Evaluation of the Slip Sinkage and its Effect on the Compaction Resistance of an Off-Road Tracked Vehicle

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Abstract: When an off-road tracked vehicle travels, shearing action and ground sinkage occur on the soil–track interface, severely affecting the tractive performance of the vehicle. Notably, ground sinkage, which is induced by the vehicle’s weight (static sinkage) and longitudinal forces in the direction of travel producing slip (slip sinkage), develops motion resistance, directly restricting the tracked vehicle’s performance. Thus, it is critical to consider both static sinkage and slip sinkage to assess the tractive performance of a tracked vehicle. In this research, model track experiments were conducted to investigate slip sinkage. The experimental results showed that the slip sinkage increased as the slip ratio increased, but the rate of increase decreased. The slip sinkage was found to increase as the density of the ground decreased and imposed vertical load increased. The experimental results were used to calculate normalized slip sinkage, and an empirical equation for slip sinkage in terms of slip ratio was developed. This equation will allow vehicle operators to predict the slip sinkage and associated motion resistance for given soil and vehicle conditions.

Keywords: slip sinkage; off-road tracked vehicle; track system; model track experiment; motion resistance

1. Introduction

A tracked vehicle uses a track system composed of track plates and grousers as its mobility system. While the track system has lower drive autonomy than the wheel system, it provides a large contact area and lowers ground contact pressure, enabling better traction [1]. The track system is generally used in heavy vehicles (e.g., excavator, crawler tractor, and armed vehicle). Unlike road vehicles, which utilize the entire engine torque through sufficient ground strength as the driving force, the tractive performance of tracked vehicles, which drive on unpaved off-roads/terrains (hereinafter “off-road tracked vehicles”), is hindered by the shearing action and ground sinkage that occur on soil–track interfaces [2]. As the ground sinkage causes motion resistance that directly interferes with driving the off-road tracked vehicle, it is important to assess and limit ground sinkage [3].

Ground sinkage is categorized into static sinkage, which is induced by the weight of a vehicle, and slip sinkage, which occurs when the track system imposes a shearing load on the ground. Since Bekker [4] pioneered the soil–track interaction theory, research for predicting ground sinkage has mainly focused on static sinkage. Bekker [4] and Kogure et al. [5] demonstrated that it is possible to calculate static sinkage through an elastic settlement assessment method, assuming the track system as the shallow foundation that supports the off-road vehicle’s weight. In addition, various researchers [2,3,6,7] have proposed empirical equations for ground contact pressure and static sinkage based on the results of bevameter
pressure–sinkage experiments performed on various types of ground. Wong and Huang [8] and Baek et al. [9] used these equations to calculate the static sinkage of off-road tracked vehicles under various ground and vehicle conditions, which were applied to assess motion resistance.

References to slip sinkage are first found in Bekker’s terramechanic book. Bekker [4] noted that the trim effect (or slip sinkage effect), first considered a result of soil pumping under the track system, was estimated at 5°. Similarly, Janosi and Hanamoto [10] stated that the sinkage is linearly proportional with the horizontal distance between an arbitrarily chosen point on the track and the leading point of the interface, and adopted the equation proposed by Weiss [11] to calculate the maximum sinkage (i.e., sinkage at the end point of the interface). These studies [4,10] imply the presence of slip sinkage, but the effect of slip ratio (or slip displacement) is not taken into account in the calculation of the sinkage. Several researchers [12–15] developed slip sinkage models that include the effect of the slip ratio. Reece [14] proposed a slip sinkage formula as a function of slip ratio and grouser height. Vasil’ev et al. [15] developed an analytical model that took into account soil deformation propagation depth, particles trajectories, and volume changes under the track system in the shearing process. Lyasko [13] extended the work of Ksenovich et al. [12], and expressed the slip sinkage using slip ratio and static sinkage based on the law of conservation energy. However, these slip sinkage models [12–15] do not account for changes in slip sinkage along the track system; the slip sinkage increases proportionally with the horizontal distance from the leading point of the soil–track interface [10].

