Critical Clearing Time Sensitivity for Differential-Algebraic Power System Model

Chetan Mishra, Member, IEEE

Abstract—Standard power systems are modeled using differential-algebraic equations (DAE). Following a transient event, voltage collapse can occur as a bifurcation of the transient load flow solutions. This results from the states’ trajectories reaching a singular surface where the voltage causality is lost. Preventive controls such as changes in AVR setpoints need to be taken to enhance stability. To determine which actions are most effective, the sensitivity of the critical clearing time to controllable parameters can be exploited. This work derives the expressions for critical clearing time sensitivity for a generic DAE model using trajectory sensitivities. The results are illustrated for multiple test systems that are then validated against computationally intensive time-domain simulations.

Index Terms—Differential-algebraic systems, Singularity, Power System Transient Stability.

I. INTRODUCTION

ONVENTIONAL large-scale power system dynamic models are represented using DAEs. It is well known that DAE models can have certain regions in state space called singular surfaces. The dynamics in those regions cannot be studied using these models [1] because the algebraic states, e.g., load bus voltages lose causal relationships with dynamic states like generator rotor angle, speed, etc. A type of local bifurcation of equilibria that is a characteristic of such models is singularity induced bifurcation [2], where one or more equilibrium points merge with a singular surface. Trajectories reaching a singular surface have been shown to have a strong tendency to result in a voltage collapse [3].

Lately, utilities all over the world are having voltage stability concerns owing to the retirements of conventional generators, which result in a loss of voltage controllability. Furthermore, owing to the difficulties in building new right-of-ways to supply the increasing demand, utilities tend to maximize the utilization of the existing transmission networks in certain regions, which makes matters worse. In the past, voltage collapse was studied as a small signal problem [4] resulting from the saddle-node bifurcation (SNB) of the power flow solutions where a stable equilibrium point (SEP) merges with an unstable equilibrium point (UEP) on its stability boundary and vanishes. However, it was shown that during transient conditions, voltage collapse can occur differently [5]. A trajectory passing through the singular surface may bifurcate and settle to an infeasible (low voltage) point. In [6], Hiskens and Hill showed that operation in the vicinity of a singular surface or trajectories intersecting it was associated with sudden reductions in voltage or voltage instability. Therefore, there has been a great focus in the past on analyzing the stability of DAE systems to incorporate voltage stability into the transient stability assessment (TSA) [7].

TSA is concerned with estimating the critical clearing time (CCT), which refers to the maximum time that can be taken to clear a fault while remaining stable. Since CCT is a function of system conditions, understanding its dependence on parameters is fairly helpful in identifying effective preventive control decisions and/or fast analysis of a range of operating conditions using sensitivities. In this regard, Ayasun [8] reduced the multi-machine system to a single machine infinite bus system to evaluate CCT sensitivities that are computationally efficient yet approximate. Nguyen [9] and Laufenberg [10] computed the sensitivity of post-fault angle and speed trajectories with respect to the fault clearing time, which is expected to grow for marginally stable trajectories. Nguyen also computed CCT sensitivities by approximating the relevant portion of the stability boundary by a constant energy surface passing through the controlling unstable equilibrium point (CUEP). In [11], the closest UEP was used to approximate the stability boundary, which is known to be extremely conservative. Furthermore, the solution of the fault-on system was represented by an approximate analytical expression, which is not possible to obtain for systems with more than a few buses. Dobson et al. [12] use the variational equations for the post-fault dynamics to approximate the stability boundary, which yields accurate CCT sensitivity estimates. The derivation proposed was for unconstrained ODE systems with an extension for DAE systems under the assumption of absence of the singular surface on the stability boundary. [13] deals with deriving the same for systems with constraints arising from protection devices and various other limits. However, none of the previous works address the derivation of the CCT sensitivity expressions for faults driving the system to singularity (voltage collapse), which is the contribution of this work.

The rest of this paper is organized as follows. In Section II, the stability theory for DAE systems with an emphasis on the role of singular surfaces is briefly discussed. The new sensitivity formulas are derived in Section III. Meanwhile, Section IV describes how to compute them for practical systems. Finally, the derived
expressions are validated against time-domain simulation (TDS) in Section V.

II. STABILITY THEORY OF DAE SYSTEMS

A. DAE System Dynamics

A generic power system DAE model can be written as,

\[
\begin{align*}
\dot{x} &= f(x, y) \\
0 &= g(x, y)
\end{align*}
\]  

(1)

Here, \( x \in \mathbb{R}^n \) are dynamic states such as generator rotor angles and generator flux linkages, and \( y \in \mathbb{R}^m \) are algebraic states such as load bus voltages and phase angles making the overall state space as \( \mathbb{R}^{m+n} \). The system evolves on a lower-dimensional constraint set \( \Gamma \) (largely \( n \) dimensional) given by,

\[
\Gamma = \{ (x, y) \in \mathbb{R}^{m+n} | g(x, y) = 0 \}
\]  

(2)

A point \((\bar{x}, \bar{y}) \in \Gamma\) is an equilibrium point if \( f(\bar{x}, \bar{y}) = 0 \). As the system undergoes discrete changes (line tripping, etc.), \( \Gamma \) undergoes discrete changes as well. This results in the system trajectory jumping to a new constraint set. From equation (1), the \( x \) coordinate of the trajectory varies smoothly with time, unlike the \( y \) coordinate.

Points where \( \partial_y g \) is invertible are called regular points. Here, the operator \( \partial_y \) represents the partial derivative with respect to the variable \( y \) i.e., \( \frac{\partial}{\partial y} \). Using implicit function theorem [14], \( y \) can be locally written as a function of \( x \) at the regular points. This enables reducing (1) at those points to an ODE system which guarantees the existence and uniqueness of its solution. The surface of points denoted by \( S \) where the matrix \( \partial_y g \) is singular is called the singular surface and is defined by,

\[
S = \{ x, y \in \Gamma | \Delta(x, y) = 0 \}
\]  

(3)

where \( \Delta \) denotes the determinant of \( \partial_y g \) i.e., \( |\partial_y g| \). Overall, \( \Gamma \) is comprised of multiple disjoint subsets of regular points (typically \( n \) dimensional submanifolds of the ambient space \( \mathbb{R}^{n+m} \) [15]) on which the dynamics exist and which are separated by components of \( S \).

B. Characterization of Quasi-Stability Boundary

Because the CCT refers to the fault clearing time at which the fault trajectory intersects the stability boundary, it is important to characterize the various components that comprise the stability boundary. This allows quantifying how the boundary and consequently the CCT will change under parameter variations.

