Gasdynamic Investigation of Rotating Gas Accretion

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Abstract

Gasdynamic features of slowly rotating gas axisymmetric accretion onto a gravitating center are investigated. The process of flow restructuring is studied as the angular velocity of accreting matter approaches the Keplerian angular velocity. For spherically-symmetric accretion, the conditions are found of the existence of a steady-state supersonic solution for various external boundary conditions. The numerical study is performed on the basis of the Lax–Friedrichs-type second order of accuracy high-resolution numerical scheme with the implicit approximation of the source term in the Euler equations.

1 Introduction.

Accretion onto neutron stars and black holes gives the main energy supply in galactic X-ray sources. Among X-ray sources with high mass companions there are long-periodic
pulsars, whose origin is not yet clear (Nagase 1989; Lipunov 1992). Owing to high-speed stellar wind, the angular momentum of the falling matter is often not sufficient for the accretion disk formation at the level of the Alfvénic radius where the magnetic pressure of the neutron star is balanced by the dynamic pressure of the falling gas. The angular momentum acquired in this case by the neutron star from the falling gas was shown (Bisnovatyi-Kogan 1991) to be such that the equilibrium rotational period of the X-ray pulsar might be large. It is rather difficult to make estimations for the equilibrium rotational period because the formation is possible of outflowing streams which carry away the angular momentum (Illarionov & Kompaneetz 1990; Lovelace 1995). The accretion picture in this case is three-dimensional and its numerical modeling is very complicated. It was performed only for the case of accretion onto a gravitating center, showing the presence of the Rayleigh–Taylor instabilities (Matsuda et al. 1989). The nature of these instabilities is unclear and their numerical origin can not be excluded (Steinolfson et al. 1994).

To investigate the formation of long-periodic pulsars, the model is necessary for the accretion onto a magnetized neutron star from the stellar wind in the binary. To reduce a 3D problem to two dimensions, we can consider either the conical accretion of nonrotating gas or the accretion of slowly rotating gas onto a stationary star. In the first case (Koide et al. 1991), the average angular momentum acquired by the neutron star is zero due to the symmetry of the flow, so it can not be applied to long-periodic X-ray pulsars.

Here we use the second approach. Our aim is to consider accretion onto a magnetized star with the Alfvénic radius $R_A \gg R_*$ (star radius) taking into account the interaction
between the gas and the magnetosphere and the possible outflow formation. As the first step, we consider accretion with the full penetration of the gas through the magnetosphere substituted by the inner boundary with the radius $R_*$ and neglect magnetogasdynamic effects.

Beskin & Pidoprygora (1995) presented the approximate solution for the accretion of nonrotating gas onto a slowly rotating black hole in the framework of the general theory of relativity. In this work, we consider the accretion of rotating gas in the Newton approximation up to the rotation velocities close to the Keplerian velocity.

Three different modes are usually considered as constituent parts of the astrophysical accretion and are fairly well investigated (Lipunov 1992; Bisnovatyi-Kogan 1989).

1. Spherically-symmetric accretion occurs if the star velocity $v_\infty$ is much smaller than the speed of sound $a_\infty$ in an accreting matter and the angular momentum is negligible.

2. Cylindrical accretion is realized when $v_\infty \geq a_\infty$ with vanishing angular momentum.

3. Accretion disk is formed if the total angular momentum of the matter is sufficient for its formation.

Real accretion is, in fact, a combination of the above-mentioned modes.

From gasdynamic viewpoint, it is of interest to investigate the process of the transition from regime 1 to regime 3 for $v_\infty \ll a_\infty$ as the rotational velocity of the accreting matter approaches the Keplerian velocity. We consider the polytropic flow of ideal, perfect gas with the polytropic index $\gamma = 1.4$. Calculations are performed using the second order of
accuracy in space high-resolution Lax–Friedrichs-type numerical scheme proposed by one of the authors (Barmin & Pogorelov 1995). Detailed description of the scheme is given by Barmin, Kulikovskiy & Pogorelov (1996). Recently, a three-dimensional Bondi–Hoyle accretion was investigated for the accretor (star) radius varying from 10 to 0.02 Bondi radii, so that both subsonic and supersonic regimes of accretion could be realized (Ruffert 1994).

