Observation of Large CP Violation and Evidence for Direct CP Violation in $B^0 \to \pi^+\pi^-$ Decays

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Abstract

We report the first observation of $CP$ violation in $B^0 \rightarrow \pi^+ \pi^-$ decays based on 152 million $\Upsilon(4S) \rightarrow B\bar{B}$ decays collected with the Belle detector at the KEKB asymmetric-energy $e^+e^-$ collider. We reconstruct a $B^0 \rightarrow \pi^+ \pi^- CP$ eigenstate and identify the flavor of the accompanying $B$ meson from its decay products. From the distribution of the time intervals between the two $B$ meson decay points, we obtain $A_{\pi\pi} = +0.58 \pm 0.15$(stat) $\pm 0.07$(syst) and $S_{\pi\pi} = -1.00 \pm 0.21$(stat) $\pm 0.07$(syst). We rule out the $CP$-conserving case, $A_{\pi\pi} = S_{\pi\pi} = 0$, at a level of 5.2 standard deviations. We also find evidence for direct $CP$ violation with a significance at or greater than 3.2 standard deviations for any $S_{\pi\pi}$ value.

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In the standard model (SM) of elementary particles, CP violation arises from the Kobayashi-Maskawa (KM) phase \(^{1}\) in the weak interaction quark-mixing matrix. In particular, the SM predicts CP asymmetries in the time-dependent rates for \(B^0\) and \(\bar{B}^0\) decays to a common CP eigenstate \(^{2}\). Comparison between SM expectations and measurements in various CP eigenstates is important to test the KM model. The \(B^0 \rightarrow \pi^+\pi^-\) decay \(^{3}\), which is dominated by the \(b \rightarrow u\bar{d}\) transition, is of particular interest, and is sensitive to the CP-violating parameter \(\phi_2\). Direct CP violation may also occur in this decay because of interference between the \(b \rightarrow u\bar{t}d\) and \(b \rightarrow d\) penguin \((P)\) amplitudes \(^{4}\).

In the decay chain \(\Upsilon(4S) \rightarrow B^0\bar{B}^0 \rightarrow (\pi^+\pi^-)f_{\text{tag}}\), where one of the \(B\) mesons decays at time \(t_{\pi\pi}\) to the CP eigenstate \(\pi^+\pi^-\) and the other decays at time \(t_{\text{tag}}\) to a final state \(f_{\text{tag}}\) that distinguishes between \(B^0\) and \(\bar{B}^0\), the decay rate has a time dependence given by \(^{5}\)

\[
P_{\pi\pi}(\Delta t) = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}} [1 + q \cdot \{S_{\pi\pi} \sin(\Delta m_d \Delta t) + A_{\pi\pi} \cos(\Delta m_d \Delta t)\}],
\]

where \(\tau_{B^0}\) is the \(B^0\) lifetime, \(\Delta m_d\) is the mass difference between the two \(B^0\) mass eigenstates, \(\Delta t = t_{\pi\pi} - t_{\text{tag}}\), and the \(b\)-flavor charge \(q = +1\) \((-1)\) when the tagging \(B\) meson is a \(B^0\) \((\bar{B}^0)\). \(S_{\pi\pi}\) and \(A_{\pi\pi}\) are mixing-induced and direct CP-violating parameters, respectively.

Belle’s previous results for \(B^0 \rightarrow \pi^+\pi^-\) \(^{6}\), based on a 78 fb\(^{-1}\) data sample \((85 \times 10^6 B\bar{B}\) pairs), suggested large direct CP asymmetry and/or mixing-induced asymmetry while the result by the BaBar collaboration based on a sample of \(88 \times 10^6 B\bar{B}\) pairs did not \(^{7}\). In this Letter, we report a new measurement with an improved analysis that incorporates an additional 62 fb\(^{-1}\) for a total of 140 fb\(^{-1}\) \((152 \times 10^6 B\bar{B}\) pairs) that confirms Belle’s previous results with much greater significance.

