An Algorithm Combining the Branch-Cut Method and Rhombus Phase Unwrapping Algorithm

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Abstract. In order to solve the problem that the wrapped phase cannot be unwrapped effectively, this paper proposes an algorithm combining the branch-cut method and rhombus phase unwrapping algorithm for unwrap phase. The algorithm combines the advantages of the two methods, at first, the Goldstein’s branch-cut method is used to find the residual points in the wrapping phase, then the rhombus phase unwrapping algorithm is used to unwrap the non-residual area phase, finally the phase of the residual area is obtained by using the cubic spline interpolation method, so as to ensure that the phase of each pixel can be unwrapped smoothly. The simulation results verify the feasibility and effectiveness of the algorithm, the combined phase unwrapping algorithm can effectively reduce the line and island phenomenon.

1. Introduction

With the development and improvement of modern industry, optics and computer graphics, people pay great attention to the 3D contour measurement. Phase unwrapping is a key step in Phase Measuring Profilometry, and its unwrapped accuracy rate will directly affect the results of the 3D measurement[¹]. The branch-cut method and the rhombus phase unwrapping algorithm are the most commonly used for the spatial phase unwrapping tasks. In the branch-cut method, the residual points are connected by establishing branch cut lines, and branch cut lines can be skipped when the phase is unwrapping, so as to achieve the purpose of preventing the cumulative of errors. However, there are many ways to connect the branch cut lines, it is difficult to determine an optimal connection mode, and the branch cut lines are easy to close themselves, so that the phase of this area cannot be unwrapped, thus island phenomenon occurs[²]. Zhang et al. made improvements to the branch-cut method. Although the improved algorithm can reduce the island phenomenon, it required twice search, which resulted in a large amount of calculation and reduced efficiency[³]. The rhombus phase unwrapping algorithm is simple and efficient, but in the actual unwrapping process, due to the influence of noise and shadow, the local error is easy to spread along the unwrapping path, resulting in the phenomenon of line and island. Li et al. improved the rhombus phase unwrapping algorithm,
which reduced the influence of phase graph boundary and invalid points on the phase unwrapped results, but the improved algorithm is only applicable to the objects with simple surface[4].

Therefore, in order to solve the problem that the wrapped phase cannot be unwrapped effectively, this paper proposes an algorithm combining the branch-cut method and rhombus phase unwrapping algorithm for unwrap phase. The algorithm combines the advantages of the two algorithms, and Goldstein's branch-cut method is used to find the residual points in the wrapping phase, then the rhombus phase unwrapping algorithm is used to unwrap the non-residual area phase, for a small number of residual point areas, finally the cubic spline interpolation method is used to obtain the true phase, so as to ensure that the phase of each pixel can be unwrapped smoothly, and the island and line phenomenon can be reduced effectively without affecting the operation efficiency.

2. Phase Measurement Profilometry

The principle of Phase Measurement Profilometry is project multiple grating stripes with sinusoidal modulation on the horizontal reference plane and the object to be measured[5]. Due to the change of the horizontal reference plane height and the object to be measured height, the phase of each pixels offset to different degrees, and the offset phase value contains the height information of the object. figure 1 shows the 5 steps of Phase Measurement Profilometry.

For step three, the four-step phase shift method is usually used to extract the wrapped phase information from the raster stripes captured by camera, and optical grating is moved by the Digital Light Projector accurately, so that the phase field of optical grating image is shifted[6]. The phase shifted of $\alpha_i$ is 0, $\pi/2$, $\pi$, and $3\pi/2$, and the deformed optical grating image can be expressed as:

$$I_i(m,n) = I_0(m,n) + r(m,n)\cos[\phi(m,n)+\alpha_i], \quad i = 1, 2, 3, 4$$

(1)

In equation (1), $I_i$ represent the gray value of these four phase shifted images, $I_0$ is the value of the stripe background image, $r(m,n)$ is the emphasis function, $\phi(m,n)$ is the required phase field, and the main phase value can be obtained by mathematical operation, the result is shown in equation (2).

$$\phi(m,n) = \arctan \left( \frac{I_4 - I_2}{I_1 - I_3} \right)$$

(2)

It can be seen from equation (2) that the wrapped phase value is obtained through the arctangent function, but the inherent properties of the arctangent function make the obtained phase value discontinuous. Therefore, in order to calculate the height of the measured object, unwrapping phase technique must be used to restore the discontinuous wrapped phase to the continuous true phase.

