Multi-center MICZ-Kepler system, supersymmetry and integrability

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We propose the general scheme of incorporation of the Dirac monopoles into mechanical systems on the three-dimensional conformal flat space. We found that any system (without monopoles) admitting the separation of variables in the elliptic or parabolic coordinates can be extended to the integrable system with the Dirac monopoles located at the foci of the corresponding coordinate systems. Particular cases of this class of system are the two-center MICZ-Kepler system in the Euclidean space, the limiting case when one of the background dyons is located at the infinity as well as the model of particle in parabolic quantum dot in the presence of parallel constant uniform electric and magnetic fields.

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I. INTRODUCTION

The Kepler system is a special one among the finite-dimensional integrable systems because of its wide applications and the existence of the hidden symmetry given by the Runge-Lenz vector \( \mathbf{L} \). Many generalizations of the Kepler system were proposed during the last century, including the Kepler systems on the spheres and hyperboloids as well as their higher dimensional generalizations \[2\]. Zwanziger, and McIntosh and Cisneros independently suggested generalization of the Kepler system describing the motion of the charged particle in the field of Dirac dyon (electrically charged Dirac monopole), which is presently known as MICZ-Kepler system \[3\]. Later on, the similar generalizations have been found also for the Kepler system on three- and higher-dimensional spheres and on the two-sheet hyperboloids \[4\]. Besides the monopole magnetic field, the MICZ-Kepler Hamiltonian contains the specific centrifugal potential term which endows it with the hidden symmetry of the Kepler system. The essential differences between the MICZ-Kepler system and the Kepler one are the lift of the possibility of the permissible dipole transitions \[3\]. Indeed, after the incorporation of the Dirac monopole at the center of spherical coordinates any spherically symmetric system (without monopole) will undergo only the minor changes mentioned above, provided the potential term to be replaced in the following way \[6\]:

\[ U_0(r) \rightarrow U_0(r) + \frac{s^2}{2Gr^2}. \] (1)

Here \( G(|\mathbf{r}|)d\mathbf{r}^2 \) is the metrics of the configuration space, and \( s = e g \) is the “monopole number” (\( e \) is the electric charge of probe particle, and \( g \) is magnetic charge of the monopole). On the other hand, in the middle of XIX century Jacobi established the integrability of the two-center Kepler system and of its limiting case when one of the forced centers is placed at infinity which leads to generalization of the homogeneous potential field (we shall refer this limiting case as the Kepler-Stark system because of its obvious relevance to the Stark effect in the hydrogen atom). Jacobi also found that these systems admit the separation of variables in the elliptic and parabolic coordinates, respectively. However, the MICZ-like extensions of these systems, as well as of their analogs on the curved spaces, were unknown until now. Respectively, the multi-center analog of the MICZ-replacement \[1\] for the potentials more general than the Kepler one was also unknown.

In this paper we define the “multi-center MICZ-system” as follows

\[ \mathcal{H} = \left( \mathbf{p} - e \mathbf{A} \right)^2 \frac{1}{2G(r)} + \frac{e^2 f^2}{2G(r)} + U_0(r) \] (2)

where \( \mathbf{A} \) is a superposition of the vector potentials of the Dirac monopoles located at the points \( \mathbf{a}_I \) (including the vector potential of the Dirac monopole located at infinity) and \( f \) is the potential of the corresponding magnetic field:

\[ \mathbf{A} = \sum_I g_I \mathbf{A}_D (\mathbf{r} - \mathbf{a}_I) + \mathbf{B}_0 \times \mathbf{r}, \]

\[ f = \sum_I \frac{g_I}{|\mathbf{r} - \mathbf{a}_I|} + \mathbf{B}_0 \cdot \mathbf{r}. \] (3)