In our study we are mainly interested in qualitative tracing the flow restructuring as its angular velocity increases. For this reason, the Euler equations are solved and the external flow is supposed to be supersonic. On the other hand, the size of the star is assumed to be sufficiently small; the inner, also supersonic, boundary is fixed at a finite distance from the gravitational center. The effect of the external boundary conditions is investigated on the existence and properties of a stationary supersonic accretion for the chosen computational region.

2 Spherically-symmetric supersonic accretion.

Let us consider the accretion gas flow between two spheres with the inner and outer radii equal to \( R_\star \) and \( R_0 \) respectively. If the flow at the distance \( R_0 \) is assumed to be supersonic, then all flow parameters should be fixed at this boundary. The question is what parameter values can be imposed at a given distance for the existence of a stationary solution. If the flow is shockless and polytropic, the following three conservation equations must be satisfied:

\[
4\pi R^2 \rho U = \dot{M}, \tag{1}
\]
\[
\frac{p}{\rho^\gamma} = K, \quad (2)
\]

\[
\frac{U^2}{2} + \frac{\gamma - 1}{\gamma - 1} \frac{p}{\rho} - \frac{GM}{R} = h_{10}, \quad (3)
\]

Here \(\rho, p, U\) are, respectively, the density, the pressure and the radial velocity, \(R\) is the current distance from the gravitating center, \(M\) is the mass of the star. The values of the accretion rate \(\dot{M}\), the constant \(K\), and the Bernoulli constant \(h_{10}\) are to be fixed at \(R = R_0\). Introducing dimensionless variables with the units of velocity, pressure, density, and length equal to \(U_0, \rho_0v_0^2, \rho_0, \) and \(R^*\), respectively, and designating \(M_0 = U_0/a_0\), \(a_0 = (\gamma\rho_0/\rho_0)^{1/2}\) and \(S = GM/U_0^2R^*\), we can rewrite system (1)–(3) in the dimensionless form using the same notations for nondimensional values of \(U, R, p, \rho, \) and \(a = (\gamma p/\rho)^{1/2}\) (indices “0” and “*” correspond to the outer and to the inner boundary, respectively):

\[
\frac{U^2}{2} + \frac{a^2}{\gamma - 1} \frac{S}{R} = \frac{1}{2} + \frac{1}{(\gamma - 1)M_0^2} - \frac{S}{R_0}, \quad (4)
\]

\[
\rho U = \frac{R_0^2}{R^2}, \quad (5)
\]

\[
\frac{p}{\rho^\gamma} = \frac{1}{\gamma M_0^2} \quad (6)
\]

Being subsonic at infinity and supersonic at \(R = R_0\), the flow becomes sonic \(U = U_B = a_B\) at some point \(R = R_B \geq R^*\), where (Bisnovatyi-Kogan 1989)

\[
U_B = a_B = \frac{1}{2} \frac{S}{R_B}, \quad (7)
\]
In the sonic point Eqs. (4)–(7) reduce to

\[ \frac{5 - 3\gamma}{4(\gamma - 1)R_B} S = h_{t0} \]  

(8)

If \( R_B \geq R_0 \) and the flow comes from infinity, the value \( h_{t0} \) must satisfy the inequality

\[ 0 < h_{t0} < \frac{5 - 3\gamma}{4(\gamma - 1)R_0} S \]  

(9)

3 Accretion with rotation.

In this section we consider the process of the axisymmetric rotating flow accretion onto a gravitating object. Analysis is performed on the basis of the numerical solution of the Euler gasdynamic equations. The gas is assumed to be ideal and perfect with the following caloric equation of state: \( \varepsilon = p/(\gamma - 1)\rho \), where \( \varepsilon \) is the internal energy per unit mass and \( \gamma = 1.4 \) is the polytropic index. The system of governing equations in the Cartesian coordinate system shown in Fig. 1 reads:

\[ \frac{\partial U}{\partial t} + \frac{\partial E}{\partial x} + \frac{\partial G}{\partial z} + H = 0, \]

