ANALYSIS OF THE RADIATIVE DECAYS AMONG THE CHARMONIUM STATES

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Abstract
In this article, we study the radiative decays among the charmonium states with the heavy quark effective theory, and make predictions for the ratios among the radiative decay widths of an special multiplet to another multiplet. The predictions can be confronted with the experimental data in the future and put additional constraints in identifying the $X$, $Y$, $Z$ charmonium-like mesons.

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1 Introduction
In recent years, the Babar, Belle, CLEO, D0, CDF and FOCUS collaborations have discovered (or confirmed) a large number of charmonium-like states and revitalized the interest in the spectroscopy of the charmonium states. Many possible assignments for those states have been suggested, such as the conventional charmonium states, the multiquark states (irrespective of the molecule type and the diquark-antidiquark type), the hybrid states, the baryonium states, the threshold effects, etc [1][2][3][4][5][6][7]. The main difficulties in identifying those new mesons as the excited charmonium states in that the observed masses do not fit in with the predictions of the potential models based on the confining potential. In this article, we will focus on the traditional charmonium scenario and study the radiative decays among the charmonium states with the heavy quark effective theory.

In 2003, the Belle collaboration observed the $X(3872)$ in the $J/\psi\pi^+\pi^−$ invariant mass distribution with a significance in excess of 10$\sigma$ in the $B \rightarrow KJ/\psi\pi^+\pi^−$ decay [11], and later the $X(3872)$ was confirmed by the D0, CDF and Babar collaborations [12][13][14]. The decay $X(3872) \rightarrow J/\psi\gamma$ observed by the Belle and Babar collaborations [15][16][17] and the decay $X(3872) \rightarrow \psi'\gamma$ observed by the Babar collaboration [17] favor the even charge conjunction assignment. The di-pion spectrum also indicates that the charge conjunction is even [11], and the studies of the Belle collaboration [15] and CDF collaboration [18][19] favor the spin-parity-charge-conjunction $J^{PC} = 1^{++}$ assignment for the $X(3872)$. However, the recent analysis of the $B \rightarrow J/\psi\omega K$ decay by the Babar collaboration indicates that the $P$-wave orbital angular momentum for the $J/\psi\omega$ system is more favored than the $S$-wave, the $X(3872)$ maybe have the spin-parity-charge-conjunction $J^{PC} = 2^{−}$ instead of the $J^{PC} = 1^{++}$ [20]. In Ref. [21], Jia et al study the radiative decays $X(3872) \rightarrow J/\psi(\psi' + \gamma$ within several phenomenological potential models, and observe that the $2^{−}$ assignment for the $X(3872)$ is highly problematic. In this article, we assume that the $X(3872)$ is the conventional charmonium $\chi_{c1}(2P)$ [22][23][24][25][26].

In 2005, the Belle collaboration observed the $Z(3930)$ in the $D\bar{D}$ invariant mass distribution near 3.93 GeV in the $\gamma\gamma$ collision with the statistical significance of 5.3$\sigma$ [27], the mass and width are $M_{Z(3930)} = (3929 \pm 5 \pm 2)$ MeV and $\Gamma_{Z(3930)} = (29 \pm 10 \pm 2)$ MeV, respectively. The production rate and the angular distribution favor the $\chi_{c2}(2P)$ assignment [27]. In the same year, the Belle collaboration observed the $X(3940)$ in the recoiling spectrum of the $J/\psi$ in the process $e^+e^- \rightarrow J/\psi + D^*\bar{D}$ [28]. Later the Belle collaboration studied it with higher statistics and determined the mass and width $M_{X(3940)} = (3942^{+6}_{-7} \pm 6)$ MeV and $\Gamma_{X(3940)} = (37^{+26}_{-15} \pm 8)$ MeV [29]. Furthermore, they observed the $X(4160)$ in the $D^*\bar{D}^*$ invariant mass distribution in the process $e^+e^- \rightarrow J/\psi + D^*\bar{D}^*$ with a significance of 5.1 $\sigma$. The mass and width of the $X(4160)$ are $M_{X(4160)} = (4156^{+25}_{-20} \pm 15)$ MeV and $\Gamma_{X(4160)} = (139^{+111}_{-61} \pm 21)$ MeV, respectively. The observation of the dominant decay mode of the $X(3940)$ being the $D^*\bar{D}$ and the lack of evidence for the
$D\bar{D}$ decay mode indicate that it is a good candidate for the $\eta_c(3S)$ state [28, 29, 30], although the mass is lower than the prediction of the potential models [31]. The dominant decay mode of the $X(4160)$ is the $D^+\bar{D}^*$ [28], and the $D\bar{D}$ and $D\bar{D}^*$ modes have not been observed, so the $X(4160)$ may be the $\eta_c(4S)$ or $\chi_{c0}(3P)$ candidate [32].

