STRING SOLUTIONS IN GENERAL BACKGROUNDS

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Motivated by the recent interest in the different aspects of the string/field theory duality, we describe an approach for obtaining exact string solutions in general backgrounds, based on two types of string embedding, allowing for separation of the worldsheet variables \( \tau \) and \( \sigma \).

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1 Introduction

Recently, there is a lot of interest in the investigation of the existing connections between different classical string configurations, their semi-classical quantization and the relevant objects in the dual gauge theories, as well as between the corresponding integrable models appearing on the string and field theory sides (see e.g. \([1] - [86]\) and the references therein). In this connection, it seems useful to formulate an approach, which will allow us to obtain exact string solutions in general enough string theory backgrounds. Here, we describe such an approach, based on two types of string embedding, which allow for separation of the worldsheet variables \( \tau \) and \( \sigma \).

2 Exact string solutions in general backgrounds

In our further considerations, we will use the Polyakov type action for the bosonic string in a \( D \)-dimensional curved space-time with metric tensor \( g_{MN}(x) \), interacting with a background 2-form gauge field \( b_{MN}(x) \) via Wess-Zumino term

\[
S^P = \int d^2\xi \mathcal{L}^P, \quad \mathcal{L}^P = -\frac{1}{2} \left( T \sqrt{-\gamma} \gamma^{mn} G_{mn} - Q \varepsilon^{mn} B_{mn} \right),
\]

\[
\xi^m = (\xi^0, \xi^1) = (\tau, \sigma), \quad m, n = (0, 1),
\]

For previously obtained string solutions in curved space-times, see e.g. \([88, 89]\) and the references in \([89, 90]\).
where
\[ G_{mn} = \partial_m X^M \partial_n X^N g_{MN}, \quad B_{mn} = \partial_m X^M \partial_n X^N b_{MN}; \]
\[ (\partial_m = \partial/\partial \xi^m, \quad M, N = 0, 1, \ldots, D − 1), \]
are the fields induced on the string worldsheet, \( \gamma \) is the determinant of the auxiliary worldsheet metric \( \gamma_{mn} \), and \( \gamma^{mn} \) is its inverse. The position of the string in the background space-time is given by \( x^M = X^M(\xi^m) \), and \( T = 1/2\pi \alpha' \), \( Q \) are the string tension and charge, respectively. If we consider the action (2.1) as a bosonic part of a supersymmetric one, we have to put \( Q = \pm T \). In what follows, \( Q = T \).

The equations of motion for \( X^M \) following from (2.1) are:
\[ -g_{LK} \left[ \partial_m \left( \sqrt{-\gamma} \gamma^{mn} \partial_n X^K \right) + \sqrt{-\gamma} \gamma^{mn} \Gamma^K_{MN} \partial_m X^M \partial_n X^N \right] = \frac{1}{2} H_{LMN} \epsilon^{mn} \partial_m X^M \partial_n X^N, \]
\[ \text{where} \]
\[ \Gamma_{LMN} = g_{LK} \Gamma^K_{MN} = \frac{1}{2} \left( \partial_M g_{NL} + \partial_N g_{ML} − \partial_L g_{MN} \right), \]
\[ H_{LMN} = \partial_L b_{MN} + \partial_M b_{NL} + \partial_N b_{LM}, \]
are the components of the symmetric connection corresponding to the metric \( g_{MN} \), and the field strength of the gauge field \( b_{MN} \) respectively. The constraints are obtained by varying the action (2.1) with respect to \( \gamma_{mn} \):
\[ \delta \gamma_{mn} S^P = 0 \Rightarrow \left( \gamma^{kl} \gamma^{mn} - 2 \gamma^{km} \gamma^{ln} \right) G_{mn} = 0. \]

In solving the equations of motion (2.2) and constraints (2.3), we will use the worldsheet gauge \( \gamma_{mn} = \text{constants} \). This will allow us to consider the tensionless limit \( T \rightarrow 0 \), corresponding to small t’Hooft coupling \( \lambda \rightarrow 0 \) on the field theory side. Of course, we can always set \( \gamma^{mn} = \eta^{mn} = \text{diag}(-1, 1) \), to turn to the usually used conformal gauge.

We will investigate the string dynamics in the framework of the following two types of embedding:
\[ X^\mu(\tau, \sigma) = \Lambda^\mu_0 \tau + \Lambda^\mu_1 \sigma + Y^\mu(\tau), \quad X^a(\tau, \sigma) = Y^a(\tau); \]
\[ X^\mu(\tau, \sigma) = \Lambda^\mu_0 \tau + \Lambda^\mu_1 \sigma + Z^\mu(\sigma), \quad X^a(\tau, \sigma) = Z^a(\sigma); \]
\[ \Lambda^\mu_m = \text{const}, \quad (m = 0, 1). \]

Here, the embedding coordinates \( X^M(\tau, \sigma) \) are divided into \( X^M = (X^\mu, X^a) \), where \( X^\mu(\tau, \sigma) \) correspond to the space-time coordinates \( x^\mu \), on which the background fields do not depend
\[ \partial_\mu g_{MN} = 0, \quad \partial_\mu b_{MN} = 0. \]

In other words, we suppose that there exist \( n_\mu \) commuting Killing vectors \( \partial/\partial x^\mu \), where \( n_\mu \) is the number of the coordinates \( x^\mu \). In this case, the ansätze (2.4) and (2.5) allow for separation of the variables \( \tau \) and \( \sigma \). By using (2.4), one obtains \( \tau \)-dependent dynamics, while by using (2.5), one obtains \( \sigma \)-dependent dynamics.
2.1 \( \tau \)-dependent dynamics

Here, we are going to use the ansatz (2.4) for the string embedding coordinates. In addition, we assume that the conditions (2.6) on the background fields hold.

As far as only two of the constraints (2.3) are independent, we have to choose consistently two of them. Our independent constraints, with which we will work in this subsection, are given by

\[ \gamma^{00}G_{00} - \gamma^{11}G_{11} = 0 \]  

and

\[ \gamma^{00}G_{01} + \gamma^{01}G_{11} = 0. \]  

2.1.1 The case \( Y^\mu(\tau) = 0 \)

Let us start with considering the particular case, when in (2.4) \( Y^\mu(\tau) = 0 \), i.e. \( X^\mu \) depend on \( \tau \) and \( \sigma \) linearly. Then, the Lagrangian density, the induced fields, the constraints (2.7) and (2.8) respectively, and the Euler-Lagrange equations for \( X^M \) (2.2), can be written as (the over-dot is used for \( d/d\tau \))

\[ L_A(\tau) = -\frac{T}{2} \sqrt{-\gamma} \left[ \gamma^{00}g_{ab} \dot{Y}^a \dot{Y}^b + 2 \left( \gamma^{0n}g_{\nu\mu} \Lambda_0^\nu - \frac{1}{\sqrt{-\gamma}} \Lambda_1^\nu b_{\nu\mu} \right) \dot{Y}^a + \gamma^{mn} \Lambda_0^\mu \Lambda_0^\nu g_{\mu\nu} - \frac{2}{\sqrt{-\gamma}} \Lambda_0^\mu \Lambda_1^\nu b_{\mu\nu} \right]; \]  

\[ G_{00} = g_{ab} \dot{Y}^a \dot{Y}^b + 2 \Lambda_0^\mu g_{\nu\mu} \dot{Y}^a + \Lambda_0^\mu \Lambda_0^\nu g_{\mu\nu}, \]  

