Modelling and Turning Characteristics Analysis of Dual-Steering Multi-Axle Vehicles

Yun-chao WANG¹, Zhi-chao HU¹,* and Heng DU²

¹College of Mechanical and Energy Engineering, Jimei University, Xiamen, China
²School of Mechanical Engineering and Automation, Fuzhou University, Fuzhou, China

*Corresponding author

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Abstract. Multi-axle vehicles with dual steering systems (DSS), which include a hydraulic-assisting steering link mechanism system for front axles and an electrohydraulic active-steering system (EHASS) for rear axles, become the development tendency of multi-axle vehicles. However, until now, no attempt has been made with regard to the mathematical model for multi-axle vehicles with DSS. By introducing two ratios of wheel steering angle to first wheel angle for the two steering systems, the model of multi-axle vehicles with DSS was deduced based on the Newton’s second law. The relevant experiment was carried and validate the model correction. The comparison between the vehicles with DSS and those with EHASS show that vehicles with DSS have an approximate mobility and stronger cornering stability.

Introduction

In recent years, there have been rapid developments in all-terrain cranes for engineering construction. Concern for the limits on axle loads for road transportation demands multiple axles in all-terrain cranes in order to distribute the loads. Because all-terrain cranes usually operate in harsh environments, they must have good performance such as trafficability, manoeuverability, and stability, which presents a great design challenge. To improve the cornering performance of all-terrain cranes, multi-axle steering technology has found wide application.

Recently, several papers were published on the mobility and maneuverability of multi-axle vehicles [1] and some 2 degrees of freedom (DOF) and 3-DOF dynamic models for multi-axle steering vehicles have been presented [2–4]. On the basis of these models, many steering control strategies have been proposed, such as a steering control strategy based on the mass centre zero side-slip angles of vehicles [4], \( H_2/H_\infty \) optimal controllers [2], \( H_\infty \) robust control based on T-S fuzzy models [5], variable-structure sliding mode control of all-wheel steering systems [6], nonlinear adaptive sliding mode control [7,8], a steering control strategy using a linear quadratic regulator technique [9], and a pole assignment method based on state feedback using full-order observers [10].

The above strategies are only adopt to the multi-axle steering vehicles with a single electrohydraulic active steering system. However, the single active steering system is poorly applied...
because it can’t guarantee the steering reliability of multi-axle vehicles. To achieve better steering reliability performance, almost multi-axle steering vehicles adopt two steering steering systems, namely, a hydraulic-assisting steering link mechanism system (HASLMS) and an electro-hydraulic active steering system (EHASS) for the front and rear axles, respectively. The type of steering system is called as dual steering systems, namely DSS, which is widely applicable to the all-terrain cranes QAY1000 from XCMG and LTM 1400.7.1 from Liebher. Figure1 shows the chassis of the all-terrain crane LTM 1400.7.1. However, there is no reported research in the literature on dynamic models for and turning characteristics of multi-axle vehicles with DSS. A methodology for the prediction of the handling properties of such vehicles is quite valuable. However, until now, no attempt has been made with regard to the dynamics performance of multi-axle steering vehicles with DSS. This motivates the current research.

Modeling Of Multi-Axle Vehicles with DSS

Description of Steady-state Cornering

Figure 2 shows the steering state and the forces acting on the multi-axle vehicles with DSS. To describe a multi-axle vehicle, the side-slip angle, vehicle velocity and yaw rate are denoted as $\beta$, $v$, $r$ respectively. The direction of $\beta$ and $r$ conforms to the right hand rule, and $v$ is positive in the $x$ direction. The steering system of the axles $1 \sim k$ is the HASLMS and that of the axles $(k+1) \sim n$ is EHASS. Point G, $O_1$, $O_i$ is the vehicle centre of gravity, the turning centre of the HASLMS and that of the EHASS, respectively. $F_{xi}$ and $F_{yi}$ is the $x$-, $y$-component of the $i$th wheel frictional forces $F_i$ respectively. $\Delta$ and $\Delta_0$ denotes the distance as shown in Fig.2. $\delta_i$ describes the steering angle of the $i$th wheel. The subscript $i$ indicates the position of each wheel from the right to left in Fig.2. $\alpha_i$ denotes the side-slip angle of the $i$th wheel.

