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Numerical-relativity simulations of long-lived remnants of binary neutron star mergers

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We analyze the properties of the gravitational wave signal emitted after the merger of a binary neutron star system when the remnant survives for more than a 80 ms (and up to 140 ms). We employ four different piecewise polytropic equations of state supplemented by an ideal fluid thermal component. We find that the post-merger phase can be subdivided into three phases: an early post-merger phase (where the quadrupole mode and a few subdominant features are active), the intermediate post-merger phase (where only the quadrupole mode is active) and the late post-merger phase (where convective instabilities trigger inertial modes). The inertial modes have frequencies somewhat smaller than the quadrupole modes. In one model, we find an interesting association of a corotation of the quadrupole mode in parts of the star with a revival of its amplitude. The gravitational wave emission of inertial modes in the late post-merger phase is concentrated in a narrow frequency region and is potentially detectable by the planned third-generation detectors. This allows for the possibility of probing not only the cold part of the equation of state, but also its dependence on finite temperature. In view of these results, it will be important to investigate the impact of various types of viscosities on the potential excitation of inertial modes in binary neutron star merger remnants.

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I. INTRODUCTION

The landmark joint detection of gravitational waves (GWs) and electromagnetic waves from the binary neutron star merger event GW170817 / GRB 170817A / AT 2017gfo is widely regarded as the start of Multi-Messenger Astrophysics (see [1] and references therein). This unique event has provided key evidence to address long-standing issues in relativistic astrophysics such as the origin of short gamma-ray bursts and kilonovae, the r-process-mediated nucleosynthesis of heavy elements, and independent measures of cosmological parameters [2–5]. Constraints on the binary parameters, such as component masses, radii, spins, and the equation of state (EOS), as derived from the detected GW signal in the inspiral phase, were first presented in [6] and subsequently improved in [7, 8]. See also [9,12] for additional estimates, [13] for a different assessment of EOS constraints, and [14,15] for future prospects. The constraints on radii and EOS are based on the analysis of the tidal interactions between the two stars during their inspiral. Additional constraints on the radius can be obtained assuming that there was no prompt collapse of the remnant (see e.g. [16] and [17,18] for reviews and references therein). Complementary information on the internal structure of neutron stars is expected to become available through the observation of the post-merger GW signal, primarily through the application of an empirical relation between the $m = 2$ f-mode frequency and the neutron star radius (see e.g. [19,31] and [17,18] for reviews and references therein). A first potential approach for inferring information on the finite-temperature dependence of the EOS, based on the long-term GW emission in the post-merger phase, was presented in [32]. Reliable theoretical models of post-merger GW emission can only be obtained through nonlinear, general-relativistic numerical simulations. Generically, such numerical simulations of binary neutron star (BNS) mergers show that the outcome depends on the component masses and the EOS (see e.g. [33,34] for reviews). Prompt collapse to a black hole is the likely outcome only if the initial masses are sufficiently large, while delayed collapse (or no collapse) is produced otherwise. In the latter cases, the resulting object may be either a hypermassive neutron star (HMNS) [37], where the remnant is temporarily supported against collapse by differential rotation and thermal gradients, or even a supramassive neutron star, when the mass of the remnant is sufficiently small to be supported by rigid rotation [38]. Either case leads to sufficiently long-lived remnants to produce large amounts
of high-frequency (∼kHz) gravitational radiation. Hypermassive remnants may avoid collapse for timescales of tens to hundreds of milliseconds. Supramassive neutron stars will avoid collapse on an immensely longer timescale set by magnetic dipole radiation. On such long timescales, additional GW emission mechanisms may become relevant, such as the CFS instability [39, 40], shear instabilities [11, 35] or convective instabilities in rotating stars with non-barotropic thermal profiles [44]. There is thus good reason to study in more detail the long-term evolution of BNS remnants.

Due to their high frequency, the GW signals produced by post-merger remnants are much more difficult to detect for the Advanced LIGO and Virgo detectors than those from the late inspiral phase, as the (quantum) shot-noise limits the sensitivity of interferometers above a few kHz. Not surprisingly, the recent searches for GWs from the post-merger remnant conducted by the LIGO/Virgo collaboration in the data of GW170817 have not revealed any detection [15, 37]. Nevertheless, post-merger GWs for GW170817-like events, in terms of distance and amplitude, seem within reach of observing runs with expected higher sensitivity. We note that a seconds-long post-merger signal candidate has been reported by [48] with a lower GW energy estimate than that computed by [15].

Simulations of the post-merger phase of HMNSs show the emission of significant amounts of gravitational radiation at distinct frequencies of a few kHz (e.g. [19, 21–24, 19, 62]), with contributions as low as ∼1 kHz, depending on the EOS [63], see [64] for a review and references therein. All of these simulations last for up to a few tens of ms after merger, which is long enough to show that the GW emission is dominated by a main peak in the frequency range ∼2-4 kHz, with secondary peaks on both sides of the main peak. Employing a mode-analysis technique it was found in [21], that the main post-merger peak is due to the excitation of the fundamental \( m = 2 \) \( f \)-mode (denoted in the literature as or \( f_{\text{peak}} \) or \( f_2 \)). In addition, it was shown in [21] that some of the secondary peaks can be explained as quasi-linear combinations between \( f_2 \) and the fundamental, quasi-radial, \( m = 0 \) mode (that is, sums and differences of these frequencies, denoted e.g. as \( f_{2-0} \) and \( f_{2+0} \)). These combination frequencies are primarily present in models that are relatively close to the threshold mass to collapse (high-mass/soft EOS models) [24]. In addition, in [24] it was found that a different secondary peak, \( f_{\text{spiral}} \), is present in models that are relatively far from the threshold mass to collapse (low-mass/stiff EOS) and it is due to a spiral deformation excited during merger. Depending on the relative strength between \( f_{2-0} \) and \( f_{\text{spiral}} \), [24] introduced a spectral classification of the post-merger GW emission (see [17, 28] for reviews). For additional studies of the properties of post-merger remnants (where some time a different label for the mode is used), see e.g. [56–61, 63–65, 70] as well as [15] and references therein.

Recently we performed numerical simulations in full general relativity of BNS systems, extending the simulation time significantly longer than those previously reported in the literature [32]. These simulations allowed us to find new features in the GW spectrum of the remnant on longer timescales. In particular, the simulations were extended up to ∼140 ms after merger and were based on a piecewise polytropic approximation for the EOS treatment, supplemented by a thermal component. We found that after an initial phase of a few tens of ms (where the dominant oscillation mode of the remnant and main GW emitter is the \( m = 2 \) \( f \)-mode), convective instabilities in the remnant may excite inertial modes, which also radiate GWs. The phase during which inertial modes of global and discrete nature can be excited, may last at least up to several tens of milliseconds and, as shown in [32], the associated GW emission may become potentially observable by the planned third-generation GW detectors at frequencies of a few kHz.

Here, we complement the study initiated in [32] by carrying out the analysis of additional simulations, which account for two new EOS, namely H4 and MS1. Within the range of validity of the input physics and assumptions employed in our simulations, the extended results presented here confirm our earlier findings in a generic way. We present a convergence check for a specific model to further validate our results.

