Generalized predictive speed control based on equivalent-input-disturbance for PMSM drive system

Gang Huang\(^1\), Jiajun Li\(^1\), Huijun Yu\(^1\), Wei Huang\(^1\) and Kaihui Zhao\(^2\)

Abstract
The speed control performance of permanent magnet synchronous motor (PMSM) drive system is degraded due to non-matching disturbances such as parameter perturbation and load torque mutation. This paper presents a nonlinear generalized predictive control method based on equivalent-input-disturbance (GPC-based-EID) to realize the fast response and strong robustness of the speed controller of the PMSM drive system. Firstly, the continuous time nonlinear system of a motor mechanical equation is established. A speed controller based on the generalized predictive theory rather than the PI controller of a traditional vector control is designed. Then, the drive system with disturbance is transformed into an EID system. An improved nonsingular fast terminal sliding-mode observer (NFTSMO) is introduced to accurately estimate the EID of total system non-matching disturbances. And the active compensation of non-matching disturbances is realized through feedforward method. This greatly enhances the robustness and the speed tracking performance of the drive system. Comparisons with PI control, traditional GPC and GPC-based-ESO methods show the effectiveness of the method.

Keywords
Permanent magnet synchronous motor, generalized predictive control, equivalent-input-disturbance, nonsingular fast terminal sliding-mode observer

Introduction
Permanent magnet synchronous motor (PMSM) has gradually replaced DC motor and induction motor due to its advantages of high efficiency, high power density, high dynamic performance, and high reliability. And it is widely used in various industrial production practices, such as new energy vehicles, industrial robots, aviation, rail transit, high-precision machine tools, and UAVs.\(^1\)–\(^3\) The performances of a fast speed dynamic response and a high-precision torque response of a PMSM drive system are very important in high-precision and high-performance engineering applications.\(^4\) At present, the PMSM drive system usually uses the field orientation control (FOC) strategy as a classical control scheme, which contains the double closed-loop control structure of a speed outer loop and a current inner loop. The PI controller is widely used in the structure because of its simplicity and good steady-state performance.\(^5\)

However, the PMSM drive system is a typical highly nonlinear system with multivariable, strongly coupling, multivariable and time-varying characteristics.\(^6\) The operating conditions of the PMSM drive system are usually complex and changeable, which increases the difficulty of its control. The control performance of the PMSM drive system will be degraded due to the existence of parameter perturbation and load torque mutation.\(^7\) Therefore, it is necessary to improve the robustness and the speed tracking performance of the PMSM drive system by adopting advanced control methods.

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and uncertain model. Moreover, the system operating conditions are complex and changeable, which are easily affected by uncertain factors such as internal disturbances (parameter perturbation and non-modeling dynamics) and external disturbances (sudden changes in load torque). This results in the speed loop control is nonlinear and time-varying. Especially, the disturbances, such as motor parameters perturbation and sudden torque change, inevitably lead to fluctuation of motor speed. This leads to the degradation of the speed tracking performance of a PI controller. Although the speed fluctuation is generally ignored in low-precision industrial applications, accurate speed control is an inevitable problem in high-precision industrial applications such as computer numerical control machine tools, industrial robots, and electric vehicles. Many scholars did a lot of researches on speed control of PMSM in recent years, and presented a large number of algorithms to improve the anti-disturbance performance of the speed controller, such as adaptive control,7 sliding-mode control,8 fuzzy control,9 active disturbance rejection control,10 model predictive control (MPC),11 neural network control12 and control based on disturbance observer (DOB),13 and so on. These methods improve the speed control performance of the PMSM drive system in different aspects.

Among the above methods, the MPC method has attracted much attention and has been considered as an optimal control method due to its advantages such as simple design, superior dynamic performance, and complete compatibility with digital controllers.14 With the development of digital signal control technology and the advancement of field programmable gate array (FPGA) technology, the MPC method has been successfully applied in PMSM control systems.15,16 The MPC uses the input and output data of a system to predict system future output, and designs an optimal control law by minimizing the cost function, which consists of the difference between a expected output and a predictive reference trajectory. Comparisons with other control methods show that it has the advantages of good control effect and low requirements for model accuracy.14 The MPC contains continuous-time MPC and discrete-time MPC. The MPC of PMSM drive system is mainly based on the linear discrete model of the motor such as deadbeat predictive control,17–19 finite control set model predictive control (FCS-MPC)20–22 and so on. However, the design process of the controllers in the above methods is greatly affected by a sampling period. A small sampling period easily leads to a larger prediction time domain, which increases the amount of calculation and affects real-time performance. While a large sampling period leads to the degradation of the system anti-disturbance ability, which increases the difficulty of controller design in practice.

