Single evolution equation in a light-matter pairing system

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Abstract
The coupled system including wave mixing and nonlinear dynamics of a nonlocal optical medium is usually studied (1) numerically, with the medium being regarded as a black box, or (2) experimentally, making use of some empirical assumptions. In this paper we deduce for the first time a single evolution equation describing the dynamics of the pairing system as a holistic complex. For a non-degenerate set of parameters, we obtain the nonlinear Schrödinger equation with coefficients being written out explicitly. Analytical solutions of this equation can be experimentally realized in any photorefractive medium, e.g. in photorefractive, liquid or photonic crystals. For instance, a soliton-like solution can be used in dynamical holography for designing an artificial grating with maximal amplification of an image.

Keywords: nonlocal Kerr-type medium, dynamic grating, light-matter pairing, parametric nonlinear Schrödinger equation, envelope soliton

(Some figures may appear in colour only in the online journal)

1. Introduction

Nonlinear interaction of coherent laser beams attracts numerous studies for their self-action processes in optical materials promising in various applications. The dynamic holography is the most famous in this respect due to its simple realizability in optical communication systems such as all-optical controllable amplifiers, switches, holographic multiplexing, adaptive data storage, optically addressed spatial light modulators, including many-channels systems and optical neural networks. The other huge area is fiber applications of dynamic gratings for different types of sensors, tunable filters and adaptive interferometers [1–6]. The following
three main effects acting simultaneously are the subject of dynamic holography: (1) creation of periodic interference pattern inside a nonlinear medium with the help of two or more laser beams; (2) modulation of the refractive index (and/or the absorption coefficient) under action of this interference pattern, or, in other words, inducing a phase dynamic grating inside a nonlinear medium; (3) self-diffraction of recording beams with the dynamic grating created by the same beams.

Practical use of waves’ diffraction on dynamical gratings is related to several specific effects which can be implemented depending on the initial parameters of beams coupling. Among these effects are energy and phase transfer between interacting waves which may occur due to the specific features of nonlinear response in optical material.

The nonlinear problem of optical two-mixing has been considered thoroughly in the following way. The coupled-wave equations were derived from Maxwell’s wave equations, see e.g. [7, 8, 10]. There exist two essentially different mathematical approaches used for solving these coupled-wave equations.

In the first approach, a general mathematical convention is used that the medium response is replaced by the modulation of the refractive index $\Delta n$ and/or by the modulation of the absorption $\Delta \alpha$, so-called the approach of the given gratings. In other words, the medium is regarded as a ‘black box’, without regarding its own intrinsic dynamics, while effects of the energy transfer and the phase coupling are explained in terms of either shift (the non-local response) or no shift (the local response) of the photoinduced gratings relative to the interference pattern. For instance, in [8] equations for two-wave mixing in the general form and their solutions have been obtained in the transmission geometry taking into account many factors, including the photoinduced response of the medium that may be both local and non-local and generates both gratings of the refractive index and of the absorption (or amplification).

In the second approach, the study of the nonlinear media takes into account its intrinsic dynamics, including the relaxation of the nonlinearity, e.g. [9, 13]. In particular, this approach has been used for detailed characterization of the stationary regime in the transmission four wave-mixing [14]. In general, it allows one to provide explicit description for many characteristics responsible for various nonlinear optical properties of the medium. For instance, explanations can be provided for redistribution of charge carriers in photorefractive crystals and semiconductors; the changes of molecules’ orientations in liquid crystals; heat-induced nonlinearity, under given conditions of moving gratings, etc.

The holographic dynamical phase grating is considered as a photoinduced spatial modulation of the refractive index appearing during the degenerate (i.e. on the same wavelengths) wave-mixing in a Kerr-type nonlinear medium. Nonlocal mechanism in the dynamical media is regarded in the relation to diffusion and drift of carriers (responsible for the induced nonlinearity); they occur under the action of time-changing spatially modulated optical field. Thus in one dimensional case, a dynamic grating is formed under the action of a sinusoidal light interference pattern. If the grating is varying in time it may be considered as a moving matter-wave of one wavelength described by the $K$-vector of the grating period.

