Model Error Compensator with Parallel Feed-Forward Filter

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Abstract: Design methods for control systems based on plant models have been developed for many years. If a mathematical model is accurately obtained from the input-output relation of a plant, then the designed controller for the model performs well for the control system connected with the plant. However, the desired control performance might not be achieved when there is an undeniable modeling error. To overcome this problem, the authors proposed the model error compensator (MEC) to minimize the effect of the modeling error between the plant and the model. The MEC works well for many control systems, such as unstable systems and non-linear systems. However applying the MEC to non-minimum-phase plants is difficult because of their control system structures. Non-minimum-phase plants are well known for being difficult to control. This paper proposes an MEC with a parallel feed-forward filter (PFF). The PFF is used to cancel the non-minimum-phase characteristics of the plant. The effectiveness of the proposed method is illustrated through numerical examples.

Key Words: internal model control, non-minimum-phase system, model error compensator.

1. Introduction

Design methods for control systems based on plant models have been developed for many decades. If there is no modeling error between the plant and its model, a desired output can be achieved using the designed controller. However, a desired output might not be achieved when there exists a modeling error because of the variation of plant parameters and approximation errors.

A model error compensator (MEC) has been proposed for overcoming the effect of the modeling error [1]. The MEC comprises a model component and an error feedback component. It works to minimize the modeling error between the plant and the model. The plant compensated by the MEC is used instead of the plant itself. If the compensated plant is similar to the model in its input-output relation, it can be expected to achieve a desired output by applying the designed controller for the model. The MECs have shown effectiveness when used for electric wheelchair control [2], vibration control [3], non-linear systems [4] and multiple-input/multiple-output (MIMO) systems [5].

In previous research [1], the MEC has minimized the modeling error when a high gain compensator is designed. Therefore, it is difficult to apply the MECs to non-minimum-phase plants having time delays or unstable zeros as this would require designing a high gain compensator, which is challenging for such systems.

This paper proposes an MEC with a parallel feed-forward filter (PFF). In the simple adaptive control (SAC), the PFF is a filter that compensates plant characteristics to allow for easy design of the controller [6]–[11]. In this paper, a PFF is used to overcome the non-minimum-phase characteristics of the plant. If the PFF that overcomes such characteristics is designed, the MEC compensator regards the extended plant comprising the plant and the PFF as a minimum-phase plant. As a result, it is expected that high model error suppression performance is achieved. The effectiveness of the proposed method is illustrated through numerical examples.

2. Model Error Compensator

2.1 Basic Idea

The basic idea of the standard MEC [1] is presented in this section.

In the model-based control systems design, control systems are designed through the following steps. First, a nominal model is derived from the output of a plant and physical laws. Next, a controller for the derived model $P_m(s)$ is designed as shown in the upper part of Fig. 1. The output $y_m$ in Fig. 1 is the ideal output. Finally, the designed controller is applied to the plant $P$ as shown in the lower part of Fig. 1.

When there is no difference between the nominal model and the plant, the ideal output $y_m$ is obtained as the actual output $y$ by using the controller designed by existing studies. However, if the dynamics of the model and the plant are not similar, the actual output $y$ is not similar to the ideal output $y_m$. Therefore, it is necessary to suppress the model error between the nominal model and the plant.

To minimize the model error, we have considered a compensated system, as shown in Fig. 2. The compensator $H(s)$ suppresses the model error between the nominal model $P_m(s)$ and the compensated system $P_c(s)$. A control system structured by using the compensated system $P_c(s)$ instead of $P(s)$ is shown in Fig. 3. If the model error between $P_m(s)$ and $P_c(s)$ is minimized by the compensator $H(s)$, then the output of the compensated system $P_c(s)$ is similar to that of the nominal model $P_m(s)$. Therefore, a similar trajectory of the ideal output can be obtained when there exists the model error between the nominal model and the plant.
error. The transfer function from the error between the compensated system and the nominal model of the plant $P(s)$.

