Imaging Dirac fermions flow through a circular Veselago lens

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Graphene charge carriers behave as relativistic massless fermions, thereby exhibiting a variety of counter-intuitive behaviors. In particular, at p-n junctions, they behave as photons encountering a negative index media, therefore experiencing a peculiar refraction known as Veselago lensing. However, the way Dirac fermions flow through a Veselago lens remains largely unexplored experimentally. Here, a novel approach to create a movable and tunable circular Veselago lens in graphene is proposed, using the polarized tip of a scanning gate microscope. Scanning the tip in the vicinity of a graphene constriction while recording the device conductance yields images related to the electron flow through a circular Veselago lens, revealing a high current density in the lens core, as well as two low current density zones along transport axis. Tight-binding simulations reveal the crucial role of the p-n junction smoothness on these phenomena. The present research adds new dimensions in the control and understanding of Dirac fermions optical elements, a prerequisite to engineer relativistic electron optics devices.

Optic-like manipulation of electron beams could in principle be achieved in p-n junctions, provided that the junction transmission is near perfect. However, in most cases, in particular in semiconductors, conventional p-n junctions are poor electron transmitters due to the inherent depletion of carriers in the vicinity of the junction. The discovery of graphene [1] changed this paradigm. Since charge carriers in graphene behave as massless Dirac fermions, Klein-like tunneling [2, 3] is at play, allowing perfect transmission for charge carriers impinging on a p-n junction perpendicularly. In addition, a diverging flow of Dirac fermions is refocused at a p-n interface, an effect known as Veselago lensing [4].

The possibility of forming such Veselago lenses using p-n junctions in graphene [5, 6] as well as the resulting caustics [7–13] have attracted a lot of attention from a theoretical point of view. Recent works suggested that Veselago lensing could be used to create highly focused electron beams [14], and even a two-dimensional scanning Dirac fermions microscope [15]. But the experimental challenges required to materialize these visionary ideas have not yet been overcome. Though few experiments reported signatures of Veselago lensing [16, 17], the lack of tunability of such devices kept these discoveries to the state of “proof of concept”. The use of a scanning tunneling microscopy (STM) tip to induce circular p-n junctions and probe the resulting local density of states unveiled beautiful phenomena such as whispering gallery modes [18, 19] and quasi-bound states [20]. However, though these approaches reveal the rich internal electronic structure of p-n nano-islands, they cannot probe the current density through these p-n junctions and in their vicinity, an essential parameter to control when designing Veselago-lensing-based devices.

In the present letter, a novel way to image current density through a tunable circular p-n junction, using a scanning gate microscope, is proposed. Scanning gate microscope (SGM) consists in scanning a polarized metallic tip, acting as a local gate above a device’s surface, and mapping out tip-induced conductance changes. Initially developed to investigate transport in III-V semiconductor heterostructures [21–23], it has been extended by several groups to investigate transport in graphene devices [24–30]. However, very few SGM experiments have been conducted yet on clean encapsulated graphene devices. Hence, it has proven difficult to observe electron-optics behavior in graphene using SGM, despite several theoretical predictions [31–33].

Here, the polarized SGM tip is used to create a movable and tunable circular Veselago lens. Applying a large voltage on the conductive tip induces circular p-n junction, whose potential can be controlled in amplitude and decay length. The tip scans in the vicinity of a constriction etched in a high mobility encapsulated graphene flake. Mapping the device conductance as a function of tip position yields images related to the electron flow through this Veselago lens. Surprisingly, transmission efficiency is drastically reduced when placing the lens next to the constriction entrances, and significantly enhanced when placing the lens at the constriction center. Tight-binding simulations indicate that SGM images partly reflect the current density distribution around the Veselago lens. The present work also emphasizes the role of the p-n interfaces sharpness and opens a door towards high precision optical elements design for Dirac fermions optics.

The sample consists of a graphene flake encapsulated between two 20 nm-thick h-BN flakes, deposited on top of a doped Si substrate covered by a 300 nm-thick SiO2 layer. A 250 nm-wide constriction is defined by...
corresponds to the tip-position-dependent value of $V_L$ Lorentzian shape centred at the constriction. This locus in the line profile mapping as a function of back-gate voltage (Fig. 2a). The locus density scanning tip. The biased tip locally changes the carriers discussed, only its relative fluctuations. on back-gate voltage $V_{bg}$ are taken into account. One device is measured between two line-contacts, and con-
ting probe microscope [35]. The conductance ber of a dilution refrigerator in front of a cryogenic scan-
tials).

