Hybrid Dark Sector: Locked Quintessence and Dark Matter

Minos Axenides* and Konstantinos Dimopoulos*

*Institute of Nuclear Physics, National Center for Scientific Research ‘Demokritos’,
Agia Paraskevi Attikis, Athens 153 10, Greece

Abstract

We present a unified model of dark matter and dark energy. The dark matter field is a modulus corresponding to a flat direction of supersymmetry, which couples, in a hybrid type potential, with the dark energy field. The latter is a light scalar, whose direction is stabilized by non-renormalizable terms. This quintessence field is kept ‘locked’ on top of a false vacuum due to the coupling with the oscillating dark matter field. It is shown that the model can satisfy the observations when we consider low-scale gauge-mediated supersymmetry breaking. The necessary initial conditions are naturally attained by the action of supergravity corrections on the potential, in the period following the end of primordial inflation.

1 Introduction

In the last few years cosmology experienced a dramatic influx of observational data [1, 2, 3, 4]. Analysis of these observations have resulted in the emergence of the so-called ‘concordance’ model of cosmology. According to the data we live in a spatially flat Universe, whose content is comprised of predominantly unknown substances, not accounted for by the standard model of particle physics. In particular, roughly 1/3 of the energy density of our Universe behaves as pressureless matter (dust), with little or no interactions with the usual baryonic matter. These weakly interacting massive particles (WIMPs) do not correspond to the luminous matter of galaxies and, hence, have been named dark matter. The remaining 2/3 of the Universe content at present, is attributed to an even more exotic substance, whose properties are such that affect the global geometry of the Universe and cause the observed current accelerated expansion of spacetime. In analogy with dark matter, this substance has been named dark energy. Therefore, despite the substantial progress of particle cosmology in the last decade, cosmologists are forced to admit that the bulk of the content of the Universe corresponds to an unknown dark sector, on whose origin and theoretical justification we can only speculate.

Modern particle physics indeed offers a number of possibilities for the explanation of this dark sector. In particular, supersymmetric theories include a zoo of unobserved particles corresponding to the superpartners of standard model particles. This so-called hidden sector may soon be accessible to collider experiments. Other candidates are offered by modified gravity theories (e.g. the Brans-Dicke scalar of scalar tensor gravity), string theories (e.g. string axions/moduli, Kaluza-Klein particles) and theories of large extra dimensions (e.g. the radion). Therefore, it seems that particle physics has available options capable of addressing the problem of the dark sector.

Since the dark matter issue has been present for some time, a number of successful candidate WIMPs exist to explain it, the most prominent of which are the lightest supersymmetric particle (e.g. the neutralino) and the axion (i.e. the phase-field of the Peccei-Quinn field, used to solve the strong CP-problem). On the other hand, the problem of dark energy is quite recent and more difficult to address, because the properties of dark energy are quite bizarre and may even threaten some of our ‘fundamental prejudices’ such as the vanishing of the cosmological constant or even the dominant energy condition (for a review see [5]).

The simplest form of dark energy is a non-zero cosmological constant $\Lambda$. Phenomenologically, this is the most appealing choice, since it provides a very nice fit to the data (the so-called $\Lambda$CDM model, whose minimal form is the famous ‘vanilla model’). However, the value of the cosmological constant has to be fine tuned to the incredible level of $\Lambda \sim 10^{-123} M_P^2$, compared to its natural value, given by the Planck mass $M_P$. Moreover, a constant non-zero vacuum density inevitably leads to eternal accelerated expansion. This results in the presence of future causal horizons, which inhibit the construction of the S-matrix in string theory and are, therefore, most undesirable [6].
For these reasons theorists have attempted to formulate alternative solutions to the dark energy problem, while keeping Λ = 0 as originally conjectured. The most celebrated such idea is the introduction of the so-called quintessence field [7]; the fifth element after cold dark matter (WIMPs), hot dark matter (neutrinos), baryons and photons. Quintessence is a light scalar field Q, which has not yet reached the minimum of its potential and, therefore, is responsible for the presence of a non-vanishing potential density V₀ today. This density currently dominates the Universe, giving rise to an effective cosmological constant Λₑff = 8πGV₀, which causes the observed accelerated expansion. Eventually, the quintessence field will reach the minimum of its potential (corresponding to the true vacuum) ending, thereby, the accelerated expansion. Hence, quintessence dispenses with the future horizon problem of ΛCDM.

However, despite its advantages, the quintessence idea suffers from certain generic problems [8]. For example, in order to achieve the correct value of V₀, one usually needs to fine-tune accordingly the quintessence potential. Also, in fairly general grounds it can be shown that the value of quintessence at present is Q ≈ M_P (if originally at zero) with a tiny effective mass m_Q ≈ 10^{-33}eV. In the context of supergravity theories such a light field is difficult to understand because the flatness of its potential is lifted by excessive supergravity corrections or due to the action of non-renormalizable terms, which become important at displacements of order M_P. Finally, quintessence introduces a second tuning problem, that of its initial conditions.

In this paper we attempt to address the dark sector problem in a single theoretical framework. Other such attempts can be found in Ref. [9]. We assume that the dark matter particle is a modulus Φ, corresponding to a flat direction of supersymmetry. The modulus field is undergoing coherent oscillations, which are equivalent to a collection of massive Φ-particles, that are the required WIMPs. Coupled to the dark matter is another scalar field ϕ. This can be thought of as our quintessence field and it corresponds to a flat direction lifted by non-renormalizable terms. Even though the ϕ-field is a light scalar, it is much more massive than the m_Q mentioned above, so as not to be in danger from supergravity corrections to its potential. Our quintessence field is coupled to our dark matter in a hybrid manner, which is quite natural in the context of a supersymmetric theory. Due to this coupling, the oscillating Φ, keeps ϕ ‘locked’ on top of a potential hill, giving rise to the desired dark energy. When the amplitude of the Φ-oscillations decreases enough, the dark energy dominates the Universe, causing the observed accelerated expansion. Much later, when the oscillation amplitude is reduced even further, the ‘locked’ quintessence field is released and rolls down to its minimum. Then, the system reaches the true vacuum and accelerated expansion ceases. Our model accounts successfully for the observations using natural mass-scales (corresponding to low-scale gauge-mediated supersymmetry breaking). In order to explain the required initial conditions we explore in detail the history of our system during the early Universe, when the supergravity corrections to the scalar potential are essential.

Our paper is organized as follows. In Sec. 2 we present and analyze the dynamics of our model, while we also determine the value of the model parameters. In Sec. 3 we demonstrate that the required initial conditions for our dark matter field Φ may be naturally attained due to the action of supergravity corrections on the scalar potential. We also investigate the disastrous possibility of the decay of the oscillating dark matter condensate into quintessence quanta. In Sec. 4 we show that the supergravity corrections may also ensure the locking of the quintessence field ϕ. Additionally, we elaborate more on the value of the tachyonic mass of ϕ and its vacuum expectation value. Finally, in Sec. 5 we discuss our results and present our conclusions.

