Optimization of Adaptive Cruise Control system Controller: using Linear Quadratic Gaussian based on Genetic Algorithm

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Abstract: In this paper, we propose a novel linear quadratic Gaussian controller whose parameters can be automatically tuned. While linear quadratic regulator method has been widely used in adaptive cruise control system design, researchers have not yet proposed a feasible and effective linear quadratic regulator. Neither did they well tackle with process and measurement noise, nor did they solve the tedious and inefficient tuning problem in adaptive cruise control system design. To compensate for the noise, the authors introduced linear quadratic Gaussian where Kalman filter is applied. And to make the tuning process more efficient, genetic algorithm is used to search for the optimal linear quadratic Gaussian parameters.

Key Words: Linear Quadratic Regulator Linear Quadratic Gaussian, Genetic Algorithm, Optimization Control

1 Introduction

Advanced Driver Assistance System (ADAS) has become a hot topic since 1990s. ADAS provides the drivers not only warning signals i.e. lane departure warning (LDW) signals when the vehicle departs from the current lane and forward collision warning signals (FCW) when there is danger of car crash, but also subjective vehicle control functions like Lane Keeping Assistant (LKA), and Adaptive Cruise Control (ACC) by controlling throttle valve, brake and steering wheel. ACC system is one of the most important active control functions in ADAS. Equipped with ACC, a vehicle can not only cruise at a preset speed, but also maintain a desired distance with a longitudinal preceding vehicle. ACC system, therefore, relax drivers in both of long-time highway driving and urban traffic congestion.

ACC system consists of measurement sensors, controller, and vehicle actuators. Measurement sensors are used to detect vehicle surroundings. Typically, smart camera (which can measure distance and velocity) and millimeter-wave radar, by data fusion method, are used as measurement sensors, providing longitudinal vehicle information like relative distance and relative velocity to controller. Desired acceleration is calculated by controller to maintain a desired distance. Finally, vehicle actuators i.e. throttle valve and brake realize the desired acceleration.

The focus of this paper is the optimization of ACC controller. Many optimization methodologies have been applied to different controllers to ensure vehicle safety, increase traffic efficiency, and improve driver comfortability. One of the most popular controllers is Proportional-Integral-Derivative(PID) controller, which use relative distance, relative velocity and relative acceleration as inputs, and desired acceleration as output.\textsuperscript{[1]}\textsuperscript{[2]} used fuzzy-PID method to optimize the PID controller, through which error between desired distance and real distance is better minimized, but the weakness of using PID controller is the large amount of time needed for parameter tuning, and PID cannot predict the future motion of vehicle. Model Predictive Controller (MPC) is also used to design ACC controller algorithm\textsuperscript{[3]}.\textsuperscript{[4]} introduced adaptive Neuro-Fuzzy Predictive Control(ANFPC) where predictive control law is derived by Fuzzy Neural Networks (FNNs) to optimize predictive control. But MPC is computationally expensive, and the tuning process is time consuming. Some researchers also used fuzzy logic as a optimization method to design ACC controller\textsuperscript{[5]}. One of the advantages of applying fuzzy logic method is that it is suitable for multi-paramaters and nonlinear control problems. Another advantage is that the transfer function of the system is not necessary, it makes use of human empirical control reaction to the surrounding environment. Due to this reason, however, a large amount of data is needed to determine the fuzzy logic rules, which again costs much time.

Linear Quadratic Regulator (LQR) has been applied to ACC upper controller design\textsuperscript{[6]}. In terms of real situation application, the advantage of using LQR controller is (1) the removal of what we consider in current MPC approaches to be tedious tuning process, that is, the control horizon N (the number of future controls moves in the current optimal control step\textsuperscript{[7]}). (2) It is also shown that LQR is less computationally expensive than MPC. However, when applying LQR method in ACC controller design, engineers still need to face the tuning problem, but few well tackles with the LQR parameters tuning issue, deals with vehicle nonlinearity, parametric uncertainty, and measurement noise in real situations. Up till now, no researchers have put forward effective and feasible tuning methods in ACC controller design that uses LQR method.\textsuperscript{[8]} used LQR method in ACC controller design, but the author directly chose constant Q and R without explaining how the constant parameters are determined.\textsuperscript{[9]} also used LQR method. The author, however, goes only through several different Q parameters, by simulation comparison, to determine the most appropriate parameters.\textsuperscript{[10]} noticed that the

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importance of driver comfortability increases with the increase of vehicle speed. The authors, therefore, fixed all parameters except the one corresponds to vehicle acceleration which indicates driver comfortability. By tuning this parameter in a constrained range, they tried to find the optimal choice under different vehicle speed. But this tuning method neither considered the effects of the other parameters, nor explained how parameter initialization is determined.