To overcome these limitations, this study conducted model track experiments using different surcharge stresses (7.0 kPa, 10.6 kPa, and 14.0 kPa) in silty sand comprising three different relative densities (40%, 60%, and 80%). This allowed a comprehensive assessment of the effect of various ground and load conditions on slip sinkage. From the experiment results, we then proposed an equation for calculating the slip sinkage of off-road tracked vehicles as a function of static sinkage, slip ratio, and the horizontal distance from the leading point of the soil–track interface. The equation is designed to assess the motion resistance of off-road tracked vehicles in silty sand. Particular attention was given to slip sinkage while assessing the motion resistance of off-road tracked vehicles.

2. Ground Sinkage and Motion Resistance of Off-Road Tracked Vehicles

The track system, namely, the mobility system of off-road tracked vehicles, transfers driving torque and vehicle weight (W) to the ground (Figure 1). The driving torque shears the ground and the vehicle’s weight exerts a vertical force. From the view point of terramechanics, driving torque and vehicle weight can be expressed by longitudinal and vertical force on the soil–track interface, respectively [2]. The tractive performance of off-road tracked vehicles is reduced by the shearing action and sinkage caused by these forces.

![Figure 1. Schematic diagram of the ground deformation (settlement and slip displacement) and motion resistances.](image-url)
In the soil–track interaction theory of Bekker [4], ground shear displacement caused by the track system is represented by slip displacement \((j)\). Slip ratio \((i)\) is defined as the relative extent of \(j\) that occurs when the track system moves longitudinally by a certain distance \((x)\). \(i\) is calculated as shown in Equation (1).

\[
i = \frac{j}{x}
\]  

(1)

\(i\) is determined by the ground’s shear characteristics. Assuming the ground is homogeneous, \(i\) of the entire soil–track interface is constant. In this case, as shown in Equation (1), \(j\) varies with \(x\) (i.e., the position of the soil–track interface in proportion to the longitudinal distance by which the track system moves).

When off-road tracked vehicles drive at non-zero slip, ground sinkage \((z)\) can be expressed as the sum of static sinkage \((z_0)\), which occurs due to vehicle weight, and slip sinkage \((z_j)\), which occurs due to ground shearing (i.e., when \(j\) occurs), as shown in Equation (2) [16].

\[
z_t = z_0 + z_j
\]  

(2)

Ground sinkage causes motion resistances such as compaction resistance \((R_c)\) and bulldozing resistance \((R_b)\) on soil–track interfaces, thereby limiting the tractive performance of off-road tracked vehicles (Figure 1).

\(R_c\) occurs because part of the force transmitted to the ground through the track system contributes to ground sinkage. The work consumed in compacting the ground equal to the area of the soil–track interface is equal to the work used in \(R_c\) when towing the track system longitudinally by the soil–track contact length. This work can be assessed as shown in Equation (3) [3].

\[
R_{cl} =wl\int_0^{z_t}(W/wl)dz
\]  

(3)

Here, \(w\) and \(l\) represent the track system width and soil–track contact length, respectively. \(R_c\) is the resistance caused by ground deposited at the front of the track system when driving the off-road tracked vehicle. Only static sinkage occurs in the front portion of the off-road tracked vehicle; here, slip displacement and slip sinkage do not occur \((j = z_j = 0)\). Thus, \(R_c\) is the force required to push the ground corresponding to \(z_0\) of the front of the track system (i.e., the force required to overcome passive earth pressure) using a plate of the same width as that of the track system. \(R_c\) can be expressed using Equation (4), which uses the Rankine passive earth pressure theory.

\[
R_c = 2w\int_0^{z_0}(\gamma K_p z + 2c\sqrt{K_p})dz
\]  

(4)

Here, \(\gamma\) is the unit weight of the soil, \(c\) is the cohesion of the soil, and \(K_p\) is the Rankine passive earth pressure coefficient determined by the internal friction angle \((\phi)\) of the soil. The total motion resistance that occurs in off-road tracked vehicle driving can be assessed as the sum of \(R_c\) and \(R_b\), which are calculated independently using Equations (3) and (4).

To sum up, the motion resistance of the off-road tracked vehicle occurs due to ground sinkage. In particular, the compaction resistance is affected by static sinkage and slip sinkage. However, due to the current lack of theoretical background research on assessing slip sinkage, most studies considered only static sinkage when calculating motion resistance. Motion resistance is severely underestimated when considering only static sinkage [14], in which case the off-road tracked vehicle’s performance is overestimated. To accurately predict the sinkage and motion resistance of a vehicle for a given soil and vehicle conditions, the slip sinkage effect should be taken into account.

