saprEMo: a simplified algorithm for predicting detections of electromagnetic transients in surveys

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ABSTRACT

The multi-wavelength detection of GW170817 has inaugurated multi-messenger astronomy. The next step consists in interpreting observations coming from population of gravitational wave sources. We introduce saprEMo, a tool aimed at predicting the number of electromagnetic signals characterised by a specific light curve and spectrum, expected in a particular sky survey. By looking at past surveys, saprEMo allows us to constrain models of electromagnetic emission or event rates. Applying saprEMo to proposed astronomical missions/observing campaigns provides a perspective on their scientific impact and tests the effect of adopting different observational strategies. For our first case study, we adopt a model of spindown-powered X-ray emission predicted for a binary neutron star merger producing a long-lived neutron star. We apply saprEMo on data collected by XMM-Newton and Chandra and during \(10^4\) s of observations with the mission concept THESEUS. We demonstrate that our emission model and binary neutron star merger rate imply the presence of some signals in the XMM-Newton catalogs. We also show that the new class of X-ray transients found by Bauer et al. in the Chandra Deep Field-South is marginally consistent with the expected rate. Finally, by studying the mission concept THESEUS, we demonstrate the substantial impact of a much larger field of view in searches of X-ray transients.

Key words: EM follow-up models – BNS mergers – rates – short gamma-ray bursts

1 INTRODUCTION

GW170817 (Abbott et al. 2017a) has just opened the era of multi-messenger astronomy (Abbott et al. 2017c). The first coincident set of gravitational-waves and electromagnetic observations has already provided an extraordinary insight into the physics of the binary neutron star mergers. Among the key results of this revolutionary discovery is the confirmation of the association between the merger of two neutron stars (NSs) and (at least some) short gamma ray bursts (SGRBs) (Abbott et al. 2017c and refs. therein). The last radio VLBI observations demonstrate that a narrow jet was formed and prove the association with a classical SGRB (see Troja et al. 2017; Mooley et al. 2018b; Margutti et al. 2018; Dâ˘A¸zAvanzo et al. 2018; Lyman et al. 2018; Dobie et al. 2018; Mooley et al. 2018a; Ghirlanda et al. 2018).

The intense multi-wavelength follow-ups of gamma ray bursts in the last decade have revealed new and unexpected features, such as early and late X-ray flares, extended emission, and X-ray plateaus (e.g., Berger 2014 and refs. therein). The challenges posed by this rich astronomical scenario motivated a growing interest of the community in investigating compact binary mergers from both the theoretical and observational points of view. Intensified theoretical efforts have been dedicated to explain these observations and coherently explore these and other possible electromagnetic signals generated by these sources.

In order to validate the variety of proposed theoretical scenarios in the context of multi-messenger astronomy with compact binary mergers, we present saprEMo.

We developed saprEMo, a Simplified Algorithm for PRedicting ElectroMagnetic Observations, to evaluate how many electromagnetic (EM) signals, characterised by a specific light curve and spectrum, should be present in a data set given some overall char-
acteristics of the astronomical survey and a cosmological rate of compact binary mergers. Predictions can be used both to validate the theoretical scenarios against already collected data and to critically examine the scientific means of future missions and their observational strategies. While we use compact binary mergers as the prime multi-messenger targets, sapremo can also be applied to other type of transients (e.g. core-collapse supernovae).

We describe the main features of sapremo in Section 2. As first case study, we use sapremo to investigate the X-ray emission from Binary NS (BNS) mergers leading to the formation of a long-lived and strongly magnetized NS, following the model of Siegel & Ciolfi 2016a,b (see Section 3.1). We apply sapremo to present X-ray surveys, collected by XMM-Newton and Chandra, and study the prospects of the mission concept THESEUS. Results are reported in Section 3.3 and discussed in Section 4. Finally, in Section 5 we draw our conclusions summarising our first results and outlining the main features and primary scopes of sapremo.

2 SAPREMO OUTLINE

sapremo is a Python algorithm designed to predict how many detectable electromagnetic signals EMe are present in a survey S. The code and more information will be publicly available at the link https://sapremo.github.io.

According to instrument limitations (such field of view and spectral sensitivity), sapremo estimates the number of signals whose emission flux F at the observer is above the flux limit FLim of the S survey. Accounting for the energy dependency of the survey sensitivity, we define detections on an instantaneous flux-based criterion: \( 3g \cdot t' \cdot F_s(t') > \text{FLim} \), where \( g \) labels the spectral band of the survey. We therefore simplify our analysis by treating detectability in each band independently, i.e., a source is considered to be detected if and only if it can be independently detected in at least one instrument band. sapremo does not directly consider the actual sky locations observed by the survey \( S \) (even when applied to archival data) and instead focuses on accounting for cosmological distances, relying on the isotropy of the Universe.

2.1 Core analysis

sapremo can be applied to any type of EM emission, from transients to continuous sources emitting in any EM spectral range. In this work, we focus on X-ray transients associated with neutron star binaries. The expected number of BNS mergers \( N_{BNS} \) in the volume enclosed within redshift \( z_{\text{max}} \), in a time \( T \) at the observer, is:

\[
N_{\text{BNS}} = T \int_{0}^{z_{\text{max}}} \frac{R_{V}(z)}{1+z} \frac{dV_{c}}{dz} dz
\]

where \( R_{V}(z) \) is the rate of BNS mergers per unit comoving volume, per unit source time. In our case \( z_{\text{max}} \) is the maximum distant at which the emission following the model of interest EMe can be detected. \( z_{\text{max}} \) is calculated considering both the spectral shift due to the redshifted rest of the instrument energy band \( E' = \left[ E_{\text{min}}, E_{\text{max}} \right] \) and the maximum luminosity distance, set by the peak luminosity \( L_{p}(E) \) of the EMe model and the sensitivity \( F_{\text{Lim}} \) of the survey. We only expect a fraction of \( N_{BNS} \) to be observed by a specific instrument, depending on the emission properties and the characteristics of the survey. The number of BNS mergers, detectable by the survey \( S \) such that the peak of the considered emission \( EMe \) falls within the observing time, is given by:

\[
N_{p} = \frac{\text{FoV}}{4\pi} \int_{0}^{z_{\text{max}}} \frac{R_{V}(z)}{1+z} \frac{dV_{c}}{dz} dz
\]

The total observing time \( T \) can be expressed in terms of the survey \( S \) as \( T = (T_{\text{obs}}) n_{\text{obs}}, \) where \( n_{\text{obs}} \) is the number of the observations and \( (T_{\text{obs}}) \) is the average exposure time for observation. In eq. (2), the field of view FoV of the instrument reduces the number \( N_{BNS} \) of signals present in the volume enclosed within \( z_{\text{max}} \) by FoV/4\pi. In the specific case of BNS mergers, the efficiency factor \( e \) typically includes the occurrence rate of a specific merger remnant (\( \epsilon_{\text{ns}} \)), which are expected to generate the emission \( EMe \), and source geometry/observational restrictions such as collimation (\( \epsilon_{c} = 1 - \cos(\theta) \), where \( \theta \) is the beaming angle), so that \( e \sim \epsilon_{\text{ns}} \cdot \epsilon_{c} \).

