Solar-type Variables

Günter Houdek

Institute of Astronomy, University of Vienna, A-1180 Vienna, Austria

Abstract. The rich acoustic oscillation spectrum in solar-type variables make these stars particularly interesting for studying fluid-dynamical aspects of the stellar interior. I present a summary of the properties of solar-like oscillations, how they are excited and damped and discuss some of the recent progress in using asteroseismic diagnostic techniques for analysing low-degree acoustic modes. Also the effects of stellar-cycle variations in low-mass main-sequence stars are addressed.

Keywords: solar-like oscillations; mode physics; stochastic excitation

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INTRODUCTION

Solar-type stars possess extended surface convection zones. The observed oscillation modes generally behave as acoustic modes and their frequencies are sensitive predominantly to the sound speed $c$ in the stellar interior. It appears that all possible oscillation modes are intrinsically stable. They are excited stochastically by the strong emission of acoustic noise by the turbulent velocity field in the upper convectively unstable layers of the star. The excitation occurs in a broad frequency range, giving rise to a rich pulsation spectrum. The amplitudes of the oscillations are small, typically 5 ppm $L_\odot/M_\odot$ (Kjeldsen & Bedding 1995). Such small amplitudes allow us to describe the pulsations with linear theory, however, they also pose a challenge to the detection limits of observing campaigns. But the recently made progress in continuously lowering the detection threshold of today’s instrumentation has allowed us to have access to these low-amplitude pulsators, providing high-precision data such as from the space mission CoRoT (Convection Rotation and planetary Transits; Baglin 2003), from the recently launched Kepler satellite (Basri et al. 2005, Christensen-Dalsgaard 2007), and from the ground-based observing campaigns, such as SONG (Grundahl et al. 2007) and those of the kind organized by Kjeldsen, Bedding and their colleagues (e.g. Kjeldsen et al. 2005). Solar-like oscillations have been convincingly observed on other stars and augur asteroseismic diagnosis that will raise stellar physics to a new level of sophistication. It will not be possible to measure either the internal structure or the internal motion of a distant star with a resolution comparable with that which we have achieved for the Sun, because high-degree modes will not be accessible in the foreseeable future; but some theoretically important properties, such as the gross structure of the energy-generating core and the extent to which it is convective, and the large-scale variation of the angular velocity, will become available. Such information will be of crucial importance for checking, and then calibrating, the theory of the structure and evolution of stars, the backbone of theoretical astrophysics.

FIGURE 1. Hertzsprung-Russell diagram of stars in which solar-like oscillations have been detected (adapted from Aerts et al. 2008; the original figure was kindly provided by Fabien Carrier).

The first indication of excess acoustic power in other stars with a frequency dependence similar to the Sun was reported by Brown et al. (1991) for the F5 star Procyon A ($\alpha$ CMi), in which the first unambiguous detection of solar-like oscillations was reported by Martić et al. (1999), confirmed later by Mosser et al. (2008). First (unconfirmed) detection of individual peaks in the acoustic power spectrum from high-precision time-resolved spectroscopic observations was published for the G5 star $\eta$ Boo by Kjeldsen et al. (1995), but it was not before 2003 that an unambiguous confirmation was established by Carrier et al. (2003) and Kjeldsen et al. (2003). The location of stars in the Hertzsprung-Russell diagram, in
FIGURE 2. Small section of a solar acoustic power spectrum. The radial order \( n \) and spherical degree \( l \) are indicated in pairs of \((n,l)\) for each mode. The large and small frequency separations, \( \Delta \nu_{n,l} \) and \( \delta \nu_{n,l} \), are in general functions of \( n \) and \( l \) and can be used to infer the mass and age of a star (adapted from Christensen-Dalsgaard 2001).

which solar-like oscillations have been detected until today, are indicated in Fig. 1 and a summary of such stars was presented recently by Bedding & Kjeldsen (2007).

