Bi-pair Neutrino Mixing

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A new type of neutrino mixing named bi-pair neutrino mixing is proposed to describe the current neutrino mixing pattern with a vanishing reactor mixing angle and is determined by a mixing matrix with two pairs of identical magnitudes of matrix elements. As a result, we predict \( \sin^2 \theta_{12} = 1 - 1/\sqrt{2} ( \approx 0.293) \) for the solar neutrino mixing and either \( \sin^2 \theta_{23} = \tan^2 \theta_{12} \) or \( \cos^2 \theta_{23} = \tan^2 \theta_{12} \) for the atmospheric neutrino mixing. We determine flavor structure of a mass matrix \( M \), leading to diagonal masses of \( m_{1,2,3} \), and find that

\[
|M_{\mu\mu} - M_{ee}/\sqrt{2}| : |M_{\mu\tau}| : |M_{\tau\tau} - M_{ee}/2|^2 = \ell_{23} : |\ell_{23}| : 1 \text{ for the normal mass hierarchy if } m_1 = 0, \text{ where } \ell_{ij} = \tan \theta_{ij} (i,j=1,2,3) \text{ and } M_{ij} (i,j=\mu,\tau) \text{ stand for flavor neutrino masses. For the inverted mass hierarchy, the bi-pair mixing scheme turns out to satisfy the strong scaling ansatz requiring that } |M_{\mu\mu}| : |M_{\mu\tau}| : |M_{\tau\tau}| = 1 : |\ell_{23}| : \ell_{23}^2 \text{ if } m_3 = 0.
\]

These predictions are consistent with the 2\( \sigma \) data although \( \sin^2 \theta_{12} \) slightly exceeds the allowed range of the 1\( \sigma \) data.

In this short note, we would like to find new mixing schemes [10], which may well describe the solar neutrino mixing. To do so, we demand that at least two of the mixing angles \( \theta_{12}, \theta_{23}, \theta_{13} \) can account for the neutrino oscillations if neutrinos are massive and are characterized by three mixing angles \( \theta_{12,23,13} \) associated with the mixings of \( \nu_e, \nu_{\mu}, \nu_{\tau} \) and \( \nu_\tau, \nu_\mu, \nu_e \), respectively. The Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [7] parameterized by these mixing angles is used to convert mass eigenstates of neutrinos into the flavor neutrinos.

The observed results of the mixing angles are summarized as [8]:

\[
\begin{align*}
\sin^2 \theta_{12} &= 0.304^{+0.022}_{-0.016} (0.27 - 0.35), \\
\sin^2 \theta_{23} &= 0.50^{+0.07}_{-0.06} (0.39 - 0.63), \\
\sin^2 \theta_{13} &= 0.01^{+0.016}_{-0.011} (\leq 0.04),
\end{align*}
\]

(1)

for the 1\( \sigma \) range, where the values in the parentheses denote the 2\( \sigma \) range. There is a theoretical prediction of these mixing angles based on the tri-bimaximal mixing scheme [6], which yields

\[
\sin^2 \theta_{12} = \frac{1}{3}, \quad \sin^2 \theta_{23} = \frac{1}{2}, \quad \sin^2 \theta_{13} = 0. \tag{2}
\]

These predictions are consistent with the 2\( \sigma \) data although \( \sin^2 \theta_{12} \) slightly exceeds the allowed range of the 1\( \sigma \) data.

More than ten years have passed since the first confirmation of the neutrino oscillations by the Super-Kamiokande collaboration, who observed the oscillation of atmospheric neutrinos [1]. Subsequent experimental observations have also confirmed the solar neutrino oscillations really occur in terrestrial observations [2, 3]. These oscillations confirm the neutrino oscillations by the Super-Kamiokande collaboration, who observed the oscillation of atmospheric neutrinos [1].

\[K^0 = \text{ diag}(1, e^{i\phi_2/2}, e^{i\phi_3/2}), \tag{4}\]

where \( c_{ij} = \cos \theta_{ij}, s_{ij} = \sin \theta_{ij} (i,j=1,2,3) \) and \( \phi_{2,3} \) are CP-violating Majorana phases.

