Analysis of $\Omega^*_c(css)$ and $\Omega^*_b(bss)$ with QCD sum rules

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Abstract

In this article, we calculate the masses and residues of the heavy baryons $\Omega^*_c(css)$ and $\Omega^*_b(bss)$ with spin-parity $\frac{3}{2}^+$ with the QCD sum rules. The numerical values are compatible with experimental data and other theoretical estimations.

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1 Introduction

Several new excited charmed baryon states have been observed by BaBar, Belle and CLEO Collaborations, such as $\Lambda_c(2765)^+$, $\Lambda_c^+(2880)$, $\Lambda_c^+(2940)$, $\Sigma_c^+(2800)$, $\Xi_c^+(2980)$, $\Xi_c^+(3077)$, $\Xi_c^0(2980)$, $\Xi_c^0(3077)$ [1, 2]. The charmed baryons provide a rich source of states, including possible candidates for the orbital excitations. They serve as an excellent ground for testing predictions of the constituent quark models and heavy quark symmetry [3]. The charmed and bottomed baryons, which contain a heavy quark and two light quarks, provides an ideal tool for studying dynamics of the light quarks in the presence of a heavy quark. The $u$, $d$ and $s$ quarks form an $SU(3)$ flavor triplet, $3 \times 3 = 3 + 6$, two light quarks can form diquarks with a symmetric sextet and an antisymmetric antitriplet. For the $S$-wave baryons, the sextet contains both spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ states, while the antitriplet contains only spin-$\frac{1}{2}$ states. By now, the $\frac{1}{2}^+$ antitriplet states ($\Lambda_c^+, \Xi_c^+, \Xi'_c^0$), and the $\frac{3}{2}^+$ and $\frac{5}{2}^+$ sextet states ($\Omega_c, \Sigma_c, \Xi'_c^0$) and ($\Omega^*_c, \Sigma^*_c, \Xi'^*_c$) have been established.

The baryon $\Omega^*_c$, a $css$ candidate for the $\frac{3}{2}$ partner of the strange baryon $\Omega(sss)$, was observed by BaBar collaboration in the radiative decay $\Omega^*_c \rightarrow \Omega_c \gamma$ [4]. The $\frac{1}{2}^+$ baryon $\Omega^*_c(css)$ was reconstructed in decays to the final states $\Omega^- \pi^+$, $\Omega^- \pi^+ \pi^0$, $\Omega^- \pi^+ \pi^- \pi^+$ and $\Xi^- K^- \pi^+ \pi^+$. It lies about 70.8$\pm$1.0$\pm$1.1MeV above the $\Omega_c$, and it is the last singly-charmed baryon with zero orbital momentum observed experimentally [5].

In this article, we calculate the mass and residue of the $\Omega^*_c$ (and $\Omega^*_b$ as byproduct, the $\Omega^*_b$ has not been observed experimentally yet) with the QCD sum rules [6, 7]. In the QCD sum rules, operator product expansion is used to expand the time-ordered currents into a series of quark and gluon condensates which parameterize the long distance properties of the QCD vacuum. Based on current-hadron duality, we can

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obtain copious information about the hadronic parameters at the phenomenological side.

The article is arranged as follows: we derive the QCD sum rules for the masses and residues of the $\Omega_c^*$ and $\Omega_b^*$ in section 2; in section 3, numerical results and discussions; section 4 is reserved for conclusion.

2 QCD sum rules for the $\Omega_c^*$ and $\Omega_b^*$

In the following, we write down the two-point correlation functions $\Pi^a_{\mu\nu}(p^2)$ in the QCD sum rules approach,

\[
\Pi^a_{\mu\nu}(p^2) = i \int d^4x e^{ip\cdot x} \langle 0 | T \{ J^a_\mu(x) J^a_\nu(0) \} | 0 \rangle , \tag{1}
\]

\[
J^a_\mu(x) = \epsilon_{ijk} s^T_i(x) C^\mu s^j(x) Q^a_k(x) , \tag{2}
\]

\[
\lambda_a N_\mu(p, s) = \langle 0 | J^a_\mu(0) | \Omega^*_a(p, s) \rangle , \tag{3}
\]

where the upper index $a$ represents the $c$ and $b$ quarks respectively; the $N_\mu(p, s)$ and $\lambda_a$ stand for the Rarita-Schwinger spin vector and residue of the baryon $\Omega^*_a$, respectively. $i$, $j$, and $k$ are color indexes, $C$ is charge conjugation matrix, and $\mu$ and $\nu$ are Lorentz indexes.

