CONSTRAINTS ON THE CARDASSIAN SCENARIO FROM THE EXPANSION TURNAROUND REDSHIFT AND THE SUNYAEV-ZELDOVICH/X-RAY DATA

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Received 2003 May 12; accepted 2003 October 25

ABSTRACT

Cosmic acceleration is one of the most remarkable cosmological findings of recent years. Although a dark energy component has usually been invoked as the mechanism for the acceleration, a modification of the Friedmann equation from various higher dimensional models provides a feasible alternative. Cardassian expansion is one of these scenarios, in which the universe is flat, matter (and radiation) dominated, and accelerating but contains no dark energy component. This scenario is fully characterized by $n$, the power index of the so-called Cardassian term in the modified Friedmann equation, and $\Omega_m$, the matter density parameter of the universe. In this work, we first consider the constraints on the parameter space from the turnaround redshift, $z_{q=0}$, at which the universe switches from deceleration to acceleration. We show that for every $\Omega_m$ there exists a unique $n_{\text{peak}}(\Omega_m)$ that makes $z_{q=0}$ reach its maximum value, $(z_{q=0})_{\text{max}} = \exp\left[1/(2 - 3n_{\text{peak}})\right] - 1$, which is nonlinearly inverse to $\Omega_m$. If the acceleration happens earlier than $z_{q=0} = 0.6$, as suggested by Type Ia supernovae measurements, we have $\Omega_m < 0.328$ no matter what the power index is, and moreover, for reasonable matter density, $\Omega_m \sim 0.3$, it is found that $n \sim (-0.45, 0.25)$. We next test this scenario using the Sunyaev-Zeldovich/X-ray data of a sample of 18 galaxy clusters with $0.14 < z < 0.83$ compiled by Reese et al. We determine $n$ and $\Omega_m$ as well as the Hubble constant $H_0$, using the $\chi^2$ minimization method. The best fit to the data gives $H_0 = 59.2$ km s$^{-1}$ Mpc$^{-1}$, $n = 0.5$, and $\Omega_m = 0.3$ ($\Omega_b$ is the baryonic matter density parameter). However, the constraints from the current Sunyaev-Zeldovich/X-ray data are weak, although a model with lower matter density is preferred. A certain range of the model parameters is also consistent with the data.

Subject headings: cosmic microwave background — cosmology: theory — distance scale — galaxies: clusters: general

1. INTRODUCTION

Recent observations of Type Ia supernovae by two independent groups, the High-z Supernova Team$^1$ ( Riess et al. 1998) and the Supernova Cosmology Project$^2$ (Perlmutter et al. 1999) suggest that our universe is presently undergoing an accelerating expansion. The highest redshift supernova observed so far, SN 1997ff, at $z = 1.755$, not only supports this accelerating view but also glimpses the earlier decelerating stage of the expansion (Riess et al. 2001). It seems that determining a convincing mechanism with a solid basis in particle physics that explains the accelerating universe is emerging as one of the most important challenges in modern cosmology.

