INVITED REVIEW

What do students gain from games? Dice games vs word problems

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Abstract
Learning the fundamentals of probability and random variables can be a struggle for many students. Games by their nature require active participation and reward mastery, lending naturally to the framework of active learning. In this study, we designed and evaluated the efficacy of a lab activity using a game where strategy hinges on sums of dice to teach about linear combinations of independent random variables. We found that both activities lead to improved understandings of course concepts, but the dice game provided increased student engagement and interest.

KEYWORDS
dice, games, probability, teaching statistics

INTRODUCTION

Learning the fundamentals of probability and random variables lays an important foundation for a thorough understanding of statistical inference, but it can be a struggle for many students. The mean and variance properties of sampling distributions that drive statistical inference are directly tied to the behavior of linear combinations of independent random variables. It is common to reinforce technically challenging topics like these with lab activities that provide context and practice with applied problems. This study compares the learning outcomes from a lab using a new dice game with a lab using a traditional word-problem worksheet on linear combinations of independent random variables. Additionally, we surveyed students to learn about their perceived learning, challenge, engagement, enjoyment, and interest during the lab. We use the learning outcomes and student surveys to answer the question: “What do students gain in a lab based on a dice game vs using word problems?”

There is precedent for exploring the benefits of incorporating games as a tool in education. Games by their nature require active participation and reward mastery, lending naturally to the framework of active learning. Statistics education literature commonly advocates using games to help teach probability. Lesser and Pearl include games as part of a general strategy to employ fun in statistics education as a means for improving student engagement and enjoyment in the classroom. Some statistics educators have explored the probabilistic structures in games of chance such as poker, blackjack and Monopoly. Others have focused on the probabilistic deconstruction of games from popular television shows such as Let’s Make a Deal and The Price is Right. The physical randomization processes built into many games can also lend to tactile learning. The Guidelines for Assessment and Instruction in Statistics Education (GAISE) guidelines have suggested using dice or prior to using simulation methods.

There are few contexts for highlighting basic concepts of probability that are more historically rooted than the outcomes in dice games. Some of the earliest foundations of formal probability were first recorded in the correspondence between Pascal and Fermat in 1654, which included a debate on gambling strategy in a dice game. Then, in 1657, Huygens published his treatise on...
Statistics educators have continued the tradition by using many dice games in teaching probability concepts, for example, using Yahtzee, Pass the Pigs, HOG, and GOLO.

In this study, we designed and evaluated the efficacy of a new dice game where strategy hinges on the behavior of sums of dice to teach about linear combinations of independent random variables. Section 2 describes the game and worksheet lab activities used with the 10 classroom sections across two introductory statistics courses and the procedure used for comparing the effectiveness of labs. Section 3 then explains the analysis of quiz scores for comparing learning outcomes and reviews the survey responses for evaluating student opinion of the activities. We conclude in 4 with a discussion of the relative merits of the two lab approaches.

2 METHODS

Stake Your Claims is a dice game that we developed to help teach and reinforce the concepts surrounding linear combinations of independent random variables for students in an introductory statistics course. The game has a central mechanic where each player has —4-, 6-, 8-, 10-, and 12-sided—that are used as game pieces that can be placed out onto the game board based on a player’s strategy. Winners in the game are decided when the group of dice, added by the players during play, are rolled and summed; thus behaving as a linear combination of independent random variables. The dice ante mechanism used by players can be guided by the expected value and variance of the group of dice, providing a concrete application of these probabilistic properties. The dice ante mechanic of the Stake Your Claims game was inspired by elements of the board game Floating Market: A Game of Diced Fruit. While Floating Market is a delightful game about collecting fruit at a Thai river market, it contains complex gaming elements that did not fit well into a classroom lab activity focused on the behavior of linear combinations of independent random variables.

2.1 Stake Your Claims rules

Objective: Be the first player to claim three points!

Game set-up:

1. Set out the game board. Notice that it is broken into three sections:
   - “Stake Your Claims!” with six spaces labeled with number ranges.
   - “Ante Up!” with one large dice pool.
   - “Rig the Game!” with two circular and two split rectangular spaces.

2. Each player gets a set of five dice of matching color (Figure 1).
   - In a set: 4-, 6-, 8-, 10-, and 12-sided.

3. There are two coins, which correspond to the circular locations in the “Rig the Game” section of the game board (Figure 2).
   - The penny corresponds to the “+2/−2” space.
   - The nickel corresponds to the “+4/−4” space.

