Radiative $\phi$ decays.

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**ABSTRACT**
Radiative $\phi$ decays give us an excellent opportunity to study scalar and pseudoscalar mesons below 1 GeV. In this paper, results from different experiments are reviewed and compared.
The puzzle regarding the scalar mesons $f_0(980)$ and $a_0(980)$

Although the existence and properties of the scalar mesons $f_0(980)$ and $a_0(980)$ are well established and have been known for about thirty years, the physical nature of these mesons is still unclear. The first evidence for the $f_0$ was in the reaction $p\pi \rightarrow n\pi\pi$. The $I = J = 0$ elastic $\pi\pi$ cross section shows a dip close to the $K\bar{K}$ threshold. Similarly, in $p\pi \rightarrow nK\bar{K}$, a sharp onset of inelasticity indicates the presence of a dynamical structure strongly coupled to $K\bar{K}$. Evidence for strange quark content in the $f_0$ is also confirmed by $J/\psi$ decays. In fact, the Mark-III, DM2, and BES collaborations measure $\text{BR}(J/\psi \rightarrow f_0\phi) > \text{BR}(J/\psi \rightarrow f_0\omega)$. However, the result obtained by the E791 collaboration \[1\] shows that 56% of the $D_s$ decay into $3\pi$ proceeds via $f_0$, which suggests that $f_0$ has both $s\bar{s}$ and $n\bar{n}$ components. The total width of the $f_0$ meson has been measured by several experiments and ranges between 40–100 MeV; moreover, the $f_0$ is weakly coupled to photons $[\Gamma(f_0 \rightarrow \gamma\gamma) \approx 0.3 \text{ KeV}]$. The first evidence for the $a_0$ was found in the $\eta\pi$ system in $Kp \rightarrow \Sigma\eta\pi$. Today, there are very accurate data from the Crystal Barrel, GAMS, Obelix and E852 collaborations \[2, 3\]. As in the case of the $f_0$, the $a_0$ also has a small width compared to ordinary mesons with the same mass, and a small $\gamma\gamma$ coupling. In short, these two scalar mesons are very close in mass, and have small widths and small $\gamma\gamma$ couplings. The standard interpretations of these states as $q\bar{q}$ mesons is not favoured. The total widths are much smaller than the $\sim 500$ MeV expected from the $q\bar{q}$ prediction, the $\gamma\gamma$ partial widths measured are a factor 10–20 smaller than the $q\bar{q}$ prediction, and the $K\bar{K}$ coupling is too large for an OZI-forbidden decay. The theoretical situation is further complicated because there are many scalars below 1.5 GeV. Many different interpretations of these states have been proposed: $K\bar{K}$ molecules \[3\], four-quark states \[4\], and, in the case of the $f_0$, glueballs. Interest in studying light scalar mesons also comes from an old suggestion by Gribov that foresees the existence of a peculiar state with the vacuum quantum numbers and a mass close to the proton mass to explain quark confinement. Most recently, Close and Törnqvist \[5\] have interpreted these mesons as the Higgs nonet of a hidden $U(3)$ symmetry. From all of the above considerations, it is obviously urgent to clarify the situation. The radiative $\phi$ decays are very useful for addressing this puzzle. In fact, the absolute rates for $\phi \rightarrow f_0\gamma$ and $a_0\gamma$ and the mass spectra of the $f_0$ and $a_0$ are very sensitive to the nature of these scalar particles, as shown in Tab. \[1\].
Table 1: Branching ratio predictions for different models.

| Channel | $qar{q}$ | $qar{q}qar{q}$ | $Kar{K}$ |
|---------|------------|------------------|----------|
| $\text{BR}(\phi \rightarrow f_0\gamma)$ | $5 \times 10^{-5}$ | $10^{-4}$ | $10^{-5}$ |
| $\text{BR}(\phi \rightarrow a_0\gamma)$ | $10^{-5}$ | $10^{-4}$ | $10^{-5}$ |