\[ G_{01} = \Lambda_1^\nu \left( g_{\nu a} \dot{Y}^a + \Lambda_0^\mu g_{\mu\nu} \right), \quad G_{11} = \Lambda_1^\mu \Lambda_1^\nu g_{\mu\nu}; \]  

\[ B_{01} = -\Lambda_1^\mu \left( b_{\mu a} \dot{Y}^a + \Lambda_0^\nu b_{\mu\nu} \right), \]  

\[ \gamma^{00}g_{ab}\dot{Y}^a\dot{Y}^b + 2\gamma^{00}\Lambda_0^\nu g_{\nu a} \dot{Y}^a + \left( \gamma^{00} \Lambda_0^\mu \Lambda_0^\nu - \gamma^{11} \Lambda_1^\mu \Lambda_1^\nu \right) g_{\mu\nu} = 0, \]  

\[ \Lambda_1^\nu \left( \gamma^{00}g_{\nu a} \dot{Y}^a + \gamma^{0n} \Lambda_0^\nu g_{\mu\nu} \right) = 0; \]  

\[ \gamma^{00} \left( g_{Lb} \dot{Y}^b + \Gamma_{L,cb} \dot{Y}^b \dot{Y}^c \right) + 2\gamma^{0n} \Lambda_0^\mu \Gamma_{L,\mu b} \dot{Y}^b + \gamma^{mn} \Lambda_0^\mu \Lambda_0^\nu \Gamma_{L,\mu \nu} \]  

\[ = -\frac{1}{\sqrt{-\gamma}} \Lambda_1^\nu \left( H_{L\nu \mu} \Lambda_0^\mu + H_{L\nu a} \dot{Y}^a \right). \]  

\( L_A(\tau) \) in (2.9) is like a Lagrangian for a point particle, interacting with the external fields \( g_{MN}, b_{\nu\mu} \) and \( b_{\mu\nu} \).

Let us write down the conserved quantities. By definition, the generalized momenta are

\[ P_L \equiv \frac{\partial L}{\partial (\partial_0 X^L)} = -T \left( \sqrt{-\gamma} \gamma^{0n}g_{LN}\partial_n X^N - b_{LN} \partial_1 X^N \right). \]
For our ansatz, they take the form:

\[ P_L = -T \left[ \sqrt{-\gamma} \left( \gamma^{00} g_{La} \dot{Y}^a + \gamma^{0n} g_{Ln} \Lambda^\nu_n \right) - b_{Ln} \Lambda^\nu_n \right]. \]

The Lagrangian (2.9) does not depend on the coordinates \( X^\mu \). Therefore, the conjugated momenta \( P_\mu \) are conserved

\[ \gamma^{00} g_{\mu a} \dot{Y}^a + \gamma^{0n} \Lambda^\nu_n g_{\mu n} - \frac{1}{\sqrt{-\gamma}} \Lambda^\nu_n b_{\mu n} = -\frac{P_\mu}{T \sqrt{-\gamma}} = \text{constants}. \quad (2.14) \]

The same result can be obtained by solving the equations of motion (2.13) for \( L = \lambda \).

From (2.12) and (2.14), one obtains the following compatibility condition

\[ \Lambda^\nu_1 P_\nu = 0. \quad (2.15) \]

This equality may be interpreted as a solution of the constraint (2.12), which restricts the number of the independent parameters in the theory.

With the help of (2.14), the other constraint, (2.11), can be rewritten in the form

\[ g_{ab} \ddot{Y}^a \dot{Y}^b = U, \quad (2.16) \]

where \( U \) is given by

\[ U = \frac{1}{\gamma^{00}} \left[ \gamma^{mn} \Lambda^\mu_m \Lambda^\nu_n + \frac{2 \Lambda^\mu_0}{T \sqrt{-\gamma}} (P_\mu - T \Lambda^\nu_1 b_{\mu n}) \right]. \quad (2.17) \]

Now, let us turn to the equations of motion (2.13), corresponding to \( L = a \). By using the explicit expressions for \( \Gamma_{a,\mu b}, \Gamma_{a,\mu \nu}, H_{a\mu \nu} \) and \( H_{a b \nu} \), one obtains

\[ g_{ab} \ddot{Y}^b + \Gamma_{a,\mu b} \dot{Y}^b \dot{Y}^c = \frac{1}{2} \partial_a U + 2 \partial_{[a} A_{b]} \dot{Y}^b. \quad (2.18) \]

In (2.18), an effective potential \( U \) and an effective gauge field \( A_a \) appeared. \( U \) is given in (2.17), and

\[ A_a = \frac{1}{\gamma^{00}} \left( \gamma^{0n} \Lambda^\mu_m g_{\mu n} - \frac{\Lambda^\mu_0 b_{\mu n}}{\sqrt{-\gamma}} \right). \quad (2.19) \]

The reduced equations of motion (2.18) are as for a point particle moving in the gravitational field \( g_{ab} \), in the potential \( U \) and interacting with the 1-form gauge field \( A_a \) through its field strength \( F_{ab} = 2 \partial_{[a} A_{b]} \). The corresponding Lagrangian is

\[ L_{\text{red}}^A (\tau) = -\frac{T}{2} \sqrt{-\gamma} \gamma^{00} \left( g_{ab} \dot{Y}^a \dot{Y}^b + 2 A_a \dot{Y}^a + U \right) + \Lambda^\nu_1 P_\nu. \]

Now our task is to find exact solutions of the nonlinear differential equations (2.16) and (2.18). It turns out that for background fields depending on only one coordinate \( x^a \), we can always integrate these equations, and the solution is ²

\[ \tau (X^a) = \tau_0 \pm \int_{x^a_0}^{X^a} dx \left( \frac{U}{g_{aa}} \right)^{-1/2}. \quad (2.20) \]

²In this case, the constraint (2.16) is first integral for the equation of motion (2.18).
Otherwise, supposing the metric \( g_{ab} \) is a diagonal one, (2.18) and (2.16) reduce to
\[
\frac{d}{d\tau} (g_{aa} \dot{Y}^a) - \frac{1}{2} \left[ \partial_a g_{aa} (\dot{Y}^a)^2 + \partial_a \mathcal{U} \right] - \frac{1}{2} \sum_{b \neq a} \left[ \partial_a g_{bb} (\dot{Y}^b)^2 + 4 \partial_a \mathcal{A}_b \dot{Y}^b \right] = 0, \quad (2.21)
\]
\[
g_{aa} (\dot{Y}^a)^2 + \sum_{b \neq a} g_{bb} (\dot{Y}^b)^2 = \mathcal{U}. \quad (2.22)
\]
With the help of the constraint (2.22), we can rewrite the equations of motion (2.21) in the form
\[
\frac{d}{d\tau} (g_{aa} \dot{Y}^a)^2 - \dot{Y}^a \partial_a (g_{aa} \mathcal{U}) + \dot{Y}^a \sum_{b \neq a} \left[ \partial_a \left( \frac{g_{aa}}{g_{bb}} \right) (g_{bb} \dot{Y}^b)^2 - 4 g_{aa} \partial_a \mathcal{A}_b \dot{Y}^b \right] = 0. \quad (2.23)
\]
To find solutions of the above equations without choosing particular background, we can fix all coordinates \( Y^a \) except one. Then the exact string solution of the equations of motion and constraints is given again by the same expression (2.20) for \( \tau (X^a) \).

