Research on Damage Identification of a Beam Based on Curvature Mode

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Abstract: To study the bridge damage location and degree identification ability based on curvature mode, with a three-span continuous steel beam as the research object, compared with the damage location identification of curvature modal difference and displacement modal difference, analyzing the influence of damage location identification on the curvature modal difference index considering the two factors of the measuring point number reduce and the noise, identifying the damage degree by the method of relative variation of the curvature modal shapes. The results show that the location damage identification effect of curvature modal difference is more obvious than that of displacement modal difference, the identification effect has higher requirements for the density of the measuring points and has low anti-noise ability; when the damage degree is recognized, the larger the damage degree is, the smaller the identification result error is. The method of curvature calculation can be incorporated into other dynamic fingerprint methods, and then the ability of damage identification will be further improved.

1. Introduction
There are many researches on strain mode since Hillary B. and Ewins D. J. put forward the theory of strain mode[1]. A. K. Pandy and Li Debao[2,3] proposed the concept of curvature mode based on the study of strain mode. The curvature mode/strain mode can reflect the change of local characteristics of structure, and can be obtained by various modes, and the sensitivity of curvature mode to local structure is much higher than that of displacement mode. Therefore, the curvature mode/strain mode is developed to meet this requirement[4,5], and has a good application prospect in structural condition monitoring. The existing results show that the local structural damage such as cracks and steel corrosion lead to the decrease of the structure local stiffness, resulting in the abrupt change of the curvature modal shape curve, which can provide an analysis basis for the bridge damage identification based on the curvature mode.

In this paper, a three-span continuous steel beam is taken as the research object, and the damage location is identified by the curvature modal difference and the displacement modal difference, and the influence factors to damage index are analyzed, and the damage degree is identified by the relative change of the curvature modal shape.

2. Theoretical analysis

2.1 Calculation of curvature modal difference
The curvature modal shapes $\varphi_i(x)$ can be obtained by calculating the displacement modes with difference approximation method,
\[ \varphi_i = \frac{\varphi_{i+1} - 2\varphi_i + \varphi_{i-1}}{(l_{i-1} - l_i)^2} \]  

(1)

where, \( \varphi_i(x) \) is for displacement modal shapes, the subscript \( i \) is for the No. \( i \) measuring point, and \( l_{i-1}, l_i \) is for the distance between the adjacent measuring points \( i-1 \) and \( i \).

The curvature modal difference \( \Delta \) before and after damage is,

\[ \Delta = \varphi_u'' - \varphi_d'' \]  

(2)

where, \( \varphi_u'' \), \( \varphi_d'' \) are respectively for curvature modal matrix of the beam before and after damage obtained by difference calculation.

\[ \delta = \max |\Delta| \]  

(3)

The damage identification of the beam can be realized by the maximum \( \delta \) of the change vector of the curvature mode, and the damage degreee can be reflected by the change value.

2.2 Normalization of curvature modal shapes

The normalized curvature modal shape \( V_i \) can be obtained through the measured curvature mode \( V''_i \), multiplied by \( c \) [6].

\[
\sum_{j=1}^{n} |V_{ij}| \\
\sum_{j=1}^{n} |V''_{ij}|
\]

(4)

2.3 Noise simulation

The noise simulation in this paper is to add noise to the modal parameters according to modal analysis[7].

\[
\tilde{\phi}_j = \phi_j (1 + r_j^p \phi_{max,j} / 100)
\]

(5)

where, \( \phi_j \) and \( \tilde{\phi}_j \) are the \( i \) component of the \( j \) modal shape before and after the noise respectively, and \( \phi_{max,j} \) is the largest component of the absolute value of the \( j \) modal shape \( \varphi_j \), and \( r_j^p \) is random numbers with mean square error of 1 and mean value 0, and \( p^\phi \) is the imposed noise(If 5% noise is imposed, then \( p^\phi / 100 = 5\% \)).

The modal shapes with simulated noise by formula (5) are the same according to the mass normalization and the maximum value normalization.

2.4 Damage degree identification

The damage degree of each point can be identified by the relative change of the curvature mode \( \Delta V_{ij} \) [6],

\[
\alpha = \frac{\Delta k}{k} = \frac{\Delta V_{ij}}{1 + V_{ij}}
\]

(6)

where, \( k \) is for the stiffness before the beam damage at the \( j \) point, and \( \Delta k \) is for the beam stiffness.
variation caused by damage.

From the formula (6), it is known that the relative change of curvature mode is only related to the relative change rate of the beam stiffness $Δk/k$, but not to the damage location of the beam, and the relative change of the curvature modal shapes with the same damage is approximately equal. The damage degree at the zero point of a certain curvature mode should be identified in other order curvature modes.

3. Numerical simulation
There is a 3-span continuous steel beam, with I section, and a span combination of 8.0m+12.0m+8.0m and a total length of 28.0m, and 4 vertical supports restraints and 1 longitudinal restraint, whose steel density is 7698 kg . m$^3$, and elastic modulus is 2.1x10$^5$MPa, and sectional area is 2.159 x 10-3m$^2$, and the inertia distance is 3.86x10$^{-6}$m$^4$. The numerical model of the beam is divided into 280 equal parts, with 281 nodes (see figure 1).

