Quark lepton complementarity and renormalization group effects

Michael A. Schmidt

Physik-Department T30, Technische Universität München, James-Franck-Straße, 85748 Garching, Germany

Alexei Yu. Smirnov

Physik-Department T30, Technische Universität München, James-Franck-Straße, 85748 Garching, Germany
The Abdus Salam International Centre for Theoretical Physics, I-34100 Trieste, Italy and
Institute for Nuclear Research, Russian Academy of Science, Moscow, Russia

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We consider a scenario for the Quark-Lepton Complementarity relations between mixing angles in which the bi-maximal mixing follows from the neutrino mass matrix. According to this scenario in the lowest order the angle \( \theta_{12} \) is \( \sim 1\sigma (1.5 - 2\sigma) \) above the best fit point coinciding practically with the tri-bimaximal mixing prediction. Realization of this scenario in the context of the seesaw type-I mechanism with leptonic Dirac mass matrices approximately equal to the quark mass matrices is studied. We calculate the renormalization group corrections to \( \theta_{12} \) as well as to \( \theta_{13} \) in the standard model (SM) and minimal supersymmetric standard model (MSSM). We find that in large part of the parameter space corrections \( \Delta \theta_{12} \) are small or negligible. In the MSSM version of the scenario the correction \( \Delta \theta_{12} \) is in general positive. Small negative corrections appear in the case of an inverted mass hierarchy and opposite CP parities of \( \nu_1 \) and \( \nu_2 \) when leading contributions to \( \theta_{12} \) running are strongly suppressed. The corrections are negative in the SM version in a large part of the parameter space for values of the relative CP phase of \( \nu_1 \) and \( \nu_2 \): \( \varphi > \pi/2 \).

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I. INTRODUCTION

Implications of the observed pattern of neutrino mass and mixing (with two large angles) for fundamental physics are still an open question. This pattern has not yet led to a better understanding of the origins of the neutrino mass as well as fermion masses and mixing in general. In contrast, it made the situation more complicated and more intriguing. In this connection, any hint from data and any empirical relation should be taken seriously and analyzed in details.

In fact, one feature has been realized recently that (if not accidental) may lead to a substantially different approach to the underlying physics. Namely, the sums of the mixing angles of quarks and leptons for the 1-2 and 2-3 generations agree within 1\( \sigma \). In other words, the quark and lepton mixings sum up to maximal mixing:

\[
\theta_{12} + \theta_C \approx \frac{\pi}{4}, \quad \theta_{23} + \theta_{cb} \approx \frac{\pi}{4}. \tag{1}
\]

Here \( \theta_C \) is the Cabibbo angle, \( \theta_{cb} = \text{arcsin} \, V_{cb} \), and \( V_{cb} \) is the element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. According to eqs. (1) which are called the quark-lepton complementarity (QLC) relations, the quark and lepton mixings are complementary to the maximal mixing. (A possibility that the lepton mixing responsible for the solar neutrino conversion equals maximal mixing minus \( \theta_C \) was first mentioned in [2], and corrections to the bi-maximal mixing from the CKM type rotations were discussed in [3].)

For various reasons it is difficult to expect exact equalities (1). However certain correlations clearly show up:

- the 2-3 leptonic mixing is close to maximal because the 2-3 quark mixing, \( V_{cb} \), is very small;
- the 1-2 leptonic mixing deviates from maximal substantially because the 1-2 quark mixing (i.e., the Cabibbo angle) is relatively large.

If not accidental coincidence, the QLC relations imply (i) a kind of quark-lepton symmetry or quark-lepton unification which propagates the information about mixing from the quark sector to the lepton sector.

(ii) existence of some additional structure which produces maximal or bi-maximal mixing.

Even within this context one expects some deviations from exact quark-lepton complementarity due to

- broken quark-lepton symmetry,
- renormalization group (RG) effects.

There is a number of attempts to reproduce the QLC relations on the basis of already existing ideas about fermion mass matrices [3, 4, 9, 10]. Usually they lead to too small deviations of \( \theta_{12} \) from \( \pi/4 \), and therefore require further corrections or deviations from the bi-maximal mixing or from the Cabibbo mixing already in the lowest order. So, in the majority of the models proposed so far, an approximate QLC relation appears as a result of an interplay of different independent factors or as a sum of several independent contributions. In these

\*Electronic address: michael.schmidt@ph.tum.de
\†Electronic address: smirnov@ictp.trieste.it
cases the QLC relation seems to be accidental. There are few attempts to construct a consistent gauge model which explains the QLC relations. The simplest possibility is the $SU(2)_L \times SU(2)_R \times SU(4)_C$ model that implements the quark-lepton symmetry in the most straightforward way \cite{8,10}. Phenomenology of schemes with QLC relations has been extensively studied \cite{1,3,11,12}.

The relation (1) is realized at some high energy scale, $M_F$, of flavor physics and quark-lepton unification. Therefore one should take into account the renormalization group effects on the QLC relations when confronting them with the low energy data. In fact, it was marked in \cite{1} that in the Minimal Supersymmetric Standard Model (MSSM) the corrections are typically positive but negative $\Delta \theta_{12}$ can be obtained from the RG effects in presence of non-zero 1-3 mixing. Also threshold corrections due to some new scale of physics, such as the low scale supersymmetry, can produce a negative shift of $\theta_{12}$, thus enhancing its deviation from $\pi/4$ \cite{13}.

The Cabibbo mixing can be transmitted to the lepton sector in a more complicated way (than via the quark-lepton symmetry). In fact, $\sin \theta_{12}$ may turn out to be a generic parameter of the theory of fermion masses - the "quantum" of flavor physics. Therefore it may appear in various places: mass ratios, mixing angles. One can consider the Cabibbo angle as an expansion parameter for mixing matrices \cite{1,7,14,15}.

In this paper we study in details the RG effects in the QLC scenario where the bi-maximal mixing is generated by the neutrino mass matrix. We calculate corrections to the angles both in the Standard model (SM) and MSSM. We analyze the dependence of the corrections on various parameters and obtain bounds on the parameters from consistency condition with QLC. In particular, we find regions where the corrections are negative. The paper is organized as follows. In sec. 2 we formulate the scenario and comment on parameterization dependence of the QLC relations as well as confront the relations with experimental data. In sec. 3 we consider realization of the scenario in the seesaw type I mechanism. The RG effects in MSSM and SM are described in secs. 4 and 5 correspondingly. We consider the RG effects on 1-3 mixing and dependence of the effects on scale of new physics in sec. 6. Conclusions are formulated in sec. 7.

II. UPDATE ON QLC

A. A scenario

A general scheme for the QLC relations is that

\begin{equation}
\text{"lepton mixing = bi-maximal mixing} - \text{CKM"}, \tag{2}
\end{equation}

where the bi-maximal mixing matrix is \cite{6}

\begin{equation}
U_{bm} = U^m_{31}U^m_{12} = \frac{1}{2} \left( \begin{array}{ccc}
\sqrt{2} & \sqrt{2} & 0 \\
-1 & 1 & \sqrt{2} \\
1 & 1 & -\sqrt{2}
\end{array} \right). \tag{3}
\end{equation}

Here $U^m_{ij}$ is the maximal mixing ($\pi/4$) rotation in the $ij$-plane.

We assume that the bi-maximal mixing is generated by the neutrino mass matrix. That is, the same mechanism which is responsible for the smallness of neutrino mass leads also to the large lepton mixing, and it is the seesaw mechanism \cite{16} that plays the role of additional structure that generates the bi-maximal mixing. Therefore

\begin{equation}
U_{PMNS} = U^t_1U_\nu = V^t_{\text{CKM}} \Gamma\alpha U_{bm}, \tag{4}
\end{equation}

where $\Gamma\alpha = \text{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3})$ is the phase matrix that can appear, in general, at diagonalization of the charged lepton or neutrino Dirac mass matrices.