We define the signals included in \( N_{p} \) as \( \text{Peaks} \). This contribution only depends on the emission model by the intensity of the light curve peaks in the energy bands of the survey. The Hubble parameter \( H_{0} \) is used as an upper limit.

There is also a contribution, which we call \( \text{Tails} \), from the mergers whose emission is detected only before (first block of eq. (3)) or after (second block of eq. (3)) the horizon peaks (i.e., \( L_{p} \) is outside the observation period). The longer the light curve is above \( F_{\text{Lim}} \), the higher the probability of it being observed. To estimate this contribution, we need to account for the evolution in time of the emission luminosity \( L(t') \), which affects the horizon of the survey:

\[
N_{t} = e n_{\text{obs}} \frac{\text{FoV}}{4\pi} \int_{0}^{t_{\text{max}}} \int_{0}^{\theta_{L}(t')} \frac{R_{V}(z)}{1+z} \frac{dV_{c}}{dz} dz dt + \int_{0}^{t_{\text{max}}} \int_{0}^{\theta_{L}(t')} \frac{R_{V}(z)}{1+z} \frac{dV_{c}}{dz} dz dt
\]

where \( t' = t/(1+z) \) are the time respectively in observer and source frames and \( t_{\text{max}} \) is the time corresponding to the peak luminosity \( L(t') \) represents the horizon of the survey, given the specific intrinsic luminosity of the source \( L(t') \). The integration time of \( (3) \) is practically limited by the duration of the emission above the flux limit. At the moment, sapremo does not correctly account for the possibility of detecting multiple times the same event. Multiple detections of the same source might occur if the survey contains repeated observations of the same sky locations and the time interval between the different exposures is shorter than the considered emission \( EMe \). While \( N_{t} \) would be unaffected, in these cases \( N_{t} \) would overestimate the expected number of events by these additional detections. Under these specific conditions, \( N_{t} \) should then be considered as an upper limit.

The relative importance of peaks and tails depends on the ratio between the duration of the observable emission and the typical exposure time. The trade off between these two contributions, as well as their different origin, can be understood with figures 1 and 2, where we report an unphysical scenario for illustrative purposes. Figure 2 shows that doubling the signal duration has no effects on the peaks, while it doubles the expected detections in the tail contribution. Given a fixed event rate, \( N_{t} \) depends on the observing time, while \( N_{p} \) depends on the duration of the events.

While eq. (2) and (3) explain the general concept behind the tool, they do not explicitly account for the energy (or wavelength \( E = h\nu/c\lambda \) dependence of light curves, instrument sensitivities and absorption. These effects are particularly important when explor-
Figure 1. Schematic representation of a peak ($t_p \in (T_{obs})$) signal (a) and tails (b, c). The solid curves represent the part of signals EM$_e$ visible during the exposure time at the observer, the dashed components are the missed (because of time or flux restrictions) part of the emissions. The upper dotted line shows the peak flux $F_p = L_p/(4\pi D^2)$, the lower line the flux limit of the survey.

Figure 2. Example of peaks and tails: we report the ideal case of perfectly peaked tails (top panel) and 2 s (bottom panel). $x$- and $y$-axes represent respectively the 10 s observation time and the emission flux $F$ above the survey sensitivity $F_{lim}$, in arbitrary units. The graph shows the peak contribution, i.e. the events whose luminosity peak falls in the observational window (green stars) and the tail contribution, i.e. the events detected only before or after the peak of luminosity (fuchsia lines). In both top and bottom cases, we detect 10 peaks, while the number of tails changes from 1 to 2 by doubling the duration of the detectable emission. The graph shows that, given a fixed event rate, the expected number of peaks only depends on the observing time, while the expected number of tails only depends on the duration of the events.

2.2 Inputs and Outputs

We now present inputs and outputs of saprEMo.

Inputs:

(i) light curves (LC), in terms of luminosity rest frame, of the EM emission EM$_e$ in different energy bins (including energies higher than the instrument band, if predicted from the model, which might be redshifted into the band after accounting for cosmology corrections;

(ii) astrophysical rate in the source frame $R_s$ (z);

(iii) efficiency $\varepsilon$ of the EM model EM$_e$ ($\varepsilon$ should account for source geometry/selection effects, such as collimation, as well as the frequency with which this type of astrophysical source generates the electromagnetic emission EM$_e$);

(iv) main instrument and survey properties:

• for each spectral band $\nu$ of the survey $S$, minimum and maximum energy included $[E_\nu, E_\nu^\prime]$;
• corresponding flux limits $F_{lim}$;
• average exposure time ($T_{obs}$);
• field of view FoV$^1$;
• number of observations $n_{obs}^1$.

Outputs:

• $N_p$ and $N_t$: numbers of peaks and tails which are expected in the survey $S$. The numbers of signals returned by saprEMo should be interpreted as the expectation value of a Poisson process. Therefore Poisson statistical errors should be considered in addition to the systematics due to rate and emission model uncertainties;

• $dN_p/dz$ and $dN_t/dz$: distributions of tail and peak numbers as a function of redshift;

• $dN_p/d\log(D)$ and $dN_t/d\log(D)$: expected distribution of signal observed durations, obtained by convolving the approximate distribution of the survey exposure times $P_{obs}$ with the light curve span LCS observable at each step in redshift. For each redshift, the LCS represents the total time of the light curve which is above the flux limits at the observer frame (for more details see appendix A3). To estimate the distribution of the signal durations, we analytically approximate the expected exposure time distribution $P_{obs}$ from the average exposure time ($T_{obs}$) (and standard deviation, when available) with a Maxwell-Boltzmann or Log-normal function, according to the user input. For each data point saved from the cosmic integrations, we simulate $N_{trials}$, for both peaks and tails) observation durations $T_{obs}$ and define for each of them the starting time $t_i$. The starting time is uniformly drawn from a time interval including both the exposure time of the specific trial $T_{obs}$ and the observable emission at observer $(t'_f - t'_i)(1 + z)$ (where $t'_i$ and $t'_f$ are respectively the last and first LC time at source satisfying our detection criteria at redshift $z$). If $t_i$ is drawn in the interval $I_p = [t_p - T_{obs}, t_p]$, where $t_p$ is the time at observer correspondent to the luminosity peak, it contributes to the peak distribution, otherwise it adds up to the tail distribution. For each simulated observation the total duration is then calculated summing only the contribution of light curve intervals whose flux is above the limit;

• $dN_p/d\log(F)$ and $dN_t/d\log(F)$: distributions of peak and tail detection numbers as a function of maximum flux. At each step in redshift, necessary to compute the integral (2), saprEMo also calculates the associated flux. The fluxes are obtained by summing the contribution of each energy and rescaling with the associated luminosity distance. From the same observations simulated for estimating the duration distributions, we obtain the expected distribution of maximum fluxes.

Distributions in redshift are useful to estimate the horizon of the survey to the emission EM$_e$ and for astrophysical interpretation. They provide a prior on the redshift distribution when a counterpart allowing $z$ measurement is missing, or constrain cosmic rate evolution of BNS mergers when multi-wavelength observations yield the source distance. Distributions of fluxes and durations are robust observables, which enable comparisons with real data$^2$.

