Trajectories of a spacecraft aiming to approach at a near-
regular cadence of the Enceladus and Dione moons

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Abstract. Saturn currently has about 62 moons already discovered. This number is uncertain
due to the numerous of objects that orbit the planet, but it guarantees the placement of the
second planet with the highest number of moons in the Solar System, each one with diverse
physical and orbital characteristics. Among all the large moons of Saturn, Dione and Enceladus
are the subject of the present work, whose objective is to evaluate strategic trajectories aiming
to approach an artificial satellite to these moons, in a near-regular cadence. In the vicinity of
Dione and Enceladus, the artificial satellite is significantly perturbed by the gravitational
potential of Saturn, which in this work is expanded in spherical harmonics ($J_2$), and also by the
gravitational field of the 13 largest moons. All simulations were done using the Spacecraft
Trajectory Simulator, an orbital simulator capable to consider continuous propulsion, and
trajectory control in closed loop.

1. Introduction
Saturn is the second planet in the Solar System with the largest number of moons, around 62, whose
exact number is still uncertain as numerous objects orbit the planet.

Enceladus is the sixth largest natural satellite of Saturn (approximately 504 km of diameter),
discovered in 1789, whose first images were sent by the Voyager probes. Voyager 1 just flew over the
moon from far away, but Voyager 2 managed to get pictures. Twenty years later, due to the interesting
images already obtained, Enceladus was considered one of the priorities of the Cassini mission, which
made three approaches, reaching 350 km away from the moon. After Titan, Enceladus was the satellite
most observed by the Cassini probe, which showed strong evidence of the presence of liquid water
in the subsurface source region [1].

Another large moon of Saturn, Dione with twice the orbital period of Enceladus and approximately
1120 km of diameter, orbits the planet at a distance of 6.3 times the radius of Saturn on the outside of
the ring. Due to the orbital position of Dione and the position of the ring, there are indications that the
moon contributes with the ring material, and can therefore be geologically active [2].

Dione was discovered in 1684, but until the early 1980s little was known about it, until the first
relevant information was sent by the Voyager probes, that visited Saturn. Some years later, in 2004,
the Cassini spacecraft flew over the moon and in 2005 the closest approach was made between a
spacecraft and Dione, at a distance of 500 km.

The present work aims to evaluate a strategic trajectory aiming at approaching a spacecraft of the
Enceladus and Dione moons in a near-regular cadence. In this work the satellite orbits Saturn with
specific initial conditions obtained by means of simple equations of the Celestial Mechanics. Because
Enceladus and Dione have a synchronous motion around Saturn, the artificial satellite approaches the moons in a near-regular cadence. The movement is not regular for a long period of time due to the intense perturbation of the gravitational potential of the bodies, especially of Saturn. A similar study presenting this approach for the case of Mars and its natural satellites, Phobos and Deimos, can be found in [3].

In the present study the gravitational potential of Saturn is expanded in spherical harmonics, where the term \( J_2 \) is considered. In addition to the gravitational potential of Saturn, simulations also consider the gravitational attraction of the 13 largest moons: Dione, Enceladus, Epimetheus, Hyperion, Iapetus, Janus, Mimas, Pandora, Phoebe, Prometheus, Rhea, Titan and Tethys. The simulations are done using the Spacecraft Trajectory Simulator (STRS), an orbital simulator capable to consider continuous propulsion, and trajectory control in closed loop [4 - 9].

2. Considered orbital perturbations

The gravitational potential of Saturn can be expressed from the expansion of the spherical harmonic coefficients, given by Equation (1) [10]:

\[
U(r, \lambda, \phi) = \frac{\mu}{r} + \sum_{n=2}^{\infty} \sum_{m=0}^{n} \left( \frac{a_e}{r} \right)^n \left( c_{nm} \cos m \lambda + s_{nm} \sin m \lambda \right) \tilde{P}_{nm}(\sin \phi) \tag{1}
\]

where \( n \) is the degree, \( m \) is the order, \( \mu \) is the gravitational constant and \( r \) is the Saturn equatorial radius. \( \tilde{P}_{nm} \) are the fully normalized associated Legendre polynomials. \( a_e \) is the reference Saturn radius, \( \phi \) is the latitude, and \( \lambda \) is the longitude.

The value of the coefficient of the spherical harmonics of Saturn is \( J_2 = 16290.71 \times 10^{-6} \) [11], and the physical and orbital parameters of the moons are presented in Table 1.

|                     | Enceladus | Dione   |
|---------------------|-----------|---------|
| Mean Radius (km)    | 252.3     | 562.5   |
| Mass \( (10^{19} \text{ kg}) \) | 10.805    | 109.572 |
| Semimajor axis, \( a \) \( (10^3 \text{ km}) \) | 238.04    | 377.42  |
| Eccentricity, \( e \) | 0.0047    | 0.0022  |
| Inclination, \( i \) \( (\text{deg}) \) | 0.009     | 0.028   |
| Orbital period (days) | 1.370218  | 2.736915 |

