Effect of Soret and Dufour numbers on double diffusive mixed convection boundary layer flow induced by a shrinking sheet

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Abstract. The numerical study of double diffusive mixed convection boundary layer flow over a shrinking sheet are presented in the presence of Soret and Dufour effects. Shrinking velocity, wall temperature and wall concentration are assumed to have exponential function forms. Non-similarity method is applied for the governing basic equations (flow, momentum, energy and concentration equations) before they are solved numerically using bvp4c program in Matlab software. The numerical results of velocity, temperature and concentration are reported graphically for various values of suction parameter, dimensionless coordinate along the plate parameter, shrinking parameter, mixed convection parameter, buoyancy ratio parameter, Soret parameter and Dufour parameter. Since dual solutions is obtained, stability analysis is performed to select the stable solution.

1. Introduction

The study of thermodynamics is beneficial as its concepts are being applied to engineering [1,2]. In engineering applications, the mixed convection in fluid flow due to a stretching/shrinking sheet will produce desired findings. In the sectors of science and engineering, mixed convection flows (combination of forced and natural convection) are applied in spray and flash dying, crystal growth, float gas production, combustion of atomized liquid fuels, thermal–hydraulics of nuclear reactors, the cooling of sophisticated electronics and heat exchanger devices, and dehydration operations in chemical processing [3]. On the other hand, the flow due to a stretching or shrinking sheet plays a role in cooling of metallic plate in a bath, glass fibre and paper production, extrusion of polymer sheet
from a die, and continuous casting [4]. Therefore, a lot of reports regarding to the exponential variations of stretching/shrinking velocity and temperature distributions have been published [5–11].

In the process in fluid flow induced by temperature and concentration differences, Soret and Dufour effects must not be neglected. Mass transfer induced by temperature differences is known as Soret effect, whereas Dufour effect refers to the heat transfer caused by concentration gradient in the fluid. From experimental reports, the diffusion of matter and heat due to Soret and Dufour effects become effective when the gradients of these two elements are very large. When the gradient of the temperature and the concentration are large, the coupled interaction between Soret and Dufour effects are sufficiently great [12]. As a results, many researchers explored the impacts of these to parameters in their model of the fluid flow in the presence of stretching/shrinking sheet [13–18]. They considered linear, nonlinear and exponential variations of stretching/shrinking velocity, temperature and concentration distributions.

In numerical solutions, the existence of multiple solutions are able to be obtained. These multiple solutions always occur in the case of convection occurred due to stretching/shrinking sheet. Stability analysis is used to determine the most stable solution, and the other is declared as unstable. This analysis can be performed by using the bvp4c programme from MATLAB. This analysis calculates the growth of disturbances for dual solutions, where the solution with initial decay of disturbance labeled as stable solution (first solution). Besides, unstable solution (second solution) owns initial growth of disturbance. The application of stability analysis in the related cases (Soret and Dufour impacts) have been done by some researchers [19–20] for unsteady state. This method also have been applied in the case of fluid flow occurred by the movement of exponential stretching or shrinking sheet [21].

This paper applies the problem formulation developed by Srinivasacharya and RamReddy [13], and stability analysis is applied in the numerical solutions. The previous investigators [13] studied the case of the sheet is stretched and impermeable, whereas this paper focuses on the sheet when it is shrieked and the fluid is allowed to pass through the sheet. The variations of velocity, temperature and concentration in this problem are exponential [13–14,15]. To the best of the authors’ knowledge, the stability analysis of the impacts of Soret and Dufour parameters in the case of mixed convection over an exponentially flat stretching/shrinking sheet is not yet to be reported in the literature. The governing equations are transformed to ordinary differential equations using dimensionless similarity transformation parameter and are solved numerically by Matlab. Computed numerical results are performed through graphs of velocity temperature and concentration profiles.