The sample is thermally anchored to the mixing cham-
ber of a dilution refrigerator in front of a cryogenic scanning probe microscope [35]. The conductance $G$ of the device is measured between two line-contacts, and contact series resistances of 2 kΩ are taken into account. One of the contacts' resistance exhibits a strong dependence on back-gate voltage $V_{bg}$, and presents a secondary Dirac cone around $V_{bg} = +2.5$ V due to one of the ohmic contacts.

We now discuss transport under the influence of the scanning tip. The biased tip locally changes the carriers density $n$, in an isotropic way, leading to a Lorentzian evolution of $n$, centered at the tip position. When a p-n junction is induced by the biased tip, this Lorentzian profile sharpness has a crucial influence on Dirac fermion semiclassical trajectories in its vicinity [36]. It is therefore important to extract this profile from our experimental data. For this, we scan the tip along a line perpendicular to the constriction transport axis (red dashed line Fig. 1a and Fig. 3a), and map the resistance $R$ as a function of back-gate voltage (Fig. 2a). The locus of maxima in the line profile mapping as a function of $V_{bg}$ draws a Lorentzian shape centred at the constriction. This locus corresponds to the tip-position-dependent value of $V_{bg}$ required to locally reach the Dirac point at the center of the constriction, hence the change in $n$ required to compensate the tip-induced local change of density. In other words, the Lorentzian shape in Fig. 2a directly maps the effect of the tip-induced potential on the local carrier density, provided that the $V_{bg}$-axis is properly scaled into the local carrier density change $\Delta n$. The Lorentzian maximum linearly scales with tip voltage $V_{tip}$, allowing its characterization by a simple lever-arm parameter that depends on tip-to-graphene distance $d_{tip}$, as shown Fig. 2b. The half width at half maximum $R_{tip}$ depends also on $V_{tip}$ and $d_{tip}$ and varies between 200 and 300 nm (see supplemental materials).

With this fully-characterized tip influence, the SGM images (i.e. conductance as a function of tip position) can be analyzed. They are obtained by scanning the tip above the sample at fixed $V_{tip}$ and $V_{bg}$. Figure 3 shows such SGM maps, in different conditions of bulk carrier densities, tip voltages and temperature $T$. Figure 3b-c shows the evolution of the SGM contrast with carrier density. $V_{bg}$ is chosen such that transport is governed by electrons. In Fig. 3a, the highest tip-induced density change ($\Delta n_{max} \sim -2.3 \times 10^{13}$ m$^{-2}$) is lower than the bulk electron density ($3 \times 10^{15}$ m$^{-2}$). This therefore corresponds to a n-n’-n configuration, inducing a quasi-isotropic decrease of $G$ when the negatively-biased tip is placed above the constriction center. This means that $G$ is governed by the constriction region, and that reducing the carriers density in the center of the constriction reduces the total transmission, as naturally expected. This prominent influence of the constriction on transport justifies the validity of the above-described method to characterize the tip-induced potential. For lower bulk densities (Fig. 3b-c), the tip bias creates a circular n-p-n junction. In this case, an important qualitative change is observed. Contrary to Fig. 3, we observe a local maximum of $G$ when the tip is placed at the very center of the constriction, surrounded by an elliptic region of reduced conductance. Interestingly, $G$ change is not

FIG. 1. Graphene constriction: (a) AFM image of the studied device (b) Scheme of the experiment: the charged AFM tip locally creates a n-p-n junction in the graphene flake. (c) Resistance as a function of backgate-voltage $V_{bg}$ recorded in 2 contacts by standard lock-in technique at 4K, using an AC excitation of 35 $\mu$V at 77 Hz. The main Dirac cone is at a gate voltage of $V_{bg} = -1$ V, and a smaller Dirac cone around $V_{bg} = +2.5$ V is due to one of the ohmic contacts.

(d) Conductance as a function of $V_{bg}$ around the main Dirac cone.

etching [34] (see Fig. 1b). The mean free path is of the order of the contacts separation (see supplemental materials).

The sample is thermally anchored to the mixing chamber of a dilution refrigerator in front of a cryogenic scanning probe microscope [35]. The conductance $G$ of the device is measured between two line-contacts, and contact series resistances of 2 kΩ are taken into account. One of the contacts’ resistance exhibits a strong dependence on back-gate voltage $V_{bg}$, and presents a secondary Dirac cone around $V_{bg} = +2.5$ V (Fig. 1c). Note that this does not play any role here since the absolute $G$ value is not discussed, only its relative fluctuations.

