Effects of Misalignment on the Sliding Rates of Parallel Line Gear Pair

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Abstract. The sliding rates of parallel line gear pair (PLGP) are equal to zero and pure rolling meshing can be achieved under ideal condition. In this paper, effects of misalignment on the sliding rates of PLGP were studied. Firstly, the geometric model of PLGP was presented. Then the contact curves of PLGP with misalignment were obtained by the tooth contact analysis (TCA) method and the sliding rates were calculated. The calculation results of the example show that PLGP under ideal condition or with only axial deviation accomplishes transmission along the theoretical contact curves, the sliding rates are equal to zero, and pure rolling meshing can be achieved. For PLGP with centre distance deviation and parallelism misalignment, the sliding rates are not equal to zero and the form of relative motion between the meshing surfaces combines rolling and sliding. This investigation provides a theoretical basis for the further study of meshing efficiency and wear prediction of line gear pair under actual operating condition.

1. Introduction
Relative sliding between the meshing surfaces occurs throughout the meshing process (except for the pitch point) of the commonly used gear pairs [1, 2]. Sliding rates are often used as a measure of the relative sliding between the meshing surfaces [3]. Researchers study the relative sliding of a gear pair by calculating its sliding rates. Xu et al. [4] derived the formula of sliding rates of a novel cycloidal gear pair and analysed the effects of various parameters on the sliding rates. Wang et al. [5] presented an internal gear pair with a high contact ratio and studied its sliding rates. Relative sliding between the meshing surfaces under dry friction and starved-oil conditions will lead to problems such as low meshing efficiency and severe wear, which can limit the application of the gear pairs.

Line gear (LG), a novel gear mechanism invented on the basis of space curve meshing theory, accomplishes transmission via point contact meshing of a pair of space conjugate curves named the driving and driven contact curves [6, 7]. By selecting the design parameters of a line gear pair, the sliding rates can be equal to zero, which means that relative sliding during the meshing process can be eliminated and pure rolling meshing can be achieved theoretically. For example, when selecting cylindrical helix as the driving contact curve of a parallel line gear pair (PLGP), the sliding rates are equal to zero throughout the meshing process [8].

However, misalignment under actual operating condition can change the meshing positions of PLGP, which might result in relative sliding between the meshing surfaces and change the sliding rates. In this paper, the effects of misalignment on the sliding rates of PLGP were studied. Firstly, the geometric model of PLGP was presented. Then the contact curves of PLGP with misalignment were...
obtained by the tooth contact analysis (TCA) method. Finally, the sliding rates were calculated and the effects of misalignment on the sliding rates were analysed through a calculation example.

2. Sliding rates of PLGP

2.1. Geometric model of PLGP

The geometric model of PLGP is shown in Figure 1. \(a - xyz\) and \(o - x_1y_1z_1\) are fixed coordinate systems. Coordinate systems \(a - x_1y_1z_1\) and \(o - x_2y_2z_2\) are fixed with the driving LG and driven LG respectively. The driving LG and driven LG rotate around axis \(z_1\) and axis \(z_2\) respectively with a centre distance of \(a\). When the theoretical driving contact curve \(r_1\) is a right-hand cylindrical helix, the theoretical driven contact curve \(r_2\) is a left-hand one according to the space curve meshing theory \([6, 7]\). \(k\) is the instantaneous meshing point. The expressions of \(r_1\) and \(r_2\) in \(a - x_1y_1z_1\) and \(o - x_2y_2z_2\) are shown as Equation (1) and Equation (2) respectively:

\[
\begin{align*}
\mathbf{r}_1^{(1)}(t_1) &= \begin{pmatrix} m_1 \cos(t_1) \\ m_1 \sin(t_1) \\ n_1(t_1 + \pi) \end{pmatrix} \quad (t_1 \leq t_1 \leq t_{10}) \\
\mathbf{r}_2^{(1)}(t_2) &= \begin{pmatrix} m_2 \cos(t_2) \\ m_2 \sin(t_2) \\ n_2(t_2 + \pi) \end{pmatrix} \quad (t_{12} \leq t_2 \leq t_{12})
\end{align*}
\]

where, subscript \(i = 1, 2\) represent the parameters of the driving LG and driven LG respectively. \(m_1\) is the helix radius of \(r_1\), \(m_2 > 0\) and \(m_2 > 0\). \(n_1\) is the pitch parameter of \(r_1\), \(n_1 > 0\) and \(n_2 < 0\). \(t_{12}\) is the parameter indicating the scope of \(r_1\), \(r_2\) and \(r_2\) start to mesh at \(t_{10}\) and separate at \(t_{12}\).