(10)

where

\[
U = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ e \end{bmatrix}, \quad E = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho v \\ \rho w \\ (e + p)u \end{bmatrix}, \quad G = \begin{bmatrix} \rho w \\ \rho uw \\ \rho v \\ \rho w^2 + p \\ (e + p)w \end{bmatrix}, \quad H = \frac{\rho}{x} \begin{bmatrix} u \\ (u^2 - v^2) + GMx \frac{\sin \theta}{R^2} \\ 2uv \\ uw + GMx \frac{\cos \theta}{R^2} \\ \frac{(e + p)u}{\rho} + GMx \frac{u \sin \theta + w \cos \theta}{R^2} \end{bmatrix} \]

Here \( e = p/(\gamma - 1) + \rho(u^2 + v^2 + w^2)/2 \) is the total energy per unit volume. As far as the gravitational field is considered spherically symmetric, the polar computational
domain \((R, \theta)\) is chosen with the inner and outer radii equal to \(R_*\) and \(R_0\), see Fig. 1. On normalizing the quantities of density, pressure, and velocity by \(\rho_0\), \(\omega_*R_*, \rho_0\omega_*^2R_*^2\), where \(\rho_0\) is the density at \(R = R_0\) and \(\omega_*\) is the gas angular velocity at \(R = R_*\) at the equator for the constant angular momentum distribution, the source term can be rewritten in the dimensionless form as

\[
\mathbf{H} = \frac{\rho}{\alpha} \begin{bmatrix}
  u \\
  (u^2 - v^2) + S_x \frac{\sin \theta}{R^2} \\
  2uv \\
  u \rho \frac{\cos \theta}{R^2} \\
  (\epsilon + p)u + S_x \frac{u \sin \theta + w \cos \theta}{R^2}
\end{bmatrix}
\]

(11)

Here \(S = GM/\omega_*^2R_*^3\). The form of the other vector components in Eq. (10) remains unchanged.

From now on we consider only nondimensional parameters.

The following procedure is used to construct the initial and boundary conditions.

1. Introducing dimensionless parameter \(\alpha = U_0/U_{K_*}\), where \(U_0\) is the radial velocity on the outer boundary and \(U_{K_*} = (GM/R_*)^{1/2}\) is the Keplerian velocity on the inner boundary, we fix the values on the outer boundary as if the flow is spherically symmetric:

\[
\rho_0 = 1; \; U_0 = \alpha S^{1/2}; \; p_0 = \alpha^2 S/\gamma M_0^2; \; W_0 = 0,
\]

where \(W_0\) is the \(\theta\)-component of the velocity and \(M_0 = U_0/a_0\).

Corresponding parameter values inside the computational region are adopted equal to those at the boundary.
We can find the entropy and the total enthalpy dimensionless values as follows:

\[
\frac{P_0}{\rho_0} = K; \quad h_{t0} = S\left[\alpha^2\left(\frac{1}{(\gamma - 1)M_0^2} + \frac{1}{2}\right) - \frac{1}{R_0}\right]
\]

2. The following initial distribution of the angular velocity is assumed:

\[
\begin{cases}
\omega = \frac{1}{x^2} = \frac{1}{R^2\sin^2\theta} & \text{if } R > 20 \text{ (constant angular momentum)} \\
\omega = 0.0025 & \text{if } R \leq 20 \text{ (constant angular velocity)}
\end{cases}
\]

The \(y\)-component \(v\) (normal to the plane of Fig. 1) of the velocity \(\mathbf{v}\) is then \(v = \omega x\).

3. The values of \(\rho\) and \(p\) in the whole computational region are modified. Assuming \(U(R, \theta) = U_0\) and \(W(R, \theta) = 0\), we find the new pressure and density values from the formulas

\[
\frac{\gamma}{\gamma - 1} \frac{p}{\rho} + \frac{U^2}{2} + \frac{v^2}{2} - \frac{S}{R} = h_{t0}; \quad \frac{p}{\rho^\gamma} = K
\]

After that all values on the external boundary are fixed because it is supersonic. No boundary conditions are necessary on the inner circle, since the flow through it is supersonic.