FIG. 2. Evaluation of the tip-induced potential: (a) Resistance as a function of tip position along red dashed line Fig. 1a (zero being the center of the constriction), and $V_{bg}$. $d_{tip} = 90$ nm, $V_{tip} = -8$ V. (b) Tip lever-arm parameter evolution with $d_{tip}$, for $V_{tip} = -8$ V.

FIG. 3a-c. The half width at half maximum $R_{tip}$ depends also on $V_{tip}$ and $d_{tip}$ and varies between 200 and 300 nm (see supplemental materials).

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isotropic anymore, and two spots of lower $G$ are clearly visible on both sides of the constriction, as indicated by arrows in Fig. 3b-c.

A similar behavior is observed for hole-type carriers, as shown in Fig. 3d-f. Here, the bulk hole density is fixed ($V_{bg} = -3$ V) and $V_{tip}$ is varied. Once again, reducing the hole density below the tip leads to a quasi-isotropic single spot of reduced $G$ (Fig. 3d). But if $V_{tip}$ is chosen such that it locally changes the charge carriers sign (i.e. a circular p-n-p region is created), a local maximum of $G$ is observed when placing the tip at the very center of the constriction, and two regions of lower $G$ are visible when placing the tip on both sides of the constriction along transport axis (Fig. 3e-f). Note that data in Fig. 3b-f are recorded at $T = 28$ mK, which does not qualitatively change the main observations compared to Fig. 3a-c, recorded at 4 K. Noteworthy, the main features are really robust to temperature changes, and still observable up to at least 100 K (see supplemental materials).

In the following, we discuss the correspondence between the SGM maps and the current density pattern inside and around a tip-induced circular Veselago lens.

To understand the contrast in SGM images, tight-binding simulations of transport through model graphene devices are performed, using a home-made recursive Green function code [37]. The charge carrier flow around a tip-induced potential is first investigated in an infinite graphene sheet without the constriction, as shown in Fig. 4a,c,e and g. The tip-induced perturbation is included by imposing a potential

$$V(r) = \frac{V_m}{1 + \left(\frac{|r-r_{tip}|}{R_{tip}}\right)^d} \tag{1}$$

where $r_{tip}$ is the tip position, $R_{tip}$ the potential decay length and $d$ the decay exponent. The current density $|\mathbf{J}|$ is computed at a given energy $E$. Note that all distances, energies and potential are scaled to match the experimental parameters. Figure 4 presents the current density around a Lorentzian potential fixed at $r_{tip} = (0,0)$, for $R_{tip} = 300$ nm and $V_m = E$ (i.e. zero density at $r_{tip}$). In this configuration, the current density is minimal around the lower density region at the tip center, as expected. Figure 4b shows the computed SGM map when the perturbation is scanned around a 250 nm-wide constriction (represented by blue dashed lines), i.e. the system transmission as a function of the Lorentzian potential position. A minimum of transmission is observed when scanning the Lorentzian potential close to the constriction. Figures 4a-d present the current density and SGM image for the same Fermi energy but with a tip potential leading to a n-p-n configuration.

These tight-binding simulations capture the essential features observed in experimental the SGM images (Fig. 3), and the comparison with current densities shines light on the mechanisms at play in the experiment. Indeed, scanning the tip in the vicinity of a constriction yields images that are qualitatively similar to the current density pattern around and through the tip-induced circular Veselago lens. This result contrasts with SGM images obtained on GaAs quantum point contacts, which take advantage of the tip-induced backscattering to image the unperturbed electron wavefunctions [38], or trajectories through the bulk disordered potential [21, 39].

In graphene, backscattering is prevented by pseudo-spin conservation [36], leading to Klein tunneling, and this reverses the paradigm of SGM experiment. In turn, this allows to study the Dirac fermion flow through the tip potential.