3. Experiment Program

3.1. Experiment Equipment

The model track experiments in this study were carried out in a cuboid soil chamber with a width of 800 mm, a length of 1000 mm, and a height of 500 mm. As shown in Figure 2, movable partition walls were installed in the soil chamber in a length and width direction, and the width and
length of the model ground were adjusted to 650 mm and 450 mm, respectively. Additionally, taking into account that the critical depth layer of the off-road tracked vehicle on coarse-grained soils and fine-grained soils empirically proposed in WES [17] is 180 mm and 360 mm, respectively, the height of the model ground was set at 400 mm.

Figure 2. Schematic diagram of the laboratory soil chamber used in this study.

The mobility system of the off-road tracked vehicle is a continuous-track system comprising several connected single-track systems (Figure 3). As tractive performance manifests independently in each single-track system [18,19], this study considers only one single-track system. A commercial single-track system, which is produced by Hyundai steel and is widely used in heavy crawler tractors, was selected as a full-scale prototype. In order to consider the size effect, Iai’s similitude law for a 1g gravitational field [20] was applied to the material properties of the single-track system to create a downscaled model. Based on the basic equations that govern the behavior of soil-structure-fluid systems, Iai [20] derived the scale factors of each engineering property using three independent parameters such as the geometric scale factor (λ), the scale factor for density of soil (λρ), and the scale factor for strain of soil (λε). In this study, little differences of density and strain between the prototype and model were expected, given the relatively small geometry scale factor (λ) of 1.6 used to fabricate the model track system; thus, both λρ and λε were assumed to be unity (= 1.0) [21–24]. Consequently, the scale factors of all engineering properties were derived in terms of λ, which are identical to the scale factors for geotechnical application suggested by Gibson [25].

Figure 3. Schematic diagram of the mobility system for heavy-weight tracked vehicles.
Figure 4 and Table 1 show the schematic diagram of the model track system and the material properties of the full-scale prototype and model single-track system, respectively. In line with Reece [16] and Park [26], the width of the track system was judged to have an insignificant effect on the occurrence of off-road tracked vehicle slip sinkage. Thus, this value was reduced from 900 mm to 247.7 mm (562.5 mm to 154.8 mm for the model track system), thereby reducing the surcharge mass and longitudinal load to be applied in the model experiment. The final material properties of the model track system were set as follows: width \((w)\) and length \((l_t)\) of 154.8 mm, thickness \((t)\) of 9 mm, and grouser height \((h)\) of 44.6 mm. The system was fabricated using steel plates.

![Figure 4. Schematic diagram of the model track system used for the model track experiments in this study.](image)

**Table 1.** Material properties of the full-scale prototype and model track system.

| Scale Factor (Prototype/Model) | Full-Scale Prototype | Model \((\lambda = 1.6)\) |
|-------------------------------|----------------------|---------------------------|
| Shape Ratio, \(l/h\)          | 1                    | 3.47                      | 3.47                      |
| Width, \(w\) (mm)             | \(\lambda\)          | 247.7 (900)               | 154.8                     |
| Length, \(l_t\) (mm)          | \(\lambda\)          | 247.7                     | 154.8                     |
| Height, \(h\) (mm)            | \(\lambda\)          | 71.4                      | 44.6                      |
| Thickness, \(t\) (mm)         | \(\lambda\)          | 14.4                      | 9.0                       |
| Vertical Stress, \(\sigma_q\) (kPa) | \(\lambda\)       | 11.2 (68.8 kg)           | 7.0 (16.8 kg)             |
|                               |                      | 17.0 (104.0 kg)          | 10.6 (25.4 kg)            |
|                               |                      | 22.4 (137.6 kg)          | 14.0 (33.6 kg)            |

We calculated the vertical stress induced when off-road tracked vehicles with ground pressures of 11.2 kPa, 17.0 kPa, and 22.4 kPa were evenly distributed across full-scale prototype track systems. Applying Iai’s similitude law [20], the vertical stresses to be applied in the model experiments were determined to be 7.0 kPa, 10.6 kPa, and 14.0 kPa. These stress values were applied to the model track system in the form of dead-weight loads of 16.8 kg, 25.4 kg, and 33.6 kg, respectively. In addition, using a displacement-controlled longitudinal loading device, longitudinal load applied to the track system (i.e., driving torque of off-road tracked vehicles) was simulated. During the model experiments, the vertical and longitudinal displacement of the model track system was measured once per second using a linear variable differential transformer (LVDT), CDP-50, manufactured by the Tokyo Sokki Kenkyujo Company. Then, vertical and longitudinal displacement were measured and used to assess slip sinkage and slip displacement, respectively.
### 3.2. Model Ground

