Comment on
"Comparison of potential models with the \textit{pp} scattering data below 350 MeV"

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Abstract

We point out two flaws in the recent test of nucleon-nucleon (NN) potentials conducted by Stoks and de Swart. First, in some cases, the neutron-proton (np) version of an NN potential was compared to the proton-proton (pp) data, which is improper and yields (large) $\chi^2$ that are essentially meaningless. Second, for a proper test of the quantitative nature of a NN potential, it is insufficient to compare to pp data only, since this leaves the T=0 potential untested. Thus, it can happen that the pp version of a potential predicts the pp data accurately, while the np version of that same potential is poor in np (where also the T=0 potential is involved). An example for this is the Nijmegen potential, which predicts the pp data well with a $\chi^2$/datum of 2.0, but yields a $\chi^2$/datum of 6.5 in np.

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In a recent paper [1], Stoks and de Swart compare some nucleon-nucleon (NN) potential models with the proton-proton (pp) scattering data below 350 MeV. The general purpose of their study is to test the quantitative nature of these NN models, since this is important when these potentials are applied in “three-nucleon elastic scattering, few-nucleon bound-states, and nuclear matter calculations” [1]. Moreover, the authors stress “that one has to be very careful in drawing conclusions regarding the importance or
unimportance of, e. g., three-nucleon forces in many-body calculations, when
these conclusions are only based on calculations where the NN interaction is
represented by an NN potential model which cannot even adequately describe
the two-nucleon scattering data” [1].

We strongly agree with the authors of Ref. [1] that a comprehensive and
reliable knowledge of the quantitative nature of an NN potential is important
to properly assess results based upon the potential. However, we are con-
cerned that the information provided by the Nijmegen group in their study
is, in part, incomplete and may, in part, be misleading to some non-experts.
Essentially, we see two flaws in the Nijmegen investigation. First, in some
cases, the Nijmegen group considers neutron-proton ($np$) potentials to cal-
culate the $\chi^2$ for the fit of the proton-proton ($pp$) data; this is improper and
yields huge $\chi^2$ values that are basically meaningless. Second, the Nijmegen
analysis is restricted to a comparison with the $pp$ data only. This leaves the
$T=0$ part of the NN potential untested. However, this part of the poten-
tial is very important in, e. g., calculations of few-nucleon bound-states and
three-nucleon scattering, which are part of the motivation for the Nijmegen
study (see quotes above).

We will use the rest of this Comment to explain our two points of concern
in more detail.

It is very important that a $\chi^2$ is calculated properly. The most important
rule here is: A $pp$ potential must only be confronted with $pp$ data, while
a $np$ potential must only be confronted with $np$ data. Though this rule is
obvious, it has been violated in Ref. [1] in the case of the Argonne [2] and
the “Bonn87” potentials [3], which are $np$ potentials by construction. Let us
briefly explain why this rule is so important. At low energies, NN scattering
takes place mainly in $S$ wave. There is well-known charge-dependence in
the $^1S_0$ state and the electromagnetic effects are very large in low energy
$pp$ scattering. Thus, $np$ and $pp$ differ here substantially. Moreover, there
exist very accurate $pp$ cross section data at low energies. Consequently, if
(improperly) a $np$ potential is applied to $pp$ scattering, a very large $\chi^2$ is
obtained. However, this large $\chi^2$ has nothing to do with the quality of the
$np$ potential; it simply reflects the fact that charge-dependence is important
and that the $pp$ data carry a very small error at low energies.

To give an example: When the $np$ versions of the Argonne [2] and
“Bonn87” [3, 4] potentials are (improperly) confronted with the $pp$ data,
a $\chi^2$/datum of 824 and 641, respectively, is obtained for the energy range
0–350 MeV; for 2–350 MeV the $\chi^2$/datum are 7.1 and 13, respectively (cf. Table II of Ref. [1]). However, if (properly) the $pp$ version of the Bonn potential is confronted with the $pp$ data, a $\chi^2$/datum of 1.9 is obtained (cf. “Bonn89” in Table II of Ref. [1]). It is now important to notice that the change in the potential, that brings about this large change in the $\chi^2$, is minimal. The main effect comes from the $^1S_0$. A $np$ potential is fitted to the $np$ value for the singlet scattering length. Now, if one wants to construct a $pp$ potential from this, one has to do essentially only two things: The Coulomb force has to be included and the singlet scattering length has to be readjusted to its $pp$ value. Since the scattering length of an almost bound state is a super-sensitive quantity, this is achieved by a very small change of one of the fit parameters; for example, a change of the $\sigma$ coupling constant by as little as 1%. This is all that needs to be done; this changes the $\chi^2$/datum from 641 to 2. It shows in a clear way how misleading $\chi^2$ can be if the reader is not familiar with the field.

