Analysis of historical data leveraging the sandpile model of self-organized criticality demonstrates the efficacy of prescribed burns in reducing risk of destructive wildfires

Running head: Prescribed burns reduce risk of extreme wildfire

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Abstract

Prescribed burns have been increasingly administered to forest management with the assumption that they help reduce the risk of wildfires; however, this hypothesis has yet to be rigorously tested. We leverage the historical data of forest fires from multiple states to show that the sandpile model of self-organized criticality accurately represents the real-world incidence of fire by describing a negative linear relationship between the logarithm of fire size and the logarithm of the fire incidence number of that size. We then investigate the association between the size of prescribed burn and the slope of the negative linear relationship which represents the relative risk of destructive wildfires. The results demonstrate that increases in the area subject to prescribed burning generally reduce the risk of destructive wildfires. This is consistent with the Florida data, which shows a trend in reduction of destructive wildfires as prescribed burns have been progressively introduced to forest management. Our study justifies the application of the sandpile model to wildfire research and establishes a novel method of the analysis of slope estimated from the sandpile model for facilitating the investigation of potential risk factors of destructive wildfires and the development of an optimal strategy for prescribed burning.

Brief summary

Analysis of historical forest fire data leveraging the sandpile model demonstrates a negative association between the logarithm of fire size and the logarithm of fire incidence number of that size. An increase in the area subject to prescribed burning generally reduces the risk of destructive wildfires.

Introduction
Wildfire is of increasing risk to people and landscapes as anthropogenic climate change progresses (IPCC 2018). The 2018 Camp Fire in Northern California was the largest and most destructive fire in California history, burning 153,336 acres (620.5 km$^2$) and responsible for eighty-five fatalities (Cal Fire 2019). Even if global warming can be limited to just 1.5°C above preindustrial levels, the incidence and intensity of wildfires is expected to increase dramatically (Settele et al. 2015).

While wildfires can be deadly and destructive, fire also plays a major role in promoting ecological diversity and maintaining sustainability in bio-systems. For example, regular fires result in the increased diversity of bat species (Steel et al. 2019), and some plant species require fire treatment of their seeds in order to germinate (Keeley 1987). Efforts to prevent as many fires as possible over the past century by federal and state agencies in the United States have resulted in unhealthy buildups of brush and dead wood littering forest floors, promoting more intense and destructive fires (Haugo et al. 2019; Roos et al. 2020). Prescribed fires, as defined by the National Park Service (2020), are intentionally set, low intensity fires, which are used to manage land. The main function of prescribed burning is to reduce the ground litter, which serves as fuel driving the most destructive wildfires. There is evidence to show that regularly prescribed burning of forests may be highly beneficial by, among other effects, maintaining critical species habitat (Russell et al. 1999). However, the hypothesis that prescribed burns can significantly reduce the risk of destructive wildfires has not been rigorously tested using mathematical models based on historical data.

The sandpile model of self-organized criticality was introduced by Bak et al (1987). The model consists of an imagined plane organized as a grid. In each site of the grid, grains of sand can be placed. Once the number of grains passes a predefined threshold, the pile in the site
collapses, transferring its collected grains to neighboring sites. If any of those sites then reach the threshold, the piles in the adjacent sites collapse and transfer their collected grains to their respective neighboring sites. The pile’s collapse can propagate across the plane until the plane reaches equilibrium. Testing this model has shown that the size of collapses follows a negative $\log_{10}$ linear pattern, with a collapse involving more sites occurring much less frequently than smaller collapses (Bak et al. 1987; Cajueiro and Andrade 2010). In examining this effect, it has been found that numerous real-world phenomena follow this same pattern, including earthquake power (Bak and Tang 1989), solar x-ray flares (Crosby et al. 1993), and even evolutionary rates (Sneppen et al. 1995). Of particular relevance, the sandpile model has been applied to wildfire size, measured by area affected (Ricotta et al. 1999). However, to date, no publication has quantified the effect of prescribed fire on the incidence of wildfires.

Ishii et al. (2002) showed that agent choice had a significant impact on the size of the avalanches in counterintuitive ways. When the agent’s intention is to create a large avalanche, the average avalanche size is decreased relative to the random deposition. On the other hand, if the agent’s intention is to cause the smallest possible avalanche, the result is a major increase in the chances of triggering exceptionally large avalanches. The relevance between the sandpile model and forest fires is that the buildup of fuel functions like the buildup of sand, releasing in critical events. To expand the conceptual model, the setting of prescribed fires can be considered analogous to an agent’s intention to cause a collapse of the sandpile. Here, we leverage historical data of forest fires from multiple states to show that the sandpile model accurately represents the real-world incidence of fire and demonstrate that prescribed burns can significantly reduce the incidence of especially destructive wildfires.
Materials and methods

