Structure and evolution of low mass W UMa type systems

Lifang Li *, Zhanwen Han and Fenghui Zhang
National Astronomical Observatories/Yunnan Observatory, Chinese Academy of Sciences, P.O.Box 110, Kunming, Yunnan Province 650011, P. R. China

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ABSTRACT
The structure and evolution of low-mass W UMa type contact binaries are discussed by employing Eggleton’s stellar evolution code \cite{Eggleton1971, Eggleton1972, Eggleton1973}. Assuming that these systems completely satisfy Roche geometry, for contact binaries with every kind of mass ratios (0.02 \textasciitilde 1.0), we calculate the relative radii ($R_1$, $R_2$, $A$, where $R_1$, $R_2$ are the radii of both stars, and $A$ the orbital separation) of both components of contact binaries in different contact depth between inner and outer Roche lobes. We obtain a radius grid of contact binaries, and can ensure the surfaces of two components lying on an equipotential surface by interpolation using this radius grid when we follow the evolution of the contact binaries. Serious uncertainties concern mainly the transfer of energy in these systems, i.e., it is unclear that how and where the energy is transferred. We assume that the energy transfer takes place in the different regions of the common envelope to investigate the effects of the region of energy transfer on the structure and evolution of contact binaries. We find that the region of energy transfer has significant influence on the structure and evolution of contact binaries, and conclude that the energy transfer may occur in the outermost layers of the common convective envelope for W-type systems, and this transfer takes place in the deeper layers of the common envelope for A-type systems. Meanwhile, if we assume that the energy transfer takes place in the outermost layers for our model with low total mass, and find that our model steadily evolves towards a system with a smaller mass ratio and a deeper envelope, suggesting that some A-type W UMa systems with low total mass could be considered as the later evolutionary stages of W-subtype systems, and that the surface temperature of the secondary excesses the of the primary during the time when the primary expands rapidly, or the secondary contracts rapidly, suggesting that W-subtype systems may be caused by expansion of the primary, or by the contraction of the secondary.

Key words: stars: binaries: close–stars: mass-loss–stars: rotation–stars: evolution

1 INTRODUCTION

It is probable that more than 50 per cent of all stars are in binary or multiple systems. An unknown, but possibly large, percentage of these systems are sufficiently close that sometime during their lifetime interacting as a result of Roche lobe overflow (RLOF). The W UMa-type contact binaries are the most common ones, comprising some 90 per cent of eclipsing variables in the solar neighborhood \cite{Shapley1948} or one stars in every 1000~2000 in the same spectral range \cite{Eggen1967}. Allowing for selection effects, W UMa stars may even contribute 1 per cent of all F and G dwarfs \cite{van't Veer1975a}. A more recent discussion of contact binaries in the solar neighborhood is carried out by \cite{Rucinski2002}. He considers the complete sample of 32 EW, EB, and ellipsoidal (ELL) variables with V < 7.5, and gives the frequency as 1 per 500 stars with $-0.5 < V < 5.5$, including a serious estimate of the uncertainty.

W UMa stars are found both in young and in old galactic clusters \cite{van't Veer1975b}. Among these clusters are: 1. NGC 2602, NGC 6383, NGC 7235 ( $\lesssim 10^7$ yr); 2. Pleiades, Coma, Praesepe ( $\lesssim 5 \times 10^8$ yr); 3. NGC188, M67 ( $\gtrsim 5 \times 10^9$ yr). For this reason it seems we should be able to construct zero-age as well as evolved contact models. Further, there would be a natural explanation for the existence of W UMa stars in young as well as in old cluster if zero-age contact models could be shown to evolve on a nuclear timescale. Structure and evolution of contact binaries are complex and by no means well understood. Although theoretical investigation on early-type systems is almost absent, our discussion will be restricted to late-type systems. The observational properties \cite{Mochnacki1981, Rucinski1993} that provide
the best clues to the nature of late-type W UMa systems are that the systems are of fairly low total mass, and are in shallow contact, that no equal mass systems exist, that the mass-luminosity relation is unusual, and that most of the W-type systems are to be unnealed or slightly evolved only, and some of them are known to be unnealed.

Lucy (1968) had the key idea of a common convective envelope in which the entropy is constant and energy is transferred from the primary to the secondary in the common convective envelope. Thermal equilibrium turned out to be usually impossible. Lucy thermal equilibrium model (violating the period-colour relation) only in a limited mass region and for extreme Population I composition ($Z \gtrsim 0.04$). Moss & Whelan (1970) and Whelan (1972) encountered similar difficulties. Indeed, thermal equilibrium was found to be in conflict with the equal entropy condition in zero age contact binaries (Kähler 1995).

