Hierarchical Neutrino Masses and Leptogenesis  
in Type I+II Seesaw Models

Debasish Borah* and Mrinal Kumar Das†

Department of Physics, Tezpur University, Tezpur-784028, India

Abstract

The baryon to photon ratio in the present Universe is very accurately measured to be $6.19 \times 10^{-10}$. We study the possible origin of this baryon asymmetry in the neutrino sector through the generic mechanism of baryogenesis through leptogenesis. We consider both type I and type II seesaw origin of neutrino masses within the framework of left right symmetric models (LRSM). Using the latest best fit global neutrino oscillation data and assuming the Dirac neutrino mass matrix to be either charged lepton (CL) or up quark (UQ) type, we compute the predictions for baryon to photon ratio keeping the lightest active neutrino mass eigenstate a free parameter for both normal and inverted hierarchical cases. We show that in inverted hierarchical scenario with type I seesaw, observed baryon asymmetry can not be generated for both CL and UQ type Dirac neutrino mass matrices. We also study the predictions for baryon asymmetry when the neutrino masses arise from a combination of both type I and type II seesaw (with one of them dominant at a time) as well as different combinations of Majorana neutrino phases and show that the observed baryon asymmetry can be generated within these models up to certain exceptions.

PACS numbers: 12.60.-i, 12.60.Cn, 14.60.Pq

*Electronic address: dborah@tezu.ernet.in
†Electronic address: mkdas@tezu.ernet.in
I. INTRODUCTION

Recent neutrino oscillation experiments have provided significant amount of evidence which confirms the existence of the non-zero yet tiny neutrino masses [1]. We know that the smallness of three Standard Model neutrino masses can be naturally explained via seesaw mechanism. In general, such seesaw mechanism can be of three types: type I [2], type II [3] and type III [4]. All these mechanisms involve the inclusion of additional fermionic or scalar fields to generate tiny neutrino masses at tree level. Although these seesaw models can naturally explain the smallness of neutrino mass compared to the electroweak scale, we are still far away from understanding the origin of neutrino mass hierarchies as suggested by experiments. Recent neutrino oscillation experiments T2K [5], Double ChooZ [6], Daya-Bay [7] and RENO [8] have not only made the earlier predictions for neutrino parameters more precise, but also predicted non-zero value of the reactor mixing angle $\theta_{13}$. The latest global fit value for 3σ range of neutrino oscillation parameters [9] are as follows:

$$\Delta m_{21}^2 = (7.00 - 8.09) \times 10^{-5} \text{ eV}^2$$

$$\Delta m_{31}^2 \text{ (NH)} = (2.27 - 2.69) \times 10^{-3} \text{ eV}^2$$

$$\Delta m_{23}^2 \text{ (IH)} = (2.24 - 2.65) \times 10^{-3} \text{ eV}^2$$

$$\sin^2 \theta_{12} = 0.27 - 0.34$$

$$\sin^2 \theta_{23} = 0.34 - 0.67$$

$$\sin^2 \theta_{13} = 0.016 - 0.030$$

$$\delta_{CP} = 0 - 2\pi$$

(1)

where NH and IH refers to normal and inverted hierarchy respectively.

The above recent data have positive evidence for non-zero $\theta_{13}$ as well, which was earlier thought to be zero or negligibly small. This has led to a great deal of activities in neutrino model building [10, 11]. These frameworks which predict non-zero $\theta_{13}$ may also shed light on the Dirac CP violating phase which is still unknown (and could have remained unknown if $\theta_{13}$ were exactly zero). Apart from predicting the correct neutrino oscillation data as well as the Dirac CP phase, the nature of neutrino mass hierarchy is also an important yet unresolved issue. Understanding the correct nature of hierarchy can also have non-trivial
relevance in the production of matter antimatter asymmetry (or baryon asymmetry) and other relevant cosmology issues. The observed baryon asymmetry in the Universe is encoded in the baryon to photon ratio measured by dedicated cosmology experiments like Wilkinson Mass Anisotropy Probe (WMAP) among others. The baryon to photon ratio in nine year WMAP data [12] is found to be

\[ Y_B \simeq (6.19 \pm 0.14) \times 10^{-10} \]  