The paper is organized as follows: in Section II we discuss our initial models and numerical setup. Section III presents our main results, with a particular attention to the mode analysis in Sections III B III C III D and III E. A summary of the main results presented in the previous sections of this work is provided in Section V. The paper closes with three appendices, where numerical issues and convergence properties of our simulations are discussed (Appendix A), along with a revision of past literature regarding long-term simulations of the post-merger phase of binary neutron star mergers (Appendix B), and the thermodynamical properties of the piecewise-polytropic EOS used in this study (Appendix C).

Unless otherwise noted, we employ units of \( c = G = M_\odot = 1 \).

II. INITIAL DATA AND SETUP

Initial data for irrotational neutron star binaries in the conformal thin sandwich approximation are generated using the LORENE code [77, 78]. We employ four EOS, namely SLy, APR4, H4, and MS1, parameterized as piecewise polytropes [79] with 7 pieces plus a thermal component with adiabatic index \( \Gamma_{\text{th}} = 1.8 \). The main properties of our initial data are a total baryonic mass of \( 2.8 \, M_\odot \) and a total mass of \( \simeq 2.55 - 2.60 \, M_\odot \) with an initial separation of \( \approx 44.3 \, \text{km} \) (four full orbits before merger). We consider equal-mass systems characterized by relatively low-mass components, below the range of the inferred masses for GW170817 [7] (which had a total
For the simulations we employ the same setting in the General Relativity based on the Cactus framework [81, 82]. The set of equations we solve numerically comprise Einstein’s gravitational field equations, the general relativistic hydrodynamics equations for a perfect-fluid stress-energy tensor and the relativistic energy conservation equation. Therefore, our simulations do not account for effects due to magnetic fields or non-thermal radiation transport associated with neutrinos. In particular, we evolve Einstein’s equations in the BSSN formulation [86, 87], as implemented in the McLachlan module [88], and the Valencia formulation of the general relativistic hydrodynamics equations [89, 90], as implemented in the public GRHydro module [91, 92]. For the matter fields we employ a finite-volume algorithm with the HLLE Riemann solver [93, 94] and the WENO reconstruction method [95, 96]. The combined use of WENO reconstruction and the BSSN formulation was found in [97] to be the best combination within the Einstein Toolkit even at low resolution. For the time evolution we used the Method of Lines with a fourth-order, conservative Runge-Kutta scheme [97, 98]. The evolved variables for both the spacetime and the hydrodynamics are discretized on a Cartesian grid with 6 levels of fixed mesh refinement, each using twice the resolution of its parent level. The outermost boundary of the grid is set at \(720M_\odot\) (\(\approx 1040\) km) from the center. The standard spatial resolution at the finest refinement level in our production runs is \(dx = 277\) m. For the SLy EOS, we also use a higher-resolution setup, with spatial resolution at the finest level of \(dx = 185\) m (this simulation is denoted as SLy HR below). A comparison of GW spectrograms for the two resolutions is provided in Appendix A.

### III. RESULTS

#### A. Matter dynamics and waveforms

The dynamics of the matter of the four models considered in this work can be followed by analyzing the time evolution of quantities such as the central density and the density distribution within the equatorial and the vertical planes. The overall behavior for the first phase (inspiral and merger) of the binary system’s evolution is similar for each model, until the two neutron stars form a bar-deformed remnant as in [69] (the appearance of a bar deformation is due to the rotating pattern of the dominant \(m = 2\) \textit{f}-mode excited after merger). During the initial post-merger phase we note some oscillations in the evolution of the maximum density, as shown in Figure 1 which have different durations for each model. For the SLy and MS1, before we terminate our simulations at \(t \approx 84\) ms after the merger, respectively, while for the APR4 and H4 \(t \approx 7\) ms respectively, until the core becomes more stable and the bar-deformed remnant is left. This configuration of a bar-deformed remnant survives for some tens of milliseconds for each model before becoming nearly axisymmetric and more stable.

We note that for the SLy and H4 EOS the density of the core grows even after the oscillations in the maximum density have dampened (see Sec. 11B for a discussion of the frequency evolution of the dominant \(m = 2\) \textit{f}-mode during this time interval). Eventually, a delayed collapse to a black hole occurs for both of these two models, at \(\sim 124\) and \(\sim 84\) ms after the merger, respectively, while we do not observe a collapse to a black hole in the case of APR4 and MS1, before we terminate our simulations.

### TABLE I. Main properties of the four equal-mass BNS systems studied in this work.
The columns report the baryonic mass, \(M_0\), the gravitational mass, \(M\), and the compactness \(C := M/R\) of the individual stars, the total ADM mass, \(M_{\text{ADM}}\), the angular momentum, \(J_{\text{ADM}}\), and the angular velocity of the binary system, \(\Omega_0\), at the start of the simulation. All systems have roughly the same initial separation of 44.3 km.

| EOS  | \(M_0\)   | \(M\)     | \(C\)     | \(M_{\text{ADM}}\) | \(J_{\text{ADM}}\) | \(\Omega_0\) (krad/s) |
|------|-----------|-----------|-----------|---------------------|---------------------|------------------------|
| SLy  | 1.40      | 1.2810    | 0.161     | 2.538               | 6.623               | 1.770                  |
| APR4 | 1.40      | 1.2755    | 0.166     | 2.528               | 6.577               | 1.767                  |
| H4   | 1.40      | 1.3004    | 0.137     | 2.576               | 6.802               | 1.783                  |
| MS1  | 1.40      | 1.3047    | 0.129     | 2.585               | 6.850               | 1.787                  |

The initial data are evolved using the Einstein Toolkit [88], an open-source, modular code for numerical relativity based on the Cactus framework [81, 82]. For the simulations we employ the same setup in the code as in [88, 69, 69]. The only difference here is the use of \(\pi\)-symmetry to reduce the computational cost by a factor 2. This allows to push the limit of the simulated time to \(\sim 150\) ms, of which the last 130 ms correspond to the post-merger phase.

The set of equations we solve numerically comprise Einstein’s gravitational field equations, the general relativistic hydrodynamics equations for a perfect-fluid stress-energy tensor and the relativistic energy conservation equation. Therefore, our simulations do not account for effects due to magnetic fields or non-thermal radiation transport associated with neutrinos. In particular, we evolve Einstein’s equations in the BSSN formulation [86, 87], as implemented in the McLachlan module [88], and the Valencia formulation of the general relativistic hydrodynamics equations [89, 90], as implemented in the public GRHydro module [91, 92]. For the matter fields we employ a finite-volume algorithm with the HLLE Riemann solver [93, 94] and the WENO reconstruction method [95, 96]. The combined use of WENO reconstruction and the BSSN formulation was found in [97] to be the best combination within the Einstein Toolkit even at low resolution. For the time evolution we used the Method of Lines with a fourth-order, conservative Runge-Kutta scheme [97, 98]. The evolved variables for both the spacetime and the hydrodynamics are discretized on a Cartesian grid with 6 levels of fixed mesh refinement, each using twice the resolution of its parent level. The outermost boundary of the grid is set at \(720M_\odot\) (\(\approx 1040\) km) from the center. The standard spatial resolution at the finest refinement level in our production runs is \(dx = 277\) m. For the SLy EOS, we also use a higher-resolution setup, with spatial resolution at the finest level of \(dx = 185\) m (this simulation is denoted as SLy HR below). A comparison of GW spectrograms for the two resolutions is provided in Appendix A.