The correlation functions $\Pi^a_{\mu\nu}(p)$ can be decomposed as follows:

\[
\Pi^a_{\mu\nu}(p) = - g_{\mu\nu} \left\{ \rho \Pi^a_1(p^2) + \Pi^a_2(p^2) \right\} + \cdots , \tag{4}
\]

due to Lorentz covariance. The first structure $g_{\mu\nu} \rho$ has an odd number of $\gamma$-matrices and conserves chirality, the second structure $g_{\mu\nu}$ has an even number of $\gamma$-matrices and violate chirality. In the original QCD sum rules analysis of the nucleon masses and magnetic moments [8], the interval of dimensions (of the condensates) for the odd structure is larger than the interval of dimensions for the even structure, one may expect a better accuracy of results obtained from the sum rules with the odd structure.

In this article, we choose the two tensor structures to study the masses and residues of the heavy baryons $\Omega^*_c$ and $\Omega^*_b$, as the masses of the heavy quarks break the chiral symmetry explicitly.

According to basic assumption of current-hadron duality in the QCD sum rules approach [6], we insert a complete series of intermediate states satisfying unitarity principle with the same quantum numbers as the current operator $J^a_\mu(x)$ into the correlation functions in Eq.(1) to obtain the hadronic representation. After isolating the pole terms of the lowest states $\Omega^*_a$, we obtain the following result:

\[
\Pi^a_{\mu\nu}(p^2) = - g_{\mu\nu} \lambda^2_a \frac{M_{\Omega^*_a} + \rho}{M_{\Omega^*_a}^2 - p^2} + \cdots , \tag{5}
\]
where we have used the relation to sum over the Rarita-Schwinger spin vector,

$$\sum_s N_\mu(p, s) N_\nu(p, s) = -(\phi + M_{\Omega^a}) \left\{ g_{\mu\nu} - \frac{\gamma_\mu \gamma_\nu}{3} - \frac{2p_\mu p_\nu}{3M_{\Omega^a}^2} + \frac{p_\mu \gamma_\nu - p_\nu \gamma_\mu}{3M_{\Omega^a}^2} \right\}. \quad (6)$$

In the following, we briefly outline operator product expansion for the correlation functions $\Pi_{\mu\nu}(p)$ in perturbative QCD theory. The calculations are performed at large space-like momentum region $p^2 \ll 0$, which corresponds to small distance $x \approx 0$ required by validity of operator product expansion. We write down the "full" propagators $S_{ij}(x)$ and $S^{ij}_Q(x)$ of a massive quark in the presence of the vacuum condensates firstly \[6\]

$$S_{ij}(x) = \frac{i\delta_{ij}}{2\pi^2 x^4} - \frac{\delta_{ij}m_s}{12} \langle \bar{s}s \rangle + \frac{i\delta_{ij}}{48} m_s \langle \bar{s}s \rangle \not{x} - \frac{\delta_{ij}x^2}{192} \langle \bar{s} s \rangle \sigma G \langle \bar{s} s \rangle \not{x} \not{y} + \cdots,$$

$$S^{ij}_Q(x) = \frac{i}{(2\pi)^4} \int d^4k e^{-ik\cdot x} \left\{ \delta_{ij} - \frac{g_s G_{ij}^{\alpha\beta}}{4} \frac{\sigma_{\alpha\beta}(k + m_Q) + (k + m_Q)\sigma_{\alpha\beta}}{(k^2 - m_Q^2)^2} \right\} \not{x} \not{y} + \cdots, \quad (7)$$

where $\langle \bar{s} s \rangle = \langle \bar{s} s \rangle \sigma G \langle \bar{s} s \rangle$ and $\langle \alpha G \rangle = \langle \alpha G \rangle \sigma G$, then contract the quark fields in the correlation functions $\Pi_{\mu\nu}(p)$ with Wick theorem, and obtain the result:

$$\Pi_{\mu\nu}(p) = 2i\epsilon_{ijk}\epsilon_{j'k'} \int d^4x e^{ip\cdot x} \text{Tr} \left\{ \gamma_\mu S_{iv}(x) \gamma_\nu C S^T_{v'j'}(x) C \right\} S^{k'k}(x). \quad (8)$$