A nonlinear generalized predictive control (GPC) method based on a continuous-time system is presented to achieve fast dynamic response and easy implementation in practical applications.23 The method uses Taylor series expansion to obtain a predictive model based on a continuous-time model. Then, the nonlinear GPC law of the system is designed by defining and optimizing the cost function composed of the predictive output and the predictive reference. The GPC method has the advantages of fast dynamic response, simple calculation steps, and convenient implementation. It effectively realizes the speed tracking control of the PMSM system. However, the designed GPC controller is still based on the system model that does not directly consider the model uncertainty, which cannot directly eliminate the influence of uncertain factors and external disturbances. To solve this problem, Errouissi et al. presented a robust nonlinear GPC method combined with an integral sliding-mode control.24 The designed composite controller ensures that the steady-state error of a class of non-matching nonlinear systems is zero. Liu et al. and Zhao et al. designed the speed loop controller of a PMSM drive system based on a nonlinear GPC method.25,26 An extended-state observer is introduced to estimate the load disturbance and parameter perturbation of the system, and the estimates are used for the feedforward compensation of the GPC. The method effectively improves the dynamic response and robustness of the motor speed control. Errouissi et al.27 presented a generalized predictive double closed-loop control method for a PMSM system and designed a disturbance observer to estimate the offset caused by parameter uncertainty and load torque changes. Shao et al.28 discussed a robust GPC method with a high-order terminal sliding-mode observer (HOTSMO) for a PMSM control system to achieve fast response, strong robustness, and high anti-disturbance performance of the system. Liu et al.29 presented a compound control method that combines nonlinear GPC with high-order terminal sliding-mode control to achieve accurate speed tracking. Tang et al.30 introduced an adaptive nonlinear GPC for a hypersonic aircraft with unknown parameter uncertainties and control surface constraints. An adaptive parameter estimator is designed to update the unknown controller parameters to enhance the robustness of the system.

Compared with the above-mentioned disturbance compensation methods, a method, which does not need the inverse model of the system (this effectively avoids the cancelation between unstable poles and zeros) and does not require external disturbance information or accurate system model, has greater advantages. She et al.31 proposed such the method of equivalent-input-disturbance (EID). The method uses a state observer and an estimator to estimate the signal on the control
input channel, which has the same impact on the system output as external disturbance has, to realize the active compensation for the disturbance. It has been widely used in various control systems such as linear systems,\textsuperscript{32} nonlinear systems,\textsuperscript{33} time-delay systems,\textsuperscript{34} repetitive control systems,\textsuperscript{35} fractional order systems.\textsuperscript{36} Huang et al.\textsuperscript{37,38} extended its application to fault-tolerant control systems. A sliding-mode observer (SMO) is introduced to replace a Luenberger observer (LO) used in conventional EID system,\textsuperscript{38,39} which achieves faster and more accurate estimation of state variables and EID. The strong robustness of the SMO is achieved by the large switching control gain of the sliding-mode function, which leads to a chattering problem. High-order sliding-mode control is widely used to solve the chattering phenomenon. Zhang et al.\textsuperscript{40} presented an adaptive second-order nonsingular fast terminal sliding-mode (SONFTSM) control scheme by combining integral terminal sliding-mode and nonsingular fast terminal sliding-mode. This realized the finite-time stability of chattering-free control input for a class of nonlinear uncertain systems.

This paper presents a nonlinear generalized predictive speed control method based on EID to solve the problem of speed control performance degradation for a PMSM drive system with parameter perturbation and load torque disturbance. Firstly, a generalized predictive speed controller is designed to realize the fast dynamic response of speed. Secondly, an EID method based on a NFTSMO is introduced to realize active compensation of the total disturbance in a feed-forward manner. The experimental results show that the method effectively improves the speed tracking performance of the PMSM drive system and has strong robustness. The main contributions of the paper are summarized as follows:

1. Unlike the work,\textsuperscript{25} an EID method instead of an extended state observer (ESO) is introduced into the design of GPC for a PMSM drive system. The estimate of the EID does not need an inverse model of the system. This effectively ensures the stability of the system. Moreover, the designed composite speed controller has strong anti-disturbance ability to parameter perturbation and load torque disturbance;

2. An NFTSMO instead of an LO in the conventional EID method and SMOs in other existing EID methods is introduced to design the EID system. This eliminates sliding-mode chattering and improves the estimation accuracy of the EID;

3. Unlike a conventional linear PI method, a conventional GPC method and a GPC method with an ESO, a nonlinear GPC method with an EID is designed. This significantly improves the robustness and the speed tracking performance of the system.

The rest of the paper is organized as follows. Section “System description” describes the mathematical model of a nominal PMSM. Then, the continuous time nonlinear system of the motor mechanical equation is formulated. Section “Design of generalized predictive speed controller for PMSM” introduces the design process of generalized predictive speed controller. The disturbance compensation based on EID and the design of an improved NFTSMO are shown in Section “Disturbance compensation based on EID.” The experimental results are compared in Section “Experimental analysis.” Then, the conclusions are drawn in Section “Conclusion.”

