Sensitometric curve of radiographic films by X-ray fluorescence

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Radiographic film exposure is traditionally measured by the transmittance of a beam of light through the film. There are many mathematical and computational models to characterize the curve behaviour and its properties, but almost none of them considers the limitations caused by the equipment used. As long as exposure in film increases, light intensity measured after the film decreases in a way that from a certain exposure, light couldn’t be distinguished from any kind of noise. This work aims to propose X-ray fluorescence as a solution for better measure high exposed films and show how it could be modelled mathematically.

1. Introduction

Radiographic film is a well known and still very used way to measure ionizing radiation exposure both in research and in individual dosimetry.[1,2] The main propriety is the metallic silver concentration adhered to film after development due reaction with photons [3-6].

As the metallic silver is opaque, its concentration increase turn the film also opaque. A way to measure this is by light transmission across the film and how it is attenuated. The main models consider radiation exposure proportional to silver concentration which is proportional to light attenuation. So, read the optical density is indirectly measure the exposure before development [5,6].

For high values of optical density, the light that crosses the film is too low to be read, and it makes a upper limit for equipment, even if the silver concentration still growing. This fact brings a question about if another way to read silver concentration could be better for high exposures [7,8].

If the new method to measure silver had no limit, the H&D curve could have a range bigger than made by light measurement [7,8].

This work aims to show a little correction in mathematical model for radiographic film due the light limitation and also suggest the X-ray fluorescence (XRF) as a way to read films avoiding this limitations.

2. Mathematical Model and instrument correction

2.1. The traditional models
Most of the models to the sensitometric curve are similar and take two considerations to make a mathematical model:

- The probability to a photon to interact to a halide silver grain in film (turning it into a black metallic silver grain) is proportional to the number of grains and the number of photons (which is proportional to the total exposure)[9,10];

- To increase the optical density, a unsensitized grain must interact with a photon, and once sensitized, no matter if a new photon hits it again. It creates a saturation behavior, once number of the unsensitized grain decreases due exposure[9,10]

This behavior could be represented by the equation:

\[ dN = -N \alpha QX \]  

(1)

Where \( N \) is the number of unsensitized grains per area, \( \alpha \) is the cross section of a single grain to x-ray photons, \( Q \) is a constant and \( X \) is the exposure. Solving this, we can find [9,10]:

\[ N_a = N_0 \left( 1 - \exp^{-\alpha QX} \right) \]  

(2)

\( N_a \) and \( N_0 \) are blackened and total number of grains per area, respectively. As optical density is defined by reading values \( I_0 \) of light intensity entering and \( I \) crossing the film [9,10]:

\[ OD = \log \left( \frac{I_0}{I} \right) \]  

(3)

If we treat light photon interaction with matter like x-ray photons,

\[ I = I_0 \exp^{\sigma s} \]  

(4)

\[ OD = n \sigma s \log(e) \]  

(5)

Where \( n \) is the grain concentration per volume, \( \sigma \) is the cross section for light and \( s \) is the emulsion thickness.

As said before, OD is proportional to \( N_a \) [7,8]:

\[ OD = N_0 \sigma \log(e) \left( 1 - e^{-\alpha QX} \right) \]  

(6)

This equation gives the OD in function of exposure \( X \). The behavior can be seen in figures (1) and (2).

**Figure 1.** OD x Exposure by equation (6). It starts increasing fastly but saturates soon.
Figure 2. OD x log(Exposure) by equation (6). It starts increasing slowly, became fastly but saturates soon. Between the slow part and the saturated part there is a linear.

2.2. Instrument correction
In equation (3), I is the light read in OD measurement after crossing the film. If we add a noise, the equation will be:

\[ OD = \log \left( \frac{I_0}{I_0 + \epsilon} \right) \]  

Doing same as did before the new final equation is:

\[ OD = \log \left( \frac{I_0}{I_0 \exp\left(\frac{-\sigma N_0 (1-e^{-\alpha Q X})}{1-e^{-\alpha Q X}} + \epsilon\right)} \right) \]  

(8)

For low values of X, \( \epsilon \) is insignificant. For high values, it gives a limitation in OD. This is shown in figure (3).

Figure 3. OD x log(Exposure) by equation (8). When \( \epsilon > 0 \), the saturation is in low values, hiding a significant range from characteristic curve.

3. Materials and Methods

3.1. Exposing the films
Odontological KodaK E type films were exposed in a 40kV 80μA x-ray been, from an Amptek mini-X tube, in different exposures controlled by time. A non exposed and a film exposed to light for a day were also made for create a “zero” and “infinite” exposures. The films were developed using a portable Kodak developing chamber and chemicals, using times and temperatures suggested by the manufacturer. The optical density by light was obtained using a X-rite densitometer, model 341.
3.2. XRF measurement
After read optical densities, each film was also read in a Amptek XRF equipment, 40 kV and 80 μA. The area of each Kα peak was taken to build a XRF sensitometric curve.

3.3. Light and X-ray measurement fitting
The next step is to fit Light and X-ray fluorescence reads, at least the low values. in the same graph , finding the proportion between them.

3.4. Finding parameters to the equation corrected.
Fitting curves and, theirs limits and proportion, its possible to find all the parameters to build the film equation corrected by instrument limits. After, the theoretical optical density for all the exposures were calculated, to plot together.

4. Results

4.1. Sentitometric curves by X-ray, light and theoretical
The graph showing the three curves is shown below.

![Graph showing three curves](image)

**Figure 4.** OD x log(Exposure) and XRF measurement. XRF behaves like the total range without limitation.

4.2. The corrected equation
The proportion between read optical density and Kα peak area (in number of counts) is β = 0,0012. Using this value, o better plot equation is given below:

\[
OD = \log \left( \frac{4495}{4495 - 22.9 \ln \left( 1 - e^{-0.0043X} \right)} + 0.0176 \right)
\]  

(9)

5. Discussion
The mathematical model was created with a small correction of pre-existing models, giving rise to a slightly larger expression because there were no simplifications previously used. Within a small statistical variation this model seems to be at least a good approximation for the relationship between the H & D curve and the limitation for optical density reading. The model still suggests ways to overcome the problem in some situations, requiring minimum and maximum values. For the light readers used in the curve construction, and consequently in the scanning of radiographic films. The model was successful in its experimental validation. The data used in this work has a pretty good fit. It
was thus possible to construct the equation of the H&D curve for the film used in this work and predict how far it would go without limitation. The use of the X-ray fluorescence peak in place of the optical density was very successful avoiding the existence of a maximum value of the concentration of the plot. A characteristic curve made through XRF are due to the adjustment of the geometry and correction, showing purely proportional silver concentration, and convenient for use, besides proving the fact that the limiting density of the H&D curve is in the measure by optical density.

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