The SGM images reveal two main features of the charge flow through the Veselago lens: (i) The local maximum of transmission, observed when the tip is placed at the center of the constriction, reflects the high current density at the lens core in the n-p-n or p-n-p configuration. In other words, Klein tunneling is directly imaged in real space. (ii) The reduced conductance spots observed
when the tip is placed slightly away from the constriction, indicated by arrows in Fig. 4d and very similar to the experimentally observed spots in Fig. 3, are the loci of all the tip positions that suppress the transmission of Dirac fermions through the constriction (more details in the supplemental materials). This is similar to the effect visible in Fig. 4c where the tip also induces low current density (dark) regions around the tip-induced p-n junction. Noteworthy, the patterns observed in the SGM maps are significantly distorted with respect to the corresponding current density patterns. For example, the distance of the reduced conductance spot to the constriction center in Fig. 4d is different from the distance of the lowest current density to the lens center in Fig. 4c. The spot of enhanced conductance also appears wider than the sharp spot of high current density. The constriction shape plays a central role in this distortion, as it alters Dirac fermions trajectories. The finite opening angle of the etched region tends to refocus the carriers toward the constriction, yielding a distortion in the SGM images compared to the current density. Noticeably, in the case of an abrupt constriction, the simulated SGM images appear really similar to the current density patterns around the Veselago lens, which offers promising perspectives for future experiments (see supplemental materials).

FIG. 4. **Tight-binding simulations:** Simulations of current density $J$ in a graphene sheet around the fixed tip potential of Eq. (1) characterized by the parameters $V_m$, $E$ and $d$ (left image in each couple), and simulated SGM maps obtained by displacing the same potential around a 250 nm-wide constriction (right image in the couple). (a-b) Lorentzian profile ($d = 2$) with $V_m = E$. (c-d) n-p-n configuration with $d = 2$ and $V_m = 2E$. (e-f) Abrupt circular p-n junction with $d = \infty$ and $V_m = 2E$. (g-h) Fast decaying potential profile ($d = 5$) with $V_m = 2E$. The chosen energy $E$ corresponds in all cases to $n = 1.5 \times 10^{14} \text{m}^{-2}$, and $\Delta n_{\text{max}} = -3 \times 10^{15} \text{m}^{-2}$ in (c) to (h). Dashed circle indicates the locus corresponding to zero charge density. Insets: schematics of potential profile vs position, Lorentzian profile is indicated as dashed blue line for comparison, when $d \neq 2$.

Interestingly, the smoothness of the p-n junctions plays a crucial role in the emergence of the low conductance and low current density points. They almost vanish and match the loci of zero charge density in the case of an infinitely sharp circular potential, as shown in Fig. 4 for the current density, and concomitantly Fig. 3 for the calculated SGM map. This can be easily understood in the framework of Klein tunneling, considering the effect of the p-n interface sharpness at the incident carriers energy [36]. Dirac fermions encountering an infinitely sharp p-n interface at normal incidence will be transmitted with perfect unitary probability, since the pseudo-spin conservation prevents backscattering. This probability decreases slowly with incident angle down to zero at $\pi/2$ incidence. However, in the case of a smooth p-n interface, though the transmission probability remains unitary at normal incidence, it decreases much faster with incident angle [17, 36, 40, 41]. Moreover, the smooth potential barrier bends electron trajectories [42] around the junction, generating the low current density zones. As visible by comparing Fig. 4g-h with Fig. 3-d, showing the current density around a potential of same amplitude but different decay exponent (respectively $1/r^5$ and $1/r^2$), the distance between low current density zones depends drastically on the smoothness of the p-n interface, which has to be considered in the future design of graphene-based electron optics devices.

In conclusion, a SGM tip scanned above a high mobility graphene device is used to create a smooth circular Veselago lens. By scanning the tip in the vicinity of a
constriction, images related to the electron flow through this circular Veselago lens are obtained, as confirmed by tight-binding simulations. In particular, a high current density is observed at the lens core, as a direct consequence of Klein tunneling. The present study also reveals the existence of low current density points away from a smooth circular p-n junction, highly sensitive to the junction smoothness. Whereas much studies investigated caustics of Veselago lenses [7–13] (i.e. points at which trajectories density diverges), the existence of such low current density points may prove very useful in the future design of Dirac fermions optical elements, adding a new possibility to the existing toolbox.

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1 Determination of the lever-arm

In order to convert the back gate voltage $V_{bg}$ into a charge carrier density $n_{2D}$, we need to evaluate the lever arm defined by $C = n / \Delta V$ where $\Delta V = V_{bg} - V_{CNP}$ with $V_{CNP} = -1$ V in our sample. This can be achieved by two different methods:

- From a simple capacitor model with $d_{h-BN} = 30$ nm and $d_{SiO_2} = 300$ nm, the lever-arm is $C = 6.1 - 6.7 \times 10^{14} m^{-2}V^{-1}$, depending on h-BN dielectric constant ($\epsilon_{h-BN} = 2 - 4$) [1].

