ANALYSIS AND PREDICTION FOR THE SPREADING OF COVID-19 PANDEMIC IN INDIA USING MATHEMATICAL MODELING

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Abstract In the present time, the biggest problem of the world is the outbreak of novel coronavirus. Novel coronavirus (COVID-19), this one name has become a part of our daily lives over the past few months. Beyond the boundaries of medical science, coronavirus is now the main subject of research in all fields like Applied Mathematics, Economy, Philosophy, Sociology, Politics up to living room. The epidemic has brought unimaginable changes in our traditional habits and daily routines. Thousands of people in our country are fighting with the rest of the world to survive in various new situations. There are different kinds of coronavirus appeared in different times. In this time, Severe Acute Respiratory Syndrome Coronavirus-2 (SARS-CoV-2) is responsible for the coronavirus disease of 2019 (COVID-19). This virus was first identified towards the end of 2019 in the city of Wuhan in the province of Hubei in China. Within very short duration of time and very fast, it has spread throughout a large part of the world. In this study, the main aim is to investigate the spreading rate, death rate, recovery rate due to corona virus infection and to study the future of the coronavirus in India by using mathematical modeling based on the previous data. Mathematical models, in this situation, are the important tools in recruiting effective strategies to fight this epidemic. India is at high risk of spreading the disease and is facing many losses in socio-economic aspects. With current infection rates and existing levels of personal alertness, the number of infected people in India will increase at least in the next three months. Proper social awareness, maintain of social distance, large rate of testing and separation may break the chain of the Coronavirus-2.
Keywords Corona Virus · Covid-19 · India · Pandemic · Mathematical modeling

1 Introduction

The COVID-19 pandemic or coronavirus is a grisly disease of 2019 and it is caused by Severe Acute Respiratory Syndrome Coronavirus-2 (SARS-CoV-2). Its first appearance was identified in Wuhan, China in December 2019. Within very short period of time, the infection of COVID-19 has spread to the other area of China and around the world. The World Health Organization declared the outbreak a Public Health Emergency of International Concern on 30 January 2020 and a pandemic on 11 March 2020. More than 6.15 million cases of COVID-19 have been identified in more than 188 countries and territories as of 1st June 2020. More than 3, 71,000 peoples have lost their lives in this pandemic. Also, more than 2.63 million people have recovered from this virus around the world. The common symptoms of this virus are fever, cough and fatigue, shortness of breath and loss of sense of smell. In several cases, pneumonia and acute respiratory distress syndrome have been found in the infected persons. Also, in several cases, there are no particular symptoms has found in the infected patients. Researchers say coronavirus may start showing symptoms in 5 days. The virus primarily spread between people during close contact, most often via small droplets produced by coughing, sneezing and talking.

In India first active case of COVID-19 was identified on 30 January 2020. As of 1 June 2020, the Ministry of Health and Family Welfare have confirmed a total of 190,535 positive cases, 91,819 recoveries (including 1 migration) and 5,394 deaths in the country. The fatality rate in India is relatively lower at 3.09% where as global rate is 6.63% as of 20 May 2020. Most of the positive cases were reported from six cities Mumbai, Delhi, Ahmadabad, Chennai, Pune and Kolkata. As of 24 May 2020, Lakshadweep is the only region in the country which has not reported a case. Government fixed some instruction for the people to reduce the chances of infection of COVID-19 such as hand washing, covering ones mouth when coughing, maintaining social distance, wearing a face mask in public setting. Also monitoring and self-isolation for the people who suspect they are infected. There is no known vaccine or specific antiviral treatment for this disease. Many countries are trying to find the vaccine for this disease.

The main aim of present study is to investigate the spreading rates, changes in mortality and recovery rates due to corona virus infection and future predictions of COVID-19 pandemic in India by mathematical modeling based on previous data. The outcomes of the present mathematical modeling may help the appropriate authorities to prepare a proper medical planning and to take relevant steps to control this pandemic. We have taken some data for this mathematical modeling from various sources[1,2,3,4,5,6,7,8,9,10,11].

2 Mathematical Model

The spreading rate or new case of infection is proportional to multiplication of the number of total infected patients and number of persons who have not caught an infectious disease. Therefore the model for an infectious diseases can be written as:

\[ \frac{dI}{dt} = KI(P - I) \]  

(1)

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where $I$ is the total infected persons, $t$ is the time (day) and $P$ is the total population of a country or a region. Therefore, $\frac{dI}{dt}$ represents new COVID-19 cases per day. The spreading rate of covid-19 is covered by the parameter $K$. To make the model more consistent with the actual data this parameter $K$ have been taken as a time dependent parameter. The rate of spread will decrease over time due to awareness and other government activities. Hence the parameter $K$ can be written as, $K(t) = k \frac{1}{\sqrt{t}}$, here $k$ is the proportional constant.