Also in 2005, the Babar collaboration observed the $Y(4260)$ in the $J/\psi\pi^+\pi^-$ invariant mass distribution in the initial state radiation (ISR) process $e^+e^- \rightarrow \gamma_{\text{ISR}} J/\psi\pi^+\pi^-$ [33]. It was confirmed by the CLEO collaboration [34] and Belle collaboration [35], and in the Belle data there is also a relative broad structure $Y(4008)$ with the mass $M_{Y(4008)} = (4008 \pm 40_{+17}^{+14})$ MeV and width $\Gamma_{Y(4008)} = (226 \pm 44 \pm 87)$ MeV, respectively. In Ref. [36], Llanes-Estrada suggests that the $Y(4260)$ may be the $\psi(4S)$ charmonium state and displaces the $\psi(4115)$.

In 2006, the Babar collaboration observed the structure $Y(4320)$ in the initial state radiation process $e^+e^- \rightarrow \gamma_{\text{ISR}} \psi' \pi^+\pi^-$ with the mass $M_{Y(4320)} = (4324 \pm 24)$ MeV and width $\Gamma_{Y(4320)} = (172 \pm 33)$ MeV, respectively [37], and later the Belle collaboration observed two relative narrow resonant structures $Y(4360)$ and $Y(4660)$ with the masses $M_{Y(4360)} = (4361 \pm 9 \pm 9)$ MeV, $M_{Y(4660)} = (4664 \pm 11 \pm 5)$ MeV and the widths $\Gamma_{Y(4360)} = (74 \pm 15 \pm 10)$ MeV, $\Gamma_{Y(4660)} = (48 \pm 15 \pm 3)$ MeV, respectively [38]. In 2008, the Belle collaboration observed the $Y(4630)$ as a threshold enhancement in the $\Lambda_c^+\Lambda_c^-$ invariant mass distribution in the initial state radiation process $e^+e^- \rightarrow \gamma_{\text{ISR}} \Lambda_c^+\Lambda_c^-$ [39]. The mass and width are $M_{Y(4630)} = (4634^{+5}_{-3_{-9}})$ MeV and $\Gamma_{Y(4630)} = (92_{-24}^{+10_{-41}}) \text{MeV}$, respectively, which are roughly in agreement with that of the $Y(4660)$. In Ref. [29], Li and Chao calculate the mass spectrum of the charmonium states based on the screened potential, and observe that the $Z(3930)$ agrees with the $\chi_{c2}(2P)$, the $\psi(4115)$ is compatible with the $\psi(5S)$ rather than the $\psi(4S)$, the $Y(4260)$, $Y(4360)$ and $Y(4660)$ may be the $\psi(4S)$, $\psi(3D)$ and $\psi(6S)$ respectively, and the $X(3940)$ and $X(4160)$ may be the $\eta_c(3S)$ and $\chi_{c0}(3P)$ respectively. In Ref. [40], Ding et al identify the $Y(3460)$ and $Y(4660)$ as the $3^3D_1$ and $5^3S_1$ charmonium states respectively (Badalian et al share the same opinion [41]), and evaluate the $e^+e^-$ lepton widths, $E1$ transitions, $M1$ transitions and the open flavor strong decays.

