Elastic–Plastic and Inelastic Characteristics of High Strength Steel Sheets under Biaxial Loading and Unloading

Mohammad Omar ANDAR,1) Toshihiko KUWABARA,2) Shigeru YONEMURA3) and Akihiro UENISHI3)

1) Department of Mechanical Systems Engineering, Graduate School of Technology, Tokyo University of Agriculture and Technology, 2-24-16 Naka-cho, Koganei, Tokyo 184-8588 Japan. E-mail: andaromar@gmail.com
2) Division of Advanced Mechanical Systems Engineering, Institute of Symbiotic Science and Technology, Graduate School of Tokyo University of Agriculture and Technology, 2-24-16 Nakacho, Koganei Tokyo 184-8588 Japan. E-mail: kuwabara@cc.tuat.ac.jp
3) Kimitsu R&D Laboratory, Nippon Steel Corporation, 1 Kimitsu, Kimitsu, Chiba 299-1141 Japan. E-mail: yonemura.shigeru@nsc.co.jp, uenishi.akihiro@nsc.co.jp

(Received on October 23, 2009; accepted on February 5, 2010)

Biaxial tensile tests followed by biaxial unloading and reloading are carried out for BH340 and DP590 steel alloys. The contours of plastic work and the directions of plastic strain rates were measured at different levels of plastic work in the first and second quadrants of stress space. The applicability of conventional anisotropic yield functions, Hill’s quadratic function and the Yld2000-2d function to the accurate prediction of the plastic deformation behavior of these steel alloys is discussed using the measured data. The measured work contours and directions of plastic strain rates were in good agreement with those calculated using the Yld2000-2d yield function with an exponent of four. The initial and subsequent elastic moduli after prestraining and the instantaneous tangent moduli during subsequent unloading after an equivalent plastic prestrain of $e_p^0=0.02$ are measured from the biaxial loading, unloading and reloading experiments. The moduli of elasticity at reloading were lower by 9 to 17% than those at initial loading. The amount of strain recovery along the rolling direction (RD) is more than that along the transverse direction (TD) for uniaxial unloading, as well as for biaxial unloading. An exponential decay model is proposed that provides good reproduction of the unloading stress–strain relations, $(\sigma/\sigma_u) - \Delta e/(\sigma/E_1)$, of both materials under different stress ratios.

KEY WORDS: biaxial tension; contours of plastic work; anisotropic yield function; unloading; inelastic strain recovery; cruciform specimen; instantaneous tangent modulus.

1. Introduction

The demand for high strength steel alloys has been increasing due to vehicle weight reduction and crash performance improvement requirements. However, a large springback occurs during the press forming of high strength steel alloys. Springback often causes serious problems in determining the optimum geometry of dies, and is the main impediment to high-efficiency production. In order to establish time- and cost-effective press-forming technologies for automotive body parts requiring the use of high strength steel alloys, it is vital to accurately predict the amount of springback using simulation techniques and determine the optimum forming conditions.

Springback is caused by the bending moment retained in press-formed parts before unloading.1–3) Furthermore, it is also affected by the unloading stress–strain response of the material, i.e., the Bauschinger effect.4–6) Therefore, in order to accurately predict the amount of springback of automotive body parts made from high strength steel alloys, it is crucial to have a highly accurate material model that is capable of predicting the work hardening behavior of the material during biaxial loading, as well as the stress–strain responses during biaxial unloading.

Regarding the measurement of flow stresses under biaxial stress states and the determination of an appropriate anisotropic yield function for a given material, the biaxial tensile testing method with a cruciform specimen is useful.7,8) On the other hand, the development of stresses during unloading of pre-loaded materials has been investigated conventionally for the uniaxial stress state. Some authors have suggested that Young’s modulus is decreased with the increase in the amount of prestrain.9–12) Moreover, accurate material modeling of the non-linearity of stress–strain curves during unloading is another important factor for accurate springback simulation.5–6) However, to our knowledge, there has been no experimental work involving the measurement and formulation of the loading and unloading behavior of steel alloy sheets under biaxial stress states.

