Asymmetric nuclear matter and neutron-skin in
extended relativistic mean field model

B. K. Agrawal

Saha Institute of Nuclear Physics, Kolkata - 700064, India.

(Dated: March 18, 2010)

Abstract

The density dependence of the symmetry energy, instrumental in understanding the behaviour of the asymmetric nuclear matter, is investigated within the extended relativistic mean field (ERMF) model which includes the contributions from the self and mixed interaction terms for the scalar-isoscalar (σ), vector-isoscalar (ω) and vector-isovector (ρ) mesons up to the quartic order. Each of the 26 different parameterizations of the ERMF model employed are compatible with the bulk properties of the finite nuclei. The behaviour of the symmetry energy for several parameter sets are found to be consistent with the empirical constraints on them as extracted from the analyses of the isospin diffusion data. The neutron-skin thickness in the $^{208}$Pb nucleus for these parameter sets of the ERMF model lie in the range of $\sim 0.20 - 0.24$ fm which is in harmony with the ones predicted by the Skyrme Hartree-Fock model. We also investigate the role of various mixed interaction terms which are crucial for the density dependence of the symmetry energy.

PACS numbers: 21.30.Fe, 21.65.Cd, 26.60.-c
I. INTRODUCTION

The accurate knowledge of the equation of state (EOS) for asymmetric nuclear matter is important in understanding the structure of finite nuclei away from the stability line and critical issues in astrophysics. The EOS for asymmetric nuclear matter is mainly governed by the density dependence of the nuclear symmetry energy. Indeed, significant progress has been made in understanding the behaviour of the symmetry energy at the subnormal densities from the analyses of the isospin diffusion data in heavy ion collision \cite{1-4} and from the available data for the neutron skin thickness of several nuclei \cite{5}. The constraints on the density dependence of the nuclear symmetry energy as extracted from the isospin diffusion data agree with the ones deduced from the isoscaling analyses of isotope ratios in intermediate energy heavy-ion collisions \cite{6}. The experimental data on the isotopic dependence of the nuclear giant monopole resonance in even-A Sn isotopes \cite{7,8} also provides some informations on the nuclear symmetry energy which is in agreement with those derived from the analyses of the isospin diffusion data. The behaviour of nuclear symmetry energy at supranormal densities is largely unknown. Theoretically, at supranormal densities, even the issue of whether the symmetry energy increases or decreases with density still remains unresolved. The precise measurements of the properties of the compact stars and the transport model analyses of the heavy-ion collisions at intermediate and high energies can provide some constraints on the high density behaviour of the symmetry energy.

The density dependence of symmetry energy obtained using the Skyrme Hartree-Fock models have been confronted with the constraints extracted by analyzing the isospin diffusion data \cite{9}. The symmetry energy and its slope and the curvature parameter at the saturation density for only four out of 21 different parameter sets of the Skyrme interactions are found to be consistent with the ones extracted from the isospin diffusion data \cite{9}. It may be pointed out that all of these four parameter sets are obtained by fitting the experimental data for the binding energies and charge radii for finite nuclei. Scenario is not the same when the similar investigation \cite{10} is carried out using three different versions of the relativistic mean field (RMF) models, namely, (i) models with meson field self interaction (ii) models with density dependent meson-nucleon couplings and (iii) point coupling models without meson fields. Out of 23 parameter sets of these RMF models, only a few are found to yield symmetry energies and their density dependence which are consistent with the empirical constraints imposed by isospin diffusion data. In particular, 10 different parameter sets were considered for the model (i), but, only two of them could yield behaviour of
the symmetry energy consistent with the empirical constraints. Both of these parameter sets are obtained using nuclear matter observables, instead of fit to the bulk properties of finite nuclei. The density dependence of the symmetry energy is studied also for several other parameter sets of the RMF model \[11\]. None of these parameter sets yield acceptable results for the density dependence of the symmetry energy. Very recently \[12\], density dependence of the symmetry energy has been studied using extended relativistic mean-field (ERMF) model which includes the contributions from self and mixed interaction terms for the $\sigma$, $\omega$ and $\rho$ mesons up to the quartic order. Even in the ERMF model, the density dependence of the symmetry energy obtained for several parameter sets are found to be inadequate. One of the existing parameter set is fine tuned in Ref. \[12\] so that the resulting behaviour of the symmetry energy can fulfill the empirical constraints on them. However, this parameter set is not capable of reproducing the experimental data on the bulk properties of finite nuclei. We would like to emphasize that the ERMF model, due to the presence of the various mixed interaction terms, can yield wide variations in the density dependence of the symmetry energy without affecting the quality of the fit to the bulk properties of finite nuclei \[13,14\]. Nevertheless, the slope of the symmetry energy at the saturation density or alternatively the neutron-skin thickness for the parameter sets of the ERMF model considered in Ref. \[12\] are either too low or quite high.

