Prospects for Parity Non-conservation Experiments with Highly Charged Heavy Ions

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Abstract

We discuss the prospects for parity non-conservation experiments with highly charged heavy ions. Energy levels and parity mixing for heavy ions with two to five electrons are calculated. We investigate two-photon-transitions and the possibility to observe interference effects between weak-matrix elements and Stark matrix elements for periodic electric field configurations.
I. INTRODUCTION

Atomic physics tests of the standard model \[1–3\] play a very special role because of the small momentum transfers involved. Comparisons between their results and high energy data are highly sensitive to radiative corrections and thus to extensions of the standard model \[4\]. With the percent precision reached in the Cs experiments described in \[3\], the effect of radiative corrections is of the order of the experimental accuracy. If a system is found for which a 0.1% accuracy can be reached the experimental results would allow most interesting and far reaching conclusions (see e.g. \[5\]). For the atoms and experimental setups studied so far this seems unluckily to be out of reach, which motivates the search for significantly different alternatives. The possibility we want to discuss is the use of highly charged heavy ions, which can be produced and stored in great variety at, e.g., Gesellschaft für Schwerionenforschung in Darmstadt, Germany. We discussed already some time ago the prospects for inducing a two-photon transition in helium-like uranium \[6\]. In this contribution we extend our studies to systems with up to five electrons and we adopt the ingenious ideas proposed by Botz, Bruß, and Nachtmann \[7\] especially suited for the investigation of parity-violating effects in storage rings.

The starting point for all such experiments is that, due to the parity-violating exchange of neutral $Z$ bosons between nucleus and electrons, every electron state is mixed with states of opposite parity. In first order perturbation theory the coefficient $\eta$ of this admixture is given by

$$\eta = \frac{\langle i | \frac{G_F}{\sqrt{2}} (1 - 4 \sin^2 \vartheta_W - \frac{N}{Z}) \rho \gamma_5 | f \rangle}{E_i - E_f},$$

where $G_F$ denotes Fermi’s constant, $\vartheta_W$ the Weinberg angle, $N$ the neutron number, $Z$ the proton number, and $\rho$ the nuclear density normalized to $Z$. From this formula we see why heavy ions with few electrons left in inner shells are good candidates for investigating
parity non-conservation effects: The admixture coefficient $\eta$ is very large (typically orders of magnitude larger than for usual, neutral atoms) due to the big overlap between the nucleus and the electron states. The other factor that can make $\eta$ large is the energy difference between the two mixing electronic states $i$ and $f$ that ought to be very small. Therefore we are especially interested in level crossings of electron states with the same spin but opposite parity.

It was pointed out in [4] that equation (1) has to be modified by radiative corrections, the weak charge $Q_W$ included in (1) changes according to

$$Q_W = Z - 4Z \sin^2 \vartheta_W - N \quad \rightarrow \quad \rho'_{PV}(Z - 4Z \kappa'_{PV} \sin^2 \vartheta_W - N) . \quad (2)$$

Here $\rho'_{PV}$ and $\kappa'_{PV}$ are constants that arise from the radiative corrections mentioned above. The crucial point is, that they depend on the masses of the particles involved in the radiative processes, especially the top quark and the Higgs boson. As it seems now that there is evidence for the top quark to exist, it should be from a theoretical point of view possible to determine from $\rho'_{PV}$ and $\kappa'_{PV}$ the value of the mass of the Higgs boson that makes the standard model renormalizable, thus giving important guidance to identify this particle in high energy experiments.

In section 2, we will discuss uranium with two to five electrons as a model for other heavy ions reaching from gold to plutonium. In section 3 we will discuss the possibility of level crossing in compound heavy ions and finally in section 4 we will investigate the possibility of polarization rotations in heavy ions.

II. HEAVY IONS WITH TWO TO FIVE ELECTRONS IN INNER SHELLS

As to an experiment with heavy ions with few inner shell electrons we have to give a criterion under which we can judge feasibility of such an experiment. As such a
criterion we should compare the $\eta$ values of the systems regarded here with the $\eta$ value of the helium-like uranium system discussed in [3], i.e., $\eta \approx 10^{-6}$, when taking the energy difference to $\Delta E = 1$ eV. Even this relatively high value of $\eta$ left the proposed experiment beyond the scope of experimental feasibility for the set up discussed there.

We get a second restriction by the following consideration. If, for example, the interesting electron states are excited during the stripping process of the ion in a stripping foil, then for any realistic experiment the experimental set up should be placed a little distance behind this foil, lets say one meter. Then the lifetime of these excited states should be long enough to survive this one meter of flight. Taking into account a time dilation factor of about 5 for an ion accelerated to 5 GeV per nucleon, the lifetime should be larger than $\approx 10^{-9}$ s. This would be an optimal value, but also a lifetime of $10^{-10}$ s would probably do, corresponding to a distance of 10 cm.

We furthermore consider only the lowest lying electron states that offer a possibility for a parity-violation experiment. As the parity admixture is proportional to the overlap of the electron states in question with the nucleus, this admixture should become very large for low lying states, if the energies are sufficiently degenerate.

We should state that these criterions do not rule out all imaginable experiments. It could be possible for example to store ions in an ion trap and to generate the excited state by a laser beam, maybe by a laser that still has to be invented or that will be available in a few years, then the question of the lifetime of the electron states may be superfluous. Also there could be other electronic configurations in the ions studied here or in ions having a few more electrons, with suitable properties. The relativistic corrections can lead to very rich structures, with level crossings and metastable states, which have just begun to be explored (see for example [8,9]) and can lead to increased sensitivity to e.g., electric quadrupole hyperfine interaction [10].
The systems we are interested in are highly charged heavy ions for which two states with equal angular momentum $J$, but opposite parity have similar energy. We have investigated the binding energies of the lower lying levels of ions with two to five electrons, to identify the most promising candidates. When not available from the literature, energies were calculated with the Multiconfiguration Dirac-Fock Program (MCDF) published by Grant et al. [11], which provide relativistic correction as well as one-electron QED corrections and approximate, although inaccurate, many-body QED corrections. For helium-like systems however, we can use very precise MCDF or relativistic Configuration-Interaction (RCI) calculations including correlation and QED effects.

For each electron configuration we show as an example the results for uranium. There are no noticeable qualitative differences for other heavy ions down to gold as is graphically shown for the interesting electron states, except for the two-electron $1s2s\,^1S_0 \rightarrow 1s2p\,^3P_0$ case for which two crossings at $Z \approx 62$ and $Z \approx 92$ occur. As in this section we do only exploratory work, we do not claim a precision much better than a few eV, except for two-electron systems. Lifetimes are calculated in the single LS configuration approximation. We take the inverse of the main transition probability to be the lifetime of the respective state, neglecting hereby other contributions of lower order. The parity admixture coefficient in this second section is determined only for the main electron state and therefore also gives only the order of magnitude.