We assume a spatially flat Universe, according to the WMAP observations [1]. Throughout our paper we use natural units such that h = c = 1 and Newton’s gravitational constant is 8πG = m_P^{-2}, where m_P = 2.4 \times 10^{18}\text{GeV} is the reduced Planck mass.

2 The model

Consider two real scalar fields Φ and ϕ with a hybrid type of potential of the form

\[ V(\Phi, \phi) = \frac{1}{2} m_Φ^2 \Phi^2 + \frac{1}{2} \lambda \Phi^2 \phi^2 + \frac{1}{4} \alpha (\phi^2 - M^2)^2, \tag{1} \]
where $\lambda \lesssim 1$. From the above we see that the tachyonic mass of $\phi$ is given by

$$m_\phi = \sqrt{\alpha} M.$$  \hspace{1cm} (2)

The above potential has global minima at $(\Phi, \phi) = (0, \pm M)$ and an unstable saddle point at $(\Phi, \phi) = (0, 0)$. Now, since the effective mass–squared of $\phi$ is

$$(m_\phi^{\text{eff}})^2 = \lambda \Phi^2 - \alpha M^2,$$  \hspace{1cm} (3)

if $\Phi > \Phi_c$ then $\phi$ is driven to zero, where

$$\Phi_c \equiv \sqrt{\frac{\alpha}{\lambda}} M.$$  \hspace{1cm} (4)

Suppose that originally the system lies in the regime, where, $\Phi \gg \Phi_c$ and $\phi \simeq 0$. With such initial conditions the effective potential for $\Phi$ becomes quadratic:

$$V(\Phi, \phi = 0) = \frac{1}{2} m_\phi^2 \Phi^2 + V_0.$$  \hspace{1cm} (5)

Hence, when $\phi$ remains at the origin, $\Phi$ oscillates on top of a false vacuum with density

$$V_0 = \frac{1}{4} \alpha M^4.$$  \hspace{1cm} (6)

The oscillation frequency is $\omega_\Phi \sim m_\phi$ and the time interval $(\Delta t)_s$ that the field spends on top of the saddle point $(\Delta \Phi \leq \Phi_c)$ is

$$\omega_\Phi \Delta t \sim \frac{\Delta \Phi}{\Phi} \Rightarrow (\Delta t)_s \sim \frac{\Phi_c}{m_\phi \Phi},$$  \hspace{1cm} (7)

where $\Phi$ is the amplitude of the oscillations. Originally this amplitude may be quite large but the expansion of the Universe dilutes the energy of the oscillations and, therefore, $\Phi$ decreases, which means that $(\Delta t)_s$ grows.

However, as long as the system spends most of this time away from the saddle and until $(\Delta t)_s$ becomes large enough to be comparable to the inverse of the tachyonic mass of $\phi$, the latter has no time to roll away from the saddle [10, 11]. Hence, the oscillations of $\Phi$ on top of the saddle can, in principle, continue until the amplitude decreases down to

$$\Phi_s \sim \frac{\Phi_c m_\phi}{m_\phi} \sim \frac{\alpha M^2}{\sqrt{\lambda} m_\phi},$$  \hspace{1cm} (8)

at which point $\phi$ has to depart from the origin and roll down toward its vacuum expectation value (VEV) $M$. However, the roll down of $\phi$ can occur earlier if $\Phi_c > \Phi_s$, even though the period of oscillation is smaller than $m_\phi^{-1}$. Indeed, when $\Phi_s < \Phi < \Phi_c$, $\phi \simeq 0$ is not possible because, were it otherwise, it would mean that the field would have had to remain on top of the saddle for the entire period of oscillation. Hence, $\phi$ departs from the origin at $\Phi_{\text{end}}$, where

$$\Phi_{\text{end}} \equiv \max \{\Phi_c, \Phi_s\}.$$  \hspace{1cm} (9)

From Eqs. (4) and (8) we find that $\Phi_{\text{end}}$ is decided by the relative magnitude of the masses of the scalar fields because

$$\frac{\Phi_c}{\Phi_s} \sim \frac{m_\phi}{m_\phi}.$$  \hspace{1cm} (10)

During the oscillations the density of the oscillating $\Phi$ is

$$\rho_\Phi = \frac{1}{2} \dot{\Phi}^2 + \frac{1}{2} m_\phi^2 \Phi^2 \simeq \frac{1}{2} m_\phi^2 \Phi^2,$$  \hspace{1cm} (11)
where the dot denotes derivative with respect to the cosmic time $t$. Comparing this with the overall potential density given in Eq. (5) we see that, the overall density is dominated by the false vacuum density given in Eq. (6), when the oscillation amplitude is smaller than,

$$\Phi_\Lambda \sim \frac{\sqrt{\alpha} M^2}{m_\Phi} \sim \left(\frac{m_\Phi}{m_\Phi}\right) M .$$

(12)

The above model can be used to account for both the dark matter and the dark energy in the Universe. Provided the initial conditions for the two fields are appropriate, it is possible that the oscillating field $\Phi$ constitutes the dark matter, whereas the field $\phi$ is responsible for eliminating the dark energy in the future, so as to avoid eternal acceleration and future horizons. The dark matter field $\Phi$ oscillates on top of the false vacuum $V_0$ in the same manner as in 'locked' inflation [10, 11]. The false vacuum is not felt until today, when the accelerated expansion begins. Eventually, at some moment in the future, the amplitude of the $\Phi$–oscillation reaches $\Phi_{\text{end}}$ and $\phi$ rolls away from the origin terminating the accelerated expansion.

We want our model to explain the dark energy responsible for the currently observed accelerated expansion of the Universe. Hence, the false vacuum density $V_0$ of our model should be comparable to the density $\rho_0$ of the Universe at present

$$V_0 \sim \rho_0 \sim 10^{-120} m_P^4 \sim (10^{-3} \text{eV})^4 .$$

(13)

In view of Eq. (6), this implies the condition

$$M \sim \alpha^{-1/4} 10^{-30} m_P .$$

(14)

We also want our model to explain the dark matter, by means of the oscillating scalar field $\Phi$. Indeed, it is well known that a scalar field oscillating in a quadratic potential has the equation of state of pressureless matter [12] (it corresponds to a collection of massive $\Phi$–particles) and, therefore, $\Phi$ can account for the dark matter necessary to explain the observations. For this, the oscillating $\Phi$ has to satisfy certain requirements. One of these is the obvious requirement that $\Phi$ should not have decayed until today. This means that the decay rate of $\Phi$ should satisfy the condition

$$\Gamma_\Phi < H_0 ,$$

(15)

where $H_0 \sim \sqrt{\rho_0}/m_P$ is the Hubble parameter at present. Using that $\Gamma_\Phi \sim g^2_\Phi m_\Phi$ we find the bound

$$m_\Phi \leq 10^{-20} m_P ,$$

(16)

where we used that the coupling $g_\Phi$ of $\Phi$ with its decay products lies in the range $\frac{m_\Phi}{m_P} \leq g_\Phi \leq 1$, with the lower bound corresponding to the gravitational decay of $\Phi$, for which $\Gamma_\Phi \sim m_\Phi^3/m_P^2$.