For initial condition, it is considered that, at $t = 0$, $I(0) = I_0$ and to compute the value of $k$ we have taken that at time $t = t_1$ the number of patient will be $I(t_1) = I_1$. Therefore, the initial value problem related to the infectious disease becomes,

$$\frac{dI}{dt} = \frac{k}{2\sqrt{t}} I(P - I) \quad (2a)$$
$$I(0) = I_0 \quad (2b)$$
$$I(t_1) = I_1 \quad (2c)$$

Solving Eq. (2a), we have,

$$I(t) = \frac{P}{1 + \frac{1}{e^{Pp\sqrt{t}}}e^{-pk\sqrt{t}}} \quad (3)$$

Using (2b), equation (3) becomes

$$I(t) = \frac{P(P - I_0)}{P - I_0(1 - e^{-pk\sqrt{t}})} \quad (4)$$

Now, from Eq. (2c) we get, $k = \frac{1}{\sqrt{t}} \ln \left( \frac{I_1(P - I_0)}{I_0(P - I_1)} \right)$ and equation (4) can be written as

$$I(t) = \frac{P(P - I_0)}{P - I_0(1 - e^{-\kappa\sqrt{t}})} \quad (5)$$

where

$$\kappa = \frac{1}{\sqrt{t}} \ln \left( \frac{I_1(P - I_0)}{I_0(P - I_1)} \right) \quad (6)$$

From equation (6), one can find the value of $\kappa$ and for this case $\kappa = 0.6459$. The additional data $t_1$ and $I_1$ can be choosen randomly. Moreover, the control parameter $\kappa$ depends on $t_1$, $I_1$ and $P$. We have assumed that the number of population in India is 1382642280[7]. We have considered that $I(t)$ is the total case of COVID-19 in India, therefore $\frac{dI}{dt}$ and $\frac{d^2I}{dt^2}$ becomes new cases and growth rate of the number of patients per day in India.

From equation (2a), one can get $\frac{d^2I}{dt^2}$ and it is given by,

$$\frac{d^2I}{dt^2} = \frac{k}{2\sqrt{t}} \left( P - 2I - \frac{1}{2t} \right) \frac{dI}{dt} \quad (7)$$

The values of $\kappa$ have been shown due to the changes of $t_1$ in the Table 1. It is observed that The values of $\kappa$ may decrease or increase due to the increment of $t_1$. 

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Table 1: Estimated values of control parameter ($\kappa$) for the model system (2)

| $t_1$ | $I_1$  | $\kappa$ | $I(t_1)$ | Date    |
|-------|--------|----------|----------|---------|
| 45    | 78003  | 0.6459   | 3763     | 13 May  |
| 30    | 29974  | 0.6165   | 1873     | 28 April|
| 40    | 56342  | 0.6337   | 3344     | 8 May   |
| 50    | 96169  | 0.6424   | 4630     | 18 May  |
| 60    | 158333 | 0.6508   | 7300     | 28 May  |

Now, we will discuss about mortality rate and recovery rate of an infectious disease. It is obvious that, the mortality rate is proportional to the number of infected patients. Thus, the model equation for death rate is given by,

$$\frac{dD}{dt} = \mu I(t) \quad (8)$$

Here $D(t)$ is the total death in a region and $I(t)$ is given by the equation (5). $\mu$ is the proportional constant. For $t = 0$ (i.e. 29 March) it can be found that $D(0) = 27$ from the actual data. Therefore, the equation (8) becomes an initial value problem.

The recovery rate is also proportional to the total cases of disease. Hence, the model for recovery rate becomes,

$$\frac{dR}{dt} = \delta I(t) \quad (9)$$

Where $R(t)$ is total recoveries in a region. $I(t)$ can be find from equation (5). $\delta$ is the proportional constant and infectious individuals either die at a rate $\mu$ or recover at a rate $\delta$. From the previous data we get $R(0) = 96$ and equation (9) also becomes a IVP. We have used 4th order Runge-kutta method to solve the equations (8) and (9).

3 Results and Simulations

Using the date 29 March 2020 as $t = 0$, $I(0) = I_0 = 1024$ patients; $t_1 = 45$ as 13 May 2020 with $I(t_1) = I_1 = 78003$ patients, let us assess the Fig. 1 where we may observe the behavior of the spread (approximate number of cases) of the corona virus - 2 upto 31 July 2020. To validate the present mathematical model, the solution ($I(t)$) has been compared with the actual data of Covid-19 in India[3]. It is observed from the Fig. 1a that the present mathematical model has achieved a good agreement with the actual data. It is also seen that before the date 18 May, 2020, the graph of actual data was narrowly passing just below the graph of $I(t)$ but after that date opposite phenomena has happened. The reason behind this phenomena, can be explained as the lockdown in India was relaxed on this day. Moreover, the model predicts the number of infected persons and it is seen from the Fig. 1b that on 31 July total cases of COVID-19 in India will be approximately 1.4 Million.