2 Radiative $\phi$ decays to $f_0$, $a_0$

Most of the data on the $\phi \rightarrow f_0\gamma$ and $\phi \rightarrow a_0\gamma$ decays come from three experiments: KLOE \cite{8, 7} at the Frascati $\phi$-factory DAΦNE\cite{9}, and SND \cite{12, 10} and CMD-2 \cite{13, 11} at VEPP-2M in Novosibirsk. The published analyses are based on $5.3 \times 10^7 \phi$ decays for the KLOE experiment, with large acceptances, and $2 \times 10^7 \phi$ decays for the VEPP-2M experiments, with smaller acceptances. The $f_0$ has been studied via its decay to $\pi^0\pi^0$. The $a_0$ has been studied via its decay to $\eta\pi^0$, with $\eta \rightarrow \gamma\gamma$ and $\eta \rightarrow \pi^+\pi^-\pi^0$. For this last channel, only data from the KLOE experiment are available.

2.1 $\phi \rightarrow \pi^0\pi^0\gamma$

Two amplitudes contribute to the $\phi \rightarrow \pi^0\pi^0\gamma$ final state: $\phi \rightarrow S\gamma; S \rightarrow \pi^0\pi^0$ and $\phi \rightarrow \rho^0\pi^0; \rho^0 \rightarrow \pi^0\gamma$, where $S$ is a scalar meson. The main backgrounds come from $e^+e^- \rightarrow \omega\pi^0 \rightarrow \pi^0\pi^0\gamma$, $\phi \rightarrow \eta\pi^0\gamma \rightarrow 5\gamma$, $\phi \rightarrow \eta\gamma \rightarrow 3\pi^0\gamma$ with two undetected photons, and $\phi \rightarrow \eta\gamma \rightarrow 3\gamma$ with two additional photons from accidentals.

The natural cross sections for the signal and background channels are listed in Tab. 2. The $\omega\pi$ background is the most important, since the background from $\eta\gamma$ into 3- and 7-photon final states is immediately reduced by exploiting the detector hermiticity and the photon timing.

Table 2: Cross sections for signal and background channels for $\phi \rightarrow \pi^0\pi^0\gamma$.

| Channel | cross section (nb) |
|---------|--------------------|
| $f_0\gamma$ | $\approx 0.4$ |
| $\omega\pi$ | $\approx 0.5$ |
| $\eta\pi\gamma$ | $\approx 0.1$ |
| $\eta\gamma$ | $\approx 17.0$ |

All three collaborations have performed similar analyses. The selection of $\phi \rightarrow \pi^0\pi^0\gamma$ events starts from the five-photon sample. A cut on the total energy
in the calorimeter rejects the $\phi \rightarrow K_S K_L$ background. The photons are paired to search for a $\pi^0$ and a cut on $|M_{\pi\gamma} - M_\omega|$ is performed to veto the $\omega\pi$ background. A kinematic fit requiring the $\pi^0$ masses further reduces the background. The numbers of signal events and the mean efficiencies are shown in Tab. 3 for each of the three collaborations. The $\pi\pi$ invariant mass spectra are shown in Figs. 1 and 2. A clear peak in the $f_0$ region is seen.

Table 3: Signal statistics and efficiencies for $\phi \rightarrow \pi^0 \pi^0 \gamma$.

| EXPERIMENT | Signal events | Mean efficiency (%) |
|-------------|---------------|---------------------|
| KLOE        | 2438 ± 61     | ∼ 40                |
| SND         | 419 ± 31      | ∼ 20                |
| CMD-2       | 268 ± 27      | ∼ 12                |

Figure 1: $\pi\pi$ invariant mass spectrum, KLOE experiment.

2.2 Model for fits to $M_{\pi\pi}$ spectrum

All the three experiments have fit the mass spectrum using the same model. This spectrum is taken as the sum of $S\gamma$, $\rho\pi$, and interference terms: $f(m) = f_{S\gamma}(m) + f_{\rho\pi}(m) + f_{\text{int}}(m)$. For the scalar term, the kaon loop model is used [14]. In this model, the radiative $\phi$ decays to a scalar proceed through a charged kaon loop, and the scalar term can be written as:

$$f_{S\gamma}(m) = \frac{2m^2}{\pi} \frac{\Gamma_{\phi S\gamma} \Gamma_{S\pi^0 \pi^0}}{|D_S|^2} \frac{1}{\Gamma_{\phi}}. \quad (1)$$
\[ \Gamma_{\phi f_0 \gamma}(m) = \frac{g_{f_0 K^+K^-}^2 - g_{\phi K^+K^-}^2}{12\pi} \frac{|g(m)|^2}{M_\phi^2} \left( \frac{M_\phi^2 - m^2}{2M_\phi} \right), \]  

where \( g_{\phi K^+K^-} \) and \( g_{f_0 K^+K^-} \) are the couplings and \( g(m) \) is the loop integral function.