Fire data

We used fire data from California, Georgia, and Florida. Fire data from California was available from CalFire (https://www.fire.ca.gov/stats-events/). Fire data from Georgia was provided by Fire Chief Frank Sorrells of the Georgia Forestry Commission (personal communication October 21, 2020). Fire data from Florida was available from the Florida Forest Service (http://fireinfo.fdacs.gov/fmis.publicreports), with additional data provided by the Prescribed Fire Manager of the Florida Forest Service, John Saddler (personal communication, December 18, 2020). The fire incidence data from each state was divided for each year into seven class sizes, A-G, as defined by the federal government. We did not use Class A fires (<0.25 acres [<0.001 km²]) in the analysis as they are heavily underreported (Ricotta et al 1999) and so the numbers associated with this class in the data severely underestimates their true incidence. The maximum fire size in each class was used to represent that class for the subsequent analysis. Classes B, C, D, E, and F were considered to be 9.9, 99, 299, 999 and 4,999 acres, respectively (0.04, 0.40, 1.21, 4.04. and 20.2 km², respectively). Class G, which included wildfires greater than 5,000 acres (20.2 km²), was set to be 10,000 acres (40.47 km²) for ease of analysis. The data available from Georgia and Florida included the number of acres subjected to prescribed burning each year, while no prescribed fire data was available for California due to a lack of reporting regulations in the state (Melvin 2018). Prescribed and wildfire data was available for Georgia from 1995 to 2019, inclusive. Prescribed and wildfire data was available for Florida from 1993 to 2019, inclusive. Wildfire data for California was available from 2008 to 2018, inclusive.

Log₁₀ transformation based on sandpile model of self-organized criticality
For Florida and Georgia, we divided yearly data into quintiles based on the number of acres subject to prescribed burning that year. Most of our results and conclusions are based on the Florida data, since it is the best documented and managed. The breakdown of the Florida data following the aforementioned data grouping strategy is shown in Table 1. As no prescribed burn data was available for California, we compared data across all eleven years of available data, divided between the two Geographic Area Coordination Centers, Northern California and Southern California, defined by the National Interagency Fire Center.

Based on the theorem described in the sandpile model of self-organized criticality by Bak et al. (1987), we took the log_{10} of the average number of fires in each class for each category and plotted them against the log_{10} of the maximum fire size in acres in that class for that category. For each plot we calculated the slope of the fitted regression line as determined by the sum of least squares, which represents the relative risk for destructive fires. The estimated values and standard errors of these slopes were analyzed for model comparisons and for hypothesis testing.

*Comparison of data with and without prescribed fires*

The Florida data consists of data without the record of prescribed fires (1981-1992) and data with the record of prescribed fires (1993-2019). To compare subsets of fire incidence data in Florida, we split the data with the record of prescribed fires into first (1993-2005) and second (2006-2019) halves, yielding three consecutive periods for comparisons, i.e., period I (1981-1992), period II (1993-2005), and period III (2006-2019). We then plotted the data from each subset in a box plot for visual examination (Fig. 1).
We used two-sample $t$ test or analysis of variance (ANOVA) to compare the data with and without the record of prescribed fires based on Florida wildfire data. The nominal $p < 0.05$ was used to claim a significant difference between any comparison.

**Analysis of slopes**

We conducted the pairwise comparison of slopes of the fitted regression lines for various categories using the pooled $t$-test. Suppose we would like to test whether two slopes are identical, i.e., $H_0: b_1 = b_2$ vs. $H_a: b_1 \neq b_2$, where $b_1$ and $b_2$ are slopes estimated from two linear regression models, respectively. Under the null hypothesis, the $t$-test statistic

$$t = \frac{b_1 - b_2}{\sqrt{s_{b_1}^2 - s_{b_2}^2}}$$

follows a $t$ distribution with degree of freedom $n_1 + n_2 - 4$, where $s_{b_1}$ and $s_{b_2}$ represent the standard errors for two slopes and $n_1$ and $n_2$ are the sample sizes for two models, respectively. Since the sample sizes were very small in this study, we used $p < 0.1$ as significance level for rejecting a null hypothesis. The R script, named ‘TwoSlope.ttest’, for implementing the comparison of two slopes of fitted regression lines is available in Supplementary File S2.

We used the simple linear regression to analyze the association between the estimated slopes (risk of destructive wildfire) and the average acres per year that were subject to prescribed burn in various categories. The significant negative slope of the regression plots indicates that increased prescribed burns will decrease the risk of destructive fires. Again, because of the limited sample size, we used $p < 0.1$ as significance level for the statistical test.