In the present work we investigate the density dependence of the symmetry energy using 26 different parameterizations of the ERMF model. All the parameterizations of the ERMF model considered are obtained by fitting the experimental data on the binding energy and charge radius for the finite nuclei. Furthermore, these parameter sets yield the neutron-skin thickness in $^{208}$Pb nucleus which vary over a wide range from 0.16 – 0.28 fm. We find that quite a few of these parameterizations can fulfill the empirical constraints on the symmetry energy. Role of various mixed interaction terms are also investigated.

The paper is organized as follows. In Sec. II we describe the ERMF model in brief. In Sec. III, we provide the expressions used to compute various quantities associated with the nuclear matter along with the empirical constraints on them. In Sec. IV, the results obtained using different parameterizations of the ERMF model are confronted with the empirical constraints on the symmetry energy as extracted from the analyses of the isospin diffusion data. The role of mixed interaction terms of the ERMF model which are important in determining the variations in the density dependence of the symmetry energy is investigated in Sec. V. In Sec. VI we state our conclusions.
II. EXTENDED RELATIVISTIC MEAN FIELD MODEL

The ERMF model includes the contributions from the self and mixed interaction terms for the scalar-isoscalar ($\sigma$), vector-isoscalar ($\omega$) and vector-isovector ($\rho$) mesons up to the quartic order. Mixed interaction terms involving $\rho$-meson field enables one to vary the density dependence of the symmetry energy coefficient and the neutron skin thickness in heavy nuclei over a wide range without affecting the other properties of finite nuclei \cite{13, 14}. The contribution from the self interaction of $\omega$-mesons plays important role in varying the high density behaviour of the EOS and also prevents instabilities in the calculation of the EOS \cite{15, 16}. On the other hand expectation value of the $\rho$-meson field is order of magnitude smaller than that for the $\omega$-meson field \cite{17}. Thus, inclusion of the $\rho$-meson self interaction can affect the properties of the finite nuclei and neutron stars only very marginally \cite{16}. The effective Lagrangian density for the ERMF model can be written as,

$$
L = L_{NM} + L_{\sigma} + L_{\omega} + L_{\rho} + L_{\sigma\omega\rho},
$$

(1)

where the nucleonic and mesonic Lagrangian $L_{NM}$ can be written as,

$$
L_{NM} = \sum_{J=n,p} \bar{\Psi}_J [i\gamma'^{\mu}\partial_\mu - (M - g_\sigma \sigma) - (g_\omega \gamma'^{\mu} \omega_\mu + \frac{1}{2} g_\rho \gamma'^{\mu} \tau_\rho \omega_\mu)] \Psi.
$$

(2)

Here, the sum is taken over the neutrons and protons. $\tau$ are the isospin matrices. The Lagrangian describing self interactions for $\sigma$, $\omega$, and $\rho$ mesons can be written as,

$$
L_{\sigma} = \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \frac{k_3}{6 M} g_\sigma m_\sigma^2 \sigma^3 - \frac{k_4}{24 M^2} g_\omega^2 m_\omega^2 \sigma^4,
$$

(3)

$$
L_{\omega} = -\frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + \frac{1}{24} \xi_0 g_\omega^2 (\omega_\mu \omega^\mu)^2,
$$

(4)

$$
L_{\rho} = -\frac{1}{4} \rho_{\mu\nu} \rho^{\mu\nu} + \frac{1}{2} m_\rho^2 \rho_\mu \rho^\mu.
$$

(5)

The $\omega_{\mu\nu}$, $\rho_{\mu\nu}$ are field tensors corresponding to the $\omega$ and $\rho$ mesons, and can be defined as $\omega^{\mu\nu} = \partial^\mu \omega^\nu - \partial^\nu \omega^\mu$ and $\rho^{\mu\nu} = \partial^\mu \rho^\nu - \partial^\nu \rho^\mu$. The mixed interactions of $\sigma$, $\omega$, and $\rho$ mesons $L_{\sigma\omega\rho}$ can be written as,

$$
L_{\sigma\omega\rho} = \frac{\eta_1}{2M} g_\omega m_\omega^2 \sigma \omega_\mu \omega^\mu + \frac{\eta_2}{4M^2} g_\omega^2 m_\omega^2 \sigma^2 \omega_\mu \omega^\mu + \frac{\eta_3}{2M} g_\sigma m_\sigma^2 \sigma \rho_\mu \rho^\mu
$$

$$
+ \frac{\eta_4}{4M^2} g_\omega^2 m_\omega^2 \sigma^2 \rho_\mu \rho^\mu + \frac{\eta_5}{4M^2} g_\omega^2 m_\omega^2 \sigma \rho_\mu \rho^\mu.
$$

(6)

The $L_{em}$ is Lagrangian for electromagnetic interactions and can be expressed as,

$$
L_{em} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - e \bar{\Psi}_p \gamma_\mu A_\mu \Psi_p,
$$

(7)
where, $A$ is the photon field and $F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$. The equation of motion for nucleons, mesons and photons can be derived from the Lagrangian density defined in Eq. (1). The contributions from Eq. (7) are included only for the case of finite nuclei.