It is well known that all known types of matter with positive pressure generate attractive forces and decelerate the expansion of the universe—conventionally, a deceleration factor is always used to describe the status of the universe’s expansion (Sandage 1988). Given this, the discovery of high-redshift Type Ia supernovae may indicate the existence of a new component with fairly negative pressure, which is now generally called dark energy. Coincidentally or not, a dark energy component could offset the deficiency of a flat universe, favored by the measurements of the anisotropy of the cosmic microwave background (de Bernardis et al. 2000; Balbi et al. 2000; Durrer, Novosyadlyj, & Apuneych 2003; Bennett et al. 2003; Melchiorri & Odman 2003; Spergel et al. 2003), but with a very subcritical matter density parameter, $\Omega_m \sim 0.3$, obtained from dynamical estimates or X-ray and lensing observations of clusters of galaxies (for a recent summary, see Turner 2002). The simplest possibility for the dark energy component is the cosmological constant $\Lambda$ (Weinberg 1989; Carroll et al. 1992; Krauss & Turner 1995; Ostriker & Steinhardt 1995; Chiba & Yoshii 1999; Futamase & Hamana 1999). Other candidates for the dark energy include a decaying vacuum energy density or a time varying $\Lambda$-term (Ozer & Taha 1987; Vishwakarma 2001; Alcaniz & Maia 2003), an evolving scalar field, referred to by some as “quintessence”; (Ratra & Peebles 1988; Wetterich 1988; Frieman et al. 1995; Coble, Dodelson, & Frieman 1997; Caldwell et al. 1998; Wang & Lovelace 2001; Wang & Garnavich 2001; Podarius & Ratra 2001; Li, Hao, & Liu 2002; Weller & Albrecht 2002; Li et al. 2002a, 2002b; Chen & Ratra 2003; Mukherjee et al. 2003), the phantom energy, in which the sum of the pressure and energy density is negative (Caldwell 2002; Hao & Li 2003a, 2003b; Dabrowski, Stochowiak, & Gabadadze 2003), the so-called X-matter, an extra component simply characterized by an constant equation of state $p_x = \omega p_x$ (XCDM; Turner & White 1997; Chiba, Sugiyama, & Nakamura 1997;
The Cardassian scenario is motivated to provide a possible mechanism for the acceleration of the universe. It is natural that one could use the observational constraints on the deceleration parameter, \(q(z) = -\dot{R}/R^2\), to check the feasibility of the model. In terms of \(E(z)\) function, we get the deceleration parameterization of the Cardassian model. In this parameterization of \((n, z_{eq})\), it can be shown that (Freese & Lewis 2002),

\[
B = H_0^2 (1 + z_{eq})^{3(1-n)} \rho_0 [1 + (1 + z_{eq})^{3(1-n)}]^{-1},
\]

where \(H_0 = \frac{8\pi G}{3}\rho_0[1 + (1 + z_{eq})^{3(1-n)}]\). (2)

Because in the Cardassian model the universe is flat and contains only matter, the matter density at present, \(\rho_0\), should be equal to the "critical density" of this scenario. From equation (2), we have

\[
\rho_0 = \rho_c, \text{Cardassian} = \rho_c \times F(n, z_{eq}).
\]

\[
F(n, z_{eq}) = [1 + (1 + z_{eq})^{3(1-n)}]^{-1},
\]

where \(\rho_c = 3H_0^2/8\pi G\) is the critical density of the standard Friedmann model. As expected for a flat universe, the "density parameter", \(\Omega_m, \text{Cardassian} \equiv \rho_0/\rho_c, \text{Cardassian},\) should be equal to 1. But we are used to the standard density parameter, \(\Omega_m \equiv \rho_0/\rho_c\), which is defined in terms of the critical density in the standard Friedmann model. From equation (3), we have

\[
\Omega_m = \Omega_{CDM} + \Omega_b = F(n, z_{eq}),
\]

and hence we could alternatively use \((n, \Omega_m)\) to fully characterize the Cardassian model.

Now we evaluate the angular diameter distance as a function of redshift \(z\) as well as the parameters of the model. Following the notation of Peebles (1993), we define the redshift dependence of the Hubble parameter \(H\) as \(H(z) = H_0 E(z)\). For the Ansatz of equation (1) and a flat universe with only matter (baryonic and cold dark matter), we get

\[
E^2(z; n, \Omega_{CDM}, \Omega_b) = \left[ (\Omega_{CDM} + \Omega_b)(1+z)^3 + (1-\Omega_{CDM} - \Omega_b)(1+z) \right].
\]

Then, it is straightforward to show that the angular diameter distance is given by

\[
D^A(z; H_0, n, \Omega_{CDM}, \Omega_b) = \frac{c}{H_0} \frac{1}{1+z} \int_0^z \frac{dz'}{E(z'; n, \Omega_{CDM}, \Omega_b)}.
\]

3. CONSTRAINTS FROM THE TURNAROUND REDSHIFT FROM DECELERATION TO ACCELERATION

The Cardassian scenario is motivated to provide a possible mechanism for the acceleration of the universe. It is natural that one could use the observational constraints on the deceleration parameter, \(q(z) = -\dot{R}/R^2\), to check the feasibility of the model. In terms of \(E(z)\) function, we get the deceleration constraints and discussion in § 5.

