Elastic turbulence in two-dimensional cross-slot viscoelastic flows

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We report evidence of irregular unsteady flow of two-dimensional polymer solutions in the absence of inertia in cross-slot geometry using numerical simulations of Oldroyd-B model. By exploring the transition to time-dependent flow versus both the fluid elasticity and the polymer concentration, we find periodic behaviour close to the instability threshold and more complex flows at larger elasticity, in agreement with experimental findings. For high enough elasticity we obtain dynamics pointing to elastic turbulence, with temporal spectra of velocity fluctuations showing a power-law decay, of exponent in between $-3$ and $-2$, and probability density functions of velocity fluctuations that weakly deviate from Gaussianity while high non-Gaussian tails characterise those of local accelerations.

INTRODUCTION

The rheological behaviour of viscoelastic flows at vanishingly small inertia can be related to strongly non-linear phenomena and includes an association of viscous and elastic effects, with the latter being typically due to the presence of flexible long-chain polymers in the solution. The elasticity of the flow can give rise to complex dynamics that are relevant for both fundamental studies and industrial applications, as e.g. efficient mixing and heat transfer in microdevices [1], or painting and coating processes [2–4].

The purely elastic instabilities marking the transitions between different flow regimes have been documented in a variety of geometrical configurations [3, 5–7], including complex ones, such as the abrupt axisymmetric contraction [8] and the lid-driven cavity [9]. The cross-slot setup, made of two perpendicularly intersecting channels with two inlets and two outlets, is, in this sense, no exception. Due to its relevance for mixing and rheology, it has been the subject of extensive studies. Indeed, experimental [10–12], theoretical [13, 14] and numerical [15–17] investigations have reported about the existence of instabilities solely driven by elasticity in this setup. It is now known that low-Reynolds-number polymeric flows in this geometry can display two types of instabilities: a first one, corresponding to a supercritical pitchfork bifurcation, to steady asymmetric flow [16, 17], and a second one leading to unsteady oscillatory behaviour [11, 16, 18]. Concerning the latter, it is interesting to recall that it has been provided numerical evidence, in a two-dimensional (2D) flow, that it occurs via a supercritical Hopf bifurcation [19]; a mechanism relying on the role of stress gradients and the existence of a stagnation point at the centre of the setup was also proposed [19].

Above a critical Weissenberg number ($\text{Wi}$), meaning for elasticity larger than a threshold, purely elastic instabilities can lead to the appearance of disordered flows corresponding to the dynamical regime known as elastic turbulence [6, 20]. As shown in the seminal work [6], where a swirling flow between counter-rotating parallel disks was considered, and in subsequent ones also employing different geometries [21, 22], such flows are reminiscent of the turbulent ones occurring in Newtonian fluids. In particular, they are characterised by a whole range of active scales, irregular temporal behaviour, growth of flow resistance and enhanced mixing properties [21]. Interestingly, however, the spectrum of velocity fluctuations displays power-law behaviours, in both the temporal ($E(f) \sim f^{-\delta}$) and spatial ($E(k) \sim k^{-\delta}$) domains, with an exponent (in absolute value) $\delta \approx 3.5 > 3$, corresponding to a smooth flow essentially dominated by the largest spatial scales. It is worth to remark that such experimental findings are supported by theoretical predictions based on a simplified uniaxial model of viscoelastic fluid dynamics in the absence of walls and in homogeneous isotropic conditions [23]. At the same time, it was recently pointed out in [24] that numerical simulations based on standard constitutive models may be dramatically affected by the polymer-stress diffusivity typically added to the evolution equations to ensure numerical stability, and that this particularly applies to flows characterised by regions of pure strain. Notably, using a cellular forcing in two dimensions, it was shown that kinetic energy spectra are considerably flatter in the absence of artificial polymeric diffusion and scale as $k^{-2.5}$ [24].