Based on Newton’s second law, yields the equilibrium equations of the vehicles:

$$\begin{align*}
\sum_{i=1}^{n} m \ddot{v} &= \sum_{i=1}^{n} F_{yi} \\
\sum_{i=1}^{n} F_{xi} l_{xi} &= 0
\end{align*}$$

(1)

According to Figure 2, the theoretical side slip angles of the $i$th tyres are expressed as

$$\alpha_i = \tan^{-1}\left(\frac{v\beta + l_i r}{v}\right) - \delta_i \approx \beta + \frac{r l_i}{v} - \delta_i$$

(2)

If the wheel steering angle of the SLMSHA is very small, the ratio $H_i$ of $\delta_i$ to $\delta_1$ can be written as
\[
\frac{l_i - \Delta_0}{l_i - \Delta} = \frac{\tan \delta_i}{\tan \delta_i} \Rightarrow H_i = \frac{\delta_i}{\delta_i} \approx \frac{l_i - \Delta_0}{l_i - \Delta}_i
\]

The wheel steering angles of the EHASS are determined by the turning centre \(O_r\), so the ratio \(K_i\) of \(\delta_i\) to \(\delta_i\) is different than the previous ratio \(H_i\), and it can be expressed as

\[
\frac{l_i - \Delta}{l_i - \Delta} = \frac{\tan \delta_i}{\tan \delta_i} \Rightarrow K_i = \frac{\delta_i}{\delta_i} \approx \frac{l_i - \Delta}{l_i - \Delta}
\]

Assuming the cornering stiffness of the \(i\)th tyre \(C_i\) is a constant. If a side-slip angle is positive, \(F_{yi}\) acts in the negative \(y\)-direction and can be written as follows

\[
F_{yi} = -C_i \alpha_i
\]

Substituting Equations (2) – (5) into Equation (1), and rearranging the equations:

\[
\begin{align*}
\sum_{i=1}^{n} C_i \beta + \left( \frac{m v^2}{\rho} + \sum_{i=1}^{n} C_i l_i \right) \frac{r}{v} &= \left( \sum_{i=1}^{k} C_i H_i + \sum_{i=k+1}^{n} C_i K_i \right) \delta_i \\
\sum_{i=1}^{n} C_i l_i^2 \frac{r}{v} + \sum_{i=1}^{n} C_i l_i \beta &= \left( \sum_{i=1}^{k} C_i l_i H_i + \sum_{i=k+1}^{n} C_i l_i K_i \right) \delta_i
\end{align*}
\]

**Side-slip Angle at the Vehicle Centre of Gravity**

Solving Equation (6) yields:

\[
\beta = 1 \frac{\sum_{j=1}^{k} \sum_{i=1}^{n} C_i C_j H_j \left(l_i - l_j\right) + \sum_{j=k+1}^{n} \sum_{i=1}^{n} C_i C_j l_j K_j \left(l_i - l_j\right) - m v^2 \left( \sum_{i=1}^{k} C_i l_i H_i + \sum_{i=k+1}^{n} C_i l_i K_i \right)}{\rho \sum_{j=1}^{k} \sum_{i=1}^{n} C_i C_j H_j \left(l_i - l_j\right) + \sum_{i=k+1}^{n} \sum_{i=1}^{n} C_i C_j K_j \left(l_i - l_j\right)}
\]

**Yaw Rate**

Solving Equation (6) yields:

\[
r = \frac{\sum_{j=1}^{k} \sum_{i=1}^{n} C_i C_j H_j \left(l_i - l_j\right) + \sum_{j=k+1}^{n} \sum_{i=1}^{n} C_i C_j K_j \left(l_i - l_j\right)}{\sum_{i=1}^{n} \sum_{j=1}^{n} C_i C_j \left(l_i - l_j\right)} \frac{v \delta_i}{v^2 \delta_i}
\]

**Lateral Acceleration**

From Equation (8), the lateral acceleration \(a\) can be expressed as

\[
a = \frac{\sum_{j=1}^{k} \sum_{i=1}^{n} C_i C_j H_j \left(l_i - l_j\right) + \sum_{j=k+1}^{n} \sum_{i=1}^{n} C_i C_j K_j \left(l_i - l_j\right)}{\sum_{i=1}^{n} \sum_{j=1}^{n} C_i C_j \left(l_i - l_j\right) + m v^2 \sum_{i=1}^{n} C_i l_i} \frac{v^2 \delta_i}{v^2 \delta_i}
\]
Experimental Validation of the Model

