Improving mathematical representation ability in solving word problems through the use of cognitive strategies: orientation, organization, and elaboration

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Abstract. The purpose of this study is to better understand the use of three cognitive strategies, orientation, organization, and elaboration by students in completing word problems. The sample consisted of 54 senior high school students from three different school levels tested twice within the period of two months. The study found progress from the initial test to the final test, which is about the frequency of use of mathematical representation when students solve word problem that is written texts, drawings and mathematical expressions. Scores increase of significantly between initial test and final test in the first and second groups, whereas the control group did not increase significantly. Each group gain a scores on the initial test of 26.7; 26.3 and 27.2, after treatment the scores on the final test became 36.1; 33.7 and 30.2. The study shows that the first group was more effective in using mathematical representations in the classroom than the second group and the control group. Students in the first group, second group, and control group tended to have different abilities in mathematical expressions and written texts when solving algebra and geometry problems, although none satisfactory. In conclusion, The Think Talk-Write + cooperative group had positive effects on students in the final test.

1. Introduction
Mathematical representation is a skill that needs to be improved in order for students to solve word problems. The National Council of Teachers of Mathematics (NCTM) [6] states that representation is a new form that results from the translation of a problem or idea or translation of a diagram or physical models into symbols or words. For example, the translation of visual presentations such as a drawing, chart or tables into mathematical expressions and writing in their own words, both formally and informally (written texts). Representations can help students explain concepts or ideas and make it easier for them to find strategic solutions. In addition, the use of representation can increase flexibility in answering mathematical word problems. But students generally do not have enough mathematical representation skills. MacGregor & Stacey [7] mention that most students still did not formulate correct equations and solutions. By Comparing their written responses in the two tests, we were able to see how students approached the task of formulating an equation for a problem, in what ways they had improved, and what the difficulties were that prevented success.
Therefore, students need strategies to overcome the above problems. Ansari [1] states that there are three student thinking strategies when solving difficult problems: attention control, organizing information and elaboration. Gagne [3] defines, attention control as an individual’s capacity to choose what to pay attention to and what to ignore. In lay terms, the strategy is known as endogenous attention or executive attention. Attention control can be described as an individual’s ability to concentrate. The attention of a student will be higher when the student encounters a question, because the answer requires several associated concepts. So, when a student is faced with a problem, he or she will analyze various ways of completion using the information he or she has. Then in the category of organizing information, students learn to remember information in several ways. Information is divided according to its categories then arranged and organized into patterns that can be used in problem solving. So, if a student is faced with a problem, he or she will organize the patterns, shapes or drawings that are eventually used to solve the problem. The elaboration thinking strategy involves integrating associated lessons learned with previous knowledge so that students are able to change complex information into simple information as if it is a new packet of knowledge. Finally, these techniques help students to create models and eliminate possibilities to solve problems.

A model of learning for improving representation skills is the Think-Talk-Write (TTW) strategy. The TTW strategy utilizes a series of steps: think, talk, and write. The first step is to read the text and take notes individually (Think). Next are discussions within the group to discuss the content of the notes (Talk), and finally is the construction of knowledge from the first two steps of thinking and talking (Write).

The students learn in three groups, the first is the experimental group (Cooperative + TTW), second is the experimental group (Individual + TTW), and finally the control group (Individual). The focal research question of this study asks how the TTW strategy affects the mathematical representation abilities in students.

2. Literature review

2.1. Think-Talk-Write

The think-talk-write strategy builds in time for thought and reflection and for the organization of ideas and testing of those ideas before students are expected to write. When assigned a writing task, students are often expected to begin writing immediately. The talk phase of the think-talk-write strategy allows for exploratory talk “the process of learning without the answers fully intact” [4]. The flow of communication progresses from students engaging in thought or reflective dialogue with themselves, to talking and sharing ideas with one another, to writing. This strategy seems to be particularly effective when students, working in heterogeneous groups of two to six, are asked to explain, summarize, or reflect. Starting first in smaller groups, the student groups then increase in size as the students become more comfortable with the think-talk-write process.

2.2. Mathematical representations

NCTM [6] and Kramarski [5], state that mathematical representation abilities are classified into four categories: (a) verbal arguments based on visual analysis of a chart; (b) verbal arguments based on formal setting; (c) numeric/algebraic arguments; and (d) arguments based on drawings that students added to a graph. Points (a) and (b) are relevant to written texts, point (c) is relevant to mathematical expressions, and point (d) is relevant to drawings. There are three aspects of mathematical representations assessed in the study: the construction of conceptual models such as drawings, tables, graphs and diagrams; the creation of mathematical models (math expressions); and development of verbal arguments based on analysis of formal images and concepts (written texts). Students worked to translate within the four representations: words, tables, graphs, and equations.

