Enhancement of Superconducting correlation due to interlayer tunneling

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Abstract

Interlayer single particle tunneling between the Cu – O layers suppress the in-plane short range magnetic order (which is modeled as spin density wave (SDW) insulator). Doping over the SDW state kills perfect nesting of the Fermi surface (FS) in certain directions and hence SDW gap reduces to zero in those directions of the FS. Coupling between the planes through interlayer tunneling ($t_\perp$) further suppresses the in-plane magnetic SDW-gap and hence becomes anisotropic. Superconductivity arises in the gapless regions of the FS under the ‘modified spin bag’ mechanism. We show that the highest $T_c$ can only be obtained for non-zero $t_\perp$ based on this mechanism.

Keywords : Spin density wave, Pseudo gap, Anisotropic superconductors, Pair breaking.

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1 Introduction.

Short range antiferromagnetic correlations in the superconducting (SC) phase of high $T_c$ materials have been confirmed by numerous neutron [1] and Raman Scattering experiments [2], although the proper understanding of the nature of the SC mechanism is far from reality. However, the normal state of the cuprates at half filling is insulating and antiferromagnetic in nature. There are two different ways of describing the insulating state of the pure (undoped) material: (a) the localised description of the electrons in the Mott–Hubbard insulator for which $U >> W$ (bandwidth), the system stabilises to the so called Resonating Valence Bond (RVB) state in comparison to the Neel ordered ground state. (b) In the limit $U \leq W$, i.e. in the itenarent description of the electrons the nesting of the FS leads to the SDW or CDW instability driving the system from normal metallic to insulating state [4]. We explore the later possibility. The mechanism of SC in the two descriptions are expected to be of different origin, as the nature of charge carriers and the effective pairing interactions are quite different.

The important features of band structure calculations for 214 and 123 compounds are the following [5]: (i) The bands show very little dispersion along the c-axis, indicating the 2-d nature of the electronic states, (ii) The Fermi surface is square planar with perfect nesting along the wave vector $\vec{Q} = (\frac{\pi}{a}, \frac{\pi}{a}, 0)$. The later fact further confirms the low dimensional character of electronic states and predicts the instability of the system against the formation of a spin density wave (SDW) state. Based on such a physical picture Schrieffer et al, [6] describes the ground state of the cuprates as SDW insulator, characterised by an isotropic SDW gap through out the Fermi surface (FS). Doping with injected charge carriers essentially suppress the SDW gap locally which acts as a potential well for the carriers, in which the injected charge carrier gets self trapped (so called spin bag). Two such holes of opposite spins find it energetically favourable to stay together digging a deeper well. This is so, because there always exists an attractive interaction
between the charge carriers mediated by the quanta of fluctuations of the amplitude mode of the SDW gap\cite{6,7} and hence superconductivity sets in. 

However, superconductivity under such a situation is restricted either to lower or the upper SDW band. Such a picture is physically reasonable strictly in very low doping limit, so that the nesting of the FS remains intact. On the other hand as one dopes over the SDW ground state, it is expected that the FS will move away from perfect nesting and thereby the the SDW gap will no longer be an isotropic one. The SDW gap will exist only in the directions of the FS, where nesting is still preserved and with no gap in other directions, resulting in a pseudo gap rather than a real gap at the Fermi level. Therefore, the above picture needs necessary modifications as will be described in more details in the next section.

Experimentally\cite{1} it is found that both the short range spin ordering as well as the dispersion of the spin excitations above the Neel temperature is predominantly 2-d in nature. This is so, because the exchange integral between the planes ( proportional to the square of the interlayer tunneling matrix element $t_\perp$) is much smaller $\sim 10^{-5}$ compared to the inplane exchange coupling. On the other hand, in our ‘modified spin bag’ scenario, there are low energy quasiparticles available in large parts of the FS, where nesting is lost. They are now available to tunnel across the planes. Hence the interplane tunneling of quasiparticles (linear in $t_\perp$) is expected to be a more relevant perturbation than the interplane exchange coupling.

In the present paper, our main aim is to incorporate a physically realistic model for high $T_c$ superconductivity modifying Schrieffer’s spin bag model\cite{4}, which holds good only in the low doping limit. Subsequently, emphasis has been given to interlayer tunneling of SDW quasiparticles in the intermediate metallic SDW state. We find that the interlayer tunneling between the planes, suppress the inplane SDW gap, which in turn enhances the superconducting correlations. In sec.II, we describe the basic ingredients of the ‘modified
spin bag’, indicating the shortcomings of the spin bag model. Sec. III is devoted to show, how the interlayer tunneling between the layers suppress the inplane magnetic SDW gap. The modified spin bag model is employed to describe superconductivity in sec. IV. Finally, we remark about our results in the concluding section V.

