The light echo (LE) systems of historical supernovae in the Milky Way and local group galaxies provide an unprecedented opportunity to reveal the effects of asymmetry on observables, particularly optical spectra. Scattering dust at different locations on the LE ellipsoid witnesses the supernova from different perspectives, and the light consequently scattered toward Earth preserves the shape of line profile variations introduced by asymmetries in the supernova photosphere. However, the interpretation of supernova LE spectra to date has not involved a detailed consideration of the effects of outburst duration and geometrical scattering modifications due to finite scattering filament dimension, inclination, and image point-spread function and spectrograph slit width. In this paper, we explore the implications of these factors and present a framework for future-resolved supernova LE spectra interpretation, and test it against Cas A and SN 1987A LE spectra. We conclude that the full modeling of the dimensions and orientation of the scattering dust using the observed LEs at two or more epochs is critical for the correct interpretation of LE spectra. Indeed, without doing so one might falsely conclude that differences exist when none are actually present.

Key words: ISM: individual objects (SN 1987A, Cas A) – ISM: supernova remnants – supernovae: general

Online-only material: color figures

1. INTRODUCTION

The first light echoes (LEs) were discovered around Nova Persei 1901 (Ritchey 1901a, 1901b, 1902; Kapteyn 1902). Since then, LEs (whereby we mean a simple scattering echo rather than fluorescence or dust re-radiation) have been seen in the Galactic Nova Sagittarii 1936 (Swope 1940) and the eruptive variable V838 Monocerotis (e.g., Bond et al. 2003). LEs have also been observed from extragalactic supernovae (SNe), with SN 1987A (Kunkel et al. 1987) being the most famous case (Croft 1888; Suntzeff et al. 1988; Bond et al. 1990; Xu et al. 1995; Sugerman et al. 2005a, 2005b; Newman & Rest 2006), but also including SNe 1991T (Schmidt et al. 1994; Sparks et al. 1999), 1993J (Sugerman & Crotts 2002; Liu et al. 2003), 1995E (Quinn et al. 2006), 1998bu (Cappellaro et al. 2001), 2002hh (Meikle et al. 2006; Welch 2007), 2003gd (Sugerman 2005; Van Dyk et al. 2006), and 2006X (Wang et al. 2008).

The suggestion that historical SNe might be studied by their scattered LEs was first made by Zwicky (1940) and attempted by van den Bergh (1965, 1966). The first LEs of centuries-old SNe were discovered in the Large Magellanic Cloud (LMC; Rest et al. 2005b). They found three LE complexes associated with three small and therefore relatively young SN remnants (SNRs) in the LMC which were subsequently dated as being between 400 and 1000 years old using the LMC distance and LE feature positions and apparent motions (Rest et al. 2005b).

These findings provided an extraordinary opportunity to study the spectrum of the SN light that reached Earth hundreds of years ago and had never been recorded visually by modern scientific instrumentation. A spectrum of one of the LEs associated with SNR 0509-675 revealed that the reflected LE light came from a high-luminosity Type Ia supernova (SN Ia; Rest et al. 2005), similar to SN 1991T (Filippenko et al. 1992; Phillips et al. 1992) and SN 1999aa (Garavini et al. 2004)—the first time that an ancient SNe was classified using its LE spectrum. Analysis of ASCA X-ray data of SNR 0509-675 led Hughes et al. (1995) to suggest that this SNR likely came from an SN Ia. Recent analysis of its Chandra X-ray spectra by Badenes et al. (2008) also supported the classification of this object as a high-luminosity SN Ia.

The interpretation of LEs, however, is fraught with subtleties. Krause et al. (2005), for example, identified moving Cas A features (called “infrared echoes” or “IR echoes”) using mid-IR imaging from the Spitzer Space Telescope. IR echoes are the result of dust absorbing the outburst light, warming, and re-radiating at longer wavelengths. The main scientific conclusion of Krause et al. (2005) was that most of the IR echoes were caused by a series of recent X-ray outbursts from the compact object in the Cas A SNR based on their apparent motions. However, the analysis was flawed because it did not account for the fact that the apparent motion strongly depends on the inclination of the scattering dust filament (Dwek & Arendt 2008; A. Rest et al. 2011, in preparation). Dwek & Arendt (2008) also showed that X-ray flares are not energetic enough to be the source of these IR echoes, but that instead the LEs must have been generated by an intense and short burst of EUV–UV radiation associated with the breakout of the shock through the surface of the Cas A progenitor star. Therefore, the most likely source of all LEs associated with Cas A, both infrared and scattered, is the Cas A SN explosion itself. The need to properly analyze the growing collection of LE spectra is what motivated the present work.

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The first scattered LEs of Galactic SNe associated with Tycho’s SN and the Cas A SN were discovered by Rest et al. (2007, 2008b). Contemporaneously, Krause et al. (2008) obtained a spectrum of a scattered optical LE spatially coincident with one of the Cas A IR echoes and identified the Cas A SN to be of Type IIb. Because Rest et al. (2008b) discovered several LEs with multiple position angles relative to the SNRs, there is a new opportunity for observational SN research—the ability to measure the spectrum of the same SN from several different directions. At any given instant after the SN light has reached an observer by the direct path, an ellipsoidal surface exists where scattered light from the SN may reach the observer, the arrival of which to the observer is delayed by the additional path length. Interstellar dust concentrations must lie at one or more locations on the ellipsoid for the scattered LE to be detectable. Since each LE is at a unique position on the ellipsoid, each LE has a unique line of sight to the SN. Therefore, the scattered spectral light can provide spectral information on the asymmetry of the SN photosphere. In the limiting case of dust on the ellipsoid opposite the observer, the spectrum of the scattered LE carries the signature of conditions on the SN hemisphere usually hidden from the observer. As the time since the explosion increases, the scattering ellipsoid surface expands and may intersect additional dust concentrations. We present a framework on how to interpret observed LE spectra depending on the scattering dust properties, seeing, and spectrograph slit width and position.

In this paper, we examine the pitfalls of previous analyses, suggest improvements for future studies, and present the application of our methods to data. In Section 2, we describe how the properties of the scattering dust can cause significant differences to observed LE spectra and show a specific example of how this has been overlooked in the past. In Section 3, we quantify how observed LE spectra depend on potential differences in dust characteristics (thickness and inclination) as well as observational characteristics (seeing and spectroscopic slit size and orientation). We then apply this framework to observations of Cas A (Section 4) and SN 1987A (Section 5). In Section 6, we discuss how differences in the dust and observing conditions can affect an analysis of LE spectra and suggest future applications of LE observations. We conclude in Section 7.

2. A CASE STUDY: THE KRAUSE ET AL. (2008) CAS A LIGHT ECHO SPECTRUM

A common assumption is that an observed LE spectrum is equivalent to the light-curve-weighted integration of the spectra at individual epochs (e.g., Rest et al. 2008a; Krause et al. 2008). However, Figure 1 illustrates how slit width, dust filament width, and inclination significantly influence the observed LE spectrum. We consider an LE from a 300 year old event 10,000 lt-yr away that lasts 150 days (yellow shaded area), with a rising and declining arm of 30 and 120 days, respectively. The peak of the event is indicated with the red line. We place the origin of our coordinate system \((\rho, z)\) at the source event. The \(z\)-axis is the distance of the dust to the source event along the line of sight. The \(\rho\)-axis is in the plane of the sky from the source event to the LE position. The quadratic relation between the parameters \(z\), \(\rho\), and \(t\) is described by the well-known LE equation (Couderc 1939; see Appendix A for details).

