Equivalent SU(3)$_f$ approaches for two-body anti-triplet charmed baryon decays

Y.K. Hsiao, Y.L. Wang and H.J. Zhao

School of Physics and Information Engineering, Shanxi Normal University, Taiyuan, 030031, China

E-mail: yukuohsiao@gmail.com, 1556233556@qq.com, hjzhao@163.com

ABSTRACT: For the two-body $\mathbf{B}_c \rightarrow \mathbf{B} \mathbf{M}$ decays, where $\mathbf{B}_c$ denotes the anti-triplet charm baryon and $\mathbf{B}(\mathbf{M})$ the octet baryon (meson), there exist two theoretical studies based on the SU(3) flavor [SU(3)$_f$] symmetry. One is the irreducible SU(3)$_f$ approach (IRA). In the irreducible SU(3)$_f$ representation, the effective Hamiltonian related to the initial and final states forms the amplitudes for $\mathbf{B}_c \rightarrow \mathbf{B} \mathbf{M}$. The other is the topological-diagram approach (TDA), where the $W$-boson emission and $W$-boson exchange topologies are drawn and parameterized for the decays. As required by the group theoretical consideration, we present the same number of the IRA and TDA amplitudes. We can hence relate the two kinds of the amplitudes, and demonstrate the equivalence of the two SU(3)$_f$ approaches. We find a specific $W$-boson exchange topology only contributing to $\Xi_c^0 \rightarrow \mathbf{B} \mathbf{M}$. Denoted by $E_M$, it plays a key role in explaining $\mathcal{B}(\Xi_c^0 \rightarrow \Lambda^0 \bar{K}_0^0, \Sigma^0 \bar{K}_0^0, \Sigma^+ K^-)$. We consider that $\Lambda_c^+ \rightarrow n \pi^+$ and $\Lambda_c^+ \rightarrow p \pi^0$ proceed through the constructive and destructive interfering effects, respectively, which leads to $\mathcal{B}(\Lambda_c^+ \rightarrow n \pi^+) \gg \mathcal{B}(\Lambda_c^+ \rightarrow p \pi^0)$ in agreement with the data. With the exact and broken SU(3)$_f$ symmetries, we predict the branching fractions of $\mathbf{B}_c \rightarrow \mathbf{B} \mathbf{M}$ to be tested by future measurements.

KEYWORDS: Branching fraction, Charm Physics, Flavour Physics, $e^+e^-$ Experiments

ArXiv ePrint: 2111.04124
1 Introduction

In recent years, more and more charmed baryon decay channels have been reanalyzed and discovered [1–8]. The most observations come from $\mathbf{B}_c \rightarrow \mathbf{B} \mathbf{M}$, where $\mathbf{B}_c$ denotes the anti-triplet charmed baryon state, and $\mathbf{B} (\mathbf{M})$ the octet baryon (meson). For example, Belle has newly reported the observation of $\Xi_0^0 \rightarrow (\Lambda^0 K^0_S, \Sigma^0 K^0_S, \Sigma^+ K^-)$ [7], and BESIII that of $\Lambda^+_c \rightarrow n \pi^+$ [8]. For investigation, theoretical attempts have been given, such as the factorization [9], pole model [10, 11], quark model [12–14], current algebra [15], and final state interaction [16, 17], whose calculations might be complicated. As the alternative approach, the SU(3) flavor symmetry can avoid the model calculation.

There exist two SU(3) approaches for the $\mathbf{B}_c$ decays. One is the irreducible SU(3) approach (IRA), where the effective Hamiltonian ($\mathcal{H}_{\text{eff}}$) for the $c$ quark decays can be presented as the irreducible SU(3) expression. As a consequence, $\mathcal{H}_{\text{eff}}$ connected to $\mathbf{B}_c$ and the final states induces the SU(3) invariant amplitudes [18–35]. The other is the topological-diagram approach (TDA) [36–40], where the W-boson emission ($W_{\text{EM}}$) and W-boson exchange ($W_{\text{EX}}$) effects can be drawn and parameterized as the topological amplitudes.

It is reasonable to regard IRA and TDA as the equivalent approaches for the heavy hadron decays [37, 39, 41–44]. However, He and Wang first point out that the previous analyses using IRA and TDA could not consistently match [42]. In the two-body $D$ and $B$ decays, one seeks the overlooked TDA amplitudes to solve the mismatch problem [42–44]. In the two-body $\mathbf{B}_c \rightarrow \mathbf{B} \mathbf{M}$ decays, the mismatch problem remains unsolved, which is due to the inconclusive TDA amplitudes involved in the decays [37, 43].

The equivalence should be in accordance with the equal number of the IRA and TDA amplitudes. For instance, one derives two SU(3) amplitudes and two topological ones for $\mathbf{B}_c \rightarrow \mathbf{B}^* \mathbf{M}$ [39], where $\mathbf{B}^*$ denotes the decuplet baryon. Without considering the singlet contributions to the formation of $\eta_1$, there can be seven independent IRA amplitudes in the $\mathbf{B}_c \rightarrow \mathbf{B} \mathbf{M}$ decays [18, 26, 43], whereas TDA leads to six, seven, eight and sixteen topological amplitudes from refs. [15, 36–38] and [43], respectively. Clearly, the unique unification of the two SU(3) approaches is unavailable. Therefore, we propose to clarify how many independent topological amplitudes can actually exist, and newly unify the IRA
2 Formalism

For the anti-triplet charmed baryon decays, we present the effective Hamiltonian for the \( c \to uqq' \) decays as \[45, 46\]
\[
\mathcal{H}_c \equiv \mathcal{H}_{\text{eff}}/(G_F/\sqrt{2}) = \lambda_{q'q} [c_1(\bar{u}q')\langle \bar{q}c \rangle + c_2(\bar{u}_\beta q'_{\alpha})\langle \bar{q}_\alpha c_\beta \rangle], \tag{2.1}
\]
where \( G_F \) is the Fermi constant, and \( \lambda_{q'q} \equiv V_{q'q}V_{eq}^* \) denotes the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. Besides, \( \langle \bar{q}_1 q_2 \rangle = q_1 \gamma_\mu(1 - \gamma_5)q_2 \) are quark currents, and \( (\alpha, \beta) \) represent the color indices. As \( q^{(i)} \) takes the values \( d \) and \( s \), the decays with \( \lambda_{q'q} = [V_{cs}V_{ud}, V_{cs(d)}V_{us(d)}, V_{cd}V_{us}] \approx [1, s_c(-s_c), -s_c^2] \) are regarded as the Cabibbo-allowed (CA), singly Cabibbo suppressed (SCS), and doubly Cabibbo suppressed (DCS) processes, respectively, where \( \theta_c \equiv \sin \theta_c \approx 0.22 \) is the Cabibo angle. Moreover, the Wilson coefficient \( c_{1/2} \) is a scale \( (\mu) \)-dependent number, and we take \( \mu = m_c \) in the \( c \) decays.