2 Modified Spin Bag Model.

The normal state of the high $T_c$ superconductors exhibits a number of anomalous properties like, linear temperature dependence of resistivity, Raman continuum due to scattering of light by charge carrier. In order to understand such puzzling features of the cuprates in the normal as well as the SC state, realistic theoretical model is necessary. One of the most important theoretical model in this direction is the spin bag model of Schrieffer et al as mentioned in the last section. According to spin bag model, on doping holes the long range Neel order is suppressed, but still short range magnetic order persists for small doping away from half filling. This can be identified with a SDW state (however in dispute). The Cu-O planes being 2-d square planar exhibits perfect nesting. Presence of high degree of nesting in the Fermi surface, together with strong in-plane spin correlation between the Cu-spins drive the system to SDW instability and in the process there is a net gain in energy, because states with higher energies come down to lower energy states resulting in a gap at the Fermi level of the electronic spectrum. Now, in order to get SC one needs to dope the system with charge carriers. In very low doping limit, doping supresses the SDW gap locally and hence Schrieffer’s spin bag picture holds good. However, for all practical purposes, superconductivity in these materials appear only at larger doping over the Neel ordered state. Hence the FS is expected to deviate from perfect nesting. As a result the SDW gap will vanish in certain directions, rather than just the local supression as in the spin bag model. Therefore, the SDW gap will exist only in certain parts of FS in which nesting still exists, where as in other directions
there is no gap, resulting in a pseudo gap for excitations. Under these circumstances the pairing takes place dominantly between the SDW quasiparticles from both the bands (lower and upper SDW bands). The pairing interaction between the quasiparticles in the regions of the FS, where the gap is suppressed is mediated by the quanta of the fluctuations of the amplitude and phase modes of the SDW gap (collective modes). The presence of SDW gap in certain parts of the FS is however essential, because it is the quanta of the fluctuations of the collective modes that mediate the pairing interaction between the quasiparticles. The main point being that here the superconductivity is not restricted either to the SDW valence or conduction band alone, but is a global phenomena, unlike in \[6\]. The low energy SDW quasiparticles, that feel the pairing interaction, come from the regions where SDW gap has gone to zero or vanishingly small, i.e., from the regions where the lower (valence) and upper (conduction) SDW bands touch other. So they come from both the valence and conduction bands. These are the basic ingredients of the “modified spin bag model” (a similar model for non cuprate systems called the “modified charge bag model” is proposed in [11]).

However, this is a purely two dimensional picture where the coupling between the planes is neglected. On the other hand, under the modified spin bag scenario, the nature of the SDW quasi-particles (above \(T_c\)) in most of the regions of the FS being free electron like, the interlayer tunneling between planes is likely to play a crucial role. The motivation for introducing the coupling between the planes through interlayer tunneling is as follows. Anisotropy in the normal state properties (resistivity, thermal conductivity) is typically of the order of \(10^3\) or more, whereas typical anisotropy of quantities like coherence length, penetration depth below \(T_c\) is \(\approx 5 - 10\). Also the analysis of the fluctuation conductivity and diamagnetic fluctuations above \(T_c\) has shown that the character of fluctuation changes from 2-d to 3-d as one approaches \(T_c\). This shows that the normal to SC transition is at the same time a dimensional crossover from 2-d to 3-d. This is taken with the fact that
the BISCO materials ($T_c \sim 10^oK$ for single layer, $\sim 80^o$ K for double layer and $\sim 110^o$ K for triple layers ) clearly indicates that the third direction coupling between the planes, is a relevant parameter that ultimately brings in the full 3-d coherence. The $T_c$ may not be a single plane property and interplanar coupling has to be ultimately incorporated in any kind of theory of superconductivity [12]. Here we explore the effect of single particle tunneling on the metallic SDW ground state (the normal state) of the $Cu - O$ planes and study the observable effects on the SC state.

Usually in coupled planar superconductors, the tunneling of single particles between the planes, suppress superconductivity, because as far as individual planes are concerned, it acts as a pair breaking perturbation, and hence $T_c$ is reduced. In contrast, here we will show that, under this novel mechanism of superconductivity exactly the opposite happens and maximum $T_c$ is got only for non zero $t_\perp$ between the planes.

