Supernova Neutrino Spectrum with Matter and Spin Flavor Precession Effects

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Abstract

We consider Majorana neutrino conversions inside supernovae by taking into account both flavor mixing and the neutrino magnetic moment. We study the adiabaticity of various possible transitions between the neutrino states for both normal and inverted hierarchy within the various solar neutrino problem solutions. From the final mass spectrum within different scenarios, we infer the consequences of the various conversion effects on the neutronization peak, the nature of final spectra, and the possible Earth matter effect on the final fluxes. This enables us to check the sensibility of the SN neutrino flux on magnetic moment interaction, and narrow down possible scenarios which depend on: the mass spectrum normal or inverted, the solution of the solar neutrino problem; and the value of $\mu B$.

1 Introduction

The neutrino signal detected on the Earth from the SN1987A explosion [1] has opened up new ways to probe the neutrino properties. Initially only constraints on the static properties of neutrino during its interstellar journey (namely from vacuum oscillations) could be acquired. Now with the entry into service of the large neutrino underground detectors like Superkamiokande (SK) [2] and the Sudbury Neutrino Observatory (SNO) [3], modification of the flavor content of the neutrinos emerging from the SN itself could be contemplated. This is helped greatly by the new quantitative progress achieved in the past few years in our knowledge of the mass squared differences and mixing parameters from the atmospheric neutrino oscillations (SK and MACRO [4] in particular), and the solar neutrino oscillations [5].

Various works have appeared studying the consequences of both the matter resonance effect (the so-called MSW effect) [6], and the spin flavor precession (the RSFP effect) [7], especially with the present data on the neutrino magnetic moment [8], due to the very large magnetic field in the pre-supernova interior [9]. These studies have been mostly confined to the case of two active neutrinos. The case of sterile neutrinos is no more rigorously pursued in the light of the strong constraints on their existence.
from the atmospheric neutrinos experiments. They used to be invoked in neutrino conversion, in particular for their potential role in enabling r-process nucleosynthesis [10].

We wish in this paper to consider the general case of three active neutrinos on the neutrino SN spectrum, namely the ones associated with the three known leptonic flavors, in the context of both the MSW effect and the RSFP effect. We have followed the seminal work of Dighe and Smirnov [11] as far as their general line of attack on the question, but generalizing it to take into account both of the above mentioned effects at the same time. Considering the various schemes of neutrino mass and mixing allowed by the data, we systematize the discussion of the various spectrum distortion effects, and thus check the discriminating power of such studies and how they could help resolve the ambiguities associated with it, notably:

- the solar neutrino solution (which one to choose?)\(^1\).
- the type of hierarchy for the neutrino masses (normal or inverted).
- the value of \(\mu B_\perp\) where \(\mu\) is the neutrino magnetic moment, and \(B_\perp\) the SN transverse magnetic field.

It follows from the existing solar and atmospheric neutrino data that neutrino mass-squared differences satisfy the hierarchy \(\Delta m^2_{31} \ll \Delta m^2_{32}\), which permits us to order the three mass-squared according to the two following cases: (i) normal (or direct) hierarchy, where \(m_1^2 \leq m_2^2 \ll m_3^2\), and thus \(\Delta m^2_{32}\) will be positive (See Fig.1-a). (ii) the inverted mass hierarchy, where \(m_3^2 \ll m_1^2 \leq m_2^2\) and thus \(\Delta m^2_{32}\) will be negative (See Fig.1-b). We will assume that \(\Delta m^2_{21}\) is relevant for the solar neutrinos oscillations, while \(\Delta m^2_{32}\) is relevant for the oscillations of the atmospheric neutrinos.

The effects of the neutrino conversions can be observed through, (i) the disappearance (partial or complete) of the neutronization peak; (ii) the interchange of original spectra and the appearance of a hard \(\nu_e\) spectrum; (iii) the modification of the \(\bar{\nu}_e\) spectrum; (iv) the Earth matter effect, which is studied taking into account neutrino mass and mixing in [11]. Note that for significantly cosmological mass-squared differences (\(\Delta m^2 = 1 \sim 100\) eV\(^2\)), the spin-flavor precession and resonant spin-flavor conversions may affect the supernova shock reheating and r-process nucleosynthesis [9].

Although the ambiguities on the neutrino spectrum could not be solved, a systematic study of their effects in the general three-generation case pave the way for further constraining the various scenarios as new limits are obtained from the detectors on the Earth.

This paper is organized as follows: we first obtain the original neutrino fluxes emerging from the SN core; we then study neutrino conversion outside the core for both normal and inverted mass hierarchy, and finally determinate the final neutrino fluxes which reach the Earth detectors. The various neutrino conversion effects are then classified according to the different parameters relevant for the Solar neutrino solutions and the magnitude of the magnetic interaction term \(\mu B_\perp\).

\(^1\)The recent results from the KamLAND experiment [12] indicate that the LOW scenario is most probably ruled out, which leaves the LMA as the most favored one.
When the inner iron core of a massive star going through a type II supernova explosion becomes unable to support the electron degeneracy pressure, most of the gravitational binding energy is released in the form of a violent burst of neutrinos of all species. These neutrinos produced both during the neutronization burst and the subsequent thermal cooling could undergo transformation of kind both inside the SN, and outside on their way to the Earth. We are however interested only in their transformation within the SN. In some scenarios these transformations could boost the energy deposition at the stalled explosion front as well as play a key role in the explosive nucleosynthesis. Yet, since in our study we are using mass differences relevant to the Solar Neutrino Problem (SNP) and the Atmospheric Neutrino Problem only, these neutrino transformations are not expected to change the dynamics of the explosion as the corresponding resonances take place outside the core.

Two effects have been widely considered: the first one is the matter effect, i.e. the interaction of \( \nu \)'s with different matter constituents which are mainly electrons, protons and neutrons. The second one consists of the interaction of the neutrino’s magnetic moment with the transverse magnetic field of the SN. For Dirac neutrinos, they flip into a right handed \( \nu_R \) kind which is known to be sterile since undetected till now. Comparing the inferred energy output of SN1987A to the theoretical expectations, one can put strong constraints on these Dirac neutrino conversions. On the other hand, when including the most general mass terms in the simplest extension of the Weinberg-Salam model, the neutrino fields, once diagonalized, turn out to be of Majorana kind. All this concurs to make the Majorana neutrinos more palatable than the Dirac kind in the context of SN studies. We will assume in what follows that the neutrinos are Majorana particles. Now since they are their own antiparticles, only flavor conversion will be allowed.

In order to study these effects together, we should identify the profile of both the density and the SN magnetic field. We take the standard parametrization:

\[
B(r) = B_{\perp o} \left( \frac{r}{r_o} \right)^{-k} ; \quad r \geq r_o
\]  

\( \text{(1)} \)
where $B_{\perp0}$ is the magnetic field strength at the distance $r_o=10$ km; and $k=2$ or $3$; in our work, we will consider $k = 3$. The value of $B_{\perp0}$ lies between $2 \times 10^{12}$ and $10^{15}$ G, where $G$ is the strength magnetic field unit. For this density profile, the important effects occur between $O(18 \text{ cm}^{-3})$ and $O(10^2 \text{ g cm}^{-3})$, i.e. this region may not be affected by the SN shock wave. We will thus take the progenitor profile for $\rho \lesssim 1 \text{ g cm}^{-3}$ as approximately given by  \cite{17} 

$$\rho Y_e \approx \rho_o \left(\frac{r_o}{r}\right)^{-3} \tag{2}$$

where $\rho_o = 2 \times 10^{13}$ g cm$^{-3}$. Of course, the exact shape of the density profile will depend on the precise composition of the star.

The first question to answer is the flavor content of this burst when reaching the conversion regions. The key point here is that they are produced in a high density medium so that their mass eigenstates can readily be inferred.

For this matter, let us rewrite the Hamiltonian in the flavored basis $\nu^f = (\nu_e, \nu_\mu, \nu_\tau, \bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau)$ which is related to the mass eigenstates basis in vacuum $\nu^v = (\nu_1, \nu_2, \nu_3, \bar{\nu}_1, \bar{\nu}_2, \bar{\nu}_3)$ by the unitary transformation $\nu^f = U \nu^v$. We can then deduce the initial mass spectrum from the evolution of neutrinos at very high density ($\rho \gtrsim 10^8$ g cm$^{-3}$).

