Most matter in the Universe is invisible and is called dark matter. A candidate of dark matter is the axion, which is an ultralight particle motivated as a solution for the 

\[ CP \]

problem. Axions form clouds in a galactic halo, and amplify and delay a part of gravitational waves propagating in the clouds. The Milky Way is surrounded by the dark matter halo composed of a number of axion patches. Thus, the characteristic secondary gravitational waves are always expected right after the reported gravitational-wave signals from compact binary mergers. In this paper, we derive a realistic amplitude of the secondary gravitational waves. Then we search the gravitational waves having the characteristic time delay and duration with a method optimized for them. We find no significant signal. Assuming the axions are a dominant component of dark matter, we obtain the constraints on the axion coupling to the parity-violating sector of gravity for the mass range, \[ [1.7 \times 10^{-13}, 8.5 \times 10^{-12}] \text{ eV}, \]

which is at most \( \sim 10 \) times stronger than that from Gravity Probe B.

I. INTRODUCTION

Most matter in the Universe is invisible and is called dark matter. Many candidates have been considered for dark matter. One candidate is a quantum chromodynamics (QCD) axion \[ \text{[1]} \] which is a pseudo Nambu-Goldstone boson introduced to resolve the \( CP \) problem. The \( CP \) problem is that many experiments, especially the measurements of the neutron electric-dipole moment \[ \text{[2]} \], prefer that the electric charge conjugate \( C \) and the parity \( P \) symmetries are conserved, which requires fine tuning in QCD theory. Furthermore, the existence of axions is also expected from the string theory \[ \text{[3]} \], and is called the string axion. The mass of the string axion ranges widely because it depends on the way of the compactification of the extra dimensions occurs. Thus, we should search for a broad mass range for both axions.

Although axions were historically introduced in the standard model of particle physics, axions have often been searched for with electromagnetic (EM) interaction in laboratory experiments \[ \text{[4, 5]} \], and in astrophysics through the observations of supernovae \[ \text{[6]} \] and active galactic nuclei \[ \text{[7]} \]. Also the cosmological evolution of axions has been studied \[ \text{[8–10]} \]. Many search methods and recent constraints are reviewed in \[ \text{[17, 19]} \].

Axions can also interact with gravity. Then, similar to the coupling to an electromagnetic field tensor in the QCD axion Lagrangian, we consider the Chern-Simons (CS) interaction term coupled to axions; that is, the axion nonminimal coupling to a Riemann tensor \[ \text{[20]} \]. The additional term is the simplest coupling of a pseudoscalar field to gravity (the one appears in the CS gravity) which is a low-energy effective theory of the parity-violating extension of GR \[ \text{[20]} \]. The CS coupling of axions have been searched for in \[ \text{[21–23]} \]. In the previous study \[ \text{[23]} \], the measurement of the frame-dragging effect by Gravity Probe B around the Earth constrained the coupling, \( \ell \lesssim 10^8 \) km.

If axions interact with gravity through the CS term, gravitational waves (GWs) induce axion decay into gravitationalons \[ \text{[24, 25]} \]. If dark matter in the Milky Way (MW) is composed of axions, the propagating GWs are amplified and delayed. Since axions are expected to be cold dark matter in the MW halo, the GWs from the axion decay are almost monochromatic. Therefore, axions through the interaction generate characteristic secondary GWs whose features depend on the axion mass and the coupling to the parity-violating sector of gravity \[ \text{[22]} \].

In this paper, we search the monochromatic secondary GWs induced by primary GWs from coalescences of binary neutron stars (BNSs) and binary black holes (BBHs) in the observational data of GW detectors \[ \text{[1]} \]. Our method is optimized for the time delay and the signal duration. Then, from no detection of the axion signal, we constrain the coupling constant, improving the upper limit from Gravity Probe B.

The organization of this paper is as follows. In Sec. \[ \text{II} \] we review the properties of the secondary GWs generated by axion decay. We show the method we analyze GW data in Sec. \[ \text{III} \] and give the results in Sec. \[ \text{IV} \]. We discuss the results and future prospects in Sec. \[ \text{V} \]. Finally, Sec. \[ \text{VI} \] is devoted to a summary.

In this paper, we use the natural units \( h = c = 1 \).

II. PROPERTIES OF SECONDARY GRAVITATIONAL WAVES

In this section, we briefly review the axion decay and enhancement of a GW in the MW halo, following Jung et
al. [32], and enumerate the characteristic quantities for axions that we use later in the search. The Lagrangians are

\[ S = S_{EH} + S_a + S_{CS}, \]  
\[ S_{EH} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R, \]  
\[ S_a = -\int d^4x \sqrt{-g} \left( \frac{1}{2} \nabla\mu a^\mu a + \frac{m_a^2}{2} a^2 \right), \]  
\[ S_{CS} = \frac{\ell^2}{16\sqrt{2}\pi G} \int d^4x \sqrt{-g} a^* R R, \]

where \( \ell \) is the axion coupling constant in the parity-violating sector of gravity, \( m_a \) is the axion mass, \( a \) is the axion field, and *\( RR \) is the Pontryagin density.

**A. Characteristic quantities**

A GW propagating through the MW halo composed of the axion dark matter with mass \( m_a \) produces secondary GWs which are almost monochromatic. The resonance frequency \( f_{\text{res}} \) is

\[ f_{\text{res}} = \frac{1}{2\pi} \frac{m_a}{2} = 12 \text{ Hz} \left( \frac{m_a}{10^{-13} \text{ eV}} \right), \]  
and its width is

\[ \Delta f_{\text{res}} = 2 f_{\text{res}} \Delta v = 2.4 \times 10^{-2} \text{ Hz} \left( \frac{m_a}{10^{-13} \text{ eV}} \right) \left( \frac{\Delta v}{10^{-3}} \right), \]

where \( \Delta v \) is a velocity dispersion of dark matter [36]. From Eq. (5), the LIGO [37, 38] sensitive band, 20 Hz – 1024 Hz, corresponds to \( 1.7 \times 10^{-13} \text{ eV} < m_a < 8.5 \times 10^{-12} \text{ eV} \). Therefore around a detected GW from compact binary coalescence (CBC), we can find the GW spectrum with a peak at only \( f_{\text{res}} \). The signal duration of the secondary GWs is roughly given by the inverse of the line width,

\[ \Delta t_{\text{duration}} = \frac{1}{2\pi f_{\text{res}}} = \frac{2\pi}{m_a \Delta v} = 41 \text{ s} \left( \frac{10^{-13} \text{ eV}}{m_a} \right) \left( \frac{10^{-3}}{\Delta v} \right). \]

The MW is composed of many coherent clouds of axion dark matter whose size is about the coherent length \( L_c \) defined by

\[ L_c := \frac{2\pi}{m_a \Delta v} = 4.0 \times 10^{-7} \text{ pc} \left( \frac{10^{-13} \text{ eV}}{m_a} \right) \left( \frac{10^{-3}}{\Delta v} \right). \]

We note that the factor of \( 2\pi \) is different from the definition in [32]. Then, the following quantities including \( L_c \) are modified from those in [32] by the factor of \( 2\pi \).