3 Effect of Interlayer tunneling on the SDW state.

A mean field theory for the SDW state starting from the original Hubbard interaction has already been well studied [3]. Perfect nesting of the FS exists due to the 2-d nature of the electrons in the Cu-O plane, and is described by the tight binding dispersion relation,

$$\epsilon_k = -\epsilon_{k+Q} = -2t(cosk_xa + cosk_ya)$$  \hspace{1cm} (1)

where $Q$ being the nesting wave vector corresponding to the 2-d square lattice. In contrast, (to [3, 4]) we consider two Cu-O layers per unit cell, where each $Cu - O$ layer can be described as SDW insulator in the ground state. Doping over the SDW state destroys FS nesting in certain directions, resulting in the suppression of the SDW gap. The behaviour of quasi particles in the regions where nesting is lost, is likely to be a nested Fermi liquid [5]. Therefore, the interlayer tunneling between the metallic $Cu - O$ layers is likely to play an important role in the subsequent SDW and hence SC state. We model our system
as two SDW planes corresponding to two layers, coupled by single particle interlayer tunneling.

\[ H = H_0 + H' \]

\[ H_0 = \sum_{k\sigma} (\epsilon_k - \mu) (c_{1k\sigma}^\dagger c_{1k\sigma} - c_{1k+Q\sigma}^\dagger c_{1k+Q\sigma}) + G \sum_{k\sigma} (\sigma_3)_{\sigma\sigma'} (c_{1k+Q\sigma}^\dagger c_{1k\sigma} + \text{h.c}) + 1 \rightarrow 2 \]  \hspace{1cm} (2)

\[ H' = \sum_{k\sigma} t_\perp (c_{1k\sigma}^\dagger c_{1k\sigma} + \text{h.c}) \]  \hspace{1cm} (3)

where the indices 1 and 2 refers to two different layers, \( t_\perp \) and \( G \) being the tunneling matrix element and the SDW order parameter respectively. The Hamiltonian \( H_0 \) in (2) describes the SDW states of different layers obtained by the mean field calculation of the Hubbard model [6, 7]. The interlayer tunneling between the layers, treated as small perturbation over the unperturbed SDW state \( (H_0) \), is described in equation (3). The SDW order parameter in different layers are defined as,

\[ G = -\frac{1}{2} U \sum_{k\sigma}^{FBZ} \langle c_{1k+Q\sigma}^\dagger (\sigma_3)_{\sigma\sigma'} c_{1k\sigma} \rangle = -\frac{1}{2} U \sum_{k\sigma}^{FBZ} \langle c_{2k+Q\sigma}^\dagger (\sigma_3)_{\sigma\sigma'} c_{2k\sigma} \rangle \]  \hspace{1cm} (4)

where \( U \) being the strength of the onsite Coulomb repulsion. Equation (4) indicates that each layer has the same SDW gap. The \( k \)-sum is over the full Brillouin zone. On the other hand the \( k \)-sums in equations (2), (3) are extended only upto the reduced magnetic Brillouin zone boundary.

Usually, for quasi 1-d systems the transition to SDW (repulsive system of electrons) or CDW (with strong electron phonon interaction) are calculated approximately within the framework of purely 1-d models, for which the exact \( T_c \) should, according to the Landau theory be exactly zero. To get a nonzero \( T_c \) one needs to introduce transverse coupling between the chains or planes, which could be either of two kinds, coulomb reulsion between the chains, or kinetic coupling, i.e single particle hopping (tunneling) term between the
chains or planes. The presence of non zero $t_\perp$ in the third direction usually stabilizes long range order at finite temperatures for quasi 1-d or quasi 2-d systems. So sufficiently close to the critical point, the 3-d character of correlation starts developing, so that a Ginzberg Landau theory of phase transition becomes valid. Two quite incompatible approaches are used to deal with anisotropic systems. One being, the existence of finite $t_\perp$ and $T_c$ justifies the use of global mean field, that neglects all lower dimensional effects. In the other scheme, one does mean field in the transverse direction only, treating the lower dimensional system more rigorously. The later approach has the merit of correctly reproducing the relevant physics in the limit $t_\perp \to 0$. However, it has very restricted domain of application. Reason is that, the transverse mean field is usually done on an effective interplane two particle hopping term, whereas the bare Hamiltonian has only single particle tunneling process. Standard perturbation theory in $t_\perp$ will lead to an effective pair hopping across the planes with amplitude proportional to $t_\perp^2/\Delta$, only if the two particles or a particle and hole are bound with a gap equal to $\Delta$, in the lower dimensional spectrum. In this way, there is no real one particle hopping, and it is through virtual processes that the transverse pair motion is possible. On the other hand, in our mechanism as we shall discuss later on, the SDW gap is zero over some parts of the Fermi surface and hence the gap becomes irrelevant. The ratio $t_\perp/\Delta$ is not small any more and perturbation theory breaks down. The effect of small $t_\perp$ in the pseudo gap situation has to be handled more carefully. Here we include and treat the single particle hopping exactly, and ignore the possible generation of particle hole pair hopping across the planes.

