A passive GHz frequency-division multiplexer/demultiplexer based on anisotropic magnon transport in magnetic nanosheets

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The emerging field of magnonics employs spin waves and their quanta, magnons, to implement wave-based computing on the micro- and nanoscale [1, 2, 3, 4, 5, 6]. Multi-frequency magnon networks allow for parallel data processing within single logic elements whereas this is not the case with conventional transistor-based electronic logic [7]. However, a lack of experimentally proven solutions to efficiently combine and separate magnons of different frequencies has impeded the intensive use of this concept. In this Letter, we demonstrate the experimental realization of a spin-wave demultiplexer enabling frequency-dependent separation of GHz signals. The device is based on two-dimensional magnon transport in the form of spin-wave beams in unpatterned magnetic nanosheets. The intrinsic frequency-dependence of the beam direction is exploited to realize a passive functioning obviating an external control and additional power consumption. This approach paves the way to magnonic multiplexing circuits [8] enabling simultaneous information transport and processing.

Wave-based computing [9, 10, 11], required for parallel data processing in single elements, has already been demonstrated by utilizing spin waves in micro- and nanostructures. Furthermore, the likewise necessary technique of simultaneous information transport in separated frequency-channels, called frequency-division multiplexing, is used for decades in various applications such as in fiber-optic networks. However, its implementation in magnonic networks has only been theoretically studied [7, 8] or discussed in preparatory works [12, 13, 14]. Here, we present the missing link by experimentally demonstrating a magnonic frequency-division multiplexer/demultiplexer enabling the realization of multi-frequency spin-wave circuits as recently designed by micromagnetic modelling [8].
The utilization of spin waves for wave-based computing offers many advantages [1, 2, 3, 4, 5]. Their typical frequencies in the range from GHz to THz together with the associated wavelengths on the micro- and nanometre scale [15, 16, 17] allows for their utilization in microchips and nanostructured devices while operating at the same frequency as many current and emerging techniques, e.g. Bluetooth, WLAN and 5G mobile networks. This makes a conversion between different frequencies redundant when alternating between data transmission via air and data processing in devices. Furthermore, the enormous tunability of spin-wave properties by various parameters such as the external magnetic field or the local magnetization [18] can be a significant advantage compared to the utilization of, e.g., surface acoustic waves.

One exceptional property utilized in the following work is the anisotropic propagation of spin waves in suitable magnetic media, which can lead to the creation of narrow spin-wave beams and caustics [8, 19, 20, 21, 22, 23, 24, 25] (see Methods). Their sub-wavelength transverse aperture [23] is an outstanding reason for their potential use in nanostructured networks, especially when considering the beam formation of high-wavevector magnons [20]. The spin-wave beams can be radiated from point-like sources into unpatterned magnetic films [21, 22, 23, 24] and they are formed due to the non-collinearity of the wavevector and the group velocity vector. Furthermore, their propagation direction can be versatility controlled by, e.g., frequency [8, 19, 21, 24, 25], magnetic field strength [19] and direction of the local magnetization [22, 23].

In this work, we experimentally demonstrate how these spin-wave beams can be used to realize a controllable information transport in two-dimensional magnetic films with thicknesses of a few tens of nanometers, so called nanosheets. The guidance of the signal is not achieved by geometrical structuring [9, 10] or magnetically induced channels [26] but by the inherent anisotropy of the magnon system. Utilizing this anisotropic signal propagation allows for the realization of passive elements without any additional energy consumption since the magnon system inherently collimates and steers the energy transport. As just one of the many possible applications of this concept, we present the realization of a passive demultiplexer exploiting the frequency-dependence of the beam direction. By inverting the geometry a multiplexer can be realised equally well. The discussed two-dimensional signal transport could be used in various logic circuits, for example to distribute information in artificial neural networks [4]. Finally, the presented concept could be transferred to other systems which can show a pronounced anisotropy of energy propagation as well like, e.g., surface acoustic waves [27] or photons in hyperbolic materials [28].

