Abstract

Actions for $D = 2$, $N = 2$ supergravity coupled to a scalar field are calculated, and it is shown that the most general power-counting renormalizable dilaton gravity action has an $N = 2$ locally supersymmetric extension. The presence of chiral terms in the action leads one to hope that non-renormalization theorems similar to those in global SUSY will apply; this would eliminate some of the renormalization ambiguities which plague ordinary bosonic (and $N = 1$) dilaton gravity. To investigate this, the model is studied in superconformal gauge, where it is found that one chiral term becomes nonchiral, so that only one term is safe from renormalization.
1. Introduction

The theory of dilaton gravity, defined by the action

\[ S_D = \int d^2 x \sqrt{-g} e^{-2\phi} \left[ R + 4(\nabla \phi)^2 + 4\lambda^2 \right], \]

(1.1)

has received much attention recently as a two-dimensional model of gravity (see e.g. [1,2]); as such it has proved to be a convenient laboratory for studying many outstanding issues in black hole physics and quantum gravity. The theory, which derives originally from string theory, shares many interesting properties with four dimensional general relativity,

\[ S_G = \int d^4 x \sqrt{-g} R. \]

For example, it has collapsing black hole solutions which radiate by the Hawking process, and it has a positive energy theorem analogous to the one for four dimensional general relativity. In addition the theory is significantly simpler to work with than general relativity, allowing many analytic results to be obtained whose four-dimensional counterparts are known approximately at best. In particular, the theory coupled to conformal matter is completely soluble classically, and among the exact solutions one finds a set describing collapse of matter to form a black hole. The black holes emit Hawking radiation, for which the energy flux and Bogoljubov coefficients may be computed analytically; and, in addition, the backreaction may be completely accounted for by a simple addition to the action [1,4].

Unfortunately, this toy model is so faithful in resembling the four dimensional general relativity, that it also shares some unwanted features with the full theory; in particular, although the theory is power-counting renormalizable, the fields are dimensionless, so an infinite number of counterterms are allowed, leading to a quantum theory which is in principle the most general power-counting renormalizable theory coupling 2D gravity with a scalar dilaton. This large ambiguity destroys predictability. One might expect that adding supersymmetry would ameliorate this problem, since in global supersymmetry certain terms (chiral terms) are known not to receive renormalization corrections. N = 1 supersymmetry (as explored in [3]), although interesting in many ways, is not sufficient for this purpose, since no chiral terms may be written down. To have chiral superfields in D = 2 one needs at least N = 2 supersymmetry; accordingly, in this paper, we begin with the most general power-counting renormalizable theory of dilaton gravity

\[ S = \int d^2 x \sqrt{-g} \left[ J(\phi) R + 2K(\phi)(\nabla \phi)^2 + W(\phi) \right] \]

and extend it to have N = 2 supersymmetry.
2. Formalism

In the next section we will write actions using formulas first calculated in [9]; however, since that paper contains errors, we reproduce here the results we need. Some of these formulas are also given in [8,9], and the method is that of [10], section 5.6. We use $N = 2$ superspace with covariant derivative

$$\nabla_A = E_A^M \partial_M + \omega_A M + \Gamma_A Y$$

where $\omega$ is the spin connection and $\Gamma$ is the $U(1)$ connection which arises due to $N = 2$. Also $M, Y$ are the Lorentz and $U(1)$ generators which act as follows on spinors $F_\alpha$ and vectors $F_a$:

\begin{align*}
[M, F_\alpha] &= \frac{1}{2} (\gamma^3)_{\alpha}^\beta F_\beta, \\
[M, F_a] &= \epsilon_a^b F_b, \\
[Y, F_\alpha] &= \frac{1}{2} i (\gamma^3)_{\alpha}^\beta F_\beta.
\end{align*}

