An ostentatious model of cosmological scalar-tensor theory

Nahomi Kan

National Institute of Technology, Gifu College, Motosu-shi, Gifu 501-0495, Japan

Kiyoshi Shiraishi

Graduate School of Sciences and Technology for Innovation,
Yamaguchi University, Yamaguchi-shi, Yamaguchi 753-8512, Japan

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We consider a novel model of gravity with a scalar field described by the Lagrangian with higher order derivative terms in a cosmological context. The model has the same solution for the homogeneous and isotropic universe as in the model with the Einstein gravity, notwithstanding the additional higher order terms. A possible modification scenario is briefly discussed lastly.

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* kan@gifu-nct.ac.jp
† shiraish@yamaguchi-u.ac.jp
I. INTRODUCTION

Basics of modern cosmology was born almost at the time when Einstein discovered his theory of gravitation, called the general theory of relativity, about one hundred years ago. The most outstanding fact is that general relativity has passed repetitive tests with high degrees of accuracy until very recently. Nevertheless, modification of general relativity or modified theory of gravity is eagerly discussed by many authors [1–9], who are willing naturally to explain the acceleration of our universe in the present epoch [10, 11] and in the very early universe [12]. To achieve the accelerating behavior, scalar or other degrees of freedom are incorporated into the sector of Lagrangian that describes gravitation.

Another motivation of exploring alternative theories of gravity is found in the theoretical pursuit of a possible consistent quantum field theory of gravity. Because the Planck mass has physical dimension, general relativity cannot be renormalisable straightforwardly as a quantum field theory. At least, inclusion of higher order terms of curvature tensors in the Lagrangian is needed to control the UV behavior of the theory [13].

The recent observation of gravitational waves from a neutron star merger restricts the difference in the velocity of the gravitational wave and the light speed [14–17] (see also earlier discussions [18–20]). Since this experimental limit is very severe, we accept giving up many modified models of gravity, or very unnatural fine tuning in the theories.

More recently, Motohashi and Minamitsuji [21] proposed a possible form of the Lagrangian for modified gravity, which leads to the exact general relativistic solutions in a certain limit. In such models, the gravitational wave propagates with the light speed.

Now, let us consider a cosmological models with a neutral scalar field. Suppose the following action of Einstein–scalar system:

\[ I_0 = \int d^4x \sqrt{-gL_0} = \int d^4x \sqrt{-g} \left[ R - \sigma \frac{1}{2} (\nabla \phi)^2 - V(\phi) \right], \quad (1.1) \]

where \( R \) is the Ricci scalar, \( (\nabla \phi)^2 = g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi \), and \( V(\phi) \) is the potential of the scalar field \( \phi \). If the constant \( \sigma \) takes one, the kinetic term is a canonical one and on the other hand, if \( \sigma = -1 \), the scalar is called phantom [22]. By taking the variation with respect to the metric, we find the Einstein equation

\[ T_{\mu\nu} = 0 \quad (1.2) \]
with

\[ T_{\mu\nu} \equiv \frac{1}{\sqrt{-g}} \frac{\delta I_0}{\delta g^{\mu\nu}} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - \sigma \frac{1}{2} \nabla_\mu \phi \nabla_\nu \phi + \frac{1}{4} (\nabla \phi)^2 g_{\mu\nu} + \frac{1}{2} V(\phi) g_{\mu\nu} \]

\[ \equiv \tau_{\mu\nu} - \frac{1}{2} \tau g_{\mu\nu} + \frac{1}{2} V(\phi) g_{\mu\nu}, \]

(1.3)

where \( R_{\mu\nu} \) is the Ricci tensor, \( \tau_{\mu\nu} \equiv R_{\mu\nu} - \sigma \frac{1}{2} \nabla_\mu \phi \nabla_\nu \phi \), and \( \tau \equiv \tau^\mu_\mu \). Now, we consider a new Lagrangian

\[ L = L_0 + F(T_{\mu\nu}), \]

(1.4)
yields the equation of motion which is satisfied with \( T_{\mu\nu} = 0 \), provided that the function \( F \) is at least the second order or higher in \( T_{\mu\nu} \). The existence of the general relativistic solution means that the existence of a massless graviton moving with the speed of light.