By omitting Lorentz structure, \( \mathcal{H}_c \) is seen as \( \mathcal{H}_c \sim (\bar{q}_1 q_2 \bar{q}_k)c \), where \( q_k = (u, d, s) \) is a triplet (3) under the \( SU(3)_f \) symmetry. In IRA, \( \mathcal{H}_c \) is decomposed as two irreducible \( SU(3)_f \) forms, \( 6_H \) and \( \bar{T}_5_H \), whereas TDA only values the flavor changes of \( c \to q_1 \bar{q}_j q_k \) in \( \mathcal{H}_c \). Thus, the effective Hamiltonian can be rewritten as \[18, 19, 26, 39, 40\]
\[
\mathcal{H}_{\text{IRA}} = c_2 \frac{\epsilon^{ijl}}{2} H(6)_{lk} + c_4 H(\bar{T}_5)^{ij}_k, \quad \mathcal{H}_{\text{TDA}} = H_f^{ki}, \tag{2.2}
\]
where \( c_2 = (c_1 \mp c_2) \), and the non-zero entries of \( H(6)_{lk} \), \( H(\bar{T}_5)^{ij}_k \) and \( H_f^{ki} \) are given by \[18, 39\]
\[
H_{22}(6) = 2, \quad H_{23}^{13}(\bar{T}_5) = H_2^{31}(\bar{T}_5) = 1, \quad H_{33}^{21}(\bar{T}_5) = -s_c, \quad H_{31}^{21} = -s_c, \quad H_{33}^{21} = s_c, \quad H_2^{31} = s_c, \quad H_2^{31} = s_c. \tag{2.3}
\]
As the final states, the octet baryon and meson \((8_B \text{ and } 8_M)\) have components
\[
B^j_2 : (n, p, , \Sigma^{\pm, 0}, \Xi^{-, 0}, \Lambda), \quad M^j_2 : (\pi^{\pm, 0}, K^{\pm}, K^0, \bar{K}^0, \eta), \tag{2.4}
\]
where the octet baryon can also be written as \( B_{ijkl} = \epsilon_{ijkl} B^j_1 \). Moreover, the \( \eta \) state mixes \( \eta_q = \sqrt{1/2}(u\bar{u} + dd) \) and \( \eta_s = s\bar{s} \) as \( \eta = \eta_q \cos \phi - \eta_s \sin \phi \), where \( \phi = (39.3 \pm 1.0)^0 \) is the mixing angle \[47, 48\]. For the anti-triplet charmed baryon states \((3_c)\), \( \Xi^0 \), \( \Xi^+_c \) and \( \Lambda^+_c \) consist of \((ds - sd)c\), \((su - us)c\) and \((ud - du)c\), respectively, and we present them in the two forms,
\[
B_c(B_c i) = (\Xi^0_c, -\Xi^+_c, \Lambda^+_c),
\]
\[
B_c(B_c ij) = \begin{pmatrix}
0 & \Lambda^+_c & \Xi^+_c \\
-\Lambda^+_c & 0 & \Xi^0_c \\
-\Xi^+_c & -\Xi^0_c & 0
\end{pmatrix}. \tag{2.5}
\]
We can hence construct the amplitudes of $B_c \to BM$ in IRA and TDA, written as \[36, 38–40, 43\]

\[
\mathcal{M}_{\text{IRA}} = \mathcal{M}_6 + \mathcal{M}_{15},
\]

\[
\mathcal{M}_6 = a_1 H_{ij}(6) T^{ik} B^i_j M^j_k + a_2 H_{ij}(6) T^{ik} M^j_k B^i_j + a_3 H_{ij}(6) B^i_j M^j_k T^{ik},
\]

\[
\mathcal{M}_{15} = a_4 H_k^{(i1)} T (5) B_{cij} M^j_k B^i_k + a_5 B^i_j M^j_k H (15)^{jk} B_{ck}
\]

\[
+ a_6 B^i_j M^j_k H (15)^{jk} B_{ck} + a_7 B^i_j M^j_k H (15)^{jk} B_{ck},
\]

\[
\mathcal{M}_{\text{TDA}} = T B_c^{ab} H^{ki}_j B^i_{ab} M^j_k + C B_c^{ab} H^{ki}_j B^i_{ab} M^j_k + C' B_c^{ab} H^{ki}_j B^i_{ab} M^j_k
\]

\[
+ E B_c^{ab} H^{ki}_j B^i_{ab} M^j_k + E' B_c^{ab} H^{ki}_j B^i_{ab} M^j_k + E'' B_c^{ab} H^{ki}_j B^i_{ab} M^j_k,
\]

\[
\text{where } T^{ij} \equiv B_{c,k} e^{ijk}, a_i (i = 1, 2, \ldots, 7) \text{ are the SU(3)$_f$ invariant amplitudes, and } (T, C', E') \text{ the topological ones.}
\]

For $\mathcal{M}_{\text{IRA}}$, $(c_-, c_+, c)$ in $H_c$ are absorbed into $a_i (i = 1, 2, \ldots, 7)$. Since one obtains $(c_-, c_+, c) = (1.65, 0.79)$ as a result of QCD corrections \[49\], $(a_1, a_2, a_3)$ with $c_-$ \[36, 38–40, 43\] with $c_+$ are regarded as QCD-(dis)favored parameters. Under the group theoretical consideration, we present $3_c \times 6_H \times 8_B \times 8_M$ and $3_c \times 15_H \times 8_B \times 8_M$ for $B_c \to BM$ \[40\], which lead to 3 and 4 SU(3) singlets to be in accordance with $a_{1,2,3}$ and $a_{4,5,6,7}$, respectively. Hence, there can be seven independent IRA amplitudes as those in \[18, 26, 43\].

For $\mathcal{M}_{\text{TDA}}$, the topological amplitudes correspond to the $W$-boson emission ($W_{EM}$) and $W$-boson exchange ($W_{EX}$) diagrams in figure 1(a–c) and figure 1(d–h), respectively. More specifically, $T$ and $C'$ represent the external and internal $W_{EM}$ processes in figure 1a and figure 1b(c), respectively. Compared to $T$ and $C'$, the $W_{EX}$ process ($E$) needs an additional gluon to connect a quark pair in $BM$. Particularly, $E_{B(M)}$ with the subscript $B(M)$ stands for the $W_{EX}$ topology, where the baryon (meson) receives the quark $q_k$ from the $c \to q_k$ transition [see figure 1c(g)]. Moreover, $E'_{B(M)}$ presents the same topology as $E_{B(M)}$ except that their baryon states have different quark orderings, that is, $B \sim (q_a - q_k)q_b [(q_i - q_a)q_b]$ for $E_{B(M)}$ and $B \sim (q_k - q_b)q_a [(q_k - q_i)q_a]$ for $E'_{B(M)}$. By contrast, $E'$ parameterizes the $W_{EX}$ process in figure 1d, where the meson receives no quark to relate to the $W$-boson.

