Stability Analysis for Equivalent Circular Cylindrical Shell

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Abstract. Ring truss antenna is an ideal structure for large satellite antenna, which can be equivalent to circular cylindrical shell model. Based on the high-dimensional nonlinear dynamic vibration and bifurcation theory, we focus on the nonlinear dynamic behavior for breathing vibration system of ring truss antenna with internal resonance. The nonlinear transformation and Routh-Hurwitz criterion are used to analyze the stability of equilibrium point after disturbance, and the theoretical analysis is verified by numerical simulation. It provides a reference to ensure the stability and control parameters of satellite antenna in complex space environment.

1. Introduction

Significant achievements in the field of aerospace indicate the advanced level of science and technology in a country. In order to effectively solve the limitation of carrying weight, cost and space of rocket during the satellite launch stage, the large deployable ring truss antenna meets the development requirements of large calibers, high precision and light weight. It has a broad prospect in the field of aerospace engineering [1-2]. In the work of exploring stability and structural performance of antenna, Reference [3] simplified the ring truss antenna into equivalent circular cylindrical shell structure. This structure has the advantages of high strength and stiffness, which is widely used as basic structural unit in aerospace, machinery manufacturing and civil construction. Therefore, it is of great value to study the nonlinear vibration and stability analysis of equivalent circular cylindrical shell structure.

In practical applications, the research on equivalent circular cylindrical shell of ring truss antenna has attracted great interest nowadays. Liu et al. [4] revealed the nonlinear vibration of composite laminated circular cylindrical shell clamped along the generatrix for the first time. Chen et al. [5] focused on the equivalent continuous cylindrical shell model based on beam elements and numerically simulated periodic and chaotic nonlinear vibrations under thermal excitation. Based on the equivalent process of cylindrical shell structure, Liu et al. [6] considered the influence of thermal excitation and damping coefficient on the nonlinear dynamics of ring truss antenna. Researching on stability analysis of cylindrical shell model, Chakraborty and Dey [7] analyzed the nonlinear stability characteristics of composite cylindrical shell panel under thermal mechanical loading. Park and Kim [8] focused on the stability of completely free circular cylindrical shell under follower force and obtained that the shell structure can be analyzed by beam model within a certain range. Karagiozis et al. [9] established the theoretical model of shells with simply supported ends and compared it with experiment to explain the nonlinear dynamics and stability of thin circular cylindrical structure. In the study of thin circular cylindrical shells without clamped boundary conditions, Paak et al. [10] found that the shell was...
unstable in supercritical Hopf bifurcation, resulting in stable periodic motion. Dey et al. [11] discussed
the parametric vibration of composite circular cylindrical shell with simply supported and revealed its
nonlinear response and chaotic phenomenon.
This overall structure shows as follows. In section 2, the equivalent cylindrical shell model of ring truss antenna is presented, and the six-dimensional average equation in rectangular coordinate is
obtained by multiple scales method. In section 3, we obtain the stable region, unstable region and
critical bifurcation conditions of the equilibrium point according to nonlinear transformation and
Routh-Hurwitz criterion. Then we analyze the stability of system after small disturbance and verify the
accuracy of theoretical analysis by numerical simulations. In section 4, conclusions in this paper are
presented.

2. System and Average Equation of Equivalent Circular Cylindrical Shell Structure
We focus on the equivalent circular cylindrical shell model of ring truss antenna with transverse
vibration as the main vibration, which can be described by the following three degrees of freedom
nonlinear dynamic system [12]:

\[
\begin{align*}
\dot{w}_i + \mu_i \dot{w}_i + \omega_i^2 w_i + f_i \cos(\Omega t) \omega_i^2 &+ \alpha_{i1} w_1^2 + \alpha_{i2} w_2^2 + \alpha_{i3} w_3^2 + \alpha_{i4} w_4^2 + \alpha_{i5} w_5^2 \\
+ \alpha_{i6} w_6^2 + \alpha_{i7} w_7 w_2 + \alpha_{i8} w_8 w_3 + \alpha_{i9} w_9 w_1 + \alpha_{i10} w_1^2 w_2 + \alpha_{i11} w_2^2 w_1 + \alpha_{i12} w_2^2 w_3 \\
+ \alpha_{i13} w_3^2 w_2 + \alpha_{i14} w_4 w_1 + \alpha_{i15} w_3 w_1 + \alpha_{i16} w_1 w_3 = F_i \cos(\Omega t)
\end{align*}
\]  

(1)

where \( i = 1, 2, 3 \), \( w_i \) are the first, second and third order modes for nonlinear breathing vibration
respectively. \( \mu_i \) represent the structure damping coefficients. \( \omega_i \) are natural frequencies of the
corresponding linear system. \( f_i \) and \( F_i \) denote parametric excitation and denote external excitation of
the ring truss antenna. \( \Omega \) is the frequency of thermal excitation. \( \alpha_{ij} (j = 1, \ldots, 16) \) are non-dimensional
coefficients.

