Ground-state atomic polarization relaxation-time measurement of Rb filled hypocycloidal core-shaped Kagome HC-PCF

T D Bradley\textsuperscript{1,2}, E Ilinova\textsuperscript{1}, J J McFerran\textsuperscript{1}, J Jouin\textsuperscript{3}, B Debord\textsuperscript{1}, M Alharbi\textsuperscript{1}, P Thomas\textsuperscript{3}, F Gérôme\textsuperscript{1} and F Benabid\textsuperscript{1,2}

\textsuperscript{1}GPPMM Group, Xlim Research Institute, UMR CNRS 7252, University of Limoges, 123 Avenue Albert Thomas, F-87060, France
\textsuperscript{2}Department of Physics, University of Bath, Claverton Down, BA2 7AY, UK
\textsuperscript{3}SPCTS UMR CNRS 7315, University of Limoges, Centre Européen de la Céramique, 12 rue Atlantis, F-87068 Limoges, France

E-mail: f.benabid@xlim.fr

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Abstract

We report on the measurement of ground-state atomic polarization relaxation time of Rb vapor confined in five different hypocycloidal core-shape Kagome hollow-core photonic crystal fibers made with uncoated silica glass. We are able to distinguish between wall-collision and transit-time effects in an optical waveguide and deduce the contribution of the atom’s dwell time at the core wall surface. In contrast with conventional macroscopic atomic cell configuration, and in agreement with Monte Carlo simulations, the measured relaxation times were found to be at least one order of magnitude longer than the limit set by atom-wall collisional from thermal atoms. This extended relaxation time is explained by the combination of a stronger contribution of the slow atoms in the atomic polarization build-up, and of the relatively significant contribution of dwell time to the relaxation process of the ground state polarization.

Keywords: photonic crystal fiber, coherent optical effects, atom optics

(Some figures may appear in colour only in the online journal)

1. Introduction

Micro-confinement of atomic alkali vapors in hollow wave-guides such as hollow-core photonic crystal fibers (HC-PCF) has gradually emerged as a timely and an important topic in several applications \cite{1–10}. Such interest is explained by the HC-PCF success in combining compact experimental schemes with enhancement in spectroscopic contrast or optical nonlinearities \cite{11}. This in turn, along with other applications requirements, has stimulated efforts in developing HC-PCF designs that overcome some of its intrinsic limitations, which led to the advent of hypocycloidal core-shaped inhibited-coupling (IC) guiding Kagome HC-PCF \cite{12}. This hollow waveguide combines ultra-low transmission loss, wide transmission bandwidth, reduced overlap with the silica core surround and larger hollow-core diameters whilst maintaining excellent single mode properties \cite{13–16}. These properties along with its extremely long interaction length make this fiber an excellent platform for atom optics, especially those where the dephasing due to atom-wall collisions is a significant drawback. In parallel, the atom-light interaction in HC-PCF departs in many ways from conventional configurations and necessitates a fresher look at the spectroscopy measurements of atoms inside the fiber. First, the HC-PCF specific modal properties, such as the guided mode field diameter $d_m$ being very close to the fiber geometrical diameter $d_l$ (typically $d_m \approx 0.7d_l$), make it difficult to discriminate the optical spectral-line broadening due to transit-time, $\tau_t = (2/\pi)d_m/\nu$ \cite{17}, from that of the atom-wall collisions $\tau_w \approx (V/A)(2\sqrt{\pi})/\nu$ \cite{18}, which reduces to $\tau_w \approx (\sqrt{\pi}/2)d_l/\nu$ for a cylindrical core-geometry. Here $\nu$, $A$ and $V$ are the atoms most probable speed, the fiber...
hollow-core total-area and volume respectively. This partly explains the measured linewidths found in the early work on electromagnetically induced transparency or saturable absorption in anti-relaxation coated hollow-core waveguides (e.g. ARROW’s, PBG HC-PCF and IC HC-PCF [1–3]), which do not show clear evidence of anti-relaxation effect. Furthermore, because of the HC-PCF micrometric confining scale and the unusually large surface-to-volume ratio of its core, the atom-wall interaction dynamics can no longer be neglected. First, because of the extremely short $\tau_{tt}$, which remain shorter than 100 ns even for a core diameter as large as 50 $\mu$m, the spectral broadening dynamics is often transit-time limited for HC-PCF confined atoms [19]. This is so even with low laser power-levels and large frequency detuning. The net result is that atom spectroscopy in HC-PCF is dominated by only the atoms that are sufficiently slow to complete a transition Rabi-cycle before leaving the interaction volume. Second, because of the close vicinity of the atoms to the large wall surface, the atom mean dwell time $\tau_{dw}$ at the trapping surface potential can no longer be neglected as its range of $\tau_{dw} \sim 80 \text{ ns} - 55 $ s [20] indicates that it can dominate the other dephasing time-scales. This new light-atom hosting configuration thus calls for spectroscopic measurements capable of assessing and discriminating the three mentioned-above time-scales.