The data were collected with the Belle detector \(^{8}\) at the KEKB asymmetric-energy \(e^+e^-\) collider \(^{9}\), which collides 8.0 GeV \(e^-\) and 3.5 GeV \(e^+\) beams. The \(\Upsilon(4S)\) is produced with a Lorentz boost of \(\beta\gamma = 0.425\) nearly along the electron beamline \((z)\). Since the \(B^0\) and \(\bar{B}^0\) mesons are approximately at rest in the \(\Upsilon(4S)\) center-of-mass system \((\text{cms})\), \(\Delta t\) can be determined from \(\Delta z\), the displacement in \(z\) between the \(\pi^+\pi^-\) and \(f_{\text{tag}}\) decay vertices:

\[
\Delta t \approx (z_{\pi\pi} - z_{\text{tag}})/\beta\gamma c \equiv \Delta z/\beta\gamma c.
\]

The reconstruction method of the vertex positions remains unchanged from the previous publication \(^{10}\).

We use oppositely charged track pairs that are positively identified as pions to reconstruct \(B^0 \rightarrow \pi^+\pi^-\) candidates. The pion efficiency is 91\% and 10.4\% of kaons are misidentified as pions. We select the \(B\) meson candidates using the energy difference \(\Delta E \equiv E_{B_{\text{beam}}}^{\text{cms}} - E_{\text{beam}}^{\text{cms}}\) and the beam-energy constrained mass \(M_{bc} \equiv \sqrt{(E_{\text{beam}}^{\text{cms}})^2 - (p_B^{\text{cms}})^2}\), where \(E_{\text{beam}}^{\text{cms}}\) is the beam energy, and \(E_{B_{\text{beam}}}^{\text{cms}}\) and \(p_B^{\text{cms}}\) are the cms energy and momentum of the \(B\) candidate. The signal region is defined as \(5.271 \text{ GeV/c}^2 < M_{bc} < 5.287 \text{ GeV/c}^2\) and \(|\Delta E| < 0.064 \text{ GeV}\), corresponding to \(\pm 3\sigma\) from the central values. To suppress the \(e^+e^- \rightarrow q\bar{q}\) continuum background \((q = u, d, s, c)\), we form signal and background likelihood functions, \(L_S\) and \(L_{BG}\), from the event topology, and impose requirements on the likelihood ratio \(LR = L_S/(L_S + L_{BG})\) for candidate events. We use the same event topology variables and the procedure that were used for the \(B(B^0 \rightarrow \pi^0\pi^0)\) measurement \(^{11}\).

The flavor of the accompanying \(B\) meson is identified from inclusive properties of particles that are not associated with the reconstructed \(B^0 \rightarrow \pi^+\pi^-\) decay. We use two parameters, \(q\) [defined in Eq. \(^{12}\)] and \(r\), to represent the tagging information. The parameter \(r\) is an event-by-event, MC-determined flavor-tagging dilution factor that ranges from \(r = 0\) for no flavor discrimination to \(r = 1\) for unambiguous flavor assignment. It is used only to sort
data into six $r$ intervals. The wrong tag fractions for the six $r$ intervals, $w_l$ ($l = 1, 6$), and differences between $B^0$ and $\bar{B}^0$ decays, $\Delta w_l$, are determined from data \[10\].

We optimize the expected sensitivity by using the improved likelihood ratio $LR$. We require $LR > 0.86$ for all $r$ intervals. We include additional candidate events with lower signal likelihood ratio cuts (0.50, 0.45, 0.45, 0.45, 0.45, and 0.20) for different $r$ intervals since the separation of continuum background from the $B$ signal varies with $r$; we accept candidate events from twelve distinct regions in the $LR-r$ plane.

![Figure 1: $\Delta E$ distribution in the $M_{bc}$ signal region for $B^0 \rightarrow \pi^+ \pi^-$ candidates with $LR > 0.86$.](image)

Figure 1 shows the $\Delta E$ distribution for the $B^0 \rightarrow \pi^+ \pi^-$ candidates that are in the $M_{bc}$ signal region with $LR > 0.86$ after flavor tagging and vertex reconstruction. In the $M_{bc}$ and $\Delta E$ signal region, we find 483 candidates with $LR > 0.86$ and 1046 candidates with $LR \leq 0.86$. The $B^0 \rightarrow \pi^+ \pi^-$ signal yield for $LR > 0.86$ is determined from an unbinned two-dimensional maximum likelihood fit to the $M_{bc}$-$\Delta E$ distribution ($5.20 \text{ GeV}/c^2 < M_{bc} < 5.30 \text{ GeV}/c^2$ and $-0.3 \text{ GeV} < \Delta E < 0.5 \text{ GeV}$) with a Gaussian signal function plus contributions from misidentified $B^0 \rightarrow K^+ \pi^-$ events, three-body $B$-decays, and continuum background. The fit yields $232^{+20}_{-19} \pi^+ \pi^-$ events and $82^{+14}_{-13} K^+ \pi^-$ events in the signal region, where the errors are statistical only. Extrapolating from the size of the continuum background in this fit, we expect 169 continuum events in the signal region. For $LR \leq 0.86$, the same procedure used in the previous publication \[5\] yields $141 \pm 12 \pi^+ \pi^-$ events, $50 \pm 8 K^+ \pi^-$ events and 855 continuum events in the signal region. The contribution from three-body $B$-decays is negligibly small in the signal region.