2. Mathematical formulation and stability analysis

2.1. Mathematical formulation

Consider the two-dimensional incompressible, viscous and electrically conducting fluid over an exponentially permeable stretching/shrinking surface with the effect of Soret and Dufour parameters (Figure 1). The $x$-axis runs along the shrinking surface in the direction opposite to the sheet motion and the $y$-axis is perpendicular to it. The viscous fluid is placed at $y > 0$. Under the assumption of Boussinesq and boundary layer approximations, the problem is governed by the following equations [13]:

$$
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
$$

(1)

$$
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g \beta_T(T - T_o) + g \beta_C(C - C_o)
$$

(2)
where \( u \) and \( v \) are the components of velocity in the \( x \) and \( y \) directions, \( \nu = \mu / \rho \) is the kinematic viscosity, \( \mu \) is the viscosity, \( \rho \) is the fluid density, \( g \) is the acceleration due to gravity, \( \beta_r \) is the coefficient of thermal expansion, \( \beta_C \) is the coefficient of solutal expansions, \( T \) is the temperature of the fluid, \( C \) is the concentration of the fluid, \( \alpha \) is the thermal diffusivity, \( D \) is the solutal diffusivity of the medium, \( K_T \) is the thermal diffusion ratio, \( C_x \) is the concentration susceptibility, \( C_P \) is the specific heat at constant pressure and \( T_m \) is the mean fluid temperature. The subscripts of \( w \) and \( \infty \) at parameters \( T \) and \( C \) indicate the conditions at the wall and at the outer edge of the boundary layer.

The appropriate boundary conditions are

\[
\begin{align*}
    u &= u_w(x) = \lambda U_0 \exp(x/L), \\
    v &= v_w(x), \\
    T_w(x) &= T_w + T_0 \exp(x/2L), \\
    C_w(x) &= C_w + C_0 \exp(x/2L), \\
    u &\to 0, \quad T \to T_w, \quad C \to C_w \\
    \text{as} \quad y &\to \infty,
\end{align*}
\]

where \( \lambda < 0 \) is the shrinking parameter and \( v_w(x) < 0 \) is the wall mass suction velocity. The term \( \exp(x/2L) \) in exponential temperature distribution \( T_w \) and exponential concentration distribution \( C_w \) are used [13-14,15] in this problem for the case of two-dimensional flow with the presence of Soret-Dufour effects and when concentration equation included in governing equations. The assumption of temperature and concentration functions are to make sure the numerical results will satisfy final boundary condition after substituting similarity variables.

**Figure 1.** Physical model and coordinate system

In this problem, the stream function is stated as \( \psi(x,y) = (2\nu U_0)^{1/2} \exp(x/2L) f(\eta) \), where

\[
u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} \quad (6)
\]

Introducing new similarity variables:
\[ \theta(\eta) = \frac{T - T_\infty}{T_n - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \quad \eta = y \left( \frac{U_0}{2 \nu L} \right)^{1/2} \exp(x/2L), \quad (7) \]

\[ u = U_0 \exp(x/L) f'(\eta), \quad v = \left( \frac{U_0}{2L} \right) \exp(x/2L) \left[ f(\eta) + \eta f'(\eta) \right], \]

where prime denotes differentiation with respect to \( \eta \).

When the governing equations (2)‒(4) and boundary conditions (5) are substituted by similarity variables, it will be transformed to the following form:

\[ f''' + ff'' - 2\left( f' \right)^2 + 2Ri \left[ \exp \left( \frac{-3X}{2} \right) \right] (\theta + N\phi) = 0. \quad (8) \]

\[ \frac{1}{Pr} \theta'' - f\theta' - f'\theta + Db \phi' = 0. \quad (9) \]

\[ \frac{1}{Sc} \phi'' + f\phi' - f'\phi + Sr \theta' = 0. \quad (10) \]

\[ f'(\eta) = \lambda, \quad f(\eta) = S, \quad \theta(\eta) = 1, \quad \phi(\eta) = 1 \quad \text{at} \quad \eta = 0, \]