(2)

Leptogenesis is a novel mechanism to generate this observed baryon asymmetry in the Universe by generating an asymmetry in the lepton sector first which is later converted into baryon asymmetry through electroweak sphaleron transitions [13]. As pointed out by Fukugita and Yanagida [14], the out of equilibrium and CP violating decay of heavy Majorana neutrinos provides a natural way to create the required lepton asymmetry. The novel feature of this mechanism is the way it relates two of the most widely problems in physics: the origin of neutrino mass and the origin of matter-antimatter asymmetry. This idea has been implemented in several interesting models in the literature [15, 16].

In view of above, the present work is planned to carry out a study of baryogenesis through leptogenesis in neutrino mass models with normal and inverted hierarchical neutrino masses within the framework of generic left-right symmetric models (LRSM) [17]. Such a work was done recently in [18] where the viability of several models were studied by fitting their predictions to the best fit neutrino oscillation data in the presence of both type I and type II seesaw. In this present work, we scan the parameter space of the same sets of models by calculating the baryon asymmetry and comparing with the experimental data. We parametrize the neutrino mass matrix using global fit neutrino oscillation data. We then keep the lightest active neutrino mass eigenstate as a free parameter and find out the allowed range of its mass for which correct baryon asymmetry can be generated. This calculation is performed by considering either type I or type II seesaw at a time and taking the Dirac neutrino mass matrix to be either CL type or UQ type. We then consider the contribution of both type I and type II seesaw terms to the neutrino mass with one of them dominating at a time. The dominating seesaw term is then used to find out the right-handed Majorana neutrino mass matrix by assuming two different types of Dirac neutrino mass matrices (CL and UQ type). Fixing the dominating seesaw term in this way, the other seesaw term is allowed to vary and prediction for baryon asymmetry is calculated for different combinations.
of Majorana neutrino phases $\pi/6, \pi/4, \pi/3, \pi/2$. We note that the variation of Majorana phases and the strength of the non-leading seesaw term not only changes the predictions for baryon to photon ratio but also leads to a sign flip (which would correspond to an antimatter dominated Universe) in some cases.

This paper is organized as follows: in section II we discuss the methodology of type I and type II seesaw mechanism in generic LRSM. In section III, we outline the details of computing baryon asymmetry in LRSM. In section IV we discuss our numerical analysis and results. We then finally conclude in section V.

II. SEESAW IN LRSM

Type I seesaw framework is the simplest possible seesaw mechanism and can arise in simple extensions of the standard model by three right handed neutrinos. There is also another type of non-canonical seesaw formula (known as type-II seesaw formula)\cite{3} where a left-handed Higgs triplet $\Delta_L$ picks up a vacuum expectation value (vev). This is possible both in the minimal extension of the standard model by $\Delta_L$ or in other well motivated extensions like left-right symmetric models (LRSM) \cite{17}. The seesaw formula in LRSM can be written as

$$m_{LL} = m_{LL}^{II} + m_{LL}^{I}$$  \hspace{1cm} (3)

where the usual type I seesaw formula is given by the expression,

$$m_{LL}^{I} = -m_{LR}M_{RR}^{-1}m_{LR}^{T}.$$  \hspace{1cm} (4)

Here $m_{LR}$ is the Dirac neutrino mass matrix. The above seesaw formula with both type I and type II contributions can naturally arise in extension of standard model with three right handed neutrinos and one copy of $\Delta_L$. However, we will use this formula in the framework of LRSM where $M_{RR}$ arises naturally as a result of parity breaking at high energy and both the type I and type II terms can be written in terms of $M_{RR}$. In LRSM with Higgs triplets, $M_{RR}$ can be expressed as $M_{RR} = v_Rf_R$ with $v_R$ being the vev of the right handed triplet Higgs field $\Delta_R$ imparting Majorana masses to the right-handed neutrinos and $f_R$ is the corresponding Yukawa coupling. The first term $m_{LL}^{II}$ in equation (3) is due to the vev of $SU(2)_L$ Higgs triplet. Thus, $m_{LL}^{II} = f_Lv_L$ and $M_{RR} = f_Rv_R$, where $v_{L,R}$ denote the vev’s and $f_{L,R}$ are symmetric $3 \times 3$ matrices. The left-right symmetry demands $f_R = f_L = f$. The
induced vev for the left-handed triplet $v_L$ can be shown for generic LRSM to be