Based on the SU(3)$_f$ symmetry, TDA and IRA should have seven independent amplitudes. Nonetheless, we derive eight TDA amplitudes. Since we find the appearance of $(E' - E_B)$ in the $\mathcal{M}_{\text{TDA}}$ expansion, either $E'$ or $E_B$ can be redundant. We thus reduce the TDA amplitudes to seven by choosing to work with the convention of $E_B = 0$. Using $\mathcal{M}_{\text{IRA}} = \mathcal{M}_{\text{TDA}}$, we find the relations as

\[
(T, C, C') = \left( a_1 + \frac{a_4 + a_6}{2}, -a_1 + \frac{a_4 + a_6}{2}, -2a_1 + 2a_3 \right),
\]

\[
(E_B, E_M, E') = (a_4, -2a_4 - 2a_7, -2a_2 + a_4 + a_7),
\]

\[
\text{where } T^{ij} \equiv B_{c,k} e^{ijk}, a_i (i = 1, 2, \ldots, 7) \text{ are the SU(3)$_f$ invariant amplitudes, and } (T, C', E') \text{ the topological ones.}
\]
we can compute the branching fraction to be used in the numerical analysis, where $m_{\pm} = m_B \mp m_M$, and $\tau_{B_c}$ stands for the $B_c$ lifetime, $\mathcal{M}(B_c \to BM)$ is from the full expansion of $\mathcal{M}_{IRA}$ and $\mathcal{M}_{TDA}$ in tables 1, 2 and 3.
\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
Decay mode & $M_{TDA}$ \\
\hline
$\Lambda_c^+ \to \Lambda^0 \pi^+$ & $-\frac{1}{\sqrt{6}} (4T + C' - E_B - E')$ \\
$\Lambda_c^+ \to \Sigma^0 \pi^+$ & $\frac{1}{\sqrt{6}} (C' + E_B - E')$ \\
$\Lambda_c^+ \to \Sigma^+ \pi^0$ & $-\frac{1}{\sqrt{2}} (C' + E_B - E')$ \\
$\Lambda_c^+ \to \Xi^+ K^+$ & $E'_{(s)}$ \\
$\Lambda_c^+ \to p K^0$ & $2C - E'_M$ \\
$\Lambda_c^+ \to \Xi^+ \eta$ & $\left[ \frac{1}{\sqrt{2}} (C' - E_B + E') c\phi - E'_M^{(s)} s\phi \right]$ \\
\hline
$\Xi^+ \to \Sigma^+ K^0$ & $-2C + C'$ \\
$\Xi^+ \to \Xi^0 \pi^+$ & $-2T - C'$ \\
\hline
$\Xi^0 \to \Lambda^0 K^0$ & $\frac{1}{\sqrt{6}} (2C + C' - E_M - 2E'_M - E')$ \\
$\Xi^0 \to \Sigma^0 K^0$ & $\frac{1}{\sqrt{6}} (2C - C' + E_M + E')$ \\
$\Xi^0 \to \Sigma^+ K^-$ & $-E_M - E'$ \\
$\Xi^0 \to \Xi^0 \pi^0$ & $\frac{1}{\sqrt{2}} (E_B + C')$ \\
$\Xi^0 \to \Xi^0 \eta$ & $\left[ \frac{1}{\sqrt{2}} (E_B - C') c\phi + \left( E'_M^{(s)} + E'_M^{(s)} + E^{(s)} \right) s\phi \right]$ \\
\hline
\end{tabular}
\caption{Cabibbo-allowed (CA) amplitudes in the expansions of $M_{TDA}$ and $M_{IRA}$, and $(s\phi, c\phi) \equiv (\sin \phi, \cos \phi)$.}
\end{table}

3 Numerical analysis

In the numerical analysis, we adopt the CKM matrix elements and $(m_{B_{s(c)}}, \tau_{B_{s(c)}})$ from PDG [1], where

$$(V_{cs}, V_{ud}, V_{us}, V_{cd}) = (1 - \lambda^2/2, 1 - \lambda^2/2, \lambda, -\lambda), \quad (3.1)$$

with $\lambda = s_c = 0.22453 \pm 0.00044$ in the Wolfenstein parameterization. Making use of

$$\chi^2 = \Sigma_i [(B_{th}^{i} - B_{ex}^{i})/\sigma_{ex}^{i}]^2, \quad (3.2)$$

we perform a minimum $\chi^2$-fit. As the theoretical input, $B_{th}$ is calculated with the equations in eq. (2.9) and $M(B_{s(c)} \to B M)$ in tables 1 and 2. As the experimental input, $B_{ex}$ can be found in tables 5 and 6, along with $\sigma_{ex}$ the experimental error. Until very recently, only the upper limit of $B_{ex}(\Lambda_c^+ \to p\pi^0) > 0.8 \times 10^{-4}$ has been reported by Belle in ref. [5], from which the likelihood distribution in figure 7 as a function of $\mathcal{B}(\Lambda_c^+ \to p\pi^0)$ can be used to estimate that $B_{ex}(\Lambda_c^+ \to p\pi^+)$ is $(0.3 \pm 0.3) \times 10^{-4}$.

We perform the global fit in the two scenarios. In the first scenario (S1), we exactly preserve the SU(3)$_f$ symmetry, such that $E_{B,M} = E_{B,M}', E_{M} = E_{M}'$ and $E_{s} = E'$. The topological amplitudes as complex numbers can be written as

$$|T|, |C|e^{i\delta_C}, |C'|e^{i\delta_{C'}}, |E_B|e^{i\delta_{E_B}}, |E_M|e^{i\delta_{E_M}}, |E'|e^{i\delta_E'}, \quad (3.3)$$

where $T$ has been set as a relatively real number, and $E_{M}' = E_{B}$ has been implied in eq. (2.7).
Table 2. Singly Cabibbo-suppressed (SCS) amplitudes in the expansions of $\mathcal{M}_{\text{TDA}}$ and $\mathcal{M}_{\text{IRA}}$.

| Decay mode | $\mathcal{M}_{\text{TDA}}$ | $\mathcal{M}_{\text{IRA}}$ |
|------------|----------------------------|----------------------------|
| $\Lambda^+ \to \Lambda^0 K^+$ | $-\frac{1}{\sqrt{2}} \left( -E_B^{(s)} + E_M^{(s)} + 2T + C^0 \right) s_c$ | $-\sqrt{2} \left( a_1 - 2a_2 + a_3 + \frac{3(s_1 - s_2) + 2a_2}{2} \right) s_c$ |
| $\Lambda^+ \to \Sigma^0 K^+$ | $\frac{1}{\sqrt{2}} \left( E_B^{(s)} + C^0 \right) s_c$ | $\sqrt{2} \left( a_1 - a_3 + \frac{3c_2}{2} \right) s_c$ |
| $\Lambda^+ \to \Sigma^+ K^0$ | $-E_M^{(s)} + C^0) s_c$ | $-2(a_1 - a_3 + \frac{3c_2}{2}) s_c$ |
| $\Lambda^+ \to n\pi^+$ | $(C^0 - E_M) s_c$ | $-2(a_2 + a_3 + \frac{3c_2}{2}) s_c$ |
| $\Lambda^+ \to p\eta^0$ | $\left( \frac{1}{\sqrt{2}}(2C - C^0 - E_B - E_M + E') \right) s_c$ | $\sqrt{2} \left( a_1 + a_2 - \frac{3(s_1 + s_2) + 2}{2} \right) s_c$ |
| $\Lambda^+ \to p\eta$ | $\left( \frac{1}{\sqrt{2}}(2C - C^0 - E_M + E - E') \right) c_\phi$ | $-\sqrt{2} \left( a_1 + a_2 - \frac{3(s_1 + s_2) + 2}{2} \right) s_c$ |