\[ f'(\eta) \to 0, \quad \theta(\eta) \to 0, \quad \phi(\eta) \to 0 \quad \text{as} \quad \eta \to \infty, \quad (11) \]

where \( Ri = Gr/Re^2 \) is the mixed convection parameter, \( Gr = g \beta_T (T_0 - T_\infty) L^3/\nu^2 \) is the thermal Grashof number, \( Re = u_0 L/\nu \) is the Reynolds number, \( X = x/L \) dimensionless coordinate along the plate parameter, \( L \) is the characteristic length of the plate, \( N = \beta_C (C_0 - C_\infty)/\beta_T (T_0 - T_\infty) \) is the buoyancy ratio, \( Pr = \nu/\alpha \) is the Prandtl number, \( Sc = \nu/D \) is the Schmidt number, \( Sr = DK_T (T_0 - T_\infty)/T_m \nu (C_0 - C_\infty) \) is the Soret number, \( Db = DK_T (C_0 - C_\infty)/C_s C_p \nu (T_0 - T_\infty) \) is the Dufour number and \( S = \left( \nu_0 (x)/\exp(x/2L) \right) (2L/\nu U_0)^{1/2} \) is the suction parameter.

### 2.2. Stability analysis

Since the numerical results obtain 2 solutions, stability analysis is performed to select the stable one. The first step for performing stability analysis is to consider an unsteady state for governing equations (2)‒(4). Then, the new dimensionless time variable \( \tau \) is introduced in similarity variables for exponential variation of sheet velocity [21]. Consequently, the following equations are obtained:

\[ \frac{\partial u}{\partial \tau} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g \beta_T (T - T_\infty) + g \beta_C (C - C_\infty) \quad (12) \]

\[ \frac{\partial T}{\partial \tau} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{DK_T}{C_s C_p} \frac{\partial^2 C}{\partial y^2} \quad (13) \]

\[ \frac{\partial C}{\partial \tau} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} + \frac{DK_T}{T_m} \frac{\partial^2 T}{\partial y^2} \quad (14) \]
\[ \theta(\eta, \tau) = \frac{T - T_w}{T_w - T_c}, \quad \phi(\eta, \tau) = \frac{C - C_w}{C_w - C_c}, \quad \tau = \frac{U_0}{2L} \left( \exp\left(\frac{x}{L}\right) \right) t, \]

\[ \eta = y \left( \frac{U_0}{2\nu L} \right)^{\frac{1}{2}} \exp\left(\frac{x}{2L}\right), \quad u = U_0 \exp\left(\frac{x}{L}\right) \frac{\partial}{\partial \eta} f(\eta, \tau), \]

\[ v = -\frac{\eta}{2L} \sqrt{2\nu L U_0} \exp\left(\frac{x}{2L}\right) \frac{\partial}{\partial \eta} f(\eta, \tau) - \frac{U_0}{2L^2} \exp\left(3x/L\right) t \sqrt{2\nu L U_0} \frac{\partial}{\partial \tau} f(\eta, \tau) \]

\[ - \frac{1}{2L} \left( \frac{U_0}{\nu} \right)^{\frac{1}{2}} \exp\left(\frac{x}{2L}\right) f(\eta, \tau). \]

By substituting equation (15) into equations (12)–(14), then these equations are transformed as below:

\[ \frac{\partial^3 f}{\partial \eta^3} + f \frac{\partial^3 f}{\partial \eta^2} - 2 \left( \frac{\partial f}{\partial \eta} \right)^2 + 2 \Re \left( \exp\left(\frac{-3X}{2}\right) \right) \phi + 2 \Re \left( \exp\left(\frac{-3X}{2}\right) \right) \phi - \frac{\partial^2 f}{\partial \eta^2} \tau = 0 \]

\[ + 2 \tau \frac{\partial}{\partial \tau} \frac{\partial^2 f}{\partial \eta^2} - 2 \tau \frac{\partial}{\partial \tau} \frac{\partial f}{\partial \eta} \tau = 0 \]

subject to boundary conditions with the existence of \( \tau \):

\[ \frac{\partial f}{\partial \eta}(\eta, \tau) = 0, \quad \theta(\eta, \tau) = 0, \quad \phi(\eta, \tau) = 0 \quad \text{as} \quad \eta \to \infty, \]

\[ f(\eta, \tau) = 0, \quad \theta(\eta, \tau) = 0, \quad \phi(\eta, \tau) = 0 \quad \text{at} \quad \eta = 0. \]

