Influence of electric charge on surface of a solar sail on dynamics of the sail which moves along Tsander's trajectories

A B Yakovlev
Saint Petersburg State University, 7/9 Universitetskaya nab., St. Petersburg, 199034, Russia
a.b.yakovlev@spbu.ru

Abstract. The application of a solar sail spacecrafts for a realization of distant flights into a space is actively discussed. However, all bodies in space plasma get electric charge which depends on density and temperature of plasma, a stream of sunlight and electrophysical characteristics of a body. But till now not enough attention has been given a problem of influence of the charging of a solar sail on its dynamics and durability characteristics. The charging of thin films, such as a solar sail has a number of features. Most important of them are: 1) a surface is bilateral and not closed, 2) partial permeability of a film for charged particles of a space environment. In this work there are the results of calculations of electric charge density on the surface of the solar sail in the space between orbits of the Earth and Jupiter and an estimate of influence of electric charge of a sail on dynamics of solar sail spacecrafts which moves along elliptic Tsander's trajectory. The problem about deflection determination is reduced to integration of one-dimensional problem of the motion of a body with variable acceleration. The obtained estimates of the influence of an electric charge induced on the surfaces of the solar sail on the motion of a space vehicle during interplanetary flights along the orbits mentioned above have shown that the influence is small.

1. Introduction
The development of space researches presumes the planning and realization of more distant flights into a space is actively discussed. Practice of flights of space vehicles to Jupiter and Saturn showed that possibility of use of alternative engine systems is required to study for reduction of their duration. For example, the application of a solar sail for such purposes is actively discussed [1, 2]. For hundred years a lot of trajectory problems and control problems of space vehicles with a sail is solved in light of various factors of the cosmic space, in particular for effect of the Earth shade. However, interaction of a sail with environment does not amount to pressure of sunlight only. All bodies in space plasma get electric charge which depends on density and temperature of plasma, a stream of sunlight, electrophysical characteristics of a body, and therefore on position and orientation of bodies during its moving. But till now not enough attention has been given a problem of influence of the charging of a solar sail on its dynamics and durability characteristics [3, 4].

The charging of space vehicles was actively studied both experimentally and theoretically. Thus, both analytical and numerical methods were applied to simulate this process [5-16]. The charging of thin films, such as a solar sail possess a number of features. Most important of them are: 1) a surface is bilateral and not closed, 2) area of surface large, but final, 3) partial permeability of a film for the
charged particles of a space environment. The research of partially permeable film is caused by presence of the charged particles with high energies in space plasma. It is especially actual for more thin films which are supposed to be used further. In this connection there is a necessity of modernization of the theory of the charging of a surface for space plasma taking into account these features. A charge density on a sail surface should become the major result of such modernized model. But knowledge of density of electric charge allows to solve problems about influence of the charging of a solar sail on its dynamics and durability characteristics. However, till now in most cases the studying of process of the charging of flat bilateral surfaces in space plasma was carried out to determine only potential of a surface concerning surrounding plasma and was made without taking into account partial permeability [17]. The first results of the model's modernization which take into account mentioned above features are considered in [4].

For the first time the charging process of partially permeable film in space plasma has been considered in [18, 19] for determination of the specific power of a high-voltage electric generator in natural radiation belts and in polar region. But in this case the object geometry has facilitated essentially a mathematical model of the process because only one component of plasma took part in the charging process.

2. A mathematical model of the charging process

In this work calculations of density of electric charge on the surface of the solar sail in the space between orbits of the Earth and Jupiter are carried out by the using of the method offered in [4]. We will notice that in this case for a description of the motion of a solar sail with accessible thickness and other characteristics that are necessary for realization of interplanetary flights we can use bilateral, infinite, aluminum plate completely absorbing particles with the given reflection factor of sunlight as a solar sail model. Also we suppose that 1) the body of space vehicle does not influence on the charging of a sail, 2) the expansion of coronal gas is adiabatic (the assumption is used for calculation of speed of a solar wind), 3) it is possible to use the conservation of a stream for determination of a solar wind density, 4) on a upper side (from the Sun) of the plate the distribution of electrostatic potential is nonmonotonic, 5) on an underside of a plate the distribution of electrostatic potential is monotonous, 6) potentials on both surfaces of a plate are equal.