Residual soil acquired from the Mt. Gwanak region (hereinafter referred to as Gwanak soil) was used for the model test. Gwanak soil is weathered granite soil that covers more than two-thirds of the Korean Peninsula [27]. Its basic material properties are shown in Table 2. The collected Gwanak soil samples showed a passing percentage of 31.9% through a #200 sieve, with a silty sand (SM) classification according to the Unified Soil Classification System (USCS). Particles with a diameter of 4.75 mm or greater (#4) were removed as they were deemed to have an excessive impact on the model experiment. Thereafter, the samples were used to compose the model ground (about 1% of the total weight of the collected samples).

| Table 2. Properties of Gwanak soil. |
|-------------------------------------|
| Unified Soil Classification System Classification | SM | Maximum Void Ratio | 1.1 |
| Coefficient of Uniformity | 29.4 | Minimum Void Ratio | 0.4 |
| Mean Grain Size (mm) | 0.3 | Specific Gravity | 2.6 |
| Percent Finer than #200 Sieve (%) | 31.9 | Optimum Moisture Content (%) | 12.7 |

With the moist tamping method [28], under an optimum moisture content of 12.7%, the model ground was constructed to be 40%, 60%, and 80% of relative densities, which represented loose, medium, and dense states of the ground [29]. Dry density of the model ground was 14.1 kN/m³ (40%), 15.2 kN/m³ (60%), and 16.6 kN/m³ (80%), respectively. And the cohesion and internal friction angle of the model ground evaluated through the direct shear tests were 6.1 kPa and 30.7° (40%), 8.8 kPa and 36.4° (60%), and 11.1 kPa and 38.3° (80%), respectively. To ensure the repeatability of model tests, cone resistance in multiple locations were measured using a portable cone penetrometer, developed by WES [17] and consisting of a 30° cone with 3.23 cm² based area. For model ground constructed with the same relative density, the consistency of the model ground was confirmed within a 5% margin of error.

### 3.3. Experiment Procedure and Conditions

As shown in Figure 5, the homogeneous model ground created through the above process was trimmed to a form of the soil block and soil channel where the target track system shears off. The model track system was then installed on the model ground. As ground shearing occurs independently due to the single-track system during off-road tracked vehicle driving, the ground after the track-loaded area in the traveling direction was removed to exclude the effects of this area [18]. A schematic drawing of the model track experiment (i.e., the shape of the trimmed model ground and experiment set-up) is presented in Figure 6.

![Figure 5. Preparation of model ground (top view): (left) before trimming, and (right) after trimming.](image-url)
The experiments were conducted in two stages, as follows.

1. A surcharge mass simulating the weight of the off-road tracked vehicle was installed on the model track system, and the vertical displacement generated in the model track system was measured to determine the static sinkage.

2. Longitudinal load (3% strain/min) simulating the driving torque of an off-road tracked vehicle was applied to the model track system, while vertical and longitudinal displacement were measured using the LVDT to determine slip sinkage and slip displacement, respectively. Vertical displacement (i.e., slip sinkage) was measured at the center point of the surcharge mass during the experiments.

In this study, the experiment program was designed to assess the effect of off-road tracked vehicle weight and ground conditions on slip sinkage. A total of nine model track experiments were conducted under three different surcharge stresses (7.0 kPa, 10.6 kPa, and 14.0 kPa) and three different relative densities of the model ground (40%, 60%, and 80%). During the experiments, vertical and longitudinal displacements, which correspond to the slip sinkage and slip displacement, respectively, were recorded at every second via a static data logger. The shape of the model ground after conducting model track experiments was also observed.

