Flow Effects on Jet quenching with Detailed Balance

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A new model potential in the presence of collective flow describing the interaction of the hard jet with scattering centers is derived based on the static color-screened Yukawa potential. The flow effect on jet quenching with detailed balance is investigated in pQCD. It turns out that the collective flow changes the opacity and the LPM destructive interference comparing to that in the static medium. Considering the collective flow with velocity \( v_2 \) along the jet direction, the energy loss is \( (1 - v_2) \) times that in the static medium to the first order of opacity. The flow dependence of the energy loss will affect the suppression of high \( p_T \) hadron spectrum and anisotropy parameter \( v_2 \) in high-energy heavy-ion collisions.

Introduction — One of the most striking features of nucleus-nucleus collisions at the Relativistic Heavy Ion Collider (RHIC) is the collective flow. This phenomenon has been subject of intensive theoretical and experimental studies\textsuperscript{1, 2, 3, 4}. It is believed that the medium produced in nucleus-nucleus collisions at RHIC equilibrates efficiently and builds up a flow field.

Gluon radiation induced by multiple scattering of an energetic parton propagating in a dense medium leads to induced parton energy loss or jet quenching. As discovered in high-energy heavy-ion collisions at RHIC, jet quenching is manifested in both the suppression of single inclusive hadron spectrum at high transverse momentum \( p_T \) region\textsuperscript{5} and the disappearance of the typical back-to-back jet structure in dihadron correlations\textsuperscript{6}. Extensive theoretical investigation of jet quenching has been widely carried out in recent years\textsuperscript{7, 8, 9, 10, 11, 12}. Based on the static color-screened Yukawa potential proposed by Gyulassy and Wang\textsuperscript{7}, an opacity expansion method was developed to obtain the non-Abelian energy loss, and it shows that the induced gluon radiation intensity is dominated by the first order opacity contribution\textsuperscript{10}. However, the medium cannot be described as being static, the collective flow has to be considered\textsuperscript{13, 14, 15}. Later, the interaction between the jet and the target partons in the presence of collective flow was modeled by a momentum shift \( q_0 \) in the Gyulassy-Wang’s static potential\textsuperscript{16}, but this assumption lacks sufficient theoretical evidence. Recently, local transport coefficient \( \hat{q} \), which is related to the squared average transverse momentum transfer from the medium to the hard parton per unit length, has been investigated in the presence of transverse flow\textsuperscript{17, 18}. However, when relating \( \hat{q} \) with the energy density \( \varepsilon \) of the medium, \( \hat{q} \approx \varepsilon \xi^{3/4} \), a problem appears that the determination of \( \varepsilon \) is different from \( \varepsilon = 2 \) to \( \varepsilon > 8, \cdots, 19 \) by various authors, the results lack consistency. The \( \hat{q} \) study with collective flow in Ref.\textsuperscript{17} based on BDMPs energy loss calculation gives only a macroscopic result for parton energy loss. Many interesting properties such as the flow effects on the non-Abelian Laudau-Pomeranchuk-Migdal(LPM) interference effect and opacity can not be studied. On the other hand, only radiative energy loss can be considered in Ref.\textsuperscript{17}, the detailed balance effect with gluon absorption cannot be included. It has been shown that the gluon absorption play an important role for the intermediate jet energy region\textsuperscript{19}.

In this letter, we report a first study of the parton energy loss with detailed balance in the presence of collective flow in perturbative Quantum Chromodynamics (pQCD). We first determine the model potential to describe the interaction between the energetic jet and the scattering target partons with collective flow of the quark-gluon medium using Lorentz boosts. Based on this new potential, we then consider both the radiation and absorption induced by the self-quenching and multiple scattering in the moving medium. We are led to the conclusion that the net medium effect without rescattering is dominated by the final-state thermal absorption, whose result is the same as that in the static medium. For the case of rescattering with targets the collective flow changes the opacity and the LPM destructive interference. Overall, it reduces (enhances) the jet energy loss induced by rescattering with stimulated emission and thermal absorption depending on the direction of the flow in the positive (negative) jet direction.

