Research Article

Prediction of Surface Subsidence Extension due to Underground Caving: A Case Study of Hemushan Iron Mine in China

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Underground caving can potentially lead to large-scale surface destruction. To test the safety conditions of the surface construction projects near the circular surface subsidence zone in the Hemushan Iron Mine, this paper proposes an analytical model to analyze the stability of the cylindrical caved space by employing the long-term strength of the surrounding rock mass, the in situ stress, and the impact of caved materials as inputs. The proposed model is valid for predicting the orientation and depth where rock failure occurs and for calculating the maximum depth of the undercut, above which the surrounding rock mass of the caved space can remain stable for a long duration of time. The prediction for the Hemushan Iron Mine from the proposed model reveals that the construction projects can maintain safe working conditions, and such prediction is also demonstrated by the records from Google Earth satellite images. This means that the proposed model is valid for conducting such analysis. Additionally, to prevent rock failure above the free surface of caved materials, backfilling the subsidence zone with waste rocks is suggested, and such a measure is implemented in the Hemushan Iron Mine. The monitoring results show that this measure contributes to protecting the surrounding wall of the caved space from large-scale slip failure. The contribution of this work not only provides a robust analytical model for predicting the stability of rock around a cylindrical caved space but also introduces employable measures for mitigating the subsequent extension of surface subsidence after vertical caving.

1. Introduction

Surface subsidence due to underground mining is commonly observed in caving-based construction projects (Figure 1), the shape of which is composed of long or circular shape generally. The associated surface destruction endangers construction near the subsidence zone [1, 2].

To predict the surface subsidence due to underground caving, numerous methods have been proposed, such as empirical models, analytical solutions [3, 4], and numerical simulations [5–7]. Laubscher [8] proposed an empirical model to predict the angle of break based on the mining rock mass rating (MRMR), depth of the mined block, minimum and maximum span of the undercut, and density and height of the caved material (Figure 2). This model is commonly utilized to conduct preliminary estimations, but further corrections are still required for the analysis of rock failure mechanisms.

Rock failure mechanisms involved in surface subsidence due to underground caving include vertical caving of the overburdened rocks, progressive caving of the surrounding rocks, and toppling of steeply dipping rock strata [9]. These rock failures can be divided into the following steps [10]: (1) overburdened rocks vertically cave into the void formed after ore excavation; (2) tension cracks appear on the surface marking the new boundary of the subsidence zone; (3) the fractured rocks move downwards on the steeply inclined shear failure surface; and (4) new tension cracks and shear surfaces form expansion of the subsidence zone. Brady and Brown [9] provided a robust analytical solution for vertical
caving. After vertical caving, the subsequent surface subsidence, i.e., progressive caving and rock strata toppling, leads to the extension of the caved space. In situ data reveal that shear failure at the surrounding wall of a caved space is the primary failure mechanism for subsequent surface subsidence [11]. Accordingly, it can be expected that neither the subsequent surface subsidence nor associated large-scale surface destruction will take place if the shear failure at the surrounding rocks of the caved space is prevented after the vertical caving of the overburdened rocks. Some analytical solutions have been provided for such shear failure. One of the most commonly cited models is Hoek’s model [12]. Hoek proposed a limiting equilibrium analysis that relates the shear failure that occurs in the process of progressive hanging-wall caving to the thrust on the shear failure plane due to the caved material, base area, and weight of the wedge of the sliding rock mass. Brown and Ferguson [13] extended Hoek’s model by involving the impact of water pressure and a sloping ground surface on the shear failure plane. Lupo [11] presented a modified solution that enables Hoek’s model to predict rock failure on both hanging walls and footwalls. Even though Hoek’s and associated extended approaches have been successfully applied in many projects [14, 15], their employment is still restricted due to a basic assumption; i.e., the deposit excavation, as well as the vertical caving and subsequent surface subsidence, takes place for a long distance compared with the cross section normal to the strike of the ore body, and the problem can be reduced to two dimensions. This means that Hoek and associated extended models are valid for projects with long-distance subsidence (e.g., Kiruna Iron Mine) rather than circular subsidence (e.g., San Manuel Copper Mine) because the basic assumption is not satisfied for circular subsidence. This means that a new model that is valid for predicting the stability of rocks around a cylindrical caved space is required because current models are unable to conduct such predictions.