We rewrite the Hamiltonian (2) in the matrix representation as follows,

$$H = \sum_{k\sigma} \Psi_{k\sigma}^\dagger \hat{H}_M \Psi_{k\sigma}$$

(5)

where the Hamiltonian matrix ( $\hat{H}_M$ ) and four component Nambu operators are obtained
as,

\[
\hat{H}_M = \begin{pmatrix}
(\epsilon_k - \mu) & G(\sigma_3)_{\sigma\sigma} & t_\perp & 0 \\
G(\sigma_3)_{\sigma\sigma} & -(\epsilon_k - \mu) & 0 & 0 \\
t_\perp & 0 & (\epsilon_k - \mu) & G(\sigma_3)_{\sigma\sigma} \\
0 & 0 & G(\sigma_3)_{\sigma\sigma} & -(\epsilon_k - \mu)
\end{pmatrix}
\]  \tag{6}

and

\[
\Psi_{k\sigma}^\dagger = (c_{1k\sigma}^\dagger, c_{1k+Q\sigma}^\dagger, c_{2k\sigma}^\dagger, c_{2k+Q\sigma}^\dagger)
\]  \tag{7}

In order to diagonalize the Hamiltonian (2) one needs to find a suitable Bogoliubov transformation. To do so, we calculate the eigenvalues and eigenvectors of the matrix \(\hat{H}_M\) by unitary transformation, such that, \(H_{diag} = \hat{U}_{k\sigma}^{-1}\hat{H}_M\hat{U}_{k\sigma}\) and hence the diagonalised SDW Hamiltonian can be obtained as,

\[
H = \sum_{k\sigma} \phi_{k\sigma}^\dagger H_{diag} \phi_{k\sigma}
\]  \tag{8}

where \(\phi_{k\sigma} = (\gamma_{1k\sigma}^c, \gamma_{1k\sigma}^v, \gamma_{2k\sigma}^c, \gamma_{2k\sigma}^v)\), are the new SDW basis states, that diagonalise the Hamiltonian (2). \(\gamma_{1,2}^c,v\) are the SDW quasiparticle annihilation operators in the conduction (valence) band for the effective nonbonding (bonding) combinations (1,2).

Now, in order to find out the explicit structure of the Hamiltonian (8) we need to find out explicit structure of \(\hat{U}_{k\sigma}\) \((\Psi_{k,\sigma} = \hat{U}_{k\sigma}\phi_{k\sigma})\) which requires the evaluation of the eigen vectors of \(\hat{H}_M\) because each column of \(\hat{U}_{k,\sigma}\) is given by the respective orthonormalised eigen vectors of \(\hat{H}_M\). We evaluate the unnormalised orthogonal eigen vectors of \(\hat{H}_M\) as,

\[
\hat{e}_1 = \begin{pmatrix}
1 \\
(E_k^0 - \epsilon_k^0)/\hat{G}(\sigma_3)_{\sigma\sigma'} \\
1 \\
-1 \\
-1
\end{pmatrix}, \quad \hat{e}_2 = \begin{pmatrix}
1 \\
-(E_k^0 + \epsilon_k^0)/\hat{G}(\sigma_3)_{\sigma\sigma'} \\
1 \\
-1 \\
-1
\end{pmatrix}, \quad \hat{e}_3 = \begin{pmatrix}
1 \\
-(E_k^0 + \epsilon_k^0)/\hat{G}(\sigma_3)_{\sigma\sigma'} \\
1 \\
-1 \\
-1
\end{pmatrix}, \quad \hat{e}_4 = \begin{pmatrix}
1 \\
(E_k^0 - \epsilon_k^0)/\hat{G}(\sigma_3)_{\sigma\sigma'} \\
1 \\
-1 \\
-1
\end{pmatrix}
\]  \tag{9}
Therefore, one can readily find the transformation matrix \( \hat{U}_{k,\sigma} \) (or the so-called appropriate Bogoliubov transformation) that connects the 4-component Nambu-operator \( \Psi_{k,\sigma} \) of the original lattice to the 4-component Nambu operator \( \phi_{k,\sigma} \) for the condensate SDW state as,