The Lagrangian of this simple model has apparently higher order terms in curvature tensors, i.e., the model describes a higher-derivative theory. A generic higher-derivative gravity contains ghost-like massive modes as a perturbative field theory [13, 23–30]. Although this fact may be a fault of the model, discussions have been made to unravel a mystery in astrophysics, such as the problem with galactic rotation curves, by considering Yukawa-like potential of the form \( e^{-mr}/r \) associated with the massive modes [31]. In addition, much theoretical interest has been found in study of higher order gravities in diverse branches [32–48].

Modern theoretical cosmologists also hate a related ghost, so-called Ostrogradsky ghost [49, 50], which appears in the canonical equation of motion of the scale factor and homogeneous fields in cosmology. Even if we restrict ourselves to considering isotropic and homogeneous cosmological models (or minisuperspace), the Lagrangian that contains the higher order terms in the the second time derivatives yields the multiple degrees of freedom, which behave as ghosts. In the simplest way to avoid the Ostrogradsky ghost in the metric theory of gravity, we have to choose the combination of curvature tensors in the Lagrangian to contain only linear order of the second time derivative of the scale factor of the universe. Such an elaborated choice has been already known for the FLRW cosmology [51, 52] and has been studied by the present authors [53–55]. Thus, in this Letter, we study the model of higher order scalar-tensor theory, which has the general relativistic solution in minisuperspace governed by the field equation which includes at most the second time derivative of the scale factor and the scalar field.
We are now standing at an intermediate position between modern theoretical cosmologists and high energy theorists. We think that the ghost-like fluctuations around the classical homogeneous background fields would yield some physical implication if we can treat them correctly. The modification of the theory will be discussed later in this Letter.

II. ACTION WITH HIGHER ORDER CURVATURE TERMS

We shall begin with introducing the Meissner–Olechowski gravity in four dimensional spacetime. The \( n \)-th order Meissner–Olechowski density is defined here by using the Schouten tensor and the generalized Kronecker delta as \([51–55]\)

\[
L^{(n)}_{MO} = -\delta_{\nu_1\cdots\nu_n}^{\mu_1\cdots\mu_n} S_{R\mu_1} \cdots S_{R\mu_n} \equiv -[S_R \cdots S_R],
\]

where the symbol \( S_{R\mu} \) denotes the Schouten tensor (in our definition, which is different from the original definition by a factor \( 1/2 \) in four dimensions), which is

\[
S_{\mu\nu} R_{\mu\nu} = R_{\mu\nu} - \frac{1}{6} R g_{\mu\nu}
\]
in four dimensions, and the generalized Kronecker delta \( \delta_{\nu_1\nu_2\cdots\nu_p}^{\mu_1\mu_2\cdots\mu_p} \) is defined as

\[
\delta_{\nu_1\nu_2\cdots\nu_p}^{\mu_1\mu_2\cdots\mu_p} \equiv \left| \begin{array}{ccc}
\delta_{\nu_1}^{\mu_1} & \delta_{\nu_2}^{\mu_1} & \cdots & \delta_{\nu_p}^{\mu_1} \\
\delta_{\nu_1}^{\mu_2} & \delta_{\nu_2}^{\mu_2} & \cdots & \delta_{\nu_p}^{\mu_2} \\
\vdots & \vdots & \ddots & \vdots \\
\delta_{\nu_1}^{\mu_p} & \delta_{\nu_2}^{\mu_p} & \cdots & \delta_{\nu_p}^{\mu_p}
\end{array} \right| (p = 2, 3, 4).
\]

The Lagrangian density that consists of a linear combination of the Meissner–Olechowski density includes at most linear order terms in the second derivative of the metric tensor in the FLRW universe \([51–55]\). Note that \( L^{(2)}_{MO} = R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2 \) in four dimensions. The theory of Meissner–Olechowski gravity perturbatively describes a ghost-like massive spin-2 mode in addition to a massless spin-2 graviton and contains no scalar mode.

