Einstein gravity as an emergent phenomenon?

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Abstract

In this essay we marshal evidence suggesting that Einstein gravity may be an emergent phenomenon, one that is not “fundamental” but rather is an almost automatic low-energy long-distance consequence of a wide class of theories. Specifically, the emergence of a curved spacetime “effective Lorentzian geometry” is a common generic result of linearizing a classical scalar field theory around some non-trivial background. This explains why so many different “analog models” of general relativity have recently been developed based on condensed matter physics; there is something more fundamental going on. Upon quantizing the linearized fluctuations around this background geometry, the one-loop effective action is guaranteed to contain a term proportional to the Einstein–Hilbert action of general relativity, suggesting that while classical physics is responsible for generating an “effective geometry”, quantum physics can be argued to induce an “effective dynamics”. This physical picture suggests that Einstein gravity is an emergent low-energy long-distance phenomenon that is insensitive to the details of the high-energy short-distance physics.

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1 Analog gravity

There is a risk that all current attempts at “quantizing gravity” are condemned to failure because they have been starting from fundamentally flawed premise, and that in reality there are no fundamental gravitational degrees of freedom to quantize — it is possible that Einstein gravity is an emergent phenomenon, in the same sense that fluid dynamics emerges from molecular physics as a low-momentum long-distance approximation. In this essay we will take a careful look at this idea, and highlight some of the possibilities, problems, and opportunities that such a situation entails.

We were led to these notions via current research on analog models of general relativity. Because of the extreme difficulty (and inadvisability) of working with intense gravitational fields in a laboratory setting, interest has now turned to investigating the possibility of simulating aspects of general relativity — though it is not a priori expected that all features of Einstein gravity can successfully be carried over to the analog models. Numerous rather different physical systems have now been seen to be useful for developing analog models of general relativity. A literature search as of March 2001 finds well over a hundred scientific articles devoted to one or another aspect of analog gravity and effective metric techniques. The sheer number of different physical situations lending themselves to an “effective metric” description strongly suggests that there is something deep and fundamental going on.

Typically these are models of general relativity, in the sense that they provide an effective metric and so generate the basic kinematical background in which general relativity resides; in the absence of any dynamics for that effective metric we cannot really speak about these systems as models for general relativity. However, as we will discuss more fully below, quantum effects in these analog models might provide of some sort of dynamics resembling general relativity.

Remember that for mechanical systems with a finite number of degrees of freedom small oscillations can always be resolved into normal modes: a finite collection of uncoupled harmonic oscillators. For a classical field theory you would also expect similar behaviour: small deviations from a background solution of the field equations will be resolved into travelling waves; then these travelling waves can be viewed as an infinite collection of harmonic oscillators, or a finite number if the field theory is truncated in the infra-red and ultra-violet, to which you can then apply a normal mode analysis. The
physically interesting question is whether this normal mode analysis for field theories can then be reinterpreted in a “geometrically clean” way in terms of some “effective metric” and “effective geometry”. In many cases the answer is definitely yes: Linearization of a Lagrangian-based dynamics, or linearization of any hyperbolic second-order PDE, will automatically lead to an effective Lorentzian geometry that governs the propagation of the fluctuations. [The most general situation (multiple scalar fields, or a multi-component vector or tensor) is quite algebraically messy — details of that situation will be deferred for now.]

Once the notion of a derived “effective metric” has been established, we can certainly consider the effect of quantizing the linearized fluctuations. At one loop the quantum effective action will contain a term proportional to the Einstein–Hilbert action — this is a key portion of Sakharov’s “induced gravity” idea [2]. In the closing segment of the essay we argue that the occurrence of not just an “effective metric”, but also an “effective geometrodynamics” closely related to Einstein gravity, is a largely unavoidable feature of the linearization and quantization process.

2 Effective metric

Suppose we have a single scalar field $\phi$ whose dynamics is governed by some first-order Lagrangian $L(\partial_\mu \phi, \phi)$. (By “first-order” we mean that the Lagrangian is some arbitrary function of the field and its first derivatives.) We want to consider linearized fluctuations around some background solution $\phi_0(t, \vec{x})$ of the equations of motion, and to this end we write

$$\phi(t, \vec{x}) = \phi_0(t, \vec{x}) + \epsilon \phi_1(t, \vec{x}) + O(\epsilon^2).$$  

(1)

Linearizing the Euler–Lagrange equations results in a second-order differential equation with position-dependent coefficients (these coefficients all being implicit functions of the background field $\phi_0$). Following an analysis developed for acoustic geometries (Unruh [3], Visser et al [4]), which also applies to this much more general situation, this can be given a nice clean geometrical interpretation in terms of a d’Alembertian wave equation — provided we define the effective spacetime metric by

$$\sqrt{-g} \ g^{\mu \nu} \equiv f^{\mu \nu} \equiv \left\{ \frac{\partial^2 L}{\partial (\partial_{\mu} \phi) \partial (\partial_{\nu} \phi)} \right\}_{\phi_0}.$$  

