Abstract

We compute the leading QCD corrections to $K\rightarrow \bar{K}$ mixing in the supersymmetric standard model with general soft supersymmetry-breaking parameters. We construct the $\Delta S = 2$ effective Lagrangian for three hierarchies of supersymmetric particle masses, namely, when the gluino mass is comparable to, much greater than, or much less than the masses of the first two generation squarks. We find that the QCD corrections tighten the limits on squark mass splittings by more than a factor of two.
1 Introduction

Low-energy supersymmetry is a leading candidate for physics beyond the standard model because it stabilizes the gauge hierarchy. However, the minimal supersymmetric extension to the standard model contains over 100 new parameters. Present experimental searches are beginning to place significant constraints on the parameter space. To date, the most important limits arise from the study of flavor-changing neutral currents (FCNC).

In general, supersymmetric particles give rise to large contributions to flavor-changing neutral currents [1]. The present limits on these processes place strict constraints on the squark and slepton mass matrices. Indeed, the discovery of supersymmetric FCNC will be an important step towards understanding the source of supersymmetry breaking.

There are several FCNC processes which constrain the parameters of supersymmetric models. For example, the rare decay $b \to s\gamma$ has been used to constrain the mass parameters of the third generation squarks [2]. In this paper, we will focus on $K^\pm\bar{K}$ mixing, which gives the most stringent limit on squark masses of the first two generations.

In what follows, we will extend the analysis of Refs. [3]-[5] to include the leading order QCD corrections to $K^\pm\bar{K}$ mixing in the minimal supersymmetric standard model (MSSM). The QCD corrections are clearly very important, and for the standard model, complete leading order [6] and next-to-leading order [7] analyses have been carried out. (Our results can be easily generalized to $B\bar{B}$ and $D\bar{D}$ mixing, in which case they also involve the third generation squarks.)

For general squark masses, the most important supersymmetric contribution to $K^\pm\bar{K}$ mixing comes from the gluino box diagrams.‡ We will, therefore, restrict our attention to the gluino diagrams, and ignore all diagrams with chargino and neutralino exchanges. These diagrams depend on the details of the Higgs sector and are generally less important than the diagrams we consider here.

As is by now a standard procedure, we define the source of FCNC in terms of dimensionless flavor-changing insertions $\delta$, which parametrize small deviations from the case of flavor-diagonal soft squark masses. We follow the notation of Ref. [5], but we omit the generation and weak isospin labels because we concentrate solely on the first two generations.

This paper is organized as follows. In Sec. 2 we review the general formalism for calculating QCD corrections in the context of effective field theory. We then consider three cases for the hierarchy of the superparticle masses: (a) $M_{\tilde{g}} \simeq M_{\tilde{q}}$, where the gluino mass, $M_{\tilde{g}}$, is on the order of the average squark mass, $M_{\tilde{q}}$, as well as (b) $M_{\tilde{g}} \ll M_{\tilde{q}}$ and (c) $M_{\tilde{g}} \gg M_{\tilde{q}}$. In each case, we construct the effective Lagrangian at the matching scale and compute the renormalization group equations for the Wilson

‡In the minimal supergravity model, the chargino and neutralino box diagrams can be of comparable importance [1]. For a recent review, see Ref. [8].
coefficients. In Sec. 3 we present the results of our numerical analysis. We find that the QCD corrections tighten the limits on the squark mass splittings by more than a factor of two. (Of course, our results are subject to the usual uncertainties associated with the hadronic matrix elements.) We reserve Sec. 4 for our conclusions.

2 QCD corrections to the effective Lagrangian

2.1 General formalism and conventions

To analyze a low energy physical process in a theory with several mass scales, it is useful to construct an effective field theory, in which particles with masses greater than the scale of interest are integrated out. This gives rise to an effective Lagrangian, which can be written as follows,

$$L_{\text{eff}} = \sum A C_A(\mu) O_A(\mu),$$

(1)

where $\mu$ is the renormalization scale, and the operators $O_A$ involve only low energy fields. The Wilson coefficients $C_A$ are obtained by matching S-matrix elements in the full and effective theories at the threshold (matching) scale. The heavy particles modify the ultraviolet behavior of the low energy theory, but the infrared physics is the same in both.