SOLAR-LIKE OSCILLATION PROPERTIES

Only modes of low degree can be observed in distant stars, however, in general, the observed modes are of high radial order, which allows us to extract diagnostic properties of the frequencies \( \nu_{n,l} \) with radial order \( n \) and spherical degree \( l \) for the asymptotic limit \( n/l \to \infty \). The diagnostic properties of this type of mode have been studied extensively in the solar case. From asymptotic theory we find for the cyclic oscillation frequencies (Gough 1986, 1993; see also Tassoul 1980 and Vandakurov 1967)

\[
\nu_{n,l} \simeq \left(n + \frac{l}{2} + \tilde{\varepsilon}\right) \nu_0 - \frac{AL^2 - B}{v_{n,l}^2} \nu_0^2 + O(\nu_0^4),
\]

where

\[
\nu_0 = \left[2 \int_0^R \frac{dr}{c}\right]^{-1}
\]

is the inverse of twice the sound travel time between the centre and surface (\( R \) is surface radius), and

\[
A = \frac{1}{4\pi^2} \left[ \frac{c(R)}{R} - \int_0^R \frac{dc}{dr} \frac{dr}{r} \right].
\]

The frequency-dependent coefficient \( \tilde{\varepsilon} \) is determined by the reflection properties of the surface layers, as is the small correction term \( B \), and \( L^2 = l(l + 1) \). The value of \( \nu_0 \) can be estimated from taking the average (over \( n \) and \( l \)) of the so-called large frequency separation \( \Delta \nu_{n,l} \equiv \nu_{n,l} - \nu_{n-1,l} \) between modes of like degree and consecutive order. The last two terms on the right-hand side of equation (1) lift the degeneracy between modes with the same value of \( n + l/2 \) and leads to the so-called small frequency separation \( \delta \nu_{n,l} \equiv \nu_{n,l} - \nu_{n-1,l+2} \). One obtains a frequency structure in which modes of odd degree fall approximately halfway between modes of even degree, which is illustrated in Fig. 2 for a solar spectrum. The mean small frequency separation (averaged over \( n \))

\[
\langle \delta \nu_{n,l} \rangle = \langle \nu_{n,l} - \nu_{n-1,l+2} \rangle \simeq 24 \left(2I + 3\right) \nu_0^2 \langle \nu_{n,l} \rangle
\]

is predominantly determined by the acoustic sound speed in the stellar core and hence is sensitive to the chemical composition there and consequently is an indicator for the stellar age (e.g., Gough 2001, Houdek & Gough 2008).

Seismic diagnostic

Acoustic modes depend predominantly on the sound speed in the stellar interior and consequently on the chemical composition. As the star evolves, the chemical composition will vary and consequently also the value of the small frequency separation \( \delta \nu_{n,l} \). So does the mean large frequency separation (averaged over \( n \) and \( l \))

\[
\langle \Delta \nu \rangle = \langle \nu_{n,l} - \nu_{n-1,l} \rangle \simeq \nu_0 \frac{M}{R^3} \frac{1}{\frac{1}{2}}
\]

mainly because of the increasing surface radius with time. A convenient way to demonstrate the dependence
Stellar evolutionary tracks (solid curves, indicated by the stellar mass) and curves of constant central hydrogen abundance (dashed curves, indicated by the central hydrogen abundance by mass, $X_c$) in terms of the average large separation $\langle \Delta \nu \rangle$ and average small separation $\langle \delta \nu \rangle \equiv \langle \delta \nu_n^0 \rangle$ [c.f. equations (5) & (4)] (from Christensen-Dalsgaard 1993).

Additional information of structural aspects of solar-like stars is obtained from the seismic signatures contained in $\Delta \nu_n^l$ and $\delta \nu_n^l$. Abrupt variation in the stratification of a star (relative to the scale of the inverse radial wavenumber of a seismic mode of oscillation), such as that resulting from the (smooth, albeit acoustically relatively abrupt) depression in the first adiabatic exponent $\gamma = (\partial \ln p / \partial \ln \rho)_s$, caused by the ionization of helium, where $p$, $\rho$ and $s$ are pressure, density and specific entropy, or from the sharp transition from radiative to convective heat transport at the base of the convection zone, induces small-amplitude oscillatory components (with respect to frequency) in the spacing of the cyclic eigenfrequencies $\nu_{n,l}$ of seismic oscillation and consequently also in $\Delta \nu_{n,l}$ and $\delta \nu_{n,l}$. We call such abrupt variations an acoustic glitch. One might hope that the variation of the sound speed $c$ induced by helium ionization might enable one to determine from the low-degree eigenfrequencies a measure that is directly related to, perhaps even almost proportional to, the helium abundance, with little contamination from other properties of the structure of the star.