Substitute the full $s$, $c$ and $b$ quark propagators into above correlation functions and complete the integral in coordinate space, then integrate over the variable $k$, we can obtain the correlation functions $\Pi_i(p^2)$ at the level of quark-gluon degree of freedom:

\[2\] One can consult the last article of Ref.\[6\] for technical details in deriving the full propagator.
\[ \Pi_1(p^2) = -\frac{1}{64\pi^4} \int_0^1 dx x(1-x)^2(x+2) \left( \tilde{m}_a^2 - p^2 \right)^2 \log \left( \tilde{m}_a^2 - p^2 \right) \\
- m_a \langle \bar{s}s \rangle \int_0^1 dx x(2-x) \log \left( \tilde{m}_a^2 - p^2 \right) \\
+ \frac{1}{192\pi^2} \left( \frac{\alpha_s GG}{\pi} \right) \int_0^1 dx x(2-x) \log \left( \tilde{m}_a^2 - p^2 \right) \\
+ m_a \langle \bar{s}g\sigma Gs \rangle \int_0^1 dx \frac{x}{\tilde{m}_a^2 - p^2} - \frac{m_a^2}{576\pi^2} \left( \frac{\alpha_s GG}{\pi} \right) \int_0^1 dx \frac{(1-x)^2(x+2)}{x^2(\tilde{m}_a^2 - p^2)} \\
+ \frac{\langle \bar{s}s \rangle^2}{3} \frac{1}{m_a^2 - p^2} + m_a \langle \bar{s}g\sigma Gs \rangle \frac{1}{12\pi^2} \frac{1}{m_a^2 - p^2} + \cdots \] 

\[ \Pi_2(p^2) = -\frac{m_a}{64\pi^4} \int_0^1 dx x(1-x)^2(x+2) \left( \tilde{m}_a^2 - p^2 \right)^2 \log \left( \tilde{m}_a^2 - p^2 \right) \\
- m_a m_a \langle \bar{s}s \rangle \int_0^1 dx x(2-x) \log \left( \tilde{m}_a^2 - p^2 \right) \\
- \frac{m_a}{576\pi^2} \left( \frac{\alpha_s GG}{\pi} \right) \int_0^1 dx \left( \frac{4}{x^2} - \frac{9}{x} - 3x^2 + 2x + 9 \right) \log \left( \tilde{m}_a^2 - p^2 \right) \\
+ \frac{m_a}{576\pi^2} \left( \frac{\alpha_s GG}{\pi} \right) \int_0^1 dx \frac{x^4 - x^3 - 3x^2 + 5x - 2}{x(1-x)} \frac{\tilde{m}_a^2}{\tilde{m}_a^2 - p^2} \\
+ \frac{m_a m_a \langle \bar{s}g\sigma Gs \rangle}{24\pi^2} \int_0^1 dx \frac{1}{\tilde{m}_a^2 - p^2} + \frac{m_a \langle \bar{s}s \rangle^2}{3} \frac{1}{m_a^2 - p^2} \\
+ \frac{m_a m_a \langle \bar{s}g\sigma Gs \rangle}{12\pi^2} \frac{1}{m_a^2 - p^2} + \cdots \] 

where \( \tilde{m}_a^2 = \frac{m_a^2}{x} \).

We carry out operator product expansion to the vacuum condensates adding up to dimension-6. In calculation, we take assumption of vacuum saturation for high dimension vacuum condensates, they are always factorized to lower condensates with vacuum saturation in the QCD sum rules, factorization works well in large \( N_c \) limit. In this article, we take into account the contributions from the quark condensate \( \langle \bar{s}s \rangle \), mixed condensate \( \langle \bar{s}g\sigma Gs \rangle \), gluon condensate \( \frac{\alpha_s GG}{\pi} \), and neglect the contributions from other high dimension condensates, which are suppressed by large denominators and would not play significant roles.

