Signals of $Z'$ boson in the Bhabha process within the LEP 2 data set

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Abstract

The LEP2 data set on the Bhabha process is analyzed with the aim to detect the signals of the heavy virtual $Z'$ gauge bosons. The state interacting with the left-handed standard-model doublets and called the Chiral $Z'$ is investigated. This particle was introduced already as the low-energy state allowed by the renormalizability of the model. The contribution of the Chiral $Z'$ state to the Bhabha process is described by two parameters: the coupling to electrons and the $Z$-$Z'$ mixing angle. The sign-definite one-parameter observable is proposed to measure the $Z'$ coupling to the electron current. The one-parameter fit of the data shows no signals of the particle. The alternative two-parameter fit of the differential cross-sections is also performed. It also shows no Chiral $Z'$ signals. The comparisons with other fits are discussed.

1 Introduction

Searching for new heavy particles beyond the energy scale of the standard model (SM) is an important problem of modern high energy physics. In particular, in recently finished experiments at LEP either model dependent or model independent searches for different heavy virtual states were carried out and the masses and couplings of numerous particles entering a number of popular models extending the SM have been restricted [1]. As a general observation it was found that the low bounds on the mass of a specific particle are strongly model dependent ones and vary in a wide energy range. For
instance, the lower limit on the mass of the $Z'$ gauge boson changes from 670 GeV to 2 TeV \cite{1}. At the accelerators of the next generation (the LHC, in particular) the signals of new heavy particles are expected to be clearly detected and some of them could be discovered. But the underlying theory beyond the SM will hardly be settled. These circumstances may serve as a motivation for the model-independent description of new heavy virtual particles with specific quantum numbers.

In Refs. \cite{2, 3} the approach to pick out uniquely the $Z'$ gauge boson in different scattering processes at low energies has been developed and applied \cite{4, 5} to analyze the data of the LEP2 experiments. It is based on two main principles: 1) the renormalizability of an underlying model extending the SM is assumed; 2) the kinematics features of the specific processes in which the $Z'$ appears as a virtual state are accounted for. The former requirement results in the specific relations between the low energy couplings of the $Z'$ with the particles of the SM called the renormalization group (RG) relations and decreasing a number of independent parameters. The latter gives a possibility to introduce the observables which uniquely determine this state in different scattering processes. It is important to notice that at each stage of this analysis one also has to take into consideration the derived correlations.

The detailed analysis of the existing in the literature \cite{6} experimental data for the annihilation $e^+e^- \to \mu^+\mu^-, e^+e^- \to \tau^+\tau^-$ processes and the Bhabha process, $e^+e^- \to e^+e^-$, having as a goal searching for signal of the Abelian $Z'$ boson, showed \cite{4, 5}: 1) In the annihilation processes, the present day data are in accordance with the $Z'$ existence, but the data set is not bulk enough to detect the signals at more than the 1\(\sigma\) confidence level (CL). 2) In the Bhabha process, the signals can be determined at the 2\(\sigma\) CL. The mass of the particle was estimated to be \(m_{Z'} \sim 1 - 1.2\) TeV. So one has to expect that the increase of the data set could make the signals more evident.

An important finding in Ref \cite{2} was the result that at low energies there are two types of the $Z'$ interactions with the SM fermions compatible with the RG equations. The first is the well known Abelian $Z'$ boson. And the second one is the “Chiral $Z'$” interacting with the left-handed species, only. The RG relations are different for these cases. Correspondingly, different observables must be constructed in order to pick out the signals of these states. So, searching for the Chiral $Z'$ is of interest as a possibility of new physics beyond the SM.

In the present paper the effects of the Chiral $Z'$ boson in the Bhabha process $e^-e^+ \to e^-e^+$ are investigated. We introduce the one-parametric
observable to pick out the low energy coupling of the $Z'$ to the left-handed electrons. Because of a few independent couplings entering the cross-section there is other interesting possibility of two-parameter fits. We find that in both cases the LEP2 data show no signals of the Chiral $Z'$ boson.

