Born-Infeld-Goldstone superfield actions
for gauge-fixed D-5- and D-3-branes in 6d

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Abstract

The supersymmetric Born-Infeld actions describing gauge-fixed D-5- and D-3-branes in ambient six-dimensional (6d) spacetime are constructed in superspace. A new 6d action is the (1,0) supersymmetric extension of the 6d Born-Infeld action. It is related via dimensional reduction to another remarkable 4d action describing the N=2 supersymmetric extension of the Born-Infeld-Nambu-Goto action with two real scalars. Both actions are the Goldstone actions associated with partial (1/2) spontaneous breaking of extended supersymmetry having 16 supercharges down to 8 supercharges. Both actions can be put into the ‘non-linear sigma-model’ form by using certain non-linear superfield constraints. The unbroken supersymmetry is always linearly realised in our construction.

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1 Introduction

As an introduction, we remind the reader about some general features of the gauge-invariant and gauge-fixed D-brane actions in components, along the lines of ref. [1] (see also refs. [2, 3, 4, 5]). This also allows us to specify our motivation to introduce superspace in some particular cases.

The good starting point is provided by a type-II D-p-brane embedded into flat 10-dimensional (10d) spacetime. The gauge-invariant D-p-brane effective action is usually written down in terms of the worldvolume fields $(X^m(ξ), θ_{αA}(ξ), A_μ(ξ))$ depending upon worldvolume coordinates $ξ^μ$, where $(X^m, θ_{αA})$ themselves can be considered as the coordinates of N=2 superspace in 10 dimensions $(m = 0, 1, \ldots, 9, \ α = 1, \ldots, 16, \ A = 1, 2)$, whereas $A_μ$ is an abelian gauge field, $μ = 0, 1, \ldots, p$. The gauge symmetries of the action comprise (i) worldvolume diffeomorphisms, (ii) a fermionic \(κ\)-symmetry, and (iii) a U(1) gauge invariance, whereas the global or rigid invariances are given by 10d, N=2 super-Poincaré symmetry. The gauge-invariant D-p-brane action is a sum of the Born-Infeld-Nambu-Goto (BING) and Wess-Zumino (WZ) terms,

$$S_p = -\int d^{p+1}ξ \sqrt{-\det(G_{μν} + F_{μν})} + \int Ω_{p+1} ,$$

where $G_{μν}$ is the supersymmetric induced metric in the worldvolume,

$$G_{μν} = η_{mn}Π^m_μΠ^n_ν , \quad Π^m_μ = ∂_μX^m - \bar{θ}\tilde{Γ}m\partial_μ\theta ,$$

$F_{μν}$ is the supersymmetric abelian field strength,

$$F_{μν} = [∂_μA_ν - \bar{θ}\tilde{Γ}m\partial_μθ (∂_νX^m - \frac{1}{2}\bar{θ}\tilde{Γ}m\partial_νθ)] - (μ ↔ ν) ,$$

and

$$\tilde{Γ} = \begin{cases} I \otimes τ_3, & p \ odd, \\ Γ_{11} \otimes I, & p \ even, \end{cases}$$

with respect to the ($αA$) indices. The WZ term in eq. (1.1) describes a coupling of the D-brane to the background Ramond-Ramond (RR) gauge fields [3], while its explicit form is fixed by the \(κ\)-symmetry of the whole action (1.1),

$$δ_κX^m = \bar{θ}\tilde{Γ}mδθ , \quad δ_κθ = \frac{1}{2}(1 + Γ)κ ,$$

where $Γ$ is a (field-dependent) projector [4]. The worldvolume diffeomorphisms (i) ensure that only the $(9 − p)$ coordinates $\{X^i\}, i = p + 1, \ldots, 9$, transverse to the D-brane worldvolume are physical, whereas the \(κ\)-symmetry (ii) effectively eliminates

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<sup>3</sup>The D-brane torsion coefficient in front of the action is chosen to be one.
half of the fermionic $\theta$'s in accordance with the BPS nature of the D-brane that breaks just half of spacetime supersymmetry. The rigid 10d, N=2 supersymmetry transformations are

$$\delta_\varepsilon X^m = \varepsilon \Gamma^m \theta, \quad \delta_\varepsilon \theta = \varepsilon.$$  \hspace{1cm} (1.6)

All physical fields in the D-brane worldvolume can be interpreted as the Goldstone fields associated with the symmetries broken by the D-brane \[6, 7\]. These spontaneously broken symmetries (including broken supersymmetry) are therefore to be non-linearly realised in the gauge-fixed D-brane action to be obtained by fixing the local symmetries and removing unphysical degrees of freedom. A covariant physical gauge for the worldvolume general coordinate transformations is given by the so-called static gauge, in which the first $(p + 1)$ spacetime coordinates are identified with the D-brane worldvolume coordinates, i.e. $X^\mu = \xi^\mu$. The remaining scalars $X^i$ representing transverse excitations of the D-brane can then be identified with the Goldstone bosons (collective modes) $\phi^i$ associated with spontaneously broken translations \[6, 7\].

The bosonic part of the induced metric in the static gauge reads

$$G_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu \phi^i \partial_\nu \phi^i, \quad \text{where} \quad i = 1, \ldots, 9 - p.$$  \hspace{1cm} (1.7)

A covariant gauge-fixing of the $\kappa$-symmetry is also possible (e.g. taking either $\theta_{a1} = 0$ and $\theta_{a2} = \psi$ in the type-IIB case, or just the opposite, $\theta_{a1} = \psi$ and $\theta_{a2} = 0$, in the type-IIA case), while the WZ term vanishes in this gauge \[\square\]. The covariant gauge-fixed D-p-brane action can therefore be identified with a supersymmetric extension of the following BING (or Goldstone)-type action:

$$S_{\text{bosonic}} = - \int d^{p+1}\xi \sqrt{-\det (\eta_{\mu\nu} + F_{\mu\nu} + \partial_\mu \phi^i \partial_\nu \phi^i)}.$$  \hspace{1cm} (1.8)

This action depends upon the abelian gauge field $A_\mu$ only via its field strength $F_{\mu\nu}$ so that the $U(1)$ gauge invariance is kept.

The number $(p - 1) + (9 - p) = 8$ of the bosonic physical degrees of freedom in the action (1.8) matches with the number of fermionic degrees of freedom $16/2 = 8$ associated with the 10d Majorana-Weyl (MW) spinor $\psi$, and it does not depend upon $p$. It is not, therefore, surprising that supersymmetric extensions of all the gauge-fixed D-p-brane actions (1.8) can be deduced by dimensional reduction from a single master 10d action \[\square\],

$$S_{\text{master}} = - \int d^{10}\xi \sqrt{-\det \left[ \eta_{\mu\nu} + F_{\mu\nu} - 2\bar{\psi} \Gamma_\mu \partial_\nu \psi + (\bar{\psi} \Gamma_\rho \partial_\mu \psi)(\bar{\psi} \Gamma_\rho \partial_\nu \psi) \right]},$$  \hspace{1cm} (1.9)

associated with the top value $p = 9$ of the 10d ‘spacetime-filling’ D-9-brane.
By construction, the component 10d super-Born-Infeld (sBI) action (1.9) is invariant under two 10d MW supersymmetries, one unbroken and another one spontaneously broken, with the 10d Maxwell supermultiplet \((A_\mu, \psi_\alpha)\) being the Goldstone vector supermultiplet associated with the second non-linearly realised supersymmetry. In particular, the spinor superpartner \(\psi_\alpha\) of the BI vector is the Goldstone fermion. It should, however, be emphasized that the first unbroken supersymmetry of the action (1.9) is not the same as the original rigid supersymmetry (1.6) since it has to be supplemented by the compensating gauge transformation needed to preserve the gauge. In other words, neither of supersymmetries is manifest in the action (1.9).

Our goal in this paper is to rewrite some of the supersymmetric gauge-fixed D-p-brane actions in superspace, in order to make their unbroken supersymmetries manifest. The superfield formulation is useful in deciphering the unique non-trivial geometry underlying the complicated Goldstone actions associated with partial supersymmetry breaking in various spacetime dimensions (see e.g., refs. [9, 10] for a recent account of non-linear realizations of supersymmetry). The superspace formulation becomes indispensable if one wants to address quantum properties of D-branes, e.g. their black-hole applications [11].

Supersymmetrizing the BI actions in various spacetime dimensions represents a challenge in supersymmetry since one has to deal with a non-polynomial field theory containing higher derivatives of all orders. Causal propagation of the physical fields is to be maintained, while the auxiliary fields needed to close the off-shell supersymmetry algebra are to be kept non-propagating (the last consistency condition was called the ‘auxiliary freedom’ in ref. [12]). Both requirements are non-trivial in supersymmetric field theories with higher derivatives. As was demonstrated e.g., in ref. [13], the naive approach based on insisting on purely algebraic equations of motion for the auxiliary fields rules out a supersymmetrization of the 4d BI action at all. In fact, it is possible to avoid propagating auxiliary fields (i.e. to achieve the auxiliary freedom) by imposing less restrictive conditions in some particular cases, with the sBI actions being the most important examples. It turns out to be possible due to the very special (Goldstone) nature of the sBI superfield actions whose physical bosonic part is free of ghosts and, hence (if consistent), the auxiliary fields should be non-propagating.

Yet another important asset of the BI action in 4d is its electric-magnetic (e.-m.) self-duality (see also ref. [14]). The self-duality and causal propagation together are responsible for the characteristic (‘square root of a determinant’) non-
polynomial structure of the 4d BI action \cite{18}. It is worth mentioning here that the fundamental motivation in favor of the non-linear BI generalization of the Maxwell electrodynamics is the well-known BI taming of Coulomb self-energy, i.e. the existence of a non-singular charged soliton with finite self-energy \cite{14, 17}.

The leading term in the expansion of the BI action with respect to the gauge field strength is the Maxwell action. As is well-known, even a covariant off-shell manifestly N-extended supersymmetrization of the 4d free (!) Maxwell theory is the difficult problem once a number (N) of supersymmetries exceeds two. The infinite number of auxiliary fields beyond the N=2 (or 8 supercharges) barrier is, in fact, required. In this paper we restrict ourselves to the cases of N=1 and N=2 supersymmetry in 4d, and, most notably, (1,0) supersymmetry in 6d too, where an off-shell formulation of the super-BI theory is still possible in the conventional superspace with finite number of auxiliary fields. Similar reasoning (for example, the need for an off-shell extended superspace formulation of a Fayet-Sohnius hypermultiplet) also restricts the number of real Goldstone bosons $\phi_i$ in the 4d (gauge-fixed) super-BING action (1.8), if one wants to achieve its off-shell superspace reformulation by using a finite set of auxiliary fields. In 4d (i.e for a D-3-brane) and N=2 unbroken supersymmetry we are thus led to restrict $i = 1, 2$, which implies the six-dimensional ambient spacetime for the D-3-brane to propagate. Accordingly, in 6d we are going to restrict ourselves to the 6d ‘spacetime-filling’ D-5-brane whose bosonic gauge-fixed action is the 6d Born-Infeld action.

It is worthy to be mentioned that the initial motivation to supersymmetrize the BI action came from the fact that it is the relevant part of the 10d open superstring effective action \cite{20}. In addition, the quartic terms in the expansion of the 4d BI action amount to the so-called Euler-Heisenberg (EH) action \cite{21}, which is known to be the one-loop bosonic contribution to the low-energy effective action of supersymmetric scalar QED.

Taken together, the above reasoning provides broad and compelling motivation for a construction of supersymmetric BI and BING actions in superspace, in terms of constrained extended superfields capable to unify Goldstone scalars and vectors. The importance of those problems, their actuality, as well as some outstanding technical difficulties, related to highly non-trivial extensions of the known N=1 supersymmetric Maxwell-Goldstone action \cite{22} to the sBI actions with extended unbroken supersymmetry, were recently emphasized from various points of view in refs. \cite{9, 10, 23, 24}.

We adopt the most straightforward (bottom-up) approach to supersymmetrize

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5This supermembrane was first considered in ref. \cite{19}.
the bosonic BI action by employing extended superspace, without using the standard coset construction underlying non-linear realizations of internal and spacetime symmetries, including supersymmetry [25]. Though being quite powerful, the general theory of non-linear realizations usually leads in practice to highly involved perturbative calculations in order to arrive at a closed form of the Goldstone action associated with partially broken supersymmetry. Moreover, the coset construction of non-linearly realised supersymmetry turns out to be incomplete since it does not automatically imply the irreducibility constraints on the Goldstone superfields [9, 10]. Just using the basic fact that, being of the Goldstone origin, the bosonic BI action is the unambiguous consequence of non-linearly realised (broken) supersymmetry, its minimal and unique completion with respect to unbroken supersymmetry can be most easily obtained in appropriate superspace by taking a massless vector supermultiplet as the Goldstone one.