Once analytical results are obtained, then we can take current-hadron duality below the threshold \( s_0 \) and perform Borel transformation with respect to the variable \( P^2 = -p^2 \), finally we obtain the following sum rules:
\[
\lambda_a^2 \exp \left\{ -\frac{M_{\Omega_a}^2}{M^2} \right\} = \frac{1}{64\pi^4} \int_{th}^{s_a} \, ds \int_{\Delta_a}^{1} \, dx \, (1-x)^2 (x+2) \left( \tilde{m}_a^2 - s \right)^2 \exp \left\{ -\frac{s}{M^2} \right\} \\
+ \frac{m_a \langle \bar{s}s \rangle}{4\pi^2} \int_{th}^{s_a} \, ds \int_{\Delta_a}^{1} \, dx \, (x-2) \exp \left\{ -\frac{s}{M^2} \right\} \\
- \frac{1}{192\pi^2} \left( \frac{\alpha_s G}{\pi} \right) \int_{th}^{s_a} \, ds \int_{\Delta_a}^{1} \, dx \, (x-2) \exp \left\{ -\frac{s}{M^2} \right\} \\
+ \frac{m_a \langle \bar{s}g_s G_s \rangle}{24\pi^2} \int_{th}^{s_a} \, ds \int_{\Delta_a}^{1} \, dx \, \exp \left\{ -\frac{\tilde{m}_a^2}{M^2} \right\} \\
- \frac{m_a^2 \langle \alpha_s G G \rangle}{576\pi^2} \int_{th}^{s_a} \, ds \int_{\Delta_a}^{1} \, dx \, \frac{(1-x)^2 (2+x)}{x^2} \exp \left\{ -\frac{\tilde{m}_a^2}{M^2} \right\} \\
+ \frac{\langle \bar{s}s \rangle^2}{3} \exp \left\{ -\frac{m_a^2}{M^2} \right\} + \frac{m_a \langle \bar{s}g_s G_s \rangle}{12\pi^2} \exp \left\{ -\frac{m_a^2}{M^2} \right\}, \\
\]
(11)

\[
M_{\Omega_a} \lambda_a^2 \exp \left\{ -\frac{M_{\Omega_a}^2}{M^2} \right\} = \frac{m_a}{64\pi^4} \int_{th}^{s_a} \, ds \int_{\Delta_a}^{1} \, dx \, (1-x)^2 (x+2) \left( \tilde{m}_a^2 - s \right)^2 \exp \left\{ -\frac{s}{M^2} \right\} \\
+ \frac{m_a m_s \langle \bar{s}s \rangle}{4\pi^2} \int_{th}^{s_a} \, ds \int_{\Delta_a}^{1} \, dx \, (x-2) \exp \left\{ -\frac{s}{M^2} \right\} \\
+ \frac{m_a \langle \phi_s G \rangle}{576\pi^2} \int_{th}^{s_a} \, ds \int_{\Delta_a}^{1} \, dx \, \frac{4}{x^2} - \frac{9}{x} - 3x^2 + 2x + 9 \exp \left\{ -\frac{s}{M^2} \right\} \\
+ \frac{m_a \langle \phi G \rangle}{576\pi^2} \int_{th}^{s_a} \, ds \int_{\Delta_a}^{1} \, dx \, \frac{x^4 - x^3 - 3x^2 + 5x - 2}{x(1-x)} \tilde{m}_a \exp \left\{ -\frac{\tilde{m}_a^2}{M^2} \right\} \\
+ \frac{m_a m_s \langle \bar{s}g_s G_s \rangle}{24\pi^2} \int_{th}^{s_a} \, ds \int_{\Delta_a}^{1} \, dx \, \exp \left\{ -\frac{\tilde{m}_a^2}{M^2} \right\} + \frac{m_a \langle \bar{s}s \rangle^2}{3} \exp \left\{ -\frac{m_a^2}{M^2} \right\} \\
+ \frac{m_a m_s \langle \bar{s}g_s G_s \rangle}{12\pi^2} \exp \left\{ -\frac{m_a^2}{M^2} \right\}, \\
\]
(12)

where \( th = (m_a + 2m_s)^2 \) and \( \Delta_a = m_s^2 \).