As long as \( \theta_{13} = 0 \) is maintained, it is not difficult to search for alternative relations to Eq. (4) for the given values of Eq. (1). There are only two possibilities, which shows

\[
\begin{align*}
|U_{12}| &= |U_{22}| = |U_{32}|, \\
|U_{12}| &= |U_{23}|, \\
|U_{12}| &= |U_{22}|, \\
|U_{32}| &= |U_{33}|,
\end{align*}
\]

(5) as the case (1), and

\[
\begin{align*}
|U_{12}| &= |U_{22}|, \\
|U_{32}| &= |U_{33}|,
\end{align*}
\]

(6) as the case (2). These equations in turn provide useful relationship among the mixing angles:

\[
\frac{\sin \theta_{12}}{\cos \theta_{12}} = \frac{\cos \theta_{12}}{\sqrt{1 + \cos^2 \theta_{12}}}. \tag{7}
\]
as well as
\[
\sin^2 \theta_{23} = \tan^2 \theta_{12}, \tag{8}
\]
for the case (1), and
\[
\cos^2 \theta_{23} = \tan^2 \theta_{12}, \tag{9}
\]
for the case (2). Numerically, these relations predict
\[
\sin^2 \theta_{12} = 1 - \frac{1}{\sqrt{2}} \approx 0.293,
\]
\[
\sin^2 \theta_{23} = \begin{cases} 
\sqrt{2} - \frac{1}{2} & \text{the case (1)} \\
2 - \sqrt{2} & \text{the case (2)}
\end{cases}, \tag{10}
\]
which are consistent with the 2σ data.

Our prediction on sin^2 θ_{23} is slightly inconsistent with the 1σ data as in the similar situation to that of sin^2 θ_{12} in the tri-bimaximal mixing. However, it is well known that the corresponding 2-3 mixing in charged leptons (labeled by θ'_{23}) can produce additional contribution to θ_{23} without affecting θ_{12} and θ_{13}. Namely, we obtain that
\[
\theta_{23} = \theta'_{23} - \theta_{23}^a,
\]
where θ_{23}^a is given by θ_{23} in Eq. (11). Therefore, if charged leptons have a mass matrix M_{ℓ} described by
\[
M_{ℓ} = \begin{pmatrix} 
m_ε & 0 & 0 \\
0 & * & * \\
0 & * & *
\end{pmatrix}, \tag{12}
\]
appropriate correction automatically comes in θ_{23} so that θ_{23} can be shifted to the 1σ region. Other corrections may arise from the renormalization effect if the bi-pair mixing is generated at a higher scale such as the seesaw scale, where the seesaw mechanism gets active.

Having understood that the bi-pair mixing is another candidate predicting the reasonable values of θ_{12} and θ_{23}, we discuss its implication of flavor structure of the neutrino masses. It has been discussed that any models with neutrino mass matrix labeled by U defined in Eq. (10), to take care of general phase structure of M_{ν} [14 15]. For U^{PDG} as the standard parameterization of U adopted by the Particle Data Group (PDG) [16], M_{ν} is shifted to a modified mass matrix M_{ν} after ρ and γ present in U are transferred to M_{ν}.

Owing to the rephasing ambiguity in the charged lepton sector, one can choose three flavor masses to be real numbers, where d can be taken to be positive without loss of generality. As a result, we obtain
\[
M_{ν} = \begin{pmatrix} 
κ_α |a| & e^{iα} |b| - t_{23} e^{iβ} |b| \\
d & f
\end{pmatrix} \tag{16}
\]
\[
f = e^{4iγ}d + e^{2iγ} \frac{1 - t_{23}^2}{t_{23}} e^\gamma,
\]
where κ_α, ε take care of the sign of a and ε. The mixing angles θ_{12} is given by
\[
\tan 2θ_{12} = \frac{2e^{iε}}{c_{23} e^{2iγ}|d| - t_{23} κ_ε |e| - κ_α e^{2iρ} |a|}, \tag{17}
\]
which determines the phase ρ expressed in terms of flavor neutrino masses for a given value of θ_{12}.

