Dynamic characteristics of ball bearing-coupling-rotor system with angular misalignment fault

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Received: 30 November 2021 / Accepted: 11 April 2022 / Published online: 25 April 2022
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Abstract The rotor misalignment fault, which occurs only second to imbalance, easily occurs in the practical rotating machinery system. Rotor misalignment can be further divided into coupling misalignment and bearing misalignment. However, most of the existing references only analyze the effect of coupling misalignment on the dynamic characteristics of the rotor system and ignore the change of bearing excitation caused by misalignment. Based on the above limitations, a five degrees of freedom nonlinear restoring force mathematical model is proposed, considering misalignment of bearing rings and clearance of cage pockets. The finite element model of the rotor is established based on the Timoshenko beam element theory. The coupling misalignment excitation force and rotor imbalance force are introduced. Finally, the dynamic model of the ball bearing-coupling-rotor system is established. The radial and axial vibration responses of the system under misalignment fault are analyzed by simulation. The results show that the bearing misalignment significantly influences the dynamic characteristics of the system in the low-speed range, so bearing misalignment should not be ignored in modeling. With the increase of rotating speed, rotor imbalance and coupling misalignment have a greater impact. Misalignment causes periodic changes in bearing contact angle, radial clearance, and ball rotational speed. It also leads to reciprocating impact and collision between the ball and cage. In addition, misalignment increases the critical speed and the axial vibration of the system. The results can provide a basis for health monitoring and misalignment fault diagnosis of the rolling bearing-rotor system.

Keywords Ball bearing-coupling-rotor system · Misalignment fault · Axial vibration · Rotor dynamics · Finite element

Abbreviations

\[ A_j \] Relative distance between curvature centers of inner and outer raceways after initial misalignment

\[ A_j' \] Relative distance between curvature centers of inner and outer raceways after vibration

\[ C_b \] Support damping matrix
Introduction

As the core component of modern aero-engine, the rotor system often uses rolling bearing as its supporting element. Misalignment faults often occur in rolling bearing-rotor systems due to improper machining and assembly, load, or temperature changes during operation. According to statistics, misalignment, as a fault form second only to rotor imbalance, can account for 70% of the whole rotor system faults [1]. Rotor misalignment can be divided into coupling misalignment and bearing misalignment according to the fault location. The coupling misalignment can be further subdivided into deflection angle misalignment,
parallel misalignment, and combination misalignment. Harris [2] subdivided the rolling bearing misalignment into out-of-line, tilted outer ring, tilted inner ring, and shaft deflection. If the assembly coaxiality error of the rotor system is large, the misalignment fault of coupling and rolling bearing may coincide. The coupling misalignment not only aggravates the system vibration but also causes other faults such as rub-impact between rotor and casing, shaft crack, and so on. It has been pointed out clearly in the standard ISO 15243 that the bearing misalignment can also lead to fatigue fracture of bearing cage [3] and finally cause bearing failure, as shown in Fig. 1. In severe cases, the misalignment fault may further lead to engine shutdown and then cause great losses of aircraft destruction and human death. Therefore, it is of significant significance to study the dynamic characteristics of the rolling bearing-rotor system under the combined action of coupling and bearing misalignment for dynamic design, fault detection, and diagnosis of the system during installation and operation.

The research literature on coupling misalignment and rolling bearing misalignment can be dated back to the 1970s. In the study of rotor-coupling misalignment, the most important thing is to establish a mathematical expression that can simulate coupling misalignment. At present, the main modeling methods of coupling misalignment can be divided into the following three types: (1) According to the geometric relationship and force analysis of misalignment, the expression of excitation force generated by coupling misalignment is obtained [5–10]. (2) Based on the Lagrange energy method, the dynamic equation of misalignment coupling-rotor system is established [11–13]. (3) The coupling is regarded as shaft element or lumped mass point, and it is grouped with the finite element model (FEM) of rotor [14–18]. Gibbons [5] established the coordinate system of coupling parallel misalignment and then proposed the calculation formulas of additional force and torque under the action of parallel misalignment. Subsequently, Sekhar and Prabhu [6] further gave the excitation force expression of coupling angular misalignment on this basis. Lee and Lee [7] gave the decomposition formula of coupling misalignment force and combined with the nonlinear rolling bearing force model of five degrees of freedom (5-DOF). The nonlinear vibration characteristics of the rolling bearing-rotor system under coupling misalignment fault were discussed by experiment and simulation. In addition, Zhao et al. [8] proposed a meshing force model of spline coupling under misalignment fault and numerically simulated and analyzed the influence law of spline coupling misalignment on the dynamic characteristics of the rotor system. Wu et al. [9] proposed the equation for calculating the excitation force of elastic coupling under dynamic spatial misalignment (DSM) and analyzed the vibration response of the rotor system with the DSM and imbalance. Al-Hussain [11] established a symmetrical double-span Jeffcott rotor with angular misalignment fault through Lagrange energy equation and analyzed the internal relationship between the misalignment degree of coupling and the stable operation of the system. Patel and Darpe [14] proposed a method to determine the magnitude of misalignment excitation and harmonic excitation based on experiments, established the FEM of a coupled rotor with the Timoshenko beam element, and simulated the coupling misalignment effect of rotor by node force vector. Then, they studied the steady-state response of the rotor at subcritical speed and compared the difference between parallel misalignment and angular misalignment in the bending-axial-torsional coupling vibration characteristics of the rotor system.

In terms of the research on the influence of coupling misalignment fault on the dynamic characteristics of the rotor system, Bouaziz et al. [19] studied the dynamic characteristics of misalignment rotor system supported by active magnetic bearing numerically. They found that the frequency spectrum of angular misalignment fault is mainly 2 times of rotating frequency (2f₀) and 4 times of rotating frequency (4f₀).

Fig. 1 Fracture failure of rolling bearing cage of aero-engine [4]
Patel and Darpe [20] carried out an experimental study on the misalignment problem of the rotor-coupling system. It was found that the axis trajectory showed outer looped orbits under the angular misalignment fault, while the axis trajectory showed inner looped orbits under the parallel misalignment fault. The full spectrum is better than the FFT spectrum for coupling misalignment fault diagnosis. Lu et al. [21] built a simple aero-engine dual-rotor test-bed, established the FEM of the test-bed system, introduced the nonlinear force of rolling bearing and coupling misalignment fault, reduced the dimension of the system by the proper orthogonal decomposition method, and discussed the spectral characteristics of misalignment fault of the dual-rotor-bearing system. In fact, once the misalignment fault occurs, other coupling faults may further occur in the rotor system, and the most common is the imbalance and coupling misalignment faults [22–24]. In addition, rubbing and coupling misalignment faults [25, 26], loosening and coupling misalignment faults [27], fluid excitation of sliding bearing and coupling misalignment faults [25, 28, 29] may also occur. For example, taking the turbine generator rotor system as the research object, Fu et al. [25] established the nonlinear dynamic model of rubbing-imbalance-misalignment coupling fault under the action of nonlinear oil-film force and verified by experiment. The results showed that the system moves periodically at low speed, and the vibration energy is mainly the frequency of \( f_r \) and \( 2f_r \). However, with the increase of rotating speed, the vibration energy of the system is gradually dominated by subharmonic frequency components under the action of oil film force and rubbing force. Jin et al. [26] further analyzed the nonlinear vibration characteristics of blade-casing rubbing caused by misalignment of the aero-engine dual-rotor-rolling bearing system. Ma et al. [28] simulated and studied the oil film instability law of the cantilever rotor system with parallel and angular misalignment during speed up and down and found that the coupling misalignment delays the occurrence of first-order oil-film instability.