- From Shubnikov-de Haas (SdH) oscillations, the Landau levels positions in the $(V_g, B)$ plane are given by the expression

$$B_{\nu} = \frac{\pi h}{2eV} C \Delta V$$

where $\nu$ is the filling factor. By fitting the Landau levels positions in the conductance map in the $(V_g, B)$ plane (Fig. S1b), measured in the bulk of the sample (see Fig. S1a for the measurement configuration), we obtain $C = 7.4 \pm 0.2 \times 10^{14} m^{-2}V^{-1}$.

2 Mobilities

Charge carriers mobilities are extracted from linear fits to the $G$ vs $n$ curve in the vicinity of the charge neutrality point (i.e. the Dirac cone). Note that we measure $G$ in the two-contacts configuration depicted in Fig. S1a, taking into account a total series resistance of 2 kΩ corresponding to the electrical filters on the measurement setup. This yields lower bounds for the mobilities: $\mu_e \sim 40 000$ cm$^2$V$^{-1}$s$^{-1}$ for electrons and $\mu_h \sim 28 000$ cm$^2$V$^{-1}$s$^{-1}$ for holes.
Figure S1: Lever-arm parameter of the back-gate. (a) AFM topography image of the graphene device. The two contacts used for the transconductance measurement shown in (b) are below the constriction, in yellow, and separated from each other by a distance of 800 nm. (b) Transconductance $\partial G/\partial V_{bg}$ as a function of $V_{bg}$ and $B$.

Figure S2: Electron and holes mobilities from the Dirac cone slope. Dirac cone measured in the same configuration as Fig. S1a. Two linear fits allow to calculate lower bounds for the electron and holes mobilities.

3 Temperature robustness

The main features of the SGM images discussed in the main text are visible at a temperature of $\approx 100$ K. Fig. S3 shows SGM maps for a tip voltage of $-8$ V and a tip-to-graphene distance of
≈100 nm. The vertical as well as the horizontal length scales are only roughly estimated since we have no calibration for our piezoelectric scanner at 100 K. These images therefore just bring a qualitative insight on the temperature robustness of the observed features.

Figure S3: Signatures of Veselago lensing at high temperature. (a) $G$ vs $V_{bg}$ across the QPC, measured in four-contacts configuration, at a temperature around 100 K. (b-f) SGM maps for a tip voltage $-8$ V and tip-to-graphene distance of ≈100 nm, for the gate voltages indicated on (a).
4 Tip characterization

Figure S4: Evaluation of the tip-induced potential. (a) Resistance map as a function of tip position along a line passing by the centre of the constriction (position 0.0) perpendicularly to the transport axis, and $V_{bg}$. $d_{tip} = 90$ nm, $V_{tip} = -8$ V. (b) Fits of the tip-induced resistance maxima as a function of tip position extracted for different values of $V_{tip}$, from $-8$ V to $+8$ V, and $d_{tip} = 90$ nm, converted into local carrier density. (c) Evaluation of maximum charge density change around the center of the tip-affected region as a function of tip voltage (red points), and linear fit giving a tip lever-arm $C_{tip} = 4.5 \times 10^{14}$ m$^{-2}$V$^{-1}$ (red line). $R_{tip}$, corresponding to half-width at half maximum of the Lorentzian fits in (b), as a function of $V_{tip}$ (blue dots). (d) Tip lever-arm parameter evolution with $d_{tip}$, for $V_{tip} = -8$ V.

Fig. S4 presents the full characterization of the tip-induced potential for different tip voltages. As in the main text, we scan the tip along a line perpendicular to the QPC transport axis (red dashed line in Fig. 1a in the main text), and map the resistance $R$ as a function of back-gate voltage. The resulting figure exhibits a Lorentzian-shape maximum, as shown in Fig. S4a.

Horizontal lines can also be noticed in Fig. S4a, around $V_{bg} = +2.3$ V and $V_{bg} = -1$ V. The one at $V_{bg} = +2.3$ V corresponds to the contribution of the series ohmic contact responsible for the second Dirac cone also visible at $V_{bg} = +2.3$ V in Fig. 1c of the manuscript. It is therefore completely insensitive to the tip position along this specific line since this contact is far away from the scan line. The horizontal conductance minimum around $V_{bg} = -1$ V corresponds to the bulk...
Dirac cone, i.e. away from the constriction.