III. EMPIRICAL CONSTRAINTS ON SYMMETRY ENERGY

The symmetry energy $E_{\text{sym}}$, slope $L$ and curvature $K_{\text{sym}}$ can be evaluated as,

\begin{align*}
E_{\text{sym}}(\rho) &= \frac{1}{2} \left. \frac{d^2 E(\rho, \delta)}{d\delta^2} \right|_{\delta=0}, \quad (8) \\
L &= 3\rho_0 \left. \frac{dE_{\text{sym}}(\rho)}{d\rho} \right|_{\rho=\rho_0}, \quad (9) \\
K_{\text{sym}} &= 9\rho_0^2 \left. \frac{d^2 E_{\text{sym}}(\rho)}{d\rho^2} \right|_{\rho=\rho_0}, \quad (10)
\end{align*}

where, $\rho_0$ is the saturation density, $E(\rho, \delta)$ is the energy per nucleon at a given density $\rho$ and asymmetry $\delta = (\rho_n - \rho_p)/\rho$. The density dependence of the symmetry energy can also be expressed in terms of $E_{\text{sym}}(\rho)$, $L$ and $K_{\text{sym}}$ as,

\begin{equation}
E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_0) + L \left( \frac{\rho - \rho_0}{3\rho_0} \right) + \frac{K_{\text{sym}}}{2} \left( \frac{\rho - \rho_0}{3\rho_0} \right)^2, \quad (12)
\end{equation}

The above equation represent very well the behaviour of the symmetry energy at subnormal densities. At supranormal densities, one needs to include in Eq. (12) the contributions of higher order terms [18]. We also evaluate,

\begin{align*}
K_0 &= 9\rho_0^2 \left. \frac{d^2 E_0(\rho)}{d\rho^2} \right|_{\rho=\rho_0}, \quad (13) \\
J_0 &= 27\rho_0^3 \left. \frac{d^3 E_0(\rho)}{d\rho^3} \right|_{\rho=\rho_0}, \quad (14) \\
K_{\text{sat,2}} &= K_{\text{asy}} - \frac{J_0}{K_0} L, \quad (15) \\
K_{\text{asy}} &= K_{\text{sym}} - 6L. \quad (16)
\end{align*}

The $E_0(\rho) = E(\rho, \delta = 0)$ is the energy per nucleon for symmetric nuclear matter. The $K_0$ is the incompressibility coefficient of the symmetric nuclear matter at the saturation density which
together with $K_{\text{sat}, 2}$ can yield the value of incompressibility coefficient for asymmetric nuclear matter [18]. The constraints on the values of $E_{\text{sym}}$, $L$, $K_{\text{asy}}$ [1–3], $K_0$ [7, 8, 19–24], $K_{\text{sat}, 2}$ [18] are,

$$E_{\text{sym}} = 30 \pm 5\text{MeV}$$
$$L = 88 \pm 25\text{MeV}$$
$$K_{\text{asy}} = -500 \pm 50\text{MeV}$$
$$K_0 = 240 \pm 20\text{MeV}$$
$$K_{\text{sat}, 2} = -370 \pm 120\text{MeV}. \quad (17)$$

IV. ASYMMETRIC NUCLEAR MATTER AND NEUTRON-SKIN

We study the properties of asymmetric nuclear matter for 26 different parameterizations of the ERMF model. Each of these parameterizations are obtained by fitting the experimental data on the binding energies and charge radii for few closed shell nuclei. Twenty one of these parameter sets were obtained in our earlier work [25] which correspond to different combinations of neutron-skin thickness $\Delta R$ in $^{208}\text{Pb}$ nucleus and the $\omega$-meson self-coupling strength $\zeta_0 = \zeta g_\omega^2$ (Eq. 4). Hereafter, we name these 21 parameter sets as BSR1 - BSR21 according to the values of $\zeta$ and $\Delta R$. The parameter sets BSR1, BSR2, ..., BSR7 correspond to $\Delta R = 0.16, 0.18, ..., 0.28$ fm with $\zeta = 0$. Similarly, the parameter sets BSR8, BSR9, ..., BSR14 and BSR15, BSR16, ..., BSR21 correspond to $\Delta R = 0.16, 0.18, ..., 0.28$ fm, but, with $\zeta = 0.03$ and $0.06$, respectively. The set of experimental data for the binding energies and charge radii used to generate the parameter sets BSR1 - BSR21 are exactly the same. It can be seen from Ref. [25] that the rms error on the total binding energy and the charge radius for all our 21 parameter sets are more or less the same. The rms errors for the total binding energy are 1.5 - 1.8 MeV and that for the charge radii lie within the $0.025 - 0.04$ fm. Other five parameter sets considered are, FSUGold [26], FSUGZ03 [27], G2 [28], TM1 [15] and TM1* [29].