Here \( \mathbf{A}_D = \frac{e \mathbf{m}_I}{(\mathbf{r} - \mathbf{m}_I)^2}, \) \( \mathbf{n}^2 = 1 \) is a vector potential of the Dirac monopole. The unit vector \( \mathbf{n}_I \) and the constant \( g_I \) determine, respectively, the singularity line and the magnetic charge of the \( I \)-th Dirac monopole whereas the constant \( \mathbf{B}_0 \) corresponds to the magnetic field of the monopole located at infinity. Its magnitude is given by the expression \( \mathbf{B}_0/G \). Analyzing the two-monopole configuration (including the monopole located at infinity) we arrive at the following conclusion:

The system (without monopoles) admitting separation of variables in elliptic/parabolic coordinates results in the separable two-center MICZ-system \[2\] with the Dirac monopoles placed at the foci of elliptic/parabolic coordinates.
In this way we obtain the integrable two-center MICZ-Kepler system and the MICZ-Kepler-Stark system. The MICZ-Kepler-Stark system is of the special importance as it describes the one-electron MICZ-Kepler system subjected to the parallel constant uniform electric and magnetic fields with some special confining potential. One can add the specific oscillator potential to this system and obtain the integrable model describing the dyon interacting with the the parallel constant uniform electric and magnetic fields in the parabolic quantum dot. Switching off the magnetic charge of the dyon and the magnetic fields in the parabolic quantum dot. It was constructed in 2003 by Ivanov and Lechtenfeld \[9\]. It is noteworthy that integrability of the systems with one- and two-center Taub-NUT metrics has been established, respectively, in \[\text{[11,12]}\] and \[\text{[13]}\].

The system defined by Eq.\((2)\) admits the \(\mathcal{N} = 4\) supersymmetric extension at the following choice of one-parametric family of potentials

\[
U_0 = \frac{\kappa}{G} \left( \sum g_i \left[ \frac{1}{|r - a_i|} + B_0 \cdot r \right] + \frac{\kappa^2}{2G} \right), \quad \kappa = \text{const.} \tag{4}
\]

It was constructed in 2003 by Ivanov and Lechtenfeld \[\text{[8]}\]. On the Euclidean space it results in the multi-center MICZ-Kepler system with the background dyons which have the same ratio of the electric and magnetic charges.

**II. TWO-CENTER SYSTEM: INTEGRABILITY**

In this Section we show that if the system (without monopoles) admits the separation of variables in elliptic/parabolic coordinates then the corresponding two-center MICZ system \[\text{[2]}\] with the Dirac monopoles placed at the foci of elliptic/parabolic coordinate system is also separable in these coordinates. Particularly, since two-center Kepler system is separable in elliptic coordinates \[\text{[1]}\] the corresponding two-center MICZ-Kepler system will be also separable in that coordinate systems (i.e. integrable).

Let us suppose that the two static monopoles with magnetic charges \(g_1\) and \(g_2\) are fixed on \(z\)-axis at the points \(a = (0, 0, a)\) and \(-a = (0, 0, -a)\). In the symmetric gauge when Dirac string goes along whole \(z\)-axis the vector potential of the monopole in spherical coordinates has the form \(A_r = A_\varphi = 0, A_z = g \cos \theta\). Thus, in the spherical coordinates the Hamiltonian of two-center MICZ-Kepler system looks as follows

\[
\mathcal{H} = \frac{p_r^2}{2} + \frac{p_\varphi^2}{2} + \frac{(p_r - s_1 \cos \theta_1 - s_2 \cos \theta_2)^2}{2(2\sin^2 \theta)} + \frac{1}{2} \left( \frac{2s_1}{r_1} + \frac{2s_2}{r_2} \right) + U(r, \theta, \varphi), \tag{5}
\]

where \(p = (p_r, p_\varphi, p_z)\) is the canonical momentum of the system and \(U\) is the appropriate potential. Now we turn to the elliptic coordinates \((2\xi = (r_1 + r_2)/a, 2\eta = (r_1 - r_2)/a)\). In these coordinates the Hamiltonian reads