A. Two-electron ions

Extensive calculations of two-electron ions binding energies have appeared in the literature over the past 10 years [12–14]. In figure 1, we plot the $1s2s\,^1S_0 \rightarrow 1s2p\,^3P_0$ energy difference as calculated in [12] and [13]. The first one is an all-order Relativistic Many-Body Perturbation theory (RMBPT) calculation, which uses Ref. [13] two-body
QED corrections. The second calculation is a MCDF calculation done along the line of [13,16], which uses the Welton model for two-body self-energy corrections, experimental nuclear size when available and includes finite-nuclear size correction to the self-energy [17]. The energy separation between $1s^2 \,^1S_0$ and $1s2p \,^3P_0$ is plotted in Fig.1 as a function of $Z$. In order to show how this level crossing happens we show in detail the contributions to the energy separation at $Z = 62$ and 92 in table I. It should be noted that this crossing mostly involves the interplay between magnetic energy and QED correction contributions.

With this new energy determination the parity admixture $|\eta| \approx 5 \times 10^{-6} \text{eV}/(\Delta E)$ [6] would be enhanced by a factor of 3. For the experimental set up discussed in [6] with the detection of a laser-induced two-photon transition, the laser intensity required would still be unrealistically large, of order $10^{21}$ W/cm$^2$ (presently only lasers up to an intensity of $10^{17}$ W/cm$^2$ $- 10^{18}$ W/cm$^2$ exist.). The main problem in this context is that the heavy ions are only available in the form of a rapid ion beam and that the only possibility to excite the interesting electron states is by means of the stripping process.

One hope to improve the situation is to study different isotopes to see if one could still reduce the energy difference. Fig. 2 shows that by choosing suitable isotopes the degeneracy can be improved. Only the Coulomb energy is modified due to the change in nuclear radius. For uranium the energy separation does cancel between isotope 233 and 234, within the present calculation. One should keep in mind, however, that the present calculation as well as the one in [12] are not precise enough for finding exactly at which atomic number and for which isotope the crossing occurs. The main uncertainty is in the self-energy screening. In table I the self-energy screening is evaluated with the Welton model [13], which has been proven to be rather accurate [19], but which is not ab-initio. In [12] Drake’s screening calculations, which are more adapted to low $Z$ are used.
one uses ab-initio QED calculations [18], one gets a larger screening. However [18] did not include relaxation, which seems to be sizable for the $1s2s\,^1S_0$ state. For uranium the Welton model with relaxation gives 4.29 eV, while the result from [18] is only 1.08 eV. It has been shown on other systems that the Welton model should not be wrong by more than 10% for this atomic number, while it can be good to 1% at lower Z [19]. One should note also that higher order radiative corrections (of order $\alpha^2$, i.e., of order $\alpha$ with respect to the one electron self-energy), and QED corrections to the two-photon exchange diagrams [20] have not been evaluated. Both corrections could be as large as 0.5 eV. The position of the crossing point as well as the smallest energy which can be obtained is thus very uncertain. Also it should be remembered that if the energy separation is too small it may be difficult to find a laser to excite the two-photon transition.

B. three to five-electron ions

The characteristic feature of the lithium-like uranium (cf. table II) is the fact, that already the ground state and the first excited state fulfill the main conditions of a parity-violation experiment, i.e., they have the same angular momentum and opposite parity. Moreover the lifetime of the first excited state lies in the range of $10^{-10}$ seconds. Very sophisticated calculation of the ionization energies in lithium-like uranium including discussion on nuclear effects can be found in [21–23]. Complete calculations with relativistic correlation energy and radiative corrections for lower atomic numbers can be found in Refs. [24–26]. Unluckily, between these two energy states there is a wide energy gap that reduces the magnitude of the parity admixture, which is in rough approximation about $\eta = 1.4 \times 10^{-8}$. We shall discuss a scheme of detecting parity-violation in lithium-like atoms in section 3.

Fig. 3 shows that the $Z$-dependence of the energy difference of the first two electron
states is nearly linear for atomic numbers ranging from $79 \leq Z \leq 92$, such that no element can be found for which the situation would be substantially different.

The beryllium-like ions case is comparable to the lithium-like case. The first two electron levels are in principle suitable for parity admixture experiments. The lifetime of the first excited state is very large and depends crucially on the spin of the nucleus. In the case of an even-even nucleus, e.g., uranium 238 the lifetime is dominated by a two-photon $E1M1$ transition which is in general very slow ($10^7$ s for $Z = 82$ [27]), and can therefore be treated to be infinity in comparison with the lifetime of the next higher levels. In the case of uranium 235 the nucleus has an angular momentum of $7/2$, and due to hyperfine mixing of electron orbitals the lifetime is severely reduced to $8.562 \times 10^{-5}$ s [27].

As a model for beryllium-like heavy ions we tabulate the energy and lifetime of the lower level of beryllium-like uranium in table III. In order to get a reasonable precision both the ground state and the $1s^22p_{1/2}^2$ are calculated as the lower and intermediate level of the $1s^22s^2 + 1s^22p_{1/2}^2 + 1s^22p_{3/2}^2 J = 0$ configuration set, because intrashell correlation is very large in that case. As in the lithium-like case the energy gap between the mixing levels is large, leading to a parity admixture of about $|\eta| \approx 2.4 \times 10^{-8}$.

For a five-electron system we examine again uranium ions (cf. table IV). The first two electronic levels are in principle usable for a parity-violation experiment, but the comparatively short lifetime of the first excited state and the small admixture of only $|\eta| \approx 9.4 \times 10^{-9}$ make this system completely unattractive. We shall discuss in the following therefore mainly lithium-like ions.

