The generic $U(1)_X$ models inspired from $SO(10)$

Tianjun Li,1,2,* Qianfei Xiang,1,† Xiangwei Yin,1,2,‡ and Han Zhou1,§

1CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China
2School of Physical Sciences, University of Chinese Academy of Sciences, No. 19A Yuquan Road, Beijing 100049, China

Abstract

We propose the family universal $U(1)_X$ models with three right-handed neutrinos by choosing the $U(1)_X$ gauge symmetry as a linear combination of $U(1)_Y \times U(1)_X$ of $SO(10)$. To be consistent with the quantum gravity effects, we introduced a Dirac fermion $\chi$ as a dark matter candidate, which is odd under the gauged $Z_2$ symmetry after $U(1)_X$ breaking. The isospin violation dark matter with $f_n/f_p = -0.7$ can be realized naturally, and thus the LUX, PANDAX, and XENON1T experimental constraints can be evaded. Moreover, we study the masses and mixings for Higgs and gauge bosons, consider the LHC constraints on the $Z'$ mass, simulate various constraints from dark matter direct and indirect detection experiments, and then present the viable parameter spaces. To study the LHC $Z'$ mass bounds on the generic $U(1)_X$ models, we considered four kinds of scenarios, where scenario II with zero $U(1)_X$ charge for right-handed up-type quarks can relax the LHC $Z'$ mass bound a little bit.

* tli@itp.ac.cn
† xiangqf@pku.edu.cn
‡ yinxiangwei@itp.ac.cn
§ zhouhan@alumni.itp.ac.cn
I. INTRODUCTION

The Standard Model (SM) has been confirmed since Higgs particle was discovered at the LHC. However, there exist some evidences for new physics beyond the SM, for example, neutrino masses and mixings, dark matter (DM), dark energy, and cosmic inflation, etc. Therefore, the SM is not the whole story, and we need to explore the new physics. There are many possible directions to go beyond the SM. For example, the fine-tuning problem such as gauge hierarchy problem leads to supersymmetry [1], technicolor [2], extra dimensions [3, 4], etc, while the aesthetic issues such as the unification of fundamental interactions and the explanation of charge quantization lead to the grand unified theories (GUTs) [5] and string theory [6, 7].

On the other hand, DM particle candidates have a very wide mass range from around $10^{-22}$ eV to about $10^7 M_\odot$ mass [8], including the weakly interacting massive particle (WIMP), the lightest supersymmetric particle (LSP), massive compact halo object (MACHO), super-heavy candidates, axino, sterile neutrino, fuzzy DM, and etc. Among these huge amount of DM candidates, WIMP is a well-motivated DM candidate. It is stable, nonrelativistic, electrically neutral, colorless, and have a mass range from about 10 GeV to few TeV. However, if the discrete symmetry, which stabilizes the DM candidate, is not a gauged discrete symmetry, it can be broken via the non-renormalizable operators due to quantum gravity effects, and then the DM candidate can decay and cannot be a valid DM candidate. Moreover, there are strong constraints from direct search experiments, for instance, the PandaX-II (2017) [9], LUX (2017) [10], and XENON1T (2018) [11] experiments. As we know, the isospin-violating dark matter (IVDM) is a kind of DM with different couplings $f_p$ and $f_n$ respectively to proton and neutron, and was originally proposed to explain the tensions among DAMA/LIBRA, CoGeNT, and XENON experiments for light DM [12, 13]. Interestingly, it can evade the LUX, PANDAX, and XENON1T experimental constraints naturally as well [14–16]. For these xenon based experiments [9–11], the ratio $f_n/f_p$ is about $-0.7$. Moreover, several IVDM models have been proposed in recent years [15, 17–24]. However, it is well known that in the $U(1)'$ models with $E_6$ origin, the vector coupling of the up-type quarks to the $Z'$ boson should be zero while their axial coupling may have nonzero value. Thus, one cannot realize the isospin violation with $f_n/f_p = -0.7$ in the $U(1)'$ model from $E_6$ unless one introduces vectorlike particles [24].