Within this context, we report, for the first time to our knowledge, on magneto-optical spectroscopy of Rb vapor confined in hypocycloidal core shaped Kagome HC-PCF and assess experimentally and theoretically their ground state polarization relaxation dynamics. The results show that the ground state population relaxation time, $\tau_{l}$, can be up to tenfold longer than $\tau_{w}$ and $\tau_{tt}$. We show that it is dominated by the strong contribution of slow atoms in the Zeeman-optical pumping induced atom polarization build-up and by the larger $\tau_{dw}$ relative to the other relaxation timescales.

2. Experimental set-up

Figure 1(a) schematically illustrates the experimental set-up. One of the five 8 cm long Rb loaded Kagome HC-PCFs placed in a high vacuum chamber is fully Zeeman optically pumped using a circularly polarized laser beam with 650 nW of power (pump beam). The fiber input ends are 1 cm away
from the vacuum chamber windows. The beam is emitted from an extended cavity diode laser whose frequency is set at 500 MHz blue-detuned from the 85Rb D2 absorption line (5S1/2F = 3 ↔ 5P3/2F′ = 2, 3, 4) (figure 1(b)). The Zeeman sub-level degeneracy is lifted by applying a dc magnetic field (≤3.5 mT) along the guidance axis of the optical fibers using a solenoid wound around the vacuum chamber. The electron spin randomization rate τ1 was measured using a variation of Franzen technique [20]. This consists in initially optical pumping the atoms into a spin oriented state and then abruptly shutting off the pump laser and measuring (while keeping the pump off) the linearly polarized probe optical rotation exponential decay, from which the atomic polarization relaxation rate can be deduced. The probe beam is extracted from the same laser as that of the pump and has a power of 60 nW. The fiber set under test consists of five hypocycloidal-like core-shaped Kagome HC-PCF’s [14, 22] with different contour curvature values and radii. Figure 2 summarizes the physical properties of the fibers. The core-contour geometry is fully parameterized by the inner radius R_in, the cup curvature parameter b, and the ratio between the inner radius and the outer radius d = R_out/R_in (figure 2(a)). The parameter b is defined as the ratio of the distance from the outer edge of the arc to the half-length of the chord transecting the largest diameter of the cup [15]. It is noteworthy that the larger b is the better the fiber transmission performance is and the closer the fiber modal content is to a single mode guidance regime [15]. Three of the HC-PCF’s have a 7-cell core defect with inner core radius R_in ranging between ~15 and ~30 μm, and outer radius of R_out ranging from ~16 to ~39 μm (see top of figure 2(b) and table 1). The remaining two fibers are 19-cell core defects with core inner radii of 38 and 48 μm (rhs of figure 2(b)). All the fibers have been fabricated for low loss guidance around 780 nm with a range of 70–300 dB km⁻¹. The fibers are loaded with natural Rb vapor for a period of 3.5 weeks. At the end of the vapor loading process the current source to the Rb getter is switched off to minimize the influence of the Rb source on the relaxation time measurement [20]. During the data collection, the Rb vapor density was measured and was found to remain steady over the corresponding time period.

