Cheng–Weyl vector field and its cosmological application

Hao Wei\textsuperscript{1,2} and Rong-Gen Cai\textsuperscript{2}

\textsuperscript{1} Department of Physics and Tsinghua Center for Astrophysics, Tsinghua University, Beijing 100084, People’s Republic of China
\textsuperscript{2} Institute of Theoretical Physics, Chinese Academy of Sciences, PO Box 2735, Beijing 100080, People’s Republic of China
E-mail: haowei@mail.tsinghua.edu.cn and cairg@itp.cn

Received 20 June 2007
Accepted 24 August 2007
Published 17 September 2007

Online at stacks.iop.org/JCAP/2007/i=09/a=015
doi:10.1088/1475-7516/2007/09/015

Abstract. Weyl’s idea on scale invariance was resurrected by Cheng in 1988. The requirement of local scale invariance leads to a completely new vector field, which we call the ‘Cheng–Weyl vector field’. The Cheng–Weyl vector field couples only to a scalar field and the gravitational field naturally. It does not interact with other known matter in the standard model of particle physics. In the present work, the (generalized) Cheng–Weyl vector field coupled with the scalar field and its cosmological application are investigated. A mixture of the scalar field and a so-called ‘cosmic triad’ of three mutually orthogonal Cheng–Weyl vector fields is regarded as the dark energy in the universe. The cosmological evolution of this ‘mixed’ dark energy model is studied. We find that the effective equation-of-state parameter of the dark energy can cross the phantom divide $w_{de} = -1$ in some cases; the first and second cosmological coincidence problems can be alleviated at the same time in this model.

Keywords: dark energy theory, cosmology of theories beyond the SM
1. Introduction

Since the discovery of present accelerated expansion of our universe [1]–[7], dark energy [8] has been an active field in modern cosmology. One of the puzzles of the dark energy problem is the (first) cosmological coincidence problem, namely, why does our universe begin accelerated expansion recently and why are we living in an epoch in which the energy densities of dark energy and dust matter are comparable? In order to give a reasonable interpretation to the (first) cosmological coincidence problem, many dynamical dark energy models have been proposed: quintessence [9,10], phantoms [11]–[13], k-essence [14,15] etc.

The equation-of-state parameter (EoS) of dark energy \( w_{de} \equiv p_{de}/\rho_{de} \) plays a central role in observational cosmology, where \( p_{de} \) and \( \rho_{de} \) are its pressure and energy density respectively. Recently, by fitting the observational data, marginal evidence for \( w_{de}(z) < -1 \) at redshift \( z < 0.2–0.3 \) has been found [16]–[18]. In addition, many best fits of the present value of \( w_{de} \) are less than \(-1\) in various data fittings with different parameterizations (see [19] for a recent review). The present observational data seem to slightly favor an evolving dark energy with \( w_{de} \) crossing \(-1\) from above to below in the near past [17]. Obviously, the EoS of dark energy \( w_{de} \) cannot cross the so-called phantom divide \( w_{de} = -1 \) for quintessence or phantom alone. Although at first glance it seems possible for some variants of k-essence to give a promising solution for crossing the phantom divide, a no-go theorem, shown in [20], shatters this kind of hope. In fact, it is not a trivial task to build a dark energy model whose EoS can cross the phantom divide. To this end, a lot of efforts [21]–[39,68] have been made: to name a few, the quintom model, string theory inspired models, vector field models, crossing the phantom divide in the braneworld models, scalar–tensor models [68]. However, to our knowledge, many of those models only provide the possibility that \( w_{de} \) can cross \(-1\). They do not answer another question, i.e., why does crossing the phantom divide occur recently? Why
are we living in an epoch $w_{de} < -1$? This can be regarded as the second cosmological coincidence problem [35, 36].

Although cosmological observations hint that $w_{de} < -1$, there is a subtle tension between observations and theory. For the canonical scalar phantom model [11], the universe has an inevitable fate of the big rip [12], and the instability is inherent [13]. For the k-essence model with EoS less than $-1$, spatial instabilities inevitably arise too [40] (see also [20]). In a more general case, it is argued that there is a direct connection between instability and the violation of the null energy condition (NEC) [41, 42]. Some models can evade this result, at the price of the lack of isotropy of the background and the presence of superluminal modes [43, 44]. Recently, two seemingly viable models that violate the NEC without instability or other pathological features have been proposed in different ways [45, 46]. In particular, in [46], a scalar field coupled with a vector field is used; and the effective Lagrangian explicitly depends on the vector field $A_\mu$, which avoids one of the assumptions of [41, 42] that the effective Lagrangian only depends on $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ and has no dependence on the vector field $A_\mu$ itself.

Motivated by the work of [46], we are interested in studying the case of a scalar field coupled with a vector field. In fact, the vector field has been used in modern cosmology in many cases; see e.g. [36, 47] and references therein. It is worth noting that comparing to the ones investigated in e.g. [69, 70], the vector fields used in [36, 47] have fairly different motivations and forms. In [30], a single dynamical scalar field is coupled with an a priori non-dynamical background vector field with a constant zeroth component. In that model, the effective EoS can cross the phantom divide $w = -1$. However, the appearance of the a priori non-dynamical vector field has no clear physical motivation.

In [48, 49], Weyl’s old idea of scale invariance [50, 51] was resurrected by Cheng in 1988, almost 60 years later (see [52] for an independent rediscovery). The requirement of local scale invariance leads to the existence of a completely new vector field, which we call the ‘Cheng–Weyl vector field’ throughout this paper, in honor of the proposers Cheng and Weyl (a great mathematician and physicist [53]). The Cheng–Weyl vector field only couples to the scalar field and the gravitational field. It does not interact with other known matter in the standard model of particle physics, such as quarks, leptons, gauge mesons, and so on. In particular, it has no interaction with photons and electrons. So, it is ‘dark’ in this sense. As mentioned above, the fact that the Cheng–Weyl vector field naturally couples to the scalar field makes it very interesting, especially when the scalar field is considered as a dark energy candidate. Required by the local scale invariance, the potential term of the scalar field has to be of $\phi^4$ form, while the form of the coupling between the Cheng–Weyl vector field and the scalar field is fixed also, and the form is different from the ones of [30, 46]. Interestingly, the effective Lagrangian also explicitly depends on the Cheng–Weyl vector field itself naturally. In section 2, we will give a brief review of the work of Cheng [48, 49], in which the Cheng–Weyl vector field was proposed.

In the present work, the (generalized) Cheng–Weyl vector field coupled with a scalar field and its cosmological application are investigated. We regard a mixture of the scalar field and a so-called ‘cosmic triad’ of three mutually orthogonal Cheng–Weyl vector fields as the dark energy in the universe. We derive the effective energy density and pressure of the ‘mixed’ dark energy, and the equations of motion for the scalar field and the Cheng–Weyl vector field respectively. The cosmological evolution of this ‘mixed’ dark energy is studied. We find that the effective EoS of dark energy can cross the phantom divide
$w_{de} = -1$ in some cases; the first and second cosmological coincidence problems can be alleviated at the same time in this model.

This paper is organized as follows. In section 2, we will briefly present the main points of the Cheng–Weyl vector field proposed in [48, 49]. In section 3, the effective energy density, pressure, and the equations of motion are obtained. In section 4, the cosmological evolution of the ‘mixed’ dark energy is investigated by means of a dynamical system [60]; the first and second cosmological coincidence problems are discussed. Finally, a brief conclusion is given in section 5.

Throughout this paper, we use the units $\hbar = c = 1$ and the notation $\kappa^2 \equiv 8\pi G$, and adopt the metric convention as $(+,-,-,-)$.