Taking into account the neutrino effective potential coming from both the MSW effect and the RSFP effect, we can cast the Hamiltonian into the following form:

$$H = \text{Vacuum term} + \text{Matter term} + \text{Spin Precession term} \tag{3}$$

where each term, after the subtractions of terms proportional to the unity, is given by:

\begin{align*}
\text{Vacuum term} &= \frac{1}{2\pi} \left( \begin{array}{cc} U & 0 \\ 0 & U \end{array} \right) \left( \begin{array}{cc} M^2 & 0 \\ 0 & M^2 \end{array} \right) \left( \begin{array}{cc} U^\dagger & 0 \\ 0 & U^\dagger \end{array} \right) \\
\text{Matter term} &= \alpha \rho \times \text{diag} \{ Y_e, 0, 0, 1-2Y_e, 1-Y_e, 1-Y_e \} \\
\text{Spin Precession term} &= \left( \begin{array}{ccc} 0 & C, & 0 \\ -C & 0 & 0 \end{array} \right), \text{where } C = \left( \begin{array}{ccc} 0 & \mu_{e\mu} & \mu_{e\tau} \\ -\mu_{e\mu} & 0 & \mu_{\mu\tau} \\ -\mu_{e\tau} & -\mu_{\mu\tau} & 0 \end{array} \right) \times B_{\perp}
\end{align*} \tag{4}

with $M^2 = \text{diag} \{ 0, \Delta m^2_{21}, \Delta m^2_{31} \}$, $\alpha = \sqrt{2} G_F n_N$, $Y_e = \frac{n_e}{n_e + n_n}$ is the electronic fraction, the $\Delta m^2$'s are the difference between squared masses; $\mu_{e\mu}$ is the transition magnetic moment between the two flavored states $\nu_e$ and $\nu_\mu$; and $U$ is the mixing matrix given by \cite{18}

$$
\begin{pmatrix}
  c_{12}c_{13} & s_{12}c_{13} & s_{13} \\
 -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\
 s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13}
\end{pmatrix} \tag{5}
$$

\footnote{This is taken to be the transversal value of the magnetic field strength at the surface of the neutron star.}

\footnote{The resonance densities are approximately given by $\sim \frac{2\pi\Delta m^2 \cos \theta}{q^3}$, where $q$ is $Y_e$ for the MSW transitions, and $1-2Y_e$ for the RSFP ones. The values of $\Delta m^2$ and $\theta$ are taken from the solar and atmospheric neutrino data. $Y_e$ is generally set to be half, and therefore $1-2Y_e$ lies between $10^{-4}$ to $10^{-7}$ \cite{16}, then all transitions occur in a range extending from a few g cm$^{-3}$ to $10^7$ g cm$^{-3}$.}
where \( s_{ij} = \sin \theta_{ij}, c_{ij} = \cos \theta_{ij} \) for \( i,j = 1,2,3 \) \((i < j)\); and \( \theta_{ij} \) is the vacuum mixing angle between \( \nu_i \) and \( \nu_j \). The mixing matrix terms satisfy the unitarity relations: \( \sum_i |U_{ij}|^2 = \sum_l |U_{il}|^2 = 1. \) The equality in the second term in Eq. (1), however, is only true to leading order since radiative corrections induce tiny differences between the neutral current potentials of \( \nu_e, \nu_\mu \) and \( \nu_\tau \), and in particular, results in a very small \( \nu_\mu|\nu_\tau \) potential difference \( |V_{\mu\tau} - V_{\tau\mu}| \sim (10^{-5} \sim 10^{-4}) \times \alpha \rho \) \text{[19]}. Performing a rotation in the non-electronic subspace as \((\nu_\mu, \nu_\tau) \rightarrow (\nu_\mu', \nu_\tau')\) which diagonalizes the vacuum term and leaves the matter term invariant, while for the magnetic term, the two values \( \mu_{\nu_\mu} \) and \( \mu_{\nu_\tau} \) become nearly maximally mixed, and so whatever their values, the rotated ones will be of the same magnitude, and therefore we have \( \mu_{\nu_\mu'} \sim \mu_{\nu_\tau'} \sim \mu \). As for the third one \( \mu_{\nu_\tau} \) it remains invariant. Since \( \nu_\mu \) and \( \nu_\tau \) are both produced via neutral currents only and are indistinguishable as far as their detection, we take the non-electronic original fluxes to be equivalent, so we can then write \( F_\mu^0 = F_\tau^0 = F_\mu' = F_\tau' \).

Comparing at very high density, the values of the various Hamiltonian terms; we notice that for energies in the range of MeV, \( \alpha \rho \gg \frac{\Delta m^2_{21}}{2E}, \frac{\Delta m^2_{32}}{2E}. \) Since we took the term \( \mu \) to be between \text{[20]} \( 10^{-12} \mu_B \) and \( 10^{-10} \mu_B \), the same thing applies when we compare the matter term with the magnetic one with respect to the chosen range of \( \mu B_{\perp \circ} \) i.e. \( \alpha \rho \gg \mu B_{\perp \circ} \). Note that the chosen values of both \( \mu \) and \( B_{\perp \circ} \) don’t affect the SNP solution. We then deduce that at the SN core, the matter term is the dominant one, and therefore the Hamiltonian is approximately diagonal

\[
H \simeq \alpha \rho \times \text{diag} \{ 0, 0, 1 - 2Ye, 1 - Ye, 1 - Ye \} \tag{6}
\]

This means that the matter eigenstates coincide with the flavored states, and so, the neutrinos emerging from the SN core are as follow:

\begin{align*}
\text{1- Normal Mass Hierarchy} & \quad \nu_1^0 \sim \nu_\mu' \quad \bar{\nu}_1^0 \sim \bar{\nu}_e \\
& \quad \nu_2^0 \sim \nu_\tau' \quad \bar{\nu}_2^0 \sim \bar{\nu}_\mu' \\
& \quad \nu_3^0 \sim \nu_e \quad \bar{\nu}_3^0 \sim \bar{\nu}_\tau'
\end{align*}

\begin{align*}
\text{2- Inverted Mass Hierarchy} & \quad \nu_1^0 \sim \nu_\mu' \quad \bar{\nu}_1^0 \sim \bar{\nu}_\tau' \\
& \quad \nu_2^0 \sim \nu_e \quad \bar{\nu}_2^0 \sim \bar{\nu}_\mu' \\
& \quad \nu_3^0 \sim \nu_\tau' \quad \bar{\nu}_3^0 \sim \bar{\nu}_e
\end{align*}

3 The Neutrino Flavor Dynamics

In order to find the final spectrum, we should study all possible transitions between the neutrino species. We note that these transitions occur in the \textit{isotopically neutral region} \textsuperscript{4}, which consists mainly of layers of \( ^4\text{He}, ^{12}\text{C}, ^{16}\text{O}, ^{28}\text{Si} \) and \( ^{32}\text{S} \) whose nucleus have \( N \simeq Z \), so that the electronic fraction \( Y_e \) will be close to half. We also consider the following parameters values which are: first, when the SNP solution is the LMA scenario, the present parameters are \( \sin^2 2\theta_{12} = \sin^2 2\theta_{13} = 0.91 \) and \( \Delta m_{21}^2 = 6.9 \times 10^{-5} \text{eV}^2 \), while for the LOW scenario, we take \( \sin^2 2\theta_{12} = 0.92 \) and \( \Delta m_{21}^2 = 1.3 \times 10^{-7} \text{eV}^2 \). Secondly, for the atmospheric data, we have \( \sin^2 2\theta_{\text{atm}} = \sin^2 2\theta_{23} = 1 \) and \( \Delta m_{32}^2 = 2.7 \times 10^{-3} \text{eV}^2 \).

\textsuperscript{4}The region characterized by \( 1 - 2Ye = 10^{-4} \sim 10^{-2} \), is called \textit{the isotopically neutral region}. 
Thirdly, for the 1-3 mixing, we choose a value just below the experimental limit, which is given by CHOOZ [21], and satisfies the adiabaticity condition at the \( H \) resonance, which leads us to take \( \sin^2 \theta_{13} = 10^{-3} \).

