Weighted second-order cone programming twin support vector machine for imbalanced data classification

Saeideh Roshanfekr¹, Shahriar Esmaeili², Hassan Ataeian³, and Ali Amiri³

¹ Department of Computer Engineering and Information Technology, Amirkabir University of Technology, 424 Hafez Avenue, 15875-4413 Tehran, Iran
² Department of Physics and Astronomy, Texas A&M University, 4242 TAMU, University Dr., College Station, TX 77840, US
³ Department of Computer Engineering, University of Zanjan, University Blvd., 45371-38791 Zanjan, Iran

Abstract

We propose a method of using a Weighted second-order cone programming twin support vector machine (WSOCP-TWSVM) for imbalanced data classification. This method constructs a graph based under-sampling method which is utilized to remove outliers and reduce the dispensable majority samples. Then, appropriate weights are set in order to decrease the impact of samples of the majority class and increase the effect of the minority class in the optimization formula of the classifier. These weights are embedded in the optimization problem of the Second Order Cone Programming (SOCP) Twin Support Vector Machine formulations. This method is tested, and its performance is compared to previous methods on standard datasets. Results of experiments confirm the feasibility and efficiency of the proposed method.

1 Introduction

The imbalanced problem for classification methods is the basic issue of research in data mining. Datasets are said to be imbalanced if the samples belonging to majority class outnumbers the data samples belonging to the minority class. Many attempts have been made to deal with this problem in various context, such as credit scoring [Brown and Mues, 2012], fraud detection [Phua et al., 2010], spam filtering [Tang et al., 2006] and anomaly detection [Pichara and Soto, 2011]. When the training dataset is imbalanced, the difference between the performance of the majority class and minority class becomes larger. To solve this problem, two methods have been proposed: one is based on the sampling method and the other one is a cost-sensitive method. Sampling method can be divided into two classes: under-sampling method and over-sampling method. In the under-sampling, the training dataset is reduced in the majority training set so this this dataset is balanced samples of the majority sets. In the over-sampling method, data from the minority class are copied multiple times or slightly changed such that the two classes are balanced. Many issues have been made in this context, such as Random under-sampling, Random over-sampling [Kotsiantis et al., 2006], SMOTE [Bowyer et al., 2011], MSMOTE [Phua et al., 2010], Random Walk over-sampling [Zhang and Li, 2014]. An hybrid method that selects features in high dimensional datasets has also been proposed by Moradkhani et al. [2015]. Recently Ataeian et al. [2019] investigated a method for large margin classifiers.

The second approach to the imbalanced data classification problem is to apply the weights of the training data points [Elkan, 2001; Ting, 2002; Zadrozny et al., Zhou and Liu, 2006]. Twin SVM is one of the extensions of SVM which constructing two classifiers in such a way that each one is close to one of the two classes. This classifier is in many ways superior to the SVM. Note
that for the imbalanced problem, the standard SVM has been modified by many researchers [Deng, 2012, Shao et al., 2014, Suykens et al., 2002, Tomar et al., 2014]. The Second-Order Cone Programming (SOCP) formulations have been proposed for SVM and Twin SVM. These formulations consider all possible choices of class-conditional densities in a way that with a given mean and covariance matrix and also with having two constraints, one for each class results in a much more efficient training [Maldonado et al., 2016, Nath and Bhattacharyya, 2007]. SOCP-TWSVM constructs two nonparallel classifiers in a way that each hyperplane is closer to one of the training patterns and at the same time as far as possible from the other. Each training pattern is represented by an ellipsoid characterized by the mean and covariance of each class.

In this paper, we attempt to extend the SOCP-TWSVM of imbalanced datasets. The proposed Weighted SOCP-TWSVM (WSOCP-TWSVM) has two phases. At first, it utilizes a graph-based under-sampling method to remove outliers and reduce the dispensable majority samples. Then, a weighted bias is introduced to decrease the impact of samples of the majority class and increase the effect of minority class in the optimization formula of the classifier. The SOCP is utilized to solve the model. The methods Twin-SVM, SOCP-TWSVM for binary classification are introduced in Section 2. The proposed approach is discussed in Section 3. The Experimental results are given in Section 4. The main conclusions and future works have also been provided in Section 5.

