Deconfining phase transition in the 3D Georgi-Glashow model with finite Higgs-boson mass

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Abstract

The (2+1)D Georgi-Glashow model is explored at finite temperature in the regime when the Higgs boson is not infinitely heavy. The resulting Higgs-mediated interaction of monopoles leads to the appearance of a certain upper bound for the parameter of the weak-coupling approximation. Namely, when this bound is exceeded, the cumulant expansion used for the average over the Higgs field breaks down. The finite-temperature deconfining phase transition with the account for the same Higgs-mediated interaction of monopoles is further analysed. It is demonstrated that in the general case, accounting for this interaction leads to the existence of two distinct phase transitions separated by the temperature region where W-bosons exist in both, molecular and plasma, phases. The dependence of possible ranges of the critical temperatures corresponding to these phase transitions on the parameters of the Georgi-Glashow model is discussed. The difference in the RG behaviour of the fugacity of W-bosons from the respective behaviour of this quantity in the compact-QED limit of the model is finally pointed out.

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1 Introduction

Although the confining properties of the (2+1)D Georgi-Glashow model are known since the second half of the seventies [1], its finite-temperature properties were addressed only recently, in refs. [2]-[6]. In ref. [3], it has been shown that at the temperature equal to $g^2/2\pi$, where $g$ stands for the electric coupling constant, the monopole plasma undergoes the Berezinsky-Kosterlitz-Thouless (BKT) phase transition [4] to the molecular phase. In refs. [3], [4], the relevance of the charged

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W-plasma to the dynamics of the deconfining phase transition in the Georgi-Glashow model has been pointed out. In particular, in ref. [3], it has been shown that the phase transition is associated with the deconfinement of W-bosons, belongs to the 2D-Ising universality class and occurs at the temperature approximately equal to $g^2/4\pi$. Further, in ref. [4], various physical aspects of this phase transition, as well as of the analogous transition in the $SU(N)$-generalization of the Georgi-Glashow model with $N > 2$ have been studied. In ref. [5], the monopole BKT phase transition has been explored in the presence of dynamical massless fundamental quarks. In this way, it has been shown that the presence of one quark flavour makes the critical temperature of this phase transition equal to $g^2/4\pi$, while in the presence of more than one flavour, the respective critical temperature becomes exponentially small.

In ref. [6], the influence of the Higgs field to the dynamics of the Georgi-Glashow model has been studied both at zero and nonzero temperature. In particular, it has been found that the finiteness of the Higgs-boson mass does not change the value of the critical temperature of the monopole BKT phase transition. However, in ref. [3] the effects of W-bosons have been disregarded, that makes the performed analysis incomplete. The aim of the present letter is to explore the influence of the Higgs-mediated interaction of monopoles to the deconfining phase transition with the account for W-bosons. The phase transition occurs when the density of monopoles becomes equal to the one of W-bosons. Up to inessential subleading corrections, this takes place when the exponent of the monopole fugacity is equal to that of the fugacity of W-bosons [3]. Another way to understand why the phase transition occurs when the two fugacities are equal to each other is to notice that once this happens, the thickness of the string confining two W’s (which is proportional to (monopole fugacity)$^{-1/2}$) becomes equal to the average distance between the W’s (proportional to (fugacity of W’s)$^{-1/2}$). This qualitative result was also confirmed by the RG analysis performed in ref. [3].] On the other hand, the average over the Higgs field in the dimensionally-reduced theory (one works with at finite temperatures) changes the monopole fugacity. Owing to this effect, the critical temperature of the deconfining phase transition changes as well. Moreover, we shall see that the average over the Higgs field makes the monopole fugacity temperature-dependent. Due to that, comparison of the exponents of two fugacities yields no more a single value of the critical temperature, but rather a quadratic equation for this temperature. Consequently, in general, one gets two distinct critical temperatures. We shall discuss possible ranges of these temperatures and also the modification of the RG behaviour of the model due to the existence of two phase transitions instead of one. Besides that, we shall see that the requirement of convergence of the cumulant expansion, one should demand in the course of the average over the Higgs field, leads to a certain upper bound for the parameter of the weak-coupling approximation.

The organization of the letter is the following. In the next Section, we shall consider the dual theory describing the (2+1)D Georgi-Glashow model at finite temperature and the peculiarities of the average over the Higgs field in that theory. In Section 3, there will be discussed the deconfining phase transition and the RG properties of the model. The main results of the letter will then be summarized in the Conclusions. In the Appendix A, some technical details necessary for the evaluation of a certain integral will be outlined.

