Topological quantum phase transition in the Ising-like antiferromagnetic spin chain BaCo$_2$V$_2$O$_8$

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Since the seminal ideas of Berezinskii, Kosterlitz and Thouless, topological quantum phase transitions have been at the heart of our understanding of a whole novel class of phase transitions. In most cases, those transitions are controlled by a single type of topological objects. There are, however, some situations, still poorly understood, where two dual topological excitations fight to control the phase diagram and the transition. Finding experimental realizations of such cases is thus of considerable interest. We show here that this situation occurs in BaCo$_2$V$_2$O$_8$, a spin-1/2 Ising-like quasi-one-dimensional antiferromagnet, when subjected to a uniform magnetic field transverse to the Ising axis. Using neutron scattering experiments, we measure a drastic modification of the quantum excitations beyond a critical value of the magnetic field. This quantum phase transition is identified, through a comparison with theoretical calculations, to be a transition between two different types of solitonic topological objects, which are captured by different components of the dynamical structure factor.

The pioneering work of Berezinskii, Kosterlitz and Thouless highlighted the role played by topological excitations in the two-dimensional classical XY model. Since then, the topological aspects have been found to be crucial not only to a host of two-dimensional classical systems, but also in a spectacular way in the one-dimensional quantum world, with, in particular, the remarkable case of spin-1 chains. Such concepts have allowed us to understand important aspects of the physics of materials such as the quantum Hall effect and even predict new classes of systems such as topological insulators. Identifying and understanding the topological aspects of matter has thus become a major focus in condensed-matter physics and quantum optics, where topological phases such as the Haldane model have been remarkably realized.

For classical and quantum critical phenomena, we have by now a good understanding of the prototypical topological phase transition in which only a single topological entity controls the transition. This was the case in the original work of Berezinskii, Kosterlitz and Thouless, where vortex–antivortex excitations deconfine in a similar way to electrical charges in the two-dimensional Coulomb gas. In the quantum world, this situation is described by the celebrated sine-Gordon model, which also plays a central role in quantum field theory. However, a richer and more difficult to understand class of topological transitions was also shown to play a major role for several systems. This situation arises when two conjugate fields, subjected to the Heisenberg uncertainty principle and plunged into different potentials, compete with each other. The phase diagram is thus controlled by the confinement/deconfinement of the corresponding dual topological excitations. These situations are considerably more difficult to analyse and need much more sophisticated field theory descriptions such as the so-called dual-field double sine-Gordon model. Thus, it is of considerable interest to find good experimental realizations of such cases, with the possibility to tune the system through the transition and in which the evolution of the excitations can be scrutinized.

Here we show that the Ising-like spin chain compound BaCo$_2$V$_2$O$_8$ yields a good realization of such a topological transition when subjected to a magnetic field transverse to the Ising axis (which points along the chains). This compound is characterized by unique features. First, it shows a strong Ising-like anisotropy. Next, because of the crystallographic peculiarities, applying a uniform field creates a staggered field perpendicular to both the Ising axis and the uniform field. This specificity allows dual topological excitations to be present. Those are solitonic excitations associated with the two angles needed to parametrize a spin and which in quantum mechanics are conjugate variables. Using a combination of neutron scattering experiments, numerical calculation of the microscopic description of the system, and a field theory analysis based on the double sine-Gordon model, we show that the transition observed in BaCo$_2$V$_2$O$_8$ at a certain critical value of the magnetic field corresponds to a quantum phase transition between two phases dominated by dual topological excitations: spinons along the chain direction (fractionalized excitations carrying a spin 1/2); their dual excitations (carrying a spin 1) along the axis of the staggered magnetic field.