Differentiate the above sum rules with respect to the variable \( \frac{1}{M^2} \), then eliminate the quantity \( \lambda_{\Omega_a} \), we obtain two QCD sum rules for the masses \( M_{\Omega_a} \):
\[
M_{\tilde{\Omega}_a}^2 = \left\{ \frac{1}{64\pi^4} \int_{th}^{s_0} ds \int_{\Delta^{0}}^1 dx (1-x)^2 (x+2) \left( \tilde{m}_a^2 - s \right)^2 s \exp \left\{ -\frac{s}{M^2} \right\} \\
+ \frac{m_s \langle \bar{s}s \rangle}{4\pi^2} \int_{th}^{s_0} ds \int_{\Delta^{0}}^1 dx (x-2) s \exp \left\{ -\frac{s}{M^2} \right\} \\
- \frac{1}{192\pi^2} \frac{\alpha_s G G}{\pi} \int_{th}^{s_0} ds \int_{\Delta^{0}}^1 dx (x-2) s \exp \left\{ -\frac{s}{M^2} \right\} \\
+ \frac{m_s m_a^2 \langle \bar{s}g_s \sigma G s \rangle}{24\pi^2} \int_0^1 dx \exp \left\{ -\frac{\tilde{m}_a^2}{M^2} \right\} \\
- \frac{m_a^4}{576\pi^2} \frac{\alpha_s G G}{\pi} \int_0^1 dx \frac{(1-x)^2 (2+x)}{x^3} \exp \left\{ -\frac{\tilde{m}_a^2}{M^2} \right\} \\
+ \frac{m_a^2 \langle \bar{s}s \rangle^2}{3} \exp \left\{ -\frac{m_a^2}{M^2} \right\} + \frac{m_s m_a^2 \langle \bar{s}g_s \sigma G s \rangle}{12\pi^2} \exp \left\{ -\frac{m_a^2}{M^2} \right\} \right\} / \\
\left\{ \frac{1}{64\pi^4} \int_{th}^{s_0} ds \int_{\Delta^{0}}^1 dx (1-x)^2 (x+2) \left( \tilde{m}_a^2 - s \right)^2 s \exp \left\{ -\frac{s}{M^2} \right\} \\
+ \frac{m_s \langle \bar{s}s \rangle}{4\pi^2} \int_{th}^{s_0} ds \int_{\Delta^{0}}^1 dx (x-2) s \exp \left\{ -\frac{s}{M^2} \right\} \\
- \frac{1}{192\pi^2} \frac{\alpha_s G G}{\pi} \int_{th}^{s_0} ds \int_{\Delta^{0}}^1 dx (x-2) s \exp \left\{ -\frac{s}{M^2} \right\} \\
+ \frac{m_s \langle \bar{s}g_s \sigma G s \rangle}{24\pi^2} \int_0^1 dx \exp \left\{ -\frac{\tilde{m}_a^2}{M^2} \right\} \\
- \frac{m_a^4}{576\pi^2} \frac{\alpha_s G G}{\pi} \int_0^1 dx \frac{(1-x)^2 (2+x)}{x^3} \exp \left\{ -\frac{\tilde{m}_a^2}{M^2} \right\} \\
+ \frac{\langle \bar{s}s \rangle^2}{3} \exp \left\{ -\frac{m_a^2}{M^2} \right\} + \frac{m_s \langle \bar{s}g_s \sigma G s \rangle}{12\pi^2} \exp \left\{ -\frac{m_a^2}{M^2} \right\} \right\} , \quad (13)
\]
and
\[
M_{a}^2 = \left\{ \frac{m_a}{64\pi} \int_{\Delta^a}^{s_0} ds \int_{-\Delta^a}^{1} dx(1-x)^2(x+2) \left( \tilde{m}_a^2 - s \right)^2 s \exp \left\{ -\frac{s}{M^2} \right\} \right.
\]
\[+ \frac{m_a m_a \langle \bar{s}s \rangle}{4\pi^2} \int_{\Delta^a}^{s_0} ds \int_{-\Delta^a}^{1} dx(2-s) \exp \left\{ -\frac{s}{M^2} \right\} \] 
\[+ \frac{m_a^2 \langle \alpha_s \overline{G}G \rangle}{576\pi^2} \int_{\Delta^a}^{s_0} ds \int_{-\Delta^a}^{1} dx \left( \frac{4}{x^2} - \frac{9}{x} - 3x^2 + 2x + 9 \right) s \exp \left\{ -\frac{s}{M^2} \right\} \]
\[+ \frac{m_a^3 m_a \langle \bar{s}g_s \sigma Gs \rangle}{24\pi^2} \int_{\Delta^a}^{s_0} ds \int_{-\Delta^a}^{1} dx \left( \frac{1}{x} \right) \exp \left\{ -\frac{\tilde{m}_a^2}{M^2} \right\} + \frac{m_a^3 \langle \bar{s}s \rangle^2}{3} \exp \left\{ -\frac{m_a^2}{M^2} \right\} \]
\[+ \frac{m_a^3 m_a \langle \bar{s}g_s \sigma Gs \rangle}{12\pi^2} \exp \left\{ -\frac{m_a^2}{M^2} \right\} \right\} \] 
\[= \left\{ \frac{m_a}{64\pi} \int_{\Delta^a}^{s_0} ds \int_{-\Delta^a}^{1} dx(1-x)^2(x+2) \left( \tilde{m}_a^2 - s \right)^2 s \exp \left\{ -\frac{s}{M^2} \right\} \right. \]
\[+ \frac{m_a m_a \langle \bar{s}s \rangle}{4\pi^2} \int_{\Delta^a}^{s_0} ds \int_{-\Delta^a}^{1} dx(2-s) \exp \left\{ -\frac{s}{M^2} \right\} \] 
\[+ \frac{m_a^2 \langle \alpha_s \overline{G}G \rangle}{576\pi^2} \int_{\Delta^a}^{s_0} ds \int_{-\Delta^a}^{1} dx \left( \frac{4}{x^2} - \frac{9}{x} - 3x^2 + 2x + 9 \right) s \exp \left\{ -\frac{s}{M^2} \right\} \]
\[+ \frac{m_a^3 m_a \langle \bar{s}g_s \sigma Gs \rangle}{24\pi^2} \int_{\Delta^a}^{s_0} ds \int_{-\Delta^a}^{1} dx \left( \frac{1}{x} \right) \exp \left\{ -\frac{\tilde{m}_a^2}{M^2} \right\} + \frac{m_a^3 \langle \bar{s}s \rangle^2}{3} \exp \left\{ -\frac{m_a^2}{M^2} \right\} \]
\[+ \frac{m_a^3 m_a \langle \bar{s}g_s \sigma Gs \rangle}{12\pi^2} \exp \left\{ -\frac{m_a^2}{M^2} \right\} \right\} \}.
\]