4. Experiment Results and Discussion

4.1. Failure Surface

As slip sinkage of the off-road tracked vehicle occurs due to shearing action from the track system, it is necessary to observe the shearing failure surface of the soil–track interface to assess slip sinkage. Given that the skin friction between the soil and steel plate (track system material) is negligible when compared to longitudinal load applied to the track system, the shearing action of model ground is only considered in this study. Figures 7 and 8 show the top view of the shape of the failure surface after conducting the model track experiments. The failure surface of the soil–track interface was identical regardless of surcharge mass and relative density of the model ground. As representatively shown in Figure 8, under all experiment conditions, a soil block of which the shape was identical to that of the track system was formed and sheared off while the slight crack was observed in the surrounding soil. That is, failure was observed along the bottom and side of the block, showing a tendency similar to that observed in past research [18].
Observation of the soil–track interface failure surface shows that the slip sinkage is caused by the vertical displacement that results from the shearing of the soil block. This was similar to the vertical displacement caused by the volumetric change of the sample during the soil direct shear test. However, unlike the direct shear sample, in which all the sides are rigidly constrained, the soil block is rigidly constrained only on the side on which the grouser is located. Therefore, we concluded that the vertical displacement of the sample during slip sinkage and the direct shear test would not show the exact same tendency.

4.2. Observation of the Failure Surface

Figure 9 shows the slip sinkage ($z_j$)-slip displacement ($j$) relationships assessed through the vertical and longitudinal displacement measured in the model experiments. (Downward slip sinkage is denoted using positive values); here, a Savitzky–Golay filter [30] was applied for the purpose of smoothing the data.
Figure 9. Slip sinkage and slip displacement relationships for (a) $D_r = 40\%$, (b) $D_r = 60\%$, and (c) $D_r = 80\%$.

Excluding the slight upward slip sinkage that occurred at the beginning of shearing for T4 (relative density of 60\% and vertical stress of 7.0 kPa), downward slip sinkage occurred under all test conditions. The extent of the downward slip sinkage increased with slip displacement. However, the rate of increase gradually decreased. This behavior is similar to the tendency of the increase in vertical displacement of the soil sample as shearing progressed in the direct shear test. However, while upward vertical displacement is normally observed in the direct shear test for dense soil due to the volumetric expansion [31], in this experiment, downward slip sinkage also occurred in soil with a
relative density of 80%. This is because unlike the sample of the direct shear test, in which all sides are rigidly constrained, the soil block is rigidly constrained only on the side on which the grouser is located, resulting in a volumetric change in all unconstrained directions.

The slip sinkage in the model test increased as the surcharge mass increased and the relative density of the soil decreased. As mentioned above, as the slip sinkage of the off-road tracked vehicle increases, compaction resistance, a type of motion resistance, also increases. Therefore, we expect that it will be difficult to ensure tractive performance when a heavy-weight off-road tracked vehicle is driven on loose soil due to the increasing effect of compaction resistance caused by slip sinkage.

4.3. Evaluation of the Normalized Slip Sinkage

To quantitatively assess the effect of surcharge mass and the relative density of the soil on slip sinkage, we assessed normalized slip sinkage, defined as the ratio of slip sinkage to static sinkage. Table 3 shows the measured static sinkage ($z_0$) under each model experiment condition. This value was used to calculate normalized slip sinkage. Figure 10 shows the relationship between normalized slip sinkage ($z_j/z_0$) and slip displacement ($j$).

Table 3. Static sinkage of model track experiments.

| Vertical Stress, $\sigma_q$ (kPa) | Static Sinkage, $z_0$ (mm) |
|-----------------------------------|----------------------------|
|                                   | $D_r = 40\%$ | $D_r = 60\%$ | $D_r = 80\%$ |
| 7.0                               | 1.96         | 1.37         | 1.04         |
| 10.6                              | 2.99         | 2.00         | 1.56         |
| 14.0                              | 3.75         | 2.69         | 2.09         |
As shown in Figure 10, normalized slip sinkage increased with slip displacement; similar behavior was shown under different surcharge stress and relative density of the soil conditions. Therefore, slip sinkage can be predicted relatively simply and accurately by applying the static sinkage amount of the off-road tracked vehicle assessed under specific conditions and normalized slip sinkage–slip displacement relationship presented in Figure 10. Many researchers [2,3,6,7] have proposed methods to calculate off-road tracked vehicle static sinkage; the total ground sinkage (static sinkage+slip sinkage) that occurs in the soil–track interface is obtained using the proposed normalized slip sinkage (= slip sinkage / static sinkage).

