An improved single particle potential for transport model simulations of nuclear reactions induced by rare isotope beams

Chang Xu\textsuperscript{1,2} and Bao-An Li\textsuperscript{1}

\textsuperscript{1}Department of Physics and Astronomy, Texas A\&M University-Commerce, Commerce, Texas 75429-3011, USA
\textsuperscript{2}Department of Physics, Nanjing University, Nanjing 210008, China

Taking into account more accurately the isospin dependence of nucleon-nucleon interactions in the in-medium many-body force term of the Gogny effective interaction, new expressions for the single nucleon potential and the symmetry energy are derived. Effects of both the spin(isospin) and the density dependence of nuclear effective interactions on the symmetry potential and the symmetry energy are examined. It is shown that they both play a crucial role in determining the symmetry potential and the symmetry energy at supra-saturation densities. The improved single nucleon potential will be useful for simulating more accurately nuclear reactions induced by rare isotope beams within transport models.

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The rapid progress in conducting nuclear reaction experiments using rare isotope beams provides a great opportunity to explore the isospin dependence of strong nuclear interactions \[1,2\]. The latter determines not only the structures of and reactions induced by rare isotopes but also the Equation of State (EOS) of neutron-rich nuclear matter relevant for understanding many interesting astrophysical phenomena \[3,4\]. One of the most important inputs for simulating nuclear reactions induced by rare isotopes is the single nucleon potential, especially its isovector part, i.e., the symmetry potential \[3\]. However, our current knowledge about the latter is still very poor despite the great efforts made in recent years by many people. In fact, a number of theoretical approaches, both microscopic and phenomenological in nature, have been used in studying the single nucleon potential, such as the relativistic Dirac-Brueckner-Hartree-Fock (DBHF) \[15,16\], the nonrelativistic Brueckner-Hartree-Fock (BHF) approaches \[20\], the chiral perturbation theory \[21\], the nonrelativistic mean-field theory using various effective interactions \[22\], and the relativistic impulse approximation coupled with the relativistic mean-field models \[23,24\]. As one expects, the single nucleon potentials obtained usually depend rather strongly on the details of the model nucleon-nucleon interactions used in various many-body approaches, especially at high densities and/or momenta \[26,29\]. For instance, within the Thomas-Fermi model, it was shown recently that the symmetry potential is essentially determined by the competition between the isospin singlet (isosinglet T=0) and isospin triplet (isotriplet T=1) channels of nucleon-nucleon interactions \[30\]. Moreover, the resulting symmetry potential is found to be very sensitive to various in-medium effects, such as the in-medium effective nuclear many-body forces and the short-range tensor force due to the in-medium $\rho$ meson exchange \[31,32\]. It is well known that in nonrelativistic models the in-medium many-body force effects can be taken into account through a density-dependent term in the two-body effective interactions, such as in the Skyrme, the M3Y and the Gogny forces \[33,35\]. In relativistic approaches, on the other hand, the dressing of the in-medium spinors introduces the density dependence in the two-body interaction leading to similar effects as in the nonrelativistic approaches \[36,37\]. Interestingly, while all effective interactions are adjusted to reproduce the saturation properties of symmetric nuclear matter, the symmetry potentials/energies calculated are rather model dependent especially at high momenta and/or densities. Therefore, it is necessary to further study in more detail the in-medium many-body force effects on the symmetry potentials/energies.

In this work, taking into account the isospin dependence of the density-dependent term in the Gogny effective interaction we derive new expressions for the single nucleon potential and the nuclear symmetry energy. By comparing with the old ones previously obtained using the original Gogny force \[22,32\], we investigate effects of the spin(isospin) and density dependence of the many-body force term on the symmetry potential and the symmetry energy. The improved single particle potential obtained with the more complete isospin dependence of the effective many-body force term will be useful for more accurately simulating nuclear reactions induced by rare isotope beams within transport models.

