Fermion Masses and Mixing Angles from $SU(3)$ Family Symmetry and Unification

S. F. King† and G. G. Ross‡
†Department of Physics and Astronomy, University of Southampton,
Southampton, SO17 1BJ, U.K.
‡CERN, 1211 Geneva 23, Switzerland

and
Department of Physics, Theoretical Physics, University of Oxford,
1 Keble Road, Oxford OX1 3NP, U.K.

Abstract
We develop a bottom-up approach to constructing a theory of fermion masses and mixing angles based on the gauge group $SU(3) \times G$ where $SU(3)$ is a family symmetry and $G$ contains a unified group such as $SO(10)$ or its Pati-Salam subgroup, together with other discrete symmetries. We construct a realistic model and show that it can provide an excellent description of quark and lepton masses and mixing angles, including almost maximal atmospheric mixing and the LMA MSW solar neutrino solution. We predict a neutrino mixing angle $\theta_{13}$ near the current limit. The model provides the basis for a new solution to the flavour problem with a characteristic soft SUSY breaking mass spectrum.

Dedicated to Ian I. Kogan
1 Introduction

The flavour problem, the problem of the origin of families and of fermion masses and mixing angles, has been a longstanding unanswered question facing the Standard Model, and remains a powerful motivation for going beyond it [1]. The recent progress in neutrino physics in fact demands new physics beyond the Standard Model, and implies that any solution to the flavour problem must also include (almost) maximal atmospheric neutrino mixing, and large mixing angle (LMA) MSW solar neutrino mixing [2]. Such a spectrum can be readily reproduced from the see-saw mechanism in a very natural way using right-handed neutrino dominance [3], but the necessary conditions required for this mechanism to work can only be understood in terms of beyond Standard Model physics. On the other hand, these conditions provide powerful clues to the nature of the new physics, which may help to unlock the whole mystery of flavour.

It is clear that any hope of a understanding the flavour problem from present data is only going to be possible if the Yukawa matrices exhibit a high degree of symmetry. A recent phenomenological analysis shows that an excellent fit to all quark data is given by the approximately symmetric form of quark Yukawa matrices [4]

\[
Y^u \propto \begin{pmatrix} 0 & \epsilon^3 & O(\epsilon^3) \\ \epsilon^2 & 0 & O(\epsilon^2) \\ 0 & \epsilon & 1 \end{pmatrix}, \quad Y^d \propto \begin{pmatrix} 0 & 1.5\epsilon^3 & 0.4\epsilon^3 \\ \epsilon^2 & 0 & 1.3\epsilon^2 \\ 0 & \epsilon & 1 \end{pmatrix} \tag{1}
\]

where the expansion parameters \(\epsilon\) and \(\bar{\epsilon}\) are given by

\[
\epsilon \approx 0.05, \quad \bar{\epsilon} \approx 0.15. \tag{2}
\]

In [5] we showed how Yukawa matrices with the structure of Eq.1, could originate from an \(SU(3)\) family symmetry. \(^1\) The \(SU(3)\) family symmetry constrains the leading order terms to have equal coefficients [5]. We also showed that, due to the see-saw mechanism [7], the neutrino Yukawa matrix \(Y^\nu\) could have a similar form to \(Y^u\) in Eq.1, providing that the heavy Majorana matrix \(M_{RR}\) has a strongly hierarchical form. In the explicit model presented [5] a new expansion parameter was invoked to describe the right-handed neutrino sector. The first right-handed neutrino was arranged to be light enough to be the dominant one, and the second one the leading subdominant one, corresponding to sequential dominance [3]. This implies that the atmospheric neutrino mixing angle is given by \(\tan \theta_{23} \approx Y^\nu_{21}/Y^\nu_{31}\), where this ratio is equal to unity at leading order due to the \(SU(3)\) symmetry. Similarly the solar neutrino angle is then given by \(\tan \theta_{12} \approx \sqrt{2}Y^\nu_{12}/(Y^\nu_{22} - Y^\nu_{32})\), where the leading order terms in the denominator cancel due to the \(SU(3)\) symmetry. We further proposed that the charged lepton Yukawa matrix \(Y^e\) has a similar form to \(Y^d\) in Eq.1, apart from a Georgi-Jarlskog factor of 3 premultiplying the \(\epsilon^2\) terms [8].

Although the above \(SU(3)\) family symmetry model is in many ways very attractive, the model presented in [5] has one major shortcoming: the proposed form of neutrino Yukawa matrix \(Y^\nu\) implies that LMA MSW solar solution cannot be reproduced. In this paper we shall construct a modified version in which the LMA MSW solution is natural. The difficulty in obtaining a large solar angle was due to the fact that \(Y^\nu_{(22,32)} \sim \epsilon^2\) are larger than \(Y^\nu_{12} \sim \epsilon^3\). In [9] it was shown that if a Grand Unified theory (GUT) such as \(SO(10)\) [10] is used to obtain the Georgi-Jarlskog

\(^1\)For reviews of \(SU(3)\) family symmetry with original references see for example [6].
factor, it simultaneously suppresses $Y_{\nu}^{(22,32)}$, permitting the LMA MSW solution. The basic idea is that the effective Yukawa couplings are generated by Froggatt-Nielsen diagrams which involve a Higgs field $\Sigma$ in the 45 of $SO(10)$ coupling the fermion line, as shown in Figure 1. The external lines are then left-handed fermions $\psi$ or charge conjugates of right-handed fermions $\psi^c$ belonging to the second or third family, the internal lines are corresponding “fermion messengers”, and $H$ in the 10 of $SO(10)$ contains the usual Higgs doublets. The left-handed fermion messengers $\chi, \bar{\chi}$ have a mass $M$, while the charge conjugates of the right-handed messengers $\chi^c, \bar{\chi}^c$ have mass $M'$. If $M' \ll M$, due to left-right $SO(10)$ breaking, then the second diagram (b) is expected to dominate. If, in addition, $\Sigma$ gets a vacuum expectation value (vev) in the hypercharge direction $Y$, so that its couplings to fermions are proportional to their hypercharge, then this results in the usual Georgi-Jarlskog factor of 3, since right-handed charged leptons have 3 times the charge and hypercharge of right-handed down quarks. In addition it leads to a suppressed coupling in the neutrino Yukawa matrix, since right-handed neutrinos have zero charge and hypercharge. This suppression then permits the LMA MSW solution.

![Froggatt Nielsen supergraphs generating fermion masses](image)

Figure 1: Froggatt Nielsen supergraphs generating fermion masses (from [9]).