No level crossing was found for $78 \leq Z \leq 96$ in any of the three, four and five-electron systems.
III. LITHIUM-LIKE HEAVY IONS WITH HIGH Z AND N

In this section we study super-heavy lithium-like ions. It is interesting to check, how the situation would change if $Z$ would be increased beyond the existing periodic system. Such high-Z systems can be formed for a short time in heavy-ion collisions. Here we treat the high-Z system as an ordinary atom with the charge $Z = Z_1 + Z_2$ being just the sum of its components. While the energy difference $E(1s^22p^2P_{1/2}) - E(1s^22s^2S_{1/2})$ is nearly linearly increasing in the range from $Z = 79$ to $Z = 94$ it again decreases in the higher $Z$ region and has a crossing point at $Z_{\text{united}} \approx 122$. This effect is due to the relativistic contraction of the $2p_{1/2}$ wave function which dominates over all other contributions for very large $Z$. For further increasing $Z$ the $2p_{1/2}$ state, being below the $2s_{1/2}$ state, reaches the the negative energy continuum \[28\]. We used Desclaux’s code to evaluate a number of systems for $104 \leq Z \leq 128$, with self-consistent magnetic interaction \[29\], vacuum polarization of order $\alpha(Z\alpha)$, $\alpha(Z\alpha)^3$ and $\alpha^2(Z\alpha)$, self-energy extrapolated from Mohr’s values and corrected for finite nuclear size. For this to be valid, however we had to limit ourselves to $Z < 137$. It happens that the interesting region lies well inside this boundary. From table \[V\] one can see how for such high $Z$ values the two interesting lithium-like states cross around the united charge number $Z_{\text{united}} \approx 122$. We analized only symmetric collision systems, which are parity even, provided their charge states are equal.

IV. POLARIZATION ROTATIONS

This section follows the analysis given in \[7\] by G. W. Botz, D. Bruß and O. Nachtmann. We follow here their notations. The energies, lifetimes, Stark and parity admixture coefficients were calculated with the Multi-Configuration Dirac Fock package from \[30\]. To make this paper self contained let us shortly repeat some of the basic arguments of
The atomic system we are interested in is a lithium-like ion that has a nonzero nuclear angular momentum. For simplicity we take the nuclear angular momentum $I = 1/2$ and look at the first four electron states (cf. Fig. 4). The situation for ions with other nuclear angular momentum is completely the same except that other numbers for the total angular momentum $F$ have to be inserted (The formalism could also be applied to the boron-like case where we look at boron-like uranium 235 that has $I = 7/2$). The experimental situation in which we like to place this system is shown in Fig. 5.

The lithium-like ion moves in the 1-direction of our coordinate system. This ion is moving through alternating electric fields of width $x_1$, at a distance of $x_2$. The electric fields point in the positive and negative 3-direction. The moving ion sees a magnetic field due to the boost, but as this field is even under parity transformation we can neglect it. The arrangement has still one symmetry operation $\hat{R}$ under which it is invariant and this is a combination of parity transformation and rotation about $\pi$ around the 2-axis. Together, this gives a reflection with respect to the 1-3 plane.

$$\hat{R} : \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \rightarrow \begin{pmatrix} x_1 \\ -x_2 \\ x_3 \end{pmatrix}$$  \hspace{1cm} (3)

$$\hat{R} = e^{i\pi \hat{F}_2} \cdot \hat{P} \hspace{1cm} (4)$$

It is clear that the angular momentum states $|F, F_3\rangle$ are in general no eigenstates of this operation. But from

$$e^{i\pi \hat{F}_2} |F, F_3\rangle = \sum_{F_3'} |F, F_3'\rangle \langle F, F_3'| e^{i\pi \hat{F}_2} |F, F_3\rangle$$

$$= \sum_{F_3'} |F, F_3'\rangle D^{(F_3)}_{F_3',F_3}(0,-\pi,0)$$

10
\[
\begin{align*}
&= \sum_{F_3'} |F, F_3'\rangle d_{F_3,F_3'}^{(F)}(\pi) \\
&= \sum_{F_3'} |F, F_3'\rangle (-1)^{F-F_3'} \delta_{F_3'-F_3} \\
&= (-1)^{F+F_3} |F,-F_3\rangle
\end{align*}
\]  

(5)

It is easily seen that states with \( F_3 = 0 \) still are eigenstates of the reflection symmetry operator, and for simplicity we will constraint our considerations to those states.

This reflection symmetry is destroyed by the weak interaction of the electron with the nucleus which adds to the atomic Hamiltonian the terms

\[
H_{PV} = H^{(1)}_{PV} + H^{(2)}_{PV},
\]

\[
H^{(1)}_{PV} = -\frac{G_F}{\sqrt{2}} \int d^3x 2g^e_{\lambda} \bar{e}(x) \gamma^\lambda \gamma_5 e(x) \left( \sum_q g^e_{\lambda q} \bar{q}(x) \gamma_q(x) \right),
\]

\[
H^{(2)}_{PV} = -\frac{G_F}{\sqrt{2}} \int d^3x 2g^e_{\lambda} \bar{e}(x) \gamma^\lambda e(x) \left( \sum_q g^e_{\lambda q} \bar{q}(x) \gamma_\lambda \gamma_5 q(x) \right).
\]

(6)

Here \( q \) runs over all quarks, \( G_F \) is Fermi's constant and \( g^e_{\lambda q} \) denotes the neutral current coupling constants for the quark flavour \( q \) or the electron \( e \), respectively. Both terms together have no defined parity and consequently no defined quantum number according to the reflection symmetry operation \( \hat{R} \).

On its flight the ion stays for the time \( t_1 \) in the Stark field and during the time \( t_2 - t_1 \) outside of it.

Following essentially the notation of \([7]\) we get for the transition amplitude during the time \( t_1 \), in the case that there is no change in angular momentum
\[ f_{F,F_3;F,F_3}(t_1) = \exp \left\{ -iE(2\hat{S}, F)t_1 - i\tilde{\kappa}_{F,F_3} \left( \frac{\sqrt{3}F}{L} \right)^2 L t_1 - \kappa_{F,F_3} \frac{1}{2} \left( \frac{\sqrt{3}F}{L} \right)^2 \Gamma t_1 \right\} \]  

(7)

In this formula we take \( E(2\hat{S}, F) \) to be the energy of the 2S hyperfine states, perturbed by the parity-violating weak interaction denoted by the hat over the \( S \). \( F \) is the electric stark field \( \mathcal{E} \) multiplied with \( e \) and the Bohr radius:

\[ \mathcal{F} = \frac{e}{Z\alpha m_e} \mathcal{E} \]  

(8)

\( L = E_{2S_{1/2}} - E_{2P_{1/2}} \) is the energy difference of the two electron states of opposite parity considered in Fig. 3 and \( \Gamma \) the decay constant of the \( 2P_{1/2} \) state mentioned above. Here the hyperfine splitting is neglected because of its relative smallness. The \( \kappa \)'s are perturbative constants that give the admixtures due to the quadratic Stark effect.