**System description**

The mathematical model of a PMSM in $dq$ reference frame is established as

\[
\begin{align*}
    u_d &= L_d \frac{di_d}{dt} + R_s i_d - \omega_L q_i_d, \\
    u_q &= L_q \frac{iq}{dt} + R_s iq + \omega_L id_d + \omega_e \phi, \\
    T_e &= \frac{3}{2} n_p [(L_d - L_q)i_d i_q + \phi i_q], \\
    \frac{d\omega_e}{dt} &= \frac{n_p}{J}(T_e - T_L - B\omega_m),
\end{align*}
\]

where $L_d$ ($L_q$) is the stator inductance in $d$ ($q$) reference frame; $i_d$ ($i_q$), $u_d$ ($u_q$) are the stator current and the stator voltage in $d$ ($q$) reference frame, respectively; $\omega_e$ is the electrical angular velocity; $\omega_m$ is the rotor mechanical angular velocity; $R_s$, $n_p$, $\phi$, $J$, $T_e$, $T_L$, and $B$ are the stator resistance, number of pole pairs, permanent magnet flux linkage, inertia, electromagnetic torque, load torque, and damping coefficient, respectively.

A PMSM drive system usually uses the vector control scheme of $i_d^r = 0$ in engineering applications, and the reference current $i_d^r$ is obtained by designing the speed controller. If regarding the load torque as a disturbance variable in this scheme, and letting the state variable $x = \omega_e$, the control input $u = i_d$, and the output variable $y = h(x) = \omega_e$. According to (2) and (3), the standard nonlinear equation form of the nominal model of the PMSM without the disturbance variable is described by

\[
\begin{align*}
    \dot{x} &= f(x) + g_1(x)u, \\
    y &= h(x) = g_2(x)x,
\end{align*}
\]

where

\[
    f(x) = -\frac{n_p B\omega_m}{J} = -\frac{B\omega_e}{J},
\]

\[
    g_1(x) = \frac{3}{2} n_p [(L_d - L_q)i_d i_q + \phi i_q],
\]

\[
    g_2(x) = \frac{n_p}{J}(T_e - T_L - B\omega_m).
\]

The rest of the paper is organized as follows. Section “System description” describes the mathematical model of a nominal PMSM. Then, the continuous time nonlinear system of the motor mechanical equation is formulated. Section “Design of generalized predictive speed controller for PMSM” introduces the design process of generalized predictive speed controller. The disturbance compensation based on EID and the design of an improved NFTSMO are shown in Section “Disturbance compensation based on EID.” The experimental results are compared in Section “Experimental analysis.” Then, the conclusions are drawn in Section “Conclusion.”
Design of generalized predictive speed controller for PMSM

Based on the above nominal system continuous-time model (4), this section focuses on the design and study of the speed outer-loop generalized predictive controller to achieve high-precision and robust control of the motor speed.

Define a cost function to be

\[
J_r = \frac{1}{2} \int_0^{T_p} \| \dot{y}(t + \tau) - \ddot{y}_r(t + \tau) \|^2 d\tau,
\]

with

\[
\begin{align*}
\dot{y}(t + \tau) &= \omega_s(t + \tau), \\
\ddot{y}_r(t + \tau) &= \omega^\text{ref}_e(t + \tau),
\end{align*}
\]

where \( T_p \) is the predictive horizon, \( \dot{y}(t + \tau) \) is the prediction output, \( \ddot{y}_r(t + \tau) \) is the prediction reference speed. The cost function ensures that the prediction output value accurately tracks the given reference input as soon as possible.

The relative order \( \rho \) of the output to the input is defined as the \( n \) th derivative of the output to time, and \( n = (0, 1, 2, \cdots) \), until the output includes the input \( u \).

Therefore, chose \( \rho = 1 \) in this paper, and the 0–1 order derivative of output \( \ddot{y}(t) \) are

\[
\ddot{y}(t) = L^0_p h(x),
\]

\[
\ddot{y}(t) = L^0_p h(x) + L^1_p h(x) u(t),
\]

where \( L^0_p h(x) = \dot{h}(x) = \omega_s(t) \), \( L^1_p h(x) = \frac{\partial h(x)}{\partial x} f(x) = \dot{f}(x) \),

Expand the prediction output \( \ddot{y}(t + \tau) \) according to Taylor series at time \( t \), we have

\[
\ddot{y}(t + \tau) = \dot{y}(t) + \tau \dot{y}(t),
\]

Rewrite (11) as

\[
\ddot{y}(t + \tau) = [1 \quad \tau] \begin{bmatrix} \dot{y}(t) \\ \dot{y}(t) \end{bmatrix}.
\]

Letting \( \Gamma(\tau) = [1 \quad \tau] \), \( \ddot{y}(t) = \begin{bmatrix} \dot{y}(t) \\ \dot{y}(t) \end{bmatrix} \). Then,

\[
\ddot{y}(t + \tau) = \Gamma(\tau) \dot{y}(t).
\]

In the same manner, the predicted reference speed \( \ddot{y}_r(t + \tau) \) is expanded by Taylor series as

\[
\ddot{y}_r(t + \tau) = \Gamma(\tau) \ddot{y}_r(t),
\]

where \( \ddot{y}_r(t) = [\ddot{y}_r(t) \quad \dot{y}_r(t)]^T \), \( \ddot{y}_r(t) \) and \( \dot{y}_r(t) \) are the given reference speed \( \omega^\text{ref}_e \) and its derivative, respectively.