Similarity of the Dirac mass matrices in the lepton and quark sectors, related to the quark-lepton symmetry, is the origin of the CKM rotations in the lepton sector. Here, there are two possibilities:

(i) In a certain ("symmetry") basis, where the theory of flavor is formulated, the neutrino mass matrix is of the bi-maximal form. So $U_\nu = U_{bm}$, and the charged lepton mass matrix is diagonalized by the CKM rotation:

\begin{equation}
U_1 = V_{\text{CKM}}. \tag{5}
\end{equation}

The problem here is that the masses of charged leptons and down quarks are different: in particular, $m_e/m_\mu = 0.0047$, whereas $m_d/m_s = 0.04 - 0.06$, and also $m_\mu \neq m_s$ at the grand unified theory (GUT) scale. Since $m_1 \neq m_d$, the equality (3) implies particular structure of the mass matrices in which mixing weakly depends on eigenvalues.

(ii) In the "symmetry" basis both the bimaximal and CKM mixings come from the neutrino mass matrix, and the charged lepton mass matrix is diagonal. That is, the symmetry basis coincides with the flavor basis. In this case the Dirac mass matrix of neutrinos is the origin of the CKM rotation, whereas the Majorana mass matrix of the right-handed (RH) neutrinos is responsible for the bi-maximal mixing. Since the eigenvalues of the Dirac neutrino mass matrix are unknown we can assume an exact equality of the mass matrices

\begin{equation}
m_\alpha = m_\nu^D, \tag{6}
\end{equation}

as a consequence of the quark-lepton symmetry. The equality (6) propagates the CKM mixing from the quark to the lepton sector precisely. In this case, however, the Gatto-Sartori-Tonin relation between the Cabibbo angle and the ratio of down quark masses \cite{17} turns out to be accidental. Furthermore, one needs to explain why in the symmetry basis both the charged lepton and down quark mass matrices are diagonal simultaneously in spite of difference of eigenvalues.

These two cases have different theoretical implications, however the phenomenological consequences and the RG effects are the same.

In the scenario under consideration, the relation (1) is not realized precisely even for zero phases $\alpha_i$ since the
mixing is large in this scenario \cite{4, 11, 12}: at the level of accuracy we will consider here. The 1-3 elements of the mixing matrix, as well as by the last term, that turn out to be relevant \delta
parameterization independent. In the standard parameterization form \cite{4}. From (4) we obtain the following expressions for the leptonic mixing angles:

\[ U_{e2} \equiv \cos \theta_{13} \sin \theta_{12} = \sin(\pi/4 - \theta_C) \]
\[ + 0.5 \sin \theta_C \left( \sqrt{2} - 1 - V_{cb} \cos(\alpha_3 - \alpha_1) \right) \]
\[ + 0.5 V_{ub} \cos(\alpha_3 - \alpha_1 - \delta_q), \]

where \delta_q is the quark CP-violating phase. This expression differs from the one derived in \cite{4} by a factor \cos \theta_{13} as well as by the last term, that turn out to be relevant at the level of accuracy we will consider here. The 1-3 mixing is large in this scenario \cite{4, 11, 12}:

\[ \sin \theta_{13} = -\frac{\sin \theta_C}{\sqrt{2}} (1 - V_{cb} \cos(\alpha_3) - \frac{V_{ub}}{\sqrt{2}} \cos(\alpha_3 - \delta_q)) \approx -\frac{\sin \theta_C}{\sqrt{2}}, \]

and, hence, the Dirac CP phase \delta is close to 180\(^\circ\). So, for the 1-2 mixing we find from (7) and (8)

\[ \sin \theta_{12} \approx U_{e2} (1 - \frac{1}{4} \sin^2 \theta_C), \]

and \( U_{e2} \) is given in (7). Expression for the 2-3 mixing reads

\[ U_{\mu 3} = \cos \theta_{13} \sin \theta_{12} \]
\[ = \cos \theta_C \left( \sin(\pi/4 - \theta_{cb}) + \frac{V_{cb}}{\sqrt{2}} (1 - \cos \alpha_3) \right). \]

The RG effect on \( V_{\text{CKM}} \) is negligible.

\section*{B. QLC and parameterization.}

The QLC relations are essentially parameterization independent. They can be expressed in terms of physical quantities (compare with \cite{18}). Indeed, the moduli of elements of the mixing matrix, \( U_{\alpha i} \), are physical quantities immediately related to observables and consequently, parameterization independent. In the standard parameterization

\[ |U_{e2}| = |\cos \theta_{13} \sin \theta_{12}|, \quad |U_{e3}| = |\sin \theta_{13}|, \]

and therefore

\[ |\sin \theta_{12}| = \frac{1}{\sqrt{1 - |U_{e3}|^2}} |U_{e2}|. \]

Notice that the presence of 1-3 mixing produces some ambiguity in formulation of the QLC relations. One can write the relations in terms of angles in the standard parameterization or in terms of matrix elements:

\[ \arcsin(V_{us}) + \arcsin(U_{e2}) = \pi/4. \]

Both forms coincide in the limit \( U_{e3} \to 0 \).

\section*{C. Experimental status}

In fig. 1 we show results of determination of \( \theta_{12} \) by different groups. SNO collaboration did analysis of the data in terms of 2\( \nu \) mixing \cite{15}, whereas in \cite{20} and \cite{21} complete 3\( \nu \) analyses have been performed. Furthermore, in \cite{21} a non-zero best fit value of 1-3 mixing has been obtained. Results of different analyses are in a very good agreement:

\[ \theta_{12} = (33.8 \pm 2.2)^\circ (\theta_{13} = 0), \]
\[ \theta_{12} = (34.2 \pm 1.5)^\circ (\theta_{13} = 7^\circ). \]

Notice that the determination of \( \theta_{12} \) follows mainly from analysis of the solar neutrino data. In this analysis \( \theta_{12} \) and \( \theta_{13} \) correlate. In particular, the CC/NC ratio that gives the most important restriction on mixing is determined by \( P \sim \cos^4 \theta_{13} \sin^2 \theta_{12} \). The best fit values (14) are along with the trajectory \( P = \text{constant} \).

In fig. 1 we show the range of QLC values of \( \theta_{12} \) obtained from eqs. \cite{7} \cite{9} by varying \( (\alpha_3 - \alpha_1) \). This variation gives

\[ \theta_{12}^{\text{QLC}} = 35.65 - 36.22^\circ, \]

or

\[ \sin^2 \theta_{12}^{\text{QLC}} = 0.340 - 0.349. \]

The smallest value of \( \theta_{12} \) corresponds to \( \alpha_3 - \alpha_1 = -24^\circ \). For the 1-3 mixing we obtain

\[ \sin^2 \theta_{13} \approx \frac{1}{2} \sin^2 \theta_C = 0.024, \]

which is at the upper 1\( \sigma \) edge from the analysis \cite{21}.

