On the large-scale instability in interacting dark energy and dark matter fluids

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Recently, Valiviita et al. (2008) have reported a large-scale early-time instability in coupled dark energy and dark matter models. We take the same form of energy-momentum exchange and specialise to the case when the interaction rate is proportional to Hubble’s parameter and the dark energy density only. Provided the coupling is made small enough for a given equation of state parameter, we show that the instability can be avoided. Expressions are derived for non-adiabatic modes on super-horizon scales in both the radiation and matter dominated regimes. We also examine the growth of dark matter perturbations in the sub-horizon limit. There we find that the coupling has almost no effect upon the growth of structure before dark energy begins to dominate. Once the universe begins to accelerate, the relative dark matter density fluctuations not only cease to grow as in uncoupled models, but actually decay as the universe continues to expand.

I. INTRODUCTION

There is good evidence to believe the present day energy density of the universe is mostly in the form of dark energy [1,2], the properties of which remain relatively unknown. Furthermore, observations of Type Ia supernovae [3,4] leave little doubt that the expansion of the universe is accelerating. Viable models of cosmology now require a large dark energy component, capable of producing the negative pressure required for accelerated expansion.

By far the simplest model of dark energy is Einstein’s cosmological constant, Λ. The cosmological constant(Λ) and cold dark matter (CDM) model, with values of today’s density parameter for the dark energy Ω_x ≈ 0.7 and dark matter Ω_m ≈ 0.3 is the current prevailing paradigm. But while consistent with observational constraints, the standard model is in many ways unsatisfactory. One such example is the ‘coincidence problem’: why are the energy densities in the dark energy and dark matter comparable today, when the redshift dependence of each is so different?

Motivated to explain the coincidence problem while deviating as little as possible from the successful ΛCDM model, a coupling between dark energy and dark matter has often been considered. An energy exchange modifies the background evolution of the dark sector, and explaining the coincidence problem can be reduced to tuning a coupling parameter to an appropriate value.

The coupling enters via the continuity equations. With energy exchange rate Q between the dark energy (subscript x) and the cold dark matter (subscript c), the dark energy obeys the continuity equation in conformal time

$$\dot{\rho}_x + 3H(1 + w_x)\rho_x = -Q,$$

while the dark matter obeys

$$\dot{\rho}_c + 3H\rho_c = Q.$$

Here we have introduced the equation of state parameter $w_A$ that gives the ratio of the pressure $P_A$ to the energy density $\rho_A$ of a fluid,

$$w_A = \frac{P_A}{\rho_A}.$$  (3)

We have also used $H = a\dot{a}$, where $a(t)$ is the expansion scale-factor and $H$ the Hubble parameter. Acceleration of the expansion rate requires the energy density of the universe to dominated by a fluid with an effective equation of state parameter $w_{eff} < -1/3$. We do not allow the phantom case of $w < -1$ in this work. Simple solutions for the background exist for couplings of the form $Q = \alpha H\rho_x + \beta H\rho_c$. These were initially investigated by Chimento [5] and then expanded upon by Barrow and Clifton [6], who provided general solutions for any cosmology with two components exchanging energy in such a fashion, provided the components were modelled as cosmological fluids with constant $w$. Quar-tin et al. [7] examined the observational constraints upon such a class of models, significantly limiting the available parameter space. Again, the equation of state parameter was treated as fixed. Non-zero values of $\beta$ were found to reduce the required fine-tuning of the initial energy density, as well as increase the observationally allowed values of $\alpha$ [7].

A coupling would influence more than just the background dynamics of the universe. In particular, the
growth of perturbations in the coupled fluids would be affected. Recent work by Valiviita et al. [8] has shown that couplings of the simple form described above, with constant $w$, exhibit extremely rapid growth of dark energy fluctuations on super-horizon scales in the early universe. In fact, the perturbations in the dark energy become unstable for any model with non-zero $\beta$, no matter how small this parameter is made. While this would appear to rule out all couplings of the above form and with constant $w$, the explicit examples in [8] included no cases where the interaction rate was proportional to the density of dark energy and not of the dark matter, i.e. with $\beta = 0$ and $\alpha \neq 0$. Here we look at just such a scenario.

