Neutron stars with outbursts from superfluid crust

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Abstract. We model heat propagation and the thermal surface luminosity $L_{\infty}^s(t)$ of a neutron star after an internal outburst in its crust. Simulations take into account superfluidity of free neutrons and the thickness of the outbursting layer (heater) in the crust. Crustal superfluidity can shorten and intensify variations of $L_{\infty}^s(t)$.

1. Introduction and simulations

Here we continue to study the effects of crustal heaters on the variable surface thermal emission \cite{1,2} of neutron stars (NSs). Our results can be useful for the interpretation of observations of old transiently accreting NSs in low-mass X-ray binaries (LMXBs) and of warm bursting magnetars. It is currently thought \cite{3} that NSs in LMXBs are reheated by nuclear transformations in the accreted crust \cite{4,5}. Magnetars, which are mainly hot, middle-aged and active NSs are most likely reheated by the processes associated with strong magnetic fields, e.g., \cite{6}.

To simulate NS thermal evolution, we use a one-dimensional generally relativistic code \cite{2} for a spherically symmetric NS. We solve the thermal evolution equations at $\rho \geq \rho_b \sim 10^9 - 10^{10}$ g cm$^{-3}$ (to the bottom of the heat blanketing envelope – a thin surface layer with a strong thermal insulation \cite{7}). For the densities $\rho < \rho_b$, we use separately calculated $T_b - T_s$ relations, where $T_b$ is the temperature at $\rho = \rho_b$ and $T_s$ is the local effective temperature of the stellar surface. We consider two NS models with the BSk21 equation of state \cite{8}. The masses of these stars are $M = 1.4$ and $1.85$ $M_\odot$; their radii are $R = 12.6$ and $12.25$ km, respectively.

For simplicity, we neglect the effects of superfluidity (SF) in NS cores (but consider SF of free neutrons in the crust). The $1.4M_\odot$ NS demonstrates standard neutrino cooling via modified direct Urca process; the $1.85M_\odot$ star undergoes much faster neutrino cooling via the direct Urca process in its central kernel.

The heater will be treated as a spherical layer where a given heat power $Q(\rho, t) [\text{erg s}^{-1} \text{cm}^{-3}]$ is released. Integrating $Q(\rho, t)$ over the heater, we calculate the total redshifted heat power $L_{\infty}^h(t) [\text{erg s}^{-1}]$. Using the cooling code, we find the redshifted effective surface temperature $T_{\infty}^s(t)$ and thermal luminosity $L_{\infty}^s(t)$. The problem is if the $L_{\infty}^s(t)$ or $T_{\infty}^s(t)$ variations are observable.

2. Thin heating shell and crustal superfluidity

In this section, following \cite{2}, we treat the heater as a thin spherical shell located at $\rho_1 < \rho < \rho_2$; $\rho_1$ and $\rho_2$ being the lowest and highest densities in the shell, respectively. We start with the case of non-superfluid NS. Initially, we let a young and hot NS to cool freely. Then we turn on
a time-independent heat power $Q(\rho, t) \equiv Q_0(\rho)$, $L_h^\infty \equiv L_{h0}^\infty$, and wait until a stationary state within the star is established. After that we vary $Q(\rho, t)$ (imitating an internal outburst) and follow the outburst dynamics of $L_h^\infty(t)$ and $L_s^\infty(t)$.

We set $Q(\rho, t)$ to be uniform inside the heater, with the time dependence \[ Q(t) = H_0 + (H_{\text{max}} - H_0) \sin^2(\pi t / \Delta t), \quad 0 \leq t \leq \Delta t, \]
where $t$ is the time from the beginning of the outburst, $\Delta t$ is the internal outburst duration and $H_{\text{max}}$ is the maximum of $Q(t)$ reached at $t = \Delta t/2$ (assuming $H_{\text{max}} > H_0$). The associated total heat power varies as $L_h^\infty(t) = L_{h0}^\infty + (L_{h,\text{max}}^\infty - L_{h0}^\infty) \sin^2(\pi t / \Delta t)$, where $L_{h,\text{max}}^\infty$ is the maximum of $L_h^\infty(t)$. When $t$ exceeds $\Delta t$, we return the heater to the stationary state, with $Q(\rho, T) = H_0$ and $L_h^\infty = L_{h0}^\infty$. The NS also begins returning to its state before the outburst.

We consider two heaters. The outer heater is located between $\rho_1 = 10^{11}$ and $\rho_2 = 10^{12}$ g cm$^{-3}$, with the inner heater, we take $\rho_1 = 10^{12}$ g cm$^{-3}$, with $\rho_2 = 1.27 \times 10^{13}$ and $1.23 \times 10^{13}$ g cm$^{-3}$ for the 1.4 and 1.85 $M_\odot$ stars, respectively (to have equal $L_h^\infty(t)$ at the same $H_0$, $H_{\text{max}}$ and $\Delta t$ for both heaters).