3. Results and discussions

The polarization of the probe transmitted through the fiber is oriented to have 45% with the PBS two ports. This is set so the signal ((P2 - P1)/(P2 + P1)) after the difference amplifier is at its minimum in the absence of the pump beam. Here, P1 and P2 are the probe-beam power of the two orthogonal polarization components transmitted through the HC-PCF and split by the PBS. Furthermore, given the negligible fiber birefringence, the probe polarization rotation is directly proportional to ((P2 - P1)/(P2 + P1)). Indeed, by virtue of the nonlinear magneto-optical effect, as the probe propagates along the fiber confined polarized atoms the two circular components of its field acquire a relative phase φ [23]. Consequently, and after a straightforward algebra, the expression of the probe power emerging from the PBS ports is

\[ P_{12} = \frac{(P_1 + P_2)}{2}[1 \pm \sin(2\varphi)]. \]

For small relative phase values, our detected signal reduces to

\[ \frac{(P_2 - P_1)}{(P_2 + P_1)} \sim 2\varphi. \]

Following Graf et al formalism (see equation (7) in [20]), the time-dependence of the probe optical rotation φ(t)−primarily due to the relaxation of the atomic polarization, takes the form of the sum of two negative exponentials:

\[ \varphi(t) = \alpha_1 e^{-\gamma_1 t} + \alpha_2 e^{-\gamma_2 t} + \varphi_{dc}, \]

where γ_1 and γ_2 are the faster and slower relaxation rates respectively. α_1 and α_2 are the amplitude of the fast and slow decaying exponential respectively, φ_{dc} is the dc rotation caused by the linear Faraday effect. Both relaxation rates are directly related to the relaxation rates of the processes involved in the ground state atom polarization (equations (2)–(6) of [20]). These relaxation rates are: (1) γ_a for atom-wall collision induced electron randomization; (2) γ_{ac} for atom–atom collision induced spin exchange and (3) γ_r for reservoir effect. Implementing equations (2)–(6) in [20] for the case of Rb, we have

\[ \gamma_{ls} = \gamma_a + \frac{1}{108}(57\gamma_a + 38\gamma_{se}) \pm \sqrt{2601\gamma_a^2 + 3684\gamma_a\gamma_{se} + 1444\gamma_{se}^2}. \]

In our experimental conditions, we ignore the Rb–Rb spin exchange collisional rate γ_{se}, because of the low atomic density we operate at (~n < 10^{10} cm⁻³). The estimated Rb–Rb spin exchange collisional relaxation time τ_{se} = γ_{se}⁻¹ ≈ 1/(nσ_{se}ν) is ~180 ms, which several orders of magnitude larger than τ_a.
Here, $\sigma_{sc}$ and $v$ are the Rb effective spin-exchange cross section, and relative velocity between Rb atoms. We also ignore $\gamma_0$ as we operate with the Rb source being off. This leads to $\gamma_t \approx \gamma_{st}$ and $\gamma_\omega \approx \gamma_{st}/18$. The relaxation time of the probe polarization signal $\varphi(t)$ is directly related to the rate of atom-wall collisions induced electron spin randomization. The rate of electron spin randomization for uncoupled fiber is given by $\gamma_{\omega} \sim \tau^{-1} = (\tau_w + \tau_{bw})^{-1}$, where $\tau_w$ is the average time between the two atom-wall collisions and $\tau_{bw}$ is the average dwell time of the atoms near the wall surface. Thus this experimental protocol applied to several HC-PCFs with different core radii can directly measure the HC-PCF atom-wall collision dynamics induced population decay and the dwell time.