In 2009, the CDF collaboration observed a narrow structure $Y(4140)$ near the $J/\psi\phi$ threshold with a statistical significance in excess of 3.8$\sigma$ in the exclusive $B \rightarrow J/\psi\phi K$ decays produced in $\bar{p}p$ collisions [42]. The mass and width are $M_{Y(4140)} = (4143.0 \pm 2.9 \pm 1.2)$ MeV and $\Gamma_{Y(4140)} = (11.7^{+5.3}_{-3.0}) \pm 3.7$ MeV, respectively [42]. The $Y(4140)$ is very similar to the charmonium-like state $Y(3940)$, which was observed by both the Belle and Babar collaborations near the $J/\psi\omega$ threshold in the exclusive $B \rightarrow J/\psi\omega K$ decays [43, 44]. The mass and width are $M_{Y(3940)} = (3943 \pm 11 \pm 13)$ MeV and $\Gamma_{Y(3940)} = (87 \pm 22 \pm 26)$ MeV respectively from the Belle collaboration [43] and $M_{Y(3940)} = (3914.6^{+3.8}_{-3.4}) \pm 2.0$ MeV and $\Gamma_{Y(3940)} = (34^{+12}_{-8}) \pm 5$ MeV respectively from the Babar collaboration [44]. In 2009, the Belle collaboration reported the observation of a significant enhancement with the mass $(3915 \pm 3 \pm 2)$ MeV and total width $(17 \pm 10 \pm 3)$ MeV respectively in the process $\gamma\gamma \rightarrow \omega J/\psi$ [45], these values are consistent with that of the $Y(3940)$. The updated values of the mass $(3919.1^{+3.8}_{-3.5} \pm 2.0)$ MeV and total width $(31^{+10}_{-8}) \pm 5$ MeV from the Babar collaboration are also consistent with the old ones [40]. The Belle collaboration measured the process $\gamma\gamma \rightarrow \phi J/\psi$ for the $J/\psi\phi$ invariant mass distributions between the threshold and 5 GeV, and observed a narrow peak $X(4350)$ with a significance of 3.2$\sigma$ [47]. The mass and width are $M_{X(4350)} = (4350.6^{+6}_{-5.1} \pm 0.7)$ MeV and $\Gamma_{X(4350)} = (13.3^{+17}_{-9.1}) \pm 4.1$ MeV, respectively; and no signal for the $Y(4140) \rightarrow J/\psi\phi$ structure was observed [47]. It is difficult to identify the $Y(3940)$, $Y(4140)$ and $X(4350)$ as the conventional charmonium states [48].

In Table 1, we list the experimental values of the charmonium states with the possible identifications compared with the theoretical predictions [20, 31, 49]. We do not mean that such assignments are correct and exclude other possibilities, and just take it for granted for the moment and study the radiative decays among the charmonium states with the heavy quark effective theory [8, 9, 10], which have been applied to identify the excited $D_s$ and $D$ mesons, such as the $D_s(3040)$, $D_s(2700)$, $D_s(2860)$, $D(2550)$, $D(2600)$, $D(2750)$ and $D(2760)$ [50, 51, 52, 53, 54].

The article is arranged as follows: we study the radiative decays among the charmonium states
Table 1: Experimental and theoretical mass spectrum of the charmonium states. The SP, NRP and GI denote the screened potential model, the non-relativistic potential model and the Godfrey-Isgur relativized potential model, respectively.
with the heavy quark effective theory in Sect.2; in Sect.3, we present the numerical results and discussions; and Sect.4 is reserved for our conclusions.

2 The radiative decays with the heavy quark effective theory

The charmonium states can be classified according to the notation \( n^{2s+1}L_j \), where the \( n \) is the radial quantum number, the \( L \) is the orbital angular momentum, the \( s \) is the spin, and the \( j \) is the total angular momentum. They have the parity and charge conjugation \( P = (-1)^{L+1} \) and \( C = (-1)^{L+s} \), respectively. The states have the same radial quantum number \( n \) and orbital momentum \( L \) can be expressed by the superfields \( J, J^\mu, J^{\mu
u} \), etc [55],

\[
\begin{align*}
J &= \frac{1 + \gamma}{2} \left\{ \psi_\mu \gamma^\mu - \eta_c \gamma_5 \right\} \frac{1 - \gamma}{2}, \\
J^\mu &= \frac{1 + \gamma}{2} \left\{ \chi_2^\mu \gamma^\mu + \frac{1}{2\sqrt{2}} \tau^\mu \chi^1 \chi^3 \right\} \frac{1 - \gamma}{2}, \\
J^{\mu\nu} &= \frac{1 + \gamma}{2} \left\{ \chi_3^{\mu\nu} \gamma^\mu + \frac{1}{\sqrt{2}} \tau^\mu \chi^1 \chi^3 \right\} \frac{1 - \gamma}{2},
\end{align*}
\]

