The London moment: what a rotating superconductor reveals about superconductivity

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Abstract
The London moment is the magnetic moment acquired by a rotating superconductor. We propose that the London moment reveals the following fundamental properties of the superconducting state: (i) superconductors (unlike normal metals) know the sign of the charge carriers; (ii) the superconducting charge carriers are free electrons; (iii) electrons are expelled from the interior to the surface in the transition to the superconducting state; (iv) superfluid electrons occupy orbits of radius $2\lambda_L$ ($\lambda_L$ = London penetration depth); and (v) a spin current exists in the ground state of superconductors. These properties are consistent with the Meissner effect, however the Meissner effect does not directly reveal the sign of the charge carriers nor the fact that the carrier’s mass is the free electron mass nor the fact that a spin current exists in superconductors. Note also that within the BCS theory of superconductivity none of the key properties of superconductors listed above are predicted. Instead, these properties are predicted by the theory of hole superconductivity.

Keywords: rotating superconductor, London moment, charge expulsion, hole superconductivity

1. Introduction
While every book on superconductivity features prominently the Meissner effect, almost none, particularly the most popular ones, discuss the London moment. For example, the books by de Gennes [1], Schrieffer [2], Ketterson and Song [3], Buckel and Kleiner [4], Tilley [5] Abrikosov [6] and Parks [7] do not even mention it, Tinkham’s [8], only in passing. As a consequence, the phenomenon appears to be unknown to a large fraction of contemporary physicists working in the field of superconductivity.

The London moment is the magnetic moment exhibited by a rotating superconductor. The phenomenon was predicted in 1933, just before the Meissner effect was discovered, in a seminal paper by Becker et al [9]: a superconducting body rotating with angular velocity $\omega_0$ develops a uniform magnetic field throughout its interior, given by

$$\mathbf{B} = -\frac{2me_e}{e} \omega_0 \mathbf{e}$$

with $e (< 0)$ the electron charge, and $m_e$ the bare electron mass. The magnetic field is parallel to the angular velocity. The resulting magnetic moment will depend on the shape of the body. For example for a sphere of radius $R$ the resulting magnetic moment $\mathbf{m}$ is

$$\mathbf{m} = -\frac{m_e e}{e} R^3 \omega_0.$$  

These results are valid for bodies of dimensions much larger than the London penetration depth. Qualitatively we can understand the physics as follows: when the superconductor is set into rotation, the electrons initially stay at rest. The electric current produced by the moving ions generates a changing magnetic field, which in turn generates an azimuthal electric field that sets the electrons into motion. Because of the inertia of the electrons they slightly ‘lag behind’ the motion of the body near the surface, giving rise to an electric current that generates the magnetic field in the interior of the superconductor.

The reason the phenomenon is called the London moment rather than the ‘Becker et al moment’ is presumably that Becker et al predicted this phenomenon only for the case when a body at rest that is already superconducting is set into rotation. Instead, London realized, after the Meissner effect had been discovered experimentally, that a rotating normal metal cooled into the superconducting state while
rotating would develop the same magnetic moment. In other words, just like for the Meissner effect, the state of a rotating superconductor is independent of its history.

It is interesting to read about a key aspect of the phenomenon in London’s own words [10]: ‘There is an implication which might be worth mentioning since it would appear quite absurd from the viewpoint of the perfect conductor concept. The uniqueness properties of the present theory provide for only one current distribution independent of the way in which the superconducting state is reached. Consequently we have to conclude that the same state as has been calculated above must also be obtained if the sphere is cooled from the normal into the superconducting state while it is rotating. The perfect conductor theory would, of course, furnish no reason for the electrons near the surface of the metal to lag suddenly behind when the rotating sphere goes into the superconducting state. It would simply lead to a state of zero magnetic moment in which all charges move in phase—the same below as above the transition point.’

This bold prediction of London (in his words, ‘there cannot be much doubt as to the outcome of this experiment’ [10]) was first verified experimentally in 1964 by Hildebrandt for Pb [11, 12]. Since then it has been tested for other ‘conventional’ superconductors [13–16] as well as for ‘unconventional’ ones including high Tc cuprates [17] and heavy fermions [18], yielding always results in agreement with equation (1), independent of history.

In this paper we propose that the London moment reveals fundamental properties of the superconducting state that have been unrecognized in the conventional understanding of superconductors.

2. Classical derivation of the London moment

We start by reviewing the classical derivation of the effect as discussed by Becker et al [9] before the Meissner effect was discovered.

Consider a perfect conductor at rest in the absence of external fields that is put into rotational motion starting at time $t = 0$. The body has angular velocity $\omega$ at time $t$ and attains a final angular velocity $\omega_0$, it does not matter how long it takes to reach the final state. We assume, following Becker et al [9], that the superfluid electrons are completely detached from the ions. As the body starts to rotate only the positive ions rotate, giving rise to an electric current and a magnetic field. The changing magnetic field generates an electric field through Faraday’s law, and this electric field acts on the superfluid electrons pushing them along to follow the ions. The direction of the fields and rotation is shown in figure 1.