The scattering dust filament (brown shaded area) is located at \((\rho, z) = (300, 0)\) lt-yr with an inclination of \(\alpha = 0\)°, where \(\alpha\) is the angle with respect to the sky (see Equation (A3)). The dust width is 0.02 lt-yr, 0.2 lt-yr, and 0.6 lt-yr from left to right, respectively. The rightmost panel of Figure 1 shows the flux of the LE that we would observe with these dust filaments versus \(\rho\). We denote these as the LE profiles, which essentially are cuts through the LE along the axis pointing toward the source event. In order to be able to directly compare the LE profiles to the dust and LE pulse geometry, we show the flux and \(\rho\) as the

![Figure 1. Illustration of how the dust filament width, \(\sigma_d\), influences the observed LE spectrum. The yellow shaded area indicates a 300 year old SN event that lasts 150 days, with a rising and declining arm of 30 and 120 days, respectively. The peak is indicated with a red line. The scattering dust filament (light gray shaded region) is located in the plane of the sky crossing the event position \((z = 0.0\ \text{lt-yr})\), with an inclination of \(\alpha = 0\)°. The dust width is 0.02 lt-yr, 0.2 lt-yr, and 0.6 lt-yr from left to right, respectively, and the left panel shows the respective LE profiles. A spectroscopic slit with a width of 1° is indicated with the thick black rectangle. Only light that is within the intersection of the event pulse, dust, and slit (indicated with the thick black rectangle) contributes to the observed spectrum. The blue dashed lines indicate which part of the light pulse is probed by the spectrum.](image-url)
the late-time portion of the SN light curve, i.e., the weighting alternative is that the LE spectrum does not contain light from SN 1993J, the very well observed and prototypical example of the SN IIb class (e.g., Filippenko et al. 1993, 1994, Richmond et al. 1994; Matheson et al. 2000), at 6, 39, 88, and 205 days after peak brightness from the bottom to the top, respectively (Jeffery et al. 1994; Barbon et al. 1995; Fransson et al. 2005). During the late phases, a significant fraction of flux is emitted in the [O i] λλ6300, 6364 doublet, which completely dominates the spectra for times later than 200 days. The bottom panel shows the light-curve-weighted integrated spectra of SN 1993J with different integration limits. The integration limits of these spectra are from 20 days before peak brightness to $P_{\text{max}}$, where $P_{\text{max}}$ ranges from 5 to 300 days after peak brightness from the bottom to the top. The most significant difference between the integrated spectra with early and late integration limits is the emergence of the [O i] doublet. We use the SN 1993J light curve from Richmond et al. (1996) as weights for the integration.

SNe IIb are not unique in this regard. The nebular spectra of all SNe are dominated by strong emission lines. Specifically, SNe Ia have strong [Fe ii] and [Fe iii] lines, SNe Ib/c have strong [O i], [Mg ii], Ca ii, and [Ca ii] lines, and SNe II have strong hydrogen Balmer lines. See Filippenko (1997) for a review of SN spectroscopy.

The perils of not properly accounting for differences in dust filament thickness are clear—the strength of features in observed LE spectra can be significantly altered by the width of the scattering dust on the sky.

In Figure 2, we display the LE spectrum of Cas A by Krause et al. (2008) in red. This spectrum has a weak [O i] doublet, consistent with the integrated spectra of SN 1993J, but only if integrated until ~80 days after peak brightness. The full integrated spectra of SN 1993J have a much stronger [O i] feature. If the LE spectrum is the full light-curve-weighted integrated spectrum of Cas A, then Cas A and SN 1993J have a significant difference at wavelengths near the [O i] feature. The alternative is that the LE spectrum does not contain light from the late-time portion of the SN light curve, i.e., the weighting function is truncated at late times.

### 3. LIGHT ECHO PROFILE MODELING

In Appendix A, we derive that the observed LE profile shape, $P$, strongly depends on the dust thickness, dust inclination, and...
seeing. We also show that this may have a significant impact on the observed LE spectra (see Appendices B and C). Here, we investigate the magnitude of each of these effects on observed LE spectra.

For this analysis, we examine a toy model of an LE that, despite its simplicity in some respects, appears to be representative of actual LE situations. The toy model is an LE from a 300 year old event with a "sail" shape as described in the text. The scattering dust filament intersects the event pulse paraboloid at $z = 0.0$ and has an inclination of $\alpha = 45^\circ$. (A color version of this figure is available in the online journal.)

**Figure 3.** Modeled LE profiles (left column), window functions (middle column), and effective light curves (right column) for our toy model as a function of dust width. All models in all rows have the same parameters except for the dust width; the separation into three rows is only for clarity. $\Delta \rho$ is with respect to the position where the dust intersects the light pulse. The toy model is a 300 year old event with a "sail" shape as described in the text. The scattering dust filament intersects the event pulse paraboloid at $z = 0.0$ and has an inclination of $\alpha = 45^\circ$. (A color version of this figure is available in the online journal.)

### 3.1. Dust Thickness

As described above, the dust thickness can significantly alter the LE spectrum. Using our toy model, we can quantify this effect.

**Figure 4.** Same as Figure 3, but for varying dust filament inclinations, $\alpha$, and constant dust width. The width of the dust filament is $\sigma_d = 0.1$ lt-yr. Note that the LE profile gets "stretched" when the inclination of the dust filament approaches the tangential angle of the LE ellipsoid, which is $\alpha = -45^\circ$ for $z = 0.0$ lt-yr. (A color version of this figure is available in the online journal.)

**Figure 5.** Same model as in Figure 3, but with varying dust filament width and seeing. The dust width presented are $\sigma_d = 0.001$ lt-yr (top row), $\sigma_d = 0.1$ lt-yr (middle row), and $\sigma_d = 0.3$ lt-yr (bottom row). Seeing of 0.0', 0.4', 1.0', and 2.0' is indicated with red, blue, yellow, and green lines, respectively. (A color version of this figure is available in the online journal.)

The associated window function (middle column) and effective light curves (right column), which are simply the light curve convolved with the window function. The different rows show different dust widths.

There are two distinct regimes where a modest change in the dust width has little effect on the effective light curve or integrated spectrum. The first one is shown in the top panels, where the dust width is so thin that the observed LE profile is effectively the projected initial light curve (corresponding to the left panel of Figure 1). The other limit is a very thick dust sheet (corresponding to the right panel of Figure 1).

In the former case (top row of Figure 3), changing the dust width from $\sigma_d = 0.001$ lt-yr to $\sigma_d = 0.01$ lt-yr only marginally modifies the effective light curve. Centering a spectroscopic slit on the LE profile results in a boxcar window function, and the effective light curve is truncated to the region near maximum brightness. In the latter case (bottom row of Figure 3), the dust width is large enough that the LE profile is dominated by the
dust width. The LE profile has a flat top, and only the edges are impacted by the original light curve profile. Centering a spectroscopic slit on the LE profile results in very wide window functions, and the effective light curve is essentially the same as original light curve (see the right panel of Figure 1 for an illustration).

As we show in Sections 4 and 5, our observations of LEs have been a mixture of both regimes. The middle row of Figure 3 shows the light curves, window functions, and effective light curves for dust widths of \( \sigma_d = 0.1 \) lt-yr (0.03 pc) and \( \sigma_d = 0.2 \) lt-yr (0.06 pc). The effective light curve is still heavily weighted to the time near maximum,\(^6\) but with increasing dust width the effective light curve slowly transitions to the full light curve.

3.2. Dust Inclination

The dust inclination has a large effect on the LE profile. This is simply a projection effect; as the dust sheet rotates, the projection of the light curve on the sky can change significantly. Turning to our toy model, we quantify this effect.