From the above bound we see that we require $\Phi$ to be a rather light field with mass $\lesssim 10$ MeV. We choose, therefore, to use a modulus field, corresponding to a flat direction of supersymmetry, whose mass is estimated as

$$m_\Phi \sim \frac{M_S^2}{m_P} ,$$

(17)

where $M_S$ is the supersymmetry breaking scale, ranging between $m_{3/2} \leq M_S \leq \sqrt{m_P m_{3/2}}$, where $m_{3/2} \sim 1$ TeV is the electroweak scale (gravitino mass) and the upper bound corresponds to gravity mediated supersymmetry breaking while the lower bound corresponds to gauge mediated supersymmetry breaking, which can give $M_S$ as low as (few) x TeV. Eqs. (16) and (17) suggest

$$1 \text{ TeV} \leq M_S \leq 10^{-10} m_P .$$

(18)

If $\phi$ were a modulus too then the natural value of $\alpha$ would have been

$$\alpha \sim \left(\frac{M_S}{m_P}\right)^4 ,$$

(19)
with $M \sim m_P$. The above, however, in view of Eq. (14), results in the condition

$$M \sim 10^{-30} \frac{m_P^2}{M_S},$$

which, combined with the range in Eq. (18), results in the following range for the vacuum expectation value (VEV) of $\phi$

$$10 \text{ MeV} \leq M \leq 1 \text{ TeV}.$$  

(21)

Thus, we see that $M \ll m_P$ in contrast to expectations. However, there are ways to reduce the VEV of $\phi$, provided the tachyonic mass $m_\phi$ remains roughly unmodified. Hence, in the following we retain the value of $\alpha$ shown in Eq. (19). We discuss the small VEV of $\phi$ in Sec. 4.2. In view of Eqs. (18) and (21) we make the following choice

$$M \sim M_S \sim 1 \text{ TeV}.$$  

(22)

With this choice, the number of parameters of the model in Eq. (1) is minimized to two natural mass scales: $m_P$ and $M_S \sim m_{3/2}$ and a coupling $\lambda \leq 1$.

Before concluding this section we notice that, from Eqs. (3), (19) and (22) we find

$$m_\phi \sim \frac{M_S^3}{m_P^2},$$

which, in view of Eqs. (9) and (10), gives

$$\frac{\Phi_c}{\Phi_s} \sim \frac{m_P}{M_S} \gg 1 \quad \Rightarrow \quad \Phi_{\text{end}} \sim \Phi_c.$$  

(24)

Now, since we need the oscillation of $\Phi$ on top of the false vacuum to continue until today, when $\Phi \sim \Phi_A$, we require

$$\frac{\Phi_A}{\Phi_{\text{end}}} \sim \sqrt{\lambda} \left(\frac{m_P}{M_S}\right) > 1 \quad \Rightarrow \quad \lambda > 10^{-30}$$

(25)

where we also used Eqs. (4) and (12).
3 Dark matter requirements

3.1 The value of \( \Phi \) at the onset of the oscillations

Another important requirement for \( \Phi \), if the latter is to account for the dark matter in the Universe, is that it has the correct energy density. This requirement is determined by the initial amplitude \( \Phi_{\text{osc}} \) of the oscillations of the field. When the oscillations begin we have [cf. Eq. (11)]

\[
\rho_{\Phi}^{\text{osc}} = \frac{1}{2} m_{\Phi} \Phi_{\text{osc}}^2,
\]

where the subscript ‘osc’ denotes the onset of the \( \Phi \)-oscillations. According to the Friedman equation we have \( \rho = 3H^2m_{\phi}^2 \), where \( H(t) \equiv \dot{a}/a \) is the Hubble parameter and \( a(t) \) is the scale factor, parameterizing the Universe expansion. Hence, since the oscillations begin when \( H_{\text{osc}} \sim m_{\phi} \), we find

\[
\rho_{\Phi} \underset{\text{osc}}{\bigg|} \sim \left( \frac{\Phi_{\text{osc}}}{m_{\phi}} \right)^2.
\]

Now, using Eq. (17) and also that, during the radiation era, \( \rho \sim T^4 \) (with \( T \) being the temperature) we obtain

\[
\frac{m_{\phi}}{H_{\text{eq}}} \sim \left( \frac{M_{\text{S}}}{T_{\text{eq}}} \right)^2 \gg 1
\]

where ‘eq’ denotes the time \( t_{\text{eq}} \) of equal matter and radiation densities, at which \( T_{\text{eq}} \sim 1 \text{ eV} \). Hence, we see that \( H_{\text{osc}} \gg H_{\text{eq}} \), which means that the oscillations begin during the radiation dominated period. During this period the density of the Universe scales as \( \rho \propto a^{-4} \), while the density of the oscillating scalar field scales as \( \rho_{\Phi} \propto a^{-3} \) [12]. Hence we have \( \rho_{\Phi}/\rho \propto a \sim H^{-1/2} \). Therefore, the density of the oscillating scalar field eventually dominates the Universe. Since, we want \( \Phi \) to be the dark matter, we require that its density dominates at \( t_{\text{eq}} \). Consequently, Eq. (27) suggests

\[
\rho_{\Phi} \underset{\text{osc}}{\bigg|} \sim \sqrt{\frac{H_{\text{eq}}}{m_{\phi}}} \Rightarrow \Phi_{\text{osc}} \sim \left( \frac{T_{\text{eq}}^2}{m_{\phi}^2 m_{\phi}} \right)^{1/4} m_{\phi} \sim \sqrt{\frac{T_{\text{eq}}}{M_{\text{S}}}} m_{\phi},
\]

where we also used Eq. (17). Putting the numbers in the above we find \( \Phi_{\text{osc}} \sim 10^{-6} m_{\phi} \). This is substantially smaller than the natural expectation for a modulus, which corresponds to an original misalignment (i.e. displacement from its VEV) of order \( m_{\phi} \). However, below we attempt to explain this reduced misalignment by means of supergravity corrections. These corrections are expected to lift the flatness of the \( \Phi \)-direction and enable \( \Phi \) to begin rolling down long before \( H \sim m_{\phi} \).