The paper is organized as follows: in sect. 2 we review the 4d bosonic BI action and then discuss its supersymmetric generalizations, namely, (i) the 4d Goldstone action associated with N=2 supersymmetry spontaneously broken to N=1 and a massless N=1 vector superfield as the Goldstone-Maxwell superfield [22, 23], and (ii) the 4d Goldstone action associated with N=4 supersymmetry spontaneously broken to N=2 with a massless N=2 vector superfield as the Goldstone-Maxwell superfield [26]. Both supersymmetric BI actions can be equally interpreted as the gauge-fixed actions of a D-3-brane either ‘filling’ 4d spacetime or propagating in six-dimensional spacetime, respectively. Our main new construction that generalizes those of sect. 2 is presented in sect. 3, where we formulate for the first time the manifestly 6d Lorentz invariant and (1,0) supersymmetric Goldstone action associated with partial breaking of (2,0) supersymmetry down to (1,0) supersymmetry in 6d, with a massless (1,0) vector superfield being the Goldstone-Maxwell superfield in 6d. The new action is simultaneously the (1,0) supersymmetric gauge-fixed 6d ‘spacetime-filling’ D-5-brane action in 6d superspace. Our conclusions are summarized in sect. 4.

2 4d (super)BI actions in N=0,1 and 2 superspace

In this section we only discuss four-dimensional supersymmetric BI actions, both in components and in superspace. We briefly review some features of the bosonic BI action, which are going to be relevant for us in what follows. Then we introduce the N=1 supersymmetric BI action [24] and generalize it further to the N=2 BING action in 4d, N=2 superspace.
2.1 The bosonic BI action

The BI action in flat four-dimensional (4d) spacetime with Minkowski metric $\eta_{\mu\nu} = \text{diag}(+, -, -, -)$,

$$S_{\text{BI}} = -\frac{1}{b^2} \int d^4x \sqrt{-\det(\eta_{\mu\nu} + bF_{\mu\nu})}, \quad (2.1)$$

was introduced \[14\] as the non-linear generalization of Maxwell electrodynamics. This action also naturally arises (i) as the bosonic part of the 4d low-energy effective action of open superstrings (together with other massless superstring modes), and (ii) as the bosonic 4d spacetime-filling D-3-brane action as well (sect. 1). In string/brane theory $b = 2\pi\alpha'$, whereas we choose $b = 1$ for notational simplicity.

The BI action (2.1) is manifestly Lorentz-invariant, it depends upon the gauge field $A_{\mu}$ only via its field strength $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$, it contains no spacetime derivatives of $F$, and, after being expanded in powers of $F$, it gives the Maxwell action as the leading contribution. In fact, the BI action shares with the Maxwell action some other physically important properties, such as

- causal propagation (no ghosts),
- positive energy density,
- electric-magnetic self-duality,

which are non-trivial in the BI case \[17, 18\]. Unlike the Maxwell action, the BI action provides a natural taming of the Coulomb self-energy, which is yet another argument in favor of quantum consistency of superstring theory!

Taking advantage of the Lorentz invariance of the BI action, it is always possible to simplify a calculation of its expansion in powers of the gauge field strength by putting $F_{\mu\nu}$ into a particular form, e.g.

$$F_{\mu\nu} = \begin{pmatrix} 0 & \lambda_1 & 0 & 0 \\ -\lambda_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_2 \\ 0 & 0 & -\lambda_2 & 0 \end{pmatrix} \quad (2.2)$$

in terms of real ‘eigenvalues’ $(\lambda_1, \lambda_2)$. Eq. (2.2) is, of course, just a manifestation of the fact that the Lorentz group $SO(1, 3)$ has merely two independent Casimir operators.

\[6\]Our notation in this paper differ from that of ref. \[26\].
In other words, it suffices to pick up two independent Lorentz-invariant \( F \)-products in order to parametrize any Lorentz-invariant function of \( F_{\mu\nu} \). For example,

\[
\det(\eta_{\mu\nu} + F_{\mu\nu}) = -1 - \frac{1}{2} F^2 + \det(F_{\mu\nu}) = -1 - \frac{1}{2} F^2 + \frac{1}{4} \left[ F^4 - \frac{1}{2}(F^2)^2 \right],
\]

where we have introduced two real independent Lorentz-invariants as follows:

\[
F^2 \equiv F_{\mu\nu} F_{\mu\nu} \quad \text{and} \quad F^4 \equiv F_{\mu\nu} F^{\nu\lambda} F_{\lambda\rho} F^{\rho\mu} .
\]

The choice (2.4) is, of course, not unique, and it is not really the most convenient one in 4d supersymmetry. So let’s introduce the 4d dual of \( F_{\mu\nu} \),

\[
\tilde{F}^{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\lambda\rho} F_{\lambda\rho},
\]

and form (anti)self-dual linear combinations,

\[
F^{\pm}_{\mu\nu} = \frac{1}{2} \left( F \pm i \tilde{F} \right)_{\mu\nu},
\]

that satisfy the identities

\[
(F^{\pm})^2 = \frac{1}{2} (F^2 \pm i F \tilde{F}) , \quad (F^2)^2 + (F \tilde{F})^2 = 4(F^+)^2(F^-)^2 .
\]

Note that

\[
\det(F_{\mu\nu}) = \frac{1}{40} (F \tilde{F})^2 .
\]

Using yet another identity

\[
F^4 = \frac{1}{2} (F^2)^2 + \frac{1}{4} (F \tilde{F})^2,
\]

it is possible to slightly simplify eq. (2.3) to the form

\[
- \det(\eta_{\mu\nu} + F_{\mu\nu}) = 1 + \frac{1}{2} F^2 - \frac{1}{10} (F \tilde{F})^2 ,
\]

which implies

\[
L_{BI} \equiv 1 - \sqrt{-\det(\eta_{\mu\nu} + F_{\mu\nu})} = -\frac{1}{4} F^2 - \frac{1}{8} \left[ \frac{1}{4}(F^2)^2 - F^4 \right] + O(F^6)
\]

\[
= -\frac{1}{4} F^2 + \frac{1}{16} \left[ (F^2)^2 + (F \tilde{F})^2 \right] + O(F^6)
\]

\[
= -\frac{1}{4} F^2 + \frac{1}{8} (F^+)^2(F^-)^2 + O(F^6) .
\]

By a complex ‘rotation’ of Lie algebra of \( SO(1, 3) \) to that of \( SL(2, C) \), it is sometimes useful (in supersymmetry) to replace \( F^+_{\mu\nu} \) by a \( 2 \times 2 \) matrix

\[
\hat{F}_{\alpha}{}^\beta = (\sigma^\mu)^{\alpha}{}_{\beta} F_{\mu\nu} , \quad \alpha, \beta = 1, 2 ,
\]
where we have introduced the two-component spinor notation,

\[ (\sigma^\mu) = \frac{1}{4} (\sigma^\mu \tilde{\sigma}^\nu - \sigma^\nu \tilde{\sigma}^\mu), \quad \sigma^\mu = (1, \vec{\sigma}), \quad \tilde{\sigma}^\mu = (1, -\vec{\sigma}), \]  

(2.13)
in terms of Pauli matrices \( \vec{\sigma} \). We find in addition that

\[ \frac{1}{4} \left| \det \hat{F} \right|^2 = 4(F^+)^2(F^-)^2 = (F^2)^2 + (F \tilde{F})^2, \]  

(2.14)
where we have introduced the chiral \((2 \times 2)\) determinant, \( \det \hat{F} \), on the left-hand-side. The right-hand-side of the identity (2.14) is often referred to as the Euler-Heisenberg (EH) lagrangian \([21]\). It arises, in particular, as the bosonic part of the one-loop effective action in N=1 supersymmetric scalar electrodynamics (= the supersymmetric quantum field theory of a massive N=1 scalar multiplet minimally coupled to an N=1 vector multiplet in 4d) with the parameter \( b^2 = e^4/(24\pi^2 m^4) \).

The single complex Lorentz invariant

\[ \frac{1}{16} \text{tr}(\hat{F}^2) = -\frac{1}{4} F^2 - \frac{i}{4} F \tilde{F} \equiv A + iB \]  

(2.15)
is another natural variable for an expansion of the BI action in terms of the field strength \( F \) (it will be used in subsect. 2.2). Yet another choice of variables to be used in subsect. 2.3 is given by the Maxwell lagrangian and the Maxwell energy-momentum tensor squared (= the EH lagrangian!),

\[ -\frac{1}{4} F^2 = A \quad \text{and} \quad \frac{1}{16} \left[ (F^2)^2 + (F \tilde{F})^2 \right] \equiv E. \]  

(2.16)
The Lorentz invariants (2.16) have natural supersymmetric extensions (subsect. 2.2 and 2.3), with the first one having the form of a chiral superspace integral while the second one being a full superspace integral. This justifies our choice (2.16). We find

\[ \det(\eta_{\mu\nu} + F_{\mu\nu}) = 1 - 2A - B^2 = (1 - A)^2 - 2E. \]  

(2.17)
This allows us to rewrite the BI lagrangian to the form

\[ L_{\text{BI}}(F) = A + E + \ldots = A + EY(A, E), \]  

(2.18)
where the function \( Y(A, E) \) has been introduced. It is not difficult to check that \( Y(A, E) \) is just a solution to the quadratic equation

\[ Ey^2 + 2(A - 1)y + 2 = 0. \]  

(2.19)
Similarly, it is straightforward to calculate \( L_{\text{BI}}(F) \) as a function of \( A \) and \( B \) (see subsect. 2.2), e.g., by using the identity \( A^2 + B^2 = 2E \), and eqs. (2.18) and (2.19).
A lagrangian ‘magnetically dual’ to the BI one is obtained via a first-order action

\begin{equation}
L_1 = L_{\text{BI}}(F) + \frac{1}{2} \tilde{A}_\mu \epsilon^{\mu\nu\lambda\rho} \partial_\nu F_{\lambda\rho},
\end{equation}

where $\tilde{A}_\mu$ is a (dual) magnetic vector potential. $\tilde{A}_\mu$ enters eq. (2.20) as the Lagrange multiplier enforcing the Bianchi identity $\epsilon^{\mu\nu\lambda\rho} \partial_\nu F_{\lambda\rho} = 0$. Varying eq. (2.20) with respect to $F_{\mu\nu}$ instead, solving the arising algebraic equation on $F_{\mu\nu}$ as a function of the magnetically dual gauge field strength $F_{\mu\nu} = \partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu$ (use the representation (2.2) for $F_{\mu\nu}$ and similarly for $*F_{\mu\nu}$!), and substituting a solution back into eq. (2.20) yields the magnetically dual action in terms of $*F_{\mu\nu}$, which has the same form as the original BI action (2.1) in terms $F_{\mu\nu}$. This is called electric-magnetic (e.-m.) self-duality [15, 16, 18], and it is connected to the classical $SL(2, \mathbb{R})$ symmetry of IIB superstrings [27]. The non-gaussian BI lagrangian (2.1) is uniquely fixed by the requirements of causal propagation and classical e.-m. self-duality if, in addition, one insists on the Maxwell low-energy limit, i.e. the (strong) correspondence principle. In general, there exists a family of e.-m. self-dual lagrangians parametrized by one variable, with all of them being solutions to a first-order Hamilton-Jacobi partial differential equation [18].

The classical BI action can be rewritten to many equivalent forms by introducing some auxiliary fields that allow one to get rid of the square root or the determinant in the action. For instance, it is possible to put the BI action to a classically equivalent form that is quadratic in the gauge field strength [28, 23]. We are not going to use this kind of tricks in what follows.

### 2.2 N=1 sBI action

The manifestly $N=1$ supersymmetric 4d Born-Infeld (or Goldstone-Maxwell) action associated with partial spontaneous breaking of rigid $N=2$ supersymmetry in terms of the Goldstone-Maxwell $N=1$ supermultiplet $(A_\mu, \psi_\alpha, D)$ was constructed in superspace in refs. [29, 22] (see also ref. [23]). Amongst the superpartners of the Maxwell gauge field $A_\mu$ are the Goldstone (Majorana) fermion $\psi_\alpha$ and the real auxiliary scalar $D$. In this subsection we briefly review some of the results of ref. [22], since the $N=1$ supersymmetric Goldstone-Maxwell action provides the basic pattern that will be subsequently generalized to extended unbroken supersymmetry in the next subsection 2.3.