In the second scenario (S2), we test the SU(3)$_f$ symmetry breaking, which is indicated by the ratio of $B(\Xi^-_c \to \Xi^- K^+)$ to $B(\Xi^-_c \to \Xi^- \pi^+)$,

$$R(\Xi^-_c) \equiv \frac{B(\Xi^-_c \to \Xi^- K^+)}{B(\Xi^-_c \to \Xi^- \pi^+)} = s_c^2 \frac{(2T - E_B^{(s)})^2}{(2T - E_B)^2}. \tag{3.4}$$

With $E_B = E_B^{(s)}$, we obtain $R(\Xi^-_c) = s_c^2 \simeq 0.05$ away from the data of $0.03 \pm 0.01$ by two standard deviation. Note that the IRA amplitudes would cause the same deviation, when one investigates $\Xi^-_c \to \Xi^- K^+$ and $\Xi^-_c \to \Xi^- \pi^+$ in IRA without considering the broken SU(3)$_f$ symmetry [22, 25-27, 32]. Since $R(\Xi^-_c)$ suggests the existence of the broken effect, we add $E_M^{(s)} = E_M^{(s)} = |E_B^{(s)}|^{1/2} s_c$ to the parameters in eq. (3.3) for the S2 global fit. We thus determine the parameters in the two scenarios (S1 and S2), given in table 4. Moreover, we present our calculations in tables 5, 6 and 7 for $B(B_c \to BM)$ to be compared with the experimental results and other theoretical calculations.
\[
\begin{array}{|c|c|c|}
\hline
\text{Decay mode} & MTDA & MIRA \\
\hline
\Lambda^+_c \rightarrow pK^+ & (C' - 2C)s_c^2 & -2\left( \frac{a_3 + \frac{a_4 + a_6}{2}}{2} \right)s_c^2 \\
\Lambda^+_c \rightarrow nK^+ & -(C' + 2T)s_c^2 & 2\left( \frac{a_3 + \frac{a_4 + a_6}{2}}{2} \right)s_c^2 \\
\Xi^+_c \rightarrow \Lambda^0 K^+ & \frac{1}{\sqrt{2}}(2C + E_B^{(s)} - 2E^{(s)} + 2T)s_c^2 & -\frac{\sqrt{2}}{2}\left( a_1 - 2a_2 - 2a_3 - \frac{a_4 + a_6 - 2a_7}{2} \right)s_c^2 \\
\Xi^+_c \rightarrow \Sigma^0 K^+ & \frac{1}{\sqrt{2}}(E_B^{(s)} - 2T)s_c^2 & \sqrt{2}\left( a_1 - \frac{a_4 - a_6}{2} \right)s_c^2 \\
\Xi^+_c \rightarrow \Sigma^+ K^0 & (2C - E_M^{(s)})s_c^2 & 2\left( a_1 - \frac{a_4 + a_6}{2} \right)s_c^2 \\
\Xi^+_c \rightarrow n\pi^+ & -E's_c^2 & 2\left( a_2 - \frac{a_4 + a_6}{2} \right)s_c^2 \\
\Xi^+_c \rightarrow p\pi^0 & \frac{1}{\sqrt{2}}(E' - E_B - E_M^{(s)})s_c^2 & \sqrt{2}\left( a_2 + \frac{a_4 + a_6}{2} \right)s_c^2 \\
\Xi^+_c \rightarrow pq & \left[ \frac{1}{\sqrt{2}}(E' + E_B - E_M^{(s)})c\phi + C's\phi \right] s_c^2 & -\sqrt{2}\phi\left( a_2 - \frac{a_4 + 2a_7}{2} \right) - 2\phi\left( a_1 - a_3 - \frac{a_5}{2} \right) s_c^2 \\
\hline
\end{array}
\]

Table 3. Doubly Cabibbo-suppressed (DCS) amplitudes in the expansions of \( \mathcal{M}_{\text{TDA}} \) and \( \mathcal{M}_{\text{IRA}} \).

| \( \chi^2 \) | TDA (S1) | TDA (S2) |
|---|---|---|
| n.d.f. | 4.5 | 5.7 |
| \(|T|\) | 0.23 ± 0.02 | 0.24 ± 0.02 |
| \(|C|\) | 0.26 ± 0.01 | 0.23 ± 0.02 |
| \(|C'|\) | 0.34 ± 0.02 | 0.32 ± 0.03 |
| \(|E_B|\) | 0.22 ± 0.03 | 0.22 ± 0.05 |
| \(|E_B'|\) | 0.37 ± 0.06 |
| \(|E_M|\) | 0.40 ± 0.03 | 0.38 ± 0.03 |
| \(|E'|\) | 0.24 ± 0.02 | 0.23 ± 0.02 |
| \(\delta_C\) | (183.2 ± 9.6)° | (179.5 ± 12.9)° |
| \(\delta_C'\) | (163.7 ± 5.0)° | (149.7 ± 6.7)° |
| \(\delta_{E_B}\) | (-100.3 ± 7.1)° | (-93.6 ± 8.2)° |
| \(\delta_{E_B'}\) | (43.3 ± 8.0)° |
| \(\delta_{E_M}\) | (100.3 ± 8.0)° | (113.2 ± 10.7)° |
| \(\delta_{E'}\) | (-71.1 ± 6.7)° | (-50.1 ± 12.3)° |

Table 4. Fit results of the topological parameters in S1 and S2 for the exact and broken SU(3)_f symmetries, respectively. Besides, \(|T|, |C'|, |E_B|, |E_M|, |E'|\) are in units of GeV^3, and n.d.f the number of degrees of freedom.
4 Discussions and conclusions

Under the group theoretical consideration, we present the seven SU(3)$_f$ singlets for $B_c \to BM$, which are in agreement with the seven independent IRA amplitudes in eq. (2.6). Since TDA also relies on the SU(3)$_f$ symmetry, there should exist seven independent TDA amplitudes. We draw and parameterize eight TDA amplitudes as those using the topological-diagram scheme [36, 37]. By finding that either $E'$ or $E'_B$ is redundant, we reduce them to seven. We hence obtain the unique relations in eqs. (2.7) and (2.8), and demonstrate that TDA and IRA are the equivalent SU(3)$_f$ approaches.

Confusingly, there can be six, seven and sixteen topological amplitudes from refs. [15, 38] and [43], respectively. In ref. [38], the less TDA amplitudes reflects the fact that one disregards the quark orderings for $B_c$. It also fails to present the isospin relation: $\mathcal{M}(\Lambda^+_c \to \Sigma^0 \pi^+) = -\mathcal{M}(\Lambda^+_c \to \Sigma^+ \pi^0)$. Although ref. [15] provides the seven TDA parameters, the equality of $\mathcal{M}(\Xi^0_c \to \Xi^- K^+) = -s_c \mathcal{M}(\Xi^0_c \to \Xi^- \pi^+)$ is not given, which disagrees with the SU(3)$_f$ approaches. In ref. [43], the sixteen amplitudes are due to that all possible quark orderings of the octet baryon are taken into account, which can be reduced with the identity $B_{ijk} + B_{kij} + B_{jki} = 0$ [36, 40].