As it was showed in [20], the distribution of photoelectrons leaving the aluminum plate has to be Maxwellian with concentration on a surface $N_{e0} = 1.891 \times 10^9 \text{ m}^{-3}$ and temperature $T_e = 0.9 \text{ eV}$ for a case of full absorption of sunlight and normal falling of radiance on a plate surface. For other values of a polar angle and a reflection factor a value of $N_{e0}$ are accordingly recalculated. For electrons and ions of a solar wind the distributions are Maxwellian with $T_e = 10 \text{ eV}$ and $N_{i0} = 9 \times 10^6 \text{ m}^{-3}$ (a quiet solar wind) [21]. Thus, for small angles of incidence $\theta$ of solar rays the condition of high photoemission $N_{e0} \gg N_{i0}$ is satisfied. Therefore, for particles above the plate it is possible to expect existence of nonmonotonic potential distribution.

The potential of electric field and particles distribution in a double layer are found by solving the system of the Vlasov-Poisson equations with corresponding boundary conditions. As the dimension of the sail is much more than solar wind’s plasma Debye length the Poisson equation becomes one-dimensional. For a bilateral plate the solve of the system of the Vlasov-Poisson equations is considered separately in two areas - above the plate (region 1) and under the plate (region 2)

$$\frac{d^2 \Phi}{dz^2} = -\frac{e}{\varepsilon_0} (N_i - N_{i1} - N_{e})$$

for the region 1 and

$$\frac{d^2 \Phi}{dz^2} = \frac{e}{\varepsilon_0} N_{e2}$$

(2)
for the region 2.
Here $\Phi$ is the potential of electric field, $z$ is the height above the lunar surface, $N_i$, $N_{el}$, $N_{e2}$, $N_{v}$ are concentrations of ions and electrons (in regions 1 and 2) of solar wind and photoelectrons, correspondingly, $e$ is the proton electric charge, $\varepsilon_0$ is the dielectric constant.

As in the papers [21-23] we use four parameters for determination of nonmonotonic potential: $\Phi_0$ is the potential on the plate surface, $\Phi_m$ is the minimum value $\Phi$ at height $z_m$, $\Phi_1$ is value of potential on external border of a double layer. To find the parameters mentioned above the following conditions are used:
- zero total current to the surface
  
  \[ j_i - j_e(z_m) - j_e(z_m) - j_e(\psi_0, \psi_m) = 0 \]

- neutrality condition on external border of a double layer ($\zeta \gg 1$)
  \[ N_{el}(\psi_1) + N_{v}(\psi_1) = N_i \]

- no electric field on external border of a double layer
  \[
  \left. \frac{d\psi}{d\zeta} \right|_{\zeta \rightarrow \infty} = -\left| Z_+(\psi_1) \right|^{1/2} = 0
  \]

Here

\[
\zeta = \frac{z}{D} \quad D = \left[ \frac{\varepsilon_0 kT_e}{N_{el} e^2} \right]^1/2 \\
\psi_0 = -\frac{e\Phi_0}{kT_e} \quad \psi_m = -\frac{e\Phi_m}{kT_e} \quad \psi_1 = -\frac{e\Phi_1}{kT_e} \quad \Psi(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2) dt
\]

\[
N_{el} = \frac{N_{el0}}{2\sqrt{\pi}} \exp(-w^2) \int_0^w \exp(-t + 2w\sqrt{t}) dt \\
N_{e1} = \frac{N_{e10}}{2\sqrt{\pi}} \exp(-w^2) \int_0^w \frac{\exp(-t + 2w\sqrt{t})}{\sqrt{t - \psi}} dt
\]

