Combining Photometry from Kepler and TESS to Improve Short-period Exoplanet Characterization

Ben Placek\(^1,3\), Kevin H. Knuth\(^1,4\), and Daniel Angerhausen\(^2\)

\(^1\)Physics Department, University at Albany (SUNY), Albany, NY 12222, USA; placekbh@sunysccc.edu, kknuth@albany.edu
\(^2\)NASA Goddard Space Flight Center, Exoplanets & Stellar Astrophysics Laboratory, Code 667, Greenbelt, MD 20771, USA; daniel.angerhausen@nasa.gov

Received 2015 November 3; accepted 2016 February 7; published 2016 June 13

Abstract

Planets emit thermal radiation and reflect incident light that they receive from their host stars. As a planet orbits its host star the photometric variations associated with these two effects produce very similar phase curves. If observed through only a single bandpass, this leads to a degeneracy between certain planetary parameters that hinder the precise characterization of such planets. However, observing the same planet through two different bandpasses gives much more information about the planet. Here we develop a Bayesian methodology for combining photometry from both Kepler and the Transiting Exoplanet Survey Satellite. In addition, we demonstrate via simulations that one can disentangle the reflected and thermally emitted light from the atmosphere of a hot-Jupiter as well as more precisely constrain both the geometric albedo and day-side temperature of the planet. This methodology can further be employed using various combinations of photometry from the James Webb Space Telescope, the Characterizing ExOplanet Satellite, or the PLATO mission.

Key words: methods: data analysis – techniques: photometric

1. Introduction

With the advent of high-precision photometry, a hidden component to the light curve of exoplanets has been revealed: the phase curve. The photometric phase curve corresponding to an exoplanet consists of four major effects: the reflection of incident stellar light (Seager et al. 2000; Jenkins & Doyle 2003; Perryman 2011; Placek et al. 2014), the emission of thermal radiation from the atmosphere (or surface) of the planet (Charbonneau et al. 2005; Borucki et al. 2009; Placek et al. 2014), the relativistic Doppler beaming of starlight as the host star orbits the system’s center of mass (Loeb & Gaudi 2003; Rybicki & Lightman 2008; Placek et al. 2014), and the tidal warping of the stellar surface due to the proximity of the massive planet known as ellipsoidal variations (Loeb & Gaudi 2003; Placek et al. 2014). Each effect has been observed for short-period hot-Jupiters in the Kepler field (Borucki et al. 2009; Welsh et al. 2010; Faigler & Mazeh 2011; Mazeh et al. 2012; Shporer et al. 2011; Mislis & Hodgkin 2012; Esteves et al. 2013; Faigler et al. 2013; Lillo-Box et al. 2014a; Placek et al. 2014, 2015; Angerhausen et al. 2015). While the Doppler beaming and tidal warping effects provide information on the planetary mass, reflection and thermal emission yield information on the planetary radius, geometric albedo, and day-side temperature.

In the case of circular orbits, the reflection and thermal emission components to the phase curve are largely indistinguishable—they both manifest as sinusoidal modulations. Placek et al. (2014) showed that it should be possible to disentangle thermal and reflected photons for sufficiently close-in eccentric orbits if the day-side temperature of the planet does not change significantly during apastron passage.

The amount of thermal flux received from an exoplanet depends on the bandpass through which the planet is observed. A significant degeneracy exists between the geometric albedo and day-side temperature when a light curve is analyzed in only a single bandpass. However, when two (potentially overlapping) bandpasses are employed and the geometric albedo of the planet does not change significantly between bandpasses, it is possible to break this degeneracy and significantly improve the characterization of such planets. Shporer et al. (2014) used a similar approach with the transiting exoplanet Kepler-13Ab in which the planetary system was observed in three separate bandpasses—Kepler, IRAC/3.6 μm, and IRAC/4.5 μm. This allowed them to greatly constrain both the albedo and day-side temperature.