Letting \( \hat{\Gamma}(T_p) = \int_0^{T_p} \Gamma(\tau) d\tau \), the elements of the matrix are expressed as

\[
\hat{\Gamma}(T_p)(i, j) = \frac{1}{(i - 1)(j - 1)(i + j - 1)} T_p^{i + j - 1}
\]

\( i, j = 1, 2, \cdots, \rho + 1 \).

Thus, according to (13) and (14), the cost function (7) is rewritten as

\[
J_r = \frac{1}{2} \| \dot{y}(t) - \ddot{y}_r(t) \|^2 \hat{\Gamma}(T_p) [\dot{y}(t) - \ddot{y}_r(t)].
\]

In order to achieve precise tracking of the speed, the cost function needs to be minimized, that is, \( \frac{\partial J_r}{\partial \dot{y}(t)} = 0 \). Thus, the generalized predictive speed control law is given as

\[
u = - G^{-1}(x)(KM_p + L^1_p h(x) - \ddot{y}_r)
\]

where

\[
G(x) = \begin{bmatrix} L^1_p & L^p \end{bmatrix} \begin{bmatrix} \omega_s + \frac{3n^2_\mu}{2J} (L_d - L_q) i_d + \phi \end{bmatrix}
\]

\[
L^1_p h(x) = \frac{\partial h(x)}{\partial x} f(x) = \omega_e - \omega^\text{ref}_e,
\]

\[
k = \hat{\Gamma}^{-1}_r \hat{\Gamma}^{-1}_r
\]

\[
\hat{\Gamma}_r \in \mathbb{R}, \hat{\Gamma}_{pr} \in \mathbb{R}, \hat{\Gamma}_{rr} \in \mathbb{R}
\]

So, we can get \( k = \frac{1}{\hat{\Gamma}_r} \).

Thus, (17) is written as

\[
u(t) = - \frac{2J}{3n^2_\mu (L_d - L_q) i_d + \phi} \begin{bmatrix} 3 \\ 2T_p \end{bmatrix}
\]

\[
(\omega_e - \omega^\text{ref}_e) - \frac{B}{J} \omega_e - \omega^\text{ref}_e.
\]

For the speed loop of the PMSM drive system, the control quantity \( u(t) \) in (18) is the output \( \omega^\text{ref}_q \) of the generalized predictive speed controller.

Note that the above speed controller is obtained according to the nominal model of the PMSM. That is, the influence of load torque, parameter perturbation and external disturbance on the system is ignored when the prediction model is established. However, external disturbance and parameter perturbation are inevitable in actual engineering applications. Therefore, how to realize the compensation of system disturbance is a key issue to improve the robustness of GPC methods, and it is also very important to improve the performance of the motor drive system.

**Disturbance compensation based on EID**

In this section, an active disturbance compensation method based on EID is presented. The basic idea of
the method is to treat load torque, parameter perturbation and unmodeled disturbance as a total disturbance, and transforms the total disturbance into an EID of the control input. Then, using a designed NFTSMO to estimate the EID, and the estimate is compensated in feed-forward manner. The specific design steps are as follows:

The mechanical equation of a PMSM with parameter perturbation and load disturbance is

\[ \dot{\omega}_c = (a + \Delta a)\omega_q - (b + \Delta b)\omega_c - (c + \Delta c)T_L, \]  

where \( a = \frac{3n_p^2(\tau_L - L_0)k_s + \phi_t}{2}, b = \frac{b}{\tau}, c = \frac{n_p}{\tau}, \Delta a, \Delta b, \) and \( \Delta e \) are the motor parameter perturbations. Letting a total disturbance

\[ f(t) = \Delta ai_q - \Delta bo_c - (c + \Delta c)T_L + a(i_q - i_q^{\text{ref}}), \]  

(20)

It is clear from (19) and (20) that

\[ \dot{\omega}_c = a\omega_c^{\text{ref}} - bo_c + f(t). \]  

(21)

Rewrite (21) as a state space expression as

\[
\begin{align*}
\dot{x} &= Ax + Bu + Ff, \\
y &= x,
\end{align*}
\]

(22)

where \( x = \omega_c \) is a state variable, \( A = -b, B = a, u = \omega_c^{\text{ref}}, F = 1. \)

According to the EID theory, we use \( f_c \) to describe system (22). That is,

\[
\begin{align*}
\dot{x} &= Ax + B[u + f_c], \\
y &= x,
\end{align*}
\]

(23)

where \( f_c \) is the EID of the total disturbance \( f \) and is a signal on the control input channel that produces the same effect on the output as the total disturbance \( d \).\textsuperscript{31}

Design of NFTSMO and EID estimator

Based on the EID theory, a state observer is used to estimate the EID. The state observer used in conventional EID system is an LO. An SMO is introduced to estimate the EID, and the estimate is compensated in feed-forward manner. The specific design steps are as follows:

The mechanical equation of a PMSM with parameter perturbation and load disturbance is

\[ \dot{\omega}_c = (a + \Delta a)\omega_q - (b + \Delta b)\omega_c - (c + \Delta c)T_L, \]  

(19)

where \( a = \frac{3n_p^2(\tau_L - L_0)k_s + \phi_t}{2}, b = \frac{b}{\tau}, c = \frac{n_p}{\tau}, \Delta a, \Delta b, \) and \( \Delta e \) are the motor parameter perturbations. Letting a total disturbance

\[ f(t) = \Delta ai_q - \Delta bo_c - (c + \Delta c)T_L + a(i_q - i_q^{\text{ref}}). \]  

(20)

Design a NFTSMO for the EID system (23) to be

\[
\begin{align*}
\dot{x} &= Ax + Bu + \nu, \\
\dot{y} &= \hat{x},
\end{align*}
\]

(24)

where \( \hat{x} \) is the estimate of \( x \), \( \hat{y} \) is the estimate of \( y \), \( \nu \) is a sliding-mode control function, \( u_f \) is a control input. A state-estimation error \( e_w \) is described as

\[ e_w = x - \hat{x} = \omega_c - \hat{\omega}_c. \]  

(25)

According to (23) and (24), the derivative of \( e_w \) is

\[ \dot{e}_w = Ae_w + Bu + f_c - u_f - \nu. \]  

(26)

Substitute (24) into (26) yields

\[ \dot{\hat{x}} = \hat{\dot{x}} + \hat{\dot{y}} \]

\[ = Ae_w + Bu + f_c - u_f - \nu + A\hat{x} + Bu + \nu \]  

(27)

According to (27), we get

\[ \dot{\hat{x}} = A\hat{x} + Bu + (Ae_w + Bf_c). \]  

(28)

Assume that there is a variable \( \Delta f_c \) that satisfies

\[ B\Delta f_c = Ae_w - \hat{e}_w. \]  

(29)

Define the estimate of \( f_c \) as

\[ \hat{f}_c = f_c + \Delta f_c. \]  

(30)

Substituting (29) and (30) into (28) yields

\[ \dot{\hat{x}} = A\hat{x} + Bu + B\hat{f}_c. \]  

(31)

Combining (31) and (24) yields

\[ \dot{\hat{f}_c} = u_f - u + B^\top \nu, \]  

(32)

where \( B^\top = (B^\top B)^{-1}B^\top \).

Moreover, a low-pass filter \( M(s) \) is introduced to reject high-frequency noise and satisfies

\[ |M(j\omega)| \approx 1, \forall \omega \in [0, \omega_c], \]  

(33)

where \( \omega_c \) is the highest frequency for estimation. Select the cutoff angular frequency of \( M(s) \) to be \( 5\sim10 \) times of \( \omega_c. \textsuperscript{39} \)

Then, the filtered EID, \( \hat{f}_c \), is given by

\[ \hat{F}_c(s) = M(s)\hat{F}_c(s), \]  

(34)

where \( \hat{F}_c(s) \) and \( F_c(s) \) are the Laplace transformations of \( \hat{f}_c \) and \( f_c \), respectively.

Finally, a compound control law \( u(t) \) for disturbance rejection is gotten as

\[ u(t) = u_f - \hat{f}_c. \]  

(35)

According to (35) and (18), we have

\[ u(t) = -\frac{2J}{3n_p^2(\tau_L - L_0)k_s + \phi_t} \left[ \frac{3}{2T_p} \left( \omega_c - \omega_c^{\text{ref}} \right) - \frac{B}{J} \omega_c - \hat{\omega}_c - \hat{\omega}_c^{\text{ref}} \right] - \hat{f}_c. \]  

(36)
That is, equation (36) is the designed PMSM generalized predictive speed controller based on EID.

Remark 1: The designed controller (36) is based on GPC and EID methods. Compared with the conventional generalized predictive controller (18), it contains an active disturbance compensation item. This effectively reject the influence of non-matching disturbance caused by parameter perturbation and load torque changes on the system, and effectively enhance the robustness of the controller.

Substitute (35) into (26) yields

\[ \dot{e}_w = Ae_{ew} + B(f_e - \tilde{f}_e) - v. \]  

(37)

Letting \( \Delta f_e = f_e - \tilde{f}_e \), Then,

\[ \dot{e}_w = Ae_{ew} + B\Delta f_e - v. \]  

(38)

Chose a non-singular fast terminal sliding-mode (NFTSM) surface as

\[ l = s + k_1|s|^\alpha \text{sign}(s) + k_2|\dot{s}|^\beta \text{sign}(\dot{s}), \]  

(39)

with

\[ s(t) = e_{ew}, \]  

(40)

where \( k_1 > 0, k_2 > 0 \), \( 1 < \beta < 2, \alpha > \beta \), \( \text{sign}(\cdot) \) is a sign function.

