Poiseuille flow of soft glasses in narrow channels: From quiescence to steady state

Pinaki Chaudhuri and Jürgen Horbach
Institut für Theoretische Physik II, Heinrich-Heine-Universität Düsseldorf, 40225 Düsseldorf, Germany

Using numerical simulations, the onset of Poiseuille flow in a confined soft glass is investigated. Starting from the quiescent state, steady flow sets in at a timescale which increases with decrease in applied forcing. At this onset timescale, a rapid transition occurs via the simultaneous fluidisation of regions having different local stresses. In the absence of steady flow at long times, creep is observed even in regions where the local stress is larger than the bulk yielding threshold. Finally, we show that the timescale to attain steady flow depends strongly on the history of the initial state.

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Introduction. The advent of microfluidic devices has led to studying the structural and dynamical properties of soft materials in narrow confinements \[1\] \[2\]. Due to a wide range of applications, understanding their rheological properties is of great interest. A typical flow pattern in microchannels is Poiseuille flow which is associated with a spatially inhomogeneous stress field. Particularly interesting in this context are glassy materials, which have a characteristic stress threshold for yielding. Their response to inhomogeneous stress fields is, however, not well studied. The main focus has so far been on studying the response to spatially uniform stress fields \[3\] \[4\].

A natural question, which has not been addressed so far, is the microscopic development of Poiseuille flow from the initial quiescent glassy state towards the approach of steady-state flow, especially in the vicinity of the yielding threshold \(\sigma_d\). So far, most studies have focused on the steady state Poiseuille flow \[10\] \[18\], which have shown that the narrow confinement of micro-channels results in the local steady state rheology significantly deviating from the bulk behaviour \[14\] \[15\] \[16\] \[17\]. In particular, the naive expectation fails that flow will only occur at regions where the local stress \(\sigma_{\text{loc}} > \sigma_d\). Dynamic fluctuations were even observed at the centre of the channel where \(\sigma_{\text{loc}} < \sigma_d\) \[15\] \[17\]. These findings have been associated with non-local processes that drive the flow in soft amorphous assemblies \[15\] \[20\].

In this Rapid Communication, we study the onset of planar Poiseuille flow in a model colloidal glass confined between rough walls. We demonstrate that the imposed inhomogeneous stress field in combination with strong confinement leads to the following observation: there is a sudden and rapid transition to steady flow, after a waiting time which depends on the applied external forcing and the history of the sample. The cooperative nature of the yielding process is expressed by the nearly simultaneous fluidization of different regions in the sample, although local stresses vary significantly across the channel (including \(\sigma_{\text{loc}} < \sigma_d\)). At small forcings, spatially resolved dynamical measurements reveal a creep flow regime, even in regions where \(\sigma_{\text{loc}} > \sigma_d\). For wider channels, we however, demonstrate that the transient dynamics is qualitatively different. Thus, our study reveals that the interplay between stress inhomogeneities and cooperative behavior results in a surprising transient flow response of glasses in the Poiseuille flow geometry.

Simulation details. We consider the model colloidal system of a 50 : 50 binary Yukawa fluid (for details, see \[21\] \[23\]). Molecular dynamics simulations were done at constant particle number density \(\rho = N/V = 0.676 d_s^{-3}\) in the canonical ensemble for samples consisting of \(N = 12800\) particles in a box of volume \(V = L_x \times L_y \times L_z\), with the dimensions \(L_x = 53.32 d_s\), \(L_y = 26.66 d_s\), and \(L_z = 13.33 d_s\) (here, \(d_s\) is the diameter of the smaller particles in the binary mixture with size ratio 1.2). The temperature \(T\) was controlled via a Lowe thermostat \[24\] \[25\]. At a high temperature of \(T_0 = 0.2\), the system was equilibrated using periodic boundary conditions. Then, \(m = 40\) independent configurations sampled at this \(T_0\) were instantaneously quenched to \(T = 0.10\) (below the mode-coupling critical temperature of \(T_c = 0.14\) \[21\] \[22\]). After each of these \(m\) configurations were aged for durations of \(t_{\text{age}} = 10^4\), the particles were frozen at \(0 < y < 2 d_s\) and \(L_y - 2 d_s < y < L_y\) to obtain glassy states confined between rough walls. Thus, the effective width of the channels (i.e. the wall-to-wall distance) was \(w = 22.66 d_s\). This width, typical to experimental micro-channels, is narrow enough to observe significant deviations from bulk rheological behavior \[10\] \[11\]. For comparison of flow response, we also consider widths of \(w/d_s = 49.33, 102.66\), using same number of particles.