2 3D Georgi-Glashow model at finite temperature beyond the compact-QED limit

The Euclidean action of the (2+1)D Georgi-Glashow model reads [1]
\[ S = \int d^3x \left[ \frac{1}{4g^2} (F_{\mu\nu}^a)^2 + \frac{1}{2} (D_\mu \Phi^a)^2 + \frac{\lambda}{4} ((\Phi^a)^2 - \eta^2)^2 \right]. \]  

(1)

Here, the Higgs field \( \Phi^a \) transforms by the adjoint representation, \( D_\mu \Phi^a \equiv \partial_\mu \Phi^a + \varepsilon^{abc} A^b_\mu \Phi^c \). Next, \( \lambda \) is the Higgs coupling constant of dimensionality [mass], \( \eta \) is the Higgs v.e.v. of dimensionality [mass]^{1/2}, and \( g \) is the electric coupling constant of the same dimensionality.

At the one-loop level, the sector of the theory (I) containing dual photons and Higgs bosons is represented by the following partition function (8):

\[ Z = 1 + \sum_{N=1}^\infty \frac{\zeta^N}{N!} \left( \prod_{i=1}^N \int d^3z \sum_{q_i=\pm1} \right) \exp \left[ -\frac{g^2}{8\pi} \sum_{a,b=1}^N \left( \frac{q_a q_b}{|z_a - z_b|} - \epsilon^{-m_H |z_a - z_b|} \right) \right] \equiv \int \mathcal{D}\chi \mathcal{D}\psi e^{-S}, \]  

(2)

where

\[ S = \int d^3x \left[ \frac{1}{2} (\nabla \chi)^2 + \frac{1}{2} (\nabla \psi)^2 + \frac{m_W^2}{2} \psi^2 - 2\zeta \epsilon \cos (g_m \chi) \right] \equiv \int d^3x \mathcal{L}[\chi, \psi; g_m, \zeta]. \]  

(3)

Clearly, this partition function describes the grand canonical ensemble of monopoles with the account for their Higgs-mediated interaction. In eqs. (2) and (3), \( \chi \) is the dual-photon field, and the field \( \psi \) accounts for the Higgs field, whose mass reads \( m_H = \eta \sqrt{2\lambda} \). Note that from eq. (2) it is straightforward to deduce that when \( m_H \) formally tends to infinity, one arrives at the conventional sine-Gordon theory of the dual-photon field (I) describing the compact-QED limit of the model.

Next, in the above equations, \( g_m \) stands for the magnetic coupling constant related to the electric one as \( g_m g = 4\pi \), and the monopole fugacity \( \zeta \) has the form: \( \zeta = \frac{m_H^{7/2}}{g} \delta \left( \frac{\lambda}{g^2} \right) e^{-4\pi m_W \epsilon / g^2} \). In this formula, \( m_W = g\eta \) is the W-boson mass, and \( \epsilon = \epsilon(\lambda / g^2) \) is a certain monotonic, slowly varying function, \( \epsilon \geq 1, \epsilon(0) = 1 \) (8), \( \epsilon(\infty) \approx 1.787 \) (9). As far as the function \( \delta \) is concerned, it is determined by the loop corrections. It is known (9) that this function grows in the vicinity of the origin [i.e., in the Bogomolny-Prasad-Sommerfield (BPS) limit (I)]. However, the speed of this growth is so that it does not spoil the exponential smallness of \( \zeta \) in the standard weak-coupling regime \( g^2 \ll m_W \) which we shall imply throughout this letter.

At finite temperature \( T \equiv 1/\beta \), one should supply the fields \( \chi \) and \( \psi \) with the periodic boundary conditions in the temporal direction, with the period equal to \( \beta \). Owing to that, the lines of the magnetic field emitted by a monopole cannot cross the boundary of the one-period region and consequently, at the distances larger than \( \beta \), should go almost parallel to this boundary, approaching it. Therefore, monopoles separated by such distances interact via the 2D Coulomb potential, rather than the 3D one. Since the average distance between monopoles in the plasma is of the order of \( \zeta^{-1/3} \), we see that at \( T \geq \zeta^{1/3} \), the monopole ensemble becomes two-dimensional. Owing to the fact that \( \zeta \) is exponentially small in the weak-coupling regime under discussion, the idea of dimensional reduction is perfectly applicable at the temperatures of the order of \( g^2 \), i.e., the critical ones (cf. the Introduction). The factor \( \beta \) at the action of the dimensionally-reduced theory, \( S_{d-\text{r.}} = \beta \int d^3x \mathcal{L}[\chi, \psi; g_m, \zeta] \), can be removed [and this action can be cast to the original form of eq. (I) with the substitution \( d^3x \rightarrow d^2x \)] by the obvious rescaling:

\[ S_{d-\text{r.}} = \int d^2x \mathcal{L} \left[ \chi^{\text{new}}, \psi^{\text{new}}; \sqrt{T} g_m, \beta \zeta \right]. \]