3. Results

As stated previously, the main objective of this work is to present a strategic trajectory that allows an artificial satellite to approach Enceladus and Dione in a near-regular cadence. For this, an approach is adopted in which the artificial satellite orbits Saturn with specific initial conditions obtained by means of simple equations of the Celestial Mechanics, in which the synchronous movement of Enceladus and Dione around Saturn was considered. Figure 1 shows the trajectory of the artificial satellite around Saturn (blue line), the orbit of the moons and the approach points.
Due to the intense disturbance of Saturn's gravitational potential, it is not effortless for an artificial satellite to orbit one of the two moons for a long period of time without the use of an orbital control system. Thus, the artificial satellite remains orbiting Saturn all the time, however, it approaches the moons in a near-regular cadence, as can be seen in Figure 1. Such an approach was initially presented in simplified form by [13] and studied by [3, 14], in the case of Mars, Phobos and Deimos. Enceladus and Dione perform a synchronous movement with Saturn, since their orbital periods are 33 and 66 hours, respectively. Thus, the moons present themselves naturally in 2:1 resonance, which allows the calculation of all possible orbits that would have medium motion resonance with Enceladus and Dione. Such orbits are called DROs (Double Resonant Orbits). These approach orbits can be used to observe the moons closely, take pictures, collect data, and perform experiments. From them it is also possible to plan a landing on the surface of Enceladus and Dione.

It can be seen that the elliptical trajectories of the spacecraft, in blue, almost touch the orbits of Enceladus and Dione. But because of the orbital disturbances generated by Saturn and the other moons, some of them illustrated in the figure by their orbits inside the orbit of Dione, the orbit of the vehicle evolves and for this reason we see a track with the trajectories of the spacecraft.

In the periapsis the spacecraft almost tangentiates the orbit of Enceladus and can approach this moon. At the apoapsis the spacecraft almost tangentiates the Dione's orbit and in turn, can approach that moon. However, approximations can occur with Tethys, since the spacecraft's orbit intercepts the Tethys orbit as shown in the Figure 3. For this reason in the simulation the perturbation generated by Tethys was quite pronounced when there was the approximation with this moon, as shown in Figure 2.
Initially, the magnitude of the perturbative forces capable of changing the trajectory of an artificial satellite was analyzed. In this case, the magnitude of the perturbations due to the gravitational potential of Saturn, Enceladus and Dione were analyzed, but also the perturbation due to the gravitational attraction of Epimetheus, Hyperion, Iapetus, Janus, Mimas, Pandora, Phoebe, Prometheus, Rhea, Titan and Tethys. The results obtained are presented in Figures 4 to 12, in which 24 orbits were simulated around Saturn, during approximately 45 terrestrial days, with the initial conditions are presented in Table 2.

| Parameter                                | Value                  |
|------------------------------------------|------------------------|
| Semimajor axis (m)                       | 3.080284815713322x10^8 |
| Eccentricity                             | 0.226009488548114      |
| Inclination (deg)                        | 0.015967875265287      |
| Right ascension of ascending node (deg)  | 1.897902316733461x10^2 |
| Argument of periapsis (deg)              | 2.492951875429249x10^2 |
| Mean anomaly (deg)                       | 3.435169584111600x10^2 |
Figure 4. Velocity increment due to Saturn perturbation.

Figure 5. Satellite altitude with respect to Saturn.

Figure 6. Velocity increment due to Enceladus perturbation.
Figure 7. Distance between the satellite and Enceladus.

Figure 8. Velocity increment due to Dione perturbation.

Figure 9. Distance between the satellite and Dione.
Figures 5, 7 and 9 show the satellite's altitude relative to Saturn, and the distance between the satellite and Enceladus and the satellite and Dione, respectively. Figure 5 shows that the artificial satellite is performing an eccentric elliptical orbit around Saturn. Figures 7 and 9 show that the satellite trajectory approaches and departs periodically from each of the moons, satisfying the objective of this work.

Figures 4, 6 and 8 are respectively in accordance to Figures 5, 7 and 9, since the moments of closer approximation between the artificial satellite and Saturn, Enceladus or Dione are the moments of greater intensity of the disturbing force. It is worth noting again the intense magnitude of the disturbance of Saturn over the satellite. Even at the closest moments between the artificial satellite and the moons (and consequently the times when the satellite is further away from Saturn), the most significant disturbance is due to the gravitational potential of Saturn.

Figures 10 to 12 present the semimajor axis, eccentricity and inclination of the satellite as a function of time, to the orbit of the artificial satellite around Saturn.

Figure 10. Semimajor axis.

Figure 11. Eccentricity.
In Figures 10 to 12 the blue line represents the real satellite trajectory and the magenta line the reference trajectory, which indicates the trajectory that the satellite would follow if it were free of the effects of the disturbing forces.

4. Conclusions
The study presents a possibility of orbit around Saturn capable of making an approximation between the artificial satellite and the moons Enceladus and Dione. In the case of an ideal situation where the gravitational attraction of bodies would not disturb the artificial satellite, the moments of approximation would occur in a perfect regular cadence. However, such forces exist and act on the artificial satellite intensively, causing the satellite to deviate from the trajectory to be followed. In this case, it becomes necessary to use a control system capable of minimizing the effects of disturbances and keeping the satellite in the determined reference trajectory.

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