The central part of the original Gogny effective interaction is \[33\]

\[
v(r) = \sum_{i=1,2} (W + BP_{\sigma} - HP_{\tau} - MP_{\sigma}P_{\tau})e^{-r^2/\mu^2}(1) + t_0(1 + x_{P_{\sigma}}\rho^\delta)(1 + r_{1} + r_{2})\delta(r_{ij}),
\]

where $W$, $B$, $H$, $M$, and $\mu$ are five parameters and $P_{\sigma}$ and $P_{\tau}$ are the spin and isospin exchange operators, respectively. The last term is an effective two-body force deduced from a three-body contact force \[33,35\]. The
$x_0$ is the spin(isospin)-dependence parameter controlling the relative contributions of the isosinglet and isotriplet contributions while the $\alpha$ is the density-dependence parameter used to mimic in-medium effects of the many-body forces. Since the $nn$, $np$, and $pp$ interactions are all assumed to depend on the same total density $\rho = \rho_n + \rho_p$, the proper isospin dependence is neglected in the in-medium effects of the many-body forces. However, as it was pointed out earlier in Refs. [38–42] there is no a priori physical justification based on first-principles for any of these interactions to have this kind of density dependence. In fact, Brueckner, Dabrowski, Haensel et al. studied in detail the nuclear symmetry energy within the Brueckner theory in the early 60’s–70’s and found already a strong dependence of the $\alpha$ parameter used to mimic in-medium effects of the many-body forces. For comparisons, it is expected that the improved version (IMDI) potential derived in Ref. [22] and used in the IBUU04 transport model [49], we expect the improved version (IMDI) single nucleon potential [22]. For example, if one considers the interaction between two neutrons near the surface of a rare neutron-rich isotope or in the neutron-skin of a heavy nucleus, it is obviously more appropriate to assume that the neutron-neutron interaction depends on the local average neutron density. Indeed, the idea of using a separate density-dependence for $nn$, $pp$ and $np$ pairs has already been implemented in various models to better understand the structure of nuclei with large isospin asymmetries. For instance, local effective interactions with an appropriate density-dependence separately for $nn$, $pp$ and $np$ pairs have been proposed a long time ago by Sprung and Banerjee [17], Brueckner and Dabrowski [42, 46] and Negele [48]. Unfortunately, to our best knowledge, similar kinds of considerations have not been applied yet in simulating heavy-ion collisions involving rare isotopes. To help remedy the situation, we adopt here the idea from nuclear structure studies. Specifically, we replace the density-dependent term in Eq. (1) with the following

$$V_D = t_0(1 + x_0 P_\sigma)(\rho_r, (r_i) + \rho_r, (r_j))\delta(r_{ij})$$

where $\rho_r(r)$ denotes the density of nucleon $\tau$ (neutron/proton) at the coordinate $r$. Using this improved density-dependent term, we shall present in the following an analytical expression for the single nucleon potential that can be used as an input for transport model simulations of nuclear reactions. Compared to the MDI (Momentum-Dependent Interaction) single nucleon potential derived in Ref. [22] and used in the IBUU04 transport model [49], we expect the improved version (IMDI) will help more accurately simulate nuclear reactions induced by rare isotope beams.

The original MDI single nucleon potential was derived from the Hartree-Fock approximation using the original Gogny effective interaction [22]. For comparisons, it is necessary to first recall the MDI single particle potential for a nucleon of momentum $p$ moving in asymmetric matter of isospin asymmetry $\delta = (\rho_n - \rho_p)/\rho$ and density $\rho$

$$U(\rho, \delta, \vec{p}, \tau) = A_u(x) \frac{\rho^\prime}{\rho_0} + A_l(x) \frac{\rho}{\rho_0}$$

$$+ B\left(\frac{\rho}{\rho_0}\right)^{(1-x/3)^2} - 8\pi x \frac{B}{\sigma + 1} \frac{\rho^{\sigma-1}}{\rho_0} \delta \rho_r$$

$$+ \frac{2C_{\tau,\tau}}{\rho_0} \int d^3\vec{p}' \frac{f_r(\vec{r}, \vec{p}')}{1 + (\vec{p} - \vec{p}'\vec{p})^2/\Lambda^2}$$