BaCo$_2$V$_2$O$_8$ as a model system

The cobalt oxide BaCo$_2$V$_2$O$_8$ indeed offers the unique opportunity to study this physics. This material exhibits screw chains of Co$^{3+}$ rotating around the four-fold caxis. The magnetic moments of the Co$^{3+}$, in a distorted octahedral environment, are described as highly anisotropic effective spins $S = 1/2$ (see Fig. 1a). In BaCo$_2$V$_2$O$_8$, the presence of a small coupling between the spin chains...
Fig. 1 | BaCo$_2$V$_2$O$_8$ at zero field. a. The structure of a single Co$^{2+}$ screw chain of BaCo$_2$V$_2$O$_8$ (blue and red spheres are Co and O respectively) along the zero-field magnetic arrangement (blue arrows). The local anisotropy axis is plotted (red dashed line) for the bottom CoO$_6$ octahedron. Note that two kinds of chain parallel to c axis are equally present in BaCo$_2$V$_2$O$_8$, rotating in opposite directions with respect to the c axis. b. A schematic diagram of the two-spinon excitations described as domain walls in the pure Ising case. The transverse (T) and longitudinal (L) modes correspond to linear superpositions of states with odd (respectively even) N numbers of flipped spins (in red) between two spins (dotted purple lines), yielding a total spin $S_z$ = ±1 (respectively, $S_z$ = 0). In both cases, the lowest energy excitation consists mostly of the state with the minimal N value, with a smaller contribution for the higher N states in the linear superposition. Neutron scattering can detect only the $\Delta S = 0$, ±1 type of excitations, hence pairs of spinons, which individually carry a spin 1/2. c. Numerically calculated magnetic excitations of BaCo$_2$V$_2$O$_8$ in zero field, which show the Zeeman ladders (details in the last section of the main text). The colour scale is in arbitrary units.

leads to a long-range ordering below the critical temperature $T_N = 5.5$ K. The order consists of an intrachain antiferromagnetic (AF) arrangement of the magnetic moments pointing along the Ising caxis$^{18-20}$. Moreover, because of the original crystallographic structure of BaCo$_2$V$_2$O$_8$, the magnetization local easy axes of the Co$^{2+}$ ions are actually tilted away from the chain caxis by $\approx 5^\circ$ and rotate by 90$^\circ$ when moving along the four-fold axis (see Fig. 1a). This leads to a fully anisotropic g-tensor, which produces additional effective fields perpendicular to an applied transverse field$^{11}$. These effective fields are different whether the applied field is along b (or equivalently a) or along a$\pm$b. In all cases, effective fields are produced along c, whereas only in the former case a staggered field is induced along a. Importantly, the critical field marking the end of the Néel phase determined from macroscopic measurements is $\mu_bH_c \approx 10$ T for H||b and $\mu_bH_c \approx 40$ T for H||a$\pm$b, the latter being much closer to the expected value corresponding to the magnetization saturation$^{21-23}$. For H||b, only a small ferromagnetic component is induced at the critical field. As we will confirm here, this indicates that the effective staggered field induced along a opens an unconventional intermediate phase above $\mu_bH_c$.

The adequate model to describe BaCo$_2$V$_2$O$_8$ along with this phenomenology is as follows:

$$H = J \sum_{\mu, \nu} \left[ c (S_{\mu, \nu} \cdot S_{\mu+1, \nu} + S_{\mu, \nu} \cdot S_{\mu+1, \nu+1}) + S_{\mu, \nu} S_{\mu+1, \nu} \right]$$

$$- \sum_{\mu, \nu} g \mu_b H \cdot S_{\mu, \nu} + J' \sum_{\mu, \nu} \sum_{\mu \neq \nu} S_{\mu, \nu} S_{\mu', \nu'}$$

(1)

The first term is the XXZ Hamiltonian where $S_{\mu, \nu}$ is a spin 1/2, $\mu$ and $\nu$ are chain and site indices, $J > 0$ is the AF intrachain interaction and c is the anisotropy parameter ($0 < c < 1$ in our case). The action of a magnetic field yields the second term with $\mu_b$ being the Bohr magneton, $g$ being the Landé tensor and H being the external magnetic field. The last term arises from the weak interchain coupling $J'$. In the specific case of H||b, the case that we shall consider below, the total (external + effective) magnetic field at site n can be written as:

$$g \mu_b H_n = H \left[ g_{\mu y} (1)^y x + g_{\mu y} y + g_{\mu z} \cos \left( \frac{2\pi n-1}{4} \right) \right]$$

with $x = a$, $y = b$ and $z = c$.