The gamma matrix conventions are $(\gamma^0)_{\alpha}^\beta = (\sigma^2)_{\alpha}^\beta$, $(\gamma^1)_{\alpha}^\beta = (-i\sigma^1)_{\alpha}^\beta$, $(\gamma^3)_{\alpha}^\beta = (\sigma^3)_{\alpha}^\beta$, and the metric is taken to have signature $(+1, -1)$. Spinors are raised from the left and lowered from the right by the spinor metric $C_{\alpha\beta} = -C^{\alpha\beta} = \sigma^2$, and the antisymmetric tensor $\epsilon_{ab}$ satisfies $\epsilon_{01} = 1$. With these conventions one has the identity

\begin{align*}
(\gamma^a)_{\alpha}^{\hat{\alpha}} (\gamma_a)_{\beta}^{\hat{\beta}} &= -\delta_{\alpha}^{\hat{\alpha}} \delta_{\beta}^{\hat{\beta}} - (\gamma^3)_{\alpha}^{\hat{\alpha}} (\gamma^3)_{\beta}^{\hat{\beta}}. 
\end{align*}

Conventions for conjugation are as in [10]; in particular, $(\psi^\alpha)\dagger = \bar{\psi}^{\dot{\alpha}}$, while there is a sign change if the indices are lowered, since the spinor metric is imaginary. Lastly, we use the standard index convention whereby indices from early in the alphabet are Lorentz indices while those from later are Einstein.

To reduce to a minimal theory of supergravity, one imposes constraints on the graded commutator $[\nabla_A, \nabla_B]$, after which one finds that the Bianchi identities may be "solved" to express all the commutations in terms of one chiral superfield, giving [7,8,9]

\begin{align*}
\{\nabla_\alpha, \nabla_\beta\} &= 2(\gamma^3)_{\alpha\beta} \Sigma (M + iY), \\
\{\nabla_\alpha, \nabla_{\dot{\alpha}}\} &= 2i(\gamma^a)_{\alpha}^{\dot{\alpha}} \nabla_a, \\
[\nabla_\alpha, \nabla_a] &= \frac{1}{2} i (\gamma_a)^{\alpha}{}_{\dot{\alpha}} [\bar{\Sigma} \nabla_{\dot{\alpha}} - (\gamma^3)^{\alpha\beta} \nabla_{\beta} \bar{\Sigma} (M + iY)], \\
[\nabla_a, \nabla_b] &= \frac{1}{4} \epsilon_{ab} [(\gamma^3)^{\alpha\beta}(\nabla_\alpha \Sigma) \nabla_\beta + (\gamma^3)^{\dot{\alpha}\dot{\beta}}(\nabla_{\dot{\alpha}} \bar{\Sigma}) \nabla_{\dot{\beta}}] \\
&\quad + \frac{1}{8} \epsilon_{ab} [\nabla^2 \Sigma + \bar{\nabla}^2 \bar{\Sigma} - 8 \Sigma \bar{\Sigma}] M - \frac{i}{8} \epsilon_{ab} [\nabla^2 \Sigma - \bar{\nabla}^2 \bar{\Sigma}] Y, \\
\end{align*}

(2.1)
where $\Sigma$ is a chiral superfield, $\nabla_\dot{\alpha} \Sigma = 0$.

Supergravity component fields are defined as $\theta = \bar{\theta} = 0$ components of $E_a^M$, $\omega_a$, and $\Gamma_a$ by

$$\nabla_a | = e_a^m \partial_m + \psi_a^\mu \partial_\mu + \bar{\psi}_a^\dot{\mu} \partial_{\dot{\mu}} + \omega_a M + A_a Y$$

$$= e_a^m D_m + \psi_a^\mu \partial_\mu + \bar{\psi}_a^\dot{\mu} \partial_{\dot{\mu}}, \quad (2.2)$$

where “$|$” stands for the $\theta = \bar{\theta} = 0$ projection. Then computing in a Wess-Zumino gauge, one finds the component content of $\Sigma$ to be

$$\Sigma | = B,$$

$$\nabla_\dot{\alpha} \Sigma | = -2\epsilon^{ab}(\gamma_3)_{\alpha}^{\beta} \psi_{ab\dot{\beta}} - 2iB(\gamma_\alpha)_{\dot{\beta}}^{\dot{\gamma}} \bar{\psi}_{b\dot{\gamma}}, \quad (2.3)$$

$$\nabla^2 \Sigma | = -2\epsilon^{ab}[r_{ab} + if_{ab}] + 4B\bar{B} + 16i\bar{\psi}_a^{\alpha} \gamma^b \psi_{ab} + 8\bar{\psi}_a \bar{\psi}_a B,$$