**Neutrino Resonances**

The transition between possible neutrino states occur at some preferred regions, called resonance regions, which are characterized by the equality between two Hamiltonian’s diagonal elements. Thus the possible transitions are of two kinds: the \textit{MSW} kind occurring due to changing electronic density when both diagonal elements corresponds to two neutrino states or two anti-neutrino ones, and the \textit{RSFP} kind corresponding to resonant spin flavor in presence of a magnetic field [7] and involving a neutrino and a anti-neutrino state of different flavors. In general we expect 4 significant resonances, two of each kind namely: the known two \textit{MSW} ones which are called \( L \) and \( H \) transitions, and the corresponding \textit{RSFP} ones which we call \( \bar{L} \) and \( \bar{H} \) transitions; in addition to a fifth one of \textit{RSFP} kind, which we call \( A \), but which doesn’t affect the neutrino dynamics. Let us see how we can deduce these four resonances from general considerations on the full matrix. Starting from the 6X6 matrix with its six diagonal elements to be equated two by two, we have \( C_6^2 = 15 \) combinations. Out of these 15 combinations, 3 can be eliminated due to the Majorana character which doesn’t allow for the like-flavor spin flip, 2 can be further eliminated due to the smallness of the \( \nu_\mu-\nu_\tau \) potential difference and likewise for \( \bar{\nu}_\mu-\bar{\nu}_\tau \). For the remaining 10 cases, they constitute five pairs of conjugate transitions, for which there could only occur a transition from each pair at a time; since the resonance density computed by equating diagonal elements can be of both signs, while it’s conjugate transition will be necessarily have the inverse sign, and since physically only the case with positive value is relevant. This leaves us with five transitions, the fifth one being as mentioned previously not significant. We will study each of the \textit{MSW} and \textit{RSFP} kinds separately.

**\textit{MSW} Resonances**

In general, \textit{MSW} transition between two neutrinos (or anti-neutrinos) is studied around its resonance and the general form of the 2X2 submatrix is:

\[
\frac{1}{2E} \begin{pmatrix} c & b \\ b & Y_e \times z \end{pmatrix}
\]

where \( z = 2\alpha \rho E \), and the resonance corresponds to \( z_{\text{res}} = \frac{c}{Y_e} \). The transition will be adiabatic, \textit{i.e.} there is no jumping between eigenstates, for \( \gamma_{\text{res}} > 1 \), where \( \gamma_{\text{res}} \) is the adiabaticity parameter [22] at the resonance given in this case by:

\[
\gamma_{\text{res}} = \left( \frac{E_{na}}{E} \right)^\frac{2}{3}
\]

with

\[
E_{na} = 2.74 \times 10^9 \times Y_e^2 \times \left( \frac{|b|}{1\text{eV}} \right)^3 \times \left( \frac{|c|}{1\text{eV}} \right)^{-2} \text{MeV}
\]
The parameters \( b \) and \( c \) are given by:

| Mass hierarchy | SNP solution | \( b \) (L) | \( c \) (L) | \( b \) (H) | \( c \) (H) |
|----------------|--------------|------------|----------|------------|----------|
| Normal         | LMA          | 3.29 \times 10^{-5} | 1.81 \times 10^{-5} | 8.46 \times 10^{-5} | 2.67 \times 10^{-3} |
|                | LOW          | 6.31 \times 10^{-8} | -2.60 \times 10^{-6} | 8.53 \times 10^{-5} | 2.69 \times 10^{-3} |
| Inverted       | LMA          | -3.29 \times 10^{-5} | 2.35 \times 10^{-5} | -8.61 \times 10^{-5} | -2.72 \times 10^{-3} |
|                | LOW          | -6.13 \times 10^{-8} | 2.80 \times 10^{-6} | 8.53 \times 10^{-5} | -2.69 \times 10^{-3} |

\( b \) and \( c \) are given in eV\(^2\). The sign (-) for the \( c \) value means that the conjugate transition occurs instead of the usual one, for example in the case of inverted mass hierarchy within LMA solution, \( c(H) = -2.7 \times 10^{-3} \) eV\(^2\) means that \( \bar{\nu}_1 \leftrightarrow \bar{\nu}_3 \) occurs instead of \( \nu_1 \leftrightarrow \nu_3 \).^5

The values of \( E_{na} \) are given in MeV at each layer within all scenarios by:

\[
\text{Mass hierarchy} \quad \text{SNP solution} \quad L \quad H
\]

| Mass hierarchy | SNP solution | \( L \) | \( H \) |
|----------------|--------------|--------|--------|
| Normal         | LMA          | 2.11 \times 10^5 | 164.6 |
|                | LOW          | 7.20 \times 10^{-2} | 166.2 |
| Inverted       | LMA          | 1.25 \times 10^5 | 167.2 |
|                | LOW          | 5.69 \times 10^{-2} | 166.2 |

**RSFP Resonances**

RSFP transitions occurring between a neutrino state and an anti-neutrino one is also studied around its resonance and is of the form:

\[
\frac{1}{2E} \begin{pmatrix} c & s_z \\ s_z & d_z \end{pmatrix}
\]

but with \( s = \frac{\mu_B}{\alpha \rho} \simeq 1.9 \times 10^{-9} \times \left( \frac{\mu_B}{\rho} \right) \). Likewise, we find that the adiabaticity parameter is given as in Eq. (8), but with

\[
E_{na} = 1.89 \times 10^{-17} \times d^{-\frac{5}{2}} \times \left( \frac{\mu_B}{\rho} \right) \times \left( \frac{\mu_B}{\rho} \right)^3 \text{MeV}
\]

where \( d = 1-Y_e \) for both \( \bar{L} \) and \( \bar{H} \), \( d = 1-Y_e \) for \( A \). Note that \( E_{na} \) is proportional to \( (\mu_B)^3 \), which shows that the adiabaticity of the RSFP resonances depends strongly on the interaction term \( \mu_B \).

The values of \( c \) is given in eV\(^2\).

| Mass hierarchy | SNP solution | \( c \) (\( \bar{L} \)) | \( c \) (\( \bar{H} \)) | \( c \) (\( A \)) |
|----------------|--------------|----------------|---------------|----------------|
| Normal         | LMA          | 1.81 \times 10^{-5} | 2.67 \times 10^{-3} | 2.65 \times 10^{-3} |
|                | LOW          | -2.60 \times 10^{-6} | 2.69 \times 10^{-3} | 2.71 \times 10^{-3} |
| Inverted       | LMA          | 2.35 \times 10^{-5} | -2.72 \times 10^{-3} | -2.74 \times 10^{-3} |
|                | LOW          | 2.80 \times 10^{-6} | -2.69 \times 10^{-3} | -2.71 \times 10^{-3} |

^5Note that the \( L \) resonance transition occurs in the antineutrino channel for the case of normal mass hierarchy within the LOW scheme (For more details see Appendix A).
We give in the following table, the values of $E_{na}$ in MeV multiplied by $(\frac{\mu_{B_{\nu}}}{\mu_{B_{\tau}}})^{-3}$ for $\bar{L}$ and $\bar{H}$, and by $(\frac{\mu_{\nu B_{\nu}}}{\mu_{B_{\tau}}})^{-3}$ for $A$, at each layer within all scenarios:

| Mass hierarchy | SNP solution | $\bar{L}$ | $\bar{H}$ | $A$ |
|----------------|--------------|-----------|-----------|-----|
| Normal         | LMA          | $3.38 \times 10^{-11}$ | $4.99 \times 10^{-9}$ | $2.83 \times 10^{-19}$ |
|                | LOW          | $4.86 \times 10^{-12}$ | $5.02 \times 10^{-9}$ | $2.90 \times 10^{-19}$ |
| Inverted       | LMA          | $4.39 \times 10^{-11}$ | $5.08 \times 10^{-9}$ | $2.93 \times 10^{-19}$ |
|                | LOW          | $5.23 \times 10^{-12}$ | $5.02 \times 10^{-9}$ | $2.90 \times 10^{-19}$ |

Note that stating that the resonance corresponds to the equality of diagonal elements of Eq. (7) is not correct strictly speaking. The resonances, which corresponds to the minima of the difference between the Hamiltonian eigenvalues, match up exactly the diagonal elements when the off-diagonal elements are $r$–independent, which is the case for the MSW effect. On the other hand, the resonances will be shifted when the off-diagonal elements are $r$-dependent as in the case of the RSFP effect. In our case however where the $r$-dependency is of the form $\rho \propto B \sim r^{-3}$, this shift is negligible as we show in Appendix B.

The Adiabaticity at Neutrino Resonances

At the resonance, jumping probability is given by

$$P_f = \exp \left\{ -\frac{\pi}{2} \left[ \frac{E_{na}}{E} \right]^{2} \right\}$$

which depends on the energy $E$ as we see from Fig. 2.