**Results**

*Comparison of data with and without record of prescribed fires*
The Florida data consists of data without the record of prescribed fires (period I, 1981-1992) and data with the record of prescribed fires (1993-2019). We split the data with the record of prescribed fires into first (period II, 1993-2005) and second (period III, 2006-2019) halves, yielding three consecutive periods for comparisons. Fig. 1a indicates that there was a significant increase in the total size of prescribed burn between period II (1998-2008) and period III (2009-2019). We assume that undocumented prescribed fires in period I (1981-1993) were significantly less than period II or III. The comparison of the total areas burnt yearly between these three periods are shown by boxplot in Fig. 1b. Although no significant difference in means was detected among these three periods, it is clear to see a trend in the reduction of giant wildfires as the prescribed burns have been progressively introduced to forest management.

Calculating slopes based on sandpile model

For the log\(_{10}\) transformed Florida data, the scatter plots with the best fitted linear regression lines for the five categories are shown in Fig. 2a-e. The intersects, the slopes estimated for these five categories, their standard errors, and p-values are shown in Table 2. The slopes of the line of best fit in all quartiles of the Georgia data was about -1.42 with an average p-value of 0.0066 (Sup. Fig. 1). The slope of the line of best fit for the Northern California data was about -0.73 with an average p-value of 0.0029 (Sup. Fig. S2a). The slope of the line of best fit for the Southern California data was about -0.75 with an average p-value of 0.00044 (Sup. Fig. S2b).

Analysis of slopes to test association between prescribed burn and wildfire incidence

We carried out a pairwise comparison among slopes of the fitted regression lines for various categories using the log\(_{10}\) transformed Florida data, the results of which are shown in
Table 3. Due to the limited sample size in each category, significant differences (p value < 0.1) have been detected only between quintile 1 with quintile 2 and between quintile 1 with quintile 4. However, the regression of these slopes on the median land areas that were subject to prescribed burn in four categories (quintiles 1, 3, 4 and 5) showed a significantly negative slope (-0.2206 with p = 0.09 shown in Fig. 2f), suggesting that an increased prescribed burn had effectively reduced the risk of destructive wildfires. Note that the slope of the data in quintile 2, labelled as red in Fig. 2f, appeared to be much lower than expected; therefore, it was not included in the regression analysis of the slopes. Possible explanations for this outlier slope associated with the data in quintile 2 were proposed in Discussion. California has no data on prescribed burning and it is believed to have only low levels (Mark Melvin, personal communication, September 14, 2020), and Georgia has consistently high levels.

Discussion

In all examined fire incidence data, Class A fires (<0.25 acres [0.001 km²]) were greatly underrepresented relative to expectations based on the modelling. This is likely because many, if not most, small fires are not reported to statewide agencies and may be put out by landowners or simply run out of fuel and sputter out without intervention.

The slopes of the line of best fit in the log_{10} transforms of the fire incidence show a correlation between an increasing number of acres subjected to prescribed burning and a more negative slope. The slope of the California data, Northern and Southern, is -0.73 and -0.75, respectively. The slope of the Georgia data averages about -1.42. The slope of the quintiles in the Florida data, however, connects the two. The slope of the Florida first quartile is -0.81, the second -1.01, the third -0.92, the fourth -0.97, and the fifth -0.94. This tracks with the increasing use of prescribed burning in Florida over the past twenty-seven years, while use has remained
essentially static in California and Georgia at low and high levels, respectively. From 1993 to 2002, an average of about 1.9 million acres (7689 km$^2$) in Florida were subject to prescribed burning, while from 2010 to 2019 an average of about 2.3 million acres (9308 km$^2$) were subject, an increase of ~20%.

Interestingly, the slope of the log$_{10}$ of fire incidence of the second quintile in Florida is far lower than would be expected based on the trend present in the other data. The slope of the second quintile is -1.01, even more than the fifth quintile, which had a slope of -0.94. It may be that the major dip indicates the presence of an optimal level of prescribed burning. The possible presence of optima requires further study to determine if judicious application of prescribed burning can be more effective, both in terms of cost and effort, than generally increasing the use of prescribed burning.

The overabundance of large fires (>1,000 acres [4.04 km$^2$]) in California, above the level predicted by the negative linear log$_{10}$ slope of the line of best fit, could indicate disruption of the normal cycles of fire. The national fire policy in the United States from the late 1800’s has been fire suppression, leading to significant fire deficits in the 20$^{th}$ century. This fire suppression policy has been demonstrably linked to major increases in fire intensity, especially in the 21$^{st}$ century (Haugo et al. 2019; Roos et al. 2020). The increased incidence of especially destructive fires could be due to buildup of debris in the forest floor. It should be noted that not all the blame for increased fire intensity is due to fire suppression; indeed, global climate change has greatly stressed forests by shifting rainfall patterns, causing prolonged droughts (Stephens et al. 2018), and opportunistic infestations, such as by bark beetles (Preisler et al. 2017), leading to large stands of dead trees, which fuel the more intense fires.
The fire regime in California can be compared to the fire regime in Florida, which has made aggressive use of prescribed burns. The results show that not only does the fire incidence track the expected negative linear log$_{10}$ line, but that in years in which more acreage was subject to prescribed burns the slope of the line of best fit of the log$_{10}$ line is more negative, indicating across the board reductions in larger and more destructive fires.

