RELATIVISTIC OUTFLOWS FROM ADVECTION-DOMINATED ACCRETION DISKS AROUND BLACK HOLES

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ABSTRACT

Advection-dominated accretion flows (ADAFs) have a positive Bernoulli parameter, and are therefore gravitationally unbound. The Newtonian ADAF model has been generalized recently to obtain the ADIOS model that includes outflows of energy and angular momentum, thereby allowing accretion to proceed self-consistently. However, the utilization of a Newtonian gravitational potential limits the ability of this model to describe the inner region of the disk, where any relativistic outflows are likely to originate. In this paper we modify the ADIOS scenario to incorporate a pseudo-Newtonian potential, which approximates the effects of general relativity. The analysis yields a unique, self-similar solution for the structure of the coupled disk/wind system. Interesting features of the new solution include the relativistic character of the outflow in the vicinity of the radius of marginal stability, which represents the inner edge of the

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quasi-Keplerian disk in our model. Hence our self-similar solution may help to explain
the origin of relativistic jets in active galaxies. At large distances the radial depend-
ence of the accretion rate approaches the unique form $\dot{M} \propto r^{1/2}$, with an associated
density variation given by $\rho \propto r^{-1}$. This density variation agrees with that implied by
the dependence of the X-ray hard time lags on the Fourier frequency for a number of
accreting galactic black hole candidates. While intriguing, the predictions made using
our self-similar solution need to be confirmed in the future using a detailed model that
includes a physical description of the energization mechanism that drives the outflow,
which is likely to be powered by the shear of the underlying accretion disk.

Subject headings: accretion — accretion disks — black hole physics — galaxies: jets

1. INTRODUCTION

Advection-dominated accretion flows (ADAFs) are attractive models for explaining the emis-
sion from underluminous black hole candidates such as the Galactic center source. The earliest
paper on this subject was by Ichimaru (1977), and interest in the subject was revived by Narayan
& Yi (1994, 1995a, 1995b) and by Abramowicz et al. (1995). Comprehensive reviews of work in
this area can be found in Kato et al. (1998) and in Narayan, Mahadevan, & Quataert (1998).
Briefly, these models describe the dynamics of gas fed onto a black hole at very low (significantly
sub-Eddington) accretion rates. The gas is optically thin and hot, and possesses separate ion
($T_i \sim 10^{12}$ K) and electron ($T_e \sim 10^9$ K) temperatures because it is too tenuous for Coulomb col-
lisions to equilibrate the species. The low density also reduces the efficiency of bremsstrahlung
emission, hence limiting the luminosity of the flows. Although some success has been achieved in
understanding underluminous systems using ADAF models, a number of issues have been raised
concerning their self-consistency. Attention has been focused for example on the two-temperature
assumption, which is one of the basic features of ADAF models. While some authors have identi-
fied conditions under which this assumption is justified (Blackman 1999; Gruzinov 1998; Quataert
1998; Quataert & Gruzinov 1999), the validity of the assumption has also been challenged based on
the possibility that dissipation via plasma instabilities may preferentially heat the electrons rather
than the protons (Bisnovatyi-Kogan & Lovelace 1997). Chakrabarti (1999) has questioned the rel-
levance of the self-similar solutions that do not incorporate boundary conditions and therefore do
not exhibit shocks.

Narayan, Kato, & Honma (1997) and Narayan & Yi (1994, 1995a) have pointed out that in
ADAF models the gas has a positive Bernoulli parameter and is therefore gravitationally unbound.
The positivity of the Bernoulli parameter reflects the fact that the viscous torque transports energy
as well as angular momentum outward through the disk, which tends to make the total local
energy per unit mass positive. This could result in the evaporation and escape of at least some of
the gas before it passes through the event horizon of the black hole. Hence the standard ADAF
models may not be self-consistent, since they assume that all of the available gas accretes onto the compact object. This idea has been further supported by the ADAF simulations of Igumenshchev & Abramowicz (1999), which indicate that bipolar outflows may be produced when the Shakura-Sunyaev (1973) viscosity parameter $\alpha \sim 1$. However, Abramowicz et al. (2000) have suggested that self-consistent advection-dominated models with a positive Bernoulli parameter and no outflow can be constructed if the self-similar approach is abandoned. Ogilvie (1999) makes a similar point and solves a simplified version of one-dimensional, quasi-spherical accretion to conclude that time-dependent accretion can take place without the need for any outflows. While this is an interesting possibility, it appears that many underluminous black hole systems such as Sgr A* may possess relativistic outflows (Falcke 1999; Donea et al. 1999 and references therein). We therefore focus on coupled disk/wind models in this paper.

To address the problem of gravitationally unbound accretion in the standard ADAF models, Blandford & Begelman (1999; hereafter BB) generalized the model to include the possibility of powerful winds that carry away mass, energy, and angular momentum in proportions that render the Bernoulli constant of the gas remaining in the disk negative, thereby allowing it to accrete. The resulting advection-dominated inflow-outflow solutions (ADIOS) are similar to the ejection-dominated accretion flows (EDAFs) analyzed by Donea et al. (1999). Beckert (2000) has also obtained related self-similar solutions for advection-dominated accretion flows that include outflows. As it stands, the formalism of BB does not address the critical question of the formation mechanism for the outflow. This mechanism must be capable of accelerating high-energy protons, thereby creating a nonthermal tail in the particle distribution. Protons in the nonthermal tail escape to form the wind whereas the low-energy protons are tightly bound and therefore accrete. As discussed by Subramanian, Becker, & Kazanas (1999; hereafter SBK), the energy transfer may be accomplished via second-order Fermi acceleration driven by the shearing magnetic field.

One of the primary reasons for studying black hole accretion is to understand the connection between accretion and the production of relativistic jets. The jets probably originate close to the black hole as relativistic winds that become collimated with increasing distance. The study of the formation of the wind/jet therefore requires a reasonably precise treatment of the inner region of the disk, where the physical conditions are most extreme. Although it provides a useful means for analyzing steady accretion at large distances from the black hole, the ADIOS model in its original form is not applicable close to the event horizon due to the utilization of the standard Newtonian form for the gravitational potential. In this paper we attempt to incorporate the effects of general relativity by utilizing the pseudo-Newtonian approximation for the gravitational potential suggested by Paczyński & Wiita (1980). Clearly, this approach is not a substitute for a full treatment of general relativity, but it does provide us with a convenient mathematical tool for exploring the structure of the inner region of the disk. Our strategy is to construct new models for self-similar, advection-dominated accretion disks and associated winds that are as faithful as possible to the original ADAF and ADIOS scaling principles, while incorporating the pseudo-Newtonian form for the gravitational potential. The solutions obtained describe the structure of the entire disk outside
the radius of marginal stability, which is the edge of the quasi-Keplerian disk in our model. One clear implication of our approach is that our model should reduce to the ADIOS form at large distances from the black hole. We focus here on gasdynamical winds that exert no torque on the underlying disk. These winds may be powered by the shear acceleration mechanism discussed by SBK. The remainder of the paper is organized as follows. In §2 we develop the pseudo-Newtonian analog to the self-similar ADIOS model of BB. The consequences for the structure of the disk/wind system are analyzed in §§3 and 4, and the implications for the interpretation of observations of underluminous black-hole systems are discussed in §5. This includes a consideration of the relevance of the SBK shear acceleration in the context of the pseudo-Newtonian disk/wind model developed here.

2. PSEUDO-NEWTONIAN INFLOW-OUTFLOW MODEL

Working in units such that $GM = c = 1$, we can modify the thin-disk, quasi-Keplerian ADIOS formalism of BB to approximate the effects of general relativity close to a Schwarzschild black hole by replacing the Newtonian potential $\Phi = -1/r$ with the pseudo-Newtonian form $\Phi = -1/(r - 2)$ introduced by Paczyński and Wiita (1980) and further discussed by Paczyński (1998). This prescription yields the correct values for the marginally bound and marginally stable orbital radii. The pseudo-Newtonian potential offers a convenient means of approximating the effects of general relativity while preserving the degree of mathematical simplicity contained in the ADIOS solutions. However, as we demonstrate below, the pseudo-Newtonian potential introduces a length scale (namely, the horizon at $r = 2$), and therefore it is no longer possible to construct self-similar models based solely on power-law variations in radius.

