A modified mathematical model for end-point carbon prediction of BOF based on off-gas analysis

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Abstract. Several models for end-point carbon prediction of BOF (Basic Oxygen Furnace) based on off-gas analysis were studied in this paper. The advantages and disadvantages of the integral model, the exponential decay model and the cubic fitting model were analyzed respectively. Based on analysis of the characteristics of the decarburization rate curve, a new exponential model was established by the introduction of a correction algorithm. The principle of the proposed model involves applying the decarburization rate curve and the descending gradient of the historical heats to obtain the average decarburization curve and reference decarburization efficiency coefficient using the regression fitting method. According to the deviation between the actual and the predicted decarburization curves, the decarburization efficiency coefficient was corrected to improve the prediction accuracy. Plant trials were carried out in a 210 t converter to compare the performance of the mentioned models. The results showed that the new model exhibited better adaptability and higher accuracy than the other ones. The hit ratio of the new model reached more than 90% for the prediction of end-point carbon content within a tolerance of ± 0.02%.

1. Introduction
Off-gas analysis technique is one of the most important methods for end-point control of converter steelmaking. Its core principle is using the information of CO and CO₂ in the off-gas to estimate the decarburization reaction and the carbon content in the bath, so as to realize the dynamic prediction of the blowing process and the control of the blowing end-point. In recent years, Dofasco [1,2] in Canada, ILVA Taranto [3,4] in Italy, Tangshan ISCO [5] in China and many other steel plants have achieved satisfied results with the application of off-gas analysis technology.

IRSID® developed a technique for continuously measuring the CO and CO₂ contents of the off-gas from the basic oxygen furnace and using these data to compute the decarburization rate of the bath [6]. Based on this technique, MacFarlane et al [7] and Meyer et al [8] built a carbon integral model to calculate the variation of carbon content in the bath. However, this model usually presents large deviations because of the poor detection precision of the off-gas flow and composition as well as the unstable accuracy of the original information of the material, and the deviations are difficult to be eliminated. In order to improve the accuracy of BOF end-point carbon prediction, the optimization of algorithms also drew increasing attention. Meyer and Glasgow et al [9] proposed an exponential correspondence between specific decarburization rate and the carbon content of the bath, and established
an exponential decay model. On this basis, Liu et al. [10] corrected the exponential model parameters by means of isometric multi-point continuous correction. Li et al. [11] established a modified exponential model by the introduction of an index of bath mixing degree, and the hit ratio of the model was improved. Zhang et al. [12] proposed a cubic fitting model by comparing the off-gas data and the sub-lance detection data at the end of the blowing, and fit them according to different oxygen lance heights. As the understanding of decarburization mechanism is still inadequate and the blowing process could be dramatically different, the documented models fail to be applied in practices due to the poor hit rate and unstable prediction results. As to be applied to control the actual operations of industrial steelmaking process, the accuracy and stability of the off-gas model needs to be further improved.

The basic principles, main advantages and disadvantages of the above mentioned algorithms for carbon prediction based on off-gas analysis are systematically studied in this paper. A new exponential model correction algorithm is proposed by analysing the characteristics of the actual off-gas curve in a steelmaking process and the prediction accuracy is improved.

2. Characteristics of off-gas during BOF blowing process

In BOF blowing process, hot metal in the furnace and high-speed oxygen jet ejecting from the top lance undergo intense reactions of decarburization and combustion. Then, molten steel, slag and off-gas are obtained as the products of these reactions. There are three main reaction zones in the furnace, which are name as the oxygen jet impact reaction zone, the slag-metal interface reaction zone and the emulsion phase reaction zone [13], as shown in figure 1.