A general solution to (1) starting from condition \((x^0, y^0)\) is,

\[
\begin{align*}
x(t) &= \varphi^x \left((x^0, y^0), t\right) \\
y(t) &= \varphi^y \left((x^0, y^0), t\right)
\end{align*}
\]  

(4)

Since the trajectories cannot cross \( S \), the region of interest for TSA is the subset of regular points which contains the SEP \((x^*, y^*)\) along with its stability region \( A(x^*, y^*) \). Let this set be denoted by \( \Gamma^* \). The stability region \( A(x^*, y^*) \) is then defined as,

\[
\begin{align*}
A(x^*, y^*) &= \{ (x^0, y^0) \in \Gamma^* | \lim_{t \to \infty} (\varphi^x((x^0, y^0), t) \\
&\quad \quad \rightarrow x^*, \varphi^y((x^0, y^0), t) \rightarrow y^*) \}
\end{align*}
\]  

(5)

For the remainder of the paper, \( S \) will be used to represent the particular component(s) of the singular set which separate \( \Gamma^* \) from the other subsets of \( \Gamma \). Venkatasubramanian et al. [16] presented a completed characterization of the stability boundary for DAE systems. This section briefly discusses important parts of their results. Broadly speaking, the stability boundary \((\partial A(x^*, y^*))\) is comprised of some components of \( S \), sets comprised of trajectories intersecting \( S \) and stable manifolds of some unstable equilibrium points (UEP). Instead of the stability boundary, which can have a very complex structure, the focus is on the characterization of the quasi-stability boundary. It is defined as the boundary of the closure of \( A(x^*, y^*) \) i.e., \( \partial A(x^*, y^*) \).

To analyze the stability of DAE systems using the existing tools for ODE systems, a regularized version of the system was proposed in [16]. This system is obtained by scaling the dynamics of (1) (and consequently \( y \)) using \( \Delta(x, y) \), resulting in

\[
\begin{align*}
\dot{x} &= \Delta \cdot f \\
y &= \kappa (x, y) = -\text{adjoint} (\partial_y g) \partial_x g \cdot f
\end{align*}
\]  

(6)

Assuming \( \Delta > 0 \) inside \( \Gamma^* \), (6) is simply a time-scaled version of (1). Thus, both systems share the same invariant sets inside \( \Gamma^* \), which also includes the stability region \( A(x^*, y^*) \).

Note that the dynamics of (6) are globally defined unlike (1) whose dynamics are not defined at/near \( S \). Thus, the stability region of (1) can be analyzed using (6) in the neighborhood of \( S \).

Understanding the dynamics of (6) in the neighborhood of the singular surface \( S \) is crucial for characterizing the stability boundary. From (6), \( S \) can be decomposed into three disjoint sets: the semi-singular surface, the pseudo-equilibrium surface, and the remaining points in \( S \).

The semi-singular surface denoted by \( \Xi \) is a surface of non-equilibrium points in \( S \) at which the solutions of (6) intersect \( S \) tangentially. It is typically \( n - 2 \) dimensional and is defined as,

\[
\Xi = \{ (x, y) \in S | \Delta = \partial_y \Delta \cdot \kappa (x, y) = 0, \kappa (x, y) \neq 0 \}
\]  

(7)

Depending on local dynamics, the semi-singular surface can further be divided into semi-saddle and semi-focus. These are denoted by \( \Xi^{sa} \) and \( \Xi^{sf} \) respectively and defined as,

\[
\begin{align*}
\Xi^{sa} &= \{ (x, y) \in \Xi | \partial_y (\partial_y \Delta \cdot \kappa) \cdot \kappa = 0 \} \\
\Xi^{sf} &= \{ (x, y) \in \Xi | \partial_y (\partial_y \Delta \cdot \kappa) \cdot \kappa < 0 \}
\end{align*}
\]  

(8)

To illustrate, in Fig. 1, the trajectories curve towards \( \Gamma^* (\Delta > 0) \) for \( \Xi^{sa} \) whereas for \( \Xi^{sf} \), they curve away. Therefore, only the connected components of semi-saddle points \( \Xi^{sa} \) can exist on the quasi-stability boundary.

The system in (6) possesses some equilibrium points in addition to those of (1). These belong to \( S \) and are called pseudo-equilibrium points with the set of these points defined as,

\[
\psi = \{ x, y \in S | f(x, y) \neq 0, \kappa (x, y) = 0 \}
\]  

(9)

\( \psi \) is typically \( n - 2 \) dimensional and therefore each point in \( \psi \) possesses an \( n - 2 \) dimensional center manifold which is the set \( \psi \) itself. Therefore, these are all non-hyperbolic equilibrium
and parameter \( t \) is the parameter(s) whose impact on the CCT is to be studied. Let \( \psi \) denote the fault clearing time, \( t^\text{cl} \) denotes the fault clearing time, \( t_0 \) represents the state value at the time of fault clearing \( \Delta(\cdot) \) denotes the fault clearing time. The stability boundary intersects with the vector field of (6) is neither tangential nor null to \( S \) is comprised of –

1) Find the base critical trajectory (using iterative TDS).
2) Derive the sensitivity of the relevant portion of the stability boundary \[ \{x^cl, y^cl\} \] i.e., one that the base critical trajectory intersects.
3) Derive the sensitivity of the state variable values \[ \{x^cl, y^cl\} \] at the time of fault clearing to \( p \) and \( t^cl \).
4) Evaluate the above two expressions at the base critical trajectory and equate to get the CCT sensitivity value.

When following the above process in practice, the following challenges need to be addressed -

i. The closed-form expression for the stability boundary is usually not available and only local approximations can be made around critical points. For example, a hyperplane normal to the unstable eigenvector of a type-1 UEP can be used to locally approximate its stable manifold.

ii. The local approximation listed above is usually not given in the neighborhood of the exit point but is available at some other point along the post-fault trajectory. Since only the first-order sensitivities are being calculated, challenge i) can be addressed by replacing the first step with calculating the sensitivity of a local approximation of the stability boundary. Challenge ii) requires modifying the second and third steps of the process discussed above. Because the whole critical post-fault trajectory lies on the stability boundary, it is possible to compute and equate the same sensitivities at whatever point on the stability boundary the approximation is available. This usually comes at an added cost of evaluating the sensitivity of the post-fault trajectory as opposed to just doing so for the fault-on trajectory.
B. Dealing With Discontinuities in y

In typical TSA studies, the system passes through at least three distinct system conditions viz. pre-fault, fault-on, and post-fault. As discussed before, these discrete changes result in the y values jumping between the different constraint surfaces \( \Gamma_{\text{pre}}, \Gamma_{\text{fault}} \) and \( \Gamma_{\text{post}} \). Let the corresponding values of \( y \) be denoted by \( y_{\text{pre}}(t), y_{\text{fault}}(t) \) and \( y_{\text{post}}(t) \). Since the focus is on the stability of the post-fault system, a knowledge of the state values immediately following the clearing of the fault is important. Therefore, even when analyzing the fault on dynamics with the state values \( x(t) \) and \( y_{\text{fault}}(t) \), there is a need to track \( y_{\text{post}}(t) \), which represents the value of \( y \) immediately following the clearing of the fault at any time \( t \). This results in the following extended state fault-on system

\[
\dot{x} = f_{\text{fault}} (x, y_{\text{fault}}, p) \\
0 = g_{\text{fault}} (x, y_{\text{fault}}, p) \\
0 = g_{\text{post}} (x, y_{\text{post}}, p)
\]

As expected, \( y_{\text{post}} \) dynamics during the fault depend on \( x \), which further is coupled with \( y_{\text{fault}} \).

