Conditional Transit Signal Priority Optimization at Stop-to-Stop Segments to Improve BRT On-Time Performance

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ABSTRACT Bus rapid transit (BRT) as a valid means of public transportation for overpopulated country of China is an especial mode of travel to relieve traffic congestion. Despite utilizing exclusive bus line, BRT constantly experiences delay at intersections when encountering a red signal. Transit signal priority (TSP) considered as a promising control strategy to improve the operation efficiency of BRT has been widely used at isolated intersections or arterials to reduce travel delay. However, as a transport with schedule, the unexpected arrival time of transit vehicle at stop lines of consecutive intersections inevitably affects the operation reliability of the BRT system. To solve the problem, this study develops an optimization model of conditional TSP for bus rapid transit with the objective of improving the on-time performance of the BRT system while mitigating adverse impacts on private vehicles. A stop-to-stop segment is established with both road sections and intersections for the consideration of the transit operation reliability. The constraints on the degree of saturation and queue overflow were considered for mitigating adverse impacts of TSP. A mixed-integer non-linear programming procedure is adopted to formulate the optimization model. The mathematical model is linearized and solved by the branch-and-bound method. Extensive numerical analyses are conducted, and the proposed conditional TSP strategy is compared with unconditional TSP and no-TSP control to evaluate its performance under various traffic and signal conditions. A case study of a BRT line in Shanghai, China, was selected to illustrate the effectiveness of the proposed model. For the tested scenarios, the transit on-time performance was improved by 21%, with an incurred cost of 3.4% increase in the delay of private vehicles. This indicates that the proposed model performs well in maintaining the reliability of the transit system with the least impact on general traffic.

INDEX TERMS Bus rapid transit (BRT), mixed-integer-linear-programing (MILP), on-time performance, stop-to-stop segment, transit signal priority (TSP)

I. INTRODUCTION
The rapid development of urbanization and the rapid growth of the city population in China has reinforced the challenging situation of traffic congestion in urban areas. Public transportation, especially transit operation, as a result of higher passenger capacity, has long been considered an effective measure for mitigating the average person road occupancy to deal with the problem. To facilitate transit service with less interference from private vehicles on road, bus rapid transit (BRT) with exclusive bus line has been considered a more effective mode of travel among public transit services. However, like conventional transit operation, BRT constantly encounter a red signal when approaching intersections which generates undesirable delay towards transit passengers. Among all valid strategies investigated in existing literature, transit signal priority (TSP) is considered one of the most operable and economical solutions for the improvement of the level of service [1, 2], and even for further benefits of BRT ridership increase [3, 4]. Starting with the first bus pre-emption experiment, various studies have been attempted to explore different types of TSP applications in the context of
China's sustainable public transportation for different real-time scenarios. Existing studies mostly focused on improving BRT operation with TSP control strategies towards delay reduction to achieve better travel experiences. But as a transport with schedule, BRT system should be more focused on the punctuality, i.e., operational reliability towards fixed timetable, except the operation efficiency. To improve the attractiveness of the BRT, operation reliability is a more significant concern. For passengers, transit on-time performance is the most direct evaluation indicator of transit operation reliability. Besides, noticing that the unexpected arrival time of transit vehicle at stop lines of consecutive intersections will inevitably affects the operation reliability of the BRT system, both intersections and the road segments in between should be considered during TSP adjustment. Therefore, a stop-to-stop segment with two bus stations, several intersections, and road sections should be taken into account when optimizing BRT operation.

In literature, the TSP can be categorized into three control strategies: passive, active, and adaptive control [5, 6]. Passive TSP control is mainly conducted offline with the information regarding historical transit messages, including route information and ridership patterns [7, 8]. Passive TSP mainly yields benefits to specific intersections and neglect transit arrival patterns and real-time traffic progression. Active TSP control, however, gives priority to the requested transit vehicles based on the detection ahead of the target intersection—and is considered to be an effective measure in dealing with the inflexibility problem in fixed-time passive control [9-11]. TSP treatments, such as early green and green extensions, are commonly used in such a scheme. Additionally, advanced techniques in connected vehicles, traffic sensing, and signal control systems allow the simultaneous optimization of the traffic performance of both transit and private vehicles [12, 13]. This enables the TSP systems to operate in real-time conditions to deal with sophisticated problems generated from earlier active ones, namely, adaptive TSP control [14-16]. This study focuses on the adaptive TSP strategy for optimizing transit vehicles with exclusive bus line in real-time conditions.

Former development process of TSP implementation has different considerations regarding the optimization objective function, the impact of TSP on private vehicles, and the solution method to tackle the specific problem. The following paragraphs are the overview of these three aspects in that order.