2. The Cheng–Weyl vector field

Following [48, 49], here we give a brief review of the so-called Cheng–Weyl vector field. The arguments are based on the local scale invariance. It is important to distinguish the scale invariance from the gauge invariance. The scale invariance is the invariance of the action under the change of the magnitude rather than the phase of the fields. To be definite, let us consider the distance between two neighboring spacetime points, $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$. We change the scale of the distance, for instance, changing the unit of length from meter to inch. With this change, the distance remains the same, but is measured in a different unit [49]. That is,

$$g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = \Lambda^2 g_{\mu\nu},$$

where $\Lambda$ is a constant for the global scale invariance, and is a function of space and time for the local scale invariance. Then we have $ds^2 \rightarrow \tilde{d}s^2 \equiv \tilde{g}_{\mu\nu} dx^\mu dx^\nu = \Lambda^2 ds^2$, and $g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = \Lambda^{-2} g_{\mu\nu}$. And, $\sqrt{g} \rightarrow \sqrt{|\tilde{g}|} = \Lambda^4 \sqrt{|g|}$, where $g$ is the determinant of the metric $g_{\mu\nu}$. So, the action $S = \int d^4x \sqrt{|g|} L$ is invariant under the scale transformations, provided that the Lagrangian density satisfies

$$L \rightarrow \tilde{L} = \Lambda^{-4} L.$$  

In this case, the forms of all equations in the theory remain the same.

Let us first see the case of the global scale invariance, i.e. $\Lambda$ is a constant. The Lagrangian density of a scalar field is given by

$$\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \lambda \phi^4,$$

where $\lambda$ is a dimensionless constant. The Lagrangian density of a gauge meson is

$$-\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma},$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, or $F_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a - g f^{abc} W_\nu^b W_\mu^c$ (here $g$ is a coupling constant) for the Yang–Mills theory. The Lagrangian density for a fermion $\Psi$ coupled with the electromagnetic field and the gravitational field is

$$\bar{\Psi} \Gamma^c_\mu \epsilon^\mu_c \left[ \partial_\mu + i e A_\mu - \frac{1}{2} \sigma_{ab} \epsilon_\mu^b \left( \partial_\mu \varepsilon_\nu^a - \Gamma^\rho_{\mu\nu} \varepsilon_\rho^a \right) \right] \Psi,$$

where $\Gamma^\mu_\nu = g^{\rho\sigma} (\partial_\mu g_{\rho\nu} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu})/2$, $\sigma^{ab} = (\gamma^a \gamma^b - \gamma^b \gamma^a)/4$, and $\epsilon_\mu^a$ is the tetrad satisfying $\eta_{abc} \epsilon_\mu^a \epsilon_\nu^b = g_{\mu\nu}$. It is easy to verify that the above Lagrangian densities satisfy...
equation (2) under the scale transformation equation (1) and
\[ \phi \rightarrow \tilde{\phi} = \Lambda^{-1} \phi, \quad A_\mu \rightarrow \tilde{A}_\mu = A_\mu, \quad W^\alpha_\mu \rightarrow \tilde{W}^\alpha_\mu = W^\alpha_\mu, \]
\[ \varepsilon^a_\mu \rightarrow \tilde{\varepsilon}^a_\mu = \Lambda \varepsilon^a_\mu, \quad \Psi \rightarrow \tilde{\Psi} = \Lambda^{-3/2} \Psi. \]

Next we consider the case with the theory being scale invariant locally, i.e. \( \Lambda \) is a function of space and time. Like with the well-known arguments used to deduce the existence of gauge fields, one can find that a completely new vector field \( \Pi^\mu \), namely the so-called Cheng–Weyl vector field, is required by the local scale invariance, while the replacements
\[ \partial^\mu g_{\nu\rho} \rightarrow (\partial^\mu + 2f \Pi^\mu) g_{\nu\rho}, \quad \partial^\mu g^{\nu\rho} \rightarrow (\partial^\mu - 2f \Pi^\mu) g^{\nu\rho}, \]
\[ \partial^\mu \varepsilon^a_\nu \rightarrow (\partial^\mu + f \Pi^\mu) \varepsilon^a_\nu, \quad \partial^\mu \varepsilon^a_\nu \rightarrow (\partial^\mu - f \Pi^\mu) \varepsilon^a_\nu, \]
\[ \partial^\mu \phi \rightarrow (\partial^\mu - f \Pi^\mu) \phi, \quad \partial^\mu \Psi \rightarrow (\partial^\mu - \frac{3}{2}f \Pi^\mu) \Psi, \]
are also required in these Lagrangian densities, where \( f \) is a dimensionless constant. One can verify that these Lagrangian densities with the replacements of equation (7) satisfy equation (2) under the scale transformation equations (1), (6) and
\[ \Pi^\mu \rightarrow \tilde{\Pi}^\mu = \Pi^\mu - \frac{1}{f} \partial^\mu \ln \Lambda. \]

With the replacements of equation (7), the Lagrangian density of the scalar field equation (3) becomes
\[ \frac{1}{2} g^{\mu\nu} (\partial^\mu - f \Pi^\mu) \phi (\partial^\nu - f \Pi^\nu) \phi - \lambda \phi^4. \]
Thus, the scalar field is coupled with \( \Pi^\mu \) naturally by the scale invariance. However, with the replacements of equation (7), the Lagrangian densities of the gauge meson and the fermion, i.e. equations (4) and (5) respectively, need not be altered, since the terms involving \( \Pi^\mu \) completely cancel one another [48, 49]. Therefore, we conclude that the gauge meson and the fermion do not couple with \( \Pi^\mu \). With identical arguments, the quarks and leptons etc do not couple with \( \Pi^\mu \) as well. Finally, we would like to mention that the Lagrangian density of \( \Pi^\mu \) itself [48],
\[ -\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} Y_{\mu\nu} Y_{\rho\sigma}, \]
also satisfies equation (2) under the transformations of equations (1), (6) and (8), where
\[ Y_{\mu\nu} \equiv \partial_\mu \Pi_\nu - \partial_\nu \Pi_\mu. \]

We close this section with some remarks. First, required by the local scale invariance, the potential term of the scalar field has to be of \( \phi^4 \) form, as in equations (3) and (9). Second, it is easy to see from equation (9) that the form of the coupling between the Cheng–Weyl vector field \( \Pi^\mu \) and the scalar field \( \phi \) is fixed naturally. Note that the form is quite different from the ones considered in [30, 46]. For more details on the Cheng–Weyl vector field, please see [48, 49]. In addition, one may also refer to [54, 55] for relevant papers on the scale invariance.
3. Applying the Cheng–Weyl vector field to cosmology

In modern cosmology, the scalar field is used extensively. Actually, the scalar field is one of the leading dark energy candidates. If Nature respects the local scale invariance, the so-called Cheng–Weyl vector field must exist, and couples to the scalar field inherently. If the scalar field is indeed the cause driving the accelerated expansion of the universe, we argue that the dark energy should be a mixture of the scalar field and the Cheng–Weyl vector field, which can be considered as the partner of the scalar field. This seems quite plausible when the fact that the Cheng–Weyl vector field does not interact with other known matter (so, it is ‘dark’ to them) is taken into account. Therefore it is quite interesting to study the cosmological consequence of the Cheng–Weyl vector.

We begin with the action

\[ S = S_{\text{grav}} + S_{\text{CW}} + S_m, \]  

where \( S_{\text{grav}} \) and \( S_m \) are the actions for the gravitational field and matter respectively, and

\[ S_{\text{CW}} = \int d^4x \sqrt{-g} \mathcal{L}_{\text{CW}}. \]

Naively, one may write the Lagrangian density \( \mathcal{L}_{\text{CW}} \) as

\[ \mathcal{L}_{\text{CW}} = \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi - f \Pi_\mu \phi) (\partial_\nu \phi - f \Pi_\nu \phi) - \lambda \phi^4 - \frac{1}{4} g^{\mu\alpha} g^{\nu\beta} Y_{\mu\nu} Y_{\alpha\beta}, \]

directly from equations (9) and (10). In order to have compatibility with homogeneity and isotropy, \( \Pi_\mu \) can be chosen as \( \Pi_\mu = (\Pi_0, 0, 0, 0) \), where \( \Pi_0 = \Pi_0(t) \) only depends on the cosmic time \( t \). However, in this case \( Y_{\mu\nu} = 0 \). From this Lagrangian density \( \mathcal{L}_{\text{CW}} \), for the case of homogeneous \( \phi \), one finds that \( \Pi_0 = \phi = 0 \) which is not dynamical, and the Lagrangian density \( \mathcal{L}_{\text{CW}} \) is zero actually. Thus, unfortunately, the naive approach is not viable.