Assume at time $t$ an electric field $E$ is acting on the electrons, exerting an azimuthal force

$$m_e \frac{dv}{dt} = eE$$

with $v$ the electron’s azimuthal speed. The body is rotating rigidly with angular velocity $\omega$, and we assume the electrons follow suit with angular velocity $\omega_e$ which is very close to $\omega$. In fact, we assume that $\omega_e$ is identical to $\omega$ in the bulk of the system, so that no volume currents exist, and only differs from $\omega$ near the surface giving rise to a surface electric current. We will see that this assumption is self-consistent. We have then $v = \omega_e r = \omega r$ for electrons in the bulk at distance $r$ from the rotation axis, hence from equation (1)

$$E = \frac{m_e}{e} \omega r.$$  (4)

This electric field is generated by the changing magnetic field which in turn results from the ionic rotation giving rise to an electric current. From Faraday’s law,

$$\oint E \cdot dl = -\frac{1}{c} \frac{\partial}{\partial t} \int B \cdot dS$$

yielding at distance $r$

$$E = -\frac{1}{2c} r B.$$  (6)

Replacing $E$ from equation (4) into equation (6) and solving for $B$

$$B = -\frac{2m_e c}{e} \frac{\omega}{\omega_0}$$

and finally, integrating and using that $\omega(t=0) = B(t=0) = 0$

$$B = -\frac{2m_e c}{e} \omega_0$$

in agreement with equation (1).

This remarkably simple derivation says remarkably little about the actual origin of the magnetic field, i.e. the nature and magnitude of the surface current created by the lagging electrons. To obtain the current density

$$j(\vec{r}, t) = en_e [\vec{v}(\vec{r}, t) - \vec{v}_0(\vec{r})]$$

with $\vec{v}_0 = \vec{\omega}_0 \times \vec{r}$ the ionic speed one uses the equation of motion (3) together with Maxwell’s equations

$$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c} j,$$  (10a)
\[ \vec{v} \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t} \]  
(10b)

and the boundary conditions that \( \vec{B} \) and its derivatives are continuous across the surface of the body. Becker et al. performed this calculation for a rotating sphere of radius \( R \), and obtained the result for the superfluid velocity field (in what follows \( r \) denotes distance to the center of the sphere, not to the rotation axis)

\[ \vec{v} = \vec{\omega}_0 \times \vec{r} \left[ 1 + 3 \frac{\lambda_L^2 R}{r^3} f(r) \right], \]  
(11a)

\[ f(r) = \frac{1}{\sinh(R/\lambda_L)} \left[ \sinh(r/\lambda_L) - \frac{r}{\lambda_L} \cosh(r/\lambda_L) \right], \]  
(11b)

from which the current equation (9) follows. Note that \( f(r) \) is very small except for \( r \) very near the surface. For \( r \gg \lambda_L \) it is given approximately by

\[ f(r) \approx -\frac{r}{\lambda_L} e^{-(r-R)/\lambda_L}, \]  
(12)

so that indeed \( \vec{v} = \vec{\omega}_0 \times \vec{r} \) for \( R-r > \lambda_L \), as assumed in the earlier derivation. In other words, the superfluid velocity equals the ion velocity except close to the surface, so there is no volume current. Near the surface, equation (11) becomes approximately

\[ \vec{v} \approx \vec{\omega}_0 \times \vec{r} \left[ 1 - 3 \frac{\lambda_L^2 R}{r^2} e^{-(r-R)/\lambda_L} \right], \]  
(13)

so that even at the surface the superfluid velocity is only slightly less than the body’s velocity, yet sufficiently different to give rise to the interior magnetic field equation (1). The full expression for the magnetic field, using spherical coordinates \((r, \theta, \varphi)\) with \( \vec{\omega}_0 \) parallel to the \( z \)-axis is [9]

\[ \vec{B} = -\frac{2 m_e c}{e} \vec{\omega}_0 + \delta B_r \hat{r} + \delta B_\theta \hat{\theta} \]  
(14)

with

\[ \delta B_r = -\frac{6 m_e c}{e} \frac{\omega_0}{\omega} \frac{\lambda_L^2 R}{r^3} f(r) \cos \theta, \]  
(15a)

\[ \delta B_\theta = -\frac{3 m_e c}{e} \frac{\omega_0}{\omega} \frac{\lambda_L^2 R}{r^3} \left[ f(r) + \frac{r^2}{\lambda_L^2} \sinh(r/\lambda_L) \right] \sin \theta, \]  
(15b)

so that the magnetic field is given by equation (1) except for small corrections very close to the surface.