Now the observable part of the supernova neutrino spectrum lies mainly between the energies 5 and 50 MeV and we will consider only energies in this range in our work. One can then divide the whole range of energy in three parts:

• Part I: $P_f \leq 0.1 \sim 0$, corresponding to $E_{na} \gtrsim 70$ MeV, and where pure adiabatic conversions occurs.
Part II: $70 \text{ MeV} \gtrsim E_{\text{na}} \gtrsim 0.2 \text{ MeV}$, for which $0.1 \leq P_f \leq 0.9$. In this range, $P_f$ increases strongly with the neutrino energy. The adiabaticity is partially broken.

Part III: $P_f \geq 0.9 \sim 1$, corresponding to $E_{\text{na}} \lesssim 0.2 \text{ MeV}$, where the flip probability is close to 1, and which leads to a strong violation of adiabaticity.

Then one can find easily that the transition at the $H$ resonance is completely adiabatic for all possible scenarios, as mentioned above. While, at the $L$ resonance, the transition is: completely adiabatic within $LMA$ scenario, and therefore $P_L \simeq 0$; and completely non-adiabatic within $LOW$ one and therefore $P_L \simeq 1$, for both normal and inverted hierarchies.

For $RSFP$ transitions, the adiabaticity depends on the value of $\mu B_{\perp o}$, and therefore we will divide the whole range of $\mu B_{\perp o}$ according to the adiabaticity at the $\bar{H}$ resonance, then the $\mu B_{\perp o}$ regions are:

1. Completely non-adiabatic if: $\mu B_{\perp o} < 340 \mu_B G$.
2. Partially broken if: $340 \mu_B G < \mu B_{\perp o} < 2400 \mu_B G$.
3. Purely adiabatic if: $\mu B_{\perp o} > 2400 \mu_B G$.

We have plotted in Fig. 3 the flip probability at the $\bar{H}$ layer as a function of the energy for different values of $\mu B_{\perp o}$. The curves from (1) to (6) correspond to $\frac{\mu B_{\perp o}}{\mu_B G} = 300, 500, 1000, 1500, 2000, 2500$ respectively.

The dependance of the flip probability at the layer $\bar{H}$, on the neutrino energy for different values of $\mu B_{\perp o}$.

At the $A$ resonance layer, the $E_{\text{na}}$ values are very small within the $\mu B_{\perp o}$ considered range, which corresponds to a complete non-adiabatic conversion, from which we deduce that the transition at the layer $A$ doesn’t affect the neutrino flavor dynamics. For the $\bar{L}$ layer, the adiabaticity depends on the $SNP$ solution and the specific mass hierarchy. Thus, if $\mu B_{\perp o}$ is larger than a given value $(\mu B_{\perp o})_2$, the transition will be purely adiabatic; if it is less than the other limiting value $(\mu B_{\perp o})_1$, the transition will be completely non-adiabatic; and finally when $\mu B_{\perp o}$ lies between these two values, the adiabaticity...
will be partially broken. We give in units of $\mu_B G$, these two limiting values for the various scenarios:

| Mass Hierarchy | SNP solution | $(\mu B_{1,0})_1$ | $(\mu B_{1,0})_2$ |
|----------------|--------------|------------------|------------------|
| **Normal**     | LMA          | 1800             | 12700            |
|                | LOW          | 3500             | 24300            |
| **Inverted**   | LMA          | 1700             | 11600            |
|                | LOW          | 3400             | 23700            |

Note that the $\mu B_{1,0}$ interval where the $\bar{L}$ ($\bar{H}$) adiabaticity is partially broken intersects with the similar one for the $\bar{H}$ ($\bar{L}$) resonance within the LMA solution, but it doesn’t within the LOW scheme.

Then the region II can be divided in two parts\(^6\): the first one corresponds to the completely non-adiabatic resonance at the $\bar{L}$ layer, i.e. $\mu B_{1,0}$ lies between $340 \mu_B G$ and $1800 \mu_B G$ ($1700 \mu_B G$ for the case inverted hierarchy); while for the remaining part from $1800 \mu_B G$ ($1700 \mu_B G$) to $2400 \mu_B G$, it corresponds to the partially broken adiabaticity at the layer $\bar{L}$.

For this purpose, we will give the level crossing for each one of different scenario, which are given in Fig’s 4, 5, 6 and 7. We will check later where the transition occur, namely whether is it in the neutrino or anti-neutrino channel (for MSW), and which one of the two RSFP conjugated transitions is involved.

The hierarchy of the densities of the resonance layers ($L$ with $H$, and therefore $\bar{L}$ with $\bar{H}$ too), leads to the factorization of the neutrino flavor dynamics, the transitions between two of resonance layers can be considered independently, and therefore each transition is reduced to a two neutrino problem\(^{[11]}\). Furthermore, the MSW-RSFP resonances regions non-overlapping condition, for $L-\bar{L}$ or $H-\bar{H}$ transitions, is given by\(^{[24]}\)

\[
L_\rho |\tan 2\theta_{\alpha\beta}| + \left| \frac{2\mu_{\alpha\beta} B_{1}(r_1)}{(\Delta m^2/2E)} \right| L_\rho \lesssim |r_2 - r_1| \tag{13}
\]

where $r_1$ is the radial position of the $\bar{L}$ ($\bar{H}$) resonance; and $r_2$ is that the $L$ ($H$) one, and $L_\rho \equiv \left| \frac{d(\ln \rho)}{dr} \right|^{-1}$, $\theta_{\alpha\beta}$ is the mixing angle between the two flavors $\alpha$ and $\beta$ ($\alpha, \beta = e, \mu, \tau$). This condition is satisfied for both $L-\bar{L}$ and $H-\bar{H}$.

**Neutrino Fluxes from Supernovae**

Having constructed the level crossing for the various scenarios, we can now extract the final $\nu$-flux in terms of $F_i$ (or $F_i^\tau$) in function of the original flux. We can then rewrite it in terms of the flavored $\nu$-flux using the identity:

\[
F_l = \sum_i |U_{li}|^2 F_i \quad \text{and} \quad F_l = \sum_i |U_{li}|^2 F_i^\tau \tag{14}
\]

where $l=e, \mu, \tau$; $i=1,2,3$; and $U_{li}$ are the mixing matrix elements. The final flux is thus given by the following general relation:

\[
\begin{align*}
F_i &= \sum_l a_{il} F_i^o + a_{il} F_i^o \\
F_i^\tau &= \sum_l a_{il} F_i^o + a_{il} F_i^o
\end{align*} \tag{15}
\]

\(^6\)We will denote them as II-a and II-b.
Fig 3: The level crossing diagram for the case of normal mass hierarchy within LMA scheme.

Fig 4: The level crossing diagram for the case of normal mass hierarchy within LOW scheme.
**Fig 5:** The level crossing diagram for the case of inverted mass hierarchy within LMA scheme.

**Fig 6:** The level crossing diagram for the case of inverted mass hierarchy within LOW scheme.
where \(a_{il}\) is the probability that a \(\nu_i\), which is produced inside SN core, leaves the supernova as a vacuum eigenstate \(\nu_i\). Inserting Eq. (14) in Eq. (13), we find for the flavored neutrino flux:

\[
\begin{pmatrix}
F_e \\
F_\bar{e} \\
4F_x
\end{pmatrix} = \begin{pmatrix}
p_{ee} & p_{e\bar{e}} & 1 - p_{ee} - p_{e\bar{e}} \\
p_{e\bar{e}} & p_{\bar{e}\bar{e}} & 1 - p_{ee} - p_{e\bar{e}} \\
1 - p_{ee} - p_{e\bar{e}} & 1 - p_{ee} - p_{e\bar{e}} & 2 + p_{ee} + p_{e\bar{e}} + p_{\bar{e}\bar{e}}
\end{pmatrix} \begin{pmatrix}
F_\nu^o \\
F_{\bar{e}}^o \\
F_x^o
\end{pmatrix}
\]

where \(p_{ee}\) is the survival probability of \(\nu_e\), i.e., the probability that a \(\nu_e\) doesn’t change during the core collapse; \(p_{e\bar{e}}\) the probability that a \(\nu_e\) leaves SN as a \(\bar{\nu}_e\); the index \((\nu)\) refers to the original fluxes. It is those various matrix elements which characterize the probability of SN neutrino conversion. Those probabilities satisfy the condition:

\[
\sum_\alpha p_{\alpha\beta} = \sum_\beta p_{\alpha\beta} = 1
\]

which are functions of the probabilities \(a_{il}\)s and \(U_{ei}\)s, and \(\alpha, \beta = e, \bar{e}, x\).