\[
\mathcal{H} = \frac{(\xi^2 - 1) p_r^2 + (1 - \eta^2) p_\varphi^2}{2a^2 (\xi^2 - \eta^2)} + U_{s_1, s_2}(\xi, \eta, \varphi), \tag{6}
\]

where

\[
U_{s_1, s_2}(\xi, \eta, \varphi) = U(\xi, \eta, \varphi) + \frac{1}{2a^2(\xi^2 - \eta^2)} \left( \frac{p_r^2 + s_\perp^2 - 2p_r s_\perp \xi}{\xi^2 - 1} + \frac{p_\varphi^2 + s_\perp^2 - 2p_\varphi s_\perp \eta}{1 - \eta^2} \right), \tag{7}
\]

and \(s_\perp \equiv s_1 \pm s_2\). The underlying system (without monopoles) admits the separation of variables in elliptic coordinates if its potential term has the following form

\[
U(\xi, \eta) = \frac{V(\xi) + W(\eta)}{\xi^2 - \eta^2}. \tag{8}
\]

In this case the system with monopoles admits the separation of variables too. If one put for the generating function \(S = p_\varphi^2 + S_1(\xi) + S_2(\eta) - Et\), then we get the following Hamilton-Jacobi equations

\[
(\xi^2 - 1)^2 \left( \frac{dS_1}{d\xi} \right)^2 + V(\xi) - 2a^2 E (\xi^2 - 1) = n, \tag{9}
\]

\[
(1 - \eta^2)^2 \left( \frac{dS_2}{d\eta} \right)^2 + W(\eta) - 2a^2 E (1 - \eta^2) = -n,
\]

where

\[
V(\xi) = \frac{p_r^2 + s_\perp^2 - 2p_r s_\perp \xi}{\xi^2 - 1} + 2a^2 V(\xi), \tag{10}
\]

\[
W(\eta) = \frac{p_\varphi^2 + s_\perp^2 - 2p_\varphi s_\perp \eta}{1 - \eta^2} + 2a^2 W(\eta),
\]

and \(n\) is separation constant. The two-center MICZ-Kepler system with arbitrary values of electric and magnetic charges belongs to this class: its potential is given by the expression \(U = eq_1 / r_1 + eq_2 / r_2\), and could be represented in the form \([8]\).

From \([9]-[10]\) we can immediately get the explicit expression for the quadratic (on momenta) constant of motion, which is responsible for the separation of variables \([1]\). That is

\[
\mathcal{I} = \frac{n^2 [(\xi^2 - 1) p_r^2 + V(\xi)] + \xi^2 [(1 - \eta^2) p_\varphi^2 - W(\eta)]}{\xi^2 - \eta^2}. \tag{11}
\]

Though we restrict ourselves to the system in the Euclidean space it is clear from the consideration that our conclusion remains valid for the systems with any metric which admits the separation of variables in elliptic coordinates.

**III. MICZ-KEPLER-STARK(-ZEEMAN) SYSTEM**

The systems considered above have important limiting case. If we move one of the background monopoles to infinity then it will generate the constant uniform magnetic
field. Similar to previous case we are going to show that if the system (without monopoles) allows the separation of variables in the parabolic coordinates then it admits the separation of variables also in the presence of the monopoles placed at the foci of parabolic coordinates.

In the spherical coordinates the vector-potential of the constant uniform magnetic field, assumed to be in z-direction, is \( A_r = A_\varphi = 0, A_z = B\varphi t^2 \sin^2 \theta \). The Hamiltonian reads:

\[
H = \frac{p_r^2}{2} + \frac{p_\varphi^2}{2r^2} + \left( \frac{p_\varphi \cos \theta - \frac{1}{2} B \varphi t^2 \sin^2 \theta}{2r^2 \sin^2 \theta} \right)^2 + \frac{1}{2} \left( \frac{z}{r} + eBz \right)^2 + U(r, \theta, \varphi).
\]  