The difference between $y$ and $y_m$ is used as the feedback signal, and the compensator $D(s)$ reduces the effect of the model error. The transfer function from $u_c$ to $y$ can be represented as follows:

$$P_c(s) = P(s)\frac{1 + P_m(s)D(s)}{1 + P(s)D(s)}.$$  

When the plant $P(s)$ is equal to the nominal model $P_m(s)$, the transfer function becomes $P_c(s)$ for any $D(s)$. This fact can be confirmed from Eq. (1).

On the other hand, if there exists a model error $\Delta P(s)$ between $P(s)$ and $P_m(s)$, the MEC suppresses the effect for outputs by the error. When $P(s)$ is represented as $P_m(s) + \Delta P(s)$, the difference between the compensated system $P_c(s)$ and $P_m(s)$ can be written as follows.

$$P_c(s) - P_m(s) = \frac{1}{1 + P(s)D(s)}\Delta P(s).$$

Following Eq. (2), the MEC minimizes the modeling error when a high gain compensator $D(s)$ is designed [1]. The design method of the compensator $D(s)$ was presented in [1].

An MEC has similar concept of a disturbance observer. In case of the disturbance observer, an inverse model of the plant is required to compensate the effect of the disturbance and the model error. It is difficult to apply a disturbance observer to such as non-linear systems, MIMO systems, and non-minimum-phase systems. On the other hand, the MEC does not need the inverse model of the plant in the compensator. Therefore, design degree of freedom is larger in the MEC. The MEC showed effectiveness for electric wheelchair control [2], vibration control [3], non-linear systems [4], and MIMO systems [5].

### 2.2 Structure of MEC

In this section, the structure of the MEC is explained.

The standard structure of the MEC [1] is shown in Fig. 4. In this figure, $P(s)$ is the plant, $D(s)$ is the differential compensator and $P_m(s)$ is the nominal model of the plant $P(s)$.

In case the plant is equivalent to the nominal model ($P(s) = P_m(s)$), the transfer function becomes $P_c(s)$ for any $D(s)$. This fact can be confirmed from Eq. (1).

On the other hand, if there exists a model error $\Delta P(s)$ between $P(s)$ and $P_m(s)$, the MEC suppresses the effect for outputs by the error. When $P(s)$ is represented as $P_m(s) + \Delta P(s)$, the difference between the compensated system $P_c(s)$ and $P_m(s)$ can be written as follows.

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An MEC has similar concept of a disturbance observer. In case of the disturbance observer, an inverse model of the plant is required to compensate the effect of the disturbance and the model error. It is difficult to apply a disturbance observer to such as non-linear systems, MIMO systems, and non-minimum-phase systems. On the other hand, the MEC does not need the inverse model of the plant in the compensator. Therefore, design degree of freedom is larger in the MEC. The MEC showed effectiveness for electric wheelchair control [2], vibration control [3], non-linear systems [4], and MIMO systems [5].

### 3. Main Result

#### 3.1 Non-Minimum-Phase Plants and Model

Plants with non-minimum-phase characteristics are described by the following equation.

$$P(s) = P_0(s)e^{-Ls} \prod_{i=1}^{N} \frac{s - z_i}{s + \bar{z}_i} + \Delta P(s).$$  

$P_0(s)$ is a transfer function of a stable minimum-phase system, $z_i$ represents unstable zeros, $\bar{z}_i$ is the complex conjugate of $z_i$, and $L$ expresses the time delay. Plants with no time delay can be expressed by $L = 0$. If plants have no unstable zeros, then $N$ is set to zero to express the plant characteristics. $\Delta P(s)$ is an additive model error. Moreover, the nominal model is given as follows:

$$P_n(s) = P_0(s)e^{-Ls} \prod_{i=1}^{N} \frac{s - z_i}{s + \bar{z}_i}. $$

#### 3.2 MEC with PFF

In this section, the proposed structure of the MEC using a PFF is explained. Figure 5 shows the proposed structure of the MEC. $D(s)$ is a compensator, and $F(s)$ is a PFF. In Fig. 4, the feedback signals to the compensator $D(s)$ are generated from the difference between $y$ and $y_m$. Meanwhile, in Fig. 5, the difference between $y$ and $y_m$ is used as the feedback signal. The transfer function from $u_c$ to $y(s)$ is described by the following equation.

