Effect of second order slip and non-linear thermal radiation on mixed convection flow of MHD Jeffrey nanofluid with double stratification under convective boundary condition

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Abstract. In this article, the effect of non-linear thermal radiation and second order slip velocity in MHD convective Jeffrey nanofluid flow over a stretching surface with convective boundary condition is investigated in the presence of thermal and concentration stratification. The ordinary differential equations are obtained by using suitable transformations in the governing equations. Homotopy analysis method is used to solve the ordinary differential equations. The velocity, temperature and concentration profile is computed for different pertinent parameters involved in the study. Also computation is done for skin friction coefficient and local Nusselt number and the results are discussed.

1. Introduction

The effect of thermal radiation on convective MHD viscoelastic fluid flow over a stretching sheet was analyzed by Eswaramoorthi et al. [1]. Stratification of the fluid is a deposition or formation of layers. The mixed convective micropolar fluid flow over a continuously moving vertical surface engrossed in a stratified thermally and solutally medium was discussed by Rashad et al. [2]. Hayat et al. [3] investigated about the unsteady nanofluid flow with magnetohydrodynamics and double stratification. The MHD nanofluid flow past a permeable stretching surface with second order velocity slip was examined by Zhu et al. [4]. The thermal radiation effect on magnetohydrodynamics stagnation point Jeffrey fluid flow of towards a stretching sheet was analyzed by Das et al. [5]. The thermal radiation effect on mixed convection Jeffrey nanofluid flow with thermal and solutal stratified medium was discussed by Abbasi et al. [6]. In this paper, the effect of second order slip and non-linear thermal radiation on magnetohydrodynamics mixed convection flow of Jeffrey nanofluid towards a stretching sheet with double stratification under convective boundary condition is investigated which is the extension of the paper Abbasi et al. [6].

2. Mathematical Formulation

The steady, incompressible and laminar Jeffrey nanofluid flow towards stretching surface with non-linear thermal radiation and convective boundary condition is considered. The stretching sheet velocity is taken as $u_{1,w} = a_1 x$ where $a_1$ are positive constants. The fluid flow is considered along $y$-
axis with second order slip and magnetic field normal to the fluid flow. The thermophoresis and Brownian motion are considered due to the presence of nanoparticles. The ambient temperature and concentration are assumed as \( T_\infty = T_0 + d_1 x \) and \( C_\infty = C_0 + d_2 x \). The fluid temperature and concentration are considered as \( T_w = T_0 + d_3 x \) and \( C_w = C_0 + d_4 x \). The thermal and concentration stratification effects are also considered. The governing equations for this flow analysis can be constructed as

\[
\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0, \\
\frac{\partial u_1}{\partial x} + v_1 \frac{\partial u_1}{\partial y} = \nu \left( \frac{1}{\lambda_1 + 1} \frac{\partial^2 u_1}{\partial y^2} + \frac{\lambda_2}{\lambda_1 + 1} \left[ \frac{\partial u_1}{\partial y} \frac{\partial^2 u_1}{\partial x \partial y} + u_1 \frac{\partial^2 u_1}{\partial x^2} - \frac{\partial u_1}{\partial x} \frac{\partial u_1}{\partial y^2} + v_1 \frac{\partial^2 u_1}{\partial y^2} \right] \right) - \frac{\sigma \beta T}{\rho} u_1 + \left[ \beta T (T - T_\infty) + \beta C (C - C_\infty) \right] g, \\
\frac{\partial T}{\partial y} + v_1 \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + D_B \frac{\partial}{\partial y} \left( \frac{\partial T}{\partial y} \right) + \left( \frac{\partial T}{\partial y} \right)^2 - \frac{\alpha q_r}{(\rho c_p) \partial y}, \\
\frac{\partial C}{\partial y} + v_1 \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{\partial T}{\partial y} \frac{\partial^2 C}{\partial y^2},
\]

where the boundary conditions are

\[
u_1 = u_1 w + u_1 \mathrm{slip} = a_1 x + A \frac{\partial u_1}{\partial y} + B \frac{\partial^2 u_1}{\partial y^2}, \quad \nu_1 = 0, -k_T \frac{\partial T}{\partial y} = h_1 (T_w - T), \quad C = C_w \text{ at } y = 0,
\]

\[
u_1 \to 0, \quad T \to T_\infty, \quad C \to C_\infty \text{ as } y \to \infty,
\]