In any realistic calculation, the Wilson coefficients and operators must be evolved from the matching scale to the scale associated with the low energy process under consideration. The renormalization group equations are given by

$$\mu \frac{dO_A}{d\mu} = -\gamma_{AB} O_B, \quad \mu \frac{dC_A}{d\mu} = (\gamma^T)_{AB} C_B,$$

(2)

where $\gamma$ is the anomalous dimension matrix. Using the one-loop RGE for the strong coupling,

$$\mu \frac{dg_s}{d\mu} = \beta_1 g_s^3,$$

(3)

where

$$\beta_1 = \frac{1}{16\pi^2} \left(-11 + \frac{2}{3} n_q + \frac{1}{6} n_{\tilde{q}} + 2 n_{\tilde{g}}\right)$$

(4)

and $n_q, n_{\tilde{q}}, n_{\tilde{g}}$ are the number of active quark, squark and gluino flavors, respectively, one can find a formal solution for the Wilson coefficients at any scale $\mu$,

$$C(\mu) = \exp \left[\frac{\gamma^T}{2\beta_1 g_s^2} \log \left(\frac{\alpha_s(\mu)}{\alpha_s(M)}\right)\right] C(M).$$

(5)

\[\text{We do our calculation in the MS-scheme.}\]
Here $M$ is the matching scale and $\gamma$ is the one-loop anomalous dimension matrix, which is of order $g_s^2$. This procedure resums the leading logarithmic QCD corrections.

In the following subsections, we will use the effective field theory formalism to construct the leading-logarithmic QCD-corrected $\Delta S = 2$ effective Lagrangian in the low energy limit of the MSSM. We will consider models with the following hierarchies between the gluino and first two generation squark masses: $M_{\tilde{g}} \simeq M_{\tilde{q}}$, $M_{\tilde{g}} \ll M_{\tilde{q}}$ and $M_{\tilde{g}} \gg M_{\tilde{q}}$.

### 2.2 $M_{\tilde{g}} \simeq M_{\tilde{q}}$

When $M_{\tilde{g}} \simeq M_{\tilde{q}}$, we choose to integrate out the squarks and gluino at the scale $M_{\text{SUSY}}$, which we define to be the geometric mean of the squark and gluino masses, $M_{\text{SUSY}} = \sqrt{M_{\tilde{g}}M_{\tilde{q}}}$. The supersymmetric contribution to the $\Delta S = 2$ effective Lagrangian is then

$$L_{\text{eff}} = \frac{\alpha_s^2(M_{\text{SUSY}})}{216 M_{\tilde{q}}^2} \sum A C_A(\mu) O_A(\mu),$$

where we have defined the operators

$$O_1 = \bar{d}_L^\mu s_i \bar{d}_L^j \gamma^\mu s_j,$$
$$O_2 = \bar{d}_R^i s_i \bar{d}_R^j s_j,$$
$$O_3 = \bar{d}_R^i s_i \bar{d}_L^j s_j,$$
$$O_4 = \bar{d}_R^i s_i \bar{d}_L^j s_j,$$
$$O_5 = \bar{d}_R^i s_i \bar{d}_L^j s_j,$$

plus other operators $\tilde{O}_{1,2,3}$, with the obvious exchanges $L \leftrightarrow R$ in $O_{1,2,3}$. In these expressions, the superscripts $i, j$ are $SU(3)$ color indices. All other operators with the correct Lorentz and color structure can be related to these by operator identities and Fiertz rearrangements.