We fix the dust width to \( \sigma_d = 0.1 \) lt-yr. Figure 4 shows modeled LE profiles, window functions, and effective light curves for several examples that only differ by the inclination of the scattering dust filament. As expected, we find that the filament inclination has a profound impact on the LE profile width. For inclinations close to 90° (see \( \alpha = 70° \) in the top row of Figure 4), the dust filament is aligned along the line of sight, while the light curve projected into \( \rho \) space is compressed. Consequently, the window function has large wings, and the effective light curve is very similar to the full light curve. However, for inclination angles corresponding to dust sheets close to the tangent plane of the LE paraboloid (for \( \alpha = 0.0 \) lt-yr \(-45° \)), the LE profile subtends a larger angle on the sky (see \( \alpha = -30° \) in the bottom row of Figure 4). As a result, a spectroscopic slit will cover a very small portion of the light curve, and the window function is approximately a boxcar with a small width.

It is clear that the determining the dust inclination is very important, but fortunately it is straightforward to do so from the observations in cases where the location of the SN and date of outburst are known. In Appendix E and in more detail in A. Rest et al. (2011, in preparation), we derive the dust inclination from the apparent motion of the LE in difference images and discuss its sources of uncertainty. We discuss the impact the dust inclination has on the possibility of sampling the spectra within different age intervals for the same SN as well as the possibility of constraining the SN light curve shape in Section 6.2 and 6.3, respectively.

3.3. Point-spread Function Size

Observations of resolved sources must always account for the point-spread function (PSF). LEs are no exception since degradation of an image by a large PSF will not only smear the LE across the sky, but will also smear the LE in \( \textit{time} \). Again, we turn to our toy model and vary the PSF to determine its effect on the interpretation of LEs.

Figure 5 shows our toy model with dust inclination of \( \alpha = 45° \). We examine a Gaussian PSF with four different full widths at half-maximum (FWHMs): 0°0 (corresponding to a \( \delta \) function PSF), 0°4, 1°, and 2°. Each row of Figure 5 represents a different dust thickness, with the top, middle, and bottom rows corresponding to \( \sigma_d = 0.001, 0.1, \) and 0.3 lt-yr, respectively. For small dust widths (\( \leq 0.1 \) lt-yr), and thus thin LE profiles, the effective light curve depends significantly on the seeing. However, for larger dust widths, the seeing has very little effect on the effective light curve. This insight provides guidance for prioritizing targets for spectroscopic follow-up observations based on the conditions at the telescope.

3.4. Slit Offset

Thus far, we have assumed for our toy model that the spectroscopic slit was centered on the peak of the LE profile. However, LEs are usually both very faint and extended. Additionally,
their apparent motion on the sky makes perfect alignment of a spectroscopic slit unlikely—especially when there is a significant delay between the time of mask preparation and observations. Figure 6 shows the effect of having the slit offset from the LE–SNR. The LE is along the ρ,u-axis if the dust filament is perpendicular to the ρ,u-plane. In this example the angle δ₁ between the LE and the u-axis is 18°, and the angle δ₂ between slit and LE is 22°. Six subslits of width 0′′8 are indicated with the blue dotted lines. The offset Δs is 1′′.

(A color version of this figure is available in the online journal.)

Figure 8. Toy model of an LE and a spectroscopic slit configuration where the slit is misaligned both in angle and offset relative to the LE. The LE is indicated with the shaded area. We define a coordinate system (ρ, u) in the plane of the sky centered at the LE, where the ρ-axis is in the direction to the source event, and u is perpendicular to ρ, with the positive direction along positive position angle of LE–SNR. The LE is along the u-axis if the dust filament is perpendicular to the (ρ, z)-plane. In this example the angle δ₁ between the LE and the u-axis is 18°, and the angle δ₂ between slit and LE is 22°. Six subslits of width 0′′8 are indicated with the blue dotted lines. The offset Δs is 1′′.

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4. CAS A LIGHT ECHOES

4.1. Observations

As a case study, we investigate LEs discovered in a multi-season campaign beginning in 2006 October on the Mayall 4 m telescope at Kitt Peak National Observatory (Rest et al. 2008b). The MOSAIC imager on the Mayall 4 m telescope, which operates at the f/2.9 prime focus at an effective focal ratio of f/3.1, was used with the Bernstein “VR” broad filter (λc = 595 nm, δλ = 212 nm; NOAO Code k1040). The imaging data were kernel- and flux-matched, aligned, subtracted, and masked using the SMSN pipeline (Rest et al. 2005a; Garg et al. 2007; Miknaitis et al. 2007). We investigate the three Cas A SN LEs LE2116, LE2523, and LE3923, for which we presented spectra in Rest et al. (2011). Figure 9 shows the difference images and the slit position for these three LEs, and Table 1 lists the positions, lengths, and position angles for each slit. LE2116 was previously reported by Rest et al. (2008b), whereas LE2521 and LE3923 were discovered on 2009 September 14 and 16, respectively. For the analysis of LE2116, we use images from 2009 September 14, while we use the discovery images for LE2521 and LE3923.

4.2. Cas A Light Echo Template Spectra

There are several LEs associated with the Cas A SN. Here, we examine three LEs for which we have spectra. Their profiles are shown in Figure 10. To fit the LE profiles, we numerically

![Figure 8](image8.png)

![Figure 9](image9.png)

![Figure 10](image10.png)
solve the model defined by Equation (A14) for the observed data. To do this, we assume that the explosion occurred in the year 1681 ± 19 (Fesen et al. 2006), determine the scattering dust coordinates $\rho_0$ and $z_0$ by using Equation (A1) with the measured angular separation between the LE and the SNR, and we determine the dust inclination from the apparent motion of the LEs (this process is defined in Appendix E and described in more detail by A. Rest et al. 2011, in preparation). The remaining free model parameters are the LE profile peak height and position and the dust filament width, which has physical implications. For the three LEs presented above, we display their best-fit model profiles in Figure 10.

To numerically fit the model parameters, we determine the best-fitting profile by calculating the reduced $\chi^2$ for a three-dimensional grid of the three fit parameters. For each parameter, we marginalize over the other parameters to obtain the best-fitting value. The initial step sizes of the parameter grid are often too large for a proper marginalization. To ensure a proper determination of the parameters, we iterate this process three times, adjusting the grid spacing, guided by the fitted parameters and their uncertainties from the previous iteration. In all but the lowest signal-to-noise ratio cases, the probability density functions (PDFs) have a well-behaved Gaussian shape. Figure 11 shows as examples the marginal PDFs for the peak position and dust width of the LE2521 LE profile at $u = 0^\circ.5$.

Table 2 shows the input and fitted parameters of these three LE profiles. These profiles are for subslits with $u = 0^\circ.5$, i.e., close to the center of the spectroscopic slit. The definition of the ($\rho, u$) coordinate system and how it relates to the LE and the subslits is explained in detail in Section 3.4, Appendix C, and Figure 8. LE2116 and LE2521 have profiles with a similar width. The profile of LE3923, however, has a significantly larger width. Note that the model fits the data very well.

![Figure 10](image_url) LE profiles for LE2116, LE2521, and LE3923 from top to bottom, respectively. The model fit is indicated with the black lines. (A color version of this figure is available in the online journal.)

![Figure 11](image_url) Probability density functions (PDFs) for the peak position (left) and dust width (right) of LE2521 at $u = 0^\circ.5$. The most likely value is indicated with a dashed line, and the yellow shaded region indicates the 1σ interval. (A color version of this figure is available in the online journal.)
Panels (A), (B), and (C) show the positions of the LE profile peaks for the different subslits for LE2116, LE2521, and LE3923, respectively. \( \Delta \rho_0(u) \) is defined as \( \rho_0(u) - \rho_0(0) \), and \( u \) is orthogonal to the \( \rho \)-axis through \( \rho_0(0) \). Red, blue, and cyan symbols indicate LE2116, LE2521, and LE3923, respectively. Panel (D) shows the fitted dust widths, \( \sigma_d \), and panel (E) shows the surface brightness.

(A color version of this figure is available in the online journal.)

