1. A nonperturbative ground state at high temperature

In spite of the innocent-looking title I will point out that the deconfining phase of an SU(2) Yang-Mills theory relies, in an essential way, on interspersed nonperturbative delicacies. As a consequence, unexpected results emerge in low-temperature, low-momentum photon propagation.

Let me first sketch the phase diagram of an SU(2) (or SU(3)) Yang-Mills theory as derived in $^1$ : There are a deconfining, high-temperature phase, a preconfining, thin intermediate phase, and a low-temperature confining phase. In the former two phases excitations are partially and exclusively massive gauge bosons, respectively. In contrast to perturbative screening temperature-dependent masses are induced by topological field configurations upon a spatial coarse-graining: Masses appear at tree-level by Higgs mechanisms in the associated effective theories. In the confining phase excitations are spin-1/2 fermions with equidistant mass spectrum set by the
Yang-Mills scale\textsuperscript{a}.

Accurate results are obtained by (i) the consideration of BPS saturated, topological configurations (calorons and anticalorons) and (ii) a self-consistent spatial coarse-graining. An essential singularity in the weight of (anti)calorons at zero coupling forbids them in weak-coupling expansions. This is at the heart of the magnetic-sector instability encountered in perturbation theory\textsuperscript{9,10}.

One writes a (unique) definition for the kernel $K$ of a to-be-determined differential operator $D$ in terms of the composite

\begin{equation}
\text{tr} \frac{\lambda^a}{2} F_{\mu\nu} \left( \tau, 0 \right) \left\{ \left( \tau, 0 \right), \left( \tau, \vec{x} \right) \right\} F_{\mu\nu} \left( \tau, \vec{x} \right) \left\{ \left( \tau, \vec{x} \right), \left( \tau, 0 \right) \right\}
\end{equation}

(1)

of fundamental field variables. In (1) $F_{\mu\nu}$ is the field strength and $\left\{ \left( \tau, 0 \right), \left( \tau, \vec{x} \right) \right\}$ denotes a fundamental Wilson line\textsuperscript{b}. $K$ contains the phase $\hat{\phi}^a$ of an emerging adjoint scalar field $\phi^a$. Due to an indefinite spatial coarse-graining $\hat{\phi}^a$ depends, in a periodic way, only on euclidean time $\tau$: No information on dimensional transmutation enters in $\hat{\phi}^a(\tau) \in K$. Therefore, it suffices to evaluate the object in (1) on absolutely stable classical configurations and the Wilson lines are along straight lines\textsuperscript{c}. But only BPS saturated configurations are absolutely stable\textsuperscript{d}. In passing I mention that adjointly transforming local composites vanish on BPS saturated configurations. For $Q = 0$ BPS saturated configurations are pure gauges, $F_{\mu\nu} \equiv 0$, and (1) vanishes identically. For $|Q| = 1$ stable BPS saturated configurations are trivial-holonomy or Harrington-Shepard (HS)\textsuperscript{11,12} (anti)calorons\textsuperscript{e}. The integration over the independent\textsuperscript{f} moduli of HS (anti)calorons (scale parameter $\rho$) must be subject to a flat measure since no scale exists which would set a 'spectral slope' for this dimensionless quantity. Also, there is

\textsuperscript{a}Except for a small range of temperatures above the critical temperature $T_c$ the pressure is positive in the deconfining phase and reaches the Stefan-Boltzmann limit in a power-like way. While the total pressure is negative in the preconfining phase it is precisely zero at $T = 0$.

\textsuperscript{b}On the level of BPS saturated configurations, see below, no scale is available for a shift $0 \rightarrow \vec{y}$. As a consequence, the definition in (1) is no restriction of generality.

\textsuperscript{c}No scale determining a curvature of a spatial path is available.

\textsuperscript{d}All other solutions have higher euclidean action: A departure from classical trajectories takes place by their decay into BPS saturated plus topologically trivial configurations.

\textsuperscript{e}Each solution enters the definition (1) separately, and the sum over $Q = \pm 1$ is taken subsequently.

\textsuperscript{f}The integral over global spatial or color rotations is contained in the spatial average because of the particular structure of the HS (anti)caloron. Nontrivial periodicity excludes the integration over time translations. The integration over space translations leaves $K$ invariant because each shift is compensated for by an according parallel transport: This integration is already performed.
no a priori cutoff for the spatial coarse-graining. It is easily checked by dimensional counting that both adding higher $n$-point functions of the field strength to (1) and BPS saturated configurations with $|Q| > 1$ are forbidden (dimensionful space and moduli integrations). Thus $\mathcal{K}$ is defined by integrating the $Q = \pm 1$ sum of the expression in (1) with the weight $\int d^3x \int \rho$.

