MLIC: A MaxSAT-Based framework for learning interpretable classification rules

Dmitry Malioutov\textsuperscript{1} \hfill Kuldeep S. Meel\textsuperscript{2}

\textsuperscript{1}IBM Research, USA
\textsuperscript{2}School of Computing, National University of Singapore

CP 2018
The Rise of Artificial Intelligence

• “In Phoenix, cars are self-navigating the streets. In many homes, people are barking commands at tiny machines, with the machines responding. On our smartphones, apps can now recognize faces in photos and translate from one language to another.” (New York Times, 2018)

• “AI is the new electricity” (Andrew Ng, 2017)
The Need for Interpretable Models

- Core public agencies, such as those responsible for criminal justice, healthcare, welfare, and education (e.g., “high stakes” domains) should no longer use “black box” AI and algorithmic systems (AI Now Institute, 2018)
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- Medical and education domains see usage of techniques such as classification rules, decision rules, and decision lists.
Prior Work

- Long history of interpretable classification models from data such as decision trees, decision lists, checklists etc with tools such as C4.5, CN2, RIPPER, SLIPPER
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- The problem of learning optimal interpretable models is computationally intractable
- Prior work, which was mostly rooted in late 1980s and 1990s, focused on greedy approaches
Objective Learn rules that are accurate and interpretable. The learning procedure is offline, so learning does not need to happen in real time.
Our Approach

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Approach
- The problem of rule learning is inherently an optimization problem
- The past few years have seen SAT revolution and development of tools that employ SAT as core engine

SAT revolution
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- The past few years have seen SAT revolution and development of tools that employ SAT as core engine
- Can we take advantage of SAT revolution, in particular progress on MaxSAT solvers?
Key Contributions

• A MaxSAT-based framework, MLIC, that **provably** trades off accuracy vs interpretability of rules
• The prototype implementation is capable of finding optimal (or high quality near-optimal) classification rules from large data sets
Part I

From Rule Learning to MaxSAT
Binary Classification

- **Features:** \( \mathbf{x} = \{x^1, x^2, \ldots x^m\} \)
- **Input:** Set of training samples\( \{\mathbf{X}_i, y_i\} \)
  - each vector \( \mathbf{X}_i \in \mathcal{X} \) contains valuation of the features for sample \( i \),
  - \( y_i \in \{0, 1\} \) is the binary label for sample \( i \)
- **Output:** Classifier \( \mathcal{R} \), i.e. \( y = \mathcal{R}(\mathbf{x}) \)
- **Our focus:** classifiers that can be represented as CNF Formulas
  \( \mathcal{R} := C_1 \land C_2 \land \cdots \land C_k \).
- **Size of classifiers:** \( |\mathcal{R}| = \Sigma_i |C_i| \)
Constraint Learning vs Machine Learning

**Input**  Set of training samples\(\{X_i, y_i\}\)

**Output**  Classifier \(\mathcal{R}\)

- **Constraint Learning:**

\[
\min_{\mathcal{R}} |\mathcal{R}| \quad \text{such that} \quad \mathcal{R}(X_i) = y_i, \quad \forall i
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- **Constraint Learning:**
  \[
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  \]

- **Machine Learning:**
  \[
  \min_{\mathcal{R}} |\mathcal{R}| + \lambda |\mathcal{E}_\mathcal{R}| \quad \text{such that} \quad \mathcal{R}(X_i) = y_i, \quad \forall i \notin \mathcal{E}_\mathcal{R}
  \]
Step 1  Discretization of Features
Step 2  Transformation to MaxSAT Query
Step 3  Invoke a MaxSAT Solver and extract $\mathcal{R}$ from MaxSAT solution
**Encoding to MaxSAT**

**Input** Features: $\mathbf{x} = \{x^1, x^2, \cdots x^m\}$; Training Data: $\{\mathbf{X}_i, y_i\}$ over $m$ features

**Output** $\mathcal{R}$ of $k$ clauses

**Key Ideas**

- $k \times m$ binary coefficients, denoted by $\{b^1_i, b^2_i, \cdots b^m_i \cdots b^m_k\}$, such that $\mathcal{R}_i = (b^1_i x^1 \lor b^2_i x^2 \cdots \lor b^m_i x^m)$

- For every sample $i$, we have noise variable $\eta_i$ to encode sample $i$ should be considered as noise or not.
Encoding to MaxSAT

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- \( k \times m \) binary coefficients, denoted by \( \{b_1^1, b_1^2, \ldots, b_m^1, \ldots, b_m^m\} \), such that \( R_i = (b_1^1 x^1 \lor b_2^2 x^2 \ldots \lor b_m^m x^m) \)

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\[ R = \bigwedge_{i=1}^{k} R_i(x \mapsto X_i) \]: Output of substituting valuation of feature vectors of \( i \)th sample
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1. $R = \bigwedge_{i=1}^k R_i(x \mapsto X_i)$: Output of substituting valuation of feature vectors of $i$th sample

2. $D_i := (\neg \eta_i \rightarrow (y_i \leftrightarrow R(x \mapsto X_i)))$; $W(D_i) = \top$
   If $\eta_i$ is False, $y_i$ is equivalent to prediction of the Rule
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4. $N_i := (\eta_i); W(N_i) = \lambda$
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Construction

Let \( Q^k = \bigwedge_i D_i \land \bigwedge_i N_i \land \bigwedge_{i,j} V^j_i \)
\( \sigma^* = \text{MaxSAT}(Q^k, W) \), then \( x^j \in R_i \) iff \( \sigma^*(b^j_i) = 1 \).