2 Preliminaries

2.1 SVM

The SVM is aimed at finding a hyperplane of the form \( f(x) = W^T x + b \) by solving the following quadratic programming problem (QPP):

\[
\begin{aligned}
\min_{W,b} & \quad \frac{1}{2} ||W||^2 + C \sum_{i=1}^{m} \xi_i (y_i (W^T x_i + b) \geq 1 - \xi_i, \\
& \quad i = 1, ..., m, \quad \xi_i \geq 0, \quad i = 1, ..., m, \\
\end{aligned}
\]

(1)

where \( W \in R^n, b \in R, \xi_i \) is slack variable and \( C > 0 \) is a regularization parameter [Deng, 2012].

2.2 Twin SVM

Twin SVM [Jayadeva et al., 2007] is a classification method which separates the instances by constructing two nonparallel hyperplanes instead of a hyperplane. The hyperplanes are obtained by solving two small size optimization problem using QPP. In the linear case, consider the following hyperplanes:

\[
W_1^T x + b_1 = 0, W_2^T x + b_2 = 0
\]

(2)

each of these hyperplanes is very close to one of the classes and far from other ones. The parameters of these hyperplanes are calculated by solving the following optimization problems:

\[
\begin{aligned}
\min_{W_1,b_1,\xi_2} & \quad \frac{1}{2} ||AW_1 + e_1 b_1||^2 + \frac{1}{2} ||W_1||^2 + b_2^T + c_3 e_2^T \xi_2 \\
& \quad -(BW_1 + e_2 b_1) \geq e_2 - \xi_2, \quad \xi_2 \geq 0, \\
\min_{W_2,b_2,\xi_1} & \quad \frac{1}{2} ||BW_2 + e_2 b_2||^2 + \frac{1}{2} ||W_2||^2 + b_2^T + c_4 e_2^T \xi_1 \\
& \quad -(AW_2 + e_1 b_2) \geq e_1 - \xi_1, \quad \xi_1 \geq 0, \\
\end{aligned}
\]

(3)

where \( c_1, c_2, c_3, c_4 \) are positive parameters, and \( e_1 \) and \( e_2 \) are vectors of ones of appropriates dimensions. Parameters \( c_3 \) and \( c_4 \) determine the tradeoff between the respective model fit and the summation of the slack variables.
2.3 SOCP-TWSVM

This classifier has combined the ideas of Twin SVM and SOCP-SVM. The reasoning behind this approach is developing two nonparallel classifiers in a way that each hyperplane is closest to one of the two classes and also in the same distance from the other class. However, ellipsoids are used to characterize each training pattern instead of the convex hulls, following the ideas of SOCP-SVM [Nath and Bhattacharyya, 2007]. This problem can be formulated as the following quadratic programming model

\[
\begin{align*}
\min_{W_1, b_1} & \quad \frac{1}{2} \| AW_1 + e_1 b_1 \|^2 + \frac{\eta_1}{2} (\| W_1 \|^2 + b_1^2) \\
\text{subject to} & \quad -(W_1^T \mu_1 - b_1) \geq 1 + \kappa_1 \| S_1^T W_1 \|. \\
\min_{W_2, b_2} & \quad \frac{1}{2} \| BW_2 + e_2 b_2 \|^2 + \frac{\eta_2}{2} (\| W_2 \|^2 + b_2^2) \\
\text{subject to} & \quad -(W_2^T \mu_2 - b_2) \geq 1 + \kappa_1 \| S_1^T W_2 \|. 
\end{align*}
\]

where \( \eta_1, \eta_2 > 0, \Sigma_i = S_i S_i^T \) and \( \kappa_i = \sqrt{\frac{\eta_i}{\eta_i - \eta_i}} \).

A kernel-based version can be derived from Eqs. (5) and (6) by rewriting weight vector \( W \in \mathbb{R}^d \) as \( W = Xs + Mr \) where \( M \) is a matrix whose columns are orthogonal to training data points. \( S \) and \( r \) are vectors of combining coefficients with the appropriate dimension.