### FIG. 1. Maximum density as a function of time for the simulations of the models with the four different EOS (notice that HR is a high-resolution run with the SLy EOS).
at $t - t_{\text{merger}} \sim 150\, \text{ms}$.

The GW signal obtained from the simulations of these four models is presented in Figure 2. We show the $h_{22}$ component of the GW amplitude (multiplied by the assumed distance to the source, $R$) (black line) and the absolute value of $h_{22}$ (blue line), assuming an ideal source orientation. The time of merging and the time of the collapse to a black hole are indicated by vertical, dotted red lines. For details on the numerical extraction of the GW signal see [63, 69, 83].

The evolution of the GW amplitude for the SLy EOS model in Figure 2 shows a relatively slow decay up to about 20 ms after merger (in this phase, the decay is due to both nonlinear hydrodynamical interactions and GW emission), followed by a second phase of somewhat faster decay, which lasts up to about 65 ms after merger, when the GW emission almost ceases and the remnant has become nearly axisymmetric. However, at about 65 ms after merger, there is a clear revival of GW emission (see [32] and the following sections, for an interpretation of this observation in terms of the excitation of inertial modes by convective instabilities in the remnant).

For the model with EOS MS1, we observe an initial decay of GW emission up to about 40 ms after merger, followed by several episodes of revival of GW emission, up to about 100 ms after merger. At even later times, the remnant radiates GWs at practically a single frequency and at a practically constant amplitude. Such a behavior is expected in the case of a specific, unstable mode saturating at a maximum amplitude, due to a balance between energy gained (by an instability) and energy lost (e.g. by GW emission).

For the model with EOS H4, Figure 2 reveals an almost linear decay of the GW amplitude for the most part of the time period up to almost 80 ms (shortly after, the remnant collapses to a black hole). However, after 45 ms from merger, a modulation appears in the amplitude evolution, indicating the presence of a second frequency, close to the dominant post-merger frequency (in [32] and the following sections, we give the same interpretation to this second frequency, as in the case of the late-time behavior of the SLy EOS model).

A similar late-time revival (more than 60 ms after merger) of GW emission can be observed for the model with the APR4 EOS. In addition, for this model we observe a strong revival of GW emission already at 25 ms after merger. We will comment on this finding in Section III F.

B. Spectrum evolution and Prony’s analysis

The general dynamics of the evolution of our four BNS systems can be neatly captured in the time-frequency plots of the $h_{22}$ component of the spherical harmonic decomposition of the GW signal shown in Figure 3. On top
of each spectrogram we also superimpose with thick gray lines the time evolution of the frequency of the main active spectral modes of the remnant. This is determined using the ESPRIT Prony’s method (employing a moving window interval of 3 ms) as discussed in [63]. The corresponding extracted amplitudes of the modes are also reported in the lower half of each panel in arbitrary units. The leftmost vertical line in each panel of Figure 3 indicates the time of merger and the rightmost vertical line (for EOS SLy and H4 only) indicates the time of black hole formation.

For the H4 EOS and the SLy EOS, the post-merger remnant survives for over 80 ms and 120 ms, respectively, before collapsing to a black hole. For the MS1 EOS and APR4 EOS, the remnant has not collapsed even after 140 ms. The findings about the mode dynamics presented in [32] continue being valid when adding the H4 and MS1 EOS to the sample. For all four EOS the dominant mode in the early post-merger phase is the $m = 2$ $f$-mode, with a frequency above 3 kHz for SLy and APR4, about 2.5 kHz for H4, and below 2 kHz for MS1. In all four cases, the evolution of this mode is characterized by a decaying amplitude. As first found by [32], distinct, lower-frequency modes appear later in the evolution, at $t - t_{\text{merger}} \sim 35$ ms for SLy and MS1, $\sim 45$ ms for H4, and $\sim 55$ ms for APR4. This new type of GW emission dominates over the initial $m = 2$ $f$-mode at late times.

A closer look at our Prony’s analysis of the behavior of the post-merger signal for each EOS reveals the following (all times are w.r.t. the time of merger):

For the SLy EOS the early post-merger phase (which includes oscillations in the maximum density and transient GW emission of secondary peaks) ends at about 5 ms. After this, the $m = 2$ $f$-mode is practically the only significant oscillation mode and we observe that its frequency changes secularly with time, at a constant rate of $\sim 1.6 \times 10^{-2}$ kHz/ms up to $\sim 20$ ms. From then on and up to about 60 ms the frequency of the $m = 2$ $f$-mode remains practically constant. The decay of the amplitude of the $m = 2$ $f$-mode is slower in the first 20 ms than in the period between 20 ms and 60 ms (at the end of this period this mode has practically faded away). At $\sim 37$ ms
an additional mode at a different (lower) frequency appears alongside with the $m = 2$ $f$-mode. The frequency of the new excited mode is about $\sim 2.93$ kHz. Within about 8 ms from its appearance, the new mode has grown more than an order of magnitude in amplitude and then its amplitude rapidly collapses. Another phase of the excitation, saturation and destruction of a distinct, low-frequency mode (with frequency of roughly $2.7$ kHz) follows in the time period between 45 ms and 60 ms. A third such phase lasts between 60 ms and 110 ms, where the dominant mode has a nearly constant frequency of about $2.5$ kHz and a nearly constant amplitude (with small variations). Notice that the saturation amplitude of the low-frequency modes in these three periods is similar in order of magnitude. In the following sections (as in [32]) we interpret the low-frequency modes as inertial modes excited by convective instabilities in the remnant (in the model with the SLy EOS we thus observe three such episodes of inertial mode excitation, with somewhat different frequencies, but comparable saturation amplitude - whether it is the same or different inertial modes that are excited in each episode remains to be determined by a detailed mode analysis). At $\sim 110$ ms it is not possible to recognize any mode clearly and at $\sim 123.6$ ms a black hole is formed.

For the APR4 EOS the early post-merger phase ends at about 3.2 ms. At this point there is only one mode with frequency $\sim 3.1$ kHz which remains practically constant up to $\sim 25$ ms. The exponential damping timescale is constant in this time interval. At $\sim 25$ ms there is a small glitch in the frequency of the main mode, which otherwise continues practically constant up to $\sim 54$ ms. Notice that at $\sim 25$ ms the amplitude of the mode increases appreciably on a dynamical timescale, before the damping continues (we will comment on this finding in Section IIIF). The frequency of the main mode continues to be visible up to $\sim 100$ ms, having a somewhat larger damping rate. However, at $\sim 54$ ms an additional mode appears, with a lower frequency of $\sim 2.70$ kHz. Whereas its frequency remains practically constant up to $\sim 100$ ms, its amplitude shows rapid variations, which can be interpreted as several episodes of excitation-saturation-destruction of an unstable mode. Between $\sim 105$ ms and $\sim 120$ ms another such episode takes place (at the same frequency), but this time the saturation amplitude rises higher and remains constant for a longer time. After $\sim 120$ ms a few more episodes are visible at a frequency slightly smaller than in the previous time period. Notice that between $\sim 98$ ms and $\sim 105$ ms another mode with smaller frequency of $\sim 2$ kHz appears in the spectrogram, but with very small amplitude (we cannot exclude that this could be an artifact of the Prony’s method).

