The strong coupling constant at large distances

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Abstract. In this paper we discuss effective strong coupling constants. Those are well behaved in the low-$Q^2$ domain, contrarily to $\alpha_s$ from pQCD. We present an extraction of an effective strong coupling constant from Jefferson Lab polarized data at intermediate and low $Q^2$. We also show how these data, together with spin sum rules, allow us to obtain the effective coupling constant over the entire $Q^2$ range. We then discuss the relation between the experimentally extracted coupling constant and theoretical calculations at low $Q^2$. We conclude on the importance of such study for the application of the AdS/CFT correspondence to QCD.

Keywords: Strong coupling constant, QCD sum rules, non-perturbative, commensurate scale relations, Schwinger-Dyson, Lattice QCD, AdS/CFT

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In QCD, the magnitude of the strong force is given by the running coupling constant $\alpha_s$. At large $Q^2$, in the pQCD domain, $\alpha_s$ is well defined and is given by the series:

$$\mu \frac{\partial \alpha_s}{\partial \mu} = 2\beta(\alpha_s) = -\frac{\beta_0}{2\pi} \alpha_s^2 - \frac{\beta_1}{4\pi^2} \alpha_s^3 - \frac{\beta_2}{64\pi^3} \alpha_s^4 - \ldots \quad (1)$$

Where $\mu$ is the energy scale, to be identified to $Q$. The first terms of the $\beta$ series are: $\beta_0 = 11 - \frac{2}{3} n$ with $n$ the number of active quark flavors, $\beta_1 = 51 - \frac{19}{7} n$ and $\beta_2 = 2857 - \frac{5033}{7} n + \frac{325}{7} n^2$. The solution of the differential equation 1 is:

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda_{QCD}^2)} \times$$

$$\left[1 - \frac{2\beta_1}{\beta_0^2} \frac{\ln[\ln(\mu^2/\Lambda_{QCD}^2)]}{\ln(\mu^2/\Lambda_{QCD}^2)} \right] + \frac{4\beta_1^2}{\beta_0^3} \frac{\ln[\ln(\mu^2/\Lambda_{QCD}^2)]}{\ln(\mu^2/\Lambda_{QCD}^2)} \left(\frac{1}{2} - \frac{1}{2} \ln[\ln(\mu^2/\Lambda_{QCD}^2)] + \frac{\beta_2}{8\beta_0^2} - \frac{5}{4} \right) \quad (2)$$

Eq. 2 allows us to evolve the different experimental determinations of $\alpha_s$ to a conventional scale, typically $M_{z_0}^2$. The agreement between the $\alpha_s$ obtained from different observables demonstrates its universality and the validity of Eq. 1. One can obtain $\alpha_s(M_{z_0}^2)$ with doubly polarized DIS data and assuming the validity of the Bjorken sum rule [1]:

$$\Gamma_{1}^{p-n} = \int_0^1 (g_1^p - g_1^n) dx = \frac{g_A}{6} \left[1 - \frac{\alpha_s}{\pi} - 3.58 \left(\frac{\alpha_s}{\pi}\right)^2 - 20.21 \left(\frac{\alpha_s}{\pi}\right)^3 + \ldots\right] + O(\frac{1}{Q^2}) \quad (3)$$

where $g_A$ is the well measured nucleon axial charge. Solving Eq. 3 using the experimental value of $\Gamma_{1}^{p-n}$, and then using Eq. 2 provides $\alpha_s(M_{z_0}^2)$.

Eq. 2 leads to an infinite coupling at large distances, when $Q^2$ approaches $\Lambda_{QCD}^2$. This is not a conceptual problem since we are out of the validity domain of pQCD on which
Eq. 2 is based. But since data show no sign of discontinuity or phase transition when crossing the intermediate $Q^2$ domain, one should be able to define an effective coupling $\alpha_s^{\text{eff}}$ at any $Q^2$ that matches $\alpha_s$ at large $Q^2$ but stays finite at small $Q^2$.

The Bjorken Sum Rule can be used again to define $\alpha_s^{\text{eff}}$ at low $Q^2$. Defining $\alpha_s^{\text{eff}}$ from Eq. (3) truncated to first order: $\Gamma_{\text{BP}}(1 - \alpha_s^{g_1}/\pi)$, offers many advantages. In particular, $\alpha_s^{\text{eff}}$ does not diverge near $\Lambda_{\text{QCD}}$ and is renormalization scheme independent since the first term in a pQCD series is the same, regardless to the choice of renormalization scheme. However, $\alpha_s^{\text{eff}}$ becomes dependent on the choice of observable employed to define it. If $\Gamma_{\text{BP}}(1 - \alpha_s^{g_1}/\pi)$ is used as the defining observable, the effective coupling is noted $\alpha_s^{g_1}$. Relations, called commensurate scale relations [2], link the different effective couplings so in principle one effective coupling is enough to describe the strong force and the theory retains its predictive power.