In the normal plane of an arbitrary point on \(r_1\), an arc \(P_i\) is selected as the generatrix and made to sweep along \(r_1\), and the tooth surface \(\Sigma_i\) is formed. The expressions of \(\Sigma_1\) and \(\Sigma_2\) in \(\alpha - x_1y_1z_1\) and \(\alpha - x_2y_2z_2\) are shown as Equation (3) and Equation (4) respectively:

\[
\begin{align*}
\Sigma_1^{(1)}(t_1, \theta) &= \begin{pmatrix} \cos(t_1)(m_1 - R_1(\cos(\theta + \phi) + \sin(\phi))) + n_1R_1 \sin(t_1)(\sin(\theta + \phi) - \cos(\phi))(m_1^2 + n_1^2)^{-\frac{1}{2}} \\
\sin(t_1)(m_1 - R_1(\cos(\theta + \phi) + \sin(\phi))) - n_1R_1 \sin(t_1)(\sin(\theta + \phi) - \cos(\phi))(m_1^2 + n_1^2)^{-\frac{1}{2}} \\
n_1(t_1 + \pi) + m_1R_1(\sin(\theta + \phi) - \cos(\phi))(m_1^2 + n_1^2)^{-\frac{1}{2}} \end{pmatrix} \\
\Sigma_2^{(1)}(t_2, \theta) &= \begin{pmatrix} \cos(t_2)(m_2 - R_2(\cos(\theta + \phi) + \sin(\phi))) + n_2R_2 \sin(t_2)(\sin(\theta + \phi) - \cos(\phi))(m_2^2 + n_2^2)^{-\frac{1}{2}} \\
\sin(t_2)(m_2 - R_2(\cos(\theta + \phi) + \sin(\phi))) - n_2R_2 \sin(t_2)(\sin(\theta + \phi) - \cos(\phi))(m_2^2 + n_2^2)^{-\frac{1}{2}} \\
n_2(t_2 + \pi) + m_2R_2(\sin(\theta + \phi) - \cos(\phi))(m_2^2 + n_2^2)^{-\frac{1}{2}} \end{pmatrix}
\end{align*}
\]

where, \(\theta\) is the parameter indicating the scope of \(P_i\). \(R_i\) is the radius of \(P_i\). \(\phi_i\) is the modification angle of the tooth profile.

\(n_{10}\) and \(n_{20}\) are the unit normal vectors of \(\Sigma_1\) and \(\Sigma_2\), of which the expressions in \(\alpha - x_1y_1z_1\) and \(\alpha - x_2y_2z_2\) are shown as Equation (5) and Equation (6) respectively:

\[
\begin{align*}
n_{10}^{(1)}(t_1, \theta) &= \frac{\partial \Sigma_1^{(1)}}{\partial \theta} \times \frac{\partial \Sigma_1^{(1)}}{\partial t_1} \\
n_{20}^{(2)}(t_2, \theta) &= \frac{\partial \Sigma_2^{(2)}}{\partial \theta} \times \frac{\partial \Sigma_2^{(2)}}{\partial t_2}
\end{align*}
\]
The direction from the tooth entity to the outside of the tooth is the positive direction of \( \mathbf{n}_{i0} \), while the direction from the outside of the tooth to the tooth entity is the positive direction of \( \mathbf{n}_{20} \).