3. Phase unwrapping algorithm

3.1. Rhombus phase unwrapping algorithm

Henri et al. proposed a rhombus phase unwrapping algorithm based on path tracking. The basic idea is to select a phase point in the wrapping phase diagram and using it as the starting point for unwrap phase firstly, and then taking the starting point as the center point, the four points phase are added or subtracted in sequence according to the order of top, left, bottom and right, so as to complete the first rhombus unwrapping path[7]. The pixels in the four adjacent areas which have completed the unwrap phase are taken as the new starting point for unwrapping, and a new round of phase unwrapping
process is carried out with each center pixels, until all the pixels have completed the phase unwrapping process.

3.2. Branch-cut method
The residual points are connected by the branch-cut method, and phase unwrapping can only be along the branch tangent, but not through. The algorithm can be roughly divided into three major steps.

(1) Identifying residual points and marking the positive and negative polarities.

(2) Selecting a residual point from the identified residual points, and searching for another unbalanced residual point in the neighborhood of the residual point and connect it[8]. If the branch cut line is balanced or it has reached the boundary of the phase diagram, marking all residual points on the branch cut lines as equilibrium state, and then repeating the above process from another unbalanced residual point until all residual points have been marked as equilibrium state.

(3) A phase point outside the branch tangent is selected as the starting point, and the phase of non-branch phase points are unwrapped with the starting point as the center, until all non-branch phases have been unwrapped, and finally unwrapping the branch cut phase point. The algorithm has high efficiency, but in the area of dense residual points, the branch tangents are easy to close themselves, which affects recover 3D surface[9]. Figure 2 shows the connection of the branch cut lines, and figure 3 shows the 3D surface recovered by the branch-cut method.

![Figure 2](image1.png)  
**Figure 2.** The connection of the branch cut lines

![Figure 3](image2.png)  
**Figure 3.** The recovered 3D surface

3.3. The combination of two algorithms
In the real measurement environment, due to the effect of noise, under-sampling and height jump of the measured object, individual pixels are disconnected, the phase cannot be unwrapped by branch-cut method and rhombus phase unwrapping algorithm effectively. In order to ensure that the phase of each pixel can be unwrapped smoothly within the full field, this article combines the branch-cut method and rhombus phase unwrapping algorithm, then using the cubic spline interpolation method to obtain the parts of real phase, so as to ensure that the phase of each pixel can be unwrapped smoothly, the improved algorithm flow is shown in figure 4.

![Figure 4](image3.png)  
**Figure 4.** The workflow of combined algorithm.
Firstly, the concept of residual in the branch-cut method is used to identify the residual in the wrapped phase diagram. The judgment equation of the residual point is:

\[
R(m, n) = \left[\phi(m,n+1) - \phi(m,n)\right] + \left[\phi(m+1,n+1) - \phi(m+1,n)\right] + \left[\phi(m+1,n+1) - \phi(m,n+1)\right] + \left[\phi(m,n) - \phi(m+1,n)\right]
\]

If the result is zero, then the point is a residual point, otherwise, it is a non-residual point. Then the wrapped phase map is divided into residual area and non-residual area by marking the location of the residual point. Then the rhombus phase unwrapping algorithm is used to unwrap the non-residual area phase, finally according to the principle of phase continuity, the phase of the residual area is obtained by using the cubic spline interpolation method, the function of spline interpolation \( S_j(x) \) should meet the following two conditions:

1. \( S_j(x) \) is polynomial in degree no more than \( m \) on each subinterval \( [x_i, x_{i+1}] \) \((i = 0, 1, 2, \ldots, n-1)\).
2. \( S_j(x) \) has \( m-1 \) continuous derivatives in interval \([a,b] \).