Spectroscopic studies in zero field showed$^{24,25}$ that BaCo$_2$V$_2$O$_8$ does not host a classical Néel state at low temperature, in the sense that it lacks conventional spin-wave excitations. Instead, the excitations consist of two-spinon bound states confined by the interchain interaction and c is the orbital excitation parameter ($0 < c < 1$ in our case). The action of a magnetic field yields the second term with $\mu_b$ being the Bohr magneton, $g$ being the Landé tensor and H being the external magnetic field. The last term arises from the weak interchain coupling $J'$. In the specific case of H||b, the case that we shall consider below, the total (external + effective) magnetic field at site n can be written as:

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magnetic structures measured at 0, 6 (below $H$) and 12 T (above $H$). With increasing $H$, the magnetic moments remain staggered but progressively rotate in the (a,c) plane, from the Ising c axis to the a axis precisely at the transition (see Supplementary Information). The field evolution of the staggered components $m_a(H)$ and $m_c(H)$ of the ordered magnetic moments along a and c, respectively, are shown in Fig. 2b. The a component increases at the expense of the c component that eventually vanishes at the transition. This evolution of the staggered moment orientation from the Ising c axis to the a axis originates from an energetic compromise, between, on the one hand, the intrachain Ising-like exchange interaction, and, on the other hand, the Zeeman energy gain due to the effective transverse fields.

It is worth noting that, in principle, the effective field along c also induces a magnetic component of 0.04 $\mu_B$ at $\mu_H = 12$ T, as deduced from the calculations. Its value, however, is extremely small and consequently has no relevant role in the phase transition.

Note that this transition is not a standard spin reorientation with a global rotation of the spins. Indeed, the relative orientations of the magnetic moments between neighbouring chains are different in the zero field and in the high-field structure, due to a different symmetry of the interchain interactions and the staggered field, respectively. As shown in Fig. 2a (see also Supplementary Information), to accommodate this competition, the spins rotate clockwise for half of the chains, and anticlockwise for the other half, yielding a non-collinear intermediate magnetic structure. This subtle modification points out the role of the staggered field, which forces the formation of a magnetic structure that competes with the interchain interactions.

The peculiar nature of the high-field phase at $\mu_H = 12$ T is further illustrated in Fig. 2c. It shows the measured temperature dependence of the staggered order parameter ($m_a$) compared to the uniform component ($M_a$) induced along c by the field. In contrast with the usual abrupt drop of the order parameter expected for a temperature becoming larger than the interactions between...
magnetic moments above a critical field (see for instance Fig. 6 of ref. 20), $m_a$ decreases smoothly up to high temperature. $m_a$ is thus induced by the staggered magnetic field as $M_f$ is induced by the uniform magnetic field. The intrachain interaction $J$ is still effective in the high-field phase, and gives rise to well-defined excitations as shown in Fig. 2d. Note that these modes disappear between 10 and 20 K, at a much lower temperature than the staggered magnetization. These excitations are actually quite unconventional as described in the following.