Enlightened by the work of [47] (see also [36]), we can describe the dark energy as a mixture of a scalar field and a so-called ‘cosmic triad’ (in the terminology of [47]) of three mutually orthogonal Cheng–Weyl vector fields. In this case, the Lagrangian density \( \mathcal{L}_{\text{CW}} \) is given by

\[ \mathcal{L}_{\text{CW}} = \sum_{a=1}^3 \left[ \frac{\epsilon}{2} g^{\mu\nu} (\partial_\mu \phi - f \Pi_\mu^a \phi) (\partial_\nu \phi - f \Pi_\nu^a \phi) - \lambda \phi^4 - \frac{\eta}{4} g^{\mu\alpha} g^{\nu\beta} Y_{\mu\nu}^a Y_{\alpha\beta}^a \right] \]

\[ = \sum_{a=1}^3 \left[ \frac{\epsilon}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \lambda \phi^4 - \epsilon f g^{\mu\nu} \Pi_\mu^a \phi \partial_\nu \phi \right. \]
\[ + \left. \frac{\epsilon}{2} g^{\mu\nu} f^2 \Pi_\mu^a \Pi_\nu^a \phi^2 - \eta \frac{1}{4} g^{\mu\alpha} g^{\nu\beta} Y_{\mu\nu}^a Y_{\alpha\beta}^a \right] \]

\[ \equiv \sum_{a=1}^3 \mathcal{L}_{\text{CW}}^{(a)}, \]

where

\[ Y_{\mu\nu}^a \equiv \partial_\mu \Pi_\nu^a - \partial_\nu \Pi_\mu^a. \]
\( \epsilon, \eta, \lambda \) and \( f \) being dimensionless constants. Latin indices label the different Cheng–Weyl vector fields \((a, b, \ldots = 1, 2, 3)\) and Greek indices label different spacetime components \((\mu, \nu, \ldots = 0, 1, 2, 3)\). Actually, the number of Cheng–Weyl vector fields is dictated by the number of spatial dimensions and the requirement of isotropy \([36, 47]\). The italic indices are raised and lowered with the flat ‘metric’ \( \delta_{ab} \). It is worth noting that the Lagrangian density \( L_{\text{CW}} \) in equation (14) satisfies the requirement of local scale invariance. Note that in equation (14), we have generalized the original scalar field to include the cases of quintessence \((\epsilon = +1)\) and phantoms \((\epsilon = -1)\), while the Lagrangian density for the Cheng–Weyl vector fields has also been generalized by introducing the constant \( \eta \).

Varying the action (13) with equation (14), one can get the energy–momentum tensor of the ‘mixed’ dark energy as

\[
(T_{\mu\nu})^{\text{(cw)}} = \frac{2}{\sqrt{-g}} \frac{\delta S_{\text{CW}}}{\delta g^{\mu\nu}} = \sum_{a=1}^{3} \left[ -g_{\mu\nu} L_{\text{CW}}^{(a)} + 2 \frac{\delta L_{\text{CW}}^{(a)}}{\delta g^{\mu\nu}} \right],
\]

where

\[
\frac{\delta L_{\text{CW}}^{(a)}}{\delta g^{\mu\nu}} = \epsilon \left( \frac{1}{2} \partial_\mu \phi \partial_\nu \phi - f \Pi_\mu^a \partial_\nu \phi + \frac{1}{2} f^2 \Pi_\mu^a \Pi_\nu^a \phi^2 \right) - \frac{\eta}{2} g^{\sigma\beta} Y_{\mu}^{a} Y_{\nu}^{a}.
\]

From the action (13) with equation (14), one can also obtain the equations of motion for \( \phi \) and \( \Pi_\mu^a \), namely

\[
\epsilon \partial_\mu \left[ \sqrt{-g} \sum_{a=1}^{3} (g^{\mu\nu} \partial_\nu \phi - f g^{\mu\nu} \Pi_\nu^a \phi) \right] = \sqrt{-g} \sum_{a=1}^{3} (-4 \lambda \phi^3 - \epsilon f g^{\mu\nu} \Pi_\mu^a \partial_\nu \phi + \epsilon f^2 g^{\mu\nu} \Pi_\mu^a \Pi_\nu^a \phi),
\]

and

\[
\eta \partial_\mu \left( \sqrt{-g} g^{\mu\nu} g^{\nu\beta} Y_{\alpha}^{a} \right) = \epsilon \sqrt{-g} \left( f g^{\mu\nu} \phi \partial_\nu \phi - f g^{\mu\nu} f^2 \Pi_\mu^a \phi^2 \right),
\]

respectively. We consider a spatially flat Friedmann–Robertson–Walker (FRW) universe with metric

\[
ds^2 = dt^2 - a^2(t) \, dx^2,
\]

where \( a(t) \) is the scale factor. In this work, we assume the scalar field is homogeneous, namely \( \phi = \phi(t) \). Like in [47], the ansatz for the Cheng–Weyl vectors is

\[
\Pi_\mu^b = \delta_\mu^b \Pi(t) \cdot a(t).
\]

Thus, the three Cheng–Weyl vectors point in mutually orthogonal spatial directions, and share the same time-dependent length, i.e. \( \Pi^2 \equiv -g^{\mu\nu} \Pi_\mu^a \Pi_\nu^a = \Pi^2(t) \). Hence, the equations of motion (18) and (19) become, respectively,

\[
\epsilon \left( \ddot{\phi} + 3H \dot{\phi} \right) + \epsilon f^2 \Pi^2 \phi + 4 \lambda \phi^3 = 0,
\]

and

\[
\eta \left[ \dddot{\Pi} + 3H \ddot{\Pi} + \left( H^2 + \frac{\ddot{a}}{a} \right) \Pi \right] + \epsilon f^2 \Pi \phi^2 = 0,
\]
where \( H \equiv \dot{a}/a \) is the Hubble parameter, and a dot denotes the derivative with respect to the cosmic time \( t \). From equations (16), (14) and (17), we find that

\[
\rho_{\text{cw}} = (\text{cw}) T^0_0 = \frac{3}{2} \epsilon \dot{\phi}^2 + 3 \lambda \phi^4 + \frac{3}{2} \epsilon f^2 \Pi^2 \phi^2 + \frac{3}{2} \eta \left( \dot{\Pi} + H \Pi \right)^2,
\]

where \( \rho_{\text{cw}} \) is the energy density of dark energy, while

\[
(\text{cw}) T^i_0 = 0,
\]

and

\[
(\text{cw}) T^i_j = \sum_{a=1}^{3} (a) T^i_j,
\]

where

\[
(a) T^i_j = - \left[ \frac{\epsilon}{2} \dot{\phi}^2 - \lambda \phi^4 - \frac{\epsilon}{2} f^2 \Pi^2 \phi^2 + \frac{\eta}{2} \left( \dot{\Pi} + H \Pi \right)^2 \right] \delta^i_j - \left[ \epsilon f^2 \Pi^2 \phi^2 - \eta \left( \dot{\Pi} + H \Pi \right)^2 \right] \delta^i_a \delta^a_j.
\]

It is worth noting here that in fact, even adopting the ‘cosmic triad’ of three mutually orthogonal Cheng–Weyl vector fields, Lagrangian (14) is not invariant under \( SO(3) \) rotation in the internal space. Thus, the energy–momentum tensor is not strictly diagonal. One can see this point by noting that \( (\text{cw}) T^0_0 \neq 0 \), due to the second term in the right-hand side of equation (17). To overcome this inconsistency with the isotropy, the spatial volume-averaging procedure has to be employed here as was done in [56,57]. In those two papers the authors considered the non-linear electromagnetic field as the source driving the accelerated expansion of the universe. There, in order to obtain an energy–momentum tensor consistent with the FRW metric, the spatial volume-averaging procedure on the large scale [58,59] has been used. By using this procedure, the spatial volume-averaged non-diagonal components of the energy–momentum tensor become zero, namely \( (\text{cw}) T^0_0 = 0 \), while the diagonal components are kept unchanged. Therefore, in our model, the energy–momentum tensor is a spatially volume-averaged one on the cosmological scale. In this way, the energy–momentum tensor is compatible with isotropy.