5. Empirical Equation for Slip Sinkage of a Track System

During off-road tracked vehicle driving, normalized slip sinkage was similar regardless of changes in surcharge mass and relative density of the soil; the normalized slip sinkage increased with slip displacement and gradually converged. Accordingly, normalized slip sinkage in this study is shown as a function of slip displacement, allowing slip sinkage to be predicted based on the static sinkage calculated according to driving conditions (Figure 11).

Considering that the normalized slip sinkage–slip displacement relationship shows a sigmoid function form, a regression analysis was conducted with various sigmoid functions (Boltzmann,
Logistic, and Weibull). The Weibull function showed the best regression results, as evidenced by the highest $R^2$ (Equation (5)), which was then used to evaluate the normalized slip sinkage–slip displacement relationship equation.

$$y = A_2(1 - e^{-(k(x-A_1))^p})$$  \hspace{1cm} (5)

Here, $A_1$ and $A_2$ represent the minimum and maximum values of the graph, respectively, while $k$ and $p$ are constants determined by the shape of the curve. As shown in Figure 11, the constants of Equation (5) were determined after assessing the best-fit curve, that is, the curve that best predicts the normalized slip sinkage–slip displacement relationship. The determined constants were substituted into Equation (5). The following equation to evaluate the slip displacement of the off-road tracked vehicle was proposed.

$$z_j = z_0(2.78 - 2.78e^{(-0.009j^{1.77})})$$  \hspace{1cm} (6)

Here, the unit of $j$ is mm. As mentioned earlier, the extent of slip displacement ($j$) occurring in one continuous-track system is proportional to the longitudinal distance traveled by the track system ($x$). As confirmed by the model experiment results, slip sinkage increases with slip displacement; it increases with the longitudinal distance traveled by the track system. By substituting Equation (1) into Equation (6), Equation (7) can be used to estimate the slip sinkage according to the longitudinal position ($x$) and slip ratio ($i$) of the track system.

$$z_j = z_0(2.78 - 2.78e^{(-0.009(x)^{1.77})})$$  \hspace{1cm} (7)

Here, the unit of $x$ is mm. That is, as shown in Figure 12, slip sinkage does not occur at the frontmost part ($x = 0$) of the soil–track interface; it occurs predominantly at the end part ($x = l$) of the soil–track interface. On the other hand, static sinkage due to vehicle weight ($z_0$) occurs evenly throughout the entire soil–track interface. Therefore, the total ground sinkage ($z_t$) increases by the amount of slip sinkage ($z_j$) induced when the track system moves longitudinally.

To verify the applicability of the slip sinkage assessment results and proposed equation, additional model experiments were carried out using two model track systems with material properties different from those used thus far in this research. The properties of model track systems for verification are shown in Table 4. Soil samples with relative densities similar to those used thus far (40% and 80%) were used for the model ground. The slip sinkage–slip displacement curve was assessed from the verification experiment results obtained using the same procedure as described above. Figure 13 shows the comparison between experimental values and those predicted using the proposed model (Equation (6)).
Table 4. Material properties of model track systems used for the verification experiments.

|                          | Model (a) | Model (b) |
|--------------------------|-----------|-----------|
| Shape Ratio, $l/h$       | 2         | 5         |
| Width, $w$ (mm)          | 154.8     | 154.8     |
| Length, $l$ (mm)         | 89.2      | 223.0     |
| Height, $h$ (mm)         | 44.6      | 44.6      |
| Thickness, $t$ (mm)      | 9.0       | 9.0       |
| Vertical Stress, $\sigma_q$ (kPa) | 7.0 (9.7 kg) | 7.0 (24.2 kg) |

Figure 13. Comparison between experimental values and the values predicted by the proposed model: (a) Model (a); and (b) Model (b).

Similar to Figure 10, downward slip sinkage occurred in all the verification experiments, and the extent of slip sinkage increased with slip displacement although the rate of increase showed a gradual reduction. In addition, the proposed equation predicted slip sinkage in the model experiment results relatively accurately, with a maximum error of 21%. Therefore, we conclude that the results of this study can be applied to the assessment of off-road tracked vehicle slip sinkage on silty sand.