**Magnetic excitations in a magnetic field**

Figure 3a–i shows the measured evolution of the lowest modes ([$1, S_z = 0, \pm 1$] and [$2, S_z = 0, \pm 1$]) of the Zeeman ladder, as a function of the transverse field $H||b$, for several scattering vectors $Q$: the AF point ($2, 0, 1$) and the two zone centre (ZC) positions ($0, 0, 2$) and ($3, 0, 1$). By increasing $H$, the zero-field [$1, S_z = \pm 1$] mode splits into two branches (see Fig. 3a–c). Note that the energy dependence of these two branches is not linear. The upper branch exhibits an upward variation up to $\mu_B H = 12$ T while the lower branch decreases...
down to $\mu_b H = 10T = \mu_H$. At this field, this branch reaches its minimum energy before increasing again, as seen, for example, at the AF position $Q = (2, 0, 1)$. The softening of the lower branch at $H_1$ thus marks the quantum phase transition, as already observed by electron spin resonance\(^4\). Note that a small energy gap of about 0.2 meV is still present (see Fig. 3a). The width of these two modes remains resolution-limited, indicating that they still must be considered as long-lived quasiparticles. The energy of the $(1, S_z = 0)$ mode is not constant with the field but increases with increasing field up to about 3T. At this field, an anti-crossing with the lowest branch of the upper $(2, S_z = \pm 1)$ mode occurs (see dashed white lines in Fig. 3a). As $H$ increases above $\mu_b H = 3T$, the lowest of the two hybridized branches broadens while its energy decreases, to finally disappear completely at the critical field.

This field dependence of the excitations is very different from the case of an external longitudinal field (parallel to the Ising axis) for which the $(j, S_z = \pm 1)$ and $(j, S_z = 0)$ excitations remain decoupled. In this case, the field produces a Zeeman splitting of the transverse excitations (linear field dependence) and has no effect on the longitudinal ones whose energy remains constant. This is indeed what is observed by electron spin resonance\(^3\) and inelastic neutron scattering (INS) in BaCo$_2$V$_2$O$_8$ (see Supplementary Information). The transverse field, on the other hand, allows the spinons to hop by one site and the $S_z = \pm 1$ and $S_z = 0$ sectors are no more independent. As a result, the field creates a quantum overlap between the $(j, S_z = \pm 1)$ and $(j, S_z = 0)$ excitations (see Fig. 1b). This hybridization process produces nonlinearities to second order in $H$ in the present case, there are two kinds of transverse field, the uniform one along $b$ and the staggered one along $a$. The influence of the latter is the strongest as shown in Fig. 3$c$ since it produces the rapid decrease of the lower branch towards the critical field compared to almost no field dependence in its absence.

A signature of the influence of the staggered field along $a$ is also visible in the field dependence of the intensity of the modes. The lowest energy mode displays a drastically different spectral weight evolution for the equivalent ZC $(0, 0, 2)$ and $(3, 0, 1)$ positions (see Fig. 3$b,c$). The latter gets more intense as the critical field is approached, while the former progressively vanishes. To understand this behaviour, we performed INS measurements using polarized neutrons and polarization analysis in a vertical magnetic field parallel to the $b$ axis on the IN12 triple-axis spectrometer. In this set-up, the non-spin-flip (NSF) and spin-flip (SF) scattering processes give information respectively about the spin fluctuations parallel to the field direction $b$, and perpendicular to it (without discriminating between the $a$ and $c$ directions). The lowest branch of the split $(1, S_z = \pm 1)$ modes is found to be SF, hence polarized within the $(a , c)$ plane, while the upper branch is NSF, hence polarized along $b$ (see Supplementary Information). Note that a geometrical factor enters the neutron cross-section, reflecting the fact that only spin components perpendicular to the scattering vector $Q$ contribute to the intensity. The decrease (respectively, increase) of the spectral weight of the lowest energy mode for the $(0, 0, 2)$ (respectively, $(3, 0, 1)$ ZC can be explained by the change of polarization of the excitation, from parallel to $a$ at low field to parallel to $a$ at the critical field and above. This result can be understood by the rotation of the ordered moment from $c$ to $a$, as established by the diffraction results, the lowest excitation branch thus conserving its transverse character in the whole field range.

**Topological nature of the transition and the excitations**

To determine the nature of the transition experimentally identified above, we performed numerical simulations of the XXZ model in the presence of an external magnetic field (see equation (1)). The effects of the interchain interactions were taken into account by a mean field theory, in which an effective staggered field induced by the Néel order of the neighbouring chains is determined self-consistently.