In this section, we present the proposed Weighted Second-Order Cone Programming Twin Support Vector Machine (WSOCP-TWSVM) for the imbalanced problem. This classifier removes outliers and reduces the unessential majority samples with a graph-based under-sampling method. Also, a weighted bias is presented to control the impact of the samples of each class. These weights define the sensitivity of the classifiers to the imbalance ratio and are considered in the mathematical model of the classifier. The SOCP is utilized to solve the model.

3 WSOCP-TWSVM: Weighted SOCP-TWSVM

In this section, we present the proposed Weighted Second-Order Cone Programming Twin Support Vector Machine (WSOCP-TWSVM) for the imbalanced problem. This classifier removes outliers and reduces the unessential majority samples with a graph-based under-sampling method. Also, a weighted bias is presented to control the impact of the samples of each class. These weights define the sensitivity of the classifiers to the imbalance ratio and are considered in the mathematical model of the classifier. The SOCP is utilized to solve the model.

3.1 Sampling method

In this method, supposing that the samples of minority class remain unchanged, and the samples of majority class are selected by constructing a proximity graph [Belkin et al., 2006; Yang et al., 2009]. In the graph, if two samples are k-nearest neighbors (KNN) of each other, an edge between the pair of samples is added. The samples with nonzero degree are in high density regions; and the samples with zero degree such as outliers are in low density regions. The adjacent matrix, \( U_i \), is defined as follows,

\[
U_{ij} = \begin{cases} 
\tau_{ij}, & x_i \in N_k(j) \text{ and } x_j \in N_k(i) \\
0, & \text{otherwise}
\end{cases}
\]
where \((N_k(j))\) is a set of the k-nearest neighbors in the majority class of the point \(x_j\). \(\tau_{ij}\) is a scalar value, and \(i, j = 1, ..., n\). Then we define the under-sampling coefficient as

\[
u_i = \begin{cases} 1, & \sum_j U_{ij} \geq k \\ 0, & \text{otherwise} \end{cases}
\]  

(10)

where all points with nonzero \(u_i\) are selected as members of the sample set, Fig. 1.

Figure 1. Decreasing the number of majority class.

3.2 Defining bias weights

In imbalanced problems, setting of the appropriate weights to the samples of training set is a critical issue in cost-sensitive approaches. Any weighting method should deliberate the following conditions. The data in the majority class have to receive lower weight than those in the minority class. Also, the weight should be in (0, 1) state. If the size of positive class is \(N_{pos}\) and that of negative set after undersampling is \(N_{neg}\), the weights are defined as

\[
D_1 = \begin{cases} 1, & N_{pos} \geq N_{neg} \\ \frac{N_{neg}}{N_{pos}}, & N_{pos} < N_{neg} \end{cases}
\]  

(11)

\[
D_2 = \begin{cases} \frac{N_{neg}}{N_{pos}}, & N_{pos} \geq N_{neg} \\ 1, & N_{pos} < N_{neg} \end{cases}
\]  

(12)

3.3 Linear Weighted SOCP twin SVM

WSOCP-TWSVM combines the graph-based under-sampling and the previous weighting methods. First, performing the under-sampling method described in Subsection 3.1 discards instances from the majority class and the remaining is demonstrated by \(B_+\). Then, the weight of the two classes will be calculated using Eq. (11) and Eq. (12). The majority and minority hyperplanes are determined by solving the following optimization equations

\[
\begin{align*}
\min_{W_1, b_1, \xi_2} \quad & \frac{1}{2} \|W_1 + e_1 b_1\|^2 + \frac{\nu}{2} (\|W_1\|^2 + b_1^2) + C_1 \xi_1 \\
\text{subject to} \quad & -(W_1^T \mu_2 - b_1) \geq 1 - \xi_2 + k_2 \|W_1\| \xi_2 \geq 0,
\end{align*}
\]

(13)
\begin{equation}
\min_{W_2, b_2, \xi_1} \frac{1}{2} \|B^T W_2 + c_2 b_2\|^2 + \frac{\theta_2}{2} (\|W_2\|^2 + b_2^2) + C_2 D_2 \xi_1
\end{equation}

subject to \(-W_2^T \mu_1 - b_2) \geq 1 - \xi_1 + \kappa_1 S_2^T W_2 \xi_1 \geq 0.