A convenient and easily evaluated measure of the oscillatory component produced by acoustic glitches is the second multiplet-frequency difference with respect to order $n$ amongst modes of like degree $l$:

$$\Delta_2 \nu_{n,l} \equiv \nu_{n-1,l} - 2 \nu_{n,l} + \nu_{n+1,l}$$

(Gough 1990). Any localized region of rapid variation of either the sound speed $c$ or the density scale height, or a spatial derivative of them, induces an oscillatory component in $\Delta_2 \nu$ (from here on the subscripts $n,l$ have been dropped for simplicity) with a ‘cyclic frequency’ approximately equal to twice the acoustic depth

$$\tau = \int_{r_{\text{glitch}}}^{R} c^{-1} \, dr$$

of the glitch, and with an amplitude which depends on the amplitude of the glitch and which decays with $\nu$ once the inverse radial wavenumber of the mode becomes comparable with or less than the radial extent of the glitch.

Various approximate formulae for the oscillatory components that are associated with the helium ionization...
have been suggested and used, by e.g., Basu et al. (1994, 2004), Monteiro & Thompson (1998, 2005) and Gough (2002), not all of which are derived directly from explicit acoustic glitches. Gough used an analytic function for modelling the dip in the first adiabatic exponent. In contrast, Monteiro & Thompson assumed a triangular form. Basu et al. have adopted a seismic signature for helium ionization that is similar to that arising from a single discontinuity; the artificial discontinuities in the sound speed and its derivatives that this and the triangular representations possess cause the amplitude of the oscillatory signal to decay with frequency too gradually, although that deficiency may not be immediately noticeable within the limited frequency range in which adequate asteroseismic data are or will imminently be available. More recently Houdek & Gough (2007) proposed a seismic diagnostic in which the variation of the eigenfunction phase with acoustic depth, thereby improving the discrepancy between the seismically inferred depths of the acoustic glitches and that of a corresponding stellar model. An example of the application of Houdek & Gough’s technique is presented in Fig. 4 which shows the resulting fit to simulated data for a solar-like star.

Mode parameters

Were solar p modes to be genuinely linear and stable, their power spectrum could be described in terms of an ensemble of intrinsically damped, stochastically driven, simple-harmonic oscillators, provided that the background equilibrium state of the star is independent of time (Fig. 5); if we assume further that mode phase fluctuations contribute negligibly to the width of the spectral lines, the intrinsic damping rates of the modes, $\Gamma/2$, could then be determined observationally from measurements of the pulsation linewidths $\Gamma$.

The power (spectral density), $P$, of the surface displacement $\xi_{nl}(t)$ of a damped, stochastically driven, simple-harmonic oscillator, satisfying

$$I_{nl} \left[ \frac{d^2 \xi_{nl}}{dt^2} + \Gamma_{nl} \frac{d \xi_{nl}}{dt} + \omega_{nl}^2 \xi_{nl} \right] = f(t), \quad (8)$$

which represents the pulsation mode of order $n$ and degree $l$, with linewidth $\Gamma_{nl}$ and frequency $\omega_{nl}$ and mode inertia $I_{nl}$, satisfies

$$P \propto P_L P_I = \frac{\Gamma_{nl}/2 \pi}{(\omega - \omega_{nl})^2 + \Gamma_{nl}^2/4} P_I, \quad (9)$$

assuming $\Gamma_{nl} \ll \omega_{nl}$, where $f(t)$ describes the stochastic forcing function. Integrating equation (9) over frequency leads to the total mean energy in the mode

$$I_{nl} V_{nl}^2 := \frac{1}{2} \omega_{nl}^2 I_{nl} \langle |A_{nl}|^2 \rangle \propto \omega_{nl}^2 I_{nl} \int_{-\infty}^{\infty} P(\omega) d\omega \propto \frac{P_I(\omega_{nl})}{\Gamma_{nl}}, \quad (10)$$

where $A_{nl}$ is the displacement amplitude (angular brackets, $\langle \rangle$, denote an expectation value) and $V_{nl}$ is the rms velocity of the displacement. The total mean energy of a mode is therefore directly proportional to the rate of work (also called energy supply rate) of the stochastic forcing $P_I$ at the frequency $\omega_{nl}$ and indirectly proportional to $\Gamma_{nl}$.