$SO(10)$ unification has other important implications for the theory. It reduces the maximum possible family symmetry from $U(3)^6$ to $U(3)$, and implies that $\psi$ and $\psi^c$ must both transform as triplets under the family symmetry which we shall take to be the gauged $SU(3)$ subgroup of $U(3)$. This immediately implies that all fermion masses vanish in the limit of unbroken $SU(3)$. However in the context of supersymmetry the presence of $SU(3)$ can be seen as desirable since it helps to ensure that the sfermion masses are approximately degenerate as required by flavour changing neutral current phenomenology. Another important implication of $SO(10)$ is related to its spontaneous breaking. We shall assume that it is broken, at least partially, by Wilson line breaking which corresponds to a higher dimensional component of a higher dimensional gauge field developing a vev. The advantage of Wilson line breaking is that since gauge fields couple universally it generates a universal mass for states in a given representation. For instance if it generates the dominant breaking of $SU(2)_R$ it leads to three universal messenger mass scales: the right-handed up masses $M^u$, the right-handed down masses $M^d$, and the left-handed doublet masses $M_L$. This observation greatly increases the predictive power of the Froggatt-Nielsen approach since the messengers in a given representation have universal masses of order the compactification scale. This motivates the idea that the physics of flavour resides at the compactification scale, presumably not too far below the string scale, with Wilson line symmetry breaking playing an important rôle.

---

Footnote: In [9] this was characterized by $B - L + 2T_{3R}$; this is proportional to hypercharge.
In this paper we shall develop a bottom-up approach to constructing a theory of fermion masses and mixing angles based on the family unification gauge group $SU(3) \times G$. The bottom-up approach means that we shall work just below the string scale, where $G = SO(10)$ has been broken either in the $SU(5)$ or the Pati-Salam direction [11]. In practice we shall start from the Pati-Salam subgroup which avoids the problems of doublet-triplet splitting [12], but still allows us to exhibit the effects associated with left-right, $SU(2)_R$, and quark-lepton symmetry breaking, which mainly concern us here. The underlying unification implies several important differences to the approach followed in [5]. In particular a new symmetry is required to enforce the presence of the $\Sigma$ field in the second and third family operators. The new symmetry should also allows us to predict $M_{RR}$ in terms of the same expansion parameters that determines the Yukawa matrices, and not rely on a new expansion parameter.

The layout of the remainder of the paper is as follows. In Section 2 we discuss our general approach involving Family Symmetry and Unification, and introduce the basic framework of our model. In Section 3 we discuss a realistic model in some detail, specify the operators allowed by the symmetries, and discuss the Yukawa and Majorana matrices which result from a particular messenger sector. In Section 4 we justify the spontaneous symmetry breaking of the symmetries assumed in the model. In Section 5 we consider some supersymmetric aspects theory, in particular for soft masses and the rôle of D-terms. In section 6 we discuss the phenomenology of the model, including the neutrino masses and mixing angles. Section 7 concludes the paper.

2 Family Symmetry and Unification

In this section we shall introduce the ideas of family symmetry and unification including the necessary ingredients needed to solve the flavour problem. We shall construct a supersymmetric version of the theory but for simplicity of presentation, we will treat the supersymmetric structure as implicit.

In a theory of family symmetry and unification based on gauged $SU(3) \times SO(10)$ all quarks and leptons originate from a single representation $\psi_i \sim (3, 16)$. The Higgs doublets are contained in the $H \sim (1, 10)$, while $\Sigma \sim (1, 45)$. In our bottom-up approach we do not construct the fully unified Grand Unified or string model but start with the models defined slightly below the GUT or string scale where the surviving maximal subgroups of $SO(10)$ may be either $SU(5) \times U(1)_X$, or the Pati-Salam subgroup $G_{PS} = SU(4)_{PS} \times SU(2)_L \times SU(2)_R$. In this paper we shall focus on the Pati-Salam symmetry breaking direction which is the most predictive case. Pati-Salam breaking down to the Standard Model gauge group is achieved by a combination of Wilson line breaking, which is dominantly responsible for $SU(2)_R$ breaking, and Higgs breaking due to a field $\Sigma$ which is an adjoint of both $SU(4)_{PS}$ and $SU(2)_R$, and will play the same rôle as the adjoint 45 of $SO(10)$ discussed above, from which it may originate. The full flavour symmetry of the model must involve some additional symmetry under which $\Sigma$ transforms, and is responsible for $\Sigma$ appearing in the leading operators in the 22,23,32 positions of the Yukawa matrices. The symmetry must allow all the necessary leading operators, while suppressing all unwanted operators, including all subleading operators which are not required. It must also lead

---

3In this paper we emphasise the possibility that the underlying theory has an underlying $G = SO(10)$ symmetry. However the approach is readily adapted to other underlying grand unified schemes.

4If the symmetry is broken by Wilson lines the doublet-triplet splitting problem can be elegantly solved [13].
Table 1: Transformation of the superfields under the $SU(3)$ family, Pati-Salam and $R \times Z_2 \times U(1)$ symmetries which restrict the form of the mass matrices for three representative examples. The continuous R-symmetry may be alternatively be replaced by a discrete $Z_{2R}$ symmetry. Also shown in the last three columns is the transformation under a $Z_5 \times Z_3 \times Z'_2$ subgroup of the $U(1)$ which is sufficient to ensure a phenomenologically viable pattern of couplings. We only display the fields relevant for generating fermion mass and spontaneous symmetry breaking.

to an acceptable heavy Majorana matrix $M_{RR}$.

The explicit bottom-up models we shall construct are based on $SU(3)$ family symmetry commuting with Pati-Salam symmetry, $SU(3) \times G_{PS}$. The transformation properties of the left-handed quarks and leptons $\psi_i$, the left-handed charge conjugates of the right-handed quarks and leptons $\psi^c_i$, the Higgs doublets $H$ and the $\Sigma$ field under the gauge group $SU(3) \times G_{PS}$ are given in Table 1. Assuming the Pati-Salam symmetry to start with has the advantage that it explicitly exhibits $SU(4)_{PS}$ quark-lepton and $SU(2)_R$ isospin symmetry, allowing Georgi-Jarlskog factors to be generated and isospin breaking to be controlled, while avoiding the Higgs doublet-triplet splitting problem [12]. The $SU(4)_{PS}$ symmetry also provides a welcome restriction of the messenger masses, providing a link between the up-quarks and neutrinos. The fields $\theta$ and $\bar{\theta}$ carry lepton number 1 and $-1$ respectively. They acquire vevs and break lepton number giving rise to the Majorana masses for the neutrino components of $\psi^c$.

The adjoint $\Sigma$ field develops vevs in the $SU(4)_{PS} \times SU(2)_R$ direction which preserves the hypercharge generator $Y = T_{3R} + (B - L)/2$, and implies that any coupling of the $\Sigma$ to a fermion and a messenger such as $\Sigma^{a \alpha}_{b \beta} \psi^c_{a \alpha} \chi^{b \beta}$, where the $SU(2)_R$ and $SU(4)_{PS}$ indices have been displayed explicitly, is proportional to the hypercharge $Y$ of the particular fermion component of $\psi^c$ times the vev $\sigma$.