![Figure 1. Three zone of oxidation reactions in BOF steelmaking [13].](image)

The main chemical reactions occurring in each reaction zone are as follows:

1. jet impact zone

\[
[C] + \frac{1}{2} O_2 = CO
\]

\[
CO + \frac{1}{2} O_2 = CO_2
\]

\[
[Si] + O_2 = SiO_2
\]

\[
[Mn] + \frac{1}{2} O_2 = MnO
\]

\[
[Fe] + \frac{1}{2} O_2 = FeO
\]

2. slag-metal interface

\[
[C] + (FeO) = CO + [Fe]
\]

3. emulsion phase reaction zone
\[ [\text{Si}] + 2(\text{FeO}) = (\text{SiO}_2) + 2[\text{Fe}] \]  
(7)

\[ [\text{Mn}] + (\text{FeO}) = (\text{MnO}) + [\text{Fe}] \]  
(8)

\[ 2[P] + 5(\text{FeO}) + 4(\text{CaO}) = (4\text{CaO} \cdot P_2\text{O}_5) + 5[\text{Fe}] \]  
(9)

Among them, the CO and CO\(_2\) generated by the carbon-oxygen reaction become the main component of the converter off-gas. Most of the oxidation products obtained by other oxidation reactions enter the slag phase, and the remaining become dust. A typical variation curve of CO and CO\(_2\) in the off-gas is shown in figure 2.

**Figure 2.** Typical variation curve of CO and CO\(_2\) in the off-gas.

In the initial blowing stage of BOF (initial 240s of oxygen blown time in figure 2), only a few part of the supplied oxygen is used for the oxidation reaction of carbon, due to the preferential oxidation of [Si] and [Mn] in the melt, and the content of CO and CO\(_2\) in the off-gas is low and tends to increase. With the decrease of [Si] and [Mn] contents and the increase of bath temperature, the carbon-oxygen reaction occurs intensely, and maintains a relatively stable and high decarburization rate in the main blowing stage (oxygen blown time 240~650 s). The oxygen supplied during the main blowing stage is mainly used for the decarburization. Once the carbon reaches a threshold carbon concentration in the bath, the carbon-oxygen reaction rate decreases sharply, and the CO and CO\(_2\) content in the off-gas also decreases sharply. It can be seen from figure 2 that the variation trend of the BOF off-gas composition curve throughout the blowing process is basically consistent with the description of the three-stage decarburization theory [14].

3. Mechanism of carbon prediction by off-gas analysis

According to the information of composition and flow rate of off-gas and oxygen blown rate, the decarburization rate and specific decarburization rate of the metal bath can be obtained using the following equations.

\[ v_C = Q_{\text{off-gas}} \times q(\text{CO} + \text{CO}_2) \times \frac{12}{22.4} \]  
(10)

\[ R = \frac{v_C}{Q_{O_2}} \]  
(11)

Where the decarburization rate \( v_C \) is defined as the decarburization quantity of the metal bath per unit
time [6], and the unit is kg/s, which is greatly affected by the oxygen blown rate $Q_{O_2}$ with the unit Nm$^3$/s; and the specific decarburization rate $R$ is defined as the quantity of carbon removed per volume of oxygen blown [9], and its unit is kg/m$^3$, which is less affected by the oxygen blown rate; and $Q_{off-gas}$ is the flow rate of off-gas, the unit is Nm$^3$/s; $\varphi(CO + CO_2)$ is the volume percentage of CO and CO$_2$ in the off-gas.

Since the composition and flow rate of the off-gas are continuously detected during the blowing process, $v_C$ and $R$ can be obtained, which allows a real-time prediction of the carbon content from the metal bath. Thus, making full use of these two parameters is the key to establish an effective end-point carbon predict model based on off-gas analysis.

4. Advantages and disadvantages of existing off-gas models

The integral model, the exponential decay model, and the cubic fitting model are the main models used for end-point carbon prediction of BOF based on off-gas analysis.

4.1. The integral model

During the blowing of oxygen into the converter bath, the carbon in the bath is oxidized to CO and CO$_2$. Under the premise of knowing the initial carbon content of the metal bath, the real-time carbon content of the bath can be predicted by integrating the cumulative decarburization during the process. The integral model expression is shown in the following formula:

$$C_t = C_{HM} - \frac{\int_0^t v_C dt}{M(t)}$$  \hspace{1cm} (12)

where $C_t$ is the carbon content at time $t$,%; $C_{HM}$ is the initial carbon content of the bath,%; $v_C$ is decarburization rate, kg/s; $M(t)$ is the total weight of liquid metal, kg.