3.2 The effect of supergravity corrections

Supergravity corrections to the potential generate an effective mass term proportional to the Hubble parameter [13]. Thus, the effective potential in Eq. (5) becomes

\[
V(\Phi) = \frac{1}{2} [\pm cH^2(t) + m_{\Phi}^2] \Phi^2,
\]

where \( c \) is a positive constant and we ignored the false vacuum contribution \( V_0 \), which is negligible at times much earlier than the present time.

We assume that, in the early stages of its evolution, the Universe underwent a period of cosmic inflation. During and after inflation, until reheating, the Universe is dominated by the density of the inflaton field. The minimum of \( V(\Phi) \), in general, is expected to be shifted by \( \Delta \Phi \sim m_{\phi} \) at the end of inflation. Hence, at the end of inflation we expect \( \Phi_{\text{inf}} \sim m_{\phi} \). After the end of inflation and until reheating \( V(\Phi) \) is given by Eq. (30) with \( c \sim O(1) \) [13]. However, after reheating, when the Universe becomes radiation dominated, one expects \( c \to 0 \) and the supergravity correction
vanishes\textsuperscript{1} \cite{14}. Now, the reheating temperature is \( T_{\text{reh}} \sim \sqrt{T_{\text{inf}} m_P} \), where \( T_{\text{inf}} \) is the decay rate of the inflaton field. Using this and Eq. (17) it is easy to show that

\[ H_{\text{reh}} \sim \Gamma_{\text{inf}} \geq m_\Phi \quad \Leftrightarrow \quad T_{\text{reh}} \geq M_S \sim 1 \text{ TeV}, \tag{31} \]

where the subscript ‘reh’ denotes the time of reheating. Typically, baryogenesis mechanisms require \( T_{\text{reh}} > 1 \text{ TeV} \). Therefore, reheating occurs before the onset of the quadratic oscillations, which we discussed in the previous subsection. As a result, after reheating, the motion of the field is overdamped by the excessive friction of a large Hubble parameter (compared to its mass) and so \( \Phi \) freezes until \( H \) is reduced enough for the quadratic oscillations to commence.

To understand this consider the Klein-Gordon equation of motion of the field, which, in view of Eq. (30), takes the form

\[ \ddot{\Phi} + 3H \dot{\Phi} + (\pm cH^2 + m_\Phi^2)\Phi = 0. \tag{32} \]

If, after reheating, \( \Phi \) is dominated by its kinetic density \( \rho_{\text{kin}} \equiv \frac{1}{2} \dot{\Phi}^2 \) then only the first two terms in the left-hand-side (LHS) of Eq. (32) are important, which results in \( \rho_{\text{kin}} \propto a^{-6} \). Thus, the kinetic density is soon depleted away and the field becomes potential density dominated.\textsuperscript{2} When this happens the first term in the LHS of Eq. (32) becomes negligible. Then, considering that \( c = 0 \) after reheating, it is easy to find the solution

\[ \Phi \simeq \Phi_{\text{reh}} \exp \left[ -\frac{1}{12} \left( \frac{m_\Phi}{H} \right)^2 \left( 1 - \frac{H^2}{H_{\text{reh}}^2} \right) \right]. \tag{33} \]

From the above it is evident that, in the interval \( m_\Phi < H < H_{\text{reh}} \), the field remains frozen. Consequently,

\[ \Phi_{\text{osc}} \sim \Phi_{\text{reh}}. \tag{34} \]

Hence, the required value of \( \Phi_{\text{osc}} \), given in Eq. (29), may be explained by the evolution of \( \Phi \) during the period after the end of inflation until reheating. Below we discuss this evolution assuming that the sign of the supergravity correction is positive.

The evolution of a scalar field under the influence of supergravity corrections has been thoroughly studied in Ref. [15], where it was found that, during a matter dominated period (such as the one after the end of inflation and before reheating, when the Universe is dominated by massive inflaton particles), the value of the field is given by the following equations:

For \( c > 9/16 \):

\[ \Phi = \Phi_{\text{inf}} \sqrt{\frac{H}{H_{\text{inf}}}} \cos \left( \sqrt{\frac{16c}{9}} - 1 \ln \sqrt{\frac{H}{H_{\text{inf}}}} \right) - \frac{1}{\sqrt{16c/9 - 1}} \sin \left( \sqrt{\frac{16c}{9}} - 1 \ln \sqrt{\frac{H}{H_{\text{inf}}}} \right) \]

For \( c = 9/16 \):

\[ \Phi = \Phi_{\text{inf}} \sqrt{\frac{H}{H_{\text{inf}}}} \left( 1 + \ln \sqrt{\frac{H_{\text{inf}}}{H}} \right) \]

\textsuperscript{1}The supergravity corrections during radiation domination are due to Kähler couplings of the scalar field with the thermal bath dominating the density of the Universe. For example, consider a scalar field \( \Psi \), which is part of the thermal bath. Then the supergravity corrections arise through the kinetic density due to terms in the Kähler potential of the form: \( K \sim \Psi^2 \Phi^2/m_P^2 \). The kinetic term \( \mathcal{L}_{\text{kin}} \equiv (\partial_m \partial_n K)\partial_m \phi_n \partial^m \phi_n \) includes a contribution of the form \( \delta \mathcal{L}_{\text{kin}} \sim (\Phi/m_P)^2 \partial_m \phi_n \partial^m \phi_n \). Now, naively one expects \( (\partial \Phi)^2 \sim \rho_\Phi \sim T^4 \), because \( \Phi \) is part of the thermal bath. Since, \( T^4 \sim \rho \sim (Hm_P)^4 \), we find that \( \delta \mathcal{L}_{\text{kin}} \sim H^2 \Phi^2 \), i.e. supergravity corrections seem to result again in an effective mass of order \( H \). However, a more careful examination of the above shows that this is not so. Indeed, \( \partial_m \Psi \partial^m \Psi = \Psi^2 - (\nabla \Psi)^2 = 0 \), because \( \Psi \) is a relativistic (effectively massless) field, whose modes correspond to plane waves of the form: \( \Psi_k = \Psi_k e^{\pm ik \cdot x} \). Similar results are obtained with fermions. Hence, the supergravity correction vanishes in the radiation dominated period. KD wishes to thank T. Moroi for clarifying this point.

