BRST and Anti-BRST Symmetries in Perturbative Quantum Gravity

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Abstract In perturbative quantum gravity, the sum of the classical Lagrangian density, a gauge fixing term and a ghost term is invariant under two sets of supersymmetric transformations called the BRST and the anti-BRST transformations. In this paper we will analyse the BRST and the anti-BRST symmetries of perturbative quantum gravity in curved spacetime, in linear as well as non-linear gauges. We will show that even though the sum of ghost term and the gauge fixing term can always be expressed as a total BRST or a total anti-BRST variation, we can express it as a combination of both of them only in certain special gauges. We will also analyse the violation of nilpotency of the BRST and the anti-BRST transformations by introduction of a bare mass term, in the massive Curci-Ferrari gauge.

Keywords BRST · Anti-BRST · Perturbative quantum gravity

1 Introduction

Three out of the four fundamental forces in nature are described by Yang-Mills theories. The fourth, being gravity, is described by gauge theory of diffeomorphism [1]. In this sense all the forces of nature can be formulated in the language of gauge theory.

However, when analysing with any gauge theory, we have to deal with the redundant degrees of freedom due to gauge invariance of that theory. We have to eliminate these redundant degrees of freedom before trying to quantize that theory. An elegant formalism called the BRST formalism is usually employed for this purpose [2]. In this formalism the sum of the classical Lagrangian density, a gauge fixing term and a ghost term (collectively called a gauge fixing Lagrangian density in this paper) is
invariant under a set of supersymmetric transformations called the BRST transformations. This total Lagrangian density is also invariant under another set of supersymmetric transformations called the anti-BRST transformations [3].

The BRST and the anti-BRST symmetries for perturbative quantum gravity in four dimensional flat spacetime have been studied by a number of authors [4–6] and their work has been summarized by N. Nakanishi and I. Ojima [7]. The BRST symmetry in two dimensional curved spacetime has been thoroughly studied [8–10]. The BRST and the anti-BRST symmetries for topological quantum gravity in curved spacetime have also been studied [11, 12]. All this work has been done in linear gauges.

However BRST and anti-BRST symmetries are known to have a richer structure in Yang-Mills theories. In case of Yang-Mills theories, it is known that in Landau gauge we can express the gauge fixing Lagrangian density as a combination of total BRST and total anti-BRST variations [13]. This is also achieved by addition of suitable non-linear terms to the gauge fixing Lagrangian density [14]. Furthermore, the addition of a bare mass term breaks the nilpotency of the BRST and the anti-BRST transformations and this leads to the violation of unitarity of the resultant theory [15].

In this paper we will try to generalize these results that are known in the context of Yang-Mills theories in four dimensional flat spacetime to perturbative quantum gravity in curved spacetime, in arbitrary dimensions. It may be noted that the violation of unitarity did not have much physical relevance in the context of Yang-Mills theories. However it is suspected that certain quantum gravitational processes might lead to violation of unitarity [16]. So this loss of unitarity, due to the addition of a bare mass term, seems to be physically more relevant to quantum gravity in curved spacetime than Yang-Mills theories in flat spacetime.

2 BRST and Anti-BRST Transformations

The Lagrangian density for pure Euclidean gravity with cosmological constant $\lambda$ is given by

$$\mathcal{L} = \sqrt{g}(R - 2\lambda),$$

where we have adopted units, such that

$$16\pi G = 1.$$ (2)

In perturbative gravity one splits the full metric $g_{ab}^F$ into the metric for the background spacetime $g_{ab}$ and a small perturbation around it being $h_{ab}$. The covariant derivatives along with the lowering and raising of indices are compatible with the metric for the background spacetime. The small perturbation $h_{ab}$ is viewed as the field that is to be quantized.

All the degrees of freedom in $h_{ab}$ are not physical as the Lagrangian density for it is invariant under a gauge transformation,

$$\delta_L h_{ab} = \nabla_a \Lambda_b + \nabla_b \Lambda_a + \xi(\Lambda) h_{ab},$$ (3)