The experimental realization of the frequency-division demultiplexer is based on 30-nm-thin films of the magnetic alloy CoFeB. Being based on our previous studies [8], we have designed a prototype by micromagnetic modelling. Subsequently, the sample has been fabricated and studied
by detecting the spin-wave intensity using micro-focused Brillouin light scattering spectroscopy (µBLS) [29]. After investigating the frequency-dependence of the spin-wave beam directions and comparing it to theoretical calculations, we verify the full functionality of the demultiplexer by two-dimensional measurements of the spin-wave intensity.

Figure 1a,b presents the simulated intensity distribution inside of the designed structure for spin waves of frequency \( f_1 = 11.2 \text{ GHz} \) and \( f_2 = 13.8 \text{ GHz} \). The dashed arrows mark the beam directions that are predicted by the developed theoretical model (see Fig. 1c and Methods). The functional part of the device comprises the unpatterned area in the centre of the structure in which the two-
dimensional energy transport takes place. Input and output waveguides, which can serve as interfaces to adjacent magnonic building blocks in larger magnonic networks, are connected to this area. To provide an input signal, magnon excitation is simulated by applying localized alternating magnetic fields $h_r$ inside the input waveguide. The field distribution is chosen in accordance with the experiment in which these fields are created by microwave currents flowing through a microstrip antenna placed across the waveguide. The spin waves propagate towards the unpatterned area and, in agreement with previous observations [8, 21, 22, 23], two symmetric spin-wave beams are emitted from the waveguide opening, which acts as a point-like source exhibiting a broad wavevector spectrum. The anisotropy of the spin-wave dispersion leads to a strong focusing of the energy. The observed frequency dependence of the beam directions is utilized to realize the demultiplexing functionality by properly adding two output waveguides after a certain propagation distance. Their positions are asymmetric with respect to the input, leading to the following consequence: at frequency $f_1 = 11.2$ GHz (Fig. 1a), the upper spin-wave beam, which propagates at an angle of $\theta_{B,1}^{\text{theo}} = 77.6^\circ$ as predicted by the theoretical model, is channelled into output 1 and transmits the information through the device. The lower one is blocked by the edge of the magnetic structure so that the signal cannot reach output 2. In contrast, the beam angles of spin waves of frequency $f_2 = 13.8$ GHz are changed to $\theta_{B,2}^{\text{theo}} = 68.9^\circ$ ($\theta_{B,2} = 180^\circ - \theta_{B,2}$, resp.) resulting in a sole signal transmission into output 2 (Fig. 1b). Hence, the device separates spin-wave signals of two frequencies into two spatially detached output waveguides, whose transition zones are especially tailored to enable an efficient channeling of the spin waves. The exploited frequency dependence of the beam directions result from the modification of the spin-wave isofrequency curve with frequency (see Fig. 1c) and the according variation of the group-velocity vectors, which are always perpendicular to this curve (see Methods). It should be mentioned that the exact operating frequencies of the device can be controlled by changing, e.g., the external field strength since this shifts the magnon dispersion. Here, the external magnetic field is set to $\mu_0 H_{\text{ext}} = 75$ mT.

Based on the design developed and optimized by micromagnetic modelling, the prototype of the passive demultiplexer has been fabricated based on a 30-nm-thin nanosheet of CoFeB as shown in Fig. 1d. To prove the functionality of the frequency-division demultiplexer, µBLS measurements of the spin-wave intensity have been performed at two positions in the centre of the output waveguides as marked in Fig. 1d. Spin waves are excited in the input in a frequency range from 10.5 GHz to 15.5 GHz in steps of 0.1 GHz by applying an according microwave signal to the microstrip antenna. Figure 2a shows the detected frequency dependence of the spin-wave intensity in the two spatially detached output waveguides. A clear separation of spin-wave signals
depending on their frequency is visible. Signals in the narrow range from 10.8 GHz to 11.8 GHz are channelled into output 1. In contrast, output 2 accepts only spin waves in the broader frequency band from 12.3 GHz to 15.3 GHz, hence, without any overlap. This measurement clearly demonstrates that spin-wave signals with appropriately chosen frequencies, e.g., the frequencies of maximum intensity in the different outputs \( f_1 = 11.2 \, \text{GHz} \) and \( f_2 = 13.8 \, \text{GHz} \), are spatially separated by the prototype device as predicted by the micromagnetic modelling.