1 Or equivalently covered sky-area $f_{sky} = n_{obs}^1 F_{lim}^\alpha$.

2 A direct comparison with the event flux distributions often reported in survey catalogs require to take into account the data analysis procedure.
3 APPLICATION TO SOFT X-RAY EMISSION FROM LONG-LIVED BINARY NEUTRON STAR MERGER REMNANTS

In the following, we consider a specific application of saprEMo to the case of spindown-powered X-ray transients from long-lived NS remnants of BNS mergers.

Depending on the involved masses and the NS equation of state (EOS), a BNS merger can either produce a short-lived remnant, collapsing to a black hole (BH) within a fraction of a second, or a long-lived massive NS. The latter can survive for much longer spindown timescales (up to minutes, hours or more) prior to collapse or even be stable forever. After the discovery of NSs with a mass of $\sim 2 M_\odot$ (Demorest et al. 2010; Antoniadis et al. 2013), different authors converged to the idea that the fraction of BNS mergers leading to a long-lived NS remnants should range from a few percent up to more than half (e.g., Piro et al. 2017). Information extracted from the multimessenger observations of the BNS merger event GW170817 did not change this view, although more stringent constraints on the NS EOS were obtained from the GW signal (Abbott et al. 2018), from various indications excluding the prompt collapse to a BH, and from the kilonova brightness and the relatively high mass of the merger ejecta (e.g., Margalit & Metzger 2017; Bauswein et al. 2017; Radice et al. 2018; Rezzolla et al. 2018).

An additional supporting element in favour of long-lived remnants is given by the observation of long-lasting (minutes to hours) X-ray transients following a significant fraction of SGRBs (e.g., Rowlinson et al. 2013; Gompertz et al. 2014; Lü et al. 2015). Given the short accretion timescale of a remnant disk onto the central BH ($\leq 1$ s), such long-lasting emission represents a challenge for the canonical BH-disk scenario of SGRBs while it can be easily explained by alternative scenarios involving a long-lived NS central engine, e.g., the magnetar (Zhang & Meszáros 2001; Metzger et al. 2008) and the time-reversal (Ciolfi & Siegel 2015; Ciolfi 2018) scenarios. According to this view, the fraction of SGRBs accompanied by long-lasting X-ray transients might reflect the fraction of BNS mergers producing a long-lived NS.

If the remnant is a long-lived NS, its spindown-powered electromagnetic emission represents an additional energy reservoir that can potentially result in a detectable transient. Recent studies taking into account the reprocessing of this radiation across the baryon-polluted environment surrounding the merger site have shown that the resulting signal should peak at wavelengths between optical and soft X-rays, with luminosities in the range $10^{43}$ – $10^{48}$ erg/s and time scales of minutes to days (e.g., Yu et al. 2013; Metzger & Piro 2014; Siegel & Ciolfi 2016a,b). Besides representing an explanation for the long-lasting X-ray transients accompanying SGRBs, this spindown-powered emission is a promising counterpart to BNS mergers (e.g., Stratta et al. 2017 and refs. therein), having the advantage of being both very luminous and nearly isotropic.

For our first direct application of saprEMo, we consider the spindown-powered transient model by Siegel & Ciolfi (2016a,b, hereafter SC16), described in the next Section 3.1, in which the emission is expected to peak in the soft X-ray band. This model cannot be excluded or constrained by GW170817. The first X-ray observations in the 2 – 10 keV band were performed by MAXY (Sugita et al. 2018) 4.6 hours after the merger with a sensitivity of $8.6 \times 10^{-9}$ erg s$^{-1}$ cm$^{-2}$, well above the flux that the model predicts at that time after the merger.

In Section 3.2, we briefly describe the model of the BNS merger rate adopted for this first study. Then, in Section 3.3 we present our results referring to three different X-ray satellites: XMM-Newton, Chandra, and the proposed THESEUS. We discuss these results in section 4.

3.1 Reference emission model

The model proposed by Siegel & Ciolfi (SC16) describes the evolution of the environment surrounding a long-lived NS formed as the result of a BNS merger. The spindown radiation from the NS injects energy into the system and interacts with the optically thick baryon-loaded wind ejected isotropically in the early post-merger phase, rapidly forming a baryon-free high-pressure cavity or “nebula” (with properties analogous to a pulsar wind nebula) surrounded by a spherical shell of “ejecta” heated and accelerated by the incoming radiation. As long as the ejecta remain optically thick, the non-thermal radiation from the nebula is reprocessed and thermalised before eventually escaping. As soon as the ejecta become optically thin, a signal rebrightening is expected, accompanied by a transition from dominantly thermal to non-thermal spectrum. The model can also take into account the collapse of the NS to a BH at any time during the spindown phase.3

Exploring a wide range of physical parameters, Siegel & Ciolfi found that the escaping spindown-powered signal has a delayed onset of $\sim 10 - 100$ s and in most cases peaks $\sim 100 - 10^3$ s after merger. Furthermore, the emission typically falls inside the soft X-ray band (peaking at $\sim 0.1 - 1$ keV) and the peak luminosity is in the range $10^{46}$ – $10^{48}$ erg s$^{-1}$. In this work, we consider only one representative case, corresponding to the “fiducial case” of SC16 (SC16f). The light curve and spectral distribution of this particular model are shown in figure 3. The main parameters of the model

\footnote{We refer to Siegel & Ciolfi 2016a,b and Ciolfi 2016 for a detailed discussion of the model and its current limitations.}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{light_curve.png}
\caption{Light curve of the spindown-powered emission from a long-lived BNS merger remnant according to the model proposed by Siegel & Ciolfi 2016a,b (corresponding to their “fiducial” case; see text). The solid curves represent the contributions of different energy bands to the total light curve (dashed line).}
\end{figure}
are as follows. The early baryon-loaded wind ejects mass isotropically at an initial rate of \( 5 \times 10^{-3} \, M_\odot \, s^{-1} \), decreasing in time with a timescale of 0.5 s. The dipolar magnetic field strength at the poles of the NS is \( 10^{13} \) G and the initial rotational energy of the NS is \( 5 \times 10^{52} \) erg (\( \sim \) ms initial spin period). Moreover, in this case the remnant evolves without collapsing to a BH. In figure 3 we can distinguish two important transitions. The first, around \( 10 \) s, marks the end of the early baryon wind phase and the beginning of the spindown phase. The second, at several times \( 10^3 \) s, corresponds to the time when the ejecta become optically thin. While the emission described by the above model is essentially isotropic, allowing us to set \( \varepsilon_{\nu} \sim 1 \), only a fraction of BNS mergers is expected to generate a long-lived neutron star. The value of this fraction mainly depends on the unknown NS EOS and distribution of component masses. Here, we assume for simplicity a one-to-one correspondence between the fraction \( \varepsilon_{\text{LLNS}} \) and the fraction of SGRBs accompanied by a long-lasting X-ray transient (i.e. extended emission and/or X-ray plateau). Following the analysis presented in Rowlinson et al. 2013, we set \( \varepsilon_{\text{LLNS}} \) to 50%.

Once we assign \( \varepsilon_{\nu} = \varepsilon_{\text{LLNS}} \), the resulting total efficiency of the emission is \( \varepsilon \sim \varepsilon_{\nu} \cdot \varepsilon_{\nu} = 50\% \).

