Areas for the existence of biquadratic transformations

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Abstract. Analysis and research carried out quadratic transformations of applied geometry shows that quadratic transformations of the plane have been studied sufficiently and have found wide application in science and technology. However, little attention has been paid to the study and application of four to four-digit correspondences to the plane. Therefore, the article is devoted to the development of the theory of biquadratic plane transformations. As a result of the consistent implementation of the proposed constructive device, each point \( A \) of plane \( \Pi_1 \) is converted to four points \( A_1, A_2, A_3, A_4 \) of plane \( \Pi_1' \). Given the two parametric set of points of the combined plane \( \Pi_1 = \Pi_1' \) we obtain a biquadratic transformation of the plane, denoted by the letter \( L \). In a similar way, it can be shown that in the opposite direction each point \( A' \) of the plane \( \Pi_1' \) is transformed into four points of the plane \( \Pi_1 \). This transformation is denoted by the letter \( L' \). Also this article is devoted to determining of the area existence in the development of the theory of biquadratic transformation plane, improvement of methods of geometric design of curves and curved surfaces in architectural and construction design. To achieve this purpose it's requiring to solve the following theoretical and practical problems: definition and development of graphic model of biquadratic transformations of the plane, determination of the existence area of the transformation. The results can be widely used in design and research institutes by designing and construction of surfaces of technical forms, also new dome shells and other surfaces in architectural design.

1. Introduction

The use of a quadratic transformation in applied geometry makes it possible to reduce many times the order of the projected curve or surface, which greatly simplifies the solution of various positional problems [1]. But the insufficient application of quadratic transformations in applied geometry is explained by the fact that graphical models are poorly developed, although dozens of works by leading experts in applied geometry are devoted to the study of this problem in applied geometry. Works [2, 3, 4, 5, 6, 7] are devoted to solving this problem. The article is devoted to the definition of the domain of existence in the development of the theory of biquadratic plane transformations and the improvement of methods of geometric design of curves and curvilinear surfaces in architectural and construction design. To achieve these goals, the following theoretical and practical problems are required to be solved in the developed graphic model of biquadratic transformations of the plane for determining the domain of existence of transformations and obtaining the basis of graphic transformation methods. The practical value of the article is to create methods for obtaining curves and constructing curvilinear surfaces using the theory of biquadratic plane transformations. These methods allow us to expand the types of curved surfaces used in construction. The developed graphic models of biquadratic plane transformations determine the existence of transformations for constructing new curves and curvilinear surfaces. Thus, opens a new scientific direction in descriptive geometry. The results of
research can be widely used in design and research institutes in the design of surfaces of technical forms, as well as in the educational process of universities. Graphic models of twelve canonical biquadratic plane transformations are developed and their domains of existence are determined, so the obtained regions can be used in applied geometry. The paper contains new scientifically grounded theoretical and applied results on the development of the theory of biquadratic plane transformations in the improvement of geometric design of surfaces in architectural design, the totality of which is an achievement in the development of applied geometry. The research results can be implemented in the design of new dome and other surfaces of shells in architectural design. And following scientific works in applied geometry implies that the quadratic transformation plane sufficiently investigated and have found wide application in science and engineering. However, little attention has been paid to the study and application of four to four-digit correspondences to the plane.

2. Area of existence of the graphic model biquadratic transformation

To achieve these goals, the following theoretical and practical problems are required to be solved in the developed graphic model of biquadratic transformations of the plane for determining the area of existence of transformations and obtaining the basis of graphic transformation methods. The practical value of the article is to create methods for obtaining curves and constructing curvilinear surfaces using the theory of biquadratic plane transformations. These methods allow us to expand the types of curved surfaces used in construction. The developed graphic models of biquadratic plane transformations determine the existence of transformations for constructing new curves and curvilinear surfaces. Thus, opens a new scientific direction in descriptive geometry. The results of research can be widely used in design and research institutes in the design of surfaces of technical forms, as well as in the educational process of universities. Graphic models of twelve canonical biquadratic plane transformations are developed and their domains of existence are determined, so the obtained regions can be used in applied geometry.

