The MSSM without \( \mu \) term

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Abstract

We propose a supersymmetric extension of the standard model which does not have a "\( \mu \)" supersymmetric Higgs mass parameter. The matter content of the MSSM is extended with three additional chiral superfields: one singlet, an \( SU(2) \) triplet and a color octet, and an approximate \( U(1)_R \) symmetry naturally guarantees that \( \tan \beta \) is large, explaining the top/bottom quark mass hierarchy. Unlike in the MSSM, there are significant upper bounds on the masses of superpartners, including an upper bound of 114 GeV on the mass of the lightest chargino. However the MSSM bound on the lightest Higgs mass does not apply.

1 The \( \mu \)-SSM and its low energy spectrum

In the Minimal Supersymmetric Model (MSSM) there is a supersymmetric Higgs mass parameter, "\( \mu \)", which must be of order of the electroweak scale for successful phenomenology. The difficulty of generating the correct mass scale for this supersymmetric mass parameter is the so-called "\( \mu \) problem". This problem is more severe in gauge mediated supersymmetry breaking (GMSB) models, since it is quite difficult in gauge mediation to induce a \( \mu \) parameter which is naturally related to supersymmetry breaking, without inducing an excessively large \( B\mu \) parameter [1].

We consider an alternative solution to the \( \mu \) problem, by building a viable model which does not have a \( \mu \) parameter. In order to obtain a spectrum of superpartner masses experimentally acceptable without \( \mu \) we have to add some matter content to the MSSM. This model, which we call the "\( \mu \)-less Supersymmetric Standard Model" (\( \mu \)-SSM), has an approximate \( U(1)_R \) symmetry which guarantees naturally large \( \tan \beta \), explaining the top/bottom quark mass hierarchy, and suppresses dangerous supersymmetric contributions to anomalous magnetic moments, \( b \to s\gamma \), and proton decay.

The \( \mu \)-SSM can naturally arise from either gauge or gravity mediation [2], if the supersymmetry breaking sector respects an approximate \( U(1)_R \) symmetry. Such an approximate symmetry can easily arise by accident, as a consequence of the absence of gauge singlet chiral superfields with \( F^- \) terms in the supersymmetry breaking or mediation sector.

We start with the principle that all mass terms arise directly either from electroweak symmetry breaking or from supersymmetry breaking. We therefore do not allow a supersymmetric \( \mu \) term or any supersymmetric mass term. The MSSM without a \( \mu \) term would have charginos lighter than the \( W \) boson, which should have been found at LEP II, so we have to extend the theory.

In the exact \( U(1)_R \) symmetric limit there are no supersymmetry breaking Majorana gaugino masses, so in order to give the gauginos Dirac masses we add three chiral superfields, namely a color octet \( O \), a triplet under the \( SU(2) \) gauge group, \( T \) and a singlet \( S \). These adjoint matter multiplets could have an extra dimensional origin, since extra dimensional theories in
which gauge bosons live in the bulk and chiral matter fields live on a three brane typically have additional matter fields in the adjoint representation when described four dimensionally, unless the extra dimension is orbifolded. The adjoint fields might be \( N = 2 \) superpartners of the gauge fields \([3]\).

We now turn to a discussion of the spectrum of the \( \mu \)SSM, from the bottom up. The charge assignments of some of the components of Higgs and electroweak gauge fields under the unbroken \( U(1)_R \) are:

| \( \Psi_{H_1} \) | \( \Psi_{H_2} \) | \( \Psi^+_T \) | \( H_1 \) | \( H_2 \) | \( \lambda^\pm \) |
|---|---|---|---|---|---|
| 1 | -1 | -1 | 2 | 0 | 1 |

Thus we can add the superpotential coupling

\[
\int d^2 \theta \ h_S H_1 H_2 + h_T H_1 T H_2 .
\]

\( U(1)_R \) charges are also assigned to quarks and leptons to allow the usual MSSM superpotential couplings.

Scalar trilinears involving the \( T \) scalar are potentially troublesome, because they could induce a tadpole for \( T \), which would get a vev and lead to a large electroweak \( T \) parameter. Sufficient suppression of this tree level contribution is provided by the approximate \( U(1)_R \) symmetry and by a heavy mass for the \( T \) scalar, which is automatic in the gauge mediated models.

Now the chargino mass matrix is

\[
\begin{align*}
\Psi_T & | \Psi^+_T \quad -i\lambda^+ \quad \Psi^+_{H_2} \\
-\Psi_T & | 0 \quad M_2 \quad -h_T v_1 \\
-i\lambda^- & | \tilde{M}_2 \quad \tilde{m}_2 \quad \sqrt{2} m_W s_\beta \\
\Psi_{H_1} & | h_T v_2 \quad \sqrt{2} m_W c_\beta \quad 0
\end{align*}
\]
where $\tilde{M}_2$ ($\tilde{m}_2$) is a soft supersymmetry breaking Dirac (Majorana) mass term. Note that all the charginos can be made heavier than 104 GeV without a $\mu$ parameter.