The density dependence of the symmetry energy $E_{\text{sym}}(\rho)$ plays central role in understanding the behaviour of the asymmetric nuclear matter. The $E_{\text{sym}}(\rho)$ at subnormal densities can be expressed in terms of the $E_{\text{sym}}(\rho_0)$, slope $L$ and the curvature $K_{\text{sym}}$ or $K_{\text{asy}}$ as given by Eq. (12). We mainly focus on the values of $E_{\text{sym}}(\rho_0)$, $L$ and $K_{\text{asy}}$ as obtained for different parameterizations of the ERMF model. In addition, we also calculate the curvature parameter $K_{\text{sat}, 2}$ (Eq. 15) which together with $K_0$ yields the values for the incompressibility coefficient for the asymmetric nuclear matter [18].
In Table I we list the values of the $\Delta R$, $B/A$, $\rho_0$, $K_0$, $E_{\text{sym}}$, $L$, $K_{\text{sym}}$, $K_{\text{asy}}$ and $K_{\text{sat,2}}$ for different parameterizations of ERMF model. For the comparison, in the last row of Table I we also provide the results obtained for most commonly used NL3 parameterization [30] of the conventional RMF model which includes the non-linear terms only for the $\sigma$ meson. For better insight, in Fig. 1, the values of $E_{\text{sym}}(\rho_0)$ and $L$ are plotted against $\Delta R$ and $\zeta$ for the parameter sets BSR1 - BSR7 (squares), BSR8 - BSR14 (circles) and BSR15 - BSR21 (triangles). Similar plots for $K_0$, $K_{\text{asy}}$ and $K_{\text{sat,2}}$ are displayed in Fig. 2. The results for the other parameter sets FSUGold, FSUGZ03, G2, TM1 and TM1* are also depicted in Figs. 1 and 2. We first focus on the results obtained for our 21 parameter sets BSR1 - BSR21. It can be seen from Figs. 1 and 2 that the values of the isovector quantities $E_{\text{sym}}(\rho_0)$, $L$ and $K_{\text{asy}}$ depends mainly on the value of $\Delta R$. The isoscalar quantity $K_0$ is more sensitive to the choice of $\zeta$. The values of $K_{\text{sat,2}}$, however, depends on both the $\zeta$ and $\Delta R$. The constraints (Eq. 17) on the values of $E_{\text{sym}}(\rho_0)$ and $L$ are satisfied by our parameter sets for which $\Delta R = 0.18 - 0.24$ fm. The constraint on the values of $K_0$ is satisfied by all the 21 parameter sets BSR1 - BSR21. The constraints on the value of $K_{\text{asy}}$ are satisfied by our parameterizations with $\Delta R = 0.22 - 0.24$ fm. On the other hand, except for three parameter sets, all of our other parameterizations satisfy the constraint on the value of $K_{\text{sat,2}}$. These three parameter sets are BSR5, BSR6 and BSR7 which correspond to $\zeta = 0$ with $\Delta R = 0.24, 0.26$ and $0.28$ fm, respectively. Other parameter sets FSUGold, FSUGZ03, G2, TM1 and TM1* are not compatible simultaneously with the various constraints summarized in Eq. (17). In short, it appears that only five parameter sets BSR4, BSR11, BSR12, BSR18 and BSR19 with $\Delta R = 0.22 - 0.24$ fm obey the empirical constraints of Eq. (17) very well. Our parameter sets BSR3, BSR10 and BSR17 with $\Delta R = 0.20$ fm also satisfy all the constraints of Eq. (17) except for the $K_{\text{asy}}$. The values of $K_{\text{asy}}$ for these parameter sets are only marginally away from the ones given by Eq. (17). So, on the basis of the these constraints, the ERMF model predicts the value of neutron-skin thickness in the $^{208}\text{Pb}$ nucleus to be $\sim 0.20 - 0.24$ fm. Similar investigations using the Skyrme Hartree-Fock models [9] predicted $\Delta R = 0.18 - 0.26$ fm. Thus, the empirical constraints extracted from the isospin diffusion data predict more or less model independent values for the neutron-skin thickness. In Ref. [12], various parameter sets of the ERMF model considered are the FSUGZ00, FSUGZ03, FSUGZ06 and G2. For these cases, $\Delta R$ is either $\sim 0.19$ or $\sim 0.26$ fm (see also Table I). These values of $\Delta R$ are seem to be either little smaller or quite larger. Consequently, none of the parameter sets considered in Ref. [12] are consistent with all the constraints of Eq. (17).