The Transiting Exoplanet Survey Satellite (TESS) will be conducting an all-sky survey of transiting exoplanets, thus providing the opportunity to further characterize short-period hot-Jupiters when it observes the Kepler field (Ricker et al. 2015). TESS will observe the nearest and brightest stars...
with magnitudes of $+4 < V < +12$ at a short, 2-minute cadence and will provide full-frame images at a cadence of 30 minutes, allowing for the study of even dimmer stars in each of the observation sectors. According to the Exoplanet Orbit Database (Han et al. 2014), there are 37 confirmed planetary systems in the *Kepler* field around stars brighter than $V = 12$ and an additional 47 systems around stars brighter than $V = 15$, which is the expected limit of the TESS full-frame images. Of these 84 systems, 20 are thought to be short-period planets with orbital periods of less than 15 days, and of the planets for which there is a published $M_p \sin i$, there are 13 hot-Jupiters. These planets and their published characteristics are listed in Table 1 along with several hot planets with no published $V$-mag. Currently, the K2 mission is observing stars brighter than those observed by *Kepler* ($V < 12$) at an expected photometric precision of $\approx 80$ ppm for a 6-hr integration and 1-minute cadence (Howell et al. 2014), increasing the number of known planets that can be further characterized with TESS.

Here, we present a study of model-generated photometric data for a transiting short-period hot-Jupiter observed through both the *Kepler* and TESS bandpasses. This synthetic planet is based on published parameter estimates from the exoplanet Kepler-13Ab, which orbits a hot A-type star in the *Kepler* field. The host star has a magnitude of $V = 9.9$, making it a great candidate to characterize with both *Kepler* and TESS. We employ Bayesian model selection to show that one can disentangle thermal and reflected light when both bandpasses are simultaneously analyzed and provide parameter estimates displaying a breaking of the albedo versus day-side temperature degeneracy. In addition, we quantify the extent to which using multiple bandpasses can aid in exoplanet characterization and provide a framework for similar studies based on future missions.

## 2. Bayesian Inference

Bayes’ Theorem, which is used to make inferences from data, is given by

$$P(\theta_M|D, M) = \frac{P(\theta_M|M)P(D|\theta_M, M)}{P(D|M)} ,$$

where $D$ represents a given data set, and $M$ represents a particular model described by a set of model parameters $\theta_M$. The prior probability, $P(\theta_M|M)$, quantifies knowledge of the parameter values $\theta_M$ before analyzing the data set $D$. Bayes’ takes the data, $D$, into account through the likelihood function $P(D|\theta_M, M)$, which represents the probability that one would observe the data given the set of model parameters $\theta_M$ of the model $M$. The denominator is the Bayesian evidence, or marginal likelihood, which quantifies the probability that the model $M$ could have produced the data $D$. Together, these three quantities yield the posterior probability $P(\theta_M|D, M)$, which quantifies knowledge of the model parameters when

### Table 1

Seventeen of the Hottest Known *Kepler* Planets with Periods of less than Fifteen Days

| Name        | Orbital Period (days) | Planetary Mass ($M_J$) | Stellar Mass ($M_\odot$) | V-Magnitude | Equilibrium Temperature (K) |
|-------------|-----------------------|------------------------|---------------------------|-------------|----------------------------|
| TrES-2b     | $2.47063 \pm 1.0 \times 10^{-5}$ | $1.201 \pm 0.052$ | $0.983 \pm 0.059$ | 11.4        | $1470 \pm 10$             |
| HAT-P-7     | $2.204737 \pm 1.7 \times 10^{-5}$ | $1.792 \pm 0.063$ | $1.50 \pm 0.03$ | 10.5        | $2200 \pm 30$            |
| Kepler-10b  | $0.837491 \pm 2.0 \times 10^{-6}$ | $0.0145 \pm 0.0046$ | $0.913 \pm 0.022$ | 10.9        | $2130^{+120}_{-60}$      |
| Kepler-12b  | $4.43796370 \pm 2.0 \times 10^{-7}$ | $0.432 \pm 0.042$ | $1.166 \pm 0.054$ | 13.8        | $1480 \pm 30$            |
| Kepler-13Ab | $1.76358799 \pm 3.7 \times 10^{-7}$ | $6.5 \pm 1.57$ | $1.72 \pm 0.1$ | 9.9         | $2750 \pm 160$           |
| Kepler-14b  | $6.7901230 \pm 4.3 \times 10^{-6}$ | $8.41 \pm 0.29$ | $1.512 \pm 0.043$ | 12.0        | NA                       |
| Kepler-17b  | $1.48571080 \pm 2.0 \times 10^{-7}$ | $2.48 \pm 0.102$ | $1.16 \pm 0.06$ | 13.8        | $1570 \pm 200$           |
| Kepler-40b  | $6.87349 \pm 6.4 \times 19^{-4}$ | $2.18 \pm 0.34$ | $1.48 \pm 0.06$ | 14.8        | $1620 \pm 30$            |
| Kepler-41b  | $1.8555580 \pm 7.1 \times 10^{-6}$ | $0.494 \pm 0.071$ | $0.94 \pm 0.09$ | 14.5        | $1770 \pm 50$            |
| Kepler-74b  | $7.3407180 \pm 1.0 \times 10^{-6}$ | $0.667 \pm 0.09$ | $1.40^{+0.14}_{-0.11}$ | 14.2        | $1078 \pm 19$           |
| Kepler-76b  | $1.5449298 \pm 4.0 \times 10^{-7}$ | $2.01^{+0.37}_{-0.35}$ | $1.2 \pm 0.2$ | 13.8$^a$ | $2140 \pm 40$            |
| Kepler-78b  | $0.355 \pm 4.0 \times 10^{-4}$ | $<8 M_J$ | $0.758 \pm 0.046$ | $11.5^b$ | $2250 \pm 750$           |
| Kepler-91b  | $6.24658 \pm 8.0 \times 10^{-5}$ | $0.81^{+0.18}_{-0.17}$ | $1.31 \pm 0.10$ | $12.9^c$ | $2040 \pm 50$            |
| Kepler-412b | $1.720891232 \pm 4.7 \times 10^{-8}$ | $0.940 \pm 0.087$ | $1.167 \pm 0.091$ | $13.7$ | $1850 \pm 30$            |
| Kepler-423b | $2.684328480 \pm 8.2 \times 10^{-8}$ | $0.723 \pm 0.100$ | $1.07 \pm 0.05$ | $14.5$ | $1605 \pm 120$           |
| Kepler-424b | $3.31186440 \pm 3.9 \times 10^{-7}$ | $0.77 \pm 0.23$ | $1.010 \pm 0.054$ | $14.5$ | NA                       |
| Kepler-425b | $3.79701816 \pm 1.9 \times 10^{-7}$ | $0.249 \pm 0.074$ | $0.93 \pm 0.05$ | $15$ | $1070 \pm 15$            |