$$P_c(s) = P(s)\frac{1 + (P_m(s) + F(s))D(s)}{1 + (P(s) + F(s))D(s)}.$$  

In case the plant is equivalent to the nominal model ($P_n(s) = P(s)$), the transfer function becomes $P(s)$ for any $D(s)$ from Eq. (5). Moreover, if $P_m(s) + F(s)$ becomes a stable minimum-phase system, the design problem of $D(s)$ can be handled in the same framework as in previous research [1]. Therefore, it is necessary to design a compensator $D(s)$ that has high model error suppression performance.

Here, the design conditions of $F(s)$ and $D(s)$ are as follows:

C1: The difference between $P_{ef}(s)$ and $P_m(s)$ is suppressed.
3.3 Design of Filters in Proposed Systems

In this section, first, a design method of the PFF $F(s)$ for overcoming the non-minimum-phase characteristics is shown. Next, a design method of compensator $D(s)$ is explained using the designed $F(s)$.

3.3.1 Design of $F(s)$

A design method of the PFF $F(s)$ is considered when the plant defined in Eq. (3) is (i) $n \neq 0$, $L = 0$, (ii) $n = 0$, $L \neq 0$, and (iii) $n \neq 0$, $L \neq 0$, respectively.

First, case (i) $n \neq 0$, $L = 0$ is considered. When $F(s)$ is added to the nominal model $P_m(s)$, the system $P_m(s) + F(s)$ must become the transfer function of the minimum-phase characteristics. Here, if $D(s)$ is set to high gain, then minimizing $W_e(s)F(s)/P_m(s) + F(s)$ is effective for the minimization problem of $J$. Consequently, $F(s)$ is designed using the following evaluation function:

$$J = \left\| \frac{W_e(s)F(s)}{P_m(s) + F(s)} \right\|_\infty.$$  

The design method of the PFF can be regarded as a minimization problem of $J$ under the minimum-phase condition. This is one of the contributions of our paper. The evaluation function is minimized by using an optimization technique such as the particle swarm optimization (PSO) [12].

Next, case (ii) $n = 0$, $L \neq 0$ is considered. When the plant has a time-delay, $F(s)$ is taken to be the Smith predictor [9], [10], [13]. Thus, $F(s)$ is represented as

$$F(s) = P_0(s)(1 - e^{-Ls}).$$  

Finally, case (iii) where $n \neq 0$, $L \neq 0$ is considered. $F(s)$ is created by combining the results (i) and (ii). First, the following $F_1(s)$ is defined to overcome the time-delay component:

$$F_1(s) = P_0(s)\prod_{i=1}^n (s - z_i)/(s + \bar{z}_i).$$

Furthermore, to overcome undershoot, $F_2(s)$ is designed using Eq. (9) for the virtual model $P_{m\theta}(s) = P_m(s) + F_1(s) = P_0(s)\prod_{i=1}^n (s - z_i)/(s + \bar{z}_i)$. Then, $F(s)$ can be created by using the sum of $F_1(s)$ and $F_2(s)$ as follows:

$$F(s) = F_1(s) + F_2(s).$$

Thus, the PFF $F(s)$ that overcomes the non-minimum-phase characteristics can be designed even when the plant has the characteristics $n \neq 0$, $L \neq 0$.

3.3.2 Design of $D(s)$

In this section, a design method for the compensator $D(s)$ based on [1] is shown. From the results of the previous section, each design condition C1 and C2 can be handled as an $H_\infty$-control problem. Therefore, it is possible to consider the following design problem regarding $D(s)$ in Fig. 5.

[Problem 1] Find $D(s)$ that minimizes the following evaluation function $J_1$:

$$J_1 = \sup_{\Delta_r(s)} \left\| \frac{W_e(s)(1 + (P(s) + F(s))D(s))}{\Delta_0(s)} \right\|_\infty,$$

where $W_e(s)$ is a weighting function. The constraint condition is as follows:

- $D(s)$ is a robustly stabilizing controller for the feedback system with the loop transfer function $(P(s) + F(s))D(s)$.

