SNe Ia AND THEIR ENVIRONMENT: THEORY AND APPLICATIONS TO SN 2014J

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ABSTRACT

We present theoretical semi-analytic models for the interaction of stellar winds with the interstellar medium (ISM) or prior mass loss implemented in our code SPICE, assuming spherical symmetry and power-law ambient density profiles and using the II-theorem. This allows us to test a wide variety of configurations, their functional dependencies, and to find classes of solutions for given observations. Here, we study Type Ia Supernova (SN Ia) surroundings of single and double degenerate systems, and their observational signatures. Winds may originate from the progenitor prior to the white dwarf (WD) stage, the WD, a donor star, or an accretion disk (AD). For $M_{\text{CH}}$ explosions, the AD wind dominates and produces a low-density void several light years across, surrounded by a dense shell. The bubble explains the lack of observed interaction in late time SN light curves for, at least, several years. The shell produces narrow ISM lines Doppler shifted by $10–100\, \text{km s}^{-1}$, and equivalent widths of $\approx 100\, \text{mA}$ and $\approx 1\, \text{mA}$ in cases of ambient environments with constant density and produced by prior mass loss, respectively. For SN2014J, both mergers and $M_{\text{CH}}$ mass explosions have been suggested based on radio and narrow lines. As a consistent and most likely solution, we find an AD wind running into an environment produced by the red giant wind of the progenitor during the pre-WD stage, and a short delay, 0.013–1.4 Myr, between the WD formation and the explosion. Our framework may be applied more generally to stellar winds and star formation feedback in large scale galactic evolution simulations.

Key words: circumstellar matter -- hydrodynamics -- stars: winds, outflows -- supernovae: general -- supernovae: individual (SN2014J)

1. INTRODUCTION

Type Ia supernovae (SNe Ia) allow us to study the universe at large and have proven invaluable in cosmological studies, the understanding of the origin of elements, and they are laboratories to study physics such as: hydrodynamics, radiation transport, non-equilibrium systems, nuclear, and high energy physics. The consensus picture is that SNe Ia result from a degenerate C/O white dwarf (WD) undergoing a thermonuclear runaway (Hoyle & Fowler 1960), and that they originate from close binary stellar systems. Potential progenitor systems may either consist of two WD, the so-called double degenerate system (DD), or a single WD along with a main sequence (MS) or red giant (RG) star, the so-called single degenerate system (SD).

Within this general picture for progenitors, three classes of explosion scenarios are discussed that are distinguished by the mechanism which triggers the thermonuclear explosion. (1) Within the Chandrasekhar mass $M_{\text{CH}}$ models, the explosion is triggered by compressional heat close to the center when the WD approaches $M_{\text{CH}}$. The accretion material may originate either from an RG or MS companion via Roche-lobe overflow, or from tidal disruption of another WD in an DD system. (2) In a second class, the explosion is triggered by heat released during the dynamical merging (DM) of two WDs of a DD system. (In some cases, the combination may result in what is known as an accretion induced collapse). For overviews, see Branch et al. (1995), Nomoto et al. (2003), Di Stefano et al. (2011), Di Stefano & Kilic (2012), Wang & Han (2012), Hoeflich et al. (2013). (3) Recently, a third trigger mechanism has been revived, known as the double detonation scenario, or He-detonation in a sub-$M_{\text{CH}}$ mass WD (Nomoto 1982a; Livne 1990; Woosley & Weaver 1994; Hoeflich & Khokhlov 1996; Kromer et al. 2010; Woosley & Kasen 2011). In this picture, a C/O WD star accretes He-rich material at a low rate to prevent burning. Explosions are triggered from ignition of the surface He layer with masses of about a few hundreds to 0.1 $M_{\odot}$. The resulting strong shock wave may trigger a detonation of underlaying C/O. Previous calculations produced a few $1/100th \ M_{\odot}$ of $^{56}\text{Ni}$, which is inconsistent with the early time spectra when the photosphere is formed within $10^{-3} \ldots 2\, M_{\odot}$ (Höflich et al. 1997). Modern recalculations have utilized a smaller He shell mass and obtain better agreement with observations (Kromer et al. 2010; Woosley & Kasen 2011), though the problem with the outer layers still persists. Recent studies of helium detonations including curvature and expansion effects may be in better agreement with the observations (Moore et al. 2013; Ruiter et al. 2014; Zhou et al. 2014). For this class of explosions, we may expect accretion disk (AD) winds similar to those in $M_{\text{CH}}$ scenarios as discussed below. (4) Another explosion path, also recently set forth by Tsebrenko & Soker (2015), is known as the core-degenerate (CD) scenario, an explosion of a WD and the core of an AGB star which may produce a very short phase of mass loss similar to a planetary nebula (PN). It is not obvious that the properties of a SNe Ia would resemble a typical SNe Ia (Tsebrenko & Soker 2015).

The stellar environment will shed light on the evolutionary history of the progenitor, supernovae light curves (LCs), and spectra, with X-rays and radio emission being the probes (e.g., Chugai & Danziger 1994; Dwarkadas & Chevalier 1998; Chandra et al. 2012; Chevalier & Irwin 2012; Fransson et al. 2014). As discussed below, the density limits for the environment of a typical SNe Ia are well below those of the solar neighborhood and one of the goals is to probe whether SD and/or DD systems may create this environment.

In the case of DD progenitors, we may expect long evolutionary timescales after the formation of the WDs compared to the accretion phase in $M_{\text{CH}}$ and the double degenerate scenarios. The timescales depend on the unknown initial separation and mass of the binary WDs and the decay...
of the orbits due to gravitational waves (possibly modified during a common envelope phase). The timescale of angular momentum loss by gravitational waves scales with the fourth power of the separation (Landau & Lifshitz 1971). For example, the orbits of two 1 \( M_\odot \) WDs at 1 \( R_\odot \) will decay in \( \approx 6 \times 10^8 \) years representing a period with no or little mass loss. However, we may expect wind just prior to the explosion when the WDs fill their Roche lobe. The size of the Roche lobe corresponds to a separation of \( \approx 13,000 \) km s\(^{-1}\) (Eggleton 1983), which translates to mass loss at most some months prior to the merging (Han & Webbink 1999), and material close to the system which will be quickly overrun by the SN material.

In DD, therefore, we expect no ongoing wind with the exception of a brief period just prior to the DM. Thus, the environment of a DD system may be dominated by the interstellar medium (ISM) the system has moved into which depends on its peculiar velocity and the delay between WD formation and explosion. Mannucci et al. (2006) argued that the observed evidence of SNe Ia rates favors a bimodal distribution of the delay times between star formation and explosion, with about 50% of all explosions taking place after \( \approx 0.1 \) Gyr and 3 Gyr, respectively. Using the star formation rates and assuming that all SNe Ia originate from DD systems, Piersanti et al. (2009) concluded that 50% of all DD systems explode within 4 \( \times 10^3 \) yr, with a long tail to about 14 Gyr. Recent studies show that the distribution of delay times is more continuous (see Maoz et al. 2014 and references therein). It is likely that the DD system moved far away from its birthplace and that the explosion happens in the (constant density) ISM. In most cases, we may expect a low density environment consistent with observations.

The environment of SD systems can be expected to consist of three main components: (1) some matter bound in the progenitor system at the time of the explosion that may originate from the AD or be shed from the donor star; (2) the wind from the WD, AD, or donor star; and (3) the ISM.

Within the scenario of \( M_{\text{CNO}} \) WD explosions, hydrodynamic calculations have shown that the expanding supernova ejecta wraps around the companion star and may pull off several tenths of a solar mass of material in case of a RG donor (Marietta et al. 2000; Kasen 2010). Besides the donor star, another source of matter is the AD material (Gerardy et al. 2003) lifted during a pulsational phase during the explosion, or debris from the merging of two WDs (Hoeftlich & Khokhlov 1996; Quimby et al. 2007). There has been some observed evidence for interaction between the explosion and the immediate environment. Although H-lines like those in SN 2003ic are rare, a common feature is a high-velocity CaII line, which first was prominently seen in events like SN1995D, SN 2001el, SN2003du, and SN 2000cx, a feature present in almost all SNe Ia (Fisher 2000; Hatano et al. 2000; Wang et al. 2003; Silverman et al. 2012; Folatelli et al. 2013). This line may even be attributed to the material that is solar metallicity bound or in close proximity to the progenitor system.\(^1\) (Gerardy et al. 2004; Quimby et al. 2007).

At intermediate distances of up to several light years, in the case of \( M_{\text{CNO}} \) explosions, the environment may be dominated by the wind from the donor star, the AD, or for high accretion rates, the wind from the WD, or the interstellar material (ISM). A number of possible interaction signatures has been studied, including X-rays, radio, and narrow H and He lines, but no evidence has been found, with an upper limit of \( 10^{-5} M_\odot \) for the mass loss (Chugai 1986; Schlegel & Petre 1993; Schlegel 1995; Cumming et al. 1996; Chomiuk et al. 2012). In late-time LCs, interaction should result in excess luminosity but, in general, is not seen. No sign of an interaction has been found even in SN1991T, which has been observed up to day 1000. This implies particle densities less than \( \approx 10^{-3} \) cm\(^{-3}\) (Schmidt et al. 1994).

At large distances, from a few tenths to several light years, the environment is determined by the ISM. It is known that SNe Ia generally explode away from star-forming regions (Wang et al. 1997). This can be partly attributed to the long stellar evolutionary lifetimes of the low-mass stars in the progenitor systems, allowing them sufficient time to move away from their place of birth. It is also known that SNe Ia occur in elliptical and spiral galaxies, including galactic disks, the bulge, and the halo. One may expect the explosion to occur in ISM particle densities of \( \approx 10^{-3} \) cm\(^{-3}\) (Ferrière 2001). Light echoes from SNe Ia have been used to probe their environments, and shown that many SNe Ia have circumstellar dust shells at distances ranging from a few up to several hundred parsecs (Hamuy et al. 2000; Rest et al. 2005; Aldering et al. 2006; Patat et al. 2007a; Croots & Younord 2008; Rest et al. 2008; Wang et al. 2008).

Most evidence for a link between SNe Ia and their environment comes from the observations of narrow, time-dependent, blueshifted NaD and KI absorption lines, which, for a significant fraction of all SNe Ia, indicates outflows (D’Odorico et al. 1989; Patat et al. 2007b; Blondin et al. 2009; Sternberg et al. 2011; Foley et al. 2012; Phillips et al. 2013). In addition, extinction laws derived from SNe Ia seem to be different from the ISM in our galaxy, suggesting a component linked to the environment of SNe Ia rather than the general host galaxy (Cardelli et al. 1989; Kriściunas et al. 2000; Elias-Rosa et al. 2006; Nordin et al. 2011; Pastorello et al. 2011). Possibly, the hydrodynamical impact of the SN ejecta will produce additional emission and may modify the outer structure of the envelope, and thus, the Doppler shift of spectra features. Light emitted from the photosphere of the supernovae may heat up matter in the environment, which in turn may change the ionization balance or the dust properties (Raymond et al. 2013; Krügel 2015; Patat et al. 2015; Slavin et al. 2015). We note that this effect for different dust formation in the host galaxy may lead to extinction laws different from the Milky Way, as commonly observed in SNe Ia (Kriściunas et al. 2003; Goobar 2008; Folatelli et al. 2010; Kawara et al. 2011; Burns et al. 2014). The dust properties may effect the light echoes, which could in turn change the extinction laws (Wang 2014).

