$K_L \to \gamma \nu \bar{\nu}$ decay beyond the standard model

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Abstract

The decay $K_L \to \gamma \nu \bar{\nu}$ is investigated beyond the standard model. Interestingly, the upper limit of the $CP$-conserving and $CP$-violating branching ratios of the decay, induced from the possible extensions of the standard model, would be larger than the corresponding branching ratios given in the standard model respectively, and it is expected that the $CP$-violating part could be enhanced.

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Rare kaon decays play an important role in studying flavor-changing neutral currents (FCNC) and $CP$-violating phenomena in modern particle physics [1]. Within the context of the standard model, they are suggested to test the Cabibbo-Kobayashi-Maskawa (CKM) [2] paradigm. They also provide an ideal place to search for new physics beyond the standard model [3,4]. Even the new physics may appear in $B$-meson decays, it is interesting to study the footprint it leaves in rare kaon decays and especially the $s \rightarrow d$ weak transition.

The decays $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ are both governed by the same $s \rightarrow d \nu \bar{\nu}$ transition. They are dominated from the short-distance loop contributions containing virtual heavy quarks. It is believed that the long-distance contributions in these decays are much smaller than the short-distance ones, and could be negligible [5–7]. Due to the absence of possible large theoretical uncertainty from the long-distance, these two modes are very useful both to test the standard model and to explore the new physics. Experimentally, the decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ has been measured by E787 group at BNL [8]

$$B(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (1.57^{+1.75}_{-0.82}) \times 10^{-10},$$

which is consistent with the prediction of the standard model [9]

$$B(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{SM} = (0.72 \pm 0.21) \times 10^{-10}. \quad (2)$$

The amplitude of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ in the standard model is proportional to the CKM factor $Im(V_{td}^* V_{ts})$ and the decay branching ratio is found to be at the level of $10^{-12}$ [10]. Whereas the current experiment limit is less than $5.9 \times 10^{-7}$ [11], and this work is ongoing. But from the experimental point of view, it is difficult to find the event, because all the final-state particles are neutral, only the $2\gamma$'s from $\pi^0$ can be detectable.

An alternative way is to search the decay $K \rightarrow \pi \pi \nu \bar{\nu}$ [12–14]. However, the decay branching ratio is small and the background for $\pi \pi$ is large. So another good choice is to study the decay of $K_L \rightarrow \gamma \nu \bar{\nu}$ [15,16], where there is only one photon at the final state.

The total branching ratio of the decay $K_L \rightarrow \gamma \nu \bar{\nu}$ can be divided into CP-conserving part and CP-violating part [16]. The contribution within the standard model to this decay
has been investigated by some authors, and it is believed that the mode is short-distance dominated [15,16]. The purpose of this paper is to examine the possible contribution to this decay from the new physics scenarios beyond the standard model. It will be shown below that there exists the significant model-independent limit, from which the branching ratio of $CP$-conserving part could be up to the same order of magnitude as the one predicted within the standard model while the $CP$-violating part could be of more than one order of magnitude larger than the one given in the standard model. As an interesting example, we also consider the possible dominant supersymmetric contributions to this decay mode.

The effective Hamiltonian for $s \rightarrow d\nu\bar{\nu}$ transition generally takes the form

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \theta_W} W_{ds} \cdot \bar{d} \gamma_\mu (1 - \gamma_5) s \ \bar{\nu} \gamma^\mu (1 - \gamma_5) \nu + H.c.,$$

(3)

where the short-distance physics is lumped in $W_{ds}$. In the standard model $W_{ds}$ is dominated by penguin and box diagrams with intermediate charm and top quark

$$W_{ds}^{\text{SM}} = \sum_{l=e,\mu,\tau} [\lambda_c X_{NL}^l + \lambda_t X(x_t)]$$

(4)

where $x_t = m_t^2/M_W^2$, and $\lambda_i = V_{is}^* V_{id}$. The functions $X_{NL}^l$ and $X(x_t)$ correspond to charm and top contributions in the loops with the next-to-leading logarithmic approximation respectively. It has been shown explicitly in Ref. [17] that the corresponding top contribution can be written as $X(x_t) = \eta_t X_0(x_t)$, with the QCD-uncorrected top quark contribution [18]

$$X_0(x_t) = \frac{x_t}{8} \left[ -\frac{2 + x_t}{1 - x_t} + \frac{3x_t - 6}{(1 - x_t)^2} \ln(x_t) \right],$$

(5)

and the QCD correction factor $\eta_t = 0.994$.

