Calculation of the contribution to muon $g - 2$
due to the effective anomalous three boson interaction and the new experimental result

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Abstract

Using the approach based on Bogoliubov compensation principle is applied to
calculation of a contribution to the muon $g - 2$. Using the previous results on spont-
aneous generation of the effective anomalous three-boson interaction we calculate
the contribution, which proves to agree with the well-known discrepancy. The cal-
culated quantity contains no adjusting parameters but the experimental values for
the muon and the W-boson masses. The result can be considered as a confirmation
of the approach.

Previous measurements of the anomalous magnetic moment of the muon $(g - 2)_\mu = a_\mu$ [1]
provided a deviation of the experiment from predictions of the Standard Model. According
to previous analysis of the problem [3] this deviation $\Delta a_\mu$ safely comprise 3 standard
deviations

\begin{align}
\Delta a_\mu &= (276 \pm 80) \times 10^{-11} ; \\
\Delta a_\mu &= (250 \pm 80) \times 10^{-11} .
\end{align}
Quite recent result reads \[\Delta a_{\mu} = (251 \pm 59) \times 10^{-11};\] (2)

Deviations (1) exceed zero a bit more than three SD, but the last result (2) may be considered as the final establishing of the effect. It should be emphasized, that the deviation from the SM calculations means the deviation from perturbative calculations in the electro-weak theory. However there quite may be non-perturbative contributions to physical quantities. In particular, the method of disclosing of the non-perturbative effects is developing starting of N.N. Bogoliubov compensation principle \[4, 5\]. In works \[6, 7, 8, 9, 10, 11, 12, 13\], this principle was applied to studies of a spontaneous generation of effective non-local interactions in renormalizable gauge theories of the Standard Model.

In the present letter we apply the previous results to the problem of the muon \(\Delta a_{\mu}\). It will come clear, that the effect under discussion is quite natural in the theory with account of the spontaneous generation of an effective interaction in the conventional electro-weak theory.

The main principle of the approach is to check if an effective interaction could be generated in the chosen variant of a renormalizable theory. In view of this one performs ”add and subtract” procedure for the effective interaction with a form-factor. Then one assumes the presence of the effective interaction in the interaction Lagrangian and the same term with the opposite sign is assigned to the newly defined free Lagrangian.

In works \[12, 13\] the approach was applied to the electro-weak interaction and a possibility of spontaneous generation of anomalous three-boson interaction of the form

\[-\frac{G}{3!} \cdot \epsilon_{abc} W^{a}_{\mu\nu} W^{b}_{\nu\rho} W^{c}_{\rho\mu};\] (3)

was studied. In the present work we continue investigation of the electro-weak theory using other approximation scheme, which will be formulated in what follows.

The notation (3) means corresponding non-local vertex in the momentum space

\[
(2\pi)^4 G \epsilon_{\mu\nu\lambda} (q_{\mu}p_{\nu}k - p_{\nu}q_{\mu}k) + g_{\nu\rho}(k_{\mu}p_{\rho} - q_{\mu}p_{\rho}) +
\]

\[
g_{\mu\nu}(p_{\nu}q_{\mu} - k_{\nu}p_{\rho}) + q_{\mu}k_{\nu}p_{\rho} - k_{\mu}p_{\nu}q_{\rho} \times
\]

\[
F(p, q, k)\delta(p + q + k) + ... ;
\] (4)
where $F(p, q, k)$ is a form-factor and $p, \mu, a$; $q, \nu, b$; $k, \rho, c$ are respectfully incoming momenta, Lorentz indices and weak isotopic indices of $W$-bosons. We mean also that there are present four-boson, five-boson and six-boson vertices according to the well-known nonlinear expression for $W_{\mu\nu}^a$. Note, that in the approximation used we maintain the gauge invariance of the approach.

