T-MOKE for nuclear resonant reflectivity

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Abstract. It has been shown that transverse magneto-optical Kerr effect (T-MOKE) can be essentially enhanced in multilayers with antiferromagnetic interlayer coupling and observable even at grazing angles. The description of this effects need the application of the exact theory of reflectivity. The performed calculations show the presence of the antiferromagnetic Bragg maximum on the delayed reflectivity curve for transverse geometry which is absent in the used at present theory of Mössbauer reflectivity. The energy or time spectra of reflectivity, calculated on the basis of the exact and approximate theory, also reveal the essential difference.

1. Introduction
Mössbauer reflectivity is observed at grazing angles because for wavelengths ~ 0.1 nm the medium susceptibility $\chi$ is very small ~ $10^{-5}$. So the general theory of reflectivity [1-2] had been simplified for this case [3-4]. This approximation was equivalent to the approach used in the further investigations of the synchrotron Mössbauer reflectivity [5-6]. However, sometimes the exact theory of reflectivity can give the different results from the used approximation. Here we show that even for Mössbauer wavelengths and grazing angles some new effects can be predicted on the basis of the general theory of reflectivity.

2. Exact theory
Transverse magneto-optical Kerr effect (T-MOKE) is characterized by the difference in reflectivity for $\pi$-polarized light from the sample, magnetized perpendicular to the scattering plane, when the direction of the magnetization changes sign. It has maximum value for the angles $\theta$ close to $45^\circ$, so for Mössbauer reflectivity at grazing angles it is supposed negligible.

The origin of T-MOKE is the specific relationship between the tangential components of the electric $E$ and magnetic $H$ fields of radiation. In the used at present theory of the nuclear resonant reflectivity [3-6] the tangential components of the radiation field in the scattering plane at grazing angles are ignored, thus T-MOKE could not appear.

![Figure 1](image-url) Figure 1. For the explanation of the T-MOKE. The tangential component of the $\pi$-polarized electric field of radiation in the sample are different for two opposite directions of the magnetization.
In the correct theory the exact 4x4-propagation matrices \[1-2\] should be used, for description of the depth evolution of the tangential components \(H_t = I H\) and \(q^x E\):

\[
M = \begin{pmatrix}
q^x q \cdot x \cos \theta & i [q^x q \cdot y \cos \theta - y \cdot x] \\
1 - \cos^2 \theta x \cdot x & 1 + q^x q
\end{pmatrix},
\]

(1)

where \(x,y,q\) is the unit vector basis (Fig. 1), a symbol \(\odot\) means the outer product of vectors, \(\theta\) is the glancing angle, \(I = 1 - q \odot q\), \(\hat{\theta} = 1 + \hat{x}\), \(\tilde{\theta}\) means a transposed tensor, \(q^x\) is a dual tensor, i.e. \(q^x a = q \times a\), which is a vector product. For dipole transitions different hyperfine transitions with the change of the magnetic quantum number \(\Delta m = \pm 1,0\) have a specific contribution to the resonant susceptibility \(h_{\Delta m} \odot h^*_{\Delta m}\) (\(h_{\Delta m}\) are the spherical orts of the hyperfine field reference system), so the permittivity tensor has a form:

\[\hat{\epsilon} = 1 + \hat{x} = 1 + \epsilon^{el} A_0 h^* + A_{+1} h^* + A_{+1} h^* + A_{-1} h^* \]

(2)

When the hyperfine magnetic field directions are perpendicular to the scattering plane (transverse geometry) we have (in \(x,y,q\) basis):

\[\hat{\epsilon} = 1 + \hat{x} = \begin{pmatrix}
1 + A + C & 0 & 0 \\
n - A & 1 + A & iB \\
- iB & 1 + A & 0
\end{pmatrix},
\]

(3)

where the notations are inserted

\[A = \epsilon^{el} + \frac{A_{+1} + A_{-1}}{2}, \quad C = A_0 - \frac{A_{+1} + A_{-1}}{2}, \quad B = \frac{A_{+1} - A_{-1}}{2}.
\]

(4)

The propagation matrix (1) for the transformation of the vector-column \(\{H_x, H_y, -E_y, E_x\}\) takes a form

\[
\hat{\epsilon} = \begin{pmatrix}
-i \cos \theta & 0 & B_{11} = 1 + A - B^2 & 0 \\
0 & 0 & 0 & \sin^2 \theta + A + C \\
\sin^2 \theta + A & 0 & iB \cos \theta \\
0 & 1 + A & 0 & 0
\end{pmatrix},
\]

(5)

In this geometry \(\sigma\) - and \(\pi\)–polarizations become the eigen polarizations of the exact propagation matrix (5), as well as that of the approximate propagation matrix. The normal component of the wave vector \(k^2 \pi\) (in \(\omega/c\) units) for \(\pi\)–polarization equals to

\[k^2 \pi = \frac{\sin^2 \theta + A + \frac{A_{+1} + A_{-1}}{2}}{\sin^2 \theta + \chi_{33}},
\]

(6)

which is the same as that obtained from the approximate theory. If the transverse external field changes sign than \(A_{-} \rightarrow A\) and \(A_{+} \rightarrow A_{-}\) and (6) is not changed. However, for calculation of reflectivity we should solve the boundary task which for \(\pi\)–polarization takes a form

\[
\begin{cases}
H^0_x + H^R_x = H^T_x, \\
\sin \theta (H^0_x + H^R_x) = \gamma^\pi H^T_x
\end{cases}
\]