Remark 2: When the system state is far from an equilibrium state, \( k_1|s|^\alpha \text{sign}(s) \) is dominant compared to \( k_2|\dot{s}|^\beta \text{sign}(\dot{s}) \). This improves the convergence rate of the observer. When the system state is close to the equilibrium state, the higher-order term \( k_1|s|^\alpha \text{sign}(s) \) is ignored. \( k_2|\dot{s}|^\beta \text{sign}(\dot{s}) \) ensures the convergence of the system in a finite time. \(^{41}\)

A NFTSMO control function is designed as

\[ v = v_{eq} + v_{sw}, \]  

(41)

where

\[ v_{eq} = Ae_{ew}, \]  

(42)

\[ v_{sw} = \int_0^t \left( \frac{1 + k_1\alpha|s|^\alpha - 1}{k_2\beta} \right) \left| s \right|^{2-\beta} \text{sign}(\dot{s}) + (k_3 + \eta)\text{sign}(l) + \mu l \right] , \]  

(43)

with \( k_3 > \text{max}(B \parallel \Delta f_e \parallel) \), \( k_3 > 0, \eta > 0, \mu > 0 \) are the parameters to be designed.

Stability analysis

The derivative of the NFTSM surface (39) is

\[ \dot{l} = \dot{s} + ak_1|s|^{\alpha - 1}\text{sign}(s) + \beta k_2|\dot{s}|^{\beta - 1}\text{sign}(\dot{s}). \]  

(44)

Proof: Choose a Lyapunov function as

\[ V = \frac{1}{2} \dot{t}^T l. \]  

(45)

The derivative of \( V \) is

\[ \dot{V} = \dot{t}^T \dot{l} \]

\[ = \dot{t}^T \left[ \dot{s} + ak_1|s|^{\alpha - 1}\text{sign}(s) + \beta k_2|\dot{s}|^{\beta - 1}\text{sign}(\dot{s}) \right] \]

\[ = \dot{t}^T \beta k_2|\dot{s}|^{\beta - 1} \left[ \left( 1 + \frac{ak_1|s|^{\alpha - 1}}{\beta k_2} \right) |\dot{s}|^{2-\beta}\text{sign}(\dot{s}) + \dot{s} \right]. \]  

(46)

Combining (38) and (41) with (42) yields

\[ \dot{e}_w = Ae_{ew} + B\Delta f_e - v - B\Delta f_e - v_{sw}. \]  

(47)

According to (40), (46) and (47), we have

\[ \dot{V} = \dot{t}^T \dot{l} \]

\[ = \dot{t}^T \beta k_2|\dot{s}|^{\beta - 1} \left[ \left( 1 + \frac{ak_1|s|^{\alpha - 1}}{\beta k_2} \right) |\dot{s}|^{2-\beta}\text{sign}(\dot{s}) + \dot{s} \right] \]

\[ = \dot{t}^T \beta k_2|\dot{s}|^{\beta - 1} [B\Delta f_e - (k_3 + \eta)\text{sign}(l) - \mu l]. \]  

(48)

Since \( k_3 > \text{max}(B \parallel \Delta f_e \parallel) \), thus,

\[ \dot{V} \leq - \dot{t}^T \beta k_2|\dot{s}|^{\beta - 1} [\eta \parallel l \parallel + \mu \parallel l \parallel^2]. \]  

(49)

Since \( 1 < \beta < 2 \), then,

\[ \left\{ \begin{array}{l} |\dot{s}|^{\beta - 1} > 0 \ldots \ldots \dot{s} \neq 0, \\ |\dot{s}|^{\beta - 1} = 0 \ldots \ldots \dot{s} = 0. \end{array} \right. \]  

(50)

Substitute (50) into (49), we have

\[ \dot{V} \leq 0. \]  

(51)

Therefore, the error equation (38) converges to zero. This completes the proof.

Remark 3: Compared with an EID system based on a conventional SMO, the NFTSMO effectively eliminates the chattering problem caused by the discontinuity term of the conventional SMO, and greatly improves the estimation accuracy of the observer for disturbances.

Experimental analysis

This section presents the example of a PMSM drive system that illustrates the validity of the designed method. Figure 1 shows the control block diagram of
whole system. Since it is difficult to simulate parameter perturbation in an actual PMSM drive system, we used an RT-LAB hardware-in-the-loop experimental setup. Figure 2 shows the RT-LAB experimental setup, which includes a DSP controller TMS320F2812, OP5600 unit, a computer host, and a motor model as software component. OP5600 unit is mainly used to simulate the rest of the system such as inverter and PMSM. The parameters of the PMSM used in the experiment are shown in Table 1. The control method \( \dot{r}_{ref} = 0 \) was adopted.