In Problem 1, the model error between the transfer functions from $u_0$ to $y_f$ and $P_m(s) + F(s)$ is evaluated. Here, each condition in Problem 1 is given as the $H_\infty$ norm. When $P_m(s) + F(s)$ is considered as the nominal model, it is a minimum-phase system owing to the designed $F(s)$ in Section 3.3.1. Furthermore, the design problem is equivalent to that in [1], because the model error $\Delta_r(s)$ is given an additive uncertainty $\Delta_r(s) = W(s)\Delta(s)|_{\Delta} < 1$. Problem 1 is the standard $\mu$ synthesis problem and can be solved numerically by MATLAB $\mu$ synthesis toolbox.

Therefore, it is possible to obtain a numerical solution of $D(s)$ by using the numerical calculation software such as MATLAB to solve the $H_\infty$ control problem. Moreover, it is expected that smaller $J_1$ will be obtained because $P_m(s) + F(s)$ is a minimum-phase plant. This leads to the suppression of the model error.

In previous research [1], when the nominal model $P_m(s)$ has a time delay, the structure of the MEC in Fig. 4 could not design a compensator $D(s)$ based on an $H_\infty$ control problem. In contrast, using the proposed structure, a compensator $D(s)$ can be designed in the case of the nominal model $P_m(s)$ with the time delay, because the dead time is ignored by the PFF $F(s)$.
3.4 Elimination of Constant Disturbances and Steady-State Errors

In this section, an elimination method of a constant disturbance and a steady state error is discussed through limiting the structures of $D(s)$ and $F(s)$. When $F(s) = 0$, the proposed structure in Fig. 5 is equivalent to the traditional one in Fig. 4; the mentioned problems are eliminated by $D(s)$ with an integrator as is shown in [1].

Here, it is assumed that the structures $D(s)$ and $F(s)$ are given as follows.

$$D(s) = \frac{1}{s}D_0(s), \quad F(s) = sF_0(s), \quad \text{(14)}$$

where $D(s)$ and $F(s)$ have a pole at $s = 0$ and a zero at $s = 0$, respectively. For example, $F(s)$ in Eq. (10) satisfies the condition shown in Eq. (14).

First, we consider the elimination of a steady-state errors when a step input $\hat{u}(s) = 1/s$ is applied. The error signal is derived as follows:

$$\hat{e}(s) = \gamma(s)\Delta_P(s)\hat{u}(s). \quad \text{(15)}$$

$\hat{e}(s)$ becomes zero at the steady state regardless of $\Delta_P(s)$, provided that $\gamma(s)$ has a zero at $s = 0$. By substituting Eq. (14) in Eq. (13), we obtain the following equation:

$$\gamma(s) = \frac{s(1 + F_0(s)D_0(s))}{s + (P(s) + sF_0(s))D_0(s)}. \quad \text{(16)}$$

From Eq. (16), $\gamma(s)$ has a zero at $s = 0$ as long as $P_0(s)$ does not have a zero at $s = 0$. Thus, it is confirmed that the steady-state error of $\hat{e}(s)$ becomes zero.

Next, the case where a step disturbance $d$ is applied to the input $u$ in Fig. 5 is considered. The transfer function $T_{yd}$ from $d$ to $y$ is represented as

$$T_{yd} = \frac{sP(s)(1 + F_0(s)D_0(s))}{s + (P(s) + sF_0(s))D_0(s)}. \quad \text{(17)}$$

From Eq. (17), if $1 + F_0(s)D_0(s)$ is not equal to zero, $T_{yd}$ has a zero at $s = 0$. Therefore, it can be seen that the influence of the step disturbance $d$ is removed.

From the above results, when $D(s)$ and $F(s)$ have an integrator and a zero at $s = 0$ respectively, the steady-state error can be eliminated. Furthermore, if $F(s)$ is designed based on the Smith predictor in Eq. (10), then it is possible to remove a steady-state error automatically because the designed $F(s)$ necessarily has a zero at $s = 0$.

