Estimation of Heterogeneous Treatment Effects Using a Conditional Moment Based Approach

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Abstract

We propose a new estimator for heterogeneous treatment effects in a partially linear model (PLM) with many exogenous covariates and a possibly endogenous treatment variable. The PLM has a parametric part that includes the treatment and the interactions between the treatment and exogenous characteristics, and a nonparametric part that contains those characteristics and many other covariates. The new estimator is a combination of a Robinson transformation to partial out the nonparametric part of the model, the Smooth Minimum Distance (SMD) approach to exploit all the information of the conditional mean independence restriction, and a Neyman-Orthogonalized first-order condition (FOC). With the SMD method, our estimator using only one valid binary instrument identifies both parameters. With the sparsity assumption, using regularized machine learning methods (i.e., the Lasso method) allows us to choose a relatively small number of polynomials of covariates. The Neyman-Orthogonalized FOC reduces the effect of the bias associated with the regularization method on estimates of the parameters of interest. Our new estimator allows for many covariates and is less biased, consistent, and $\sqrt{n}$-asymptotically normal under standard regularity conditions. Our simulations show that our estimator behaves well with different sets of instruments, but the GMM type estimators do not. We estimate the heterogeneous treatment effects of Medicaid on individual outcome variables from the Oregon Health Insurance Experiment. We find using our new method with only one valid instrument produces more significant and more reliable results for heterogeneous treatment effects of health insurance programs on economic outcomes than using GMM type estimators.

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1 Introduction

Heterogeneous treatment effects have been a focus in literature since Imbens and Angrist (1994). Heterogeneous treatment effects models are those where the treatment effects of policies, programs, or other variables on some outcome of interest may vary across individuals with different characteristics. To measure heterogeneous treatment effects, many papers add interaction terms between the treatment variable and some covariates to the standard linear model (see Wooldridge (2010) and Imbens and Rubin (2015)). The basic linear model becomes linear in the treatment, the interaction term, and the covariates. The parameters of interest are the parameter for the treatment and that for the interaction term.

Unfortunately, the true model is unknown. If the model is nonlinear in the covariates, OLS, 2SLS, or GMM with a linear form will suffer from model misspecification, and estimates will not be consistent and normally distributed. With the correct model form, the OLS, 2SLS, or GMM will work. However, if the nonlinear part of the model is unknown, we will need to use other methods to estimate the parameters. There are at least three challenges associated with the estimation of both parameters in the model with a parametric part and unknown nonlinear part, i.e., the partially linear model (PLM).

One challenge happens when the treatment variable is endogenous, we need to use valid instruments to estimate parameters for the treatment and for the interaction term. The usual approach would be to partial out the unknown and nonlinear part of the model via a Robinson (Robinson (1988)) transformation and then use the GMM estimator with one instrument and the interaction term between the instrument and the covariate to estimate both parameters. When we only have one valid instrument and the covariate inside the interaction term has little variation or is not very informative, the GMM estimator will not be consistent and its variance will have a large magnitude. This is similar to a weak identification problem. Additionally, the selection of instruments will affect the interpretation of the treatment (see Dieterle and Snell (2016) for related discussions).

A second challenge is that traditional estimation methods do not work when the number of covariates is possibly larger than the number of observations. This challenge exists if there are a large number of covariates in the structural model or if many covariates must be included to ensure that the instrument is valid.

The third challenge is that if the covariates enter the model in an unknown (and possibly nonlinear) way, estimation methods assuming linearity will suffer from specification bias.

Our new estimator solves the first challenge by using only the valid instrument (e.g., the random assignment) to estimate both parameters. Our method does not use the interaction term between the instrument and the covariate to identify model parameters. We do this by first applying the Robinson transformation to remove the nonlinear and unknown part, and then transforming the conditional moment restriction: $E(\epsilon_i | Z_i) = 0$ where $\epsilon_i$ is the error term
inside the model and $Z_i$ is the instrument. Usually, GMM uses the unconditional moments generated by $E(\epsilon_i|Z_i) = 0$ to construct objective functions. But, with the i.i.d assumption we can also generate objective function as $E(\epsilon_i|Z_i)E(\epsilon_j|Z_j)$. As written, this type of objective function is hard to use. To make it tractable, we apply both the i.i.d. assumption and the Fourier transform to obtain a new objective function related with $E(\epsilon_i|Z_i)E(\epsilon_j|Z_j)$. This is tractable. In technical terms, the Fourier transform and the new objective function are based on the Smooth Minimum Distance (SMD) approach \cite{Lavergne and Patilea 2013}. The SMD-based approach exploits all the information of the conditional mean independence restriction without having to specify the first stage, or to select a finite number of unconditional moments (e.g., using transformations of $Z_i$).

To tackle the last two challenges, our new estimator uses regularized Machine Learning (i.e., the Lasso method) to estimate the nuisance parameters generated by Robinson transformation \cite{Robinson 1988} and a Neyman-Orthogonalized \cite{Chernozhukov et al. 2018} First-Order Condition (FOC). With the Robinson transformation, we do not need to specify how these covariates enter the model; it partials out the unknown (and nonlinear) part of the partially linear model, but it also introduces nuisance parameters which need to be estimated. The Neyman-Orthogonalized FOC reduces the effect of the bias associated with the estimation of these nuisance parameters. When there are many covariates, assuming sparsity and using regularized machine learning methods, such as the Lasso method, allows us to choose a relatively small number of covariates for the estimation of nuisance parameters. Thus, our new estimator is the D-RSMD estimator (Debiased Robinson-SMD).

To apply the new estimator we only need one tuning parameter. If we are using the Lasso method, the tuning parameter is inside the penalty term for the estimation of the nuisance parameter. In practice, choosing Lasso tuning parameters is well-understood \cite{van de Geer 2016} and we use cross-validation \cite{Chernozhukov et al. 2018}. When there are few covariates (and the model is sparse by design), the Nadaraya–Watson estimator is an alternative method to estimate the nuisance parameter. This estimation procedure also needs one tuning parameter, that is, the bandwidth. The bandwidth can be chosen by the rule of thumb or cross-validation in practice. We show that our D-RSMD estimator is consistent and $\sqrt{n}$ asymptotically normal under mild regularity conditions. Monte-Carlo simulations show that the D-RSMD estimator outperforms the R-SMD and GMM-type estimators in terms of much smaller bias and lower standard errors.

To illustrate the value of our method, we estimate both parameters to check the effects of Medicaid from the Oregon Health Insurance Experiment\footnote{I would like to thank A. Colin Cameron for bringing the Oregon Health Insurance Experiment to my attention and providing me with the data set. The data set is from his and Pravin K. Trivedi’s book in 2022 and is also available at the website \url{https://www.stata-press.com/data/mus2.html}.} and to extend the estimation results in \cite{Card 1993}. In the Oregon Health Insurance Experiment, using our estimator
with only one valid instrument produces statistically significant results for heterogeneous
treatment effects when the GMM estimator does not. We use the Card application to show
the differences of estimators using various sets of instruments. It shows that our new method
generates more precise estimates in comparison to GMM.

Our work is an extension of Antoine and Sun (2021) who propose the R-SMD estimator
which combines a Robinson transformation and a smooth minimum distance estimator in a
partially linear model. However, when we use the Nadaraya-Watson estimator to estimate
the conditional means, the dimension of covariates has to be less than four (see chapter 7 of
Li and Racine (2006)). To extend the R-SMD estimator to the big data setting and relax the
restriction on the dimensions of covariates, our work applies machine learning methods, such
as the Lasso. To reduce the effect of bias introduced by Lasso, our work employs the Neyman-
orthogonal first-order condition. The method also extends the Neyman-orthogonal estimator
(Chernozhukov et al. (2018)) to the U-statistic setting. Our new estimator contributes to
the literature on the partially linear model and conditional moment restriction models in the
same way as the R-SMD estimator does. Both the partially linear model and conditional
moment restriction model involve the problem of choosing the model form. For a partially
linear model, some researchers use sieves for the nonlinear part. For conditional moment
restriction models, some generate a finite number of unconditional moments. In comparison,
our method does not need to choose the number of polynomials for the sieve method or the
number of unconditional moments. Our method only needs to choose one tuning parameter.

The paper is organized as follows. Section 2 introduces our framework, motivation, and
our estimator. Section 3 states the large sample properties for our estimator. Section 4
presents the simulation results for finite samples with and without a large number of covariates.
Section 5 uses the D-RSMD estimator to estimate the effects of education on earnings
and effects of Enrollment in Medicaid on outcomes of interest, based on the Oregon Medicaid
health experiment. Additional results and proofs are in the Appendix and Supplementary
Appendix.

2 Framework and Motivation

Our framework is based on a partially linear model with a treatment variable $W_i$ which is
binary. The heterogeneous treatment effects enter the model through the linear term on $W_i$
and some interaction terms between $W_i$ and $X_i$, a vector of $P$ covariates with $X_i = [X_{i1}, X_{i2}]'.$
The other part of the model is an unknown function of $X_i$.

$$y_i = \theta_{w0}W_i + W_i \cdot X_{i1}^t \theta_{wx0} + f_{0,1}(X_i) + \epsilon_i$$ (2.1)
The treatment is not always exogenous. For example, in the Oregon Medicaid health experiment, the treatment is the enrollment in Medicaid. Enrollment in Medicaid is endogenous because it is based on the choices of the lottery winners. Only a fraction of lottery winners decided to enroll in Medicaid.

$W_i \cdot X_{i1}$ is the interaction term. The formal definition for $W_i \cdot X_{i1}'\theta_{w x 0}$ is

$$\sum_{k=1}^{p_1} W_i X_{i1,k} \times \theta_{w x 0,k}$$

where $X_{i1,k}$ is a $k$-th element inside the vector $X_{i1}$. The treatment part of interest is $W_i + W_i \cdot X_{i1}$. The parameters that measure heterogeneous treatment effects, i.e. $\theta_{w0}$ and $\theta_{w x 0}$, are our key parameters. The control vector $X_i$ in our setting is exogenous and appears in an unknown function $f_{0,1}(X_i)$. $f_{0,1}(\cdot)$ is a nuisance parameter that we are not interested in.

$X_{i1}$ is a subvector of $X_i$ of dimension $p_1$ with $p_1 \leq P$. We can have $X_{i1} = X_i$ if the dimension of $X_{i2}$, $p_2$, is zero. That is, $W_i \cdot X_{i1}$ can be the interaction term using only a small set of covariates, while $f_{0,1}(X_i)$ is the unknown function of the whole set of exogenous controls. The dimension of $X_i$ is not restricted and can be either high or low-dimensional. However, we restrict the dimension of $X_{i1}$. And we also maintain the exogeneity of all the controls $X_i$. The outcome variable $y_i$ is not necessarily binary. For a binary outcome variable, a large number of covariates, and similar restricted dimension setting on estimating heterogeneous treatment effects, see Nekipelov et al. (2018).

The unknown form of $f_{0,1}(X_i)$ allows $X_i$ to enter the model in a flexible way. To avoid estimating the nuisance parameter $f_{0,1}(X_i)$, a Robinson transformation is introduced to eliminate the unknown $f_{0,1}(X_i)$. The Robinson transformation consists of subtracting the conditional expectation of $y_i$ with respect to the controls $X_j$. After such a Robinson transformation, $f_{0,1}(X_i)$ will disappear. In this case, we do not need to assume the form of $f_{0,1}(X_i)$. It can be sparse and may be nonlinear. With Robinson transformation and the exogenous assumption on $X_i$, Equation 2.1 becomes

$$y_i - E(y_i | X_i) = (P_i - E(P_i | X_i))'\theta_0 + \epsilon_i$$

where $P_i = [W_i, W_i \cdot X_{i1}']'$ and $\theta_0 = [\theta_{w0}, \theta_{w x 0}]'$.

Denote

$$\epsilon_j(\theta, g_0) = \tilde{y}_j - \tilde{P}_j \theta$$

where $\tilde{y}_j \equiv y_j - E(y_j | X_j)$ and $\tilde{P}_j \equiv P_j - E(P_j | X_j)$. $g_0$ stands for all of the unknown real values of nuisance parameters. $g_0$ is a vector of $g_{0,y}(X_j)$ (or $E(y_j | X_j)$) and $g_{0,p}(X_j)$ (or $E(P_j | X_j)$) at this stage.

As in the previous discussion, $W_i$ and the interaction term between $W_i$ and $X_{i1}$ are all endogenous. Following the classic endogenous variable estimation, a vector of instruments, including the random assignment $Z_i$, is introduced. For instance, in the Oregon Medicaid
health experiment, the instrument is the lottery outcome. We maintain the conditional mean independence assumption for the random assignment. With valid instrument and exogenous control variables, the conditional moment restriction is

\[ E(\epsilon_i | X_i, Z_i) = 0 \text{ a.s.} \quad (2.3) \]

In our setting, we look at the single treatment case where the controls may be correlated with the instruments. With traditional estimators such as GMM or 2SLS, \(1 + p_1\) instruments are needed to identify (and estimate) the slope parameters associated with \(W_i\) and the interaction terms \(W_i \cdot X_{i1}\). GMM uses \(Z_i\) (the random assignment) and the interaction terms \(Z_i \cdot X_{i1}\) to estimate the key parameters.

However, if there is little variation in \(X_{i1}\), the \(Z_i \cdot X_{i1}\) is less informative, or if we use a second instrument that is (highly) correlated with the valid \(Z_i\), the GMM method will fail to provide reliable estimates: intuitively, it is similar to a weak instrument problem. With only one valid instrument, GMM encounters the problem of how to generate new moments when we need to estimate more than one parameter. With only one valid instrument and two parameters to estimate, there is under-identification, and GMM cannot be implemented. We will show that our estimation strategy, which only relies on using one valid instrument (e.g. the random assignment \(Z_i\)), delivers reliable inference on both parameters.

Further, when it comes to the interpretation of the heterogeneous treatment effects, when using two or more instruments, additional assumptions are needed because the traditional monotonicity assumption (as in Imbens and Angrist (1994)) is only for one instrument. For our setting, in the simplest case where \(X_{i1}\) is one dimensional (with values -1 or 1), if we use instruments \(Z_i\) (with values 0 or 1) and \(Z_i \cdot X_{i1}\), then there are two types of compliers. The first kind of compliers are the individuals who will accept the treatment if and only if \(Z_i\) is one regardless of the values of \(X_{i1}\). The second kind of compliers are the individuals who will accept the treatment if and only if \(Z_i\) is one and \(X_{i1}\) is positive.

Hence, if we want to use LATE interpretation as in Imbens and Angrist (1994) then we cannot include the interaction term to measure the heterogeneous treatment effects. If we need to estimate the interaction term, then we need to use the interaction term between the instrument and covariates as extra instruments. As a consequence, the types of compliers will affect the interpretation of LATE. This interpretation problem is discussed further after our formal Assumption \(I(v)\) is introduced.

This dilemma can be solved by employing the conditional moment restriction directly. The conditional moment restriction contains all of the information no matter the number of instruments inside. Hence, the conditional moment based approach enables us to use one instrument \(Z_i\) (e.g. the random assignment) only to identify and estimate slope parameters as-

\footnote{In the experiment, the lottery winners are allowed to enroll in the Medicaid program. Not every lottery winner enrolled. The treatment variable is the actual enrollment status.}
associated with both $W_i$ and $W_i \cdot X_{i1}$. The Bierens type estimator is a conditional moment based approach which transforms the conditional moment into an infinite number of unconditional moments using complex exponential functions. See Bierens (1982), Antoine and Lavergne (2014), and Antoine and Sun (2021).

$$E[\epsilon_j(\theta_0, g_0)e^{itZ_j}] = 0 \quad \forall t \in \mathbb{R}^{q_z} \quad \iff \quad E(\epsilon_j(\theta_0, g_0)|Z_j) = 0 \text{ a.s.} \quad (2.4)$$

The population objective function defined in Equation (2.5) below is based on the previous equation. Inside the objective function, $\mu(t)$ is a strictly positive measure on the vector $t$. $Z_i$ stands for the vector of instruments.

$$M_\infty(\theta, g) = \int_{\mathbb{R}^{q_z}} |E[\epsilon_j(\theta, g)e^{itZ_j}]|^2 d\mu(t) \quad (2.5)$$

The objective function involves the norm of a complex function. To estimate $\theta_0$, we need to find its derivative, which is difficult to compute. Under the independent assumption for the population, the objective function has an alternative expression.

$$\forall j \neq l,$$

$$M_\infty(\theta, g) = E[\epsilon_j(\theta, g)e_{i}(\theta, g)\kappa_{j,l}] \text{ with } \kappa_{j,l} = \int_{\mathbb{R}^{q_z}} e^{it(Z_j-Z_l)} d\mu(t) \quad (2.6)$$

The objective function defined in Equation (2.6) is a function of the parameters $\theta$ we are interested in and the nuisance parameters $g$ as in the Equation (2.2) after the Robinson transformation. With Regularity Assumptions provided in the following, $\theta_0$ is the unique minimizer of the objective function $M_\infty(\theta, g_0)$ where $g_0$ is a vector of $g_{0,g}(X_j)$ (or $E(y_j|X_j)$) and $g_{0,\mathcal{P}}(X_j)$ (or $E(P_j|X_j)$) at this stage.

**Assumption 1.** *(Regularity assumptions)*

(i) $E(\epsilon_i|X_i, Z_i) = 0$.

(ii) $E(\tilde{P}_i|Z_i) \neq 0$ a.s. *(with probability 1)* with $\tilde{P}_i = P_i - E(P_i|X_i)$.

(iii) $E(\tilde{P}_i\tilde{P}_i^\prime)$ is nonsingular.

(iv) Let $f_Z(.)$ denote the density function of $Z_j$. We assume that $E(\tilde{P}_j|Z_j = .)f_Z(.)$ is $L_q$ for some $1 \leq q \leq 2$.

(v) for all possible $z_1, z_2$, either $E(W_i|Z_i = z_1) \geq E(W_i|Z_i = z_2)$ for all $i$, or $E(W_i|Z_i = z_1) \leq E(W_i|Z_i = z_2)$ for all $i$.

(vi) $(y_i, W_i, X_i, Z_i)$ is an independent and identical copy of $(y_j, W_j, X_j, Z_j)$.

(vii) Let $\mu$ be a given strictly positive measure on $\mathbb{R}^{q_z}$. Let $k(.)$ be the Fourier transform induced by $\mu$, $k(Z_j - Z_l) = \int_{\mathbb{R}^{q_z}} e^{it(Z_j-Z_l)} d\mu(t)$. We assume that $k(.)$ is a symmetric bounded density function on $\mathbb{R}^{q_z}$ and that its Fourier transform is strictly positive.

$Z_i$ denotes the general vector of instruments. If we only use one instrument, $Z_i$ will be the random assignment. If we use a vector of instruments, $Z_i$ will be the random assignment and the interaction term between random assignment and covariates.
Assumption $1(i)$ is the exogeneity assumption for the controls and instruments. In the potential outcomes setting, Assumption $1(i)$ is also the Random Assignment Assumption. The model form implies the Exclusion Restriction Assumption as in Imbens and Rubin (2015), that is, the value of the instrument does not affect the potential outcomes directly. Assumption $1(ii)$ is the relevant instrument assumption. Assumptions $1(iii)$ and $1(iv)$ guarantee the identification of the parameters that we are interested in. Robinson (1988) also imposes the same assumption as Assumption $1(iii)$. Assumption $1(vii)$ is for the measure $\mu(.)$. These conditions in Assumption $1(vii)$ are not very restrictive. There are many different available measures. We use the CDF of Gaussian distribution in simulations and applications.

The Monotonicity Assumption (or Assumption $1(v)$) needs more investigation. Assumption $1(v)$ is a multivariate extension of the Monotonicity Assumption from Imbens and Angrist (1994). It is a condition that assumes all individuals make the same choice if they are given the same options. That is, even if there are two types of compliers in reality, Assumption $1(v)$ assumes that only one type exists. Recall that when individuals accept the treatment based on the value of the covariate inside the interaction term, they are different compliers. If we only use one instrument, Assumption $1(v)$ will simply be the Monotonicity Assumption. The interpretation of the parameters $\theta_{w0}$ and $\theta_{wx0}$ are associated with the average treatment effect of the compliers who make their choices based on the result of the random assignment. If $Z_i$ is a vector of instruments, Assumption $1(v)$ still ensures that there is only one type of compliers. The interpretation of the parameters $\theta_{w0}$ and $\theta_{wx0}$ is connected with the average treatment effect of that kind of compliers. We will show that our estimator can be reliably implemented with only one instrument. In our simulations and applications, we will provide results using one and multiple instruments for comparison.

Under Assumption $1$, $\theta_0$ is the unique minimizer of

$$M_\infty(\theta, g) = 0$$

when $g = g_0$, because $E(\epsilon_i | Z_i) = 0$ with probability 1. Without replacing the nuisance parameter $g$ with its true value $g_0$ the First Order Condition of $M_\infty(\theta, g)$ is

$$E((P_j - g_P(X_j))[y_l - g_y(X_l) - (P_l - g_P(X_l))'\theta]\kappa_{j,l}) = 0 \quad (2.7)$$

When $g = g_0$, $g_P(X_j)$ becomes $g_{0,P}(X_j)$ or $E(P_j | X_j)$ and $g_y(X_l)$ is $g_{0,y}(X_l)$ or $E(y_l | X_l)$, the FOC becomes:

$$E[\tilde{P}_j(\tilde{y}_l - \tilde{P}_l'\theta)\kappa_{j,l}] = 0 \quad (2.8)$$

$\theta_0$ is identified under a strong identification assumption and Assumption $1$. We leave the formal discussion to Proposition $1$. The FOC defined in Equation (2.8) provides an explicit form for $\theta_0$. The sample analog of the explicit form for $\theta_0$ under Equation (2.8) is a direct extension from Antoine and Sun (2021) which allows for an interaction term inside the
parametric part of the model. The expression for $\theta_0$ is provided in Section E of the Appendix. Antoine and Sun (2021) provide the estimator for the parameter in front of the treatment. It is a special case of Lavergne and Patilea (2013) when the bandwidth inside $\kappa_{j,l}$ is fixed. Lavergne and Patilea (2013) show that such an estimator is consistent and asymptotically normal. However, since the nuisance parameter $g_0$ is unknown, the estimator based on the sample analogue of Equation (2.8) is infeasible and denoted as $\tilde{\theta}_{n,u}$. To obtain the feasible estimator, we need to estimate the nuisance parameters in the first step. Antoine and Sun (2021) replace their nuisance parameters with Nadaraya-Watson estimators, and show that with proper assumption on the bandwidth, the infeasible and feasible estimators share the same asymptotic properties. To satisfy the assumption on bandwidth, the Nadaraya-Watson estimator imposes one constraint on the number of covariates. We will discuss this constraint in the later sections about feasible estimators. Using Nadaraya-Watson estimators or other non-parametric estimators, with regularized conditions, such as the Lasso method, the bias introduced in estimating the nuisance parameters may cause bias in the second stage where we estimate the key parameters.

We propose a new FOC to estimate the $\theta_0$, which extends the Neyman-orthogonal estimator (Chernozhukov et al. (2018)) to the U-statistic setting. Our original FOC defined in Equation (2.7) is not orthogonal to the nuisance parameter, so the bias in the first step will affect the estimate of the key parameter. This is shown in the Appendix. The Neyman-orthogonal method constructs a new FOC which is orthogonal to the nuisance parameters introduced in the first step. The new FOC has all partial derivatives with respect to the nuisance parameters equal to zero, in this way it is orthogonal to the bias of the nuisance parameter (or function). The definition of the partial derivative with respect to a function is provided in Chernozhukov et al. (2018). A similar method is used in Nekipelov et al. (2018).

After Neyman-orthogonalization, the new FOC for $\theta_0$ becomes insensitive to the bias of the estimator for nuisance parameters. It is sensitive to the square of the bias. As long as the bias is $o_p(n^{-1/4})$, we still have a $\sqrt{n}$-asymptotically normally distributed estimator for the key parameter. This order for the bias is not an issue, because there are still many estimation methods to choose from, for instance, the Lasso, Sieves, Random forest, and so on.

The new FOC for $\theta_0$ corresponding to Equation (2.7) is

$$E[\Psi(D; \theta, g_0)] = E\left[ \left. \left( P_j - g_0, P(X_j) - \frac{g_0, P_m(X_j)}{g_0, \kappa_{m,l}(X_j)} \right) \left( y_l - g_0, y(X_l) - (P_l - g_0, P(X_l))' \theta \right) \kappa_{j,l} \right| X_j \right] = 0$$

with $g_0, P_m(X_j) \equiv E[(P_m - g_0, P(X_m)) \kappa_{m,l} | X_j]$ and $g_0, \kappa_{m,l}(X_j) \equiv E[\kappa_{m,l} | X_j]$.

$g_0, P_m(X_j)$ and $g_0, \kappa_{m,l}(X_j)$ are two additional parameters inside the nuisance parameter vector. At this stage, there are four nuisance parameters inside the vector. With these two extra nuisance parameters, the FOC defined in Equation (2.9) has a partial derivative with
respect to all nuisance parameters equal to zero. This is shown in the Appendix.

Equation (2.9) gives us the identification of the true key parameter and the forms of the infeasible and feasible estimators.

**Proposition 1.** *(Identification of \( \theta_0 \) using the orthogonalized FOC)*

Under Assumption 1 and FOC defined in Equation (2.9)

\[
\theta_0^* = E \left[ \kappa_{j,l} \left( \tilde{P}_j - \frac{g_0, \tilde{P}_m(X_l)}{g_0, \kappa_{m,l}(X_l)} \right) \tilde{P}_l^t \right]^{-1} E \left[ \kappa_{j,l} \left( \tilde{P}_j - \frac{g_0, \tilde{P}_m(X_l)}{g_0, \kappa_{m,l}(X_l)} \right) \tilde{y}_l \right]
\]

In Section D of the Appendix, we illustrate the identification issue for the group of SMD estimators, such as SMD, RSMD, and D-RSMD. We consider cases involving various types of instruments and models.

If we plug Equation (2.2) into the formula for \( \theta_0^* \) with error term \( \epsilon_l \), we will have \( \theta_0^* = \theta_0 \). The sample analog of \( \theta_0^* \) under Equation (2.9) (in Proposition 1) delivers an estimator for \( \theta_0 \). Because we have two distinct individuals inside the expectation (e.g., \( j \) and \( l \)), we need to replace the expectation by the average of a double summation to obtain the infeasible estimator under Equation (2.9). The closed-form expression for the infeasible estimator, \( \tilde{\theta}_{n,o} \), is:

\[
\tilde{\theta}_{n,o} = \left[ \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j}^{n} \kappa_{j,l} \left( \tilde{P}_j - \frac{g_0, \tilde{P}_m(X_l)}{g_0, \kappa_{m,l}(X_l)} \right) \tilde{P}_l^t \right]^{-1} \left[ \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j}^{n} \kappa_{j,l} \left( \tilde{P}_j - \frac{g_0, \tilde{P}_m(X_l)}{g_0, \kappa_{m,l}(X_l)} \right) \tilde{y}_l \right]
\]

To distinguish the infeasible estimators under two different FOCs, the infeasible estimator under orthogonal FOC (Equation (2.9)) is denoted as \( \tilde{\theta}_{n,o} \). Recall that the infeasible estimator under Equation (2.8) (from Antoine and Sun (2021)) is \( \tilde{\theta}_{n,u} \) and the expression is shown in Section E of the Appendix.

### 3 Large Sample Theory

In this section, we show the asymptotic properties of the infeasible estimators under the Neyman-Orthogonalized FOC. Then, we introduce the D-RSMD estimator (Debiased Robinson-SMD) as the feasible estimator under the Neyman-Orthogonalized FOC, which shares the same asymptotic properties as the infeasible estimator. The asymptotic properties of the infeasible and feasible estimators under the non-orthogonal FOC from Antoine and Sun (2021) are in the Appendix.

#### 3.1 Asymptotic Properties of the Infeasible Estimators

The infeasible estimators under both FOCs have explicit forms and are linear in \( \tilde{y}_l \). The SMD estimator introduced in Lavergne and Patilea (2013) has a general form for the asymptotic
properties even if there are no explicit forms for these infeasible estimators. Our work here extends Lavergne and Patilea (2013) to allow for nuisance parameters and introduces the debiased method to limit the impact of their estimation.

**Proposition 2.** (Consistency and Asymptotic normality of \( \tilde{\theta}_{n,o} \))

Under Assumption \( \mathbb{A} \) and iid assumption for the sample, \( \tilde{\theta}_{n,o} \) is consistent for \( \theta_0 \), that is \( \tilde{\theta}_{n,o} \xrightarrow{p} \theta_0 \), and asymptotically normally distributed,

\[
\sqrt{n}(\tilde{\theta}_{n,o} - \theta_0) \xrightarrow{d} N \left( 0, A^{-1} \text{Var}[h_1(\tilde{P}_1, \epsilon_1, Z_1, X_1)] \left( A^{-1} \right)' \right)
\]

with

\[
A = E \left[ \kappa_{j,l} \left( \tilde{P}_j - \frac{g_0 \tilde{P}_m(X_l)}{g_0 \kappa_{m,j}(X_l)} \right) \tilde{P}_l' \right]
\]

and

\[
\text{Var} \left[ h_1(\tilde{P}_j, \epsilon_j, Z_j, X_j) \right] \equiv \text{Var} \left[ \int_{\mathbb{R}^q} e^{-it'Z_i} \left( E[e^{it'Z_i} \tilde{P}_l] - \frac{g_0 \tilde{P}_m(X_l)}{g_0 \kappa_{m,j}(X_l)} E[e^{it'Z_l}] \right) d\mu(t) \right]
\]

\( \text{Var} \) stands for the variance-covariance matrix for the vector \( h_1(\tilde{P}_1, \epsilon_1, Z_1, X_1) \), which is a conditional mean function defined in Hoeffding (1948). The expression of \( h_1(\tilde{P}_1, \epsilon_1, Z_1, X_1) \) is shown in the proof section of the Appendix. The asymptotic properties of \( \tilde{\theta}_{n,o} \) are based on the corresponding properties of U-statistics introduced by Hoeffding (1948). See Theorem 7.1. The expression for the middle term of the asymptotic variance looks complicated because the \( \kappa_{j,l} \) is expressed explicitly. If we show the asymptotic variance with \( \kappa_{j,l} \), the form will be simple. The form for the asymptotic variance is shown in the Appendix.