2. BASIC EQUATIONS

We provide here the important equations resulting from the modified Friedmann equation, equation (1) (for more details, see Freese & Lewis 2002). In the usual Friedmann equation, \(B = 0\). To be consistent with the standard cosmological result, one should take \(A = 8\pi G/3\). It is convenient to use the redshift \(z_{eq}\), at which the two terms of equation (1) are equal, as the second parameter of the Cardassian model. In this parameterization of \((n, z_{eq})\), it can be shown that (Freese & Lewis 2002),

\[
B = H_0^2 (1 + z_{eq})^{3(1-n)} \rho_0 [1 + (1 + z_{eq})^{3(1-n)}]^{-1},
\]

where \(\rho_0\) is the matter density of the universe at the present time and \(H_0 = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1}\) is the Hubble constant.

Evaluating the Ansatz of equation (1) at the present time, we have (Freese & Lewis 2002)

\[
H_0^2 = \frac{8\pi G}{3} \rho_0 [1 + (1 + z_{eq})^{3(1-n)}].
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\[
B = H_0^2 (1 + z_{eq})^{3(1-n)} \rho_0 [1 + (1 + z_{eq})^{3(1-n)}]^{-1},
\]
is the current optimistic matter density, upper bounds for a wider range, respectively. The two dotted lines show the thick dot-dashed line depicts the Cardassian model to pass the turnaround-redshift test. Fujimoto 2003): term, evolution to acceleration, as a function of the power index of the Cardassian turnover (see Turner 2002 for the argument), parameter vanishes, as follows: 

\[ 1 + z_{\text{eq}} = (2 - 3n)^{1/3(1-n)} (1 + z_{\text{eq}}) \]

In Figure 1 we plot this redshift as a function the power index of the Cardassian term, \( n \), for several values of \( \Omega_m \). While \( \Omega_m = 0.330 \pm 0.035 \) is the current optimistic matter density (see Turner 2002 for the argument), \( \Omega_m = 0.2 - 0.4 \) is a wider range. The dotted lines of Figure 1 correspond to the present observational constraints on the turnaround redshift (at the 1 \( \sigma \) level), \( 0.6 < z_{\text{eq}} < 1.7 \), from the latest supernova data (Perlmutter et al. 1999; Riess et al. 1998, 2001; Turner & Riess 2002; Avelino & Martins 2002). As Figure 1 shows, for every value of \( \Omega_m \), there exists a value for the power index of the Cardassian term, \( n_{\text{peak}}(\Omega_m) \), satisfying

\[ \frac{1}{2 - 3n_{\text{peak}}} \exp \left[ \frac{3(1 - n_{\text{peak}})}{2 - 3n_{\text{peak}}} \right] = \frac{1}{\Omega_m} - 1, \]

which makes the turnaround redshift \( z_{q=0} \) reach the maximum value,

\[ (z_{q=0})_{\text{max}} = \exp \left\{ 1/\left[ 2 - 3n_{\text{peak}}(\Omega_m) \right] \right\} - 1. \tag{9} \]

This \( n_{\text{peak}}(\Omega_m) \)-\( (z_{q=0})_{\text{max}} \) relation is illustrated in Figure 1 by the thick dotted-dashed line. For example, for \( \Omega_m = 0.2 \), we have from equation (8) \( n_{\text{peak}} \sim 0.20 \) and hence \( (z_{q=0})_{\text{max}} \sim 1.05 \); in other words, a universe with \( \Omega_m = 0.2 \) cannot switch from deceleration to acceleration at redshift higher than 1.05, no matter what the power index of the Cardassian term is. We could restate the constraints in an alternative way: for every value of the turnaround redshift \( z_{q=0} \), the matter density of the universe must satisfy

\[ \Omega_m \leq \left[ \ln (1 + z_{q=0}) (1 + z_{q=0})^{(\ln (1 + z_{q=0})^2 + 1)/\ln (1 + z_{q=0})} + 1 \right]^{-1}, \tag{10} \]

