Diffusive Origin of the Cosmic-Ray Spectral Hardening

Nicola Tomassetti
INFN – Sezione di Perugia, 06122 Perugia, Italy
E-mail: nicola.tomassetti@pg.infn.it

Abstract. Recent data from ATIC, CREAM and PAMELA revealed that the energy spectra of cosmic
ray (CR) nuclei above 100 GeV/nucleon experience a remarkable hardening with increasing energy. This
effect cannot be recovered by the conventional descriptions of CR acceleration and diffusive propagation
processes. Using analytical calculations, I show that the hardening effect can be consequence of a spatial
change of the CR diffusion properties in different regions of the Galaxy. I discuss the implications of this
scenario for the main CR observables and its connections with the open issues of the CR physics.

1. Introduction
Understanding the origin of the cosmic ray (CR) energy spectrum is central to astrophysics and has long
been the focus of intensive study. The spectrum of CR nuclei at \(\sim 10^1–10^6\) GeV/nucleon is thought to be
the result of diffusive shock acceleration (DSA) mechanisms in supernova remnants (SNR), followed by
diffusive propagation in the interstellar medium (ISM) [1]. The conventional descriptions predict source
spectra such as \(E^{-\nu}\) for primary nuclei (e.g., H, He) which are steepened as \(E^{-\nu-\delta}\) by diffusion. The
data constrain \(\nu + \delta \approx 2.7\) (depending on the element) and \(\delta \sim 0.2–0.7\), whereby \(\nu \sim 2.0–2.5\). In DSA
calculations, the source spectral slope should be \(\nu < 2.2\), which implies \(\delta > 0.5\). On the other hand, large
values for \(\delta\) are at odds with the anisotropy observations at TeV energies, which favor \(\delta < 0.3\). On top
of that, recent experiments ATIC-2, CREAM, and PAMELA, reported a remarkable spectral hardening
at energies above \(\sim 100\) GeV/nucleon which cannot be explained by conventional mechanisms [2, 3, 4].
Proposed explanations of this effect deal with acceleration mechanisms [5, 6], nearby SNRs [7] or multi-
source populations [8, 9]. I propose here that the hardening originates from a spatial change of the CR
diffusion properties in the different regions of the Galactic halo. The key hypotesis is that the diffusion
coefficient \(K\) is not separable into energy and space terms as usually assumed. As I will show, this
hypotesis leads to a pronounced change in slope for the energy spectra of CR nuclei at Earth, giving a
good description of the present data. Remarkably, the model proposed here has also positive impacts on
several open problems in the CR acceleration/propagation physics.

2. The two-halo model
I use a one-dimensional inhomogeneous diffusion model for CR transport and interactions [10]. The
Galaxy is modeled to be a disc (half-thickness \(h \approx 100\) pc) containing the gas (number density \(n \approx 1\) cm\(^{-3}\)) and the CR sources. The disc is surrounded by a diffusive halo (half-thickness \(L \approx 5\) kpc)
with zero matter density. For each stable CR nucleus, the transport equation is:

\[
\frac{\partial N}{\partial t} = \frac{\partial}{\partial z} \left( K(z) \frac{\partial N}{\partial z} \right) - 2h\delta(z)\Gamma_{\text{inel}}N + 2h\delta(z)Q, \tag{1}
\]
where $N(z)$ is its number density as function of the $z$-coordinate, $K(z)$ is the position-dependent diffusion coefficient and $\Gamma^\text{inel} = \beta c n_{\text{cr}} \sigma^\text{inel}$ is the destruction rate in the ISM at velocity $\beta c$ and cross section $\sigma^\text{inel}$. The source term $Q$ is split into a primary term $Q_{\text{pri}}$, from SNRs, and a secondary production term $Q_{\text{sec}} = \sum_j \Gamma^\text{spall}_j N_j$, from spallation of heavier $(j)$ nuclei with rate $\Gamma^\text{spall}_j$. The quantities $N$, $K$, $Q$ and $\Gamma^\text{inel}$ depend on energy too. Equation 1 can be solved in steady-state conditions ($\partial N/\partial t = 0$) with the boundary conditions $N(z,L) = 0$ [10]. For each CR nucleus, the differential energy spectrum is given by:

$$J(z,E) \equiv \frac{\beta c}{4\pi} N(z,E) = \frac{\beta c}{4\pi} \frac{Q(E)}{\lambda(E)} \left[ 1 - \frac{\lambda(z,E)}{\Lambda(E)} \right],$$

where $K_0 = K(z \equiv 0)$, $\lambda = K_0 \int_0^{z \equiv 0} \frac{dz}{K(z)}$, and $\Lambda = \lambda(z \equiv L)$. In CR propagation studies, the diffusion coefficient is usually assumed to be separable as $K(z,E) \equiv f(z)K_0(E)$ (in homogenous models $f(z) \equiv 1$, whereby $\Lambda = L$). In these models, the functions $\lambda$ and $\Lambda$ are independent on energy, so that the predictions at Earth ($z=0$) are spectrally un influenced by the choice of $f(z)$. Conversely, I employ a non-separable $K(z,E)$, which reflects the existence of different CR transport properties in the propagation volume, depending on the nature and scale distribution of the magnetic-field irregularities. In fact, while SNR explosions may generate large irregularities in the region near the Galactic plane, the situation in the outer halo is different because that medium is undisturbed by SNRs. From these considerations, the authors of Ref. [11] found that the turbulence spectrum in the halo should be flatter than that in the Galactic plane. This implies a strong latitudinal dependence for the parameter $\delta$, which suggests spatial variations of the CR energy spectra. Noticeably, new data reported by Fermi/LAT on the diffuse $\gamma$-ray emission at $\sim 10$–$100$ GeV of energy seem to support these suggestions: the $\gamma$-ray spectra observed near the Galactic plane (latitude $|b|<8^\circ$) are found to be harder than those at higher latitudes [12]. Following the above arguments, I adopt a simple two-halo model consisting in two diffusive zones. The inner halo is taken to surround the disk for a typical size $\xi L$ of a few hundred pc ($\xi \sim 0.1$). Its medium properties are influenced by SNRs, which produce a steep turbulence spectrum in terms of energy density per wave number, $w(\kappa) d\kappa \sim \kappa^{-2+\delta} d\kappa$, presumably close to the Kolmogorov regime $\delta \sim 1/3$. The outer halo represents a wider region, $\xi L < |z| < L$, which is undisturbed by SNRs. Its turbulence spectrum is driven by CRs and should be flatter. For instance, CRs with rigidity spectrum $R^{-\nu'}$ (where $R = p/Z$) excite turbulent modes of wave-numbers $\kappa \propto 1/R$, giving a spectrum $\sim \kappa^{-2+\delta'}$, where $\delta' \sim \nu - 1$ is of the order of the unity. This situation can be realized by a rigidity dependent diffusion coefficient of the type

$$K(z,R) = \begin{cases} k_0 \beta (R/R_0)^{\delta} & \text{for } |z| < \xi L \text{ (inner halo)} \\ k_0 \beta (R/R_0)^{\delta+\Delta} & \text{for } |z| > \xi L \text{ (outer halo)}, \end{cases}$$

where $k_0 \equiv 0.04$ kpc/Myr specifies its normalization at the reference rigidity $R_0 \equiv 5$ GV. For $R > R_0$, the diffusion coefficient of Eq. 3 produces a higher CR confinement in the inner halo (with $\delta \sim 1/3$), whereas the outer halo (with $\Delta = \delta' - \delta \sim 0.5 - 1$) represents a reservoir from which CRs leak out rapidly and can re-enter the inner halo. The non-separability of $K$ has a remarkable consequence on the model predictions at $z = 0$. It can be understood if one neglects the term $\Gamma^\text{inel}$ and takes a source term $Q_{\text{pri}} \propto R^{-\nu}$. From Eq. 2 and using Eq. 3, one finds

$$J_0 \equiv J(z = 0) \sim \frac{L}{k_0} \left\{ \xi (R/R_0)^{-\nu - \delta} + \left( 1 - \xi \right) (R/R_0)^{-\nu - \delta - \Delta} \right\},$$