2 $Z'$ couplings at low energies

To be self-contained, let us remind the necessary results on the description of the $Z'$ interactions at low energies. Suppose that all new heavy particles are decoupled and the $Z'$ boson is associated with some low-energy gauge subgroup. Then, as it was shown in Ref [2], its couplings to the SM particles satisfy so-called RG relations required by the renormalizability of the underlying theory beyond the SM. By accounting for the relations the number of independent parameters describing cross-sections can be significantly reduced. Notice that all popular $SO(10)$-based $Z'$ models are embedded into the mentioned relations.

To describe the types of the $Z'$ boson predicted by the RG analysis let us introduce the phenomenological parametrization of the low-energy $Z'$ couplings to the SM particles [2]. The couplings to the SM fermion left-handed doublets and right-handed singlets $f$ are described by the Lagrangian

$$L_f = \frac{\tilde{g} g'}{2} Z_{0\mu} \sum_f \bar{f} \gamma^\mu \tilde{Y}(f) f,$$

where $Z_{0\mu}$ denotes the eigenstate of the $Z'$ gauge group, $\tilde{g}$ stands for the charge corresponding to the group, and $\tilde{Y}(f)$ are the unknown generators characterizing the model beyond the SM. The generators are diagonal $2 \times 2$ matrices for the fermion doublets and numbers for the fermion singlets. In the same manner, the $Z'$ couplings to the scalar doublet $\phi$ are derived by adding the appropriate term to the standard electroweak covariant derivative:

$$L_s = \left| \left( D^\text{ew}_{\mu} \phi - \frac{i\tilde{g} g'}{2} \tilde{Y}(\phi) Z_{0\mu}^\prime \right) \phi \right|^2,$$

where the generator $\tilde{Y}(\phi)$ is the diagonal $2 \times 2$ matrix.

The RG relations predict two types of the $Z'$ boson – the Abelian and the Chiral one. The couplings of the Abelian $Z'$ boson is restricted by the relations

$$\tilde{Y}(f_L) = \tilde{Y}_{fL} \cdot \hat{1}, \quad \tilde{Y}(\phi) = \tilde{Y}_{\phi} \cdot \hat{1}, \quad \tilde{Y}(f_R) - \tilde{Y}_{fL} = 2 T_{3,f} \tilde{Y}_{\phi},$$

where $T_{3,f}$ are the third components of the weak isospin.
where $\hat{1}$ is the $2 \times 2$ unit matrix, and $T_f^3$ is the third component of the fermion weak isospin. As is seen, the set of independent couplings consists of one number for each SM doublet ($\tilde{Y}_{fL}$ and $\tilde{Y}_\phi$). Introducing the $Z'$ couplings to the vector and axial-vector currents,

$$v_f = \frac{\tilde{Y}(f_R) + \tilde{Y}_{fL}}{2} = \tilde{Y}_{fL} + T_3 Y_f,$$

$$a_f = \frac{\tilde{Y}(f_R) - \tilde{Y}_{fL}}{2} = T_3 Y_f, \quad (2)$$

we see that the $Z'$ couplings to the axial-vector currents have the universal absolute value, related to the $Z'$ coupling to scalars. So, we can also use as an independent set of parameters the $Z'$ couplings to the vector fermion currents ($v_f$) and the $Z'$ coupling to the electron axial-vector current ($a_e = a$).

The second possibility is the Chiral $Z'$. It is characterized by the constraints

$$\tilde{Y}(f_L) = -\tilde{Y}_{fL} \cdot \hat{\sigma}_3, \quad \tilde{Y}(f_R) = 0, \quad \tilde{Y}(\phi) = -\tilde{Y}_\phi \cdot \hat{\sigma}_3 \quad (3)$$

where $\hat{\sigma}_3$ is the third Pauli matrix. The Chiral $Z'$ interacts with the SM doublets only that can be described by one parameter for each doublet ($\tilde{Y}_{fL}$ and $\tilde{Y}_\phi$).