where the equal sign comes into existence if and only if the power index of the Cardassian term is \( n_{\text{peak}} = [2 \ln (1 + z_{q=0}) - 1]/[3 \ln (1 + z_{q=0})] \). For example, for \( z_{q=0} = 0.6 \), as we are considering, we have from equation (10) \( \Omega_m \leq 0.328 \), where the equal sign happens at \( n = n_{\text{peak}} = -0.04 \). The problem is now apparent: a very low matter density would be always necessary for the Cardassian expansion scenario if the turnaround redshift is larger than 0.6, especially for the case of \( n \) deviating from \( n_{\text{peak}} \).
In order to illustrate how the turnaround redshift efficiently constrains the Cardassian parameter space, we explore the model with different values in the \( n-\Omega_m \) plane. The results are shown in Figure 2. The thick solid curve delimits the parameter space of accelerating/decelerating universe as the top-left/down-right area of the curve. While the present observational constraints on the turnaround redshift with \( 0.6 < z_{q=0} < 1.7 \) restrict the parameter space to the narrow shaded area, the observed matter density of the universe limits them further (see the overlap part of the shaded and the cross-hatched areas). For instance, if the density parameter of the universe takes value around \( \Omega_m \sim 0.3 \), a reasonable value suggested by various observations, the power index should be around \( n \sim (-0.45, 0.25) \) to satisfy \( z_{q=0} > 0.6 \). Last but not least, we would like to note that in the accelerating area a horizontal line with \( n = \frac{1}{2} \) will delimit it into two parts, \( n > \frac{1}{2} \) or \( n < \frac{1}{2} \), corresponding to a universe that switches from deceleration to acceleration later or earlier than the moment at which the Cardassian term equals to the conventional term of the Friedmann equation, respectively. This can be seen from the relation of equation (7). For clarity, we do not show it in Figure 2. Therefore, the expansion turnaround redshift is generally not equal to \( z_{q=0} \), the redshift at which the Cardassian term reaches the normal matter density term in the Friedmann equation. Only for \( n = \frac{1}{2} \) does the acceleration happen exactly when the Cardassian term starts to dominate. However, for \( n > \frac{1}{2} (n < \frac{1}{2}) \), the universe switches from deceleration to acceleration after (before) the Cardassian term dominates.

It is widely believed that the universe switched from deceleration to acceleration only recently. This is based on two observational arguments (Amendola 2003): (1) acceleration at high redshift might be in contrast with observed large-scale structure, because it makes gravitational instability ineffective, and (2) the supernova at the highest redshift so far discovered, SN 1997ff, at \( z = 1.755 \), seems to provide a glimpse of the epoch of deceleration (Riess et al. 2001; Turner & Riess 2002). However, a cosmological model with a dark energy component strongly coupled to dark matter (Amendola 2000, 2003) could be consistent with the above two observations but allow acceleration at high redshift, \( z_{q=0} \in (1, 5) \). We can see from above analysis that the Cardassian expansion model with reasonable matter density, \( \Omega_m \in (0.2, 0.4) \), can hardly explain an acceleration that happened earlier than \( z_{q=0} = 1 \). In this sense, a direct observation of the acceleration might be one of the most crucial and efficient tests for discriminating different mechanisms for acceleration. There have so far been two proposals for this kind of measurement. One is to monitor the redshift change of quasars during 10 yr or so (Loeb 1998). The other is to measure the gravitational wave phase of neutron star binaries at \( z > 1 \) for 10 yr using a decihertz interferometer gravitational wave observatory (DE\textsc{CI}GO: Seto, Kawamura, & Nakamura 2001). These might provide efficient tests for various acceleration mechanisms.