Let us choose the parabolic coordinates given by the relations \( \xi = r + z, \eta = r - z \), and suppose that the potential \( U \) is separable in parabolic coordinates \([1]\):

\[
U(\xi, \eta) = \frac{U(\xi) + V(\eta)}{\xi + \eta}.
\]  

Then the Hamiltonian could be presented as follows

\[
H = \frac{4\xi^2 p_\xi^2 + 4p_\eta^2}{2(\xi + \eta)} + \frac{4\xi^2 p_\eta^2 + V(\eta) + W(\eta)}{2(\xi + \eta)} - \frac{p_\varphi eB}{2},
\]  

where

\[
V(\xi) = \frac{(p_\varphi + x)^2}{\xi} + 3xeB\xi + \frac{e^2 B^2 \xi^2}{4} + 2V(\xi),
\]

\[
W(\eta) = \frac{(p_\varphi - x)^2}{\eta} - 3xeB\eta + \frac{e^2 B^2 \eta^2}{4} + 2W(\eta).
\]  

The separability of the system is obvious. The constant of motion which is responsible for the separability of variables looks as follows

\[
I = \frac{4\xi^2 p_\xi^2 - p_\varphi^2}{\xi + \eta} + \eta V(\xi) - \xi W(\eta).
\]  

Notice that similar to elliptic case, our conclusions concerning integrability remains valid also for the systems with non-constant metric \( G \) which admit the separation of variables in parabolic coordinates.

The important particular case of the systems under consideration is the Jacobi problem when the potential is the sum of the Coulomb potential and the potential of the constant uniform electric field parallel to the magnetic one (Kepler-Stark system): \( U = eq/r - eE \). In that case the monopole placed at infinity, generates the constant uniform magnetic field \( B_0 \) which is parallel to the electric one: \( B_0 \parallel E \). In that case the system could be viewed as the MICZ-Kepler-Stark system. Its Hamiltonian looks as follows:

\[
\mathcal{H} = \frac{(p_x - eA_0)^2}{2B_0} \cdot \frac{r^2}{r^2} + \frac{e^2}{2r^2} + \left( \frac{-eB_0}{2r} \right)^2 + \left( \frac{-eB_0}{2} \right)^2 + \left( \frac{eB_0}{2} \right)^2.
\]  

Hence, the two-center MICZ-Kepler-Stark system can be interpreted as the (one-centered) MICZ-Kepler system in the parallel constant uniform electric and magnetic fields with some additional potential. Due to this additional correction the system becomes integrable, in contrast to the MICZ-Kepler system in the constant uniform electric and magnetic fields.

One may also extend the Hamiltonian \([17]\) with the additional oscillator potential of the type \([13]\):

\[
V_{\text{conf}} = \frac{\omega_y^2(\xi^2 + 3\zeta^2)}{2} = \frac{\omega_\varphi^2(\xi^2 + \eta^2)}{2(\xi + \eta)}.
\]  

In this case putting \( s = 0 \) we obtain the Coulomb particle in the parallel electric and magnetic fields placed in the axially symmetric quantum dot with confining potential of parabolic type with the frequencies \( \omega_x = \omega_y = \omega_0, \omega_z = 2\sqrt{\omega_0^2 + (eB_0/2)^2} \). The similar integrable system without electric field has been recently proposed in the context of condensed matter physics as a model of two-electron quantum dot \([10]\).

