General uncertainty analysis and synthesis for a precision boomed centrifuge

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Abstract: The general precision uncertainty of a centrifuge is suggested here to determine whether a precision centrifuge is suitable to calibrate inertial sensors. The possible centrifuge errors that affect the accuracy of the specific input forces are analyzed with three kinds of centrifuge errors identified: displacement error, angular rate error, and attitude error. Taking an accelerometer’s calibration on a centrifuge as an example, the specific force components for the three axes of a tested accelerometer are acquired using a homogeneous transformation algorithm. The impact of the uncertainty for each kind of error source on the specific force is calculated, and their contributions to the general uncertainty of the centrifuge are acquired. Through simulation, by limitation of the tolerances towards the centrifuge errors, the general uncertainty of a precision centrifuge is designed to satisfy the requirements of the pre-design in $6 \times 10^{-6}$. And the influence of the general uncertainty on calibration accuracy of error model coefficients of accelerometer is calculated, mostly on $C_{\text{sp}}$. The influence of the error which accounts for a large part of the general uncertainty on the calibration accuracy is mainly studied. Simulation results show with the increase of the general uncertainty of centrifuge, its influence on the calibration accuracy of error model coefficient is greater. And when the general uncertainty is constant, the influence of the general uncertainty on the calibration accuracy can be reduced by increasing the input specific force.

1. Introduction
A boomed centrifuge with counter-rotating platform is widely used in calibration of inertial measurement unit (IMU). As the structure of such centrifuge is complicated, the centrifuge errors are so many to be researched on the impact on calibration accuracy of error model coefficients. Much work has been done on the analyses of impact of centrifuge errors on boomed centrifuge. While most of the analyses of centrifuge errors are given by several errors, which have not been thoroughly considered, the calibration accuracy can not be satisfied$^{1-6}$.

The general uncertainty of a centrifuge is determined by gathering all the influential tolerances of the centrifuge errors for the specific forces after compensating for those caused by the centrifuge errors. As the calibration accuracy of the inertial sensors (especially the higher-order coefficients) is determined from the accuracy or general uncertainty of the precision centrifuge, an analysis of the general uncertainty is an effective way to improve the calibration performance of the high-order error model coefficients and further improve the INS accuracy.

There has been little research on the general uncertainty of centrifuges as the focus has been on the influence of centrifuge errors on the calibration accuracy. A guide to the performance and analysis of precision centrifuge tests with linear accelerometers was provided in Ref. 1, but only the misalignment...
and radius errors were analyzed. A revised error calibration model was established to include the misalignments, radius errors, and other errors by analyzing the mechanisms of the main error sources. Different calibration methods for the inertial sensor on a centrifuge were studied and several centrifuge errors were considered to increase the calibration accuracy, but the error analysis was not complete and the calibration accuracy could not be guaranteed. The error compensation for calibration has also been studied. Most methods are concerned with several errors for compensation while few quantify the influence of errors on the calibration accuracy. Furthermore, there is no literature on the general uncertainty or its impact on the calibration accuracy.

This paper comprehensively analyzes centrifuge errors. The influencing mechanisms of the centrifuge errors on the input specific forces during calibration are studied based on error transfer. Calculating the contributions of the tolerance is possible after the error compensation, and errors needed to be compensated for should be measured during calibration. The general uncertainty of the boomed centrifuge is given based on the contributions of the considered errors to the input specific force to guide the design of centrifuges with different calibration accuracies. The impact of general uncertainty on calibration accuracy of error model coefficients is studied, and how to lower the impact of general uncertainty is also studied.

2. Accuracy of precision centrifuge
The accuracy of a precision centrifuge is represented by the general uncertainty of the specific forces produced by the system. The accuracy is based on the ratio of the specific force errors and the total specific force on the input axis of the tested accelerometer. The uncertainty mainly includes the centripetal acceleration uncertainty of the centrifuge and various additional acceleration uncertainties. Thus, the general uncertainty of a precision centrifuge can be expressed as:

\[
\delta_{\text{in}} = \frac{|\Delta a|}{A} = \left[ (\delta_a)^2 + \frac{1}{A^2} \sum_{i=1}^{n} (\sigma_{\text{in}})^2 \right]^{\frac{1}{2}}
\]

(1)

where \(\delta_a\) is the relative uncertainty in the centripetal acceleration of the centrifuge, which is expressed as follows based on the impacting factor:

\[
\delta_a = \frac{\sigma_a}{A} = \left[ \left( \frac{\sigma_a}{\omega_0} \right)^2 + \left( \frac{2 \sigma_a}{\omega_0} \right)^2 \right]^{\frac{1}{2}}
\]

(2)

where \(A\) is the nominal value of the centripetal acceleration with \(A = R_0 \omega_0^2\), \(\sigma_a / \omega_0\) is the relative error of the working radius, \(\sigma_a / \omega_0\) is the relative error of the angular velocity, and \(\sigma_{\text{in}}\) represents the various additional acceleration errors along the input axis of the accelerometer as caused by the misalignment angles.