We used an infinite time-evolving block decimation (iTEBD)\(^3\) with the infinite boundary condition\(^1\) (see the Supplementary Information for details). Using the parameters $J = 5.8$ meV, $\epsilon = 0.53$ and $J' = 0.17$ meV, close to those reported in the literature\(^3,15\), the results show excellent agreement with the experimental data and thus validate the model. In zero magnetic field (Fig. 1c), as discussed above, we reproduce the Zeeman ladders corresponding to the bound spinons\(^2\). With the magnetic field, the results are shown in Fig. 2b for the order parameter and in Fig. 3j–l for the excitation spectrum. In these calculations, we used the ratios $g_{\parallel}/g_{\perp} \approx 0.40$ and $g_{\parallel}/g_{\perp} \approx 0.14$ determined in ref. \(^21\), along with $g_{\parallel} \approx 2.35$, a value slightly different from the value of 2.75 determined in ref. \(^21\), to agree with the measured critical field. The numerics describe extremely well the field dependence of the (staggered) magnetization along the chains. For the component perpendicular to the chains, the overall trend of the data is correctly given by the numerics but a global scaling factor seems to exist with the experimental data. The reason for this discrepancy could be due to factors such as: effect of temperature; higher sensitivity of this quantity on small uncertainties in the parameters; treatment of the interchain interaction in the mean field theory. On the other hand, the results for the excitation spectrum (Fig. 3j–l) show a very good agreement with the data (Fig. 3a–c) accounting for the main modes observed experimentally. The numerics further validate the polarization of the modes and, in particular, the transverse nature of the lowest energy one. The rapid energy lowering of the lowest $(1, S_z = \pm 1)$ branch when the field is applied along $b$ is confirmed numerically to be a consequence of the additional effective staggered field along $a$ due to non-diagonal components of the $g$-tensor\(^2\). The validation of the model of equation (1) from the numerics allows us to use field theory to describe the transition qualitatively, thereby determining its nature in a more transparent way. Using the bosonization technique\(^1\), we obtain a dual-field double sine-Gordon model describing BaCo$_2$V$_2$O$_8$ in an external field along $b$:

$$
H_{\text{eff}} = \frac{v}{2\pi} \int dz \left[ \frac{1}{K} \left( \frac{d\phi(z)}{dz} \right)^2 + \frac{K}{\alpha} \left( \frac{d\theta(z)}{dz} \right)^2 \right] + \frac{2\lambda}{(2\pi\alpha)^2} \int dz \cos 4\phi(z) \tag{3}
$$

(see Supplementary Information) where $v$ is the spinon velocity, $K$ is the Luttinger parameter, $\lambda$ is a constant having a dimension of energy and $\alpha$ is a dimensionless constant\(^4\). These parameters ($v$, $K$, $\lambda$, $\alpha$) are a function of $\epsilon$ and $H$, but there is no analytical representation. The effect of the Zeeman coupling with the uniform field along the $b$ axis is renormalized into these parameters\(^4\). Since the Zeeman term of the four-site periodic field along the $b$ axis is irrelevant, it does not appear in equation (3). The effect of this field is actually negligibly small. $\phi(z)$ and $\theta(z)$ are dual bosonic fields that can be qualitatively identified with the polar and azimuthal angles of a staggered magnetization vector (see Fig. 4a). In equation (3), the potential terms $\cos 4\phi(z)$ and $\cos \theta(z)$ compete with each other and pin the fields $\phi(z)$ (respectively, $\theta(z)$) for the low (respectively, high)-field phase. The expectation values $\langle \cos 2\phi(z) \rangle$ and $\langle \cos \theta(z) \rangle$ correspond to the staggered magnetization along the $c$ and $a$ axes, respectively.