In terms of the research on the misalignment of rolling bearing, Hinton’s literature [30] showed that in the late 1940s, the Royal Air Force had many cases of cage fracture caused by misalignment of bearing assembly, which led to the failure of ball bearing in aero-engine. Crawford [31] found through experimental tests that when the misalignment angle of rolling bearing increased from 0.26° to 0.61°, the dynamic stress of the cage soared from 0.172 MPa to 3.45 MPa, which increased by about 2006%. This further indicated that abnormal cage stress caused by bearing misalignment is the main cause of cage fatigue fracture. Xu et al. [32] analyzed the failure of the ground bench test of an aero-engine and found that the first broken part of bearing was the cage, and the bearing tilt after engine assembly was the main cause of bearing failure. Ertas and Vance [33] built a liquid hydrogen fuel turbopump test-bed. The test found that the static misalignment can increase the radial stiffness of angular contact ball bearing, while the dynamic misalignment can reduce the bearing radial stiffness. At present, the modeling methods of bearing misalignment can be mainly divided into stiffness model calculated by static and quasi-static methods [34–42], and nonlinear force model [43, 44]. Zhang et al. [35] derived an improved quasi-static angular contact ball bearing model, discussed the influence of misalignment on bearing load distribution, stiffness, and life under axial load and combined load. The influence of two kinds of bearing misalignment on the natural frequency of the bearing–rotor system was compared and analyzed. At present, many references give the misalignment value directly. But in fact, the value of bearing misalignment is uncertain. Yang et al. [37] studied the influence of angular contact ball bearing on raceway wear in spindle system under initial uncertain misalignment and found that the wear depth of inner raceway is significantly greater than that of the outer raceway. Yi et al. [43] took the machine tool spindle-angular contact ball bearing as the research object. A relatively simple 2-DOF nonlinear force model of rolling bearing with parallel misalignment was proposed. The research found that bearing misalignment significantly impacts the dynamic characteristics and amplitude of the system. Taking the double-row angular contact ball bearing as the research object, Parmar et al. [44] considered the influence of bearing outer ring defects and bearing angular misalignment and proposed the expression of a 3-DOF nonlinear restoring force model considering bearing angular misalignment. The vibration response of bearing was studied, and relevant experimental research was carried out.

From the current research situation, the research on the dynamics and vibration of the misaligned rotor system mainly focuses on coupling misalignment.
Many scholars have carried out a lot of theoretical and experimental researches in this field and achieved fruitful research results. However, there is relatively little research on the misalignment of rolling bearing. The existing reference mainly focuses on the impact of bearing misalignment on its static performance, such as the stiffness, contact problem, and fatigue life of bearing itself. However, there is relatively little research on its dynamic performance, especially in the impact of bearing misalignment on the dynamic characteristics of the rotor system. Besides, because the interference fit is generally adopted between the rolling bearing and the rotating shaft, once the coaxiality error of the rotor assembly is too large, the rotating shaft tilt may also cause a certain tilt of the bearing inner ring relative to the outer ring and then induce the bearing misalignment fault. However, in the dynamic response study of misalignment fault of ball bearing-coupling-rotor system, only misalignment excitation of coupling is considered, and the change of bearing excitation caused by misalignment is ignored. There are few reports on the dynamic characteristics of rolling bearing and coupling misalignment.

Based on the limitations of existing research, this paper firstly deduces a 5-DOF nonlinear bearing force model based on the Hertz contact theory, which can take angular misalignment into account. At the same time, the model also includes uneven ball distribution caused by the cage pocket clearance. Secondly, the finite element dynamic model of the rotor system is constructed based on the Timoshenko beam element theory. The coupling misalignment force and the rolling bearing misalignment force are introduced into the rotor dynamic model. Finally, the radial and axial vibration responses of the rotor system under the combined action of angular misalignment of coupling and rolling bearing are analyzed. The research results can provide theoretical support for dynamic monitoring and fault diagnosis of misalignment fault in the rolling bearing-rotor system.

2 Dynamic model of ball bearing-coupling-rotor system with angular misalignment

In the actual aero-engine rotor system, since the mass is mainly concentrated in the compressor and turbine, the low-pressure rotor of the aero-engine is simplified into a single-span double-disc rotor system supported by rolling bearings. The structural diagram is shown in Fig. 2a. Due to improper installation and other reasons, the coaxiality error between the rear support bearing seat and the rotor center exceeds the standard, and there is a height difference \( \Delta \) between the two ends of the bearing seat. This paper assumes that the height difference is relatively small compared to the length of the rotor. Therefore, the shaft bending deformation caused by the rotor misalignment can be ignored. The misalignment angle \( \phi \) can be approximated by the relation between the length of the rotor \( l \) and the height difference \( \Delta \). Besides, the interference fit is generally used for bearing inner ring and the rotating shaft, so in this case, the system should have two fault forms: angle misalignment of coupling and inner ring tilt misalignment of bearing. Given that the total length of the rotor is 597 mm, it is divided into 27 shaft segments by the finite element method, and each shaft segment is simulated by the Timoshenko beam element, as shown in Fig. 2b. There are two discs at the front and rear of the rotor, which are arranged asymmetrically. The diameter of each disc is 190 mm, and the thickness is 10 mm. The disc is simplified as a lumped mass point, as shown in Fig. 2c, and it is placed at nodes 11 and 18 of the rotor, respectively. Both the rotor ends are supported by the 6205 deep groove ball bearings at nodes 7 and 24 of the rotor.

2.1 Misalignment model of bearing

The schematic of the deep groove ball bearing is shown in Fig. 3. It is assumed that the number of bearing balls is \( N_b \). The inner and outer raceway radii are \( r_b \) and \( R_b \), respectively. The ball diameter is \( d_b \). The bearing pitch diameter \( d_m \) and its size is the sum of the inner and outer raceway radii. The initial radial clearance of the bearing is \( c_r \), and the clearance of the cage pocket is \( c_c \). For the convenience of research, the following assumptions are made: (1) The ball in the bearing makes pure rolling between the inner and outer raceways. (2) The effects of lubricating oil, temperature, and ball centrifugal force are not considered.