The Potential Model — To calculate the induced radiation energy loss of jet, the interaction potential is assumed in the Gyulassy-Wang’s static model\textsuperscript{7} that the quark-gluon medium can be modeled by N well-separated color screened Yukawa potentials.

\begin{equation}
V_i^a(q_i) = \frac{4\pi\alpha_s}{q^2 + \mu^2} e^{-i\mathbf{q}\cdot\mathbf{x}_i} T_a(j) T_a(i),
\end{equation}

where \( \mu \) is the Debye screening mass, \( T_a(j) \) and \( T_a(i) \) are the color matrices for the jet and target parton at \( x_i \). In this potential, each scattering has no energy transfer \((q_0 = 0)\) but only a small momentum \( q \) transfer with the medium. If using the four-vector potential, the Gyulassy-Wang’s static potential can be denoted as \( A^\mu = (V_i^a(q_i), A^a_i(q_i) = 0) \).

As is well known in Electrodynamics, the static charge produces a static Coulomb electric field, while a moving charge produces both electric and magnetic field. In analogy a moving target parton in the quark-gluon medium...
will produce color-electric and color-magnetic fields simultaneously due to the collective flow. Therefore, the static potential model should be reconsidered.

In the quark-gluon medium with collective flow, the rest frame fixed at target parton moves with a velocity \( \mathbf{v} \) relative to the observer’s system frame \( \Sigma' \), as illustrated in Fig. 1. We first take a Lorentz transformation for four-momentum \( q \), and then for four-vector potential \( A^\mu \), we can then write \( A^\mu = (V_{(flow)}^\mu(q_i), A_{(flow)}(q_i)) \) in the observer’s system frame \( \Sigma' \) as

\[
\left\{ \begin{align*}
V_{(flow)}^\mu(q_i) &= 2\pi \delta(q_i^0 - \mathbf{v} \cdot \mathbf{q}) e^{-i\mathbf{q} \cdot \mathbf{x}} \tilde{v}(\mathbf{q}) T_a(j) T_{a_i(i)}, \\
A_{(flow)}(q_i) &= 2\pi \delta(q_i^0 - \mathbf{v} \cdot \mathbf{q}) e^{-i\mathbf{q} \cdot \mathbf{x}} \tilde{A}(\mathbf{q}) T_a(j) T_{a_i(i)},
\end{align*} \right.
\]

where \( \tilde{v}(\mathbf{q}) = 4\pi \alpha_s/(q^2 - (\mathbf{v} \cdot \mathbf{q})^2 + \mu^2) \). The new potential differs from Gyulassy-Wang’s static potential in that the collective flow of the quark-gluon medium produces a color-magnetic field and the flow leading to non-zero energy transfer \( q_0 = \mathbf{v} \cdot \mathbf{q} \), which will affect jet energy loss as we will show below.

Elastic cross section for small transverse momentum transfer between jet and target partons can be deduced as

\[
\frac{d\sigma_{el}}{d^2 q} = \frac{C_RC_2}{d_A} \left| \frac{i\delta}{2\pi} \right|^2 (1 - v_z)^2,
\]

where \( v_z \) is the velocity of the target parton along the jet direction as shown in Fig. 1, \( C_R \) and \( C_2 \) are the Casimir of jet and target parton in fundamental representation in \( d_R \) dimension, respectively. \( d_A \) is the dimension of corresponding adjoint representation. Our result agrees with the GLV elastic cross section in static potential when the flow velocity goes to zero [7].

**Flow Effect on Gluon Radiation** — Consider a hard parton produced at \( z_0 = (z_0, x_{1\perp}) \). The hard parton has initial energy \( E \), interacts with the target parton at \( z_1 = (z_1, x_{1\perp}) \) with flow velocity \( \mathbf{v} \) by exchanging gluon with four-momentum \( q \), radiates a gluon with four-momentum \( k \) and polarization \( \epsilon(k) \), and emerges with final four-momentum \( p \). In the light-cone components,

\[
k = \left[ 2\omega, \frac{k_z^2}{2\omega}, k_{\perp} \right], \quad \epsilon(k) = \left[ 0, 2\epsilon_{\perp} \cdot \frac{k_{\perp}}{x E^+}, \epsilon_{\perp} \right], \quad p = \left[ (1 - x)E^+ + 2\mathbf{v} \cdot \mathbf{q}, \mathbf{p}_{\perp} \right],
\]

where \( \omega = x E, E^+ = 2E \gg \mu \).

At zeroth order in opacity, the jet has no interaction with the target parton, we obtain the same radiation amplitude

\[
R^{(0)} = 2igT_c \frac{k_{\perp} \cdot \epsilon_{\perp}}{k_{\perp}^2},
\]

as that in the static medium in Ref. [10], where \( T_c \) is the color matrix. As shown in Ref. [19], the net energy gain without rescattering can be expressed as

\[
\Delta E^{(0)}_{abs} \approx -\frac{\pi \alpha_s C_R T^2}{3 E} \ln \left[ \frac{4E^2 + 2 - \gamma_E + \frac{6\zeta(2)}{\pi^2}}{E^2} \right],
\]

where \( \gamma_E \approx 0.5772, \zeta(2) \approx -0.9376 \) and \( T \) is the thermal finite temperature.

