Origin and formation process of a submerged vortex in a pump sump

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Abstract. The origin and formation process of a submerged vortex have been fully clarified by large-eddy simulation (LES) that used approximately of 250 million hexahedral elements with maximum resolution of 0.450 mm and was applied to the internal flows of a model pump sump. The model pump sump is composed of a 2,500 mm-long water channel of rectangular cross section with a width of 300 mm and a water depth of 100 mm and a vertical suction pipe with a 100 mm diameter installed at its downstream end. At the upstream end of the channel, a uniform velocity of 0.37 m/s is given. LES with different wall boundary conditions have revealed that the origin of a submerged vortex is the mean shear of the approaching boundary layers that develop on the bottom and side walls of the pump sump. Detailed investigations of LES computed for a long time period of 16 seconds have revealed that deviation of the mean flow that approaches the suction pipe triggers conversion of the axis of the vorticity that was originally aligned to the lateral direction in the approaching boundary layers to that aligned to the vertical direction. The local acceleration of the vertical flow stretches the afore-mentioned vertical vortex, which results in the formation of a strong submerged vortex. A computation with a different height of the vermouth inlet has supported the above mentioned formation process of a submerged vortex.

1. Introduction

Submerged vortices and air-entrained vortices (hereafter, they are referred to as suction vortices) that appear in a pump sump may cause noise and vibrations, and at the worst case, damages of the pump system. Therefore, at an early stage of its design, occurrence of the suction vortices has to be taken into account for a pump sump. Occurrence of suction vortices have been conventionally evaluated by a model-sump experiment. By recent speedup of high-end computers, computational methods have also been applied to the prediction of the suction vortices of a pump sump. In particular, Reynolds Averaged Navier-Stokes simulation (RANS), which is a method on a time-average basis, has been mainly adopted [1-4]. In such computations based on RANS, occurrence of suction vortices is judged typically by visualizing streamlines, distribution of the static pressure and/or that of the vorticity for the computed steady flow field. Ansar et al measured the velocity fields in a test pump sump by laser Doppler
velocimeter (LDV) [1] and they showed that a potential-flow analysis could predict the flow patterns in the test pump sump [2]. Constantinescu et al investigated accuracy of the flow fields in a test pump sump predicted by several RANS models all based on two equations [3]. Okamoto et al performed benchmark tests by experiments and computational fluid dynamics (CFD) methods. Figure 1 shows the model pump sump and suction vortices visualized by their experiments. By changing the water speed (inflow velocity) and the water depth, they investigated those conditions under which suction vortices took place, and evaluated capability of the CFD methods based on RANS to predict suction vortices. Figure 2 summaries the conditions under which suction vortices took place in terms of the water depth and water speed.

A time-averaged method may become a useful tool for evaluating a possibility of suction-vortices occurrence because it can compute relatively large-scale flow structures that depend on the geometries of a pump sump and occurrence of suction vortices are presumably and fundamentally determined by such flow structures. However, it is less likely that a time-averaged CFD method to be able to accurately predict suction vortices, which are essentially in unsteady motion, and it is obvious that appearances nor disappearances of suction vortices cannot be predicted by such a method. In particular, neither the fundamental origin of suction vortices nor their formation/dissipation processes have been clarified in the previous reports.

The objectives of the present study are to identify the fundamental origins of suction vortices and to clarify the formation processes of such vortices. To this end, wall-resolved Large Eddy Simulation that used up to two billion grids [5, 6] was applied to the computation of internal flows in a test pump sump. In particular, case studies that prescribed different wall boundary conditions were made to identify the origin of a submerged vortex and a long period of LES computation was preformed to see appearance and disappearance of suction vortices to investigate the formation and dissipation processes of a submerged vortex. Although the origin and formation process of an air-entrained vortex have also been clarified, this paper will focus only on those of a submerged vortex due to the limit space available. The results of investigations on the air-entrained vortex will be presented elsewhere.