II. BACKGROUND EVOLUTION

Friedmann’s equation relates the evolution of the scale-factor $a(t)$ to the background energy density $\rho$. We make use of conformal time, $\tau$, which is related to cosmic time via $d\tau = adt$. Overdots indicate derivatives with respect to conformal time.

Friedmann’s equation reads

$$H^2 \equiv \left(\frac{a'}{a}\right)^2 = \frac{8\pi G}{3} \rho a^2.$$ (4)

With the choice of $Q = \alpha H \rho_x$, the continuity equations can be solved to yield [8, 9]

$$\rho_x = \rho_{x,0} a^{-(3(1+w)+\alpha)},$$ (5)

$$\rho_c = -\frac{\alpha \rho_{x,0}}{3w+\alpha} a^{-(3(1+w)+\alpha)} + \left(\rho_{c,0} + \frac{\alpha \rho_{x,0}}{3w+\alpha}\right) a^{-3}.$$ (6)

We follow the standard notation where a subscript zero indicates today’s value. We normalise the scale-factor so that $a_0 = 1$. The ratio of dark energy to dark matter density $r$ can then be written

$$\frac{1}{r} = \frac{\rho_c}{\rho_x} = \left(\frac{\rho_{c,0}}{\rho_{x,0}} + \frac{3w+\alpha}{3w+\alpha}\right) a^{3w+\alpha} - \frac{\alpha}{3w+\alpha}.$$ (7)

With $|3w| < \alpha$, the dark energy and dark matter approach a constant ratio as the universe expands. The coincidence problem can be said to be solved if this ratio is of order unity, but this requires a value of $\alpha$ already observationally excluded [7]. Nevertheless, as argued in [7], non-zero values of $\alpha$ can still be said to alleviate the problem. We restrict ourselves to positive values of $\alpha$.

III. PERTURBED FRW COSMOLOGY

We assume a flat FRW cosmology and work in Newtonian gauge,

$$-ds^2 = dt^2(1 + 2\Psi) - a^2(1 - 2\Phi)\delta_{ij}dx^idx^j,$$ (8)

with metric signature $(+, -, -, -)$. We work in Fourier space, using comoving Fourier wave-vectors $k^i = k_i$, so that $\partial_i \varphi \rightarrow k^2/a^2$.

The four-velocity of fluid $A$ is given by

$$U^\mu_{(A)} = \left(1 - \Psi, a^{-1}v^i_{(A)}\right).$$ (9)

The peculiar velocity three-vector $v^i = v_i$ are small. We define the velocity perturbation $\theta \equiv \partial_i v^i$.

A. Energy-momentum tensors

The energy-momentum tensor for fluid $A$ is given by:

$$T^\mu_{\nu} = \left(\rho^{(A)} + P^{(A)}\right) U^\mu_{(A)} U^\nu_{(A)} - \delta^\mu_{\nu} P^{(A)}.$$ (10)

The total energy-momentum tensor is simply the sum of the components,

$$T^\mu_{\nu} = \sum_A T^\mu_{\nu}^{(A)}.$$ (11)

We define the density perturbation in fluid $\delta_A$ using $\rho_A \equiv (1 + \delta_A)\bar{\rho}_A$. An overbar denotes the background quantity, though we will usually leave this implicit.

Energy and momentum conservation for fluid $A$ implies

$$\nabla_{\mu} T^\mu_{\nu}^{(A)} = Q^\nu_{(A)},$$ (12)

and conservation for the entire system requires

$$\sum_A Q^\mu_{(A)} = 0.$$ (13)

The four-vector $Q^\mu_{(A)}$ governs the energy exchange between components, and it is to this we now turn our attention.