(7)
and one needs the exact relation $\gamma^\pi$ between $\mathbf{E}_t$ and $\mathbf{H}_t$:

$$E_y = \gamma^\pi H_x \equiv -\frac{k^\pi}{1+A} \left(1 + i \frac{(A_{+1} - A_{-1})}{2} \cos 0ight) H_x. \quad (8)$$

In approximate theory we have only

$$E_y = -k^\pi H_x. \quad (9)$$

Comparing (8) and (9), we see that in the exact theory the relationship between $\mathbf{E}_t$ and $\mathbf{H}_t$ contains the additional term, which changes sign when the direction of the hyperfine field changes sign. That is the origin of the T-MOKE effect. But the absolute value of the additional term is quite small (the first term in (8) $\sim \sqrt{\chi} \sim 10^{-3}$, the second term $\sim \chi \sim 10^{-5}$), so in the most cases it can be neglected for Mössbauer reflectivity. Notice also that the resonant 14.4 keV transition in $^{57}$Fe is M1 type and we should determine the polarization with respect to the magnetic field of radiation, so in (8,9) we should change $\mathbf{H} \leftrightarrow \mathbf{E}$. It occurs also that “$\pi$–polarization” is exactly the polarization of the synchrotron beam.

3. Some example calculations

Since T-MOKE is a consequence of the boundary conditions (the continuity of $\mathbf{E}_t$ and $\mathbf{H}_t$) we can enhance the effect at grazing angles by considering the model with many boundaries. For calculations we have chosen the periodic multilayer with antiferromagnetic interlayer coupling

$[^{57}\text{Fe}\uparrow(6\,\text{Å})/\text{V}(6\,\text{Å})/^{57}\text{Fe}\downarrow(6\,\text{Å})/\text{V}(6\,\text{Å})]_{30}/\text{Si}$ (Fig. 2). Hyperfine magnetic field in $^{57}$Fe layers ($B_{\text{hf}} = 33$ T with broadening 1T) has been put perpendicular to the scattering plane and antiferromagnetically aligned in sequent $^{57}$Fe layers.

![Figure 2. Model structure.](image)

![Figure 3. Delayed nuclear resonant reflectivity, calculated on the basis of the correct and approximate theory.](image)

Reflections from the two $^{57}$Fe layers in one magnetic period, magnetized perpendicular to the scattering plane but in opposite directions, are different due to T-MOKE, so the half-order magnetic Bragg reflection appears in the delayed nuclear resonant reflectivity curve at ~ 18 mrad (Fig. 3).

![Figure 4. Mössbauer (a) and time (b) spectra of reflectivity at the antiferromagnetic Bragg maximum in transverse geometry, calculated on the basis of the exact and approximate theory.](image)
Notice that calculations on the basis of the approximate theory are perfectly coincided with the results of the exact theory with exception of the small angular region in vicinity of the antiferromagnetic Bragg peak. The energy or time reflectivity spectra for the magnetic Bragg peak, calculated on the basis of the correct theory, are different from that calculated by the used at present theory (Fig. 4).

It is known that in longitudinal geometry when the hyperfine magnetic fields are aligned in the scattering plane the antiferromagnetic Bragg maximum perfectly described on the basis of the approximate theory and influence of the additional term in (8) disappears. However, at the other azimuth angles the exact theory can give the different results from the approximate theory. The most noticeable difference is presented on the shape of the energy or time spectra (Fig. 5). For this geometry σ- and π- polarizations are not the eigen polarizations so the calculations need the application of the general propagation matrix [1-2].

![Figure 5](image)

**Figure 5.** Mössbauer spectra of reflectivity at the antiferromagnetic Bragg maximum at the azimuth angle 45°, calculated on the basis of the exact and approximate theory.

5. Conclusions
The magnetic/nonmagnetic superlattices with antiferromagnetic interlayer coupling play an important role in modern technology. The detailed investigations of the peculiarities of the magnetic ordering in such systems are very interesting from the physical point of view. Resonant reflectivity can supply us with the very specific information. So the correct interpretation of the reflectivity data is desirable. Here we show that the exact theory of reflectivity leads sometimes to the essential difference in the shapes of the energy or time spectra of reflectivity in vicinity of the antiferromagnetic Bragg peak, existence of which has been here predicted by the exact theory of reflectivity in the transverse geometry.

We hope that this new effect will be observed for the nuclear resonant reflectivity in the nearest future. Notice that the antiferromagnetic Bragg reflections in T-MOKE geometry were successfully used for resonant X-rays (e.g. near L3 edge of Ni in superlattice [Ag(1.1 nm)/Ni(1.75 nm)]n [7]).

References
[1] Azzam R M A and Bashara N M 1977 *Ellipsometry and polarized light* (North-Holland publishing company)
[2] Borzdov G N, Barkovskii L M and Lavrukovich V I 1976 *Zhurnal Prikladnoi Spectrosk.* 25 526
[3] Andreeva M A and Rosete K 1986 *Vestnik Moskovskogo Universiteta, Fizika* 41 65
Andreeva M A and Rosete K 1986 *Poverkhnost’* (In Russian) No.9 145
[4] Irkaev S M, Andreeva M A, Semenov V G, Belozerskii G N and Grishin O V 1993 *Nuclear Instrum. and Methods in Phys. Res. B* 74 554
[5] Deák L, Bottyán L, Nagy D L and Spiering H 1996 *Phys. Rev. B* 53 6158
[6] Röhlsberger R 1999 *Hyperfine Interactions* 123/124 301
[7] Tonnerre J M, Sève L, Raoux D, Soullié G, Rodmacq B, and Wolfers P 1995 *Phys. Rev. Lett.* 75 740

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