Order and mobility of solid vortex matter in oscillatory driving currents

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We study numerically the evolution of the degree order and mobility of the vortex lattice under steady and oscillating applied forces. We show that the oscillatory motion of vortices can favor an ordered structure, even when the motion of the vortices is plastic when the same force is applied in a constant way. Our results relate the spatial order of the vortex lattice with its mobility and they are in agreement with recent experiments. We predict that, in oscillating applied forces, the lattice orients with a principal axis perpendicular to the direction of motion.

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Two limiting cases have been distinguished in the depinning process of a steady driven vortex lattice (VL) in type-II superconductors. For a high density of weak short-range pinning centers, the motion of the VL is inhomogeneous only in a narrow region near the critical force, $F_c$ (which is determined by the collective pinning theory [1]), whereas for strong pinning centers (or a small concentration of them) plastic deformations become important and the motion is disordered [2, 3]. In the last case, the VL orders and shows elastic flow only for high enough driving forces [4, 5, 6] (above a threshold force, $F_T$).

Recently, there has been much interest in the evolution of the order and mobility of the VL for different thermomagnetic histories and different temporal dependencies of the driving force, $F^L$, which is provided by a flowing current. This is motivated by the observation of a variety of puzzling phenomena that include history-dependent dynamic response and memory effects [1, 2, 3]. The nature of the ac response observed in transport and ac susceptibility measurements has made clear that it is not possible to directly extrapolate the force-dependent evolution of a steady driven VL to explain the new experimental findings for oscillatory forces. Some results are intrinsic to the oscillatory dynamics. In particular, recent experiments suggest that while for a given intensity of the dc current the VL disorders, an ac current of the same magnitude assists the VL in ordering [2, 3]. This conclusion, however, is indirectly drawn from changes in the VL mobility. A key issue that naturally surges is the microscopic dynamics of vortices and the relation between VL’s order and mobility with different temporal evolutions of the driving force. Nevertheless, in spite of the growing amount of experimental results on the ac dynamics of vortices, this relation has not been investigated numerically up to now.

In this Letter, we present a numerical study that addresses simultaneously the order and mobility of the VL in dc and ac driving forces. We show that many of the experimental observations can be reproduced for a VL around the crossover between plastic and elastic regimes. For $F^L < F_c$, the VL is at rest whereas for $F^L > F_T$ the vortex motion is ordered. For $F_c < F^L < F_T$, if the force is applied steadily, the vortex motion is always plastic and disordered but, if it is applied in an alternating way, it can bring the VL to a more ordered state. We also find that a lower density of defects in the VL is concomitant with a higher mobility and that the VL tends to order with its principal lattice vector aligned perpendicular to the direction of motion in contrast with the orderly array of vortices in the steady force case [6]. All of these observations may have important implications on the interpretation of recent neutron scattering experiments [1].