The main contribution of this paper is: (1) improvement of dynamics state space which considers time delay and gain K effect. (2) Due to the real situations where there are measurement noise and process noise, the state can be inaccurately calculated or measured, the “optimal” result, accordingly, may be suboptimal. The authors, therefore, introduced LQG, where noise is simply considered as Gaussian white noise. By using LQG, real situation noise is greatly compensated by Kalman filter. (3) In order to better improve the parameter combination of Q and R parameters, the authors used Genetic Algorithm (GA) to automatically tuning the Q and R parameters to minimize cost function. Automatically-tuning Linear Quadratic Gaussian (ALQG) method, to a great extent, compensate for these effects by automatically tuning Q and R parameters and by considering real situation noise. In simulation analysis, ALQG has better performance over LQR and MPC, and it is also less computationally expensive than MPC.

The paper is organized as follows. The vehicle dynamic model is described in section II. The weakness of some current vehicle dynamic models in use are also discussed. In section III, the design of ACC controller based on LQR and LQG methods are explored. ALQG method is given in section IV. The simulation results which compares between LQR, MPC, and ALQG are verified in section V. Finally, future work and conclusion are given in section VI.

2 Vehicle Modeling

ACC system modeling is illustrated in Fig. 1. ACC controller can be divided into upper controller and lower controller. Upper controller calculates desired acceleration based on current road situations, and lower controller translates the desired acceleration into brake and throttle so that vehicle actuator can realize. In Fig.1, the preceding vehicle is denoted by lowercase p, and the following vehicle is denoted by lowercase f. The inputs of upper controller are following vehicle velocity $v_f$, following vehicle acceleration $a_f$, real distance between two vehicles $d$, desired distance $d_{des}$, preceding vehicle velocity $v_p$, and preceding vehicle acceleration $a_p$. The output of upper controller is desired acceleration $a_{f,des}$. The input of lower controller is desired acceleration $a_{f,des}$, and it is translated to brake and throttle as outputs. The objective of ACC system is to minimize the error between desired distance and real distance, relative velocity, and acceleration of the following vehicle.

For the sake of simplicity, desired distance with fixed headway time $\tau_h$ and constant safe distance $d_0$ are used:

$$d_{desire} = \tau_h v_f + d_0$$  \hspace{1cm} (1)

This paper deals mainly with the ACC upper controller optimization, the lower controller and vehicle, therefore, is combined together and simplified as a first order system with time constant $T_L$ and gain $K_L$. Time constant $T_L$ and gain $K_L$ well simulate real vehicle dynamics which always has a time delay and different between real acceleration and desired acceleration. The relationship between desire acceleration $a_{f,des}$ calculated by upper controller, and real acceleration $a_f$ actuated by lower controller and vehicle is as follows:

$$a_f = \frac{K_L}{\tau_f T_L^2 + 1} a_{f,des}$$  \hspace{1cm} (2)

To rewrite the vehicle dynamic system in a state space form, distance error $d_{error}$, relative velocity $v_{rel}$ and acceleration of following vehicle $a_f$ are chosen as the three states, Distance error is a parameter indicating safety. Relative velocity is directly related to traffic efficiency, and acceleration of following vehicle denotes driver comfortability. They are as follows:

$$d_{error} = d_{desire} - d$$  \hspace{1cm} (3)

$$v_{rel} = v_p - v_f$$  \hspace{1cm} (4)

$$a_f = \frac{d}{dt} v_f$$  \hspace{1cm} (5)

Then the state space is written as follows:

$$\dot{x} = Ax + Bu + Fw$$  \hspace{1cm} (6)

Where

$$x = \begin{bmatrix} d_{error} \\ v_{rel} \\ a_f \end{bmatrix}, A = \begin{bmatrix} 0 & -1 & \tau_h \\ 0 & 0 & -1 \\ 0 & 0 & -\frac{1}{\tau_L} \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \frac{K_L}{\tau_L} \end{bmatrix}, F = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$  \hspace{1cm} (7)

Preceding vehicle acceleration $a_p$ is seen as disturbance to the system, and the output $y$ is the three states defined above:

$$y = Cx$$  \hspace{1cm} (8)
where 
\[
C = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

It is noted that, in previous paper, researchers also used similar state space form before applying other controller design methods, but some of them made obvious mistakes in developing vehicle dynamics, thus resulting unreasonable and unreliable results. [10] chose the same states as this paper, i.e. distance error \(d_{\text{error}}\), relative velocity \(v_{\text{rel}}\) and acceleration of following vehicle \(a_f\). But in developing control matrix \(A\), the author made obvious mathematical mistakes. [11] also used LQR methods in upper controller design. But the author chose only two states, distance error \(d_{\text{error}}\) and relative velocity \(v_{\text{rel}}\). This vehicle dynamic model considers neither the time delay and gain between desired acceleration and actual acceleration, nor driver comfortability which can be represented by acceleration change rate of following vehicle \(a_f\).