Pick a first player by rolling the 12-sided die. The player with the highest roll goes first (reroll ties).
Playing the game:

1. The first player places one of their dice into a space or the Dice Pool. Players continue placing their dice, one at a time in clockwise order.
2. The round ends immediately once a player adds the seventh die to the Dice Pool.
3. At the end of the round, the dice in the Dice Pool are rolled.
4. Find the winning number: add up the rolled dice, then make the adjustments from the “Rig the Game!” section.
5. The player who claimed the range containing the winning number earns a point.
6. Restart the next round with the player who earned the point.

Placing the dice: You have four options for placing your dice.

1. Claim an unclaimed number range.
2. Add a die to the dice pool.
3. Rig the game by placing on a coin flip space (Note: The coin is flipped at the end of the round, Heads = +, Tails = −).
4. Rig the game by choosing a number on the split spaces (Note: Only one player can go on a split space and must choose as the place the die).

2.2 Case study details

For this study, we wanted to assess the impact of the Stake Your Claims dice game on student learning outcomes and student opinions. To provide comparison, we administered two different short lab activities; one using the dice and the other using an analogous worksheet of word problems. There were 10 lab sections from two introductory statistics courses at a midsize public university in the Midwest region of the United States. There were four lab sections from an introductory statistics course designed for statistics and engineering majors (course A) and six lab sections from an introductory statistics course designed for business majors (course B); all lab sections were taught by the same instructor within each course. Lab activity treatments were randomly assigned to an equal number of lab sections within each course. The instructor from course A taught two game labs and two worksheet labs, while the instructor from course B taught three game labs and three worksheet labs. All other curricular materials, activities, and assessments were the same across all sections within each course.

The dice game lab activity included a short period where students were taught the Stake Your Claims game and allowed to play through a few rounds of the game in groups of four, asked only to record the type of dice in the dice pool and the winning numbers for each round. The activity of learning and playing the game took ~25 to 30 minutes of the 50 minutes of lab time. Then the groups worked through a set of problems related to expected values and variance for the dice pool sum as a linear combination of independent random variables. Following these questions, the students were asked to use the expected values to guide their game strategy.

To provide meaningful comparisons for learning outcomes and engagement to a more traditional lab activity, we developed a lab activity of word problems involving analogous sums of dice for student groups to work through. The questions involved an expanded set of probability calculations that ran parallel to the dice scenarios in the game lab, but the context of a game and physical dice were not included. This included written problems where dice rolls and coin tosses are treated as discrete random variables, explored the associated probability mass functions, and computing expected values and variances of linear combinations based on sums of dice rolls and coin flips. The full set of lab activities and problems can be found in Supporting information.

We can look to an example from the labs to better understand the type of parallels and differences from the two activities. Both labs reminded students of previous lessons about the properties of linear combinations of independent random variables \(X_1, X_2, ..., X_n\) and constants \(a_0, a_1, a_2, ..., a_n\), where

\[
E[a_0 + a_1X_1 + ... + a_nX_n] = a_0 + a_1E[X_1] + ... + a_nE[X_n]
\]

\[
\text{Var}[a_0 + a_1X_1 + ... + a_nX_n] = a_1^2\text{Var}[X_1] + ... + a_n^2\text{Var}[X_n]
\]

Then students were asked to consider that a sum of independent random variables is special case, where \(a_1 = a_2 = ... = a_n = 1\). Both labs used dice rolls (4-, 6-, 8-, 10-, and 12-sided) and coin flips (+4 for heads, −4 for tails) as examples of random variables for their problems. The students in both labs were required to calculate the expected value and variance from the sum a group of five rolled dice plus flipped coins. The primary difference in the activity was in how this linear combination of random variables was contextualized. In the game lab, the students had been taught the game and played several (typically highly rambunctious) rounds before being asked these questions. To these students, the sum that was being discussed was directly related to how the winning number was generated in each round of play. The
The games lab also asked questions about how the calculation of the expected values and variances could be used to guide their strategies in picking ranges of numbers on which to “Stake Their Claims” or to decide which dice they should add to the ante pool to improve their chances of winning. The game provided a context in which the answers about linear combinations of random variables would be applicable. In the worksheet lab, the students simply were asked to compute the mean and variance without a guiding motivation or application for these quantities.