Additionally, we show the similar consistency and asymptotic normality properties of \( \tilde{\theta}_{n,u} \) in the Appendix. Because \( \tilde{\theta}_{n,u} \) is a direct extension from Antoine and Sun (2021), we only list the expressions and theorems. The detailed proofs for the properties of \( \tilde{\theta}_{n,u} \) are shown in the Appendix of Antoine and Sun (2021). The comparison between Neyman orthogonal estimators and non-orthogonal estimators is discussed in the next section.

### 3.2 Feasible Estimator

The infeasible estimator \( \tilde{\theta}_{n,o} \) depends on unknown nuisance parameters: \( g_0 \tilde{P}_m(X_l) \), \( g_0 \kappa_{m,i}(X_l) \), \( E(y_l|X_l) \), and \( E(P_l|X_l) \). All of the nuisance parameters are conditional expectation functions on covariates. Hence, in practice, we need to find estimators for these conditional expectation functions to obtain the feasible estimator. Replacing every nuisance parameter \( g_0 \) with estimators \( \hat{g} \) such that \( \hat{g} \) converges to \( g_0 \) at a rate of \( o_p(n^{-1/4}) \) will deliver the feasible estimator on \( \theta_0 \) with \( \sqrt{n} \) asymptotic normality.

In this section, we are concerned with models with \( P \) variables inside \( X_l \) where \( P \) can be large. If \( P \) is large, we need to assume sparsity for the nuisance parameters, that is, the conditional means can be described with only a few non-zero parameters in front of \( X_l \). The number of non-zero parameters, \( s_0 \), is allowed to grow at the rate of \( o_p(n^{1/2}/\log(P)) \). Under
the assumed rate for $s_0$, the Lasso estimation has the following property (see van de Geer (2016)) on the order of mean square error

$$||X(\hat{\beta} - \beta_0)||^2/n = O_p\left(\frac{s_0 \log(P)}{n}\right)$$

where $\hat{\beta}$ is the Lasso estimator (Chernozhukov et al. (2018) and van de Geer (2016)) and $||.||_2$ is the $L_2$ norm. When $3 < P < n$, we can still use the Lasso method with a higher assumed rate for $s_0$ to select variables. If $P \leq 3$, we can use Nadaraya–Watson estimator to estimate the nuisance parameter with a second degree kernel. $P \leq 3$ is a constraint which is unlikely to hold in practice. Our estimation procedure allows us to handle larger values of $P$ unlike previous literature: for instance, Li and Racine (2006), Robinson (1988), and Antoine and Sun (2021).

After replacing every nuisance parameter with its estimate, we have the feasible estimator $\hat{\theta}_{n,o}$, that is, D-RSMD estimator:

$$\hat{\theta}_{n,o} = \left[ \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j}^{n} \kappa_{j,l} \left( \hat{P}_j - \frac{\hat{\theta}_0 \hat{p}_{n}(X_l)}{\hat{g}_{0,m,l}(X_l)} \right) \right]^{-1} \left[ \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j}^{n} \kappa_{j,l} \left( \hat{P}_j - \frac{\hat{g}_0 \hat{p}_{n}(X_l)}{\hat{g}_{0,m,l}(X_l)} \right) \hat{y}_l \right]$$  \hspace{1cm} (3.1)

since $E[\kappa_{j,l} \left( \hat{P}_j - \frac{\hat{g}_0 \hat{p}_{n}(X_l)}{\hat{g}_{0,m,l}(X_l)} \right) \hat{y}_l]$ is invertible.

This leads to the algorithm of our D-RSMD estimation procedure.

**Algorithm 3.1. (Implementation of the D-RSMD estimation procedure)**

(i) conduct Robinson transformation. This step is to estimate the conditional means inside $\hat{P}_j$ and $\hat{y}_l$. Any estimators $\hat{g}$ with $\hat{g}$ converges to $g_0$ at a rate of $o_p(n^{-1/4})$ can be applied, for instance, the Lasso method.

(ii) calculate the $\kappa_{j,l}$. Inside $\kappa_{j,l}$, $\mu(.)$ is the CDF of the Gaussian distribution. Because the Fourier transform of the Gaussian is still Gaussian, $\kappa_{j,l}$ is easy to compute.

(iii) estimate the nuisance parameters $\hat{g}_0 \hat{p}_{n}(X_l)$ and $\hat{g}_{0,m,l}(X_l)$ inside the orthogonal FOC. The orthogonal FOC is also orthogonal to the nuisance parameters, so we use the Lasso method to estimate these parameters.

(iv) calculate the estimate based on Equation (3.1) for $\hat{\theta}_{n,o}$.

In the approach from the algorithm, all of the nuisance parameters for D-RSMD estimators in the later sections are estimated by the Lasso method with cross validation. The maximum degree of the polynomial for the nuisance parameters is 5, which guarantees that the nuisance parameters can be approximated by 5 degree polynomials in all controls and Lasso with cross validation helps us select the controls and their polynomials.
Assumption 2. \( \hat{g} \) converges to \( g_0 \) at a rate of \( o_p(n^{-1/4}) \). For the Lasso method, the number of non-zero parameters \( s_0 \) grows at the rate of \( o_p(n^{1/2}/\log(P)) \). For the Nadaraya-Watson estimator, \( \sqrt{n} \left( \sum_{s=1}^{q} h_s^4 + \left[ \frac{1}{nh_{1,...,hn}} \right] \right) = o(1) \) where \( h \) is the bandwidth.

Theorem 3.2. (Consistency and Asymptotic normality of the D-RSMD estimator: \( \hat{\theta}_{n,o} \)) Under Assumptions 1 - 2 and iid assumption for the sample, \( \hat{\theta}_{n,o} \) is consistent, and has an asymptotically normal distribution, that is,
\[
\sqrt{n}(\hat{\theta}_{n,o} - \theta_0) \xrightarrow{d} N \left( 0, A^{-1} \text{Var}[h_1(P_1, \epsilon_1, Z_1, X_1)](A^{-1})' \right)
\]

From Theorem 3.2, the feasible and infeasible estimators share the same asymptotic distribution and are consistent. It is because under Assumption 2, the bias introduced in the first step will not affect the second step substantially. This is the same convergence rate assumption used in Chernozhukov et al. (2018). We expand these properties to Bierens type estimators.

The asymptotic properties for the feasible version of \( \hat{\theta}_{n,u} \) or the R-SMD estimator from Antoine and Sun (2021) are in Section E of the Appendix.

The asymptotic distributions of the D-RSMD and R-SMD estimators are not the same. When there are many covariates, the D-RSMD estimator using the Lasso for the nuisance parameters works. R-SMD is generated under the condition that the number of covariates \((P)\) is small. When \( P \) is small, the D-RSMD estimator is less biased than the R-SMD estimator if they both estimate the nuisance parameters using the same Nadaraya-Watson estimator. There are no analytical results for comparing the asymptotic distributions. In Chernozhukov et al. (2018), the comparison between Neyman orthogonal estimators and non-orthogonal estimators is shown with simulations. We also present the results on bias in Section 3, i.e., the simulation section.

Under heteroskedasticity, the estimator for the variance of the D-RSMD estimator is
\[
[n(n-1)C_n]^{-1} \sum_{j=1}^{n} \left( \sum_{l=1}^{n} \kappa_{j,l} \left( \hat{P}_l - \frac{g_0 \rho_l(X_l)}{g_0 \rho_{m,l}(X_l)} \right) \right) \left( \sum_{l=1}^{n} \kappa_{j,l} \left( \hat{P}_l - \frac{g_0 \rho_l(X_l)}{g_0 \rho_{m,l}(X_l)} \right) \right)' \hat{e}_j^2 [n(n-1)C_n]^{-1} \quad (3.2)
\]

with \( C_n = \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j}^{n} \kappa_{j,l} \left( \hat{P}_j - \frac{g_0 \rho_{m,l}(X_l)}{g_0 \rho_{m,l}(X_l)} \right) \hat{P}_l' \)

4 Simulation Study

In this section, we conduct 5000 Monte-Carlo replications for two types of Data Generating Process (DGP) to investigate the properties of our D-RSMD estimators for the parameters in the heterogeneous treatment effects of a partially linear model when the number of controls in \( X_i \) is greater than or equal to 1. Recall that the partially linear model with heterogeneous
treatment effects is in the following.

\[ y_i = \theta_{w0}W_i + W_i \cdot X_{1i}^t + f_{0,1}(X_i) + \epsilon_i \]  
\[ W_i = I(f_{0,2}(X_i, Z_i) > v_i) \]  

(4.1)

\[ W_i \cdot X_{1i} \] is the interaction term between the treatment and a subvector of covariates. The benchmark model has a nonlinear \( f_{0,1}(X_i) \) and a nonlinear function \( f_{0,2}(X_i, Z_i) \). \( I(.) \) is the indicator function. It will take the value one if the statement inside is true and zero otherwise.

In all of the simulations, the key parameters \( \theta_{w0} \) and \( \theta_{wx0} \) are 2 and 3 respectively, and we use the same benchmark model.

With the chosen parameter values, the benchmark model is

\[ X_i = X_i^* + 0.4Z_i \]
\[ W_i = I(3Z_i + 4Z_i^3 + \sum_{q=1}^P \alpha_{1q}X_{qi} + \sum_{q=1}^P \alpha_{3q}X_{qi}^3 > -v_i) \]
\[ y_i = 2W_i + 3W_iX_{1i} + \sum_{q=1}^P \beta_{1q}X_{qi} + \sum_{q=1}^P \beta_{2q}X_{qi}^2 + \epsilon_i \]

where \( \alpha_{3q} = 2, \beta_{2q} = -3, \alpha_{1q} = \beta_{1q} = 1 \) for \( q \leq S \) with \( S \) the number of non-zero parameters and \( P \) the number of covariates. Because there are nonlinear terms in the model, \( S \) is chosen to be a half of \( s_0 \), the sparsity level. \( \alpha_{3q} = \beta_{2q} = \alpha_{1q} = \beta_{1q} = 0 \) when \( q > S \).

If \( \beta_{2q} \) is 0 for all \( q \), \( y_i \) is linear in \( X_i \). Otherwise, the model is partially linear, because \( X_i \) enters nonlinearly. The covariate \( X_i \) is the sum of random variables \( X_i^* \) generated by a standard normal distribution (or multivariate standard normal distribution) and a fraction of the instrument because our model allows for the correlation between covariate \( X_i \) and \( Z_i \).

In the benchmark model, we choose the fraction level to be 0.4. The errors \( (\epsilon, v) \) are bivariate normal distributed with mean 0 and covariance matrix \( \Sigma_1 \) such that \( \Sigma_1 = \begin{pmatrix} 1 & 4/9 \\ 4/9 & 1 \end{pmatrix} \).

The correlation between \( \epsilon \) and \( v \) makes the treatment \( W_i \) an endogenous variable. In the DGP, \( y_i \) is not dependent on \( Z_i \) directly. The exclusion restrictions assumption is satisfied. Also, we generate \( X_i^* \) and \( Z_i \) separately and independently because \( X_i^* \) is continuous and \( Z_i \) is categorical.

When there is only one control variable \( X_i \), the two nuisance parameters are

\[ f_{0,1}(X_i) = \beta_{11}X_i + \beta_{21}X_i^2 \]
\[ f_{0,2}(X_i, Z_i) = 3Z_i + 4Z_i^3 + \alpha_{11}X_i + \alpha_{21}X_i^3 \]

We consider two types of DGPs which mainly differ in how the treatment variable is generated, e.g. using either a categorical instrument with three values, or a binary instrument. Both are motivated by our applications. The first kind of DGP uses a categorical instrument.
to generate a treatment variable. Specifically, the instrument variable has three values, 0, 1, and 2, and can be interpreted as the sum of two binary variables. This is motivated by our application based on Card (1993) where we construct a similar instrument by adding an indicator for proximity to a four-year college to an indicator for two-year college.

The second type of DGP employs a binary instrument to generate a treatment variable. Correspondingly in Card (1993), this is the indicator for proximity to a four-year college. We generate the binary instrument based on the distribution of the indicator for proximity to a four-year college.

For each DGP, the benchmark case considers 3000 observations and 30 control variables. This is once again in line with both applications. For instance, in Card (1993), there are 3010 valid observations and 27 covariates. We will also consider different sample sizes and a small number of controls.

We report and compare the simulation results for the following estimators.

(i) the D-RSMD estimator proposed in this paper.

(ii) the R-SMD estimator (from Antoine and Sun (2021)) when the number of controls is less than 3.

(iii) the R-GMM estimator which combines Robinson Transformation with GMM. In the Application section, it is called GMM-Lasso, where we use the Lasso method to estimate the nuisance parameters after the Robinson transformation.

(iv) the GMM estimator that treats $f_{0,1}(X_i)$ as a linear function in $X_i$.

(v) the GMM (Oracle) estimator that uses the true $f_{0,1}(X_i)$.

The nuisance parameters in the first step of the Algorithm for the D-RSMD estimator are estimated by the Lasso method because we will use the same estimator when the number of controls is 30. The tuning parameter of the Lasso method (e.g. the penalty term) is selected by cross-validation (see e.g. van de Geer (2016) and Chernozhukov et al. (2018)).

The R-SMD estimator can only be implemented in the first simulation design when the number of control variables is small (e.g. equal to 1). We apply the Nadaraya-Watson estimator, with the rule of thumb bandwidth $h = \sigma_x n^{-0.2}$ to estimate the nuisance parameters in the first step of the R-SMD estimator. D-RSMD and R-SMD estimators both use the CDF of a standard Gaussian distribution as $\mu(.)$ inside $\kappa(Z_j - Z_l) = \int_{R^d} e^{it'(Z_j - Z_l)} d\mu(t)$. The choice of $\mu(.)$ satisfies Assumption 1(vii).

For each DGP, we report the results for the D-RSMD estimator using only one instrument and two instruments in the following subsection. For the R-SMD estimator, we also report

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5In the Oregon Health Insurance experiment the lottery variable is also a binary instrument.

6Analyzing the Oregon Health insurance experiment, we split the data set into three groups based on age. Each group has about 6000 observations and 21 covariates.
the results for both cases. For GMM type estimators, to estimate two parameters, we need to use at least two instruments. The following is the list of available instrument sets we use for estimators.

(i) \(Z_1\): the binary instrument.

(ii) \(Z_2\): the sum of \(Z_1\) and another binary variable. There are three values in \(Z_2\).

(iii) \((Z_1, Z_1X_1)\): \(Z_1\) and the interaction term between \(Z_1\) and the covariate.

(iv) \((Z_2, Z_2X_1)\): \(Z_2\) and the interaction term between \(Z_2\) and the covariate.

(v) \((Z_1, Z_2)\): \(Z_1\) and \(Z_2\). The correlation between \(Z_1\) and \(Z_2\) is around 0.7.

In all simulation designs, we report the Monte-Carlo Median Bias (Med.Bias), Median Absolute Deviation (MAD), the median of asymptotic standard error under heteroskedasticity (Med.SE), and the Rejection Rate (RR).

### 4.1 Results for the Model with a Categorical Instrument

In this section, instrument \(Z\) is a categorical instrument with three values. That is, for the benchmark model, we generate the treatment by using \(Z_2\). In the simulation, we set the probability distribution for the instrument \(Z_2\) equal to the observed distribution of the instrument in the Card (1993) data to provide insights for the empirical results. Additionally, we use consistent notations are throughout the simulation and application parts.

In Table 1, we report the simulation results of four D-RSMD, four R-SMD, and six GMM type estimators for \(\theta_{w0}\) and \(\theta_{wx0}\). The sample size is 3,000 and there are 5,000 replications with only one control, that is, \(P = 1\). The table contains the results for D-RSMD and R-SMD estimators using one instrument, that is, \(Z_1\) or \(Z_2\), or two instruments, i.e., \((Z_1, Z_1X_1)\) or \((Z_2, Z_2X_1)\). \(Z_2\) is the instrument used in the DGP, so R-SMD using \(Z_2\) or \((Z_2, Z_2X_1)\) will produce better results than it with \(Z_1\) or \((Z_1, Z_1X_1)\). This is also shown in the table. The Nadaraya-Watson estimators for nuisance parameters inside R-SMD and R-GMM estimators when \(n = 3000\) and \(P = 1\) use a bandwidth of \(0.202\hat{\sigma}_x\). GMM type estimators are generated using two instruments \((Z_1, Z_1X_1)\) or \((Z_2, Z_2X_1)\).

Recall that \(Z_2\) is the instrument generating the treatment variable. Intuitively, estimators applying \(Z_2\) or \((Z_2, Z_2X_1)\) will produce better results than those using \(Z_1\) or \((Z_1, Z_1X_1)\) do. Indeed, this is the case for D-RSMD with \((Z_2, Z_2X_1)\), R-SMD, R-GMM, GMM, and GMM (Oracle). From Table 1 comparing D-RSMD using \((Z_2, Z_2X_1)\) and GMM (Oracle), we find that all results are close. It suggests that D-RSMD with \((Z_2, Z_2X_1)\) is as good as GMM (Oracle). D-RSMD with \((Z_2, Z_2X_1)\) has a lower median bias than R-SMD with \((Z_2, Z_2X_1)\), which means that the Debiased part in D-RSMD works. Applying the debiased procedure
| Estimator  | Instrument | $\theta_{w0}$ | | | | $\theta_{wx0}$ | | |
|------------|------------|---------------|---|---|---|---|---|
| D-RSMD     | $Z_1$      | -0.007        | 0.069 | 0.103 | 0.047 | 0.019 | 0.036 | 0.052 | 0.067 |
| D-RSMD     | $Z_2$      | -0.017        | 0.129 | 0.211 | 0.035 | 0.034 | 0.313 | 0.523 | 0.035 |
| D-RSMD     | $(Z_1, Z_1X_1)$ | 0.002 | 0.077 | 0.112 | 0.050 | -0.008 | 0.089 | 0.130 | 0.044 |
| D-RSMD     | $(Z_2, Z_2X_1)$ | 0.001 | 0.045 | 0.065 | 0.048 | -0.006 | 0.055 | 0.077 | 0.054 |
| R-SMD      | $Z_1$      | -0.207        | 0.072 | 0.236 | 0.007 | -0.424 | 0.065 | 0.449 | 0.000 |
| R-SMD      | $Z_2$      | 0.027         | 0.172 | 0.280 | 0.022 | -0.060 | 0.416 | 0.693 | 0.018 |
| R-SMD      | $(Z_1, Z_1X_1)$ | 0.064 | 0.077 | 0.127 | 0.052 | -0.112 | 0.083 | 0.220 | 0.003 |
| R-SMD      | $(Z_2, Z_2X_1)$ | 0.028 | 0.039 | 0.072 | 0.031 | -0.053 | 0.048 | 0.076 | 0.079 |
| R-GMM      | $(Z_1, Z_1X_1)$ | -0.044 | 0.076 | 0.182 | 0.003 | -0.239 | 0.089 | 0.245 | 0.026 |
| R-GMM      | $(Z_2, Z_2X_1)$ | -0.015 | 0.042 | 0.105 | 0.001 | -0.172 | 0.053 | 0.156 | 0.043 |
| GMM        | $(Z_1, Z_1X_1)$ | 2.571 | 0.318 | 0.472 | 1.000 | -6.003 | 0.415 | 0.618 | 1.000 |
| GMM        | $(Z_2, Z_2X_1)$ | 2.176 | 0.231 | 0.322 | 1.000 | -5.233 | 0.319 | 0.454 | 1.000 |
| GMM (Oracle) | $(Z_1, Z_1X_1)$ | 0.000 | 0.076 | 0.113 | 0.048 | -0.003 | 0.093 | 0.137 | 0.045 |
| GMM (Oracle) | $(Z_2, Z_2X_1)$ | 0.000 | 0.042 | 0.061 | 0.049 | 0.000 | 0.053 | 0.078 | 0.057 |

Table 1: Categorical instrument when $P = 1$ (n=3000)

Note: Simulation Results for $\theta_{w0}$ and $\theta_{wx0}$ in the benchmark model using D-RSMD estimator 5000 replications. We report the Monte-Carlo Median Bias (Med.Bias), Median Absolute Deviation (MAD), the median of asymptotic standard error under heteroskedasticity (Med.SE), and Rejection Rate (RR) using a 5% t-test.

reduces the effect of bias on the estimate introduced by the nuisance parameters. In the table, D-RSMD and R-SMD with $Z_2$ also work. MAD and Med.SE for D-RSMD are smaller than those for R-SMD. RR for D-RSMD is closer to 5% than R-SMD. The Med.Bias is higher for D-RSMD with $Z_2$, which suggests that the bias introduced by the new nuisance parameters in the debiased procedure has a larger effect on the estimate than D-RSMD with $(Z_2, Z_2X_1)$. This is reasonable because the new nuisance parameters are conditional expectations of the $\kappa(Z_j - Z_l)$, which is a function of the instruments.

When we use the sets of instruments not in the DGP, e.g., D-RSMD with valid $Z_1$ or $(Z_1, Z_1X_1)$, the D-RSMD type estimators still work. Some of the results are better than the D-RSMD with $Z_2$ or $(Z_2, Z_2X_1)$. For instance, The MAD and Med.SE for D-RSMD with $Z_1$ are smaller than those for D-RSMD with $Z_2$ for both parameters. This property disappears in the later sections, so it can be case-specific. This also implies that D-RSMD works with various instruments. With valid $Z_1$ or $(Z_1, Z_1X_1)$, R-SMD or GMM type estimators for $\theta_{w0}$ and $\theta_{wx0}$ have much higher median biases than the same estimators with valid $Z_2$ or $(Z_2, Z_2X_1)$. This suggests that choosing the instruments outside of the DGP causes problems for R-SMD or GMM type estimators.

In the empirical application, there are more than 20 control variables. Hence, the benchmark model in the simulation considers 30 controls to mimic the situation. When there are 30 controls, the controls are generated by a multivariate normal distribution with mean 0 and the covariance matrix set to the identity matrix. Following the sparsity assumption in
Assumption 2, when $P = 30$, we choose sparsity level to be 10, that is, $\alpha_3q = 2$, $\beta_2q = -3$, $\alpha_1q = \beta_1q = 1$ when $q$ is less than or equal to 5. We still use the benchmark model in the DGP.

We report the table of robustness check when the model is not sparse, that is, all $\beta_2q$ are not zero, in Supplementary Appendix.

Table 2: Instrument with 3 values ($P = 30$)

| $n$ | $\theta_{w0}$ | $\theta_{wx0}$ | Instrument with 3 values ($P = 30$) | Estimator | Instr | Med.Bias | MAD | Med.SE | RR |
|-----|----------------|----------------|-----------------------------------|-----------|-------|----------|-----|--------|----|
| 2000 | $\theta_{w0}$ | $\theta_{wx0}$ | D-RSMD $Z_2$ | -0.041 | 0.115 | 0.146 | 0.064 |
|       |                 |               | D-RSMD ($Z_2, Z_2X_1$) | -0.059 | 0.119 | 0.144 | 0.072 |
|       |                 |               | GMM (Oracle) ($Z_2, Z_2X_1$) | -0.006 | 0.110 | 0.160 | 0.050 |
| 2000 | $\theta_{w0}$ | $\theta_{wx0}$ | D-RSMD $Z_2$ | -0.019 | 0.106 | 0.158 | 0.064 |
|       |                 |               | D-RSMD ($Z_2, Z_2X_1$) | -0.033 | 0.079 | 0.101 | 0.064 |
|       |                 |               | GMM (Oracle) ($Z_2, Z_2X_1$) | 0.001 | 0.059 | 0.087 | 0.053 |
| 2000 | $\theta_{w0}$ | $\theta_{wx0}$ | D-RSMD $Z_2$ | -0.018 | 0.119 | 0.158 | 0.044 |
|       |                 |               | D-RSMD ($Z_2, Z_2X_1$) | -0.115 | 0.557 | 0.843 | 0.025 |
|       |                 |               | D-RSMD ($Z_2, Z_2X_1$) | -0.028 | 0.100 | 0.118 | 0.055 |
|       |                 |               | RSMD ($Z_1, Z_2$) | -0.019 | 0.189 | 0.267 | 0.036 |
|       |                 |               | RSMD $Z_2$ | -0.476 | 0.773 | 1.173 | 0.024 |
|       |                 |               | RSMD ($Z_2, Z_2X_1$) | -0.134 | 0.178 | 0.313 | 0.083 |
|       |                 |               | RSMD ($Z_2, Z_2X_1$) | 0.082 | 0.790 | 1.158 | 0.015 |
|       |                 |               | RSMD ($Z_2, Z_2X_1$) | -0.202 | 0.111 | 0.190 | 0.157 |
|       |                 |               | RSMC ($Z_1, Z_2$) | -0.128 | 0.267 | 0.411 | 0.073 |
|       |                 |               | GMM ($Z_2, Z_2X_1$) | 3.379 | 0.897 | 1.354 | 0.715 |
|       |                 |               | GMM ($Z_1, Z_2$) | 12.082 | 4.798 | 7.355 | 0.230 |
|       |                 |               | GMM (Oracle) ($Z_2, Z_2X_1$) | 0.000 | 0.089 | 0.130 | 0.056 |
|       |                 |               | GMM (Oracle) ($Z_1, Z_2$) | 0.003 | 0.506 | 0.802 | 0.004 |

Table 2: Instrument with 3 values

Note: Simulation Results for $\theta_{w0}$ and $\theta_{wx0}$ in the benchmark model using D-RSMD estimator 5000 replications. We report the Monte-Carlo Median Bias (Med.Bias), Median Absolute Deviation (MAD), median of asymptotic standard error under heteroskedasticity (Med.SE), and Rejection Rate (RR) using a 5% t-test.

Table 2 reports the simulation results of D-RSMD, RSMD, and GMM type estimators for the key parameters $\theta_{w0}$ and $\theta_{wx0}$ when the sample sizes are 2,000, 3000, or 5000. All of
the simulation studies have 5000 replications. The table contains the results for D-RSMD estimators using one instrument \(Z_1\) or \(Z_2\) or two instruments, i.e., \((Z_1, Z_1X_1)\), \((Z_2, Z_2X_1)\), and \((Z_1, Z_2)\). For the GMM type estimators, we report the results using two instruments, that is, \((Z_2, Z_2X_1)\) or \((Z_1, Z_2)\).

When relying on one instrument (e.g., the categorical instrument) in the estimation procedure, only D-RSMD type estimators work. We expect that estimators using \(Z_2\) will have generally better performance than the same estimators using \(Z_1\) in terms of smaller Med.Bias, MAD and Med.SE. RR should be closer to 5% for estimators using \(Z_2\). Indeed, for \(\theta_{w0}\), D-RSMD with \(Z_2\) follows this expectation. Although for \(\theta_{w0}\), D-RSMD using \(Z_1\) has lower Med.Bias, MAD and Med.SE, the RR is downward bias and not that close to 5%.

When we work with two instruments, i.e., \((Z_1, Z_1X_1)\), \((Z_2, Z_2X_1)\), or \((Z_1, Z_2)\), results for D-RSMD and GMM type estimators are in the 3000 observations panels of Table 2. The GMM (Oracle) estimator is a GMM estimator using the correct second degree polynomial for the nonlinear \(f_{01}(X_i)\). Thus, it has relatively better performance with instrument \((Z_2, Z_2X_1)\), in terms of much smaller bias, lower standard errors and higher t-statistic. It is also shown in the table. However, RR is slightly higher. This higher RR disappears when we have a larger number of observations. See the panel with \(n = 5000\).

It is no surprise to see that using another set of instruments \((Z_1, Z_2)\), the GMM (Oracle) estimator does not work so well. It is because the instruments in \((Z_1, Z_2)\) have a higher correlation. If we check the raw estimation results generated by GMM (Oracle) with \((Z_1, Z_2)\), we will see many extreme cases. The same issue happens to GMM as well. However, it is not a big problem for the D-RSMD estimator. D-RSMD estimators using any instrument has relatively stable performance. The only problem with the instrument selection problem happens to the \((Z_2, Z_2X_1)\) where the RR is higher than 5%. This rejection rate decreases when we have a bigger dataset. See the last six rows with \(n = 5000\) in Table 2.

With a categorical instrument in the DGP, the D-RSMD estimators have the proper size, much smaller bias than GMM estimators. As the sample size grows, the size distortions decrease for \((Z_2, Z_2X_1)\). D-RSMD allows us to use any possible instrument set to estimate heterogeneous treatment effects. When the correlation between the instruments inside the instrument vector is high, D-RSMD still works, while GMM generates extreme results.

### 4.2 Results for the Model with a Binary Instrument

The DGP in this section also follows the benchmark model for the categorical instrument variable. The only difference is that when we generate the treatment variable, \(Z_i\) is a binary instrument instead of a categorical instrument. Hence, in this section, \(Z_1\) is the true instrument variable. Instrument sets \((Z_1, Z_1X_1)\) and \((Z_1, Z_2)\) contain the true instrument. We report results in Table 3.

The first two panels of Table 3 contain the preliminary results when \(P = 3\). D-RSMD
using $Z_1$ has the lowest MAD and Med.SE among all of the estimators. Its RR are close to 5% for $\theta_{w0}$ and slightly oversized for $\theta_{wx0}$. Using the instrument $Z_2$ (not in the DGP) for D-RSMD will generate higher MAD and Med.SE than utilizing the correct instrument. D-RSMD estimator also works when employing $(Z_1, Z_1X_1)$ and $(Z_2, Z_2X_1)$. When comparing D-RSMD estimators, we see that using the instrument from the DGP or the instrument set including the instrument will generate results with lower MADs and Med.SEs. It is reasonable because working with the correct variable will increase the precision of the estimate. The Med.Bias column shows that for $\theta_{w0}$ the D-RSMD estimators using $(Z_1, Z_1X_1)$ and $(Z_2, Z_2X_1)$ have a higher bias than the other D-RSMD estimators. It suggests that using the instrument from the DGP directly will generate a lower bias. This property of D-RSMD also shows in the case where we have 30 controls in the model.

In the simulation, when $P = 30$, we see similar results as the case with a categorical instrument inside DGP. For D-RSMD estimators, using $Z_1$ or $Z_2$ already provides similar or better results than the GMM (Oracle) in terms of lower MAD and Med.SE. Because the DGP in this section uses the binary instrument, the D-RSMD results with $Z_1$ should be better than the other D-RSMD estimators in terms of the magnitude of the Med.Bias, MAD, Med.SE. Indeed this is the case. For the RR column, if we compare the $Z_1$ results with $Z_2$ ones, we see that the RR is higher in $Z_1$. It is reasonable because the instrument $Z_1$ has only two values and $Z_2$ has three values and $Z_2$ is the sum of $Z_1$ and another binary instrument. $Z_2$ contains more information. The size distortion problem decreases when we generate 5000 observations for each replication.