4. Numerical Examples

To show the effectiveness of the proposed method, a non-minimum-phase plant is considered. The transfer function of the plant is described as follows.

$$P(s) = \frac{-(s - 3)}{(s + 1)(s + 2)} + \frac{0.1}{(s + 1)}\Delta(s)e^{-0.5s} \quad \text{(18)}$$

$\Delta_P(s)$ in the simulation are shown in Fig. 6.

The nominal model is

$$P_m(s) = \frac{-(s - 3)}{(s + 1)(s + 2)}e^{-0.5s}. \quad \text{(19)}$$

First, the PFF $F(s)$ is designed. From Eq. (11) and Eq. (19), the filter $F_1(s)$ is designed as follows:

$$F_1(s) = \frac{-(s - 3)}{(s + 1)(s + 2)}(1 - e^{-0.5s}). \quad \text{(20)}$$

The filter $F_2(s)$ is designed by minimizing Eq. (9). The PSO algorithm is used to minimize Eq. (9) under the condition that $P_m + F$ is a stable minimum-phase system. Detailed design sequence is presented in Appendix A. As a result of the PSO algorithm, the following filter $F_2(s)$ is obtained:

$$F_2(s) = \frac{11.4s^2 + 0.518s}{11.4s^3 + 34.9s^2 + 24.2s + 1}. \quad \text{(21)}$$

Here, to confirm the effectiveness of $F(s)$, the step responses of $P_m(s)$, $F(s)$, and $P_m(s) + F(s)$ are shown in Fig. 7. In this figure, dashed, dotted, and solid lines show the step responses of $P_m(s)$, $F(s)$, and $P_m(s) + F(s)$, respectively. $F(s)$ compensates any undershoot and time delay of $P_m(s)$. Therefore, the input-output relation of $P_m(s) + F(s)$ is regarded as a minimum-phase system.

Figure 8 shows the Bode diagrams of $P_m$ and $P_m(s) + F(s)$. The gain diagram of $F(s)$ is also shown in Fig. 8. We can find that phase delay of $P_m(s) + F(s)$ is smaller than that of $P_m(s)$.

Next, the compensator $D(s)$ is designed based on the design method of [1]. When this method is used, it is necessary to set the weighting function $W_e(s)$. To suppress the model error, especially in the steady state, the function $W_e(s)$ is set as follows:

$$W_e(s) = \frac{100}{(10s + 1)^2}. \quad \text{(22)}$$

As a result, the following compensator $D(s)$ is obtained:

$$D(s) = \frac{D_{num}(s)}{D_{den}(s)}. \quad \text{(23)}$$

Fig. 6 Gain diagram of $\Delta_P(s)$.  
Fig. 7 Effect of PFF for $P_m(s)$.  

D_{num}(s) = 1.41 \times 10^3 s^7 + 1.32 \times 10^3 s^5 + 4.35 \times 10^3 s^2 + 4.11 \times 10^2 s^5 \\
+8.10 \times 10^2 s^4 + 8.40 \times 10^2 s^3 + 3.46 \times 10^3 s^2 \\
+1.11 \times 10^3 s + 4.08 \times 10^3 \\
D_{den}(s) = s^8 + 1.98 \times 10^2 s^7 + 1.07 \times 10^4 s^6 \\
+4.31 \times 10^5 s^5 + 5.94 \times 10^5 s^4 + 3.18 \times 10^5 s^3 \\
+5.36 \times 10^3 s^2 + 3.30 \times 10^2 s + 6.65

From Fig. 9, the high gain compensator $D(s)$ can be designed in the low-frequency domain which is influenced substantially by the model error. The above design results have been verified by simulations. The effectiveness of the proposed method is confirmed here by inputting step and sine waves.

[Case A] Performance for a step input

First, the step responses of $P(s)$ without a controller are shown in Fig. 10. Solid black and dashed red lines denote the responses of $P(s)$ and $P_m(s)$, respectively. It can be seen that the responses of $P(s)$ are changed by the influence of the model error.