Case of Normal Mass Hierarchy

For the **LMA scheme**: We find from the level crossing for the case of the normal mass hierarchy (Fig. 4), that the final flux is given by

\[
F_1 = (1 - p_L)p_H F_\bar{e}^o + (1 - (1 - p_L)p_H) F_\nu^o, F_2 = (1 - p_H) F_\bar{e}^o + p_H F_\nu^o \\
F_3 = F_\nu^o, F_1 = p_H p_L F_\nu^o + (1 - p_H p_L) F_x^o, F_2 = F_\nu^o, F_3 = F_x^o
\]

then the SN \(\nu\)-conversion probabilities are given by

\[
p_{ee} \simeq 10^{-3}, p_{e\bar{e}} \simeq 0.65 \times (1 - p_L) p_H + 0.35 (1 - p_H) \\
p_{\bar{e}\bar{e}} \simeq 0, p_{\bar{e}\bar{e}} \simeq 0.65 \times p_L p_H
\]

For the **LOW scheme**: According to Fig. 5, the final flux with the LOW scheme is given by

\[
F_1 = p_H F_\nu^o + (1 - p_H) F_\bar{e}^o, F_2 = F_\nu^o, F_3 = p_H F_\nu^o + (1 - p_L) F_x^o \\
F_1 = p_H F_\nu^o + (1 - p_H) F_\bar{e}^o, F_2 = (1 - p_L) F_\nu^o + p_H F_x^o, F_3 = F_x^o
\]

and therefore the SN \(\nu\)-conversion probabilities are

\[
p_{ee} \simeq 10^{-3} \times p_L, p_{e\bar{e}} \simeq 0.64 \times (1 - p_H) \\
p_{\bar{e}\bar{e}} \simeq 0.36 \times (1 - p_L), p_{\bar{e}\bar{e}} \simeq 0.64 \times p_H
\]

Case of Inverted Mass Hierarchy

For the **LMA scheme**: The final flux is given in this case (See Fig. 6) by:

\[
F_1 = p_H F_\nu^o + (1 - p_H) F_\bar{e}^o, F_2 = F_\nu^o, F_3 = p_L F_\nu^o + (1 - p_L) F_x^o \\
F_1 = p_H F_\nu^o + (1 - p_L) F_\bar{e}^o, F_2 = (1 - p_H) F_\nu^o + p_H F_x^o, F_3 = F_x^o
\]

the SN \(\nu\)-conversion probabilities are given by

\[
p_{ee} \simeq 10^{-3} \times p_L, p_{e\bar{e}} \simeq 0.65 \times (1 - p_H) \\
p_{\bar{e}\bar{e}} \simeq 0.65 \times (1 - p_L), p_{\bar{e}\bar{e}} \simeq 0.35 \times p_H
\]
For the \textit{LOW} scheme: We find for the final flux from Fig. 7:

\begin{equation}
F_1 = p_L F_{x}^o + (1 - p_L) F_{x}^o, \quad F_2 = F_{x}^o, \quad F_3 = (1 - p_H) F_{x}^o + p_H F_{x}^o
\end{equation}

and the \textit{SN} \nu-conversion probabilities are given by:

\begin{align}
    p_{\nu e} &\simeq 0.64 \times p_L, \quad p_{\bar{\nu}e} \simeq 10^{-3} \times (1 - p_H) \\
p_{\bar{\nu}e} &\simeq 0.64 \times (1 - p_L), \quad p_{\nu \bar{\nu}} \simeq 0.36 \times p_H
\end{align}

Note that any significant value for \(p_{\bar{\nu}e}\) or \(p_{\nu \bar{\nu}}\), is a strong signature of the spin flavor precession effect on the \textit{SN} neutrino dynamics.

4 Neutrino Conversion Effects on the Mass Spectrum

Let us discuss the signatures of the various scenarios as far as the emerging neutrino signal, as well as their discriminative power at the future neutrino detectors.

For future supernova neutrino burst, the present detectors can gives, for a typical supernova, at Super-Kamiokande [2] about 5000 \(\nu_e\) events, and a few hundred events can be detected in both SNO [3], LVD [25] and MACRO [4]. In this paper, we are interested on the features of the final neutrino spectra that are required for the identification of the neutrino mass spectra. The effects of neutrino mixing and magnetic moment on the final neutrino spectra can be observed through: a) the partial or complete disappearance/appearance of the \(\nu_e\) neutronization peak, b) the appearance of soft, hard or composite spectra of \(\nu_e\) and \(\bar{\nu}_e\); and c) the Earth matter effects on both \(\nu_e\) and \(\bar{\nu}_e\) spectra. Let us estimate these effects on the observed spectra at the Earth detectors.

\textbf{a) Neutronization Peak:} It comes at the first stage of core collapse and corresponds to a and the observed signal during the of the neutrino burst first few milliseconds duration. In the absence of neutrino conversion, the dominant signal are \(\nu_e\)s, are produced by the electron capture on protons and nuclei while the shock wave passes through the neutrinosphere [14]. Since the original flux is made of \(\nu_e\), the final observed fluxes give a direct measurement of the extent of conversion of \(\nu_e\) into the other neutrino species. It is thus clear that the neutronization peak is proportional to the \(\nu_e\)'s survival probability.

\textbf{b) The Nature of the Final Spectra:} due to the difference between the interactions strengths of the various neutrino species with matter, their average energies differ and are given by [26]:

\begin{equation}
\langle E_{\nu_e}^o \rangle = 10 \sim 12 \text{ MeV}, \quad \langle E_{\bar{\nu}_e}^o \rangle = 14 \sim 17 \text{ MeV}, \quad \langle E_{\nu_x}^o \rangle = 24 \sim 27 \text{ MeV}
\end{equation}

\(E\) means the original average energy \(i.e.\) in the absence \(\nu\)-conversion. Thus finding for example \(\langle E_{\nu_e} \rangle > \langle E_{\bar{\nu}_e} \rangle\), would be is a signature of neutrino conversion, since it implies that the contribution of the converted original hard \(\nu_x\) spectrum to the final \(\nu_e\) flux is significantly larger than its contribution to the final \(\bar{\nu}_e\) flux. On the other hand there is another effect which has to be accounted for: the fact that the neutrino interaction cross section increases with energy, the neutrino spectra from the
cooling stage won’t be exactly thermal, but will get pinched. We can account easily for this pinching effect by parametrizing the original spectra as \[ \text{\cite{26, 27}}: \]

\[ F_i^0(E) \propto \frac{E^2}{1 + \exp(E/T_i - \eta_i)} \]  

(27)

where \(T_i\) and \(\eta_i\) are given by

\[ T_e \approx 3 \sim 4 \text{ MeV}, \ T_x \approx 5 \sim 6 \text{ MeV}, \ T_x \approx 7 \sim 9 \text{ MeV} \]

\[ \eta_e \approx 3 \sim 5, \ \eta_e \approx 2 \sim 2.5, \ \eta_x \approx 0 \sim 2 \]

(28)

The final \(\nu_e (\bar{\nu}_e)\) spectrum can be qualitatively divided into three types:

1) The original “soft” spectrum of the \(\nu_e (\bar{\nu}_e)\) (corresponding to the survival probability \(p_{ee} = 1\) \((P_{\bar{e}\bar{e}} = 1))\).

2) The “hard” spectrum of \(\nu_x\) (corresponding to the survival probability \(p_{ee} = p_{\bar{e}\bar{e}} = 0\) \((P_{ee} = P_{\bar{e}\bar{e}} = 0))\) which would be the case when there is a complete interchange of spectra, \(i.e.\) \(\nu_e \leftrightarrow \nu_x\) \((\bar{\nu}_e \leftrightarrow \bar{\nu}_x)\); and not \(\bar{\nu}_e \leftrightarrow \nu_e\).

3) The “composite” spectrum, which is a mixture of the original soft and the hard spectra is comparable in proportions. There are other cases which are difficult to distinguish like the case where the \(\nu_e (\bar{\nu}_e)\) spectrum contains only \(\nu_e\) and \(\nu_x\), \(\bar{\nu}_e\) and \(\nu_x\), or all species. In order to distinguish among them, we denote the spectrum containing \(\nu_e\) and \(\nu_x\) by \(\text{compo-1}\), the one containing \(\bar{\nu}_e\) and \(\nu_x\) by \(\text{compo-2}\); and the one with all the species by \(\text{compo-3}\). In some cases, the appearance of one of these case is a strong signature of the spin flavor precession effect, namely when the \(\nu_e\)-spectrum contains \(\bar{\nu}_e\) and vice-versa.

c) the Earth Matter Effect:

The SN neutrinos, in order to reach the detector, have to go through various amount of the Earth material. This amount depends on the direction of the SN with respect to the Earth as well as the time of the day. This effect which could modify significantly the flavor composition of the flux may be sought by comparing the signal of a future SN from detectors at different geographical locations. Certain features of the energy spectra could also reveal this effect even from the observations at one detector.