Regarding the optimization objectives of TSP, previous studies mainly focused on reducing bus travel time through intersections for the purpose of reducing passenger travel time to their destinations. In this regard, unconditional priority strategies were formulated to facilitate preferential transit operations [17]. Al-Deek, Sandt, Alomari and Hussain [18] also combined unconditional and conditional TSP with a BRT system separately to evaluate the operations of both transit and pedestrian traffic. Furthermore, vehicle delay is selected as a vital variable in the optimization of TSP control for the same purpose. Xu, Ye, Sun and Wang [19] proposed an analytical model that considers priority strategies, route levels, and traffic patterns to minimize transit vehicle delays under conflicting TSP requests. Ding, Yang, Wang, Xu and Bao [20] adopted a fuzzy optimization approach to minimize both the average passenger delay and vehicle queue length with the estimation of transit arrival time. Yu, Gayah and Christofa [21] considered the variables of cycle length and uncertainty in transit arrival time to reduce both transit and private vehicle delays. Considering the unstable character of the transit system, studies have also focused on the improvement of transit vehicle reliability, which is believed to be a more appropriate performance measurement for system-level optimization [22]. Ma, Liu and Han [23] formulated a ruled-based model to optimize transit punctuality, bus fuel consumption, and emissions simultaneously. Some studies have optimized the dwelling time at bus stops to improve transit reliability [24-26]. These methods, though, have been remarkably successful, and are mostly effective for low-frequency transit request control. In addition to the aspects mentioned above, many TSP applications benefit only from the target individual intersections [27]. Transit arrival times at downstream intersections are often overlooked [28]. Such operations may result in ineffective transit priority which greatly affecting system reliability, and the extra green duration at the upstream intersection may be wasted [29]. However, few studies have focused on the problem of arterial TSP design. Hu, Park and Lee [30] considered adjacent intersection coordination to minimize the total passenger delay. Christofa, Ampountolas and Skabardonis [31] used both transit and private vehicle occupancy along the arterial to optimize the punctuality of the transit system. Cheng and Yang [32] further adopted a signal coordination approach by considering the bus dwell time and capacity constraints to optimize arterials accommodating heavy bus flows. To facilitate promising progression between sequential or coordinated intersections along the entire arterial, stop-to-stop background conditions are paid increasing attention and are considered for analyzing the performance of different TSP applications [33].

Regarding the impact of TSP, the main problem of applying TSP is the potential negative impact on private vehicles. With the allocation of extra green time to transit vehicles, TSP strategies facilitate parallel private vehicle progression sharing the same phase. However, these procedures inevitably reduce the green time from non-priority phases with unpleasant extra traffic delay, especially for conflicting or cross directions, which may generate blockage conditions, and vehicles may have to wait for several extra signal cycles to finally clear the
intersection. To mitigate such adverse impacts, reference [1] considered both transit delay savings and competing movement capacity losses to quantify the influence of TSP at coordinated intersections. Nevertheless, these procedures evaluate the operation of transit and private vehicles separately without the simultaneous optimization of both traffic modes. Considering two or more objectives, normally conflicting, with different weighting coefficients into a single composite mathematical function, this procedure provides trade-off solutions according to the relative importance of different traffic modes. Reference [29] proposed an arterial-based progression to reduce transit travel time while decreasing the forfeiture arising from conflicting directions by minimizing ineffective transit priority time. Li, Yin, Zhang, Zhou and Nakamura [34] proposed a bi-objective model that considers both transit vehicle delay and general traffic delay. Liu and Qiu [35] minimized the weighted sum of the signal control delay and transit vehicle delay through trade-off optimizations. Li and Jin [36] adopted a simulation method with the objective of reducing both the total passenger travel delay and passenger waiting time at downstream bus stops within a network level. To date, multi-objective optimization is an appropriate method to deal with the trade-off between transit and private vehicles in TSP problems. Lin, Yang and Wang [37] proposed an operational framework for BRT that can prevent queue spillbacks at stations considering the limited storage space and the fluctuant bus arrival patterns.

Another problem with TSP optimization applications is the complexity of the solution method. With elaborate consideration of the realistic traffic environment, the model framework proposed to tackle the problem becomes increasingly more difficult to solve. Genetic algorithms and further developed NSGA-II approaches are widely used in the delay estimation of transit vehicles, private vehicles, and even passengers because of the nonlinear characteristic nature such performance indexes experienced [8, 38]. Dynamic programming approaches have also been adopted for high-frequency multiple-TSP request optimization to maximize the available green time for transit vehicles [39], [40]. Furthermore, to minimize the weighted sum of two modes of transportation delay (transit vehicle and general traffic), Shi, Yu, Ma, Wang and Nie [41] employed a simulated annealing algorithm to optimize exclusive bus lane settings, lane markings, and TSP signal timing settings concurrently. Despite the significant benefits of the former solution methods obtained in TSP optimization, most of the previous solution algorithms are formulated as nonlinear and computationally complex. Other approaches, including quadratic programming [34, 42] and mixed-integer linear programming [5, 30, 43] are recognized as more elegant measures to solve the problem.

In view of the literature summarized above, the two major factors influencing the practical implementation of TSP control strategies—the ineffective transit priority and side effects of private vehicles—have been considered in many studies, but separately. Moreover, the computational complexity further increases when a combined consideration is considered.

Therefore, this study proposes an optimization framework to simultaneously consider the operational efficiency of both transit and private vehicles at a stop-to-stop segment. The main purpose is to improve the on-time performance of BRT system, while minimizing the total priority time along the target segment to reduce ineffective transit priority for the purpose of facilitate general traffic movement. Both the degree of saturation and queue overflow are considered to further reduce the side effects on private vehicles. The contributions of the study are twofold: (1) improving transit on-time performance and mitigating side effects on private vehicles are simultaneously considered; (2) a mixed-integer linear programming procedure is established followed by its solution algorithm to ensure the computational time requirement of the online control.

The remainder of this paper is organized as follows. In Section 2, the fundamental control logic of the proposed model, including the stop-based segment, and the conditional priority time strategy, are described. The bi-objective mixed-integer-linear-program optimization model with an explicit explanation of the constraints is proposed in Section 3. Then, the performance of the proposed model is evaluated through extensive numerical experiments and a field case study in Section 4. Finally, conclusions and suggestions are presented.

II. CONTROL CONCEPT

To comprehensively consider both the intersection-based and road-based performance optimization problems, this study mainly formulates the situation comprising an arterial that includes major components of two successive bus stops and intersections in between, as depicted in Fig. 1. The detector is set close to the upstream bus stop before the transit vehicle arrives at the first intersection. The main purpose of this research is to improve the on-time performance of the bus service of two successive bus stops while minimizing side effects on private vehicles.