Furthermore, Ansari [1] asserts that features of the attention control include: (1) the ability to remember concepts relevant to a problem and (2) the presentation of various alternative solutions before choosing the one considered the most accurate. Features of the organization of information are:
(1) organization of all facts in order to systematize information and (2) the creation of patterns and drawings associated with those in the problem. Features of elaboration are: (1) the transformation of complex information into simple information via models and (2) the construction of a hypothesis to implement proper methods to problem solving.

Based on the above, the characteristics of attention control thinking are relevant to writing in one’s own words, both formally and informally (written text), Organizing information thinking is relevant to visual presentations such as drawings, charts and tables and elaboration thinking is relevant to arranging models (mathematical expressions). Attention control is termed orientation, organization of information is called organization, and elaboration of knowledge is referred to as elaboration.

3. Methodology

3.1. Subject
The subject sample was 54 students of senior high school (class X, ages 15-16 years old) chosen according to school level and basic knowledge level. Eighteen students were chosen at each school level and divided into 3 groups (first experimental group, second experimental group, and control group). Each group consisted of six students. Each group included two students with high initial test scores (H), two with medium initial test score (M), and two with low initial test score (L). Subjects were given an initial test, a treatment, and a final test. All classes involved were mixed-ability classes.

3.2. Test
The test comprised three algebra word problem and three geometry problems varying in difficulty. For the final test, students were divided into 2 groups. The first group consisted of 27 students solving algebra problems and the second group consisted of 27 student solving geometry problems.

| School levels/ Final Test | First group experimental | Second group experimental | Control Group | Total |
|---------------------------|--------------------------|---------------------------|---------------|-------|
|                           | Initial Test             | Initial Test              | Initial Test  |       |
|                           | H | M | L | H | M | L | H | M | L |       |
| High (H)                  | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 18    |
| Middle (M)                | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 18    |
| Low (L)                   | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 18    |
| Total                     | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 54    |
| Algebra test              | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 27    |
| Geometry                  | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 27    |

3.3. Procedure
The students were from three separate senior high schools with different levels in the District of Aceh. Three classes were randomly chosen from each school in order to establish three groups i.e. two experimental groups and one control group consisting of six students each. The first experimental group was treated using the Think-Talk-Write (TTW) strategy + cooperative and the second experimental group was treated using the TTW strategy individually. The control group was treated without any strategy.

3.4. Treatment
All students studied the topics of algebra and geometry. In particular, all classroom students studied: (a) the concept of equations and square equations and the solution of equations, (b) three dimension, and (c) quantitative and qualitative methods of answering problems. The TTW strategy used in the present study was based on the techniques suggested by Huinker & Laughlin [4]. Students working in cooperative groups, translated within four representations: words, tables, graphs, and equations.
Initially, the groups were given one of the four representations for a problem and asked to produce the other three. The groups were encouraged to work together and to discuss the problem. For example, they may be given a table of $x$- and $y$-values and asked to create a story for these data and then draw the graph and write the equation. In contrast, they may be given a graph and asked to create a story, make a table, and write an equation. Students engaged in this activity once or twice a week until they were comfortable translating among the multiple representations.

Examples of the items on the Final Test are (adaptive from, MacGregor & Stacey [7]).

- A group of scouts did a 3-day walk on a long weekend. On Sunday, they walked 7 km farther than they had walked on Saturday. On Monday, they walked 13 km farther than they had walked on Saturday. The total journey was 80 km. How far did they walk on Saturday?
- Jefri washes three cars. The second car takes 7 minutes longer than the first one. The third car takes 11 minutes longer than the first one. Jefri works for 87 minutes altogether. How many minutes does he take to wash the first car?
- Both ABCD and PQRS are congruent cones. The base radius and the height of ABCD cone are $a$ cm and $b$ cm respectively. While $PR = 2 \times AC$, height $RS = x$ cm.
  Investigate whether $x = 2b$? and $QS = 2a$?
  Investigate whether the volume of the bigger cone is two times than the smaller one.

