Constrained non-linear waves for offshore wind turbine design

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Abstract. Advancements have been made in the modelling of extreme wave loading in the offshore environment. We give an overview of wave models used at present, and their relative merits. We describe a method for embedding existing non-linear solutions for large, regular wave kinematics into linear, irregular seas. Although similar methods have been used before, the new technique is shown to offer advances in computational practicality, repeatability, and accuracy. NewWave theory has been used to constrain the linear simulation, allowing best possible fit with the large non-linear wave. GH Bladed was used to compare the effect of these models on a generic 5 MW turbine mounted on a tripod support structure.

1. Introduction
The offshore environment offers great potential for wind energy: high annual mean wind speeds and minimal visual impact; however there is an additional source of environmental loading, namely waves. Water waves offer a challenge to the computational modeler as the governing equations are non-linear. Fully non-linear computational models do exist, but are yet to enter regular engineering usage. At present wave loads are calculated using one of two simplifying assumptions:

1) that the waves are regular, i.e. each wave that passes is identical to the last. With this restriction, near exact computational solutions are well established, known as stream function or regular non-linear waves. Validity of the method does not diminish as wave height is increased, so it is often used for extreme wave events. Figure 1 shows an example surface elevation.

2) that the waves are linear. By this we mean that an irregular sea state can be composed by superimposing regular waves of many frequencies. The regular waves used in this method are of a simplified kind where the surface elevation is assumed to take sinusoidal form. This method is fairly accurate when the waves are not too steep, so considered satisfactory for fatigue load calculations. Figure 2 shows an example surface elevation.

Fixed offshore structures in the oil and gas industries usually experience their highest loads as the extreme wave passes. This design situation is usually modelled using a regular non-linear wave model; the deterministic nature of regular waves is not seen as a problem. For offshore wind turbines, which are more dynamically active, the ultimate loads are more likely to occur with a particular combination of wind gust, rotor position and large waves. Of great benefit therefore would be a wave model that
both included a stochastic element and remained valid at large wave height. As mentioned earlier, models that do this explicitly are generally confined to research applications.

The draft offshore wind turbine standard IEC 61400-3 [1] includes an informative appendix which proposes various methods that consider the stochastic and non-linear effects separately. The first method is to run regular non-linear waves in combination with a steady wind whose speed has been increased above the design value using a gust reaction factor. A second proposed method is to combine a stochastic wind with a linear stochastic sea, and then augment the loads by a non-linearity factor; this approach we will use later for comparison.

1.1. Embedded non-linear waves
The third approach proposed in IEC 61400-3 is to “cut and paste” a large stream function wave into a linear, stochastic sea. This has the advantage of a stochastic element, so many simulations can be made exploring various combinations of gusts and waves, yet for the water kinematics around the extreme wave itself an accurate large wave model is used. The extreme wave can be placed anywhere, at the choice of the modeler; pasting the stream function wave over a linear wave of similar height should maintain a more realistic sea overall.

The challenges in implementing such a method are twofold: firstly finding a large wave to paste over, and secondly blending smoothly between the linear waves and the stream function. Previously this approach has been implemented by running long irregular wave simulations and picking out a suitable large wave. The large wave would then be cut out and replaced with a stream function wave. To prevent any discontinuities short blending regions are used; near the extreme crest the non-linear
solution for water kinematics is used; in the blending region all the water properties are calculated as a weighted average of the non-linear solution and the irregular linear sea; and beyond a certain period, the pure linear sea is used. The weighting function (blending parameter) used varies from unity around the crest to zero beyond a certain distance, and changes smoothly across the blending region. We have used a cosine function as set out in Table 1; this was chosen because the resulting surface and its time-derivative are both continuous over the blending region.

| Time                          | Stream-function | Stochastic time history |
|-------------------------------|-----------------|-------------------------|
| \(|t - T_0| > 0.75 \times T\)     | \(0\)           | \(1\)                   |
| \(0.5 \times T \leq |t - T_0| \leq 0.75 \times T\) | \(0.5 + 0.5 \cos\left(4\pi \left(\frac{|t - T_0|}{T} - 0.5\right)\right)\) | \(0.5 - 0.5 \cos\left(4\pi \left(\frac{|t - T_0|}{T} - 0.5\right)\right)\) |
| \(|t - T_0| < 0.5 \times T\)   | \(1\)           | \(0\)                   |

Table 1. Blending parameter for the stream-function solution, and the linear stochastic sea. The Blending parameter is a function of time (t), relative to the time of the constrained peak (T₀), and the time period of the extreme wave (T).