Experiments were conducted to verify the speed response performance and robustness to non-matching disturbances of the presented GPC + EID speed control method and compare it with three other methods: a PI control method, a conventional GPC control method without disturbance observer, and a GPC method based on an ESO (GPC + ESO).25

A second-order linear filter \( \frac{1}{s^2 + 2\xi \omega_n s + \omega_n^2} \) is considered to ensure that the reference speed is smooth when designing the speed controller,27 and select \( \xi = 1 \), \( \omega_n = 100 \). PI controller is used in the current loop in all above four methods, and the same control parameters are used. The proportional coefficients are \( k_{pd} = 500 \), \( k_{pqr} = 1.5 \). The integral coefficients are \( k_{id} = 2 \), \( k_{iq} = 5 \). The sampling period is set to be \( T_s = 1 \mu s \). Choose the proportional coefficient and the integral coefficient of the speed loop to be \( k_{ps} = 0.15 \) and \( k_{is} = 50 \) in the PI control method, separately. Set the predictive horizon is to be \( T_p = 0.00001 \text{s} \) in the GPC method. The predictive horizon is set to be \( T_p = 0.00001 \text{s} \) in the GPC + ESO method and the ESO parameter is selected to be \( p = 50 \). The predictive horizon is set to be \( T_p = 0.00001 \text{s} \) in the designed GPC + EID method, and the parameters of the NFTSMO are \( k_1 = k_2 = 1 \), \( \alpha = \frac{5}{2} \), \( \beta = \frac{7}{2} \), \( k_3 + \eta = 3000 \), \( \mu = 2000 \). Select a low-pass filter as \( M(s) = \frac{1}{0.01s + 1} \).

The reference speed is \( n = 1000 \text{r/min} \). Four cases are discussed.

**Case 1: Experiment with no load**

There is no load in the PMSM drive system. Figure 3 shows the speed response curves of the system. Figure 4 shows the \( dq \) axis currents of GPC + EID method and \( q \) axis current compensation of GPC + EID and GPC + ESO methods. It is clear from Figure 3 that the time of the four control methods from motor

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**Table 1. Parameters of PMSM.**

| Parameters       | Unit | Value     |
|------------------|------|-----------|
| Stator resistance | \( R_s \) | 2.875 \( \Omega \) |
| \( q \)-Axis \( L_q \) | \( H \) | 0.0075 \( H \) |
| \( d \)-Axis \( L_d \) | \( H \) | 0.0025 \( H \) |
| Magnetic flux \( \phi \) | \( Wb \) | 0.175 \( Wb \) |
| Inertia \( J \) | \( kg \cdot m^2 \) | 0.0008 \( kg \cdot m^2 \) |
| Number of pole pairs \( P \) | pairs | 4 pairs |
| Damping coefficient \( B \) | \( Nm \cdot s/\text{rad} \) | 0.00001 \( Nm \cdot s/\text{rad} \) |
| DC-bus voltage \( V_{dc} \) | \( V \) | 380 \( V \) |
| Rated current \( I_N \) | \( A \) | 3.5 \( A \) |
| Rated torque \( T_N \) | \( Nm \) | 18.6 \( Nm \) |
| Rated speed \( n \) | \( r/min \) | 1000 \( r/min \) |
starting to reaching the reference speed are $0.12\,s$ (GPC + EID), $0.55\,s$ (GPC + ESO), $1.4\,s$ (GPC), and $1.5\,s$ (PI), respectively. Moreover, the speed response results of the GPC + EID method and the traditional GPC method basically have no overshoot. While the speed response of the GPC + ESO method has a slight overshoot of about $0.06\%$, and the speed overshoot of the PI control is about $0.24\%$.

**Case 2: Experiment with load torque disturbance**

When the motor starts with no-load and the speed reaches a stable state, a load torque disturbance of $2\,N\cdot m$ is suddenly injected at $t = 2\,s$. Figure 5 shows the speed response curves. Figure 6 shows the $dq$ axis currents of GPC + EID method and $q$ axis current compensation of GPC + EID and GPC + ESO methods. When the load of $2N\cdot m$ is injected at $2\,s$, it is clear from Figure 5 that the time of the four control methods to reach stability are $0.02\,s$ (GPC + EID), $0.07\,s$ (GPC + ESO), $0.4\,s$ (GPC), and $0.5\,s$ (PI), respectively. Moreover, the speed fluctuations of the four control methods are $4.3\,r/min$ (GPC + EID), $4.5\,r/min$ (GPC + ESO), $4.6\,r/min$ (GPC), and $33.5\,r/min$ (PI), respectively.

**Case 3: Experiment with flux-linkage parameter disturbance**

When the motor starts with no-load and the speed reaches a stable state, a permanent magnet flux-linkage
\( \phi \) decreases from 0.175 to 0.1 Wb at \( t = 2 \text{s} \). Figure 7 shows the speed response curves. Figure 8 shows the \( dq \) axis currents of GPC + EID method and \( q \) axis current compensation of GPC + EID and GPC + ESO methods. When the flux-linkage disturbance occurs at 2s, it is clear from Figure 7 that the time for the four control methods to reach stability are 0.05 s (GPC + EID), 0.3 s (GPC + ESO), 1 s (GPC), and 1.2 s (PI), respectively. Moreover, the speed fluctuations of the four control methods are 2 \( r/min \) (GPC + EID), 1.8 \( r/min \) (GPC + ESO), 1.8 \( r/min \) (GPC), and 84 \( r/min \) (PI), respectively.