3 Numerical results and discussions

The input parameters are taken to be the standard values \( \langle \bar{q}q \rangle = -(0.24\pm 0.01) \text{GeV}^3 \), \( \langle \bar{s}s \rangle = (0.8 \pm 0.2) \langle \bar{q}q \rangle \), \( \langle \bar{s}g_s \sigma Gs \rangle = m_0^2 \langle \bar{s}s \rangle \), \( m_0^2 = (0.8 \pm 0.2) \text{GeV}^2 \), \( \langle \alpha_s \overline{G}G \rangle = (0.33\text{GeV})^4 \), \( m_s = (0.14 \pm 0.01) \text{GeV} \), \( m_c = (1.4 \pm 0.1) \text{GeV} \) and \( m_b = (4.8 \pm 0.1) \text{GeV} \) [3] [7] [9]. The contribution from the gluon condensate \( \langle \alpha_s \overline{G}G \rangle \) is less than 4%, and the uncertainty is neglected here.

For the octet baryons with \( I(J^P) = \frac{1}{2}(1^+) \), the mass of the proton (the ground state) is \( M_p = 938 \text{MeV} \), and the mass of the first radial excited state \( N(1440) \) (the Roper resonance) is \( M_{1440} = (1420 - 1470) \text{MeV} \approx 1440 \text{MeV} \) [10]. For the decuplet baryons with \( I(J^P) = \frac{3}{2}(3^+) \), the mass of the \( \Delta(1232) \) (the ground state) is \( M_{1232} = (1231 - 1233) \text{MeV} \approx 1232 \text{MeV} \), and the mass of the first radial excited
Table 1: The contributions from different terms in the sum rules for the $\Omega_c^*$ with the central values of the input parameters.