Renouncing the restriction to thermal equilibrium, Lucy (1973; Flannery 1974; and Robertson & Eggleton 1977) obtained contact binary solution evolving in thermal cycles about a state of marginal contact. The solutions are in agreement with the period-colour relation, but they have bad light curves because of a large temperature difference between the two components for a considerable part of the time in a cycle. Some authors encountered the same difficulty, the so-called light curve paradox, also for systems evolving without loss of contact. Hazlehurst (2001) pointed out that the present theory of cyclic contact binaries (thermal relaxation oscillation, TRO) is not in a position to resolve the light curve paradox. The difficulty of explaining the EW-type light curves (i.e. light curve paradox) reflects a basic conflict between the treatment of the internal structure and the observed properties of the outermost layers.

Kähler (2002a,b) has discussed the structure equations of contact binaries, and assumed that the energy sources/sinks caused by the interaction of the components occurs only in the secondary’s/primary’s outer layers to obtain the models which are applied to the typical late-type systems. First he imposed the restriction that the fractional extent in mass of the sources/sinks in the layers above the critical surface is the same in the both components, and found solutions evolving in thermal cycles. And in both cases a large potential difference exists between the surfaces of both components for a considerable part of the time in a cycle. Meanwhile, the maximum contact degree is extremely small ($\sim 2\%$) in both cases, suggesting that late-type contact binaries must be shallow contact. However, the contact degree of most of real W UMa systems which were observed to be in good thermal contact is about 10%, only a little of real late-type contact systems (ER Cep, AO Cam, Maceroni & van’t Veer, 1996) is observed to be in good thermal contact with a smaller contact degree ($\sim 2$ per cent).

In a series of papers, Shu and his colleagues have extended the unequal entropy model by putting an equal entropy common envelope on top of interiors with the entropy difference, $\Delta S = 0$, Shu, Lubow & Anderson 1976, 1979, 1980; Lubow & Shu 1977, 1978). These models satisfy the light curve constraint but requires a temperature discontinuity region between the common envelope and the interior of the secondary component. This so-called DSC model has been strongly attacked by other investigators (Hazlehurst & Rofael 1978; Papaloizou & Pringle 1974; Smith, Robertson & Smith 1980) since it was thought to be violating the second law of thermodynamics.

The structure and evolution of W UMa stars still comprises many unsolved questions although many progresses have been achieved. The most difficult problem concerns the energy transfer between the components, i.e., it is not clear where and how the energy is transferred. The mechanism causing energy transfer is still a largely unsolved problem. In addition, we do not confirm that the transfer occurs in the base, or the outermost layers, or the whole of common envelope although it seems probable that the transfer occurs in the common envelope, above the inner Roche critical surface, where the stars are in good contact.

In present paper, we have calculated a relative-radius grid of contact binaries with different mass ratios and different contact degrees according to the Roche potential to ensure the surfaces of two components lying on the same equipotential by interpolation using this radius grid. In addition, we assume that the energy transfer occurs in the different regions of common envelope to investigate the effect of region of energy transfer on the structure and evolution of contact binaries. As result of our investigation, we find that the region of energy transfer has a significant influence on the structure and evolution of the contact binaries, and that the energy transfer may take place in the outermost layers of the common envelope for W-type systems, and this probably takes place in the deeper layers of the common envelope for A-type systems.

2 STRUCTURE EQUATIONS AND BOUNDARY CONDITION

2.1 Differential equations

Eggleton’s stellar evolution code used in present work has considered the effect of stellar rotation on stellar structure. Let $\omega$ be the angular velocity, $m_i$ the mass variable, $g_i$ the effective gravity in component $i$, $\sigma_{e\nu,i}$ the source (when positive) or sink (when negative) of energy per unit of mass caused by the interaction of the components. Employing standard notation, the basic differential equations are the following

$$\frac{\partial \ln T_i}{\partial m_i} = \frac{1}{4\pi r_i^2} \frac{1}{g_i} \left( g_i - 4\pi r_i^2 P_i \frac{1}{T_i} \right) (1 - \frac{2\omega^2 r_i^3}{3Gm_i})$$  (2)

$$\frac{\partial \ln P_i}{\partial m_i} = \frac{1}{4\pi r_i^2} \frac{1}{g_i} \left( g_i - 4\pi r_i^2 P_i \frac{1}{T_i} \right) (1 - \frac{2\omega^2 r_i^3}{3Gm_i})$$  (2)

$$\frac{\partial \ln T_i}{\partial m_i} = \frac{\partial \ln P_i}{\partial m_i} \nabla_i$$  (4)

where $\nabla_i$ is the temperature gradients of both components, and given by
\[ \nabla_i = \begin{cases} 
\nabla_{a,i} & (\nabla_{r,i} > \nabla_{a,i}), \\
\nabla_{r,i} & (\nabla_{r,i} < \nabla_{a,i}), 
\end{cases} \]

where \( \nabla_{r,a} \) are the radiative and adiabatic temperature gradients. The radiative temperature gradient is given by

\[ \nabla_{r,i} = \frac{3}{16\pi G} \frac{k_i L_i P_i^2 (1 - 2\omega^2 r_i^3)}{3Gm_i} \cdot \]  