With the $U(1)_{R}$ symmetry unbroken, $\tilde{m}_2 = v_1 = c_\beta = 0$. This will get modified slightly by small $U(1)_{R}$ breaking effects, which will get us away from the limit $\tan \beta \to \infty$ and set $\tan \beta$ to a moderate value $\sim 60$. In this limit there is one chargino with mass $\tilde{M}_2$ and another chargino whose mass decreases with $\tilde{M}_2$. To obtain masses for all charginos heavier than 104 GeV, while assuming $h_T < 1.2$, $\tilde{M}_2$ must be in the range 104-120 GeV. Moreover, the requirement, that all charginos should be heavier than 104 GeV leads to a lower bound on the Yukawa coupling, $h_T > \sim 1$. Note that $\sqrt{2}m_W = 114$ GeV is an upper bound on the mass of the lightest chargino. Thus in the region where all charginos are heavier than 104 GeV we have two charginos with mass between 104 and 120 GeV and one heavier one. We show in Fig. 1 the lighter chargino masses as a function of $\tilde{M}_2$.

The neutralino mass matrix is:

$$
\begin{array}{cccccc}
\Psi^\dagger & \Psi_S & -i\lambda' & -i\lambda^3 & \Psi^\dagger_{H_1} & \Psi^\dagger_{H_2} \\
\Psi^\dagger_1 & 0 & 0 & 0 & h_T v_2/\sqrt{2} & h_T v_1/\sqrt{2} \\
\Psi^\dagger_2 & 0 & 0 & \tilde{M}_1 & 0 & h_S v_2/\sqrt{2} & h_S v_1/\sqrt{2} \\
-i\lambda' & 0 & \tilde{m}_1 & 0 & -m_Z s_W c_\beta & m_Z s_W s_\beta \\
-i\lambda^3 & \tilde{M}_2 & 0 & 0 & \tilde{m}_2 & m_Z c_\beta & -m_Z c_\beta \\
\Psi^\dagger_{H_1} & h_T v_2/\sqrt{2} & h_S v_2/\sqrt{2} & -m_Z s_W c_\beta & m_Z c_\beta & 0 & 0 \\
\Psi^\dagger_{H_2} & h_T v_1/\sqrt{2} & h_S v_1/\sqrt{2} & m_Z s_W s_\beta & -m_Z c_\beta & 0 & 0 \\
\end{array}
$$

(4)

In the large $\tan \beta$, $U(1)_R$ symmetric limit the masses become approximately Dirac. There is always a nearly Dirac neutralino with mass lighter than the $Z$. In Fig. 2 we show the neutralino

Figure 2: Neutralino masses as a function of $\tilde{M}_1$, for $\tilde{M}_2 = 104$ GeV, $h_T = 1$, $h_S = 0.1$, $\tan \beta = 60$ and $\tilde{m}_1 = \tilde{m}_2 = 5$ GeV.
masses as a function of the soft mass term $\tilde{M}_1$, for $h_T = 1$ and $h_S = 0.1$. In principle the Yukawa coupling $h_S$ is a free parameter, but large values are disfavored by electroweak precision measurements.

Similarly, gluinos get a supersymmetry breaking Dirac mass term by mixing with the fermionic component of the color octet $O$. The scalar superpartners receive soft supersymmetry breaking masses as usual. A very small scalar $\mu B$ term of order a few GeV$^2$

$$\mu BH_1H_2$$

will be needed in order to induce a small vev for $H_1$, which gives the leptons and down-type quarks mass. It is natural for this term to be small as it breaks the approximate $U(1)_{R}$ symmetry. Because the symmetry is explicitly broken rather than spontaneously broken, there is no light pseudoscalar.

The MSSM bound on the lightest Higgs mass does not apply, though, since the scalar sector is also enlarged by the scalar components of the $SU(2)$ triplet and scalar chiral superfields, and there are new, $F-$component contributions to the Higgs quartic coupling. There will still be some upper bounds, computed in general models with Higgs triplets in refs. [4].

2 $U(1)_R$ Symmetric Gauge Mediation

We assume that supersymmetry breaking is transmitted to the $\tilde{\mu}$SSM by Gauge Mediated Supersymmetry Breaking (GMSB), and a messenger sector of heavy supermultiplets in a vector-like representation of the standard gauge group. In conventional gauge mediation, the messengers learn about supersymmetry breaking from coupling to a gauge singlet with an $F-$term. This transmits both supersymmetry breaking and $U(1)_{R}$ symmetry breaking to the MSSM. Since we want an approximately $U(1)_{R}$ symmetric $\tilde{\mu}$SSM, we will assume the messenger sector does not contain any singlet. Instead supersymmetry breaking in the messenger sector is primarily mediated by some new gauge group also carried by the messengers. Such mediation will primarily induce nonholomorphic scalar supersymmetry breaking masses in the messenger sector [5, 6].