6. Evaluation of Compaction Resistance Considering Slip Sinkage

Slip sinkage affects compaction resistance, a type of off-road tracked vehicle motion resistance that occurs across the entire soil–track interface. The slip sinkage equation proposed in this study was used to assess off-road tracked vehicle compaction resistance, taking into account the effect of slip sinkage. As bulldozing resistance is caused by ground deposited at the front of the track system where slip sinkage does not occur, its dependency on the slip sinkage is not dealt with in this section.
As mentioned earlier (refer to Figure 1), slip sinkage is a function of the longitudinal distance \(0 \leq x \leq l\) traveled by the track system, which is expressed in Equation (3) as a double integral.

\[
R_c l = w \int_0^l \int_0^{2l} (W/wl) \, dz \, dx
\]  

(8)

According to Bekker [3], \((W/wl)\) is a function of \(k\) (sinkage modulus) and \(n\) (sinkage exponent), which are empirical pressure–sinkage parameters determined using the bevameter test.

\[
W/wl = kz^n
\]  

(9)

Substituting Equations (7) and (9) in Equation (8) results in the following formula:

\[
R_c = \frac{wkz_0^{n+1}}{l(n+1)} \int_0^l \left(3.78 - 2.78e^{-0.009(iz)^{1.77}}\right)^{n+1} \, dx
\]  

(10)

Moreover, assuming that \(R_{c0}\) represents compaction resistance when only static sinkage occurs, the integration range of Equation (3) can be defined as static sinkage, as follows.

\[
R_{c0} l = w \int_0^{z_0} (W/wl) \, dz
\]  

(11)

Substituting Equation (9) into Equation (11) provides the following formula:

\[
R_{c0} = \frac{wkz_0^{n+1}}{n + 1}
\]  

(12)

By substituting Equation (12) into Equation (10), the compaction resistance of the off-road tracked vehicle considering slip sinkage can be expressed as a function of \(R_{c0}\), sinkage exponent \(n\), soil–track contact length \(l\), and slip ratio \(i\). The following equation can be numerically integrated with respect to given driving conditions to quantitatively assess the compaction resistance of the off-road tracked vehicle.

\[
R_c = \frac{R_{c0}}{l} \int_0^l \left(3.78 - 2.78e^{-0.009(iz)^{1.77}}\right)^{n+1} \, dx
\]  

(13)

Based on the proposed evaluation method for compaction resistance, we parametrically investigated the effects of \(n\), \(l\), and \(i\) on compaction resistance considering the slip sinkage. In this study, \(R_c\) was normalized with respect to \(R_{c0}\) by dividing both sides of Equation (13) by \(R_{c0}\). A parametric study was then conducted. Considering that \(n\) of silty sand ranges from approximately 0.5 (loose state) to 2.0 (dense state) \([2,32–34]\), we set \(n\) as 0.5, 1.0, 1.5, and 2.0. \(l\) of a commercial crawler tractor approximately ranges from 2000–5000 mm. Thus, \(l\) was set to 2000, 3000, 4000, and 5000 mm for the parametric study. According to Wong \([2]\), it is preferable to maintain an off-road tracked vehicle slip ratio of below 0.2. Moreover, a slip ratio of 0.2 is usually used as a common basis to compare the performances of off-road tracked vehicles. Accordingly, we considered \(i\) from 0–0.2. The parameter ranges considered in this study are summarized in Table 5.

| Parameter | Range          |
|-----------|----------------|
| \(n\)     | 0.5, 1.0, 1.5, 2.0 |
| \(l\) (mm) | 2000, 3000, 4000, 5000 |
| \(i\)     | 0–0.2          |

Figure 14 presents plots of the normalized compaction resistance considering the slip sinkage \(R_c/R_{c0}\) with respect to \(i\) when \(l = 2000, 3000, 4000\), and 5000 mm, with \(n = 0.5\) (Figure 14a), 1.0 (Figure 14b), 1.5 (Figure 14c), and 2.0 (Figure 14d). Under all conditions, \(R_c/R_{c0}\) increased gradually from 1 as \(i\) increased. This is because as \(i\) increases, the function in the integral sign of Equation (13) increases. In particular, at \(i = 0.2\), a common basis to compare the performances of off-road tracked vehicles \([2]\), \(R_c/R_{c0}\) converges to 3.78\(n + 1\). Accordingly, under the analytical conditions applied in this study, overall compaction resistance \(R_c\) was 7.35 (\(n = 0.5\)) to 54.01 (\(n = 2.0\)) times greater than compaction...
resistance $R_c$ due to static sinkage. These results indicate that the effect of slip sinkage should be considered when assessing the tractive performance of off-road tracked vehicles and that tractive performance can be overestimated when compaction resistance is calculated considering only static sinkage. When excessive slip sinkage is expected with a large $i$, it is necessary to apply a proper method to suppress the effect of slip sinkage such as unconventional track suspension systems [35–38].
Figure 14. Normalized compaction resistance considering slip sinkage $R_c / R_{c0}$: (a) $n = 0.5$, (b) $n = 1.0$, (c) $n = 1.5$, and (d) $n = 2.0$.

