Theoretical Aspects of Semileptonic $B$ decays

Nikolai Uraltsev

INFN, Sezione di Milano, Milan, Italy

Strong interactions are addressed in connection to extracting $|V_{cb}|$ and determining heavy quark parameters. A comprehensive approach allows a robust analysis not relying on a $1/m_b$ expansion; a percent defendable accuracy in $|V_{cb}|$ becomes realistic. Some of the heavy quark parameters are already accurately known. We have at least one nontrivial precision check of the OPE at the nonperturbative level in inclusive decays. The alleged controversy between theory and BaBar data on $\langle M_\mu^2 \rangle$ is argued to be an artefact of oversimplifying the OPE for high cuts in lepton energy. Consequences of the proximity to the ‘BPS’ limit are addressed and their accuracy qualified. It is suggested that theory-wise $B \rightarrow D \ell \nu$ near zero recoil offers an accurate way for measuring $|V_{cb}|$.

Total semileptonic decay rate $\Gamma_{\ell\ell}(B)$ is now one of the best measured quantities in $B$ physics. This opens an accurate way to extract the CKM angle $|V_{cb}|$ where systematic uncertainties can be thoroughly studied. Nonperturbative effects are theoretically controlled by the QCD theorem [1] which established absence of the leading $A_{\text{QCD}}/m_b$ power corrections to total decay rates. It is based on OPE and applies to all sufficiently inclusive decay probabilities, semileptonic as well as nonleptonic or radiatively induced. The theorem relates the inclusive $B$ widths to (short-distance) quark masses and expectation values of local $b$-quark operators in actual $B$ mesons. To order $1/m_b^3$ they are kinetic and chromomagnetic averages $\langle \mu^2\rangle$, $\langle \mu^2_L \rangle$ having a straightforward quantum-mechanical meaning. To order $1/m_b^4$ two new operators, Darwin and LS, appear for $b \rightarrow c \ell \nu$ with the averages $\langle \rho^2_D \rangle$, $\langle \rho^2_L \rangle$, also analogous to those well known from atomic theory. The general expansion parameter for inclusive decays is energy release, in $b \rightarrow c \ell \nu$ it constitutes $m_b - m_c \approx 3.5 \text{ GeV}$. For a recent review, see Ref. [2].

Within the OPE approach calculating everything sufficiently inclusive has become a technical matter (although not necessary simple as illustrated by perturbative corrections). Yet care must be taken to treat properly certain sub-tleties; some of them will be discussed.

Heavy quark masses are full-fledged QCD parameters entering various hadronic processes. The expectation values like $\langle \mu^2 \rangle$ likewise enjoy the status of observables. The masses and relevant nonperturbative parameters can be determined from the $B$ decay distributions themselves [3-5]. Nowadays this strategy is being implemented in a number of experimental studies.

Some progress has been recently made on the theoretical side as well. A resummation of the dominant perturbative corrections in the context of Wilsonian OPE was performed for the semileptonic width and showed safe convergence [5-6], eliminating one of the major theoretical uncertainties. Higher-order nonperturbative corrections were studied and a new class of nonperturbative effects identified associated with the nonvanishing expectation values of the four-quark operators with charm field, $(B|\bar{b}c\bar{c}|B)$ [5]. The so-called “BPS” regime was proposed as an approximation based on the hierarchy $\langle \rho^2_D \rangle - \langle \rho^2_L \rangle \ll \langle \mu^2 \rangle$ suggested by experiment, and its immediate consequences analyzed [7]. The new results extending them will be addressed in the end.

The new generation of data provides accurate measurements of many inclusive characteristics in $B$ decays. At the same time, the proper theoretical formalism gradually finds its way into their analyses. To summarize the current stage:

- We have an accurate and reliable determination of some heavy quark parameters directly from experiment.
- Extracting $|V_{cb}|$ from $\Gamma_{\ell\ell}(B)$ has good accuracy and solid grounds.
- We have at least one nontrivial precision check of the OPE at the nonperturbative level.

The latter comes through comparison of the average lepton energy and the invariant hadronic mass, as explained in the following section.

Present theory allows to aim a percent accuracy in $|V_{cb}|$. Such a precision becomes possible owing to a number of theoretical refinements. The low-scale running masses $\tilde{m}_b(\mu)$, $\tilde{m}_c(\mu)$, the expectation values $\langle \rho^2 \rangle(\mu)$, $\langle \mu^2 \rangle(\mu)$, ... are completely defined and can be determined from experiment with an in principle unlimited accuracy. Violation of local duality potentially limiting theoretical predictability, has been scrutinized and found to be negligibly small in total semileptonic $B$ widths [5]; it can be controlled experimentally with dedicated high-statistics measurements. Present-day perturbative technology makes computing $\alpha_s$-corrections to the Wilson coefficients of nonperturbative operators feasible. It is also understood how to treat higher-order power corrections in a way which renders them suppressed [6].

High accuracy can be achieved in a comprehensive ap-
proach where many observables are measured in $B$ decays to extract necessary ‘theoretical’ input parameters. It is crucial that here one can do without relying on charm mass expansion at all, i.e. do not assume charm quark to be really heavy in strong interaction mass scale. For reliability of the $1/m_c$ expansion is questionable. Already in the $1/m_c^2$ terms one has $\frac{1}{m_c} > 14 \frac{1}{m_t}$; even for the worst mass scale in the width expansion, $\frac{m_{c_M}}{m_{c_T}}$ is at least 8 times smaller than $\frac{1}{m_t}$. There are indications [7] that the nonlocal correlators affecting meson masses can be particularly large – a pattern independently observed in the ‘t Hooft model [9]. This expectation is supported by the pilot lattice study [10] which – if taken at face value – suggests a very large value of a particular combination $\rho_{LS}^3/\rho_3^2$ entering in the conventional approach. On the other hand, non-local correlators are not measured in inclusive $B$ decays.

The approach which is totally free from relying on charm mass expansion [11] was put forward at the previous CKM-2002 Workshop at CERN. It allows to utilize the full power of the comprehensive studies, and makes use of a few key facts [1,3]:

- Total width to order $1/m_c^3$ is affected by a single new Darwin operator; the moments also weakly depend on $\rho_{LS}^3$.
- No nonlocal correlators ever enter per se.
- Deviations from the HQ limit in the expectation values are driven by the maximal mass scale, $2m_b$ (and are additionally suppressed by proximity to the BPS limit); they are negligible in practice.
- Exact sum rules and inequalities which hold for properly defined Wilsonian parameters.

