Effect of Thermal Gradient on Steady State Creep in a Rotating Disc of Variable Thickness

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Abstract

The steady state creep in a rotating FGM disc having linearly varying thickness has been analyzed in the presence of linear thermal gradient. The disc is made of composite containing silicon carbide particles (SiC\textsubscript{p}) in a matrix of pure aluminium. The content of SiC\textsubscript{p} in the disc decreases non-linearly from the inner to the outer radius. The distributions of stresses and strain rates, resulting due to disc rotation, have been estimated by imposing various forms of linear thermal gradient in the FGM disc. The results are also estimated from a similar FGM disc but operating at a constant temperature, estimated by averaging the thermal gradient imposed in the disc. The results indicate that the tangential stress increases near the inner radius but decreases towards the outer radius and the radial stress increases throughout when the FGM disc is assumed to operate under a linear thermal gradient. However, the strain rates in the FGM disc decreases significantly in the presence of thermal gradient as compared to those observed in the FGM disc operating at a constant average temperature.

1. Introduction

Rotating disc is a widely used component in many engineering applications involving severe thermomechanical loadings where the conventional materials alone may not survive. Therefore, the concept of Functionally Graded Materials (FGMs) has been developed. In FGMs the constituents or their contents vary with respect to position coordinates to achieve desired properties [1-3].

Singh and Ray [4] analyzed steady state creep in a constant thickness rotating FGM disc by using Norton’s power law. They observed that the steady state creep response of the FGM disc is significantly superior to a similar disc having uniform distribution of SiC\textsubscript{p}. Gupta et al [1] also investigated steady state creep in constant thickness rotating FGM disc but operating under a radial thermal gradient. They also noticed that the FGM disc is superior to a uniform composite disc. Bayat et al [5] obtained exact solutions for a variable thickness rotating disc made of FGM. The study reveals that the FGM discs having parabolic and hyperbolic convergent thickness profiles exhibit smaller stresses and displacements as compared to a constant thickness disc. Deepak
et al [6] investigated steady state creep in a rotating FGM disc having linearly varying thickness. It is revealed that the strain rates in the FGM disc are significantly lower than that observed in a similar disc having uniform distribution of reinforcement.

The literature reveals that a number of studies have been conducted to investigate the creep behaviour of rotating FGM disc having constant thickness and operating at a constant temperature. In the light of this, it is decided to investigate the creep response of a rotating FGM disc having linearly varying thickness and operating under a radial thermal gradient.

2. Disc profile and distribution of reinforcement

Considering a disc made of Al-SiCp with inner and outer radii as ‘a’ (=31.75 mm) and ‘b’ (=152.4 mm) respectively and rotating at 15000 rpm. The thickness $h(r)$ of the disk is given by,

$$h(r) = h_b + 2k(b - r) \quad (1)$$

where $k = \frac{(h_a - h_b)}{2(b - a)}$ and $h_a (= 43.22 \text{ mm})$ and $h_b (= 13.97 \text{ mm})$ are thickness of the disc at the inner and outer radii respectively.

The distribution of reinforcement (SiCp) in the disc is assumed to vary non-linearly from the inner to the outer radius. Therefore, the density and creep constants will also vary radially. The content of SiCp in the disc at any radial location, $V(r)$, is given by,

$$V(r) = V_{\text{max}} - \frac{(r-a)^2}{(b-a)^2} (V_{\text{max}} - V_{\text{min}}) \quad (2)$$

where $V_{\text{max}}$ and $V_{\text{min}}$ are respectively the maximum and minimum SiCp contents at the inner and the outer radii respectively.

Following the rule of mixtures, the density $\rho(r)$ of the FGM disc at a radius $r$ is given by,

$$\rho(r) = \rho_m + \frac{(\rho_d - \rho_m)}{100} \left[ V_{\text{max}} - \frac{(r-a)^2}{(b-a)^2} (V_{\text{max}} - V_{\text{min}}) \right] \quad (3)$$

where $\rho_m (= 2698.9 \text{ kg/m}^3)$ and $\rho_d (= 3210 \text{ kg/m}^3)$ are respectively the densities of pure Al and SiCp [7-8].