Neutrinos are expected to be arriving at the surface of the Earth as mass eigenstates (vacuum eigenstates); where they oscillate in the Earth matter, \(i.e.\) they lose their coherence. The possibility of the Earth matter effect’s observation depends strongly on the differences between the \(a_{il}\) and also the differences between the \(P_{ei}\) parameters (for more details, see Appendix C).

Let us now summarize the results within all previous scenarios in the following table:

## 5 Conclusion

We attempted in this paper to check the sensibility of the supernovae final flux of neutrinos to both the magnetic field and matter effect assuming three active neutrinos. We have taken the value of \(|U_{e3}|^2\)
to be just below the experimental upper bound given by CHOOZ [21], which makes the transition at the $H$ layer completely adiabatic. We then divided the whole range of values considered for $\mu B_{\perp o}$ into three regions $I$, $II$, and $III$ according to the adiabaticity at the $\bar{H}$ layer. We could then make some predictions on the conversion effects for supernova neutrinos. The predictions differ for the different schemes, which opens up the possibility of discriminating between them using data from future neutrino bursts. We studied the possibility of observing the conversion effects through: (i) The partial or complete change of the flavor of the neutronization peak, (ii) The appearance of a hard or composite $\nu_e$ and/or $\bar{\nu}_e$ spectra due to the conversion effect instead of their original soft spectra. (iii) the Earth matter effect from the conversion in the Earth material. We found that indeed neutrino conversion does change significantly the spectrum shape. Let us now summarize the salient features of the final flux as it appears from the last table:

1. The appearance of a hard $\bar{\nu}_e$ makes the mass hierarchy to be normal.
2. A hard $\nu_e$ spectrum makes $\mu B_{\perp o}$ to be in the first region, while a hard $\bar{\nu}_e$ spectrum exclude the first region.
3. The absence of the Earth matter effect in the $\nu_e$ channel implies that $\mu B_{\perp o}$ to be in the first region; while the absence in $\bar{\nu}_e$ one implies that it must be in the third region.
4. If $\bar{\nu}_e$ contains all spieces (i.e. $\nu_e$, $\bar{\nu}_e$ and $\nu_x$), the SNP solution must be LMA, the mass hierarchy is inverted and $\mu B_{\perp o}$ lies between $1700 \mu B_G$ and $2400 \mu B_G$.
5. If both spectra of $\nu_e$ and $\bar{\nu}_e$ contain only $\nu_e$ and $\nu_x$, the mass hierarchy will be inverted, the SNP

| Region of $\mu B_{\perp o}$ | SNP solution | Neutro-Peak | Spectrum of $\nu_e$ | $\bar{\nu}_e$ | Earth effects |
|-----------------------------|--------------|-------------|--------------------|---------------|---------------|
| $I$                         | LMA          | $\nu_x$     | hard               | compo-2       | $\times$      |
|                             | LOW          | $\nu_x$     | hard               | compo-2       | $\times$      |
|                             |              | $\nu_x$     | hard               | compo-2       | $\times$      |
|                             |              | $\nu_e, \nu_x$ | compo-1        | compo-2       | $\checkmark$ |
| $II$                        | -a LMA       | $\bar{\nu}_e, \nu_x$ | compo-2 | hard | $\checkmark$ |
|                             | -b LMA       | $\bar{\nu}_e, \nu_x$ | compo-2 | compo-2 | $\checkmark$ |
|                             | LOW          | $\bar{\nu}_e, \nu_x$ | compo-2 | compo-3 | $\checkmark$ |
|                             |              | $\bar{\nu}_e, \nu_x$ | compo-2 | compo-2 | $\checkmark$ |
|                             |              | $\bar{\nu}_e, \nu_x$ | compo-1 | compo-2 | $\checkmark$ |
| $III$                       | LMA          | $\bar{\nu}_e, \nu_x$ | compo-2 | hard | $\checkmark$ |
|                             | LOW          | $\bar{\nu}_e, \nu_x$ | compo-2 | compo-2 | $\checkmark$ |
|                             |              | $\bar{\nu}_e, \nu_x$ | compo-1 | compo-2 | $\checkmark$ |

Table 1: The dependance of the final spectra on: 1) the SNP solution, 2) the mass hierarchy and 3) the range of $\mu B_{\perp o}$. “soft” in the column refers to the original $\nu_e$ ($\bar{\nu}_e$) spectrum and “hard” refers to the original $\nu_x$ spectrum. In the Earth matter effect column, $\checkmark$ and $\times$ indicate the possibility of significant effects.
solution will be LOW, and $\mu B_{\perp o}$ will be larger than $2400 \mu B G$.

Notice that some possible observations can rule out all of the various scenarios above, like the appearance of a soft $\nu_e$ (or $\bar{\nu}_e$) signal, which would exclude all the above scenarios, since then the value of $|U_{e3}|^2$ would be less than $10^{-3}$.

The final neutrino spectra can thus help in resolving three main kinds of ambiguities that remain to be resolved with the current data: (i) the solution of the SNP, (ii) the type of mass hierarchy (sign of $\Delta m^2_{32}$), and (iii) probe the magnitude of the $\mu B_{\perp o}$ value, assuming $|U_{e3}|^2$ to be $10^{-3}$. The implications of the results of this work will depend on when will the next neutrino burst from a Galactic supernova be detected. On the other hand, there is a good chance that within the next few years the present, ongoing, or future planned experiments will allow us to identify the specific solution of the solar neutrino problem considered in this work. This will significantly diminish the number of possible schemes and will allow us to further sharpen the predictions of the effects for supernova neutrinos.

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A In which Channel does the $L$ Resonance Occur?

Taking the Hamiltonian in the $(\nu_e, \nu_\alpha)$ basis:

$$\frac{1}{2E} \begin{pmatrix} W & K \\ K & Qz \end{pmatrix}$$

where $\nu_\alpha$ is any combination of the non-electronic neutrinos, $K$ and $W$ are given in function of the various $\Delta m^2$'s; and $Q$ is a function of $\theta_{ij}$'s and $Y_e$. We know that the resonance occurs at $z_{res} = \frac{W}{Q}$; if $z_{res} < 0$, then the transition occurs between $(\bar{\nu}_e, \bar{\nu}_\alpha)$ instead of $(\nu_e, \nu_\alpha)$. In general $Q$ is always positive, thus we should check the sign of $W$.

$W$ represents the eigenvalues of the Hamiltonian submatrix, which corresponds to the non-electronic states $(\nu_\mu, \nu_\tau)$. (see. Sec II); In order to probe the sign of these two values in the general case, we take a $2 \times 2$ symmetric matrix, similar to the mass matrix in the subspace $(\nu_\mu, \nu_\tau)$, as follow

$$\begin{pmatrix} \varepsilon & \lambda \\ \lambda & \sigma \end{pmatrix}$$

where $\varepsilon$, $\lambda$ and $\sigma$ are reels; it’s eigenvalues are: $\frac{1}{2}(\varepsilon + \sigma \pm \sqrt{(\varepsilon - \sigma)^2 + 4\lambda^2})$; then one can see that the two eigenvalues have:

- the same sign if $|\varepsilon \sigma| > \lambda^2$.
- different signs if $|\varepsilon \sigma| < \lambda^2$.

The small value, in absolute value, leads the resonance at the $L$ layer, while the largest one leads to the resonance at the $H$ layer. Thus, we are led to look at the sign of the smallest value, replacing $\varepsilon$, $\lambda$ and $\sigma$ by their values in our case, and neglecting terms multiplied by $\frac{\Delta m^2_{31}}{\Delta m^2_{32}}$, then one can find the
condition that the resonances at both $L$ and $H$ layers, occurs in the same channel, it can be written as

$$
\sin^2 \theta_{13} \lesssim \frac{\Delta m^2_{32}}{\Delta m^2_{12}} \cos 2\theta
$$  \hspace{1cm} (31)

this condition is satisfied in the case of inverted mass hierarchy with both LMA and LOW scenarios. But for the case of normal mass hierarchy, it is satisfied only within LMA scheme.