At the matching scale $M_{\text{SUSY}}$, we determine the Wilson coefficients to be

$$C_1(M_{\text{SUSY}}) = (24 x f_6(x) + 66 \tilde{f}_6(x)) \delta_{LL}^2,$$
$$C_2(M_{\text{SUSY}}) = 204 x f_6(x) \delta_{RL}^2,$$
$$C_3(M_{\text{SUSY}}) = -36 x f_6(x) \delta_{RL}^2,$$
$$C_4(M_{\text{SUSY}}) = (504 x f_6(x) - 72 \tilde{f}_6(x)) \delta_{LR}\delta_{RR} - 132 \tilde{f}_6(x) \delta_{LR}\delta_{RL},$$
$$C_5(M_{\text{SUSY}}) = (24 x f_6(x) + 120 \tilde{f}_6(x)) \delta_{LL}\delta_{RR} - 180 \tilde{f}_6(x) \delta_{LR}\delta_{RL},$$
where \( x = M_\tilde{g}^2/M_\tilde{q}^2 \) and the \( \delta \)'s come from insertions of the squark mass matrix. The functions \( f_6 \) and \( \tilde{f}_6 \)

\[
f_6(x) = \frac{6(1 + 3x) \log x + x^3 - 9x^2 - 9x + 17}{6(x - 1)^5},
\]

\[
\tilde{f}_6(x) = \frac{6x(1 + x) \log x - x^3 - 9x^2 + 9x + 1}{3(x - 1)^5}
\]

(9)

arise from momentum integrals in the gluino-squark box diagrams. They have the limits

\[
x f_6(x) \to \mathcal{O}(x), \quad \tilde{f}_6(x) \to -\frac{1}{3} \quad (x \ll 1)
\]

\[
x f_6(x) \to \frac{1}{6x}, \quad \tilde{f}_6(x) \to \mathcal{O}(x^{-2}) \quad (x \gg 1),
\]

(10)

so the leading behavior of the effective Lagrangian goes as \( 1/M_\tilde{g}^2 \) for \( x \ll 1 \) and \( 1/M_\tilde{q}^2 \) for \( x \gg 1 \). The values of the coefficients \( \tilde{C}_{1,2,3} \) are obtained by replacing \( L \leftrightarrow R \) in eq. (8). Note that our result disagrees with that of Ref. [4], but confirms that of Ref. [5].

The operator \( O_1 \) is of \( V - A \) type. Its anomalous dimension is well known [10]:

\[
\gamma(O_1) = \frac{g_5^2}{4\pi^2}.
\]

(11)

The one-loop anomalous dimensions for the other operators are

\[
\gamma(O_2O_3) = \frac{g_5^2}{12\pi^2} \begin{pmatrix} -7 & 1 \\ 4 & 8 \end{pmatrix},
\]

(12)

\[
\gamma(O_4O_5) = \frac{g_5^2}{8\pi^2} \begin{pmatrix} -8 & 0 \\ -3 & 1 \end{pmatrix}.
\]

(13)

With these anomalous dimensions, we evolve the Wilson coefficients to the scale \( \mu_{\text{had}} \), where the hadronic observables are defined. We find

\[
C_1(\mu_{\text{had}}) = \eta_1 C_1(M_{\text{SUSY}}),
\]

\[
C_2(\mu_{\text{had}}) = \eta_{22} C_2(M_{\text{SUSY}}) + \eta_{23} C_3(M_{\text{SUSY}}),
\]

\[
C_3(\mu_{\text{had}}) = \eta_{32} C_2(M_{\text{SUSY}}) + \eta_{33} C_3(M_{\text{SUSY}}),
\]

\[
C_4(\mu_{\text{had}}) = \eta_4 C_4(M_{\text{SUSY}}) + \frac{1}{3}(\eta_4 - \eta_5) C_5(M_{\text{SUSY}}),
\]

\[
C_5(\mu_{\text{had}}) = \eta_5 C_5(M_{\text{SUSY}}),
\]