To find solutions depending on more than one coordinate, we have to impose further conditions on the background fields. Let us first consider the simpler case, when the last two terms in (2.23) are not present. This may happen, when
\[
\partial_a \left( \frac{g_{aa}}{g_{bb}} \right) = 0, \quad \mathcal{A}_a = 0. \quad (2.24)
\]
Then, the first integrals of (2.23) are
\[
(g_{aa} \dot{Y}^a)^2 = D_a (Y^{b \neq a}) + g_{aa} \mathcal{U}, \quad (2.25)
\]
where \( D_a \) are arbitrary functions of their arguments. These solutions must be compatible with the constraint (2.22), which leads to the condition
\[
\sum_a D_a g_{aa} = (1 - n_a) \mathcal{U},
\]
where \( n_a \) is the number of the coordinates \( Y^a \). From here, one can express one of the functions \( D_a \) through the others. To this end, we split the index \( a \) in such a way that \( Y^r \) is one of the coordinates \( Y^a \), and \( Y^\alpha \) are the others. Then
\[
D_r = -g_{rr} \left( n_a \mathcal{U} + \sum_a D_a g_{aa} \right) ,
\]
and by using this, one rewrites the first integrals (2.25) as
\[
(g_{rr} \dot{Y}^r)^2 = g_{rr} \left[ (1 - n_a) \mathcal{U} - \sum_a D_a g_{aa} \right] \geq 0, \quad (g_{aa} \dot{Y}^a)^2 = D_a (Y^{a \neq a}) + g_{aa} \mathcal{U} \geq 0, \quad (2.26)
\]
where \( n_a \) is the number of the coordinates \( Y^\alpha \). Thus, the constraint (2.22) is satisfied identically.
Now we turn to the general case, when all terms in the equations of motion (2.23) are present. The aim is to find conditions, which will allow us to reduce the order of the equations of motion by one. An example of such sufficient conditions, is given below:

\[ A_\alpha \equiv (A_r, A_\alpha) = (A_r, \partial_r f), \quad \partial_\alpha \left( \frac{g_{\alpha\alpha}}{g_{aa}} \right) = 0, \]

\[ \partial_\alpha \left( g_{\alpha\alpha} \dot{Y}^\alpha \right)^2 = 0, \quad \partial_r \left( g_{\alpha\alpha} \dot{Y}^\alpha \right)^2 = 0. \]

By using the restrictions given above, one obtains the following first integrals of the equations (2.23), compatible with the constraint (2.22)

\[ \left( g_{\alpha\alpha} \dot{Y}^\alpha \right)^2 = g_{\alpha\alpha} \left[ (1 - n_\alpha) U - \sum \frac{D_\alpha}{g_{\alpha\alpha}} - 3n_\alpha (A_r - \partial_r f) \dot{Y}^r \right] = E_\alpha (Y^\alpha) \geq 0, \quad (2.27) \]

\[ \left( g_{\alpha\alpha} \dot{Y}^\alpha \right)^2 = g_{\alpha\alpha} \left[ U + 2 (A_r - \partial_r f) \dot{Y}^r \right] = E_\alpha (Y^\alpha) \geq 0, \quad (2.28) \]

where \( D_\alpha, E_\alpha \) and \( E_r \) are arbitrary functions of their arguments.

Further progress is possible, when working with particular background configurations, allowing for separation of the variables in (2.26), or in (2.27) and (2.28).

Our results obtained so far are not applicable to tensionless (null) strings, because the action (2.1) is proportional to the string tension \( T \).

Allowing for separation of the variables in (2.26), or in (2.27) and (2.28).

The reduced equations of motion and constraints (2.18) and (2.16) have the same form, corresponding to the zero tension limit \( T \to 0 \).

An example of such independent constraints. In these notations, the constraints (2.11) and (2.12), the equations of motion (2.13), and the conserved momenta (2.14) take the form

\[ g_{\alpha\beta} \dot{Y}^\alpha \dot{Y}^\beta + 2 \Lambda_0^\nu g_{\nu\alpha} \dot{Y}^\alpha + \left\{ \left( \Lambda_0^\mu \Lambda_0^\nu + \left[ (2\lambda_0^0 T)^2 - (\lambda_1^0)^2 \right] \Lambda_1^\mu \Lambda_1^\nu \right) g_{\mu\nu} = 0, \right. \]

\[ \Lambda_1^\nu \left[ g_{\nu\alpha} \dot{Y}^\alpha + \left( \Lambda_0^\mu - \lambda_1^1 \Lambda_1^\mu \right) g_{\mu\nu} \right] = 0; \]

\[ g_{\beta\nu} \dot{Y}^\beta + \Gamma_{L,\nu} \dot{Y}^\nu \dot{Y}^\nu + 2 \left( \Lambda_0^\mu - \lambda_1^1 \Lambda_1^\mu \right) \Gamma_{L,\mu\nu} \dot{Y}^b \]

\[ + \left[ \left( \Lambda_0^\mu - \lambda_1^1 \Lambda_1^\mu \right) \left( \Lambda_0^\nu - \lambda_1^1 \Lambda_1^\nu \right) - (2\lambda_0^0 T)^2 \Lambda_1^\mu \Lambda_1^\nu \right] \Gamma_{L,\mu\nu} = 2\lambda_0^0 T \Lambda_1^\nu \left( H_{L,b} \dot{Y}^b + \Lambda_0^\mu H_{L,\mu} \right); \]

\[ g_{\mu\alpha} \dot{Y}^\alpha + \left( \Lambda_0^\alpha - \lambda_1^1 \Lambda_1^\alpha \right) g_{\mu\nu} = 2\lambda_0^0 T \Lambda_1^\nu b_{\mu\nu} = 2\lambda_0^0 P_\mu. \]

The reduced equations of motion and constraints (2.18) and (2.16) have the same form, but now, the effective potential (2.17) and the effective gauge field (2.19) are given by

\[ U^\Lambda = \left[ \left( \Lambda_0^\mu - \lambda_1^1 \Lambda_1^\mu \right) \left( \Lambda_0^\nu - \lambda_1^1 \Lambda_1^\nu \right) - (2\lambda_0^0 T)^2 \Lambda_1^\mu \Lambda_1^\nu \right] g_{\mu\nu} - 4\lambda_0^0 \Lambda_0^\mu \left( P_\mu - T \Lambda_1^\mu b_{\mu\nu} \right), \]

\[ A^\Lambda_\alpha = \left( \Lambda_0^\alpha - \lambda_1^1 \Lambda_1^\alpha \right) g_{\mu\nu} + 2\lambda_0^0 T \Lambda_1^\nu b_{\mu\nu}. \]

If one sets \( \lambda_1^1 = 0 \) and \( 2\lambda_0^0 T = 1 \), the results in conformal gauge are obtained, as it should be. If one puts \( T = 0 \) in the above formulas, they will describe tensionless strings.
2.1.2 The case \( Y^{\mu}(\tau) \neq 0 \)