C. Sensitivity of Dynamic State Values at the Time of Fault Clearing \( \partial_p x^{cl} \mid p \)

Here, the sensitivity of the state value at the time of fault clearing \( (x^{cl}, y^{cl}) \) is derived. A typical assumption in TSA studies is that the pre-fault system is at its SEP. Therefore, the pre-fault states at time zero \( (x^0, y^0_{\text{pre}}) \) satisfy,

\[
f_{\text{pre}} (x^0, y^0_{\text{pre}}, p) = 0 \\
g_{\text{pre}} (x^0, y^0_{\text{pre}}, p) = 0
\]

(12)

Differentiating (12) and evaluating at the starting point of the base critical trajectory \( (x^0, y^0_{\text{pre}}) \),

\[
\partial_p x^0 \mid p = \left( \partial_x f_{\text{pre}} - \partial_{y_{\text{pre}}} f_{\text{pre}} \partial_{y_{\text{pre}}} g_{\text{pre}} - \partial_y g_{\text{pre}} \right) \\
\cdot \left( \partial_{y_{\text{pre}}} f_{\text{pre}} \partial_{y_{\text{pre}}} g_{\text{pre}} - \partial_y g_{\text{pre}} - \partial_g f_{\text{pre}} \right) \mid (x^0, y^0_{\text{pre}})
\]

(13)

Note that the above expression is constant. Now, let \( y^0_{\text{fault}} \) be the \( y \) value immediately following the onset of the fault.

Extending (4) to parametric systems, the state value at the time of fault clearing \( t^{cl} \) can be written as follows,

\[
x^{cl} = \varphi_{\text{fault}} (x^0, y^0_{\text{fault}}, t^{cl}, p) \\
y^{cl}_{\text{fault}} = \varphi_{\text{fault}}^y (x^0, y^0_{\text{fault}}, t^{cl}, p)
\]

(14)

It is known that \( (x^0, y^0_{\text{fault}}) \) cannot be a singular point on \( \Gamma_{\text{fault}} \), otherwise, the fault-on trajectory cannot exist. Therefore, using implicit function theorem [14], \( y^0_{\text{fault}} \) can locally be written as a function of \( x^0 \). Finally, computing the sensitivity of \( x^{cl} \) to \( p \) by differentiating (14) and evaluating at the base case system’s CCT \( t^{cl} \), results in

\[
\partial_p x^{cl} \mid p = \partial_x \varphi_{\text{fault}} \mid p \cdot \partial_{x^0} x^{cl} + \partial_y \varphi_{\text{fault}} \mid p \cdot \partial_{y^0} x^{cl} + \partial_{t^{cl}} x^{cl} \cdot \partial_{t^{cl}} x^{cl} \mid p
\]

(15)

To clarify, \( \partial_x \varphi_{\text{fault}} \mid p \) and \( \partial_y \varphi_{\text{fault}} \mid p \) are used to denote \( \partial_x \varphi_{\text{fault}} (x^0, y^0_{\text{fault}}, t^{cl}, p) \) and \( \partial_y \varphi_{\text{fault}} (x^0, y^0_{\text{fault}}, t^{cl}, p) \). These are obtained using trajectory sensitivity analysis [18] for the fault-on variational equations that will be discussed later. Substituting (13) in (15), \( \partial_p x^{cl} \mid p \) can be expressed as a linear function of \( \partial_p x^{cl} \mid p \) and a constant.

D. Derivation for Singularity Immediately Following the Fault Clearing

As discussed in Section II-B, the stability boundary has some components of the singular surface itself. CCT sensitivity derivation for faults that drive the system to those components is discussed next. Here, the CCT represents the time it takes for the sustained fault trajectory to reach the singular surface of the post-fault system denoted by \( S_{\text{post}} \). This sensitivity gives an insight into what control parameters \( p \) are effective in pushing away the singular surface, thereby reducing the likelihood of voltage instability. Therefore, there is a greater value to computing this sensitivity for each fault regardless of the actual instability phenomenon.

Since the fault-on system is assumed to have no singularities within the region of interest, one can define this phenomenon as one where the fault-on trajectory immediately on clearing the fault intersects \( S_{\text{post}} \). Using the definition of \( S_{\text{post}} \) given in (3),

\[
\Delta_{\text{post}} (x^{cl}, y^{cl}_{\text{post}}, p) = 0 \\
g_{\text{post}} (x^{cl}, y^{cl}_{\text{post}}, p) = 0
\]

(16)

Here \( y^{cl}_{\text{post}} \) denotes the \( y \) value right after clearing the fault at the time \( t^{cl} \). Differentiating the above equation and evaluating at the base critical trajectory gives the following linear equations,

\[
\begin{bmatrix}
\partial_x \Delta_{\text{post}} & \partial_{y^{cl}_{\text{post}}} \Delta_{\text{post}} & \partial_p \Delta_{\text{post}} \\
\partial_x g_{\text{post}} & \partial_{y^{cl}_{\text{post}}} g_{\text{post}} & \partial_p g_{\text{post}}
\end{bmatrix}
\begin{bmatrix}
(x^{cl}, y^{cl}_{\text{post}}, p)
\end{bmatrix}
= 0
\]

(17)

Since \( \partial_{y^{cl}_{\text{post}}} g_{\text{post}} |_{x^{cl}, y^{cl}_{\text{post}}, p} \) is singular on \( S_{\text{post}} \), let there be a left eigenvector \( v^* \) corresponding to the zero eigenvalue of this matrix. Pre-multiplying the second set of equations above with \( v^* \) eliminates the \( \partial_{y^{cl}_{\text{post}}} g_{\text{post}} |_{x^{cl}, y^{cl}_{\text{post}}, p} \) term yielding,

\[
v^* \partial_x g_{\text{post}} |_{x^{cl}, y^{cl}_{\text{post}}, p} = -v^* \partial_p g_{\text{post}} |_{x^{cl}, y^{cl}_{\text{post}}, p}
\]

(18)

Substituting (15) in (18) yields the final expression for CCT sensitivity,

\[
\partial_p x^{cl} \mid p = - (v^* t(\partial_{y^{cl}_{\text{post}}} f_{\text{fault}}, \partial_{y^{cl}_{\text{post}}} \cdot \partial_{x^0} x^{cl} \\
+ \partial_{t^{cl}} x^{cl} \cdot \partial_{t^{cl}} x^{cl} ) \cdot \frac{1}{v^* \cdot \partial_{y^{cl}_{\text{post}}} f_{\text{fault}} \mid p}
\]

(19)

E. Sensitivity of Dynamic State Values at the End of the Post-Fault Trajectory \( \partial_p x^{end} \mid p \)

For the remaining instability phenomena, the local characterization of the stability boundary is not available at the exit point.
but some other point along the post-fault trajectory. Let this point be defined as the endpoint of the post-fault trajectory. The base critical post-fault trajectory is simulated through then. Denoting that point by \((x^{end}, y^{end})\), it can be expressed as a function of the general solution of the post-fault system’s DAE as follows,

\[
\begin{align*}
\dot{x}^{end} &= \varphi^{x}_{post}(x^{cl}, y^{cl}_{post}, \tau^{end}, p) \\
\dot{y}^{end} &= \varphi^{y}_{post}(x^{cl}, y^{cl}_{post}, \tau^{end}, p)
\end{align*}
\tag{20}
\]

where \((\varphi^{x}_{post}, \varphi^{y}_{post})\) is the solution to the post-fault DAE system with the initial condition at time zero as \((x^{cl}, y^{cl})\). Here, \((x^{cl}, y^{cl})\) is the state value immediately following the clearing of the fault. From the results in Section III-A, it is only necessary to equate the sensitivity of the local characterization of the stability boundary available at \((x^{end}, y^{end})\) to that of \((x^{end}, y^{end})\) itself.