where the \( \mu^\mu \) denotes the four-velocity associated to the superfields, the fields \( \chi_3^{\mu\nu}, \chi_2^\mu, \chi^1, \chi^0, \eta \) have the total angular momentum \( j = 3, 2, 1, 0, 2 \) respectively, and belong to different multiplets. The fields in a definite superfield have the same \( n \), and form a multiplet. We multiply the charmonium fields \( \chi_3^{\mu\nu}, \chi_2^\mu, \chi^1, \chi^0, \eta \) with a factor \( \sqrt{M_c}, \sqrt{M_s}, \cdots \), and they have dimension of mass \( \frac{3}{2} \). The superfields \( J^{\mu_1 \cdots \mu_L} \) have the following properties under the parity, charge conjugation, heavy quark spin transformations,

\[
\begin{align*}
J^{\mu_1 \cdots \mu_L} &\xrightarrow{P} \gamma^0 J^{\mu_1 \cdots \mu_L} \gamma^0, \\
J^{\mu_1 \cdots \mu_L} &\xrightarrow{C} (-1)^{L+1} \gamma C J^{\mu_1 \cdots \mu_L} C, \\
J^{\mu_1 \cdots \mu_L} &\xrightarrow{S} SJ^{\mu_1 \cdots \mu_L}, \\
\mu^\mu &\xrightarrow{P} \mu^\mu,
\end{align*}
\]

where \( S, S' \in SU(2) \) heavy quark spin symmetry groups, and \( [S, \gamma] = [S', \gamma] = 0 \).

The radiative transitions between the \( m \) and \( n \) charmonium states can be described by the following Lagrangians,

\[
\begin{align*}
\mathcal{L}_{SS} &= \sum_{m,n} \delta(m,n) \text{Tr} \left[ \bar{J}(m) \sigma_{\mu\nu} J(n) \right] F^{\mu\nu}, \\
\mathcal{L}_{SP} &= \sum_{m,n} \delta(m,n) \text{Tr} \left[ \bar{J}(m) J_\mu(n) + \bar{J}_\mu(n) J(m) \right] V^\mu, \\
\mathcal{L}_{PD} &= \sum_{m,n} \delta(m,n) \text{Tr} \left[ \bar{J}_{\mu\nu}(m) J^\nu(n) + \bar{J}^\nu(n) J_{\mu\nu}(m) \right] V^{\mu\nu},
\end{align*}
\]

where \( \bar{J}_{\mu_1 \cdots \mu_L} = \gamma^0 J^{\dagger}_{\mu_1 \cdots \mu_L} \gamma^0, V^{\mu\nu} = F^{\mu\nu} \mu^\nu \), the \( F^{\mu\nu} \) is the electromagnetic tensor, and the \( \delta(m,n) \) are the coupling constants, which have different values in the Lagrangians \( \mathcal{L}_{SS}, \mathcal{L}_{SP}, \mathcal{L}_{PD} \), we use the same notation for simplicity. The Lagrangians \( \mathcal{L}_{SP} \) and \( \mathcal{L}_{PD} \) preserve parity, charge
conjugation, gauge invariance and heavy quark spin symmetry, while the Lagrangian $L_{SS}$ violates the heavy quark symmetry. There are two tensors $\sigma_{\mu\nu}F^{\mu\nu}(=\frac{2}{3}\epsilon^{\mu\nu\alpha\beta}\sigma_{\mu\nu}F_{\alpha\beta})$ and $\sigma_{\mu\nu}F^{\mu\nu}$ can be used to construct the spin-breaking Lagrangian $L_{SS}$. They both have the terms $\bar{E} \cdot E$ and $\bar{B} \cdot B$, and can turn into each other with the interchange $E \leftrightarrow B$, we choose the parity-conserving structure $\sigma_{\mu\nu}F^{\mu\nu}$ in constructing the spin-breaking Lagrangian. We take the Lagrangian $L_{SP}$ from Ref.[8] and construct the Lagrangians $L_{SS}$ and $L_{PD}$, and later Dr. F. De Fazio draws my attention to Ref.[56], where the Lagrangian $L_{PD}$ is introduced for the first time.