In the radial ($r$) part of space integral a logarithmic divergence occurs for the magnetic-to-magnetic correlation of the field strength $^{1,3}$. At the same time, the azimuthal angular integration yields zero. The former divergence can be regularized in a rotationally invariant way (dimensional regularization). This is not true for the latter zero: an apparent breaking of rotational symmetry is required for regularization. Namely, a defect (or surplus) angle needs to be defined with respect to a fixed direction in the azimuthal plane. Since distinct directions are connected by global gauge rotations no breaking of rotational symmetry is detected in a physical quantity. Thus the angular regularization is admissible.

Performing the integrals, undetermined normalizations appear for each contribution (caloron or anticaloron). Moreover, there are undetermined global phase shifts $\tau \to \tau + \tau_{C,A}$. The convergence towards $\mathcal{K} = \left\{ \hat{\phi}^a \big| D\hat{\phi}^a = \left[ \partial_x^2 + \left( \frac{2\pi}{\alpha} \right)^2 \right] \hat{\phi}^a = 0; \text{ fixed ang. reg.} \right\}$ is extremely fast. That is, with finite upper limits $\rho_u$ and $r_u$ in both the $\rho$- and the $r$-integration the $\tau$-dependence of the results resembles the limiting behavior ($\rho_u = r_u = \infty$) within a small error already for $\rho_u$ and $r_u$ a few times $\beta \equiv 1/T^{1,3}$. This, however, makes the introduction of a finite cutoff $|\phi|^{-1}$ self-consistent: At fixed global gauge the infinite-volume coarse-graining, determining the $\tau$-dependence of $\hat{\phi}^a$, is saturated on a finite ball of radius $\sim |\phi|^{-1}$.

How large is $|\phi|^{-1}$? Since a sufficiently large cutoff $|\phi|^{-1}$ saturates $\mathcal{K}$, since $D\hat{\phi}^a = 0$ is a linear equation, and since $|\phi|$ is $\tau$-independent$^5$ we also have $D\phi = 0$. Moreover, since a (finite) coarse-graining over noninteracting, BPS saturated configurations implies the BPS saturation of the field $\phi$ we need to find an appropriate square root of $D\phi = 0$. Assuming the existence of a scale $\Lambda$, which together with $\beta$ determines the scale $|\phi|$, the right-hand side of the BPS equation must not depend on $\beta$ explicitly and must be analytic and linear in $\phi$. The only consistent option (up to global gauge rotations) is $\partial_\tau \phi = \pm i\lambda_3 \Lambda^3 \phi^{-1}$ where $\phi^{-1} \equiv \sqrt{|\phi|}$. Solutions

$^8$Composed of coarse-grained, large quantum fluctuations $\Rightarrow$ no finite Matsubara frequencies.
are \( \phi(\tau) = \sqrt{\frac{\Lambda^3}{2\pi}} \lambda_1 \exp \left( \pm \frac{2\pi i}{\beta} \lambda_3 (\tau - \tau_0) \right) \) where \( \tau_0 \) is a physically irrelevant integration variable (global gauge rotation). A critical temperature \( 2\pi T_c = 13.867 \Lambda \) exists, see \(^1\). Thus, expressing the cutoff \(|\phi|^{-1} = \sqrt{\frac{2\pi}{\Lambda^3/\beta}} \) in units of \( \beta \), yields 8.22 at \( T_c \); for \( T > T_c \) this number grows as \( (T/T_c)^{3/2} \). But for \( \rho_u \sim r_u \geq 8.22 \) the kernel \( K \) is practically that of the infinite-volume limit, see also \(^1,14,3\).

Coarse-graining the \( Q = 0 \) sector alone, leaves the Yang-Mills action form-invariant\(^b\). One can shown that the field \( \phi \) is inert: Quantum fluctuations of resolution \( \langle \phi \rangle \) do not deform \( \phi \) making it a background for the coarse-grained \( Q = 0 \)-dynamics\(^1\). The gauge-invariant extension of the kinetic term \( \text{tr} (\partial_\tau \phi)^2 \) in the (gauge-dependent) action for the field \( \phi \) alone is \( \partial_\tau \rightarrow D_\tau \) (\( D_\tau \) the adjoint covariant derivative): A unique effective action emerges. The equations of motion for the \( Q = 0 \) sector (subject to the coarse-grained \( \langle Q \rangle = 1 \)-background) possess a pure-gauge solution \( a_{\mu}^{bg} \): The ground-state energy density \( \rho^{gs} \) and pressure \( P^{gs} \) then are

\[
\rho^{gs} = -P^{gs} = 4\pi \Lambda^3 T^3.
\]

2. Constraints on resolution in the effective theory

Two color directions acquire mass (adjoint Higgs mechanism). In unitary gauge, \( \phi = \lambda_3 |\phi| \), \( a_{\mu}^{bg} = 0 \), one has \( m_{1,2} \propto |\phi| \). Fixing the remaining \( U(1) \) by \( \partial_i a_i^{a=3} = 0 \) a given mode’s momentum is physical. To distinguish between quantum and thermal fluctuations we work in the real-time formalism when integrating out gauge-field fluctuations in the effective theory.