The general accuracy of a precision centrifuge is limited to \(10^{-3} - 10^{-5}\), which is relatively lenient based on the known errors. The accuracy requirement can be met in consideration of several of the primary error terms. The aim here is to design and research high-precision centrifuges with accuracies reaching \(10^{-6}\) to calibrate the high-order coefficients. Therefore, all the errors that may affect the accuracy of a centrifuge should be comprehensively considered.

3. Calibration method for error model coefficients of accelerometers
A boomed precision centrifuge with an accuracy of \(10^{-6}\) is taken as an example to facilitate the error source analysis. The specific force errors on the input axis of the tested accelerometer caused by centrifuge errors are analyzed in combination with their test method on a centrifuge. A boomed precision centrifuge is shown in Fig. 1 with a nominal working radius of 2.5 m and a maximum acceleration of 30 g. The three axes are given in Fig. 1, where “1” represents the main axis, “2” represents the horizontal axis, “3” represents the azimuth axis of the counter-rotating platform, and “D” represents the tested quartz accelerometer. “IA” is the input axis of accelerometer, “OA” is the output axis of accelerometer, “PA” is the pendulum axis of accelerometer.
The classical installation method for a calibrated accelerometer is shown in Fig. 1-(a), where \( R_0 \) is the nominal working radius of the centrifuge, \( \omega_0 \) is the angular velocity of the main axis, and \( a_I, a_P, \) and \( a_O \) are the input specific forces along the input axis, pendulum axis, and output axis, respectively. As shown in Fig. 1, the main specific force of the accelerometer along the input axis is the acceleration generated from the rotation about the main axis. Without considering the errors of the centrifuge, the expression for the centripetal acceleration is \( A = R_0 \omega_0^2 \), where the gravity vector shown in Fig.1-(a) is the specific force vector generated by the gravitational acceleration and is in the upward direction.

![FIG. 1 Schematic diagram for a precision centrifuge and the defined coordinate systems.](image)

**TABLE 1. Coordinate systems of the boomed centrifuge**

| No | coordinate system | Name | No | coordinate system | Name | No | coordinate system | Name |
|----|-------------------|------|----|-------------------|------|----|-------------------|------|
| 1  | \( o_0x_0, y_0, z_0 \) | geographic coordinate system | 4  | \( o_2x_2, y_2, z_2 \) | sleeve coordinate system of the horizontal axis |
| 2  | \( o_1x_1, y_1, z_1 \) | sleeve coordinate system of the main axis | 5  | \( o_2x_2, y_2, z_2 \) | coordinate system of the horizontal axis |
| 3  | \( o_1x_1, y_1, z_1 \) | coordinate system of the main axis | 6  | \( o_3x_3, y_3, z_3 \) | sleeve coordinate system of the counter-rotating platform axis |
| 7  | \( o_3x_3, y_3, z_3 \) | coordinate system of the counter-rotating platform |
| 8  | \( o_4x_4, y_4, z_4 \) | coordinate system of the working base |
| 9  | \( o_5x_5, y_5, z_5 \) | coordinate system of the accelerometer |

Tab. 1 shows the coordinate systems according to Fig. 1-(b), The coordinate system of serial number \( i + 1 \) is based on the coordinate system of serial number \( i \).

For precision applications, a relatively complex model equation with high-order nonlinear and cross-axis terms is usually required. According to the IEEE-836 standard, a normalized model equation for the accelerometer input-output function is often expressed as:

\[
a_j = B_0 + \sum_{i=0,p} S_i a_i + \sum_{i=0,p} D_i a_i^2 + \sum_{i=0,p} T_{ii} a_i^3 + \frac{1}{2} \sum_{i,j=0,p} C_{ij} a_i a_j + \varepsilon
\]

where \( a_j \) is the indicated sensor output, \( B_0 \) is the bias, \( S_i \) is the scale factor along each axis, \( D_i \) is the second-order coefficient, \( T_{ii} \) is the third-order coefficient, \( C_{ij} \) is the cross-coupling coefficient, and \( \varepsilon \) is the random error.