Furthermore, it has been shown theoretically that in the presence of energy transfer, the amplitude of the grating becomes nonuniform in the medium along the $z$-direction of the light wave propagation [17, 19]. The dynamics of grating can be described by the damped sine-Gordon equation in the transmission geometry or by the damped tanh-Gordon equation in the reflection geometry. In the steady state, the grating amplitude admits the form of either a bright-soliton or a dark-soliton solution along the wave-propagation direction [16, 18, 24]. The first experimental observation of the localization of the dynamical grating was found in a bulk photo-refractive crystals $LiNbO_3$ during degenerate four-wave mixing [15].
In this paper we regard the dynamical system containing the coupled-wave equations for the two-wave mixing in the reflection geometry, with pure non-local response and neglected absorption. In section 2 we present the basic mathematical model for this case originating from Maxwell’s equations. In section 3 we apply the second approach described above and demonstrate for the first time that the complete dynamical system (which includes both the coupled-wave equations and the dynamical equation for the medium) can be reduced to a single nonlinear evolution equation. In section 4 we present briefly the procedure allowing to reduce this evolution equation to the nonlinear Schrödinger (NLS) equation with coefficients depending on parameters. As the solutions of the NLS are fairly well studied in the nonlinear physics, one can operate them for prediction of some effects in the nonlinear dynamical media in optical system. Especially it concerns the solutions for the transient processes such as regular self-oscillation, periodic doubling and others, as well as their dependence on the parameters of the system. Main theoretically plausible physical phenomena appearing in the frame of our novel evolution equation are briefly discussed at the end of the paper.

2. Basic mathematical model

Classical theory of the interaction of two waves in optical anisotropic medium originates from the well known paper of Kogelnik, [11]. In this paper, the coupled wave equations have been deduced from Maxwell’s equations in a general form taking the form

\[ \partial_z E_1(z, t) = -i Q E_2(z, t), \]
\[ \partial_z E_2^*(z, t) = s i Q E_1^*(z, t). \]

These two equations describe the coupling of two light beams \( E_1(z, t) = E_1(z, t) \exp\{i \phi_1\} \) and \( E_2(z, t) = E_2(z, t) \exp\{i \phi_2\} \) in a nonlinear medium. Here \( s \) is an integer, \( s = 1 \) for transmission geometry and \( s = -1 \) for reflection geometry and the coupling coefficient \( Q \) is a modulation of the refractive index. As a rule, \( Q \) is regarded as an empirical constant depending on specific nonlinear mechanism in a medium. In this setting, the initial intensities of the waves do not change which means that no energy exchange between two beams takes place.

Later on it was observed experimentally in photorefractive crystals, [12], that energy transfer exists and yields changes in the properties of medium, i.e. \( Q \neq \text{const} \) and there exists a nonlocal response of the medium. The evolution equation for \( Q = Q(z, t) \) being a function of time and space variables has been deduced in [9, 16, 17], under the assumptions that (a) the coupling coefficient \( Q(z, t) \) describes the evolution of the medium, (b) the medium exhibits a nonlocal response, i.e. a phase shifts on \( \pi/2 \) relative to local points of the action of light intensity, and (c) this is a real-valued function of two scalar variables given by

\[ \partial_t Q(z, t) = \gamma_N \frac{I_m(z, t)}{I_0(z, t)} - \frac{1}{\tau} Q(z, t). \]

Here, \( \tau \) and \( \gamma_N \) are the relaxation constant and the amplification coefficient of the nonlocal nonlinear response of the medium correspondingly; both parameters are real. The total intensity \( I_0 \) and the intensity of the interference pattern \( I_m \) are defined by the beams amplitudes as follows:

\[ I_0 = I_1(z, t) + I_2(z, t) = |E_1|^2 + |E_2|^2, \]
\[ I_m = (E_1 E_2^* + E_1^* E_2) = 2E_1 E_2 \cos(\phi_1 - \phi_2). \]
The numerical study of the equations (1)–(3) and experimental studies of the two-wave mixing, e.g. [9, 16–19], revealed two different regimes observable in the nonlinear medium: localization, and regular oscillations. No analytical study of the equations (1)–(3) is presently known.

Our main goal in this paper is twofold: firstly, we rewrite the system the equations (1)–(3) as a single equation which is equivalent to the initial system of equations; secondly, we make use of some additional assumptions allowing us to reduce this equation to the NLS equation for the case of reflection geometry of wave interaction. Below we present the main schema of our calculations; more details can be found in the the extended version of this paper, [20].