We assume the usual messenger matter content of chiral superfields $L, \bar{L}, D, \bar{D}$ where $L, \bar{L}$ transform under $SU(2) \otimes U(1)$ in conjugate representations and $D, \bar{D}$ carry color. In order to obtain Dirac gaugino masses, $S, T$ and $O$ must couple to the messengers. The messenger superpotential is

$$\lambda_S S \bar{L}L + \lambda'_S S \bar{D}D + \lambda_T T \bar{L}L + \lambda_O O \bar{D}D + M_L \bar{L}L + M_D \bar{D}D .$$

(6)

The supersymmetric mass parameters $M_L$ and $M_D$, which can be dynamically generated [4], are much heavier than the weak scale.

The mass matrix for, e.g. the $L, \bar{L}$ scalar fields will have the following form

$$\begin{pmatrix}
M_L^2 + \tilde{m}_L^2 & 0 \\
0 & M_D^2 + \tilde{m}_D^2
\end{pmatrix}$$

(7)

where $\tilde{m}_L^2, \tilde{m}_D^2$ are soft supersymmetry breaking masses. With no messenger singlet, to leading order the messenger sector will accidentally have unbroken $U(1)_{R}$ symmetry, and no Majorana gaugino masses will be produced. However, at one loop, the gauginos couple to the fermionic components of $T, O$ and $S$ and get a Dirac supersymmetry breaking mass.

Note also that provided the $D-$type masses are generated by new gauge interactions whose generators are orthogonal to electroweak hypercharge, i.e. $\text{Tr } T_Y T_{\text{new}} = 0$, the disaster of generating a $D$-term for hypercharge at one loop is avoided.
There are two diagrams contributing to Dirac gaugino masses, which cancel in the limit that $\tilde{M}_{L,D}^2 = \tilde{M}_{\tilde{L},\tilde{D}}^2$. In the limit that the supersymmetry breaking terms are much smaller than $M_L$, the Dirac masses $\tilde{M}_{2,3}$ are

$$\tilde{M}_{2,3} = S_{L,D} \frac{g_{2,3} \lambda_{T,O} \tilde{m}_{L,D}^2 - \tilde{m}_{L,D}^2}{4\pi^2 M_{L,D}}. \quad (8)$$

where $S_{L,D}$ are the Dynkin indices of the $L, D$ representations respectively. Similarly, $\tilde{M}_1$ will receive contributions from both $L$ and $D$.

The masses of scalar $\tilde{\mu}$SSM particles may be found as a special case of the general expressions computed in [7]. Note that obtaining positive squark and slepton masses will require negative supertrace in the messenger sector. As a consequence, the scalar components of $T, S, O$ will receive a large positive mass squared at one loop and will therefore be significantly heavier than the other superpartners. This mass is of order a loop factor times the soft masses in the messenger sector, and is not suppressed by the messenger mass scale. The $T$ and $O$ scalar masses should not be much larger than $10^4$ GeV, or they will give excessive two loop contributions to squark and slepton masses. The supersymmetry breaking terms in the messenger sector should therefore not be larger than of order $M_S \sim 10^5$ GeV. Since squark and slepton masses will be of order $(\alpha/\pi)(M_S/\mu)$, the messenger mass scale $M$ must be below $10^6$ GeV.

The $\tilde{\mu}$SSM avoids the gauge mediated $\mu$ problem, because a $\mu B$ parameter can be induced which is proportional to a small coupling, and it is not a problem that the resulting $\mu$ parameter will be much smaller than the weak scale.

### 3 Contribution to precision electroweak parameters

#### 3.1 $T$ parameter

The superpotential couplings $h_T T H_1 H_2$ and $h_S S H_1 H_2$ break custodial $SU(2)$ symmetry and thus can lead to potentially large one-loop effects in the $T$ parameter. Although the oblique approximation is not appropriate for light superpartners, we shall interpret our results for the $T$ parameter as an order of magnitude estimate of the radiative corrections expected in the $\tilde{\mu}$SSM.

We find that the leading contribution to the $T$ parameter grows as $h_T^2 \log(h_T^2 \nu^2/\mu^2)$ and it is therefore very sensitive to the exact value of the coupling $h_T$. Recall that there is a lower limit on this coupling from chargino masses. Although the singlet coupling $h_S$ also contributes to the $T$ parameter, its contribution is negligible provided $h_S \lesssim 0.1$.