(14)
where

\[ \eta_1 = \left( \frac{\alpha_s(m_c)}{\alpha_s(\mu_{\text{had}})} \right)^{6/27} \left( \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right)^{6/25} \left( \frac{\alpha_s(M_{\text{SUSY}})}{\alpha_s(m_t)} \right)^{6/23}, \]

\[ \eta_{22} = 0.983 \eta_2 + 0.017 \eta_3, \quad \eta_{23} = -0.258 \eta_2 + 0.258 \eta_3, \]

\[ \eta_{32} = -0.064 \eta_2 + 0.064 \eta_3, \quad \eta_{33} = 0.017 \eta_2 + 0.983 \eta_3, \]

\[ \eta_2 = \eta_{1}^{2.42}, \quad \eta_3 = \eta_{1}^{2.75}, \quad \eta_4 = \eta_{1}^{-4}, \quad \eta_5 = \eta_{1}^{1/2}. \]  

(15)

2.3 \( M_{\tilde{g}} \ll M_{\tilde{q}} \) \((x \ll 1)\)

When \( M_{\tilde{g}} \ll M_{\tilde{q}} \) \((x \ll 1)\), we proceed in two steps. We first integrate out the heavy squarks at their own mass scale, \( M_{\tilde{q}} \) (see Fig. [1]). This gives an effective Lagrangian with \( \Delta S = 1 \) and \( \Delta S = 2 \) four-fermion operators. The \( \Delta S = 1 \) operators are of the form

\[ g_s^2 \left( T^a T^b \right)_{ij} \tilde{T}^a \tilde{T}^b \delta_{ij}, \]

(16)

together with similar operators with the obvious exchanges \( L \leftrightarrow R \) and/or \( d \leftrightarrow s \).

The \( \Delta S = 1 \) operators give rise to one-loop \( \Delta S = 2 \) diagrams which are suppressed by \( M_{\tilde{g}}^2/M_{\tilde{q}}^4 \), so we ignore them in what follows. The \( \Delta S = 2 \) terms in the effective Lagrangian are given by

\[ L_{\text{eff}} = \frac{\alpha_s^2(M_{\tilde{q}})}{216 M_{\tilde{q}}^2} \left\{ -22 \delta_{LL}^2 \mathcal{O}_1 - 22 \delta_{RR}^2 \tilde{\mathcal{O}}_1 + \delta_{LL} \delta_{RR} \left( 24 \mathcal{O}_4 - 40 \mathcal{O}_5 \right) \right. \]

\[ + \left. \delta_{LR} \delta_{RL} \left( 44 \mathcal{O}_4 + 60 \mathcal{O}_5 \right) \right\}, \]

(17)

where the matching is performed at the scale \( M_{\tilde{q}} \).

To construct the effective Lagrangian, these \( \Delta S = 2 \) operators must first be evolved to the gluino scale, and then to the hadronic scale.\(^\dagger\) The one-loop anomalous dimensions for operators \( \mathcal{O}_{1,4,5} \) and \( \tilde{\mathcal{O}}_1 \) were calculated previously, so the leading \( \Delta S = 2 \) effective Lagrangian at the hadronic scale can be obtained by scaling eq. (17).

We find

\[ L_{\text{eff}} = \frac{\alpha_s^2(M_{\tilde{q}})}{216 M_{\tilde{q}}^2} \left\{ -22 \delta_{LL}^2 \kappa_1 \mathcal{O}_1 - 22 \delta_{RR}^2 \kappa_1 \tilde{\mathcal{O}}_1 \right. \]

\[ + \left. \delta_{LL} \delta_{RR} \left( \frac{8}{3} (4 \kappa_4 + 5 \kappa_5) \mathcal{O}_4 - 40 \kappa_5 \mathcal{O}_5 \right) \right\}, \]