The physically more interesting and relevant question is what difference it makes in microscopic nuclear structure calculations whether the NN potential used is adjusted to $pp$ or $np$. To give two examples: In nuclear matter, the binding energy per nucleon at normal nuclear matter density comes out 0.61 MeV smaller for a $pp$ potential as compared to an $np$ potential [7]. The correct charge-dependent calculation is 0.27 MeV above the $pp$ value for the binding energy. This must be compared to the total nuclear matter binding energy per nucleon of 16 MeV. With regard to the large uncertainties that nuclear matter theory is beset with, charge-dependence is a negligible effect in nuclear matter at the present time.

The situation is different for the three-nucleon problem, where rigorous Faddeev calculations are performed. Here, $np$ potentials predict about 0.3 MeV more binding energy for the triton than $pp$ potentials. The correct charge-dependent calculation is 0.1 MeV above the $pp$ result. Since the gap between predictions from two-body forces and the experimental value for the triton binding of 8.48 MeV is between 0.2 and 1 MeV, charge-dependence is important in three-nucleon bound state calculations. For three-nucleon scattering (e. g., $n - d$ elastic and breakup) charge-dependence may even be crucial, as shown by the Bochum group [8, 9].

In summary, one should in general carefully distinguish between the $np$ and the $pp$ version of an NN potential. This distinction is absolutely crucial for the calculation of the $\chi^2$ of the fit of the NN scattering data. In most
nuclear structure calculations, charge-dependence is not important at the present time; a remarkable exception occurs, however, in ‘exact’ few-nucleon calculations, for which charge-dependence is crucial in some cases.

The second point which we would like to explain in this Comment is the fact that for testing the quantitative nature of a NN potential, it is insufficient to make a comparison with $pp$ data only. Proton-proton states are $T=1$ (where $T$ denotes the total isospin of the two-nucleon system) and, thus, a comparison with the $pp$ data tests only the $T=1$ potential. However, there is also the $T=0$ potential, which is equally important for applications in few-nucleon physics and nuclear structure; in fact, one may well argue that the $T=0$ potential is more crucial, since the important $^3S_1$ state is $T=0$. This isoscalar potential is tested only in a comparison with $np$ data (in which both isospin states are involved).

Since $T=0$ is excluded from $pp$, it may happen that the $pp$ version of a potential is very successful in $pp$, while the $np$ version of that same potential fails in $np$. To illustrate this point, we show in Table I the $\chi^2$/datum for three modern potentials which describe the $pp$ data about equally well ($\chi^2$/datum $\approx 2$ in all three cases, in agreement with the findings of Ref. [1]) [10]. In the second row of Table I, the $\chi^2$/datum for the fit of the $np$ data (using the $np$ version of the potentials) is given. It is clearly seen that, in some cases, this $\chi^2$ is substantially different (factor 2-3 larger) from $pp$. In the case of the Paris potential (and in part for the Nijmegen potential) the large $np$ $\chi^2$ is essentially due to the fact that the $np$ total cross sections ($\sigma_{tot}$) are predicted too large, which in turn is due to too large $^3D_2$ phase shifts (see Fig. 1b).

While the $T=1$ phase-shift predictions by modern potentials (e. g., Nijmegen, Paris, and Bonn) are so close that in conventional graphs they are almost indistinguishable, the situation is very different for $T=0$. To demonstrate this point, we show in Fig. 1 some $T=0$ phase shifts. Clearly there are substantial differences among the predictions by the three models considered. There are corresponding differences in the predictions for $np$ spin observables of which we show two very recent measurements in Fig. 2 and 3. Differences in the predictions for the spin-correlation parameter $A_{yy}$ (Fig. 2) can be clearly traced to $^3D_2$ and $^3D_3$, while in $A_{zz}$ (Fig. 3) the differences in the predictions for the $^1P_1$ phase shifts show up.

In summary, the largest differences between modern NN potentials occur in the $T=0$ states. Thus, a pure $pp$ investigation (restricted to $T=1$) misses important information that may have serious implications for calculations of
few-nucleon bound-states, three-nucleon scattering, and other applications.