**FIG. 1.** The signal delay \( \Delta t_{\text{delay}} \) (dashed line) and the time to merger for various equal-mass binaries (solid lines) as a function of \( m_a \) for \( \ell_{\text{eff}} = 10^8 \text{ km} \), \( R = 100 \text{ kpc} \), \( \Delta v = 10^{-3} \), and \( \rho_{\text{DM}} = 0.3 \text{ GeV/cm}^3 \). Solid lines are plotted up to the ISCO frequency of a Schwarzschild BH for BNS of 1.4 \( M_\odot \) – 1.4 \( M_\odot \) and their merger frequencies for other BBH.

The time-averaged group velocity of a GW in the axion clouds, \( \bar{v}_g \), is

\[ \bar{v}_g = 1 - \frac{1}{3} \left( \frac{\sqrt{\pi G \rho_a \ell^2 m_a^2}}{2} \right)^2. \]

Assuming that the MW halo is spherical symmetric and its radius is \( R \sim 100 \text{ kpc} \), we can derive the delay time of the secondary GWs from the primary GW,

\[ \Delta t_{\text{delay}} = \frac{R}{\bar{v}_g} - \frac{R}{c} \approx \frac{\pi^3 R G \rho_a \ell^4 m_a^2}{3 \Delta v^2} \]

\[ = 1.1 \times 10^3 \text{ s} \left( \frac{R}{100 \text{ kpc}} \right) \left( \frac{10^{-3}}{\Delta v} \right)^2 \left( \frac{\ell_{\text{eff}}}{10^8 \text{ km}} \right)^4 \times \left( \frac{m_a}{10^{-13} \text{ eV}} \right)^2 \left( \frac{\rho_{\text{DM}}}{0.3 \text{ GeV/cm}^3} \right). \]

where \( \rho_a \) and \( \rho_{\text{DM}} \) are the energy densities of axions and dark matter. We defined the effective coupling parameter,

\[ \ell_{\text{eff}} := \ell_{\text{DM}}^{1/4}, \]

including the fraction of the amount of axions to the total amount of dark matter, \( f_{\text{DM}} := \rho_a / \rho_{\text{DM}} \).

Figure 1 is an example for the signal delay and duration compared with the time to merger for various compact binaries. When the delay is longer than the time to merger, the axion signal is observed after the binary merger (\( m_a \gtrsim 10^{-13} \text{ eV} \) in Fig. 1).
B. Amplitude enhancement of secondary GWs

We are surrounded by the MW dark matter halo with size $R$. Since axions form clouds with the $L_c$, the MW halo contains many axion patches. The number of patches, $N$, along the propagation path of a GW is estimated to be

$$N = \frac{R}{L_c} = 2.5 \times 10^{11} \left( \frac{R}{100 \text{ kpc}} \right) \left( \frac{\Delta v}{10^{-3}} \right) \left( \frac{m_a}{10^{-13} \text{ eV}} \right),$$  

where $\Delta v$ is the width of the resonance, $R$ is the halo size, and $m_a$ is the axion mass.

In the MW halo, GWs from CBC induce the decay of axions into monochromatic GWs at the frequency, $f_{\text{res}}$. In other words, the propagating GW is amplified in each axion patch by a factor of $1 + \delta_{\text{patch}}$, where

$$\delta_{\text{patch}} = \frac{1}{2} \left( \frac{\sqrt{\pi} \rho_a c^2 m_a^2}{2} \right) \sin^2 \left( 2\pi \frac{f - f_{\text{res}}}{\Delta f_{\text{res}}} \right).$$

The authors of neglect the phase rotation of the GW propagating through each axion patch because it is tiny. In this case, the amplification in each patch is treated coherently and the GW amplitude grows exponentially, giving the amplification factor of $\langle 1 + \delta_{\text{patch}} \rangle e^{\theta} \approx e^{N\delta_{\text{patch}}}$ for sufficiently large $N$ and tiny phase shift $\theta$ for the secondary GW (see Supplemental material). However, the statement is not true for the entire MW halo; because the number of patches $N$ is very large and the tiny phase shift $\theta$ is accumulated during propagation in the MW halo, the total phase shift finally reaches the limit beyond which the GW is no longer enhanced coherently. Such imperfect amplification has been numerically simulated in the time domain in [39], but for the first time we derive the analytical formula below.

Since a secondary GW is almost monochromatic with the width $\Delta f_{\text{res}}$, let us consider the two frequency modes at the edges of the resonance, that is, at $f_{\text{res}} + \Delta f_{\text{res}}$ and $f_{\text{res}} - \Delta f_{\text{res}}$. If the phase difference accumulates over $\pi$, those modes suppress each other incoherently. Then, the phase shift is accumulated coherently during the period $\Delta t_{\text{coh}}$ defined by $2\pi(2\Delta f_{\text{res}})\Delta t_{\text{coh}} = \pi$; that is, $\Delta t_{\text{coh}} = 1/(4\Delta f_{\text{res}}) = L_c/4$. When propagating in $\Delta t_{\text{coh}}$, a secondary GW passes the number of patches $N_c$:

$$N_c := N \frac{\Delta t_{\text{coh}}}{\Delta t_{\text{delay}}},$$

because the delay from one axion patch is $\Delta t_{\text{delay}}/N$. If $N_c > N$, the enhancement occurs coherently in all patches. However, in the case of $N_c < N$, the resonant growth stops after passing $N_c$ patches and the total enhancement is given by an incoherent superposition of $N/N_c$ enhanced GWs. The total enhancement factor is given by

$$F_{\text{halo}} \simeq \begin{cases} e^{N\delta_{\text{patch}}} & \text{for } N_c \geq N \\ \frac{e^{N\delta_{\text{patch}}}}{N_c} & \text{for } N_c < N \end{cases}.$$  