Here, \( \chi^{\text{new}} = \sqrt{\beta} \chi, \psi^{\text{new}} = \sqrt{\beta} \psi \), and in what follows we
shall denote for brevity $\chi^{\text{new}}$ and $\psi^{\text{new}}$ simply as $\chi$ and $\psi$, respectively. Averaging then over the field $\psi$ with the use of the cumulant expansion we arrive at the following action:

$$S_{\text{d.-t.}} \simeq \int d^2 x \left[ \frac{1}{2} (\nabla \chi)^2 - 2 \xi \cos \left( g_m \sqrt{T} \chi \right) \right] - 2 \xi^2 \int d^2 x d^2 y \cos \left( g_m \sqrt{T} \chi(x) \right) \mathcal{K}(x-y) \cos \left( g_m \sqrt{T} \chi(y) \right).$$

(4)

In this expression, we have disregarded all the cumulants higher than the quadratic one, and the limits of applicability of this so-called bilocal approximation will be discussed below. In eq. (4), $\mathcal{K}(x) \equiv e^{g_m^2 T_D m_H (x)} - 1$ with $D_{m_H}(x) \equiv K_0(m_H|x|)/2\pi$ being the 2D Yukawa propagator ($K_0$ here is the modified Bessel function), and $\xi \equiv \beta \xi e^{g_m^2 T_D m_H(0)}$ denotes the monopole fugacity modified by the interaction of monopoles via the Higgs field. Clearly, in the compact-QED limit (when $m_H$ formally tends to infinity) $D_{m_H}(0)$, being equal to $\int \frac{e^{g_m^2 T_D}}{(2\pi)^2} \frac{1}{p^2+m_H^2}$, vanishes already before doing the integration, and $\xi \to \zeta$, as it should be. In the general case, when the mass of the Higgs field is moderate and does not exceed $m_W$, which in the weak-coupling regime plays the role of the UV cutoff, $\xi \propto \exp \left[ -\frac{4\pi}{e} \left( m_W^2 \epsilon + T \ln \left( \frac{g^2}{\sqrt{\pi}} c \right) \right) \right]$. Here, we have introduced the notation $c \equiv m_H/m_W$, $c < 1$, and $\gamma \simeq 0.577$ is the Euler constant, so that $\frac{\gamma}{2} \simeq 0.890 < 1$. We see that the modified fugacity remains exponentially small, provided that

$$T < -\frac{m_W \epsilon}{\ln \left( \frac{\gamma}{2} c \right)}.$$  

(5)

This constraint should be updated by another one, which would provide the convergence of the cumulant expansion applied in the course of the average over $\psi$. In order to get this new constraint, notice that the parameter of the cumulant expansion reads $\xi I$, where $I \equiv \int d^2 x \mathcal{K}(x)$. The integral $I$ is evaluated in the Appendix A and has the following form:

$$I \simeq \frac{2\pi}{m_H^2} \left[ \frac{1}{2} \left( c^2 - 1 + \left( \frac{2}{e^\gamma} \right)^{\frac{8\pi T}{g^2}} \frac{1 - c^{2 - \frac{8\pi T}{g^2}}}{1 - \frac{8\pi T}{g^2}} \right) + e^{\frac{a}{e}} - 1 + \frac{a}{e} \right].$$

(6)

(Note that at $T \to g^2/4\pi$, $\frac{1}{2} - \frac{2 - \frac{8\pi T}{g^2}}{1 - \frac{8\pi T}{g^2}} \to -2 \ln c$, i.e., $I$ remains finite.) In the derivation of this expression, the parameter $a \equiv 4\pi \sqrt{2\pi T}/g^2$ was assumed to be of the order of unity. That is because the critical temperature of the deconfining phase transition, we are interested with, cannot exceed the critical temperature of the monopole BKT phase transition, $g^2/2\pi$. In fact, above the point of the BKT phase transition, the monopole ensemble passes to the molecular phase and loses its confining properties (in particular, with respect to $W$’s).