The original motivation for the precision control of strong interaction effects in $B$ decays was accurate determination of $|V_{cb}|$ and $|V_{ub}|$ as means to cross check the Standard Model. For theorists it is clear, however that interesting physics lies not only in the CKM matrix; knowledge of heavy quark masses and nonperturbative parameters of QCD is of high importance as well.

Some of the HQ parameters like $\mu_G^2$ are known beforehand. Proper field-theoretic definition allows its accurate determination from the $B^*-B$ mass splitting: $\mu_G^2 (1\, \text{GeV}) = 0.35^{+0.03}_{-0.02}\, \text{GeV}^2$ [7]. A priori less certain is $\mu_{LS}^2$. However, the inequality $\mu_{LS}^2 > \mu_G^2$ valid for any definition of kinetic and chromomagnetic operators respecting the QCD commutation relation $[D_j, D_k] = -ig_{ij} G_{jk}$, and the corresponding sum rules essentially limit its range: $\mu_{LS}^2 (1\, \text{GeV}) = 0.45\pm0.1\, \text{GeV}^2$.

**Lepton and hadron moments.** Moments of the charged lepton energy in the semileptonic $B$ decays are traditional observables to measure heavy quark parameters. Moments of the invariant hadronic mass squared $M_X^2$ in semileptonic decays is another useful set of observables. Their utility follows from the observation [2] that, at least if charm were heavy enough the first, second and third moments would more or less directly yield $X$, $\mu_G^2$ and $\rho_{LS}^3$. Precision measurements of the $B \to X_c + \gamma$ decays also may yield a number of constraints. Details of the present situation can be found in the experimental review talks [12,13,14,15]. Let me briefly illustrate how this strategy works number-wise. Lepton energy moments, for instance, are given by the following approximate expressions ($b \to u$ decays are neglected):

$$
\langle E_l \rangle = 1.38 \, \text{GeV} + 0.38 [(m_b - 4.6 \, \text{GeV}) - 0.7 (m_c - 1.15 \, \text{GeV})]
+ 0.03 (\mu_G^2 - 0.4 \, \text{GeV}^2) - 0.09 (\rho_{LS}^3 - 0.12 \, \text{GeV}^3),
$$

$$
\langle (E_l - \langle E_l \rangle)^2 \rangle = 0.18 \, \text{GeV}^2 + 0.1 [(m_b - 4.6 \, \text{GeV}) - 0.6 (m_c - 1.15 \, \text{GeV})] + 0.045 (\mu_G^2 - 0.4 \, \text{GeV}^2)
- 0.06 (\rho_{LS}^3 - 0.12 \, \text{GeV}^3),
$$

$$
\langle (E_l - \langle E_l \rangle)^3 \rangle = -0.033 \, \text{GeV}^3 - 0.03 [m_b - 4.6 \, \text{GeV}]
- 0.8 (m_c - 1.15 \, \text{GeV}) + 0.024 (\mu_G^2 - 0.4 \, \text{GeV}^2)
- 0.035 (\rho_{LS}^3 - 0.12 \, \text{GeV}^3)
$$

(1) (all dimensionful factors are given in the corresponding powers of GeV). The moments depend basically on one and the same combination of masses $m_b - 0.65m_c$; dependence on $\mu_G^2$ is rather weak. To even larger extent this applies to the CLEO’s cut moments $R_1, R_2$ and the ratio $R_3$ – they depend practically on a single combination $m_b - 0.63m_c + 0.5\mu_G^2$. The effect of the spin-orbital average $\rho_{LS}^3$ is negligible. The uniform dependence can be understood noting that the spectrum is mainly determined by the parton expression, with nonperturbative effects playing a relatively insignificant role.

In spite of this ‘inefficiency’ of higher moments in constraining heavy quark parameters, lepton moments already allow a decent determination of $|V_{cb}|$. Indeed, its value extracted from $\Gamma_{b}(B)$ has the following dependence:

$$
|V_{cb}| = 0.042 \pm 0.013 [(m_b - 4.6 \, \text{GeV}) - 0.61 (m_c - 1.15 \, \text{GeV})]
+ 0.013 (\mu_G^2 - 0.4 \, \text{GeV}^2) + 0.11 (\rho_{LS}^3 - 0.12 \, \text{GeV}^3)
+ 0.06 (\mu_G^2 - 0.35 \, \text{GeV}^2) - 0.01 (\rho_{LS}^3 + 0.15 \, \text{GeV}^3) =
1 - \frac{6.5}{12} (\langle E_l \rangle - 1.38 \, \text{GeV}) - 0.06 (m_c - 1.15 \, \text{GeV})
- 0.07 (\mu_G^2 - 0.4 \, \text{GeV}^2) - 0.05 (\rho_{LS}^3 - 0.12 \, \text{GeV}^3) - 0.08 (\mu_G^2 - 0.35 \, \text{GeV}^2) - 0.005 (\rho_{LS}^3 + 0.15 \, \text{GeV}^3); (2)
$$

a combination of the parameters has been replaced by the first lepton moment in Eq. 1, and the residual sensitivity to $\mu_G^2$ and $\rho_{LS}^3$ is illustrated. The combination determining the semileptonic width is also very close! We see that the precise value of charm mass is irrelevant, but reasonable accuracy in $\mu_G^2$ and $\rho_{LS}^3$ is required.

The first hadronic moment takes the form

$$
\langle M_X^2 \rangle = 4.54 \, \text{GeV}^2 - 5.0 [(m_b - 4.6 \, \text{GeV}) - 0.62 (m_c - 1.15 \, \text{GeV})]
- 0.66 (\mu_G^2 - 0.4 \, \text{GeV}^2) + (\rho_{LS}^3 - 0.12 \, \text{GeV}^3), (3)
$$
i.e., again given by nearly the same combination $m_b - 0.7m_c + 0.1\mu_a^2 - 0.2\rho_D^3$ as the lepton moment. Not very constraining, this provides, however a highly nontrivial check of the HQ expansion. For example, taking the DELPHI’s central value for $\langle M^2_X \rangle$ we would predict $\langle E_\ell \rangle = 1.377$ GeV, while experimentally they obtain $\langle E_\ell \rangle = (1.383 \pm 0.015)$ GeV [14]. In this respect such a comparison is more critical than among the lepton moments themselves. In particular, these two first moments together verify the heavy quark sum rule for $M_B - m_b$ with the accuracy about 40 MeV!