\section{2.2 Contact condition}

In hydrostatic equilibrium the surface of each star should coincide with the same equipotential surface which, in Roche approximation, requires

\[ \frac{R_2}{R_1} = \left( \frac{M_2}{M_1} \right)^{\beta}, \]

where \( R_{1,2} \) are the radii of both components, \( M_{1,2} \) the masses of both components, \( \beta \) the index of mass-radius relation of contact binaries. The value of \( \beta \) varies not only with mass ratio, but also with the depth of contact. It covers a range of 0.45 \( \sim \) 0.50 for the marginal contact binaries, and will cover a larger range if the contact depth varies. But most of investigators takes \( \beta = 0.46 \) approximately, which is very close to a value of a marginal contact binary with a mass ratio of \( q \approx 0.5 \). Since the mass ratio of each contact binary varies during its evolution, some investigators use a boundary condition as the following

\[ \frac{R_1}{R_{\text{crit}1}} = \frac{R_2}{R_{\text{crit}2}}, \]

where \( R_{1,2} \) are radii of two components, \( R_{\text{crit}1,2} \) the Roche critical radii. Eqs. (7) and (8) only approximately ensure the surfaces of both components lying on the same equipotential surface. In order to let the surfaces of the two stars lying on the same equipotential accurately, we calculate a radius grid of contact binaries which have different fixed mass ratios (0.02, 0.04, 0.06, \ldots, 1.0) and different contact degrees (0\%, 2.5\%, 5\%, \ldots, 100\%) for each fixed mass ratio using a dimensionless Roche potential

\[ \Phi = \frac{1}{r_1} + 1 + q \frac{x - \sqrt{1 + q}}{2} + \frac{(x - \sqrt{1 + q})^2 + y^2}{2(1 + q)} \]

where \( \Phi \) is the potential at an arbitrary point \( P, r_{1,2} \) the distances of point \( P \) from the centers of gravity of the two stars, \( x \) and \( y \) the Cartesian coordinates of point \( P \) (the origin of the rectangular system of Cartesian coordinates at the center of gravity of mass \( M_1 \), and the \( x \)-axis of which coincides with the line joining the centers of the two stars, the \( y \)-axis in the orbital plane), \( r_{1,2} \), \( x \), and \( y \) are in unit of orbital separation of the binary, \( q = \frac{M_2}{M_1} \) the mass ratio. We can accurately ensure the surfaces of two stars lying on the same equipotential by interpolation using our radius grid. Eggleston’s stellar evolution code uses the most modern physics as reported by Pols et al. (1995), and it can calculate the evolution of a single star or a binary (not contact). For evolution of a single star, just choose an orbital period so large that there is no prospect of RLOF. For evolution of a detached binary, the evolution of each star is similar to that of a single star. For evolution of a semi-detached binary, RLOF has been treated within the code, so that the mass above Roche lobe is transferred to its companion, and the mass lost at a rate proportional to the cube of the fractional excess of the star’s radius over its lobe’s radius \( \frac{dn}{dt} = -C \cdot \text{Max}[0, (\frac{\pi R_3^2}{\frac{4}{3} \pi R_2^2})^3] \), where \( dn/dt \) gives the rate at which the mass of the star changes, \( r \) the radius of the star, and \( R_{\text{crit}} \) the radius of its Roche lobe), it has been tested thoroughly and works very reliably. With \( C = 1000 M_\odot/yr \), RLOF proceeds steadily (Han et al. 2003).

When a close binary evolves into a contact one, the mass which is transferred is no longer the mass above its Roche lobe. If the surface of star 2 is at a higher potential than star 1, so that an unbalanced pressure gradient will force mass transferred from star 2 to star 1, then this boundary condition should be modified as

\[ \frac{dn_2}{dt} = -C \cdot \text{Max}[0, (\frac{R_2}{R_{20}})^3] \]

in which

\[ R_{20} = \frac{\pi}{8} \sqrt{\frac{g_1 R_1^2 + g_2 R_2^2}{g_1 R_1^2 + g_2 R_2^2}} \]

