Electroinduction disk sensor of electric field strength

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Abstract. Measurement of the level of electric fields exposure to the technical and biological objects for a long time is an urgent task. To solve this problem, the required electric field sensors with specified metrological characteristics. The aim of the study is the establishment of theoretical assumptions for the calculation of the flat electric field sensors. It is proved that the accuracy of the sensor does not exceed $3\%$ in the spatial range $0<a<0.2$ and in the entire spatial range of measurement is negative. Using the methods of calculation of electrostatic fields, this article examines single-axis electromotive disk sensor radius $R$, and the estimation errors caused by the inhomogeneity of the field. The maximum of this error is $3\%$ in the spatial range from $0$ to $5R$ to the source field that allows you to design better quality sensors used in different measuring systems of wide application.

1. Introduction
Around us the electric field causes an adverse impact both on technical and biological objects. To study the effect of electric fields on these objects the sensors and measuring instruments on their basis are necessary. The development of such measuring instruments began with the 70-ies of the last century. Analyzing manufactured and designed devices with a flat sensitive elements such as [1], [2] and [3] we see that the shape of the sensing elements is a form of a petal [1] and [2], or the shape of a flat square [3], which is made of conductive material. According to the information about the approved types of measurements of Federal information fund by providing a single measurement of the Russian Federation [4] referred to such measurements as IESP-01, which are sensitive elements made in the form of a flat electrode. However, the error of such measurements is 20%, which is unacceptable in accordance with the new regulations on electromagnetic safety of production and life companies.

In this labor, the study is subjected to single-axis flat panel detector of the electric field made in the form of a conductive disc. The reason for this study was that currently on the market there are a large number of devices for measuring the strength of the electric field [4], that are not supported by research, articles, patents for utility model and invention. In this regard, on the market of instruments for measuring electric field strength than does not confirm its metrological characteristics. Therefore, they ascribe errors reaching 20 %, which is not always acceptable.

2. Formulation of the problem
In the physical basis of most sensors is the phenomenon of electrostatic induction, that is, the appearance of electric charges on the conductor surface under the action of an electric field. Such charges are called induction. In a constructive-based sensor can lie as dielectric and conductive substrate. The difference in these sensors is that the sensors with a dielectric substrate have floating potential of the midpoint, and the sensors of the conductive substrate have the potential mid-point equal to the potential of a point in space place of the sensor. Our study will be single-axis electromotive sensor with a conductive substrate in the form of a disc. The aim of the study is the creation of electric field intensity sensor with an accuracy not more than 5 %

3. Theory
Theory of the sensor operation is based on the consideration of a conducting disk of thickness $h$ and radius $R$ ($h << R$) is placed in a uniform and non-uniform quasi-static electric field $E_0 = E \cdot \sin \omega t$. Hereinafter simply $E_0$.

Consider the interaction of the sensor tension with electric fields of different discontinuity. As these fields will choose a uniform electric field and a point source. A uniform field will be considered
as the reference model field. In relation to it will be evaluated the accuracy of the sensor working in real conditions, where the fields are often uneven. The non-uniform of the fields to be considered as one part in assessing the error limits of the sensor. Further, we assume that the sensor accuracy in the other uniform fields less than the field of the point source. Field of the point source is chosen from the condition of maximum that can be modeled in the analysis of the behavior of the sensor in a non-uniformly field. It should be noted that there are other sources of electric fields with greater heterogeneity than the point source. These include the electric field of the dipole and quadruple [5]. However, the electric field of these sources rapidly decreases inversely proportional to the third and fourth degree, respectively, of the distance from the investigated point of the field. At distances where all these fields exist, measurement techniques, based on the phenomenon of electrostatic induction, unsuitable.

The sensor is a conductive circular plate 1, of radius R and thickness \( h \) \( (h \ll R) \). On two opposite surfaces of the plate are conductive sensitive elements 2 and 3, as shown in Figure 1.

**Figure 1.** The sensor of the electric field with the conductive substrate in the form of a circular plate.

The sensitive elements 2 and 3, a thin conductive layer with thickness \( \delta \), are the same size and shape, and located at a distance \( l \) from the surface of conductive plate 1 (Figure 2).

**Figure 2.** Design parameters of the sensor and its interaction with an electric field.
Adopted assumptions $\delta \ll h$ and $l \ll R$ give reason to believe that the potential of the sensitive elements 2 and 3 is equal to the potential of the conductive plate 1, and the sensing element is something other than the surface of the plate. Further other measures will be taken, which allows considering conductive sensing element surface to a conducting plate. Thus, the sensor in General is something else, like a flat conductive plate in the form of a disk.