Next, the step responses of the compensated system $P_c(s)$ using the proposed control system are shown in Fig. 11. Solid black and dashed red lines denote the responses of $P_c(s)$ and $P_m(s)$, respectively. In comparison with Fig. 10, it is clear that the influence of the model error can be suppressed, especially in the steady state.

Here, to verify whether the designed compensator $D(s)$ suppresses the model error between the transfer function $P_c(s)$ from $u_c$ to $y_f$ and $P_m(s) + F(s)$ in Fig. 5, the step responses of $P_c(s)$ are shown in Fig. 12. Solid black and dashed red lines represent the outputs of $P_c(s)$ and $P_m(s) + F(s)$, respectively. From Fig. 12, the designed compensator $D(s)$ suppresses the model error between $P_c(s)$ and $P_m(s) + F(s)$. This shows that the designed compensator $D(s)$ satisfies the design conditions in Problem 1.

From this result, we can confirm the effectiveness of the proposed control system.

[Case B] Performance for a sine wave

The output responses for the sine wave $(\sin \pi t/7)$ which is used as an input for $P(s)$ are shown in Fig. 10. Solid black and dashed red lines show the responses of $P(s)$ and $P_m(s)$, respectively. The responses $y$ are different from the output of the nominal model $y_m$ because of the model error. The responses of the compensated system $P_c(s)$ are shown in Fig. 14. Solid black and dashed red lines are responses of $P_c(s)$ and $P_m(s)$, respectively. By comparing Fig. 14 with Fig. 13, we confirm that the variation from the output of the nominal model $y_m$ is suppressed.

In the same manner as in Case A, in order to confirm that the designed compensator $D(s)$ satisfies the design conditions in Problem 1, the step responses of $P_c(s)$ are shown in Fig. 15. Solid black and dashed red lines show outputs of $P_c(s)$ and $P_m(s) + F(s)$, respectively. Figure 15 shows that the designed compensator $D(s)$ suppresses the model error between $P_c(s)$ and $P_m(s) + F(s)$.

Therefore, the proposed control system is effective for a variety of input signals.

5. Design of Proposed Compensator for MIMO Plants

The proposed system is applied to MIMO systems in this section. The block diagram of the compensated system $P_{CF}(s)$ is shown in Fig. 16 for the case where the plant has $n$ inputs and $n$ outputs.

The PFF $F(s)$ can be designed using the design method shown in Section 3.3.1. The designed $F(s)$ must satisfy the condition that $P_{MF}(s) + F(s)$ has minimum-phase characteristics.

The design method of $D(s)$ is considered. In Problem 1, the designed $D(s)$ suppresses the model error between the transfer function from $u_c$ to $y_f$ and $P_m(s) + F(s)$ in Fig. 5. Here, in Fig. 16, the difference between the transfer function $P_{CF}(s)$ from $u_c$ to $y_f$ and $P_m(s) + F(s)$ is given as follows, with the subscript $s$ omitted:

$$P_{CF} - (P_M + F) = (I + (P + F)D)^{-1}(P + F + PD(P + F) + FDP)$$
$$- (P_M + F) = (I + (P + F)D)^{-1} \Delta \rho$$
$$= \epsilon \Delta \rho$$ (24)

where $I$ is an $n \times n$ unit matrix. $\epsilon \Delta \rho$ in Eq. (24) is the matrix representation of an element in Eq. (25). As a result, $D(s)$ for MIMO systems can be designed because the design method of the MEC for MIMO systems has been shown in [5]. Therefore, the following design problem regarding $D(s)$ for MIMO systems in Fig. 16 can be considered.

[Problem 2] Find $D(s)$ that minimizes the following evaluation function $\Gamma_2$:

$$\Gamma_2 = \sup_{\Delta \rho(s)} \|W_e(s)(I + (P(s) + F(s))D(s))^{-1}\|_\infty$$ (25)

where $W_e(s)$ is a weighting function. The constraint condition is given as follows:

![Fig. 8 Bode diagram of $P_m(s)$ and $P_m(s) + F(s)$.](image)

![Fig. 9 Gain diagram of $D(s)$.](image)
• $D(s)$ is the robust stabilization controller for the feedback system with the loop transfer function $(P(s) + F(s))D(s)$.