The following picture of the environment emerges: SNe Ia are surrounded by a cocoon with a much lower environment than the ISM. Sometimes, narrow ISM lines indicate clumps or surrounding shells. However, the diversity of supernova and progenitor channels leaves open a huge parameter space which cannot be covered by numerical simulations. To cover the parameter space, we use a semi-analytic approach similar to those developed by Weaver et al. (1977) for stellar wind/ISM interaction, Chevalier & Imamura (1983) for stellar wind/ISM wind interaction, and Chevalier (1982) for supernova remnants. In our study, we make use of the Π theorem (Buckingham 1914; Sedov 1959) to study the classes of self-similar solutions for the environment of SNe Ia.

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\(^1\) H\(_\alpha\) emission is suppressed by collisions, charge exchange, and low ionization of hydrogen in the absence of a strong, ongoing interaction with a red giant wind; see also Section 4.
The current state of the research leaves some important questions unresolved. How can we understand the ubiquitously low density environment, their general structure, and their link to the progenitor systems? Do SNe Ia all originate from merging WDs? Which of the wide variety of progenitor systems are compatible with the observations and the range of parameters? What other possible signatures might be seen due to the interaction of the explosion within the possible progenitor systems? For SN 2014J, can we find a class of progenitor systems which is consistent with the lack of X-rays and radio (Margutti et al. 2014; Pérez-Torres et al. 2014), which favor dynamical merger scenarios, and the narrow ISM lines, which favor $M_{\text{Ch}}$, mass explosions (Graham et al. 2015b)?

To address the questions, we developed a parameterized model in Section 2 using fluid mechanics. In Section 3, we present the application of semi-analytic models and our code SPICE$^2$ as an analysis tool in the framework of the environment of SNe Ia. We evaluate the imprint of different environments and wind properties. In Section 4, we apply the framework to SN 2014J as an example (see Figure 17) and discuss the results. In Section 5, final discussions and conclusions are presented.

2. THEORY AND ASSUMPTIONS

We develop here a model for wind-environment interaction from basic fluid mechanics. In the case of spherical symmetry and adiabatic flows, the hydrodynamic equations take the form:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial r} \left( r^2 \rho u \right) = 0$$

(1)

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} = 0$$

(2)

$$\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial r} + \frac{\gamma p}{r^2} \frac{\partial}{\partial r} \left( r^2 u \right) = 0,$$

(3)

where $u$ is the fluid velocity, $\rho$ is the mass density, $p$ is the pressure, and $\gamma = 5/3$ is the adiabatic index. The structures have four characteristic regions (Figure 1): (I) undisturbed wind emanating from the source between $r = 0$ and an inner shock front $R_2$, (II) the inner shocked region of accumulated wind matter between $R_2$ and the contact discontinuity $R_C$, (III) an outer region of swept-up interstellar gas between $R_C$ and the outer shock $R_1$, and (IV) the outermost, undisturbed, ambient medium. The solutions for regions I and IV are trivial. The solution for region III was found by Parker (1963), who implemented a self-similar ansatz and the following transformations:

$$u = r \frac{R}{t} U(\eta)$$

(4)

$$\rho = \Omega(\eta) \rho_0 r^{-\lambda}$$

(5)

$$p = r^{2 - \lambda - 2} \rho_0 P(\eta)$$

(6)

$$\eta = tr^{-\lambda},$$

(7)

where $\rho_0 (r) = \rho_0 r^{-\lambda}$ for the outer density. The similarity exponent $\lambda$ can be found by dimensional analysis.

$$R_C \propto \left( \frac{\rho_0}{\rho_0(0)} \right)^s \left( \frac{P(0)}{P(\eta)} \right)^{(1/\lambda)} \frac{1}{\lambda^3},$$

(8)

where $\lambda = 5/3$ for $s = 0$ and $\lambda = 1$ for $s = 2$ (See Equations (16) and (24) for exact expressions of $R_C$). Substituting these into Equations (1)–(3), making the substitution $\chi = \frac{\rho_0}{\rho_0(0)}$, and re-arranging gives us the following:

$$\frac{d \chi}{d \log(\eta/\eta_0)}$$

$$\gamma \chi (1 - \lambda U) [2 + U (1 - 3\lambda - 3\gamma + 2\lambda\gamma + 3\lambda U) + 2\lambda \gamma U^2]$$

$$+ \lambda \chi^2 (s\gamma - 2\gamma + 2 - s - 3\gamma + 2\lambda \gamma U)$$

$$\frac{(1 - \lambda U) (1 - \lambda U^2 - \lambda^2 \chi)}{\gamma (1 - \lambda U) \gamma (1 - \lambda U^2 - \lambda^2 \chi)}$$

(9)

$$\frac{d U}{d \log(\eta/\eta_0)}$$

$$= \chi U (1 - U) (1 - \lambda U) + \chi (2\lambda + s - 2 - 3\lambda U)$$

$$\frac{(1 - \lambda U) (1 - \lambda U^2 - \lambda^2 \chi)}{\gamma (1 - \lambda U^2 - \lambda^2 \chi)}$$

(10)

$$\frac{d \log(P/P_0)}{d \log(\eta/\eta_0)}$$

$$= \frac{2 + U (s - 2 - 2\lambda + \lambda \gamma - 3\gamma)}{1 - \lambda U^2 - \lambda^2 \chi}$$

(11)

$^2$ Supernovae Progenitor Interaction Calculator for parameterized Environments, available on request.
The outer boundary conditions are obtained from the Rankine–Hugoniot jump conditions for large Mach numbers:

$$
\chi_1 = \frac{2\gamma (\gamma - 1)}{\lambda^2 (\gamma + 1)^2},
$$

$$
P_1 = \frac{2}{\lambda^2 (\gamma + 1)},
$$

$$
U_1 = \frac{2}{\lambda (\gamma + 1)}.
$$

Integration is carried out with respect to $U$ from the outer shock $R_1$ where $U = U_1$ to the contact discontinuity $R_C$ where the velocity $u$ is equal to $dR_C/dt = \frac{R_C}{\lambda}$ and hence $U_c = 1/\lambda$.

The solution for region III is self-similar because the only relevant parameters are the mechanical luminosity of the wind $\frac{1}{2} m_{w,i}^2$ emanating from the origin and the outer density constant $\rho_0$. There is no way to obtain a parameter with dimensions of length or time using those parameters. This is not the case in region II, where the relevant parameters are $\rho_c$, as well as $m$ and $v_w$, individually. Therefore, in the case of $s = 0$ a self-similar solution is not possible in region II, as was shown by Weaver et al. (1977). They did, however, obtain useful analytic solutions directly from the hydrodynamic equations by assuming that region II was isobaric. Their results for the locations of the inner shock, contact discontinuity, outer shock, the velocity, pressure, and density are the following:

$$
R_2(t) = 0.748 \left( \frac{m}{\rho_0} \right)^{3/10} v_w^{1/10} t^{2/5},
$$

$$
R_C(t) = 0.660 \left( \frac{m_{w,i}}{\rho_0} \right)^{1/5} v_w^{3/5},
$$

$$
R_C(t) = 0.769 \left( \frac{m_{w,i}}{\rho_0} \right)^{1/5} v_w t^{1/5},
$$

$$
u(r, t) = \frac{11 R_C(t)}{25 r^2} + \frac{4 r}{25 t},
$$

$$
p(t) = 0.126 \left( \frac{m_{w,i}}{\rho_0} \right)^{1/5} v_w^{-4/5} t^{-4/5},
$$

$$
\rho(r, t) = 0.628 \left( \frac{m_{w,i}}{v_w^3 \rho_0^3} \right)^{1/5} v_w t^{-4/5} (1 - \frac{r^3}{R_C(t)^3})^{-8/33},
$$

after correcting a small typographical error in their given expression for the velocity. This is the solution for the structure of the interaction region for the $s = 0$ (constant IS density) case as the Mach number goes to infinity.

One notable feature about these structures is that the density goes to zero as $r$ approaches $R_C$ from above but diverges to infinity upon approach from below. Pressure and fluid velocity are finite and continuous across the boundary. Using the analytic expression for $\rho(r)$ (Equation (20)) we may define a characteristic width of the density peak in the inner region by $\Delta \rho_0 \equiv R_C - r(2\rho_2) = R_C[1 - 2^{-3/8}(1 - R_2/R_C)^{1/3}]$, where $r(\rho)$ is the inverse of expression $\rho(r)$ as given in 20. The width of region III is given by the integration of the Equations (9) and (10); it is $\Delta^2_{III} = 0.165 R_C$. The temperature varies with the inverse of the density. In reality the extreme temperature discontinuity will smooth out due to finite heat conduction.

2.1. Self-similar Solutions for $s = 2$

The case for $s = 2$ allows us self-similar solutions because all characteristic scales $R$ are proportional to $t$ (Equation (8)) and thus the time-dependence cancels out. Chevalier & Imamura (1983) found self-similar solutions for the interaction regions of colliding winds. Their work is similar to what we do here. The density in region IV is of the form $\rho(t) = \frac{m_i}{v_{w,i}^2 t^{3/5}}$ assuming it is of a prior stellar wind with parameters $m_i$ and $v_{w,i}$. The boundary conditions become:

$$
\chi_i = \frac{2\gamma (\gamma - 1)}{(\gamma + 1)^2} \left( \frac{v_{w,i}}{R_i} - 1 \right)^2,
$$

$$
P_i = \frac{2}{\gamma + 1} \left( \frac{v_{w,i}}{R_i} - 1 \right)^2,
$$

$$
U_i = \frac{2}{\lambda (\gamma + 1)} \left[ 2 + \frac{1}{(\gamma - 1) v_{w,i}} \right],
$$

where the subscript $i$ is either 1 or 2, referring to the outer and inner shock front boundaries, respectively ($v_{w,2} = v_w$). Using the Buckingham Pl theorem (Buckingham 1914; Sedov 1959), we obtain the following expression for $R_C$:

$$
R_C = K_{2C}(\Pi_m, \Pi_w) v_w t,
$$

$$
K_{2C,1,2} = \frac{2K_{II}(\Pi_m, \Pi_w)}{K_{II}(\Pi_m, \Pi_w)},
$$

$$
\Pi_m = \frac{m_i}{v_{w,1}^2} - 1.
$$

$K_{2C,1,2}$ are functions to be determined numerically. By requiring pressure continuity across $R_C$, we acquire an analytic expression for the inner shock radius as a function of time:

$$
R_1 = (\eta_{+}^{-1} / \eta_{-}(t_2/\eta_{-} R_1) R_2,
$$

and likewise:

$$
R_C = (\eta_{+}/\eta_{-} R_1) R_2,
$$

The quantities $(P_2/P_c), (P_c^+/P_c), (\eta_{+}/\eta_{+})$ and $(\eta_{-}/\eta_{+})$ are found from integrating Equations (9)–(11) in either region III (+) or II (–). However, in order to calculate them, initial guesses of $R_1, R_2,$ and $R_C$ are required. A consistent solution is obtained by damped fixed-point iteration. As Figure 1 shows, the structures are qualitatively different for $s = 2$ than for when $s = 0$. The density goes to infinity as one approaches $R_C$ from either side while the pressure is finite. Formally, the temperature therefore goes to zero.

2.2. Self-similar Solutions for $s = 0$ and Boundary Conditions

As discussed above, the self-similar solution depends on the ambient density, $\rho_0$, and the kinetic energy flux, $1/2 m_{w,i}^2$ at the inner boundary. For constant density ISM ($s = 0$), the solutions are not valid for $t \rightarrow 0$ and $t \rightarrow \infty$. This is because physical assumptions break down and the results are unphysical solutions.