When the hard parton goes through the quark-gluon medium, it will suffer multiple scattering with the parton target inside the medium. Here we investigate the rescattering-induced radiation by considering the flow effect resulting from the moving parton target. We will work in the framework of opacity expansion developed by Gyulassy, Lévai and Vitev (GLV) [10] and Wiedemann [11]. It was shown by GLV that the higher order corrections contribute little to the radiative energy loss. So we will only consider the contributions to the first order in the opacity expansion. The opacity is defined as the mean number of collisions in the medium, \( \bar{n} = \frac{L/l}{N \sigma_{el}/A_\perp} \). Here \( N, L, A_\perp \) and \( l \) are the number, thickness, transverse area of the targets, and the average mean-free-path for the jet, respectively.

Based on our new potential in Eq. (2) by considering collective flow of the quark-gluon medium, assuming the flow velocity \( |\mathbf{v}| \ll 1 \), we obtain the radiation amplitude associated with a single rescattering,

\[
R^{(S)} = 2ig \left( H T_c T_{\epsilon} + B_1 e^{\frac{i\epsilon_{\perp} \cdot \mathbf{v}}{x E^+}} [T_c, T_\alpha] - 2v_z T_{\epsilon} T_c H(1 - e^{\frac{i\omega \tau_0}{x E^+}}) + C_1 e^{\frac{i\omega \tau_0}{x E^+}} [T_c, T_\alpha] \right) \cdot \epsilon_{\perp} (1 - v_z),
\]

where \( \tau_0 = z_1 - z_0 \),

\[
\omega_0 = \frac{k_z^2}{2\omega}, \quad \omega_1 = \frac{(k_{\perp} - q_{\perp})^2}{2\omega}, \quad H = -\frac{k_{\perp}}{k_{\perp}} \cdot C_1 - \frac{k_{\perp} - q_{\perp}}{(k_{\perp} - q_{\perp})^2}, \quad B_1 = H - C_1.
\]

Different from the static medium case, the single rescattering amplitude depends on the collective flow of the quark-gluon medium.
The interference between the process of double scattering and no rescattering should also be taken into account to the first order in opacity $\alpha$. Assuming no color correlation between different targets, the double rescattering corresponds to the "contact limit" of double Born scattering with the same target. Assuming the flow velocity $|v| \ll 1$, with our new potential in Eq. (2), the radiation amplitude can be expressed as

$$ R^{(D)} = 2i g T e^{\frac{ie\omega}{\lambda s}} \left( -\frac{C_R + C_A}{2} \right) \mathcal{H} e^{\frac{ie\omega}{\lambda s}} + \frac{C_A B_1}{2} + 2v_z C_R - C_A \mathcal{H} (1 - e^{-\frac{ie\omega}{\lambda s}}) \right) \cdot \epsilon_{\perp} (1 - v_k)^2, \quad (12) $$

where $C_A$ is the Casimir of the target parton in adjoint representation in $d_A$ dimension. The double rescattering amplitude also depends on the collective flow.

To the first order in opacity, we then derive the induced radiation probability including both the stimulated emission and thermal absorption as

$$ \frac{dP^{(1)}}{d\omega} = \frac{C_2}{8\pi d_A d_R A_1} \int \frac{d^2 k_1}{(2\pi)^2} \int \frac{d^2 q_{\perp}}{(2\pi)^2} P_{\perp}(\omega) \left\{ \begin{array}{c} (1 - v_k)^2 v^2(q_{\perp}) \left( T \left[ R^{(S)} \right]^2 + 2R \left[ R^{(0)} \right] \right) \\ (1 + N_g(xE)) \delta(\omega - \omega E)/\theta(1 - x) + N_g(xE) \delta(\omega + xE) \end{array} \right\} $$

$$ \approx \frac{\alpha_s C_R C_A}{d_A \pi} \int \frac{d^2 k_1}{(2\pi)^2} \int \frac{d^2 q_{\perp}}{(2\pi)^2} P_{\perp}(\omega) \left\{ \begin{array}{c} (1 - v_k)^2 v^2(q_{\perp}) \left( \frac{k_\perp \cdot q_{\perp}}{(k_\perp - q_{\perp})^2} \right) \\ (1 + N_g(xE)) \delta(\omega - \omega E)/\theta(1 - x) + N_g(xE) \delta(\omega + xE) \end{array} \right\}, \quad (13) $$