The sums of angles equal

\[ \theta_{12} + \theta_C = 46.7^\circ \pm 2.4^\circ \quad (1\sigma) \]
\[ \theta_{23} + \theta_{cb} = (43.9 \pm 5.1)^\circ \quad (1\sigma). \]
The QLC prediction is slightly larger than the experimental best fit point:

$$\theta_{12}^{QLC} - \theta_{12}^{exp} = 1.5^\circ - 2.0^\circ. \quad (20)$$

The difference is well within 1\(\sigma\) of experimental measurements. The exact complementarity value, \(45^\circ - \theta_C\), is \((1.8 - 2.0)^\circ\) below the best fit value. To disentangle these possibilities one needs to measure the 1-2 angle with accuracy better than 1 degree: \(\Delta \theta_{12} < 1^\circ\), that is translated into

$$\frac{\Delta \sin^2 \theta_{12}}{\sin^2 \theta_{12}} = \frac{2}{\tan \theta_{12}} \Delta \theta_{12} \sim 5\% (\Delta \theta_{12}/1^\circ), \quad (21)$$

or

$$\frac{\Delta \sin^2 2 \theta_{12}}{\sin^2 2 \theta_{12}} = \frac{4}{\tan 2 \theta_{12}} \Delta \theta_{12} \sim 2.7\% (\Delta \theta_{12}/1^\circ). \quad (22)$$

Forthcoming results from SNO phase-III (He) will improve determination of the CC/NC ratio, and consequently, \(\theta_{12}\). Future low energy solar neutrino experiments aimed at measurements of the pp-neutrino flux will have a (1 - 2 \% ) sensitivity to \(\sin^2 2 \theta_{12}\), provided that degeneracy with 1-3 mixing is resolved. Similar sensitivity could be achieved in dedicated reactor neutrino experiments with a large base-line [22].

D. QLC and tri-bimaximal mixing

The QLC prediction [13] is practically indistinguishable from the tri-bimaximal mixing [23] prediction \(\sin^2 \theta_{12} = 1/3\). So, it turns out that almost the same values of 1-2 mixing are obtained from two different and independent combinations of matrices

$$U_{23}^m U_{12}(\arcsin(1/\sqrt{3})) \quad \text{and} \quad U_{12}(\theta_C) U_{23}^m U_{12}^m. \quad (23)$$

There are two possible interpretations of this fact:

The coincidence is accidental, which means that one of the two approaches (QLC or tri-bimaximal mixing) does not correspond to reality. To some extend that can be tested by measuring the 1-3 mixing. In the QLC-scenario one obtains [17], whereas the tri-bimaximal mixing implies \(\sin^2 \theta_{31} = 0\) unless some corrections are introduced.

The coincidence is not accidental, and therefore it implies a non-trivial expression for the Cabibbo angle. Indeed, from the equality \(\sin \theta_{12}^{QLC} = \sin \theta_{12}^{bimax}\) we obtain

$$\sin \theta_C = \frac{2}{3\sqrt{3}} \left( \sqrt{\frac{5}{2}} - 1 \right). \quad (24)$$

III. SEESEAW AND QLC

The seesaw mechanism [16] that provides a natural explanation of smallness of neutrino mass can also be the

origin of the difference of the quark and lepton mixings, and in particular, the origin of bimaximal mixing. In the context of seesaw type-I this implies a particular structure of the RH neutrino mass matrix.

We assume that the type-I seesaw gives the dominant contribution to neutrino masses since it can provide the closest relation between the quark and lepton mass matrices, as required by the QLC. The relevant terms of the Lagrangian are

$$-\mathcal{L} = t^c Y_e L H_d + N^T Y_\nu L H_u + \frac{1}{2} N^T M_R N + h.c., \quad (25)$$

where \(L = (\nu, l)\) is the leptonic doublet, \(N = (\nu_R)^c\), \(H_d\) and \(H_u\) are two different Higgs doublets in the MSSM and \(H_d = i\tau_2 H_d\) in SM; \(Y_e\) and \(Y_\nu\) are the charged lepton and neutrino Yukawa coupling matrices. We consider \(M_R\) as a bare mass matrix of the RH neutrinos formed already at the GUT or even higher scale. It can be generated by some new interactions at higher scales.

Decoupling of \(N\) leads to the low energy effective \(D = 5\) operator

$$\nu^c Y_\nu^T M_R^{-1} Y_\nu H_u H_u. \quad (26)$$

After the electroweak symmetry breaking this operator generates the mass term for light neutrinos, \(\nu^T m_\nu\nu\) with

$$m_\nu = -m_D^T M_R^{-1} m_D, \quad (27)$$

where \(m_D = Y_e \langle H_u \rangle\).

Let us consider the basis where the neutrino Dirac mass matrix is diagonal:

$$Y_\nu = Y_\nu^{\text{diag}} \equiv \text{diag}(y_1, y_2, y_3). \quad (28)$$

Then the light neutrino mass matrix equals

$$m_\nu = -m_D^{\text{diag}} M_R^{-1} m_D^{\text{diag}}, \quad (29)$$

and \(m_D^{\text{diag}} \equiv Y_\nu^{\text{diag}} \langle H_u \rangle\).

According to our assumption, the matrix [20] should generate the bimaximal rotation:

$$m_\nu = m_{bm}, \quad (30)$$

where in general,

$$m_{bm} = \Gamma_\delta U_{bm} \Gamma_{\varphi/2} m_\nu^{\text{diag}} \Gamma_{\varphi/2} U_{bm}^T \Gamma_\delta. \quad (31)$$

Here

$$\Gamma_\delta \equiv \text{diag}(e^{i\delta_1}, e^{i\delta_2}, e^{i\delta_2}), \quad (32)$$

is the phase matrix,

$$m_\nu^{\text{diag}} \equiv \text{diag}(m_1, m_2, m_3) \quad (33)$$

is the diagonal matrix of the light neutrinos, and

$$\Gamma_\varphi \equiv \text{diag}(e^{i\varphi_1/2}, e^{i\varphi_2/2}, 1), \quad (34)$$

is the $\varphi$-phase matrix.
with $\varphi_i$ being the Majorana phases of light neutrinos.

According to our assumption, the CKM rotation follows from diagonalization of the charged lepton matrix

$$V_{\text{CKM}}^\dagger Y_e Y_e V_{\text{CKM}} = \text{diag} (y_e^2, y_\mu^2, y_\tau^2), \quad (35)$$

and we will parameterize it as

$$V_l = \Gamma_\phi V_{\text{CKM}}(\theta_\ell, \delta_\ell). \quad (36)$$

Here the diagonal matrix of the phase factors on the RH side has been absorbed in the charged lepton field redefinition; $V_{\text{CKM}}$ is the CKM matrix in the standard parameterization, $\theta_\ell$ and $\delta_\ell$ are the quark (CKM) mixing angles, and

$$\Gamma_\phi \equiv \text{diag} (e^{i \phi_1}, e^{i \phi_2}, e^{i \phi_3}). \quad (37)$$

Thus, in general, there are three matrices of phases, $\Gamma_\delta$, $\Gamma_\gamma$, and $\Gamma_\phi$, relevant for relations between the mixing angles. Finally, from (36) and (31) we obtain

$$U_{PMNS} = V_{\text{CKM}}^\dagger (\theta_\ell, \delta_\ell) \Gamma (\delta_l - \phi_l) U_{bm}, \quad (38)$$

and therefore in (31) $\alpha_j = (\delta_j - \phi_j)$.