The dependence on heavy quark parameters expectedly changes for higher hadronic moments:

$$\langle (M^2_X - \langle M^2_X \rangle)^2 \rangle = 1.2 \text{ GeV}^4 - 0.003 (m_b - 4.6 \text{ GeV})$$

$$- 0.68 (m_t - 1.15 \text{ GeV}) + 4.5 (\mu_a^2 - 0.4 \text{ GeV}^2)$$

$$- 5.5 (\rho_D^3 - 0.12 \text{ GeV}^3)$$,

$$\langle (M^2_X - \langle M^2_X \rangle)^3 \rangle = 4 \text{ GeV}^6 + (m_b - 4.6 \text{ GeV})$$

$$- 3 (m_t - 1.15 \text{ GeV}) + 5 (\mu_a^2 - 0.4 \text{ GeV}^2)$$

$$+ 13 (\rho_D^3 - 0.12 \text{ GeV}^3).$$ (4)

Ideally, they would measure the kinetic and Darwin expectation values separately. At present, however, we have only an approximate evaluation and informative upper bound on $\rho_D^3$. The current sensitivity to $\mu_a^2$ and $\rho_D^3$ is about 0.1 GeV$^2$ and 0.1 GeV$^3$, respectively.

The experimental constraint on the combination driving $\Gamma_{\ell \ell}(B)$ in the reported DELPHI measurements appears stronger for the hadronic moment. Using it instead of $\langle E_\ell \rangle$ we would arrive at

$$\frac{|V_{cb}|}{0.042} = 1 + 0.14 \langle (M^2_X - 4.54 \text{ GeV}^2) - 0.03 (m_b - 1.15 \text{ GeV})$$

$$+ 0.1 (\mu_a^2 - 0.4 \text{ GeV}^2) + 0.1 (\rho_D^3 - 0.12 \text{ GeV}^3).$$ (5)

We see that measuring the second and third hadronic moments is an essential step in implementing the comprehensive program of extracting $|V_{cb}|$ (see figures in M. Calvi’s talk, [14], and in [16]). Neglecting possible theoretical uncertainties in the above relations, we get, for example,

$$|V_{cb}| = 0.0421 \left(1 \pm 0.01 \text{SL width} \pm 0.015 \text{HQPpar} \right)$$ (6)

from only DELPHI hadronic moments. Incorporating into the fit the full set of moments they arrived at even smaller error interval.

Clearly, more work – both theoretical and experimental – is required to fully use the potential of inclusive semileptonic decays. It is crucial that this extraction carries no hidden assumptions, and at no point we rely on $1/m_c$ expansion. Charm quark could be either heavy, or light as strange or up quark, without deteriorating – and rather improving – the accuracy.

A similar analysis can be applied to the moments with a cut on lepton energy; in particular, CLEO has measured a few lepton energy moments for $E_\ell > 1.5$ GeV with unprecedented accuracy. As mentioned above, they also fix more or less the same combination of masses and nonperturbative parameters. The value comes out close, but does not literally coincide with that obtained by DELPHI. This clearly deserves further scrutiny. However, since the accuracy of theoretical expressions is limited, in particular for relatively high cut employed, at the moment I do not see reasons to worry. As we heard at this Workshop [12], CLEO was able to do similar measurements lowering the cut down to about 1 GeV. We are looking forward to the new data – they can be treated more reliably by theory and are expected to provide critical checks.

Due to space and time limitations I have to omit discussion of an important question of convergence of power expansion we employ for the analysis of inclusive decays, and closely related to it theoretically problem of possible violations of quark-hadron duality. Referring the reader to the dedicated publications [8] (an interesting complementary discussion can also be found in the recent CKM proceedings [17]. Chapter 3. Sect. 2.3), I only mention that the answer radically depends on what concretely is studied, and how. Using the proper approach the effects are under good control for total semileptonic widths, but not necessary so for higher hadronic moments where some improvements may be required to achieve desirable precision.

**Experimental cuts and hardness.** There is a problem, however, which should not be underestimated. The intrinsic ‘hardness’ of the moments deteriorates when the cut on $E_\ell$ is imposed. As a result, say the extraordinary experimental accuracy of CLEO’s $R_\ell$ cannot be even nearly utilized by theory, whether or not the expressions we use make this explicit.\(^2\)

For total widths the effective energy scale parameter is generally $Q = m_b - m_c$. Where OPE applies we can go beyond purely qualitative speculations about hardness. Then it is typically given by $Q \lesssim \omega_{\text{max}}$, with $\omega_{\text{max}}$ the threshold energy at which the decay process kinematically disappears once $m_b$ is replaced by $m_b - \omega$. With the $E_\ell > E_{\text{min}}$ cut then

$$Q \approx m_b - E_{\text{min}} - \sqrt{E_{\text{min}}^2 + m_c^2}$$ (7)

constituting only meager 1.25 GeV for $E_{\text{min}} = 1.5$ GeV, and falls even below 1 GeV for the decays with $E_\ell > 1.7$ GeV. A closer look reveals that such a limitation appears relevant for understanding the ‘unexpected’ behavior of the first hadronic moment with respect to the cut on $E_\ell$ reported by BaBar last summer [19].

\(^2\)An instructive example of how naive analysis can miss such effects was given in [13]. Sect. 5.
In $b \to s + \gamma$ decays one has $Q \approx m_b - 2E_{\text{min}}$, once again a rather soft scale 1.2 GeV if the lower cut is placed at $E_{\gamma} = 2$ GeV. Hence, the reliability of theory can be questioned when one aims at maximum precision. For higher moments the hardness further deteriorates in either decays. A high premium should then be put on lowering the cuts [18].