### 3 LQG and LQG Application

#### 3.1 LQR and LQG

In real cases, the controller is designed in discrete form. The continuous state space is therefore discretized, by zero order hold method, into discrete form with sampling time \(T\):
\[
x_{k+1} = f(x_k,u_k,w_k)
\]
which is supposed to have equilibrium state at \(x = 0\), \(u = 0\), and \(w = 0\). In order to simplify this nonlinear problem, the system is linearized at equilibrium state, and rewritten as:
\[
\tilde{x}_{k+1} = A\tilde{x}_k + Bu_k + g_k
\]
In (9), by checking the observability matrix of \(A\) and \(C\), system states \(\tilde{x}\) are ensured observable and measurable (full rank). System inputs are constrained in a safe range. According to national regulation on ACC system, the input \(u\) is constrained in a range of \([-0.25g,0.25g]\). Process noise \(g\) is considered as Gaussian white noise with zero mean.

After linearization, the problem of search for an optimal feedback controller can be seen as a linear quadratic optimization problem. The control objective, as mentioned in previous section, is to minimize the error between desired distance and real distance, relative velocity, and acceleration of the following vehicle. Even though the cost function, theoretically, is infinite horizon, a finite yet long enough time period is chosen for feasibility of cost function calculation in real cases. The cost function, therefore, in discrete form, is given:
\[
J = \frac{1}{N} \sum_{k=0}^{N-1} [x^T(k)Qx(k) + u^T(k)Ru(k)]
\]

where \(Q\) matrix is positive semi-definite, and \(R\) matrix is positive definite. The two weighting matrices are written as:
\[
Q = \begin{bmatrix}
\rho_1 & 0 & 0 \\
0 & \rho_2 & 0 \\
0 & 0 & \rho_3
\end{bmatrix}, R = [r]
\]

The optimal output is then written in the form of \(u = -Kx\), where \(K\) can be obtained by:
\[
K = R^{-1}B^T P
\]

\(P\) can be obtained by solving the famous Algebraic Riccati Equation (ARE), which is written as:
\[
A^TP + PA + PBR^{-1}B^TP + Q = 0
\]

Once the \(Q\) and \(R\) matrices are determined, optimal \(u\) can be achieved. Only when the states are perfectly measured, the optimal \(u\) is globally optimal. In real scenarios, however, due to the process gaussian noise \(g\), the optimal \(u\) is only locally optimal. In order to compensate for the process noise effect, the input states of upper controller should be well estimated and filtered by Kalman Filter(KF).

As shown in Fig. 2, state \(y\) is first filtered by Kalman filter and then be future used. LQR with KF is what we call LQG controller. How Kalman Filter works is shown in the next subsection.

#### 3.2 Kalman Filter

Based on the state space of dynamic model, initial states and covariance matrices at previous step \(k-I\), a new state and covariance matrix can be updated at current step \(k\), which is written as:
\[
x_{kp} = Ax_{k-1} + Bu_k + w_k
\]
Then Kalman gain \(K\) can be calculated with process covariance, and measurement covariance matrices, which is written as:
\[
P_{kp} = AP_{k-1}A^T + Q_k
\]
\[
K = \frac{P_{kp}H}{HP_{kp}H^T + \bar{R}}
\]

where \(P\) is process covariance, \(Q\) is process noise covariance, \(R\) is sensor noise covariance matrix (measurement error).

Then new states \(X_k\) can be updated by using Kalman gain \(K\), previous step states \(X_{kp}\), and measurement input \(Y\), which is written as:
\[
Y_k = CX_km + Z_k
\]
\[
X_k = X_{kp} + K(Y - HX_{kp})
\]
Finally, process covariance matrix $P_k$ is updated by Kalman gain $K$ and previous step process covariance matrix $P_{kp}$, which is written as:

$$P_k = (I - KH)P_{kp}$$  \hspace{1cm} (18)

With updated states $X$ and process covariance matrix $P_k$, the same filtering process can be repeated continuously.

### 4 ALQG Based on Genetic Algorithm

One of the most important work in designing LQG is tuning. Usually, manual tuning method is: (1) choose $Q$ and $R$ values as the inverse of the square of the maximum value for the corresponding $x$ and $u$. (2) modify the elements to obtain a compromise among response time, damping ratio, and control effort. Most researchers used manual tuning method in LQG design. But the disadvantage of manual tuning is it is very time consuming. In this section, GA is used to optimize LQG tuning process. By name, genetic algorithms try to solve an optimization problem by mimicking the evolutionary process in nature. Over generations, natural populations evolve according to the principles of natural selection and “survival of the fittest”.