From the heavy quark effective Lagrangians $L_{SS}$, $L_{SP}$ and $L_{PD}$, we can obtain the radiative decay widths $\Gamma$,

$$\Gamma = \frac{1}{2j + 1} \sum k_{\nu} \frac{1}{8\pi M_{i}^{2}}|T|^{2},$$

(4)

where the $T$ denotes the scattering amplitude, the $k_{\nu}$ is the momentum of the final states in the center of mass coordinate, the $\sum$ denotes the sum of all the polarization vectors, the $j$ is the total angular momentum of the initial state, and the $M_{i}$ is the mass of the initial state. Cho and Wise study the radiative decays of the heavy quarkonia using the multipole expansion and heavy quark symmetry.

3 Numerical Results

We calculate the radiative decay widths $\Gamma$ using the FeynCalc to carry out the sum of all the polarization vectors. In calculations, the masses of the charmonium states are taken as the experimental values from the Review of Particle Physics [49], see Table 1; for the unobserved charmonium states, we take the values from the screened potential model [26], and assume that the charmonium states $\psi(5^{3}S_{1})$ and $\eta_{c}(5^{1}S_{0})$ have degenerate masses. In this article, we identify the $\psi(4415)$ as the $\psi(5^{3}S_{1})$ charmonium state, and expect that the $\eta_{c}(5^{1}S_{0})$ charmonium state has slightly smaller mass. The predictions of the screened potential model are $M_{\psi(4S)} = 4463\text{MeV}$ and $M_{\eta_{c}(4S)} = 4446\text{MeV}$, respectively, so we take the approximation $M_{\psi(4S)} = M_{\eta_{c}(4S)} = 4421\text{MeV}$.

The numerical values of the radiative decay widths are presented in Tables 2-4, where we retain the unknown coupling constants $\delta(m,n)$ among the multiplets of the radial quantum numbers $m$ and $n$. In general, we expect fitting the parameters $\delta(m,n)$ to the precise experimental data, however, in the present time the experimental data are far from enough. In Tables 4-6, we present the ratios of the radiative decay widths among the charmonium states.

The radiative decay widths listed in the Review of Particle Physics are

$$\begin{align*}
\Gamma(\psi(2S) \rightarrow \eta_{c}(1S)\gamma) &= 1.0336\text{KeV}, \\
\Gamma(\psi(2S) \rightarrow \chi_{c0}(1P)\gamma) &= 29.2448\text{KeV}, \\
\Gamma(\psi(2S) \rightarrow \chi_{c1}(1P)\gamma) &= 27.968\text{KeV}, \\
\Gamma(\psi(2S) \rightarrow \chi_{c2}(1P)\gamma) &= 26.5969\text{KeV}, \\
\Gamma(\chi_{c0}(1P) \rightarrow J/\psi\gamma) &= 119.48\text{KeV}, \\
\Gamma(\chi_{c1}(1P) \rightarrow J/\psi\gamma) &= 295.84\text{KeV}, \\
\Gamma(\chi_{c2}(1P) \rightarrow J/\psi\gamma) &= 384.15\text{KeV}, \\
\Gamma(\psi(3770) \rightarrow \chi_{c0}(1P)\gamma) &= 199.29\text{KeV}, \\
\Gamma(\psi(3770) \rightarrow \chi_{c1}(1P)\gamma) &= 79.17\text{KeV}, \\
\Gamma(\psi(3770) \rightarrow \chi_{c2}(1P)\gamma) &= 24.57\text{KeV}, \\
\Gamma(\psi(4040) \rightarrow \chi_{c1}(1P)\gamma) &< 0.88\text{MeV}, \\
\Gamma(\psi(4040) \rightarrow \chi_{c2}(1P)\gamma) &< 1.36\text{MeV},
\end{align*}$$

(5)
where we have neglected the uncertainties [49]. From those radiative decay widths, we can obtain the following ratios,
\[
\begin{align*}
\Gamma(\psi(2S) \to \chi_c(1P)\gamma) &= 1.046(1.151), \\
\Gamma(\psi(2S) \to \chi_c(1P)\gamma) &= 1.101(1.649), \\
\Gamma(\psi(2S) \to \chi_c(1P)\gamma) &= 0.311(0.363), \\
\Gamma(\chi_c(1P) \to J/\psi) &= 0.770(0.756), \\
\Gamma(\chi_c(1P) \to J/\psi) &= 2.517(3.167), \\
\Gamma(\chi_c(1P) \to J/\psi) > 8.111(82.02),
\end{align*}
\]
where the values in the bracket are the theoretical calculations based on the heavy quark effective theory. The agreements between the experimental data and the theoretical calculations are rather good, and the heavy quark effective theory works rather well. The ratios presented in Tables 4-6 can be confronted with the experimental data in the future at the BESIII, KEK-B, RHIC, PANDA and LHCb, and put additional constraints in identifying the X, Y, Z charmonium-like mesons.