3. Single evolution equation

An equation for the dynamical medium \( Q(z, t) \) can be deduced by introducing an auxiliary function \( J_m = J_m(z, t) \). It defines modulation depth of the intensity field and is given by the following ratio:

\[
J_m = \frac{I_m}{I_0} = \frac{E_1 E_2^* + E_1^* E_2}{E_1^* E_1 + E_2^* E_2}.
\]

Taking derivative over \( z \) of the equation (3), we can rewrite it via new variable \( J_m \) as

\[
\frac{\partial}{\partial t} \frac{\partial}{\partial z} Q + \frac{1}{\tau} \frac{\partial}{\partial z} Q - \gamma N \frac{\partial J_m}{\partial z} = 0.
\]

Straightforward calculation making use of the equations (1) and (2) allows to compute \( \frac{\partial}{\partial z} J_m \) as

\[
\frac{\partial}{\partial z} J_m = \frac{2}{\tau} (Q - Q J_m J_m).
\]

The equation (8) is a nonlinear evolutionary PDE of the second order which is equivalent to the initial system of the equations (1)–(3).

4. Reduction to a parametric NLS

4.1. Multi-scale method

The multi-scale method (MSM) is used below used for deducing a parametric NLS. This method has been developed in the 1960s and successfully applied in hydrodynamics, physics of atmosphere, mechanics, optics, oceanography, etc. The main idea is illustrated below on a simple example. Let us regard weakly nonlinear equation of the form

\[
L(\psi) = \varepsilon N(\psi), 0 < \varepsilon \ll 1
\]

where \( L \) is a linear operator with constant coefficients, \( N \) is a nonlinearity and \( \varepsilon \) is a small parameter. Then linear part of the equation (9) has a solution in the form of Fourier harmonics,

\[
L(\phi) = 0, \phi = A \exp[i(kx - \omega t)], A = \text{const.}
\]

As nonlinearity \( \varepsilon N(\psi) \) in the equation (9) is very small, we assume that a solution \( \psi \) of the equation (9) differs not too much from the solution \( \phi \) of the linear equation (10). Namely, in the simplest case,
\[ \psi \approx \tilde{A} \exp[i(kx - \omega t)], \tilde{A} \neq \text{const}, \tilde{A} = \tilde{A}(T), T = \epsilon t \]  

(11)
i.e. the exponential part is the same as in linear case but the amplitude is a slowly changing function of time. In other words, we have two time scales: fast time \( t \) (also called linear time) and slow time \( T \) depending on the small parameter.

The solution of the equation (9) is represented as the following expansion:

\[ \psi = \psi_0(x, t, T) + \epsilon\psi_1(x, t, T) + \epsilon^2\psi_2(x, t, T) + \ldots \]  

(12)

and zero approximation \( \psi_0(x, t, T) \) can be computed as a sum of a few modes with slowly changing amplitudes,

\[ \psi_0(x, t, T) = \sum_{k=1}^{k_o \rightarrow \infty} \tilde{A}_k(T) \exp[i(kx - \omega t)]. \]  

(13)

Substituting (13) into the equation (10) and grouping the terms with the same powers of the small parameter \( \epsilon \), we deduce a sequence of different nonlinear equations on \( \psi_1, \psi_2, \ldots \). As we are interested in bounded solutions, we have to exclude unbounded, or secular, terms appearing due to resonances and induced by the nonlinearity playing role of an external force acting on a linear wave (10) or their combination.

The most important result of this approach is that e.g. \( \psi_0 \) and \( \psi_1 \) can be regarded as independent, and approximate solution \( \psi \) at different time scales. For instance, for planetary waves in the Earth’s atmosphere linear time scale \( T_0 \) is of the order of 8–10 days and describe linear oscillations in the atmospheric pressure while the next time scale \( T_1 \) for this case is of order 50–80 days and describe the intra-seasonal oscillations in the Earth’s atmosphere, [22]. For experimental optical set-up, applications of the MSM can be found e.g. in [23, 24]. Analogously, we can introduce more time scales \( T_0 = t, T_1 = \epsilon t, T_2 = \epsilon^2 t, \ldots \), and also space scales.