where \( g_{1,2} \) are the surface densities of both components, \( R_1 \) the radius of star 1 (known), \( R_2 \) the radius of an equipotential surface in star 2, whose potential is equal to the surface potential of star 1, and it is obtained by the interpolation according to our radius grid. \( R_2 \) the real radius of star 2 because of its evolution. (The mass which is transferred to star 1 is the mass above \( R_{20} \) rather than that above \( R_2 \), because the surface potential of star 1 will be higher than that of star 2 if the mass above \( R_2 \) is transferred to Star 1.) Assuming that the radii of the two stars 1 and 2 become \( R_{10} \) and \( R_{20} \) respectively after mass transfer, they should satisfy a very accurate condition \( \frac{R_{10}}{R_{20}} = \frac{M_1}{M_2} \). Meanwhile, the mass gained by star 1 equals to that lost by star 2, i.e., \( (\frac{4}{3} \pi R_2^3 - 4 \pi R_1^3) g_1 = (\frac{4}{3} \pi R_1^3 - 4 \pi R_2^3) g_2 \). Then Eq. (11) can be easily derived from the two conditions mentioned above. Eq. (10) gives a mass loss rate from star 2 to star 1. If the mass is lost by star 1, gained by star 2, the mass loss rate can be derived in the same way.

For interacting binaries, we put the two stars ‘side by side’ in the computer, so that 22 differential equations (11 equations should be solved for each stars), in 22 variables, have to be solved. The direct coupling between the sets of equations for the two components is via the boundary conditions and energy transfer discussed in sect. 3.

\section{3 A MODEL FOR ENERGY TRANSFER}

Struve (1938) first recognized the unusual mass-luminosity relationship of the secondary components of W Ursae Majoris systems, suggesting that it might be causally related to a possible common envelope. Osaki (1966) noted that von Zeipel’s theorem would require the observed approximate constancy of radiative flux over the surface of a system with a radiative common envelope in order to maintain hydrostatic equilibrium. The fact that most W UMa systems appear to have convective envelopes, however, led Lucy (1968) to propose that the over-luminosity of the secondaries is directly attributable to energy transfer between the two stars within a convective envelope.

It is not clear that where and how the energy is transferred from the primary (more massive star) to the secondary. It seems probable that the transfer occurs in the
common envelope, above the inner Roche lobe, where the stars are in good contact, but we could not confirm that the energy is transferred in the base or the outermost layers or the whole of the common convective envelope. Meanwhile, the mechanism causing energy transfer between the two components remains uncertain. Moses (1974, 1976) and Moses & Smith (1974) have argued that the large-scale circulation envisaged by Hazlehurst & Meyer-Hofmeister (1973) or Nariai (1974) is likely to be destroyed by both normal, vertical convection and coriolis forces. They consider that a more appropriate picture would involve horizontally moving eddies, again driven by horizontal pressure gradients, which would travel only a short distant horizontally before of convection; it differs fundamentally from the mechanism of Hazlehurst & Meyer-Hofmeister in that the eddies are not in thermal equilibrium with the surroundings, and will therefore dissipate on a thermal time-scale even if no other process acts to break them up. On the contrary, Webbink (1977) argues that a large-scale circulation can be maintained, and has returned to the model of Hazlehurst & Meyer-Hofmeister. Therefore, the mechanism causing energy transfer is still largely an unsolved problem. Because of these uncertainties, most numerical models of contact binaries have been phenomenological, simply inserting an artificial energy source $\Delta L$ in the secondary, usually in the adiabatic part of the common envelope, and a corresponding energy sink in the primary, the value of $\Delta L$ being chosen to satisfy the requirements that the system be both in contact and in equilibrium. As Robertson & Eggleton (1977), let $L_1$ and $L_2$ be the luminosities (nuclear plus thermal luminosities), and $m_1$ and $m_2$ the masses of the components. Although the W UMa-type systems are relatively much more common, $\sim$ 1 per 10 in the equivalent magnitude range, we still assume that the effect of fully efficient energy exchange is to equalize the light-to-mass ratio of the stars, since this seems to obtain approximately among the observed systems. If $\Delta L_0$ is lost by the primary and gained by the secondary, we require that

$$\frac{L_1 - \Delta L_0}{m_1} = \frac{L_2 + \Delta L_0}{m_2},$$

(12)
since transfer is not fully efficient at all phases, an arbitrary factor $f$ is introduced, which varies through the cycle and goes to zero with the depth of contact. Thus we take:

$$\Delta L = f \cdot \Delta L_0, \quad (0 \leq f \leq 1),$$

(13)

where $f$ is the efficient factor of energy transfer. We take

$$f = \text{Min}[1, \alpha(d^2 - 1)]$$

(14)
in which

$$d = \text{Max}[1, \min\left(\frac{r_1}{R_{\text{crit}1}}, \frac{r_2}{R_{\text{crit}2}}\right)]$$

(15)

where $r_1, r_2$ are the radii of both stars, $R_{\text{crit}1,2}$ the Roche critical radii of both stars. The parameter $\alpha$ is expected to be moderately large, so that heat transfer becomes fully efficient for stellar radii exceeding the Roche radii by some standard small amount.