Notes. Data for planets with asterisks were obtained from the NASA exoplanet archive. All other data were retrieved from the exoplanet orbit database unless otherwise stated. Note that the listed temperature of Kepler-13Ab is its measured temperature, not the calculated equilibrium temperature.

$^a$ (Faigler et al. 2013).

$^b$ (Sanchis-Ojeda et al. 2013).

$^c$ $M_{Kep}$

$^d$ (Lillo-Box et al. 2014b).
considering the data. Therefore, Bayes’ Theorem acts as an updating rule. The posterior probability is essential for obtaining parameter estimates through summary statistics. The Bayesian evidence is more important for determining to what extent a model $M$ describes the observed data.

### 2.1. Bayesian Model Selection

Bayesian Model Selection relies on the Bayesian evidence, $P(D|M)$, which is commonly denoted $Z$, and is calculated by marginalizing the likelihood:

$$Z = \int P(\theta_M|M)P(D, |\theta_M, M)d\theta_M.$$  \hfill (2)

This integration is performed over each model parameter, which typically makes this integral analytically intractable and requires employing numerical integration techniques. It can be shown using Bayes’ theorem that the ratio of posterior probabilities between two competing models, $M_1$ and $M_2$, with equal prior probabilities, is equal to the ratio of the evidence for each model. Therefore the model with the larger evidence value will be considered to be more favorable for describing the data (Knuth et al. 2014). The amount by which one model with evidence $Z_1$ is favored over another model with evidence $Z_2$ is typically quantified using the Bayes’ Factor, which is given by

$$O = \frac{Z_1}{Z_2}. \hfill (3)$$

It is typical to focus on the logarithm of the Bayes’ factor, which is given by

$$\ln O = \ln Z_1 - \ln Z_2. \hfill (4)$$

Guidelines for interpreting Bayes’ Factors were given both by Jeffreys (1939) and Kass & Raftery (1995). There is overwhelming evidence for a particular model being favored over a competing model if the log-Bayes’ factor, $\ln O$, is greater than 5, strong evidence if it is between $2.5 < \ln O < 5.0$, positive evidence if between $1.0 < \ln O < 2.0$, and hardly significant evidence below 1 (von der Linden et al. 2014).

Since this integral is performed over the entire parameter space, there is an inherent Occam’s razor effect. Complex models typically have large parameter spaces where the probability of the model being correct is spread out over a larger region of parameter values. Alternatively, simpler models with smaller parameter spaces have the probability spread out over a smaller region of parameter space. Therefore, if both a simple and complex model describe the data equally well, the simple model will have more evidence and thus be the preferred model.