On the theoretical side, the higher hadronic moments can be affected by nonperturbative physics formally scaling as powers of $1/m_b$ greater than 3. At the same time, these moments are instrumental for truly model-independent comprehensive studies of $B$ mesons; improvement is needed already for the third moment, its expression given above is not too accurate. Considering alternative kinematic variables will help to improve the convergence. This is related to the peculiarity of the kinematic definition of the invariant mass $M_X^2$:

$$M_X^2 = (P_B - q)^2 + 2(M_B - m_b)(m_b - q_0) + (M_B - m_b)^2$$

where $p_c \equiv m_b v - q$ is the $c$-quark momentum as it emerges at the quark level and $m_b - q_0 \equiv E_c$ is its energy; as always $q$ is the lepton pair four-momentum. In the standard OPE we compute separately $\Delta \equiv p_c^2 - m_c^2$ having the meaning of the final state quark virtuality, and $E_c$. Power expansion in terms of local heavy quark operators applies to both of them, or any their product. Convergence of $\Delta$ or its powers is satisfactory; large corrections in higher moments emerge from the product $2(M_B - m_b)E_c$. Moreover, it is powers of $\Delta$ that are directly related to the higher-dimension expectation values. They also determine the distribution over the off-shellness of the final-state $c$ quark, much in the same way as the moments of the light-cone heavy quark distribution function shape the photon energy spectrum in $b \to s + \gamma$.

Therefore, it is advantageous to trade the traditional hadronic mass $M_X^2$ for the observable more closely corresponding to the quark virtuality, defined as

$$N_X^2 = M_X^2 - 2\Delta E_X,$$  

where $E_X = M_B - q_0$ is the total hadronic energy in the $B$ restframe, and $\Delta$ a fixed mass parameter (a constant in $N_X^2$ does not affect higher moments). Preferred values are about $\Delta \approx M_B - m_b(1$ GeV) and can be taken 500–700 MeV. The higher moments $\langle (N_X^2 - \langle N_X^2 \rangle)^2 \rangle$, $\langle (N_X^2 - \langle N_X^2 \rangle)^4 \rangle$... should enjoy better theoretical stability. It may be possible to further improve it dividing $N_X^2$ by certain kinematic factors.

The kinematic variable $N_X^2$ is not well constrained inclusively at LEP experiments. Is it possible to measure it at $B$-factories? This question was raised at ICHEP 2002, and it seems the answer is positive; I’m grateful to colleagues from BaBar for clarifying this during the $V_{ub}$ and $V_{ts}$ Workshop at SLAC. This possibility should be explored.

**Problems for HQE?** So far I was seemingly quite optimistic confronting OPE-based heavy quark expansion for inclusive semileptonic decays to experiment. There are two problems, however which are often mentioned as constituting problems for the OPE. One is a stronger than predicted [20] dependence of $\langle M_X^2 \rangle$ on the lepton cut $E_\gamma$ reported last summer by BaBar [13]. The second is a possible deviation of the heavy quark parameters from $b \to s + \gamma$ spectrum reportedly deteriorating the global fit when these data are included [20].

I am not sure we really witness here the problem for the theory based on the OPE. Clearly, future confirmation of experimental results would be necessary to draw any radical conclusion, as has been emphasized in a number of talks. Yet, I think even the present data taken at face value do not indicate problems with applicability of the OPE, but may rather be attributed to certain unjustified simplifications made in the theoretical analyses.

The corresponding problems with the $b \to s + \gamma$ spectrum actually had been discussed in paper [18] contributed to the CKM 2002 Workshop. Sect. 5 specifically pointed out the complications arising when the cut on $E_\gamma$ is not low enough. In this case the routinely used expressions for the moments in the form of simple sum of perturbative and nonperturbative terms, are not justified. This is another side of the same deteriorating ‘hardness’ I discussed earlier. Theory would become more reliable if the cut could be lowered by even 100 MeV.

The similar problem plagues usefulness of the ‘global fits’ like one reported in Ref. [20] – they combine accurate and well justified theoretical expressions with those of an imprecise nature and treat them on parallel footing. The literal outcome like the overall quality of the fit may then be not too meaningful, and some conclusions may happen to be misleading. The employed approach to treat theoretical uncertainty was also criticized by some experimental colleagues. While such fits are a reasonable starting point when new data are just coming, clearly a more thoughtful and sophisticated way to analyze the whole set of data has to be elaborated to confront data and theory seriously and to target the ultimate precision.

Here I would like to dwell in more detail on the first alleged problem, the $E_\gamma$-cut dependence of $\langle M_X^2 \rangle$, since it attracted much attention and caused various speculations. There may be a certain pure experimental controversy which will eventually lead to some change in numbers. Its concise discussion has been given by M. Luke [21]. I abstract here from such possibilities and accept the data at face value.

On one hand, I have mentioned that, say comparing the DELPHI’s leptonic and hadronic moments one gets an impressive agreement at the nonperturbative level. On
the other hand, the experimental measurement of $\langle M_\chi^2 \rangle$ at different $E_\ell$ performed by BaBar apparently shows a far stronger dependence than the curve quoted as a theoretical prediction, Fig. 1. This may look as a controversy: do we observe a triumph or failure of the OPE?

I would advocate the opinion that the discrepancy – which may look serious – in fact is hardly established since the theoretical uncertainties in the evaluation of $\langle M_\chi^2 \rangle$ have been essentially underestimated here, in particular at the higher end of $E_\ell$. (Claimed theory error bars ranged from $\pm 0.035$ GeV$^2$ to $\pm 0.05$ GeV$^2$ for low and high $E_\ell$, respectively.)

To give a sense of numbers without submerging into technicalities, I recall a few relevant facts.

- OPE computes in first instance not the plotted $\langle M_\chi^2 - M_{\ell b}^2 \rangle \approx 0.4$ GeV$^2$ for which the relative discrepancy looks of order unity, but rather $\langle M_\chi^2 - (m_\ell + M/2)^2 \rangle \approx 1.2$ GeV$^2$ – and all of it is the effect of strong interactions, mostly nonperturbative.
- The actual sensitivity of $\langle M_\chi^2 \rangle$ to heavy quark parameters is illustrated by the fact that variation of $m_b$ by only $\pm 20$ MeV changes $\langle M_\chi^2 \rangle$ by $\pm 0.1$ GeV$^2$, the scale of the alleged discrepancy.\footnote{Z. Ligeti claimed he and collaborators did not find such a strong dependence in the theoretical expressions implying that this should undermine the proposed explanation. I insist that this dependence is true.}

Rephrasing Ben Bradley, one may then suspect that the problem originates from “Dealing in the expressions, not necessarily in (OPE) truths”.