\textsuperscript{2}We make the conservative assumption that the value of \( \Phi \) is not dramatically reduced until its density becomes potential dominated. The solution of Eq. (32) is: \( \Phi \simeq \Phi_{\text{reh}} - \sqrt{\rho_{\text{kin}}^{\text{reh}}/\rho_{\text{reh}}} \left( 1 - \sqrt{H/H_{\text{reh}}} \right) \sqrt{6} m_P \). Hence, \( \Phi \simeq \Phi_{\text{reh}} \) if \( \rho_{\text{kin}}^{\text{reh}} \ll \rho_{\text{reh}} \).
For $c < 9/16$:

$$\Phi = \Phi_{\text{inf}} \sqrt{\frac{H}{H_{\text{inf}}}} \left[ \left( 1 + \frac{1}{\sqrt{1 - 16c/9}} \right) \left( \frac{H_{\text{inf}}}{H} \right)^{\frac{1}{2} \sqrt{1 - 16c/9}} \right. \\
\left. + \left( 1 - \frac{1}{\sqrt{1 - 16c/9}} \right) \left( \frac{H_{\text{inf}}}{H} \right)^{-\frac{1}{2} \sqrt{1 - 16c/9}} \right], \quad (37)$$

which are solutions of Eq. (32) for $m_\Phi^2 \ll cH^2$. From the above we see that, if $\sqrt{c} \leq \frac{3}{4}$, then the field gently rolls toward the origin. On the other hand, if $\sqrt{c} > \frac{3}{4}$, then the field oscillates with decreasing amplitude and frequency $\propto \log \sqrt{H}$. In all cases, the value (or the amplitude) of the field scales as

$$\Phi = \Phi_{\text{inf}} \left( \frac{H}{H_{\text{inf}}} \right)^{\frac{1}{2} \left( 1 - \sqrt{1 - 16\hat{c}/9} \right)}, \quad (38)$$

where $\hat{c} \equiv \min\{c, 9/16\}$. \quad (39)

Using Eqs. (29) and (38) [cf. also, Eq. (34)] one finds

$$\sqrt{T_{\text{eq}}} \sim \frac{\Phi_{\text{osc}}}{M_S} \sim \left( \frac{\Gamma_{\text{inf}}}{H_{\text{inf}}} \right)^{\frac{1}{2} \left( 1 - \sqrt{1 - 16\hat{c}/9} \right)}, \quad (40)$$

where $H_{\text{inf}}$ is the Hubble parameter at the end of inflation and we considered $\Phi_{\text{inf}} \sim m_P$. From the above, it is easy to obtain

$$\frac{T_{\text{reh}}}{V_{\text{inf}}^{1/4}} \sim 10^{-6} \left( 1 - \sqrt{1 - 16\hat{c}/9} \right)^{-1}, \quad (41)$$

where we used that $\sqrt{T_{\text{eq}}/M_S} \sim 10^{-6}$ and also that $V_{\text{inf}}^{1/4} \sim H_{\text{inf}} m_P$ and $T_{\text{reh}} \sim \sqrt{\Gamma_{\text{inf}} m_P}$. The amplitude of the density perturbations (given by the COBE satellite observations), if they are due to the amplification of the quantum fluctuations of the inflaton field, determines the energy scale of inflation as follows [16]

$$V_{\text{inf}}^{1/4} = 0.027 \epsilon^{1/4} m_P, \quad (42)$$

where $\epsilon$ is one of the, so-called, slow-roll parameters, associated with the rate of change of $H$ during inflation. Typically, $\epsilon \sim 1/N$ where $N \simeq 60$ is the number of the remaining e-foldings of inflation when the cosmological scales exit the causal horizon. Hence, we see that the energy scale of inflation is determined by the COBE observations to be given by the energy of grand unification: $V_{\text{inf}}^{1/4} \sim 10^{16}$GeV.

Inserting this value into Eq. (41) we find that, for $\sqrt{c} > \frac{3}{4}$, we obtain $T_{\text{reh}} \sim 10^{10}$GeV. A reheating temperature this high is in danger of violating the well-known gravitino constraint, which requires $T_{\text{reh}} \leq 10^9$GeV. Enforcing this constraint we find the following allowed range for $c$

$$1 \text{ TeV} \leq T_{\text{reh}} \leq 10^9 \text{GeV} \quad \Leftrightarrow \quad \frac{2}{5} \leq c \leq \frac{5}{9}. \quad (43)$$

This is a rather narrow range for the value of $c$, albeit quite realistic. However, this does not necessarily imply any tuning. Indeed, different values of $c$ result in different values of $\Phi_{\text{osc}}$, which, with $\Phi$ being the dark matter, would give different values of $T_{\text{eq}}$. The latter is determined observationally and has no fundamental origin. Hence, one can view the above result as an observational determination of $c$.

Still, we can expand the allowed range of $c$ even above 9/16 if we break loose from the COBE condition in Eq. (42). This is possible if we consider alternative scenarios for structure formation. For example, if we assume that the primordial spectrum of density perturbations is due to the
amplification of the quantum fluctuations of some curvaton field other than the inflaton, as suggested in Ref. [17], then the COBE constraint on $V_{inf}^{1/4}$ becomes relaxed into an upper bound [18]. Assuming $c \geq 9/16$, Eqs. (39) and (41) give

$$T_{reh} \sim 10^{-6} V_{inf}^{1/4}.$$  \hspace{1cm} (44)

Hence, for the allowed range of $T_{reh}$ we find

$$1 \text{ TeV} \leq T_{reh} \leq 10^{9} \text{GeV} \quad \Leftrightarrow \quad 10^{9} \text{GeV} \leq V_{inf}^{1/4} \leq 10^{15} \text{GeV}. \hspace{1cm} (45)$$

From the above it is clear that the necessary initial conditions for the quadratic oscillations of $\Phi$, in order for the latter to be the dark matter particle, can be naturally attained by considering the action of supergravity corrections on $V(\Phi)$ after the end of inflation and until reheating.

### 3.3 Avoiding the decay of $\Phi$ into $\phi$-particles

One final requirement for our dark matter field $\Phi$ is that it should not decay into $\phi$-particles until the present time. Indeed, the coupling between the two fields suggests that, in principle, such a decay is possible. Here we find the appropriate constraint on the coupling constant $\lambda$, which ensures that such a decay does not take place.

Let us consider first the perturbative decay of the $\Phi$ condensate. The decay rate for the decay: $\Phi \rightarrow \phi \phi$ is estimated as

$$\Gamma_{\Phi \rightarrow \phi\phi} \simeq \frac{\lambda^2 \bar{\Phi}^2}{8\pi m_{\Phi}}. \hspace{1cm} (46)$$

In order to avoid the decay we need to have $\Gamma_{\Phi \rightarrow \phi\phi} < H$ until today. Now, since $\bar{\Phi} \propto a^{-3/2}$, it is easy to find that

$$\frac{\Gamma_{\Phi \rightarrow \phi\phi}}{H} \propto H^{1+w}, \hspace{1cm} (47)$$

where $w$ is the barotropic parameter corresponding to the equation of state of the dominant component of the content of the Universe ($w = 0 \{w = \frac{1}{3}\}$ for the matter (radiation) dominated epoch). Since $w \leq 1$, we see that the constraint on $\Gamma_{\Phi \rightarrow \phi\phi}$ relaxes with time. Hence, the tightest constraint corresponds to the earliest time when the decay $\Phi \rightarrow \phi \phi$ can occur. Now, this decay is possible only when $m_{\Phi} \geq 2 m_{\phi}^{\text{eff}} \sim \sqrt{\lambda} \bar{\Phi}$, where we considered that, during most of the oscillation period, $\bar{\Phi} \sim \bar{\Phi}_m$. Hence, the decay can take place only after the amplitude of the oscillations becomes $\bar{\Phi} < \bar{\Phi}_m$, where

$$\bar{\Phi}_m \equiv \frac{m_{\Phi}}{\sqrt{\lambda}}. \hspace{1cm} (48)$$

From Eqs. (46) and (48) we find

$$\Gamma_m \equiv \Gamma_{\Phi \rightarrow \phi\phi}(\bar{\Phi}_m) \sim \lambda m_{\Phi}. \hspace{1cm} (49)$$