The beam angle \( \theta_B \) shown in Fig. 2b is experimentally determined by measuring the spin-wave intensity along a line in the input transition zone (left green dotted line, Fig. 1d) and along a line in the second third of the unpatterned area (right green dotted line). From the observed shift of the intensity maxima, the angle \( \theta_B \) is calculated. No beams are detected below 10.8 GHz since the dispersion relation is very flat until the frequency of ferromagnetic resonance \( f_{\text{FMR}} = 10.94 \, \text{GHz} \) is
reached. The measured data agrees well with the theoretically predicted curve (solid line in Fig. 2b). This is due to the fact that, in addition to the peculiarities of anisotropic magnon propagation [19], our model considers such important parameters like the excited wavevector spectrum and the distance between the observation point and the source. The optimum angles $\theta^\text{exp}_B,1 = 77.9^\circ$ and $\theta^\text{exp}_B,2 = 69.6^\circ$, leading to maximum intensity in the outputs, are indicated in Fig. 2b together with the shaded acceptance intervals, which result from the assumption that beams are channelled into the outputs if their direction deviates less than 2.5° from the optimum angles. These acceptance intervals in combination with the curve $\theta_B(f)$ can explain the differing widths of the accepted frequency bands of the two outputs. The angle variation is quite large in the region of $f_1 = 11.2$ GHz. Hence, only frequencies within a small range around $f_1$ create beams whose directions lie inside the angular acceptance interval of output 1. In contrast, the slope of the mentioned curve is significantly lower at $f_2 = 13.8$ GHz. Thus, a larger variation of the frequency is permitted around $f_2$ until the occurring beam angles are out of the acceptance interval of output 2.

For a detailed visualisation, the spin-wave intensity was measured in the entire demultiplexer area at the frequencies $f_1$ and $f_2$ by performing two-dimensional µBLS scans. A comparison of these experimental results (Fig. 3) with the results of the micromagnetic modelling (Fig. 1a,b) reveals a remarkably good agreement. This confirms that the whole demultiplexing mechanism works exactly as predicted and is based on the creation of spin-wave beams, their frequency-dependent propagation though the unstructured area, and the channelling of the signals into different outputs.

Furthermore, these measurements can be used to determine the input-output ratio $\kappa$ of the device to reveal the influence of the beam formation on signal losses. Considering the transmission from the input waveguide into output 1 at frequency $f_1$ yields a value of $\kappa^\text{exp}_1 = (18.1 \pm 2.1)$ dB whereas a ratio of $\kappa^\text{exp}_1 = (18.2 \pm 1.9)$ dB is obtained for a signal at frequency $f_2$ channelled into output 2. For comparison, the ratio $\kappa^\text{theo}$ can be calculated analytically considering the energy splitting and the propagation losses only (see Methods). The resulting values $\kappa^\text{theo}_1 = (18.95 \pm 1.84)$ dB and $\kappa^\text{theo}_2 = (17.64 \pm 1.69)$ dB show that the major fraction of the losses arise from the intrinsic spin-wave damping during their propagation and not due the demultiplexing mechanism based on beam formation and channelling (the splitting is suppressible [8]).

Finally, the scalability and advantages of the presented concept should be discussed. The employed spin-wave beams are created due to the anisotropy of the magnonic system and, for
limited length scales, their width is primary determined by the size of the source [23]. In the presented case, the anisotropy is induced by the dipole-dipole-interaction, which is dominant in the low-wavevector range of the magnon spectrum. However, a beam formation is also possible for high-wavevector spin waves whose dispersion characteristics are mainly dominated by the exchange interaction [20]. Hence, the presented concept to guide two-dimensional energy transport by spin-wave beams, formed due to the intrinsic anisotropy of the system, can be scaled down to significantly smaller length scales. Furthermore, the passive character of the signal steering is an important advantage over other concepts requiring energy-consuming external control. This is the case, e.g., with the previously demonstrated spin-wave time-division multiplexer, which relies on external charge currents [30]. In addition, the technique of time-division multiplexing transmits multiple signals through a single line one after the other. Hence, this does not enable simultaneous data processing in single devices.

