Investigation on the Nonlinear Control System of High-Pressure Common Rail (HPCR) System in a Diesel Engine

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Abstract. This study first constructed the nonlinear mathematical model of the high-pressure common rail (HPCR) system in the diesel engine. Then, the nonlinear state transformation was performed using the flow’s calculation and the standard state space equation was acquired. Based on sliding-mode variable structure control (SMVSC) theory, a sliding-mode controller for nonlinear systems was designed for achieving the control of common rail pressure and the diesel engine’s rotational speed. Finally, on the simulation platform of MATLAB, the designed nonlinear HPCR system was simulated. The simulation results demonstrate that sliding-mode variable structure control algorithm shows favorable control performances and overcome the shortcomings of traditional PID control in overshoot, parameter adjustment, system precision, adjustment time and ascending time.

1. Introduction

Bosch from Germany, Denso from Japan, Delphi from USA and Continental AG from Germany are four main global suppliers of common-rail electronically controlled fuel injection systems. Additionally, Caterpillar and Cummins from USA, and LIEBHERR and L’orange from Germany are also four manufacturers of common-rail electronically controlled fuel injection systems for self-use. Wärtsilä from Swiss and MAN DIESEL & TURBO from Germany launched low-speed marine diesel engines with electronically-controlled injection systems. Taking RT-Flex diesel engine manufactured by Wärtsilä Company as an example, 100MPa fuel in high-pressure common rail and 20MPa servo oil in medium-pressure common rail are transported directly by the plunger-type high-pressure fuel injection pump, which is driven by the transmission gear via a crankshaft; the pressure in two fuel rails can be adjusted by the electronic actuator and the connecting rod according to the preset pressure signal in Wärtsilä Electric Control System. MAN DIESEL & TURBO Company uses medium-pressure common-rail (for servo oil) hydraulic-booster electronically controlled fuel injection system, in which the hydraulic unit provides the medium servo oil (i.e., lubricating oil) at 20MPa, and low-pressure fuel is boosted to 100MPa via a boosting piston and fuel injection is controlled by a FIVA valve.

Many control systems show nonlinear properties. Sometimes, the nonlinearity is inherent because of the object’s dynamic properties. Although some objects themselves are linear, nonlinear laws are consciously introduced for achieving high-quality control. Strictly speaking, nonlinearity is ubiquitous and nonlinear systems are most general systems, while linear systems are only special cases. A
common-rail electrically-controllable fuel injection system involves multiple parameters such as common-rail pressure, fuel-injection quantity and the rotational speed of the engine, which is undoubtedly a complex nonlinear system.

Variable-structure control is an advance control algorithm based on accurate mathematical model[1]. As a comprehensive method for nonlinear control systems, variable-structure control is practical and can acquire a series of favorable properties and qualities. Sliding mode variable structure control (SMVSC) is a control strategy for variable-structure control systems, which is also a comprehensive method in modern control theory based on phase plane method. The basic idea is that the system’s motion first reaches a certain switching surface, then, the motion follows a sliding mode and gradually approaches the origin, and finally, the control goal can be realized when the sliding motion is endowed with good character.

At present, the main control method of diesel engine running parameters is PID[2]. This paper used Sliding mode variable structure control algorithm to study the nonlinear diesel engine.

2. Control of diesel engine common rail
HPCR system in a diesel engine is a hybrid linear system combining discrete events and continuous processes, which mainly consists of low-pressure fuel transfer pump, high-pressure fuel pump, high-pressure fuel pipe, fuel common-rail pipe, pressure regulator, filter, fuel injector and rail-pressure control valve (i.e., PCV value). At a high rotational speed, fuel was fed more frequently in unit time, and therefore, the common-rail system maintained a high common-rail fuel pressure. It means that the fuel pressure in a HPCR system mainly depends on the opening degree of the PCV value. Additionally, the opening degree can also regulate the fuel transfer quantity so as to achieve the regulation of common-rail pressure[3].

2.1. Establishment of the nonlinear model of high-press common rail (HPCR) system
HPCR oil pressure determines both fuel injection quantity and fuel return quantity. On the other hand, the opening degree of the PCV value depends on the pulse width and starting time of the PCV value in the electronic circuit. In fact, the fuel quantity that is transported to fuel rail by the fuel transfer pump depends on the correct timing of the current on the PCV valve. Therefore, we should clarify the nonlinear relationships among fuel pressure, fuel injection flow, fuel supply flow and fuel return flow. The common-rail pressure is subjected to the effects of fuel supply quantity of the high-pressure fuel pump, fuel return flow and fuel injection quantity of the injector.