In this paper, to explore the new physics beyond the SM, we choose a conservative approach by neglecting the fine-tuning and aesthetic problems, and concentrate on the low energy new physics [25]. In particular, we consider the family universal $U(1)_X$ models. If we only have SM fermions, the only $U(1)_X$ model, which one can build, is the top hypercharge model or its variation [26, 27]. Thus, we study the family universal $U(1)_X$ models with
TABLE I: The \( U(1)_\chi \) charges of the SM particles.

| SM particles | \( 2\sqrt{10}Q_\chi \) | SM particles | \( 2\sqrt{10}Q_\chi \) |
|--------------|-----------------|--------------|-----------------|
| \( Q_i, U^c_i, E^c_i \) | –1              | \( D^c_i, L_i \) | 3               |
| \( N^c_i \)              | –5               | \( H \)       | –2              |

three right-handed neutrinos in general, and then the neutrino masses and mixings can be explained via type I seesaw mechanism [28, 29]. As we know, one family of the SM fermions plus the right-handed neutrino forms a spinor representation 16 in \( SO(10) \) model, and \( SO(10) \) has a subgroup \( SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\chi \). Therefore, we construct this kind of the \( U(1)_X \) models by choosing the \( U(1)_X \) gauge symmetry as a linear combination of \( U(1)_Y \times U(1)_\chi \). For more discussions about generic and particular \( U(1) \) models see [30].

Also, we introduce another Dirac fermion \( \chi \) as a DM candidate, which is odd under the gauged \( Z_2 \) symmetry after \( U(1)_X \) breaking. Thus, \( \chi \) is a viable DM candidate consistent with the quantum gravity effects. Interestingly, we show that the isospin violation DM with \( f_n/f_p = -0.7 \) can be realized naturally in our models without introducing any vectorlike particles. Moreover, we study the masses and mixings for Higgs and gauge bosons, consider the LHC constraints on the \( Z' \) mass, simulate various constraints from DM direct and indirect detection experiments, and present the viable parameter spaces. To study the LHC \( Z' \) mass bounds on the generic \( U(1)_X \) models, we consider four kinds of scenarios: scenarios I, II, and III have zero \( U(1)_X \) charges, respectively, for quark doublets, right-handed up-type quarks, and right-handed down-type quarks, and scenario IV has approximately equal charges for all the quarks. We find that the low bounds on the \( Z' \) masses are about 4.94, 4.87, 5.34, and 5.09 TeV for scenarios I, II, III, and IV, respectively. Thus, scenario II can relax the LHC \( Z' \) mass bound a little bit, but not too much.

The this paper is organized as follows. In Sec II, we describe the \( SO(10) \)-inspired \( U(1)_X \) model, and calculate the Higgs mass and other parameters. In Sec III, we study the constraints from the LHC, dark matter direct and indirect detection experiments by considering isospin violation effects. In Sec IV, we discuss the LHC bounds on the \( Z' \) masses in four kind of scenarios. Finally, we conclude in Sec V.

II. THE GENERIC \( U(1)_X \) MODELS INSPIRED FROM \( SO(10) \)

First, let us explain the convention. We denote the left-handed quark doublets, right-handed up-type quarks, right-handed down-type quarks, left-handed lepton doublets, right-handed neutrinos, right-handed charged leptons, and Higgs particle as \( Q^c_i, U^c_i, E^c_i, D^c_i, L_i, N^c_i, \)
To break $U(1)_X$ gauge symmetry and give masses to the right-handed neutrinos, we introduce a SM singlet Higgs $S$ with $U(1)_X$ charge 10. So the neutrino masses can be explained via the type I seesaw mechanism. The particles and their quantum numbers under the $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$ gauge symmetry [31–38], where $U(1)_X$ charges for the SM particles are given in Table I.

We shall propose the generic $U(1)_X$ model inspired from $SO(10)$, which is the mixing between the previous $U(1)_X$ gauge symmetry and $U(1)_Y$ gauge symmetry. Thus, we have
\[ Q_X = \cos \alpha \ Q_Y + \sin \alpha \ Q_X \, . \tag{1} \]

To break $U(1)_X$ gauge symmetry and give masses to the right-handed neutrinos, we introduce a SM singlet Higgs $S$ with $U(1)_X$ charge 10. So the neutrino masses can be explained via the type I seesaw mechanism. The particles and their quantum numbers under the $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$ gauge symmetry are given in Table II.