Without loss of generality, this paper first assumes that there is an initial misalignment in the installation process of the rotating shaft, which leads to the bearing inner ring tilting along the X-axis at an angle of \( \phi_x \), as shown in Fig. 4a. The geometric relationship after
misalignment of the bearing inner ring is shown in Fig. 4b, where the angular position of the \( j \)-th ball is \( \theta_j \), and the central position point of the ball is \( O_{bj} \). The contact point between the \( j \)-th ball and the curvature center of the outer raceway is \( O_{oj} \), and the contact point with the curvature center of the inner raceway is \( O_{ij} \). The curvature center trajectory of the inner raceway also changes when the inner raceway tilt is misaligned. The contact point between the \( j \)-th ball and the inner raceway becomes \( O_{ij}^' \), and the center point of the ball becomes \( O_{bj}^' \).

In Fig. 4b, the radius of curvature center of the inner and outer raceways is \( \overline{OO_{ij}} \) and \( \overline{OO_{oj}} \), respectively. Combined with the geometric relationship shown in Fig. 5a, they can be expressed as:

\[
\overline{OO_{ij}} = r_b + r_i, \quad \overline{OO_{oj}} = R_b - r_o. \tag{1}
\]

In Eq. (1), \( r_i \) and \( r_o \) are curvature radii of inner and outer raceways, respectively, and the curvature radius coefficient \( f_{i/o} \) of the inner/outer raceway is introduced. Therefore, the curvature radius of inner/outer raceway of the bearing can be expressed as the product of their curvature radius coefficient and ball diameter:

\[
r_i = f_id_b, \quad r_o = f_od_b. \tag{2}
\]

Therefore, the geometric relationship can be obtained according to Fig. 4b as follows:

\[
\begin{align*}
\overline{D_iO_{ij}^'} &= (r_b + r_i) \sin \theta_j \sin \varphi_x, \tag{3} \\
\overline{D_iO_{ij}^2} &= \overline{D_iO_{ij}^1} - \overline{G_iD_i} \\
&= (r_b + r_i) \sin \theta_j (1 - \cos \varphi_x), \tag{4} \\
\overline{O_{oj}N_o} &= \overline{O_{oj}O_{ij}^1} \cos \theta_j = (r_i + r_o - d_b - c_t) \cos \theta_j, \tag{5} \\
\overline{N_oD_i} &= \overline{N_oO_{ij}^1} - \overline{D_iO_{ij}^1} \\
&= [(r_i + r_o - d_b - c_t) - (r_b + r_i)(1 - \cos \varphi_x)] \sin \theta_j, \tag{6}
\end{align*}
\]
\[
|O_{ij}D_i| = \sqrt{\left(\left|O_{ij}N_0\right|\right)^2 + \left(\left|N_0D_i\right|\right)^2}
= \sqrt{(r_i + r_o - d_b - c_i)^2 \cos^2 \theta_j + [(r_i + r_o - d_b - c_i) - (r_b + r_i)(1 - \cos \varphi_y)]^2 \sin^2 \theta_j}.
\]

(7)

According to Figs. 4b and 5b, the contact angle \(\alpha_j\) and normal clearance \(\Delta_j\) generated at the angular position of the \(j\)-th ball due to bearing misalignment can be expressed as:

\[
\alpha_j = \arctan\left(\frac{D_iO_{ij}}{O_{ij}D_i}\right),
\]

(8)

\[
\Delta_j = r_i + r_o - d_b - \sqrt{\left(D_iO_{ij}\right)^2 + \left(O_{ij}D_i\right)^2}.
\]

(9)

Since the curvature radius of the ring is much larger than the normal clearance, combined with Fig. 5(b), the bearing radial clearance \(c_{ij}'\) at the \(j\)-th ball position can be approximately expressed as:

\[
c_{ij}' = \frac{\Delta_j}{\cos \alpha_j} = \frac{r_i + r_o - d_b - \sqrt{\left(D_iO_{ij}\right)^2 + \left(O_{ij}D_i\right)^2}}{\cos \left(\arctan\left(\frac{D_iO_{ij}}{O_{ij}D_i}\right)\right)}.
\]

(10)

Substituting Eqs. (3) and (7) into Eqs. (8)-(10), the expressions of the contact angle and bearing clearance of the \(j\)-th ball caused by misalignment can be derived.

On this basis, the misalignment angle \(\varphi_y\) of the rotating axis along the \(Y\)-axis is introduced. Then Eqs. (3) and (4) can be further rewritten as:

\[
|D_iO_{ij}| = (r_b + r_i)\left[\sin \theta_j \sin \varphi_x - \cos \theta_j \sin \varphi_y\right],
\]

(11)

\[
|D_iO_{ij}| = (r_b + r_i)\sqrt{\sin^2 \theta_j(1 - \cos \varphi_y)^2 + \cos^2 \theta_j(1 - \cos \varphi_y)^2}.
\]

(12)

Substituting Eq. (12) back into Eqs. (6) and (7), the result is as follows:

\[
|O_{ij}D_i| = \sqrt{\left[(r_i + r_o - d_b - c_i) \cos \theta_j\right]^2 + \left[(r_i + r_o - d_b - c_i) \sin \theta_j - (r_b + r_i)\sqrt{\sin^2 \theta_j(1 - \cos \varphi_y)^2 + \cos^2 \theta_j(1 - \cos \varphi_y)^2}\right]^2}.
\]

(13)

Then substitute Eqs. (2), (11), and (13) into Eqs. (8) and (10). Therefore, when the inner ring of the bearing tilts along any direction, the generalized expression of the contact angle and bearing clearance is as follows:

\[
\alpha_j = \arctan\left(\frac{r_b + f_d h_0 \xi}{\sqrt{(x - c_j)^2 \cos^2 \theta_j + [(x - c_j) \sin \theta_j - (r_b + f_d h_0 \eta)]^2}}\right)
\]

(14)

\[
\Delta_j = \sqrt{(r_b + f_d h_0 \xi)^2 + (x - c_j)^2 \cos^2 \theta_j + [(x - c_j) \sin \theta_j - (r_b + f_d h_0 \eta)]^2}.
\]