with

$$N\delta_{\text{patch}} = \frac{\pi^2 G R m_a^4 \rho_a}{4} \left( \frac{\Delta v}{\Delta f_{\text{res}}} \right) \left( \frac{\sin^2 \left( 2\pi \frac{f - f_{\text{res}}}{\Delta f_{\text{res}}} \right)}{2} \right),$$

$$\Delta f_{\text{res}} = 9.3 \left( \frac{10^{-3}}{\Delta v} \right) \left( \frac{R}{100 \text{ kpc}} \right) \left( \frac{m_a}{10^{-13} \text{ eV}} \right)^3 \left( \frac{L_{\text{eff}}}{10^8 \text{ km}} \right) \left( \frac{\rho_{\text{DM}}}{0.3 \text{ GeV}/\text{cm}^3} \right)^{1/4},$$

where the sinc factor is averaged for $|f - f_{\text{res}}| < \Delta f_{\text{res}}$ because GWs are amplified in the narrow range, $\langle \sin^2 \left( 2\pi \frac{f - f_{\text{res}}}{\Delta f_{\text{res}}} \right) \rangle = 0.24$. The critical case is $N = N_c$, i.e., from Eqs. (17) and (18), the critical coupling is

$$L_{\text{eff}} = 4.5 \times 10^7 \text{ km} \left( \frac{\Delta v}{10^{-3}} \right)^{1/4} \left( \frac{100 \text{ kpc}}{R} \right)^{1/4} \left( \frac{10^{-13} \text{ eV}}{m_a} \right)^{3/4} \left( \frac{0.3 \text{ GeV}/\text{cm}^3}{\rho_{\text{DM}}} \right)^{1/4}.$$  

In the left panel of Fig. 2 the coherent $(\theta = 0)$ and realistic enhancement factors are plotted as a function of $N/N_c$ for illustration. It is shown that neglecting the phase shift $\theta$ in each axion patch significantly overestimates the enhancement factor for $N > N_c$. In the right panel of Fig. 2 the realistic enhancement factor in Eq. (15) is plotted as a function of $m_a$ and $L_{\text{eff}}$. Even for the realistic case, the enhancement factor can be significantly large for large $N/N_c$ (the upper-right region).

III. METHOD

In this section, a search method optimized for axion signals is explained. If an axion signal is in data, it should be almost a monochromatic wave at the frequency of $f_{\text{res}}$ for a time duration $\Delta t_{\text{duration}}$ starting at $\Delta t_{\text{delay}} - \tau(f_{\text{res}})$ from the coalescence time, where the time to merger is $\tau(f) = (5/256)(\pi f)^{-8/3} (G M/c^3)^{-5/3}$ and $M$ is the chirp mass. For the axion mass range that we search, $1.7 \times 10^{-13} \text{ eV} < m_a < 8.5 \times 10^{-12} \text{ eV}$, the time to merger is negligible compared with the delay time so that the starting time of the secondary GW is almost at $\Delta t_{\text{delay}}$ from the coalescence time. The Fourier amplitude of the secondary signal is given by

$$|\tilde{h}_{\text{axion}}(f_{\text{res}})| = |F_{\text{halo}}(f_{\text{res}}) - 1| \left| \tilde{h}_{\text{CBC}}(f_{\text{res}}) \right|,$$
FIG. 2. Left: the coherent ($\theta = 0$) (blue) and realistic (orange) enhancement factors as a function of $N/N_c$. Right: the realistic enhancement factor as a function of $m_a$ and $\ell_{\text{eff}}$. The white diagonal lines from the bottom to the top are the coupling strengths of $N/N_c = 1$ (the critical coupling), $10^4$, and $10^5$.

the waveform is not accurate enough for high frequencies because of tidal deformation and the high-frequency cut-off is set to the innermost stable circular orbit (ISCO) frequency for a Schwarzschild black hole (BH). When obtaining the CBC amplitude, we need the distance to the GW source and the antenna responses at the time of each event. For conservative constraints, the farthest distance within the $1\sigma$ error is used [12]. As discussed in Supplemental material, the difference of the antenna responses hardly affects the results. The events used in this paper are enumerated later, and the data are downloaded from [12]. The other waveform parameters (e.g. the chirp mass, inclination angle, right ascension, declination, and so on) are set to the maximum-likelihood values.

To search the axion signals, we use the following steps:

1. Make the $\chi^2_{\text{obs}}$ map on the $m_a$-$\ell_{\text{eff}}$ plane from whitened data:
   $$\chi^2_{\text{obs}} := \sum_{I \in \text{IFO}} |\tilde{d}_I(f_{\text{res}})\sqrt{\Delta f}/\sqrt{S_{n,I}(f_{\text{res}})}|^2,$$
   where $\tilde{d}_I$ is the Fourier amplitude of the data starting at $\Delta t_{\text{delay}}$ from the coalescence time with the chunk size of $\Delta t_{\text{duration}}$, $S_{n,I}(f)$ is the power spectral density (PSD) for the $I$th detector, and $\Delta f$ is the width of a frequency bin.

   (a) Take a chunk of whitened data with the duration equal to $\Delta t_{\text{duration}}(m_a)$ from $\Delta t_{\text{delay}}(m_a, \ell_{\text{eff}})$ for a set of the parameters $(m_a, \ell_{\text{eff}})$.

   (b) Calculate the $\chi^2_{\text{obs}}$ at $f_{\text{res}}$.

   (c) Repeat (1a) and (1b) for other $(m_a, \ell_{\text{eff}})$.

2. Make a $p$-value map on the $m_a$-$\ell_{\text{eff}}$ plane from the $\chi^2_{\text{obs}}$ map. The $p$-value to reject the $\ell_{\text{eff}}$ is
   $$p(m_a, \ell_{\text{eff}}) := \int_0^{\chi^2_{\text{obs}}} p_\chi(x|\chi^2) \, dx.$$  

   where $\chi^2 := \sum_{I \in \text{IFO}} |\tilde{h}_{\text{axion}}(f_{\text{res}})\sqrt{\Delta f}/\sqrt{S_{n,I}(f_{\text{res}})}|^2$. For example, the $p$-value is tiny when the expectation value of the noncentral $\chi^2$-distribution is much larger than the actually observed one.

3. Search for parameter sets, $(m_a, \ell_{\text{eff}})$, for which the $p$-value is larger than a threshold, that is, judge if the axion signal exists. We set the threshold to $5 \times 10^{-3}$ or at 0.5% credible level.