The analysis developed by Weidman et al. [22] are stated as below, to determine the stability of the solution

\[ f(\eta, \tau) = f_0(\eta) + \left( \exp(-\gamma \tau) \right) F(\eta, \tau), \]

\[ \theta(\eta, \tau) = \theta_0(\eta) + \left( \exp(-\gamma \tau) \right) G(\eta, \tau), \]

\[ \phi(\eta, \tau) = \phi_0(\eta) + \left( \exp(-\gamma \tau) \right) H(\eta, \tau), \]

where \( \gamma \) denotes an unknown eigenvalue while \( F(\eta, \tau), G(\eta, \tau) \) and \( H(\eta, \tau) \) are small relative to \( f_0(\eta), \theta_0(\eta) \) and \( \phi_0(\eta) \). Substituting equation (20) into equations (16)–(18), and take \( \tau = 0 \), the following linear eigenvalue problem are obtained:

\[ F_0'' + f_0 F_0' + f_0'' F_0 - 4 f_0' F_0' + 2 \Re \left( \exp\left(\frac{-3X}{2}\right) \right) (G_0 + NH_0) = \gamma F_0' = 0 \]

\[ \frac{1}{Pr} G_0'' + f_0 G_0' + \theta_0' F_0 - \theta_0 F_0' - G_0 f_0' + D b H_0'' + \gamma G_0' = 0 \]
\[
\frac{1}{Sc} H''_0 + f_0 H'_0 + \phi'_0 F_0 - \phi F'_0 - H_0 f'_0 + Sr G''_0 + \gamma H'_0 = 0
\]  
(23)

along with the boundary conditions:
\[
F'_0(\eta) = 0, \quad F_0(\eta) = 0, \quad G_0(\eta) = 0, \quad H_0(\eta) = 0 \quad \text{at} \quad \eta = 0,
\]
\[
F'_0(\eta) \to 0, \quad G_0(\eta) \to 0, \quad H_0(\eta) \to 0 \quad \text{as} \quad \eta \to \infty,
\]  
(24)

According to Harris et al. [23], the range of possible eigenvalues can be determined by relaxing a boundary condition on \( F_0(\eta), G_0(\eta) \) and \( H_0(\eta) \). For the present problem, we relax the boundary condition \( F'_0(\eta) \to 0 \) as \( \eta \to \infty \). Finally, the system of equations (21)–(23) subject to equation (24) along with the new boundary condition \( F''_0(\eta) = 1 \) is solved.

3. Results and discussion

Numerical computations have been performed for the velocity profile \( f'(\eta) \), temperature profile \( \theta(\eta) \) and concentration profile \( \phi(\eta) \) for various values of physical parameters, such as suction \( S \), dimensionless coordinate along the plate \( X \), shrinking \( \lambda \), mixed convection \( Ri \), buoyancy ratio parameter \( N \), Soret \( Sr \) and Dufour \( Db \). In the present study, the following values are adopted for the numerical computations: \( S = 3.1, \quad X = 1, \quad \lambda = -0.4, \quad Ri = 1, \quad N = 0.5, \quad Sr = 1.8 \) and \( Db = 0.1 \). These values are used throughout the calculations unless otherwise mentioned. In addition, the appropriate values of \( S, X, \lambda, Ri, N, Sr \) and \( Db \) are used in MATLAB routine bvp4c, to obtain the profiles which satisfy the boundary conditions. First solution is presented by solid line, whereas dashed line denoted as second solution.