The luminosity transferred by circulation currents from the primary to the secondary adopted by Kühler (2002b) is

$$\Delta L = \int_0^{M_2} \sigma_{\text{ex,2}} dm_2 = - \int_0^{M_1} \sigma_{\text{ex,1}} dm_1,$$

(16)
naries. Then we consider the rotation of both components of the binary, but the energy transfer between the two components, and take a orbital period of 0.3732 days, and orbital angular momentum, $J$, of $5.283 \times 10^{35}$ erg s ($\log J = 51.723$).

The initial separation is about $2.65R_\odot$. The initial model is a detached binary, and the surface of the primary lies only a short way inside its Roche lobe which it fills after $2.8 \times 10^6$ yr of nuclear evolution. Thereafter it loses mass to its companion at a rather slow rate which rises approximately $\sim 1.4 \times 10^{-5}M_\odot/yr$. At the beginning of the mass transfer, the mass loss rate is very slow, the radius of the secondary is smaller than that of a ZAMS star with the same mass because the secondary with a convective envelope gains mass and its radius decreases. When the mass loss rate rises to a higher value, the secondary is too late to adjust its structure thermally after it gains mass from the primary and its radius increases so that the radius of the secondary is larger than that of a ZAMS star with the same mass (see Figure 2).

The addition mass on to the secondary causes it to expand and its effective temperature and luminosity to increase, and after a total of $3.5 \times 10^7$ yr of evolution from the main sequence it has swollen to fill its Roche lobe, so that the system evolve into a contact system. The contact binary formed at this point has masses of 1.0 and $0.8M_\odot$ and a mass ratio of 0.8. Since we do not consider the energy transfer between the two components, the two stars evolve steadily towards equal masses on a thermal timescale (see Figure 2). Although some of W UMa-type systems (at least V348 Car, Hilditch & Bell, 1987) with closely equal masses are consistent with this scenario, however most of the real W UMa-type systems are not equal-mass ones. This can be regarded as sufficient proof that the majority of contact binaries never approach the equal-mass state, suggesting that the transfer of significant amount of energy through the common envelope surrounding the components in direction from the primary to the secondary has the effect of preventing mass transfer so that most of the real binaries do not evolve towards contact systems with equal masses. Therefore, the following discussions only restrict to the possible consequence of energy transfer.

4.2 Contact Evolution With Energy Transfer

4.2.1 The regions of energy transfer

We consider now the evolution of the binary system from the initial contact model with energy transfer included in the manner described in sect. 3. Nuclear evolution is included in the models, although the influence of the composition changes is negligible in the early stage. The differential equations describing both the structure and the chemical composition for both stars are solved simultaneously. The luminosity increment $\Delta L$ is applied to each component with appropriate sign in a relevant region of both components. As the uncertainty of the region of the energy transfer, we take $\alpha = 45$, and assume that the energy transfer takes place in the different regions of the common envelope to investigate the effect of the energy transfer region on the structure and evolution of the contact systems. At first, we assume that the energy transfer takes place in the adiabatic part in the base of common convective envelope, and energy increment is applied...
plied in 10 meshpoints just above the Roche lobe, so that more stellar material takes part in energy transfer, and the significant energy (∼ 0.5\(L_\odot\)) can be transferred between the two components. Then we assume that the energy transfer takes place in the whole of the common envelope surrounding the two components, and energy increment is applied in all meshpoints in the common envelope. Finally, we assume that the energy transfer takes place in the outermost layers in the common envelope of the two stars, and the luminosity increment is applied in 10 meshpoints near the surface of common envelope. The evolution of the surface effective temperature of both components are plotted in Figures 3, 4, and 5, respectively. As seen from Figs. 3 and 4, The same result is obtained when the energy transfer is assumed to take place in the base or the whole of the common envelope. In these cases, there is no possibility of the effective temperature of the secondary (less massive star) exceeding that of the primary at any time of a cycle even though energy transfer is fully efficient, and a large temperature difference between the two components occurs in a considerable time of a cycle when the energy transfer takes place in the base or the whole of the common envelope. This would usually be interpreted as implying that these contact models represent the A-subtype W UMa systems only, but the W-subtype W UMa systems. It is seen in Figure 5 that the temperature of the secondary can exceed that of the primary for a considerable time of a cycle, and that a small temperature difference (\(\Delta T < 300\text{K}\)) between the two components occurs in a large part of the time of a cycle. In this case, the model represents the structure and evolution of a W-subtype W UMa system during a considerable time of a cycle when the energy transfer takes place in the outermost layers (10 meshpoints) near the surface of the common envelope. Therefore, the region of the energy transfer has a significant influence on the structure and evolution of the contact binaries. The same results are obtained if the energy transfer is assumed to take place in the base or in the whole of the common envelope, because most of energy is still transferred in the base of the common envelope due to a higher density of the base of the common envelope even if the energy transfer is assumed to take place in the whole of the common envelope. However, it is very different from the result based on the assumption that the energy transfer in the outermost layers. Therefore, we conclude that the energy transfer in W-type systems may take place in the outermost layers near the surface of common envelope, and energy transfer in A-type systems probably takes place in the deeper layers of their common envelope.