The elasticity-driven transition to turbulent-like states was experimentally investigated in cross-slot devices of different aspect ratio (vertical size over channel width), for more and less concentrated polymer solutions [25]. Independently of the aspect ratio, it was found that the more concentrated solution undergoes a transition to unsteady flows that become progressively more irregular when the Weissenberg number is increased. The power spectra of velocity fluctuations, obtained from single-point time series of the streamwise component measured in the outlet channel at midway from the lateral walls (both in the horizontal and vertical directions), were characterised by the presence of marked peaks (a fundamental frequency plus some harmonics), and by a power-law behaviour of exponent smaller than $-3$, at small and large $\text{Wi}$ values, respectively. In particular, for the smaller aspect ratio, continuous spectra and features typical of elastic turbulence were observed when $\text{Wi} \gtrsim 25$. For the more dilute solution, although the phenomenology of the transitions was similar, the chaotic flow observed at high $\text{Wi}$ did not show similar spectral proper-
ties.

In this letter we explore the unsteady viscoelastic flow regime occurring in a 2D cross-slot geometry at high elasticities and vanishing Reynolds number ($Re$) by means of extensive numerical simulations, for different polymer concentrations. For this purpose we adopt Oldroyd-B model, i.e. the simplest possible one, to describe the dynamics of the viscoelastic fluid. As in [26], where elastic turbulence was simulated in a 2D Taylor-Couette system, we integrate the model evolution equations using the open-source code OpenFOAM® [27, 28], which allows control of the numerical instabilities associated with large Weissenberg numbers [29]. We provide numerical evidence of the emergence of turbulent-like features for quite concentrated solutions when $Wi$ is large enough. We analyse the transition to irregular dynamics and we characterise the statistical properties of the high-$Wi$ flows, discussing the similarities and differences with experimental results.

**MODEL AND METHODS**

We consider an isothermal, incompressible, inertialess, 2D viscoelastic fluid flow in a cross-slot geometry. The latter consists of two perpendicular and bisecting channels of identical width $d$, with opposing inlets (here, along the $x$ direction) and outlets (along the $y$ direction), as shown schematically in fig. 1. The velocity field $u(x, t) = (u_x(x, t), u_y(x, t))$ at position $x$ and time $t$ evolves according to the momentum conservation equation

$$
\rho \left[ \frac{\partial u}{\partial t} + (u \cdot \nabla) u \right] = \nabla \cdot T - \nabla p \quad (1)
$$

and the incompressibility condition $\nabla \cdot u = 0$, where $T$ is the total (viscous plus elastic) stress tensor, $p$ the pressure and $\rho$ the density.

In the framework of Oldroyd-B model [30, 31], the stress tensor $T$ is the sum of a viscous component $\sigma = \eta_s \dot{\gamma}$, with $\eta_s$ the zero-shear dynamic viscosity of the solvent and $\dot{\gamma} = \nabla u + (\nabla u)^T$ the strain-rate tensor, and an elastic one $\tau$ due to polymers. The constitutive equation for the extra-stress tensor $\tau$ reads:

$$
\tau + \lambda \left[ \frac{\partial \tau}{\partial t} + \nabla \cdot (u \tau) - (\nabla u)^T \cdot \tau - \tau \cdot \nabla u \right] = \eta_p \dot{\gamma},
$$

(2)

where $\lambda$ represents the largest polymer relaxation time and $\eta_p$ the polymer contribution to viscosity. An important parameter is the viscosity ratio $\beta = \eta_s / (\eta_s + \eta_p)$, which is inversely proportional to the polymer concentration. Let us remark that in the limit $\beta \to 0$, one recovers the upper-convected Maxwell (UCM) model [31], accounting for the dynamics of very concentrated solutions. At fixed $\beta$, the control parameters of the dynamics specified by eqs. (1) and (2) are the Reynolds number $Re = \rho U_b d / (\eta_s + \eta_p)$ and Weissenberg $Wi = \lambda U_b / d$ numbers, where $U_b$ is the (uniform) velocity at the inlet.

In spite of important limitations, such as the infinite extensibility of polymers - and the consequent unbounded nature of extensional viscosity at strain rates $\geq 1 / (2\lambda)$ - or the absence of shear-dependent viscosity, Oldroyd-B model corresponds to the simplest differential constitutive equation for viscoelastic fluids, and it exhibits normal stress differences. Furthermore, it has been successfully employed to numerically reproduce the basic phenomenology of elastic turbulence in different 2D configurations [7, 26, 32].