2.2. Contact curves with misalignment

Misalignment under the actual operating condition can change the meshing positions, thus the actual contact curves might deviate from the theoretical ones. As shown in Figure 2, misalignment of PLGP is defined as: centre distance deviation, namely, an extra displacement \( \delta a \) in the direction of axis \( x_f \); axial deviation, namely, an extra displacement \( \delta b \) in the direction of \( z_f \); parallelism misalignment of the two rotating shafts, namely, the angle \( \gamma_a \) between the projections of axis \( z_i \) and axis \( z_a \) in the plane \( y_i o z_f \) (crossed axes misalignment), and the angle \( \gamma_b \) in the plane \( x_i o z_f \) (intersected axes misalignment). The values of \( \delta a, \delta b, \gamma_a \) and \( \gamma_b \) shown in Figure 2 are all positive.

![Figure 1. Geometric model of PLGP.](image1)

![Figure 2. Misalignment of PLGP.](image2)

Transformation matrix \( \mathbf{M}_{f1} \) from \( o_1 - x_1 y_1 z_2 \) to \( o_f - x_f y_f z_f \) and \( \mathbf{M}_{f2} \) from \( o_2 - x_2 y_2 z_2 \) to \( o_f - x_f y_f z_f \) are shown as Equation (7) and Equation (8) respectively:

\[
\mathbf{M}_{f1} = \begin{bmatrix}
\cos(\gamma_a) \cos(\phi_1) & \cos(\gamma_b) \sin(\phi_1) & \sin(\gamma_a) & a + \delta a \\
\sin(\gamma_a) \cos(\phi_1) - \cos(\gamma_b) \sin(\phi_1) & \sin(\gamma_b) \sin(\phi_1) + \cos(\gamma_a) \cos(\phi_1) & -\sin(\gamma_a) \cos(\gamma_b) & \delta b \\
-\cos(\gamma_a) \sin(\phi_1) - \sin(\gamma_b) \cos(\phi_1) & -\cos(\gamma_b) \sin(\phi_1) + \sin(\gamma_a) \cos(\phi_1) & \cos(\gamma_a) \cos(\gamma_b) & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(7)

\[
\mathbf{M}_{f2} = \begin{bmatrix}
\cos(\phi_2) & -\sin(\phi_2) & 0 & 0 \\
\sin(\phi_2) & \cos(\phi_2) & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(8)

where \( \phi_1 \) and \( \phi_2 \) are the rotational angles of the driving LG and driven LG respectively.

Utilizing coordinate transformation, \( \mathbf{S} \) and \( \mathbf{n}_i \) in \( o_f - x_f y_z z_f \) are obtained as Equation (9) and Equation (10):

\[
\mathbf{S}^{(1)}(t_1, \theta_1, \phi_1) = \mathbf{M}_{f1}(\phi_1) \cdot \mathbf{S}^{(1)}(t_1, \theta_1)
\]

\[
\mathbf{S}^{(2)}(t_2, \theta_2, \phi_2) = \mathbf{M}_{f2}(\phi_2) \cdot \mathbf{S}^{(2)}(t_2, \theta_2)
\]

(9)

\[
\mathbf{n}_i^{(1)}(t_1, \theta_1, \phi_1) = \mathbf{L}_{f1}(\phi_1) \cdot \mathbf{n}_i^{(1)}(t_1, \theta_1)
\]

\[
\mathbf{n}_i^{(2)}(t_2, \theta_2, \phi_2) = \mathbf{L}_{f2}(\phi_2) \cdot \mathbf{n}_i^{(2)}(t_2, \theta_2)
\]

(10)

where \( \mathbf{L}_{f1} \) and \( \mathbf{L}_{f2} \) are the 3-order submatrices of \( \mathbf{M}_{f1} \) and \( \mathbf{M}_{f2} \) respectively.