Comparing the results for instrument sets $(Z_1, Z_1X_1)$ and $(Z_1, Z_2)$, we see similar stories to before. For GMM type estimators, $(Z_1, Z_1X_1)$ is the better choice. For instance, the GMM (Oracle) using $(Z_1, Z_1X_1)$ is the best results for both $\theta_{w0}$ and $\theta_{wx0}$ with lowest Med.Bias. GMM type estimators using $(Z_1, Z_2)$ still generate scattered results and many extreme cases. The RR for GMM and GMM (Oracle) with $(Z_1, Z_2)$ is close to 0 for $\theta_{wx0}$. Even with the GMM (Oracle) estimator, the correlation between the instruments inside $(Z_1, Z_2)$ still causes GMM estimators a big problem.

If we compare D-RSMD results with GMM (Oracle) estimator, we find that D-RSMD results with $Z_1$ are close to the Oracle ones for $\theta_{w0}$ in terms of MAD, Med.SE, and RR, that suggests that with $Z_1$ D-RSMD is good enough. For $\theta_{wx0}$ estimation, D-RSMD results with $Z_1$ is better than the GMM (Oracle) using $(Z_1, Z_1X_1)$ in terms of lower MAD and Med.SE. It is reasonable, because we use all of the information from the conditional moment, while GMM (Oracle) only uses two unconditional moments.

Using one binary instrument for D-RSMD generates better results than using two instruments for D-RSMD estimators. It suggests that for D-RSMD, the number of instruments is not a big problem.
Table 3: Binary Instrument

| Estimator | Ins | Med.Bias | MAD | Med.SE | RR |
|-----------|-----|----------|-----|--------|----|
| D-RSMD | $Z_1$ | -0.006 | 0.146 | 0.215 | 0.048 |
| D-RSMD | $Z_2$ | -0.004 | 0.212 | 0.314 | 0.043 |
| D-RSMD | $(Z_1, Z_1 X_1)$ | -0.026 | 0.164 | 0.247 | 0.048 |
| D-RSMD | $(Z_2, Z_2 X_1)$ | -0.035 | 0.250 | 0.340 | 0.049 |
| D-RSMD | $(Z_1, Z_1 X_1)$ | -0.004 | 0.164 | 0.250 | 0.048 |

$P = 3$

| Estimator | Ins | Med.Bias | MAD | Med.SE | RR |
|-----------|-----|----------|-----|--------|----|
| D-RSMD | $Z_1$ | -0.005 | 0.039 | 0.068 | 0.052 |
| D-RSMD | $Z_2$ | -0.004 | 0.110 | 0.186 | 0.056 |
| D-RSMD | $(Z_1, Z_1 X_1)$ | -0.004 | 0.124 | 0.186 | 0.056 |
| D-RSMD | $(Z_2, Z_2 X_1)$ | -0.001 | 0.120 | 0.191 | 0.047 |
| D-RSMD | $(Z_1, Z_1 X_1)$ | -0.004 | 0.126 | 0.191 | 0.047 |

| Estimator | Ins | Med.Bias | MAD | Med.SE | RR |
|-----------|-----|----------|-----|--------|----|
| D-RSMD | $Z_1$ | 0.028 | 0.288 | 0.427 | 0.043 |
| D-RSMD | $Z_2$ | 0.033 | 0.524 | 0.773 | 0.030 |
| D-RSMD | $(Z_1, Z_1 X_1)$ | -0.259 | 0.291 | 0.431 | 0.082 |
| D-RSMD | $(Z_2, Z_2 X_1)$ | 0.041 | 0.333 | 0.430 | 0.041 |
| D-RSMD | $(Z_1, Z_1 X_1)$ | 0.128 | 0.408 | 0.563 | 0.072 |
| D-RSMD | $(Z_2, Z_2 X_1)$ | 0.154 | 0.555 | 0.790 | 0.039 |
| D-RSMD | $(Z_1, Z_1 X_1)$ | 0.230 | 0.463 | 0.572 | 0.110 |
| D-RSMD | $(Z_2, Z_2 X_1)$ | 0.140 | 0.438 | 0.572 | 0.060 |
| D-RSMD | $(Z_1, Z_1 X_1)$ | 7.471 | 2.968 | 4.368 | 0.356 |
| D-RSMD | $(Z_2, Z_2 X_1)$ | 6.857 | 5.691 | 11.948 | 0.043 |
| D-RSMD | $(Z_1, Z_1 X_1)$ | 0.007 | 0.266 | 0.392 | 0.047 |
| D-RSMD | $(Z_2, Z_2 X_1)$ | -0.011 | 0.126 | 0.191 | 0.047 |

Note: Simulation Results for $\theta_{w0}$ and $\theta_{wx0}$ in the benchmark model using D-RSMD estimator 5000 replications. We report the Monte-Carlo Median Bias (Med.Bias), Median Absolute Deviation (MAD), median of asymptotic standard error under heteroskedasticity (Med.SE), and Rejection Rate (RR) using a 5% t-test.

5 Empirical Application

5.1 Estimating the Returns of Education on Wages

To illustrate the proposed process, we use the data set from Card (1993) to estimate the heterogeneous returns to education. Many works have used the same data set, for instance, Yanagi (2019), Kitagawa (2015), and Ashenfelter and Rouse (1998). The data is from the
National Longitudinal Survey for young men. The detailed information for the variables is in Card (1993). In this paper, we use the same dependent variable, log hourly wages and the same covariates in the baseline model. The education variable in the original work is the years of education. To investigate the treatment effect of college education, we construct the college indicator based on whether the years of education are higher than 14 years. The treatment can be considered as a two-year college degree or higher. It is used in Yanagi (2019) as well. We also conduct the same analysis for years of education. In Kitagawa (2015), the author also treats the education variable as an indicator.

The instrument variable used in Card (1993) is a dummy for growing up near a local four-year college ($Z_1$). It is used as an instrument because it is not correlated with the individual’s ability and increases the probability of attending college. From Kitagawa (2015), the validity of the instrument is not rejected once the covariates are in the estimation. In our models and estimators, all of the covariates are included.

We use the covariates in the original study by Card, for instance, experience, experience squared, age, and so on. In Card (1993), the variables on the family background are an important group of control variables, including the parents’ years of education, classes of education, and two indicators for family structure. We incorporate those variables in the analysis. To illustrate the heterogeneous treatment effects of interest, we use the parents’ education to generate the interaction term. The parents’ education is the average of father’s and mother’s years of education. A similar control variable is in Ashenfelter and Rouse (1998). The minimum of parents’ education is 0. Three individuals’ parents have no education. Twenty-four fathers and fifteen mothers have zero years of education. The average value of parents’ education is 10.16.

To investigate the properties of different instrument sets, we construct five instrument sets. In three of them, we utilize the information from the dummy of proximity to a two-year college. For instance, $Z_2$ is the sum of two dummies: growing up near a four-year college and a two-year college. $Z_2$ generally implies that the more local colleges are, the higher the value is. There are three values, 0, 1, and 2, in $Z_2$. There are two reasons why $Z_2$ is generated. The first is that we will use a categorical instrument $Z_2$ to demonstrate the properties of the new estimator. We also use $Z_1$ and $Z_2$ as an instrument set to show that the new method works well with highly correlated instruments. The sets of instruments we considered in this subsection are as follows:

(i) $Z_1$: the indicator for proximity to a four-year college.

(ii) $Z_2$: the sum of the indicator for proximity to a two-year college and $Z_1$. There are 3 values in $Z_2$.

(iii) $(Z_1, Z_1 X_1)$: $Z_1$ and the interaction term between parents’ education and $Z_1$. 

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(iv) \((Z_1, Z_2): Z_1\) and \(Z_2\). The correlation between \(Z_1\) and \(Z_2\) is around 0.729.

We examine four models in the following. Because we have two types of D-RSMD estimators in this section, we denote the one used in simulations as DRSMD-Lasso. The other one is DRSMD-2SOLS.

(i) DRSMD-Lasso estimators:
\[
y_i = \theta_{w_0} W_i + W_i \cdot X_i' \theta_{wx0} + f_{0,1}(X_i) + \epsilon_i
\]
\[
W_i = I(f_{0,2}(X_i, Z_i) > v_i)
\]

(ii) DRSMD-2SOLS estimators:
\[
y_i = \theta_{w_0} W_i + W_i \cdot X_i' \theta_{wx0} + \sum_{p=1}^{27} \beta_p X_i + \epsilon_i
\]
\[
W_i = I(f_{0,2}(X_i, Z_i) > v_i)
\]

(iii) GMM estimators:
\[
y_i = \theta_{w_0} W_i + W_i \cdot X_i' \theta_{wx0} + \sum_{p=1}^{27} \beta_p X_i + \epsilon_i
\]
\[
W_i = I(\alpha Z_i + \sum_{p=1}^{27} \alpha_p X_i > v_i)
\]

(iv) GMM-Lasso estimators:
\[
y_i = \theta_{w_0} W_i + W_i \cdot X_i' \theta_{wx0} + f_{0,1}(X_i) + \epsilon_i
\]
\[
W_i = I(\alpha Z_i + \sum_{p=1}^{27} \alpha_p X_i > v_i)
\]

Our new method, the DRSMD-Lasso method, allows the covariates \(X_i\) to enter the model through an unknown function \(f_{0,1}(\cdot)\) and \(f_{0,2}(\cdot)\). Because the true function form is unknown, our method will generate more reliable results. The DRSMD-2SOLS estimator allows a nonlinear and unknown \(f_{0,2}(\cdot)\), but \(f_{0,1}(\cdot)\) needs to be linear. The GMM estimator assumes both \(f_{0,1}(\cdot)\) and \(f_{0,2}(\cdot)\) to be linear in \(X_i\). The GMM-Lasso estimator imposes that \(f_{0,2}(\cdot)\) is linear. All of the results in the subsection are generated with the college indicator as treatment.

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7The treatment effects of years of education are reported in Table A1 in Appendix A. The estimation results for 2SLS from Card (1993) are around 0.132 to 0.140. The DRSMD-Lasso estimator produces similar results. Table A3 in Appendix A reports heterogeneous treatment effects of years of education.
Table 4 reports the results for heterogeneous treatment effects of college ($W_i$). $X_{i1}$ is the parents’ education. If we use GMM estimates with $(Z_1, Z_1X_1)$ and $(Z_1, Z_2)$, no results are statistically significant. The GMM results for $(Z_1, Z_2)$ have a large magnitude. There are two moments in $(Z_1, Z_2)$ including the valid instrument. It suggests that using the extra moment is not a good idea. In Table 3 of Section 4, the GMM estimators with and without the oracle features using $(Z_1, Z_2)$ also generate unreliable results. In this application, the results for GMM and GMM-Lasso using $(Z_1, Z_1X_1)$ show that the heterogeneous treatment effects are not statistically significant at 5% level. The results from GMM are affected by instruments.

For DRSMD-Lasso estimators, we have obtained different results. DRSMD-Lasso and DRSMD-2SOLS methods with $(Z_1, Z_2)$ as instruments generate more reliable results. Using $(Z_1, Z_1X_1)$ the DRSMD-Lasso and DRSMD-2SOLS produce similar statistically significant results for $\theta_{w0}$ and $\theta_{wx0}$ at 5% level. The estimate changes across different sets of instruments, but the magnitude does not change too much. It is reasonable because different instruments imply distinct local average treatment effects on the corresponding groups of compliers. When we compare all of the estimates for the overall treatment effect, in Table 5, we see that for DRSMD type estimators, the treatment effects are higher than those estimated by GMM type estimators. This difference suggests that the nonlinear part in the $f_{0,2}(\cdot)$ is responsible for the gap between DRSMD and GMM type estimators.

The interpretation of estimates of the parameter $\theta_w0$ and $\theta_{wx0}$ is straightforward. Recall that three individuals’ parents have no education. For DRSMD-Lasso with $(Z_1, Z_1X_1)$, when the parents’ education is 0, having a college degree increases the average hourly earnings by 299 log points holding other variables constant. When parents’ education is 10, having a college degree increases the average hourly earnings by 108 log points. This is not surprising. It means that before 1981, having a college degree or higher almost doubled the average hourly wage ($e^{1.08} - 1$). It is also rare that parents’ education is 0 and the child has a college degree. If that is the case, having a college degree helps find a job with higher payments significantly. When we compare the overall effects of heterogeneous treatment effect with parents’ education being the mean with the homogeneous treatment effect in Appendix B, we find that they are very close, which means that the new method generates reliable results.

5.2 Oregon Health Insurance Experiment

In early 2008, Oregon used a lottery to select low-income uninsured adults to expand Medicaid enrollment (health coverage). In this process, low-income uninsured adults first register for the waiting list. Then, from the waiting list, the names will be drawn by a lottery. The draw (or the lottery) was random. The randomness provided by the lottery allows researchers to analyze the effects of health insurance on medical, financial, and labour market outcomes. This experiment is a well-known Randomized Control Trial and is studied in many articles, for instance, Baicker et al. (2014) and Finkelstein et al. (2012).
Table 4: Heterogeneous Treatment Effects of college education on the log wage

Note: *** Significant at 1%, ** at 5%, * at 10%. $\theta_{w0}$ is the parameter in front of the treatment and $\theta_{wx0}$ is the parameter for College $\times$ Parents' education. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used. Every regression contains 3010 observations.

Table 5: Average Treatment Effects of college education on the log wage

Note: Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used. Every regression contains 3010 observations.
Many studies focus on the treatment effects of health insurance (Medicaid) on health care, employment, debt for health, and many other outcome variables. In those works, the treatment effects are assumed to be homogeneous. In Finkelstein et al. (2012), authors conduct their analysis assuming homogeneity and also use regression analysis to check whether there are heterogeneous treatment effects. They are unable to make precise inferences using traditional estimators like GMM or 2SLS. Studying the heterogeneous treatment effects using only one valid instrument variable needs a more advanced method.

The new estimator, the D-RSMD estimator, is generated to estimate the heterogeneous treatment effects using a conditional moment restriction directly. With an instrument variable that satisfies the conditional moment restriction, the estimator uses all of the information from the restriction to allow us to estimate more than one key parameter inside the model. The new estimator is designed for a partially linear model—that is, a model that contains a linear part and a nonparametric part.

In this empirical study, we include the interaction terms inside the model to account for heterogeneity. The new estimator is used to estimate both parameters within the heterogeneous treatment effects using the only valid instrument variable without generating new variables. The traditional estimation procedure, such as GMM or 2SLS, needs two moments to estimate both parameters within the heterogeneous treatment effects. Without the generated instrument variable, the traditional estimator will face an under-identification problem; that is, one of the key parameters will not be identified. The new estimator with the only valid instrument can identify both. With the new estimation method, we compare the results between the new and traditional estimation methods using or not using the generated instrument variable with reliable inference.

To illustrate the existence of heterogeneous treatment effects in some studies, we revisit the Oregon Health Insurance Experiment and estimate both parameters within the heterogeneous treatment effects to check the impacts of Medicaid from the Oregon Health Insurance Experiment. The public website for this Oregon Medicaid Health Experiment contains all of the related work, data sets, and data descriptions. The original data sets contain several files. In this paper, we use the data set derived from the original data sets. In the Oregon Health Insurance Experiment, using our estimator with only one valid instrument produces statistically significant results for heterogeneous treatment effects when the GMM estimator does not. Also, if the generated instrument variable is not reliable, using our estimator with only one valid instrument generates more reliable results, and the GMM estimator cannot.

The section contributes another empirical study for the new D-RSMD estimator and pro-

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8See https://www.nber.org/research/data/oregon-health-insurance-experiment-data
9I would like to thank A. Colin Cameron for the data set and for bringing the Oregon Health Insurance Experiment to my attention.
vides a clear illustration of the advantages of the new estimator in estimates and standard errors. It also contributes to the literature on the Oregon Health Insurance Experiment. With a new estimation procedure, we identify both parameters inside the heterogeneous treatment effects and verify that there are heterogeneous treatment effects, while the traditional estimation method does not.

The original data sets are available on the public website, and the detailed description is on the website and in Finkelstein et al. (2012). There are 18572 observations in the dataset we use. Using the dataset, we want to estimate the effects of Medicaid (health coverage) on various outcomes. Because the health conditions, employment, and other outcomes are closely related to the age of the individuals, we believe that age is one of the sources of heterogeneity. We generate the age group variable from the year of the birth variable. First, we calculate the age of each individual. The range of ages is from 21 to 64. Second, we split the original dataset into three or five subsets based on age to create an age group variable. After the split, there are around 5900 to 6900 observations in each subsample. Individuals in the same age group have more similarities. In Section 5.2.1, we report the results under three age groups (e.g., 21 to 35, 35 to 50, and 50 to 64).

Additionally, because age is a source of heterogeneity, we can include the age group variable inside the linear part of the partially linear model. The results for this regression framework are provided in Section 5.2.2.

There are also many control variables inside the dataset. We use similar controls to Baicker et al. (2014) and Finkelstein et al. (2012). These controls include household controls, lottery and survey wave indicators, and individual characteristic. We are interested in the heterogeneous treatment effects from the interaction terms generated by an indicator for household income above 50% of the federal poverty line in 2008, household income (hhincome), TANF (cash welfare assistance to low-income families), and cigarette smoking level (smoke). The dependent variables are the current employment indicator, constructed by three indicators on hours of employment (employment), the total out-of-pocket spending on medical care (Out of Pocket Cost), and whether the individual is currently owing money to a health care provider (Debt for Health). All of the dependent variables are from a mail survey starting in July 2009 and ending in March 2010. They are outcomes obtained approximately one year after the treatment. The out-of-pocket cost is in dollars, and hhincome is the household income as a percent of the federal poverty line.

The D-RSMD estimator is an estimation method that combines regularized machine learning methods, Smooth Minimum Distance (SMD) estimation, and Robinson transformation. With the Robinson transformation, the D-RSMD estimator provides the estimation results without assuming the function form for the nonparametric part of the partially linear model. For the linear part of the model, we introduce an interaction term between the treatment and

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10See Section B of the Appendix for tables with five age groups.
and a covariate to account for the heterogeneity of the treatment. With the SMD estimation method, the D-RSMD estimator delivers the results of both parameters inside the heterogeneous treatment effects. The parameters we are interested in are the one in front of the treatment Medicaid and the one in front of the interaction term between the treatment and the control variable $X_1$.

In this model, $X_1$ represents the vector of control variables that are the sources of the heterogeneity. In Section 5.2.1, it is the indicator of the income level, and the estimation results are provided. Section 5.2.2 presents the D-RSMD estimation results when $X_1$ is a vector of control variables. The detailed information will be discussed in the later sections.

To create a better illustration, we compare the D-RSMD estimator and the traditional estimator, for instance, the GMM estimator, in terms of framework. The D-RSMD estimator is reliable in a nonparametric first stage and a partially linear second stage. The framework is in the following:

$$
Debt_i = \theta_{w0}Medicaid_i + \theta_{wz0}Medicaid_i \times X_{i1} + f_{0,1}(X_i) + \epsilon_i
$$

$$
Medicaid_i = I(f_{0,2}(X_i, Lottery_i) > v_i)
$$

For one of the traditional estimators, for example, GMM estimators, the framework for the GMM estimator in this section is linear first stage (for the treatment: Medicaid) and linear second stage (for the dependent variable: debt for health). The framework is shown as follows.

$$
Debt_i = \theta_{w0}Medicaid_i + \theta_{wz0}Medicaid_i \times X_{i1} + X_i'\beta_x + \epsilon_i
$$

$$
Medicaid_i = I(\alpha_z Lottery_i + X_i'\alpha_x > v_i)
$$

The GMM estimator is not the only estimator that we compare the D-RSMD estimator to. There are other estimators. For instance, the GMM-Lasso estimator combines the Robinson transformation, a regularized machine learning method, and the traditional GMM method. The GMM-Lasso estimator is reliable when the first stage is linear and the second stage is partially linear. The framework for this estimator is as follows.

$$
Debt_i = \theta_{w0}Medicaid_i + \theta_{wz0}Medicaid_i \times X_{i1} + f_{0,1}(X_i) + \epsilon_i
$$

$$
Medicaid_i = I(\alpha_z Lottery_i + X_i'\alpha_x > v_i)
$$

The expression of the D-RSMD is shown in Section 3. In this estimator, the regularized machine learning method we choose to use is still Lasso. Since the D-RSMD estimation method can utilize many kinds of machine learning methods when the convergence rate of these methods satisfy the assumption we provided in the Section 3 we add ”Lasso” to the name of the estimator to indicate we use the Lasso method here.
5.2.1 Results

In this section, we discuss the possible heterogeneous treatment effects of Medicaid on debt for health when household income is the source of the heterogeneity, after splitting the dataset in 3 subsets by age (see Page 32 for specific details). This procedure allows for straightforward intuition and interpretation. The heterogeneous treatment effects inside the model is presented by the non-zero coefficient for the interaction term between the indicator of household income above 50% federal poverty line and Medicaid. The whole table that contains all of the results for the effects of the treatment (with covariate household income above 50% federal poverty line inside the interaction term) is contained at the end of the section. The additional results for the other dependent variables (or with other covariates inside the interaction term) are in the Appendix. Some necessary robustness checks are contained in Section 5.2.2.

We look at two sets of instruments: the lottery (only for D-RSMD) and the lottery and its interaction with $X_1$ (income greater than 50% of the poverty line). Because D-RSMD is the only estimator that uses the lottery only to estimate both parameters, the D-RSMD results will contain two parts. The first part of the results uses the lottery only, and the second part uses the lottery and its interaction term with $X_1$. In this way, we can compare the results of the two different instrument sets for the D-RSMD estimation procedure. The GMM and GMM-Lasso estimators can only use the lottery and its interaction term as instruments, which suggests that we can compare the results from D-RSMD and GMM estimators with the lottery and its interaction term as instruments.

Age, in this example, is also considered a source of the heterogeneity. In this section, we split our sample by age into three age groups. In Section 5.2.2 we run the regressions for the whole dataset for D-RSMD and GMM estimators, with the age variable agegroup serving as a categorical variable with three values to mimic the three age groups. In this way, we can compare the results in Section 5.2.1 and Section 5.2.2 to check the properties of estimators with more than one interaction term inside.

The following table (Table 6) contains the results for individuals between 36 and 50 years old. In this age group, the total number of observations is 6693. Please keep in mind that $X_1$ in this table of results stands for "Income above the 50% Federal Poverty Line." Table 6 is a subset of Table 9. Notice that DRSMD-Lasso is the D-RSMD estimator. We find heterogeneous treatment effects for individuals between 36 and 50. Estimates obtained by D-RSMD (with the lottery as an instrument or the lottery and its interaction with $X_1$ as instruments) are very similar. They are also very close to the GMM estimator (with the lottery and its interaction with $X_1$ as instruments). It suggests that the interaction

\[^{11}\text{We also consider the full sample (not split by age) and consider possible heterogeneous treatment effects based on income and age: see Page 33 or Section 5.2.2.}\]
between the lottery and $X_1$ is strong and reliable. One important difference between D-RSMD and GMM is that heterogeneous effects are found to be statistically significant only with D-RSMD estimation procedure. In Table 6, the age group reports statistically significant results for both $\theta_{w0}$ and $\theta_{wx0}$ when we use our new estimator with the valid lottery instrument. It suggests the effect of Medicaid on debt for health depends on the income level. Hence, there is heterogeneity.

The interpretation of the estimates is that for people with an income 50% below federal poverty line, Medicaid enrollment decreases their probability of owning money to health providers by 23.2 log points on average, ceteris paribus. For people with an income above 50% federal poverty line, Medicaid enrollment reduces their probability of owning money, but not by that much. This is reasonable because enrollment in Medicaid is not that critical in reducing the debt for individuals with higher incomes compared with individuals with low incomes.

Comparing the results between the two instrument sets (the lottery as an instrument set and the lottery and its interaction with $X_1$ as the other instrument set), we find that the estimates are close for both parameters, but the standard errors for the $\theta_{wx0}$ using the new method are substantially lower, suggesting that using one valid instrument works better. Table 3 in the Simulation Section also shows that the standard errors of the DRSMD-Lasso estimator for $\theta_{wx0}$ are the lowest among all of the estimators.

Table 7 has the results for individuals aged 21 to 35. In this age group, the number of observations is 5962. The covariate remains $X_1$, indicating whether income is greater than 50% of the Federal Poverty Line.

For the age group 21–35, all estimators generate similar results using the second instrument set, that is, the lottery and its interaction term with the indicator for household income. The coefficients for $\theta_{w0}$ are not statistically significant, and the ones for $\theta_{wx0}$ are statistically significant. It suggests that when households’ incomes are above 50% federal poverty line, individuals with Medicaid will be less likely to own money to the health providers. It also suggests that Medicaid helps people with higher income levels more than it helps people with lower incomes. Using only the lottery as the instrument, our new procedure generates

| Debt for Health | Estimator for $\theta_{w0}$ | Estimator for $\theta_{wx0}$ |
|-----------------|-----------------------------|-----------------------------|
|                 | Lottery (Lottery, Lottery $\times X_{11}$) | Lottery (Lottery, Lottery $\times X_{11}$) |
| GMM             | -0.213*** (0.041) | 0.108 (0.085) |
| DRSMD-Lasso     | -0.232*** (0.062) | 0.091*** (0.031) |

Table 6: results for Individuals Aged 36 to 50

Note: *** Significant at 1%, ** at 5%, * at 10%.
the opposite results. The interpretation is that for people with lower incomes, Medicaid enrollment decreases their probability of owing money to health providers by 19.9 log points on average, holding other variables constant. For people with higher incomes, the effect of Medicaid decreases.

Based on the D-RSMD results using only the lottery, we do not find support in the data to say that there are heterogeneous treatment effects for individuals between 21 and 35 years old. Furthermore, we discover that the results for individuals aged 21–35 differ significantly between the two instrument sets. It suggests that the generated interaction $Lottery \times X_1$ is invalid.

Next, we re-estimate a homogeneous model. In a homogeneous model, because we only want to estimate the homogeneous treatment effects, the interaction term is not included in the regression model. The traditional estimation method only needs one instrument to estimate one parameter. When we look at the homogeneous treatment effects using both instrument sets in Table 8, all estimators generate similar results as the DRSMD-Lasso using only the lottery as the instrument in the heterogeneous treatment effects panel. It also suggests that using a valid instrument to estimate both parameters provides a more reliable outcome. DRSMD-Lasso is reliable under both heterogeneous and homogeneous conditions. Since it is unclear in practice whether the model is homogeneous or not, DRSMD-Lasso appears to be extremely valuable.

Table 7: Results for Individuals Aged 21 to 35

| Estimator for $\theta_{w0}$ | Lottery (Lottery, Lottery$\times X_1$) | Lottery (Lottery, Lottery$\times X_1$) |
|-----------------------------|---------------------------------------|---------------------------------------|
| GMM                         | -0.069                                | -0.204**                              |
|                             | (0.055)                               | (0.096)                               |
| DRSMD-Lasso                 | -0.199***                             | -0.053                                |
|                             | (0.071)                               | (0.065)                               |

Table 8: Homogeneous Treatment Effects of Medicaid (Age: 21 - 35)

| Estimator for $\theta_{w0}$ | GMM       | DRSMD-Lasso |
|-----------------------------|-----------|-------------|
| Lottery                     | -0.161*** | -0.172***   |
|                             | (0.049)   | (0.063)     |

Note: *** Significant at 1%, ** at 5%, * at 10%.
### Panel A: Heterogeneous treatment effects

| Debt for Health | Age: 21 - 35 | Age: 36 - 50 | Age: 51 - 64 |
|-----------------|--------------|--------------|--------------|
| **Estimator for $\theta_{w0}$** | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ |
| GMM             | -0.069       | -0.213***    | -0.193***    |
|                 | (0.055)      | (0.041)      | (0.046)      |
| GMM-Lasso       | -0.091       | -0.259***    | -0.245***    |
|                 | (0.093)      | (0.078)      | (0.090)      |
| DRSMD-Lasso     | -0.199***    | -0.053       | -0.232***    |
|                 | (0.071)      | (0.062)      | (0.050)      |
| **Estimator for $\theta_{w0}$** | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ |
| GMM             | -0.204**     | 0.108        | 0.069        |
|                 | (0.096)      | (0.085)      | (0.094)      |
| GMM-Lasso       | -0.184       | 0.008        | -0.081       |
|                 | (0.131)      | (0.128)      | (0.153)      |
| DRSMD-Lasso     | 0.058        | -0.252**     | 0.091***     |
|                 | (0.036)      | (0.105)      | (0.031)      |

### Panel B: Homogeneous treatment effects

| **Estimator for $\theta_{w0}$** | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ |
| GMM             | -0.161***    | -0.130***    | -0.170***   | -0.180***    | -0.164***   | -0.177***    |
|                 | (0.049)      | (0.046)      | (0.040)      | (0.037)      | (0.044)      | (0.041)      |
| GMM-Lasso       | -0.180*      | -0.113       | -0.256***   | -0.260***    | -0.279**     | -0.244***    |
|                 | (0.104)      | (0.092)      | (0.097)      | (0.078)      | (0.112)      | (0.089)      |
| DRSMD-Lasso     | -0.172***    | -0.187***    | -0.198***   | -0.198***    | -0.150**     | -0.145**     |
|                 | (0.063)      | (0.066)      | (0.056)      | (0.057)      | (0.063)      | (0.064)      |

| $N$             | 5962         | 6693         | 5917         |

Table 9: Heterogeneous Treatment Effects of Medicaid on Debt for Health

Note. *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used and the age groups. The interaction term is Medicaid $\times$ Above 50% Federal Poverty Line.