**Numerical simulations**

Equations (1) and (2) are integrated by means of the open-source numerical solver RHEOTOOL®, which was developed in the framework of the OpenFOAM® simulation code [28]. This solver is based on a finite-volume discretisation and makes use of the log-conformation technique [33] to control the numerical instabilities appearing at high $Wi$ values. We remark that no polymer-stress diffusion is included.

The cross-slot configuration has recently been proposed as a benchmark problem [34], for its geometrical characteristics and the existence of the instability leading to asymmetric flow at appropriate $\beta$, $Re$ and $Wi$ values. Similarly to the reference studies with this setup, here we set a length to width ratio of $10 : 1$ for each of the four “arms”, which was previously shown to be enough to ensure a fully developed flow away from the inlet in a channel [35]. The global mesh adopted for the
The complete stability portrait, obtained by spanning the \((\beta, \text{Wi})\) plane with a large number of simulations, is shown in fig. 3, where the different point types correspond to the different dynamical regimes observed; here we only show a limited subset of the results from the simulations performed. By measuring the amplitude and frequency of the time series of \(u(x^{(2)}_t, t)\) at the fixed location \(x^{(2)}_t\) (corresponding to point 2 in fig. 1) for Wi close to the onset of the unsteady regime and for different concentration values, we could assess that the second instability is a supercritical Hopf bifurcation (see inset of fig. 3 for \(\beta = 1/9\)), as also suggested by [19] using a
FENE-P model at non-zero $Re$ and large $\beta$. Indeed, the velocity signal displays a growth of its amplitude that is fairly well described by $(Wi - W_i^{(II)})^{1/2}$, with $W_i^{(II)}$ the critical Weissenberg number, and an approximately linear decrease of its frequency with $Wi$. For both the first and the second instability, the critical Weissenberg number, $W_i^{(I)}$ and $W_i^{(II)}$, respectively, grows with growing $\beta$, which is reasonable since increasing $\beta$ corresponds to decreasing polymer concentration. The faster growth of $W_i^{(I)}(\beta)$ causes the shrinking of the region of steady asymmetric flow. Determining the functional dependencies $W_i^{(i)}(\beta)$ (with $i = I, II$) from stability analysis is not an easy task, due to the formation of a birefringent strand and a diverging base state associated with the infinite extensibility of polymers [14]. Since here we are mainly interested in characterising the boundaries, in the $(\beta, Wi)$ plane, of the regions where elastic turbulence could be excited, we proceed heuristically, especially focusing on $W_i^{(II)}(\beta)$. In order to account for non-zero $\beta$ effects, we conjecture that $W_i^{(II)}(\beta) = W_i^{(II)}(0)f(\beta)$, where $f(\beta)$ is a positive analytic function, except for $\beta \rightarrow 1$ where a divergence is expected, since the fluid becomes Newtonian and no purely elastic instability should occur; clearly $f(0) = 1$. Our numerical results suggest that the data are compatible with a Laurent expansion at second order around the point $\beta = 1$. Somehow more surprisingly, we find that the same functional shape can also be used to fit the $W_i^{(I)}(\beta)$ data, indicating that:

$$W_i^{(I)} = W_i^{(I)}(0) \left[ a_0^{(i)} + \frac{a_1^{(i)}}{1 - \beta} + \frac{a_2^{(i)}}{(1 - \beta)^2} \right],$$

where $a^{(i)}_{-2} = 1 - a_0^{(i)} - a_{-1}^{(i)}$ using the constraint $f(0) = 1$, and $i = I, II$. In fig. 3 we report a comparison between a fit with function (3) (dashed and continuous lines for $i = I, II$, respectively) and the numerical data; the agreement is rather good for both instability types, confirming our conjecture.

To conclude this discussion, we mention that in our calculations with a more refined grid or at $Re = 0.1$ (see the previous section for the details about simulations) we did not observe any qualitative difference in the dynamical regimes occurring for different values of $\beta$ and $Wi$.