**Case 4: Experiment with load and flux-linkage parameter disturbance**

Experiment with load and flux-linkage parameter disturbance. Set a load torque to be 2N \( \cdot \) m. When the motor speed reaches stability, a flux-linkage disturbance occurs suddenly at \( t = 2 \text{s} \), and the flux-linkage \( \phi \) decreases from 0.175 to 0.1 Wb. Figure 9 shows the speed response curves. Figure 10 shows the \( dq \) axis currents of GPC + EID method and \( q \) axis current compensation of GPC + EID and GPC + ESO methods. When the flux-linkage disturbance occurs at 2s, it is clear from Figure 9 that the time for the four control methods to reach stability are 0.1 s (GPC + EID), 0.07 s (GPC + ESO), 0.9 s (GPC), and 1.2 s (PI), respectively. Moreover, the speed fluctuations of the four control methods are 2 \( r/min \) (GPC + EID), 1.8 \( r/min \) (GPC + ESO), 2 \( r/min \) (GPC), and 68 \( r/min \) (PI), respectively.

It can be seen from Figures 4(b), 6(b), 8(b) and 10(b) that the \( q \)-axis current of GPC + EID method is compensated faster than that of the GPC + ESO method under no-load, loaded, and disturbance conditions of...
Figure 8. $dq$ Axis currents of GPC + EID method and $q$ axis current compensation of GPC + EID and GPC + ESO methods: (a) $dq$ axis currents of GPC + EID control method and (b) Comparison of compensation values of GPC + EID and GPC + ESO methods.

Figure 9. Experimental results of speed response: (a) speed response of PI control method, (b) speed response of GPC control method, (c) speed response of GPC + ESO control method, and (d) speed response of GPC + EID control method.

Figure 10. $dq$ axis currents of GPC + EID method and $q$ axis current compensation of GPC + EID and GPC + ESO methods: (a) $dq$ axis currents of GPC + EID control method and (b) Comparison of compensation values of GPC + EID and GPC + ESO methods.
the motor. This ensures that the designed method has better motor speed dynamic response performance for motor speed than other three methods.

Table 2 shows the comparisons of the experimental results for the four control methods under the case 1 and the case 2. Table 3 shows the comparisons of the experimental results for the four control methods under the case 3 and the case 4. By comparing the above experimental results, we conclude that the presented GPC + EID speed control method has faster response speed, smaller overshoot and stronger anti-disturbance capability than those of the PI control method, GPC control method, and GPC + ESO control method. That is, our GPC + EID control method has better robustness and superiority.

Table 2. Comparisons of the experimental results under case 1 and case 2.

| Control method | Performance comparison |
|----------------|------------------------|
|                | Overshoot (case 1) | response time (case 1) | Maximum change of speed (case 2) | Time to reach stability (case 2) | Steady state error (case 2) |
| PI             | 0.24%                | 1.5 s                   | 33.5 r/min                     | 0.5 s                       | 0                         |
| GPC            | 0                    | 1.4 s                   | 4.6 r/min                      | 0.4 s                       | 0.5 r/min                 |
| GPC + ESO      | 0.06%               | 0.55 s                  | 4.5 r/min                      | 0.07 s                      | 0                         |
| GPC + EID      | 0                    | 0.12 s                  | 4.3 r/min                      | 0.02 s                      | 0                         |

Table 3. Comparisons of the experimental results under case 3 and case 4.

| Control method | Performance comparison |
|----------------|------------------------|
|                | Maximum change of speed (case 3) | Time to reach stability (case 3) | Steady state error (case 3) | Maximum change of speed (case 4) | Time to reach stability (case 4) | Steady state error (case 4) |
| PI             | 84 r/min               | 1.2 s                     | 0                           | 68 r/min                      | 1.2 s                       | 0                           |
| GPC            | 1.8 r/min              | 1 s                       | 0                           | 2 r/min                       | 0.9 s                       | 0.5 r/min                    |
| GPC + ESO      | 1.8 r/min              | 0.3 s                     | 0                           | 4.1 r/min                     | 0.07 s                      | 0                           |
| GPC + EID      | 2 r/min                | 0.05 s                    | 0                           | 2 r/min                       | 0.1 s                       | 0                           |

Conclusion

This paper studied the speed tracking problem of a PMSM drive system with parameter perturbation and load torque disturbance. A new GPC + EID speed control method is presented. A PMSM speed controller under the nominal state is given based on the continuous-time generalized prediction theory. Then, an active disturbance compensation EID method based on a NFTSMO is designed to solve the problem of speed tracking performance degradation caused by parameter perturbation, load torque, and other non-matching disturbances. This method greatly improves the anti-disturbance ability of the speed loop without sacrificing the nominal control performance of the system. The experimental comparison results showed the validity of the presented method and its advantages over PI one, GPC one, and GPC + ESO one. In addition, the coupling effect of the parameter perturbation and permanent magnet flux linkage will be studied in the future.

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