Optimal design of self-retaining full complement cylindrical roller bearings

C Ursache¹, A Barili¹,², L Tudose¹,² and C Tudose³

¹Technical University of Cluj-Napoca, Faculty of Machine Building, Bd. Muncii 103-105, 400641 Cluj-Napoca, Romania
²RKB Europe SA, Via Primo Agosto 1, 6828 Balerna, Switzerland
E-mail: lucian.tudose@omt.utcluj.ro

Abstract: Due to the presence of the ribs on both sides of the outer ring, the self-retaining full complement (cageless) cylindrical roller bearings have a very interesting particularity: the outer ring must be heated to create enough room between the already mounted rollers to allow the last roller to be inserted in the row. In addition, after cooling down, the rollers must support each other with a certain small clearance. For this reason, the internal geometry of the bearing as well as the precision of execution are more than critical. On the other hand, one of the most important problems in rolling bearing design is to establish the optimal set of internal dimensions in order to obtain the maximum value of the basic radial dynamic load rating. Pursuing this goal, in this paper, the optimal design of certain self-retaining cylindrical roller bearing is performed by means of the PSO (Particle Swarm Optimization) algorithm. In this approach two design variables were considered (the roller diameter and the diameter of the inner ring raceway) and both geometrical and thermal constraints were taken into account. A study referring to the sensitivity of the design to the small variation of the geometrical parameters is also presented.

1. Introduction
Rolling bearings are one of the most interesting and, at the same time, complex mechanical component. There are many types of rolling bearings, starting from the simplest one, the single row ball bearings till most complex ones, like multi-row tapered roller bearings. Every bearing type has an unique internal geometry related to the shape of the rolling elements and to the afferent raceways. Apart the bearing internal geometry, there are also many other important features and variants. For example, for cylindrical roller bearings there are two main variants, with or without cage (full complement). Most of the rolling bearings are provided with a cage that guides and evenly angularly divide the rolling elements but, on the contrary, full complement bearings do not feature a cage, meaning that each rolling element is touching the next and the previous one, during rotation. The main benefit of the full complement bearing type is related to an increased number of rolling elements that, compared to an equivalent design with cage, leads to increased dynamic [1] and static [2] load ratings. Anyway, considering the applications where this types of bearings are used (e.g. planetary speed reducers), they have mandatory to reach a high value of fatigue life. As per [1], the maximization of the bearing fatigue life can be obtained only by maximizing its basic dynamic load rating due to an optimal internal bearing geometry. To achieve this goal, in the next paragraphs, it has been described geometrically a self-retaining full complement cylindrical roller bearing, in order to obtain the necessary equations to permit the mathematical design of the optimization problem. The main innovation of this work is represented by the optimization of the internal bearing geometry of a self-retaining full complement roller bearing.
2. Literature review
The study of cylindrical roller bearing has been managed by many researchers. Dragoni [3] developed a design procedure to maximize the basic load ratings values of cylindrical roller bearings mounted inside a gearbox. Tudose et al. [4] considered two objective functions (the basic dynamic radial load rating and the minimum thickness of the elastohydrodynamic lubrication film between rollers and raceways) and performed the multi-objective optimization using Cambrian, a very powerful optimization platform. In order to describe completely and unique the bearing geometry, three design variables were considered: the geometric factor $\gamma$, roller diameter and roller length, respectively.

Considering full complement roller bearing, it is not easy at all to find similar work in the field. In fact, as it is in the real world, the focus of all the authors is to study the caged bearings that clearly represents the biggest part of the market. Anyway, about single objective optimization, it has been found some studies that consider the bearing mounting, see for example [5], where, in order to maximize the basic dynamic load rating, it has been imposed a constraint on the bearing diameter and number of balls. Then a constraint regarding the assembly angle was imposed to find a feasible set of designing variables that are compatible with the assembling method of the bearing.

Some works see for example [6], were focused on the maximization of the dynamic load rating of cylindrical roller bearing considering the influence of the interference between the rings and the afferent seats. Regarding other works, related to different bearing types (see, for example, [7]), it becomes clear that the maximization of the bearing fatigue life, i.e. of the basic dynamic load rating, can be defined as the most common and diffused objective function in the optimal design of rolling bearings. Nevertheless, considering the particular geometry and assembling method of the bearing in discussion here, the authors of this paper did not find anything in the open literature regarding the optimal design of the self-retaining full complement cylindrical roller bearings.