2.1. Self-Similar Forms

Our goal here is to use the same physical principles underlying the ADIOS scalings to obtain new self-similar forms that are valid in the inner region of a disk governed by the pseudo-Newtonian potential. The advantage of the self-similar approach is that the solutions obtained may be valid over several orders of magnitude in radius. As with any self-similar model, there is a certain degree of arbitrariness in the imposed scalings. We therefore base our approach on the same physical principles employed in the ADIOS model, as applied in the context of the pseudo-Newtonian potential. We believe that the self-similar solutions obtained can yield useful insights into possible behaviors that should be investigated subsequently using models based on rigorous descriptions of the relevant physics.

BB assumed that the angular velocity of the disk $\Omega$ satisfies the quasi-Keplerian scaling $\Omega \propto \Omega_K$, where $\Omega_K \propto r^{-3/2}$ is the Keplerian angular velocity in a Newtonian potential. In the pseudo-Newtonian case the appropriate expression for the Keplerian angular velocity is (Paczyński & Wiita
and therefore the variation of $\Omega$ in a quasi-Keplerian accretion disk now becomes

$$\Omega(r) = \frac{v_{\phi}}{r} = \Omega_0 \frac{r^{-1/2}}{r-2},$$

(2.2)

where $\Omega_0$ is a positive constant and $v_{\phi}$ is the azimuthal velocity. Hence $\Omega$ scales as $\Omega_K$ throughout the disk. As in the ADIOS model, we assume that $0 < \Omega_0 < 1$ so that the disk is sub-Keplerian. Simulations carried out by Narayan et al. (1997) indicate that the behavior of $\Omega$ is well represented by equation (2.2) over several orders of magnitude in radius (outside the radius of marginal stability) for values of the Shakura-Sunyaev viscosity parameter $\alpha \sim 0.1-0.3$. Based on these results, we shall assume in our work that the variation of $\Omega$ is given by equation (2.2). However, this assumption does not apply in the region below the radius of marginal stability, where the gas passes through a sonic point before crossing the horizon. We provide further discussion of this point in § 5.

Proceeding by analogy with the ADAF/ADIOS models, we shall suppose that the outwardly-directed energy transport rate $E$ and the inwardly-directed angular momentum transport rate $I$ in the disk scale as

$$E(r) = \epsilon \frac{\dot{M}(r)}{r-2}, \quad I(r) = \frac{\lambda}{\Omega_0} \ell \dot{M}(r),$$

(2.3)

respectively, where $\epsilon$ and $\lambda$ are positive constants, $\ell = r^2 \Omega$ is the angular momentum per unit mass, and $\dot{M}(r)$ is the positive mass accretion rate. We will also assume that the internal energy per unit mass (i.e., the ion temperature) scales as the gravitational potential energy, implying that

$$a^2(r) \equiv \frac{P}{\rho} = \frac{a_0^2}{r-2},$$

(2.4)

where $a$ is the isothermal sound speed, $P$ is the gas pressure, $\rho$ is the mass density, and $a_0$ is a positive constant. All of the physical quantities denote vertical averages at a given cylindrical radius $r$. The assumed variations of $E$, $I$, $a$, and $\Omega$ are the most natural extensions of the scalings used by BB into the pseudo-Newtonian regime. In § 3 we shall see that these scalings impose definite constraints on the mechanism responsible for producing the energy and mass loss that powers the outflow.

### 2.2. Disk-Wind Coupling

Our ultimate goal is to understand the possible role of the shear acceleration mechanism discussed by SBK in launching ADIOS-type outflows. Consequently, we focus here on gasdynamical outflows (winds) that exert no local torque on the disk, and therefore we stipulate that $\Omega = \Omega_W$, where

$$\Omega_W \equiv \frac{1}{r^2} \frac{dI}{dM} = \frac{1}{r^2} \frac{dI/dr}{dM/dr}$$

(2.5)
denotes the angular velocity of the wind at the point of contact with the disk. Substituting for I using equation (2.3) then yields

\[ r^2 \Omega \frac{d\dot{M}}{dr} = \frac{d}{dr} \left( \lambda \dot{M} r^2 \Omega \frac{\Omega}{\Omega_0} \right), \]  

(2.6)

which immediately implies that the variation of \( \dot{M} \) is given by

\[ \dot{M} \propto (r^2 \Omega)^{\lambda/(\Omega_0 - \lambda)} , \]  

(2.7)

or, using equation (2.2) for \( \Omega(r) \),

\[ \dot{M} \propto \left( \frac{r^{3/2}}{r - 2} \right)^{\lambda/(\Omega_0 - \lambda)} . \]  

(2.8)

We can relate the variation of \( \dot{M} \) to the other physical quantities in the disk by writing

\[ \dot{M} = 4\pi r H v \rho , \]  

(2.9)

where \( H \) is the half-thickness of the disk and \( v \) is the radial velocity, defined to be positive for inflow. The value of \( H \) can be characterized by employing the equation of vertical hydrostatic equilibrium to write

\[ \frac{P}{z} \approx - \frac{\rho}{(r - 2)^2} \frac{z}{r} , \]  

(2.10)

which is valid provided \( z \ll r \). A scale analysis of this expression yields the usual approximation (e.g., Narayan et al. 1997)

\[ \left( \frac{H}{r} \right)^2 = a^2 \left( \frac{r - 2}{r} \right)^2 = a_0^2 \left( \frac{r - 2}{r} \right) , \]  

(2.11)

which reduces to equation (20) of BB in the limit \( r \rightarrow \infty \). Since the disk must be thin for self-consistency, we require that \( a_0 < 1 \).

2.3. Radial Momentum Conservation

Our self-similar approach is based on the scalings for \( E \), \( I \), and \( a \) given by equations (2.3) and (2.4), respectively. We conjecture that the associated radial velocity \( v \) scales as

\[ v(r) = \frac{v_0}{\sqrt{r - 2}} , \]  

(2.12)

where \( v_0 \) is a positive constant, implying that the radial velocity is proportional to the local freefall velocity, \( v_{ff}(r) = \sqrt{2/(r - 2)} \). We shall verify this hypothesis by confirming that the resulting variation of the energy transport rate \( E(r) \) satisfies equation (2.3). Note that in order to avoid
inflow faster than freefall, we require that \( v_0^2 < 2 \). In a steady state, the radial equation of motion (eq. [6] of BB) can be written as

\[
\frac{dv}{dr} = r \Omega^2 - \frac{1}{(r-2)^2} - \frac{1}{\rho} \frac{dP}{dr} .
\]

We can obtain a differential equation for the pressure \( P \) by substituting for \( \Omega, \rho = P/a^2 \), and \( v \) in equation (2.13) using equations (2.2), (2.4), and (2.12), respectively, which yields

\[
\frac{d\ln P}{dr} = \frac{v_0^2 + 2 \Omega_0^2 - 2}{2a_0^2(r-2)} ,
\]

with solution

\[
P(r) = P_0 (r-2)^{(v_0^2+2\Omega_0^2-2)/(2a_0^2)} ,
\]

where \( P_0 \) is a positive constant. The associated density variation is given by

\[
\rho(r) = \rho_0 (r-2)^{(v_0^2+2\Omega_0^2+2a_0^2-2)/(2a_0^2)} ,
\]

where

\[
\rho_0 \equiv \frac{P_0}{a_0^2} .
\]

Using equations (2.11), (2.12), and (2.16) to substitute for \( H, v, \) and \( \rho \), respectively, in equation (2.9) for the accretion rate, we obtain

\[
\dot{M} = \dot{M}_0 r^{3/2} (r-2)^{(v_0^2+2\Omega_0^2+2a_0^2-2)/(2a_0^2)} ,
\]

where

\[
\dot{M}_0 \equiv 4\pi a_0 v_0 \rho_0 .
\]