Therefore, to predict the extension of the circular surface subsidence in the Hemushan Iron Mine, we first propose an analytical solution for analyzing the stability of the surrounding wall of the cylindrical caved space by employing the in situ stress, rock strength, and the impact of the caved materials as inputs. Then, the prediction of the proposed model is compared with that from the in situ monitoring to test the validity of the model. Additionally, to protect the surrounding rock above the free surface of the caved materials, it is suggested to backfill the caved space with waste rocks. Finally, the limitations regarding the proposed model and associated measures to enhance the stability of caved space walls are discussed.

2. Engineering Background

The Hemushan Iron Mine, located in Anhui, is one of the most representative caving-based projects in the Middle-Lower Yangtze River Valley iron ore clusters of China [16]. The thickness of the magnetic deposit in the Hemushan Iron Mine varies between 10 m and 75 m, and the length is 130 m. The ore body and surrounding rock are composed of magnetite and limestone, respectively. This project is conducted by the sublevel caving method. Due to the requirement of protecting the construction near the subsidence zone (less than 80 m, Figure 1(c)), the extension
of the subsidence zone is predicted to test whether these construction projects will maintain safe operating conditions during the mining process. Figure 3 shows the distribution of the subsidence zone and the constructions on the ground surface, as well as the caved space in three-dimensional (3D) space.

Figure 3 shows that the shape of the caved space in the Hemushan Iron Mine is nearly cylindrical. This means that Hoek’s model and associated extended models are unable to predict the extension of surface subsidence in the Hemushan Iron Mine because the basic assumptions for Hoek’s solution, i.e., that deposit excavation, vertical caving, and subsequent surface subsidence occur for a long distance compared with the cross section normal to the strike of the ore body, are not satisfied. An analytical model for the stability of the surrounding rocks of such a cylindrical caved space is required to predict the extension of the surface subsidence zone due to rock failure in the surrounding rocks.

To conduct such modeling, the caved space in the Hemushan Iron Mine is assumed to be a cylinder filled with caved materials, whose axis is normal to the plane consisting of the maximum and minimum in situ stress, as illustrated in Figure 4. The following inputs are involved for modeling, including the in situ stresses and associated orientations, the long-term strength of the rock mass at the caved space wall, and the impact of the caved materials on the surrounding rocks, including the horizontal stress from the caved materials on the caved space wall, and the friction between the caved materials and caved space wall.

### 2.1. In Situ Stress

The redistribution of the in situ stress is the major cause of rock failure [17]. The in situ stress is commonly assumed to be three mutually orthogonal principal stresses [18]: vertical stress \( \sigma_V \), maximum horizontal stress and minimum horizontal stress \( \sigma_H \) and \( \sigma_h \). Conventionally, the in situ stress can be obtained by in situ tests [19], such as overcoring and hydraulic fracturing (HF), and core-based tests, such as anelastic strain recovery (ASR), differential strain curve analysis (DSCA), and acoustic emission analysis (AE). Numerous tests and literature demonstrate that the vertical, maximum, and minimum horizontal in situ stresses linearly vary with the depth [20, 21], and such relationships can be expressed as follows:

\[
\begin{align*}
\sigma_H &= a_H z + b_H, \\
\sigma_h &= a_h z + b_h, \\
\sigma_V &= a_V z + b_V, 
\end{align*}
\]