Mathematical justification of this method is given in the excellent monograph [21] and illustrated by numerous examples. In particular, it is demonstrated that standard direct perturbation methods of searching for a solution in the form of an infinite series on a small parameter \( \epsilon \) give the same results as the MSM, for each power of the small parameter \( \epsilon^0, \epsilon^1, \epsilon^2, \ldots \). The only difference is that in the former case a solution \( \psi \) is perturbed while in the last case variables \( t \) and \( x \) are perturbed.

4.2. Auxiliary function

Let us present the function \( Q \) in the following form: \( Q = \frac{1}{2} (Q + Q^*) \) where an auxiliary complex-valued function \( Q \) is introduced and \( Q^* \) is its complex conjugate. Further we apply the procedure of the standard perturbation technique to the equation (8). First we expand the auxiliary function \( Q \) into the series

\[ Q(z, t) = \delta F_0 + \delta^2 F_1 + \delta^3 F_2 + \ldots, \]  

(14)
in curved coordinates, assuming that the coordinates \( t \) and \( z \) are also is expanded into series

\[ T_0 = t, T_1 = \delta t, T_2 = \delta^2 t, \ldots, Z_0 = z, Z_1 = \delta z, Z_2 = \delta^2 z, \ldots \]  

with \( \delta \) being a small parameter, and functions \( F_j \) depending on all coordinates,

\[ F_j = F_j(T_0, T_1, T_2, \ldots, Z_0, Z_1, Z_2, \ldots). \]  

(15)
Substituting the series above into the equation (8) and summing up terms with the same power of the small parameter \( \delta \) we deduce the equation (11):

\[
\delta \left[ \frac{\partial}{\partial T_0} \frac{\partial}{\partial Z_0} F_0 + \frac{1}{\tau} \frac{\partial}{\partial Z_0} F_0 - 2\gamma N F_0 \right] \\
+ \delta^2 \left[ \frac{\partial}{\partial T_0} \frac{\partial}{\partial Z_0} F_1 + \left( \frac{\partial}{\partial T_0} \frac{\partial}{\partial Z_1} + \frac{\partial}{\partial T_1} \frac{\partial}{\partial Z_0} \right) F_0 + \frac{1}{\tau} \left( \frac{\partial}{\partial Z_0} F_1 + \frac{\partial}{\partial Z_1} F_0 \right) - 2\gamma N F_1 \right] \\
+ \delta^3 \left[ \frac{\partial}{\partial T_0} \frac{\partial}{\partial Z_0} F_2 + \left( \frac{\partial}{\partial T_0} \frac{\partial}{\partial Z_1} + \frac{\partial}{\partial T_1} \frac{\partial}{\partial Z_0} + \frac{\partial}{\partial T_2} \frac{\partial}{\partial Z_0} + \frac{\partial}{\partial T_1} \frac{\partial}{\partial Z_1} \right) F_0 \right] \\
+ \delta^3 \left[ \frac{1}{\tau} \left( \frac{\partial}{\partial Z_0} F_2 + \frac{\partial}{\partial Z_1} F_1 + \frac{\partial}{\partial Z_2} F_0 \right) - 2\gamma N F_2 \right] + \delta^3 \frac{2}{\gamma N} \bar{R}(F_0) + \ldots + \text{c.c.} = 0
\]  

(16)

where

\[
\bar{R}(F_0) = Q \left( \partial_\tau Q + \frac{1}{\tau} Q \right) \left( \partial_\tau Q + \frac{1}{\tau} Q \right).
\]  

(17)

4.3. First order terms, \( \delta \)

The first order terms generate the equation for the function \( F_0 \):

\[
\frac{\partial}{\partial T_0} \frac{\partial}{\partial Z_0} F_0 + \frac{1}{\tau} \frac{\partial}{\partial Z_0} F_0 - 2\gamma N F_0 + \text{c.c.} = 0
\]  

(18)

which can be rewritten in the operator form as

\[
\hat{L}F_0 = 0, \quad \hat{L} = \frac{\partial}{\partial T_0} \frac{\partial}{\partial Z_0} + \frac{1}{\tau} \frac{\partial}{\partial Z_0} - 2\gamma N.
\]  

