Regression functions for determining the expression of surface Roughness at plane grinding of ceramic

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Abstract. The aim of the work is to determine, through mathematical modelling using regression functions, the dependency expression between the surface roughness Ra at grinding with diamond wheels of aluminum oxide ceramic (Al2O3) for plain grinding with longitudinal advance, and the values of the grinding processing.

1. Mathematical model
Mathematical modelling of the data gathered through experiments is used to determine the expression of roughness $Ra=f(S_l,t,V_d)$, at plain grinding with longitudinal advance where: $S_l$ (m/min) - longitudinal advance; $t$ (mm) - depth of cut; $V_d$ (m/s) - peripheral speed of diamond wheel, tested on the parameters of grinding, as function of multiple linear regression.

For determination of roughness expression at plain grinding, tested on the parameters of grinding, as a power function of the form like:

$$Y = e^c \cdot S_l^x \cdot t^y \cdot V_d^z$$

(1)

where: $c$ an constant, $x$, $y$, $z$ the exponents of the variables, the expression (1) is logarithmic, obtaining:

$$\ln Y = \ln (e^c \cdot X_1^x \cdot X_2^y \cdot X_3^z)$$

(2)

which leads to obtaining the relationship:

$$\ln Y = c \cdot \ln e + x \cdot \ln X_1 + y \cdot \ln X_2 + z \cdot \ln X_3$$

(3)

which by identification with the general expression of multiple linear regression [1]:

$$Y = b_0 + b_1 \cdot X_1 + b_2 \cdot X_2 + b_3 \cdot X_3$$

(4)

leads to the relation:

$$c \cdot \ln (e) = b_0$$

which give $c = b_0$, $x = b_1$, $y = b_2$, $z = b_3$

(5)

where: $b_0$, $b_1$, $b_2$, $b_3$, $X_1$, $X_2$, $X_3$ [1] have the meanings given previously, and the function can be written as:

$$Y = e^{b_0} \cdot X_1^{b_1} \cdot X_2^{b_2} \cdot X_3^{b_3}$$

(6)
2. Experimental components

The material used is a ceramic material with a high content of aluminum oxide, whose properties are [2]: Al$_2$O$_3$: 99.5 (%), density: 3450 (kg/m$^3$), porosity: 1 (%), breaking resistance: 250 (MPa), compression strength: 1180 (MPa), flexion resistance 175 (MPa), resiliens: 2.9 (N/mm$^2$), hardness: 2000 (Vikers), modulus of elasticity: 210 (MPa), thermal expansion module: 8.3 \times 10^{-6} \, (K^{-1}).

Processing was carried out on a grinding machine RPO-200, and measurements was performed on a roughness score FORM-Talysurf-120, and the diamond disk was 1-A1-150-10-4-D125-R-C75. Given the possibility of kinematic machine tool, the parameters of the system have changed in next order: [3]

- depth of cut $t$ (mm): 0.02 (mm); 0.03 (mm); 0.04 (mm); 0.05 (mm); 0.06 (mm); 0.07 (mm)
- longitudinal advance $S_l$ (m/min): 1 (m/min); 2 (m/min); 2.5 (m/min); 3 (m/min)
- peripheral speed of diamond wheel $V_d$ (m/s): 15 (m/s); 23 (m/s); 28 (m/s); 31 (m/s).

