GRAND UNIFICATION IN EXTRA DIMENSIONS AND PROTON DECAY

FERRUCCIO FERUGLIO
Department of Physics, University of Padova and I.N.F.N, Sezione di Padova, Via Marzolo 8
Padova - Italy

We discuss baryon and lepton violation in the context of a simple 5-dimensional grand unified model, based on the orbifold $S^1/(Z_2 \times Z_2')$. While gauge and Higgs degrees of freedom live in the bulk, matter is located on the boundaries of the space-time. We show that proton decay is naturally suppressed or even forbidden by suitable implementations of the parity symmetries in the matter sector. The corresponding mechanism does not affect the SU(5) description of fermion masses also including neutrinos.

The idea that strong and electroweak interactions may possess a common description in the framework of a grand unified theory (GUT) is very attractive. Supersymmetric GUTs, with superpartners of the ordinary particles at the TeV scale as required by the solution of the hierarchy problem, predict a successful gauge couplings unification\(^1\) at a very large mass scale, close to the gravitational scale and possibly coinciding with it\(^2\). Moreover, the violation of the lepton number in the vicinity of the GUT scale would provide an elegant description of the observed smallness of neutrino masses. One of the distinctive features of GUTs, the violation of baryon number (B) is also one of the necessary conditions to generate the baryon asymmetry of the universe starting from symmetric conditions.

Despite their beauty, GUTs suffer from several difficulties that render rather cumbersome their specific realization in the context of conventional, four dimensional models. Perhaps the most serious problem is represented by the doublet-triplet (DT) splitting that in realistic GUTs can only be achieved at the cost of a quite complicated Higgs sector. Furthermore, minimal GUTs predict a proton lifetime that, although affected by large theoretical uncertainties, is on the verge of being experimentally excluded\(^3\). These drawbacks provide a strong motivation to look for alternative formulations of GUTs, also by going beyond the conventional framework.

It has been recently observed\(^4\) that the DT splitting problem can find an economic and elegant solution if the GUT gauge symmetry is realized in 5 (or more) space-time dimensions and
broken down to the Standard Model (SM) gauge symmetry by the compactification of the extra dimension(s). Of course the idea that extra dimensions may offer a natural framework for grand unification is an old one. Already in the late seventies, extended gauge symmetries were found by building supergravity theories in higher dimensions and, subsequently, by looking for non-anomalous supergravity/superstring theories. Furthermore it was soon suggested that in models with extra dimensions the grand unification scale could be set by the inverse compactification radius. It was also clear that the compactification process could offer new ways of breaking the gauge symmetry, in particular with the help of singular manifolds. Indeed the model of ref. consists of a 5-dimensional N=2 supersymmetric GUT where the compactification of the fifth dimension on $S^1/(Z_2 \times Z'_2)$ breaks at the same time N=2 down to N=1 and SU(5) down to SU(3)\times SU(2)\times U(1). The novelty of this model is the specific mechanism employed to obtain the DT splitting.

The fifth dimension is spanned by a coordinate $y$ parameterizing a circle $S^1$ with the identification of points related by the discrete symmetries $Z_2$ and $Z'_2$. These are reflections symmetries about orthogonal diameters of the circle (see fig.1): $y \rightarrow -y$ and $y' \rightarrow -y'$, respectively ($y' = y - \pi R/2$). The resulting orbifold $S^1/(Z_2 \times Z'_2)$ can be thought as the arc going from $y = 0$ to $y = \pi R/2$. The space-time has a 5-dimensional bulk, $0 < y < \pi R/2$, and two 4-dimensional boundaries at $y = 0$ and $y = \pi R/2$. The metric is everywhere flat. The generic bulk field $\phi(x,y)$, depending on all 5-dimensional coordinates, has well-defined ($P, P'$) ($P, P' = \pm 1$) parities under ($Z_2, Z'_2$). There are only four possible cases: $\phi_{++}, \phi_{+-}, \phi_{-+}$ and $\phi_{--}$, whose Fourier expansions give rise to 4-dimensional modes of masses $2n/R$, $(2n + 1)/R$, $(2n + 1)/R$ and $(2n + 2)/R$, ($n \geq 0$), respectively. Only the bulk field of the type $\phi_{++}$ has a zero mode.

The theory contains a bulk vector supermultiplet that includes a set of gauge bosons $A_\mu^A$ ($\mu = 0, ..., 3$) ($A = 1, ..., 24$) together with their 5-dimensional completions ($\mu = 5$) and their supersymmetric partners. The index $A$ will be denoted by $a$ when referring to SU(3)\times SU(2)\times U(1) and by $\hat{a}$ when indicating the coset SU(5)/SU(3)\times SU(2)\times U(1). In the Higgs sector there are bulk N=1 chiral multiplets $H_u$ and $H_d$ transforming as 5 and 5 under SU(5) and belonging to distinct hypermultiplets of N=2 supersymmetry. They contain SU(2) doublets $H^D_{u,d}$ and color triplets $H^T_{u,d}$. The ($Z_2, Z'_2$) parities of the relevant bulk fields are shown in table 1. The only massless vector bosons of the theory are the zero modes of $A_\mu^a$, which are identified with the gluons and the electroweak gauge bosons. The other vector bosons have masses of order $1/R$: SU(5) is broken down to SU(3)\times SU(2)\times U(1) (see fig. 2). The length of the radius $R$ is of or-
Figure 2: Modes in the Fourier expansion of the bulk fields. Crossed levels are eliminated by the orbifold projection thus breaking 4-dimensional SU(5) multiplets into disjoint components.