where \(\theta_1, \theta_2, C_1, C_2 > 0, \Sigma_2 = S_2 \Sigma_2^T\). The optimization functions associated with Eqs. (13) and (14) are given by

\begin{align*}
\frac{1}{2} \|AW_1 + c_1 b_1\|^2 + \frac{\theta_1}{2} (\|W_1\|^2 + b_1^2) + C_1 D_1 \xi_2 &= \frac{1}{2} V_1^T (H^T H + \theta I) V_1 + C_1 D_1 \xi_2, \\
\frac{1}{2} \|B^T W_2 + c_2 b_2\|^2 + \frac{\theta_2}{2} (\|W_2\|^2 + b_2^2) + C_2 D_2 \xi_1 &= \frac{1}{2} V_2^T (G^T G + \theta I) V_2 + C_2 D_2 \xi_1,
\end{align*}

where

\(V_i = [W_i^T, B_i]^T \in \mathbb{R}^d + 1\)

\(H = [A, e_1] \in \mathbb{R}^{m \ast (k + 1)}\)

\(G = [B, e_1] \in \mathbb{R}^{n \ast (k + 1)}\).

Lagrangian function under Karush-Kuhn-Tucker (KKT) condition and associated with Eqs. (13) and (14) can also be rewritten as

\begin{align*}
\mathcal{L}(W_1, b_1, \lambda_1, \rho_1, \xi_2, u_2) &= \frac{1}{2} \|AW_1 + c_1 b_1\|^2 + \frac{\theta_1}{2} (\|W_1\|^2 + b_1^2) + C_1 D_1 \xi_2 = \lambda_1 (W_1^T \mu_2 + b_1 + 1 - \xi_2 + \kappa_2 W_1^T S_2 \xi_2 - \rho_1 \xi_2, \\
\mathcal{L}(W_2, b_2, \lambda_2, \rho_2, \xi_1, u_1) &= \frac{1}{2} \|B^T W_2 + c_2 b_2\|^2 + \frac{\theta_2}{2} (\|W_2\|^2 + b_2^2) + C_2 D_2 \xi_1 = \lambda_2 (-W_2^T \mu_1 + b_2 + 1 - \xi_1 + \kappa_2 W_2^T S_1 \xi_1 - \rho_2 \xi_1),
\end{align*}

which the Karush-Kuhn-Tucker (KKT) conditions for the eqs. (16) and (17) can be written as

\begin{align*}
\frac{\partial \mathcal{L}}{\partial W_1} &= A^T (AW_1 + c_1 b_1) + \theta_1 W_1 + \lambda_1 (\mu_2 + \kappa_2 W_1^T S_2 \xi_2) = 0, \\
\frac{\partial \mathcal{L}}{\partial W_2} &= e_1^T (AW_1 + c_1 b_1) + \theta_1 W_1 + \lambda_1 = 0, \\
\frac{\partial \mathcal{L}}{\partial \xi_1} &= C_2 D_2 - \lambda_2 - \rho_2 = 0.
\end{align*}

The decision function is similar to the one used for the Twin SVM method [Maldonado et al., 2016], where the class label of a new instance \(x\) is calculated as

\(k' = \arg\min_{i = 1, 2} \frac{W_i^T x + b_i}{\|W_i\|}\).

So Fig. (2) shows the geometrical interpolation of SOCP-TWSVM and WSOCP-TWSVM in a two-dimensional dataset. The blue points are negative samples and yellow points are removed samples in under-sampling method. The red points are positive samples. The dashed lines represent the hyperplanes constructed with SOCP-TWSVM. Similarly, the dot-dash lines correspond to the hyperplanes defined by WSOCP-TWSVM. Both methods construct a decision rule that classifies all training points correctly for the dataset. Although, the decision rules are slightly different. The method has the advantage that it optimizes both twin hyperplanes in the same optimization problem, leading to better predictive performance.
3.4 Nonlinear Weighted SOCP twin SVM

The kernel-based nonlinear WSOCP-TWSVM is presented here. The weight vectors for each of the twin hyperplanes can be written as \( W_k = X_s + M_r \), where \( M \) is a matrix that the columns are orthogonal to the training data points. Variables \( s \) and \( r \) are vectors of combining coefficients with the appropriate dimension. And, \( X = [A^T, B^T] \in R^{k \times (m+n-\text{neg})} \) is the data matrix containing both training patterns [Maldonado et al., 2016].