If the average SiCp content in the FGM disc is $V_{\text{avg}}$ and $t$ is the average thickness of the disc, then one may write,

$$V_{\text{av}} = \int_a^b 2\pi r h(r) V(r) dr$$

$$\pi(b^2 - a^2)t$$

Substituting $h(r)$ and $V(r)$ respectively from eqs. (1) and (2) into eq. (4), we may get,

$$V_{\text{min}} = \frac{L * V_{\text{av}} - V_{\text{max}} * (M - K)}{K} \quad (5)$$
where,
\[
L = \left( \frac{b^2 - a^2}{2} \right) t,
\]
\[
M = h_b \left( \frac{b^2 - a^2}{2} \right) + \left( \frac{b^3 - a^3}{3} \right) (b - a)^2 * (b + 2 * a)
\]
\[
K = h_b \left[ \left( \frac{b^4 - a^4}{4} \right) + \left( \frac{b^2 - a^2}{2} \right) a^2 - \frac{2 * a * (b^3 - a^3)}{3} \right] + \frac{2 * k}{3} \left[ \frac{b * (b - a)^4}{4} - \frac{(b - a)^7}{10} \right]
\]

3. Creep law and creep parameters

The material of the disc is assumed to undergo steady state creep according to a well known threshold stress based law as given below [6],
\[
\dot{\varepsilon} = [M(r) \{ \bar{\sigma} - \sigma_0(r) \}]^n
\]

where \( M(r) = \left[ \frac{1}{E} \left( A' \exp \frac{-Q}{RT} \right)^{\frac{1}{n}} \right] \) and \( \sigma_0(r) \) are the creep parameters. The symbols \( A' \), \( n \), \( Q \), \( E \), \( R \) and \( T \) denote respectively the structure dependent parameter, true stress exponent, true activation energy, temperature-dependent Young’s modulus, gas constant and operating temperature. The value of true stress exponent \( (n) \) is taken as 5 in this study, which corresponds to creep controlled by high temperature dislocation climb [6].

The values of creep parameters \( M(r) \) and \( \sigma_0(r) \) depend on particle size \( (P) \), particle content \( (V) \) and operating temperature \( (T) \). This dependence is expressed in terms of the following regression equations, as developed earlier [6],
\[
M(r) = 0.0288 - \frac{0.0088}{P} - \frac{14.0267}{T(r)} + \frac{0.0322}{V(r)}
\]
\[
\sigma_0(r) = -0.084 P - 0.023 T(r) + 1.185 V(r) + 22.207
\]

In this study, the disc is assumed to operate under a radial thermal gradient as given by,
\[
T(r) = T_a + (T_b - T_a) \times \frac{(r - a)}{(b - a)}
\]

where \( T_a \) and \( T_b \) are respectively the temperatures at the inner and the outer radii of the disc.

4. Mathematical formulation

The analysis is based on the following assumptions:

(i) Material of the disc is locally isotropic and incompressible.
(ii) Steady state condition of stress is assumed.
(iii) Elastic deformations in the disc are small and hence neglected.
(iv) The axial stress \( (\sigma_z) \) is assumed to be zero throughout the disc.

