Constraints on lepton flavor violation in the two Higgs doublet model

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Abstract
Constraints on the whole spectrum of lepton flavor violating vertices are shown in the context of the standard two Higgs doublet model. The vertex involving the $e-\tau$ mixing is much more constrained than the others, and the decays proportional to such vertex are usually very suppressed.

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1 Introduction
Many extensions of the standard model leads to flavor changing neutral currents (FCNC) naturally. It is the case of models with an extended Higgs sector. However, owing to the high suppression imposed by experiments, several mechanisms has been used to get rid of them, such as discrete symmetries [1], permutation symmetries [2], and different textures of Yukawa couplings [3]. Notwithstanding, the increasing evidence on neutrino oscillations seems to show the existence of mass terms for the neutrinos as well as of family lepton flavor violation (LFV) [4]. Such fact has inspired the study of many scenarios that predict LFV processes as in the case of SUSY theories with R-parity broken [5], SU(5) SUSY models with right-handed neutrinos [6], models with heavy Majorana neutrinos [7], and multi-Higgs doublet models with right-handed neutrinos for each lepton generation [8]. On the other hand, LFV in the charged sector has been also examined in models such as SUSY GUT [9], and the two Higgs doublet model (2HDM) [3, 10, 11].

Experimental upper limits for the branching ratios of these processes have been obtained from several collaborations, in the case of the charged lepton...
sector, searches for them have been carried out through leptonic and semileptonic decays of $K$ and $B$ mesons \cite{12}, as well as purely leptonic processes \cite{13}. On the other hand, some collaborations plan to improve current upper limits of some LFV decays by several orders of magnitude, by increasing the statistics \cite{14}. Other possible sources of improvement lies on the Fermilab Tevatron and LHC by means of LFV Higgs boson decays. In particular, Ref. \cite{15} shows that the flavor changing mode $h \rightarrow \mu \tau$ in the context of MSSM and of an $E_6$-inspired multi-Higgs model with an abelian flavor symmetry; can be sizable at the CERN-LHC and the Fermilab-Tevatron.

Further, one of the most promising source to look for Higgs mediated flavor changing neutral currents, lies on the muon colliders. It is because they have the potentiality to produce Higgs bosons in the $s-$channel, with substantial rate production at the Higgs mass resonance \cite{16}. Some of the main advantages of muon colliders consists of its negligible synchrotron radiation and bremsstrahlung, as well as the small beam energy spread \cite{16}. From the theoretical point of view, since Higgs Yukawa couplings are usually proportional to the lepton mass, they give an important enhancement to cross sections with Higgs mediated $s-$channels, respect to the ones in an $e^+ e^-$ collider. The small spread in the center of mass energy would permit a precision measurements of narrow resonances, that in turn allow a good determination of the Higgs mass and Higgs decay width. In particular, for the process $\mu^+ \mu^- \rightarrow h \rightarrow \mu \tau$, Ref. \cite{17} have found that in the context of the 2HDM III, hundreds of such events are expected if $m_h \leq 140$ GeV with a total integrated luminosity of $1 fb^{-1}$ over a negligible background, providing useful information about the $\mu - \tau$ mixing. Besides, Ref. \cite{17} also found that the process $\mu \mu \rightarrow e \tau$ could be observable as well, though only some few events are expected. Notwithstanding, for Higgs boson masses above the $h \rightarrow WW^*, ZZ^*$ threshold ($m_h \gtrsim 150$ GeV) the opening of these new channels decrease dramatically the production of such FCNC decays.

In a recent previous work \cite{11}, some constraints on LFV have been found in the framework of the two Higgs doublet model with flavor changing neutral currents. Specifically, bounds on the vertices $\xi_{\mu \tau}, \xi_{e \tau}, \xi_{\mu \mu}, \xi_{\tau \tau}$, where obtained based on the $g - 2$ muon factor and the leptonic decays $\mu \rightarrow e \gamma, \tau \rightarrow \mu \mu \mu, \tau \rightarrow \mu \gamma$. Additionally, upper limits on the decays $\tau \rightarrow e \gamma$ and $\tau \rightarrow eee$ were estimated, finding them to be highly suppressed respect to the present experimental sensitivity.

The purpose of this work is to complete the information about the spectrum of the LFV matrix. With this in mind, we shall use the leptonic processes $\tau^- \rightarrow \mu^- e^+, \tau^- \rightarrow \mu^+ \mu^- e^-$ and $\tau^- \rightarrow \mu^- e^- e^+$ as the inputs for our constraints.