4 Experimental Results

To show the effectiveness of the proposed WSOCP-TWSVM, we compare it with WSVM, OverSVM, UnderSVM, SMOTESVM, TWSVM and SOCP-TWSVM on 11 standard benchmark datasets from the UCI repository (https://archive.ics.uci.edu/ml/index.php). All methods are implemented in MATLAB R2014a environment.

4.1 Evaluation Criteria

The performance of different classifiers is evaluated using confusion matrix. Confusion matrix stores the details of actual and predicted class in tabular as shown in table. This paper evaluates the performance of proposed methodology for class imbalance using accuracy and AUC. Accuracy of a classifier is estimated by the correct prediction made by the classifier in proportion to total number of prediction,

\[
\text{Accuracy} = \frac{TP + TN}{TP + FP + TN + FN}.
\]

Sensitivity of a classifier is evaluated by the percentage of positive values that are recognized accurately. It is also known as true positive rate and formulated as,

\[
\text{Sensitivity} = \frac{TP}{TP + FN}.
\]

Specificity of a classifier is estimated by the percentage of negative values that are recognized correctly by the classifier. It is also known as true negative rate and formulated as,

\[
\text{Specificity} = \frac{TN}{TN + FP}.
\]

Generally, the G-mean [Bowyer et al., 2011] can characterize trade-off between sensitivity and specificity; so, in our experiments, we use two common performance measures associated with classifier, defined as follows

\[
G\text{-mean} = \sqrt{\frac{TP}{TP + FN} \times \frac{TN}{TN + FP}}.
\]
4.2 Description of datasets and validation procedure

These benchmark datasets represent a wide range of fields, size and imbalanced ratios. Table 2 gives the characteristics of these datasets and minority class for each dataset is shown in the table. The rest of data are classified as the majority one. A grid search is also performed to study the influence of the parameters $\eta$ and $k$ in KNN method for the new approach. In this case, we studied all combinations of the following data, $\eta_1 = \{0.2, 0.4, 0.6, 0.8\}$ and $\eta_2 = \{0.2, 0.4, 0.6, 0.8\}$. The K parameter is selected from the set $\{3, 5, 10, 15\}$, and all other parameters for WSVM, TWSVM, SOCP-TWSVM and our WSOCP-TWSVM is selected from the set $\{10^{-9}, ..., 10^{-6}\}$, also we consider $c_1 = c_2$ and $c_3 = c_4$. For the above procedure, we employ libsvm [Chang and Lin, 2011] to be the base classifier of SVM and the SeDuMi MATLAB toolbox as the SOCP-based classifiers [Sturm, 1999].

4.3 Test using UCI database

We study the following classification approaches; the Weighted SVM, OverSVM, UnderSVM, SMOTESVM, TWSVM, and SOCP-TWSVM, and WSOCP-TWSVM. For each dataset, seven different classifiers were trained and tested by using nested 10 cross-validation technique. The accuracy and G-mean of this cross-validation process are averaged for ten runs. The average accuracy of the compared classifiers with linear kernel and the nonlinear kernel is summarized in Tables 3 and 5. Average G-mean of the compared classifiers with linear kernel and the nonlinear kernel is summarized in Tables 4 and 7. The Friedman test [Friedman, 1939] is a non-parametric statistical test that is used to detect differences in treatments across multiple test attempts by considering their ranking. This test is employed here to detect differences by our method.

The Friedman test confirms that our strategy is better than other comparable methods in terms of accuracy. For linear cases, the result of the test is presented in Table 7(a) and (b) and for nonlinear cases, the result of the test is presented in Table 7(c) and (d). The ranking results from the Friedman test show that WSOCP-TWSVM performs better than other methods. Based on results, for linear cases, though the accuracy of the WSOCP-TWSVM is similar to that of SMOTESVM, the accuracy of WSVM is a little worse than both. Also, the G-mean of WSOCP-TWSVM and SMOTESVM are similar and both have the best performance. The results on nonlinear classifiers have shown that accuracy of the SMOTESVM is a little worse than our WSOCP-TWSVM, the G-mean of SOCP-TWSVM is lower than WSOCP-TWSVM. We can see that our proposed approach obtains the best-imbalanced classification performance than the others in most cases, specifically, it enhances the performance of SOCP-TWSVM.