Accordingly, for the convenience in the design of control law and the related stability analysis, the common-rail pressure model for a diesel engine can be simplified. The relationship among common-rail pressure, fuel injection flow, fuel supply flow and fuel return flow can be written as:

\[ \frac{dP_{cr}}{dr} = \frac{K}{V} (Q_1 - Q_2 - Q_3) \]  

where \(Q_1\) denotes the fuel supply flow of a HVCR pump, \(Q_2\) denotes the injector’s fuel injection flow, \(Q_3\) denotes fuel return flow of the PCV valve, \(P_{cr}\) denotes the fuel pressure in the HVCR pipe, \(K\) denotes the diesel’s elastic modulus and \(V\) denotes the high-pressure common rail volume.

For accurately acquiring the quantitative relations among common rail pressure, the engine’s rotational speed and fuel injection quantity, a 6L16/24 diesel was selected for the experiments on a HPCR testing platform. The fuel supply and return data at different common-rail pressures of 80MPa, 100MPa, 120MPa and 140MPa were recorded for further analysis.

Fuel supply flow mainly depends on the rotational speed of the high-pressure fuel pump which equals to half of the diesel’s rotational speed \(n\) and the common rail pressure \(P_{cr}\). Therefore, the following expression can be derived:
\[ Q_1 = k_1 n + c_1 \]  
(2)

Then, a quadratic polynomial of \( P_{cr} \) relative to \( k_1 \) and \( c_1 \) can be acquired through fitting, which can be written as:

\[ k_1 = -0.0000000392 P_{cr}^2 + 0.00000273 P_{cr} + 0.001065 \]  
(3)

\[ c_1 = 0.0000581 P_{cr}^2 - 0.00356 P_{cr} - 0.072 \]  
(4)

If the injector’s fuel injection flow \( q \) is set as the circulating fuel injection flow, the following expression can be acquired:

\[ Q_2 = q \]  
(5)

The fuel return quantity increases with the rising of HPCR pressure. In this study, by neglecting the other factors of the equipment, fuel return quantity is directly determined by HPCR pressure:

\[ Q_3 = 0.00416 P_{cr} \]  
(6)

According to equations (1)-(6), the mathematical model of a HPCR nonlinear system can be described as:

\[ \frac{dP_{cr}}{dt} = (a_1 P_{cr}^2 + a_2 P_{cr} + a_3)n + a_4 P_{cr}^2 + a_5 P_{cr} + a_6 + a_7 q \]  
(7)

where \( a_1 = -0.00392, a_2 = 0.273, a_3 = 106.5, a_4 = 5.81, a_5 = -35.6, a_6 = -720, a_7 = 929.6 \)

Obviously, the differential equation not only includes the nonlinear quadratic component of HPCR pressure but also connects with the diesel’s rotational speed and fuel injection quantity.

The differential equation of the rotational speed control system is also required. Similarly, using curve fitting method, the circulating fuel injection quantity data and average indicating pressure \( p_i \) at different rotational speeds (900 r/min, 1000 r/min, 1100 r/min and 1200 r/min, respectively) were analyzed. The average indicating pressure can be expressed as:

\[ p_i = k_2 q + c_2 \]  
(8)

Then, we conducted linear fitting on the coefficients and polynomial fitting on the constant, and the following expression can be acquired:

\[ k_2 = 3.9 \]  
(9)

\[ c_2 = 0.00003346 n^2 - 0.107 n - 71.5294 \]  
(10)

The variation of diesel rotating speed directly depends on the difference between output torque \( M \) and load torque \( M_l \). According to the D’Alembert fundamental equation of motion[4], the following expression can be acquired:

\[ J \frac{dw}{dt} = M - M_l \]  
(11)

Assuming that the characteristic of load torque \( M_l \) is the vehicle’s stable road characteristic, the following expression can be derived:

\[ M_l = l \cdot n^2 \]  
(12)