B The Adiabaticity of Neutrino Conversion

For neutrinos traveling through a non-constant density medium, it is proved that the adiabaticity is always satisfied except for the cases around the resonance layer, where it must be studied more thoroughly. The resonance, in general, can be characterized by the matter density value $\rho$, and therefore the distance $r$ from the SN center, when the difference between the two Hamiltonian eigenvalues, $\Delta H_{eff} = \left| \frac{M^2_2}{2E} - \frac{M^2_1}{2E} \right|$, is minimal, i.e.

$$
\frac{\partial}{\partial f} (\Delta H_{eff})_{res} = 0
$$  \hspace{1cm} (32)

where $f$ is either the density $\rho$ or the travelled distance $r$. Considering the 2-$\nu$ scheme (Like $\nu_e, \nu_\mu$) case for the MSW effect, $H$ can be written as:

$$
H = \frac{1}{2E} U \left( \begin{array}{cc}
m^2_1 & 0 \\
0 & m^2_2
\end{array} \right) U^\dagger + \frac{\alpha}{2} \rho \left( \begin{array}{ccc}
3Y_e - 1 & 0 \\
0 & Y_e - 1
\end{array} \right)
$$  \hspace{1cm} (33)

and the mixing matrix $U$, is given by

$$
U = \left( \begin{array}{cc}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{array} \right)
$$  \hspace{1cm} (34)

Neglecting the terms proportional to the unity matrix, we find

$$
H = U \left( \begin{array}{cc}
0 & 0 \\
0 & \Delta
\end{array} \right) U^\dagger + \alpha \rho \left( \begin{array}{cc}
1 & 0 \\
0 & 0
\end{array} \right) = \left( \begin{array}{cc}
\alpha \rho + \Delta \sin^2 \theta & \frac{\Delta}{2} \sin 2\theta \\
\frac{\Delta}{2} \sin 2\theta & \Delta \cos^2 \theta
\end{array} \right)
$$  \hspace{1cm} (35)

where $\Delta = \frac{m^2_2 - m^2_1}{2E}$. Then the difference between the two eigenvalues is:

$$
\sqrt{(\alpha \rho - \Delta \cos 2\theta)^2 + \Delta^2 \sin^2 2\theta}
$$  \hspace{1cm} (36)

which takes its minimum at

$$
\rho = \frac{\Delta}{\alpha} \cos 2\theta
$$  \hspace{1cm} (37)

This corresponds to the equality of the diagonal elements.

If we try to redo the same operation but writing this time $H$ in the vacuum state basis ($\nu_1, \nu_2$), we find

$$
H = U^\dagger H U = \left( \begin{array}{cc}
0 & 0 \\
0 & \Delta
\end{array} \right) + \alpha \rho \left( \begin{array}{cc}
\cos^2 \theta & \frac{1}{2} \sin 2\theta \\
\frac{1}{2} \sin 2\theta & \sin^2 \theta
\end{array} \right)
$$  \hspace{1cm} (38)
The difference between the eigenvalues takes its minimum at the same value of \( \rho \), given in Eq. (37), because they are different representations of the same operator \( H \), which doesn’t correspond to the diagonal elements equality

\[ \rho = \frac{\Delta}{\alpha \cos 2\theta} \]  

(39)

We will obtain the same result if we write \( H \) in any other basis, i.e., the minimum of \( \Delta H_{\text{eff}} \) doesn’t correspond the diagonal elements equality.

One can deduce that the minimum of \( \Delta H_{\text{eff}} \) corresponds to the diagonal elements equality only when the off-diagonal elements are constant, i.e. \( \rho \)-independent which is realized only in the flavored basis. There are some cases, like the one in the presence of magnetic moment interaction, where the flavored basis is not the best basis to deduce the resonance density directly, i.e. by equating the diagonal elements.

Let us consider our case, from Eq. (10), the minimum of \( \Delta H_{\text{eff}} \) is given by

\[ \frac{1}{2E} \sqrt{(b - cz)^2 + 4s^2z^2} \]  

(40)

which corresponds to

\[ z_{\text{res}} = \frac{bc}{c^2 + 4s^2} \]  

(41)

but we take it \( \frac{b}{c} \); since \( s \ll c \), then one can write

\[ z_{\text{res}} \simeq \frac{b}{c} \left( 1 - 7.46 \times 10^{-14} \times \left( \frac{\mu B_{\perp}}{\mu_B \alpha} \right)^2 \right) \]  

(42)

thus the layer radius \( r_{\text{res}} \) is corrected by \( \Delta r_{\text{res}} \), where \( \frac{\Delta r_{\text{res}}}{r_{\text{res}}} = -4.21 \times 10^{-5} \times \left( \frac{\mu B_{\perp}}{\mu_B \alpha} \right)^{\frac{3}{2}} \), then the adiabaticity parameter will be corrected by the factor

\[ \xi = 1 - 1.24 \times 10^{-13} \times \left( \frac{\mu B_{\perp}}{\mu_B \alpha} \right)^2 \]  

(43)

and the jumping probability becomes

\[ P_{f}^{\text{(corrected)}} = [P_{f}]^{\xi} \]  

(44)

suppose that \( P_{f} = 0.4 \), in order to be corrected by 0.1, i.e. \( P_{f}^{\text{(corrected)}} = 0.5 \), we need that the quantity \( \mu B_{\perp} \) to be larger than \( 1.4 \times 10^{6} \mu_B G \); which is a huge value non-available in the supernovae. We conclude that this correction doesn’t affect the neutrino conversion in our work, and we have thus neglected it.

C  The Earth Matter Effect

If neutrinos reach the Earth detectors without interacting with matter, the signal at the detectors is given by Eq. (16). Since they reach detectors through the Earth matter, the flux given by Eq. (16) will be modified in general.

\footnote{We are here closely following the treatment of [11], adopting it to our case by taking into account the possibility of the \( \nu - \bar{\nu} \) transition. This effect is studied with more details for difference scenarios in [28].}
Let $P_{ie}$ ($P_{\bar{e}i}$) be the probability that a vacuum mass eigenstate $\nu_i$ ($\bar{\nu}_i$) entering the Earth reaches the detector as a $\nu_e$ ($\bar{\nu}_e$). The flux of $\nu_e$ ($\bar{\nu}_e$) at the detector is

$$F^{D}_e = \sum_i P_{ie} F_i, \quad F^{D}_{\bar{e}} = \sum_i P_{\bar{e}i} F_i$$

(45)

Inserting $F_i$, we get

$$F^{D}_e = F^0_e \sum_i P_{ie} a_{ie} + F^0_\bar{e} \sum_i P_{\bar{e}i} a_{\bar{e}i} + F_x^0 \sum_i P_{ie} a_{ie} + F_x^0 \sum_i P_{\bar{e}i} a_{\bar{e}i} (1 - a_{ie} - a_{\bar{e}i})$$

$$F^{D}_{\bar{e}} = F^0_\bar{e} \sum_i P_{\bar{e}i} a_{\bar{e}i} + F^0_e \sum_i P_{ie} a_{ie} + F_x^0 \sum_i P_{ie} a_{ie} + F_x^0 \sum_i P_{\bar{e}i} a_{\bar{e}i} (1 - a_{ie} - a_{\bar{e}i})$$

(46)

where the $a$’s are defined in Sec. 3. Then one can write

$$p^{D}_{ee} = \sum_i P_{ie} a_{ie}, \quad p^{D}_{\bar{e}e} = \sum_i P_{\bar{e}i} a_{\bar{e}i}, \quad p^{D}_{e\bar{e}} = \sum_i P_{ie} a_{\bar{e}i}, \quad p^{D}_{\bar{e}\bar{e}} = \sum_i P_{\bar{e}i} a_{ie}$$

(47)

Similarly we have

$$F_e = p_{ee} F^0_e + p_{\bar{e}e} F^0_{\bar{e}} + (1 - p_{ee} - p_{\bar{e}e}) F'_x$$

$$F_{\bar{e}} = p_{e\bar{e}} F^0_e + p_{\bar{e}\bar{e}} F^0_{\bar{e}} + (1 - p_{e\bar{e}} - p_{\bar{e}\bar{e}}) F'_x$$

(48)

which enables us to write:

$$F^{D}_e = P^{D}_{ee} F^0_e + P^{D}_{e\bar{e}} F^0_{\bar{e}} + (1 - P^{D}_{ee} - P^{D}_{e\bar{e}}) F'_x$$

$$F^{D}_{\bar{e}} = P^{D}_{\bar{e}e} F^0_e + P^{D}_{\bar{e}\bar{e}} F^0_{\bar{e}} + (1 - P^{D}_{\bar{e}e} - P^{D}_{\bar{e}\bar{e}}) F'_x$$

