Collective flow from AA, pA to pp collisions
– Toward a unified paradigm

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Abstract

I give an overview of the latest development in understanding collective phenomena in high-multiplicity hadronic final state from relativistic nucleus-nucleus, proton-nucleus and proton-proton collisions. Upon reviewing the experimental data and confronting them with theoretical models, a unified paradigm in describing the observed collectivity across all hadronic collision systems is emerging. Potential future paths toward addressing key open questions, especially on collectivity in small systems (pp, pA), are discussed.

Keywords: heavy ions, collectivity, flow, ridge, small systems, quark-gluon plasma

1. Introduction

Collective phenomena have been a central theme in the research of strongly correlated, interacting many-body systems in nearly all fields of physics, from the astrophysical scale like the neutron stars to the most fundamental scale, such as the “quark-gluon plasma” (QGP). They form the “complex frontier” in physics, which addresses issues of the emergence, or emergent phenomena. The most important, key question to ask is what the fundamental laws are in describing the emergent phenomena, or taking a different point of view, how the emergent phenomena can be understood from the fundamental forces, if possible at all.

In high-energy proton-proton (pp), proton-nucleus (pA) and nucleus-nucleus (AA) collisions, an emergent phenomenon of a novel anisotropy azimuthal correlation is observed in high-multiplicity events [1–10]. As shown in Fig. 1, a cos 2(Δφ)-like azimuthal structure is observed in two-particle Δη–Δφ correlation functions, most notably in pPb and PbPb collisions but also in high-multiplicity pp collisions.

There are two most striking features of this novel correlation, which have imposed strong constraints on the possible interpretations of its underlying physics mechanisms: (1) first, this azimuthal structure is long-range in rapidity, extending over at least 5 even up to 10 units, thus known as the “ridge”; (2) secondly, the correlation is found to be collective involving nearly all particles produced in the event. This is evidenced
by the fact that the strengthen of elliptic flow, \( v_2 \), is nearly invariant when extracted by correlating no matter how many particles at a time. Also, the \( v_2 \) values in pp and pPb are observed to be roughly independent of multiplicity \([11, 12]\), indicating that as more particles are incorporated into the system, they immediately follow the same behavior of collectivity as the rest of the system. The mass dependence of identified particle yield and \( v_2 \) also suggests the production of particles from a collectively moving source \([12, 15]\).

The collective property of the ridge indicates the presence of strong interactions either during the (final) stage of system evolution, or at the initial stage before particles are produced. The long-range nature in rapidity then implies that the anisotropies must be rooted at an early time constrained by causality, \( \tau_0 \leq \tau_{fa} \exp \left( -\frac{1}{2} |y_a - y_b| \right) \), where \( y_{a,b} \) is the momentum-space rapidity of particles \( a, b \) \([17]\). Namely, before a time-scale of \( \sim 0.1 \) fm/c, an initial anisotropy must be present either in the position space or in the momentum space. For the first scenario of position-space anisotropy, strong final-state interactions are necessary to transpose it into the final observed anisotropy in the momentum space. This scenario includes models of hydrodynamics, parton transport and/or escape. For the second scenario, momentum space anisotropies are already present before the collision occurs via initial interactions of gluons inside the projectile proton or nucleus. Models belonging to this category include color glass condensate (CGC) glasma model, color-field domains and etc. Any theoretical model proposed must fall into one scenario or the other. Otherwise, it is immediately ruled out by the experimental data (see a review in Ref. \([18]\) and references therein).