(5)
Further, according to Refs. [45–47], since the outer ring of bearing is matched with the bearing seat, the outer ring does not rotate. At this time, the rotational angular velocity of the bearing cage can be written as:
\[
\omega_c = \frac{\omega_r r_b}{R_b + r_b},
\]
where \(\omega_r\) is the rotational angular velocity of the rotor. Due to the existence of pocket clearance in the cage, the angular position of each ball bearing is nonuniform distribution. Therefore, the angular position \(\theta_j\) of the \(j\)-th ball bearing can be expressed as:
\[
\theta_j = \omega_c t + \frac{2\pi}{N_b} (j - 1) + \theta_{01} + 2\tau_j \arcsin\left(\frac{c_c}{2d_m}\right).
\]
In Eq. (21), the initial position angle of the first ball is \(\theta_{01}\), and \(\tau_j\) is a random number evenly distributed between -1 and 1 (\(j = 1,2,\ldots,N_b\)). Further, in the initial misalignment state, the geometric relationship after bearing movement is shown in Fig. 6. The contact point between the \(j\)-th ball and the inner raceway changes from \(O_{ij}\) to \(O'_{ij}\). The distance between the curvature center of the inner and outer raceway after initial misalignment is \(A_j\), and the distance between the curvature center of the inner and outer raceway after vibration is \(A'_j\). Then the contact
deformation $\delta_j$ between the $j$-th ball and the raceway can be written as:

$$\delta_j = |O_{yj}O_{yj}'| - |O_{yj}O_{yj}'| = A'_j - A_j,$$

where

$$A_j = z - \Delta_j,$$

$$A'_j = \sqrt{(A_j \cos z_j + \delta_{ij})^2 + (A_j \sin z_j + \delta_{ij})^2}.$$  

The contact angle $z'_j$ of the bearing after vibration can be written as follows:

$$z'_j = \arctan\left(\frac{A_j \sin z_j + \delta_{ij}}{A_j \cos z_j + \delta_{ij}}\right).$$  

In Eqs. (24) and (25), $\delta_{ij}$ and $\delta_{ij}'$ are radial and axial contact deformations, respectively, which can be expressed as follows:

$$\delta_{ij} = x \cos \theta_j + y \sin \theta_j - c'_{ij},$$

$$\delta_{ij}' = z + r_{ij}(\theta_x \sin \theta_j - \theta_y \cos \theta_j).$$

Therefore, according to the Hertz contact theory, the expression of 5-DOF nonlinear bearing force considering deep groove ball bearing misalignment and cage pocket clearance is [48, 49]:

$$\left\{ \begin{align*}
F_{bx} & = - \sum_{j=1}^{N_b} k_{bj} \delta_j^{1.5} H(\delta_j) \cos z'_j \cos \theta_j \\
F_{by} & = - \sum_{j=1}^{N_b} k_{bj} \delta_j^{1.5} H(\delta_j) \cos z'_j \sin \theta_j \\
F_{bc} & = - \sum_{j=1}^{N_b} k_{bj} \delta_j^{1.5} H(\delta_j) \sin z'_j \\
M_{hx} & = - r_{ij} \sum_{j=1}^{N_b} k_{bj} \delta_j^{1.5} H(\delta_j) \sin z'_j \sin \theta_j \\
M_{hy} & = r_{ij} \sum_{j=1}^{N_b} k_{bj} \delta_j^{1.5} H(\delta_j) \sin z'_j \cos \theta_j
\end{align*} \right.$$  

where $H(\delta_j)$ is the Heaviside function. $r_{ij}$ represents the radial distance of the curvature center of the inner raceway at the $j$-th ball position. $k_{bj}$ is the Hertz contact stiffness between the $j$-th ball and raceway. Their expressions are shown as follows [2, 46–49]:

$$H(\delta_j) = \begin{cases} 1, & \delta_j > 0 \\ 0, & \text{else} \end{cases},$$

$$r_{ij} = 0.5d_m + (f_i - 0.5)d_n \cos \theta_j,$$

$$k_{bj} = \left\{ \begin{align*}
& \left[ 2.1458 \times 10^5 \sum \rho_{e^{-0.5}(\lambda^*)^{-1.5}} \right]^{-2} \\
& + \left[ 2.1458 \times 10^5 \sum \rho_{e^{-0.5}(\lambda^*)^{-1.5}} \right]^{-1.5} \end{align*} \right\}, e = \text{in/out},$$

$$\sum \rho_{\text{in}} = \frac{1}{d_n} \left( 4 - \frac{1}{f_i} + \frac{2\gamma}{1 - \gamma} \right), \quad \gamma = \frac{d_n \cos \theta_j}{d_m}.$$  

In Eqs. (31) and (32), $\sum \rho_{\text{k}}$ represents the sum of inner/outer raceway curvature, and the parameters can also be obtained according to Ref [2].

2.2 Misalignment model of coupling

The diagram of coupling misalignment is shown in Fig. 7a, in which points $O_1$ and $O_2$ are the rotation centers of right half-coupling and left half-coupling, respectively, and points $O$ and $P$ are the static center and dynamic center of the coupling housing, respectively. $\delta$ represents the misalignment of the coupling. It is assumed that the coordinate at the point $P$ is $(x, y)$. According to the geometric relationship

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diagram shown in Fig. 7b, x and y can be written as follows:

\[
\begin{align*}
\dot{x} &= \delta \cos \alpha \sin \alpha = \frac{\delta}{2} \sin 2\alpha \\
\dot{y} &= \delta \cos \alpha \cos \alpha = \frac{\delta}{2} (2 \cos^2 \alpha - 1) = \frac{\delta}{2} \cos 2\alpha.
\end{align*}
\]

(33)

Substituting \( \alpha = \omega t \) into Eq. (33) and calculating the second derivative for time \( t \) to obtain the acceleration at point \( P \) \([50]\):

\[
\begin{align*}
\ddot{x} &= \delta \dot{\omega} (-\sin 2\alpha) + 2\dot{\omega} \cos 2\alpha = -2\delta \omega^2 \sin 2\alpha = -2\delta \omega^2 \sin(2\omega t) \\
\ddot{y} &= -\delta \dot{\omega} \cos 2\alpha + 2\dot{\omega} \sin 2\alpha = -2\delta \omega^2 \cos 2\alpha = -2\delta \omega^2 \cos(2\omega t).
\end{align*}
\]

(34)

When misalignment fault exists in the coupling, an additional exciting force, \( F_c \), will be applied to the rotor system. Further introducing the coupling mass \( m_c \) and the acceleration calculated in Eq. (34), the misaligned excitation force of the coupling can be calculated according to the Newton’s second law. If the exciting force \( F_c \) is projected onto the OXY coordinate system, its component can be expressed as \([21, 23, 26, 50]\):

\[
\begin{align*}
F_{cx} &= -2m_c \delta \dot{\omega} \sin(2\omega t) \\
F_{cy} &= -2m_c \delta \dot{\omega} \cos(2\omega t).
\end{align*}
\]

(35)

The coupling misalignment \( \delta \) can be expressed as follows:

\[
\delta = \Delta l \tan \varphi + \Delta e,
\]

(36)

where \( \Delta l \) is the distance between two coupling halves, and \( \Delta e \) represents the parallel misalignment of the coupling. In this paper, \( \Delta e = 0 \).

