Coherent electronic transport through a superconducting film

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We study coherent quantum transport through a superconducting film connected to normal-metal electrodes. Simple expressions for the differential conductance and the local density of states are obtained in the clean limit and for transparent interfaces. Quasiparticle interference causes periodic vanishing of the Andreev reflection at the energies of geometrical resonances, subgap transport, and gapless superconductivity near the interfaces. Application of the results to spectroscopic measurements of the superconducting gap and the Fermi velocity is analyzed.

I. INTRODUCTION

Conducting properties of normal metal/superconductor/normal metal (NSN) heterostructures have attracted considerable interest.\(^\text{1}\) Electronic transport through a normal metal/superconductor (NS) junction, with an insulating barrier of arbitrary strength at the interface, was studied by Blonder, Tinkham, and Klapwijk (BTK)\(^\text{5}\) and the Andreev reflection\(^\text{4}\) was recognized as the mechanism of normal-to-supercurrent conversion. The BTK model can be used when electrons pass incoherently through the interfaces of NSN double junctions.\(^\text{2}\)

However, in clean superconducting heterostructures coherent quantum transport is strongly influenced by size effects due to the interplay between geometrical resonances and the Andreev reflection\(^\text{8,9,10}\). Well-known examples are the current-carrying Andreev bound states\(^\text{11,12}\) and multiple Andreev reflections\(^\text{13,14,15}\) in superconductor/normal metal/superconductor (SNS) junctions. Since early experiments by Tomasz\(^\text{16}\) the geometric resonance nature of the differential conductance oscillations in SNS and NSN tunnel junctions has been attributed to the electron interference in the central layer\(^\text{17,18,19,20}\). Recently, McMillan-Rowell oscillations were observed in SNS tunnel junctions of \(d\)-wave superconductors, and used in measurements of the superconducting gap and the Fermi velocity.\(^\text{21}\)

In this paper, we focus on size and coherency effects in NSN double junctions with a clean conventional (\(s\)-wave) superconductor and transparent interfaces. In that case, instead of a fully numerical treatment,\(^\text{8,22}\) a simple analytic form for the differential conductance and the local quasiparticle density of states (DOS) can be obtained from the solution of the Bogoliubov-de Gennes equation. The proximity effect, which is significant in thin superconducting films, is taken into account through the self-consistent calculation of the pair potential. Striking consequence of the quasiparticle interference in the clean metallic NSN junctions include periodic vanishing of the Andreev reflection at the energies of geometrical resonances and the characteristic changes of DOS with respect to the bulk superconductor.

II. SCATTERING PROBLEM

We consider an NSN double junction consisting of a clean superconducting layer of thickness \(d\), connected to normal-metal electrodes by transparent interfaces. The quasiparticle propagation is described by the Bogoliubov-de Gennes equation

\[
\begin{pmatrix}
H_0(r) & \Delta(r) \\
\Delta^*(r) & -H_0(r)
\end{pmatrix}
\begin{pmatrix}
u(r) \\
v(r)
\end{pmatrix}
=
E
\begin{pmatrix}
u(r) \\
v(r)
\end{pmatrix},
\]

with \(H_0(r) = -\hbar^2\nabla^2/2m - E_F\), where \(E_F\) is the Fermi energy. For simplicity, the magnitude of the Fermi wave vector, \(k_F = \sqrt{2mE_F/\hbar^2}\), is assumed to be constant through the junction. The superconducting pair potential is taken in the form \(\Delta(r) = \Delta \Theta(z) \Theta(l - z)\), where \(\Theta(z)\) is the Heaviside step function, and the \(z\)-axis is perpendicular to the layers. In Eq. \(\text{I}\), \(E\) is the quasiparticle energy with respect to the Fermi level. The electron effective mass \(m\) is assumed to be the same for the entire junction. The parallel component of the wave vector \(k_{||}\) is conserved, and the wave function

\[
\begin{pmatrix}
u(r) \\
v(r)
\end{pmatrix}
=
\exp(i k_{||} \cdot r)
\psi(z),
\]