The $Z'$ couplings to the SM scalars necessarily lead to the mixing of physical $Z$ and $Z'$ states. It is given by the relation [2]

$$\theta_0 = \frac{\sin \theta_W \cos \theta_W \frac{m_Z^2}{m_{Z'}^2} \tilde{g} Y_f + O \left( \frac{m_Z^4}{m_{Z'}^4} \right)}{\sqrt{\frac{4\pi\alpha}{m_{Z'}^2}}} \quad (4)$$

for both the Abelian and the Chiral $Z'$ bosons, where $\theta_W$ is the SM value of the Weinberg angle. As estimates fulfilled at LEP2 energies showed, in spite of the smallness, the $Z$-$Z'$ mixing affects the $Z$-boson couplings and can produce the effects of the order of that induced by the $Z'$ exchange diagrams. So it cannot be neglected.

Due to a reduced number of independent couplings it is possible to pick out the $Z'$ signals in leptonic processes and efficiently constraint the parameters of this particle. In our previous papers [3-5] we have analyzed the signals of the Abelian $Z'$ for $e^-e^+ \rightarrow e^-e^+$, $\mu^-\mu^+$, and $e^-e^+ \rightarrow \tau^-\tau^+$ processes. We constructed the one-parametric sign-definite observables responsible for $a^2$ coupling for each of the processes. It was also possible to provide the one-parametric sign-definite observable for $v_e^2$ in the Bhabha process $e^-e^+ \rightarrow e^-e^+$. The LEP2 data on $e^-e^+ \rightarrow \mu^-\mu^+$, $\tau^-\tau^+$ processes show
the signal for $a^2$ at the $1\sigma$ CL. The Bhabha process is mainly sensitive to the vector coupling $v_e^2$, for which the signal is found at the $2\sigma$ CL.

Below we turn to the analysis of the Bhabha process with the aim to search for the Chiral $Z'$ gauge boson.

### 3 The differential cross-section

In four-fermion scattering processes $e^-e^+ \rightarrow f\bar{f}$ there are two leading-order (in the improved Born approximation) effects related to the $Z'$-boson existence. The first one is the amplitude with the virtual $Z'$ exchange $e^-e^+ \rightarrow Z' \rightarrow f\bar{f}$. In the lowest order in $m_{Z'}^2$ it is resulted in four-fermion contact couplings, which are various quadratic combinations of $\tilde{g}\tilde{Y}_f/m_{Z'}$ for all possible generators $\tilde{Y}_f$ for the considered type of the $Z'$-boson.

The second effect is originated by the $Z$-$Z'$ mixing. The mixing affects the $Z$-boson exchange diagram, $e^-e^+ \rightarrow Z \rightarrow f\bar{f}$, by terms of order $\theta_0 \tilde{g}\tilde{Y}_f$. Since the mixing angle is proportional to $m_{Z'}/m_Z$, in the amplitude the leading-order terms are described by the products $\tilde{g}^2\tilde{Y}_f\tilde{Y}_\phi m_{Z'}^2$ for all generators $\tilde{Y}_f$.

In what follows we will use the dimensionless coupling constants

$$\bar{l}_f = \frac{m_Z}{\sqrt{4\pi m_{Z'}}}\tilde{g}\tilde{Y}_{fL}, \quad \bar{r}_f = \frac{m_Z}{\sqrt{4\pi m_{Z'}}}\tilde{g}\tilde{Y}(f_R), \quad \bar{\phi} = \frac{m_Z}{\sqrt{4\pi m_{Z'}}}\tilde{g}\tilde{Y}_\phi,$$

as well as the couplings to the vector and axial-vector fermion currents, $\tilde{v}_f$ and $\tilde{a}_f$

$$\tilde{a}_f = \frac{m_Z}{\sqrt{4\pi m_{Z'}}}\tilde{g}a_f, \quad \tilde{v}_f = \frac{m_Z}{\sqrt{4\pi m_{Z'}}}\tilde{g}v_f.$$

Then, the $Z'$-induced contact couplings are just the quadratic combinations of $\bar{l}_f$, $\bar{r}_f$ (or, alternatively, $\bar{a}_f$ and $\bar{v}_f$), and the $Z$-$Z'$ mixing effects are described by $\bar{l}_f\bar{\phi}$, $\bar{r}_f\bar{\phi}$ (or $\bar{a}_f\bar{\phi}$, $\bar{v}_f\bar{\phi}$).