For the present purpose, we can consider the terms \([SS], [SSS]\) and \([SSSS]\) in addition to \( L_0 \) in the Lagrangian, where

\[
S_{\mu\nu} \equiv T_{\mu\nu} - \frac{1}{3} T g_{\mu\nu},
\]
in four dimensions. These candidate terms, however, yields \( O(h_{\mu\nu}^2), O(h_{\mu\nu}^3) \) and \( O(h_{\mu\nu}^4) \) contributions in terms of the metric fluctuation \( h_{\mu\nu} = g_{\mu\nu} - \bar{g}_{\mu\nu} \), if the background metric \( \bar{g}_{\mu\nu} \) satisfies the Einstein equation \( T_{\mu\nu} = 0 \). Therefore, we consider here the Lagrangian

\[
L = \alpha L_0 + \beta [SS] + \gamma [S\tau SS] + \delta [S\tau S\tau SS], \quad (\alpha, \beta, \gamma \text{ and } \delta \text{ are constants})
\]
with $S_{\tau \mu \nu} \equiv \tau_{\mu \nu} - \frac{1}{6} \tau g_{\mu \nu}$, which has the $O(h_{\mu \nu}^2)$ contribution in each term and reduces to the Meissner–Olechowski gravity in the limit of vanishing $\phi$ and $V(\phi)$. Because the explicit form of the full Lagrangian in terms of the individual field content is lengthy, we only exhibit $\alpha L_0 + \beta [SS]$ below:

$$L = \alpha \left[ R - \sigma \frac{1}{2} (\nabla \phi)^2 - V(\phi) \right] - \beta \left[ \left( R_{\mu \nu} - \sigma \frac{1}{2} \nabla_{\mu} \phi \nabla_{\nu} \phi \right)^2 - \frac{1}{3} \left( R - \sigma \frac{1}{2} (\nabla \phi)^2 \right)^2 
+ \frac{1}{3} V(\phi) \left( R - \sigma \frac{1}{2} (\nabla \phi)^2 \right) - \frac{1}{3} V^2(\phi) \right]. \quad (2.5)$$

We find that the expansion of the Lagrangian (2.4) around the background metric $\bar{g}_{\mu \nu}$ satisfying the field equation $T_{\mu \nu} = 0$ yields

$$\sqrt{-g} L = \sqrt{-\bar{g}} \left[ \alpha \frac{1}{4} h^{\mu \nu} \nabla^2 h_{\mu \nu} - \left\{ \beta + \frac{\gamma V(\bar{\phi})}{3} + \frac{\delta V^2(\bar{\phi})}{18} \right\} \frac{1}{4} h^{\mu \nu} \nabla^2 \nabla^2 h_{\mu \nu} + \cdots \right], \quad (2.6)$$

where $\bar{\phi}$ is the classical homogeneous solution of the general relativistic field equation and $\nabla^2$ denotes the covariant d’Alembertian in terms of the background metric $\bar{g}_{\mu \nu}$. Thus, the present model theory described the Lagrangian (2.4) has the ghost-like massive spin-2 mode of mass squared $m^2(\bar{\phi}) = \alpha (\beta + \gamma V(\bar{\phi})/3 + \delta V^2(\bar{\phi})/18)^{-1}$, if the time evolution of the scalar field is sufficiently slow. This mode causes Yukawa-type static potential $\sim e^{-m(\bar{\phi})r} / r$ [23]. This field-dependent potential is interesting from an astrophysical perspective, which can be concerned with the galactic length scale, as an extension of the discussion in [31].