In order to compute model evidence the MultiNest algorithm (Feroz & Hobson 2008; Feroz et al. 2009, 2013), which is a variant of the Nested Sampling algorithm (Sivia & Skilling 2006), was utilized. Given a model, prior probability assignments, and a likelihood function, MultiNest provides estimates of the Bayesian log-evidence for a particular model as well as posterior samples from which parameter estimation can be performed.

### 2.2. Priors and Likelihood Function

As described in Section 2, making inferences using Bayes’ Theorem requires assigning prior probabilities for each model parameter. For this study, each model parameter is assigned a uniform prior probability over a reasonable range as shown in Table 2. The uniform prior probability assignment for the log of the signal variance is equivalent to Jeffreys Prior, which is an uninformative prior for scale parameters (Jeffreys 1946; Sivia & Skilling 2006). Each of these assignments can always be modified to incorporate additional information.

Since there are two data sets in this study, there must be two likelihood functions, one for the Kepler time series ($L_{Kep}$) and another for TESS ($L_{TESS}$). The joint probability for observing a data set $D = \{D_{Kep}, D_{TESS}\}$, given a set of model parameters $\theta_m$, corresponding to a model $M$ can be written as the product of two likelihood functions

$$P(D|\theta_m, M) = P(\{D_{Kep}, D_{TESS}\}|\theta_m, M)$$

$$= P(D_{Kep}|\theta_m, M)P(D_{TESS}|\theta_m, M)$$

$$\equiv L_{Kep}L_{TESS}, \hfill (7)$$

where

$$L_{Kep} = P(D_{Kep}|\theta_m, M) \hfill (8)$$

and

$$L_{TESS} = P(D_{TESS}|\theta_m, M). \hfill (9)$$

In practice, due to the magnitude of the likelihood functions, it is common to use the log-likelihood function so that

$$\log P(D|\theta_m, M) = \log L_{Kep} + \log L_{TESS}. \hfill (10)$$

where $\log L_{Kep}$ and $\log L_{TESS}$ independently take the observed data from Kepler and TESS into account, respectively. These two log-likelihood functions are chosen to be Gaussian based on the expected nature of the noise in both observations. However, this can be changed to account for a variety of

| Parameter | Variable | Distribution |
|-----------|----------|--------------|
| Day-side Temp. (K) | $T_d$ | U(0, 6000) |
| Night-side Temp. (K) | $T_n$ | U(0, 6000) |
| Orbital Inclination | $\cos i$ | U(0, 1) |
| Planetary Radius ($R_p$) | $R_p$ | U(0, 3) |
| Planetary Mass ($M_p$) | $M_p$ | U(0, 10) |
| Geometric Albedo | $A_\chi$ | U(0, 1) |
| Standard Dev. of noise (Kepler, ppm) | $\log \sigma_K$ | U(−15, 0) |
| Standard Dev. of noise (TESS, ppm) | $\log \sigma_T$ | U(−15, 0) |
situations such as data with apparent correlated (red) noise (Sivia & Skilling 2006; Placek et al. 2015). When the mean value of the signal and the signal variance are the only relevant parameters, it can be shown by the principle of maximum entropy that a Gaussian likelihood is the least biased choice (Sivia & Skilling 2006) of likelihood function. This yields a log-likelihood function of the form

$$\log L = -\frac{\chi^2_{Kep}}{2} - \frac{N_{Kep}}{2} \log 2\pi \sigma^2_{Kep}$$

$$- \frac{\chi^2_{TESS}}{2} - \frac{N_{TESS}}{2} \log 2\pi \sigma^2_{TESS},$$

(11)

where \(N_{Kep}, \sigma_{Kep}, \sigma_{TESS}\) and \(N_{TESS}\) are the total number of data points and expected deviation of the noise in the \(Kepler\) and \(TESS\) data sets, respectively. The model-dependent \(\chi^2\) terms are given by

$$\chi^2 = \sum_{i=1}^{N} (F(t_i) - D_i)^2,$$

(12)

where \(F(t_i)\) represents the forward model evaluated at the times \(t_i\), at which the data were observed.