B. Covariant energy exchange

The energy exchange in the background does not determine a fully covariant form of energy exchange [8, 9]. Instead, energy exchange four-vector must be specified. We adopt the approach of [8] and consider two scenarios; aligning the four-vector with the dark energy four-velocity,

$$Q^\mu_{(A)} = Q_A U^\mu_{x}.$$ (14)

or with the four-vector of the dark matter four-velocity,

$$Q^\mu_{(A)} = Q_A U^\mu_{c}.$$ (15)

These choices produce slightly different outcomes, and the differences are noted as we proceed. To produce the desired changes to the continuity equations, we see that $aQ_c = -aQ_x = \alpha H \rho_x$ in both cases. We also make the common assumption that $\alpha H$ gives an interaction rate that has no spatial dependence. We therefore perturb only $\rho_x$, not $H$, in the coupling.
C. Sound speed of dark energy

The speed of sound of a fluid or scalar field \( A \) is denoted by \( c_s A \). For a barotropic fluid with a constant value of \( w_\Lambda \), then \( c_s^2 A = w_\Lambda \). This leads to an imaginary speed of sound for the dark energy (\( c_{sx}^2 = w_x < 0 \)). An imaginary sound speed leads to instabilities in the dark energy; the problem is commonly remedied by imposing a real sound speed by hand. A common choice (and the one we make here) is the scalar field value of \( c_{sx} = 1 \).

This choice leads to an intrinsic non-adiabatic pressure perturbation in the dark energy. This contains a term, highlighted recently in [8], that arises due to the coupling between dark energy and dark matter. We include this term, and refer the interested reader to [8].

D. Perturbation equations of motion

Conservation of the energy-momentum tensor, combined with results of the previous sections and our choice of energy exchange four-vector, implies the following. For the dark energy density perturbation:

\[
\delta_x' + 3(1 - w_x)\delta_x + (1 + w_x)\theta_x + 9\mathcal{H}^2(1 - w_x^2)\frac{\theta_x}{k^2} - 3(1 + w_x)\Phi' = -\alpha\mathcal{H}'\left[\Psi + 3\mathcal{H}(1 - w_x)\frac{\theta_x}{k^2}\right].
\]

(16)

For the dark energy velocity perturbation, the right-hand side differs slightly depending on our choice of energy exchange four-vector.

\[
\theta_x' - 2\mathcal{H}\theta_x - \frac{k^2}{1 + w_x}\delta_x - k^2\Psi
= \frac{(1 + b)\alpha\mathcal{H}}{1 + w_x}\theta_x,
\]

(17)

where

\[
b = \begin{cases} 0 & \text{if } Q^\mu_{(A)} = Q A U^\mu_x, \\ 1 & \text{if } Q^\mu_{(A)} = Q A U^\mu_c. \end{cases}
\]

(18)

For the dark matter, the density perturbation obeys

\[
\delta_c' + c_s^2 \delta_c' - 3\Phi' = \alpha\mathcal{H}\frac{\rho_c}{\rho_c} [\delta_x - \delta_c],
\]

(19)

while the velocity perturbation is governed by

\[
\theta_c' + \theta_c\mathcal{H} - k^2\Psi
= (1 - b)\alpha\mathcal{H}\frac{\rho_c}{\rho_c} [\theta_x - \theta_c].
\]

(20)

The perturbed Einstein equations are well known, and we do not reproduce them here. They can be found in [10], whose notation for the scalar metric perturbations we share.

IV. INITIAL CONDITIONS IN THE EARLY RADIATION ERA

In [8] it was shown that models with \( \beta \neq 0 \) suffered from an early time large-scale instability no matter how small the value of \( \beta \). This was driven by a term proportional to \( \beta \) on the right-hand side of equation (17). A term proportional to \( \alpha \) also exists, which can be large if \( w \) is close to \(-1\) or \( \alpha \) is made very large. In this section we examine how large this term needs to be to cause the non-adiabatic mode to be a growing one.