Vehicle Configuration

The experimental vehicle with 4WS and 4WD was controlled by a wireless remote controller as shown in Figure 3. The remote controlled vehicle was developed by our research team. There is a steering linkage mechanism between the left and right wheels for each axle. The steering angle of each axle was measured with external KTC-200 position sensors installed on the rack-and-pinion steering gear, which was driven by a servo motor. The vehicle was equipped with four 195/60R15 tyres driven by in-wheel motors, an electronic gyroscope Gyro-3 made by Murata, a differential global positioning system (with receivers’ sample rate of 10 Hz) for the vehicle velocity and heading direction, and an ARK-5261 industrial personal computer—including a DMC2C80 motion control card for the two steering servo motors, and a PCI-1712 data acquisition card to acquire information on the wheel steering angle, wheel velocity, and moment torque.

The parameters of the experimental vehicle are as follow: $l_f=0.55\text{m}$, $l_r=-0.55\text{m}$, $l=1.1\text{m}$, $\delta_f=-\delta_r=0.238\text{rad}$, $H_1=K_1=1$, $C_f=C_r=25000\text{N/\text{rad}}$, and vehicle weight $m=277\text{kg}$.

Experimental Validation

A cornering scenario was conducted on level ground (friction coefficient=0.8). The steering angles of the front and rear steering axles were an average of the inner and outer wheels and were equal to $13.637^\circ$. The vehicle speed accelerated from 0km/h to 10km/h at an approximate fixed acceleration during cornering.
Figures 4-5 illustrate that the simulation results were a good fit with the experimental results. Therefore, the dynamic model of vehicles with DSS is correct and can estimate the vehicle steady state.

Performance Analysis of Multi-axle Vehicles with DSS

Setting of Simulation Parameters

A sample five-axle vehicle was used for a simulation with HASLMS for the front three axles and EHASS for the rear two axles. To discuss the characteristics of vehicles with DSS, another sample of a five-axle vehicle with EHASS were employed. The configuration of the vehicle was follows: \(l_1=4.07\text{m}, l_2=1.32\text{m}, l_3=-0.3\text{m}, l_4=-1.92\text{m}, l_5=-3.54\text{m}\), vehicle mass \(m=58000\text{kg}\), cornering stiffness \(C=612\ \text{kN/rad}\) for all tyres, and \(\Delta=\Delta_0=0\) for the vehicle with HASLMS. The turning radius of vehicle gravity centre and those of every axle can be calculated according to the turning centre \(O\).

Simulation Results

To discuss the effect of \(\Delta\), three kinds of variation laws were adopted: \(\Delta=-v^{2.5}/1200\), \(\Delta=-v^{2.8}/1200\), and \(\Delta=-v^{2.8}/600\), as shown in Figure 6. A comparison of the turning radii of the two vehicles is presented in Figure 7, corresponding to the three kinds of variation laws of \(\Delta\). The turning radius of the vehicle with DSS is represented by the \(d\)-radius, and that of the vehicle with EHASS is denoted by the \(e\)-radius. The comparison results illustrate that the turning radii of the vehicle with DSS are greater than those of the vehicle with EHASS.

In Figure 8, the lateral acceleration of the vehicles with DSS is denoted by \(d\)-acc, and that of the vehicles with EHASS is represented by \(e\)-acc. As the wheel steering angle increases, the lateral acceleration of the vehicles with DSS is clearly lower than that of the vehicles with EHASS for the same conditions. Therefore, the stability of the vehicle with DSS is higher than that of the vehicle with EHASS at high speeds. However, at lower speeds, the lateral acceleration of the vehicle with DSS is close to that of the vehicle with EHASS. Therefore, the vehicle with DSS has a mobility that approximates that of the vehicle with EHASS at lower speeds. In summary, compared to the vehicle with EHASS, the turning characteristics of the vehicle with DSS are in accordance with the demand for higher stability of vehicles at high speeds and a higher mobility of vehicles at lower speeds.
Conclusions

A steady-state 2-DOF cornering model for multi-axle vehicles with DSS was developed by introducing the ratio $H_i$ of the steering angles of wheels steered by HASLMS to the steer angles of the first wheel, and the ratio $K_i$ of the steering angles of wheels steered by EHASS to the steer angles of the first wheel and by using Newton’s second law. The simulation results are in good agreement with the experimental data. Compared to the vehicle with EHASS, the turning characteristics of the vehicle with DSS are in accordance with the demand for higher stability of vehicles at high speeds and a higher mobility of vehicles at lower speeds.

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