The corresponding pressure of dark energy is given by

\[
p_{\text{cw}} = - (\text{cw}) T^i_i = \frac{3}{2} \epsilon \dot{\phi}^2 - \frac{3}{2} \lambda \phi^4 - \frac{\epsilon}{2} f^2 \Pi^2 \phi^2 + \frac{\eta}{2} \left( \dot{\Pi} + H \Pi \right)^2.
\]

The Friedmann equation and Raychaudhuri equation read, respectively,

\[
H^2 = \frac{\kappa^2}{3} \rho_{\text{tot}} = \frac{\kappa^2}{3} (\rho_{\text{cw}} + \rho_m),
\]

and

\[
\dot{H} = - \frac{\kappa^2}{2} (\rho_{\text{tot}} + p_{\text{tot}}) = - \frac{\kappa^2}{2} (\rho_{\text{cw}} + \rho_m + p_{\text{cw}} + p_m),
\]

where \( p_m \) and \( \rho_m \) are the pressure and energy density of matter, respectively.
From equations (24) and (28), we obtain
\[
\rho_{\text{CW}} + p_{\text{CW}} = 3 \epsilon \dot{\phi}^2 + \epsilon f^2 \Pi^2 \phi^2 + 2 \eta \left( \dot{\Pi} + H \Pi \right)^2.
\] (31)

Obviously, the EoS of dark energy \( w_{\text{CW}} \equiv p_{\text{CW}} / \rho_{\text{CW}} \) is always larger than \(-1\) for the case of \( \epsilon > 0 \) and \( \eta > 0 \), while \( w_{\text{CW}} < -1 \) for the case of \( \epsilon < 0 \) and \( \eta < 0 \). Crossing the phantom divide is impossible for both cases. However, for the case of \( \epsilon \) and \( \eta \) having opposite signs, \( w_{\text{CW}} \) can be larger than or smaller than \(-1\). Of course, for this case, crossing the phantom divide is possible.

In addition, from equations (24) and (28), we have
\[
\dot{\rho}_{\text{CW}} + 3H (\rho_{\text{CW}} + p_{\text{CW}}) = 3 \epsilon \left( \dot{\phi} + 3H \phi \right) + \epsilon f^2 \Pi^2 \phi + 4 \lambda \phi^3 \right] \\
+ 3 \left( \dot{\Pi} + H \Pi \right) \left\{ \eta \left[ \dot{\Pi} + 3H \Pi + \left( 2H^2 + \dot{H} \right) \Pi \right] + \epsilon f^2 \Pi \phi^2 \right\}.
\] (32)

Noting that \( \ddot{a}/a = H^2 + \dot{H} \) and the equations of motion for \( \phi \) and \( \Pi \), i.e. equations (22) and (23), it is easy to see that the energy conservation equation of dark energy holds, namely, \( \dot{\rho}_{\text{CW}} + 3H (\rho_{\text{CW}} + p_{\text{CW}}) = 0 \).

4. Dynamical system and cosmological evolution

In this section, we investigate the cosmological evolution of the ‘mixed’ dark energy by means of dynamical system [60]. Our main aim is to see whether this model can alleviate the coincidence problems. Like many considerations in the literature, we allow the existence of interaction between the dark energy and the background matter (usually the cold dark matter). The cases of the scalar field and vector field interacting with background matter are studied extensively; see, for example, [36], [61]–[67]. Although the Cheng–Weyl vector field does not interact with the known matter in the particle physics standard model, nothing precludes the possibility of the Cheng–Weyl vector field interacting with the cold dark matter, since the nature of cold dark matter is also unknown so far.

4.1. Dynamical system

We assume that the dark energy and background matter interact through interaction terms \( C \) and \( Q \), namely
\[
\dot{\rho}_{\text{CW}} + 3H (\rho_{\text{CW}} + p_{\text{CW}}) = -C - Q,
\] (33)
\[
\dot{\rho}_m + 3H (\rho_m + p_m) = C + Q,
\] (34)
which keep the total energy conservation equation \( \dot{\rho}_{\text{tot}} + 3H (\rho_{\text{tot}} + p_{\text{tot}}) = 0 \). The background matter is described by a perfect fluid with barotropic equation of state
\[
p_m = w_m \rho_m \equiv (\gamma - 1) \rho_m,
\] (35)
where the barotropic index \( \gamma \) is a dimensionless constant and satisfies \( 0 < \gamma \leq 2 \). In particular, \( \gamma = 1 \) and \( 4/3 \) correspond to dust matter and radiation, respectively. Due to
With the help of equations (29), (30), (24) and (28), the evolution equations (36) and (34) can be rewritten as a dynamical system [60], i.e.

\[ x' = \Theta_1 - 3 \sqrt{2} \lambda \epsilon^{-1} u \theta_2 - \sqrt{2} f^2 v^2 \theta_2^{-1} - C_1, \]

\[ y' = \Theta_1 - 3 \epsilon \eta^{-1} f^2 u^2 - 2 \eta v - Q_1, \]

\[ z' = \Theta_1 - \frac{3}{2} \gamma z + C_2 + Q_2, \]

\[ u' = u \left( \Theta_1 + \sqrt{2} xu \theta_2^{-1} \right), \]

\[ v' = v, \]

where

\[ C_1 = \frac{\kappa^2 C}{6 \epsilon H^3 x}, \quad Q_1 = \frac{\kappa^2 Q}{6 \eta H^3 (y + v)}, \quad C_2 = \frac{z C}{2H \rho_m}, \quad Q_2 = \frac{z Q}{2H \rho_m}, \]

and

\[ \Theta_1 \equiv -\frac{\dot{H}}{H^2} = 3 \epsilon x^2 + \epsilon f^2 u^2 v^2 + 2 \eta (y + v)^2 + \frac{3}{2} \gamma z^2, \]

\[ \Theta_2 \equiv \left[ 1 - z^2 - \epsilon x^2 + \epsilon f^2 u^2 v^2 - \eta (y + v)^2 \right]^{1/2}, \]

and a prime denotes the derivative with respect to the so-called e-folding time \( N \equiv \ln a \).

The fractional energy densities of the background matter and dark energy are given by

\[ \Omega_m = \frac{\kappa^2 \rho_m}{3H^2} = z^2, \]

and

\[ \Omega_{cw} = \frac{\kappa^2 \rho_{cw}}{3H^2} = \epsilon x^2 + \lambda \kappa^2 H^2 u^4 + \epsilon f^2 u^2 v^2 + \eta (y + v)^2, \]

respectively. On the other hand, from equation (29), one has

\[ \Omega_{cw} = 1 - z^2. \]

Hence, from equations (47) and (48), one can find out

\[ \kappa H = \frac{\Theta_2}{u^2}, \]

where \( \Theta_2 \) is given by equation (45). The effective EoS of the whole system is

\[ w_{\text{eff}} \equiv \frac{p_{\text{tot}}}{\rho_{\text{tot}}} = \Omega_{cw} w_{\text{cw}} + \Omega_m w_m, \]
where \( w_{cw} \equiv p_{cw}/\rho_{cw} \) and \( w_m \equiv p_m/\rho_m = \gamma - 1 \) are the EoS of dark energy and background matter, respectively.