After redundant phases are removed from U, U becomes U^{PDG} and, accordingly, M_{ν} is shifted to
\[
M_{ν} = \begin{pmatrix} 
é^{2iρκ_α} |a| & e^{iε} |b| - t_{23} e^{iε} |b| \\
e^{2iγ} |d| & κ_ε |e| \\
e^{-2iγ} f
\end{pmatrix} \tag{18}
\]
We finally reach M_{ν} given by
\[
M = e^{-i(α - β)} |d| I + κ_ε |e| \begin{pmatrix} 
t_{23} & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 - t_{23}^2
\end{pmatrix} + e^{i(α + β)} |b| \begin{pmatrix} 
A & 1 & t_{23} \\
1 & 0 & 0 \\
t_{23} & 0 & 0
\end{pmatrix}, \tag{19}
\]
where t_{12} = tan θ_{12} and I is the unit matrix and |a| in Eq. (18) is eliminated by Eq. (16) to yield Eq. (19). This mass matrix certainly gives M_{ν} for the tri-bimaximal neutrino mixing if t_{23}^2 = 1 and t_{12} = 1/2 giving A = -1. The bi-pair neutrino mixing gives A = -1/|t_{12}| with t_{23}^2 = 1/\sqrt{2} and sin θ_{23} = σ tan θ_{12} for the case (1), where σ(±1) takes care of the sign of sin θ_{23} and A = -t_{23}^2 with t_{23}^2 = \sqrt{2} and cos θ_{23} = tan θ_{12} for the case (2). More transparent flavor structure for the bi-pair neutrino mixing can be found if neutrino mass hierarchies are taken into account.
Neutrino masses are calculated to be

\begin{align*}
m_{1}e^{-i\varphi_{1}} & = \kappa_{a}e^{2i\rho}|a| - \tan\theta_{12}e^{i\xi}|b|,\\
m_{2}e^{-i\varphi_{2}} & = \kappa_{a}e^{2i\rho}|a| + \frac{1}{\tan\theta_{12}}e^{i\xi}|b|,\\
m_{3}e^{-i\varphi_{3}} & = e^{-i(\alpha-\beta)}|d| + \frac{1}{t_{23}}\kappa_{e}|c|,
\end{align*}

(21)

where the CP-violating Majorana phases \(\varphi_{2,3}\) are given by \(\varphi_{2} = \varphi_{2} - \varphi_{1}\) and \(\varphi_{3} = \varphi_{3} - \varphi_{1}\). Let us consider that neutrinos exhibit either \(m_{1}=0\) leading to the normal mass hierarchy or \(m_{3}=0\) leading to the inverted mass hierarchy as in the minimal seesaw model \([17]\), where \(\det(M)=0\) is satisfied. We, then, find that, for the normal mass hierarchy,

\[
M = \begin{pmatrix}
Be^{i(\alpha+\beta)}|b| & e^{i(\alpha+\beta)}|b| & -t_{23}e^{i(\alpha+\beta)}|b| \\
\frac{t_{12}^{2}}{t_{23}}e^{i(\alpha+\beta)}|b| & \kappa_{e}|e| & t_{23}e^{i(\alpha+\beta)}|b| \\
\frac{t_{12}^{2}}{t_{23}}e^{i(\alpha+\beta)}|b| & \kappa_{e}|e| & \frac{1}{t_{23}}e^{i(\alpha+\beta)}|b|
\end{pmatrix},
\]

\[
B = \frac{\tan\theta_{12}}{c_{23}} = \begin{pmatrix}
|t_{23}| \cdots (\sin \theta_{23} = \sigma \tan \theta_{12}) \\
1 \cdots (\cos \theta_{23} = \tan \theta_{12})
\end{pmatrix},
\]

(22)

where \(\rho = (\alpha + \beta)/2 \mod \pi\) from \(m_{1} = 0\) and, for the inverted mass hierarchy,

\[
M = \begin{pmatrix}
\kappa_{a}e^{2i\rho}|a| & e^{i\xi}|b| & -t_{23}e^{i\xi}|b| \\
\frac{1}{t_{23}}\kappa_{e}|e| & \kappa_{e}|e| & -t_{23}\kappa_{e}|e|
\end{pmatrix},
\]

(23)

where \(e^{-i(\alpha-\beta)}|d| = -\kappa_{e}/t_{23}|e|\) from \(m_{3}=0\), thus, leading to \(\alpha=\beta \mod \pi\) and \(\rho\) is determined so as to satisfy Eq.\([16]\), which is used to express this mass matrix in terms of \(|b|\) and \(|e|\).