Substituting the formula (10) into the formula (12), the carbon content of the bath at any time $t$ can be obtained by integration during the blowing process. In theory, the carbon integral model can be used to calculate the real-time carbon content of the bath from the start to the end of the process. However, Meyer et al [8] and Cheng et al [15] found that the calculation error of the integral model caused by detection error of the initial carbon content, off-gas flow rate and component is much larger than the requirement of the end point carbon prediction.

Therefore, the carbon integral model is only applicable if the raw material data and instrumentation detection data are very accurate. Since the integral carbon model usually has large deviations that are difficult to eliminate, it is rarely used in industrial practices.

4.2. The exponential decay model

According to previous studies, the oxidation reaction of carbon in main blowing stage is mainly controlled by oxygen blown rate. The decarburization rate is almost constant, and the decarburization efficiency reaches the maximum. In the end blowing stage, carbon content continues to decline and once the carbon reaches a threshold carbon concentration in the bath, the decarburization rate is limited by mass transport of carbon in the melt [16].

Meyer and Glasgow et al [9] studied typical paths of decarburization versus the carbon concentration which are shown in figure 3, and found that a family of curves of the type shown can be represented by a function in the following form:

$$R = A + Be^{KC}$$  \hspace{1cm} (13)

The parameters $A$, $B$ and $K$ have unique values for any particular curve. $C$ is the carbon concentration corresponding to the specific decarburization rate. And that $A$, the asymptotic value of the curve, can be determined during actual refining. The value of the variable $K$ can then be determined from the ratio of the first derivative to the second derivative of the equation of the rate curve.

4
Figure 3. Typical end-point decarburization curves[9].

An exponential decay model was proposed based on equation (13). The main method of Meyer model is to average the decarburization curves of the historical heats to obtain a “historical average curve”. Taking the “historical average curve” as a reference curve, the decarburization curve parameters of the current heat can be calculated with a single point correction method.

As shown in figure 4, when the maximum decarburization efficiency of the current heat differs greatly from the reference curve, the Meyer model calculation result will have a large calculation error. And due to the single point correction method, small fluctuations near the correction point can also cause large changes in the shape of the calculated curve.

Figure 4. Comparison of Meyer curve and actual curve.

Based on the Meyer model, Liu et al [10] proposed a modified algorithm to perform isometric multipoint continuous correction on the calculation results by using the deviation between the actual decarburization curve and the calculated curve. This algorithm can eliminate the instability of single point correction method to a certain extent, but its main disadvantage is that some characteristics of the historical decarburization curves are not fully utilized. Therefore, the accuracy of this model is still not very high.

Tu et al [17] considered that there is a certain relationship between the decarburization rate and the
carbon content in the end blowing stage of BOF process. The decarburization rate is used as an independent variable of the exponential function to predict the endpoint carbon content in their model. However, the stability of the model is not satisfactory, because the effect of the actual oxygen blowing flow rate on the decarburization rate is not considered. Li et al [11] considered the influence of operating parameters such as lance height, oxygen blowing rate and bottom blowing rate, and used the bath mixing degree [18] to modify the exponential model. Although the calculated hit ratio has been improved, the model fails to take the actual decarburization curve characteristics of current heat into consideration.

4.3. The cubic model
Zhang et al [12] collected 100 heats of the off-gas and sub-lance measurement data at the end-point. The relationship between carbon content and decarburization rate of these heats were fitted by the exponential model, inverse scale model, quadratic model and cubic model, respectively. The cubic model was found to be the best, and the function is as follows:

$$w_{[C]} = C_0 + a_1 \cdot v_C + a_2 \cdot (v_C)^2 + a_3 \cdot (v_C)^3$$

where $w_{[C]}$ is carbon concentration of the molten steel, %; $v_C$ is the decarburization rate, kg/s; $a_1$, $a_2$, $a_3$ are fitting coefficients of the model; $C_0$ indicates the ultimate carbon content when the decarburization rate is zero during oxygen blowing, normally about 0.02%.

