Spontaneous Emission Spectrum In Semiconductor Quantum Well Under Coherent Control

Haiming Li\textsuperscript{1}, Ping Zhang\textsuperscript{1} and Li Deng\textsuperscript{2,*}
\textsuperscript{1}School of Science, East China Jiaotong University, Nanchang, China
\textsuperscript{2}School of Science, Zhejiang Sci-Tech University, Hangzhou, China

*Corresponding author e-mail: lideng75@zstu.edu.cn

Abstract. In this paper, we study spectra of spontaneous emission in a semiconductor quantum well which is driven by a controlling field and a driving field. Under the quantum coherent manipulation, we analyze the properties of emission spectra and our results show the spectra can present some novel physical phenomena, such as narrowing, fluorescence quenching, suppression and enhancement. Semiconductor medium, being artificial atom, has great potential applications in some fields.

1. Introduction

In recent years, more and more researches are beginning to focus on quantum well systems. We know the structure of semiconductor quantum well has discrete levels due to confinement of motion of electrons. Under the condition of coupling of laser field, one can manipulate the electron transitions between different levels, so lots of researchers have pay attention to its optical properties. Under the control of quantum coherence, a weak pulse light propagating in the quantum well can show novel quantum coherent phenomena, for example electromagnetically induced transparency [1-3] and coherent population oscillation [4-8] and so on. Comparing with atomic system, structure of energy levels of semiconductor quantum well are easily designed, so people can prepare various quantum well structure according to different requirement. Because of easy design and easy operation of semiconductor quantum well, its nonlinear optical properties have been attracted wide attention. For example, we investigate optical multistability and optical bistability in an asymmetric quantum well system, where we obtain switching between optical multistability and optical bistability by choosing appropriate physical parameters, and realize the controllable phenomena of optical bistability [9]. In a semiconductor quantum well with four subbands, Yang’s group reports a scheme in which they give an efficient method to enhance optical four-wave mixing signals, and their scheme provides potential applications in design of quantum device [10]. In addition, Semiconductor quantum well has many inherent advantages due to its interband transition based devices, for example strong electric dipole moments because of small effective electron masses, and its transformation energy, dipole and symmetry can be realized in engineering as people hope. Based on these advantages, semiconductor quantum well structure can be used as quantum information processors, so people can use it to construct quantum logic gate and prepare quantum entangled state.

In the last few years, study of spontaneous emission spectra in atomic medium are reported [11,12]. Controlling effectively and modulating precisely the spectra of spontaneous emission are able to push the development of magmetometry and precision measurement. In the system of semiconductor quantum well, longitudinal and transverse relaxation can occur, so people can easily obtain
spontaneous emission spectra of electrons. In our work, we consider a quantum well structure with multilevel and we use three light fields with different frequency to interact with intersubband transitions. Under the condition of quantum coherent modulation, spontaneous emission spectra of quantum well can show some interesting phenomena. The advantages of flexibility and easy integration make semiconductor quantum well become more practical value. Our paper includes four parts. In this section, we give an introduction about this research topic. In Section 2, theoretical model is shown. Some interesting results numerical analysis are obtained in section 3. Finally a conclusion is shown in section 4.

2. Physical model and dynamical equations

Here we study a coupling quantum well system with a shallow well and a deep well. Due to quantum tunneling effect, levels of conduction band and valence band reconstruct. The levels of system which we choose are shown in Fig.1.

![Figure 1](https://example.com/figure1.png)

**Figure. 1** Schematic levels structure diagram of double semiconductor quantum well for (a), and schematic of coupling between quantum well and light fields for (b).

A probe field with Rabi frequency $\Omega_p$ and frequency $\omega_p$ is applied to interact with transition between levels $|1\rangle$ and $|2\rangle$. A controlling field with Rabi frequency $\Omega_c$ and frequency $\omega_c$ couples with levels $|2\rangle$ and $|3\rangle$. In addition, we add a driving field with Rabi frequency $\Omega_d$ and frequency $\omega_d$ to couple with levels $|4\rangle$ and $|2\rangle$. $\Delta_p, \Delta_c, \Delta_d$ stands for detuning of the probe, the controlling and the driving fields, respectively. There is a ground state $|g\rangle$ in the deep well. We suppose the decay rate from level $|2\rangle$ to $|g\rangle$ is $\Gamma$, the decay frequency is $\omega_k$, the coupling coefficient of decay is $g_k$, where $k$ denotes the $k$th mode of photon.

