Supplementary Materials

Design and construction of a Maxwell-type induction coil for magnetic nanoparticle hyperthermia

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Reactive Power

In case of a direct current (DC) circuit with potential, \( V \) (Volts), and current, \( I \) (Amperes), voltage and current are, by definition, in phase because \( I=V/R \); and, Power, \( P \) (Watts) = \( V \times I \). For AC applications with inductive and capacitive loads in the circuit the current and voltage are out of phase, i.e. the current lags (inductive load) or current leads (capacitive load) the voltage. The analogous quantity is ‘reactive power’ in AC circuits having sinusoidal voltage and current that are out of phase. Reactive power, \( Q \) which is the maximum value of the instantaneous power absorbed by the reactive component of the load and the real power dissipated are given by

\[
Q = V_{rms} \cdot I_{rms} \cdot \sin \varphi \quad \text{Supplementary Equation 1}
\]

\[
P = V_{rms} \cdot I_{rms} \cdot \cos \varphi \quad \text{Supplementary Equation 2}
\]

where \( \varphi \) is the phase angle between the current and voltage and reactive power units are VAR. A related quantity, the apparent power, \( S \) measured in units of Volt-Amperes (VA), is the vector sum of the reactive power \( Q \) (VAR) and the real power \( P \) (W). Inductor design should minimize reactive power and power demand.

Analytical estimation of AMF electrical parameters

Analytical calculations for predicting approximate values of electrical parameters including inductance, resistance, capacitance, current and voltage were conducted by using a solenoid
approximation, where the solenoid is assumed to have equivalent dimensions as a single turn of the modified Maxwell coil (fig 1(b)). The formulae used in calculations were previously shown to be accurate in estimating electrical parameters of circular coils, as they were obtained through curve fitting of nomograms [54].

From a total of 80 kW from the power supply, 21 kW power losses occur in the applicator, with most of the power lost among busbars, cables, inverters and capacitors. Assuming input power \( P_{in}; \) coil diameter \( D_c = 20 \) cm; coil length (3 turns in parallel plus gaps assumed equivalent to 1 turn of a solenoid) \( h = 22 \) cm; applicator efficiency = \( \eta; \) frequency, \( f = 150 \) kHz; and, number of turns, \( N=1, \) \( P_{loss} = 21 \) kW = \( P_{in} \ast (1 - \eta), \) the inductance is

\[
L = 10^{\beta} N^2 D_c ,
\]

Supplementary Equation 3

where \( \beta = -6.17 + 0.67x - 0.19x^2 - 0.09x^3; \) and \( x = \log_{10} \left( \frac{D_c}{h} \right) = \log_{10} \left( \frac{20}{22} \right) = -0.04. \)

Therefore, \( \beta = -6.17 + 0.67x - 0.19x^2 - 0.09x^3 = -6.19. \) Using these values, inductance is calculated to be \( L = 10^{\beta} N^2 D_c = 0.126 \mu H. \)

The unloaded \( Q \) factor, \( Q_u, \) is defined as

\[
Q_u = AD_c\sqrt{f},
\]

Supplementary Equation 4

where \( A = 3.95 - 2.9x - 0.62x^2 + 0.7x^3 = 4.06. \) Therefore, \( Q_u = AD_c\sqrt{f} = 314.40. \) The equivalent series resistance, \( R_e = \frac{\omega L}{Q_u} = \frac{2 \ast 3.14 \ast 150000 \ast 0.126 \ast 10^{-6} - 6}{314.40} = 0.0004 \, \Omega. \) The voltage across the coil is analytically given by

\[
V = \sqrt{P_{in} \omega Q_u (1 - \eta)}
\]

Supplementary Equation 5

\[
= \sqrt{(0.21 \ast 10^5 \ast 0.126 \ast 10^{-6} \ast 2 \ast \pi \ast 0.15 \ast 10^6 \ast 314.4)} = 885.46 \, V
\]

\[
I_C = \sqrt{\frac{P_{in} Q_u (1 - \eta)}{L \omega}},
\]

Supplementary Equation 6

\[
= \sqrt{0.21 \ast 10^5 \ast 314.4 / (0.126 \ast 10^{-6} \ast 2 \pi \ast 150000)} = 7.45 \, kA \, \text{rms.}
\]
Comparing this with a basic Ampere's law calculation $H \times l \times k = I$, where $k$ is a shape factor (to account for deviation from infinite length approximation) given by $1+0.44(D_c/I) = 1+0.44(20/22) = 1.4$, $I = 35 \text{ kA/m} \times 0.22 \text{ m} \times 1.4 = 10.8 \text{ kA peak}$, or $7.63 \text{ kA rms}$. Thus, the analytical estimation of current generally agrees with $H$-$f$ specifications and finite element analysis.

Using these values, the capacitance required to achieve the resonant condition then is

$$C = \frac{1}{\omega^2 L} = 8.93 \mu F,$$

Supplementary Equation 7

**Analytical calculations comparing power deposited by iron oxide nanoparticles vs eddy currents**

One significant challenge for clinical application of magnetic hyperthermia is to achieve and sustain an effective thermal dose in the tumor, while minimizing thermal dose to the surrounding normal tissues [3-7]. Significant power deposition to tissues can occur via Joule heating by eddy currents when large volumes of tissue are exposed to high amplitude AMF. Magnetic nanoparticle heating is largely dominated by magnetic hysteresis losses, which also depend on amplitude and frequency of the magnetic field [5, 21]. For a comparison of the relative heating potential using a previously describe iron oxide nanoparticle system, we calculated total power deposited in a mouse model vs human deep tumor in the torso (e.g. liver). For this comparison, we used the previously reported specific loss power (SLP) data for JHU iron oxide nanoparticles (IONPs) [21].