\(^\dagger\)Integrating out the gluinos does not induce any new \( \Delta S = 2 \) operators. However, it changes the beta function for the strong coupling. We have assumed that the third generation squarks have masses of order \( M_{\tilde{g}} \), in which case the one-loop beta function coefficient is \(-13/3\) between \( M_{\tilde{g}} \) and \( M_{\tilde{q}} \).
\[ \Delta S = 1 \quad \quad = \quad + \ldots \quad O \left( \frac{1}{M^2_{\tilde{q}}} \right) \]

\[ \Delta S = 2 \quad \quad = \quad + \quad O \left( \frac{1}{M^2_{\tilde{q}}} \right) \quad + \quad O \left( \frac{M^2_{\tilde{g}}}{M^4_{\tilde{q}}} \right) \quad + \ldots \]

Figure 1: The matching of the S-matrices in the full and effective theories, at the scale \( M_{\tilde{q}} \), for the case of \( M_{\tilde{g}} \ll M_{\tilde{q}} \). At one loop, the \( \Delta S = 1 \) operator generates a subleading \( \Delta S = 2 \) operator, of order \( M^2_{\tilde{g}}/M^4_{\tilde{q}} \).

\[ + \delta_{LR} \delta_{RL} \left( (64\kappa_4 - 20\kappa_5)\mathcal{O}_4 + 60\kappa_5\mathcal{O}_5 \right) \],

\[ (18) \]

where

\[ \kappa_1 = \left( \frac{\alpha_s(m_c)}{\alpha_s(\mu_{\text{had}})} \right)^{6/27} \left( \frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right)^{6/25} \left( \frac{\alpha_s(m_t)}{\alpha_s(m_b)} \right)^{6/23} \left( \frac{\alpha_s(M_{\tilde{g}})}{\alpha_s(m_t)} \right)^{6/21} \left( \frac{\alpha_s(M_{\tilde{q}})}{\alpha_s(M_{\tilde{g}})} \right)^{6/13}, \]

\[ \kappa_4 = \kappa_1^{-4}, \quad \kappa_5 = \kappa_1^{1/2}. \quad (19) \]

2.4 \( M_{\tilde{g}} \gg M_{\tilde{q}} \quad (x \gg 1) \)

When \( M_{\tilde{g}} \gg M_{\tilde{q}} \) \( (x \gg 1) \), we must again proceed in two steps. We first integrate out the gluino at \( M_{\tilde{g}} \) (see Fig. 3). The effective Lagrangian between \( M_{\tilde{g}} \) and \( M_{\tilde{q}} \),

\[ \mathcal{L}_{\text{eff}} = \sum_A D_A(\mu) \mathcal{Q}_A(\mu), \quad (20) \]

contains the following leading order operators

\[ \mathcal{Q}_1 = \bar{d}^j_{R} s^i_{L} \bar{d}^j_{R} \tilde{c}^i_{L} \]

\[ \mathcal{Q}_2 = \bar{d}^j_{R} \tilde{c}^i_{L} \bar{d}^j_{R} s^i_{L} \]

\[ \mathcal{Q}_3 = \tilde{d}^i_{L} s^j_{L} \tilde{c}^i_{L} \tilde{s}^j_{L} \tilde{s}^i_{L} \quad (21) \]
Figure 2: The matching of the S-matrices in the full and effective theories, at the scale \( \mu = M_\tilde{g} \), for the case \( M_\tilde{g} \gg M_\tilde{q} \). The \( \Delta S = 0 \) operator generates a leading \( \Delta S = 2 \) operator, whose coefficient exactly matches the \( x \gg 1 \) limit of the corresponding box diagram. This implies that the Wilson coefficient of the first \( \Delta S = 2 \) operator on the right-hand side is zero.

as well as other operators with obvious exchanges, \( L \leftrightarrow R \) and/or \( s \leftrightarrow d \). The superscript \( c \) in \( Q_3 \) stands for the charge conjugated field.