2. Collective flow in large systems – AA

Based on extensive studies over the past couple of decades, the community has converged to the first scenario of strong final-state interactions in describing the collective anisotropy flow observed in large AA collision systems. A paradigm of a nearly perfect QGP fluid formed has been established. Deformations present in a lumpy initial energy density profile are fully transposed, by nearly ideal hydrodynamics on an event-by-event basis, into anisotropies in the final-state particle momentum distribution. Other components of the model are also introduced to describe pre-equilibrium, freeze-out and hadronic transport stages. Fourier bases have been applied to characterize the final-state anisotropy flow in momentum space. For the first scenario of position-space anisotropy, strong final-state interactions are necessary to transpose it into the final observed anisotropy in the momentum space. This scenario includes models of hydrodynamics, parton transport and/or escape. For the second scenario, momentum space anisotropies are already present before the collision occurs via initial interactions of gluons inside the projectile proton or nucleus. Models belonging to this category include color glass condensate (CGC) glasma model, color-field domains and etc. Any theoretical model proposed must fall into one scenario or the other. Otherwise, it is immediately ruled out by the experimental data (see a review in Ref. \([18]\) and references therein).
sity function \((p.d.f.)\). Practically, experimentalists construct the “new” flow observables to extract various orders of moments and cumulants of the \(p.d.f\). Two main recent directions are discussed below.

### 2.1. Mixed-order harmonic correlations \(-\left\{\vec{V}_n, \vec{V}_m\right\}\)

Correlations of flow harmonic magnitudes between different orders have been studied by ALICE in PbPb collisions using a four-particle cumulant approach (or symmetric cumulant) \(20\), \(S C(n,m) = \langle \vec{v}_n^2 \vec{v}_m^2 \rangle - \langle \vec{v}_n^2 \rangle \langle \vec{v}_m^2 \rangle\). An anti-correlation between \(v_2^2\) and \(v_3^2\) is observed, while \(v_2^2\) and \(v_4^2\) are positively correlated. The general feature of the data is captured by hydrodynamic models. As \(v_2^2\) and \(v_3^2\) are linearly proportional to the initial eccentricities, after normalizing by the \(v_2^2\) and \(v_3^2\) magnitudes, \(S C(2,3)/\langle \vec{v}_2^2 \rangle\) is shown to be insensitive to the medium transport properties, and thus can directly probe properties of initial-state geometry fluctuations, more specifically the fluctuating granularities.

Fig. 2. The non-linear response coefficients for various orders of harmonic mixings in PbPb collisions as a function of centrality, measured by ALICE and CMS. Calculations from hydrodynamic and parton transport models are also shown.

Correlation between \(v_2^2\) and \(v_4^2\) is more complicated, showing a strong dependence on both the initial state and transport properties. The reason has been understood as the breakdown of linear responses to the initial-state geometry for higher-order harmonics, \(v_n^2\) \((n \geq 4)\), which receive a non-linear contribution from mixings of lowest-order \(v_2^2\) and \(v_3^2\) harmonics, as described by following equations,

\[
\vec{V}_4 = \vec{V}_{4L} + \chi_{422} \vec{V}_2^2 \\
\vec{V}_5 = \vec{V}_{5L} + \chi_{523} \vec{V}_2 \vec{V}_3 \\
\vec{V}_6 = \vec{V}_{6L} + \chi_{6222} (\vec{V}_2^2)^2 + \chi_{633} (\vec{V}_3^2)^2 \\
\vec{V}_7 = \vec{V}_{7L} + \chi_{7223} (\vec{V}_2^2 \vec{V}_3)^2 \\
\]

(1)

The non-linear terms are sensitive to final-state dynamics of the QGP, as opposed to the leading terms, which are approximately linear with initial eccentricities. As leading and non-linear terms are orthogonal to each other, they can be decomposed by a projection of higher-order flow vector onto the lowest-order ones and extract the so-call non-linear response coefficients, \(\chi\), for various mixing combinations \(21, 22\).

New measurements of non-linear response coefficients are performed by the ALICE and CMS in PbPb collisions, shown in Fig. 2, averaged over the full \(p_T\) range as a function of centrality for five different mixing coefficients. Overall, weak centrality dependence for all coefficients is observed. The data are compared to predictions of hydrodynamic and parton transport (AMPT) models. Impressively, the highest-order mixing coefficient, \(\chi_{7223}\), is well described by the AMPT model. The hydrodynamic model can capture qualitative features of the data but has a strong sensitivity to both initial-state models and \(\eta/s\) values. As the harmonic mixings are mainly determined at the freeze-out stage, these high precision new data show great promise in providing unique constraints on the modeling of freeze-out dynamics of the QGP evolution.

