ELECTROWEAK BARYOGENESIS AND THE MINIMAL SUPERSYMMETRIC STANDARD MODEL

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ABSTRACT

In principle, the baryon asymmetry of the Universe can be generated at the electroweak phase transition but the experimental lower limit on the Higgs mass seems to rule out a Standard Model scenario. However, it has been shown recently that in the Minimal Supersymmetric Standard Model, the electroweak phase transition can be a strong enough first order one for baryogenesis if the mass of one top squark is close to or smaller than the top mass.

INTRODUCTION

Why the Universe is dominated by matter and not by antimatter is one of the most intriguing characteristic of the Nature. Astrophysical observations imply in fact a very small value for the Baryon Asymmetry of the Universe (BAU):

$$\frac{n_B - n_{\bar{B}}}{n_{\gamma}} \simeq 4 - 7 \times 10^{-10}$$ (1)

with $n_B, n_{\bar{B}}$ and $n_{\gamma}$ are respectively the densities of baryons, antibaryons and photons.

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In 1967, Sakharov was the first to point out that the BAU could be explained in terms of high energy physics. He showed that, in order to generate it, a particle theory has to satisfy 3 conditions:
- Baryon number cannot be conserved.
- C and CP (C for Conjugaison Charge and P for parity) have to be violated. Otherwise, the rate of reactions with particles and the rate of reactions with antiparticles should be the same.
- Departure from thermal equilibrium is needed. At some stage of its history, the Universe had to be out of thermal equilibrium. During these phases, the state of Universe was non-stationary and some macroscopic variables as the baryonic charge was time-dependent.

This letter is divided in 2 parts. In the first, properties of the Standard Model are discussed. In the second, it will be shown that in a simplified scenario described by the Minimal Supersymmetric Standard Model (MSSM) the baryogenesis constraint can be satisfied. The allowed range of the parameters of the model is consistent with the present experimental bounds.

**THE STANDARD MODEL**

In the electroweak Standard Model (SM), the Sakharov’s conditions are full-filled. The SM has 2 sources of CP-violation. The first is the CP-violation coming from the phase $\delta_{CKM}$ in the Cabibbo-Kobayashi-Maskawa quark mixing matrix with 3 generations of quarks. A basis-invariant measure of this is given by the Jarlskog’s determinant:

$$\Delta_{CP} = \text{det}[M_u M_u^\dagger, M_d M_d^\dagger]$$ (2)

where $M_u$ and $M_d$ are respectively the $3 \times 3$ up and down quark mass matrices.

Some recent attempts to calculate the BAU using the CKM CP-violating phase led to the conclusion that the $\delta_{CKM}$ is not efficient enough to produce the right order of magnitude.

The second source of CP-violation in the SM is the $\theta_{strong}$-angle. The most general QCD lagrangian contains the following 4-dimensional term:

$$\mathcal{L}_\theta = \theta_{QCD} \frac{g_s^2 N_f}{32\pi^2} G_\mu^a \tilde{G}_\mu^a$$ (3)

with $N_f$ is the flavor number, $g_s$ the QCD gauge coupling, $G_\mu^a$ is the gluon field strength tensor and $\tilde{G}_\mu^a$, its dual. This term is T-violating (T for time reversal) and its related to axial $U(1)$ anomaly.

Indeed, at the classical level, the QCD lagrangian for $N_f$ massless quarks is invariant under the chiral $U(N_f)_L \times U(N_f)_R$ symmetry. But at the quantum level, the flavor-singlet axial current $J^a_{\mu\delta}(\bar{q}^\gamma_5 \gamma^\mu q)$ is not conserved.

$$\partial_\mu J^a_{\mu\delta} = \frac{g_s^2 N_f}{16\pi^2} G_\mu^{a\nu} \tilde{G}_\mu^{a\nu}$$ (4)
In the procedure to diagonalize the arbitrary mass matrices of the standard electroweak model, a chiral redefinition of the right-handed fields is necessary.

\[ q_R \rightarrow e^{-i\frac{\theta}{N_f}} q_R \]  

(5) 

with \( \theta = \arg \det M_u M_d \).

The anomalous effect of this chiral transformation is to induce the following modification to the lagrangian:

\[ \delta \mathcal{L}_a = \theta \frac{g_s^2 N_f}{32 \pi^2} G_a^{\mu
u} \tilde{G}_a^{\mu\nu} \]  

(6) 

Therefore, the physical \( \theta \) is the sum of both contributions: one from QCD (Eq.[3]) and the other from the mass matrices (Eq.[6]).

The strongest constraint on the physical \( \theta \) is coming from the neutron electric dipole momentum [11]:

\[ \theta_{\text{physical}} < 10^{-10} \]  

(7) 

It is interesting to note that this limit is of the same order of magnitude than the BAU.