Figure 12. Panels (A), (B), and (C) show the positions of the LE profile peaks for the different subslits for LE2116, LE2521, and LE3923, respectively. \( \Delta \rho_0(u) \) is defined as \( \rho_0(u) - \rho_0(0) \), and \( u \) is orthogonal to the \( \rho \)-axis through \( \rho_0(0) \). Red, blue, and cyan symbols indicate LE2116, LE2521, and LE3923, respectively. Panel (D) shows the fitted dust widths, \( \sigma_d \), and panel (E) shows the surface brightness.

(A color version of this figure is available in the online journal.)

above. The spectroscopic slit is applied to the fitted LE profile using Equations (B1)–(B4) as described in Appendix B. It is important to note that now the seeing from the spectroscopic observation is used instead of the seeing from the image used to determine the LE profile. For each slit, the weighted average of all subslits is calculated using Equations (C1)–(C4) in Appendix C.

The top panel of Figure 13 shows the window functions for the three LEs for which we have spectra. The middle panel shows the SN 1993J light curve and the effective light curves for each LE (using the LE profiles and assuming that their light curves are the same as SN 1993J).

Even though LE2521 and LE2116 have comparable dust widths, the dust inclination for LE2521 is significantly larger, resulting in a wider window function than LE2116. This also causes the projected SN light-curve shape to be “squashed.” LE3923 has a similar inclination to that of LE2116, but its scattering dust is thicker, also leading to a wider window function. Note that the three window functions, and therefore also the effective light curves (see the middle panel in Figure 13), have significantly dropped off after 100 days past maximum. However, the decline time of SN is significantly longer, and consequently the contribution of spectra with phases \( \geq 100 \) days is cut off—a fact illustrated in the bottom panel of Figure 13, which shows the integrated spectra for SN 1993J. The late-type spectra of SN IIb, like SN 1993J, are dominated by \([\text{O} \text{i}] \lambda \lambda 6300, 6363, \) \([\text{Ca} \text{ii}] \lambda \lambda 7291, 7324, \) and \([\text{Ca} \text{ii}] \) NIR triplet features in the late-phase spectra.

(A color version of this figure is available in the online journal.)

Figure 13. Top: window function for the different LEs. Middle: effective light curves of SN 1993J for the LEs. The unmodified light curve of SN 1993J is shown in black. Bottom: integrated SN 1993J spectra, where the integration is weighted by the respective effective light curve. Red, cyan, and blue curves represent LE2116, LE2521, and LE3923, respectively. The black spectrum is weighted by the original, unmodified SN 1993J light curve. The differences in the spectra are mainly due to the strong \([\text{O} \text{i}] \lambda \lambda 6300, 6363, \) \([\text{Ca} \text{ii}] \lambda \lambda 7291, 7324, \) and \([\text{Ca} \text{ii}] \) NIR triplet features in the late-phase spectra.

(A color version of this figure is available in the online journal.)

4.3. Light Echo Spectra: Systematics

We have shown in Section 3 with simulated light curves and dust filaments how the LE spectra are influenced by the dust properties and observing conditions, and in Section 4.2 we have applied this to the real-world example of the Cas A LEs. In this section, we investigate how changing these properties impacts the LE spectra. We start with the fit to the Cas A LE profiles of LE2521 described in Section 4.2 and create a window function, effective light curve, and LE spectrum using the spectroscopic and photometric library of SN 1993J as a template. We then vary dust width, dust inclination, and seeing while keeping all other parameters fixed.

Figure 14 shows the effective light curves (left panels) and integrated SN 1993J spectra (right panels). In the top panel,
we vary the dust width, $\sigma_d$. We find that, consistent with the simulations in Section 3, the nebular [O\textsc{i}] and [Ca\textsc{ii}] lines in the integrated spectra have a strong dependence on the dust width. For $\sigma_d \lesssim 0.1$ lt-yr, the [O\textsc{i}] and [Ca\textsc{ii}] emission lines are weak, whereas for larger values the integrated spectrum quickly approaches the limit of the integrated spectrum without any window function applied with strong [O\textsc{i}] and [Ca\textsc{ii}] emission lines. We find similar results for the dust inclination: the difference between the integrated spectra is small and the [O\textsc{i}] and [Ca\textsc{ii}] lines are weak for $\alpha \lesssim 60^\circ$, but then rapidly approaches the integrated spectrum without any window function applied for $\alpha$ close to $90^\circ$. We find that the seeing (bottom panel) only has a very strong impact on the observed LE spectrum if the dust width is very small as discussed in Section 6.2 and 6.3.

We also investigate how uncertainties in the dust inclination impact the derived integrated spectra template. To do this, we fixed the dust inclination from $0^\circ$ to $80^\circ$ for LE2521 and fit the LE profile. Figure 15 shows the fitted dust width, $\sigma_d$, with respect to the input dust inclination, $\alpha$. Three examples of the fitted LE profiles are shown in the top panel of Figure 16. The bottom panels show the corresponding effective light curves (left) and integrated spectrum (right).

Note that while the LE profile fit is very good in all cases, the effective light curves are different, and thus the integrated spectra differ significantly in [O\textsc{i}] and [Ca\textsc{ii}] line strengths. The reason for these differences is the degeneracy between the dust inclination and the dust width. For decreasing dust inclination (i.e., the dust inclination gets closer to the tangential of the LE paraboloid), the projected light curve is “stretched.” This effect can be nullified by decreasing the dust width, resulting in a similar fit to the LE profile with significantly different parameters (see Figure 15). For inclinations close to $90^\circ$, the LE profile is dominated by the dust profile and not by the projected light-curve profile. Such a situation manifests itself in an LE profile that is symmetrical (see the green line in the top panel of Figure 16) in contrast to the asymmetrical LE profile shape (see the red line in the top panel of Figure 16) that still reveals indications of the original light curve. Similarly, the fitted $\sigma_d$ is essentially constant for $\alpha \approx 90^\circ$.

The tests in this section have shown that an incorrect dust inclination can systematically bias the interpretation of SN
properties from LE spectra. We discuss the sources of uncertainty in the determination of the inclination in Appendix E.

5. SN 1987A LIGHT ECHOES

5.1. Observations

Beginning in 2001, the SuperMACHO Project microlensing survey employed the CTIO 4 m Blanco telescope with its 8K × 8K MOSAIC imager (plus its atmospheric dispersion corrector) to monitor the central portion of the LMC every other night for five seasons (September through December). The images were taken through our custom “VR” filter (λc = 625 nm, δλ = 220 nm; NOAO Code c6027). SN 1987A and its LEs are in field sm77, which was observed with exposure times between 150 and 200 s. We have continued monitoring this field since the survey ended. The difference imaging reduction process was identical to the KPNO Cas A imaging.

5.2. Profile Modeling

The ring-shaped LEs associated with SN 1987A have previously been studied in great detail, mapping the dust structure surrounding the SN (e.g., Sugerman et al. 2005b). Unlike historical SNe, a complete spectral and photometric history of the SN is available for SN 1987A. This historical database allows an observed LE spectrum to be modeled unambiguously against a spherically symmetric model for the explosion. In addition, the high signal-to-noise ratio of the SN 1987A LEs make them ideal for our tests.

We consider an example LE from SN 1987A corresponding to a unique position angle on the sky. The spectrum associated with this LE was taken using the Gemini Multi-Object Spectrograph (GMOS; Hook et al. 2004) on Gemini-South, using the R400 grating. The spectroscopic slit was 1″0 wide and 2′′16 long, with the slit tilted to be tangential to the observed LE ring. The upper panel of Figure 17 shows the apparent motion of the LE monitored over more than a year. The motion on the sky is constant on such timescales (4′09 ± 0′′02 year−1), indicating constant inclination of the dust structure. To determine the angle of inclination, α, we translate (ρ, t) into (z, ρ) as described in Appendix E, resulting in a dust inclination of α = 30.4 ± 0.8° (see the lower panel of Figure 17).