2.3. Forward Model

The likelihood function depends explicitly on the forward model \(F(t_i)\), as shown in (12). The forward model computes the observed flux originating from a planetary system modeled by the parameters \(\theta_M\) of model \(M\). In addition to transits and secondary eclipses, there are four photometric effects that are potentially significant. These include the reflection of incident stellar light off of the planetary surface and/or atmosphere, thermal emission from the planetary surface and/or atmosphere, relativistic Doppler beaming of light as the host star orbits the center of mass, and the ellipsoidal variations caused by tidal forces induced by the planet onto the host star. The stellar-normalized reflected light component of the flux is given by

$$\frac{F_F(t)}{F_\star} = \frac{A_g}{\pi} \frac{R_p^2}{r(t)^2} (\sin(\theta(t)) + (\pi - \theta(t)) \cos(\theta(t))),$$

(13)

where \(A_g\) is the geometric albedo, \(R_p\) is the planetary radius, \(r(t)\) is the planet–star separation distance, and \(\theta(t)\) is the angle between the vector connecting the planet and star and the line of sight. The thermal emission from both the day-side and night-side of the planetary atmosphere is given by

$$\frac{F_D(t)}{F_\star} = \frac{1}{2} \frac{B(T_{D})}{B(T_{eff})} \left( \frac{R_p}{R_\star} \right)^2 (1 + \cos(\theta(t))),$$

(14)

and

$$\frac{F_N(t)}{F_\star} = \frac{1}{2} \frac{B(T_{N})}{B(T_{eff})} \left( \frac{R_p}{R_\star} \right)^2 (1 + \cos(\theta(t))),$$

(15)

where \(R_\star\) is the stellar radius, and \(B(T)\) is Planck’s law, which is evaluated at the day-side temperature of the planet \(T_{D}\) and the effective temperature \(T_{eff}\) of the host star. The beaming effect is given by

$$\frac{F_B(t)}{F_\star} = (3 - \alpha_b) \beta_r(t),$$

(16)

where \(\beta_r(t)\) is the radial velocity of the host star and directly depends on the radial velocity semi-amplitude

$$K = 28.435 \left( \frac{T}{1 \text{ year}} \right)^{1/2} \frac{M_\star \sin i}{M_J} \left( \frac{M_\star}{M_\odot} \right)^{1/2},$$

(17)

and \(\alpha_b\) is the photon-weighted bandpass-integrated beaming factor given by

$$\alpha_b = \frac{\int K(\lambda) B(\lambda) F_{\lambda,\star} d\lambda}{\int K(\lambda) B(\lambda) d\lambda}.$$

(18)

Here, \(K(\lambda)\) is the \(Kepler\) response function, \(\lambda\) is the wavelength, \(F_{\lambda,\star}\) is the stellar spectrum, and \(B\) is given by

$$B = 5 + \frac{d \ln F_{\lambda,\star}}{d \ln \lambda},$$

(19)

where the derivative is averaged over the observed wavelengths. The tidal effect is approximated as

$$\frac{F_t(t)}{F_\star} = \alpha \left( \frac{R_\star}{M_\star} \right)^3 \sin^2 i \cos(2\theta(t)),$$

(20)

where \(\alpha\) is given by

$$\alpha = \frac{0.15(15 + u)(1 + g)}{3 - u},$$

(21)

where \(u\) and \(g\) are the linear limb-darkening coefficient and gravity darkening coefficient for the host star and can be determined from the tables in Claret & Bloemen (2011).

Together, the forward model \(F(t)\) can be written as the sum of all four components of the photometric flux and the flux during transit and secondary eclipse, which is computed using the method of Mandel & Agol (2002).

3. Results on Model-generated Data

\(TESS\) will observe 26 sectors of the sky over a 2-year period and it will observe each sector for approximately 27 days. It is expected that \(TESS\) will achieve a photometric precision of approximately 200 ppm (Kraft Vanderspek et al. 2010). \(Kepler\), on the other hand, observed a patch of sky roughly 1/400th the size that \(TESS\) will observe but achieved a photometric precision roughly an order of magnitude lower than \(TESS\) due in large part to an order of magnitude larger telescope aperture. The observation sectors of \(TESS\) will overlap the \(Kepler\) field and provide a unique opportunity to study a subset
of the Kepler planets in a slightly different bandpass. The Kepler bandpass ranges from 420 to 900 nm (Koch et al. 2010), whereas the TESS bandpass overlaps that of Kepler ranging from 600 to 1000 nm (Ricker et al. 2015). The spectral response functions for both Kepler and TESS are displayed in Figure 1, with each function normalized to have a maximum of unity.