To build a viable model we also need spontaneous breaking of the family symmetry

$$SU(3) \longrightarrow SU(2) \longrightarrow \text{Nothing}$$

To achieve this symmetry breaking we introduce additional Higgs fields $\phi_3, \bar{\phi}_3, \phi_{23}$ and $\bar{\phi}_{23}$ in the representations given in Table 1. The largeness of the third family fermion masses implies that $SU(3)$ must be strongly broken by new Higgs antitriplet fields $\phi_3$ which develop a vev in the third $SU(3)$ component $< \phi_3 >^T = (0, 0, a_3)$ as in [5]. However, for reasons discussed later,
we assume that $\phi_3^i$ transforms under $SU(2)_R$ as $3 \oplus 1$ rather than being $SU(2)_R$ singlets as we assumed in [5], and develops vevs in the $SU(3) \times SU(2)_R$ directions

$$< \phi_3 > = \langle \phi_3^a \rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} a_u^a \\ 0 \\ a_d^a \end{pmatrix}. \quad (4)$$

The symmetry breaking also involves the $SU(3)$ antitriplets $\phi_{23}$ which develop vevs [5]

$$< \phi_{23} > = \begin{pmatrix} 0 \\ 1 \\ e^{i\theta} \end{pmatrix} b, \quad (5)$$

where, as in [5], vacuum alignment ensures that the vevs are aligned in the 23 direction. Due to D-flatness there must also be accompanying Higgs triplets such as $\phi_{23}$ which develop vevs [5]

$$< \phi_{23} > = \begin{pmatrix} 0 \\ 1 \\ e^{-i\theta} \end{pmatrix} e^{i\phi}. \quad (6)$$

In Section 4 we will show how this pattern can be achieved through the introduction of the additional triplet field $\phi_2$ given in Table 1. With the spectrum shown in this Table there are residual $SU(3)$ and $U(1)$ anomalies but no mixed anomalies involving the Standard Model gauge group. These anomalies can be cancelled by the addition of Standard Model singlet fields all of which can acquire a mass at the scale of breaking of $SU(3)$. We do not list these fields here as they play no role in the low energy theory but note that in a more unified model such anomaly cancellation can happen in an elegant manner [14].

### 3 A Realistic Model

#### 3.1 Operators and Additional Symmetries

In building a phenomenologically viable scheme it is necessary to constrain the allowed Yukawa couplings through additional symmetries. There is considerable freedom in implementing such symmetries, the resultant models differing in their detailed phenomenology. In this paper we present a simple example in which the $SU(3)$ family symmetry is augmented by a $Z_2 \times U(1)$ gauge symmetry. It will ensure that the quark and lepton Dirac masses have an acceptable form and also order the Majorana mass matrix of the right handed neutrinos so that the see-saw mechanism gives to large mixing angles. The assignment of the $Z_2 \times U(1)$ charges is shown in Table 1. The symmetries of the model are completed through the addition of an R-symmetry (or a discrete version of it $Z_{2R}$).

In practice it is not necessary that the full $U(1)$ symmetry be present and a discrete subgroup can be sufficient to limit the allowed Yukawa couplings. For example, the discrete group $Z_5 \times Z_3 \times Z'_2$ with charges given in Table 1 gives the same leading operators discussed in the next Section and hence approximately the same mass matrices. Note that the charges under the discrete symmetry look simpler than those for the $U(1)$ showing that is not necessary to have an exotic choice of charges to achieve a realistic model.
The leading operators allowed by the symmetries are

\[
P_{\text{Yuk}} \sim \frac{1}{M^2} \psi_i \bar{\phi}^i_3 \psi_j \phi^j_3 H
\]
\[
+ \frac{1}{M^3} \psi_i \bar{\phi}^i_{23} \phi^j_3 \phi^j_{23} H
\]
\[
+ \frac{1}{M^5} \left( (e^{ijk} \psi_i \bar{\phi}^i_{23} \bar{\phi}^j_{23}) \bar{\psi}_l \phi^l_{23} \right) H(\phi^m_{23} \bar{\phi}^m_{23})
\]
\[
+ \frac{1}{M^3} (e^{ijk} \psi_i \bar{\phi}^i_{23} \psi_k) H(\phi^l_{23} \bar{\phi}^m_{23}) H(\phi^m_{23} \bar{\phi}^m_{23})
\]
\[
+ \frac{1}{M^4} \left( \psi_i \phi^i_{23} \psi_j \phi^j_{23} + \psi_i \phi^i_3 \psi_j \phi^j_{23} \right) H.S
\]

\[
P_{\text{Maj}} \sim \frac{1}{M} \psi_i \theta^i \nu_j
\]
\[
+ \frac{1}{M^{11}} \psi_i \phi^i_{23} \nu_j \phi^j_{23} (\theta^k \phi^k_{23} \phi^m_{23})^3
\]
\[
+ \frac{1}{M^{13}} (e^{ijk} \psi_i \phi^i_{23} \psi_k \phi^k_{23})^2 (\theta^k \phi^k_{23} \phi^m_{23}) (\phi^m_{23} \phi^m_{23})
\]

where, as discussed below, the operator mass scales, generically denoted by \( M \) may differ and we have suppressed couplings of \( O(1) \). The field \( S \) is involved in symmetry breaking as discussed in Section 4. Its quantum numbers are given in Table 2. It acquires a vev of \( O(\phi_{23} \bar{\phi}_{23}) \).

### 3.2 Messengers and the \((2,3)\) Yukawa block

The leading Yukawa operators which contribute to the \((2,3)\) block of the Yukawa matrices are given in Eqs.7 and 8. These operators arise from Froggat-Nielsen diagrams similar to Figure 1, but generalized to include insertions of the \( \phi_3, \phi_{23} \) fields. \( M \) represents the right-handed up and down messenger mass scales \( M^{u,d} \), corresponding to the dominance of diagram (b), which applies if \( M < M^L \) where \( M^L \) represents the left-handed messenger mass scale. We shall not specify the messenger sector explicitly, but characterize it by the messenger mass scales

\[
M^d \approx \frac{1}{3} M^u \ll M^L.
\]

Such a universal structure is to be expected in theories with Wilson line breaking in which the breaking is due to the \((4D)\) scalar component of a higher dimension gauge field because it couples universally to fields in the same representation of gauge group factors left unbroken by the Wilson line. The Wilson line breaking is associated with the compactification and so the splitting induced is naturally of order the compactification scale. Thus, if Wilson line breaking is responsible for breaking \( SU(2)_R \), the messenger states (Kaluzo-Klein modes or vectorlike states obtaining mass on compactification) must have masses of order the compactification scale.