$$+ \frac{2C_{\tau,\tau^*}}{\rho_0} \int d^3\vec{p}' \frac{f_r(\vec{r}, \vec{p}')}{1 + (\vec{p} - \vec{p}'\vec{p})^2/\Lambda^2}.$$
nuclear symmetry energy \[^{[30]}\]. However, we emphasize here that the parameter \(x\) or \(x_0\) does not affect the EOS of symmetric nuclear matter because the \(x(x_0)\) related contributions from \(T=0\) and \(T=1\) channels cancel out exactly, i.e., \(x(1 + x_0)\rho^{\sigma+1} + (1 - x_0)\rho^{\sigma+1} = 2\rho^{\sigma+1}\).

The potential energy density corresponding to the improved density-dependent term of Eq. \([2]\) is \[^{[4]}\]

\[
\xi(\rho) = t_0[1 + \frac{x_0}{2}\rho^\sigma \rho_n \rho_p + \frac{1}{16}(1 - x_0)(2\rho_n)^{\alpha+2} + (2\rho_p)^{\alpha+2})]
\]

The corresponding contribution to the single nucleon potential obtained from taking the partial derivative of the potential energy density with respect to the neutron/proton density is

\[
U_D(\rho, \tau) = t_0[(1 + \frac{x_0}{2})(1 + \alpha \frac{\rho_\tau}{\rho}) \rho_\tau \rho^\alpha + \frac{1}{8}(1 - x_0)(\alpha + 2)(2\rho_\tau)^{\alpha+1}]
\]

Replacing properly the parameters \(t_0\), \(x_0\), and \(\alpha\) used in the Gogny force with the \(B\), \(x\), and \(\sigma\) used in the MDI interaction \[^{[3]}\], the complete expression of an improved MDI (IMDI) single particle potential can be written as

\[
U'(\rho, \delta, \vec{p}, \tau) = A'_u(x) \frac{\rho_\tau}{\rho_0} + A'_i(x) \frac{\rho_\tau}{\rho_0}
\]

\[
+ \frac{2B}{\sigma + 1} \left[ (1 + x)(1 + (\sigma - 1)\frac{\rho_\tau}{\rho}) \rho_\tau \rho^\sigma \right]
\]

\[
+ \frac{B}{2}(1 - x)(\frac{2\rho_\tau}{\rho_0})^\sigma
\]

\[
+ \frac{2C_{\tau, \tau}}{\rho_0} \int d^3\rho' \frac{f_{\tau}(\vec{r}, \vec{p}')}{1 + (\vec{p} - \vec{p}')^2/\Lambda^2}
\]

\[
+ \frac{2C_{\tau, \tau}}{\rho_0} \int d^3\rho' \frac{f_{\tau}(\vec{r}, \vec{p}')}{1 + (\vec{p} - \vec{p}')^2/\Lambda^2}.
\]

Some of the parameters have to be re-adjusted to fit the saturation properties of symmetric nuclear matter and the symmetry energy of \(E_{sym}(\rho_0) = 30\) MeV at the normal nuclear matter density of \(\rho_0 = 0.16/fm^3\). More specifically, the \(A'_u\) and \(A'_i\) are respectively

\[
A'_u(x) = -95.98 - \frac{2B}{\sigma + 1} [1 - 2\sigma^{-1}(1 - x)]
\]

\[
A'_i(x) = -120.57 + \frac{2B}{\sigma + 1} [1 - 2\sigma^{-1}(1 - x)]
\]

with \(B = 106.35\) MeV. In the case of symmetric nuclear matter, one expects, the IMDI single particle potential reduces to the original MDI one, i.e., \(U(\rho, 0, \vec{p}, \tau) = U'(\rho, 0, \vec{p}, \tau)\). The symmetry energy corresponding to the IMDI is

\[
E_{sym}(\rho) = \frac{h^2}{6m} \left[ \frac{3\pi^2\rho}{2} \right]^\frac{\sigma}{2} + \frac{B}{4\rho_0} (A'_u(x) - A'_i(x))
\]