The standard 4d, $N=1$ superspace is parametrized by the coordinates $Z^M = (x^\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}})$, where $\theta_\alpha$ and $\bar{\theta}_{\dot{\alpha}}$ are (Majorana) spinor anticommuting coordinates in
the 2-component notation, \((\theta_\alpha)^* = \bar{\theta}_\alpha\) and \(\alpha = 1,2\). An abelian vector \(N=1\) supermultiplet is described in \(N=1\) superspace by the irreducible chiral spinor superfield \(W_\alpha\) satisfying the off-shell constraints \[30\]

\[\bar{D}_\alpha W_\alpha = 0, \quad D^\alpha W_\alpha = \bar{D}_\alpha \bar{W}^\alpha.\]

As a result of these constraints, the bosonic components of the \(N=1\) superfield strength \(W_\alpha\) can be introduced as follows \[30\]:

\[D^\alpha W_\beta | = (\sigma^{\mu\nu})^\alpha_\beta F_{\mu\nu} + i\delta^\alpha_\beta D,\]

where \(F_{\mu\nu}\) is the Maxwell field strength of the gauge field \(A_\mu\).

The superfield constraints (2.21) can be solved in terms of a real gauge superfield pre-potential \(V(x, \theta, \bar{\theta})\) as \[30\]

\[W_\alpha = \bar{D}^2 D_\alpha V,\]

subject to abelian gauge transformations \(\delta V = i(\Lambda - \bar{\Lambda})\) where \(\Lambda\) is a chiral superfield gauge parameter, \(\bar{D}_\alpha \Lambda = 0\). This gives the necessary input for a superfield quantization in terms of the unconstrained superfield \(V\).

The 4d, \(N=2\) supersymmetry algebra can be decomposed with respect to unbroken \(N=1\) supersymmetry as \[22\]

\[\{Q_\alpha, Q_\beta\} = 2\sigma^{\mu}_\alpha_\beta P_\mu, \quad \{S_\alpha, S_\beta\} = 2\sigma^{\mu}_\alpha_\beta P_\mu,\]

\[\{Q_\alpha, S_\beta\} = 0, \quad \{Q_\alpha, S^\beta_\alpha\} = 0,\]

where \(Q\)'s stand for the unbroken \((N=1)\) supersymmetry generators, \(S\)'s stand for the broken \((N=1)\) supersymmetry generators, while \(P_\mu\) are 4d translation generators. It is worth mentioning that a non-vanishing central charge does not appear in the \(N=2\) algebra (2.24). The vanishing central charge is, in fact, required for a consistency of the Goldstone-Maxwell action. In particular, the BPS nature of this action also implies the vanishing vacuum expectation values for composites of the physical fields, while the auxiliary field \(D\) should vanish on-shell too. In general, vanishing vacuum expectations for the physical (Goldstone) composites protect the auxiliary fields from becoming propagating due to interacting terms in all supersymmetric BI actions provided that pure kinetic terms for the auxiliary fields do not appear. The latter turns out to be the case for the manifestly supersymmetric (superfield) Born-Infeld-Goldstone actions considered in this paper.

A generalization of the \(N=1\) supersymmetric constraints (2.21), which would be invariant under the second \((S)\) non-linearly realised supersymmetry, is possible, in
principle, by using the standard perturbative approach of non-linear realizations \[25\], though the full answer in a closed form is still unknown in this case \[22\]. It is, nevertheless, possible to determine the full and manifestly N=1 supersymmetric Goldstone-Maxwell action by a direct (and unique) N=1 supersymmetrization of the BI action, as in ref. \[29\]. The result is given by the sBI action \[29, 22, 23\]

\[
S_{N=1 \text{ GM}} = \left[ \frac{1}{4} \int d^4 x d^2 \theta W^2 + \text{h.c.} \right] + \frac{1}{8} \int d^4 x d^2 \theta d^2 \bar{\theta} f(A, B) W^2 \bar{W}^2
\]

\[
= \frac{1}{4} \int d^4 x d^2 \theta \left\{ W^2 + \frac{1}{4} D^2 \left[ f(A, B) W^2 \bar{W}^2 \right] \right\} + \text{h.c.} \quad \quad (2.25)
\]

\[
\equiv \frac{1}{4} \int d^4 x d^2 \theta \ W_{\text{improved}}^2 + \text{h.c.}
\]

where the structure function \( f(A, B) \) is given by

\[
f(A, B) = \frac{1}{1 - A + \sqrt{1 - 2A - B^2}}, \quad (2.26)
\]

whereas \( A \) and \( B \) stand for the N=1 superfields

\[
A = \frac{1}{4} D^2 W^2 + \text{h.c.}, \quad (2.27)
\]

\[
iB = \frac{1}{4} D^2 W^2 - \text{h.c.}
\]

respectively, whose leading \((F\text{-dependent})\) components \((\text{at } \theta_\alpha = \bar{\theta}^\alpha = 0)\) are just given by \( A \) and \( B \) of eq. (2.15).\footnote{It is customary (in supersymmetry) to denote both a superfield and its first component by the same letter. This slight abuse of notation, hopefully, does not lead to a confusion.}

The Goldstone-Maxwell action \((2.25)\) is thus given by a sum of the chiral N=1 superspace integral (= super-Maxwell or super-\(A\) invariant) and the full N=1 superspace integral (= super Euler-Heisenberg or super-\(E\) invariant), with the latter being modified by the ‘formfactor’ \( f(D^2 W^2, \bar{D}^2 \bar{W}^2) \). The only quartic (higher derivative) combination, \( \frac{1}{4} (F^2)^2 - F^4 \), that can be supersymmetrized up to the full (EH) N=1 superinvariant, was earlier identified in ref. \[31\] by using helicity conservation of four-particle scattering amplitudes in N=1 supersymmetric scalar QED.

In terms of our ‘smart’ variables (2.16) the bosonic BI lagrangian in the form (2.18) can be immediately supersymmetrized to the form (2.25). The N=1 supersymmetric Goldstone-Maxwell action is therefore given by the N=1 supersymmetric Born-Infeld action. As was recently argued in ref. \[23\], the same sBI action emerges from the N=2 supersymmetric non-linear \( APT \text{ model} \) \[32\], where N=2 supersymmetry is partially broken to N=1 supersymmetry due to the non-linearity of the Seiberg-Witten-type action for an N=2 vector supermultiplet in the presence of ‘electric’ and ‘magnetic’
Fayet-Iliopoulos (FI) terms, after ‘integrating out’ (or decoupling) the massive N=1 scalar superfield component of the N=2 vector superfield.

It is worth mentioning that a positivity of the ‘discriminant’ (under the square root in the denominator of eq. (2.26)) is ensured by a positivity of the BI determinant on the left-hand-side of eq. (2.17). A causal (no ghosts) propagation of the physical fields in the sBI theory is achieved due to the Goldstone nature of the whole N=1 vector multiplet and its irreducibility with respect to unbroken supersymmetry. The auxiliary field $D$ does not propagate, with $D = 0$ being an on-shell solution to its equation of motion.

The whole non-linear structure of the N=1 sBI action (2.25) is dictated by the hidden non-linearly realised $S$-supersymmetry whose transformation laws can be found in ref. [22]. It is, therefore, not very surprising that the same action (2.25) can be nicely represented as the ‘non-linear sigma-model’ [22]

$$S_{N=1 \text{ GM}} = \frac{1}{4} \int d^4 x d^2 \theta \, X + \text{h.c.} \ ,$$  

where the chiral $N = 1$ superspace lagrangian $X$ obeys a non-linear N=1 superfield constraint [22],

$$X = \frac{1}{4} XD^2 X + W^2 \ .$$  

The uniqueness of the N=1 Goldstone-Maxwell action (2.28) now becomes apparent because of the identity $X^2 = 0$. The N=1 chiral superfield $X$ can be interpreted as the chiral N=1 superfield component in the N=1 superspace description of the N=2 vector superfield [23] (see also subsect. 2.3).

The e.-m. self-duality of the BI action is also naturally generalized to the N=1 supersymmetric e.-m. self-duality of the N=1 sBI action, when using the N=1 supersymmetric analogue

$$S_{N=1} = S_{N=1 \text{ GM}} + \left[ \frac{i}{2} \int d^4 x d^2 \theta \, \tilde{W}^\alpha W_\alpha + \text{h.c.} \right]$$  

of the bosonic first-order action (2.20). Here the N=1 chiral Lagrange multiplier superfield $\tilde{W}^\alpha$ has been introduced to enforce the $N = 1$ Bianchi identity given by the second equation (2.21) on $W_\alpha$ that is merely an N=1 chiral superfield in eq. (2.30). Hence, on the one side, varying the action (2.30) with respect to $\tilde{W}^\alpha$ gives us back the action (2.25), whereas, on the other side, varying the action (2.30) with respect to $W_\alpha$ instead, solving the arising equation on $W_\alpha$ in terms of $\tilde{W}^\alpha$, and substituting the result back into the action (2.30), yield the same sBI action (2.25) in terms of $\tilde{W}^\alpha$. This is the $N = 1$ supersymmetric e.-m. self-duality in terms of N=1 superfields [22, 23].
2.3 N=2 sBI action

A manifestly N=2 supersymmetric and e.-m. self-dual extension of the 4d BING action (1.8) with two real scalars can be constructed in N=2 superspace as the N=2 generalization of the bosonic BI action (2.1) \[26\]. Two massless Goldstone bosons and Maxwell vector can be unified into a single massless N=2 vector supermultiplet. The N=2 sBI action \[26\] can be considered either as the Goldstone action associated with partial breaking of N=4 supersymmetry down to N=2 in 4d, with the Goldstone-Maxwell N=2 supermultiplet with respect to unbroken N=2 supersymmetry, or, equivalently, as the gauge-fixed N=2 superfield action of a D-3-brane in flat six-dimensional ambient spacetime (sect. 1). This action can be most easily constructed in the standard N=2 superspace parametrized by the coordinates

\[ Z^M = (x^\mu, \theta^\alpha_i, \bar{\theta}^{\dot{\alpha}i}) \]

where \( \mu = 0, 1, 2, 3 \), \( \alpha = 1, 2 \), \( i = 1, 2 \), and \( \theta^\alpha_i = \bar{\theta}^{\dot{\alpha}i} \). The Goldstone-Maxwell N=2 supermultiplet is described in this N=2 superspace by a restricted chiral (complex scalar) N=2 superfield \( W \) \[33, 34\]. The N=2 superspace approach automatically implies manifest (linearly realised) N=2 extended supersymmetry. One cannot, however, use a similar N=4 superspace approach, in order to construct a 4d sBI/BING action with manifest N=4 supersymmetry, since a 4d gauge field theory with linearly realised N=4 supersymmetry merely exists in its on-shell version, in the standard N=4 superspace in 4d.

The restricted chiral N=2 superfield \( W \) is an off-shell irreducible N=2 superfield satisfying the N=2 superspace constraints

\[ \bar{D}_{a\dot{i}} W = 0 \, , \quad D^4 W = \Box W \, , \quad (2.31) \]

where we have used the following realisation of the supercovariant N=2 superspace derivatives (with vanishing central charge) \[34\]:

\[ D^i = \partial + i\theta^a_i \partial_{a\dot{\alpha}} \, , \quad \bar{D}_{a\dot{i}} = -\partial - i\theta^{\dot{\alpha}i} \partial_{a\alpha} \, ; \quad D^4 = \frac{1}{12} D^{\dot{\alpha}a} D^\alpha D^\beta D_j D_j^\beta \, . \quad (2.32) \]

The first constraint in eq. (2.31) is just the N=2 generalization of the usual N=1 chirality condition, whereas the second one can be considered as the generalized reality condition \[33, 34\] that has no analogue in N=1 superspace. A component solution to eq. (2.31) in the N=2 chiral superspace (parametrized by the coordinates \( y^\mu = x^\mu - \frac{i}{2} \theta^\alpha_i \sigma_{\alpha\beta} \bar{\theta}^{\dot{\alpha}i} \) and \( \theta^j_\beta \)) reads

\[ W(y, \theta) = a(y) + \theta^\alpha_i \psi^i_\alpha(y) - \frac{1}{2} \theta^\alpha_i \bar{\psi}^j_\alpha \theta^j_\beta \cdot \bar{D}(y) \]

\[ + \frac{i}{8} \theta^\alpha_i (\sigma^{\mu\nu})_{\alpha\beta} \theta^j_\beta F_{\mu\nu}(y) - i (\theta^3)^{i\dot{\alpha}} \partial \psi^b_\dot{\alpha}(y) + \theta^4 \Box a(y) \, , \quad (2.33) \]
where we have introduced a complex scalar $a$, a chiral spinor doublet $\psi$, a real isovector $\vec{D} = \frac{1}{2}(\vec{\tau}^i) j_i \equiv \frac{1}{2} \text{tr}(\vec{D})$, $\text{tr}(\tau_m \tau_n) = 2 \delta_{mn}$, and a real antisymmetric tensor $F_{\mu\nu}$ as the field components of $W$, while $F_{\mu\nu}$ has to satisfy the ‘Bianchi identity’

$$
\varepsilon^{\mu\nu\lambda\rho} \partial_\nu F_{\lambda\rho} = 0 ,
$$

(2.34)

whose solution is just given by $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ in terms of the vector gauge field $A_\mu$ subject to the gauge transformations $\delta A_\mu = \partial_\mu \lambda$. The N=2 supersymmetry transformation laws for the components can be found e.g., in ref. [34].