Model-generated synthetic data were created for a hot-Jupiter in the Kepler field observed by both Kepler and TESS. The Kepler light curve spans 16 quarters of observations (~1500 days), whereas the synthetic TESS light curve spans only 27 days, corresponding to the duration that data will be taken from each observation sector. The orbital parameters used to create the synthetic data for this planet were based on Kepler-13Ab (Shporer et al. 2014), and were taken to be $e = 0$, $i = 84^\circ.3$ (cos $i = 0.1$), $P = 1.7637$ days, and $\omega = 0$, where $e$, $i$, $P$, and $\omega$ are the orbital eccentricity, inclination, period, and argument of periastron, respectively. Model parameters that describe the planetary characteristics were assumed to be $M_p = 7.5 M_J$, $R_p = 1.4 R_J$, $T_d = 3500$ K, $T_n = 1500$ K, and $A_g = 0.2$, where $M_p$ and $R_p$ are the planetary mass and radius, $T_d$ and $T_n$ are the day-side and night-side temperatures of the planet, and $A_g$ is the geometric albedo of the planet. Stellar parameters, which were held fixed, include the stellar mass $M_* = 1.72 M_\odot$, radius $R_* = 1.71 R_\odot$, and effective temperature $T_{\text{eff}} = 7650$ K.

### 3.1. Model Selection

A series of six simulations were conducted on the two data sets. The results of these simulations are displayed in Table 3.

![Spectral Response Function](image)

**Figure 1.** Spectral response functions for both Kepler (solid curve; Koch et al. 2010) and TESS (dashed curve; Ricker et al. 2015). Both are normalized so that their peaks occur at unity.

| Parameter | Kepler Only RBE | TESS Only RBE | Kepler + TESS RBE |
|-----------|-----------------|---------------|-------------------|
| $\cos i$  | 0.117 ± 0.010   | 0.116 ± 0.015 | 0.117 ± 0.010     |
| $R_p$     | 1.414 ± 0.017   | 1.408 ± 0.015 | 1.409 ± 0.015     |
| $M_p$     | 7.48 ± 0.17     | 7.36 ± 1.00   | 7.36 ± 0.96       |
| $A_g$     | 0.58 ± 0.01     | 0.56 ± 0.072  | 0.82 ± 0.07       |
| $T_d$     | ...             | ...           | ...               |
| $T_n$     | ...             | ...           | ...               |
| $\sigma_k$| 104.0 ± 6.0     | 108.0 ± 6.0   | 106.0 ± 6.0       |
| $\sigma_f$| ...             | ...           | 206.0 ± 13.0      |
| log Z     | 500.9 ± 0.4     | 1004.3 ± 0.3  | 949.0 ± 0.2       |
| log $L_{\text{max}}$ | 516.1 | 1021.6 | 969.9 |

**Table 3**
Parameter Estimates and Log-evidence for the Two Models (RBE, RBET) that Were Applied to the Simulated* Kepler and TESS Light Curves

Note. For these simulations the photometric precisions of the model-generated Kepler and TESS data were assumed to be 60 ppm and 200 ppm, respectively.

The goal was to determine whether or not one could disentangle thermal and reflected light by employing Bayesian model selection. Using data from both Kepler and TESS, the Bayesian log-evidence was computed for a model that included the reflection, Doppler beaming, and ellipsoidal variations effects but neglected thermal emission (labeled RBE in Table 3), and another model that included all four photometric effects (labeled RBET in Table 3). For these simulations, the photometric precision of the simulated data from Kepler and TESS were 60 and 200 ppm, respectively. The two models were first applied to the simulated* Kepler and TESS data individually. As shown in Table 3, the log-evidence for the RBE and RBET models was equal to within uncertainty. This indicates that with only a single data set (TESS or Kepler individually) one cannot disentangle reflected and thermally emitted light as expected. Also note that the maximum log-likelihoods are equal when considering data from only a single instrument, meaning that both models yield the same overall best fit. When considering data from both instruments simultaneously, the log-evidence corresponding to the RBE model was log $Z = 949.0 ± 0.2$ and for the RBET model it

---

*Publications of the Astronomical Society of the Pacific, 128:074503 (9pp), 2016 July Placek, Knuth, & Angerhausen*
was determined to be \( \log Z = 950.6 \pm 0.2 \). This indicates that the model including thermal emission (RBET) was favored by a factor of approximately \( \exp(1.6) \approx 5 \) over the RBE model, which neglected thermal emission. It should be noted that this planet was assumed to be in a circular orbit, which results in the thermal flux and reflected light variations having approximately the same sinusoidal wave-form. Observed through a single bandpass, this configuration is highly degenerate between the two effects. This would result in the equivalent log-evidence, or the RBE model being favored since the RBET model has a significantly higher penalty than the RBE model due to the addition of the day-side and night-side temperature parameters. The maximum log-likelihood values also indicate that the RBET model is a better fit to the data than the RBE model.