### 5.2.2 Robustness Check

In this section, we look into the two different kinds of variations of the previous estimation. In Section 5.2.1 the covariate inside the interaction term is the indicator of whether the household income is higher than 50% Federal Poverty Line. To provide more information about these regression results in Section 5.2.1, we need to conduct several different robustness checks. In the first subsection, we check the estimation results when the covariate is a dummy. The dummy has a value of 1 when the household income is higher than 100% or 150% Federal Poverty Line. In the second subsection, we will not split the dataset into three groups, and we will include the age group variable and its interaction term with the treatment in the model directly. In the last subsection, we will briefly discuss the other robustness checks.
### Average Treatment Effects

| Debt for Health | Age: 21 - 35 | Age: 36 - 50 | Age: 51 - 64 |
|----------------|-------------|-------------|-------------|
| Estimator for LATE | $Z_1$ ($Z_1 X_1$) | $Z_1$ ($Z_1 X_1$) | $Z_1$ ($Z_1 X_1$) |
| GMM | -0.190*** | -0.150*** | -0.149*** |
| | (0.055) | (0.048) | (0.055) |
| GMM-Lasso | -0.201* | -0.255** | -0.296** |
| | (0.112) | (0.110) | (0.133) |
| DRSMD-Lasso | -0.164*** | -0.202*** | -0.179*** | -0.181*** | -0.135** | -0.129* |
| | (0.061) | (0.069) | (0.054) | (0.067) | (0.060) | (0.075) |
| N | 5962 | 6693 | 5917 |

Table 10: Heterogeneous Treatment Effects of Medicaid on Debt for Health

Note: *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used and the age groups. The interaction term is Medicaid $\times$ Above 50% Federal Poverty Line. The expression for LATE is $\theta_{w0} + \theta_{wx0}E(X)$.

- **Estimation Results with Other Covariates**

The results are included in Table 11 when $X_1$ is an indicator for "Income Above 100% Federal Poverty Line". Comparing Table 11 with Table 9, we find that the results are quite similar. For individuals between 36 and 50 years old, there are heterogeneous treatment effects. The estimates show that they benefit more from Medicaid health coverage when they have lower incomes. We do not find support in the data to say that there are heterogeneous treatment effects for individuals between 21 and 35 years old, and the DRSMD-Lasso estimator using only one valid instrument variable generates more reliable results.

| Heterogeneous Treatment Effects of Medicaid on Debt for Health | Age: 21 - 35 | Age: 36 - 50 |
|----------------|-------------|-------------|
| Estimator for $\theta_{w0}$ | Lottery ($\text{Lottery} \times X_{i1}$) | Lottery ($\text{Lottery} \times X_{i1}$) |
| GMM | -0.123** | -0.179*** |
| | (0.048) | (0.038) |
| DRSMD-Lasso | -0.178*** | -0.123*** | -0.202*** | -0.200*** |
| | (0.063) | (0.061) | (0.057) | (0.049) |
| Estimator for $\theta_{w0}$ | Lottery ($\text{Lottery} \times X_{i1}$) | Lottery ($\text{Lottery} \times X_{i1}$) |
| GMM | -0.200 | 0.072 |
| | (0.148) | (0.159) |
| DRSMD-Lasso | 0.023 | -0.212 | 0.102*** | 0.032 |
| | (0.033) | (0.154) | (0.038) | (0.192) |
| N | 5962 | 6693 |

Table 11: Income above 100% Federal Poverty Line

Note: *** Significant at 1%, ** at 5%, * at 10%.
Table 12 reports the outcomes when $X_1$ is an indicator for “Income Above 150% Federal Poverty Line”. Results in Table 12 and 11 are similar in estimates but different in standard errors. In the table, estimation results from using a dummy for income above 150% Federal Poverty Line show that the treatment effects of Medicaid on debt for people between 36 and 50 are not supported by the data to be heterogeneous, because the estimate for the parameter in front of the interaction term is not statistically significant at the 5% significance level.

The effects of Medicaid on debt for people between 21 and 35 are heterogeneous because the estimate is statistically significant. The difference in the results using two instrument sets for the DRSMD-Lasso estimators suggests that using only the lottery variable as an instrument generates more reliable results. The difference between Table 12 and 11 shows that the covariate inside the interaction is important for the estimation results. It can be explained by the fact that there are only 808 individuals with incomes above 150% Federal Poverty Line for people between 36 and 50, and 794 individuals for people between 21 and 35 in this data set.

| Heterogeneous Treatment Effects of Medicaid on Debt for Health | Age: 21 - 35 | Age: 36 - 50 |
|---------------------------------------------------------------|-------------|-------------|
| **Estimator for $\theta_w$**                                 | Lottery     | Lottery     |
| **GMM**                                                      | -0.145***   | -0.179***   |
|                                                             | (0.048)     | (0.038)     |
| **DRSMD-Lasso**                                              | -0.181***   | -0.149**    |
|                                                             | (0.062)     | (0.062)     |
|                                                             | -0.207***   | -0.205***   |
|                                                             | (0.056)     | (0.053)     |

| **Estimator for $\theta_{w0}$**                              | Lottery     | Lottery     |
| GMM                                                          | -0.204      | 0.195       |
|                                                             | (0.245)     | (0.340)     |
| **DRSMD-Lasso**                                              | 0.095*      | -0.254      |
|                                                             | (0.055)     | (0.292)     |
|                                                             | 0.067       | 0.030       |
|                                                             | (0.061)     | (0.453)     |

| **$N$**                                                      | 5962        | 6693        |

Table 12: Income above 150% Federal Poverty Line

Note: *** Significant at 1%, ** at 5%, * at 10%.

- **Estimation Results Using the Whole Data Set**

This subsection provides the estimation results with an age group variable included in the model, instead of splitting the data set by the age group variable. The difference between the framework in Section 5.2.1 and the framework in this subsection lies in the covariate $X_1$. In this section, $X_1$ has more than one variable inside. In Section 5.2.1, age is also a source of heterogeneity, so the data set is split into 3 groups based on the age of the individuals in the data. In this subsection, through an interaction term, the heterogeneity from age is included in the framework. For the D-RSMD estimation method, including more than two interactions will still work.
The framework in this section also means that we move the $X_1$ from the nonparametric part of the model to the linear part. Hence, there are three kinds of coefficients in the linear part of the model. The first type of coefficient is the coefficient in front of the treatment. The second type of coefficients is the parameters in front of $X_1$, such as the parameters measuring the effects of age and income. And the third type of parameters is the parameters in front of the interaction terms. The first and third types of parameters are our key parameters since we want to measure the heterogeneous treatment effects of the treatment.

The lottery is still the instrument in this framework. For the traditional method, the GMM estimators, it uses the instrument set, including lottery, income, age, the interaction term between lottery and income, and the interaction term between lottery and age. That is, to estimate five parameters, the GMM estimators use the instrument set, which includes five instruments, for instance, exogenous variables, lottery, and generated instruments.

The framework for DRSMD-Lasso estimators is summarised in the following equation.

$$
Debt_i = \theta_{w0}Medicaid_i + \theta_{wx0}Medicaid_i \times income_i + \theta_{x10}income_i \\
+ \theta_{x20}agegroup_i + \theta_{x30}Medicaid_i \times agegroup_i + f_{0,1}(X_i) + \epsilon_i
$$

$$
Medicaid_i = I(f_{0,2}(X_i, Lottery_i) > v_i)
$$

The framework for GMM estimators includes a linear first stage within the indicator function, as well as a linear second stage for the dependent variable.

$$
Debt_i = \theta_{w0}Medicaid_i + \theta_{wx0}Medicaid_i \times income_i + \theta_{x10}income_i \\
+ \theta_{x20}agegroup_i + \theta_{x30}Medicaid_i \times agegroup_i + X'_i\beta_x + \epsilon_i
$$

$$
Medicaid_i = I(\alpha_x Lottery_i + X'_i\alpha_x > v_i)
$$

The model for GMM-Lasso estimators includes a linear first stage for the indicator function and a partially-linear second stage for the dependent variable.

$$
Debt_i = \theta_{w0}Medicaid_i + \theta_{wx0}Medicaid_i \times income_i + \theta_{x10}income_i \\
+ \theta_{x20}agegroup_i + \theta_{x30}Medicaid_i \times agegroup_i + f_{0,1}(X_i) + \epsilon_i
$$

$$
Medicaid_i = I(\alpha_x Lottery_i + X'_i\alpha_x > v_i)
$$
In Table 13, we show the results for three estimators from the regressions when we extract income outside the nonparametric part of the model. For DRSMD-Lasso estimator, the estimates for Medicaid, the interaction between Medicaid and income are statistically significant at 5% significance level when using only one instrument, and the estimates for income and the interaction term between Medicaid and age are statistically significant at 15% significance level when using only one instrument. These results allow us to draw similar conclusions as in Section 5.2.1. That is, using only the valid instrument, the model shows that the treatment of Medicaid is heterogeneous, and the heterogeneity comes from age and income. After analyzing the outcomes from the GMM estimator or the GMM-Lasso using the instrument set \((Z_1, Z_1X_1)\), we would incorrectly conclude that there is no heterogeneity in the treatment effects. This shows that the generated instruments are problematic.

| Variables          | Debt for Health | GMM   | GMM-Lasso | DRSMD-Lasso |
|--------------------|-----------------|-------|-----------|-------------|
|                    | \((Z_1, Z_1X_1)\) | \((Z_1, Z_1X_1)\) | \((Z_1, Z_1X_1)\) | \((Z_1, Z_1X_1)\) |
| Medicaid           | -0.148***       | -0.206* | -0.187*** | -0.178***   |
|                    | (0.044)         | (0.115) | (0.037)   | (0.046)     |
| Medicaid*income    | 0.003           | -0.045  | 0.074***  | -0.018      |
|                    | (0.053)         | (0.068) | (0.020)   | (0.056)     |
| income             | 0.003           | 0.014   | -0.025    | 0.005       |
|                    | (0.020)         | (0.062) | (0.016)   | (0.025)     |
| agegroup           | -0.007          | -0.010  | -0.006    | -0.007      |
|                    | (0.010)         | (0.021) | (0.005)   | (0.009)     |
| Medicaid*agegroup  | -0.019          | -0.016  | -0.018    | -0.016      |
|                    | (0.032)         | (0.054) | (0.011)   | (0.028)     |

Table 13: Heterogeneous Treatment Effects for Robustness Check (5 parameters)

Note: *** Significant at 1%, ** at 5%, * at 10%, , at 15%.

- **Other Robustness Checks**

For other robustness check, we report the results when the covariate \(X_1\) is TANF in the appendix. TANF variable is known as a variable with little variation. For instance, there are only 2% of individuals on TANF. We want to see whether this will affect our results. Table B5 reports the case where \(y_i\) is still Debt for Health and there are five age groups. The DRSMD-Lasso with \(Z_1\) estimate for \(\theta_{w0}\) is -0.215 and for \(\theta_{wX0}\) is 0.307. Both of them are statistically significant at 5% level. It suggests that for an individual between 21 and 29 years old, TANF will decrease the negative effect of Medicaid on the probability of owning money. We also see that the new method using the valid instrument \(Z_1\) still generates reliable results with the lowest standard error.
Table B1 contains the results after splitting the sample into 5 age groups. Table S3 and Table S14 report the results for the effects of treatment on Employment and Out of Pocket Cost correspondingly. Both of the two tables show that there are heterogeneous treatment effects for young individuals (between 21 and 29 years old) when $X_1$ is a dummy for “Income Above 50% Federal Poverty Line”.
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A Tables for the Returns of Education

| Homogeneous treatment effects | $Z_1$ | $Z_2$ | $(Z_1, Z_1 X_1)$ | $(Z_1, Z_2)$ |
|------------------------------|-----|-----|-----------------|--------------|
| **Estimator**                |     |     |                 |              |
| GMM                          | 0.138** | 0.223** | 0.138** | 0.144*** |
|                             | (0.055) | (0.092) | (0.055) | (0.055) |
| GMM-Lasso                   | 0.142 | 0.244 | 0.156 | 0.064 |
|                             | (0.108) | (0.207) | (0.111) | (0.082) |
| DRSMD-Lasso                 | 0.187* | 0.574 | 0.128* | 0.203** |
|                             | (0.104) | (0.728) | (0.078) | (0.098) |
| DRSMD-2SOLS                | 0.214 | 0.738 | 0.137* | 0.234** |
|                             | (0.131) | (1.226) | (0.075) | (0.121) |

Table A1: Homogeneous Treatment Effects of years of education on the log wage

Note: *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used. Every regression contains 3010 observations.

| Homogeneous treatment effects | $Z_1$ | $Z_2$ | $(Z_1, Z_1 X_1)$ | $(Z_1, Z_2)$ |
|------------------------------|-----|-----|-----------------|--------------|
| **Estimator**                |     |     |                 |              |
| GMM                          | 0.788** | 1.025** | 0.656** | 0.867** |
|                             | (0.349) | (0.447) | (0.309) | (0.349) |
| GMM-Lasso                   | 0.793 | 1.085 | 1.017 | 0.648 |
|                             | (0.676) | (0.967) | (0.715) | (0.609) |
| DRSMD-Lasso                 | 1.757 | 2.474 | 0.914 | 1.394* |
|                             | (1.457) | (2.646) | (0.620) | (0.820) |
| DRSMD-2SOLS                | 2.194 | 3.677 | 0.985 | 1.642 |
|                             | (2.269) | (5.765) | (0.728) | (1.084) |

Table A2: Homogeneous Treatment Effects of education on the log wage

Note: *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used. Every regression contains 3010 observations.
Table A3: Heterogeneous Treatment Effects of years of education on the log wage

Note: *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used. Every regression contains 3010 observations.

Table A4: Heterogeneous Treatment Effects of years of education on the log wage

Note: Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used. Every regression contains 3010 observations.
B Results of Oregon Health Insurance Experiment

Panel A: Heterogeneous treatment effects

| Debt for Health | Age: 21 - 29 | Age: 30 - 38 | Age: 39 - 47 | Age: 48 - 56 | Age: 57 - 64 |
|-----------------|--------------|--------------|--------------|--------------|--------------|
| Estimator for $\theta_{w0}$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ |
| GMM             | -0.088       | -0.122       | -0.200***    | -0.226***    | -0.020       |
|                 | (0.076)      | (0.077)      | (0.062)      | (0.053)      | (0.116)      |
| GMM-Lasso       | -0.101       | -0.121       | -0.205**     | -0.318***    | -0.251       |
|                 | (0.141)      | (0.122)      | (0.088)      | (0.095)      | (0.159)      |
| DRSMD-Lasso     | -0.203***    | -0.108       | -0.177**     | -0.146**     | -0.241***    |
|                 | (0.075)      | (0.252)      | (0.080)      | (0.071)      | (0.063)      |
|                 |               | (0.148)      |               | (0.078)      | (0.0691)     |
| Estimator for $\theta_{w20}$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ |
| GMM             | -0.002*      | -0.001       | 0.002        | 0.001        | -0.003       |
|                 | (0.001)      | (0.001)      | (0.001)      | (0.001)      | (0.003)      |
| GMM-Lasso       | -0.002       | -0.001       | 0.000        | -0.001       | -0.001       |
|                 | (0.001)      | (0.001)      | (0.001)      | (0.001)      | (0.004)      |
| DRSMD-Lasso     | 0.000        | -0.007       | 0.001**      | -0.004*      | 0.000        |
|                 | (0.000)      | (0.006)      | (0.000)      | (0.002)      | (0.000)      |
|                 |               | (0.002)      |               | (0.002)      | (0.002)      |
|                 |               |               |               | (0.000)      | (0.0034)     |
|                 |               |               |               | (0.000)      | (0.005**     |
|                 |               |               |               |               |               |
| $N$             | 3504          | 3596          | 3941          | 4599         | 2932         |

Table B1: Heterogeneous Treatment Effects of Medicaid on Debt for Health

Note: *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used and the age groups. The interaction term is Medicaid $\times$ hhincome.
| Estimator for LATE     | Age: 21 - 29 | Age: 30 - 38 | Age: 39 - 47 | Age: 48 - 56 | Age: 57 - 64 |
|-----------------------|--------------|--------------|--------------|--------------|--------------|
| GMM                   | -0.216***    | -0.196**     | -0.078       | -0.166**     | -0.251*      |
|                       | (0.072)      | (0.079)      | (0.069)      | (0.066)      | (0.144)      |
| GMM-Lasso             | -0.248**     | -0.195       | -0.223*      | -0.362***    | -0.318       |
|                       | (0.149)      | (0.145)      | (0.124)      | (0.133)      | (0.281)      |
| DRSMD-Lasso           | -0.211**     | -0.613**     | -0.124       | -0.427**     | -0.170***    |
|                       | (0.090)      | (0.276)      | (0.084)      | (0.174)      | (0.062)      |
|                       |              |              |              |              | (1.886)      |
|                       |              |              |              |              | (0.103)      |
|                       |              |              |              |              | (0.163)      |
|                       | 3504         | 3596         | 3941         | 4599         | 2932         |

Table B2: Heterogeneous Treatment Effects of Medicaid on Debt for Health

Note: *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used and the age groups. The interaction term is Medicaid × hhincome. The expression for LATE is $\theta_{w0} + \theta_{wx0}E(X)$. 
Panel A: Heterogeneous treatment effects

| Age: 21 - 29 | Age: 30 - 38 | Age: 39 - 47 | Age: 48 - 56 | Age: 57 - 64 |
|-------------|-------------|-------------|-------------|-------------|
| **Estimator for $\theta_{w0}$** | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ |
| GMM | -0.048 | -0.125* | -0.210*** | -0.190*** | -0.244*** |
| | (0.072) | (0.069) | (0.053) | (0.046) | (0.077) |
| GMM-Lasso | -0.130 | -0.087 | -0.271*** | -0.274*** | -0.489*** |
| | (0.121) | (0.122) | (0.101) | (0.091) | (0.156) |
| DRSMD-Lasso | -0.251*** | -0.064 | -0.156*** | -0.085 | -0.155** | -0.218*** | -0.269*** | -0.215*** | -0.141 | -0.237*** |
| | (0.098) | (0.086) | (0.086) | (0.076) | (0.066) | (0.071) | (0.055) | (0.112) | (0.102) |

| **Estimator for $\theta_{w0}$** | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ |
| GMM | -0.330** | -0.109 | 0.223** | 0.004 | 0.192 |
| | (0.135) | (0.124) | (0.104) | (0.106) | (0.137) |
| GMM-Lasso | -0.312 | -0.112 | 0.070 | -0.158 | 0.327 |
| | (0.196) | (0.150) | (0.155) | (0.169) | (0.204) |
| DRSMD-Lasso | 0.059 | -0.409*** | 0.025 | -0.119 | 0.039 | 0.196* | 0.137*** | -0.013 | 0.027 | 0.237 |
| | (0.045) | (0.158) | (0.039) | (0.139) | (0.039) | (0.118) | (0.039) | (0.116) | (0.047) | (0.164) |

Panel B: Homogeneous treatment effects

| **Estimator for $\theta_{w0}$** | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ |
| GMM | -0.188*** | -0.138*** | -0.175*** | -0.156*** | -0.120*** | -0.154*** | -0.191*** | -0.190*** | -0.151*** | -0.184*** |
| | (0.065) | (0.062) | (0.063) | (0.059) | (0.049) | (0.046) | (0.046) | (0.042) | (0.068) | (0.064) |
| GMM-Lasso | -0.265* | -0.148 | -0.150 | -0.096 | -0.242** | -0.271*** | -0.334*** | -0.269*** | -0.315* | -0.481*** |
| | (0.140) | (0.120) | (0.142) | (0.121) | (0.120) | (0.101) | (0.114) | (0.090) | (0.183) | (0.156) |
| DRSMD-Lasso | -0.228*** | -0.246*** | -0.144* | -0.152* | -0.140** | -0.138** | -0.219*** | -0.219*** | -0.128 | -0.104 |
| | (0.087) | (0.090) | (0.085) | (0.089) | (0.070) | (0.071) | (0.063) | (0.063) | (0.107) | (0.112) |

$N$ | 3504 | 3596 | 3941 | 4599 | 2932

Table B3: Heterogeneous Treatment Effects of Medicaid on Debt for Health

Note: *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used and the age groups. The interaction term is Medicaid × Above 50% Federal Poverty Line.
### Average Treatment Effects

| Debt for Health | Age: 21 - 29 | Age: 30 - 38 | Age: 39 - 47 | Age: 48 - 56 | Age: 57 - 64 |
|----------------|--------------|--------------|--------------|--------------|--------------|
| Estimator for LATE | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ |
| GMM | -0.237*** | -0.193*** | -0.082 | -0.192*** | -0.116 |
|      | (0.076) | (0.073) | (0.058) | (0.060) | (0.080) |
| GMM-Lasso | -0.308* | -0.158 | -0.231* | -0.365*** | -0.271 |
|      | (0.160) | (0.148) | (0.134) | (0.138) | (0.200) |
| DRSMD-Lasso | -0.217*** | -0.298*** | -0.140* | -0.160* | -0.132* | -0.106 | -0.189*** | -0.222*** | -0.123 | -0.078 |
|      | (0.083) | (0.102) | (0.085) | (0.094) | (0.068) | (0.082) | (0.059) | (0.079) | (0.107) | (0.123) |

Table B4: Heterogeneous Treatment Effects of Medicaid on Debt for Health

Note: *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used and the age groups. The interaction term is Medicaid \times Above 50% Federal Poverty Line. The expression for LATE is $\theta_{u0} + \theta_{ux0}E(X)$. 
### Panel A: Heterogeneous treatment effects

#### Debt for Health

| Estimator for $\theta_{x=0}$ | Age: 21 - 29 | Age: 30 - 38 | Age: 39 - 47 | Age: 48 - 56 | Age: 57 - 64 |
|-------------------------------|--------------|--------------|--------------|--------------|--------------|
| GMM                           | -0.193***    | -0.199***    | -0.113**     | -0.196***    | -0.157**     |
|                               | (0.065)      | (0.061)      | (0.049)      | (0.046)      | (0.068)      |
| GMM-Lasso                     | -0.270*      | -0.216       | -0.211*      | -0.351***    | -0.295       |
|                               | (0.147)      | (0.136)      | (0.110)      | (0.112)      | (0.182)      |
| DRSMD-Lasso                   | -0.215**     | -0.217**     | -0.137*      | -0.148**     | -0.135**     |
|                               | (0.087)      | (0.103)      | (0.082)      | (0.068)      | (0.068)      |

| Estimator for $\theta_{x=0}$ | Age: 21 - 29 | Age: 30 - 38 | Age: 39 - 47 | Age: 48 - 56 | Age: 57 - 64 |
|-------------------------------|--------------|--------------|--------------|--------------|--------------|
| GMM                           | 0.212        | 1.959        | -0.791       | 2.059        | 3.074        |
|                               | (0.900)      | (1.353)      | (0.652)      | (2.040)      | (3.827)      |
| GMM-Lasso                     | 0.983        | 1.904        | -0.785       | 1.083        | 0.818        |
|                               | (1.099)      | (1.348)      | (0.956)      | (1.844)      | (1.164)      |
| DRSMD-Lasso                   | 0.307**      | 0.777        | 0.042        | -0.320***    | -1.448       |
|                               | (0.142)      | (4.239)      | (0.153)      | (19.627)     | (0.085)      |

| N                             | 3504         | 3596         | 3941         | 4599         | 2932         |

**Note:** *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used and the age groups. The interaction term is Medicaid $\times$ TANF.
Average Treatment Effects

| Estimator for LATE | Debt for Health | Age: 21 - 29 | Age: 30 - 38 | Age: 39 - 47 | Age: 48 - 56 | Age: 57 - 64 |
|-------------------|----------------|-------------|-------------|-------------|-------------|-------------|
|                   |                | Z₁          | Z₁          | Z₁          | Z₁          | Z₁          |
| GMM               |                | -0.184**    | -0.114      | -0.131***   | -0.177***   | -0.147**    |
|                   |                | (0.074)     | (0.083)     | (0.050)     | (0.050)     | (0.068)     |
| GMM-Lasso         |                | -0.230      | -0.133      | -0.228**    | -0.341***   | -0.293      |
|                   |                | (0.156)     | (0.151)     | (0.111)     | (0.115)     | (0.182)     |
| DRSMD-Lasso       |                | -0.203**    | -0.185      | -0.135      | -0.155**    | -0.194***   |
|                   |                | (0.087)     | (0.146)     | (0.083)     | (0.068)     | (0.061)     |
|
| N                 |                | 3504        | 3596        | 3941        | 4599        | 2932        |

Table B6: Heterogeneous Treatment Effects of Medicaid on Debt for Health

Note: *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used and the age groups. The interaction term is Medicaid × TANF. The expression for LATE is $\theta_{w0} + \theta_{wx0}E(X)$. 
Table B7: Heterogeneous Treatment Effects of Medicaid on Debt for Health

Note: *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used and the age groups. The interaction term is Medicaid × smoke.
### Average Treatment Effects

| Estimator for LATE | Debt for Health | Age: 21 - 29 | Age: 30 - 38 | Age: 39 - 47 | Age: 48 - 56 | Age: 57 - 64 |
|--------------------|----------------|--------------|--------------|--------------|--------------|--------------|
|                    | $Z_1$ | $(Z_1, Z_1 X_1)$ | $Z_1$ | $(Z_1, Z_1 X_1)$ | $Z_1$ | $(Z_1, Z_1 X_1)$ | $Z_1$ | $(Z_1, Z_1 X_1)$ |
| GMM                | -0.190*** | -0.175*** | -0.100** | -0.192*** | -0.141** |
|                    | (0.066) | (0.064) | (0.051) | (0.047) | (0.068) |
| GMM-Lasso          | -0.267*  | -0.185 | -0.211* | -0.363*** | -0.289 |
|                    | (0.155) | (0.138) | (0.114) | (0.115) | (0.184) |
| DRSMD-Lasso        | -0.210** | -0.220** | -0.135* | -0.136* | -0.146** | -0.123* | -0.197*** | -0.201*** | -0.138 | -0.123 |
|                    | (0.087) | (0.087) | (0.082) | (0.082) | (0.067) | (0.069) | (0.061) | (0.062) | (0.107) | (0.104) |
| $N$                | 3504 | 3596 | 3941 | 4599 | 2932 |

Table B8: Heterogeneous Treatment Effects of Medicaid on Debt for Health

Note: *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used and the age groups. The interaction term is Medicaid × smoke. The expression for LATE is $\theta_{w0} + \theta_{wx0}E(X)$. 
C Proofs of the Theoretical Results

C.1 Equivalence between the Objective Functions (2.5) and (2.6)

Proof. The objective function (2.5) can be written as

\[ M_\infty(\theta, g) = \int_{\mathbb{R}^q} E[\epsilon_j(\theta, g)e^{itZ_j}] E[\epsilon_l(\theta, g)e^{-itZ_l}] d\mu(t) \]

From Assumption 1(vi), for all \( j \neq l \), \( \text{Cov}(\epsilon_j(\theta, g)e^{itZ_j}, \epsilon_l(\theta, g)e^{-itZ_l}) = 0 \). Thus, for all \( j \neq l \), we have:

\[ M_\infty(\beta) = \int_{\mathbb{R}^q} E(\epsilon_j(\theta, g)e^{itZ_j}\epsilon_l(\theta, g)e^{-itZ_l}) d\mu(t) \]

Thus, the objective function becomes

\[ M_\infty(\beta) = E(\epsilon_j(\theta, g)\epsilon_l(\theta, g)\kappa_{j,l}) \]

where \( \kappa_{j,l} = k(Z_j - Z_l) = \int_{\mathbb{R}^q} e^{it(Z_j - Z_l)} d\mu(t) \). And \( k(u) \) is the inverse Fourier transform of \( d\mu(t) \) with \( u = Z_j - Z_l \). \( \square \)

C.2 Proof of Proposition 1

The orthogonal FOC in Equation (2.9) implies that

\[ E \left[ \left( \tilde{P}_j - \frac{g_0, \tilde{P}_m(X_l)}{g_0, \kappa_{m,l}(X_l)} \right) (\tilde{y}_l - \tilde{P}_l^\prime \theta) \kappa_{j,l} \right] = 0 \]

\[ E \left[ \kappa_{j,l} \left( \tilde{P}_j - \frac{g_0, \tilde{P}_m(X_l)}{g_0, \kappa_{m,l}(X_l)} \right) \tilde{y}_l - \kappa_{j,l} \left( \tilde{P}_j - \frac{g_0, \tilde{P}_m(X_l)}{g_0, \kappa_{m,l}(X_l)} \right) \tilde{P}_l^\prime \theta \right] = 0 \]

\[ \theta_0 = E \left[ \kappa_{j,l} \left( \tilde{P}_j - \frac{g_0, \tilde{P}_m(X_l)}{g_0, \kappa_{m,l}(X_l)} \right) \tilde{P}_l^\prime \right]^{-1} E \left[ \kappa_{j,l} \left( \tilde{P}_j - \frac{g_0, \tilde{P}_m(X_l)}{g_0, \kappa_{m,l}(X_l)} \right) \tilde{y}_l \right] \]

The proof for invertibility of \( E \left[ \kappa_{j,l} \left( \tilde{P}_j - \frac{g_0, \tilde{P}_m(X_l)}{g_0, \kappa_{m,l}(X_l)} \right) \tilde{P}_l^\prime \right] \) follows the same steps of proofs for identification in Antoine and Sun (2021).
C.3 Proof of Orthogonal Properties of Equation (2.7) and (2.9)

In this section, we check the orthogonal properties of the two FOCs.

\[ M_{\infty}(\theta, g) = -\frac{1}{2} E[\epsilon_j(\theta)\epsilon_l(\theta)\kappa_{j,l}] = -\frac{1}{2} E[(\tilde{y}_j - \tilde{P}_j^\theta)(\tilde{y}_l - \tilde{P}_l^\theta)\kappa_{j,l}] \]

where \( g \) stands for all nuisance parameters.

The FOC defined in Equation (2.7) is not orthogonal

The key parameter is \( \theta \), the true value is \( \theta_0 \). The nuisance parameters are \( g_P(X_i) \) and \( g_y(X_i) \). Their true values are \( E(P_i|X_i) = g_{0,P}(X_i) \) and \( E(y_i|X_i) = g_{0,y}(X_i) \).

The first order condition is written as follows:

\[ E[(P_j - g_P(X_j))(y_l - g_y(X_l)) - [P_l - g_P(X_l)]\theta]\kappa_{j,l} = 0 \]

If \( P_i = [W_i, f(W_i, X_i)] \) and there are only one variable in \( X_i \), \( \partial_\theta \varphi(D; \theta, g) \) becomes:

\[ \partial_\theta \varphi(D; \theta, g) = \left( (P_{1j} - g_{0_P}(X_j))[y_l - g_y(X_l)] - (P_{1l} - g_{0_P}(X_l))\theta_1 - (P_{2l} - g_{0_P}(X_l))\theta_2 \right)\kappa_{j,l} \]

\[ \left( (P_{2j} - g_{0_P}(X_j))[y_l - g_y(X_l)] - (P_{1l} - g_{0_P}(X_l))\theta_1 - (P_{2l} - g_{0_P}(X_l))\theta_2 \right)\kappa_{j,l} \]

Prove that \( \partial_\theta E[\partial_\theta \varphi(D; \theta_0, g_0)] \neq 0 \).