The neutrino mass matrix in the flavor basis equals

$$m^f_\nu = V_{\text{CKM}}^T m_{\nu} V_{\text{CKM}}. \quad (39)$$

From (29) and (31) we find an expression for the RH neutrino mass matrix:

$$M_R = \Gamma_\delta m^v_{\nu} \text{diag} U_{bm} \Gamma_{\varphi/2} (m^v_{\nu})^{-1} \Gamma_{\varphi/2} U^T_{bm} m^v_{\nu} \text{diag} \Gamma_\delta. \quad (40)$$

Omitting the phase factor $\Gamma (\delta_j)$ (that can be absorbed in the definition of $M_R$) and including the CP phases $\varphi_i$ into masses of light neutrinos $\Gamma_{\varphi/2} (m^v_{\nu})^{-1} \Gamma_{\varphi/2} = (m^v_{\nu})^{-1}$, we obtain

$$M_R = m^v_{\nu} \text{diag} U_{bm} (m^v_{\nu})^{-1} U^T_{bm} m^v_{\nu}. \quad (41)$$

Explicitly

$$M_R = \frac{1}{4} m^v_{\nu} \text{diag} \begin{pmatrix} 2A - \sqrt{2B} & -\sqrt{2B} \\ ... & C + A \\ ... & C - A \end{pmatrix} m^v_{\nu}, \quad (42)$$

where

$$A \equiv \frac{1}{m_1} + \frac{1}{m_2}, \quad B \equiv \frac{1}{m_2} - \frac{1}{m_1}, \quad C \equiv \frac{2}{m_3}, \quad (43)$$

(with phases $\varphi_i$ included). We can parameterize $m^v_{\nu}$ as

$$m^v_{\nu} = m_t \text{diag} (\epsilon^2, \epsilon, 1), \quad (44)$$

with $m_t$ being the mass of top quark and $\epsilon' \approx \epsilon \sim 3 \cdot 10^{-3}$. Using smallness of $\epsilon$'s it is easy to estimate the mass eigenvalues:

$$M_3 \approx \frac{m_t^2}{4} (A + C), \quad M_2 \approx \frac{m_t^2 \epsilon^2}{A + C}, \quad M_1 \approx \frac{m_t^2 \epsilon''^2}{2A}. \quad (45)$$

Furthermore, the 1-2 and 2-3 mixing angles are of the order $\epsilon$, whereas 1-3 mixing is of the order $\epsilon'^2$.

In the case of normal mass hierarchy, $m_1 \ll m_2 \ll m_3$, eqs. (45) lead to

$$M_3 \approx \frac{m_t^2}{4 m_1}, \quad M_2 \approx \frac{2 m_t^2 \epsilon^2}{m_3}, \quad M_1 \approx \frac{2 m_t^2 \epsilon'^2}{m_2}, \quad (46)$$

in agreement with results of [24]. Notice a permutation character of these expressions: the masses of RH neutrinos 1, 2, 3 are determined by light masses 2, 3, 1. With $m_1 \to 0$, apparently, $M_3 \to \infty$. For $\epsilon' = \epsilon \sim 3 \cdot 10^{-3}$ and $m_1 = 10^{-3}$ eV values of masses equal $M_3 = 9 \cdot 10^{15}$ GeV, $M_2 = 1 \cdot 10^{10}$ GeV, $M_1 = 5 \cdot 10^5$ GeV.

So, masses have a “quadratic” hierarchy.

In the case of inverted mass hierarchy, $m_3 \ll m_2 \ll m_1$, and the same CP phases of $\nu_1$ and $\nu_2$ we obtain from (45)

$$M_3 \approx \frac{m_t^2}{4 m_3}, \quad M_2 \approx \frac{2 m_t^2 \epsilon^2}{m_3}, \quad M_1 \approx \frac{2 m_t^2 \epsilon'^2}{m_2}, \quad (48)$$

where $m_A \equiv \sqrt{\Delta m^2_{13}}$. This leads again to a strong mass hierarchy. Notice that now the mass of the lightest RH neutrino is determined by the atmospheric mass scale. Thus, apart from special regions in the parameter space that correspond to level crossings (see sect. 5) the QLC implies generically a very strong (“quadratic”) mass hierarchy of the RH neutrinos and very small mixing: $\delta_{ij} \sim \epsilon$. As we will see, this determines substantially the size of the RG effects.

Let us introduce the unitary matrix, $U_R$, which diagonalizes the right-handed neutrino mass matrix

$$U_R^\dagger M_R U_R = M_R^\text{diag} \equiv \text{diag} (M_1, M_2, M_3), \quad (49)$$

and the mixing matrix can be parameterized as

$$U_R = \Gamma_{\Delta} V_{\text{CKM}}(\Theta_{ij}, \Delta) \Gamma_{\xi/2}, \quad (50)$$

where $\Theta_{ij}$ and $\Delta$ are the angles and CP-phase of the RH neutrino mixing matrix.

In what follows we will not elaborate further on the origin of particular structures of $M_R$ (42), just noticing that it can be related to the double (cascade) seesaw mechanism [25] with the “screening” of Dirac structure [26, 27].
IV. RG EFFECTS: GENERAL CONSIDERATION AND THE MSSM CASE

A. General consideration

The quark-lepton symmetry implied by the QLC relation means that physics responsible for this relation should be realized at some scale $M_F$ which is at the quark-lepton unification scale, $M_{GUT}$, or even higher scales. An alternative possibility would be the quark-lepton relation due to the Pati-Salam symmetry [28] broken below the GUT scale. Consequently, there are three different regions of RG running:

(i) below the seesaw scales, $\mu < M_1$, where $M_1$ is the lightest RH neutrino mass. In this region all three neutrinos decouple and the $D=5$ operator (26) is formed;

(ii) between the seesaw scales, $M_1 < \mu < M_3$, where $M_3$ is the heaviest RH neutrino mass;

(iii) above the seesaw scales $M_3 < \mu < M_F$. If $M_F > M_{GUT}$ new features of running can appear above $M_{GUT}$.

\[ 32\pi^2 \theta_{12} = Q_{12} \left[ \sin 2\theta_{12} \left( P_{11} - c_{23} P_{22} - s_{23} P_{33} + s_{23} \Re P_{23} \right) + 2 \cos 2\theta_{12} \left( c_{23} \Re P_{21} - s_{23} \Re P_{31} \right) \right] + 4 S_{12} \left( c_{23} \Im P_{21} - s_{23} \Im P_{31} \right), \tag{53} \]

where $s_{23} \equiv \sin \theta_{23}$, $c_{23} \equiv \cos \theta_{23}$, etc.,

\[ Q_{ij} \equiv \frac{|m_i e^{i\varphi_i} + m_j e^{i\varphi_j}|^2}{\Delta m_{ji}^2}, \tag{54} \]

and

\[ S_{12} \equiv \frac{m_1 m_2 \sin (\varphi_1 - \varphi_2)}{\Delta m_{21}^2}. \tag{55} \]

Above the seesaw scale one needs to consider renormalization of couplings of the full Lagrangian [29]. The evolution of the effective operator which gives masses to neutrinos after the electroweak symmetry breaking is determined by evolution of the neutrino Yukawa couplings $Y$ and the mass terms of right-handed neutrinos.

Below the seesaw scales, running is dominated by $P_{33}$ in the flavor basis which results in an increase of $\theta_{12}$ in MSSM and a slight decrease in the SM due to different signs of $C_e$:

\[ 32\pi^2 \theta_{12} \approx -Q_{12} \sin 2\theta_{12} s_{23}^2 P_{33}. \tag{56} \]

Above the seesaw scales, the leading contribution is again given by $P_{33}$, and the next-to-leading contribution comes from $P_{22}$. This yields an increase of $\theta_{12}$ when running to low scales both in the MSSM and in SM.