Consider the case of 1:1:1 internal resonance for the circular cylindrical shell. The resonant
relations are given as follows:

\[
\omega_1^2 = \Omega_1^2 + \epsilon \sigma_1, \quad \omega_2^2 = \Omega_2^2 + \epsilon \sigma_2, \quad \omega_3^2 = \Omega_3^2 + \epsilon \sigma_3
\]  

(2)

where \( \sigma_i \) are three detuning parameters. \( \epsilon \) is a small parameter. In order to simplify the analysis, we
assume that \( \Omega = \Omega_1 = \Omega_2 = \Omega_3 = 1 \) here to study and introduce the scale transformations for variables

\( \alpha_{ij} \rightarrow \epsilon \alpha_{ij}, \quad \mu_i \rightarrow \mu_i, \quad F_i \rightarrow \epsilon F_i \)

The average equation is obtained by using the methods of multiple scales

\[
\dot{x} = Lx + \hat{X}(x)
\]  

(3)

where \( x = (x_1, x_2, x_3, x_4, x_5, x_6) \) in \( \mathbb{R}^6 \), \( \hat{X} = (H_{1,1}, H_{1,2}, H_{2,1}, H_{2,2}, H_{3,1}, H_{3,2}) \) is the vector valued
polynomials in variables of \( x_k (k = 1, \ldots, 6) \). \( L \) is the matrix and can be shown as

\[
L = \frac{1}{2} \sum_{i=1}^{3} \left[ \partial_{2i-1,1}^{6,6} (\sigma_i) + \partial_{2i-1,2}^{6,6} (\sigma_i) \right]
\]

\( \partial_{p,q}^{u,v} (M) \) denotes \( u \times v \) block matrix with \( (p, q) \)-th block \( M \) and all other blocks are zero matrices [13].
Assuming that

\[
d_{i,j} = \begin{cases} 
3 \alpha_{ij} / 2, & j < 10 \\
\alpha_{ij} / 2, & j \geq 10 
\end{cases} (j = 1, \ldots, 16)
\]

Then \( H_{i,l} (i = 1, 2, 3; l = 1, 2) \) of equation (3) can be expressed as
\[ H_{i,l} = -\mu_i x_{2i+l-2} / 2 + (AB + CD) x_l - (l - 1) F / 4 \]

where

\[ x_l = (-1)^l (x_{3-l}, x_{5-l}, x_{7-l})^T \]

and

\[ A = \sum_{n=1}^{3} \xi_{1,n}^3 (x_{2n}^2 + x_{2n}^2) \]

\[ B = \sum_{n=1}^{3} \xi_{1,n}^3 (d_{i,n+1}) + \sum_{n=1}^{2} \xi_{1,n}^3 (d_{i,n+8} + \xi_{1,n}^3 (d_{i,2n+9}) + \xi_{1,n}^3 (d_{i,2n+13})) \]

\[ C = \sum_{n=1}^{3} \xi_{1,n}^3 (x_{n',-5n+7} x_{3n-n'+3} + x_{n',-5n+8} x_{3n-n'+4}) \]

\[ D = \sum_{n=1}^{3} \xi_{1,n}^3 (d_{i,16}) + 2 \sum_{n=1}^{2} \xi_{1,n}^3 (d_{i,3n+9}) + \xi_{1,n}^3 (d_{i,17-3n}) + \xi_{1,n}^3 (d_{i,16-3n}) \]

3. Stability Analysis for Equivalent Circular Cylindrical Shell Structure

In actual operation, the control system may be interfered by external or internal factors such as the fluctuation of load or energy, the change of environmental conditions and system parameters. If the system deviates from the equilibrium state after being disturbed and returns to the original equilibrium state with certain accuracy through self-regulation when the disturbance disappears, then the system is said to be stable. Usually, the structural performance and stability of satellite antenna will change after disturbance. In order to ensure the effective operation of satellite antenna in complex space environment, it is one of the important tasks of dynamics and control theory to analyze and ensure the stability of the system.