Equations (8)–(10) are discussed in terms of the displacement $\xi$ (from here on we omit the subscripts $n$ and $l$), but in order to have a direct relation between the observed velocity signal $v(t) = d\xi/dt$ and the modelled excitation rate $P_I$ we shall first take the Fourier transform $\hat{V}(\nu)$ of $v(t)$ ($\nu = \omega/2\pi$). It follows that the total mean

![Figure 5. Power spectral density of a randomly excited, damped, harmonic oscillator. $P_I$ represents the spectral density of the random force and $P$ is the product of the Lorentzian $P_L$ and $P_I$ (adapted from Kosovichev 1995).](image)
energy $E$ of the harmonic signal of a single pulsation mode is then given by (Chaplin et al. 2005)

$$E = IV^2 = I\delta \int_{-\infty}^{\infty} |\hat{V}(v)|^2 \, dv = \frac{1}{4} I\Gamma H, \quad \text{(11)}$$

in which

$$H := \int_{v-\delta/2}^{v+\delta/2} |\hat{V}(v)|^2 \, dv, \quad \text{(12)}$$

is the maximum power density - which corresponds to the ‘height’ of the resonant peak in the frequency domain (see Fig. 2). The height $H$ is the maximum of the discrete power, i.e. the integral of power spectral density over a frequency bin $\delta = 1/T_{\text{obs}}$, where $T_{\text{obs}}$ is the total observing time. The following expressions

$$V^2 := \frac{P_t}{\Gamma I} = \frac{1}{4} \Gamma H \quad \text{(13)}$$

and

$$H := \frac{P_t}{(\Gamma/2)^2 I} \quad \text{(14)}$$

provide a direct relation between the observed height $H$ (in cm$^2$ s$^{-2}$ Hz$^{-1}$), the modelled energy supply rate $P_t$ (in erg s$^{-1}$), and damping rate $\eta = \Gamma/2$.

**PULSATION COMPUTATIONS AND AMPLITUDE RATIOS**

The linearized pulsation equations for nonadiabatic radial oscillations can be presented as (e.g. Balmforth 1992a):

$$\frac{\partial}{\partial m} \left( \frac{\delta p}{p} \right) = f \left( \frac{\delta T}{T}, \frac{\delta \rho}{\rho}, \frac{\delta \rho_t}{\rho_t}, \frac{\delta \Phi}{\Phi} \right),$$

$$\frac{\partial}{\partial m} \left( \frac{\delta r}{r} \right) = -\frac{1}{4\pi r^4 \rho} \left( \frac{\delta r}{r}, \frac{\delta \rho}{\rho} \right),$$

$$\frac{\partial}{\partial m} \left( \frac{\delta L}{L} \right) = -i\omega c_p T \left( \frac{\delta T}{T} - \nabla_{ad} \frac{\delta \rho}{\rho} \right), \quad \text{(15)}$$

where $\delta$ is the Lagrangian perturbation operator, and for simplicity the right hand side of the perturbed momentum equation is formally expressed by the function $f$ (the full set of equations can be found in, e.g., Balmforth 1992a). Equations (15) are solved subject to boundary conditions to obtain the eigenfunctions and the complex angular eigenfrequency $\omega = \omega_0 + i\eta$, where $\omega_0$ is the (real) pulsation frequency and $\eta = \Gamma/2$ is the damping rate in (s$^{-1}$). The turbulent flux perturbations of heat and momentum, $\delta L_c$ and $\delta p_t$, and the fluctuating anisotropy factor $\delta \Phi$ are obtained from the non-local, time-dependent convection formulation by Gough (1977a,b).

From the linearized nonadiabatic pulsation equations (15) theoretical intensity-velocity amplitude ratios

$$\frac{\Delta L}{\Delta V} := \frac{\delta L/L}{\omega r \delta r/r} \quad \text{(16)}$$

TABLE 1. Absorption lines and their wavelengths $\lambda$ of various helioseismic instruments. Also listed are the optical depths $\tau_{5000}$ at 5000 Å and the corresponding approximate heights above the photosphere ($h = 0$ at $T = T_{\text{eff}}$) at which the lines are formed.

| Instrument | line | $\lambda$ (Å) | $\tau_{5000}$ | height (km) |
|------------|------|---------------|---------------|-------------|
| BBSO       | Ca   | 6439          | 0.05          | $\sim 129^*$          |
| BiSON      | K    | 7699          | 0.013         | $\sim 250^*$          |
| MDI        | NiI  | 6708          | 9$\times10^{-3}$ | $\sim 300^{**}$     |
| GOLF       | Na D1/D2 | 5690 | 5$\times10^{-4}$ | $\sim 500^{**}$     |

* from Libbrecht (1988)  
† from Christensen-Dalsgaard & Gough (1982)  
** from Toutain et al. (1997, but see also Baudin et al. 2005)

can be compared with observations, without the need of a specific excitation model and all its uncertainties in describing the turbulent spectra.