Acquiring mappings similar to Fig. S4a, for different tip voltages at a fixed tip-to-graphene
distance of 90 nm, yields the set of Lorentzian fits to the data displayed Fig. S4b.

In figure S4c we summarize these fits by plotting the maximum density change on top of
the Lorentzian \( \Delta n_{\text{max}} \) (red dots, right axis) and its half-width at half maximum \( R_{\text{tip}} \) (blue dots,
left axis). The maximum tip-induced density change as a function of tip voltage scales almost
linearly with tip voltage, which allows us to characterize it by a simple lever-arm parameter \( C_{\text{tip}} = 4.5 \times 10^{14} \text{ m}^{-2}\text{V}^{-1} \) for this tip-to-graphene distance.

5 Tight-binding simulations

In this section, we introduce briefly the tight-binding calculations [2] used to simulate the current
density and the SGM maps presented in the main text (Fig. 4). Similar simulations have also been
performed with the Kwant package [3] and led to the same conclusions.

The electronic properties of the considered graphene system are modeled by a first-nearest-
neighbor tight-binding Hamiltonian [4]:

\[
H = \sum_n V_n c_n^\dagger c_n + t \sum_{\langle n,m \rangle} c_n^\dagger c_m, \tag{1}
\]

where \( t \) represents the hopping energy between first-nearest-neighbor \( n^{\text{th}} \) and \( m^{\text{th}} \) atoms and \( V_n \)
corresponds to the potential energy induced by gate voltages at the \( n^{\text{th}} \) side. To compute the
transport quantities, this tight-binding Hamiltonian is solved using the Green’s function technique
[2]. In particular, the retarded Green’s function is determined as:

\[
G(E) = \left[ E + i0^+ - H - \Sigma_L - \Sigma_R \right]^{-1}, \tag{2}
\]

where \( \Sigma_{L,R} \) are the self-energies describing the left and right device-to-lead couplings, respectively.
This Green’s function equation is solved using the recursive method [5]. The transport quantities
such as conductance and bond currents injected from the left lead are then computed using the
following formulas:

\[
G(E_F) = \frac{2e^2}{h} \text{Tr} \left[ \Gamma_L G \Gamma_R G^\dagger \right], \tag{3}
\]

\[
J_{\text{L}nm}(E_F) = -\frac{2e}{h} H_{nm} \text{Im} \left( \Gamma_L G \Gamma_R G^\dagger \right)_{nm}, \tag{4}
\]

with \( \Gamma_{L,R} = i \left( \Sigma_{L,R} - \Sigma_{L,R}^\dagger \right) \).

In order to simulate graphene devices of similar sizes (i.e., at the \( \mu \text{m} \) scale) as in the experi-
ments, there is however a numerical challenge. In particular, the simulation of such large graphene
system (typically, more than 8000 lattice sites in transverse direction for \( W = 1 \mu \text{m} \)) includes sev-
eral million carbon orbitals. Hence, to avoid such numerical challenge, we employed the scaling
technique as presented in [6], that has been demonstrated to work very well for graphene de-
vice simulations. In particular, we increase the bond length \( a_{\text{CC}} \) between the carbon atoms by
a factor of \( s_f = 24 \) (i.e., \( a_{\text{CC}} = s_f a_0 \) where \( a_0 = 0.142 \text{ nm} \) is the well known \( \text{C} - \text{C} \) bond length
in graphene) and simultaneously decreases the nearest neighbor hopping energy with the same
factor (i.e., \( t = t_0 / s_f \) where \( t_0 = 2.7 \text{ eV} \) is the typical tight-binding hopping energy [4]).

5
6 Anti-focusing with a constriction

In the main text, we study the appearance of low conductance spots (low \( G \) spots) in the experimental SGM maps. In order to understand the origin of these spots, we compare simulated SGM maps (Fig. S5a) with current density maps simulated in a graphene sheet without constriction (Fig. S5b). In this section, we detail how to interpret the SGM map in relation to the current density map. In particular, we will explain why the low \( G \) spots (blue arrows in Fig. S5a) do not have exactly the same shape as the low current density spots (low \( \vec{J} \) spots, red arrows in Fig. S5b). Section 7 presents some perspectives towards getting a direct correspondence between the SGM maps and the current density around a p-n junction.