In the case that there is a transition between the angular momentum states the amplitude is proportional to the applied electric field i.e., e.g.

\[ f_{1,0,0,0} \sim \mathcal{F} \]  

(9)

The total transition amplitude for an ion flying through one capacitor and the subsequent free drift length is given by

\[ g_{F',F_3;F,F_3} = e^{-iE(2\hat{S}, F)(t_2-t_1)} f_{F',F_3;F,F_3}(t_1) \]  

(10)

For an experimental set up with \( K \) capacitors the amplitude for the \( R \) symmetry violating transition \( |F = 0 F_3 = 0 \rangle \rightarrow |F = 1 F_3 = 0 \rangle \) is:

\[ f^{(K)}_{1,0,0,0} = g_{1,0,0,0} \sum_{k=0}^{K-1} g_{0,0,0,0}^k g_{1,0,1,0}^{K-k-1} \]

\[ = g_{1,0,0,0} g_{1,0,1,0}^{K-1} \frac{1 - \left( \frac{g_{0,0,0,0}}{g_{1,0,1,0}} \right)^K}{1 - \left( \frac{g_{0,0,0,0}}{g_{1,0,1,0}} \right)} \]  

(11)
The basic idea is now to make the absolute value of such a transition amplitude large. To this end, with the definitions given before one can express first

\[
\frac{g_{0,0,0,0}}{g_{1,0;1,0}} = \exp \left\{ +i \left[ A t_2 - (\tilde{\kappa}_{0,0} - \tilde{\kappa}_{1,0}) \left( \frac{\sqrt{3} F}{L} \right)^2 L t_1 \right] - \frac{1}{2} (\kappa_{0,0} - \kappa_{1,0}) \left( \frac{\sqrt{3} F}{L} \right)^2 \Gamma t_1 \right\} .
\]

(12)

Here \( A = E(2\hat{S}, 1) - E(2\hat{S}, 0) \) denotes the energy difference due to hyperfine splitting of the 2S electron orbitals. This very expression can be made real by a suitable choice of the length of the free drift space so that the condition

\[
A t_2 - (\tilde{\kappa}_{0,0} - \tilde{\kappa}_{1,0}) \left( \frac{\sqrt{3} F}{L} \right)^2 L t_1 = 2\pi n
\]

(13)

holds. We will come back to this later. With the above choice of \( t_2 \) we can get for the absolute value of the amplitude \( f_{1;0,0,0}^{(K)} \) the expression

\[
|f_{1;0,0,0}^{(K)}| \sim (\sqrt{3} F t_1) |g_{1;0,1,0}| \frac{1 - \left( \frac{g_{0,0,0,0}}{g_{1,0;1,0}} \right)^K}{1 - \left( \frac{g_{0,0,0,0}}{g_{1,0;1,0}} \right)^K}
\]

\[
= \frac{1}{2\sqrt{Q}} .
\]

(14)

Here we have assumed \( K \gg 1 \). Now the aim is to maximize \( |f_{1;0,0,0}^{(K)}| \) which is the same as to minimize \( Q \). This quantity \( Q \) plays an important role in this connection because as is shown in [7] \( Q \) is a measure for the polarization rotation of the ion flying through the capacitor arrangement as at \( t = 0 \) there is no component of angular momentum \( F \) parallel to the direction of flight.

\[
|e_1 \cdot \mathbf{F}(K t_2)| \sim \frac{1}{2\sqrt{Q}} .
\]

(15)

For definiteness we discuss the case of a pair of states with \( F = 0 \) and \( F = 1 \). We abbreviate
\[
x = \frac{1}{2}(\kappa_{0,0} - \kappa_{1,0}) \left( \frac{\sqrt{3}F}{L} \right)^2 \Gamma t_1 K,
\]

\[
\kappa = \frac{2\kappa_{1,0}}{\kappa_{0,0} - \kappa_{1,0}},
\]

(16)

and use as independent variables \(x\) and \(K\). We get up to factors independent of \(K\) and \(x\)

\[
Q \sim \frac{K}{x} e^{\kappa x} \cdot \frac{(1 - e^{-x/K})^2}{(1 - e^{-x})^2}.
\]

(17)

Let us assume \(K\) to be large, then \(Q\) is antiproportional to the number of capacitors \(K\).

We now treat \(K\) as a fixed number and then look for the minimum of \(Q\) as a function of \(x\). As \(K \gg 1\) the formal minimum of \(Q\) is obtained for \(x \ll 1\) such that in the vicinity of the minimum one has

\[
Q \sim e^{\kappa x} \rightarrow x_{\text{min}} \sim \frac{1}{\kappa}.
\]

(18)

At the minimum the quantity \(F\), essentially the electric field \(E\), is determined by

\[
\left( \frac{\sqrt{3}F}{L} \right)^2 = \frac{1}{\kappa_{1,0} K t_1}.
\]

(19)

We shall discuss below that this optimal situation cannot be reached for the ions considered here. The derivation of these equations has been done for a pair of atomic states \(F = 0, F = 1\). But there is no principal difference for other combinations like \(F = 3, F = 4\), which is considered here for boron-like uranium.

While the formulae are just the same as derived in [7] the quantities involved are quantitatively very different. Various large factors appear both in favor and in disfavor of the
heavy-ion system and there is no simple way to estimate the relative size of the effect. We shall present the numerical results for \(U^{235}\) in table VI. It will turn out that also some light ions might be of interest. Therefore we also add to table VI the results for the three lithium-like systems \(Be^+, B^{2+}\) and \(C^{3+}\). Their atomic properties are shown in tables VII, VIII and IX. The atomic properties of \(235U\) are shown in tables X, XI and XII. For the calculation of the \(\kappa\) coefficients we use perturbation theory:

\[
\tilde{\kappa}_{F,F_3} \left( \frac{\sqrt{3}F}{L} \right)^2 L = \sum_{n \neq m} \frac{|\langle n|eEz|m\rangle|^2}{E_m - E_n},
\]

\[
\kappa_{F,F_3} \left( \frac{\sqrt{3}F}{L} \right)^2 \Gamma = \sum_{n \neq m} \frac{|\langle n|eEz|m\rangle|^2}{E_m - E_n} \Gamma_n.
\] (20)

Here \(m\) denotes the state with the quantum numbers \(F, F_3\) and \(n\) the other admixing states. Solving this for the \(\kappa'\)'s and using the Wigner Eckart, 6j- and 9j- theorems one gets

\[
\tilde{\kappa}_{F,F_3} = \frac{1}{3} (2F + 1) \sum_n (2F_n + 1) \frac{L}{E_m - E_n} \frac{\langle n|j_3|z|m_j_m\rangle|^2}{r_B(Z)^2} \left( \begin{array}{cc} F_n & 1 & F \\ -F_3 & 0 & F_3 \end{array} \right)^2 \left( \begin{array}{c} F_n \\ j \end{array} \right),
\]

\[
\kappa_{F,F_3} = \frac{1}{3} (2F + 1) \sum_n (2F_n + 1) \frac{L}{E_m - E_n} \left( \begin{array}{cc} -F_3 & 0 & F_3 \\ F_n & 1 & F \end{array} \right)^2 \left( \begin{array}{c} j \\ F \end{array} \right).
\] (21)