Inside the channel, Poiseuille flow is set up by applying a body force, \(F_0 \hat{z}\), on each particle \[26\]. For this external forcing, the local stress in the velocity-gradient plane is given by \(\sigma(y) = \rho F_0 |y - w/2|\) \[27\]. Since \(\sigma(y)\) is a monotonic function, all forcings are quantified by the stress at the wall, \(\sigma_w = \frac{1}{2} \rho F_0 w\), scaled by the dynamic yield stress \(\sigma_d\) \[28\].

Velocity profiles. Once steady state is reached, we obtain the spatial profiles of the local flow velocities, \(\langle u(y) \rangle\), as shown in Fig. 1(a), with \(\langle \cdots \rangle\) denoting a steady state average. Unlike the plug-like profiles observed for Poiseuille flow of glassy systems in wider channels \[8\] \[10\] \[26\], the velocity profiles, shown here, have a more rounded central region, similar to earlier studies in narrow channels \[11\] \[12\] \[17\].

The time evolution of the velocity profiles, for a single initial configuration, from the quiescent glass state at \(t = 0\) to the steady state is monitored in Fig. 1(b) for
Thus, for $\sigma_w = 2.37\sigma_d$ there is no flow in the channel for a long time and then suddenly, there is a burst of flow leading to a quick approach of the steady state. The velocity profiles shown in Fig. 1(b) are sampled during the narrow time window over which this transition occurs (as marked with circles in Fig. 1(c)). This indicates that the fluidization occurs in almost all regions inside the channel within a short time-window.

**Mobility maps.** In order to observe how the flow develops at a local scale, we construct maps of local transverse displacements $\mathbf{x}$, which are non-affine motions caused by local structural rearrangements (see Supp. Mat. [28]).

In Fig. 1(d), we illustrate how the local displacements evolve during the time period over which flow suddenly sets in, as marked in Fig. 1(c). We begin at $t = 1.23 \times 10^5$, when the first signs of a spatially varying velocity profile are seen, albeit at regions shifted from the walls. At this time, the velocity profile is somewhere between $\sigma_w/\sigma_d = 2.09$ and $2.65$, MSD for the latter three layers.

In Fig. 2, (a) During flow onset, layer-resolved MSD $\Delta^2_0(t)$ for $\sigma_w = 1.67\sigma_d$, marking data for the central layer (dotted), and layers centred $1d_e$ away from the two walls (dashed). (b) for $\sigma_w/\sigma_d = 2.09$ and $2.65$, MSD for the latter three layers.

FIG. 1: (a) Steady state velocity profiles for $\sigma_w/\sigma_d = 2.65$ (magenta), $2.37$ (blue), $2.09$ (green). (b) Transient evolution of $v_x(y)$ before onset of steady state, for $\sigma_w = 2.37\sigma_d$. (c) Variation of the spatially averaged flow velocity indicating the sudden outbreak of flow at $\sigma_w = 2.37\sigma_d$. (d) Maps of transverse displacements at $t/10^5 = 1.23, 1.46, 1.59$. The colorbar shows the displacement scales (in units of $d_e$).

$\sigma_w = 2.37\sigma_d$. Each of the velocity profiles are averaged over a short time period, $\Delta t = 415$, centered around successively increasing times of measurement. We observe that, at first, no flow is visible, until a point in time when for regions near the walls, the local shear rates (i.e. $\partial v_x/\partial y$) become finite and first signs of a spatially varying $v_x(y)$ are seen, albeit small in magnitude. With increasing time, the magnitude of the local velocities increases until the steady state profile finally sets in.