Also from table I in Ref. [17], one has

$$X_{NL}^{e,\mu} = 11.00 \times 10^{-4},$$

(6)

and

$$X_{NL}^{\tau} = 7.47 \times 10^{-4}.$$
with the central value of the QCD scale \( \Lambda = \frac{\Lambda^{(4)}_{\overline{MS}}}{\Lambda^{(4)}_{\overline{MS}}} = 325 \pm 80 MeV \), and the charm quark mass \( m_c = m_c(m_c) = 1.30 \pm 0.05 GeV \).

From the effective Hamiltonian in eq.(3), one can evaluate the hadronic matrix element \( \langle \gamma | J_\mu | K^0 \rangle \), where \( J_\mu = \bar{d} \gamma_\mu (1 - \gamma_5) s \). Then the corresponding matrix elements are parameterized as

\[
\langle \gamma | \bar{d} \gamma_\mu s | K^0(p + q) \rangle = -e \frac{F_A}{M_k} \left[ \epsilon^\mu (p \cdot q) - (\epsilon^* \cdot p) q^\mu \right],
\]

\[
\langle \gamma | \bar{d} s | K^0(p + q) \rangle = -ie \frac{F_V}{M_k} \epsilon^{\mu \alpha \beta \gamma} \epsilon^*_\alpha p_\beta q_\gamma,
\]

where \( q \) and \( p + q \) are photon and \( K \)-meson four momenta, \( F_A \) and \( F_V \) are form factors of axial-vector and vector respectively, and \( \epsilon^* \) is the photon polarization vector. The form factors \( F_V \) and \( F_A \) have been evaluated in [16] using light front quark model. We find, it is enough to use momentum independent form factors, \( F_A(0) = 0.0429 \) and \( F_V(0) = 0.0915 \) given by the authors of [16] to illustrate the numerical estimates in the present paper.

Following the similar way to study \( K_{L,S} \rightarrow \pi \pi \nu \bar{\nu} \) in Ref. [14], here we also consider two representative cases to probe possible new physics effects of \( K_L \rightarrow \gamma \nu \bar{\nu} \): one is a model-independent way; the other is a special case–low energy supersymmetry, which however represents one of the most interesting and consistent extensions of the standard model.

i) Effective flavor-changing neutral currents (FCNC) interaction. As formulated by Nir and Silverman [19], it reads

\[
\mathcal{L}^{(s)} = -\frac{g}{4 \cos \theta_W} U_{ds} \bar{d} \gamma_\mu (1 - \gamma_5) s Z^\mu.
\]

Combining with the coupling of the \( Z \) boson to neutrino-antineutrino pair, one finds that

\[
W_{ds}^{NP} = \frac{\pi^2}{\sqrt{2} G_F M_W^2} U_{ds} = 0.93 \times 10^2 U_{ds}
\]

as the new piece of \( W_{ds} \) in the effective Hamiltonian contributing to the basic transition of \( s \rightarrow d \nu \bar{\nu} \).

The upper bounds on \( U_{ds} \) have been determined by other processes involving \( K \)-meson and were summarized in Ref. [20]. After translating them into the bounds on \( W_{ds} \), we obtain
\[ |Re(W_{ds})| \leq 0.93 \times 10^{-3}, \]
\[ |W_{ds}| \leq 2.8 \times 10^{-3}, \]
\[ |Re(W_{ds})Im(W_{ds})| \leq 1.1 \times 10^{-5}, \]
\[ |Im(W_{ds})| \leq 0.93 \times 10^{-3}. \] (11)

The bound on \(|W_{ds}|\) can be improved using the most recent measurement \([8]\) of \(K^+ \to \pi^+ \nu \bar{\nu}\), which is \(|W_{ds}| = 1.0^{+0.46}_{-0.31} \times 10^{-3}\).

ii) Supersymmetry: Possible interesting supersymmetric effects arise from penguin diagrams involving charged-Higgs plus top-quark intermediate states or squark and chargino intermediate states. These give additional pieces to the effective Hamiltonian of the form \([3]\)

\[ W_{ds}^{NP} = \lambda_t \frac{m_H^2}{M_W^2 \tan^2 \beta} H(x_{tH}) + \frac{1}{96} \tilde{\lambda}_t, \] (12)

where \(\tan \beta\) is the ratio of the two Higgs vacuum expectation values and \(x_{tH} = m_t^2/M_{H^\pm}^2\).