Figure 1(c) shows typical traces of the recorded optical polarization rotation signal (circled points) for the case of two different Kagome HC-PCFs from the fiber set. It is noteworthy that $\alpha_1$ and $\alpha_2$ are opposite in signs plaining partly the relative flatness of the experimental traces for very short times after the pump is switched off. The solid lines in figure 1(c) are the associated fit to the experimental traces. Here, the fit was limited to the slow component of $\varphi(t)$ (i.e. $\varphi(t) \approx \alpha_1 e^{-\gamma_{\omega}t} + \varphi_{bk}$) by considering the experimental data points taken at times (measured from the moment of the pump switch off) longer than 5 $\mu$s. This is justified by our limited experimental temporal resolution, which set by the finite transit time of the chopper vane through the pump beam (estimated to be 7 $\mu$s).

Table 1 lists the measured atomic density $n$ along with the inner and outer radius, the $b$ parameter, the transit time $\tau_t$, the wall-atom collision time $\tau_w$, and the measured ground-state atomic polarization relaxation time $\tau_\text{pol}$. Because of the non-circular geometry of the core-contour, $\tau_w$ takes a different expression from the circular contour and is given in the appendix). The salient and common feature of the measured $\tau_{\text{pol}}$, which is in contrast with macroscopic atomic cell configurations is that they are all more than one order of magnitude larger than the atom-wall collisional time. Below we show that these longer $\tau_{\text{pol}}$ values occur because of the combination of the enhanced initial transit time of polarized atoms and the effect of the dwell time of the atoms in the wall trapping potential. The increased transit-time results from the slow polarization build-up rate at our given experimental conditions. This is in turn means that only the transversally slow atoms are initially polarized. This is explained in figure 3, which summarizes the atomic polarization build-up dynamics as deduced from solving the optical Bloch equations. Figure 3(a) shows the seven Zeeman ground-state population evolutions when the atoms are excited by the pump at our experimental conditions. The atomic population is fully optically-pumped to the ground state Zeeman sublevel $|F = 3, m = \pm 3\rangle$ after a certain build-up time $\tau_{\text{pol}}$, following a time evolution $e^{-1/\tau_{\text{pol}}}$, with $\tau_{\text{pol}}$ found to be approximately 2 ms. This is more than three orders of magnitude larger than transit-time limit $\tau_t$. It is noteworthy that the build-up time $\tau_{\text{pol}}$ grows when increasing the detuning from the resonance or decreasing the pump intensity $\lambda/\tau_{\text{pol}} \sim I/\Delta^2$ [24]), as numerically calculated and shown in figures 3(b) and (c). In our experiment, the blue detuning of 500 MHz from $F = 3$ to $F' = 2$, 3, 4 line is set to optimize the polarization rotation amplitude [23], and the pump power value (coupled intensity in the range of 10–100 mW cm$^{-2}$, and optical density on resonance of $\sim$2.3) is set to produce a sufficient signal to noise ratio. As a consequence of such a long polarization build-up time, only atoms whose time-off-flight through the beam without depolarization is longer than $T_{\text{bp}}$ significantly contribute to the ground-state atomic polarization (i.e. populating $|F = 3, m = +3\rangle$) [19]. The velocity-distribution $f(v)$ of these polarized atoms is illustrated by the red curve in figure 3(d) along with the Maxwellian distribution (blue filled curve) for comparison (see appendix for details).

The results readily show that the polarized atom velocity distribution significantly deviates from the Boltzmann–Maxwell thermal distribution, and peaks at much smaller velocity subclasses. The calculations for the fiber #1 experimental conditions show that the most probable initial speed of polarized atoms is $v_{\text{pol}} \approx 8$ m s$^{-1}$, which is $\sim$20 times lower than the most probable speed $v$ for thermal distribution. Equivalently, when the pump laser is turned off, the polarized atoms before their depolarization and thermalization by wall-collision cross the interaction volume with a transit time 20 slower than the one experienced by thermal atoms. In order to assess the impact of such a nonthermal velocity distribution on the atom-wall collisional rate and the effect of the dwell time, we carried out a Monte Carlo (with $10^5$ realizations) based calculations. The detailed description of the model is given in the appendices section.