Here \(r_B(Z) = 1/(Z\alpha m_e)\). The point is now that the kappa coefficients only deviate by the small energy differences that are due to the hyperfine splitting. In table XII we show the \(\kappa\) values for \(Be^+, B^{2+}, C^{3+}\) and \(U^{87+}\). Together with the numerical values of
the hyperfine splitting and the Stark matrix elements, which are given in tables VII–XII, we can calculate the interesting expressions for the polarization rotation effects. Let us first start in the same way as in [7] and analyze the situation for the minimal $Q$. It will turn out, that this assumption would imply unrealistically large electric fields resulting from:

$$t_2 = \frac{\bar{\kappa}_{0,0} - \bar{\kappa}_{1,0}}{\kappa_{1,0}} \frac{L}{\Gamma A} \left[ \frac{\hbar}{e} \right] \frac{1}{K} \quad (n = 0) \quad . \tag{22}$$

From (12) we get the requirement for the individual effects to add.

$$\left[ At_2 - \frac{\bar{\kappa}_{0,0} - \bar{\kappa}_{1,0}}{\kappa_{1,0}} \frac{L}{K} \right] = 2\pi n \quad . \tag{23}$$

This implies that the deviation $\delta t_2$ in $t_2$ should be smaller than

$$\delta t_2 < \frac{1}{100} \frac{1}{A} \left[ \frac{\hbar}{e} \right] \quad . \tag{24}$$

For the time $t_1$, which gives the length of the capacitor we are required to take $t_1 \leq t_2$, but there are no other constraints. To make the required electric field small one has to chose $t_1$ large [see Eq. (25) below], so we take $t_1 = t_2/2$. Finally from the relation

$$\left( \frac{\sqrt{3} F}{L} \right)^2 = \frac{1}{\kappa_{1,0} \Gamma t_1} \left[ \frac{\hbar}{e} \right] \frac{1}{K} \quad . \tag{25}$$

one can well calculate the electric field. The terms in [...] give always the necessary factors for the translation into SI units. The resulting numbers are given in table XIV. Here we always set $K = 1$. The values for other $K$ can easily be determined from the formulas above. Note that $K$ has to be chosen very large and that the electric field $E$ is for the choice $t_1 = t_2/2$ or for any choice $t_1 \sim t_2$ independent of $K$. Table XIV shows the results for $Be^+, B^{2+}, C^{3+}$ and $U^{87+}$. The values for the electric field $E$ are so unrealistically large that such an experiment cannot be realized. The reason for the large values of $E$ is the fact that in atoms with more than one electron the energy difference between
the $2p_{1/2}$ and the $2s_{1/2}$ states is orders of magnitude larger than for hydrogen-like atoms because the $2s_{1/2} - 2p_{1/2}$ degeneracy is removed by the electron-electron interaction.

We now proceed in the opposite direction. We take a realistic field $\mathcal{E}$ and other realistic values

$$K = 1000 \ ,$$
$$\mathcal{E} = 1000 \frac{V}{m} \ ,$$
$$t_1 = 1.0 \times 10^{-8} s \ .$$

We then calculate

$$x = \frac{1}{2}(\kappa_{0,0} - \kappa_{1,0}) \left( \frac{\sqrt{3} F}{L} \right)^2 \Gamma t_1 K \left[ \frac{e}{\hbar} \right] .$$

(27)

As $x$ is very small we approximate

$$Q = \frac{(\kappa_{0,0} - \kappa_{1,0}) \Gamma K}{8L^2t_1} \frac{1}{x} (1 - e^{-x})^2 (1 - e^{-x})^{-2} \exp \left( \frac{2\kappa_{1,0}}{\kappa_{0,0} - \kappa_{1,0}} x \right)$$

$$\rightarrow \frac{\kappa_{0,0} - \kappa_{1,0}}{8L^2t_1} \frac{1}{xK} \left[ \frac{\hbar}{e} \right] .$$

(28)

In this way we get the values of table XV. These values must be compared to that obtained in [7] for hydrogen $Q_{\text{min}} = 6.6 \times 10^{-9}$.

V. CONCLUSIONS

In principle it is obvious that heavy ions with few inner shell electrons offer a possibility to test the effects of parity admixture. This admixture is in heavy ions orders of magnitude larger than in neutral atoms.
The ideal case is a parity-violation effect to a sizable extent without applying any of the elaborated methods used in the cesium experiment [3]. Then, the only chance to get measurable parity admixtures is to find a pair of energy states near the ground state with equal angular momentum but opposite parity that is nearly degenerated with respect to its energy. Unfortunately there is no such pair of orbitals in uranium with two to five electrons except for the already known degeneracy in helium-like uranium. As the electron levels do only change very slowly with $Z$ the same is true for the neighboring heavy ions.

The next step will consequently be a very detailed analysis of the degeneracy in helium-like heavy ions including nuclear and isotopic effects because here a level crossing must exist. Level crossing also exists for compound nuclear reactions but here the lifetime of the compound nucleus is too short to allow for atomic physics experiments. Looking for parity-violating spin rotations opened another perspective. We showed, however, that the net effect (value of $1/\sqrt{Q}$) for heavy ions is about thirty times weaker than for hydrogen.
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TABLE I. Contributions to the $1s2s^1S_0 \rightarrow 1s2p^3P_0$ separation near the two crossing points.