The power-law indices in this expression must match those appearing in equation (2.8), and therefore we require that

\[
\lambda = \frac{\Omega_0}{2} , \quad v_0^2 = 2 - 2\Omega_0^2 - 4a_0^2 ,
\]

which together imply that the self-similar variation of the accretion rate is given by the unique solution

\[
\dot{M}(r) = \dot{M}_0 \frac{r^{3/2}}{r-2} .
\]

This solution for \( \dot{M}(r) \) has an interesting behavior, in that it possesses a minimum at \( r = r_{\text{ms}} = 6 \), which is the radius of marginal stability for a Schwarzschild black hole. The rate at which matter is fed into the wind therefore approaches zero as \( r \to r_{\text{ms}} \). This is physically reasonable since below this radius we expect that most of the remaining material will be swept directly into the black hole rather than expelled into the wind. For \( r > 6 \), \( \dot{M} \) decreases monotonically with decreasing radius in response to the loss of mass into the wind, and as \( r \to \infty \), we find that \( \dot{M} \propto r^{1/2} \), in agreement with BB provided their parameter \( p \equiv d\ln \dot{M}/d\ln r = 1/2 \). Our sub-Keplerian assumption \( \Omega_0 < 1 \)
can be combined with the requirement that $v_0^2 > 0$ to conclude that $a_0^2 < (1 - \Omega_0^2)/2$, confirming that the disk is thin (see eq. [2.11]). Incorporating equations (2.20) into our previous results, we conclude that the pressure and density vary as

$$P(r) = \frac{P_0}{(r - 2)^2}, \quad \rho(r) = \frac{\rho_0}{r - 2},$$

(2.22)

which both increase monotonically with decreasing radius as required. In §5 we discuss the possible relevance of this density distribution in accounting for the observations of time lags between the soft and hard X-rays in accreting galactic sources.

### 2.4. Energy and Angular Momentum Conservation

Following BB, we express conservation of angular momentum in the disk by writing the inwardly-directed angular momentum transport rate as

$$I(r) = \dot{M} r^2 \Omega - \mathcal{G},$$

(2.23)

where $\mathcal{G}(r)$ is the torque exerted by the disk interior to radius $r$ on the outer portion of the disk. The outwardly-directed energy transport rate in the disk is likewise given by

$$E(r) = \mathcal{G} \Omega - \dot{M} \left( \frac{1}{2} v^2 \Omega^2 + \frac{1}{2} v^2 + \frac{5}{2} a^2 - \frac{1}{r - 2} \right),$$

(2.24)

which incorporates the pseudo-Newtonian gravitational potential. Note that equation (2.24) includes a term describing the radial kinetic energy of the gas, whereas BB ignored this term in their energy transport equation. The gas in the disk is assumed to have adiabatic index $\gamma = 5/3$. We can solve for the torque by combining equations (2.3) and (2.23) and setting $\lambda = \Omega_0/2$ in compliance with equation (2.20), which yields

$$\mathcal{G}(r) = I(r) = \frac{1}{2} \dot{M} r^2 \Omega.$$  

(2.25)

The physical implications of this torque can be investigated by combining equation (2.25) with the expressions for $\dot{M}$ and $\Omega$ to determine the implied variation of the dynamic viscosity $\nu$. We discuss this issue further in §3. By using equation (2.25) to eliminate $G$ in equation (2.24) and substituting for $\Omega$ and $a$ using equations (2.2) and (2.4), respectively, we can reexpress the energy transport rate as

$$E(r) = \epsilon \frac{\dot{M}(r)}{r - 2},$$

(2.26)

where

$$\epsilon = 1 - \frac{5}{2} a_0^2 - \frac{1}{2} v_0^2.$$  

(2.27)
Equations (2.20) and (2.27) establish three connections between the five parameters $\lambda$, $\Omega_0$, $\epsilon$, $a_0$, and $v_0$. Hence knowledge of any two is sufficient to calculate the remaining three. We shall therefore treat $\Omega_0$ and $\epsilon$ as free parameters in our model and compute $\lambda$, $a_0$, and $v_0$ using the relations

$$
\lambda = \frac{\Omega_0}{2}, \quad a_0^2 = 2\Omega_0^2 - 2\epsilon, \quad v_0^2 = 8\epsilon - 10\Omega_0^2 + 2,
$$

(2.28)

obtained using equations (2.20) and (2.27).

The energy transport rate given by equation (2.26) does satisfy equation (2.3), and therefore we have confirmed the validity of our solution for $v(r)$ given by equation (2.12). The establishment of the self-similar form for $E$ stems from the fact that in our model, the local torque is equal to the local angular momentum transport rate (see eq. [2.25]), and consequently the outward energy transport due to the torque balances the inward energy transport due to the accretion of azimuthal kinetic energy. BB obtain the same result for gasdynamical winds when $p = 1/2$. The net energy transport is in the outward direction provided $\epsilon > 0$. As $r \to \infty$, we find that $E(r) \to \epsilon \dot{M} r^{-1}$ in agreement with the ADIOS model of BB. Substituting for $\dot{M}$ using equation (2.21), our results for the energy and angular momentum transport rates can be rewritten as

$$
E(r) = \epsilon \dot{M}_0 \frac{r^{3/2}}{(r - 2)^2}, \quad I(r) = \lambda \dot{M}_0 \frac{r^3}{(r - 2)^2}, \quad G(r) = \lambda \dot{M}_0 \frac{r^3}{(r - 2)^2}.
$$

(2.29)

Note that the torque $G$ possesses a minimum at the radius of marginal stability $r = r_{ms} = 6$, but it does not vanish there. This behavior is consistent with simulations performed by Hawley & Krolik (2000), who find that the viscous stress is finite at $r = r_{ms}$. Moreover, Narayan et al. (1997) suggest that the stress actually vanishes at the sonic point (between the horizon and the radius of marginal stability), where the radial velocity becomes supersonic and the gas therefore loses pressure support. It follows that the torque is finite at $r = r_{ms}$, as implied by equation (2.29). However, it must be pointed out that this issue is a subject of ongoing debate. While Hawley & Krolik (2000), Agol & Krolik (2000), Gammie (1999) and Krolik (1999) contend that there is a finite stress at the radius of marginal stability, Armitage et al. (2000) and Paczynski (2000) argue that this should not be the case, at least for thin disks. In § 3 we examine the implications of our self-similar model for the variation of the viscosity and for the associated dissipation of kinetic energy in the disk.

3. Viscosity and Dissipation

Equation (2.25) for the torque $G$ in our self-similar RADIOS model has definite implications for the variation of the kinematic viscosity $\nu$. We can calculate the viscosity by writing the shear stress as (e.g., Frank, King, & Raine 1985)

$$
\frac{G}{4\pi r^2 H} = -\rho \nu r \frac{d\Omega}{dr},
$$

(3.1)
and eliminating $\mathcal{G}$ using equation (2.25), which yields

$$\nu = -\frac{v \Omega}{2} \left(\frac{d\Omega}{dr}\right)^{-1}.$$  \hspace{1cm} (3.2)

Substituting for $\Omega$ and $v$ using equations (2.2) and (2.12), respectively, we obtain

$$\nu = \frac{v_0 r (r - 2)^{1/2}}{3r - 2}.$$  \hspace{1cm} (3.3)

We can evaluate the variation of $\nu$ most straightforwardly by calculating an equivalent value for the associated Shakura-Sunyaev $\alpha$-parameter, defined by writing the shear stress as

$$\alpha P \equiv -\rho \nu r \frac{d\Omega}{dr}.$$  \hspace{1cm} (3.4)

Setting $P = \rho a^2$ and combining the result with equations (2.2), (2.4), (2.28), and (3.3) yields

$$\alpha = \alpha_0 \left(\frac{r}{r - 2}\right)^{1/2},$$  \hspace{1cm} (3.5)

where

$$\alpha_0 \equiv \frac{\Omega_0 (8 \epsilon - 10 \Omega_0^2 + 2)^{1/2}}{4 (\Omega_0^2 - \epsilon)}. $$  \hspace{1cm} (3.6)

We plot $\alpha_0$ as a function of $\epsilon$ and $\Omega_0$ in Figure 1. Equation (3.5) indicates that $\alpha$ has a very weak radial dependence, increasing by a factor of 1.2 as $r$ decreases from infinity to $r = r_{\text{ms}} = 6$. Hence the $\alpha$-parameter associated with our self-similar RADIOS model is practically constant within the hot inner region of the disk, which is consistent with the conventional assumption that the shear stress is approximately proportional to the pressure (e.g., Narayan et al. 1997). Based upon the observed behavior of $\alpha$, we conclude that our model has an implied viscosity variation that is consistent with fundamental notions regarding the transport of angular momentum in the shear flow. This tends to further support our fundamental assumptions regarding the scalings of $E$, $I$, and $a$.