where $r_{ab}, f_{ab}$ and $\psi_{ab}^\alpha$ are respectively the curvature, the $U(1)$ field strength, and the gravitino curl:

$$r_{ab} = e_a^m e_b^n \partial_{[m} \omega_{n]},$$

$$f_{ab} = e_a^m e_b^n \partial_{[m} A_{n]},$$

$$\bar{\psi}_a^{\alpha} = e_a^m e_b^n D_{[m} \psi_{n]}^{\alpha}.$$  

Our index suppression convention for spinor indices is that they are in matrix multiplication order, e.g. $\bar{\psi}_a \gamma_3 \psi_a$.

We will want to write down chiral terms in the action, which have the generic form

$$\mathcal{L} = \int d^2\Theta \Delta_c \mathcal{L}_{\text{chiral}}. \quad (2.4)$$

In this formula, $\mathcal{L}_{\text{chiral}}$ is some chiral superfield, $\nabla_\dot{\alpha} \mathcal{L}_{\text{chiral}} = 0$. The integration is over the two $\Theta$ variables of a chiral representation of superspace; these are variables defined such that a chiral superfield $\Phi$ has the $\Theta$ expansion

$$\Phi = \Phi | + \Theta^{\alpha} \nabla_\alpha \Phi | - \frac{1}{4} \Theta^2 \nabla^2 \Phi |. \quad (2.5)$$

One arrives at such a representation through a superspace coordinate transformation whose parameters must depend on the fields in the supergravity multiplet (since $\nabla_\alpha$ does). To make the integral covariant under coordinate transformations one needs the chiral density $\Delta_c$; it serves the same purpose as the ordinary spacetime density $\sqrt{-g}$. To calculate $\Delta_c$ there are various strategies, two of which are given in [7,11]. With either, one finds

$$\frac{1}{c} \Delta_c = 1 + 2i\Theta^{\alpha} \bar{\psi}_a^\alpha - \frac{1}{4} \Theta^2 [-4\bar{B} - 8\epsilon^{ab} \bar{\psi}_a^\gamma \bar{\psi}_b^{\dot{\gamma}}] \quad (2.6)$$
where \( e = \sqrt{-g} \), and the factor of \(-\frac{1}{4}\) is for convenience since \( \int d^2\Theta \Theta^2 = -4 \). Lastly, chiral actions may be obtained from nonchiral with the projection

\[
\mathcal{L}_{\text{chiral}} = \bar{\nabla}^2 \mathcal{L}_{\text{nonchiral}}.
\]

### 3. Dilaton Gravity Action

Now we would like to use this technology to write actions which extend the bosonic and \( N = 1 \) dilaton gravity models to models with \( N = 2 \), for the reasons stated in the Introduction. Since renormalization of dilaton gravity may produce in principle any power-counting renormalizable term coupling a scalar dilaton to two-dimensional gravity, we attempt to write down the most general power-counting renormalizable action coupling a scalar dilaton to two-dimensional \( N = 2 \) supergravity. For \( N = 2 \) supersymmetry the dilaton must become a complex scalar in order to equalize bosonic and fermionic degrees of freedom. One minimal multiplet for a complex scalar field is the chiral scalar multiplet, consisting of the components of one chiral superfield, which we call \( \Phi \). We find three renormalizable terms can be written, corresponding to the three terms in the original dilaton gravity action (1.1).

\[
\mathcal{L} = \int d^2\Theta \Delta c [\Sigma S(\Phi) + \bar{\nabla}^2 T(\Phi, \bar{\Phi}) + U(\Phi)] + \text{h.c.} \tag{3.1}
\]

The functions \( S, T, \) and \( U \) are arbitrary.