The equivalent SU(3)$_f$ approaches are able to provide more information on the hadronization in the $B_c \to BM$ decays. For instance, we derive $E_M^l = E_B = a_4$ to reduce the topological diagrams involved in the decays, and $a_1 = (T - C)/2$ indicates that $a_1$ connects the two $W$-emission topological amplitudes. With $a_5 = 0$, $a_5$ has no topological correspondence. The QCD-disfavored parameters have been commonly neglected in the numerical analyses [21, 22, 24–30], such as $(a_4, a_5, a_6, a_7)$ in the $B_c \to BM$ decays. According to eq. (2.8), we find that $(a_4, a_6, a_7)$ are associated with the topological parameters which can be sizeable. Therefore, it is unlikely that the QCD disfavored parameters are negligible, whereas they were commonly discarded.

In eq. (3.4), since $\mathcal{R}(\Xi^0_c)$ has indicated that one cannot explain $B_{ex}(\Xi^0_c \to \Xi^- K^+)$ with the exact SU(3)$_f$ symmetry, it has been excluded in the S1 global fit. As a result, $\chi^2/n.d.f = 0.9$ presents a reasonable fit. We also include $B_{ex}(\Xi^0_c \to \Xi^- K^+)$ for a test, which causes $\chi^2/n.d.f = 5.5$. We hence add $E_B = E_M^l = |E_B^*|e^{i\delta_{Bq}}$ as the new parameters in the S2 global fit, in order to accommodate $B_{ex}(\Xi^0_c \to \Xi^- K^+)$. It turns out to be a reasonable fit with $\chi^2/n.d.f = 1.4$, and $B_{th}(\Xi^0_c \to \Xi^- K^+) = (4.1 \pm 2.8) \times 10^{-4}$ can explain the data. Moreover, we extract $|n_q| = 1.2 \pm 0.4$ and $\delta_{nq} = (50.3 \pm 11.5)^\circ$ in $E_B^* = |n_q| e^{i\delta_{nq}} E_B$ as the measure of the SU(3)$_f$ symmetry breaking. One can study the broken SU(3)$_f$ symmetry with IRA [19, 23]. Without the topological indication in eq. (3.4), three IRA amplitudes should be introduced for the broken effects in $B_c \to BM$ [23].

As can be seen in tables 5 and 6, the pole model [10, 11], IRA with the neglecting of $(a_4, a_5, a_7)$ [25], and TDA of this work (S1) all lead to $\mathcal{R}(\Xi^0_c) \simeq 0.05$. It seems that TDA of ref. [38] gives the consistent $\mathcal{R}(\Xi^0_c) \simeq 0.03$; it is, however, based on $\mathcal{M}(\Xi^0_c \to \Xi^- K^+) \neq -s_c \mathcal{M}(\Xi^0_c \to \Xi^- \pi^+)$ inconsistent with the other SU(3)$_f$ approaches. By contrast, the recent global fit using IRA without neglecting the QCD-disfavored parameters can present $\mathcal{R}(\Xi^0_c) \simeq 0.04$ close to the data [31], where the amplitudes are considered to depend on the actual particle masses.
According to our numerical results, the equivalent SU(3)$_f$ approaches are able to interpret the new data, that is, $\mathcal{B}(\Xi^- \to \Lambda^0 K^0, \Sigma^0 K^0, \Sigma^+ K^-)$ and $\mathcal{B}(\Lambda_c^+ \to n\pi^+, p\pi^0, pn)$. Particularly, we find that the topology $E_M$ only contributes to $\Xi^0 \to BM$, but neglected

\[ \mathcal{B}(\Xi^0 \to \Lambda^0 K^0, \Sigma^0 K^0, \Sigma^+ K^-) \]

\[ \mathcal{B}(\Lambda_c^+ \to n\pi^+, p\pi^0, pn) \]

Table 6. Singly Cabibbo-suppressed (SCS) branching fractions.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
Branching fraction & pole model [10, 11] & IRA [25]$^1$ & IRA [31] & TDA [38] & TDA [This work] (S1, S2) & Data \\
\hline
$10^6\mathcal{B}(\Lambda_c^+ \to \Lambda^0 \pi^0)$ & 1.30 & $1.27 \pm 0.07$ & $1.307 \pm 0.069$ & $1.32 \pm 0.34$ & $(1.27 \pm 0.22, 1.24 \pm 0.30)$ & $1.31 \pm 0.09$ \cite{8} \\
$10^6\mathcal{B}(\Lambda_c^+ \to \Sigma^0 \pi^0)$ & 2.24 & $1.26 \pm 0.06$ & $1.272 \pm 0.056$ & $1.26 \pm 0.32$ & $(1.22 \pm 0.23, 1.24 \pm 0.30)$ & $1.22 \pm 0.11$ \cite{8} \\
$10^6\mathcal{B}(\Lambda_c^+ \to \Sigma^+ \pi^0)$ & 2.24 & $1.26 \pm 0.06$ & $1.283 \pm 0.057$ & $1.23 \pm 0.17$ & $(1.22 \pm 0.23, 1.24 \pm 0.30)$ & $1.25 \pm 0.10$ \cite{1} \\
$10^6\mathcal{B}(\Lambda_c^+ \to \Xi^0 K^-)$ & 0.73 & $0.57 \pm 0.09$ & $0.548 \pm 0.063$ & $0.59 \pm 0.17$ & $(0.54 \pm 0.07, 0.51 \pm 0.07)$ & $0.55 \pm 0.07$ \cite{1} \\
$10^6\mathcal{B}(\Lambda_c^+ \to pK^0)$ & 2.11 & $3.14 \pm 0.15$ & $3.174 \pm 0.154$ & $3.14 \pm 1.00$ & $(3.18 \pm 0.64, 3.10 \pm 0.80)$ & $3.18 \pm 0.16$ \cite{1} \\
$10^6\mathcal{B}(\Lambda_c^+ \to \Sigma^+ \eta)$ & 0.74 & $0.29 \pm 0.12$ & $0.45 \pm 0.19$ & $0.47 \pm 0.22$ & $(0.42 \pm 0.18, 0.57 \pm 0.26)$ & $0.44 \pm 0.20$ \cite{1} \\
\hline
$10^6\mathcal{B}(\Xi^0 \to \Sigma^0 K^0)$ & 2.0 & $7.8^{+10.2}_{-7.8}$ & $10.6 \pm 14.0$ & $24.1 \pm 7.1$ & $(12.7 \pm 7.0, 14.7^{+10.7}_{-6.4})$ & $16.0 \pm 8.0$ \cite{7} \\
$10^6\mathcal{B}(\Xi^0 \to \Xi^0 \pi^0)$ & 17.2 & $4.2 \pm 1.7$ & $5.4 \pm 1.8$ & $9.3 \pm 3.6$ & $(7.0^{+1.3}_{-2.6}, 5.1 \pm 6.2)$ & $26.7 \pm 3.0$ \cite{7} \\
\hline
$10^6\mathcal{B}(\Xi^- \to \Lambda^0 K^0)$ & 13.3 & $14.2 \pm 0.09$ & $6.68 \pm 1.30$ & $8.3 \pm 5.0$ & $(9.85 \pm 2.26, 10.0 \pm 2.9)$ & $8.24 \pm 2.44$ \cite{7} \\
$10^6\mathcal{B}(\Xi^- \to \Sigma^0 K^0)$ & 0.4 & $9.8^{+3.6}_{-1.9}$ & $1.38 \pm 0.48$ & $7.9 \pm 4.8$ & $(1.48^{+1.27}_{-0.92}, 1.46^{+1.57}_{-1.09})$ & $1.38 \pm 0.48$ \cite{7} \\
$10^6\mathcal{B}(\Xi^- \to \Sigma^+ K^0)$ & 4.6 & $7.6 \pm 1.4$ & $2.21 \pm 0.68$ & $22.0 \pm 5.7$ & $(2.21^{+3.67}_{-1.97}, 2.25^{+1.74}_{-1.37})$ & $2.21 \pm 0.68$ \cite{7} \\
$10^6\mathcal{B}(\Xi^- \to \Xi^- \eta)$ & 18.2 & $10.0 \pm 1.4$ & $2.56 \pm 0.93$ & $4.7 \pm 0.9$ & $(6.0 \pm 1.2, 3.6 \pm 1.2)$ & $18.0 \pm 5.2$ \cite{7} \\
$10^6\mathcal{B}(\Xi^- \to \Xi^- \pi^0)$ & 64.7 & $29.5 \pm 1.4$ & $12.1 \pm 2.1$ & $19.3 \pm 2.8$ & $(24.5 \pm 3.7, 23.3 \pm 4.5)$ & $18.0 \pm 5.2$ \cite{7} \\
$10^6\mathcal{B}(\Xi^- \to \Xi^- \eta)$ & 26.7 & $13.0 \pm 2.3$ & $8.3 \pm 2.3$ & $(4.2^{+11.1}_{-3.1}, 7.3 \pm 3.2)$ & $18.0 \pm 5.2$ \cite{7} \\
\hline
\end{tabular}
\caption{Cabibbo-allowed (CA) branching fractions.}
\end{table}
in the pole model of ref. [11] and IRA of ref. [25]. $E_M$ plays a key role in explaining $B(\Xi^0_c \to \Lambda^0 K^0, \Sigma^0 K^0, \Sigma^+ K^-)$. For example, since $E_M = -2a_4 + 2a_7$ largely cancels $E' = -2a_2 + a_4 + a_7$ in $\mathcal{M}(\Xi^0_c \to \Sigma^+ K^-) = (E_M + E') s_c$, $B(\Xi^0_c \to \Sigma^+ K^-)$ can be as small as $2.0 \times 10^{-3}$. As a test, we set $E_M = 0$, which results in $B(\Xi^0_c \to \Lambda^0 K^0, \Sigma^0 K^0, \Sigma^+ K^-) \simeq (19.3, 4.6, 4.4) \times 10^{-3}$ deviating from the data.