where \( \sigma_H \) and \( \sigma_h \) are the maximum and minimum horizontal in situ stresses in kPa, respectively; \( \sigma_V \) is the vertical in situ stress in kPa; \( z \) is the depth from the local ground surface in m; \( a_H, b_H, a_h, b_h, \) and \( a_V \) are constants and can be obtained by in situ or core-based tests; \( a_H, a_h, \) and \( a_V \) are in kPa/m, respectively; \( b_H, b_h, \) and \( b_V, \) are in kPa, respectively. To obtain the relationship between the three in situ stresses and the depth in the Hemushan Iron Mine, we regressed 21 sets of data near the project, provided by the Fundamental Database of Crustal Stress Environment in Continental China. The orientation of the minimum horizontal in situ stress is N70°E [22]. The relationship between the in situ stress and the depth from the local ground surface in the Hemushan Iron Mine can be expressed as follows:

\[
\begin{align*}
\sigma_H &= 0.0274z + 8.7993, \\
\sigma_h &= 0.0134z + 5.2201, \\
\sigma_V &= 0.0148z + 2.3576. 
\end{align*}
\]

### 2.2. Strength of Rocks

The rock strength is a key input for analyzing the stability of caved space walls. Numerous methods have been provided for the rock strength, among which the laboratory core test is preferable [23]. However, due to a lack of core data, indirect methods, such as point load tests, are sometimes employed. The result from the
point load test can be converted into the result for the uniaxial compressive strength (UCS) by proposed linear relationships [24]. The ISRM [25] provides a correlation between the point load strength index ($I_p$) and the uniaxial compressive strength ($\sigma_c$):

$$\sigma_c = \xi \cdot I_p,$$

where $\sigma_c$ is the uniaxial compressive strength of rock in kPa; $\xi$ is the conversion factor from the results from the point load test to the uniaxial compressive strength of rock, and $\xi = 20 \sim 50$; and $I_p$ is the point load strength index obtained by the point load test in kPa.

Laboratory tests reveal that the rock strength decreases as time increases due to rock relaxation or creep [26, 27]. This means that rock failure will eventually occur after a time delay, even though the constant stress applied to the rocks is smaller than the strength of the rocks. Therefore, a long-term strength [28] value is employed to analyze the stability of the surrounding rock of a caved space over a long duration of time. The long-term strength ($\sigma_{cd}$) is linearly related to the uniaxial compressive strength without the time effect:

$$\sigma_{cd} = \alpha \sigma_c',$$

where $\sigma_{cd}$ is the long-term strength of rock in kPa; $\sigma_c'$ is the uniaxial compressive strength of rock without a time effect in kPa; and $\alpha$ is a constant that equals 0.4–0.6 for intact rocks without preexisting fractures [29].

Additionally, the impact of discontinuities (e.g., faults and beddings) on the rock strength should be addressed. The significant reduction in the rock strength due to inherent discontinuities has been discussed in the literature [30–33]. The rock mass integrity index [34, 35] is employed to consider such a reduction in the rock strength due to inherent discontinuities. The relationship between the strength of the rock mass with discontinuities and the strength of intact rocks is provided as follows:

$$\sigma_{mc} = K_v \sigma_c'',$$

where $\sigma_{mc}$ is the strength of the rock mass with discontinuities in kPa; $\sigma_c''$ is the strength of intact rock without discontinuities in kPa; $K_v$ is the rock mass integrity index, which can be obtained by an in situ test; and $K_v = (\nu_m / \nu_d)^2$, where $\nu_m$ and $\nu_d$ are the speeds of the elastic waves in the rock mass with discontinuities and intact rocks, respectively.

To obtain the properties of the rock mass in the Hemushan Iron Mine, we conducted a point load test and an in situ geological survey, and the associated parameters are listed in Table 1.