Actual data of daily new cases in India and plot of the function $\dot{I}(t) = \frac{dI}{dt}$ has also been compared in Fig. 2a. Since the actual data of COVID-19 fluctuates everyday, $\dot{I}(t)$ not matched properly with the actual data. However, seven day averaged bar diagram of actual and computed data are almost same and it is shown in Fig. 3. It can be seen from the Fig. 2b that if the situation in India does not improve according to the present model daily new case may rise 40,000 on 31st July, 2020. Seven day averaged
bar diagram of computed data of $\dot{I}(t)$ and daily new cases in India have been shown in Fig. 3a and 3b respectively. From Fig. 3, it is observed that, daily new cases has been increased in compare to that of $I(t)$ after the 7th week from 29 March. The effect of control parameter $\kappa$ has been shown in Fig. 4a and Fig. 4b on computed infected patients ($I(t)$) and daily new COVID-19 cases ($\dot{I}(t)$) respectively. It is seen that with the increment of kappa number of total cases and daily new cases both are increased. The values of $\kappa$ are taken from the Table 1. This represents the fact that when the number of patients decreases as $t_1$ increases, total cases and daily new cases both will be decreased. The variation of growth rate of daily patients ($\ddot{I}(t)$) with time has been plotted in Fig. 5. Although, this data does not match with the actual data due to fluctuations of actual data. But, from this figure, we can get the idea of
average growth rate of daily infected persons. In Fig. 6a, computed result of the model equation (8) and the total death due to COVID-19 in India has been compared for $\mu = 0.0016$. It is seen that in the time interval 8 April to 27 May plot of the computed data was passing under the graph of actual data and after that time interval opposite scenario has been arrived. The effect of the parameter $\mu$ on the computed total deaths($D(t)$) has been shown in Fig. 6. It is noticed that a very little change in the value of $\mu$ reduces the total death significantly. Since, $\mu$ depends on the development of medical science, medical facilities in the affected area and different government activities to protect their people from corona virus, it’s value may be reduced. Thus, when the value of $\mu$ decreases to zero, the number of total death will become constant. In other words, if a suitable vaccine or medicine for corona virus is discovered, $\mu$
tends to almost zero and consequently, the number of total death also tends to zero. The computed results of model equation (9) has also been compared with the actual total recovered patients in India for $\delta = 0.023$. It is interesting to note that when the real time data of total deaths is greater than computed results of equation (8), on the same time interval actual data of total recoveries is less than or equal to the calculated data from equation (9). The parameter $\delta$ depends on the different factors as in the parameter $\mu$. It is observed that when the value of $\delta$ increases, total recoveries also increase.

![Fig. 5: Number of patients growth rate according to the model](image)

![Fig. 6](image)

(a) Comparison of computed data and actual data on total deaths with $\mu = 0.0016$
(b) Effect of the parameter $\mu$ on total deaths for COVID-19 in India
4 Social Awareness

In this time there is an outrageous situation of COVID-19 in India. several steps such as a period of 68 days lockdown, social distancing, curfew etc, has been taken by the Government of India. But due to the depression in economical condition in India as well as throughout the world and due to the large number of population, the Government of India takes the decision to relax the lockdown gradually. From 1 June 2020, the Government of India gives a large number of relaxation except in the containment zone through Unlock-1. To resolve the problem due to COVID-19, the citizen of India maintains some rules such as keep social distance, use mask when go outside home, washing hands frequently, avoid gathering etc. Also, peoples are advised to avoid touching mouth, nose and eyes. Most positive cases were initially reported from the metropolitans, but the infection has spread to rural areas since migrant workers began arriving at their homes. If the infection will spray rapidly in the rural areas, the situation will be out of control. So, to make a fight against COVID-19 peoples of India follow the guidelines which are fixed by the Government of India. The Government of India advised to all of the citizens to aware themselves to protect their families and society. It is expected that man will overcome the trouble of COVID-19 proficiently. One day world will be free from COVID-19 pandemic.

5 Conclusions

Through the present mathematical modeling, we have tried to analyze, how the coronavirus has spread in India, how mortality has decreased, how the total recovery has increased. We also have predicted the future of COVID-19 in India with the help of our model. The computed results of the present mathematical model has achieved a good agreement with the actual data of total number of infected patients, deaths and recoveries due to covid-19. Also, it is clear from the figure 1b that the disaster in India will increase exponentially in the coming two months (June and July, 2020). The reason behind it
are as follows: the relaxation in the lockdown in unlockdown-1, less number of testing in different places and arriving of migrant workers from the Red zone area to the villages. One glimmer of hope, however, is that the recovery rate of infected people in India has already increased significantly. If, we can follow the various precaution measures and advices even after the unlockdown period, the spreading rate of the corona virus can be controlled.

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7 Declarations

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Conflict of Interest: The authors declare that there is a no conflict of interest.

Availability of data and material: The data that support the findings of this study are available.

Code availability: We have created the programming code.

Authors’ contributions: Authors have equally contributed for this work.

Research involving human participants and/or animals: This study does not involve any re-search with humans or animals.

Informed consent: Informed consent was received from all the independent partner covered in the study.

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