For $\Lambda^+_c \to n\pi^+$ and $\Lambda^+_c \to p\pi^0$, the pole model predicts the destructive interferences between the factorizable and non-factorizable amplitudes, leading to $B_{el}(\Lambda^+_c \to n\pi^+, p\pi^0) \simeq (2.7, 1.3) \times 10^{-4}$ [10], whereas $B_{ex}(\Lambda^+_c \to n\pi^+) \gg B_{ex}(\Lambda^+_c \to p\pi^0)$ [5, 8]. Here, we can present $\mathcal{M}(\Lambda^+_c \to n\pi^+, p\pi^0) \sim (A+B, A-B)$ with $(A, B) = (a_2 + a_3 - a_7, 2a_4/2)$, such that $\Lambda^+_c \to n\pi^+$ and $\Lambda^+_c \to p\pi^0$ can be viewed to proceed through the constructive and destructive interferences, respectively. Since the SU(3)$_f$ symmetry predicts $\mathcal{M}(\Xi^0_c \to \Xi^0 K^+)$ = $\mathcal{M}(\Lambda^+_c \to n\pi^+)$, one should have $B(\Xi^0_c \to \Xi^0 K^+) \simeq R_d B_{ex}(\Lambda^+_c \to n\pi^+) \simeq (12 - 18) \times 10^{-4}$ with $R_d = 2.4$ the ratio of the dynamical factors in eq. (2.9), which is consistent with our predictions in table 6. Therefore, $B(\Xi^0_c \to \Xi^0 K^+)$ compared to future measurements can be used to test the validity of the SU(3)$_f$ approaches.

In summary, we have studied the two-body anti-triplet charmed baryon decays using the irreducible SU(3)$_f$ approach (IRA) and topological-diagram approach (TDA). Due to the group theoretical consideration, we have presented that there can be the same number of the IRA and TDA amplitudes. We have hence found out the unique relations, and demonstrated that IRA and TDA are the equivalent SU(3)$_f$ approaches. We have explained the recently measured branching fractions, that is, $B(\Xi^0_c \to \Lambda^0 K^0, \Sigma^0 K^0, \Sigma^+ K^-)$ and $B(\Lambda^+_c \to n\pi^+, p\pi^0, \eta)$. Moreover, we have predicted the branching fractions of $B_c \to BM$ under the exact and broken SU(3)$_f$ symmetries, which can be tested by future measurements.

Table 7. Doubly Cabibbo-suppressed (DCS) branching fractions.

| Branching fraction | pole model [10, 11]  | IRA [25] | IRA [31] | TDA [38] | TDA [This work] (S1, S2) | Data |
|--------------------|----------------------|----------|----------|----------|--------------------------|------|
| $10^5 B(\Lambda^+_c \to pK^0)$ | 1.2$^{+1.4}_{-1.2}$ | 1.5 $\pm$ 5.6 | 3.7 $\pm$ 1.1 | (1.8$^{+1.0}_{-0.8}, 2.1^{+1.2}_{-1.0}$) | (1.0 $\pm$ 0.5, 2.2 $\pm$ 0.9) |
| $10^5 B(\Lambda^+_c \to nK^0)$ | 0.4 $\pm$ 0.2 | 4.8 $\pm$ 2.2 | 1.4 $\pm$ 0.5 | (1.0 $\pm$ 0.5, 2.2 $\pm$ 0.9) |
| $10^5 B(\Xi^+_c \to \Lambda^0 K^+)$ | 3.3 $\pm$ 0.8 | 3.65 $\pm$ 0.53 | 7.5 $\pm$ 1.9 | (1.3$^{+0.7}_{-0.6}$, 4.6 $\pm$ 2.1) | |
| $10^5 B(\Xi^+_c \to \Sigma^0 K^+)$ | 11.9 $\pm$ 0.7 | 12.23 $\pm$ 0.57 | 7.2 $\pm$ 1.8 | (10.3 $\pm$ 1.6, 3.6 $\pm$ 1.4) | |
| $10^5 B(\Xi^+_c \to \Sigma^+ K^0)$ | 19.5 $\pm$ 1.7 | 34.6 $\pm$ 2.2 | 16.9 $\pm$ 5.4 | (18.1 $\pm$ 3.6, 41.9 $\pm$ 8.1) | |
| $10^5 B(\Xi^+_c \to n\pi^+)$ | 12.1 $\pm$ 2.8 | 13.2 $\pm$ 3.5 | 5.2 $\pm$ 1.5 | (4.7 $\pm$ 0.6, 4.5 $\pm$ 0.6) | |
| $10^5 B(\Xi^+_c \to p\pi^0)$ | 6.0 $\pm$ 1.4 | 40.0 $\pm$ 49.0 | 1.5 $\pm$ 1.5 | (2.8$^{+1.6}_{-1.3}, 4.2^{+3.2}_{-2.4}$) | (7.0 $\pm$ 0.8, 7.9 $\pm$ 1.0) |
| $10^5 B(\Xi^+_c \to \eta\pi)$ | 20.4 $\pm$ 8.4 | 50.0 $\pm$ 120.0 | 16.6 $\pm$ 3.1 | (6.0 $\pm$ 0.8, 3.4 $\pm$ 0.9) | |
| $10^5 B(\Xi^0_c \to \Lambda^0 K^0)$ | 0.6 $\pm$ 0.2 | 0.37 $\pm$ 0.84 | 2.4 $\pm$ 1.4 | (0.1$^{+0.7}_{-0.6}, 0.8^{+0.5}_{-0.6}$) | |
| $10^5 B(\Xi^0_c \to \Sigma^0 K^0)$ | 2.5 $\pm$ 0.2 | 4.12 $\pm$ 0.19 | 2.3 $\pm$ 1.4 | (3.0 $\pm$ 0.9, 7.0 $\pm$ 1.4) | |
| $10^5 B(\Xi^0_c \to \Sigma^- K^+)$ | 6.1 $\pm$ 0.4 | 3.28 $\pm$ 0.58 | 5.5 $\pm$ 0.7 | (6.9 $\pm$ 1.0, 2.4 $\pm$ 0.9) | |
| $10^5 B(\Xi^0_c \to n\pi^0)$ | 1.5 $\pm$ 0.4 | 2.6 $\pm$ 2.7 | 3.3 $\pm$ 0.9 | (1.1$^{+0.7}_{-0.5}, 1.1^{+1.1}_{-0.9}$) | |
| $10^5 B(\Xi^0_c \to p\pi^0)$ | 3.1 $\pm$ 0.7 | 1.4 $\pm$ 43.0 | 7.6 $\pm$ 2.0 | (0.1$^{+0.7}_{-0.6}, 0.8^{+0.5}_{-0.6}$) | |
| $10^5 B(\Xi^0_c \to n\eta)$ | 5.2 $\pm$ 2.1 | 4.2 $\pm$ 8.8 | 4.2 $\pm$ 0.8 | (1.1$^{+0.4}_{-0.2}, 0.8^{+0.4}_{-0.1}$) | |