In conclusion, we have developed and experimentally realized a passive frequency-division demultiplexer, which is based on intrinsically steered two-dimensional signal transport in unpatterned magnetic nanosheets. The prototype device enables the spatial separation of spin-wave signals of different frequencies. It exploits the frequency-dependence of the direction of narrow spin-wave beams, which are formed in in-plane magnetized films due to the anisotropic

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**Figure 3 | Experimental verification of the beam-based demultiplexing mechanism.** μBLS measurements of the spin-wave intensity are performed to visualize the two-dimensional spin-wave transport in the nanosheets. The separation of spin-wave signals from one shared input waveguide by frequency-dependent channelling into spatially detached output waveguides occurs due to the intrinsically controlled beam formation. a, Spin wave guidance into output 1 at \( f_1 = 11.2 \) GHz and b, into output 2 at \( f_2 = 13.8 \) GHz.
spin-wave dispersion. A theoretical approach to predict the beam directions has been developed and verified. The utilised beam steering requires no external control or additional power consumption and it can be used to realize a whole multi-frequency circuit which enables simultaneous data transport through single transmission lines by employing the technique of frequency-division multiplexing. This is the basis for exploiting one of the main advantages of wave-based computing, namely the technique of parallel data processing in single devices, which can now be implemented in an energy-efficient way building on the presented prototype.

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Author contributions

P.P., T.B., A.A.S. and B.H. supervised the project. F.H. and P.P. developed the concept and planned the experiment. F.H. performed the micromagnetic modelling. G.T., F.C. and C.A. produced the sample. F.H. and T.M. carried out the µBLS measurements. B.He. and M.G. performed the FMR measurements. F.H. and K.Y. developed the theoretical model. All authors discussed the results and wrote the manuscript.

Competing interests

The authors declare no competing interests.

Data availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.
Methods

**Micro-focused Brillouin light scattering (µBLS) measurements.** The spin-wave intensity is experimentally measured by utilising the inelastic scattering of photons from magnons, which is called Brillouin light scattering. The annihilation (or creation) of magnons in the sample leads to a respective frequency shift and change of the wavevector of the incident photons in accordance with the conservation laws. To use this phenomenon, light of a probing laser with a wavelength of 532 nm is focused onto the sample and the reflected light is send through a (3 + 3) pass Tandem-Fabry-Pérot-Interferometer (JRS Scientific Instruments) to analyse the photons with respect to frequency and intensity. The intensity of the light which is shifted in frequency is directly proportional to the intensity of spin waves relating to this frequency shift. Due to the focusing of the incident laser light by means of a microscope objective, a spatial resolution of 250 nm can be reached leading to the term micro-focused Brillouin light scattering (µBLS) spectroscopy. By changing the measuring position, two-dimensional intensity maps can be recorded. The shown spin-wave intensity of frequency \( f \) is the detected time-averaged µBLS intensity integrated over the frequency interval \([-0.25 \text{ GHz}, 0.25 \text{ GHz}]\). The error of the measured beam angles \( \theta_B \) shown in Fig. 2b results from the spatial resolution of the setup and is calculated to 3.5°. For reasons of clarity, error bars are omitted in the figure. The spin-wave intensity shown in Fig. 3 is normalized along the \( y \)-direction to the interval \([0,1]\) to compensate for the background noise of the setup and for the spin-wave damping so that a clear comparison of the two output signals is easily possible. Due to this normalization, the visibility of spin-wave beams, which are reflected from the edge of the structure and which are much weaker than the incident beams, is reduced.