It is, however, important to realize that various instruments observe in different absorption lines and consequently at different heights in the atmosphere. This property has to be taken into account not only when comparing observations between various instruments (e.g. Christensen-Dalsgaard & Gough 1982), but also when comparing theoretical amplitude estimates with observations (Houdek et al. 1995). Table 1 lists some of the relevant properties of various instruments.

In the top panel of Fig. 6 the theoretical amplitude ratios (equation (16)) of a solar model are plotted as a function of height for several radial pulsation modes. The mode energy density (which is proportional to $r \rho^{1/2} \delta r$) increases rather slowly with height; the density $\rho$, however, decreases very rapidly and consequently the displacement eigenfunction $\delta r$ increases with height. This leads to the results shown in the upper panel of Fig. 6, where the decrease in the amplitude ratios with height is particularly pronounced for high-order modes for which the eigenfunctions vary rapidly in the evanescent outer layers of the atmosphere. It is for that reason why solar velocity amplitudes from, e.g., the GOLF instrument have larger values than the measurements from the BiSON instrument (by about 25%, Kjeldsen et al. 2005).

The lower panel of Fig. 6 compares the estimated solar amplitude ratios (curves) with observed ratios (symbols) as a function of frequency. The model results are depicted for velocity amplitudes computed at different atmospheric levels. The observations are obtained from accurate irradiance measurements from the IPHIR instrument of the PHOBOS 2 spacecraft with contemporaneous low-degree velocity data from the BiSON instrument at Tenerife (Schrijver et al. 1991). The thick solid curve represents a running-mean average, with a width of 300 µHz, of the observational data. The theoretical ratios
FIGURE 6. Top: Calculated amplitude ratios (see equation (16)) as a function of height in a solar model for modes with different frequency values. Bottom: Theoretical amplitude ratios (surface luminosity perturbation over velocity) for a solar model compared with observations by Schrijver et al. (1991). Computed results are depicted at different heights above the photosphere ($h=0$ km at $T=T_{\text{eff}}$). The thick, solid curve indicates a running-mean average of the data (from Houdek et al. 1995).

for $h = 200$ km (dashed curve) show reasonable agreement with the observations.

In Fig. 7 model results for the F5 star Procyon A are compared with observations (horizontal dotted line) by Arentoft et al. (2008). Theoretical results are shown for two stellar atmospheres: a VAL-C (Vernazza et al. 1981) atmosphere scaled with the model’s effective temperature $T_{\text{eff}}$ (thin curves), and for an Eddington atmosphere (thick curves). For both stellar atmospheres the agreement with the observations is less satisfactory than in the solar case, indicating that we may not represent correctly the shape of the pulsation eigenfunctions. Consequently there is need for adopting more realistically computed atmospheres in the equilibrium models, particularly for stars with much higher surface temperatures than the Sun. It should, however, be mentioned that the current photometric observations are still uncertain.

FIGURE 7. Calculated amplitude ratios (see equation (16)) for a model of Procyon A are compared with observations by Arentoft et al. (2008; horizontal dotted line). Theoretical results are shown for a scaled VAL-C atmosphere (black, thin curves) and for an Eddington atmosphere (red, thick curves). Top: The theoretical amplitude ratios are shown as a function of height and for three different pulsation modes. The frequencies of the three pulsation modes are indicated. Bottom: The theoretical amplitude ratios are shown as a function of frequency at three different heights in the stellar atmosphere. The heights above the photosphere $h = 0$ km are indicated.

### DAMPING RATES

Damping of stellar oscillations arises basically from two sources: processes influencing the momentum balance, and processes influencing the thermal energy equation. Each of these contributions can be divided further according to their physical origin, which was discussed in detail by Houdek et al. (1999).

Important processes that influence the thermal energy balance are nonadiabatic processes attributed to the modulation of the convective heat flux by the pulsation. This contribution is related to the way that convection modulates large-scale temperature perturbations induced by the pulsations which, together with the conventional κ-mechanism, influences pulsational stability.