The RG equation for the neutrino mass matrix is given by [29, 30, 31]

\[ 16\pi^2 \dot{m}_\nu = P^T m_\nu + m_\nu P + \kappa m_\nu, \tag{51} \]

where $\dot{m}_\nu \equiv \mu \frac{d m_\nu}{d\mu}$, $\mu$ is the renormalization scale, and $\kappa m_\nu$ includes the gauge interaction terms that can influence the flavor structure in the SM case (see below);

\[ P \equiv C_e Y_e^T Y_e + C_\nu Y_\nu^T Y_\nu, \tag{52} \]

$C_e = -3/2$, $C_\nu = 1/2$ in the SM and $C_e = C_\nu = 1/2$ in the MSSM. From the evolution equation (51), for the mass matrix we can obtain the equations [42] for observables (masses and mixings) [32, 33, 34, 35]. Above and below the seesaw scales the gauge interactions produce a flavor universal effect and all contributions (from all RH neutrinos) to the neutrino mass matrix have the same renormalization group equation.

In the limit of vanishing 1-3 mixing the evolution of $\theta_{12}$ is described approximately by

Explicitly the corresponding evolution equation can be written as

\[ 32\pi^2 \dot{\theta}_{12} = -Q_{12} C_e \sin 2\theta_{12} \sin \theta_{23} \left[ \sin \theta_{23} - V_{cb} \cos \theta_{23} \cos (\phi_2 - \phi_3) \right], \tag{57} \]

and the phases $\phi_i$ are determined in Eq. [36, 37].

Effect of running between the seesaw scales (about 10-orders of magnitude in $\mu$) is more complicated. In this range the Yukawa coupling matrix has two terms (contributions):

(1) $D=5$ effective operators for the light neutrinos formed after decoupling of one or two RH neutrinos,

(2) Dirac type couplings and mass terms for undecoupled RH neutrinos given by the Lagrangian [29].

These terms are renormalized differently. In particular, for the terms of second type the neutrino Yukawa couplings are important. The difference, however, cancels (between the seesaw scales) in the case of MSSM in which only the wave function renormalization takes place due to the non-renormalization theorem. In contrast, in the SM due to vertex corrections to the $D=5$ operators the difference does not cancel, and, as we will see, produces a significant effect. So, in the MSSM, the RG equations are the same for both contributions and eq. [53] is valid. In the SM they are not equal and
After the heaviest right–handed neutrino is integrated out, the right–handed neutrino mixing at the threshold influences running of $\theta_{12}$. In the second order of $\sin\theta_C$, the expression for $\theta_{12}$ reads:

$$
32\pi^2\dot{\theta}_{12} = -\frac{1}{4}Q_{12}C_{\nu}(s_{23} - V_{cb}c_{23}\cos(\phi_2 - \phi_3))
$$

$$(3 - 2\cos 2\Theta_{23}\cos^2 \Theta_{13} - \cos 2\Theta_{13}) \sin 2\theta_{12}s_{23}, \quad (58)$$

where $\Theta_{ij}$ are the right-handed neutrino mixing angles at the scale at which the heaviest neutrino is integrated out. The unitary rotation of the right–handed neutrino fields is done at the threshold of the heaviest right–handed neutrino, and the exact definitions of the angles are given in Eq. (53) [53].

### B. RG evolution and scales of flavor physics

We have performed running from the $M_F$ scale down to the electroweak scale and calculated $\Delta\theta_{12} \equiv \theta_{12}(M_Z) - \theta_{12}(M_F)$. For that we solved numerically a complete set of the RG equations including sub-leading effects due to the non-zero 1-3 mixing. In most of our calculations we take for definiteness $M_F = M_{GUT} = 2 \cdot 10^{16}$ GeV. We consider separately dependence of the results on $M_F$. Notice that the renormalization of $\theta_{12}$ in the bimaximal scheme has been studied in [36].

The following free parameters determine the RG effects substantially: the absolute scale of light masses $m_1$, the CP (Majorana) phases of light neutrinos, $\varphi_1$, and the phases $\alpha_i$. We studied dependence of the RG effects on these parameter. For each set of the parameters we have calculated the RH mass matrix and running effects. The angles are fixed by the QLC relation at $M_F$, and the mass squared differences are adjusted to lie in the experimentally allowed region at the electroweak scale. For the neutrino Yukawa couplings we take $y_1 : y_2 : y_3 = \epsilon^2 : \epsilon : 1$, ($\epsilon = \epsilon'$) and $\epsilon = 3 \cdot 10^{-3}$.

We consider RG evolution in the MSSM with a unique SUSY threshold $1$ TeV. The RG effects depend on the absolute mass scale, $m_1$, tan $\beta$, and the relative phase between the first and second mass eigenstates $\varphi \equiv \varphi_2 - \varphi_1$. Dependence on other parameters (e.g., other phases) is rather weak. Still we will use explicitly the phase $\varphi_2$ keeping everywhere $\varphi_1 = 0$.

In what follows we will describe results of our numerical calculations. We give an interpretation of the results using approximate formulas presented in Secs. III and IV.1.

### C. RG effect in MSSM with normal mass hierarchy

In fig. [2] we show some examples of the scale dependence of $\theta_{12}$ for various values of parameters. With increase of $m_1$ two factors enhance the RG effects:

1. The largest mass $M_3$ decreases according to $\theta_{12}$. Correspondingly, the region above the seesaw scale, $M_3 - M_{GUT}$ increases.

2. Corrections to the mass matrix elements are proportional to values of elements: $\Delta m_{\alpha\beta} \propto m_{\alpha\beta}$ and since with increase of $m_1$ the masses, $m_{\alpha\beta}$, generically increase, the corrections increase correspondingly.

For relatively small $\tan\beta$ $\sim (3 - 10)$, the dominant contribution follows from region above the seesaw scales due to large $(\nu_e)_{33}$. Evolution below $M_3$ is mainly due to the Yukawa couplings $Y_\nu$ which are relatively small. The effect increases fast with $m_1$:

$$
\Delta\theta_{12} \propto Q_{12}\log(M_{GUT}/M_3).
$$

(59)

Notice that $M_3 \ll 1/m_1$. Therefore for $m_1 \sim 10^{-3}$ eV the running of $\theta_{12}$ is mainly related to increase of region above the seesaw scale. For $m_1 > 10^{-2}$ eV the spectrum of light neutrinos becomes degenerate and $\Delta\theta_{12} \propto Q_{12} \propto m_1^2$ (fig. 2(a)). For large $\tan\beta$ and small $m_1$ the dominant contribution to $\Delta\theta_{12}$ comes from the region below $M_3$ where $\Delta\theta_{12} \propto \tan\beta$ (see fig. 2(b)).

A combined dependence of corrections, $\Delta\theta_{12}$, on $m_1$ and $\tan\beta$ is presented in fig. [3] where we show contours of constant $\Delta\theta_{12}$ in the $(m_1 - \tan\beta)$ plane. The change of behavior of contours at $m_1 = 8 \cdot 10^{-4}$ eV is a consequence of our boundary condition: At $m_1 < 8 \cdot 10^{-4}$ eV we have $M_3 > M_{GUT}$, and therefore the region above seesaw scale disappears.

In fig. [4] we show the correction $\Delta\theta_{12}$ as functions of $m_1$ for different values of $\varphi_2$. The dependence of $\Delta\theta_{12}$ on $\varphi_2$, given essentially by the factor $Q_{12}$, is weak for the hierarchical spectrum, $m_1 \ll 8 \cdot 10^{-3}$ eV, and very strong for the degenerate spectrum: $\Delta\theta_{12} \propto (1 + \cos \varphi)$.

The corrections are strongly suppressed for the opposite CP-parities, $\varphi_2 = 180^\circ$, (fig. [4]) that agrees with the results of previous studies of corrections in the degenerate case [37, 38, 39].