Another experiment was performed to determine the photometric precision of \( \mathrm{TESS} \) required for the reflected light and thermal emission signals to be distinguishable when both \( \mathrm{TESS} \) and \( \mathrm{Kepler} \) data are used together. The RBE and RBET models were applied to nine pairs of simulated time series. The photometric precision of the model-generated \( \mathrm{Kepler} \) data was kept fixed at 60 ppm, while the photometric precision for the simulated \( \mathrm{TESS} \) observations were varied from 100 to 300 ppm in steps of 25 ppm. Figure 2 displays the log-Bayes’ factor for each pair of simulations. Based on these results, thermal emission and reflected light should be definitively distinguishable for \( \mathrm{TESS} \) observations that achieve a photometric precision of <250 ppm, while for noise levels greater than approximately 250 ppm the two signals become indistinguishable once again.

Finally, simulations were conducted to determine the extent to which one could disentangle thermal and reflected light as a function of planetary day-side temperature. The same process of applying RBE and RBET models was repeated. The day-side temperature of the planet was varied from 1500 to 3500 K in steps of 200 K, while the photometric precision of \( \mathrm{TESS} \) was held fixed at 100 and 200 ppm. The log-Bayes’ factors are displayed in Figure 3 and show that this methodology should work for any planet with a day-side temperature greater than \( 2300 \) K assuming \( \mathrm{TESS} \) can achieve a photometric precision of 100 ppm, and greater than \( 2500 \) K if it can achieve 200 ppm. A planet such as \( \mathrm{Kepler}-13\text{Ab} \) satisfies this requirement, as it has a published day-side temperature of 2750 K Shporer et al. (2014), and there are five additional planets that are close to this cutoff (see Table 1).

### 3.2. Parameter Estimation

The third aim of this study was to determine if one could constrain either the day-side temperature or geometric albedo when using data from two separate bandpasses. First, the RBET model was applied to the simulated \( \mathrm{Kepler} \) data alone to determine how well one could constrain these two parameters without \( \mathrm{TESS} \). In this case, the day-side temperature and geometric albedo were found to be \( T_d = 2315.8 \pm 1014.4 \) K and \( A_g = 0.46 \pm 0.16 \), respectively. With such large uncertainties, the results do not precisely constrain these two parameters, as the true values of the day-side temperature and geometric albedo were set to 3500 K and 0.2, respectively. The uncertainties in the estimates of both of these parameters are large, especially in the case of the day-side temperature. The reason for this can easily be seen in Figure 4, where the posterior samples for \( T_d \) and \( A_g \) are displayed as gray dots. Note the significant ridge-like degeneracy among these two parameters when the planet is observed through a single bandpass. This indicates that one could observe the same flux from a more reflective and cool planet, or a dark and hot planet.

Figure 4 illustrates the risk in choosing the most probable value of \( A_g \) and \( T_d \). Also displayed are the posterior samples from the RBET model applied to both simulated \( \mathrm{Kepler} \) and \( \mathrm{TESS} \) (200 ppm) data sets (black + signs). In this case, the day-side temperature was much better constrained, yielding \( T_d = 3457.6 \pm 217.9 \) K, as indicated by the lack of posterior samples in the 0–2000 K range. Even with two bandpasses it was not possible to precisely estimate the geometric albedo, which was estimated to be \( A_g = 0.27 \pm 0.17 \). Although the uncertainty is still high, the mean is much closer to the true value of 0.2 since many of those samples corresponding to bright, cool objects could not explain the light curves.

### 4. Doppler Beaming

The amplitude of the observed beaming effect depends on the bandpass through which the planetary system is observed (Equations (16)–(19)). Therefore, it may be possible to further constrain the mass of exoplanets using more than one photometric channel. Assuming a blackbody spectrum for the
Figure 3. Two sets of Log-Bayes’ factors for a series of eleven simulations on model-generated Kepler and TESS data. The day-side temperature of the planet is varied from 1500 to 3500 K while keeping the photometric precision of TESS fixed at an assumed 100 (dashed line) and 200 ppm (solid line). This plot shows that one should be able to disentangle thermal and reflected light for planets with day-side temperatures \( \geq 2300 \) K at a photometric precision of 100 ppm, and \( \geq 2500 \) K at a photometric precision of 200 ppm.