\[
+ \frac{B}{\sigma + 1} \left[ \frac{\rho}{\rho_0} \right] [2\sigma^{-1}(1 - x) - 1]
\]

\[
+ \frac{C_{\tau, \tau}}{9\rho_0 \rho} \left( \frac{4\pi\Lambda}{h^3} \right)^2 \left[ 4p^*_f - \Lambda^2 p^*_f \ln \frac{4p^*_f + \Lambda^2}{\Lambda^2} \right]
\]

\[
+ \frac{C_u}{9\rho_0 \rho} \left( \frac{4\pi\Lambda}{h^3} \right)^2 \left[ 4p^*_f - p^*_f (4p^*_f + \Lambda^2) \ln \frac{4p^*_f + \Lambda^2}{\Lambda^2} \right].
\]

![FIG. 1: Momentum dependence of the nuclear isoscalar potentials from both the MDI interaction and the IMDI interaction at a density of \(\rho=0.32\) fm\(^{-3}\).](image)

To evaluate quantitatively effects of the isospin and density dependence of the in-medium many-body force term, we compare in the following the symmetry potential and the symmetry energy calculated with the MDI and the IMDI single nucleon potentials. In heavy-ion reactions induced by rare isotopes, the global reaction dynamics is mostly controlled by the isoscalar nuclear potential \(U_0\) because of its overwhelming strength compared to the isovector potential \(U_{sym}\). The latter, nevertheless, determines all the isospin effects that can be observed by using some delicate experimental observables mostly involving ratios and differences of neutrons and protons, see, e.g., ref. \[^{[3]}\]. The \(U_0\) and \(U_{sym}\) are related to the single nucleon potential by the well-known Lane relationship \[^{[50]}\], namely, \(U_{n/p} \approx U_0 \pm U_{sym}\). Thus, in terms of the neutron \((U_n)\) and proton \((U_p)\) single particle potentials, the nucleon isoscalar and isovector potential can be approximated by \(U_0 = (U_n + U_p)/2\) and \(U_{sym} = (U_n - U_p)/2\alpha\), respectively. Both the \(U_0\) and \(U_{sym}\), especially their momentum dependence, influence the density dependence of the nuclear symmetry energy via \[^{[30, 15, 40]}\].

\[
E_{sym}(\rho) \approx \frac{1}{3} \left[ \left( k_F \right)^0 \frac{1}{6\partial k} \left| k_F \cdot k_F \right| + \frac{1}{2} U_{sym}(k_F) \right]
\]
where \( t(k) \) is the nucleon kinetic energy and \( k_F = (3\pi^2 \rho/2)^{1/3} \) is the Fermi momentum of nucleons in symmetric nuclear matter. While the symmetry energy is calculated exactly using Eqs. 1 and 11 in this work, the above relationship is useful for checking the consistency and understanding the behaviors of the symmetry potential and the symmetry energy. We notice here that with the parameter \( x = 1 \), by design, the \( U_0 \) and \( U_{sym} \) obtained using the IMDD are the same as those obtained using the MDI and they both reduce to the predictions using the original Gogny force 22. This can be clearly seen from Eq. 8 as the fourth term is zero with \( x = 1 \) while other terms stay unchanged as in the original MDI potential.

![FIG. 2: Momentum dependence of the nuclear symmetry potentials from both the MDI interaction and the IMDD interaction at the total nucleon density of \( \rho = 0.16, 0.32, \) and 0.48 fm\(^{-3} \), respectively.](image)