All units are given in eV.

|                  | $Z = 62$                  | $Z = 92$                  | Diff   | $Z = 62$                  | $Z = 92$                  | Diff   |
|------------------|---------------------------|---------------------------|--------|---------------------------|---------------------------|--------|
|                  | 1$s2p^3P_0$ | 1$s2s^1S_0$ | Diff   | 1$s2p^3P_0$ | 1$s2s^1S_0$ | Diff   |
| Coulomb          | -68868.56                | -68861.61                | -6.948 | -165518.05                | -165487.55                | -30.50 |
| Magnetic         | 38.30                    | 17.12                    | 21.17  | 151.30                    | 66.36                     | 84.91  |
| Retardation      | -3.26                    | 1.30                     | -4.56  | -10.09                    | 5.56                      | -15.65 |
| Mass Pol         | -0.029                   | 0.00                     | -0.03  | -0.04                     | 0.00                      | -0.04  |
| Correlation      | -0.39                    | -0.59                    | 0.20   | -1.02                     | -1.18                     | 0.16   |
| 1e- Self-energ   | 82.66                    | 95.16                    | -12.50 | 364.88                    | 420.68                    | -55.80 |
| 2e- Self-energ   | -0.18                    | -1.24                    | 1.06   | -1.15                     | -5.44                     | 4.29   |
| Uehling          | -13.52                   | -15.18                   | 1.67   | -96.13                    | -108.71                   | 12.59  |
|                 | -0.02                    | -0.10                    | 0.087  | -0.28                     | -0.89                     | 0.61   |
| Wichman & Kroll  | 0.38                     | 0.42                     | -0.04  | 4.75                      | 5.28                      | -0.53  |
| Kallen & Sabry   | -0.11                    | -0.12                    | 0.01   | -0.73                     | -0.83                     | 0.09   |
| Nuclear Pol.     | -1.10                    | -1.28                    | 0.18   | -1.10                     | -1.28                     | 0.18   |
| Total energy     | -68764.71                | -68764.83                | 0.11   | -165107.70                | -165108.00                | 0.30   |
TABLE II. Electron configuration of lithium-like uranium.

| main conf.       | parity | energy (eV)     | lifetime (s) |
|------------------|--------|-----------------|--------------|
| $1s^22s^2 2S_{1/2}$ | $+$    | $-2.9424 \times 10^5$ | $\infty$    |
| $1s^22p^2 2P_{1/2}$ | $-$    | $-2.9395 \times 10^5$ | $1.0 \times 10^{-10}$ |
| $1s^22p^2 2P_{3/2}$ | $-$    | $-2.8978 \times 10^5$ | $1.1 \times 10^{-14}$ |
| $1s^23s^2 2S_{1/2}$ | $+$    | $-2.7545 \times 10^5$ | $4.9 \times 10^{-15}$ |
| $1s^23p^2 2P_{1/2}$ | $-$    | $-2.7537 \times 10^5$ | $4.6 \times 10^{-16}$ |

TABLE III. Electron configuration of beryllium-like uranium.

| main conf.       | parity | energy (eV)     | lifetime (s) |
|------------------|--------|-----------------|--------------|
| $1s^22s^2 1S_0$ | $+$    | $-326604$       | $\infty$    |
| $1s^22s2p^3P_0$  | $-$    | $-326345$       | $\infty$ for $^{238}U$ |
|                  |        |                 | $8.56 \times 10^{-5}$ for $^{235}U$ |
| $1s^22s2p^3P_1$  | $-$    | $-326305$       | $1.00 \times 10^{-10}$ |
| $1s^22p^3P_0$    | $+$    | $-325894$       | $7.87 \times 10^{-12}$ |
| $1s^22s2p^3P_2$  | $-$    | $-322224$       | $3.37 \times 10^{-12}$ |
### TABLE IV. Electron configuration of boron-like uranium.

| main conf.          | parity | energy (eV)      | lifetime (s) |
|---------------------|--------|------------------|--------------|
| $1s^22s^22p^2\,^2P_{1/2}$ | $-$    | $-3.5826 \times 10^5$ | $\infty$       |
| $1s^22s^22p^2\,^4P_{1/2}$ | $+$    | $-3.5785 \times 10^5$ | $5.2 \times 10^{-11}$ |
| $1s^22s^22p^2\,^2P_{3/2}$ | $-$    | $-3.5417 \times 10^5$ | $3.3 \times 10^{-12}$ |
| $1s^22s^22p^2\,^4P_{3/2}$ | $+$    | $-3.5389 \times 10^5$ | $7.5 \times 10^{-13}$ |
| $1s^22s^22p^2\,^2D_{5/2}$ | $+$    | $-3.5384 \times 10^5$ | $6.6 \times 10^{-11}$ |

### TABLE V. Energies of the first two electron states in lithium-like heavy ions for high nuclear charges.

| Name    | Z  | A | $E(1s^22s^2\,^2S_{1/2})\ (J^P = \frac{1}{2}^+) \ (eV)$ | $E(1s^22p^2\,^2P_{1/2})\ (J^P = \frac{1}{2}^-) \ (eV)$ | $\Delta \ (eV)$ |
|---------|----|---|--------------------------------------------------|--------------------------------------------------|----------------|
| Te+Te   | 104 | 260 | -396234.8 | -395910.3 | 324.6 |
| Ce+Ce   | 116 | 280 | -528168.1 | -527979.6 | 188.5 |
| Nd+Nd   | 120 | 288 | -581273.8 | -581267.7 | 6.1  |
| Sm+Sm   | 124 | 304 | -640357.1 | -640692.8 | -335.7 |
| Gd+Gd   | 128 | 316 | -706756.4 | -707698.3 | -941.9 |
TABLE VI. Hyperfine splitting (A), level width (Γ) and level separation (L) for selected ions.

| Ion     | $Be^+$ | $B^{2+}$ | $C^{3+}$ | $U^{87+}$ |
|---------|---------|----------|----------|-----------|
| Isotope | Be 9    | B 11     | C 13     | U 235     |
| lower state | 2S1/2    | 2S1/2     | 2S1/2     | 1s2 2s2 2p J=1/2 |
| upper state | 2P1/2    | 2P1/2     | 2P1/2     | 1s2 2s 2p2 J=1/2 |
| I       | 3/2     | 3/2       | 1/2       | 7/2       |
| A [eV]  | $1.71414 \times 10^{-6}$ | $1.04438 \times 10^{-5}$ | $8.44760 \times 10^{-6}$ | $1.796 \times 10^{-2}$ |
| Γ [eV]  | $7.79467 \times 10^{-8}$ | $1.30629 \times 10^{-7}$ | $1.81121 \times 10^{-7}$ | $3.11949 \times 10^{-5}$ |
| L [eV]  | $3.98910 \times 10^{+0}$ | $6.05385 \times 10^{+0}$ | $8.07181 \times 10^{+0}$ | $4.0302 \times 10^{+2}$ |
|                  | Value                      |                      |
|------------------|----------------------------|----------------------|
| $2p_{1/2} - 2s_{1/2}$ PNC matrix element | $-5.6810174 \times 10^{-15}$ eV |                      |
| $2p_{1/2} - 2s_{1/2}$ energy difference | $3.9891026 \times 10^{+00}$ eV |                      |
| $2p_{1/2} - 2s_{1/2}$ lifetime (length) | $8.4443912 \times 10^{-09}$ sec |                      |
| $2p_{1/2} - 2s_{1/2}$ lifetime (velocity) | $7.9463913 \times 10^{-09}$ sec |                      |
| $2p_{1/2} - 2s_{1/2}$ Stark–element | $.76491 \times 10^{+00}$ a.u. |                      |
| $2p_{1/2} F=2$ | total hyperfine matrix element: $-1.1844518555 \times 10^{-07}$ eV | Bohr-Weisskopf correction: $5.9033102475 \times 10^{-15}$ eV | total: $-1.1844517964 \times 10^{-07}$ eV |
| $2p_{1/2} F=1$ | total hyperfine matrix element: $1.9740864258 \times 10^{-07}$ eV | Bohr-Weisskopf correction: $-9.8388504126 \times 10^{-15}$ eV | total: $1.9740863274 \times 10^{-07}$ eV |
| $2s_{1/2} F=2$ | total hyperfine matrix element: $-6.4289041847 \times 10^{-07}$ eV | Bohr-Weisskopf correction: $8.8568061986 \times 10^{-11}$ eV | total: $-6.4280185040 \times 10^{-07}$ eV |
| $2s_{1/2} F=1$ | total hyperfine matrix element: $1.0714840308 \times 10^{-06}$ eV | Bohr-Weisskopf correction: $-1.4761343664 \times 10^{-10}$ eV | total: $1.0713364173 \times 10^{-06}$ eV |
TABLE VIII. Atomic structure for lithium-like $^{11}B$.