We consider super-horizon scales (\( k/\mathcal{H} \ll 1 \)) and assume adiabatic initial conditions. The gravitational potentials are dominated by fluctuations in the dominant fluid (radiation or matter). The well known result is that \( \Phi \propto \Psi = \text{constant} \). The constant of proportionality in the radiation era is determined by the anisotropic stress generated by the neutrinos. In the absence of neutrinos or in the matter dominated era, the potentials are equal. These assumptions will be invalid only if perturbations in the dark energy are large enough to influence the gravitational potentials. As the dark energy has a very low background density in the radiation era, this can only happen if \( \delta_x \) grows extremely large.

Neglecting time derivatives of the gravitational potential, and keeping only leading order terms in \( k/\mathcal{H} \), the dark energy equations (16) – (17) can be combined into a second order equation:

\[
\delta_x'' + \mathcal{H}(1 - 3w)\left(1 + \frac{b\alpha}{1 + w}\right)\delta_x' - \frac{2\mathcal{H}'\mathcal{H}}{\mathcal{H}^2}\delta_x + 3\mathcal{H}^2\left(1 - \frac{b\alpha}{1 + w}\right) - \mathcal{H}'\left(1 - w\right)\delta_x
= \left(A\mathcal{H}^2 + B\mathcal{H}'\right)\Psi,
\]

(21)

The constants \( A \) and \( B \) have values unimportant for our analysis.

In the radiation era, \( \mathcal{H} = \tau^{-1} \). The adiabatic mode is therefore an obvious solution: \( \delta_x \propto \Psi = \text{constant} \). To find the remaining solutions, we define a new variable \( \delta_x = \delta_x + C\Psi \), with the constant \( C \) chosen such that the right-hand side of (21) is equal to zero. In the radiation dominated era, we can then write:

\[
\tau^2 \delta_x'' + \left(3 - 3w - \frac{b\alpha}{1 + w}\right)\tau\delta_x' + 3(1 - w)\left(2 - \frac{b\alpha}{1 + w}\right)\delta_x = 0
\]

(22)

When \( b = 1 \), equation (22) becomes formally the same equation found by He et al. [11], despite the differing assumptions made about the physics involved. In their investigation of perturbations given a background coupling of the form \( Q = \alpha\mathcal{H}\rho_c \), they choose to set the net momentum exchange to zero \( (Q^\mu_{(A)} = 0) \), in contrast to our adoption of the form of momentum exchange used in [8]. The differences between the \( b = 1 \) choice of momentum
exchange and zero net momentum exchange arise in the equations for the dark matter perturbations, which are not used in the above analysis, nor in the analysis by He et al. This leads to the same behaviour of dark energy perturbations. This is not true when \( b = 0 \), and can result in different behaviour (oscillatory or non-oscillatory) for the same choice of parameters (see the remainder of this section). Note also that the simplifying assumptions, and their justifications, made in \([11]\) differ to those made here: we have neglected terms that will be small due to choice of initial conditions, and simplified the result by extracting the adiabatic mode. In \([11]\), terms are instead neglected that are found to be small from a numerical analysis.

Solutions of equation (22) are power laws, \( \hat{\delta}_x \propto \tau^{n_{\pm}} \). The index is given by:

\[
 n_{\pm} = \frac{\Gamma}{2(1 + w)} \pm \frac{\sqrt{\Delta}}{2(1 + w)}, \tag{23}
\]

where we follow the notation of \([11]\) and have defined the quantities

\[
\Gamma = 3w^2 + w + (1 + b)\alpha - 2, \tag{24}
\]

and

\[
\Delta = 9w^4 + 30w^3 + (13 - 6b - 1)\alpha w^2 + 2w [(1 + b)\alpha - 14] + 4(2b - 1)\alpha + (1 + b)\alpha w - 20. \tag{25}
\]

