Soft supersymmetry breaking and finiteness

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We calculate the two-loop $\beta$-functions for soft supersymmetry-breaking interactions in a general supersymmetric gauge theory, emphasising the role of the evanescent couplings and masses that are generic to a non-supersymmetric theory. We also show that a simple set of conditions sufficient for one-loop finiteness renders a theory two-loop finite, and we speculate on the possible significance of finite softly broken theories.
Recently we addressed the issue of the applicability of regularisation by dimensional reduction (DRED) \[1\] \[2\] to non-supersymmetric theories\[3\] \[4\].

The conclusion of this analysis was that DRED is indeed a consistent procedure. Moreover, for calculations performed at a given choice of renormalisation scale $\mu$, the usual procedure of ignoring the “evanescent couplings” (for an account of these see below or Refs. \[3\], \[4\]) is perfectly valid. However we indicated that care must be exercised if the object is to relate calculations at two different values of $\mu$ by means of appropriate $\beta$-functions.

DRED has been used in non-supersymmetric theories because certain calculations are simplified if the Dirac matrix algebra can be carried out in 4 rather than $d$ dimensions. Another class of theories where use of DRED rather than DREG (conventional dimensional regularisation) is clearly indicated is softly broken supersymmetric theories. Here DRED is preferable to DREG because the dimensionless couplings in a softly broken supersymmetric theory renormalise (using minimal subtraction) exactly as in the corresponding supersymmetric one.

The supersymmetric standard model consists of the minimal supersymmetric extension of the standard model plus all possible soft-breaking terms. A popular constraint on the large number of parameters thus invoked is the assumption of a simple form for the soft-breaking terms at a scale corresponding to or near the Planck mass. It is then necessary to evolve the parameters using their $\beta$-functions to determine the low energy theory. The one loop $\beta$-functions have been known for some time\[5\], and the two-loop $\beta$-functions have been calculated recently for a general theory\[6\]–\[9\].

The method used in Refs. \[7\], \[8\] is somewhat different to that used in Refs. \[6\], \[9\]. The calculations of Refs. \[6\], \[9\] are performed using the superfield formalism together with DRED, the supersymmetry breaking terms being managed by the use of spurion superfields. The authors of Refs. \[7\], \[8\] work using component fields, and they start with results obtained using dimensional regularisation, obtaining the results corresponding to DRED by making a coupling constant redefinition, after which their results agree with those of Refs. \[6\], \[9\]. We have independently calculated the same set of $\beta$-functions, obtaining a slightly different result. As we shall explain below, we ascribe this difference to our treatment of the evanescent $\epsilon$-scalar mass.

The difference between DRED and DREG resides in the treatment of the gauge fields as one passes from 4 to $4 - \epsilon$ dimensions in order to regulate the theory. In DREG the number of gauge fields is simply continued to $4 - \epsilon$, while in DRED the theory in $4 - \epsilon$
dimensions is obtained by compactification: the number of fields is unaltered, but $\epsilon$ components of a vector multiplet become scalars, called “$\epsilon$-scalars”\[^2\]. It can be shown that the $\beta$-functions obtained by DRED (when properly defined–see later) can be obtained by redefinition of coupling constants from the results for DREG–indeed, this is a crucial requirement for the consistency of DRED\[^3\][^10][^11]. In order to show this in general, it is important to realise\[^3\][^10][^11] that the terms in the Lagrangian involving $\epsilon$-scalars will not in general renormalise in the same way as the terms involving the corresponding vector fields, since they are not related by $d$-dimensional Lorentz invariance. We should introduce new couplings–called “evanescent”–for the terms with $\epsilon$-scalars. However, for a supersymmetric theory the evanescent couplings are nevertheless related by supersymmetry to the corresponding real ones, and so in practice may be identified with them\[^11\]. To put it another way, if the evanescent couplings are set equal to their “natural” supersymmetric values at one value of the renormalisation scale $\mu$, then they remain equal at all values of $\mu$. For a non-supersymmetric theory, on the other hand, it is crucial to distinguish the evanescent couplings from the real ones. Moreover, one must be careful to include all the evanescent couplings which are allowed–not only those which appear upon compactification of the original 4-dimensional theory–since they will in general be generated in the renormalisation process. The softly broken $N = 1$ supersymmetric theory under discussion is a case in point. Here the dimensionless evanescent couplings may still be identified with the supersymmetric gauge couplings, since their evolution as a function of $\mu$ is not affected by the soft-breaking terms. Furthermore, the compactification to $4 - \epsilon$ dimensions does not generate any dimensionful $\epsilon$-scalar couplings. Nevertheless, there are, for instance, one-loop diagrams generating divergent contributions to the $\epsilon$-scalar mass, and hence to discuss the renormalisation of the theory using DRED, it behoves us to include an $\epsilon$-scalar mass term. This is essential in order to obtain the $\beta$-functions for DRED from those for DREG by coupling constant redefinition, as was realised by the authors of Ref. \[^8\]. However, at the end of their calculations they set the $\epsilon$-scalar mass to zero.