(19)

Let us look for the solution of the equation (19) in the form of a two-dimensional plane wave in the coordinates \( T_0 \) and \( Z_0 \), but with the amplitude depending on all other coordinates:

\[
F_0 = A(T_1, T_2, ..., Z_1, Z_2, ...) \exp \left[ i \omega T_0 - q Z_0 \right]
\]  

(20)

(physical meaning of \( q \) is grating period). Substituting (20) into the equation (19) and taking into account that

\[
\frac{\partial F_0}{\partial Z_0} = -iqF_0, \quad \frac{\partial}{\partial T_0} \frac{\partial F_0}{\partial Z_0} = \omega q F_0,
\]  

(21)

it is easy to deduce the form of the dispersion function

\[
\omega = i + 2\gamma N \frac{1}{q}
\]  

(22)

and the general solution \( F_0 \) to the equation (19):

\[
F_0 = A(T_1, T_2, ..., Z_1, Z_2, ...) \times \exp \left[ i \frac{2\gamma N}{q} T_0 - q Z_0 + i \frac{1}{\tau} T_0 \right] = A \exp i \Phi_0
\]  

(23)

with the complex phase
\[ \Phi_0 = 2\gamma N \frac{1}{q} T_0 - qZ_0 + i \frac{1}{\tau} T_0. \]  

(24)

4.4. Second order terms, \( \delta^2 \)

These terms generate an equation connecting two functions \( F_0 \) and \( F_1 \):

\[ \hat{L} F_1 + \hat{M} F_0 = 0 \]  

(25)

where

\[ \hat{M} = \frac{\partial}{\partial T_0} \frac{\partial}{\partial Z_1} + \frac{\partial}{\partial T_1} \frac{\partial}{\partial Z_0} + \frac{1}{\tau} \frac{\partial}{\partial Z_1} \]  

(26)

and operator \( \hat{L} \) as above.

With substituting the solution (23) for \( F_0 \) in the equation (25), we obtain a linear equation for \( F_1 \) which is driven by terms of the form \( \Phi_0 e^{i\Phi_0} \) which are resonant, or secular, terms for the operator \( \hat{L} \). These terms lead to the divergence of the solution and the MSM, as well as any perturbation method, cannot be used, [21, 23]. To avoid the appearance of the secular terms we may assume that

\[ F_1 = 0 \]  

(27)

thus reducing the equation (25) to

\[ \hat{M} F_0 = 0. \]  

(28)

The use of the equation (23) yields the evolution equation for the amplitude \( A \):

\[ \left( \frac{\partial}{\partial T_1} - \frac{\gamma N}{q^2} \frac{\partial}{\partial Z_1} \right) A = 0. \]  

(29)

It follows that the variables \( T_1 \) and \( Z_1 \) are not independent and can be replaced by one variable

\[ \zeta = T_1 - v_g Z_1, \quad v_g = \frac{\gamma N}{q^2} \]  

(30)

(the physical meaning of \( v_g \) is the group velocity).

4.5. Third order terms, \( \delta^3 \)

Following the same lines as above, we deduce the equation in the operator form

\[ \hat{L} F_2 + \hat{M} F_1 + \hat{N} F_0 + \frac{4}{\gamma N} \hat{R} (F_0) = 0 \]  

(31)

where the operator \( \hat{N} \) reads

\[ \hat{N} = \frac{\partial}{\partial T_0} \frac{\partial}{\partial Z_2} + \frac{\partial}{\partial T_2} \frac{\partial}{\partial Z_0} + \frac{1}{\tau} \frac{\partial}{\partial Z_2} + \frac{\partial}{\partial T_1} \frac{\partial}{\partial Z_1}. \]  

(32)

Again, we avoid the secular terms by assuming that

\[ F_2 = 0 \]  

(33)

yielding
\[ N F_0 + 4^\gamma N R(F_0) = 0. \] (34)

We introduce new variable \( \eta = T_2 - v_g Z_2 \), compute all necessary derivatives and substitute the results into the equation (34):
\[ i \frac{\partial A}{\partial \eta} + \frac{v_g}{q} \frac{\partial A^2}{\partial \zeta^2} - \frac{2}{q^\gamma N} e^{-i\Phi_0} R = 0. \] (35)