Proof. The first row of \( \partial_\theta \varphi(D; \theta_0, g_0 + r(g - g_0)) \) is defined as \( I \) where

\[ I = [P_{1j} - g_{0_P}(X_j)] - r(g_{P}(X_j) - g_{0_P}(X_j)) \]

\[ [y_l - g_y(X_l)] - r(g_y(X_l) - g_{0_y}(X_l)) \]

\[ - (P_{1l} - g_{0_P}(X_l))\theta_1 - (P_{2l} - g_{0_P}(X_l))\theta_2 \]

According to the definition for \( \partial_\theta E[\partial_\theta \varphi(D; \theta_0, g_0)] \neq 0 \) in Chernozhukov et al. (2018), to show that \( \partial_\theta E[\partial_\theta \varphi(D; \theta_0, g_0)] \neq 0 \) we need to show \( \partial_\theta E[I]\|_{r=0} \neq 0 \).

\[ \partial_\theta E[I]\|_{r=0} = -I_1 - I_2 + I_3 + I_4 \]

\[ I_1 = E [(g_{P}(X_j) - g_{0_P}(X_j))(y_l - g_y(X_l)) - (P_{1l} - g_{0_P}(X_l))\theta_{0,1} - (P_{2l} - g_{0_P}(X_l))\theta_{0,2})\kappa_{j,l}] \]

\[ = E [(g_{P}(X_j) - g_{0_P}(X_j))\epsilon_l\kappa_{j,l}] = 0 \]

\[ I_2 = E [(P_{1j} - g_{0_P}(X_j))(g_y(X_l) - g_{0_y}(X_l))\kappa_{j,l}] \neq 0 \]

\[ I_3 = E [(P_{1j} - g_{0_P}(X_j))(g_{P}(X_l) - g_{0_P}(X_l))\theta_1\kappa_{j,l}] \neq 0 \]

\[ I_4 = E [(P_{1j} - g_{0_P}(X_j))(g_{P}(X_l) - g_{0_P}(X_l))\theta_2\kappa_{j,l}] \neq 0 \]

Hence, \( \partial_\theta E[\partial_\theta \varphi(D; \theta_0, g_0)] \neq 0 \)
The FOC defined in Equation (2.9) is orthogonal

With \( E[(P_{1m} - g_0, P_1(X_m)\kappa_{m,l}|X_l)] = g_0, \tilde{P}_{1,j}(X_l), \ E[\kappa_{m,l}|X_l] = g_0, \kappa_{m,l}(X_l), \)

\[
\Psi(D; \theta, g) = \begin{bmatrix}
(P_{1j} - g_{P_1}(X_j) - \frac{g_{P_1,j}(X_j)}{g_{r,j,l}(X_l)}) [y_l - g_y(X_l) - (P_{1l} - g_{P_1}(X_l))\theta_1 - (P_{2l} - g_{P_2}(X_l))\theta_2] \kappa_{j,l} \\
(P_{2j} - g_{P_2}(X_j) - \frac{g_{P_2,j}(X_j)}{g_{r,j,l}(X_l)}) [y_l - g_y(X_l) - (P_{1l} - g_{P_1}(X_l))\theta_1 - (P_{2l} - g_{P_2}(X_l))\theta_2] \kappa_{j,l}
\end{bmatrix}
\]

Prove that \( \partial_g E[\Psi(D; \theta_0, g_0)] = 0. \)

**Proof.** The first row of \( \Psi(D; \theta_0, g_0 + r(g - g_0)) \) is \( I' \).

\[
I' = \left( P_{1j} - g_0, P_1(X_j) - r(g_{P_1}(X_j) - g_0, P_1(X_j)) \right) - \frac{g_0, P_1(X_j) + r [g_{P_1,j}(X_j) - g_0, P_1,j(X_j)]}{g_0, \kappa_{m,l}(X_l) + r [g_{r,j,l}(X_l) - g_0, \kappa_{m,l}(X_l)]}
\]

\[
[y_l - g_y(X_l) - r(g_y(X_l) - g_0, y(X_l))] - (P_{1l} - g_0, P_1(X_l) - r(g_{P_1}(X_l) - g_0, P_1(X_l))\theta_1 - (P_{2l} - g_{P_2}(X_l))\theta_2] \kappa_{j,l}
\]

\[
\partial_g E[I'|_{g=0}] = -I_1 - I_2' + I_3' + I_4' - I_5
\]

\[
I_1 = E [ (g_{P_1}(X_j) - g_0, P_1(X_j)) (y_l - g_y(X_l) - (P_{1l} - g_0, P_1(X_l))\theta_0, 1 - (P_{2l} - g_0, P_2(X_l))\theta_0, 2) \kappa_{j,l} ] = 0
\]

\[
I_2' = E \left[ (P_{1j} - g_0, P_1(X_j)) - \frac{E[(P_{1m} - g_0, P_1(X_m)\kappa_{m,l}|X_l)]}{E[\kappa_{m,l}|X_l]} (g_y(X_l) - g_0, y(X_l)) \right] \kappa_{j,l}|X_l
\]

\[
= E \left[ (P_{1j} - g_0, P_1(X_j)) \kappa_{j,l} - \frac{E[(P_{1m} - g_0, P_1(X_m)\kappa_{m,l}|X_l)]}{E[\kappa_{m,l}|X_l]} (g_y(X_l) - g_0, y(X_l)) \right] \kappa_{j,l}|X_l
\]

\[
= 0
\]

because \( E \left[ (P_{1j} - g_0, P_1(X_j)) \kappa_{j,l} - \frac{E[(P_{1m} - g_0, P_1(X_m)\kappa_{m,l}|X_l)]}{E[\kappa_{m,l}|X_l]} \kappa_{j,l}|X_l \right] = 0. \)

\[
I_3' = E \left[ (P_{1j} - g_0, P_1(X_j)) - \frac{E[(P_{1m} - g_0, P_1(X_m)\kappa_{m,l}|X_l)]}{E[\kappa_{m,l}|X_l]} (g_{P_1}(X_l) - g_0, P_1(X_l))\theta_1 \kappa_{j,l} \right] = 0
\]

\[
I_4' = E \left[ (P_{1j} - g_0, P_1(X_j)) - \frac{E[(P_{1m} - g_0, P_1(X_m)\kappa_{m,l}|X_l)]}{E[\kappa_{m,l}|X_l]} (g_{P_2}(X_l) - g_0, P_2(X_l))\theta_2 \kappa_{j,l} \right] = 0
\]

\[
I_5 = E \left[ \frac{g_{P_{1,j}}(X_l) - g_0, \tilde{P}_{1,j}(X_l)}{g_0, \kappa_{m,l}(X_l)^2} \frac{g_0, \kappa_{m,l}(X_l) (g_{\kappa_{j,l}}(X_l) - g_0, \kappa_{j,l}(X_l))}{g_0, \kappa_{m,l}(X_l)^2} \theta_1 \kappa_{j,l} \right] = 0
\]

Since \( E(\epsilon|X_l, Z_l) = 0. \)

Hence, \( \partial_g E[\Psi(D; \theta_0, g_0)] = 0. \)

\( \square \)
C.4 Proof of Proposition 2

Proof.

\[ \hat{\theta}_{n,o} = \left[ \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j} \kappa_{j,l} \left( \tilde{P}_j - \frac{g_0 P_m(X_l)}{g_0 \kappa_{m,l}(X_l)} \right) \tilde{P}_l \right]^{-1} \]

\[ = \theta_0 + \left[ \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j} \kappa_{j,l} \left( \tilde{P}_j - \frac{g_0 P_m(X_l)}{g_0 \kappa_{m,l}(X_l)} \right) \tilde{P}_l \right]^{-1} \]

Denote \( A_n = \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j} \kappa_{j,l} \left( \tilde{P}_j - \frac{g_0 P_m(X_l)}{g_0 \kappa_{m,l}(X_l)} \right) \tilde{P}_l \) and

\[ B_n = \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j} \kappa_{j,l} \left( \tilde{P}_j - \frac{g_0 P_m(X_l)}{g_0 \kappa_{m,l}(X_l)} \right) \epsilon_l. \]

We first show that \( A_n \) is a U-statistic and find its probability limit. Then we show that \( B_n \) is also a U-statistic and find its probability limit.

To show that \( A_n \) is a U-statistic, notice that:

\[ A_n = \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j} \kappa_{j,l} \left( \tilde{P}_j - \frac{g_0 P_m(X_l)}{g_0 \kappa_{m,l}(X_l)} \right) \tilde{P}_l \]

\[ = \frac{1}{2n(n-1)} \sum_{j<l}^{n} \left( \kappa_{j,l} \left( \tilde{P}_j - \frac{g_0 P_m(X_l)}{g_0 \kappa_{m,l}(X_l)} \right) \tilde{P}_j + \kappa_{l,j} \left( \tilde{P}_l - \frac{g_0 P_m(X_j)}{g_0 \kappa_{m,j}(X_j)} \right) \tilde{P}_j \right) \]

According to WLLN for U-statistics under Assumption 1.

\[ A_n \xrightarrow{p} A \quad \text{with} \quad A \equiv E \left[ \kappa_{j,l} \left( \tilde{P}_j - \frac{g_0 P_m(X_l)}{g_0 \kappa_{m,l}(X_l)} \right) \tilde{P}_l \right] \]

and under Assumption 1(iii) \( A \) is nonsingular.

To show that \( B_n \) is also a U-statistic, notice that:

\[ B_n = \frac{1}{n(n-1)} \sum_{j<l}^{n} \left( \kappa_{j,l} \left( \tilde{P}_j - \frac{g_0 P_m(X_l)}{g_0 \kappa_{m,l}(X_l)} \right) \epsilon_l + \kappa_{l,j} \left( \tilde{P}_l - \frac{g_0 P_m(X_j)}{g_0 \kappa_{m,j}(X_j)} \right) \epsilon_j \right) \]

Define \( h(\tilde{p}_1, e_1, z_1, x_1; \tilde{p}_2, e_2, z_2, x_2) = \kappa_{1,2}\tilde{p}_1 e_2 + \kappa_{2,1}\tilde{p}_2 e_1 - \kappa_{1,2}\frac{g_0 P_1(x_2)}{g_0 \kappa_{1,2}(x_2)} e_2 - \kappa_{2,1}\frac{g_0 P_2(x_1)}{g_0 \kappa_{2,1}(x_1)} e_1. \)

Since \( h \) is a symmetric function of observations 1 and 2, a U-statistic with kernel \( h \) is defined as

\[ B'_n = \frac{2}{n(n-1)} \sum_{j<l}^{n} h(\tilde{P}_j, \epsilon_j, Z_j, X_j; \tilde{P}_l, \epsilon_l, Z_l, X_l) \quad \text{and} \quad B_n = \frac{1}{2} B'_n \]
And we have:

\[
E(B'_n) = E(\kappa_{j,l} \tilde{P}_j \epsilon_l + \kappa_{l,j} \tilde{P}_l \epsilon_j) - \left( E \left( \kappa_{j,l} \frac{g_0 \tilde{F}_m(X_l)}{g_0, \kappa_{m,l}}(X_l) \epsilon_l \right) + E \left( \kappa_{l,j} \frac{g_0 \tilde{F}_m(X_j)}{g_0, \kappa_{m,l}}(X_j) \epsilon_j \right) \right)
\]

\[
= 2E(\kappa_{j,l} \tilde{P}_j \epsilon_l) - 2E \left( \kappa_{j,l} \frac{g_0 \tilde{F}_m(X_l)}{g_0, \kappa_{m,l}}(X_l) \epsilon_l \right)
\]

\[
= 2 \int_{\mathbb{R}^q} E[\tilde{P}_j \epsilon_l e^{it' (Z_j - Z_l)}] d\mu(t) - 2 \int_{\mathbb{R}^q} E \left[ \frac{g_0 \tilde{F}_m(X_l)}{g_0, \kappa_{m,l}}(X_l) \epsilon_l e^{it' (Z_j - Z_l)} \right] d\mu(t)
\]

\[
= 2 \int_{\mathbb{R}^q} E[\tilde{P}_j e^{it' Z_j} \epsilon_l e^{-it' Z_l}] d\mu(t) - 2 \int_{\mathbb{R}^q} E \left[ \frac{g_0 \tilde{F}_m(X_l)}{g_0, \kappa_{m,l}}(X_l) e^{it' Z_j} \epsilon_l e^{-it' Z_l} \right] d\mu(t)
\]

\[
= 2 \int_{\mathbb{R}^q} E[\tilde{P}_j e^{it' Z_j}] E[\epsilon_l e^{-it' Z_l}] d\mu(t) - 2 \int_{\mathbb{R}^q} E \left[ \frac{g_0 \tilde{F}_m(X_l)}{g_0, \kappa_{m,l}}(X_l) \epsilon_l e^{-it' Z_l} \right] d\mu(t)
\]

\[
= 0 \quad \text{since} \quad E[\epsilon_l e^{-it' Z_l}] = 0 \quad \text{and} \quad E[\epsilon_l | X_l, Z_l] = 0.
\]

Hence, \(E(B_n) = 0\). According to WLLN for U statistics, we have \(B_n \xrightarrow{P} 0\), and we conclude that \(\hat{\beta}_n\) is a consistent estimator of \(\beta_0\).

To derive the asymptotic normality, we first need to compute the asymptotic variance for the U-statistic \(B'_n\), which means that we need to find the variance for \(E(h(\tilde{P}_1, \epsilon_1, Z_1, X_1; \tilde{P}_2, \epsilon_2, Z_2, X_2) | \tilde{P}_1 = \tilde{p}_1, \epsilon_1 = \epsilon_1, Z_1 = z_1, X_1 = x_1)\).

Let \(h(\tilde{p}_1, \epsilon_1, z_1, x_1) \equiv E(h(\tilde{P}_1, \epsilon_1, Z_1, X_1; \tilde{P}_2, \epsilon_2, Z_2, X_2) | \tilde{P}_1 = \tilde{p}_1, \epsilon_1 = \epsilon_1, Z_1 = z_1, X_1 = x_1)\). We have:

\[
h(\tilde{p}_1, \epsilon_1, z_1, x_1; \tilde{p}_2, \epsilon_2, z_2, x_2) = \kappa_{1,2} \tilde{p}_1 \epsilon_2 + \kappa_{2,1} \tilde{p}_2 \epsilon_1 - \kappa_{1,2} \frac{g_0 \tilde{F}_m(x_2)}{g_0, \kappa_{1,2}}(x_2) \epsilon_2 - \kappa_{2,1} \frac{g_0 \tilde{F}_m(x_1)}{g_0, \kappa_{2,1}}(x_1) \epsilon_1
\]

and

\[
h_1(\tilde{p}_1, \epsilon_1, z_1, x_1) = E \left[ \int_{\mathbb{R}^q} e^{it' (z_1 - Z_2)} d\mu(t) \tilde{p}_1 \epsilon_2 + \int_{\mathbb{R}^q} e^{it' (Z_2 - z_1)} d\mu(t) \tilde{P}_2 \epsilon_1 \right]
\]

\[
- \int_{\mathbb{R}^q} e^{it' (z_1 - Z_2)} d\mu(t) \frac{g_0 \tilde{F}_m(X_2)}{g_0, \kappa_{1,2}}(X_2) \epsilon_2 - \int_{\mathbb{R}^q} e^{it' (Z_2 - z_1)} d\mu(t) \frac{g_0 \tilde{F}_m(X_1)}{g_0, \kappa_{2,1}}(X_1) \epsilon_1
\]

\[
= E \left[ \int_{\mathbb{R}^q} e^{it' (z_1 - Z_2)} d\mu(t) \tilde{p}_1 \epsilon_2 + \int_{\mathbb{R}^q} e^{it' (Z_2 - z_1)} d\mu(t) \tilde{P}_2 \epsilon_1 \right]
\]

\[
- \int_{\mathbb{R}^q} e^{it' (z_1 - Z_2)} d\mu(t) \frac{g_0 \tilde{F}_m(X_2)}{g_0, \kappa_{1,2}}(X_2) \epsilon_2 - \int_{\mathbb{R}^q} e^{it' (Z_2 - z_1)} d\mu(t) \frac{g_0 \tilde{F}_m(X_1)}{g_0, \kappa_{2,1}}(X_1) \epsilon_1
\]

The first element of the right hand side is

\[
E \left[ \int_{\mathbb{R}^q} e^{it' (z_1 - Z_2)} d\mu(t) \tilde{p}_1 \epsilon_2 \right] = \int_{\mathbb{R}^q} E[e^{it' z_1} e^{-it' Z_2} \tilde{p}_1 \epsilon_2] d\mu(t)
\]

\[
= \int_{\mathbb{R}^q} e^{it' z_1} \tilde{p}_1 E[e^{-it' Z_2} \epsilon_2] d\mu(t) = 0
\]
The third term is

\[
E\left[\int_{\mathbb{R}^n} e^{it'(z_1 - z_2)} d\mu(t) \frac{g_0, \tilde{P}_1(X_2)}{g_0, \kappa_1, 2(X_2)} e_2\right] = \int_{\mathbb{R}^n} E\left[ e^{it'z_1} e^{-it'z_2} \frac{g_0, \tilde{P}_1(X_2)}{g_0, \kappa_1, 2(X_2)} e_2\right] d\mu(t)
\]

\[
= \int_{\mathbb{R}^n} e^{it'z_1} \tilde{P}_1 E\left[ e^{-it'z_2} \frac{g_0, \tilde{P}_1(X_2)}{g_0, \kappa_1, 2(X_2)} e_2\right] d\mu(t) = 0
\]

The second term of the right hand side is

\[
E\left[\int_{\mathbb{R}^n} e^{it'(Z_2 - z_1)} d\mu(t) \tilde{P}_2 e_1\right] = \int_{\mathbb{R}^n} E\left[ e^{it'Z_2} e^{-it'z_1} \tilde{P}_2 e_1\right] d\mu(t)
\]

\[
= \int_{\mathbb{R}^n} e^{-it'z_1} e_1 E\left[ e^{it'Z_2} \tilde{P}_2\right] d\mu(t)
\]

The fourth term is

\[
E\left[\int_{\mathbb{R}^n} e^{it'(Z_2 - z_1)} d\mu(t) \frac{g_0, \tilde{P}_2(x_1)}{g_0, \kappa_2, 1(x_1)} e_1\right] = \int_{\mathbb{R}^n} E\left[ e^{it'Z_2} e^{-it'z_1} \frac{g_0, \tilde{P}_2(x_1)}{g_0, \kappa_2, 1(x_1)} e_1\right] d\mu(t)
\]

\[
= \int_{\mathbb{R}^n} e^{-it'z_1} e_1 \frac{g_0, \tilde{P}_2(x_1)}{g_0, \kappa_2, 1(x_1)} E\left[ e^{it'Z_2}\right] d\mu(t)
\]

Hence, \( h_1(\tilde{P}_1, \epsilon_1, z_1, x_1) = \int_{\mathbb{R}^n} e^{-it'z_1} e_1 E\left[ e^{it'Z_2} \tilde{P}_2\right] d\mu(t) - \int_{\mathbb{R}^n} e^{-it'z_1} e_1 \frac{g_0, \tilde{P}_2(x_1)}{g_0, \kappa_2, 1(x_1)} E\left[ e^{it'Z_2}\right] d\mu(t). \)

Since \( E(h_1(\tilde{P}_1, \epsilon_1, Z_1, X_1)) = 0, \) we have:

\[
Var[h_1(\tilde{P}_1, \epsilon_1, Z_1, X_1)] = Var \left[ \int_{\mathbb{R}^n} e^{-it'z_1} e_1 \left( E\left[ e^{it'Z_2} \tilde{P}_2\right] - \frac{g_0, \tilde{P}_2(x_1)}{g_0, \kappa_2, 1(x_1)} E\left[ e^{it'Z_2}\right]\right) d\mu(t) \right]
\]

Following \textbf{Hoeffding} [1948], the asymptotic distribution for U-statistics yields:

\[
\sqrt{n}(B'_n - 0) \xrightarrow{d} N(0, 4Var[h_1(\tilde{P}_1, \epsilon_1, Z_1, X_1)])
\]

Thus,

\[
\sqrt{n}(\tilde{\theta}_{n,o} - \theta_0) \xrightarrow{d} N \left( 0, A^{-1} Var[h_1(\tilde{P}_1, \epsilon_1, Z_1, X_1)] (A^{-1})' \right)
\]

with \( A = E \left[ \kappa_{j,l} \left( \tilde{P}_j - \frac{g_0, \tilde{P}_m(X_i)}{g_0, \kappa_m, j(X_i)} \tilde{P}_j \right) \right]. \)

\section*{C.5 Proof of Theorem \ref{thm_table}}

\textbf{Proof}. Recall that

\[
\tilde{y}_i = y_i - \tilde{g}_y(X_i) = y_i - E(y_i|X_i) + E(y_j|X_i) - \tilde{g}_y(X_i)
\]
For the first two terms of the right hand side, we have

\[ y_i - E(y_i|X_i) = (P_i - E(P_i|X_i))'\theta_0 + \epsilon_i \]

\[ = (P_i - \hat{g}_P(X_i))'\theta_0 + (\hat{g}_P(X_i) - E(P_i|X_i))'\theta_0 + \epsilon_i \]

In matrix form, the feasible estimator writes:

\[
\hat{\theta}_{n,o} = \left[ \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j}^{n} \kappa_{j,l} \left( \tilde{P}_j - \frac{g_0,\bar{P}_m(X_i)}{g_0,\kappa_{m,l}(X_i)} \right) \right]^{-1} \left[ \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j}^{n} \kappa_{j,l} \left( \tilde{P}_j - \frac{g_0,\bar{P}_m(X_i)}{g_0,\kappa_{m,l}(X_i)} \right) \right] \tilde{y}_l
\]

Define \( C_n = \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j}^{n} \kappa_{j,l} \left( \tilde{P}_j - \frac{g_0,\bar{P}_m(X_i)}{g_0,\kappa_{m,l}(X_i)} \right) \). We get:

\[
\hat{\theta}_{n,o} = [C_n]^{-1} \left[ \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j}^{n} \kappa_{j,l} \left( \tilde{P}_j - \frac{g_0,\bar{P}_m(X_i)}{g_0,\kappa_{m,l}(X_i)} \right) \right] \left[ y_l - E(y_l|X_l) + E(y_l|X_l) - \hat{g}_y(X_l) \right]
\]

\[
\quad = [C_n]^{-1} \left[ \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j}^{n} \kappa_{j,l} \left( \tilde{P}_j - \frac{g_0,\bar{P}_m(X_i)}{g_0,\kappa_{m,l}(X_i)} \right) \right] \left[ (P_l - \hat{g}_P(X_l))'\theta_0 + \epsilon_l + E(y_l|X_l) - \hat{g}_y(X_l) \right]
\]

\[
\quad = \theta_0 + [C_n]^{-1} \left[ \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j}^{n} \kappa_{j,l} \left( \tilde{P}_j - \frac{g_0,\bar{P}_m(X_i)}{g_0,\kappa_{m,l}(X_i)} \right) \right] \left[ (\hat{g}_P(X_l) - E(P_l|X_l))'\theta_0 + \epsilon_l + E(y_l|X_l) - \hat{g}_y(X_l) \right]
\]

Consider now,

\[
A_n - C_n = \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j}^{n} \kappa_{j,l} \left( \tilde{P}_j - \frac{g_0,\bar{P}_m(X_i)}{g_0,\kappa_{m,l}(X_i)} \right) \tilde{P}_l'
\]

\[
\quad - \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j}^{n} \kappa_{j,l} \left( \tilde{P}_j - \frac{g_0,\bar{P}_m(X_i)}{g_0,\kappa_{m,l}(X_i)} \right) \tilde{P}_l'
\]

\[
\quad = \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j}^{n} \kappa_{j,l} \left( \tilde{P}_j - \frac{g_0,\bar{P}_m(X_i)}{g_0,\kappa_{m,l}(X_i)} \right) \tilde{P}_l'
\]

\[
\quad - \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j}^{n} \kappa_{j,l} \left( \tilde{P}_j + \tilde{P}_j - \tilde{P}_j - \tilde{P}_j + \frac{g_0,\bar{P}_m(X_i)}{g_0,\kappa_{m,l}(X_i)} \right) \tilde{P}_l'
\]

\[
\quad = \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j}^{n} \kappa_{j,l} \left[ \tilde{P}_j - \frac{g_0,\bar{P}_m(X_i)}{g_0,\kappa_{m,l}(X_i)} \right] \left( \tilde{P}_j - \frac{g_0,\bar{P}_m(X_i)}{g_0,\kappa_{m,l}(X_i)} \right) \tilde{P}_l'
\]

\[
\quad + \left( \tilde{P}_j - \frac{g_0,\bar{P}_m(X_i)}{g_0,\kappa_{m,l}(X_i)} \right) (-\tilde{P}_l + \tilde{P}_l)
\]

\[
P \to 0
\]

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Hence, we have \( \text{Plim} C_n = A \), since we showed in the proof of Proposition \( 2 \) that \( \text{Plim} A_n = A \).

Define now the following quantities:

\[
D_n = \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j}^n \kappa_{j,l} \left( \hat{P}_j - \frac{\hat{g}_0 \hat{P}_m(X_l)}{g_{0,\kappa,m,l}(X_l)} \right) \left[ (\hat{g}_P(X_l) - E(P|X_l))\' \theta_0 + \epsilon_l + E(y_l|X_l) - \hat{g}_y(X_l) \right]
\]

\[
E_n = \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j}^n \kappa_{j,l} \left( \hat{P}_j - \frac{\hat{g}_0 \hat{P}_m(X_l)}{g_{0,\kappa,m,l}(X_l)} \right) \left( \hat{g}_P(X_l) - E(P|X_l) \right)\' \theta_0
\]

\[
F_n = \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j}^n \kappa_{j,l} \left( \hat{P}_j - \frac{\hat{g}_0 \hat{P}_m(X_l)}{g_{0,\kappa,m,l}(X_l)} \right) \epsilon_l
\]

\[
G_n = \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j}^n \kappa_{j,l} \left( \hat{P}_j - \frac{\hat{g}_0 \hat{P}_m(X_l)}{g_{0,\kappa,m,l}(X_l)} \right) \left[ E(y_l|X_l) - \hat{g}_y(X_l) \right]
\]

We have, \( D_n = E_n + F_n + G_n \). For consistency, we show that the probability limits for \( E_n \),
\( F_n \) and \( G_n \) are all zero.

\[
E_n = \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j} \kappa_{j,l} \left( \hat{P}_j - \frac{g_{0,\hat{P}_m}(X_l)}{g_{0,\kappa_m,l}(X_l)} \right) (\hat{g}_P(X_l) - E(P_l|X_l))' \theta_0
\]
\[
= \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j} \kappa_{j,l} \left( \hat{P}_j + \hat{\tilde{P}}_j - \hat{P}_j - \frac{g_{0,\hat{P}_m}(X_l)}{g_{0,\kappa_m,l}(X_l)} + \frac{g_{0,\hat{P}_m}(X_l)}{g_{0,\kappa_m,l}(X_l)} \right) (\hat{g}_P(X_l) - E(P_l|X_l))' \theta_0
\]
\[
= \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j} \kappa_{j,l} \left( \hat{P}_j - \frac{g_{0,\hat{P}_m}(X_l)}{g_{0,\kappa_m,l}(X_l)} \right) (\hat{g}_P(X_l) - E(P_l|X_l))' \theta_0
\]
\[
+ \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j} \kappa_{j,l} \left( \hat{P}_j - \frac{g_{0,\hat{P}_m}(X_l)}{g_{0,\kappa_m,l}(X_l)} + \frac{g_{0,\hat{P}_m}(X_l)}{g_{0,\kappa_m,l}(X_l)} \right) (\hat{g}_P(X_l) - E(P_l|X_l))' \theta_0
\]
\[
P \rightarrow 0
\]

\[
F_n = \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j} \kappa_{j,l} \left( \hat{P}_j - \frac{g_{0,\hat{P}_m}(X_l)}{g_{0,\kappa_m,l}(X_l)} \right) \epsilon_l
\]
\[
= \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j} \kappa_{j,l} \left( \hat{P}_j - \frac{g_{0,\hat{P}_m}(X_l)}{g_{0,\kappa_m,l}(X_l)} \right) \epsilon_l
\]
\[
+ \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j} \kappa_{j,l} \left( \hat{P}_j - \frac{g_{0,\hat{P}_m}(X_l)}{g_{0,\kappa_m,l}(X_l)} + \frac{g_{0,\hat{P}_m}(X_l)}{g_{0,\kappa_m,l}(X_l)} \right) \epsilon_l
\]
\[
P \rightarrow 0
\]

\[
G_n = \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j} \kappa_{j,l} \left( \hat{P}_j - \frac{g_{0,\hat{P}_m}(X_l)}{g_{0,\kappa_m,l}(X_l)} \right) [E(y|X_l) - \hat{g}_y(X_l)]
\]
\[
+ \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j} \kappa_{j,l} \left( \hat{P}_j - \frac{g_{0,\hat{P}_m}(X_l)}{g_{0,\kappa_m,l}(X_l)} + \frac{g_{0,\hat{P}_m}(X_l)}{g_{0,\kappa_m,l}(X_l)} \right) [E(y|X_l) - \hat{g}_y(X_l)]
\]
\[
P \rightarrow 0
\]

All in all, we have \( \text{Plim} D_n = \text{Plim}(E_n + F_n + G_n) = 0 \), so \( \hat{\theta}_{n,o} \xrightarrow{P} \theta_0 \).

In addition, we have:

\[
\sqrt{n}(\hat{\theta}_{n,o} - \theta_0) = [C_n]^{-1} \sqrt{n}[E_n + F_n + G_n]
\]
Under heteroskedasticity, the estimator for variance of the feasible-RSMD is

$$\sqrt{n}F_n = \sqrt{n}B_n + \sqrt{n} \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j} \kappa_{j,l} \left( \hat{P}_j - \check{P}_j + \frac{g_0 \hat{P}_m(X_j)}{g_0 \kappa_{m,j}(X_j)} - \frac{g_0 \check{P}_m(X_j)}{g_0 \kappa_{m,j}(X_j)} \right) \epsilon_j$$

$$\sqrt{n}E_n = \sqrt{n} \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j} \kappa_{j,l} \left( \hat{P}_j - \check{P}_j + \frac{g_0 \hat{P}_m(X_j)}{g_0 \kappa_{m,j}(X_j)} \right) (\hat{g}_P(X_j) - E(P_l|X_l))' \theta_0 \right)$$

$$\sqrt{n}G_n = \sqrt{n} \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j} \kappa_{j,l} \left( \hat{P}_j - \check{P}_j + \frac{g_0 \hat{P}_m(X_j)}{g_0 \kappa_{m,j}(X_j)} \right) E(y_l|X_l) - \hat{g}_y(X_l)$$

Hence, 

$$\sqrt{n}[\hat{\theta}_{n,o} - \theta_0] = [C_n]^{-1} \sqrt{n}B_n + o_p(1)$$

$$\hat{\theta}_{n,o}$$ and $$\check{\theta}_{n,o}$$ share the same asymptotic distribution.