Assuming the interactions between the DM and nucleons are mediated by the $U(1)_X$ gauge boson, we obtain the coupling ratio $f_n/f_p$ in our model
\[ f_n/f_p = \frac{b_u + 2b_d}{2b_u + b_d} = \frac{(q_Q - q_U) + 2(q_Q - q_D)c}{2(q_Q - q_U) + (q_Q - q_D)c} \, , \tag{2} \]

where $q_f$ represents the corresponding $U(1)_X$ charge for particle $f$, which is given in Table II. Thus, we have
\[ f_n/f_p = \frac{\sqrt{10} \cos \alpha - 8 \sin \alpha}{3\sqrt{10} \cos \alpha - 4 \sin \alpha} \, . \tag{3} \]

To have $f_n/f_p = -0.7$, we obtain $\tan \alpha = \frac{31}{108} \sqrt{10}$. Therefore, the $U(1)_X$ charges of the SM particles can be calculated, and are presented in the Table III. To be consistent with GUTs, we choose $g_X \simeq \sqrt{5/3} g_Y$, and assume that the contribution to the one-loop beta function of $U(1)_X$ from one family of the SM fermions in the supersymmetric $U(1)_X$ models is equal to 2 as in the supersymmetric SMs or GUTs. Thus, we obtain the following normalization factor of $U(1)_X$ charge
\[ N = \sqrt{\frac{Q_i^2 \times 2 \times 3 + U_i^2 \times 1 \times 3 + D_i^2 \times 1 \times 3 + L_i^2 \times 2 + E_i^2 \times 1 + N_i^2 \times 1}{2}} \, . \tag{4} \]
\[ Q_i \quad (3, 2, 1/6, 1, -1) \quad U_i^c \quad (\overline{3}, 1, -2/3, -35, -1) \]

\[ D_i^c \quad (\overline{3}, 1, 1/3, 33, 3) \quad L_i \quad (1, 2, -1/2, -3, 3) \]

\[ E_i^c \quad (1, 1, 1, 37, -1) \quad N_i^c \quad (1, 1, 0, -31, -5) \]

\[ H \quad (1, 2, -1/2, -34, -2) \quad S \quad (1, 1, 62, 10) \]

\[ \chi \quad (1, 1, 0, 31/2, 5/2) \]

| TABLE III: The particles and their quantum numbers under the \( SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X \) and \( U(1)_\chi \) gauge symmetry. Here, the correct \( U(1)_X \) and \( U(1)_X \) charges are their charges in the above table divided by \( 2\sqrt{10} \) and \( 2\sqrt{1162} \). Moreover, we introduce a Dirac fermion \( \chi \) as DM candidate, whose \( U(1)_X \) charge is a half integer. Of course, there are many choices for its \( U(1)_X \) charge, and we take \( 31/2 \) for simplicity, which is given in the table as well. The Lagrangian is given by

\[-L = m_s^2|S|^2 + m_H^2|H|^2 + \frac{\lambda_s}{2}|S|^4 + \frac{\lambda_H}{2}|H|^4 + \lambda_{SH}|S|^2|H|^2 + \left( y_{ij}^U Q_i U_j \tilde{H} + y_{ij}^D Q_i D_j H + y_{ij}^E L_i E_j H + y_{ij}^N L_i N_j \tilde{H} + y_{ij}^{MN} S N_i^c N_j^c + H.C. \right), \]

where \( \tilde{H} = i\sigma_2 H^* \).

We parametrize the Higgs fields as follows:

\[ S = v_s + S_1 + iS_2 \quad , \quad H = \begin{pmatrix} H^+ \\ v_h + H_1 + iH_2 \end{pmatrix} . \]

After gauge symmetry breakings, the vacuum expectation values (VEVs) of these Higgs fields are given by

\[ <S> = v_s \quad , \quad <H> = \begin{pmatrix} 0 \\ v_h \end{pmatrix} . \]