In order to see what models of dilaton gravity are generated by this theory, we study its classical bosonic solutions; for this purpose we need only the bosonic part of the action, which comes out to be

\[
\frac{1}{e} \mathcal{L} = -4R \text{Re} S(\varphi) + 4F \text{Im} S(\varphi) + 32\text{Re} [\bar{\partial} \partial T(\varphi, \bar{\varphi})] \nabla^a \bar{\varphi} \nabla_a \varphi
\]

\[
+ 2\text{Re} \left[ U'(\varphi) G - 4U(\varphi) \bar{B} + S'(\varphi) BG \right]
\]

\[
+ 2\text{Re} \left[ \bar{\partial} \partial T(\varphi, \bar{\varphi}) \right] G \bar{G},
\]

where

\[
\Phi| = \varphi, \quad \nabla^2 \Phi| = G.
\]

and \( R = \epsilon^{ab} r_{ab} \) is the scalar curvature, while \( F \equiv \epsilon^{ab} f_{ab} \) is the equivalent quantity for the \( U(1) \) field.
Next we eliminate the auxiliary fields from the action, beginning with $B$. Variation of $B$ leads to the equation

$$G = \frac{4U}{Sr};$$

when this is substituted back into the action, all terms with $B$ cancel - as expected, since $B$ appeared only linearly. Variation of $G$ leads to an equation for $B$, which is irrelevant, since $B$ has been eliminated.

Next we consider the equation of motion of the $U(1)$ field $A_m$. This field appears only once in the bosonic action, in the field strength $F$. Its variation yields

$$\partial_m \text{Im}[S(\varphi)] = 0,$$

i.e., $\text{Im}S = \text{const.} \equiv c$. This we solve parametrically, introducing a real parameter $\phi$ and setting $\varphi = \varphi(\phi)$ such that $\text{Im}S(\varphi(\phi)) = c$. Then for functions $F(\varphi)$ we have

$$F(\varphi) \to F(\phi) \equiv F(\varphi(\phi))$$

and

$$F'(\varphi) \to F'(\phi) \left(\frac{d\varphi}{d\phi}\right)^{-1}.$$

Lastly, we must satisfy equation of motion of the degree of freedom which was eliminated with $\varphi = \varphi(\phi)$; this is equivalent to varying $c$, which just fixes $F = 0$.

Making these substitutions in the action, and redefining

$$J(\phi) = 4S(\phi),$$
$$K(\phi) = -16 \left|\frac{d\varphi}{d\phi}\right|^2 \text{Re}[\partial\partial^\dagger T(\phi, \phi)],$$
$$L(\phi) = 8U(\phi),$$

one finds that $c$ disappears, so that $J$ may be taken to be real. Furthermore, $L$ appears only in the forms $|L|^2$ or $\text{Re}(L^\dagger \bar{L})$, which are unchanged if $L \to |L|$; therefore we may take $L$ to be real. The action we finally obtain is

$$\frac{1}{c} \mathcal{L} = -R J - 2K(\nabla \phi)^2 + \frac{LL'}{2J'} - \frac{KL^2}{2(J')^2},$$

which is almost exactly the action obtained in the $N = 1$ case \[3\]; the only difference is the sign of the first two terms, which is just due to our opposite-signature metric. The stationary points of this action are in one to one correspondence with the bosonic classical solutions of the full action \[3.1\]. It is curious that the bosonic solutions of this $N = 2$ theory are the same as those of the $N = 1$ theory, even though here there is initially an extra bosonic degree of freedom.
4. Superconformal Gauge

In this section we study the theory (3.1) in superconformal gauge in superspace, both because it illuminates the question of non-renormalization of chiral terms, which we wish to understand, and because it is interesting in its own right.

The action (3.1) as it stands is not ready for quantization, since no gauge has been chosen and no superfields have been identified for expansion (curved space chiral superfields such as Φ above are not suitable for quantization, since the chirality condition is a constraint which depends on the supergravity fields.) A good gauge for quantizing bosonic dilaton gravity is conformal gauge (see e.g. [6], $g_{ab} = e^{2\rho} \eta_{ab}$; this completely fixes the gauge freedom, and the fields $\phi$ and $\rho$ are the quantum fields (along with some ghosts). One expands the action in $\phi$ and $\rho$ about the free action and quantizes as usual. In the present case we adopt the analogous solution, choosing a superconformal gauge in superspace, which was shown to exist in [9]. This gauge corresponds to the component field conditions

\[ e^m_a = e^{-\rho} \delta^m_a, \]
\[ \psi^a = \gamma^a \chi, \]
\[ \partial^m A_m = 0, \]