By matching at the scale \( M_\tilde{g} \), we determine the Wilson coefficients to be

\[
D_1 = -3D_2 = 6D_3 = -\frac{g_s^2(M_\tilde{g})}{M_\tilde{g}}.
\]  

The one-loop anomalous dimensions are

\[
\gamma(Q_1Q_2) = \frac{g_s^2}{8\pi^2} \begin{pmatrix} -8 & 0 \\ -3 & -\frac{7}{2} \end{pmatrix}, \quad \gamma(Q_3) = -\frac{3g_s^2}{8\pi^2}.
\]  

Therefore the Wilson coefficients \( D_A (A = 1, 2, 3) \) at the scale \( M_\tilde{q} \) are given by:

\[
D_1(M_\tilde{q}) = \varepsilon_1D_1(M_\tilde{g}) + \frac{1}{3}(\varepsilon_1 - \varepsilon_2)D_2(M_\tilde{g})
\]

\[
D_A(M_\tilde{q}) = \varepsilon_AD_A(M_\tilde{g}), \quad A = 2, 3
\]  

where

\[
\varepsilon_1 = \left( \frac{\alpha_s(M_\tilde{g})}{\alpha_s(M_\tilde{q})} \right)^{-8/5}, \quad \varepsilon_2 = \varepsilon_1^{7/16}, \quad \varepsilon_3 = \varepsilon_1^{3/8}.
\]
We now evolve the effective Lagrangian to the scale \( M_{\tilde{q}} \), where we integrate out all the squarks. At that scale, the \( \Delta S = 2 \) effective Lagrangian is given by

\[
\mathcal{L}_{\text{eff}} = \frac{\alpha_s^2(M_{\tilde{q}})}{216 M_{\tilde{g}}^2} \left\{ \delta_{LL} \delta_{RR} \left( \frac{4}{3} (64 \varepsilon_1^2 - \varepsilon_2^2) \mathcal{O}_4 + 4 \varepsilon_2^2 \mathcal{O}_5 \right) \\
+ \left[ \frac{2}{3} (64 \varepsilon_1^2 - \varepsilon_2^2) - 8 \varepsilon_3^2 \right] \mathcal{O}_2 + (2 \varepsilon_2^2 - 8 \varepsilon_3^2) \mathcal{O}_3 \right\}.
\]

(26)

This effective Lagrangian matches the \( M_{\tilde{q}} \ll M_{\tilde{g}} \) limit of the full theory.

Finally, this Lagrangian must be evolved to the hadronic scale. We find

\[
\mathcal{L}_{\text{eff}} = \frac{\alpha_s^2(M_{\tilde{q}})}{216 M_{\tilde{g}}^2} \left\{ \delta_{LL} \delta_{RR} \left( \frac{4}{3} (64 \varepsilon_1^2 - \varepsilon_2^2) \mathcal{O}_4 + 4 \varepsilon_2^2 \mathcal{O}_5 \right) \\
+ \left[ \frac{2}{3} (64 \varepsilon_1^2 - \varepsilon_2^2) - 8 \varepsilon_3^2 \right] \mathcal{O}_2 + (2 \varepsilon_2^2 - 8 \varepsilon_3^2) \mathcal{O}_3 \\
+ 4 \varepsilon_3^2 \delta_{LL}^2 \mathcal{O}_1 + (L \leftrightarrow R, \mathcal{O} \rightarrow \tilde{\mathcal{O}}) \right\}.
\]

(27)

where the \( \eta' \)'s have the same form as the \( \eta \)'s, with the scale \( M_{\text{SUSY}} \) replaced by \( M_{\tilde{q}} \) in eq. (15).

3 Numerical results

In this section, we will use the \( \Delta S = 2 \) effective Lagrangians to compute the supersymmetric corrections to the \( K \)-meson mass difference,

\[
\Delta m_K = 2 \text{Re} \langle K | \mathcal{L}_{\text{eff}} | \overline{K} \rangle.
\]

(28)

We shall see that the measured value, \( \Delta m_K = 3.5 \times 10^{-12} \text{ MeV} \), places tight limits on the parameters \( \delta \).