...der \( (10^{16} \text{ GeV})^{-1} \). In \( H_{u,d} \), color triplets are automatically splitted from SU(2) doublets, since the only massless scalars are the zero modes of \( H^D_{u,d} \), while the remaining modes have masses of \( O(1/R) \). The 5-dimensional parameters of gauge transformations, \( \alpha^a(x,y) \) and \( \alpha^\hat{a}(x,y) \) have \( (P,P') \) parities equal to \((+,+)\) and \((+,-)\), respectively. This means that in \( y = \pi R/2 \) only \( \alpha^a \) is non-vanishing and the transformations reduce to those of a (5-dimensional) SU(3)×SU(2)×U(1) group. On the boundary at \( y = 0 \) fields feel both \( \alpha^a \) and \( \alpha^\hat{a} \) parameters. Matter fields cannot be bulk fields and can only live on the boundaries, either in \( y = 0 \) or in \( y = \pi R/2 \). To motivate the introduction of matter in the SU(5) representations 10 and \( \bar{5} \) (and N=1 chiral multiplets), the natural choice is \( y = 0 \), where the whole (5-dimensional) gauge group is active. To preserve the orbifold construction, \( 10 \equiv (Q,U^c,E^c) \) and \( \bar{5} \equiv (L,D^c) \) should be even under \( Z_2 \), because \( y = 0 \) is a fixed point under \( Z_2 \), and should possess appropriate \( Z'_2 \) parities. The only \( Z'_2 \) parities that are compatible with SU(5) are [3]:

\[
(Q,U^c,E^c) = \pm (+,-,-) \quad (L,D^c) = \pm (+,+) .
\]

None of these choices leads however to realistic SU(5)×\( Z_2 \)×\( Z'_2 \) invariant Yukawa couplings, if the coupling constants are independent from \( y \) over \( S^1 \). Indeed, the choice in eq. (1) implies, for a single generation of matter fields, that 1010\( H_u \) is odd under \( Z'_2 \). If such a term were present in \( y = 0 \) with a Yukawa coupling \( y_u \), then the \( Z'_2 \) symmetry would require the same term in \( y = \pi R \) with the opposite coupling \(-y_u \). Thus the coupling of 1010\( H_u \), defined on the whole circle \( S^1 \),

Table 1: Parity assignment and masses \( (n \geq 0) \) for gauge vector bosons and Higgs supermultiplets.

| \((P,P')\) | field | mass |
|-----------|-------|------|
| \((+,+)\) | \( A^a_\mu, H^D_u, H^D_d \) | \( \frac{2n}{R} \) |
| \((+,-)\) | \( A^\hat{a}_\mu, H^T_u, H^T_d \) | \( \frac{(2n+1)}{R} \) |
would behave like a $Z'_2$ odd field. Barring this interesting possibility, $y_u$ should vanish and no mass for the up quark is obtained. Also with 3 generation no realistic spectrum in the up sector can be recovered.

Realistic masses for matter fields are instead obtained from the superpotential

$$w = y_u QU^c H_u^D + y_d QD^c H_d^D + y_e LE^c H_d^D, \tag{2}$$

provided each term is invariant under $(Z_2 \times Z'_2)$. If we bound ourselves to the case of Yukawa couplings $y_u$, $y_d$ and $y_e$ constant over $S^1$, the orbifold symmetry implies that $Q$, $U^c$ and $D^c$ should have equal $Z'_2$ parities and similarly for $L$ and $E^c$:

$$(Q, U^c, E^c, L, D^c) = (P_{q_1}^d, P_{q_2}^d, P_{l_1}^d, P_{l_2}^d, P_{l_3}^d) \quad (P_{q_1}^d, P_{l_1}^d = \pm 1). \tag{3}$$