In this study, the biaxial loading and unloading behavior of two types of steel alloys, a bake-hardenable steel alloy with a tensile strength of 340 MPa (BH340) and a dual-phase steel alloy with a tensile strength of 590 MPa (DP 590), was precisely measured using biaxial stress tests with cruciform specimens. The effect of pre-loading on the elastic modulus and the inelastic strain recovery behavior for different stress ratios are discussed. An exponential decay model that can well reproduce the unloading stress–strain
relations of both materials for different stress ratios is proposed.

2. Experimental Method

2.1. Test Materials

Two types of high strength steel alloy test materials were used in this study: 0.7 mm thick BH340 and 1 mm thick DP590. BH and DP denote bake-hardenable and dual-phase, respectively. The numbers indicate the tensile strength of the materials. The work hardening characteristics and the r-values of the test materials are listed in Table 1.

2.2. Test Specimens

Figures 1(a), 1(b) and 1(c) show the geometry of the specimens used in this study. Figure 1(a) is the specimen for the uniaxial tensile test (JIS 13 B-type). Figure 1(b) is the specimen for the biaxial tensile test of sheet metals, which was originally proposed by Kuwabara et al. and is the same as those used for the biaxial tensile tests of steel alloys, aluminum alloys, austenitic stainless steel SUS304 and pure titanium. Figure 1(c) is the specimen for the combined tension–compression test of sheet metals, which was originally proposed by Kuwabara et al. The x and y-axes are in the rolling (RD) and transverse directions (TD) of the materials, respectively. Seven slits have been fabricated in the arms of the specimens at intervals of 7.5 mm in order to exclude geometric constraint on the 60 × 60 mm square gauge section.

The normal strain components, $e_x$ and $e_y$, were measured for the specimens shown in Figs. 1(a) and 1(b) using uniaxial strain gauges (Tokyo Sokki Kenkyujo, YFLA-2). A biaxial strain gauge (Tokyo Sokki Kenkyujo, FCA-11-1L) was used at the center of the specimen shown in Fig. 1(c).

2.3. Testing Apparatus

Figure 2 shows the biaxial testing apparatus used in this study. It was originally designed and fabricated by Kuwabara et al. The hydraulic pressure of each pair of hydraulic cylinders is servo-controlled independently, so that the stress-paths or strain-paths applied to a specimen can be arbitrarily controlled using a closed-loop circuit. The displacements of the rams of the opposing hydraulic cylinders are equalized using the pantograph-type link mechanism proposed by Shiratori and Ikegami, so that the centre of the specimen is always maintained at the centre of the testing apparatus during the tests. A load cell is included in each loading direction. Biaxial strain components in the RD and TD of the specimen are measured continuously using strain gauges.

For combined tension–compression tests, the specially designed comb-shaped jig shown in Fig. 3 was used. The comb-shaped jig was originally designed and fabricated by Kuwabara et al.

2.4. Procedure for Biaxial Experiments

The true stress components, $\sigma_x$ and $\sigma_y$, were determined by dividing the measured loads $F_x$ and $F_y$ by the current cross-sectional area of the gauge section, which was determined from the measured values of plastic strain compo-

---

Table 1. Mechanical properties of the test materials.

| Test material | Tensile direction | $\sigma_{0.2}$ / MPa | $\sigma_{0.5}$ / MPa | $c$ | $n^1$ | $\varepsilon_{0.2}^*$ | $\varepsilon^*$
|---------------|-------------------|----------------------|----------------------|-----|-----|------------------|-----
| BH340         | 0°                | 246                  | 595                  | 0.216 | 0.013 | 1.52             |     |
|               | 22.5°             | 254                  | 600                  | 0.213 | 0.015 | 1.33             |     |
|               | 45°               | 264                  | 601                  | 0.207 | 0.015 | 1.15             |     |
|               | 67.5°             | 265                  | 598                  | 0.200 | 0.016 | 1.36             |     |
|               | 90°               | 260                  | 577                  | 0.203 | 0.016 | 1.64             |     |
| DP590         | 0°                | 359                  | 1157                 | 0.251 | 0.007 | 0.85             |     |
|               | 22.5°             | 363                  | 1163                 | 0.251 | 0.007 | 0.81             |     |
|               | 45°               | 372                  | 1152                 | 0.243 | 0.007 | 0.84             |     |
|               | 67.5°             | 375                  | 1160                 | 0.245 | 0.008 | 0.96             |     |
|               | 90°               | 372                  | 1167                 | 0.247 | 0.008 | 1.02             |     |