As one expects, introducing the isospin dependence in the density-dependent term of the effective interactions does not affect much the isoscalar potential \( U_0 \). As an example, shown in Fig. 1 is the \( U_0 \) as a function of momentum at twice the normal nuclear matter density with the MDI and IMDD single particle potentials. It is seen that there is very little difference between the results obtained using the IMDD or MDI. On the other hand, there are significant effects on the symmetry potential \( U_{sym} \) and consequently the symmetry energy \( E_{sym}(\rho) \). Shown in Fig. 2 is the momentum dependence of the symmetry potentials at density \( \rho = 0.16, 0.32, \) and 0.48 fm\(^{-3} \), respectively. Three typical values of the spin(isospin)-dependence parameter \( x = 1, 0, \) and \(-1\) are used. It is seen from the left panel that the symmetry potentials with the MDI or IMDD are indeed the same using \( x = 1 \). For the cases with \( x = 0 \) and \(-1\), one can see from the middle and right panels that the symmetry potentials with the IMDD begin to deviate significantly from the ones with the MDI as the density increases. As a result, one expects that the symmetry energy will be significantly different at supersaturation densities with the MDI and IMDD potentials using \( x = 0 \) and \(-1\). This expectation is confirmed in Fig. 3 where the density dependence of the symmetry energy is compared using the MDI and IMDD interactions. Because the \( U_{sym}(k) \) remains unchanged for \( x = 1 \) (see Fig. 2), it is not surprising that the symmetry energy is the same with both the MDI and IMDD interactions using \( x = 1 \). With \( x = 0 \) and \(-1\), however, the symmetry energy with the IMDD becomes significantly stiffer compared to the MDI case. This effect is consistent with the variations of the symmetry potential \( U_{sym}(k) \) obtained using the MDI and IMDD interactions.

![FIG. 3: Density dependence of the symmetry energies calculated using the MDI interaction and the IMDD interaction with \( x = 1, 0 \) and \(-1\).](image)

From all the expressions for the effective interactions, the single nucleon potentials and the symmetry potentials/energies it is clear that effects of the parameter \( x \) (or \( x_0 \)) depend on the choice of the parameter \( \sigma \) (or \( \alpha \)) originally introduced to mimic the in-medium effects of many-body forces. We thus examine next effects of this parameter. Firstly, it is important to note that the choice of the parameter \( \sigma \) and \( x \) is not arbitrary. Besides the correlation between them, there are existing experimental constraints that have to be respected, especially the incompressibility \( K_0 = 9\rho_0^2(\partial^2 E/\partial \rho^2)_{\rho_0} \) of symmetric nuclear matter and the slope of the symmetry energy \( L = 3\rho_0 \frac{\partial E_{sym}(\rho)}{\partial \rho} \big|_{\rho_0} \) at normal density. While both the \( K_0 \) and \( L \) still have some uncertainties mostly because their extraction from experimental data is model dependent, we use here \( K_0 = 210 \pm 20 \) MeV which is consistent within error bars with the ones recently used in the literature 51 52. For the slope parameter we use \( L = 88 \pm 25 \) MeV extracted from isospin diffusion data within the IBUU04 transport model using the MDI with \( \sigma = 4/3 \). This range of \( L \) is also consistent with the ones extracted by using other models 53. However, how the use of the IMDD within transport models may affect the extraction of the parameter \( L \) from experimental data remains an interesting question to be investigated.
Shown in Fig. 4 are correlations between the $K_0$ and $L$ calculated with the MDI and IMDI with $x=1, 0$ and $-1$ and three values of $\sigma$, i.e., $\sigma_{1,2,3}$ of $\frac{1}{3} - \frac{1}{100}$, $\frac{1}{10}$, and $\frac{2}{3} + \frac{1}{100}$, respectively. As already indicated in Fig. 4 calculations with the MDI and IMDI result in symmetry energies with significantly different slopes at the saturation density except with $x=1$. With the $\sigma$ between $\frac{1}{3} - \frac{1}{100}$ and $\frac{2}{3} + \frac{1}{100}$, both the MDI and IMDI with $x=0$ as well as the MDI with $x=-1$ fall into the area constrained jointly by the available constraints on the $K_0$ and $L$. It is worth emphasizing that the $K_0$ and $L$ only constrains the behaviors of the EOS and symmetry energy around the saturation point, but not at densities far away from the saturation point.