The above introduced coupling constants are related to the traditional four-fermion contact couplings (usually marked as $\epsilon$) through the expressions

$$\epsilon_{LL} = -\bar{l}_f\bar{l}_f m_Z^{-2}/4, \quad \epsilon_{RR} = -\bar{r}_f\bar{r}_f m_Z^{-2}/4, \quad \epsilon_{LR} = -\bar{l}_f\bar{r}_f m_Z^{-2}/4, \ldots$$

and

$$\epsilon_{VV} = -\bar{v}_f\bar{v}_f m_Z^{-2}/4, \quad \epsilon_{AA} = -\bar{a}_f\bar{a}_f m_Z^{-2}/4, \quad \epsilon_{AV} = -\bar{a}_f\bar{v}_f m_Z^{-2}/4, \ldots$$
Finally, the $Z$-$Z'$ mixing angle is determined by $\bar{\phi}$ as follows,

$$\theta_0 \simeq \frac{m_W \sin \theta_W}{\sqrt{\alpha m_{Z'}}} \bar{\phi},$$

where $\alpha$ is the fine structure constant.

The leading effects in the cross-section are originated from the interference of the SM amplitude with the new physics amplitude. For the Bhabha process, the generic $Z'$-inspired deviation of the cross-section from its SM prediction is

$$\Delta d\sigma_{\text{Bhabha}}/dz = F_L\bar{l}_e^2 + F_R\bar{r}_e^2 + F_{LR}\bar{l}_e\bar{r}_e + F_{L\phi}\bar{l}_e\bar{\phi} + F_{R\phi}\bar{r}_e\bar{\phi}. \quad (5)$$

where $z$ is the cosine of the scattering angle, and $F = F(\sqrt{s}, z)$ are known functions of the center-of-mass energy and the scattering angle. It can be also expressed in terms of the vector and axial vector couplings:

$$\Delta d\sigma_{\text{Bhabha}}/dz = F_v\bar{v}_e^2 + F_a\bar{a}_e^2 + F_{av}\bar{a}_e\bar{v}_e + F_{v\phi}\bar{v}_e\bar{\phi} + F_{a\phi}\bar{a}_e\bar{\phi}. \quad (5)$$

For a fixed $z$ the factors $F(\sqrt{s}, z)$ are smooth functions of $\sqrt{s}$. However, each factor $F(\sqrt{s}, z)$ grows infinitely at $z \rightarrow 1$. This is caused by the photon exchange in the $t$-channel. Due to this singular behavior the experimental values and the uncertainties change significantly with increasing of $z$. As it was shown in [5], it is possible to make the factors finite for all the values of the scattering angle by dividing the differential cross-section by some known monotonic function. For instance, the factor $F_v$ is a positive monotonic function of $z$. So it can be chosen for normalization of the differential cross-section:

$$F^{-1}_v \Delta d\sigma_{\text{Bhabha}}/dz = F_L\bar{l}_e^2 + F_R\bar{r}_e^2 + F_{LR}\bar{l}_e\bar{r}_e + F_{L\phi}\bar{l}_e\bar{\phi} + F_{R\phi}\bar{r}_e\bar{\phi},$$

$$F^{-1}_v \Delta d\sigma_{\text{Bhabha}}/dz = \bar{v}_e^2 + F_a\bar{a}_e^2 + F_{av}\bar{a}_e\bar{v}_e + F_{v\phi}\bar{v}_e\bar{\phi} + F_{a\phi}\bar{a}_e\bar{\phi}. \quad (5)$$

This normalization gives us two benefits. First, the obtained factors $F(\sqrt{s}, z)$ are finite for all values of the scattering angle $z$. Second, the experimental errors for different bins become equalized that provides the statistical equivalence of different bins. The latter is important for the construction of integrated cross-sections.
Figure 1: The factors $F_L$ and $F_{L\phi}$ describing the Chiral $Z'$ effects in the differential cross-section at $\sqrt{s} = 200$ GeV.

Figure 2: The normalized factors $F_L$ and $F_{L\phi}$ describing the Chiral $Z'$ effects in the Bhabha process at $\sqrt{s} = 200$ GeV (solid lines) and $\sqrt{s} = 500$ GeV (dashed lines).