where $N_g(|k|) = 1/\exp(|k|/T) - 1$ is the thermal gluon distribution, $v(q_{\perp}) = 4\pi \alpha_s/\langle q_{\perp}^2 + \mu^2 \rangle$, $\alpha_s = g^2/4\pi$ is strong coupling constant. We have also included the splitting function $P_{q_{\perp}}(x) \equiv P(x)/x = \left[ 1 + (1 - x)^2/x \right]$ for $q \to gg$.

The jet energy loss can be divided into two parts. The zero-temperature part corresponds to the radiation induced by rescattering without detailed balance effect and can be expressed as

$$ \Delta E^{(1)}_{rad} = \int d\omega \omega \left. \frac{dP^{(1)}}{d\omega} \right|_{T=0} = \frac{\alpha_s C_R L}{\pi l_g^2} \int dx \int \frac{d^2 k_1}{k_1^2} \int d^2 q_{\perp} |\bar{v}(q_{\perp})|^2 P(x) \left\{ \begin{array}{c} (1 - v_k)^2 (1 + 2v_k L) \left( \frac{k_\perp \cdot q_{\perp}}{(k_\perp - q_{\perp})^2} \right) \\ (1 + N_g(xE)) \delta(\omega - \omega E)/\theta(1 - x) + N_g(xE) \delta(\omega + xE) \end{array} \right\}, \quad (14) $$

where $l_g = C_R l/C_A$ is the mean-free path of the gluon.

The temperature-dependent part of energy loss induced by rescattering at the first order of opacity comes from thermal absorption with partial cancellation by stimulated emission, in the presence of flow it can be written as

$$ \Delta E^{(1)}_{abs} = \int d\omega \omega \left( \frac{dP^{(1)}}{d\omega} - \frac{dP^{(1)}}{d\omega} \right|_{T=0} = \frac{\alpha_s C_R L}{\pi l_g^2} \int dx \int \frac{d^2 k_1}{k_1^2} \int d^2 q_{\perp} |\bar{v}(q_{\perp})|^2 N_g(xE) \left\{ \begin{array}{c} (1 - v_k)^2 (1 + 2v_k L) \left( \frac{k_\perp \cdot q_{\perp}}{(k_\perp - q_{\perp})^2} \right) \\ (1 + N_g(xE)) \delta(\omega - \omega E)/\theta(1 - x) + N_g(xE) \delta(\omega + xE) \end{array} \right\}, $$

where $|\bar{v}(q_{\perp})|^2$ is defined as the normalized distribution of momentum transfer from the scattering centers,

$$ \int d\omega \omega \left\{ \begin{array}{c} \frac{1}{\sigma_{cl}} d\sigma_{cl} = \frac{1}{\pi \left( \frac{q_{\perp}^2 + \mu^2}{q_{\perp}^2 + \mu^2} \right)^2} \right\}, \quad (15) $$

The gluon formation factor $1 - \exp(\frac{i\omega z_10}{1 - v_k})$ reflects the destructive interference of the non-Abelian LPM effect. The formation time of gluon radiation $\tau_f \equiv (1 - v_k)/\omega_1$ becomes shorter (longer) as an exponential Gaussian form $\rho(z) = \exp(-z/L_e)/L_e$ with $L_e = L/2$, the gluon formation factor can be deduced as

$$ \left\{ \begin{array}{c} \int d\omega \omega \left\{ \begin{array}{c} (1 - v_k)^2 (1 + 2v_k L) \left( \frac{k_\perp \cdot q_{\perp}}{(k_\perp - q_{\perp})^2} \right) \\ (1 + N_g(xE)) \delta(\omega - \omega E)/\theta(1 - x) + N_g(xE) \delta(\omega + xE) \end{array} \right\}, \quad (16) $$

$$ \frac{1}{\mu^2} = \frac{1}{\rho_{max}^2 + \mu^2}, \quad \rho_{max} = 3E\mu. \quad (17) $$

The opacity factor $L/l_g$ in Eqs. (16) and (17) arises from the sum over the N distinct targets. The mean-free-path $l_g$ of the gluon can be expressed as

$$ l_g^{-1} = \langle \sigma_{gg} \rho_g \rangle + \langle \sigma_{gg} \rho_g \rangle \approx (1 - v_k)^2 \lambda_g^{-1}, \quad (19) $$

where $\lambda_g$ is the mean-free-path of the gluon in the static medium. It is shown that collective flow increases (decreases) mean free path, leading to a decreasing (increasing) mean number of collisions in the medium in the presence of collective flow in the positive (negative) jet direction, respectively.