Given that \( SU(4)_{PS} \) remains after compactification, its subsequent breaking will be a small effect so that the right-handed lepton messenger masses are \( M^\nu \approx M^u \), and \( M^e \approx M^d \). The splitting of the messenger mass scales relies on left-right and \( SU(2)_R \) breaking effects which we shall assume to be due to the Wilson line symmetry breaking mechanism. Eq.15 implies that diagrams of type (b) in Figure 1 dominate, and the expansion parameters associated with \( \phi_{23} \)
are then generated as in [5]

\[ \epsilon \equiv \frac{b}{M^u}, \quad \bar{\epsilon} \equiv \frac{b}{M^d} \] (16)

Unlike the previous model [5], we shall construct a model in which the \( \phi_3 \) vev \( a_3 \) is less than the messenger mass scale \( M \). The reason for this is twofold. Firstly, an underlying \( SO(10) \) leads us to consider fermion messengers, and if \( a_3 > M \) this then implies an undesirably massive third family fermion \( \psi_3 \) from the coupling \( \phi_3 \psi_3 \bar{\chi} \), and a light fermion messenger \( \chi \). Secondly, wavefunction insertions of the invariant operator \( \phi_3 \phi_3^\dagger / M^2 \) on an third family fermion propagator can spoil the perturbative expansion if \( a_3 > M \). Therefore we shall assume here that \( a_3 < M \). If \( \phi_3 \) were a \( SU(2)_R \) singlet as in [5] then Eq.15 would imply that the top quark Yukawa coupling is much smaller than the bottom quark Yukawa coupling by a factor of \( 1/9 \). This explains why \( \phi_3 \) cannot be a \( SU(2)_R \) singlet. For the case that \( \phi_3 \) transforms as \( 2 \times 2 \) under \( SU(2)_R \) it may acquire vevs \( a^u_3, a^d_3 \) in the up and down directions. Then with \( a^u_3/M^u \approx a^d_3/M^d < 1 \) we have comparable top and bottom Yukawa couplings, as required. For definiteness we shall consider the case that

\[ \frac{a^u_3}{M^u} = \frac{a^d_3}{M^d} = \sqrt{\epsilon}. \] (17)

It remains to specify the expansion parameter associated with \( \sigma \), the vev of \( \Sigma \). For phenomenological reasons we take it to be

\[ \frac{\sigma Y(d)}{M^d} = \bar{\epsilon} \] (18)

where \( Y(d) = 1/3 \) is the hypercharge of \( d^c \). From Eqs.15,18, we find

\[ \frac{\sigma Y(u)}{M^u} = -\frac{2}{3} \bar{\epsilon} \] (19)

where \( Y(u) = -2/3 \) is the hypercharge of \( u^c \).

The operators in Eqs.7-14 with the expansion parameters in Eqs.16, 17, 18, 19, and the vevs in Eqs.4, 5 lead to the approximate form of the quark Yukawa matrices for the \( (2, 3) \) block given by:

\[ Y^u \approx \begin{pmatrix} \epsilon^2 \left(\frac{2}{3}\right) & \epsilon^2 \left(\frac{2}{3}\right) & 1 \\ \epsilon^2 \left(\frac{2}{3}\right) & 1 & \epsilon \end{pmatrix} \bar{\epsilon}, \quad Y^d \approx \begin{pmatrix} \epsilon^2 & 1 \\ \epsilon^2 & 1 \end{pmatrix} \bar{\epsilon} \] (20)

which is of the form in Eq.1.

The charged lepton Yukawa matrix \( Y^e \) has a similar form to \( Y^d \) since the charged lepton operators are generated by messengers with the quantum numbers of \( e^c \), with the same messenger mass scale as for \( d^c \) messengers, \( M^e = M^d \), due to the \( SU(4)_{PS} \) symmetry. However due to \( \Sigma \) the 22, 23, 32 elements of \( Y^e \) are multiplied by the Georgi-Jarlskog factor of \( Y(e)/Y(d) = 3 \), where \( Y(e) \) is the hypercharge of \( e^c \), giving

\[ Y^e \approx \begin{pmatrix} \epsilon^2(3) & 1 \\ \epsilon^2(3) & 1 \end{pmatrix} \bar{\epsilon}. \] (21)

The above results apply in the limit that the Froggatt-Nielsen diagrams are dominated by the right-handed messengers. In this limit the neutrino Yukawa matrix \( Y^\nu \) has zeroes in the 22, 23, 32 positions due to the fact that these elements are proportional to the hypercharge of
the right-handed neutrino which is zero. This leads to the desired suppression of these elements of \( Y^\nu \). To characterise this suppression we define the expansion parameter in the left-handed neutrino sector

\[
\frac{\sigma Y(\nu_L)}{M_L} \equiv -\alpha \bar{\epsilon}.
\]

(22)

Then the 22, 23, 32 elements of \( Y^\nu \) are of order \( \alpha \epsilon^2 \), once the overall factor of \( \bar{\epsilon} \) has been factored out,

\[
Y^\nu \approx \begin{pmatrix}
\epsilon^2(-\alpha) & \epsilon^2(-\alpha) \\
\epsilon^2(-\alpha) & 1
\end{pmatrix} \bar{\epsilon}.
\]

(23)

### 3.3 The complete Yukawa Matrices

The leading elements in the 12, 13, 21, 31 positions contain contributions from two different leading order operators, namely those in Eqs. 9 and 10. These contributions depend on the vevs of \( \phi_{23} \) in Eq.5 and \( \bar{\phi}_{23} \) in Eq.6 with the 12, 13 contributions being of order \( \epsilon^3 \) in the up and neutrino sector, and \( \epsilon^3 \) in the down and charged lepton sector, each multiplied by an overall factor of \( \bar{\epsilon} \). However, due to the antisymmetric \( SU(3) \) invariant, the relative coupling of the (1, 2) and (1, 3) elements have opposite signs. Note that in a full \( SO(10) \) theory the operators in Eq.10 are forbidden due to antisymmetry since \( \psi \) and \( \psi^c \) are unified into a single 16 representation. However, since \( SO(10) \) breaking effects are required in any case, we must allow for the presence of such operators. The sum of the contributions gives a factor \( g + h + h' \) in the 12 entry and \( g - h \) in the 13 entries of \( Y^d, Y^e \), a factor \( g + h/3 + h'/3 \) in the 12 entry and \( g - h/3 \) in the 13 entries of \( Y^u, Y^\nu \), where we allow for the fact that the \( SO(10) \) symmetry breaking effects which are responsible for the existence of the second term are controlled by the same messenger masses \( M^u, M^d \) as in Eq.15. The corresponding operators in the 21, 31 positions have an independent coefficient \( g' \) due to the contribution from the two operators in Eq. 9.