![FIG. 4: Correlations between the $K_0$ and $L$ with the different spin(isospin)-dependence parameter $x$ and the density-dependence parameter $\sigma$. The filled round symbols denote the results using the MDI interaction while the open squares are from the IMDI with $x=1, 0,$ and $-1$, respectively.](image)

![FIG. 5: Momentum dependence of the nuclear symmetry potentials from the IMDI interaction with different density-dependence at a total density of $\rho=0.16$, 0.32, and 0.48 fm$^{-3}$, respectively.](image)

![FIG. 6: Symmetry energies calculated by the IMDI interaction with different spin(isospin)-dependence and density-dependence.](image)

Shown in Fig. 5 are the symmetry potentials with the three different values of the density-dependence parameter $\sigma_{1,2,3}$ and $x=1, 0$ and $-1$, respectively. It is seen that the variation of the $U_{sym}$ with $\sigma$ is relatively small except at high densities with $x=-1$. As the density increases, the symmetry potential from the IMDI interaction with $x=1$ starts to deviate from the MDI ones. Meanwhile, the symmetry potential also shifts downwards with the larger $\sigma$ values. However, this is not the case for $x=-1$. In this case, the third term in the IMDI interaction disappears (see Eq.(8)) and only the fourth term contributes to the symmetry potential. Opposite to the case of $x=1$, the $U_{sym}$ moves upwards and becomes more positive with increasing $\sigma$. The corresponding symmetry energies with the different $x$ and $\sigma$ parameters are shown in Fig. 4. It is seen that the variation of $\sigma$ can alter appreciably the high density behavior of the symmetry energy for any given values of $x$. Unlike the parameter $x$, however, the variation of $\sigma$ has negligible effects around and below the saturation density.

In summary, using different density-dependences for the like and unlike nucleon pairs within the Gogny effective interaction, we derived new expressions for the single nucleon potential and the nuclear symmetry energy. Effects of both the spin(isospin) and the density dependence of the nuclear effective interaction are examined. It is found that they play a crucial role in determining the symmetry potentials and symmetry energies at supra-saturation densities. The improved single nucleon potential will be used to simulate more accurately nuclear reactions induced by rare isotope beams within transport models.

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[1] J. M. Lattimer, M. Prakash, Science 304 (2004) 536.
[2] A. W. Steiner et al., Phys. Rep. 411 (2005) 325.
[3] B. A. Li, L. W. Chen and C. M. Ko, Phys. Rep. 464 (2008) 113.
[4] B. A. Li, C. M. Ko and W. Bauer, Int. Jour. Mod. Phys. E 7 (1998) 147.
[5] B. A. Brown, Phys. Rev. Lett. 85 (2000) 5296.
[6] Isospin Physics in Heavy-Ion Collisions at Intermediate Energies, Eds. Bao-An Li and W. Udo Schröer (Nova Science Publishers, Inc, New York, 2001).
[7] P. Danielewicz, R. Lacey and W.G. Lynch, Science 298 (2000) 1592.
[8] V. Baran et al., Phys. Rep. 410 (2005) 335.
[9] K. Sumiyoshi and H. Toki, Phys. Rev. Lett. 85 (2000) 5296.
[10] M. B. Tsang, Yingxun Zhang, P. Danielewicz, M. Famiano, Zhuxia Li, W. G. Lynch, and A. W. Steiner, Phys. Rev. Lett. 102 (2009) 122701.
[11] E. N. E. van Dalen, C. Fuchs, and A. Faessler, Nucl. Phys. A 744 (2004) 227.
[12] S. Ulrych and H. Mäther, Phys. Rev. C 56 (1997) 1788.
[13] J. Decharge and D. Gogny, Phys. Rev. C 21 (1980) 1965.
[14] C. B. Das, S. Das Gupta, C. Gale, B.A. Li, Phys. Rev. C 67 (2003) 034611.
[15] J. R. Stone, J.C. Miller, R. Koncwiecz, P.D. Stevenson, M.R. Strayer, Phys. Rev. C 68 (2003) 034324.
[16] V. R. Pandharipande, V.K. Garde, Phys. Lett. B 39 (1972) 608.
[17] B. A. Li, L. W. Chen, C. M. Ko, B. A. Li, and H. R. Ma, Phys. Rev. C 74 (2006) 044613.