Absolute frequency measurement at $10^{-16}$ level based on the international atomic time

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Abstract. Referring to International Atomic Time (TAI), we measured the absolute frequency of the $^{87}$Sr lattice clock with its uncertainty of $1.1 \times 10^{-15}$. Unless an optical clock is continuously operated for the five days of the TAI grid, it is required to evaluate dead time uncertainty in order to use the available five-day average of the local frequency reference. We homogeneously distributed intermittent measurements over the five-day grid of TAI, by which the dead time uncertainty was reduced to low $10^{-16}$ level. Three campaigns of the five (or four)-day consecutive measurements have resulted in the absolute frequency of the $^{87}$Sr clock transition of $429,228,004 \pm 229,872.85$ (47) Hz, where the systematic uncertainty of the $^{87}$Sr optical frequency standard amounts to $8.6 \times 10^{-17}$.

1. Introduction
Recent developments of optical frequency standards have led to the achievement of stabilities and accuracies at the $10^{-18}$ level [1–4]. Thus, it is expected that the SI second is redefined by an optical transition in the future. Towards this goal, it is essential to confirm the reproducibility of the frequency by comparing the frequencies among laboratories. From this point of view, direct frequency comparisons of optical clocks have been demonstrated mostly on-campus in the initial stage. Recently, as the next step, the agreement of distant optical clock frequencies has been intensively studied using an optical fiber [5, 6] or a satellite link [7, 8]. These include our activities; direct comparison of our $^{87}$Sr optical lattice clock against a distant clock at the University of Tokyo (UT) using an optical fiber link [5, 9] as well as that against one at Physikalisch-Technische Bundesanstalt (PTB) using a two-way carrier phase (TWCP) satellite frequency transfer [7, 10]. Another requirement for the redefinition is the consistency of the duration “one second” before and after the redefinition. This is realized by rigorous absolute frequency measurements of the new standard frequency. Thus, we performed absolute frequency measurements of an $^{87}$Sr lattice clock in reference to International Atomic Time (TAI) [11] in 2012 [12] and 2015 [13]. The recent measurement was performed to improve its uncertainty by carrying out homogeneously distributed intermittent measurements over the five-day grid of TAI [13]. The two measurements contributed to the update of the recommended frequency of an $^{87}$Sr clock transition by the Comité International des Poids et Mesures (CIPM) since 2012. These efforts will certainly push forward the optical redefinition of the SI second.
2. Direct frequency comparison

2.1. Comparison with UT using an optical fiber link

Using a telecommunication optical fiber link between NICT and UT [9], we performed a direct comparison of two $^{87}$Sr clocks at each site [5]. Figure 1(a) shows a schematic diagram of the frequency comparison. The clock frequency at NICT is optically transferred to UT using an optical fiber link through the urban region of Tokyo. At NICT, the Ti:sapphire-based optical frequency comb (Ti:Sa comb) is phase-locked to the clock laser at 698 nm. The telecom laser at 1538 nm is phase-locked to the Ti:Sa comb through its frequency-doubled light at 769 nm and transferred to UT through a 60-km-long optical fiber, to which a phase noise cancellation method is applied. The Ti:Sa comb at UT is phase-locked to the frequency-doubled transferred light. The frequency offset between the two Sr clocks is detected at UT as a beat signal between the clock laser and the nearest component of the Ti:Sa comb.

The observed frequency difference is caused by different systematic shifts of the two clocks, in which the gravitational red shift of 2.62 Hz is the largest. The weighted mean of eleven measurements resulted in the fractional frequency difference between the two distant Sr lattice clocks to be $(1.0 \pm 7.3) \times 10^{-16}$, demonstrating the direct confirmation of the reproducibility of the frequency in distant optical clocks. The uncertainty of the frequency comparison is predominantly limited by the uncertainties of both Sr clocks. Details of the optical fiber link system and the frequency comparison of two distant clocks are described in [9] and [5], respectively.