At each realization we trace the trajectory of initially polarized atom, considering the pump laser is off. We assume that, after each collision with the wall, the atomic velocity-vector stochastically changes, and its newly acquired absolute value obeying the Maxwell–Boltzmann distribution. Each
collision is followed by the atom dwell period (sticking on the wall) \( \tau_{dW} \), which is taken randomly over the range from 0 to \( 2 \tau_{dW} \), where \( \tau_{dW} \) is the trial average dwell time, which we keep fixed during all the realizations. Each collision involves a single occurrence of electron spin projection spontaneous change. The nucleus spin projection stays unchanged [24].

Furthermore, in our model we associate the atomic spin state with the pseudo spin \( S \). We consider that the pseudospin \( S \) can take only two values: \( S = 0 \)-corresponding to thermal distribution of the probabilities of different Zeeman levels and \( S = 1 \)-corresponding to the atom in a superposition of Zeeman states producing the initial average orientation in ensemble. Thus all the atoms are initially prepared in the state corresponding to \( S = 1 \). The probability of the drop of the value of \( S \) from 1 to 0 is \( P = 1/18 \), where the factor 18 arises due to the nuclear slow down factor [20]. Based on these assumptions we calculate the atomic total spin relaxation time \( \tau_{S} \), corresponding to a chosen trial value of the average dwell time \( \tau_{dW} \). One can show (based on the central limit theorem) that the result for the average total spin relaxation time \( \tau_{S} \), obtained within this simplified two-states model, is the same as if we considered all the variety of the total spin states, which the atom can acquire during a single collision with the wall, taking to account that the electron spin state spontaneously changes only one time per collision. The procedure described above is repeated for a series of trial average dwell time values \( \tau_{S} \), for each of the fiber geometries considered here. Based on these calculations we obtain the dependence \( \tau_{S} (\tau_{dW}) \) for each geometry. Finally we determine the optimal fitting parameter \( \tau_{dW} \), to minimize the discrepancy between the experimental and theoretically calculated values of \( \tau_{S} \) at each geometry. Here we notice that, because of our off-resonance optical pumping scheme and the very low atomic density we neglect the radiation trapping, which is another source of polarization relaxation [25, 26].

Results from the Monte-Carlo simulations are summarized with the experimental results in figure 4. Figure 4(a) shows an excellent agreement between the measured and the calculated relaxation time \( \tau_{S} \). For comparison, the calculated \( \tau_{W} \) and \( \tau_{H} \) are also shown, and one clearly notices the significantly longer polarization relaxation time \( \tau_{S} \) than the limit set by the atom-wall collision rate of thermal Rb. The results give a fitted \( \tau_{dW} \) of 0.7 \( \mu \)s, which is consistent with the measured one reported by Zhao et al [21] for coated Pyrex glass surface. The very good agreement between experiment and simulation is further illustrated in figure 4(b), which shows the atom polarization time-evolution in the dark for the case of fiber \# 1 as measured (solid points) and Monte Carlo calculated (red dashed curve). The figure also shows the single-time exponential decay fit to the experiment (solid green curve). To complete the study, figures 4(c) and (d) show how the polarization
decay scales with the fiber geometrical parameters. We readily observe that at fixed curvature (Figure 4(c)), the relaxation time increases linearly with radius as expected from conventional macroscopic cylindrical vapor cells [27]. On the other hand, the relaxation time dependence with curvature (Figure 4(d)) shows a decrease of the relaxation time with increasing $b$. This may be expected since an increase in $b$ implies an increase in the surface-to-volume ratio. This behavior is in quantitative agreement with the experimental data for the five different fibers.

4. Conclusion

In conclusion we have demonstrated using a combination of experimental protocol and Monte-Carlo simulation the major dephasing time scales in the ground state of HC-PCF confined Rb. Unlike in conventional cells, the atom polarization dynamics is no longer dominated by the transit-time or by atom-wall collisions but by the stronger contribution of the slow atoms in the atom polarization build-up and to the dwell time of the atom at the core wall surface.