In this study, green extension and early green signal adjustment schemes were considered for the TSP implementation. The conditional priority time strategy was adopted, and it contains three aspects:

1) Provide priority time only to buses whose arrival time is later than the schedule.
2) Provide priority time only when the on-time performance of buses can be improved.
3) Provide priority time only when the constraints on the degree of saturation and queue overflow can be satisfied.
III. INTEGRATED OPTIMIZATION MODEL

A. OBJECTIVE FUNCTION

To coordinate the competitive relationship between transit vehicles along arterial and private vehicles and to comprehensively consider positive/negative effects generated from transit priority simultaneously, two optimization objectives with different priority levels are proposed. The first priority level objective is to minimize the transit vehicle delay against the schedule at the downstream bus stop. This can eliminate the ineffective transit signal priority problem. The second priority level objective is to minimize the total transit priority time assigned to intersections, designed to minimize the side effects of TSP on private vehicles. Further constraints on the degree of saturation and queue length will be considered to ensure the operational efficiency of private vehicles. In summary, the overall objective function is given by Eq. (1).

\[ \min P_1 \Delta T + P_2 \sum_{i=1}^{N} (G_{ib} + G_{ie}), \quad \forall i = 1, 2, \ldots, n \]  

(1)

where \( P_1, P_2 \) are weight coefficients to differentiate the priority levels of the two control objectives, \( P_1 > P_2; \Delta T \) is the schedule deviation at the downstream bus stop; \( G_{ib} \) and \( G_{ie} \) are the decision variables represent transit priority time generated by early green and green extension strategies at intersection \( i \), respectively—and \( N \) is the number of intersections at the stop-to-stop segment.

B. CONSTRAINTS

To reflect the real traffic operation conditions, the model proposed in this study considers the constraints in terms of schedule deviation, transit arrival time, transit passing time, beginning/ending time of transit phase, and priority time.

1) DELAY AGAINST THE SCHEDULE CONSTRAINTS

When the actual arrival time of the transit vehicle exceeds the predefined arrival time of the schedule table at the downstream stop, the delay time against the schedule equals the difference between the actual arrival time and scheduled arrival time of the transit vehicle at the downstream stop. When the actual arrival time of the transit vehicle is ahead of the schedule at the downstream stop, there is no delay for passengers aboard or for those waiting at the station, and the delay against the schedule is equal to 0, as shown in Eq. (2).

\[ \Delta T = \begin{cases} t_f - t_f^0, & t_f > t_f^0 \\ 0, & t_f \leq t_f^0 \end{cases} \]  

(2)

where \( t_f \) and \( t_f^0 \) represent the actual/scheduled arrival time of the transit vehicle at downstream stop, respectively.

2) TRANSIT ARRIVAL TIME CONSTRAINTS

Owing to the phase sequence and signal status of traffic light at each intersection, the moment the transit vehicle arrives at intersection \( i \) determines whether it encounters a red light or a green light, which should obey the following set of constraints.

a: TRANSIT ARRIVAL TIME AT INTERSECTION

Starting from the upstream stop, the moment the transit vehicle arrives at the first intersection is mainly related to the departure time from the upstream bus stop, and the travel time between the upstream bus stop and the first intersection, as shown in Eq. (3). The moment the transit vehicle arrives at other intersections along the arterial depends on the departure time from the upstream intersection and the travel time between the two adjacent intersections, as shown in Eq. (4).

\[ t_{ia} = t_s + \frac{d_{(i-1)i}}{v_{(i-1)i}}, \quad \forall i = 1 \]  

(3)

\[ t_{ia} = t_{(i-1)p} + \frac{d_{(i-1)i}}{v_{(i-1)i}}, \quad \forall i = 2, 3, \ldots, n \]  

(4)

where \( t_{ia} \) is the transit arrival time at intersection \( i; t_s \) is the departure time from upstream stop, \( t_{(i-1)p} \) is the transit passing time at intersection \( i - 1; d_{(i-1)i} \) and \( v_{(i-1)i} \) represent the distance and transit velocity of each segment along the arterial, respectively; \( d_{(i-1)i} / v_{(i-1)i} \) denotes the transit travel time from intersection \( i - 1 \) to intersection \( i \).

b: THE STATE OF SIGNAL WHEN THE TRANSIT VEHICLE ARRIVES

Since the signal at intersections runs in a cycle, the transit vehicle always arrives within a certain signal cycle, as shown in Eqs. (5) and (6). If the arrival time of a bus at intersection \( i \) is earlier than the end of the transit phase, the bus arrives during effective green time. Otherwise, the bus arrives during the red time, as shown in Eq. (7). Since the effective green time concept is used, the yellow time is not counted independently in the study.

\[ t_{ia} \geq g_{ib} + a_i C_i, \quad \forall i = 1, 2, \ldots, n \]  

(5)

\[ t_{ia} < g_{ib} + (a_i+1) C_i, \quad \forall i = 1, 2, \ldots, n \]  

(6)

\[ \beta_i = \begin{cases} 1, & t_{ia} \leq g_{ie} + a_i C_i \\ 0, & t_{ia} > g_{ie} + a_i C_i \end{cases}, \quad \forall i = 1, 2, \ldots, n \]  

(7)

where \( g_{ib} \) is the start of the effective green time of the transit phase at intersection \( i; a_i \) is an integer variable indicating the number of signal cycles transit vehicles experienced at intersection \( i; C_i \) is the signal cycle length at intersection \( i; g_{ie} \) is the end the effective green time of the
transit phase at intersection \( i \); \( \beta_i \) is a binary variable deciding whether the bus arrives during green light at intersection \( i \), 1-yes, 0-no.