### 3.2 BNS merger rate model

The BNS merger rate dependence on redshift is poorly observationally constrained, hence for these investigations we assume the simplified model of a constant cosmological rate per unit comoving volume per unit source time in the range \( R(z) = [100 - 10,000] \) Gpc\(^{-3}\)yr\(^{-1} \) which extends up to a maximum redshift of \( z \sim 6 \). As a representative case, we report distributions and results for \( R(z) = 1000 \) Gpc\(^{-3}\)yr\(^{-1} \). Because we consider a constant \( R(z) \), the results for the upper (lower) bound of the whole range \( R(z) = [100 - 10,000] \) Gpc\(^{-3}\)yr\(^{-1} \), can be obtained by scaling up (down) the output quantities by one order of magnitude. This wide range of the BNS merger rate includes the local rate interval inferred from the detection of GW170817 (1540–1220 Gpc\(^{-3}\)yr\(^{-1} \) median and uncertainties at 90% probability, Abbott et al. 2017b) and is broadly consistent with estimates obtained using Galactic BNS observations and population synthesis models (Abadie et al. 2010; Chruslinska et al. 2017; Paul 2017).

We test the impact of the assumed model for BNS merger rate by considering three further astrophysically-motivated scenarios:

- **D2013** the Monte Carlo population synthesis model of Dominik et al. 2013 (their cosmological standard model, high-end metallicity evolution scenario K et al. 2013.)
- **G2016** the analytic approximation based on SGRB observations described in eq. (12) of Ghirlanda et al. 2016 (adopting the average value of the parameter reported for case \( a \) with an opening angle of 4.5 deg);
- **MD2014** the analytic prescription for the star formation history proposed by Madau & Dickinson 2014 convolved with a probability distribution of delay times between formation and merger given by the power law \( P(z_{\text{del}}) \propto z_{\text{del}}^{-1} \), with a minimum delay time of \( 20 \times 10^7 \) yr, normalised to the local BNS merger rate of 1540 Gpc\(^{-3}\)yr\(^{-1} \), as estimated with GW170817.

We test and discuss the impact of applying different BNS merger rate models in Section 4. The different BNS merger rates are reported in Figure 4. We note that D2013 and G2016, as proposed, are inconsistent with the local rate range obtained with GW170817. However usually binary rate models relay on assumptions which allow significant rescaling: e.g. Dominik et al. 2013 present 4 different scenarios of which 3 present very similar dependence on redshift, but with considerable differences in magnitude. We decide to adopt these rate models for illustrative purposes to test the impact of different (both in shape and in magnitude) redshift dependences.

Our analyses assume a flat cosmology with: \( H_0 = 70 \) km s\(^{-1} \) Mpc\(^{-1} \), \( \Omega_M = 0.3 \) and \( \Omega_L = 0.7 \).

### 3.3 Results

We now proceed using saprEMo and the X-ray transient model described in Section 3.1 to evaluate the expected number of detections of this type of signal in three present and future surveys by XMM-Newton, Chandra and THESEUS. In order to emphasise the impact of the survey properties, we keep the light curve, the assumed astrophysical rate and the efficiency fixed throughout our calculations. In particular, we consider:

- two present surveys, collected during the decade of operation of XMM-Newton, to predict the expected number of detectable signals in these archived data;
- the Chandra Deep Field - South (CDF-S) data set to verify whether the transient class discovered by Bauer et al. 2017 is statistically consistent with the SC16 model;
- 10 ks of THESEUS observations, to explore the sensitivity of this mission concept to transients associated with BNS mergers, such as SC16f.

The main properties of the surveys are summarised in appendix B.

#### 3.3.1 XMM-Newton

We apply saprEMo to two different collections of data obtained by XMM-Newton; we call them SLEW and PO (Pointed Observations), their characteristics are presented in the following.

![Figure 4. BNS merger rate as a function of redshift for different models: D2013 Dominik et al. 2013 in green, G2016 Ghirlanda et al. 2016 in blue, MD2014 Madau & Dickinson 2014 convolved with \( P(z_{\text{del}}) \propto z_{\text{del}}^{-1} \) in orange and our default constant model in violet: the solid line represents the reference rate of \( R(z) = 1000 \) Gpc\(^{-3}\)yr\(^{-1} \), while the light shadowed area includes the whole interval \( R(z) = [100 - 10,000] \) Gpc\(^{-3}\)yr\(^{-1} \). The range in redshift is divided into \( z \leq 1 \) and \( z > 1 \) to enhance the visibility of the constraints set by the observation of GW170817 (gray area, 90% probability as reported in Abbott et al. 2017a), which only apply to the local Universe.](image-url)
number of signals predicted by saprENo are reported in table 1. In the case of XMM-Newton surveys, the sky locations of observations have been used to estimate the impact of the absorption due to the Milky Way (see appendix A2 for more details on our adopted absorption model).

The P.O. survey is a collection of pointed observations made between 3/2/2000 and 15/12/2016 (XMM-Newton SSC Consortium 2017a; Rosen et al. 2016a). The data belong to the XMM-Newton Serendipitous Source Catalog (3XMM DR7). PO exposures are longer (typically $10^{-3} - 10^{-5}$ s) compared to the SLEW catalog (see following paragraph). This implies an extension to lower fluxes (down to $10^{-15} - 10^{-16}$ erg s$^{-1}$ cm$^{-2}$), as figure 5 (a) shows. The same figure shows that such low fluxes are however reached only by tails. This is because the luminosity of the model makes the flux higher than the survey flux limits up to $z = 6$, which is the artificial cut of our BNS merger rate. The distribution of source redshift is represented in the bottom graph of figure 5 (a) and implies that, under the assumption of a constant cosmic BNS merger rate, the median redshift of detectable signals is $z \sim 2$. The double bump in the tail distribution of the PO survey is explained by the blue and purple curves which respectively represent the light curve span above the threshold at a fixed redshift $z$. LCS(z), and the time-shifted rate of events per unit redshift, $R(z) dV/dz$. Given our simplified BNS merger rate model, the purple and black-solid lines scale like the redshift derivative of the comoving volume, since in both cases only constants multiply the element $dV/dz$. The blue curve has indeed a very peculiar behaviour which depends on the specific emission light curve compared to the limit fluxes of the survey. The distinct trends in the blue lines of figure 5, are due to features very peculiar to the adopted light curve (figure 3). When the flux from the non-thermal second peak drops below the limit, the overall visible duration sharply decreases: this happens at $z \sim 0.5$ and $z \sim 0.05$ for PO and SLEW, respectively. The second turn-over at $z = 4$ in the LCS, evident in PO tails (gray area and dashed-black line), is instead due to the disintegration of the light curves in energy bins and the relatively soft energy spectrum of these transients. In particular the lowest energy bin characterising the light curve (see figure 3) exits the band of the instrument.