The $\Delta t$ resolution function $R_{\pi\pi}$ for $B^0 \rightarrow \pi^+ \pi^-$ signal events is formed by convolving four components: the detector resolutions for $z_{\pi\pi}$ and $z_{\text{tag}}$, the shift in the $z_{\text{tag}}$ vertex position due to secondary tracks originating from charmed particle decays, and the smearing due to
the kinematic approximation used to convert $\Delta z$ to $\Delta t$. We assume $R_{\pi\pi} = R_{K\pi}$ and denote them collectively as $R_{\text{sig}}$.

$A_{\pi\pi}$ and $S_{\pi\pi}$ are obtained from an unbinned maximum likelihood fit to the observed $\Delta t$ distribution. The probability density function (PDF) for $B^0 \rightarrow \pi^+\pi^- \text{ signal events (} P_{\text{sig}}^{\pi\pi}(\Delta t) \text{)}$ is given by Eq. (1) modified to incorporate the effect of incorrect flavor assignment. The PDF for $B^0 \rightarrow K^+\pi^-$ background events is $P_{\text{bg}}^{\pi\pi}(\Delta t, w_1, \Delta w_1) = \frac{1}{4B_{\pi\pi}} e^{-\Delta t/\tau_{\pi\pi}} \{1 - q\Delta w_1 + q \cdot (1 - 2w_1) \cdot A_{K\pi} \cdot \cos(\Delta m_q \Delta t)\}$. We use $A_{K\pi} = 0$ as a default and include an effect of possible non-zero value for $A_{K\pi}$ in the systematic error. The PDF for continuum background events is $P_{\text{cont}}(\Delta t) = (1 + q \cdot A_{\text{bkg}}) \{\frac{1}{2\tau_{\text{bkg}}} e^{-\Delta t/\tau_{\text{bkg}}} + (1 - f_r)\delta(\Delta t)\}/2$, where $f_r$ is the fraction of the background with effective lifetime $\tau_{\text{bkg}}$ and $\delta$ is the Dirac delta function. We use $A_{\text{bkg}} = 0$ as a default. A fit to sideband events yields $A_{\text{bkg}} = 0.010 \pm 0.005$. This uncertainty is included in the systematic error for $A_{\pi\pi}$ and $S_{\pi\pi}$. All parameters of $P_{\text{cont}}(\Delta t)$ and $R_{\text{cont}}$ are determined from the events in the sideband region.

We define the likelihood value for each ($i$th) event as a function of $A_{\pi\pi}$ and $S_{\pi\pi}$:

$$P_i = (1 - f_{\text{ol}}) \int_{-\infty}^{+\infty} \{f_{\text{sig}} P_{\text{sig}}^{\pi\pi}(\Delta t', w_i; A_{\pi\pi}, S_{\pi\pi})$$

$$+ f_{\text{bg}} P_{\text{bg}}^{\pi\pi}(\Delta t, w_i)\} \cdot R_{\text{sig}}(\Delta t_i - \Delta t')$$

$$+ f_{\text{cont}} P_{\text{cont}}(\Delta t') \cdot R_{\text{cont}}(\Delta t_i - \Delta t')d\Delta t' + f_{\text{ol}} P_{\text{ol}}(\Delta t_i).$$

Here the probability functions $f_k^m$ ($k = \pi\pi$, $K\pi$ or $q\bar{q}$) are determined on an event-by-event basis as functions of $\Delta E$ and $M_{bc}$ for each $LR-r$ interval (m = 1, 2) [3]. The small number of signal and background events that have large values of $\Delta t$ are accommodated by the outlier PDF, $P_{\text{ol}}$, with fractional area $f_{\text{ol}}$. In the fit, $S_{\pi\pi}$ and $A_{\pi\pi}$ are the only free parameters determined by maximizing the likelihood function $L = \Pi_i P_i$, where the product is over all $B^0 \rightarrow \pi^+\pi^-$ candidates.