4.2. Interaction terms and critical points

In this subsection, we obtain all critical points of the dynamical system (38)–(42). A critical point \((\bar{x}, \bar{y}, \bar{z}, \bar{u}, \bar{v})\) satisfies the conditions \( \bar{x}' = \bar{y}' = \bar{z}' = \bar{u}' = \bar{v}' = 0 \). Before giving the particular interaction terms \( C \) and \( Q \), let us first find the general features of the critical points of dynamical system (38)–(42). From equation (42) and \( \bar{v}' = 0 \), it is easy to see that

\[
\bar{y} = 0. \tag{51}
\]

If this dynamical system has some critical points, their corresponding \( \bar{x}, \bar{y}, \bar{z}, \bar{u}, \) and \( \bar{v} \) should be constants. Therefore, from equations (49) and (45), the corresponding Hubble parameter \( H = \dot{H} = \text{const} \). From equation (44),

\[
\bar{\Theta}_1 = 0 \tag{52}
\]

follows. Substituting into equation (41), \( \bar{u}' = 0 \) requires

\[
\bar{x} = 0, \tag{53}
\]

since \( \bar{u}^2 \bar{\Theta}_2^{-1} = (\kappa \dot{H})^{-1} \neq 0 \). Hence, from equations (52), (44), (51) and (53), we have

\[
(\epsilon f^2 \bar{u}^2 + 2\eta) \bar{v}^2 + \frac{3}{2} \gamma \bar{z}^2 = 0. \tag{54}
\]

So, \( \epsilon f^2 \bar{u}^2 + 2\eta < 0 \) is required for non-vanishing real \( \bar{z} \) and \( \bar{v} \). By using equations (38)–(40) and equations (51)–(54), \( \bar{x}' = \bar{y}' = \bar{z}' = 0 \) become, respectively,

\[
\sqrt{2} \bar{u} (\epsilon \bar{\Theta}_2)^{-1} \left( 2 - \frac{\gamma}{2} \bar{z}^2 \right) + \bar{C}_1 = 0, \tag{55}
\]

\[
(\epsilon f^2 \bar{u}^2 + 2\eta) \bar{v} + \eta \bar{\Theta}_1 = 0, \tag{56}
\]

\[
\bar{C}_2 + \bar{Q}_2 - \frac{3}{2} \gamma \bar{z} = 0, \tag{57}
\]

where

\[
\bar{\Theta}_2 = \left[ \frac{1 + (\frac{3}{2} \gamma - 1) \bar{z}^2 + \eta \bar{v}^2}{\lambda} \right]^{1/2}, \tag{58}
\]

which comes from equations (45), (51), (53), and (54). Then, one can find out the remaining \( \bar{z}, \bar{u} \) and \( \bar{v} \) from equations (54)–(57). Obviously, only three of them are independent of each other.

So far, the above results are independent of particular interaction terms \( C \) and \( Q \). To find out \( \bar{z}, \bar{u} \) and \( \bar{v} \), we have to choose proper \( C \) and \( Q \) here. The interaction forms extensively considered in the literature (see [34, 36], [60]–[67] for instance) are

\[
C \propto H \rho_m, H \rho_{tot}, H \rho_{cw}, \kappa \rho_m \dot{\phi}, \ldots
\]

\[
Q \propto H \rho_m, H \rho_{tot}, H \rho_{cw}, \kappa \rho_m \ddot{\Pi}, \ldots
\]
Noting equations (53), (37) and the definition of $C_1$ in equation (43), we have to choose
\[ C = \alpha \kappa \rho_m \dot{\phi}, \] (59)
to avoid the divergence of $\bar{C}_1$ in equation (55), where $\alpha$ is a dimensionless constant. In this case, from equation (43), one has
\[ C_1 = \frac{\alpha z^2}{\sqrt{2} \epsilon}, \quad C_2 = \frac{\alpha x z}{\sqrt{2}}. \] (60)

From equations (53) and (60), we find that $\bar{C}_2 = 0$ in equation (57). Noting equations (51), (53), (46) and the definition of $Q_2$ in equation (43), we cannot choose $Q \propto H \rho_m$ or $\kappa \rho_m \dot{\Pi}$, to avoid the solution $\bar{z} = 0$ for equation (57), since our main aim is to alleviate the coincidence problems. Therefore, we choose Case (I) $Q = 3\beta H \rho_{cw}$ or Case (II) $Q = 3\sigma H \rho_{tot}$, where $\beta$ and $\sigma$ are dimensionless constants.

**Case (I) $Q = 3\beta H \rho_{cw}$.** In this case, from equation (43), one has
\[ Q_1 = \frac{3\beta (1 - z^2)}{2\eta (y + v)}, \quad Q_2 = \frac{3}{2} \beta \left( z^{-1} - z \right). \] (61)

Noting that $\bar{C}_2 = 0$ in equation (57), we find out
\[ \bar{z} = \sqrt{\frac{\beta}{\beta + \gamma}}. \] (62)

One can check that equation (56) is equivalent to equation (57) for this case. Then, one can find out $\bar{u}$ and $\bar{v}$ from equations (54) and (55), by using equations (51), (53) and (62). We do not present them here, since the final results are involved and tedious. One can work them out with the help of Mathematica. Instead we would like to give several particular examples to support our statement. **Example (I.1):** for parameters $\gamma = 1$, $\epsilon = 1$, $\eta = -1$, $\alpha = 5$, $\beta = 3/5$, $\lambda = 0.1$ and $f = 5$, we have $\bar{u} = -0.246341$, $\bar{v} = \pm 1.07927$. **Example (I.2):** for parameters $\gamma = 1$, $\epsilon = 1$, $\eta = -1$, $\alpha = 5$, $\beta = 1/2$, $\lambda = 0.1$ and $f = 5$, we find that $\bar{u} = -0.249803$, $\bar{v} = \pm 1.06605$.

From equations (46), (48), and (62), the fractional energy densities of background matter and dark energy are given by
\[ \Omega_m = \frac{\beta}{\beta + \gamma}, \quad \Omega_{cw} = \frac{\gamma}{\beta + \gamma}, \] (63)
respectively. For reasonable $\Omega_m$ and $\Omega_{cw}$, it is easy to see that $\beta > 0$ is required. As mentioned above, at the critical point $(\bar{x}, \bar{y}, \bar{z}, \bar{u}, \bar{v})$, the Hubble parameter $H = \dot{H} = \text{const.}$ From equation (30), this means
\[ w_{\text{eff}} = -1. \] (64)

From equation (50), we find that the EoS of the dark energy is given by
\[ w_{cw} = -1 - \beta. \] (65)

Obviously, $w_{cw} < -1$. 

Journal of Cosmology and Astroparticle Physics 09 (2007) 015 (stacks.iop.org/JCAP/2007/i=09/a=015)
\textit{Case (II)} $Q = 3\sigma H\rho_{\text{tot}}$. In this case, the corresponding $Q_1$ and $Q_2$ read
\[ Q_1 = \frac{3\sigma}{2\eta(y + v)}, \quad Q_2 = \frac{3}{2} \sigma z^{-1}, \tag{66} \]
respectively. Solving equation (57) with $\bar{C}_2 = 0$, we get
\[ \ddot{z} = \sqrt{\frac{\sigma}{\gamma}}. \tag{67} \]
Again, one can check that equation (56) is equivalent to equation (57) for this case. Then, one can find out $\bar{u}$ and $\bar{v}$ from equations (54) and (55), by using equations (51), (53) and (67). Once again, we do not present the long and involved expressions here. We only give some particular examples. \textit{Example (II.1)}: for parameters $\gamma = 1, \epsilon = 1, \eta = -1, \alpha = 4, \sigma = 1/3, \lambda = 0.1$ and $f = 3$, we find that $\bar{u} = -0.410797, \bar{v} = \pm 1.01934$. \textit{Example (II.2)}: for parameters $\gamma = 1, \epsilon = 1, \eta = -1, \alpha = 5, \sigma = 0.3, \lambda = 0.1$ and $f = 7$, we have $\bar{u} = -0.180804, \bar{v} = \pm 1.06307$.