5 Conclusion

A new method of imbalanced data classification named WSOCP-TWSVM is proposed in the present paper. This method uses the under-sampling procedure for training the dataset and gives the weights for each class. The results of numerical tests performed on datasets show that the proposed methodology is feasible and effective on generalization ability. The WSOCP-TWSVM method is better than the others in the kernel case. Introducing this method provides opportunities to continue the future works. Our method can be extended to multi-class classifications and used for some practical application. In addition, the employment of some different weight setting methods can improve the performance of the WSOCP-TWSVM method.
Table 1. Confusion matrix

| Actual positive | Predicted Positive | Predicted Negative |
|-----------------|--------------------|-------------------|
| Actual negative | True Positive (TP) | False negative (FN) |
|                 | False Positive (FP)| True Negative (TN) |

Table 2. Characteristics of the benchmark datasets.

| Dataset  | IR     | #Features | Minority class | Data size |
|----------|--------|-----------|----------------|-----------|
| Yeast3   | 0.1098 | 8         | ME3            | 1484      |
| Vehicle  | 0.2362 | 18        | VAN            | 946       |
| Transfusion | 0.2380 | 4         | Yes            | 748       |
| Wine     | 0.3315 | 13        | Class I        | 178       |
| PimaIndian | 0.3490 | 8         | Diabetes       | 768       |
| Ionosphere | 0.3590 | 34        | Bad            | 351       |
| Haberman | 0.2647 | 3         | Died           | 306       |
| German   | 0.3000 | 20        | Bad            | 1000      |
| CMC      | 0.2261 | 9         | Lon-term       | 1473      |
| Yeast4   | 0.0340 | 8         | ME2            | 1484      |
| Wisconsin| 0.1021 | 9         | Rest           | 683       |
Table 3. The training accuracy of linear classifiers on benchmark datasets

| Dataset   | WSVM     | OverSVM  | UnderSVM | SMOTESVM | TWSVM    | SOCPTWSVM | WSOCPTWSVM |
|-----------|----------|----------|----------|----------|----------|-----------|------------|
| Yeast3    | 90.79±1.36 | 90.22±0.15 | 83.78±0.19 | 91.92±0.39 | 89.17±0.41 | 90.56±3.05 | 92.35±3.25 |
| Vehicle   | 95.80±0.86 | 94.67±0.10 | 93.83±0.85 | 97.03±0.24 | 96.59±0.23 | 95.65±3.98 | 94.23±1.04 |
| Transfusion | 62.73±4.28 | 54.86±2.21 | 55.69±2.67 | 65.06±3.73 | 49.43±2.27 | 59.45±4.26 | 65.23±8.79 |
| Wine      | 92.12±0.50 | 95.72±0.55 | 91.20±0.73 | 96.04±0.19 | 94.81±0.50 | 93.38±10.10 | 92.92±7.03 |
| PimaIndian | 75.64±0.79 | 75.26±0.90 | 72.79±0.53 | 73.01±0.62 | 76.10±0.25 | 71.23±7.83 | 74.42±0.36 |
| Ionosphere | 88.75±2.29 | 89.11±0.67 | 83.62±0.44 | 73.01±0.30 | 84.93±1.21 | 83.73±4.52 | 84.90±3.27 |
| Haberman  | 70.68±0.92 | 64.82±0.51 | 58.96±1.74 | 74.77±0.28 | 75.81±0.46 | 75.89±6.67 | 76.53±6.72 |
| German    | 72.30±1.28 | 71.68±1.58 | 49.99±0.47 | 69.66±0.63 | 74.84±0.30 | 73.70±3.12 | 72.90±6.15 |
| CMC       | 73.46±0.35 | 71.25±0.17 | 51.94±0.78 | 75.83±0.41 | 74.36±0.21 | 76.64±1.17 | 77.39±0.26 |
| Yeast4    | 94.29±0.25 | 84.98±0.18 | 87.06±1.73 | 96.50±0.26 | 75.10±0.16 | 90.89±2.37 | 92.12±2.07 |
| Wisconsin | 96.72±0.73 | 93.37±0.26 | 93.10±0.24 | 95.92±0.17 | 95.34±0.25 | 94.13±2.75 | 94.02±9.35 |
Table 4. The training G-mean of linear classifiers on benchmark datasets.