4. CONSTRAINTS FROM THE SZ/X-RAY DATA

It has long been suggested that a measurement of the thermal SZ effect (Sunyaev & Zeldovich 1972) of a cluster of galaxies, combined with X-ray observations, can be used to determine the cluster cosmological angular-diameter distance and hence the Hubble constant \( H_0 \) and the deceleration parameter \( q_0 \) (Cavaliere, Danese, & de Zotti 1977; Silk & White 1978; Birkinshaw 1979). Benefiting from the improvements to the traditional single-dish observations (Myers et al. 1997) and thanks to the new bolometer technology (Holzapfel et al. 1997) and interferometry technique (Calstrom, Joy, & Grego 1996), several tens of clusters of galaxies have been selected for the SZ measurements and have been used to determine the Hubble constant (for recent summaries, see Birkinshaw 1999; Calstrom, Holder, & Reese 2002). Recently, Reese et al. (2002) published the SZ measurements of a sample of 18 galaxy clusters with redshifts ranging from 0.14 to 0.83 observed by The Owens Valley Radio Observatory and the Berkeley-Illinois-Maryland Association interferometers. The authors used the maximum likelihood joint analysis method to analyze their SZ measurements with archival ROSAT X-ray imaging observations, which provide the largest homogeneously analyzed sample of the SZ/X-ray clusters with angular diameter distance determinations so far (Reese et al. 2002). The database is shown in Figure 3. Because the redshift range of the cluster sample is comparable with the distant Type Ia supernovae data compiled by Riess et al. (1998) and Perlmutter et al. (1999) that led to the discovery of accelerating expansion, it provides a good independent cross check of the acceleration mechanism. We use this sample to give an observational constraint on the Cardassian model parameters, \( n \) and \( \Omega_m \) (or \( \Omega_{\text{CDM}} \)).

We determine the model parameters \( n \) and \( \Omega_{\text{CDM}} \) through a \( \chi^2 \) minimization method. The range of \( n \) spans the interval \([-1, 1]\) in steps of 0.01, while the range of \( \Omega_{\text{CDM}} \) spans the interval \([0, 1-\Omega_b]\) in 101 points.

\[
\chi^2(n, H_0, \Omega_{\text{CDM}}, \Omega_b) = \sum_i \frac{[D_A^i(z_i; H_0, \Omega_{\text{CDM}}, \Omega_b) - D_A^i]^2}{\sigma_i^2}
\]

(11)

FIG. 3.—Diagram of angular diameter distance vs. redshift for 18 X-ray galaxy clusters compiled by Reese et al. (2002), in which the distances are determined by the Sunyaev-Zeldovich/X-ray route. The solid curve corresponds to the best fit of the Cardassian expansion model to the subsample in which the outlier, cluster A370 (open diamond), is excluded. The values of \((n, \Omega_m)\) for the other two curves are taken from the edges of the allowed regions shown in Fig. 4. We assume the baryonic matter density to be \( \Omega_b = 0.0205 \, h^{-2} \).
where \( z_i \) are the redshifts of the galaxy clusters and \( D^i(z_i; H_0, \Omega_{CDM}, \Omega_b) \) are theoretical predictions from equation (5). Values of \( D^i \) are the angular diameter distances of the cluster sample determined by the SZ/X-ray route, and values of \( \sigma_i \) are the symmetric rms errors of the cluster distance measurements (Reese et al. 2002; Allen, Schmidt, & Fabian 2002). The summation is over all the observational data points.

In order to reduce the number of parameters to be determined, we specify the baryonic matter density parameter to be \( \Omega_b h^2 = 0.0205 \) (O'Meara et al. 2001; Allen et al. 2002). In order to make the analysis independent of choice of the Hubble constant, we minimize equation (11) for \( H_0, n, \) and \( \Omega_{CDM} \) simultaneously, which gives \( H_0 = 59.6 \) km s\(^{-1}\) Mpc\(^{-1}\), \( n = 0.42 \), and \( \Omega_{CDM} = 0.0 \) as the best fit with \( \chi^2 = 16.21 \) and 15 degrees of freedom (dof). An inspection of terms of \( \chi^2 \) = 16.21 for the above best fit shows that in this summation a large amount, namely, \( \sim 5.8 \), is given by one cluster, A370, which has a redshift of 0.374 and thus should be considered an outlier. Following Reese et al. (2002), we also analyze a subsample of 17 galaxy clusters that excludes the outlier, A370. From the point of view of statistics, outliers appear when a member of a data set is inconsistent with the remainder of the data set and is best removed from the sample (Mészáros 2002). A glimpse at Figure 3, immediately shows that A370 might be an outlier, because it is far beyond the distance expected from the trend given by other objects.; in other words, it is “too far away.” Our best fit to the remaining 17 clusters occurs for \( H_0 = 59.2 \) km s\(^{-1}\) Mpc\(^{-1}\), \( n = 0.50 \), and \( \Omega_{CDM} = 0.0 \), with \( \chi^2 = 10.55 \) and 14 dof. In Figure 4 we show contours of constant likelihood (68% and 95% C.L.) in the \( n-\Omega_m \) plane for the best fit. The fitting results for samples with and without A370 are very similar, but the \( \chi^2 \) per dof for the latter is significantly reduced and indicates that no large statistical errors remain unaccounted for. The similarity of the fitting results also indicates the robustness of our analysis. The best fits of the Cardassian expansion model to the current SZ/X-ray data lead to a universe containing baryonic matter only, leaving no space for the huge amount of dark matter whose existence has been widely accepted among the astronomical community. However, note from Figure 4 that the allowed range for both \( n \) and \( \Omega_{CDM} \) is reasonably large, showing that the constraints on Cardassian expansion from the considered SZ/X-ray cluster sample are not restrictive. For example, the models with reasonable matter density (\( \Omega_m \sim 0.3 \)) but some negative values of \( n \) are within 1 \( \sigma \) of the best fit.