Further information about the evolution to steady state is provided by the centre-of-mass velocity in $x$-direction, $v_x^{cm}(t)$, corresponding to spatial average of the instantaneous velocity profile $v_x(y)$. When flow occurs, $v_x^{cm}$ displays a finite steady-state value; see Fig. 1(c).
particles located in the central slab and the slabs near the two walls.

For $\sigma_w = 1.67\sigma_d$ (Fig. 2(a)), the dynamics is sub-diffusive in all the three layers over the time of observation. This suggests the existence of a creep flow \[ 2 \text{ cm} \]. Thus, even for regions where $\sigma_{\text{loc}} > \sigma_d$, i.e. for $6.7d_s < |y - w/2| < w/2$, we observe creep. Such motion is not seen in athermal jammed systems, where the material quickly reaches either a completely stuck or a flowing state \[ 15 \text{ cm} \], indicating the significant of thermal fluctuations to the occurrence of creep.

In Fig. 2(b), we show the data for $\sigma_w/\sigma_d = 2.09$ and 2.65, focusing again on the three layers mentioned above. In both cases, there is the identical initial caging regime, followed by the dramatic burst into flow at a timescale which decreases with increasing $\sigma_w$. Therefore, whenever there is steady flow at long times, diffusive dynamics is observed for particles originating from each of these layers, including those where $\sigma_{\text{loc}} < \sigma_d$ (see Suppl. Matt. 22). For $\sigma_w = 2.09\sigma_d$, e.g., this corresponds to $0 < |y-w|/2 < 5.3d_s$, i.e. nearly half the channel width. Also, for all the layers, the jump in the dynamics occurs at nearly the same timescale. Thus, this complements the scenario of complete fluidisation across the channel as illustrated in the displacement maps of Fig. 1. The displacement data conclusively demonstrates the correlated processes underlying the yielding of such materials. Regions having different local stresses either fluidize (i.e. attain steady flow) or continue to creep, depending on the imposed stress gradient which links and constrains the motion across the channel-width.

**Time-scales for onset of flow.** Until now, we have studied the spatio-temporal evolution of the flow for a single initial configuration. Now, we expand the analysis to the ensemble of $m = 40$ initial configurations. In Fig. 3(a)-(b), we show the flow velocity $v(x,t)$ for all the $m$ trajectories within the ensemble at imposed stresses of $\sigma_w/\sigma_d = 2.79, 2.23$. For $\sigma_w = 2.23\sigma_d$, in all cases, steady flow is observed at long times and the onset time-scale depends on the initial condition. However, for a decreased forcing of $\sigma_w = 2.23\sigma_d$, we observe that not only does the onset time-scales become larger, but also a certain fraction of the trajectories undergo the transition to steady flow during the period of our observation. This is further illustrated for different $\sigma_w$ in Fig. 3(c), where we plot the time evolution of the fraction of states that have not yielded, $n_o(t)$. Now, $n_o(t)$ is related to the distribution of time-scales $\Pi(t_o)$ for the onset of flow: $n_o(t) = 1 - \int_0^t \Pi(t_o)dt_o$. Thus, with decreasing stress, $\Pi(t_o)$ broadens. In Fig. 3(d), we show (using green triangles) the variation of the mean ($\tau_o$) of the distribution of these time-scales with $\sigma_w$. The data can be fitted with the function $A/(\sigma_w/\sigma_d - x_c)^\beta$, where $A = 3136.35$, $x_c = 2.01$, $\beta = 3.83$. We also consider another ensemble of $m = 40$ configurations which were quenched from $T^0 = 0.16$ and aged for $t_{\text{age}} = 10^4$. The variation of $\tau_o$ with $\sigma_w/\sigma_d$ for this set is also shown in Fig. 3(d) (using stars). For each $\sigma_w/\sigma_d$, the typical values are higher than those obtained for the ensemble quenched from $T^0 = 0.2$. It is known that quenches from lower supercooled temperatures generate glassy states with lower energies \[ 21 \]. Thus, we observe that longer timescales are necessary to fluidise these low-lying states. Further, a fit of the timescales provides a threshold of $x_c = 2.05$. Thus, for both ensembles, there is apparent divergence at $\sigma_c \gg \sigma_d$ depending upon the history of quenching. This is also different from Couette flow where the divergence occurs at $\sigma_c \approx \sigma_d$ \[ 5, 28 \].