The TCA equations at the meshing points is expressed as Equation (11) [9]:

\[
\text{Eq 11}
\]
$$\begin{align*}
\Sigma^{(i)}_1(t_1, \theta_1, \varphi_1) &= \Sigma^{(i)}_2(t_2, \theta_2, \varphi_2) \\
n_i^{(i)}_1(t_1, \theta_1, \varphi_1) &= n_i^{(i)}_2(t_2, \theta_2, \varphi_2)
\end{align*}$$

(11)

A meshing period from meshing starting to meshing ending of a pair of teeth of the PLGP is divided evenly into \( K \) meshing positions. Rotational angle of the driving LG is given as \( \varphi_{k1} (k = 1, ... , K) \), and the other parameters \( t_{i1}, t_{2i}, \theta_{i1}, \theta_{2i} \) and \( \varphi_{2k} \) can be obtained by solving Equation (11). \( (t_i, \theta_i) \) represents the coordinate of the meshing point on \( \Sigma_i \). \( \theta_i = \theta_i(t_i), \theta_2 = \theta_2(t_2) \) and \( t_2 = t_2(t_i) \) can be obtained by fitting the TCA results. Submitting \( \theta_i = \theta_i(t_i) \) into Equation (3), the driving contact curve with misalignment can be obtained as \( r_{ia} = r_{ia}(t_i) \). Submitting \( \theta_2 = \theta_2(t_2) \) and \( t_2 = t_2(t_i) \) into Equation (4), the driven contact curve with misalignment can be obtained as \( r_{ia} = r_{ia}(t_i) \).

2.3. Sliding rates of PLGP

PLGP accomplishes transmission via point contact meshing of a pair of contact curves \( r_{ia} \) and \( r_{ia} \). As shown in Figure 3, \( r_{ia} \) and \( r_{ia} \) mesh at point \( k \) initially. After a period of \( \Delta t \), point \( k_1 \) on \( r_{ia} \) meshes with point \( k_2 \) on \( r_{ia} \). \( \Delta s_1 \) and \( \Delta s_2 \) are the arcs on \( r_{ia} \) and \( r_{ia} \) respectively when \( \Delta t \) approaches zero. Sliding rates of \( r_{ia} \) are defined as the limit value of the ratio of the length difference between \( \Delta s_1 \) and \( \Delta s_2 \) to the length \( \Delta s_1 \), as expressed in Equation (12) and Equation (13):

$$\sigma_1 = \lim_{\Delta s_1 \to 0} \frac{\Delta s_1 - \Delta s_2}{\Delta s_1} = \frac{ds_1}{dr_1} - \frac{ds_2}{dr_1} = 1 - \frac{ds_2}{dr_1} \quad (12)$$

$$\sigma_2 = \lim_{\Delta s_2 \to 0} \frac{\Delta s_1 - \Delta s_2}{\Delta s_2} = \frac{ds_1}{dr_1} - \frac{ds_2}{dr_1} = 1 - \frac{ds_2}{dr_1} \quad (13)$$

\( r_{ia} \) and \( r_{ia} \) can be denoted as \( r_{ia} = [x_i(t_i), y_i(t_i), z_i(t_i)]^T \) and \( r_{ia} = [x_j(t_i), y_j(t_i), z_j(t_i)]^T \), then \( ds_i/dr_1 = ((x'_i(t_i))^2 + (y'_i(t_i))^2 + (z'_i(t_i))^2)^{0.5} \), \( ds_2/dr_1 = ((x'_i(t_i))^2 + (y'_i(t_i))^2 + (z'_i(t_i))^2)^{0.5} \). Thus Equation (12) and Equation (13) can be rewritten as:

$$\sigma_1 = 1 - \frac{((x'_i(t_i))^2 + (y'_i(t_i))^2 + (z'_i(t_i))^2)^{0.5}}{((x'_i(t_i))^2 + (y'_i(t_i))^2 + (z'_i(t_i))^2)^{0.5}} \quad (14)$$

$$\sigma_2 = 1 - \frac{((x'_i(t_i))^2 + (y'_i(t_i))^2 + (z'_i(t_i))^2)^{0.5}}{((x'_i(t_i))^2 + (y'_i(t_i))^2 + (z'_i(t_i))^2)^{0.5}} \quad (15)$$

![Figure 3](image-url)

**Figure 3.** Relative sliding between the \( r_{ia} \) and \( r_{ia} \).

3. Calculation example and analysis

A PLGP was given as an example. The design parameters and misalignment values of the PLGP are given in Table 1 and Table 2 respectively.
The sliding rates of PLGP were calculated by the method mentioned in Section 2. The sliding rates under ideal condition can be obtained as $\sigma_1 = \sigma_2 = 0$, which means that PLGP under ideal condition accomplishes transmission along the theoretical contact curves and pure rolling meshing can be achieved throughout the meshing process.