A high signal-to-noise ratio difference image is used to model the effect of the dust on the spectroscopic slit. Input parameters are the inclination (α = 30.4°), observed seeing for the image (1′′22), and delay time since explosion (ct = 19.9 years). The photometric data of SN 1987A (Hamuy et al. 1988; Suntzeff et al. 1988; Hamuy & Suntzeff 1990) are used to model the LE profile observed on the sky, with the best-fitting model shown in the left panel of Figure 18. The dust structure model is able to recreate the observed LE profile exceedingly well. The window function and resulting effective light curve associated with the spectroscopic slit (convolved to the time of spectroscopy) are shown in the middle and right panels of Figure 18, respectively. The average dust width associated with the spectroscopic slit is σd = 4.26 ± 0.06 lt-yr (1.31 ± 0.02 pc). We note that at the inclination and dust widths observed here, the model is less sensitive to uncertainties in α. Testing of the model shows that at such inclinations, uncertainties in α on the order of 5° have little impact on the window function and resulting integrated spectrum (see Section 3.2).

The impact of the scattering dust is clear when fitting the model to the observed spectrum. To integrate the spectral history of SN 1987A, we take advantage of the extensive spectral database from SAAO and CTIO observations after the explosion (Menézies et al. 1987; Catchpole et al. 1987, 1988, 1989; Whitelock et al. 1988, 1989; Phillips et al. 1988, 1990). Figure 19 shows the observed LE spectrum (black), the model spectrum (red), and the full light-curve-integrated spectrum (blue dashed). Note that the observed LE spectrum includes strong nebular emission lines that are not associated with the SN. Both the model spectrum and the full light-curve-weighted integrated spectrum show a good fit for features blueward of 5500 Å. However, the lower panel of Figure 19 shows that the spectrum redward of 5500 Å, and in particular the Hα feature, are best fit by the spectrum corresponding to the best-fit LE profile model. From Figure 18, it is clear that the strength of the Hα feature in the integrated spectrum is highly dependent on the window function. At late times, the spectra of SN 1987A are dominated by strong Hα emission. The excellent fit to the LE spectrum of SN 1987A validates our method of determining the effective light curve from the LE profile.

5.3. Impact of Dust Parameters on Observed Spectrum

Similar to the analysis performed in Section 4.3, we vary the three free parameters of our model to determine their impact on the observed LE spectrum for SN 1987A. Figure 20 shows the effective light curves (left column) and integrated SN 1987A spectra (right column). The top, middle, and bottom rows show the model varying the dust width (σd), dust inclination (α),
Rest et al.

**Figure 18.** Left: observed profile for the SN 1987A LE, with the best-fit model profile overplotted. Middle: resulting window function associated with the 1″ wide spectroscopic slit. Right: effective light curve from the model compared with the original light curve of SN 1987A.

(A color version of this figure is available in the online journal.)

**Figure 19.** Top: observed LE spectrum of SN 1987A (black) with the best-fit model (using the window function from Figure 18) integrated spectrum (red) and the full light-curve-weighted integrated spectrum (blue). Bottom: detail of the spectra near the Hα feature. Note that the narrow lines dominating the emission peak of the profile are strong nebular emission lines and are not associated with the SN. The best-fit model-integrated spectrum fits the observed LE spectrum well in this region, while the full light-curve-weighted integrated spectrum is a poor fit.

(A color version of this figure is available in the online journal.)

and seeing, respectively, while holding the other parameters constant. To emphasize the differences in the integrated spectra, the figure focuses on the Hα feature. The advantage of the SN 1987A observations over those of Cas A is that there is a complete linking between the LE and the final integrated spectrum via observation, since we have a priori knowledge of the SN creating the observed LE profile.

The dependence on the integrated spectra of SN 1987A are similar to that of SN 1993J (see Figure 14), where increasing the dust width, inclination, or seeing all share the result of stretching the weight function closer to the $w_f = 1$ case, resulting in an effective light curve approaching that of the original SN 1987A light curve. Figure 20 allows us to rank the impact of the dust parameters on the observed LE spectrum. Seeing has the least significance on the spectrum, although we note that poor image quality should still be taken into account. Dust inclination has a large effect, although the sensitivity is small if the dust is close to perpendicular to the observer. The dust width has the greatest impact on the spectrum, which is particularly noteworthy since this quantity has been ignored until now. For SN 1987A, the Hα peak height increases by about 20% if the dust thickness is 10 lt-yr instead of 5 lt-yr, indicating a sensitivity that exceeds that of the other parameters.

Figure 20, as well as the SN 1987A spectrum presented in Section 5.2, clearly show the danger in interpreting
spectroscopic data from LEs without proper modeling. Any interpretation of LE spectra based on relative line strengths must be accompanied by a careful analysis of the dust properties associated with the LE.

6. DISCUSSION

In the previous sections, we have presented the LE model, the dependency of the model on various parameters, and have applied that model to multiple real-world cases. In this section, we discuss how we can use our new understanding of LEs and the scattering dust to perform additional observations that will provide new insight into historical SNe.

6.1. Verification of Model Parameters and Systematic Biases

The analysis of SN 1987A LEs has the added benefit that the validity of our dust models and their impact on the observed LE profiles and spectra can be tested, since the spectrophotometric evolution of the SN itself has been monitored extensively. We show in Section 5 and in more detail in B. Sinnott et al. (2011, in preparation) that the observed LE profile, apparent motion, and spectra can all be brought in excellent accordance with what is predicted using the SN 1987A spectrophotometric library, and our dust model locally parameterized as a planar filament with a Gaussian density profile. This shows that at least in the case of SN 1987A, the Gaussian dust model is a good approximation of the true dust profile.

In general, dust structures are filamentary in nature, which is easily visible in its effect on the shapes of LEs, which often show twists and furcations. However, on the few arcsecond scale, most of the shape and profiles of LEs change slowly over months and even years. For example, the dust width, \( \sigma_d \), for LE3923 and LE2521 is nearly constant over 20" (see the second panel from the bottom in Figure 12) and smoothly increasing, respectively. In order to account for small changes in the dust parameters, we split the slit into 1" sections and fit the dust parameters separately for each of these sections.

If there are significant deviations from the Gaussian dust density model (e.g., if there is substructure in the density on small scales), the window function can potentially be biased, which in turn can cause spectral differences that would be interpreted as intrinsic to the SN. We can test and guard against these systematic biases: LEs often come in “groups,” i.e., there are often several distinct LEs separated by tens of arcseconds for LEs of Galactic SNe. The scattering dust filaments likely belong to the same dust structure, but are tens of light years apart. Therefore any substructure in these filaments is uncorrelated. For LEs of the same source event only tens or hundred of arcseconds apart, the physical properties of the source event are the same. An upper limit on the systematic bias can be set by the differences in the observed spectra which are not accounted for by the differences in fitted dust parameters. In all data obtained by our team, we have not found any evidence that there are significant biases in our analysis due to substructure in the dust or deviation from the dust model. This is particularly important for the two novel methods that we describe in the next two sections.

If LEs from two different SNe are reflected off the same dust, then the dust thickness measured from our method should be the same for both sets of LEs. Such a scenario is a direct test of the model fits. Fortunately, this situation occurs for LEs from Cas A and Tycho, and a comparison of the derived values will be presented in a future work. Since this situation was discovered in a relatively small and low filling factor survey, it is reasonable to expect other such regions of the sky exist where dust is at the intersection of two historical SN LE ellipsoids.

6.2. Temporally Resolving Light Echo Spectra

One of the most exciting opportunities LEs present is to use them to temporally resolve the SN spectrum: if a slit is aligned perpendicular to an LE profile instead of parallel, different parts of the slit probe different epochs of the SN light curve.