4 One parametric fit for the Chiral $Z'$

The Chiral $Z'$ boson does not interact with the right-handed species. So, only the factors $F_L$ and $F_{L\phi}$ survive in Eq. 1. They are plotted in Fig. 1 for the center-of-mass energy 200 GeV. As it was mentioned above, the factors are singular at $z \rightarrow 1$. On the other hand, the normalized deviation of the differential cross-section from its SM prediction,

$$\mathcal{F}^{-1} \Delta \sigma^{\text{Bhabha}}/dz = F_L \bar{p}_e^2 + F_{L\phi} \bar{p}_e \bar{\phi},$$

is determined by two finite factors, $F_L$ and $F_{L\phi}$, which are shown in Fig. 2.
As it is seen, the four-fermion contact coupling $\bar{l}^2$ contributes mainly to the forward scattering angles, whereas the $Z-Z'$ mixing term affects the backward angles. At the LEP energies they can be of the same order of magnitude. The contribution of the mixing vanishes with the energy growth.

The key problem of the data treatment is the number of independent parameters which should be determined in an experiment. A large number of them disperses the experimental statistics leading to significant uncertainties. This difficulty usually prompts one to consider various restricted models, supposing some couplings to be zero. In those cases the loss of generality is the price of the statistical adequacy of fits. The effects of the Chiral $Z'$ boson in the Bhabha process are described by two unknown parameters, only. So, we have a possibility to derive the effective experimental constraints on them without any additional restrictions.

First, let us construct a one-parametric observable which is most preferred by the statistical treatment of data. As is clear, it is impossible to separate the couplings $\bar{l}^2_e$ and $\bar{l}_e\bar{\phi}$ in any observable which is an integrated cross-section over some interval of $z$. However, the mixing contribution can be eliminated in the cross-section of the form (which is inspired by the forward-backward asymmetry)

$$\Delta\sigma(z^*) = \int_{z^*}^{z_{\text{max}}} \mathcal{F}^{-1}_v(z) \frac{d\sigma}{dz} dz - \int_{-z_{\text{max}}}^{z_{\text{min}}} \mathcal{F}^{-1}_v(z) \frac{d\sigma}{dz} dz,$$

where the boundary value $z^*$ should be chosen to suppress the coefficient at $\bar{l}_e\bar{\phi}$. The maximal value of the scattering angle $z_{\text{max}}$ is determined by a particular experiment. In this way we introduce the one-parametric sign-definite observable sensitive to $\bar{l}^2_e$.

The LEP Collaborations DELPHI and L3 measured the differential cross-sections with $z_{\text{max}} = 0.72$ [6]. The set of boundary angles $z^*$ as well as the theoretic and experimental values of the observable are collected in Table 1. The other LEP Collaborations – ALEPH and OPAL – used $z_{\text{max}} = 0.9$ [6]. The corresponding data are presented in Table 2.

The standard $\chi^2$-fit gives the following constraints for the coupling $\bar{l}^2_e$ at the 68% CL:

- ALEPH: $\bar{l}^2_e = -0.00304 \pm 0.00176$
- DELPHI: $\bar{l}^2_e = -0.00054 \pm 0.00086$
- L3: $\bar{l}^2_e = 0.00109 \pm 0.00184$
Table 1: The boundary angles $z^*$ and the theoretic values of the observable $\Delta \sigma(z^*)$ at $z_{\text{max}} = 0.72$. The experimental values of the observable are marked by (D) for DELPHI Collaboration and by (L) for L3 Collaboration.

| $\sqrt{s}$, GeV | $z^*$ | $\Delta \sigma(z^*)$ | $\Delta \sigma^{\text{ex}}(z^*)$ |
|-----------------|-------|----------------|-------------------------------|
| 183             | -0.245| 1742 $l_e^2$  | -2.38 ± 7.03 (L)             |
| 189             | -0.252| 1775 $l_e^2$  | -4.28 ± 3.36 (D)             |
|                 |       | 1771 $l_e^2$  | 3.05 ± 3.65 (L)              |
| 192             | -0.255| 1788 $l_e^2$  | 4.57 ± 7.32 (D)              |
| 196             | -0.259| 1806 $l_e^2$  | 3.77 ± 4.33 (D)              |
| 200             | -0.263| 1823 $l_e^2$  | -1.95 ± 4.05 (D)             |
| 202             | -0.265| 1831 $l_e^2$  | 1.31 ± 5.54 (D)              |
| 205             | -0.267| 1843 $l_e^2$  | -4.09 ± 3.89 (D)             |
| 207             | -0.269| 1851 $l_e^2$  | 0.40 ± 3.33 (D)              |