Effective interaction (3) is usually called anomalous three-boson interaction and it is considered for long time on phenomenological grounds [14]. Note, that the first attempt to obtain the anomalous three-boson interaction in the framework of Bogoliubov approach was done in work [15]. Our interaction constant $G$ is connected with conventional definitions in the following way

$$G = - \frac{g \lambda}{M_W^2}. \quad (5)$$

The current limitations for parameter $\lambda$ read [16, 17],

$$\begin{align*}
\lambda &= -0.016^{+0.021}_{-0.023}; \quad -0.059 < \lambda < 0.026 \ (95\% \ C.L.) , \\
\lambda_\gamma &= -0.022 \pm 0.019; \quad (6)
\end{align*}$$

where the last number (6) is obtained recently by joint analysis of LEP data by the four experimental groups: ALEPH, DELPHI, L3, OPAL. We assume no difference in anomalous interaction for $Z$ and $\gamma$, i.e. $\lambda_Z = \lambda_\gamma = \lambda$ according to standard relation $W^0 = \sin \theta_W A + \cos \theta_W Z$.

In works [12, 13] the approach was applied to the electro-weak interaction and a possibility of spontaneous generation of anomalous three-boson interaction of the form (3) was studied. In the present work we continue investigation of the electro-weak theory using other approximation scheme, which will be formulated in what follows.

The goal of a study is a quest of an adequate approach, the first non-perturbative approximation of which describes the main features of the problem. Improvement of a precision of results is to be achieved by corrections to the initial first approximation.

This first approximation, corresponding to one-loop equation, is described in works [12, 13], where we have studies the possibility of the existence of a non-trivial solution with the following simple dependence on all three variables

$$F(p_1, p_2, p_3) = F\left(\frac{p_1^2 + p_2^2 + p_3^2}{2}\right); \quad (7)$$
Let us present the expression for four-boson vertex

\[
V(p, m, \lambda; q, n, \sigma; k, r, \tau; l, s, \pi) = \frac{i}{(2\pi)^4} gG(\epsilon^{amn}\epsilon^{ars}\left(U(k, l; \sigma, \tau, \pi, \lambda) - U(k, l; \lambda, \tau, \pi, \sigma) - U(l, k; \sigma, \pi, \tau, \lambda) + U(l, k; \lambda, \pi, \tau, \sigma) \right) + U(p, q; \pi, \lambda, \sigma, \tau) - U(p, q; \tau, \lambda, \sigma, \pi) - U(q, p; \pi, \sigma, \lambda, \tau) + U(q, p; \tau, \sigma, \lambda, \pi) - U(p, l; \sigma, \pi, \tau, \lambda) + U(p, l; \lambda, \pi, \tau, \sigma) - U(l, p; \sigma, \tau, \lambda, \pi) + U(l, p; \tau, \sigma, \lambda, \pi)) - \epsilon^{arn}\epsilon^{ars}\left(U(p, l; \sigma, \tau, \pi, \lambda) - U(p, l; \lambda, \tau, \pi, \sigma) - U(l, p; \sigma, \tau, \lambda, \pi) + U(l, p; \tau, \sigma, \lambda, \pi) - U(p, l; \sigma, \pi, \tau, \lambda) + U(p, l; \lambda, \pi, \tau, \sigma) - U(l, p; \sigma, \tau, \lambda, \pi) + U(l, p; \tau, \sigma, \lambda, \pi)) \right)
\]

Here triad \(p, m, \lambda\) etc means correspondingly momentum, isotopic index, Lorentz index of a boson.

Now in the way of studying the problem \[12, 13\] we get convinced, that there exists a non-trivial solution of the compensation equation, which has the following form for \(0 < z < z_0\)

\[
F(z) = \frac{1}{2} G^{a_1}_{15} \left( z | 0 \right) - \frac{85 g \sqrt{N}}{512 \pi} G^{a_1}_{15} \left( z | 1/2, 1/2, -1/2, -1 \right) + C_1 G^{10}_{04} \left( z | 1/2, 1, -1/2, -1 \right) + C_2 G^{10}_{04} \left( z | 1, 1/2, -1/2, -1 \right),
\]

\[
z = \frac{N G^2 x^2}{1024 \pi^2}; \quad x = p^2;
\]

where

\[
G^{am}_{qp} \left( z | a_1, \ldots, a_q \right); \quad b_1, \ldots, b_p
\]
is a Meijer function \[18\]. In case \( q = 0 \) we write only indices \( b_i \) in one line. Constants \( C_1, C_2 \) are defined by boundary conditions. For \( z > z_0 \)

\[ F(z) = 0. \tag{10} \]