4. Combine the search results from multiple GW events to check consistency among all GW events. Logical OR of nondetection is used to combine the results. That is, if the parameter sets are rejected even by one GW event, they are also rejected in the combined result. Then we derive upper limits on $\ell_{\text{eff}}$ for each $m_a$.

As the constraints are combined for multiple GW events, the constraint on the $\ell_{\text{eff}}$ becomes always tighter with more events. However, if the axion signal exists, the true parameter set cannot be rejected even if the number of the combined GW events is large enough.

Since the GW detectors, LIGO and Virgo [18], are sensitive at $20\text{ Hz}$–$1024\text{ Hz}$, we search the corresponding mass range, $1.7 \times 10^{-13}\text{ eV} < m_a < 8.5 \times 10^{-12}\text{ eV}$, in this paper. However, the axion signal does not exist beyond the peak frequency because no primary GW exists as in Eq. (20). Thus, for each GW event, we search the axion signal up to the peak frequency. However, since we use the IMRPhenomD waveform up to the ISCO frequency for BNS, the cutoff frequency GW170817 is given by the ISCO frequency.

For this search to constrain the wide parameter range, we need the data that last longer after the amplitude peak of primary GWs and that are available from all detectors (LIGO-Hanford, LIGO-Livingston, and Virgo). We require the length of the data to be about $5.3 \times 10^4\text{ s}$ for GW170814 [44], GW170817 [45] [46].
TABLE I. Properties of primary GWs analyzed in this paper. The frequency cutoff is set to 1024 Hz for the peak frequency > 1024 Hz.

| Event name      | Primary mass [M⊙] | Secondary mass [M⊙] | Frequency cutoff [Hz] | Network SNR of a primary GW | Duty cycle of the data used | References |
|-----------------|-------------------|---------------------|-----------------------|----------------------------|-----------------------------|------------|
| GW170814        | 31                | 25                  | 5.9 × 10^2            | 16                         | 92%                         | [43, 44]  |
| GW170817        | 1.5               | 1.3                 | 8.0 × 10^2            | 33                         | 79%                         | [45, 46]  |
| GW190728_064510 | 12                | 8.1                 | 1.0 × 10^3            | 14                         | 61%                         | [47]      |
| GW200202_154313 | 10                | 7.3                 | 1.0 × 10^3            | 11                         | 97%                         | [47]      |
| GW200316_215756 | 13                | 7.8                 | 1.0 × 10^3            | 10                         | 100%                        | [47]      |

FIG. 3. Constraint on the ℓ_eff as a function of m_a. The yellow region is rejected at more than 99.5% level, the purple one is not excluded, and the white one is not searched. The green line is an upper limit and the red dashed one is the previous constraint from Gravity Probe B for f_{DM} = 1.

and GW190728_064510 [43], and about 1.2 × 10^4 s for GW200202_154313 and GW200316_215756 [47], which are determined from computational time and the lower fraction of lacking data (see Supplemental material). Furthermore, for the third observing run (O3) events, we require the network signal-to-noise ratio (SNR) to be higher than 10 and the peak frequency higher than 1024 Hz. The GW events satisfying the conditions are listed in Table II.

FIG. 4. Minimum values of ℓ_eff constrained at more than 99.5% level from each GW event in Table II. The red dashed line is the previous constraint from Gravity Probe B for f_{DM} = 1.

The constraint is improved by at most one order of magnitude from the Gravity Probe B. The upper shape of the yellow region is like a saw. This is because the search is performed from weaker to stronger couplings until the unsearched ℓ_eff or Δt_{delay} is encountered. For larger ℓ_eff or longer time delay, the data chunk searched is far from the event time of the primary GW, and the detector data are more likely to be in the nonscience mode.

IV. RESULTS

Figure 3 shows the search result from the events listed in Table I. The yellow region is excluded at the 99.5% credible level, but the purple is not. The white region is not searched—the reason is discussed in the next section. The green line is an upper limit of the constraint, which is the maximum of ℓ_eff not excluded in the coupling parameter searched. Since there are many points sparsely distributed on the (m_a, ℓ_eff) plane that were not ruled out due to accidental noise fluctuations, we consider the envelope to obtain conservative upper limits on the coupling ℓ_eff for each m_a. The upper limit has some peaks; those are caused by detector line noises. The red dashed line is the previous constraint from Gravity Probe B [33].

Figure 4 shows the minimum values of ℓ_eff that were ruled out from each GW event in Table II. In Fig. 4 there are many step-functionlike jumps of the lines. This is due to short duration noises. Since Δt_{delay} ∝ m_a^2 ℓ_eff^4, the search with the mass heavier than that at which the jump exists always encounters the short duration noise and gives almost the same largest ℓ_eff not excluded. On the other hand, in the search with lighter masses, we can neglect the short duration noise because the contribution is diluted enough due to longer duration of the chunks. Although the highest SNR event is GW170817, the strongest constraint is obtained from GW170814. This is because, even though the SNR is lower, the amplitude of a primary GW is larger for BBH events, producing a larger secondary GW.
V. DISCUSSIONS

The constraints are obtained for the MW dark matter halo parameters, $R = 100\text{ kpc}$, $\Delta v = 10^{-3}$, and $\rho_{\text{DM}} = 0.3\text{ GeV/cm}^3$, but these measurement values should have large uncertainties. Nevertheless, the effects on our results are expected to be small, because the dependence of $\ell_{\text{eff}}$ on the halo parameters is the power of $1/4$ from Eq. (19). Given the uncertainties of $O(10\%)$ [49] in the MW dark matter halo parameters, they modify $\ell_{\text{eff}}$ by the order of $O(1\%)$. Although we assume that the halo is homogeneous in this paper, we could consider a realistic halo density profile, which is more dense at the center and coarse at the edge [35, 50]. However, it is beyond the scope of this paper.

Our constraint is stronger than that in [32]. Their method is simply to set the condition, $\chi_{\text{obs}} > 100$, for detection. The threshold corresponds to $F_{\text{total}} \sim 100$ in Fig. 2. Our detection threshold is roughly $F_{\text{total}} \sim 5–50$ and much smaller thanks to statistically evaluating the $\chi^2$-value and combining multiple events. Thus the difference of the constraints is from that of the detection criteria.

Also, there is another constraint from the observation of a neutron star with NICER [51]. Although the constraint looks much stronger than ours, it is not obvious for some assumptions in [51] to be satisfied for $\ell \sim 10^8\text{ km}$. Differences between our and their constraints are discussed in Supplemental material.