Table 2: The boundary angles $z^*$ and the theoretic values of the observable $\Delta \sigma(z^*)$ at $z_{\text{max}} = 0.9$. The experimental values of the observable are marked by (A) for ALEPH Collaboration and by (O) for OPAL Collaboration.

| $\sqrt{s}$, GeV | $z^*$ | $\Delta \sigma(z^*)$ | $\Delta \sigma^{\text{ex}}(z^*)$ |
|-----------------|-------|----------------|-------------------------------|
| 130             | -0.217| 2017 $l_e^2$  | -12.40 ± 19.24 (A)           |
|                 |       |                | -4.13 ± 29.29 (O)            |
| 136             | -0.266| 2092 $l_e^2$  | -50.21 ± 16.64 (A)           |
|                 |       |                | -34.18 ± 31.58 (O)           |
| 161             | -0.370| 2311 $l_e^2$  | -15.90 ± 13.24 (A)           |
|                 |       |                | -14.02 ± 22.32 (O)           |
| 172             | -0.400| 2398 $l_e^2$  | -12.11 ± 12.50 (A)           |
|                 |       |                | 13.71 ± 17.84 (O)            |
| 183             | -0.424| 2474 $l_e^2$  | -1.51 ± 5.18 (A)             |
|                 |       |                | 11.04 ± 5.57 (O)             |
| 189             | -0.435| 2512 $l_e^2$  | -0.63 ± 3.28 (O)             |
| 192             | -0.441| 2531 $l_e^2$  | -3.48 ± 9.85 (O)             |
| 196             | -0.447| 2554 $l_e^2$  | 2.96 ± 5.09 (O)              |
| 200             | -0.454| 2577 $l_e^2$  | 0.35 ± 4.68 (O)              |
| 202             | -0.457| 2587 $l_e^2$  | -2.87 ± 9.00 (O)             |
| 205             | -0.461| 2604 $l_e^2$  | 5.88 ± 4.67 (O)              |
| 207             | -0.464| 2614 $l_e^2$  | -1.42 ± 3.46 (O)             |
OPAL: $\bar{l}_e^2 = 0.00051 \pm 0.00064$

Combined: $\bar{l}_e^2 = -0.00004 \pm 0.00048$

Hence it is seen that the most precise data of DELPHI and OPAL collaborations give no signal of the Chiral $Z'$ at the 1σ CL. The combined value also shows no signal at the 1σ CL. From the combined fit the 95% CL bound on the value of $\bar{l}_e^2$ can be derived, $\bar{l}_e^2 < 9 \times 10^{-4}$. Supposing the $Z'$ coupling constant $\tilde{g}$ to be of the order of the electroweak one, $\tilde{g} \simeq 0.6$, the corresponding $Z'$ mass has to be larger than 0.5 TeV.

5 Two parametric fit for the Chiral $Z'$

Now, let us carry out a complete two parametric fit of experimental data based directly on the differential cross-sections. Since in case of the Chiral $Z'$ there are only two independent couplings, one has to expect that this fit has to be reliable.

In the fitting we used the available final data for the differential cross-sections of the Bhabha process. The data set consists of 299 bins including the data of ALEPH at 130-183 GeV, DELPHI at 189-207 GeV, L3 at 183-189 GeV, and OPAL at 130-207 GeV.

The $\chi^2$ function reads

$$\chi^2(\bar{l}_e, \bar{\phi}) = \sum_i \left( \frac{\sigma^i_{\text{ex}} - \sigma^i_{\text{th}}(\bar{l}_e, \bar{\phi})}{\delta \sigma^i_{\text{ex}}} \right)^2,$$

where $\sigma^i_{\text{ex}}$ and $\delta \sigma^i_{\text{ex}}$ are the measured deviation from the SM value of the differential cross-section for the $i$th bin (we used the SM predictions, given by the collaborations) with the corresponding error, and $\sigma^i_{\text{th}}$ is the theoretical prediction for the deviation from the SM due to the Chiral $Z'$ effects. The sum runs over all the bins.