The well-known N=2 supersymmetric extension of the Maxwell lagrangian $A = -\frac{1}{4} F_{\mu\nu}^2$ is given by

$$
\frac{1}{2} \int d^4 \theta W^2 = -a \Box \bar{a} - \frac{i}{2} \psi^\alpha \partial_\alpha \bar{\psi}^{\ast j} - \frac{1}{2} (F^+)^2 + \frac{1}{2} \bar{D}^2 .
$$

(2.35)

The Maxwell energy-momentum tensor squared or, equivalently, the EH lagrangian $E = \frac{1}{8} (F^+)^2 (F^-)^2$, is also easily extended in N=2 superspace,

$$
\int d^4 \theta d^4 \bar{\theta} W^2 \bar{W}^2 = (F^+)^2 (F^-)^2 + (\bar{D}^2)^2 - \bar{D}^2 F^2 + \ldots .
$$

(2.36)

This N=2 supersymmetric generalization of the Euler-Heisenberg lagrangian also arises as the leading (one-loop) non-holomorphic (non-BPS) contribution to the N=2 gauge low-energy effective action in the interacting N=2 supersymmetric quantum field theory of a charged hypermultiplet minimally coupled to an N=2 Maxwell supermultiplet [36, 37].

The gauge-invariant N=2 superfield strength squared, $W^2$, is an N=2 chiral but not a restricted N=2 chiral superfield. As is clear from eq. (2.35), the first component of the N=2 anti-chiral superfield $K \equiv D^4 W^2$ takes the form

$$
K \equiv D^4 W^2 = -2a \Box \bar{a} - (F^+)^2 + \bar{D}^2 + \ldots .
$$

(2.37)

It is now straightforward to N=2 supersymmetrize the BI lagrangian (2.11) by engineering the proper N=2 superspace invariant,

$$
L = \frac{1}{2} \int d^4 \theta W^2 + \frac{1}{8} \int d^4 \theta d^4 \bar{\theta} \mathcal{Y}(K, \bar{K}) W^2 \bar{W}^2 ,
$$

(2.38)

whose ‘formfactor’ $\mathcal{Y}(K, \bar{K})$ is dictated by the known bosonic structure function $Y(A, E)$ in eq. (2.18). Note that the vector-dependent contributions to the first scalar components of the N=2 superfields $K$ and $\bar{K}$ are simply related to $A$ and $E$ as

$$
K + \bar{K} = 4A , \quad K \bar{K} = 8E ,
$$

(2.39)
i.e. they are just the roots of a quadratic equation

\[ k^2 - 4Ak + 8E = 0 . \]  

(2.40)

We find

\[ \mathcal{Y}(K, \bar{K}) = \frac{1 - \frac{1}{4}(K + \bar{K}) - \sqrt{(1 - \frac{1}{4}K - \frac{1}{4}\bar{K})^2 - \frac{1}{4}K\bar{K}}}{K\bar{K}} \]

\[ = 1 + \frac{1}{4}(K + \bar{K}) + O(K^2) . \]  

(2.41)

The proposed N=2 sBI action [26]

\[ S[W, \bar{W}] = \frac{1}{2} \int d^4x d^4\theta W^2 + \frac{1}{8} \int d^4x d^4\theta d^4\bar{\theta} \mathcal{Y}(K, \bar{K}) W^2 \bar{W}^2 \]

\[ = \frac{1}{2} \int d^4x d^4\theta \left\{ W^2 + \frac{1}{4}D^4 \left[ \mathcal{Y}(K, \bar{K}) W^2 \bar{W}^2 \right] \right\} \]

\[ = \frac{1}{2} \int d^4x d^4\theta W^{2\text{improved}} , \]  

(2.42)

can be nicely rewritten to the ‘non-linear sigma-model’ form

\[ S[W, \bar{W}] = \frac{1}{2} \int d^4x d^4\theta X , \]  

(2.43)

where the N=2 chiral superfield \( X \equiv W^{2\text{improved}} \) has been introduced as a solution to the non-linear N=2 superfield constraint

\[ X = \frac{1}{4}X \bar{D}^4 \bar{X} + W^2 . \]  

(2.44)

Eq. (2.43) is the ‘improved’ non-linear extension of the N=2 Maxwell lagrangian in N=2 chiral superspace. The existence of the ‘non-linear sigma-model’ form of our action (2.42) implies its uniqueness and supports its interpretation as the Goldstone action associated with partial breaking of N=4 supersymmetry down to N=2, with the N=2 vector multiplet as a Goldstone multiplet, in a remarkable similarity to the N=1 supersymmetric Goldstone-Maxwell theory discussed in the previous subsection 2.2. In particular, eq. (2.44) can be considered as the N=2 superfield generalization of the N=1 superfield non-linear constraint (2.29).

Like the N=1 sBI action, our N=2 action (2.42) does not lead to the propagating auxiliary fields \( \bar{D} \), despite of the presence of higher derivatives to all orders. Though the equations of motion for the auxiliary fields do not seem to be algebraic, the kinetic terms for them do not appear, with \( \bar{D} = 0 \) being an on-shell solution. Non-vanishing expectation values for fermionic and scalar composite operators in front of the ‘dangerous’ interacting terms that could lead to a propagation of the auxiliary fields are also forbidden because of the vanishing N=2 central charge and unbroken
Lorentz- and N=2 super-symmetries. We recall that the N=2 central charge $Z$ in abelian N=2 supersymmetric gauge field theories can be identified with a (complex constant) vacuum expectation value $\langle a \rangle$ of the first scalar component of the Maxwell N=2 superfield strength, $\langle W \rangle = \langle a \rangle = Z$ (see e.g., ref. [37]).

To verify that eq. (2.42) is the N=2 supersymmetric extension of the N=1 sBI action indeed, it is useful to rewrite it in terms of N=1 superfields by integrating over a half of the N=2 superspace anticommuting coordinates. The standard identification of the N=1 superspace anticommuting coordinates, $\theta^\alpha \rightarrow \theta^\alpha$, and $\bar{\theta}^\dot{\alpha} \rightarrow \bar{\theta}^\dot{\alpha}$, (2.45) implies the N=1 superfield projection rule

$$G = G(Z)|$$

(2.46)

where $|$ means taking a $(\theta^\alpha, \bar{\theta}^\dot{\alpha})$-independent part of an N=2 superfield $G(Z)$. As regards the N=2 restricted chiral superfield $W$, its N=1 superspace constituents are given by N=1 complex superfields $\Phi$ and $W_\alpha$,

$$W| = \Phi, \quad D^2\alpha W| = W_\alpha, \quad \frac{1}{2}(D^2\alpha)(D^2)_{\dot{\alpha}} W| = D^2\Phi,$$

(2.47)

which follow from the N=2 constraints (2.31). The reality condition given by the second equation (2.31) implies the N=1 superfield Bianchi identity

$$D^\alpha W_\alpha = \bar{D}_\dot{\alpha} \bar{W}^\dot{\alpha},$$

(2.48)

as well as the relations

$$K| = D^2 \left( W^\alpha W_\alpha + 2\Phi D^2\Phi \right).$$

$$\left(D^2\right)^\alpha\dot{\alpha} K| = 2iD^2\partial^\beta (W_\beta\Phi),$$

$$\left(D^2\right)^\alpha(D^2)\dot{\alpha} K| = -4D^2\partial_\mu \Phi (\Phi \partial^\mu \Phi),$$

(2.49)

together with their conjugates. Eqs. (2.47), (2.48) and (2.49) are enough to perform a reduction of any N=2 superspace action depending upon $W$ and $\bar{W}$ into N=1 superspace by differentiation,

$$\int d^4\theta \rightarrow \int d^2\theta \frac{1}{2}(D^2\alpha)(D^2)_{\dot{\alpha}},$$

$$\int d^4\theta d^4\bar{\theta} \rightarrow \int d^2\theta d^2\bar{\theta} \frac{1}{2}(D^2\alpha)(D^2)_{\dot{\alpha}} \frac{1}{2}(D^2\dot{\alpha})(D^2)\alpha.$$

(2.50)

\footnote{We underline particular values $i = 1, 2$ of the internal SU(2) indices, and use the N=1 notation $D^2 = \frac{1}{2}(D_\alpha)(D_\alpha)$ and $\bar{D}^2 = \frac{1}{2}(\bar{D}_\dot{\alpha})(\bar{D}_\dot{\alpha})$ here.}
It is now straightforward to calculate the N=1 superfield form of the N=2 action (2.42). For our purposes, it is enough to notice that the first term in eq. (2.42) gives rise to the kinetic terms for the N=1 chiral superfields $\Phi$ and $W_\alpha$,

$$\text{Re} \int d^2 \theta \left( \frac{1}{2} W^\alpha W_\alpha + \Phi \bar{D}^2 \bar{\Phi} \right),$$

(2.51)

whereas the N=1 vector multiplet contribution arising from the second term in eq. (2.42) is given by

$$\frac{1}{8} \int d^2 \theta d^2 \bar{\theta} \mathcal{Y}(K\), \bar{K}\)W^\alpha W_\alpha \bar{W}^\alpha + \ldots,$$

(2.52)

where the dots stand for $\Phi$-dependent terms. The $W$-dependent contributions of eqs. (2.51) and (2.52) exactly coincide with the N=1 supersymmetric extension (2.25) of the BI action after taking into account that the vector field dependence in the first component of the N=1 superfield $K|$ is given by

$$K| = D^4 W^2| = 2D^2(\frac{1}{2} W^\alpha W_\alpha + \Phi \bar{D}^2 \bar{\Phi})| = -(F^+)^2 + D^2 + \ldots,$$

(2.53)

and similarly for $\bar{K}|$.