The study is devoted to the development of the theory of four-four-digit correspondences between non-aligned planes and the development of the theory of biquadratic plane transformations.

As a result of the consistent implementation of the above constructive apparatus [5], each point \( A \) of the plane \( \Pi_1 \) is converted into four points \( A'_1, A'_2, A'_3, A'_4 \) plane \( \Pi'_2 \). Given two parameter set points combined plane \( \Pi'_2 = \Pi_1 \) biquadratic obtain transform plane, indicated by the letter \( L \). Similarly, it can be shown that in the opposite direction each point \( A'_1 \) plane \( \Pi'_2 \) is converted into four points of the plane \( \Pi_1 \) it the transformation is denoted by the letter \( L' \) [8].

Theorem: If two surfaces of rotation are given \( \Phi_0^1 \) and \( \Phi_0^2 \), which respectively undergo spatial transformations \( p_1 \) and \( p_2 \) they are displayed by directions \( s \) and \( s' \) on the plane \( \Pi'_2 = \Pi_1 \), then the biquadratic transformation is set \( L \) and \( L' \) between aligned planes \( \Pi'_2 = \Pi_1 \).

Using the spatial design scheme proposed above [5], various types of canonical biquadratic transformations of the plane are obtained \( L, L' \) [3, 4, 5].

By considering of three cases, binary mapped x n s by two second-order plane of surfaces Recipients and m: a) coupling e nonlinear surfaces of the second order; b) coupling e conical and cylindrical surfaces of the second order; c) a combination of second-order unipolar hyperboloids. As a result of the implementation of these cases, we received three subgroups of biquadratic transformations of the plane [5].

To obtain the first subgroup of biquadratic transformations of the plane, when the combination of binary mapped surfaces of the second order are nonruled bubbled surfaces of the 2nd order, such as a sphere and two-sheeted hyperboloid. For the second subgroup plane biquadratic transformations consider the case when the binary combination of the display surface of the second order are ruled surfaces of holes and 2nd order, such as conical and cylindrical surfaces of revolution. And for obtaining the third subgroup of biquadratic plane changes, consider the case when the binary combination of the display surface of the second order are nonlinear second order surfaces such as sphere and bipolar hyperboloid [5].
Thus, the theoretical position created by modeling transformations of biquadratic plane and also a method for obtaining designed plane of biquadratic transformations generated by binary mapping second order surfaces on two combined plane, allowed to get twelve types of biquadratic plane transformations $L_1, L_2 \ldots L_{12}$ [8].

Y alignment considered biquadratic transform plane is given as:

$$\begin{align*}
x_1 &= \sqrt{f_1(x_1, x_2)} \\
x_2 &= \sqrt{f_2(x_1, x_2)}.
\end{align*}$$

The system of equations (1) has solutions if the radical expressions are not less than zero. This condition, we write in the form:

$$\begin{align*}
f_1(x_1, x_2) &\geq 0 \\
f_2(x_1, x_2) &\geq 0.
\end{align*}$$

Thus, we construct graphical areas corresponding to expression (2), which the intersection of them determines the existence of this region biquadratic transform plane.

2.1. Graphic model of the area of existence biquadratic transformation $L_1$

To develop a graphical model of the region of existence biquadratic transformation of $L_1$ in the following sequence:

a) we write the equations of the considered biquadratic transformation of the plane $L_1$ in the form:

$$\begin{align*}
x_1 &= \sqrt{x_1^2 + x_2^2 + R^2} \\
x_2 &= \sqrt{x_1^2 - x_2^2 - R^2}.
\end{align*}$$

The system of equations has a solution if the radicals are not less than zero. This condition, we write in the form:

$$\begin{align*}
\sqrt{x_1^2 + x_2^2 + R^2} &\geq 0 \\
\sqrt{x_1^2 - x_2^2 - R^2} &\geq 0.
\end{align*}$$

b) we construct graphic areas corresponding to expressions whose intersection determines the region of existence of the considered biquadratic transformation of the plane $L_1$ in accordance with figure 1.

2.2. Graphic model of the area of existence biquadratic transformation $L_2$

To develop a graphical model of the region of existence biquadratic converting $L_2$ in the following sequence.