According to the coordinate translation principle, the non-zero equilibrium point of differential equation can be translated to the equilibrium point of zero solution, so we only need to consider the stability near the equilibrium point. The characteristic polynomial corresponding to Jacobian matrix of system can be computed as follows:

\[ f(\lambda) = \sum_{s=0}^{6} a_s \lambda^{6-s} \]  \hspace{1cm} (4)

In this section, we discuss the case that the characteristic polynomial has four zeros and a pair of pure imaginary complex eigenvalues at the equilibrium point. For convenience of analysis, we choose parameters of equation (3) as follows: \( \sigma_1 = \sigma_2 = 0, \sigma_3 = 2 \). Then select \( \mu_i \) as perturbation parameters and introduce perturbation transformation \( \mu_i = \varepsilon_i \). According to Routh-Hurwitz stability criterion, if the equilibrium point is stable, the following conditions (as shown in Table 1) need to be satisfied. The symbol \( \Delta_k (k = 1, \cdots, 6) \) represent leading principle minors of Hurwitz determinant, where \( \Delta_3 = (\varepsilon_1 + \varepsilon_2) \eta_1 / 16 \) and \( \Delta_4 = (\varepsilon_1 + \varepsilon_2) \eta_2 / 256 \). The critical surface \( \eta_1, \eta_2 \) are shown in Figure 1-3. Then we can obtain the stable and unstable region of the equilibrium point.

Table 1. The conditions of Routh-Hurwitz stability criterion.

| Leading principle minors | Determinant conditions |
|--------------------------|------------------------|
| \( \Delta_1 \Delta_2 \)  | \( \Delta_1 > 0, \Delta_2 > 0 \) |
| \( \Delta_3 \Delta_4 \)  | \( \varepsilon_1 + \varepsilon_2 > 0, \eta_1 > 0, \eta_2 > 0 \) |
| \( \Delta_5 \Delta_6 \)  | \( \varepsilon_1 \varepsilon_2 \varepsilon_3 > 0, \varepsilon_1 + \varepsilon_2 \neq 0 \) |
In order to verify the validity of theoretical analysis results, we conduct numerical analysis on the average equation. By selecting parameters $\varepsilon_i = 10$ in the stable region, we can see that the trajectory rotates from outside to inside and close to the initial point (as shown in Figure 4). In the same way, we can see that the trajectory rotates from inside to outside and far away from the initial point by selecting parameters $\varepsilon_i = -10$ in the unstable region (as shown in Figure 5). The movement trend can be perceived more intuitively by drawing the phase portraits of plane $(x_1, x_2)$ in different intervals of stable and unstable regions (as shown in Figure 6). $(x_3, x_4)$ plane and $(x_5, x_6)$ plane are similar.
Figure 4. Trajectory of stable region on different planes

Figure 5. Trajectory of unstable region on different planes.

Figure 6. Phase portraits of \((x_1, x_2)\) plane: (a) \(x_1 \in [-2, 2]\) in stable region; (b) \(x_1 \in [-3, 3]\) in stable region; (c) \(x_1 \in [-2, 2]\) in unstable region; (d) \(x_1 \in [-3, 3]\) in unstable region.

We discuss the influence of time \(t\) on the system by time series plot. As shown in Figure 7, two figures (a) and (b) are the trend of \(x_1\) and \(x_2\) direction over time in stable region. It can be seen that \(x_1\) and \(x_2\) tend to zero with time increases gradually. The right (c) and (d) graphs are the trend of \(x_1\) and \(x_3\) direction over time in unstable region. We can see that as time \(t\) increases, \(x_1\) and \(x_2\) don’t approach to stable state in the region and the amplitude of vibration increases. The corresponding Figure 4-6 verify the one-to-one correspondence between time and phase portraits.
When we know the determined dynamic equation, the behavior of system, namely its motion and change, may be determined by initial conditions. Many nonlinear systems are sensitive to initial conditions under certain conditions. It is a general behavior of nonlinear dynamical systems and a reflection of the inherent randomness. Considering the sensitivity of the system in stable and unstable regions to the selection of different initial conditions, we get Figure 8-9. With the increase of time, the initial conditions have certain disturbances on the system, and then form periodicity in the region, which makes us have reference for the selection of different initial conditions in solving practical problems.

4. Conclusion
Due to the interaction between different vibration modes, there are many internal resonances in high-dimensional nonlinear dynamic systems. This mutual transfer and conversion of energy may lead to violent nonlinear vibration, which will have incalculable consequences when serious. Therefore, it is very important to study the stability of satellite antenna in operation. We use the local bifurcation
theory of high-dimensional nonlinear system to research the complex dynamic behavior that may occur near the critical point of large deployable antenna, and discuss the situation that the characteristic polynomial has four zeros and a pair of pure imaginary complex eigenvalues at the equilibrium point through theoretical analysis. According to the nonlinear transformation and Routh-Hurwitz criterion, the critical condition of ring truss antenna is obtained, so as to determine the stable region and unstable region of the equilibrium point. We select different parameters in different regions to obtain the corresponding trajectories of phase space, which further verifies the consistency of numerical calculation and theoretical analysis. The results of numerical calculation and theoretical analysis can optimize the structural design and control parameters of the system, so as to prevent and avoid the large vibration of ring truss antenna in operation.

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