Due to the exponential term in eq. (6), the violation of the cumulant expansion may occur at high enough temperatures [that parallels the above-obtained constraint (5)]. The most essential, exponential, part of the parameter of the cumulant expansion thus reads

$$\xi I \propto \exp \left[ -\frac{4\pi}{g^2} \left( m_W^2 \epsilon + T \ln \left( \frac{g^2}{\sqrt{\pi}} c \right) - T \frac{\sqrt{2\pi}}{e} \right) \right].$$

\[1\]In another words, $\xi$ vanishes together with $\zeta$ above the BKT critical temperature. This is another reflection of the fact that confining strings disappear (i.e., become infinitely thick) in that phase, since their thickness is proportional to $\xi^{-1/2}$. 
Therefore, the cumulant expansion converges at the temperatures obeying the inequality

\[ T < \frac{m_W \epsilon}{\sqrt{2\pi \epsilon} - \ln \left( \frac{\epsilon}{2} c \right)}, \]

which updates the inequality (5). Since, as it has been just discussed in the preceding paragraph, the temperatures we are working with do not exceed \( g^2 / 2\pi \), it is enough to demand the following upper bound on the parameter of the weak-coupling approximation, \( g^2 / m_W \):

\[ \frac{g^2}{m_W} < \frac{2\pi \epsilon}{\sqrt{2\pi \epsilon} - \ln \left( \frac{\epsilon}{2} c \right)}. \]

Note that although this inequality is satisfied automatically at \( \frac{\epsilon}{2} c \sim 1 \), since it then takes the form \( \frac{g^2}{m_W} < \sqrt{2\pi \epsilon} \), this is not so for the BPS limit, \( c \ll 1 \). Indeed, in such a case, we have \( \frac{g^2}{m_W} \ln \left( \frac{\epsilon}{2c} \right) < 2\pi \epsilon \), that owing to the logarithm is however quite feasible.

3 Critical temperatures of the deconfining phase transition

We are now in the position to explore the influence of the Higgs-mediated interaction of monopoles to the critical temperature of the deconfining phase transition. As it has already been discussed in the Introduction, this phase transition occurs when the density of monopoles, approximately equal to \( 2\xi \), becomes of the same order of magnitude as the density of W-bosons [3]. The latter can be evaluated as follows (see e.g. ref. [12]):

\[ \rho_W = -\frac{\partial}{\partial \bar{\mu}} \left[ 6T \int \frac{d^2 p}{(2\pi)^2} \ln \left( 1 - e^{\beta(\bar{\mu} - \varepsilon(p))} \right) \right] \bigg|_{\bar{\mu} = 0} = 6 \int \frac{d^2 p}{(2\pi)^2} \frac{1}{e^{\beta \varepsilon(p)} - 1} = \]

\[ = \frac{3m_W^2}{\pi} \int_1^\infty \frac{dz z}{e^{m_W \beta z} - 1} \simeq \frac{3m_W^2}{\pi} \int_1^\infty dz z e^{-m_W \beta z} = \frac{3m_W T}{\pi} \left( 1 + \frac{T}{m_W} \right) e^{-m_W \beta}. \]

Here, \( \bar{\mu} \) stands for the chemical potential, \( \varepsilon(p) = \sqrt{p^2 + m_W^2} \), and the factor "6" represents the total number of spin states of \( W^+ \) and \( W^- \)-bosons. We have also denoted \( z \equiv \varepsilon(p) / m_W \) and took into account that the temperatures of our interest are much smaller than \( m_W \) in the weak-coupling regime, since they should not exceed \( g^2 / 2\pi \).

Also, as it has been mentioned in the Introduction, in the evaluation of the critical temperature(s), it is enough to compare the exponents of \( \xi \) and \( \rho_W \), since the preexponential factors yield only the subleading corrections. Then, in the compact-QED limit, \( \xi \rightarrow \zeta \) (cf. the preceding Section), and \( T_c = \frac{g^2}{4\pi\epsilon(\infty)} \) [3]. In the general case under discussion, \( c < 1 \), we obtain the two following distinct values of critical temperatures:

\[ T_{1,2} = g^2 \epsilon \frac{1}{2b} \sqrt{1 - \frac{b}{4\pi^2}}. \]