$$v_L = \frac{\gamma M_W^2}{v_R}$$

with $M_W \sim 80.4$ GeV being the weak boson mass such that

$$|v_L| << M_W << |v_R|$$

In general $\gamma$ is a function of various couplings in the scalar potential of generic LRSM and without any fine tuning $\gamma$ is expected to be of the order unity ($\gamma \sim 1$). type-II seesaw formula in equation (3) can now be expressed as

$$m_{LL} = \gamma (M_W/v_R)^2 M_{RR} - m_{LR} M_{RR}^{-1} m_{LR}^T$$  \hspace{1cm} (5)$$

With above seesaw formula (5), the neutrino mass matrices are constructed by considering contributions from both type I and type II terms. The choice of $v_R$ however, remains ambiguous in the literature where different choices of $v_R$ are made according to convenience [16, 19-22]. However, in this present work we will always take $v_R$ as $v_R = \gamma \frac{M_W}{v_L} \simeq \gamma \times 10^{15}$ GeV [16, 22]. It is worth mentioning that, here $SU(2)_R \times U(1)_{B-L}$ gauge symmetry breaking scale (as in generic LRSM) $v_R$ is the same as the scale of parity breaking [19]. Using this form of $v_R$, the seesaw formula (5) becomes

$$m_{LL} = \gamma \left(\frac{M_W}{\gamma \times 10^{15}}\right)^2 M_{RR} - m_{LR} M_{RR}^{-1} m_{LR}^T$$  \hspace{1cm} (6)$$

Quantitatively, either of the two terms on the right hand side of equation (6) can be dominant or both the terms can be equally dominant. However, for generic choices of symmetry breaking scales (mentioned above) as well as the Dirac neutrino mass matrices (generically to be of same order as corresponding charged lepton masses), both type I and type II term can be equally dominant only when the dimensionless parameter $\gamma$ is fine tuned to be very small. We check this by equating both the terms to the best fit value of $m_{LL}$. We skip such a fine-tuned case here and consider two other possible cases in our work: one in which type I term dominates whereas type II term is present as a small perturbation and the other in which type II term dominates with type I term as a small perturbation.

A. Case I: Dominating Type I Seesaw

In this case, the second term in the equation (6) gives the leading contribution to $m_{LL}$ and hence we compute the right-handed neutrino mass matrix $M_{RR}$ by using the inverse
type I seesaw formula $M_{RR} = m_{LR}^T m_{LL}^{-1} m_{LR}$ where we use the best fit $m_{LL}$ and generic $m_{LR}$ as will be shown in the section IV. Here we hold $M_{RR}$ fixed, so the first term in equation (6) is dependent on the value of $\gamma$ while second term is fixed. For $\gamma \sim 1$, the first term has minimum contribution whereas for smaller values of $\gamma$, the contribution of the type II term will increase. We vary the dimensionless parameter $\gamma$ from 0.0001 to 1.0 and check the survivability of neutrino mass models with contributions from type I and type II terms.

B. Case II: Dominating Type II seesaw

This is the case where the first term in the equation (6) gives the leading contribution to $m_{LL}$. This scenario within grand unified models like $SO(10)$ have been discussed in [23]. In this case, we compute $M_{RR}/\gamma$ from the inverse type II seesaw formula

$$\frac{M_{RR}}{\gamma} = \left( \frac{1 \times 10^{15}}{M_W} \right)^2 m_{LL}$$

(7)

It should be noted that here we are keeping $M_{RR}/\gamma$ constant instead of just $M_{RR}$ as in the case I. Thus, although the first term in equation (6) remains constant, the second term varies as we vary $\gamma$ and have the minimal contribution for $\gamma \sim 1$. Similar to the case I, here also we vary $\gamma$ between 0.0001 and 1.0 and calculate the predictions for baryon asymmetry. The details of both these case will be presented in details in section IV

III. LEPTOGENESIS IN LRSM

Study of Leptogenesis in LRSM has received lots of attention in last two decades and a large number of works dedicated to this topic can be found in the literature. Here we closely follow the notations of [16] to carry out our analysis.