Current models suggest that an important contribution...
that influences the momentum balance is the exchange of energy between the pulsation and the turbulent velocity field through dynamical effects of the fluctuating Reynolds stress. In fact, it is the modulation of the turbulent fluxes by the pulsations that seems to be the predominant mechanism responsible for the driving and damping of solar-type acoustic modes. It was first reported by Gough (1980) that the dynamical effects arising from the turbulent momentum flux perturbation \( \delta p_t \) contribute significantly to the damping \( \Gamma \). Detailed analyses (Balmforth 1992a) reveal how damping is controlled largely by the phase difference between the momentum perturbation and the density perturbation. Therefore, turbulent pressure fluctuations must not be neglected in stability analyses of solar-type p modes.

A comparison between the latest linewidth measurements (full-width at half-maximum) \( \Delta_{nl} = \Gamma_{nl}/2\pi \) and theoretical damping rates is given in Fig. 8. The observational time series from BiSON (Chaplin et al. 2005) was obtained from a 3456-d data set and the linewidths of the temporal power spectrum extend over many frequency bins \( \delta = 1/2T_{obs} \). In that case the linewidth in units of cyclic frequency is related to the damping rate according to

\[
\Delta_{nl} = \pi^{-1}\eta_{nl}.
\]

(17)

Recently Dupret et al. (2004) performed similar stability computations for the Sun using the time-dependent mixing-length formulation by Gabriel et al. (1975, 1998) and Grigahcène et al. (2005), which is based on the formulation by Unno (1967). Their results are illustrated in the right panel of Fig. 8 which also shows the characteristic plateau near 2.8 mHz. It is, however, interesting to note that their findings suggest the fluctuating convective heat flux to be the main contribution to mode damping, whereas for the model results shown in the left panel of Fig. 8 which are based on Gough’s (1977a,b) convection formulation, it is predominantly the fluctuating Reynolds stress that makes all modes stable.

Houdek et al. (1999) computed damping rates \( \eta \) of solar-like oscillations in about 160 stars with masses between 0.9 \( M_\odot \) and 2.0 \( M_\odot \) in the vicinity of the main sequence, and for various metallicities and convection parameters. Recently Chaplin et al. (2009) conducted a similar theoretical study of estimating mode lifetimes \( \tau = \eta^{-1} \) in low-mass stars and compared the theoretical estimates with the latest linewidth measurements in twelve solar-type variables. They found that the mean mode lifetimes of the five most prominent solar-like p modes \( \langle \tau \rangle \) scale like

\[
\langle \tau \rangle \propto T_{eff}^{-4}.
\]

(18)

Their results are depicted in Fig. 9 where the diamond symbols are the theoretical estimates of \( \langle \tau \rangle \) obtained from a grid of models computed in the manner of Chaplin et al. (2005) and the crosses with error bars are the linewidth measurements of twelve main-sequence, sub-giant and red-giant stars.

**STOCHASTIC EXCITATION**

Because of the lack of a complete model for convection, the mixing-length formalism still represents the main method for computing the turbulent fluxes in the convectively unstable layers in a star. One of the assumptions in the mixing-length formulation is the Boussinesq approximation, which results in neglecting the acoustic wave generation by assuming the fluid to be incompressible.
right-hand-side of equation \([19]\) arise from the fluctuating Reynolds stresses

\[ F(u) = \nabla \cdot (\rho uu - \langle \rho uu \rangle) \] (21)

and from the fluctuating gas pressure (due to the fluctuating buoyancy force), represented by \(G(s')\), where \(s'\) is the Eulerian entropy fluctuation (Bohn 1984; Osaki 1990; Goldreich & Kumar 1990; Balmforth 1992b; Goldreich, Murray & Kumar 1994; Samadi & Goupil 2001). The latest numerical simulations by Stein et al. (2004) suggest that both forcing terms in equation \([19]\) contribute to the energy supply rate \(P_1\) by about the same amount, a result that was also reported by Samadi et al. (2003) using the turbulent velocity field and anisotropy factors from numerical simulations (Stein & Nordlund 2001). In this paper we consider only the term of the fluctuating Reynolds stresses and because we use only radial modes, only the vertical component \(F_3\) of \(F\) is important,

\[ F_3(u_3) \simeq \frac{\partial}{\partial r}(\rho u_3^2 - \langle \rho u_3^2 \rangle). \] (22)