### 3. London’s derivation of the London moment

London derived the same results discussed in the previous section through a slightly different route [10]. He started from the London equation

\[ \vec{\nabla} \times \vec{\nabla} \times \vec{\nabla} = -\frac{e}{m_e c} \vec{B}, \]  
(16)

A perfect conductor with initial conditions \( \vec{v} = \vec{B} = 0 \) of course obeys London’s equation. This can be seen as follows: in the presence of an electric field \( \vec{E} \) the equation of motion for an electron in the perfect conductor is (first London equation)

\[ \frac{\partial \vec{v}}{\partial t} = \frac{e}{m_e} \vec{E}. \]  
(17)

Taking the curl on both sides and using Faraday’s law,

\[ \frac{\partial \vec{v}}{\partial t} \times \vec{v} = -\frac{e}{m_e c} \frac{\partial \vec{B}}{\partial t}. \]  
(18)

Integrating and using that initially \( \vec{v} = 0 \) and \( \vec{B} = 0 \), equation (16) follows.

This derivation, which is found in essentially all superconductivity textbooks, is actually incorrect for two reasons: (i) the correct equation of motion should use the total time derivative rather than the partial time derivative in equation (17); and (ii) the right-hand side of equation (17) should include the Lorentz force on the electron due to a magnetic field \( \vec{B} \).

The correct derivation of equation (16) for a perfect conductor is as follows [10]: we start from the equation of motion

\[ \frac{d^2 \vec{v}}{dt^2} = \frac{e}{m_e} \vec{E} + \frac{e}{m_e} \vec{v} \times \vec{B}. \]  
(19)

Using the relation between total and partial time derivatives

\[ \frac{d\vec{v}}{dt} = \frac{\partial \vec{v}}{\partial t} + \vec{v} \times \left( \frac{\partial \vec{v}}{\partial t} \right) \]  
(20)

and taking the curl on both sides of equation (19) yields

\[ \frac{\partial \vec{w}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{w}). \]  
(21a)

with

\[ \vec{w} = \vec{\nabla} \times \vec{v} + \frac{e}{m_e c} \vec{B}. \]  
(21b)

Before the perfect conductor is set into rotation, \( \vec{w} = 0 \) since both velocity and magnetic fields are zero. According to equation (21a), if \( \vec{w} = 0 \) initially it remains equal to zero at all times. \( \vec{w} = 0 \) is equivalent to London’s equation (16).

Assuming that the superfluid electrons in the bulk rotate with the same angular velocity of the body

\[ \vec{v} = \vec{\omega}_0 \times \vec{r} \]  
(22)

replacement in equation (16) immediately yields

\[ 2 \vec{\omega}_0 = -\frac{e}{m_e c} \vec{B}, \]  
(23)

which is the same as equation (1). To obtain the full solution for the current and magnetic field distribution we use equation (9) for the current together with equation (16) and Maxwell’s equation (10a) and the boundary conditions for the magnetic field at the surface of the sphere, from which the same solution as obtained by Becker et al. results, as expected.

The fundamental difference between the London and Becker et al.’s approaches is that London’s equation (16) applies to a superconductor independent of its history. Thus London’s derivation implies that if a rotating normal metal is cooled into the superconducting state, the resulting velocity field and magnetic field will be given by the expressions derived by Becker et al. and reviewed in section 2. Instead, Becker et al. would predict that a rotating metal that becomes suddenly a perfect conductor would develop no current, since the suddenly free electrons would continue rotating at the same speed as the body because of inertia.
4. Inconsistency of London–BCS theory with measurements of the London moment

Consider for simplicity a superconductor where the transport of electric current occurs predominantly in a single band, all other bands are either completely full or completely empty. BCS (Bardeen–Cooper–Schrieffer)–London theory assumes that London’s equation

$$\mathbf{\nabla} \times \mathbf{\tilde{v}} = -\frac{q}{m_q c} \mathbf{\tilde{B}}$$

(24)

holds, where $\tilde{v}$ is the superfluid velocity and $q$ and $m_q$ the charge and mass of the superfluid charge carriers in the band. The electric current density is given by

$$\mathbf{j} = q n_s \mathbf{\tilde{v}}$$

(25)

with $n_s$ the density of charge carriers. From equations (24) and (25)

$$\mathbf{\nabla} \times \mathbf{j} = -\frac{n_s q^2}{m_q c} \mathbf{\tilde{B}}.$$  

(26)

Applying the curl operator to both sides of Maxwell’s equation

$$\mathbf{\nabla} \times \mathbf{B} = \frac{4\pi}{c} \mathbf{j},$$

(27)

and replacing the right-hand side by equation (26) yields

$$\mathbf{\nabla}^2 \mathbf{\tilde{B}} = \frac{4\pi n_s q^2}{m_q c^2} \mathbf{\tilde{B}} = \frac{1}{\lambda_L^2} \mathbf{\tilde{B}},$$

(28)

which describes the exponential decay of a magnetic field in a superconductor: the magnetic field decays to zero over a distance from the surface given by the London penetration depth

$$\frac{1}{\lambda_L^2} = \frac{4\pi n_s q^2}{m_q c^2}.$$  

(29)