We have four Nambu-Goldstone bosons from \( H^\pm, H_2, \) and \( S_2, \) as well as two neutral physical scalars \( s \) and \( h \) from the mixings of \( S_1 \) and \( H_1 \) via their following mass matrix

\[ M(S_1, H_1) = \begin{bmatrix} 2v_s^2\lambda_S & 2v_h v_s \lambda_{SH} \\ 2v_h v_s \lambda_{SH} & 2v_h^2 \lambda_H \end{bmatrix} . \]

The physical scalars \( s \) and \( h \) can be written as the linear combination of \( H_1 \) and \( S_1 \)

\[ \begin{pmatrix} s \\ h \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} S_1 \\ H_1 \end{pmatrix} , \]

5
where the mixing angle \( \theta \) is

\[
\tan 2\theta = \frac{2v_h v_s \lambda_{SH}}{v_H^2 \lambda_H - v_s^2 \lambda_S}.
\]  

(10)

The Higgs masses are, respectively,

\[
m^2_{h/s} = \lambda_h v_h^2 + \lambda_s v_s^2 \pm \sqrt{4\lambda_h^2 v_h^2 v_s^2 - 2\lambda_s \lambda_h v_h^2 v_s^2 + \lambda_h^2 v_h^4 + \lambda_s^2 v_s^4}.
\]  

(11)

Because \( h \) is the SM Higgs field, we should have \( v_h = 174 \text{ GeV} \) and \( m_h = 125 \text{ GeV} \).

Next we shall discuss the gauge boson masses. The covariant derivative in our model is

\[
D_\mu = \partial_\mu - ig_2 T^i A^i_\mu - ig_Y B_\mu - ig_X X C_\mu
\]  

(12)

where \( i = 1, 2, 3 \), \( T^i \) are the three generators of \( SU(2)_L \), \( Y \) and \( X \) are the charges, respectively for \( U(1)_Y \) and \( U(1)_X \), \( g_2, g_Y \) and \( g_X \) are the gauge couplings, respectively, for \( SU(2)_L, U(1)_Y \), and \( U(1)_X \), \( W^{\pm}_\mu \) are the SM charged gauge bosons \( W^{\pm}_\mu = \frac{A^{\pm}_\mu + i A^3_\mu}{\sqrt{2}}, A^i_\mu \) and \( B_\mu \) are the SM gauge fields, and \( C_\mu \) represents for the \( U(1)_X \) gauge field.

After gauge symmetry breaking, we obtain the gauge boson masses from the kinetic terms of the Higgs fields

\[
L_{GM} = (D_\mu H)^\dagger D_\mu H + (D_\mu S)^\dagger D_\mu S
\]  

(13)

where \( v_h = 125 \text{ GeV} \) is the electroweak vacuum expectation value.

The gauge boson mass matrix in the basis \( (A^3_\mu, B_\mu, C_\mu) \) is

\[
\begin{pmatrix}
\frac{1}{2}g_2^2 v_h^2 & \frac{1}{2}g_2 g_Y v_h^2 & -g_2 g_X v_h^2 X_h \\
\frac{1}{2}g_2 g_Y v_h^2 & \frac{1}{2}g_2^2 v_h^2 & -g_2 g_X v_h^2 X_h \\
-g_2 g_X v_h^2 X_h & -g_Y g_X v_h^2 X_h & 2g^2_X v_h^2 X_h^2 + 2g^2_X v_s^2 X_s^2
\end{pmatrix},
\]  

(14)

and then the gauge boson mass are given by

\[
m_1^2 = 0 \quad , \quad m_{2,3}^2 = \frac{1}{4} \left( g_2^2 v_h^2 + g_Y^2 v_h^2 + 4g_X^2 v_h^2 X_h^2 + 4g_X^2 v_s^2 X_s^2 \right)
\]  

\[
\pm \sqrt{(-g_2^2 v_h^2 - g_Y^2 v_h^2 - 4g_X^2 v_h^2 X_h^2 - 4g_X^2 v_s^2 X_s^2)^2 - 4(4g_2^2 g_X^2 v_h^2 v_s^2 X_s^2 + 4g_Y^2 g_X^2 v_h^2 v_s^2 X_s^2)}
\]  

(15)

which obviously are the masses for photon, \( Z \), and \( Z' \) gauge bosons, respectively. Because \( Z' \) is much heavier than \( Z \), we have \( m_3^2 = m_Z^2 \). With these calculations, we can give these parameters the proper values to satisfy the experimental constraints. For example, we can
have $v_s = 8000$ GeV, $m_H = 1884$ GeV, $m_S = 3465$ GeV, $\lambda_S = 0.17$, $\lambda_H = 0.27$, $\lambda_{SH} = 0.05$, $g_X = 0.50$, and then get $m_h = 125$ GeV, $m_s = 4900$ GeV, and $m_{Z'} = 5500$ GeV.