For convenience of calculation, we choose rotation frame and rewrite the Hamiltonian. We can get the interaction Hamiltonian. at the time $t$, we assume the wavefunction of the system can be expressed as

$$|\psi(t)\rangle = C_1(t)|1\rangle + C_2(t)|2,0_k\rangle + C_3(t)|3\rangle + C_4(t)|4\rangle + \sum_k C_k(t)|g,1_k\rangle$$

According to the Schrödinger’s Equation, we obtain the evolution equations of the system

$$\frac{d}{dt} C_1(t) = i\Omega_p C_2(t)$$
$$\frac{d}{dt} C_2(t) = \Omega_c^* C_3(t) + i\Omega_d C_4(t) + i\Omega_p C_1(t) - i(\Delta_p - \frac{\Gamma}{2}) C_2(t)$$
$$\frac{d}{dt} C_3(t) = i\Omega_c C_2(t) - i(\Delta_p + \Delta_c) C_3(t)$$
$$\frac{d}{dt} C_4(t) = -i(\Delta_p + \Delta_d) C_4(t) + i\Omega_d C_2(t)$$
$$\frac{d}{dt} C_g(t) = -i(\Delta_p - \delta_k) C_g(t) + ig_k C_2(t)$$

Where $\Gamma = 2m|g_k|^2 D(\omega_k)$, and $D(\omega_k)$ is vacuum mode density of free space.

In order to get the spontaneous emission spectra, we need to perform Laplace transform $a_j(s) = \int_0^\infty e^{-st} a_j(t) dt$ on Eqs. (4). The integral of Laplace transform

$$a_j(s) = \frac{1}{s + \delta_k}$$

where $\delta_k$ is the decay rate of low levels. The Laplace transform of spontaneous emission spectrum

$$S_j(s) = \int_0^\infty \frac{1}{s + \delta_k} e^{-st} dt = \frac{1}{s}$$

The integral of Laplace transform
\[ a_j(t) = \int_0^\infty e^{-st} a_j'(t) \, dt = -\frac{1}{s} \left[ a_j(t) e^{-st} \bigg|_0^\infty - \int_0^\infty e^{-st} d(a_j(t)) \right], \]

and its simplified form can be written \( s a_j(s) - a_j(0) = \int_0^\infty e^{-st} a_j'(t) \, dt. \) After performing Laplace transform on Eqs. (4), we obtain

\[ S \tilde{C}_1(s) - C_1(0) = i \Omega_p \tilde{C}_1(s) \]
\[ S \tilde{C}_2(s) - C_2(0) = i \Omega_p \tilde{C}_1(s) - i \left( \Delta_p - i \frac{1}{2} \right) \tilde{C}_2(s) + i \Omega_d \tilde{C}_3(s) + i \Omega_d \tilde{C}_4(s) \]
\[ S \tilde{C}_3(s) - C_3(0) = i \Omega_d \tilde{C}_2(s) - i \left( \Delta_p + \Delta_d \right) \tilde{C}_3(s) \]
\[ S \tilde{C}_4(s) - C_4(0) = i \Omega_d \tilde{C}_2(s) - i \left( \Delta_p + \Delta_d \right) \tilde{C}_4(s) \]
\[ S \tilde{C}_g(s) = i g_k \tilde{C}_2(s) + i \left( \Delta_p - \delta_k \right) \tilde{C}_g(s) \]

Next we apply the initial condition probability amplitude \( C_1(0)=1, \) and \( C_i=2,3,4(0)=0, \) and obtain the expression of \( \tilde{C}_2(s), \) according to the general method for spectra, we can get the expression of spectra of the spontaneous emission \( S(\delta_k) \)

\[ S(\delta_k) = \frac{\Gamma}{2\pi |g_k|^2} \left| C_g(t \to \infty) \right|^2 = \frac{\Gamma}{2\pi} \left| \tilde{C}_2(s = -id) \right|^2 \]

where \( f_1(\delta_k) = (e - d - id) \Omega_p (d - b) \), \( f_2(\delta_k) = id(e - d)(d - b) \), \( f_3(\delta_k) = (-id)(e - d)(d - b) \), \( f_4(\delta_k) = d\Omega_d (b - d) \).