2.3 Finite element model of rotor system

In this paper, the flexibility of the shaft is considered, and the dynamic model of the rotating shaft is established by the finite element method. According to the Timoshenko beam element theory, each divided shaft element contains two nodes, as shown in Fig. 2b. The \( i \)-th node has 6 DOFs, and the displacement and rotation angles of node \( i \) in X-, Y-, and Z-directions are denoted by \( x_i, y_i, z_i, \theta_{xi}, \theta_{yi}, \theta_{zi} \), respectively. Therefore, the displacement vector of the \( q \)-th shaft element can be written as:

\[
U_q = [x_i, y_i, z_i, \theta_{xi}, \theta_{yi}, \theta_{zi}]^T.
\]

(37)

According to Refs. [51, 52], the mass matrix \( M_{sq} \), stiffness matrix \( K_{sq} \), and gyro matrix \( J_{sq} \) of the \( q \)-th shaft element can be further determined. The mass matrix \( M_s \), stiffness matrix \( K_s \), and gyro matrix \( J_s \) of the rotating shaft can be obtained through the assembly of elements. Because the material of the rotor studied in this paper is single and its property can be regarded as isotropic. Thus, the damping characteristics of the system are distributed evenly. In addition, due to the simplicity and convenience of proportional damping, it can well simulate the internal damping of the structure. Therefore, it has been widely used in

Table 1 Parameters of the rotating shaft

| Number of shaft element | Length of shaft element/mm | Diameter of shaft element/mm | Number of shaft element | Length of shaft element/mm | Diameter of shaft element/mm |
|-------------------------|-----------------------------|------------------------------|-------------------------|-----------------------------|------------------------------|
| 1                       | 6                           | 16                           | 13–15                   | 40                          | 30                           |
| 2                       | 15                          | 16                           | 16                      | 40                          | 30                           |
| 3                       | 20                          | 16                           | 17                      | 25                          | 50                           |
| 4                       | 20                          | 16                           | 18                      | 5                           | 50                           |
| 5                       | 16.5                        | 25                           | 19                      | 23                          | 30                           |
| 6                       | 15                          | 25                           | 20–22                   | 40                          | 30                           |
| 7                       | 7.5                         | 25                           | 23                      | 7.5                         | 25                           |
| 8                       | 25                          | 30                           | 24                      | 15                          | 25                           |
| 9                       | 27                          | 30                           | 25                      | 16.5                        | 25                           |
| 10                      | 5                           | 50                           | 26                      | 15                          | 18                           |
| 11                      | 25                          | 50                           | 27                      | 10                          | 18                           |
| 12                      | 30                          | 30                           |                         |                             |                              |

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engineering practice. In this paper, the proportional damping is also used to simulate the structure damping $C_s$ of the shaft [51–53]. In addition, gyro matrix $D_{dp}$ and mass matrix $M_{dp}$ of the disc element at the $p$-th node can be written as follows:

$$D_{dp} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & J_{pp} \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix},$$

$$M_{dp} = \text{diag}(m_{dp}, m_{dp}, m_{dp}, J_{dp}, J_{dp}, J_{pp}),$$

$p = 11, 18$.

In Eq. (38), the mass, diameter moment of inertia, and polar moment of inertia of the disc are $m_{dp}$, $J_{dp}$, and $J_{pp}$, respectively, which can be calculated according to the disc material density and the geometry (inner radius $r_d$, outer radius $R_d$, and thickness $h_d$) in Fig. 2c:

$$m_{dp} = \frac{\pi \rho h_d (R_d - r_d)^2}{4}, \quad J_{dp} = \frac{\pi \rho h_d (R_d - r_d)^4}{4}, \quad J_{pp} = \frac{\pi \rho h_d (R_d - r_d)^4}{2}.$$ (39)

By assembling the disc mass matrix and gyro matrix at rotor nodes 11 and 18, the overall disc mass matrix $M_d$ and gyro matrix $J_d$ are obtained. Therefore, the differential equation of motion of the rotor system has the following form:

$$(M_s + M_d)\ddot{U} + [C_s + C_b + \omega_b (J_s + J_d)]\dot{U} + K_s U = F_b + F_c + F_e - G,$$ (40)

where $U$ is the displacement vector of the rotor system. $C_b$ is the support damping matrix of the system. $F_c$ is the misalignment force vector of the coupling. $F_e$ and $F_b$ represent the imbalance force and rolling bearing force vector, respectively. $G$ is the gravity vector of the rotor system. Assuming that the imbalance of the system exists only on two discs, the rotor imbalance force can be written as:

$$\begin{cases}
F_{epx} = m_{dp} e_{dp} \omega_b^2 \cos \omega_b t, \\
F_{epy} = m_{dp} e_{dp} \omega_b^2 \sin \omega_b t,
\end{cases} \quad p = 11, 18,$$ (41)

where $F_{epx}$ and $F_{epy}$ are the components of the imbalance force in the $X/Y$-direction, respectively, and $e_{dp}$ is the eccentric distance at the disc $P$. It is assumed that the relative phase angle between the misalignment force and the imbalance force at both discs is $0^\circ$. The first and second modal damping ratios are 0.03 and 0.06, respectively. The support damping coefficient at the $i$-th node is: $c_{bx} = c_{by} = 1500 \text{ Ns/m}$, $c_{bx} = 800 \text{ Ns/m}$ ($i = 7, 24$). In addition, the length and diameter of each shaft element of the rotor are shown in Fig. 8.
in Table 1. The parameters of the rolling bearings supported at both ends of the rotor are shown in Table 2.

3 Dynamic response analysis of the system

In this paper, the Newmark-$\beta$ numerical integration method is used to solve Eq. (40), and each calculation period is further divided into 512 integration steps. In order to ensure that the transient free vibration response has been attenuated and only the steady-state forced vibration response exists in the numerical calculation process, the data of 300 cycles are calculated in this paper. The results of the first 200 cycles are discarded and the data of the last 100 cycles are taken for analysis. The dynamic characteristics of the ball bearing-coupling-rotor system with angular misalignment fault are analyzed through the rotor orbit, time-domain waveform, FFT frequency spectrum, and Poincaré section map.

3.1 Comparison of system vibration response between Case 1, Case 2, and Case 3

In order to facilitate comparison and analysis, this paper assumes that the healthy case (without misalignment) is Case 0. In the traditional research on rotor misalignment, most of the references only consider coupling misalignment. It is assumed that the rotor system only has the coupling misalignment fault in Case 1, as shown in Fig. 8a. The single consideration of bearing misalignment is Case 2, as shown in Fig. 8b. Case 2 is a hypothetical case for comparison with coupling misalignment. At this time, the axes of the left halve-coupling and right halve-coupling are collinear. Because of interference fit between bearing inner ring and rotor, there is only bearing misalignment and no coupling misalignment in this case. As considered in this paper, the combined misalignment between coupling and bearing is Case 3, as shown in Fig. 2a. It is assumed that there is only misalignment $\varphi = \varphi_x = 0.2^\circ$ in the direction around the $X$-axis. In the whole speed-raising process ($n \in [2000, 15000]$ r/min) of the system under the four cases, the spectrum cascade at the left disc (node 11) is shown in Fig. 9. It can be seen from the figure that in all four cases, the spectrum cascade at the left disc (node 11) is shown in Fig. 9. It can be seen from the figure that in all four cases, the system is mainly excited by rotating frequency ($f_r$) caused by rotor imbalance. For Case 1 and Case 3, there is also a tiny amount of $2f_r$ caused by coupling misalignment excitation in the spectrum, while Case 2 has no such frequency. By comparison, it can also be found that the critical speed of the rotor system in Case 0 and Case 1 is about 12,300 r/min, while the critical speed is increased to about 12,700 r/min in Case 2 and Case 3. This is because after considering the bearing misalignment, the support stiffness of bearing to the rotor increases, thus increasing the natural frequency.
of the system. The coupling misalignment has no effect on the critical speed of the rotor system.