At low energy, we have the SM particles, $Z'$, and DM $\chi$. Therefore, we can use the simplified $U(1)_X$ model whose the interactions are given by

$$-\mathcal{L} = \sum_q g_u \bar{u} \gamma^\mu u Z'_\mu + g_{uA} \bar{u} \gamma^\mu \gamma^\nu d Z'_\mu + g_{dA} \bar{d} \gamma^\mu \gamma^\nu d Z'_\mu + g_\chi \bar{\chi} \gamma^\mu \chi Z'_\mu, \quad (16)$$

Where $g_f = g_X q_f$ with $q_f$ the $U(1)_X$ charge for the fermion $f$, and we have

$$g_u : g_d : g_{uA} : g_{dA} : g_\chi = 36 : -32 : 34 : -34 : 31, \quad g_u = 18/\sqrt{1162}. \quad (17)$$

III. THE $U(1)_X$ MODEL WITH THE IVDM $\chi$

First, we would like to study the LHC constraints on the $Z'$ mass via the code $Z'$ explorer [39]. For simplicity, we take $g_X = \sqrt{5/3} g_Y = 0.46$, and then we have $g_{uL} = g_{dL} \approx 0.0067$, $g_{uR} \approx 0.236$, and $g_{dR} \approx -0.223$. Further parameter settings can be found in the Appendix A. Because the $U(1)_X$ charge of $Q_i$ is much smaller than $U_c$ and $D_c$, the coupling between the left-handed quarks and $Z'$ is much smaller than the right-handed quarks.

We present the LHC simulation results in Fig. 1. $S = \frac{\sigma_{\text{pred}}}{\sigma_{\text{lim}}}$ is the signal strength for each channel, where $\sigma_{\text{pred}}$ is the predicted $Z'$ production cross section times branching ratio times acceptance and $\sigma_{\text{lim}}$ is the corresponding predicted experimental upper limit at the 95% confidence level (C.L). If $S > 1$, then the corresponding point in the parameter space is experimentally excluded. If $S < 1$ for all channels, then the corresponding point is viable. Therefore, the channels with the $e\bar{e}$ and $\mu\bar{\mu}$ final states give the strong constraints on $Z'$ mass, and we obtain that the low bound on $Z'$ mass in our model is around 5.03 TeV. Because the current LHC mass bound on generic $Z'$ is about 5 TeV [40], our numerical result is consistent with LHC searches. For simplicity, we shall take $M_{Z'} = 5.5$ TeV in the following study.

Second, we shall consider the direct and indirect experimental constraints on the DM $\chi$, and present the simulation results in the $g_X$ versus $m_\chi$ plane with $M_{Z'} = 5.5$ TeV and in the $M_{Z'}$ versus $m_\chi$ plane with $g_X = 0.46$, respectively, in the left and right panels of Fig. 2. In the left panel, to have the decay width of $Z'$ smaller than $Z'$ mass from the $Z'$ particle point of view, we obtain that the upper bound on $g_X$ is roughly 2, i.e., $g_X < 2$. In the right panel, we have the low bound on $Z'$ mass from the LHC constraints, i.e., $m_{Z'} > 5.03$ TeV. These two conditions are shown as the solid black lines in Fig. 2. Also, the dark-green line indicates the parameter space with the observed dark matter relic density, i.e., $\Omega_\chi h^2 = 0.12$ and the relic density is calculated by the popular code MicrOMEGAs [45, 46].
FIG. 1: The signal strengths of the SM fermion final states versus $Z'$ mass for the $Z'$ searches at the LHC. The LHC bound on the $Z'$ mass is about 5.03 TeV.