Figure 1 compares the generated heat power $L_h^\infty(t)$ with the NS surface luminosity $L_s^\infty(t)$ at $L_{h,\text{max}}^\infty = 10 L_{h0}^\infty$. The cases (a), (b) and (c) refer to the three values of $H_0 = 5 \times 10^{17}$, $5 \times 10^{18}$ and $5 \times 10^{19}$ erg cm$^{-3}$ s$^{-1}$, respectively. The corresponding steady-state heat powers are $L_{h0}^\infty = 1.7 \times 10^{35}$ (a), $1.7 \times 10^{36}$ (b) and $1.7 \times 10^{37}$ erg s$^{-1}$ (c). The figure refers to two outburst durations, $\Delta t=10$ and 100 yr, produced by the outer and inner heaters. The two solid curves on the upper-left part (a)–(c) show $L_h^\infty/L_{h0}^\infty$ which are the same for the outer and inner heaters. Any two pairs of curves on panels (a), (b) and (c) show $L_h^\infty(t)/L_{h0}^\infty$ produced either by the outer (dash-dot lines) or the inner (dashed lines) heater. At the same $L_h^\infty$ and $\Delta t$, the surface luminosities $L_s^\infty(t)$ depend on the heater’s position; the deeper the heater, the lower $L_s^\infty(t)$. In all the cases, only a small fraction of the generated heat is emitted through the NS surface ($L_s^\infty \ll L_h^\infty$), while the rest is emitted by neutrinos [2].

Now we notice that NSs are likely superfluid inside [9]. We consider only singlet-state SF of free neutrons in the inner crust (at $\rho \gtrsim 4 \times 10^{11}$ g cm$^{-3}$). Let $T_c(\rho)$ be the critical temperature...
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\[ \log L^\infty_s \left( \frac{t}{\text{yr}} \right) \]

\[ \log L^\infty_h \left( \frac{t}{\text{yr}} \right) \]

\[ \log L^\infty_h \left( \frac{t}{\text{yr}} \right) \]

\[ \log L^\infty_s \left( \frac{t}{\text{yr}} \right) \]

**Figure 2.** The heater’s power \( L^\infty_h(t) \) (‘Heater’, the same for the 1.4\( M_\odot \) and 1.85\( M_\odot \) NSs) and the surface luminosity \( L^\infty_s(t) \) (‘Surface’) for two models [GIPSF – long dashes, AWP2 – dots] of crustal SF as well as for non-superfluid crust (No SF) at \( L^\infty_{h,\text{max}}/L^\infty_{h0} = 500 \) and \( \Delta t = 1 \) yr. The left and right panels correspond to the outer and inner heaters, respectively. The circles show maxima of \( L^\infty_h(t) \); the squares and triangles position maxima of \( L^\infty_s(t) \) for the 1.4 and 1.85\( M_\odot \) NSs, respectively. **Inset:** Critical temperature \( T_c(\rho) \) for the two models of crustal SF.

that depends on \( \rho \). Calculations of \( T_c(\rho) \) give a large scatter of results depending on microphysics of the matter. For example, the inset in figure 2 shows \( T_c(\rho) \) for two SF models. The ‘GIPSF’ model [10] is close to basic the BCS model and gives a high and broad peak of \( T_c(\rho) \). The ‘AWP2’ model [11] is stronger affected by the in-medium effects; the \( T_c(\rho) \) peak is lower and shifted to higher \( \rho \).

SF strongly reduces the heat capacity in the inner crust, \( C(\rho,T) \) [12]. In the absence of SF (\( T > T_c \)), the main contribution to the heat capacity of the inner crust is provided by free neutrons, but at \( T \ll T_c \) the neutron heat capacity is suppressed, and the heat capacity is determined by much smaller contribution of ions and electrons.

Figure 2 is similar to figure 1 and compares the variations of \( L^\infty_h(t) \) and \( L^\infty_s(t) \) (for the same 1.4 and 1.85\( M_\odot \) stars) but for the two models of crustal SF (see the inset) in a powerful outburst, \( L^\infty_{h,\text{max}}/L^\infty_{h0} = 500 \) and \( \Delta t = 1 \) yr. With the suppressed crustal heat capacity, one needs less energy to warm up the heater and its vicinity. Then more energy is directed to heat the surface. Therefore, for the heaters with \( \rho_1 > \rho_{\text{drip}} \) (the neutron drip density), the \( L^\infty_s(t) \) peaks are more pronounced in the presence of superfluid neutrons. The initial luminosity \( L^\infty_{h0} \) for the 1.85\( M_\odot \) star with fast neutrino cooling is about two orders of magnitude lower than for the 1.4\( M_\odot \) star.

3. **Deep crustal heating and afterburst relaxation**

Here we outline the crust-core relaxation in NSs which enter quasi-persistent LMXBs (six sources known at the moment [13]). Such an NS transiently accretes matter from its low-mass companion during long (months–years) and intense (about the Eddington level) accretion episodes (outbursts). During an outburst, the crustal heater is so strong that it makes the crust hotter than the core, destroying the isothermality of NS interiors.