| $t$ (mm) | $S_l$ (m/min) | $R_a$ (µm) |
|----------|---------------|------------|
|          | $V_d=15$ (m/s) | $V_d=23$ (m/s) | $V_d=28$ (m/s) | $V_d=31$ (m/s) |
| 0.02     | 1             | 0.8431     | 0.7485     | 0.6524     | 0.6475     |
| 0.03     | 1             | 0.8564     | 0.7623     | 0.6595     | 0.6491     |
| 0.04     | 1             | 0.8827     | 0.7761     | 0.6687     | 0.6511     |
| 0.05     | 1             | 0.8912     | 0.7922     | 0.6732     | 0.6572     |
| 0.06     | 1             | 0.8953     | 0.7981     | 0.6996     | 0.6832     |
| 0.07     | 1             | 0.9058     | 0.8024     | 0.7036     | 0.6934     |
| 0.02     | 2             | 0.8614     | 0.7628     | 0.6727     | 0.6398     |
| 0.03     | 2             | 0.8725     | 0.7745     | 0.6812     | 0.6413     |
| 0.04     | 2             | 0.8963     | 0.7896     | 0.6983     | 0.6487     |
| 0.05     | 2             | 0.8975     | 0.7951     | 0.7075     | 0.6528     |
| 0.06     | 2             | 0.9012     | 0.8076     | 0.7181     | 0.6669     |
| 0.07     | 2             | 0.9427     | 0.8154     | 0.7234     | 0.7104     |
| 0.02     | 2.5           | 0.8763     | 0.7717     | 0.6793     | 0.6654     |
| 0.03     | 2.5           | 0.8824     | 0.7897     | 0.7021     | 0.6797     |
| 0.04     | 2.5           | 0.8979     | 0.7984     | 0.7153     | 0.6821     |
| 0.05     | 2.5           | 0.9233     | 0.8057     | 0.7334     | 0.7169     |
| 0.06     | 2.5           | 0.9357     | 0.8238     | 0.7645     | 0.7483     |
| 0.07     | 2.5           | 0.9421     | 0.8345     | 0.7892     | 0.7651     |
| 0.02     | 3             | 0.8862     | 0.7851     | 0.6859     | 0.6712     |
| 0.03     | 3             | 0.8998     | 0.7978     | 0.6908     | 0.6748     |
| 0.04     | 3             | 0.9371     | 0.8026     | 0.6993     | 0.6875     |
| 0.05     | 3             | 0.9547     | 0.8264     | 0.7367     | 0.7247     |
| 0.06     | 3             | 0.9772     | 0.8415     | 0.7782     | 0.7323     |
| 0.07     | 3             | 0.9964     | 0.8597     | 0.7956     | 0.7767     |

The parameter of surface roughness $R_a$ was measured in three different areas of the workpiece surface processed, and the values taken into account is the average of the three measured values.

With each longitudinal advances $S_l$ and depth of machining $t$, having grinding to the four peripheral speed $V_d$ of diamond disc, every time the measured parameter $R_a$ under the conditions present in the previous chapter, the results are presented in Table 1 (roughness values representing the average of the measurements at the three points of measurement) [3].
3. Determination the Expression of Roughness at Plain Grinding Regression

To determine the expression of roughness Ra µm depending on the parameters of the cutting regime as a linear function of multiple regression as:

\[ Ra = e^{c \cdot S_l^x \cdot t^y \cdot V_d^z} \]  

(7)

where: \( x, y, z, S_l, t, V_d \) have the meaning mentioned, it solves the normal equations, are constituting the matrix \( B[4] \) of variables at the following form:

the expression (7) is logarithmed, obtaining:

\[ \ln (Ra) = \ln (e^{c \cdot S_l^x \cdot t^y \cdot V_d^z}) \]  

(8)

which leads to the relation:

\[ \ln(Ra) = c \cdot \ln e + x \cdot \ln S_l + y \cdot \ln t + z \cdot \ln V_d \]  

(9)

which by identification with the general expression of the multiple linear regression (4) leads to the relation:

\[ c \cdot \ln(e) = b_0 \]  

(10)

where: \( b_0, b_1, b_2, b_3 \), \( X_l, X_2, X_3[4] \): have the meanings given previously, and the function can be written as: the matrix of the variables being of the form:

\[
B = \begin{bmatrix}
\ln (S_{l11}) & \ln (t_{11}) & \ln (V_{d11}) & \ln (R_{a11}) \\
\ln (S_{l21}) & \ln (t_{21}) & \ln (V_{d21}) & \ln (R_{a21}) \\
\vdots & \vdots & \vdots & \vdots \\
\ln (S_{ln1}) & \ln (n_{1}) & \ln (V_{dn1}) & \ln (R_{an1})
\end{bmatrix}
\]  

(11)

With next notation:

\( X_1 := B<0> \) - column with the values of logarithm of longitudinal advance \( S_l \) in the Table 1
\( X_2 := B<1> \) - column with the values of logarithm of depth of cut \( t \) in the Table 1
\( X_3 := B<2> \) - column with the values of logarithm of peripheral speed of wheel \( V_d \) in the Table 1
\( Y := B<3> \) - column with the values of logarithm of roughness \( Ra \) in the Table 1

Determination of \( Ra = e^{c \cdot S_l^x \cdot t^y \cdot V_d^z} \) have been made for next peripheral speed of the diamond disc: \( V_d = 15 \) (m/s), \( V_d = 23 \) (m/s), \( V_d = 28 \) (m/s), \( V_d = 31 \) (m/s) and are presented in the following subsections.

The term roughness \( Ra = e^{c \cdot S_l^x \cdot t^y \cdot V_d^z} \) to the plane of the correction \( V_d = 15 \) (m/s).

For the determination of multiple linear regression coefficients, with the data are made matrix of the logarithm of variables and by solving the normal equation [1] were obtain the following values for multiple linear regression coefficients [5]:

\[
\begin{bmatrix}
b_0 \\
b_1 \\
b_2 \\
b_3
\end{bmatrix} = \begin{bmatrix}
1.943 \\
0.054 \\
0.070 \\
-0.680
\end{bmatrix}
\]

and the regression expression would have the form:

\[ Ra = e^{1.943 \cdot S_l^{0.054} \cdot t^{0.07} \cdot V_d^{-0.68}} \text{ (µm)}. \]  

(12)

The term roughness \( Ra = e^{c \cdot S_l^x \cdot t^y \cdot V_d^z} \) to the plane of the correction \( V_d=28 \) (m/s).
For the determination of multiple linear regression coefficients, with the data from Table 3 are made matrix of the logarithm of variables and by solving the normal equation [1] were obtained the following values for multiple linear regression coefficients:

\[
\begin{bmatrix}
  b_0 \\
  b_1 \\
  b_2 \\
  b_3
\end{bmatrix} = \begin{bmatrix}
  0.014 \\
  0.040 \\
  0.061 \\
  -0.019
\end{bmatrix}
\]

and the regression expression would have the form:

\[ Ra = e^{0.014 \cdot SI^{0.04} \cdot t^{0.061} \cdot Vd^{0.19}} (\mu m). \]  
\( (13) \)

The term roughness \( Ra = e^{c \cdot SI \cdot t \cdot Vd} \) to the plane of the correction \( Vd = 28 \) (m/s).

For the determination of multiple linear regression coefficients, with the data from Table 3 are made matrix of the logarithm of variables and by solving the normal equation [1] were obtained the following values for multiple linear regression coefficients:

\[
\begin{bmatrix}
  b_0 \\
  b_1 \\
  b_2 \\
  b_3
\end{bmatrix} = \begin{bmatrix}
  0.152 \\
  0.074 \\
  0.091 \\
  -0.730
\end{bmatrix}
\]

and the regression expression would have the form:

\[ Ra = e^{0.152 \cdot SI^{0.074} \cdot t^{0.091} \cdot Vd^{0.73}} (\mu m). \]  
\( (14) \)

The term roughness \( Ra = e^{c \cdot SI \cdot t \cdot Vd} \) to the plane of the correction \( Vd = 31 \) (m/s).

For the determination of multiple linear regression coefficients, with the data from Table 1 are made matrix of the logarithm of variables and by solving the normal equation [1] were obtained the following values for multiple linear regression coefficients:

\[
\begin{bmatrix}
  b_0 \\
  b_1 \\
  b_2 \\
  b_3
\end{bmatrix} = \begin{bmatrix}
  2.600 \\
  0.064 \\
  0.109 \\
  -0.768
\end{bmatrix}
\]

and the regression expression would have the form:

\[ Ra = e^{2.6 \cdot SI^{0.064} \cdot t^{0.019} \cdot Vd^{0.768}} (\mu m). \]  
\( (15) \)
4. Evaluation of the roughness

Figure 1. The dependence of the Ra at grinding with Vd =15 (m/s), and Sl =2.5 (m/min)