The constitutive equations for creep in an isotropic disc under biaxial state of stress \( (i.e. \sigma_z = 0) \) are given by [1].
\[
\dot{\varepsilon}_r = \frac{\dot{\varepsilon}}{2\bar{\sigma}} [2\sigma_r - \sigma_\theta]
\]
\[
\dot{\varepsilon}_\theta = \frac{\dot{\varepsilon}}{2\bar{\sigma}} [2\sigma_\theta - \sigma_r]
\]
\[
\dot{\varepsilon}_z = \frac{\dot{\varepsilon}}{2\bar{\sigma}} [-\sigma_r - \sigma_\theta]
\]

where \(\dot{\varepsilon}_r, \dot{\varepsilon}_\theta, \dot{\varepsilon}_z\) and \(\sigma_r, \sigma_\theta, \sigma_z\) are respectively the strain rates and the stresses in \(r\), \(\theta\) and \(z\) directions.

The effective stress \((\bar{\sigma})\) in the disc under biaxial state of stress may be expressed as,

\[
\bar{\sigma} = \frac{1}{\sqrt{2}} \left[ \sigma_\theta^2 + \sigma_r^2 + (\sigma_r - \sigma_\theta)^2 \right]^{1/2}
\]

The equilibrium equation for a variable thickness disc, rotating with angular velocity \(\omega\), is given as [6],

\[
\frac{d}{dr} [h(r)r\sigma_r] - h(r)\sigma_\theta + \rho(r)\omega^2 r^2 h(r) = 0
\]

The disc is assumed to be fitted on a splined shaft where small axial movement is permitted, for which we may use the following free-free boundary conditions [3],

\[
\sigma_r = 0 \text{ at } r = a \text{ and } \sigma_r = 0 \text{ at } r = b
\]

The equilibrium eq. (11) is solved along with set of constitutive eqs. (9) by following the procedure given in [6] to obtain the distribution of stresses and strain rates in the FGM disc.

5. Results and discussions

The numerical computations are carried out to obtain the stresses and steady state creep rates in the FGM disc having three different radial thermal gradients (Table-1). The results obtained are compared with a similar FGM disc (D1) but operating at a uniform average temperature, obtained by averaging the thermal gradient imposed on the FGM disc.

| Notations used | Temperature (K) | Temp. Gradient | (TG) = (T_e - T_i), K |
|----------------|-----------------|----------------|-----------------------|
| D1             | 623 623 623     | 0              |
| D2             | 608 638 623     | 30             |
| D3             | 593 653 623     | 60             |
| D4             | 578 668 623     | 90             |
The radial stress in all the discs, Fig. 1a, increases from zero at the inner radius, reaches a maximum, before dropping to zero again at the outer radius. With the increase in thermal gradient (TG), the radial stress is observed to increase over the entire disc. The radial stress is the highest in FGM disc-D4 and the lowest in FGM disc-D1. As TG in the FGM disc increases, the tangential stress (Fig. 1a) increases near the inner radius but decreases towards the outer radius. The strain rates, tangential as well as radial, decrease significantly with the increase in thermal gradient in the FGM disc (Fig. 1b). Though, near the inner radius the tangential stress in the FGM disc-D4 is highest due to relatively higher density, still the FGM disc-D4 exhibits lower tangential creep rates near the inner radius. It is attributed to respectively lower and higher values of parameters $M$ and $\sigma_0$, [eqs. (7) and (8)], near the inner radius of the FGM disc-D4 than the FGM discs D1-D3. On the other hand, the tangential strain rate near the outer radius of the FGM disc-D1 is lower due to lower tangential stress towards the outer radius. The effect of imposing various kinds of TG on the radial strain rate is similar to that observed for tangential strain rate (Fig. 1b).

![Fig. 1. Effect of imposing various thermal gradients on (a) Stress and (b) Strain rates in the FGM.](image)

### 6. Conclusions

The present study has led to the following conclusions:
- The radial stress in the rotating disc increases throughout with the increase in thermal gradient. The increase in thermal gradient in the FGM disc leads to increase the tangential stress near the inner radius but decreases its values towards the outer radius.
- The creep response of the FGM disc operating under a radial thermal gradient is superior to a similar FGM disc operating under a uniform temperature. The creep rates in the FGM disc further reduce by imposition of relatively steeper thermal gradient.

### References

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