The operators in Eq.11 give an important sub-leading contribution to the 23, 32 elements of the Yukawa matrices. This, together with the structure discussed above, and allowing for the corrections due to wavefunction insertions of the invariant operator \( \phi_3 \phi_3^1 / M^2 \sim \bar{\epsilon} \) on a third family fermion leg, gives the final form of the Yukawa matrices

\[
Y^u \approx \begin{pmatrix}
0 & \epsilon^3(g + \frac{h}{3} + \frac{h'}{3}) & \epsilon^3(g - \frac{h}{3})(1 + O(\bar{\epsilon})) \\
\epsilon^3(g' - \frac{h}{3} - \frac{h'}{3})(1 + O(\bar{\epsilon})) & \epsilon^2(-\frac{2}{3}) & \epsilon^2(-\frac{2}{3}) + c\epsilon^3\bar{\epsilon}^{\frac{1}{2}} \\
\epsilon^3(g' + \frac{h}{3})(1 + O(\bar{\epsilon})) & \epsilon^2(\frac{2}{3}) + c\epsilon^3\bar{\epsilon}^{\frac{1}{2}} & 1 + O(\bar{\epsilon})
\end{pmatrix} \bar{\epsilon},
\]

(24)

\[
Y^d \approx \begin{pmatrix}
0 & \epsilon^3(g - h - h') & \epsilon^3(g - h)(1 + O(\bar{\epsilon})) \\
\epsilon^3(g' - h - h')(1 + O(\bar{\epsilon})) & \epsilon^2 & \epsilon^2 + c\epsilon^3\bar{\epsilon}^{\frac{1}{2}} \\
\epsilon^3(g' + h)(1 + O(\bar{\epsilon})) & \epsilon^2 + c\epsilon^3\bar{\epsilon}^{\frac{1}{2}} & 1 + O(\bar{\epsilon})
\end{pmatrix} \bar{\epsilon},
\]

(25)

\[
Y^e \approx \begin{pmatrix}
0 & \epsilon^3(g + h + h') & \epsilon^3(g - h)(1 + O(\bar{\epsilon})) \\
\epsilon^3(g' - h - h')(1 + O(\bar{\epsilon})) & \epsilon^2(3) & \epsilon^2(3) + c\epsilon^3\bar{\epsilon}^{\frac{1}{2}} \\
\epsilon^3(g' + h)(1 + O(\bar{\epsilon})) & \epsilon^2(3) + c\epsilon^3\bar{\epsilon}^{\frac{1}{2}} & 1 + O(\bar{\epsilon})
\end{pmatrix} \bar{\epsilon},
\]

(26)

\[
Y^\nu \approx \begin{pmatrix}
0 & \epsilon^3(g + \frac{h}{3} + \frac{h'}{3}) & \epsilon^3(g - \frac{h}{3})(1 + O(\bar{\epsilon})) \\
\epsilon^3(g' - \frac{h}{3} - \frac{h'}{3})(1 + O(\bar{\epsilon})) & \epsilon^2(-\alpha) & \epsilon^2(-\alpha) + c\epsilon^3\bar{\epsilon}^{\frac{1}{2}} \\
\epsilon^3(g' + \frac{h}{3})(1 + O(\bar{\epsilon})) & \epsilon^2(-\alpha) + c\epsilon^3\bar{\epsilon}^{\frac{1}{2}} & 1 + O(\bar{\epsilon})
\end{pmatrix} \bar{\epsilon}.
\]

(27)
3.4 Heavy Majorana Masses

The leading heavy right-handed neutrino Majorana mass arises from the operator of Eq.12 where the θ fields defined in Table 1 are further Higgs superfields whose vevs break lepton number. It is spontaneously broken when the right-handed sneutrinos develop vevs in the third $SU(3)$ direction. This operator gives the Majorana mass,

$$M_3 \approx <\theta >^2 / M,$$

(28)

to the third family, where $M^\nu = M^u$ is the same messenger mass scale as in the up sector due to $SU(4)_{PS}$. Operators involving Σ do not contribute since it does not couple to right-handed neutrinos which have zero hypercharge.

The operator in Eq.13 gives Majorana mass, $M_2 = \epsilon^6 \bar{\epsilon}^2 < \theta >^2 / M$, to the second family, and the operator in Eq.14 gives Majorana mass, $M_1 = \epsilon^6 \bar{\epsilon}^3 < \theta >^2 / M$, to the first family, giving the final form

$$M_{RR} \approx \begin{pmatrix} \epsilon^6 \bar{\epsilon}^3 & 0 & 0 \\ 0 & \epsilon^6 \bar{\epsilon}^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} M_3.$$  

(29)

4 Spontaneous Symmetry Breaking

The pattern of $SU(3)$ family symmetry breaking explored here is as in Eq.3. We start with a discussion of the first stage of breaking, $SU(3) \rightarrow SU(2)$, induced by the vevs of the $\phi_3$, $\bar{\phi}_3$ Higgs. The structure of the effective potential is very sensitive to the field content as well as the additional symmetries. In the context of the model of interest here the additional symmetry is $U(1)$ or some discrete subgroup of it which still maintains the structure of the leading operators given in Eqs.7-14. To illustrate the mechanisms that can lead to a phenomenologically acceptable pattern of symmetry breaking we consider a simple case in which that discrete subgroup of $U(1)$ is the $Z_5 \times Z_3 \times Z_2$ symmetry introduced in the last two columns of Table 1.

As we have discussed it is necessary for there to be a hierarchy in the vevs of the fields $\phi_3$, $\bar{\phi}_3$ and $\phi_{23}$, $\bar{\phi}_{23}$. One way such an hierarchy can develop is through radiative breaking in which, due to radiative corrections, the running mass squared of a field becomes negative at some scale, triggering a vev close to this scale. Gauge interactions increase the mass squared while Yukawa interactions decrease it so it is likely that the field undergoing radiative breaking has a reduced gauge symmetry. For this reason we suppose that a $SO(10) \times SU(3) \times Z_{2R}$ singlet field $S$ acquires a vev due to (unspecified) Yukawa interactions\(^5\). Its charge is given in Table 2. We expect the symmetry breaking will be communicated to the other fields of the theory via heavy messenger fields. Due to the $Z_{2R}$ symmetry the superpotential does not contain terms involving the $\phi$ superfields on their own. The only way the superpotential will generate a potential for these fields is if there are additional fields carrying $Z_{2R}$ charge 2. Allowing for such a $SO(10) \times SU(3)$ singlet field $U$ carrying the $Z_5$ charge as in Table 2 we find the relevant superpotential term $P_1$ given by

$$P_1 = U((\phi_{23}\bar{\phi}_{23})^2 + S^2)$$

(30)

\(^5\)It is easy to add a Yukawa interaction involving $X$ and additional fields to drive radiative breaking. When $X$ acquires a vev these additional field acquire a large mass and need play no role in low energy phenomenology.
Table 2: The charges of the messenger sector fields communicating symmetry breaking to the fields in Table 1.

The potential corresponding to the $|F_U|^2$ term triggers a vev $-<S^2> = b^4$ for the combination $(\phi_{23}\bar{\phi}_{23})^2$, where $b$ was defined in Eq.5. Note that in Eq.30 and in the following equations we have not written the Yukawa couplings which are expected to be of $O(1)$ and do not change to overall ordering in terms of the expansion parameter of the mass matrix. However they are expected to be complex and can generate the phases as in Eqs.5,6 needed for CP violation and the precise description of the quark masses. Given that the magnitude of the phases are not determined by the symmetries of the model we do not write them explicitly here. The vacuum alignment of $\phi_{23}$ will be discussed below.