\[
\begin{pmatrix}
c_{1k}\ 
c_{1k+Q}\ 
c_{2\sigma}\ 
c_{2k+Q}\end{pmatrix} =
\begin{pmatrix}
u_k^1 & v_k^1 & v_k^2 & u_k^2 \\
u_k^1 & v_k^1 & v_k^2 & u_k^2 \\
-(\sigma^3)_{\sigma\sigma'}v_k^1 & -(\sigma^3)_{\sigma\sigma'}u_k^1 & (\sigma^3)_{\sigma\sigma'}v_k^2 & (\sigma^3)_{\sigma\sigma'}u_k^2 \\
-u_k^1 & -v_k^1 & v_k^2 & u_k^2 \
\end{pmatrix}
\begin{pmatrix}
\gamma_{1k}\ 
\gamma_{1k}\ 
\gamma_{v\sigma}\ 
\gamma_{v\sigma}\end{pmatrix}
\tag{10}
\]

Where \( u_k^1(v_k^1) = \frac{1}{2}(1 \pm \frac{\epsilon_k}{E_k})^{1/2} \) and \( u_k^2(v_k^2) = \frac{1}{2}(1 \pm \frac{\epsilon_k}{E_k})^{1/2} \), with \( E_k^0 \) and \( \tilde{E}_k^0 \) being the SDW quasiparticle energies for the antibonding and bonding bands, and are given by,

\[
E_k^0 = \sqrt{(\epsilon_k)^2 + G^2} = \sqrt{(\epsilon_k - \mu - \frac{t_\perp}{2})^2 + G^2} \tag{11}
\]

\[
\tilde{E}_k^0 = \sqrt{(\epsilon_k)^2 + G^2} = \sqrt{(\epsilon_k - \mu + \frac{t_\perp}{2})^2 + G^2} \tag{12}
\]

Hence the mean field SDW Hamiltonian for the system can be obtained using equations (5 - 10) as,

\[
H_{SDW} = \sum_{i=1}^{2} \sum_{k\sigma} (E_{ik}^c \gamma_{i\sigma k}^c \gamma_{i\sigma k}^c + E_{ik}^v \gamma_{i\sigma k}^v \gamma_{i\sigma k}^v)
\tag{13}
\]

where \( i=1,2 \), corresponds to the bonding and antibonding SDW bands and

\[
E_{1k}^{c(v)} = -\frac{t_\perp}{2} \pm E_k^0 \quad \text{and} \quad E_{2k}^{c(v)} = \frac{t_\perp}{2} \pm \tilde{E}_k^0 \tag{14}
\]

are the quasi particle energies in the respective bands. Note that \( E_{1k}^{c(v)} \) and \( E_{2k}^{c(v)} \) are symmetric about \(-\frac{t_\perp}{2}\) and \( \frac{t_\perp}{2} \) respectively. In absence of \( t_\perp \), both the layers are degenerate with SDW quasiparticle energies \( E_k = \pm \sqrt{(\epsilon_k - \mu)^2 + G^2} \) and both the SDW planes are equally gapped. On the other hand, for any finite \( t_\perp \) the hybridized effective bands are non degenerate, being symmetric only about the shifted Fermi energy. This clearly indicates that the interlayer tunneling have non trivial observable effects on the SDW ground state.
The remaining part of this section will be devoted to showing how the SDW gap gets modified in presence of interlayer tunneling.

Now the actual nature of the SDW gap parameter can be found out from the self consistent calculation of the SDW order parameter given in eqn (4) using eqn (10).

\[
G = -\frac{U}{2N} \sum_{k\sigma} \sum_{i=1}^{2} \left( u_i^k v_i^k \right) \left( \langle \gamma_{ik\sigma}^c \gamma_{ik\sigma}^c \rangle - \langle \gamma_{ik\sigma}^v \gamma_{ik\sigma}^v \rangle \right) \tag{15}
\]

Where the thermal averages \( \langle ... \rangle \) have to be evaluated with the mean field Hamiltonian (13). The resulting gap equation can be written as,

\[
1 = -\frac{U}{2E_0^k} \sum_k \left[ f(E_{1k}^1) - f(E_{2k}^2) \right] - \frac{U}{2E_0^k} \sum_k \left[ f(E_{3k}^3) - f(E_{4k}^4) \right] \tag{16}
\]