![Figure S5: Comparison between simulated current density and SGM maps. (a-b)](image)
(a) Same SGM maps as Fig. 4d and 4c in the main text. Different zones have been delimited on top of the maps for the discussion exposed in the text. Blue arrows indicate the position of the low \( G \) spots and red arrows indicate the position of the low \( \vec{J} \) spots. (c) Qualitative sketch illustrating the electron trajectories (i, ii and iii) impinging the circular smooth p-n junction in an area corresponding to the orange square in (b). A forbidden zone is associated to each trajectory (blue boundaries for trajectory ii and green for iii).

To properly understand the SGM maps, we performed simulations using our home-made code presented in section 5. Figure S6 shows simulations of the current density in a large graphene ribbon containing a constriction whose width is progressively lowered. The tip potential used in each figure is the same as in Fig. 4c-d of the main text. All the current density maps are normalised with the same factor \( J_{\text{max}} \) so they can be compared together.

Figure S6a shows a map of the current density in a ribbon without constriction that exhibits the two characteristic low \( \vec{J} \) spots. These spots are caused by the smooth p-n junction that bends the electrons trajectories. When the constriction width is lowered (Fig. S6b-e), the edges of the constriction also bend electrons trajectories towards the centre of the constriction. The conductance is therefore given by the combined influence of the tip potential and the constriction. The important feature observed in Fig. S6b-e is the reduction of the low \( \vec{J} \) spot size as the constriction width is lowered. This explains why the low \( G \) spots do not directly map the low \( \vec{J} \) spots.
Figure S6: Simulation of current density when varying the constriction width. A Lorentzian potential ($V_m = 2EF$) creates a circular p-n junction (red dashed line, $R_{tip} = 300$ nm). (a) Current density computed without constriction. Two spots of low current density appear, similarly to figure S5a. (b-e) Current density computed for constriction widths of respectively: (b) 1450 nm, (c) 1050 nm, (d) 650 nm and (e) 250 nm. The distance between the tip and the constriction is 600 nm.

Figure S7: Simulation of current density when varying the tip position. A Lorentzian potential ($V_m = 2EF$) creates a circular p-n junction (red dashed line, $R_{tip} = 300$ nm). The tip position is moved along $x$ and is located at a distance from the constriction of (a) 800 nm (b) 600 nm, (c) 400 nm, (d) 200 nm and (e) 0 nm. The width of the constriction is 250 nm.
With the bending effect of the edges in mind, we can discuss the origin of the low $G$ spots. Figure S7 illustrates the influence of the tip potential on the conductance for a constriction of 250 nm. The tip potential is the same as the one used in Fig. S6 and Fig. 4c-d. We can distinguish four situations:

1. When the tip is far from the centre of the constriction (Fig. S7a), electrons flow along the edges. The low $\vec{J}$ spot at the constriction side disappears and the conductance remains high.

2. When the tip starts to get closer to the constriction (Fig. S7b), the trajectories along the edges encounter the p-n junction with a parallel incidence. Due to the junction smoothness, a forbidden zone\(^1\) (illustrated in Fig. S5c) appears, whose width depends on the incident angle of the electron trajectories. For parallel incidence (trajectory iii in Fig. S5c), the forbidden zone (delimited by green boundaries) is wide. By approaching the tip from the edge, the forbidden zone starts to block edge trajectories, reducing the conductance.

3. When further approaching the tip (Fig. S7c and d), the majority of trajectories will be blocked due to the effect of the forbidden zone around the p-n junction. Only the trajectories favoured by Klein tunnelling will go through the constriction. In this configuration, the transmission mainly depends on the constriction width and the p-n junction shape, as it will be illustrated in section 7.

4. Finally, when the tip is located at the centre of the constriction (Fig. S7e), the conductance is dominated by Klein tunnelling so that the transmission increases again. This is illustrated by trajectory i in Fig. S5c.

Since we have a good insight about the mechanisms at play when the tip approaches the constriction, we can summarise by establishing the correspondence between the SGM map of Fig. S5a and the current density map of Fig. S5b. To do so, the SGM map and the current density map have been divided into different zones in Fig. S5.

- **Zone 1** corresponds to an increase of the conductance. It is a direct consequence of the high current density in the lens core indicated by zone I. This region is therefore an image of Klein tunnelling in real space.

- **Zone 2** and **zone II** are a consequence of the smoothness of the junction. Zone 2 has its origin in the forbidden zone that blocks electron trajectories in the vicinity of the constriction, as discussed above in point 3. Zone II is also related to the presence of the forbidden zone. Indeed, only perpendicular trajectories cross the p-n junction. The smoothness of the potential also bends the trajectories, leading to the stretched shape of zone II along transport axis.