A. Sensor in a homogeneous field

Place a conductive circular plate in a uniform electric field perpendicular to its force lines. Then, according to the theory, the plate will occur the separation of electric charges: positive charges will move in the direction of the field, the negative against of this direction. As a result, on one surface of the plate induced by a negative and the other positive charges, separated by a plane of the electrical and geometrical neutral (see Fig. 2, b). Induced on the surfaces of the plates of the external electric field electric charges create their own electric field directed opposite to the external field. As a result, within plate resultant field will be zero. In this case all points of the plate will have the same potential equal to the potential of a point in space the place of the plate.

Value of these charges can be determined from the interaction between a conducting plate with a uniform electric field. The solution of this problem shows that the surface of the conducting plane, there is only the normal part $E_n$ of the electric field intensity equal to the intensity of the external uniform field $E_0$. This electric field according to the Gauss theorem, and determines the surface density of electric charge

$$\sigma = -2\varepsilon_0 E_n = -2\varepsilon_0 E_0,$$

where $\varepsilon$ – dielectric permeability of the medium surrounding the conductive disk; $\varepsilon_0$ – dielectric constant. As can be seen from the formula (1) surface charge density on a conducting disk in a uniform field constant.

Therefore, in this case the charge is distributed on the surface of the disc evenly.

The total charge on the plane of the conductive disk can be determined from the formula

$$Q = \int \sigma \cdot dS$$

Then the size of the charges according to the formula (2), induced on the two planes of the conductive disk can be calculated by introducing polar coordinates

$$Q_0 = \pm \int \sigma \cdot dS = \pm 2\varepsilon_0 E_0 \int_0^\pi \int_0^\rho \rho d\rho d\varphi = \pm 2\pi\varepsilon_0 R^2 E_0,$$

where $Q_0$ – the charge induced by a homogeneous field; the sign ”-” – refers to the upper plane of the disk, and the sign "+" to the lower plane of the disk (see Fig. 2, b).

It follows from formula (3) that the charges on the surfaces of the conducting disk are proportional to the electric field strength. Therefore, they can act as a measure of a strength. If you remove them from the surfaces of the plate and measure, you can get an electric field strength sensor. Such a sensor was described at the beginning of the article. Since the sensor has two sensing elements, a dual sensor is obtained. Therefore, when the sensor is switched on in differential mode, it is total, that is, the differential charge will be doubled, according to expression

$$Q_0^{\text{dif}} = (+Q_0) - (-Q_0) = 2Q_0 = 4\pi\varepsilon_0 R^2 E_0.$$
surfaces of a conducting disk shows their dependence on the radius $R$ of the disk plate. For further investigations, it is more convenient to have a certain value that does not depend on the dimensions of the plate and the value of the electric field strength, and such a value can be the normalized value of the charge

$$Q_{nor} = \frac{Q_0}{\pi \varepsilon_0 R^2 E_0} = 2.$$ (5)

**B. A sensor in an inhomogeneous field of a point source**

We place the conducting disk plate in the field of the point source. As a point source, we will consider a positive point charge $q$ at a distance $d$ from the conductive plate. And in this case negative plates are induced on one side of the conducting plate, and positive charges on the other. Field lines and induced on the conducting plate charges is presented in figure 3, and the equipotential surface of a point source in the presence of the flat electrode obtained by simulation in the Elcut program, shown in Fig. 3, b.

![Figure 3. View of power lines a), and equipotential surfaces, b) in the system of a point charge - conducting disk plate.](image)

Using the method of images in the plane it is possible to get the potential near the surface of the conducting plane, the normal part of the electric field and surface charge density on the surface of the conductive disk

$$\sigma = \frac{2 \pi \varepsilon_0 d^3}{r^3} E_{it},$$ (6)

where $r$ - the distance from the charge $q$ to the observation point; $E_{it}$ - the intensity of the non-uniform electric field at the observation point.

Using formula (6), one can know the surface charge density at each point of the surface of the conducting plate. Consider the point of the surface of the conducting plate at a distance $\rho$ from its
center (see Fig. 3), provided that the point charge \( q \) is located above the center of the plate. Since \( r \) in expression (6) is the distance from the point charge to the observation point, we express it in terms of the characteristic \( \rho \)

\[
    r = (d^2 + \rho^2)^{\frac{1}{2}}. \tag{7}
\]

Then the expression for the surface charge density as a function of the distance \( \rho \) from the center of the conducting plate will have the form

\[
    \sigma(\rho) = -\frac{2 \varepsilon \rho d'^1}{(d^2 + \rho^2)^\frac{5}{2}} E_x = -\frac{2 \varepsilon \rho \rho'^1}{1 + \left(\frac{\rho}{d}\right)^2} E_x \tag{8}
\]

As can be seen from formula (8), the surface density of charges on a conducting disk, unlike a homogeneous field, is not constant over the entire area of the conducting plate, and depends not only on the distance \( \rho \) from the center of the plate, but also on the distance of the plate to the source of the field \( d \). Consequently, in this case the charge is distributed unevenly over the surface of the plate.