There is a relative $P$-wave between the final-state charmonium and the photon, the radiative decay widths $\Gamma \propto k_s^2$, where $k_s = \frac{M_i^2 - M_f^2}{2M_i}$, the $M_i$ and $M_f$ denote the masses of the initial and final charmonium states respectively. The numerical values of the decay widths shown in Tables 2-4, where the effective coupling constants $\delta(m,n)$ have been factorized out, reflect the corresponding processes are facilitated or suppressed in the phase-space. If the energy gap $M_i - M_f$ is small (or large), small variations of the masses $M_i$ and $M_f$ can (or cannot) lead to remarkable changes for the decay width. For example, we plot the radiative decay widths $\Gamma(\chi_{c0}(2P) \to J/\psi\gamma)$ versus the mass $M_X$ in Fig.1. From the experimental data of the Babar collaboration $\text{Br}(B^+ \to X(3872)K^+) \times \text{Br}(X(3872) \to J/\psi\gamma) = (2.8 \pm 0.8 \pm 0.2) \times 10^{-6}$ and $\text{Br}(B^+ \to X(3872)K^+) \times \text{Br}(X(3872) \to J/\psi\gamma) = (9.9 \pm 2.9 \pm 0.6) \times 10^{-6}$ [17], we can obtain the central value of the ratio $\Gamma(\chi_{c0}(2P) \to J/\psi\gamma) = 0.283$, which means $\frac{\delta(2,1)}{\delta(2,2)} = 0.00576$. The large hierarchy $\delta(2,1) \ll \delta(2,2)$ is compatible with the phenomenological expectation that the $\chi_{c1}(2P)$ state potentially decays to the $\psi\gamma$ rather than to the $J/\psi\gamma$.

The $E_1$ and $M_1$ transitions among the charmonium states are usually calculated by the formula [58 59 60 61],
\[
\begin{align*}
\Gamma_{E1} \left( n^{2s+1}L_j \to n^{2s'+1}L'_{j'} + \gamma \right) &= \frac{4}{3}\alpha\alpha_e E_j^2 M_i \delta_{ss'} C_{f_1} \left| \langle n^{2s+1}L'_{j'} | r | n^{2s+1}L_j \rangle \right|^2, \\
\Gamma_{M1} \left( n^{2s+1}L_j \to n^{2s'+1}L'_{j'} + \gamma \right) &= \frac{4}{3}\alpha\alpha_e E_j^2 M_i \frac{2j'+1}{2L+1} \delta_{LL'}\delta_{ss'} \left| \langle n^{2s'+1}L'_{j'} | n^{2s+1}L_j \rangle \right|^2,
\end{align*}
\]
(7)
where the $E_j$ is the photon energy, the $E_j$ is the energy of final state charmonium, the $M_i$ is the mass of the initial state charmonium, and the angular matrix factor $C_{f_1}$ is
\[
C_{f_1} = \max(L, L') \left( \frac{L'}{j} \right)^2 \left( \frac{L}{j} \right)^2.
\]
(8)
The values of the matrix elements $\langle n^{2s'+1}L'_{j'} | r | n^{2s+1}L_j \rangle$ and $\langle n^{2s'+1}L'_{j'} | n^{2s+1}L_j \rangle$ depend on the details of the wave-functions which are evaluated using a special potential model, for example,
the Cornell potential model, the logarithmic potential model, the power-law potential model, the QCD-motivated potential model [62], the relativized Godfrey-Isgur model, the non-relativistic potential model [31], the screened potential model [26], the relativistic quark model based on a quasipotential approach in QCD [63, 64], etc. All predictions should be confronted with the experimental data. In this article, we intend to make estimations based on the heavy quark effective theory, and prefer the ratios among the radiative decay widths of an special multiplet to another multiplet, where the unknown parameters \( \delta(m, n) \) are canceled out with each other.