2.3. The Impact of Caved Materials. In Hoek’s approach, the caved materials contribute to the stability of the caved space wall by providing a thrust force on the shear failure plane. However, the impact of the caved materials on the caved space wall in the Hemushan Iron Mine cannot be described by Hoek’s solution because the basic assumption of Hoek’s solution (i.e., that the vertical caving and subsequent surface subsidence occur for a long distance compared with the cross section normal to the strike of ore body) is not satisfied. However, scaled laboratory tests [36] have demonstrated that Janssen’s solution [37] is valid for describing the horizontal stress of caved materials at varying depths. In Janssen’s solution, the vertical and horizontal stress in the granular caved materials and the surrounding wall can be analytically calculated by the following equations:

$$\sigma_{zz} = \frac{\rho b g D_e}{4 K \tan \phi} \left[ 1 - \exp \left( -\frac{4 K \tan \phi \omega \sqrt{z}}{D_e} \right) \right],$$

$$\sigma_{rr} = K \sigma_{zz} = \frac{\rho b g D_e}{4 \tan \phi} \left[ 1 - \exp \left( -\frac{4 K \tan \phi \omega \sqrt{z}}{D_e} \right) \right],$$

$$\tau_w = \sigma_{rr} \tan \phi = \frac{\rho b g D_e}{4} \left[ 1 - \exp \left( -\frac{4 K \tan \phi \omega \sqrt{z}}{D_e} \right) \right],$$

where $\sigma_{zz}$ and $\sigma_{rr}$ are the vertical and horizontal stresses of the granular caved materials, respectively; $\tau_w$ is the friction between the granular caved materials and caved space wall; $D_e$ is the diameter of the caved space filled with granular caved materials; $K$ is the Janssen coefficient, and $K = (1 + \sin \phi_b)/(1 - \sin \phi_b)$; $\phi_b$ is the internal friction angle in the granular caved materials; $\phi$ is the friction angle between the granular caved materials and the surrounding wall; $z$ is the depth of the granular caved materials from the free surface of the caved materials, and $z = z - z_0$; and $z_0$ is the depth of the free surface of the caved materials measured from the local ground surface. To analyze the stability of the caved space wall in the Hemushan Iron Mine, we tested the in situ properties of the caved materials, and the parameters of the caved materials are summarized in Table 2.

### Table 1: Properties of the rock mass in the Hemushan Iron Mine. | $I_p$ (kPa) | $\xi$ | $\alpha$ | $K_v$ | $\phi$ (°) |
|---|---|---|---|---|
| 3150 | 40 | 0.55 | 0.66 | 37 |

3. Analytical Prediction for the Surface Subsidence Extension in the Hemushan Iron Mine

When the iron deposits are excavated, stress redistribution occurs near the caved space. The modeling of the stability of the caved space wall is primarily performed to compare such redistributed stress with the strength of the rock mass at the caved space wall with an appropriate failure criterion. Therefore, Kirsch’s elastic solution is employed to obtain the redistributed stress in the element near the surrounding wall on the cross section of the cylindrical caved space (Figure 4), and the redistributed stress can be expressed as follows:
\[ \sigma_r = \frac{\sigma_H + \sigma_h}{2} \left( 1 - \frac{R^2}{r^2} \right) + \frac{\sigma_H - \sigma_h}{2} \left( 1 - 4 \frac{R^2}{r^2} + 3 \frac{R^4}{r^4} \right) \cos 2\theta + \frac{R^2}{r^2} \tau_{rr}, \]

\[ \sigma_\theta = \frac{\sigma_H + \sigma_h}{2} \left( 1 + \frac{R^2}{r^2} \right) - \frac{\sigma_H - \sigma_h}{2} \left( 1 + 3 \frac{R^4}{r^4} \right) \cos 2\theta - \frac{R^2}{r^2} \tau_{rr}, \]

\[ \sigma_z = \sigma_v - 2v(\sigma_H - \sigma_h) \frac{R^2}{r^2} \cos 2\theta, \]

\[ \tau_{rz} = \frac{R^2}{r^2} \tau_{rz} \]

\[ \tau_{r\theta} = \tau_{r\theta} = 0, \]

where \( \sigma_r, \sigma_\theta, \) and \( \sigma_z \) are the radial, tangential, and axial normal stresses near the cave space wall, respectively; \( \tau_{r\phi}, \tau_{r\theta}, \) and \( \tau_{r\phi} \) are the radial, tangential, and axial shear stresses near the cave space wall, respectively; \( \theta \) is the angle representing the orientation of the element on the cross section of the cave space, measured from the direction of the maximum horizontal stress (\( \sigma_H \)); \( \theta = 0^\circ \) represents the direction of the maximum horizontal in situ stress (\( \sigma_H \)); \( \theta = 90^\circ \) represents the direction of the minimum horizontal in situ stress (\( \sigma_h \)); and \( v \) is Poisson’s ratio of the surrounding rock mass.