For the H4 EOS the early post-merger phase ends at $\sim 6.7$ ms when only one mode is present. This mode has an initial frequency of $\sim 2.37$ kHz and changes constantly with a rate of $\sim 7.1 \times 10^{-3}$ kHz/ms up to $\sim 46$ ms after the merger, after which it remains practically constant at $\sim 2.64$ kHz. The exponential damping timescale is constant in this time interval (and becomes somewhat shorter after that). At $\sim 46$ ms a second mode appears at a practically constant, lower frequency of $\sim 2.11$ kHz. This new modes rises in amplitude on a dynamical timescale, saturates quickly and stays at a nearly constant amplitude until black hole formation at $\sim 84$ ms.

For the MS1 EOS the early post-merger phase ends at $\sim 6.7$ ms, when only one single mode is active, with an initial frequency of $\sim 1.88$ kHz that stays practically constant up to $\sim 35$ ms. The exponential damping timescale is constant in this time interval. From $\sim 35$ ms to $\sim 100$ ms one can observe several episodes of the excitation-saturation-destruction of another, lower-frequency mode. As time goes on, the frequency of this second mode decreases. The final episode takes place at $\sim 100$ ms, after which the lower-frequency mode maintains a nearly constant (but slightly decreasing) frequency of $\sim 1.55$ kHz and a nearly constant saturation amplitude.

To summarize, for all four models the post-merger phase can be subdivided into three distinct periods:

- **Early post-merger phase** ($0 < t - t_{\text{merger}} \lesssim 3.7$ ms): the main $m = 2$ $f$-mode frequency and the subdominant frequencies $f_2 - 0$ and/or $f_\text{spiral}$ are active.

- **Intermediate post-merger phase** ($3.7$ ms $\lesssim t - t_{\text{merger}} \lesssim 35.50$ ms): only the $m = 2$ $f$-mode is active.

- **Late post-merger phase** ($t - t_{\text{merger}} \gtrsim 35.50$ ms): low-frequency modes appear in several episodes of excitation-saturation-destruction.

In addition, we notice a sudden revival of the $m = 2$ $f$-mode in the intermediate phase (see discussion in Section IIIF).

In the following sections, as in [32], we will identify the low-frequency modes in the late post-merger phase as inertial modes, driven by convective instabilities.

### C. Spectral content in the different stage of the dynamics

According to the temporal subdivisions discussed in the previous section, it is meaningful to single out the particular contributions of different time windows in the GW spectra for each of the EOS. The corresponding spectra are displayed in Figure 1. This figure shows the scaled PSD $2\hbar^2 f$ of the GW signal for a maximally-aligned source at a distance of 50 Mpc (where $\hbar$ is the Fourier transform of the GW signal). From top to bottom each row corresponds to the spectra of the models with the SLy, APR4, H4, and MS1 EOS, respectively. The thin-dashed-black lines in all of the panels display the entire spectra computed taking into account the complete temporal evolution of the simulations, i.e. $t - t_{\text{merger}} \in [-20, 140]$ ms. On the other hand, the
FIG. 4. GW spectra for a BNS merger at 50 Mpc with optimal orientation (one row for each EOS model). The spectra are shown for the entire GW signal (thin-dashed lines) and for different restricted time windows (indicated at the top of each frame) in order to emphasize the contribution of the dominant spectral components at different times. The design sensitivities of Advanced Virgo [99], Advanced LIGO [100], Einstein Telescope [101], and Cosmic Explorer [102] are shown for reference.

colored thick-solid lines indicate the corresponding spectra for the particular time window reported in the top of each frame. To obtain a cleaner separation of the contributions to the spectra due to the various time intervals considered, we do not apply any windowing functions. As a result, the FFT includes small artifacts due to the finite size of the time intervals. As the time window changes from left to right, the more prominent features in the spectra can be seen to readjust accordingly.

The vertical red dashed lines in the first column (roughly corresponding to the early post-merger phase) of Figure 4 mark the position of the initial frequency $f_{2i}$ of the dominant $m = 2$ $f$-mode and of the subdominant peaks $f_{2-0}, f_{2+0}$ (for the models with EOS SLy and APR4) and $f_{\text{spiral}}$ (for the models with EOS H4 and MS1). These peaks can be obtained directly from the GW spectrum, but also using the ESPRIT Prony algorithm. Our identification of the subdominant peaks is
in agreement with the spectral classification of the post-merger GW emission in [24], see also [17]. Notice that in other works, the \( f_{2-0} \) and \( f_{\text{s}} \) peaks have been named, collectively, \( f_1 \) and the \( f_{2+0} \) peak has been named \( f_3 \).

In the second column (roughly corresponding to the intermediate post-merger phase) of Figure 4, the vertical red dashed lines mark the position of the \( f_2 \) frequency peaks of the dominant \( m = 2 \) \( f \)-mode (notice that \( f_2 \) can remain very close to its initial value \( f_{2i} \), or it can display a secular time variation).

For all models, the spectra corresponding to the entire duration of the simulations is clearly dominated by the \( m = 2 \) \( f \)-mode (\( f_2 \)) in the EOS-dependent frequency range \( \sim 2-3.5 \) kHz. On the other hand, when selecting the signal in different time periods displayed in Figure 4, the dominant peaks in the spectra begin to change. In the final time interval, \( t - t_{\text{merger}} \in [60, 140] \) ms, the spectrum is dominated by a peak at frequency \( \sim 2.38 \) kHz in the case of the SLy EOS and well below 2 kHz for the MS1 EOS. Moreover, in the case of the APR4 EOS there are several nearby lower-frequency peaks contributing to the GW spectrum in the late post-merger phase.

In the intermediate post-merger phase, the spectra for the EOS SLy model (5 ms to 35 ms) and the EOS H4 model (7 ms to 45 ms) are consistent with the Prony’s method results that, for these models in these time periods, the dominant \( f_2 \) frequency is subject to a slow drift to higher frequencies. This is due to the fact that in this time period the remnant experiences an almost steady drift to higher maximum densities (see Figure 4).

It is well known that the frequency of the fundamental fluid modes of a star scale as the square root of the average density (see e.g. [103]). Notice that the remnants for these two models have a mass relatively close to the threshold mass for collapse to a black hole (and indeed they do collapse within the duration of our simulations). The panels in Figure 4 also show the design sensitivities of Advanced Virgo [99] and Advanced LIGO [100] as well as those of third-generation detectors Einstein Telescope [101] and Cosmic Explorer [102]. In [52] we discussed the prospects of detectability of the low-frequency modes excited at sufficiently long times after merger for models SLy and APR4. In the current work our simulations account for additional models with the H4 and MS1 EOS. In all cases we find that there is sufficient power in the lower-frequency modes to render them potentially observable by third-generation detectors. For a source at 50 Mpc, the expected Cosmic Explorer [102] S/N ratios for optimal use of matched filtering techniques for the signal emitted in the time intervals used in the last two columns of Figure 4 are \( \sim 1.3 \) and \( \sim 3.5 \) for SLy, \( \sim 9.1 \) and \( \sim 3.7 \) for APR4, \( \sim 11.6 \) and \( \sim 5.1 \) for H4, \( \sim 9.7 \) and \( \sim 8.3 \) for MS1 EOS, respectively (for the Einstein Telescope-D [101] they are \( \sim 1.2 \) and \( \sim 2.5 \), \( \sim 8.0 \) and \( \sim 2.9 \), \( \sim 8.7 \) and \( \sim 3.7 \), \( \sim 6.4 \) and \( \sim 5.5 \), but enhancement can be expected due to the triangular arrangement with three non-aligned interferometers). We note that the S/N ratio is significantly higher in the first \( \sim 40 \) ms after merger for each model, while it becomes almost two orders of magnitude smaller when the inertial modes appear and the signal can only be observed by the next generation detectors, as mentioned above.