So far, we have constructed an ostentatious model of higher order scalar-tensor gravity, which has the general relativistic solution. This model has a massless graviton as well as a ghost-like massive spin-2 mode, which may affect some astrophysical phenomena.

In the first place, however, can we find the modification or new perspective in cosmological evolution? How can we tell our model from general relativity in cosmology at a large scale? We will serve an answer later, and we first illustrate a possible moderate modification of our present model.

### III. A NOT-SO-OSTENTATIOUS COSMOLOGICAL MODEL OF GRAVITY

We start with the simplest Lagrangian $L = \alpha L_0 + \beta [SS]$, i.e., the case $\gamma = \delta = 0$. One can seen that $L$ includes the pure higher order term of curvature tensors $R_{\mu \nu}^\rho R^\rho_{\mu \nu} - \frac{1}{3} R^2$. As far as we consider the FLRW cosmological model, this term has no effect on the equation
for the scale factor, because the FLRW metric is conformally flat [56] and the term is just the Weyl tensor squared modulo the Euler density in four dimensions. Thus, we consider a modified model described by the Lagrangian density $L' = L + \beta \left( R^\mu_\nu R^\nu_\mu - \frac{1}{3} R^2 \right)$, which still leads to the same FLRW solution as in the Einstein gravity described by $I_0$, aside from the frame dependence. More explicitly, the modified Lagrangian can be written as

$$L' = \alpha \left[ R - \sigma \frac{1}{2} (\nabla \phi)^2 - V(\phi) \right] + \beta' \left[ -\frac{1}{6} V(\phi)(\nabla \phi)^2 + \sigma^2 \frac{1}{6} [(\nabla \phi)^2]^2 - \frac{1}{3} V^2(\phi) \\
+ \frac{1}{3} (V(\phi) + \sigma (\nabla \phi)^2) R - \sigma R^\mu_\nu \nabla_\mu \phi \nabla_\nu \phi \right],$$

(3.1)

where we use $\beta' \equiv -\beta$ for convenience. Note that if $\beta' \geq 0$, the total scalar potential $V + \beta' V^2/3$ can be positive definite. Incidentally, this model can be regarded as an extension of the $f(R, T, R^\mu_\nu T^\nu_\mu)$ gravity [57, 58]. The model Lagrangian (3.1) does not yield the spin-2 ghost, because the higher derivative terms for the metric has been discarded. The massless mode, however, does not propagate with the speed of light in this model described by $L'$. To see this, we follow the method used in Refs. [59–61]. Thus, we assume the line element with the perturbation $\Phi$:

$$ds^2 = -dt^2 + a^2(t)(1 + 2\Phi)dx^2.$$

(3.2)

Then, the action can be read as

$$\int d^4x \sqrt{-g} L' = \int d^4xa^3 \left[ w_4 \left( \partial_i \Phi \right)^2 - 3w_1 \Phi^2 + \cdots \right],$$

(3.3)

with

$$w_4 = 2 \left( \alpha + \frac{\beta'}{3} V(\bar{\phi}) \right) - \frac{2}{3} \sigma \beta' \dot{\bar{\phi}}^2, \quad w_1 = 2 \left( \alpha + \frac{\beta'}{3} V(\bar{\phi}) \right) + \frac{4}{3} \sigma \beta' \dot{\bar{\phi}}^2,$$

(3.4)

where the dot denotes the time derivative. The speed of propagation of gravitational wave $c_{GW}$ is then given by [59–61]

$$c_{GW}^2 = \frac{w_4}{w_1} = 1 - \frac{2\sigma \beta' \dot{\bar{\phi}}^2}{w_1} \phi.$$

(3.5)

The stringent bound from the observation of GW170817 $|c_{GW}^2 - 1| < 10^{-15}$ [14] implies very strict fine tuning in this model parameters.