According to Eq. (5), the theoretic predictions $\sigma^i_{\text{th}}$ are linear combinations of two products of $Z'$ couplings

$$\sigma^i_{\text{th}} = \sum_{j=1}^{2} C_{ij} A_j, \quad A_j = \{\bar{l}_e, \bar{l}_e \bar{\phi} \},$$

where $C_{ij}$ are known coefficients. Introducing matrix notations $\sigma^\text{th} = \sigma^\text{th}_i$, $\sigma^\text{ex} = \sigma^\text{ex}_i$, $C = C_{ij}$, $A = A_j$, the $\chi^2$-function can be rewritten as follows

$$\chi^2(A) = (\sigma^\text{ex} - CA)^T D^{-1} (\sigma^\text{ex} - CA),$$

10
where superscript $T$ denotes the matrix transposition, and $D$ is the covariance matrix. The diagonal elements of $D$ are the experimental errors squared, $D_{ii} = (\delta \sigma_{i}^{\text{ex}})^2$, whereas the non-diagonal elements of $D$ are responsible for possible correlations of the observables.

The $\chi^2$-function has a minimum, $\chi^2_{\text{min}}$, at the maximum-likelihood values of the $Z'$ couplings,

$$
\hat{A} = (C^T D^{-1} C)^{-1} C^T D^{-1} \sigma^{\text{ex}}.
$$

From Eqs. (7), (8) we obtain

$$
\chi^2(A) - \chi^2_{\text{min}} = (\hat{A} - A)^T \hat{D}^{-1} (\hat{A} - A),
$$

$$
\hat{D} = (C^T D^{-1} C)^{-1}.
$$

Usually, the experimental values $\sigma^{\text{ex}}$ are normal-distributed quantities with the mean values $\sigma^{\text{th}}$ and the covariance matrix $D$. The quantities $\hat{A}$, being the superposition of $\sigma^{\text{ex}}$, also have the same distribution. It is easy to show that $\hat{A}$ has the mean values $A$ and the covariance matrix $\hat{D}$.

The inverse covariance matrix $\hat{D}^{-1}$ is the symmetric $2 \times 2$ matrix, which can be diagonalized. The number of non-zero eigenvalues is determined by the rank of $\hat{D}^{-1}$, which equals to the number of linear-independent terms in the differential cross-section $\sigma^{\text{th}}$. In the case of Chiral $Z'$-boson, the rank of $\hat{D}^{-1}$ equals to 2. So, the right-hand-side of Eq. (9) is a quantity distributed as $\chi^2$ with 2 degrees of freedom (d.o.f.). Since this random value is independent of $A$, the confidence area in the parameter space $A = \{\bar{l}_e, \bar{\phi}\}$ corresponding to the probability $\beta$ can be defined as [7]:

$$
\chi^2 \leq \chi^2_{\text{min}} + \chi^2_{\text{CL},\beta},
$$

where $\chi^2_{\text{CL},\beta}$ is the $\beta$-level of the $\chi^2$-distribution with 2 d.o.f.

The parameter space of the Chiral $Z'$ is the plane ($\bar{l}_e$, $\bar{\phi}$). The minimum of the $\chi^2$-function, $\chi^2_{\text{min}} = 237.29$, is reached at zero value of $\bar{l}_e$ ($\simeq 10^{-4}$) and almost independent of the value of $\bar{\phi}$ (the maximal-likelihood values of the couplings). The 95% CL area ($\chi^2_{\text{CL}} = 5.99$) is shown in Figure 3.

As one can see, the zero point, $\bar{l}_e = \bar{\phi} = 0$ (the absence of the Chiral $Z'$ boson) is inside the confidence area. The value of $\chi^2$ in this point (238.62) is indistinguishable from the $\chi^2_{\text{min}}$. In other words, the set of experimental data cannot determine the signal of the Chiral $Z'$-boson.