Here, \( b \equiv -\frac{g^2}{m_W} \ln \left( \frac{\epsilon}{2} c \right), b \geq 0 \), and the indices 1,2 refer to the smaller and the larger temperatures, respectively. The degenerate situation \( T_1 = T_2 = g^2 / 2\pi \epsilon \) then corresponds to \( b = \pi \epsilon^2 \), and, since
increases until the temperature is not equal to \( g = b \mu \) for some value for the existence of a new (metastable) phase at anotherwords, accounting for the interaction of monopoles via the Higgs field opens a possibility one of the monopole BKT phase transition. In the same way, for any \( b \) \( T \), one can see from this equation that if the evolution starts at \( b = \pi (2\epsilon - 1) \). At the values of \( b \) lying in this interval, the phase transition corresponding to the critical temperature \( T_2 \) thus may occur. In the BPS limit, \( T_2 \) can only be equal to \( g^2/2\pi \), that corresponds to the above-discussed case when both critical temperatures coincide with the one of the monopole BKT phase transition. In the same way, for any \( b \leq \pi \epsilon^2 \), \( T_1 \leq g^2/2\pi \), and, in particular, \( T_1 = g^2/\pi \) only in the BPS limit, when \( \epsilon = 1 \). Therefore, the phase transition corresponding to the temperature \( T_1 \) always takes place. Also an elementary analysis shows that for any \( \epsilon > 0 \) (and, in particular, for the realistic values \( \epsilon \geq 1 \)) and \( b < \pi \epsilon^2 \), \( T_1 > g^2/4\pi \epsilon \) (and consequently \( T_2 > g^2/4\pi \) as well). Since \( \epsilon \leq \epsilon(\infty) \), we conclude that both phase transitions always occur at the temperatures which are larger than that of the phase transition in the compact-QED limit.

Obviously, the RG analysis, performed in ref. [3] for the compact-QED limit remains valid, but with the replacement \( \zeta \to \xi \). In particular, the deconfining phase transition corresponds again to the IR unstable fixed point, where the exponent of the W-fugacity, \( \mu \propto \rho_W \), is equal to the exponent of \( \xi \) [that yields the above-obtained critical temperatures (7)]. One can further see that the initial condition \( \mu_{in} < \xi_{in} \) takes place, provided that the initial temperature, \( T_{in} \), is either smaller than \( T_1 \) or lies between \( T_2 \) and \( g^2/2\pi \). For these ranges of \( T_{in} \), the temperature starts decreasing according to the RG equation \( d\lambda / d\lambda = \pi^2 a^4 (\mu^2 - t^2 \xi^2) \). In this equation, \( t = 4\pi T/g^2 \), \( \lambda \) is the evolution parameter, \( \bar{a} \) is some parameter of the dimensionality [length], and for the comparison of \( \mu \) and \( \xi \) the preexponent \( t^2 \) is again immaterial. Then, in the case \( T_{in} < T_1 \), the situation is identical to the one discussed in ref. [3], namely \( \mu \) becomes irrelevant and decreases to zero. Indeed, from the evolution equation for \( \mu \) there follows the equation for \( d\mu / dt \), by virtue of which one can determine the sign of this quantity. It reads

\[
\frac{d\mu}{dt} = \frac{\mu \left(2 - \frac{1}{\xi}\right)}{\pi^2 \bar{a}^4 (\mu^2 - t^2 \xi^2)}.
\]

One can see from this equation that if the evolution starts at \( T_{in} \in (g^2/8\pi, T_1) \), \( \mu \) temporaly increases until the temperature is not equal to \( g^2/8\pi \), but then nevertheless starts vanishing together with the temperature. However, by virtue of the same evolution equations we see that at \( T_{in} \in (T_2, g^2/2\pi) \), the situation is now different. Indeed, in that case, \( \mu \) is not decreasing, but rather increasing with the decrease of the temperature (since \( d\mu / dt < 0 \) at \( T > T_2 \)), until it reaches some value \( \mu_* \sim e^{-\mu_W / T_2} \). Once the temperature becomes smaller than \( T_2 \), the temperature starts increasing again, that together with the change of the sign of \( d\mu / dt \) causes the increase of \( \mu \), and so on. Thus, we see that \( \mu_* \) is the stable local maximum of \( \mu \) for such initial conditions.
4 Conclusions

In this letter, we have explored the consequences of accounting for the Higgs field to the deconfining phase transition in the finite-temperature (2+1)D Georgi-Glashow model. To this end, this field was not supposed to be infinitely heavy, as it takes place in the compact-QED limit of the model. Owing to that, the Higgs field starts propagating and, in particular, causes the additional interaction of monopoles in the plasma. This effect modifies the monopole fugacity, making it temperature-dependent, and leads to the appearance of the novel terms in the action of the dual-photon field. The cumulant expansion applied in the course of the average over the Higgs field is checked to be convergent, provided that the weak-coupling approximation is implied in a certain sense. Namely, the parameter of the weak-coupling approximation should be bounded from above by a certain function of masses of the monopole, W-boson, and the Higgs field.