The sensitivity to the precise value of $m_b$ refers to varying only $m_b$ while keeping all other heavy quark parameters we use, in particular $m_\ell$ and $\mu^2$ fixed. Since, as mentioned earlier, charm mass expansion has a questionable accuracy and this is the only independent way to get a direct con-

\[ \langle M_\chi^2 \rangle - \langle M_{\ell b}^2 \rangle, \] 

straing on $m_b - M_c$, this is an appropriate yardstick; even having $m_c$ known exactly we a priori cannot pinpoint the value of $m_b$ with better than 30 to 40 MeV.

Of course, this example helps only to illustrate the scale of the involved theoretical uncertainties in more familiar terms. By itself varying $m_b$ by such a small amount can shift overall theoretical prediction up or down, but cannot noticeably change the variation of $\langle M_\chi^2 \rangle$ with increase of the $E_\ell$ cut up to 1.5 GeV. Whatever $m_b$ is, it is a constant, the actual $b$ quark mass. This might suggest the above consideration is not relevant for reconciling theory with the data. Such a conclusion, however would tacitly assume that other theoretical uncertainties are likewise independent on the placement of the cut. This would be grossly wrong. In fact, we know that the OPE expansion deteriorates fast when $E_\ell$ increases and must completely blow up when $E_\ell$ approaches the borderline at a fraction of GeV below 2.15 GeV.

The numbers following Eq. (4) characterizing the hardness qualitatively suggest the breakdown scale can be just around 1.7 GeV. It can be concluded in a more quantitative fashion that the usual power expansion of $\langle M_\chi^2 \rangle$ loses sense at least for cuts above just $E_\ell = 1.7$ GeV. To illustrate this, let us assume the chromomagnetic operator vanishes, and neglect the perturbative effects. The leading term in the OPE is then given by the kinetic average $\mu^2_{\ell \gamma}$ and it contributes with the negative sign to $\langle M_\chi^2 - M_{\ell b}^2 \rangle$. There is, however a positive leading parton-like contribution $2(M_{\ell b} - m_b)(E_\gamma)$. With the canonical values of parameters, $m_b = 4.6$ GeV and $\mu^2_{\ell \gamma} = 0.4$ GeV$^2$ we would obtain reasonable positive $\langle M_\chi^2 \rangle - M_{\ell b}^2$ at low lepton cut, it would vanish for the cut near 1.7 GeV and, retaining only the leading terms $\langle M_\chi^2 \rangle$ becomes smaller than $M_{\ell b}^2$ for higher cuts (the shift in the meson masses when playing with $\mu^2_{\ell \gamma}$ and $\mu^2_{\ell b}$ should be accounted for). Clearly this would be physically meaningless, signifying that higher order corrections must become 100% important, whatever are our naive estimates of the next-to-leading terms.

This qualitative behavior of the OPE is quite natural. The question then arises – why the increasing uncertainties were not revealed in the corresponding theoretical calculations? This is related to a certain subtlety in applying the OPE which was missed already in the original estimates [22]. Those early evaluations, representing a natural first step in the analysis, are conceptually incomplete in this respect. The underlying reason was addressed in Ref. [13], and I briefly reiterate it here.

In $b \rightarrow c \ell \nu$ decays this is somewhat obscured by three-body kinematics with massive $c$ quark. It is more transparent for $b \rightarrow s + \gamma$ which we then can look at first. Let us consider a constrained fraction of the $B \rightarrow X_s + \gamma$ events

\[ 1 - \Phi_s(E) = \frac{1}{\Gamma_{b s}} \int_E^{\infty} dE' \frac{d\Gamma_{b s}}{dE'}, \quad (10) \]
Ignoring the perturbative corrections, the usual $1/m_b$ expansion always yields the spectrum in the form of expanding around the free-quark kinematics:

$$\frac{1}{m_b} \frac{d \Gamma_{bs}^{\ell \nu}}{d E_{\ell \nu}} = a \delta(E_{\gamma} - \frac{m_b}{2}) + b \delta'(E_{\gamma} - \frac{m_b}{2}) + c \delta''(E_{\gamma} - \frac{m_b}{2}) + ...$$

where $a, b, ...$ are given by the expectation values of local $b$-quark operators over $B$. Naively computing $1 - \Phi_q(E)$, or spectral moments over the restricted domain in this way would yield unity for any $E > \frac{m_b}{2}$ - the result clearly unjustified. The actual behavior of the rates is described by the heavy quark distribution function. Its tail is indeed exponentially suppressed by a typical factor $e^{-Q/\mu_{hadr}}$ at $Q(E_{\gamma}) \gg \mu_{hadr}$, however for $Q(E_{\gamma}) - \mu_{hadr}$ the true integral differs by terms of order 1.

This is an expected behavior. The point is rather that this is totally missed in the naive OPE and in the way to gauge the theoretical uncertainty based on it! Conceptually this is related, as illustrated below, to the limited range of convergence of the OPE for the width, determined in this case by the support of the heavy quark distribution function.

The OPE for the semileptonic decays is similar in this respect. Both the lepton spectrum itself and the moments of, say $M_{b \ell}^2$, have similar end-point singularities in the power expansion, their resummation likewise yields a certain distribution function strongly affecting moments at $Q \lesssim \mu_{hadr}$. In $b \to u \ell \nu$ this is the domain $2E_{\ell} \gtrsim m_b - \mu_{hadr}$.

The situation turns out more deceptive for $b \to c \ell \nu$. In contrast to $b \to u$, charm mass leads to an apparent softening of the end-point behavior where some of $\delta(E_{\ell} - E_{\ell \max})$, $\delta'(E_{\ell} - E_{\ell \max})$, ... are replaced by regular functions. The latter, in fact represent the finite-width realization of the same distributions, with the effective width $\sim m_c^2/2m_b$. If $m_c^2$ were much larger than $\mu_{hadr} m_b$ this would indeed damp the end-point nonperturbative corrections. In practice, however, it appears that qualitatively $m_c^2 \lesssim 2\mu_{hadr} m_b$ and the effect of the nonperturbative smearing may still dominate. Then it is not reflected in the size of the naive OPE terms for the decay distributions.