(49)

From those expressions we deduce:

$$F^{D}_e - F^0_e = (F^0_e - F^0_{\bar{e}}) (P^{D}_{ee} - p_{ee}) + (F^0_{\bar{e}} - F^0_e) (P^{D}_{e\bar{e}} - p_{e\bar{e}}) + (F'_x - F^0_e) (P^{D}_{ee} - p_{ee})$$

$$F^{D}_{\bar{e}} - F^0_\bar{e} = (F^0_\bar{e} - F^0_e) (P^{D}_{e\bar{e}} - p_{e\bar{e}}) + (F^0_e - F^0_\bar{e}) (P^{D}_{\bar{e}e} - p_{\bar{e}e})$$

(50)

so the Earth matter effect can be quantified by the various differences $(P^{D}_{ee} - p_{ee})$, $(P^{D}_{e\bar{e}} - p_{e\bar{e}})$, $(P^{D}_{\bar{e}e} - p_{\bar{e}e})$ and $(P^{D}_{\bar{e}\bar{e}} - p_{\bar{e}\bar{e}})$, which equal;

$$P^{D}_{ee} - p_{ee} = \sum a_{ie} (P_{ie} - |U_{ei}|^2), \quad P^{D}_{e\bar{e}} - p_{e\bar{e}} = \sum a_{\bar{e}i} (P_{\bar{e}i} - |U_{ei}|^2)$$

$$P^{D}_{\bar{e}e} - p_{\bar{e}e} = \sum a_{ie} (P_{\bar{e}i} - |U_{ei}|^2), \quad P^{D}_{\bar{e}\bar{e}} - p_{\bar{e}\bar{e}} = \sum a_{\bar{e}i} (P_{\bar{e}i} - |U_{ei}|^2)$$

(51)

One can finally write $P^{D}_{ee} - p_{ee}$ as

$$P^{D}_{ee} - p_{ee} = a_{1e} (P_{e1} - |U_{ei}|^2) + a_{2e} (P_{e2} - |U_{e2}|^2) + a_{3e} (P_{e3} - |U_{e3}|^2)$$

(52)

We obtain similar expressions for the other difference terms. We notice that the last term in Eq. 52 is negligible due to the very small depth oscillation of $\nu_3$ inside the Earth.

$$P_{3e} - |U_{e3}|^2 \leq 10^{-3}$$

(53)

Taking into account the relations $\sum |U_{ei}|^2 = 1$ and $\sum_i P_{ie} = \sum P_{i\bar{e}} = 1$, one can write

$$P^{D}_{ee} - p_{ee} = (a_{2e} - a_{1e}) (P_{e2} - |U_{e2}|^2), \quad P^{D}_{e\bar{e}} - p_{e\bar{e}} = (a_{2\bar{e}} - a_{1\bar{e}}) (P_{e2} - |U_{e2}|^2)$$

$$P^{D}_{\bar{e}e} - p_{\bar{e}e} = (a_{2e} - a_{1e}) (P_{e2} - |U_{e2}|^2), \quad P^{D}_{\bar{e}\bar{e}} - p_{\bar{e}\bar{e}} = (a_{2\bar{e}} - a_{1\bar{e}}) (P_{e2} - |U_{e2}|^2)$$

(54)
finally, the differences $F_{e}^{D} - F_{e}^{o}$ and $F_{\bar{e}}^{D} - F_{\bar{e}}^{o}$ can be written as

$$
F_{e}^{D} - F_{e}^{o} \approx [(F_{o}^{e} - F_{o}^{x}) (a_{2e} - a_{1e}) + (F_{e}^{o} - F_{x}^{o}) (a_{2\bar{e}} - a_{1\bar{e}})] (P_{2e} - |U_{e2}|^{2})
$$

$$
F_{\bar{e}}^{D} - F_{\bar{e}}^{o} \approx [(F_{o}^{e} - F_{o}^{x}) (a_{2e} - a_{1e}) + (F_{e}^{o} - F_{x}^{o}) (a_{2\bar{e}} - a_{1\bar{e}})] (P_{2\bar{e}} - |U_{e2}|^{2})
$$

In general, when the signal from two detectors $D_1$ and $D_2$ are compared, we get the following flux differences:

$$
F_{e}^{D_{1}} - F_{e}^{D_{2}} \approx [(F_{o}^{e} - F_{o}^{x}) (a_{2e} - a_{1e}) + (F_{e}^{o} - F_{x}^{o}) (a_{2\bar{e}} - a_{1\bar{e}})] (P_{2e}^{(1)} - P_{2e}^{(2)})
$$

$$
F_{\bar{e}}^{D_{1}} - F_{\bar{e}}^{D_{2}} \approx [(F_{o}^{e} - F_{o}^{x}) (a_{2e} - a_{1e}) + (F_{e}^{o} - F_{x}^{o}) (a_{2\bar{e}} - a_{1\bar{e}})] (P_{2\bar{e}}^{(1)} - P_{2\bar{e}}^{(2)})
$$

According to this relation, the difference $F_{e}^{D_{1}} - F_{e}^{D_{2}}$ ($F_{\bar{e}}^{D_{1}} - F_{\bar{e}}^{D_{2}}$) is factorized: it is proportional to the difference of the Earth oscillation probability $P_{2e}$ ($P_{2\bar{e}}$) at the two detectors and a function of supernovae oscillation probabilities $a$'s and the difference in the original fluxes of $\nu_e$ ($\bar{\nu}_e$) and $\nu_x$.

Let us consider these factors separately:

1) $P_{2e}^{(1)} - P_{2e}^{(2)}$ and $P_{2\bar{e}}^{(1)} - P_{2\bar{e}}^{(2)}$: if the neutrino trajectory go through only the Earth’s mantle, one can use a constant density approximation, which gives [11]

$$
P_{2e}^{(1)} - P_{2e}^{(2)} = \sin 2\theta_{12} \sin 2(\theta_{12} + \theta_{13}) \sin^{2}\left(\frac{\pi d_{1}}{L_{m}}\right) - \sin^{2}\left(\frac{\pi d_{2}}{L_{m}}\right)
$$

$$
P_{2\bar{e}}^{(1)} - P_{2\bar{e}}^{(2)} = \sin 2\theta_{12} \sin 2(\theta_{12} + \theta_{13}) \sin^{2}\left(\frac{\pi d_{1}}{L_{m}}\right) - \sin^{2}\left(\frac{\pi d_{2}}{L_{m}}\right)
$$

here $\theta_{m}$ and $L_{m}$ are the mixing angle and the oscillation length inside the Earth respectively, and $d_i$ is the distance travelled by neutrinos inside the Earth before reaching the detector $D_i$.

2) The second factors is a summing of two-factor terms. Let us try to see how they behave:

* $(F_{o}^{e} - F_{x}^{o})$ and $(F_{o}^{\bar{e}} - F_{x}^{\bar{e}})$: Since the $\nu_e$ ($\bar{\nu}_e$) spectrum is softer than the $\nu_x$ spectrum, and the luminosities of both the spectra are similar in magnitude [23], the term $(F_{o}^{e} - F_{x}^{o})$, (and therefore $(F_{o}^{\bar{e}} - F_{x}^{\bar{e}})$), is positive at LOW energies and becomes negative at higher energies where the $\nu_x$ flux overwhelms the $\nu_e$ ($\bar{\nu}_e$) flux. Therefore, the Earth effect has a different sign for LOW and high energies, and there exists a critical energy $E_c$ ($\bar{E}_c$), such that $F_{e}^{o}(E_c) = F_{x}^{o}(E_c)$, $(F_{\bar{e}}^{o}(\bar{E}_c) = F_{x}^{o}(\bar{E}_c))$, where this change of sign takes place. Since the cross section of the neutrino interactions increases with energy, the Earth effect is expected to be more significant at higher energies (if all the other factors are only weakly sensitive to the neutrino energy).

* $(a_{2e} - a_{1e})$, $(a_{2\bar{e}} - a_{1\bar{e}})$ or $(a_{2e} - a_{1\bar{e}})$: They represent the differences between the $a_{it}$ probabilities and can be extracted from Eq. [19], Eq. [21], Eq. [22] and Eq. [23]. They are seen to be dependent on the mass hierarchy as well as the specific solar neutrino solution (LMA or LOW). Non vanishing values for this difference make the observation of the Earth matter effect possible.

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