Corrections $\Delta\theta_{12}$ are positive. This fact is essentially a consequence of strong hierarchy of the Yukawa couplings $Y_e$ and $Y_\nu$. The evolution is given approximately by eq. [53], where $P_{33} \propto 1/2((|Y_\nu|)_{33}^2 + |(Y_e)_{33}|^2) > 0$. The off-diagonal couplings $P_{13}$ are much smaller. Since $Q_{12} > 0$ we obtain $\theta_{12} < 0$, that is, the angle $\theta_{12}$ increases with decrease of $\mu$.

Condition that the QLC prediction for $\theta_{12}$ is within $1\sigma$ of the best fit experimental value requires $\Delta\theta_{12} < 0.5^\circ - 1^\circ$. This, in turn, leads to bounds on parameters of neutrino spectrum and $\tan\beta$. In particular, according to fig. [4] the degenerate neutrino spectrum is excluded for the same CP parities ($\varphi_2 = 0$). In the case of large $\tan\beta$ it requires strongly hierarchical spectrum: $m_1 < 10^{-3}$ eV that eliminates the running region above seesaw scale. However, a degenerate spectrum is allowed for $\varphi \sim 180^\circ$.

Taking $2\sigma$ upper bound $\Delta\theta_{12} < 2^\circ$ we find that the quasi-degenerate spectrum with $m_1 \sim 10^{-2}$ eV is allowed even for the same parities. For normal mass hierarchy with $m_1 < 10^{-3}$ eV and $\tan\beta \sim (3 - 10)$ the
The dependence of the RG correction, $\Delta \theta_{12}$, on $m_1$ and $\tan \beta = 10$: (b) on $\tan \beta$ for $m_1 = 10^{-3}$ eV. All the CP-phases are taken to be zero.

In the case of inverted mass hierarchy, the states $\nu_1$ and $\nu_2$ associated to 1-2 mixing are strongly degenerate. Therefore, the RG effects are similar to those in the normal hierarchy case for $m_1 = m_A \sim 5 \cdot 10^{-2}$ eV. So, the corrections, $\Delta \theta_{12}$, are enhanced by the factor

$$\frac{(\Delta \theta_{12})^{IH}}{(\Delta \theta_{12})^{NH}} = \frac{(m_2^{IH})^2}{(m_2^{NH})^2},$$

where subscripts NH and IH stand for normal and inverted mass hierarchy. This factor equals

$$\frac{\Delta m_{13}^2}{\Delta m_{21}^2} \text{ or } \frac{(m_1^{IH})^2}{(m_1^{NH})^2}$$

for the strong normal hierarchy and normal ordering ($m_1 \approx m_2$) correspondingly.

In fig. 4 we show examples of running of $\theta_{12}$ for different values of masses and phases. Dependences of $\theta_{12}$ are well described by $Q_{12}$, as in the case of normal mass hierarchy. Notice that now the heaviest RH neutrino mass is determined by $m_3$, and two others by $m_A$. With increase of $m_3$ (now the lightest neutrino mass) (fig. 5[a]) the range above the seesaw scales, where the evolution of $\theta_{12}$ is most strong, increases. The change of $\theta_{12}$ below $M_3$ is slower being of the same size for different values of $m_3$ (until $m_3 \ll m_A$). In this range the evolution is essentially due to $Y_\ell$ couplings, so that $\Delta \theta_{12} \propto \tan^2 \beta$ (fig. [5(b)]). The correction can be strongly suppressed for the opposite CP-parities of $\nu_1$ and $\nu_2$: $\Delta \theta_{12} \propto (1 + \cos \varphi)$.

As in the case of normal hierarchy (ordering), in a large part of the parameter space the correction is positive, $\Delta \theta_{12} > 0$, due to dominant effect of $P_{33}$. For $\varphi_2 = 0$, consistency of the QLC prediction with data taken as $\Delta \theta_{12} < 2^\circ$, implies $\tan \beta < 10$ and $m_3 < 8 \cdot 10^{-4}$ eV.
According to this equation for \( \varphi_2 = 180^\circ \), \( \varphi_1 = 0^\circ \) and \( \delta = 180^\circ \), the dominant contribution is determined by the combination \( -\frac{m_3}{m_1} \sin^2 \theta_{12} \sin 2\theta_{23} \), that is positive in the inverted hierarchy case, and therefore \( \dot{\theta}_{12} \) decreases from high to low energies.

\[
\frac{C_v \theta_{13}}{32 \pi^2} \sin 2\theta_{23} \left[ (Q_{12} \cos 2\theta_{12} + Q_{13} s_{12}^2 + Q_{23} c_{12}^2) \cos \delta \right. \\
+ 2 \left( \frac{m_1 m_2}{\Delta m_{31}^2} \sin (\varphi_1 - \varphi_2) + \frac{m_1 m_3}{\Delta m_{31}^2} \sin \varphi_1 s_{12}^2 + \frac{m_2 m_3}{\Delta m_{32}^2} \sin \varphi_2 c_{12}^2 \right) \sin \delta \bigg] \quad (62)
\]

V. RG EFFECTS IN THE STANDARD MODEL

In general, for non-zero \( \theta_{13} \), the contribution to \( \dot{\theta}_{12} \) is given by \( \Delta \theta_{12} \).

For \( \varphi_2 \sim \pi \) corrections can be strongly suppressed, so that a larger region of the parameter space becomes allowed. The corrections become negative for \( \varphi_2 = \pi \) (see figs. 6-7) when the leading RG effects are strongly suppressed and the running is mainly due to sub-leading effect related to non-zero 1-3 mixing. This possibility has been mentioned in [4]. The sign of contribution due to non-zero \( \theta_{13} \) to the RG running of \( \dot{\theta}_{12} \) due to non-zero \( \theta_{13} \) depends on the parameter (masses, phases) region.

SUSY case [41]. Furthermore, the vertex diagrams with the gauge bosons become important: their contribution to running between the seesaw scales influences the flavor structure of mass matrix and therefore changes \( \theta_{12} \) [32, 34, 35]. The point is that individual RH neutrinos, \( N_i \), have the flavor dependent couplings with the left handed components of neutrinos. Therefore the gauge boson corrections to the corresponding couplings will influence the flavor structure. Above the seesaw scales (where all RH neutrinos are operative) and below the seesaw scales (where all RH neutrinos decouple), flavor universality of the gauge interaction corrections is restored. There is no simple analytic formula for the \( \theta_{12} \) renormal-
the both above and between the seesaw scales (see fig. 8(b)).

Those diagrams are finite and do not lead to logarithmic

erator due to the gauge (and also Yukawa) interactions

interactions,

Above the seesaw scales the running is due to the Yukawa

θ

12

is formed. The vertex diagram corrections to this op-
erator due to the gauge (and also Yukawa) interactions produce running. Notice that for other RH neutrinos

12

that do not decouple, the corresponding couplings pro-
duce box diagrams with propagators of the RH neutrinos.

Those diagrams are finite and do not lead to logarithmic
corrections. The gauge interaction effect dominates since

12

are small. The corrections increase with

m

1.

The most interesting dependence of ∆θ₁₂ is the one on

the CP-violation phase φ2 (fig. 8(b)). The corrections are positive, ∆θ₁₂ > 0, for φ₂ = 0. They are strongly

suppressed for φ₂ ∼ π/2, in contrast to the SUSY case

where suppression is realized for φ ∼ π. The corrections are negative for φ₂ > π/2. The angle of zero corrections, φ₂(0), depends on m₁ and in general deviates from π/2.

The deviation is due to the Yukawa interaction effects

that produce the positive shift for strong Yukawa cou-
pling hierarchy as we discussed before. The shift occurs

both above and between the seesaw scales (see fig. 8(b)).