which describes the CR spectrum as a result of two components. Its differential log-slope as a function of rigidity reads

$$\gamma(R) = -\frac{\partial \log J_0}{\partial \log R} \approx \nu + \delta + \frac{\Delta}{1 + \xi R/R_0^\Delta},$$
Thus, the two-halo model predicts a gradual flattening also connected with the global large-scale anisotropy amplitude, which should increases as 

which shows a clear transition between two regimes. In practice the low-energy regime \((\gamma \approx \nu + \delta + \Delta)\) is never reached due to spallation, that becomes relevant and even dominant over escape \((r_{\text{inel}} \gtrsim K_0/h\)). In this case the log-slope is better approximated by \(\gamma \approx \nu + \frac{1}{2}(\delta + \Delta)\) \cite{22}. The hard high-energy regime \((\gamma \approx \nu + \delta)\) is determined by the diffusion properties of the inner halo only. In this limit one has \(\Lambda \approx \xi L\). The effect vanishes at all rigidities when passing to the homogeneous limit of \(\xi \rightarrow 1\) (one-halo) or \(\Delta \rightarrow 0\) (identical halos), where one recovers the usual relation \(\gamma = \nu + \delta\). From Eq. 2, it can be seen that the intensity of the harder component diminishes gradually with increasing \(|z|\), \(i.e.,\) the CR spectra at high energies are steeper in the outer halo, as suggested by the new Fermi/LAT data.

Figure 1 shows the \(H\) and \(He\) spectra at Earth, from Eq. 2, at kinetic energies above \(10\, \text{GeV}\). The implementation of the model follows Refs.\cite{10, 23}. The source spectra are taken as \(Q \propto R^{1/3}\), while its low-energy \(Q \propto R^{-\nu}\), with \(\nu \approx 2.3\) for \(H\) and \(\nu \approx 2.16\) for \(He\) \cite{24}. The two halos are defined by \(L \approx 5\, \text{kpc}\) and \(\xi L \approx 0.5\, \text{kpc}\). A Kolmogorov-type diffusion is adopted for the inner halo \((\delta \approx 1/3)\), while \(\Delta\) is taken as \(0.55\), consistently with Ref.\cite{11}. Results from the two-halo model (solid lines) are compared with those from the homogeneous model (dashed lines), which uses \(\delta \approx 0.6\), and with recent CR data. Below \(\approx 10\, \text{GeV/nucleon}\), the solar modulation is apparent and it is described using a force-field modulation potential \(\phi \approx 400\, \text{MV}\) \cite{25}. Note that my model may be inadequate in this energy region due to approximations. Remarkably, my model reproduce well the observed changes in slope at \(\gtrsim 100\, \text{GeV}\), in good agreement with the trend indicated by the data. It should be noted, however, that the sharp spectral structures suggested by the PAMELA data at \(\sim 300\, \text{GeV}\) cannot be recovered. Figure 1c shows the mean logarithmic mass, \(<\ln(A)\rangle\), which is described well by the two-halo model. Figure 1d shows the B/C ratio from the two-halo model, which is predicted to harden as \(\propto \Lambda(E)/K_0(E)\), while its low-energy behavior is similar to that of the homogeneous model (with \(\delta = 0.6\) in the whole halo). Interestingly, a slight flattening for the B/C ratio is also suggested by recent data, \(e.g.,\) from TRACER. A multi-channel study of the AMS data will allow to resolutely test these features. The B/C ratio hardening is also connected with the global large-scale anisotropy amplitude, which should increases as \(\eta \propto K_0/\Lambda\) \cite{26, 10}. Thus, the two-halo model predicts a gradual flattening of \(\eta\), as indicated by the present data. It
is also interesting noticing that the anisotropy may be furtherly reduced at all energies if one accounts for a proper radial dependence for \( K \) [27]. In summary, the model I proposed is potentially able to reconcile a weak energy dependent of the anisotropy amplitude (as suggested by observations) with relatively hard source spectra \( \nu \approx 2.2 \) (as preferred by the DSA theory), giving good fits to the B/C ratio. I recall that plain diffusion models employ \( \nu \approx 2.2 \) and \( \delta \approx 0.5 \) in the whole halo (which gives a too strong energy dependence for \( \eta \)), whereas diffusive-reacceleration models use \( \nu \approx 2.4 \) and \( \delta \approx 1/3 \) (which are too challenging for the DSA and require strong reacceleration to match the B/C data).