We observe that flavor structure of \(M\) for the bi-pair neutrino mixing shows

\[
|M_{ep}| : |M_{e\tau}| = 1 : |t_{23}|,
\]

(24)

\[
\arg(M_{e\mu}) = \arg(M_{e\tau}) \mod \pi,
\]

(25)

and

- for the normal mass hierarchy,

\[
|M_{ee}| = \begin{pmatrix}
|M_{e\tau}| \cdots (\sin \theta_{23} = \sigma \tan \theta_{12}) \\
|M_{e\mu}| \cdots (\cos \theta_{23} = \tan \theta_{12})
\end{pmatrix},
\]

(26) \hspace{1cm}

\[
|M_{\mu\mu}| - \frac{M_{ee}}{t_{12}^{2}} : |M_{\mu\tau}| : |M_{e\tau}| - \frac{M_{ee}}{t_{12}^{2}} = t_{23}^{2} : |t_{23}| : 1,
\]

(27)

\[
\arg(M_{ee}) = \arg(M_{e\mu}) \mod \pi,
\]

(28)

- for the inverted mass hierarchy,

\[
|M_{\mu\mu}| : |M_{\mu\tau}| : |M_{e\tau}| = 1 : |t_{23}| : t_{23}^{2},
\]

(29)

It is noted that Eq.\((23)\) may satisfy the strong scaling ansatz \([18]\) since \(\sin \theta_{13} = 0\) and \(m_{3} = 0\). It is evident that the resulting mass matrix does satisfy the strong scaling ansatz requiring the relation of Eq.\((29)\) (as well as Eq.\((24)\)). Therefore, when \(m_{3} \neq 0\), the bi-pair neutrino mixing provides an example of the approximate strong scaling ansatz, where Eq.\((23)\) is approximately satisfied. If \(m_{1} = 0\) for the normal mass hierarchy, the relation Eq.\((27)\) including \(M_{ee}\) can be predicted. \(^1\)

In summary, we have found that the bi-pair mixing well reproduces the current neutrino mixings and is described by \(U_{BP}\):

\[
U_{BP} = \begin{pmatrix}
c_{12} & s_{12} & 0 \\
-t_{12} & t_{12} & t_{12} \\
s_{12}t_{12} & -s_{12} & t_{12}/c_{12}
\end{pmatrix},
\]

(30)

for the case (1), and

\[
U_{BP} = \begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12}t_{12} & s_{12} & t_{12}/c_{12} \\
t_{12} & -t_{12} & t_{12}/c_{12}
\end{pmatrix},
\]

(31)

for the case (2), where \(s_{12}^{2}\) is predicted to be: \(s_{12}^{2} = 1 - 1/\sqrt{2}\). The bi-pair mixing scheme turns out to be complementary to the tri-bimaximal mixing scheme in a sense that

- the bi-pair mixing predicts \(\sin^{2}\theta_{12} = 0.293\), which well describes the solar neutrino mixing while it predicts \(\sin^{2}\theta_{23} = 0.414/0.586\), which gives slight deviation of the atmospheric neutrino mixing angle from the \(1\sigma\) region, and

- the tri-bimaximal mixing predicts \(\sin^{2}\theta_{23} = 0.5\), which well describes the atmospheric neutrino mixing while it predicts \(\sin^{2}\theta_{12} = 0.333\), which gives slight deviation of the solar neutrino mixing angle from the \(1\sigma\) region.

We have clarified the flavor structure of the neutrino mass matrix giving \(\sin \theta_{13} = 0\), which is described by Eq.\([19]\) as long as the parameterization of \(U_{PDG}\) is adopted. For the bi-pair mixing, in the simplest cases with \(m_{1} = 0\) for the normal mass hierarchy and \(m_{3} = 0\) for the inverted mass hierarchy, the phase structure is subject to Eqs.\([25]\) and \([28]\). We have also predicted Eq.\([27]\) for the normal mass hierarchy, which should be compared with Eq.\([29]\) for the strong scaling ansatz valid in the inverted mass hierarchy.

Finally, we point out that the results of Eqs.\([23]\) and \([24]\) for both normal and inverted mass hierarchies and of Eq.\([27]\) for the normal mass hierarchy and Eq.\([29]\) for the inverted mass hierarchy are not only valid in the bi-pair mixing scheme and but also valid for any models

\(^1\) If \(m_{3}^{2} \gg m_{2}^{2}\) is further imposed, the condition of \(b \approx 0\) leading to \(M_{ee} \approx 0\) should be satisfied and Eq.\([27]\) becomes \(|M_{e\mu}| : |M_{\mu\tau}| : |M_{e\tau}| \approx t_{23}^{2} : |t_{23}| : 1\).
We will discuss the detailed feature of our flavor neutrino parameters. The bi-pair mixing scheme provides a good example of these properties of the flavor neutrino masses. We will discuss the detailed feature of our flavor neutrino mass matrix as well as phenomenological implication of the bi-pair mixing scheme based on Majorana CP violation from Eq. (21) and on leptogenesis in a future publication.

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