In order to validate the present numerical method used in this paper, we have compared the present results with those obtained by Magyari and Keller [5] and Srinivasacharya and RamReddy [13] for values of wall-temperature gradient \(-\theta'(0)\). This physical parameter is calculated by using the formulation \( Nu_x/\sqrt{Re_x} = -\sqrt{X/2} \theta'(0) \). The comparison is tabulated in Table 1. The bvp4c programme is used to calculate the values of wall-temperature gradient. As a conclusion, the present values of \(-\theta'(0)\) are in good agreement with those reported by the previous investigators. Therefore, the good comparison proves that our bvp4c MATLAB is applicable to present our numerical computation.

Figures 2–8 shows the velocity profiles \( f'(\eta) \) for different values of \( S, X, \lambda, Ri, N, Sr \) and \( Db \). From Figures 4–8, the magnitude of velocity is increased due to the increment of \( \lambda, Ri, N, Sr \) and \( Db \) for first solution. The instantaneous velocity is reduced by the increment of \( S \) and \( X \), as shown in Figures 2–3. For second solution and for high parameters \( S \) and \( X \), the instantaneous negative velocity decreases and reaches minimum peak, then it increases until it reaches zero value. The decrement of negative velocity denotes the increment of velocity magnitude. After that, velocity magnitude is slowing down in opposite direction, until it reaches stationary state. In addition, the velocity values decrease at the point far from shrinking sheet for the addition of parameters \( \lambda, Ri, N, Sr \) and \( Db \) (Figures 4–8).
The temperature profiles \( \theta(\eta) \) for various values of \( S, X, \lambda, Ri, Nr, \) and \( Db \) are presented by Figures 9–15, respectively. For variation of temperature of first solution is continuously decrease until it reaches zero value, controlled by the distance from the shrinking sheet. Moreover, the temperature of second solution increases until it achieves maximum temperature. The maximum temperature is gained at the near point. Subsequently, temperature is measured as zero at the point far from shrinking sheet. The decrement of temperature for first solution is contributed by an augmentation of \( S, Ri, N, \) and \( Nr \) (Figures 9, 12–14). The effect of Dufour parameter \( Db \) on temperature, which showed in Figure 15 causes the pattern of temperature variation is non-uniform (decreases at close point and subsequently increases at far point).

Table 1. Comparison between wall-temperature gradient \( -\theta'(0) \) calculated by the present method and that of Magyari and Keller [5] and Srinivasacharya and RamReddy [13] for \( S = 0, \lambda = 1 \) and \( Ri = Sc = Nr = Db = N = X = 0 \).

| \( Pr \) | Magyari and Keller (1999) | Srinivasacharya and RamReddy (2011) | Present |
|---------|--------------------------|-----------------------------------|---------|
| 0.5     | -0.59434                 | -0.59438                          | -0.59466 |
| 1.0     | -0.95478                 | -0.95478                          | -0.95478 |
| 3.0     | -1.86908                 | -1.86908                          | -1.86907 |
| 5.0     | -2.50014                 | -2.50015                          | -2.50013 |
| 8.0     | -3.24213                 | -3.24218                          | -3.24212 |
| 10.0    | -3.66038                 | -3.66043                          | -3.66037 |
Figure 4. Variation of velocity for various $\lambda$.

Figure 5. Variation of velocity for various $Ri$.

Figure 6. Variation of velocity for various $N$.

Figure 7. Variation of velocity for various $Sr$.

Figure 8. Variation of velocity for various $Db$. 
Figure 9. Variation of temperature for various $S$.

Figure 10. Variation of temperature for various $X$.