The explicit dependence of \( R \) on \( F_0 \) can be deduced from the equations (8), (23) and (24)
\[ -\hat{R} \frac{8q^2}{\gamma N} = AAA \exp\{3i\Phi_0\} - AAA^* \exp\{i\Phi_0\} \exp\{i(\Phi_0 - \Phi_0^\ast)\} \] \[ -AA^*A^* \exp\{-i\Phi_0^\ast\} \exp\{i(\Phi_0 - \Phi_0^\ast)\} + A^*A^*A^* \exp\{-3i\Phi_0^\ast\} \] (36)

and for the phase difference \( \Phi_0 - \Phi_0^\ast = 2i \frac{\tau}{T_0} \). Their substitution into the main equation yields finally the well known parametric nonlinear Schrödinger equation (pNLS):
\[ i \frac{\partial A}{\partial \eta} + \frac{2\gamma N}{q^2} \frac{\partial A^2}{\partial \zeta^2} - \frac{\gamma N}{q^2} \exp\{-2T_0/\tau\}|A|^2 A = 0. \] (37)

This equation describes the evolution of the envelope for the amplitude \( A \) of photoinduced grating. The pNLS includes actual parameters of the experimental process, which are the following: \( \gamma_N \) is the nonlocal nonlinear response of the medium, \( \tau \) is the time relaxation constant of the nonlinearity, \( q \) is the grating period, and \( T_0 \) is the real time.

The pNLS is of course, a simplified version of the equation (8) describing the nonlinear coupling between the material lattice \( Q \) (i.e. refractive index grating) and the light lattice \( J_m \).
In the steady state the both envelopes $Q$ and $J_m$ have similar shapes differing only in a constant: $Q = \tau \gamma N J_m$ and $J_m \propto \frac{1}{\tau N \gamma} \frac{1}{2} \left( A + A^* \right)$. The envelope in the shape of a dark soliton is formed along the longitudinal direction of wave propagation, for both lattices. Accordingly, the energy of the output waves depends on $J_m$. An example of nonlinear coupling of these two lattices is shown in the figure 1.

Numerical simulations with the initial dynamical system equation (1)-(3) clearly demonstrate this effect thus confirming our theoretical findings, [18]. It is also demonstrated there that if we omit the assumption of $Q$ being a real-valued function and proceed formally with the MSM, the deduced evolution equation turns into the Ginzburg-Landau equation with complex variables, with no known physical interpretation.

5. Discussion

In this paper it was demonstrated for the first time that evolution of the nonlocal optical media under the action of the two-wave mixing can be described by the (pNLS) equation. The beauty of the universal models appearing in various fields of physics lies in the fact that one can use immediately all the results available in other disciplines to the current problem. For instance, the experimentally observed localization and regular oscillations may be attributed to the periodical, [25], and soliton-like solutions of the NLS. The fact that the coefficients in front of dispersive and nonlinear terms have opposite signs tells us that the equation (37) possess modulation instability [26]. Accordingly the Akhmediev breathers [27], dissipative solitons [28], dynamical energy cascades [29] are to be expected. For sufficiently small nonlinearity and suitable initial conditions [30, 31], resonance clusters might occur [32], etc.

These theoretical findings can be experimentally realized in any photorefractive medium, either in traditional photorefractive crystals, liquid crystals, media with thermal or saturable absorption nonlinearity, or in modern photonic crystals. In particular, the properties of wave-mixing are used in holography where a beam going through an image mask carries the information about the image. During the dynamical holography, two beams - the image beam and a pump beam, are mixed in a nonlinear medium resulting in various many nonlinear effects of image processing including amplification of an image, see e.g. [2, 4, 10, 12].

This happens due to the energy transfer from the pump beam to the image beam yielding a dynamical grating and self-diffraction of beams. The equation (37) allows to determine the refractive index grating in a soliton-like shape to provide the energy transfer between interacting beams. In other words, we can construct an artificial grating with the same or similar refractive index profile in such a way that the image beam diffracted on this grating will have maximal amplification.

A simple way to create an artificial grating is to use several holographic gratings with different refractive modulation depths. These gratings should be put consecutively by such a way that the envelope of their combined modulation depth has a soliton-like shape previously computed from the equation (37). Currently, we plan experimental study of energy transfer in nonlinear liquid crystal cells aiming to identify individual scenarios predicted by this equation.