*1 Approximated using $\sigma = c(e_x + e_y)^n$ for $e^* = 0.002-0.093$

*2 Measured at nominal uniaxial strain $\varepsilon_{0.2} = 0.1$
corresponding to particular values of principal stress space to construct contours of plastic work. Tests and (1.2–9.9) be practically viewed as a yield locus. As sufficiently small, the corresponding work contour can be practically viewed as a yield locus. Tour23,24) in the stress space was introduced. This was motivated by simulation of the metal forming, in which the flow stress was modeled as the average behavior of a material over a deformation range is more important than determining the initial yield locus of the material. The method for the construction of plastic work contours is given as follows. A initial yield locus of the material. The method for the construction of yield locus, as has been customarily used by a number of other authors. The materials under test are anisotropic; therefore, the plastic work definition of yielding, which has a definite physical meaning, appears to be more appropriate than the offset plastic strain definition, which is certainly equivalent to the former only for von Mises-type materials.

2.4.1. Biaxial Tensile Tests and Combined Tension–Compression Tests

The nominal stress ratios chosen for the biaxial tensile tests were $\sigma_{N_x}: \sigma_{N_y}=1:0, 4:1, 2:1, 4:3, 1:1, 3:4, 1:2, 1:4$ and $0:1$ and those for the combined tension–compression tests were $\sigma_{N_x}: \sigma_{N_y}=-1:2, -1:1$ and $-2:1$. The strain rates were $(0.9–3.9) \times 10^{-4}$ s$^{-1}$ for the biaxial tensile tests and $(1.2–9.9) \times 10^{-3}$ s$^{-1}$ for the combined tension–compression tests. The gauge area of the combined tension–compression specimen (Fig. 1(c)) was fully covered with Teflon sheet using Vaseline in order to reduce friction between the specimen and the dies.

To evaluate the work hardening behavior of the test material under biaxial tension, the concept of a plastic work contour$^{21,24}$ in the stress space was introduced. This was motivated by simulation of the metal forming, in which the flow stress modeled as the average behavior of a material over a deformation range is more important than determining the initial yield locus of the material. The method for the construction of plastic work contours is given as follows. A uniaxial tension test in the RD of the material ($\sigma_x: \sigma_y=1:0$) is first conducted, and the uniaxial true stress $\sigma_x$ and plastic work $W$ dissipated per unit volume are determined for particular values of the uniaxial logarithmic plastic strain $\varepsilon_y^p$. Uniaxial tension tests in the TD of the material $\sigma_x: \sigma_y=0:1$ and biaxial tension tests with the stress ratio $\sigma_x: \sigma_y$ held at specific proportions are also carried out. Finally, groups of stress points ($\sigma_x, \sigma_y$) for which the same amount of plastic work $W$ is required, are plotted in the principal stress space to construct contours of plastic work corresponding to particular values of $\varepsilon_y^p$. When $\varepsilon_y^p$ is taken as sufficiently small, the corresponding work contour can be practically viewed as a yield locus. It should be noted that the offset plastic strain was not used for the definition of the yield locus, as has been customarily used by a number of other authors. The materials under test are anisotropic; therefore, the plastic work definition of yielding, which has a definite physical meaning, appears to be more appropriate than the offset plastic strain definition, which is certainly equivalent to the former only for von Mises-type materials.