**c: ACTUAL TRANSIT ARRIVAL TIME AT THE DOWNSTREAM BUS STOP**

The actual arrival time at the downstream stop, which is mainly related to the passing time of the transit vehicle at the last intersection of the arterial. \( t_f \) can be calculated using Eq. (8).

\[
t_f = t_{ip} + \frac{d_{if}}{v_{if}}, \quad \forall i = n
\]  

(8)

where \( d_{if} \) and \( v_{if} \) represent the distance and transit velocity of the segment between the last intersection and the downstream stop.

**3) TRANSIT DEPARTURE TIME CONSTRAINTS**

According to the arrival time, the transit departure time at each intersection can be divided into two conditions. If a transit vehicle arrives during the green light at intersection \( i \), it can pass through the intersection directly without any delay. If the transit vehicle arrives during the red light at intersection \( i \), it has to wait for the beginning time of the transit phase at the next signal cycle. The transit departure time can be calculated using Eq. (9).

\[
t_{ip} = \left\{ \begin{array}{ll}
t_{ia} & \beta_i = 1 \\
(t_{ib} + (\alpha_i + 1)c_i, \beta_i = 0, & \forall i = 1, 2, ..., n
\end{array} \right.
\]  

(9)

where \( t_{ip} \) is the transit passing time at intersection \( i \).

**4) BEGINNING/ENDING TIME OF TRANSIT PHASE CONSTRAINTS**

To reduce the waiting time of the transit vehicle at each intersection and improve the on-time performance of the transit vehicle at the downstream stop, TSP strategies are introduced into the proposed model to grant priority to the approaching transit vehicle.

**a: ACTUAL STARTING TIME OF THE TRANSIT PHASE AT INTERSECTION UNDER TSP STRATEGIES**

If the transit vehicle arrives during the red light, and the arrival time is close enough to the beginning time of the transit phase in the next signal cycle, an early green strategy is introduced into the proposed model to generate transit signal priority. To accommodate such a situation, the actual starting time of the transit phase should be advanced by a few seconds from the original signal timing plan, which can be calculated using Eq. (10).

\[
g_{ib} = g^0_{ib} - G_{ib}, \quad \forall i = 1, 2, ..., n
\]  

(10)

where \( g_{ib} \) is the actual starting time of the transit phase at intersection \( i \); \( g^0_{ib} \) is the original starting time of the transit phase at intersection \( i \); \( G_{ib} \) is the priority time assigned for the early green strategy at intersection \( i \).

**b: ACTUAL ENDING TIME OF THE TRANSIT PHASE AT INTERSECTION UNDER TSP STRATEGIES**

While the arrival time of the transit vehicle is behind, but close enough to the ending time of the transit phase in the present signal cycle, a green extension strategy is adopted to ensure that the transit vehicle passes through the intersection without stopping. This can be calculated using Eq. (11).

\[
g_{ie} = g^0_{ie} + G_{ie}, \quad \forall i = 1, 2, ..., n
\]  

(11)

where \( g_{ie} \) is the actual ending time of the transit phase at intersection \( i \); \( g^0_{ie} \) is the original ending time of the transit phase at intersection \( i \); and \( G_{ie} \) represents the priority time generated by the green extension strategy at intersection \( i \).

**5) TRANSIT PRIORITY TIME CONSTRAINTS**

The extra green time assigned to the priority phase is actually at the cost of decreasing the green duration of the other phases at intersection \( i \). The maximum transit priority time generated under early green and green extension strategies should be restricted appropriately to avoid undue adjustment of background signal timing. In view of the fact that TSP green provided to transit vehicles may result in adverse impacts on private vehicles from other approaches (conflict movements), both the degree of saturation and queue overflow situations should be taken into consideration to restrict the upper limit of transit priority time at intersection \( i \).

Transit priority time under early green and green extension strategies assigned to intersections along the arterial should be restricted by Eqs. (12) and (13), respectively:

\[
0 \leq G_{ib} + G_{ie} \leq \Delta g^P_i, \quad \forall i = 1, 2, ..., n
\]  

(12)

\[
0 \leq G_{ib} + G_{ie} \leq \Delta g^P_i, \quad \forall i = 1, 2, ..., n
\]  

(13)

where \( \Delta g^P_i \) is the upper limit of transit priority time under the degree of saturation constraints at intersection \( i \); and \( \Delta g^Q_i \) is the upper limit of transit priority time under the queue overflow constraints at intersection \( i \).

**6) DEGREE OF SATURATION CONSTRAINTS**

According to the definition of saturation, the minimum required green duration of phase \( j \) at intersection \( i \) under the maximum accepted saturation can be calculated using Eq. (14).

\[
g_{ij}^N = \frac{q_{ij}c_i}{s_{ij}X_i}, \quad \forall i = 1, 2, ..., n; \quad j = 2, 3, 4
\]  

(14)

where \( g_{ij}^N \) is the minimum required green duration of phase \( j \) at intersection \( i \) under the maximum accepted saturation, \( q_{ij} \) is the traffic flow of phase \( j \) at intersection \( i \); \( s_{ij} \) is the saturation flow rate of phase \( j \) at intersection \( i \); \( c_i \) is the cycle length of intersection \( i \); and \( X_i \) is the maximum accepted saturation at intersection \( i \).