| Dataset    | WSVM    | OverSVM | UnderSVM | SMOTESVM | TWSVM    | SOCPTWSVM | WSOCP-TWSVM |
|------------|---------|---------|----------|----------|----------|-----------|-------------|
| Yeast3     | 78.02±1.20 | 76.23±0.15 | 71.75±1.69 | 84.93±0.73 | 83.07±0.96 | 91.48±4.85 | 93.23±3.16  |
| Vehicle    | 90.38±0.74 | 91.69±0.41 | 86.91±0.80 | 96.05±0.27 | 93.74±0.43 | 89.17±3.26 | 90.33±1.62  |
| Transfusion| 58.68±2.75 | 61.31±2.72 | 56.97±2.66 | 62.06±3.18 | 52.72±0.36 | 56.82±4.26 | 63.24±3.02  |
| Wine       | 76.57±1.64 | 82.07±2.56 | 80.20±2.73 | 84.72±2.19 | 86.87±1.36 | 88.02±8.66 | 90.52±0.52  |
| PimaIndian | 66.39±2.16 | 71.24±0.92 | 68.79±0.53 | 72.01±1.51 | 70.43±2.80 | 63.75±3.25 | 71.52±8.03  |
| Ionosphere | 75.59±1.26 | 80.54±0.44 | 62.09±0.41 | 77.31±0.93 | 75.67±0.97 | 76.47±8.73 | 79.49±3.82  |
| Haberman   | 60.84±2.85 | 62.90±0.67 | 52.15±3.48 | 66.92±0.50 | 52.21±4.90 | 52.73±11.51 | 58.96±4.24  |
| German     | 59.09±3.62 | 67.09±3.50 | 61.96±3.37 | 64.24±1.83 | 65.47±2.51 | 71.95±4.71 | 70.03±5.04  |
| CMC        | 52.91±1.08 | 50.58±2.17 | 45.56±1.29 | 60.43±1.76 | 54.61±2.22 | 59.83±5.22 | 64.91±5.28  |
| Yeast4     | 73.48±2.01 | 82.02±0.19 | 67.93±1.97 | 84.35±0.18 | 18.78±6.20 | 80.93±6.79 | 85.36±8.85  |
| Wisconsin  | 96.49±0.36 | 91.40±0.21 | 89.14±0.24 | 93.54±0.18 | 92.04±0.52 | 90.71±5.72 | 89.86±7.07  |
Table 5. The training accuracy of nonlinear classifiers on benchmark datasets.

| Dataset      | WSVM       | OverSVM    | UnderSVM   | SMOTESVM   | TWSVM      | SOCPTWSV   | WSOCP-TWSVM |
|--------------|------------|------------|------------|------------|------------|------------|--------------|
| Yeast3       | 91.39±1.77 | 92.91±1.04 | 87.61±3.42 | 93.52±1.49 | 92.31±0.16 | 91.91±2.01 | 92.31±0.78   |
| Vehicle      | 78.54±0.89 | 80.60±1.89 | 74.16±2.56 | 82.80±1.52 | 79.47±1.65 | 94.10±4.32 | 93.89±2.43   |
| Transfusion  | 73.25±2.35 | 74.85±1.38 | 68.59±1.17 | 75.60±2.59 | 60.16±3.29 | 77.14±4.76 | 72.31±3.51   |
| Wine         | 84.28±2.18 | 95.73±3.48 | 77.62±1.17 | 92.51±2.64 | 92.15±2.07 | 92.16±5.70 | 92.19±3.67   |
| PimaIndia    | 62.79±1.61 | 65.10±0.37 | 64.42±2.28 | 73.42±1.78 | 72.23±1.72 | 73.43±3.24 | 73.53±4.69   |
| Ionosphere   | 93.67±1.59 | 91.63±2.64 | 87.45±2.36 | 94.32±1.71 | 90.30±0.47 | 92.90±3.26 | 90.31±5.35   |
| Haberman     | 73.55±3.14 | 74.24±1.15 | 58.96±2.68 | 72.77±2.41 | 69.39±1.56 | 75.83±6.28 | 76.03±5.33   |
| German       | 71.24±3.01 | 68.45±0.39 | 60.43±1.58 | 71.53±0.29 | 70.47±1.86 | 70.00±3.43 | 72.50±4.37   |
| CMC          | 62.25±2.07 | 68.13±1.65 | 60.27±1.89 | 71.74±2.20 | 73.62±1.55 | 78.34±2.34 | 78.12±7.14   |
| Yeast4       | 92.63±1.39 | 94.07±2.86 | 92.49±1.73 | 95.48±1.82 | 92.16±2.79 | 94.55±0.20 | 95.67±3.02   |
| Wisconsin    | 93.65±2.25 | 94.26±0.86 | 91.28±2.58 | 96.16±0.81 | 96.02±0.72 | 95.85±1.70 | 96.56±2.28   |
Table 6. The training G-mean of nonlinear classifiers on benchmark datasets.