Parameters of solution \([9]\) are the following

\[
g = g(z_0) = 0.60366; \quad z_0 = 9.61750; \\
C_1 = -0.035096; \quad C_2 = -0.051104. \tag{11} \]

We would draw attention to the fixed value of parameter \( z_0 \). The solution exists only for this value \([11]\) and it plays the role of eigenvalue. As a matter of fact from the beginning the existence of such eigenvalue is by no means evident. The definite value for \( g(z_0) \) is also worth mentioning.

Emphasize, that an existence of a non-trivial solution of a compensation equation is extremely restrictive. In the most cases such solutions do not exist at all. When we start from a renormalizable theory we have arbitrary value for its coupling constant. Provided there exists stable non-trivial solution of a compensation equation the coupling is fixed as well as the parameters of this non-trivial solution.

Now let us consider a contribution of interaction \([3]\) with form-factor defined by relations \([7,9,11]\) to the anomalous magnetic moment of the charged spin one half particle. The first approximation described by the simplest diagram presented in Fig. gives zero. However, the next approximation, presented by diagrams Fig\[2\] leads to an interesting result. The calculations are performed in the unitary gauge and give the following result after normalization of the gauge coupling \( G \). In doing this we choose the part of the vertex \([3]\), which gives contribution to usual gauge structure of triple vertex whereas the structure of the anomalous vertex \([3]\) gives zero contribution to the magnetic moment.

\[
\Delta a = \frac{meg^2G^2N}{6(16\pi^2)^2M_W^2} \\
\int_0^\infty dt F^2(t) \left( \int_0^t \frac{4ty^2 \, dy}{(6t - 3y)(y + M_W^2)^2} + \int_{4t/3}^t \frac{4ty(16t^3 - 48t^2y + 48ty^2 - 15y^3) \, dy}{3(2t - y)(y + M_W^2)^2} \right). \tag{12} \]
Figure 1: One loop diagram for calculation of new contribution to the muon magnetic moment. Vertical line represents the photon, simple lines – W bosons, black spot – triple vertex \(3\) with corresponding form-factor. Double line represents the muon.

From (12) with definitions of variable \(z\) and of the form-factor (9, 11) we obtain the following result for contribution to the magnetic moment

\[
\Delta a_\mu = \frac{g(z_0)^2 m^2}{3 \pi^2 M^2_W} \left(20 \ln \left[\frac{4}{3}\right] - \frac{13}{3}\right) \times \int_0^{z_0} F^2(z) \, dz = 2.775 \times 10^{-9} .
\]  

(13)

where we have used only values of the muon mass and the \(W\)-boson mass. All other parameters are defined by solution (9) with parameters (11). Let us draw attention to the disappearance of the effective interaction coupling constant \(G\) from expression (13). This is due to entering of factor \(G^2\) into the denominator according to definition of variable \(z\). Thus the main result does not depend on \(\lambda\). This parameter influence only the next approximations. Let us estimate possible corrections due to \(M_W \neq 0\). They are defined by the following parameter

\[
\frac{\sqrt{2} g |\lambda|}{32 \pi} = 0.0005 ;
\]

(14)

with the maximal value of \(|\lambda| = 0.059\) from restrictions (9). Thus this correction may
Figure 2: Two loop diagrams for calculation of new contribution to the muon magnetic moment. Vertical line represents the photon, simple lines – W bosons, black spots – triple vertex (3) and four leg vertex (8) with corresponding form-factors. Double line represents the muon.
comprise only 0.05%.

We conclude that our calculations quite agree the last decisive measurement \[2\] \[2\]. Note, that earlier our approach to the problem was discussed in work \[19\] and in book \[20\]. It also would be advisable to look for other effects of interaction \[3\] e.g. \[21\] \[22\].

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