The symmetry energy and its density dependence in the ERMF model is mainly governed by
values of the coupling strengths $g_\rho$, $\eta_\rho$, $\eta_{1\rho}$ and $\eta_{2\rho}$ (Eqs. 2 and 6). The strengths $\eta_\rho$, $\eta_{1\rho}$ and $\eta_{2\rho}$ determines the contributions of the mixed interaction terms which account for the coupling of the isoscalar $\sigma$ and $\omega$ mesons to the isovector $\rho$ mesons. It may be emphasized, in the conventional RMF models, the contributions of these mixed interaction terms are ignored (i.e., $\eta_\rho = \eta_{1\rho} = \eta_{2\rho} = 0$). We have used our parameter sets BSR1 - BSR21 to look into the variations of the $g_\rho$, $\eta_\rho$, $\eta_{1\rho}$ and $\eta_{2\rho}$ with $\Delta R$. As an illustration, in Fig. 3, we plot the values of the $g_\rho/4\pi$, $\eta_\rho$, $\eta_{1\rho}$ and $\eta_{2\rho}$ for the parameter sets BSR8 – BSR14 which correspond to different values of $\Delta R$ with $\zeta = 0.03$. Scenario for the parameter sets BSR1- BSR7 and BSR15- BSR21 (not shown here) is analogous to that of BSR8 – BSR14. We can see from Fig. 3, there is an overall decrease in the values of these coupling strengths with increase in $\Delta R$. It is interesting to note that all the coupling strengths are nearly unity for $\Delta R \sim 0.22$ fm. Coincidently, for $\Delta R \sim 0.22$ fm, the empirical constraints on the behaviour of the symmetry energy as extracted from the isospin diffusion data are also satisfied very well. The strengths $\eta_\rho$, $\eta_{1\rho}$ and $\eta_{2\rho}$ tend to vanish for higher values of $\Delta R$. In other words, if the contributions of the mixed interaction terms in question are ignored, as in the case of the conventional RMF model, the parameters obtained by fit to the experimental data on the binding energy and charge radius would give rise to $\Delta R \geq 0.26$ fm. The values of $\Delta R \sim 0.22$ fm, as favoured by the isospin diffusion data, can be achieved within the conventional RMF model only at the expense of the quality of fit to the bulk properties of the finite nuclei.

V. ROLE OF MIXED INTERACTIONS

We would like to investigate the role of various mixed interaction terms which are crucial in determining the density dependence of the symmetry energy. In particular, we investigate the effects of the terms which account for the coupling of the isoscalar $\sigma$ and $\omega$ mesons to the isovector $\rho$ mesons. The coupling constants for these terms are $\eta_\rho$, $\eta_{1\rho}$ or $\eta_{2\rho}$ (Eq. 6). The term with coupling constant $\eta_\rho$ is of the cubic order in the meson fields. Whereas, the terms with $\eta_{1\rho}$ and $\eta_{2\rho}$ are of quartic order in the meson fields. Our objective is to delineate the effects of these cubic and quartic order mixed interaction terms. For this purpose, we generate two different families of interactions. For the first family of interactions F1, we put $\eta_{1\rho} = \eta_{2\rho} = 0$ and fit the remaining coupling constants using appropriate set of experimental data for the bulk properties of finite nuclei. But, the coupling constants $\eta_{1\rho}$ and $\eta_{2\rho}$ are also included in the fit for the second family F2. Thus, the behaviour of the symmetry energy for the F1 family is governed by the coupling constants $g_\rho$ and $\eta_\rho$ only. In
case of the F2 family, the behaviour of the symmetry energy depends additionally on \( \eta_{1\rho} \) and \( \eta_{2\rho} \). For both the families we obtain the parameter sets corresponding to different values of \( \Delta R \) in the range of 0.18 – 0.26 fm with fixed \( \zeta = 0.03 \). The procedure for calibrating the parameters of the model as well as the set of experimental data for the binding energies and charge radii used here are exactly the same as in Ref. [27]. In addition, the parameters are also subjected to the empirical constraints of Eq. (17).

In Fig. 4 we plot our results for the rms errors on the total binding energies and charge radii. We see that quality of the fits to the total binding energies and charge radii are more or less the same for both the F1 and F2 families of interactions, except for \( \Delta R \leq 0.2 \) fm. Therefore, it appears that the quartic order mixed interaction terms with coupling constants \( \eta_{1\rho} \) and \( \eta_{2\rho} \) are redundant. More precisely, one might say that the values of \( \eta_{1\rho} \) and \( \eta_{2\rho} \) can not be appropriately determined by the bulk properties of the finite nuclei. These quartic order terms might play important role in fixing the behaviour of the symmetry energy at supranormal densities which is largely unknown at present. It can be easily concluded from Fig. 4 that the most preferred value for the neutron-skin thickness in the \( ^{208}\text{Pb} \) nucleus within the ERMF model is \( \Delta R \sim 0.22 \). In Table II we give the parameter sets for the F1 family of interactions obtained for different values of \( \Delta R \). These parameter sets are named as BKA20, BKA22 and BKA24 which correspond to \( \Delta R = 0.20 \), 0.22 and 0.24 fm, respectively. In Table III we present the results for the various quantities associated with the symmetric and asymmetric nuclear matter calculated at the saturation density using the parameter sets BKA20, BKA22 and BKA24. For the sake of completeness, we have repeated our calculation for \( \Delta R = 0.22 \) fm with \( \eta_{\rho} = \eta_{1\rho} = \eta_{2\rho} = 0 \). In this case, the rms errors \( \delta B = 4.8 \) MeV and \( \delta r_{ch} = 0.05 \) fm are significantly higher compared to the ones obtained for the F1 and F2 families. Thus, the contributions of the mixed interaction terms seem indispensable in order to satisfy simultaneously the empirical constraints on the density dependence of the symmetry energy as well as the experimental data on the bulk properties of the finite nuclei.