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Appendix

A.1. Calculating the wall collision rate for the 7 and 19 cell geometry fibers

The rate of the atom wall collisions is determined as

$$\gamma_{\text{col}} = \frac{A}{V}$$

where $A$ is the collisional surface and $V$ is the volume of the atomic container and $v_t$ is the transverse speed of the atoms. To determine the ratio $A/V$ for the 7 and 19 cell Kagome fibers we used the approximate expression for the
perimeter $P$ of the half ellipse describing the shape of the inner core cups

$$P(r, \eta) \sim \pi a^2 + b^2,$$

where $a$ and $b$ are the half axes of the ellipse. The ratio of collision area $A/V$ to the given geometries are given then by

$$(A/V)_{cell} = 4\sqrt{2}\pi(n_{\|} - 2\sqrt{3}) \left( \frac{(b^2 - 4\sqrt{3}b + 12)\theta^2 + 7(b - 2\sqrt{3})\theta + 13 + \sqrt{b^2 + 1}}{R_{in}(2(\sqrt{3}b - 6)\theta + 4b + 7\sqrt{3}) - 30\sqrt{3}} \right),$$

$$(A/V)_{bulk} = 4\sqrt{2}\pi(b^2 - 6\sqrt{3}b + 28) \left( \frac{(b^2 - 6\sqrt{3}b + 28)\theta^2 - 11b^2 - 6\sqrt{3}b + 28\theta + 31 + 2\sqrt{b^2 + 1}}{R_{in}(2(\sqrt{3}\pi\theta + 16)b^2 - 6\sqrt{3}b + 28 + (2 - \pi)(8b + 11\sqrt{3}))} \right),$$

where $\theta = R_{out}/R_{in}$ is the ratio of the outer and inner core radiuses of the fiber as it is shown on figure 2 of the main text, and $b$ is the ratio between the lengths of the axes of half ellipse approximating the shape of the bigger cup.

### A.2. Initial velocity distribution of polarized atoms

The velocity selective optical pumping leads to the non Maxwellian initial transverse velocity distribution of polarized atoms. This can be understood as follows. The dependence of the atomic polarization on the time $t_{pump}$ spent inside the pump laser beam is given by

$$P(t) = 1 - e^{-t/T_{p}},$$

where the parameter $T_{p}$ is the characteristic polarization build up time which depends on the de-tuning and the intensity of the pump laser.

The velocity dependence of the atomic polarization created during a single pass of the atom through the pump beam can be obtained then as

$$P_{sp}(v) = 1 - e^{-\sqrt{2}R_{init}/(vT_{p})},$$

where $\sqrt{2}R_{init}$ is the average length of the path through the interaction cross section.

After a single collision with the wall the atom can keep its total momentum projection unchanged. Consequently the pumping can occur during several passes of the atom through the interaction volume. To calculate the resulting velocity dependence of the atomic polarization $P(v)$, one needs to average over the all possible trajectories of the atom during the pump process, taking to account the probability of depolarization and the velocity change after each collision with the wall. At the condition when the pumping time is much longer than the typical polarization build up time, the final atomic polarization velocity dependence can be parameterized as:

$$P(v) = p_0 - (1 - p_0) \left( 1 - e^{-\sqrt{2}R_{init}/vT_{p}} \right),$$

where $p_0$ is constant. One can show that when the polarization build up time is much larger then the time between the two following collisions with the wall, so that $T_{bp} \gg ncT_{p}$, where $nc$ is the average number of collisions experienced by the atom before it depolarizes, the value of $p_0$ is much less than 1 and one can approximate $P(v)$ as

$$P(v) \approx 1 - e^{-\sqrt{2}R_{init}/vT_{p}}.$$

The effective transverse velocity distribution of polarized atoms can be introduced then as