### 4 Conclusion

In this article, we study the radiative decays among the charmonium states with the heavy quark effective theory, and make predictions for ratios among the radiative decay widths of a special multiplet to another multiplet, where the unknown coupling constants \( \delta(m, n) \) are canceled out with each other.

| \( \chi_c \rightarrow \psi \gamma \) | \( \chi_c \rightarrow \chi_c \gamma \) | \( \eta_c \rightarrow \eta_c \gamma \) | \( \eta_c \rightarrow \eta_c \gamma \) |
|-----------------|-----------------|-----------------|-----------------|
| 2S \( \rightarrow \) 1P | 3.546 | 5.082 | 5.849 | 4.092 |
| 3S \( \rightarrow \) 1P | 146.4 | 112.1 | 57.15 | 174.9 |
| 3S \( \rightarrow \) 2P | 2.198 | 4.488 | 2.388 | 0.122 |
| 4S \( \rightarrow \) 1P | 405.8 | 287.4 | 126.3 | 768.9 |
| 4S \( \rightarrow \) 2P | 53.97 | 50.43 | 20.43 | 103.5 |
| 4S \( \rightarrow \) 3P | 0.285 | 0.620 | 0.407 | 0.880 |
| 5S \( \rightarrow \) 1P | 685.4 | 471.2 | 193.7 | 1323 |
| 5S \( \rightarrow \) 2P | 158.3 | 128.3 | 48.69 | 317.5 |
| 5S \( \rightarrow \) 3P | 15.13 | 13.26 | 5.647 | 36.96 |
| 6S \( \rightarrow \) 1P | 1283 | 858.8 | 328.0 | 2062 |
| 6S \( \rightarrow \) 2P | 466.5 | 341.7 | 122.7 | 695.3 |
| 6S \( \rightarrow \) 3P | 130.6 | 93.49 | 34.93 | 175.4 |
| 1D \( \rightarrow \) 1P | 0.258 | 6.691 | 21.19 | |
| 2D \( \rightarrow \) 1P | 4.205 | 78.91 | 147.5 | |
| 2D \( \rightarrow \) 2P | 0.288 | 8.305 | 14.63 | |
| 3D \( \rightarrow \) 1P | 9.075 | 164.7 | 277.1 | |
| 3D \( \rightarrow \) 2P | 1.821 | 38.97 | 60.39 | |
| 3D \( \rightarrow \) 3P | 0.097 | 2.438 | 4.506 | |

Table 2: The radiative decay widths of the \( S \)-wave and \( D \)-wave charmonium states to the \( P \)-wave charmonium states. The unit is \( 10^{-4} \delta^2(m, n) \).
\[
\begin{array}{c|c|c|c}
\text{Transition} & \Gamma(\psi \rightarrow \eta_c \gamma) & \Gamma(\eta_c \rightarrow \psi \gamma) & \frac{\Gamma(\eta_c \rightarrow \psi \gamma)}{\Gamma(\psi \rightarrow \eta_c \gamma)} \\
\hline
2S \rightarrow 1S & 0.089 & 0.136 & 1.519 \\
3S \rightarrow 1S & 0.244 & 0.430 & 1.762 \\
3S \rightarrow 2S & 0.021 & 0.018 & 0.848 \\
4S \rightarrow 1S & 0.384 & 0.919 & 2.392 \\
4S \rightarrow 2S & 0.071 & 0.161 & 2.281 \\
4S \rightarrow 3S & 0.012 & 0.011 & 0.911 \\
5S \rightarrow 1S & 0.502 & 1.273 & 2.536 \\
5S \rightarrow 2S & 0.127 & 0.325 & 2.550 \\
5S \rightarrow 3S & 0.035 & 0.057 & 1.614 \\
5S \rightarrow 4S & 0.002 & 0.005 & 2.384 \\
6S \rightarrow 1S & 0.712 & 1.692 & 2.375 \\
6S \rightarrow 2S & 0.253 & 0.561 & 2.221 \\
6S \rightarrow 3S & 0.106 & 0.160 & 1.505 \\
6S \rightarrow 4S & 0.024 & 0.039 & 1.616 \\
6S \rightarrow 5S & 0.005 & 0.006 & 1.143 \\
\end{array}
\]