In the limit of \( w \) very close to \(-1\) (and assuming \( \alpha \) is reasonably small), we can expand as a series in \((1 + w)\),

\[
\frac{\Gamma}{2(1 + w)} \approx -\frac{5}{2} + \frac{(1 + b)\alpha}{2(1 + w)} + \frac{3(1 + w)}{2} + O(1 + w)^2, \tag{26}
\]

\[
\Delta \approx (1 + 3b)\alpha^2 + 2(7b - 5)(1 + w)\alpha + (6 - b)\alpha - 23)(1 + w)^2 + O(1 + w)^3. \tag{27}
\]

When \( \alpha = 0 \), the non-adiabatic mode is decaying. But when the coupling is switched on, the second term in \( \Gamma \) can become very large, resulting in \( n_+ \gg 1 \). For a range of \( \alpha \) and \( w \), which is much larger in the \( b = 0 \) case, oscillatory behaviour can also result (due to \( \Delta \) becoming negative). The instability means these coupled models suffer from all the problems outlined in \([8]\) for \( \beta \neq 0 \) models, unless the value of \( \alpha \) is made small enough. The closer \( w \) is to \(-1\), the smaller \( \alpha \) must be made to avoid the instability. This is in contrast to \( \beta \neq 0 \) models, which are unstable no matter how small the parameter \( \beta \) is made.

In the matter dominated era, we can carry out the same procedure, this time with \( \mathcal{H} = 2\tau^{-1} \). We find,

\[
\frac{\Gamma}{2(1 + w)} \approx -9/2 + \frac{(1 + b)\alpha}{1 + w} + 3(1 + w) + O(1 + w)^2, \tag{28}
\]

\[
\Delta \approx 4(1 + 3b)\alpha^2 + 12(5b - 3)\alpha(1 + w) + O(1 + w)^3. \tag{29}
\]

Once again, the second term in \( \Gamma \) can result in a rapidly growing dark energy fluctuation.

We have solved equations (10) – (20) numerically in the matter dominated regime (Figure 1), where we need not worry about the radiation fluid and its perturbations. The analytical agreement is excellent until the mode leaves the horizon \((k\tau \sim 1)\). Numerically we see that when this happens the mode begins to oscillate with a growing amplitude.

V. SUB-HORIZON EVOLUTION IN THE MATTER AND RADIATION DOMINATED ERAS

In the sub-horizon limit, \( \mathcal{H}^2/k^2 \ll 1 \), equation (10) yields,

\[
\delta_x'' + 3\mathcal{H}(1 - w_x)\delta_x + (1 + w_x)\theta_x - 3(1 + w_x)\Phi' = 0. \tag{30}
\]

Note the two terms on the right-hand side of equation (10) scale as \( \mathcal{H}^2/k^2 \). As these are the only two terms containing the coupling parameter \( \alpha \), the simplified equation above does not contain the coupling parameter.

One of the perturbed Einstein equations simplifies to Poisson’s equation in comoving coordinates,

\[
-k^2\Psi = 4\pi G\alpha^2 (\rho_x\delta_x + \rho_c\delta_c). \tag{31}
\]

Without the coupling, the dark energy perturbations are significantly suppressed on small-scales in comparison to dark matter perturbations, primarily due to its large
speed of sound [12]. The coupling does nothing to alter this fact unless the right-hand side of equation (17) makes a significant contribution. If the early time instability has been avoided this cannot be the case, as \( \alpha/(1+w) \) will be small. Thus it is reasonable to expect the dark energy to remain suppressed on sub-horizon scales. We therefore neglect dark energy perturbations for the remainder of this section.

By combining equations (19) and (20), we eliminate \( \theta_c \) and find a second-order equation for the growth of the matter density perturbation. From the above argument, we have neglected dark energy perturbations. Keeping only the dominant gravitational terms,

\[
\delta_c'' + \mathcal{H} \left( 1 + 2 \alpha \frac{\rho_c}{\rho_c} \right) \delta_c' + \frac{\alpha \rho_c}{\rho_c} \left( \mathcal{H}' - \mathcal{H}^2 (\alpha + 3w - 1) \right) \delta_c = -k^2 \Psi. \tag{32}
\]