As an example, we imagine an SN LE where the SN is a carbon copy of SN 1993J, and the dust filament is at \( z = 400 \text{ lt-yr} \) with an inclination of 0° and a width of 0.02 lt-yr. With a Hubble Space Telescope (HST)-like PSF FWHM of 0.05°, the projected light curve would have minimal distortion. If a spectrum is taken with the slit perpendicular to the LE arc (i.e., parallel to the line from the LE to the SNR) with 0°05 seeing, then the spatially resolved spectrum (i.e., moving along the axis perpendicular to the wavelength direction) corresponds to a temporally resolved spectrum. The left panel of Figure 21 shows the effective light curves for this situation if spectra are extracted from 1° long parts of the slit. The offset from the peak of the LE are 1°0,
0′.0, and −1′.0 along the ρ-axis. The right panel shows the corresponding integrated spectra. The three spectra show clear differences, in particular, the spectra with Δρ_{offset} = 1.0 is very blue since it is dominated by the first peak of the light curve.

Aligning the slit perpendicular to the LE can significantly reduce the amount of light within the slit, and therefore, this method is best applied to the brightest LEs. To maximize the temporal resolution of the LE spectra, the LE should have the following properties.

1. The dust filament is very thin—on the order of 0.01 lt-yr.
2. The inclination is close to the LE ellipsoid tangential. This “stretches” the projected light curve along the ρ-axis.
3. The LE is bright.
4. The PSF of the spectroscopic observation is small.

Even though there are no known Galactic LEs that fulfill all of these criteria, we note that the existing surveys are far from complete. Additionally, even in non-ideal conditions, some temporal information could be obtained. The ability to temporally resolve the SN spectra will allow us to follow the evolution of the explosion, specifically measure velocity gradients of spectral features, and potentially probing the abundances of elements in the ejecta.

6.3. Constraining Supernova Light-curve Shapes

Under similar circumstances needed to temporally resolve the LE spectra (see Section 6.2), we will also be able to spatially resolve the temporal characteristics of the source event light curve. The most important requirements are that the dust width has to be thin, the inclination close to the LE ellipsoid tangential, and the PSF needs to be significantly smaller than the width of the LE profile. Note that in contrast to temporally resolving the LE spectra, constraining the LE shape can be done with LEs that have a significantly lower signal-to-noise ratio.

Unfortunately, there is a degeneracy between the dust width and the light curve parameters (e.g., peak width and height; peak ratio if double peaked) which can be broken if different areas of the LE have different dust widths. Since the light curve must be the same for all LEs in a given direction, any difference in the shape must be from the dust. If this degeneracy cannot be broken, LEs can still provide interesting constraints. First, the light curve can only be broadened by the dust, so any profile will provide some constraint on the light-curve shape. Additionally, by comparing the profile to known SN light curves, the properties of the SN can be further constrained.

The light-curve shape of an SN can give important insights into the nature of the SN explosion. SNe Ia, for example, show a clear relationship between the width of their light curves and their intrinsic brightness (e.g., Phillips 1993). Another example is if the shock breakout of a core-collapse SN is resolved in the light curve, which would provide a measurement (or strict constraints) on the radius of the progenitor star. We will discuss this in more detail in the following paragraphs.

After the gravitational collapse of the core of a massive star, the core bounces, sending a shock wave out through the outer layers of the star, exploding the star (Woosley & Weaver 1986). When the shock wave reaches the surface of the star, a flash (at optical wavelengths, it has a thermal spectrum) is generated that decays over hours to days (Colgate 1974; Klein & Chevalier 1978). This has been seen for SNe IIP (Gezari et al. 2008; Schawinski et al. 2008), but was best observed recently with SN Ib 2008D (Soderberg et al. 2008; Modjaz et al. 2009). As the stellar envelope cools, the optical emission fades away. This subsequent fading was first detected in SN 1987A (e.g., Woosley et al. 1987; Hamuy et al. 1988), which had a shock breakout cooling phase that was unusually long. This cooling phase has been detected for a handful of other SNe, notably SN 1993J (e.g., Richmond et al. 1994), an SN Ib similar to Cas A.

The duration, luminosity, and spectral energy distribution of the shock breakout phase depend on a handful of parameters such as the presence of a stellar wind and the ejecta mass (Matzner & McKee 1999), but is most dependent on the progenitor radius (Calzavara & Matzner 2004). The peak brightness of the subsequent fading is dependent on the ejecta mass, kinetic energy, and progenitor radius, while the timescale for the fading is proportional to both the radius of the photosphere and its temperature (Waxman et al. 2007). One caveat is that interaction of the shock with shells of material outside the star can produce prompt emission similar to a shock breakout.

The disadvantage of using LEs to observe the shock breakout is that if the resolution of the image is poor or the dust thickness is too large, it is impossible to distinguish between a shock breakout, subsequent fading, and normal radioactive-decay-powered SN light curve, and a somewhat broader radioactive-decay-powered SN light curve. For Cas A, where we might expect to see a shock breakout, we are unable to separate the light curve into multiple components in ground-based images with moderate seeing (<1″). However, as shown in Figure 22, if the dust thickness is sufficiently small, we should detect a luminous shock breakout with the resolution of HST, if it exists. This will allow us to constrain the radius of the progenitor and its mass loss history, linking the physical properties of a star with its SNR.

7. CONCLUSIONS

In recent years, LEs of SNe have offered the rare opportunity to spectroscopically classify the SN type hundreds of years after the light of the explosion first reached the Earth (e.g., Rest et al. 2008a). Recently, we have obtained the first spectroscopic observations of a historical SN from different lines of sight using LEs (Rest et al. 2011). These observations provide a unique opportunity to examine an SN from different directions, with the light for each LE coming from slightly different hemispheres of the SN. In this paper, we have demonstrated that to properly...
separate differences in the observed LE spectra caused by
intrinsic asymmetries in the SN from those caused by scattering
dust, seeing, and spectroscopic slit position, it is necessary to
properly model the latter.
Throughout this paper, we have shown how to determine the
properties of the scattering dust, and what their influence, as well
as that of the PSF and spectroscopic slit position, on the observed
LE spectra is. We found that the dust width and inclination are
the dominant factors, especially when the dust filament is thin.
These two factors can be degenerate with respect to the LE
profile shape; therefore, it is essential to determine the dust
filament inclination independently using the LE proper motion
from images taken at two (or more) epochs. The slit width and its
misalignment with the major axis of the LE is also a significant
factor. The image PSF smears out the observed LE and thus also
impacts the observed LE spectrum. Taking all of these factors
into account, we found that the observed LE spectrum is not the
light-curve-weighted integration of the spectra at all epochs,
but rather the integration of the spectra weighted by an effective
light curve. This effective light curve is the original light curve
convolved with a window function, and it is different for every
LE location. In general, the most significant difference between
the effective light curves and the original light curve was that
latter epochs tend to be truncated in the effective light curves.
We used fits to the LE profiles of Cas A and SN 1987A to test
the consistency and veracity of our model. We fit the model to the
observed LE profiles, where the three free parameters were the
LE profile peak height, LE position, and—physically the most
important—the dust filament width. The model reproduced the
observed LE profiles very well, and we found that it was possible
to determine all factors influencing the observed LE spectrum a
priori just from images, without using the observed spectrum.
We constructed window functions and effective light curves
for the Cas A LEs using SN 1993J as a template. We found a
much better match of the SN 1993J spectral template to the
observed Cas A LE spectrum if we used the effective light curve
instead of the real light curve for weighting. The most significant
difference was in the late-phase emission lines [O I] λλ 6300,
6363, [ Ca II] λλ 7291, 7324, and the Ca II NIR triplet, consistent
with our expectation from the model. Although we do not have
access to the Krause et al. (2008) images, we find that their Cas
A spectrum is consistent with an integrated spectrum weighted
with a light curve truncated at 80 days. Our expectation is that
the majority of differences between the Cas A LE spectrum
and the full light-curve-weighted SN 1993J spectrum are due to
scattering dust properties.
We also constructed a window function and effective light
curve for an SN 1987A LE which constitutes a special test case
since the light curve and spectral data of the SN are available.
With this data, we constructed an LE spectrum from the original
SN and compared it to the observed LE spectrum. We found that
the constructed LE spectrum correctly predicted the line strength
of the Hα line, while a simple integration weighted by the light
curve was a poor fit.
We presented several additional investigations that can be
attempted with LEs in the near future. Two of these studies,
temporally resolved spectra and spatially resolved light curves,
require thin dust filaments. Until now, the assumption of thick
scattering dust prevented such studies.