There are two problems associated with this method. Firstly it is computationally inefficient in the way that a suitable large linear wave is found. Long simulations are typically required before a suitably large wave appears ‘by chance’. Secondly, the large wave is generally asymmetric, with the preceding trough of different depth to the subsequent trough. This makes the blending process quite unphysical. Figure 3 shows an example surface elevation for the linear sea, non-linear regular wave and the final combination blending between the two. The preceding trough is considerable lower than the non-linear wave, so the blended solution has an unphysical sharp corner. We have used constrained waves to overcome both of these problems.

![Figure 3. Embedded non-linear wave, previous method](image-url)
1.2. NewWave theory and Constrained waves

Linear sea states can be viewed as an example of a Gaussian process; thereby conforming to one of the most fundamental and well studied stochastic processes. At any given time the surface elevation could be found at any height, however it is clearly more likely to be around the still water level than to be greatly displaced. The surface elevation must therefore have a probability distribution, and for the linear sea state, being a Gaussian process, it is a normal distribution. The standard deviation is related to the significant wave height; Figure 4 overlays the probability distribution onto a typical time history.

![Figure 4](image-url)

**Figure 4.** Probability distribution of the surface elevation is a linear stochastic sea.

If, at one instant, the sea surface is at a specified elevation high above the mean water level then a short time later (much less than a typical wave period) the surface elevation is expected to be closer to this specified elevation than zero. The conditional probability distribution of the new surface elevation (conditional on the surface having passed through the specified elevation) will be different from the original normal distribution and will depend on the frequency content of the sea state as well as the significant wave height. NewWave theory [2] was developed by investigating the surface elevation around a large crest, and deriving the conditional probability distribution i.e. the mean and standard deviation of the surface elevation given that there was a stationary point of elevation X at time T, as illustrated in Figure 5.
Figure 5. Conditional probability distribution for a constrained linear sea.

Not only does this give the most likely shape of a large crest, but it also allows a constrained simulation to be run. Linear sea state computer simulations can be adapted so that they conform to the conditional probability distribution and are in effect constrained to have a stationary point at any predetermined height and time. This method can be used instead of running many simulations in search of a wave of the required height, saving much computing time in the process.

2. Our method

We have used GH Bladed to implement our new wave modeling approach. GH Bladed [3] is an integrated software package which allows the user to carry out the full range of performance and loading calculations required for the design and certification of wind turbines, both onshore and offshore. Bladed supports calculations of combined wind and wave loading, with full aeroelastic and hydroelastic modeling.

A key part of our method is the use of constrained linear waves to simulate a suitable large wave in the background sea-state which is subsequently replaced, in part, by a non-linear regular wave. We believe we are the first to integrate this method into a wind turbine design tool, and not only that, we have extended the use of constrained waves to mitigate the problem of blending the linear sea into the regular wave. As well as constraining the water surface elevation to pass through a specified peak, constraints are placed in the troughs at either side. New wave theory was developed to study peaks but mathematically all that is stated is that the time derivative of the surface elevation is zero at the point of interest. A constrained trough can therefore be created using exactly the same methods except that a large negative surface elevation is specified.

In GH Bladed the user specifies the constrained wave height, period and time of occurrence. The nonlinear regular wave model is run first, so that the elevation of the troughs is ascertained. Six constraints are then set: three surface elevations and three zeros of surface elevation time derivative. The linear sea state model is then run and blended with the non-linear wave in the same manner as the older method. Although the blending method is the same, the constraints on the troughs ensure that the linear sea state will match very closely with the non-linear wave in the blending regions. For the surface elevation itself the blending could be done away with altogether, however we are also simulating the water velocities, accelerations and pressures through the entire water column. These are...
not considered in the constraining process, so a degree of blending is still required. The resulting time history is illustrated in Figure 6.