By using the ansatz (2.4), one obtains that the Lagrangian density, the induced fields, the constraints (2.7) and (2.8) respectively, and the Euler-Lagrange equations for \( X^{M} \) (2.2) are given by

\[
\mathcal{L}^{GA}(\tau) = -\frac{T}{2} \sqrt{-\gamma} \left[ \gamma^{00} g_{MN} \dot{Y}^{M} \dot{Y}^{N} + 2 \left( \gamma^{0n} \Lambda_{n}^{\mu} g_{\mu\nu} - \frac{\Lambda_{1}^{\nu} b_{\mu\nu}}{\sqrt{-\gamma}} \right) \dot{Y}^{M} + \gamma^{mn} \Lambda_{m}^{\mu} \Lambda_{n}^{\nu} g_{\mu\nu} - \frac{2 \Lambda_{0}^{\mu} \Lambda_{n}^{\nu} b_{\mu\nu}}{\sqrt{-\gamma}} \right];
\]

\[
G_{00} = g_{MN} \dot{Y}^{M} \dot{Y}^{N} + 2 \gamma^{0\nu} \Lambda_{0}^{\mu} g_{\nu\mu} \dot{Y}^{N} + \Lambda_{\mu}^{\nu} \Lambda_{0}^{\nu} g_{\mu\nu}, \quad G_{01} = \Lambda_{1}^{\nu} \left( g_{\nu N} \dot{Y}^{N} + \Lambda_{0}^{\nu} g_{\mu\nu} \right), \quad G_{11} = \Lambda_{1}^{\mu} \Lambda_{1}^{\nu} g_{\mu\nu};
\]

\[
B_{01} = -\Lambda_{1}^{\mu} \left( b_{\mu N} \dot{Y}^{N} + \Lambda_{0}^{\nu} b_{\mu\nu} \right);
\]

\[
\gamma^{00} g_{MN} \dot{Y}^{M} \dot{Y}^{N} + 2 \gamma^{0\nu} \Lambda_{0}^{\mu} g_{\nu N} \dot{Y}^{N} + \left( \gamma^{00} \Lambda_{0}^{\mu} \Lambda_{1}^{\nu} - \gamma^{11} \Lambda_{1}^{\mu} \Lambda_{1}^{\nu} \right) g_{\mu\nu} = 0, \quad (2.30)
\]

\[
\Lambda_{1}^{\nu} \left( \gamma^{00} g_{\nu N} \dot{Y}^{N} + \gamma^{0\nu} \Lambda_{n}^{\mu} g_{\mu\nu} \right) = 0; \quad (2.31)
\]

\[
\gamma^{00} \left( g_{LN} \dot{Y}^{N} + \Gamma_{L,MN} \dot{Y}^{M} \dot{Y}^{N} \right) + 2 \gamma^{0\nu} \Lambda_{n}^{\mu} \Gamma L_{\mu\nu} \dot{Y}^{N} + \gamma^{mn} \Lambda_{m}^{\mu} \Lambda_{n}^{\nu} \Gamma L_{\mu\nu} = -\frac{1}{\sqrt{-\gamma}} \Lambda_{1}^{\nu} \left( H_{LM\nu} \dot{Y}^{M} + \Lambda_{0}^{\mu} H_{L\mu\nu} \right). \quad (2.32)
\]

The conserved momenta \( P_{\mu} \) can be found as before, and now they are

\[
\gamma^{00} g_{\mu N} \dot{Y}^{N} + \gamma^{0\nu} \Lambda_{n}^{\mu} g_{\mu\nu} - \frac{\Lambda_{1}^{\nu} b_{\mu\nu}}{\sqrt{-\gamma}} = -\frac{P_{\mu}}{T \sqrt{-\gamma}} = \text{constants}. \quad (2.33)
\]

The compatibility condition following from the constraint (2.32) and from (2.34) coincides with the previous one (2.15). With the help of (2.34), the equations of motion (2.33) corresponding to \( L = a \) and the other constraint (2.31), can be rewritten in the form

\[
g_{aN} \dot{Y}^{N} + \Gamma_{a,MN} \dot{Y}^{M} \dot{Y}^{N} = \frac{1}{2} \partial_{a} \mathcal{U} + 2 \partial_{[a} \mathcal{A}_{N]} \dot{Y}^{N}, \quad (2.35)
\]

\[
g_{MN} \dot{Y}^{M} \dot{Y}^{N} = \mathcal{U}, \quad (2.36)
\]

where \( \mathcal{U} \) is given by (2.17) and

\[
\mathcal{A}_{N} = \frac{1}{\gamma^{00}} \left( \gamma^{0\nu} \Lambda_{n}^{\mu} g_{N\mu} - \frac{\Lambda_{1}^{\nu} b_{N\mu}}{\sqrt{-\gamma}} \right); \quad (2.37)
\]

coincides with (2.19) for \( N = a \).
Now we are going to eliminate the variables \( \dot{Y}^\mu \) from (2.35) and (2.36). To this end, we express \( \dot{Y}^\mu \) through \( \dot{Y}^a \) from the conservation laws (2.34): 

\[
\dot{Y}^\mu = -\frac{\gamma_0}{\gamma_{00}} A^\mu_n - (g^{-1})^{\mu\nu} \left[ g_{\nu a} \dot{Y}^a + \frac{1}{\gamma_{00} T \sqrt{-\gamma}} (P_\nu - T \Lambda_1^0 b_{\nu \rho}) \right]. 
\] (2.38)

After using (2.38) and (2.15), the equations of motion (2.35) and the constraint (2.36) acquire the form

\[
h_{ab} \ddot{Y}^b + \Gamma^h_{a, bc} \dot{Y}^b \dot{Y}^c = \frac{1}{2} \partial_a U^h + 2 \partial_a [\dot{a} h^b] \dot{Y}^b, \quad (2.39)
\]

\[
h_{ab} \dot{Y}^a \dot{Y}^b = U^h, \quad (2.40)
\]

where a new, effective metric appeared

\[
h_{ab} = g_{ab} - g_{a\mu} (g^{-1})^{\mu\nu} g_{\nu b}.
\]

\( \Gamma^h_{a, bc} \) is the symmetric connection corresponding to this metric

\[
\Gamma^h_{a, bc} = \frac{1}{2} (\partial_b h_{ca} + \partial_c h_{ba} - \partial_a h_{bc}).
\]

The new effective scalar and gauge potentials, expressed through the background fields, are as follows

\[
U^h = \frac{1}{\gamma (\gamma_{00})^2} \left[ \Lambda_1^\mu \Lambda_1^\nu g_{\mu\nu} + \frac{1}{T^2} (P_\mu - T \Lambda_1^0 b_{\mu \rho}) (g^{-1})^{\mu\nu} (P_\nu - T \Lambda_1^0 b_{\nu \rho}) \right],
\]

\[
A^h_a = -\frac{1}{\gamma_{00} T \sqrt{-\gamma}} \left[ g_{a\mu} (g^{-1})^{\mu\nu} (P_\nu - T \Lambda_1^0 b_{\nu \rho}) + T \Lambda_1^0 b_{a\rho} \right].
\]

We point out the qualitatively different behavior of the potentials \( U^h \) and \( A^h_a \), compared to \( U \) and \( A_a \), due to the appearance of the inverse metric \((g^{-1})^{\mu\nu}\). The corresponding Lagrangian is

\[
L^{GA}_{red}(\tau) = -\frac{T}{2} \sqrt{-\gamma} \gamma_{00} \left( h_{ab} \dot{Y}^a \dot{Y}^b + 2 A^h_a \dot{Y}^a + U^h \right) + \frac{d}{d\tau} P_\mu (Y^\mu + \Lambda^0_1 \tau). \]