where $\chi$ is a Dirac spinor. Note that the last equation is the integrability condition for the existence of $\alpha$ satisfying $A_m = \epsilon_m^\alpha \partial_n \alpha$. The fields $\rho$, $\chi$, and $\alpha$, plus the complex auxiliary field $B$, have altogether $4 + 4$ real degrees of freedom, suggesting that they might fit into one chiral superfield. So one guesses that in this theory superconformal gauge is realized by a chiral scalar field $\Lambda$, which is the “superconformal factor” extending the ordinary conformal factor $\rho$. Alternatively, one can observe as in [9] that the Bianchi identities were solved in terms of a single chiral superfield $\Sigma$, which must therefore contain all the gauge invariant information in the theory.

The form of the super-Weyl transformations in $N = 2$ superspace was worked out in [9]; in terms of the quantities $H_A^B \equiv E_A^M \delta E_M^B$ they are

\[ H_a^b = (\delta \Lambda + \delta \bar{\Lambda}) \delta_a^b, \]
\[ H_\alpha^\beta = \delta \bar{\Lambda} \delta_\alpha^\beta, \]
\[ H_a^\alpha = i(\gamma_a)^{\alpha\beta} D_\beta \delta \Lambda. \]  

(4.1)

Here $D_\alpha$ is the usual flat superspace covariant derivative $D_\alpha = \partial_\alpha + i \bar{\theta}^\dot{\alpha}(\gamma^a)_{\alpha\dot{\alpha}} \partial_a$. 

6
Already from here one can derive the interesting result that $E \equiv \text{sdet}(E_{M}^{A}) = 1$ in conformal gauge; i.e., the superdeterminant of the vielbein is pure gauge. This follows from $\delta E = E \text{str}(E^{-1} \delta E) = E \text{str}H = 0$ (see e.g. \cite{10}, section 3.7). So the chiral action $\int d^{2}\Theta \Delta c \Sigma$ cannot be written as a full superspace integral $\int d^{2}\theta d^{2}\bar{\theta} E$; in this respect the theory differs from $D = 4$, $N = 1$ supergravity. This leads one to hope that the $S$ term in the action (3.1) will in fact be protected from renormalization. Unfortunately, the conformal gauge analysis doesn’t bear this out.

Working from the variations (4.1), one can construct the superconformal gauge forms of all superfields in the theory. We give the ones we need in the form

$$\nabla_{\alpha} = e^{-\bar{\Lambda}}D_{\alpha} - e^{-\bar{\Lambda}}(\gamma^{3})_{\alpha}^{\beta}D_{\beta}\Lambda(M + iY),$$
$$\bar{\nabla}_{\dot{\alpha}} = e^{-\Lambda}\bar{D}_{\dot{\alpha}} - e^{-\Lambda}(\gamma^{3})_{\dot{\alpha}}^{\dot{\beta}}\bar{D}_{\dot{\beta}}\bar{\Lambda}(M - iY),$$
$$\Sigma = -\frac{1}{2}e^{-2\Lambda}D^{2}\bar{\Lambda}. \tag{4.2}$$

From here one finds $\nabla_{a}$ using the constraint on $\{\nabla_{\alpha}, \bar{\nabla}_{\dot{\alpha}}\}$ (eq. (2.1)); it comes out to

$$\nabla_{a} = e^{-\Lambda - \bar{\Lambda}}\left[\partial_{a} - \frac{i}{2}(\gamma_{a})^{\beta\dot{\beta}}\left(D_{\beta}\Lambda\bar{D}_{\dot{\beta}} + \bar{D}_{\dot{\beta}}\bar{\Lambda}D_{\beta}\right)\right] + \text{connections}, \tag{4.3}$$

and one can go on to deduce the vielbein, inverse vielbein, etc. These equations can be checked by verifying they satisfy the constraints (2.1).

A useful observation is that the form of $\nabla_{\alpha}$ given above, along with $[(M + iY), \psi^{\alpha}] = 0$, implies

$$\nabla^{2}P = e^{-2\bar{\Lambda}}D^{2}P,$$

where $P$ stands for any scalar superfield; and analogously for $\bar{\nabla}^{2}P$.