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[4] In Ref. [1] the “Bonn87” model is identified with a local r-space OBEP. This is very incorrect and misleading. In Ref. [2] the Bonn full model is presented with which, therefore, the Bonn model of 1987 is to be identified. Clearly, a Physics Reports article published in 1987 does not serve the purpose to present a simplistic model appropriate for the 1960’s. Since Ref. [3] is also a review article, it contains also some discussion of other (simpler) models. In particular, comparison is made with the local r-space concept (denoted by OBEPR in Ref. [3]) to point out the deficiencies of such simple models in fitting certain phase shifts and to discuss to which (limited) extend such models may still be usefull in some nuclear structure applications. But clearly, OBEPR is not ‘the Bonn potential’ and the $\chi^2$ of OBEPR is of no interest.

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[10] The $\chi^2$ given in our Table I are calculated for the range 10–300 MeV (in terms of the kinetic energy of the incident nucleon in the laboratory system). We find this energy range more appropriate than the range 0–350 MeV, for the following reasons. For energies below 10 MeV, the effective range expansion applies and the quantitative nature of a potential can be tested by checking how well it reproduces the empirical effective range parameters. This is physically more reasonable than calculating (possibly, huge) $\chi^2$. The low energy parameters as predicted by the Nijmegen, Paris, and Bonn potential are compared to the empirical values in Table 2 of Ref. [15] and we will not repeat this here. Concerning the upper end of the energy range to be considered for testing real potentials, one has to keep in mind that pion production starts around 280–290 MeV. This implies that the real potential concept is strictly speaking wrong above these energies. It may be o. k. to stretch the limit by a few MeV up to 300 MeV, but not beyond that. Not surprisingly, for most potentials, the $np$ $\chi^2$datum is already very bad for the range 300–350 MeV, e. g., 18.0, 7.2, and 6.2 for the Nijmegen, Paris, and Bonn potentials, respectively. Thus, including this energy range will increase the over-all $\chi^2$ substantially, due to contributions that are rather irrelevant.

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Table 1. $\chi^2$/datum for fit of world NN data by some current models for the NN interaction.

|                    | Nijmegen [11] | Paris [12] | Bonn full model$^a$ |
|--------------------|---------------|------------|---------------------|
| all $pp$ data      | 2.06          | 2.31       | 1.94                |
| all $np$ data      | 6.53          | 4.35       | 1.88                |
| ($np$ without $\sigma_{tot}$) | (3.83)       | (1.98)     | (1.89)              |
| all $pp$ and $np$  | 5.12          | 3.71       | 1.90                |

The $\chi^2$/datum are obtained from the computer software SAID of R. A. Arndt and L. D. Roper (VPI&SU) [13]. The world NN data set in the range 10–300 MeV as of September 1992 is used [10]; it includes 1070 data for $pp$, 2158 data for $np$ without total cross sections ($\sigma_{tot}$), and 2322 data for $np$ with $\sigma_{tot}$.

$^a$ The $np$ version of the Bonn full model is published in the original paper [3], the $pp$ version can be found in Refs. [14, 15]; the phase shifts for both $np$ and $pp$ are available from SAID [13].
FIGURE CAPTIONS

Figure 1. Some T=0 phase shifts. (a) $^1P_1$, (b) $^3D_2$, and (c) $^3D_3$. Predictions are shown by the Nijmegen potential [11] (dotted line), Paris potential [12] (dashed), and the Bonn full model [3] (solid line). The solid dots represent the energy-independent phase shift analysis by Arndt et al. [16].

Figure 2. Neutron-proton spin correlation parameter $A_{yy}$ at 181 MeV. Predictions by the Nijmegen potential [11] (dotted line), Paris potential [12] (dashed), and Bonn full model [3] (solid line) are compared with the data (solid dots) from Indiana [17]. The $\chi^2$/datum for the fit of these data is 54.4 for Nijmegen, 3.22 for Paris, and 1.78 for Bonn [13]. The experimental error bars include only systematics and statistics; there is also a scale error of ±8%. In the calculations of the $\chi^2$, all three errors have been taken into account [13].

Figure 3. Neutron-proton spin correlation parameter $A_{zz}$ at 67.5 MeV. Predictions by the Nijmegen potential [11] (dotted line), Paris potential [12] (dashed), and Bonn full model [3] (solid line) are compared with the data (solid dots) taken by the Basel group [18]. The $\chi^2$/datum for the fit of these data is 47.7 for Nijmegen, 1.6 for Paris, and 1.2 for Bonn [13]. The experimental error bars include only systematics and statistics; there is also a normalization uncertainty of ±6%. In the $\chi^2$ calculations, all three errors have been taken into account [13].
The figures, which are crucial for a proper understanding of this Comment, are available upon request from

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