The quantity \(H(x)\) is given by

\[ H(x) = \frac{x^2}{8} \left[ -\frac{\ln x}{(x-1)^2} + \frac{1}{x-1} \right]. \] (13)

The definition of \(\tilde{\lambda}_t\) is shown in Ref. \([3]\). The \(\tilde{\lambda}_t\) part in eq. (12), which comes from chargino-squark diagrams, represents the dominant supersymmetric effect \([3]\) [Only small value of \(\tan \beta\) might enhance the first term in eq. (12), which however is not in favor with the present phenomenologies]. The bound on \(\tilde{\lambda}_t\) can be imposed by similar considerations that were used for \(W_{ds}\). In Ref. \([3]\), the upper limits \(|Re\tilde{\lambda}_t| \leq 0.21\) and \(|\tilde{\lambda}_t| \leq 0.35\) have been obtained from the observed branching ratios of decays \(K_L \to \mu^+\mu^-\) and \(K^+ \to \pi^+ \nu \bar{\nu}\).

Using the most recent experiment data \([8]\), the last limit could be updated as \(|\tilde{\lambda}_t| \leq 0.14\).

Using the effective Hamiltonian for \(K^0 \to \gamma \nu \bar{\nu}\) in eq.(1) together with \(K_L \simeq K_2 = (K^0 + \bar{K}^0)/\sqrt{2}\), the amplitude of \(K_L \to \gamma \nu \bar{\nu}\) can be divided into the \(CP\)-conserving and \(CP\)-violating parts as follows

\[ \mathcal{M}(K_L \to \gamma \nu \bar{\nu}) = \mathcal{M}_{CP} + \mathcal{M}_{CPV}, \]
where

\[ M_{CP C} = -\frac{G_F}{\sqrt{2}} \frac{\alpha}{2 \pi \sin^2 \theta_W} \frac{2}{\sqrt{2}} Re(W_{ds}) \epsilon^{\mu \rho \alpha \beta} F_V \epsilon^*_{\mu} q_\alpha p_\beta v(\hat{p}_\nu) \gamma_\nu (1 - \gamma_5) v(\nu), \quad (14) \]

and

\[ M_{CP V} = -\frac{G_F}{\sqrt{2}} \frac{\alpha}{2 \pi \sin^2 \theta_W} \frac{2}{\sqrt{2}} Im(W_{ds}) \frac{F_A}{M_K} \epsilon^*_{\mu} (-p' \cdot q g^{\mu \nu} + p'^\nu q_\nu) u(\hat{p}_\nu) \gamma_\nu (1 - \gamma_5) v(\nu) \quad (15) \]

In \( K_L \) rest frame the partial decay rate of \( K_L \to \gamma \nu \bar{\nu} \) is given by

\[ d^2 \Gamma = \frac{1}{(2\pi)^3} \frac{1}{8M_K} |M| dE_\gamma dE_\nu. \quad (16) \]

By integrating the \( E_\gamma \) and \( E_\nu \), we obtain

\[ \Gamma_{CP C} = \frac{\alpha}{15} \left( \frac{G_F \alpha}{16 \pi^2 \sin^2 \theta_W} \right)^2 |Re(W_{ds})|^2 F_V^2 M_K^5, \quad (17) \]

\[ \Gamma_{CP V} = \frac{\alpha}{15} \left( \frac{G_F \alpha}{16 \pi^2 \sin^2 \theta_W} \right)^2 |Im(W_{ds})|^2 F_A^2 M_K^5. \quad (18) \]

Using \( \sin^2 \theta_W = 0.23 \) and \( M_K = 0.5 \text{ GeV} \) the decay branching ratios are found to be

\[ B(K_L \to \gamma \nu \bar{\nu})_{CP C} = 6.7 \times 10^{-8} |Re(W_{ds})|^2, \quad (19) \]

\[ B(K_L \to \gamma \nu \bar{\nu})_{CP V} = 1.4 \times 10^{-8} |Im(W_{ds})|^2. \quad (20) \]

The decay branching ratios of \( K_L \to \gamma \nu \bar{\nu} \), within the standard model, have been evaluated by Geng, Lih, and Liu [16]

\[ B(K_L \to \gamma \nu \bar{\nu})_{CP C}^{SM} = 1.0 \times 10^{-13}, \quad (21) \]

and

\[ B(K_L \to \gamma \nu \bar{\nu})_{CP V}^{SM} = 1.5 \times 10^{-15}. \quad (22) \]

For the representative scenarios of physics beyond the standard model, the model-independent upper limits of the \(|Re(W_{ds})|\) and \(|Im(W_{ds})|\) have been given in eq.(11). Thus the maximal values of the branching ratios of the decay could be

\[ B(K_L \to \gamma \nu \bar{\nu})_{CP C} = 1.8 \times 10^{-13}, \quad (23) \]
and

\[ B(K_L \rightarrow \gamma \nu \bar{\nu})_{CPV} = 0.42 \times 10^{-13}. \]  

(24)

Note that a factor 3 has been multiplied in deriving eqs. (23) and (24) since all three flavor neutrinos should be taken into account. Also from the previous discussions on the limits of \( \tilde{\lambda}_t \), it is easy to see that supersymmetric extensions of the standard model could allow \(|\text{Re}(W_{ds})|\) and \(|\text{Im}(W_{ds})|\) to reach the upper limit \(0.93 \times 10^{-3}\) without conflict with other constraints.