### C.6 Consistent Estimator of the Asymptotic Variance

$$Var \left[ \int_{\mathbb{R}^{q_x}} e^{-it'Zj} \epsilon_j \left( E[e^{it'Zj} \hat{P}_l] - \frac{g_0 \hat{P}_m(X_j)}{g_0 \kappa_{m,j}(X_j)} E[e^{it'Zj}] \right) d\mu(t) \right]$$

$$= Var \left[ E_l \left[ \int_{\mathbb{R}^{q_x}} e^{it'(Z_l - Z_j)} \epsilon_j \left( \hat{P}_l - \frac{g_0 \hat{P}_m(X_j)}{g_0 \kappa_{m,j}(X_j)} \right) d\mu(t) \right] \right]$$

$$= Var \left[ E_l \left[ k(Z_l - Z_j) \left( \hat{P}_l - \frac{g_0 \hat{P}_m(X_j)}{g_0 \kappa_{m,j}(X_j)} \right) \epsilon_j \right] \right]$$

$$= E \left[ \left( E_l \left[ k(Z_l - Z_j) \left( \hat{P}_l - \frac{g_0 \hat{P}_m(X_j)}{g_0 \kappa_{m,j}(X_j)} \right) \epsilon_j \right] \right) \left( E_l \left[ k(Z_l - Z_j) \left( \hat{P}_l - \frac{g_0 \hat{P}_m(X_j)}{g_0 \kappa_{m,j}(X_j)} \right) \epsilon_j \right] \right) \right] \epsilon_j^2$$

Under heteroskedasticity, the estimator for variance of the feasible-RSMD is

$$[n(n-1)C_n]^{-1} \sum_{j=1}^{n} \sum_{l \neq j} \kappa_{j,l} \left( \hat{P}_l - \frac{g_0 \hat{P}_m(X_j)}{g_0 \kappa_{m,j}(X_j)} \right) \epsilon_j \left( \hat{P}_l - \frac{g_0 \hat{P}_m(X_j)}{g_0 \kappa_{m,j}(X_j)} \right) \epsilon_j^2 [n(n-1)C_n]^{-1}$$
D Identification

D.1 Summary

In this section, I provide a summary to illustrate the identification issue for SMD (Smooth Minimum Distance), RSMD, and DRSMD using a single instrument to estimate two parameters. If there are constants in the model, the conditions for identification will be slightly different, but the logic will remain the same.

I will start with the simplest model first. Then move on to the complicated models. Models are provided with coherent simulation results. Those results are included in the Table B9.

1. Model (1):
\[
y_i = \beta_1 X_{i,1} + \beta_2 X_{i,2} + u_i \\
X_{i,1} = \pi_1 W_i + \nu_{i,1} \\
X_{i,2} = \pi_2 W_i + \nu_{i,2}
\]

SMD cannot identify \(\beta_1\) and \(\beta_2\) with continuous or discrete \(W_i\). To identify \(\beta_1\) and \(\beta_2\) with SMD, the model need to have a nonlinear part of \(W_i\) (continuous) or another control variable \(Z_i\) inside one of the equations for \(X_i\).

2. Model (2):
\[
y_i = \beta_1 X_{i,1} + \beta_2 X_{i,2} + u_i \\
X_{i,1} = \pi_{1,1} W_{i,1} + \pi_{1,2} W_{i,2} + \nu_{i,1} \\
X_{i,2} = \pi_{2,1} W_{i,1} + \pi_{2,2} W_{i,2} + \nu_{i,2}
\]

When (continuous or discrete) \(W_{i,1}\) and \(W_{i,2}\) are independent, we are unable to identify both parameters using SMD with only one instrument \(W_{i,1}\) under some specific conditions for the mean of \(E(W_{i,2})\), \(\pi_{1,1}\), \(\pi_{1,2}\), \(\pi_{2,1}\), and \(\pi_{2,2}\). We are able to identify both parameters \(\beta_1\) and \(\beta_2\) when these conditions are met.

3. Model (3):
\[
y_i = \beta_1 X_{i,1} + \beta_2 X_{i,1} * Z_i + u_i \\
X_{i,1} = \pi_1 W_i + \nu_{i,1} \\
X_{i,1} * Z_i = \pi_1 W_i * Z_i + \nu_{i,1} * Z_i
\]

If \(W_i\) and \(Z_i\) are dependent, when \(W_i\) is continuous, both parameters are identified, and when \(W_i\) is binary, using SMD we can identify 0 or 1 parameter based on the value of \(E(Z_i|W_i)\). If \(W_i\) and \(Z_i\) are independent, using SMD we can identify 0 or 1 parameter based on the value of \(E(Z_i)\). This is the same case for a continuous or discrete \(W_i\).
4. Model (4):

\[ y_i = \beta_1 X_{i,1} + \beta_2 X_{i,2} * Z_i + \gamma_1 Z_i + u_i \]

\[ X_{i,1} = \pi_1 W_i + v_{i,1} \]

\[ X_{i,1} * Z_i = \pi_1 W_i * Z_i + v_{i,1} * Z_i \]

If \( W_i \) and \( Z_i \) are dependent, RSMD identifies both parameters using a continuous or discrete \( W_i \). If \( W_i \) and \( Z_i \) are independent, we can use RSMD to identify 0 or 1 parameter based on the value of \( E(Z_i) \) with continuous or discrete \( W_i \).

5. DRSMD has extra terms in its formula and it is not symmetric. It will identify both parameters when RSMD cannot.

D.2 Proofs and Simulation Examples

Following that, we move on to sections with more detailed explanations, proofs, and simulation studies to provide more information on the identification issue for methods incorporating SMD (smooth minimum distance approach), such as SMD, RSMD, and DRSMD.

1. Prove the identification of the key parameters using the SMD method for the simplest model in the following (Model 1).

\[ y_i = \beta_1 X_{i,1} + \beta_2 X_{i,2} + u_i \]

\[ X_{i,1} = \pi_1 W_i + v_{i,1} \]

\[ X_{i,2} = \pi_2 W_i + v_{i,2} \]

Proof. The SMD estimator is

\[
\begin{pmatrix}
\beta_1 \\
\beta_2
\end{pmatrix}
= [E(\kappa_{j,t}X_jX_i')]^{-1}E(\kappa_{j,t}X_jy_t) = \left[ E \left( \kappa_{j,t} \begin{pmatrix} X_{j,1} \\ X_{j,2} \end{pmatrix} \begin{pmatrix} X_{i,1} \\ X_{i,2} \end{pmatrix}' \right) \right]^{-1} E(\kappa_{j,t}X_jy_t)
\]

We need to prove that \( E(\kappa_{j,t}X_jX_i') \) is nonsingular. To show that \( E(\kappa_{j,t}X_jX_i') \) is nonsingular, we consider the associated quadratic form, and show that it is positive definite.
For any $a$ real vector of size $p$, we have:

\[ E(a' X_j X'_i | \kappa_{j,l}) = E(\kappa_{j,l} a' E(X_j | W_j) E(X'_i | W_i)) \]

\[ = E(\int_{R^{qw}} e^{it'(W_j - W_i)} d\mu(t) a' E(X_j | W_j) E(X'_i | W_i) a) \]

\[ = \int_{R^{qw}} E[e^{it'(W_j - W_i)} a' E(X_j | W_j) E(X'_i | W_i) a] d\mu(t) \]

\[ = \int_{R^{qw}} E[a'e^{itW_j} E(X_j | W_j)] E[E(X'_i | W_i) a e^{-it'W_i}] d\mu(t) \]

\[ = \int_{R^{qw}} \left( \int_{R^{qw}} a'e^{itW_j} E(X_j | W_j) f_W(W_j) dW_j \right)^2 d\mu(t) \]

\[ = (2\pi)^{2q} \int_{R^{qw}} \left( \mathcal{F}[a' E(X_j | W_j) f_W(W_j)](t) \right)^2 d\mu(t) \]

\[ \geq 0 \]

with $\mu$ strictly positive on $\mathbb{R}^p$ and $\mathcal{F}[g]$ the Fourier transform of a well-defined function $g(.)$ on $\mathbb{R}^{qw}$ is formally defined as,

\[ \mathcal{F}[g](t) = \frac{1}{(2\pi)^{qw}} \int \exp{it'u} g(u) du. \quad (D.1) \]

We then have:

\[ a' E(\kappa_{j,t} X_j X'_i) a = 0 \iff \exists a \neq 0 \text{ s.t. } a' E(X_j | W_j) f(W_j) = 0 \text{ a.s.} \]

\[ \iff \exists a \neq 0 \text{ s.t. } a' E(X_j | W_j) = 0 \text{ a.s.} \]

In the specific case,

\[ \iff \exists a \neq 0 \text{ s.t. } a' E(X_j | W_j) = a_1 \pi_1 W_i + a_2 \pi_2 W_i = (a_1 \pi_1 + a_2 \pi_2) W_i = 0 \text{ a.s.} \]

$\beta_1$ and $\beta_2$ are not identified. To identify $\beta_1$ and $\beta_2$, the model must contain a nonlinear part of $W_i$ or another control variable $Z_i$ inside one of the equations for $X_i$. \hfill \Box

2. Prove the identification of the key parameters using the SMD method for the model with an interaction term (Model (3)).

\[ y_i = \beta_1 X_{i,1} + \beta_2 X_{i,1} * Z_i + u_i \]

\[ X_{i,1} = \pi_1 W_i + v_{i,1} \]

\[ X_{i,1} * Z_i = \pi_1 W_i * Z_i + v_{i,1} * Z_i \]
Proof. We need to prove that $E(\kappa_{j,l}X_jX'_l)$ is nonsingular. Follow the same logic as in the previous proof.

$$a' E(\kappa_{j,l}X_jX'_l) = 0 \iff \exists a \neq 0 \text{ s.t. } a' E(X_j|W_j) = 0 \ a.s.$$

$$E(X_{i,1}|W_i) = \pi_1 W_i$$

$$E(X_{i,1} * Z_i|W_i) = E((\pi_1 W_i + v_{i,1}) * Z_i|W_i) = E(\pi_1 W_i * Z_i + v_{i,1} * Z_i|W_i) = \pi_1 W_i E(Z_i|W_i) + E(v_{i,1} * Z_i|W_i)$$

$$a' E(X_i|W_i) = a_1 \pi_1 W_i + a_2 \pi_1 W_i E(Z_i|W_i) + a_2 E(v_{i,1} * Z_i|W_i)$$

If $v_{i,1} \perp (Z_i, W_i)$,

$$E(v_{i,1} * Z_i|W_i) = 0$$

$$a' E(X_i|W_i) = a_1 \pi_1 W_i + a_2 \pi_1 W_i E(Z_i|W_i)$$

When $W_i$ and $Z_i$ are dependent, and $W_i$ is continuous, there are no $a_1 \neq 0$ and $a_2 \neq 0$, s.t. $a' E(X_j|W_j) = 0 \ a.s.$. Both parameters are identified.

When $W_i$ is binary, there will be a problem.

For instance,

$$a' E(X_i|W_i = 0) = a_1 \pi_1 * 0 + a_2 \pi_1 * 0 * E(Z_i|W_i = 0) = 0$$

$$a' E(X_i|W_i = 1) = a_1 \pi_1 * 1 + a_2 \pi_1 * 1 * E(Z_i|W_i = 1) = 0$$

When $E(Z_i|W_i = 1) \neq 0$ we can find a $a_1 \neq 0$ and $a_2 \neq 0$, s.t. $a' E(X_j|W_j) = 0 \ a.s.$

When $W_i$ is 3-value instrument, both parameters will be identified.

Here are 2 examples for simulation illustration when using the SMD estimator. I consider the case that $W_i$ and $Z_i$ are independent for the second example.

(a)

$$y_i = 2X_{i,1} + 3X_{i,2} + u_i$$

$$X_{i,1} = 4W_{i,1} + W_{i,2} + v_{i,1}$$

$$X_{i,2} = W_{i,1} + 3W_{i,2} + v_{i,2}$$

When $W_{i,1}$ and $W_{i,2}$ are independent, we are not able to identify both parameters inside the model for $y_i$ using SMD with only one instrument $W_{i,1}$ or $W_{i,2}$ when
\( E(W_{i,2}) \) or \( E(W_{i,1}) \) are zero. Indeed, identification of SMD with \( W_{i,1} \) yields the following equation.

\[
a' E(X_i|W_{i,1}) = a_1 E(4W_{i,1} + W_{i,2} + v_{i,1}|W_{i,1}) + a_2 E(W_{i,1} + 3W_{i,2} + v_{i,2}|W_{i,1})
\]
\[
= a_1 4W_{i,1} + a_1 E(W_{i,2}|W_{i,1}) + a_2 W_{i,1} + a_2 E(3W_{i,2}|W_{i,1})
\]

When \( W_{i,1} \) and \( W_{i,2} \) are independent,

\[
a' E(X_i|W_{i,1}) = a_1 4W_{i,1} + a_1 E(W_{i,2}) + a_2 W_{i,1} + a_2 E(3W_{i,2})
\]
\[
a' E(X_i|W_{i,1}) = (4a_1 + a_2)W_{i,1} + (a_1 + 3a_2)E(W_{i,2})
\]

i. When \( W_{i,1} \) and \( W_{i,2} \) are continuous, we need \( E(W_{i,2}) \neq 0 \) to identify both parameters.

ii. When \( E(W_{i,2}) = 0 \), there are infinite possible combinations for \( a_1 \) and \( a_2 \) to have

\[
a' E(X_i|W_{i,1}) = 0
\]

. In this case, neither parameter is identified.

iii. When \( W_{i,1} \) and \( W_{i,2} \) are binary 0, 1, \( E(W_{i,2}) \neq 0 \). \( a_1 = 0 \) and \( a_2 = 0 \) to have

\[
a' E(X_i|W_{i,1}) = 0
\]

. Thus, we will identify both parameters.

(b)

\[
y_i = 2X_{i,1} + 3X_{i,1} \ast Z_i + u_i
\]
\[
X_{i,1} = 2W_i + v_{i,1}
\]
\[
X_{i,1} \ast Z_i = 2W_i \ast Z_i + v_{i,1} \ast Z_i
\]

When \( W_i \) and \( Z_i \) are independent and \( E(Z_i) = 0 \), we are able to identify the first parameter. When \( W_i \) and \( Z_i \) are independent and \( E(Z_i) \neq 0 \), there is no identification for both parameters. Indeed,

\[
a' E(X_i|W_i) = a_1 E(2W_i + v_{i,1}|W_i) + a_2 E(2W_i \ast Z_i + v_{i,1} \ast Z_i|W_i)
\]
\[
= 2a_1 W_i + 2a_2 W_i \ast E(Z_i|W_i)
\]

When \( W_i \) and \( Z_i \) are independent,

\[
a' E(X_i|W_i) = 2a_1 W_i + 2a_2 W_i \ast E(Z_i)
\]

i. When \( E(Z_i) = 0 \), \( a_1 \) must be 0 for a continuous or discrete \( W_i \). \( a_2 \) can be any value. Thus, we can only identify the first parameter.
ii. When $E(Z_i) = 1$, there are infinite combinations for $(a_1, a_2)$ to make

$$a'E(X_i|W_i) = 0$$

Both parameters are not identified.

All of these examples are verified used simulation studies. The simulation results are included in the Table B9.
3. Prove the identification of the key parameters using the RSMD method for the following model (Model 4).

\[ y_i = \beta_1 X_{i,1} + \beta_2 X_{i,1} \ast Z_i + Z_i + u_i \]

\[ X_{i,1} = \pi_1 W_i + v_{i,1} \]

\[ X_{i,1} \ast Z_i = \pi_1 W_i \ast Z_i + v_{i,1} \ast Z_i \]

**Proof.** We need

\[ a' E(\kappa_j, \tilde{X}_j, \tilde{X}_j')a = 0 \iff \exists a \neq 0 \text{ s.t. } a' E(\tilde{X}_j | W_j) = 0 \text{ a.s.} \]

to show that the parameters are not identified.

\[
E(\tilde{X}_{i,1} | W_i) = E(X_{i,1} - E(X_{i,1} | Z_i) | W_i) = \pi_1 W_i - \pi_1 E(E(W_i | Z_i) | W_i)
\]

\[
E(X_{i,1} Z_i - E(X_{i,1} Z_i | Z_i) | W_i) = E(X_{i,1} Z_i - E(X_{i,1} | Z_i) Z_i | W_i)
\]

\[
= E(\pi_1 W_i Z_i + v_{i,1} Z_i - E(X_{i,1} | Z_i) Z_i | W_i)
\]

\[
= E(\pi_1 W_i Z_i - E(\pi_1 W_i + v_{i,1} | Z_i) Z_i | W_i)
\]

\[
= E(\pi_1 W_i Z_i - E(\pi_1 W_i | Z_i) \ast Z_i | W_i)
\]

\[
= \pi_1 W_i E(Z_i | W_i) - \pi_1 E(E(W_i | Z_i) Z_i | W_i)
\]

\[
a' E(\tilde{X}_i | W_i) = a_1 (\pi_1 W_i - \pi_1 E(E(W_i | Z_i) | W_i)) + a_2 (\pi_1 W_i E(Z_i | W_i) - \pi_1 E(E(W_i | Z_i) Z_i | W_i))
\]

Thus, when \( a' E(\tilde{X}_i | W_i) = 0 \),

\[
a_1 W_i - a_1 E(E(W_i | Z_i) | W_i) + a_2 W_i E(Z_i | W_i) - a_2 E(E(W_i | Z_i) Z_i | W_i) = 0 \quad \text{(D.2)}
\]

(a) When the instrument \( W_i \) is binary, if \( W_i = 0 \), Equation (D.2) becomes:

\[-a_1 E(E(W_i | Z_i) | W_i = 0) - a_2 E(E(W_i | Z_i) Z_i | W_i = 0) = 0 \]

If \( W_i = 1 \), Equation (D.2) becomes:

\[ a_1 - a_1 E(E(W_i | Z_i) | W_i = 1) + a_2 E(Z_i | W_i = 1) - a_2 E(E(W_i | Z_i) Z_i | W_i = 1) = 0 \]
We need to solve for $a_1$ and $a_2$. If both of them are 0, both parameters will be identified. If one of them is 0, only one parameter will be identified.

\[
\begin{pmatrix}
    E(E(W_i|Z_i)|W_i = 0) & E(E(W_i|Z_i)|W_i = 0) \\
    1 - E(E(W_i|Z_i)|W_i = 1) & (E(Z_i|W_i = 1) - E(E(W_i|Z_i)Z_i|W_i = 1))
\end{pmatrix}
\begin{pmatrix}
    a_1 \\
    a_2
\end{pmatrix}
= \begin{pmatrix}
    0 \\
    0
\end{pmatrix}
\]

We will have a problem if $W_i \perp Z_i$. The determinant of the above matrix is 0. In that scenario, there are many possible solutions for $a_1$ and $a_2$. We cannot identify both parameters.

Another straightforward way to illustrate that is in the following.

\[
a' E(\tilde{X}_i|W_i) = a_1 E(\tilde{X}_{i,1}|W_i) + a_2 E(\tilde{X}_{i,1} \ast Z_i|W_i)
\]
\[
a' E(\tilde{X}_i|W_i = 0) = a_1 E(\tilde{X}_{i,1}|W_i = 0) + a_2 E(\tilde{X}_{i,1} \ast Z_i|W_i = 0) = 0
\]
\[
a' E(\tilde{X}_i|W_i = 1) = a_1 E(\tilde{X}_{i,1}|W_i = 1) + a_2 E(\tilde{X}_{i,1} \ast Z_i|W_i = 1) = 0
\]
\[
\begin{pmatrix}
    E(\tilde{X}_{i,1}|W_i = 0) & E(\tilde{X}_{i,1} \ast Z_i|W_i = 0) \\
    E(\tilde{X}_{i,1}|W_i = 1) & E(\tilde{X}_{i,1} \ast Z_i|W_i = 1)
\end{pmatrix}
\begin{pmatrix}
    a_1 \\
    a_2
\end{pmatrix}
= \begin{pmatrix}
    0 \\
    0
\end{pmatrix}
\]

If matrix $A$ is invertible or $\det(A)$ is not 0, then $a_1$ and $a_2$ are zeros. Both parameters are identified. If $\det(A)$ is 0, we cannot identify both parameters. $\det(A) = E(\tilde{X}_{i,1}|W_i = 0)E(\tilde{X}_{i,1} \ast Z_i|W_i = 1) - E(\tilde{X}_{i,1} \ast Z_i|W_i = 0)E(\tilde{X}_{i,1}|W_i = 1)$.

Additionally, with a binary instrument, the RSMD identifies 2 or less than 2 parameters. For instance, if we want to identify 3 parameters with 2 parameters for 2 interaction terms, RSMD can only identify 2 parameters at most.

\[
a' E(\tilde{X}_i|W_i) = a_1 E(\tilde{X}_i|W_i) + a_2 E(\tilde{X}_iZ_{i,1}|W_i) + a_3 E(\tilde{X}_iZ_{i,2}|W_i)
\]
\[
a' E(\tilde{X}_i|W_i = 0) = a_1 E(\tilde{X}_i|W_i = 0) + a_2 E(\tilde{X}_iZ_{i,1}|W_i = 0) + a_3 E(\tilde{X}_iZ_{i,2}|W_i = 0)
\]
\[
a' E(\tilde{X}_i|W_i = 1) = a_1 E(\tilde{X}_i|W_i = 1) + a_2 E(\tilde{X}_iZ_{i,1}|W_i = 1) + a_3 E(\tilde{X}_iZ_{i,2}|W_i = 1)
\]

4. Prove the identification of the key parameters using the DRSMD method for Model 4.

The DRSMD estimator relies on the invertibility of the following matrix. Because it is not symmetric, we cannot use the same method to prove the identification problem. There is an alternative proof.
\[
E \left[ \kappa_{j,l} \left( \tilde{P}_j - \frac{g_0, P_{m_0}(X_i)}{g_0, \kappa_{m,l}(X_i)} \tilde{P}_t \right) \tilde{P}_t^\prime \right] = E \left[ \kappa_{j,l} \tilde{P}_j \tilde{P}_t^\prime \right] - E \left[ \kappa_{j,l} \frac{g_0, P_{m_0}(X_i)}{g_0, \kappa_{m,l}(X_i)} \tilde{P}_t \right] \]

The invertibility of the matrix is illustrated by a simple example. \( X_i \) and \( W_i \) are random variables. \( P_i = [W_i, W_i X_i]' \). \( W \) is the treatment. \( X \) is the control. \( Z \) is the instrument.

\[
\tilde{P}_j = \left( \begin{array}{c} \tilde{W}_j \\ \tilde{W}_j X_j \end{array} \right) = \left( \begin{array}{c} W_j - E(W_j | X_j) \\ W_j X_j - E(W_j | X_j) X_j \end{array} \right)
\]

\[
g_{0, \tilde{P}_{1,m}}(X_i) = E[(P_{1m} - g_0, P_1(X_m)) \kappa_{m,l}(X_i)] = E[\tilde{W}_j \kappa_{j,l}(X_i)] = a(X_i),
\]

\[
g_{0, \tilde{P}_{2,j}}(X_i) = E[(P_{2j} - g_0, P_2(X_j)) \kappa_{j,l}(X_i)] = E[\tilde{W}_j X_j \kappa_{j,l}(X_i)] = b(X_i),
\]

\[
g_{0, \kappa_{m,l}}(X_i) = E[\kappa_{m,l}(X_i)] = c(X_i).
\]

Based on the definition of \( \tilde{P}_j \), \( E(\tilde{P}_j) = 0 \).

\[
E(\kappa_{j,l} \tilde{W}_j \tilde{W}_l) = E[\kappa_{j,l} E(\tilde{W}_j | Z_j) E(\tilde{W}_l | Z_l)]
\]

\[
= E[\kappa_{j,l} E(\tilde{W}_j | Z_j) E(\tilde{W}_l | Z_l) | Z_j = 1, Z_l = 1] P_r(Z_j = 1, Z_l = 1)
+ E[\kappa_{j,l} E(\tilde{W}_j | Z_j) E(\tilde{W}_l | Z_l) | Z_j = 1, Z_l = 0] P_r(Z_j = 1, Z_l = 0)
+ E[\kappa_{j,l} E(\tilde{W}_j | Z_j) E(\tilde{W}_l | Z_l) | Z_j = 0, Z_l = 1] P_r(Z_j = 0, Z_l = 1)
+ E[\kappa_{j,l} E(\tilde{W}_j | Z_j) E(\tilde{W}_l | Z_l) | Z_j = 0, Z_l = 0] P_r(Z_j = 0, Z_l = 0)
\]

In the simulations, we use Gaussian CDF for \( \mu(t) \) inside \( \kappa_{j,l} \). \( \kappa_{j,l} = 1 \) when instrument variables \( Z_j = Z_l \) and \( \kappa_{j,l} = a \sim 0.61 \) when \( Z_j \neq Z_l \).

\[
E(\kappa_{j,l} \tilde{W}_j \tilde{W}_l) = E(\tilde{W}_j | Z_j = 1) E(\tilde{W}_l | Z_l = 1) P_r(Z_j = 1) P_r(Z_l = 1)
+ a E(\tilde{W}_j | Z_j = 1) E(\tilde{W}_l | Z_l = 0) P_r(Z_j = 1) P_r(Z_l = 0)
+ a E(\tilde{W}_j | Z_j = 0) E(\tilde{W}_l | Z_l = 1) P_r(Z_j = 0) P_r(Z_l = 1)
+ E(\tilde{W}_j | Z_j = 0) E(\tilde{W}_l | Z_l = 0) P_r(Z_j = 0) P_r(Z_l = 0)
\]

Because \( E(\tilde{W}_j | Z_j = 1) E(\tilde{W}_l | Z_l = 1) P_r(Z_j = 1) P_r(Z_l = 1) \)
\[
+ E(\tilde{W}_j | Z_j = 1) E(\tilde{W}_l | Z_l = 0) P_r(Z_j = 1) P_r(Z_l = 0)
= E(\tilde{W}_j | Z_j = 1) P_r(Z_j = 1) [E(\tilde{W}_l | Z_l = 1) P_r(Z_l = 1) + E(\tilde{W}_l | Z_l = 0) P_r(Z_l = 0)]
= 0,
\]
\[ E(\kappa_{j,l}\tilde{W}_j\tilde{W}_l) = (1 - a)E(\tilde{W}_j|Z_j = 1)E(\tilde{W}_l|Z_l = 1)P_r(Z_j = 1)P_r(Z_l = 1) \]
\[ + (1 - a)E(\tilde{W}_j|Z_j = 0)E(\tilde{W}_l|Z_l = 0)P_r(Z_j = 0)P_r(Z_l = 0) \]

Denote \( E(\tilde{W}_l|Z_l = 1) \) as \( \theta_1 \), \( E(\tilde{W}_l|Z_l = 0) \) as \( \theta_2 \), and \( P_r(Z_j = 1) \) as \( b \). We have
\[ E(\kappa_{j,l}\tilde{W}_j\tilde{W}_l) = (1 - a)(\theta_1^2 b^2 + \theta_2^2 (1 - b)^2) \]

Follow the same steps,
\[ E[\kappa_{j,l}\tilde{W}_j\tilde{W}_l X_l] = (1 - a)E(\tilde{W}_j|Z_j = 1)E(X_l\tilde{W}_l|Z_l = 1)P_r(Z_j = 1)P_r(Z_l = 1) \]
\[ + (1 - a)E(\tilde{W}_j|Z_j = 0)E(X_l\tilde{W}_l|Z_l = 0)P_r(Z_j = 0)P_r(Z_l = 0) \]