| Dataset     | WSVM     | OverSVM  | UnderSVM | SMOTESVM | TWSVM    | SOCPTWSVM | WSOCP-TWSVM |
|-------------|----------|----------|----------|----------|----------|-----------|-------------|
| Yeast3      | 81.05±1.46 | 80.73±0.45 | 74.14±1.25 | 85.32±0.83 | 78.68±2.57 | 91.99±2.01 | 91.60±3.37  |
| Vehicle     | 81.05±1.39 | 84.31±0.42 | 69.41±2.44 | 90.61±1.29 | 63.08±3.30 | 90.56±7.26 | 91.99±4.56  |
| Transfusio  | 57.55±1.83 | 50.89±2.89 | 43.97±1.50 | 60.52±2.04 | 51.90±1.21 | 61.21±6.99 | 63.96±5.14  |
| Wine        | 7.52±18.94 | 71.93±4.62 | 0.00±0.00  | 67.44±2.81 | 60.87±2.68 | 91.73±5.75 | 92.39±7.57  |
| PimaIndia   | 48.63±2.73 | 54.89±3.11 | 19.74±8.51 | 58.42±3.20 | 57.16±2.68 | 70.54±3.31 | 71.04±4.12  |
| Ionosphere  | 93.93±1.60 | 94.07±2.56 | 72.44±2.96 | 93.35±2.95 | 90.73±3.14 | 93.67±3.13 | 93.84±8.11  |
| Haberman    | 35.52±1.47 | 43.78±11.1 | 29.26±2.18 | 51.61±1.58 | 41.37±2.36 | 60.77±10.24| 63.00±10.53 |
| German      | 59.25±0.73 | 58.79±1.31 | 0.00±0.00  | 65.12±0.47 | 54.12±3.25 | 59.74±7.17 | 65.23±7.33  |
| CMC         | 64.04±7.38 | 62.23±2.98 | 54.34±2.00 | 41.05±5.40 | 53.64±2.19 | 66.18±3.91 | 66.41±5.19  |
| Yeast4      | 31.13±3.01 | 51.24±1.05 | 40.17±4.15 | 48.08±1.63 | 46.66±2.54 | 60.36±8.91 | 62.64±10.21 |
| Wisconsin   | 75.98±0.53 | 84.26±8.29 | 90.48±2.51 | 94.44±1.32 | 93.85±3.20 | 95.22±2.04 | 96.85±1.90  |
| Method           | Mean Rank | Mean Rank | Mean Rank | Mean Rank |
|------------------|-----------|-----------|-----------|-----------|
| WSVM             | 4.73      | 3.18      | 3.00      | 3.09      |
| OverSVM          | 3.36      | 4.45      | 4.18      | 3.82      |
| UnderSVM         | 1.55      | 1.64      | 1.27      | 1.55      |
| SMOTESVM         | 4.91      | 5.64      | 5.55      | 4.45      |
| TWSVM            | 4.36      | 3.55      | 3.14      | 2.64      |
| SOCPTWSVM        | 4.18      | 3.91      | 5.18      | 5.73      |
| WSOCP-TWSVM      | **4.91**  | **5.64**  | **5.68**  | **6.73**  |

(a) (b) (c) (d)
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