From equations (46), (48), and (67), the fractional energy densities of background matter and dark energy are given by, respectively,
\[ \Omega_m = \frac{\sigma}{\gamma}, \quad \Omega_{cw} = 1 - \frac{\sigma}{\gamma}, \tag{68} \]
which requires $0 < \sigma < \gamma$. Following a similar argument, we have $w_{\text{eff}} = -1$ also. And then, from equation (50), we find that the EoS of the dark energy is given by
\[ w_{cw} = -1 - \frac{\sigma \gamma}{\gamma - \sigma}, \tag{69} \]
which is also smaller than $-1$.

4.3. Stability analysis

In this subsection, we discuss the stability of these critical points. An attractor is one of the stable critical points of the autonomous system. To study the stability of these critical points, we substitute linear perturbations $x \rightarrow \bar{x} + \delta x$, $y \rightarrow \bar{y} + \delta y$, $z \rightarrow \bar{z} + \delta z$, $u \rightarrow \bar{u} + \delta u$ and $v \rightarrow \bar{v} + \delta v$ about the critical point $(\bar{x}, \bar{y}, \bar{z}, \bar{u}, \bar{v})$ into the dynamical system equations (38)–(42) and linearize them. We get the evolution equations for the fluctuations as
\begin{align*}
\delta x' &= (\bar{\Theta}_1 - 3) \delta x + \bar{x} \delta \Theta_1 - 2\sqrt{2} \lambda \epsilon^{-1} (\bar{u} \delta \Theta_2 + \bar{\Theta}_2 \delta u) + \sqrt{2} f^2 \bar{v}^2 \bar{u}^3 \Theta_2^{-2} \delta \Theta_2 \\
&\quad - \sqrt{2} f^2 (3 \bar{u}^2 \bar{v}^2 \delta u + 2 \bar{u}^3 \bar{v} \delta v) \bar{\Theta}_2^{-1} - \delta C_1, \tag{70} \\
\delta y' &= (\bar{\Theta}_1 - 3) \delta y + \bar{y} \delta \Theta_1 + (\bar{\Theta}_1 - \epsilon \eta^{-1} f^2 \bar{u}^2 - 2) \delta v + (\delta \Theta_1 - 2 \epsilon \eta^{-1} f^2 \bar{u} \delta u) \bar{v} - \delta Q_1, \tag{71} \\
\delta z' &= (\bar{\Theta}_1 - \frac{3}{2} \gamma) \delta z + \bar{z} \delta \Theta_1 + \delta C_2 + \delta Q_2, \tag{72} \\
\delta u' &= \bar{u} \left[ \delta \Theta_1 - \sqrt{2} \bar{x} \bar{u} \bar{\Theta}_2^{-2} \delta \Theta_2 + \sqrt{2} (\bar{x} \delta u + \bar{u} \delta x) \bar{\Theta}_2^{-1} \right] + \left( \bar{\Theta}_1 + \sqrt{2} \bar{x} \bar{u} \bar{\Theta}_2^{-1} \right) \delta u, \tag{73} \\
\delta v' &= \delta y. \tag{74} 
\end{align*}
where \( \delta \Theta_1, \delta \Theta_2, \delta C_1, \delta C_2, \delta Q_1 \) and \( \delta Q_2 \) are the linear perturbations coming from \( \Theta_1, \Theta_2, C_1, C_2, Q_1 \) and \( Q_2 \), respectively. The five eigenvalues of the coefficient matrix of the above equations determine the stability of the corresponding critical point.

Now, we work out \( \delta \Theta_1, \delta \Theta_2, \delta C_1, \delta C_2, \delta Q_1 \) and \( \delta Q_2 \) one by one. From equation (44), we get

\[
\delta \Theta_1 = 6 \epsilon \dot{x} \delta x + 2 \epsilon f^2 (\ddot{\alpha} \ddot{\beta} \delta v + \ddot{\alpha} \dot{\beta} \delta u) + 4 \eta (\ddot{\alpha} + \ddot{\beta}) (\delta y + \delta v) + 3 \gamma \ddot{z} \delta z. \tag{75}
\]

From equation (45), we have

\[
\delta \Theta_2 = (\lambda \Theta_2)^{-1} \left[ \ddot{z} \delta z + \epsilon \dot{x} \delta x + \epsilon f^2 (\dddot{\alpha} \dot{\beta} + \dddot{\beta} \dot{\alpha}) \delta v + \eta (\ddot{\alpha} + \ddot{\beta}) (\delta y + \delta v) \right]. \tag{76}
\]

From equation (60), it is easy to find that

\[
\delta C_1 = \frac{\sqrt{2} \alpha}{\epsilon} \ddot{z} \delta z, \quad \delta C_2 = \frac{\alpha}{\sqrt{2}} (\dddot{z} \delta z + \dot{z} \delta x). \tag{77}
\]

Then, we obtain \( \delta Q_1 \) and \( \delta Q_2 \) for Case (I) and Case (II) respectively, since they depend on the particular form of \( Q \). For the Case (I), from equation (61), we obtain

\[
\delta Q_1 = -\frac{3 \beta (1 - \dot{z}^2)}{2 \eta (\ddot{\alpha} + \ddot{\beta})^2} (\delta y + \delta v) - \frac{3 \beta \dot{z}}{\eta (\ddot{\alpha} + \ddot{\beta})} \delta z, \quad \delta Q_2 = -\frac{3}{2} \beta (\dot{z}^2 - 1) \delta z. \tag{78}
\]

For the Case (II), from equation (66), it is easy to get

\[
\delta Q_1 = -\frac{3 \sigma}{2 \eta} (\ddot{\alpha} + \ddot{\beta}) \delta v, \quad \delta Q_2 = -\frac{3}{2} \sigma \dot{z} \delta z. \tag{79}
\]

We substitute the critical point \((\ddot{x}, \ddot{y}, \ddot{z}, \ddot{u}, \ddot{v})\) and \( \dot{\Theta}_1 = 0 \) as well as \( \dot{\Theta}_2 \) given by equation (58) into equations (70)–(74) with equations (75)–(79). The five eigenvalues of the coefficient matrix of these equations determine the stability of the corresponding critical point. We find that for both Case (I) \( Q = 3 \beta H \rho_{\text{cw}} \) and Case (II) \( Q = 3 \sigma H \rho_{\text{tot}} \), the critical points can exist and are stable in proper parameter spaces, respectively. In other words, they are late time attractors. Needless to say, the particular parameter spaces for the existence and stability of these critical points are considerably involved and tedious. Since our main aim here is just to point out the fact that it can exist and is stable, we do not present those very involved expressions for the corresponding parameter space. Of course, one can work them out with the help of Mathematica. Here, we just give several particular examples to support our statement. For the Example (I.1) of Case (I) mentioned above, the corresponding eigenvalues are \{-3.10805 + i3.37335, -3.10805 - i3.37335, -3.43776, -1.49442, -0.134622\}; for the Example (I.2), the corresponding eigenvalues are \{-3.10108 + i3.12107, -3.10108 - i3.12107, -2.95079, -1.67829, -0.108711\}; for the Example (II.1) of Case (II) mentioned above, the corresponding eigenvalues are \{-2.68308 + i2.6312, -2.68308 - i2.6312, -2.42269, -1.6597, -0.032674\}; for the Example (II.2), the corresponding eigenvalues are \{-2.68735 + i3.12848, -2.68735 - i3.12848, -1.97246 + i0.723607, -1.97246 - i0.723607, -0.0785649\}. Obviously, they are all stable.
4.4. The first and second cosmological coincidence problems

Here, following the argument in section VI of [36], we briefly show that the first and second cosmological coincidence problems can be alleviated at the same time in our model. As is well known, the approach most frequently used to alleviate the cosmological coincidence problem is the scaling attractor(s) in the dynamical system (see [34, 36], [60]–[67] for example). The most desirable feature of dynamical system is that the whole system will eventually evolve to its attractors, having nothing to do with the initial conditions. Therefore, fine-tuning is unnecessary.