**Micromagnetic modelling.** The micromagnetic modelling is carried out by using the GPU-accelerated simulation program MuMax3 [31]. A finite-difference-method is used to numerically calculate the time-dependent magnetisation dynamics in microstructures by solving the Landau–Lifshitz–Gilbert equation (LLG) in the cells. The LLG is the fundamental equation describing the magnetisation dynamics. The simulated structure of the size 31.5 \( \mu \text{m} \times 15 \mu \text{m} \times 30 \text{ nm} \) is discretized into 2048 x 1024 x 1 cells and the experimentally determined material parameters of CoFeB (\( M_S = 1558 \text{ kA m}^{-1}, A_{ex} = 17.6 \text{ pJ m}^{-1}, \alpha = 0.0043 \)) are used. On the outer vertical edges of the waveguides (shape of the structure as shown in Fig. 1), the damping is incrementally increased (25 steps over a distance of 0.5 \( \mu \text{m} \)) to a value of \( \alpha = 0.5 \) to suppress reflections of the spin-wave energy. Furthermore, periodic boundary conditions are used along the horizontal edges of the unstructured film area. A spin-wave excitation and propagation is simulated for 7.5 ns by applying an alternating magnetic field inside the input waveguide. The used field distribution is calculated
according to Biot-Savart’s law for a 0.5 µm wide and 100 nm thick microstrip antenna (as used in the experiment) through which a current of 5 mA flows. The resulting raw data of the modelling is the time-dependent vector of the magnetisation inside every cell, which has been saved every 25 ps. A temporal Fourier-transformation of the data from 2.5 ns to 7.5 ns and a squaring of the resulting amplitude has been performed to calculate the frequency-dependent, time-averaged spin-wave intensity. This intensity is integrated over the frequency interval \([f - 0.25 \text{ GHz}, f + 0.25 \text{ GHz}]\), so that a direct comparison with the experimental results obtained by µBLS measurements is possible. Furthermore, the simulated spin-wave intensity distribution shown in Fig. 1 is normalized along the \(y\)-direction to its sum to compensate for the spin-wave damping and to allow for a clear comparison of the two output signals.

**Sample fabrication.** The basis of the sample fabrication [32] is a 30-nm-thin nanosheet of the magnetic alloy CoFeB, which is deposited as part of the layer stack Ta(3 nm)/CoFeB(30 nm)/Ta(3 nm) onto a Si/SiO\(_2\)(300 nm) wafer. First, the material parameters of this unpatterned sample have been measured using two methods. The saturation magnetisation \(M_s = 1558 \text{ kA m}^{-1}\) and the Gilbert damping constant \(\alpha = 0.0043\) have been determined by measurements of the ferromagnetic resonance (FMR). Furthermore, the exchange constant \(A_{\text{ex}} = 17.6 \text{ pJ m}^{-1}\) results from µBLS measurements of the thermal magnon spectrum and a comparison to analytical calculations according to the theory [33]. Afterwards, the magnetic layer is structured by e-beam lithography of a hydrogen silsesquioxane (HSQ) hard mask and subsequent ion beam etching, leaving around 70 nm of the resist on top of the structure. To reduce the height differences, a planarization step using Spin-on Carbon (SoC) follows with a subsequent deposition of a 30-nm-thin layer of SiN (150°C, Chemical Vapor Deposition). Finally, the microstrip antenna is fabricated by creating a mask using polymethylmethacrylat (PMMA) resist and e-beam lithography followed by sputtering deposition of Ti (10 nm)/Au (100 nm) and the removal of the unwanted parts by a lift-off process in acetone. The resulting sample is shown in Fig. 1c. The magnetic structure consists of an unpatterned film area and three 1.5 µm wide waveguides. The light shading on the edge of the transition zones is due to some residual material, which, however, doesn’t influence the functionality of the device. The 0.5 µm wide and 100 nm thick microstrip antenna, which is placed on top of the input waveguide, is connected to a microwave setup consisting of a microwave generator and an amplifier which provide the high-frequency current with a nominal output power of \(P = +7 \text{ dBm}\). The final power reaching the sample is very likely significantly reduced. The rectangular structures around the actual device are used to stabilize the positioning of the µBLS microscope. It should be mentioned that the dimensions of the presented prototype has been chosen to allow for an optical investigation using µBLS. Further miniaturization is possible as discussed in the main text.
Calculation of the input-output ratio $\kappa$. To calculate the experimental values $\kappa^{\text{exp}}$ of the input-output ratio, the spin-wave intensity at the position 0.5 $\mu$m in front of the input transition zone and the intensity 5.1 $\mu$m behind the output transition zone is integrated over the waveguide width and both results are divided by each other. The mentioned error results from the deviations around the mean value in case the surrounding measuring points are taken into account. For both frequencies, the detected intensity inside the output, which is not hit by the spin-wave beam, is equal or only marginally above the noise-level of the $\mu$BLS setup. This highlights the pronounced demultiplexing functionality.