Since the equations (2.15), (2.16) and (2.39), (2.40) have the same form, for obtaining exact string solutions, we can proceed as before and use the previously derived formulas after the replacements \((g, \Gamma, U, A) \rightarrow (h, \Gamma^h, U^h, A^h)\). In particular, the solution depending on one of the coordinates \(X^a\) will be

\[
\tau (X^a) = \tau_0 \pm \int_{X_0^a}^{X^a} dx \left( \frac{U^h}{h_{aa}} \right)^{-1/2}.
\] (2.41)

In this case by integrating (2.38), and replacing the solution for \( Y^\mu \) in the ansatz (2.4), one obtains the solution for the string coordinates \(X^\mu\):

\[
X^\mu (X^a, \sigma) = X_0^\mu + \Lambda_1^\mu \left[ \sigma - \frac{\gamma_{01}}{\gamma_{00}} \tau (X^a) \right] - 
- \int_{X_0^a}^{X^a} (g^{-1})^{\mu\nu} \left[ g_{\nu a} \pm \frac{P_\nu - T \Lambda_1^0 b_{\nu \rho}}{\gamma_{00} T \sqrt{-\gamma}} \left( \frac{U^h}{h_{aa}} \right)^{-1/2} \right] dx.
\] (2.42)
To be able to take the tensionless limit $T \to 0$ in the above formulas, we have to use the $\lambda$-parametrization \((2.29)\) of $\gamma^{mn}$. The quantities that appear in the reduced equations of motion and constraint \((2.39)\) and \((2.40)\), which depend on this parametrization, are $U^{h}$ and $A^{h}_{a}$. Now, they are given by

$$U^{h,\lambda} = -(2\lambda^{0})^{2} \left[ T^{2} \Lambda^{\mu}_{\lambda} \Lambda^{\nu}_{\rho} g_{\mu \rho} + (P_{\mu} - T \Lambda^{\mu}_{\rho} b_{\rho \lambda}) (g^{-1})^{\mu \nu} \left( P_{\nu} - T \Lambda^{\lambda}_{\nu} b_{\nu \lambda} \right) \right],$$

$$A^{h,\lambda}_{a} = 2\lambda^{0} \left[ g_{a \mu} (g^{-1})^{\mu \lambda} (P_{\lambda} - T \Lambda^{\lambda}_{\rho} b_{\rho \lambda}) + T \Lambda^{\rho}_{b} b_{a \rho} \right].$$

If one sets $\lambda^{1} = 0$ and $2\lambda^{0}T = 1$, the conformal gauge results are obtained. If one puts $T = 0$ in the above equalities, they will correspond to tensionless strings.

### 2.2 $\sigma$-dependent dynamics

In this subsection, we will use the ansatz \((2.5)\) for the string embedding coordinates. Of course, the conditions \((2.6)\) on the background fields are also fulfilled.

Our independent constraints, with which we will work in this subsection, are given by

$$\gamma^{00}G_{00} - \gamma^{11}G_{11} = 0, \quad (2.43)$$

and

$$\gamma^{01}G_{00} + \gamma^{11}G_{01} = 0. \quad (2.44)$$

#### 2.2.1 The case $Z^{\mu}(\sigma) = 0$

Taking into account the conditions \((2.6)\), one obtains the following reduced Lagrangian density, arising from the action \((2.1)\) (the prime is used for $d/d\sigma$)

$$\mathcal{L}^{A}(\sigma) = -\frac{T}{2} \sqrt{-\gamma} \left[ \gamma^{11} g_{ab} Z^{a} Z^{b} + 2 \left( \gamma^{1m} \Lambda^{\mu}_{m} g_{\mu a} - \frac{1}{\sqrt{-\gamma}} \Lambda^{\mu}_{0} b_{\mu a} \right) Z^{a} + \gamma^{mn} \Lambda^{\mu}_{m} \Lambda^{\nu}_{n} g_{\mu \nu} - \frac{2}{\sqrt{-\gamma}} \Lambda^{\mu}_{0} \Lambda^{\nu}_{0} b_{\mu \nu} \right], \quad (2.45)$$

where the fields induced on the string worldsheet are given by

$$G_{00} = \Lambda^{\mu}_{0} \Lambda^{\nu}_{0} g_{\mu \nu}, \quad G_{01} = \Lambda^{\mu}_{0} \left( g_{\mu a} Z^{a} + \Lambda^{\nu}_{1} g_{\mu a} \right), \quad G_{11} = g_{ab} Z^{a} Z^{b} + 2 \Lambda^{\mu}_{1} g_{\mu a} Z^{a} + \Lambda^{\mu}_{1} \Lambda^{\nu}_{1} g_{\mu \nu}; \quad (2.46)$$

$$B_{01} = \Lambda^{\mu}_{0} \left( b_{\mu a} Z^{a} + \Lambda^{\nu}_{1} b_{\mu a} \right);$$

The constraints \((2.43)\) and \((2.44)\) respectively, and the equations of motion for $X^{\mu}$ \((2.2)\), can be written as

$$\gamma^{11} \left( g_{ab} Z^{a} Z^{b} + 2 \Lambda^{\mu}_{1} g_{\mu a} Z^{a} \right) - \left( \gamma^{00} \Lambda^{\mu}_{0} \Lambda^{\nu}_{0} - \gamma^{11} \Lambda^{\mu}_{1} \Lambda^{\nu}_{1} \right) g_{\mu \nu} = 0, \quad (2.47)$$

$$\Lambda^{\mu}_{0} \left( \gamma^{11} g_{\mu a} Z^{a} + \gamma^{1n} \Lambda^{\nu}_{n} g_{\mu \nu} \right) = 0; \quad (2.48)$$

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\[(\gamma_{ab}Z^m_{\mu} + \Gamma_{L,\mu bc}Z^bZ^c) + 2\gamma_{a}^{m} \Lambda_{\mu m}^{\rho} \Gamma_{L,\mu \rho} + \gamma_{mn} \Lambda_{\mu n}^{\rho} \Gamma_{L,\mu \rho} \quad (2.49)\]

\[= -\frac{1}{\sqrt{-\gamma}} \Lambda_{\mu 0}^{\rho} (H_{L,\mu a}Z_{\rho a} + \Lambda_{\mu}^{\rho} H_{L,\mu \rho}).\]

Let us write down the conserved quantities. By definition, the generalized momenta are

\[P_{L} \equiv \frac{\partial L_{P}}{\partial (\partial_{0}X_{L})} = -T \left( \sqrt{-\gamma} \gamma_{0a}^{a} g_{LN} \partial_{n}X_{N} - b_{LN} \partial_{1}X_{N} \right).\]

For our case, they take the form:

\[P_{L} = -T \sqrt{-\gamma} \left[ (\gamma_{01}^{a} g^{ab} - \frac{1}{\sqrt{-\gamma}} b_{ab}^{b1}) Z^{b} + \gamma_{0n}^{a} \Lambda_{n}^{\rho} g_{L,\rho \nu} - \frac{1}{\sqrt{-\gamma}} \Lambda_{0}^{\rho} b_{L,\rho \nu} \right].\]