The solid purple, orange and yellow lines, respectively, correspond to the constraints from the PandaX-II (2017) [9], Xenon1T (2018) [11], and DEAP3600 (2019) [41] experiments. The first two experiments are xenon-based DM direct detections, while the last one is argon-based. On account of isospin-violating affects, they have the rescale factors around 7600 and 235 [42], respectively. Compared to the two black lines, the DM direct detection experiments barely give additional constraints, and then we have escaped these experimental constraints.

The dashed blue and red lines correspond to the constraints from the Fermi-dSph (6 year) [43] and HESS (254h) [44], respectively. In addition, we should clarify that the DAMA and GoGeNT are not shown, since they have less constrains. The interesting parameter spaces for our simulations are distributed in the resonant regions, which have $m_\chi \sim \frac{1}{2} m_{Z'}$, and these regions together with the regions below the black lines constitute the main parameter spaces in our model.

Comparing the direct and indirect experiments, we find that the indirect detection have much better sensitivity near the resonant regions with $m_\chi \sim \frac{1}{2} m_{Z'}$ due to resonant enhancement. Beyond these regions, the direct detection experiments have better sensitivities. In short, there are still some viable parameter spaces in our $U(1)_X$ model.
IV. THE GENERIC $U(1)_X$ MODELS WITH LHC BOUNDS ON THE $Z'$ MASSES

In the following, we shall consider four scenarios and study the LHC low bounds on the $Z'$ mass.

A. Scenario I: The $U(1)_X$ model with zero $U(1)_X$ charge for the quark doublets $Q_i$

In scenario I, we consider that the $U(1)_X$ charge for the quark doublets $Q_i$ is equal to 0, and then we obtain

$$\tan \alpha = \frac{\sqrt{10}}{3}.$$  \hfill (18)

The particles and their quantum numbers under the $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$ gauge symmetry are given in Table IV. We present the LHC simulation results in Fig. 3, and obtain that low bound on $Z'$ boson mass is about 4.94 TeV.

B. Scenario II: The $U(1)_X$ model with zero $U(1)_X$ charge for the right-handed up-type quarks $U^c_i$

In scenario II, we consider that the $U(1)_X$ charge for the right-handed up-type quarks $U^c_i$ is equal to 0, and then we obtain

$$\tan \alpha = -\frac{4\sqrt{10}}{3}.$$  \hfill (19)
TABLE IV: The particles and their quantum numbers under the 
$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$ and $U(1)_\chi$ gauge symmetry in scenario I. Here, the 
correct $U(1)_\chi$ and $U(1)_X$ charges are their charges in the above table divided by $2\sqrt{10}$ and 2.

| $Q_i$ | $(3, 2, 1/6, 0, -1)$ | $U^c_i$ | $(\overline{3}, 1, -2/3, 1, -1)$ |
|-------|---------------------|---------|-------------------------------|
| $D^c_i$ | $(\overline{3}, 1, 1/3, -1, 3)$ | $L_i$ | $(1, 2, -1/2, 0, 3)$ |
| $E^c_i$ | $(1, 1, 1, -1, -1)$ | $N^c_i$ | $(1, 1, 0, 1, -5)$ |
| $H$ | $(1, 2, -1/2, 1, -2)$ | $S$ | $(1, 1, 0, -2, 10)$ |

FIG. 3: The signal strengths of the SM fermion final states versus $Z'$ mass for the $Z'$ 
searches at the LHC in scenario I. The low bound on $Z'$ mass is around 4.94 TeV.

The particles and their quantum numbers under the $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$
gauge symmetry are given in Table V. We present the LHC simulation results in Fig. 4, and
obtain that low bound on $Z'$ boson mass is about 4.87 TeV.
TABLE V: The particles and their quantum numbers under the \( SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X \) and \( U(1)_\chi \) gauge symmetry in scenario II. Here, the correct \( U(1)_\chi \) and \( U(1)_X \) charges are their charges in the above table divided by \( 2\sqrt{10} \) and \( 2\sqrt{7} \).

