Stopping and Isospin Equilibration in Heavy Ion Collisions

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Abstract

We investigate the density behaviour of the symmetry energy with respect to isospin equilibration in the combined systems Ru(Zr) + Zr(Ru) at relativistic energies of 0.4 and 1.528 AGeV. The study is performed within a relativistic framework and the contribution of the iso-vector, scalar $\delta$ field to the symmetry energy and the isospin dynamics is particularly explored. We find that the isospin mixing depends on the symmetry energy and a stiffer behaviour leads to more transparency. The results are also nicely sensitive to the “fine structure” of the symmetry energy, i.e. to the covariant properties of the isovector meson fields.

The isospin tracing appears much less dependent on the in medium neutron-proton cross sections ($\sigma_{np}$) and this makes such observable very peculiar for the study of the isovector part of the nuclear equation of state.

Within such a framework, comparisons with experiments support the introduction of the $\delta$ meson in the description of the iso-vector equation of state.

Key words: Asymmetric colliding nuclear matter, Heavy ion collisions at relativistic energies, Isospin transparency, Isovector channel in effective field models.

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determined for densities beyond saturation and actually it is very differently predicted from the various nuclear many-body models [4]. On the other hand, the iso-vector EOS is of crucial importance in extrapolating structure calculations away from the valley of stability and in astrophysical processes, such as neutron star cooling and supernova explosions.

So far asymmetric nuclear matter has been only poorly studied for extreme conditions beyond saturation. Studies on finite nuclei [5] refer only to ground state nuclear matter, while at supra-normal densities one has to rely on extrapolations. Thus, more useful information could be achieved in studies of heavy ion collisions due to the formation of hot and dense asymmetric matter for short time scales during the reaction dynamics. Theoretical studies of asymmetric nuclear matter beyond saturation have been recently started. They can be divided into two groups, non-relativistic using phenomenological Skyrme-interactions [6] and relativistic ones based on the Relativistic Mean Field (RMF) theory of the Quantum Hadrodynamics (QHD) [7–10].

In this study we use a common relativistic framework for the static description of asymmetric nuclear matter [8] and for the dynamic case of heavy ion collisions [10]. This is particularly appropriate for heavy ion collisions at intermediate energies, since such a covariant formulation intrinsically predicts the main features of the density and momentum dependence of the nuclear EOS due to the relative importance of the scalar and vector fields in the reaction dynamics. The relativistic structure is of great interest in the iso-vector sector for the competition between Lorentz vector (ρ) and Lorentz scalar (δ) iso-vector fields, which show a similar cancellation and similar dynamic effects as in the iso-scalar (σ, ω) sector. While structure calculations seem to be rather insensitive to the presence of a δ field, we expect clearly distinguishable effects at higher densities. Indeed the high density dependence of the symmetry energy $E_{\text{sym}}$ is particularly affected by the different treatment of the microscopic Lorentz structure of the iso-vector part of the nuclear mean field [8–10].

$E_{\text{sym}}$ becomes stiffer due to the introduction of the attractive scalar δ field, for a pure relativistic mechanism. The iso-vector EOS can be characterized by a vector ρ meson or, as in the iso-scalar case, by a competition of vector ρ and scalar δ field contributions. As discussed in detail in Refs. [8,11], with the same fixed bulk asymmetry parameter in both cases, one has to increase the ρ meson coupling in the presence of the attractive δ field. On the other hand, the latter field, due to its Lorentz scalar character, is proportional to an invariant scalar density $\rho_s \approx \frac{\pi}{2} \rho_B^2$, suppressed for baryon densities above saturation $\rho_B >> \rho_{\text{sat}}$. Thus, this competition between Lorentz vector (ρ) and Lorentz scalar (δ) iso-vector fields leads finally to a stiffer behavior of the symmetry energy at high densities. The scalar nature of the δ-field will also naturally imply a definite neutron/proton effective mass splitting [8,11], with a more reduced neutron mass at high density.
Fig. 1. (Top) Density dependence of the symmetry energy $E_{sym}$ for the Non-Linear Walecka-model ($NL$) including only the $\rho$ field ($NL\rho$) and both, the $\rho$ and $\delta$ fields ($NL\rho\delta$) in the iso-vector sector. The panels in the bottom show the kinetic ($E_{kin}$) and potential ($E_{pot}$) contributions to the total symmetry energy $E_{sym}$ separately.