The larger $n$ and $l$, the larger $R_c / R_{c0}$. Moreover, the extent of compaction resistance $R_{c0}$ induced by static sinkage decreases as $n$ and $l$ increase. In this case, an increase in $R_c / R_{c0}$ does not translate into an increase of the absolute extent of overall compaction resistance, but this result indicates that the effect of slip sinkage on compaction resistance increases. That is, even as the soil becomes dense and the soil–track contact length extends (i.e., as $n$ and $l$ increase), thereby reducing off-road tracked vehicle static sinkage ($z_0$) and $R_{c0}$, compaction resistance induced by slip sinkage ($z_i$) can still greatly affect tractive performance. Considering that slip sinkage is caused by ground shearing, not only ground sinkage characteristics but also shear characteristics should be included when calculating the compaction resistance of off-road tracked vehicles.

7. Conclusions

In this study, we conducted model track experiments to assess the slip sinkage induced during an off-road tracked vehicle executing its mission. The model track system was installed on silty sand of different relative densities (40%, 60%, and 80%), while three different vertical stresses (7.0 kPa, 10.4 kPa, and 14.0 kPa) and longitudinal loads were applied, thereby comprehensively assessing the effects of various ground and load conditions on slip sinkage. We derived the following conclusions.

1. In the model track experiments, the soil block was formed and failure occurred along the bottom and side ground of the block regardless of the ground density and vertical stress conditions. That is, slip sinkage of the off-road tracked vehicle was induced by vertical displacement caused by shearing action in the soil block formed on the soil–track interface. This is similar to the phenomenon in which vertical displacement of the sample occurs in the soil direct shear test. However, as the soil block is rigidly constrained on only the side with grousers, it did not show the exact same tendencies.

2. Downward slip sinkage occurred in the soil–track interface under all experimental conditions. The extent of slip sinkage gradually increased with slip displacement, though the rate of increase gradually decreased. In addition, slip sinkage increased as surcharge mass increased and relative density of the soil decreased. We expect that it will be difficult to ensure tractive performance for heavy-weight off-road tracked vehicles driving on loose ground.

3. Normalized slip sinkage, defined as the ratio of slip sinkage to static sinkage, which is caused by vehicle weight, increases with slip displacement. However, it is not greatly affected by surcharge mass or relative density of the soil. From the experiment results, we proposed an equation for assessing the slip sinkage of off-road tracked vehicles. We used the proposed equation to predict the results of the additional model experiments for verification and confirmed that the slip sinkage of off-road tracked vehicles can be predicted relatively accurately using the proposed formula.
We proposed a method for assessing the compaction resistance of off-road tracked vehicles considering the effect of slip sinkage using the proposed slip sinkage equation. A parametric study was conducted taking into account the driving conditions of the off-road tracked vehicle. Overall compaction resistance was estimated to be about 7.35–54.01 times higher than when static sinkage alone was considered. These results indicate that the effect of slip sinkage should be considered when assessing the tractive performance of off-road tracked vehicles, and that tractive performance can be overestimated when compaction resistance is calculated considering only static sinkage.

We expect that the results of this study can be used to predict the slip sinkage of off-road tracked vehicles driving on silty sand, which can then be used as a basis to assess motion resistance. By evaluating the static sinkage of off-road tracked vehicles using existing research, the extent of slip sinkage can be predicted quantitatively through the proposed formula. However, as the above conclusions are based on the results of experiments conducted only on silty sand, the results of this study should be verified and supplemented by additional studies under various soil conditions. Especially for clayey soils, slip sinkage is manifested in different ways because the shape of the failure surface on the soil–track interface (i.e., shear failure mode) is completely different from that of silty sand [39]. Also, caution should be exercised when applying the results of this study to fast-moving tracked vehicles that exert dynamic loads to the ground surface; slip sinkage may differently appear. Further experiments conducted under various conditions are required to verify the outcomes of this study, through a full-scale test.

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