We now consider the transition to turbulent-like flow. In the following we will present the results of the analysis performed for increasing $Wi$ at $\beta = 1/9$. Notwithstanding some quantitative differences, the phenomenology holds similar in the whole range ($\beta \lesssim 0.56$) of concentrated solutions, including for UCM ($\beta = 0$). In the case of more diluted solutions, while we observed some hints of the onset of irregular flow, we could not reach a fully developed regime and we cannot conclude about the emergence of elastic turbulence. Notice that for such large values of $\beta$, the critical Weissenberg number $W_i^{(II)}$ grows very rapidly, making the simulations more and more delicate.

Our analysis is based on the measurement of time series of the velocity components at two different positions marked as probe 1 ($x^{(1)}_*$, entrance) and probe 2 ($x^{(2)}_*$, exit) (see fig. 1), over long durations corresponding to at least $800\lambda$, and up to $1000\lambda$. As for the experiments reported in [25], we choose to focus on the axial component $u_y(x^{(2)}_*, t)$ at the exit probe, whose behaviour is presented in fig. 4 for several values of $Wi$. Remark that in this figure the initial transient was removed and only a subset of the data record is shown.

The spectra of $u_y(x^{(2)}_*, t)$ are shown in fig. 5. All those corresponding to the developed regime are averages over ten spectra computed from consecutive subintervals of
the velocity time series obtained for a given value of $Wi$ (after the transient). For $Wi \geq W_i (II)$, time dependency manifests in the form of regular oscillations with a single frequency close to $0.4/\lambda$ (see inset of fig. 5). At slightly higher Weissenberg number ($Wi = 3$ in fig. 4) the flow is still periodic but it is now characterised by more discrete frequencies; correspondingly, the spectrum shows several distinct peaks associated with a fundamental frequency and some harmonics (inset of fig. 5). The occurrence of a transitional periodic regime was also found in different setups [36, 37]. Above $Wi \approx 5$, the flow loses periodicity and the velocity spectra become continuous. Indeed, starting from $5 \lesssim Wi \lesssim 10$ they result to be quite well described by a power-law function (fig. 5). When elasticity is increased in the range $Wi > 10$, the faster fluctuating behaviour of the flow is accompanied by quite wide and irregular oscillations, over longer durations. The flow now loses its spatial asymmetry to alternatively select the outlet in the positive/negative $y$-direction. Such a phenomenon has a strong impact on the statistics of the transversal velocity component $u_x (x^2, t)$ at the outlet (and similarly on $u_y (x^1, t)$ at the inlet), whose fluctuations are accompanied by irregular jumps between two mean values of opposite sign (see fig. 6), thus complicating their analysis. A detailed investigation of the behaviour of such a two-state system goes beyond the scope of the present work.

In the turbulent-like regime ($Wi > 5$), the spectrum of velocity fluctuations displays a power-law behaviour $E_y (f) \sim f^{-\delta}$ beyond a frequency that, as in experimental studies [25], slightly increases with $Wi$. The absolute value of the exponent is found to be in the range $2 \lesssim \delta \lesssim 3$ and shows some tendency to decrease at higher $Wi$; the latter feature is also detected in experiments [25, 38]. In particular, we find $\delta \simeq (2.8, 2.5, 2.2, 2.1) \pm 0.4$ for $Wi = 6, 12, 20, 25$, respectively. Inset: similar spectra at lower elasticity. For $Wi = 1.55 \lesssim W_i (II)$ a single frequency peak is found; at larger $Wi = 3$ more discrete frequencies are present.

To further characterise the statistical properties of

FIG. 5. (Colour online) Temporal spectra of fluctuations of the axial velocity at the outlet $u_y (x^2, t)$, normalised by their integral $E_y^{tot}$ in the elastic turbulence regime for $Re = 0$ and $\beta = 1/9$; the curves have been vertically shifted to ease readability. The dashed black curves stand for $E_y (f) \sim f^{-\delta}$, the fitted values of $\delta$ are $\delta \simeq (2.8, 2.5, 2.2, 2.1) \pm 0.4$ for $Wi = 6, 12, 20, 25$, respectively. Inset: similar spectra at lower elasticity. For $Wi = 1.55 \lesssim W_i (II)$ a single frequency peak is found; at larger $Wi = 3$ more discrete frequencies are present.