$$\tilde{f}(v) = \frac{1}{N} f(v) \left( 1 - e^{-\sqrt{2}R_{init}/vT_{p}} \right),$$

where

$$N = \int_0^{\infty} f(v) \left( 1 - e^{-\sqrt{2}R_{init}/vT_{p}} \right) dv.$$

The slowing factor $fs$ can be introduced as the ratio of most probable velocities of sub ensemble of polarized atoms and the whole ensemble of thermal atoms: $fs = \bar{v}_p \cdot \sqrt{2m/kT}$, where $\bar{v}_p$ is the solution of the equation

$$T_{bp}v^2 - v_p^2\left( e^{\frac{R}{\sqrt{2}v}} - 1 \right) + Rv_p^2 = 0,$$

where $v_p = \sqrt{2m/kT}$, $R$ is the laser beam radius. For $T_{bp} = 1$ ms one has $\bar{v}_p \approx 8$ m s$^{-1}$. The ratio $v_p/\bar{v}_p = 169/8 = 21$.

To find the average transverse velocity of the polarized atoms, one needs to calculate the value:

$$\bar{v}_{pol} = \left( \int_0^{\infty} f(v)(1 - e^{-Rv^2/vT_{pol}})vdv \right)/\left( \int_0^{\infty} f(v)(1 - e^{-Rv^2/vT_{pol}})dv \right).$$

### A.3. Monte-Carlo simulations

To calculate the time dependence of the polarization rotation signal we calculate the evolution of the number of polarized atoms inside the interaction volume.

It is worth noting that the number of polarized atoms inside the interaction volume $N(t)$ at given moment of time is different than in the whole volume. To calculate $N(t)/N(0)$ we trace the trajectories of $N(0) = 100000$ initially polarized atoms. We associate the polarization (total momentum projection state) with the pseudo spin $S$, which can be 1 or 0. We say that at the moment of time $t = 0$ there are $N(0)$ atoms which have the pseudo spin value equal to 1.
The red-zone is the calculated area.

Figure A1. The fibre core-contour geometry. Rin is the inner radius of the contour cups. Rint is the interaction radius and is taken to be equal to the mode field radius. The yellow region correspond to the laser-atom interaction area.

After each collision with the wall the atom can change its pseudo spin value to zero with the probability 1/18 (we assume that each collision completely randomize the electron spin of the atom). After the pseudo spin of the atom becomes zero it remains zero until the end of the probe period. The number \( N_{\text{int}}(t) = \sum_{i=1}^{N_{\text{int}(0)}} p_i(t) \), where \( p_i = 1 \), if \( S_i = 1 \) and the atom is inside the interaction volume, otherwise \( p_i = 0 \).

We consider the atom (at the moment of the pump switch off) can be at the arbitrary position in the fiber, have the random direction and module of the transverse velocity. Taking to account the symmetry of the fiber its enough to consider the dynamics of the atoms which are initially in the sector marked red on the figure A1. Each time the atom collides with the wall it changes it is velocity randomly (according to Maxwell distribution).

The scope of this paper does not include calculating the average dwell time \( T_{dw} \) which the atom spends near the wall surface after each collision (without being depolarized) and we take it as unknown parameter. We perform the Monte-Carlo simulations for a set of values of \( T_{dw} \) in the interval from 0 to 2 \( \mu s \). During the MC simulations of atomic spin dynamics (at given fiber geometry and fixed value of the average dwell time \( T_{dw} \)), for each collision with the wall we take the current value the dwell time \( t_{dw} \) randomly from the interval \( 0 < t_{dw} < 2 \ T_{dw} \), assuming that it is uniformly distributed on this interval.

Based on the values of \( T_i \) calculated for different fixed values of \( T_{dw} \) for each fiber geometry we determine the dependence \( T_i(T_{dw}) \). After that we determine the optimal value of \( T_{dw} \) corresponding to the least discrepancy of the calculated and experimental values of \( T_i \) considering all set of six fiber geometries.

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