Future GW detectors are more sensitive than or have the sensitive frequency bands different from that of the current GW detectors. However, as discussed in Supplemental material, we cannot expect a significant improvement of the constraint on the coupling due to weak dependence on GW amplitude.

VI. CONCLUSIONS

GWs from CBCs are delayed and amplified during the propagation in an axion dark matter halo. In this paper, we have derived a realistic enhancement of the secondary GW amplitude, taking into account the accumulation of a phase delay during propagation and searched such signals with characteristic duration and time delay in the GW observational data. Since we know the signal duration and the time delay of axion signal, we take the data chunk whose length is the same as the duration at the time delay after a binary merger. In the search, we use the data right after the five reported GWs from CBCs. Then, since the signal duration and the time delay depend on the axion mass and the CS coupling, we analyze the data chunk and constraint the coupling by comparing the search result with the expected amplitude of the secondary GW. The constraint on the effective coupling constant $\ell_{\text{eff}}$ for axion mass in the range of $[1.65 \times 10^{-13}, 8.47 \times 10^{-12}]\text{ eV}$ is at most $\sim 10$ times improved from the previous study, Gravity Probe B [33].

ACKNOWLEDGMENTS

We thank S. Morisaki for fruitful discussions and valuable comments on the draft of the paper. T. T. is supported by JSPS KAKENHI Grant No. 21J12046. A. N. is supported by JSPS KAKENHI Grants No. JP19H01894 and No. JP20H04726 and by Research Grants from the Inamori Foundation. The authors are grateful for computational resources provided by the LIGO Laboratory and supported by National Science Foundation Grants No. PHY-0757058 and No. PHY-0823459. This material is based upon work supported by NSF’s LIGO Laboratory which is a major facility fully funded by the National Science Foundation.

Appendix A: Brief review of the phase shift of a secondary GW

By axion decaying, secondary GWs are generated and superposed the observational GWs has large amplitude and a tiny phase shift [32]. However, it is useful to explain more about the tiny phase shift, and then we review it briefly in this section. You can refer [32] for details.

If a GW propagate along to $z$-coordinate, it can be written as

\[ h_{ij}(z,t) = h^{R}_{ij}(z,t) + h^{L}_{ij}(z,t) \]  

(A1)

where the both helicity modes are expressed as

\[ h^{R/L}_{ij}(z,t) = e^{i\psi}(e^{i(kz-\frac{ma}{c}t)} - i e^{L/R}_{ij} h^{R/L}_{ij}(t)e^{i(kz-\frac{ma}{c}t)} + c.c., \]  

(A2)

where $h_{F}$ and $h_{B}$ are complex amplitudes for the forward and the backward waves, $e^{L/R}_{ij}$ is the polarization tensor, and $k$ is the wave number. By solving the equation of motion for $h_{ij}$ from Eq. (11), we obtain a solution after propagating in one axion patch:

\[ h^{R/L}_{F}(t = L_{c}) = h^{R/L}_{F}(0)(1 + \delta_{\text{patch}})e^{-i\psi(t = L_{c})} \]  

(A3)
\[ \psi(t) \simeq \frac{m_a}{2} e^{\epsilon t} \left\{ 1 + \frac{1}{2} \text{sinc}(m_a \epsilon t) - 1 \right\} \left( \frac{\gamma}{\epsilon} \right)^2 \]  
(A4)

\[ \epsilon = \frac{k - m_a/2}{m_a/2} \]  
(A5)

\[ \gamma = \sqrt{\pi G \rho_a \ell^2 m_a} \]  
(A6)

\[ = 5.7 \times 10^{-9} \left( \frac{\ell}{10^5 \text{ km}} \right)^2 \left( \frac{m_a}{10^{-18} \text{ eV}} \right) \left( \frac{\rho_a}{0.3 \text{ GeV/cm}^3} \right)^{1/2} \]  
(A7)

and \( \delta_{\text{patch}} \) is Eq. (13). \( L_c \) is Eq. (6). \( h_{ij}^{R/L} (t) \sim \gamma h_{ij}^{R/L} (t) \), that is, the backward waves are much smaller than the forward waves, and then we neglect the backward waves \[32\]. In the case, the GW waveform is

\[ h_{ij}^{R/L} (z, t = L_c) = e_{ij}^{R/L} (1 + \delta_{\text{patch}}) e^{i \theta} e^{ik(z-t)} + \text{c.c.} , \]  
(A8)

\[ \theta = - \frac{m_a}{2} \epsilon L_c \left( \frac{1}{2} \text{sinc}(m_a \epsilon L_c) - 1 \right) \left( \frac{\gamma}{\epsilon} \right)^2 . \]  
(A9)

Then, the total amplification factor from \( N \) patches is

\[ F_{\text{halo}} = \left[ (1 + \delta_{\text{patch}}) e^{i \theta} \right]^N \]  
(A10)

Because of \( \gamma \ll 1 \), the phase shift \( \theta \) is tiny, so that the total amplification factor is \( (1 + \delta_{\text{patch}})^N \) in \[32\]. However, in our paper, we consider the effect of the tiny phase shift.

**Appendix B: Antenna responses**

In this search, we do not consider the antenna response effects, that is, we assume that all GW detectors have the same antenna responses. For the unconstrained parameters or the data chunks, the antenna responses vary in \( O(10\%) \), and then the errors in the \( \ell_{\text{eff}} \) are in \( O(1\%) \). Thus our constraint is practically consistent with more realistic analysis without the assumption of the constant response.

**Appendix C: Tricks for an efficient search**

In this paper, the constraints for \((m_a, \ell)\) are obtained with the method in Sec. III. However, there is a problem how to divide the parameter space \((m_a, \ell)\). For a non-optimal division, the calculations are super heavy and cannot be done in reasonable run time. We should consider efficient way to take bins of the parameters.

\[ a. \text{ For effective coupling constant} \]

If bins for \( \ell_{\text{eff}} \) are sampled (log-)uniformly, the chunks to calculate \( \chi^2_{\text{obs}} \) exist on data densely for the lower \( \ell_{\text{eff}} \) but coarsely for the higher \( \ell_{\text{eff}} \) because of \( \Delta t_{\text{delay}} \propto \ell_{\text{eff}}^4 \). For the dense case, those chunks search almost same targets for an axion mass, but for the coarse case, almost different targets, which is inefficient. Hence, the efficient sampling should be that the intervals between the chunks is constant. Since the SNR of an axion signal is proportional to an overlap between the axion signal and the chunk, the interval is \( 0.2 \Delta t_{\text{duration}} \) when we approve a 10% SNR loss. That is, we search data at \( \alpha \Delta t_{\text{duration}} \) after a binary merger where \( \alpha \in \mathbb{R}_{>0} \) is uniformly sampled. In this paper, we search a range \( \alpha \in [0.002, 0.2] \) with less than 0.1% SNR loss and \( \alpha \in [0.2, 4 \times 10^4] \) with less than 10% SNR loss.