2 The decays

We shall work in the context of the two Higgs doublet model (2HDM) with flavor changing neutral currents, the so called model type III. The lepton vertices are
described by the following Yukawa Lagrangian

\[ -\mathcal{L}_Y = \sum_{E} \frac{g}{2M_W} M_{E}^{\text{diag}} E \left( \cos \alpha H^0 - \sin \alpha h^0 \right) \]

\[ + \frac{1}{\sqrt{2}} \sum_{E} E \xi^E \left( \sin \alpha H^0 + \cos \alpha h^0 \right) \]

\[ + \bar{E} \xi^E P_R E H^+ + \frac{i}{\sqrt{2}} \sum_{E} \bar{E} \xi^E \gamma_5 E A^0 + \text{h.c.} \] (1)

where $H^0 (h^0)$ denote the heaviest (lightest) neutral $CP$-even scalar, and $A^0$ is a $CP$-odd scalar. $E$ refers to the three charged leptons $E \equiv (e, \mu, \tau)^T$ and $M_E, \xi_E$ are the mass matrix and the LFV matrix respectively, $\alpha$ is the mixing angle in the $CP$-even sector. We use the parametrization in which one of the vacuum expectation value vanishes.

The decays needed to obtain our bounds are given by

\[ \Gamma (\tau^- \rightarrow \mu^- \mu^- e^+) = \frac{m_e^5}{4096\pi^3 \xi_{\mu e}^2} \left[ \left( \frac{\sin^2 \alpha}{m_{H^0}^2} + \frac{\cos^2 \alpha}{m_{h^0}^2} - \frac{1}{m_{A^0}^2} \right)^2 \right. \]

\[ \left. + \frac{8}{3m_{A^0}^2} \left( \frac{\sin^2 \alpha}{m_{H^0}^2} + \frac{\cos^2 \alpha}{m_{h^0}^2} \right) \right] \]

\[ \Gamma (\tau^- \rightarrow \mu^+ \mu^- e^-) = \frac{m_e^5}{6144\pi^3 \xi_{\mu e}^2} \left[ \left( \frac{\sin^2 \alpha}{m_{H^0}^2} + \frac{\cos^2 \alpha}{m_{h^0}^2} + \frac{1}{m_{A^0}^2} \right)^2 \right] \]

\[ \Gamma (\tau^- \rightarrow \mu^- e^- e^+) = \frac{m_e^5}{6144\pi^3 \xi_{\mu e}^2} \left\{ \left( \frac{\sin (2\alpha)}{\sqrt{2}} \left( \frac{1}{m_{H^0}^2} - \frac{1}{m_{h^0}^2} \right) \right) m_e \right. \]

\[ \left. + \xi_{ee} \left( \frac{\sin^2 \alpha}{m_{H^0}^2} + \frac{\cos^2 \alpha}{m_{h^0}^2} \right)^2 + \frac{\xi_{ee}^2}{m_{A^0}^4} \right\}. \]

Observe that the decays containing two identical particles in the final state possess interferences involving the pseudoscalar Higgs boson, while the decays with no identical leptons in the final state do not contain interference terms with the pseudoscalar. On the other hand, in the calculation of the decay width $\Gamma (\tau^- \rightarrow \mu^+ \mu^- e^-)$ we neglect the diagrams containing the vertex $\xi_{\mu e}$ and keep only the ones proportional to $\xi_{\mu e}$, we make this approximation because previous phenomenological analysis shows a strong hierarchy between these mixing vertices \cite{11} ($|\xi_{\mu e}| << |\xi_{\mu e}|$ by at least five orders of magnitude).

The corresponding experimental upper limits for these rare processes are \cite{18}

\[ \Gamma (\tau^- \rightarrow \mu^- \mu^- e^+) \leq 3.4 \times 10^{-18} \text{ GeV}, \]

\[ \Gamma (\tau^- \rightarrow \mu^+ \mu^- e^-) \leq 4.07 \times 10^{-18} \text{ GeV}, \]

\[ \Gamma (\tau^- \rightarrow \mu^- e^- e^+) \leq 3.85 \times 10^{-18} \text{ GeV}. \] (2)
2.1 Bounds on $\xi_{\mu e}$ and $\xi_{ee}$