The $Z_2 \times Z'_2$ invariant Yukawa interactions can be defined by first considering, at the boundary $y = 0$, the superpotential

$$w(y) = y_u \, 10 \, 10 \, H_u + y_d \, 10 \, 5 \, H_d + ... \tag{4}$$

where dots stand for R-parity violating terms. Given the decompositions

$$10 \, 10 \, H_u = QU^c H_u^D + \frac{1}{2} QQ H_u^T + U^c E^c H_u^T, \tag{5}$$

$$10 \, 5 \, H_d = QD^c H_d^D + LE^c H_d^D + QLH_d^T + U^c D^c H_d^T, \tag{6}$$

and the $Z'_2$ parity assignments, we can separate in $w(y)$ an even part and an odd part under $Z'_2$: $w(y) = w_E(y) + w_O(y)$. The odd part is projected out by requiring, at the $Z'_2$ mirror point $y = \pi R$, the same interactions as in $y = 0$:

$$w^{(4)} = \int dy \left[ \delta(y) + \delta(-y + \pi R) \right] w(y)$$

$$= \int dy \left[ \delta(y) + \delta(-y + \pi R) \right] w_E(y) \tag{7}$$

Eq. (7) is taken as the definition of $[\delta(y) + \delta(-y + \pi R)] w_E(y)$, the 5-dimensional Yukawa interaction. The SU(5) gauge symmetry is thus explicitly violated by both the gauge and the Yukawa interactions at $y = 0$. As a further consequence of the parity choice in (3), the B-violating terms $\overline{\psi}_Q \sigma^\mu T^A \psi_U A^\mu_M$ (where $\psi_M$ stands for the fermion member of the $M$ chiral multiplet and $T^A$ are the SU(5) generators), $QQH_u^T$ and $U^c D^c H_d^T$ are odd under $Z'_2$ and vanish. Hence the tree-level amplitudes from gauge boson or Higgsino exchange contributing to proton decay also vanish. Dangerous combinations of the dimension 4 operators $QLD^c$, $LE^c$ and $U^c D^c D^c$ can be avoided by particular $Z'_2$ assignments like

$$(Q, U^c, E^c, L, D^c) = (+, +, -, -) \tag{8}$$

Actually, with the choice of eq. (8) the proton is stable. Neutrino masses can be generated either by operators living on the boundaries like $LH_d^D H_d^D$ or by the presence of a $Z'_2$-odd, SU(5) singlet $\nu^c$. In the latter case both Dirac and Majorana neutrino mass terms are allowed and the see-saw mechanism is viable. A large mixing for atmospheric neutrinos can be driven by a large mixing between right-handed $s$ and $b$ quarks via the relation $y_e = y_{d}^T$. In conclusion, realistic fermion masses are obtained with parities that break SU(5) and that do not allow for tree-level proton decay amplitudes, via gauge boson or Higgsino exchange. Suitable parities can make the proton stable, while allowing for the desired terms that describe the observed neutrino oscillations.
The use of discrete symmetries to remove operators providing dangerous contributions to the proton decay has been advocated long ago. The interesting feature of the model under discussion is that here such symmetries are not introduced appositely to tame proton decay but they are essential to the construction of the space-time orbifold underlying the theory. The explicit SU(5) breaking due to the parity choice in (8) is not welcome. It prevents a common evolution of the gauge couplings after the unification scale. It weakens the motivation for introducing matter in GUT representations. However these unpleasant features might be peculiar of the exploratory 5-dimensional model presented here and they could hopefully be avoided in alternative constructions based on different orbifold symmetries or on higher space-time dimensions.

Acknowledgments

I would like to thank Guido Altarelli for the enjoyable collaboration on which this talk is based. Many thanks go also to Andrea Brignole and Yasunori Nomura for useful discussions. This work was partially supported by the European Program HPRN-CT-2000-00148 (network Across The Energy Frontier).

References

1. L. E. Ibanez and G. G. Ross, Phys. Lett. B 105 (1981) 439; S. Dimopoulos and H. Georgi, “Supersymmetric GUTs”, p 285, Second Workshop on Grand Unification, University of Michigan, Ann Arbor, April 24-26, 1981, eds. J. Leveille, L. Sulak, D. Unger; Birkhauser, 1981; S. Dimopoulos and H. Georgi, Nucl. Phys. B 193 (1981) 150.
2. P. Horava and E. Witten, Nucl. Phys. B 475 (1996) 94 [hep-th/9603142]; E. Witten, Nucl. Phys. B 471 (1996) 135 [hep-th/9602070].
3. Y. Totsuka, talk at the “Susy 2K” conference, CERN, June 2000.
4. See, for instance, G. Altarelli, F. Feruglio and I. Masina, JHEP11 (2000) 040 [hep-ph/0007254].
5. Y. Kawamura, [hep-ph/0012125].
6. See, for instance, E. Cremmer and B. Julia, Phys. Lett. B 80 (1978) 48.
7. M. B. Green and J. H. Schwarz, Phys. Lett. B 149, 117 (1984).
8. P. Fayet, Phys. Lett. B146 (1984) 41.
9. L. Dixon, J. A. Harvey, C. Vafa and E. Witten, Nucl. Phys. B 261 (1985) 678 and Nucl. Phys. B 274 (1986) 285.
10. R. Barbieri, L. J. Hall and Y. Nomura, Phys. Rev. D 63 (2001) 105007 [hep-ph/0011311].
11. G. Altarelli and F. Feruglio, [hep-ph/0102301] to appear on Phys. Lett. B.
12. L. Hall and Y. Nomura, [hep-ph/0103122].
13. N. Sakai and T. Yanagida, Nucl. Phys. B197 (1982) 533.