Assume the energy band under consideration is nearly full. In the normal state, transport can be formally described using either electrons or holes [19]. However because the effective mass for electrons is negative when the band is more than half-full it is customary to describe the transport in this case as being carried by holes, with positive charge $q > 0$ and positive effective mass $m_q > 0$. The simple expression for the current equation (25) is correct for an almost full band only if $n_s$ denotes the density of holes, not of electrons [19]. Furthermore, when a band is almost full, the London penetration depth is found to diverge, which is consistent with the number of superfluid charge carriers going to zero in equation (29), i.e. with $n_s$ describing the density of holes rather than of electrons in the band. In the same way, the number of carriers that enters the normal state conductivity in the normal state within Drude theory is the number of holes rather than the number of electrons when the band is almost full, using the fact that a full band carries no current (nor supercurrent). Note that the London penetration depth is independent of the sign of the charge carriers.

For example, consider an attractive Hubbard model with Hamiltonian

$$H = -t \sum_{i,o} \left[ c_{i\sigma}^\dagger c_{o\sigma} + \text{h.c.} \right] + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

(30)

with $U < 0$ as a simple model to describe conventional superconductivity in a real system. Straightforward application of BCS theory [8] yields for the London Kernel describing the diamagnetic response of the system, at zero temperature [20]

$$K = \frac{1}{\lambda_L^2} = \frac{8\pi e^2 t}{\hbar^2 c^2 a_0} \sum_k \cos k \left[ 1 - \frac{e_k - \mu}{E_k} \right]$$

(31)

with $a_0$ the lattice spacing in the $\delta$ direction and $E_k = \sqrt{(e_k - \mu)^2 + \Delta^2}$ the BCS quasiparticle energy, with $\Delta$ the energy gap. This can also be written as

$$K = \frac{1}{\lambda_L^2} = \frac{8\pi e^2 t}{\hbar^2 c^2 a_0} \sum_k \cos k n_k$$

(32)

with $n_k$ the occupation of state $k$. When the band is almost full we have simply

$$K = \frac{1}{\lambda_L^2} = \frac{8\pi e^2 t}{\hbar^2 c^2 a_0} \sum_k \cos (k a_0) \sim n_h$$

(33)

with $n_h$ the number of holes per site. The effective mass for holes near the top of the band in this model, with energy dispersion relation

$$\epsilon_k = -2 \sum_\delta \cos (k a_0)$$

(34)

is

$$\frac{1}{m_q} = \frac{1}{\hbar^2} \frac{\partial^2 \epsilon_k}{\partial k^2} \bigg|_{k=0} = \frac{2 \pi a_0^2}{\hbar^2}.$$  

(35)

Hence from equations (32), (33) and (35) and assuming an isotropic model so that the density of superfluid holes (of positive charge $q = -e$ per unit volume) is

$$n_s = \frac{n_h}{a_0^2},$$

(36)

we find

$$\frac{1}{\lambda_L^2} = \frac{4\pi n_s q^2}{m_q c^2}.$$  

(37)

The result equation (37) is identical to equation (29) obtained through the electrodynamic equations. Therefore we conclude that equation (24) applies to this model for the superfluid hole carriers, with $q > 0$ and effective mass $m_q$ given by equation (35). Following then London’s derivation, for a rotating superconductor the velocity of the hole carriers in the interior of the body is identical to the ionic velocity, i.e. $\mathbf{\tilde{v}} = \mathbf{\tilde{\omega}}_0 \times \mathbf{\tilde{r}}$ and we obtain

$$\mathbf{\tilde{B}} = -\frac{2m_q c}{q} \mathbf{\tilde{\omega}}_0,$$

(38)

which is different from the measured value equation (1) both in sign and in magnitude. In particular, equation (38) predicts that the magnetic field of a rotating superconductor described by this model is antiparallel to the angular velocity rather than parallel as observed, since $q > 0$.

The reader may argue that an attractive Hubbard model with an almost full band is not an accurate description of any real superconductor. We argue that in fact superconductors
are almost always found to have positive Hall coefficients in the normal state [21], or at least to have one band with hole carriers in a multi-band situation [22], and that conduction of one particular band often dominates the transport, so that an approximate description with a model with a single almost full band is appropriate in many cases. The use of an instantaneous attractive interaction instead of a retarded electron–phonon one is a simplifying approximation that may be quantitatively appropriate only for high frequency phonons but in any event does not qualitatively affect the results, as can be seen for example in the calculation of London penetration depth in a lattice electron–phonon model using Eliashberg theory as in [23]. Whether the superconductor is s- or d-wave as believed to be for certain unconventional superconductors would not change our results qualitatively either, as can be seen from [24, 25].

5. The unexplained and counterintuitive features of the London moment

In this section we summarize the unexplained features of the London moment within the conventional BCS–London theory of superconductivity.