This poses an obvious problem, as follows. In a previous paper, we demonstrated that it is perfectly possible to assign arbitrary values to the evanescent couplings for calculations performed using DRED at a given momentum scale in a non-supersymmetric theory; and so, for example, zero for the $\epsilon$-scalar mass is a possible and indeed the obvious choice. But it does not follow that if one is interested in studying the running of the couplings (in other words relating calculations at one value at $\mu$ to those at another), one is simply entitled to set the $\epsilon$-scalar mass identically zero throughout. Suppose the $\epsilon$-scalar mass
was chosen to be zero at the unification scale, and we want to calculate parameters at the weak scale in terms of those at the unification scale. The $\epsilon$-scalar mass $\beta$-function contains contributions proportional to the soft scalar and gaugino masses, and consequently the $\epsilon$-scalar would develop a mass as we run down to the weak scale. Moreover, the two-loop $\beta$-function for the soft-breaking mass $m^2$ acquires contributions from a non-zero $\epsilon$-scalar mass within DRED. We thus obtain at the weak scale a theory with a non-zero $\epsilon$-scalar mass, and a certain set of values for the physical couplings. As we discussed in Ref. [4], this theory is equivalent to one with a vanishing $\epsilon$-scalar mass, and a different set of values for the physical couplings, at the same scale; but this is not the same theory as would be obtained by simply setting the $\epsilon$-scalar mass identically zero by fiat throughout the process of running the couplings.

We now turn to presenting the details of our calculations of the $\beta$-functions for the softly broken $N = 1$ theory. These are computed using DRED. We should stress here that our definition of DRED includes a particular prescription for the construction of subtraction diagrams. In fact, two versions of DRED have been discussed in the literature [2] [3] [4] [10] [12], which differ in their use of counterterms within subtraction diagrams. In the first version of DRED [2] [3] [4], counterterms are calculated by requiring that all graphs, including those with external $\epsilon$-scalars, are finite—leading to evanescent couplings being assigned counterterms different from those for physical couplings; in the second version [10], on the other hand, evanescent couplings are assigned the same counterterms as the physical gauge coupling. It is easy to see that the first version corresponds to the normal process of diagram-by-diagram subtraction, in which one subtracts diagrams in which divergent subdiagrams are replaced by counterterm insertions with the same pole structure. Moreover, it can be shown [4] that in general it is the results obtained using the first version of DRED which can be obtained by the coupling constant redefinition from those of DREG. The results obtained using the second prescription cannot [10] in general be obtained by coupling constant redefinition from those of DREG. In addition, it is clear, as argued in Ref. [10], that the second version of DRED will in general lead to loss of unitarity. For a fully supersymmetric theory the distinction between the two versions of DRED disappears [12]; however, in the present case of the softly broken theory we should be more careful, and henceforth, we shall use the term DRED to refer to the first version alone, unless otherwise stated. It is this version of DRED which we use for our calculations.
In computing the $\beta$-functions for the softly broken $N = 1$ supersymmetric theory using DRED we must include all interactions with $\epsilon$-scalars consistent with $d$-dimensional gauge invariance and with $O(\epsilon)$ global invariance. The Lagrangian is given by

$$L = L_{\text{SUSY}} + L_{\text{SB}} + L_{\epsilon}. \tag{1}$$

Here $L_{\text{SUSY}}$ is the Lagrangian for the $N = 1$ supersymmetric gauge theory, containing the gauge multiplet $\{A_\mu, \lambda\}$ ($\lambda$ being the gaugino) and a chiral superfield $\Phi_i$ with component fields $\{\phi_i, \psi_i\}$ transforming as a (in general reducible) representation $R$ of the gauge group $G$ (which we assume to be simple). The superpotential is given by

$$W = \frac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} \mu^{ij} \Phi_i \Phi_j + L^i \Phi_i. \tag{2}$$