Figure 10 presents the radial (X/Y-direction) spectrum cascade of the left disc under three cases when the rotor is operating in the low-speed range \((n \in [2000, 4500] \text{ r/min})\). For Case 1, the frequency components of \(f_r\) and its super-harmonic are appeared in both X- and Y-directions. With the increase of rotating speed, the amplitude of \(2f_r\) increases most obviously. For Cases 2 and 3, it can be obviously found that in addition to \(f_r\) and its super-harmonic components, the frequency component of bearing varying compliance vibration (\(f_{vc}\)) also appears, which indicates that the misalignment can aggravate the bearing varying compliance (VC) vibration in the low-speed range. In addition, under the combined action of VC vibration of bearing and vibration of the rotor imbalance, the combined frequency components of \(f_{vc}\) and \(f_r\) appear in the spectrum, such as \(f_{vc} \pm f_r, 2f_r, 2f_{vc}, 3f_r, f_{vc}, 4f_r, 4f_{vc}\), and so on. However, with the gradual increase of rotating speed, the influence of rotor imbalance excitation increases gradually, and the vibration frequency of \(f_{vc}\) weakens and disappears gradually when the rotating speed is higher than 4000 r/min. In addition, it can also be found that for Cases 2 and 3 with bearing misalignment, the frequency amplitude in the X-direction is larger than that in the Y-direction. This is mainly because when there is a bearing misalignment around the X-axis of the system, the constraint of the bearing on the rotor in the Y-direction enhances and the rotor deflection increases. It can be seen that the bearing misalignment has a strong directivity. For Case 3, under the combined action of misalignment of coupling and bearing, it can be found that compared with Case 1, bearing misalignment can cause the occurrence of \(f_{vc}\) and the combined frequency components of \(f_{vc}\) and \(f_r\) in the system. By comparing Case 3 with bearing misalignment (Case 2), it is found that although the high-order harmonic component of frequency conversion including \(2f_r\) is generated under the action of the nonlinear factors of the bearing, the coupling misalignment further increases the frequency amplitude of \(2f_r\).
Taking the vibration response of the rotor system under the operation of 2000 r/min and 6000 r/min as an example, the results are shown in Figs. 11 and 12, respectively. Through further comparison, the similarities and differences of the three misalignments are shown as follows:

1) When the rotating speed is $n = 2000$ r/min, the vibration responses of Cases 1, 2, and 3 are pretty different. For radial vibration, combined with Fig. 11a and b, it can be found that in Case 1, the rotating speed is low, and the effect of rotor imbalance is weak. Due to the existence of gravity, the vibration center of the rotor is located in the negative direction of the Y-axis. After considering the bearing misalignment, the motion trajectory of the rotor has an apparent
upward trend. This is due to the misalignment assumed in this paper as shown in Fig. 2, and the right end of the rotor is “lifted” upward. Therefore, further consideration of the misalignment of the rotor support can better simulate the misalignment problem assumed in this paper. As can be seen from Fig. 11d, the Poincaré map is a closed curve surrounded by several points, indicating that the rotor system shows the quasi-periodic response in all three cases. For Case 1, in addition to rotating frequency $f_r$ and its high-order harmonic frequency components, there are also the varying compliance (VC) vibration frequency $f_{vc}$ of rolling bearing and its combined frequency with $f_r (n f_r \pm m f_{vc}, n$ and $m$ are positive integers), as well as many incommensurate frequencies (see Fig. 11e). The amplitude of these frequencies is relatively low. Case 2 and Case 3 are similar to Case 1 (see Fig. 11g, i). Figure 11e, g, and i clearly shows that, compared with Case 2 of bearing misalignment, considering Case 1 and Case 3 of coupling misalignment can make the frequency component of $2 f_r$ in the system more obvious. Compared with Case 1 of coupling misalignment, considering Cases 2 and 3 of bearing misalignment can make the frequency component of $f_{vc}$ in the system more obvious. In addition, there exist abundant combination frequency components.

(2) For axial vibration, only the coupling misalignment cannot generate axial excitation, so the axial vibration in Case 1 is very small and can be almost negligible (see Fig. 11c, f). When the bearing misalignment is further considered, the misalignment makes the bearing produce a component force along the axial direction, which leads to the increase of the axial vibration of the system under the action of this excitation. In addition, misalignment also results in a constant component of rotor gravity along $Z$-direction, so the balance position of vibration waveform in this direction is located in the negative direction of the $Z$-axis, as shown in

![Fig. 12](image-url)
Fig. 11c. Under the action of bearing misalignment, except that the frequency components of $f_{vc}$ and $nf_r \pm mf_{vc}$ are still generated in Cases 2 and 3, the rotor imbalance and the coupling misalignment generated in the radial direction can also generate frequency components such as $f_r$ and $2f_r$ in the axial direction, as shown in Fig. 11h, j.

(3) As can be seen from Fig. 12, when the speed $n = 6000$ r/min, the rotor orbits of Cases 1 and 3 with coupling misalignment fault both show the motion law of two ellipses nested with each other, and the time-domain waveforms present the feature of alternating high and low peaks, and the amplitudes of $f_r$ and $2f_r$ in the spectrum are prominent. However, for Case 2, the rotor orbit presents an elliptical shape, the vibration waveform is approximately a sine wave, and the frequency $f_{vc}$ gradually weakens and disappears in the end. These characteristics show that the system vibration is mainly excited by rotor imbalance and coupling misalignment after the speed increases, and the bearing misalignment has little effect. This is because the rotor imbalance force and coupling misalignment force are related to the square term of rotating speed according to Eqs. (35) and (41), while the bearing misalignment force is only related to the first power of speed. Therefore, the influence of bearing misalignment decreases gradually with the increase of rotating speed. The frequency components of $f_{vc}$ and the combined frequency gradually weaken or even disappear. In contrast, the influence of rotor imbalance and coupling misalignment enhances gradually, and the frequency components of $f_r$ and $2f_r$ are obvious. At this time, there is little difference in the system dynamic characteristics between Cases 1 and 3.