3. Bearing geometry
Since the focus of the present work is to study the maximization of the basic dynamic radial load rating considering the mounting of rollers, in Figure 1 it is presented only a bearing cross section perpendicular to the bearing axis. In this way, the objective function will not consider the roller length, which has nothing to do with the mounting procedure. As one can see, the rollers centres are staying on the bearing pitch circumference of diameter $D_{pw}$ and $\delta$ is the space to insert the last roller. If the last roller is somehow mounted, $\delta$ is enough small that the all rollers support each other. But to mount the last roller the outer ring has to be heated and the roller is inserted radially. Note that the outer ring has ribs on both sides and the rollers cannot be mounted from lateral.

![Figure 1. Bearing scheme.](image-url)
4. Optimization problem

For radial cylindrical roller bearings, the basic dynamic radial load rating is given by the following equation:

$$C_r = b_m f_c (i L_{we} \cos \alpha)^{\frac{7}{9}} Z^{\frac{3}{8}} D_{we}^{\frac{29}{8}}$$  \hspace{1cm} (1)

where $b_m = 1.1$ is the rating factor for contemporary, commonly used, high quality hardened bearing steel in accordance with good manufacturing practices, $i$ represents the number of rows, $L_{we}$ is the effective roller length, $\alpha$ is the bearing contact angle, $Z$ is the number of rollers per row, and $D_{we}$ is the roller diameter. The remaining $f_c$ is a factor which depends on the geometry of the bearing components, the accuracy to which the various components are made, and the material. An accurate equation of this factor is given in [8]:

$$f_c = 172.455 \frac{\gamma^2 (1 - \gamma)^{\frac{27}{8}}}{(1 + \gamma)^{\frac{1}{4}}} \left[ 1 + \left( \frac{1 - \gamma}{1 + \gamma} \right)^{\frac{14}{8}} \right]^{\frac{2}{9}}$$  \hspace{1cm} (2)

where

$$\gamma = \frac{D_{we}}{D_{pw}}$$  \hspace{1cm} (3)

4.1. Design variables

In order to describe completely and unique the bearing geometry, it has been found that only two design variables are necessary. Therefore, the design variable vector has the following form:

$$X = (D_{we}, F)$$  \hspace{1cm} (4)

where $F$ is the diameter of the inner ring raceway, and obviously

$$D_{pw} = F + D_{we}$$  \hspace{1cm} (5)

It was preferred $F$ instead of the pitch diameter $D_{pw}$, because the outer diameter of the inner ring raceway is controlled easier during the manufacturing process. As one will see in the followings, for both design variables (roller diameter and inner ring raceway diameter) constraints connected to the available manufacturing precision are taken into account.

The problem here is how to consider the length of the roller. In the clear majority of the cases the application where the bearing will be used is unknown and therefore the thickness of the ring ribs can be established only based on the manufacturer experience and feedback from the field. For this reason, the authors did not consider the roller length as a design variable of the optimization problem.

4.2. Optimization function

The main goal of an optimal bearing design is to achieve the maximum basic dynamic load rating. Obviously, in the present case $i = 1$ and $\cos \alpha = 1$, and since the roller length can be considered as a given value, the objective function can be expressed as:

$$f(X) = \frac{C_r}{b_m (L_{we})^{\frac{9}{8}}} = f_c Z^{\frac{3}{8}} D_{we}^{\frac{29}{8}}$$  \hspace{1cm} (6)

where

$$Z = \text{Int} \left[ \frac{\pi}{\text{asin}(\gamma)} \right]$$  \hspace{1cm} (7)
4.3. Constraints

As it has been discussed in section 2, the usual constraints that shall be provided to the optimization problems are trivial, since the last roller can be inserted only after the heating of the outer ring. At the same time, once the last roller has been inserted, when the bearing is at environmental temperature, all the rolling elements must remain together, in contact with the outer ring raceway, and without falling (when the inner ring is not inserted). So, special constraints must be developed for this uncommon optimization problem.