4. Result and discussion

4.1. Recapitulation and explanation of the final test result

| Items Test | Mathematical Representation | Groups of |        |        | Total |
|------------|-----------------------------|-----------|--------|--------|-------|
|            |                             | First     | Second  | Control|       |
|            |                             | Experimental | Experimental |       |       |
| Algebra (1)| Written Texts               | 24        | 23     | 24     | 71    |
|            | Drawing                     | -         | -      | -      | -     |
|            | Math. Express.              | 9         | 7      | 6      | 22    |
| Algebra (2)| Written Texts               | -         | -      | -      | -     |
|            | Drawing                     | 27        | 27     | 26     | 80    |
|            | Math. Express.              | 12        | 11     | 6      | 29    |
| Algebra (3)| Written Texts               | 23        | 21     | 21     | 65    |
|            | Drawing                     | -         | -      | -      | -     |
|            | Math. Express               | 13        | 10     | 7      | 30    |
| Geometry (1)| Written Texts              | 26        | 24     | 22     | 72    |
|            | Drawing                     | -         | -      | -      | -     |
|            | Math. Express               | 20        | 20     | 17     | 57    |
| Geometry (2)| Written Texts              | -         | -      | -      | -     |
|            | Drawing                     | 27        | 27     | 24     | 78    |
|            | Math. Express               | 7         | 5      | 2      | 14    |
| Geometry (3)| Written Texts              | 27        | 25     | 25     | 77    |
|            | Drawing                     | 21        | 24     | 22     | 67    |
|            | Math. Express               | 18        | 13     | 10     | 41    |
| Total      |                             | 254       | 237    | 212    | 703   |
According to Table 2, the frequency of the use of mathematical representations for each group is given in Table 3.

| Mathematical Representation | Experimental: 1 | Experimental: 2 | Control |
|-----------------------------|-----------------|-----------------|---------|
| Written Texts               | 100 (35.1%)     | 93 (32.6%)      | 92 (32.3%) |
| Drawing                     | 75 (33.3%)      | 78 (34.7%)      | 72 (32.0%) |
| Math. Exp.                  | 79 (40.9%)      | 66 (34.2%)      | 48 (24.9%) |
| Total                       | 254 (36.1%)     | 237 (33.7%)     | 212 (30.2%) |

The percentage of using representative aspects in each group can be seen as the groups’ abilities in those aspects. For example, the percentage of experimental group 1 using written texts, drawing, and mathematical expressions is 36.1%, 33.7% and 30.2% respectively so the ability of mathematical representation of this group was 36; 33; 7; and 30,2 respectively. The table of above shows that experimental group 1, experimental group 2, and the control group tend to have different abilities in mathematical expression and written text when solving algebra and geometry problems, whereas drawing ability tended to be the same for all groups.

4.2. Attempts to integrate problem information
Several student wrote the equation for problem 1 (algebra) as,

Students wrote the equation for problem 1 (algebra) as,

(i) \( x \rightarrow x + 7 \rightarrow x + 13 = 80 \)
(ii) \( X \)
(iii) \( 1 = y \)
(iv) \( x, x + 7, x + 13, 80 \)
(v) \( x = ?; \quad b = a+7; \quad c = a + 13 \)

Figure 1. The students answered

4.3. Equations as descriptions of procedures used for calculating
Several students calculated answers to each problem by arithmetic reasoning, and then tried to represent these calculations as equations. Some researchers [7] have observed this method of dealing with algebra word problems. These equations were not representations of problem structure, but descriptions of the procedure used to calculate a value for one of the unknowns. For example, a student wrote the equation for Problem 2 (algebra) as, \( x = (87-18) \div 3 \) while others wrote \( x = 87-18 = 63 \div 3 = 23 \) where \( x \) stands for the time taken to wash the first car. It can be argued that technically, the first of these is an acceptable equation. However in the harder problems on the test, which were too difficult to be solved by mental reasoning and arithmetic, students who were limited to writing a description of the solution had no chance of success. Lacking the support of an algebraic representation of the problem, they were unable to devise solutions. Furthermore, students wrote the equation for Problem 3 (Geometry) as,
One student, from the first group with high basic knowledge and high school level appropriately formulated the behaviour of congruent shapes as shown in the above figure. The student showed that $x = 2b$ and $QS = 2a$, by recognizing that $\sin \angle B = \sin \angle Q$ and $\cos \angle B = \cos \angle Q$. Then the student showed that the volume ratio of the tall cone to the small cone is 1:8 using:

$$\frac{1/3 \pi a^2 b}{1/3 \pi QS^2 x} = \frac{a^2 b}{(2a)^2 x} = \frac{a^2 b}{4a^2 2b} = \frac{1}{8}$$

Explanation of the answer is as follow: volume ratio of bigger cone to smaller cone is not two times, but eight times. The volume is proportional to the radius squared, so that the volume ratio of both cones is 1:4 then multiplied by the height. Notice that the bigger radius can result in a greater cone volume.