For the inverse propagator, \( D_S \), the finite width corrections [14] are taken into account. The parametrization of Ref. [15] has been used for the \( \rho \pi \) and interference terms. A recent measurement [16, 17] reports the existence of a scalar \( \sigma \) with \( M_\sigma = (478^{+24}_{-23} \pm 17) \) MeV and \( \Gamma_\sigma = (324^{+42}_{-40} \pm 21) \) MeV. The contribution of this meson [18] has also been included in fits to the mass spectrum, giving different results as discussed below.

The observed mass spectrum, \( S_{\text{obs}}(M_{\pi\pi}) \), is fit by folding the experimental efficiency and resolution into the theoretical shape after properly normalizing for the cross section for \( \phi \) production, \( \sigma(\phi) \), and the integrated luminosity, \( L_{\text{int}} \). The KLOE collaboration has performed a fit including \( \sigma \) and \( \rho \pi \) contributions. They find that the data prefer a negligible \( \rho \pi \) contribution and a negative interference between the \( f_0 \) and \( \sigma \) amplitudes at \( M_{\pi\pi} < 700 \) MeV. The CMD-2 collaboration has done a similar fit and they find a negligible \( \sigma \) contribution. SND has fit the data considering the \( f_0 \) contribution only. It is worth noticing that, due to the lack of statistics, the VEPP-2M experiments do not have much sensitivity in the \( \sigma \) region.

The values of the coupling constant obtained by KLOE [8], SND [12], and CMD-2 [13] are listed in Tab. 4 and are compared with the results from the WA102 and E791 collaborations. The coupling constants obtained from \( \phi \) decays agree with each other, and differ from the WA102 result on \( f_0 \) production in central \( pp \) collisions [19] and from results obtained when the \( f_0 \) is produced in \( D_s^+ \to \pi^+\pi^-\pi^0 \) decays [1], in which \( g_K \) is consistent with zero. In the same table, measurements of \( \text{BR}(\phi \to f_0 \gamma) \) for \( M_{\pi\pi} > 700 \) MeV are also listed.
Table 4: Comparison of $f_0$ parameters from different experiments.

| Parameter                  | KLOE          | SND           | CMD-2         | WA102         | E791          |
|---------------------------|---------------|---------------|---------------|---------------|---------------|
| $M_{f_0}$ (MeV)           | 973 ± 1       | 969 ± 5       | 975 ± 7       | 987 ± 8       | 977 ± 4       |
| $g_{f_0K^+K^-}/(4\pi)$ (GeV$^2$) | 2.79 ± 0.12   | 2.47 ± 0.73   | 1.48 ± 0.32   | 0.40 ± 0.06   | 0.02 ± 0.05   |
| $g_{f_0K^+K^-}/g_{f_0\pi^+\pi^-}$ | 4.00 ± 0.14   | 4.6 ± 0.8     | 3.61 ± 0.62   | 1.63 ± 0.46   | —             |
| $g_{\phi\sigma\gamma}$   | 0.060 ± 0.008 | —             | —             | —             | —             |
| BR($\phi \rightarrow \pi^0\pi^0\gamma) \times 10^4$ | 0.96 ± 0.05   | 1.03 ± 0.09   | 0.92 ± 0.09   | —             | —             |
| $M_{\pi\pi} > 700$ MeV   |               |               |               |               |               |

2.3 $\phi \rightarrow \eta\pi^0\gamma$

Production of the $a_0$ meson followed by $a_0 \rightarrow \eta\pi^0$ dominates the final state $\eta\pi^0\gamma$ in $\phi$-decays. A small contribution from $\phi \rightarrow \rho^0\pi^0$, $\rho \rightarrow \eta\gamma$ is present.