Now, similar to the previous derivation, because \((x^{cl}, y^{cl})\) is a regular point, \(y^{end}\) can locally be written as a function of \(x^{cl}\). Taking the derivative of \((20)\) and evaluating at the base critical trajectory

\[
\partial_{p}x^{end}\big|_{p} = \partial_{x^{cl}}^{x} \varphi^{x}_{post}\big|_{p} \partial_{x^{cl}} x^{cl}\big|_{p} + \partial_{p^{x}x^{end}} \varphi^{x}_{post}\big|_{p} + \int_{\Delta_{ps}} \partial_{\tau^{end}}^{x} \varphi^{x}_{fault}\big|_{p} \partial_{\tau^{end}} x^{cl}\big|_{p} + \partial_{p^{x}x^{end}} \varphi^{x}_{fault}\big|_{p} + \partial_{x^{cl}}^{x} \varphi^{x}_{cl}\big|_{p} + \partial_{p^{x}x^{end}} \varphi^{x}_{cl}\big|_{p} + \gamma_{1} \partial_{p^{x}x^{end}}\big|_{p} + \gamma_{2} \partial_{p^{y}x^{end}}\big|_{p} + \gamma_{3}
\tag{21}
\]

The arguments needed for evaluating \(\partial_{p^{x}x^{end}}\) at the endpoint of the base critical post-on trajectory are shortened from \((x^{cl}, y^{cl}, \tau^{end}, p)\) to \(p\). Here, \(\partial_{x^{cl}} \varphi^{x}_{post} \) and \(\partial_{p^{x}x^{end}}\) are solutions to the post-fault variational equations. Substituting \((15)\) in \((21)\) yields \(\partial_{p^{x}x^{end}}\big|_{p}\) as a linear combination of \(\partial_{x^{cl}}^{x} \varphi^{x}_{post}\big|_{p}\) and \(\partial_{p^{x}x^{end}}\big|_{p}\)

\[
\partial_{p^{x}x^{end}}\big|_{p} = \partial_{x^{cl}}^{x} \varphi^{x}_{fault}\big|_{p} \partial_{x^{cl}} x^{cl}\big|_{p} + \partial_{p^{x}x^{end}} \varphi^{x}_{fault}\big|_{p} + \partial_{x^{cl}}^{x} \varphi^{x}_{cl}\big|_{p} + \partial_{p^{x}x^{end}} \varphi^{x}_{cl}\big|_{p}
\]

The coefficients in the above equation are substituted for \((\gamma_{1}, \gamma_{2}, \gamma_{3})\) to make the final CCT sensitivity expressions concise.

F. Derivation for Singularity in the Post-Fault Trajectory

Here, the CCT sensitivity is derived for faults for which the post-fault trajectory intersects the singular surface \(S_{post}\) after some finite amount of time. For this to happen, \((x^{end}, y^{end})\) should either lie on an \(n - 2\) dimensional connected component of semi-saddle points \(\Xi^{sa}_{post}\) or on that of transversal saddle pseudo-equilibrium points \(\psi^{trsa}_{post}\). Now, both \(\Xi^{sa}_{post}\) and \(\psi^{trsa}_{post}\) satisfy \((3)\) and are \(n - 2\) dimensional. Therefore, \((x^{end}, y^{end})\) lying on them also satisfy \(n + m + 2\) equality constraints of the form,

\[
\begin{align*}
\Delta_{post} (x^{end}, y^{end}, p) &= 0 \\
g_{post} (x^{end}, y^{end}, p) &= 0 \\
\lambda_{post} (x^{end}, y^{end}, p) &= 0
\end{align*}
\tag{23}
\]

Taking the derivative of the above equations

\[
\begin{bmatrix}
\partial_{x^{end}} \Delta_{post} & \partial_{y^{end}} \Delta_{post} \\
\partial_{x^{end}} g_{post} & \partial_{y^{end}} g_{post} \\
\partial_{x^{end}} \lambda_{post} & \partial_{y^{end}} \lambda_{post}
\end{bmatrix}\begin{bmatrix} x^{end} \\ y^{end} \end{bmatrix} + \begin{bmatrix}
\partial_{p^{x}x^{end}} \big|_{p} \\
\partial_{p^{y}x^{end}} \big|_{p} \\
\partial_{p^{y}y^{end}} \big|_{p}
\end{bmatrix} = -\begin{bmatrix}
\partial_{g_{post}} \Delta_{post} \\
\partial_{g_{post}} g_{post} \\
\partial_{g_{post}} \lambda_{post}
\end{bmatrix}
\tag{24}
\]

Finally, combining \((24)\) with \((22)\) gives a set of linear equations that can easily be solved to obtain the value of CCT sensitivity, i.e., \(\partial_{\text{cl}}^{x}\big|_{p}\).

\[
\begin{bmatrix}
\partial_{x^{end}} \Delta_{post} & \partial_{y^{end}} \Delta_{post} \\
\partial_{x^{end}} g_{post} & \partial_{y^{end}} g_{post} \\
\partial_{x^{end}} \lambda_{post} & \partial_{y^{end}} \lambda_{post}
\end{bmatrix}\begin{bmatrix} x^{end} \\ y^{end} \end{bmatrix} + \begin{bmatrix}
\partial_{p^{x}x^{end}} \big|_{p} \\
\partial_{p^{y}x^{end}} \big|_{p} \\
\partial_{p^{y}y^{end}} \big|_{p}
\end{bmatrix}
= \begin{bmatrix}
\partial_{g_{post}} \Delta_{post} \\
\partial_{g_{post}} g_{post} \\
\partial_{g_{post}} \lambda_{post}
\end{bmatrix}
\tag{25}
\]

For scenarios with the base case critical post-fault trajectory intersecting a semi-saddle point lying on \(\Xi^{sa}_{post}, \lambda_{post} = \partial_{p^{cl}} \Delta_{post}\). As for intersection with a transversal saddle pseudo EP on \(\psi^{trsa}_{post}, \lambda_{post} \neq \partial_{p^{cl}} \Delta_{post}\), which is not a scalar but an \(m\) dimensional vector function. However, since only \(n - 2\) dimensional connected components in \(\psi^{trsa}_{post}\) exist on the quasi-stability boundary \([16]\),

\[
\begin{bmatrix}
\partial_{x^{end}} \Delta_{post} & \partial_{y^{end}} \Delta_{post} \\
\partial_{x^{end}} g_{post} & \partial_{y^{end}} g_{post} \\
\partial_{x^{end}} \lambda_{post} & \partial_{y^{end}} \lambda_{post}
\end{bmatrix}
\begin{bmatrix} x^{end} \\ y^{end} \end{bmatrix} = \begin{bmatrix}
\partial_{p^{x}x^{end}} \big|_{p} \\
\partial_{p^{y}x^{end}} \big|_{p} \\
\partial_{p^{y}y^{end}} \big|_{p}
\end{bmatrix}
\tag{26}
\]