(2)
that is,
\[
g_{\mu\nu}(\phi_0) = \left( -\det \left\{ \frac{\partial^2 \mathcal{L}}{\partial (\partial_{\mu} \phi) \partial (\partial_{\nu} \phi)} \right\} \right)^{1/(d-1)} \left\{ \frac{\partial^2 \mathcal{L}}{\partial (\partial_{\mu} \phi) \partial (\partial_{\nu} \phi)} \right\}^{-1} \bigg|_{\phi_0} \bigg|_{\phi_0} \]. \quad (3)

The equation of motion for the linearized fluctuations can then be written in the geometrical form
\[
[\Delta(g(\phi_0)) - V(\phi_0)] \phi_1 = 0, \quad (4)
\]
where \(\Delta\) is the d’Alembertian operator associated with the effective metric \(g(\phi_0)\), and \(V(\phi_0)\) is a background-field-dependent potential. It is important to realise just how general the result is: it works for any Lagrangian depending only on a single scalar field and its first derivatives. The linearized PDE will be \textit{hyperbolic} (and so the linearized equations will have wave-like solutions) if and only if the effective metric \(g_{\mu\nu}\) has Lorentzian signature \([-\, (+)^d]\). Note that \(d = 1\) space dimensions is special, and the present formulation does not work unless \(\det(f^{\mu\nu}) = 1\). This observation can be traced back to the conformal covariance of the Laplacian in \(1+1\) dimensions, and implies (perhaps ironically) that the only time the procedure risks failure is when considering a field theory defined on the world sheet of a string-like object.

Indeed, even if you do not have a Lagrangian, it is still possible to extract an “effective metric” for a system with one degree of freedom. (More precisely, we can define a conformal class of effective metrics. The analysis is not as geometrically “clean”.) While several of the technical details are different from the Lagrangian-based analysis, the basic flavor is the same: The key point is that hyperbolicity of the linearized PDE is defined in terms of the presence of a matrix of indefinite signature \([-\, (+)^d]\). This matrix is enough to define a conformal class of Lorentzian metrics, and picking the “right” member of the conformal class is largely a matter of taste — do whatever makes the “geometrized” equation look cleanest.

### 3 Effective dynamics

At this stage we have derived the existence of a background metric \(g_{\mu\nu}(\phi_0)\) and linearized fluctuations governed by the equation \((4)\). We shall now try to see how and to what extent it is possible to define a \textit{dynamics} for this
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metric. Of course it would be not particularly useful to search for such dynamics in the equations of motion of the “fundamental system” [those derived from \( \mathcal{L}(\phi, \partial \phi) \)]. It is from this perspective obvious that the dynamical equations of the effective metric should also be regarded as an “emergent” phenomenon. The idea of “induced gravity”, proposed several years ago by Andrei Sakharov [2], provides a natural framework for such an emergence of an effective geometrodynamics. Along these lines we shall now try to derive Einstein-like equations from the one-loop action of the \( \phi \) field.

Expand the field \( \phi \) into a background \( \phi_b \), which does not necessarily satisfy any classical equation of motion, and a quantum fluctuation \( \phi_q \) so that \( \phi = \phi_b + \phi_q \). Integrate out the quantum fluctuations; then at one loop

\[
\Gamma[g(\phi_b), \phi_b] = S[\phi_b] + \frac{1}{2} \hbar \text{tr} \ln \left[ \Delta(g(\phi_b)) - V(\phi_b) \right] + O(\hbar^2). \tag{5}
\]

Here the determinant of the differential operator may be defined in terms of zeta functions or heat kernel expansions [3]. Note also that the effective action depends on the background field in two ways: explicitly through \( \phi_b \), and implicitly through \( g(\phi_b) \). The key point is that defining the determinant requires both regularization and renormalization, and that doing so introduces counterterms proportional to the first \( d/2 \) Seeley-DeWitt coefficients [3]. The zeroth Seeley–DeWitt coefficient \( a_0 \) induces a cosmological constant, while \( a_1 \) induces an Einstein–Hilbert term, and there are additional terms proportional to \( a_2 \). All in all:

\[
\Gamma[g(\phi_b), \phi_b] = S[\phi_b] + \hbar \int \sqrt{-g} \kappa \left[ -2\Lambda + R(g) - 6V(\phi_b) \right] d^{d+1}x \\
+ \hbar X[g(\phi_b), \phi_b] + O(\hbar^2). \tag{6}
\]

Here \( X[g(\phi_b), \phi_b] \) denotes all other finite contributions to the renormalized one-loop effective action. It is the automatic emergence of the Einstein–Hilbert action as part of the one-loop effective action that is the salient point. Note that our approach is not identical to Sakharov’s idea — in his proposal the metric was put in by fiat, but without any intrinsic dynamics; all the dynamics was generated via one loop quantum effects. In our proposal the very existence of the effective metric itself is an emergent phenomenon. In Sakharov’s approach the metric was free to be varied at will, leading precisely to the Einstein equations (plus quantum corrections); in our approach the metric is not a free variable and the equations of motion will be a little trickier.
The quantum equations of motion are defined in the usual way by varying \( \Gamma[\phi_b] \) with respect to the background field \( \phi_b \). It is important to remember that the metric is a function of the background field so that it does not make sense to vary the metric independently — we must always evaluate variations using the chain rule.