This model does not consider the effect of the lance height and oxygen blowing rate on the decarburization rate. However, even if there is the same carbon content in the bath in the actual blowing process, the decarburization rate is different due to the difference in lance height and oxygen blowing rate. Generally, when using the cubic model, it is necessary to obtain different fitting coefficients for the decarburization data under different lance height to ensure the calculated hit ratio of the model. However, it has been found that the model coefficients fitted at different lance height are sometimes very different, even different in magnitude.

![Figure 5. Fitting results at different lance heights using the Cubic model.](image)

The fitting results of literature [12] are shown in figure 5, it can be seen that there is no uniform law to indicate the relationship between $w_{[C]}$ and $v_C$ according to the three fitting curves, and the cubic model fails to get reliable results when the carbon content is higher than 0.10%. So it can be considered that the cubic model does not actually have universal applicability, although it may achieve good fitting effects in a few specific conditions.

5. Correction algorithm for exponential decay model
5.1. Development of the correction algorithm

In this section, the experiments based on above off-gas models are performed by introducing actual production data from a 210-ton BOF converter. The experimental results indicate that the exponential model in [11] reveals the highest hit ratio in comparison to other models. In summary, the exponential model is also the better candidate in terms of the overall performance.

On the basis of the morphology of the fitting curves shown in figure 6, better fitting degree of the exponential model is also found compared with cubic fitting model. Hence, a further research will be carried out based on the exponential model. To obtain a better hit ratio, a correction algorithm is proposed.

In the literature, the study of the exponential decay model commonly includes two parts of modes: a) Decarburization rate is regarded as the relevant variable; b) Decarburization oxygen efficiency (also called specific decarburization rate by Meyer and Glasgow et al [9]) is regarded as the relevant variable. The details are displayed in equations (15) and (16).

\[ v_C = k_1 (1 - e^{-k_2 (C - C_0)}) \]  \hspace{1cm} (15)

\[ R = \alpha + \beta e^{\gamma C} \]  \hspace{1cm} (16)

where \( C_0 \) indicates the ultimate carbon content when the decarburization rate is zero during oxygen blowing; \( k_1, k_2, \alpha, \beta, \gamma \) represent the fitting coefficients related to the decarburization rate or decarburization oxygen efficiency, in which the physical significances of \( k_1, \alpha \) are the maximum decarburization rate, and the maximum decarburization oxygen efficiency respectively.

In previous methods, only the decarburization rate and decarburization oxygen efficiency of historical heats are used to obtain the model coefficients. In this paper, the indexes of descending gradient for the decarburization rate and decarburization oxygen efficiency are also considered, and the detailed expressions are shown as follows.

\[ \nabla v_C = k_1 \cdot k_2 \cdot e^{-k_2 (C - C_0)} \]  \hspace{1cm} (17)

\[ \nabla R = \beta \cdot \gamma \cdot e^{\gamma C} \]  \hspace{1cm} (18)

Apart from the thermodynamic factors, the dynamic conditions, such as combined blowing of converter, also dramatically influence the limit decarburization of converter. Thus, the ultimate carbon content, \( C_0 \), is also taken as one of the regression parameters.
Then, the values of $k_1, k_2, C_0, \beta, \gamma$ could be obtained according to the decarburization rate curve and descending gradient data of historical heats.

\[
\alpha = -\beta e^{\gamma C_0} \tag{19}
\]

Using the values of $k_1, k_2, C_0, \alpha, \beta, \gamma$, the average decarburization curve of historical heats can be achieved following equations (15) and (16). Subsequently, the isometric multi-point continuous correction is performed according to the deviation degree between the decarburization results of current heat and the ones of historical heats, as shown in figure 7.