IV. SUPERSYMMETRY

As it was mentioned in Introduction, the MICZ-system with the potential \([11]\) admits the \( \mathcal{N} = 4 \) supersymmetry, the corresponding system was constructed in \([8]\) (for earlier work devoted to supersymmetrization of (one-center) MICZ-Kepler system and related issues see \([7]\) and refs therein). In the Hamiltonian approach it is described by the canonical Poisson brackets \( \{x^n, p_m\} = \delta^n_m, \{\chi^a, \bar{\chi}_b\} = \delta^a_b \) and by the following Hamiltonian

\[
\mathcal{H}_{\text{SUSY}} = \frac{\pi^2}{2G} + \frac{\epsilon^2(f + \kappa)^2}{2G} + eB \Lambda + \left( \frac{\partial f + \partial \kappa}{2G} \right) (\chi \bar{\chi})^2.
\]  

The supercharges look as follows

\[
Q_\alpha = \frac{1}{\sqrt{G}} \left[ (\pi \chi + 2(\chi \bar{\chi}) \bar{\chi})_\alpha - e(f + \kappa) \chi_\alpha \right].
\]  

Here we use the notations \( \pi \equiv p - eA \), and

\[
B \equiv \frac{\partial \times A}{G}, \quad \Gamma \equiv \frac{\partial \log G}{2}, \quad \Lambda \equiv \chi \bar{\chi}.
\]  

One can see that \( B \) is the magnitude of the external magnetic field which emerged in the system and \( \Lambda \) defines the spin matrices. The third term in the Hamiltonian may be interpreted as Zeeman energy. To quantize the system one has to replace the odd variables \( \chi^a \) by four-dimensional Euclidean gamma-matrices \( \gamma^a + i\eta^{a+2}/\sqrt{2} \) and \( \chi_a \) by their Hermitian conjugates, and \( (\chi \bar{\chi})^2 \) by \( \gamma_5 \) matrix. The momentum will be quantized as follows \( \pi \rightarrow -i\hbar \partial - eA \).

In the Euclidean space the bosonic part of potential looks as follows

\[
\frac{\epsilon^2(f + \kappa)^2}{2G} = \sum_{I,J} \frac{\epsilon B_{IJ}}{2G r_{IJ}} + \sum_{I} \frac{\epsilon B_{I0}}{r_{I}} + \epsilon E_0 r + \mathcal{E}_0 + eB_0 \cdot r \left( \sum_{I} \frac{\epsilon B_{I0}}{2G r_{I}} \right), \quad r_I \equiv |r - a_I|.
\]
where $s_I \equiv e g_I$, $\epsilon_I \equiv \kappa s_I$, $E_0 \equiv e \kappa B_0$, $\mathcal{E}_0 \equiv e^2 \kappa^2/2$. One can obviously interpret $\epsilon_I$ as the electric charge of the $I$-th monopole, $E_0$ and $B_0$ as the magnitudes of the constant uniform electric and magnetic fields which are parallel to each other, and $\mathcal{E}_0$ as a ground state energy of the system. Thus, the constant $\kappa$ provides the background monopoles with electric charges, i.e. turns them into dyons. All these dyons have the same ratio of the electric and magnetic charges $\epsilon_I/g_I = |E_0|/|B_0| = e \kappa$ and therefore obey trivial Dirac-Schwinger-Zwanziger charge quantization condition:

$$s_{IJ} = \epsilon_I g_J - \epsilon_J g_I = 0, \quad s_{I\infty} = 0.$$  \hfill (23)

One should remind that in the general case one have $s_{IJ}/(2\pi h) \in \mathbb{Z}$. Notice, that this system can have not only repulsive Coulomb potential but an attractive one as well: the presence of the “ground state energy” term $\mathcal{E}_0$ makes the Hamiltonian positive in the whole range.

Hence, on the Euclidean space $(G = 1)$ the Hamiltonian [19] defines $\mathcal{N} = 4$ supersymmetric multi-center MICZ-Kepler system with the background dyons satisfying the condition (23). In the case of single background dyon (MICZ-Kepler system) there is no restriction on the permissible values of electric and magnetic charges.

Unfortunately, this parametric family of Hamiltonians in case of sphere and hyperboloid does not yield the corresponding MICZ-Kepler counterpart. Let us remind that the conformal flat metrics on the sphere and hyperboloid is defined by the factor $G(r) = 4(1 \pm r^2)^{-2}$ and the Coulomb potential is given by the expression $U_0 = \gamma(1 \pm r^2)/r$ (upper sign corresponds to the sphere, and lower-to the hyperboloid; we assume that sphere/hyperboloid has unit radius).