For our purposes, it is convenient to define the component content of $\Lambda$ by comparing the expressions for $\Sigma$ in (1.2) and in (2.3); this gives

$$\Lambda| = \lambda \equiv \frac{1}{2}(\rho - i\alpha),$$
$$D_{\alpha}\Lambda| = i e^{\bar{\Lambda}}(\gamma^{b})_{\alpha}^{\dot{\alpha}}\bar{\psi}_{b\dot{\alpha}},$$
$$D^{2}\Lambda| = -2\bar{\Lambda}e^{2\bar{\Lambda}}. \tag{4.4}$$

(Note that this is not what one finds by comparing (1.3) and (2.2); the reason is that the computations of section 2 were in WZ gauge, which is not compatible with superconformal gauge.)
In order to write the action (3.1) in conformal gauge, we need the form of \( \Delta_c \). The simplest guess is \( e^{2\Lambda} \), but this doesn’t satisfy \( \Delta_c| = e \). The correct form is

\[
\Delta_c = Fe^{2\Lambda}
\]

with \( F = \exp \left[ 2\bar{\lambda} \left( x^\mu + i\bar{\theta}\gamma^\mu\theta \right) \right] \).

The variable substitution in \( F \) is the familiar transformation leading to the chiral representation in global supersymmetry. It gives \( F \) the convenient properties

\[
F| = e^{2\bar{\lambda}},
\]

\[
\bar{D}_\alpha F = 0,
\]

\[
D_\alpha F| = D^2 F| = 0.
\]

This formula for \( \Delta_c \) can be checked explicitly using the component forms (2.6) and (4.4).

Finally we note using (2.5) that (2.4) may be written in the representation independent form

\[
\mathcal{L} = \nabla^2 \Delta_c \mathcal{L}_{\text{chiral}},
\]

which can be immediately translated into conformal gauge using the above formulas. Applying this procedure to the dilaton gravity action (3.1) yields

\[
\mathcal{L} = -\frac{1}{2} D^2 \bar{D}^2 (\bar{\Lambda} \bar{S})| + D^2 \bar{D}^2 T| + D^2 e^{2\Lambda} U| + \text{h.c.}
\]

\[
= \int d^2\theta d^2\bar{\theta} \left( -\frac{1}{2} \bar{\Lambda} \bar{S} - \frac{1}{2} \Lambda S + T + \bar{T} \right) + \int d^2\theta e^{2\Lambda} U + \int d^2\bar{\theta} e^{2\bar{\Lambda}} \bar{U}.
\]

Note that in computing the component action from this and comparing with (3.3) one must remember that components of \( \Phi \) were defined previously by \( \nabla_\alpha \Phi| \), etc., and are related by factors of \( e^\lambda \), \( e^{\bar{\lambda}} \) to components defined with \( D_\alpha \). To this action must be added of course Fadeev-Popov ghosts; we do not compute those terms here.

An unfortunate feature of this action is that the \( S \) term is now a full superspace integral, i.e. it is not protected from renormalizations as we had hoped. Only the \( U \) term remains chiral, and should remain unaffected by renormalizations.

Note that we cannot get (4.6) by simply \( N = 2 \) supersymmetrizing the bosonic \( \sigma \)-model action,

\[
\mathcal{L} = \int d^2 x \left[ 8\partial_+ \partial_- \rho J(\phi) - 8K(\phi) \partial_+ \phi \partial_- \phi + e^{2\rho} W(\phi) \right]
\]

because the target space \((\rho, \phi)\) does not have a metric with definite sign and we need a Kähler target space to \( N = 2 \) supersymmetrize and Kähler metrics have definite sign.
5. Discussion

As mentioned in the Introduction, quantization of the dilaton gravity action (3.3) is ambiguous because of the infinite number of allowed counter-terms. Our attempted cure is to extend the action to have $N = 2$ supersymmetry where we can write chiral action terms which we hope won’t suffer renormalizations. Unfortunately this program meets with only partial success, since only one term remains chiral when we put the theory in conformal gauge for quantization purposes. But for that term the the global supersymmetry non-renormalization theorem should apply.