Constraint $g_1$

As it was mentioned above, before the last roller is inserted, the initial distance $\delta$ (Figure 1) must be lower than the roller diameter:

$$ g_1 = \frac{1}{2\gamma} \sin[(Z - 2) \sin(\gamma)] - 1 < 0 $$ (8)

Constraint $g_2$

The second constraint has been developed considering the mounting method for the rollers: the bearing outer ring temperature is increased in order to mount the entire number of rollers. Considering that, when the outer ring is heated, the bearing pitch diameter $D_{pw}$ increases to $D_{pw}'$ and the factor $\gamma$ decreases to $\gamma'$ (as per the thermal linear expansion law) and taking into account that the rollers are not heated, it yields:

$$ \gamma' \frac{D'_{we}}{D_{pw}} = \frac{D_{we}}{D_{pw}(1 + \alpha_T \Delta T)} = \frac{\gamma}{1 + \alpha_T \Delta T} < \gamma $$ (9)

where $\alpha_T$ the linear expansion coefficient of the bearing steel [9]. The new factor $\gamma'$ must be the root of the following equation (the new distance $\delta'$ between the mounted rollers is equal to the roller diameter):

$$ 2 \gamma' - \sin[(Z - 2) \sin \gamma'] = 0 $$ (10)

The necessary difference of temperature to achieve this thermal expansion is given by:

$$ \Delta T = \frac{1}{\alpha_T} \cdot \left( \frac{\gamma}{\gamma'} - 1 \right) $$ (11)

Now it is possible to define the constraint $g_2$, considering that the maximum difference of temperature that can be applied to the bearing outer ring is equal to 90°C. In fact, considering an environmental temperature of 20°C, $\Delta T = 90°C$ leads to a heating temperature 110°C, perfectly compatible with the standard bearing steel [10]. Therefore:

$$ g_2 = \frac{\Delta T}{90} - 1 < 0 $$ (12)

Constraints $g_3$ and $g_4$

To avoid unfeasible designs and after studying many existing bearing designs, other two simple constraints were used to complete the optimization problem and make it reliable and close to the real-world applications. The first one stipulates that the thickness of the inner ring

$$ \delta_i = \frac{F - d}{2} $$ (13)

and the thickness of the outer ring

$$ \delta_e = \frac{D - (F + D_{we})}{2} $$ (14)

cannot be very dissimilar:
\[ g_3 = \frac{|\delta_e - \delta_i|}{0.05 \min(\delta_e, \delta_i)} < 0 \] (15)

The last constraint refers to the minimum allowable thickness of the bearing rings:

\[ g_4 = 1 - \frac{\min(\delta_e, \delta_i)}{\delta_{min}} < 0 \] (16)

Again, the minimum allowable thickness of the bearing rings (as the thickness of ring ribs) is a very delicate issue and is rather connected to bearing manufacturer experience and feedback from the field than to rigorous mechanical calculations (which are possible only in concrete loading conditions of the bearings). Based on author experience, the minimum thickness was set to 18% of the bearing section height, i.e. 0.09 \((D - d)\).

5. Optimization method

To solve the presented optimization problem and maximize (6), it was used a Particle Swarm Optimization algorithm that, in literature, has been widely applied and proved to be a solid, simple and reliable optimization tool. The scope of the present work was not a study of the performance of the PSO algorithm itself and, because the problem requests nothing special, a basic PSO with global best position update has been used (gbest algorithm). The main steps of this algorithm are the following ones:

1. Define the objective function \(f(X)\) and the space of the possible solutions;
2. Set PSO parameters: population number, maximum number of iterations, inertia weight and acceleration coefficients;
3. Create, randomly, the initial population;
4. Calculate the fitness value \(f(X_i(t))\) of each particle \(X_i\) at time \(t\);
5. Calculate pbest for each particle;
6. Calculate gbest;
7. Update the velocity \(v_i\) of each particle;
8. Update the position \(X_i\) of each particle;
9. If the stopping condition is met, return gbest, otherwise restart from step 4.

Within the gbest algorithm, it is possible to define the velocity update of each particle as:

\[ v_i(t + 1) = w v_i(t) + C_1 r_1 (pbest_i(t) - X_i(t)) + C_2 r_2 (gbest(t) - X_i(t)) \] (17)

where the particle velocity is composed by many contributes:

- \(w v_i(t)\) provides the exploration ability of the optimization algorithm
- \(C_1 r_1 (pbest_i(t) - X_i(t))\) represents the cognitive part of PSO, where the change of the speed of the single particle is due to the relationship with its own best position
- \(C_2 r_2 (gbest(t) - X_i(t))\) represents the social part of the PSO, where the particles change their own velocity considering the information that are coming from other particles.

To enhance the exploration ability in the initial stages of the optimization process and to increase the local search in the last phases, the inertia weight \(w\) was modified according to the following rule:

\[ w(t) = w_{min} + \frac{(t_{max} - t)(w_{max} - w_{min})}{t_{max} - 1} \] (18)

The constraint handling has been done by means of the penalty method: if one of the introduced constraints is violated, the value of the objective function is greatly penalized so that the new position of the particle will not be chosen in the future.