The expected upper limit of the transit priority time under the degree of saturation constraints for both the early green
and green extension strategies at intersection $i$ can be calculated using Eq. (15). However, if the current saturation is higher than the pre-set maximum saturation, no priority treatment is provided, and the upper limit of the transit priority is 0.

$$\Delta g_i = \begin{cases} (g_{ij} - g_{ij}^N) X_{\text{current}} < X_{\text{pre-set}}, \\ 0 \quad X_{\text{current}} \geq X_{\text{pre-set}} \end{cases} \quad \forall i = 1,2,\ldots,n; j = 2,3,4 \tag{15}$$

where $g_{ij}$ is the green duration of phase $j$ at intersection $i$ under the background signal plan, and $(g_{ij} - g_{ij}^N)$ denotes the maximum extra green time phase $j$ may contribute.

Besides, it should be noticed that the degree of saturation considered in this study is less than 1(under saturated). Nevertheless, an oversaturated situation may be caused by the application of the transit priority implementation; therefore, a queue overflow constraint is introduced to consider the circumstance that the saturation is temporarily greater than 1.

7) QUEUE LENGTH CONSTRAINTS
The TSP may lead to overflowing queue problems on side-street arms due to oversaturated conditions, and for the determination of the transit priority time, analysis of the situation is required so that each direction does not overflow onto the adjacent downstream intersection. Based on the time queue length of each non-priority traffic flow used to restore to the initial state, the maximum queued vehicles from phase $j$ at intersection $i$ were considered to define the upper limit of transit priority time under the queue overflow constraints.

In terms of the temporary oversaturation of the intersection caused by the transit signal priority, queued vehicles may not fully discharge within the current signal cycle but must wait for the next or the further behind signal cycle to dissipate completely, as shown in Fig. 2. It can be seen that the delayed vehicles always accumulate the most during the first signal cycle after the activation of TSP, i.e., $\Delta N_{ij}^1 \geq \Delta N_{ij}^2$, under the circumstance that non-priority phase(s) takes three signal cycles to restore to the initial state, and of course, the same with the circumstances that the non-priority phase(s) used more signal cycles to restore to the initial state. Hence, the maximum queued vehicles will only emerge in the second signal cycle. Therefore, the queued vehicle in the second signal cycle is used as the judgment standard to limit the maximum transit priority time at the intersection, which can be calculated using Eqs. (16) and (17), respectively.

$$N_{ij(\text{max})} = C_i \cdot q_{ij} + \Delta N_{ij}^1, \quad \forall i = 1,2,\ldots,n; j = 2,3,4 \tag{16}$$

$$\Delta N_{ij}^1 = C_i \cdot q_{ij} - (C_i - t_{Rij} - \Delta t_{ij}) \cdot s_{ij}, \quad \forall i = 1,2,\ldots,n; j = 2,3,4 \tag{17}$$

where $N_{ij(\text{max})}$ is the maximum queued vehicle of phase $j$ at intersection $i$; $\Delta N_{ij}^1$ is the number of vehicles stuck in the first signal cycle; $\Delta t_{ij}$ is the green loss of phase $j$ at intersection $i$ under transit signal priority; and $t_{Rij}$ is the red duration of phase $j$ at intersection $i$ under the background signal plan.

To prevent queued vehicles of non-priority flow from overflowing onto the adjacent intersection, the maximum queue length must be no more than the minimum distance between the two intersections. This can be constrained by Eq. (18).

$$N_{ij(\text{max})} \leq l_{ij}^{\text{max}}, \quad \forall i = 1,2,\ldots,n; j = 2,3,4 \tag{18}$$

where $l_{ij}^{\text{max}}$ is the queue length per standard vehicle; $l_{ij}^{\text{max}}$ is the queue length limitation in space of the upstream segment in phase $j$ at intersection $i$.

If we insert Eqs. (16) and (17) to Eq. (18), we can get the maximum green loss limit of a phase, as shown in Eq. (19).

Then, the upper limits of transit priority time under queue-overflow constraints can be determined by Eq. (20).

$$\Delta t_{ij(\text{max})} = \frac{\Delta t_{ij}^{\text{max}}}{l_{ij}^{\text{max}}} = \frac{2C_i q_{ij}}{s_{ij}} + (C_i - t_{Rij}), \quad \forall i = 1,2,\ldots,n; j = 2,3,4 \tag{19}$$

$$\Delta g_i = \sum_{j=2}^{3} \Delta t_{ij(\text{max})}, \quad \forall i = 1,2,\ldots,n \tag{20}$$

where $\Delta t_{ij(\text{max})}$ represents the maximum green loss of phase $j$ at intersection $i$.

![FIGURE 2. Maximum queued vehicles under the over-saturated conditions.](image)

C. SOLUTION ALGORITHM
The proposed optimization model is a mixed-integer nonlinear programming problem with the objective function of Eq. (1), with the constraints in Eqs. (2) – (20). In solving the program, seven aspects of constraint regarding delay against schedule, transit arrival time, transit departure time, beginning/ending time of transit phase, transit priority time, degree of saturation, and queue length constraints should be addressed. After that, the presented optimization model could be transformed into a mixed-integer linear program, which can be solved by the standard branch-and-bound technique with the aid of binary indicators $\delta$, $\beta_i$, $\gamma_i$, and a non-negative variable $M$ introduced into the model. Then, the problem can be reduced to minimize the aggregate objective function with weighted factors. By dividing the two goals into different priority levels, the integrated optimization model can be
solved using a simplified pre-emptive goal programming procedure.

1) LINEARIZATION OF THE DELAY AGAINST THE SCHEDULE CONSTRAINTS

A binary control variable $\delta$ is introduced into Eq. (2) for the linearization of the delay against the schedule calculation. $\delta$ indicates the scenario of a transit vehicle arriving late from the schedule table at the downstream stop (1-Yes, 0-No). $M$ is a large positive constant.