On the other hand, here in $b \to c \ell \nu$ decays these singular Wilson coefficients do depend on the lepton cut through the ‘tails’ of the end-point spikes. In contrast to $b \to s + \gamma$ where total absence of the $E_{\ell \max}$-dependence alarms one that something is missing, the residual dependence in theoretical expressions for $1/m_b^3$ terms on $E_{\ell \max}$ masks this, and their suggested size strongly underestimated the actual corrections in the upper part of the spectrum.

This becomes rather obvious in the limit where $m_c^2 \ll 2\mu_{hadr} m_b$ [3]. Here the strong nonperturbative effects completely dominate the rates or moments at $E_{\ell} \gtrsim \frac{m_c^2 - m_b^2}{2m_b} - \frac{\mu_{hadr}}{2}$, while the naive OPE terms seem to suggest the corresponding effects are still suppressed by powers of $m_c/m_b$ there since are given by the tails of the $\delta$-like distributions localized in the domain of $m_c^2/2m_b$ near the end point.

This understanding reveals the proper approach to judging the convergence, as well as the actual uncertainty of the OPE series for the decay distributions. This follows from the consistent way of deriving the OPE for them [3]: one should analyze the power expansion for the whole forward transition amplitude in question, rather than the expansion obtained for its absorptive part one routinely is interested in.

Say, for $b \to s + \gamma$ we would need to take $T(q_0; 0)$, expand it in $1/(m_b - 2q_0)$ and study convergence of the power expansion for all $|m_b - 2q_0| > m_b - 2E_{\gamma \max}$. For the semileptonic width we would select [8]

$$\int dq^2 \left[ q^2 h_1(q_0; q^2) + \frac{1}{3} (2E_{\ell}(q_0-q_0)-q^2) h_2(q_0; q^2) + q^2 (2E_{\ell}-q_0) h_3(q_0; q^2) \right], \quad (11)$$

e etc. [4] Computing the width directly amounts to integrating these hadronic functions over a closed contour, which may lead to certain kinematic cancellations of individual coefficients even if the overall result is not convergent at all. A simple illustration is provided by the toy function

$$T(q_0; 0) = \frac{2}{m_b - \mu_{hadr} - 2q_0} \quad (12)$$

for which

$$\frac{1}{\pi} \int_{E_{\min}} dq_0 \text{Im} T(q_0; 0) = \delta \left( \frac{\mu_{hadr}}{2} - E_{\min} \right), \quad (13)$$

while its naive power expansion in $1/m_b$ yields

$$\frac{1}{\pi} \int_{E_{\min}} dq_0 \text{Im} T(q_0; 0) = \int_{E_{\min}} dq_0 \delta(q_0 - \frac{m_b}{2})$$

$$+ \frac{\mu_{hadr}}{2} \int_{E_{\min}} dq_0 \delta'(q_0 - \frac{m_b}{2}) + \cdots = \delta \left( \frac{m_b}{2} - E_{\min} \right), \quad (14)$$

i.e. exactly 1 for any cut below $m_b/2$. (The actual $b \to s + \gamma$ is not much different - one just needs to integrate this over $\mu_{hadr}$ with the light-cone distribution function $F(\mu_{hadr})$.) The flaw in relying on expansion (14) is obvious: this is not an expansion in $\mu_{hadr}/m_b$, but in powers of $\mu_{hadr}/(m_b - 2q_0)$ (cf. with energy release), and it can be used only at $1/(m_b - 2q_0)$ below the radius of convergence of the function $T(q_0; 0)$ itself, which is just $m_b - 2q_0 > \mu_{hadr}$. The power series for $\int dq_0 \text{Im} T(q_0; 0)$ which consists of the single, first term, of course converge for any $q_0$, but may yield the wrong answer.

[4] Convergence for hadronic $\tau$ decay widths also should be analyzed in the similar way.
Let me parenthetically note that the degree of convergence with a cut in \( E_t \) can be also assessed by studying the OPE for moments containing additional powers of \( E_t \). By approaching the endpoint \( E_{\text{max}} \) beyond just \( \langle M_X^2 \rangle \) we are directly interested in (here \( E_{\text{max}} \) is the end-point energy). However, this is a less straightforward and somewhat more ambiguous way.

Returning to our problem, we conclude that the intrinsic uncertainties of the straightforward theoretical expressions were clearly underestimated in [20], in particular for significant cuts in \( E_t \).

It is more difficult to quantify this. We know that \( \langle M_X^2 - M_D^2 \rangle \) must end up at a low value \(-0.3\,\text{GeV}^2\) when one approaches the endpoint \( E_{\text{max}} = \frac{M_X^2-M_D^2}{2M} \) since there only \( D \) can be produced, and \( \langle M_X^2 \rangle = M_D^2 \). Since actual \( \langle M_X^2 \rangle \) is a smooth function of \( E_{\text{cut}} \), we would a priori expect a behavior similar to the curve given by BaBar, rather than what they showed as the theoretical prediction, at least in the right-most part of the plot. However, at present it is difficult to evaluate the actual difference: the steep fall-off may start around 1.7 GeV or even a little higher. Or a more gradual decrease over a wider domain can show up for \( E_t \) even below 1.5 GeV, depending on the actual behavior of higher-order power corrections. Likewise it is hard to quantify, say the numerical difference between \( \langle M_X^2 \rangle_{E_{\text{cut}}=0} - \langle M_X^2 \rangle_{E_{\text{cut}}=1.5 \, \text{GeV}} \) and its value as given by the first three terms in its naive \( 1/m_{\text{PS}} \) expansion. It can naturally be as large as 0.15 GeV, but may turn out both smaller or even larger.