**Stress quenches.** For glasses, the transient response depends upon the preparation of the initial quiescent states \[ 5, 20, 22, 34 \]. Such dependence on sample history can lead to practical advantages for Poiseuille flow. As noted above, for $\sigma_w = 2.09\sigma_d$, most states remain non-flowing even at long time-scales. Thus, at lower $\sigma_w$, yielding would be difficult to observe within experimental timescales. Now, we demonstrate how steady flow can still be attained at $\sigma_w < 2.09\sigma_d$. We take configurations from the steady state at $\sigma_w = 2.79\sigma_d$ and then suddenly decrease $\sigma_w$ to a smaller value. In the left panel of Fig. 4, we show the resultant mean flow velocities averaged over all the $m = 40$ independent states. First, we show that for $\sigma_w = 2.51\sigma_d$, we recover the same mean flow velocity as observed in the steady-state obtained from the quiescent state. On the other hand, if $\sigma_w = 1.67\sigma_d$ after the quench, no discernible mean flow is observed at long times, similar to the case when the stresses were switched on. However, for $\sigma_w = 1.95\sigma_d$, we do see a sustained

![FIG. 3: (Top): Flow velocity, $v^{\infty}(t)$, for $m = 40$ initial states ($t_{\text{age}} = 10^4$), prepared via quench from $T_0 = 0.20$, at $\sigma_w/\sigma_d = 2.79$ (left), 2.23 (right). (Bottom): (left) For the same initial states, time evolution of fraction of non-flowing states $n_o(t)$ for different $\sigma_w$. (right) Variation of $\tau_o$ with $\sigma_w$ for initial states, prepared via quench from $T_0 = 0.2$ (green triangles) and 0.16 (purple star). The dashed line is a fit with $A/(\sigma_w/\sigma_d - x_c)^\beta$; $x_c = 2.01, 2.05$ for the two respective $T_0$.](attachment:fig3.png)
steady flow after quenching, in contrast to the situation after start-up, and a stable velocity profile is also developed. In Fig. 4(b), we plot the variation of steady state average of the flow velocity, \( \langle v_x^{cm} \rangle \), with \( \sigma_w/\sigma_d \). The set of data points can be fitted with \( B(\sigma_w/\sigma_d - x_0)^\theta \), obtaining \( x_0 = 1.61 \). Thus, even though this threshold for obtaining finite \( \langle v_x \rangle \) is smaller than the threshold where \( \tau_o \) apparently diverges, it is still larger than \( \sigma_d \).

**Wider channels.** We also explored how \( x_0 \) varies with increasing channel width. In Fig 4(b), \( \langle v_x^{cm} \rangle \) is shown for \( w/d_0 = 49.33, 102.67 \) and similar fits as above provides \( x_0 = 1.27, 1.11 \) respectively, i.e. the threshold decreases with increasing channel width and approaches the bulk limit. A consequence of this decrease in \( x_0 \) with increasing \( w \) is that the timescale for onset of flow also decreases for a fixed \( \sigma_w/\sigma_d \) (cf. \( n_0(t) \) for \( \sigma_w = 2.09\sigma_d \) shown in Fig 4(c)). The local dynamics during the onset of flow also seems to change with increasing \( w \). In Fig 4(d), we plot the layer-resolved MSD for \( \sigma_w = 1.67\sigma_d \) in a channel of width \( w = 49.33d_0 \). We see that particles in the central layer undergo much smaller displacements compared to those near the boundaries. Thus, although a weaker stress gradient due to the increasing channel width allows for flow to occur for this \( \sigma_w \), the transverse mixing of particle layers are distinctly different from what is observed for \( w = 22.66d_0 \).