The dependence of the N=2 sBI action upon the N=1 chiral part $\Phi$ of the N=2 vector multiplet is clearly of most interest, since it is entirely dictated by N=2 extended supersymmetry and electric-magnetic self-duality. Let’s now take $W_\alpha = 0$ in the action (2.42), and calculate merely the leading terms depending upon $\Phi$ and $\bar{\Phi}$ there. After some algebra one gets the following N=1 superspace action:

$$S[\Phi, \bar{\Phi}] = \int d^4 x d^2 \theta d^2 \bar{\theta} \left[ \Phi \bar{\Phi} - 4(\Phi \partial^\mu \Phi)(\bar{\Phi} \partial_\mu \bar{\Phi}) + 4 \partial^\mu (\Phi \bar{\Phi}) \partial_\mu (\Phi \bar{\Phi}) \right] + \ldots,$$

(2.54)

where the dots stand for the higher order terms depending upon the derivatives of $\mathcal{Y}$. The field components of the N=1 chiral superfield $\Phi$ are conveniently defined by the projections

$$\Phi| = \frac{1}{\sqrt{2}} \phi \equiv \frac{1}{\sqrt{2}} (P + iQ), \quad D_\alpha \Phi| = \psi_\alpha, \quad D^2 \Phi| = F,$$

(2.55)

where $P$ is a real physical scalar, $Q$ is a real physical pseudo-scalar, $\psi_\alpha$ is a chiral physical spinor, and $F$ is a complex auxiliary field. It is not difficult to check that the kinetic terms for the auxiliary field components $F$ and $\bar{F}$ cancel in eq. (2.54), as they should. This allows us to simplify a calculation of the quartic term in eq. (2.54) even further by going on-shell, i.e. assuming that $\Box \phi = F = 0$ there, even though it is not really necessary. A simple calculation now yields

$$S[\phi, \bar{\phi}] = \int d^4 x \left\{ \partial^\mu \phi \partial_\mu \bar{\phi} - 2(\partial_\mu \phi \partial_\nu \phi \bar{\phi})^2 + (\partial_\mu \phi \partial^\nu \bar{\phi})^2 \right\},$$

(2.56)
which exactly coincides with the leading terms in the derivative expansion of the Nambu-Goto (NG) action

\[ S = - \int d^4x \sqrt{-\det(\eta_{\mu\nu} + \partial_\mu P \partial_\nu P + \partial_\mu Q \partial_\nu Q)}. \]  

(2.57)

Eq. (2.57) yields the effective action of a (static-gauge) 3-brane in flat six-dimensional ambient spacetime, with the Goldstone scalars \((P, Q)\) being two collective coordinates corresponding to a ‘transverse’ motion of the 3-brane.\(^9\) A 3-brane solution to \((1, 0)\), 6d super-Maxwell theory coupled to chiral (scalar) multiplets was constructed in ref. \([19]\). The solution of ref. \([19]\) breaks translational invariance in two spacial directions and half of the 6d supersymmetry. This observation strongly indicates on a possible six-dimensional origin of our four-dimensional \(N=2\) supersymmetric BI action that should be derivable by dimensional reduction from a supersymmetric BI action in six spacetime dimensions after identifying the extra two components of a six-dimensional abelian vector potential with the scalars \(P\) and \(Q\). The very existence of the super-BI action in six dimensions is enough to ensure the Goldstone nature of scalars in eq. (2.56) and (2.57), as well as the \textit{off-shell} invariance of our 4d, \(N=2\) action under constant shifts of these scalars. Finding this 6d, manifestly \((1,0)\) supersymmetric BI action, which can be considered as the top or \textit{master} sBI action in superspace, is one of our main results in this paper (see sect. 3).

To indicate on the possibility of adding some additional structure given by a magnetic FI term into our 4d, \(N=2\) theory, it is worth mentioning here that the \(N=2\) superspace constraints (2.31) imply

\[ \Box \left( D^{ij} W - \bar{D}^{ij} \bar{W} \right) = 0. \]  

(2.58)

This means that the function \(\text{Im}(D^{ij} W)\) is harmonic, and, therefore, it should be constant, \(^{10}\) i.e.

\[ D^{ij} W - \bar{D}^{ij} \bar{W} = 4i M^{ij}. \]  

(2.59)

Taking into account a constant (FI) vector \(\bar{M}\) in the constraint (2.59) is equivalent to adding a ‘\textit{magnetic}\’ Fayet-Iliopoulos (FI) term to the dual action \([32]\). The FI term can be formally removed from the constraint (2.59) by a field redefinition of \(W\), i.e. at the expense of adding a constant imaginary part to the auxiliary scalar triplet \(\bar{D}\) of the \(N=2\) vector multiplet in eq. (2.33). Let’s recall that the APT model \([32]\) is defined by adding the usual (electric) and magnetic FI terms to the general (Seiberg-Witten-type) \(N=2\) chiral action in terms of \(W\) \([32, 34]\).

\( ^9\)An \(N=1\) superspace description of the two transverse 3-brane coordinates in terms of \(N=1\) chiral, complex linear and real linear Goldstone superfields was recently obtained in ref. \([24]\).

\( ^{10}\)We assume that all components of the superfield \(W\) are regular in spacetime.
The 4d, N=2 ‘Bianchi identity’ can be enforced by introducing an unconstrained real N=2 superfield Lagrange multiplier (known as the Mezincescu pre-potential \[38\])

\[ \vec{L} = \frac{1}{2} (\vec{\tau})^i_j L^j_i \equiv \frac{1}{2} \text{tr}(\vec{\tau} L) \]

in the first-order N=2 superspace action (cf. ref. \[39\])

\[
S[W, \bar{W}] \to S[W, \bar{W}; L] = S[W, \bar{W}] + i \int d^4x d^4\theta d^4\bar{\theta} L_{ij} \left(D^{ij}W - \bar{D}^{ij}\bar{W}\right),
\]

\[ (2.60) \]

where the N=2 superfield \(W\) is now a chiral (unrestricted) N=2 superfield, while

\[ W_{\text{magn.}} \equiv \bar{D}^4 D^{ij} L_{ij} \]

is the dual or ‘magnetic’ N=2 superfield strength that automatically satisfies the N=2 constraints (2.31) due to its defining equation (2.61). Varying the action (2.60) with respect to \(W\), solving the resulting algebraic equation on \(W\) in terms of \(W_{\text{magn.}}\), and substituting the result back into the action (2.60), results in the dual N=2 action \(S[W_{\text{magn.}}, \bar{W}_{\text{magn.}}]\) that takes exactly the same form as eq. (2.42). In other words, it is self-dual with respect to the N=2 supersymmetric electric-magnetic duality. Of course, the (1,0) supersymmetric BI action in six spacetime dimensions (sect. 3) cannot be e.-m. self-dual since the dual to a vector is again going to be a vector in four spacetime dimensions only.

The 4d, N=2 Maxwell multiplet considered in this subsection can be interpreted as the Goldstone multiplet associated with partial spontaneous breaking of rigid N=4 supersymmetry down to N=2 supersymmetry, with the action (2.42) being the corresponding Goldstone-Maxwell N=2 superfield action (cf. ref. \[10\]). The whole non-linear structure of this action dictated by the non-linear superfield constraint (2.44) should therefore be entirely determined by hidden (non-linearly realised, or broken) N=2 supersymmetry (cf. refs. \[1, 22\]). The transformation laws of the spontaneously broken N=2 supersymmetry can be deduced \[10\] from the general theory of non-linear realizations \[3, 25\], despite of the fact that the general theory \[3, 25\] does not give us any clues about a non-perturbative construction of the Goldstone actions beyond the Noether (trials and errors, order-by-order) method.

The invariance of our action (2.42) under constant shifts of the N=2 superfield strength \(W\) is, of course, a necessary condition for its Goldstone interpretation. It is easy to verify this symmetry if \(W\) is subject to the on-shell condition \(\Box W = 0\). The 6d super-BI action (sect. 3) dimensionally reduced down to four dimensions automatically implies this symmetry off-shell.
3 Gauge-fixed ‘spacetime-filling’ D-5-brane action

In this section we generalize the results of sect. 2 by constructing a (1,0) supersymmetric Born-Infeld-Goldstone action in 6d superspace.

3.1 Group theory: $SU(4)$ versus $SO(1,5)$

Let’s now consider flat six-dimensional (6d) spacetime with a Minkowski signature $\eta_{\mu\nu} = \text{diag}(+, -, -, -, -, -)$, where the vector indices take values $\mu, \nu = 0, 1, 2, 3, 4, 5$. The 6d Lorentz group $SO(1,5)$ has rank three, so that there are three independent Casimir eigenvalues in 6d instead of two in the 4d case (sect. 2). The obvious choice of the independent Lorentz-invariant products of a 6d abelian gauge field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is given by

$$
F^2 \equiv F_{\mu\nu} F^{\mu\nu},
$$

$$
F^4 \equiv F_{\mu\nu} F^{\nu\lambda} F^{\lambda\rho} F^{\rho\mu},
$$

$$
F^6 \equiv F_{\mu\nu} F^{\nu\lambda} F^{\lambda\rho} F^{\rho\sigma} F^{\sigma\tau} F^{\tau\mu}.
$$

(3.1)

It is straightforward to calculate any Lorentz-invariant function of $F_{\mu\nu}$ in terms of the invariants (3.1). In the BI case, we find

$$
- \det (\eta_{\mu\nu} + F_{\mu\nu}) = 1 + \frac{1}{2} F^2 + \frac{1}{2^3} (F^2)^2 - \frac{1}{2^2} F^4
$$

$$
\quad + \frac{1}{3 \cdot 2^4} (F^2)^3 - \frac{1}{2^3} F^2 F^4 + \frac{1}{2 \cdot 3} F^6,
$$

(3.2)

and

$$
- \sqrt{- \det (\eta_{\mu\nu} + F_{\mu\nu})} = -1 - \frac{1}{2^2} F^2 - \frac{1}{2^3} \left[ \frac{1}{4} (F^2)^2 - F^4 \right]
$$

$$
\quad + \frac{1}{2^5} \left[ F^2 F^4 - \frac{1}{2^2} \frac{1}{3} (F^2)^3 - \frac{2^3}{3} F^6 \right]
$$

$$
\quad - \frac{1}{2^8} \left[ 3^2 (F^2)^2 F^4 - \frac{2^4}{3} F^2 F^6 - 2 (F^4)^2 - \frac{7}{2^3} \frac{3}{3} (F^2)^4 \right] + O(F^{10}),
$$

(3.3)

where we have used the expansion

$$
\sqrt{1 + x} = 1 + \frac{1}{2} x - \frac{1}{2^3} x^2 + \frac{1}{2^4} x^3 - \frac{5}{2^7} x^4 + O(x^5).
$$

(3.4)

The leading term in the expansion (3.3) of the BI action is given by the 6d Maxwell lagrangian, $-\frac{1}{4} F^2$, as expected. The quartic (of the fourth order in spacetime derivatives) terms in eq. (3.3) occur in the same combination as in the 4d case (see the
right-hand-side of the first line of eq. (2.11) in subsect. 2.1), \( \frac{1}{8} \left[ \frac{1}{4} (F^2)^2 - F^4 \right] \), equal to the 6d Maxwell energy-momentum tensor squared. It agrees with (i) earlier perturbative calculations of the gauge low-energy effective action of open superstrings [41], and (ii) restrictions implied by supersymmetry in 10d [42] and 6d [43]. In order to make these restrictions manifest in 6d, it is useful to switch to a four-component (spinor) \( SU(4) \) notation in 6d, which is similar to the two-component spinor notation in 4d, by using a complex ‘rotation’ of Lie algebra of \( SO(1,5) \) to that of \( SU(4) \) [44].

Let \( \Gamma^m \) be 8 \times 8 gamma matrices in 6d, that satisfy a Clifford algebra \( \{ \Gamma^m, \Gamma^n \} = 2\eta^{mn} \). Let’s choose a representation of these matrices in the form

\[
\Gamma^m = \begin{pmatrix}
0 & (\Sigma^m)_{\alpha\beta} \\
(\bar{\Sigma}^m)_{\dot{\alpha}\dot{\beta}} & 0
\end{pmatrix}, \quad \alpha = 1, 2, 3, 4,
\]

where the 4 \times 4 matrices \( \Sigma^m \) and \( \bar{\Sigma}^m \) have been introduced. They have to satisfy the relations

\[
\Sigma^m \bar{\Sigma}^n + \Sigma^n \bar{\Sigma}^m = 2\eta^{mn},
\]

\[
\bar{\Sigma}^m \Sigma^n + \bar{\Sigma}^n \Sigma^m = 2\eta^{mn}.
\]

A solution to eq. (3.7) exists in the form

\[
\Sigma^m = (1, \gamma^i), \quad \bar{\Sigma}^m = (1, -\gamma^i), \quad m = (0, i), \quad i = 1, 2, 3, 4, 5,
\]

where \( \gamma^i \) are hermitian 4 \times 4 gamma matrices in five euclidean dimensions,

\[
\{ \gamma^i, \gamma^j \} = 2\delta^{ij}.
\]

An explicit representation of \( \gamma^i = (\bar{\gamma}, \gamma_4, \gamma_5) \), with \( \bar{\gamma} \) standing for \( (\gamma_1, \gamma_2, \gamma_3) \) and \( \gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4 \), is given by [45]

\[
\bar{\gamma} = \begin{pmatrix}
0 & -i\vec{\sigma} \\
i\vec{\sigma} & 0
\end{pmatrix}, \quad \gamma_4 = \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} 0 & -I_2 \\ -I_2 & 0 \end{pmatrix},
\]

where \( \vec{\sigma} \) are Pauli matrices, \( \sigma_1\sigma_2\sigma_3 = iI_2 \).