In conclusion, it is shown that the upper limit value of \(CP\)-conserving branching ratio in eq.(23) can be of the same order of magnitude as the result given in the standard model, while the upper limit value of \(CP\)-violating branching ratio in eq.(24) is nearly a factor of 30 larger than the value within the standard model. Thus it seems that there exists room to look for new physics beyond the standard model by observing the decay \(K_L \rightarrow \gamma \nu \bar{\nu}\) although this is not a very easy task. In addition to investigations on the decays of \(K \rightarrow \pi \nu \bar{\nu}\) and \(K \rightarrow \pi \pi \nu \bar{\nu}\), it is therefore expected that both experimental and theoretical studies of \(K_L \rightarrow \gamma \nu \bar{\nu}\) could provide complementary information on the basic weak transition \(s \rightarrow d \nu \bar{\nu}\).

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REFERENCES

[1] A.R. Baker and S.H. Kettell, Ann. Rev. Nucl. Part. Sci. 50, 249 (2000); G. Buchalla, arXiv:hep-ph/0103166; A.J. Buras, arXiv:hep-ph/0101336.

[2] N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).

[3] G. Colangelo and G. Isidori, JHEP 9809, 009 (1998), arXiv:hep-ph/9808487.

[4] G. Colangelo, G. Isidori, and J. Portoles, Phys. Lett. B 470, 134 (1999); A.J. Buras, G. Colangelo, G. Isidori, A. Romanino, and L. Sivestri, Nucl. Phys. B 566, 3 (2000); C.S. Huang, W.J. Huo, and Y.L. Wu, Phys. Rev. D 64, 016009 (2000); A.J. Buras, arXiv:hep-ph/0109197; G. D’Ambrosio and G. Isidori, Phys. Lett. B 530, 108 (2002); C.H. Chen, J. Phys. G 28, L33 (2002); G. D’Ambrosio and D.N. Gao, JHEP 07, 068 (2002); A. Messina, Phys. Lett. B 538, 130 (2002); R. Fleischer, arXiv:hep-ph/0207108.

[5] D. Rein and L. M. Sehgal, Phys. Rev. D 39, 3325 (1989); J. Hagelin and L. S. Littenberg, Prog. Part. Nucl. Phys. 23, 1(1989).

[6] M. Lu and M. B. Wise, Phys. Lett. B 324, 461(1994).

[7] C.Q. Geng, I.J. Hsu and Y.C. Lin, Phys. Lett. B 355, 569 (1995), arXiv:hep-ph/9506313; C.Q. Geng, I.J. Hsu and C.W. Wang, Prog. Theor. Phys. 101, 937 (1999).

[8] S. Adler et al. [E787 Collaboration], Phys. Rev. Lett. 88, 041803 (2002), arXiv:hep-ex/0111091.

[9] A.J. Buras and R. Fleischer, Phys. Rev. D 64, 115010 (2001), arXiv:hep-ph/0104238.

[10] G. Belanger and C.Q. Geng, Phys. Rev. D 43, 140 (1991).

[11] A. Alavi-Harati et al. [The E799-II/KTeV Collaboration], Phys. Rev. D 61, 072006 (2000) arXiv:hep-ex/9907014.

[12] C.Q. Geng, I.J. Hsu and Y.C. Lin, Phys. Rev. D 54, 877 (1996) arXiv:hep-ph/9604228.
[13] L.S. Littenberg and G. Valencia, Phys. Lett. B 385, 379 (1996).

[14] C.W. Chiang and F.J. Gilman, Phys. Rev. D 62, 094026 (2000), arXiv:hep-ph/0007063.

[15] E. Ma and I. Okada, Phys. Rev. D 18, 4219 (1978); S. Richardson and C. Picciotto, Phys. Rev. D 52, 6342 (1995); W. Marciano and Z. Parsa, Phys. Rev. D 53, 1 (1996).

[16] C.Q. Geng, C.C. Lih and C.C. Liu, Phys. Rev. D 62, 034019 (2000), arXiv:hep-ph/0004164.

[17] G. Buchalla and A.J. Buras, Nucl. Phys. B 548, 309 (1999), arXiv:hep-ph/9901288.

[18] T. Inami and C. S. Lim, Prog. Theor. Phys. 65, 297 (1981).

[19] Y. Nir and D.J. Silverman, Phys. Rev. D 42, 1477 (1990).

[20] Y. Nir, arXiv:hep-ph/9911321.