We note that in the limit of \( \alpha \to 0 \), this reduces to the standard growth equation, with the well known growing mode \( \delta_c \propto \tau^2 \) in both matter and radiation eras. The additional terms are proportional to \( \alpha r \) (recall \( r \) is the ratio of dark energy to dark matter). In the matter dominated regime, then \( \alpha r \ll 1 \), and these terms will be negligible. Even when \( r \sim 1 \), the terms will be suppressed by the size of \( \alpha \), which will be small itself. The dominant effect causing a deviation from standard linear growth of structure in the matter dominated regime will therefore be, as in an uncoupled cosmology, the influence of the dark energy upon the expansion rate. The growth of structure in a coupled model can therefore be treated in the matter dominated regime simply as an uncoupled model with an effective dark energy equation of state parameter \( w_{\text{eff}} = w + \alpha/3 \). This will cease to be true only when the background energy density of matter is no longer well approximated by its usual \( \rho_c \propto a^{-3} \) dependence, and the late time scaling behaviour becomes apparent.

VI. SUB-HORIZON EVOLUTION IN THE DARK ENERGY DOMINATED ERA

The coupling between dark energy and dark matter eventually leads to a constant ratio between the two dark components. With a small value of \( \alpha \), the dark energy still dominates. We consider the evolution of structure once this equilibrium has been reached.

Friedmann’s equation solves to yield

\[
\mathcal{H} = 2(\alpha + 3w + 1)^{-1}\eta^{-1}, \tag{33}
\]

with the new time variable \( \eta = \tau - \tau_\infty \). Note that as \( \eta \) increases (\( \tau \) decreases and approaches \( \tau_\infty \)), the scale-factor increases. The constant of integration, \( \tau_\infty \), is the radius of the de Sitter event-horizon in the uncoupled case with a cosmological constant. The growth equation can then be written as,

\[
\eta^2 \delta_c'' + \frac{2 - 12w - 4\alpha}{\alpha + 3w + 1} \delta_c' + \frac{2(3\alpha + 9w - 1)(3w + \alpha) + \alpha w}{(\alpha + 3w + 1)^2} \delta_c = 0. \tag{34}
\]

This admits power law solutions, \( \delta_c \propto \eta^m \), where

\[
m = \frac{5}{2} - \frac{3}{1 + 3w + \alpha} \pm \frac{1}{2} \sqrt{1 - \frac{8\alpha}{w(1 + 3w + \alpha)^2}}. \tag{35}
\]

In the range of \( \alpha \) and \( w \) relevant to the problem, then \( m > 0 \). Recalling that \( \eta \) decreases with increasing scale-factor, we see that the universe becomes steadily more homogeneous as it expands. We interpret this to be a combination of two effects. The first is the accelerating expansion, which slows and (without the coupling) eventually stops structure formation. This occurs, for example, in ΛCDM cosmology when the cosmological constant becomes dominant. The second effect is that dark energy is constantly being transformed into dark matter, via the coupling. As the rate is proportional to the density of the dark energy, and the dark energy density is essentially uniform, new dark matter is also created uniformly. This rising ‘background’ of dark matter reduces the relative value of the fluctuations, reducing \( \delta_c \).

We have also investigated both numerically and analytically the extreme late time behaviour, where \( k\eta \ll 1 \) and the modes can be thought of as leaving the horizon. We find the tend toward homogeneity continues, but with a much milder rate of decay.