VI. CONCLUSIONS

The density dependence of symmetry energy and the incompressibility coefficient for the asymmetric nuclear matter are studied using 26 different parameterizations of the ERMF model. The model includes the contributions from self and mixed interaction terms for \( \sigma, \omega \) and \( \rho \) mesons upto the quartic order. Each of the parameterizations considered are compatible with the bulk
properties of the finite nuclei. Furthermore, these parameter sets yield the neutron-skin thickness in $^{208}$Pb nucleus which vary over a wide range from 0.16 – 0.28 fm. The behaviour of symmetry energy at subnormal densities for several parameterizations of the ERMF model corresponding to $\Delta R \sim 0.20 – 0.24$ fm are found to be consistent with the empirical constraints on them as extracted from the analyses of the isospin diffusion data. The ERMF model prediction for $\Delta R \sim 0.20 – 0.24$ fm is in reasonable agreement with the ones obtained in the similar way, but, for the Skyrme Hartree-Fock model [9]. We have investigated the role of the cubic and quartic order mixed interaction terms which are crucial for the density dependence of the symmetry energy. It is utmost important to include the contributions at least from the cubic order term to incorporate the empirical constraints on the density dependence of the symmetry energy without affecting the quality of the fit to the bulk properties of the finite nuclei. The mixed interaction terms of the quartic order might be important to obtain the appropriate behaviour of the symmetry energy at supranormal densities which is largely unknown.
[1] M. B. Tsang, T. X. Liu, L. Shi, P. Danielewicz, C. K. Gelbke, X. D. Liu, W. G. Lynch, W. P. Tan, G. Verde, A. Wagner, et al., Phys. Rev. Lett. 92, 062701 (2004).
[2] L.-W. Chen, C. M. Ko, and B.-A. Li, Phys. Rev. Lett. 94, 032701 (2005).
[3] B.-A. Li and L.-W. Chen, Phys. Rev. C 72, 064611 (2005).
[4] B.-A. Li, L.-W. Chen, and C. M. Ko, Phys. Rep. 464, 113 (2008).
[5] M. Centelles, X. Roca-Maza, X. Vias, and M. Warda, Phys. Rev. Lett. 102, 122502 (2009).
[6] D. V. Shetty, S. J. Yennello, and G. A. Souliotis, Phys. Rev. C 75, 034602 (2007).
[7] U. Garg and et al, Nucl. Phys. A788, 36 (2007).
[8] T. Li and et al, Phys. Rev. Lett. 99, 162503 (2007).
[9] L.-W. Chen, C. M. Ko, and B.-A. Li, Phys. Rev. C 72, 064309 (2005).
[10] L.-W. Chen, C. M. Ko, and B.-A. Li, Phys. Rev. C 76, 054316 (2007).
[11] I. Vidana, C. Providencia, A. Polls, and A. Rios, Phys. Rev. C 80, 045806 (2009).
[12] A. Sulaksono and Kasmudin, Phys. Rev. C 80, 054317 (2009).
[13] R. Furnstahl, Nucl. Phys. A706, 85 (2002).
[14] T. Sil, M. Centelles, X. Vinas, and J. Piekarewicz, Phys. Rev. C 71, 045502 (2005).
[15] Y. Sugahara and H. Toki, Nucl. Phys. A579, 557 (1994).
[16] H. Müller and B. D. Serot, Nucl. Phys. A606, 508 (1996).
[17] B. D. Serot and J. D. Walecka, Int. J. Mod. Phys. E 6, 515 (1997).
[18] L.-W. Chen, B.-J. Cai, C. M. Ko, B.-A. Li, C. Shen, and J. Xu, Phys. Rev. C 80, 014322 (2009).
[19] D. H. Youngblood, H. L. Clark, and Y. W. Lui, Phys. Rev. Lett. 82, 691 (1999).
[20] Y. W. Lui, D. H. Youngblood, Y. Tokimoto, H. L. Clark, and B. John, Phys. Rev. C 70, 014307 (2004).
[21] Z. yu Ma, A. Wandelt, N. V. Giai, D. P. Ring, and L. gang Cao, Nucl. Phys. A703, 222 (2002).
[22] D. Vretenar, T. Niksic, and P. Ring, Phys. Rev. C 68, 024310 (2003).
[23] G. Colo, N. V. Giai, J. Meyer, K. Bennaceur, and P. Bonche, Phys. Rev. C 70, 024307 (2004).
[24] S. Shlomo, V. M. Kolomietz, and G. Colo, Eur. Phys. J. A 30, 23 (2006).
[25] S. K. Dhiman, R. Kumar, and B. K. Agrawal, Phys. Rev. C 76, 045801 (2007).
[26] B. G. Todd-Rutel and J. Piekarewicz, Phys. Rev. Lett 95, 122501 (2005).
[27] R. Kumar, B. K. Agrawal, and S. K. Dhiman, Phys. Rev. C 74, 034323 (2006).
[28] R. Furnstahl, B. D. Serot, and H.-B. Tang, Nucl. Phys. A615, 441 (1997).
[29] M. D. Estal, M. Centelles, X. Vias, and S. K. Patra, Phys. Rev. C 63, 024314 (2001).

