Development of the measuring complex with reduced regulator

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Abstract. In this paper the measuring complex of a prospective aircraft is investigated. The accuracy of navigation definitions is improved by compensating for errors in the inertial navigation system structure. In this paper based on a reduced nonlinear regulator performed the correction of navigation information, and the state dependent coefficient (SDC) representation of the nonlinear model of inertial navigation system errors is used to obtain the regulator.

1. Introduction

Information about navigation parameters is used in aircraft controlling (AC), and measuring complexes (MC) are used to determine the navigation parameters of AC [1-7]. The basic MC system as an inertial navigation system (INS) [1], and modern INSs have errors that accumulate over time which reduce the accuracy of system [1,2]. To improve the accuracy of the INS, various correction schemes are applied using as well as external measuring systems [7], estimation algorithms [5], and regulators [4].

While the aircraft operates its functional during long time intervals, a correction in the structure of INS is applied to the prevention of increase errors in INS using a reduced regulator [4]. MC with INS correction in this structure is presented in Fig.1.

![Fig. 1. The structure of measuring complex with a correction in frame of INS](image-url)

**Fig. 1.** The structure of measuring complex with a correction in frame of INS
In Fig.1, the following notation is introduced: AIC - algorithm for integration and comparison; AE - algorithm of estimation; AMB – algorithm for model building; AC – algorithm of control; \( \emptyset_k \) - true navigation information; \( \mathbf{x}_k \) - vector of INS errors; \( \mathbf{z}_k \) - measurement vector; \( \mathbf{\hat{x}}_k \) - vector of INS error estimation; \( \mathbf{\hat{x}}_k^p \) - vector of prediction of INS errors; \( u_k \) - control vector.

The AIC block contains an ensemble of selection criteria [4]. Using these criteria, measuring systems are selected that allow to build models with the maximum degrees of observability and controllability. Also, the synthesis of control algorithm is considered, which used to compensate for the errors of INS.

As shown In the AC block, the output is obtained as control vector \( u_k \) and the control algorithm is implemented. In this structure, we took using nonlinear INS error models, as it is reasonable to obtain higher accuracy of MC.

2. Development of nonlinear regulator

In this section, the synthesis of the control algorithm for the nonlinear model of INS errors in continuous form is carried out. The nonlinear error model of INS can be described as follows:

\[
\begin{align*}
\frac{d}{dt} x(t) &= f(t, x) + g_1(t, x)w(t) + g_2(t, x)u(t), \quad x(t_0) = x_0, \\
y(t) &= h(t, x).
\end{align*}
\]  

where, \( f(t, x), g_1(t, x), g_2(t, x), h(t, x) \) are valid and continuous. And we could imagine that equations (1) are in an equivalent form: the model has the structure of linear differential equations with parameters which depend on the state (SDC) [8].

With the help of the SDC representation method, the equations (1) could be transformed in the following form:

\[
\begin{align*}
\frac{d}{dt} x(t) &= A(t, x)x(t) + D(t, x)w(t) + B(t, x)u(t), \quad x(t_0) = x_0, \\
y(t) &= H(t, x)x(t).
\end{align*}
\]  

The nonlinear system (2) is controllable, if it satisfies:

\[
\begin{align*}
\text{rank} \begin{bmatrix} D(t, x), A(t, x)D(t, x), A^2(t, x)D(t, x), \ldots, A^n(t, x)D(t, x) \end{bmatrix} &= n, \\
\text{rank} \begin{bmatrix} B(t, x), A(t, x)B(t, x), A^2(t, x)B(t, x), \ldots, A^n(t, x)B(t, x) \end{bmatrix} &= n.
\end{align*}
\]  

Here, \( n \) is the corresponding order of the nonlinear error model system of INS, which is defined previously in (1).

As the Gramians of observability \( P_o(t, x) \) and \( P_s(t, x) \) can be found and defined, and they are solutions of the Lyapunov equations:

\[
\begin{align*}
A(t, x)P_o(t, x) + P_o(t, x)A^T(t, x) + D(t, x)D^T(t, x) &= 0, \\
A(t, x)P_s(t, x) + P_s(t, x)A^T(t, x) + B(t, x)B^T(t, x) &= 0.
\end{align*}
\]  

Accordingly, the nonlinear system (2) is observable and controllable, if the following conditions hold:

\[
\begin{align*}
\text{rank} \begin{bmatrix} H^t(t, x), H^t(t, x)A^t(t, x), \ldots, H^n(t, x)A^n(t, x) \end{bmatrix} &= n.
\end{align*}
\]  

At the meantime, the Gramian observability \( P_o(t, x) \) exists and is considered the solution of Lyapunov equation:

\[
A^T(t, x)P_o(t, x) + P_o(t, x)A(t, x) + H^T(t, x)H(t, x) = 0.
\]  

When criteria (4) and (5) are satisfied, the system (2) will be characterized as observable and controllable.