4 Numerical scheme.

Point clustering towards the internal boundary circle is made to obtain a sufficiently fine flow resolution in the vicinity of the gravitating center. The following formula is used:

\[
R = R_* + (R_0 - R_*) \frac{e^{\beta \xi} - 1}{e^\beta - 1}
\]

with the clustering parameter \(\beta\).
If we introduce a polar mesh

\[ \xi_l = (l - 1)\Delta \xi, \ l = 1, 2, \ldots, L; \ \theta_n = (n - 2.5)\Delta \theta, \ n = 1, 2, \ldots, N; \]

\[ R_l = R(\xi_l), \ \Delta \xi = 1/(L - 1), \ \Delta \theta = \pi/(2N - 8) \]

with the center in the accretor position, then for each cell the system (1) in the finite-volume formulation acquires the form

\[
R_l \Delta R_l \Delta \theta \frac{\partial U_{ln}^k}{\partial t} + (R_{l+1/2} \bar{E}_{l+1/2,n}^k + R_{l-1/2} \bar{E}_{l-1/2,n}^k) \Delta \theta +
(\bar{E}_{l,n+1/2}^k + \bar{E}_{l,n-1/2}^k) \Delta R_l + R_l \Delta R_l \Delta \theta H_{l,n}^{k+1} = 0 \quad (12)
\]

Here \( \Delta R_l = R_{l+1/2} - R_{l-1/2} \) and \( \bar{E} \) is the flux normal to the boundary, defined as:

\[
\bar{E} = n_1 E + n_2 G,
\]

where \( n = (n_1, n_2) \) is a unit outward vector normal to the cell surface.

The assumption is made of a piecewise-parabolic distribution of the primitive gas-dynamic parameters \( q \) inside the cells to specify values on their boundaries and slope delimiters are used to obtain the nonoscillatory property.

The averaged slope inside the cell in the radial direction is determined as (Sawada et al. 1989)

\[
\Delta q_l = [(2\Delta R_{l-1} + \Delta R_l)\mu + (2\Delta R_{l+1} + \Delta R_l)\nu] \Delta R_l / \chi,
\]

where

\[
\chi = \Delta R_{l+1} + \Delta R_l + \Delta R_{l-1}, \ \mu = \frac{\delta q_{l+1/2}}{\Delta R_{l+1} + \Delta R_l}, \ \nu = \frac{\delta q_{l-1/2}}{\Delta R_l + \Delta R_{l-1}}
\]

The left and the right boundary value are then estimated as

\[
q_{l+1/2}^L = q_l + \frac{1}{2} \Delta q_l, \ q_{l-1/2}^R = q_l - \frac{1}{2} \Delta q_l,
\]
\[ \tilde{\Delta} q_l = \minmod(\Delta q_l, 2\mu \Delta R_l, 2\nu \Delta R_l) \]

Here

\[
\minmod(a, b, c) = \begin{cases} 
\text{sign}(a) \min(|a|, |b|, |c|) & \text{if } bc > 0 \\
0 & \text{otherwise}
\end{cases}
\]

Found the values of \( q^L \) and \( q^R \) on the both sides of the cell boundary, the appropriate fluxes \( \tilde{E} \) are calculated using the modified Lax–Friedrichs formula (Barmin & Pogorelov 1995):

\[
\tilde{E}(U^R, U^L) = \frac{1}{2}[\tilde{E}(U^L) + \tilde{E}(U^R) - \hat{R}(U^R - U^L)],
\]

where the matrix \( \hat{R} \) is a positive diagonal matrix with the entries equal to the spectral radius (the maximum of eigenvalue magnitudes) of \( \frac{\partial \tilde{E}}{\partial U} \). This scheme gives a drastic simplification of the numerical algorithm comparing to the methods based on precise characteristic splitting of the Jacobian matrices, while preserving nonoscillatory property. Having the second order of accuracy, it is less dissipative than the original Lax–Friedrichs scheme. Similarly, the fluxes through another pair of cell surfaces are obtained.