| Term                   | Eq.(11)  | Eq.(12)  |
|------------------------|----------|----------|
| perturbative term      | $+80\%$ | $+83\%$ |
| $\langle \bar{s}s \rangle$ | $+12\%$ | $+10\%$ |
| $\langle \bar{s}g_s\sigma G s \rangle$ | $-4\%$  | $-2\%$  |
| $\langle \bar{s}s \rangle^2$ | $+12\%$ | $+7\%$  |
| $\langle \bar{s}G G \pi \rangle$ | $+1\%$  | $+2\%$  |

Table 2: The contributions from different terms in the sum rules for the $\Omega_b^*$ with the central values of the input parameters.

| Term                   | Eq.(11)  | Eq.(12)  |
|------------------------|----------|----------|
| perturbative term      | $+78\%$ | $+80\%$ |
| $\langle \bar{s}s \rangle$ | $+10\%$ | $+10\%$ |
| $\langle \bar{s}g_s\sigma G s \rangle$ | $-4\%$  | $-3\%$  |
| $\langle \bar{s}s \rangle^2$ | $+15\%$ | $+12\%$ |
| $\langle \bar{s}G G \pi \rangle$ | $+1\%$  | $+1\%$  |

The state $\Delta(1600)$ is $M_{1600} = (1550 - 1700)\text{MeV} \approx 1600\text{MeV}$ $[10]$. The separation between the ground states and first radial excited states is about 0.5GeV. So in the QCD sum rules for the baryons with the light quarks, the threshold parameters $s_0$ are always chosen to be $\sqrt{s_0} = M_{gr} + 0.5\text{GeV}$ $[8,11]$, here $gr$ stands for the ground states. The threshold parameters for the heavy baryons $\Omega_c^*$ and $\Omega_b^*$ can be chosen to be $s_0^{\Omega_c^*} = (2.8 + 0.5)^2\text{GeV}^2$ and $s_0^{\Omega_b^*} = (6.1 + 0.5)^2\text{GeV}^2$, respectively. The mass of the bottomed baryon $\Omega_b^*$ with spin-parity $\frac{3}{2}^+$ is about $M_{\Omega_b^*} = (6.04 - 6.09)\text{GeV}$, which is predicted by the quark models and lattice QCD $[12,13]$.

In this article, the threshold parameters and Borel parameters are taken as $s_0^{\Omega_c^*} = 11.0\text{GeV}^2$ and $M^2 = (2.5 - 3.5)\text{GeV}^2$ for the charmed baryon $\Omega_c^*$, and $s_0^{\Omega_b^*} = 45.0\text{GeV}^2$ and $M^2 = (5.0 - 6.0)\text{GeV}^2$ for the bottomed baryon $\Omega_b^*$. The contributions from different terms for the central values of the input parameters are presented in Table.1 and Table.2, respectively. From the two tables, we can expect convergence of the operator product expansion. In the two sum rules in Eqs.(11-12), the contributions from the terms proportional to the quark condensate $\langle \bar{s}s \rangle$ and mixed condensate $\langle \bar{s}g_s\sigma G s \rangle$ are suppressed due to the small mass $m_s$, comparing with the terms proportional to the $\langle \bar{s}s \rangle^2$. Furthermore, from the 'full' propagator of the $s$ quark, we can see that the mixed condensate $\langle \bar{s}g_s\sigma G s \rangle$ is accompanied with additional large denominators, its contribution is even smaller. In the right-hand side of Eqs.(11-12), the terms proportional to the $\langle \bar{s}s \rangle^2$ are suppressed...
by the exponents $\exp[-m_a^2/M^2]$, which is balanced by the factor $\exp[-M_{\Omega^*_a}^2/M^2]$ in the left-hand side. Although the masses of the $c$ quark and $\Omega_c^*$ baryon are much smaller than the corresponding ones of the $b$ quark and $\Omega_b^*$ baryon, the Borel parameters $M^2$ are different, for the central values of the Borel parameters $M^2$, $\exp[M_{\Omega^*_b}^2/M^2 - m_b^2/M^2] > \exp[M_{\Omega^*_c}^2/M^2 - m_c^2/M^2]$. It is not unexpected, the contributions from the $(\bar{s}s)^2$ are larger in the sum rules for the $\Omega_b^*$ baryon than the ones for the $\Omega_c^*$ baryon.