In a previous work [11], the LFV vertices coming from the 2HDM type I II, were constrained by using several pure leptonic processes, the following bounds for the LFV vertices were found

$$
\xi_{\mu \tau}^2 \lesssim 2.77 \times 10^{-14},
$$

$$
|\xi_{\mu \mu}| \lesssim 1.3 \times 10^{-1},
$$

$$
7.62 \times 10^{-4} \lesssim \xi_{\mu \tau}^2 \lesssim 4.44 \times 10^{-2},
$$

$$
|\xi_{\tau \tau}| \lesssim 2.2 \times 10^{-2}.
$$

Such constraints are valid in most of the region of parameters. Since we intend to complete the analysis made in [11], we shall make the same assumptions which we summarize here for completeness. We settle $m_{h^0} \approx 115$ GeV, and $m_{A^0} \gtrsim m_{h^0}$. In order to cover a very wide region of parameters, we examine five cases for the remaining free parameters of the model [11]

1. When $m_{H^0} \approx 115$ GeV.
2. When $m_{H^0} \approx 300$ GeV and $\alpha = \pi/2$.
3. When $m_{H^0}$ is very large and $\alpha = \pi/2$.
4. When $m_{H^0} \approx 300$ GeV and $\alpha = \pi/4$.
5. When $m_{H^0}$ is very large and $\alpha = \pi/4$.

For all those cases the value of the pseudoscalar mass is swept in the range of $m_{A^0} \gtrsim 115$ GeV.

The vertex $\xi_{\mu e}^2$ can be constrained by combining the existing limits on $\xi_{\mu \tau}^2$ given in Eqs. (3), and the upper limit on the decay width $\Gamma (\tau^{-} \rightarrow \mu^{-} \mu^{+} e^{-})$ given by Eq. (2). Alternatively, we can constrain the same vertex from the decay $\Gamma (\tau^{-} \rightarrow \mu^{+} \mu^{-} e^{-})$. The upper limits on $\xi_{\mu e}^2$ obtained from both decays are illustrated in table (1) for the five cases explained above. We should observe

| case | from $\tau^{-} \rightarrow \mu^{-} \mu^{+} e^{+}$ | from $\tau^{-} \rightarrow \mu^{+} \mu^{-} e^{-}$ |
|------|--------------------------------|--------------------------------|
| 1    | $\xi_{\mu e}^2 \lesssim 5.59 \times 10^{-3}$ | $\xi_{\mu e}^2 \lesssim 1.0 \times 10^{-2}$ |
| 2    | $\xi_{\mu e}^2 \lesssim 1.5 \times 10^{-1}$ | $\xi_{\mu e}^2 \lesssim 2.7 \times 10^{-1}$ |
| 3    | unconstrained | unconstrained |
| 4    | $\xi_{\mu e}^2 \lesssim 1.35 \times 10^{-2}$ | $\xi_{\mu e}^2 \lesssim 2.43 \times 10^{-2}$ |
| 5    | $\xi_{\mu e}^2 \lesssim 1.67 \times 10^{-2}$ | $\xi_{\mu e}^2 \lesssim 3.0 \times 10^{-2}$ |

Table 1: Bounds on the mixing vertex $\xi_{\mu e}^2$, based on the processes $\tau^{-} \rightarrow \mu^{-} \mu^{+} e^{+}$ and $\tau^{-} \rightarrow \mu^{+} \mu^{-} e^{-}$ for the five cases cited in the text.

that the upper limits obtained from $\tau^{-} \rightarrow \mu^{+} \mu^{-} e^{-}$ are less restrictive than the
ones coming from $\tau^- \rightarrow \mu^- e^+ e^-$. However, both sets of constraints lie roughly on the same order of magnitude. From table 1 we can extract a quite general bound for the vertex $\xi_{\mu e}$

$$\xi_{\mu e}^2 \leq 1.5 \times 10^{-1},$$
valid for most of the region of parameters\(^1\). It worths to say that other restrictions on this vertex can be gotten from $\mu \rightarrow e \gamma$ or $\tau \rightarrow e \gamma$ assuming that only the diagrams with a muon in the loop contribute, instead of the tau as customary. Notwithstanding, bounds obtained this way are much less restrictive.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{contourplots.png}
\caption{Contourplots for the five cases cited in the text in the $\xi_{ee} - m_{A^0}$ plane, based on the process $\tau^- \rightarrow \mu^- e^+ e^-$. On left: Case 1 (dotted line), case 4 (dashed line) and case 5 (solid line). On right: Case 2 (solid line) and case 3 (dashed line).}
\end{figure}