Total power deposited by JHU IONPs in representative tumor volumes were calculated and compared with total power deposited by eddy currents in cylindrical volume approximations of mice and humans, respectively. Power deposited by IONPs was calculated using reported experimental values of specific loss power (SLP) and assuming fixed tumor volumes. Equation 3 was used to estimate total power deposited by eddy currents in cylindrical approximations for
mice and humans. For a cylindrical volume with radius $r$ and height $h$, the total power deposited is given by:

$$P_{tot} (W) = \int_0^r P(r) 2\pi h \, r \, dr = \int_0^r \sigma \times (\pi \mu_0)^2 \times (Hf r)^2 \times 2\pi h \, r \, dr$$

Supplementary Equation 8

$$P_{tot} (W) = \sigma(\pi \mu_0)^2 \times (Hf)^2 \times \pi h r^4/2.$$  
Supplementary Equation 9

For these calculations, the following assumptions were made: $\sigma = 0.1 \, \text{S/m}$ [55], $\mu_0 = 1.25 \times 10^{-6} \, \text{H/m}$, $f = 150 \, \text{kHz}$. For mice, tumor volume = 0.2 ml (mean volume for xenograft tumor studies) [6, 7], Fe concentration in tumor = 5 mg Fe/ml, torso radius = 1 cm, $h = 5$ cm. For humans, tumor volume = 23 ml (diameter = 3.5 cm) [58], Fe concentration in tumor = 50 mg Fe/ml, torso radius = 15 cm, $h = 5$ cm target region length. The Atkinson-Brezovich field-frequency criteria [22] was applied to mark clinically acceptable levels of AMF against the total non-specific power deposited in the tissue.

Analytical calculations comparing eddy current power deposited in a cylindrical tissue load by MFH300F coil vs JHU modified Maxwell coil.

Here, through analytical calculations, we directly compare power deposited by induced eddy currents in a 30 cm diameter cylindrical tissue load placed in the JHU Maxwell coil vs the MFH300F coil [38]. The following assumptions are made about the cylindrical tissue load, tumor and AMF parameters:

1) Cylindrical tissue load radius, $r = 15$ cm

2) Spherical tumor volume, $V_{tum} = 2 \, \text{cm}^3$
3) Tumor location is at center of symmetry of the cylindrical tissue section, on the z-axis (r = 0).

4) Target magnetic field at tumor location, $H_{\text{center}} = 4 \text{ kA/m}$ (Atkinson-Brezovich limit for human torso [22] at 150 kHz).

5) AMF frequency, $f = 150 \text{ kHz}$.

6) Electrical conductivity for tissue, $\sigma = 0.1 \text{ S/m}$.

7) Magnetic permeability, $\mu_0 = 1.25 \times 10^{-6} \text{ H/m}$.

Power deposited by eddy currents is given by

$$P_{\text{eddy}}(r) = \sigma.(\pi . \mu_0)^2.(H. f. r)^2 \text{ W/m}^3 \quad [21, 22],$$

where $r$ is the radial distance from central z-axis.

**MFH300 coil**

Based on the MFH300F prototype description in Wust et al [38], the magnetic field strength, $H$ (in A/m), in the cylindrical load can be calculated by the following expression:

$$H = (a * |COD|^d + (b * AD) + c) * CC$$

Supplementary Equation 11

Where,

a, b, c and d are parameters fitted to experimentally measured data in the MFH300; the reported values are $a = 0.001$, $b = -0.12$, $c = 60$ and $d = 2$.

$AD = $ aperture distance in mm.

$CC = $ coil current in amperes (A).

$COD = $ center offset distance in mm, i.e., distance between measuring point and center of aperture.

For $AD = 300 \text{ mm}$ and $CC = 150 \text{ A}$, $H = 4 \text{ kA/m}$ (target) is measured at the center (tumor location).
To calculate the field at the cylindrical tissue surface, i.e., at \( r = 15 \) cm, corresponding COD = 150 mm.

Using Supplementary Equation 11, \( H \) at 150 mm COD is calculated to be \( H_{\text{edge}} = 6.975 \) kA/m.

Thus, to attain \( H_{\text{center}} = 4 \) kA/m, \( H_{\text{edge}} = 6.975 \) kA/m.

Using Supplementary Equation 10,

\[ P_{\text{eddy}} \text{ (W/ml)} = 0.0379 \text{ W/ml}. \]

**Modified Maxwell (JHU) Coil.**

For the human scaled modified Maxwell, a uniform field of 4 kA/m was used (Fig 5). Thus,

when \( H_{\text{center}} = 4 \) kA/m, \( H_{\text{edge}} = 4 \) kA/m.

Therefore, using Supplementary Equation 10,

\[ P_{\text{eddy}} \text{ (W/ml)} = 0.0124 \text{ W/ml}. \]

Note that in the Modified Maxwell coil, power (W/ml) deposited by eddy currents is 32.8% of non-specific power deposited by MFH300F (approximately one-third).