The theoretical reference values $\kappa^{\text{theo}}$ are calculated from the distances $L$ which the spin waves have to travel from the position in the input to the position in the output, where the intensity has been measured to determine the experimental value. The calculations are carried out according to the formula

$$\kappa^{\text{theo}} = 10 \log_{10}[1/2 \exp\left(-\frac{L_\ell}{(0.5 \nu_\ell \tau_\ell)} - \frac{L_w}{(0.5 \nu_w \tau_w)}\right)],$$

in which the occurring energy splitting into two spin-wave beams is taken into account by the factor 1/2. It should be mentioned that the splitting can be suppressed by changing the design of the demultiplexer as shown in [8]. Furthermore, the factor 0.5 has to be introduced since the spin-wave intensity is measured instead of the amplitude.

The group velocity $\nu$ of the spin waves differs inside the waveguides ($\nu_w$) and in the unpatterned film area ($\nu_\ell$) and can be calculated according to [33, 34]. To determine $\nu_\ell$, the spin-wave wavevector relating to the occurring beam angle is used for the calculations. Furthermore, $L_w$ is the spin-wave propagation distance in the waveguides whereas $L_\ell$ is the corresponding distance in the film area. Finally, the spin-wave lifetime $\tau$ results from the damping parameter $\alpha$ and is approximated by the value $\tau(k = 0)$ at ferromagnetic resonance [34]. For all calculations, the effective values of the magnetic field $H_{\text{eff}}$ (75 mT inside the film, 50 mT inside the waveguide) and the waveguide width $w_{\text{eff}} = 1.3$ $\mu$m are used. These effective values result from the demagnetising field at the edge of the waveguides and can be extracted from the micromagnetic modelling or calculated as shown in [34].

The following values are obtained for the different frequencies $f_1 = 11.2$ GHz and $f_2 = 13.8$ GHz: $L_{1,w} = 5.48$ $\mu$m, $L_{1,\ell} = 13.05$ $\mu$m, $\nu_{1,w} = 11.65$ $\mu$m ns$^{-1}$, $\nu_{1,\ell} = 7.08$ $\mu$m ns$^{-1}$, $\tau_w = 1.284$ ns, $\tau_\ell = 1.254$ ns, and $L_{2,w} = 5.51$ $\mu$m, $L_{2,\ell} = 13.90$ $\mu$m, $\nu_{2,w} = 9.70$ $\mu$m ns$^{-1}$, $\nu_{2,\ell} = 8.93$ $\mu$m ns$^{-1}$ resulting in the values for the input-output ratio $\kappa^{\text{theo}}$ as mentioned in the main text. The error of $\kappa^{\text{theo}}$ is calculated by assuming a slightly larger Gilbert damping parameter ($\alpha = 0.0048$), which is very likely to occur due to, e.g., the structuring process of the device.
Theoretical description of spin-wave beams and caustics. The basis of the developed theoretical model is the anisotropic isofrequency curve (see Fig. 1c), which describes the dependence of the wavevector component \( k_y \) on the component \( k_x \) at a particular frequency \( f \). The dispersion calculations are based on [33] taking into account \( k_y || H_{\text{ext}}, k_x \perp H_{\text{ext}}, \mu_0 H_{\text{ext}} = 75 \text{ mT} \) and the material parameters of CoFeB. The direction of the anisotropic spin-wave propagation can be derived from the curve since the spin-wave group velocity vector is perpendicular to it at every point. For a precise prediction of the resulting spin-wave beam orientation, the excited wavevector spectrum \( A_k \) and the distance \( d \) between the beam source and the observation point is taken into account as explained in the following. This is the main difference to a previous theoretical approach [19] and our results reveal, that the observed direction of maximum focusing is not always given by the caustic direction (According to [19], caustics occur at the points of the isofrequency curve where its curvature is zero). Due to this reason, we refer to the observed beams resulting from the focused energy transport as (caustic-like) spin-wave beams instead of caustics.