Therefore, the constraint for the avoidance of the decay $\Phi \rightarrow \phi \phi$ reeds

$$\Gamma_m < H_m, \hspace{1cm} (50)$$

where $H_m = H(\bar{\Phi} = \bar{\Phi}_m)$. It can be checked that, in all cases, $H_m > H_{eq}$, which means that the amplitude of the $\Phi$-oscillations becomes smaller than $\bar{\Phi}_m$ during the radiation epoch, when $\bar{\Phi} \propto \sqrt{\rho} \propto a^{-3/2} \propto H^{3/4}$. Thus, it is easy to find that

$$H_m \sim H_{eq} \left( \frac{\bar{\Phi}_m}{\bar{\Phi}_{eq}} \right) \frac{4}{3} \sim \lambda^{-2/3} \frac{T_{eq}^2}{m_P} \left( \frac{m_{\Phi}}{T_{eq}} \right)^{8/3}, \hspace{1cm} (51)$$

where we used that $m_{\Phi} \bar{\Phi}_{eq} \sim \sqrt{T_{eq}} \sim T_{eq}^2 \sim m_P H_{eq}$. 


Using Eqs. (49) and (51) and enforcing the constraint in Eq. (50) we obtain

$$\lambda < \frac{m_\phi}{m_P} \left( \frac{m_P}{t_{eq}} \right)^{2/5} \sim 10^{-19}. \quad (52)$$

Apart from the perturbative decay of $\Phi$ it is possible that $\phi$-particle production occurs in an explosive manner due to parametric resonance effects [19]. This process takes place during the small fraction of each oscillation when $\Phi$ is close enough to the origin that $m_\phi \geq 2m_\phi^{\text{eff}} \simeq 2\sqrt{\lambda \Phi(t)}$ even though $\Phi > \Phi_m$. The efficiency of the resonance is determined by the so-called $q$-factor:

$$q \sim \frac{\lambda \Phi^2}{m_\phi^2} \sim \left( \frac{\Phi}{\Phi_m} \right)^2. \quad (53)$$

When $q \gg 1$ we are in the broad resonance regime and the production of $\phi$-particles is quite efficient. However, despite this fact, their energy is only a fraction of the total energy in the oscillating $\Phi$. Consequently, the evolution of $\Phi$ is hardly affected by the resonant production of $\phi$-particles. The produced $\phi$-particles are expected to eventually thermalize and become a (negligible) component of the thermal bath. The resonance becomes narrow when $q \lesssim 1$, which occurs deep into the radiation epoch. Soon afterwards, backreaction and rescattering effects are expected to shut down the resonance and terminate the non-perturbative production of $\phi$-particles. Hence, the resonant decay of $\Phi$ does not really impose any additional constraints.\(^3\)

In view of Eqs. (25) and (52) we see that the allowed range for $\lambda$ is

$$10^{-30} < \lambda < 10^{-19}. \quad (54)$$

Such a small coupling between flat directions can be naturally realized through being determined by the Planck suppressed expectation value of some other field.

### 4 Dark energy requirements

#### 4.1 Locking conditions for $\phi$

Our results also depend on the initial conditions for our quintessence field $\phi$, which has to find itself near the origin in order to become locked, when the $\Phi$ oscillations begin. The required condition, in fact, is

$$\phi_{\text{osc}} \ll M. \quad (55)$$

Recall, here, that the subscript ‘osc’ denotes the onset of the oscillations of the field $\Phi$ and not of $\phi$.

Firstly, due to the interaction term in Eq. (1), it is evident that, if $V_{\text{inf}}^{1/4}$ is small, one may not be able to have both $\Phi$ and $\phi$ of the order of $m_P$ after the end of inflation (despite the fact that $\lambda < 10^{-19}$ [cf. Eq. (54)]) because we need $V(\Phi, \phi) \ll V_{\text{inf}}$, otherwise the inflationary dynamics would be disturbed. As we have chosen $\Phi_{\text{inf}} \sim m_P$, we obtain the following bound for the value of $\phi$ at the end of inflation\(^4\)

$$\phi_{\text{inf}} \leq \min \left\{ \frac{1}{\sqrt{\lambda}} \left( \frac{V_{\text{inf}}^{1/4}}{m_P} \right)^2, 1 \right\} m_P. \quad (56)$$

Assuming that $\phi$ is also subject, like $\Phi$, to supergravity corrections, which provide a contribution $c' H^2$ to its mass-squared, we can estimate $\phi_{\text{reh}}$ using the analog of Eq. (38)

$$\phi_{\text{reh}} = \phi_{\text{inf}} \left( \frac{\Gamma_{\text{inf}}}{H_{\text{inf}}} \right)^{1/2} \left( \frac{1 - \sqrt{1 - 16c'/9}}{9} \right), \quad (57)$$

\(^3\)Note that because of the absence of a quartic term $\sim \Phi^4$ in the scalar potential in Eq. (1), we do not expect the resonant decay of the zero mode of the oscillating $\Phi$-condensate into $\Phi$ bosons of larger momenta [19].

\(^4\)According to this bound and Eq. (25) the quartic term is $\alpha \phi_{\text{inf}}^4 < (\alpha / \lambda^2) V_{\text{inf}}^2 / m_P^4 \ll V_{\text{inf}}$. 

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\[ \hat{c'} \equiv \min\{c', 9/16\} . \] (58)

Using Eq. (40) it is easy to obtain
\[ \phi_{\text{reh}} \sim \phi_{\text{inf}} \left( \frac{T_{\text{eq}}}{M_S} \right)^{\frac{1}{2}} \left( \frac{1 - \sqrt{1 - 16c'/9}}{1 - \sqrt{1 - 16c'/9}} \right) . \] (59)

If both \( c, c' \geq 9/16 \) then the above gives \( \phi_{\text{reh}} \sim 10^{-6}\phi_{\text{inf}} \). However, one can achieve a substantially smaller \( \phi_{\text{reh}} \). For example, with \( c' \geq 9/16 \) and \( c \approx 0.4 \) one finds \( \phi_{\text{reh}}^{\text{min}} \sim 10^{-13}\phi_{\text{inf}} \).