The SLEW survey is composed by data collected while changing the target in the sky, according to the XMM-Newton observation program (Smale, AP and others 2017). The tested observations are collected in the XMM-Newton Slew Survey Clean Source Catalog, v2.0. The SLEW survey is characterised by typical exposure time of only few seconds, and consequent flux limits $\geq 10^{-15}$ erg s$^{-1}$ cm$^{-2}$. Given the properties of the model SC16, this yields a predominance of tails over peaks (see first point of section 4). Our results show that SLEW observations, assuming correct identification (see sec. 4), could already reveal a population of BNS merger events. The flux limits of the survey determines the distance of most of the X-ray sources at $z < 3$. Although the data have been collected by the same instrument, PO and SLEW considerably differ in terms of exposure time (and therefore sensitivity), sky coverage and energy responses (as shown by tables in appendix B1). The SLEW survey is less sensitive, but it scans a much wider area of the sky compared to the PO survey, so that the total number of expected signals is actually considerably larger (see tab. 1).

### Table 1. Average expected values for peaks ($N_p$) and tails ($N_t$) for different surveys. XMM-Newton PO and SLEW surveys\( ^6 \), Chandra Deep Field - South (CDF-S) and 10 ks of THESEUS operation for a single exposure, case a, and 10 distinct exposures case b considering $N_H = 5 \times 10^{-22}$ cm$^{-2}$ ($N_H = 5 \times 10^{-20}$ cm$^{-2}$).\(^6\) For the XMM-Newton surveys the total observing time was inferred from $T = n_{obs} \times (T_{obs}) = \frac{4 \pi f_d \times \text{FoV}^{-1}}{T_{obs}}$, using the properties reported in tables B1 and B3.

| Survey          | PO     | SLEW   | CDF-S  | THESEUS |
|-----------------|--------|--------|--------|---------|
|                | $N_p$  | $N_t$  | $n_{obs}$ | Case a | Case b |
| XMM-Newton PO   | 1      | 0      | 0.14   | 5 (4)   | 3 (2)   |
| XMM-Newton SLEW | 25     | 165    | X      | 5 (3)   | 20 (11) |

$^6$ For both the surveys, the statistical uncertainties due to the assumed Poisson distribution are almost negligible compared to the systematics due to uncertainty in the signal production efficiency $\epsilon$ and the BNS merger rate.
Figure 5. Comparison between expected distributions of detectable signals in PO and SLEW surveys. The upper-left panel shows the total expectations on the observed maximum flux distribution obtained by adding peak (dark green) and tail (green) contributions. The upper-right panel shows the expected distributions of peak and tail durations. The redshift distributions in the bottom panel represent the differential contribution of peaks and tails throughout the scanned comoving volume of Universe. The violet and blue curves have been added to explain the trend of the tail distribution and represent respectively the tail distribution and its redshift (bottom) of the observations in XMM-Newton PO survey. (a) Expected distributions of observed maximum flux (upper-left), duration of signals above the flux limits (upper-right) and redshift (bottom) of the observations in XMM-Newton PO survey. (b) Expected distributions of observed maximum flux (upper-left), duration of signals above the flux limits (upper-right) and redshift (bottom) of the observations in XMM-Newton SLEW survey. Because of the low expected number of detections for peaks compared to tails (see tab. 1), the black solid line in bottom graph is not visible.
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probability of 0 signals, correspondent to \( R_v(z) = 100 \text{ Gpc}^{-3}\text{yr}^{-1} \) and \( \sim 35\% \) (with \( \sim 25\% \) probability of 0 signals, correspondent to \( R_v(z) = 10000 \text{ Gpc}^{-3}\text{yr}^{-1} \)).

Despite the broad consistency of the transient revealed by Bauer et al. 2017 with the rescaled SC16 emission model, our analysis shows that a real association between the two is rather disfavoured, although not inconsistent with the uncertainties over rate and emission model. Conversely, assuming that the detected transient is in fact the rescaled SC16 signal adopted in the above calculation, the constant BNS merger rate value is constrained to \( (6000)_{-3700}^{+6000} \text{ Gpc}^{-3}\text{yr}^{-1} \) (median with 90\% credible interval, as in Abbott et al. 2017a), higher than the median inferred from GW170817, though still consistent with the claimed interval (cf. Section 3.2).

### 3.3.3 Future observations with THESEUS

In the last few years, different wide-FoV X-ray missions have been proposed to monitor the X-ray sky, and specifically to follow up GRBs and GWs (Feng et al. 2016; Barcons et al. 2012; Yuan et al. 2015; Merloni et al. 2012). In particular, the mission concept THESEUS has been recently selected by ESA for assessment studies (Bauer & Colangeli 2018) to explore the transient high-energy sky and contribute to multi-messenger astronomy (Amati et al. 2017; Stratta et al. 2017; Frontera et al. 2018). We apply sapreEffo to test the sensitivity of the THESEUS mission to BNS mergers emitting in the X-ray according to the SC16f model. On the THESEUS payload, the Soft X-ray Imager (SXI, O’Brien et al. 2018) would be the instrument sensitive to such emission. SXI flux sensitivities for sources in the Galactic plane (\( N_{FP} = 5 \times 10^{33} \text{ cm}^{-2} \)) and well outside it (\( N_{FP} = 5 \times 10^{30} \text{ cm}^{-2} \)) are taken from figure 4 of Amati et al. 2017. With sapreEffo, we predict detection numbers and properties for two cases of gathering THESEUS observations, each having a total observing duration \( T \) of 10 ks, acquired with:

- **case a** a single exposure of \( (T_{\text{obs}}) = 10^4 \text{ s} \)
- **case b** 10 exposures of non-overlapping sky regions, each lasting \( (T_{\text{obs}}) = 10^5 \text{ s} \).

The last 2 columns of table 1 show the expectation values for \( N_p \) and \( N_t \) for both case a and b. In less than 3 hours of total observing time \( T \), we expect THESEUS to detect a number of SC16-like transients comparable to the ones inferred for the former and the ones in Chandra and PO XMM-Newton surveys (see also table 1). The main reason why THESEUS is capable of reaching these numbers of detections in such a short observing time \( T \) (about 4 orders of magnitude shorter than for CDF-S and PO/SLEW), is the 4 orders of magnitude difference between its FoV and the ones in Chandra and XMM-Newton. Albeit specific for the SC16f emission, our tabulated results demonstrate that the characteristics of the mission concept THESEUS suit the search for X-ray emission generated during BNS mergers. We predict that THESEUS/SXI with case a sensitivity would detect on the order of hundreds to thousands of BNS mergers in only a few years, assuming an emission with peak luminosity in the range \( L_p \sim [10^{38} \text{ to } 10^{38}] \text{ erg cm}^{-2} \text{ s}^{-1} \) and a spectral distribution similar to SC16f. Our analysis therefore shows that proposed large FoV instruments, such THESEUS, offer an incredible opportunity compared to present deep surveys, for the detection of rare but bright transients, such as those of the presented model.