The consideration of the $\delta$ meson in the description of asymmetric nuclear matter gives then rise to a larger repulsion (attraction) for neutrons (protons) for baryon densities $\rho_B$ above saturation. Moreover the larger $\rho$-meson coupling is further increasing the repulsion seen by high momentum neutrons due to the Lorentz boosting of the charged vector field, as already observed in flow studies, see the discussion in ref. [9]. These effects together with the effective mass splitting are responsible for new interesting transport phenomena which could help to determine the high density behavior of the symmetry energy and its microscopic structure.

The relativistic features of the iso-vector sector are shown in Fig. 1 in terms of the symmetry energy $E_{sym}$ within the non-linear RMF model including only the iso-vector, vector $\rho$ ($NL\rho$) and both, the iso-vector, vector $\rho$ and iso-vector, scalar $\delta$ mesons ($NL\rho\delta$). The choice of the corresponding coupling constants is discussed in detail in the refs. [9,10]. The related kinetic equations are used for transport simulations of the reaction dynamics of the relativistic heavy ion collisions presented here [10]. We solve the covariant transport equation of the Boltzmann type within the Relativistic Landau Vlasov (RLV) method [12] (for the Vlasov part) and applying a Monte-Carlo procedure for the collision term, including inelastic processes involving the production/absorption of nucleon resonances [13].

In order to reach definitive conclusions on the high density behavior of the symmetry energy one should first select the key signals and then simultaneously compare with experimental data. Collective isospin flows [9,14] and particle production, i.e. pion [6,10,14] and kaon spectra [15] seem to be good
candidates. Here we continue the previous studies by considering another interesting aspect, which has been extensively investigated by experiments of the FOPI collaboration: the correlation between the degree of stopping and the isospin equilibration (or transparency).

The idea is to study colliding systems with the same mass number but different N/Z ratio, in particular a combination of $^{96}_{44}$Ru, N/Z = 1.18 and $^{96}_{40}$Zr, N/Z = 1.4 has been used as projectile/target in experiments at intermediate energies of 0.4 and 1.528 AGeV [16,17]. The degree of stopping or transparency has been determined by studying the rapidity dependence of the imbalance ratio for the mixed reactions Ru(Zr) + Zr(Ru): \[ R(y^{(0)}) = \frac{N^{RuZr}(y^{(0)})}{N^{ZrRu}(y^{(0)})}. \]

The observable \( R \) can be measured for different particle species, like protons, neutrons, light fragments such as \( t \) and \( ^3He \) and produced particles such as pions (\( \pi^0, \pm \)), etc. It characterizes different stopping scenarios.

In the proton case, moving from target to cm rapidity, \( R(p) \) rises (positive slope) for partial transparency, falls (negative slope) for full stopping/rebound scenarios and it is flat when total isospin mixing is achieved in the collision. An opposite behavior will appear for neutrons. Indeed we remind that in the full transparency limit \( R \) should approach the initial value of \( R(p) = \frac{Z^{Zr}}{Z^{Ru}} = \frac{40}{44} = 0.91 \) and \( R(n) = \frac{N^{Zr}}{N^{Ru}} = \frac{56}{52} = 1.077 \) for protons and neutrons at target rapidity, respectively. Therefore, \( R(p) \) can be regarded as a sensitive observable with respect to isospin diffusion, i.e. to properties of the symmetry term as we will show later.

In order to proceed to a direct comparison with experiments, in our kinetic equation simulations we have used a coalescence method to select nucleon and light ion emissions. The algorithm is quite standard. As described in ref.[18], the coalescence radii in coordinate (\( < R >_c = 4fm \)) and in momentum (\( < R >_p = 1.3fm^{-1} \)) space are fixed from the experimental charge distributions. In the present calculations we have also checked the stability of the yields by varying the coalescence parameters (\( < R >_{c,p} \)) up to a 20%.

For the investigation of isospin equilibration we have analyzed the transport results in the same way as carried out in the FOPI experiment, in particular selecting central events through the observable ERAT which measures the ratio of the mean transverse to the mean longitudinal kinetic energy. To do so, we first apply the coalescence method for fragment production in the final state and then we calculate the observable ERAT. A detailed description of this analysis can be found in ref. [18], where it was shown that the charged particle multiplicity and ERAT distributions fit well the experimental data, as an important check of the phenomenological phase space model. At this point we can use the same centrality cuts as in the FOPI exps. [17].
We start the discussion of the results showing, in the case of a central $Ru + Zr$ collision at $E = 0.4 AGeV$, the time evolution of the asymmetry parameter $\langle n > - \langle p > \rangle / \langle n > + \langle p > \rangle$, with $\langle \cdots \rangle$ being the average number of emitted particles (protons, neutrons), for different rapidity regions as indicated (see Fig. 2). The compression/expansion time scales are given in terms of the temporal development of the $\Delta$ resonances and the produced pions ($\pi$).