As explained above, the PFF $F(s)$ and the compensator $D(s)$ can be designed even when the plant is a MIMO system.

6. Conclusion

This paper proposed an MEC with a PFF. A design method that uses the PFF to overcome non-minimum-phase characteristics was shown. Moreover, a design method for the compensator using the PFF was described. Simulations confirmed that the compensator with high model error suppression performance can be designed. The effectiveness of the MEC with the PFF was illustrated through numerical simulations.

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Appendix PSO Algorithm

In this section, a concrete design procedure to design $F_2(s)$ which minimize Eq. (9) is presented. The filter $F_2(s)$ is designed using the PSO algorithm. The form of $F_2(s)$ is given as follows:

$$F_2(s) = \frac{a_1 s^3 + a_2 s}{a_3 s^3 + a_4 s^2 + a_5 s + 1}.$$  \hfill (A.1)

The parameter position is denoted as $p = [a_1, a_2, a_3, a_4, a_5]^T$. The parameter velocity is denoted as $\Delta p = [\Delta a_1, \Delta a_2, \Delta a_3, \Delta a_4, \Delta a_5]^T$.

The optimal parameter vector $p$ is required to be obtained from an optimization algorithm. Particle swarm optimization (PSO) is a computation method for optimizing a problem by iteratively trying to improve a solution. Multiple particles $p_1, \ldots, p_m$ are used in the PSO algorithm where $m$ denotes the number of particles. $m = 100$ is used in this paper.

To minimize $J$ in Eq. (9), the following objective function $E(p)$ is determined.

$$E(p) = \left\{ \begin{array}{ll} J(p) & \text{Minimum phase condition satisfied} \\ E_{pen} & \text{otherwise} \end{array} \right. \hfill (A.2)$$

The penalty term is used if the stable minimum-phase condition is not satisfied. The penalty $E_{pen}$ is a positive value larger than a feasible value of $J(p)$. $E(p)$ becomes large if the minimum phase condition of $P_m + F$ is not satisfied.

The position and the velocity of $i$-th particle are denoted as $p_i$ and $\Delta p_i$, respectively. $p_i$ is updated based on the following update laws:

$$p_i^{t+1} = p_i^t + \Delta p_i^{t+1},$$  \hfill (A.2)

$$\Delta p_i^{t+1} = \omega_1 \Delta p_i^t + \omega_2 \text{rand} I_{2i} (p_{i,gbest}^t - p_i^t) + \omega_2 \text{rand} I_{2i} (p_{i,gbest}^t - p_i^t).$$  \hfill (A.3)

The iteration number is denoted as $t$ and its initial value is $t = 0$. The maximum iteration number $t_{max}$ is given as 200 in this paper. $\omega_1, \omega_2$ and $\omega_3$ are positive weighting coefficients. Random numbers rand $I_{2i}$ and rand $I_{2i}$ are selected in the range $[0, 1]$. In (A.3), $p_{i,gbest}^t$ means the personal best solution, which is determined as follow:

$$p_{i,gbest}^t := \arg \min_{x \in [p_i^t, |j=1...t|]} E(p).$$  \hfill (A.4)

Then, the PSO algorithm is given as following steps:

**PSO algorithm**

- **Step 1**: Set $t = 0$. For $i = 1, \ldots, m$, select the initial position $p_i^0$ and the velocity $\Delta p_i^0$ randomly and evaluate the corresponding objective function $E$ at each position.

- **Step 2**: Update $p_{i,gbest}^t$ and $p_{i,gbest}^t$ by (A.4) and (A.5), respectively. Then, apply the update laws (A.2) and (A.3) for all the particles, and go to Step 3.

- **Step 3**: Evaluate all the position $p_i^t$ by (A.2). Set $t = t + 1$ and go to Step 2 if $t < t_{max}$. Else, update $p_{i,gbest}^t$ and $p_{i,gbest}^t$.

By using the PSO algorithm, Eq. (21) is obtained.

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