3.2 Effect of misalignment degree

3.2.1 Process of speed-up

Based on the discussion in Sect. 3.1, this section takes Case 3 under the action of misalignment of coupling and bearing as an example to further discuss the influence of different misalignment degrees on the dynamic characteristics of the system in the process of speed-up. When the system is in a healthy state ($\phi = 0^\circ$) and misaligned state ($\phi = 0.2^\circ$), respectively, radial and axial amplitude curves at the left rotor disc (node 11) are shown in Fig. 13. It is not
difficult to find the following interesting phenomena from the figure:

(1) For the radial vibration of the system, as shown in Fig. 13a, misalignment increases the critical speed of the system from about 12,300 r/min to about 12,700 r/min, indicating that bearing misalignment increases the support stiffness of the system, which is consistent with the conclusion obtained from the test results in Ref. [33]. In the subcritical speed range, the system amplitude decreases due to misalignment. The misalignment causes the system amplitude to increase slightly in the range of critical and supercritical speeds.

(2) For the axial vibration of the system, as shown in Fig. 13b, the axial vibration is slightly in a healthy state. Misalignment intensifies the axial vibration of the system, and the resonance peak appears at about 12,700 r/min. It can be seen that misalignment has a great influence on the axial vibration.
vibration of the system. This shows that axial vibration is another basis for judging whether there is angular misalignment fault in the deep groove ball bearing-rotor system.

(3) By comparing the amplitude–frequency curves of the healthy and misalignment cases in Fig. 13, it can be found that the rotor system has rich nonlinear dynamic behaviors. Regardless of radial vibration or axial vibration, the healthy rotor system and the misaligned rotor system have a jumping phenomenon, and the jumping discontinuity occurs in the first-order resonance region. At this time, the system shows the hardening-type nonlinear behavior. For the rotor system with misalignment fault, another jumping phenomenon appears at 10,100 r/min, and subcritical super-harmonic resonance occurs at 5500 r/min. The phenomenon of super-harmonic resonance is mainly caused by the coupling misalignment, which is reflected in the axial vibration under the action of the bearing misalignment.

3.2.2 Process of steady speed

Taking the No. 1 ball of the left bearing with rotating speed \( n = 3000 \) r/min as an example, the influence laws of different misalignment on bearing parameters, such as contact angle, radial clearance, and ball revolution angular velocity, are shown in Fig. 14. When the system is in a healthy state \( (\varphi = 0^\circ) \), the contact angle \( \alpha \) of deep groove ball bearing is always \( 0^\circ / C_{176} \). It can be seen from Table 2 and Fig. 14b that the

![Fig. 15](image-url)  
**Fig. 15** Effect of misalignment on contact force of ball bearing

![Fig. 16](image-url)  
**Fig. 16** Spectrum cascade of rotor system under different misalignment:
(a) 3000 r/min, Y-direction,
(b) 3000 r/min, Z-direction,
(c) 12,700 r/min, Y-direction,
(d) 12,700 r/min, Z-direction,
(e) 15,000 r/min, Y-direction, and
(f) 15,000 r/min, Z-direction.
bearing radial clearance $c_r$ is constant at 8 µm. The approximate calculation formula of the angular velocity of ball revolution is given as follows:

$$\omega_{bj} = \left(1 - \frac{d_b}{d_m} \cos \alpha_j \right) \frac{\omega_r}{2}.$$  \hspace{1cm} (42)

By calculation, the rotational angular velocity $\omega_{bj}$ of each ball bearing in this state is constant at 125.1 rad/s. When a misalignment fault appears in the system, the misalignment makes the contact angle and radial clearance of each ball bearing no longer constant, but shows a law of periodic change. At each moment, all the ball contact angles of the bearing and the radial clearance at the position of the ball are not equal. Although the average value of the contact angle within one revolution is still 0°, the contact angle of the ball fluctuates with approximate sinusoid. The bearing radial clearance fluctuates twice within the range of one rotation of the ball, which indicates that misalignment causes the bearing to have two contact areas with a central phase difference of 180°. With the increase of misalignment degree, the amplitude of this periodic fluctuation also increases gradually, and the average radial clearance decreases gradually. Under a significant misalignment, the bearing clearance may even be negative at a certain position of the ball rolling. This indicates that the bearing misalignment has squeezed some of the balls in the bearing. Combined with Eq. (42), the contact angles of the balls are not equal at all places on the circumference of the raceway, so the rotational angular velocity of each ball is constantly changing and unequal. The degree of difference between the angular velocity of each ball increases with the increase of misalignment. Taking the misalignment angle $\varphi = 0.21°$ as an example, according to Fig. 14c, d, the maximum and minimum values of the ball angular velocity on the circumference are about 90° apart. In other words, when the ball is rotated 1/4 turn, the angular velocity increases by 1.5 rad/s. According to this calculation, the maximum velocity difference of ball revolution in the bearing can reach about 0.06 m/s. It can be seen that the relative tilt of bearing caused by rotor misalignment may lead to the unequal angular velocity of the ball and cage, and the center of the ball and cage pocket do not coincide. When the distance between the two centers is greater than the clearance of cage pocket $c_c$, an elastic collision occurs between the ball and the
cage pocket. Moreover, the more serious the misalignment, the greater the impact strength of the ball on the cage along the circumferential direction, which improves the possibility of fatigue fracture of the cage.

According to Eqs. (22), (31), and (32), the contact force of the \( j \)-th ball can be calculated as 
\[
F_{b_j} = k_b \delta_j^{1.5}
\]
[54, 55]. The maximum contact force \( \max(F_{b_1}) \) of the No. 1 ball in the time range of one revolution under different misalignment conditions is extracted, respectively. It is also marked that the maximum contact force of the ball bearing without misalignment is \( \max(F_{b_1,0}) \). The relative variation of the maximum contact force of the ball under different misalignment degrees can be calculated according to the following equation:
\[
\varepsilon = \frac{\max(F_{b_1}) - \max(F_{b_1,0})}{\max(F_{b_1,0})} \times 100\%.
\]

The maximum contact force variation curve of the ball is calculated according to Eq. (43), as shown in Fig. 15. With the misalignment angle increases, the contact force presents a trend of nonlinear increase. The more serious the misalignment, the more obvious the increase of the contact force between the ball and the raceway. When the misalignment angle \( \varphi = 0.21^\circ \), the maximum contact force even increases by 183.6%. This may further lead to contact deformation between the ball and the raceway, which is not conducive to the stable operation of the bearing.