![Figure 1](image1.jpg)  
Figure 1 Model pump sump and visualized submerged and air-entrained vortex (Okamoto, et al., 2007)

![Figure 2](image2.jpg)  
Figure 2 Suction vortices generated region in terms of flow-rate and water depth (Okamoto, et al., 2007)
2. Test Pump Sump

The model pump sump that Okamoto et al examined [4] was set as the benchmark case in the present study. Table 1 shows the specifications of the model pump sump and its operation conditions studied in the present paper. Among the cases studied by Okamoto et al [4], the operation condition with the inlet velocity of 0.37 m/s and the water depth of 150 mm was selected, for which both submerged and air-entrained vortices are confirmed to take place in their experiment [4]. In addition to this case, an LES with a different height of the vermouthe inlet was performed in order to confirm the formation process of a submerged vortex, proposed in this study.

Table 1 Specifications of the model pump sump

| Specification              | Value                        |
|---------------------------|------------------------------|
| Diameter of the inlet pipe| 100 mm                       |
| Height of the vermouthe    | 100 mm                       |
| Water depth                | 150 mm                       |
| Inlet velocity             | 0.37 m/s                     |

3. Numerical method

LES of the incompressible fluid flows in the model pump sump was performed by FrontFlow/blue (FFB) flow solver [7, 8]. This flow solver is an in-house-made general-purpose LES code based on a finite element method (FEM), which enables accurate predictions of turbulent flows by directly computing dynamics of streamwise vortices in the turbulent boundary layers. Dynamic Smagorinsky Model (DSM) [9, 10] is adopted as the subgrid-scale model of LES. A large-scale LES of various industrial flows, such as turbomachinery internal flows [5, 6, 11, 12], automobile flows [13, 14], and hull boundary layers for ship hydrodynamics [15], can be performed with up to 100 billion grids by using a unique function implemented in this code to automatically refine the computational grids at run time [16].

4. Computational conditions

4.1 Computational model

To identify the fundamental origin of a submerged vortex and to clarify its formation process, the internal flows in the model pump sump shown in Figure 3 were computed by wall-resolved LES. The x-axis is set in the upstream direction with its origin placed at the projected center of the intake pipe, the y-axis is set in the lateral direction, and the z-axis is set in the vertical direction with its origin placed on the bottom wall of the pump sump. The inlet boundary is located at X = 2,496 mm where a uniform velocity of 0.37 m/s without freestream turbulence is given. The outlet boundary is set at the exit of the intake pipe located at Z = 2,000 mm where the fluid traction is assumed to be zero and the static pressure is assumed to be the atmospheric pressure. At all the other boundaries of the computational domain, such as the bottom wall, the side wall, inner and outer walls of the vermouthe and intake pipe, the non-slip boundary condition is given. To prompt transition of the boundary layers on the bottom as well as side walls of the model pump sump, a small step with a height of 5 mm and a streamwise length of 5 mm is implemented at 300 mm downstream of the inlet boundary as shown in Figure 4.

4.2 Grid resolutions and Computed cases

In our previous study [5], the effects of the grid resolutions were first investigated by comparing the results computed by two computational grids with different grid resolutions: one with a horizontal
resolution of 0.225 mm, resulting in a total number of the computational grids of 2.05 billion, the other with a horizontal resolution of 0.450 mm, resulting in that of 250 million. The former grid is expected to fully resolve the streamwise vortices in the turbulent boundary layers on the bottom and side walls of the pump sump. As a result, no essential difference between the two cases were confirmed in terms at least of the formation and dissipation of submerged vortices nor those of air-entained vortices. Therefore, those results computed only by the coarser grids will be presented in this paper.