The graphs of the concentration profiles $\phi(\eta)$ are depicted by Figures 16–22, respective to the controlling parameters $S, X, \lambda, Ri, N, Sr$ and $Db$. From Figures 17, 18 and 21, the concentration values of first solution are ascended due to the improvement of the following: $X, \lambda$ and $Sr$. On the other hand, the concentration values of first solution become lessen due to the impact of $Ri$ and $N$ (Figures 19 and 20). However, the effect of $S$ and $Db$ are to increase the concentration $\phi(\eta)$ of first solution at the point close to the sheet (Figures 16 and 22). Furthermore, concentration profiles of second solution consistently increase due to the effect of $Ri$ and $Db$ (Figures 19 and 22). Moreover, the impact of $X, N$ and $Sr$ are to reduce the concentration value of second solution. These unchanging pattern of decrement can be seen in Figures 17, 20 and 21. Refer to the Figure 16, the concentration values of second solution are decrease when parameter suction $S$ is increased. However, these feature changes when the distance of the concentration point is far from the shrinking sheet ($\phi(\eta)$ is increased due to addition of $S$). The impact of shrinking parameter $\lambda$ on concentration values second solution (Figure 18) shows a reverse trend, compared to the impact of suction parameter $S$ (Figure 16).
Figure 11. Variation of temperature for various $\lambda$.

Figure 12. Variation of temperature for various $Ri$.

Figure 13. Variation of temperature for various $N$. 
In the case of second solution for all profiles, the effects of controlling parameters in this problem mostly depends on the distance from the shrinking sheet. For example: the impact of suction parameter $S$ is to reduce concentration rate, at the point close to the sheet (Figure 16). However, the condition at the point far from sheet is not supposed to follow the previous decrement of concentration affected by the same parameter. Furthermore, inconsistent pattern can be seen for all velocity profiles (Figures 2–8), temperature profiles (Figures 9, 11) and concentration profiles (Figures 18).

Figure 14. Variation of temperature for various $Sr$.

Figure 15. Variation of temperature for various $Db$.

Figure 16. Variation of concentration for various $S$.

Figure 17. Variation of concentration for various $X$. 
Stability analysis is tabulated in Table 2, which reports the smallest eigenvalues for first solution (positive eigenvalues) and second solution (negative eigenvalues). Therefore, the first solutions with positive smallest eigenvalues indicate that the solution is stable and physically reliable. Moreover, second solution absolutely has reverse characteristics compared to the first one.
Figure 21. Variation of concentration for various Sr.

Figure 22. Variation of concentration for various Db.

Table 2. Smallest eigenvalues $\gamma$ for several values of $S$ and $Ri$.

| $S$  | $Ri$      | $\gamma$        |         |
|------|-----------|-----------------|---------|
|      |           | First solution  | Second  |
|      |           |                  | solution|
| 1.75 | -0.01001  | 0.07766         | -0.07915|
|      | -0.01     | 0.07955         | -0.08097|
|      | -0.009999 | 0.08180         | -0.08341|
| 1.80 | -0.01001  | 0.28926         | -0.31038|
|      | -0.01     | 0.28978         | -0.31098|
|      | -0.009999 | 0.29031         | -0.31157|
4. Conclusion
The characteristics of the fluid flow due to a shrinking sheet with the exponential variation of temperature and concentration are reported, by the presence of Soret and Dufour effects. From the findings in the form of table and graphs, the main conclusions are as follows:

a) The effect of increasing values of parameters $\lambda$, $Ri$, $N$, $Sr$ and $Db$ are to increase velocity distribution for the first solution.

b) The temperature distribution by first solution in flow region increases with increasing the magnitude of parameters $X$, $\lambda$ and $Db$. The effect of Dufour parameter $Db$ to increase the temperature rate is effective when the instantaneous point is far from shrinking sheet.

c) It can be seen that larger $X$, $\lambda$ and $Sr$ values always imply higher values of concentration of first solution.

d) The variation of velocity, temperature and concentration values of second solution which affected by governed parameters in this problem show inconsistent pattern (increment and decrement). This pattern is depends on the location of the measured point from the shrinking sheet.

e) The calculation of positive eigenvalues for the first solution proved that it is stable and reliable.

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