Equation (3) indicates that the accelerometer output is obtained when the input specific force for each axis is accurately given at the special position or time. Thus, the error model coefficients can be calculated when more specific forces and outputs are given in Eq. (3) and solved using a least squares approach. As the real specific forces produced from the centrifuge are influenced by the centrifuge errors, the real specific forces inserted into Eq. (3) are not the correct values and are inconsistent with
the accelerometer’s output; therefore, the calibration results will not be satisfactory. The centrifuge
errors should be analyzed in detail and the contributions of each error to the input specific force of the
accelerometer should be calculated by propagating these errors to the accelerometer coordinate system.
Therefore, the corresponding error sources and propagation are studied in the following.

4. Error sources for precision centrifuge and their propagation

As shown by \( A = R_0 \omega_0^2 \), the main factors that affect the acceleration accuracy of a centrifuge are the
working radius error and the angular velocity error, as described in Eq. (2). Furthermore, several
additional acceleration errors along the three axes are caused by misalignment angle errors along the
gravitational acceleration, Coriolis acceleration, etc. Besides the centripetal acceleration, these errors
should be corrected during data processing. The described errors all contribute to the general uncertainty of the centrifuge, as shown in Eq. (1). Thus, the input specific force error vectors can be
expressed as,

\[
\Delta a = \Delta a_R + \Delta a_\omega + \Delta a_\theta
\]

where \( \Delta a_R \) is the input specific force due to the working radius error, \( \Delta a_\omega \) is the impact of the
angular velocity, and \( \Delta a_\theta \) is the contribution of the misalignment angle errors to the input specific
force. Thus, the centrifuge error sources can be divided into three categories: displacement error,
angular rate error, and attitude error. Detailed analyses of these three errors are given as follows.

4.1 Displacement error sources

The displacement error sources that affect the working radius of the centrifuge mainly include the
static radius error \( \Delta R_s \) and dynamic radius error \( \Delta R_d \) as well as the associated measurement errors.
The analysis of the two main error sources of the working radius is shown, and the expression for the
input specific force along each axis is given. An illustration of the displacement error sources that
affect the working radius is shown in Fig. 2, which is the top-down view of Fig. 1.

![Diagram of centrifuge error sources](image)

**FIG. 2 Vertical view of the displacement errors that affect the working radius.**

The static working radius of the centrifuge \( R_s \) is the distance from the effective center of mass for
the measured inertial device to the main axis for a stationary centrifuge. The errors involved in all the
connecting parts of the mechanical components are treated as error sources that affect the accuracy of the
static radius. These include the concentricity error of the main axis line \( \Delta R_m \), which is the bias
between the shaft center and the instantaneous axis of rotation; the concentricity error of the azimuth
axis \( \Delta R_a \); runout of the main axis \( \Delta R_m \); runout of the azimuth axis \( \Delta R_a \); alignment error \( \Delta \theta \);
errors caused by the misalignment angle, including between the horizontal axis and main axis \( \Delta \theta \);
misalignment angle between the azimuth and horizontal axes \( \Delta \theta_{mh} \); parallelism between the installed
plane and azimuth axis \( \Delta \theta_{p} \); intersection errors including for the horizontal and main axes \( \Delta y_{h} \);
error of the azimuth and horizontal axes \( \Delta y_{h} \); error due to temperature \( \Delta R_t \); stability error \( \Delta R_s \).
repeatability error $\Delta R_x$; and measurement error $\Delta R_y$.

All the described errors are defined in vector form. These errors are propagated to the coordinate system that is fixed on the calibrated accelerometer using the homogeneous transformation. The homogeneous transformation is a common technique to simultaneously specify translations and rotations between coordinate systems in three dimensions. Suppose $\omega t$ is the rotation angle of the main axis, $\gamma$ is the rotation angle of the counter-rotating platform, $U_1 = [\cos \omega t \sin \omega t 0]^T$, $U_2 = [-\sin \omega t \cos \omega t 0]^T$, $U_3 = [-\sin \gamma \cos \gamma 0]^T$, and $L$ is the distance from the origin of the coordinate system for the azimuth axis to the surface of the installed plane.

The expression for the acceleration errors through the projection of the centripetal acceleration as affected by each error is:

\[
\begin{bmatrix}
\Delta a_x \\
\Delta a_y \\
\Delta a_z
\end{bmatrix} = \begin{bmatrix}
\tilde{\Delta} x \\
\tilde{\Delta} y \\
\tilde{\Delta} z
\end{bmatrix}
\]

where $\mathbf{T}_{\omega t}$ is the attitude component of $\mathbf{T}$, $\begin{bmatrix}
\tilde{\Delta} x \\
\tilde{\Delta} y \\
\tilde{\Delta} z
\end{bmatrix}$ is the second derivative of the displacement component of $\mathbf{T}$, and $\mathbf{T}$ represents the specific homogeneous transformation that contains the centrifuge errors. Ignoring the detailed derivation process, the contribution of each static radius error to the acceleration error produced by the centrifuge is expressed as:

\[
\Delta a = (\Delta R_x + \Delta R_y) + (\Delta \theta_{mx} + \Delta \theta_{my})L
\]

Thus, the contribution to the general uncertainty for the acceleration is:

\[
\delta a = \frac{|\Delta a|}{R_0 \Omega^2} = \frac{|\sigma_a|}{R_0}
\]

where

\[
\sigma_a = (\Delta R_x + \Delta R_y) + (\Delta \theta_{mx} + \Delta \theta_{my})L
\]

When taking the test method for full-circle rotation, the full-circle average value of the term is zero.