### 3.1. Implications for Dissipation and Cooling

Next we consider the implications of our self-similar model for the dissipation on energy in the shear flow, and for the rate of cooling from the disk due to the loss of energetic particles into the wind. The equation governing the internal energy of the matter in the disk is

$$\frac{DU}{Dt} = \gamma \frac{U}{\rho} \frac{D\rho}{Dt} + \dot{U}_{\text{viscous}} - \dot{U}_{\text{wind}},$$  \hspace{1cm} (3.7)

where $D/Dt$ represents the comoving (Lagrangian) time derivative following the gas, $\gamma$ is the adiabatic index, $U = P/(\gamma - 1)$ is the internal energy density, and $\dot{U}_{\text{viscous}}$ and $\dot{U}_{\text{wind}}$ denote the
(positive) rates of viscous heating and cooling into the wind, respectively. Using the standard expression for the viscous heating rate (Frank, King, & Raine 1985),

$$\dot{U}_{\text{viscous}} = \rho \nu r^2 \left( \frac{d\Omega}{dr} \right)^2,$$

we obtain in a steady state

$$\frac{v \rho}{1 - \gamma} \frac{da^2}{dr} + v a^2 \frac{d\rho}{dr} = \rho \nu r^2 \left( \frac{d\Omega}{dr} \right)^2 - \dot{U}_{\text{wind}}. \quad (3.9)$$

This is equivalent to equation (2.12) from Narayan et al. (1997), except that we have chosen $v$ to be positive for inflow. In the standard ADAF theory, a factor $f$ is introduced to represent the fraction of the energy dissipated via viscosity that goes into heating the gas, with the remainder escaping as radiative losses. We can define an analogous heating efficiency factor $f$ in the context of our disk/wind model by writing

$$f \rho \nu r^2 \left( \frac{d\Omega}{dr} \right)^2 \equiv \rho \nu r^2 \left( \frac{d\Omega}{dr} \right)^2 - \dot{U}_{\text{wind}}. \quad (3.10)$$

Hence $1 - f$ gives the fraction of the energy dissipated via viscosity that is expelled into the wind via the internal energy of the escaping particles. Narayan et al. and other authors focusing on the standard ADAF scenario usually assume that $f = 1$ throughout the disk. However, in our RADIOS model we can calculate $f$ self-consistently by solving for $\dot{U}_{\text{wind}}$ and then utilizing equation (3.10). Setting $\gamma = 5/3$ and substituting for $v, a^2, \rho, \nu,$ and $\Omega$ using our self-similar model, we find that equation (3.9) yields for the implied wind cooling rate

$$\dot{U}_{\text{wind}} = \frac{\rho_0 v_0}{4} \left[ \frac{\Omega_0^2 (6 - r) + 4 \epsilon (r - 2)}{(r - 2)^{9/2}} \right]. \quad (3.11)$$

The corresponding result for the heating efficiency factor obtained using equation (3.10) is

$$f(r) = 4 \left( \frac{\Omega_0^2 - \epsilon}{\Omega_0^2} \right) \left( \frac{r - 2}{3r - 2} \right). \quad (3.12)$$

Since the energy dissipated from the shear flow via viscosity is divided between heating of the thermal particles and acceleration of the nonthermal (wind) particles, it follows that $f$ must lie in the range $0 < f < 1$ if we are to obtain a physically reasonable solution. In order to satisfy this condition as $r \to \infty$, we must stipulate that

$$\epsilon > \frac{\Omega_0^2}{4}. \quad (3.13)$$

Note that the heating efficiency $f$ displays a weak radial dependence, increasing by a factor of $4/3$ as $r$ varies from $r = 6$ to infinity. This indicates that a nearly fixed fraction of the energy supplied via the dissipation of the shear goes into heating of the disk, with the remainder powering the outflow. This seems to be a reasonable result, and one cannot say more in the absence of a detailed model for the acceleration mechanism that energizes the wind particles. We plan to address this issue by developing an advection-dominated disk/wind model that incorporates the Fermi-shear acceleration process discussed by SBK in future work.
3.2. Parameter Constraints

We have assumed that the disk is thin and sub-Keplerian, and that the inflow velocity is less than the local freefall velocity. These assumptions can be combined to constrain the range of values for the various parameters appearing in the model. In order for the disk to be thin, we require that \( a_0^2 < 1 \). Equation (2.28) for \( a_0^2 \) then implies that for a given value of \( \Omega_0 \), the parameter \( \epsilon \) must satisfy

\[
\max\left(0, \frac{\Omega_0^2 - 1}{2}\right) < \epsilon < \Omega_0^2,
\]

where the upper bound follows from the requirement that \( a_0^2 > 0 \). By combining equation (3.14) with our sub-Keplerian restriction \( \Omega_0 < 1 \), we find that \( \epsilon \) cannot exceed unity in general. We must also stipulate that \( 0 < v_0^2 < 2 \) in order to keep \( v^2 \) positive while avoiding inflow faster than the free-fall velocity. Equation (2.28) for \( v_0^2 \) then implies that \( \epsilon \) must fall within the range

\[
\max\left(0, \frac{5\Omega_0^2 - 1}{4}\right) < \epsilon < \frac{5\Omega_0^2}{4}.
\]

We plot the constraints represented by equations (3.14) and (3.15) in the \((\Omega_0, \epsilon)\) parameter space in Figure 2. The intersection of the constraints is expressed by the condition

\[
\max\left(0, \frac{5\Omega_0^2 - 1}{4}\right) < \epsilon < \Omega_0^2.
\]

We can combine this condition with equation (3.13) to conclude that in order for our model to be fully self-consistent, \( \epsilon \) and \( \Omega_0 \) must satisfy

\[
\max\left(\frac{\Omega_0^2}{4}, \frac{5\Omega_0^2 - 1}{4}\right) < \epsilon < \Omega_0^2, \quad 0 < \Omega_0^2 < 1.
\]

The allowed region of the \((\Omega_0, \epsilon)\) parameter space is the curved, shaded region depicted in Figure 2. For convenience, we have also included the shaded region of self-consistency in Figure 1 for \( \alpha_0 \). Note that we can obtain values for \( \alpha_0 \) in the range \( 0 < \alpha_0 < 1 \) within the region of the parameter space satisfying the conditions given by equations (3.17). In §4 we present a quantitative discussion of the implications of the pseudo-Newtonian potential for the coupled disk/wind structure.