Table 4: The ratios of the radiative decay widths of the $S$-wave charmonium states to the $S$-wave charmonium states. The unit of the widths is $\delta^2(m, n)$.

\[
\begin{array}{c|c|c|c}
\text{Transition} & \Gamma(\psi \rightarrow \chi_{c1} \gamma) & \Gamma(\psi \rightarrow \chi_{c0} \gamma) & \Gamma(\eta_c \rightarrow h_c \gamma) \\
\hline
2S \rightarrow 1P & 1.433 & 1.649 & 1.154 \\
2S \rightarrow 1P & 1.053 & 1.101 & \\
3S \rightarrow 1P & 0.766 & 0.390 & 1.195 \\
3S \rightarrow 2P & 2.042 & 1.087 & 0.056 \\
4S \rightarrow 1P & 0.708 & 0.311 & 1.895 \\
4S \rightarrow 2P & 0.934 & 0.379 & 1.918 \\
4S \rightarrow 3P & 2.176 & 1.428 & 3.090 \\
5S \rightarrow 1P & 0.688 & 0.283 & 1.931 \\
5S \rightarrow 2P & 0.810 & 0.308 & 2.006 \\
5S \rightarrow 3P & 0.876 & 0.373 & 2.443 \\
6S \rightarrow 1P & 0.669 & 0.256 & 1.607 \\
6S \rightarrow 2P & 0.732 & 0.263 & 1.490 \\
6S \rightarrow 3P & 0.716 & 0.267 & 1.343 \\
1D \rightarrow 1P & 25.91 & 82.02 & \\
1D \rightarrow 1P & > 3.222 & > 8.111 & \\
2D \rightarrow 1P & 18.77 & 35.08 & \\
2D \rightarrow 2P & 28.85 & 50.81 & \\
3D \rightarrow 1P & 18.15 & 30.54 & \\
3D \rightarrow 2P & 21.40 & 33.17 & \\
3D \rightarrow 3P & 25.27 & 46.71 & \\
\end{array}
\]

Table 5: The ratios of the radiative decay widths of the $S$-wave and $D$-wave charmonium states to the $P$-wave charmonium states. There we normalize $\Gamma(\psi \rightarrow \chi_{c2} \gamma) = 1$, the wide-hat denotes the experiential values.
Table 6: The ratios of the radiative decay widths of the $P$-wave charmonium states to the $S$-wave charmonium states. There we normalize $\Gamma_{\chi_{c2} \rightarrow \psi\gamma} = 1$, the wide-hat denotes the experiential values.

| Process       | $\Gamma(\chi_{c1} \rightarrow \psi\gamma)$ | $\Gamma(\chi_{c0} \rightarrow \psi\gamma)$ | $\Gamma(h_c \rightarrow \eta_c\gamma)$ |
|---------------|--------------------------------------------|--------------------------------------------|---------------------------------------|
| $1P \rightarrow 1S$ | 0.756                                      | 0.363                                      | 1.549                                 |
| $1P \rightarrow 1S$ | 0.770                                      | 0.311                                      |                                       |
| $2P \rightarrow 1S$ | 0.842                                      | 0.744                                      | 1.262                                 |
| $2P \rightarrow 2S$ | 0.462                                      | 0.279                                      | 1.360                                 |
| $3P \rightarrow 1S$ | 0.952                                      | 0.866                                      | 1.148                                 |
| $3P \rightarrow 2S$ | 0.854                                      | 0.748                                      | 1.124                                 |
| $3P \rightarrow 3S$ | 0.566                                      | 0.342                                      | 2.803                                 |

Figure 1: The radiative decay widths of the $X(3872)$ versus its mass, the $A$ and $B$ denote the processes $X(3872) \rightarrow J/\psi\gamma$ and $X(3872) \rightarrow \psi'\gamma$, respectively; the units of the decay widths are $10^{-2}\delta^2(2,1)$ and $10^{-3}\delta^2(2,2)$, respectively, $\delta(2,2) \gg \delta(2,1)$.
each other. The predictions can be confronted with the experimental data in the future at the BESIII, KEK-B, RHIC, PANDA and LHCb, and put additional constraints in identifying the $X$, $Y$, $Z$ charmonium-like mesons.

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