Thus, the contributions of the working radius to the general uncertainty of the acceleration generated by the centrifuge is:

\[
\delta R = \frac{|\sigma_R|}{R_0} = \sqrt{\delta a^2 + \delta \rho^2}
\]

4.2 Angular velocity error sources

The angular velocity error of the centrifuge results from changes in the angular velocity for the main axis with time. According to the IEEE-836 standard, the angular velocity error of the centrifuge can be expressed as:

\[
\Delta \omega = \omega_p \sin(2\pi f_w t + \varphi) + \omega_C t + \omega_z
\]

where $\omega_p \sin(2\pi f_w t + \varphi)$ represents the periodic variations with amplitude $\omega_p$, frequency $f_w$, and phase $\varphi$ at the beginning of data sampling; $\omega_C t$ is the constant drift error with the angular velocity at the initial time of data sampling $\omega_C$, and $\omega_z$ is the random error.
4.3 Attitude error sources

The misalignment angle error is the primary error source that affects the testing accuracy when calibrating the accelerometer on a precision centrifuge. This impacts the input specific force of the three acceleration axes through the projection of the gravitational acceleration and centripetal acceleration along their non-sensitive directions. The misalignment angle error is composed of both static and dynamic errors.

4.3.1 Impact of static misalignment angle error

The static angular misalignment error sources are caused primarily by the static characteristics of the three bearings used in the centrifuge, including the initial installation angle error of the azimuth axis on the horizontal axis \( \Delta \theta_{ah} \), the initial installation angle error of the accelerometer on the platform of the azimuth axis (including the testing fixture) \( \Delta \theta_{oi} \), verticality of the main axis \( \Delta \theta_{mv} \), misalignment angle between the main and horizontal axes \( \Delta \theta_{hm} \), misalignment angle between the azimuth and horizontal axes \( \Delta \theta_{ah} \), misalignment angle between the installed plane and the azimuth axis \( \Delta \theta_{pm} \), and the initial positioning error of the surface \( \Delta \theta_{iz} \). These errors are propagated to the coordinate system that is fixed on the calibrated accelerometer using the homogeneous transformation. So,

\[
\begin{bmatrix}
T_4 & \cos \gamma & \sin \gamma & 0 \\
T_5 & 0 & 1 & 0 \\
T_6 & 1 & 0 & 0 \\
T_7 & \sin(\omega t + \gamma) & \cos(\omega t + \gamma) & 0 \\
T_8 & \cos(\omega t + \gamma) & \sin(\omega t + \gamma) & 0 \\
T_9 & 0 & 0 & 1 \\
T_{10} & 0 & 0 & \sin \gamma \\
T_{11} & 0 & 0 & -\cos \gamma \\
\end{bmatrix}
\]

The static misalignment angle error primarily affects the specific force through the projection of the gravitational acceleration on the X and Y axes, and the corresponding expression is:

\[
\Delta \mathbf{a}_{ix} = \Delta \theta_{ah} \mathbf{a}_x + \Delta \theta_{oi} \mathbf{a}_y + \Delta \theta_{mv} \mathbf{a}_z + \Delta \theta_{hm} \mathbf{a}_{zh} + \Delta \theta_{ah} \mathbf{a}_{zh} + \Delta \theta_{pm} \mathbf{a}_{zh} + \Delta \theta_{iz} \mathbf{a}_z
\]

When considering the test method for the full-circle rotation, there is no contribution from the term \( \Delta \theta_{pv} \) in Eq. (11) to the acceleration error of the centrifuge. The contribution of the static misalignment angle error of the centrifuge to the general uncertainty of the system acceleration is then expressed as:

\[
\delta_{ay} = \frac{\Delta \mathbf{a}_{ay}}{R_y \omega_y^2}
\]