The practical recommendation can be suggested as stemming from this analysis. To extract heavy quark parameters or \( |V_{cb}| \) experiments should use as low cut on \( E_t \) as possible in their data sets where they are still sufficiently accurate and reliable. It is then instructive to compare the theoretically predicted \( \langle M_X^2 \rangle \) at higher cuts on \( E_t \) based on these robust low-\( E_t \) determinations, with direct experimental data. Since the overall shift is very sensitive already to quark masses themselves, and the data may be strongly correlated, the particular quantity of interest is

\[
\Delta M_X^2(E_{\text{cut}}) = \langle M_X^2 \rangle_{E_{\text{cut}}} - \langle M_X^2 \rangle_{E_{\text{cut}}}.
\]

Theory must properly describe its slope at lower \( E_{\text{cut}} \), but possibly not approaching 1.5 GeV. Confronting \( \Delta M_X^2 \) with theoretical predictions in a wider range we may infer a nontrivial qualitative information about strong dynamics of heavy quarks, similar to studying the photon spectrum in \( b \rightarrow s + \gamma \).

**BPS limit.** An intriguing theoretical environment opens up if \( \mu^2_\pi(1\,\text{GeV}) \) is eventually confirmed to be close enough to \( \mu^2_G(1\,\text{GeV}) \) as currently suggested by experiment, say it does not exceed 0.45 GeV. If \( \mu^2_\pi - \mu^2_G \ll \mu^2_\pi \) it is advantageous to analyze strong dynamics expanding around the point \( \mu^2_\pi = \mu^2_G \). This is not just one point of a continuum in the parameter space, but a quite special ‘BPS’ limit where the heavy flavor ground state satisfies functional relations \( \delta \pi(B) = 0 \). This limit is remarkable in many respects, for example, it saturates the bound \( \lim q^2 \geq \frac{1}{3} \) for the slope of the IW function. I hasten to recall that already quite some time ago there were dedicated QCD sum rules estimates of both \( \hat{q}^2 \) [24] and \( \mu^2_\pi \) [25] yielding literal values nearly at the respective lower bounds, supporting this limit.

The SV heavy quark sum rules place a number of important constraints on the nonperturbative parameters. For instance, they yield a bound on the IW slope

\[
\mu^2_\pi - \mu^2_G = 3 \hat{\delta} \langle q^2 - \frac{1}{4} \rangle,
\]

so that \( \hat{q}^2 \) can barely reach 1 being rather closer to 0.85. It is interesting that this prediction [27] turned out in a good agreement with the recent lattice calculation [26] \( \hat{q}^2 = 0.83^{+0.15}_{-0.11} \). This leaves only a small room for the slope of the actual \( B \rightarrow D^* \) formfactor, excluding values of \( \hat{q}^2 \) in excess of 1.15–1.2. This would be a very constraining result for a number of experimental studies, in particular for extrapolating the \( B \rightarrow D^* \) rate to zero recoil. Since there is a strong correlation between the extrapolated rate and the slope, this may change the extracted value of \( |V_{cb}| \).

Therefore, it is advantageous to analyze the \( B \rightarrow D^* \ell \nu \) data including the above constraint as an option, and I suggest experimental colleagues to explore this in future analyses.

The experimentally measured slope \( \hat{\rho}^2 \) differs from \( \hat{q}^2 \) by heavy quark symmetry-violating corrections. The estimate by Neubert that \( \hat{\rho}^2 \) is smaller than \( \hat{q}^2 \), \( \hat{\rho}^2 \approx \hat{q}^2 - 0.09 \) seems to be ruled out by experiment. It is not clear if a better estimate can be made in a trustworthy way.

The whole set of the heavy quark sum rules is even more interesting. Their constraining power depends strongly on the actual value of \( \mu^2_\pi \). When it is at the lower end of the allowed interval, the BPS expansion appears the most effective way to analyze them.

**Miracles of the BPS limit.** A number of useful relation for nonperturbative parameters hold in the limit \( \mu^2_\pi = \mu^2_G \): they include \( \hat{q}^2 = \frac{3}{4}, \frac{\overline{\Delta}}{N} = 2 \sum_i \rho^2_{i3} = -\rho^2_D \), relations for nonlocal correlators \( \rho^2_{\pi G} = -2 \rho^2_{\pi^0}, \rho^2_A + \rho^2_D = -\rho^2_{\pi^0} + \rho^2_{\pi^0} \), etc.

This limit also extends a number of the heavy flavor symmetry relations for the ground-state mesons to all orders in \( 1/m \):

- There are no formal power corrections to the relation \( M_p = m_0 + \overline{\Delta} \) and, therefore to \( m_b - m_0 = M_B - M_D \).
- For the \( B \rightarrow D \) amplitude the HQ limit relation between the two formfactors

\[
f_+(q^2) = -\frac{M_B - M_D}{M_B + M_D} f_-(q^2)
\]

(17)
does not receive power corrections.

- For the zero-recoil $B \to D$ amplitude all $\delta_{1/m}\beta$ terms vanish.
- For the zero-recoil amplitude $f_\pi$ with massless leptons

$$f_\pi((M_B-M_D)^2) = \frac{M_B+M_D}{2\sqrt{M_B M_D}}$$

(18)

to all orders in $1/m_Q$.
- At arbitrary velocity power corrections in $B \to D$ vanish,

$$f_\pi(q^2) = \frac{M_B+M_D}{2\sqrt{M_B M_D}} \xi\left(\frac{M_B^2+M_D^2-q^2}{2M_B M_D}\right)$$

(19)

so that the $B \to D$ decay rate directly yields Isgur-Wise function $\xi(w)$.

It is interesting that experimentally the slope of the $B \to D$ amplitude is indeed smaller centering around $\hat{\beta}(D) \approx 1.15$ [17] indicating qualitative agreement with the BPS regime.

What about the $B \to D^*$ amplitude, are the corrections suppressed as well? Unfortunately, the answer is negative.

The structure of power corrections indeed simplifies in the BPS limit, however $\delta_{1/m}\beta$, $\delta_{1/m}\beta$ are still very significant [7], and the literal estimate for $F_{D^*}(0)$ falls even below 0.9. Likewise, we expect too significant corrections to the shape of the $B \to D^*$ formfactors. Heavy quark spin symmetry controlling these transitions seems to be violently affected by strong interactions for charm.

A physical clarification must be added at this point. Absence of all power corrections in $1/m_Q$ for certain relations may be naively interpreted as implying that they would hold for arbitrary, even small quark masses, say in $B \to K$ transitions. This is not correct, though, for the statement refers only to a particular fixed order in $1/m_Q$ expansion in the strict BPS limit. In fact the relations become more and more accurate approaching this limit only above a certain mass scale of order $\Lambda_{QCD}$, while below it their violation is of order unity regardless of proximity of the heavy quark ground state to BPS.