For the calculation of baryon asymmetry, we go to the basis where the right handed Majorana neutrino mass matrix is diagonal

$$U^*_R M_{RR} U^+_R = \text{diag}(M_1, M_2, M_3)$$

(8)

In this diagonal $M_{RR}$ basis, the Dirac neutrino mass matrix also changes to

$$m_{LR} = m_{LR}^d U_R$$

(9)
FIG. 1: Predictions for baryon to photon ratio as a function of lightest active neutrino mass eigenstate (in eV) for both CL and UQ type $m_{LR}$ with normal hierarchy where $m_{LR}^d$ is the assumed choice of the Dirac neutrino mass matrix in our calculation which is either CL or UQ type diagonal matrix. The lepton asymmetry arising from the decays of the lightest right handed Majorana neutrino $N_1 \rightarrow \Phi L^c$ and $N_1 \rightarrow \Phi^* L^*$:

\[
\epsilon = \frac{\Gamma(N_1 \rightarrow \Phi L^c) - \Gamma(N_1 \rightarrow \Phi^* L)}{\Gamma(N_1 \rightarrow \Phi L^c) + \Gamma(N_1 \rightarrow \Phi^* L)} = \frac{1}{8\pi v^2 (m_{LR}^d m_{LR})_{11}} \sum_{j=2,3} \text{Im}(m_{LR}^d m_{LR})_{1j}^2 f(M_j^2/M_1^2) \tag{10}
\]

where $v = 174$ GeV is the vev of the Higgs bidoublets responsible for breaking the electroweak symmetry and the function $f(x) = -\frac{3}{2\sqrt{x}}$ for $x \gg 1$. After determining the lepton asymmetry $\epsilon$, the corresponding baryon asymmetry can be obtained by

\[
Y_B = \frac{c \epsilon}{g_*} \tag{11}
\]

through electroweak sphaleron processes [13]. Here the factor $c$ is measure of the fraction
FIG. 2: Predictions for baryon to photon ratio as a function of lightest active neutrino mass eigenstate (in eV) for both CL and UQ type $m_{\text{LR}}$ with inverted hierarchy of lepton asymmetry being converted into baryon asymmetry and is approximately equal to $-0.55$. $\kappa$ is the dilution factor due to wash-out process which erase the produced asymmetry and can be parametrized as \([24]\)

\[
-\kappa \simeq \sqrt{0.1} K \exp[-4/(3(0.1K)^{0.25})], \quad \text{for } K \geq 10^6
\]

\[
\simeq \frac{0.3}{K (\ln K)^{0.6}}, \quad \text{for } 10 \leq K \leq 10^6
\]

\[
\simeq \frac{1}{2\sqrt{K^2 + 9}}, \quad \text{for } 0 \leq K \leq 10.
\]  

(12)

where $K$ is given as

\[
K = \frac{\Gamma_1}{H(T = M_1)} = \frac{(m_{\text{LR}}^\dagger m_{\text{LR}})M_1}{1.66\sqrt{g_*M_1^2}}
\]

Here $\Gamma_1$ is the decay width of $N_1$ and $H(T = M_1)$ is the Hubble constant at temperature $T = M_1$. The factor $g_*$ is the effective number of relativistic degrees of freedom at $T = M_1$. 

\[\text{FIG. 2: Predictions for baryon to photon ratio as a function of lightest active neutrino mass eigenstate (in eV) for both CL and UQ type } m_{\text{LR}} \text{ with inverted hierarchy.}\]
and is approximately 110. It should be noted that the above estimate for baryon asymmetry is valid only in the absence of wash-out effects responsible for erasing the asymmetry created. In left right symmetric models, right handed neutrinos also have $SU(2)_R$ gauge interactions and hence can give rise to sizable wash-out effects. However, in our present work we neglect these wash-out effects assuming tiny $SU(2)_R$ gauge coupling ($g_R \ll 1$). We leave a detailed analysis of wash-out effects for order one gauge coupling to future investigations.