4. EFFECTS OF THE RELATIVISTIC POTENTIAL

In this section we compare the results obtained using our self-similar, pseudo-Newtonian, relativistic advection-dominated inflow-outflow solution (RADIOS) with the ADIOS results derived by BB using the standard Newtonian potential. Before proceeding, it is important to point out that the ADIOS model includes the possibility of a mismatch between \( \Omega \) and \( \Omega_W \), i.e., there may be a torque between the disk and the wind. Since the winds treated here are gasdynamical, we
have assumed that no such torque exists, and consequently we have set $\Omega = \Omega_W$. This corresponds to case (iv) of BB, with $\lambda = \lambda_{\text{crit}}$, where

$$\lambda_{\text{crit}} = 2p \left[ \frac{(10\epsilon + 4p - 4ep)}{(2p + 1)(4p^2 + 8p + 15)} \right]^{1/2}.$$  \hfill (4.1)

Furthermore, as we have already pointed out, our utilization of the pseudo-Newtonian potential imposes a length scale (i.e., the horizon at $r = 2$) that implies a definite behavior for $\dot{M}$ as given by equation (2.21). This in turn requires that we set $p = 1/2$ in the ADIOS model in order to match the behavior of our $\dot{M}$ solution as $r \to \infty$. Incorporating this value for $p$ into equation (4.1) then yields

$$\lambda = \frac{1}{2} \left( \frac{1 + 4\epsilon}{5} \right)^{1/2},$$  \hfill (4.2)

which can be combined with equations (19) and (20) of BB to obtain

$$\Omega_0 = \left( \frac{1 + 4\epsilon}{5} \right)^{1/2}, \quad a_0 = \left( \frac{2 - 2\epsilon}{5} \right)^{1/2}.$$  \hfill (4.3)

These results are consistent with our equations (2.28) in the special case $v_0 = 0$, which is due to the fact that BB neglected the radial kinetic energy term in their energy transport equation. We shall use the subscript “$A$” to denote quantities related to the ADIOS model. Unsubscripted quantities will continue to refer to our RADIOS model. The self-similar ADIOS expressions for the angular velocity and the isothermal sound speed are then given by

$$\Omega_A(r) \equiv \Omega_0 r^{-3/2}, \quad a_A(r) \equiv a_0 r^{-1/2},$$  \hfill (4.4)

respectively. The corresponding ADIOS solutions for the energy, angular momentum, and mass transport rates reduce when $p = 1/2$ to

$$E_A(r) \equiv \epsilon \dot{M}_0 r^{-1/2}, \quad I_A(r) \equiv \lambda \dot{M}_0 r, \quad M_A(r) \equiv \dot{M}_0 r^{1/2},$$  \hfill (4.5)

respectively. These solutions obviously agree with our equations (2.21) and (2.29) in the limit $r \to \infty$ as required. In Figures 3 and 4 we compare our results for the energy and mass transport rates $E(r)$ and $\dot{M}(r)$ with the corresponding ADIOS quantities $E_A(r)$ and $\dot{M}_A(r)$ for radii in the range $r_{\text{ms}} < r < 10$, where $r_{\text{ms}} = 6$. Note that for the same value of $\epsilon$, energy is transported outward through the disk at a substantially greater rate in the relativistic (pseudo-Newtonian) model due to the steepness of the potential in the inner region of the disk.

4.1. Disk/Wind Structure

The modifications to the ADIOS disk structure due to the incorporation of the pseudo-Newtonian gravitational potential in RADIOS can be summarized by writing

$$\frac{\dot{M}}{\dot{M}_A} = \frac{\Omega}{\Omega_A} = \left( \frac{a}{a_A} \right)^2 = \left( \frac{I}{I_A} \right)^{1/2} = \left( \frac{E}{E_A} \right)^{1/2} = \frac{r}{r - 2},$$  \hfill (4.6)
which together suggest that no dramatic changes in the disk properties occur until one approaches the horizon at \( r = 2 \). In fact, the importance of the subtle variations in structure between the relativistic and nonrelativistic models really becomes clear only when one examines the properties of the associated outflows. Due to the symmetry across the disk midplane, identical outflows emanate from the upper and lower surfaces of the disk. For brevity, we shall refer to these two outflows collectively as the “wind.” The total amounts of mass and energy emitted into the wind per unit time between radii \( r_{\text{ms}} \) and \( r \) are given by

\[
\dot{m}(r) = \dot{M}(r) - \dot{M}(r_{\text{ms}}) , \quad L(r) = E(r_{\text{ms}}) - E(r) ,
\]

for the pseudo-Newtonian model. The corresponding expressions in the Newtonian case are

\[
\dot{m}_A(r) = \dot{M}_A(r) - \dot{M}_A(r_{\text{ms}}) , \quad L_A(r) = E_A(r_{\text{ms}}) - E_A(r) .
\]

In the absence of any outflows (\( \epsilon = 0 \)), the energy and momentum transport rates are conserved and therefore \( L(r) = L_A(r) = 0 \), as in the ADAF models of Narayan & Yi (1994). At large radii, we find that the total power emitted into the wind approaches

\[
L(\infty) \equiv E(r_{\text{ms}}) = \epsilon \dot{M}_0 \frac{6^{3/2}}{16} ,
\]

and

\[
L_A(\infty) \equiv E_A(r_{\text{ms}}) = \epsilon \dot{M}_0 6^{-1/2} ,
\]

for the pseudo-Newtonian and Newtonian models, respectively. If we interpret the radius of marginal stability \( r_{\text{ms}} = 6 \) as the inner edge of the disk, then the values \( L(\infty) \) and \( L_A(\infty) \) represent the total power emerging from the disk into the wind from all radii \( r > 6 \). Equations (4.9) and (4.10) indicate that \( L(\infty)/L_A(\infty) = 2.25 \), so that the relativistic potential roughly doubles the total power in the outflow for fixed \( \epsilon \) and \( \dot{M}_0 \). In an actual accretion disk, we expect that the wind will terminate at some outer radius \( r_{\text{out}} > r \) at which the hot inner region transitions into the optically thick, cool outer disk. We are not particularly concerned with the value of \( r_{\text{out}} \) here, although we note that most hot disk models give values in the range \( r_{\text{out}} \sim 10^{2-4} \) (Narayan, McClintock, & Yi 1996; Narayan, Barret, & McClintock 1997).

It is also interesting to compute the amount of mass and energy emitted into the wind per unit radius per unit time. Utilizing our pseudo-Newtonian RADIOS expressions yields

\[
\frac{dL}{dr} = \epsilon \dot{M}_0 \frac{r^{1/2} (r+6)}{2 (r-2)^3} , \quad \frac{d\dot{m}}{dr} = \dot{M}_0 \frac{r^{1/2} (r-6)}{2 (r-2)^2} .
\]

The corresponding expressions obtained using the standard ADIOS model of BB are

\[
\frac{dL_A}{dr} = \epsilon \dot{M}_0 \frac{r^{-3/2}}{2} , \quad \frac{d\dot{m}_A}{dr} = \dot{M}_0 \frac{r^{-1/2}}{2} .
\]

These results are compared graphically in Figures 5 and 6 for \( r \) in the range \( 6 < r < 10 \). For a given value of \( \epsilon \), the power emitted into the wind is significantly greater in the pseudo-Newtonian
case than in the standard Newtonian model due to the steeper potential gradient. Note that in the pseudo-Newtonian case, \( dm/dr = 0 \) at the radius of marginal stability \( r_{ms} = 6 \). As explained earlier, we therefore interpret \( r_{ms} \) as the inner edge of the disk, and assume that essentially all of the matter remaining at that radius falls directly into the black hole.

4.2. Bernoulli Parameters

Another useful way of contrasting the properties of the Newtonian and pseudo-Newtonian inflow-outflow models is to calculate the specific energy (i.e., the Bernoulli parameter) in the disk and the wind as functions of radius. In the pseudo-Newtonian case, the disk Bernoulli parameter is given by

\[ B(r) = \frac{1}{2} v^2 + \frac{1}{2} r^2 \Omega^2 + \frac{5}{2} a^2 - \frac{1}{r - 2} = \frac{20 \Omega_0^2 - 12 \epsilon - 4 + (2 + 6 \epsilon - 9 \Omega_0^2) r}{2(r - 2)^2}, \]

where we have utilized equations (2.28) to arrive at the final result. The corresponding expression in the Newtonian case is

\[ B_A(r) = \frac{1}{2} r^2 \Omega_A^2 + \frac{5}{2} a_A^2 - \frac{1}{r} = \frac{1 - 6 \epsilon}{10 r}, \]

obtained using equations (4.3) and (4.4). In equations (4.13) and (4.14), \( r \) is interpreted as the cylindrical “starting radius” at the base of the wind in the disk. After leaving the disk, the wind is expected to expand to larger cylindrical radii as the outflow proceeds. Note that equation (4.13) for \( B \) includes the contribution due to the radial kinetic energy, whereas the expression for \( B_A \) employed by BB (eq. [4.14]) assumes that \( v_0 = 0 \).