### 2.2. Correlations of harmonics at different \((\eta, p_T)\) – \(\left\{\vec{V}_a(\eta_1, p_{T1}), \vec{V}_b(\eta_2, p_{T2})\right\}\)

Much of studies on collective flow have previously been focusing on the midrapidity region, whereas the QGP evolution takes place in 3-D. Several important questions related to the longitudinal dynamics of
QGP have not yet been well addressed, e.g., (1) how is the initial entropy deposited in 3-D space? How does it fluctuate event-by-event?; (2) What is the role of the longitudinal pressure gradient? These issues start being explored in recent couple of years by studying correlation of flow harmonic vectors at different rapidities and transverse momentum.

Novel rapidity-dependent event plane twist or de-correlation has been predicted, as illustrated in Fig. 3. In the picture of wounded nucleon model [23], particles produced at midrapidity receive about equal contribution from participants of both nuclei. However, if going to forward rapidity region, particles will be predominantly produced from one of the projectile nuclei. As a result, the flow orientation angle (or event plane) at forward and backward rapidities can be slightly twisted event-by-event, creating a torqued QGP along rapidity direction. Additionally, in the CGC glasma model [24], fluctuating granularity of the gluon field is rapidity dependent. As moving toward larger rapidity and smaller x value, the initial configuration of gluon fields tends to become smoother. Both effects of participant nucleon and gluon field fluctuations can induce rapidity-correlated flow fluctuations.

The rapidity-dependent flow de-correlations have been observed by CMS by constructing a ratio of flow correlators between two flow vectors measured at different rapidity regions, $r_n \equiv \langle \Psi_a^{(\eta)}(q^\tau) \rangle \langle \Psi_a^{(-\eta)}(\omega^\tau) \rangle / \langle \Psi_b^{(\eta)}(q^\tau) \rangle \langle \Psi_b^{(-\eta)}(\omega^\tau) \rangle$, designed to approximate the de-correlation between two event plane angles separated by a gap of $2\eta_n$, $\langle \cos n \left[ \Psi_a^{(\eta)}(q^\tau) - \Psi_a^{(-\eta)}(\omega^\tau) \right] \rangle$, as shown in Fig. 4(right) for elliptic flow in 0–5% central PbPb collisions. The data are compared to several models of initial states including torqued QGP model, AMPT initial state followed by a 3-D hydrodynamics and 3-D CGC glasma model, which all qualitatively reproduce the data. It is worth noting that almost all the rapidity de-correlation effect is determined from the initial state, addition of 3-D hydrodynamic evolution is found to have little impact on the $r_n$ ratio. This underlines the importance to incorporate a rapidity dependent modeling of initial-state fluctuations in hydrodynamic calculations.

New studies on flow de-correlations in rapidity in PbPb collisions are performed by ATLAS, using a new ratio of correlators among four flow vectors, $R_n \equiv \langle \Psi_{a}^{(-\eta)}(\omega^\tau)\Psi_{b}^{(-\eta)}(q^\tau)\Psi_{b}^{(\eta)}(\omega^\tau)\Psi_{a}^{(\eta)}(q^\tau) \rangle / \langle \Psi_{a}^{(-\eta)}(\omega^\tau)\Psi_{b}^{(-\eta)}(q^\tau)\Psi_{b}^{(\eta)}(\omega^\tau)\Psi_{a}^{(\eta)}(q^\tau) \rangle$. The $R_n$ ratio has the advantage of being more sensitive to the effect of event plane angle twist, while the originally proposed $r_n$ ratio is sensitive to both event plane and participant eccentricity fluctuations in rapidity. Comparing the $r_n$ and $R_n$ data from ATLAS in PbPb collisions, one can conclude that event plane twist accounts for about 50% of de-correlation effect previously observed in $r_n$.