In fig. 8(b) we show contours of constant corrections in

the m₁ – φ₂ plane, and in fig. 10 – an explicit depen-
dence of ∆θ₁₂ on m₁ for different values of φ₂. The line

Δθ₁₂ = 0, is close to φ₂ = π/2, 3π/2 for m₁ → 0, and

it approaches π with increase of m₁ when spectrum be-
comes strongly degenerate. The pattern is nearly sym-
metric with respect to φ = π for small m₁, the asymme-
try appears for m₁ > 3 · 10⁻³ eV.

The line Δθ₁₂ = 2° restricts the region consistent with

the QLC relation. Along the contours Δθ₁₂ = −1.5° the

best fit experimental value for θ₁₂ can be reproduced.

This corresponds to m₁ > 2 · 10⁻³ eV and φ₂ ∼ 5π/6 −

7π/6. Large negative corrections appear in the region

m₁ > 5 · 10⁻³ eV and φ₂ ∼ π.

VI. RENORMALIZATION OF 13 MIXING.
LEVEL CROSSING. EVOLUTION ABOVE M_GUT

A. Renormalization of 13 mixing

In the scenario discussed in this paper, the 1-3 mixing

is non-zero and relatively large at the boundary (17).

Notice that θ₁₃ (i) interferes with 1-2 mixing in the QLC

relation as we discussed before; (ii) produces sub-leading

effects in renormalization of θ₁₂, (iii) can provide further

bounds on the considered scenario if RG corrections are

positive and large.

The dominant contribution to the renormalization of

θ₁₃ is given by [33, 34, 35]

64π²δθ₁₃ = C₂ sin 2θ₁₂ sin 2θ₂₃(A₁₃ − A₂₃),

where

A₁₃ ≡ \frac{1}{\Delta m^2_{31}} \left[(m^2_1 + m^2_2) \cos \delta + 2m_1 m_2 \cos(\delta - \phi_1)\right].

(65)

In our case sin 2θ₁₂ > 0, sin 2θ₂₃ > 0, δ ≈ 180° and for

vanishing Majorana CP phases, φ₁ = 0, the dominant

correction can be approximated to

64π²δθ₁₃ = C₂ sin 2θ₁₂ sin 2θ₂₃(Q₂₃ − Q₁₃),

(66)

and the last factor in (66): (Q₂₃ − Q₁₃) = (A₁₃ − A₂₃) is

negative, irrespective of the mass hierarchy. Consequently θ₁₃ increases when running to low energies. For

non-vanishing phases φ₁ this factor can be positive, thus

leading to a decrease of θ₁₃ when φ decreases.

In the case of strong mass hierarchy eq. (66) gives

64π²δθ₁₃ = −2 sin 2θ₁₂ sin 2θ₂₃ \cos(δ - φ₂) \sqrt{\frac{\Delta m^2_{21}}{\Delta m^2_{31}}}. \quad (67)

The running is suppressed by small mass ratio. Therefore

only a small RG effect on 1-3 mixing appears for the

hierarchical (normal as well as inverted) case. For in-

stance, we find that for the parameter sets used in figure

2 (MSSM), the correction ∆θ₁₃ is always smaller than

0.2°. In the SM, it is smaller than 0.3°.

For the degenerate spectrum, there can be a larger

effect which strongly depends on the CP-phases. From
FIG. 8: Examples of running of $\theta_{12}$ in the case of SM and normal mass hierarchy. The dependence of $\theta_{12}$ on $\mu$ (a) for different values of $m_1$, and $\varphi_2 = 0$, (b) on $\varphi_2$ for $m_1 = 10^{-3}$ eV.

FIG. 9: Contours of constant RG corrections to $\theta_{12}$ (figures at the curves) in the $\varphi_2 - m_1$ plane in the case of SM and normal mass hierarchy.

FIG. 10: The dependence of the RG correction $\Delta\theta_{12}$ on $m_1$ for different values of $\varphi_2$ (figures at the curves) in the SM with the normal mass hierarchy.

[64] we find

$$64\pi^2\hat{\theta}_{13} \approx 2\sin 2\theta_{12} \sin 2\theta_{23} \frac{m_1^2}{\Delta m^2_{3\bar{1}}}[\cos(\delta - \varphi_1) - \cos(\delta - \varphi_2)].$$

(68)

Notice that for zero CP phases the cancellation occurs again. In the MSSM for $m_1 = 0.03$ eV and $\tan \beta = 50$, we find $\Delta\theta_{13} \sim 0.5^\circ$. In contrast, for $\delta = \varphi_1 = \pi$ and $\varphi_2 = 0$ the two terms in (68) sum up and we obtain positive running: $64\pi^2\hat{\theta}_{13} \approx 4\sin 2\theta_{12} \sin 2\theta_{23}$. Consequently $\theta_{13}$ becomes smaller at low energies.

B. Level crossing points

As we have established in sect. 3 the spectrum of the right–handed Majorana neutrinos is generically hierarchical. However, there are the level crossing points, where two of the RH neutrino masses become equal [24]. The case of degeneracy of two lighter RH neutrino states, $M_1 \approx M_2$, is of special interest from the point of view of generation of the baryon asymmetry in the Universe. In this case the resonance leptogenesis becomes possible which produces large enough asymmetry in spite of smallness of the masses and consequently, large wash out effect.

From (69) we find

$$M_1 = \frac{2m_\tau^2 \epsilon^4}{m_1 + m_2}, \quad M_2 = \frac{2m_e^2 \epsilon^2 (m_1 + m_2)}{(m_1 + m_2)m_3 + 2m_1m_2}. \quad (69)$$

(Here the Majorana phases are included in $m_1$). It is easy to see that due to smallness of $\epsilon$ the condition $M_1 \approx M_2$ can be satisfied only in the case of strong mass degeneracy $|m_1| \approx |m_2| \approx m_0$ when

$$m_1 + m_2 = \frac{\Delta m^2_{21}}{2m_0} \approx 0. \quad (70)$$

Then from the condition $M_1 \approx M_2$ we find

$$m_0 = \sqrt{\frac{\Delta m^2_{21}}{2\epsilon}} \approx 0.1 \text{ eV}. \quad (71)$$
In this special case the mass

$$M_1 \approx M_2 \frac{4m_1^2 e^4 m_0}{\Delta m_{21}^2} = M_1^{NH} \frac{2m_0}{\sqrt{\Delta m_{21}^2}}$$  \hspace{1cm} (72)

is enhanced by factor $2m_0/\sqrt{\Delta m_{21}^2} \sim 20$ and the third mass is much smaller than in the hierarchical case:

$$M_3 \approx \frac{m_1^2}{2m_3},$$  \hspace{1cm} (73)

that is, smaller by factor $m_1^{NH}/m_3 < 10^{-3}$.

The level crossing condition (70) implies the opposite CP-violating phases; it coincides with the condition of strong suppression of the RG effects. It also implies smallness of the 11-element of $m_{\nu \nu \nu}$ matrix. The condition for level crossing differs from that in [24] since here we require the neutrino Dirac matrix to be diagonal in the basis where the mass matrix of light neutrinos has exactly bimaximal form. If instead we use a generic matrix with non-maximal 1-2 mixing the level crossing condition can be realized for the hierarchical spectrum [24].