To identify the fundamental origin of a submerged vortex, three different boundary conditions were given at the bottom and side walls of the pump sump: turbulent boundary layers (referred to as Case A), laminar boundary layers (referred to as Case B), no boundary layer (referred to as Case C). The underlining assumptions for setting these boundary conditions are explained below. If their origin was the streamwise vortices in the turbulent boundary layers, submerged vortices should appear only in Case A. If it was the mean shear of the approaching boundary layers, submerged vortices should appear both in Case A and Case B. If it was the global shear caused by the asymmetry of the model pump sump, submerged vortices should appear in all cases. Note that the intake pipe is offset to the center line of the pump sump as shown in Figure 3 (c). The computation for Case A was performed as already described. Streamwise vortices, typical of a turbulent boundary layer, were formed near the bottom and side walls of the model pump sump. The computation for Case B was performed by removing the step implemented at 300 mm downstream of the inlet boundary. In this case, the velocity profiles of the approaching boundary layers were close to the Blasius solution although they started to deviated from it near the projected center of the vermouth due to the adverse pressure gradient caused by the upward flow directed to the inlet of the vermouth. The computation for Case C was done by setting the moving-wall boundary condition at the bottom and side walls with the moving velocity identical to the inlet flow velocity. For this case, the small step was also removed. Table 2 presents the cases computed in the present study. Case D in table 2 was done to confirm the formation process that has been clarified in the present study and it will be described later in detail. The computation for Case A was continued for a long period of 16 seconds to see formation/dissipation of the suction vortices.
Table 2 Computed cases

| Case Name | Num. of grids | Grid resolution | Approaching B.L. | h₁ [mm] | Cal. time |
|-----------|---------------|-----------------|------------------|---------|-----------|
| Case A    | 250 million   | 0.45 mm         | Turbulent        | 100     | 16.00     |
| Case B    | 250 million   | 0.45 mm         | Laminar          | 100     | 3.20      |
| Case C    | 250 million   | 0.45 mm         | No B.L. (moving wall) | 100 | 3.20 |
| Case D    | 250 million   | 0.45 mm         | Turbulent        | 150     | 0.65      |

5. Results

5.1 Origin of a submerged vortex

Figure 5 shows typical instantaneous flow fields computed by Cases A, B and C, respectively, where suction vortices are visualized by contour surfaces of the Laplacian of the static pressure colored by the vertical vorticity. In order to judge formation of a submerged vortex, we used the Laplacian of static pressure which corresponds to the second invariant in a high Reynolds number flow and therefore presents vortex structure. No submerged vortex appeared in Case C throughout the entire computational time of 3.2 second, which corresponded to 12 non-dimensional time normalized by the inlet velocity and the vermouth height of the model pump sump. On the other hand, submerged vortices appeared and existed at least for a certain period of time in Case A and Case B. It is therefore concluded that the vorticity in the mean shear of the approaching boundary layers, with its axis aligned to the lateral direction of the pump sump, is the fundamental origin of a submerged vortex, and that the streamwise vortices in the approaching turbulent boundary layers play no fundamental role for the formation of a submerged vortex. In all the cases, air-entrained vortices appeared as shown in this figure because their origin is the vorticity formed in the separated flow in the wake of the intake pipe and it has nothing to do with the approaching boundary layers, as will be presented elsewhere in detail.

![Instantaneous flow fields visualized by iso-surfaces of Laplacian of static pressure colored by vertical vorticity computed for Case A, Case B and Case C](image)

5.2 Formation process of a submerged vortex

As identified in the previous subsection, the fundamental origin of a submerged vortex is the vorticity with its axis aligned in the lateral direction of the pump sump in the mean shear in the approaching boundary layers on the bottom and side walls. Unsteady effects of the approaching flow is not important because once generated, a submerged vortex existed stably for a certain period of time. On the other hand, for a submerged vortex to be formed, the vorticity with its axis originally aligned in the lateral direction of the pump sump, $\omega_y$, has to change its direction to the vertical direction, $\omega_z$. In addition, the vortex with its axis aligned to the vertical direction, hereafter referred to as “vertical vortex”, has to be stretched for a strong submerged vortex to be formed. In this subsection, formation process of a submerged vortex will be investigated in detail. To this end, the flow fields computed in Case A for a long period of time were analysed based on the time-averaged vorticity-transport equation shown below.
\[(\vec{u} \cdot \nabla)\vec{\omega} = (\vec{\omega} \cdot \nabla)\vec{u} + \nu \nabla^2 \vec{\omega} \]  