For small times, this can be seen as follows: as shown in Table 1, $R_C \propto t^{3/5}$ and $R_2 \propto t^{2/5}$. This implies the velocity of
the reverse shock would go against infinity. No interaction is possible if the wind cannot overrun $R_{C}$, therefore the description becomes unphysical. Also, the reverse shock $R_{2}$ is greater than the contact discontinuity $R_{C}$ for small enough $t$. Therefore, no self-similar solutions exist for $R_{2} \approx v_{w}$ and $R_{3} \approx R_{C}$, i.e., times shorter than $t \sim \frac{m}{\rho_{0} v_{w}^{2}}$. For our application to SN environments at times greater than when interaction takes place within the progenitor system, this hardly poses a limitation. For our reference model in the AD wind case, the MS wind, and the RG-like wind (see Tables 3–5) the critical times are 3.8 year 0.17 and 1.2 $\times 10^{5}$ yr, respectively. The times where no-similar solutions are short compared to the duration of the winds from the progenitor system.

For large times, the solution depends on the outer boundary condition, namely the pressure of the ambient medium. In the following, we want to consider the validity of solutions at large times, and develop approximations which allow us to study environmental properties. We will compare solutions with and without ambient pressure. We will refer to those as zero-ambient pressure (ZAP) and finite-ambient pressure models (FAP), respectively.

For $t \to \infty$, $R_{C}$ goes out indefinately according to the self-similar solution as external pressure is neglected. In reality, the outer pressure will increasingly confine the expansion of the structure and thus $R_{C}$. In the self-similar solution without ambient pressure, $p_{1}$ decreases with $R_{C}^{2} / t^{2} \propto t^{-4/5}$ (Equation (39)) and eventually it will drop below the ambient pressure of the physical medium. As a reference, we define the pressure-equilibration time $t_{p}$ as the time at which the pressure just inside $R_{1}$ equals the ambient (constant) pressure, $p_{0}$. It is given by

$$t_{p} = 1.23 \sqrt{\frac{m v_{w}^{2}}{\rho_{0}}} \times \left[ \frac{\gamma (\gamma + 1)}{\rho_{0}} \right]^{5/4}. \quad (32)$$

In Figure 2, we give the evolution of the basic physical quantities as a function of time for models with parameters typically for AD-, RG-like, and MS-star winds. For our reference models (see Tables 3, 4, and 6), assuming an ideal gas ambient pressure given by temperature $T_{0} = 10^{4}$ K, we obtain in the AD wind case, the RG-like wind, and the MS wind $t_{p} = 7.45 \times 10^{5}$ year, $t_{p} = 2.36 \times 10^{5}$ year, and $t_{p} = 124$ day, respectively.

As an extreme case and benchmark for modifications, we use an MS wind with parameters similar to the Sun and an evolutionary time $t = 4 \times 10^{7}$ yr (see the reference model in Table 6). $t_{p}$ is some 124 years only, i.e., smaller than $t$ by a factor of $3 \times 10^{5}$. In the ZAP model, the contact discontinuity $R_{C}$ and the location of the reversed shock are about 23 and 20 yr, respectively. The solar wind has similar properties but the termination shock is at about 75–95 AU based on Voyager 1 (Shiga 2007). The discrepancies can be understood due to the ambient pressure not being taken into account. Moreover, for times much larger than $t_{p}$, $R_{C} \ll c_{i}$, where $c_{i}$ is the ambient sound speed. We would therefore expect turbulent instabilities which results in mixing. Ignoring ambient pressure for the MS (solar) model results in the contact discontinuity overriding the the heliopause in about 10 years. Therefore, it is imperative that we consider how to account for finite ambient density in order to get realistic solutions.

A first order estimate for the solution may be obtained by stopping the time integration at $t_{p}$ for a model without ambient pressure, and neglecting the further evolution. For $R_{C}$ and $R_{2}$ and with this crude approximation, we obtain the right order of magnitude with $R_{2} = 190$ AU compared to the solar value of 75–95 AU.

In the following, we will construct physically motivated boundary conditions for moderate log($t/t_{p}) < 1 \ldots 3$, and discuss the uncertainties estimated by a comparison between the FAP and zero ambient pressure model (ZAP).

Besides simply truncating the solution at $t_{p}$, there is a way to approximately incorporate the ambient pressure in a way that retains the self similar solution at any time, although with a modified ambient pressure profile. In order to see how, we first notice the Rankine–Hugoniot jump conditions (in shock rest frame):

$$p' = \frac{2 \rho_{0} u_{0}^{2} - (\gamma - 1) p_{0}}{\gamma + 1} \quad (33)$$

$$u' = \frac{2 \gamma \rho_{0} + (\gamma - 1) \rho_{0} u_{0}^{2}}{(\gamma + 1) \rho_{0} u_{0}} \quad (34)$$

$$\rho' = \frac{(\gamma + 1) \rho_{0}^{2} u_{0}^{2}}{2 \rho_{0} + (\gamma - 1) \rho_{0} u_{0}^{2}}. \quad (35)$$

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Numerical results are given for parameters typical for those winds for the FAP model (see the text and Table 1). In addition, we give the velocity dispersion of the shell $\sigma_n$ and the optical depth $\tau$ and equivalent width EW of the NaID line. Our reference model is marked by $^*$. 

\begin{table}[h]
\centering
\begin{tabular}{ccccccccc}
\hline
No. & $n_0$ (cm$^{-3}$) & $v_0$ (km s$^{-1}$) & $M$ (M$_e$ yr$^{-1}$) & $t$ (Myr) & $t_\sigma$ (Myr) & $R_C$ (ly) & $R_1$ (ly) & $R_2$ (ly) \\
\hline
1 & 0.1 & 3000 & $10^{-8}$ & 0.3 & 2.36 & 33.34 & 41.94 & 4.89 \\
2$^*$ & 1.0 & 3000 & $10^{-8}$ & 0.3 & 0.745 & 20.11 & 28.53 & 2.29 \\
3 & 10.0 & 3000 & $10^{-8}$ & 0.3 & 0.236 & 11.39 & 22.02 & 0.98 \\
4 & 1.0 & 3000 & $10^{-9}$ & 0.3 & 0.236 & 11.39 & 22.02 & 0.98 \\
5 & 1.0 & 3000 & $10^{-10}$ & 0.3 & 0.0745 & 5.80 & 20.47 & 0.36 \\
6 & 1.0 & 3000 & $10^{-8}$ & 0.15 & 0.745 & 13.70 & 17.84 & 1.82 \\
7 & 1.0 & 3000 & $10^{-8}$ & 0.4 & 0.745 & 23.44 & 35.07 & 2.50 \\
\hline
\end{tabular}
\caption{Interaction of Winds of an Accretion disk (AD) with a Constant ISM ($s = 0$)}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{cccccccc}
\hline
No. & $u_1$ (km s$^{-1}$) & $u_2$ (km s$^{-1}$) & $\sigma_n$ (km s$^{-1}$) & $n_2$ (cm$^{-3}$) & $\tau_m$ (g cm$^{-2}$) & log($\tau$) & EW$_{NaI}$ (mÅ) \\
\hline
1 & 16.56 & 19.99 & 0.85 & $2.07 \times 10^{-5}$ & $2.60 \times 10^{-6}$ & 1.34 & 157 \\
2$^*$ & 9.45 & 12.06 & 0.62 & $9.45 \times 10^{-5}$ & $1.91 \times 10^{-5}$ & 2.34 & 165 \\
3 & 5.53 & 6.83 & 0.32 & $5.20 \times 10^{-5}$ & $1.71 \times 10^{-4}$ & 3.59 & 192 \\
4 & 5.53 & 6.83 & 0.32 & $5.20 \times 10^{-5}$ & $1.71 \times 10^{-4}$ & 2.59 & 98.4 \\
5 & 4.50 & 3.48 & 0.62 & $3.94 \times 10^{-5}$ & $1.84 \times 10^{-5}$ & 2.33 & 163 \\
6 & 13.34 & 16.42 & 0.76 & $1.50 \times 10^{-4}$ & $1.13 \times 10^{-5}$ & 2.03 & 180 \\
7 & 8.16 & 10.54 & 0.56 & $7.95 \times 10^{-5}$ & $2.42 \times 10^{-5}$ & 2.49 & 157 \\
\hline
\end{tabular}
\caption{Same as Table 3 but for an “RG-like” Wind}
\end{table}

Note. Our Reference Model is marked by $^+$. 

where the 0 subscripts denote pre-shock and the primed variables are post-shock quantities. After applying the substitutions in (4)–(6), we see that the boundary conditions can remain constant with respect to space and time if $p_0$ follows a spacial power law with $r^{-4/3}$ (or, equivalently, a time power-law of $r^{-4/5}$). An
effective pressure power-law environment can be defined by requiring that, at a certain final time \( t_f \), the thermal energy contained within \( R_1(t_f) \) in our effective environment is equal to the thermal energy in the physical, constant ambient density at the same radius. We find it by volume integration along \( R_1(t) \): 
\[
P_{\text{eff}}(t) = \frac{n_0}{\rho_0} t_f^{4/5} \left( \frac{t_f}{t} \right)^{4/5}.
\]
Although \( t_f \) can be thought of as a constant parameter used to define the environment, in practice \( t_f = t \) and \( P_{\text{eff}}(t) = \frac{n_0}{\rho_0} \). However, we note that, as the solution advances forward in time, this means that \( t_f \) and the boundary condition will vary as well, meaning the solution will not be truly “self-similar,” i.e., the morphology changes with time. The solution obtained in this way is, in fact, a series of snapshots of self-similar solutions where \( t_f \) is equal to the instantaneous time \( t \). Using this parameterization, the Buckingham \( \Pi \) theorem gives us the following:
\[
R_C = K_{0C}(\Pi_0) \left( \frac{\dot{m} v_0^2 t_f}{\rho_0} \right)^{1/5}
\]
\[
R_i = K_{0i}(\Pi_0) \left( \frac{\dot{m} v_0^2 t_f^3}{\rho_0} \right)^{1/5}
\]
\[
\Pi_0 = \left( \frac{\dot{m} v_0^2}{\rho_0 (R_t T_0)^{5/2} t_f^2} \right)^{1/5},
\]
where the \( K \)’s are to be determined numerically and the ideal gas relation was used: \( R_t \) is the gas constant divided by the mean molecular weight and \( T_0 \) is the physical ambient, constant temperature. \( T_0 \) will be taken to be \( 10^4 \) K typical for ISM gas (Osterbrock & Ferland 2006). Note that using Equations (38) and (32) (for \( \gamma = 5/3 \)), we have \( \Pi_0 = 2.50(t/t_p)^{-2/5} \).