On the other hand, we can get constraints on the vertex $\xi_{ee}$ by combining the already mentioned bounds on $\xi_{\mu \tau}^2$ and the upper experimental constraints for the decay $\Gamma (\tau^- \rightarrow \mu^- e^+ e^-)$ of Eq. 2. Since the factor $\xi_{ee}$ cannot be factorized in contrast to the case of $\xi_{\mu e}$, we extract its bounds in the form of contourplots in the $\xi_{ee} - m_{A^0}$ plane, see Fig. 1. Additionally, we write in table 2, the constraints obtained for $m_{A^0}$ very heavy and for $m_{A^0} \approx 115$ GeV. From table 2 we can extract general constraints for $\xi_{ee}$, the general bounds read

$$|\xi_{ee}| \lesssim 5.1 \times 10^{-1}; \ |\xi_{ee}| \lesssim 7.54 \times 10^{-2}$$

for $m_{A^0} \approx 115$ GeV and for $m_{A^0}$ very heavy respectively. We emphasize again that this prediction is valid in most of the region of parameters but fails in the case 3 cited above, i.e. when $m_{H^0}$ is very large and $\alpha = \pi/2$.

Finally, we make a prediction about the upper limit for the decay width of the process $\tau^- \rightarrow \mu^+ e^- e^-$, based on the limits on $\xi_{\mu \mu}$ shown in table 1 and the

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\(^1\)We should bear in mind however, that none of the restrictions obtained here, are valid for the third case explained in the text.
Table 2: Bounds for the mixing matrix element $\xi_{ee}$, for $m_{A^0} \simeq 115$ GeV and for $m_{A^0}$ very heavy. Such constraints are based on the bounds on $\xi_{\mu\tau}$ and the upper limit for the decay width $\Gamma (\tau^- \rightarrow \mu^- e^+ e^-)$.

| Case | $\xi_{ee}$ ($m_{A^0}$ very heavy) | $\xi_{ee}$ ($m_{A^0}$ $\sim 115$ GeV) |
|------|---------------------------------|-------------------------------------|
| 1    | $\lesssim 9.75 \times 10^{-2}$  | $\lesssim 6.89 \times 10^{-2}$     |
| 2    | $\lesssim 5.1 \times 10^{-1}$  | $\lesssim 7.41 \times 10^{-2}$     |
| 3    | unconstrained                   | unconstrained                       |
| 4    | $\lesssim 1.5 \times 10^{-1}$  | $\lesssim 7.54 \times 10^{-2}$     |
| 5    | $\lesssim 1.7 \times 10^{-1}$  | $\lesssim 7.53 \times 10^{-2}$     |

Table 3: Upper limits for the decay width $\Gamma (\tau^- \rightarrow e^- e^- \mu^+)$, based on the contraints obtained for the LFV vertices $\xi_{\mu e}$ and $\xi_{e\tau}$. The experimental upper limit is $3.39 \times 10^{-18}$ GeV

| Caso | $\Gamma (\tau^- \rightarrow \mu^+ e^- e^-)$ |
|------|---------------------------------------------|
| 1    | $\lesssim 2.15 \times 10^{-29}$ GeV        |
| 2    | $\lesssim 7.16 \times 10^{-29}$ GeV        |
| 3    | Unconstrained                              |
| 4    | $\lesssim 2.91 \times 10^{-29}$ GeV        |
| 5    | $\lesssim 3.21 \times 10^{-29}$ GeV        |

Table 4: Upper limits predicted for some lepton decays. All of them are highly suppressed respect to the current experimental upper limit.
3 Conclusions

We have found constraints on the whole spectrum of the mixing matrix of leptons, by using purely leptonic processes. Gathering the information of all the contraints for LFV vertices, we get the following bounds

$$
|\xi_{ee}| \lesssim 5.1 \times 10^{-1} ; |\xi_{\mu e}| \leq 3.9 \times 10^{-1}
$$

$$
|\xi_{e\tau}| \lesssim 1.66 \times 10^{-7} ; |\xi_{\mu\mu}| \lesssim 1.3 \times 10^{-1} ;
$$

$$
2.76 \times 10^{-2} \lesssim |\xi_{\mu\tau}| \lesssim 2.1 \times 10^{-1}
$$

$$
|\xi_{\tau\tau}| \lesssim 2.2 \times 10^{-2}
$$

In which a strong hierarchy between the vertices $\xi_{\mu\tau}$ and $\xi_{e\tau}$ is manifest. Additionally, we get predictions for some leptonic decays shown in table 4, in which we also include the experimental upper limit for the sake of comparison. From table 4 we see that such decays are highly suppressed respect to the current instrumental sensitivity. It owes mainly to its dependence on the $\xi_{e\tau}$ vertex which is much more restricted than the others.

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