If we approximate the phenomenological spectral density with the perturbative term, the contribution from the pole term is as large as $(28 - 54)\%$ for the charmed baryon $\Omega_c^*$ and $(33 - 50)\%$ for the bottomed baryon $\Omega_b^*$. We can choose smaller Borel parameter $M^2$ or larger threshold parameters $s_0^a$ to enhance the contributions from the ground states. However, if we take larger threshold parameter $s_0^a$, the contribution from the first radial excited state maybe included in; on the other hand, for smaller Borel parameter $M^2$, the sum rules are not stable enough, the uncertainty with variation of the Borel parameter is large. In the case of the multiquark states, the standard criterion of the lowest pole dominance cannot be satisfied, we have to resort to new criterion to overcome the problem, for detailed discussions about this subject, one can consult Ref.[14].

Taking into account all uncertainties of the input parameters, finally we obtain the values of the masses and residues of the heavy baryons $\Omega_c^*$ and $\Omega_b^*$, which are shown in Figs.1-4 respectively,

$$
M_{\Omega_c^*} = (2.72 \pm 0.12) \text{GeV}, \\
M_{\Omega_b^*} = (6.04 \pm 0.13) \text{GeV}, \\
\lambda_{\Omega_c^*} = (0.047 \pm 0.008) \text{GeV}, \\
\lambda_{\Omega_b^*} = (0.057 \pm 0.011) \text{GeV},
$$

(15)
Figure 2: $\lambda_{Q_c}$ with Borel parameter $M^2$, A from Eq.(11) and Eq.(13), and B from Eq.(12) and Eq.(14).

Figure 3: $M_{Q_b}$ with Borel parameter $M^2$, A from Eq.(13) and B from Eq.(14).
from A from Eq.(11) and Eq.(13), and
\[
M_{\Omega_c^*} = (2.80 \pm 0.08)\text{GeV}, \\
M_{\Omega_b^*} = (6.08 \pm 0.12)\text{GeV}, \\
\lambda_{\Omega_c^*} = (0.046 \pm 0.007)\text{GeV}, \\
\lambda_{\Omega_b^*} = (0.060 \pm 0.011)\text{GeV},
\]
(16)
from Eq.(12) and Eq.(14). The average values are about
\[
M_{\Omega_c^*} = (2.76 \pm 0.10)\text{GeV}, \\
M_{\Omega_b^*} = (6.06 \pm 0.13)\text{GeV}, \\
\lambda_{\Omega_c^*} = (0.047 \pm 0.008)\text{GeV}, \\
\lambda_{\Omega_b^*} = (0.058 \pm 0.011)\text{GeV}.
\]
(17)

The value of the mass $M_{\Omega_c^*}$ is compatible with the experimental data $M_{\Omega_c^*} = (2.768 \pm 0.003)\text{GeV}$ [10], the interpolating current $J_{\mu}^c(x)$ can couple with the charmed baryon $\Omega_c^*$ and give reasonable mass. The value of the mass $M_{\Omega_b^*}$ for the bottomed baryon $\Omega_b^*$ with $\frac{3}{2}^+$ is compatible with other theoretical calculations, $M_{\Omega_b^*} = (6.04 - 6.09)\text{GeV}$, such as the quark models and lattice QCD [12, 13]. Once reasonable values of the residues $\lambda_{\Omega_c^*}$ and $\lambda_{\Omega_b^*}$ are obtained, we can take them as basic input parameters and study the hadronic processes [15], for example, the radiative decay $\Omega_c^* \to \Omega_c \gamma$, with the light-cone QCD sum rules or the QCD sum rules in external field.

4 Conclusion

In this article, we calculate the masses and residues of the heavy baryons $\Omega_c^*(c\bar{s}s)$ and $\Omega_b^*(b\bar{s}s)$ with the QCD sum rules. The numerical values are compatible with
the experimental data and other theoretical estimations. Once reasonable values of the residues $\lambda_{\Omega_c}$ and $\lambda_{\Omega^*_b}$ are obtained, we can take them as basic parameters and study the hadronic processes, for example, the radiative decay $\Omega^*_c \to \Omega_c \gamma$, with the light-cone QCD sum rules or the QCD sum rules in external field.

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