Furthermore, from the results in \([16]\), \(\partial_{x^{end}} g_{post}, \partial_{y^{end}} g_{post}\) is full ranked. Therefore, there exists a scalar function \(\kappa_{post}\), which is an element of vector function \(\kappa_{post}\) S.L.,

\[
\begin{bmatrix}
\partial_{x^{end}} \Delta_{post} & \partial_{y^{end}} \Delta_{post} \\
\partial_{x^{end}} g_{post} & \partial_{y^{end}} g_{post} \\
\partial_{x^{end}} \lambda_{post} & \partial_{y^{end}} \lambda_{post}
\end{bmatrix}
\begin{bmatrix} x^{end} \\ y^{end} \end{bmatrix} \in \text{span} \left( \begin{bmatrix}
\partial_{x^{end}} \Delta_{post} & \partial_{y^{end}} \Delta_{post} \\
\partial_{x^{end}} g_{post} & \partial_{y^{end}} g_{post} \\
\partial_{x^{end}} \lambda_{post} & \partial_{y^{end}} \lambda_{post}
\end{bmatrix} \right)
\tag{27}
\]

Thus, \(\lambda_{post} = \kappa_{post}\). Now, to ensure the non-singularity of equation \((25)\), \(\kappa_{post}\) is chosen as the scalar component function of the vector function \(\kappa_{post}\) that maximizes the smallest singular value of the matrix on the RHS above.
G. Sensitivity Result for the Traditional Loss of Instability

The CCT sensitivity derivation for the traditional instability phenomenon that involves the critical post-fault trajectory eventually converging to a type-1 UEP has been fully explored in previous literature. Here, the singular surface does not play any role and therefore, only the key points of the derivation are mentioned for completeness. The reader is advised to consult [12] for an in-depth treatment.

Let \((x^{cu}, y^{cu}_{post})\) be the controlling unstable equilibrium point (CUEP) for the fault under study. By definition, this is a regular point and so the post-fault DAE system can be reduced into an ODE system locally in the neighborhood of \((x^{cu}, y^{cu}_{post})\). The reduced state matrix obtained by further linearizing this ODE system is expressed as 
\[
[\partial_{x} f_{post} - \partial_{y_{post}} f_{post} \cdot (\partial_{y_{post}} g_{post})^{-1} \cdot \partial_{x} g_{post}] |_{x^{cu}, y^{cu}_{post}, p^{}}.
\]

Since \((x^{cu}, y^{cu}_{post})\) is a type-1 UEP, its stable manifold can locally be approximated by a hyperplane normal to \(v^{cu}\), the only unstable eigenvector of this matrix. Since \((x^{end}, y^{post})\) should continue to lie on this hyperplane under variations in \(p\), differentiating and evaluating at the base critical trajectory yields the following variational equation,
\[
(t^{cu} T |_{x^{cu}, y^{cu}_{post}, p^{}} \cdot \left(\partial_{p} x^{end} |_{p^{}} - \partial_{x} x^{cu} |_{x^{cu}, y^{cu}_{post}, p^{}}\right) + (\partial_{p} v^{cu} T |_{x^{cu}, y^{cu}_{post}, p^{}} \cdot \left(x^{end} - x^{cu}\right) = 0
\]
(28)

The expression for \(\partial_{p} v^{cu} |_{x^{cu}, y^{cu}_{post}, p^{}}\) can easily be obtained similar to (13). It is known that \((x^{end} |_{y^{post}}) \rightarrow (x^{cu}, y^{cu}_{post})\) as \(t^{end} \rightarrow \infty\) and therefore, for large enough \(t^{end}\), the second term in (28) vanishes. Also, since \((x^{cu}, y^{cu}_{post})\) is an equilibrium point, the value for \(\gamma_{2}\) in (22) also tends to zero as \(t^{end} \rightarrow \infty\). Finally, substituting the expression for \(\partial_{p} x^{end} |_{p^{}}\) from (22) and evaluating it as \(t^{end} \rightarrow \infty\), the final expression for CCT sensitivity is given by
\[
\partial_{p} t^{cu} |_{p^{}} = \frac{v^{cu} T |_{x^{cu}, y^{cu}_{post}, p^{}} \cdot \left(\partial_{p} x^{cu} |_{x^{cu}, y^{cu}_{post}, p^{}} - \gamma_{3}\right)\}
\]
(29)

IV. NUMERICAL IMPLEMENTATION AND COMPUTATIONAL ASPECTS

A. Precision Requirements

For the traditional mode of instability, i.e., the loss of synchronism in the base case system, it is nearly impossible to ensure that the fault is cleared precisely on the stable manifold of the type-1 CUEP. Furthermore, it is highly likely for the same trajectory to eventually intersect the singular surface. Therefore, for correct identification of the instability mechanism to select the correct CCT sensitivity expression, it is advisable to first check whether the unstable trajectory passes close to a UEP by evaluating the value of \(\|f_{post}\|\) along the trajectory. Once the value goes below a threshold, the simulation is stopped to obtain \(t^{end}\) and \((x^{cu}, y^{cu}_{post})\).

When the mode of instability involves intersecting the singular surface \(S_{post}\) in the post-fault phase, the simulation immediately stops converging. Depending on the precision of the underlying solver, the simulation may stop considerably far from \(S_{post}\), which could introduce errors when evaluating CCT sensitivity expression in (25). This can be solved by extrapolating \(\Delta_{post}\) to detect the zero-crossing. Similarly, to precisely detect the fault clearing time at which the base system’s fault-on trajectory directly intersects \(S_{post}\), the extended state fault-on system given in (11) can be simulated, which will stop converging close to the \(S_{post}\). However, this approach adds additional \(m\) algebraic equations corresponding to the post fault constraint surface that slow down the overall simulation. Since the post-fault algebraic system has no impact on the fault-on trajectory, it is advisable to compute \(\Delta_{post}\) at only a few points until it changes sign. Thereafter, the precise time \(t^{post}\) and state values \((x^{post}, y^{post})\) at the zero-crossing of \(\Delta_{post}\) can be found by interpolating \(x^{post}(t)\) values and finding the corresponding \(y_{post}(t)\) values using the post-fault constraint surface \(y_{post}(x^{post}, y^{post}, p^{}) = 0\).

To obtain an accurate estimate from (25) and (29), it is desirable that \((x^{end}, y^{post})\) be sufficiently close to the appropriate critical element on the stability boundary. However, a high precision comes at the cost of a greater number of iterations to get the precise \(t^{post}\) and also a lengthier post-fault simulation. In practice, a precise base critical trajectory is only required if slight variations in \(p\) can result in drastically different relevant portions of the stability boundary resulting in a non-differentiable CCT vs \(p\) curve.