As is explicitly shown in this work, all stable attractors have the desirable properties, namely, their corresponding $\Omega_m$ and $\Omega_{cw}$ are comparable, while $w_{cw} < -1$. As mentioned at the end of section 3 of the present paper, for the case of $\epsilon$ and $\eta$ having opposite signs, $w_{cw}$ can be larger than or smaller than $-1$. Thus, crossing the phantom divide is possible. For a fairly wide range of initial conditions with $w_{cw} > -1$, the universe will eventually evolve to the scaling attractor(s) with $w_{cw} < -1$, while the corresponding $\Omega_m$ and $\Omega_{cw}$ are comparable. Thus, it is not strange that we are living in an epoch when the densities of dark energy and matter are comparable, and the EoS of dark energy is smaller than $-1$. In this sense, the first and second cosmological coincidence problems are alleviated at the same time in our model.

5. Conclusion and discussion

In summary, the (generalized) Cheng–Weyl vector field coupled with a scalar field and its cosmological application are investigated in the present work. In our model, the dark energy is described as a mixture of a scalar field and a so-called ‘cosmic triad’ of three mutually orthogonal Cheng–Weyl vector fields. We derive the effective energy density and pressure of the ‘mixed’ dark energy, and the equations of motion for the scalar field and the Cheng–Weyl vector field, respectively, by using the spatial volume-averaging procedure. The cosmological evolution of this ‘mixed’ dark energy is studied. We find that the effective EoS can cross the phantom divide $w_{de} = -1$ in the case of $\epsilon$ and $\eta$ having opposite signs. The first and second cosmological coincidence problems can be alleviated at the same time in our model. On the other hand, it is easy to see that all stable attractors have $w_{eff} = -1$. Although the EoS of dark energy $w_{cw}$ can be smaller than $-1$, the big rip never appears in this model. The fate of our universe is an inflationary phase, in which the Hubble parameter $H = \dot{H} = \text{const}.$

We finish this paper with some remarks. In this work, to cross the phantom divide, $\epsilon$ and $\eta$ should have opposite signs. This means that either the scalar field or the vector field has a negative sign of its kinetic term. As is argued in [71], these ghost-type fields are possible and viable. Second, one might notice that from equations (63), (65), (68) and (69), at the attractors, the appropriate value $\Omega_m \sim 0.25$ at $\beta \sim 1/3$ or $\sigma \sim 1/4$ results $w_{cw} \sim -4/3$, for $\gamma = 1$. However, this is not inconsistent with the observations [72]. The situation becomes more comfortable when one is aware of the possibility that the universe perhaps nears but has not reached the attractors. Finally, at the attractors, $\Omega_m$ is constant and not equal to zero, thanks to the interaction between the vector fields and the background matter. In fact, the interaction between dark energy and dark matter can be constrained by the cosmological observations; see [37, 73] for example. However, there is still a long way to go to obtain some strict constraints on this interaction.
Acknowledgments

We thank the anonymous referee for useful comments and suggestions, which helped us to improve this paper. We are grateful to Professor Hung Cheng for his impressive talk on [48, 49] given at ITP, July 2004, which inspired the present work. We also thank Professor Xin-Min Zhang for drawing our attention to the work of Rubakov [46], and thank Professor Yue-Liang Wu for comments on this paper. HW is grateful to Zong-Kuan Guo, Yun-Song Piao, Bo Feng, Yan Chai, Ding-Fang Zeng, Xun Su, Wei-Shui Xu, Hui Li, Li-Ming Cao, Da-Wei Pang, Xin Zhang, Qing-Guo Huang, Yi Zhang, Qi Guo, Hong-Sheng Zhang, Hong-Bao Zhang, Hua Bai, Jia-Rui Sun, Ding Ma, Ya-Wen Sun, Xin-Qiang Li and Hao Ma for kind help and useful discussions.

This work was supported in part by China Postdoctoral Science Foundation, and grants from the Chinese Academy of Sciences (No. KJCX3-SYW-N2), and grants from NSFC, China (Nos. 10325525, 90403029 and 10525060).

References

[1] Riess A G et al (Supernova Search Team Collaboration), 2004 Astrophys. J. 607 665 [SPIRES]
[astro-ph/0402512]
Knop R A et al (Supernova Cosmology Project Collaboration), 2003 Astrophys. J. 598 102 [SPIRES]
[astro-ph/0309368]
Tonry J L et al (Supernova Search Team Collaboration), 2003 Astrophys. J. 594 1 [SPIRES]
[astro-ph/0305008]
Riess A G et al (Supernova Search Team Collaboration), 1998 Astron. J. 116 1009 [SPIRES]
[astro-ph/9805201]
Perlmutter S et al (Supernova Cosmology Project Collaboration), 1999 Astrophys. J. 517 565 [SPIRES]
[astro-ph/9812133]

[2] Bennett C L et al (WMAP Collaboration), 2003 Astrophys. J. Suppl. 148 1 [astro-ph/0302207]
Spergel D N et al (WMAP Collaboration), 2003 Astrophys. J. Suppl. 148 175 [astro-ph/0302209]
Spergel D N et al (WMAP Collaboration), 2006 Preprint astro-ph/0603449
Page L et al (WMAP Collaboration), 2006 Preprint astro-ph/0603450
Hinshaw G et al (WMAP Collaboration), 2006 Preprint astro-ph/0603451
Jarosik N et al (WMAP Collaboration), 2006 Preprint astro-ph/0603452

[4] Tegmark M et al (SDSS Collaboration), 2004 Phys. Rev. D 69 103501 [SPIRES] [astro-ph/0310723]
Tegmark M et al (SDSS Collaboration), 2004 Astrophys. J. 606 702 [SPIRES] [astro-ph/0310725]
Seljak U et al, 2005 Phys. Rev. D 71 103515 [SPIRES] [astro-ph/0407372]
Adelman-McCarthy J K et al (SDSS Collaboration), 2005 Preprint astro-ph/0507711
Abazajian K et al (SDSS Collaboration), 2004 Preprint astro-ph/0410239
Abazajian K et al (SDSS Collaboration), 2004 Preprint astro-ph/0403325
Abazajian K et al (SDSS Collaboration), 2004 Preprint astro-ph/0305492

[5] Allen S W, Schmidt R W, Ebeling H, Fabian A C and van Speybroeck L, 2004 Mon. Not. R. Astron. Soc. 353 457 [astro-ph/0405340]

[6] Astier P et al (SNLS Collaboration), 2006 Astron. Astrophys. 447 31 [SPIRES] [astro-ph/0510447]
Neill J D et al (SNLS Collaboration), 2006 Preprint astro-ph/0605148

[7] Jassal H K, Bagla J S and Padmanabhan T, 2005 Phys. Rev. D 72 103503 [SPIRES] [astro-ph/0506748]
Jassal H K, Bagla J S and Padmanabhan T, 2006 Preprint astro-ph/0601389

[8] Peebles P J E and Ratra B, 2003 Rev. Mod. Phys. 75 559 [SPIRES] [astro-ph/0207347]
Carroll S M, 2003 Preprint astro-ph/0310342
Bean R, Carroll S and Trodden M, 2005 Preprint astro-ph/0510059
Copeland E J, Sami M and Tsujikawa S, 2006 Preprint hep-th/0603057
Padmanabhan T, 2003 Phys. Rep. 380 235 [SPIRES] [hep-th/0212290]
Sahni V and Starobinsky A A, 2000 Int. J. Mod. Phys. D 9 373 [SPIRES] [astro-ph/9904398]
Carroll S M, 2001 Living Rev. Rel. 4 1 [astro-ph/0004075]
Padmanabhan T, 2005 Curro. Sci. 88 1057 [astro-ph/0411044]
Cheng–Weyl vector field and its cosmological application