FIG. 3: Variation of baryon to photon ratio with $\gamma$ and Majorana phases for CL type $m_{LR}$ and dominant type I seesaw with normal hierarchy
FIG. 4: Variation of baryon to photon ratio with $\gamma$ and Majorana phases for CL type $m_{LR}$ and dominant type II seesaw with normal hierarchy

IV. NUMERICAL ANALYSIS AND RESULTS

For the purpose of numerical analysis we parametrize the neutrino mixing matrix as

$$U_L = \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix} \text{diag}(1, e^{i\alpha}, e^{i(\beta+\delta)})$$  (13)

where $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$. $\delta$ is the Dirac CP phase and $\alpha, \beta$ are the Majorana phases. Since we know only the two mass squared differences for the active neutrinos, we consider a hierarchical pattern of neutrino masses where the heavier mass eigenstates are of the order of the mass squared differences, whereas the the lightest one can be much lighter than the other two. We take the best fit central values of the mixing angles and take the Dirac CP phase to be $\pi$ [25].
FIG. 5: Variation of baryon to photon ratio with $\gamma$ and Majorana phases for CL type $m_{LR}$ and dominant type II seesaw with inverted hierarchy.

After parameterizing the neutrino mass matrix using oscillation data, we consider the two cases I and II mentioned in the previous section one by one. First, we consider the case I i.e., type I dominance and calculate the right-handed Majorana neutrino matrix $M_{RR}$ using the inverse type I seesaw formula

$$M_{RR} = m_{RR}^T m_{LL}^{-1} m_{LR},$$

To calculate the $M_{RR}$ for each case, we need to have the Dirac neutrino mass matrix ($m_{LR}$). We take the Dirac neutrino mass matrix $m_{LR}$ to be diagonal with either charged lepton mass structure or up quark mass structure. The general form of Dirac neutrino mass matrix is

$$m_{LR}^{(0)} = \begin{pmatrix} \lambda^n & 0 & 0 \\ 0 & \lambda^n & 0 \\ 0 & 0 & 1 \end{pmatrix} m_f$$
where \( m_f \) corresponds to \( m_\tau \tan \beta \) for \((m,n) = (6,2)\), \( \tan \beta = 40 \) in case of charged lepton and \( m_t \) for \((m,n) = (8,4)\) in the case of up-quarks [26, 27]. \( \lambda = 0.22 \) is the standard Wolfenstein parameter. Now, using \( m_{LR} \) in the inverse type I seesaw formula above, we calculate \( M_{RR} \). For case II that is type II seesaw dominating scenario, we calculate \( M_{RR} \) by using the type II seesaw formula for \( \gamma = 1.0 \). We then diagonalize \( M_{RR} \) as shown in (8) for the calculation of baryon asymmetry. The diagonalizing matrix of \( M_{RR} \) effectively makes \( m_{LR} \) non-diagonal as can be seen from (9). Thus, we are studying a model with diagonal \( M_{RR} \) and non-diagonal \( m_{LR} \). This kind of specific Yukawa structure can be naturally explained by using additional flavor symmetries, either global or gauged. A possible abelian gauge symmetry that can constrain the Dirac neutrino mass matrix to the diagonal form was outlined recently in one of our works (first reference in [18]). Similar approach can be followed to constrain the right handed neutrino mass matrix to the diagonal form while keeping the Dirac neutrino mass matrix in its general non-diagonal form.

After calculating \( M_{RR} \) for both type I and type II seesaw dominating cases, we calculate the predictions for baryon asymmetry by varying the lightest active neutrino mass eigenstates for both types of neutrino mass hierarchies (Normal and Inverted) as well as both types of Dirac neutrino mass matrices (CL and UQ). The results are shown in figure 1 and 2. We notice that, for the entire range of lightest active neutrino mass eigenstate we scan through, inverted hierarchy with type I seesaw domination do not give rise to the observed baryon asymmetry for both CL and UQ type Dirac neutrino mass matrices. Whereas, for normal hierarchy with either type I or type II seesaw as well as inverted hierarchy with type II seesaw are found to produce the observed baryon asymmetry for certain ranges of lightest active neutrino masses as seen from figure 1 and 2.