**D. Mode eigenfunctions**

To analyze and show the difference of the various oscillation modes we have performed the FFT transform in time segments of \( 5 \) ms of the density distribution and extracted the Fourier amplitude at selected fixed frequencies (this correlates with the eigenfunction of a given mode, as shown in [24 [101]]). For each segment we assign the central time as a label. In Figures 5, 6, 7, 8, and 9 we summarize the behavior of the density eigenfunction of
The APR4 EOS shows an excited mode already after $\sim 30 \text{ ms}$ in the post-merger phase, which is not visible in the spectrogram of Figure 9 produced using the Prony’s method, but is clearly visible from the density eigenfunction profiles in Figure 5. This is due to the fact that this excited mode is extremely weak and cannot be distinguished using the Prony’s method. In Figure 9 we note that the $m = 2$ $f$-mode for the MS1 does show a nodal line far from the center, in the extended, low-density envelope. This behavior is not surprising for differentially rotating stars in rapid rotation, where the rotational forces can modify the structure of the eigenfunction in the low-density regions near the surface, see e.g. [104].

The SLy EOS presents only an $m = 2$ $f$-mode until $\sim 35 \text{ ms}$ after the merger, where we note the appearance of an excited mode at lower frequency. The main mode is then suppressed and only the excited one will survive until the remnant forms a black hole in the late post merger phase. For the H4 EOS we note that the main $m = 2$ $f$-mode is the only mode present until $\sim 45 \text{ ms}$ when the excited mode at lower frequency appears. The two modes then survive $\sim 30 \text{ ms}$ until the HMNS collapses to a black hole.

The excited low-frequency modes in the late post-merger phase have the characteristics of inertial modes. Their frequency is in the right range, with the dominant mode always having a frequency somewhat smaller than the maximum rotational frequency in the star (although our sample of models is small, it appears that the frequency of the dominant mode in the late post-merger phase correlates with the rotational frequency of the star, as expected from inertial modes). Their eigenfunction has a peak in the low-density outer parts of the stars, having more nodal lines than the fundamental quadrupole mode. We note that inertial modes in rapidly rotating stars can be axial-led (reducing to axial $r$-modes in the slow-rotation limit) or polar-led (reducing to $g$-modes in the slow-rotation limit) [105] [106]. The latter are also called gravito-inertial modes. A precise identification of the character of the excited modes would require a detailed comparison with linear oscillations of quasi-equilibrium models. Because the inertial modes observed in the post-merger phase are excited by convective instabilities in the outer layers of the remnant, it is likely that at least some (or most) of the inertial modes we observe are polar-led, i.e. gravito-inertial modes (but the presence of axial-led inertial modes cannot be excluded without further detailed study).
FIG. 10. Snapshots of $A_r$ in the equatorial plane. Each row corresponds to a different EOS model and each column to different times after merger. Dark color indicates regions where $A_r > 0$, which corresponds to convective instability (see text for details).

E. Convective instability and inertial modes

The appearance of lower-frequency modes in the late stage of our simulations can be interpreted as inertial modes, for which the Coriolis force is the dominant restoring force. The growth of such modes is caused by a convective instability that appears in the nonisentropic remnant just before the excited modes start to grow from
a small amplitude.

As we note in [32], the local convective stability depends on the Schwarzschild discriminant, which is defined as

\[ A_\alpha = \frac{1}{\varepsilon + p} \nabla_\alpha \varepsilon - \frac{1}{\Gamma_1 p} \nabla_\alpha p, \]  

(1)

where

\[ \Gamma_1 := \frac{\varepsilon + p}{p} \left( \frac{dp}{d\varepsilon} \right) = \left( \frac{d\ln p}{d\ln \rho} \right)_s \]  

(2)

is the adiabatic index around a pseudo-barotropic equilibrium and where \( \varepsilon \) is the energy density. Regions where \( A_\alpha < 0 \) are convectively stable, while regions where \( A_\alpha > 0 \) are convectively unstable. Since in this work we are using a piecewise polytropic approximation for the construction of our models, we calculate \( \Gamma_1 \) as

\[ \Gamma_1 = \Gamma_{th} + (\Gamma_i - \Gamma_{th}) \frac{K_i \rho^{\Gamma_i}}{p} \]  

(3)

where \( K_i \) and \( \Gamma_i \) are the polytropic constant and exponent in the \( i \)-th piece of the EOS, respectively, and \( \Gamma_{th} \) is the adiabatic index added for the thermal treatment (see Appendix C for further details). In order to determine the convective stability in the long-living remnant, we calculate \( A_r \) and \( A_\theta \) in the equatorial and vertical planes at different times.

The results of the computation of the Schwarzschild discriminant in the equatorial plane for each model considered in this work are displayed in Figure 10 at selected evolution times. In this figure convectively unstable regions (\( A_r > 0 \)) are represented using a dark color. The
first row of Figure 10 shows the case of the SLy EOS and we note that the remnant is stable in the first $\sim 35$ ms of the post-merger phase and the GW spectrum is dominated by the $m = 2$ $f$-mode. After $\sim 39$ ms we observe the formation of a convectively unstable ring, which corresponds to the outer layer of the hot ring which is visible in the second panel of the third row of Figure 11 depicting temperature isocontours. This coincides with the first appearance of inertial modes. At $\sim 65$ ms the convectively unstable ring has expanded to the lower density regions of the remnant and appears fragmented, which correspond to the strong growth of an inertial mode at 2.38 kHz. The amplitude of the mode remains almost constant throughout the following phase, while the convectively unstable regions keep expanding in the envelope until the remnant finally collapses to a black hole.

In the second row of Figure 10 we display the Schwarzschild discriminant for the H4 EOS, which presents a slightly different situation compared to the SLy EOS. Around $\sim 32$ ms we note that there are unstable regions in the outer layer of the core which are due to the presence of hot spots (not shown in this work) generated by the shock heating produced during the merger. At $\sim 45$ ms an unstable ring appears just outside the core, corresponding to the growth of an inertial mode with frequency 2.11 kHz. The unstable regions expand and become fragmented at a later time whereas the amplitude of the inertial mode continues to grow until the remnant collapses to a black hole after $\sim 84$ ms.