![Figure 6. Non-linear wave embedded in stochastic linear sea using the new method of constraining the linear sea at three points.](attachment:figure6.png)

The benefits of constraining the troughs of the extreme wave can be seen when comparing vertical profiles of water particle kinematics for the linear sea, and the non-linear wave. The blending starts at the troughs of the non-linear wave, so accuracy is aided by the linear sea having a similar velocity profile at this time. As an example, Figure 7 shows vertical profiles of water particle horizontal velocity. The velocity profiles shown were taken from the stream function solution and six constrained linear simulations into which the non-linear wave was being placed. Three of the linear seas were constrained only at the peak and three were additionally constrained at the neighbouring troughs. The figure shows that the extra constrained points produce simulations where the velocity profiles are considerably better matched to the stream function solution.
3. Comparison with the wave non-linearity factor approach

Both the wave non-linearity factor approach and the "cut and paste" methods obtain time saving benefits from the use of constrained waves. The draft IEC 61400-3 standard requires at least six, one hour periods to be simulated for each combination of extreme turbulent wind speed and extreme stochastic sea state. However this can be reduced to six 600 second simulations when constrained waves are used. This represents a factor of six saving on computing time, which can be considerable.

GH Bladed was used to carry out design load case simulations for a generic 5MW turbine mounted on a tripod support structure. The extreme wave was modelled using both the wave non-linearity factor approach (including constrained linear waves) and our new constrained non-linear wave method.

Results showed that the linear extreme wave simulations generally led to higher structural loading. This result seems surprising at first, and in fact highlights a possible weakness in the application of constrained linear wave models. As the linear constrained wave is still an irregular wave, the two troughs neighbouring the highest crest are not of equal elevation. If the height of the extreme wave is defined as the vertical distance from the crest to the mean of the two adjacent trough elevations, one of the faces of the wave will in general indicate an apparently higher wave height (often by a considerable fraction) than the other. The water particle kinematics associated with this higher face of the wave can, in turn, lead to higher loads than would have resulted if the modelled wave height was controlled more rigorously. An alternative method would be to define the wave height on the higher of the two faces of the wave, but this would be non-conservative. Using the new method, however, the constrained non-linear wave is symmetric, so there is no such confusion in the definition of wave height.

When the linear constrained wave is used, IEC 61400-3 suggests that the ultimate loads are multiplied by corresponding wave non-linearity factors. These are calculated by running regular wave simulations with linear and non-linear waves. When the loads are higher in the non-linear case, they are divided by the loads from the linear case to calculate the non-linearity factor. The ultimate loads from the linear constrained wave simulation are then multiplied by this factor. These factors have to be calculated for each load component; however each load component generally experiences its ultimate
load at a different time, and since the non-linearity factor should be calculated with the wave height and wind speed that lead to the ultimate load, each load component requires its own two simulations. For tripod and jacket type structures this adds considerably to computing time. Moreover determining the wind speed and wave height at the time of the ultimate load is not as straightforward as it might seem; in a dynamic structure the resultant loads are dependant on the time history of the external loads from some period in the past; the ultimate load effects generally occur some time after the extreme external loads that caused them. In contrast, our non-linear constrained wave approach avoids the need to calculate any additional load factors.

4. Conclusions
In this paper we have presented a new method of modelling the extreme sea state for the design of offshore wind turbines. We believe that the proposed method offers a practical and convenient approach to modeling the extreme sea state, by incorporating a non-linear steep wave model within a stochastic sea.

NewWave theory has been used to constrain a linear stochastic wave train to the crest and adjacent trough elevations of a stream function solution for the extreme wave. The stream function wave may then be blended into the background sea state using an appropriate blending function.

The use of constrained waves offers two advantages. Firstly, long simulations are not required to identify large waves in the background sea state. Secondly, the blending of the stream function wave into the background sea state is improved.

The new method offers considerable savings in computational time and design effort relative to existing methods.

References
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[3] Bossanyi, E. A., 2006, *GH Bladed Theory Manual*, GH report 282/BR/09.