After calculating the baryon asymmetry for type I seesaw and type II seesaw separately, we consider the contribution of both type I and type II seesaw terms to the neutrino mass by considering one of them dominating at a time. The contribution of the non-leading seesaw term can be adjusted by varying the dimensionless parameter \( \gamma \) with minimum non-leading contribution for \( \gamma \sim 1 \). We vary this dimensionless parameter \( \gamma \) from 0.001 to 1.0 and check the variations in the predictions for baryon asymmetry. We do this exercise for different Majorana phases \( \alpha = \beta = \pi/6, \pi/4, \pi/3, \pi/2 \) as shown in the figures 3, 4, 5, 6, 7, 8. These figures show that the variations of Majorana phases as well as \( \gamma \) can not only change the predictions for baryon asymmetry from the observed value but also can give rise to a flip in
FIG. 6: Variation of baryon to photon ratio with $\gamma$ and Majorana phases for UQ type $m_{LR}$ and dominant type I seesaw with normal hierarchy.

We observe that, in the presence of both type I and type II seesaw contribution, all the viable models shown in figures 1, 2 can give rise to the correct baryon asymmetry for specific values of the dimensionless parameter $\gamma$ and Majorana neutrino phases $\alpha, \beta$. From such an analysis, we can discriminate between different Majorana phases, which are presently unconstrained from neutrino oscillation experiments.

V. CONCLUSION

We have studied the possibility of producing the observed baryon asymmetry in the Universe within the framework of generic left-right symmetric models by considering both types of hierarchies (normal and inverted) and two types of Dirac neutrino mass matrices: charged
lepton type and up quark type. In generic LRSM, neutrino mass can get contributions from both type I as well as type II seesaw terms. We consider two possible cases: one in which type I seesaw dominates and the other in which type II seesaw dominates.

We use the generic parametrization of neutrino mixing matrix and find the numerical value of these parameters using the global fit neutrino oscillation data. We consider the heavier mass eigenstates of active neutrinos to be of the order of measured mass squared differences and keep the lightest mass eigenstate as a free parameter. We then compute the predictions for baryon to photon ratio by considering either type I or type II seesaw terms for both the types of hierarchies as well as both CL and UQ type Dirac neutrino mass matrices. We show that inverted hierarchical models with type I seesaw are disfavored as their predicted baryon asymmetry falls below the observed value by around two orders of magnitudes.
We then keep the dominating seesaw term fixed in the seesaw formula and vary the other term by varying the dimensionless parameter $\gamma$. We then calculate the predictions for baryon asymmetry for different values of $\gamma$ as well as four different Majorana phases $\pi/6, \pi/4, \pi/3, \pi/2$. We show that the models which give rise to correct baryon asymmetry with either type I or type II seesaw can also give rise to the correct baryon asymmetry for the combination of type I and type II seesaw provided the dimensionless parameter $\gamma$ takes appropriate values. We can also constrain the Majorana neutrino phases from this study as can be seen from the figures 3, 4, 5, 6, 7, 8.

In view of above, the neutrino mass models considered in our study can survive in nature and produce the observed baryon asymmetry within the framework of type I and type II seesaw mechanism. Inverted hierarchy with type I seesaw was found to be disfavored in our analysis as it falls short of the observed baryon asymmetry by two orders of magnitudes.
Whereas, normal hierarchy can give rise to the observed baryon asymmetry for both type I and type II seesaw separately and both CL and UQ type Dirac neutrino mass matrices. In the presence of both type I and type II seesaw (with one of them dominating at a time), most of the models included in our study are found to produce the observed baryon asymmetry provided the Majorana neutrino phases and the dimensionless parameter $\gamma$ are chosen appropriately. More precise data from neutrino oscillation experiments as well as cosmology should be able to reduce the number of available possibilities presented in our analysis.