To relate the search results with \( \ell_{\text{eff}} \), we solve \( \Delta t_{\text{delay}} = \alpha \Delta t_{\text{duration}} \):

\[ \ell_{\text{eff}} = \left( \frac{6 \alpha}{\pi^2 G R \rho_{\text{DM}} m_a^2} \right)^{1/4} \]  
(C1)

\[ = (4.4 \times 10^7 \text{ km}) \left( \frac{100 \text{ kpc}}{R} \right)^{1/4} \left( \frac{\Delta v}{10^{-3}} \right)^{1/4} \left( \frac{0.3 \text{ GeV/cm}^3}{\rho_{\text{DM}}} \right)^{1/4} \left( \frac{10^{-13} \text{ eV}}{m_a} \right)^{3/4} \left( \frac{\alpha}{1} \right)^{1/4} . \]  
(C2)

We can know from Eq. (C2) how to divide the range of \( \ell_{\text{eff}} \). Because the effective coupling constant \( \ell_{\text{eff}} \) is proportional to the shift ratio \( \alpha^{1/4} \), the bins on the \( m_a - \ell_{\text{eff}} \) plane is highly denser for larger \( \ell_{\text{eff}} \). Thus, this division is not good to search for larger \( \ell_{\text{eff}} \) because of the computational costs.
b. For axion mass

The bin width of the axion mass $m_a$ is considered in this sub-section. Since the axion mass is related to the resonance frequency $f_{res}$ with Eq. (5), the mass bin width is interpreted as a frequency bin width. Since the frequency bin width should be equal to $\Delta f_{res}$, the bin width of the axion mass is log-uniform: $\Delta \log m_a = \Delta v$. However, to obtain the $\chi^2_{obs}$, we have to do Fourier transform for the each mass bin, because the length of the data chunk is assigned with $m_a$. It needs a large calculation cost although the analyzed chunks are almost same between $m_a$ and $m_a + \Delta m_a$. This is inefficient, and then we group the some mass bins. When the axion signal duration is 10% shorter than the chunk size, the SNR is 10% diluted. That is, approving 10% SNR loss, we can use the same chunk for a group $g_m = \{0.1m + n \Delta \log m_a | n, m \in \mathbb{Z}, 0 \leq n \Delta \log m_a < 0.1\}$. The length of the chunk is the longest one in $g_m$. By this grouping, we can make the calculation $10%/\Delta v = 100 \left( 10^{-3}/\Delta v \right)$ times rapider.

Appendix D: Computational costs

Since the upper limit from the Gravity Probe B \cite{33} is $\ell \sim 10^8$ km, we should search all the region below $\ell_{\text{eff}} \lesssim 10^8$ km. However, in Fig. 3 the stronger region of $\ell_{\text{eff}}$ for $m_a \gtrsim 10^{-12}$ eV is not searched because of computational cost. Since the delay is larger in the unsearched region because of $\Delta t_{\text{delay}} \propto \ell_{\text{eff}}^4 m_a^2$, We have to take finer bins for larger $\ell_{\text{eff}}$ not to lose SNR of an axion signal. Concretely, to obtain the constraint up to $\ell_{\text{eff}} = 10^8$ km at the heaviest axion mass of $8.5 \times 10^{-12}$ eV, we need the computational time of $\sim 20$ years, which is $10^5$ times longer than the current calculation (see Supplemental material). Also, the other problem is that there exists no data at large time delay, because the delay for $\ell_{\text{eff}} = 10^8$ km and $m_a = 8.5 \times 10^{-12}$ eV is about 9.3 months but the GW observations have already ended at the time. Thus, searching all the unsearched region below $\ell_{\text{eff}} = 10^8$ km is not feasible with state-of-the-art technology and data.

Appendix E: Constraints from NICER

There is another constraint from the observation of a neutron star (NS) with NICER \cite{51}. For massless CS gravity or at the massless limit of the axion-CS coupling \cite{51}, the NS observation gives the significantly stronger constraint, $\ell \lesssim 10$ km. The constraint for the massless CS gravity might be applied also to the massive or axion CS case with the mass range as low as the de Broglie wavelength is much longer than the size of a NS. However, the applicability is nontrivial because the constraint is obtained under the small coupling approximation \cite{52}, which is obviously invalid for the coupling range we searched, $10^9 \lesssim \ell \lesssim 10^8$ km. Also, there is another study for scalar-tensor theories \cite{53}, which states that a constraint for a massive scalar field is much worse than that for a massless one. This is because the effects of spontaneous scalarization and finite mass are compensated each other in the equation of motion of a scalar field. As a consequence of the degeneracy between the constrained coupling parameter and the mass of the scalar field, the posterior distribution is elongated toward the stronger regime of the coupling parameter. The scalar-tensor theory is, of course, different from the CS gravity. However, from the similarity of the equation of motions between the scalar field in \cite{52} and in \cite{53}, we expect that the statement of the worse constraint might be true also in the case of the CS gravity.