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APPENDIX A
LIGHT ECHO PROFILES

The LE equation (Couderc 1939),
\[
z = \frac{\rho^2}{2ct} - \frac{ct}{2},
\]
relates the depth coordinate, \(z\), the LE–SN distance projected
along the line of sight, to the LE distance, \(\rho\), perpendicular to
the line of sight, and the time, \(t\), since the initial (undelayed)
arrival of SN photons. Then the distance \(r\) from the scattering
dust to the source event is \(r^2 = \rho^2 + z^2\), and \(\rho\) is
\[
\rho = (D - z) \sin \gamma,
\]
where \(D\) is the distance from the observer to the source event
and \(\gamma\) is the angular separation between the source event and
the scattering dust, which yields the three-dimensional position
of the dust associated with the arclet.
Tylneda (2004) elegantly derives the apparent expansion of an
LE ring caused by a thin plane-parallel sheet of dust. In a very
similar manner, we derive how the dust filament inclination influences
the observed LE profile. Because of the symmetry
around the \(z\)-axis, we define our coordinate system so that the
dust sheet is normal in the \((\rho, z)\)-plane. This allows us to do all
our calculations in the two-dimensional \((\rho, z)\) space. We define
the dust sheet as
\[
z = z_d - a\rho,
\]
where \(a = \tan \alpha\). We define \(\alpha\) as the angle of the dust sheet with
respect to the \(\rho\)-axis, where positive angles go from the positive
\(\rho\)-axis toward the negative \(z\)-axis. The dust sheet intersects with the
\(z\)-axis at \(z = z_d\). The two intersections of the dust sheet with the
reflection paraboloid defined in Equation (A1) are then
\[
\rho_\pm(t, z_d, a) = -ac t \pm \xi, \text{ where (A4)}
\]
\[
\xi = \sqrt{(1 + a^2)(ct)^2 + 2z_0ct}.
\]

Now consider an LE caused by a source event with a peak at \(t_0\)
at an angular separation \(\gamma\) from the source. Assuming a distance
\(D\) to the source event, we can then calculate the position of the
scattering dust \((\rho_0, z_0)\) using Equations (A1) and (A2). The dust
sheet defined in Equation (A3) goes through \((\rho_0, z_0)\) if
\[
z_d = z_0 + a\rho_0.
\]
There are two special dust inclinations. We denote the inclination angle of the dust sheet tangential and orthogonal to the LE ellipse as $\alpha_\parallel$ and $\alpha_\perp$, respectively. Then the slopes of the dust sheets are $a_\parallel = \tan \alpha_\parallel$ and $a_\perp = \tan \alpha_\perp$, and we can calculate them as

$$a_\parallel = -\left(1 + \frac{2z_0}{ct}\right)^{1/2} = -\frac{\rho_0}{ct}; \quad (A7)$$

$$a_\perp = -a_\parallel^{-1} = \left(1 + \frac{2z_0}{ct}\right)^{-1/2} = \frac{ct}{\rho_0}. \quad (A8)$$

The intersection of the dust sheet with the LE ellipsoid is then at

$$\rho_0 = \begin{cases} 0 & \text{for } \alpha = 90^\circ \\ \rho_+ & \text{for } \alpha > a_\parallel \\ \rho_- & \text{for } \alpha < a_\parallel. \end{cases} \quad (A9)$$

Now consider a dust sheet with a finite width defined by a density $D(\Delta \rho)$ and a light curve $f_{\text{source}}(\Delta t)$, where $\Delta t$ is the distance orthogonal to the dust sheet, and $\Delta t = t - t_0$ (see Appendix D for a more detailed description of the dust model $D(\Delta t)$). The flux from the source event at time $t$ is reflected by the part of the dust sheet at a distance $\Delta t$ to the center of the dust sheet and makes a relative contribution to the LE signal of

$$f_{\text{LE}} = D(\Delta t)f_{\text{source}}(\Delta t) \quad (A10)$$

at the position

$$\rho_{\text{LE}} = \begin{cases} \rho_+(t_0 + \Delta t, z_d + \Delta r/\cos \alpha, a) & \text{if } \rho_0 = \rho_+ \\ \rho_-(t_0 + \Delta t, z_d + \Delta r/\cos \alpha, a) & \text{if } \rho_0 = \rho_- \end{cases}. \quad (A11)$$

The reason for choosing $+$ and $-$ is based on the choice made for $\rho_0$. For all practical purposes, $\Delta t$ and $\Delta r$ are small, and therefore the sign should be the same. Exceptions are $\alpha \approx 0.0$ and $a \approx a_\parallel$. We define

$$\Delta \rho = \rho_{\text{LE}} - \rho_0. \quad (A12)$$

By examining the relative contribution for all values of $\Delta t$ and $\Delta r$ and using Equation (A10)–(A12), we can calculate a cumulative grid of fluxes, $F_{\text{LE}}(\Delta t, \Delta \rho)$.

The model must now account for seeing. We describe the data PSF as a Gaussian with an FWHM of $x$ arcsec and Gaussian width of $\sigma_{\text{PSF}} = x/\left(2\sqrt{2}\ln(2)\right) \approx x/2.35$. Then the cumulative grid of fluxes convolved with the seeing is

$$F_{\text{LE}}(\Delta t, \Delta \rho) = \sum_{\Delta \rho'} F_{\text{LE}}(\Delta t, \Delta \rho') \frac{1}{\sqrt{2\pi \sigma_{\text{PSF}}^2}} \exp\left(-\frac{(\Delta \rho - \Delta \rho')^2}{2\sigma_{\text{PSF}}^2}\right). \quad (A13)$$

Integrating over $\Delta t$ returns the LE flux profile with respect to $\rho$:

$$P_{\text{LE}}(\Delta \rho) = \int F_{\text{LE}}(\Delta t, \Delta \rho) d\Delta t. \quad (A14)$$

APPENDIX B

DEPENDENCE OF THE LIGHT ECHO SPECTRUM ON LIGHT ECHO PROFILE

We now examine the LE flux for a spectroscopic observation. Consider a slit positioned at $\rho_0$ with an effective width of $\sigma_{s,\text{eff}}$. The reason that this is the effective slit width, and its relation to the real, physical slit width $\sigma_s$, is explained below in Appendix C. We define the offset between the slit and the LE measured parallel to the $\rho$-axis as $\Delta \rho_{\text{offset}} = \rho_s - \rho_0$. The slit then subtends from

$$\Delta \rho_{\text{min}} = \Delta \rho_{\text{offset}} - \sigma_{s,\text{eff}}/2 \quad (B1)$$

$$\Delta \rho_{\text{max}} = \Delta \rho_{\text{offset}} + \sigma_{s,\text{eff}}/2. \quad (B2)$$

We can then calculate the relative contribution, $H(\Delta t)$, of the source event at time $\Delta t$ to the spectra with

$$H(\Delta t) = \int_{\Delta \rho_{\text{min}}}^{\Delta \rho_{\text{max}}} F_{\text{LE}}(\Delta t, \Delta \rho) d\Delta \rho. \quad (B3)$$

$H(\Delta t)$ is also the effective light curve. We define the window function $w(\Delta t)$ as the fractional difference between the effective light curve and the real light curve,

$$w(\Delta t) = f_{\text{source}}(\Delta t)/H(\Delta t). \quad (B4)$$

The observed LE spectrum $S_{\text{LE}}(\lambda)$ is then

$$S_{\text{LE}}(\lambda) = \int S(\lambda, \Delta t)H(\Delta t) d\Delta t, \quad (B5)$$

where $S(\lambda, \Delta t)$ is the spectrum of the source event at the given phase $\Delta t$.