[1] S. Fukuda et al. (Super-Kamiokande), Phys. Rev. Lett. 86, 5656 (2001), hep-ex/0103033; Q. R. Ahmad et al. (SNO), Phys. Rev. Lett. 89, 011301 (2002), nucl-ex/0204008; Phys. Rev. Lett. 89, 011302 (2002), nucl-ex/0204009; J. N. Bahcall and C. Pena-Garay, New J. Phys. 6, 63 (2004), hep-ph/0404061; K. Nakamura et al., J. Phys. G37, 075021 (2010).

[2] P. Minkowski, Phys. Lett. B67, 421 (1977); M. Gell-Mann, P. Ramond, and R. Slansky (1980), print-80-0576 (CERN); T. Yanagida (1979), in Proceedings of the Workshop on the Baryon Number of the Universe and Unified Theories, Tsukuba, Japan, 13-14 Feb 1979; R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett 44, 912 (1980); J. Schechter and J. W. F. Valle, Phys. Rev. D22, 2227 (1980).

[3] R. N. Mohapatra and G. Senjanovic, Phys. Rev. D23, 165 (1981); G. Lazarides, Q. Shafi and C Wetterich, Nucl. Phys. B181, 287 (1981); C. Wetterich, Nucl. Phys. B187, 343 (1981); B. Brahmachari and R. N. Mohapatra, Phys. Rev. D58, 015001 (1998); R. N. Mohapatra, Nucl. Phys. Proc. suppl. 138, 257 (2005); S. Antusch and S. F. King, Phys. Lett. B597, (2), 199 (2004).

[4] R. Foot, H. Lew, X. G. He and G. C. Joshi, Z. Phys. C44, 441 (1989).

[5] K. Abe et al. [T2K Collaboration], Phys. Rev. Lett. 107, 041801 (2011), arXiv:1106.2822 [hep-ex]].

[6] Y. Abe et al., Phys. Rev. Lett. 108, 131801 (2012), arXiv:1112.6353 [hep-ex]].

[7] F. P. An et al. [DAYA-BAY Collaboration], Phys. Rev. Lett. 108, 171803 (2012), arXiv:1203.1669 [hep-ex]].

[8] J. K. Ahn et al. [RENO Collaboration], Phys. Rev. Lett. 108, 191802 (2012),
[arXiv:1204.0626][hep-ex]]

[9] M. C. Gonzalez-Garcia, M. Maltoni, J. Salvado and T. Schwetz, [arXiv:1209.3023 [hep-ph]].

[10] Y. Shimizu, M. Tanimoto and A. Watanabe, Prog. Theor. Phys. **126**, 81 (2011); S. F. King and C. Luhn, JHEP **1109**, 042 (2011); S. Antusch, S. F. King, C. Luhn and M. Spinnrath, Nucl. Phys. **B856**, 328 (2012); S. F. King and C. Luhn, JHEP **1203**, 036 (2012); S. Gupta, A. S. Joshipura and K. M. Patel, Phys. Rev. **D85**, 031903 (2012); S-F. Ge, D. A. Dicus and W. W. Repko, Phys. Rev. Lett. **108**, 041801 (2012); S-F. Ge, D. A. Dicus and W. W. Repko, Phys. Lett. **B702**, 220 (2011).

[11] G. Altarelli, F. Feruglio, L. Merlo and E. Stamou, JHEP **1208**, 021 (2012).

[12] C. L. Bennett et. al., arXiv:1212.5225; G. Hinshaw et. al., arXiv:1212.5226.

[13] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. **B155**, 36 (1985).

[14] M. Fukugita and T. Yanagida, Phys. Lett. **B174**, 45 (1986).