(1)

where \(\vec{u}\) and \(\vec{\omega}\) are, respectively, the time-averaged velocity and vorticity vectors. The left hand side of equation (1) represents the change in the vorticity along a streamline. The first term in the right hand side in equation (1) represents generation and/or destruction of the vorticity while the second term represents the diffusion of the vorticity due to viscous effects. As mentioned above, directional change of the axis of the vorticity vector from the lateral direction to the vertical direction is needed for a submerged vortex to be formed. The diffusion term will be neglected in this analysis because the diffusion of the vorticity vector does not cause the directional change of the axis of the vorticity vector. Z-component of the equation (1) without the diffusion term can be written as follows;

\[
\begin{align*}
&\frac{u}{u} \frac{\partial \omega_z}{\partial x} + v \frac{\partial \omega_z}{\partial y} + w \frac{\partial \omega_z}{\partial z} = \omega_x \frac{\partial u}{\partial x} + \omega_y \frac{\partial u}{\partial y} + \omega_z \frac{\partial u}{\partial z} \\
&\frac{u}{u} \frac{\partial \omega_z}{\partial x} + v \frac{\partial \omega_z}{\partial y} + w \frac{\partial \omega_z}{\partial z} = \omega_x \frac{\partial v}{\partial x} + \omega_y \frac{\partial v}{\partial y} + \omega_z \frac{\partial v}{\partial z} \\
&\frac{u}{u} \frac{\partial \omega_z}{\partial x} + v \frac{\partial \omega_z}{\partial y} + w \frac{\partial \omega_z}{\partial z} = \omega_x \frac{\partial w}{\partial x} + \omega_y \frac{\partial w}{\partial y} + \omega_z \frac{\partial w}{\partial z} \\
\end{align*}
\]

(2)

where \(u\), \(v\) and \(w\) are, respectively, the streamwise (X), spanwise (Y) and vertical (Z) velocity components. During the long period of the entire computation, a submerged vortex existed stably for 0.8 seconds. Each term in the right hand side of equation (2) represents the source of the vertical vorticity, and they are visualized in Figure 6 where distributions of each term averaged over the 0.8 seconds on the cross section of a height of 0.1 mm are shown. Hereafter, the vorticity and the velocity gradient are normalized by the width of the pump sump of 300 mm and the inlet velocity of 0.37 m/s.

The vorticity components \(\omega_x\) and \(\omega_y\) indicate that there is radial vorticity around the projected center of the vermouth (hereafter, simply referred to as “the vermouth center”). On the other hand, the velocity gradients \(\partial w/\partial x\) and \(\partial w/\partial y\) indicate that the vertical velocity \(w\) increases to the vermouth center. Note that X-axis is directed from the projected center of the vermouth to the inlet boundary, and therefore, the main-stream velocity is negative in this computational model. From these results, directional change and stretching of the vortex takes place in the following manner. Firstly, the vorticity \(\omega_y\) which is originally in the approaching boundary layers changes its direction to the radial direction due to the incoming flow rotating around the vermouth center. Secondary, the radial vorticity \(\omega_r\) changes its direction to the vertical direction due to the radial gradient of the vertical velocity \(\partial w/\partial r\). Finally, the vertical vorticity \(\omega_z\) increases due to the stretching of vortex by the vertical velocity gradient \(\partial w/\partial z\).

The reason for the directional change of the lateral vorticity \(\omega_y\) in the approaching boundary layer is investigated further in detail. As shown in Figure 6, streamwise vorticity \(\omega_x\) becomes large at the right side (viewed from the upstream) of the vermouth center. To understand the reason for this large \(\omega_x\), relationship between \(\omega_x\) and \(\omega_y\) in the approaching boundary layer is investigated. Their relationship is represented by the X-component of equation (1) with the diffusion term neglected as previously;

\[
\begin{align*}
&\frac{u}{u} \frac{\partial \omega_x}{\partial x} + v \frac{\partial \omega_x}{\partial y} + w \frac{\partial \omega_x}{\partial z} = \omega_x \frac{\partial u}{\partial x} + \omega_y \frac{\partial u}{\partial y} + \omega_z \frac{\partial u}{\partial z} \\
&\frac{u}{u} \frac{\partial \omega_x}{\partial x} + v \frac{\partial \omega_x}{\partial y} + w \frac{\partial \omega_x}{\partial z} = \omega_x \frac{\partial v}{\partial x} + \omega_y \frac{\partial v}{\partial y} + \omega_z \frac{\partial v}{\partial z} \\
&\frac{u}{u} \frac{\partial \omega_x}{\partial x} + v \frac{\partial \omega_x}{\partial y} + w \frac{\partial \omega_x}{\partial z} = \omega_x \frac{\partial w}{\partial x} + \omega_y \frac{\partial w}{\partial y} + \omega_z \frac{\partial w}{\partial z} \\
\end{align*}
\]