In the case of $\phi_3$ and $\bar{\phi}_3$ there are two possibilities:

(i) The first employs the same mechanism as used above, triggering the vevs by a $SO(10) \times SU(3) \times Z_2$ singlet field, $T$, which also acquires a vev through radiative breaking. The superpotential

$$P_2 = V(\phi_3\bar{\phi}_3 + T)$$

then drives $<\phi_3\bar{\phi}_3> = -T$. Finally the relative magnitudes of the vacuum expectation values of $\phi_3$ are fixed by the following superpotential

$$P_3 = W S^2 \phi_3\bar{\phi}_3$$

which, once the messenger mass scales of the operator are taken into account, leads to a potential proportional to a sum of squares $(a_3^u/M^u)^2 + (a_3^d/M^d)^2$, where the vevs were defined in Eq.4. The potential is minimised by $a_3^u/M^u \simeq a_3^d/M^d$.

(ii) The second possibility applies if there is only a discrete subgroup of the $U(1)$ in which case, in the absence of the $T$ field, $P_2$ takes the form

$$P_2 = V\phi_3\bar{\phi}_3(1 + (\phi_3\bar{\phi}_3)^5 + ...)$$

The resulting potential has a minimum for $<\phi_3\bar{\phi}_3> = O(1)$ corresponding to the vanishing of the term in brackets which is a polynomial in $(\phi_3\bar{\phi}_3)^5$. In both cases, when combined with the potential from $P_3$ we can generate $<\phi_3^u/M^u> \simeq <\phi_3^d/M^d> = \sqrt{\epsilon} = O(1)$, as assumed Eq.17.

There remains the question of the relative vacuum alignment of $\phi_3$ and $\phi_{23}$. This is readily done in the manner proposed in [5] through the superpotential

$$P_4 = X \phi_3\bar{\phi}_2 + [Y(\phi_{23}\bar{\phi}_{23}\bar{\phi}_3 + (\phi_3\bar{\phi}_3)(\phi_3\bar{\phi}_{23})^2)(\phi_{23}\bar{\phi}_{23})]$$

| Field | SU(3) | SU(4) | SU(2)_L | SU(2)_R | R | U(1) | Z_5 | Z_3 | Z_2 |
|-------|-------|-------|---------|---------|---|------|-----|-----|-----|
| S     | 1     | 1     | 1       | 1       | 0 | -1   | 1   | -1  | -1  |
| T     | 1     | 1     | 1       | 1       | 0 | +6   | 1   | -6  | -1  |
| U     | 1     | 1     | 1       | 1       | 2 | -2   | 2   | -2  | -2  |
| V     | 1     | 1     | 1       | 1       | 2 | +6   | 1   | +6  | -1  |
| W     | 1     | 1     | 1       | 1       | 2 | -2   | 1   | -2  | 1   |
| X     | 1     | 1     | 1       | 1       | 2 | -1   | 1   | -1  | 1   |
| Y     | 1     | 1     | 1       | 1       | 2 | -7   | 2   | 7   | -1  |
| A     | 1     | 1     | 1       | 1       | 2 | -7   | 1   | 7   | 1   |
| \Sigma_X | 1 | 15 | 1 | 3 | 2 | -1 | 1 | 2 | 1 |
| \theta' | \bar{3} | 4 | 1 | 2 | 2 | -1 | 1 | -1 | 1 |
where the $SO(10) \times SU(3)$ singlet fields $X, Y$ have the discrete symmetry charges given in Table 2. The potential from $|F_X|^2$ forces $\phi_3$ to be orthogonal to $\overline{\phi_2}$ while the potential following from $|F_Y|^2$ requires that the vev of $\phi_{23,2}\phi_{23,3}$ is non-zero. Including soft and D-terms and minimising the potential then leads to the vacuum alignment of Eqs. 5,6 in the manner discussed in [5]. The alignment is due to the underlying $SU(3)$ symmetry which requires the soft mass terms of the triplet components to be degenerate.

Finally we consider the $SO(10)$ breaking. As discussed above, gauge invariant combinations of fields carrying the same discrete quantum numbers can readily acquire vevs of the same magnitude. Thus, with the introduction of an additional messenger field, we can readily have $<Tr(\Sigma^2)> \simeq <\phi_3 \overline{\phi_3}>$. Taking account of the uncertainty in the messenger mass scale for the $\Sigma$ field this is quite compatible with the magnitude used in Eq.18. Similarly we can readily construct a messenger sector that drives $<\theta \overline{\theta}> \simeq <\phi_3 \overline{\phi_3}>$. Finally the alignment of the $\Sigma$ vev can be arranged through a term in the potential proportional to $|\Sigma \overline{\theta}|^2$ and again such a term can readily be generated via a suitable messenger sector. An explicit example (for the discrete symmetry case) of how this can be done is given by the superpotential $P_4$ with the additional messenger fields given in Table 2,

$$P_4 = A(\phi_3 \overline{\phi_3} + \theta \overline{\theta} + Tr(\Sigma^2)) + \phi_3 \Sigma_X \overline{\phi_3} + Tr(\Sigma_X \Sigma^2) + \theta' \Sigma \overline{\theta}. \quad (35)$$

## 5 SUSY Breaking Soft Terms

A major problem with a continuous gauged family symmetry is that the D-terms split the degeneracy needed between the squarks and sleptons. This may give rise to unacceptably large flavour changing neutral currents (FCNC) due to flavour dependent contributions to sparticle masses of the form $\Delta \tilde{m}_i^2 = c_i D^2$ where $c_i$ is a family dependent coefficient and $D$ is the magnitude of the D-term. In the model discussed above the magnitude of the D-terms are small and the FCNC are within experimental limits. Consider first the D-term which is generated at the highest scale of symmetry breaking by the fields $\phi_3$ and $\overline{\phi_3}$. The term in the superpotential $P_2$ driving this breaking is symmetric between the two fields. As a result the magnitude of the $D-$term obtained from $\overline{\phi_3} \partial V / \partial \phi_3 - \phi_3 \partial V / \partial \overline{\phi_3}$, where $V$ is the full potential involving D-terms, scalar mass terms and F-terms, is given by

$$D_3^2 = \frac{g^2}{3} \left| \phi_3^4 \overline{\phi_3} - \overline{\phi_3} \phi_3 \right|^2 \simeq \frac{1}{16} \left( m_3^2 - m_3^2 \right)^2 \quad (36)$$

If the soft masses are driven by supergravity coupling to a hidden supersymmetry breaking sector and the modular weights of the two fields are the same, then the masses will be equal at the Planck scale. Since the breaking triggered by $\phi_3$ and $\overline{\phi_3}$ is close to the Planck scale the splitting induced in the soft masses of these fields by radiative corrections involving Yukawa couplings are likely to be small as there are no large logarithms involved. As a result the D-term is likely to be very small.