5.1. The sign

The measured direction of the magnetic field generated by rotation is always parallel, never antiparallel, to the angular velocity [11–17]. According to our discussion in the previous section, if the carriers in the normal state have dominant hole character the resulting magnetic field should be antiparallel to the angular velocity according to conventional theory. In other words, experiments tell us that superfluid carriers are always negatively charged electrons, while normal state transport experiments in superconductors most often yield positive Hall coefficient indicating that normal state carriers are hole-like, positive. BCS theory does not predict a change in the character of electronic states from hole-like to electron-like when a system goes superconducting, hence the observations are inconsistent with BCS theory.

5.2. The magnitude

The magnitude of the measured magnetic field is given by equation (1) with \( m_e \) the bare electron mass. This is inconsistent with the prediction of BCS theory as reviewed in the previous section, that the mass that enters in the expression for the London penetration depth equation (37) is the effective rather than the bare mass. The same mass that enters in the expression for the London penetration depth should enter equation (24) from which the London field equation (1) directly follows upon applying the curl. Note that the effective mass in heavy fermion systems can be up to three orders of magnitude larger than the bare electron mass, and the London moment measured in heavy fermion systems is no different from that measured in conventional superconductors [17].

We can see the problem most directly in the simple derivation equations (3)–(8). Equation (3) is valid for an electron in free space but not for an electron in the conduction band of a solid. In a solid, semiclassical transport theory [19] tells us that the equation of motion is, instead of equation (3)

\[
\hbar \kappa = eE
\]

and

\[
v = \frac{1}{\hbar} \frac{\partial \epsilon}{\partial k}
\]

from which

\[
\frac{\partial v}{\partial t} = \frac{1}{\hbar^2} \frac{\partial^2 \epsilon_k}{\partial k^2} eE = \frac{e}{m^*} E,
\]

\[
\frac{1}{m^*} = \frac{1}{\hbar^2} \frac{\partial^2 \epsilon_k}{\partial k^2},
\]

so that equations (4) to (8) follow with \( m^* \) replacing \( m_e \), and the result instead of equation (8) is

\[
B = -\frac{2m^*c}{e} \omega_0
\]

in contradiction with experiment, since the effective mass is never identical to the bare mass. In other words, the response of an electron in the conduction band of a solid, whether the electric field is an externally applied field derived from an applied voltage difference or an electric field arising from Faraday’s law and a changing magnetic field generated by the ionic current of the rotating body, is described in terms of the effective mass and not the bare mass.

Thus, the London moment measurements tell us that the conduction electrons in the solid become completely ‘free’ of interactions with the ionic lattice in the superconducting state. This point of view was prevalent in the early days of superconductivity. In particular, Becker et al [9] start their paper by stating ‘Man pflegt den supraleitenden Zustand dadurch zu deuten, dass man den Leitungselektronen relativ zu den Gitterionen eine unendlich grosse Beweglichkeit zuschreibt’ (It is customary to construe the superconducting state by attributing an infinite mobility to the electrons relative to the lattice of ions). It was pointed out for example by Rudnitzkij [26] that the London moment experimental measurement proves that the electron becomes free in the superconducting state. BCS theory does not describe any change in the character of the electronic states other than the pairing correlations, and the electric current in the normal state is carried by carriers with effective mass \( m^* \) rather than bare mass \( m_e \), so BCS theory is in disagreement with experiment. Furthermore, BCS theory attributes the pairing of the carriers to the electron–phonon interaction, which is completely inconsistent with the evidence that superfluid carriers become free of interactions with the ionic lattice.

5.3. The slowing down of electrons near the surface

In a rotating normal metal the electrons move together with the ions, there is no current and no magnetic field. When the metal is cooled into the superconducting state, electrons near the surface spontaneously slow down to give rise to the electric current

\[
\vec{j} = 3\epsilon n e \omega_0 \times \frac{\lambda^2 F}{R^3} f(r)
\]
with \( f(r) \) given by equation (11b)) that gives rise to the magnetic field equation (1). What is the physical mechanism that causes the electrons to slow down? As recounted in the introduction, London predicted this confidently even though he viewed it as ‘quite absurd from the viewpoint of the perfect conductor concept’ and offered no physical explanation for why this would occur.

There is another associated puzzle. The angular momentum of the entire system cannot change upon cooling. The electrons near the surface move slower and consequently carry smaller angular momentum than in the normal state. Hence we have to conclude that the angular velocity of the body slightly increases in the transition from the normal to the superconducting state. What is the physical explanation for this phenomenon?

BCS theory does not provide us with an explanation of these puzzles, it merely agrees with London theory in that this is the unique state of the rotating superconductor. But how does the rotating normal metal get there? BCS theory does not offer any clues.

If it is ever experimentally observed that a rotating normal metal cooled into the superconducting state does not develop a magnetic field, one could not say that this would falsify BCS theory, one could simply say that the system has remained in an infinitely long-lived metastable state which is not the lowest energy state according to BCS theory. By the same token it can be said that BCS theory does not predict nor explain the development of the London moment.