Two classes of constraints emerge: (i) Only propagating modes of resolution \( \Delta p \leq |\phi| \) need to be considered. (ii) Since coarse-graining generates (quasi)particle masses for \( \Delta p \leq |\phi| \) we need assure that the exchange of unresolved massless particles contributing to an effective, local vertex does not involve momentum transfers larger than \( |\phi| \). Condition (i) reads

\[
|p^2 - m^2| \leq |\phi|^2 \quad \text{(massive mode)}, \quad |p^2| \leq |\phi|^2 \quad \text{(massless mode)} \quad (2)
\]

where \( |\phi| = \sqrt{\frac{\Lambda^3}{2\pi}} \). For a three-vertex (ii) is contained in (i) by momentum conservation. For a four-vertex condition (ii) distinguishes \( s, t, \) and \( u \) chan-

\(^b\)By all-order perturbative renormalizability interaction effects are absorbed into redefinitions of the parameters of the bare action \(^8\).

\(^1\)A negative ground-state pressure is expected microscopically due to the dominating dynamics of small-holonomy calorons leading to finite life-time cycles of magnetic dipoles (a magnetic monopoles attracts its antimonopole, the pair annihilates, and is recreated)\(^12\).
nels in the scattering process. Labelling the ingoing (outgoing) momenta by \( p_1 \) and \( p_2 \) (\( p_3 \) and \( p_4 = p_1 + p_2 - p_3 \)), we have

\[
| (p_1 + p_2)^2 | \leq |\phi|^2 \quad (s) \quad | (p_3 - p_1)^2 | \leq |\phi|^2 \quad (t) \quad | (p_2 - p_3)^2 | \leq |\phi|^2 \quad (u). \tag{3}
\]

Notice conditions (3) reduce to the first condition if one computes the one-loop tadpole contribution to the polarization tensor or the four-vertex induced two-loop contribution to a thermodynamical quantity\(^1\). The pressure was computed up to two loops in \(^1,13,6\): Two-loop corrections are smaller than (depending on temperature) \( \sim 0.1\% \) of the one-loop result.

We expect that the contribution of \( N \)-particle irreducible (NPI) polarizations to the dressing of propagators vanishes for \( N > N_{\text{max}} < \infty \) since (2) and (3) then impose more independent conditions than there are independent loop-momentum components. It is instructive to analyze the two bubble diagrams in Fig. 1. While, due to (2) and (3), the two-dimensional region of integration for \(|k_1|\) and \(|k_2|\) in diagram (a) is non-compact the three-dimensional region of integration for \(|k_1|, |k_2|, \) and \(|k_3|\) is compact in diagram (b)\(^15\). In one-particle reducible diagrams so-called pinch-singularities arise in the real-time dressing of propagators (powers of delta functions).

But a re-summation of 1PI polarizations modifies \(^k\) the scalar part of the tree-level propagators by momentum dependent screening functions with finite imaginary parts. This makes powers of spectral functions well-defined.

To summarize, the effective loop expansion should be given by infinite re-summations of a finite number of NPI polarizations\(^15\). Because the latter

\(^1\)The \( t \)-channel condition is then trivially satisfied while the \( u \)-channel condition reduces to the \( s \)-channel condition by letting the loop momentum \( k \rightarrow -k \) in \(|(p-k)^2| \leq |\phi|^2\), see \(^1,13,6\).

\(^k\)To avoid a logical contradiction the 1PI polarizations are first computed in real time subject to the constraints (2) and (3). Subsequently, a continuation to imaginary time in the external momentum variable \( p^0 \) is performed. Then the re-summation is carried out, and finally the result is continued back to real time.
dramatically decrease with $N$ radiative effects are reliably approximated at the two-loop level.