Denote \( E(X_l\tilde{W}_l|Z_l = 1) \) as \( \theta_3 \), \( E(X_l\tilde{W}_l|Z_l = 0) \) as \( \theta_4 \).
\[ E[\kappa_{j,l}\tilde{W}_j\tilde{W}_l X_l] = (1 - a)(\theta_1 \theta_3 b^2 + \theta_2 \theta_4 (1 - b)^2) \]

\[ E[\kappa_{j,l}\tilde{W}_j\tilde{W}_l X_j] = (1 - a)E(X_j\tilde{W}_j|Z_j = 1)E(X_l\tilde{W}_l|Z_l = 1)P_r(Z_j = 1)P_r(Z_l = 1) \]
\[ + (1 - a)E(X_j\tilde{W}_j|Z_j = 0)E(X_l\tilde{W}_l|Z_l = 0)P_r(Z_j = 0)P_r(Z_l = 0) \]
\[ E[\kappa_{j,l}\tilde{W}_j\tilde{W}_l X_j] = (1 - a)(\theta_3^2 b^2 + \theta_4^2 (1 - b)^2) \]
\[ E[\kappa_{j,l}\tilde{P}_j\tilde{P}_l] = \begin{bmatrix} (1 - a)(\theta_1^2 b^2 + \theta_2^2 (1 - b)^2) & (1 - a)(\theta_1 \theta_3 b^2 + \theta_2 \theta_4 (1 - b)^2) \\ (1 - a)(\theta_1 \theta_3 b^2 + \theta_2 \theta_4 (1 - b)^2) & (1 - a)(\theta_3^2 b^2 + \theta_4^2 (1 - b)^2) \end{bmatrix} \]

\[ E \left[ \frac{a(X_l)}{c(X_l)} \tilde{W}_l \right] = E \left[ \kappa_{j,l} E \left( \frac{a(X_l)}{c(X_l)} \tilde{W}_l | Z_l \right) \right] \]
\[ = E \left[ \kappa_{j,l} E \left( \frac{a(X_l)}{c(X_l)} \tilde{W}_l | Z_l \right) | Z_j = 1, Z_l = 1 \right] P_r(Z_j = 1, Z_l = 1) \]
\[ + E \left[ \kappa_{j,l} E \left( \frac{a(X_l)}{c(X_l)} \tilde{W}_l | Z_l \right) | Z_j = 1, Z_l = 0 \right] P_r(Z_j = 1, Z_l = 0) \]
\[ + E \left[ \kappa_{j,l} E \left( \frac{a(X_l)}{c(X_l)} \tilde{W}_l | Z_l \right) | Z_j = 0, Z_l = 1 \right] P_r(Z_j = 0, Z_l = 1) \]
\[ + E \left[ \kappa_{j,l} E \left( \frac{a(X_l)}{c(X_l)} \tilde{W}_l | Z_l \right) | Z_j = 0, Z_l = 0 \right] P_r(Z_j = 0, Z_l = 0) \]
\[
E \left[ \kappa_{j,l} \frac{a(X_l)}{c(X_l)} \tilde{W}_l \right] = E \left( \frac{a(X_l)}{c(X_l)} \tilde{W}_l | Z_l = 1 \right) P_r(Z_j = 1) P_r(Z_l = 1) \\
+ aE \left( \frac{a(X_l)}{c(X_l)} \tilde{W}_l | Z_l = 0 \right) P_r(Z_j = 1) P_r(Z_l = 0) \\
+ aE \left( \frac{a(X_l)}{c(X_l)} \tilde{W}_l | Z_l = 1 \right) P_r(Z_j = 0) P_r(Z_l = 1) \\
+ E \left( \frac{a(X_l)}{c(X_l)} \tilde{W}_l | Z_l = 0 \right) P_r(Z_j = 0) P_r(Z_l = 0)
\]

\[
E \left( \frac{a(X_l)}{c(X_l)} \tilde{W}_l \right) = E \left( \frac{a(X_l)}{c(X_l)} E(\tilde{W}_l | X_l) \right) = 0
\]

with \( \tilde{P}_j = [\tilde{W}_j, \tilde{W}_j, X_j]' = [W_j - E(W_j | X_j), W_j X_j - E(W_j | X_j) X_j] \)

\[
E \left[ \kappa_{j,l} \frac{a(X_l)}{c(X_l)} \tilde{W}_l \right] = P_r(Z_j = 1)(1-a)E \left( \frac{a(X_l)}{c(X_l)} \tilde{W}_l | Z_l = 1 \right) P_r(Z_l = 1) \\
+ aP_r(Z_j = 1)E \left( \frac{a(X_l)}{c(X_l)} \tilde{W}_l | Z_l = 1 \right) P_r(Z_l = 1) \\
+ aP_r(Z_j = 1)E \left( \frac{a(X_l)}{c(X_l)} \tilde{W}_l | Z_l = 0 \right) P_r(Z_l = 0) \\
+ aE \left( \frac{a(X_l)}{c(X_l)} \tilde{W}_l | Z_l = 1 \right) P_r(Z_j = 0) P_r(Z_l = 1) \\
+ E \left( \frac{a(X_l)}{c(X_l)} \tilde{W}_l | Z_l = 0 \right) P_r(Z_j = 0) P_r(Z_l = 0)
\]

\[
= P_r(Z_j = 1)(1-a)E \left( \frac{a(X_l)}{c(X_l)} \tilde{W}_l | Z_l = 1 \right) P_r(Z_l = 1) \\
+ P_r(Z_j = 0)(1-a)E \left( \frac{a(X_l)}{c(X_l)} \tilde{W}_l | Z_l = 0 \right) P_r(Z_l = 0)
\]

When we have \( P_r(Z_j = 1) = 0.5, \ d_1 = 0. \)

Denote \( E \left( \frac{a(X_l)}{c(X_l)} \tilde{W}_l | Z_l = 1 \right) \) as \( \delta_1 \) and \( E \left( \frac{a(X_l)}{c(X_l)} \tilde{W}_l | Z_l = 0 \right) \) as \( \delta_2. \)

\[
E \left[ \kappa_{j,l} \frac{a(X_l)}{c(X_l)} \tilde{W}_l \right] = (1-a)(b^2 \delta_1 + (1-b)^2 \delta_2)
\]

Follow the same steps,
\[
E \left[ \kappa_{j,l} \frac{a(X_l)}{c(X_l)} \tilde{W}_l X_l \right] = P_r(Z_j = 1)(1 - a) E \left( \frac{a(X_l)}{c(X_l)} \tilde{W}_l X_l | Z_l = 1 \right) P_r(Z_l = 1)
+ a P_r(Z_j = 1) E \left( \frac{a(X_l)}{c(X_l)} \tilde{W}_l X_l | Z_l = 1 \right) P_r(Z_l = 0)
+ a P_r(Z_j = 0) E \left( \frac{a(X_l)}{c(X_l)} \tilde{W}_l X_l | Z_l = 1 \right) P_r(Z_l = 1)
+ a E \left( \frac{a(X_l)}{c(X_l)} \tilde{W}_l X_l | Z_l = 0 \right) P_r(Z_j = 0) P_r(Z_l = 0)
+ P_r(Z_j = 1)(1 - a) E \left( \frac{a(X_l)}{c(X_l)} \tilde{W}_l X_l | Z_l = 1 \right) P_r(Z_l = 1)
+ P_r(Z_j = 0)(1 - a) E \left( \frac{a(X_l)}{c(X_l)} \tilde{W}_l X_l | Z_l = 0 \right) P_r(Z_l = 0)
\]

Denote \( E \left( \frac{a(X_l)}{c(X_l)} \tilde{W}_l X_l | Z_l = 1 \right) \) as \( \delta_3 \) and \( E \left( \frac{a(X_l)}{c(X_l)} \tilde{W}_l X_l | Z_l = 0 \right) \) as \( \delta_4 \).

\[
E \left[ \kappa_{j,l} \frac{a(X_l)}{c(X_l)} \tilde{W}_l X_l \right] = (1 - a)(b^2 \delta_3 + (1 - b)^2 \delta_4)
\]

Denote \( E \left( \frac{b(X_l)}{c(X_l)} \tilde{W}_l | Z_l = 1 \right) \) as \( \delta_5 \) and \( E \left( \frac{b(X_l)}{c(X_l)} \tilde{W}_l | Z_l = 0 \right) \) as \( \delta_6 \).

\[
E \left[ \kappa_{j,l} \frac{b(X_l)}{c(X_l)} \tilde{W}_l \right] = (1 - a)(b^2 \delta_5 + (1 - b)^2 \delta_6)
\]

Denote \( E \left( \frac{b(X_l)}{c(X_l)} \tilde{W}_l X_l | Z_l = 1 \right) \) as \( \delta_7 \) and \( E \left( \frac{b(X_l)}{c(X_l)} \tilde{W}_l X_l | Z_l = 0 \right) \) as \( \delta_8 \).

\[
E \left[ \kappa_{j,l} \frac{b(X_l)}{c(X_l)} \tilde{W}_l X_l \right] = (1 - a)(b^2 \delta_7 + (1 - b)^2 \delta_8)
\]

\[
E \left[ \kappa_{j,l} \left( \tilde{P}_j - \frac{g_0, P_m(X_l)}{g_0, \sigma_m, l(X_l)} \right) \tilde{P}_l' \right]
\]

becomes:

\[
\begin{bmatrix}
(1 - a)(\theta_1^2 b^2 + \theta_2^2 (1 - b)^2) - (1 - a)(b^2 \delta_1 + (1 - b)^2 \delta_2) & e_1 \\
(1 - a)(\theta_1 \theta_3 b^2 + \theta_2 \theta_4 (1 - b)^2) - (1 - a)(b^2 \delta_5 + (1 - b)^2 \delta_6) & e_2
\end{bmatrix}
\]

\(e_1 = (1 - a)(\theta_1 \theta_3 b^2 + \theta_2 \theta_4 (1 - b)^2) - (1 - a)(b^2 \delta_3 + (1 - b)^2 \delta_4)\)

\(e_2 = (1 - a)(\theta_3^2 b^2 + \theta_4^2 (1 - b)^2) - (1 - a)(b^2 \delta_7 + (1 - b)^2 \delta_8)\)

Since \( a \) is not 1, the determinant of \( E \left[ \kappa_{j,l} \left( \tilde{P}_j - \frac{g_0, P_m(X_l)}{g_0, \sigma_m, l(X_l)} \right) \tilde{P}_l' \right] \) depends on the exact values for these numbers.
5. Simulation illustration for RSMD and DRSMD

Here are one simulation example for illustration (Model (4)).

\[ y_i = X_{i,1} + X_{i,1} * Z_i + Z_i + u_i \]
\[ X_{i,1} = 2W_i + v_{i,1} \]
\[ X_{i,1} * Z_i = 2W_i * Z_i + v_{i,1} * Z_i \]

(a) For RSMD,

\[ a'E(\tilde{X}_i|W_i) = 2a_1W_i - 2a_1E(E(W_i|Z_i)|W_i) \]
\[ + 2a_2W_iE(Z_i|W_i) - 2a_2E(E(W_i|Z_i)Z_i|W_i) \]
\[ = 0 \]

When \( W_i \) and \( Z_i \) are independent and \( E(W_i) \) is a number (for instance, \( E(W_i) = 1 \)),

\[ a_1W_i - a_1E(W_i) + a_2W_iE(Z_i) - a_2E(Z_i)E(W_i) = 0 \]

i. When \( E(Z_i) = 0 \), \( a_1 \) must be 0 when \( W_i \) is continuous or discrete. \( a_2 \) can be any value. Hence, the first parameter is identified.

ii. When \( E(Z_i) = 1 \), there are infinite combinations for \( (a_1, a_2) \) to make \( a'E(X_i|W_i) = 0 \). Both parameters are not identified.

(b) For DRSMD, because we have extra terms, it should identify both parameters when RSMD cannot.

All of the simulation results are included in Table B9.

E R-SMD Estimator and its Properties from Antoine and Sun (2021)

Under Assumption 1 and FOC defined in Equation (2.8)

\[ \theta_0 = [E(\kappa_{j,l}\tilde{P}_j\tilde{P}'_l)]^{-1}E(\kappa_{j,l}\tilde{P}_j\tilde{y}_l) \]

\[ \tilde{\theta}_{n,u} = \left[\frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{i \neq j} \kappa_{j,l}\tilde{P}_j\tilde{P}'_l\right]^{-1} \left[\frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{i \neq j} \kappa_{j,l}\tilde{P}_j\tilde{y}_l\right] \]
| Estimator | Ins | Med.Bias | MAD | Med.SE | RR | Mean.Bias | Mean.SE | Mean.AHSE |
|-----------|-----|----------|------|--------|----|-----------|---------|-----------|
| SMD $\beta_1$ | $(W_{i,1}, W_{i,2})$ | 0.000 | 0.005 | 0.007 | 0.055 | 0.000 | 0.007 | 0.007 |
| SMD $\beta_2$ | $(W_{i,1}, W_{i,2})$ | 0.000 | 0.006 | 0.009 | 0.052 | 0.000 | 0.009 | 0.009 |
| SMD $\beta_1$ | $W_{i,1}$ | -0.011 | 0.034 | 0.065 | 0.026 | -0.021 | 1.598 | 37.825 |
| SMD $\beta_2$ | $W_{i,1}$ | 0.042 | 0.132 | 0.256 | 0.041 | 0.087 | 6.394 | 149.586 |
| SMD $\beta_1$ | $W_{i,2}$ | 0.018 | 0.106 | 0.209 | 0.022 | -0.050 | 18.259 | 783.448 |
| SMD $\beta_2$ | $W_{i,2}$ | -0.007 | 0.036 | 0.070 | 0.014 | 0.008 | 5.521 | 238.702 |
| TSLS $\beta_1$ | $(W_{i,1}, W_{i,2})$ | 0.000 | 0.004 | 0.006 | 0.063 | 0.000 | 0.006 | 0.006 |
| TSLS $\beta_2$ | $(W_{i,1}, W_{i,2})$ | 0.000 | 0.006 | 0.008 | 0.055 | 0.000 | 0.008 | 0.008 |
| SMD $\beta_1$ | $(W_{i,1}, W_{i,2})$ | 0.000 | 0.008 | 0.012 | 0.052 | 0.000 | 0.012 | 0.012 |
| SMD $\beta_2$ | $(W_{i,1}, W_{i,2})$ | 0.000 | 0.010 | 0.015 | 0.047 | 0.000 | 0.015 | 0.015 |
| SMD $\beta_1$ | $W_{i,1}$ | 0.000 | 0.011 | 0.017 | 0.053 | 0.000 | 0.017 | 0.017 |
| SMD $\beta_2$ | $W_{i,1}$ | -0.001 | 0.017 | 0.026 | 0.049 | 0.000 | 0.026 | 0.026 |
| SMD $\beta_1$ | $W_{i,2}$ | 0.000 | 0.013 | 0.020 | 0.046 | 0.000 | 0.020 | 0.021 |
| SMD $\beta_2$ | $W_{i,2}$ | 0.000 | 0.014 | 0.021 | 0.046 | 0.000 | 0.021 | 0.021 |
| TSLS $\beta_1$ | $(W_{i,1}, W_{i,2})$ | 0.000 | 0.008 | 0.013 | 0.054 | 0.000 | 0.013 | 0.013 |
| TSLS $\beta_2$ | $(W_{i,1}, W_{i,2})$ | 0.000 | 0.011 | 0.017 | 0.048 | 0.000 | 0.017 | 0.017 |
| SMD $\beta_1$ | $W_i$ | 0.000 | 0.013 | 0.019 | 0.036 | 0.000 | 0.024 | 0.027 |
| SMD $\beta_2$ | $W_i$ | 0.000 | 0.043 | 0.077 | 0.039 | 0.019 | 0.738 | 0.596 |
| SMD $\beta_1$ | $W_i$ | 0.000 | 0.046 | 0.077 | 0.039 | -0.020 | 0.742 | 0.602 |
| SMD $\beta_2$ | $W_i$ | 0.000 | 0.043 | 0.077 | 0.039 | 0.019 | 0.738 | 0.596 |
| RSMD $\beta_1$ | $W_i$ | -0.001 | 0.009 | 0.013 | 0.032 | 0.000 | 0.171 | 1.937 |
| RSMD $\beta_2$ | $W_i$ | 0.000 | 0.056 | 0.138 | 0.006 | 0.020 | 4.528 | 77.709 |
| DRSMD $\beta_1$ | $W_i$ | -0.001 | 0.009 | 0.013 | 0.041 | -0.001 | 0.021 | 0.114 |
| DRSMD $\beta_2$ | $W_i$ | 0.000 | 0.041 | 0.066 | 0.029 | 0.026 | 1.693 | 7.282 |
| RSMD $\beta_1$ | $W_i$ | -0.002 | 0.057 | 0.137 | 0.008 | -0.039 | 7.188 | 120.469 |
| RSMD $\beta_2$ | $W_i$ | 0.001 | 0.055 | 0.137 | 0.005 | 0.038 | 7.331 | 122.732 |
| DRSMD $\beta_1$ | $W_i$ | -0.001 | 0.043 | 0.067 | 0.031 | -0.002 | 0.970 | 2.922 |
| DRSMD $\beta_2$ | $W_i$ | 0.002 | 0.041 | 0.066 | 0.029 | 0.001 | 0.970 | 2.919 |

Table B9: Identification Illustration

Notes: Mean.AHSE stands for Mean.Asympt.Heterosk.SE. There are 5000 rounds of simulation and 2000 observations for each simulation study.
Additionally, we show the similar consistency and Asymptotic normality for \( \tilde{\theta}_{n,u} \). Under Assumption 1 and iid assumption for the sample, \( \tilde{\theta}_{n,u} \) is consistent for \( \theta_0 \), and the asymptotic distribution is:

\[
\sqrt{n}(\tilde{\theta}_{n,u} - \theta_0) \overset{d}{\to} N \left( 0, E \left[ \kappa_{j,l} \tilde{P}_j \tilde{P}_l' \right]^{-1} \text{Var}[h'_1(\epsilon_1, Z_1, X_1)] \left( E \left[ \kappa_{j,l} \tilde{P}_j \tilde{P}_l' \right]^{-1} \right)' \right)
\]

where \( h'_1(\epsilon_1, z_1, x_1) = E[\int_{R_{\epsilon_1}} e^{it(z_1-Z_2)}d\mu(t)\tilde{p}_1(\epsilon_2) + \int_{R_{Z_2}} e^{it(Z_2-z_1)}d\mu(t)\tilde{p}_2(\epsilon_1)] \).

Replace every nuisance parameter with its estimate, we have the feasible estimator \( \hat{\theta}_{n,u} \), the R-SMD estimator is in the following.

\[
\hat{\theta}_{n,u} = \left[ \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j}^{n} \kappa_{j,l} \tilde{P}_j \tilde{P}_l' \right]^{-1} \left[ \frac{1}{n(n-1)} \sum_{j=1}^{n} \sum_{l \neq j}^{n} \kappa_{j,l} \tilde{P}_j \tilde{P}_l' \right]
\]

since \( E[\kappa_{j,l} \tilde{P}_j \tilde{P}_l'] \) is invertible.

Under Assumptions 1-2, \( \hat{\theta}_{n,u} \) is consistent for \( \theta_0 \), and the asymptotic distribution is:

\[
\sqrt{n}(\hat{\theta}_{n,u} - \theta_0) \overset{d}{\to} N \left( 0, E \left[ \kappa_{j,l} \tilde{P}_j \tilde{P}_l' \right]^{-1} \text{Var}[h'_1(\epsilon_1, Z_1, X_1)] \left( E \left[ \kappa_{j,l} \tilde{P}_j \tilde{P}_l' \right]^{-1} \right)' \right)
\]

where \( h'_1(\epsilon_1, z_1, x_1) = E[\int_{R_{\epsilon_1}} e^{it(z_1-Z_2)}d\mu(t)\tilde{p}_1(\epsilon_2) + \int_{R_{Z_2}} e^{it(Z_2-z_1)}d\mu(t)\tilde{p}_2(\epsilon_1)] \).

**Supplementary Appendix**

S1. Robustness Check: When the Model is not Sparse

All of the other simulation results are based on the sparsity assumption. In this section, Table S1 reports the results when the sparsity assumption is unsatisfied. We choose the DGP where all of the \( \beta_1q \) and \( \beta_2q \) are not 0. When \( q \leq 5 \), \( \beta_1q = 1 \) and \( \beta_2q = 3 \). When \( q > 5 \), \( \beta_1q = \beta_2q = 0.05 \). The D-RSMD with the Lasso method as the nuisance parameter estimator is affected by the violation of the assumption. It is no surprise since we need the sparsity assumption to conduct the Lasso method. The magnitude of the Med.Bias, MAD and Med.SE are not changing. RRs are higher.

S2. Robustness Check: Other Results of Oregon Health Insurance Experiment
| Estimator         | Ins            | Med.Bias | MAD  | Med.SE | RR  |
|------------------|----------------|----------|------|--------|-----|
| D-RSMD           | $Z_2$          | -0.045   | 0.123| 0.158  | 0.063|
|                  | ($Z_2, Z_2X_1$)| -0.019   | 0.102| 0.119  | 0.057|
|                  | GMM (Oracle)   | (Z_2, Z_2X_1) | 0.000| 0.089  | 0.130| 0.056|
|                  |                |          |      |        |     |
| D-RSMD           | $Z_2$          | -0.032   | 0.162| 0.238  | 0.076|
|                  | ($Z_2, Z_2X_1$)| -0.053   | 0.069| 0.084  | 0.094|
|                  | GMM (Oracle)   | (Z_2, Z_2X_1) | 0.000| 0.048  | 0.071| 0.052|

Table S1: Instruments with 3 values

Note: Simulation Results for $\theta_{w0}$ and $\theta_{wx0}$ in the benchmark model using D-RSMD estimator 5000 replications. We report the Monte-Carlo Median Bias (Med.Bias), Median Absolute Deviation (MAD), median of asymptotic standard error under heteroskedasticity (Med.SE), and the Rejection Rate (RR) using a 5% t-test.
### Panel A: Heterogeneous treatment effects

| Estimator for $\theta_{w0}$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ |
|-----------------------------|------|-----------------|------|-----------------|------|-----------------|
| GMM                         | 0.048| -0.001          | -0.029|                 |      |                 |
|                             | (0.053)| (0.037)          | (0.039)|                 |      |                 |
| GMM-Lasso                   | 0.042| -0.030          | -0.062|                 |      |                 |
|                             | (0.086)| (0.067)          | (0.075)|                 |      |                 |
| DRSMD-Lasso                 | 0.095| 0.016           | -0.029| 0.091           | 0.030|                 |
|                             | (0.067)| (0.057)         | (0.045)| (0.067)         | (0.049)|                 |

| Estimator for $\theta_{w10}$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ |
|-------------------------------|------|-----------------|------|-----------------|------|-----------------|
| GMM                          | -0.080| 0.046          | 0.073|                 |      |                 |
|                             | (0.092)| (0.083)         | (0.090)|                 |      |                 |
| GMM-Lasso                    | -0.039| 0.086          | 0.191|                 |      |                 |
|                             | (0.123)| (0.122)         | (0.148)|                 |      |                 |
| DRSMD-Lasso                  | -0.147*** | -0.027   | 0.092 | 0.074           | 0.074|                 |
|                             | (0.034)| (0.029)         | (0.092)| (0.037)         | (0.100)|                 |

### Panel B: Homogeneous treatment effects

| Estimator | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ |
|-----------|------|-----------------|------|-----------------|------|-----------------|
| GMM       | 0.012| 0.024           | 0.017| 0.009           | 0.001| -0.012           |
|           | (0.047)| (0.045)         | (0.038)| (0.034)         | (0.042)| (0.036)         |
| GMM-Lasso | 0.022| 0.037           | 0.006| -0.033          | 0.019| -0.065           |
|           | (0.098)| (0.086)         | (0.089)| (0.067)         | (0.103)| (0.065)         |
| DRSMD-Lasso | 0.027| 0.025           | 0.006| 0.007           | 0.059| 0.064           |
|           | (0.06)| (0.063)         | (0.052)| (0.053)         | (0.059)| (0.061)         |

| $N$ | 5962 | 6693 | 5917 |

Table S2: Heterogeneous Treatment Effects of Medicaid on Employ

Note: *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used and the age groups. The interaction term is Medicaid $\times$ Above 50% Federal Poverty Line.
Table S3: Heterogeneous Treatment Effects of Medicaid on Employment

Note: *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used and the age groups. The interaction term is Medicaid \times Above 50\% Federal Poverty Line. The expression for LATE is $\theta_{w0} + \theta_{wX_0}E(X)$.
### Panel A: Heterogeneous Treatment Effects

| Employ | Age: 21 - 29 | Age: 30 - 38 | Age: 39 - 47 | Age: 48 - 56 | Age: 57 - 64 |
|--------|--------------|--------------|--------------|--------------|--------------|
| **Estimator for \( \theta_{w0} \)** | \( Z_1 \) \((Z_1, Z_{1}X_1)\) | \( Z_1 \) \((Z_1, Z_{1}X_1)\) | \( Z_1 \) \((Z_1, Z_{1}X_1)\) | \( Z_1 \) \((Z_1, Z_{1}X_1)\) | \( Z_1 \) \((Z_1, Z_{1}X_1)\) |
| GMM | 0.134* | -0.042 | 0.007 | -0.063 | 0.022 |
| | \( (0.071) \) | \( (0.067) \) | \( (0.048) \) | \( (0.040) \) | \( (0.064) \) |
| GMM-Lasso | 0.143 | -0.125 | -0.069 | -0.105 | -0.063 |
| | \( (0.112) \) | \( (0.118) \) | \( (0.089) \) | \( (0.077) \) | \( (0.123) \) |
| DRSMD-Lasso | 0.255*** | 0.159** | 0.014 | -0.067 | 0.033 | -0.058 | 0.086 | 0.072 |
| | \( (0.093) \) | \( (0.081) \) | \( (0.085) \) | \( (0.071) \) | \( (0.060) \) | \( (0.066) \) | \( (0.049) \) | \( (0.103) \) | \( (0.086) \) |
| **Estimator for \( \theta_{w0} \)** | \( Z_1 \) \((Z_1, Z_{1}X_1)\) | \( Z_1 \) \((Z_1, Z_{1}X_1)\) | \( Z_1 \) \((Z_1, Z_{1}X_1)\) | \( Z_1 \) \((Z_1, Z_{1}X_1)\) | \( Z_1 \) \((Z_1, Z_{1}X_1)\) |
| GMM | -0.070 | 0.066 | -0.025 | 0.159 | -0.046 |
| | \( (0.128) \) | \( (0.120) \) | \( (0.100) \) | \( (0.104) \) | \( (0.127) \) |
| GMM-Lasso | -0.014 | 0.198 | 0.120 | 0.304* | 0.157 |
| | \( (0.179) \) | \( (0.145) \) | \( (0.150) \) | \( (0.162) \) | \( (0.193) \) |
| DRSMD-Lasso | -0.233*** | -0.003 | -0.092** | 0.073 | 0.006 | 0.040 | -0.083** | 0.169 | -0.018 | 0.012 |
| | \( (0.043) \) | \( (0.145) \) | \( (0.039) \) | \( (0.135) \) | \( (0.037) \) | \( (0.113) \) | \( (0.036) \) | \( (0.113) \) | \( (0.041) \) | \( (0.153) \) |

### Panel B: Homogeneous Treatment Effects

| | Age: 21 - 29 | Age: 30 - 38 | Age: 39 - 47 | Age: 48 - 56 | Age: 57 - 64 |
|---|--------------|--------------|--------------|--------------|--------------|
| **GMM** | 0.105 | 0.115 | -0.012 | -0.023 | -0.003 | 0.001 | -0.007 | -0.035 | -0.001 | 0.007 |
| | \( (0.063) \) | \( (0.060) \) | \( (0.061) \) | \( (0.058) \) | \( (0.047) \) | \( (0.043) \) | \( (0.044) \) | \( (0.038) \) | \( (0.063) \) | \( (0.056) \) |
| **GMM-Lasso** | 0.135 | 0.142 | -0.013 | -0.110 | -0.018 | -0.068 | 0.011 | -0.115 | 0.021 | -0.059 |
| | \( (0.132) \) | \( (0.112) \) | \( (0.137) \) | \( (0.118) \) | \( (0.111) \) | \( (0.089) \) | \( (0.104) \) | \( (0.076) \) | \( (0.166) \) | \( (0.124) \) |
| **DRSMD-Lasso** | 0.162** | 0.158* | -0.032 | -0.026 | 0.013 | 0.015 | 0.003 | 0.005 | 0.078 | 0.079 |
| | \( (0.083) \) | \( (0.086) \) | \( (0.085) \) | \( 0.088 \) | \( 0.066 \) | \( 0.067 \) | \( 0.059 \) | \( 0.060 \) | \( 0.099 \) | \( 0.105 \) |

| \( N \) | 3504 | 3596 | 3941 | 4599 | 2932 |

Table S4: Heterogeneous Treatment Effects of Medicaid on Employ

Note: *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used and the age groups. The interaction term is Medicaid \( \times \) Above 50% Federal Poverty Line.
### Table S5: Heterogeneous Treatment Effects of Medicaid on Employ

| Estimator for LATE | Age: 21 - 29 | Age: 30 - 38 | Age: 39 - 47 | Age: 48 - 56 | Age: 57 - 64 |
|--------------------|--------------|--------------|--------------|--------------|--------------|
| GMM                | 0.094        | (0.071)      | 0.135        | (0.146)      | 0.122        |
|                    | -0.001       | (0.071)      | 0.000        | (0.143)      | 0.157*       |
|                    | -0.007       | (0.056)      | 0.000        | (0.127)      | -0.044       |
|                    | 0.029        | (0.060)      | 0.071***     | (0.013)      | -0.021       |
|                    | -0.009       | (0.077)      | 0.043        | (0.185)      | 0.014        |
| GMM-Lasso          | 0.135        | (0.146)      | 0.000        | (0.143)      | 0.122        |
|                    | 0.000        | (0.056)      | 0.071***     | (0.013)      | 0.021        |
|                    | 0.029        | (0.077)      | 0.043        | (0.185)      | 0.014        |
| DRSMD-Lasso        | 0.122        | (0.079)      | 0.014        | (0.064)      | 0.157*       |
|                    | 0.157*       | (0.095)      | 0.021        | (0.078)      | -0.044       |
|                    | -0.044       | (0.085)      | 0.015        | (0.056)      | -0.021       |
|                    | -0.021       | (0.093)      | 0.040        | (0.076)      | 0.014        |
|                    | 0.071        | (0.098)      | 0.074        | (0.099)      | 0.021        |
|                    | 0.080        | (0.116)      | 0.080        | (0.116)      | 0.014        |
| N                  | 3504         | 3596         | 3941         | 4599         | 2932         |

Note: *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used and the age groups. The interaction term is Medicaid × Above 50% Federal Poverty Line. The expression for LATE is $\theta_{w0} + \theta_{wz0}E(X)$. 
Panel A: Heterogeneous treatment effects

| Estimator for $\theta_{w0}$ | $Z_1$ | $(Z_1, Z_1 X_1)$ | $Z_1$ | $(Z_1, Z_1 X_1)$ | $Z_1$ | $(Z_1, Z_1 X_1)$ | $Z_1$ | $(Z_1, Z_1 X_1)$ | $Z_1$ | $(Z_1, Z_1 X_1)$ |
|-----------------------------|-------|-----------------|-------|-----------------|-------|-----------------|-------|-----------------|-------|-----------------|
| GMM                         | 0.211*** | -0.072 | 0.056 | -0.059 | -0.171 |
|                            | (0.073) | (0.075) | (0.057) | (0.050) | (0.106) |
| GMM-Lasso                   | 0.196 | -0.150 | 0.011 | -0.154 | -0.331** |
|                            | (0.133) | (0.116) | (0.076) | (0.087) | (0.156) |
| DRSMD-Lasso                 | 0.183** | 0.081 | -0.028 | -0.321** | -0.011 | 0.566 | 0.013 | 0.001 |
|                            | (0.072) | (0.176) | (0.070) | (0.138) | (0.065) | (0.061) | (0.059) | (0.107) | (0.106) |

| Estimator for $\theta_{w0}$ | $Z_1$ | $(Z_1, Z_1 X_1)$ | $Z_1$ | $(Z_1, Z_1 X_1)$ | $Z_1$ | $(Z_1, Z_1 X_1)$ | $Z_1$ | $(Z_1, Z_1 X_1)$ | $Z_1$ | $(Z_1, Z_1 X_1)$ |
|-----------------------------|-------|-----------------|-------|-----------------|-------|-----------------|-------|-----------------|-------|-----------------|
| GMM                         | -0.002* | 0.001 | -0.001 | 0.001 | 0.003 |
|                            | (0.001) | (0.001) | (0.001) | (0.001) | (0.002) |
| GMM-Lasso                   | -0.001 | 0.002 | 0.000 | 0.003 | 0.006 |
|                            | (0.001) | (0.001) | (0.001) | (0.001) | (0.002) | (0.004) |
| DRSMD-Lasso                 | 0.000 | -0.003 | 0.000 | 0.000 | 0.007*** |
|                            | (0.000) | (0.004) | (0.000) | (0.002) | (0.000) | (0.002) | (0.000) | (0.026) | (0.000) | (0.002) |

| $N$                         | 3504 | 3506 | 3941 | 4599 | 2932 |

Table S6: Heterogeneous Treatment Effects of Medicaid on employment

Note: *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used and the age groups. The interaction term is Medicaid × hhincome.
Table S7: Heterogeneous Treatment Effects of Medicaid on employment