4.3.2 Impact of dynamic misalignment angle and integration

The dynamic misalignment angle error that may cause additional acceleration includes the wobble errors of the main axis \( \Delta \theta_{mv}(\omega t) \), horizontal axis \( \Delta \theta_{mv}(\beta t) \), and azimuth axis \( \Delta \theta_{mv}(\gamma t) \), and the random error \( \Delta \theta_{im} \). These errors are caused primarily by the dynamic characteristics of the three bearings used in the centrifuge. By homogeneous transformation method,

\[
\mathbf{U}_{12} = \begin{bmatrix} \sin \gamma & \cos \gamma & 0 \end{bmatrix}^T \\
\mathbf{U}_{13} = \begin{bmatrix} -\cos \gamma & \sin \gamma & 0 \end{bmatrix}^T
\]

Equations (10) and (5) are adopted simultaneously when calculating the projection of the gravitational acceleration and the centripetal acceleration on each axis of the accelerometer as affected by the dynamic misalignment angle error. Therefore, the contribution of the dynamic misalignment angle error to the general uncertainty of the centrifuge acceleration is expressed as:
\[ \Delta a_{\phi} = \left[0 \quad U_x \quad 0 \right] \Delta \theta_{\phi, \omega} + \left[0 \quad U_y \quad 0 \right] \Delta \theta_{\phi, \omega} + \left[U_{12} \quad U_{13} \quad 0 \right] \Delta \theta_{\omega, \phi} \]  
where \( \Delta \theta_{\phi, \omega} \) is the total random error term for the dynamic misalignment angle, which is monitored in real-time during the practical testing period.

If taking the test method of the full-circle rotation, there is no contribution of the \( \Delta \theta_{\phi, \omega}(\omega \tau) \) term in Eq. (13) to the acceleration error. The contribution of the dynamic misalignment angle error of the centrifuge to the general uncertainty of the system acceleration is then expressed as:

\[ \delta_{\phi,d} = \frac{\left| \Delta a_{\phi,d} \right|}{R_{0}\omega_0^2} \]  

The contribution of the total misalignment angle error to the general uncertainty is then:

\[ \delta_{\phi} = \frac{\left| \Delta a_{\phi} \right|}{R_{0}\omega_0^2} = \sqrt{\delta_{\phi,s}^2 + \delta_{\phi,d}^2} \]  

where \( \Delta a_{\phi} = \Delta a_{\phi,s} + \Delta a_{\phi,d} \).

### 5. Influence of centrifuge errors on the accelerometer input

The above analyses for the three kinds of error sources in the precision centrifuge testing allow expressing the comprehensive input specific force errors of the tested accelerometer as:

\[ \Delta a = \Delta a_s + \Delta a_w + \Delta a_{\phi} \]  

\[ = \left[ \psi_1(\Delta \theta_{\phi, \omega} + \Delta \theta_{\omega, \phi} + \Delta \theta_{\phi, \omega}) + \psi_2(\Delta \theta_{\phi, \omega} + \Delta \theta_{\phi, \omega} + \Delta \theta_{\phi, \omega}) + \psi_3(\Delta \theta_{\phi, \omega} + \Delta \theta_{\phi, \omega} + \Delta \theta_{\phi, \omega}) + \psi_4(\Delta \theta_{\phi, \omega} + \Delta \theta_{\phi, \omega} + \Delta \theta_{\phi, \omega})(\omega \tau) \right] \]  

where \( \psi_1 = \left[0 \quad U_x \quad 0 \right], \psi_2 = \left[U_{12} \quad U_{13} \quad 0 \right], \psi_3 = \left[U_{10} \quad U_{11} \quad 0 \right], \psi_4 = \left[U_{1} \quad 0 \quad 0 \right], \psi_5 = \left[U_{12} \quad U_{13} \quad 0 \right], \psi_6 = \left[U_{10} \quad U_{11} \quad 0 \right], \psi_7 = \left[U_{1} \quad 0 \quad 0 \right], \psi_8 = \left[U_{12} \quad U_{13} \quad 0 \right], \psi_9 = \left[U_{10} \quad U_{11} \quad 0 \right], \psi_{10} = \left[U_{1} \quad 0 \quad 0 \right], \psi_{11} = \left[U_{1} \quad U_{2} \quad 0 \right], \psi_{12} = \left[U_{1} \quad 0 \quad 0 \right], \psi_{13} = \left[U_{1} \quad 0 \quad 0 \right]. \]

Taking the average of the full-circle rotation of the centrifuge, Eq. (16) can be simplified as:

\[ \Delta a = \Delta a_s + \Delta a_w + \Delta a_{\phi} \]  

\[ = \left[ \psi_1(\Delta \theta_{\phi, \omega} + \Delta \theta_{\omega, \phi} + \Delta \theta_{\phi, \omega}) + \psi_2(\Delta \theta_{\phi, \omega} + \Delta \theta_{\phi, \omega} + \Delta \theta_{\phi, \omega}) + \psi_3(\Delta \theta_{\phi, \omega} + \Delta \theta_{\phi, \omega} + \Delta \theta_{\phi, \omega}) + \psi_4(\Delta \theta_{\phi, \omega} + \Delta \theta_{\phi, \omega} + \Delta \theta_{\phi, \omega})(\omega \tau) \right] \]  

where \( \omega_0 \) is the linear rotation rate of the centrifuge, and \( \omega_\phi \) is the local latitude.