where \( A & B, \alpha, \beta_C, \beta_T, c_p, D_B, D_T, g, k_T, \lambda_1 & \lambda_2, v, q_r, \rho & \sigma \) and \( \tau \) are first and second order slip velocity factor, thermal diffusivity, concentration expansion co-efficient, thermal expansion co-efficient, specific heat, Brownian motion co-efficient, thermophoresis co-efficient, acceleration due to gravity, fluid thermal conductivity, ratio of relaxation to retardation & retardation time parameter, kinematic viscosity, radiative heat flux, fluid density and electrical conductivity and ratio between the effective nanoparticle materials and fluid heat capacity. The similarity transformations are

\[
\eta = y \sqrt{\frac{a_1}{v}}, \quad u_1 = a_1 x s'(\eta), \quad v_1 = -\sqrt{a_1 \nu s(\eta)}, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_0}, \quad \varphi(\eta) = \frac{C - C_\infty}{C_w - C_0}
\]

Substituting (6) in (1), (1) is satisfied identically. Substituting (6) in (2) to (5), we get

\[
\begin{align*}
s'' &= (\lambda_1 + 1) \left[ s'' - s'' - s'' \right] - \beta \left[ s \ s'' - s'' \right] + (\lambda_1 + 1) \left[ Ri (\theta + N \varphi) - M^2 s' \right] = 0, \\
\left(1 + \frac{4}{3} \frac{Rd}{Ri} \right) \theta'' + \frac{2}{3} \frac{Rd}{Ri} \left[ (\theta'' \theta'' + \eta^2 \theta'') + 3(\theta'' \theta'' + \theta'' \eta^2) \right] &+ 4 Rd \left[ (\theta'' \theta'' + \eta^2 \theta'') \right] + Pr [s \theta' + Nb \theta' \varphi' + Nt (\theta'' + s' \theta' - S_T s')] = 0, \\
\varphi'' &+ Pr Le s \varphi'' + \frac{Nt}{Nb} \theta'' - Pr Le s \varphi' - Pr Le Sc s' = 0,
\end{align*}
\]

\[
\begin{align*}
s(0) &= 0, s'(0) = 1 + \epsilon s'(0) + \epsilon_2 s''(0), \quad \theta'(0) = -\alpha_T (1 - S_T - \theta(0)), \\
\varphi(0) &= 1 - Sc, \quad s'(\eta) = 0, \theta(\eta) = 0, \varphi(\eta) = 0, \text{as } \eta \to \infty,
\end{align*}
\]

where \( \alpha_T, N, \beta, \epsilon & \epsilon_2, \text{Le & Pr, M}\), \( \text{Nd & Nr, Rd, Re}, \text{Sc, S_T & } \theta_w \) are thermal Biot number, buoyancy ratio parameter, Deborah number, first & second order slip velocity parameter, Lewis & Prandtl number, Hartmann number, Brownian motion and thermophoresis parameters, non-linear thermal radiation, local Reynolds number, Richardson number, nanoparticle volume fraction stratification parameter, thermal stratification parameter and temperature ratio parameter are defined as
The skin-friction co-efficient & local Nusselt number are

\[ Re_x^{1/2}C_{f, x} = \left( \frac{2}{1 + \lambda_1} \right) (1 + \beta) s''(0), \]

\[ Re_x^{-1/2}Nu = -\left( 1 + \frac{4}{3} Rd(\theta_w)^3 \right) \theta'(0). \]

3. **Method of the solution**

The equations (7) to (10) are solved using HAM by choosing the initial approximation and auxiliary linear operators (refer Jagan et al. [7]) as

\[ s_0(\eta) = \eta e^{-\eta} + \left( \frac{3 \epsilon_2 - 2 \epsilon}{\epsilon_2 - 1 - \epsilon} \right) e^{-\eta} - \left( \frac{3 \epsilon_2 - 2 \epsilon}{\epsilon_2 - 1 - \epsilon} \right), \]

\[ \theta_0(\eta) = -\alpha T \left[ S_T \eta - \left( \frac{1}{\alpha T + 1} \right) \exp(-\eta) \right], \varphi_0(\eta) = -(S_C - 1) \exp(-\eta). \]

\[ L_s(s) = \frac{d^3s}{d\eta^3} - \frac{d^2s}{d\eta^2}, L_\theta(\theta) = \frac{d^2\theta}{d\eta^2} - \theta, L_\varphi(\varphi) = \frac{d^2\varphi}{d\eta^2} - \varphi, \]

which satisfies the property

\[ L_s[Q_4 + Q_2 \exp(-\eta) + Q_3 \exp(\eta)] = 0, \]

\[ L_\theta[Q_4 \exp(-\eta) + Q_5 \exp(\eta)] = 0, \]

\[ L_\varphi[Q_6 \exp(-\eta) + Q_7 \exp(\eta)] = 0 \]

where \( Q_1, Q_2, ..., Q_7 \) are the arbitrary constants. The h-curve is plotted for \( \lambda_1 = 0.3, \beta = 0.2, M = 0.7, \)