If we define the vertical component of the velocity correlation as \(R_{33} = \langle u_3 u_3 \rangle\), its Fourier transform \(\tilde{R}_{33}\) can be expressed in the Boussinesq-quasi-normal approximation (e.g. Batchelor 1953) as a function of the turbulent energy spectrum function \(E(k, \omega)\):

\[ \tilde{R}_{33} = \frac{\Psi E(k, \omega)}{12\pi k^2}, \] (23)

where \(k\) is a wavenumber and \(\Psi\) is an anisotropy parameter given by

\[ \Psi = [2\Phi/(3(\Phi - 1))]^{1/2}, \] (24)

which is unity for isotropic turbulence (Chaplin et al. 2005). This factor was neglected in previously published excitation models but it has to be included in a consistent computation of the acoustic energy supply rate. Following Stein (1967) we factorize the energy spectrum function into \(E(k, \omega) = \tilde{E}(k) \Omega(\omega; \tau_\lambda)\), where \(\tau_\lambda = \lambda/ku_\lambda\) is the correlation time-scale of eddies of size \(\pi/k\) and velocity \(u_\lambda\); the correlation factor \(\lambda\) is of order unity and accounts for uncertainties in defining \(\tau_\lambda\).

The energy supply rate is then given by (see Chaplin et al. 2005 for details)

\[ P_1 = \frac{\pi}{9} \int_0^R \int_0^{\ell^3} R \Phi \Psi \rho_{\text{hr}} \frac{\partial}{\partial r}(\rho \xi) \cdot \mathcal{S}(r, \omega) \, dr, \] (25)

with

\[ \mathcal{S}(r, \omega) = \int_0^{\infty} \kappa^{-2} \tilde{E}^2(\kappa) \tilde{\Omega}(\tau_\lambda, \omega) \, d\kappa, \] (26)

where \(\kappa = k\ell/\pi\), \(\ell\) is the mixing length, \(R\) is surface radius, and \(\xi_r\) is the normalized radial part of \(\xi\). The spectral function \(\mathcal{S}\) accounts for contributions to \(P_1\) from the
small-scale turbulence and includes the normalized spatial turbulent energy spectrum $\bar{E}(k)$ and the frequency-dependent factor $\Omega(k;\tau)$. For $\bar{E}(k)$ it has been common to adopt either the Kolmogorov (Kolmogorov 1941) or the Spiegel spectrum (Spiegel 1962). The frequency-dependent factor $\Omega(k;\tau)$ is still modelled in a very rudimentary way and we adopt two forms:

- the Gaussian factor (Stein 1967),

$$\Omega_G(\omega;\tau) = \frac{\tau}{\sqrt{2\pi}} e^{-\left(\omega\tau_0/\sqrt{2}\right)^2}; \quad (27)$$

- the Lorentzian factor (Gough 1977b; Samadi et al. 2003; Chaplin et al. 2005),

$$\Omega_L(\omega;\tau) = \frac{\tau_0}{\pi\sqrt{2\ln2}} \frac{1}{1 + (\omega\tau_0/\sqrt{2\ln2})^2}. \quad (28)$$

The Lorentzian frequency factor is a result predicted for the largest, most-energetic eddies by the time-dependent mixing-length formulation of Gough (1977b). Recently, Samadi et al. (2003) reported that Stein & Nordlund’s hydrodynamical simulations also suggest a Lorentzian frequency factor, which decays more slowly with depth $z$ and frequency $\omega$ than the Gaussian factor. Consequently, a substantial fraction to the integrand of equation (25) arises from eddies situated in the deeper layers of the Sun, resulting in a larger acoustic excitation rate $P_f$.