Others students from experimental group-2 appropriately formulated the behaviour of congruent shapes as shown in above figure. However, there was a mistake in the calculations. The formula should be written $2a^2$ instead of $(2a)^2$ and the ratio becomes 1:4. In other words, the big cone volume is four times bigger than the volume of the small one.

Others students from control group, didn’t use congruent shape behaviour appropriately as implemented in the above figure. However, the answer in no.2 is without calculation and leads to an erroneous answer. The students answered that the big cone is two times bigger than the small cone because all the cone sizes are two times bigger than the smaller one. In the other words, the big cone volume is two times greater than the small one.

The results of this experiment are relevant to previous research by Esty & Teppo [2]. They state that students are able to solve problems given in forms of narrative drawings but students failed to build a problem solving formula.

According to Table 3, the frequency of representative for using in problem solving among the 3 groups does not greatly vary, but conceptually, the first group conducted better consecutive cognitive processes which lead to attaining relatively better results. According to the experiment on the 54 students, the first group predominantly used written texts and mathematical expressions, whereas the second group predominantly used the drawing aspect. The results show that representation that is treated using the TTW strategy positively affects students’ success in the final test.

According to students’ answers, there are three cognitive processes conducted by students in selecting and choosing the rules of solving algebra and geometry problems. The students use drawing as a model, formulating math equations (associating ideas), describing in their own words (written text), and a combination of all those aspects. In this case, the focal cognitive process is students’ ability at math representation. Drawing was more frequently used by the second experimental group, whereas formulating math models and explaining based on students’ own expression (written text) were more frequently used by the first group. The combination of drawing, formulating a math model, and constructing their own expressions were more frequently used by the first experimental group.
In contrast, students of the control group more frequently used explanations based on their own expressions than the other two representative aspects. Drawing and model creation were more relevant in the first group and second group than in the control group. Relevant drawing and model are first from student that has high initial knowledge. Drawing and math models used by both groups in the study were combined with information contained in the problems and the “package of knowledge” about algebra and geometry that was previously learned. The “Package of knowledge” is related to properly employed geometry rules. Based on drawings, students forecast the next steps and select rules to correspond with the drawing. Students who have medium and low initial knowledge also conducted processes like those conducted by students with high initial knowledge. Because of the limitations of “packet knowledge,” students did not understand the associated concepts in the problems, so that drawings were not predictive of the results and could not be used to arrange further steps. Finally, students had difficulties in selecting the rules to attain accurate results. These results are comparable to those of Gagne [3], who found that students are not able to understand the associated concepts because they do not save information in meaningful proposition, instead in open proposition.

Our data indicate that major difficulties in formulating equations in the test did not lie in students’ failure to comprehend the written information, to understand the problem structure, or to see how the parts were related to each other and to the whole. Most students could solve the problems by non-algebraic methods, providing that they understood the problem situation. For the students who tried to use algebra, the main obstacles to success were (a) incorrect use of algebraic syntax and (b) failure to integrate the given information as an equation or set of equations. Others wrote correct equations that could be used to solve the problems, but did not use their equations, apparently not knowing how to use the notation as a tool for deductive reasoning.

According to our data, for most students in the sample, naming variables and understanding relations were not difficult for the simple test problems. Most students who tried algebra could name quantities and there was little difficulty related to expressing several quantities in terms of one variable. However there were several instances of students who named the three parts in a problem appropriately (e.g., $x$, $x + 7$, $x + 13$) but did not try to relate them to the total. As we have shown, some students used unconventional formats such as arrow-diagrams, vertical addition, or invented notations to try to denote the idea that the sum of the parts is equal to the total given but they were unable to write an equation to express the structure of the problem. Others had not learned that an equation is written to represent the problem; they wrote a description of the calculation procedure they had used to solve the problem. These students perceived the equations as a formula for calculating. They need to know that algebra can also be used to extend and support logical reasoning; its purpose is not to describe a solution procedure that has already been constructed mentally.

Conclusions
The study shows that cooperative learning + Think-Talk-Write Strategy (First Group) was more effective in creating mathematical representations in the classroom than the Think-Talk-Write Strategy + individually (Second Group) and the Think-Talk-Write Strategy (Control Group). The first experimental group, second experimental group, and control group tended to have different abilities in mathematical expressions and written texts when solving algebra and geometry problems, whereas drawing ability tended to be the same for all groups. Mathematical representation ability of the first group was 36, of the second group 33.7 and of the control group 30.2. Therefore, the results suggest that the ability of mathematical representation when treated using the TTW strategy has a positive, though not completely satisfactory, effect on student success in the final test. The study suggests that future research tests other learning strategies to improve the ability of mathematical representation.

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