### 3. Discussion about Results

We, in this section, discuss the distribution about spectra of spontaneous emission by choosing the appropriate physical parameters. According to the Eqs.(8), we use Matlab software and obtain spontaneous emission spectra for different cases.
Firstly, through equation (8), we find that the change of spontaneous emission spectrum is related to the frequency detuning of driving field and the Rabi frequency of driving field. The two parameters simultaneously affect the spontaneous emission spectrum. Under the above parameter setting, we can get the expression by substituting each parameter into equation (8). By analyzing the expression, we can know that the quenching point is related to the frequency detuning of driving field. In the meantime, we can see that the spectral value is zero, which implies that the detuning of driving field affects the position of fluorescence quenching. We let \( \Omega_p = 0.4\Gamma, \Omega_c = 0.3\Gamma, \Omega_d = 0.2\Gamma, C_1 = 1, C_2, 3, 4 = 0, \Delta_p = \Delta_c = 0. \) (a) \( \Delta_d = 0, \) (b) \( \Delta_d = 0.2\gamma, \) (c) \( \Delta_d = 0.3\gamma, \) and (d) \( \Delta_d = 0.4\gamma. \) The curves about spectra of spontaneous emission are plotted in Fig. 1. When we let the driving field interact resonantly with quantum well structure, it is clear that two peaks of equal height appear as spectral lines and two peaks are symmetrical located both sides of the zero point. In this case, there is no fluorescence quenching point, the height of the peaks is equal, and the width of the peaks is limited by decay rate. Adding the detuning of the driving field (see Fig. 1 (b)), the effect of the driving field causes more splitting of each dynamic Stark component, which is characterized by three peaks, i.e. an ultra-narrow center line and two lines with normal-width edge occur, and the quenching point is located at \( \delta_k = -0.2\gamma. \) From Fig. 1 (b) to (d), we find that, increasing the frequency detuning of the driving field, spectral line with ultra-narrow shape in the middle becomes wider and the height decreases slowly, and the peak value of the left spectral line decreases gradually and the width becomes narrower, meanwhile, the peak value on the right increases and the width of the spectral line increases slowly. The frequency detuning of the driving field affects the width of the spectral line and the location about the quenching point of fluorescence emission. The causes of these phenomena are attributed to the weakening of the quantum interference.

Figure. 3 Curves of spontaneous emission spectra \( S(\delta_k) \) for different initial condition.
To further study the influences of the initial state on the spectrum of spontaneous emission, considering the condition of different initial state we show the shapes of emission spectrum (see Fig. 3). The In Fig. 3 (a), $C_1(0)=1$, and $C_{i=2,3,4}(0)=0$, the fluorescence quenching point on the spontaneous emission spectrum is located at position $\delta_k = 0$. In addition, an ultra narrow spectral line can be observed at the fluorescence quenching point. For another case where, $C_1(0)=0.707$, $C_3(0)=0$, by substituting the above parameters into the expression of spectra, we get the intensity of spectra is zero point that is located at the left side of the origin of coordinates, which is shown in Fig. 3 (b). In this case we can see that there are still ultra-narrow spectral lines at zero point, and the peak value is gradually weakening, but also see that there are sideband lines in the gradually strengthened spectral lines. In Figure 3 (c) where $C_1(0)=C_4(0)=0.707$ and $C_3(0)=C_2(0)=0$, and the quenching point can be obtained by substituting the parameters and it is located at point with $\delta_k = 0.06\gamma$. And compared with the previous figure, we can see that the peak value of the ultra-narrow spectral line will decrease, and the sideband lines on both sides will be strengthened. Finally we choose initial condition with $C_2(0)=1$ and $C_1(0)=C_3(0)=C_4(0)=0$ (see Fig. 3 (d)), and we find two quenching points occur: one is located at point $\delta_k = 0$, and another is located at $\delta_k = \Delta_p = 0.2\gamma$. At the same time, it can be clearly seen that this figure includes an ultra-narrow spectral line, and there are two band lines with equal height and equal width which distribute on both sides of the ultra-narrow line.

4. Conclusion

In Summary, by method of numerical analysis, we studied the spectra of spontaneous emission in a multilevel type semiconductor quantum coherent medium which is derived by a deriving field and controlling field. Our studies find that when we effectively adjust the intensity and frequency detuning of the controlling field, novel physical phenomena about the spectra of spontaneous emission, such as fluorescence quenching, narrowing, suppression and enhancement, can been shown. The phenomena observed in spectra of spontaneous emission can be attributed to the multiple decay coherent effects. Finally, we also analyze the influences of different initial state on the emission spectrum. Semiconductor medium being artificial atom, has great potential applications in some fields.

Acknowledgments

This work was financially supported by by the National Natural Science Foundation of China (NSFC) (11775190, 11365009) and by Science Foundation of Zhejiang SCI-TECH University (17062071-Y).

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