satisfies the continuity boundary conditions for \(\psi(z)\) and \(\psi'(z)\) at \(z = 0\) and \(z = d\). The four independent solutions of Eq. \(\text{I}\) correspond to the four types of injection: an electron or a hole, from either the left or the right electrode.\(^\text{2}\)
For the injection of an electron from the left, with energy \( E > 0 \) and angle of incidence \( \theta \) (measured from the \( z \)-axis), solution for \( \psi(z) \) has the following form

\[
\psi(z) = \begin{cases} 
\exp(ik^+z) \left( \frac{1}{0} \right) + a(E,\theta) \exp(ik^-z) \left( \frac{0}{1} \right) & z < 0, \\
b_1(E,\theta) \exp(iq^+z) \left( \frac{\bar{u}}{\bar{v}} \right) + b_2(E,\theta) \exp(iq^-z) \left( \frac{\bar{v}}{\bar{u}} \right) & 0 < z < d, \\
c(E,\theta) \exp(ik^+z) \left( \frac{1}{0} \right) & z > d.
\end{cases}
\]

Here, \( \bar{u} = \sqrt{(1 + \Omega/E)/2} \) and \( \bar{v} = \sqrt{(1 - \Omega/E)/2} \) are the BCS coherence factors, and \( \Omega = \sqrt{E^2 - \Delta^2} \). The \( z \)-components of the wave vectors are \( k^\pm = \sqrt{(2m/\hbar^2)(E_F \pm E) - k_0^2} \) and \( q^\pm = \sqrt{(2m/\hbar^2)(E_F \pm \Omega) - k_0^2} \), where \( k_0 = \sqrt{(2m/\hbar^2)(E_F + E)} \sin \theta \). The coefficients \( a \) and \( c \) are, respectively, the probability amplitudes of the Andreev reflection as a hole of the opposite spin (AR) and transmission to the right electrode as an electron (TE). The amplitudes of the Bogoliubov electron-like and hole-like quasiparticles, propagating in the superconducting layer, are given by the coefficients \( b_1 \) and \( b_2 \). The normal reflection of electrons and the transmission to the right electrode as a hole are absent. Solutions for the general case of insulating interfaces and the Fermi velocity mismatch are given in Ref. 7.

By neglecting the small terms \( E/E_F \ll 1 \) and \( \Delta/E_F \ll 1 \) in the wave vectors, except in the exponent

\[
\zeta = d(q^+ - q^-),
\]
solutions of the boundary-condition equations can be written in a simple form:

\[
an(E,\theta) = \bar{u}\bar{v}[1 - \exp(i\zeta z/d)]/[\bar{u}^2 - \bar{v}^2 \exp(i\zeta z/d)], \\
b_1(E,\theta) = \bar{u}/[\bar{u}^2 - \bar{v}^2 \exp(i\zeta z/d)], \\
b_2(E,\theta) = -\bar{v} \exp(i\zeta z/d)/[\bar{u}^2 - \bar{v}^2 \exp(i\zeta z/d)], \\
c(E,\theta) = -i(\bar{u} - \bar{v}) \exp(i\zeta z/d)/[\bar{u}^2 - \bar{v}^2 \exp(i\zeta z/d)].
\]

From the probability current conservation, the probabilities of outgoing particles satisfy the normalization condition \( A(E,\theta) + C(E,\theta) = 1 \), where \( A(E,\theta) = |a(E,\theta)|^2 \) and \( C(E,\theta) = |c(E,\theta)|^2 \). Explicitly, the AR probability is

\[
A(E,\theta) = \frac{\Delta \sin(\zeta/2)}{E \sin(\zeta/2) + i\Omega \cos(\zeta/2)}.
\]

Solutions for the other three types of injection can be obtained by the same procedure. In particular, if a hole with energy \( -E \) and angle of incidence \( \theta \) is injected from the left, the substitution \( q^+ \leftrightarrow q^- \) holds, and the scattering probabilities are the same as for the injection of an electron with \( E \) and \( \theta \). Therefore, in order to include the description of both electron and hole injection, the calculated probabilities should be regarded as even functions of \( E \). Also, for an electron or a hole injected from the right, the probabilities are the same as for the injection from the left.