In general, the pseudo-Newtonian disk Bernoulli parameter \( B \) may be either positive or negative depending on the values of \( \epsilon \), \( \Omega_0 \), and \( r \). For given values of \( \Omega_0 \) and \( r \), we find that \( B < 0 \) if and only if \( \epsilon < \epsilon_* \), where

\[ \epsilon_* \equiv \frac{4 - 2r + (9r - 20) \Omega_0^2}{6r - 12}. \]

The roots of \( B \) occur at radius \( r = r_* \), where

\[ r_* \equiv \frac{12 \epsilon - 20 \Omega_0^2 + 4}{2 + 6 \epsilon - 9 \Omega_0^2}, \]

with \( B < 0 \) for \( r > r_* \) and \( B > 0 \) for \( r < r_* \). The situation is complicated by the fact that \( \epsilon \) and \( \Omega_0 \) must also satisfy the self-consistency conditions given by equations (3.17). However, we can make a few general statements: (i) If \( \Omega_0^2 < 1/3 \), then \( B > 0 \) for all radii \( r > 6 \); (ii) If \( 1/3 < \Omega_0^2 < 4/5 \), then there is an allowed range of \( \epsilon \) values for which \( B \) changes sign at \( r = r_* \), where \( r_* > 6 \); (iii) If \( \Omega_0^2 > 4/5 \), then \( B < 0 \) for all radii \( r > 6 \). Hence, in the pseudo-Newtonian case, the inner region of the disk may be unbound even when the outer disk is bound.

In Figure 7 we plot \( B \) and \( B_A \) as functions of radius for various values of \( \epsilon \), with \( v_0 = 0 \) in order to facilitate a direct comparison between the RADIOS and ADIOS model results. Recall
that setting \( v_0 = 0 \) implies that \( \Omega_0 = \sqrt{(4 \epsilon + 1)/5} \) according to the last of equations (2.28). In the Newtonian case, we find that \( B_A > 0 \) for all values of \( r \) if \( \epsilon < 1/6 \), and \( B_A < 0 \) for all radii if \( \epsilon > 1/6 \). In the pseudo-Newtonian case with \( v_0 = 0 \), we also find that \( B > 0 \) for all values of \( r \) if \( \epsilon < 1/6 \), but if \( \epsilon > 1/6 \), then \( B \) changes sign at \( r_* = 20 \epsilon / (6 \epsilon - 1) \). Note that \( B \) exceeds \( B_A \) at all radii, which is due to the effect of the pseudo-Newtonian potential.

A positive disk Bernoulli parameter violates self-consistency in standard ADAF models because these do not include outflows. By contrast, in the ADIOS and RADIOS models considered here, outflows are included and therefore we can obtain self-consistent solutions even when the Bernoulli parameter is positive, although the most plausible scenario is one in which it is driven to a negative value as a result of the outflow. Specifically, the most energetic outflows are obtained when \( \epsilon = 1 \), which yields a negative (bound) value for the disk Bernoulli parameter for all values of \( r \). Conversely, the most positive values for the Bernoulli parameter are obtained when \( \epsilon = 0 \), which corresponds to the no-wind solution since in this case the energy transport rate in the disk vanishes. The source of the change in behavior of the RADIOS results plotted in Figure 7 relative to those obtained using the ADIOS model can be traced to the quasi-Keplerian form for \( \Omega \) operative in the pseudo-Newtonian case, expressed by equation (2.2). As a consequence of equation (2.2), the azimuthal kinetic energy term \((1/2)r^2 \Omega^2\) in equation (4.13) for \( B \) varies as \( r/(r - 2)^2 \), whereas the terms describing the internal and gravitational potential energies each vary as \( 1/(r - 2) \). At sufficiently small radii, the azimuthal kinetic energy (which is positive) therefore overwhelms the remaining terms, which have the weaker \( 1/(r - 2) \) radial dependence. The vigorous azimuthal motion is due to the stronger gradient in the pseudo-Newtonian potential relative to the Newtonian potential. In the ADIOS model all three terms in equation (4.14) for \( B_A \) have the same scaling, and therefore \( B_A \) has the same sign at all radii.

The Bernoulli parameter for the wind particles is given in the RADIOS model by

\[
\begin{align*}
\frac{dL}{d\dot{m}} = \frac{dE}{dM} = \frac{\epsilon (r + 6)}{(r - 6) (r - 2)}.
\end{align*}
\] (4.17)

The equivalent expression obtained using the ADIOS model results is

\[
\begin{align*}
\frac{dL_A}{d\dot{m}_A} = \frac{dE_A}{dM_A} = \frac{\epsilon}{r}.
\end{align*}
\] (4.18)

Note that \( b \) and \( b_A \) are always positive, and that they each exceed the associated disk Bernoulli parameters \( B \) and \( B_A \), respectively. This implies that the energy per unit mass in the wind is higher than that in the disk, which is consistent with our disk/wind scenario. As the wind flows away from the black hole, we expect the gas to expand adiabatically, passing through a critical point at which the flow velocity equals the sound speed as discussed by SBK. In this situation, the specific energy of the wind particles is conserved and resides ultimately in the kinetic energy of the outflow. It must be emphasized that the wind Bernoulli parameter describes only the kinetic, potential, and internal energy per unit mass, since it is based on the energy transport equation in the disk, which is classical in form (see eq. [2.24]). Hence the rest mass energy of the wind particles
is not included in either $b$ or $b_A$. It follows that if the outflowing gas expands adiabatically, then the total energy per unit mass in the cold, terminal wind is given in the pseudo-Newtonian and Newtonian cases by (e.g., SBK)

$$\Gamma \approx b + 1, \quad \Gamma_A \approx b_A + 1,$$

respectively, where $\Gamma$ and $\Gamma_A$ are the corresponding asymptotic Lorentz factors of the outflow in the two models.

Without considering the structure of the outflow in any detail, we can use equation (4.19) to determine the asymptotic nature of the wind originating at radius $r$. In the pseudo-Newtonian RADIOS case we can combine equations (4.17) and (4.19) to obtain

$$\Gamma(r) \approx b(r) + 1 = \frac{\epsilon (r + 6)}{(r - 6)(r - 2)} + 1.$$

In the Newtonian ADIOS case the corresponding result obtained using equations (4.18) and (4.19) is

$$\Gamma_A(r) \approx b_A(r) + 1 = \frac{\epsilon}{r} + 1.$$

According to equations (3.17), the maximum value of $\epsilon$ satisfying our self-consistency conditions is $\epsilon = 1$, corresponding to the limiting case $\Omega_0 = 1$. We find that in the pseudo-Newtonian RADIOS model with $\epsilon = 1$, the asymptotic Lorentz factor $\Gamma \gtrsim 2$ for $r \lesssim 8.3$, and $\Gamma \gtrsim 3$ for $r \lesssim 7.3$. This indicates that the inner region of the disk can produce a relativistic wind/outflow that may subsequently transition into a relativistic jet at large distances from the black hole, provided the gas expands adiabatically. Conversely, in the corresponding ADIOS model with $\epsilon = 1$, the largest possible value for $\Gamma_A$ is 1.17 (obtained for starting radius $r = r_{ms} = 6$), and therefore the standard ADIOS outflow is clearly nonrelativistic. These results are depicted in Figure 8.

5. DISCUSSION AND CONCLUSION

Our relativistic, advection-dominated inflow-outflow solution (RADIOS) incorporates a pseudo-Newtonian gravitational potential, and therefore it provides an approximate representation of the effects of general relativity on the properties of the accretion disk and the associated outflow, which is assumed here to be a gasdynamical wind. Consideration of mass, momentum, and energy conservation yields a new, self-similar solution for the disk/wind structure that is rather different from that obtained by BB. In particular, our results show that a powerful relativistic outflow originates from the inner region of the disk, close to the radius of marginal stability. The appearance of this outflow is a consequence of the steep gradient in the pseudo-Newtonian potential. Consequently, the RADIOS model may provide a better description of the inner region of the disk than the standard ADIOS model, which is based on Newtonian gravitation. Although it utilizes a pseudo-Newtonian potential, our model nonetheless incorporates the same fundamental physical scaling principles as the ADIOS model. Namely, we assume that the angular velocity $\Omega$ scales as the Keplerian value
\( \Omega_K \) (see eqs. [2.1] and [2.2]), and we assume that the energy transport rate per unit mass \( E/\dot{M} \) and the specific internal energy \( a^2 \) each scale as the gravitational potential energy \( \Phi = -1/(r-2) \). We also assume that the angular momentum transport rate per unit mass \( I/\dot{M} \) scales as the specific angular momentum \( r^2 \Omega \).