\textit{Moses (1974)} has proposed a small-scale eddy model underlying the energy transfer which is similar to mixing-length theory, and concluded that the energy transfer in contact binaries must be characterized by a scale of a complicated eddy structure which is much shorter than the separation between the two stars, and that it appears possible to divide the envelope into a surface layer where convective mixing determines the scale and a deeper layer where Coriolis effects dominate. Meanwhile, each A-type system has a radiative envelope, and the energy transfer is caused by the eddies due to Coriolis effects, and should take place in the deeper layers (base) of the common envelope; However each W-type system has a convective envelope, the energy transfer is attributed to eddies due to convection, and should take place in the surface layers of the common envelope. Since the surfaces of both stars are radiative rather than convective, it follows that convection is by no means essential to heat transport in contact envelopes, although it may well have an important influence. In our model with low total mass, we assume the energy transfer takes place in the outermost layers of the common envelope. The thermal structure of the two components of the binary (\(\log J = 51.723, \alpha = 45\)) during a phase of contact evolution and of semi-detached evolution is shown in Figure 6. In contact phase, energy transfer takes place in the outermost layers of the common envelope, and the outermost layers of the envelopes of both components are so similar that the difference is slight.

It is seen in Figure 6a that the luminosity of the secondary’s outer layers below the energy sources is very low, and in these layers the core luminosity (includes thermal and nuclear luminosities) has almost completely been exhausted by negative values of the \(\epsilon_\alpha\)-terms in the energy balance, i.e. by the expansion of the secondary (\textit{Kahler (2002b)}).

\subsection{4.2.2 Cyclic evolution}

We consider now the evolutionary behavior of the binary, also beginning at the contact model with a mass ratio of about 0.8. The evolution of the binary, with energy transfer included and the luminosity increment is applied in the outermost layers (10 meshpoints) of the common envelope, is shown in Figures 7, 8 and 9 for \(\alpha = 45\). The system undergoes thermal cycles on a thermal timescale. At the early stage, the evolution of the contact phase resemble the properties in the case of no energy transfer, since the efficiency of energy transfer is extremely low and equilibrium configuration remains at the equal-mass state. However, the expansion of the secondary is hastened by the energy which is being deposited in its envelope, and the growth in the depth of the common envelope which these causes lead to the continued increase of energy transfer. The added mass onto the secondary stops when the significant energy is transferred from the primary to the secondary. This followed by a rapid rise in the rate of mass transfer back to the primary, and the rate of mass transfer rises rapidly to a higher value because of the rapid increase in the heat transfer, then decreases rapidly to stable value (see Figure 8a), however these could not prevent a further increase in the depth of contact and in the luminosity transfer (see Figure 8d and Figure 9c) until the decrease in radius of the secondary caused by mass loss can not be compensated by its increase caused by luminosity transfer. If this has occurred, the binary evolves rather slowly towards smaller mass ratios as each star attempts to obtain thermal equilibrium. Throughout this phase the secondary contracts rapidly and the depth of contact decreases so that this phase will be rapidly terminated when full efficient energy transfer can no longer be maintained. This phase of good thermal contact is characterized by a remarkable constancy of most of properties of both stars.

Once full efficiency is lost, the secondary, no longer adequately supported, contracts very rapidly and the direction of mass transfer reverses, the rate of mass transfer rising to its highest value in the cycle. The secondary breaks contact with its Roche lobe, and continues to collapse towards a main-sequence equilibrium state, its temperature and luminosity falling rapidly. The luminosity of the primary also decreases during semi-detached phase, since the Roche lobe
Figure 6. The thermal structure of the primary (solid line) and the secondary (dashed line) during a phase of semidetached evolution (SD) and of contact evolution (C) with efficient energy transfer. In contact, luminosity transfer takes place in the outermost layers of common envelope.

again contracts the star to prevent its free expansion towards thermal equilibrium, and the process of raising matter up through the star, for transfer to the secondary, requires significant quantities of energy, at the expense of the surface luminosity. Meanwhile, the luminosity produced by nuclear sources also falls because of the core of the primary is also expanding and cooling during semidetached phase (Robertson 1977). Since the thermal timescales of the two components are not equal, the radius of the secondary will increase before the secondary collapses to be a main-sequence equilibrium state during semi-detached phase, so that the system evolves into a contact binary again. Therefore, the two stars of the system are unlikely in thermal equilibrium at any time of a cycle.