### 2.3 Measurements for student learning and opinions

A pair of quizzes was administered pre- and post-lab to evaluate the students’ understanding of the learning outcomes from each lab. The pre- and post-lab quiz structure and problem tasks were developed by the authors to assess student ability to mathematically express and calculate the expected value and variance for linear combinations of independent random variables. Each quiz consisted of five questions that were graded with one point per question.

In addition to assessing learning outcomes through the quizzes, a survey was given to assess student opinions of the labs. The survey was identical for both groups and used a Likert scale such that 1 = strongly disagree, 2 = disagree, 3 = neutral, 4 = agree, and 5 = strongly agree to gather responses to the following statements:

- “I enjoyed working through the lab activity.”
- “I felt challenged during the lab activity.”
- “I felt engaged with the lab activity.”
- “I feel a better understanding of how to combine sums of random variables after the lab activity.”
- “I feel more interested in the class after the lab activity.”

The shorthand enjoyed, challenged, engaged, understanding, and interest will be used to refer to these survey items, respectively. The survey also included an open response area to allow students to provide any additional comments they had about their experience in the lab.

### 3 RESULTS

We begin with an analysis of the learning outcomes following the lab activities using the quiz scores as a measure of understanding. Figure 3 shows the distribution of grade changes from the pre-lab and post-lab score. We see a general improvement to student scores after both learning activities in both of the courses, with a slightly stronger improvement seen in the sections using the worksheet.

Next, we look to the survey to explore the student opinions related to the lab. Figure 4 displays the

![Figure 3: Distribution of score changes pre/post-lab activity](image-url)
responses for each of the five survey items listed in Section 2. Here, we see some distinct differences in student sentiment between the labs and, in some cases, also differences across students of the two courses. Lab differences for student responses were similar across the two courses for engagement and interest survey items, where the game lab was associated with stronger engagement and stronger interest. Classes differed in their feelings about the challenge and enjoyment from lab activities. Students of course A—primarily college sophomore statistics and engineering majors—showed a decrease in perceived challenge with games lab but no change in enjoyment when compared to their counterparts from course A who did the worksheet lab. Whereas the course B students—primarily college freshmen business majors—showed little change in perceived challenge with the game lab, but increased enjoyment compared to their counterparts in course B with the worksheet lab. There appeared to be little difference in feelings of understanding across both courses, with most students perceiving a better understanding of how to combine sums of random variables after both lab activities.

4 | DISCUSSION

Did students gain from using the Stake Your Claims dice game instead of a worksheet? Our results found in our student survey showed stronger engagement and interest in both courses when using the dice game. The survey also revealed that students in the course for business majors found the dice game more enjoyable, and the students in the course for statistics and engineering majors found the dice game less challenging. Students from both courses expressed that they generally understood the course concepts better following either lab and both groups did see a general improvement from pre-lab to post-lab scores.

The score improvements were, however, slightly stronger for the students who completed the worksheet lab, so we worry that the learning outcomes from the dice game activity were weakened somewhat from by the time-investment required for learning the rules and playing the game; a problem not faced by the word problem worksheet. From the prospective of cognitive load theory, learning the somewhat challenging rules (extraneous load) could have also interfered with intended learning about theoretical properties (intrinsic load) underlying the behavior of the dice.

There are limitations and concerns regarding the interpretation of lab effects and the generalizability to other topics and groups of students. The limitation for generalizability of our results about the impact of classroom games on engagement and learning depends on the extent to which other game activities are similar to the topic, game type, and student groups involved in this
study. Here, we have a group of mostly statistics, engineering, and business undergraduate students at a mid-size public Midwestern university that was learning about random variables using a dice game that was not previously known to them; generalization should be made accordingly. Additionally, we recognize that differences that were seen between courses may not be due to the differences in student backgrounds, because of the confounding presented by having different instructors in each course.

We plan to continue using the dice game in the introductory courses, primarily because we value the improvement to student engagement and interest that have historically been abysmally low during the unit on probability. We do however plan to adapt the activity to include a video tutorial on the rules of the game that students will be required to watch before coming to lab. This will allow more time-on-task to focus on linear combinations of independent random variables during the dice game lab to more closely resemble the amount of time-on-task in the worksheet lab. Hopefully we will retain the gains in student engagement and interest from playing the game, while spending more time thinking about the connection to random variables.

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SUPPORTING INFORMATION
Additional supporting information may be found online in the Supporting Information section at the end of this article.

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