\[11\] In order to distinguish between 6d vector and spinor indices both denoted by greek letters, we also introduce here latin letters to denote tangent space vector components. They are trivially related to spacetime vector components, \( V^\mu = e^\mu_m V^m \), via a flat 6-bein \( e^\mu_m = \delta^\mu_m \). This notation shall, however, be abandoned in the next subsections where all vector indices, if any, are hidden, while latin indices are used to denote internal \( SU(2) \) symmetry.
The 6d Lorentz generators in a non-chiral spinor representation are given by
\[
\Gamma_{mn} = \frac{1}{4} \left[ \Gamma^m, \Gamma^n \right] = \begin{pmatrix}
(\Sigma_{mn})^\alpha_\beta & 0 \\
0 & (\Sigma_{mn})^\alpha_\beta
\end{pmatrix},
\]
where
\[
\Sigma_{mn} = \frac{1}{4} \left( \Sigma^m \Sigma^n - \Sigma^n \Sigma^m \right),
\]
and similarly for \( \hat{\Sigma}_{mn} \). The chiral spinor generators \( \Sigma_{mn} \) satisfy the Lorentz algebra
\[
[\Sigma_{mn}, \Sigma_{pq}] = \eta_{mq} \Sigma_{np} + \eta_{np} \Sigma_{mq} + \eta_{mp} \Sigma_{qn} + \eta_{np} \Sigma_{mq},
\]
while they can be used to convert any antisymmetric tensor \( F_{mn} \) into a traceless \( 4 \times 4 \) matrix \( \hat{F}^{\alpha \beta} \) as follows:
\[
\hat{F}^{\alpha \beta} = (\Sigma_{mn})^\alpha_\beta F_{mn}, \quad \text{tr} \hat{F} = 0.
\]

As a preparation for (1,0) supersymmetrization of 6d BI action (subsect. 3.3), let's introduce another set of independent Lorentz-invariant \( F \)-products:
\[
\text{tr} \hat{F}^2, \quad \text{tr} \hat{F}^4, \quad \text{tr} \hat{F}^6.
\]
They are, of course, linearly related to those of eq. (3.1). We find
\[
\text{tr} \hat{F}^2 = -2F^2, \\
\text{tr} \hat{F}^4 = 3(F^2)^2 - 4F^4, \\
\text{tr} \hat{F}^6 = -32F^6 - \frac{15}{2} F^2 \left[ (F^2)^2 - 4F^4 \right].
\]
It is straightforward to verify that e.g., \( \text{tr} \hat{F}^8 \) is not independent, being a function of those in eq. (3.16), namely,
\[
\text{tr} \hat{F}^8 = \frac{1}{4}(\text{tr} \hat{F}^4)^2 + \frac{4}{3} \text{tr} \hat{F}^6 \text{tr} \hat{F}^2 - \frac{3}{4} \text{tr} \hat{F}^4 (\text{tr} \hat{F}^2)^2 + \frac{5}{36} (\text{tr} \hat{F}^2)^4.
\]

After some algebra, we find
\[
- \det (\eta_{mn} + F_{mn}) = 1 - \frac{1}{2^2} \text{tr} \hat{F}^2 + \frac{1}{2^4} \left[ \text{tr} \hat{F}^4 - \frac{1}{4} (\text{tr} \hat{F}^2)^2 \right]
+ \frac{1}{2^8} \left[ \text{tr} \hat{F}^2 \text{tr} \hat{F}^4 - \frac{1}{6} (\text{tr} \hat{F}^2)^3 - \frac{4}{3} \text{tr} \hat{F}^6 \right],
\]
and
\[
\sqrt{- \det (\eta_{mn} + F_{mn})} = 1 - \frac{1}{2^3} \text{tr} \hat{F}^2 + \frac{1}{2^5} \left[ \text{tr} \hat{F}^4 - \frac{1}{2} (\text{tr} \hat{F}^2)^2 \right]
- \frac{1}{3 \cdot 2^7} \text{tr} \hat{F}^6 + \frac{3}{2^9} \text{tr} \hat{F}^2 \text{tr} \hat{F}^4 - \frac{7}{3 \cdot 2^{10}} (\text{tr} \hat{F}^2)^3
- \frac{1}{2^{11}} (\text{tr} \hat{F}^4)^2 + \frac{5}{2^{12}} \text{tr} \hat{F}^4 (\text{tr} \hat{F}^2)^2 - \frac{1}{3 \cdot 2^{10}} \text{tr} \hat{F}^6 \text{tr} \hat{F}^2
- \frac{3 \cdot 5}{2^{15}} (\text{tr} \hat{F}^2)^4 + O(F^{10}).
\]
The easiest way to get the key equations (3.2) and (3.16) is to take advantage of their 6d Lorentz invariance by choosing $F_{mn}$ in the form

$$F_{mn} = \begin{pmatrix}
-\lambda_1 & \lambda_2 \\
\lambda_1 & -\lambda_2 \\
-\lambda_2 & \lambda_3 \\
\lambda_3 & -\lambda_1
\end{pmatrix}$$

(3.20)

similar to that of eq. (2.2), in terms of three independent real ‘eigenvalues’ $\vec{\lambda}$. Eq. (3.2) then amounts to a classical Miura transform in terms of symmetric polynomials of $\vec{\lambda}$, whereas the linear transform (3.16) becomes apparent when using a basis comprising all independent antisymmetric products of $\gamma$-matrices. The coefficients are, of course, Lorentz-invariant and independent upon the representation of $\gamma$-matrices used to calculate them. Similar techniques were used for a calculation of perturbative anomalies in chiral 6d supersymmetric gauge field theories and 6d supergravity [46, 47].

3.2 6d Maxwell (1,0) supermultiplet in superspace

The (1,0) superspace techniques in 6d were proposed e.g., in refs. [48, 44]. In this subsection we briefly review the construction of ref. [44], and then extend it for a later use in the next subsect. 3.3.

Chiral 6d spinors can be equivalently represented by symplectic Majorana-Weyl (MW) spinors $\psi^\alpha_i$ carrying an extra $SU(2) \cong Sp(1)$ index $i = 1, 2$ and obeying the MW-type condition [49]

$$(\psi^\alpha_i)^\ast \equiv \bar{\psi}^{\dot{\alpha}i} = \varepsilon^{ij} B^{\dot{\alpha} \beta} \psi^\beta_j,$$

(3.21)

where $\varepsilon^{ij}$ is antisymmetric Levi-Civita symbol, $\varepsilon^{ij} \varepsilon_{kj} = \delta_i^k$, and the matrix $B$ can be chosen to be unitary and antisymmetric, $BB^\dagger = 1$ and $B^T = -B$. The existence of the matrix $B$ follows from the uniqueness of a non-trivial representation of 6d Clifford algebra (3.5). Using symplectic MW spinors in 6d allows one to directly compare 6d chiral supersymmetry to N=2 extended supersymmetry in 4d (subsect. 2.3) where symplectic MW spinors also naturally appear and make the internal $SU(2)$ symmetry manifest [50].

The matrix $B$ relates dotted and undotted 6d spinor indices, while any 6d vector index ($\mathbf{6}$ of $SO(1,5)$) can be traded for a pair of undotted spinor indices ($\mathbf{6}$ of $SU(4)$) by using a matrix

$$(\Sigma^m)_{\alpha\dot{\beta}} \equiv (\Sigma^m)_{\alpha\dot{\beta}} = -(\Sigma^m)_{\dot{\beta} \alpha}.$$

(3.22)
We use here the same notation as in ref. [44]. The $SU(2)$ indices are raised and lowered according to the ‘North-West/South-East’ rule,

$$V^i = \varepsilon^{ij} V_j, \quad V_i = V^j \varepsilon_{ji},$$

(3.23)

whereas antisymmetric pairs of spinor indices are raised and lowered by using totally antisymmetric Levi-Civita symbols,

$$V^{\alpha\beta} = \frac{1}{2} \varepsilon^{\alpha\beta\gamma\delta} V_{\gamma\delta}, \quad V_{\alpha\beta} = \frac{1}{2} \varepsilon_{\alpha\beta\gamma\delta} V^{\gamma\delta}.$$  

(3.24)

Note the identities [44]

$$\left(\Sigma^m\right)_{\alpha\beta} \left(\Sigma^n\right)^{\beta\alpha} = 4\eta^{mn}, \quad \left(\Sigma^m\right)_{\alpha\beta} (\Sigma_m)_{\gamma\delta} = -2\varepsilon_{\alpha\beta\gamma\delta}.$$  

(3.25)

The supercovariant derivatives $D^i_{\alpha}$ in flat 6d, (1,0) superspace $Z^A = (x^m, \theta^\alpha_i)$, with the Grassmann anticommuting coordinates $\theta^\alpha_i$, being a symplectic MW spinor, satisfy a (1,0) supersymmetry algebra

$$\{D^i_{\alpha}, D^j_{\beta}\} = i\varepsilon^{ij} \partial_{\alpha\beta}.$$  

(3.26)

It is clear from eq. (3.26) that imposing the $SU(2)$-covariant chirality condition, $D^i_{\alpha} \Phi = 0$, on a 6d superfield $\Phi$ implies $\partial_{\alpha\beta} \Phi = 0$, i.e. $\Phi = \text{const}$. Introducing 6d chiral superfields is, nevertheless, possible at the expense of breaking the $SU(2)$ symmetry [48] .

A massless (1,0) vector multiplet in 6d superspace is described by a symplectic MW spinor superfield strength $W^\alpha_i$ that satisfies some additional off-shell constraints. The superspace constraints in supersymmetric gauge field theories are usually imposed on the invariant field strengths $F_{AB}$ defined by an algebra of the gauge- and supercovariant superspace derivatives $D_A = D_A + iA_A$ [43],

$$[D_A, D_B] = t^{\ C}_{\ AB} D_C + iF_{AB},$$  

(3.27)

where $t^{\ C}_{\ AB}$ is the flat 6d superspace torsion tensor. In particular, one has

$$\{D^i_{\alpha}, D^j_{\beta}\} = i\varepsilon^{ij} D_{\alpha\beta} + iF^\alpha_{ij}.$$  

(3.28)

The off-shell (1,0) vector supermultiplet constraints read [44]

$$F^\alpha_{ij} = 0.$$  

(3.29)

It follows from eq. (3.29) and the Bianchi identities $D_{[A} F_{BC]} = 0$ related to the defining eq. (3.27) that

$$F_{\alpha\beta} = (\Sigma^m)_{\alpha\beta} W^\beta_i.$$  

(3.30)
where $W^\beta_i$ is further constrained by

$$D^i_\alpha W^\beta_j = \hat{F}^\alpha_\beta \delta^i_j + \delta^\alpha_\beta Y^i_j, \quad \text{while} \quad \text{tr} \hat{F} = Y^i_i = 0. \quad (3.31)$$

The leading superfield component of $\hat{F}^{\alpha_\beta}$ can be identified with the 6d Maxwell field strength (3.14), whereas $Y^i_j$ is just the $SU(2)$ triplet of the scalar auxiliary fields. The fermionic superpartners are given by $W^\alpha_i = |\psi^\alpha_i|$.

The 6d superspace constraints on $W^\alpha_i$ can be solved \[14\] in terms of an unconstrained superfield $V_{ij}$ that is the 6d analogue of the 4d, N=2 Mezincescu prepotential in eqs. (2.60) and (2.61). For example, in a WZ gauge, one finds \[10\]

$$V_{ij}(x, \theta) = \theta^\alpha_i \theta^\beta_j A_{[\alpha\beta]}(x) + \theta^\alpha_i \theta^\beta_j \theta^\gamma_k \epsilon_{\alpha\beta\gamma\delta} \psi^{\delta k}(x) + (\theta^4)_{ijkl} Y^{kl}(x), \quad (3.32)$$

where $A_{[\alpha\beta]} = (\Sigma^m)_{\alpha\beta} A_m$ is the abelian vector gauge field. It is, therefore, possible to quantize the (1,0) vector multiplet directly in 6d superspace. When being interpreted as the 6d Goldstone-Maxwell multiplet, it should be associated with partial spontaneous breaking of 6d extended chiral (2,0) supersymmetry down to (1,0) supersymmetry since the Goldstone fermion $\psi^\alpha_i(x)$ and the (1,0) anticommuting spinor coordinate $\theta^\alpha_i$ in eq. (3.32) have the same chirality \[10\].