(3)

Since \(\omega_x\) and \(\omega_y\) are small in the approaching boundary layer, the first term and third term in the right hand side of equation (3) can be neglected. The second term in the right hand side of equation (3) consists of lateral vorticity \(\omega_y\) and the lateral gradient of the streamwise velocity \(\partial u/\partial y\). Figure 7 shows \(\omega_y\) and \(\partial u/\partial y\), which are time-averaged during the period from 0.0 second to 0.8 second on the cross section at a height of 0.1 mm as in Figure 6. The visualized zones in Figure 6 are larger than those in Figure 7.
to clearly visualize the change in vorticity $\omega_y$ in the approaching boundary layer. The time period for the averaging and the height of the cross section are same in Figures 6 and 7.

Since the approaching boundary layer is turbulent, streaky structures can be seen in the distribution of velocity gradient $\partial u/\partial y$ (right in Figure 7) while it becomes positive at the right side of the vermouth center as shown by the square in Figure 7. This indicates that the incoming flow changes its direction to negative Y, and therefore the mainstream velocity is larger in the negative Y region and smaller in the positive Y region. Streamwise vorticity $\omega_x$ is generated there (right side of the vermouth center) due to the term $\omega_y \times \partial u/\partial x$ in equation (3). Generation of positive $\omega_x$ at the right side of the vermouth center was explained by the equation (3). The other vorticity components, such as positive $\omega_y$ at the upper side, negative $\omega_x$ at the left side, and negative $\omega_y$ at the lower side of the vermouth center can also be explained by the incoming flow rotating around the vermouth center.

Figure 6 Distributions of vorticity components ($\omega_i$) and gradients of axial velocity ($\partial w/\partial x$) on cross section with a height of 1mm, averaged during 0 sec and 0.8 sec when a clock-wise submerged vortex stably existed

Figure 7 Distributions of vorticity components $\omega_x, \omega_y$ and gradient of mainstream velocity ($\partial u/\partial y$) on cross section with a height of 1mm, averaged during 0 sec and 0.8 sec when a clock-wise submerged vortex stably existed

Temporal evolution of vertical vorticity $\omega_z$ and velocity gradient $\partial w/\partial z$ near the vermouth when a submerged vortex is being formed are investigated to confirm the above-mentioned formation process.
of a submerged vortex. Figure 8 shows temporal evolution of short-time-averaged $\omega_z$ and $\partial w/\partial z$ on the cross section at a height of 0.1 mm. Isosurface of $\omega_z$ is also visualized in order to judge formation of a submerged vortex. A submerged vortex is not formed during the time period from 4.8 second to 5.6 second (left in Figure 8). But, it starts to be formed during the time period from 5.6 second to 6.4 second (center in Figure 8) and becomes stronger during the time period from 6.4 second to 7.2 second (right in Figure 8). Vertical vorticity $\omega_z$ and velocity gradient $\partial w/\partial z$ become larger when the submerged vortex is formed. This is consistent with the formation process mentioned above.

Figure 8 Temporal evolution of time-averaged $\omega_z$ (top) and $\partial w/\partial z$ (bottom) on cross section with a height of 1 mm during $t = 4.8 \text{ sec}$ to $t = 7.2 \text{ sec}$

5.3 Effects of vermouth height on the formation of a submerged vortex

To confirm the formation process of a submerged vortex clarified in the previous subsection, computation of the flow in a model pump sump with a modified geometry, referred to as Case D, was performed. In the modified model pump sump, the vermouth height $h_1$ (see (b) in Figure 3) was changed from its original height of 100 mm to 150 mm. The vertical acceleration of the flow becomes smaller near the bottom wall due to a longer distance between the bottom wall and the inlet of the vermouth. As a result, the stretch of the vertical vortex, which was formed near the bottom wall due to the change of the axis of the vorticity detailed in the previous subsection, becomes weaker. A submerged vortex is therefore less likely to appear in Case D. As expected, no submerged vortex appeared in Case D for the time period between 0 second to 0.65 second as shown in Figure 9.