| Transition                  | Value                           |
|-----------------------------|---------------------------------|
| $2p_{1/2} - 2s_{1/2}$ PNC matrix element | $-2.7316505 \times 10^{-14}$ eV   |
| $2p_{1/2} - 2s_{1/2}$ energy difference     | $6.0538537 \times 10^{+00}$ eV   |
| $2p_{1/2} - 2s_{1/2}$ lifetime (length)   | $5.0387842 \times 10^{-09}$ sec   |
| $2p_{1/2} - 2s_{1/2}$ lifetime (velocity) | $4.6813678 \times 10^{-09}$ sec   |
| $2p_{1/2} - 2s_{1/2}$ Stark--element      | $.52970 \times 10^{+00}$ a.u.    |

| Transition                  | Value                           |
|-----------------------------|---------------------------------|
| 2p 1/2 F=2 total hyperfine matrix element | $8.7104273520 \times 10^{-07}$ eV   |
| Bohr-Weisskopf correction:  | $-8.9772008878 \times 10^{-14}$ eV |
| total:                      | $8.7104264543 \times 10^{-07}$ eV   |

| Transition                  | Value                           |
|-----------------------------|---------------------------------|
| 2p 1/2 F=1 total hyperfine matrix element | $-1.4517378920 \times 10^{-06}$ eV   |
| Bohr-Weisskopf correction:  | $1.4962001480 \times 10^{-13}$ eV |
| total:                      | $-1.4517377424 \times 10^{-06}$ eV   |

| Transition                  | Value                           |
|-----------------------------|---------------------------------|
| 2s 1/2 F=2 total hyperfine matrix element | $3.9170586600 \times 10^{-06}$ eV   |
| Bohr-Weisskopf correction:  | $-6.4395425787 \times 10^{-10}$ eV |
| total:                      | $3.9164147057 \times 10^{-06}$ eV   |

| Transition                  | Value                           |
|-----------------------------|---------------------------------|
| 2s 1/2 F=1 total hyperfine matrix element | $-6.5284311000 \times 10^{-06}$ eV   |
| Bohr-Weisskopf correction:  | $1.0732570965 \times 10^{-09}$ eV |
| total:                      | $-6.5273578429 \times 10^{-06}$ eV   |
| Transition                      | Value                                |
|--------------------------------|--------------------------------------|
| $2p_{1/2} - 2s_{1/2}$ PNC matrix element | $-8.7153650 \times 10^{-14}$ eV       |
| $2p_{1/2} - 2s_{1/2}$ energy difference | $8.0718138 \times 10^{+00}$ eV       |
| $2p_{1/2} - 2s_{1/2}$ lifetime (length) | $3.6341015 \times 10^{-09}$ sec       |
| $2p_{1/2} - 2s_{1/2}$ lifetime (velocity) | $3.3515826 \times 10^{-09}$ sec       |
| $2p_{1/2} - 2s_{1/2}$ Stark–element    | $0.40514 \times 10^{+00}$ a.u.        |

**2p 1/2 F=0**

- Total hyperfine matrix element: $-1.5529296692 \times 10^{-06}$ eV
- Bohr-Weisskopf correction: $3.0240473822 \times 10^{-13}$ eV
- Total: $-1.5529293668 \times 10^{-06}$ eV

**2p 1/2 F=1**

- Total hyperfine matrix element: $5.176432307 \times 10^{-07}$ eV
- Bohr-Weisskopf correction: $-1.0080157941 \times 10^{-13}$ eV
- Total: $5.1764312227 \times 10^{-07}$ eV

**2s 1/2 F=0**

- Total hyperfine matrix element: $-6.3369865130 \times 10^{-06}$ eV
- Bohr-Weisskopf correction: $1.2851809538 \times 10^{-09}$ eV
- Total: $-6.3357013320 \times 10^{-06}$ eV

**2s 1/2 F=1**

- Total hyperfine matrix element: $2.1123288377 \times 10^{-06}$ eV
- Bohr-Weisskopf correction: $-4.2839365127 \times 10^{-10}$ eV
- Total: $2.1119004440 \times 10^{-06}$ eV
TABLE X. Atomic level structure for boron–like $^{235}\text{U}$.

| level | binding energy (eV) | excitation energy (eV) |
|-------|---------------------|------------------------|
| ground state | $-358233.01$ | —— |
| $1s^2 2s 2p^2 J=1/2$ | $-357829.98$ | $403.02$ |
| $1s^2 2s 2p^2 J=3/2$ | $-353861.72$ | $4371.29$ |
| $1s^2 2s 2p^2 J=5/2$ | $-353818.09$ | $4414.92$ |
| $1s^2 2s^2 2p^3/2$ | $-354139.11$ | $4093.90$ |
| $1s^2 2s^2 2p^2 J=1/2$ 2nd | $-353712.14$ | $4520.87$ |

TABLE XI. $1s^2 2s^2 2p J=\frac{1}{2} - 1s^2 2s 2p^2 J=\frac{1}{2}$ matrix elements in boron–like $^{235}\text{U}$.