The task for the synthesis of control algorithm is formulated within the framework of the theory about differential games. Then the quality function of the differential games [4] will be:
Symmetric matrices $F$ and $Q$ are positive semidefinite, $R$ and $P$ are positive definite matrices.

Optimal control actions that minimize the function (7) are:

$$w(t) = P^T D^T (x) \left[ \hat{S}(x)x(t) + \hat{q}(x) \right],$$

$$u(t) = -R^T B^T (x) \left[ \hat{S}(x)x(t) + \hat{q}(x) \right].$$

In order to find the matrix $S(x)$ and $q(x)$ in the equations (8), we can use the inverse sweep method [4]. Matrix $S(x)$ and $q(x)$ estimates are determined by solving the equations:

$$\frac{d}{dt} \hat{S}(x) + A'(x)\hat{S}(x) + \hat{S}(x)A^T(x) - \hat{S}(x)R(x)\hat{S}(x) + H^T QH = 0, \hat{S}(x_o) = S_o,$$

$$\frac{d}{dt} \hat{q}(x) + \left[ A'(x) - \hat{S}(x)R(x) \right] H(x)\hat{q}(x) = 0, \hat{q}(x_o) = q_o,$$

where, $R(x) = B(x)R^{-1}B^T(x) - D(x)P^{-1}D^T(x)$.

Under the control of (8), model equation (1) could take the form as:

$$\frac{d}{dt} x(t) = f(t,x) - \Pi(x) \left[ \hat{S}(x)x(t) + \hat{q}(x) \right], x(t_0) = x_o,$$

$$y(t) = h(t,x).$$

When implementing the found control, it is necessary to use the estimate obtained using a non-linear Kalman filter instead of its estimate [5]. Taking the equivalence theorem into account, if the state vector is replaced with its estimate, the structure of the control algorithm does not change.

In practice, the control vector has a simplified form according to formula (8):

$$u(t) = -R^T B^T (\hat{x}) S_o \hat{x}(t).$$

Where $S_o$ is a positive definite matrix, which is determined by solving the following equation:

$$S_o A_o + A_o^T S_o - S_o B_o R^{-1} B_o^T S_o + H^T QH = 0.$$
Another solution is to create a database of matrices $S(x)$, which is shown in Fig.2. It is proposed for each state of the system to find $S(x)$ in advance. According to the system state, use the corresponding matrix $S(x)$ from the existing database during the flight.

3. Development of a nonlinear error correction algorithm INS

The INS error equations are considered as the errors equations of orientation and the error equations of horizontal accelerometers. These equations take the following form [9]:

$$
\delta V = -g \psi + B, \\
\psi = \frac{\delta V}{R} \frac{\partial V}{R} + \epsilon, \\
\epsilon = -\mu \epsilon + \eta.
$$

(13)

Here $\delta V$ - the error in determining the speed; $\psi$ - the deviation angle of the gyro-stabilized platform (GSP); $\epsilon$ - the drift velocity GSP; $B$, $\eta$ - Markov random processes; $R$ - Earth radius; $g$ - the acceleration of gravity; $\mu$ - the average frequency of random changes in the drift.

Equations (13) in the matrix form can be written in the following forms:

$$
\dot{x}(t) = f(t, x(t)) + w(t)
$$

(14)

Here $x(t) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $f(t, x) = \begin{bmatrix} \delta V \\ \psi \\ \epsilon \end{bmatrix} = \begin{bmatrix} -g x_2 \\ \frac{x_1}{R} + \frac{x_2}{R} + x_3 \\ -\mu x_3 \end{bmatrix}$;

and $w(t) = \begin{bmatrix} B \\ \eta \end{bmatrix}$.

In accordance with equation (2), the SDC representation of equation (15) is obtained:

$$
\dot{x}(t) = A(t, x) x(t) + w(t)
$$

(15)

Here $A(t, x)x(t) = \begin{bmatrix} 0 & -g & 0 \\ \frac{1}{R} & x_2 & 1 \\ 0 & 0 & -\mu \end{bmatrix} x(t)$.

