieve since it is the ground state. Following this initialization, we illuminate QDs $b$ and $c$ with a radiation pulse of frequency $\omega$ given by $\xi(t) = A \exp(-i\omega t)$ (see Fig. 2(b)); here $A$ includes the electron-photon coupling and the electric field strength, and the time-dependence of $\xi$ gives the pulse shape. As an example we consider the case of ZnSe-based QDs. The band gap $\epsilon = 2.8$ eV, hence the resonance optical frequency $\omega = 4.3 \times 10^{15}$ s$^{-1}$. In units of $\epsilon$, $W = 0.1$ and $A = \frac{1}{2\pi}$. For a 0 or $2\pi-$pulse, the density of probability for finding the QDs $b$ and $c$ in the Bell state $\frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$ shows that a time $\tau_{Bell} = 7.7 \times 10^{-15}$ s is required (see Fig. 3(a)). This time $\tau_{Bell}$ hence corresponds to the realization of the first two gates of the circuit in Fig. 1(a), i.e. the Hadamard transformation (or $\frac{\pi}{4}$-rotation) over QD $b$ followed by the $CN$ gate between QDs $b$ and $c$. After this, the information in qubit $c$ is sent to Bob and Alice keeps in her memory the state of QD $b$ (Fig. 1(a)). Next, we need to perform a $CN$ operation between QDs $a$ and $b$ and, following that, a Hadamard transform over the QD $a$: this procedure then leaves the system in the entangled state
\[
\frac{1}{2} \left\{ |00\rangle (\alpha |0\rangle + \beta |1\rangle) + |01\rangle (\beta |0\rangle + \alpha |1\rangle) + |10\rangle (\alpha |0\rangle - \beta |1\rangle) + |11\rangle (-\beta |0\rangle + \alpha |1\rangle) \right\}.
\]

The last step is possible in practice using the masking technique of exciting and detecting the dot luminiscence through micrometer-sized apertures in an aluminum mask \cite{6}; this allows for selection of a few QDs within the broad distribution of dots in the sample. Combining spatial and spectral resolutions, it therefore becomes possible to excite and probe only one individual QD with the corresponding dephasing time being $\tau_d = 4 \times 10^{-11}$ s \cite{6}. Hence we have the possibility of coherent optical control of the quantum state of a single dot.
Furthermore, this mechanism can be extended to include more than one excited state: since \( \frac{\tau_{\text{Bell}}}{\tau_d} \simeq 1.8 \times 10^{-4} \), several thousand unitary operations can in principle be performed in this system before the excited state of the QD decoheres. This fact together with the experimental feasibility of applying the required sequence of laser pulses on the femtosecond time-scale [17] leads us to conclude that we do not need to worry unduly about decoherence occurring whilst performing the other four unitary operations that Bob needs in order to obtain the final state \( \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_{a} \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_{b} |\Psi\rangle_{c} \), thereby completing the teleportation process. In the case of Fig. 3(b), a similar analysis shows that \( \tau_{\text{GHZ}} = 1.3 \times 10^{-14} \text{ s} \), and hence \( \frac{\tau_{\text{GHZ}}}{\tau_d} \simeq 3.3 \times 10^{-4} \): this also makes the circuit in Fig. 1(b) experimentally feasible.

Although this discussion refers to ZnSe-based QDs, other regions of parameter space can be explored by employing semiconductors of different bandgap \( \epsilon \). A more detailed description of the implementation of the CN and the Hadamard operations will be discussed elsewhere [10]. Even though the structures that we are considering have a dephasing time of order \( 10^{-11} \text{ s} \), QDs with stronger confinement are expected to have even smaller coupling to phonons giving the possibility for much longer intrinsic coherence times.

In summary, we have proposed a practical implementation of a quantum teleportation device, exploiting current levels of optical control in coupled QDs. Furthermore the analysis suggests that several thousand quantum computation operations may in principle be performed before decoherence takes place.

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presented here, where the measurement is performed between the first and the \((n - 1)\)th qubits.