Appendix F: Future detectors

1. ground-based

Although the future ground-based GW detectors \cite{54,57} are about ten times sensitive, it improves the constraint of $\ell_{\text{eff}}$ by a factor of a few because of $\ell_{\text{eff}} \propto \text{SNR}^{-1/2}$, where SNR is that for the primary GW, from Eqs. (15) and (20) for an incoherent case, which is the regime of the current upper limit. On the other hand, as the sensitivities of GW detectors are improved, many new GW events will be detected. From Fig. 4 combining the search results from multiple events is obviously important to improve the constraint.
2. space-based

With space-based GW detectors \cite{58,60} and pulsar timing array \cite{61}, we can search lower frequencies or lighter axion masses. However, the sensitivities to the \( \ell_{\text{eff}} \) are much worse than those of ground-based ones at lower frequencies. This is because the scaling, \( \ell_{\text{eff}} \propto m_a^{-3/4} |\langle h_{\text{CBC}}(f_{\text{res}}) \rangle |^{-1/2} \), is obtained from Eqs. \cite{15} and \cite{20}. Using \( h_{\text{CBC}}(f) \propto f^{-7/6} \) and given the same PSD of a detector, we have the scaling with the axion mass,

\[
\ell_{\text{eff}} \propto m_a^{-3/4} f_{\text{res}}^{1/12} \propto m_a^{-2/3}.
\]

Therefore, the constraint would be worse than that from Gravity Probe B at low frequencies.

\[ \text{(F1)} \]

\begin{thebibliography}{99}
\bibitem{1} R. D. Peccei and Helen R. Quinn. CP Conservation in the Presence of Pseudoparticles. \textit{Phys. Rev. Lett.}, 38:1440–1443, Jun 1977. \[ \square \]
\bibitem{2} C. A. Baker, D. D. Doyle, P. Geltenbort, K. Green, M. G. D. van der Grinten, P. G. Harris, P. Iaydjiev, S. N. Ivanov, D. J. R. May, J. M. Pendlebury, J. D. Richardson, D. Shiers, and K. F. Smith. Improved Experimental Limit on the Electric Dipole Moment of the Neutron. \textit{Phys. Rev. Lett.}, 97:131801, Sep 2006. \[ \square \]
\bibitem{3} Asimina Arvanitaki, Savas Dimopoulos, Sergei Dubovsky, Nemanja Kaloper, and John March-Russell. String axiverse. \textit{Phys. Rev. D}, 81:123530, Jun 2010. \[ \square \]
\bibitem{4} Igor G. Irastorza and Javier Redondo. New experimental approaches in the search for axion-like particles. \textit{Prog. Part. Nucl. Phys.}, 102:89–159, 2018. \[ \square \]
\bibitem{5} Pierre Sikivie. Invisible Axion Search Methods. \textit{Rev. Mod. Phys.}, 93(1):015004, 2021. \[ \square \]
\bibitem{6} Jack W. Brockway, Eric D. Carlson, and Georg G. Raffelt. SN 1987A gamma-ray limits on the conversion of pseudoscalars. \textit{Physics Letters B}, 383(4):439–443, 1996. \[ \square \]
\bibitem{7} Christopher S. Reynolds, M. C. David Marsh, Helen R. Russell, Andrew C. Fabian, Robyn Smith, Francesco Tombesi, and Sylvain Veilleux. Astrophysical Limits on Very Light Axion-like Particles from Chandra Grating Spectroscopy of NGC 1275. \textit{The Astrophysical Journal}, 890(1):59, Feb 2020. \[ \square \]
\bibitem{8} John Preskill, Mark B. Wise, and Frank Wilczek. Cosmology of the invisible axion. \textit{Physics Letters B}, 120(1):127–132, 1983. \[ \square \]
\bibitem{9} L.F. Abbott and P. Sikivie. A cosmological bound on the invisible axion. \textit{Physics Letters B}, 120(1):133–136, 1983. \[ \square \]
\bibitem{10} Michael Dine and Willy Fischler. The not-so-harmless axion. \textit{Physics Letters B}, 120(1):137–141, 1983. \[ \square \]
\bibitem{11} Zurab Berezhiani and Maxim Yu. Khlopov. Cosmology of spontaneously broken gauge family symmetry with axion solution of strong CP-problem. \textit{Zeitschrift für Physik C Particles and Fields}, 49:73–78, 1991. \[ \square \]
\bibitem{12} Z. G. Berezhiani, A. S. Sakharov, and M. Yu. Khlopov. Primordial background of cosmological axions. \textit{Soviet Journal of Nuclear Physics}, 55(7):1063–1071, July 1992. \[ \square \]
\bibitem{13} A. S. Sakharov, D. D. Sokoloff, and M. Yu. Khlopov. Large-scale modulation of the distribution of coherent oscillations of a primordial axion field in the universe. \textit{Physics of Atomic Nuclei}, 59(6):1005–1010, June 1996. \[ \square \]
\bibitem{14} M.Yu. Khlopov, A.S. Sakharov, and D.D. Sokoloff. The nonlinear modulation of the density distribution in standard axionic CDM and its cosmological impact. \textit{Nuclear Physics B - Proceedings Supplements}, 72:105–109, 1999. Proceedings of the 5th IFT Workshop on Axions. \[ \square \]
\bibitem{15} A.S. Sakharov and M.Yu. Khlopov. The nonhomogeneity problem for the primordial axion field. \textit{Physics of Atomic Nuclei}, 57:485–487, 1994. \[ \square \]
\bibitem{16} Fabio Moretti, Flavio Bombacigno, and Giovanni Montani. gravitational Landau damping for massive scalar modes. \textit{The European Physical Journal C}, 80(12), Dec 2020. \[ \square \]
\bibitem{17} Francesca Chadha-Day, John Ellis, and David J. E. Marsh. Axion dark matter: What is it and why now? \textit{Sci. Adv.}, 8(8):abj3618, 2022. \[ \square \]
\bibitem{18} Luca Di Luzio, Maurizio Giannotti, Enrico Nardi, and Luca Visinelli. The landscape of QCD axion models. \textit{Phys. Rept.}, 870:1–117, 2020. \[ \square \]
\bibitem{19} Giorgio Galanti and Marco Roncadelli. Axion-like Particles Implications for High-Energy Astrophysics. \textit{Universe}, 8(5):253, 2022. \[ \square \]
\bibitem{20} Stephon Alexander and Nicolás Yunes. Chern–Simons modified general relativity. \textit{Physics Reports}, 480(1-2):1–55, Aug 2009. \[ \square \]
\bibitem{21} Renée Hložek, David J E Marsh, and Daniel Grin. Using the full power of the cosmic microwave background to probe axion dark matter. \textit{Monthly Notices of the Royal Astronomical Society}, 476(3):3063–3085, 02 2018. \[ \square \]
\bibitem{22} David J. E. Marsh and Pedro G. Ferreira. Ultralight scalar fields and the growth of structure in the Universe. \textit{Phys. Rev. D}, 82:103528, Nov 2010. \[ \square \]
\bibitem{23} Andrei Khmelnitsky and Valery Rubakov. Pulsar timing signal from ultralight scalar dark matter. 2014(02):019–019, Feb 2014. \[ \square \]
\bibitem{24} N. K. Porayko and K. A. Postnov. Constraints on ultralight scalar dark matter from pulsar timing. \textit{Phys. Rev. D}, 90:062008, Sep 2014. \[ \square \]
\bibitem{25} Arata Aoki and Jiro Soda. Pulsar timing signal from ultralight axion in \( f(R) \) theory. \textit{Phys. Rev. D}, 93:083503, Apr 2016. \[ \square \]
\bibitem{26} Arata Aoki and Jiro Soda. Detecting ultralight axion dark matter wind with laser interferometers. \textit{International Journal of Modern Physics D}, 26(07):1750063, Dec 2016. \[ \square \]
\bibitem{27} Arata Aoki and Jiro Soda. Nonlinear resonant oscillation of gravitational potential induced by ultralight axion in \( f(R) \) gravity. \textit{Phys. Rev. D}, 96:023534, Jul 2017. \[ \square \]
\end{thebibliography}
[37] J Aasi, B P Abbott, R Abbott, T Abbott, M R Abernathy, K Ackley, C Adams, T Adams, P Addesso, et al. Advanced LIGO. *Classical and Quantum Gravity*, 32(7):074001, Mar 2015. 2