\[
N_{v}(\psi) = \cos \theta \frac{N_{v0}}{2} \exp(\tau \psi_0 - \tau \psi) \left[ 1 \pm \Psi(\sqrt{\psi - \psi_m}) \right]
\]

\[
\psi_0 = \frac{V_e}{2\sqrt{\pi}} \left\{ \exp\left[-(w - \sqrt{\psi_m})^2\right] + \sqrt{\pi} w \left[ 1 + \Psi(w - \sqrt{\psi_m}) \right] \right\}
\]

\[
\psi_{e1} = \frac{V_e}{2\sqrt{\pi}} \left[ 1 - \sqrt{\pi} w \right]
\]

\[
j_{1,2}(\psi_m) = -N_{v0} \frac{V_e}{2\sqrt{\pi}} \exp(\tau \psi_0 - \tau \psi_m)
\]

\[
j_{1,2}(\psi_0) = -N_{v0} \frac{V_e}{2\sqrt{\pi}} \exp(\tau \psi_0 - \tau \psi_m)
\]

\[
A = \frac{N_{el0} \cos \theta \exp(\tau \psi_0)}{N_{v0}} \quad F_{1,2}(\psi, \psi_m) = \frac{4}{\sqrt{\pi}} e^{-\psi} \left[ \int_0^\psi \exp(-x^2) \sqrt{x^2 + \psi} dx + \int_0^{\sqrt{\psi_0 - \psi}} \exp(-x^2) \sqrt{x^2 + \psi} dx \right]
\]
Here the top sign is pertinent to the area $0 \leq \zeta \leq \zeta_m$, the bottom sign - to area $\zeta_m < \zeta < \infty$ of the region 1. $V_0$ is the speed of the solar wind, ion density $N_{i0} = N_{e0}$, $m_e$ is mass of electron, $k$ is Boltzmann’s constant. In new variables the equation (1) is solved by integration

$$
\zeta = \zeta_m \mp \int_{\psi}^{\psi_{\infty}} \frac{d\psi}{\sqrt{Z_{1,2}(\psi)}}
$$

where

$$
\zeta_m = \int_{\psi_0}^{\psi_{\infty}} \frac{d\psi}{\sqrt{Z_1(\psi)}}.
$$

The equation (2) is solved similarly. If we know the potential distribution near to each of plate surfaces, we can find electric field strength near to surfaces. Then we determine the electric charge density on both surfaces by using the theorem of Gauss. It allows to find a value of the total charge induced on the surface of a solar sail. The dependences of the total charge $\sigma(r)$ which is on unit of a surface of one side of a solar sail for two different reflection factors are shown in the figure 1.

![Figure 1](image_url)

**Figure 1.** Dependencies of the total charge which is on unit of a surface of one side of a solar sail (in $10^{-11} \ C \cdot m^{-2}$) on distance from Sun for two different reflection factors. Solid line for the case of reflection factor $\delta = 0 \%$. Dashed line for a case of reflection factor $\delta = 90 \%$.

### 3. An estimate for influence of electric charge of a sail on its dynamics

The presence of electric charge induced on the surface of the solar sail which moves in an interplanetary magnetic field leads to the existence of Lorentz force which changes a trajectory of the solar sail motion. For simplification of calculations and an estimate of influence we have chosen elliptic Tsander’s trajectories [1] as undisturbed trajectories. They are the trajectory from orbit of the Earth to an orbit of Mars and the trajectory from an orbit of the Earth to an orbit of Jupiter.
The trajectory of space flight which is carried out along a tangential ellipse between circular co-planar planetary orbits by means of a mirror which is oriented normal to sun rays [1] is called Tsander’s trajectory. As the vector of an induction of an interplanetary magnetic field lies in an orbit plane the action of Lorentz force leads to motion which is perpendicularly to the plane of an undisturbed motion. We will consider this motion as perturbation. This assumption is well borne by the obtained results. Calculation of Lorentz force was made taking into account the change of a value and a direction of the magnetic field in an equatorial plane at removal from a surface of the Sun [24]