6. Case study

The optimization problem has been applied for the bearing NJG 2340. The main boundary dimensions of this bearing follow the standard prescriptions [11] and the roller chamfer form and dimensions, and
the roller length were kept as in the already existing design. The bearing geometry is summarized in Table 1.

**Table 1. NJG 2340.**

| Dimension       | Value [mm] |
|-----------------|------------|
| Bore diameter, \(d\) | 200        |
| Outer diameter, \(D\) | 420        |
| Width, \(B\)     | 138        |
| Roller length, \(L_w\) | 97         |
| Roller chamfer, \(r_{ch}\) | 2         |

Obviously, the useful roller length \(L_{we}\) (necessary for the final \(C_r\) calculation) is,

\[
L_{we} = L_w - 2r_{ch} = 93 \text{ mm}
\]  

(19)

The range of the design variables, together with their requested precision are given in Table 2.

**Table 2. Design variables range.**

| Variable | Range, mm | Precision, mm |
|----------|-----------|---------------|
| \(D_{we}\) | 60.0 … 71.5 | 0.001         |
| \(F\)    | 207.5 … 291.5 | 0.010        |

To solve the above defined optimization problem, the parameters of the PSO algorithm are enlisted in Table 3 and the optimization results are presented in Table 4.

**Table 3. PSO parameters.**

| Parameter | Value |
|-----------|-------|
| Population | 200   |
| Max iteration | 50   |
| \(w_{\text{max}}\) | 0.6   |
| \(w_{\text{min}}\) | 0.3   |
| \(C_1\) | 1.2   |
| \(C_2\) | 1.2   |

**Table 4. Optimization results.**

| Design variable/Parameter | Value |
|---------------------------|-------|
| \(D_{we}\)               | 69 mm |
| \(F\)                    | 242 mm|
| \(Z\)                    | 14    |
| \(\delta_i\)             | 21 mm |
| \(\delta_e\)             | 20 mm |
| \(\Delta T\)             | 75.2 °C|
| \(C_r\)                  | 2249.87 kN |
The obtained geometrical set of data was represented with a CAD software in both conditions, before (Figure 2a) and after (Figure 2b) heating of the outer ring.

Figure 2a. Distance δ before the heating of the outer ring (δ < \(D_{we}\)).

Figure 2b. Distance δ′ after the heating of the outer ring (δ′ = \(D_{we}\)).

Despite being a single objective optimization process with only 2 designing variables, the constraints handling of this problem was not easy to deal with. In fact, the equation (10) is not so easy to be solved numerically, because it has been found that it is very sensitive to the provided inputs. In fact, a small perturbation of one of the particle components, for example \(D_{we}\), can greatly modify \(\Delta T\) value. In Table 5 one can find an example of the numerical criticality related with the temperature variation calculation.

| Parameter | Set 1    | Set 2     |
|-----------|----------|-----------|
| \(D_{we}\) | 69 mm   | 69.01 mm  |
| \(F\)     | 242 mm   | 242 mm    |
| \(\Delta T\) | 75.2°C  | 84.7 °C   |

As it can be seen, with a slight variation of \(D_{we}\), it resulted an important variation of \(\Delta T\). It is worth to notice that such behaviour is not at all strange in mechanics, especially when it is necessary to slightly modify some dimensions to obtain a specific bearing clearance or to adjust the roller size as per some manufacturing requirements.

7. Conclusion and future improvements
In this article, it has been described a novel approach in the optimization of the design of a self-retaining full complement cylindrical roller bearing, considering its peculiar assembly method. The maximization of the objective function, the basic dynamic load rating, has been obtained by means of a basic PSO, but modified to implement the innovative set of constraints.

Considering the mounting process and the self-retaining ability of the bearing geometry, the heating of the outer ring to fit the last roller also, lead to a feasible result in terms of internal geometry. The idea to embed this new type of constraint it can be considered solid and of extreme interest in many other cases.

The only criticality found, it is related to the sensitivity of the equation (10): it has been demonstrated that a small variation (acceptable if considering a normal mechanical manufacturing process) of only
one of the two components of the best individual has a great influence on the heating temperature of the outer ring.

Another interesting and mandatory future development of the present work consists in taking into account of the inherent tolerances of the $F$ dimension, and then to optimize the internal geometry considering the required range of the bearing radial internal clearance.

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