Note that the \( \Delta S = 2 \) effective Lagrangians can also be used to compute the supersymmetric contribution to the CP-violating parameter \( \epsilon_K \),

\[
|\epsilon_K| = \frac{\text{Im} \langle K | \mathcal{L}_{\text{eff}} | \overline{K} \rangle}{2 \text{Re} \langle K | \mathcal{L}_{\text{eff}} | \overline{K} \rangle}.
\]

(29)

If the phases of the \( \delta \)'s are of order one, the experimental limits on \( \epsilon_K \) place very tight constraints on the squark mass matrices (see, for example, Ref. [8]).
Table 1: Limits on the $\text{Re}\delta$’s, for squark mass $M_{\tilde{q}} = 500$ GeV and different $x = M_{\tilde{g}}^2/M_{\tilde{q}}^2$. In each case, the left (right) numbers are limits without (with) leading order QCD corrections.

| $x$  | $\sqrt{|\text{Re}\delta_{LL}^2|}$ | $\sqrt{|\text{Re}\delta_{LR}^2|}$ |
|------|----------------------------------|----------------------------------|
| 0.3  | $2.3 \times 10^{-2}$            | $3.0 \times 10^{-2}$            | $5.5 \times 10^{-3}$ | $2.8 \times 10^{-3}$ |
| 1.0  | $5.0 \times 10^{-2}$            | $6.6 \times 10^{-2}$            | $6.3 \times 10^{-3}$ | $3.2 \times 10^{-3}$ |
| 4.0  | $0.12$                           | $0.16$                          | $9.2 \times 10^{-3}$ | $4.6 \times 10^{-3}$ |

| $\sqrt{|\text{Re}\delta_{LL}\delta_{RR}|}$ | $\sqrt{|\text{Re}\delta_{LR}\delta_{RL}|}$ |
|-----------------------------------------------|-----------------------------------------------|
| 0.3 $3.1 \times 10^{-3}$                      | $1.0 \times 10^{-3}$ $4.2 \times 10^{-3}$ | $1.4 \times 10^{-3}$ |
| 1.0 $3.6 \times 10^{-3}$                      | $1.2 \times 10^{-3}$ $7.2 \times 10^{-3}$ | $2.4 \times 10^{-3}$ |
| 4.0 $5.3 \times 10^{-3}$                      | $1.7 \times 10^{-3}$ $1.7 \times 10^{-2}$ | $5.6 \times 10^{-3}$ |

In each of these expressions, the effective Lagrangian $L_{\text{eff}}$ contains the operators $O_1$ through $O_5$, with coefficients evaluated at the hadronic scale, which we define to be the scale where $\alpha_s = 1$. We evaluate the hadronic matrix elements and find

$$
\langle K | O_1 | \overline{K} \rangle = \frac{1}{3} m_K f_K^2 B_1,
$$

$$
\langle K | O_2 | \overline{K} \rangle = -\frac{5}{24} \left( \frac{m_K}{m_s + m_d} \right)^2 m_K f_K^2 B_2,
$$

$$
\langle K | O_3 | \overline{K} \rangle = \frac{1}{24} \left( \frac{m_K}{m_s + m_d} \right)^2 m_K f_K^2 B_3,
$$

$$
\langle K | O_4 | \overline{K} \rangle = \left[ \frac{1}{24} + \frac{1}{4} \left( \frac{m_K}{m_s + m_d} \right)^2 \right] m_K f_K^2 B_4,
$$

$$
\langle K | O_5 | \overline{K} \rangle = \left[ \frac{1}{8} + \frac{1}{12} \left( \frac{m_K}{m_s + m_d} \right)^2 \right] m_K f_K^2 B_5.
$$

The coefficients $B_1 \ldots 5$ characterize the long-distance hadronic physics; we take $B_i = 1$, as determined in the vacuum insertion approximation.