Table 5
Same as Table 3 but for the Combination of an AD- and “RG-like” Wind (See the Text)

| No. | \( n_0 \) (cm\(^{-3} \)) | \( v_0 \) (km s\(^{-1} \)) | \( \dot{m} \) (M\(_{\odot}\) yr\(^{-1} \)) | \( t \) (Myr) | \( t_f \) (Myr) | \( R_C \) (ly) | \( R_i \) (ly) | \( R_p \) (ly) |
|-----|-----------------|-----------------|-----------------|---------|---------|-------------|-------------|-------------|
| 21  | 1.0             | 300             | \( \times 10^{-7} \)  | 0.30    | 0.247   | 11.68       | 22.17       | 3.21        |
| 22  | 1.0             | 60              | \( \times 10^{-6} \)  | 0.30    | 0.150   | 8.85        | 21.03       | 4.73        |
| 23  | 1.0             | 33              | \( \times 10^{-5} \)  | 0.30    | 0.082   | 6.15        | 20.50       | 3.70        |

| No. | \( u_1 \) (km s\(^{-1} \)) | \( u_c \) (km s\(^{-1} \)) | \( \sigma_u \) (km s\(^{-1} \)) | \( n_2 \) (cm\(^{-3} \)) | \( \tau_\infty \) (g cm\(^{-2} \)) | \( \log(r) \) | \( EW_{\text{NaI}} \) (mÅ) |
|-----|-----------------|-----------------|-----------------|---------|-------------|-------------|----------------|
| 21  | 5.62            | 7.00            | 0.33            | 5.33 \( \times 10^{-3} \) | 1.71 \( \times 10^{-5} \) | 2.57        | 100           |
| 22  | 4.88            | 5.30            | 0.33            | 1.17 \( \times 10^{-1} \) | 1.75 \( \times 10^{-5} \) | 2.59        | 100           |
| 23  | 4.52            | 3.69            | 0.58            | 3.49 \( \times 10^{-1} \) | 1.83 \( \times 10^{-5} \) | 2.36        | 154           |

Figure 2. Structure feature comparisons of ZAP and FAP models for given sets of parameters as a function of \( t/t_p \) for the reference models of the AD-, RG-like, and MS wind (right to left). Dependence of the radii of the outer and inner shock \( R_{1,2} \), of the contact discontinuity \( R_C \), and the ram pressure \( P_{\text{ram}}/\rho_0 \) are shown as a function of the duration of the wind \( t \) normalized to the time \( t_p \) at which the inner and outer pressure are equal. We show the functions for the zero ambient and finite pressure model indicated by the small and large symbols, respectively.
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Table 6
Same as Table 3 but for an “MS-like” Wind for the Solution Including the Ambient Pressure (FAP Models)

| No. | $n_0$ (cm$^{-3}$) | $v_∞$ (km s$^{-1}$) | $m$ ($M_⊙$ yr$^{-1}$) | $t$ (Myr) | $t_p$ (yr) | $R_C$ (ly) | $R_2$ (ly) | $R_{2, mp}$ (ly) |
|-----|-----------------|-------------------|-------------------|---------|--------|--------|--------|-----------|
| 1   | 0.1             | 500               | 10$^{-14}$        | 40      | 393    | 0.921  | 0.00477 | 0.00390   |
| Rem.|                 |                   |                   |         |        |        |        |           |
| 2+  |                 |                   |                   |         |        |        |        |           |
| 3   |                 |                   |                   |         |        |        |        |           |
| Rem.|                 |                   |                   |         |        |        |        |           |
| 4   |                 |                   |                   |         |        |        |        |           |
| 5   |                 |                   |                   |         |        |        |        |           |
| 6   |                 |                   |                   |         |        |        |        |           |
| 7   |                 |                   |                   |         |        |        |        |           |
| 8   |                 |                   |                   |         |        |        |        |           |
| 9   |                 |                   |                   |         |        |        |        |           |
| 10  |                 |                   |                   |         |        |        |        |           |
| 11  |                 |                   |                   |         |        |        |        |           |
| 12  |                 |                   |                   |         |        |        |        |           |

Note. Our reference model is marked by +. Values are given for the radius of the contact discontinuity and the inner shock using parameters typical for the wind of a main sequence star, and using the solution including the ambient pressure (FAP models). For times $t$ much larger than $t_p$, the location of the reversed shock becomes stationary and close to the distance $R_{2, mp}$ expected from equilibrating the pressure at the reversed shock with the ambient medium. In addition, models without outer pressure are given and marked by “Rem.” This shows the importance of the pressure term for $t/t_p \gg 1$ in the extreme case. However, for winds relevant in SNe, the difference is of the order of several% (see the text, and Figure 2).

The outer boundary conditions of the ODE’s are then given as

$$\chi_i = \frac{(2\gamma - (\gamma - 1)/M^2)(2/M^2 + \gamma - 1)}{\lambda^2(\gamma + 1)^2}$$ (39)

$$P_i = \frac{(2\gamma - (\gamma - 1)/M^2)}{\lambda^2(\gamma + 1)}$$ (40)

$$U_i = 2\frac{1 - 1/M^2}{\lambda(\gamma + 1)}$$ (41)

where the Mach number $M$ is given by:

$$M = \frac{R(t)}{\lambda} \sqrt{\frac{9\rho}{5(m/\rho_0)}}$$ (42)

An initial guess of $R_1$ is required in order to numerically solve the ODE’s and obtain the structure, therefore iteration is necessary in order to obtain a consistent solution. Following the method of Weaver et al. (1977) using our modified boundary conditions, we obtain the following expressions for the radii and inner structure profile:

$$R_2(t) = 1.13(\beta \alpha^{3/2})^{3/10} v_w^{1/10} t^{7/5}$$ (43)

$$R_1(t) = \alpha/2^{3/5} \left(\frac{m v_w^2}{\rho_0}\right)^{1/5} t^{3/5}$$ (44)

$$R_{\infty}(t) = \beta R_1$$ (45)

$$u(r, t) = \frac{11}{25} R_2^3 + \frac{r}{25t}$$ (46)

$$p(r, t) = \frac{5}{22\pi(\beta \alpha)^3} \left(1 + m/4\rho_0 v_w^2 \right)^{1/5} t^{-4/5}$$ (47)

$$\rho(r, t) = \frac{0.274}{(\alpha \beta)^{3/5}} \left(\frac{m^2 \rho_0^3}{v_w^6}\right)^{1/5} t^{-4/5} \left(1 - \frac{r^3}{R_C^3}\right)^{8/33}$$ (48)

where $\alpha = \frac{1}{\beta} \left(\frac{5}{2\pi \rho_0^3}\right)^{1/5}$, $\beta = \left(\frac{\eta}{\rho_0}\right)^{-1/3}$, and $P_c^+$ and $\eta_c$ are evaluated at the contact discontinuity. Comparison of Equations (44) and (45) with Equations (37) and (36) give $K_{\alpha 1} = \frac{\alpha}{2\pi}$ and $K_{\alpha \infty} = \frac{3\alpha}{2\pi}$. We can then define a proportionality for the reverse shock:

$$R_2 = K_{\alpha 2}(\Pi_0) \left(\frac{m}{\rho_0}\right)^{3/10} v_w^{1/10} t^{2/5}$$ (49)

where

$$K_{\alpha 2} = 1.39K_{\alpha \infty}^{3/2}.$$ (50)

For $\rho_0 \to 0$, $M \to \infty$, $\alpha \to 0.88$, $\beta \to 0.86$, and Equations (15)–(20) from the last section are reproduced. For the reference models for an AD-, RG-like and MS-wind, a comparison of the basic properties between the ZAP and FAP models as a function of $t/t_p$ is shown in Figure 2. Qualitatively, the main differences are as follows: for FAP models, $R_2$ and $R_{\infty}$ are smaller and $R_1$ is larger than for ZAPs, and $R_2$ goes to a constant value for large $t/t_p$. For the range shown and for the RG-like wind, $R_2$ becomes larger than $R_{\infty}$ at about $t/t_p \approx -0.5$, marking the regime of “unphysical” solutions already discussed above for the ZAP model. The functional relations appear to be similar, and in fact they are identical as a consequence of the II theorem. The relative shifts between the various quantities are given by proportionality factors which, in turn, depend only on the basic parameters, namely $v_w$, $m$ and $n_0$. For the FAP model, the proportionality constants have to be determined numerically (Figure 3). A further consequence of the II theorem is that the differences are only a function of $t/t_p$ and do not depend individually on $m$, $v_w$ and $n_0$ (Figure 4). For a constant density medium, the characteristic parameters can be directly obtained using Figure 3.

The detailed solutions for our reference models are shown in Figures 5 and 6. The morphology of the envelopes does not
change for a wide range of parameters and time. As discussed above in case of the MS star wind, however, we must expect strong mixing for $t/\tau_p \gg 1$ in FAP models. The solution becomes “un-physical” for $r \lesssim R_C$ in the regime of a weak shock.

2.3. Existence of Solutions

Here, we want to provide the range for which self-similar solutions exist using the Π theorem.

Case ($s = 0$): for FAP and $s = 0$, there is one Π group given by Equation (38). Solutions do not exist for Mach numbers less than 1.3849.

Case ($s = 2$): two Π groups exist and thus possible solutions are a combination of Π groups with $K_{\Pi} (\Pi_m, \Pi_n)$ given by Equations (25) and (26). For $r^{-2}$ ambient density profiles, the shell velocity range is sufficient to determine the relation between $\dot{m}_i$ and $v_{\infty,i}$ for the prior mass loss. In Figures 7–9, we show $K_{\Pi_{\infty}}$, $K_{\Pi_1}$ and $K_{\Pi_2}$ as a function of the wind parameters covering the entire range discussed in this paper.

Regime I: for high mass loss rates, we have no power law relation between the wind and environmental parameters, and the values of $K_{\Pi_{\infty}}$, $K_{\Pi_1}$ and $K_{\Pi_2}$ need to be interpolated in the figures or can be calculated by SPICE.

Regime II: if a low mass loss wind runs into a high mass loss wind, $\Pi_{\infty} < 0.1$, $K_{\Pi_{\infty}}$, $K_{\Pi_1}$ and $K_{\Pi_2}$ hardly vary with the ratio $\dot{m}_i/\dot{m}_1$. Thus, the contour lines in Figures 7–9 are horizontal. Their value can be approximated therefore as a function of only the relative wind velocities. In Figure 10, the variation of the cuts for various $\Pi_{\infty}$, $K_{\Pi_{\infty}}$ and $K_{\Pi_1}$ can be well described by single functions. $K_{\Pi_2}$ needs two descriptions valid at low and high ratios of $v_{\infty}/v_{\infty,1}$ separated at $\log \Pi_{\infty} \approx 0.5$. The resulting power law dependencies are given in Table 2.

3. APPLICATIONS: ENVIRONMENTS OF SNE IA PROGENITORS

We will explore winds emanating from the progenitor system and interacting with the ISM of mass loss of the system prior to the supernova explosion. We consider winds from each source separately, and we address the question of which component is mostly responsible for the formation of the environment, and the typical structure to be expected. Subsequently, we discuss the link between observables and progenitor systems, and analyze SN2014J.

We employ our spherical, semi-analytical models constructed by piecewise, scale-free analytic solutions. Scales enter the system via the equation of state, the boundary, and the jump conditions. The free parameters are: (1) the velocity $v_{\infty}$, (2) mass loss rate $\dot{m}$ from the central object, and the (3) $n_0 = \text{const}$ or a mass loss rate $\dot{m}_1$ with $v_{\infty,1}$, i.e., $n \propto r^{-s}$, and (4) the duration of the wind interaction $t$. As a result, we obtain the density, velocity, and pressure as a function of time, namely $p(r, t)$, $\rho(r, t)$, and $p(r, t)$ which can be linked to observables. We use typical parameters to discuss the different regimes which may occur in nature. For actual fits of observations, appropriate solutions can be constructed by tuning these parameters with SPICE.

The wind may originate from the AD, the donor star which may be an MS or an RG-, horizontal- and asymptotic-branch star, the WD during a phase of over-Eddington accretion, or a combination of AD with a RG-like wind. As shown below, the timescale for the accretion and thus the progenitor, is an important factor in formation of the environment. The timescales $t$ vary widely depending on the scenario and chemical composition of the accreted material and the initial mass of the progenitor (e.g., Sugimoto & Nomoto 1980; Piersanti et al. 2003; Wang & Han 2012 and reviews cited in the introduction). To reach $M_{\text{Ch}}$, about 0.2 to 0.8 $M_\odot$ of material needs to be accreted. For hydrogen accretion, the rates for stable hydrogen burning are between $M \approx 2 \times 10^{-8}$–$10^{-6} M_\odot$ year$^{-1}$ depending on the metallicity (Nomoto 1982b; Hachisu et al. 2010). The upper and lower limits for $M$ are set by the Eddington limit for the luminosity and the minimum amount of fuel needed for steady burning, respectively.