For T=0 the above gap equation (16) simplifies to,

\[
1 = -\frac{U}{4} \sum_k \left( \frac{1}{E_{1k}^0} + \frac{1}{E_{2k}^0} \right) \tag{17}
\]

Exact solution of the gap equation (17) results,

\[
G(T = 0) = 2\sqrt{E_s^2 - t_\perp^2/4} \ \text{Exp}(-\frac{1}{N(0)U}) \tag{18}
\]

where \( E_s \) is some cut off energy below which the system undergoes SDW transition. Equation (18) clearly shows that the SDW gap reduces with the interlayer tunneling matrix element. The SDW gap at zero temperature without interlayer tunneling can be reproduced from equation (18) by setting \( t_\perp = 0 \). For a small but finite value of \( t_\perp \), \( G(k)^{t_\perp \neq 0}/G(k)^{t_\perp = 0} = 0 \) is plotted (Fig.1) for different values of \( k_x \) and \( k_y \). It is shown that the SDW gap reduces upto 98% of its original value (in absence of \( t_\perp \)) in certain directions of the FS and thereby the SDW gap becomes anisotropic. The parameter values chosen are, \( E_s = 0.3eV, N(0)V = 0.5 \) and \( t_\perp \) varies from zero to 0.1eV. With the above parameter values we get a SDW transition temperature (mean field ) of 400°K. The SDW order parameter \( G \) signifies the net amount of magnetic moment (in the z-direction for longitudinal SDW) at each site. Therefore, the interlayer tunneling have similar effect.
as magnetic impurity in ordinary superconductors, i.e the interlayer tunneling will cause SDW pair breaking leading to the suppression of SDW order parameter. The suppression of the inplane SDW ground state in turn enhances the superconducting correlations, simply because the more the number of SDW pair breaks the more number of quasiparticles becomes available for superconducting pairing, which we shall discuss in the next section.

The $k$-dependence of the SDW gap essentially depends on the anisotropic nature of the $t_\perp(k)$. It is known from electronic structure calculations \cite{[14]} of high-$T_c$ materials (each $Cu-O$ layers being two-dimensional) that the two layers touch along the $\Gamma M$ line. The point $M$ corresponds to $(\pi/a, \pi/a)$. The largest splitting of the hybridized bands of the two layers is seen to be at the point $X$, which is $(\pi/a, 0)$. Following the above symmetry, Chakravarty \cite{[15]} et al, chosen the form of $t_\perp(k)$ as

$$t_\perp(k) = \frac{t_\perp}{4} [\cos(k_xa) - \cos(k_ya)]^2 \quad (19)$$

We use the same form of $t_\perp(k)$. Now, combining (18) and (19) it is easy to see that the SDW gap will be maximum along the $k_x = k_y$ line whereas it will be minimum along the directions where $t_\perp(k)$ is maximum (i.e, the points $(0, \pm \pi/a)$, $(\pm \pi/a, 0)$) (cf. Fig.2). Therefore, the SDW gap will become smaller and smaller in certain regions of the FS, whereas it will retain its unperturbed magnitude in certain other directions. Further as we have already mentioned in sec.2. that the SC-pairing will take place predominantly between the SDW quasi particles in the gap free regions of the FS. So the SC gap peaks up in the regions where the SDW gap gets suppressed and vice-versa.

4 Superconducting State

The effective interaction between the SDW quasi particles and the fluctuations of the collective modes of the SDW state will give rise to a new kind of electron-amplitudon (phason) interaction This will be responsible for superconductivity under the present
mechanism. Such an effective interaction considering the phase and amplitude fluctuations of the order parameter had already been constructed by Behera et al., \[7\] using linear response theory of density fluctuations. In the present picture similarly, the interaction Hamiltonian due to the amplitude and phase fluctuations of the SDW order parameter in the $H_0$ part of the Hamiltonian in (2) is developed. However, the SDW order parameter $G$ in eqn(2) has to be replaced by the self-consistently calculated $G$ in eqn(18), such that the essential effect of the interlayer tunneling is taken in to account in intermediate SDW state. The interaction Hamiltonian can be written as [see ref.6],

$$H^I = U \sum_{k,q,\sigma} \sum_{i=1}^2 \hat{\Psi}^\dagger_{k+q,\sigma} \hat{\sigma}_i \hat{\Psi}_{k,\sigma} (d^i_q + d^i_{-q})$$