### 6. Simulation and tolerance allocation for precision centrifuge errors

#### 6.1 Tolerance allocation for precision centrifuge errors

The expressions for each type of error that comprehensively affect the centrifugal acceleration are shown in Eq. (19), which are based on the general uncertainty of Eq. (1) as:

\[ \delta_{\phi} = \frac{\left| \Delta a_{\phi} \right|}{A} = \sqrt{\delta_{\phi,s}^2 + \delta_{\phi,d}^2} \]  

where \( \omega_\phi \) is the horizontal component, and \( \omega_\phi \) is the vertical component, and \( \lambda \) is the local latitude.
When designing a centrifuge in practice, the tolerances towards various centrifuge errors should be allocated to meet the accuracy requirements for its general uncertainty. The following content sets the requirement to allocate the tolerance towards each error source while considering the economic factor and machining difficulties so that the accuracy of the general uncertainty can satisfy the requirements.

Assuming the general uncertainty of the centrifuge applied in the test is $6 \times 10^{-6}$ ($3\sigma$) and the corresponding displacement error source is allocated as $4 \times 10^{-6}$ ($3\sigma$), the angular velocity error source is $2 \times 10^{-6}$ ($3\sigma$) and the remaining $4 \times 10^{-6}$ ($3\sigma$) is assigned to attitude error source. Thus, the three kinds of error sources constitute the general uncertainty valued at $22 \times 2 \times 3 \times 10^{-6} = 6 \times 10^{-6}$. The detailed allocations of the error tolerances for a boomed centrifuge are given in Tab. 2. The goal is to have a general uncertainty less than $3 \times 10^{-6}$ and each kind of error should be limited to the corresponding tolerance allocation.

### TABLE 2. Tolerance allocations of the centrifuge errors

| Error vector | Relative accuracy of tolerance (10^{-6}) | Error vector | Relative accuracy of tolerance (10^{-6}) | Error vector | Relative accuracy of tolerance (10^{-6}) |
|--------------|-----------------------------------------|--------------|-----------------------------------------|--------------|-----------------------------------------|
| $\Delta R_{mc}$ | 0.8 | $\Delta R_{s}$ | 1 | $\Delta \theta_{pm}$ | 1.2 |
| $\Delta R_{ac}$ | 0.8 | $\Delta R_{s}$ | 0.4 | $\Delta \theta_{pa}$ | 1.2 |
| $\Delta R_{mr}$ | 1.6 | $\Delta R_{r}$ | 0.3 | $\Delta \theta_{pa}$ | 1.5 |
| $\Delta \theta_{\text{rad}}(R)$ | 0.2 | $\omega$ | 0.4 | $\Delta \theta_{\text{rad}}(\beta t)$ | 1 |
| $\Delta \theta_{\text{rad}}(R)$ | 0.2 | $\Delta \omega t$ | 0.5 | $\Delta \theta_{\text{rad}}(\gamma t)$ | 1 |
| $\Delta \alpha_{\text{rad}}$ | 0.2 | $\Delta \alpha_{\text{rad}}$ | 1 | $\Delta \theta_{\text{rad}}$ | 2 |
| $\Delta \theta_{\text{rad}}$ | 0.2 | $\Delta \theta_{\text{rad}}$ | 1 | $\Delta \theta_{\text{rad}}$ | 2 |
| $\Delta R_{T}$ | 0.8 | $\Delta \theta_{\text{rad}}$ | 1 | $\Delta \theta_{\text{rad}}$ | 1 |

The contribution of the centrifuge radius error to the general uncertainty of the system acceleration is:

$$\frac{\delta a}{R_0} = \sqrt{\delta a_{x}^2 + \delta a_{y}^2} = \sqrt{3^2 + 1.5^2 \times 10^{-6}} = 3.35 \times 10^{-6} < 4 \times 10^{-6}$$

Accordingly, the contribution of the angular velocity error is:

$$\frac{2\delta \omega}{\omega_0} = 2 \times 2 \times 0.4^2 + 0.5^2 \times 10^{-6} = 1.5 \times 10^{-6} < 2 \times 10^{-6}$$

and the contribution of the misalignment angle error is:

$$\delta \alpha = \sqrt{\delta a_{x}^2 + \delta a_{y}^2 + \delta a_{z}^2} = \sqrt{3.22^2 + 2^2 \times 10^{-6}} = 3.79 \times 10^{-6} \text{ rad}$$