[30] G. A. Lalazissis, J. König, and P. Ring, Phys. Rev. C 55, 540 (1997).
TABLE I: Some bulk properties of the nuclear matter at the saturation density ($\rho_0$): binding energy per nucleon ($B/A$), incompressibility coefficient for symmetric nuclear matter ($K_0$), symmetry energy ($E_{\text{sym}}(\rho_0)$), linear density dependence of the symmetry energy ($L$) and various quantities ($K_{\text{sym}}$, $K_{\text{asy}}$) and ($K_{\text{sat2}}$) as given by Eqs. (13-15). The values for the neutron-skin thickness $\Delta R$ for the $^{208}$Pb nucleus are also listed. The superscript ‘a’ on several parameter sets indicate that they satisfy the constraints of Eq. (17) very well.

| Force     | $\Delta R$ (fm) | $B/A$ (MeV) | $\rho_0$ | $K_0$ (MeV) | $E_{\text{sym}}(\rho_0)$ (MeV) | $L$ (MeV) | $K_{\text{sym}}$ (MeV) | $K_{\text{asy}}$ (MeV) | $K_{\text{sat2}}$ (MeV) |
|-----------|----------------|-------------|----------|-------------|-------------------------------|-----------|------------------------|------------------------|-------------------------|
| BSR1      | 0.16           | 16.0        | 0.148    | 240         | 31.0                          | 60        | 13                     | -345                   | -335                    |
| BSR2      | 0.18           | 16.0        | 0.149    | 240         | 31.4                          | 62        | -4                     | -376                   | -363                    |
| BSR3      | 0.20           | 16.1        | 0.150    | 231         | 32.6                          | 71        | -8                     | -431                   | -395                    |
| BSR4$^a$  | 0.22           | 16.1        | 0.150    | 239         | 33.0                          | 73        | -20                    | -460                   | -460                    |
| BSR5      | 0.24           | 16.1        | 0.151    | 236         | 34.3                          | 83        | -13                    | -513                   | -509                    |
| BSR6      | 0.26           | 16.1        | 0.149    | 236         | 35.4                          | 86        | -48                    | -562                   | -557                    |
| BSR7      | 0.28           | 16.2        | 0.149    | 232         | 37.0                          | 99        | -15                    | -608                   | -598                    |
| BSR8      | 0.16           | 16.0        | 0.147    | 231         | 31.0                          | 60        | -1                     | -363                   | -286                    |
| BSR9      | 0.18           | 16.1        | 0.147    | 233         | 31.6                          | 64        | -12                    | -396                   | -313                    |
| BSR10     | 0.20           | 16.1        | 0.147    | 228         | 32.6                          | 71        | -17                    | -442                   | -361                    |
| BSR11$^a$ | 0.22           | 16.1        | 0.147    | 227         | 33.6                          | 79        | -25                    | -497                   | -387                    |
| BSR12$^a$ | 0.24           | 16.1        | 0.147    | 232         | 33.8                          | 78        | -44                    | -511                   | -412                    |
| BSR13     | 0.26           | 16.1        | 0.147    | 229         | 35.6                          | 91        | -40                    | -585                   | -466                    |
| BSR14     | 0.28           | 16.2        | 0.147    | 236         | 36.1                          | 94        | -41                    | -601                   | -474                    |
| BSR15     | 0.16           | 16.0        | 0.146    | 227         | 30.9                          | 62        | -22                    | -393                   | -252                    |
| BSR16     | 0.18           | 16.1        | 0.146    | 225         | 31.2                          | 62        | -25                    | -399                   | -258                    |
| BSR17     | 0.20           | 16.1        | 0.146    | 222         | 31.9                          | 68        | -32                    | -437                   | -287                    |
| BSR18$^a$ | 0.22           | 16.1        | 0.146    | 221         | 32.6                          | 73        | -42                    | -478                   | -317                    |
| BSR19$^a$ | 0.24           | 16.1        | 0.147    | 221         | 33.6                          | 79        | -50                    | -526                   | -350                    |
| BSR20     | 0.26           | 16.1        | 0.146    | 223         | 34.3                          | 88        | -39                    | -565                   | -365                    |
| BSR21     | 0.28           | 16.1        | 0.145    | 220         | 35.7                          | 93        | -45                    | -600                   | -402                    |
| FSUGold   | 0.21           | 16.3        | 0.148    | 229         | 32.5                          | 60        | -52                    | -412                   | -276                    |
| FSUGZ03   | 0.19           | 16.1        | 0.147    | 233         | 31.6                          | 64        | -11                    | -396                   | -312                    |
| G2        | 0.26           | -16.1       | 0.153    | 215         | 36.4                          | 100       | -7                     | -611                   | -404                    |
| TM1       | 0.27           | 16.3        | 0.145    | 281         | 13.6                          | 111       | 34                     | -632                   | -518                    |
| TM1$^*$   | 0.27           | 16.3        | 0.145    | 281         | 37                            | 102       | -14                    | -625                   | -429                    |
TABLE II: Parameter sets for the F1 family of interactions obtained for different values of neutron-skin thickness in the $^{208}$Pb nucleus. These parameter sets are named as BKA20, BKA22 and BKA24 which correspond to the neutron-skin thickness 0.20, 0.22 and 0.24 fm, respectively. The couplings $\eta_1\rho$ and $\eta_2\rho$ are taken to be zero. The masses for $\omega$ and $\rho$ mesons are $m_\omega/\sqrt{M} = 782$ MeV and $m_\rho/\sqrt{M} = 770$ MeV with nucleon mass $M = 939$ MeV.