2.4.2. Biaxial Loading–Unloading Tests

The nominal stress ratios chosen for the uniaxial and biaxial loading–unloading tests were $\sigma_{N_x}: \sigma_{N_y}=1:0, 2:1, 1:2, 1:1$ and $0:1$. The stress ratios were kept constant in each test during loading and unloading. The strain rates during unloading were $(1–8) \times 10^{-3}$ s$^{-1}$. Biaxial loading–unloading tests were performed as follows. As a first loading, the specimen was initially pre-strained up to approximately $\varepsilon_y^p=0.02$ and then unloaded with the same stress ratio as in loading. After confirming that $F_x$ and $F_y$ became zero, the specimen was further deformed up to a total equivalent plastic strain of $\varepsilon_y^p=0.03$ (second loading). The specimen was then unloaded and finally reloaded up to fracture (third loading).

Figure 4(a) shows typical biaxial stress–strain curves measured in an equi-biaxial loading–unloading test $E, E_1$ and $E_3$ are the elastic moduli measured during the initial, second and third loadings, respectively. (b) A typical measured stress–strain curve during unloading and reloading. $\sigma_x$ and $\varepsilon_x$ are the stress and strain values at unloading, $\Delta\varepsilon^p$ is the nonlinear strain recovery, and $\Delta\varepsilon$ is the linear elastic strain recovery.
using the linear fitting function installed in Origin 7.5 software, which is based on a linear regression method. The range of linear fitting was taken to be from approximately 20 MPa up to half of the apparent yield stress. For \( \sigma^p_1 : \sigma^p_2 = 2 : 1 \) and \( 1 : 2 \), \( E_1 \) and \( E_2 \) were measured only in the maximum stress direction.

Figure 4(b) shows a typical measured stress–strain curve during unloading and reloading. The straight line in the figure indicates the elastic modulus \( E \), as determined from the initial loading. The amounts of linear strain recovery \( \Delta e^l \), and the nonlinear (inelastic) strain recovery \( \Delta e^{nl} \) were measured for each stress ratio, as shown in Fig. 4(b). The instantaneous tangent modulus during unloading \( d\sigma/d\epsilon \), simply referred to as the tangent modulus henceforth, was measured for each stress ratio, as shown in Fig. 4(b). The nonlinear (inelastic) strain recovery \( \Delta e^{nl} \) were measured only in the maximum stress direction. As depicted in Figs. 5(a) and 5(b), the von Mises yield function overestimates the normalized work contours of BH340 for stress ratios \( \sigma^p_1 : \sigma^p_2 = 3 : 4, 1 : 1 \) and \( 4 : 3 \), while the work contours of DP590 are only slightly underestimated for the same stress ratios. The Yld2000-2d yield function with an exponent of \( M=4 \) is generally in good agreement with the measured work contours for both materials.

3. Results and Discussion

3.1. Comparison of Work Contours and Directions of Plastic Strain Rates with Those Predicted Using Conventional Yield Functions

Figure 5 shows the measured stress points, \((\sigma^p_1, \sigma^p_2)\), comprising contours of plastic work for particular values of \( e^p_0 \). The stress values were normalized by \( \sigma^p_1 \) corresponding to each \( e^p_0 \). Also depicted in the figure are the theoretical yield loci based on the von Mises, \(^{27}\) Hill’s quadratic (Hill’ 48)\(^{28}\) and the Yld2000-2d non-quadratic\(^{30}\) yield functions. The anisotropic parameters of the Hill’s quadratic yield function were determined using \( \sigma^p_1, r_0 \) and \( r_90 \), and those of the Yld2000-2d were determined using \( \sigma^p_1, \sigma^p_{45}, \sigma^p_{90}, r^p_0, r^p_{45}, r^p_{90}, \sigma^p_b \) and \( r_b \), where \( \sigma^p_b \) and \( r_b \) are the yield stress and the strain hardening exponent, respectively, measured for the uniaxial tensile test inclined by \( \alpha \)-degrees from the RD, and \( \sigma^p_b \) and \( r_b \) are the flow stress and the plastic strain rate ratio, \((d\sigma^p_x/dt)/(d\sigma^p_y/dt)) \), respectively, at a given \( e^p_0 \) under equi-biaxial tension. As depicted in Figs. 5(a) and 5(b), the von Mises yield function overestimates the normalized work contours of both materials. The Hill’s quadratic yield function overestimates the work contours of BH340 for stress ratios \( \sigma^p_1 : \sigma^p_2 = 3 : 4, 1 : 1 \) and \( 4 : 3 \), while the work contours of DP590 are only slightly underestimated for the same stress ratios. The Yld2000-2d yield function with an exponent of \( M=4 \) is generally in good agreement with the measured work contours for both materials.