For a recent review, see Maoz et al. (2014). In this study, we use the wide range of accretion rates to avoid restricting possible solutions. Thus, we consider timescales $t$ between $10^5$ and $4 \times 10^7$ years. Larger rates of mass overflow result in over-Eddington luminosity and a strong wind from the progenitor WD with properties typical of RG winds (Hachisu et al. 1996, 2008). Subsequently, we refer to the high-density, low velocity winds as “RG-like.” Accretion of He and C/O-rich matter allow much shorter timescales down to the dynamical times of merging WDs.

For AD winds the mass loss rate ranges from $10^{-8}$ to $10^{-10}$ solar masses per year; and the wind velocities originating from the disks are believed to be from $2000$ to $5000$ km s$^{-1}$ (Kafka & H Nesscutt 2004). Mass loss in “RG-like” stars is typically between $10^{-6}$ and $10^{-8} M_\odot$ yr$^{-1}$ with wind velocities between 10 and 60 km s$^{-1}$ (Reimers 1977; Judge & Stencil 1991; Ramstedt et al. 2009).

MS star winds are similar to the solar wind (Wood et al. 2002). For solar wind the velocity is between 400 and 750 km s$^{-1}$ and the mass loss is $2.3 \times 10^{-14} M_\odot$ yr$^{-1}$ (Noci et al. 1997; Feldman et al. 2005; Marsch 2006).
Figure 4. Fractional difference in $R_C$ between models of zero ambient pressure ($R_{C,p}$) and finite pressure models ($R_{C,p}$) in the parameter space of the mass loss $\dot{m}$, the wind velocity $v_w$, and the environment density $n_0$ as a function of $\log(t/t_p)$. The plots visualize the (II-theorem) as discussed in Section 3: the difference depends on $(t/t_p)$ only. The ratios between scale-free variables are constant throughout the parameter space. The II-theorem applies also to $R_1$. In the lower right, we show the fractional difference of $R_C$ (red), $R_1$ (blue), and $R_2$ (magenta) between the ZAP and FAP models as a function of $\log(t/t_p)$.

Figure 5. Structure of model 3 for AD wind (see Table 3). We show the ZAP (left) with $t = 3 \times 10^5$ yr and at $t = t_p$ (middle), and the FAP model at $t = 3 \times 10^5$ yr. The overall structure is similar within the parameter range.

Figure 6. Same as Figure 5 but for RG-like winds (see Table 4).
has been measured to be $10^{-9} M_\odot \text{yr}^{-1}$. A wind velocity of 3000 km s$^{-1}$ is used. For the duration of the winds, we consider $r$ between $1.5 \times 10^5$ and $6 \times 10^5$ years, with $3 \times 10^5$ years for the references.

The outer environment of the system depends on its history including the delay time between the formation of the WD and the onset of the accretion phase. For long delays, we assume an ISM with constant density, i.e., $s = 0$. For short delay times and small peculiar velocities of the system, the outer environment may be created during the final stage of the progenitor evolution, namely the red giant branch (RGB), horizontal giant branch (HGB), and the asymptotic giant branch (AGB) phase.

### 3.2. Results

#### 3.2.1. Case I: Constant ISM Density

We first consider scenarios where the wind blows out into a medium of constant density for a wide range of parameters (Tables 3–5). The structures are characterized by (I) an undisturbed, inner layer dominated by the stellar wind, (II) an inner, shocked region with almost constant, low density, and a velocity declining with distance, (III) a slowly expanding shell of high density “swept up” material, and (IV) the ISM. The overall solution is representative for all cases as has been shown in Section 2. For the estimate of the equivalent width EW of the NaI doublet at $\lambda = 5890/5896$ Å, we use solar abundances, $X_{\text{Na}} = 3.34 \times 10^{-5}$. EW is estimated according to Spitzer (1968) and Draine (2011). For potassium lines, the corresponding equations apply.

**Case Ia:** Fast winds from an AD: Table 3 contains calculated results from several cases with different parameters but the same timescales $t$. Our reference, model 2 of Table 3, is shown in Figure 11. It has a mass loss $\dot{m}$ of $10^{-8} M_\odot \text{yr}^{-1}$ and a wind velocity $v_w = 3,000$ km s$^{-1}$. For the duration of the wind, we choose a duration of $3 \times 10^5$ yr which, within the SD scenario, corresponds to the evolutionary time for a low mass WD to grow to $M_{\text{Ch}}$ at an accretion rate of $\approx 2 \times 10^{-5} M_\odot \text{yr}^{-1}$. Up to about 2.3 ly, the environment is dominated by the ongoing wind. In this region, the particle density drops below $100$ cm$^{-3}$ at a distance of $0.0005(0.0005)$ lys $\approx 1000$ (100) AU, $\approx 10^{16}(15)$ cm, which will be overrun by the SN ejecta within 5(0.5) days. Particle densities below 100 cm$^{-3}$ in this region will hardly affect the LCs or spectra because the swept up mass will be small. Using the same argument, the low densities within 20 ly are too low to affect the hydrodynamics of the SN envelope. A high density outer shell expands at a velocity of 11 km s$^{-1}$ with a velocity dispersion of $\approx 13\%$. This shell would produce a narrow line Doppler shifted by about 11 km s$^{-1}$. For an ISM, the equivalent width would be about 165 mÅ for the NaID line, well comparable to values found by (Phillips et al. 2013), who found between 27 and 441 mÅ in a sample of some 20 SNe Ia.

The morphologies of the structures are hardly affected by variations in the wind parameters, as expected from the P theory. However, the actual size of the regions and the densities vary (see Table 3) with dependencies as expected from Table 1. Within the framework of SNe Ia, the ram pressure dominates the ambient pressure, which therefore hardly affects the solution. Typically, wind from an AD will produce an inner cavity between 5 and 30 lys surrounded by an expanding shell with a velocity of $\approx 5...20$ km s$^{-1}$. EW (NaID)
is $\approx 100$–200 mA. The combination of line shifts and strength allows us to derive the wind parameters.

In our table, we assumed an accretion rate of $\approx 2 \times 10^{-6} M_\odot$ yr$^{-1}$, which is at the upper limit allowed for stable accretion of hydrogen rich matter. Higher mass loss can be expected for over-Eddington accretion or He or C/O accreting WDs (see Section 1). Higher and lower mass loss rates will increase/decrease the size of the cavity but the dependency is relatively weak, $\propto m^{1/3}$.

However, the actual size of the region does depend sensitively on the time $t$ of the progenitor evolution as the size of the cavity goes like $\propto t^{1/5}$. In hydrogen accreters, the rate of accretion may be smaller by a factor of 100 and thus $t$ may be larger by the same factor, which in turn will increase the size of the cavity by 16 and decrease shell velocities by about a factor of 6. Shorter durations can be realized if we start with a WD of $1.2 M_\odot$, the upper end of the mass range for a C/O WD (see Introduction). Indeed, there is some evidence and theoretical arguments that the progenitors originate close to the upper end of that mass range (Hoeflich 2006; Nomoto et al. 2007; Sadler 2012) which may reduce the amount of accreted material from $\approx 0.8$ to $0.2 M_\odot$. Thus, the duration of the

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**Figure 10.** $K_{c(1,2)}$ as a function of the ratio between inner and outer wind velocity described by $\Pi_{w,\text{in}}$ as obtained by cuts at $\log(\Pi_{w,\text{in}})$ of $10^{-4}$, $10^{-2}$, $10^{-1}$, indicated by progressively thicker lines. The exact solutions are given in red. We give the solutions for $K$ in comparison to the fits. The approximations are given (magenta and blue, dotted).

**Figure 11.** Hydrodynamic profile for a wind typical for an accretion disk (left), $v_w = 3000$ km s$^{-1}$ and mass-loss rate of $10^{-8} M_\odot$ yr$^{-1}$, and a RG-like wind (right), $30$ km s$^{-1}$ and mass-loss rate of $10^{-7} M_\odot$ yr$^{-1}$, running into a constant interstellar medium density of 1 particle per cm$^3$ after a time of 300,000 years. The contact discontinuity is at 21.7 and 5.45 light-years, respectively. Fluid velocity $u$ (magenta) is normalized to 100 km s$^{-1}$, pressure $p$ (blue) is normalized to the pressure $p_1$ just inside the outer shock, and particle density $n$ is unnormalized.
accretion $t$ may be correspondingly shorter, which in turn reduces the size of the cocoon and increases the shell velocities by ≈2 and 1.5, respectively.

On the other hand, rates for He and C/O accretion can be larger than hydrogen accreters by at least the same factor of 100 (Whelan & Iben 1973; Pieri et al. 2003; Wang et al. 2009a, 2009b), reducing the size of the shell and increasing the velocity of the shell lines by the same factors. High shell velocities may indicate He- or C/O accreters. Despite the line shifts, systems with high shell velocities can be expected to have smaller low density regions. The SN ejecta have velocities up to about 10%–20% of the speed of light. For such SNe Ia, we may expect interaction on timescales from 1 to 10 years for our set of parameters.

**Case Ib:** MS star winds: MS star winds produced by a donor star are expected to show low velocities $v_w = 500$ km s$^{-1}$ and a very low mass loss $m = 10^{-15}$ $M_\odot$ yr$^{-1}$. Although the morphologies of the shells remain the same as above, the corresponding regions will be overrun and dominated by wind from the AD. The radius of the reverse shock $R_2$ show little change for timescales much longer than $t_p$ (see Table 6). As discussed above, $R_C$ becomes unphysical due to mixing in a regime of weak shocks. For SNe Ia, the MS wind component can be neglected.

**Case Ic:** Slow, RG-like winds may be produced during the RG phase of the donor star or the progenitor prior to the formation of the progenitor WD, or as a result of over-Eddington mass overflow (Table 4). We refer to those as RG-like winds. The resulting structures are very similar to Case Ia but the densities are higher by an order of magnitude (Figure 11). In particular, the density contrast $n_2/n_0 \approx 0.1$ (see also Table 1). The cocoons are smaller and less pronounced in this case.

For typical properties of the environment, we have to distinguish between winds for an RG donor, RG-like winds from over-Eddington accretion, and the prior RG-phase of the progenitor.

In Table 4, models are shown for RG-like winds for various $n_0$, $m$ and $t$. As expected, the size of the cavity decreases with $n_0$ (model 8 versus 9). We choose $t$ in the range for stable hydrogen accreters (models 8–14). As duration $t$ increases, the location of $R_2$ “stalls” and $n_2$ is hardly affected because the outer and inner pressure equiliibrates ($t/t_p \gg 1$) as discussed in the Section 2.2. See also Table 1. For the same reason, the size of the cavity increases with $m$ but $n_2/n_0$ hardly changes. Typically, RG-like winds will produce an inner cavity between 1 and 10 lys surrounded by an expanding shell with a velocity of $\approx 5$ km s$^{-1}$. EW (NaI D) is a few hundred m Å. Note that the scales of RG winds are smaller by an order of magnitude compared to those of AD winds unless those have much lower mass loss than considered in our example above. An RG wind is more likely to form a combined AD-RG wind as discussed below.

RG winds from the progenitor prior to the WD phase (models 15–20) will produce a similar structure as an RG donor in a system with high accretion rates ($\approx 10^{-6}$ $M_\odot$ yr$^{-1}$) but are systematically larger because the longer duration $t$. This wind may form an environment for subsequent winds. Progenitor system winds may run into this environment if the delay time between the formation of the WD and the onset of the accretion is sufficiently short, and if the peculiar velocity of the system is small. The peculiar velocities of stars show a wide range with a typical value of 25–50 km s$^{-1}$ for the Galactic plane (Griv et al. 2009, and references therein). Here, the duration of the wind is given by the evolutionary timescale of the RG phase rather than the time to reach $M_{\rm n, WD}$. For models 15–18, we use $t$ corresponding to the post-main sequence lifetime of 5 and 8 $M_\odot$ stars with a mass loss rates of $10^{-7}$ $M_\odot$. The resulting total mass loss is 1.3 and 0.3 $M_\odot$ for the 5 and 8 $M_\odot$ stars, respectively (Schaller et al. 1992; Chieffi et al. 2001).