Eq. (2) could be rewritten as Eqs. (21) – (24). According to Eqs. (21) – (22), $\delta$ is equal to 1 when $t_f \geq t_f^0$ and $\delta$ is equal to 0 when $t_f < t_f^0$. When $\delta = 1$, Eq. (23) is activated. The schedule deviation is defined as $\Delta T = t_f - t_f^0$. When $\delta = 0$, Eq. (24) is activated. The schedule deviation is defined as $\Delta T = 0$. Through this procedure, the schedule deviation can be calculated to satisfy the linear characteristics.

$$t_f > t_f^0 - M(1 - \delta)$$

$$t_f \leq t_f^0 + M\delta$$

$$t_f - t_f^0 - M(1 - \delta) \leq \Delta T \leq t_f - t_f^0 + M(1 - \delta)$$

$$-M\delta \leq \Delta T \leq M\delta$$

2) LINEARIZATION OF THE TRANSIT ARRIVAL TIME CONSTRAINTS

A binary variable $\beta_i$ is introduced into Eqs. (7) for the linearization of the transit arrival status calculation, which indicating whether the transit vehicle arrives during the green light at intersection $i$ (1-Yes, 0-No). Such an operation enables linear treatments to calculate transit arrival time. If the arrival time of a bus at intersection $i$ is earlier than the end of the transit phase, the bus arrives during green time ($\beta_i = 1$), as shown in Eq. (25). Otherwise, the bus arrives during the red time ($\beta_i = 0$), as shown in Eq. (26). Therefore, Eqs. (7) could be rewritten as Eqs. (25) and (26).

$$t_{ia} \leq g_{ia} + \alpha_i C_i + M(1 - \beta_i), \quad \forall i = 1, 2, ..., n$$

$$t_{ia} > g_{ia} + \alpha_i C_i - M\beta_i, \quad \forall i = 1, 2, ..., n$$

3) LINEARIZATION OF THE TRANSIT DEPARTURE TIME CONSTRAINTS

A binary control variable $\beta_i$ is also introduced into Eqs. (9) for the linearization of the transit departure time calculation. If the transit vehicle encountering a green light at intersection $i$ ($\beta_i = 1$), there is no delay generated and the transit departure time is defined as $t_{ip} = t_{ia}$. Otherwise, transit departure time is green start time at the next signal cycle, defined as $t_{ip} = g_{ib} + (\alpha_i + 1)C_i$. Eqs. (9) could be rewritten as Eqs. (27) and (28). Eqs. (27) and (28) can be activated when $\beta_i$ equals 1 and 0, respectively.

$$t_{ia} - M(1 - \beta_i) \leq t_{ip} \leq t_{ia} + M(1 - \beta_i), \quad \forall i = 1, 2, ..., n$$

$$g_{ib} + (\alpha_i + 1)C_i - M\beta_i \leq t_{ip} \leq g_{ib} + (\alpha_i + 1)C_i + M\beta_i, \quad \forall i = 1, 2, ..., n$$

4) LINEARIZATION OF THE DEGREE OF SATURATION CONSTRAINTS

A binary control variable $\gamma_i$ is introduced into Eq. (15) for the linearization of the degree of saturation constraints, which indicating whether the current saturation is lower than the pre-set maximum saturation (1-Yes, 0-No).

Eq. (15) could be rewritten as Eq. (29). Please note this equation be activated only with $\gamma_i = 1$. When $\gamma_i = 0$, the upper limit of the priority time is 0, and no priority treatment is provided. In addition, Eq. (30) is added to distinguish the situation. If $\sum (g_{ij} - g_{ij}^0) > 0$, which means there is still room for TSP green generation, the upper limit of the priority time is considered effective and $\gamma_i = 1$. Otherwise, $\gamma_i = 0$.

$$\Delta g_i^0 = \gamma_i \sum (g_{ij} - g_{ij}^0), \quad \forall i = 1, 2, ..., n; j = 2, 3, 4$$

$$\gamma_i \sum (g_{ij} - g_{ij}^0) \geq 0, \quad \forall i = 1, 2, ..., n; j = 2, 3, 4$$

IV. SENSITIVITY ANALYSES AND CASE STUDY

This section aims to evaluate the performance of the proposed model using both numerical examples and a case study. A main traffic scene of an arterial consisting of several intersections was first established, and the impacts of different performance indexes were evaluated through sensitivity analyses. Subsequently, the proposed model was further introduced into a field case study to test its effectiveness.

A. SENSITIVITY ANALYSES

An arterial with three intersections, two bus stop stations, and an exclusive bus lane was used to test the validity of the proposed model. The configuration of the numerical example is illustrated in Fig. 3. The basic vehicular traffic demands are summarized in Table 1. The saturation flow rate of the three intersections for all lane groups was 1800 veh/h. All intersections had the same signal cycle of 100 seconds, and the existing phase sequence and signal timings are listed in Table 2. The transit vehicle leaves the upstream stop at the 100th second, and the schedule table transit vehicle arrives at the downstream stop at the 150th second, and the background degree of saturation is 0.5. The maximum acceptable degree of saturation was set to 1. The distance from the upstream stop to intersection 1, and from intersection 3 to the downstream stop was 150 m; the distance between every adjacent intersection was 300 m, and the queue length limitation on the cross streets was 200 m. The travel speed of the transit vehicle was set to 50 km/h.