The test conditions are shown in table 1.

| conditions | damage units         | damage   |
|------------|----------------------|----------|
| cond. 1    | damage at each midspan | 50%,50%,50% |
| cond. 2    | damage at each midspan | 30%,50%,20% |

4. Damage location identification

4.1 Damage location identification
The damage location identification of the 3-span continuous steel beam is carried out by means of displacement modal difference and curvature modal difference.

With no noise and mass normalized modal shapes, displacement modal difference and curvature modal difference are respectively calculated in the 1 to 2 order modes, and the modal displacement curve and modal curvature difference curve are achieved by MATLAB program(see figure 2~figure 5), so the damage location can be reflected by mutation position curves.
Fig. 2 Displacement modal difference identification under cond. 1

Fig. 3 Displacement modal difference identification under cond. 2

Fig. 4 Curvature modal difference identification under cond. 1
As shown in figure 2 and figure 3, the displacement modal difference damage are reflected by the sharp point of the smooth curve, and the first and second order displacement modal difference can approximately reflect the damage position under the condition 1 and 2.

It can be seen from figure 4 and figure 5 that the damage results are reflected by obvious mutant lines of curvature modal difference curve, and the curvature modal difference at first and second order can clearly reflect the damage position under the condition 1 and 2, and the effect at the second order is better than the first order for the curvature modal difference damage identification.

The damage identification effect of displacement modal difference is not as obvious as that of curvature modal difference.

4.2 Influence factors analysis of damage index
In practical application, the influence of related factors on the damage index should be considered. This paper mainly considers two aspects, that is, the influence of reducing the measuring points number and that of the noise. The influence of reducing the measuring points number on the accuracy of damage index belongs to incomplete modal testing information, and because of adding a certain level of noise, there is a certain deviation between the modal parameters from analysis and the real modal parameters of the beam, which is considered to be the test deviation from modal testing\cite{9}.

4.2.1 Decreasing of measuring points
The above analysis uses the information of each node, and the distance between each node is 10cm. The influence of the measuring points number on the damage location identification results with the curvature modal difference method is studied by every 2 nodes (n/2) and every 3 nodes (n/3), and so on.

The damage location identification results under the condition 1 with the measuring points by 12, 13, 14 and 15 nodes respectively, that is, the measuring points distance are 120cm, 130cm, 140cm and 150cm are shown in Figure6 (a)~(e).
4.2.2 The noise

The white noise $r_i^\phi$ is added with the mean value of 0, the variance of 1, and the value between [-1,1].

The noise level $p^\phi$ starts at 0.5% and increases by 0.5%. The anti-noise ability under different conditions is numerically analyzed by MATLAB program. The results are shown in table 2.

| Condition | 1st order | 2nd order |
|-----------|-----------|-----------|
| cond.1    | 1%        | 1%        |
| cond.2    | 0.5%      | 0.5%      |

The numerical analysis results show that the anti-noise ability of the curvature modal difference index for the damage identification is low.
5. Damage degree identification
According to the formula (6), the damage degrees of the 3-span continuous steel beam are identified under condition one and two, and the results are shown in table 3.

| Damage location | 1st order | 2nd order | 3rd order | 1st order | 2nd order | 3rd order | 1st order | 2nd order | 3rd order |
|-----------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| Cond.1 Result a| 47.12%    | 52.01%    | 46.07%    | 49.33%    | 47.42%    | 53.08%    | 47.66%    | 46.87%    | 53.06%    |
| error           | -5.76%    | 4.02%     | -7.86%    | -1.34%    | -5.16%    | 6.16%     | -4.68%    | -6.26%    | 6.12%     |
| Cond.2 Result a| 27.53%    | 32.01%    | 32.95%    | 47.51%    | 51.03%    | 49.08%    | 18.66%    | 19.37%    | 22.01%    |
| error           | 8.23%     | 6.70%     | 9.80%     | 4.98%     | 2.06%     | 1.84%     | 6.70%     | 3.15%     | 10.05%    |

As shown in table 3, the damage degree under condition 1 and 2 can be basically identified by formula (6). The smaller the damage degree is, the bigger the identification error is, and the maximum identification error is about 10%. The relative change of curvature shape is not related to the order of the curvature mode and the damage location, but only depends on the rate of stiffness change, which is basically consistent with the formula (6).

6. Conclusion
In this paper, the 3-span continuous beam is taken as the research object, and the ability of damage identification based on the curvature mode is studied with numerical experiments. The main conclusions are as follows:

1. the location damage identification effect of curvature modal difference is more obvious than that of the displacement modal difference,
2. the curvature modal difference for location damage identification requires a higher density of the measurement points,
3. the anti-noise ability of curvature modal difference for location damage identification is low,
4. the relative variation of curvature modal shapes can be used to identify the damage degree. The larger the damage degree is, the smaller the identification error is, and the maximum identification error is about 10%.

Although the identification accuracy of curvature mode is slightly worse in small damage, the calculation method of curvature can be incorporated into other dynamic fingerprint methods, which will further improve the ability of damage identification.

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