| Particle | Quantum Numbers |
|----------|-----------------|
| \( Q_i \) | \((3, 2, 1/6, 1, -1)\) |
| \( D_i^c \) | \((\bar{3}, 1, 1/3, -2, 3)\) |
| \( E_i^c \) | \((1, 1, 1, 2, -1)\) |
| \( H \) | \((1, 2, -1/2, 1, -2)\) |
| \( U_i^c \) | \((\bar{3}, 1, -2/3, 0, -1)\) |
| \( L_i \) | \((1, 2, -1/2, -3, 3)\) |
| \( N_i^c \) | \((1, 1, 0, 4, -5)\) |
| \( S \) | \((1, 1, 0, -8, 10)\) |

FIG. 4: The signal strengths of the SM fermion final states versus \( Z' \) mass for the \( Z' \) searches at the LHC in scenario II. The low bound on \( Z' \) mass is around 4.87 TeV.

C. Scenario III: The \( U(1)_X \) model with zero \( U(1)_X \) charge for the right-handed down-type quarks \( D_i^c \)

In scenario III, we consider that the \( U(1)_X \) charge for the right-handed down-type quarks \( D_i^c \) is equal to 0, and then we obtain

\[
\tan\alpha = -\frac{2\sqrt{10}}{9}.
\]

(20)

The particles and their quantum numbers under the \( SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X \) gauge symmetry are given in Table VI. We present the LHC simulation results in Fig. 5, and obtain that the low bound on the \( Z' \) boson mass is about 5.34 TeV.
In scenario IV, we consider that the $U(1)_X$ charge for the quark doublets $Q_i$, right-handed up-type quarks $U^c_i$, and right-handed down-type quarks $D^c_i$ are approximately equal, and then we obtain
\[ \tan \alpha = -\frac{\sqrt{13}}{5}. \] (21)

The particles and their quantum numbers under the $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$ gauge symmetry are given in Table VII. We present the LHC simulation results in Fig. 6.
and obtain that low bound on $Z'$ boson mass is about 5.09 TeV. Therefore, scenario II can relax the LHC $Z'$ mass bound a little bit.

| $Q_i$ | $(3, 2, 1/6, 1, -1)$ | $U^c_i$ | $(\frac{3}{2}, 1, -2/3, 317 - 75\sqrt{26}/67, -1)$ |
|------|-------------------|---------|--------------------------|
| $D^c_i$ | $(3, 1, 1/3, 75\sqrt{26}/67 - 451/67, 3)$ | $L_i$ | $(1, 2, -1/2, -3, 3)$ |
| $E^c_i$ | $(1, 1, 1, 75\sqrt{26}/67 - 183/67, -1)$ | $N^c_i$ | $(1, 1, 0, 585 - 75\sqrt{26}/67, -5)$ |
| $H$ | $(1, 2, -1/2, 384 - 75\sqrt{26}/67, -2)$ | $S$ | $(1, 1, 0, 150\sqrt{26}/67 - 1170/67, 10)$ |

**TABLE VII:** The particles and their quantum numbers under the $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$ and $U(1)_{\chi}$ gauge symmetry in scenario IV. Here, the correct $U(1)_{\chi}$ and $U(1)_X$ charges are their charges in the above table divided by $2\sqrt{10}$ and $\frac{16\sqrt{5010 - 900\sqrt{26}}}{67}$.

**FIG. 6:** The signal strengths of the SM fermion final states versus $Z'$ mass for the $Z'$ searches at the LHC in scenario IV. The low bound on $Z'$ mass is around 5.09 TeV.

**V. CONCLUSION**

We constructed the family universal $U(1)_X$ models with three right-handed neutrinos by choosing the $U(1)_X$ gauge symmetry as a linear combination of $U(1)_Y \times U(1)_X$ of $SO(10)$. To be consistent with the quantum gravity effects, we introduced a Dirac fermion $\chi$ as a DM candidate, which is odd under the gauged $Z_2$ symmetry after $U(1)_X$ breaking. To satisfy
the LUX, PANDAX, and XENON1T experimental constraints, we found that the isospin violation DM with \( f_n/f_p = -0.7 \) can be realized naturally. In addition, we studied the masses and mixings for Higgs and gauge bosons, considered the LHC constraints on the \( Z' \) mass, simulated various constraints from DM direct and indirect detection experiments, and then presented the viable parameter spaces. To study the LHC \( Z' \) mass bounds on the generic \( U(1)_X \) models, we considered four kinds of scenarios: scenarios I, II, and III have zero \( U(1)_X \) charges, respectively, for quark doublets, right-handed up-type quarks, and right-handed down-type quarks, and scenario IV has approximately equal charges for all the quarks. We found that the LHC low bounds on the \( Z' \) masses are about 4.94, 4.87, 5.34, and 5.09 TeV for scenarios I, II, III, and IV, respectively. Thus, scenario II can relax the LHC \( Z' \) mass bound a little bit.