From Eq. 5, it follows that \( A(E,\theta) = 0 \), and consequently \( C(E,\theta) = 1 \), when

\[
\zeta = 2n\pi
\]

for \( n = 0, \pm 1, \pm 2, \ldots \). Therefore, the Andreev reflection vanishes, and the electron transmission becomes complete (without creation or annihilation of Cooper pairs), at the energies of geometrical resonances in the quasiparticle spectrum, given by Eq. 9.

For thin superconducting films, the proximity effect is important and the self-consistent treatment of the pair potential is required. However, the spatial variation of the pair potential is negligible if \( d/\xi_0 \lesssim 1 \), where \( \xi_0 = \hbar v_F/\pi \Delta_0 \) is the BCS coherence length in the bulk superconductor, and we can use the previous solution with the step-like form for \( \Delta \). The pair potential is given by the self-consistency equation

\[
\Delta(z) = 2\lambda N(0) \int d^2k \int_0^{\hbar\omega_D} d\Omega \ u(\mathbf{r}) v^*(\mathbf{r}) \tanh(E/2k_B T),
\]

where \( \lambda \) is the BCS coupling constant and \( N(0) = mk_F/2\pi^2\hbar^2 \) is the normal-metal density of states (per spin) at the Fermi level. We calculate the spatial average of \( \Delta(z) \) using the standard iteration procedure

\[
\Delta_{i+1} = \frac{1}{d} \int_0^d \Delta_i(z) dz,
\]

for the resulting \( \Delta \) as a function of \( d \), at zero temperature.
III. DIFFERENTIAL CONDUCTANCE AND DENSITY OF STATES

The normalized differential conductance at zero temperature for a planar double junction is

$$\frac{G(E)}{G_N} = \int_0^{\pi/2} d\theta \sin \theta \cos \theta \left[ 1 + A(E, \theta) \right],$$

where $G_N = e^2/h$. In the one-dimensional (1D) case, the normalized differential conductance is simply

$$\frac{g(E)}{G_N} = 1 + A(E, 0).$$

Coherent transport through an NSN double junction is influenced by considerable changes of the quasiparticle density of states with respect to the bulk superconductor. Adapting the method introduced by McMillan, Ishii and Furusaki and Tsukada evaluate the Green functions by combining the solutions of the Bogoliubov-de Gennes equation. From the imaginary part of the retarded Green function, we obtain the local value of the partial density of states (PDOS) for the superconducting film, normalized with respect to the 1D normal-metal density of states per spin at the Fermi level,

$$\tilde{N}(z, \theta, E) = \frac{1}{\Gamma(E) \cos \theta} \text{Re} \left\{ 2E^2(E^2 + \Omega^2) + 2E^2 \Delta^2 \cos \zeta 
+ \left[ \cos(\zeta(z/d - 1)) + \cos(\zeta z/d) \right][\Delta^4 - \Delta^2(E^2 + \Omega^2) \cos \zeta] 
+ 2E^2 \Delta^2 \sin(\zeta(z/d - 1)) - \sin(\zeta z/d) \sin \zeta \right\},$$

(11)

where

$$\Gamma(E) = [(E^2 + \Omega^2) \cos \zeta - \Delta^2]^2 + 4E^2\Omega^2 \sin^2 \zeta.$$

For a planar junction, the local DOS for the superconducting film is given by

$$\frac{N(z, E)}{N(0)} = \int_0^{\pi/2} d\theta \sin \theta \cos \theta \tilde{N}(z, \theta, E).$$

(13)

Here, $\tilde{N}(z, \theta, E) = N(z, E)/N(0) = 1$ in the normal-metal electrodes ($z < 0$ and $z > d$).