Figure 6 shows a comparison between the measured directions of the plastic strain rates \( \theta \) at \( e^p_0=0.01 \) and 0.02 for all stress ratios, and those of the local outward vectors normal to the theoretical yield loci, based on the von Mises, Hill’s quadratic and the Yld2000-2d yield functions; note that the stress ratio is represented by the angle of the stress vector \( \varphi \), in stress space. The experimental data points are generally in good agreement with those predicted using the Yld2000-2d yield function with an exponent of \( M=4 \) for both materials. This implies that the normality flow rule is well satisfied choosing the Yld2000-2d yield function with \( M=4 \) as the plastic potential for both materials.
3.2. Effect of Prestrain on the Elastic Modulus

Figure 7 shows the measured true stress–strain curves for initial and subsequent reloading. It is clear that for a given stress ratio, the slope of the curve in the elastic range, i.e., the elastic modulus, is lower for the subsequent reloading than that for the initial loading.

Table 2 shows the results of the measured initial and subsequent elastic moduli for each stress ratio. Table 2 does not include the data for $E_3$ of BH340 for $\sigma_y : \sigma_y = 2 : 1 - x$ and $1 : 2 - y$, because the specimens fractured prior to reaching $e^p_0 = 0.03$. It was found that $E_2$ and $E_3$ were 10 to 20% lower than the initial elasticity modulus $E$, under the stress states considered in this study, and there was no difference between $E_2$ and $E_3$, which were within the scatter of data.

3.3. Tangent Modulus during Unloading

Figure 8 shows the behavior of the tangent modulus $d\sigma / de$, during unloading. Each curve was obtained by averaging the experimental data measured for three specimens. For comparison, $E$ in the maximum stress direction for each stress ratio is also included in the figure as a horizontal line.

For better comparison of the behavior of the tangent modulus between different stress ratios, the data in Fig. 8 were re-plotted in Fig. 9 to show the relationship between $(d\sigma / de)/E$ and $\sigma / \sigma_p$. It is clear that the value of $d\sigma / de$ at the commencement of unloading is nearly the same as the initial elastic modulus for all stress ratios, while $d\sigma / de$ decreases with $\sigma$.

Cleveland and Ghosh [10] suggested that the general trend of uniaxial unloading stress–strain curves can be categorized into three stages:

Stage I: The tangent modulus decreases rapidly at the beginning of unloading.

Stage II: The rate of decreasing tangent modulus becomes almost constant.

Stage III: The tangent modulus decreases rapidly again with decreasing stress.

In Fig. 9, stages I and II are clearly observed for all stress ratios. Stage III is also clearly observed for $\sigma_y : \sigma_y = 1 : 1 - x$ and $2 : 1 - x$ for both materials, but is not so obvious for other stress ratios.

Table 3 tabulates the measured nonlinear strain recovery $\Delta e^{nl}$, and the linear strain recovery, $\Delta e^l$. The nonlinear...
strain recovery as a percentage of the elastic strain recovery \( \Delta \varepsilon^\text{nl} / \Delta \varepsilon^e \), is slightly larger for the biaxial case than the uniaxial case; i.e., approximately 25% for BH340 and 30% for DP590, while under uniaxial unloading it was approximately 20% for both materials. The results of elastic strain recovery for the uniaxial experiments are in good agreement with the results obtained for a high strength steel as reported by Cleveland and Ghosh,\(^{10}\) which showed that the net unloading strain recovery (linear elastic and non-linear elastic) exceeded the expected linear elastic strain recovery by approximately 19%.