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Appendix A: \( Z' \) Decay Widths

We present the \( Z' \) decay widths in details for Sec.III. The vacuum value \( v_s \) can be written as

\[
v_s^2 = \frac{2M_Z^4 - M_Z^2 v_h^2 (g_2^2 + g_5^2 + 4g_X^2 X_h^2)}{2g_X^2 (2M_Z^2 - (g_2^2 + g_5^2) v_h^2) X_h^2}.
\]  
\( (A1) \)

The decay widths of \( Z' \) to \( W^+W^- \) and to \( Zh \) are, respectively,

\[
\Gamma(Z' \rightarrow W^+W^-) = \frac{g_2^2}{192\pi} \cos^2(\theta_W) \sin^2(\theta_X) M_{Z'} (M_{Z'} \sqrt{\lambda} (4M_Z^2/M_{Z'}^2)^{3/2} (1 + 20M_W^2/M_{Z'}^2 + 12M_{Z'}^4/M_{Z'}^2), \)  
\( (A2) \)

\[
\Gamma(Z' \rightarrow Zh) = \frac{g_2^2 M_Z^2}{192\pi M_W^2} M_{Z'} \sqrt{\lambda} (1 + 12M_Z^2/M_{Z'}^2) [ (4M_Z^2/M_{Z'}^2 - g_X^2) \sin(2\theta_X) + (4M_Z^2 g_X^2/M_{Z'}^2) \cos(2\theta_X) ] \)  
\( (A3) \)
where
\[
\lambda = 1 + \left( \frac{M_Z^2}{M_{Z'}^2} \right)^2 + \left( \frac{M_h^2}{M_{Z'}^2} \right)^2 - 2 \left( \frac{M_Z^2}{M_{Z'}^2} \right) - 2 \left( \frac{M_h^2}{M_{Z'}^2} \right) \left( \frac{M_{Z'}^2}{M_{Z'}^2} \right). \tag{A4}
\]

For our model, \(\theta_X\) is the \(Z-Z'\) mixing angle, which is very small, so we have \(\sin(\theta_X) \approx \theta_X\), \(\cos(\theta_X) \approx 1\), and
\[
\sin(\theta_X) = \sqrt{\frac{A - B}{2A}}, \tag{A5}
\]
where \(A\) and \(B\) are defined by
\[
A = \sqrt{v_h^4 (g_2^2 + g_Y^2 + 4g_X^2X_s^2)^2 - 8g_X^2 v_h^2 v_s^2 (g_2^2 + g_Y^2 - 4g_X^2X_s^2) X_s^2 + 16g_X^4 v_s^4 X_s^4}
\]
\[
B = 4g_X^2 v_s^2 X_s^2 - v_h^2 (g_2^2 + g_Y^2 - 4g_X^2X_s^2). \tag{A6}
\]

For DM annihilation cross sections calculation, we also need \(Z'\) decay widths to \(\chi\chi\) and \(q\bar{q}\)
\[
\Gamma_{Z'} = \Gamma(Z' \to \chi\bar{\chi}) + \sum_q c_q \Gamma(Z' \to q\bar{q}), \tag{A7}
\]
with
\[
\Gamma(Z' \to q\bar{q}) = \frac{m_{Z'}}{12\pi} g_X^2 \xi_q (1 + \frac{2m_q^2}{m_{Z'}^2}) + g_{q', q}^2 \xi_{q'}^3, \tag{A8}
\]
\[
\Gamma(Z' \to \chi\bar{\chi}) = \frac{m_{Z'}}{12\pi} g_X^2 (\xi_{\chi} (1 + \frac{2m_{\chi}^2}{m_{Z'}^2}) + \xi_{\chi}^3). \tag{A9}
\]
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