The unbinned maximum likelihood fit to the 1529 $B^0 \rightarrow \pi^+\pi^-$ candidates (801 $B^0\tau$- and 728 $\not{B}^0\tau$-tags), containing 372$^{+32}_{-31}$ $\pi^+\pi^-$ signal events, yields $A_{\pi\pi} = +0.58 \pm 0.15(\text{stat}) \pm 0.07(\text{syst})$ and $S_{\pi\pi} = -1.00 \pm 0.21(\text{stat}) \pm 0.07(\text{syst})$. The correlation between $A_{\pi\pi}$ and $S_{\pi\pi}$ is 0.286. As in our previous publication [3], we quote the rms values of the $A_{\pi\pi}$ and $S_{\pi\pi}$ distributions of the MC pseudo-experiments as the statistical errors of our measurement [11].

The usual fit errors from the likelihood functions, called the MINOS errors in the previous publication [3], are $+0.15$ and $-0.16$ and $+0.22$ and $-0.20$ for $A_{\pi\pi}$ and $S_{\pi\pi}$, respectively, in good agreement with the rms values above [12]. In Figs. 2(a) and (b), we show the $\Delta t$ distributions for the 264 $B^0\tau$- and 219 $\not{B}^0\tau$-tagged events in the subset of data with $LR > 0.86$. We define the raw asymmetry in each $\Delta t$ bin by $A \equiv (N_+ - N_-)/(N_+ + N_-)$, where $N_{+(-)}$ is the number of observed candidates with $q = +1(-1)$. Figures 2(c) and (d) show the raw asymmetries for two regions of the flavor-tagging parameter $r$. The effective tagging efficiency and signal purity is much larger in the $0.5 < r \leq 1.0$ region.

We test the goodness-of-fit from a $\chi^2$ comparison of the results of the unbinned fit and the $\Delta t$ projections for $B^0 \rightarrow \pi^+\pi^-$ candidates. We obtain $\chi^2/\text{DOF} = 12.5/12 (7.6/12)$ for the $\Delta t$ distribution of the $B^0$ ($\not{B}^0$) tags.

An ensemble of MC pseudo-experiments indicates a 26.7% probability of measuring $CP$ violation at a level above the one we observe when the input values are $A_{\pi\pi} = +0.55$ and $S_{\pi\pi} = -0.84$, which correspond to the values at the point of maximum likelihood in the physically allowed region ($S_{\pi\pi}^2 + A_{\pi\pi}^2 \leq 1$); in this measurement it is located at the physical boundary ($A_{\pi\pi}^2 + S_{\pi\pi}^2 = 1$).
FIG. 2: The $\Delta t$ distributions for the 483 $B^0 \rightarrow \pi^+\pi^-$ candidates with $LR > 0.86$ in the signal region: (a) 264 candidates with $q = +1$, i.e. the tag side is identified as $B^0$; (b) 219 candidates with $q = -1$. (c) Asymmetry, $A$, in each $\Delta t$ bin with $0 < r \leq 0.5$ and (d) with $0.5 < r \leq 1.0$. The solid curves show the results of the unbinned maximum likelihood fit to the $\Delta t$ distributions of the 1529 $B^0 \rightarrow \pi^+\pi^-$ candidates.

The systematic error is primarily due to uncertainties in the vertexing ($\pm 0.04$ for $A_{\pi\pi}$ and $\pm 0.05$ for $S_{\pi\pi}$) and the background fractions ($\pm 0.03$ for $A_{\pi\pi}$ and $\pm 0.02$ for $S_{\pi\pi}$). We include the effect of tag side interference [13] on $A_{\pi\pi}(\pm 0.03)$ and $S_{\pi\pi}(\pm 0.01)$ in this analysis. Other sources of systematic error are uncertainties in the wrong tag fraction, physics parameters ($\Delta m_d$, $\tau_{B^0}$, and $A_{K\pi}$), resolution function, background modeling, and fit bias. We add each contribution in quadrature to obtain the total systematic errors. The effect of the 3% charge asymmetry in the kaon misidentification rate is negligibly small.