The out of equilibrium condition will be satisfied if the Electroweak Phase transition (EWPT) is a first order one. It means that the vacuum of the symmetric phase is metastable. The Phase Transition (PT) proceeds by nucleation. In such a case, 3 temperatures can be defined: one when both vacua are degenerated (\( T_1 \)), a second when the PT occurs (\( T_c \)) and the last one is when the potential is flat at the origin (\( T_0 \)). These temperatures follow this hierarchy:

\[ T_0 < T_c < T_1 \]  

(8) 

In the SM, the first order phase transition is induced by the weak gauge bosons. It is a consequence of the cubic term in the finite temperature bosonic effective potential[5].

\[ V_{\text{bosons}}(m, T) = -\frac{T^4 \pi^2}{90} + \frac{m^2 T^2}{24} - \frac{m^3 T}{12 \pi} + \ldots \]  

(9) 

\[ V_{\text{fermions}}(m, T) = -\frac{7}{180} \pi^2 T^4 + \frac{m^2 T^2}{12} + \frac{m^4}{16 \pi^2 \ln \frac{T}{T_c}} + \ldots \]  

(10) 

with \( m \) is the field-dependent mass of the bosons or the fermions.

As we can see, the first order character of the EWPT in the SM is proportional to the weak gauge coupling. The phase transition is not expected to be strong enough for baryogenesis as we shall see later.

The last Sakharov’s condition is the non-conservation of the baryonic charge. In the SM, only the B-L current is conserved (B and L are respectively the Baryonic and the Leptonic currents) but the divergence of the B+L current is given by the electroweak anomaly induced by the chiral structure of the weak gauge symmetry.

\[ \partial_\mu J^\mu_{B+L} = \frac{g_w^2 N_g}{16 \pi^2} W^\mu_a \tilde{W}_a^{\mu\nu} \]  

\[ \partial_\mu J_{B+L}^\mu \]  

(11)
with $N_g$ the generation number, $g_w$ the weak gauge coupling, $W^a_{\mu \nu}$ is the weak field strength tensor and $\tilde{W}^a_{\mu \nu}$, its dual.

But at zero temperature, the rate of the anomalous B-violating reactions ($\Gamma_B$) is strongly suppressed by an exponential factor \[12\]:

$$\Gamma_B \propto e^{-\frac{1}{\alpha_w}} \approx 10^{-100}$$

(12)

This suppression can be avoided at high temperature. In that case, $\Gamma_B$ is proportional to a Boltzmann factor and the B-violating transition is induced by an unstable solution of the equation of motion called "Sphaleron" \[13\].

$$\Gamma_B \propto e^{-\frac{M_{sph}}{T}}$$

(13)

where $M_{sph} = 4\pi v(T)/g_w B(\lambda/g^2)$ is the sphaleron mass, $B$ is a constant which in the standard model ranges between $1.5 \leq B \leq 2.7$ for $0 \leq \lambda/g^2 < \infty$ and $v(T)$ is the Higgs expectation value at the temperature $T$.

To avoid a wash-out of the BAU by the sphalerons after the phase transition, the B-violating processes have to be out of equilibrium. A way to impose this property is to ask that $\Gamma_B$ has to be smaller than the expansion rate of the Universe ($\Gamma_H$):

$$\Gamma_B < \Gamma_H$$

(14)

Using Eq.\[13\] and the expression of the $M_{sph}$, the last condition can be written as follows:

$$\frac{v(T_c)}{T_c} \geq 1$$

(15)

In the SM, this baryogenesis constraint implies an upper bound on the Higgs mass\[4\]:

$$m_H \leq 60 GeV$$

(16)

This value is experimentally ruled out\[14\].

In conclusion of this first part, we have to mention that the SM effectively fills the 3 Sakharov’s conditions but it fails on 2 main points. First, even if the CP-violating processes producing the BAU are not well known and understood, the SM CP-violation seems to be too small. Secondly, at the Electroweak Phase Transition, the jump in the Higgs expectation value is too weak. So, in order to explain the BAU, we need to go beyond the SM.

**THE MINIMAL SUPERSYMMETRIC STANDARD MODEL (MSSM)**

A simple extension of the SM is the MSSM which not only predicts a light Higgs boson but also contains new CP-violating phases. In the MSSM, there are 2 complex scalar doublets. But we shall assume that at low-energy, only one neutral scalar remains light while all the other Higgs bosons and supersymmetric partners of the SM particles have a mass of the order of the global supersymmetry breaking scale.
The tree level scalar potential for the real component \( h \) of the lightest Higgs boson reads
\[
V_{\text{tree}} = \frac{1}{2} \mu^2 h^2 + \frac{1}{32} \tilde g^2 \cos^2 2\beta \, h^4
\]  
(17)
where \( h = h_1 \cos \beta + h_2 \sin \beta \), \( \tilde g^2 = (g_y^2 + g_w^2) \) and \( g_y, w \) are the \( U(1) \) and \( SU(2) \) gauge couplings respectively.