The values of the charges induced on the two planes of the plate are formed by an external inhomogeneous field and are determined by the expression (2). Using the tables of integrals [6] we find these charges

\[
    Q_{H} = \pm \iint \sigma_{\rho} \cdot dS = \int_0^{\pi} \int_0^{2\pi} \frac{2 \varepsilon \rho d'}{(d^2 + \rho^2)^\frac{5}{2}} \rho \rho' \cdot d\varphi = \pm 2 \pi \varepsilon \rho R^2 E_{H} \left[ \frac{2}{(\frac{R}{d})^2} \left(1 - \frac{1}{1 + \left(\frac{R}{d}\right)^2}\right) \right] \tag{9}
\]

where the sign "-" refers to the upper plane of the disk, and the sign "+" to the lower plane of the disk (see Fig. 2, b).

When the sensor is switched on in differential mode, its total differential charge will be equal to

\[
    Q_{H}^{\text{total}} = (-Q_{H}) - (-Q_{H}) = 2Q_{H} = \pm 4 \pi \varepsilon \rho R^2 E_{H} \left[ \frac{2}{(\frac{R}{d})^2} \left(1 - \frac{1}{1 + \left(\frac{R}{d}\right)^2}\right) \right]. \tag{10}
\]

For the convenience of further research, we introduce the normalization of the size of the charge, similarly, as in the investigation of a homogeneous electric field
Analyze the differences in the behavior of the electric field strength sensor in extreme cases - in a homogeneous (reference) field and in the field of a point source with a significant heterogeneity.

First of all, let us consider how electric charges are distributed on the surfaces of a conducting plate-sensor. We use the mathematical editor MathCAD and construct graph of the electric charge density on the surface of sensor, that in a homogeneous and inhomogeneous field, depending on the relative distance to the source of the field $a = R/d$, where $R$ – the radius of the sensor plate; $d$ – the distance from the center of the sensor to the source of the field (Fig. 4).

It follows from Fig. 4 that the electric charge density on the sensor surface in a uniform field is constant across the entire sensitive element, and in an inhomogeneous field it decreases to the edge of the sensitive element. And this decrease is the stronger the higher the heterogeneity of the field, that is, the closer the sensor to the source of the field.

We estimate the infelicity of the sensor from the inhomogeneity of the electric field. To this end, we use formulas (4) and (10) and a normalization $a = R/d$, where $R$ – the radius of the sensor plate; $d$ – the distance from the center of the sensor to the source of the field. The normalizing measure $a$ characterizes the proximity of the sensor to the source of the field. Thus, the smaller $a$, further the sensor is from the source of the field, and the field becomes more homogeneous. The most acceptable range for changing parameter $a$ is from 0 to 1.

Taking into account this normalization, the formula for the error from the inhomogeneity of the electric field takes the form

$$\delta = \frac{Q_{\text{inh}}^{\text{exp}} - Q_{\text{inh}}^{\text{ref}}}{Q_{\text{inh}}^{\text{ref}}} = \left[ \frac{2}{a^2} \left[ 1 - \frac{1}{\sqrt{1 + a^2}} \right] - 1 \right] \times 100 \%. \quad (12)$$
We use the mathematical editor MathCAD and build graphs of the infelicity from the inhomogeneity of the electric field as a function of the characteristic $a$. The infelicity graph is shown in Fig. 5.

4. The results of the surveys
Surveys have shown (see Fig. 5) that the accuracy of the sensor in all spatial measurement range of negative and already at $a > 0.2$ beyond 3%. In this regard, the sensor is suitable for measurements at distances from the source $d$ field that is five times the radius of the conductive disk of the sensor ($d=5R$).

5. Conclusion
From the survey results it follows that the considered tension sensor has an infelicity from the inhomogeneity of the field up to 3% in the spatial range from 0 to $5R$ from the field source, where $R$ - the radius of the disk plates of the sensor. The sensor gives lower values of charges in an inhomogeneous field, this may lead to a biased assessment of the influence of the electric field on the technical and biological objects. The advantage of sensor is its simplicity, allowing to make its methods of deposition of conductive and dielectric layers on a thin conducting substrate. In addition, because of its small dimensions, the sensor allows you to place it inside the device.

In conclusion, we assume that the solution of the problem of minimizing the error from the inhomogeneity of the field lies in the optimization of the sensor sensitive elements size. Further research will be conducted in this direction.

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