[15] J. Ellis, S. Lola and D. V. Nanopoulos, Phys. Lett. **B452**, 87 (1999); G. Lazarides and N. D. Vlachos, Phys. Lett. **B459**, 482 (1999); M. S. Berger and B. Brahmachari, Phys. Rev. **D60**, 073009 (1999); M. S. Berger, Phys. Rev. **D62**, 013007 (2000); W. Buchmüller and M. Plumacher, Int. J. Mod. Phys. **A15**, 5047 (2000); K. Kang, S. K. Kang and U. Sarkar, Phys. Lett. **B486**, 391 (2000); H. Goldberg, Phys. Lett. **B474**, 389 (2000); R. Jeannerot, S. Khalil and G. Lazarides, Phys. Lett. **B506**, 344 (2001); D. Falcone and F. Tramontano, Phys. Rev. **D63**, 073007 (2001); D. Falcone and F. Tramontano, Phys. Lett. **B506**, 1 (2001); H. B. Nielsen and Y. Takanishi, Phys. Lett. **B507**, 241 (2001); E. Nezri and J. Orloff, JHEP **0304**, 020 (2003).

[16] A. S. Joshipura, E. A. Paschos and W. Rodejohann, Nucl. Phys. **B611**, 227 (2001).

[17] J. C. Pati and A. Salam, Phys. Rev. **D10**, 275 (1974); R. N. Mohapatra and J. C. Pati, Phys. Rev. **D11**, 2558 (1975); G. Senjanovic and R. N. Mohapatra, Phys. Rev. **D12**, 1502 (1975); R. N. Mohapatra and R. E. Marshak, Phys. Rev. Lett. **44**, 1316 (1980); N. G. Deshpande, J. F. Gunion, B. Kayser and F. I. Olness, Phys. Rev. **D44**, 837 (1991).

[18] M. K. Das, D. Borah and R. Mishra, Phys. Rev. **D86**, 095006 (2012), [arXiv:1209.0280[hep-ph]]; D. Borah and M. K. Das, Nucl. Phys. **B870**, 461 (2013), [arXiv:1210.5074].

[19] B. Bajc, G. Senjanovic and F. Vissani, Phys. Rev. **D70**, 093002 (2004), hep-ph/0402140; S. Bertolini, M. Frigerio and M. Malinsky, Phys. Rev. **D70**, 095002 (2004), hep-ph/0406117; T. Hambye and G. Senjanovic, Phys. Lett. **B582**, 73 (2004); N. Sahu and S. Uma Sankar, Phys.
Rev. **D71**, 013006 (2005), hep-ph/0406065.

[20] B. Dutta, Y. Mimura and R. N. Mohapatra, Phys. Lett. **B603**, 35 (2004), hep-ph/0406262.

[21] C. H. Albright and S. M. Barr, Phys. Rev. **D70**, 033013 (2004), hep-ph/0404095; S. M. Barr, Phys. Rev. Lett. **92**, 101601 (2004), hep-ph/0309152.

[22] A. S. Joshipura, E. A. Paschos and W. Rodejohann, JHEP **0108**, 029 (2001), hep-ph/0105175.

[23] B. Bajc, G. Senjanovic and F. Vissani, Phys. Rev. Lett. **90**, 051802 (2003); H. S. Goh, R. N. Mohapatra and S-P. Ng, Phys. Rev. **D68**, 115008 (2003); H. S. Goh, R. N. Mohapatra and S. Nasri, Phys. Rev. **D70**, 075022 (2004); R. N. Mohapatra and M. K. Parida, Phys. Rev. **D84**, 095021 (2011).

[24] E. W. Kolb and M. S. Turner, The Early Universe, Frontiers in Physics, Vol. 69, Addison-Wesley, Redwood, 1990; A. Pilaftsis, Int. J. Mod. Phys. **44**, 1811 (1999); E. A. Paschos and M. Flanz, Phys. Rev. **D58**, 113009 (1998).

[25] G. L. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo and A. M. Rotunno, Phys. Rev. **D86**, 013012 (2012).

[26] K. S. Babu, B. Dutta and R. N. Mohapatra, Phys. Lett. **B458**, 93 (1999), hep-ph/9904366; D. Falcone, Phys. Rev. **D65**, 077301 (2002), hep-ph/0111176; D. Falcone, Phys. Lett. **B479**, 1 (2000), hep-ph/0204335.

[27] M. K. Das, M. Patgiri and N. N. Singh, Pramana **65**, 995 (2006).