![Image](305x292 to 560x452)

**Figure 6.** Expected redshift distribution for peak signals in 10 ks of THESEUS/SXI observations (opaque lines), assuming \( N_{FP} = 5 \times 10^{30} \text{ cm}^{-2} \). In the default configuration of constant BNS merger rates we test two different observing strategies: **case a** and **case b**, as described in Section 3.3.3. They are shown by the solid-violet and dashed-fuchsia lines, which completely overlap for \( z \leq 2 \) and only differ in the maximum achieved redshift, \( z \sim 3 \) for the former and \( z \sim 2 \) for the latter. We also report the peak distribution as a function of redshift for other tested BNS merger models: MD2014 in orange, D2013 in green and G2016 in blue. For comparison with figure 4, we show the results in the two different regimes: \( z > 1 \) in linear and \( z \leq 1 \) in logarithmic scale. For the same reason, we also add in transparency the distribution of peaks missed because of sensitivity constraints.

**Table 2.** For XMM-Newton PO (top) and THESEUS case a surveys, comparison of expectation values assuming different BNS merger rate models, from left to right (i) analytic prescription proposed by Madau & Dickinson 2014, assuming a power-law distribution of delay times between formation and merger \( P(t_{\text{del}}) \sim t_{\text{del}}^{-1} \), (ii) cosmological rate derived by the population synthesis study Dominik et al. 2013, standard model at high-end metallicity scenario (D2013) and (iii) model based on SGRB statistics Ghirlanda et al. 2016 with assumed opening angle of 4.5 deg (G2016).

| Rate model   | MD2014 | D2013 | G2016 |
|--------------|--------|-------|-------|
| \( N_p \)    | 17 (15)| 0.54 (0.46) | 1.2 (1.0) |
| \( N_t \)    | 16 (10) | 0.54 (0.34) | 1.0 (0.6) |

\[ 6 \] To estimate the posterior on the constant rate value, we assume a flat prior in the range \([100, 10000] \text{ Gpc}^{-3}\text{yr}^{-1} \).
of detecting tails. We find that THESEUS/SXI would make more detections during a fixed total observing time with an observing strategy that increased sky coverage at the cost of shorter exposures.

4 DISCUSSION

With saprEMo, we tested the sensitivity of different astronomical surveys to the emission model SC16.

Given a fixed emission model, the tail contribution becomes more important for surveys with shorter exposures \( T_{\text{obs}} \). The analysis has confirmed that when the exposure as \( T_{\text{obs}} \) is considerably shorter than the duration of the emission model, as is the case of XMM-Newton SLEI observations, detections of pre-peak rises or post-peak decays will be more common than observations of peak flux. Therefore the relation between \( T_{\text{obs}} \) and the typical duration of the EM model sets the relative number of peaks and tails. Given a fixed amount of total observing time, the number of expected tails increases with the number of pointings of different sky positions. This is shown, for example, by the two cases tested with THESEUS (see tables 1 and 2).

Inferences on BNS merger models

- A few detections can already constrain the lower limit of cosmic BNS merger rate. For example, assuming the emission model proposed by Siegel & Ciolfi 2016b, the probability of detecting more than 3 peaks in XMM-Newton PO assuming a constant merger rate of 100 Gpc\(^{-3}\)yr\(^{-1}\) is < 1%; thus, a few detections could set a lower limit on the BNS merger rate. The proposed emission model also offers the unique possibility of exploring mergers in the high-redshift Universe: with peak luminosity as high as \( \sim 10^{48} \) erg/s, XMM-Newton could detect signals generated at redshifts as high as \( \sim 15 \) with PO sensitivities.

Constraints on BNS merger rate can be also obtained by assuming the association of the Chandra X-ray transient with the SC16 emission model. In this case our analysis can produce a lower limit on the BNS merger rate of \( \sim 2300 \) Gpc\(^{-3}\)yr\(^{-1}\) at 90\% confidence interval, assuming a constant rate up to \( z \sim 6 \). The peak luminosity of the rescaled SC16 emission is indeed bright enough to be detectable by Chandra up to \( z \sim 6 \), even after accounting for the Milky Way absorption.

- Larger PO-like or THESEUS surveys will probe \( R_v(z) \) and likely constrain also EM emission models. In the case of bright emission, as expected by SC16, saprEMo predicts that the PO campaign can detect all BNS merger peaks in the Universe, localised within XMM-Newton’s field of view. However, \( T_{\text{obs}} \) must be obtained with longer time window \( T \) and/or larger FoV. Large field of view instrument, such as THEMES/SXI, can also detect many of these events. Although limited to smaller redshift, cosmological distances could still be achieved if we assume bright emission models. This is shown for the case of SC16 in figure 6. These campaigns should therefore yield the BNS merger rate distribution as a function of redshift, providing that multi-wavelength observations will allow redshift associations. These observations would then enable us to constrain the proposed BNS merger rate scenarios, which indeed predict different \( z \) distributions (see figure 6).

- Redshift measurement play a fundamental role for breaking the degeneracy between emission parameters and rate models. We apply saprEMo to the PO and THESEUS cases, case a, for the three scenarios introduced in sec. 3.2: D2013, G2016 and MD2014 (see figure 4). The absolute expectation values reported in table 2 and figure 6 reflect the trend of the rate models reported in figure 4. The overall results show that, without perfect knowledge of light curve and spectrum of the emission, measurements of source distances are necessary to constrain the redshift dependence of the BNS merger rate.

Considerations on saprEMo’s results. In this paragraph we highlight some general considerations, to realistically interpreting saprEMo results. The main output of the analysis consists in the number of peaks and tails expected for a specific emission model in a selected survey of data. However, depending on the purpose of the analysis, other information, such as more accurate requirements for detectability and classification, should be taken into account. In the following we give some examples.

Challenges for detectability: despite this does not concern the results presented in the previous session, short transients in long exposure observations can be lost in the integrated background flux. To overcome this issue, targeted analyses might be required (see for e.g. the work of EXTraS group De Luca 2014 De Luca et al. 2016 for detection of \( \sim 10^2 \) s-lasting transients in PO).

Challenges for classifications: because of their definition, we generally expect durations and fluxes of tails to extend to lower values compared to the peak ones (as shown in figure 5 (a)). This generally worsen the performances of signal classifications and identification among more common phenomena. In particular for our analyses, given the shape of SC16’s light curve, tails should mostly appear as simple decaying signals, which can be challenging to distinguish from other X-ray transients (e.g., tails of tidal disruption events Lodato & Rossi 2011 or supernovae Dwarkadas & Gruszko 2011).

The correct classification of X-ray events can also be challenged by short exposure times. This is for example the case of the SLEW survey, where observations typically last only few seconds. Indeed some emission models, as SC16, predict a long-scale time evolution of the emission properties which would likely result in detections of dissimilar signals, challenging their association to a common origin. Campaigns characterised by typically longer observations, such as the XMM-Newton PO and Chandra CDF-S, are less affected by classification problems. The extension of the typical exposure time to thousands of seconds and improved spectral resolution, allow for the acquisition of more informative data, simplifying transient identification.