The $\phi \rightarrow \eta\pi^0\gamma$ decay is studied by KLOE [7], CMD-2 [11] and SND [10] using the $\eta \rightarrow \gamma\gamma$ decay mode; the analysis starts from the same 5$\gamma$ sample as for the $f_0$ selection. The main backgrounds come from the $\phi \rightarrow \pi^0\pi^0\gamma$ channel, which is dominated by $\phi \rightarrow f_0\gamma$, the non-resonant $e^+e^- \rightarrow \omega\pi^0$ interaction with $\omega \rightarrow \pi^0\gamma$, and $\phi \rightarrow \eta\gamma$ with $\eta \rightarrow \gamma\gamma$ and $\eta \rightarrow \pi^0\pi^0\pi^0$. The numbers of signal events and the mean efficiencies are summarized in Tab.5. The KLOE collaboration also makes use of the $\phi \rightarrow \eta\pi^0\gamma$, $\eta \rightarrow \pi^-\pi^+\pi^0$ channel. In this case, the lower statistics are compensated for by the fact that there are no backgrounds with the same final state. The KLOE collaboration selects 197 events with an estimated background of $4 \pm 4$ events and an efficiency of $\approx 19\%$.

The fit to the mass spectrum for the $a_0$ is performed using the kaon loop model as in the case of the $f_0$. The $\phi \rightarrow \rho\pi^0$, $\rho \rightarrow \eta\gamma$ contribution is also considered by the KLOE collaboration. The SND collaboration fits the data assuming only the $a_0$ contribution. The KLOE collaboration performs a combined fit using the two $\eta$ decay modes and setting $M_{a_0}=984.8$ MeV, from Ref. [10]. The free parameters of the fit are the branching ratio for the $\phi \rightarrow \rho\pi^0$ contribution and the two coupling constants.

The CMD-2 collaboration does not attempt to fit the mass spectrum due
to lack of statistics; only $\text{BR}(\phi \to \eta\pi^0\gamma)$ is quoted. In Fig.3, the mass spectrum from the KLOE experiment is shown for the two $\eta$ decay channels. The values of the coupling constant obtained by KLOE, SND, and CMD-2 are listed in Tab. 6 and are compared with the results from BNL-E852 [2] and with the various Crystal Barrel results [3].

Table 6: Comparison of $a_0$ parameters from different experiments.

|                  | KLOE | SND  | CMD-2 | E852  | Crystal Barrel |
|------------------|------|------|-------|-------|----------------|
| $M_{a_0}$ (MeV)  | 984.8| 995+52−10 | —     | 991 ± 3 | 1000 ± 2       |
| $g_{a_0K^+K^-}/(4\pi)$ (GeV$^2$) | 0.40 ± 0.04 | 1.4±0.9 | —     | 1.05 ± 0.06 | 0.93 – 1.07 |
| $g_{a_0\eta\pi}/g_{f_0K^+K^-}$ | 1.35 ± 0.09 | 0.75 ± 0.52 | —     | —     | —              |
| $\text{BR}(\phi \to \eta\pi^0\gamma) \times 10^5$ | 7.4 ± 0.7 | 8.8 ± 1.7 | 9.2 ± 2.6 | — | — |

Figure 3: KLOE result for combined fit: comparison of data (exp. points) vs. fit (histogram) for $\phi \rightarrow \eta\pi^0\gamma$ with (a) $\eta \rightarrow \gamma\gamma$ and (b) $\eta \rightarrow \pi^+\pi^0\pi^0$.  

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2.4 Comparison with theoretical expectations

The measured coupling constants for the $a_0$ and $f_0$ can be compared with the predictions for the $qq$ and $qqq\bar{q}$ models. These two models make predictions for the ratio of the coupling constants, $R$, based on SU(3) algebra assuming no OZI-violation. Qualitative predictions are available for the absolute values of the couplings. Tab. 7 shows these predictions compared with KLOE results. The predictions have been evaluated using a value of $\theta_P = -13.7^\circ$ for the pseudoscalar mixing angle. From this comparison, it is evident that:

1. The $f_0$ parameters are not compatible with the predictions for the $qq = u\bar{u} + d\bar{d}$ model. $R_{f_0}$ and $g_{f_0KK}$ are both too large.