The second stage of breaking is triggered by the fields $\phi_{23}$ and $\overline{\phi_{23}}$. Their D-terms can split the first two generations and so must be very small if unacceptable flavour changing neutral currents are to be avoided. In their case the superpotential term, $P_1$, triggering their vevs is also symmetric. One one must also include the effect of the term coming from $P_3$ which spoils the
symmetry between $\phi_3$ and $\bar{\phi}_3$, and between $\phi_{23}$ and $\bar{\phi}_{23}$. Together this gives for the $D$–term associated with the second stage of symmetry breaking the form

$$D_{23} = \frac{1}{4} \left( m_{23}^2 - m_{23}^2 \right) - 2\bar{\phi}_2 \phi_3 \phi_{23} \phi_{23} \left( \phi_2 \bar{\phi}_3 \phi_{23} \phi_{23} - \mu^4 \right)$$  \quad (37)$$

Since the vev of $F_Y = (\bar{\phi}_2 \phi_3 \phi_{23} - \mu^4)$ is of order the soft mass squared the second term is small, being suppressed by the small vevs of $\phi_2$ and $\phi_{23}$. The first term is also expected to be small if the soft masses of $\phi_{23}$ and $\bar{\phi}_{23}$ are degenerate at the Planck scale because $\phi_{23}$ and $\bar{\phi}_{23}$ also have vevs close to the Planck scale and so the radiative corrections splitting these masses are likely to be small.

The general property that keeps D-terms small is the fact that to a good approximation the potential is symmetric in the conjugate fields. This immediately leads to the form given by Eq.(36). For breaking close to the Planck scale radiative corrections splitting the soft masses will be suppressed by the one loop expansion parameter and may readily be small. In this case any supersymmetry breaking mechanism giving degenerate soft SUSY breaking masses at the Planck scale to the conjugate fields involved in the family symmetry breaking leads to small D-terms consistent with the bounds from FCNC. This mechanism can be applied more generally to family symmetry models and removes one of the major obstacles to implementing such a symmetry.

6 Phenomenology

The model we have constructed gives excellent agreement with the quark and lepton masses and mixing angles. For the up and down quarks the form of $Y^u$ and $Y^d$ given in Eq.24, 25 is consistent with the phenomenological fit in Eq.1, with the expansion parameters as in Eq.2, for parameters such as $g \sim 1$, $g' \sim -1$, $h \sim h' \sim 0.3$, $c' \sim 1$, $c \sim -1$. The charged lepton mass matrix is of the Georgi Jarslkog form which, after including radiative corrections, gives an excellent description of the charged lepton masses. In the neutrino sector the parameters satisfy the conditions of sequential dominance, with the lightest right-handed neutrino giving the dominant contribution to the heaviest physical neutrino mass, and the second right-handed neutrino giving the leading subdominant contribution, providing that $\alpha \sim \epsilon$.

Analytic estimates of neutrino masses and mixing angles for sequential dominance were derived in [3], and for the special case here of light sequential dominance, with the 11 neutrino Yukawa coupling equal to zero, they are summarized recently in [15], from which we readily extract the analytic estimates below for the neutrino masses,

$$m_1 \sim \frac{\epsilon^2 v_2^2}{M_3}$$  \quad (38)$$
$$m_2 \approx \frac{(g + \frac{4}{3} + \frac{h'}{3})^2 v_2^2}{s_{12}^2} \sim 5.8 \frac{v_2^2}{M_3}$$  \quad (39)$$
$$m_3 \approx \frac{[(g' - \frac{4}{3} - \frac{h}{3})^2 + (g' + \frac{4}{3})^2] v_2^2}{\bar{\epsilon}} \sim 15 \frac{v_2^2}{M_3}$$  \quad (40)$$
and neutrino mixing angles:

\[
\tan \theta_{23}^\nu \approx \frac{(g' - \frac{h}{3} - \frac{h'}{3})}{(g' + \frac{h}{3})} \sim 1.3 \tag{41}
\]

\[
\tan \theta_{12}^\nu \approx \frac{(\bar{\epsilon})^{1/2}(g + \frac{h}{3} + \frac{h'}{3})}{-cs_{23}} \sim 0.66 \tag{42}
\]

\[
\theta_{13}^\nu \approx -(\bar{\epsilon}) \left( (g + \frac{h}{3} + \frac{h'}{3})(-\frac{\alpha}{c}) + (g' + \frac{h}{3})(-\frac{\alpha}{c} + c\bar{\epsilon}^{-1/2}) \right) \left[ (g' - \frac{h}{3} - \frac{h'}{3})^2 + (g' + \frac{h}{3})^2 \right]^{3/2} \sim 1.6\bar{\epsilon} \tag{43}
\]

where the numerical estimates correspond to \( g \sim 1, g' \sim -1, h \sim h' \sim 0.3, c' \sim -c \sim 1, \alpha \sim \epsilon \sim 0.05, \bar{\epsilon} \sim 0.15 \). Note that the physical lepton mixing angle \( \theta_{13} \) receives a large contribution from the neutrino sector \( \theta_{13}^\nu \sim 0.3 \) at the high energy scale, for this choice of parameters, compared to the current CHOOZ limit \( \theta_{13} \leq 0.2 \) [16]. However the physical mixing angles will receive charged lepton contributions [3] and all the parameters are subject to radiative corrections in running from the high energy scale to low energies, although in sequential dominance models these corrections are only a few per cent [17]. We conclude that our model predicts that \( \theta_{13} \) is close to the current CHOOZ limit, and could be observed by the next generation of long baseline experiments such as MINOS or OPERA.

Any model of flavour must be sure to avoid large FCNC. In the previous Section we showed that the D-terms were under control in this respect. However there is also a problem in supergravity models which use the Froggatt Nielsen mechanism to order fermion masses due to the fact that the Froggatt Nielsen fields typically acquire a F-term vevs which cause the A terms to be misaligned by \( O(m_3/2) \) relative to the Yukawa couplings, generating FCNC and potentially large electric dipole moments [18] (see also [19]). The effect is somewhat ameliorated here because the mass matrices are symmetric. However we note that lepton number violating processes are still expected to occur at a rate close to the present limits.