Excitations in a given phase correspond to the soliton of a pinned field (tunnelling from one minimum of the cosine to the next), and carry a topological index (see Fig. 4). In the low-field phase, $4\phi(z)$ is fixed to $2\pi p$ ($p$ is an integer). Thus, a low-energy excitation corresponds to the creation of a soliton–antisoliton pair in which $\phi(z)$ changes from 0 to $2\pi/4$ (for the soliton) and from $2\pi/4$ back to 0 (for
would require nonlocal measurements as recently performed in cold atom systems\cite{NIST, Cold}. How to conduct such measurements for quantum spin systems in solid state is a challenging question.

The analysis of the properties of the modes in the experimental data and the agreement with numerics confirm that the quantum transition of the dual-field double sine-Gordon model is indeed what is observed in BaCo$_2$V$_2$O$_8$, providing an explanation for the mysterious field-induced transition and highlighting its topological nature. From a theoretical point of view, the study of the transition itself is a challenging problem. The nature of the transition depends on the precise periodicity of the cosines\cite{Kadanoff}. For the purely one-dimensional Hamiltonian (3), special solvable points suggest an Ising transition\cite{Ising}, as also confirmed by a numerical calculation of the central charge. A complete study, in particular taking into account the effective 3D coupling beyond the mean field, is still lacking. BaCo$_2$V$_2$O$_8$ thus provides a remarkable experimental system in which this transition can be tuned and studied in a controlled way.

More generally, quantum spin systems have been a steady reservoir of experimental realizations of topological phases and transitions, with, in particular, several realizations of the sine-Gordon model, or of exotic phases such as Tomonaga–Luttinger liquids. Our analysis of the transition in a uniform transverse magnetic field in BaCo$_2$V$_2$O$_8$ shows that they are also able to provide excellent and controlled realizations of more complex and yet challenging models from a theoretical point of view, confirming their place as quantum simulators of quantum correlated systems.

**Methods**

Methods, including statements of data availability and any associated accession codes and references, are available at https://doi.org/10.1038/s41567-018-0126-8.

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**Fig. 4 | Two dual topological objects.** a. A qualitative interpretation of the two fields $\phi(z)$ and $\theta(z)$ entering into the field theory description. The quantum nature of a spin 1/2 makes it impossible to determine both angles with an infinite accuracy, hence the fact that the two angles play a role similar to canonically conjugate variables. b. Topological excitations in the low-field phase (well below the transition). These excitations correspond to solitons in the field $4\phi(z)$ linking one of the minima of the $\cos 4\phi(z)$ to the next. Using the bosonization representation of the spin (see Supplementary Information), they can be identified with the spinon excitations (see also Fig. 1b). They carry a spin $S_z = \pm 1/2$ corresponding to the topological index of the excitation. c. In the high-field phase (far above the transition but still well below the saturation), the elementary excitations now correspond to the solitons of the $\cos \theta(z)$, and are thus the dual of the low-field phase excitations. They carry an index of $S_z = \pm 1$. 

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Because there are only two indices, the two solitons (the soliton and the anti-soliton) (see Fig. 4b). The soliton and antisoliton are domain walls of the Néel order, with spins reversed with respect to the ground state between them. The solitons themselves can be identified with the spinons (see Fig. 1b). The soliton–antisoliton would be deconfined in a single chain while confined in BaCo$_2$V$_2$O$_8$ due to the linear potential produced by interchain interaction. In the high-field phase, the $\cos \theta(z)$ term dominates over $4\cos 4\phi(z)$ and fixes the $\theta(z)$ field. The corresponding soliton carries a spin $S_z = 1$ (instead of 1/2) since $\theta(z)$ changes from 0 to $2\pi$ (instead of 0 to $\pi$ for the field $2\phi$) (see Fig. 4c). Note that important differences between the low- and the high-field phases exist. In the low-field phase, the Hamiltonian contains $\cos (4\phi)$ while a physical observable such as the staggered part of $S^z$ depends on $\cos (2\phi)$. Thus, the two parts of the soliton $2\phi = 0$ and $2\phi = \pi$ can be distinguished by local measurements of $S^z$, such as the string of overturned spins separating the two objects. However, in the high-field phase, both the Hamiltonian and the physical observable $S^z$ depend on $\cos \theta$, making both sides of the solution identical far from the soliton. In terms of a local measurement of $S^z$, the corresponding excitations would thus be local and would not carry a topological index. There is, however, a true topological order present since $\theta$ orders. It could be detected through a quantity such as $\langle e^{i\theta(z)/2} \rangle$, but...
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Author contributions