- **Zone 3** in Fig. S5a is given by potential positions that are far from the constriction center so that the perturbation has only a weak effect on the electron transmission.

Even if they are both due to the smoothness of the potential, zone 2 does not directly map zone II. Indeed, as highlighted in Fig. S6 and the related discussion, the bending of the electron trajectories, due to the constriction, suppresses the low $\vec{J}$ spots. In section 7, we will see how the constriction geometry can be modified to map zone II in a SGM map.

\(^1\)By forbidden zone, we mean a zone that electrons can only cross by tunnelling (with a decaying exponential transmission), as illustrated by trajectory ii in Fig. S5c [7]
7 Constriction as a current density detector

In section 6, the origin of the low G spots in the SGM maps has been discussed. They are due to the effect of the smooth p-n interface that blocks the transmission of electrons when it is in the vicinity of the constriction. Due to the bending of the electron trajectories by the constriction edges, the low G spots do not have exactly the same shape as the low $\vec{J}$ spots. In this section, we will detail how the design of the constriction can be modified in order to obtain a direct correspondence between the SGM map and the current density map without constriction. To do so, a constriction defined by vertical (i.e.; perpendicular to the transport axis) boundaries is studied, as depicted in Fig. S8a. The constriction opening width, along $y$, is 250 nm and its length, along $x$, is 60 nm.

![Figure S8: Effect of the constriction shape on the SGM maps.](image)

(a) Current density map in a constriction defined by vertical boundaries (yellow dashed line) in presence of a Lorentzian potential ($V_{m} = 2E_F$) that creates a circular p-n junction (red dashed line, $R_{tip} = 300$ nm). (b) SGM map in the same constriction and for the same Lorentzian potential as (a). The yellow dashed line shows the limits of the low current density zone shown in (d). (c) Blue curve: conductance at $y_{tip} = 0$, obtained along the white dash-dotted line of (b). Red curve: conductance curve at $y_{tip} = 0$, obtained along the white dash-dotted line of Fig. S5a. The arrows show three characteristic zones of the SGM map. (d) Same figure as Fig. S5 and Fig. 4c in the main text. The yellow dashed line delineates the low current density zone.

The SGM map obtained in the new constriction geometry is shown in Fig. S8b. It can be compared to the map "of Fig. S5a obtained for the previous constriction geometry. Fig. S8c shows
line cuts at $y_{tip} = 0$ in the two SGM maps. The obtained conductance curves highlight the key differences between the two geometries. Three zones can be distinguished, indicated by arrows in Fig. S8b and c:

- **The gold arrow** indicates the zone of increased conductance caused by Klein tunnelling. This is the zone 1 discussed in section 6.

- **Light blue arrows** indicate the low $G$ spots described by zone 2 in section 6. The position and the shape of these spots are rather similar for both constriction geometries, as highlighted in Fig. S8c. The shape of the low $G$ spots therefore mainly depend on the constriction width and the effect of the p-n interface. The slope of the constriction boundaries will also slightly modify the shape. Indeed, the p-n interface will not reach the constriction edges at the same distance from the constriction center (see Fig. S7 and the related discussion - point 2). The conductance will therefore not fall at the same position, as illustrated in Fig. S8c.

- **Dark blue arrow** indicate plateaus of reduced conductance only visible for the vertical boundaries (blue curve in Fig. S8c). By comparing Fig. S8b and d, it appears that these zones of reduced conductance are directly related to the low $\vec{J}$. To emphasise this correspondence, the delimitation of the low $\vec{J}$ zone (yellow dotted line in Fig. S8d) is reported on Fig. S8b.

The appearance of this last zone, for the vertical boundaries, can be explained by a less efficient bending of electron trajectories toward the center of the constriction, compared to the oblique edge (see section 6). Indeed, the vertical edges will mostly back-scatter electrons instead of bending their trajectories as illustrated by Fig. S8a (in comparison to Fig. S7). The low $\vec{J}$ spots (zone III in Fig. S5) have therefore a real influence when the potential is in front of the constriction. This explains the good correspondence between Fig. S8b and d.

To conclude, the shape of the constriction plays an important role on the SGM data. By designing properly this constriction, it can be optimised to yield features that directly correspond to the current density around a tip-induced potential. This opens the way to new devices for SGM characterisation.
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