Remember, \( R_i = (b^1_i x^1 \lor b^2_i x^2 \ldots \lor b^m_i x^m) \)
Provable Guarantees

Theorem (Provable trade off of accuracy vs interpretability of rules)

Let $R_1 \leftarrow \text{MLIC}(X, y, k, \lambda_1)$ and $R_2 \leftarrow \text{MLIC}(X, y, k, \lambda_2)$, if $\lambda_2 > \lambda_1$ then $|R_1| \leq |R_2|$ and $|\mathcal{E}_{R_1}| \geq |\mathcal{E}_{R_2}|$. 
• \((y = S(x)) \leftrightarrow \neg(y = \neg S(x))\).

• And if \(S\) is a DNF formula, then \(\neg S\) is a CNF formula.

• To learn rule \(S\), we simply call MLIC with \(\neg y\) as input and negate the learned rule.
Part II

Experimental Results
• Iris Classification:
• Features: sepal length, sepal width, petal length, and petal width
• MLIC learned $\mathcal{R}:=
\begin{align*}
\text{①} & \quad (\text{sepal length} > 6.3 \lor \text{sepal width} > 3.0 \lor \text{petal width} \leq 1.5) \land \\
\text{②} & \quad (\text{sepal width} \leq 2.7 \lor \text{petal length} > 4.0 \lor \text{petal width} > 1.2) \land \\
\text{③} & \quad (\text{petal length} \leq 5.0)
\end{align*}
| Dataset       | Size  | # Features | RIPPER | Log Reg | NN     | RF    | SVM     | MLIC     |
|---------------|-------|------------|--------|---------|--------|-------|---------|----------|
| TomsHardware  | 28170 | 830        | 0.968  | 0.976   | 0.977  | 0.976 | Timeout | 0.969    |
|               |       |            | (92.8) | (0.2)   | (3.4)  | (64.9 )|         | (2000)   |
| Twitter       | 49990 | 1050       | 0.938  | 0.963   | 0.965  | 0.962 | 0.962   | 0.958    |
|               |       |            | (187.3)| (0.2)   | (6.8)  | (250.9 )| (1010.0)| (2000)   |
| adult-data    | 32560 | 262        | 0.852  | 0.801   | 0.866  | 0.844 | Timeout | 0.755    |
|               |       |            | (0.5)  | (0.3)   | (3.0)  | (41.8 )|         | (2000)   |
| credit-card   | 30000 | 334        | 0.811  | 0.781   | 0.822  | 0.82  | Timeout | 0.82     |
|               |       |            | (0.7)  | (0.1)   | (3.9)  | (25.5 )|         | (2000)   |
| ionosphere    | 350   | 564        | 0.886  | 0.909   | 0.926  | 0.909 | 0.886   | 0.889    |
|               |       |            | (0.1)  | (0.1)   | (1.2)  | (1.3 )| (0.1 )  | (15.04)  |
| PIMA          | 760   | 134        | 0.774  | 0.749   | 0.764  | 0.761 | 0.77    | 0.736    |
|               |       |            | (0.1)  | (0.1)   | (1.3)  | (1.3 )| (21.4 ) | (2000)   |
| parkinsons    | 190   | 392        | 0.868  | 0.884   | 0.921  | 0.895 | 0.879   | 0.895    |
|               |       |            | (0.1)  | (0.1)   | (1.2)  | (1.1 )| (1.6 )  | (245)    |
| Trans         | 740   | 64         | 0.78   | 0.759   | 0.788  | 0.788 | 0.765   | 0.797    |
|               |       |            | (0.0)  | (0.0)   | (1.2)  | (1.2 )| (372.3 )| (1177)   |
| WDBC          | 560   | 540        | 0.961  | 0.936   | 0.961  | 0.943 | 0.955   | 0.946    |
|               |       |            | (0.1)  | (0.0)   | (1.3)  | (1.4 )| (3.0 )  | (911)    |
## Interpretability

| Dataset       | Size  | # Features | RIPPER | MLIC |
|---------------|-------|------------|--------|------|
| TomsHardware  | 28170 | 830        | 57.5   | 4    |
| Twitter       | 49990 | 1050       | 78.5   | 15   |
| adult-data    | 32560 | 262        | 74.5   | 51.5 |
| credit-card   | 30000 | 334        | 7.5    | 4    |
| ionosphere    | 350   | 564        | 3      | 5.5  |
| PIMA          | 760   | 134        | 5      | 9    |
| parkinsons    | 190   | 392        | 6.5    | 6    |
| Trans         | 740   | 64         | 6      | 4    |
Figure: Plot demonstrating behavior of training and test accuracy vs Size of Training data for WDBC.
Figure: Plot demonstrating monotone behavior of training accuracy vs $\lambda$ for CNF and DNF rules with $k = 1$ and 2.
Part III

Conclusion
• Need for interpretable machine learning systems for usage of AI in core public functions
• The learning task is offline, so allows usage of formal reasoning tools that can provide certificate of correctness
• Long history of prior work: Heuristics to work around combinatorial hardness of optimization problems
• The success of MaxSAT solvers offers opportunity to design techniques with rigorous formal guarantees
• MLIC introduces an approach to use MaxSAT solvers to compute small CNF/DNF rules
Call to the MaxSAT community

Incremental Solving

• The performance of MaxSAT solvers degrade as the problem size increases.
  • For training data of size $|D|$ MLIC constructs a query of size $|D| \times k$ to learn k-clause rules
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Encodings

• Boolean formulas can express any function.
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