The normal and shear stress in the element at the cave space wall can be obtained from equation (7):

\[ \sigma_r = \sigma_{rr}, \]

\[ \sigma_\theta = \sigma_H + \sigma_h - 2(\sigma_H - \sigma_h) \cos 2\theta - \sigma_{rr}, \]

\[ \sigma_z = \sigma_v - 2v(\sigma_H - \sigma_h) \cos 2\theta, \]

\[ \tau_{rz} = \tau_\phi = \tan \phi_\omega \sigma_{rr}, \]

\[ \tau_{r\theta} = \tau_{r\phi} = 0. \]

It should be noted that the tangential stress (\( \sigma_\theta \)) is always one of the three principal stresses because \( \tau_{r\phi} = \tau_{r\phi} = 0. \) However, the geological properties of the Hemushan Iron Mine, i.e., the in situ stress (equation (2)), the properties of the surrounding rock mass (Table 1), and the caved materials (Table 2) indicate that the three normal stresses on the caved space satisfy \( \sigma_H > \sigma_z > \sigma_r \) in the depth range where the deposits are distributed. The three principal stresses at the caved space wall can thus be calculated from equation (8) and can be expressed as follows:

\[ \sigma_{r_{max}} = \frac{\sigma_v - 2v(\sigma_H - \sigma_h) \cos 2\theta + \sigma_{rr}}{2} \left( \frac{\sigma_v - 2v(\sigma_H - \sigma_h) \cos 2\theta - \sigma_{rr}}{2} \right)^2 + (\tan \phi_\omega \sigma_{rr})^2, \]

\[ \sigma_{r_{min}} = \frac{\sigma_v - 2v(\sigma_H - \sigma_h) \cos 2\theta - \sigma_{rr}}{2} \left( \frac{\sigma_v - 2v(\sigma_H - \sigma_h) \cos 2\theta + \sigma_{rr}}{2} \right)^2 + (\tan \phi_\omega \sigma_{rr})^2, \]

\[ \sigma_{r_{0}} = \sigma_H + \sigma_h - 2(\sigma_H - \sigma_h) \cos 2\theta - \sigma_{rr}, \]

\[ \tan 2\alpha_0 = \frac{2\tau_\phi}{\sigma_{rr} - \sigma_v + 2v(\sigma_H - \sigma_h) \cos 2\theta}, \]

where \( \sigma_{r_{max}} \) and \( \sigma_{r_{min}} \) are the maximum and minimum principal stresses on the plane consisting of \( \sigma_{r_{max}} \) and \( \sigma_{r_{min}} \) respectively; \( \alpha_0 \) is the angle between the plane consisting of \( \sigma_r \) and \( \sigma_\theta \) and the plane consisting of \( \sigma_{r_{max}} \) and \( \sigma_{r_{min}} \).

If the strength of the rock is not sufficient to maintain the rock mass stability in the stress state, rock failure starts to occur. The Mohr–Coulomb criterion is valid for analyzing this type of rock failure. The Mohr–Coulomb criterion assumes that the maximum and minimum principal stresses satisfy the following relationship before shear failure occurs:

\[ \frac{2\sigma_c + (q + 2)\sigma_{rr} + q\sigma_v + 2(2 - qv)(\sigma_H - \sigma_h) \cos 2\theta - 2(\sigma_H + \sigma_h) \frac{2}{q} \left( \sigma_v - 2v(\sigma_H - \sigma_h) \cos 2\theta - \sigma_{rr} \right)^2 + (\sigma_{rr} \tan \phi_\omega)^2. \]
Because both $\sigma_y - 2\nu(\sigma_H - \sigma_z) \cos 2 \theta - \sigma_y > 0$ (i.e., $\sigma_z > \sigma_y$) and $\sigma_z \tan \phi_w > 0$ are satisfied, equation (10) can be simplified into

$$\sigma_i - (\sigma_H + \sigma_b) + 2(\sigma_H - \sigma_b) \cos 2\theta + (1 + q - q \tan \phi_w) \sigma_y \geq 0.\tag{12}$$