5.4. The radial electric field

In the rotating superconductor the electrons experience a Lorentz force due to the magnetic field equation (1):

\[
F_B = \frac{e}{c} v B
\]

For electrons moving at the same speed as the ions this force is, using equation (1)

\[
F_B = 2m_e \omega_0^2 r.
\]

Here and in what follows we use \( r \) to denote distance to the rotation axis rather than radius in spherical coordinates. This force points inward in the direction of the rotation axis. To sustain the rotational motion, a centripetal force is required

\[
F_c = m_e \frac{\omega^2}{r} = m_e \omega_0^2 r,
\]

which is only half of equation (45). Thus, the Lorentz force will pull electrons slightly inward, until a radial electric field is created to balance the radial forces. The resulting electric field is

\[
E_r = \frac{m_e}{e} \omega_0^2 r,
\]

giving rise to an outward force on electrons

\[
F_E = e E_r = m_e \omega_0^2 r
\]

thus achieving radial force balance

\[
F_B = F_c + F_E.
\]

The situation is depicted schematically in figure 2.

Assuming the electrons become more ‘free’ from the ions in the superconducting state, one would expect that they move out rather than in in the rotating superconductor due to the centrifugal force. This was the expectation of Becker et al [9], they stated: ‘Wir beschränken unsere Betrachtungen von vornherein auf die Rotationsbewegung der Elektronen, vernachlässigen also die durch die Zentrifugalkraft bedingte negative Aufladung der Kugelloberfläche’, meaning ‘We restrict our considerations from the beginning to the rotational motion of the electrons, neglecting the negative charging of the surface caused by the centrifugal force’. In [27] and [28] it was also concluded that negative charges would move out because of an incorrect analysis of the radial forces involved [29], and the authors presumably did not double-check their result relying on the same physical intuition as Becker et al.

That physical intuition is in fact flawed for the case of a perfect conductor. A perfect conductor starting from rest would indeed develop a magnetic field as the electrons near the surface lag behind the motion of the ions, and the Lorentz force due to this magnetic field would pull the electrons slightly inward. This physical effect is well known in plasma physics under the name ‘theta-pincha’. However, here we want to focus on the London scenario.

The question whether negative charges move inward or outward in a rotating superconductor has never been explored experimentally. According to London–BCS theory, the answer has to be the same whether the superconductor is set into motion or whether the rotating normal metal is cooled into the superconducting state. For the latter case it certainly defies physical intuition that the superfluid charges would spontaneously move inward, just as it defies intuition that the azimuthal motion would spontaneously slow down. Thus, we argue that this prediction of the conventional theory involving inward radial motion as depicted in figure 2 is likely to be incorrect.

6. A simple way to explain the key mystery of the London moment

The most puzzling observation related to the London moment, ‘quite absurd from the viewpoint of the perfect conductor concept’ according to London, is that electrons near the surface spontaneously slow down when a rotating metal is cooled and becomes superconducting.
Superconductivity cannot suspend the laws of mechanics and electromagnetism. Inertia compels bodies to continue their motion unless a force acts to modify the motion. What is the azimuthal force causing electrons near the surface to spontaneously slow down? There is no mechanical force nor electromagnetic force to do this task. BCS theory does not offer any insight into the process of slowing down of electrons near the surface.

There is however a very simple, intuitive explanation for this phenomenon: assume that a radial outflow of electrons takes place when a metal goes superconducting. Then, electrons from the interior that reach the surface will be moving at a slower tangential velocity than the surface, reflecting the fact that the tangential velocity of the body is slower in the interior. This is shown schematically in figure 3. The slower velocity of electrons near the surface reflects the fact that they were originally deep in the interior of the body and following the body’s rotation. As viewed in the framework of the rotating body, electrons moving radially outward with velocity \( \vec{v}_r \) experience a Coriolis force \( \vec{F}_C = -2m_e \vec{\omega}_0 \times \vec{v}_r \) in azimuthal direction opposite to the body’s motion.

As the electrons move out, they will decrease their tangential velocity if they keep their angular momentum constant. Alternatively, they could end up with any azimuthal velocity up to the azimuthal velocity they had in the interior, in which case the body as a whole will slightly slow down to conserve total angular momentum. This will depend on the details of the process, but in any case the azimuthal velocity of electrons that came from the interior will be smaller than the azimuthal velocity of the body at the surface, thus explaining the origin of the surface current for the case when a rotating normal metal is cooled into the superconducting state.

Figure 3. Electrons that move from the interior to the surface have smaller tangential velocity than the ions near the surface because they came from a region where the tangential velocity was smaller, giving rise to an electric current near the surface which creates a magnetic field parallel to the body’s angular momentum.

7. The London moment within the theory of hole superconductivity

The theory of hole superconductivity\(^1\) proposes that superconductivity in solids originates in the fundamental charge asymmetry of matter [30], namely that positive charge (protons) is heavier than negative charge (electrons). It predicts that superconductivity can only occur when the charge carriers in the normal state are holes. Thus, within this theory superconductors know very well the difference between electrons and holes, or between negative and positive charge, in contrast to conventional BCS theory that can be formulated with models that are electron–hole symmetric. As discussed earlier, the London moment (by being always of one sign) reveals that real superconductors know very well the difference between positive and negative charge [31], in agreement with the theory of hole superconductivity and in disagreement with BCS theory.