The radius-mass diagram is shown in Figure 7. It is seen in Figure 7 that the radius of the secondary is larger than that of a ZAMS star with the same mass, and the radius of the primary is smaller than that of a ZAMS star with the same mass even though the system evolves in semidetached phase. It suggests that the two components of the system are not in thermal equilibrium at any time of a cycle. As seen from Figures 8 and 9, the system undergoes cyclic evolution on a thermal timescale, with a period of about $10^7$ yr, and the system spends a large part of the time of a cycle in contact evolution (lasting about 70 percent of the time), only a small part of time in semidetached evolution. Meanwhile, in a long phase (lasting a bout 60 percent of the time of a cycle), the system is in good thermal contact with a temperature difference between the two components not larger than 300 K (Kahler 2002b) had given a model in which the temperature difference between two components is less than 300K during 80 percent time of a cycle, but the maximum contact degree of his model is extremely low (about 2%) which is much lower than the mean contact degree of the real W-type systems of about 13% (Smith 1984), suggesting that his model could not evolve in high contact depth. However, our model can evolve in a maximum contact degree of about 7% (see Figure 8d) which is very close to the mean value of W-type systems. The evolution of the binary in any cycle does not completely repeat the evolutionary track of
the previous cycle, although the evolution of the binary undergoes the thermal cycles. As seen from the Figure 8a, the mean mass of the primary in any cycle is larger than that in the previous cycle, so that the mean mass ratio of the system becomes smaller and smaller along with the evolution of the binary. It indicates that the system will evolve to a typical W-type system with a mass ratio of about 0.5 although a contact system which originates from a initially detached or semi-detached binary has a higher mass ratio of about 0.7 (Mochinaki 1981). Meanwhile, the maximum contact depth of the binary becomes higher and higher because of the evolution of the system (see Figure 8d). The evolution of the luminosities of the two stars are not synchronous, also are the radii of them, i.e., the luminosity (radius) of the secondary does not rise to the maximum when the luminosity (radius) of the primary reaches the minimum, and vice versa. It is caused by the different thermal timescales of both components. Because the thermal timescales of both components are unequal, the two stars unlikely reach the thermal equilibrium, but the two components of the system attempt to reach the thermal equilibrium, and the attempt of the system to reach the nonexistent thermal equilibrium, coupled with Roche geometry, is the driver for the cycling behavior (Rahunen & Vilhu 1981). The evolution of the temperatures and the orbital period of the two components of the system is shown in Figure 9b,d. It is seen in Figure 9 that the mean period in any cycle of the system becomes longer and longer as the evolution of the binary, and that the temperature of the secondary excesses that of the primary in a large part of the time in any cycle.

\[ 1.5\log T_{\text{eff}} - \log P = 5.975...6.15, \] (17)

4.2.3 The cycles in the period-colour diagram

As shown by Eggen (1961, 1967), the observed W UMa systems are located in a strip of the period-colour diagram, the two boundaries of the period-colour diagram are written by K"ahler (2002a) as

\[ \log T_{\text{eff}} - \log P = 51_{-723}^{+723}, \] (15)

\[ \alpha_{975}(15,17) \]

\[ \text{with} \]

\[ \log J = 51.723 \text{ and } \alpha = 45 \]

\[ \text{with} \]

\[ \text{the} \]

\[ \text{the} \]

\[ \text{the} \]

\[ \text{the} \]
Figure 8. The evolution of some quantities [such as the masses, radii, luminosities, mass loss rates, and contact degrees \( F = \frac{\Omega - \Omega_{in}}{\Omega_{out} - \Omega_{in}} \)] of the primary (solid lines) and the secondary (dashed lines) of the binary as a function of Age. A steady cycle appears to be set up after two or three initial loops.

of the primary if the energy transfer takes place in the outermost layers of the common envelope, suggesting that during contact phase of the cycle the models are reasonably good agreement with the observed properties of A-type W UMa systems when the energy transfer is assumed to take place in the base or the whole of common envelope, and the model is very consistent with the observed properties of W-type systems when the energy transfer is assumed to take place in the outermost layers of the common envelope. Therefore, we conclude that the energy transfer may take place in the base of the common envelope for A-type systems, and the transfer takes place in the outermost layers for W-type systems. Since the surfaces of both components are radiative rather than convective, it follows that convection is by no means essential to heat transport in contact envelopes, although it may well have an important influence.