2.2. Comparison with PTB using a TWCP link

Two $^{87}$Sr lattice clocks located at NICT in Japan and at PTB in Germany are directly compared using a satellite-based frequency transfer using the TWCP [7]. The baseline of 9000 km is the longest ever realized in the direct comparison of optical clocks to the best of our knowledge. Figure 1(b) shows a schematic diagram of the comparison, which involves a link system based on the TWCP technique, optical clocks, and frequency combs at each site. The technical details of the TWCP link are described in [10]. The frequency of the lattice clock at each site is counted with reference to a local hydrogen maser (H-maser). A $^{171}$Yb$^+$ clock based on the E3 transition at PTB [14, 15] was employed as a stable transfer oscillator to extend the measurement time.
when the lattice clock at PTB is offline. A total measurement time resulted in 83640 s including the extension by the Yb$^+$ clock. Both H-masers are linked by the TWCP system. The frequency ratio of the two $^{87}\text{Sr}$ clocks is derived from the ratio of the two local H-masers in real time. We measured the clock frequencies with reference to each local H-maser for several hours per day over four days. Taking into account the uncertainty of the $^{87}\text{Sr}$ lattice clock at NICT and PTB, we concluded that a fractional difference of two distant Sr clocks is $(1.1 \pm 1.6) \times 10^{-15}$. Details of the frequency comparison are described in [7].

3. Absolute frequency measurement based on TAI

In order to maintain the continuity of the scale interval “one second”, the absolute frequencies of optical frequency standards need to be rigorously evaluated to the limit of the SI second. It is straightforward to measure the optical frequency by referring to locally available cesium primary frequency standards. The evaluation of the frequency based on TAI, however, is an alternative for laboratories where highly accurate primary frequency standards are not yet available. TAI is accessible even at isolated locations via the Global Navigation Satellite System (GNSS). Thus, H-masers with a satellite-based link to TAI, which we call a TAI link hereafter, have often been employed in optical frequency measurements [12, 16–21].

Every month, the Bureau International des Poids et Mesures (BIPM) reports in Circular T the result of calculated time difference between Coordinated Universal Time (UTC) and the local realization of UTC (UTC(k)) of every fifth day on the TAI grid. The scale interval of UTC is identical to that of TAI. Consequently, only a five-day average of the TAI frequency is available as a reference to which we evaluate optical frequencies. It is ideal to operate optical clocks all through the five-days of the TAI grid. Instead of the continuous operation, however, we utilize an H-maser and pursued a possibility to estimate the difference of the H-maser frequency between the limited time of clock operation and the five-day average. Note that a five-day average of the H-maser frequency can be accurately evaluated from the time difference reported in Circular T.

Figure 2 depicts the scheme of an absolute frequency measurement employing TAI as a frequency reference. Here, the horizontal and vertical axes are time and the fractional frequency relative to TAI, respectively. In addition, $\nu_{\text{Sr}}/\nu_{\text{Sr}0}$ is the fractional frequency of Sr relative to a certain reference frequency of Sr based on TAI. This ratio is as stable as TAI. $\nu_{\text{UTC(k)}}/\nu_{\text{TAI}}$ indicates the temporal frequency variation of UTC(k) with reference to TAI. In the case of UTC(NICT), the frequency is adjusted every 8 h for the $10^{-15}$ level with reference to Cs ensemble time. Therefore the frequency of UTC(NICT) at the moment of measurement is unknown. On the other hand, the area A turns out to be the time difference between 0:00 (UTC) of day 0 and that of day 5 since that is the integrated fractional frequency difference between UTC(k) and TAI during this period. Thus, it is possible to evaluate only a five-day average of the fractional
frequency of UTC(NICT) referring to TAI for this period. Each bar in figure 2 represents the operation period of an optical clock which does not cover the whole five days, requiring an additional uncertainty to estimate the frequency of UTC(k) at the instant of measurement. We call this uncertainty the dead-time uncertainty. As shown in figure 2(b), employing a more stable predictable oscillator as an LTO moderates this error. Since the LTO oscillates continuously for five days, the sum of areas A and B in figure 2(b) can be obtained by a time-interval counter. Here, frequency measurement is affected not by the fluctuation of UTC(k) but by that of the LTO. In addition, it is known that carrying out homogeneously distributed measurements over five days reduces the dead time error [13].