5 SUMMARY AND OUTLOOK

In this study we showed some applications of our tool saprEMo; we applied it on few present and possible future surveys, assuming a specific emission model and cosmological BNS merger rate \( R_v(z) \). In terms of multi-messenger astronomy, our results show that the luminosities predicted by the SC16 emission model can be detected up to cosmological distances which extend much further than the horizon of present (Aasi et al. 2016) and future gravitational wave detectors (Sathyaprakash et al. 2012) to binary neutron star mergers, both in the cases of current surveys, such as CDF-S and XMM-Newton PO and SLEW, and proposed missions, such as THESEUS. saprEMo provides theoretical predictions allowing us:

- to compare predictions with actual data. E.g. we proved that
some signals consistent with the model could already be detected in present surveys of data such as XMM-Newton PO and SLEW;

- to test potential associations. E.g. we proved that the new transient found by Bauer et al. 2017 is marginally consistent with the model;
- to assess the effectiveness of proposed mission concepts for a specific type of signal. We illustrate the utility of saprEMo for evaluating proposed missions with a case study of THESEUS. We demonstrate that, with few years of operation, the large FoV of THESEUS/SXI could allow for the detection of up to thousands of SC16-like signals, enabling considerable constrains on both the BNS merger rate and the emission models;
- and to compare different observational strategies. saprEMo can be used to determine advantages and disadvantages compared to a particular emission, of adopting different observational strategies. With the case of THESEUS, we indeed demonstrate that saprEMo can compare observations characterised by different values of typical exposure time ($T_{\text{ex}}$) and point out the main properties of the relative detections. In general, given a total observing time $T$, different observational strategies can be applied; increasing the exposure time to increase the sensitivity or decreasing the exposure time to enlarge the sky coverage. The effect of adopting different exposures depends on several parameters, including both source and instrumental properties (such as rate, luminosity, flux limit dependence on exposure time, etc). In this paper, we specifically proved that, given THESEUS/SXI sensitivity as a function of exposure time, 10 observations of disjoint sky areas lasting 1 ks would en-captured more SC16f-like transients than an extended single pointing of 10 ks.

In general saprEMo allows us to test both survey and astrophysical properties. This study has mainly focused on the former, exploring the impact of different trade offs among such properties (including exposure time, sky localisation, and spectral sensitivity), assuming a single light curve model from SC16. However saprEMo can also test (and be used for inference on) astrophysical quantities such as emission duration, peak luminosity and spectra.

Once the design sensitivity of Advanced gravitational-wave interferometers is achieved, GW detections of EM bright sources, such as GW170817, will occur more and more often and very likely at lower SNRs. In this context of multi-messenger astronomy, saprEMo can be used to optimize the analysis by identifying specific emission model. We conclude remarking that the flexibility of the implemented methodology allows considerations of emission model spanning the whole electromagnetic spectrum (e.g. kilonova models can also be tested). Moreover our analysis include no priors on nature of EM sources, so that it can be applied to a wide range of astrophysical phenomena. With its analysis dedicated to treat high redshift effects, saprEMo particularly suits studies on emission of cosmological origin.

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APPENDIX A: SAPREMO FEATURES

A1 Redshift/K-correction

To evaluate if the emission is visible in the g band of the instrument, we need to calculate the fraction of source light curve which contributes to the flux in the g band at the observer. Here we denote each energy band \([E_{\text{min}}^g, E_{\text{max}}^g]\) with the label \(g\); we use \([E_{\text{min}}^h, E_{\text{max}}^h]\) without subscripts for the whole range of operation.

We assume a set of light curves \([L_h(t')]_{j=0}^{\text{max}}\) - where each element \(L_h(t')\) represents the emission in the source frame within a fixed \(h\) energy bin \([E_{\text{min}}^h, E_{\text{max}}^h]\). The redshifted energy bin \(h\), associated to the light-curve element \(L_h(t')\), might only partially overlap with an instrumental energy bin \(g\). To consider only the part of the light-curve which contributes to the emission visible in the \(g\) energy bin, we calculate the fraction of the \(h\) energy bin that falls into the \(g\) band and assume the energy is uniform across its intrinsic spectrum. Therefore for each numerical step in \(z\), we calculate the emission contribution to each \(g\):

\[
L_g(t', z) = \sum_{h=0}^{\text{max}} L_h(t') \omega_g(z) \tag{A1}
\]

When the redshifted \(h\) bin and the \(g\) band of the instrument overlap, the \(L_g(t', z)\) emission contributes to the total observable emission in the \(g\) band \(L_g(t', z)\) with a weight defined by the ratio between the amount of overlap and the width of the light curve energy bin:

\[
\omega_g(z) = \begin{cases} 
\alpha & \text{if } \alpha > 0 \\
0 & \text{otherwise}
\end{cases} \tag{A2}
\]

where

\[
\alpha = \min \left( (1+z)^{-1} E_{\text{max}}^g (1+z)^{-1} E_{\text{min}}^h, (1+z)^{-1} E_{\text{max}}^h (1+z)^{-1} E_{\text{min}}^g \right) \tag{A3}
\]

An increased resolution in the energy bins of the emission model will result in more precise estimates.

A2 Absorption

sapREMo can account for both host and Galactic absorptions. The host-galaxy absorption is included by substituting \(L_h(t')\) with \(L_h(t') e^{-\sigma_g} \), where \(n_{H_{\text{MW}}} \) is a typical value of the effective hydrogen column density and \(\sigma_g\) is the average of the absorption cross-section in the \(h\) energy band in the source frame. Both of these quantities may depend on the type of the host galaxy. Similarly, the Milky-Way absorption is accounted adopting \(L_h(t', z) e^{-\sigma_{H_{\text{MW}}}} \). We estimate an effective hydrogen column density as a function of the observed sky-locations (Galactic latitudes), adopting the sky-map of HI emission-line brightness \(T_3\) released by Kalberla et al. 2005. For each position in the sky, the Galactic column density along the line of sight \(n_{H_{\text{MW}}} \) is calculated adopting equation (4) of Chengalur et al. 2013 (valid for negligible total opacity). These values then have to be averaged along Galactic longitudes \(l\) \((n_{H_{\text{MW}}})_l\), and finally associated to the relative frequency of observations in the survey to calculate an effective column density \(n_l^H\):

\[
n_{H_{\text{MW}}}[cm^{-2}] = \frac{1}{n_{\text{obs}}} \sum_{j=1}^{n_{\text{obs}}} (n_{H_{\text{MW}}})_l \delta(j, l) \tag{A4}
\]

where \(h\) is the galactic longitude.

To establish the detectability of the light curves \(\tilde{L}_g(t', z)\), we
calculate the corresponding fluxes:

$$F_\delta(t', \gamma) = \frac{L_\delta(t', \gamma)}{4\pi D_L^2(z)}$$  \hspace{1cm} (A5)

### A2.1 X-ray absorption model

Different specific absorption models can be implemented accordingly to the energy range of interest. In this paper we only consider the effect of X-ray absorption at the observer. In Willingale et al. 2013, the authors investigate hundreds of GRB afterglows detected by Swift to model the effective total Galactic column density in X-ray $n_{H_{tot, MW}}$. Atomic and molecular hydrogen represents the dominant component of $n_{H_{tot, MW}} = n_{H_{MW}} + 2n_{H_2, MW}$. To estimate the molecular hydrogen component from the atomic contribution, we adopt the model proposed by Willingale et al. 2013:

$$n_{H_2, MW} = n_{H_{MW}} \left[ 1 - \exp\left(\frac{n_{H_{MW}}}{n_c}\right)\right]^{1/3}$$  \hspace{1cm} (A6)

where $n_{H_{MW}} = 7.5 \times 10^{20}$ molecules/cm$^2$, $n_c = 2.37 \times 10^{21}$ atoms/cm$^2$ and $a = 2$. We apply this effective total Galactic column density $n_{H_{tot, MW}}$ to both XMM-Newton surveys, PO and SLEW.