The proposed model, denoted as the conditional transit signal priority (CTSP) strategy, aims to improve the on-time performance, and reduce the adverse impacts of private vehicles. To demonstrate the superiority of the proposed model, two other strategies were adopted for comparison: (1)
Unconditional Transit Signal Priority Strategy (UTSP). Improves transit punctuality without considering the side effects induced in the private vehicles of non-priority phases. (2) No transit signal priority strategy (NTSP).

The models mentioned above were solved in MATLAB (2018) on an Intel Core i7 2.7 GHz processor with 16.0 GB RAM, running on MacOS Sierra. The computing time for any model operation was less than 1 second (average of approximately 0.5 s).

Three impact factors were selected to perform the sensitivity analysis for the models mentioned above: transit departure time from upstream stop, scheduled arrival time at downstream stop, and green split of transit phase at intersections. To exhibit the calculated results more intuitively and comprehensively, every impact factor changes simultaneously with the degree of saturation to examine the variation in the transit schedule deviation. Meanwhile, the performance of CTSP is compared with that of UTSP and NTSP to illustrate the superiority of the proposed model.

FIGURE 3. Geometric configuration of the numerical example.

| TABLE I | TRAFFIC DEMAND OF THE NUMERICAL EXAMPLE |
|---------|----------------------------------------|
| Direction Movement | Intersection 1 | Intersection 2 | Intersection 3 |
| EB / WB LT | 126 | 126 | 126 |
| TH + RT | 270 | 270 | 270 |
| SB / NB LT | 108 | 108 | 108 |
| TH + RT | 216 | 216 | 216 |

Note: EB, WB, SB and NB represent eastbound, westbound, southbound and northbound respectively. TH, RT and LT represent through, right-turn and left-turn respectively.

| TABLE II | SIGNAL TIMING OF THE NUMERICAL EXAMPLE |
|---------|--------------------------------------|
| Phase sequence | Phase 1 | Phase 2 | Phase 3 | Phase 4 |
| Intersection 1 Start of green (s) | 69 | 104 | 123 | 152 |
| Duration of green (s) | 30 | 12 | 24 | 12 |
| End of green (s) | 99 | 118 | 147 | 164 |
| Intersection 2 Start of green (s) | 56 | 91 | 110 | 139 |
| Duration of green (s) | 30 | 14 | 24 | 12 |
| End of green (s) | 86 | 105 | 134 | 151 |
| Intersection 3 Start of green (s) | 23 | 58 | 77 | 106 |
| Duration of green (s) | 30 | 14 | 24 | 12 |
| End of green (s) | 53 | 72 | 101 | 118 |

1) IMPACT OF THE DEPARTURE TIME FROM UPSTREAM STOP

Departure time of the transit vehicle from the upstream stop was set from the 100th second, to the 200th second with an interval of 10 seconds—while the degree of saturation was set from 0.1 to 1 with an interval of 0.2 (degree of saturation of three intersection changes simultaneously), as shown in Fig. 4. The variation in the transit schedule deviation at the downstream stop according to the optimization results from the proposed model is illustrated in Fig. 4 (a). The comparison results of the transit schedule deviation and average delay of private vehicles among the three strategies are illustrated in Fig. 4 (b) and Fig. 4 (c), respectively. The following observations can be made.

(1) The transit schedule deviation changes cyclically with the change in the departure time of the upstream bus stop when the intersections between the two bus stops have the same cycle, as illustrated in Fig. 4 (a). In this case, the frequency is 100 seconds because the signal cycle of the three intersections is 100 seconds.

(2) Under a high degree of saturation, the variation in schedule deviation is particularly sensitive to the departure time of the upstream bus stop. With saturation over 0.8, the schedule deviation varies from 106 to 186 seconds. It is because the higher the saturation, the smaller the adjustment range of the transit signal priority. Whether the transit vehicle can arrive on time is mainly affected by the departure time of the upstream bus stop.

(3) The comparison results of the transit schedule deviation with the variation of the transit departure time under the three different TSP strategies are shown in Fig. 4 (b). Under a low degree of saturation, CTSP and UTSP perform similarly. When the degree of saturation reaches a relatively high standard, UTSP outperforms CTSP in schedule deviation reduction, which is mainly due to extra passing time provided to transit vehicles unconditionally. More specifically, the CTSP control generates a statistically significant improvement in the transit schedule deviation of approximately 51% on average compared to the NTSP control, whereas UTSP benefits by approximately 79% under the same circumstances.

(4) However, as illustrated in Fig. 4 (c), the benefits of UTSP control on transit vehicles always come at a cost with serious side effects on private vehicles. The UTSP control almost doubles the average private vehicle delay compared to the NTSP control, particularly in high-saturated traffic conditions. Specifically, compared to the NTSP control, the UTSP control creates more than 90% of the average private vehicle delay in general, whereas the average private vehicle delay caused by CTSP is about 6.5%.
2) IMPACT OF THE SCHEDULED ARRIVAL TIME AT DOWNSTREAM STOP

Inheriting the same degree of saturation from above, the arrival time of the transit vehicle at the downstream stop was set from the 100th, to the 340th second—with an interval of 10 seconds. The departure time of the transit vehicle was fixed as the initial state ($t_s = 100$), as shown in Fig. 5. The following observations can be made.

1) The transit schedule deviation decreases with an increase in scheduled arrival time at the downstream bus stop, as shown in Fig. 5 (a). It is due to more travel time being provided for the transit vehicle. The variation changes in steps as the degree of saturation increases mainly because the implementation of the CTSP strategy ranges from granting priority time at all three intersections to two intersections, to one intersection, and eventually no intersection being activated with CTSP.