As seen from Eq. (12) the source term is approximated implicitly for better stability of the numerical scheme. A proper linearization of this term is made to realize the numerical procedure of obtaining the time-converging steady-state solution:

\[ H^{k+1} = H^k + \frac{\partial H^k}{\partial U^k} (U^{k+1} - U^k). \]

The promotion of the solution in time is performed with the first order of accuracy by the formula:

\[
(I + \Delta t \frac{\partial H^k}{\partial U^k})(U^{k+1}_{t,n} - U^k_{t,n}) = -\Delta t[(R_{t+1/2} E^k_{t+1/2,n}/R_t + R_{t-1/2} E^k_{t-1/2,n}/R_t)/\Delta R_t +
\]
\[(E_{l,n+1/2}^k + E_{l,n-1/2}^k) / R_l \Delta \theta + H_{l,n}^k].\]

for \(t = k \Delta t, k = 0, 1, \ldots\), and \(\Delta t\) defined by the CFL condition (\(I\) is the identity matrix).

5 Analysis of numerical results.

All results presented in this section were obtained in the ring region with the dimensionless inner and outer circle radii being \(R_0 = 1\) and \(R_0 = 100\), with 56 cells in the angular direction and 104 cells in the radial direction, and with the clustering parameter \(\beta = 4\). The calculations were performed until their full time-convergence in a quarter of the ring with the appropriate reflection conditions applied in the planes of the flow symmetry.

The way of presentation the results obtained is the following. In Figs. 2–18 the isolines of different gasdynamic parameters and the streamlines of the flow are presented in the lower and upper parts of the figure divided by the rotational axis. Figures 2–3, 7–9, and 13–15 correspond to the whole computational region. In these figures 18 isolines are presented with the constant step between the maximum and the minimum value of functions indicated in the corners, that is, the isoline value for any function \(f\) can be found by the formula \(f_i = f_{\text{min}} + i \times (f_{\text{max}} - f_{\text{min}}) / 19\). In the regions with no captions in the corner, the streamlines are shown. Figures 4–6, 10–12, and 16–18 present the magnified central part (50 computational zones) of the corresponding figures related to the whole computational region.

The following dimensionless parameters are chosen, at first, to study the accretion flow behavior for the case of slow rotation (this choice is consistent with the considerations
from Section 2):

\[ \alpha = 0.1; \quad \gamma = 1.4; \quad M_0 = 2; \quad S = 250 \]

The steady state results for this case are presented in Figs. 2–6 (the axis of rotation is aligned with the \( z \)-axis (see Fig. 1). In Fig. 2, the isolines of the pressure and the density decimal logarithm are presented above and below the rotational axis, respectively, in the whole computational region. The isolines of the velocity components \( v \) and \( W (\theta \text{-direction}) \) are not shown in the full computational region since their main variation is located in the vicinity of the inner boundary. The isolines of the velocity component \( U \) (radial direction) and the streamlines are given in Fig. 3.

In Figs. 4–6 the same lines are shown in the magnified subregion close to the accreting center (50 computational zones). It is clearly seen in these figures that for long distances from the accreting center and far enough from the rotational axis slow rotation does not affect the flow significantly and it is, in fact, the superposition of a spherically symmetric accretion and an axisymmetric constant angular momentum rotation. Closer to the \( z \)-axis and to the accreting center, however, the pressure and the density are greater at the equator than near the poles. The deviation from the purely constant angular momentum rotation appears only in the vicinity of the rotational axis and near the accreting object. The streamlines and the radial velocity \( U \) isolines behave quite like in the spherically symmetric case.

The similar pictures of the flow are presented, for \( S = 25 \) with the rest of dimensionless parameters unchanged, in Figs. 7–12. The effect of rotation in this case is quite definite in the whole region surrounding the rotational axis. The gas displacement from the poles
is much more pronounced comparing with the variant of $S = 250$ even at large distances from the accretion center, see Fig. 7. The streamlines deflect from the poles to the equator region (Fig. 12) and the size of the domain with substantial values of $W$ is larger (Figs. 8, 11).