The fermionic superfield $W^\alpha_i$ transforms in a representation $4 \times 2$ under the symmetry group $SU(4) \times SU(2)$. It is convenient to describe an irreducible product of $W$'s in terms of a Young tableau (cf. refs. [14, 34]). We shall use

$$W^\alpha_i \sim \begin{array}{c} \hline \end{array} \sim 4 \times 2 ,$$

$$W^{\alpha\beta}_{ij} \sim \begin{array}{c} \hline \hline \end{array} \sim 6 \times 3 ,$$

$$W_{ijkl} \sim \begin{array}{c} \hline \hline \hline \end{array} \sim 1 \times 5 ,$$

$$\left(W^6\right)^{ij}_{\alpha\beta} \sim \begin{array}{c} \hline \hline \hline \hline \end{array} \sim 6 \times 3 ,$$

$$\left(W^8\right) \sim \begin{array}{c} \hline \hline \hline \hline \hline \end{array} \sim 1 \times 1 ,$$

where each Young tableau describes an $SU(4)$ irreducible representation (irrep). The corresponding $SU(2)$ irrep is obtained by a reflection of the Young tableau about the main diagonal \[14\]. In their explicit form, the products defined by eq. (3.33) are given

\[12\] The same applies to the products of anticommuting superspace coordinates $\theta^\alpha_i$. 

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by
\[ W^{\alpha \beta}_{ij} = W^{[\alpha}_{(i} W^{\beta]}_{j)} , \]
\[ W_{ijkl} = \varepsilon_{\alpha \beta \gamma \delta} W^{\alpha}_{(i} W^{\beta}_{j} W^{\gamma}_{k} W^{\delta}_{l)} , \]
\[ (W^6)^{ij}_{\alpha \beta} = \frac{\partial}{\partial W^{\alpha}_{(i}} \frac{\partial}{\partial W^{\beta}_{j)} (W^8) , \] (3.34)
\[ (W^8) = \prod_{\alpha, i} W^\alpha_i , \]
where all (anti)symmetrizations are defined with unit weight. Note the identities
\[ W^{(\alpha}_{i} W^{\beta}_{j} W^{\gamma}_{k}) = 0 , \quad (W^9) = 0 . \] (3.35)

A similar description applies to the irreducible products of the supercovariant derivatives in flat 6d superspace. We find (cf. refs. [14, 34])
\[ D^i_{\alpha} \sim \begin{array}{c} \square \\ \hline \hline \end{array} \sim \frac{1}{4} \times 2 , \]
\[ D_{(\alpha \beta)} \sim \begin{array}{c} \square \\ \hline \hline \end{array} \sim \frac{10}{4} \times 1 , \]
\[ D^{ij}_{(\alpha \beta)} \sim \begin{array}{c} \square \\ \hline \hline \end{array} \sim \frac{6}{3} \times 3 , \]
\[ (D^3)^{\alpha}_{ijk} \sim \begin{array}{c} \square \\ \hline \hline \end{array} \sim \frac{1}{4} \times 4 , \]
\[ (D^3)^i_{\alpha \beta \gamma} \sim \begin{array}{c} \square \\ \hline \hline \end{array} \sim \frac{20}{2} \times 2 , \]
\[ D^{ijkl} \sim \begin{array}{c} \square \\ \hline \hline \end{array} \sim \frac{1}{5} \times 5 , \]
\[ (D^4)^{ij}_{\alpha \beta \gamma \delta} \sim \begin{array}{c} \square \\ \hline \hline \end{array} \sim \frac{15}{3} \times 3 , \] (3.36)
\[ (D^4)_{\alpha \beta \gamma \delta} \sim \begin{array}{c} \square \\ \hline \hline \end{array} \sim \frac{20'}{1} \times 1 , \]
\[ (D^5)^i_{\alpha \beta \gamma} \sim \begin{array}{c} \square \\ \hline \hline \end{array} \sim \frac{20}{2} \times 2 , \]
\[ (D^5)^{ijk}_{\alpha} \sim \begin{array}{c} \square \\ \hline \hline \end{array} \sim \frac{1}{4} \times 4 , \]
\[ (D^6)^{\alpha \beta} \sim \begin{array}{c} \square \\ \hline \hline \end{array} \sim \frac{10}{1} \times 1 , \]
\[ (D^6)^{(ij)}_{\alpha \beta} \sim \begin{array}{c} \square \\ \hline \hline \end{array} \sim \frac{6}{3} \times 3 , \]
\[ (D^7)^{\alpha}_{i} \sim \begin{array}{c} \square \\ \hline \hline \end{array} \sim \frac{4}{2} \times 2 , \]
\[ (D^8) \sim \begin{array}{c} \square \\ \hline \hline \end{array} \sim \frac{1}{1} \times 1 , \]
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where we have used the boxes with dots, as in ref. [44], in order to distinguish these Young tableaux from those of eq. (3.33). Yet another reason is the fact that the fundamental representation \( 4 \) of \( SU(4) \) is complex, so that the positioning of the \( \alpha \)-index in \( W_{\alpha} \) and \( D_{\alpha} \) matters. The irreps \( 10 \) and \( 20 \) of \( SU(4) \) are also complex, whereas the irreps \( 6, 15 \) and \( 20' \) are real.

Since the supercovariant derivatives do not just anticommute but satisfy the algebra (3.26), there are ambiguities in defining their products according to eq. (3.36). For instance, eq. (3.26) implies the identity [44]

\[
D_i \alpha D_j \beta = \frac{i}{2} \varepsilon^{ij} \partial_{\alpha \beta} + \varepsilon^{ij} D_{\alpha \beta} + D_{ij}^{\alpha \beta} \, ,
\]

where

\[
D_{\alpha \beta} = \frac{i}{2} D_{(\alpha} D_{\beta)} \, , \quad D_{ij}^{\alpha \beta} = D_{[\alpha}^i D_{\beta]}^j \, .
\]

In other words, when the products of \( D \)'s are defined in the same way as those of \( W \)'s, all their ambiguities are just total derivatives in spacetime. Hence, we can use the same explicit definitions as in eq. (3.34), viz.

\[
D^{ijkl} = \varepsilon^{\alpha \beta \gamma \delta} D_{(\alpha}^i D_{\beta)}^j D_{(\gamma}^k D_{\delta)}^l \, , \\
(D^6)^{\alpha \beta}_{ij} = \frac{\partial}{\partial D_{[\alpha}^i} \frac{\partial}{\partial D_{\beta]}^j} (D^8) \, , \\
(D^8) = \prod_{\alpha, i} D_{\alpha}^i 
\]

as long as they are going to be integrated over all spacetime coordinates. It is just the case in all the equations to be introduced in the next subsect. 3.3. Note also the identity [44]

\[
D_{(\alpha}^i D_{\beta)}^j D_{\gamma)}^k = 0 \, .
\]

3.3 6d sBI action in (1,0) superspace

A (1,0) supersymmetric Maxwell action in 6d superspace is known \([48, 44]\), and it reads in our notation as

\[
-\frac{1}{4!} \int d^6x \, D_{\alpha \beta} W_{ij}^{\alpha \beta} = \int d^6x \, \left\{ -\frac{1}{4} F^2 - \frac{i}{2} \varepsilon^{ij} \psi_{\alpha}^i \partial_{\alpha} \psi_{\beta}^j - \frac{1}{2} D_{ij} D^{ij} \right\} \, ,
\]

\(^{13}\)The products defined by eq. (3.38) and the first line of eq. (3.39) are unambiguous since all \( D \)'s effectively anticommute there.
where $-\frac{1}{4} \int d^6x F^2$ is the 6d Maxwell action. The superfield action on the left-hand-side of eq. (3.41) is supersymmetric because of the superfield constraint

$$D_{\alpha}^{(i}W_{\beta\gamma j)^k} = 0 \quad (3.42)$$

that follows from the defining superspace constraints (3.29) on the off-shell (1,0) Maxwell supermultiplet [44]. Because of eq. (3.42) the superfield $W_{\alpha\beta}^{ij}$ is independent upon some of the anticommuting superspace coordinates (they are linear combinations of $\theta$’s). Hence, after integrating over the rest of 6d superspace coordinates, as in eq. (3.41), one arrives at a supersymmetric invariant. The Lorentz and $SU(2)$ invariances are manifest in eq. (3.41). This construction is similar to the projective $N=2$ superspace in 4d [51, 52].

The superfield $W_{\alpha\beta}^{ij}$ was identified in ref. [44] with the (1,0) Maxwell ‘spin-2’ supercurrent superfield. Amongst its $40 + 40$ components are the energy-momentum tensor, the (1,0) supersymmetry current and the triplet of conserved $SU(2)$ currents.

The same reasoning based on eq. (3.42) further implies that

$$\int d^6x D^{ijkl}\hat{W}_{ijkl} = 4! \int d^6x \det \hat{F} + \ldots, \quad (3.43)$$

where the dots stand for the fermionic and $D$-dependent component terms, is also a superinvariant that is quartic in the Maxwell field strength. The $4 \times 4$ determinant in eq. (3.43) can be expanded in terms of the Lorentz invariants (3.15) as

$$\det \hat{F} = -\frac{1}{4} \left[ \text{tr} \hat{F}^4 - \frac{1}{4} (\text{tr} \hat{F}^2)^2 \right]. \quad (3.44)$$

Eq. (3.44) is the natural 6d generalization of the 4d Euler-Heisenberg (EH) lagrangian (2.14), while eq. (3.43) represents its unique 6d supersymmetric generalization. Since the EH lagrangian is just the Maxwell energy-momentum tensor squared, its supersymmetric generalization can be naturally understood as the Maxwell supercurrent superfield squared, both in 4d and 6d.

Eq. (3.44) is the same quartic combination that appears in the expansion of the 6d BI action in eq. (3.19), up to an overall normalization. Hence, eqs. (3.41) and (3.43) with proper normalization are just the leading term and the ‘next-to-leading-order-correction’, respectively, in the 6d sBI action that we are looking for. The next (of the 6-th order in $F$) correction to the BI action in eq. (3.19) also has a unique (1,0) supersymmetric extension that follows the same pattern, namely,

$$-\frac{1}{3} \cdot \frac{28}{28} \int d^6x (D^{6})_{ij}^{\alpha\beta} (W^{6})_{ij}^{\alpha\beta}. \quad (3.45)$$
This correction is also supersymmetric due to the constraint (3.42). It may be not accidental that the number (3) of special (non-universal) superinvariants given by eqs. (3.41), (3.43) and (3.45) coincides with the rank (3) of the 6d Lorentz group. This is related to the fact that the next superinvariant generalizing the 8-th order terms (in $F$) in the BI action (3.19) is universal, being given by a full superspace integral. We find the remarkably simple answer given by

$$\int d^6 x (D^8)(W^8) = -\frac{5}{2^7} \int d^6 x d^8 \theta W^8 .$$  \hspace{1cm} (3.46)

We thus have a (1,0) supersymmetric generalization of the 6d BI action

$$S_{\text{bosonic}} = -\int d^6 x \sqrt{-\det(\eta_{mn} + F_{mn})}$$ \hspace{1cm} (3.47)

in the form

$$S[W] = \int d^6 x \left\{ -\frac{1}{4!} D^{ij}_a W^{a\beta}_{ij} - \frac{1}{2^3 \cdot 4!} D^{ijkl} W_{ijkl} \right.$$ \hspace{1cm} (3.48)
$$\left. - \frac{1}{3 \cdot 2^8} (D^6)^{a\beta}_{ij} (W^6)^{ij}_{a\beta} - \frac{5}{2^7} (D^8)(W^8) + \ldots \right\} ,$$

where the dots stand for the higher order terms whose component equivalents are of the 10-th order or higher in $F$.

From the group-theoretical viewpoint, the Lorentz invariance of eq. (3.48) is manifest, being related to the existence of a trivial representation in the $SU(4)$ product

$$\mathbf{6} \times \mathbf{6} = \mathbf{1} + \mathbf{15} + \mathbf{20}' ,$$  \hspace{1cm} (3.49)

and similarly for the $SU(2)$ irreps,

$$\mathbf{3} \times \mathbf{3} = \mathbf{1} + \mathbf{3} + \mathbf{5} ,$$  \hspace{1cm} (3.50)
$$\mathbf{4} \times \mathbf{4} = \mathbf{1} + \mathbf{3} + \mathbf{5} + \mathbf{7} .$$

Being the full superspace integral, the ‘top’ superinvariant of eq. (3.46) is already manifestly supersymmetric without the use of the constraint (3.42). Hence, similarly to the previously considered cases (sect. 2), it can be further generalized by inserting a structure function $Z(K,L,M)$ at our disposal into the superfield lagrangian,

$$\int d^6 x d^8 \theta W^8 \rightarrow \int d^6 x d^8 \theta Z(D^2 W^2, D^4 W^4, D^6 W^6) W^8 ,$$  \hspace{1cm} (3.51)

where we have introduced the new bosonic scalar superfields $K$, $L$ and $M$ as the supercovariant derivatives of $W$:

$$D^2 W^2 \sim -\frac{1}{4!} D^{ij}_a W^{a\beta}_{ij} \equiv K ,$$
$$D^4 W^4 \sim -\frac{1}{2^3 \cdot 4!} D^{ijkl} W_{ijkl} \equiv L ,$$ \hspace{1cm} (3.52)
$$D^6 W^6 \sim -\frac{1}{3 \cdot 2^8} (D^6)^{a\beta}_{ij} (W^6)^{ij}_{a\beta} \equiv M .$$
The $W^8$ factor in eq. (3.51) ‘soaks up’ the anticommuting spinor derivatives in the full superspace measure, so that the structure function $Z(K, L, M)$ subject to the ‘initial’ condition $Z(0, 0, 0) = 1$ can only affect the terms of the order $F^{10}$ or higher in the component expansion of the action (3.51).