We perform a number of crosschecks. We measure the $B^0$ lifetime with the $B^0 \rightarrow \pi^+\pi^-$ candidate events. The result, $\tau_{B^0} = 1.46 \pm 0.09$ ps, is consistent with the world-average value [14]. A comparison of the event yields and $\Delta t$ distributions for $B^0$- and $\bar{B}^0$-tagged events in the sideband region reveals no significant asymmetry. We select $B^0 \rightarrow K^+\pi^-$ candidates by positively identifying the charged kaons. A fit to the 2358 candidates (1198 signal events) yields $A_{K\pi} = -0.02 \pm 0.08$, in agreement with the counting analysis [15], and $S_{K\pi} = 0.14 \pm 0.11$, which is consistent with zero. With the $K^+\pi^-$ event sample, we
determine $\tau_{B^0} = 1.52 \pm 0.06$ ps and $\Delta m_d = 0.53^{+0.04}_{-0.07}$ ps$^{-1}$, which are in agreement with the world average values $[1]$. We check the measurement of $A_{\pi \pi}$ using time-independent fits to the $M_{bc}-\Delta E$ distributions for the $B^0$ and $\overline{B}^0$ tags. We obtain $A_{\pi \pi} = +0.73 \pm 0.19$, which is consistent with the time-dependent $CP$ fit result. We also perform an independent analysis based on a binned maximum-likelihood fit to the $\Delta t$ distribution. The result is consistent with that of the unbinned maximum-likelihood fit quoted here.

The statistical significance of our measurement is determined from the same approach used in the previous publication $[5]$. Figure 3 shows the resulting two-dimensional confidence regions in the $A_{\pi \pi}$ vs. $S_{\pi \pi}$ plane. The case that $CP$ symmetry is conserved, $A_{\pi \pi} = S_{\pi \pi} = 0$, is ruled out at the 99.999976% confidence level (CL), i.e., $1 - CL = 2.5 \times 10^{-7}$, equivalent to 5.2$\sigma$ significance for Gaussian errors. The case of no direct $CP$ violation, $A_{\pi \pi} = 0$, is also ruled out with a significance at or greater than 3.3$\sigma$ for any $S_{\pi \pi}$ value. If the source of $CP$ violation is only due to $B$-$\overline{B}$ mixing or $\Delta B = 2$ transitions as in so-called superweak scenarios $[16]$, then $(S_{\pi \pi}, A_{\pi \pi}) = (-\sin 2\phi_1, 0)$. $1 - CL$ at this point is $8.4 \times 10^{-4}$, equivalent to 3.3$\sigma$ significance.

Adopting the notation of Ref. $[17]$, the range of $\phi_2$ that corresponds to the 95.5% CL region for $A_{\pi \pi}$ and $S_{\pi \pi}$ in Fig. 3 is $90^\circ \leq \phi_2 < 146^\circ$ for $0.15 < |P/T| < 0.45$ as used in the previous publication $[5]$ and $\sin 2\phi_1 = 0.736$ $[18]$. The result is in agreement with constraints on the unitarity triangle from other indirect measurements $[19]$. The 95.5% CL region for $A_{\pi \pi}$ and $S_{\pi \pi}$ excludes $|P/T| < 0.17$.

In summary, we have performed a new measurement of $CP$ violation parameters in $B^0 \rightarrow \pi^+\pi^-$ decays. We obtain $A_{\pi \pi} = +0.58 \pm 0.15$ (stat) $\pm 0.07$ (syst), and $S_{\pi \pi} = -0.53^{+0.04}_{-0.06}$. The curves show the contours for $1-CL = 3.17 \times 10^{-1}$ (solid), $4.55 \times 10^{-2}$ (dot-dashed), $2.70 \times 10^{-3}$ (dotted), $6.34 \times 10^{-5}$ (dashed), and $5.96 \times 10^{-7}$ (thick solid).
We rule out the $CP$-conserving case, $A_{\pi\pi} = S_{\pi\pi} = 0$, at the $5.2\sigma$ level. We find evidence for direct $CP$ violation with a significance at or greater than $3.2\sigma$. The constraints on $\phi_2$ from our result are consistent with indirect measurements that assume the correctness of the SM.

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