(20)

where $d_i$ is the annihilation operator for the amplitude and phase fluctuation modes of the SDW state. The above interaction Hamiltonian can be reduced to an attractive effective electron-electron interaction by usual second order perturbation theory. As is clear from eqn(20) that the form of the effective interaction between the SDW quasi particles will be quartic in $\Psi_{k,\sigma}$. In deducing the effective interaction (second order process) between the SDW quasi particles from both (the upper and the lower SDW) bands the intraband interaction terms like $\gamma^\nu \gamma^\nu \gamma^\nu \gamma^\nu$ and $\gamma^c \gamma^c \gamma^c \gamma^c$ are neglected. The details of the mean field theory leading to pairing Hamiltonian in case of charge density wave (CDW) superconductors (non-cuprate high $T_c$ superconductors) is discussed in [11]. We obtain the mean field pairing Hamiltonian derived from the effective electron-electron interaction as,

$$H_{eff} = \sum_{k,\sigma} \sum_{i=1}^2 E_k [\gamma^\dagger_{ik,\sigma} \gamma^c_{ik,\sigma} + \gamma^\dagger_{ik,\sigma} \gamma^c_{ik,\sigma}] + \sum_k \sum_{i=1}^2 \Delta(k) [\gamma^c_{i-k,\downarrow} \gamma^c_{ik,\uparrow} + \gamma^c_{i-k,\downarrow} \gamma^c_{ik,\uparrow} + h.c]$$

(21)

where $\Delta(k)$ appears as the effective SC-gap in the electronic energy spectrum as

$$e_k = \sqrt{E_k^2 + \Delta_{SC}(k)^2}$$
The anisotropic SC order parameter $\Delta_{SC}(k)$ is obtained as,

$$\Delta_{SC}(k) = -\frac{1}{N} \sum_{k'} [\lambda \sin^2(\phi_k + \phi_{k'}) + \lambda' \cos^2(\phi_k + \phi_{k'})] \langle \gamma^c_{ik'} \gamma^c_{ik} \gamma^\dagger_{ik'} \gamma^\dagger_{ik} \rangle$$  \hspace{1cm} (22)

where $\lambda(\lambda')$ are the dimensionless effective electron-electron coupling constants (proportional to $U^2$) and is equal to $\lambda, \lambda' = \frac{\Omega_{AM}}{(E_k \pm E_{k'})^2 - \Omega_{AM}^2}$, where $\Omega_{AM}$ being the maximum frequency of the SDW gap fluctuations (amplitudon) given by $\Omega_{AM} = 2G$. The order parameter $\Delta_{SC}$ is assumed to be real and invariant under the transformation $v \leftrightarrow c$. Self-consistent evaluation of the correlation function in the above equation yields the SC-gap equation as an integral equation given by,

$$\Delta_{SC}(k) = \sum_{k'} \left[ \lambda_1 + \lambda_2 \frac{(\epsilon_k - \mu)(\epsilon_{k'} - \mu) - G(k)G(k')}{E_kE_{k'}} \right] \frac{\Delta_{SC}(k')/e_{k'}}{\tanh(\beta e_{k'}/2)}$$  \hspace{1cm} (23)

where $\lambda_{1(2)} = \lambda \pm \lambda'$. The gap equation in general is very difficult to solve analytically. We solve the gap equation in the low doping limit, i.e $\epsilon_k - \mu \rightarrow 0$, implying that the injected carriers will stick to the band gap edges with very low mobility (see Schrieffer et al [6]). With this approximation, the gap equation becomes,

$$\Delta_{sc}(k) = \sum_{k'} [(\lambda_1 - \lambda_2) + \lambda_2 \frac{(\epsilon_k - \mu)(\epsilon_{k'} - \mu)}{G(k)G(k')} \frac{\Delta_{SC}(k')/e_{k'}}{\tanh(\beta e_{k'}/2)}]$$  \hspace{1cm} (24)