An interesting feature of the bosonic action (4.7) is that classically it really only contains one arbitrary function; $J$ may be varied at will, and $K$ may be eliminated, by appropriate (non-derivative) redefinitions of the fields $\phi$ and $\rho$ [12]. One wonders whether this operation can be extended supersymmetrically to the whole theory; from the superconformal gauge action (4.6) one sees that it cannot, since a shift $\Lambda \rightarrow \Lambda + Y(\Phi)$ causes a change $T + \bar{T} \rightarrow T + \bar{T} - \frac{1}{2} \bar{Y}S - \frac{1}{2} Y\bar{S}$, and it is not possible in general to cancel $T$ in this way. The obstacle is the chirality of $\Lambda$ and $\Phi$ (which must be preserved so that the $U$ term remains supersymmetric); this problem is absent in the $N = 1$ theory of [3], so in that theory the field redefinitions should be possible. Furthermore due to supersymmetry there is an extra bonus, namely that such non-derivative transformations produce trivial Jacobian in the functional integral ([10], section 3.8); however, they will alter the measures for the $\Phi$ and ghost path integrals, and will also alter any additional matter action which is coupled to the theory. So their significance is unclear.

Finally we note that in $D = 2$, there is another $N = 2$ superspace geometry [8,9], in which the $U(1)$ action on spinors involves $\delta_\alpha^\beta$ instead of $(\gamma^3)_\alpha^\beta$, and another scalar multiplet (the twisted chiral multiplet). We could have attempted to use a different combination of these than the one we chose; however, it appears that only the fermionic sector would be changed.

Possible directions which are not explored in this paper include further pursuing the quantization of these models, and extending the supersymmetry to $N = 4$.

Acknowledgments

We would like to thank M. Rocek and P. Townsend, for useful conversations; S. J. Gates, for a helpful e-mail comment; and S. Giddings and A. Strominger for discussions and comments on earlier drafts. This work was supported in part by the grants DOE-91ER40618 and NSF PYI-9157463.
References

[1] C.G. Callan, S.B. Giddings, J.A. Harvey, and A. Strominger, “Evanescent Black Holes,” Phys. Rev. D45 (1992) R1005.
[2] For recent reviews see J. A. Harvey and A. Strominger, “Quantum Aspects of Black Holes” preprint EFI-92-41, hep-th@xxx/9209055, to appear in the proceedings of the 1992 TASI Summer School in Boulder, Colorado, and S. B. Giddings, “Toy Models for Black Hole Evaporation” preprint UCSBTH-92-36, hep-th@xxx/9209113, to appear in the proceedings of the International Workshop of Theoretical Physics, 6th Session, June 1992, Erice, Italy.
[3] Youngchul Park and Andrew Strominger, “Supersymmetry and Positive Energy Theorems in Classical and Quantum Two Dimensional Dilaton Gravity,” to appear in Phys. Rev. D, hep-th@xxx/9210017.
[4] S. B. Giddings and W. M. Nelson, “Quantum Emission from Two Dimensional Black Holes”, Phys. Rev. 46D, 2486 (1992).
[5] A. Strominger, “Fadeev-Popov Ghosts and 1 + 1 Dimensional Black Hole Evaporation”, UCSB-TH-92-18, hep-th@xxx/9205028.
[6] S. Giddings and A. Strominger, “Quantum Theories of Dilaton Gravity”, UCSB-TH-92-28, hep-th@xxx/9207034.
[7] Aziz Alnowaiser, “Supergravity with \( N = 2 \) in two dimensions,” Class. Quantum Grav. 7 (1990) 1033.
[8] S. J. Gates Jr., L. Lu, and R. N. Oerter, “Simplified SU(2) Spinning String Superspace Supergravity”, Phys. Lett. 218B, 33 (1989).
[9] P. S. Howe and G. Papadopoulos, “\( N = 2, d = 2 \) Supergeometry”, Class. Quantum Grav. 4, (1987) 11.
[10] S. J. Gates Jr., M. T. Grisaru, M. Rocek and W. Siegel, “Superspace”, The Benjamin/Cummings Publishing Company, Reading (1983).
[11] J. Wess and J. Bagger, “Supersymmetry and Supergravity”, 2nd edition, Princeton University Press, Princeton (1992).
[12] J. G. Russo and A. A. Tseytlin, “Scalar Tensor Quantum Gravity in Two Dimensions”, Nucl. Phys. 382B, 259 (1992).