\[ B_r(r, \theta, \varphi) = B(\theta, \varphi_0) \left( \frac{r}{r_0} \right)^2 \]

\[ B_\varphi(r) = B(\theta, \varphi_0) \frac{r^2 \Omega_\odot}{V_r r} \]

where \( B(\theta, \varphi_0) \) is the value of an induction of a magnetic field in Sun photosphere, \( \varphi_0 \) is azimuthal angle in a stream point on distance \( r = r_0 \), \( \Omega_\odot = 2.7 \cdot 10^{-6} \text{rad} \cdot \text{s}^{-1} \), \( V_r = 300 \text{km} \cdot \text{s}^{-1} \) and \( r_0 = 7 \cdot 10^8 \text{m} \).

The known solution of a problem about the motion along Tsander’s trajectory (in quadratures) [25]

\[ \int_0^q \frac{d\vartheta}{(1 + \varepsilon \cdot \cos \vartheta)^2} = \frac{\sqrt{\gamma M_\odot}}{a^{3/2} \cdot (1 - e^2)^{1/2}} t \]

allows to determine the co-ordinates and the speed of space vehicle with a solar sail for each moment of time \( t \). In (3) \( \vartheta \) is true anomaly, \( \gamma \) is a gravitational constant, \( M_\odot \) is the mass of the Sun, \( \varepsilon \) and \( a \) are the eccentricity and the semimajor axis of the orbit.

The values of sailness (area-to-mass ratio) have been calculated in [1] for Tsander’s flights from an orbit of the Earth to the orbit of Mars and from an orbit of the Earth to an orbit of Jupiter. They allow to determine the sail area \( S_s \) for the given value of a reflection factor of a film. Thus, for each moment of time we know the total electrical charge of the sail \( Q = S_s \sigma(r) \), speed of space vehicle and a vector of an induction of a magnetic field. These values allow us to calculate the value of Lorentz force which is perpendicularly to the plane of the undisturbed trajectory. Thus, the problem about deflection determination is reduced to integration of a one-dimensional problem of the motion of a body with variable acceleration. The value of normal deflection during the motion of solar sail spacecraft with disposable load which is equal 0.1 of full mass along elliptic Tsander’s a trajectory from an orbit of the Earth to an orbit of Mars is equal to 44 m (for a reflection factor \( \delta = 90\% \)) and 308 m (for \( \delta = 0\% \)), and for a trajectory from an orbit of the Earth to the orbit of Jupiter it is equal 157 m (for \( \delta = 0\% \)).

4. Discussion and conclusions.

The obtained estimates of the influence of an electric charge induced on the surfaces of the sail on the motion of a space vehicle during interplanetary flights have shown that the influence is very small. The causes of it are very small value of an interplanetary magnetic field and a normal orientation of a sail concerning a direction on the Sun. Normal orientation of a sail is necessary for realization of Tsander’s trajectory. However, if a sail will be used in magnetospheres of big planets, then in a shadow of planets the influence of electric charge can be significant.

References
[1] Polyakhova E N 1988 Cosmic Flight with Solar Sail (Moscow: Nauka) (in Russian)
[2] Korolev V S, Polyakhova E N and Pototskaya I Yu 2020 Problem of control motion of solar sail
spacecraft in the photogravitational fields. *Nonlinear Systems. Theoretical Aspects and Recent Applications* ed W Legnons and T E Moschandreaou (London: IntechOpen) p 205

[3] Tikhonov A A and Yakovlev A B 2021 The influence of an electric charge induced on the surfaces of the solar sail on its durability characteristics. *Proc. Int. Conf. on Mechanics - Ninth Polyakhov's Reading* (Saint-Petersburg, March 9-12, 2021) (Saint-Petersburg: VVM) p 172 (in Russian)