The evolution of the Schwarzschild discriminant for the model with the APR4 EOS, which is shown in the third row of Figure 10 appears at first glance to be similar to that of the SLy EOS. However, it does show a crucial difference, visible in the first panel of the figure. This panel displays a generally stable remnant despite, as we have already observed in Sec. IIID, the presence of an additional excited mode by this evolution time ($\sim 35$ ms)). The appearance of this mode is not triggered by the expansion of the convectively unstable region, since in this phase the remnant has a corotating region which is visible in Figure 13. This figure shows that the mode frequency is lower than the peak of the rotational profile at $t = 25$ ms, which corresponds to the maximum rotation frequency for the APR4 EOS model. As in the case of the SLy EOS, a convectively unstable ring forms at a later time, a few milliseconds before the appearance of inertial modes. The unstable regions start expanding in the lower density regions of the remnant, leading to the growth in amplitude of such modes.

Finally, in the case of the MS1 EOS, the different snapshots in the last row of Figure 10 show convectively unstable regions fairly spread over the entire post-merger remnant from earlier times than for the other three models. Notice that for this EOS, the episodes of excitation-saturation-destruction of a lower-frequency inertial mode already start at earlier times than shown in Figure 10.

Figure 11 shows snapshots of the density and temperature in the equatorial plane for the model with the SLy EOS, at different times relative to the time of merger. In the first two rows of the figure we display the density and temperature profiles of the post-merger remnant, up to a time of 29 ms, when the $m = 2$ $f$-mode is still dominant. We note that in this phase there are two hot spots where the remnant is deformed and which start to diffuse. This is the expected behavior, as found in simulations with many different numerical codes (see e.g. [18] and references therein). In the last row of the figure we show snapshots of only the temperature at times between 33 ms and 84 ms. A hot shell forms around the cold core at $\sim 33$ ms. Later, at around 65 ms, a second hot ring is forming in the equatorial plane and by 84 ms the temperature of the inner shell has risen visibly, reaching about $T \sim 50$ MeV.

In our simulations, convection is exciting specific, global inertial modes. This is not unexpected, see e.g. [107–110]. We note that the inertial modes are excited when the temperature profile in the remnant has become nearly axisymmetric (by that time, the amplitude of the peak $m = 2$ $f$-mode oscillation has diminished). In contrast, the initial strongly nonaxisymmetric temperature pattern (essentially in the form of two hot spots) does not seem to lead to the excitation of global modes. This is true for all four models considered: inertial modes start growing once the temperature distribution has become sufficiently circular in the equatorial plane.

We stress that the convective instability found in our simulations is solely triggered by the thermal gradient created by shock heating at the time of merger. Hence, in our setup no additional physics (e.g. radiative cooling) is necessary to create a thermal gradient. As shown in Fig. 11 for $t > 30$ ms after merger, the core of the remnant is cold and is surrounded by a shell of hot matter, with the temperature reaching a peak at about 5 km from the center. Beyond that distance, the temperature decreases steeply with radius and it is this thermal gradient that is responsible for triggering the convective instability. A similar situation has been recently reported by [44] who study rotating stars with non-barotropic thermal profiles. This work shows that convection sets in without requiring additional physics, as long as there is a negative entropy gradient when the density is below a critical density (see Eq. C7 in [44]), which is the case in our simulations.

In particular, numerical dissipation does not play a role in triggering the convective instability. It can only dampen the instability, once it sets in, or entirely prevent it, if it is too strong. In our simulations, the numerical dissipation is sufficiently small to allow for the instability to grow and saturate. Fig. 16 in Appendix A displays the evolution of different norms of the Hamiltonian constraint violations, at three different resolutions (for EOS SLy as a representative case). The violations converge with resolution and remain sufficiently small throughout the duration of the simulation.
F. Corotation and mode revival

As noted in the introduction, the state of the system in the post-merger phase is a single (excited) differentially rotating neutron star with a non trivial rotational profile, which is shown in Figures 12 and 13. Moreover, there is also an non-trivial temperature profile (and indeed a non-trivial entropy profile, in which the temperature has an off-center maximum (a circle in the equatorial plane at late times). The rotational and thermal structure of the post-merger remnants in our simulations is in agreement with results in the literature (see e.g. [70, 73, 111]) where similar assumptions were made (only when assuming a very strong physical viscosity different profiles are found, see [75]). The remnant may be subject to various instabilities, namely the dynamical bar-mode instability (see [72, 114] and references therein), the secular Chandrasekhar–Friedman–Schutz (CFS) instability [39, 40] and shear instabilities that can develop in differentially rotating stars (see e.g. [13] and references therein). In addition, our results reveal the possibility of the excitation of additional modes in the post-merger phase for sufficiently long times. We discuss next how the frequency of the excited inertial mode is related to the rotational profile of the neutron star at the late stage of the post-merger dynamics.

In Figure 12 we show the rotational profile (angular velocity vs. distance in the equatorial plane) of the four models at 40 ms after merger, along with the pattern speed frequency for the two main modes in each model, indicated by the horizontal lines. For $m=2$ modes the pattern speed frequency is half of the associated GW frequency. We note that at this stage of the evolution only the APR4 and MS1 models are still in corotation with the mode pattern speed.

In Figure 13, we focus on the model with the APR4 EOS and show the rotational profile at different times, along with the pattern speed of the $m=2$ $f$-mode (1.55 kHz) and of the main inertial mode at the start (1.35 kHz) and at the end (1.26 kHz) of the convectively unstable phase.

IV. CONCLUSIONS

We have presented the results of new numerical simulations of BNS systems in full general relativity, where the simulations have been extended up to $\sim$140 ms after merger. In addition to the results for models based on the SLy and APR4 EOS we already discussed in [32], and whose analysis has been significantly extended here, we have also studied models equipped with two more EOS, namely H4 and MS1 EOS. The main focus of this
work has been the analysis of the spectrum of the post-merger GW signal. Our investigation has been driven, in particular, by our aim to provide further evidence in support of our claim in [32] about the potential existence of new families of oscillation modes excited in long-lived post-merger remnants (in addition to the main fluid quadrupole $f$-mode) at sufficiently late times of the evolution.

In [32] we already observed that the appearance of convectively unstable regions and the excitation of inertial modes in remnants that survive for a long time after merger depend on the rotational and thermal state of the remnant and affect the HMNS dynamical evolution. In all further cases investigated in the present work, the late-time excitation of inertial modes to large amplitudes has been corroborated. Indeed, inertial modes become the dominant modes for the GW emission in the post-merger phase (as in the case of binary neutron star remnants) and affect the HMNS dynamical evolution. In all further cases investigated in the present work, the late-time excitation of inertial modes to large amplitudes has been corroborated. Indeed, inertial modes become the dominant modes for the GW emission in the post-merger phase (as in the case of binary neutron star remnants) and affect the HMNS dynamical evolution.

Recently [44] have analyzed convectively unstable rotating neutron stars with non-barotropic thermal profiles (as in the case of binary neutron star remnants) reporting a growth timescale of convection of $\mathcal{O}(10)$ ms close to the center and $\mathcal{O}(0.1)$ ms at lower densities (close to the surface). These timescales are compatible with those presented in [32] and further corroborated in this work. Our study has neglected the effects of shear and bulk viscosity in the star as well as the effective viscosity due to MHD turbulence. In view of our results, it will be important to investigate the influence of viscosity on the lifetime of the remnant and their impact on the rotational profiles of the HMNS, as this might affect the potential excitation of inertial modes. Recent work by [75, 76], in which effective shear viscosity was modelled through the so-called viscous $\alpha$-parameter, has shown that the degree of differential rotation in the post-merger remnant is significantly reduced in the viscous timescale, $\leq 5$ ms, for $\alpha \sim \mathcal{O}(10^{-2})$. It remains to be found if such (high) values of the $\alpha$-parameter are reached in MHD simulations of BNS mergers involving pulsar-like magnetic-field strengths in the inspiral phase.