Evidence supporting our assumption regarding the variation of \( \Omega \) is provided by the simulations of Narayan et al. (1997), who recover this behavior over several orders of magnitude in radius (outside the radius of marginal stability) for \( \alpha \sim 0.1 - 0.3 \). Although Narayan et al. (1997) assume that there is no outflow from the disk, we can still utilize our equation (2.2) based on their work because we assume here that there is no relative torque between the disk and the wind. Hence we expect little change in the angular momentum distribution as a result of the outflow. This also supports our assumption regarding the variation of the angular momentum transport rate \( I \) since in a self-similar model such as ours, the scaling of \( I \) follows directly from the variation of \( \Omega \) by virtue of equation (2.23).

The pseudo-Newtonian potential introduces a length scale into the problem due to the existence of the horizon at \( r = 2 \). No such length scale appears in the Newtonian ADIOS model of BB. The imposition of a length scale in our model results in a definite behavior for the variation of the accretion rate given by equation (2.21), which approaches \( \dot{M} \propto r^{1/2} \) as \( r \to \infty \). When coupled with our assumption of a torque-free (gasdynamical) wind, the number of free parameters in our model is reduced to two, which we take to be \( \Omega_0 \) and \( \epsilon \). The remaining constants \( \lambda, a_0, \) and \( v_0 \) can be computed in terms of \( \Omega_0 \) and \( \epsilon \) using equations (2.28). The self-similar solutions we obtain asymptotically approach the corresponding, Newtonian ADIOS results for a gasdynamical wind as \( r \to \infty \) if we set \( p \equiv d \ln \dot{M} / d \ln r = 1/2 \) and \( \lambda = \lambda_{\text{crit}} \) in the ADIOS model (see eq. [4.1]). Hence the gravitational physics near the horizon forces the flow structure to satisfy \( \dot{M} \propto r^{1/2} \) at large radii. In the Newtonian ADIOS model satisfying this constraint, the outflow is always nonrelativistic. In our model, the outflow can be highly relativistic, implying a possible connection with large-scale jets.

The asymptotic Lorentz factor in the wind, \( \Gamma \), formally diverges as \( r \to 6 \) because \( d\dot{m}/dr \) vanishes at the radius of marginal stability \( r_{\text{ms}} = 6 \). This is due to the fact that for \( r < 6 \), the disk is unstable and all of the remaining material is swept directly into the black hole (see, however, our discussion of the implications of transonic flow below). Conversely, the differential power in the outflow, \( dL/dr \), remains finite even at \( r = 6 \) because energy continues to be transported in the outward radial direction by the self-similar torque, which has a radial dependence given by equation (2.25). Even with our self-consistency conditions (eqs. [3.17]) taken into consideration, equation (4.20) indicates that the asymptotic Lorentz factor for the outflow \( \Gamma \) can still greatly exceed unity for the portion of the wind emitted close to the radius of marginal stability. By contrast, when the same parameters are used in the ADIOS model of BB, the outflow is subrelativistic from all radii in the disk (cf. eq. [4.21]).

The radial Mach number is conserved in our self-similar model, and therefore the flow remains
subsonic in the entire computational domain \((r > r_{\text{ms}})\). Of course, in reality the gas must certainly achieve a supersonic velocity before it crosses the event horizon. The transition to supersonic flow requires that the material pass smoothly through a sonic point, which is a critical point for the flow. The existence of the sonic point therefore imposes critical conditions that the overall dynamical structure must satisfy. This does not appear to be a major problem for our model because the sonic point is expected to lie between \(r = r_{\text{ms}}\) and the horizon \(r = 2\), which is outside the computational domain of our disk model. However, the issue can be addressed quantitatively by constructing a \textit{global} model in which the subsonic region \((r > r_{\text{ms}})\) is described by our self-similar RADIOS solution and the transonic region \((2 < r < r_{\text{ms}})\) is described by the solution of Narayan et al. (1997; see also Chen et al. 1997). A more sophisticated alternative would be to modify the Narayan et al. calculation to include mass loss such as that envisioned here. We expect that in the resulting model, matter will actually be expelled into the wind all the way down to the sonic radius (located between the radius of marginal stability and the horizon), where the pressure support for the wind essentially vanishes. Hence, there should still be some minimal amount of outflow at \(r = r_{\text{ms}}\), and consequently \(\Gamma\) should remain finite at the radius of marginal stability, in contrast to our results. Nonetheless, we expect that the dynamical structure of such a generalized model will be similar to that of the RADIOS model because the \(\Omega(r)\) distribution that we utilize is consistent with that obtained by Narayan et al. We therefore believe that our model provides an intriguing glimpse into the possible behavior to be expected when fully relativistic calculations that include particle acceleration and mass loss become available.

### 5.1. Shear Acceleration and the Origin of the Outflows

In the treatments of advection dominated inflow-outflow solutions that have appeared to date in the literature (BB; Donea et al. 1999), the existence of outflows is generally assumed without any elaboration on the physical mechanisms that could give rise to them. That is also the case in the work presented here, since the outflow power is calculated by differentiating the disk energy transport rate \(E\), without making any reference to a microphysical model. Although several mechanisms for producing outflows have been proposed (see references in SBK), few make a connection between the outflow and the underlying accretion disk. Das & Chakrabarti (1999) and Das (1999) describe pressure-driven winds from centrifugally-supported boundary layers and shocks in the inner regions of disks. Xu & Chen (1997) proposed an advection-dominated flow where the central object effectively acts as a scatterer which redirects the inward flow at low latitudes into an outflow at high latitudes. The above-mentioned models compute the mass outflow rates self-consistently, but the associated winds are subrelativistic and therefore the possible connection with large-scale relativistic jets is unclear.

The formation of a relativistic outflow requires the operation of a mechanism in the disk that preferentially accelerates high energy particles. The resulting particle distribution comprises a thermal component that is bound to the disk and a nonthermal component that is unbound, repre-
senting the particles that are destined to escape into the wind. SBK have described a second-order Fermi acceleration mechanism powered by the shearing magnetic field that is capable of producing a relativistic outflow. Some related conceptual development has also been carried out by Blackman (1999) and by Gruzinov & Quataert (1999). The physical scenario in which the acceleration of the wind protons takes place is the same as that in which hybrid viscosity (Subramanian, Becker, & Kafatos 1996) is operative. The azimuthal kinetic energy in the shear flow is dissipated partly by viscous heating of the thermal protons, and partly by second-order Fermi acceleration of the nonthermal protons, which in turn form a relativistic outflow. SBK demonstrated that this mechanism can lead to the energization of protons via shear acceleration in a tenuous corona overlying a hot accretion disk. The resulting specific energy in the corona can be large enough to drive a relativistic outflow. In their work, SBK focused on shear acceleration occurring in a tenuous corona because the density in the underlying (luminous) disk is so large that collisional losses overwhelm shear acceleration there. However, in the advection-dominated accretion flows envisioned here, the low density environment of the (underluminous) disk itself is expected to be a favorable site for this acceleration mechanism.