Next, the influence of misalignment on dynamic characteristics of the ball bearing-coupling-rotor system is further discussed. The variation range of misalignment angle is set as \( \varphi \in [0, 0.21]^\circ \). The vibration response of the left disc of the rotor under the subcritical speed range \( (n = 3000 \text{ r/min}) \), critical speed range \( (n = 12,700 \text{ r/min}) \), and supercritical speed range \( (n = 15,000 \text{ r/min}) \) is selected. The results are shown in Figs. 16, 17, 18 and 19. It is not difficult to find the following phenomena:

1. When the rotor operates in the subcritical speed range \( (n = 3000 \text{ r/min}) \), combined with Figs. 16a, b and 17, it can be seen that under the healthy state \( (\varphi = 0^\circ) \), the vibration center of the rotor under the action of gravity is in the negative direction of \( Y \)-direction due to the weak imbalance excitation of the rotor. The rotating frequency \( (f_r) \) and its higher harmonic

Fig. 18 System vibration response at \( n = 12,700 \text{ r/min} \): a rotor orbit, b time-domain waveform in \( Y \)-direction, c time-domain waveform in \( Z \)-direction, d Poincaré section map in \( Y \)-direction, and e Poincaré section map in \( Z \)-direction
components are the main components in the spectrum. With the increase of misalignment angle, the rotor orbit and time-domain waveform of $Y$-direction show the trend of upward translation. The radial amplitude decreases slightly, and the amplitude of frequency $f_r$ and its super-harmonic components also decrease. The frequency amplitude of $f_r$ decreases from $5.135 \mu m (\phi = 0^\circ)$ to $2.913 \mu m (\phi = 0.21^\circ)$. This is because the misalignment fault enhances the constraint of bearing on the rotor. With the further increase of misalignment angle, the frequency amplitude of $2f_r$ increases, resulting from the enhancement of coupling misalignment. The radial vibration of the rotor system always keeps quasi-periodic motion under the condition of health and misalignment. In a healthy state, the axial vibration of the deep groove ball bearing-rotor system almost does not exist, and it increases gradually with the increase of misalignment degree. Specifically, the amplitude of $f_r$ and its super-harmonic components increase, and the increase of $f_r$ is the most obvious in the axial spectrum. In addition, the amplitude of the VC frequency ($f_{vc}$) of bearing increases with the enhancement of misalignment, whether radial vibration or axial vibration. This indicates that misalignment intensifies the VC vibration of bearing.

(2) When the rotor operates in the critical speed range ($n = 12,700$ r/min), it can be seen from Figs. 16c, d and 18 that the system amplitude increases sharply in the resonance state, and the system vibration is mainly excited by rotor imbalance. Under the action of imbalance excitation, the rotor orbit shows an elliptic curve before and after misalignment, and the time-domain waveforms of radial direction are approximately sine waves. The rotor system shows in the form of period-one (P-1) motion in the radial direction. When the misalignment angle is greater than $0.17^\circ$, the frequency amplitude of $f_r$ in the radial spectrum increases suddenly, and the amplitude of $3f_r$, $4f_r$, and $5f_r$ in the axial spectrum increases sharply. It indicates that the axial and radial vibrations of the system are suddenly intensified. The reason is that with the increase of misalignment angle, the support

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**Fig. 19** System vibration response at $n = 15,000$ r/min: a rotor orbit, b time-domain waveform in $Y$-direction, c time-domain waveform in $Z$-direction, d Poincaré section map in $Y$-direction, and e Poincaré section map in $Z$-direction
stiffness of bearing to rotor increases and the natural frequency of the system enhances. When the natural frequency of the system increases close to the excitation frequency of the system (rotating frequency), the resonance of the system is triggered. In addition, when the misalignment fault occurs, the system changes from the P-1 motion to the quasi-periodic motion in the axial direction.

(3) When the rotor speed is further increased to the supercritical speed range \( n = 15,000 \) r/min, combined with Figs. 16e, f and 19, it can be seen that after passing the critical speed range, the amplitude of system decreases although it is still in the P-1 motion state. In the radial vibration, the system is still dominated by rotor imbalance excitation. With the increase of misalignment degree, both the axial and radial amplitudes of the system increase, and the frequency amplitudes of \( f_r \) and its higher harmonic enhance. In the radial spectrum, the amplitude of \( f_r \) rises from 81.25 \( \mu m \) (\( \varphi = 0^\circ \)) to 88.08 \( \mu m \) (\( \varphi = 0.21^\circ \)). Under the influence of coupling misalignment excitation, the rotor orbit changes gradually from regular ellipse in the healthy state to triangle. The axial vibration waveform presents the characteristics of alternating vibration of high and low peaks. In the axial spectrum, the frequency amplitude of \( 2f_r \) increases even more than that of \( f_r \).

4 Conclusions

In this paper, a dynamic model of the ball bearing-coupling-rotor system is established based on the finite element method. Based on the Hertz contact theory, a nonlinear force model of 5-DOF rolling bearing is proposed, which can consider the misalignment of bearing rings and the clearance of cage pockets. The coupling misalignment excitation force model is further introduced. The radial and axial vibration responses of the system under the action of misalignment faults are analyzed. Some conclusions are as follows:

(1) When the rotor is running at low speed, the bearing misalignment greatly affects the dynamic characteristics of the rotor system, and the frequency component of bearing varying compliance vibration \( f_{vc} \) is obvious. Therefore, the influence of bearing misalignment cannot be ignored in the study of rotor misalignment. With the increase of operating speed, the VC vibration is relatively weakened, the rotor imbalance and coupling misalignment have a greater impact, and the component of \( 2f_r \) is more obvious. The reason is that the rotor imbalance force and coupling misalignment force are related to the square term of rotating speed, while the bearing misalignment force is only related to the power of rotating speed.

(2) Misalignment leads to periodic changes in the contact angle and radial clearance of each ball in the rolling bearing, and the velocity of each ball in the bearing is not equal. With the aggravation of misalignment, there may be two loading areas of the bearing. The contact force of the ball also increases, and the increase of the contact force can even reach 183.6%, which may lead to extrusion deformation between the ball and the raceway. In addition, the difference in ball rotational speed is more significant. The rotational speed of the ball and cage is inconsistent, which leads to the reciprocating extrusion and collision of the ball in the cage pocket, and increases the possibility of fatigue fracture of the cage.

(3) The bearing misalignment increases the support stiffness of the rotor, resulting in the enhancement of the critical speed of the system. The system vibration caused by bearing misalignment has strong directivity. In the subcritical speed range, the bearing misalignment reduces the amplitude of the radial vibration of the
system, which is due to the increased constraint of the bearing on the rotor and the increased deflection of the rotor. The radial vibration of the system is aggravated by misalignment in critical and supercritical speed range.

(4) Due to the angular misalignment, the bearing produces a component of restoring force along the axial direction, so the axial vibration of the system increases under the action of this excitation. The more serious the misalignment, the greater the axial vibration of the rotor system. Therefore, axial vibration can be used as an essential basis to measure the angular misalignment fault of the deep groove ball bearing-rotor system.

Acknowledgements The authors would like to acknowledge the support of the China North Vehicle Research Institute on the project. This work was supported by the Basic Research Project (Grant No. 20195208003).

Funding Professor Hui Ma has China North Vehicle Research Institute (20195208003).

Data availability The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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