We see that the colliding system emits more neutrons (increase of the asymmetry parameter) in the transport calculations with the $NL\rho\delta$ interaction. The effect is more pronounced during the compression phase ($15 < t < 30 \text{ fm/c}$), in particular in the mid-rapidity region, whereas in the spectator regions the trend becomes opposite for late time scales.

The observed differences between $NL\rho$ and $NL\rho\delta$ models can be understood in terms of the density dependence of the symmetry energy, Fig. 1. Its stiff character in the $NL\rho\delta$ model generates stronger (smaller) pressure gradients being responsible for more repulsion (attraction) for neutrons (protons). This is also consistent with the effective mass splitting, where the neutrons (protons) experiences a smaller (larger) effective mass for densities beyond saturation. Thus, neutrons are emitted earlier than protons increasing the asymmetry of the emitted particles. The isospin effects of the $\delta$ meson are more pronounced only during and just after the compression phase, since the different treatment of the microscopic Lorentz structure of the symmetry energy affects it only for supra-normal densities, see also Fig. 1. Moreover for high momentum particles we have a pure relativistic boosting of the vector $\rho$-meson repulsion for neutrons, and attraction for protons, see the discussion of ref.[9].
Fig. 3. The imbalance ratio $R(y^{(0)}) = \frac{N_{RuZr}(y^{(0)})}{N_{ZrRu}(y^{(0)})}$ as function of the normalized rapidity $y^{(0)} = y/y_{proj}$ of free protons (left columns) and free neutrons (right columns) for the models of Fig. 1, for central ($b \leq 2\ fm$) $Ru(Zr) + Zr(Ru)$-collisions at 0.4 (top) and 1.528 $A GeV$ beam energies. The experimental data are taken from the FOPI collaboration [17].

The opposite late trends (expansion phase) in the asymmetry parameter of emitted particles at projectile/target rapidity can be consistently accounted for by the low density behaviour of the symmetry energy. In this region the $\delta$-coupling in neutron-rich matter leads to more (less) attraction for neutrons (protons) responsible for a reduction of neutron emission. However, it is rather a moderate effect as expected from the low density dependence of the symmetry energy shown in Fig. 1.

In Fig. 3, we report the rapidity dependence of the imbalance ratio in central collisions of the mixed system $Ru(Zr) + Zr(Ru)$ for free protons ($R(p)$) and free neutrons ($R(n)$) at the two energies 0.4 and 1.528 $A GeV$.

First of all, the imbalance ratio correctly approaches unity at mid-rapidity for all particle types considered. This is a obvious check of the calculation since the symmetry of the two collisions naturally implies a full isospin mixing in the $c.m.$ rapidity region.

Going from target- to mid-rapidity it nicely rises for protons, and decreases for neutrons, a good isospin transparency signature. The effect is much more evident for the $NL\rho\delta$ interaction. The observed difference between the two models is obvious since within the $NL\rho\delta$ picture neutrons experience a more
Fig. 4. Proton imbalance ratio as function of the normalized rapidity for central ($b \leq 2 \text{ fm}$) Ru(Zr) + Zr(Ru)-collisions at 0.4 AGeV like the top-left part of Fig. 3. A curve is added (empty squares) corresponding to a $NL\rho$ calculation with reduced nucleon-nucleon cross sections (half the free values, $\sigma_{NN}(E) = \frac{1}{2}\sigma_{\text{free}}$).

repulsive iso-vector mean field, particularly at high densities, than protons, with a consequent much less nucleon stopping in the colliding system. This picture is consistent with the previous Fig. 2. It can be also considered as a high density effect since the particle emission takes mainly place during and just after the compression phase.

The observed effect of the EOS on the imbalance ratio of protons and neutrons is not very large. At low intermediate energies (0.4 AGeV) one has to deal with moderate compressions of $\rho_B < 2 \cdot \rho_{\text{sat}}$ where the differences in the iso-vector EOS arising from the $\delta$ meson and thus from the different treatment of the Lorentz structure are small. On the other hand, at higher incident energies (1.528 AGeV) a larger difference is observed between the two models. Indeed, one observes a slightly higher isospin effect on the imbalance ratios. However, with increasing beam energy the opening of inelastic channels via the production/decay of $\Delta$ resonances through pions and additional secondary scattering (pion absorption) with isospin exchange, e.g. $nn \rightarrow n\Delta^0$, $\Delta^0 \rightarrow p\pi^-$, also contributes as a background effect to the final result. This interpretation is confirmed by other studies [6].