The W UMa systems have been divided by Binnendijk (1970) into A-type and W-type systems according as the primary minimum in the light curve is a transit, in which the smaller star partially eclipses the larger, or an occultation, when the larger star is in front. The observations of W UMa stars have been reviewed by Mochnacki (1981). The main differences between these subclasses are as the followings: the A-type systems have longer periods, are hotter, smaller mass ratio, and in better contact (i.e. the higher contact depth). Meanwhile, Wilson (1978) has found that the A-type systems are likely evolved stars. For a low total mass contact binary, we assumed that the energy transfer takes place in the outermost layers of the common envelope. Our model indicates that the mass ratio of the binary becomes smaller and smaller, and the contact depth becomes higher and higher as the evolution of the system, that is to say, the system steadily evolves towards a contact binary with a smaller mass ratio and a deeper common envelope. It suggests that some A-type systems with low total mass could be considered as later evolutionary stages of W-subtypes, although W-type systems show greater activity in the form of period and light curve changes, which were originally thought to occur on a thermal timescale (Rucinski 1974) but now their periods appear to consist of abrupt
changes typically in a timescale of a few months; And A-type systems possess more stable light curves and less rapidly varying periods. However, in this scenario the system evolution was towards smaller mass ratio, longer periods and deeper contact, and the better physical contact will lead to the gradual disappearance of the W-type peculiarities (hotter secondary, light curve perturbations, and frequent period changes). Therefore, some of low total mass systems can be regarded as the later evolutionary stages of W-type systems.

In order to obtain a model of W-subtype W UMa system in which the temperature of the less massive star is higher than that of the massive one, the investigators assumed that the energy transfer takes place in the superadiabatic part or even in extreme superadiabatic region of the envelope of the secondary. However, Hazlehurst (1974) suggested that the extreme superadiabatic transfer must be excluded because the heat capacity of the secondary’s subphotospheric layers is too small. In our model, we do not assume that the energy occurs in the superadiabatic or extreme superadiabatic part of the envelope of the secondary, but the surface temperature of the secondary can exceed that of the primary during the time when the radius of the primary increases rapidly, or the radius of the secondary (less massive component) decreases rapidly (see figure 9a, b). The temperature of the secondary exceeding that of the primary in our model can be attributed to exhaustion of a part of the nuclear luminosity of the primary due to the expansion of the primary, or the release of the gravitation energy of the secondary because of its contraction. Meanwhile, it suggests that the two subtype W UMa systems are probably caused by the energy exchange between the gravitational (potential) energy and thermal one. Wang (1994) has showed that the two types of contact binaries are in two different TRO states: the less massive component of W-types are shrinking whereas the less massive star of A-types are swelling. The contraction of the secondary in W-type systems releases some of its gravi-

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**Figure 9.** The evolution of the radii and temperatures of the primary (solid line) and the secondary (dashed line), and also the orbital period (in days) and the transferred luminosity of the system.
tational energy; therefore, it makes the effective temperature of the secondary higher than the primary. However, in our model, the contraction of the secondary is one of the mechanisms which can make the temperature of the secondary higher than the primary, but it only lasts a very short period, so the surface temperature of the secondary higher than the primary is mainly caused by the depletion of a part of the luminosity of the primary due to its rapid expansion.

Our models for contact binary systems exhibit cyclic behavior about a state of marginal contact, with a period of $10^7$ yr. In cyclic evolution, the two components of the system are unlikely in thermal equilibrium because of the difference in their thermal timescales, but they attempt to reach the thermal equilibrium. The attempt of the system to reach an nonexistent thermal equilibrium, coupled with Roche geometry, is driver for cycling behavior (Rahunen & Vilhu 1981). A larger temperature difference ($\Delta T_{\text{eff}} > 300$K) between the two components occurs in a part of the time of a cycle (lasting about 30–35 percent time of a cycle). Almost all of the previous investigators thought that this requires there to be as many short-period binary with EB light curves as with EW light curves, and that the models for contact binaries encounter a difficulty, so called light curve paradox. Rucinski (2002) gives 13 EWs and 5 EBs (and 14 ELLs, which have too small an amplitude to be classified as EWs or EBs). It is reasonable to identify the EWs as contact binaries and the EBs as semi-detached. The ratio of 5/13 is not much out of line with TRO theory, so we are not sure there is any light curve paradox. However, the W UMa systems indeed undergo angular momentum loss without doubt, and the most likely angular momentum loss mechanism is magnetic braking (Huang 1966, Mestel 1968). The Einstein X-ray observations and IUE ultraviolet observations (Eaton 1983) showed that W UMa systems are strong sources, suggesting surface activity of the kind we observe on the Sun, and so the presence of the magnetic fields. The stellar wind would cause magnetic braking, and we will discuss the evolution of W UMa systems included the angular momentum loss in our future work. Meanwhile, we do not consider the energy source at secondary’s atmosphere provided by the accreting matter from the primary at our present work. This energy source can hasten the expansion of the secondary, and shorten the time spent in the semi-detached evolution. We refer to a forthcoming work included this extra energy source for the secondary.

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