$^a$We thank the authors of ref. [31] for providing $B(\Lambda^+_c \to pK^0)$, $(\Xi^+_c \to \Sigma^+ K^0)$, and $B(\Xi^0_c \to \Lambda^0(\Sigma^0)K^0)$. 

– 10 –
Acknowledgments

We would like to thank Prof. X. G. He, Prof. C. P. Shen and Dr. Yang Li for useful discussions. We would also like to thank Prof. Wei Wang for valuable comments, and Dr. Jin Sun for pointing out the typos. This work was supported in part by National Science Foundation of China (Grants No. 11675030 and No. 12175128).

Open Access. This article is distributed under the terms of the Creative Commons Attribution License (CC-BY 4.0), which permits any use, distribution and reproduction in any medium, provided the original author(s) and source are credited. SCOAP³ supports the goals of the International Year of Basic Sciences for Sustainable Development.

References

[1] Particle Data Group collaboration, Review of Particle Physics, PTEP 2020 (2020) 083C01 [inSPIRE].

[2] Belle collaboration, Measurement of the branching fraction of $\Lambda_c^+ \rightarrow p\omega$ decay at Belle, Phys. Rev. D 104 (2021) 072008 [arXiv:2108.11301] [inSPIRE].

[3] Belle collaboration, Evidence for the decay $\Omega_c^0 \rightarrow \pi^+\Omega(1200)^- \rightarrow \pi^+ (\bar{K}\Xi)^-$, Phys. Rev. D 104 (2021) 052005 [arXiv:2106.00892] [inSPIRE].

[4] Belle collaboration, Measurements of branching fractions and asymmetry parameters of $\Xi_c^0 \rightarrow \Lambda\bar{K}^0$, $\Xi_c^0 \rightarrow \Sigma\bar{K}^0$, and $\Xi_c^0 \rightarrow \Sigma^+\bar{K}^-$ decays at Belle, JHEP 06 (2021) 160 [arXiv:2104.10361] [inSPIRE].

[5] Belle collaboration, Measurements of the branching fractions of $\Lambda_c^+ \rightarrow p\eta$ and $\Lambda_c^+ \rightarrow p\pi^0$ decays at Belle, Phys. Rev. D 103 (2021) 072004 [arXiv:2102.12226] [inSPIRE].

[6] Belle collaboration, First test of lepton flavor universality in the charmed baryon decays $\Omega_c^0 \rightarrow \Omega^-\ell^+\nu_\ell$ using data of the Belle experiment, Phys. Rev. D 105 (2022) L091101 [arXiv:2112.10367] [inSPIRE].

[7] Belle collaboration, Measurements of the branching fractions of $\Xi_c^0 \rightarrow \Lambda K^0_S$, $\Xi_c^0 \rightarrow \Sigma^0 K^0_S$, and $\Xi_c^0 \rightarrow \Sigma^+ K^-$ decays at Belle, Phys. Rev. D 105 (2022) L011102 [arXiv:2111.08981] [inSPIRE].

[8] BESIII collaboration, Observation of the Singly Cabibbo Suppressed Decay $\Lambda_c^+ \rightarrow n\pi^+$, Phys. Rev. Lett. 128 (2022) 142001 [arXiv:2201.02056] [inSPIRE].

[9] T. Gutsche, M.A. Ivanov, J.G. Körner and V.E. Lyubovitskij, Nonleptonic two-body decays of single heavy baryons $\Lambda_Q$, $\Xi_Q$, and $\Omega_Q$ ($Q = b,c$) induced by $W$ emission in the covariant confined quark model, Phys. Rev. D 98 (2018) 074011 [arXiv:1806.11549] [inSPIRE].

[10] H.-Y. Cheng, X.-W. Kang and F. Xu, Singly Cabibbo-suppressed hadronic decays of $\Lambda_c^+$, Phys. Rev. D 97 (2018) 074028 [arXiv:1801.08625] [inSPIRE].

[11] J. Zou, F. Xu, G. Meng and H.-Y. Cheng, Two-body hadronic weak decays of antitriplet charmed baryons, Phys. Rev. D 101 (2020) 014011 [arXiv:1910.13626] [inSPIRE].

[12] J.G. Körner and M. Krämer, Exclusive nonleptonic charm baryon decays, Z. Phys. C 55 (1992) 659 [inSPIRE].
[13] P.-Y. Niu, J.-M. Richard, Q. Wang and Q. Zhao, Hadronic weak decays of $\Lambda_c$ in the quark model, *Phys. Rev. D* **102** (2020) 073005 [arXiv:2003.09323] [inSPIRE].

[14] Y.-K. Hsiao, L. Yang, C.-C. Lih and S.-Y. Tsai, Charmed $\Omega_c$ weak decays into $\Omega$ in the light-front quark model, *Eur. Phys. J. C* **80** (2020) 1066 [arXiv:2009.12752] [inSPIRE].

[15] S. Groote and J.G. Körner, Topological tensor invariants and the current algebra approach: analysis of 196 nonleptonic two-body decays of single and double charm baryons — a review, *Eur. Phys. J. C* **82** (2022) 297 [arXiv:2112.14599] [inSPIRE].