However, there is another important case, $G = (f + \kappa)$, which describes the motion of the probe particle in the background of well-separated BPS monopoles/dyons [11]. In this case the potential looks as follows $U = e^2 B r + \sum_s e^2 g_I r_I + \kappa$. Hence, similar to the Euclidean case, the electric charges are proportional to the magnetic ones. Moreover, this system admits further extension from $\mathcal{N} = 4$ to $\mathcal{N} = 8$ supersymmetry. Indeed, the constant $e$ in [19] can be considered as a momentum $p_0 \equiv e$ conjugated to some cyclic variable $\phi$ [9]. Then the system may be viewed as the free particle moving in the four-dimensional space endowed with the metric

$$ds^2 = G(r)d\theta^2 + \frac{G(r)(d\phi + A d\rho)^2}{(f + \kappa)^2},$$  \hfill (24)

where $\partial \times A = -\partial(f + \kappa)$. At $G = f + \kappa$ the metric [21] possesses the hyper-Kähler structure. For $f$ given by the expression [3] it yields the multi-center Taub-NUT metric. Thus, the initial $\mathcal{N} = 4$ supersymmetry can be extended to the $\mathcal{N} = 8$ one by adding a proper number of fermionic degrees of freedom [14].

The Euclidean and Taub-NUT systems are related to each other. Indeed, the energy surface of the Taub-NUT system $\mathcal{H}_{\text{Taub}} = \mathcal{E}_{\text{Taub}}$ can be written as follows $\pi^2/2 + e^2(f + \kappa)^2/2 - (f + \kappa)E_{\text{Taub}} = 0$. Taking into account the expression [4] we arrive at the energy level of the multi-center MICZ-Kepler system with the background dyons obeying the condition (23). It is clear that there is no such a simple correspondence between supersymmetric versions of these systems.

V. CONCLUSION

Let us emphasize the main statements of this paper:

- We presented the multi-center MICZ-Kepler system which describes the motion of the probe electrically charged particle in multi-dyon background.
- We proved the exact solvability of the two-center MICZ-Kepler and of the MICZ-Kepler-Stark systems.
- We extended the MICZ-Kepler-Stark system to an integrable model of the particle interacting with the parallel constant uniform electric and magnetic fields in the parabolic quantum dot.
- We have shown that the $\mathcal{N} = 4$ supersymmetric mechanics, constructed by Ivanov and Lechtenfeld, [8] describes (in the Euclidean space) the supersymmetric extension of multi-center MICZ-Kepler system with the same ratio of the electric and magnetic charges of background dyons.

The results listed above are more general because they embrace a wide range of system and potentials including important cases of curved configuration spaces. The direct relation of the MICZ-Kepler-Stark system with the problems arising in the physics of nanostructures can receive further development and applications especially in the quantum-mechanical context. The quantum-mechanical description of MICZ-Kepler and MICZ-Kepler-Stark systems, as is known, has much in common with the Hamilton-Jacobi formalism presented in this paper. We intend to study this issue in our further publications.

Finally, generalizing the construction considered in the paper we suggest an anzats which could describe the few-body MICZ-Kepler system in a proper way:

$$\mathcal{H} = \sum_J \left[ \frac{1}{2} (p_I - \sum_J s_{IJ} A_D(r_{IJ}))^2 + \frac{1}{2} \sum_{I,K} \frac{s_{IJ} s_{IK}}{r_{IJ} r_{IK}} + \sum_J \frac{s_{IJ}}{r_{IJ}} \right], \quad r_{IJ} \equiv r_I - r_J.$$  \hfill (25)

Obviously, this system is not exactly solvable. But, apparently, there should exist the static configurations of the particles analogous to those well-known from the celestial mechanics of three-body systems.

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