B. Scalability

The obtained sensitivities are based on the characterization of the stability boundary for DAE systems, which hold in general [15]. The same characterizations also enable the use of Direct methods [7], which are used for real-time TSA of large-scale systems [19]. When evaluating the derived formulas for large-scale systems, the main computational burden comes from obtaining \(\partial_{p} x^{cu} |_{x^{cu}, y^{cu}_{post}, p^{}}\) in (15) and \(\partial_{p} y^{cu}_{post} |_{p^{}}\) in (21). Given a parameter varying DAE system in (10), the sensitivity of the solution to any generic parameter \(\alpha\) can be obtained by integrating the following variation equations
\[
\frac{d}{dt}(\alpha_{\varphi^{x}}) = \partial_{\alpha} f_{\varphi^{x}} + \partial_{y} f_{\varphi^{x}} \cdot \partial_{\alpha} \varphi^{y} + \partial_{\alpha} g_{\varphi^{y}} \cdot \partial_{\alpha} g_{\varphi^{y}} + \partial_{\alpha} g_{\varphi^{y}}
\]
(30)

For \(\partial_{p} x^{cu} |_{x^{cu}, y^{cu}_{post}, p^{}}\) and \(\partial_{p} y^{cu}_{post} |_{p^{}}\), the initial conditions for integration are \(I(\alpha, x, y)\) and \(I(\alpha, 0, 1)\) respectively with \(f = f_{fault}\) and \(g = g_{fault}\). Similarly, for \(\partial_{p} x^{cu} |_{x^{cu}, y^{cu}_{post}, p^{}}\) and \(\partial_{p} y^{cu}_{post} |_{p^{}}\), these are also \(I(\alpha, x, y)\) and \(I(\alpha, 0, 1)\) respectively, but with \(f = f_{post}\) and \(g = g_{post}\). Integrating the variational equations in (30) in conjunction with the original state equation in (10) adds \(n + m\) new state variables for each scalar \(\alpha\) and therefore is intractable for large-scale systems. However, since the above sensitivity variables do not influence the state dynamics of \((x, y)\), solving (30) separately as a linear time-varying system is much more efficient and feasible for large-scale systems [20]. The time-varying state matrices \((\partial_{\alpha} f, \partial_{\alpha} \varphi^{x}, \partial_{\alpha} g)\) and the inputs \((\partial_{\alpha} f, \partial_{\alpha} g)\) are obtained at each step from the solution of the original state equations (10). Furthermore, these state matrices are a byproduct of integrating the state equations in (10) and therefore do not
need to be computed again. The overall approach is referred to as the Staggered Direct Method (SDM) and is adopted here. It is to be noted that for the case of multiple $\alpha$, the state matrices are the same and only the inputs differ. Thus, an efficient way of numerically integrating (30) for multiple $\alpha$’s is to construct a sensitivity matrix with columns representing the state sensitivity vector with respect to each $\alpha$. This is then solved as a linear matrix equation of the form $A(t)X(t) = B(t)$ by inverting a single state matrix $A(t)$ at each time; noting that the size of $A(t)$ is independent of the number of $\alpha$’s. Solving these linear equations can be further optimized by using sparsity techniques as shown in [21]. There are also other efficient approaches [20] for solving (30) which are outside the scope of this work.

V. Results

This section uses a few standard test systems from [15] to demonstrate the validity of the derived expressions in (19) and (25). These models are low dimensional and therefore easy to visualize. Furthermore, to illustrate the computational burden when evaluating these expressions for larger systems, the 39 bus 10 generator system is used.

A. Case 1: Example 7-5 [15]

To start, a non-power system model that has been adopted from Example 7-5 of [15] is discussed. This system has both a transverse saddle pseudo-equilibrium point as well as a semi-saddle point on the stability boundary, which makes it a good choice for illustration purposes.

$$ f_{\text{post}}(x, y, p) = \begin{bmatrix} y^2 + py - 2x_1 + x_2 + 1 \\ -x_2 \\ x_1y - py - x_2 + y^3 \\ 3y^2 - p + x_1 \\ 2 - y(p - x_1 + 1) - x_2 \\ 12y + 6x_1y^2 - (6x_2y + 6py^2 + 6y^2) \end{bmatrix} $$

$$ \kappa_{\text{post}}(x, y, p) = 12y + 6x_1y^2 - (6x_2y + 6py^2 + 6y^2) $$

$$ g_{\text{fault}}(x, y, p) = \begin{bmatrix} x_2 \\ -1 \end{bmatrix} $$

$$ g_{\text{fault}} = g_{\text{post}} $$

The true CCT vs $p$ is plotted in Fig. 3 using a black line. A few points on that curve are marked using bold magenta star-shaped markers, which represent operating conditions differing in the value of $p$. The marker type represents the mode of instability as being the base critical post-fault trajectory intersecting a semi-saddle point. At each of these points, a dotted straight line is drawn with a slope equal to the CCT sensitivity value computed using (25). These serve as local estimates to the CCT vs $p$ curve. It can be seen that these estimates are tangential to the actual curve which validates the derived expression in (25).

For a visual insight, the phase portrait of this system is shown in Fig. 4 for $p^* = 0$. The SEP of interest is marked in blue, the relevant semi-saddle point at (0, 0, 0) is marked pink and a transverse saddle pseudo EP is marked in red at (−3, −2, 1). The singular surface is traced using a bold yellow line. The base critical trajectory, which is shown by the bold black curve, clearly intersects the singular surface tangentially at the semi-saddle point and thus validates the mode of instability.

B. Case 2: One Machine One Bus System

Next, the remaining sensitivity expressions are obtained for a one bus one machine model with the bus voltage angle taken as a reference.

$$ f_{\text{post}}(x, y, p) = \begin{bmatrix} x_2 + \frac{P_m - E_x \sin(x_1)}{D_l} \\ \frac{E_y}{X} \cos(x_1) - \frac{y^2}{X} - Q_l \end{bmatrix} $$

$$ g_{\text{post}}(x, y, p) = \frac{E_y}{X} \cos(x_1) - \frac{y^2}{X} - Q_l $$

$$ g_{\text{fault}}(x, y, p) = y $$

where $x_1$ is the deviation of generator rotor angle from bus phase angle, $x_2$ is the generator angular speed deviation and $y$ is the bus voltage magnitude. The parameters $p$ comprise of the generator inertia constant $M$, mechanical power input to the generator $P_m$, generator damping $D_l$, internal emf of the generator $E$, the reactive power load at the bus $Q_l$, the load damping factor $D_l$ and $X$, the total series impedance (internal impedance of generator plus the transformer and transmission line impedances).

The fault under study is a 3 phase to ground fault on the bus, which is cleared without changing the network topology, i.e., pre-fault and post-fault systems are the same. The variation of CCT with generator dispatch $P_m$ is analyzed first for which the CCT sensitivity expressions are obtained. Let $p^* = [X = 0.5, P_m = 0.3 : 0.5, E = 1, M = 1, D_l = 1, D_q = 1, Q_l = 0.1]$.