Let \( l = 0.00007 \) and \( J = 5 \text{kg} \cdot \text{m}^2 \), the nonlinear mathematic model for the diesel’s rotational speed controlling can be written as:

\[ \frac{dn}{dt} = b_1 n^2 + b_2 n + b_3 + b_4 q \]  
(13)

where \( b_1 = -0.0001, b_2 = -0.144, b_3 = -114.7, b_4 = 4.2 \)
According to equations (7) and (13), the mathematic model of a nonlinear HPCR control system can be written as:

\[
\begin{align*}
\frac{dP_{cr}}{dt} &= (a_1 P_{cr}^2 + a_2 P_{cr} + a_3) n + a_4 P_{cr}^2 + a_5 P_{cr} + a_6 + a_7 q \\
\frac{dn}{dt} &= b_1 n^2 + b_2 n + b_3 + b_4 q
\end{align*}
\]

(14)

This model is characterized by the quadratic component with nonlinearity and includes two state variables to be controlled, namely, common rail pressure and the diesel’s rotational speed.

2.2. Design of a sliding mode controller (SMC)

For the system as described in equation (14), the state variable can be set as:

\[x_1 = \int_0^t e_1 \, dt, \quad x_2 = e_1 = P_{cr0} - P_{cr},\]

where \(P_{cr0}\) denotes the preset common rail pressure and \(e_1\) denotes the pressure difference.

\[x_3 = \int_0^t e_2 \, dt, \quad x_4 = e_2 = n_0 - n,\]

where \(n_0\) denotes the preset value of the diesel’s rotational speed and \(e_2\) denotes the difference of rotational speed.

Therefore, the state equation of the system can be rewritten as:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -(a_1 P_{cr}^2 + a_2 P_{cr} + a_3) n - (a_4 P_{cr}^2 + a_5 P_{cr} + a_6) - a_7 q \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= -(b_1 n^2 + b_2 n + b_3) - b_4 q
\end{align*}
\]

(15)

In a standard form of state equation, the above expression can be rewritten as:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -d_1 x_2^2 x_4 + d_2 x_2^2 + d_3 x_2 x_4 + d_4 x_2 + d_5 x_4 + d_6 + d_7 u \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= m_1 x_4^2 + m_2 x_4 + m_3 + m_4 u
\end{align*}
\]

(16)

where \(d_1 = a_1\), \(d_2 = -(a_1 n_0 + a_4)\), \(d_3 = -(a_2 P_{cr0} + a_2)\), \(d_4 = 2a_1 P_{cr0} n_0 + a_2 n_0 + 2a_4 P_{cr0} + a_5\), \(d_5 = a_4 P_{cr0}^2 + a_2 P_{cr0} + a_3\), \(d_6 = -(a_1 P_{cr0}^2 n_0 + a_2 P_{cr0} n_0 + a_3 n_0 + a_4 P_{cr0}^2 + a_5 P_{cr0} + a_6)\), \(d_7 = -a_7\), \(m_1 = -b_1\), \(m_2 = 2b_1 n_0 + b_2\), \(m_3 = -(b_1 n_0^2 + b_2 n_0 + b_1)\), \(m_4 = -b_4\).

Next, the system input equation with \(m\) inputs was considered (in practical applications, a type of affine systems were considered).

\[
\frac{d}{dt} x = f(x) + \sum_{i=1}^{m} g_i(x) u_i \quad x \in \mathbb{R}^n
\]

(17)
where \( f(x) = \begin{bmatrix} x_2 \\ -d_1x_2^2x_4 + d_2x_2^2 + d_3x_2x_4 \\ + d_4x_2 + d_5x_4 + d_6 \\ x_4 \\ m_1x_4^2 + m_2x_4 + m_3 \end{bmatrix} \), \( g(x) = \begin{bmatrix} 0 \\ d_7 \\ 0 \\ m_4 \end{bmatrix} \).

If the above expression can be rewritten as the following expression using a nonlinear state transformation \( z = \lambda(x) \):

\[
\begin{bmatrix}
\frac{dz_1}{dt} \\
\frac{dz_2}{dt} \\
\frac{dz_3}{dt} \\
\frac{dz_4}{dt}
\end{bmatrix} = \begin{bmatrix}
p_1(z_1, \ldots, z_n) \\
p_2(z_1, \ldots, z_n) + q_2(z_1, \ldots, z_n)u \\
p_3(z_1, \ldots, z_n) \\
p_4(z_1, \ldots, z_n)
\end{bmatrix}
\]

the analysis of the system will be simpler.