**Conclusions and outlook.** Our study shows that the onset timescales required for steady Poiseuille flow increases with decrease in applied forcing, and varies strongly from sample to sample as well as quench history. For narrow channels, the onset of the flow occurs simultaneously in all regions of the confined material, although the local stress varies significantly across the system. When steady flow does not set in within observational timescales, the material creeps; this happens even when local stresses are larger than \( \sigma_d \). This implies that rather than the individual local stresses, it is the full inhomogeneous stress map, characterised by the stress gradient, that determines the development of steady flow. We also demonstrated that the weakening of the gradient by widening the channel width influences the flow threshold and also impacts the nature of local dynamics during yielding.

The rapid onset of flow observed here for the model soft glass is reminiscent of granular avalanches [35][36], albeit in the absence of frictional forces and also with confining boundaries (rather than free surfaces). Understanding how such boundary conditions impact the flow start-up process is necessary [37][39], in particular for even narrower channels [40]. Further work should also explore the yieldling process under similar stress gradients for not only other soft materials like gels, but also for polymeric and metallic glasses; this would have significance for practical applications.

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**Supplementary Information**

A. Measurement of dynamic yield stress $\sigma_d$

The dynamic yield stress was estimated via strain-rate controlled simulations. The shear stress ($\sigma$) of the confined glass is measured for different imposed shear-rates ($\dot{\gamma}$). The $\sigma$ vs $\dot{\gamma}$ data is fitted with the Herschel-Bulkley function: $\sigma = \sigma_d + B\dot{\gamma}^\alpha$, which provides us with an estimate of $\sigma_d$.

B. Constructing mobility maps

Before we apply the external forcing (at time $t = 0$), we divide the $xy$ plane of the simulation box into small square cells (of length $l_c = 1.1 d_s$) and identify the particles in each cell. Next, after time $t$, we calculate the transverse displacement of each particle $\Delta y_i(t) = |y_i(t) - y_i(0)|$. Then, we construct the maps by calculating for each cell the local mobility, $\mu_{lm}(t) = \langle \Delta y_i(t) \rangle_{lm}$, where $\langle \cdot \rangle_{lm}$ is the average over all the particles in the cell $\{lm\}$ at $t = 0$.

![FIG. S1: During onset of flow, layer-resolved MSD $\Delta^2_y(t)$ for $\sigma_w = 2.09$ and 2.65.](image)

C. Mean squared displacements: spatially resolved

At $t = 0$, we subdivide the $xy$ plane into slabs which are parallel to the two confining walls. Each such slab has a thickness of $l_c = 2.05 d_s$. We identify the particles in each such slab at $t = 0$. Subsequently, we calculate the average MSD of the particles originating in each slab: $\Delta^2_y(m,t) = \langle (y_i(m,t) - y_i(m,0))^2 \rangle_{nm}$, where $\Delta^2_y$ is the MSD calculated for motions in the direction of the stress.
gradient and the averaging $\langle \cdots \rangle_{n_m}$ is done for the particles populating the $n_m$th slab at $t = 0$. In Fig-S1, we show the transient MSD data, calculated in this manner, for $\sigma_w = 2.09$ and 2.65, for channel width of $w/d_s = 22.66$.

D. Onset timescales: Couette and Poiseuille flow

For the same ensemble of initial states, quenched from $T_0 = 0.2$ and confined within a channel width of $w/d_s = 22.66$, we compare the timescale for onset of flow for two different stress fields, viz. Couette (spatially uniform) and Poiseuille (spatially non-uniform). In Fig-S2, we show the onset timescales, $\tau_{\text{onset}}$, for different applied forcings in the two cases. The plot shows that the thresholds for apparent divergence of $\tau_{\text{onset}}$ are different, with the Poiseuille flow corresponding to a larger value.

FIG. S2: Onset timescales for Couette and Poiseuille flow, for different applied forcings $\sigma_w$ (in units of $\sigma_d$).