We measured the absolute frequency of an $^{87}$Sr optical lattice clock in 2012 using a scheme as shown in figure 2(a), which had the uncertainty of $3.3 \times 10^{-15}$ [12]. In this first measurement, a large dead time uncertainty of $2.6 \times 10^{-15}$ dominated since we did not take care of the distribution of measurement time over the five-day grid. Furthermore, the inaccuracy of our Sr system has been recently reduced to $8.6 \times 10^{-17}$, as briefly described in [13]. Consequently, it is crucial to reduce the TAI link uncertainty in order to evaluate the absolute frequency more accurately. Thus, on the occasion of the recent measurement in 2015 [13], we employed the homogeneously distributed intermittent measurement scheme as shown in figure 2(b).

Three sets of the five-day frequency measurements were performed. The total weighted average of three campaigns was calculated on the basis of the statistical weighting. The uncertainty comprises of the statistical uncertainty, the clock and TAI systematic uncertainties, the link uncertainty of TAI–UTC(NICT), and the dead time uncertainties of H-maser–TAI and TAI–TT (Terrestrial Time) which are estimated by numerical analysis as described in [13]. The total uncertainty is predominantly limited by the uncertainty due to the dead time of TAI–TT frequency difference. The frequency of the TAI that we employed as a reference in this work is the five-day average, whereas the calibration of the TAI with reference to TT is provided in Circular T on basis of one-month average, requiring dead time uncertainty of $1.1 \times 10^{-15}$ per one measurement of the five-day campaign. Consequently, we have not yet reached the uncertainty at $10^{-16}$ level. The effort to reduce the link uncertainty performed in this work, however, will pave the way to the measurement at the $10^{-16}$ level since a few measurement campaigns performed in one month may reduce the TAI–TT dead time uncertainty. This prospect is supported by numerical simulation of the same manner as [13].

Finally, figure 3 summarizes the absolute frequency measurements performed in various institutes worldwide. The result reported here is consistent with other measurements within the uncertainty, indicating the validity of the uncertainty evaluation investigated here.
4. Conclusion
We measured the absolute frequency of the $^{87}\text{Sr}$ clock transition with its uncertainty of $1.1 \times 10^{-15}$ by reducing the TAI link uncertainty. Three sets of five (or four)-day measurements, where the frequency was measured for $10000 - 24000$ s per day, reduced the uncertainty due to the frequency link between TAI and the LTO. In addition, the uncertainties owing to the dead time are evaluated by numerical simulations. These efforts reduced the link uncertainty. Note that the H-maser we employed as the LTO is the most stable in the H-maser array of the Japan Standard Time system. The instability reaches $1 \times 10^{-15}$ at $1000 - 2000$ s and stays a few $10^{-16}$ for $10^4 - 10^5$ s according to the extended three-corner-hat method using several H-masers. The predictability of the H-maser frequency is another key for the measurement.

The intermittent measurement of an optical frequency standard can be applied to the estimation of the scale interval of TAI, similarly to the role currently played by microwave fountain standards. Cesium or rubidium fountain frequency standards have so far contributed to the calibration of TAI frequency by their continuous operation over 15 days or more. On the other hand, optical clocks together with a stable LTO may not require continuous operation since they reach the same level of instability in a few hours. While a frequency link in principle requires the prolonged signal integration of an LTO, the homogeneously distributed intermittent evaluations and likewise the management of dead time uncertainty may be sufficient for the estimation of TAI.

Acknowledgments
The authors are deeply grateful to T. Takano, M. Takamoto and H. Katori for their collaboration in the direct frequency comparison using an optical fiber link and to St. Falke, N. Huntemann, C. Grebing, B. Lipphardt, Ch. Lisdat and D. Piester for their collaboration in a direct international frequency comparison with the TWCP link. We would also like to thank F. Nakagawa and H. Ito for providing the record of JST. We are also grateful to Y. Hanado and T. Gotoh for the discussion regarding the TAI link. Parts of the Sr system were built by A. Yamaguchi, A. Nogami, S. Nagano, and Y. Li in the early stage of its construction. H. Ishijima, S. Ito, and M. Mizuno provided the necessary technical assistance in the measurements.

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