The environment formed by a wind consists of inner region with $s \approx 2$ with a size $R_2$ and a low density void ($s \approx 0$) of size $R_C$. The resulting size of the void, $R_C$, is typically 10–20 lys. The duration $t$ can be increased by lower main sequence masses for the progenitor. However, then, the amount of mass lost prior to forming a WD of $M(\rm WD) \approx 0.6$ $M_\odot$. Using a mass loss of $10^{-7}$ $M_\odot$ and $v_w = 30$ km s$^{-1}$, the maxima in $R_2$ and $R_C$ are produced by a 3.6 $M_\odot$ star: $\approx 33$ lys and $\approx 3.7$ lys, respectively (models 19, 20, Table 4). Models with durations of 3, 13, and 50 Myr correspond to progenitor stars of 8, 5, and 3.6 $M_\odot$.

We note that long durations may also be produced during the evolution of the progenitor system if the hydrogen accretion rate is close to the lower limit of $\approx 10^{-8}$ $M_\odot$ yr$^{-1}$, though this low of a rate may not allow for stable accretion for WDs close to $M_{\rm n, WD}$ (Piersi et al. 2003). Models with RG-like winds may correspond to systems with over-Eddington accretion and an MS donor star (models 15–20). For those long duration RG-like winds, the resulting cavities can be $\approx 15$ to 33 lys and high EW of $\approx 500$ mÅ.

**Case Id:** Fast wind from an AD combined with mass loss from RG donor star or super-Eddington accretion: if we have a system with both a dense RG donor wind and an AD wind, the two will combine. The inner interaction region will be RT unstable and mix fast (Figure 12). They can be expected to
form a uniform wind with an acceleration region. Assuming momentum conservation and our reference models for the AD wind and “RG-like” winds, we obtain a total mass loss rate of $\approx 10^{-5} - 10^{-6} \, M_\odot$ and wind velocities of $\approx 33, 60,$ and $300 \, \text{km s}^{-1}$ (see models 21–23, Table 5).

This case applies in a system with high peculiar velocity. There, the system has moved out of the environment formed during the stellar evolution of the progenitor. The cavities are somewhat larger by about a factor of 2 compared to the RG-like wind cavities, surrounded by an expanding shell of a similar velocity and EW (NaID). The larger cavity will result in a slower evolution of the narrow lines.

**Extremely low Density Environments**: In case of long delay times of SNe Ia, the progenitor system may have wandered away from the galactic disk into the halo, in a regime of very low densities Graham et al. (2015a), or in a hot-ISM Fesen et al. (2007), or in the constant density region II of the cavity from prior mass loss. In Table 7, some examples are given for constant density with particle densities between $10^{-2}$ and $10^{-3} \, \text{cm}^{-3}$. Most likely, the donor star will not be a RG star. The inner, low-density region is expected to be of the order of $\approx 100 \, \text{ly}$. EW (NaID) are between 20 to 50 mA and show increased Doppler shifts of $\approx 30-50 \, \text{km s}^{-1}$ or larger for even lower densities than found in the vicinity of SN1885 Fesen et al. (2007).

In conclusion, of all wind components analyzed separately, the AD wind component will dominate the formation of cocoons.
3.2.2. Case II: Environments Produced by Winds

Now we consider the scenario where the wind of the progenitor system runs into a nearby environment produced by a prior mass loss ($s = 2$). “Nearby” means that the reverse shock $R_2$ produced by the prior mass loss must be larger than the size of the region produced by the wind of the system. Otherwise the interaction region will move into a region of constant density, shocked RG wind, or the ISM. Moreover, the speed of the ongoing wind must exceed that of the outer wind. For our discussion, we disregard the effect of prior mass loss by a MS wind because it produces on an cocoon of pressure $p$ just inside the outer shock, and particle density $n$ is normalized to $10^{-6} \text{ cm}^{-3}$. Note that the particle density normalization used is much smaller than in the constant density case.

**Case Ia:** Fast wind from an AD and a non-RG donor star: If the donor star is a compact object like an MS or He-star, the AD wind will dominate. Examples are shown in Table 8. As a reference, some typical parameters are $\dot{m}_1 = 10^{-6} \text{M}_\odot \text{yr}^{-1}$, $v_{w,1} = 30 \text{ km s}^{-1}$, $\dot{m}_2 = 10^{-10} \text{M}_\odot \text{yr}^{-1}$, $v_w = 3000 \text{ km s}^{-1}$, and a total run time of $1.5 \times 10^5$ yr. Again, we can identify the different zones as above (Figure 13). The density contrast between the inner bubble within $R_2$ is smaller than in the constant density case, i.e., a factor of $10^{-2}$, but the density at $R_C$ is significantly lower than the ISM. This results in an “ultralow” density bubble of $n \approx 4 \times 10^{-8} \text{ cm}^{-3}$.

Moreover, this solution shows a qualitatively different feature compared to the constant density ISM: we see a very thin and dense shell with particle densities $\lesssim 10^2 \text{ cm}^{-3}$ and a thickness of light weeks to months (Figure 13). The resulting shell produces a narrow, optically thick NaID line with a small equivalent width (EW) of $0.24 \text{ mÅ}$, Doppler shifted by about $u_c = 240 \text{ km s}^{-1}$ and a width of $\approx 23 \text{ km s}^{-1}$. We note that, to first order, EW $\propto \dot{m}$ (model 1, Table 8).

**Case Ib:** Fast wind from an AD combines with a “RG-like” wind: if we have a system with a dense RG donor and wind plus an AD wind, they would form a uniform wind with an acceleration region (Figure 14). We use the same parameters as in Section 3.2.1 but omit the mixed wind with the highest mass loss because its velocity will be comparable to the wind velocity of the surrounding medium. As expected, the cavities are smaller than the AD-wind case, and EW(NaID) and its Doppler shift are larger.

Comparing interaction regions of environments created by winds with constant density ISM, the typical scales are larger in the former and the resulting narrow lines show a larger blueshift with smaller EW.

### 3.3. Application to SN2014J

We will now explore the application of our semi-analytic models to the case of SN2014J.

**Background:** This object was discovered in 2014 January in a high density region of the nearby M82. The higher than average ISM densities and the small proximity of the host make SN2014J an excellent candidate for investigating the
interaction of the SN with the environment. This gives us a premier opportunity to implement our models and make predictions.

Observational constraints for the environment and progenitor are obtained from searches for X-rays (Margutti et al. 2014), time-dependence of high-resolution spectra of narrow potassium lines formed in the environment (Graham et al. 2015b), IR imaging (Kelly et al. 2014), and radio (Pérez-Torres et al. 2014). X-rays and radio provided the most stringent constraints on the average density of ions in the environment. In the case of SN 2014J, X-rays and radio luminosities at maximum light probe the wind from the progenitor system (region I in Figure 1).

When the SN material propagates into the circumstellar surrounding, a shock is formed and leads to the acceleration of partially or fully relativistic electrons with a power-law distribution $n_e(\gamma) = n_0\gamma^{-p}$, with $p$ between 2 and 3, which produce X-ray and radio emission. From radio observations of SNe Ib/c, Chevalier & Fransson (2006) find $p \approx 3$. Because the outer layers of an SNe Ia and SN Ib/c have similar structure, abundances, and velocities, we use this value in the following.

For X-rays (Margutti et al. 2012), $L_\nu$ is given by

$$L_\nu = 16500 \epsilon_X^2 \left( \frac{\dot{m}/v_w}{M_\odot \text{yr}^{-1} \text{km}^{-1} \text{s}} \right)^{0.64} \times \left( \frac{t}{\text{days}} \right)^{-1.36} \nu^{-1} \left( \frac{L_{\text{bol}}}{\text{erg s}^{-1}} \right) \text{erg Hz}.$$  (51)

For the radio (Pérez-Torres et al. 2014), $L_\nu$ is given by

$$L_\nu = 5.81 \times 10^{-9} \epsilon_R^{1.71} \epsilon_B^{1.07} \left( \frac{\dot{m}/v_w}{M_\odot \text{yr}^{-1} \text{km}^{-1} \text{s}} \right)^{1.37} \times \left( \frac{t}{\text{days}} \right)^{-1.55} T_{\text{bright}} \nu^{-1} \text{erg Hz}.$$  (52)

The parameters are the fraction of relativistic electrons $\epsilon_e$, the energy fraction in the magnetic field $\epsilon_B$, and the brightness temperature $T_{\text{bright}}$, which, based on observations, can be expected to be $\approx 10^{11}$K (Readhead 1994). Margutti et al. (2014) and Pérez-Torres et al. (2014) use a value of 0.1 for $\epsilon_e$ and $\epsilon_B$.

If the supernova shell runs into a constant density environment, the X-ray luminosity is given by

$$L_\nu = 6.44 \times 10^{-5} \epsilon_X^2 \left( \frac{n_0}{\text{cm}^{-3}} \right)^{0.5} \left( \frac{t}{d} \right)^{-0.45} \nu^{-1} \left( \frac{L_{\text{bol}}}{\text{erg s}^{-1}} \right) \text{erg Hz}.$$  (53)

For late times when the SN shock runs into region II (Figure 1), the radio luminosity is given by

$$L_\nu \propto T_{\text{bright}}^{0.86} \epsilon_X^{1.07} \epsilon_B^{1.27} n_{\text{ISM}}^{0.88}.$$  (54)

In the case of SN2014J running into a constant density medium and from radio observations, Pérez-Torres et al. (2014) found an upper limit $n_0 \lesssim 1.3$ particles per cm$^3$. The same luminosity can be produced in a wind with $\dot{m} \lesssim 7.0 \times 10^{-10} \times v_w/(100 \text{ km s}^{-1})M_\odot \text{yr}^{-1}$. The neutral lines and the IR emission indicate shells at distances between 10 and 20 ly expanding at velocities between 120 and 140 km s$^{-1}$. Margutti et al. (2014) and Pérez-Torres et al. (2014) concluded from their X- and radio observations that a DD system was the likely progenitor. However the findings of (Graham et al. 2015b) led them to favor a SD progenitor, although excluding an RG as donor.

We note that region II has a constant density. Thus, we expect the radio luminosity to increase with time. Densities in our models of region II (see Figure 1) are two to four orders of magnitude smaller than limits discussed above. However, radio must be expected when the SN shockfront hits the shell after 10–100 years after the explosion.

In light of the different conclusions, we will apply our analytic description to explore the wide range of parameters within the observational limits from the radio- and X-rays. For the outer environment, we consider both an ISM with constant density, $s = 0$, and one consisting of a wind, $s = 2$, produced during the stellar evolution history.

The X-ray and radio observations provide limits on the far-inside region (region I) within the contact discontinuity (Figures 11 and 13). In order to apply this constraint to our parameter space we will consider our three cases: RG-like wind, MS star wind, and wind originating from the AD. Case I: if the environment is produced by an ongoing RG-like wind with a velocity of 30 km s$^{-1}$, the mass loss would have to be $\dot{m} \lesssim 2.1 \times 10^{-10} M_\odot \text{yr}^{-1}$, which is much too low for RG wind. Ongoing RG-like wind from the donor star is also very unlikely to be consistent with the IR imaging (Kelly et al. 2014). If the environment is formed by a RG-like wind, it must originate from the progenitor star prior to the formation of the WD. Case II: if the wind originates from a MS star with 500 km s$^{-1}$, radio limits would allow $\dot{m} \lesssim 3.5 \times 10^{-9} M_\odot \text{yr}^{-1}$. Case III: for an AD wind with 3000 km s$^{-1}$, this means $\dot{m} \lesssim 2.1 \times 10^{-8} M_\odot \text{yr}^{-1}$, well within limits discussed in the introduction.