The Lagrangian (2.53) does not depend on the coordinates \(X^{\mu}\). Therefore, the conjugated momenta \(P_{\mu}\) do not depend on the proper time \(\tau\)

\[P_{\mu}(\sigma) = -T \sqrt{-\gamma} \left[ (\gamma_{01}^{a} g_{\mu b} - \frac{1}{\sqrt{-\gamma}} b_{\mu b}) Z^{b} + \gamma_{0n}^{a} \Lambda_{n}^{\rho} g_{L,\rho \nu} - \frac{1}{\sqrt{-\gamma}} \Lambda_{0}^{\rho} b_{L,\rho \nu} \right], \quad \partial_{0}P_{\mu} = 0. \quad (2.50)\]

In order for our ansatz to be consistent with the action (2.1), the following conditions must be fulfilled

\[\partial_{1}P_{\mu} \equiv \frac{\partial P_{\mu}}{\partial \sigma} = 0, \quad (2.51)\]

where

\[P_{M} \equiv \frac{\partial L_{P}}{\partial (\partial_{1}X_{M})} = -T \left( \sqrt{-\gamma} \gamma_{1n}^{m} g_{MN} \partial_{n}X_{N} + b_{MN} \partial_{0}X_{N} \right)\]

\[= -T \sqrt{-\gamma} \left[ \gamma_{11}^{a} g^{ab}Z^{b} + \gamma_{1n}^{a} \Lambda_{n}^{\rho} g_{L,\rho \nu} + \frac{1}{\sqrt{-\gamma}} \Lambda_{0}^{\rho} b_{L,\rho \nu} \right]. \quad (2.52)\]

This is because the equations of motion (2.22) can be rewritten as

\[\frac{\partial P_{M}}{\partial \tau} + \frac{\partial P_{M}}{\partial \sigma} - \frac{\partial L_{P}}{\partial X_{M}} = 0.\]

Hence, for \(M = \mu\), these equations take the form (2.51). Therefore, \(P_{\mu}\) are constants of the motion

\[\gamma_{11}^{a} g_{\mu a}Z^{a} + \gamma_{1n}^{a} \Lambda_{n}^{\rho} g_{L,\rho \nu} + \frac{1}{\sqrt{-\gamma}} \Lambda_{0}^{\rho} b_{L,\rho \nu} = - \frac{P_{\mu}}{T \sqrt{-\gamma}} = \text{constants}. \quad (2.53)\]

From (2.48) and (2.53), one obtains the following compatibility condition

\[\Lambda_{0}^{\rho} P_{\rho} = 0. \quad (2.54)\]

\(^{3}\)Actually, all momenta \(P_{M}\) do not depend on \(\tau\), because there is no such dependence in (2.53).
This equality may be interpreted as a solution of the constraint (2.48), which restricts the number of the independent parameters in the theory.

With the help of (2.53), the other constraint, (2.47), can be rewritten in the form

\[ g_{ab} Z^a Z^b = U^s, \]  

where \( U^s \) is given by

\[ U^s = \frac{1}{\gamma^{11}} \left[ \gamma^{mn} \Lambda_m^\mu \Lambda_n^\nu g_{\mu\nu} + \frac{2\Lambda_1^\mu}{T\sqrt{-\gamma}} (P_\mu + T\Lambda_0^\nu b_{\mu\nu}) \right]. \]  

(2.56)

Now, let us turn to the equations of motion (2.13), corresponding to \( L = a \). In view of the conditions (2.6),

\[ \Gamma_{a,\mu b} = -\frac{1}{2} \left( \partial_a g_{\mu\nu} - \partial_b g_{\mu\nu} \right), \quad \Gamma_{a,\mu\nu} = -\frac{1}{2} \partial_a g_{\mu\nu}, \]

\[ H_{a\mu\nu} = \partial_a b_{\mu\nu}; \quad H_{ab\nu} = \partial_a b_{\mu\nu} - \partial_b b_{a\nu} = 2\partial_a b_{\mu\nu}. \]

By using this, one obtains

\[ g_{ab} Z^{\prime\prime b} + \Gamma_{a,bc} Z^b Z^c = \frac{1}{2} \partial_a U^s + 2 \partial_{[a} A_{b]}^s Z^b. \]  

(2.57)

In (2.57), an effective scalar potential \( U^s \) and an effective 1-form gauge field \( A_a^s \) appeared. \( U^s \) is given in (2.56) (and is the same as in the effective constraint (2.55)), and

\[ A_a^s = \frac{1}{\gamma^{11}} \left( \gamma^{1m} \Lambda_m^\mu g_{\alpha\mu} + \frac{1}{\sqrt{-\gamma}} \Lambda_0^\mu b_{\alpha\mu} \right). \]  

(2.58)

The corresponding Lagrangian is

\[ \mathcal{L}_{\text{red}}^A = -\frac{T}{2} \sqrt{-\gamma^{11}} \left( g_{ab} Z^{\prime a} Z^{\prime b} + 2 A_a^s Z^{\prime a} + U^s \right) + \Lambda_1^\mu P_\mu. \]

Now our task is to find exact solutions of the nonlinear differential equations (2.55) and (2.57).

If the background seen by the string depends on only one coordinate \( x^a \), the general solution for the string embedding coordinate \( X^a(\tau, \sigma) = Z^a(\sigma) \) is given by

\[ \sigma(X^a) = \sigma_0 + \int_{X_0^a}^{X^a} \frac{U^s}{g_{aa}} \left( U^s \right)^{-1/2} d\sigma. \]

When the background felt by the string depends on more than one coordinate \( x^a \), the first integrals of the equations of motion for \( Z^a(\sigma) = (Z^r, Z^\alpha) \), which also solve the constraint (2.55), are

\[ (g_{rr} Z^{\prime r})^2 = g_{rr} \left[ (1 - n_\alpha) U^s - 2n_\alpha (A_r^s - \partial_r f) Z^{\prime r} - \sum_\alpha \frac{D_\alpha (Z^{\prime\alpha} - \partial_r f) Z^{\prime\alpha}}{g_{\alpha\alpha}} \right] = F_r (Z^r) \geq 0, \]

\[ (g_{aa} Z^{\prime a})^2 = D_\alpha (Z^{\prime\alpha} - \partial_r f) + g_{aa} [U^s + 2 (A_r^s - \partial_r f) Z^{\prime r}] = F_\alpha (Z^\beta \alpha) \geq 0, \]
where $Z^r$ is one of the coordinates $Z^a$, $Z^a$ are the remaining ones, $n_a$ is the number of $Z^a$, and $D_a$, $F_a$ are arbitrary functions of their arguments. The above expressions are valid, if the $g_{ab}$ part of the metric is diagonal one, and the following integrability conditions hold

\[
A_a^\nu \equiv (A_a^\nu, A_a^\nu) = (A_a^\nu, \partial_a f), \quad \partial_a \left( \frac{g_{\alpha \alpha}}{g_{\mu \mu}} \right) = 0, \quad \partial_r (g_{\alpha \alpha} Z^\alpha) = 0.
\]

In the parametrization (2.29) of $\gamma^{nm}$, the action (2.41) becomes

\[
S_\lambda = \int d^2 \xi \left\{ \frac{1}{4 \lambda^0} \left[ G_{00} - 2 \lambda^1 G_{01} + (\lambda^1)^2 G_{11} - (2 \lambda^0 T)^2 G_{11} \right] + T B_{01} \right\}.
\]