The soft breaking is incorporated in $L_{\text{SB}}$, given by

$$L_{\text{SB}} = (m^2)^i j \phi^i \phi_j + \left( \frac{1}{6} h^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} b^{ij} \phi_i \phi_j + \frac{1}{2} M \lambda \lambda + \text{h.c.} \right) \tag{3}$$

(Here and elsewhere, quantities with superscripts are complex conjugates of those with subscripts; thus $\phi^i \equiv (\phi^i)$.*) Aside from the terms included in $L_{\text{SB}}$ in Eq. (3), one might in general have $\phi^2 \phi^*$-type couplings, $\psi \psi$ mass terms or $\lambda \psi$-mixing terms (as long as they satisfy a constraint that quadratic divergences are not produced). However, the soft-breaking terms we have included are those which would be engendered by an underlying supergravity theory (for a review, see [13]) and which are therefore considered most frequently in the literature.

Finally, $L_{\epsilon}$ contains the evanescent couplings and is given by

$$L_{\epsilon} = g^2 \phi^* R_A R_B \phi a_{\alpha,A} a_{\alpha,B} + \frac{1}{4} g^2 f_{ABC} f_{CDE} a_{\alpha,A} a_{\beta,B} a_{\gamma,C} a_{\beta,D} + \frac{1}{2} m^2 a_{\alpha,A} a_{\alpha,A} \tag{4}$$

$$+ g(\overline{\psi} \sigma_\alpha R_A \psi + f_{ABC} \overline{\lambda}_B \sigma_\gamma \lambda_C) a_{\alpha,A},$$

where $a_{\alpha,A}$ are the $\epsilon$-scalars and $f_{ABC}$ are the structure constants of $G$. The index $\alpha$ in $a_{\alpha,A}$ therefore runs from 0 to $\epsilon$. The $\epsilon$-scalars transform under the adjoint representation of $G$. There is another possible soft term of the form $\xi^{AB} a_{\alpha,A} a_{\alpha,B} \phi_i$ which will be present if $\phi$ includes a representation contained in the symmetric product of two adjoints, but we will ignore this possibility for simplicity.

The non-renormalisation theorem tells us that the superpotential $W$ undergoes no infinite renormalisation so that we have, for instance

$$\beta_{Y}^{ijk} = Y^{ijp} Y^{k} p + (k \leftrightarrow i) + (k \leftrightarrow j), \tag{5}$$

...
where $\gamma$ is the anomalous dimension for $\Phi$. The one-loop results for the gauge coupling $\beta$-function $\beta_g$ and for $\gamma$ are given by

$$16\pi^2 \beta_g^{(1)} = g^3Q \quad \text{and} \quad 16\pi^2 \gamma^{(1)} = P^i_j,$$

where

$$Q = T(R) - 3C(G), \quad \text{and} \quad P^i_j = \frac{1}{2} Y^i_{klt} Y^l_{jkl} - 2g^2 C(R)^i_j.$$

(6)

Here

$$T(R)\delta_{AB} = \text{Tr}(R_A R_B), \quad C(G)\delta_{AB} = f_{ACD} f_{BCD} \quad \text{and} \quad C(R)^i_j = (R_A R_A)^i_j.$$

(8)

The two-loop $\beta$-functions for the dimensionless couplings were calculated in Ref. [14], and it was noticed that they can be written in the form

$$16\pi^2 \beta_g^{(2)} = 2g^5 C(G)Q - 2g^3 r^{-1} C(R)^i_j P^i_j,$$

$$16\pi^2 \gamma^{(2)} = -[Y^i_{jmn} Y^{mnp} + 2g^2 C(R)^k_j \delta^i_n] P^{np} + 2g^4 C(R)^i_j Q,$$

(9)

where $Q$ and $P^i_j$ are given by Eq. (7), and $r = \delta_{AA}$. It is therefore clear that imposing the conditions for vanishing of the one-loop $\beta$-functions for the dimensionless couplings, namely

$$Q = 0 \quad \text{and} \quad P^i_j = 0$$

(10)

also guarantees that the two-loop $\beta$-functions vanish. One of our central results in this paper will be that a similar phenomenon happens in the case of the $\beta$-functions for the soft breaking couplings. Note that the conditions Eq. (10) imply that the gauge group should have no $U(1)$ factors and that there should be no gauge singlet fields.