As is seen from Figure 3, the value of $\bar{l}_e$ is constrained as $\bar{l}_e < 0.02$ at the 95% CL. This upper bound is in an agreement with the corresponding result.
Figure 3: The 95% CL area in $\bar{\ell}_e - \bar{\phi}$ plane. The final data of ALEPH 130-183 GeV, DELPHI GeV 189-207, L3 183-189 GeV, and OPAL 130-207 GeV are combined.

of one-parameter fit ($\bar{\ell}_e < 0.03$). Thus, the $Z'$ mass has to be larger than 0.75 TeV, if the $Z'$ coupling constant $\tilde{g}$ is again supposed to be of the order of the electroweak one, $\tilde{g} \approx 0.6$.

The fit of the differential cross-sections leads to the better accuracy on $\bar{\ell}_e$ than the fit of the integrated cross-sections based on the same data. Due to a few number of independent parameters the increase of the dispersion (inevitable in case if an extra parameter is added) is compensated by the increase in the bulk of available data. Without accounting for the model-independent relations between the $Z'$ couplings it is impossible to obtain such results.

6 Discussion

Let us summarize the results of our investigation. The LEP2 data for the Bhabha process are adjusted to no signals of the Chiral $Z'$ both for the one-parameter fit and for the two-parameter one.

We stress once again that the key point of our consideration is the RG relations between the low energy couplings of the $Z'$ with the SM fermions. They were systematically used through the analysis. Therefore only $Z'$ virtual states are responsible for the deviations of the investigated cross-sections.
from the SM values. This is the main feature of the developed approach. If the relations were not accounted for, no significant constraints have been found. In contrast, in the “helicity model fits” applied by the LEP collaborations \[1\] the quantum numbers of a specific virtual state, which could be responsible for the signal, remains unknown basically. The such type fits are based on the effective Lagrangian of the fermion contact interactions. They are mainly intended to detect the signals of any type virtual states contributing to a specific “helicity model” with one non-zero contact coupling (AA-model, VV-model, etc.). The more general four-parametric analysis based on the helicity models of the fermion contact interactions was carried out in Ref. \[8\] with the aim to describe the all possible deviations from the SM. The permissible domains in the parametric space of the models have been derived. But, in this analysis it is also in principle impossible to distinguish the specific states responsible for the deviations. This is because, as it follows from the above analysis, either the RG relations or the kinematics features of specific processes were not accounted for.

At LEP1 experiments \[9\] carried out at Z-boson peak the Z-boson coupling constants \(g_V, g_A\) are precisely measured. The Bhabha process shows the \(1\sigma\) deviation from the SM values for Higgs boson masses \(m_H \geq 114\) GeV. It is interesting to estimate the bounds on the \(Z-Z'\) mixing following from these experiments. To do that, let us express the measured parameters \(g_V, g_A\) through \(\bar{l}_e, \bar{\phi},\)

\[
g_V - g_V^{\text{SM}} = g_A - g_A^{\text{SM}} = 12.2647 \bar{l}_e \bar{\phi},
\]

and assume that a total deviation of theory from experiments follows due to the \(Z-Z'\) mixing. This gives the low bound on the mixing. If one assumes other possibilities for deviation the mixing is increased. In this way one can check is this mixing excluded by the experiments or not.

In Fig. 4 we reproduce the \(1\sigma\) CL area for the Bhabha process from Ref. \[9\]. Let us take the SM values of the couplings corresponding to the top quark mass \(m_t = 178\) GeV and the Higgs scalar mass \(m_H = 114\) GeV. According to Eq. (11), the deviation of \(g_V, g_A\) due to the Chiral \(Z'\) boson has to be on the straight line shown in Fig. 4. This line is completely outside the \(1\sigma\) CL area. So, the Chiral \(Z'\) is excluded in the Bhabha process by the LEP1 experiments. On the other hand, Fig. 4 demonstrates the \(1\sigma\) deviation from the SM. The signal could be compatible with another \(Z'\) state – the Abelian \(Z'\) boson. This possibility will be considered in detail in a separate publication.
As it follows from the results of the present paper, there is no light Chiral $Z'$ gauge boson with the mass $\sim 1$ TeV. Taking into account the previous investigations [4, 5], we can conclude that the Abelian $Z'$ boson is the most perspective neutral vector particle to be searched at the LHC.

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