3. Application: SU(2)$_{\text{CMB}}$

A (falsifiable, see below) postulate emerges$^{6,1,4,5}$: SU(2) Yang-Mills dynamics of scale $T_{\text{CMB}} \sim \Lambda_{\text{CMB}} = 1.065 \times 10^{-4}$ eV (thus the name SU(2)$_{\text{CMB}}$) masquerades today as the U(1)$_Y$ of the Standard Model (SM). $\Lambda_{\text{CMB}}$ derives from the b.c. that light propagates in an unadulterated way today$^1$. All low-temperature ($T \ll 0.5$ MeV) dynamics of the SM with momentum transfers considerably below the ‘electroweak scale’ $\sim 200$ GeV is unaffected by this assignment if one distinguishes between propagating (the massless excitations of SU(2)$_{\text{CMB}}$) and interacting photons$^m$. An exception takes place for $T$ a few times $T_{\text{CMB}} = 2.351 \times 10^{-4}$ eV $\sim 2.73$ K$^n$. Namely, the photon’s dispersion law modifies, see $6$:

$$\omega^2(\vec{p}) = \vec{p}^2 \rightarrow \omega^2(\vec{p}, T) = \vec{p}^2 + G(\omega(\vec{p}, T), \vec{p}, T).$$

(4)

At photon momenta $p > 0.2 \ldots 0.3 T$ small antiscreening takes place ($G < 0$ in Eq. (4)) which dies off exponentially with $p$. There is a power suppression of $|G|$ with increasing $T$. For $p$ being a small fraction of $T$ ($T \sim 5$ K $\Rightarrow p \leq 0.2 T$) antiscreening converts into screening ($G > 0$) which rapidly grows for decreasing $p$. The effect is negligible for photon-gas temperatures sufficiently above $T_{\text{CMB}}$, say $T > 80$ K, and it is absent at $T = T_{\text{CMB}}$, see $^6$. A modification of the black-body spectrum emerges at low temperatures and low momenta$^7$: At $T = 10$ K the spectral intensity vanishes for frequencies $0 < \omega \leq 0.12 T$. For $0.12 T \leq \omega \leq 0.25 T$ there is excess of spectral power. This prediction tests the postulate SU(2)$_{\text{CMB}} \equiv \text{U(1)}_Y$. The relative deviation to the U(1) black-body pressure peaks at $T \sim 2 T_{\text{CMB}}$ on the $10^{-3}$-level coinciding with the strength of the CMB dipole$^o$. The observation of cold, old, and dilute clouds of atomic hydrogen in between the spiral arms of our galaxy$^{17}$ hints to SU(2)$_{\text{CMB}} \equiv \text{U(1)}_Y$ being true$^6$$^7$$^p$. The suppression of low-momentum photons could be the reason for the missing power in TT

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$^1$This is the case only for $T_{\text{CMB}} = T_c$.

$^m$Interaction with electroweak matter dynamically invokes the Weinberg angle by a rotation of the propagating to the interacting photon, for a discussion see $^5$.

$^n$Due to interactions with the massive excitations of SU(2)$_{\text{CMB}}$ referred to as $V^\pm$ in the following.

$^o$We expect that besides the contribution due to the Doppler-effect$^{16}$ also a dynamical part to generate the CMB dipole.

$^p$The forbidden wavelengths at brightness temperature $T = 5$ K$^{17}$, range from 2.1 cm to 8.8 cm. This is comparable to the mean distance between H-atoms$^{17}$: The dipole force,
CMB spectra at low $l_{18q}$. Also there is the result of the PVLAS experiment $^{19r}$. To quantitatively investigate this the two one-loop diagrams for the photon polarization involving the full $V^\pm$-propagator in the external magnetic field must be calculated. We hope to tackle this task in the near future.

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which would cause all atoms to convert into H$_2$ molecules within a time two orders of magnitude lower than the inferred age of the cloud, is switched off.

$^9$The suppression of ‘messenger’ photons weakens the correlation between temperature fluctuations at large angular separation in the sky.

$^*$A dichroism induced by a 5 Tesla homogeneous magnet on linearly polarized laser light with the temperature of the apparatus being $\sim 4.2\,\text{K}$. When fitted to an axion model the inferred axion mass is about 1 meV with a too large coupling (contradicting solar bounds on axion-induced X-ray emission $^{20}$). The observation making contact with SU(2)$_{\text{CMB}}$ is that at $T = 4.2\,\text{K}$ one has $m_{V^\pm} = 0.4\,\text{meV}$: A value comparable to the axion mass. It is the mass of the propagating intermediary particle which is a nearly model independent quantity in non-standard theories of photon-photon coupling. Thus $m_{V^\pm} = 0.4\,\text{meV}$ is an encouraging observation. Moreover, the small photon-to-photon coupling would be explained by the smallness of a kinematically strongly constrained loop-propagation of $V^\pm$ excitations, see Sec. 2.