The $\beta$-functions for the physical soft-breaking couplings are most easily computed using the general results for DRED given in Ref. [12] (being careful to select the results corresponding to the first version of DRED described above). The $\beta$-functions for the $\epsilon$-scalar mass, however, must either be computed ab initio or else by coupling constant redefinition starting from the results for DREG (see later for more details on this process).

The one-loop $\beta$-functions for the soft-breaking couplings are given by

$$16\pi^2 \beta_h^{(1)ijk} = U^{ijk} + U^{jki} + U^{kji},$$

$$16\pi^2 \beta_b^{(1)ij} = V^{ij} + V^{ji},$$

$$16\pi^2 [\beta_{m^2}]^i_j = W^i_j + 2g^2 (R_A)^i_j \text{Tr}[R_A m^2],$$

$$16\pi^2 \beta_M^{(1)} = 2g^2 Q M,$$

(11)
where
\[ U^{ij} = h^{ij} P^k + Y^{ij} X^k, \]
\[ V^{ij} = b^{ij} P^j + \frac{1}{2} Y^{ij} Y_{lmn} b^{mn} + \mu^i X^j, \]
\[ W_i = \frac{1}{2} Y_{ipq} Y^{pqn} (m^2)^j_n + \frac{1}{2} Y^{jpr} Y_{pqn} (m^2)_{ij}^n + 2 Y_{ipq} Y^{ijr} (m^2)_{qr} + h_{ipq} h^{jpr} - 8 g^2 M M^* C(R)^j_i, \] (12)

with
\[ X^i_j = h^{ikl} Y_{jkl} + 4 g^2 M C(R)^i_j. \] (13)

The one-loop $\beta$-function for the $\epsilon$-scalar mass is given by
\[ 16 \pi^2 \beta^{(1)}_{\tilde{m}^2} = 4 g^2 S + 2 g^2 Q \tilde{m}^2, \] (14)

where
\[ S \delta_{AB} = (m^2)^i_j (R_A R_B)^j_i - M M^* C(G) \delta_{AB}. \] (15)

The two-loop $\beta$-functions for the physical soft-breaking scalar couplings can be written in the following suggestive form:
\[ (16 \pi^2)^2 \beta^{(2)}_{h}^{i j k} = \left[ -h^{i j} Y_{lmn} Y^{mpk} + 2 Y^{ij} Y_{lmn} h^{mpk} - 4 g^2 M Y^{ij} C(R)^k_n \right] P^n_p - 2 g^2 U^{ij} C(R)^k_l + g^4 (2 h^{ij} - 8 M Y^{ij}) C(R)^k_l Q - Y^{ij} Y_{lmn} Y^{p m k} X^n_p + (k \leftrightarrow i) + (k \leftrightarrow j), \]
\[ (16 \pi^2)^2 \beta^{(2)}_{b}^{i j} = \left[ b^{il} Y_{lmn} Y^{mpj} - 2 \mu^i Y_{lmn} h^{mpj} - Y^{ij} Y_{lmn} b^{mp} \right] \left[ -h^{i j} Y_{lmn} Y^{mpk} + 2 Y^{ij} Y_{lmn} h^{mpk} - 4 g^2 M Y^{ij} C(R)^k_n \right] P^n_p - 2 g^2 C(R)^i_k V^{kj} + g^4 (2 h^{ij} - 8 M Y^{ij}) C(R)^k_j Y_{lmn} b^{mn} + 2 g^4 (b^{ik} - 4 M \mu^i) C(R)^j_k Q + (i \leftrightarrow j), \]
\[ (16 \pi^2)^2 \beta^{(2)}_{m^2}^{i j} = \left[ -\frac{1}{2} Y_{ilm} Y^{j l p} (m^2)^i_n + 2 Y_{ilm} Y^{j l p} (m^2)^i_n + \frac{1}{2} Y_{ilm} Y^{j l m} (m^2)^i_p \right] P^n_p + Y_{ilm} Y^{j l p} (m^2)^i_n + h_{ilm} h^{j lp} + 4 g^2 M M^* C(R)^j_i \delta^n_p + 2 g^2 (R_A)^j_i (R_A m^2)^p_n \right] P^n_p + 2 g^2 (R_A)^j_i (R_A m^2)^p_n \right] P^n_p + 2 g^2 (R_A)^j_i (R_A m^2)^p_n \right] P^n_p + 2 g^2 C(R)^j_i \delta^n_p + 12 g^4 M M^* C(R)^j_i Q + 8 g^4 S C(R)^j_i + g^2 \tilde{m}^2 \left[ 4 g^2 Q C(R)^j_i - Y_{ikl} Y^{mkl} C(R)^j_m - 2 Y_{ikl} C(R)^k_m Y^{lmj} \right] \right] + \text{h.c.} \]
\[ (16 \pi^2)^2 \beta^{(2)}_{M} = g^4 \left( 8 C(G) Q M - 4 r^{-1} C(R)^i_j P^j_i M + 2 r^{-1} X^i_j C(R)^j_i \right), \] (16)
and the two-loop $\beta$-function for the $\epsilon$-scalar mass has the form