The picture of the flow for the rotational velocity close to the Keplerian velocity ($S = 1.5$) is shown in Figs. 13–18. Almost all the gas is removed in this case from the pole regions. The density near the equator is $\sim 10^4$ times greater than that near the poles and almost all accreting matter is involved in the motion towards $\theta = \pi/2$. The structure of the flow is very close to the picture of the accretion disk formation, see Fig. 18 showing the streamline distribution. The regions with the dense distribution of the pressure and density isolines (Fig. 16) represent the oblique transverse shock where supersonic flow of the gas in $\theta$-direction, declined by the centrifugal force from the poles to the equator, decelerates to become zero at $\theta = \pi/2$. Behind these shocks the region of a fast rotation is observed (Figs. 14, 17). This region of a highly rotating dense matter will form an accretion disk if $S < 1$.

6 Discussion.

Numerical modeling is performed for axisymmetric rotating gas accretion onto the star. The second order of accuracy in space high-resolution Lax–Friedrichs-type scheme showed good performance for the calculation of flows with an extremely great variation of the pressure and density values throughout the computational region. The flow restructuring is investigated as the rotation velocity approaches the Keplerian velocity. The variants
are presented for different cases beginning from a slow rotation and up to the case, for which the steady state centrifugal and gravitational forces near the stellar equator are rather close. The values of the angular momentum, however, were sufficiently small, so that at the inner boundary the centrifugal force was smaller than gravitational, and the stationary accretion picture could be attained. If the angular momentum of the falling matter reaches the value, for which the centrifugal force balances the gravitation before the matter falls onto the star (the inner boundary in our calculations), the matter stops near the equatorial plane and forms a disk. In the absence of viscosity the mass of this disk increases in time. In the presence of viscosity, which is especially efficient in the turbulent case, the matter in the disk looses its angular momentum and moves slowly towards the center forming a stationary accretion disk (Lynden-Bell 1969; Shakura 1972). We are interested here in the problem of the long-periodic pulsar formation, for which the angular momentum is not sufficient for the accretion disk origin near the Alfvénic surface where the dynamic pressure of the flow is balanced by the magnetic surface at the magnetosphere of the neutron star. To find the angular momentum of the matter falling onto the star during accretion from the stellar wind penetrating through the Alfvénic surface, we need to include into consideration the magnetic field. This work is now in progress.

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Figure captions.

Fig. 1. Computational region.

Fig. 2. Logarithm pressure and density isolines. Full computational region, $S = 250$.

Fig. 3. Streamlines and $U$ isolines. Full computational region, $S = 250$.

Fig. 4. Logarithm pressure and density isolines. Inner subregion, $S = 250$.

Fig. 5. Isolines of $v$ and $W$. Inner subregion, $S = 250$.

Fig. 6. Streamlines and $U$ isolines. Inner subregion, $S = 250$.

Fig. 7. Logarithm pressure and density isolines. Full computational region, $S = 25$.

Fig. 8. Isolines of $v$ and $W$. Full computational region, $S = 25$.

Fig. 9. Streamlines and $U$ isolines. Full computational region, $S = 25$.

Fig. 10. Logarithm pressure and density isolines. Inner subregion, $S = 25$.

Fig. 11. Isolines of $v$ and $W$. Inner subregion, $S = 25$.

Fig. 12. Streamlines and $U$ isolines. Inner subregion, $S = 25$.

Fig. 13. Logarithm pressure and density isolines. Full computational region, $S = 1.5$.

Fig. 14. Isolines of $v$ and $W$. Full computational region, $S = 1.5$.

Fig. 15. Streamlines and $U$ isolines. Full computational region, $S = 1.5$.

Fig. 16. Logarithm pressure and density isolines. Inner subregion, $S = 1.5$.

Fig. 17. Isolines of $v$ and $W$. Inner subregion, $S = 1.5$.

Fig. 18. Streamlines and $U$ isolines. Inner subregion, $S = 1.5$. 
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