[4] Yakovlev A B 2021 Model of the charging of thin aluminum film in the space plasma. *Proc. Int. Conf. on Mechanics - Ninth Polyakhov's Reading* (Saint-Petersburg, March 9-12, 2021) (Saint-Petersburg: VVM) p 440 (in Russian)

[5] Whipple E C 1981 Potentials of surfaces in space. *Reports on Progress in Physics* 44 1997-2250

[6] Kats I, Parks D E, Mandell M J, Harvey J M, Brownell, Wang S S and Rotenben M 1977 A three dimensional dynamic study of electrostatic charging in materials. *NASA CR-135256*

[7] Prokopenko S M L and Lafromboise J G 1980 High-voltage differential charging of geostationary spacecraft. *J. Geophys. Res.* 85 A8 4125-31

[8] Parker L W and Murphy B L 1967 Potential buildup on an electron-emitting ionospheric satellite. *J. Geophys. Res.* 72 6131-6

[9] Kolesnikov E K and Yakovlev A B 1996 Procedure for calculating the electric field strength induced near the surface of an infinite cylinder that rests in a collisionless plasma in a homogeneous magnetic field, the charge flow from surface being fixed. *Cosmic Research* 34 6 615–6

[10] Fedorov V A 2005 Neutralization of negative electric charge of a satellite by ionospheric plasma ions. *Cosmic Research* 43 7–16

[11] Samir U 1970 A possible explanation of an order of magnitude discrepancy in electron-wake measurements. *J. Geophys. Res.* 75 855

[12] Tikhonov A A and Yakovlev A B 2019 On dependence of equilibrium characteristics of the space tethered system on environmental parameters. *International Journal of Plasma Environmental Science and Technology* 13 1 49–52

[13] Kolesnikov E K and Yakovlev A B 1995 Damping of plasma waves arising in the neighbourhood of a cylindrical body injecting an electron beam. *Vestnik Sankt-Peterburgskogo Universiteta. Ser I. Matematika Mekhanika Astronomiya* 4 107-10

[14] Myers N B, Raitt W J and White A B 1990 Vehicle charging effect during electron beam emission from the CHARGE-2 experiment. *J. Spacecr. Rockets* 27 25-37

[15] Banks P M, Gilchrist B E, Neubert T et al. 1990 CHARGE-2 rocket observation of vehicle charging and charge neutralization. *Adv. Space Res.* 10 133-6

[16] Gringauz K I, Mishin E V, Shute N M and Volokitin A S 1981 Rocket potential measurements during electron beam injection into the ionosphere. *Adv. Space Res.* 1 69-76

[17] Hill J R and Wipple E C 1985 Charging of large structures in space with application to the solar sail spacecraft. *J. Spacecraft Rockets*. 22 3 245-53

[18] Kolesnikov E K 2006 Evaluation of the specific power of a high-voltage generator based on the use of energy resources of the earth's radiation belts. *Cosmic Research* 44 6 486-92

[19] Kolesnikov E K and Yakovlev A B 2009 Harnessing power from solar wind particles captured in the Van Allen belts. *Acta Futura* 3 81-8

[20] Grard R J L 1973 Properties of the satellite photoelectron sheath derived from photo-emission laboratory measurements. *J. Geophys. Res.* 78 16 2885-906

[21] Moskalenko A M 1992 The electrostatic potential near the lunar surface. *Kinematics and Physics of Heavenly Bodies* 5 5 31–40 (in Russian)

[22] Fu J H M 1971 Surface potential of a photoemitting plate. *J. Geophys. Res.* 76 10 2506–9

[23] Yakovlev A B 2015 The corrected method for calculation of electrostatic potential near to surface of nonatmospheric space body and the analysis of possible modes of dust particles motion. *Proc. Int. Conf. on Mechanics - Seventh Polyakhov's Reading* (Saint-Petersburg,
[24] Tien J Y, Akasofu S I and Chapman S. 1972 *Solar-Terrestrial Physics* (Oxford: Clarendon Press)

[25] Markeev A P 1990 *Theoretical Mechanics* (Moscow: Nauka) (in Russian)