In any scheme such as this in which the neutrino Dirac mass is equal to the up Dirac mass and the dominant right handed neutrino exchange is in the 1 direction we have a prediction for the lightest Majorana state given by \( M_1 = m_u m_e/m_3 \sim 10^8 \) GeV. In this model the expansion parameter is \( \propto \epsilon^6 \) so a very small difference in the neutrino expansion parameter, coming from \( SU(4) \) breaking, can readily increase this by more than a factor of 10, bringing it into the range that thermal leptogenesis is possible. Assuming that this is the case the resultant CP asymmetry in the decay of the heavy lepton may be readily estimated giving

\[
\epsilon_1 \approx -\frac{3}{8\pi^2} Im (YY^\dagger)_{11}
\]

\[
\approx -\frac{3}{8\pi^2} \epsilon^6
\]

This gives the asymmetry \( \epsilon_1 = 6.10^{-10} \) or somewhat larger if we allow for a slightly larger expansion parameter in the neutrino sector. However washout effects can significantly reduce the asymmetry. An analysis of these effects in this class of model is given in [20]. Note that in such models there is a link between the leptogenesis CP violating phase and the neutrino mixing phase measurable in neutrino oscillation experiments [15].
7 Summary and Conclusions

The main message of this paper is that a coherent description of all quark, charged lepton and neutrino masses and mixing angles can be constructed in a model having a very high degree of symmetry. The large mixing angles in the lepton sector follow naturally from two ingredients, the see-saw mechanism with sequential right handed neutrino dominance and a non-Abelian family symmetry. We have constructed a simple implementation of these ideas in which there is an underlying stage of $SO(10)$ type Unification and the Family Symmetry is $SU(3)$ together with a further discrete symmetry needed to restrict the allowed Yukawa couplings. The resultant model gives an excellent description of all data including lepton mixing consistent with almost maximal atmospheric mixing and the solar LMA MSW solution. The model naturally satisfies the bounds on flavour changing neutral currents. Indeed the $SU(3)$ symmetry provides a new mechanism for making the families of squarks and sleptons of a given flavour degenerate. Note that this does not require that all squarks and sleptons (and Higgs) be degenerate as in the SUGRA solution of the flavour problem. Indeed it provides a new solution to the flavour problem with different expectations for the SUSY spectrum from SUGRA, gauge and anomaly mediated schemes. In particular the strong breaking of $SU(3)$ in the third family direction, needed to give the large top and bottom quark masses, will give large splitting to the third generation through the terms $|\psi_i \phi_{3i}|^2$, $|\psi^c_i \phi_{3i}|^2$ [5]. This can considerably affect the allowed region of parameter space in a constrained model fit and the spectrum of supersymmetric states [21]. As we discussed above there is still the need for the fields $\phi_3$ and $\bar{\phi}_3$ and the fields $\phi_{23}$ and $\bar{\phi}_{23}$ to have equal initial masses respectively, but even this condition can be relaxed if one modifies the model to use a discrete subgroup of $SU(3)$ instead of the full family group, because then the D-terms associated with the family symmetry are absent.

Acknowledgement

One of us (GGR) would like to thank A.Ibarra, L. Velasco-Sevilla, S. Pokorski, R. Rattazzi, P. Ramond and particularly O. Vives for helpful discussions. SFK thanks T. Blazek for discussions, I. Peddie for carefully reading the manuscript and correcting some errors, and is grateful to PPARC for the support of a Senior Fellowship. This work was partly supported by the EU network, ”Physics Across the Present Energy Frontier” HPRV-CT-2000-00148.

References

[1] For a reviews of theories fermion masses and further references, see G. G. Ross, “Models of fermion masses,” TASI lectures, Boulder, 2000; H. Fritzsch and Z. z. Xing, Prog. Part. Nucl. Phys. 45 (2000) 1 [arXiv:hep-ph/9912358]. See also S.Raby, Phys.Rev.D66:010001,2002 (page 142)

[2] For a recent review of neutrino physics, see W. M. Alberico and S. M. Bilenky, arXiv:hep-ph/0306239.

[3] S. F. King, Phys. Lett. B 439 (1998) 350 [arXiv:hep-ph/9806440]; S. F. King, Nucl. Phys. B 576 (2000) 85 [arXiv:hep-ph/9912492]; S. F. King, JHEP 0209 (2002) 011 [arXiv:hep-ph/0204360].
[4] R. G. Roberts, A. Romanino, G. G. Ross and L. Velasco-Sevilla, Nucl. Phys. B **615** (2001) 358 [arXiv:hep-ph/0104088].

[5] S. F. King and G. G. Ross, Phys. Lett. B **520** (2001) 243 [arXiv:hep-ph/0108112].

[6] Z. Berezhiani and A. Rossi, Nucl. Phys. Proc. Suppl. **101** (2001) 410 [arXiv:hep-ph/0107054]; M.Yu.Khlopov. Cosmoparticle physics. World Scientific, 1999.

[7] M. Gell-Mann, P. Ramond and R. Slansky, *Proceedings of the Supergravity Stony Brook Workshop*, New York 1979, eds. P. Van Nieuwenhuizen and D. Freedman; Gell-Mann, Ramond, Slansky in: THE FAMILY GROUP IN GRAND UNIFIED THEORIES. By Pierre Ramond (Caltech). CALT-68-709, Feb 1979. 21pp. Invited talk given at Sanibel Symposium, Palm Coast, Fla., Feb 25 - Mar 2, 1979 [arXiv:hep-ph/9809459]; T. Yanagida, *Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe*, Tsukuba, Japan 1979, eds A. Sawada and A. Sugamoto; R. N. Mohapatra, G. Senjanovic, *Phys.Rev.Lett.* **44** (1980)912, *ibid. Phys.Rev.* **D23** (1981) 165; S. L. Glashow, *In *Cargese 1979, Proceedings, Quarks and Leptons*, 687-713 and Harvard Univ.Cambridge - HUTP-79-A059 (79,REC.DEC.) 40p.

[8] H. Georgi and C. Jarlskog, Phys. Lett. B **86** (1979) 297.

[9] G. G. Ross and L. Velasco-Sevilla, Nucl. Phys. B **653** (2003) 3 [arXiv:hep-ph/0208218].

[10] H.Georgi, Particles and Fields, Proceedings of the APS Div. of Particles and Fields, ed. C.Carlson; H. Fritzsch and P. Minkowski, Annals Phys. **93** (1975) 193.

[11] J. C. Pati and A. Salam, Phys. Rev. D **10** (1974) 275.

[12] S. F. King, Phys. Lett. B **325** (1994) 129 [Erratum-ibid. B **325** (1994) 538].

[13] E. Witten, arXiv:hep-ph/0201018 and references therein.

[14] F. S. Ling and P. Ramond, Phys. Rev. D **67** (2003) 115010 [arXiv:hep-ph/0302264].

[15] S. F. King, arXiv:hep-ph/0211228.

[16] M. Apollonio et al. [CHOOZ Collaboration], Phys. Lett. B **466** (1999) 415 [arXiv:hep-ex/9907037].

[17] S. F. King and N. N. Singh, Nucl. Phys. B **591** (2000) 3 [arXiv:hep-ph/0006229].

[18] G. G. Ross and O. Vives, Phys. Rev. D **67** (2003) 095013 [arXiv:hep-ph/0211279].

[19] S. Abel, S. Khalil and O. Lebedev, Phys. Rev. Lett. **89** (2002) 121601 [arXiv:hep-ph/0112260]; S. F. King and I. N. Peddie, arXiv:hep-ph/0307091.

[20] A. Ibarra and G.G. Ross, arXiv:hep-ph/0307051

[21] M. R. Ramage and G. G. Ross, arXiv:hep-ph/0307389.