\( S_j(x) \) is called the cubic spline interpolation function in row \( j \) when \( m = 3 \), which can be expressed as:

\[
S_j(x) = S_{j,0}(x) = a_jx^3 + b_jx^2 + c_jx + d_j, x \in \left(x_{i-1}, x_i\right) \ , \ i = 0, 2, \ldots, n-1
\]

In equation (4), \( a_j, b_j, c_j, d_j \) are parameter to be determined and should meet the interpolation condition. According to the definition of cubic spline interpolation function, the four undetermined parameters must be solved separately on each interval \( [x_i, x_{i+1}] \). It can be known from the interpolation method that there are \( n \) intervals in \( S_j(x) \), so there are \( 4n \) unknowns to be solved. Since \( S_j(x) \) on interval \([a,b] \) is continuous with the second derivative, the following continuity conditions should be met:

\[
\begin{align*}
S_j(x_i - 0) &= S_j(x_i + 0) \\
S'_j(x_i - 0) &= S'_j(x_i + 0), \quad (i = 1, 2, \ldots, n-1) \\
S''_j(x_i - 0) &= S''_j(x_i + 0)
\end{align*}
\]

Since \( S_j(x) \) satisfies the above interpolation condition, there are \( 4n-2 \) conditions in total. In addition, there are also two boundary conditions that need to be taken into account in order to determine the function \( S_j(x) \), this paper uses natural boundary conditions.

\[
S''_0(x_0) = S''_{n-1}(x_n) = 0
\]

Finally, the cubic spline interpolation function on the row \( j \) can be obtained combined with natural boundary conditions.

4. Simulation results and analysis

In order to verify the effectiveness of combination algorithm, the algorithm is analyzed and discussed through simulation experiments. Figure 5(a) shows the three-dimensional object generated by Malab simulation, figure 5(b) shows the unwrapped phase of the object, and the functional relationship is:

\[
f(x,y) = 20\exp\left(-\left(x^2 + y^2\right)/4\right) + 2x + y
\]

Noise and invalid regions are added to the wrapped phase in order to simulate the noise interference of the real environment, figure 5(c) shows the wrapped phase with added noise, and figure 5(d) shows the wrapped phase with invalid region.
Figure 5. (a) Simulate 3D objects by Matlab. (b) Unwrapped phase diagram (c) Wrapped phase with noise (d) Wrapped phase with invalid region

The combined algorithm firstly identifies the residual in the wrapped phase diagram of figure 5(c), and the result is shown in figure 6(a). The wrapped phases of figure 5(c) is unwrapped by the rhombus phase unwrapping algorithm and the combined algorithm respectively. It can be seen from figure 6(b), due to the influence of noise, individual pixels are disconnected and no longer continuous, and the correct unwrapped phase value cannot be obtained according to the rhombus phase unwrapping algorithm, finally the line phenomenon occurs. Figure 6(c) shows the phase unwrapped result of the combined algorithm, it can be seen that the lines are reduced significantly by comparing figure 6(b).

Figure 6. (a) Residue map with noise (b) The phase unwrapped result of rhombus phase unwrapping algorithm (c) The phase unwrapped result of combined algorithm

Then this study verifies the effect of the combined algorithm on the island. Similarly, the combined algorithm firstly identifies the residual in the wrapped phase diagram of figure 5(d), and the result is shown in figure 7(a). The wrapped phases of figure 5(d) is unwrapped by branch-cut method and the combined algorithm respectively. Due to the dense residual points, some of the branch cut lines are self closing when the branch cut method unwrap the phase, so that the phase of this area cannot be unwrapped, thus island phenomenon occurs. Figure 7(c) shows the phase unwrapped result of the
combined algorithm, it can be seen that the island phenomenon is reduced significantly by comparing figure 7(b).

![Figure 7](image)

(a) Residue map with invalid region (b) The phase unwrapped result of branch-cut method (c) The phase unwrapped result of combined algorithm

5. Conclusions
Due to the influence of the environment and the height jump of the surface, it is easy to generate lines and islands by using branch-cut method and rhombus phase unwrapping algorithm. In order to solve this problem, this paper proposes a phase unwrapping algorithm combines these two algorithms. The combined algorithm divides the wrapped phase map into two parts, the rhombus phase unwrapping algorithm is used to unwrap the non-residual point areas, for a few residual point areas, the cubic spline interpolation method is used to obtain the real phase, so as to ensure that each pixel point can be unwrapped smoothly. Through Matlab simulation, the experimental results show that the combined algorithm can obtain reliable unwrapped phase result without affecting the operating efficiency, which effectively reduces the line and island phenomenon.

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