2. $R_{f_0}$ is compatible both with $qqq\bar{q}$ model parameters and with the expectation for $s\bar{s}$.

3. The $a_0$ coupling, $g_{a_0KK}$, is too small with respect to predictions for the $qqq\bar{q}$ model.

4. $R_{a_0}$ seems to indicate better agreement with $q\bar{q}$ model.

Predictions for the values of the branching ratios have also been made in the framework of the linear sigma model [20, 21] and in the unitarized chiral model approach [22], and show rather good agreement.

3 Pseudoscalar: $\phi \rightarrow \eta \gamma$, $\phi \rightarrow \eta' \gamma$

Radiative decays of light vector mesons to pseudoscalars have been used as a testing ground since the early days of the quark model [23]. The branching ratio of the

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
Experiment & $g_{f_0KK}^2/4\pi$ & $g_{a_0KK}^2/4\pi$ & $R_{f_0}$ & $R_{a_0}$ \\
\hline
KLOE & 2.79 ± 0.12 & 0.40 ± 0.04 & 4.00 ± 0.14 & 0.55 ± 0.07 \\
qqq\bar{q} & “superallowed” & “superallowed” & 4 ÷ 8 & 1.2 \\
qq (a) & “OZI-allowed” & “OZI-forbidden” & 4 & 0.43 \\
qq (b) & “OZI-forbidden” & “OZI-forbidden” & 0.5 & 0.43 \\
\hline
\end{tabular}
\caption{Comparison of KLOE results with predictions from $qq$ and $qqq\bar{q}$ models. $qq$ refers to either $f_0 = (u\bar{u} + d\bar{d})/\sqrt{2}$ (b) or $f_0 = s\bar{s}$ (a). Here, $R_{f_0}$ is $g_{f_0KK}^2/K^+K^-/g_{f_0\pi^+\pi^-}$ and $R_{a_0}$ is $g_{a_0KK}^2/K^+K^-/g_{a_0\eta\pi}^2$.}
\end{table}
decay \( \phi \to \eta'\gamma \) is particularly interesting since its value can probe the \( |s\bar{s}\rangle \) and gluonium content of the \( \eta' \). In particular, the ratio \( R \) of the branching ratios for \( \phi \to \eta'\gamma \) and \( \phi \to \eta\gamma \) can be related to the \( \eta-\eta' \) mixing parameters \(24, 25, 27, 28, 29\) and determine the mixing angle in the flavor basis \( \varphi_P \), which has been identified as the best suited parameter for a process-independent description of the mixing. In fact, within the two-mixing-angles scenario, which has emerged from an extended chiral perturbation theory framework \(30\) as well as from phenomenological analyses \(31\), it has been demonstrated that the two mixing parameters in the flavor basis are equal, apart from terms which violate the OZI rule \(32, 33\). It is therefore safe to use a single mixing angle in this basis. The measurements available to date on \( \mathrm{BR}(\phi \to \eta'\gamma) \) come from the KLOE \(34\), CMD-2 \(38\), and SND \(35\) collaborations.

KLOE has analyzed a sample of \( 5.3 \times 10^7 \phi \) decays, looking for \( \phi \to \eta'\gamma \) via the \( \eta' \to \pi^+\pi^-\eta \) and \( \phi \to \eta\gamma \), \( \eta \to \pi^+\pi^-\pi^0 \) decay chains. For both decay chains, the \( \pi\pi3\gamma \) final state is used; therefore, many common systematic effects approximately cancel out in the ratio \( R = \frac{\mathrm{BR}(\phi \to \eta'\gamma)}{\mathrm{BR}(\phi \to \eta\gamma)} \). After the analysis, KLOE finds 128 \( \eta' \) events with an efficiency of \( \approx 23\% \) and a background level of 6\%. The value \( R = (4.7 \pm 0.5 \pm 0.3) \times 10^{-3} \) has been obtained.