In addition, several essential features of the theory of hole superconductivity explain the puzzles of rotating superconductors discussed earlier.

7.1. Negative charge expulsion

The theory predicts that metals expel negative charge from the interior to the surface in the transition to superconductivity [32]. The prediction follows from the microscopic Hamiltonian used in the theory [33, 34] (dynamic Hubbard model [35]) and the resulting form of the gap function [33, 36], as well as from alternative electrodynamic equations proposed within the theory [37]. This is predicted to occur independent of whether the body is rotating or not and independent of whether or not an external magnetic field is applied. In the presence of an external magnetic field it provides a dynamical explanation for the Meissner effect [38], and in the presence of body rotation it provides an explanation for the slowing down of electrons near the surface discussed in the previous section.

7.2. Superconductivity from ‘undressing’

The theory predicts that in superconductors in the normal state charge carriers are heavily ‘dressed’ by both electron–electron interactions and electron–ion interactions [39]. Both ‘dressings’ are largest when the electronic energy band is almost full. In particular the electron–ion ‘dressing’ is what changes the sign of the effective mass from its bare value (positive) to its dressed value (negative). The theory furthermore predicts that in the transition to superconductivity carriers ‘undress’ from both the electron–electron and the electron–ion interaction [40, 41]. The microscopic calculations performed so far give rise only to partial ‘undressing’ of the carriers [40], however physical considerations lead us to conclude that complete undressing occurs in the transition to superconductivity and the superfluid carriers behave as free electrons [41].

\( ^1\) See references in http://physics.ucsd.edu/~jorge/hole.html.
is consistent with the observation that the bare mass of the electron enters in the expression for the London moment rather than the effective mass.

7.3. Orbit expansion

The theory predicts that electronic orbits expand in the transition to superconductivity, from microscopic radius $k_F^{-1}$ ($k_F$ = Fermi wavevector) to mesoscopic radius $2\lambda_L$ [42]. In particular, this describes the increase in the diamagnetic susceptibility from Landau’s value for normal metals to $-1/4\pi$ for superconductors [38].

Clearly, if superconducting electrons reside in orbits of radius several hundred Å ($2\lambda_L$) it is to be expected that they do not ‘see’ the microscopic ionic structure which changes over a scale of a few Å and instead see an average continuous positive charge distribution. As a consequence they are ‘undressed’ from the electron–ion interaction and behave as free electrons, which is reflected in the fact that their bare rather than their effective mass enters into the London field expression equation (1).

7.4. Holes becoming electrons

The theory of hole superconductivity predicts that a redistribution of occupation of single electron states occurs in the transition to superconductivity [43]. Namely, that the holes that reside on top of the electronic energy band migrate to the bottom of the electronic energy band. This gives rise to an enlargement of the wavelength associated with the charge carriers and thus explains the ‘undressing’ from the electron–ion interaction. Furthermore, the holes at the bottom of the band have now dispersion relation of opposite curvature as at the top of the band, hence they respond as electrons rather than as holes. In particular, their Hall coefficient, if it could be measured, would be negative. However, the number of charge carriers has not changed, it is still the number of holes in the band, going to zero as the band becomes full. This explains the puzzle of why the London moment appears to be generated by electrons with negative charge, yet the number of charge carriers $n_s$ as reflected in the London penetration depth is the number of holes in the band.

7.5. Radial motion and existence of spin current

Assume that in the absence of body rotation electrons are moving in large orbits centered at the rotation axis, with azimuthal speed $v_s$. The centripetal acceleration for this motion is provided by the outward pointing electric field resulting from charge expulsion discussed in A. When the body is set into rotation, the azimuthal speed will change by $\Delta v_s = \alpha_0 r \omega$ in the interior. The centripetal acceleration will change by

$$\Delta \left( \frac{m_e v_s^2}{r} \right) = 2m_e v_s R - \Delta v_s = 2m_e \alpha_0 v_s \frac{e}{c} v_s B. \quad (50)$$

In the last equality we have used the expression for the London field equation (1). Thus, we obtain that the increase in centripetal force when the body rotates is exactly canceled by the inward Lorentz force due to the magnetic field in the rotating superconductor. In other words, no radial redistribution occurs when a superconductor at rest is put into rotation. When a rotating normal metal is cooled into the superconducting state of course radial redistribution occurs because electrons are expelled from the interior to the surface. In any event, the unphysical radial inward motion predicted by the conventional viewpoint (figure 2) does not take place in this scenario.

If in the superconductor electrons are rotating in large orbits in the absence of applied fields and rotation, this has to occur in the absence of charge currents. Thus, we conclude that for every electron orbiting to the right there has to be another electron orbiting to the left. This is consistent with absence of a charge current but it allows for the existence of a spin current, where electrons of opposite spin–orbit in opposite directions. This is what happens in the superconducting state according to the theory of hole superconductivity [29, 42, 44]. Thus, within this theory there is no puzzle associated with inward radial motion of electrons in the rotating superconductor.