In these notations, the constraints (2.47) and (2.48), the equations of motion (2.49), and the conserved quantities (2.50), (2.53) take the form

\[
g_{ab} Z^a Z^b + 2 \Lambda_1^a g_{ab} Z^b + \left[ \frac{\Lambda_0^a \Lambda_0^b}{(2 \lambda^0 T)^2 - (\lambda^1)^2} + \Lambda_1^a \Lambda_1^b \right] g_{\mu \nu} = 0,
\]

\[
\Lambda_0^a \left\{ g_{a \mu} Z^a + \left[ \frac{\lambda^1 \Lambda_0^a}{(2 \lambda^0 T)^2 - (\lambda^1)^2} + \Lambda_1^a \right] g_{\nu \mu} \right\} = 0;
\]

\[
g_{Lb} Z^b + \Gamma_{L, bc} Z^b Z^c + 2 \left[ \frac{\lambda^1 \Lambda_0^a}{(2 \lambda^0 T)^2 - (\lambda^1)^2} + \Lambda_1^a \right] \Gamma_{L, \mu b} Z^b + \left[ \frac{\Lambda_0^a (2 \lambda^1 \Lambda_1^a - \Lambda_0^a)}{(2 \lambda^0 T)^2 - (\lambda^1)^2} + \Lambda_1^a \Lambda_1^a \right] \Gamma_{L, \mu b} Z^b + \left[ \frac{2 \lambda^0 T}{(2 \lambda^0 T)^2 - (\lambda^1)^2} \right] \Lambda_0^a (H_{L a \mu} Z^a + \Lambda_1^a H_{L \mu a}) = 0.
\]

\[
P_\mu (\sigma) = \frac{1}{2 \lambda^0} \left[ \left( - \lambda^1 g_{\mu a} + 2 \lambda^0 T b_{\mu a} \right) Z^a + \left( \Lambda_0^a - \lambda^1 \Lambda_1^a \right) g_{\mu \nu} + 2 \lambda^0 T \Lambda_1^a b_{\mu \nu} \right],
\]

\[
P_\mu = -\frac{1}{2 \lambda^0} \left\{ \left[ (2 \lambda^0 T)^2 - (\lambda^1)^2 \right] (g_{a \mu} Z^a + \Lambda_0^a g_{\mu \nu}) + \Lambda_1^a \left( \lambda^1 g_{\mu \nu} + 2 \lambda^0 T b_{\mu \nu} \right) \right\}.
\]

The reduced equations of motion and constraint (2.57) and (2.58) have the same form, but now, the effective potential (2.54) and the effective gauge field (2.58) are given by

\[
U^{a, \lambda} = \left[ \frac{\Lambda_0^a (2 \lambda^1 \Lambda_1^a - \Lambda_0^a)}{(2 \lambda^0 T)^2 - (\lambda^1)^2} + \Lambda_1^a \Lambda_1^a \right] g_{\mu \nu} + \frac{4 \lambda^0}{(2 \lambda^0 T)^2 - (\lambda^1)^2} \Lambda_0^a (P_\mu + T \Lambda_1^a b_{\mu \nu}),
\]

\[
A^{a, \lambda}_\alpha = \left[ \frac{\lambda^1 \Lambda_0^a}{(2 \lambda^0 T)^2 - (\lambda^1)^2} + \Lambda_1^a \right] g_{\mu \nu} + \frac{2 \lambda^0 T}{(2 \lambda^0 T)^2 - (\lambda^1)^2} \Lambda_0^a b_{\mu \nu}.
\]

If one sets $\lambda^1 = 0$ and $2 \lambda^0 T = 1$, this will correspond to conformal gauge, as it should be. If one puts $T = 0$ in the above formulas, they will describe tensionless strings.
\subsection{The case \( Z^\mu(\sigma) \neq 0 \)}

Taking into account the ansatz (2.5), one obtains that the induced fields \( G_{mn} \) and \( B_{mn} \), the Lagrangian density, the constraints (2.43) and (2.44) respectively, and the Euler-Lagrange equations for \( X^M (2.2) \) are given by

\[
\begin{align*}
G_{00} &= \Lambda_0^\mu \Lambda_0^{-\mu} g_{\mu\nu}, \quad G_{01} = \Lambda_0^\mu \left( g_{\mu N} Z^{\nu N} + \Lambda_1^\nu g_{\mu\nu} \right), \\
G_{11} &= g_{MN} Z^{IM} Z^{IN} + 2 \Lambda_1^\nu g_{\mu N} Z^{IN} + \Lambda_1^\mu \Lambda_1^{-\mu} g_{\mu\nu}; \\
B_{01} &= \Lambda_0^\mu \left( b_{\mu N} Z^{\nu N} + \Lambda_1^\nu b_{\mu\nu} \right);
\end{align*}
\]

\[
\mathcal{L}_{GA}^{\gamma}(\sigma) = -\frac{T}{2} \sqrt{-\gamma} \left[ \gamma^{11} g_{MN} Z^{IM} Z^{IN} + 2 \left( \gamma^{1m} \Lambda_m^\mu g_{\mu N} - \frac{\Lambda_0^\mu b_{\mu N}}{\sqrt{-\gamma}} \right) Z^{IN} + \right.
\]

\[
\left. + \gamma^{mn} \Lambda_m^\mu \Lambda_n^\nu g_{\mu\nu} - \frac{2 \Lambda_0^\mu \Lambda_1^\nu b_{\mu\nu}}{\sqrt{-\gamma}} \right];
\]

\[
\gamma^{11} g_{MN} Z^{IM} Z^{IN} + 2 \gamma^{11} \Lambda_1^\mu g_{\mu N} Z^{IN} - \left( \gamma^{00} \Lambda_0^\mu \Lambda_0^\nu - \gamma^{11} \Lambda_1^\mu \Lambda_1^\nu \right) g_{\mu\nu} = 0, \quad (2.60)
\]

\[
\Lambda_0^\mu \left( \gamma^{11} g_{MN} Z^{IN} + \gamma^{1n} \Lambda_n^\nu g_{\mu\nu} \right) = 0; \quad (2.61)
\]

\[
\gamma^{11} \left( g_{LN} Z^{\nu N} + \Gamma_{L,MN} Z^{IM} Z^{IN} \right) + 2 \gamma^{1m} \Lambda_m^\mu \Lambda_0^\nu \Gamma_{L,MN} Z^{IN} + \gamma^{mn} \Lambda_m^\mu \Lambda_n^\nu \Gamma_{L,MN} =
\]

\[
= -\frac{1}{\sqrt{-\gamma}} \Lambda_0^\mu \left( H_{LMN} Z^{IN} + \Lambda_1^\rho H_{LM\rho} \right). \quad (2.62)
\]

The quantities \( P_L, P_L \) can be found as before, and now they are

\[
\left( \gamma^{01} g_{LN} - \frac{b_{LN}}{\sqrt{-\gamma}} \right) Z^{IN} + \gamma^{0n} \Lambda_n^\nu g_{L\nu} - \frac{\Lambda_0^\mu b_{L\mu}}{\sqrt{-\gamma}} = -\frac{P_L}{T \sqrt{-\gamma}}, \quad \partial_0 P_L = 0, \quad (2.63)
\]

\[
\gamma^{11} g_{LN} Z^{IN} + \gamma^{1n} \Lambda_n^\nu g_{L\nu} + \frac{\Lambda_0^\mu b_{L\mu}}{\sqrt{-\gamma}} = -\frac{P_L}{T \sqrt{-\gamma}}, \quad \partial_0 P_L = 0, \quad \partial_\mu P_L = 0. \quad (2.64)
\]

The compatibility condition following from the constraint (2.61) and from (2.64) coincides with the previous one (2.54).