This value for \( R \) can be related directly to the mixing angle in the flavor basis. The Bramon \( et \ al. \) \(27\) and Feldmann \(29\) parametrizations are used to extract the mixing angle; essentially the same result is obtained using both approaches, i.e., \( \varphi_P = (42.2 \pm 1.7)^\circ \), which gives in a mixing angle in the octet-singlet basis \( \vartheta_P = (-12.9 \pm 1.7)^\circ \). Moreover, using the value in \(40\) for \( \mathrm{BR}(\phi \to \eta\gamma) \), the most precise determination of \( \mathrm{BR}(\phi \to \eta'\gamma) \) to date has been extracted (see Fig. 4, left): \( \mathrm{BR}(\phi \to \eta'\gamma) = (6.1 \pm 0.6 \text{ (stat.)} \pm 0.4 \text{ (syst.)}) \times 10^{-5} \).

This value for the mixing angle has been obtained neglecting OZI-rule violation and possible contributions from gluonium to the \( \eta \) and \( \eta' \) mesons. Allowing for a gluonium component, \(23\) we write:

\[
|\eta\rangle = X_\eta |u\bar{u} + d\bar{d}\rangle / \sqrt{2} + Y_\eta |s\bar{s}\rangle + Z_\eta |\text{glue}\rangle,
\]

\[
|\eta'\rangle = X_{\eta'} |u\bar{u} + d\bar{d}\rangle / \sqrt{2} + Y_{\eta'} |s\bar{s}\rangle + Z_{\eta'} |\text{glue}\rangle.
\]

A non-zero gluonium component of the \( \eta' \) would correspond to \( Z_{\eta'}^2 > 0 \), or equivalently \( X_{\eta'}^2 + Y_{\eta'}^2 < 1 \). The following constraints on \( X_{\eta'} \) and \( Y_{\eta'} \) can be obtained in a nearly model-independent way \(25, 28, 39\):

\[
\frac{\Gamma(\eta' \to \rho\gamma)}{\Gamma(\omega \to \pi^0\gamma)} \simeq 3 \left( \frac{m_\rho^2 - m_\omega^2}{m_\omega^2 - m_\eta^2 m_{\eta'}} \right)^3 X_{\eta'}^2
\]
and

\[
\frac{\Gamma(\eta' \rightarrow \gamma\gamma)}{\Gamma(\pi^0 \rightarrow \gamma\gamma)} = \frac{1}{9} \left( \frac{m_{\eta'}}{m_{\pi^0}} \right)^3 \left( 5X_{\eta'} + \sqrt{2}Y_{\eta'} \frac{f_\pi}{f_s} \right)^2.
\] (5)

The consistency of the assumption of $\eta - \eta'$ mixing without gluonium can be checked as follows: if $Z_{\eta'} = 0$, one has $|Y_{\eta'}| = \cos \phi_P$. This remains a reasonable approximation if the gluonium component is small. In Fig. 4 (right), the allowed bands corresponding to equations (4) and (5) and the KLOE measurement of $\cos \varphi_P$ are plotted in the $X_{\eta'}$, $Y_{\eta'}$ plane, as well as the circumference $X_{\eta'}^2 + Y_{\eta'}^2 = 1$, corresponding to zero gluonium in the $\eta'$. $Z_{\eta'}^2$ is seen to be $0.06^{+0.09}_{-0.06}$, which is compatible with zero within $1\sigma$ and consistent with a gluonium fraction of less than 15%.

4 Conclusions

The data coming from radiative $\phi$ decays are fundamental in clarifying the nature of the scalar mesons. The branching ratios for $\phi \rightarrow \pi^0\pi^0\gamma$ and $\phi \rightarrow \eta\pi^0\gamma$ and the $f_0$ and $a_0$ coupling constants have been measured with the best accuracy by the KLOE collaboration and are in agreement with CMD-2 and SND results. There is still much to do in this field and more data are expected from KLOE and from other experiments ($D$ decays etc). In the pseudoscalar sector there is a new, precise measurement of $\text{BR}(\phi \rightarrow \eta'\gamma)$ and the $\eta-\eta'$ mixing angle.
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