The first step of the developed approach is to include the wavevector spectrum \( A_k \) of the beam source in the calculations. It can be assumed that the spin waves propagate through the input in the form of the first waveguide mode having a sinusoidal shape along the short axis of the waveguide and only one maximum in the centre. If the input waves reach the transition zone, the waveguide mode acts as a source for secondary spin waves propagating into the unpatterned film area and forming beams due to their anisotropic propagation. The confinement of the mode across the waveguide leads to broad spectrum \( A_k \) of the respective wavevector component \( k_y \), which can be calculated to \( A_k(k_y) = \cos(k_y w_{\text{eff}}/2)/(\pi^2/w_{\text{eff}}^2 - k_y^2) \) by spatial Fourier-transformation of the mode profile (\( w_{\text{eff}} \) is the effective width of the input waveguide. See explanation above.). This spectrum is taken into account by using the following procedure. First, the angle \( \theta_{vg} \) between the group velocity vector and the external field direction is calculated from the isofrequency curve as a function of the wavevector component \( k_y \). Second, the wavevector spectrum \( A_k(k_y) \) is projected onto the curve \( \theta_{vg}(k_y) \) by a numerical integration method to determine the amount of excited spin waves propagating into the direction \( \theta_{vg} \). The result of this procedure is the spin-wave amplitude \( A_{SW}^{\text{initial}}(\theta_{vg}) \), which reflects the angle distribution of the spin-wave flow under consideration of the initial wavevector spectrum \( A_k \). Maxima of \( A_{SW}^{\text{initial}}(\theta_{vg}) \) reveal a wave focusing into the respective directions. Their occurrence can be explained by the fact that the anisotropy of the system leads to isofrequency curves with expanded regions where the group velocity direction \( \theta_{vg} \) is (nearly) unchanged for a broad range of wavevectors \( k_y \). If the excited spectrum \( A_k \) has a significant magnitude in these ranges of \( k_y \), an amplitude concentration occurs into the respective directions.
\( \theta_v \) leading to the maxima of \( A_{SW}^{\text{initial}}(\theta_v) \). Hence, these maxima indicate the beam creation.

However, a second step is necessary to precisely determine the beam directions. Real beam sources have a certain extent which leads to the fact that the overall spin-wave amplitude at the observation point results from a superposition of spin waves which are generated at different positions within the source. If the observation point is close, the source appears under a large angular range and spin waves from considerably different directions superpose. In this case, the focusing pattern can be significantly influenced. This effect is included in the further calculations by taking into account both the initial angle distribution of the spin-wave amplitudes \( A_{SW}^{\text{initial}}(\theta_v) \) and the sinusoidal mode profile in the waveguide opening, which represents the source of the beams. Based on these quantities, the final spin-wave amplitude \( A_{SW}^{\text{final}} \) at a given observation point is calculated by integrating the amplitudes of all spin waves which originate at different points of the source and reach the observation point from different directions. Finally, this amplitude is squared to obtain the overall spin-wave intensity \( I_{SW}^{\text{final}}(\theta_v, d) \) occurring at a distance \( d \) from the source and in the direction \( \theta_v \) in relation to the external magnetic field.

Spin-wave beams occur if the curve \( I_{SW}^{\text{final}}(\theta_v, d) \) exhibits pronounced peaks into certain directions \( \theta_B \). The angles \( \theta_B \) of the theory curve shown in Fig. 2b belong to the maxima of the calculated intensity distribution \( I_{SW}^{\text{final}}(\theta_v) \) when assuming a distance of \( d = 10 \, \mu\text{m} \) and using the parameters of the investigated system (material parameters of CoFeB, \( \mu_0 H_{\text{ext}} = 75 \, \text{mT} \), \( w_{\text{eff}} = 1.3 \, \mu\text{m} \), \( f \in [10.8,15.4] \, \text{GHz} \)).
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