Note that further removing the non-minimal terms made of the product of the curvature tensors and the scalar field and its derivatives makes the model very similar to the energy-momentum-squared gravity [62–65]. Unfortunately, the renovated model obtained by the further removal of the terms cannot have the same general relativistic solution as previously.
Note also that the present model can be considered as a model of induced gravity [66–68], if we consider the limit \( \alpha \to 0 \); nevertheless, a classical FLRW solution is given by the solution of the equation of motion derived from \( I_0 \). In an academic and theoretical perspective, this is an interesting possibility and is worth studying further.

Next, we will turn to consider the modified scenario of the higher order model including higher curvature terms, after summarizing discussions.

### IV. SUMMARY AND DISCUSSION – A POSSIBLE “MODIFICATION” OF THE MODEL

We have proposed a seemingly complicate and ostentatious model of higher order scalar-tensor theory with general covariance, whose solution is given by that of the simple Einstein gravity. The model generally has massive spin-2 modes in addition to a massless graviton mode. Although, in the simplest case, removing the massive mode is possible by discarding pure curvature polynomials, this omission bring about the change in the speed of the massless graviton mode.

The conformally non-flat metric, especially the metric around compact objects is the object concerned with future study in the present higher order model. The extension of our model can be studied by incorporating the ideas of using \( F(T_{\mu\nu}, \phi, \nabla^\mu \phi, \ldots) \), induced gravity, quantum cosmology, higher-dimensional extension, and supersymmetry, etc.

Now, we will go back to the first place, and ask ourselves: where is the “modified” gravity in our higher derivative model, in which the same solution holds as in the Einstein gravity? We propose a scenario of “ghost condensation” here. Let us consider the simplest model with a massive spin-2 mode, which is governed by the Lagrangian

\[
L = \alpha L_0 - \beta \left( T^{\mu\nu} T_{\mu\nu} - \frac{1}{3} T^2 \right).
\]

This Lagrangian can be replaced by an equivalent Lagrangian, using an auxiliary field \( \tilde{S}_{\mu\nu} \) as [54, 55, 70–72]

\[
\tilde{L} = \alpha L_0 - \beta (2T_{\mu\nu} \tilde{S}^{\mu\nu} - \tilde{S}_{\mu\nu} \tilde{S}^{\mu\nu} + \tilde{S}^2),
\]

where \( \tilde{S} \equiv \tilde{S}_{\mu}^{\mu} \). Note that the Fierz–Pauli equation [73] for \( \tilde{S}_{\mu}^{\nu} \) with mass squared \( \alpha/\beta \) can be derived from \( \tilde{L} \) for a small fluctuation around \( \tilde{S}_{\mu}^{\nu} = 0 \) [55].
We conjecture that the ghost-like tensor mode condensate, most simply as $\langle \tilde{S}_\mu^\nu \rangle = \Lambda \delta_\mu^\nu$.

Then, the Lagrangian $\tilde{L}$ becomes

$$\tilde{L} = (\alpha + 2\beta \Lambda) \left( R - \frac{1}{2} \sigma (\nabla \phi)^2 \right) - (\alpha + 4\beta \Lambda) V(\phi) - \beta \Lambda^2,$$

which apparently contains the cosmological term. Therefore, the condensation of $\tilde{S}_\mu^\nu$ brings about the modification of the whole theory. We should add a comment, however. For a positive cosmological constant, $\beta$ should be positive. To obtain a “natural” condensation, the mass squared $\alpha/\beta$ around $\tilde{S}_\mu^\nu = 0$ might be negative for obtaining $\langle \tilde{S}_\mu^\nu \rangle \neq 0$. Thus, detailed investigation for the mechanism of the ghost condensation are needed, with incorporating higher order terms and/or quantum dynamics of the scalar and tensor fields in the flat or the de Sitter spacetimes. At the same opportunity, the possibility of the field-dependent condensate, such as $\Lambda(\tilde{\phi})$, could be found by further study.

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