The effective coupling definition in term of pQCD evolution equations truncated to first order was proposed by Grunberg [3]. Following this definition, effective couplings have been extracted from different observables and have been compared to each other using the commensurate scale relations [7], see Fig. 1. There is good agreement between the effective couplings $\alpha_s^{g_1}$, $\alpha_s^{F_3}$ and $\alpha_s^{\tau}$. The GDH and Bjorken sum rules can be used to extract $\alpha_s^{g_1}$ at small and large $Q^2$ respectively [7]. This, together with the JLab data at intermediate $Q^2$, provides for the first time a coupling at any $Q^2$. A striking feature of Fig. 1 is that $\alpha_s^{g_1}$ becomes scale invariant at small $Q^2$. This was predicted by a number of calculations but it is the first time it is seen experimentally.
The effective coupling $\alpha_s$ extracted from JLab data, its fit, and its extraction using the Burkert and Ioffe [11] model to obtain $\Gamma_{\pi}^{p,n}$. The $\alpha_s$ calculations are: Top left: Schwinger-Dyson equations (Cornwall [10]); Top right: Schwinger-Dyson equations (Bloch) [12] and $\alpha_s$ used in a quark constituent model [13]; Bottom left: Schwinger-Dyson equations (Maris-Tandy [14]), Fischer, Alkofer, Reinhardt and Von Smekal [15] and Bhagwat et al. [16]; Bottom right: Lattice QCD [17].

A fit of the $\alpha_{s,g_1}$ data and sum rule constraints has been performed with a form based on Eq. 2 at first order:

$$\alpha_{s,g_1}^{fit} = \frac{\gamma n(Q)}{\Lambda^2} \log\left(\frac{Q^2 + m_g^2(Q)}{\Lambda^2}\right)$$

(4)

where $\gamma = 4/\beta_0 = 12/(33 - 8)$, $n = \pi (1 + \frac{1}{\log(m^2/\Lambda^2)} + (bQ)^c - 1)$ and $m_g = m/(1 + (aQ)^d)$. The values of the parameters are: $\Lambda = 0.349 \pm 0.009$ GeV, $a = 3.008 \pm 0.081$ GeV$^{-1}$, $b = 1.425 \pm 0.032$ GeV$^{-1}$, $c = 0.908 \pm 0.025$, $m = 1.204 \pm 0.018$ GeV, $d = 0.840 \pm 0.051$. $m_g$ has been interpreted as an effective gluon mass [10]. The fit is shown on Fig. 2 (continuous black line). Eq. 4, used in $\Gamma_{\pi}^{p,n} = \frac{1}{6}(1 - \alpha_{s,g_1}/\pi)$, can also be employed to parametrize the generalized Bjorken and GDH sums.

On Fig. 2, $\alpha_{s,g_1}$ is compared to theoretical results. There are several techniques used to predict $\alpha_s$ at small $Q^2$, e.g. lattice QCD, solving the Schwinger-Dyson equations, or choosing the coupling in a constituent quark model so that it reproduces hadron spectroscopy. However, the connection between these $\alpha_s$ is unclear, in part because of the different approximations used. In addition, the precise relation between $\alpha_{s,g_1}$ (or any effective coupling defined using [3] or [2]) and these computations is unknown. Nevertheless, one can still compare them to see if they share common features. The calculations and $\alpha_{s,g_1}$ present a similar behavior. Some calculations, in particular the lattice one, are in excellent agreement with $\alpha_{s,g_1}$.

These works show that $\alpha_s$ is scale invariant (conformal behavior) at small and large $Q^2$ (but not in the transition region between the fundamental description of QCD in terms of quarks and gluons degrees of freedom and its effective one in terms of baryons.
and mesons). The scale invariance at large $Q^2$ is the well known asymptotic freedom. The conformal behavior at small $Q^2$ is essential to apply a property of conformal field theories (CFT) to the study of hadrons: the Anti-de-Sitter space/Conformal Field Theory (AdS/CFT) correspondence of Maldacena [18], that links a strongly coupled gauge field to weakly coupled superstrings states. Perturbative calculations are feasible in the weak coupling AdS theory. They are then projected on the AdS boundary, where they correspond to the calculations that would have been obtained with the strongly coupled CFT. This opens the possibility of analytic non-perturbative QCD calculations [19].

To sum up, thanks to the data on nucleon spin structure and to spin sum rules, an effective strong coupling can be extracted in any regime of QCD. The question of comparing it with theoretical calculations of $\alpha_s$ at low $Q^2$ is open, but such comparison exposes a similarity between these couplings. Apart for the parton-hadron transition region, the coupling shows that QCD is approximately a conformal theory. This is a necessary ingredient to the application of the AdS/CFT correspondence that may make analytical calculations possible in the non-perturbative domain of QCD.

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