APPENDIX C

DEPENDENCE OF THE LIGHT ECHO SPECTRUM ON SLIT POSITION

The position and inclination of the slit with respect to the LE has a major impact on the observed spectrum. We illustrate this in Figure 8. We define a coordinate system $(\rho, u)$, where $u$ is orthogonal to $\rho$ through $\Delta \rho = 0$ in the plane of the sky. For a dust filament that is locally planar and perpendicular to the $(\rho, z)$-plane (i.e., parallel to $u$), the LE is aligned with the $u$-axis. However, if it is tilted with respect to $u$ by an angle of $\delta_1$, then the LE will also be tilted by the same angle (see gray shaded area in Figure 8).

Now consider a slit (red box in Figure 8) that is misaligned with the LE by an angle $\delta_2$ and also offset along the $\rho$-axis by $\Delta \rho$. We split the slit into subslits, indicated with the dotted blue lines in Figure 8. For each of these subslits, we determine the LE profile in Figure 8. The offset $\Delta \rho_{\text{offset},i}$ of the subslit $i$ centered at $u_i$ to the LE is

$$\Delta \rho_{\text{offset},i} = u_i \sin(\delta_2) + \Delta \rho, \quad (C1)$$

where $\delta_2$ is the angle between the slit and the LE and $\Delta \rho$ is the offset of the center of the slit to the LE (see Figure 8). The effective slit width $\sigma_{s,\text{eff}}$ depends on the physical slit width $\sigma_s$ and is

$$\sigma_{s,\text{eff}} = \sigma_s / \cos(\delta_2). \quad (C2)$$

The effective light curve $H_i$ and window function $w_i$ for each subslit can be calculated plugging $\Delta \rho_{\text{offset},i}$ into Equations (B1) and (B2). The total window function $w_{\text{full}}$ for the full slit is then the weighted average of the subslit window functions, where the weight of the flux $f_{\text{w}}$ that falls into the subslit is

$$f_{\text{w},i} = \int_{\Delta \rho_{\text{min}}}^{\Delta \rho_{\text{max}}} P_{\text{LE}}(\Delta \rho) d\Delta \rho \quad (C3)$$
\[ w_{\text{slit}} = \sum_i w_i f_{w,i}^{-2} \sum_j f_{w,j}/ \] (C4)

The integrated spectrum can then be calculated by using \( w_{\text{slit}} \) in Equations (B4) and (B5).

**APPENDIX D**

**DUST MODEL**

For simplicity in our simulations, we choose a boxcar dust density profile \( D(\Delta r) \). A more realistic density profile is a Gaussian, and therefore when fitting observed LE profiles we use a profile

\[ D(\Delta r) = \frac{c}{\sqrt{2\pi} \sigma_d^2} \exp\left(\frac{\Delta r^2}{2\sigma_d^2}\right), \] (D1)

where \( \sigma_d \) is the width of the Gaussian and \( c \) is a normalization factor. Note that the physical dust density profile width \( \sigma_{d,\text{phys}} \) is different from the nominal fitted \( \sigma_d \) if \( \delta_1 \neq 0 \). They are related through

\[ \sigma_{d,\text{phys}} = \sigma_d \cos(\delta_1). \] (D2)

**APPENDIX E**

**DUST FILAMENT INCLINATION**

We refer the reader to a more thorough and detailed discussion about the apparent motion and its dependence on the dust filament inclination by A. Rest et al. (2011, in preparation). However, we present an initial treatment of the process here. We define the dust filament inclination \( \alpha \) as

\[ \alpha = \tan^{-1}\left(\frac{z(t_2) - z(t_1)}{\rho(t_2) - \rho(t_1)}\right), \] (E1)

with \( t_2 > t_1 \) and with \( z \) calculated from Equation (A1) for a measured \( \rho \) and given \( t \). Note that a dust filament in the plane of the sky has \( \alpha = 0^\circ \), and dust filaments tilted away from the observer have positive \( \alpha \). If more than two epochs are available, fitting a straight line through \( (\rho, z) \) increases the accuracy of \( \alpha \).

There are several sources that can introduce errors into the measurement of the dust inclination, specifically,

1. Poisson uncertainty in the apparent motion of the LE, especially for the fainter LEs.
2. A small number of epochs.
3. Intrinsic differences in the filament inclination with \( \rho \).

The first two mainly increase the Poisson noise in the inclination. The last one, however, is the most significant one since it can introduce systematic uncertainties. If the dust filament has additional substructure, such as twisted dust on small scales, then the inclination of the scattering dust changes on small scales and a correct constraint on the dust inclination at the time of spectroscopy can only be done if there are several epochs close to the time of the spectroscopy.

**REFERENCES**

Badenes, C., Hughes, J. P., Cassam-Chenai, G., & Bravo, E. 2008, ApJ, 680, 1149

Barbon, R., Benetti, S., Cappellaro, E., Patat, F., Turatto, M., & Iijima, T. 1995, A&AS, 110, 513

Bond, H. E., Gilmozzi, R., Meakes, M. G., & Panagia, N. 1990, ApJ, 354, L49

Bond, H. E., et al. 2003, Nature, 422, 405

Calzavara, A. J., & Matzner, C. D. 2004, MNRAS, 351, 694

Cappellaro, E., et al. 2001, ApJ, 549, L215

Catchpole, R. M., et al. 1987, MNRAS, 229, 15P

Catchpole, R. M., et al. 1988, MNRAS, 231, 75P

Catchpole, R. M., et al. 1989, MNRAS, 237, 55P

Colgate, S. A. 1974, ApJ, 187, 333

Couderc, P. 1939, Ann. d’Astrophys., 2, 271

Crotts, A. 1988, IAU Circ., 4561, 4

Dwek, E., & Arendt, R. G. 2008, ApJ, 685, 976

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...
Suntzeff, N. B., Heathcote, S., Weller, W. G., Caldwell, N., & Huchra, J. P. 1988, Nature, 334, 135
Swope, H. H. 1940, Harvard College Obs. Bull., 913, 11
Tylenda, R. 2004, A&A, 414, 223
van den Bergh, S. 1965, PASP, 77, 269
van den Bergh, S. 1966, PASP, 78, 74
Van Dyk, S. D., Li, W., & Filippenko, A. V. 2006, PASP, 118, 351
Wang, X., Li, W., Filippenko, A. V., Foley, R. J., Smith, N., & Wang, L. 2008, ApJ, 677, 1060
Waxman, E., Mészáros, P., & Campana, S. 2007, ApJ, 667, 351
Welch, D. L. 2007, ApJ, 669, 525
White, P. A., et al. 1988, MNRAS, 234, 5P
Whitelock, P. A., et al. 1989, MNRAS, 240, 7P
Woosley, S. E., Pinto, P. A., Martin, P. G., & Weaver, T. A. 1987, ApJ, 318, 664
Woosley, S. E., & Weaver, T. A. 1986, ARA&A, 24, 205
Xu, J., Crotts, A. P. S., & Kunkel, W. E. 1995, ApJ, 451, 806
Zwicky, F. 1940, Rev. Mod. Phys., 12, 66