Table 3. Measured values of the nonlinear strain recovery \( \Delta \varepsilon^\text{nl} \), and the linear strain recovery, \( \Delta \varepsilon^l \).

| Material  | \( \sigma \) : \( \sigma_u \) | \( \Delta \varepsilon^l \) | \( \Delta \varepsilon^\text{nl} \) | \( \Delta \varepsilon^\text{nl} / \Delta \varepsilon^l \times 100 / \% \) |
|-----------|----------------------------|-------------------------|-----------------------------|---------------------------------|
| BH340     | 1:0                        | 0.0013                  | 0.00024                     | 18                              |
|           | 0:1                        | 0.0013                  | 0.00025                     | 19                              |
|           | 2:1-x                      | 0.0014                  | 0.00037                     | 27                              |
|           | 1:2-y                      | 0.0013                  | 0.00033                     | 26                              |
|           | 1:1-x                      | 0.0009                  | 0.00023                     | 24                              |
|           | 1:1-y                      | 0.0010                  | 0.00016                     | 16                              |
| DP590     | 1:0                        | 0.0022                  | 0.00042                     | 19                              |
|           | 0:1                        | 0.0021                  | 0.00045                     | 21                              |
|           | 2:1-x                      | 0.0021                  | 0.00064                     | 31                              |
|           | 1:2-y                      | 0.0020                  | 0.00060                     | 30                              |
|           | 1:1-x                      | 0.0014                  | 0.00050                     | 35                              |
|           | 1:1-y                      | 0.0014                  | 0.00029                     | 20                              |

4. Approximating the Unloading Behavior Using an Exponential Function

Figure 10 shows the normalized unloading stress \( \sigma / \sigma_u \) versus the normalized recovered strain \( \Delta \varepsilon / (\sigma_u / E_2) \), after 2% equivalent prestrain for all stress states. All of the \( (\sigma / \sigma_u) - \Delta \varepsilon / (\sigma_u / E_2) \) curves nearly fall on a single curve and are independent of the stress ratio. The \( (\sigma / \sigma_u) - \Delta \varepsilon / (\sigma_u / E_2) \) curves can be approximated by the following analytical functions:

\[
\frac{\sigma}{\sigma_u} = -3.10 + 4.10 \exp \left\{-0.27 \frac{\Delta \varepsilon}{\sigma_u / E_2} \right\} \quad \text{for BH340} \\
\frac{\sigma}{\sigma_u} = -3.43 + 4.42 \exp \left\{-0.24 \frac{\Delta \varepsilon}{\sigma_u / E_2} \right\} \quad \text{for DP590}
\]

These equations will be useful for taking into account the nonlinearity of the unloading stress–strain curves in spring-back simulations.

Figure 11 shows the calculated \( (d \sigma / d \varepsilon) - \sigma \) relations based on Eqs. (1) and (2) with the experimentally observed results shown in Fig. 8. The calculated results are generally in fair agreement with the experimental data during stage II, where \( d \sigma / d \varepsilon \) is almost constantly decreasing.
5. Conclusions

The work hardening behavior and unloading stress–strain relations for two types of steel alloys, BH340 and DP590, were precisely measured under uniaxial and biaxial stress states. The following conclusions were obtained.

(1) The Yld2000-2d non-quadratic yield function with an exponent of 4 is an appropriate material model for reproducing the anisotropic plastic deformation behavior of the examined test materials.

(2) For the stress states considered in this study, the modulus of elasticity at the second loading $E_2$, after a prestrain of $\varepsilon_{0}^p=0.02$, was 10 to 20% lower than the initial elasticity modulus, $E$.

(3) The inelastic strain recovery, as a percentage of elastic strain recovery for the biaxial unloading, was approximately 25% for BH340 and 30% for DP590, while under uniaxial unloading it was approximately 20% for both materials.

(4) $(\sigma/\sigma_0) - \Delta \varepsilon/(\sigma_0/E_2)$ curves measured during unloading of the uniaxially and biaxially prestrained test materials almost fall on a single curve; they are almost independent of the amount of prestrain and the stress ratio. Therefore, the analytical expressions of the $(\sigma/\sigma_0) - \Delta \varepsilon/(\sigma_0/E_2)$ curves (Eqs. (1) and (2)) will be useful for taking into account the nonlinearity of the unloading stress–strain behaviors in springback simulations.