Rapidity dependence of flow harmonics has been proposed as a means to probe the temperature dependence of $n/s$ value as temperature of the QGP is expected to be rapidity dependent [27]. Therefore, it is crucial to have a clear understanding of rapidity-correlated initial-state effect, which has significant contributions to the observed rapidity dependence of flow harmonics.
3. Collective flow in small systems – pp and pA

A big question under intense debate in the field of heavy-ion physics is: how small a QGP fluid system can be in size? In general, hydrodynamics is applicable when the characteristic system size is much larger than its interaction mean free path, $L \gg \lambda_{m.f.p.}$, where the mean free path is inversely related to the system temperature and coupling strength, $\lambda_{m.f.p.} \sim \frac{1}{T}$. In the case of strong coupling of the order of 1, hydrodynamics requires a condition of $LT \gg 1$. On the other hand, in the limit of extremely strong coupling as for the holographic principle, this criteria could be significantly loosened to $LT \sim 1$ so that a QGP fluid may be realized with a much smaller size ($\sim 1/T$) at a given temperature. Therefore, the question on how small a QGP fluid can be has important implications to the most fundamental property of the QGP medium.

To emphasize again, it is $LT$ that determines the system’s fluid behavior, instead of just the absolute size. So what is the corresponding experimental condition then? As entropy density, $s$, scales as $T^3$ for a thermalized QGP system, and also $s$ is approximately proportional to event multiplicity, $N_{\text{trk}}$ over $L$, a qualitative relation can be derived that $LT \sim (N_{\text{trk}})^{\frac{1}{3}}$, where $N_{\text{trk}}$ is the total number of tracks. Therefore, the most relevant question to ask may not be about absolute size of the system but, instead, what is the smallest multiplicity or total entropy the system has to produce to exhibit hydrodynamic behavior. Lots of experimental evidence also suggest that total event multiplicity does seem to play a special role in driving the collective effects of produced particles.

Several key features of collectivity have recently been observed in high-multiplicity pp collisions as well, similar to those in pPb and PbPb collisions. These include, as shown in Fig. 4 for data from CMS, mass ordering of $v_2$ (comparing charged particles, $K^0_S$ and $\Lambda$), multi-particle cumulant $v_2 (v_2^{[4]} \approx v_2^{[6]})$ that is independent of multiplicity, and mass dependent of identified particle spectra (not shown). Experimental observation of collective behavior across all hadronic collision systems with high-multiplicity final state has been established, although this does not necessarily imply a hydrodynamic origin of collectivity.

Progress has also been made recently for the scenario of initial interaction models. By incorporating the Lund string model in PYTHIA to fragment gluons into final state hadrons, the CGC glasma model is
able to make direct quantitative comparisons to experimental observables for the first time. For instance of Ref. [30], mass ordering of \(v_2\) in high-multiplicity pp is reproduced by the CGC glasma model. Next important step is to examine other observables relevant to collectivity such as multi-particle cumulants.

The key to further differentiate between the initial- and final-state interaction scenarios (both may give rise to collective particle correlations) is to investigate the connection between final-state collective anisotropies and initial-state geometry (or eccentricity), which has been well established in AA but not yet for small systems. Smallness in absolute size is not the limitation in hydrodynamics. This has been convincingly demonstrated in a series of geometry-controlled experiments on small systems at RHIC by colliding light ions like dAu [31] and He\(^3\)Au [28]. For those systems, where each nucleus contains at least two nucleons, initial eccentricities are still largely determined by the position of wounded nucleons, which has been well understood in the Glauber picture. As illustrated in Fig. 5a (top), a dAu or He\(^3\)Au collision has the configuration of two-blob or three-blob deposited energy in its initial state. Later on, each blob will expand and generate a shock wave. Collisions of shock waves will lead to \(v_2\) and \(v_3\) anisotropy. Fig. 5a (bottom) shows the \(v_2\) and \(v_3\) data in 0–5% central He\(^3\)Au collisions from PHENIX [28]. The data are in good agreement with hydrodynamic calculations using either Glauber or IP-glasma initial state, providing clear evidence for the applicability of hydrodynamics in system as small as a couple of \(fm\) in size.