[38] Gregory M Harry. Advanced LIGO: the next generation of gravitational wave detectors. *Classical and Quantum Gravity*, 27(8):084006, apr 2010. 2

[39] Tomohiro Fujita, Ippei Ohashi, Takahiro Tanaka, and Kei Yamada. Resonant gravitational waves in dynamical Chern–Simons–axion gravity. *Classical and Quantum Gravity*, 38(4):045010, dec 2020. 2

[40] Sascha Husa, Sebastian Khan, Mark Hannam, Michael Pürrer, Frank Ohme, Xisco Jiménez Forteza, and Alejandro Bohé. Frequency-domain gravitational waves from nonprecessing black-hole binaries. I. New numerical waveforms and anatomy of the signal. *Phys. Rev. D*, 93:044006, Feb 2016. 3

[41] Sebastian Khan, Sascha Husa, Mark Hannam, Frank Ohme, Michael Pürrer, Xisco Jiménez Forteza, and Alejandro Bohé. Frequency-domain gravitational waves from nonprecessing black-hole binaries. II. A phenomenological model for the advanced detector era. *Phys. Rev. D*, 93:044007, Feb 2016. 3

[42] Gravitational Wave Open Science Center. https://gw-openscience.org 4

[43] R. Abbott, T. D. Abbott, et al. GWTC-2: Compact Binary Coalescences Observed by LIGO and Virgo during the First Half of the Third Observing Run. *Phys. Rev. X*, 11:021053, Jun 2021. 5

[44] B. P. Abbott, R. Abbott, T. D. Abbott, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, V. B. Adya, et al. GW170814: A Three-Detector Observation of Gravitational Waves from a Binary Black Hole Coalescence. *Physical Review Letters*, 119(14), Oct 2017. 4 5

[45] B. P. Abbott, R. Abbott, T. D. Abbott, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, V. B. Adya, et al. GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral. *Physical Review Letters*, 119(16), Oct 2017. 4 5

[46] B. P. Abbott et al. Multi-messenger Observations of a Binary Neutron Star Merger. *Astrophys. J.*, 848(2):L12, 2017. 4 5

[47] R. Abbott et al. GWTC-3: Compact Binary Coalescences Observed by LIGO and Virgo During the Second Part of the Third Observing Run. 11 2021. 5

[48] F Acernese, M Agathos, K Agatsuma, D Aisa, N Almondou, A Allocca, J Amarni, P Astone, G Balestri, G Ballardin, et al. Advanced Virgo: a second-generation interferometric gravitational wave detector. *Classical and Quantum Gravity*, 32(2):024001, Dec 2015. 4

[49] Elisa G. M. Ferreira. Ultra-light dark matter. *The Astronomy and Astrophysics Review*, 29(1), sep 2021. 6

[50] Julio F. Navarro, Carlos S. Frenk, and Simon D. M. White. The Structure of Cold Dark Matter Halos. *The Astrophysical Journal*, 462:563, may 1996. 6

[51] Hector O. Silva, A. Miguel Holgado, Alejandro Cárdenas-Avendaño, and Nicolás Yunes. Astrophysical and Theoretical Physics Implications from Multimessenger Neutron Star Observations. *Phys. Rev. Lett.*, 126:181101, May 2021. 3 5

[52] Nicolás Yunes, Dimitrios Psaltis, Feryal Özel, and Abrah Loeb. Constraining parity violation in gravity with measurements of neutron-star moments of inertia. *Phys. Rev. D*, 81:064020, Mar 2010. 5

[53] Semih Tuna, Kivanç İ. Ünlütürk, and Fethi M. Ramazanoglu. Constraining scalar-tensor theories using neutron star mass and radius measurements, 2022. 9

[54] M Punturo et al. The Einstein Telescope: a third-generation gravitational wave observatory. *Classical and Quantum Gravity*, 27(19):194002, sep 2010. 5

[55] Michele Maggiore, Chris Van Den Broeck, Nicola Bartolo, Enis Belgacem, Daniele Bertacca, Marie Anne Bizouard, Marica Branchesi, Sebastien Clesse, Stefano Foffa, Juan García-Bellido, et al. Science case for the Einstein telescope. *Journal of Cosmology and Astroparticle Physics*, 2020(03):050–050, Mar 2020.

[56] B P Abbott et al. Exploring the sensitivity of next generation gravitational wave detectors. *Classical and Quantum Gravity*, 34(4):044001, Jan 2017.

[57] Sheila Dwyer, Daniel Sigg, Stefan W. Ballmer, Lisa Barstott, Nergis Mavalvala, and Matthew Evans. Gravitational wave detector with cosmological reach. *Phys. Rev. D*, 91:082001, Apr 2015. 5

[58] Stanislav Babak, Martin Hewitson, and Antoine Petitou. LISA Sensitivity and SNR Calculations, 2021. 9

[59] Seiji Kawamura, Takashi Nakamura, et al. The Japanese space gravitational wave antenna—DECIGO. *Classical and Quantum Gravity*, 23(8):S125–S131, mar 2006.

[60] Seiji Kawamura, Masaki Ando, et al. The Japanese space gravitational wave antenna: DECIGO. *Classical and Quantum Gravity*, 28(9):094011, apr 2011. 9

[61] G. Hobbs, A. Archibald, Z. Arzoumanian, et al. The International Pulsar Timing Array project: using pulsars as a gravitational wave detector. *Classical and Quantum Gravity*, 27(8):084013, April 2010. 9