The effective cross section of the interstellar medium, for each of the energy bands in the range 0.03 – 10 keV, is analytically estimated as a function of energy $E$, following Morrison & McCammon 1983.

### A3 Detailed integrations over cosmical scales

To estimate the peak contribution, we need to account for the redshift dependence of all the quantities involved in the cosmic integration of equation 2. At each step in $z$, the portion of light curve still present (after the redshift) in each of the instrumental energy band is calculated, the absorption is applied and the new maximum redshift $z_{max, g}$ is computed. The contribution of the correspondent step in redshift $z$ is considered if at least on one energy band $g$, $z_{max, g} > z$.

Similarly, we compute the number of tails with:

$$N_g = \epsilon \bar{n} \frac{F_\text{FoV}}{4\pi} \int_{z_{max}}^{z_{\text{max}}} R_v(z) \frac{dV}{dz} \frac{LCS}{dz} dz$$  \hspace{1cm} (A7)

where $z_{\text{max}}$ is the maximum redshift which contributes to peak count and $LCS = \sum_i dL_i(z_i)$ represents the union of source time-intervals of the light curve $dL_i(z_i)$ which are visible at $z$ in at least one g band, i.e.

$$\forall g \hspace{0.2cm} dL_i(z_i) = l_i(z_i) - t_i(z_i) \geq [F_g(t_i', z) + F_g(t_i', z)] > 2 F_{\text{lim, g}}$$  \hspace{1cm} (A8)

This contribution does not include peaks, since the peak duration can be considered infinitesimal. The time dependence for tail calculation is only set by the signal duration, while is completely unaffected by the survey exposures, which instead determine the peak contribution.

### APPENDIX B: SURVEY PROPERTIES

#### B1 XMM-Newton parameters

We report the parameters adopted to apply saprETo to XMM-Newton surveys: PO and SLEW in tables B1, B2, B3 and B4. The data used to characterise PO and SLEW are collected in source catalogs. From there, relative papers (Rosen et al. 2016b; Saxton et al. 2008) and webpages (XMM-Newton SSC Consortium 2017a; Read & Saxton 2016; Saxton et al. 2017; ESA 2017), we extract the general properties necessary to apply saprETo. In the case of PO and SLEW we estimate the impact of absorption from the locations of the sources collected in the catalogs Rosen et al. 2016b$^7$, $^8$. Since no flux limits have been quoted for PO, for that case we use as a proxy the medians of the fluxes available from the catalog.

#### B2 CHANDRA

To apply saprETo to the CDF-S we use data from A. P. Smale 2017; Lehmer et al. 2005 and Luo et al. 2016. As quoted in the same references, we adopted $n_{H_{tot, MW}} = 8.8 \times 10^{20}$ cm$^{-2}$.

The adopted survey properties are reported in the tables B5 and B6. Tables B5 and B6 report conservative values extracted from figures 2, 28 and 29 of Luo et al. 2016, approximating the main characteristic a region of the sky described by roughly homogeneous properties $^9$. More details on the set of observations are available at A. P. Smale 2017.

#### B3 THESEUS

The data concerning THESEUS have been extrapolated from Amati et al. 2017. We analyse 10 ks of exposure collected with two different strategies:

- a) 1 single exposure of 10 ks;
- b) 10 distinct exposures of 1 ks each.

We report in tables B7, B8 the properties adopted for the results presented in this paper.

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7. We adopt clean data, requiring: CLEAN = OBS_CLASS < 3.
8. Data concerning average and standard deviation of observation durations are based on total band (Exp_Map_B8). Locations adopted for the absorption model are inferred from source locations.
9. To account for homogeneity, we decide to only consider a 285 arcmin$^2$ region, out of the 484.2 arcmin$^2$ of the entire survey, which corresponds to the ACIS FoV and roughly to the region observed by at least 6 million seconds (see figure 2 of Luo et al. 2016). A similar sky area is also characterised by flux limits in the total energy band $< 2 \times 10^{-16}$ erg cm$^{-2}$s$^{-1}$.
### Table B2. PO: spectral bands of pointed observations contributing to the XMM-Newton Serendipitous Source Catalog. Medians in each band from catalog as reported in website (AP Smale 2017), catalog available at (XMM-Newton SSC Consortium 2017c) cleaned with the same criteria used for calculating average and variance of exposure times.

| ENERGY BAND [keV] | SENSITIVITY [erg cm\(^{-2}\) s\(^{-1}\)] |
|-------------------|-----------------------------------------|
| 0.2 – 0.5         | 5.8 × 10^{-16}                          |
| 0.5 – 1.0         | 1.7 × 10^{-15}                          |
| 1.0 – 2.0         | 2.7 × 10^{-15}                          |
| 2.0 – 4.5         | 3.8 × 10^{-15}                          |
| 4.5 – 12.0        | 6.6 × 10^{-15}                          |

### Table B3. SLEW: general characteristics of slew data contributing to the XMM-Newton Slew Survey Catalogue, a: from clean observations according to `xmmstl2_clean.fits` file at (Smale, AP and others 2017); b: percentage when overlaps are excluded from (Smale, AP and others 2017).

| ENERGY BAND [keV] | SENSITIVITY [erg cm\(^{-2}\) s\(^{-1}\)] |
|-------------------|-----------------------------------------|
| 0.2 – 2.0         | 0.57 × 10^{-12}                         |
| 2.0 – 12.0        | 3.7 × 10^{-12}                          |

### Table B4. SLEW: spectral bands of slew data contributing to the XMM-Newton Slew Survey Catalogue. Flux limits from (ESA 2017). The energy range in SLEW catalog is divided just in 2 bands.

| PARAMETER | VALUE |
|-----------|-------|
| Minimum energy | 0.2 keV |
| Maximum energy | 12 keV |
| \(\langle T_{\text{obs}} \rangle\) a | 6.9 s |
| \(\sigma_{T_{\text{obs}}}^2\) a | 2.4 s |
| Covered Sky area b | 84% of the sky |

### Table B5. CDF-S: general properties of CDF-S (roughly homogeneous region) from (Lehmer et al. 2005), and (Luo et al. 2016). a: the maximum cleaned exposure is 6.727 × 10^6 s.

| PARAMETER | VALUE |
|-----------|-------|
| Minimum energy | 0.5 keV |
| Maximum energy | 7 keV |
| FoV | 285 arcmin\(^2\) |
| \(\langle T_{\text{obs}} \rangle\) a | \(~6 \times 10^6\) s |
| \(n_{\text{obs}}\) | 1 |

### Table B6. CDF-S: conservative estimates of flux limits from figures 28 and 29 of Luo et al. (2016) (at ~50% completeness).

| ENERGY BANDS [keV] | SENSITIVITY [erg cm\(^{-2}\) s\(^{-1}\)] |
|-------------------|-----------------------------------------|
| 0.5 – 2.0         | \(~6 \times 10^{-17}\) |
| 2.0 – 7.0         | \(~4 \times 10^{-16}\) |