Figure 2. The dependence of the Ra at grinding with Vd =15 (m/s), and Sl =3 (m/min)

Figure 3. The dependence of the Ra at grinding with Vd =23 (m/s), and Sl =1 (m/min)

Figure 4. The dependence of the Ra at grinding with Vd =23 (m/s), and Sl =2.5 (m/min)

Figure 5. The dependence of the Ra at grinding with Vd =28 (m/s), and Sl =2 (m/min)

Figure 6. The dependence of the Ra at grinding with Vd =28 (m/s), and Sl =2.5 (m/min)
Figure 7. The dependence of the Ra at grinding with $V_d = 31$ (m/s), and $S_L = 1$ (m/min)

Figure 8. The dependence of the Ra at grinding with $V_d = 31$ (m/s), and $S_L = 3$ (m/min)

5. Conclusions
The analysis of the diagrams presented in Figures 1 … 8 can lead to the following conclusions:

1. The variation curves of the roughness Ra analytically deducted, approximate better the experimental results for the peripheral speed of the diamond wheel of $V_d = 28$ (m/s) and $V_d = 15$ (m/s) than for the speeds of $V_d = 31$ (m/s) $V_d = 23$ (m/s);
2. The variation curves of the roughness Ra analytically deducted, approximate better the experimental results for the longitudinal advance $S_L = 2$ (m/min) and $S_L = 2.5$ (m/min);
3. The variation curves of the roughness Ra analytically deducted, approximate better the experimental results for the depth of cut $t = 0.02$ mm;
4. The variation of the roughness for the analytical curves is between 0.87-0.99 µm for the peripheral speed of the diamond wheel of $V_d = 15$ (m/s) is between 0.8-0.88 µm for the peripheral speed of the diamond wheel of $V_d = 23$ (m/s), between 0.65-0.78 µm for the peripheral speed of the diamond wheel of $V_d = 28$ (m/s) and between 0.66-0.77 µm for the peripheral speed of the diamond wheel of $V_d = 31$ (m/s);
5. The variation of the roughness of the analytical roughness decreasing when the peripheral speed of the diamond wheel increase from $V_d = 15$ (m/s) to $V_d = 31$ (m/s);
6. The variation of the analytical roughness increases steadily with increasing of depth of cut of processing;
7. The variation of the analytical roughness increases steadily with increasing of longitudinal advance $S_L$ of processing;
8. It is recommended to be used the following expressions to determine the roughness at the plain grinding of the aluminum oxide:

$$Ra = e^{1.943 \cdot S_L^{0.054} \cdot t^{0.07} \cdot V_d^{2-0.68}} \text{ (µm)}$$ to the plane of the correction $V_d = 15$ (m/s);

$$Ra = e^{0.014 \cdot S_L^{0.04} \cdot t^{0.061} \cdot V_d^{2-0.19}} \text{ (µm)}$$ to the plane of the correction $V_d = 23$ (m/s);

$$Ra = e^{0.152 \cdot S_L^{0.074} \cdot t^{0.091} \cdot V_d^{2-0.73}} \text{ (µm)}$$ to the plane of the correction $V_d = 28$ (m/s);

$$Ra = e^{2.6 \cdot S_L^{0.064} \cdot t^{0.019} \cdot V_d^{2-0.768}} \text{ (µm)}$$ to the plane of the correction $V_d = 31$ (m/s).

6. References
[1] Montgomery D C 2012 Introduction to linear regression analysis pp 13-20.
[2] Bulea H 2019 Mathematical Model Using Regression Functions for Determining the Expression of Surface Roughness at Plain Grinding with Diamond Wheels of Aluminum Oxide Ceramic $\text{Al}_2\text{O}_3$ Jurnal BDI Recent pp 64-69
[3] Bulea H 1999 Optimization Technology for processing of minerals and ceramic parts materials Transilvania University of Brasov (Brasov) p144-150
[4] Constantinescu L 1980 Processing of experimental data with numerical computersanother reference (Bucureşti: Ed Tehnică) pp 121-123
[5] *** 1998 Mathcad 7 Professional Chapter 1