Equation (12) reveals that rock failure is most likely to occur in the orientation of the minimum horizontal in situ stress, i.e., $\theta = 90^\circ$. Substituting $\theta = 90^\circ$, the in situ stress ($\sigma_H$ and $\sigma_b$, in equation (1)), the impact of the caved materials ($\phi_w$ and $\sigma_y$, in equation (6)), and the long-term strength of the rock mass with the consideration of inherent discontinuities ($\sigma_v$ in equation (3), $\sigma_{cd}$ in equation (4), and $\sigma_{mc}$ in equation (5)) into equation (12), we can obtain the following relationship for analyzing whether rock failure will occur at the surrounding wall of the caved space with the consideration of inherent discontinuities over a long duration of time:

$$\frac{3a_H - a_h}{1 + q - q \tan \phi_w} z + \frac{\rho_b g D_e}{4 \tan \phi_w} \exp \left[ -\frac{4K \tan \phi_w}{D_e} \right] (z - z_0) \leq \frac{\xi a K_I_a - 3b_H + b_h}{4 \tan \phi_w} + \frac{\xi a K_I_a - 3b_H + b_h}{4 \tan \phi_w} + \frac{\xi a K_I_a - 3b_H + b_h}{4 \tan \phi_w} , \tag{13}$$

Equation (13) enables the calculation of the critical depth of the undercut, above which the surrounding rock mass at the caved space wall remains stable for a long duration of time. The solutions for such a depth (i.e., $z$ in equation (13)) can be expressed as follows:

$$\frac{3a_H - a_h}{1 + q - q \tan \phi_w} z_1 + \frac{\rho_b g D_e}{4 \tan \phi_w} \exp \left[ -\frac{4K \tan \phi_w}{D_e} \right] (z_1 - z_0) = \frac{\xi a K_I_a - 3b_H + b_h}{4 \tan \phi_w} + \frac{\xi a K_I_a - 3b_H + b_h}{4 \tan \phi_w} + \frac{\xi a K_I_a - 3b_H + b_h}{4 \tan \phi_w} , \tag{14}$$

If the implemented undercut is located above $z_1$, no rock failure will take place at the caved space wall under the free surface of the caved materials. It can be accordingly expected that no subsequent surface subsidence will take place after vertical caving.

If $z_2 > (\xi a K_I_a + b_h - 3b_H)/(3a_H - a_h)$, there are two solutions where $z = z_2$ and $z_3 (z_3 > z_4 > z_2 > z_0)$.

$$\frac{3a_H - a_h}{1 + q - q \tan \phi_w} - K \rho g b < 0, \tag{15}$$

$$\frac{3a_H - a_h}{1 + q - q \tan \phi_w} z_4 + \frac{\rho_b g D_e}{4 \tan \phi_w} \exp \left[ -\frac{4K \tan \phi_w}{D_e} \right] (z_4 - z_0) < \frac{\xi a K_I_a - 3b_H + b_h}{4 \tan \phi_w} + \frac{\xi a K_I_a - 3b_H + b_h}{4 \tan \phi_w} + \frac{\xi a K_I_a - 3b_H + b_h}{4 \tan \phi_w} , \tag{16}$$