The $F$-products (3.15) are simply related to those defined by eq. (3.52), namely,

\[
\begin{align*}
\frac{1}{2} \text{tr} \hat{F}^2 & = - K |, \\
\frac{1}{2} \text{tr} \hat{F}^4 & = (L + K^2) |, \\
\frac{1}{3 \cdot 2} \text{tr} \hat{F}^6 & = - (M + \frac{2}{3} KL + \frac{1}{3} K^3) |,
\end{align*}
\]

where $|$ means taking merely the $F$-dependent terms in the first component of a superfield. It is now straightforward to rewrite eqs. (3.18) and (3.19) to another form, in terms of the new variables $K$, $L$ and $M$. We find

\[
- \det(\eta_{mn} + F_{mn}) = 1 + 2(K + L + M) + 2KL + K^2,
\]

and

\[
\sqrt{- \det(\eta_{mn} + F_{mn})} = 1 + (K + L + M) - \frac{1}{2}(L^2 + 2KM) Z.
\]

Hence, the structure function $Z(K, L, M)$ reads

\[
Z = \frac{1 + (K + L + M) - \sqrt{1 + 2(K + L + M) + 2KL + K^2}}{KM + \frac{2}{3} L^2} = 1 - K + \ldots,
\]

where the dots stand for the higher order terms (quartic in $F$ or higher) in the expansion of the exact (non-perturbative) formula on the left-hand-side.

We are now in a position to write down the full (1,0) supersymmetric Born-Infeld-Goldstone action describing the gauge-fixed 6d ‘spacetime-filling’ D-5-brane in 6d superspace as

\[
S[W] = \int d^6x \left\{-\frac{1}{4!} D_{\alpha\beta}^{ij} W^\alpha_{ij} - \frac{1}{2^3 \cdot 4!} D^{ijkl} W_{ijkl} - \frac{1}{3 \cdot 2^8} (D^6)_{ij}^{\alpha\beta} (W^6)_{ij}^{\alpha\beta}\right\}
\]

\[
- \frac{5}{3 \cdot 2^7} \int d^6x d^8\theta \ Z(D^2 W^2, D^4 W^4, D^6 W^6) W^8,
\]

where the structure function $Z$ is given by eq. (3.56). The 6d action (3.57) has linearly realised (1,0) supersymmetry, whereas the non-linear structure of the function $Z$ in eq. (3.56) is supposed to be dictated by yet another non-linearly realised (1,0) supersymmetry which is hidden, being spontaneously broken (cf. sect. 2). The (2,0) supersymmetry algebra in 6d reads

\[
\{Q^I_\alpha, Q^J_\beta\} = i \Omega^{IJ}_{\alpha\beta} \partial_{\alpha\beta},
\]

31
where $\Omega^{IJ}$ is the invariant metric of $USp(4)$ and $I, J = 1, 2, 3, 4$. It is straightforward to calculate the transformation laws of the hidden, spontaneously broken (1,0) supersymmetry by using the general theory of non-linear realisations \([39]\). We would like to emphasize here that the corresponding Goldstone-Maxwell action (3.57) was found by a direct supersymmetrization of the 6d bosonic Born-Infeld action that already ‘knows’ about partial supersymmetry breaking due to its BPS nature. The action (3.57) also gives the 6d superspace action of the gauge-fixed D-5-brane whose worldvolume is given by the whole 6d spacetime (sect. 1).

A plain dimensional reduction of the 6d super-Born-Infeld action (3.57) down to 4d leads to the 4d, $N=2$ supersymmetric Born-Infeld-Nambu-Goto-type action considered in subsect. 2.3,

$$W_\alpha^i \rightarrow D_\alpha^i W,$$  \hspace{1cm} \alpha = 1, 2. \hspace{1cm} (3.59)

Since the 6d superspace action (3.57) is written down entirely in terms of the 6d superfield strength $W_\alpha^i$, the same action in components is going to be dependent upon the 6d Maxwell vector field $A_m$ only via its field strength $F_{mn} = \partial_m A_n - \partial_n A_m$. Hence, after the dimensional reduction,

$$\partial_4 = \partial_5 = 0, \quad \text{and} \quad A_4 + iA_5 = P + iQ,$$ \hspace{1cm} (3.60)

the resulting 4d, $N=2$ super-BING action will be dependent upon the 4d scalars $(P,Q)$ only via their 4d spacetime derivatives, $\partial_\mu P$ and $\partial_\mu Q$. It guarantees the rigid off-shell symmetry

$$P \rightarrow P + \text{const.}, \quad \text{and} \quad Q \rightarrow Q + \text{const.},$$ \hspace{1cm} (3.61)

which is necessary for the Goldstone interpretation of the real scalars $(P,Q)$ as the Goldstone bosons associated with spontaneously broken translations. The symmetry (3.61) is not manifest in our 4d, $N=2$ super-BING action (2.42), but it becomes manifest after rewriting it to the more symmetric 6d form as above (cf. ref. [23]).

When using the identities

$$W_{ijkl} = 2W^{\alpha\beta}_{ij}W_{k\alpha\beta},$$

$$W^6_{ij} = \frac{1}{6}W_{\alpha\beta\delta\mu}W^{\alpha\beta\delta\mu}_{ij},$$

$$W^8 = \frac{1}{18}W^{\alpha\beta}_{ij}W^6_{ij\alpha\beta},$$ \hspace{1cm} (3.62)

and similarly for the products (3.39) of the 6d superspace supercovariant derivatives $D_{\alpha\beta}^{ij}$ (modulo total derivatives in 6d spacetime), it is possible to rewrite the action
(3.57) to the form of a ‘non-linear sigma-model’, viz.

\[
S[W] = - \frac{1}{4!} \int d^6x \, D^{ij}_{\alpha\beta} \left\{ W^{\alpha\beta}_{ij} + \frac{1}{2} D_{kl}^{\alpha\beta} \left( W_{ij}^{\gamma\delta} W_{kl}^{\gamma\delta} \right) \right. \\
\left. + \frac{1}{2^6 \cdot 3^2} D_{ijkl}^{\alpha\beta} \left( W^{\alpha\beta}_{mn} W^{mn\gamma\delta} W_{kl}^{\gamma\delta} \right) + \frac{5}{2^{12} \cdot 3^4} (D^6)^{\alpha\beta}_{ij} \left[ W_{kl}^{\gamma\delta} W_{kl}^{\sigma\tau} W_{mn}^{\gamma\delta} W_{mn}^{\sigma\tau} Z \right] \right\} \\
\equiv - \frac{1}{4!} \int d^6x \, D^{ij}_{\alpha\beta} X^{\alpha\beta}_{ij},
\]

(3.63)

where we have introduced the improved Maxwell-Goldstone (1,0) supercurrent superfield \( X^{\alpha\beta}_{ij} \equiv \left( W^{\alpha\beta}_{ij} \right)_{\text{improved}} \). The latter seems to satisfy to the very simple non-linear superfield constraint

\[
X^{\alpha\beta}_{ij} = \frac{1}{2} D_{kl}^{\alpha\beta} \left( X^{\gamma\delta}_{ij} X^{kl}_{\gamma\delta} \right) + W^{\alpha\beta}_{ij}.
\]

(3.64)

This off-shell superfield constraint is the 6d superspace generalization of the similar 4d superspace constraints in eqs. (2.29) and (2.44).

4 Conclusion

In this paper we advocate the extended superspace approach for constructing some gauge-fixed supersymmetric D-brane actions that are given by superextensions of the Born-Infeld (or Born-Infeld-Nambu-Goto) actions. Those actions are the Goldstone actions associated with partial spontaneous breaking of extended supersymmetry with 16 supercharges down to 8 supercharges in four and six spacetime dimensions. We believe that the extended superspace approach is adequate for these purposes, since it is (i) simple, (ii) transparent, (iii) universal and (iv) powerful. The number (8) of unbroken supercharges is the maximal one allowed in the conventional off-shell superspace formulation of supersymmetric field theories.

The main results of our investigation are given by the 6d, (1,0) supersymmetric Born-Infeld-Goldstone action (subsect. 3.3) and its 4d, N=2 supersupersymmetric counterpart (subsect. 2.3), as well as their ‘non-linear sigma-model’ representations given by eqs. (3.63) and (3.64), and eqs. (2.43) and (2.44), respectively. In our approach, the irreducibility of the Goldstone vector supermultiplet is ensured by the standard ‘linear’ off-shell superfield constraints on the Maxwell-Goldstone superfield, whereas the non-linearity of the Born-Infeld-Goldstone action is represented by the off-shell superfield structure functions or, equivalently, the ‘non-linear sigma-model’

\[\text{We checked it in a few leading orders in } X.\]

\[\text{The 4d, N=2 supersymmetric BING action (2.42) was found for the first time in ref. 29.}\]
off-shell superfield constraints (cf. refs. \[10, 23\]). Despite of the presence of higher
derivatives to all orders, all the actions considered have no ghosts and lead to causal
propagation of the physical fields. Moreover, they enjoy the auxiliary freedom, i.e.
their auxiliary fields do not propagate, being vanishing on-shell. The 4d, N=2 super-
BING action is self-dual with respect to the N=2 supersymmetric electric-magnetic
duality.

Other Goldstone actions associated with different patterns of partial supersymmetry
breaking $N = 2 \rightarrow N = 1$ in 4d are known to be related to other massless
Goldstone (chiral or tensor) N=1 supermultiplets \[53, 24, 23\]. The Goldstone action
associated with partial supersymmetry breaking $(1,1) \rightarrow (1,0)$ in 6d, with a tensor
$(1,0)$ supermultiplet of Goldstone fields, is known to be the (gauge-fixed) effective field
theory action in the M-5-brane worldvolume \[54, 55\]. Amongst the components of
the $(1,0)$ tensor multiplet in 6d, there is a gauge two-form whose field strength is self-
dual. After dimensional reduction on a torus, this self-dual field yields a 4d Maxwell
gauge field, while the dimensionally reduced M-5-brane effective action appears to
be the Born-Infeld-Goldstone action in 4d \[54\]. This way of doing also allows one to
make manifest the electric-magnetic self-duality of the gauge-fixed D-3-brane action,
and extend it further to a full classical $SL(2, \mathbb{R})$ duality, with the background axion- 
dilaton fields being taken into account \[27\]. The $SL(2, \mathbb{Z})$ self-duality expected to
survive in quantum theory then appears to be related to the compactification (torus)
geometry, being identified with the invariance of the torus under the $SL(2, \mathbb{Z})$ trans-
formations of its complex structure \[57, 58\]. It also implies that the supersymmetric
version of the dimensionally reduced (and truncated) theory to be obtained from the
6d self-interacting $(1,0)$ tensor multiplet action, when only two Goldstone bosons are
kept, should be given by our 4d, N=2 supersymmetric BING (or Goldstone-Maxwell)
action (subsect. 2.3).

The self-duality condition attached to the on-shell 6d tensor $(1,0)$ multiplet makes
it difficult to find its 6d Lorentz-invariant and supersymmetric action that could be
useful for doing quantum calculations. Our 6d Goldstone-Maxwell superfield action is
manifestly 6d Lorentz invariant and $(1,0)$ supersymmetric, while it can be quantized
directly in 6d superspace without obstructions. As a by-product of our 6d superfield
analysis, we found the full list of non-universal $(1,0)$ superinvariants on the $(1,0)$
Maxwell superfield constraints in 6d superspace. Those higher-derivative supersym-
metric and gauge invariants are likely to be forbidden as possible UV counterterms,
but they may still appear as finite local corrections to the low-energy effective actions
of quantum supersymmetric interacting gauge field theories in six dimensions.
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