\[
\frac{\delta \Gamma[g(\phi_b), \phi_b]}{\delta \phi_b(x)} = \left. \frac{\delta \Gamma[g(\phi_b), \phi_b]}{\delta \phi_b(x)} \right|_{g_b} + \left. \frac{\delta \Gamma[g(\phi_0), \phi_0]}{\delta g_{\mu\nu}} \right|_{\phi_b} \frac{\delta g_{\mu\nu}(\phi_b)}{\delta \phi_b(x)}. \tag{7}
\]

In this way we see that the equations of motion have a part coming from the variation with respect to the background field plus a part proportional to the variation with respect to the metric. It is this second part which provides an Einstein-like dynamics. The presence of the terms generated by variation with respect to \( \phi_b \) leads to the interesting conclusion that one needs to assume, in order to get a dynamics as close as possible to that of Einstein, that these terms satisfy a special constraint: there should exist a functional \( Y[g] \), depending only on the effective metric, such that

\[
\begin{align*}
\left\{ \frac{\delta S[\phi_b]}{\delta \phi_b} - 6\hbar \kappa \frac{\delta f \sqrt{-g} V(\phi_b)}{\delta \phi_b} + \hbar \left. \frac{\delta X[g(\phi_b), \phi_b]}{\delta \phi_b} \right|_{g_b} \right\} &= \hbar \delta Y[g] \frac{\delta g_{\mu\nu}(\phi_b)}{\delta \phi_b(x)} \\
&+ O(\hbar^2). \tag{8}
\end{align*}
\]

In this case the background geometry decouples from the effective metric and we have

\[
\kappa (G^{\mu\nu}(g) + \Lambda g^{\mu\nu}) + \frac{1}{\sqrt{g}} \frac{\delta \{X[g(\phi_b)] + Y[g(\phi_b)]\}}{\delta g_{\mu\nu}} \frac{\delta g_{\mu\nu}(\phi_b)}{\delta \phi_b(x)} = O(\hbar). \tag{9}
\]

The \( \delta X[g(\phi_b)]/\delta g \) term denotes the type of “curvature squared” correction to the Einstein equations that is commonly encountered in string theory (indeed in almost any candidate theory for quantum gravity), and also in the usual implementation of Sakharov’s approach. However it must be emphasised that because of the contraction with the \( \delta g_{\mu\nu}(\phi_b)/\delta \phi_b(x) \) these are not the usual Einstein equations, though they are certainly implied by the (curvature enhanced) Einstein equations. It is in this sense that we can begin to see the structure of Einstein gravity emerging from this field-theoretic normal mode analysis.
4 Prospects

In this essay we have argued that the emergence of an “effective metric”, in the sense that this notion is used in the so-called “analog models” of general relativity, is a rather generic feature of the linearization process. While the existence of an effective metric by itself does not allow you to simulate all of Einstein gravity, it allows one to do quite enough to be really significant — in particular it seems that the existence of an effective Lorentzian metric is really all that is in principle needed to obtain simulations of the Hawking radiation effect

By invoking one-loop quantum effects, we have argued that something akin to Sakharov’s induced gravity scenario is operative: in particular we can generically argue that there is a term in the quantum effective action proportional to the Einstein–Hilbert action. However because of the technical assumption that the effective metric depends on the background only via the scalar field $\phi_0(x)$ we have not been able to reproduce full Einstein gravity, though certainly have some extremely suggestive results along this line.

The major steps that are needed to extend this idea to a full fledged theory are

1. The question of what happens when many fields are present in the problem: the major piece of additional physics is the possible presence of birefringence, or more generally “multi-refringence”, with different normal modes possibly reacting to different metrics. The Eötvös experiment [the observational universality of free fall to extremely high accuracy] indicates that all the physical fields coupling to ordinary bulk matter “see” to high precision the same metric, allowing us to formulate the Einstein Equivalence principle and speak of the metric of spacetime.

2. Whether the addition of extra fields helps one to obtain a better approximation to full Einstein gravity — this because you would get one equation of motion per background field, so with six or more fields you would expect to be able to explore the full algebraic structure of the metric. So adding extra fields, which is technically a hindrance in the kinematical part of the program (developing the effective metric formalism), should in compensation allow one to more closely approach the dynamics of Einstein gravity.

In summary: The full generality of the situations under which effective
metrics are encountered is truly remarkable, and the extent to which the resulting analog models seem able to reproduce key aspects of Einstein gravity is even more remarkable. The physics of these systems is fascinating, and the potential for laboratory investigation of models close to (but not necessarily identical to) Einstein gravity is extremely encouraging.

Our interpretation of these results is that they provide suggestive evidence that what we call Einstein gravity (general relativity) is an almost automatic low-energy consequence of almost any well behaved quantum field theory: the occurrence of an effective metric is almost automatic (even in the classical theory), while the presence of Einstein-like dynamics is almost guaranteed by one-loop quantum effects.

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