The sliding rates with different kinds of misalignment are shown from Figure 4 to Figure 7. As shown in Figure 4, for PLGP with only axial deviation, the sliding rates are also equal to zero throughout the meshing process, which means that PLGP still accomplishes transmission along the theoretical contact curves and the form of relative motion between the meshing surfaces is pure rolling.

However, as shown from Figure 5 to Figure 7, when the two rotating shafts of PLGP have centre distance deviation or parallelism misalignment, the contact curves deviate from the theoretical ones, the sliding rates are not equal to zero, and the relative motion between the meshing surfaces combines both rolling and sliding. As shown in Figure 5, $\sigma_1 > 0$ and $\sigma_2 < 0$ when $\delta a > 0$, while $\sigma_1 < 0$ and $\sigma_2 > 0$ when $\delta a < 0$. $\sigma_1$ and $\sigma_2$ keep constant during the meshing process. As shown in Figure 6, $\sigma_1 > 0$ and $\sigma_2 < 0$ when $\gamma_a > 0$, while $\sigma_1 < 0$ and $\sigma_2 > 0$ when $\gamma_a < 0$. $\sigma_1$ and $\sigma_2$ approximately keep constant during the meshing process. As shown in Figure 7, $\sigma_1 > 0$ and $\sigma_2 < 0$ when $\gamma_c > 0$, while $\sigma_1 < 0$ and $\sigma_2 > 0$ when $\gamma_c < 0$. $|\sigma_1|$ and $|\sigma_2|$ increase approximately linearly from meshing starting to meshing ending. For this calculation example, $|\sigma_1|$ varies from approximately 0.00008 to 0.00014 when $\gamma_c = 0.01$, and from approximately 0.00015 to 0.00028 when $\gamma_c = 0.02$. The absolute values of the sliding rates increase with the increase of the absolute values of these three kinds of misalignment.

**Table 1.** Design parameters of PLGP.

| Parameters | Driving LG | Driven LG |
|------------|------------|-----------|
| $(m_1, n_r)$ /mm | (10, 20) | (30, -60) |
| $R_i$ /mm | 4 | 4 |
| $\phi_1$ /° | 30 | 150 |
| $(t_a, t_n)$ | (-$\pi$, -$\pi$/2) | (0, -$\pi$/6) |
| Transmission ratio | 3 |

**Table 2.** Misalignment values of PLGP.

| misalignment | values |
|--------------|--------|
| Centre distance deviation /mm | ± 0.01, ± 0.02 |
| Axial deviation /mm | ± 0.01, ± 0.02 |
| Crossed axes misalignment /° | ± 0.01, ± 0.02 |
| Intersected axes misalignment /° | ± 0.01, ± 0.02 |

Figure 4. Sliding rates with only axial deviation

Figure 5. Sliding rates with only centre distance deviation
4. Conclusions
By analysing the effects of misalignment on the sliding rates of PLGP, the main results can be concluded as follows:

1. PLGP under ideal condition or with only axial misalignment accomplishes transmission along the theoretical contact curves, the sliding rates are equal to zero throughout the meshing process, and pure rolling meshing can be achieved.

2. PLGP with centre distance deviation and parallelism misalignment accomplishes transmission along a pair of contact curves deviating from the theoretical ones, the sliding rates are not equal to zero, and the relative motion between the meshing surfaces combines both rolling and sliding. The sliding rates of PLGP with intersected axes misalignment vary during the meshing process, which indicates that intersected axes misalignment can affect the stability of the gear transmission.

3. The sliding rates of PLGP are close to zero, which indicates approximate pure rolling meshing can be achieved under actual operating condition.

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