After the outburst, the surface accretion energy release stops and the crust undergoes gradual (a few years) relaxation with the core. An afterburst (quiescent) stage is characterized by much weaker, but observable surface emission which is likely supported by deep crustal heating. It
Figure 3. Left: Temporal relaxation $T_s^\infty(t)$ after an outburst in the $1.4M_\odot$ NS with the pre-outburst redshifted internal temperature $T_0 = 10^8$ K; $t$ is time after the return to quiescence. Solid curves correspond to $\rho_{\text{acc}} = 10^{11.5}$, $10^{12}$ and $10^{13}$ g cm$^{-3}$ and to the GIPSF superfluidity. The dashed curves refer to the same $\rho_{\text{acc}}$ but without SF. Dotted line – no heater. Right: Long-term relaxation time $\tau$ versus $\rho_{\text{acc}}$ extracted from the theoretical cooling curves. Six filled circles and triangles with their errorbars correspond to a non-superfluid and superfluid crust, respectively. Inset: Heat power $L_h^\infty(\rho)$, integrated over a spherical layer from the surface up to a current density $\rho$, in the crust at $M = 10^{-8} M_\odot$ yr$^{-1}$. The grey horizontal line shows the modification of this curve for the partly accreted crust with $\rho_{\text{acc}} = 10^{12}$ g cm$^{-3}$.

operates in the NS crust during outbursts, when the freshly accreted matter compresses the underlying crust. Since the accreted matter is different from the fully equilibrium cold catalyzed matter, the compression triggers nuclear transformations in the accreted matter (e.g., [4, 5, 14]). As a result, the energy is generated deeply in the crust with the rate $Q(\rho,t)$ distributed from the outer NS layer to the maximum density $\rho_{\text{acc}}$ occupied by the accreted matter in the crust.

One can consider the deep crustal heating with fully accreted and partly accreted crusts (e.g., [15]). For example, we simulate the long-term cooling of the $1.4M_\odot$ NS in a quasi-persistent LMXB. We start with a hot and passively cooling NS. In about one century, it becomes thermally relaxed inside, with the same redshifted internal temperature $\tilde{T}(t)$ over the internal region. When $\tilde{T}(t)$ reaches the value $\tilde{T}_0 = 10^8$ K, we switch on the distributed heat sources within the accreted crust (the inset in figure 3) assuming a mass accretion rate $M = 10^{-8} M_\odot$ yr$^{-1}$. In $\Delta t = 1$ yr after the outburst onset, we abruptly switch off the deep crustal heating [$L_h^\infty(t) = 0$] to simulate the long-term cooling in quiescence for the cases of normal (No SF) and highly superfluid (GIPSF) crust.

The left panel of figure 3 shows the cooling curves $T_s^\infty(t)$ (in energy units) for six models of long-term relaxation after the return to quiescence. The three solid lines are calculated for the GIPSF superfluidity assuming the partially accreted crust with $\rho_{\text{acc}} = 10^{11.5}$, $10^{12}$ and $10^{13}$ g cm$^{-3}$. The other three dashed curves are the same but neglecting neutron SF. A thinner accreted crust reduces $T_s^\infty$. SF considerably shortens the relaxation by reducing the heat capacity in the inner crust; this is seen as a faster decay of $T_s^\infty(t)$ at large $t$.

In first few months of quiescence the relaxation can be affected by poorly known shallow heating (e.g. [13, 16]), which we neglect. In late long-term relaxation this effect is unimportant.
so that the observable and calculated cooling curves $T_s^\infty(t)$ are well described (e.g., [17, 18]) by a function $T_s^\infty(t) = A \exp(-t/\tau) + B$, where $\tau$, $A$ and $B$ are the fit parameters, with $\tau$ being a relaxation time. Taking the sets of our theoretical cooling curves for normal and SF crusts, we infer two branches of $\tau(\rho_{\text{acc}})$ plotted on the right panel of figure 3. If $\rho_{\text{acc}} \gtrsim 10^{11}$ g cm$^{-3}$, $\tau$ is greatly reduced by SF and independent of $\rho_{\text{acc}}$. Our preliminary analysis demonstrates that the reduction agrees with the observations of quasi-persistent LMXBs and gives evidence for crustal SF. The same conclusion has been made earlier from more complicated simulations based on similar microphysics and SF effects; see, e.g., [16, 19–21] and references therein. If $\rho_{\text{acc}} \lesssim 10^{11}$ g cm$^{-3}$, the superfluid and normal cooling curves merge into one curve.

To summarize, we confirm the results of [2] that only a small amount of heat is emitted by photons through the surface. The surface outburst $L_s^\infty(t)$ is delayed, broadened and asymmetric with respect to the internal outburst $L_h^\infty(t)$. The long-term afterburst relaxation in NSs entering quasi-persistent LMXBs depends on crustal SF and on the maximum density $\rho_{\text{acc}}$ occupied by the accreted matter. At $\rho_{\text{acc}} \gtrsim 10^{11}$ g cm$^{-3}$ superfluidity can greatly shorten the relaxation and amplify $T_s^\infty(t)$.

These results are important for theoretical interpretation of observed thermal evolution of NSs in quasi-persistent LMXBs. They can also be useful for modeling of afterburst relaxation in magnetars where the relaxation can have much in common with that in LMXBs but microphysics input is greatly complicated by strong magnetic fields (e.g., [22]).

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