In Fig. 5, the CCT is plotted vs $P_m$. As expected, the CCT reduces with an increase in generator loading due to the SEP.
moving closer to the stability boundary and also the fault-on trajectory having a larger accelerating. Here, the points corresponding to various values of $p^* = P^*_m$ are highlighted in bold using different markers to distinguish between the underlying mode of instability. Points marked using green diamond-shaped markers represent operating conditions for which the critical fault trajectory results in a loss of synchronism of the machine. On the other hand, the points shown using red triangle-shaped markers have the base critical post-fault trajectory intersecting a transverse saddle pseudo EP. This means that the mechanism for instability changes as the generator dispatch $P_m$ goes beyond 0.4 per unit and therefore, the appropriate CCT sensitivity expressions are used, i.e., (25) for red triangular points and (29) for green diamond-shaped points. The local estimates for the actual curve obtained using those expressions are shown using dotted lines as done previously. It can be seen that the estimates are tangential to the real curve which validates both (25) and (29).

To visualize the transition in the instability mechanism due to variations in $P_m$, the phase portraits for $P_m = 0.3$ and 0.5 per unit are shown in Fig. 6 and Fig. 7, respectively. The relevant portion of the singular surface in these cases can be seen as a one-dimensional component (yellow line) forming the nose of the constraint surface shown in grey. In both the figures, the pseudo EP and the UEP of interest are marked.

In Fig. 6, since the UEP is beyond the singular surface, it cannot lie on the stability boundary. The base critical trajectory, which is given by the bold black curve, can be seen intersecting the singular surface transversally at the pseudo EP marked in red-point. As $P_m$ increases to 0.5, the UEP crosses the singular surface and now lies on the stability boundary as seen from Fig. 7. Therefore, instability now happens through a loss of synchronism as opposed to voltage collapse, which is seen from the path of the base critical trajectory. Another very important observation that can be made by comparing Fig. 6 with Fig. 7 is that what was earlier the stable manifold of transverse saddle pseudo EP now becomes the stable manifold of the type-1 UEP. Therefore, the structure/governing equations of the relevant portion of the stability boundary effectively stay the same, and only the critical element lying on it changes. Now, this manifold changes smoothly with $p$, which explains why the CCT vs $p$ curve remains smooth even at the point of transition of instability mechanism at $P_m = 0.4$.

As the pseudo EP becomes irrelevant with increasing $P_m$, its dynamic nature also changes. This can be understood by plotting the dynamics in its vicinity for $P_m = 0.3$ and 0.5 as shown in Fig. 8. In the top subplot, one can notice a single-dimensional stable and unstable manifold for this point, and thus it is classified as a saddle type. In the lower subplot which corresponds to $P_m = 0.5$, the same point changes to a source type (both unstable eigenvalues) and thus repels trajectories in its vicinity. Furthermore, from the oscillatory nature of the unstable dynamics, it is safe to assume that the two unstable eigenvalues are complex conjugates of each other as opposed to both being real for lower values of $P_m$. 
Next, the load model is modified to include a term dependent on the local generator speed deviation $x_2$. This modifies the algebraic constraint as $g_{\text{post}}(x, y, p) = E_y x_1 - y^2 - Q_l(1 + x_2)$. This distorts the constraint surface in a way that a major portion of the singular surface now lies on the stability boundary, which will be seen later.

The effects of a change in inertia $M$, which has become relevant in the present scenario with increasing penetration of renewable generation, are studied. Let, $p^* = [X = 0.5, P_m = 0.5, E = 1, M = 0.1: 0.4, D_1 = 1, D_2 = 1, Q_1 = 0.1]$. The CCT is plotted against $M$ in Fig. 9. Since the mode of instability changes from the fault-on trajectory directly intersecting the singular surface to loss of synchronism of the generator, the CCT sensitivity estimates are obtained using (19) and (29) respectively. These are shown by yellow and green dotted lines along with their corresponding operating conditions. Once again, the estimates are tangential to the actual curve which validates those expressions.

Unlike the previous case, the CCT vs $p = M$ curve is not differentiable at the point of transition of the instability mechanism, which is somewhere between $M = 0.2$ and $M = 0.25$. To explain this, the phase portrait for $M = 0.2$ is shown in Fig. 10. Here, the relevant portion of the local stable manifold of a type 1 UEP (green) lying on the stability boundary is marked with a bold green line. This intersects the singular surface (yellow curve) transversally [14]. From (27), it can be seen that $g_{\text{post}}$ and consequently the singular surface itself are functions of the network topology and therefore do not depend on $M$. On the other hand, the stable manifold of the type-1 UEP is sensitive to it. Therefore, as $M$ varies, the relevant portion of the stability boundary switches from a manifold not sensitive to $M$ to one that is sensitive. This results in a sudden change in the slope of the CCT vs $M$ curve at the point of transition, and hence, it is non-differentiable. As discussed in Section IV-A, the CCT sensitivity expressions could suffer from ill-conditioning problems when evaluating close to the transition point, thus requiring high precision in computing the base critical trajectory.

### C. Case 3: 39 Bus 10 Machine System

This section considers the New England 39 bus model. The network parameters and the values for constant PQ loads are taken from [22]. Since the singular surface is a function of the network, for simplicity, the classical machine models are used to represent the generator dynamics while preserving the full network structure. The generator model parameters are given below.

Since this is a large-scale system with $n = 18$ (machine 2 is taken as a reference) and $m = 78$, it is difficult to gain a visual insight into the stability region. However, the purpose of this section is to estimate the computational burden when evaluating the CCT sensitivity expressions once the base critical trajectory is obtained as a part of the transient stability assessment. The computations are performed on a consumer PC with a 2.6 GHz Intel Core i7 processor in the MATLAB R2019a environment.

A three-phase to ground fault is studied at bus 21, which is cleared by tripping the line 21–22. The effect of unit 6’s field voltage $E_{fd6}$ is studied. The CCT vs $E_{fd6}$ curve along with the CCT sensitivity estimates are plotted in Fig. 11. For the range of $E_{fd6}$ values studied, the instability happens through a voltage collapse at bus 21 in the post-fault phase. The base critical post-fault trajectory in each of those cases intersects the singular surface at a transverse saddle pseudo EP and therefore, the expression in (25) is used for computing the CCT sensitivities.
Now, since unit 6 is fairly close to bus 21 at which the voltage collapse occurs, overexciting this generator has a net positive impact of increasing the voltage stability margin. This explains the CCT trend observed in Fig. 11.

Once the base critical trajectory is obtained as a part of the transient stability analysis using time-domain simulations, the state values along the trajectory are used to directly compute the trajectory sensitivities of the fault-on and post-fault system using SDM. Computing the sensitivities along a 5s trajectory with respect to $x(0)$ and 30 other parameters, i.e., a total of 48 parameters, takes about 1s. As a reference, integrating the original DAE system for the same amount of time using the ode15s solver takes around 3s. This appears surprising at first since the number of sensitivity variables is $96 \times 48 = 4608$, which is much larger than the number of state variables, which is 96. However, recalling the results in Section IV-B, the sensitivity matrices are obtained at each time step by solving a linear set of equations of size $96 \times 96$. On the other hand, numerically solving the state equations requires iteratively solving nonlinear equations with a $96 \times 96$ Jacobian matrix, which is computed and inverted multiple times and thus, much slower. This demonstrates the scalability of the proposed approach to larger systems.