Narrow circumstellar lines have also been used to probe the environment close to the contact discontinuity (e.g., Figure 1). Using ionization models, Graham et al. (2015b) attributed the blueshifted KI and NaI absorption features to circumstellar shells at ~10–20 ly at velocities of ~120–140 km s$^{-1}$. A further reduction of the parameter space comes from the limits on X-rays and radio. Using our models, we can infer what kind of progenitor wind (and system) could have produced such shells expanding at the proper velocity and distance with the duration of the wind as a free parameter.

**Analysis of SN2014J:** In the following, we want to apply our analysis by combining the tools of the last section with the observations of SN2014J. For finding allowed parameters, we use the given distance and velocity of the shells in combination with wind parameters for the AD-, MS-, RG-like and “RG-like plus AD” winds.

(1) As a first step, we want to determine the allowed duration of the wind $t$ using the shell velocities and distance as given by (Graham et al. 2015b). Assuming the shells are at $R_C$ and the expansion velocity at $R_C$, we can use Equation (8) to estimate the allowed range for the duration of the wind (Figure 15).

(2) In a second step, we make use of the II-theorem to determine combinations of $n_0$, $\dot{m}$, and $v_w$. Because the solutions depend on one and two II groups for constant ISM and wind-environments, respectively, we have to consider two cases.

Case I: For the constant density environment ($s = 0$), the solutions can be described in the II space by a unique parameter $\mu = \dot{m}v_w/n_0$. We have to find $\mu$ from the observations and step one as described above (Figure 16). Assuming a particle density of the ISM between 0.1 and 10 cm$^{-3}$, the allowed range for $s = 0$ in the $M$–$v_w$-space is shown in
and $R_c$ gives the ratio between the ongoing and the outer wind velocity. We give the lines of constant mass loss rates and velocities of the interacting winds. Possible solutions are linear in time (Equation (8)). Thus, $K_C = R_C/(\mu v_w)$, Possible solutions depend only on the shell velocity, which can be measured directly from the spectra Graham et al. (2015b). Note that the velocity $R_C$ equals the matter velocity. Because it varies slowly in this region (see Figure 13), the Doppler shift of lines is a good measure of $R_C$.

Knowing $v_w$, the original wind velocity is given for any $\Pi_{v_w}$. $\Pi_{v_w}$ gives the ratio between the ongoing and the outer wind mass loss rates. We assume that the progenitor wind has the parameters given by a RG star as discussed in Section 3. To solve for the implicit condition imposed given by the observations and possible model solutions, a standard rejection technique is applied. We use a Monte Carlo Scheme to sample the parameter space of the ongoing wind for acceptable solutions for $\Pi_{v_w}$. For a given wind parameter of the outer wind, we test each position in Figure 7 (or the lookup table) for whether the resulting wind velocity $v_w$ is consistent with an RG wind. If it is, we accept the possible solution. The lines of solutions found in this manner are shown in Figure 17. The line widths are given by the range of RG wind properties accepted to be consistent with a solution.

(3) In a third step, finally, we compare the allowed range in parameter space with the ranges consistent with our three cases. The ranges of $\dot{m}$ and $v_w$ for each type of wind is presented as a box.

A comparison between the predicted ranges for various winds and the allowed range for SN2014J constrains the possible progenitor properties. In addition, we marked the parameter range excluded from the lack of X-rays and radio detections. The possible wind parameters for SN 2014J and the location of typical RG, MS, and AD winds are shown in Figure 17. In the figure, we use the quantity $\delta = \log_{10}(\dot{m}/v_w)$ as an abbreviation. In addition, we show the allowed region for an environment produced by an RG wind of the progenitor prior to the WD formation.

For constant ISM environments (Case I), there is obviously no overlap between regions allowed from the observations and...
the parameter space of solutions. Constant particle density environments ranging from 0.1 to 10 cm$^{-3}$ can be ruled out. However, a very low density ISM of \( \lesssim 0.01 \) cm$^{-3}$ is consistent.

We find two other possible solutions, both of which require an environment formed by a prior RG-phase of the progenitor (Case II). In either case, we will need a short delay time between the formation of the WD and the explosion.

As discussed above, a wind from the progenitor prior to the WD phase will produce an inner structure with a density close to \( n \approx 2 \) (region I, Figure 1), and an outer cocoon of constant, low density \((s = 0, \text{region II})\) that is lower than the ISM by about an order of magnitude. The lower limit for the particle density in region II is \( \approx 0.009 \) cm$^{-3}$, which widens the regime of solutions for \( s = 0 \).

The AD wind may either interact in the \( r^{-2} \) or constant density region. In either case, we can exclude RG or RG-like winds from the progenitor system. An AD wind as the donor star can be ruled out, too, by the X-rays and radio. This finding is also consistent with the lack of a RG remnant at the location of SN 2014J.

Within the scenario of \( M_{\text{Ch}} \) explosions, AD winds are consistent with both interaction in region I and II. AD winds are also supported by the high Doppler shifts of the narrow lines reported by (Kelly et al. 2014).

For further constraints, we want to discuss the formation of the necessary wind environment formed from prior mass loss, which created the RG-cocoon to, at least, 10–20 ly. As discussed above, we use \( v_{\infty} = 20–60 \) km s$^{-1}$, mass loss rates between \( 10^{-5} \) and \( 10^{-3} \) km s$^{-1}$, and particle densities \( n_0 \) between 0.1 and 10 cm$^{-3}$ for the initial ISM. Using the relation for \( R_2 \) of Table 1 and the reference model for normalization, we obtain for the duration needed for the wind

\[
f_{\text{RG}} = \text{const} \left( R_2/v_\infty n_0 \right)^{2/3}.
\]

We obtain wind durations between \( 1.3 \times 10^4 \) and \( 1.4 \times 10^5 \) yr prior to the explosion.

This provides an upper limit for the duration of the AD disk wind and hence the accretion. Within the picture of an \( M_{\text{Ch}} \) mass explosion starting from an 0.6 to 1.1 \( M_\odot \) C/O WD, some 0.7...0.25 \( M_\odot \) must be accreted. As discussed above, hydrogen is only stable between \( 2 \times 10^{-3}–0.1 \) \( M_\odot \) yr$^{-1}$. Thus, only the long timescale is consistent with a hydrogen accretor. This makes it more likely that the donor was a He star or a C/O AD. Assuming an ISM density of \( 0.1–1 \) cm$^{-3}$, between 0.5 and 10 \( M_\odot \) would have been swept up by the ISM. We note that, with this constraint, the region within \( R_2 \), with \( s = 2 \) is much too small in all our models (see Table 4). The most likely solution is therefore an AD wind interaction within region II.

For SN2014J, the following picture emerges as our most likely solution: an environment is produced during the RG phase of the WD progenitor, consisting of a low density cavity surrounded by a shell, or a very low density ISM which would obviously not be expected for the starburst galaxy M82. Wind from the AD (AD-wind) runs into this environment and produces a shell structure responsible for the variable narrow lines observed.

Consistent solutions can be obtained if the AD wind interacts either with region I (case A, \( s = 2 \)) or region II (case B, \( s = 0 \)) of the prior RG wind. This wind runs into an ISM of 0.03 and 0.1 cm$^{-3}$, respectively. The RG winds have a mass loss rate, wind velocity and duration which are \( 8 \times 10^{-7}–10^{-5} \) \( M_\odot \) yr$^{-1}$, \( 30 \) km s$^{-1}$, \( 30 \) yr, \( 10^{-7}–10^{-5} \) \( M_\odot \) yr$^{-1}$, \( 60 \) km s$^{-1}$, and \( 2 \times 10^5 \) yr, respectively. For case A, the location of the reversed shock is larger than 20 ly. For case B, the densities in region II are about \( 0.01 \) cm$^{-3}$. Obviously, a lower ISM density by a factor of 10 is a possible solution without the need for an RG wind.

The most likely solutions for case A and B are an AD wind with \( (\dot{m}[M_\odot \text{yr}^{-1}], v_\infty, \text{[km s}^{-1}]) \) of \( (10^{-8}, 3000) \) and \( (1.5 \times 10^{-8}, 5000) \) with a duration \( t \) of 35,000 years and 20,000 years, respectively. The inner corresponding void contains \( \approx 3.5 \times 10^{-4} M_\odot \) and \( \approx 2.9 \times 10^{-4} M_\odot \) of material at a density of \( \approx 4 \times 10^{-5} \) and \( 3 \times 10^{-5} \) cm$^{-3}$. The corresponding density is \( \approx 4 \times 10^{-5} \) cm$^{-3}$, the outer shell width is \( \approx 2 \) ly and its density is \( \approx 0.02 \) cm$^{-3}$. For case B, the shell has a thickness of about 2 ly and a density of \( \approx 0.03 \) cm$^{-3}$. For case A, a thin shell is formed (Figure 1) with densities decreasing from 10 to 0.1 cm$^{-3}$ over some 0.015 ly. The equivalent width for NaI D is \( 27 \) mA with a velocity dispersion of 14 km s$^{-1}$ for case A, and 27 mA and 4.2 km s$^{-1}$ for case B.

Other analyses of SN 2014J include that of Soker (2015), who concluded that the core-degenerate model alone could account for the observations discussed. Winds of CDs are one solution among many that may be more favorable because of the consistency with other observational properties of SN2014J.

In this context, it may be interesting to apply our method to SN2011fe, one of the best observed SNe Ia at a distance of \( \approx 6.4 \) Mpc (Nugent et al. 2011). This is another nearby, well-observed, very normal SN Ia. Similar to SN2014J, environmental density constraints have been set for SN2011fe using X-rays by Margutti et al. (2012) and radio by Chomiuk et al. (2012). They find upper limits, for a wind CSM environment, of \( m \lesssim 6 \times 10^{-10} \times v_\infty/(1100 \text{ km s}^{-1}) \) \( M_\odot \) yr$^{-1}$. For an AD wind of \( 3000 \) km s$^{-1}$ this limit corresponds to \( m \lesssim 1.8 \times 10^{-9} M_\odot \) yr$^{-1}$. These parameters are consistent with the wind from an AD with SD progenitors as discussed above. For constant CSM environment, Chomiuk et al. (2012) found \( n_0 \lesssim 6 \) cm$^{-3}$ which is above the typical density found in our models within the bubble.

Unlike for SN2014J, no time varying narrow lines have been observed for this event (Patat et al. 2013), which prevents us from specifying shell velocities or distances in conducting our analysis. The narrow lines may either be attributed to the ISM or distances significantly larger than in SN2014J, which remains unaffected by the light front (see, e.g., Figure 5 in Graham et al. 2015b). If there is a shell produced in a CSM, the bubble must be larger than \( \approx 40 \) ly. For typical AD wind parameters and a constant density environment of 1 cm$^{-3}$, this requires durations of \( t \gtrsim 10^6 \) year or an environment produced by prior mass loss and a duration larger than in SN2014J by a factor of 2–4.

4. DISCUSSION AND CONCLUSIONS

We presented theoretical, semi-analytic models for the interaction of stellar winds with the ISM and its implementation in our code SPICE.\(^5\) We assumed spherical symmetry and power-law ambient density profiles. Our free parameters are: the (a) mass loss \( \dot{m} \), (b) wind velocity \( v_\infty \), (c) the particle density \( n_0 \) of the in case of a constant density ISM or the mass loss and wind velocity for environments produced by a prior stellar wind, and (d) the duration of the wind from the progenitor system.