In the rest of this section, we use these results to illustrate the size of the leading
Figure 3: Limits on the Re $\delta$'s as a function of the common first two generation squark mass, $M_{\tilde{q}}$, for a light gluino mass of $M_{\tilde{g}} = 200$ GeV. The solid lines describe our effective field theory result. The dashed lines correspond to decoupling the supersymmetric particles at $M_{\text{SUSY}} = \sqrt{M_{\tilde{q}} M_{\tilde{g}}}$, without including the leading order QCD corrections.

order QCD corrections to $K$-$\overline{K}$ mixing, for each of the three cases discussed above. For our numerical work, we take $f_K = 160$ MeV, $m_K = 498$ MeV \[\text{(1)}\] and the current mass $m_s = 150$ MeV \[\text{(1)}\]. We use the strong coupling as determined from electroweak measurements, $\alpha_s(M_Z) = 0.118$ \[\text{(1)}\]. This gives $\alpha_s(m_c) = 0.35$ and $\alpha_s(m_b) = 0.22$.

We first consider the case $M_{\tilde{g}} \simeq M_{\tilde{q}}$. The QCD-corrected $\Delta S = 2$ effective Lagrangian gives rise to the limits shown in Table 1. In the table, we have taken the first two generation squark mass $M_{\tilde{q}} = 500$ GeV and varied $x = M_{\tilde{g}}^2 / M_{\tilde{q}}^2$. The left (right) numbers correspond to the limits without (with) the leading order QCD corrections.

The most stringent limit in the table comes from $\sqrt{|\text{Re} \delta_{LL} \delta_{RR}|}$. For a given mass splitting, we see the QCD corrections increase the squark lower bound by a factor of three! Note that the corrections are generally more than 70%, so they cannot be ignored in computing limits on the squark masses.

We next turn to models with very different squark and gluino masses. In Fig. 3, we

\[\text{(1)}\] This mass is not well determined; see Ref. \[\text{(2)}\] for recent analyses. Decreasing $m_s$ to 100 MeV tightens the bounds by at most 30%.

\[\text{**} \] Our numbers agree with those of Ref. \[\text{(3)}\] if we follow their procedure and decouple the squarks and gluino at $M_Z$ and neglect the QCD corrections.
consider the case of a light gluino, and plot the limits on $\delta$'s versus the squark mass. The solid lines describe our results, while the dashed lines correspond to decoupling the supersymmetric particles at $M_{\text{SUSY}} = \sqrt{M_{\tilde{q}} M_{\tilde{g}}}$, with no QCD corrections.

From the figure we see that the QCD corrections tighten the most stringent bounds by about 50%. For $M_{\tilde{g}} = 200$ GeV, with $\delta$ of order 1, the bound on the squark mass is $M_{\tilde{q}} \gtrsim 200$ TeV. Such a heavy squark can drive the third generation squark/slepton mass squared negative if supersymmetry breaking is transmitted by gravitational interactions [13].

Alternatively, one can make the gluino heavy while keeping the squarks light. We show limits for this type of model in Fig. 4. For $M_{\tilde{q}} = 200$ GeV, and $\delta$ of order 1, the bound on the gluino is $M_{\tilde{g}} \gtrsim 200$ TeV. (Note, however, that in this scenario, the charginos and neutralinos must also be heavy to suppress FCNC's from electroweak diagrams.) In a unified model, a large gluino mass drives up the squark masses through their renormalization group evolution, so this scenario can only be natural in models with TeV-scale supersymmetry breaking.

4 Conclusion

In this paper we have calculated the leading order QCD corrections to supersymmetric contributions to $K-\bar{K}$ mixing. We find the corrections to be significant. Indeed, for
the case $M_\tilde{q} \simeq M_\tilde{g}$, the QCD corrections increase the lower bound on the first two generation squark masses by a factor of three.

For the case $M_\tilde{q} \gg M_\tilde{g}$, we find that QCD corrections also increase the squark lower bound. This exacerbates the naturalness problems associated with such a hierarchy, and can destabilize the effective potential when supersymmetry breaking is transmitted by supergravity interactions. Similar results hold when $M_\tilde{g} \ll M_\tilde{g}$. In each case, the QCD corrections are important and must be included when deriving constraints from FCNC processes.

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