\( n - m = 3 - 2 = 1 \) base vectors were added to \( G(x) \), and a matrix \( G^*(x) \in \mathbb{R}^{m \times n} \) was formed.

\[
G^*(x) = [g_1, g_2, g_3, g_4] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_7 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & m_4 \end{bmatrix}
\]

When \( x = x^0 = (0, 0, 0) \), \( G^*(x^0) \) is nonsingular. The flows, \( \Phi^{r_1}_{z_1}, \Phi^{r_2}_{z_2}, \Phi^{r_3}_{z_3} \) and \( \Phi^{r_4}_{z_4} \) were calculated using the method of flow calculation. The integral flow of the vector field, \( g_1(x) \), was first calculated.

\[
\begin{bmatrix}
\dot{x}_1 = 1 \\
\dot{x}_2 = 0 \\
\dot{x}_3 = 0 \\
\dot{x}_4 = 0
\end{bmatrix}
= \begin{bmatrix}
x_1(t) = t + x_1^0 \\
x_2(t) = x_2^0 \\
x_3(t) = x_3^0 \\
x_4(t) = x_4^0
\end{bmatrix}
\]

\[
\Phi^{r_1}_{z_1}(x) = \begin{bmatrix}
x_1 + x_1^0 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix}
\]

Similarly, the integral flows of the vector fields \( g_2(x) \) and \( g_3(x) \) were calculated, and the following solutions were obtained:

\[
\begin{align*}
\Phi^{r_2}_{z_2}(x) &= \begin{bmatrix}
x_1 \\
z_2 + x_2 \\
x_3 \\
x_4
\end{bmatrix}, \\
\Phi^{r_3}_{z_3}(x) &= \begin{bmatrix}
x_1 \\
x_2 \\
z_3 + x_3 \\
x_4
\end{bmatrix}, \\
\Phi^{r_4}_{z_4}(x) &= \begin{bmatrix}
x_1 \\
d_7z_4 + x_2 \\
x_3 \\
m_4z_4 + x_4
\end{bmatrix}
\end{align*}
\]

Then, we made the composition of flow mapping and acquired the transformation function of coordinates:

\[
x = F(z_1, z_2, z_3, z_4) = \Phi^{r_1}_{z_1} \circ \Phi^{r_2}_{z_2} \circ \Phi^{r_3}_{z_3} \circ \Phi^{r_4}_{z_4}(x^0)
\]
\[
\begin{bmatrix}
  x_1 \\
  x_2 \\
  x_3 \\
  x_4 
\end{bmatrix} = \Phi_{z_1} \circ \Phi_{z_2} \circ \Phi_{z_3} \circ \Phi_{z_4} = \begin{bmatrix}
  z_1 + x_1^0 \\
  d_7 z_4 + z_2 + x_2^0 \\
  z_3 + x_3^0 \\
  m_z z_4 + x_4^0
\end{bmatrix}
\]

Set \( \mathbf{x}^0 = (0, 0, 0) \), the following expression can be acquired:

\[
\begin{aligned}
  x_1 &= z_1 \\
  x_2 &= d_4 z_4 + z_2 \\
  x_3 &= z_3 \\
  x_4 &= m_z z_4 
\end{aligned}
\]

The inverse transformation can be written as:

\[
\begin{aligned}
  z_1 &= x_1 \\
  z_2 &= x_2 - d_4 x_4 / m_z \\
  z_3 &= x_3 \\
  z_4 &= x_4 / m_z 
\end{aligned}
\]

in which it was assumed that

\[
\lambda(\mathbf{x}) = \begin{bmatrix}
  \lambda_1(\mathbf{x}) \\
  \lambda_2(\mathbf{x}) \\
  \lambda_3(\mathbf{x}) \\
  \lambda_4(\mathbf{x})
\end{bmatrix} = \begin{bmatrix}
  x_1 \\
  x_2 - d_4 x_4 / m_z \\
  x_3 \\
  x_4 / m_z
\end{bmatrix}
\]