Quantifying deviations from BPS. The BPS limit cannot be exact in actual QCD, and it is important to understand the scale of violations of its predictions. The dimensionless parameter describing the deviation from BPS is

$$\beta = ||\sigma_0^{-1}(\cF)\langle B|\rangle|| = \sqrt{3(|\theta^2 - \frac{1}{2}| + \sum |\tau^{(n)}_{1/2}|^2)}.$$ (20)

Numerically $\beta$ is not too small, similar in size to generic $1/m_c$ expansion parameter, and the relations violated to order $\beta$ may in practice be more of a qualitative nature only. However, $\mu^2 - \mu^2_{QCD} \propto \beta^2$ can be good enough. Moreover, we can count together powers of $1/m_c$ and $\beta$ to judge the real quality of a particular HQ relation. Therefore understanding at which order in $\beta$ the BPS relations get corrections is required. In fact, we need classification in powers of $\beta$ to all orders in $1/m_Q$.

Relations [17] and [19] for the $B \to D$ amplitudes at arbitrary velocity can get first order corrections in $\beta$. Thus they may not be very accurate. The same refers to equality of $\beta_{QCD}$ and $-2\beta_{QCD}$. The other relations mentioned for heavy quark parameter are accurate up to order $\beta^2$. The other important BPS relations hold up to order $\beta^3$ as well:

- $M_B-M_D = m_b-m_c$ and $M_D = m_c + \Lambda$
- Zero recoil matrix element $\langle D|\gamma_0|B\rangle$ is unity up to $O(\beta^2)$
- Experimentally measured $B \to D$ formfactor $f_\pi$ near zero recoil receives only second-order corrections in $\beta$ to all orders in $1/m_Q$:

$$f_\pi((M_B-M_D)^2) = \frac{M_B+M_D}{2\sqrt{M_B M_D}} + O(\beta^2).$$ (21)

This is an analogue of the Ademollo-Gatto theorem for the BPS expansion.

The similar statement then applies to $f_\pi$ as well, and the HQ limit prediction for $f_{\pi}/f_{\pi}$ must be quite accurate near zero recoil. It can be experimentally checked in the decays $B \to D \tau\nu$. As a practical application of the results based on the BPS expansion, one can calculate the $B \to D$ decay amplitude near zero recoil to use this channel for the model-independent extraction of $|V_{cb}|$ in future high-luminosity experiments. For power corrections we have

$$\frac{M_B+M_D}{2\sqrt{M_B M_D}} f_\pi((M_B-M_D)^2) = 1 + \frac{\Lambda}{2\Sigma} \left(\frac{1}{m_c - m_b} - \frac{1}{m_c - m_b} \frac{M_B M_D}{M_B + M_D} - O(\frac{1}{m_Q})\right).$$ (22)

We see that this indeed is of the second order in $\beta$. Moreover, $\Lambda - \Sigma$ is well constrained through $\mu^2 - \mu^2_{QCD}$ by spin sum rules. Including perturbative corrections (which should be calculated in the proper renormalization scheme respecting BPS regime) we arrive at the estimate

$$f_{\pi}(0) = 1.03 \pm 0.025$$ (23)

It is valid through order $\beta^2/m_c$ accounting for all powers of $1/m_Q$ to order $\beta^3$. Assuming the counting rules suggested above this corresponds to the precision up to $1/m_Q^3$, essentially better than "gold-plated" $B \to D^*$ formfactor where already $1/m_Q^2$ terms are large and not well known. Therefore, the estimate [23] must be quite accurate. In fact, the major source of the uncertainty seems to be perturbative corrections, which can be refined in the straightforward way compared to decade-old calculations.

Conclusions. Experiment has entered a new era of exploring $B$ physics at the nonperturbative level, with qualitative
improvement in $|V_{cb}|$. The comprehensive approach will allow to reach a percent level of reliable accuracy in translating $\Gamma_{\ell\ell}(B)$ to $|V_{cb}|$. Recent experiments have set solid grounds for dedicated future studies at $B$ factories. We have observed a nontrivial consistency between quite different measurements, and between experiment and QCD-based theory, at the nonperturbative level.

There are obvious lessons to infer. Experiment must strive to weaken the cuts in inclusive measurements used in extracting $|V_{cb}|$. Close attention should be paid to higher moments or their special combinations, as well as exploring complementary kinematic observables.

The theory of heavy quark decays is now a mature branch of QCD. Recent experimental studies of inclusive decays yielded valuable information crucial – through the comprehensive application of all the elements of the heavy quark expansion – for a number of exclusive decays as well. This signifies an important new stage of the heavy quark theory, since only a few years ago exclusive and inclusive decays were often viewed as largely separated, if not as antipodes theory-wise.

Generally speaking, there is ample evidence that heavy quark symmetry undergoes significant nonperturbative corrections for charm hadrons. However, there appears a class of practically relevant relations which remain robust. They are limited to the ground-state pseudoscalar $B$ and $D$ mesons, but do not include spin symmetry in the charm sector.

The accuracy of these new relations based on the proximity to the “BPS limit” strongly depends on the actual size of the kinetic expectation value in $B$ mesons, $\mu^2_{\rho}(1\text{ GeV})$. The experiment must verify it with maximal possible accuracy and fidelity, without relying on ad hoc assumptions often made in the past with limited data available. This can be performed through the inclusive decays already in current experiments. If its value is confirmed not to exceed 0.45 GeV$^2$, the $B \to D$ decays can be reliably treated by theory, and the estimate $\hat{F}_+(0) \simeq 1.03$ can provide a good alternative element in the comprehensive program of model-independent extraction of $|V_{cb}|$.

There are many other important consequences of the BPS regime. The slope of the IW function must be close to unity, and actually below it. The related constraints on the slope $\hat{\rho}^2$ of the experimentally observed combination of $B \to D^*$ formfactors should be incorporated in the fits aiming at extrapolating the rate to zero recoil. Finally, I think that the semileptonic decays with $\tau$ lepton and $\nu_\tau$ have an interesting potential for both inclusive and exclusive decays at high-luminosity machines.

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