2) The comparison results of the transit schedule deviation with the variation of the scheduled arrival time are shown in Fig. 5 (b). CTSP and UTSP perform similarly in under-saturated traffic conditions. The UTSP control outperforms the CTSP control in schedule deviation reduction, which becomes more significant with a higher degree of saturation. More specifically, the CTSP control generates a statistically significant improvement in the transit schedule deviation of 55% on average compared to the NTSP control.

3) However, as illustrated in Fig. 5 (c), the side effects of UTSP control on private vehicles are also much more significant than those of the CTSP control. The CTSP always outperforms UTSP in terms of average private vehicle delay saving, and this phenomenon is obvious when the degree of saturation is relatively high. Specifically, compared to the NTSP control, the UTSP control creates more than 56% of the average private vehicle delay in general, whereas the private vehicle delay caused by CTSP is only 7.4%.

FIGURE 4. Impact of transit departure time. (a) Transit schedule deviation under the CTSP control. (b) Comparison of transit schedule deviation under three different strategies. (c) Comparison of average private vehicle delay under three different strategies.

FIGURE 5. Impact of transit schedule deviation. (a) Transit schedule deviation under the CTSP control. (b) Comparison of transit schedule deviation under three different strategies. (c) Comparison of average private vehicle delay under three different strategies.
(3) However, in terms of the average private vehicle delay, as illustrated in Fig. 6 (c), with the increase in the green split of the transit phase, the average private vehicle delay caused by the CTSP first increases and then decreases. This is because when the green split of the transit phase increases at the beginning, the green split of the non-priority phases decreases accordingly, and the side effects of the private vehicles add on the account of the CTSP green time occupation. With a further increase in the green split of the transit phase, owing to the reduction in the available CTSP control activation, the impact on private vehicles declines. However, the average private vehicle delay under the UTSP control always increases owing to the unconditional transit priority generation, which leads to its disadvantage in private vehicles compared to the CTSP control. Specifically, compared to the NTSP control, the UTSP control creates an average private vehicle delay of approximately 83%, while the CTSP control only generates approximately 5% on average.
SP and UTSP
nsit departure time
sacrificing too much green time for the non
in Fig. 8(a). Consequently,
obtained at the cost of granting excessive priority time to
21% under the
the CTSP control generates less benefits of approximately
schedule deviation reduction of approximately 57%, whereas
strategies result in transit punctuality improvement
Compared to the NTSP control, both CT
TSP strategies under the study case scenario are shown in
Fig. 8(b) and 8(c), respectively.

The comparison results of the transit schedule deviation under the CTSP and UTSP strategies are shown in Fig. 8(b) and 8(c), respectively.

The comparison results of the transit schedule deviation under three different TSP strategies under the study case scenario are shown in Fig. 8(b) and 8(c), respectively.

The side effects of the UTSP control on private vehicles are also more significant than those of the CTSP control, which may lead to a queue overflow of the side-street arms, resulting in blockage of the adjacent intersections. Even though, inevitably, both of the TSP strategies create additional traffic delay compared to the NTSP control, the effectiveness of the CTSP control is much more evident in delay reduction. Specifically, as shown in Fig. 8(c), compared to the NTSP control, the UTSP control creates more than 30% of the average private vehicle delay in general, whereas the private vehicle delay caused by the CTSP control is 3.4%.

Overall, the results show that compared to UTSP, owing to the consideration of the degree of saturation and queue overflow constraints, the proposed CTSP control could provide a significant reduction in private vehicle delay, while simultaneously improving transit punctuality.

B. CASE STUDY

According to the field survey, a stop-to-stop segment with three intersections on Xian’pu Road of Fengpu express bus line in Shanghai, China, was selected as the study case to evaluate the effectiveness of the proposed model, as shown in Fig. 7. The vehicular traffic demands are summarized in Table 3. All three intersections have the same signal cycle of 90 seconds, phase sequence, and signal timings, as shown in Table 4. The travel speed of the transit vehicle was 30 km/h. To overcome the stochastic nature of the transit vehicle operation from the upstream bus stop, transit departure time in one day (05:30-19:00) of the Xianpu Road bus stop was analyzed to verify the model efficiency. The departure times of the upstream bus stop and scheduled arrival time of the downstream bus stop are listed in Table 5.

The measurements of the model effectiveness include the transit schedule deviation and the average private vehicle delay. Furthermore, three different transit signal priority strategies, CTSP, UTSP, and NTSP, are compared to demonstrate the superiority of the proposed model, as shown in Fig. 8. Fig. 8(a) demonstrates the total priority time along the stop-to-stop segment from both the CTSP and UTSP strategies. The comparison results of the transit schedule deviation and average private vehicle delay for three different TSP strategies under the study case scenario are shown in Fig. 8(b) and 8(c), respectively.

The comparison results of the transit schedule deviation under three different TSP strategies are shown in Fig. 8(b). Compared to the NTSP control, both CTSP and UTSP strategies result in transit punctuality improvement. Statistically, the UTSP control generates an improvement in schedule deviation reduction of approximately 57%, whereas the CTSP control generates less benefits of approximately 21% under the same circumstances.

However, such a significant improvement in UTSP is obtained at the cost of granting excessive priority time to transit vehicles, as illustrated in Fig. 8(a). Consequently, sacrificing too much green time for the non-priority phases at
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in the on-time performance increases with a decrease in the degree of saturation and an increase in the green split of the transit phase. The transit schedule deviation changes cyclically with the change in the departure time of the bus at the upstream bus stop when the intersections between the two bus stops have the same cycle.

This study optimizes the signal priority for buses under the assumption that the travel time between intersections is given. The combined optimization of signal priority and bus trajectory control is a promising research field. Moreover, further research should address high-frequency transit systems with multi-request demands.

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