Figure 10 compares the vertical vorticity $\omega_z$ and the velocity gradient $\partial w/\partial z$ on the cross section at a height of 1 mm together with iso-surfaces of the vertical vorticity $\omega_z$ computed for Case A and Case D. Note that the periods during which the time average was taken were different for Case A and Case D: flow fields were averaged during the period from 6.4 second to 7.2 second for Case A while they are averaged during the period from 0.34 second 0.64 second for Case D. As mentioned in the previous subsection, strong acceleration in the vertical velocity $\partial w/\partial z$ is observed near the bottom wall at the projected center of the vermouth in Case A while no such flow structure is observed in Case D. As compared to Case A, the vertical-velocity gradient $\partial w/\partial z$ became 1.5 times smaller in Case D due to the 1.5 times longer distance between the inlet of the vermouth and the bottom wall. As a result, the stretch of the vertical vortex formed near the bottom wall becomes weaker and no submerged vortex was formed in Case D.
The new findings on the fundamental origin and the formation process of a submerged vortex clarified and presented in the present investigation will serve as a basis for designing a better pump sump. For instance, it is presumably beneficial to take a longer distance between the inlet of the vermouth and the bottom wall only under the vermouth. Longitudinal slits implemented in the approaching boundary layers is also likely to lower the possibility of the submerged-vortex occurrence because they presumably prevent the directional change of vorticity originally sitting in the mean shear of the approaching boundary layers.

![Figure 9 Instantaneous flow fields visualized by iso-surfaces of Laplacian of static pressure colored by vertical vorticity computed for Case A and Case D](image)

![Figure 10 Time-averaged $\omega_z$ and $\partial w/\partial z$ on cross section with a height of 1mm for Case A and Case D](image)

### 6. Conclusions

To identify the origin of the suction vortices that take place in a pump sump, wall-resolved LES was performed with three different wall boundary conditions for the approaching boundary layers. In addition, the computation was continued for a long time period and the formation and the dissipation of the suction vortices were observed several times. The temporal change in the computed flow fields, which led to formation of a submerged vortex, was analysed based on the vorticity transport equations. Finally, to confirm the clarified formation process of a submerged vortex, a computation, for which the vermouth was set at a longer distance from the bottom wall, was performed. The following conclusions have been drawn from the present numerical investigations.
1) The fundamental origin of a submerged vortex is the vorticity in the mean shear of the approaching boundary layers, with its axis aligned to the lateral direction of the pump sump.

2) A submerged vortex is formed in the following process: the original vorticity with its axis aligned in the lateral direction changes its axis to the radial direction around the projected center of the vermouth due to the geometrical asymmetry of the pump sump, and then, it further changes its axis in the vertical direction in the flow converged to the vermouth inlet, and finally, by the vertical acceleration of the flow converged to the vermouth inlet, the vertical vortex is stretched and, as a result, a strong submerged vortex is formed.

3) In the computation for which the vermouth was set at a longer distance from the bottom wall, no submerged wall vortex with its root at the bottom appeared, which supports the proposed formation process of a submerged vortex.

The origin and formation process of an air-entrained vortex have also been clarified by the present numerical investigations. But, they could not be presented in this paper due to the limited available space. They will be presented elsewhere together with the results of detailed investigations of the viscous-core dynamics of a suction vortex.

Acknowledgments
A portion of this research was supported by the grant for “Strategic Programs for Innovative Research” Field No. 4: Industrial Innovations, from the Ministry of Education, Culture, Sports, Science, and Technology (MEXT)’s “Development and Use of Advanced, High-Performance, General-Purpose Supercomputers Project.” We also express our thanks to all parties involved.

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