| Force | $\bar{g}_\sigma/\sqrt{M}$ | $\bar{g}_\omega/\sqrt{M}$ | $\bar{g}_\rho/\sqrt{M}$ | $\kappa_3$ | $\kappa_4$ | $\xi_0$ | $\eta_1$ | $\eta_2$ | $\eta_\rho$ | $m_\sigma/\sqrt{M}$ |
|-------|-----------------|-----------------|-----------------|--------|--------|--------|--------|--------|--------|--------|
| BKA20 | 0.8042          | 1.0102          | 0.9812          | 1.1523 | 1.9892 | 4.8344 | 0.0005 | 0.0657 | 3.6164 | 0.5430 |
| BKA22 | 0.8462          | 1.1089          | 1.0302          | 1.5500 | 2.13451 | 5.8253 | 0.1555 | 0.0697 | 3.9294 | 0.5302 |
| BKA24 | 0.8593          | 1.1463          | 0.9381          | 0.9381 | 1.7719 | 3.1064 | 6.2247 | 0.2700 | 0.1159 | 2.4900 | 0.5248 |

TABLE III: Same as Table II but, for the forces BKA20, BKA22 and BKA24.

| Force | $\Delta R$ (fm) | $B/A$ (MeV) | $\rho_0$ (fm$^{-3}$) | $K_0$ (MeV) | $E_{sym}$ (MeV) | $L$ (MeV) | $K_{sym}$ (MeV) | $K_{asy}$ (MeV) | $K_{sat2}$ (MeV) |
|-------|-----------------|-------------|---------------------|-------------|-----------------|---------|------------------|------------------|------------------|
| BKA20 | 0.20            | 16.1        | 0.146               | 240         | 32.3            | 76      | -15              | -469             | -320             |
| BKA22 | 0.22            | 16.1        | 0.148               | 227         | 33.3            | 79      | -9               | -483             | -382             |
| BKA24 | 0.24            | 16.1        | 0.148               | 228         | 34.3            | 85      | -15              | -525             | -420             |
FIG. 1: (colour online) The symmetry energy $E_{\text{sym}}(\rho_0)$ and its slope $L$ plotted against the neutron-skin thickness $\Delta R$ in the $^{208}\text{Pb}$ nucleus for 26 different parameterizations of the ERMF model (see also Table I). The open squares, circles and triangles represent the results for the parameter sets BSR1 - BSR7, BSR8 - BSR14 and BSR15 - BSR21, respectively. The symbol plus with different colours depict the results for the FSUGold (green), FSUGZ03 (black), G2 (purple), TM1 (maroon) and TM1$^*$ (orange). The dashed lines represent the constraint on $E_{\text{sym}}(\rho_0)$ and $L$ as given in Eq. (17).
FIG. 2: (colour online) Same as Fig. 1 but, for $K_0$, $K_{asy}$ and $K_{sat.2}$
FIG. 3: (colour online) The coupling strengths $g_p/4\pi$, $\eta_p$, $\eta_{1p}$ and $\eta_{2p}$ for the parameter sets BSR8 - BSR14 plotted against $\Delta R$. 
FIG. 4: (colour online) Plots for the rms errors on the total binding energy $\delta B$ (upper panel) and the charge radius $\delta r_{ch}$ (lower panel) for the F1 and F2 families of the interactions.