\[(16\pi^2)^2\beta^{(2)}_{\tilde{m}^2}\delta_{AB} = -2g^2W^i_j(R_AR_B)^i_j + 8g^4C(G)S\delta_{AB} + 8g^4C(G)QM^* \delta_{AB}
\]

\[= -g^2\tilde{m}^2\left(2Y_{ijk}Y_j^{kl}(R_AR_B)^i_j + \frac{63}{2}g^2C(G)^2\delta_{AB} - 8g^2C(G)T(R)\delta_{AB}\right),\]

where $P^i_j, Q, W^i_j, X^i_j$ and $S$ are as defined in Eqs. (7), (12), (13) and (15). The results for the soft-breaking couplings agree with Refs. [8] and [9], apart from $\beta^{(2)}_{\tilde{m}^2}$. Our result for this does not even agree with Refs. [8] and [9] when we set $\tilde{m}^2 = 0$, since the term $8g^4SC(R)^j_i$ is absent from the results given in these references. We shall discuss this discrepancy in some detail later; however we pause to note a remarkable feature of the results which is at once apparent: if we impose the conditions Eq. (10), together with the conditions

\[X^i_j = W^i_j = S = 0,\]

which guarantee that the one-loop $\beta$-functions for the soft-breaking couplings and the $\epsilon$-scalar mass vanish [15], and if furthermore the $\epsilon$-scalar mass is zero, then the two-loop $\beta$-functions for the soft-breaking couplings and the $\epsilon$-scalar mass also vanish. (We need to use the fact that $Q = 0$ implies that $G$ has no $U(1)$ factors, which means that terms in $\beta_{\tilde{m}^2}$ proportional to $\text{Tr}[R_A \tilde{m}^2]$ vanish, and also the fact that $P^i_j = 0$ implies the absence of gauge singlets which in turn guarantees $Y_{ijk}b^{jk} = 0$.) Hence the softly broken $N = 1$ supersymmetric theory can be made finite up to two loops by imposing a very simple set of conditions. Moreover, if the $\epsilon$-scalar mass is chosen to be zero at some scale, then in these particular circumstances the $\epsilon$-scalars will not develop a mass under running.

Now let us return to the question of the differences between our results, and those obtained previously. The origin of the discrepancy appears to be that the results of Refs. [8] and [9] correspond to the second version of DRED described earlier. As we mentioned earlier, this version of DRED is not, in general, equivalent to DREG—an exception being fully supersymmetric theories, for which in fact it gives the same results as the version of DRED which we use. To support our contention that the results of Refs. [8] and [9] indeed correspond to this version of DRED, and to investigate whether this version of DRED is viable for the softly broken supersymmetric theory in the sense of giving results equivalent to those of DREG, we need to explain the process of coupling constant redefinition in more detail. Let us start by considering the general case of a theory with a set of couplings (both
physical and evanescent) represented generically by $\lambda^i$. We now imagine the $\epsilon$-scalars to have multiplicity $E$, rather than $\epsilon$, and following Refs. [10], [4], we consider the $\beta$-functions evaluated using DREG. If the $\beta$-function $\beta^i_{\text{DREG}}$ for $\lambda^i$, evaluated using DREG, has the form

$$\beta^i_{\text{DREG}}(E) = \beta^i_{\text{DREG}}(E = 0) + E\xi^i + O(E^2), \quad (19)$$