7.6. Kinetic energy lowering

When an electron expands its orbit its kinetic energy decreases, since quantum kinetic energy is given by $\hbar^2/(2m_e r^2)$. The theory of hole superconductivity says that superconductivity is associated with lowering of kinetic energy [45], in contrast with BCS theory that says that kinetic energy is increased in the transition to the superconducting state.

Kinetic energy lowering is associated with quantum pressure [46], the tendency of a quantum particle to expand its wave function to lower its kinetic energy. It can also be understood from the uncertainty principle. This quantum pressure is associated with a radial force (= -change in kinetic energy) with radial expansion. Thus we propose that this is the force behind the predicted negative charge expulsion, and it is the force that ultimately explains why electrons near the surface ‘slow down’ when the rotating normal metal becomes superconducting through the mechanism discussed in section 6.

8. $2\lambda_L$ orbits and slowing down near the surface

Within the theory of hole superconductivity in the transition from the normal to the superconducting state, electronic orbits expand from radius $k_F^{-1}$ to radius $2\lambda_L$. This is accompanied by outward motion of charge, and if there is a magnetic field present it is expelled as the orbits expand, giving rise to the Meissner effect.

We can similarly understand the slowing down of electrons near the surface when the rotating normal metal is cooled into the superconducting state. For the sphere, the slowing down is given by

$$\vec{v}_s - \vec{v}_0 = -\frac{3\lambda_L}{R} \vec{\omega} \times \vec{r} \quad (51)$$

and for a cylinder by [47]

$$\vec{v}_s - \vec{v}_0 = -\frac{2\lambda_L}{R} \vec{\omega} \times \vec{r}. \quad (52)$$
Just like for the Meissner effect, it is cylindrical rather than spherical geometry that allows for the simplest understanding. This is illustrated by the fact that the critical magnetic field for a cylindrical geometry is $H_c$, while it is $(2/3)H_c$ for a sphere due to demagnetization.

The orbit expansion is shown schematically in figure 4. The electrons at the surface have their center of motion at distance $2\lambda_L$ from the surface. Thus, their rotation velocity at the surface corresponds to the tangential velocity of rotation at radius $R - 2\lambda_L$, i.e.

$$v = \omega(R - 2\lambda_L)$$  \hspace{1cm} (53)

in agreement with equation (52). In other words, electrons at the surface move slower because they were originally at radius $R - 2\lambda_L$, with slightly slower tangential velocity.

9. Summary and discussion

Perhaps the reason that the London moment is not discussed in standard superconductivity textbooks is because of the difficulty in explaining the observations with the conventional theory of superconductivity discussed in those textbooks, as argued in this paper.

Let us summarize the main points made in this paper.

1. Superconductors know the difference between positive and negative charge, unlike normal metals, as demonstrated by the universal sign of the London moment. We argue that this is a strong point against electron–hole symmetric theories like conventional London–BCS theory and in favor of the theory of hole superconductivity that has as its foundation electron–hole asymmetry.

2. The London moment shows that superfluid electrons behave as bare electrons, free of interactions with the ionic lattice. This is inconsistent with conventional London–BCS theory and particularly with the assumed electron–phonon origin of pairing for conventional superconductors. It is consistent with the theory of hole superconductivity that predicts undressing of carriers as the system enters the superconducting state.

We should mention that there is other experimental evidence for this undressing: (i) the change in sign of the Hall coefficient from positive to negative as a metal is cooled into the superconducting state [48]; and (ii) for high $T_c$ cuprates the observation in photoemission experiments that the quasiparticle weight strongly increases upon entering the superconducting state [40, 49], an effect not predicted by BCS theory.

3. The key puzzle of the London moment experiment, that electrons near the surface ‘slow down’ when a rotating normal metal becomes superconducting, and as a consequence the body as a whole speeds up to conserve angular momentum, is unexplained within London–BCS theory. The fact that electrons slow down near the surface is simply explained by the hypothesis that electrons move from the interior to the surface of the sample in the transition to superconductivity, in which case it is not necessary for the body to speed up to conserve angular momentum. This is predicted by the theory of hole superconductivity.

4. Quantitatively, the magnitude of the slowing down near the surface is consistent with electrons at $r = R$ coming from an interior position at $r = R - 2\lambda_L$. This is in turn consistent with the prediction of the theory of hole superconductivity that superfluid electrons had their orbits enlarged from a microscopic scale to orbits of radius $2\lambda_L$.

We suggest that the arguments presented in this paper should lead to questioning the validity of conventional London–BCS theory to explain superconductivity [50], given that the London moment is a universal effect in superconductors just as the Meissner effect. Alternatives to conventional London–BCS theory that are consistent with the London moment observations should be proposed and critically examined. The theory of hole superconductivity is suggested as one possibility.

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