However, when moving to pA (and pp) system, the agreement among different models and data does not hold anymore, as shown in Fig. 5b (bottom) for \(v_2\) data in pPb collisions from CMS and ATLAS. The main issue here is that if one of the projectiles is a single nucleon, the initial geometry of the overlap region is highly sensitive to the details of event-by-event shape of a nucleon, which is poorly known. Different approaches in modeling the initial state in pA collisions are demonstrated in Fig. 5b (top). The IP-glasma model significantly underestimates the \(v_2\) data in pPb because the geometry of the overlap zone is largely determined by the proton’s shape, which is spherical. In the Glauber model, it assumes that the entire nucleon of those wounded ones would contribute to the initial geometry. This leads to a much larger eccentricity because of more degree of freedom for fluctuations. Modeling of event-by-event proton shape fluctuations in the framework of CGC model has been recently attempted and shown promising results in generating a much larger final-state \(v_2\) value, closer to the experimental data, as one can see in Fig. 5b (bottom) for IP-glasma model with an eccentric proton shape. Therefore, small systems like pA and pp collisions provide us an exciting opportunity of imaging subnucleonic-scale quantum fluctuations over yoctoseconds for the first time, a unique opportunity that does not exist in large systems.

To probe the connection to initial-state fluctuations in pp and pA collisions, new flow observables can be employed, as learned from the studies of AA collisions. As an example, new measurements of \(v_2–v_3\) and

![Figure 5](image_url)

Fig. 5. Left: The \(v_2\) and \(v_3\) data measured in 0–5% central He\(^3\)Au collisions at 200 GeV by PHENIX as a function of \(p_T\) [28]. Right: the \(v_2\) data measured in high-multiplicity pPb collisions at 5.02 TeV by CMS and ATLAS as a function of \(p_T\) [1, 29]. Comparison to hydrodynamic model calculations is also shown with different modeling of initial-state fluctuations as illustrated in the cartoons.
Can we observe jet quenching in small systems? If the observed collectivity in small systems is suggestive of strong final-state interactions, quenching of high \( p_T \) jets should also be present. The energy loss from a pQCD approach is expected to follow a dependence on \( \hat{q} \) and path length (or system size), \( L \), as \( \Delta E \sim \alpha_s(T)\hat{q}(T)L^2 \). As \( \hat{q} \) is expected to go as \( T^3 \), one arrives at \( \Delta E \sim T^3L^2 \). If comparing at similar multiplicities, a smaller (in \( L \)) system possesses a higher entropy density or temperature (\( T \)). Therefore, sizeable parton energy loss should also be expected for high-multiplicity pp and pA systems, comparable to that for peripheral AA collisions. In search for jet quenching in small systems, the main complication lies in the non-trivial correlation between underlying event multiplicity and hard probes, making it difficult to construct a proper in-vacuum reference. With a high-luminosity pPb run delivered by the LHC in 2016, the non-trivial correlation between underlying event multiplicity and hard probes, making it difficult to construct a proper in-vacuum reference. With a high-luminosity pPb run delivered by the LHC in 2016,
a promising measurement to pursue would be the $v_2$ at very high $p_T$ ($\gtrsim 10\text{ GeV/c}$) using multi-particle cumulants, which is related to the path length dependence of energy loss.

Does the collectivity extend to non-hadronic collisions (e.g., $ep$, $eA$, UPC and $e^+e^-$)? What are the fundamental requirements for creating a microscopic fluid? While high-multiplicity final state seems to be a necessary condition, initial colliding projectiles may not have to be hadronic but universal also for electromagnetic probes like electrons and photons. If collective flow behavior can be observed in $ep$, $eA$, UPC and even $e^+e^-$ collisions with high-multiplicity hadronic final state, it will open a new exciting avenue of research in many-body QCD system. After all, once the QCD vacuum is excited by collision of strong fields to generate sufficient initial entropy, it may then flow collectively like a perfect fluid.

5. Summary

In summary, clear evidence of long-range collective phenomena has been observed, and it is universal in all high-multiplicity hadronic collisions. Possible interpretations have to fall into only two scenarios distinguished by whether the interactions are at final or initial stage. There is a wide consensus that collectivity in AA collisions is driven by strong final-state interactions of a fluid-like QGP medium. However, collectivity seen in small systems raised debates on whether the “perfect” fluid paradigm is still valid or not. The key to address this question is to seek for the connection to the initial-state geometry. New direction in studying new flow observables, such as flow harmonic correlations, in small systems shows good promises and may provide unique opportunities of probing subnucleonic-scale quantum fluctuations for the first time. Future paths toward addressing several open questions on collectivity in small systems are discussed.

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