It has been demonstrated that although in the compact-QED limit there exists only one critical temperature of the phase transition, in general there exist two distinct critical temperatures. We have discussed the dependence of these temperatures on the parameters of the Georgi-Glashow model. In particular, both critical temperatures turn out to be larger than the one of the phase transition in the compact-QED limit. Besides that, it has been demonstrated that the smaller of the two critical temperatures always does not exceed the critical temperature of the monopole BKT phase transition. As far as the larger critical temperature is concerned, there has been found the range of parameters of the Georgi-Glashow model, where it also does not exceed the monopole one. The situation when there exist two phase transitions implies that at the smaller of the two critical temperatures, W-molecules start dissociating, while at the larger one all of them are dissociated completely. This means that in the region of temperatures between the critical ones, the gas of W-molecules coexists with the W-plasma.

From the RG equations, it follows that the presence of the second (larger) critical temperature leads to the appearance of a novel stable value of the W-fugacity. This value is reached if one starts the evolution in the region where the temperature is larger than the above-mentioned critical one, and the density of W’s is smaller than the one of monopoles. The resulting stable value is nonvanishing (i.e., W’s at that point are still of some importance), that is the opposite to the standard situation, which takes place if the evolution starts at the temperatures smaller than the first critical one.

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Appendix A. Evaluation of the integral $\int d^2x K(x)$. 

The desired integral can be written as follows:
\[ I = \frac{2\pi}{m_H^2} \int_c^\infty dx \left[ \exp \left( \frac{8\pi T}{g^2} K_0(x) \right) - 1 \right] \simeq \]
\[ \simeq \frac{2\pi}{m_H^2} \left\{ \int_c^1 dx \left[ \exp \left( -\frac{8\pi T}{g^2} \ln \left( \frac{e^\gamma}{2} x \right) \right) - 1 \right] + \int_1^\infty dx \left[ \exp \left( a e^{-x} \right) - 1 \right] \right\} \equiv \]
\[ = \frac{2\pi}{m_H^2} \left[ \frac{1}{2} \left( e^2 - 1 + \left( \frac{2}{e^\gamma} \frac{8\pi T}{g^2} \frac{1 - e^{-\frac{8\pi T}{g^2}}}{1 - \frac{4\pi T}{g^2}} \right) \right) + J \right] , \quad (A.1) \]

where the notations \( a \) and \( c \) were introduced in the main text. The integral \( J \) here can further be evaluated as

\[ J = \sum_{n=1}^\infty \frac{a^n}{n!} \int_1^\infty dx e^{-nx} x^{1-\frac{n}{2}} = \sum_{n=1}^\infty \frac{a^n}{n!} n^{\frac{n}{2}-2} \Gamma \left( 2 - \frac{n}{2}, n \right) \simeq \sum_{n=1}^\infty \frac{a^n}{nn!} e^{-n} . \quad (A.2) \]

Here, \( \Gamma(a, x) = \int_x^\infty dt e^{-t} t^{a-1} \) is the incomplete Gamma-function, and we have used its asymptotics \( \Gamma(a, x) \simeq x^{a-1} e^{-x} \) at \( x \geq 1 \). One can further evaluate the sum \( (A.2) \) as follows:

\[ (A.2) = \int_0^\infty dt \sum_{n=1}^\infty \frac{a^n e^{-(1+t)n}}{n!} \simeq \int_0^1 dt \sum_{n=1}^\infty \frac{a^n e^{-n}}{n!} + \int_1^\infty dt \sum_{n=1}^\infty \frac{a^n e^{-tn}}{n!} = e^{a/e} - 1 + \int_1^\infty dt \left[ \exp \left( ae^{-t} \right) - 1 \right] \simeq \]
\[ \simeq e^{a/e} - 1 + a \int_1^\infty dt e^{-t} = e^{a/e} - 1 + \frac{a}{e} . \]

Inserting this expression into eq. (A.1) we arrive at eq. (6) of the main text.
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