The general process of genetic algorithm is shown in Fig.3. In the design of automatically-tuning LQG controller, each individual consists of four parts, i.e. the three $q$’s parameters in $Q$ matrix, and one $r$ parameter in $R$ matrix. Each parameter is denoted by seven 16-bits binary strings. Therefore, each individual has twenty eight 16-bits binary strings which is written as:

$$S = \{s_1, s_2, \ldots, s_n\} \hspace{1cm} (n=28)$$  \hspace{1cm} (19)

Population set $S$ is first initialized with a size of 1000. The fitness of each individual is calculated with its cost function:

$$f(s) = \frac{1}{1+f(s)}$$  \hspace{1cm} (20)

By using Weighted Roulette Wheel (WRW) method, integers between 1 and 1000 are randomly generated. $S$ is then replaced with the $n$ individuals corresponding to the $n$ generated random numbers. In reproduction, two individuals $s$ and $s'$ are selected, the crossover $(s, s')$ are inserted into $S$, and the original $s$ and $s'$ is deleted from $S$. In this design, we suppose there is no mutation. The same iteration is repeated for 500 times, then we will get the optimal individual(s) which can minimize the cost function.

### 5 Simulation Studies

In this section, system performances of different controller, under different working scenarios, are studied and verified with MATLAB/Simulink and Carsim. Distance error, relative velocity and following vehicle acceleration are selected as the main criteria to judge the performance of different controllers. In the first subsection, LQR, ALQG, and MPC are selected. First, ideal scenarios where there is no noise are studies. Then, in real-situation scenarios, high frequency Gaussian white noise is introduced so that we can see how different controllers react to system noise. In both scenarios, preceding vehicle velocity is designed to vary in a sinusoid wave, which is shown in Fig.4. In the second subsection, comparisons between (1) LQR and ALQG and (2) MPC and ALQG are studied.

#### 5.1 Ideal and Real Scenarios

In ideal scenarios, noise is ignored. LQR, ALQG and MPC controllers are individually studied under the same initial condition. Distance error, relative velocity and following vehicle velocity are separately displayed in three figures. Fig.5 is system performance of LQR. Fig.6 is system performance of MPC, and Fig.7 is system performance of ALQG.

(a) distance error

(b) following vehicle acceleration
Similarly, under real scenarios where noise is introduced, system performances of the three controllers are also separately studied. Due to the high frequency noise that we deliberately introduced, following vehicle acceleration varies more frequently when comparing with the ideal scenarios. Fig.8 shows system performance of LQR, Fig.9 MPC, and Fig.10 ALQG. According to Fig.8, LQR cannot well tackle with disturbance, following vehicle acceleration is interfered with noise. According to Fig.9 and Fig.10, even though MPC, to some extent, compensate for noise effect by prediction, ALQG still has better performance using Kalman filter and automatically tuning parameters.
5.2 Comparison of Different Controllers

In this part, only real scenarios with noise are selected for comparison. We first compare system performance between LQR and ALQG in Fig.11, then compare system performance between MPC and ALQG in Fig.12.

In Fig.11(a), ALQG has less distance error than LQR. As preceding vehicle velocity varies, ALQG can quickly respond to the distance error, and constrains the error in a relatively small range. Basically, ALQG has 2m less distance error than LQR. In terms of following vehicle acceleration shown in Fig.11(b), ALQG also has better performance. The absolute value of following vehicle acceleration and acceleration change rate of ALQG are smaller than that of LQR. Meanwhile, due to Kalman filter, the effect of real situation noise is greatly reduced. Therefore, the following vehicle acceleration of ALQG is much smoother than LQR. As for relative velocity shown in Fig.11(c), LQR, in most of the time, is at least twice as big as ALQG.

In Fig.12(a), ALQG still has slightly better performance than MPC, but usually, ALQG has only 1m less distance error than MPC. In terms of following vehicle acceleration shown in Fig.12(b), MPC output is very similar with the output of ALQG. The value of following vehicle acceleration of ALQG and MPC are almost in the same range. But it is obvious that MPC has bigger overshoots. According to Fig.12(b), at time 11.5s and 16.0s, there are two obvious overshoots in MPC. While in ALQG, at the same time period, the following vehicle acceleration varies much smoother. As for relative velocity, ALQG also keeps a relatively smaller value than MPC.

6 Future Work and Conclusion

This paper used LQR method to design ACC upper controller. In consideration of process noise and measurement noise, the authors introduced LQG which applies Kalman Filter to compensate for noise. Then GA is used to automatically tune LQG parameters so that tedious manual tuning is replaced by efficiency and intelligent tuning method.

Stability study is still necessary to check the robustness of the newly developed and optimized controller. In addition to simulation, real vehicle tests are also needed to show the feasibility of the controller.

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