From a global fit of the electroweak precision data one obtains $T = -0.02 \pm 0.13 (+0.09)$, where the central value assumes $M_H = 115$ GeV and the parentheses shows the change for $M_H = 300$ GeV. This bound can be relaxed for larger $M_H$, leading to $T \lesssim 0.6$ at 95% CL. If we impose the kinematic limit from LEP II that charginos should be heavier than 104 GeV, $h_T \sim 1$ and the contribution to the $T$ parameter is huge, $\sim 2.7$. However if the actual bound on chargino masses in this model were somewhat lower, say 90 GeV, we would obtain $h_T \gtrsim 0.6$ which leads to $T \sim 0.6$. Therefore, given the large sensitivity of the $T$ parameter to the value of $h_T$, a careful calculation of the chargino mass bounds is crucial to determine the viability of the $\tilde{\mu}$SSM model.

There is also a $T$ parameter contribution from the scalar sector, but it is not enhanced by the log $\mu^2$ term, and can be made very small by the soft supersymmetry breaking scalar masses.
3.2 Muon anomalous magnetic moment

The supersymmetric contributions to $a_\mu$ include loops with a chargino and a muon sneutrino and loops with a neutralino and a smuon. Explicit formulae can be found in [2]. In the $U(1)_R$ symmetric limit, the contributions to $a_\mu$ proportional to the neutralino and chargino masses exactly vanish, and there is only a tiny effect proportional to $m_\mu$. However, once we take into account the small $U(1)_R$ symmetry breaking effects, the leading contribution comes from the terms with the neutralino and chargino masses, much as in the MSSM. The contribution from chargino loops is typically dominant, except for $m_L \gg m_R$.

![Figure 3](image)

Figure 3: Maximum value of $\delta a_\mu \times 10^{10}$ as a function of $m_L$ and $\tan \beta$, for $\tilde{m}_1 = \tilde{m}_2 = 0$ (dashed-dotted), 5 GeV (dashed) and 10 GeV (solid). We have taken $A = 0$, $m_R = 100$ GeV, $M_1 = 100$ GeV, $M_2 = 110$ GeV, $h_T = 0.8$, $h_S = 0.1$, $\tan \beta = 60$ (left) and $m_L = 100$ GeV (right). The shadowed areas correspond to $1\sigma$ (dark-green) and $2\sigma$ (light-yellow) allowed regions from the $g - 2$ collaboration result.

In Fig. 3 we show the maximum possible value of $\delta a_\mu$ in the $\mu$SSM model as a function of the soft supersymmetry breaking mass term $m_L$ (left) and as a function of $\tan \beta$ (right), for several values of the gaugino Majorana masses. Although the contribution to $a_\mu$ in the $\mu$SSM model is also enhanced for large $\tan \beta$, due to the approximate $U(1)_R$ it is suppressed by the small gaugino Majorana masses and therefore much smaller than in the MSSM.

4 Unification of couplings

One rational for supersymmetry is coupling constant unification. If we add matter to the MSSM in incomplete multiplets under the unifying group the usual successful prediction of $s^2_W \approx .23$ may be lost. In the $\mu$SSM we have added matter in the adjoint representation of
$U(1) \otimes SU(2) \otimes SU(3)$, which will not preserve the usual prediction. It is, however a simple matter to embed the $T, S$ and $O$ fields into a complete adjoint multiplet of a GUT such as $SU(3)^3$ or $SU(5)$.

Although it is not necessary, the other fields of the multiplet can serve as the messenger fields of a gauge mediated model \[2\]. If one assumes all the $\mu$SSM superpartners are at the weak scale, and computes the one-loop running neglecting threshold effects, one can fit the scales of the new matter multiplet and GUT to the low energy gauge coupling constants. The result is

$$M_{\text{new}} = M_{\text{weak}} e^{\frac{2\pi}{3} \left( \frac{1}{\alpha_2} - \frac{5}{\alpha_1} - \frac{7}{\alpha_3} \right)}$$

(9)

$$M_{\text{GUT}} = M_{\text{weak}} e^{\frac{2\pi}{3} \left( \frac{1}{\alpha_2} - \frac{1}{\alpha_1} - \frac{2}{\alpha_3} \right)}.$$  

(10)

By taking values for the coupling constants at the edge of their allowed ranges, e.g. $\alpha(M_Z) = 1/127.7$, $\alpha_s(M_Z) = 0.122$, and $s_W^2 = 0.233$ the additional matter fields can be as heavy as $3 \times 10^7$ GeV and the GUT scale as high as $10^{18}$ GeV. Threshold effects at the GUT, messenger and $\mu$SSM scales and higher loop corrections make order one changes in these predictions. This constraint is less stringent than the upper bound on the messenger scale found in section \[3\].

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