As in the case of \( \Phi \), the supergravity corrections disappear (they cancel out) after reheating. Consequently, during the radiation dominated epoch, the effective mass of \( \phi \), according to Eq. (3), is given by
\[ m_{\phi}^{\text{eff}} \sim \sqrt{\Phi_{\text{osc}}} , \] (60)

where we considered that \( \Phi_{\text{reh}} \sim \Phi_{\text{osc}} \gg \Phi_c \) [cf. Eq. (34)]. Since, in the interval \( m_\phi < H < \Gamma_{\text{inf}} \), \( \Phi \) remains frozen, the above effective mass remains constant after reheating and until the oscillations of \( \Phi \) begin. Comparing this effective mass with \( \Gamma_{\text{inf}} \) one finds
\[ m_{\phi}^{\text{eff}} > \Gamma_{\text{inf}} \iff T_{\text{reh}} < \lambda^{1/4}10^{-3}m_p , \] (61)

where we also used Eq. (40). In view of Eq. (25) we find \( \lambda^{1/4}10^{-3}m_p > 10^8\text{GeV} \). Hence, considering the gravitino constraint \( T_{\text{reh}} \leq 10^8\text{GeV} \), we expect that \( m_{\phi}^{\text{eff}} > \Gamma_{\text{inf}} \) and, therefore, the oscillations of \( \phi \) begin immediately after reheating.

During these oscillations we have \( \phi \propto \sqrt{\rho_\phi} \propto a^{-3/2} \propto H^{3/4} \), which results in
\[ \phi_{\text{osc}} \sim \phi_{\text{reh}} \left( \frac{m_\phi}{\Gamma_{\text{inf}}} \right)^{3/4} \sim \phi_{\text{reh}} \left( \frac{M_S}{T_{\text{reh}}} \right)^{3/2} \] (62)

For the allowed range of \( T_{\text{reh}} \) the above corresponds to \( 10^{-9} \leq \phi_{\text{osc}}/\phi_{\text{reh}} \leq 1 \).

Putting Eqs. (56), (59) and (62) together we obtain:
\[ \phi_{\text{osc}} \leq \left( \frac{M_S}{T_{\text{reh}}} \right)^{3/2} \left( \frac{T_{\text{eq}}}{M_S} \right)^{\frac{1}{2}} \left( \frac{1 - \sqrt{1 - 16c'/9}}{1 - \sqrt{1 - 16c'/9}} \right) \min \left\{ \frac{1}{\sqrt{\lambda}}, \frac{V_{\text{inf}}^{1/4}}{m_p} \right\}^{2}, 1 \} m_p . \] (63)

The first factor on the right-hand-side of the above can be as low as \( 10^{-9} \), the second one can be as low as \( 10^{-13} \), while the last factor in front of \( m_p \) cannot be larger than unity. Hence, it is evident that the requirement in Eq. (55), which demands \( \phi_{\text{osc}} < 10^{-15}m_p \), may well be satisfied.

Let us demonstrate this with a small example. Suppose that \( V_{\text{inf}}^{1/4} \sim 10^{16}\text{GeV} \) and we choose \( c \approx 0.4 \) and \( c' \geq 9/16 \). Using this and in view also of Eq. (54), Eq. (56) suggests that \( \phi_{\text{inf}} \lesssim m_p \). Hence, Eq. (63) suggests that \( \phi_{\text{osc}} < 10^{-15}m_p \) can be achieved if \( T_{\text{reh}} \geq 10M_S \), which allows almost the entire range of \( T_{\text{reh}} \).

As a result of the above, our assumption \( \phi \simeq 0 \) in Sec. 2 is well justified.

### 4.2 The mass and VEV of \( \phi \)

In order to achieve a false vacuum density as small as \( \rho_0 \) we not only require a small tachyonic mass for our locked quintessence field \( \phi \) but also a small VEV according to Eq. (22). One way to achieve this is to stabilize the \( \phi \)-direction by means of some high-order non-renormalizable term, of the form
\[ V(\Phi = 0, \phi) = V_0 - \frac{1}{2}m_{\phi}^2\phi^2 + \frac{\phi^{2n}}{Q^{2n-4}} , \] (64)

where \( n > 2 \) and \( Q \) is an appropriate large cut-off scale, which is linked to the VEV \( M \) as
\[ Q \sim \alpha^{-\frac{1}{2n-4}}M , \] (65)
where we also considered Eq. (2). The most natural choice is \( Q = m_P \), which gives \( n = 4 \). Hence, the action of non-renormalizable terms may well reduce the VEV of \( \phi \) naturally\(^5\).

The important issue here is that we need to preserve the smallness of the tachyonic mass. For a flat direction one expects the dominant contribution to the mass to be of the form \( (M_S/m_P)^p M_S \) with \( p = 1 \). This is the case, for example, of the dark matter field \( \Phi \) as shown in Eq. (17). However, in the case of \( \phi \), we need to suppress this contribution and consider \( p = 2 \) instead, according to Eq. (23). It is conceivable that this may occur due to accidental cancellations in the Kähler potential, or due to some symmetry, which protects \( m_\phi \). In any case, even if this requirement corresponds to a certain level of fine-tuning, this tuning is much less stringent than what is required in most quintessence models (with typical effective mass \( m_Q \sim H_0 \)), because \( m_\phi \sim 10^{15} H_0 \). Moreover, since \( m_\phi \sim 10^9 H_{eq} \), supergravity corrections, during the matter era after \( t_{eq} \), are negligible, in contrast to the usual quintessence models [8]. Note, however, that there exist some dark energy models corresponding to particles with mass much larger than \( H_0 \). For example this is possible in scalar tensor theories of gravity [21], which can account for both quintessence and dark matter (e.g. see Refs. [22] and [23] respectively). Another recent such example is dark energy from mass varying neutrinos [24].

A marginal increase of \( m_\phi \) may be achieved if we consider that the VEV of \( \phi \) is reduced by the action of loop corrections (instead of non-renormalizable terms). These are of the form

\[
V(\Phi = 0, \phi) = V_0 - \frac{1}{2}m_\phi^2 \phi^2 + Cm_\phi^2 \ln(\phi/Q)\phi^2,
\]

where \( C \ll 1 \). In this case we have

\[
M \sim \exp(-1/2C)Q.
\]

The above setup can increase the tachyonic mass by a factor \( 1/\sqrt{C} \). The best case, however, corresponds to \( Q = m_P \), which gives \( C \simeq 0.015 \), i.e. \( m_\phi \) is increased at most by a factor of 8.