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[14] In the case of an \(m\)–qubit circuit as presented here, the output is an entangled state which contains the \((m - 2)\)–maximally entangled state \(\frac{1}{\sqrt{2}}(|00...0\rangle + |11...1\rangle)_{2,3,...,m-1} \).

[15] From (2) it follows that such “appropriate unitary transformations” are

\[
U_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad U_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad U_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \text{and} \quad U_4 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.
\]

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FIGURE CAPTIONS

FIG. 1. Circuit schemes to teleport an unknown quantum state from Alice to Bob using an arrangement of (a) 3, (b) 4 and (c) n qubits (coupled quantum dots). The methods employ (a) Bell, (b) GHZ, and (c) Schrödinger’s cat states respectively. The operator subscripts denote the qubits being addressed. For simplicity the output is not shown in (c).

FIG. 2. Practical implementation of teleportation using optically-driven coupled quantum dots. (a) Initial state of the system. (b) Intermediate step: radiating the system with the pulse $\xi(t)$. (c) Final state. Typical values for the dots are diameter $d_1 = 30 \text{ nm}$, thickness $d_2 = 3 \text{ nm}$ and separation $d_3 = 100 \text{ nm}$.

FIG. 3. Generation of (a) Bell and (b) GHZ states. These pulses correspond to the realization of the Hadamard gate followed by the quantum $CN$ gates (see Figs. 1(a) and 1(b)). $\epsilon = 2.8 \text{ eV}$, $W = 0.1$, $\phi = 0$ and $A = \frac{1}{27}$. Here $\psi(t)$ denotes the total wavefunction of the system in the laboratory frame at time $t$. 
Figure 1: Reina and Johnson

(a) ALICE

\[ \Psi_0 \Rightarrow H_b \rightarrow CN_b \rightarrow CN_{\alpha b} \rightarrow H_{\alpha} \Rightarrow CN_{\beta c} \rightarrow H_{\beta} \rightarrow CN_{\gamma c} \rightarrow H_{\gamma} \Rightarrow |\Psi_\phi\rangle \]

(b) BOB

\[ \frac{1}{\sqrt{2}}(|0_b\rangle|0_c\rangle + |1_b\rangle|1_c\rangle) \]

Input \[ |\Psi_00\rangle \Rightarrow H_b \rightarrow CN_b \rightarrow CN_{\alpha b} \rightarrow H_{\alpha} \Rightarrow CN_{\beta c} \rightarrow H_{\beta} \rightarrow CN_{\gamma c} \rightarrow H_{\gamma} \Rightarrow |\phi\rangle|\text{Bell}\rangle |\Psi\rangle \]

(c) ALICE

\[ \frac{1}{\sqrt{2}}(|0_b\rangle|0_c\rangle|0_d\rangle + |1_b\rangle|1_c\rangle|1_d\rangle) \]

Input \[ |\Psi_000\rangle \Rightarrow H_b \rightarrow CN_b \rightarrow CN_{\alpha b} \rightarrow CN_{\beta c} \rightarrow H_{\beta} \rightarrow CN_{\gamma c} \rightarrow H_{\gamma} \Rightarrow |\phi\rangle|\text{Bell}\rangle |\Psi\rangle \]

(c) BOB

\[ \frac{1}{\sqrt{2}}(|0_b\rangle|0_c\rangle \ldots |0_n\rangle + |1_b\rangle|1_c\rangle \ldots |1_n\rangle) \]

Input \[ |\Psi_0000\rangle \Rightarrow H_b \rightarrow CN_{\beta c} \rightarrow CN_{\gamma c} \rightarrow \ldots \rightarrow CN_{n-1,n} \rightarrow CN_{\alpha n} \rightarrow H_{\alpha} \Rightarrow CN_{n-1,n} \rightarrow H_n \rightarrow CN_{\alpha n} \rightarrow H_n \]
Figure 2

(a) QD a \[|\Psi_a\rangle\]

QD b \[|0_b\rangle\]
QD c \[|0_c\rangle\]

\(d_1\)
\(d_2\)
\(d_3\)

(b) \(\xi(t)\)

(c) \[|\varphi_a\rangle|\varphi_b\rangle|\Psi_c\rangle\]
Figure 3: Reina and Johnson

(a) 

(b)