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Appendix A: Comparing simulations at different resolutions

The convergence properties of the numerical code and the accuracy of the simulations were discussed in [69]. Here, we compare the long-term evolution of the model with EOS SLy using three different resolutions of the innermost grid, namely $dx = 369$ m, $dx = 277$ m (the standard resolution in this work), and $dx = 185$ m (which is about five times more computationally expensive). Figure 15 shows the results of the Prony analysis for the three resolutions. From this figure we conclude that the appearance of the convective instabilities, and the associated excitation of inertial modes, happens at somewhat later times as the resolution is increased. Likewise, black hole formation is delayed in all three cases, taking longer times for higher resolutions, as discussed in [69]. Also, there are three (instead of two) episodes of excitation-saturation-destruction before the final excitation of a long-lasting inertial mode with stable saturation amplitude. At the lowest resolution, the appearance of an inertial mode (dashed blue line in Fig. 15) is rapidly...
FIG. 14. Time-frequency analysis for the model with EOS SLy at two different resolutions (see text for details).

FIG. 15. Comparison of the Prony analysis for the model with EOS SLy at three different resolutions.

FIG. 16. Hamiltonian constraint violation for different resolutions of the model with EOS SLy.

FIG. 16. Hamiltonian constraint violation for different resolutions of the model with EOS SLy.

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FIG. 15. Comparison of the Prony analysis for the model with EOS SLy at three different resolutions.

FIG. 16. Hamiltonian constraint violation for different resolutions of the model with EOS SLy.
straint violations are attained at black hole formation, as usual in this type of simulations. Overall, this figure shows that the simulations at different resolutions are consistent and that the violations converge with resolution and remain sufficiently small throughout the duration of the simulation.

Appendix B: Notes on previous results for simulations lasting 100 ms after merger.

Our study is not the first one in which the post-merger phase of a binary neutron star merger is evolved for timescales of about 100 ms. Previous works, all of them fairly recent, include [115-119]. Because of the very small amplitude of the GWs emitted in the late convective phase (as opposed to the large amplitude emitted by hydrodynamical motions during the early phase) one can easily miss the presence of convectively-excited modes in the late post-merger phase, or one can simply dismiss these as artifacts, without further analysis. To the best of our knowledge, other groups have not specifically examined long-term simulations in terms of eigenfunctions or indicators of convective instability. The studies of [110] and [117] report the results of simulations of binary neutron star mergers with initial data constructed with a simple polytropic \( \Gamma = 2 \) EOS and evolved as an ideal fluid with \( \Gamma_{th} = 2 \), extending up to 140 ms. Similarly long-lasting simulations for the same EOS are discussed in [118]. None of those investigations report the late-time excitation of inertial modes in the remnant. This indicates that such modes are likely not easily excited when the initial data and the ideal fluid EOS have the same index \( \Gamma \) (notice e.g. that in [110] that main \( l = m = 2 \) f-mode still has a large amplitude at the end of the simulation, whereas in our simulations the convective instabilities always set in only after the amplitude of \( l = m = 2 \) f-mode has diminished).

The work of [119] shows the presence of a second mode at late post-merger times. In the top panel of their Figure 16 they show the amplitude of the \( h_{22} \) GW-mode for a simulation of a long-lived magnetized neutron star merger described by the APR4 EOS and masses \( 1.35 + 1.35 M_\odot \). This model is slightly more massive than the one with the same EOS used in this work. The simulation is followed up to \( \approx 100 \) ms after merger. This figure shows the presence of an unphysical gauge mode with a frequency of \( \approx 1 \) kHz around 70 ms after merger. On the other hand the spectrogram shown in the bottom panel of their figure 16 displays the presence of a single mode up to 30 ms after merger while two modes become visible at a later time. This suggests a similar situation to the one described in the present work. Indeed, while [119] do not arrive at an explanation for this second frequency, they notice the similarity to our results in [32]. We note, however, that an unambiguous comparison would only be possible by extracting the associated eigenfunctions of the late-time modes found by [119].

Appendix B of [115] presents a study of convergence for a model very similar to the one discussed in the present work, namely a model with EOS SLy4 and masses \( 1.30 + 1.30 M_\odot \). We recall that here we have considered a model described by the piecewise-polytropic representation of the SLy EOS with masses \( 1.28 + 1.28 M_\odot \). The resolutions used by [115] are \( dx = 415 \) m (VLR), 246 m (LR), 185 m (SR), and 136 m (HR). The GW strain and time-frequency diagram are shown in their Figure 13. In the post-merger phase the convergence properties of their simulations are poor (they had to use a rather small Courant-Friedrichs-Lewy factor of 0.075) with the VLR and SR simulations behaving differently to the LR run. Moreover, their HR simulation shows the formation of a black hole within less than 30 ms after merger (while this does not happen at the other resolutions) and that the phase of the waveform is not in the convergent regime. Nevertheless, the visual inspection of their results at VLR and SR resolutions seem compatible with our results. The discrepancy might be resolved by a closer examination of the numerical data of [115].

Finally [44] present a study of convectively unstable rotating neutron stars with non-barotropic thermal profiles (as in the case of binary neutron star remnants). In their simulations, the thermal gradients naturally trigger convective motions in the star and the authors estimate (see their equation C8) that the growth timescale of convection is of order tens of ms close to the center, but reduces to the order of 0.1 ms at lower densities (close to the surface in their models). As [44] note referring to the rapid growth of convective instabilities in our simulations in the low-density envelope of the remnant, these timescales are compatible with those we find in [32] (and here). This is additional evidence in favor of our explanation of the late post-merger phase in our simulations.

Appendix C: Piecewise polytropic EOS

The numerical simulation of gravitational systems that include neutron stars requires a description of the behavior of matter at the very high density involved inside such compact objects. A full description would require a detailed understanding of the properties of matter at densities exceeding nuclear density, which is not yet fully accomplished. For this reason and for computational convenience, we here choose to model thermal effects similar to an ideal fluid. In this approximation, the energy momentum tensor is written as \( T^{\mu \nu} = [\rho (1 + \epsilon) + p] u^\mu u^\nu + pg^{\mu \nu} \) where \( \rho \) is the baryon density, \( \epsilon \) is the specific internal energy and \( p \) is the pressure. As a consequence, the energy density is \( \epsilon = \rho (1 + \epsilon) \). At this point, the thermal description of the system is achieved assuming (where \( \epsilon_{th} \) is an arbitrary function of thermodynamical state that has the properties of being zero at zero temperature):

\[
\epsilon = \epsilon_0 (\rho) + \epsilon_{th}, \quad (C1)
\]