To obtain a simple analytic result, we take the kinematic boundaries limit $q_{\perp max} \to \infty$, the angular integral can be carried out by partial integration. In the limit of $EL \gg 1$ and $E \gg \mu$, we obtain the approximate asymptotic behavior of the energy loss,

$$ \Delta E^{(1)}_{rad} = (1 - v_k) \frac{\alpha_s C_R L^2}{4\lambda_g^2} \left[ \ln \frac{2E}{\mu^2 L} - 0.048 \right] + O(|v|^2), \quad (20) $$
collective flow velocity 
duced (enhanced) in the presence of collective flow. With 
iation becomes shorter (longer), and LPM effects is re-
(positive) jet direction, the formation time of gluon ra-
leads to a decreasing (increasing) opacity in the positive 
static medium case. The reason is that collective flow 
potential results when the velocity of the collective flow 
goes to zero. Since the parton energy loss is dominated 
by the first order opacity contribution, our result also 
agrees with the result of $\hat{q}$ calculation in Refs. $[17,18]$

Shown in Fig. 2 are the energy gain via gluon absorp-
tion with rescattering for $v_z = 0, 0.1, 0.2, 0.3$ and with-
out rescattering as functions of $E/\mu$. For comparison, 
we take the same values for the medium thickness, the 
mean free path, and the Debye screen mass as in Refs. $[10]$ 
and $[19]$. The energy gain without rescattering at very 
small $E/\mu$ region is larger than that with rescattering if 
$v_z > 0.2$, but at smaller flow velocity or at higher jet 
energies, it becomes smaller than that with rescattering.

Conclusion — In summary, we have derived a new po-
tential for the interaction of a hard jet with the parton 
target. It can be used to study the jet quenching phenom-
ena in the presence of collective flow of the quark-gluon 
medium. With this new potential, we have investigated 
the effect of collective flow on jet energy loss with detailed 
balance. Collective flow along the jet direction leads to 
a reduced opacity, $(1 - v_z)^2$ times that in static medium, 
and an increased LPM gluon formation factor, $(1 + v_z)$ 
times that in static medium. The energy gain without 
rescattering is the same as in the static medium, but the 
total energy loss to the first order of opacity is $(1 - v_z)$ 
times that in the static medium. Compared to calcula-
tions for a static medium, our results will affect the sup-
pression of high $p_T$ hadron spectrum and anisotropy pa-
parameter $v_3$ in high-energy heavy-ion collisions. Our new 
potential can also be used for heavy quark energy loss 
calculation and will alter the dead cone effect of heavy 
quark jets. Our results shall have implications for com-
parisons between theory and experiment in the future.

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\begin{figure}
\centering
\includegraphics[width=0.8\textwidth]{fig2.png}
\caption{The energy gain via gluon absorption with rescatter-
ing for $v_z = 0, 0.1, 0.2, 0.3$ and without rescattering as 
fractions of $E/\mu$.}
\end{figure}

\begin{equation}
\frac{\Delta E^{(1)}_{\text{abs}}}{E} = -(1 - v_z) \frac{\pi \alpha_s C_R}{3} \frac{LT^2}{\lambda q E^2} \left[ \ln \frac{\mu^2 L}{T} - 1 + \gamma_E - \frac{6\zeta(2)}{\pi^2} \right] + O(|v|^2). \tag{21}
\end{equation}

Our analytic result implies, to the first order in opac-
ity, that the energy loss is changed by a factor $(1 - v_z)$ 
for rescattering case with collective flow compared to the 
static medium case. The reason is that collective flow 
leads to a decreasing (increasing) opacity in the positive 
(negative) jet direction, the formation time of gluon radia-
tion becomes shorter (longer), and LPM effects is re-
duced (enhanced) in the presence of collective flow. With 
collective flow velocity $|v| = 0.1 - 0.3$ in the positive jet 
direction, jet energy loss decreases by $10 - 30\%$. This 
result implies that the collective flow has observable in-
fluence on the effective parton energy loss in the quark-
gluon medium. Our results is consistent with GLV static 
potential results when the velocity of the collective flow

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