5. CONCLUSIONS AND DISCUSSION

We have explored the constraints on the parameter space of the Cardassian scenario from the turnaround redshift, \( z_{q=0} \), at which the universe switches from deceleration to acceleration. We demonstrated that, for every matter density, \( \Omega_m \), there exists a unique power index, \( n_{\text{peak}}(\Omega_m) \), which makes \( z_{q=0} = 0 \) reach its maximum value. If the acceleration happens earlier than \( z_{q=0} = 0.6 \), we find that \( n \sim (0.45, 0.25) \) for a reasonable matter density, \( \Omega_m \sim 0.3 \) and that the Cardassian scenario can hardly explain an acceleration that happened earlier than \( z = 1 \).

We further analyzed the scenario using the largest homogeneous sample of SZ/X-ray clusters, compiled by Reese et al. (2002). All best-fitting results seem to give a universe with very low matter density (\( \Omega_m = \Omega_b = 0.0205 h^2 \)). Our analysis is consistent with previous observational constraints (Sen & Sen 2003a, 2003b; Wang et al. 2003; Cao 2003; Zhu & Fujimoto 2002, 2003). However, at present both the statistical and systematic uncertainties in the SZ/X-ray route of distance determination are very large (Birkinshaw 1999; Reese et al. 2002; Calstrom et al. 2002), which loosens the constraints of the Cardassian expansion scenario very much (Fig. 4). Therefore, the Cardassian expansion scenario with reasonable matter density is also consistent with the current SZ/X-ray data within 1 \( \sigma \). The projection effect of aspherical clusters modeled with a spherical geometry is a large systematic error in the angular diameter distance determinations based on the SZ/X-ray route (Hughes & Birkinshaw 1998). Other large systematic uncertainties are due to the departures from isothermality, the possibility of clumping, and the possible point-source contamination of the SZ observations (Calstrom et al. 2002; Reese 2002; Reese et al. 2002). The systematic uncertainties for both SZ and X-ray observations promise to improve within a few years. The absolute calibration of the SZ observation is now underway to improve significantly (Calstrom et al. 2002). The Chandra and XMM-Newton X-ray telescopes not only reduce the uncertainty in the X-ray intensity scale but are also providing temperature profiles of galaxy clusters (Markevitch et al. 2000; Nevalainen, Markevitch, & Forman 2000). Both ongoing and planned large surveys of SZ clusters will perhaps provide a SZ/X-ray sample of a few hundred clusters with redshift extending to 1 and beyond, which will improve our analysis and pin down the Cardassian model parameters more accurately.

We would like to thank S. W. Allen, J. M. Bai, A. G. Riess, and R. G. Vishwakarma for helpful discussions. Our thanks go...
to the anonymous referee for valuable comments and useful suggestions, which improved this work very much. This work was supported by a Grant-in-Aid for Scientific Research on Priority Areas (No. 14047219) from the Ministry of Education, Culture, Sports, Science and Technology. Z.-H. Zhu acknowledges support from the National Natural Science Foundation of China and the National Major Basic Research Project of China (G2000077602). He is also grateful to all TAMA300 members and the staff of NAOJ for their hospitality and help during his stay.

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