\[
p = p_0 (\rho) + (\Gamma_{th} - 1) \rho \epsilon_{th}, \quad (C2)
\]
and $\epsilon_0(\rho)$ and $p_0(\rho)$ are the specific internal energy and pressure at $T = 0$. For a fixed number of fermions the volume is $V = (1/\rho)$, and the energy $E = e/\rho$. The first law of thermodynamics is:

$$dQ = dE + pdV = TdS. \tag{C3}$$

Assuming thermodynamic consistency, Eq. (C3), Frobenius theorem uniquely fixes (up to an arbitrary function, i.e., any function $\Sigma(s)$ gives other possible definitions of “temperature” and “entropy” $(s,t)$ as $S = \Sigma(s)$ and $T = t/(d\Sigma(s)/ds)$) the “temperature”-like and “entropy”-like function as:

$$T = (\Gamma_{th} - 1)\epsilon_{th}, \tag{C4}$$

$$S = \log \left[ \frac{(\epsilon_{th})^{1/(\Gamma_{th} - 1)}}{\alpha \rho} \right], \tag{C5}$$

where $\alpha$ is an arbitrary constant with dimensions of the inverse of density. The conversion factor from the above dimensionless “temperature”-like values $\epsilon_{th}$ and the real temperature $T$ is given by the specific heat at constant volume and the function $\Sigma(s)$. The choice of $\Sigma(s) = s$ corresponds to having an “ideal-fluid” thermal behavior, and, as discussed in [120], the dimension of the temperature $T$ can be found by multiplying Eq. (C4) by a factor $m/k_B$, where $m$ is the value of the baryon mass. At the same time, the condition of thermodynamic consistency at $T = 0$ determines the functional relation between $\epsilon_0(\rho)$ and $p_0(\rho)$ ($dQ = 0$ at $T = 0$) that reads:

$$p_0(\rho) = \rho^2 \cdot \frac{d\epsilon_0(\rho)}{d\rho}. \tag{C6}$$

The zero-temperature piecewise polytropic approximate EOS amounts to the assumption that $p_0(\rho)$ and $\epsilon_0(\rho)$ are continuous polytropic functions of $\rho$. These conditions, and Equation (C6) imply that $p_0(\rho)$ and $\epsilon_0(\rho)$ are locally of the type:

$$p_0(\rho) = K_i \rho^{\Gamma_i},$$

$$\epsilon_0(\rho) = \epsilon_i + \frac{K_i}{\Gamma_i - 1} \rho^{\Gamma_i - 1} \tag{C7} \quad \text{(C7)}$$

and all coefficients are set once the polytropic index $\Gamma_i$ ($i = 0,\ldots,N - 1$), the transition density $\rho_i$ ($i = 1,\ldots,N - 1$) and $K_0$ are chosen. This defines an $N$-piece piecewise polytropic EOS with an “ideal-fluid” thermal component.

The specific enthalpy, in turn, is given by:

$$h = \frac{e + p}{\rho} = \frac{1 + \epsilon_0(\rho) + p}{\rho} = 1 + (1 - \Gamma_i)\epsilon_i + \Gamma_{th}\epsilon_{th} + \Gamma_i\epsilon_0(\rho) \tag{C8}$$

Since the relativistic speed of sound can be written as

$$c_\rho^2 = \frac{1}{h} \left( \frac{dp}{d\rho} \right)_s = \frac{1}{h} \left[ \left( \frac{dp}{d\rho} \right)_\epsilon + \frac{de}{d\rho} \left( \frac{d\rho}{d\rho} \right)_\rho \right]$$

$$= \frac{1}{h} \left[ \left( \frac{dp}{d\rho} \right)_\epsilon + \frac{p^2}{\rho^2} \left( \frac{d\rho}{d\rho} \right)_\rho \right] \tag{C9} \quad \text{for a piecewise polytropic EOS we have:}$$

$$c_\rho^2 = \frac{\Gamma_{th}(\Gamma_{th} - 1)\epsilon_{th} + \Gamma_i(\Gamma_i - 1)(\epsilon_0(\rho) - \epsilon_i)}{1 + (1 - \Gamma_i)\epsilon_i + \Gamma_{th}\epsilon_{th} + \Gamma_i\epsilon_0(\rho)} \tag{C10}$$

where $\epsilon_0$ is the same as in Equation (C7) and the index $i$ refers to the $i$-th piece. At the transition point we have imposed the condition that the pressure and energy density are continuous, but that does nod hold true for the speed of sound that can be discontinuous, i.e., in general $(c_\rho^2)_i(\rho_i) \neq (c_\rho^2)_{i+1}(\rho_i)$.

For a piecewise polytropic EOS we calculate the adiabatic index ($\Gamma_1$) for adiabatic perturbations as:

$$\Gamma_1 = \frac{e + p}{\rho} \left( \frac{dp}{d\rho} \right)_s = \frac{d\ln p}{d\ln \rho} \tag{C11}$$

$$= \Gamma_{th} + (\Gamma_i - \Gamma_{th}) \frac{K_i \rho^{\Gamma_i - 1}}{p} \tag{C12}$$

where $\epsilon$ is the specific internal energy, $K_i$ and $\Gamma_i$ are the polytropic constant and exponent in the $i$-th piece of the EOS.

The construction presented here has been considered as a 2D equation of state in [14]. They chose to assume as “entropy” the quantity $s$ defined by $S = \Sigma(s) = 2/(\Gamma_{th} - 1)\log(s)$ and to fix the value of the constant $s$ so that it corresponds to having $\epsilon_{th} \propto s^2 \rho^{1/\Gamma_{th} - 1}$. Indeed we should expect, as shown in [14], that non-barotropic stellar configurations (including binary neutron star merger remnants) may be subject to convective instabilities.

Another possibility is to assume as “entropy” the quantity $s$ defined by $S = \Sigma(s) = \log(s)$. With this choice the expressions for the main thermodynamical variables, as function of $\rho$ and $s$, are:

$$\epsilon = \epsilon_0(\rho) + s^{\Gamma_{th} - 1} \cdot (\alpha \rho)^{\Gamma_{th} - 1} \tag{C13}$$

$$p = p_0(\rho) + (\Gamma_{th} - 1) \cdot \rho \cdot s^{\Gamma_{th} - 1} \cdot (\alpha \rho)^{\Gamma_{th} - 1} \tag{C14}$$

$$t = (\Gamma_{th} - 1) \cdot s^{\Gamma_{th} - 2} \cdot (\alpha \rho)^{\Gamma_{th} - 1} \tag{C15}$$

One should note that this choice is of difficult physical interpretation in the case of $\Gamma_{th} = 2$ since it would imply that we should interpret the density as the temperature of the system. The “entropy” $(s)$ and the “temperature” $(t)$ in terms of the internal energy and density are expressed by:

$$\epsilon_{th} = (\epsilon - \epsilon_0(\rho)) \tag{C16}$$

$$t = (\Gamma_{th} - 1) \cdot (\epsilon_{th})^{(\Gamma_{th} - 2)/(\Gamma_{th} - 1)} \cdot (\alpha \rho) \tag{C17}$$

$$s = (\epsilon_{th})^{1/(\Gamma_{th} - 1)} / (\alpha \rho) \tag{C18}$$

This concludes the discussion of the thermodynamic properties of the EOSs used in the present work.
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