| matrix element | value |
|----------------|-------|
| PNC matrix element | $3.79 \times 10^{-6}$ eV |
| lifetime velocity gauge | $3.06 \times 10^{-11}$ sec |
| lifetime length gauge | $2.11 \times 10^{-11}$ sec |
| Stark–element | $.29048 \times 10^{-3}$ a.u. |
TABLE XII. Hyperfine structure in boron–like $^{235}U$.

| Configuration | Hyperfine | Bohr-Weisskopf | Total |
|---------------|-----------|----------------|-------|
| 1s2 2s 2p $j=1/2$ F=3 I=7/2 | $1.02 \times 10^{-2}$ eV | $-1.14 \times 10^{-4}$ eV | $1.01 \times 10^{-2}$ eV |
| 1s2 2s 2p $j=1/2$ F=4 I=7/2 | $-7.95 \times 10^{-3}$ eV | $8.83 \times 10^{-5}$ eV | $-7.86 \times 10^{-3}$ eV |
| 1s2 2s 2p $j=1/2$ F=3 I=7/2 | $3.03 \times 10^{-2}$ eV | $-9.83 \times 10^{-4}$ eV | $2.94 \times 10^{-2}$ eV |
| 1s2 2s 2p $j=1/2$ F=4 I=7/2 | $-2.36 \times 10^{-2}$ eV | $7.65 \times 10^{-4}$ eV | $-2.28 \times 10^{-2}$ eV |
### TABLE XIII. $\kappa$ values.

| Ion   | $\kappa$-constants | numerical values                          |
|-------|---------------------|------------------------------------------|
| $Be^+$ | $\kappa_{1,0}, \tilde{\kappa}_{1,0}$ | $5.2007794253 \times 10^{-1}; -5.2007778741 \times 10^{-1}$ |
|       | $\kappa_{2,0}, \tilde{\kappa}_{2,0}$ | $5.2007741321 \times 10^{-1}; -5.2007752275 \times 10^{-1}$ |
| $B^{2+}$ | $\kappa_{1,0}, \tilde{\kappa}_{1,0}$ | $3.8969640353 \times 10^{-1}; -3.8969687978 \times 10^{-1}$ |
|       | $\kappa_{2,0}, \tilde{\kappa}_{2,0}$ | $3.8969804714 \times 10^{-1}; -3.8969770158 \times 10^{-1}$ |
| $C^{3+}$ | $\kappa_{0,0}, \tilde{\kappa}_{0,0}$ | $3.2827626620 \times 10^{-1}; -3.2827654492 \times 10^{-1}$ |
|       | $\kappa_{1,0}, \tilde{\kappa}_{1,0}$ | $3.2827712174 \times 10^{-1}; -3.2827697269 \times 10^{-1}$ |
| $U^{87+}$ | $\kappa_{3,0}, \tilde{\kappa}_{3,0}$ | $3.9683205819 \times 10^{-5}; -3.9679966597 \times 10^{-5}$ |
|       | $\kappa_{4,0}, \tilde{\kappa}_{4,0}$ | $3.9669391256 \times 10^{-5}; -3.9673059448 \times 10^{-5}$ |

### TABLE XIV. Characteristic values for selected ions.

| Ion   | $Be^+$ | $B^{2+}$ | $C^{3+}$ | $U^{87+}$ |
|-------|-------|--------|--------|-------|
| $t_{2}[s]$ | $1.0000 \times 10^{-08}$ | $6.1594 \times 10^{-09}$ | $4.5248 \times 10^{-09}$ | $8.2441 \times 10^{-11}$ |
| $\delta t_{2}[s]$ | $3.8399 \times 10^{-12}$ | $6.3024 \times 10^{-13}$ | $7.7917 \times 10^{-13}$ | $3.6649 \times 10^{-16}$ |
| $t_{1}$ | $5.0000 \times 10^{-09}$ | $3.0797 \times 10^{-09}$ | $2.2624 \times 10^{-09}$ | $4.1221 \times 10^{-11}$ |
| $x$ | $5.0888 \times 10^{-07}$ | $-2.1088 \times 10^{-06}$ | $-1.3031 \times 10^{-06}$ | $1.7412 \times 10^{-04}$ |
| $Q_{\min}$ | $2.2789 \times 10^{-16}$ | $2.0174 \times 10^{-16}$ | $1.8042 \times 10^{-16}$ | $8.2689 \times 10^{-20}$ |
| $E[V_{m}]$ | $3.1371 \times 10^{+11}$ | $6.7668 \times 10^{+11}$ | $1.1688 \times 10^{+12}$ | $4.5952 \times 10^{+16}$ |
### TABLE XV. $x$ and $Q$ for realistic values of $K$, $E$ and $t_1$.  

| Ion  | $Be^+$     | $B^{2+}$    | $C^{3+}$    | $U^{87+}$   |
|------|------------|-------------|-------------|-------------|
| $x$  | $1.0341 \times 10^{-20}$ | $-1.4954 \times 10^{-20}$ | $-4.2159 \times 10^{-21}$ | $2.0004 \times 10^{-26}$ |
| $Q$  | $2.0629 \times 10^{-06}$ | $3.2232 \times 10^{-06}$ | $4.6414 \times 10^{-06}$ | $1.0912 \times 10^{-03}$ |
FIGURES

FIG. 1. Energy difference of the two nearly degenerated electron states as a function of atomic number.

FIG. 2. Energy difference of the two nearly degenerated electron states as a function of the mean-square nuclear radius, for $Z = 92$. Value of the splitting for experimental nuclear size are represented by square dots.

FIG. 3. Energy difference of the first excited state and the ground state in lithium-like heavy ions from gold to plutonium $\Delta E = E(1s^22p^2P_{1/2}) - E(1s^22s^2S_{1/2})$.

FIG. 4. Hyperfine-splitting for the parity-mixed states.

FIG. 5. The experimental setup studied for possible parity-violation measurement.
$2^3P_0 - 2^1S_0$ separation

- MCDF (SE screening by Welton model)
- All order RMBPT (SE screening from Drake)
Figure 2

Radius (Fm) vs. Splitting

- A=233
- A=234
- A=238
- A=235
Figure 3

\[ \Delta E \text{ [eV]} \]

\[ Z \]

210 - 300
220 - 290
230 - 280
240 - 270
250 - 260
260 - 250
270 - 240
280 - 230
290 - 220
300 - 210

78 80 82 84 86 88 90 92 94 96
Figure 4

$2P_{1/2}$

$2S_{1/2}$

electron states

hyperfine states

$F=0$

$F=1$

$F=0$

$F=1$
Figure 5

3

ion
capacitor
capacitor