Characteristic features of the coherence in NSN double junctions are subgap transport of electrons and oscillations of the conductance. Due to the interference effect, the Andreev reflection is suppressed for $E < \Delta$, whereas the AR probability oscillates with $E$ and $d$, for $E > \Delta$. The subgap transport is dominant in thin superconducting films, and the oscillatory behavior is apparent in thick films. These oscillations are more pronounced in the 1D case, which is shown in Fig. 2. It can be clearly seen that the positions of the minima in $g(E)$ exactly match the geometrical resonances imposed by Eq. (10). Results can be applied to spectroscopic measurements of $\Delta$ and $v_F$. From Eq. (10) for $\theta = 0$, the energies of the geometrical resonances, $E_n$, satisfy

$$E_n^2 = \Delta^2 + \left( \frac{\pi \hbar v_F}{d} \right)^2 n^2,$$

(14)

where $n = 1, 2, \ldots$ counts the conductance minima. Therefore, the plot of $E_n^2$ vs $n^2$ has the intercept equal to $\Delta^2$, and the slope equal to $\tan^{-1} \left( [\pi \hbar v_F/d]^2 \right)$. An example is shown in the Inset of Fig. 2. Note that the points obtained from the minima of $G(E)$ follow practically the same linear law as those of $g(E)$.

The obtained results are in a strong contrast with the conductance spectrum and DOS of tunnel junctions. In the latter case, $g(E)$ and PDOS reduce to $\delta$-function-like spikes at the bound state energies of the isolated superconducting film, given by $lq_n^+ = n_1 \pi$ and $lq_n^- = n_2 \pi$, where $n_1 - n_2 = 2n$, with $n$ given in Eq. (10). The conductance spectra $G(E)/G_N$ and the local DOS for the superconducting film in a planar NSN double junction are illustrated in Figs. 3 and 4, for $d/\xi_0 = 1$ and $d/\xi_0 = 10$, using the self-consistent pair potential $\Delta/\Delta_0 = 0.816$ and $\Delta/\Delta_0 = 1$, respectively. In Fig. 3, the conductance spectra are compared to the BTK result describing the incoherent transport. It can be seen in Fig. 4(a) that the energy spectrum of a thin superconducting film is practically gapless. For a thick superconducting film, this is the case only near the interfaces, Fig. 4(b). The characteristic oscillations of the local DOS become more apparent for a thick film. The oscillations of $G(E)$ and of the local DOS are not completely smeared in the 3D case. The positions of the minima in $G(E)$ or the maxima of the local DOS are still close to the positions of the resonant quasiparticle states in the 1D case.
IV. CONCLUSION

We have derived simple expressions describing the finite size and coherency effects in the metallic NSN double junctions, suitable for experimental data analysis. The main features of the coherent quantum transport through a superconducting film are subgap transport and oscillations of the differential conductance. The Andreev reflection is suppressed below the gap, especially in thin films. The conductance oscillates as a function of the layer thickness and of the quasiparticle energy above the superconducting gap. Periodic vanishing of the Andreev reflection at the energies of geometrical resonances is an important consequence of the quasiparticle interference; the conductance minima correspond to the maxima in the density of states. Measurements of the conductance spectra of metallic NSN double junctions could be used for accurate determination of the pair potential and the Fermi velocity in superconductors.

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FIG. 1: The pair potential $\Delta$ as a function of the film thickness $d$, at zero temperature.

FIG. 2: (a) Partial ($\theta = 0$) conductance spectra, $g(E)/G_N$, and (b) corresponding PDOS, $\tilde{N}(d/2,0,E)$, for $d/\xi_0 = 10$. Inset: square of resonant energies, $E^2_n$, vs $n^2$ obtained from the minima of $g(E)$ (open circles) and $G(E)$ (full squares).

FIG. 3: Differential conductance spectra $G(E)/G_N$ for $d/\xi_0 = 1$ and 10 (solid curves). The BTK result is shown for comparison (dashed curve).

FIG. 4: Local DOS, $N(z,E)/N(0)$, for $z = d/2$ (solid curves) and $z = 0$ or $d$ (dotted curves) as a function of $E/\Delta_0$ is shown for $d/\xi_0 = 1$ (a), and $d/\xi_0 = 10$ (b).