In fig. 11 we show scale evolution of the mixing angles for parameters that correspond to the level crossing point $M_1 = M_2$. In this point $M_1 = M_2 = 8 \cdot 10^6$ GeV, $M_3 = 8 \cdot 10^{13}$ GeV, $\varphi_1 = 0$, $\varphi_2 = \pi$, $m_1 = 0.13$ eV. The angle $\theta_{12}$ evolves very weakly due to cancellation $Q_{12} = S_{12} \approx 0$ related to (70). In contrast, the 1-3 mixing evolves substantially above thresholds: $\Delta \theta_{13} = 7^\circ$. The 2-3 mixing shows relatively weak evolution, that, however, can influence the second QLC relation.

We find that in this crossing point the solar mass squared difference becomes large even if it is very small at the boundary. So, the 1-2 split has the radiative origin. The 1-3 split decreases by factor $\sim 2$.

C. Evolution above the GUT scale

For $M_F > M_GUT$ one should perform running also above the GUT scale. Restoration of the GUT symmetry and unification of the gauge couplings does not prevent from different running of the Yukawa couplings, and therefore, from change of mixing angles. Renormalization of mixing angles would stop after possible unification of the Yukawa couplings which can be related, e.g., to restoration at $M_F$ a non-Abelian flavor symmetry. An alternative is the boundary at the string or Planck scale, where the Yukawa couplings are formed and their properties are determined immediately by some symmetry or/and string selection rules.

For illustration we performed the running in the MSSM up to the Planck scale (ignoring possible GUT effects). In fig. 12 we show the dependence of $\Delta \theta_{12}$ on $m_1$ for the same (QLC) initial conditions at the Planck scale: $M_F = M_{Pl} = 1.2 \cdot 10^{19}$ GeV. The RG effect becomes much larger. In particular the contribution from the region above the seesaw scale due to large Yukawa coupling $Y_\nu$ increases substantially. It is enhanced in comparison with the case of running up to $M_{GUT}$ by the factor

$$\frac{\log(M_{Pl}/M_3)}{\log(M_{GUT}/M_3)}$$  \hspace{1cm} (74)

that can be as large as 3 - 5 in some cases. Still for $\varphi_2 = \pi$ or for small $m_1$ the RG effects are suppressed and can be consistent with the QLC relations.

Similar RG effects are expected in the SU(5) model with the RH singlet neutrinos. In fact, no new diagrams with large $Y_\nu$ appear. Effect of the charged lepton couplings $Y_\ell$ is enhanced by factor 4 above $M_{GUT}$ due to the loop diagrams with down quarks (squarks) and $H^{1/3}$ charged Higgs bosons (Higgsinos).

The flavor-diagonal parts of the RG equations do influence the angles only indirectly through the change of the mass eigenvalues. Thus, the main effect of these interactions is due to the evolution of $\Delta m_{12}^2$. 

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FIG. 11: Examples of running of mixing angles in the case of $M_1 \approx M_2$ in MSSM and normal mass ordering. We show the dependence of $\theta_{12}, \theta_{13}, \theta_{23}$ on $\mu$ for $\tan \beta = 10$, $\varphi_1 = 0$, $\varphi_2 = \pi$ and $m_1 = 0.13$ eV.

FIG. 12: The dependence of the RG correction $\Delta \theta_{12}$ on $m_1$ for different values of $\varphi_2$ (figures at the curves) in the MSSM with the normal mass hierarchy and $\tan \beta = 10$. The boundary condition is at $M_{Pl}$. 

---

\[
\log_{10}(\frac{M_{Pl}/M_3}{M_{GUT}/M_3})
\]
VII. CONCLUSION

Experimental results on the 1-2 and 2-3 mixing in the quark and lepton sectors show certain correlations that can be interpreted as the quark-lepton complementarity. We considered the QLC scenario in which the bimaximal mixing follows from diagonalization of the neutrino mass matrix. In the lowest order of the perturbation theory, the value of angle $\theta_{12}$ predicted in this scenario is about $\sim 1\sigma$ larger than the best fit experimental value. It coincides practically with the value given by the tri-bimaximal mixing. We commented on implications of this equality as well as on perspectives of future tests of the QLC relations.

In this paper, we assumed that the QLC relations are not accidental coincidences, but consequence of the quark-lepton symmetry and additional structure in the theory that produces the bimaximal mixing. Here the accidental coincidences would mean that values of mixing angles are the result of interplay of two or more independent contributions. In this connection, we proposed a realization of the QLC scenario that provides the closest relation between quarks and leptons. It is based on
- the seesaw type-I mechanism that generates the bimaximal mixing due to specific structure of the RH neutrino mass matrix;
- approximate equalities of the Dirac mass matrices: $m_u \approx m_D$, $m_l \approx m_d$ that follow from the approximate quark-lepton symmetry or unification. A certain small violation of equalities of these matrices produces difference of mass hierarchies but does not affect substantially the mixing.

The only additional (and, in fact, unavoidable) factor that can affect the QLC relations is RG corrections.

One of the consequences of the proposed scenario is a very strong hierarchy of the RH neutrino masses, apart from several particular level crossing points. The latter are realized for strongly degenerate spectrum of light neutrinos, and particular values of the CP-violating phases. This determines substantially the RG effects.

We performed a systematic study of the RG effects in the SM and MSSM. We find that in the MSSM, the RG corrections to $\theta_{12}$ are generally positive due to a dominant effect of the Yukawa coupling $Y_{33}$. So, these corrections worsen agreement of the predicted $\theta_{12}$ with data.

In the MSSM small negative corrections, $|\Delta \theta_{12}| < 0.5^\circ$, can appear for the opposite CP parities of $\nu_1$ and $\nu_2$ and inverted mass hierarchy, in which case the main terms in the RG equations are strongly suppressed and running is due to the sub-leading effects related to the non-zero 1-3 mixing. The correction increases with $m_1$ and strongly depends on the relative Majorana phase. For $\varphi_2 = 0$ the consistency of the QLC prediction for $\theta_{12}$ with data implies strong mass hierarchy of light neutrinos and small tan $\beta$. For $\varphi_2 = \pi$ corrections are suppressed and even the degenerate spectrum becomes allowed. For the inverted mass hierarchy the corrections are generically enhanced by larger values of masses of $\nu_1$ and $\nu_2$.

The situation is qualitatively different in the SM. Here important contributions follow from the vertex corrections to the D=5 operator in the range between the seesaw scales. The Yukawa couplings (especially for small $m_1$) give sub-leading contribution. The RG corrections are negative in the interval $\varphi_2 = \pi/2 - 3\pi/2$ for small $m_1$ and the range of negative corrections becomes narrower, $\varphi_2 = (0.9 - 1.2)\pi$, for the degenerate neutrinos $\nu_1$ and $\nu_2$.

Corrections depend substantially on the boundary scale $M_F$. The value $\Delta \theta_{12}$ can be enhanced by factor 2-5 if $M_F$ increases from $M_{\text{GUT}}$ to $M_{\text{Pl}}$.

For the hierarchical mass spectrum renormalization of the 1-3 mixings is, in general, small: $\Delta \theta_{13} \sim 0.2^\circ - 0.3^\circ$. The correction can be large, $\Delta \theta_{13} \sim \theta_{13}$, for the degenerate spectrum. The sign of correction depends on values of CP-violating phases.

In conclusion, in a large part of the parameter space especially for the strong mass hierarchy and opposite CP phases of $\nu_1$ and $\nu_2$, the RG corrections to the QLC relation are small. The corrections are positive in the MSSM apart from small region of parameter space that corresponds to the degenerate spectrum of light neutrinos and their opposite CP parities. The corrections are negative in the SM for $\varphi_2 > \pi/2$. In the considered QLC scenario the RG corrections allow one to reproduce the best fit experimental value of $\theta_{12}$ exactly.

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