VI. CONCLUSIONS AND FUTURE WORK

In this paper, given the simulation results of a transient stability assessment study for a power system DAE model, expressions for the sensitivity of CCT to parameter variations are derived. Besides the traditional instability mechanism of loss of synchronism of generator(s), DAE models also exhibit a phenomenon where the system trajectory reaches a region in state space marked by the singularity of algebraic constraints, which is closely related to voltage collapse. This is particularly relevant to the study of systems operating in weak conditions with some unmodeled load dynamics, which results in such singularities influencing the size of the stability region. Due to multiple possible mechanisms of instability, a separate CCT sensitivity expression is derived for each. The derived expressions were shown to be valid when compared with the more computationally intensive time-domain simulations. It was also observed that when studying the effect of parameters that unequally impact neighboring components of the stability boundary, there can be situations where the instability mechanism changes under parameter variations, yielding a non-differentiable CCT vs parameter curve. These scenarios were found to require a high precision due to the ill-conditioning of the derived CCT sensitivity expressions when evaluated at parameter values close to the transition points. Furthermore, since a major part of the overall computation involves calculating the trajectory sensitivities, an efficient way for computing these using the Staggered Direct Method was presented. This enables large-scale systems applications.

A potential application of this work is for identifying effective controls for enhancing CCT for faults suffering from voltage collapse. Another application is for the dynamic security assessment of networks having uncertainties in operating conditions due to high renewable penetration. These will be explored in the future.

REFERENCES

[1] I. Dobson, H. D. Chiang, J. S. Thorp, and L. Fekih-Ahmed, “A model of voltage collapse in electric power systems,” in Proc. 27th IEEE Conf. Decis. Control, Dec. 1988, pp. 2104–2109.
[2] V. Venkatasubramanian, H. Schättler, and J. Zaborszky, “Analysis of local bifurcation mechanisms in large differential-algebraic systems such as the power system,” in Proc. 52nd IEEE Conf. Decis. Control, Dec. 1993, pp. 3727–3733.
[3] A. Hiskens and D. J. Hill, “Failure modes of a collapsing power system,” in Proc. NSF/ECC Workshop Bulk Power Syst. Voltage Phenomena II, 1991, pp. 53–63.
[4] T. van Cutsem and C. Vournas, Voltage Stability of Electric Power Systems. Vienna, Austria: Springer-Verlag, 1998.
[5] K. L. Praprotnik and K. A. Loparo, “An energy function method for determining voltage collapse during a power system transient,” IEEE Trans. Circuits Syst. Fundam. Theory Appl., vol. 41, no. 10, pp. 655–651, Oct. 1994.
[6] I. A. Hiskens and D. J. Hill, “Energy functions, transient stability and voltage behaviour in power systems with nonlinear loads,” IEEE Trans. Power Syst., vol. 4, no. 4, pp. 1525–1533, Nov. 1989.
[7] H.-D. Chiang, Direct Methods for Stability Analysis of Electric Power Systems, Theoretical Foundation, BCU Methodologies, and Applications. Hoboken, NJ, USA: Wiley, 2011.
[8] S. Ayasun, Y. Liang, and C. O. Nwankpa, “A sensitivity approach for computation of the probability density function of critical clearing time and probability of stability in power system transient stability analysis,” Appl. Math. Comput., vol. 176, no. 2, pp. 563–576, May 2006.
[9] T. B. Nguyen, M. A. Pai, and I. A. Hiskens, “Sensitivity approaches for direct computation of critical parameters in a power system,” Int. J. Electr. Power Energy Syst., vol. 24, no. 5, pp. 337–343, Jun. 2002.
[10] M. J. Laffenberg and M. A. Pai, “A new approach to dynamic security assessment using trajectory sensitivities,” in Proc. 20th Int. Conf. Power Industry Comput. Appl., May 1997, pp. 272–277.
[11] L. G. W. Roberts, A. R. Champneys, K. R. W. Bell, and M. di Bernardo, “Analytical approximations of critical clearing time for parametric analysis of power system transient stability,” IEEE Emerg. Sel. Top. Circuits Syst., vol. 5, no. 3, pp. 465–476, Sep. 2015.
[12] S. Sharma, S. Pushpaka, V. Chinde, and I. Dobson, “Sensitivity of transient stability critical clearing time,” IEEE Trans. Power Syst., vol. 33, no. 6, pp. 6476–6486, Nov. 2018.
[13] C. Mishra, R. S. Biswas, A. Pal, and Y. A. Centeno, “Critical clearing time sensitivity for inequality constrained systems,” IEEE Trans. Power Syst., vol. 35, no. 2, pp. 1572–1583, May 2020.
[14] J. M. Lee, Introduction to Smooth Manifolds. New York, NY, USA: Springer-Verlag, 2003.
[15] H.-D. Chiang and L. F. Alberto, Stability Regions of Nonlinear Dynamical Systems: Theory, Estimation, and Applications. Cambridge, MA, USA: Cambridge Univ. Press, 2015.
[16] V. Venkatasubramanian, H. Schättler, and J. Zaborszky, “Stability regions for differential-algebraic systems,” in Systems, Models and Feedback: Theory and Applications: Proceedings of a U.S.-Italy Workshop in Honor of Professor Antonio Ruberti, Capri, 15–17, Jun. 1992, A. Isidori and T.-J. Tam, Eds. Boston, MA, USA: Birkhäuser Boston, 1992, pp. 385–402.
[17] C. Mishra, J. S. Thorp, V. A. Centeno, and A. Pal, “Estimating relevant portion of stability region using Lyapunov approach and sum of squares,” in Proc. IEEE Power Energy Soc. Gen. Meeting, Aug. 2018, pp. 1–5.
[18] I. A. Hiskens and M. A. Pai, “Trajectory sensitivity analysis of hybrid systems,” IEEE Trans. Circuits Syst. Fundam. Theory Appl., vol. 47, no. 2, pp. 204–220, Feb. 2000.
[19] H. Chiang, J. Tong, and Y. Tada, “On-line transient stability screening of 14,000-bus models using TEPCO-BCU: Evaluations and methods,” IEEE PES Gen. Meeting, Jul. 2010, pp. 1–8, doi: 10.1109/PES.2010.559026.
[20] D. Chaniotis, M. A. Pai, and I. Hiskens, “Sensitivity analysis of differential-algebraic systems using the GMRES method-application to power systems,” in Proc. 2001 IEEE Int. Symp. Circuits Syst. (Cat. No.01CH37196), May 2001, pp. 117–120.
[21] G. Hou, V. Vital, G. Heydt, D. Tylavsky, J. Si, and A. S. University, “Trajectory sensitivity based power system dynamic security assessment,” in ASU Electronic Theses and Dissertations, Arizona State Univ., Tempe, AZ, USA, 2012.
[22] M. A. Pai, Energy Function Analysis For Power System Stability. Vienna, Austria: Springer-Verlag, 1989.