It is clear that, the $T_c$ expression would have been like usual BCS if only the first term in the r.h.s. was present, but it will vary with SDW gap parameter $G(k)$ because of the second term. We obtain,

$$K_B T_c = 1.14 \hbar \Omega_{AM} \left[ \exp\left(-\frac{1}{(\lambda_1 - \lambda_2)N(0)} + \frac{\lambda_2 \delta^2}{4G^2N(0)}\right) \right]$$  \hspace{1cm} (25)

where $\delta$ is the doping concentration, satisfies $\frac{1}{N} \sum_{k\sigma} \langle c^\dagger_{k\sigma} c_{k\sigma} \rangle = 1 - \delta$ and fixes the chemical potential $\mu$. From equation (25) we see that, as $t_\perp$ increases, the SDW gap $G$ decreases, which in turn increases the superconducting transition temperature. For larger $t_\perp$, the SDW gap becomes smaller and smaller, and hence greater number of low energy quasi-particles becomes available for pairing. This leads to the increase of superconducting
transition temperature with increasing $t_\perp$ (cf. Fig. 3). Therefore, maximum $T_c$ is obtained only for non zero $t_\perp$ in this novel mechanism. This is contrary to the situation where, single particle tunneling operates between two planar superconductors. There $T_c$ decreases with increase of $t_\perp$.

5 Conclusions

We propose a physically realistic theoretical model (called ‘modified spin bag model’) for high temperature superconductivity — an appropriate modification over Schrieffer’s spin bag model \[6\]. Inclusion of single particle interlayer tunneling in the intermediate metallic SDW state incorporates a more realistic three dimensional situation. We presented in detail an effective perturbative approach for the transverse coupling and thereby the lower dimensional fluctuations are treated more rigorously. We also provide a quantitative description of the SC-pairing mechanism (modified spin bag scenario).

In the stoichiometric compounds, the CuO layers are considered as SDW insulators due to the presence of perfect nesting of Fermi surface and intermediate strength coulomb repulsion. In order to get SC one needs to dope the system with free charge carriers over the SDW state. The Fermi surface deviates from perfect nesting with doping, and SDW gap vanishes in most part of the Fermi surface, leaving small pockets of gapped regions where nesting is still preserved. The collective modes of the surviving SDW gap (amplitude and phase modes) mediates the pairing interaction between the low energy SDW quasiparticles available near the gapless regions. This leads to superconductivity. We point out, that in the normal state with a pseudo gapped Fermi surface, there are low energy quasiparticles available in the gap free regions. These are almost like free charge carriers and are available for tunneling across the planes. The single particle tunneling between the layers, essentially acts as a SDW pair breaking perturbation, leading to further suppression of the SDW gap. Hence the number of low energy quasiparticles
available for superconducting pairing increases, leading to an increase in the transition temperature. So the maximum $T_c$ is obtained for a nonzero value of $t_{\perp}$. The increase of $T_c$ with single particle tunneling amplitude, is exactly opposite to what happens in the case of, two planar superconductors coupled by a tunneling term. There $T_c$ decrease with increase in $t_{\perp}$.

As mentioned in Sec.2, there exists two distinct regions of the FS in the metallic SDW state — the gapless regions due to loss of perfect nesting in certain directions and the gapped region where nesting of the FS survives. The gapless regions of the FS is nothing but the nested Fermi liquid [8]. It is possible to show that in the large doping limit most of the regions of the FS is free electron like in the normal state and the nature of the SDW quasi particles e.g, life time etc. in the gapfree regions are nested Fermi liquid like. As a result scattering of light by the charge carriers is very large in the normal state. This will naturally lead to a large constant background intensity in the Raman spectrum. The calculated electronic Raman scattering intensity shows suppression on going from normal to SC state in accordance with the experimental results and will be published separately elsewhere [17].

Furthermore in our mechanism, since the SDW gap survives in some parts of the Fermi surface, even below $T_c$, this would explain the origin of short range antiferromagnetic correlations observed in the cuprates below the $T_c$. It is observed in high $T_c$ materials [14], that several phonon modes shows anomalous softening and narrowing of line width, even above the $T_c$. In NMR relaxation experiments it is observed to show a spin gap like feature, leading to decrease of relaxation rate much above the transition temperature. These can be identified with the existence of SDW gap in some parts of FS in our model.

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FIGURE CAPTIONS

Figure 1: The ratio of $G(t_\perp)$ and $G(t_\perp = 0)$ is plotted as a function of momenta $k_x$ and $k_y$. $G(t_\perp)$ reduces to 98% of $G(t_\perp = 0)$ along $\Gamma X$ points.

Figure 2: The SDW and superconducting gaps in arbitrary units are plotted versus $t_\perp$ along $\Gamma X$ points. The SDW gap shows suppression whereas the superconducting gap increases with $t_\perp$.

Figure 3: The superconducting transition temperature is plotted against $t_\perp$. The $T_c$ increases with $t_\perp$. 