Figure 4. Posterior samples for the RBET model applied to both the Kepler and TESS data (black + signs), and the same model applied to only the Kepler data (gray circles). The one-dimensional histograms for the day-side temperature (B) and the geometric albedo (C) are also displayed. Note that the posterior for the day-side temperature, \( T_d \), is much more peaked in the case of utilizing data from both bandpasses and that while the uncertainty in \( A_g \) is has not improved by using Kepler and TESS, the mean is much closer to the true value.
shown in Table 1.

Figure 5. Amplitude differences for the beaming effect observed with Kepler and TESS. Each curve displays the $M_{\text{c}}^{-2/3}$ dependence for Doppler beaming as shown in (17). Also displayed (as squares) are the thirteen Kepler hot-Jupiters likely to be detectable with TESS from Table 1.

With such a small difference even in the case of a massive planet, simultaneously analyzing TESS and Kepler observations is unlikely to significantly increase the accuracy or precision with which one can estimate the planetary mass. However, the amplitude difference also depends on the stellar mass as shown in Figure 5, which would make it possible to detect such a difference for large hot-Jupiters around low-mass stars. The difference in beaming amplitudes was calculated for each of the nine bright Kepler hot-Jupiters for which there was a published planetary mass ($M_p \sin i$). Figure 5 displays the amplitude difference between Kepler and TESS for each planetary system (shown as black squares) along with four synthetic planets around stars with masses varying from 0.1 $M_\odot$ to 3.0 $M_\odot$. Each of the systems yields amplitude differences less than 2 ppm, which is unlikely to significantly increase the level of exoplanet characterization given the expected photometric precision of TESS.

5. Discussion

Using both Bayesian model selection and parameter estimation we have shown that TESS should be able to further characterize a subset of the Kepler planets where its observation sector overlaps the Kepler field. By employing two sets of simulated observations that were taken through different bandpasses, one can disentangle thermal emission and reflected light originating from the atmosphere of the planet. This can be done even in the case of circular orbits, which yield the same flux variations for the two effects. Tests on simulated data indicate that if TESS can achieve a photometric precision less than approximately 250 ppm for planets in the Kepler field, then the thermal and reflected flux can be disentangled. Additionally, such a planet would need a day-side temperature greater than \(\sim2500\) K in order for these two effects to be distinguished. Currently, there are 6 confirmed Kepler planets that are close to or surpass this temperature limit and there are 48 Kepler Objects of Interest (KOIs) with temperatures greater than 2500 K, 2 of which have been confirmed.

It was also shown that by incorporating more than just a single bandpass, the degeneracy among the day-side temperature and the geometric albedo of the planets decreases, allowing for more accurate characterization of such planets. In the case of the Kepler bandpass alone, the estimated day-side temperature of a simulated short-period hot-Jupiter was found to be $T_d = 2315.8 \pm 1014.4$ K, with a geometric albedo of $A_g = 0.46 \pm 0.16$. Using both bandpasses yielded a day-side temperature of $T_d = 3457.6 \pm 217.9$ K and a geometric albedo of $A_g = 0.27 \pm 0.17$, which are much closer to the correct values of $T_d = 3500$ K and $A_g = 0.2$. In addition to potentially characterizing known hot planets, applying this methodology to the phase curves of the aforementioned set of KOIs may reveal details about their planetary (or stellar) nature by better estimating their albedos. Albedos found to be $\gtrsim1$ would imply a stellar, rather than planetary companion.

Photometric time series data obtained from the TESS mission that overlaps with the Kepler field will allow for more precise characterization of the brightest Kepler systems, and may even shed light on current planetary candidates. Although this study was specific to Kepler and TESS, the methodology is applicable to any multi-channel (Hubble Space Telescope, James Webb Space Telescope, etc.) exoplanet observations and may be utilized to plan future missions and mission target lists with the goal of exoplanet characterization.

This research has made use of the Exoplanet Orbit Database and the Exoplanet Data Explorer at exoplanets.org. This research has also made use of the NASA Exoplanet Archive, which is operated by the California Institute of Technology, under contract with the National Aeronautics and Space Administration under the Exoplanet Exploration Program. Finally, the authors would like to thank the anonymous reviewer for the very constructive comments on the paper.

References

Angerhausen, D., DeLarme, E., & Morse, J. A. 2015, PASP, 127, 1113
Borucki, W. J., Koch, D., Jenkins, J., et al. 2009, Sci, 325, 509
Charbonneau, D., Allen, L. E., Megeath, S. T., et al. 2005, ApJ, 626, 523
Claret, A., & Bloemen, S. 2011, A&A, 529, A75