Acknowledgement

The authors wish to acknowledge the help of Mr. Satoshi Shirakami, who improved the software program for controlling the biaxial tensile testing machine used in this study.

REFERENCES

1) T. Kuwabara, N. Seki and S. Takahashi: J. Jpn. Soc. Technol. Plast., 39 (1998), 1081.
2) T. Kuwabara, S. Ikeda and Y. Asano: Proc. 5th Int. Conf. on Numerical Methods in Industrial Forming Processes, American Institute of Physics, New York, (2004), 887.
3) T. Kuwabara: Proc. Int. Symp. Automotive Sheet Metal Forming, Tata McGraw-Hill, New Delhi, (2008), 19.
4) F. Yoshida, T. Uemori and K. Fujisawa: Int. J. Plast., 18 (2002), 633.
5) K. Chung, M.-G. Lee, D. Kim, C. Kim, M. L. Wenner and F. Barlat: Int. J. Plast., 21 (2005), 861.
6) T. Kuwabara, R. Saito, T. Hirano and N. Oohashi: Tetsu-to-Hagané, 95 (2009), 732.
7) T. Kuwabara: Int. J. Plast., 23 (2007), 385.
8) A. Hannon and P. Tiernan: J. Mater. Proc. Technol., 198 (2008), 1.
9) S. Shima and M. Yang: J. Soc. Mater. Sci. Jpn., 44 (1995), 578.
10) R. M. Cleveland and A. K. Ghosh: J. Appl. Mech., 80–81 (1976), 1.
11) T. Iwata, H. Tsutamori, N. Suzuki, H. Ishihara, M. Masao and M. Gotoh: J. Jpn. Soc. Technol. Plast., 43 (2002), 1178.
12) T. Kuwabara, S. Ikeda and T. Kuroda: J. Mater. Proc. Technol., 80–81 (1998), 517.
13) T. Kuwabara, M. Kuroda, V. Varga and K. Nomura: Acta Mater., 48 (2000), 2071.
14) T. Kuwabara, A. V. Bael and E. Ilizuka: Acta Mater., 50 (2002), 3717.
15) T. Kuwabara, T. Uemori and K. Fujiwara: J. Jpn. Inst. Light Met., 58–59 (2007), 760.
16) M. Yang: J. Jpn. Soc. Technol. Plast., 45 (2004), 991.
17) T. Kuwabara, M. Umemura, K. Yoshida, M. Kuroda, S. Hirano and Y. Kikutani: J. Jpn. Inst. Light Metal, 56 (2006), 323.
18) S. Murakosho and T. Kuwabara: J. Solid Mech. Mater. Eng. (to be published)
19) M. Ishiki, T. Kuwabara, M. Yamaguchi, Y. Masuda, Y. Hayashida and Y. Isumi: Trans. Jpn. Soc. Mech. Eng., (4), 75 (2009), 491.
20) T. Kuwabara, Y. Horuchi, N. Uema and J. Ziegelheimova: J. Jpn. Soc. Technol. Plast., 48 (2007), 630.
21) E. Shiratori and K. Ikegami: J. Mech. Phys. Solids, 16 (1968), 373.
22) R. Hill and J. W. Hutchinson: J. Appl. Mech., 59 (1992), S1.
23) R. Hill, S. S. Hecker and M. G. Stout: Int. J. Solids Structures, 31 (1994), 2999.
24) S. S. Hecker: Constitutive Equations in Viscoelasticity: Computational and Engineering Aspects, ed. by J. A. Stricklin and K. H. Saczalski, ASME, New York, (1976), 1.
25) A. Phillips: Int. J. Plast., 2 (1986), 315.
26) R. Yon Mises: Göttingen Nachrichten Math.-Phys. Klasse, (1913), 582.
27) R. Hill: Proc. R. Soc. (London), A193 (1948), 281.