\( Ri = 0.3, N = 0.3, \Pr = 1, S_T = 0.1, S_C = 0.2, \theta_w = 0.1, \alpha_T = 0.1, \epsilon = 0.2 \) and \( \epsilon_2 = 0.3 \). The range for admissible values of \( h_s, h_\theta \) and \( h_\varphi \) are \(-1.4 \leq h_s \leq -0.2, -1.4 \leq h_\theta \leq 0 \) and \(-1.2 \leq h_\varphi \leq -0.1 \) (see Figure 1).

![h-curve graph](image)

4. **Results and Discussion**

Here the pertinent parameters are discussed with their different combinations which are involved in this study. From Figure 2, while increasing \( \epsilon \), the velocity profile gets decreased. It is also evident that the velocity profile enlarges with raise in second order slip parameter \( (\epsilon_2) \) for \( 0 \leq \eta < 1.675 \) whereas it decreases with increase in \( \epsilon_2 \) for \( 1.675 < \eta \leq 5 \), but, it remains same \((0.12)\) for all the values of \( \epsilon_2 \) at \( \eta = 1.675 \). The velocity profile increases with increase in ratio of relaxation to retardation time \( (\lambda_1) \)
for $0 \leq \eta < 0.7558$ whereas it decreases with increase in $\varepsilon_2$ for $0.7558 < \eta \leq 5$. But it remains same $(0.50)$ for all the values of $\lambda_2$ at $\eta = 0.7558$. Also the velocity profile diminishes first and then rises with enlargement in $\beta$. In Figure 3, the temperature profile enlarge with rise in radiation parameter $(R_d)$ and thermal Biot number $(\alpha_T)$ but decreases with increase in $S_T$. The concentration profile diminishes with rise in $S_C$. From Figure 4, the skin friction co-efficient enlarge with rise in $\varepsilon$ and $\beta$ whereas it diminishes with increase in $\varepsilon_2$ and $M$. It is concluded from Figure 5 that when $\varepsilon_2$ and $\alpha_T$ are increased, the local Nusselt number increases. Also the local Nusselt number diminishes with rise in $\varepsilon$ and $R_d$.

5. Conclusion
The study of mixed convection MHD Jeffrey nanofluid flow with non-linear radiation and second order slip with thermal and solutal stratification under convective boundary condition is presented. The thickness of thermal boundary layer enlarges with rise in thermal radiation parameter. When thermal radiation parameter value is raised, the heat transfer rate diminishes where as local mass transfer rate enlarges. The skin friction enhances (diminishes) when first (second) order slip velocity is raised. While rising the second (first) order slip velocity, the heat & mass transfer rate enhances (decreases). The thickness of thermal boundary layer and local Nusselt number enlarges with rise in thermal Biot number. When the thermal (solutal) stratification is increased, the thickness of thermal (solutal) boundary layer diminishes.

References
[1] Eswaramoorthi S, Bhuvaneswari M, Sivasankaran S and Rajan S 2015 Procedia Engineering 127 916-923.
[2] Rashad A.M, Abbasbandy S and Chamkha A.J 2014 Journal of the Taiwan Institute of Chemical Engineers 45 2163-2169.
[3] Hayat T, Imtiaz M and Alsaedi A 2016 International Journal of Heat and Mass Transfer 92 100-109.
[4] Zhu J, Zheng L and Zhang X 2015 Applied Mathematics and Mechanics 36 1131-1146.
[5] Das K, Acharya N and Kundu P.K 2015 Alexandria Engineering Journal 54 815-821.
[6] Abbasi F.M, Shehzad S.A, Hayat T and Alhuthali M.S 2016 Journal of Hydrodynamics, Ser. B 28 840-849.
[7] Jagan K, Sivasankaran S, Bhuvaneswari M and Rajan S 2017 International Journal of Pure and Applied Mathematics 117(13) 43-51.
Figure 2: Influence of $\epsilon$, $\epsilon_2$, $\lambda_1$ and $\beta$ on $s'(\eta)$

Figure 3: Influence of $R_d$, $S_T$ and $\alpha_T$ on $\theta(\eta)$ and $S_c$ on $\varphi(\eta)$
Figure 4: Influence of $\epsilon$, $\epsilon_2$, $\beta$ and $M$ on $Re_x^{1/2}C_f$.

Figure 5: Influence of $\epsilon$, $\epsilon_2$, $R_d$ and $\alpha_T$ on $Re_x^{1/2}Nu$. 