3. Conclusions

I have shown that the spectral hardening observed in CRs may be consequence of a Galactic diffusion coefficient that is not separable in energy and space terms. From the model presented here, the hardening arises naturally as a local effect and vanishes gradually in the outer halo, where the CR spectra are also predicted to be steeper. This effect must be experienced by all CR nuclei as well as by secondary-to-primary ratios. Recent data from \emph{Fermi}/LAT and TRACER seem to support this scenario, but the change in slope predicted by this model is more gradual than that suggested by PAMELA data. All these features can be tested with the data forthcoming by AMS, so that the present model will be discriminated against other interpretations. This scenario has an interesting impact on the CR physics. As shown, a Kolmogorov diffusion for the inner halo (\( \delta \sim 1/3 \)) is consistent with relatively hard source spectra (\( \nu \sim 2.2 \)), giving good descriptions of both the B/C ratio and the high-energy trend of the anisotropy amplitude.

I acknowledge the support of the \emph{Italian Space Agency} under contract ASI-INFN I/075/09/0.

References

[1] Strong A W, Moskalenko I V, and Ptuskin V S 2007 \emph{Ann. Rev. Nucl. & Part. Sci.} \textbf{57} 285
[2] Panov A D et al 2009 \emph{Bull. Russ. Acad. Sci.: Phys.} \textbf{73-5} 564
[3] Ahn H S et al 2010 \emph{ApJ} \textbf{715} 1400
[4] Adriani O, et al 2011 \emph{Science} \textbf{332} 69
[5] Biermann P L, et al 2010 \emph{ApJ} \textbf{725} 184
[6] Ptuskin V, et al 2011, \emph{Proc. 32nd ICRC, 0367, Beijing}
[7] Thoudam S and Hörandel J R 2012 \emph{MNRAS} \textbf{421} 1209
[8] Zatsepin V and Sokolskaya N V 2006 \emph{A&A} \textbf{458} 1
[9] Yuan Q, Zhang B, and Bi X J 2011 \emph{PRD} \textbf{84} 043002
[10] Tomassetti N 2012 \emph{ApJ Letters} \textbf{715} L13
[11] Erlykin A D and Wolfendale A W 2002 \emph{J. Phys. G: Nucl. Part. Phys.} \textbf{28} 2329
[12] Ackermann M, et al 2012 \emph{ApJ} \textbf{750} 3
[13] Asakimori K, et al 1998 \emph{ApJ} \textbf{502} 278
[14] Derbina A V, et al 2005 \emph{ApJ} \textbf{628} L41
[15] Aglietta M, et al 2004 \emph{Astropart. Phys} \textbf{20} 641
[16] Hörandel J R 2004 \emph{Astropart. Phys} \textbf{21} 241
[17] Obermeier A, et al 2011 \emph{ApJ} \textbf{742} 14
[18] Aguilar M, et al 2010 \emph{ApJ} \textbf{724} 329
[19] Panov A D, et al 2007, \emph{Proc. 30th ICRC, 2, 3, Mérida}
[20] Engelmann J J, et al 1990 \emph{A&A} \textbf{233} 96
[21] Ahn H S, et al 2008 \emph{Astropart. Phys} \textbf{30} 133
[22] Blasi P and Amato E 2012 \emph{JCAP} \textbf{01} 010
[23] Tomassetti N and Donato F 2012 \emph{A&A} \textbf{544} A16
[24] Malkov M A, Diamond P H, and Sagdeev R Z 2012 \emph{PRL} \textbf{108} 081104
[25] Gleeson L J and Axford W I 1968 \emph{ApJ} \textbf{154} 1011
[26] Shibata T, et al 2006 \emph{ApJ} \textbf{612} 238
[27] Evoli C, et al 2012, \emph{Preprint} astro-ph/1203.0570