then up to two loops the corresponding $\beta$-functions $\beta^i_{\text{DRED}}$ evaluated using DRED are obtained by making the coupling constant redefinitions

$$\lambda^i \rightarrow \lambda^i + \zeta^{(1)i} \quad (20)$$

(where the superscript (1) indicates the loop order) and have the form[10]

$$\beta^i_{\text{DRED}} = \beta^{(1)i} + \beta^{(2)i}_{\text{DREG}}(E = 0) + \beta^{(1)j} \frac{\partial}{\partial \lambda} \zeta^{(1)i} - \zeta^{(1)j} \frac{\partial}{\lambda \partial} \beta^{(1)i}. \quad (21)$$

(Note that at one loop, the DREG $\beta$-function has no $E$-dependence and is equal to the DRED $\beta$-function.) A fuller discussion of these results is given in Ref. [4]. Returning to the softly broken $N = 1$ theory, we must be careful when calculating the $\beta$-functions within DREG as described above, since the results obtained using DREG are not manifestly supersymmetric. We should replace $L_{\text{SUSY}}$ by a general Lagrangian for which we have not imposed the relations between the couplings corresponding to supersymmetry. The two-loop $\beta$-function for $m^2$ is then given by

$$[\beta^{(2)j}_{m^2 i}]_{\text{DRED}} = [\beta^{(2)j}_{m^2 i}]_{\text{DREG}} + \left( \zeta^{(1)}_{\lambda} \frac{\partial}{\partial \lambda} + \zeta^{(1)l}_{m^2} k \frac{\partial}{\partial (m^2)^{l} k} \right) \beta^{(1)j}_{m^2 i}$$

$$- \left( \beta^{(1)j}_{m^2} \frac{\partial}{\partial \lambda} + \beta^{(1)j}_{\lambda} \right) \zeta^{(1)j}_{m^2 i}. \quad (22)$$

Here $\lambda$ stands for all the couplings except for $m^2$ and $\tilde{m}^2$. After performing the derivatives with respect to the couplings as indicated in Eq. (22), one sets all these couplings to their supersymmetric values (so that $[\beta^{(2)j}_{m^2 i}]_{\text{DREG}}$ can be calculated using the supersymmetric values for the couplings from the outset). It is straightforward, using

$$\zeta^{(1)j}_{m^2 i} = \frac{g^2}{8\pi^2 \tilde{m}^2 C(R)^j i} \quad (23)$$

and Eq. (14), together with the results for DREG for a general theory given in Ref. [12], and the coupling constant redefinitions as given in Ref. [7], to verify that Eq. (22) does indeed reproduce the result for the DRED $\beta$-function for $m^2$ given in Eq. (16). One can
also easily check that the result of Refs. [8], [9] for $\beta^{(2)j}_{m^2_i}$ corresponds to replacing $\zeta^{(1)j}_{m^2_i}$ by $-\zeta^{(1)j}_{m^2_i}$ in Eq. (22), and then setting $\tilde{m}^2 = 0$. This means that the results of Refs. [8], [9] are equivalent up to field redefinition to those of DREG for zero $\epsilon$-scalar mass. However, as we argued earlier, if one is interested in the running of the couplings, the $\epsilon$-scalar will in general develop a mass and so one must be able to compute $\beta$-functions consistently when $\tilde{m}^2 \neq 0$. It is not clear how to extend the calculations of Refs. [8] or [9] away from $\tilde{m}^2 = 0$ in a convenient fashion. One could simply define a new regularisation scheme to be given by making the coupling constant redefinitions as in Eq. (22), but with the opposite sign for $\zeta^{(1)j}_{m^2_i}$; but this would lose all the advantages of DRED since one would be forced to calculate with the general, non-supersymmetric theory using DREG before making the coupling constant redefinitions. This would be particularly awkward for actual $S$-matrix computations. It would be preferable if one could specify a regularisation scheme at the diagrammatic level which would produce the same results as those obtained from the modified version of Eq. (22). As it happens, the second version of DRED described above does give the result for $\beta^{(2)j}_{m^2_i}$ obtained in Refs. [8], [9]. It is easy to see how one might be led to use this second prescription in the present case, since this second version of DRED only differs from ours in omitting a counterterm for the $\epsilon$-scalar mass. However, if one computes using this prescription for non-zero $\tilde{m}^2$, one finds that it gives exactly the same result for $\beta^{(2)j}_{m^2_i}$ as Eq. (10), except for the absence of the term $8g^4 SC(R)^j_i$—in other words the $\tilde{m}^2$ terms are unaltered. This is not the same as the result of the modified version of Eq. (22) for $\tilde{m}^2 \neq 0$. So it seems there is no diagrammatic prescription which reproduces the result of Refs. [8], [9] for $\tilde{m}^2 = 0$, and which is equivalent to DREG for $\tilde{m}^2 \neq 0$. We believe that if one wishes to perform the running coupling analysis one is ineluctably driven to the prescription for DRED which we have adopted.