However, if the inputs from the geological conditions cannot satisfy Solution (1) listed above, rock failure will eventually occur at the caved space wall. By substituting the in situ stresses ($\sigma_H$ and $\sigma_b$, in equation (1)), the impact of the caved materials ($\phi_w$ and $\sigma_y$, in equation (6)), and the long-term strength of the rock mass with the consideration of inherent discontinuities ($\sigma_v$ in equation (3), $\sigma_{cd}$ in equation (4), and $\sigma_{mc}$ in equation (5)) into equation (12), we can

| $\rho_b$ (t/m³) | $\phi_i$ (°) | $\phi_w$ (°) | $D_e$ (m) | $z_0$ (m) |
|----------------|-------------|-------------|----------|----------|
| 1.969          | 18          | 15          | 273      | 15       |

**Table 2:** Properties of the caved materials in the Hemushan Iron Mine.
obtain the following relationship for predicting the position, i.e., the orientation ($\theta$) and depth ($z$), where rock failure takes place at the caved space wall:

$$
\xi a K I_z - [(a_H + a_h) - 2(a_H - a_h)\cos 2\theta] z 
+ 2(b_H - b_h)\cos 2\theta - (b_H + b_h) 
+ \frac{(1 + q - q \tan \phi_w) \rho_b g D_e}{4 \tan \phi_w} \exp \left[ -\frac{4K \tan \phi_w}{D_e} (z - z_0) \right] \geq 0.
$$

Equation (17) enables the prediction of the extension of the surface subsidence. Such a prediction can be conducted by the following steps: (1) test whether the input from the geological conditions can satisfy Solution (1); (2) if Solution (1) is satisfied, neither rock failure nor associated extension of the surface subsidence will occur; (3) if Solution (1) is not satisfied, calculate the orientation ($\theta$) and depth ($z$) where rock failure occurs; and (4) the surface boundary of the extended subsidence zone can be obtained based on the caving angle, which can be calculated by analytical or empirical approaches (e.g., Laubscher [8], illustrated in Figure 2).

By employing the proposed approach, the subsequent surface subsidence in the Hemushan Iron Mine is predicted based on the geological parameters listed in equation (2), Tables 1 and 2. The provided inputs can satisfy the relationship in Solution (1), i.e., $z_0 \leq (35a K I_z + b_h - 3b_H)/ (3a_H - a_h)$, and the critical depth of the undercut is accordingly calculated by equation (14). Figure 5 shows the distribution of such a critical depth of undercut and the implemented undercut on a lengthwise section of the deposit. The implemented undercut (243.8 m) is located above the calculated critical depth of the undercut (324.8 m). This means that the surrounding rock mass between the free surface of the caved materials and the implemented undercut can remain stable, and the construction can remain safe from the associated subsequent surface subsidence.

4. Implication from the Prediction of the Surface Subsidence in the Hemushan Iron Mine

To test the validity of the proposed analytical model, the predictions are compared with the in situ observations for the Hemushan Iron Mine. Surface subsidence in the Hemushan Iron Mine was first observed on November 30, 2008. Google Earth satellite images recorded the variation in the subsidence zone, as shown in Figure 6.

Figure 6 shows that no large-scale extension of surface subsidence is observed for a long duration of time after vertical caving, and the construction on the ground surface near the subsidence zone remains safe during the period of underground mining. This phenomenon demonstrates that the proposed analytical solution is valid for predicting surface subsidence after vertical caving. However, slip failure occurs at the caved space wall above the free surface of the caved materials (Figure 6(c)). To mitigate the impact of this slip failure on the surface construction projects, backfilling the caved space is suggested for the Hemushan Iron Mine. The discussion of the impact of the rock mass strength and caved materials on the stability of the caved space walls can address the contribution of the backfilled caved materials on the stability of caved space walls. Equation (12) shows that the stability of the caved space wall heavily depends on the properties of the rock mass, i.e., the rock strength ($I_z$) and inherent discontinuities ($K_u$). However, rare measures can be utilized to economically improve the properties of the rock mass around the caved space [38, 39]. Equation (12) shows that the caved materials facilitate the stability of the caved space wall, and more details are provided in equation (13). Because $0 < \exp[-(4K \tan \phi_w/D_e) (z - z_0)] < 1$, the increase in the density of the caved materials (i.e., $\rho_b$ increases) contributes to the stability of the surrounding rock mass. Because $\tan \phi_w > 0$, the increase in the height of the caved materials (i.e., $z_0$ decreases) can produce the same contribution. This means that backfilling the caved space with excavated waste rocks contributes to preventing rock failure at the caved space wall. Therefore, such measure is suggested for the Hemushan Iron Mine, and the in situ observation shows the mitigation of the rock failure due to the proposed measure, as illustrated in Figures 6(b)–6(d).