FIG. 6. (Colour online) Temporal evolution (subset of the total record) of $u_y (x^2, t)$, normalised by its time average over the whole time series, after the transient, for $Wi = 11, 12, 15, 20$ (from top to bottom), $Re = 0$ and $\beta = 1/9$. The occurrence of a transitional periodic regime was also found in different setups [36, 37]. Above $Wi \approx 5$, the flow loses periodicity and the velocity spectra become continuous. Indeed, starting from $5 \lesssim Wi \lesssim 10$ they result to be quite well described by a power-law function (fig. 5). When elasticity is increased in the range $Wi > 10$, the faster fluctuating behaviour of the flow is accompanied by quite wide and irregular oscillations, over longer durations. The flow now loses its spatial asymmetry to alternatively select the outlet in the positive/negative $y$-direction. Such a phenomenon has a strong impact on the statistics of the transversal velocity component $u_x (x^2, t)$ at the outlet (and similarly on $u_y (x^1, t)$ at the inlet), whose fluctuations are accompanied by irregular jumps between two mean values of opposite sign (see fig. 6), thus complicating their analysis. A detailed investigation of the behaviour of such a two-state system goes beyond the scope of the present work.

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As it can be seen in fig. 7b, the corresponding pdf’s display high tails that are indicative of non-Gaussian statistics, as is typical in turbulent flows and as observed in elastic turbulence experiments [42]. This finding highlights the intermittent behaviour of local accelerations, likely due to the passage through the system of transient intense filamentary structures (fig. 2c,d, but see also [37, 43, 44] about the role of elastic propagating wavy patterns).

**CONCLUSIONS**

We investigated numerically the dynamics of Oldroyd-B fluids in a 2D cross-slot geometry for broad ranges of the Weissenberg number and the polymer concentration, focusing on the possibility to obtain elastic turbulence. We detected two instabilities: the first one, present only for rather concentrated solutions (see also [12]), leads to steady asymmetric flow; the second one, less documented, manifests for all viscosity ratios $\beta < 1$ and corresponds to a supercritical Hopf bifurcation. By characterising the dependence of the critical Weissenberg number $Wi_{c}$ on the viscosity ratio, we found a heuristic expression that allows to quantitatively delimit the regions $Wi > Wi_{c}^{(11)}(\beta)$ where elastic turbulence may be excited.

Close to the onset of the second instability, the flow of quite concentrated solutions displays regular oscillations in time, while at larger elasticities its dynamics appear more irregular. The frequency spectra measured in one of the outlets and far from the walls show distinct peaks for $Wi \gtrsim Wi_{c}^{(11)}$, while for $Wi \gtrsim 5$ they are well described by continuous power-law functions, of exponent $-\delta$, pointing to elastic turbulence. As in experiments [25, 38], the scaling range occurs beyond a frequency that moderately increases with $Wi$, and $\delta$ decreases with $Wi$. However, we obtain values $2 < \delta < 3$, somehow smaller than the experimental ones and the theoretical prediction for the homogeneous isotropic case [23]. While we cannot exclude an impact of the 2D nature of our flow here, and we recall the influence of the inlets/outlets’ length on the results for $Wi \gtrsim 25$, we remark that the symmetries assumed in the theory clearly do not hold for our setup. Similarly energetic spectra have been recently found in simulations of 2D Oldroyd-B cellular flows without polymer-stress diffusion [24].

Further, the statistics of axial velocity components are found to be weakly non-Gaussian in the developed regime, while those of transversal ones also exhibit a bi-modal pdf for $10 < Wi < 20$ due to the alternations of the spatial flow asymmetry occurring in this range of $Wi$. The pdf’s of both components of the local accelerations, instead, present high non-Gaussian tails indicative of intermittency. Such a phenomenology agrees with that observed in experiments (see e.g. [42]).

In summary, we reproduced the different dynamical regimes experimentally observed in cross-slot devices, and we obtained turbulent-like states bearing good statistical resemblance with elastic turbulence. The quanti-

![Graph](image-url)
tative differences highlighted call for further theoretical and numerical developments. In the future it would be interesting to explore such dynamics in three-dimensional flows.

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