We write the equations of the considered biquadratic transformation $L_2$ in the form:

$$\begin{align*}
x_1 &= \sqrt{x_1^2 + x_2^2 + R^2} \\
x_2 &= \sqrt{x_1^2 - x_2^2 - R^2}.
\end{align*}$$

This system of equations has solutions if the radical expressions are at least zero. This condition, we write in the form:

$$\begin{align*}
(x_1^2 + x_2^2 + R^2) &\geq 0 \\
(x_1^2 - x_2^2 - R^2) &\geq 0.
\end{align*}$$

We construct graphic regions corresponding to the above expressions, the intersection of which defines the region of existence of the considered biquadratic transformation $L_2$ in accordance with figure 2.

2.3 Graphic model of the area of existence biquadratic transformation $L_3$

Define the area of existence biquadratic transformation $L_3$. To do this, we write the equations of the biquadratic transformation $L_3$ in the form:
This system of equations has solutions if the radical expressions are at least zero. This condition, we write in the form:

\[
\begin{align*}
(x_i^2 - x_j^2 - R^2) &\geq 0 \\
(x_i^2 + x_j^2 + R^2) &\geq 0
\end{align*}
\]

We construct graphic regions corresponding to these expressions, the intersection of which defines the region of existence of the biquadratic transformation \(L_3\) in accordance with figure 3.

2.4. Graphic model of the area of existence biquadratic transformation \(L_4\)

To determine the region of existence of the biquadratic transformation \(L_4\), we write its equations in the form:

\[
\begin{align*}
(x_i^2 - x_j^2 - R^2) &\geq 0 \\
(x_i^2 + x_j^2 + R^2) &\geq 0
\end{align*}
\]

We construct graphic regions corresponding to these expressions, the intersection of which defines the region of existence of the considered biquadratic transformation \(L_4\) in accordance with figure 4.

Thus, the proposed area of their existence, which makes it possible to apply them in solving various problems in applied geometry.

2.5. Graphic model of the area of existence biquadratic transformation \(L_5\)

Development of graphic models of biquadratic transformations obtained by a binary display of a pair of conical and cylindrical surfaces.

\[
\begin{align*}
(x_i^2 - x_j^2 - R^2) &\geq 0 \\
(x_i^2 + x_j^2 + R^2) &\geq 0
\end{align*}
\]

We construct graphic regions corresponding to these expressions, whose intersection determines the region of existence of the considered biquadratic transformation \(L_5\) in accordance with figure 5.

2.6. Graphic model of the area of existence biquadratic transformation \(L_6\)

To determine the region of existence of the biquadratic transformation \(L_6\), we write its equations in the form:

\[
\begin{align*}
(x_i^2 - x_j^2 - R^2) &\geq 0 \\
(x_i^2 + x_j^2 + R^2) &\geq 0
\end{align*}
\]

This system of equations has solutions if the root expressions are not less than zero.
\[(x_1^2 - x_1^2) \geq 0, \quad (x_1^2 + x_1^2) \geq 0\]

We construct graphic regions corresponding to these expressions, whose intersection determines the region of existence of the considered biquadratic transformation \(L_6\) in accordance with figure 6.

2.7. Graphic model of the area of existence biquadratic transformation \(L_7\)
Consider the construction of a graphical model of the biquadratic transformation \(L_7\).

We write the equations of the biquadratic transformation of the plane \(L_7\):
\[
L_7: \begin{cases} \quad x_1' = \sqrt{x_1^2 + x_2^2} \\ x_2' = \sqrt{x_1^2 - x_2^2} \end{cases}
\]

This system of equations has solutions if the root expressions are not less than zero:
\[
(x_1^2 + x_2^2) \geq 0, \quad (x_1^2 - x_2^2) \geq 0
\]

We construct graphic regions corresponding to these expressions, whose intersection defines the region of existence of the considered biquadratic transformation \(L_7\) in accordance with figure 7.