In order to simplify our discussion, we shall assume that the stop masses are given by the following relations:
\[
m_{L,R}^2 = m_{L,R}^2 + m^2
\]  
(18)
with \( m \equiv \frac{g_h}{\sqrt{2}} \). We have neglected the D-term contribution to the stop masses as well as the left-right mixing effects. The top and the stop loops are the dominant contributions to the effective 1-loop potential. For the Higgs mass, one obtains
\[
m_h^2 = m_Z^2 \cos^2 2\beta + \frac{3}{4\pi^2} \frac{m_t^4}{v_0^2} \ln \left[ \left( 1 + \frac{m_t^2}{m_T^2} \right) \left( 1 + \frac{m_R^2}{m_T^2} \right) \right]
\]  
(19)
where \( m_Z = \tilde g v_0/2 \) is the Z-boson mass and \( m_t = g_t v_0/\sqrt{2} \) is the top quark mass, \( v_0 = 246 \) GeV. As we can see from Eq.[9], the strength of the phase transition can be enhanced by the right- or left-handed stop field contribution if \( m_L \) or \( m_R \) is close to zero. A scenario with \( m_R \ll m_L \) is naturally implemented even if at GUT scale \( m_R = m_L \) (universality of the soft masses)[1]. This effect is due to the 3:2:1 hierarchy in the renormalisation group equations for the Higgs scalar \( h_2 \), right-handed and left-handed squared masses respectively[7]. Finally, assuming \( m_R \ll m_L \) and \( m_t \ll m_L \) and keeping only the relevant terms in the effective potential, we obtain
\[
V(h,T) = M^2(T) h^2 - \delta(T) h^3 - a(T)(h^2 + b^2)^{3/2} + \lambda(T) h^4 ,
\]  
(20)
where
\[
M^2(T) = -\frac{1}{4} m_Z^2 \cos^2 2\beta - \frac{3}{16\pi^2} \frac{m_t^2}{v_0^2} \left\{ m_t^2 \ln \left[ \left( 1 + \frac{m_t^2}{m_T^2} \right) \left( 1 + \frac{m_R^2}{m_T^2} \right) \right] \right.
\]  
\[ + \left. m_L^2 \ln \left( 1 + \frac{m_L^2}{m_T^2} \right) + m_R^2 \ln (m_t^2 + m_R^2) + \frac{1}{2} m_R^2 \right\}
\]  
\[ + \frac{m_t^2}{2 v_0^2} T^2 + \frac{3}{16\pi^2} \frac{m_t^2}{v_0^2} m_R^2 (2 \ln T + C_B) ,
\]  
(21)
\[
\delta(T) = \frac{2 m_W^2}{6\pi v_0^2} + \frac{m_z^2}{y_0} T ,
\]  
(22)
\[
a(T) = \frac{m_t^3 T}{2\pi v_0^3} , \quad b = \frac{m_R v_0}{m_t} ,
\]  
(23)
\[
\lambda(T) = \frac{1}{8} \frac{m_Z^2}{v_0^2} \cos^2 2\beta - \frac{3}{16\pi^2} \frac{m_t^4}{v_0^2} \left( \ln \frac{T}{m_L} - C_F - \frac{1}{2} C_B - \frac{3}{4} \right) ,
\]  
(24)
\( C_B \) and \( C_F \) are constants coming from the high temperature expansion[3] (\( C_B = 5.41 \) and \( C_F = -2.64 \)). The form of the potential is simple and can be analytically studied
as done in [8]. As said before, the phase transition occurs at a temperature $T_c$ between $T_0$ and $T_1$ and these relations remain valid for the $\frac{v(T)}{T}$:

$$\frac{v(T_1)}{T_1} \lesssim \frac{v(T_c)}{T_c} \lesssim \frac{v(T_0)}{T_0}.$$  \hspace{1cm} (25)

To conclude, the $\frac{v(T)}{T}$ ratios are plotted as a function of tan$\beta$ for $m_{\tilde{t}_R} = m_t$ and $m_L = 500$ GeV in Fig.1. We can see that from the point of view of the baryogenesis the favourite value for tan$\beta$ is between 0.5 and 1.5. The maximum value for the ratio is reached for tan$\beta$ = 1. This corresponds to the lower value for the Higgs mass ($\approx 60$ GeV) which is consistent with the present experimental data on MSSM [14].

Under the assumption that the right-handed stop mass is close to or smaller than the top mass, we have shown that the baryogenesis constraint (Eq. [15]) can be satisfied for low value of tan$\beta$ in the MSSM. In [15], similar results were obtained using a numerical analysis of the effective potential. A 2-loop numerical analysis of this potential [16] and lattice calculations [17] confirm the enhancement of the first order phase transition in the range of the MSSM parameters studied in this letter.

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Figure 1: The curves $v(T_0)/T_0$ (solid line) and $v(T_1)/T_1$ (bold-dashed line) as functions of $\tan \beta$ for $m_R = 0$, $m_L = 500$ GeV and $m_t = 174$ GeV.