From Eq. (2), the general uncertainty of the centrifuge is calculated as:

$$\delta a = \sqrt{\left(\frac{\delta a_{x}}{R_0}\right)^2 + \left(\frac{2\delta \omega}{\omega_0}\right)^2 + \sum_{h} \left(\frac{\delta a_{y}}{A_h}\right)^2} = \sqrt{3.35^2 + 1.5^2 + 3.79^2 \times 10^{-6}} = 5.28 \times 10^{-6} < 6 \times 10^{-6}$$

These allocations indicate that the general uncertainty of the centrifuge satisfies the established requirements. The accuracy requirement for the centrifuge can also be satisfied by adopting the tolerance allocations towards the above error sources for practical design and manufacturing processes. It is noted that the relative accuracy of the tolerance given in Tab. 2 is corrected by compensating for the determined errors.

The tolerance for the distribution shown in Tab. 2 is from the influence of the input specific force after compensation. The calibration of an accelerometer is taken as an example to further discuss the influence of the centrifuge errors on the calibration accuracy of the error model coefficients and analyze the influence of the typical error sources on the input specific forces. Therefore, the influence of the compensated error for each high-error model coefficient is obtained.

### 6.2 Impact of general uncertainty on calibration accuracy

A boomed centrifuge shown in Fig.1 is assumed with a radius of 2.5 m that supplies a 20 g input specific force into the accelerometer. The 10-positional calibration method from Ref. 7 is taken, and
the effects of the errors listed in Tab. 2 on the high-order error term of the accelerometer are given in Tab. 3. The calibration accuracy of the accelerometer is on the order of $10^{-8}$.

**TABLE 3. Relationships between the centrifuge errors and the model coefficients of the accelerometer**

| Error vector | Measurement $(10^{-6})$ | $D_{ii} (v/g^2)$ | $D_{pp} (v/g^2)$ | $C_{io} (v/g^2)$ | $C_{pi} (v/g^2)$ | $C_{op} (v/g^2)$ | $T_{iii} (v/g^3)$ |
|--------------|------------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| $\Delta \theta_p$ | 1.2 | $-8.625 \times 10^{-10}$ | $9.4875 \times 10^{-8}$ | $1.3359 \times 10^{-8}$ | $3.9193 \times 10^{-9}$ | $-1.4034 \times 10^{-7}$ | $-1.3363 \times 10^{-9}$ |
| $\Delta \theta_h$ | 1 | $-8.625 \times 10^{-10}$ | $9.4875 \times 10^{-8}$ | $1.3359 \times 10^{-8}$ | $3.9193 \times 10^{-9}$ | $-1.4034 \times 10^{-7}$ | $-1.0201 \times 10^{-9}$ |
| $\Delta \theta_p$ | 1.5 | $-2.8980 \times 10^{-8}$ | $7.4520 \times 10^{-9}$ | $1.1334 \times 10^{-8}$ | $3.9193 \times 10^{-9}$ | $-1.3844 \times 10^{-7}$ | $-1.3437 \times 10^{-9}$ |
| $\Delta \theta_p$ | 1.6 | $-8.6250 \times 10^{-10}$ | $9.4875 \times 10^{-8}$ | $1.3359 \times 10^{-8}$ | $3.9193 \times 10^{-9}$ | $-1.3844 \times 10^{-7}$ | $-7.4980 \times 10^{-10}$ |

In Tab. 3, the $D_{ii}$, $D_{pp}$, $C_{io}$, $C_{pi}$, and $C_{op}$ are the second-order nonlinear error model coefficients of the accelerometer according to Eq. (3). $D_{ii}$, $D_{pp}$ are the axial quadratic coefficient drift coefficients, $C_{io}$, $C_{pi}$, and $C_{op}$ are the cross-coupled drift coefficients. $T_{iii}$ is the third-order error model coefficient. The main errors listed in Tab. 2 impact the high-order coefficients. In real calibrations, the errors should be measured precisely and compensated for. The errors from Tab 3 all impact the $C_{op}$, so its precise calibration is more complicated, and more errors are needed to be compensated for. The $\Delta R_{w}$ is taken as an example to highlight the influence of a single error on the different error model coefficients. The influence of $\Delta R_{w}$ on each high-order error model coefficient is discussed in Fig. 3, which shows that $\Delta R_{w}$ significantly influences $C_{op}$, and a large compensation is required for real calibrations.