**Oscillation Amplitudes**

Model predictions of the mode height $H$ were computed according to equation (14). As in the solar case damping rates for other stars are obtained from solving the eigenvalue problem (15) and the energy supply rates are calculated from expression (25). With these estimates for $\eta$ and $P_f$ Houdek & Gough (2002) predicted velocity amplitudes for several stars, using equation (13). Results for stochastically excited oscillation amplitudes and linear damping rates in the solar-like star $\alpha$ Cen A and in the sub-giant $\xi$ Hydrae are illustrated in Figs 10 and 11. Kjeldsen et al. (2005) reported mode lifetimes for
for the intensity amplitudes inferred from a narrow-band observation of wavelength \( \lambda \), i.e. \( \langle \delta L/L \rangle \sim (L/M)/T_{\text{eff}}^2 \), leading to

\[
H \propto g^{-2}
\]

for the maximum mode height. Since the surface gravity changes fairly slowly along the main sequence, the new scaling relation \((29)\) for \( H \), which assumes narrow-band intensity observations, suggests that stars notably cooler than the Sun might have mode heights that are comparable to those solar-like pulsators that are hotter than the Sun. Moreover, Hekker et al. (2009) reported that the intensity heights in about 780 red giant stars, observed in broadband photometry with the CoRoT satellite, follow the scaling law in about 780 red giant stars, observed in broadband photometry with the CoRoT satellite, follow the scaling law \((29)\) by Chaplin et al. 2009.

### STELLAR-CYCLE EFFECTS

From helioseismic data, such as those provided by the BiSON, we have learnt that not only the oscillation frequencies change with time over the 11-year solar cycle but also the height \( H \) and width \( \Gamma \). In the solar case the absolute fractional change from solar activity minimum to maximum in \( \langle H \rangle \) (angular brackets \( \langle \rangle \) denote an average over the five most prominent \( p \) modes) is about 40%. That in \( \langle \Gamma \rangle \) is about 20% (Chaplin et al. 2000). We therefore expect also in solar-like oscillators not only the mode frequencies but also the mode heights and linewidths to show stellar-cycle variations.

Chaplin et al. (2007, 2008) estimated stellar-cycle frequency shifts and mode height variations for a grid of 31 models with masses between 0.7 \( M_\odot \) and 1.3 \( M_\odot \) and stellar ages, \( t_* \), in the range from the ZAMS to 9 Gyr, using the Padova isochrones (e.g., Bonatto et al. 2004) to specify mass, radius, effective temperature, \( T_{\text{eff}} \), and luminosity. The evolutionary tracks of these models are presented in the left panel of Fig. 13 in terms of a \((L/*T_{\text{eff}})\) plot. To predict the stellar-cycle changes Chaplin et al. used the Ca II H&K index for surface activity in stars. This index is usually expressed as \( R_{\text{HK}} \), the average fraction of the star’s total luminosity that is emitted in the H&K line cores. The authors then used the data of 22 main-sequence stars collected by the Mount Wilson Ca II H&K programme (e.g., Saar & Brandenburg 2002) from which the \( R_{\text{HK}} \) cycle amplitude values, \( \Delta R_{\text{HK}}' \), were determined. The \( \Delta R_{\text{HK}}' \) values were then simply scaled against the 0.4 \( \mu \)Hz frequency shift seen for the most prominent low-\( l \) modes in the Sun, assuming that the frequency shifts \( \delta v_{\text{cyc}} \) scale approximately linearly with \( \Delta R_{\text{HK}}' \). In order to estimate the frequency shifts of the 31 grid models, it was necessary to calculate first the \( R_{\text{HK}}' \) values for the 31 models. This was done according to the procedure by Noyes (1983) and Noyes et al. (1984). The \( R_{\text{HK}}' \) values so obtained were then used to estimate the corresponding cycle frequency shifts by

\[ \frac{\delta v_{\text{cyc}}}{v_{\text{cyc}}} = \frac{\Delta R_{\text{HK}}'}{R_{\text{HK}}'} \]
interpolating linearly in the \((\delta \nu_{\text{cyc}}, R'_K)\) table of the 22 main-sequence stars observed by the Mount Wilson Ca II H\&K programme (for a detailed discussion see Chaplin et al. 2007). The outcome of this procedure for estimating the stellar-cycle frequency shifts of the most prominent p modes in the stellar models is depicted in the right panel (left ordinate) of Fig. 13. Also shown in this figure are the predicted fractional changes in \(<H>\) (right ordinate), obtained in a similar way as the frequency shifts and assuming a fractional change of \(<H>\) of 40\% for the Sun. These results suggest that, in the considered age range of 1 – 9 Gy, the variation of p-mode frequencies and heights over the stellar cycle can be up to 1.5 – 2 times larger than in the Sun, and that these variations depend predominantly on stellar age and less on \(T_{\text{eff}}\) or stellar mass.

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