As in the previous subsection, the equations (2.62) for \( L = \lambda \) lead to \( \partial_1 P_L = 0 \). Consequently, our next task is to consider the equations (2.62) for \( L = a \) and the constraint (2.60). First of all, we will eliminate the variables \( Z^{\mu} \) from them. To this end, we express \( Z^{\mu} \) through \( Z^{\nu} \) by using (2.64):

\[
Z^{\mu} = -\frac{\gamma^{1m}}{\gamma^{11}} \Lambda_m^\mu - \left( g^{-1} \right)^{\mu\nu} \left[ g_{\nu a} Z^{\alpha a} + \frac{1}{T \sqrt{-\gamma}} \left( P_{\nu} + \Lambda_0^\rho b_{\nu\rho} \right) \right]. \quad (2.65)
\]

With the help of (2.65) and (2.54), the equations (2.62) for \( L = a \) and the constraint (2.60) acquire the form

\[
h_{ab} Z^{\nu b} + \Gamma_{a,bc}^{\nu} Z^{\nu b} Z^{\nu c} = \frac{1}{2} \partial_a U^g + 2 \partial_{[a} A_b^g Z_b^{\nu b}, \quad (2.66)
\]

\[
h_{ab} Z^{\mu a} Z^{\rho b} = U^g. \quad (2.67)
\]
The effective scalar and gauge potentials, expressed through the background fields, are as follows

$$U^g = \frac{1}{\gamma (\gamma^{11})^2} \left[ \Lambda_0^\mu \Lambda_0^\nu g_{\mu \nu} + \frac{1}{T^2} (P_\mu + T \Lambda_0^\alpha b_{\nu \rho}) (g^{-1})^{\mu \nu} (P_\nu + T \Lambda_0^\lambda b_{\nu \lambda}) \right],$$

$$A^g_a = -\frac{1}{T \sqrt{-\gamma^1 \gamma^{11}}} \left[ g_{a \mu} (g^{-1})^{\mu \nu} (P_\nu + T \Lambda_0^\alpha b_{\nu \rho}) - T \Lambda_0^\alpha b_{\alpha \rho} \right].$$

The corresponding Lagrangian is

$$\mathcal{L}^{G A}_{\text{red}}(\sigma) = -\frac{T}{2} \sqrt{-\gamma^1 \gamma^{11}} \left( h_{ab} Z^a Z^b + 2 A^g_a Z^a + U^g \right) + \frac{d}{d\sigma} P_\mu (Z^\mu + \Lambda_0^\lambda \sigma).$$

Since the equations \((2.57), (2.56)\) and \((2.66), (2.67)\) have the same form, for obtaining exact string solutions, we can proceed as before and use the derived formulas after the replacements \((g, \Gamma, U^a, A^a) \rightarrow (h, \Gamma^b, U^g, A^g)\). In particular, the solution depending on one of the coordinates \(X^a\) will be

$$\sigma (X^a) = \sigma_0 + \int_{X_0^a}^{X^a} dx \left( \frac{U^g}{h_{aa}} \right)^{-1/2}.$$

In this case by integrating \((2.65)\), and replacing the solution for \(Z^\mu\) in the ansatz \((2.66)\), one obtains solution for the string coordinates \(X^\mu\) of the type \(X^\mu (\tau, X^a)\):

$$X^\mu (\tau, X^a) = X_0^\mu + \Lambda_0^\mu \left[ \tau - \frac{\gamma^0}{\gamma^{11}} \sigma (X^a) \right] - \int_{X_0^a}^{X^a} (g^{-1})^{\mu \nu} \left[ g_{\nu a} + \frac{(P_\nu + T \Lambda_0^\alpha b_{\nu \rho})}{T \sqrt{-\gamma^1 \gamma^{11}}} \left( \frac{U^g}{h_{aa}} \right)^{-1/2} \right] dx.$$

To write down a solution of the type \(X^\mu (\tau, \sigma)\), one have to invert the solution \((2.68)\): \(\sigma (X^a) \rightarrow X^a (\sigma)\). Then, \(X^\mu (\tau, \sigma)\) are given by

$$X^\mu (\tau, \sigma) = X_0^\mu + \Lambda_0^\mu \left( \tau - \frac{\gamma^0}{\gamma^{11}} \sigma \right) - \int_{\sigma_0}^{\sigma} (g^{-1})^{\mu \nu} \left[ \frac{(P_\nu + T \Lambda_0^\alpha b_{\nu \rho})}{T \sqrt{-\gamma^1 \gamma^{11}}} + g_{\nu a} \left( \frac{U^g}{h_{aa}} \right)^{1/2} \right] d\sigma.$$

Let us also give the expression for \(P_\mu\) after the elimination of \(Z^\mu\) from \((2.63)\)

$$P_\mu (\sigma) = T \left[ b_{\mu a} - b_{\mu \nu} \left( g^{-1} \right)^{\nu \lambda} g_{\lambda a} \right] Z^a + \frac{1}{\gamma^{11}} \left\{ \gamma^0 \mathcal{P}_\mu + \frac{1}{\sqrt{-\gamma}} \left[ T \Lambda_0^\nu g_{\mu \nu} - b_{\mu \nu} \left( g^{-1} \right)^{\nu \lambda} (P_\lambda + T \Lambda_0^\rho b_{\nu \rho}) \right] \right\}.$$

These equalities connect the conserved momenta \(P_\mu\) with the constants of the motion \(P_\mu\).

To be able to take the tensionless limit \(T \rightarrow 0\) in the above formulas, we must use the \(\lambda\)-parametrization \((2.29)\) of \(\gamma^{mn}\). The quantities, which depend on this parametrization,
and appear in the reduced equations of motion and constraint (2.66), (2.67), and therefore in the solutions, are $U^g$ and $A^g_a$. Now, they read

$$U^g = -\frac{(2\lambda^0)^2}{[(2\lambda^0 T)^2 - (\lambda^1)^2]^2} \left[ T^2 \Lambda^\mu_0 \Lambda^\nu_0 g_{\mu\nu} + (P_\mu + T \Lambda^0_0 b_{\mu\nu}) (g^{-1})^{\mu\nu} \left( P_\nu + T \Lambda^0_0 b_{\nu\lambda} \right) \right],$$

$$A^g_a = -\frac{2 \lambda^0}{(2\lambda^0 T)^2 - (\lambda^1)^2} \left[ g_{\mu\nu} (g^{-1})^{\mu\nu} (P_\mu + T \Lambda^0_0 b_{\mu\nu}) - T \Lambda^0_0 b_{\mu\nu} \right].$$

If one sets $\lambda^1 = 0$ and $2\lambda^0 T = 1$, the conformal gauge results are obtained. If one puts $T = 0$ in the above equalities, they will correspond to tensionless strings. For instance, the solution $X^\mu(\tau, X^a)$ reduces to

$$X^\mu(\tau, X^a)_{T=0} = X^\mu_0 + \Lambda^\mu_0 \left[ \tau + \frac{\sigma(X^a)_{T=0}}{\lambda^1} \right] - \int_{X^a_0}^{X^a} (g^{-1})^{\mu\nu} \left[ g_{\nu a} - \frac{2 \lambda^0}{(\lambda^1)^2} P_\nu \left( U^g_{h^a a} \right)^{-1/2} \right] dx.$$

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