**TABLE 4. Impact of the general uncertainty on the high-order error model coefficients of the accelerometer**

| General uncertainty $(10^{-6})$ | $D_{ii} (v/g^2)$ | $D_{pp} (v/g^2)$ | $C_{io} (v/g^2)$ | $C_{pi} (v/g^2)$ | $C_{op} (v/g^2)$ | $T_{iii} (v/g^3)$ |
|-------------------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| 5.28                          | $-1.8113 \times 10^{-7}$ | $-7.7626 \times 10^{-8}$ | $-3.8044 \times 10^{-4}$ | $3.9193 \times 10^{-9}$ | $-1.2422 \times 10^{-6}$ | $1.9289 \times 10^{-8}$ |
| 3.14                          | $-1.8113 \times 10^{-7}$ | $-7.7626 \times 10^{-8}$ | $-3.8044 \times 10^{-4}$ | $3.9193 \times 10^{-9}$ | $-1.2422 \times 10^{-6}$ | $1.5598 \times 10^{-8}$ |
| 1.6                           | $-1.8113 \times 10^{-7}$ | $-7.7626 \times 10^{-8}$ | $-3.8044 \times 10^{-4}$ | $3.9193 \times 10^{-9}$ | $-1.2422 \times 10^{-6}$ | $1.3683 \times 10^{-8}$ |

**FIG. 3 Impacts of $\Delta R_{w}$ on the high-order coefficients.**

Simulink models are performed to illustrate how the general uncertainty impacts the calibration accuracy of the high-order coefficients. Tab. 4 shows how three different general uncertainties affect the calibration accuracy. As the general uncertainty varies, there is a greater impact on the $T_{iii}$. To illustrate the impact on $T_{iii}$, the general uncertainty is assumed to vary from $3 \times 10^{-6}$ to $4 \times 10^{-3}$, and the impacts of the different general uncertainties on the calibration accuracy of $T_{iii}$ is shown in Fig. 4.
It is seen from Fig. 4, that as the general uncertainty increases, the impact on the calibration accuracy of $T_{iii}$ becomes greater with an approximately linear increase. Under the same general uncertainty, different input specific forces have various impacts on the calibration accuracy. Tab. 2 indicates that the general uncertainty is $5.28 \times 10^{-6}$, and the associated impact on the calibration accuracy is shown in Tab. 4 under a 20 g input specific force. As the input specific force varies, the associated impact also varies. Taking $T_{iii}$ as an example, the impact of different input specific forces on the calibration accuracy of $T_{iii}$ is as shown in Fig. 5.

**FIG. 4 Variations in the impact of the general uncertainty on the calibration accuracy of $T_{iii}$.**

**FIG. 5 Impact of different input specific forces on the calibration accuracy of $T_{iii}$.**

Fig. 5 shows that the general uncertainty is $5.28 \times 10^{-6}$ as the input specific force changes from 5 g to 30 g. The larger the input specific force, the smaller the effect on the calibration accuracy of $T_{iii}$. And in 30 g, the effect on the calibration accuracy of $T_{iii}$ is smaller than $1 \times 10^{-8}$.

This type of influence gradually decays and tends to become stable. It is concluded that to lower the impact of the general uncertainty of the centrifuge, the input specific force should be larger under the permitted conditions. A small specific force may significantly impact the calibration accuracy, and more precise error compensation should be considered to ensure calibration accuracy.

**7. Conclusion**

The general uncertainty of a boomed centrifuge is given based on the contributions of the considered errors to the input specific force to guide the design of centrifuges with different calibration accuracies. The impact of general uncertainty on calibration accuracy of error model coefficients is studied. The main tasks are listed as follows:
1. The calculation method for the general uncertainty of a precision centrifuge is given. Three kinds of error sources (displacement errors, angular velocity errors, and attitude errors) that affect the centrifuge acceleration accuracy are analyzed for a quartz accelerometer calibration on a boomed centrifuge.

2. Combined with the given tolerance towards each centrifuge error, their contribution to the general uncertainty was calculated to guide the method and improve the calibration accuracy of the accelerometer coefficients. Synthesizing the errors allowed determining the size of the general uncertainty. Thus, the requirements for the pre-design can be satisfied by considering the tolerances towards the centrifuge errors, which lays a theoretical foundation to design and manufacture the centrifuge.

3. From real calibration analyses, all errors impact the high-order error model coefficients of the accelerometer, which should be measured precisely and compensated for. Neally all the considered errors significantly impact $C_{op}$.

4. The general uncertainty impacts the calibration accuracy of high-order coefficients. As the general uncertainty varies, there is a greater impact on the $T_{ii}$. To reduce the impact of the general uncertainty of the centrifuge, the input specific force should be larger under the permitted conditions.

**Acknowledgement**

This work was funded by and the Natural Science Foundation of Tianjin City (18JCQNJC74700) and the Scientific Research Project of Tianjin Educational Committee (2018KJ103).

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