All authors contributed significantly to this work. In detail, sample preparation was performed by P.L., neutron scattering experiments and analysis were carried out by Q.F., B.G., S.P. and V.S. with the support of S.R., L.-P.R., M.B., J.S.W., M.M. and C.R., calculations were performed by S.T., S.C.F. and T.G., inputs for the discussion of the physical results were provided by C.R., B.C. and T.L.; the manuscript was written by V.S., S.P., B.G., Q.F., T.G. and S.T. with constant feedback from the other co-authors.

Competing interests

The authors declare no competing interests.

Additional information

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Methods

Sample preparation and experimental set-up. A BaCo$_2$V$_2$O$_8$ single crystal was grown at Institut Neel by the floating zone method$. A 5$-cm-long cylindrical crystal rod, of about 4 mm diameter, was obtained by imposing the growth axis to be along the $b$ crystallographic axis. One crystal piece of 10 mm long was cut from the rod for the diffraction experiment, while two crystal pieces of about 18 mm long were cut for the INS experiments.

The diffraction experiment was performed on the CEA-CRG D23 single-crystal two-axis diffractometer with a lifting arm detector at Institut Laue-Langevin (ILL). The sample, previously aligned with the $b$ axis vertical on the Laue diffractometer OrientExpress at ILL, was installed on D23 in the CEA 12 T vertical field cryomagnet. A maximum transverse magnetic field of 12 T ($H||b$ and thus $H\perp c$ Ising axis) could be reached with a base temperature of 1.5 K. An incident wavelength of 1.28 Å was used, from a copper monochromator, thus allowing us to measure $h0l$ and $h1l$ Bragg peaks with a maximum value of 17 for $h$ and 11 for $l$.

The INS experiments under a transverse magnetic field were performed on two cold-neutron triple-axis spectrometers, ThALES and FZI-CRG IN12 at ILL. On ThALES, a PG(002) monochromator (respectively, analyser) was used to select (respectively, analyze) the initial (respectively, final) wavevector of the unpolarized neutron beam. On IN12, we used polarized neutrons, from a cavity transmission polarizer located far upstream in the guide, with an initial wavevector selected by a PG(002) monochromator, and polarization analysis, from a Heusler analyser (see ref. $^{40}$ for a more detailed description of the standard polarized neutron set-up on IN12). On both spectrometers, the energy resolution was of the order of 0.15 meV and the higher-order contamination was suppressed by a velocity selector.

The same cryomagnet as on D23 was used on both instruments, thus providing a maximum transverse field of 12 T at a base temperature of 1.5 K. Due to the high applied vertical magnetic field (up to 12 T), the vertical current of the Mezei spin flipper, placed just before the monochromator on IN12, was calibrated for every used value of the incident wavevector and of the magnetic field. The horizontal current was checked to be non-sensitive to the applied field. The flipping ratios were ranging between 12 and 23, depending on the incident wavevector and magnetic field values. One of the two 200 mm$^3$ crystal pieces was used in both experiments and previously aligned with the $b$ axis vertical on the triple-axis spectrometer IN3 at ILL, yielding a ($a^*\perp c$) horizontal scattering plane. Once the sample glued, the alignment was checked to be better than $1\degree$ on the neutron Laue diffractometer OrientExpress at ILL. All of the INS data presented here were measured at a fixed final wavevector of 1.3 Å$^{-1}$.

Data availability. All relevant data are available from the corresponding authors. INS data collected at the ILL are available at https://doi.org/10.5291/ILL-DATA.4-03-1719 for the experiment on ThALES and https://doi.org/10.5291/ILL-DATA.4-01-1503 for the experiment on IN12.

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