2.8. Graphic model of the area of existence biquadratic transformation \(L_8\)
To determine the region of existence of the biquadratic transformation \(L_8\), we write its equations in the form:
\[
L_8: \begin{cases} \quad x_1' = \sqrt{x_1^2 + x_2^2} \\ x_2' = \sqrt{x_1^2 - x_2^2} \end{cases}
\]

This system of equations has solutions if the root expressions are not less than zero:
\[
(x_1^2 + x_2^2) \geq 0, \quad (x_1^2 - x_2^2) \geq 0
\]

We construct graphic regions corresponding to these expressions, whose intersection determines the region of existence of the considered biquadratic transformation \(L_8\) in accordance with figure 8.

2.9. Graphic model of the area of existence biquadratic transformation \(L_9\)
Development of graphical models of biquadratic transformations obtained by the binary display of two ruled hyperbolic surfaces.

To determine the region of existence of the biquadratic transformation \(L_9\), we write its equations in the form:
\[
L_9: \begin{cases} \quad x_1' = \sqrt{x_1^2 + x_2^2 - R^2} \\ x_2' = \sqrt{x_1^2 - x_2^2 + R^2} \end{cases}
\]

This system of equations has solutions if the radical expressions are not less than zero:
We construct graphic regions corresponding to these expressions, whose intersection determines the region of existence of the considered biquadratic transformation $L_9$ in accordance with figure 9.

2.10. Graphic model of the area of existence biquadratic transformation $L_{10}$

To determine the region of existence of the biquadratic transformation $L_{10}$, we write its equations in the form:

\[
L_{10} : \begin{cases} 
\bar{x}_1 = \sqrt{x_1^2 + x_2^2 - R^2}, \\
\bar{x}_2 = \sqrt{x_1^2 - x_2^2 + R^2}.
\end{cases}
\]

This system of equations has solutions if the radical expressions are not less than zero:

\[
\begin{align*}
(x_1^2 + x_2^2 - R^2) \geq 0, \\
(x_1^2 - x_2^2 + R^2) \geq 0.
\end{align*}
\]

We construct graphic regions corresponding to these expressions, whose intersection determines the region of existence of the considered biquadratic transformation $L_{10}$ in accordance with figure 10.

2.11. Graphic model of the area of existence biquadratic transformation $L_{11}$

To determine the region of existence of the biquadratic transformation $L_{11}$, we write its equations in the form:

\[
L_{11} : \begin{cases} 
\bar{x}_1 = \sqrt{x_1^2 - x_2^2 + R^2}, \\
\bar{x}_2 = \sqrt{x_1^2 + x_2^2 - R^2}.
\end{cases}
\]

This system of equations has solutions if the radical expressions are not less than zero:

\[
\begin{align*}
(x_1^2 - x_2^2 + R^2) \geq 0, \\
(x_1^2 + x_2^2 - R^2) \geq 0.
\end{align*}
\]

We construct graphic regions corresponding to these expressions, whose intersection defines the region of existence of the considered biquadratic transformation $L_{11}$ in accordance with figure 11.

2.12. Graphic model of the area of existence biquadratic transformation $L_{12}$

Consider the construction of a graphical model of the biquadratic transformation $L_{12}$.

We write the equations of the biquadratic transformation of the plane $L_{12}$:

\[
L_{12} : \begin{cases} 
\bar{x}_1 = \sqrt{x_1^2 - x_2^2 + R^2}, \\
\bar{x}_2 = \sqrt{x_1^2 + x_2^2 - R^2}.
\end{cases}
\]

This system of equations has solutions if the radical expressions are not less than zero:
We construct graphical regions corresponding to these expressions, whose intersection determines the region of existence of the considered biquadratic transformation $L_{12}$ in accordance with figure 12.

3. Conclusion

Thus, the algorithm proposed above allows to determine the region of their existence which makes it possible to use them in new modeling of curves and surfaces. The article contains new scientifically theoretical results on the development of the theory of biquadratic plane transformations in the improvement of the geometric design of surfaces in architectural and building design, the totality of which is an achievement in the development of applied geometry. Results can be implemented in the design of new dome and other surfaces of shells in architectural design. The resulting graphic models of area and existence of biquadratic transformation can be used in solving the positional problems in descriptive geometry and the determination of new higher-order curves.

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