We discovered above that a simple set of conditions sufficed to make the full set of $\beta$-functions for the softly broken $N = 1$ supersymmetric theory vanish up to two loops. This result generalises what was known already for the ordinary, unbroken $N = 1$ theory. As we mentioned earlier, we have only considered the soft-breaking terms which arise in low-energy supergravity. In fact, our results do not appear to generalise to the case when other possible soft-breaking terms, such as $\phi^2 \phi^*$ interactions, are added; it then appears that two-loop finiteness requires a completely new set of conditions in addition to those required for one-loop finiteness. It is very interesting that finite softly broken supersymmetric theories appear thus to be linked to the existence of an underlying supergravity theory. In this
context, we recall that a solution to the conditions for one-loop finiteness of a softly broken $N = 1$ theory was found in Ref. [15], namely
\begin{equation}
  h^{ijk} = -MY_{ijk}, \quad (m^2)^i_j = \frac{1}{3}MM^*\delta^i_j
\end{equation}
together with $Q = P^i_j = 0$. These relations are consistent with supergravity with a minimal Kähler potential [13] (with the gravitino mass $m_3$ given by $m_3 = \frac{1}{\sqrt{3}} M$). Remarkably, we find that the solution Eq. (24), together with $Q = 0$, also guarantees the vanishing of the one-loop $\epsilon$-scalar mass $\beta$-function $\beta^{(1)}_{\tilde{m}^2}$ given by Eqs. (14), (15). Thus Eq. (24), together with $Q = P^i_j = \tilde{m}^2 = 0$, gives a two-loop finite softly broken theory. Now in the supersymmetric case, it has been argued that from any two-loop finite theory one finite to all orders can be constructed [16]. It is tempting to suppose that the same is true in the softly broken case.

Would such theories be of phenomenological as well as theoretical interest? There has been occasional interest [17] [18] in the phenomenology of finite $N = 1$ theories, centering chiefly on a particular $SU_5$ example first explored in Ref. [17]. Now the low energy effective field theory derived from such a theory is of course not itself finite, because of the deletion of heavy fields. It would therefore appear that given that the unified model (the $SU_5$ one, say) is in turn presumably a low energy effective field theory then there is no reason to expect it to be finite. This is a compelling argument, and forced in Ref. [17] the brave conjecture that the $SU_5$ model might indeed be the ultimate theory, with gravitation interactions being dynamically generated. A less outré possibility is the following: evidently “integrating out” gravity from a theory is very different from the process of integrating out the heavy fields to produce a low energy theory; the graviton is after all massless. Perhaps the nature of gravity is such that the resulting theory does indeed remain finite. Clearly, however, substantive evidence for one or other of these conjectures is required before we take seriously a postdiction for, say, the top mass based on a finite model.

Note Added

We understand that the authors of Refs. [8] and [9] now agree that the DRED result for $\beta^{(2)j}_{\tilde{m}^2 i}$ is indeed given by our Eq. (16). It is worth noting that (as has been independently observed by Martin and Vaughn) the $\tilde{m}^2$ dependence in $\beta^{(2)j}_{\tilde{m}^2 i}$ can be removed by a simple redefinition of the form
\begin{equation}
  (\delta m^2)^j_i = -\frac{g^2}{8\pi^2}\tilde{m}^2 C(R)^j_i.
\end{equation}
The only other effect of this redefinition is to change the coefficient of $g^4SC(R)^j_i$ in $\beta^{(2)j}_{m^2} i$ from 8 to 4. It may well be that, as emphasised to us by Martin and Vaughn, this modified scheme is the most convenient for practical calculations.

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