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References

[1] Hesselink L, Feinberg J and Roosen G 2008 J. Phys. D: Appl. Phys. 41 224001–7
[2] Naydenova I (ed) 2011 Holograms—Recoding Materials and Applications (Rijeka: InTech) (https://doi.org/10.5772/751)
[3] Nakazawa M, Kikuchi K and Miyazaki T (ed) 2010 Optical and Fiber Communications Reports vol 6 (Berlin: Springer)
[4] Barbosa E A, Mariano da Silva D M and Ferreira M S 2013 Holography—Basic Principles and Contemporary Applications ed E Mihaylova (Rijeka: InTech) pp 203–25
[5] Harun S W and Arof H (ed) 2013 Current Developments in Optical Fiber Technology (Rijeka: InTech) (https://doi.org/10.5772/46191)
[6] Galisteo-Lopez J P and Lopez C 2004 Phys. Rev. B 70 035108
[7] Yeh P 1989 Two-wave mixing in nonlinear media IEEE J. Quantum Electron. 25 484–519
[8] Chi M, Huignard J-P and Petersen P M 2009 A general theory of two-wave mixing in nonlinear media J. Opt. Soc. Am. B 26 1578–84
[9] Bedowski A, Krolikowski W and Kujawski A 1989 Temporal instabilities in single-grating photorefractive four-wave mixing J. Opt. Soc. Am. B 6 1544–7
[10] Gunter P and Huignard J P 2006 Photorefractive Materials and their Applications, 1, 2, and 3 (New York: Springer)
[11] Kogelnik H 1969 Coupled wave theory for thick hologram gratings Bell Syst. Tech. J. 48 2909
[12] Gunter P and Huignard J-P (ed) 1988 Photorefractive Materials and Applications, TAP 61 and 62 (Heidelberg: Springer)
[13] Odoulov S, Soskin M and Khyzhnyak A 1991 Oscillators with Degenerate Four-Wave Mixing (New York: Harwood Academic)
[14] Bugaychuk S A and Khizhnyak A I 1998 Steady state and dynamic grating in photorefractive four-wave mixing J. Opt. Soc. Am. B 15 2107–13
[15] Bugaychuk S, Kóvacs L, Mandula G, Polgár K and Rupp R A 2003 Phys. Rev. E 67 0466031–8
[16] Jeganathan M, Bashaw M C and Hesselink L, 1995 J. Opt. Soc. Am. B 12 1370
[17] Stasel ko D I and Sidorovich V G 1974 J. Tech. Phys. 44 580–7
[18] Bugaychuk S and Conte R 2012 Phys. Rev. E 86 026603
[19] Hong J H and Saxena R 1991 Diffraction efficiency of volume holograms written by coupled beams Opt. Lett. 16 180–2
[20] Bugaychuk S and Tobisch E 2017 arXiv:1702.06853
[21] Nayfeth A H 1973 Perturbation Methods (New York: Wiley)
[22] Kartashova E and L'vov V S 2007 Phys. Rev. Lett. 98 198501
[23] Dauxois T and Peyrard M 2006 Physics of Solitons (Cambridge: Cambridge University Press)
[24] Bugaychuk S and Conte R 2009 Phys. Rev. E 80 0666031–7
[25] Akhmediev N, Eleonskii V M and Kulagin N E 1985 Sov. Phys. JETP. 62 894–9
[26] Zakharov V E and Ostrovsky L A 2009 Physica D 238 540–8
[27] Ankiewicz A, Kedziora D J, Chowdury A, Bandelow U and Akhmediev N 2016 Phys. Rev. E 93 012206
[28] Akhmediev N and Ankiewicz A 2005 Dissipative Solitons, LNP vol 661 (Berlin: Springer)
[29] Kartashova E 2012 Phys. Rev. E 86 0411291–9
[30] Kartashova E 2010 Nonlinear Resonance Analysis: Theory, Computation, Applications (Cambridge: Cambridge University Press)
[31] Kartashova E 2013 Europhys. Lett. 102 440051–6
[32] Kartashova E 2007 Phys. Rev. Lett. 98 2145021–4