On the other hand, some limitations regarding the proposed model and associated measures for enhancing the stability of the caved space wall should be noted. The analytical model employs Kirsch’s solution to obtain the redistributed stress near the caved space. Because Kirsch’s solution is developed for the stress on an infinite plate with a circular hole.
the proposed model is only valid for the cylindrical caved space. Once the anisotropic extension of the caved space takes place, the proposed model is unable to predict the stability of the caved space wall because the redistributed stress no longer matches Kirsch’s solution. Additionally, the proposed model is developed to predict the stability of the rock around the caved space after vertical caving, which means that it is not valid until surface subsidence due to vertical caving occurs. The proposed analytical solution demonstrates that increasing the height of the caved materials contributes to enhancing the stability of the caved space wall. It can be observed from equation (6) that such contribution has a maximum value at deep depths, which can be expressed as \( \lim_{z \to +\infty} \sigma_{rr} = \rho g D_j / 4 \tan \phi_w \). Additionally, equation (6) also indicates that the contribution due to the caved materials can be near its maximum value at shallow depths. For instance, because \( K \tan \phi_w = 0.5076 \), \( \sigma_{rr} \) (\( z = 420 \text{ m} \)) > 95\% with \( \lim_{z \to +\infty} \sigma_{rr} \) (i.e., \( z = 1.48 D_j \)) can be obtained for the Hemushan Iron Mine. This means that the proposed measured rock failure will eventually occur if the undercut depth continues to increase.

5. Conclusion

The stability analysis of caved space walls is critically important for the prediction of surface subsidence. To analyze the safety of the construction projects near the circular surface subsidence in the Hemushan Iron Mine, a new analytical model is proposed for predicting the rock failure at the surrounding wall of a cylindrical caved space, which is rarely studied in the literature. The long-term strength of the surrounding rock mass, the impact of the caved materials, and the in situ stress are involved as primary inputs. The proposed model is valid either for predicting the orientation and position where rock failure occurs at the caved space wall or for calculating the critical depth of the undercut, above which the surrounding rock of the caved space can remain stable for a long duration of time.

The proposed analytical model reveals that the stability of the caved space wall strongly depends on the properties of the surrounding rock mass, and the mechanism regarding the contribution of caved materials to the stability of the caved space wall is also addressed. Either increasing the density of the caved materials or reducing the depth of the free surface of the caved materials from the local ground surface facilitates an improvement in the stability of the caved space wall. Backfilling the caved space with waste rocks is accordingly introduced to improve the stability of the caved space wall.

The safety of the constructions near the surface subsidence in the Hemushan Iron Mine is predicted based on the proposed model. The depth of the implemented undercut does not exceed the critical depth, above which the
surrounding rocks of the caved space can remain stable, which means that the surface construction projects can maintain safe working conditions for a long duration of time. The in situ monitoring with Google Earth satellite images shows that no subsequent surface subsidence is observed after vertical caving, which demonstrates the validity of the prediction from the proposed model. Additionally, backfilling the caved space with excavated waste rocks is implemented to prevent rock failure above the free surface of the caved materials, and the results show that such measures contribute to protecting the surrounding rock of the caved space.

The contribution of this work is both theoretical and practical. This work is valid for predicting the orientation and the depth where rock failure occurs around a caved space, which fills the research gap due to the absence of analytical models for conducting such predictions. In addition, this work provides some employable measures for mitigating the subsequent extension of surface subsidence after vertical caving, e.g., implementing the undercut above the calculated critical depth and backfilling the caved space with waste rocks.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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