VII. CONCLUSIONS

We have shown that constant \( w \) models with the same form of energy-momentum exchange considered by [8] suffer from an instability with \( \alpha \neq 0 \), even if \( \beta = 0 \). However the instabilities in these models are not as severe as those facing models with \( \beta \neq 0 \). There is at least some non-trivial region of parameter space where the instability can be avoided, although the value of \( \alpha \) is now constrained both from background observables [2] and from stability requirements to be extremely small. Despite this, any non-zero value of \( \alpha \) will lead to a late-time scaling regime, alleviating (even if not solving) the coincidence problem. It is unfortunate that with \( \alpha \) constrained to such small values, we find any observable trace upon the growth of CDM structure will be negligible. Detecting a coupling of this form from measurements of large-scale structure is extremely doubtful, even with the precision promised by future experiments.
We have said nothing up to this point of models of dark energy with a variable equation of state parameter, such as scalar-field (quintessence) models. The same caveats in [8] apply here. Much of the above analysis will not apply in variable $w$ models, although some parameterisations such as the often used $w = w_0 + (1 - a)w_a$ lead to fixed $w$ over large periods of time. Our analysis will apply during those epochs of constant $w$. We refer interested readers to recent work on quintessence with couplings of this or similar form (such as recent work [13, 14]).

The future decay of dark matter fluctuations is an interesting result. It implies observers today find themselves close to the time of maximum inhomogeneity. The more the coincidence problem is alleviated, the closer to the late time scaling regime today becomes, and thus the closer to the peak of inhomogeneity. Without the coupling, observers find themselves at the time of the end of structure growth. The root cause in both cases is the acceleration of the universe only beginning today. Our position as apparently privileged observers in this fashion remains difficult to explain in any satisfactory way.

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Note added in proof: after submission we became aware of the work by [15] in which an independent analysis of general instabilities in coupled models was carried out, as well as more specifically the model considered in this work. They provide cosmological parameter constraints for negative values of $\alpha$ and find a non-trivial region consistent with observations.

[1] G. Hinshaw, J. L. Weiland, R. S. Hill, N. Odegard, D. Larson, C. L. Bennett, J. Dunkley, B. Gold, M. R. Greason, N. Jarosik, et al., ArXiv e-prints (2008), astro-ph/0803.0732.

[2] D. J. Eisenstein, I. Zehavi, D. W. Hogg, R. Scoccimarro, M. R. Blanton, R. C. Nichol, R. Scranton, H.-J. Seo, M. Tegmark, Z. Zheng, et al., Astrophys. J. 633, 560 (2005), astro-ph/0501171.

[3] S. Perlmutter, G. Aldering, G. Goldhaber, R. A. Knop, P. Nugent, P. G. Castro, S. Deustua, S. Fabbro, A. Goobar, D. E. Groom, et al., Astrophys. J. 517, 565 (1999), astro-ph/9812133.

[4] A. G. Riess, A. V. Filippenko, P. Challis, A. Clocchiatti, A. Diercks, P. M. Garnavich, R. L. Gilliland, C. J. Hogan, S. Jha, R. P. Kirshner, et al., American Astronomical Journal 116, 1009 (1998), astro-ph/9805201.

[5] L. P. Chimento, Journal of Mathematical Physics 38, 2565 (1997), physics/9702029.

[6] J. D. Barrow and T. Clifton, Phys. Rev. D 73, 103520 (2006), gr-qc/0604063.

[7] M. Quartin, M. O. Calvão, S. E. Jorás, R. R. Reis, and I. Waga, Journal of Cosmology and Astro-Particle Physics 5, 7 (2008), astro-ph/0802.0546.

[8] J. Valiviita, E. Majerotto, and R. Maartens, JCAP 0807, 020 (2008), astro-ph/0804.0232.

[9] T. Koivisto, Phys. Rev. D 72, 043516 (2005), astro-ph/0504571.

[10] C.-P. Ma and E. Bertschinger, Astrophys. J. 455, 7 (1995), astro-ph/9401007.

[11] J.-H. He, B. Wang, and E. Abdalla (2008), gr-qc/0807.3471.

[12] R. Bean and O. Doré, Phys. Rev. D 69, 083503 (2004), astro-ph/0307100.

[13] S. Chongchitnan (2008), astro-ph/08105411.

[14] P. S. Corasaniti, Physical Review D 78, 083538 (2008), astro-ph/08081646.

[15] M. B. Gavela, D. Hernandez, L. L. Honorez, O. Mena, and S. Rigolin (2009), astro-ph/09011611.