\[
\frac{d}{dt} z = \frac{\partial \lambda}{\partial \mathbf{x}} \frac{d}{dt} x = \frac{\partial \lambda}{\partial \mathbf{x}} f(\mathbf{x}) + \frac{\partial \lambda}{\partial \mathbf{x}} g(\mathbf{x})u = \begin{bmatrix}
  x_2 \\
  -d_4 x_4 + d_4 x_2^2 + d_4 x_3 x_4 + d_4 x_2 x_2^2 + d_4 x_3 x_3 + d_4 x_2 \\
  -d_4 x_4 + d_4 x_2^2 + d_4 x_2 x_3 + d_4 x_3 x_3 + d_4 x_2 x_2^2 + d_4 x_3 x_3 + d_4 x_2 + d_4 x_3 \\
  -d_4 x_4 + d_4 x_2^2 + d_4 x_3 x_4 + d_4 x_2 x_2^2 + d_4 x_3 x_3 + d_4 x_2 + d_4 x_3 \\
  -d_4 x_4 + d_4 x_2^2 + d_4 x_3 x_4 + d_4 x_2 x_2^2 + d_4 x_3 x_3 + d_4 x_2 + d_4 x_3
\end{bmatrix}
\begin{bmatrix}
  0 \\
  0 \\
  0 \\
  1
\end{bmatrix} + \begin{bmatrix}
p_1(\mathbf{z}) \\
p_2(\mathbf{z}) \\
p_3(\mathbf{z}) \\
p_4(\mathbf{z})
\end{bmatrix} + q(\mathbf{z})u
\]

Using the linear switch function, the above expression can be rewritten as:

\[
s(\mathbf{z}) = c_1 z_1 + c_2 z_2 + c_3 z_3 + z_4
\]

\[
\frac{d}{dt} s = c_1 p_1(\mathbf{z}) + c_2 p_2(\mathbf{z}) + c_3 p_3(\mathbf{z}) + p_4(\mathbf{z}) + q(\mathbf{z})u
\]

Based on the generalized sliding mode conditions, the sliding mode control can be described as:
\[ u(z) = \begin{cases} 
- q^{-1}(z)[c_1 p_1(z) + c_2 p_2(z) + c p(z)] & \text{when } s > 0 \\
- q^{-1}(z)[c_1 p_1(z) + c_2 p_2(z) + c p(z)] & \text{when } s < 0 
\end{cases} \]  

(28)

3. Software simulations

Next, using the CAGE toolbox in MATLAB, the original performance data were interpolated for solving the engine's performance data at the other operating points and determining the MAP chart of the simulation model. These MAP were then deposited in advance in the engine operating module on simulation platform.

In the first case, the diesel engine's rotational speed and fuel injection quantity were used as the input variables while the average indicating pressure was used as the output response. In the second case, the diesel engine's HPCR pressure and fuel injection quantity were used as the input variables while the fuel injection pulse width, the opening degree of PCV valve and the actual common rail pressure served as the output response. The diesel engine's HPCR pressure, rotational speed and fuel injection quantity data were then input to MAP charts for acquiring the calibrated target values.

The simulation program of the system is shown in Figure 1. Simulation results of the common rail pressure and speed of the diesel engine are shown in Figures 2-7.
According to the simulation results, for SMVSC, the rotational speed overshoot was much smaller than PID control, especially in the initial stage of diesel engine starting, and speed fluctuated within 10 r/min in the stable stage. Additionally, the diesel engine’s transient speed-regulating rate was smaller than 10%, the stable speed-regulation rate was about 0%, and the fluctuation ratio of rotational speed...
was smaller than 1%. Under a sudden load-up of 100%, the stabilization time of 5% error was smaller than 4 seconds.

For the common rail pressure control, SMVSC varies within a range of 5%. The simulation results fully proved the feasibility of variable structure control theory in investigating the common rail and rotational speed systems of nonlinear diesel engine. Using variable structure control method, the system’s nonlinear characteristics were well maintained.

4. Conclusions
In previous studies, the diesel engine’s rotational speed, common rail pressure, fuel viscosity temperature and cooling water temperature were generally controlled by traditional PID strategy [5]. Although the control structure was simple, the control precision, stability, robustness and control performances were far from ideal.

The fundamental difference between sliding-mode variable structural control and conventional control lies in the control’s discontinuity, i.e., a kind of switching characteristics that can make the system structure change at any time. Using sliding model variable structure control method, system shows a series of advantages including fast response, small overshoot and strong robustness.

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