Reduced in medium Nucleon-Nucleon ($NN$) cross sections, in particular the $\sigma_{np}$, will also increase the isospin transparency in the direction shown by the data. We have tested this possibility considering a factor two reduction, $\sigma = \frac{1}{2}\sigma_{\text{free}}$, of the free $NN$-cross section values used before. We note that such a reduction represents a rather strong in-medium effects as compared to recent microscopic Dirac-Brueckner estimations [19].

In Fig.4 we report the results, open squares, for the proton imbalance ratios.
at 0.4 $A GeV + 0.5\sigma_{free}$ in the $NL\rho$ case. We see a overall slightly increasing transparency but not enough to reproduce the trend of the experimental values in the target rapidity region. On the other hand the reduction of the $NN$ cross sections, and in particular of the $\sigma_{np}$, implies a too large transparency in the proton rapidity distributions for central collisions of the charge symmetric $Ru + Ru$ case; effect clearly seen in our simulations and already remarked in previous $IQMD$ calculations, see ref.[17]. So we can exclude such and further reductions of the $NN$ cross sections.

Since the calculations are performed with the same $EOS$ for the symmetric nuclear matter, same compressibility and momentum dependence [20], the observed transparency appears to be uniquely related to isovector $-EOS$ effects, i.e. to the isospin dependence of the nucleon self-energies at high baryon densities.

In Fig.5 we report the rapidity dependence of the imbalance ratio for other particle emissions in central collisions of the mixed system $Ru(Zr) + Zr(Ru)$ at 0.4 $A GeV$ beam energy: (left) for the ratio of triton to $He$-fragments ($R(t/3He)$) and (right) for the ratio of negative $\pi^-$ to positive $\pi^+$ charged pions ($R(\pi^-/\pi^+)$).

The fact that protons and neutrons exhibit an opposite behavior for the corresponding imbalance ratios at target rapidity, clearly suggests that the detection of the imbalance observable $R(t/3He)$ for the $t/3He$ ratio should reveal a larger sensitivity. We thus have determined the observable $R(t/3He)$, using the above discussed phase space coalescence procedure to extract the light ion emissions.
Fig. 5 (on the left) shows the $R(t/3He)$ results. The isospin effect originating from the appearance of the iso-vector, scalar $\delta$ meson in the $NL\rho\delta$ model turns out to be here crucial near target rapidities. We note that the effect indeed can be hardly seen from the separate imbalance ratios for protons and neutrons at the same rapidity, see Fig. 3 top panels, apart the difficulties of neutron emission detections. It would be therefore of great interest to experimentally measure directly this quantity.

Finally, another sensitive observable appears to be the imbalance ratio of charged pions $R(\pi^-/\pi^+)$ (Fig. 5, right panel). At variance with the previous results for neutrons and light isobars, this ratio is reduced at target rapidity with the $NL\rho\delta$ model. Such effect strongly supports our understanding of the transport properties of the various effective interactions. Pions are produced from the decay of $\Delta$ resonances formed during the high density phase, see Fig.1(top-left). The $\pi^-$ abundance is then linked to the neutron-excess of the high density matter, that can form negative charged resonances, as detailed described in [10]. We remind that the contribution of the $\delta$ meson leads to a more repulsive field for neutrons at supra-normal densities and consequently to less neutron collisions and finally to a smaller $\pi^-/\pi^+$ ratio.

In conclusion, we have studied the density dependence of the isovector part of the nuclear EOS, which is still poorly known experimentally, controversially predicted by theory, but of great interest in extreme nuclear systems. Here we discuss its high density behavior that can be tested in relativistic heavy ion collisions.

We have analyzed the stopping and isospin transparency in relativistic collisions in terms of the imbalance ratio of different particle types in projectile/target rapidity regions in mixed collisions. We show that this observable is even sensitive to the microscopic Lorentz structure of the symmetry term. Effective interactions with symmetry energies not much different at $2-3$ times the normal density $\rho_{sat}$ are predicting large differences in the isospin – transparencies, depending on the relative contribution of the various charged vector and scalar fields. The interest is also in the fact that this appears to be a genuine relativistic effect. Such result, combined to a weak dependence on the in-medium $NN$ cross sections for hard nucleon collisions, indicates the importance of imbalance ratio measurements for a better knowledge of the nuclear EOS, and its fine – structure, at high baryon density.

We have found moderate isospin effects on the degree of stopping for protons and neutrons, but important effects in the imbalance observable for the ratio of tritons to $^3He$ fragments and for the ratio of negative to positive charged pions. A comparison with the few preliminary available data indicates a stiff behavior of the symmetry energy, which in our formulation means that the inclusion of $\delta$-like fields is favoured. However, more experimental information,
as proposed here, would be necessary in order to achieve a clear definitive conclusion.

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