The above system is nonlinear so if considered in discrete form, the SDC representation of the system (15) will be written in the next expression:

$$
x_k = Fx_k + w_k
$$

(16)

$$
x_k = \begin{bmatrix} \delta V_k \\ \psi_k \\ \epsilon_k \end{bmatrix}, w_k = \begin{bmatrix} B_k \\ 0 \\ \eta_k \end{bmatrix}, F = \begin{bmatrix} 1 & -T \delta V_k & 0 \\ \frac{T}{R_k} & 1 + \frac{T \delta V_k}{R_k} & T \\ 0 & 0 & 1 - T \mu \end{bmatrix}, \text{and } T - \text{ sampling period.}
$$

We represent the state vector as the sum of the vectors $z_k$ and $y_k$. In the $z_k$ vector, only the components needed to control are selected. And $y_k$ vector includes all the remaining components of the state vector. Then the equation of the object takes the form as follows:

$$
x_k = Fz_{k-1} + Gy_{k-1} + w_{k-1} + u_{k-1}
$$

(17)

and denoted:
\[ w_k + Gy_k = \xi_k \]  
(18)

As the parameters \( z_{k-1} \) and \( \xi_{k-1} \) in equations (17) and (18) are estimated, the control equation written by \( z_{k-1} \) and \( \xi_{k-1} \) will be as follows:

\[ u_k = -(K_{k-1}z_k + \xi_{k-1}) \]  
(19)

In the regulator, the use of the state vector assessment assumes its preliminary assessment using an estimation algorithm. At the output of the estimation algorithm, we have a signal of the form:

\[ \hat{x}_k = x_k - \bar{x}_k \]  
(20)

where \( \bar{x}_k \) is the error of the state vector estimation.

Substituting expression (19) into equation (17) and taking expression (20) into account, we can get:

\[ x_k = (F-K_{k-1})z_k + K_{k-1}\bar{x}_{k-1} + \bar{\xi}_{k-1} \]  
(21)

Optimal control is determined by finding a regulator matrix such that the function

\[ J = M[x_k^T x_k] \]  
(22)

accepts the minimum value.

The covariance matrix of the state vector is written:

\[ M[x_k^T x_k] = M\left\{ [(F-K_{k-1})x_{k-1}+K_{k-1}\bar{x}_{k-1}+\bar{\xi}_{k-1}]^T \right\} \]  
(23)

Considering the principle of orthogonality and apply it to the whole covariance matrix, then we can take the equation (23) transforming to the following expression, as showed in equation (24):

\[ M[x_k^T x_k] = (F-K_{k-1})M[x_{k-1}x_{k-1}^T] + M[x_{k-1}x_{k-1}^T](F-K_{k-1})^T \]
\[ + (F-K_{k-1})M[x_{k-1}x_{k-1}^T] + K_{k-1}M[x_{k-1}\xi_{k-1}^T] + K_{k-1}M[\xi_{k-1}x_{k-1}^T] \]
\[ + M[\xi_{k-1}x_{k-1}^T](F-K_{k-1})^T + M[\xi_{k-1}^T x_{k-1}^T](F-K_{k-1})^T + K_{k-1}M[\xi_{k-1}^T x_{k-1}^T] \]
\[ + K_{k-1}M[\xi_{k-1}^T x_{k-1}^T]K_{k-1} \]

(24)

And the sum of variances of the state vector is determined as J:

\[ J = spM[x_k x_k^T] = M[x_k^T x_k] \]  
(25)

We could find the optimal value of the controller matrix from the condition of equality to zero gradient:

\[ \frac{\partial J}{\partial K_{k-1}} = 0 \]  
(26)

With the help of the matrix differentiation rules, the optimality condition is obtained, which leads to the minimum of the function:

\[ K_{k-1} = F \]  
(27)

After simulation using the formulated algorithm system, we can obtain corresponding results of mathematical modeling of MC operation using test models of INS errors, which are presented in Fig. 3-4 as follows:
Fig. 3. MC with the intellectual component for correction in the structure of INS (errors in determining velocity)

Fig. 4. MC with the intellectual component for correction in the structure of INS (errors in determining the angles of deviation of GSP)

In figures 3 and 4, where solid lines indicate the MC errors with the known linear control algorithm, and dotted line indicates the MC errors with the developed nonlinear control algorithm.

According to the results of mathematical modeling, the accuracy of calculation errors in determining the velocity with the help of developed MC for correction in the structure of INS increases by an average of 7%, and errors in determining the angles of deviation of GSPs increases by 10%. A comparison of the developed nonlinear control algorithm was carried out with the well-known linear control algorithm [2].

The simulation results according to the laboratory experiment demonstrated high efficiency of the developed method. The accuracy of the navigation determination for velocity of aircraft increases on average by 8%; GPS deviation angle - 10-12%; GSP drift velocity - 15%. And the simulation results with real data of the INS Ts060K demonstrated operability of the used nonlinear control algorithm, which based on SDC representation of the nonlinear INS error model. Using the developed control algorithm, it is possible to significantly improve the accuracy of navigation definitions of aircraft.

4. Conclusion
MC of modern high-precision aircrafts are considered in this paper, and MC is investigated with correction in the structure of INS. In order to improve the accuracy of navigation definitions of an aircraft, the MC use a non-linear control algorithm and a developed control algorithm based on the SDC representation of the non-linear INS error model.

5. References
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