Finally, the smallness of the tachyonic mass of our locked quintessence may result in the appearance of a fifth-force [25] because the associated Compton wavelength is

\[
\ell_\phi \sim m_\phi^{-1} \sim (10^{-27} \text{GeV})^{-1} \sim 1 \text{ A.U.}
\]

From the above we see that such a fifth-force cannot bias the formation of large structures like galaxies and galactic clusters. It is conceivable, though, that it may affect the generation of population III stars and stellar formation in general. However, the fifth-force is strongly constrained by the solar system tests on the equivalence principle [26]. Hence, we require that \( \phi \) is some hidden sector field with suppressed interactions with ordinary baryonic matter.

## 5 Discussion and conclusions

We have analyzed a unified model for the dark matter and the dark energy. As dark matter we used a modulus field \( \Phi \), which corresponds to a flat direction of supersymmetry. The field undergoes coherent oscillations that correspond to massive particles (WIMPs), constituting pressureless matter. Our \( \Phi \) field is weakly coupled with another scalar \( \phi \), through a hybrid type potential, very common in supersymmetric theories. The scalar \( \phi \) corresponds to a flat direction lifted by non-renormalizable terms. Due to the above coupling the oscillating \( \Phi \) keeps \( \phi \) ‘locked’ on top of the saddle point of the potential, resulting in a non-zero false vacuum contribution \( V_0 \). The amplitude of the oscillations decreases in time due to the Universe expansion. Below a certain value \( \Phi_\Lambda \) the Universe becomes dominated by the false vacuum density and a phase of accelerated expansion begins. Acceleration continues until the amplitude of the oscillations decreases down to a critical value \( \Phi_c \), when the ‘locked’ quintessence field \( \phi \) is released and rolls down to its VEV. At this point the system reaches the true vacuum and the accelerated expansion ceases.

\(^5\)The absence of a quartic term can be understood in a similar manner as with the so-called flaton fields discussed in Ref [20].
We have shown that it is possible to explain both dark matter and dark energy by taking the supersymmetry breaking scale to be \( M_S \sim \text{TeV} \), which corresponds to low-scale gauge-mediated supersymmetry breaking. The VEV of \( \phi \) has to be given also by \( M_S \), which is possible to achieve by stabilizing its potential with the use of a non-renormalizable term of the 8th order. Hence, using only two natural mass scales, \( m_P \) and \( M_S \sim m_{3/2} \) we are able to achieve cosmic coincidence, in the sense that we manage to obtain comparable densities for dark matter and dark energy at present without severe fine-tuning. In order to successfully account for both dark matter and dark energy our scalar fields need to have the correct initial conditions. By studying the dynamics of our scalar fields in the early Universe, we have demonstrated that the required initial conditions are naturally attained when considering the action of supergravity corrections to the scalar potential during the period following the end of primordial inflation and until reheating.

The advantages of our model are the following. Firstly, it uses a theoretically well motivated framework to address, in a unified manner, both the open issues of dark matter and dark energy. Also, coincidence is achieved with the use of only natural energy scales and initial conditions. The observational consequences of our model are similar to those of \( \Lambda \text{CDM} \) because, during most of the evolution of the Universe, the model is reduced to Eq. (5) (i.e. it corresponds to a collection of \( \Phi \)-particles (WIMPs) plus an effective cosmological constant \( \Lambda_{eq} = V_0/m_P^2 \)). Hence, our model enjoys all the successes of \( \Lambda \text{CDM} \) but it does not suffer from its disadvantages, namely the extreme fine-tuning of \( \Lambda \) and also the conceptual blunders of eternal acceleration and future causal horizons. Our model avoids eternal acceleration because the ‘locked’ quintessence field terminates false vacuum domination, when it is released from the origin. The only tuning problem that our model suffers from is the smallness of the tachyonic mass \( m \), which may be due to some approximate symmetry. Still, we have \( m_\phi \gg H_{eq} \gg H_0 \), which means that the supergravity corrections to the \( \phi \)-direction are negligible even after \( t_{eq} \), in contrast to the generic problem of most quintessence models, which have \( m_Q \sim H_0 \).

It is interesting to estimate how long the late period of accelerated expansion lasts. After domination by the false vacuum we have \( H_0 \sim \sqrt{V_0}/\sqrt{3} m_P = \text{constant} \). Hence, a phase of (quasi) de Sitter expansion begins, with \( a \approx a_0 \exp(H_0 \Delta t) \), where \( \Delta t = t - t_0 \). Now, for the oscillating \( \Phi \) we have \( \Phi \propto \sqrt{\rho_\Phi} \propto a^{-3/2} \). Thus, we obtain

\[
\Phi \sim \Phi_\Lambda \exp(-\frac{3}{2} H_0 \Delta t) \Rightarrow \Delta t_\phi \approx \frac{2}{3} \left[ \ln \left( \frac{m_P}{M_S} \right) + \ln \sqrt{\lambda} \right] H_0^{-1},
\]

where we have used Eqs. (4), (12) and (17). In view of Eqs. (22) and (54) we see that the period of acceleration may last up to 8 Hubble times (e-foldings) depending on the value of \( \lambda \). Another interesting point regards the coupling \( g_\Phi \) of the dark matter particle to its decay products. Eqs. (15), (17) and (22) suggest that \( g_\Phi \) should lie in the range: \( 10^{-30} \leq g_\Phi < 10^{-15} \), with the lower bound corresponding to gravitational decay, when \( g_\Phi \sim m_\phi/m_P \). Thus, \( \Phi \) is truly a WIMP [cf. also Eq. (54)].

We should also point out here that our oscillating \( \Phi \)-condensate does not have to be the dark matter necessarily. Indeed, it is quite possible that \( \phi \) remains locked on top of the false vacuum while \( \rho_\Phi \) is negligible at present. That way, our model can account for the dark energy requirements even if the initial conditions for \( \Phi \) (e.g. the value of \( c \)) are not appropriate for the latter to be the dark matter. Indeed, the locking of quintessence requires that \( \rho_\Phi(t_0) \geq \rho_\Phi^{\text{min}} \), where \( \rho_\Phi^{\text{min}} \sim m_\phi^2 \Phi_c^2 \) corresponds to the minimum energy for the oscillations. In view of Eqs. (4), (13), (17) and (22) it is easy to find: \( \rho_\Phi^{\text{min}} / \rho_0 \sim 10^{-30} \lambda^{-1} \). Hence, depending on \( \lambda \), \( \Phi \) may contribute only by a small fraction to dark matter, while still being able to lock quintessence and cause the observed accelerated expansion at present. However, we feel that using \( \Phi \) to account also for the dark matter renders our model much more effective and economical, without any additional tuning requirements (in the sense that the required value of \( c \) is natural).

To summarize we have presented a unified model of dark matter and dark energy in the context of low-scale gauge-mediated supersymmetry breaking. Our model retains the predictions of \( \Lambda \text{CDM} \), while avoiding eternal acceleration and achieving coincidence without significant fine-tuning. The initial conditions of our model are naturally attained due to the effect of supergravity corrections to the scalar potential in the early Universe, following a period of primordial inflation.
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