Note: *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used and the age groups. The interaction term is Medicaid × hhincome. The expression for LATE is $\theta_{w0} + \theta_{wx0}E(X)$. 
### Panel A: Heterogeneous treatment effects

| Estimator for $\theta_{w0}$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ |
|-----------------------------|-------|-----------------|-------|-----------------|-------|-----------------|-------|-----------------|
| **GMM**                     | 0.113*| -0.014          | -0.007| -0.008          | 0.000 |                 |       |                 |
|                             | (0.064)| (0.060)        | (0.047)| (0.044)        | (0.063)|                 |       |                 |
| **GMM-Lasso**               | 0.139 | -0.017          | -0.010| 0.029          | 0.009 |                 |       |                 |
|                             | (0.140)| (0.130)        | (0.101)| (0.103)        | (0.166)|                 |       |                 |
| **DRSMD-Lasso**             | 0.172**| 0.199          | -0.042| -0.097        | -0.019| -0.019          | 0.033 | 0.033          |
|                             | (0.084)| (0.129)        | (0.081)| (0.181)        | (0.063)| (0.063)        | (0.058)| (0.099)    |
| **Estimator for $\theta_{w+x0}$** | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ |
| **GMM**                     | 0.323 | -1.006         | 0.199 | -0.513        | -1.173***|                 |       |                 |
|                             | (0.876)| (1.033)        | (0.558)| (1.510)        | (0.378)|                 |       |                 |
| **GMM-Lasso**               | 0.361 | -1.490         | 0.160 | -0.432        | -0.574***|                 |       |                 |
|                             | (0.979)| (1.113)        | (0.882)| (1.703)        | (0.049)|                 |       |                 |
| **DRSMD-Lasso**             | -0.029| -3.115         | 4.228 | 0.333**       | -0.331| 0.070           | 0.294 | -0.475        |
|                             | (0.129)| (7.497)        | (9.724)| (1.291)        | (0.480)| (1.229)        | (0.456)| (0.492) |

| $N$                          | 3504  | 3596           | 3941  | 4599          | 2932  |                 |       |                 |

Table S8: Heterogeneous Treatment Effects of Medicaid on employment

Note: *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used and the age groups. The interaction term is Medicaid $\times$ TANF.
Table S9: Heterogeneous Treatment Effects of Medicaid on employment

Note: *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used and the age groups. The interaction term is Medicaid × TANF. The expression for LATE is $\theta_{w0} + \theta_{wx0}E(X)$. 

| Estimator for LATE | Age: 21 - 29 | Age: 30 - 38 | Age: 39 - 47 | Age: 48 - 56 | Age: 57 - 64 |
|--------------------|-------------|-------------|-------------|-------------|-------------|
| GMM                | Z_1 (Z_1, Z_1X_1) | Z_1 (Z_1, Z_1X_1) | Z_1 (Z_1, Z_1X_1) | Z_1 (Z_1, Z_1X_1) | Z_1 (Z_1, Z_1X_1) |
| Z_1 (Z_1, Z_1X_1) | 0.126*      | -0.057      | -0.003      | -0.012      | -0.003      |
|                    | (0.073)     | (0.074)     | (0.048)     | (0.045)     | (0.063)     |
| GMM-Lasso          | Z_1 (Z_1, Z_1X_1) | Z_1 (Z_1, Z_1X_1) | Z_1 (Z_1, Z_1X_1) | Z_1 (Z_1, Z_1X_1) | Z_1 (Z_1, Z_1X_1) |
| Z_1 (Z_1, Z_1X_1) | 0.154       | -0.081      | -0.006      | 0.025       | 0.008       |
|                    | (0.148)     | (0.140)     | (0.102)     | (0.106)     | (0.166)     |
| DRSMD-Lasso        | Z_1 (Z_1, Z_1X_1) | Z_1 (Z_1, Z_1X_1) | Z_1 (Z_1, Z_1X_1) | Z_1 (Z_1, Z_1X_1) | Z_1 (Z_1, Z_1X_1) |
| Z_1 (Z_1, Z_1X_1) | 0.171**     | 0.070       | -0.042      | 0.087       | 0.003       |
|                    | (0.084)     | (0.231)     | (0.082)     | (0.275)     | (0.063)     |
| |                     |                     |                     |                     |                     |
| N                  | 3504         | 3596         | 3941         | 4599         | 2932         |
Panel A: Heterogeneous treatment effects

| Estimator for $\theta_{w0}$ | $Z_1$ ($Z_1, Z_1 X_1$) | $Z_1$ ($Z_1, Z_1 X_1$) | $Z_1$ ($Z_1, Z_1 X_1$) | $Z_1$ ($Z_1, Z_1 X_1$) | $Z_1$ ($Z_1, Z_1 X_1$) |
|-----------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| **GMM**                    | 0.121                   | -0.047                  | -0.046                  | -0.008                  | -0.030                  |
|                            | (0.170)                 | (0.146)                 | (0.103)                 | (0.098)                 | (0.163)                 |
| **GMM-Lasso**              | 0.185                   | -0.089                  | -0.017                  | -0.040                  | -0.040                  |
|                            | (0.267)                 | (0.216)                 | (0.138)                 | (0.153)                 | (0.263)                 |
| **DRSMD-Lasso**            | 0.282***                | 0.193                   | 0.000                   | -0.054                  | -0.026                  |
|                            | (0.107)                 | (0.192)                 | (0.099)                 | (0.168)                 | (0.088)                 |
|                            |                         |                         |                         |                         |                         |
| **Estimator for $\theta_{w00}$** | $Z_1$ ($Z_1, Z_1 X_1$) | $Z_1$ ($Z_1, Z_1 X_1$) | $Z_1$ ($Z_1, Z_1 X_1$) | $Z_1$ ($Z_1, Z_1 X_1$) | $Z_1$ ($Z_1, Z_1 X_1$) |
| **GMM**                    | -0.002                  | 0.009                   | 0.020                   | -0.001                  | 0.012                   |
|                            | (0.070)                 | (0.064)                 | (0.048)                 | (0.046)                 | (0.068)                 |
| **GMM-Lasso**              | -0.017                  | 0.021                   | 0.004                   | 0.033                   | 0.021                   |
|                            | (0.102)                 | (0.087)                 | (0.063)                 | (0.068)                 | (0.101)                 |
| **DRSMD-Lasso**            | -0.047**                | -0.022                  | -0.018                  | 0.005                   | -0.005                  |
|                            | (0.022)                 | (0.074)                 | (0.020)                 | (0.070)                 | (0.020)                 |

| $N$                         | 3504                    | 3596                    | 3941                    | 4599                    | 2932                    |

Table S10: Heterogeneous Treatment Effects of Medicaid on employment

Note: *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used and the age groups. The interaction term is Medicaid $\times$ smoke.
Average Treatment Effects

| Estimator for LATE | Age: 21 - 29 | Age: 30 - 38 | Age: 39 - 47 | Age: 48 - 56 | Age: 57 - 64 |
|--------------------|--------------|--------------|--------------|--------------|--------------|
|                    | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ |
| GMM                | 0.118* | -0.025 | -0.003 | -0.009 | -0.001 |
|                    | (0.065) | (0.063) | (0.048) | (0.045) | (0.064) |
| GMM-Lasso          | 0.144 | -0.040 | -0.007 | 0.031 | 0.011 |
|                    | (0.142) | (0.131) | (0.105) | (0.104) | (0.168) |
| DRSMD-Lasso        | 0.171** | 0.142* | -0.043 | -0.007 | -0.010 | -0.020 | -0.024 | 0.032 | 0.006 |
|                    | (0.084) | (0.083) | (0.081) | (0.080) | (0.062) | (0.064) | (0.058) | (0.058) | (0.099) | (0.096) |
| $N$                | 3504 | 3596 | 3941 | 4599 | 2932 |

Table S11: Heterogeneous Treatment Effects of Medicaid on employment

Note: *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used and the age groups. The interaction term is Medicaid $\times$ smoke. The expression for LATE is $\theta_{w0} + \theta_{wx0}E(X)$. 
Table S12: Heterogeneous Treatment Effects of Medicaid on Out of Pocket Cost

Note: *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used and the age groups. The interaction term is Medicaid × Above 50% Federal Poverty Line.
## Average Treatment Effects

Out of Pocket Cost

| Estimator for LATE | Age: 21 - 35 | Age: 36 - 50 | Age: 51 - 64 |
|--------------------|--------------|--------------|--------------|
|                    | $Z_1$        | $(Z_1, X_1)$ | $Z_1$        | $(Z_1, X_1)$ | $Z_1$        | $(Z_1, X_1)$ |
| GMM                | -173.233**   | -157.835**   | -50.003      |              |
|                    | (79.133)     | (74.153)     | (88.032)     |              |
| GMM-Lasso          | -289.877*    | -273.403*    | -160.41      |              |
|                    | (156.242)    | (156.520)    | (217.11)     |              |
| DRSMD-Lasso        | -233.213***  | -212.36**    | -102.415     | -102.415     |
|                    | (89.090)     | (100.468)    | (94.184)     | (126.818)    |

$N$ = 5962, 6693, 5917

Table S13: Heterogeneous Treatment Effects of Medicaid on Out of Pocket Cost

Note: *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used and the age groups. The interaction term is Medicaid × Above 50% Federal Poverty Line. The expression for LATE is $\theta_{w0} + \theta_{wx0}E(X)$. 
### Panel A: Heterogeneous treatment effects

| Out of Pocket Cost | Age: 21 - 29 | Age: 30 - 38 | Age: 39 - 47 | Age: 48 - 56 | Age: 57 - 64 |
|--------------------|--------------|--------------|--------------|--------------|--------------|
| Estimator for $\theta_{w0}$ | $Z_1$ | $(Z_1, Z_1 X_1)$ | $Z_1$ | $(Z_1, Z_1 X_1)$ | $Z_1$ | $(Z_1, Z_1 X_1)$ | $Z_1$ | $(Z_1, Z_1 X_1)$ | $Z_1$ | $(Z_1, Z_1 X_1)$ |
| GMM | -238.166** | -294.092*** | -32.148 | -149.770*** | -153.110* |
| | (94.947) | (92.333) | (69.362) | (53.074) | (87.341) |
| GMM-Lasso | -545.537*** | -569.684*** | -276.342** | -321.387*** | -465.763** |
| | (148.534) | (141.359) | (131.711) | (91.886) | (189.849) |
| DRSMD-Lasso | -373.155*** | -250.372** | -153.656 | -376.780*** | -55.406 | 18.780 |
| | (136.072) | (119.078) | (99.185) | (84.691) | (101.883) | (68.418) |
| Estimator for $\theta_{w0}$ | $Z_1$ | $(Z_1, Z_1 X_1)$ | $Z_1$ | $(Z_1, Z_1 X_1)$ | $Z_1$ | $(Z_1, Z_1 X_1)$ | $Z_1$ | $(Z_1, Z_1 X_1)$ |
| GMM | -101.939 | 384.290** | -269.238* | 155.502 | 173.800 |
| | (193.546) | (181.372) | (156.752) | (165.046) | (211.235) |
| GMM-Lasso | 226.006 | 558.392** | -77.050 | 307.058 | 477.385 |
| | (245.546) | (243.326) | (209.610) | (276.655) | (370.165) |
| DRSMD-Lasso | 104.666* | -179.667 | 46.142 | 496.881** | -72.394* | -242.417 |
| | (62.996) | (214.131) | (53.675) | (214.081) | (43.318) | (167.287) |

### Panel B: Homogeneous treatment effects

| Out of Pocket Cost | Age: 21 - 29 | Age: 30 - 38 | Age: 39 - 47 | Age: 48 - 56 | Age: 57 - 64 |
|--------------------|--------------|--------------|--------------|--------------|--------------|
| Estimator for $\theta_{w0}$ | $Z_1$ | $(Z_1, Z_1 X_1)$ | $Z_1$ | $(Z_1, Z_1 X_1)$ | $Z_1$ | $(Z_1, Z_1 X_1)$ | $Z_1$ | $(Z_1, Z_1 X_1)$ |
| GMM | -281.393*** | -265.709*** | -118.835 | -183.904** | -140.436* | -100.078 |
| | (89.944) | (82.886) | (90.944) | (82.150) | (72.634) | (64.576) |
| GMM-Lasso | -447.522*** | -532.224*** | -255.052 | -527.148*** | -308.891* | -276.433** |
| | (176.244) | (142.177) | (209.610) | (209.610) | (163.024) | (89.334) |
| DRSMD-Lasso | -331.473*** | -330.392*** | -130.819 | -98.119 | -83.996 | -81.152 |
| | (119.712) | (123.117) | (138.478) | (145.535) | (91.477) | (93.811) |

### Table S14: Heterogeneous Treatment Effects of Medicaid on Out of Pocket Cost

Note: *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used and the age groups. The interaction term is Medicaid $\times$ Above 50% Federal Poverty Line.
### Table S15: Heterogeneous Treatment Effects of Medicaid on Out of Pocket Cost

| Estimator for LATE | Out of Pocket Cost | Age: 21 - 29 | Age: 30 - 38 | Age: 39 - 47 | Age: 48 - 56 | Age: 57 - 64 |
|--------------------|--------------------|-------------|-------------|-------------|-------------|-------------|
|                    | \( Z_1 \)          | \( Z_1, Z_1X_1 \) | \( Z_1 \)   | \( Z_1, Z_1X_1 \) | \( Z_1 \)   | \( Z_1, Z_1X_1 \) |
| GMM                | -296.380***        | -52.748     | -187.091**  | -59.762     | -36.809     |
|                    | (104.514)          | (108.950)   | (89.257)    | (95.636)    | (128.929)   |
| GMM-Lasso          | -416.474**         | -218.127    | -320.684*   | -143.655    | -146.312    |
|                    | (197.893)          | (220.931)   | (179.511)   | (212.015)   | (347.923)   |
| DRSMD-Lasso        | -313.384***        | -124.606    | -97.068     | -158.234*   | 187.576     |
|                    | -352.973***        | -63.950     | -120.728    | -143.646    | 286.854     |
|                    | (113.461)          | (138.286)   | (88.592)    | (110.318)   | (183.656)   |
|                    | (137.349)          | (154.905)   | (87.948)    | (122.867)   | (227.628)   |
| N                  | 3504               | 3596        | 3941        | 4599        | 2932        |

Note: *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used and the age groups. The interaction term is Medicaid \( \times \) Above 50% Federal Poverty Line. The expression for LATE is \( \theta_{w0} + \theta_{wx0}E(X) \).
### Panel A: Heterogeneous treatment effects

| Out of Pocket Cost | Age: 21 - 29 | Age: 30 - 38 | Age: 39 - 47 | Age: 48 - 56 | Age: 57 - 64 |
|-------------------|--------------|--------------|--------------|--------------|--------------|
| **Estimator for $\theta_{w0}$** | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ |
| GMM | -157.173* | -338.762*** | 74.768 | -197.276** | -120.243 |
| | (95.566) | (115.896) | (99.214) | (76.924) | (198.875) |
| GMM-Lasso | -438.811*** | -605.793*** | -260.345** | -526.900*** | -483.119 |
| | (163.018) | (177.487) | (123.764) | (122.174) | (305.685) |
| DRSMD-Lasso | -309.745*** | -145.185 | -187.617 | -237.79 | -81.096 | 58.587 | -182.139** | -949.979 | 213.045 | -201.021 |
| | (102.007) | (260.928) | (125.011) | (204.272) | (88.358) | (87.128) | (90.764) | (663.448) | (195.506) | (123.915) |

| **Estimator for $\theta_{wX}$** | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ |
|-------------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| GMM | -2.089 | 3.889* | -4.594* | 2.34 | 0.94 |
| | (1.294) | (2.183) | (2.486) | (1.992) | (4.800) |
| GMM-Lasso | -0.208 | 5.190* | 0.064 | 6.601** | 8.209 |
| | (1.349) | (2.858) | (2.175) | (3.138) | (8.435) |
| DRSMD-Lasso | -0.268 | -6.179 | 0.737* | -1.490 | -0.374 | -2.399 | 0.140 | 37.031 | -1.034** | 5.204** |
| | (0.543) | (5.497) | (0.428) | (3.529) | (0.582) | (2.276) | (0.377) | (31.714) | (0.496) | (2.432) |

| $N$ | 3504 | 3596 | 3941 | 4599 | 2932 |

Table S16: Heterogeneous Treatment Effects of Medicaid on Out of Pocket Cost

Note: *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used and the age groups. The interaction term is Medicaid $\times$ hhincome.
### Average Treatment Effects

| Out of Pocket Cost | Age: 21 - 29 | Age: 30 - 38 | Age: 39 - 47 | Age: 48 - 56 | Age: 57 - 64 |
|-------------------|--------------|--------------|--------------|--------------|--------------|
| **Estimator for LATE** | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ |
| GMM               | -311.890**  | -28.504      | -260.520**  | -26.410      | -31.122      |
|                   | (100.413)   | (121.165)    | (113.466)   | (260.191)    |
| GMM-Lasso         | -454.203**  | -191.738     | -255.659    | -22.092      | 252.781      |
|                   | (186.231)   | (166.964)    | (218.677)   | (558.109)    |
| DRSMD-Lasso       | -329.612*** | -602.840**   | -128.813    | -356.685*    | -171.432*    | 1881.963    | 120.352    | 265.458    |
|                   | (126.950)   | (132.003)    | (194.470)   | (92.768)     | (90.212)     | (1774.876)  | (172.107)  | (215.602)  |
| $N$               | 3504         | 3596         | 3941         | 4599         | 2932         |

Table S17: Heterogeneous Treatment Effects of Medicaid on Out of Pocket Cost

Note: *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used and the age groups. The interaction term is Medicaid × hhincome. The expression for LATE is $\theta_{w0} + \theta_{wx0}E(X)$. 
## Panel A: Heterogeneous treatment effects

|                       | Out of Pocket Cost | Age: 21 - 29 | Age: 30 - 38 | Age: 39 - 47 | Age: 48 - 56 | Age: 57 - 64 |
|-----------------------|--------------------|--------------|--------------|--------------|--------------|--------------|
| **Estimator for \( \theta_{\omega 0} \)** | \( Z_1 \) | \( (Z_1, Z_1 X_1) \) | \( Z_1 \) | \( (Z_1, Z_1 X_1) \) | \( Z_1 \) | \( (Z_1, Z_1 X_1) \) | \( Z_1 \) | \( (Z_1, Z_1 X_1) \) | \( Z_1 \) | \( (Z_1, Z_1 X_1) \) |
| GMM                   | -274.917***        | -127.619     | -144.510     | -103.605     | -69.921      |
|                       | (91.821)           | (88.876)     | (73.094)     | (66.604)     | (100.480)    |
| GMM-Lasso             | -408.080**         | -217.524     | -248.498*    | -158.129     | -16.772      |
|                       | (187.386)          | (194.537)    | (146.186)    | (160.247)    | (302.841)    |
| DRSMD-Lasso           | -328.977*** -361.671** | -144.073     | -142.501     | -92.484 -115.336 | -174.760* -171.099* | 158.324 158.070 |
|                       | (122.904) (145.690) | (129.297) (169.190) | (88.281) (89.513) | (89.460) (90.094) | (180.997) (181.520) |
| **Estimator for \( \theta_{\omega 0} \)** | \( Z_1 \) | \( (Z_1, Z_1 X_1) \) | \( Z_1 \) | \( (Z_1, Z_1 X_1) \) | \( Z_1 \) | \( (Z_1, Z_1 X_1) \) | \( Z_1 \) | \( (Z_1, Z_1 X_1) \) | \( Z_1 \) | \( (Z_1, Z_1 X_1) \) |
| GMM                   | -360.531           | 438.043      | 341.411      | 3124.245     | 160.364      |
|                       | (595.971)          | (1474.901)   | (692.758)    | (3527.129)   | (681.382)    |
| GMM-Lasso             | -2695.988*         | -1430.592    | -751.087     | 3216.942     | 620.170      |
|                       | (1567.443)         | (2019.704)   | (1111.768)   | (4538.626)   | (833.799)    |
| DRSMD-Lasso           | 142.752 2647.014   | 67.49 3.327  | -429.745 1630.753 | -178.469 -1515.266 | 741.691 1129.482 |
|                       | (97.306) (5599.135) | (196.456) (6734.098) | (364.850) (1734.534) | (358.992) (2158.009) | (699.283) (1056.448) |

## Panel B: Homogeneous treatment effects

|                       | GMM | GMM-Lasso | DRSMD-Lasso |
|-----------------------|-----|-----------|-------------|
|                       | -280.010*** | -276.059*** | -326.593*** |
|                       | (91.172) | (91.62) | (122.625) |
|                       | -121.970 | -126.700 | -143.056 |
|                       | (90.067) | (88.798) | (129.513) |
|                       | -140.874* | -143.404* | -101.027 |
|                       | (72.829) | (72.905) | (88.614) |
|                       | -94.666 | -101.783 | -175.448** |
|                       | (67.076) | (66.588) | (89.326) |
|                       | -69.635 | -69.778 |
|                       | (100.400) | (100.438) | (181.422) |
|                       | -454.731** | -389.029** | -326.593*** |
|                       | (187.884) | (186.352) | (122.35) |
|                       | -239.756 | -211.541 | -129.513 |
|                       | (198.492) | (194.269) | (128.689) |
|                       | -256.614* | -253.578* | -88.614 |
|                       | (145.706) | (145.766) | (88.326) |
|                       | -156.003 | -176.968 | (89.209) |
|                       | (158.649) | (158.521) | (89.209) |
|                       | -15.345 | -22.411 |
|                       | (303.056) | (302.061) | (181.422) |
|                       | -175.448** | -175.753** | -175.448** |
|                       | (160.566) | (162.147) | (160.566) |
|                       | -15.345 | -22.411 |
|                       | (160.566) | (162.147) | (160.566) |
|                       | 3044 | 3596 | 3941 |
|                       | 4599 | 2932 |

**Table S18:** Heterogeneous Treatment Effects of Medicaid on Out of Pocket Cost

Note: *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used and the age groups. The interaction term is Medicaid× TANF.
| Estimator for LATE | Out of Pocket Cost | Age: 21 - 29 | Age: 30 - 38 | Age: 39 - 47 | Age: 48 - 56 | Age: 57 - 64 |
|-------------------|--------------------|--------------|--------------|--------------|--------------|--------------|
| GMM               | \( Z_1 \)          | -280.837***  | -108.616     | -136.800*    | -75.073      | -69.429      |
|                   | \((Z_1, Z_1X_1)\)  | (92.479)     | (107.224)    | (73.492)     | (73.867)     | (100.347)    |
| GMM-Lasso         | \( Z_1 \)          | -519.644***  | -279.585     | -265.460*    | -128.750     | -14.868      |
|                   | \((Z_1, Z_1X_1)\)  | (202.286)    | (217.774)    | (146.215)    | (171.223)    | (303.081)    |
| DRSMD-Lasso       | \(-323.070***\)    | -252.134     | -141.147     | -142.357     | -102.189     | -176.390**   |
|                   | \((Z_1, Z_1X_1)\)  | (122.184)    | (197.461)    | (130.029)    | (221.220)    | (89.195)     |
|                   | \(-140.195\)       | (89.177)     | (93.360)     | (89.035)     | (181.463)    | (181.433)    |
|                   | \(-162.357\)       |             |             |             |             |             |
|                   | \((Z_1, Z_1X_1)\)  |             |             |             |             |             |
|                   | \(-181.357\)       |             |             |             |             |             |
|                   | \((Z_1, Z_1X_1)\)  |             |             |             |             |             |
|                   | \(-181.433\)       |             |             |             |             |             |
| \( N \)           | 3504               | 3596         | 3941         | 4599         | 2932         |

Table S19: Heterogeneous Treatment Effects of Medicaid on Out of Pocket Cost

Note: *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used and the age groups. The interaction term is Medicaid × TANF. The expression for LATE is \( \theta_{w0} + \theta_{wx0} E(X) \).
### Panel A: Heterogeneous treatment effects

| Estimator for $\theta_{wx}$ | $Z_1$ | $(Z_1, Z_1 X_1)$ | $Z_1$ | $(Z_1, Z_1 X_1)$ | $Z_1$ | $(Z_1, Z_1 X_1)$ | $Z_1$ | $(Z_1, Z_1 X_1)$ |
|-----------------------------|-------|------------------|-------|------------------|-------|------------------|-------|------------------|
| **Out of Pocket Cost**      |       |                  |       |                  |       |                  |       |                  |
| Age: 21 - 29                |       |                  |       |                  |       |                  |       |                  |
| GMM                         | -534.801* | (278.120)   | -285.780 | (144.724)     | -263.638* | (146.954)     | -241.214 | (241.115)     |
| GMM-Lasso                   | -555.430 | (361.855)   | -374.526 | (167.480)     | -440.806*** | (230.489)     | -566.475** | (427.954)     |
| DRSMD-Lasso                 | -425.270*** | (159.063)   | -685.989** | (138.335)    | -165.238 | (125.410)     | -236.941 | (105.255)     |
| **Estimator for $\theta_{wx0}$** |       |                  |       |                  |       |                  |       |                  |
| Age: 30 - 38                |       |                  |       |                  |       |                  |       |                  |
| GMM                         | -534.801* | (278.120)   | -285.780 | (144.724)     | -263.638* | (146.954)     | -241.214 | (241.115)     |
| GMM-Lasso                   | -555.430 | (361.855)   | -374.526 | (167.480)     | -440.806*** | (230.489)     | -566.475** | (427.954)     |
| DRSMD-Lasso                 | -425.270*** | (159.063)   | -685.989** | (138.335)    | -165.238 | (125.410)     | -236.941 | (105.255)     |
| **Estimator for $\theta_{wx0}$** |       |                  |       |                  |       |                  |       |                  |
| Age: 39 - 47                |       |                  |       |                  |       |                  |       |                  |
| GMM                         | -534.801* | (278.120)   | -285.780 | (144.724)     | -263.638* | (146.954)     | -241.214 | (241.115)     |
| GMM-Lasso                   | -555.430 | (361.855)   | -374.526 | (167.480)     | -440.806*** | (230.489)     | -566.475** | (427.954)     |
| DRSMD-Lasso                 | -425.270*** | (159.063)   | -685.989** | (138.335)    | -165.238 | (125.410)     | -236.941 | (105.255)     |
| **Estimator for $\theta_{wx0}$** |       |                  |       |                  |       |                  |       |                  |
| Age: 48 - 56                |       |                  |       |                  |       |                  |       |                  |
| GMM                         | -534.801* | (278.120)   | -285.780 | (144.724)     | -263.638* | (146.954)     | -241.214 | (241.115)     |
| GMM-Lasso                   | -555.430 | (361.855)   | -374.526 | (167.480)     | -440.806*** | (230.489)     | -566.475** | (427.954)     |
| DRSMD-Lasso                 | -425.270*** | (159.063)   | -685.989** | (138.335)    | -165.238 | (125.410)     | -236.941 | (105.255)     |
| **Estimator for $\theta_{wx0}$** |       |                  |       |                  |       |                  |       |                  |
| Age: 57 - 64                |       |                  |       |                  |       |                  |       |                  |
| GMM                         | -534.801* | (278.120)   | -285.780 | (144.724)     | -263.638* | (146.954)     | -241.214 | (241.115)     |
| GMM-Lasso                   | -555.430 | (361.855)   | -374.526 | (167.480)     | -440.806*** | (230.489)     | -566.475** | (427.954)     |
| DRSMD-Lasso                 | -425.270*** | (159.063)   | -685.989** | (138.335)    | -165.238 | (125.410)     | -236.941 | (105.255)     |

| $N$                          | 3504       | 3596       | 3941       | 4599       | 2932       |

Table S20: Heterogeneous Treatment Effects of Medicaid on Out of Pocket Cost

Note: *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used and the age groups. The interaction term is Medicaid×smoke.
### Average Treatment Effects

| Estimator for LATE | Age: 21 - 29 | Age: 30 - 38 | Age: 39 - 47 | Age: 48 - 56 | Age: 57 - 64 |
|-------------------|--------------|--------------|--------------|--------------|--------------|
|                   | $Z$          | $(Z_1, Z_1X_1)$ | $Z_1$        | $(Z_1, Z_1X_1)$ | $Z_1$        | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ | $Z_1$ | $(Z_1, Z_1X_1)$ |
| GMM               | -276.588***  | -112.372     | -133.077*    | -87.451      | -62.898      |
|                   | (90.822)     | (92.812)     | (76.291)     | (68.967)     | (102.980)    |
| GMM-Lasso         | -452.537**   | -240.252     | -238.778     | -142.495     | 13.630       |
|                   | (188.316)    | (198.422)    | (155.644)    | (165.046)    | (313.414)    |
| DRSMD-Lasso       | -326.325***  | -330.648***  | -142.732     | -167.472     | -100.227     | -76.031        | -177.557**        | -187.052** | 154.507 | 169.926 |
|                   | (122.502)    | (120.503)    | (129.249)    | (127.050)    | (87.164)     | (89.525)       | (89.010)          | (91.782)    | (180.620) | (168.551) |

$N$  
| 3504 | 3596 | 3941 | 4599 | 2932 |

Table S21: Heterogeneous Treatment Effects of Medicaid on Out of Pocket Cost

Note: *** Significant at 1%, ** at 5%, * at 10%. Each row shows the estimates and robust standard errors for the same type of estimator. In the columns, we present the instruments these estimators used and the age groups. The interaction term is Medicaid $\times$ smoke. The expression for LATE is $\theta_{w0} + \theta_{wx0}E(X)$. 