![Figure 7. Carbon prediction use isometric multi-point continuous correction method.](image)

The values of $\alpha$ in the two curves are the same through the normalization treatment, and meanwhile two curves both go through point $(C_0,0)$.

\[
R_A = \alpha + \beta_1 e^{\gamma_1 C}
\]

\[
R_B = \alpha + \beta_2 e^{\gamma_2 C} \tag{21}
\]

\[
\beta_2 e^{\gamma_2 C_0} = \beta_1 e^{\gamma_1 C_0} \tag{22}
\]

When the coordinates of $A_1, A_2, B_1, B_2$ are set as $(C_1^1, R_1), (C_2^1, R_2), (C_1^2, R_1), (C_2^2, R_2)$, these values can be substituted into equations (20) and (21).

\[
R_1 = \alpha + \beta_1 e^{\gamma_1 C_1^1} \tag{23}
\]

\[
R_2 = \alpha + \beta_1 e^{\gamma_1 C_2^1} \tag{24}
\]

\[
R_1 = \alpha + \beta_2 e^{\gamma_2 C_1^2} \tag{25}
\]

\[
R_2 = \alpha + \beta_2 e^{\gamma_2 C_2^2} \tag{26}
\]

The following equation can be achieved by combing equations form equation (23) to equation (26):

\[
\gamma_2 \cdot (C_2^1 - C_2^2) = \gamma_1 \cdot (C_1^1 - C_1^2) \tag{27}
\]

Then:
\[ \gamma_2 = \gamma_1 \cdot \frac{\Delta C_1}{\Delta C_2} \]  

(28)

where, \(\Delta C_1\) is the decarburization amount of the reference curve, which can be obtained from known parameters; \(\Delta C_2\) is the decarburization amount of actual curve, which can be calculated according to the off-gas composition and flow.

After obtaining, \(\gamma_2\), put it into equation (22) to calculate \(\beta_2\), and then put \(\gamma_2, \beta_2\) and \(\alpha\) into equation (26) to obtain the X-axis value of point \(B_2\), as in follow.

\[ C_2^2 = \ln \left( \frac{R_2}{\beta_2} + e^{\gamma_2 C_0} \right) / \gamma_2 \]  

(29)

The same \(\Delta R\) was used to analogize \(B_3, B_4, \ldots, B_n\), until the target carbon content was achieved.

\[ \Delta R = R_1 - R_2 = R_2 - R_3 = R_3 - R_4 = \ldots = R_{n-1} - R_n \]  

(30)

5.2. Validation of the model

Compared to previous algorithms, the main characteristics of the proposed correction algorithm involves three aspects: (1) Both the decarburization rate falling gradient and decarburization rate itself are used to calculate the model parameters by the method of regression fitting; (2) The ultimate carbon content of \(C_0\) is also regarded as one of the regression parameters, rather than just to set a fixed value of 0.02% or other fixed values; (3) Since the method of isometric multi-point continuous correction is adopted, the information referring to historical heats and current heat could be utilized efficiently. Owing to these advantages of the proposed algorithm, a higher calculation accuracy and hit ratio can be realized.

The mentioned models were validate d using actual production data from a steel plant of 210-ton converter, and the results are as follows. As shown in figure 8, the deviation between actual heats data and the historical average curve obtained by current method is smaller than the other curve obtained by previous method. And the hit ratio of the current model is the highest of the four mentioned models, as shown in table 1.

![Figure 8. Comparison of the two historical average curves and actual heats data.](image)

**Table 1.** Obtained hit ratios using the four models in the current study.

| Model                              | Hit Ratio |
|------------------------------------|-----------|
| Integral Model                     | 72.30%    |
| Cubic Model by Zhang *et al* [12]  | 85.90%    |
| Modified Exponential Model by Li *et al* [11] | 88.20%    |
| Current Model                      | 90.70%    |
6. Conclusions

- By the analysis on the characteristics of the carbon integral model, and cubic model, the exponential model is a better choice to reveal the changing rule of carbon content in the end blowing stage of BOF process, and the experimental results indicate improved accuracy on predicting the actual production data.
- This exponential model with the correction algorithm can obtain a better historical average decarburization curve by introducing the decarburization rate falling gradient and decarburization rate as the reference for regression fitting of model parameters, as well as selecting $C_0$ as the regression parameter.
- Due to the application of isometric multi-point continuous correction, the reference value of historical heats is fully excavated along with the sufficient utilization of the curve feather of current heat. The hit ratio of the new model reached more than 90% for the prediction of end-point carbon prediction within a tolerance of ±0.02%.

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