Weinberg S, 1989 Rev. Mod. Phys. 61 1 [SPIRES]
Nobbenhuis S, 2004 Preprint gr-qc/0411093
[9] Caldwell R R, Dave R and Steinhardt P J, 1998 Phys. Rev. Lett. 80 1582 [SPIRES]
Wetterich C, 1988 Nucl. Phys. B 302 668 [SPIRES]
Peebles P J E and Ratra B, 1988 Astrophys. J. 325 L17 [SPIRES]
Ratra B and Peebles P J E, 1988 Phys. Rev. D 37 3406 [SPIRES]
[10] Steinhardt P J, Wang L M and Zlatev I, 1999 Phys. Rev. D 59 123504 [SPIRES] [astro-ph/9812313]
Zlatev I and Steinhardt P J, 1999 Phys. Lett. B 459 570 [SPIRES] [astro-ph/9906481]
[11] Caldwell R R, 2002 Phys. Lett. B 545 23 [SPIRES] [astro-ph/9906168]
Caldwell R R, Kamionkowski M and Weinberg N N, 2003 Phys. Rev. Lett. 91 071301 [SPIRES]
[astro-ph/0302506]
[12] Carroll S M, Hoffman M and Trodden M, 2003 Phys. Rev. D 68 023509 [SPIRES] [astro-ph/0301273]
Cline J M, Jeon S Y and Moore G D, 2004 Phys. Rev. D 70 043543 [SPIRES] [hep-th/0311132]
[13] Armendariz-Picon C, Mukhanov V and Steinhardt P J, 2000 Phys. Rev. Lett. 85 4438 [SPIRES]
[astro-ph/0004134]
Armendariz-Picon C, Mukhanov V and Steinhardt P J, 2001 Phys. Rev. D 63 103510 [SPIRES]
[astro-ph/0006373]
Chiba T, Okabe T and Yamaguchi M, 2000 Phys. Rev. D 62 023511 [SPIRES] [astro-ph/9912463]
Malquarti M, Copeland E J and Liddle A R, 2003 Phys. Rev. D 68 023512 [SPIRES] [astro-ph/0304277]
Malquarti M, Copeland E J, Liddle A R and Trodden M, 2003 Phys. Rev. D 67 123503 [SPIRES]
[astro-ph/0302297]
[15] Chimento L P, 2004 Phys. Rev. D 69 123517 [SPIRES] [astro-ph/0311613]
[16] Huterer D and Cooray A, 2005 Phys. Rev. D 71 023506 [SPIRES] [astro-ph/0404062]
Alam U, Sahni V and Starobinsky A A, 2004 J. Cosmol. Astropart. Phys. JCAP06(2004)008 [SPIRES]
[astro-ph/0403687]
Wang Y and Tegmark M, 2005 Phys. Rev. D 71 103513 [SPIRES] [astro-ph/0501351]
Lazkoz R, Nesersis S and Perivolaropoulos L, 2005 J. Cosmol. Astropart. Phys. JCAP11(2005)010
[SPIRES] [astro-ph/0503230]
Nesersis S and Perivolaropoulos L, 2004 Phys. Rev. D 70 043531 [SPIRES] [astro-ph/0401556]
Bassett B A, Corasaniti P S and Kunz M, 2004 Astrophys. J. 617 L1 [SPIRES] [astro-ph/0407364]
Wang Y and Mukherjee P, 2006 Astrophys. J. 650 1 [SPIRES] [astro-ph/0604051]
Cabra A, Gaztanaga E, Manera M, Fosalba P and Castander F, 2006 Mon. Not. R. Astron. Soc. Lett.
372 L23 [astro-ph/0603690]
[17] Feng B, Wang X L and Zhang X M, 2005 Phys. Lett. B 607 35 [SPIRES] [astro-ph/0404224]
[18] Xia J Q, Zhao G B, Feng B, Li H and Zhang X M, 2006 Phys. Rev. D 73 063521 [SPIRES]
[astro-ph/0511625]
Xia J Q, Zhao G B, Feng B and Zhang X M, 2006 J. Cosmol. Astropart. Phys. JCAP09(2006)015
[SPIRES] [astro-ph/0603939]
Zhao G B, Xia J Q, Feng B and Zhang X M, 2006 Preprint astro-ph/0603621
Xia J Q, Zhao G B, Li H, Feng B and Zhang X M, 2006 Phys. Rev. D 74 083521 [SPIRES]
[astro-ph/0605360]
[19] Upadhye A, Ishak M and Steinhardt P J, 2005 Phys. Rev. D 72 063501 [SPIRES] [astro-ph/0411803]
Salni V and Starobinsky A, 2006 Int. J. Mod. Phys. D 15 2105 [SPIRES] [astro-ph/0610026]
[20] Vikman A, 2005 Phys. Rev. D 71 023515 [SPIRES] [astro-ph/0407107]
[21] Hu W, 2005 Phys. Rev. D 71 043501 [SPIRES] [astro-ph/0410680]
Caldwell R R and Doran M, 2005 Phys. Rev. D 72 043527 [SPIRES] [astro-ph/0501104]
[22] Elizalde E, Nojiri S and Odintsov S D, 2004 Phys. Rev. D 70 043539 [SPIRES] [hep-th/0405034]
McInnes B, 2005 Nucl. Phys. B 718 55 [SPIRES] [hep-th/0502209]
Stefanic H, 2005 Phys. Rev. D 71 124036 [SPIRES] [astro-ph/0504518]
Nojiri S and Odintsov S D, 2005 Phys. Rev. D 72 023003 [SPIRES] [hep-th/0505215]
Nojiri S and Odintsov S D, 2006 Phys. Lett. B 637 139 [SPIRES] [hep-th/0603062]
[23] Guo Z K, Piao Y S, Zhang X M and Zhang Y Z, 2005 Phys. Lett. B 608 177 [SPIRES] [astro-ph/0410654]
Zhang X F, Li H, Piao Y S and Zhang X M, 2006 Mod. Phys. Lett. A 21 231 [SPIRES] [astro-ph/0501652]
Zhao G B, Xia J Q, Li M, Feng B and Zhang X, 2005 Phys. Rev. D 72 123515 [SPIRES]
[astro-ph/0507482]
[24] Zhang X and Wu F Q, 2005 Phys. Rev. D 72 043524 [SPIRES] [astro-ph/0506310]
Zhang X, 2005 Int. J. Mod. Phys. D 14 1597 [SPIRES] [astro-ph/0504586]
[70] Kiselev V V, 2004 Class. Quantum Grav. 21 3323 [SPIRES] [gr-qc/0402095]
[71] Krause A and Ng S P, 2006 Int. J. Mod. Phys. A 21 1091 [SPIRES] [hep-th/0409241]
[72] Alam U, Sahni V and Starobinsky A A, 2007 J. Cosmol. Astropart. Phys. JCAP02(2007)011 [SPIRES] [astro-ph/0612381]
Gong Y G and Wang A Z, 2007 Phys. Rev. D 75 043520 [SPIRES] [astro-ph/0612196]
Nesseris S and Perivolaropoulos L, 2007 J. Cosmol. Astropart. Phys. JCAP02(2007)025 [SPIRES] [astro-ph/0612653]
Ichikawa K and Takahashi T, 2007 J. Cosmol. Astropart. Phys. JCAP02(2007)001 [SPIRES] [astro-ph/0612739]
Davis T M et al., 2007 Preprint astro-ph/0701510
[73] Guo Z K, Ohta N and Tsujikawa S, 2007 Phys. Rev. D 76 023508 [SPIRES] [astro-ph/0702015]