5.2. Model Predictions

The unique solution for the $\dot{M}$ variation given by our equation (2.21) and the associated density distribution $\rho \propto (r-2)^{-1}$ have definite consequences for the timing properties of accretion-powered sources. It has been proposed (Kazanas, Hua & Titarchuk 1997; Hua, Kazanas & Titarchuk 1997; Hua, Kazanas, & Cui 1999; Kazanas & Hua 1999) that the timing properties of these sources (light curves, power spectra, coherence, and time lags between soft and hard X-rays) can be easily understood as the result of Comptonization of soft photons produced in the vicinity of the compact object by a hot ($10^8$ K) corona that extends to $r \sim 10^3$ Schwarzschild radii. In particular, it was pointed out that such an interpretation provides a means for probing the density structure of the Comptonizing corona, mainly as a consequence of the dependence of the X-ray hard lags on the Fourier frequency. Hua, Kazanas, & Cui (1999) analyzed and fit the lags from a number of accreting galactic black hole candidate sources. In a few cases, the lag data were consistent with the density distribution $\rho \propto r^{-3/2}$ implied by the ADAF model (Narayan & Yi 1994). However, in most cases the lag data were consistent with a corona density profile $\rho(r) \propto r^{-1}$, which is the behavior that is uniquely determined by our self-similar RADIOS model for $r \gg 2$. Hence our pseudo-Newtonian, self-similar RADIOS model may provide the first explanation for a ubiquitous observational conclusion regarding the apparent radial density variation in accreting compact sources.
5.3. Conclusion

We have described the overall framework of a pseudo-Newtonian RADIOS model that associates relativistic outflows with advection-dominated accretion disks. The self-similar solutions obtained for the physical properties of the disk extend over several decades in radius and therefore provide a meaningful description of the global disk structure. By extending our model to large radii, we are able to constrain the parameters in the associated Newtonian ADIOS model. Our analysis yields unique radial dependences for the accretion rate, the density, and the pressure. For the special case of a torque-free, gasdynamical wind, which is the focus here, we find that the density varies as $\rho \propto r^{-1}$ as $r \to \infty$. This density variation is consistent with that implied by observations of X-ray time lags in accreting galactic sources. Moreover, the Shakura-Sunyaev $\alpha$-parameter implied by our model remains nearly constant, independent of the viscosity model (see eq. [3.5]). This is consistent with the basic notion that the shear stress should be roughly proportional to the pressure. When combined with the fact that our assumed variation of $\Omega$ is consistent with published numerical simulations, we conclude that our fundamental scaling assumptions are reasonable. Nonetheless, these assumptions cannot be fully justified a priori, and need to be tested by global calculations. In our RADIOS model, the strength of the outflow and the value of the mass loss index $p \equiv d \ln \dot{M} / d \ln r$ are determined by the shape of the non-Newtonian potential close to the black hole. However, it is possible that the behavior $p \to 1/2$ as $r \to \infty$ implied by observations is determined, not by the shape of the potential, but rather by the details of the mechanism driving the outflow (e.g., second order Fermi acceleration, magneto-centrifugal forces, convection). If the latter is true, then the self-similar solution presented in this paper, though mathematically attractive, may have limited physical relevance. More work on this issue is needed.

The RADIOS model we have developed can produce highly relativistic outflows, whereas the outflow is always nonrelativistic in the corresponding ADIOS model. Furthermore, like ADIOS, our model does not suffer from the strong sensitivity to the value of the adiabatic index $\gamma$ that is displayed by the standard ADAF models, which require $\gamma < 5/3$ (e.g., Narayan & Yi 1994, 1995a). In contrast to the results for bipolar ADAF outflows obtained by Igumenshchev & Abramowicz (1999), which require $\alpha \sim 1$, Figures 1 and 2 suggest that in our model outflows can occur for values of $\alpha$ well below unity. This work may therefore help to establish a physical connection between advection-dominated accretion flows and the relativistic outflows possibly emanating from underluminous black hole systems such as Sgr A* (e.g., Falcke 1999; Donea et al. 1999). The model may also be applicable to the mildly relativistic outflows observed from galactic black hole candidates (Mirabel & Rodriguez 1999; Rodriguez & Mirabel 1999). However, the nature of the fundamental acceleration mechanism that energizes the wind particles is still an open question. One possibility discussed here is second-order Fermi acceleration driven by the shear of the magnetic field lines. Additional work needs to be done in this area before the complete disk/wind/jet scenario can be adequately understood.

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FIGURE CAPTIONS

Fig. 1. – Contour plot of the Shakura-Sunyaev viscosity normalization parameter $\alpha_0$ (eq. [3.6]) as a function of $\epsilon$ and $\Omega_0$. The contour values are $\alpha_0 = 1$ (solid line), $\alpha_0 = 0.5$ (dashed line), and $\alpha_0 = 0.01$ (dotted line). In the region between the solid lines, $\alpha_0$ exceeds unity. Also shown for reference is the shaded region of self-consistency for our model, given by equations (3.17). Note that this region contains values for $\alpha_0$ in the physically acceptable range $0 < \alpha_0 < 1$.

Fig. 2. – Sections of the ($\Omega_0, \epsilon$) parameter space corresponding to the various constraints given by eqs. (3.17). The region above the solid line satisfies $\epsilon > \Omega_0^2/4$, the region above the dashed line satisfies $\epsilon > (5 \Omega_0^2 - 1)/4$, and the region below the dotted line satisfies $\epsilon < \Omega_0^2$. The curved, shaded area is the intersection of these constraints and therefore it represents the region of self-consistency for our model, which is also indicated in Fig. 1. See the discussion in the text.

Fig. 3. – Comparison of the pseudo-Newtonian and Newtonian energy transport rates $E$ (eq. [2.29]; solid line) and $E_A$ (eq. [4.5]; dashed line) in units of $c\dot{M}_0$ as functions of the radius in the disk $r$. In the inner region, $E$ exceeds $E_A$ due to the steepness of the pseudo-Newtonian potential. The two results converge as $r \to \infty$.

Fig. 4. – Pseudo-Newtonian and Newtonian accretion rates $\dot{M}$ (eq. [2.21]; solid line) and $\dot{M}_A$ (eq. [4.5]; dashed line) in units of $\dot{M}_0$ plotted as functions of the radius in the disk $r$. The accretion rate $\dot{M}$ has a minimum value at $r = 6$, which is the radius of marginal stability, and the inner edge of our disk model. At large radii, $\dot{M}$ approaches the standard ADIOS result given by $\dot{M}_A$.

Fig. 5. – Differential luminosities $dL/dr$ (eq. [4.11]; solid line) and $dL_A/dr$ (eq. [4.12]; dashed line) emitted into the wind are plotted in units of $c\dot{M}_0$ as functions of the starting radius in the disk $r$. These two functions correspond to the pseudo-Newtonian and Newtonian models, respectively. The power emitted into the wind is greatest in the pseudo-Newtonian case.

Fig. 6. – Comparison of the pseudo-Newtonian and Newtonian differential mass-loss rates $d\dot{m}/dr$ (eq. [4.11]; solid line) and $d\dot{m}_A/dr$ (eq. [4.12]; dashed line) emitted into the wind, plotted in units of $\dot{M}_0$ as functions of the starting radius in the disk $r$. In the pseudo-Newtonian case, this rate approaches zero close to the radius of marginal stability, and any remaining material is swept directly into the black hole.

Fig. 7. – Pseudo-Newtonian and Newtonian disk Bernoulli parameters $B(r)$ (eq. [4.13]; solid line) and $B_A(r)$ (eq. [4.14]; dashed line) plotted as functions of the radius in the disk $r$ for $v_0 = 0$. In panel (a), $\epsilon = 0.1$, and both $B$ and $B_A$ remain positive at all radii. In panel (b), $\epsilon = 0.3$, and $B$ changes sign at $r = 7.5$, whereas $B_A$ remains negative at all radii. This difference in behavior
is due to the strong variation of the quasi-Keplerian angular velocity $\Omega$ in the inner region of the disk in the pseudo-Newtonian model.

Fig. 8. – Asymptotic Lorentz factors $\Gamma$ (eq. [4.20]; solid line) and $\Gamma_A$ (eq. [4.21]; dashed line) for the outflows in the pseudo-Newtonian and Newtonian models, respectively, plotted as functions of the starting radius in the disk $r$. The Newtonian result is nonrelativistic for all values of $r$, whereas the pseudo-Newtonian result is strongly relativistic in the inner region of the disk, and formally diverges as $r$ approaches the radius of marginal stability $r_{ms} = 6$. 
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