[16] Y. Yu and Y.-K. Hsiao, Cabibbo-favored $\Lambda_c^+ \to \Lambda a_0(980)^+$ decay in the final state interaction, *Phys. Lett. B* **820** (2021) 136586 [arXiv:2012.14575] [inSPIRE].

[17] H.-W. Ke and X.-Q. Li, A natural interpretation on the data of $\Lambda_c \to \Sigma \pi$, *Phys. Rev. D* **102** (2020) 113013 [arXiv:2008.12163] [inSPIRE].

[18] M.J. Savage and R.P. Springer, SU(3) Predictions for Charmed Baryon Decays, *Phys. Rev. D* **42** (1990) 1527 [inSPIRE].

[19] M.J. Savage, SSU(3) violations in the nonleptonic decay of charmed hadrons, *Phys. Lett. B* **257** (1991) 414 [inSPIRE].

[20] K.K. Sharma and R.C. Verma, SU(3) flavor analysis of two-body weak decays of charmed baryons, *Phys. Rev. D* **55** (1997) 7067 [hep-ph/9704391] [inSPIRE].

[21] C.-D. Lü, W. Wang and F.-S. Yu, Test flavor SU(3) symmetry in exclusive $\Lambda_c$ decays, *Phys. Rev. D* **93** (2016) 056008 [arXiv:1601.04241] [inSPIRE].

[22] C.Q. Geng, Y.K. Hsiao, C.-W. Liu and T.-H. Tsai, Charmed Baryon Weak Decays with SU(3) Flavor Symmetry, *JHEP* **11** (2017) 147 [arXiv:1709.00808] [inSPIRE].

[23] C.Q. Geng, Y.K. Hsiao, C.-W. Liu and T.-H. Tsai, SU(3) symmetry breaking in charmed baryon decays, *Eur. Phys. J. C* **78** (2018) 593 [arXiv:1804.01666] [inSPIRE].

[24] C.-Q. Geng, C.-W. Liu, T.-H. Tsai and Y. Yu, Charmed Baryon Weak Decays with Decuplet Baryon and SU(3) Flavor Symmetry, *Phys. Rev. D* **99** (2019) 114022 [arXiv:1904.11271] [inSPIRE].

[25] C.Q. Geng, C.-W. Liu and T.-H. Tsai, Asymmetries of anti-triplet charmed baryon decays, *Phys. Lett. B* **794** (2019) 19 [arXiv:1902.06189] [inSPIRE].

[26] C.Q. Geng, Y.K. Hsiao, Y.-H. Lin and L.-L. Liu, Non-leptonic two-body weak decays of $\Lambda_c(2286)$, *Phys. Lett. B* **776** (2018) 265 [arXiv:1708.02460] [inSPIRE].

[27] C.Q. Geng, Y.K. Hsiao, C.-W. Liu and T.-H. Tsai, Antitriplet charmed baryon decays with SU(3) flavor symmetry, *Phys. Rev. D* **97** (2018) 073006 [arXiv:1801.03276] [inSPIRE].

[28] C.Q. Geng, Y.K. Hsiao, C.-W. Liu and T.-H. Tsai, Three-body charmed baryon Decays with SU(3) flavor symmetry, *Phys. Rev. D* **99** (2019) 073003 [arXiv:1810.01079] [inSPIRE].

[29] Y.K. Hsiao, Y. Yao and H.J. Zhao, Two-body charmed baryon decays involving vector meson with SU(3) flavor symmetry, *Phys. Lett. B* **792** (2019) 35 [arXiv:1902.08783] [inSPIRE].

[30] C.Q. Geng, C.-W. Liu and T.-H. Tsai, Charmed Baryon Weak Decays with Vector Mesons, *Phys. Rev. D* **101** (2020) 053002 [arXiv:2001.05079] [inSPIRE].

[31] F. Huang, Z.-P. Xing and X.-G. He, A global analysis of charmless two body hadronic decays for anti-triplet charmed baryons, *JHEP* **03** (2022) 143 [arXiv:2112.10556] [inSPIRE].
[32] C.-P. Jia, D. Wang and F.-S. Yu, Charmed baryon decays in SU(3)$_F$ symmetry, *Nucl. Phys. B* **956** (2020) 115048 [arXiv:1910.00876] [inSPIRE].

[33] D. Wang, P.-F. Guo, W.-H. Long and F.-S. Yu, $K^0_S - K^0_L$ asymmetries and CP-violation in charmed baryon decays into neutral kaons, *JHEP* **03** (2018) 066 [arXiv:1709.09873] [inSPIRE].

[34] D. Wang, Sum rules for CP asymmetries of charmed baryon decays in the SU(3)$_F$ limit, *Eur. Phys. J. C* **79** (2019) 429 [arXiv:1901.01776] [inSPIRE].

[35] D. Wang, Generation of SU(3) sum rule for charmed baryon decay, arXiv:2204.05915 [inSPIRE].

[36] Y. Kohara, Quark diagram analysis of charmed baryon decays, *Phys. Rev. D* **44** (1991) 2799 [inSPIRE].

[37] L.-L. Chau, H.-Y. Cheng and B. Tseng, Analysis of two-body decays of charmed baryons using the quark diagram scheme, *Phys. Rev. D* **54** (1996) 2132 [hep-ph/9508382] [inSPIRE].

[38] H.J. Zhao, Y.-L. Wang, Y.K. Hsiao and Y. Yu, A diagrammatic analysis of two-body charmed baryon decays with flavor symmetry, *JHEP* **02** (2020) 165 [arXiv:1811.07265] [inSPIRE].

[39] Y.K. Hsiao, Q. Yi, S.-T. Cai and H.J. Zhao, Two-body charmed baryon decays involving decuplet baryon in the quark-diagram scheme, *Eur. Phys. J. C* **80** (2020) 1067 [arXiv:2006.15291] [inSPIRE].

[40] J. Pan, Y.-K. Hsiao, J. Sun and X.-G. He, SU(3) flavor symmetry for weak hadronic decays of $B_{bc}$ baryons, *Phys. Rev. D* **102** (2020) 056005 [arXiv:2007.02504] [inSPIRE].

[41] D. Zeppenfeld, SU(3) Relations for B Meson Decays, *Z. Phys. C* **8** (1981) 77 [inSPIRE].

[42] G. Buchalla, A.J. Buras and M.E. Lautenbacher, Weak decays beyond leading logarithms, *Rev. Mod. Phys.* **68** (1996) 1125 [hep-ph/9512380] [inSPIRE].

[43] T. Feldmann, P. Kroll and B. Stech, Mixing and decay constants of pseudoscalar mesons, *Phys. Rev. D* **58** (1998) 114006 [hep-ph/9802409] [inSPIRE].

[44] T. Feldmann, P. Kroll and B. Stech, Mixing and decay constants of pseudoscalar mesons: The Sequel, *Phys. Lett. B* **449** (1999) 339 [hep-ph/9812269] [inSPIRE].

[45] H.-n. Li, C.-D. Lu and F.-S. Yu, Branching ratios and direct CP asymmetries in $D \to PP$ decays, *Phys. Rev. D* **86** (2012) 036012 [arXiv:1203.3120] [inSPIRE].

[46] H.-Y. Cheng and C.-W. Chiüang, Revisiting CP-violation in $D \to PP$ and VP decays, *Phys. Rev. D* **100** (2019) 093002 [arXiv:1909.03063] [inSPIRE].