\(^5\) The code SPICE can be obtained by request.
Our approach provides an efficient approach to study a wide range of parameters well beyond what is feasible with complex numerical simulations. Using the $\Pi$-theorem allows us to test a wide variety of configurations, properties of the solutions along with their sensitivity and dependencies on the wind and environment parameters. Using these dependencies, we showed how to use observations to find possible solutions. The formalism presented and SPICE may be used for a wide variety of objects, including stellar winds. The speed of the semi-analytic approach produces solutions with low computational overhead. This allows us to evaluate a large parameter space for individual objects, and to include realistic feedback from many objects in star formation and galactic large-scale simulations.

Here, the formalism has been applied to study SNe Ia and to constrain the properties of progenitor systems. As discussed in the introduction, SNe Ia may originate from a wide range of scenarios and progenitor channels, which leads to an ongoing discussion about the nature of these objects, and their diversity. Most of the channels are not well understood and thus our range may be wider than realized in nature.

We studied a variety of winds within the scenario of $M_{Ch}$ explosions. The winds may originate from the AD, MS, and RG donor star, and over-Eddington accretion of $H$/He rich matter within this scenario.

The environment considered may be produced by the ISM or may be produced by winds during the stages of the evolution of the progenitor prior to becoming a WD for both double or single degenerate systems. Within the $M_{Ch}$ scenarios, we studied wind from the AD, MS, and RG donor stars, and over-Eddington accretion of $H$/He rich matter within parameters suggested from the literature as discussed in Section 3.

Within the $M_{Ch}$ explosions, we find that the wind from the AD dominates the environment, or the combined wind from an RG donor and the AD. Such wind leads to a low-density “cocoon” of the order of light years in size. The actual size depends on the duration of the progenitor evolution. The calculations reveal that these cocoons are characterized by interior regions with particle densities often as low as $10^{-4}$ cm$^{-3}$ and which are surrounded by a thin shell. This explains why most SNe Ia appear to explode in low density environments, although SN Ia are observed in the galactic halo, the disk, and the bulge. The lack of ongoing interaction in SNe Ia may well be understood in the framework of $M_{Ch}$ mass explosions, whether they originate from accretion from a MS, RG, He-star or a tidally disrupted WD. If the wind of the progenitor system interacts with a constant, ISM, we expect narrow lines produced by the shell. Our calculations show shell velocities ranging from 10 to 100 km s$^{-1}$. For the $s = 0$ models, the narrow lines are expected to have an equivalent width of $\approx 100$ mÅ and are Doppler shifted by about 10–20 km s$^{-1}$ (see Tables 3–5). In contrast, an environment dominated by a prior stellar wind will result in weaker lines with EW lower by one to two orders of magnitude, i.e., 0.5–5 mÅ (see Table 6), which is beyond current observational limits for most SNe Ia. For small cocoons, the narrow lines may show variations in strength and velocity on timescales of months due to the radiation from a SNe Ia. Radiation pressure of the SNe Ia light may accelerate nearby shells (as seen in, e.g., SN1993J).

As a separate effect, the SN ejecta may interact with the shell as discussed in Section 1. The outer layers of an SN Ia expands with velocities of 10%–30% of the speed of light and we may expect some interaction on timescales between years and several decades (see Tables 3, 5 and 6). For more details, see Hoeflich et al. (2013) and Dragulin (2015). Note the possible implications for SN-remnants and their evolution.

We studied the effect of winds in ultra-low density environments. If there is a long delay time between the formation of the WD and the explosion, the progenitor system may have moved into a low density environment. Within $M_{Ch}$ mass explosions, the size of the “cocoon” will be larger by an order of magnitude and we would neither expect narrow lines nor interaction (Section 3.2.1). In general, dynamical mergers are expected to often explode in low density environments due to their long delay times but, also, are unlikely to produce a long-duration wind. Thus, some of these objects are expected to show ongoing interaction for sufficiently large samples of SNe Ia.

A step-by-step scheme has been developed to use the narrow lines for the analysis of SNe Ia progenitors. We applied our models to SN2014J using both the limits from the X-ray and radio, and the observation of narrow lines (Section 3.3), and discussed the allowed range of progenitor properties using the analytic relations (Figure 17). We found the observations to be consistent with an environment produced by a stellar wind. We applied both our analytic solutions to produce the environment and the wind from the progenitor system. We require an environment which has been created recently with times between $1.3 \times 10^7$ and $1.4 \times 10^8$ yr prior to the explosion. Within the picture of an $M_{Ch}$ explosion starting from an initial 0.6 to 1.1 $M_{Ch}$ C/O WD, some 0.7...0.25 $M_{Ch}$ must be accreted. As discussed in the reviews quoted in the introduction, hydrogen is only stable between $2 \times 10^{-8}...-6 M_{\odot}$ yr$^{-1}$. Thus, only the upper limit for the evolutionary times is consistent with a hydrogen accretor. This makes it more likely that the donor was a He star or a C/O AD as a result of a tidally disrupted WD in a DD system. As discussed in the introduction, He-triggered or double degenerate explosions may show winds similar to AD and the $M_{Ch}$ models. However, to build up a sufficiently large a He-layer seems to require low accretion rates inconsistent with the short delay times between the RG phase and the explosion. We note that a future generation of double degenerate models may void this argument.

Our analysis of SN2014J is consistent with other constraints: no RG donor star has been found (Kelly et al. 2014), optical to MIR LCs and spectra are consistent with an Branch-normal $M_{Ch}$ mass explosion and 0.6 $M_{Ch}$ of $^{56}$Ni (Churazov et al. 2014; Diamond et al. 2015; Isern et al. 2014; Marion et al. 2015; Telesco et al. 2015), and the lack of polarization (Patat et al. 2015).

However, within an increasing number of well observed SNe Ia, it also becomes increasingly obvious that SNe Ia are not “all the same.” Our conclusion on SN 2014J should not be generalized. Better observation of narrow line systems are key to the environment of SNe Ia and the diversity.

Within the class of single degenerate systems we found configurations for SN 2014J that are consistent with both the limits from X-rays and radio and the narrow lines. Our solutions invoke a wind from the AD running into an unusually low density ISM environment, $\leqslant 0.01$ cm$^{-3}$, or a low density cavity created by a RG wind some 20,000 to 35,000 years prior to the explosion (see Section 3.3).
We want to put our findings in context of the possible Hα emission discussed in the introduction. Its detectability depends on the amount of hydrogen, the mechanism of ionization, and the sensitivity and timing of the observations. Possible mechanisms include hard radiation from the shock breakout, $^{56}$Ni-decay, surface burning of the WD prior to the explosion, interaction in the forward shock with the CSM, and the reversed shock as a result of the interaction between the SN and the CSM. The results are very model dependent with various interaction in the forward shock with the CSM, and the material in the progenitor system. The Hα emission in the outer supernovae ejecta ionized by the reversed shock interacting with the progenitor wind. They parameterized their study in terms of $M/\dot{v}_w [M_\odot \, yr^{-1} \, km^{-1} \, s]$ with $\dot{v}_w$ being the progenitor wind velocity. Cumming et al. (1996) put their results into context for the observability of Hα emission for local SNe Ia at distances similar to SN 2014J. For $M/\dot{v}_w \lesssim 1.9 \times 10^{-5}$, supersoft X-ray sources dominate the reversed shock mechanism. We note that the actual flux from supersoft X-ray sources may be significantly lower because of material in the progenitor system (Gerardy et al. 2004). The Hα emission in the reversed shock region depends on the ionization fraction of hydrogen. Cumming et al. (1996) found the ionization fraction to be less than 0.01, at which the emission becomes inefficient for $\lesssim M/\dot{v}_w = 1.5 \times 10^{-8}$, and full ionization for $M/\dot{v}_w = 1.5 \times 10^{-6}$. For local supernovae with a reversed shock in a H-rich region, Hα should be observable in high quality spectra for an RG wind with $M \sim 10^{-5} M_\odot \, yr^{-1}$ and $\dot{v}_w = 10 \, km \, s^{-1}$. Note that the result of Cumming et al. (1996) is consistent with the finding of Gerardy et al. (2004). Using the observed LCs, Gerardy et al. (2004) put strong limits on the ongoing interaction. Without a strong interaction, there is reversed shock or too weak of one to ionize hydrogen. We note that the dominance of the ionization mechanism will depend on the class of explosion. For example, excitation by $^{56}$Ni-decay or the shock breakout can be expected to be small for deflagration models such as W7 or pulsating delayed-detonation models with a significant C/O shell reducing the γ emission; however, the role of $^{56}$Ni may become important in double-detonation/HeD explosions, which show some $^{56}$Ni in the outer layers and may produce He-lines instead (Hoeftich & Khokhlov 1996). For the progenitor wind and environment we found for SN 2014J, we do not expect early, observational Hα emission. The mass loss from ADs is smaller by some 3 to 4 orders of magnitude compared to the limits by Cumming et al. (1996) for local SNe Ia, and the forward shock runs into low density material of some $10^{-3} \ldots 10^{-4} M_\odot$ in the cavity or the ISM.

However, we may be able to see late-time, narrow Hα emission if the the supernova ejecta runs into the shell of the cavity. Due to the low masses of the cavity, the SN ejecta will remain largely unmodified as it travels through the void and produce Hα emission on impact by both the forward and the reversed shock if we have H in the outer layers of the SN. The velocities of the outer layers are about 1/3 c, and for SN 2014J, we may expect emission in about 50 years. The impact will be earlier for higher density, more compact shells or shells with inner clumps. While we wait to see hydrodynamic interaction for SN2014J, we could observe young SN remnants for this signature. One such example is the Branch-normal SNe Ia SN1972E in NGC 5253, 3.3 Mpc away (Kirshner & Oke 1975).

Here, our method has been applied to SNe Ia but studies of other types of stellar explosions, namely core-collapse SNe, and SNe interacting with Wolf–Rayet star mass loss, are under way. SPICE may also be used in modeling other hydrodynamic phenomena and has the potential to include feedback from stellar winds on a subgrid scale in star formation and large scale galactic evolution simulations.

This brings us also to the limit of our analysis. Although our analytic models provide a practical tool for individual shells or shells from well-separated phases of mass loss. In reality, nature may be more complicated. The mass loss can be expected to be time dependent. If an environment is formed by this wind, it will deviate from an $r^{-2}$ law. The mass loss may come in phases, namely brief periods of “superwinds” during the RG phase. Moreover, mass loss from the progenitor system may change over time. In fact, multiple narrow lines have been observed, e.g., for SN 2014J (Graham et al. 2015b). However, the narrow lines may be produced both in the vicinity of the SN or at any distance by unrelated clouds in the ISM. Without time evolution of the narrow lines or measurements of dust components, the origin of the systems of narrow lines remain unclear. Obviously, detailed calculations for spectra and LCs are needed to quantify the intensity of the narrow lines.

Moreover, multi-dimensional effects need to be considered for a full analysis, for example, in instabilities and mixing, non-symmetric winds, and motion of the progenitor system through the ambient medium. A combination of semi-analytic solutions provides a good starting point for more complex, multi-dimensional calculations with more detailed physics, which are under way, to be presented in forthcoming papers. What role and influence dust may have, especially in the outer layers, is another question deserving close attention.

In light of the wide range of progenitor scenarios and properties of the resulting environment, our approach has been justified in order to find the right ballpark in the vast sea of parameters. In reality, however, multi-dimensional affects such as asymmetric winds will become important, including variable winds. Moreover, proper cooling functions and equation of states for the gas are to be taken into account for detailed analysis of high quality data such as SN2014J.

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