For the first time, we applied the sandpile model of self-organized criticality to the analysis of historical data to demonstrate that increase of prescribed burns can significantly reduce the risk of destructive wildfires. Our data and conclusions warrant the future research involving the use of this model to develop the optimal strategy for prescribing fires which can achieve the maximum efficacy of suppressing giant wildfires with minimum possible human efforts and operative costs.

**Data Availability**

Complete wildfire and prescribed fire data can be found in Supplementary File S1. The R script, named ‘TwoSlope.ttest’, for implementing the comparison of two slopes of fitted regression lines is available in Supplementary File S2.

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**Conflicts of Interest**
The authors declare no conflicts of interest.

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Table 1. The average number of wildfires of each class across subgroups in Florida defined based on the acreage of prescribed burns. 1 acre = 0.00404 km$^2$.

| Class | Class Size (Max acres) | Category (based on acres of prescribed burn) | Quintile 1 | Quintile 2 | Quintile 3 | Quintile 4 | Quintile 5 |
|-------|-------------------------|---------------------------------------------|------------|------------|------------|------------|------------|
| A     | 0.24                    | 1145.17                                    | 680.00     | 986.60     | 764.80     | 534.00     |
| B     | 9.9                     | 2572.67                                    | 1621.60    | 2381.20    | 1964.40    | 1392.50    |
| C     | 99                      | 795.83                                     | 449.60     | 734.40     | 592.80     | 357.50     |
| D     | 299                     | 109.50                                     | 46.80      | 97.20      | 73.80      | 36.83      |
| E     | 999                     | 61.83                                      | 20.40      | 41.60      | 30.60      | 17.33      |
| F     | 4999                    | 23.17                                      | 7.20       | 13.80      | 10.20      | 5.33       |
| G     | 10000                   | 10.50                                      | 1.20       | 4.40       | 2.20       | 2.33       |

Table 2. Estimation of slopes and relevant metrics for quintiles of Florida wildfire data.

| Category   | Estimate of Slope | Standard Error | P value | $R^2$ |
|------------|-------------------|----------------|---------|-------|
| Quintile 1 | -0.8060           | 0.0714         | 0.00035 | 0.9696|
| Quintile 2 | -1.0139           | 0.1034         | 0.00061 | 0.9601|
| Quintile 3 | -0.9152           | 0.0769         | 0.00029 | 0.9726|
| Quintile 4 | -0.9687           | 0.0904         | 0.00043 | 0.9663|
| Quintile 5 | -0.9448           | 0.0803         | 0.00030 | 0.9719|

Table 3. The pairwise comparison of slopes among five categories in Florida data.
The diagonal entries (shaded cells) are the estimated slopes; the upper triangle show the p values for the pairwise comparisons; the ‘*’ and ‘-’ in the lower triangle represent significant results and insignificant results, respectively based on p value < 0.1.

| Category   | Quintile 1 | Quintile 2 | Quintile 3 | Quintile 4 | Quintile 5 |
|------------|------------|------------|------------|------------|------------|
| Quintile 1 | -0.8060    | 0.0683     | 0.1641     | 0.0978     | 0.1163     |
| Quintile 2 | *          | -1.0139    | 0.2329     | 0.3754     | 0.3061     |
| Quintile 3 | -          | -          | -0.9152    | 0.3321     | 0.3984     |
| Quintile 4 | *          | -          | -          | -0.9687    | 0.4241     |
| Quintile 5 | -          | -          | -          | -          | -0.9448    |
Figure 1. Comparison of Florida wildfire data between various periods. a: The areas of prescribed fires in period III (2006-2019) were significantly greater than those in period II (1993-2005), with $p$ value = 0.0007. b: No significant difference in means were detected between three periods ($p$ values $> 0.05$). There is a trend in the reduction of destructive wildfires (extreme values along y axis) along three consecutive periods. 1 acre = 0.00404 km$^2$. 
Figure 2: Analysis of slopes based on Florida wildfire data. **a-e**: Fitted regression models for the data in quintiles, i.e., quintiles 1 through 5. **f**: Fitted regression model for the slopes vs. the median sizes of prescribed burns in four out of five quintiles, with quintile 2 (labelled with red) being excluded.