We consider a transverse 2D slice (in the $x - y$ plane) of an infinite superconducting slab that contains rigid vortices that are parallel to the sample surface ($\mathbf{H} = H\hat{z}$). The overdamped equation of motion of a vortex in position $\mathbf{r}_i$ is $\dot{\mathbf{r}}_i = \sum_{j \neq i}^{N_v} \mathbf{F}^{vv}(\mathbf{r}_i - \mathbf{r}_j) + \sum_k^{N_p} \mathbf{F}^{vp}(\mathbf{r}_i - \mathbf{r}_k^p) + \mathbf{F}^L + \mathbf{F}^T_i = \eta \frac{d\mathbf{r}_i}{dt}$, where $\mathbf{F}_i$ is the total force on vortex $i$ due to vortex-vortex interactions ($\mathbf{F}^{vv}$), pinning centers ($\mathbf{F}^{vp}$), the driving current $\mathbf{J}(\mathbf{F}^L = \phi_0 J \times \hat{z})$ and thermal fluctuations ($\mathbf{F}^T_i$). Here, $\eta$ is the Bardeen-Stephen friction coefficient, $N_v$ the number of vortices, $N_p$ the number of pinning sites and $\mathbf{r}_k^p$ the location of the $k$th pinning center. The vortex-vortex interaction is modelled by a modified Bessel function $\mathbf{F}^{vv}(\mathbf{r}_i - \mathbf{r}_j) = \frac{\phi_0^2}{8\pi^2}\lambda^2 A_v K_1((\mathbf{r}_i - \mathbf{r}_j)/\lambda)\hat{r}_{ij}$ where the dimensionless prefactor, $A_v$, tunes the rigidity of the VL [2]. We cut off this interaction at $4\lambda$. Quenched random disorder is modelled by pinning centers in uncorrelated random positions that exert an attractive force on vortices: $\mathbf{F}^{vp}(\mathbf{r}_i - \mathbf{r}_k^p) = -A_p e^{-\left(\frac{r_{ik}}{\sigma_p}\right)^2} \mathbf{r}_{ik}$. $A_p$ tunes the strength of this force and $\sigma_p$ its range. The thermal force was implemented with a Box-Muller random number generator and has the properties $\langle F_{ip}^T(t) F_{jp}^T(t') \rangle = 2 \eta k_B T \delta_{ij} \delta_{\mu\nu} \delta(t - t')$ for a given temperature $T$. We normalize length scales by $\lambda$ and forces by $f_0 = \frac{\phi_0^2}{8\pi^2\lambda^2}$. We consider a sample with periodic boundary conditions in the $x - y$ plane and size $L_x \times L_y$.
with \( L_y/L_x = \sqrt{3}/2 \). The normalized vortex density is \( n_v = N_v X^2/L_x L_y = B X^2/\phi_0 \). We choose \( n_v = 1 \) and a much higher density of pinning centers \( n_p = 25 \) with \( r_p = 0.2 \lambda \) so that the resulting random pinning potential varies smoothly on a length scale of the order of, or higher than, the coherence length, \( \xi \) (< \( \lambda/10 \)). \( A_p \) has a Gaussian distribution with central value \( A_p^m = 0.02 f_0 \) and a standard deviation of 0.1\( A_p^m \). System sizes from 400 to 1600 vortices have been investigated. We show results for 1600 vortices and \( 4 \times 10^8 \) pinning sites and we set \( T \sim 0 \) (no thermal noise) unless noted to the contrary.

History effects have been frequently associated with changes in the density of defects in the VL. If this were the case, information about the history of the sample could not be retained either by a perfectly ordered VL or by a completely amorphous one, i.e. it would be required a balance between the elastic and pinning energies. In dynamical terms, this implies that, in samples that present a path dependent response, the VL would be in the region of the plastic to elastic crossover \( \mathbb{P} \mathbb{E} \mathbb{I} \).

We start by identifying this crossover by looking at the degree of order of the VL which are attained by changing \( A_v \). Moving vortices induce a total electric field \( \mathbf{E} = \hat{\mathbf{z}} \times \mathbf{v} \), with \( \mathbf{v} = \frac{1}{2} \sum_i \frac{\partial r_i}{\partial t} \). To quantify the degree of order of the VL we compute the structure factor, \( S(k) = \frac{1}{N} \sum_{i=1}^N e^{i k r_i} \), and determine the average concentration of vortices with coordination number not equal to 6, \( n_{\text{def}} \), using the Delaunay triangulation procedure. As stated in Ref. \( \mathbb{4} \), in the plastic regime there is a marked upward curvature in the \( V-I \) curve which is not present in the elastic region. In addition, the threshold force \( F_r \) diminishes with \( A_v \) and when approaching to the elastic regime it mingles with \( F_c \). This provides a simple criterion to identify the crossover that, in our case, takes place at \( A_v \sim 0.85-0.9 \). Following this result, we perform the simulations at fixed \( A_v = 0.8 \), which is in the region of the crossover but in the plastic side \( \mathbb{P} \mathbb{E} \mathbb{I} \).

In order to study the oscillatory dynamics and, at the same time, to be able to directly compare the obtained results with those of steady forces, we choose square oscillating forces of strength \( F^L \) and frequency \( \omega = 2 \pi / P \) \( (F^L = F^L \tilde{\kappa}) \). For steady driving forces between \( F_c \sim 0.0085 f_0 \) and \( F_T = 0.012 f_0 \) the vortex flow is always disordered with a high density of defects, \( n_{\text{def}} \) (of the order of 20%). For higher driving forces, \( n_{\text{def}} \) diminishes and the movement is ordered with all of the vortices moving at the same average velocity \( \mathbb{P} \mathbb{E} \mathbb{I} \mathbb{E} \mathbb{I} \). Experimental results \( \mathbb{4} \) suggest that when vortices perform an oscillatory motion whose amplitude is of the order of the lattice constant, the healing of defects should be important. For this reason, we initialize vortex motion \( F^L = 0.0118 f_0 \) and \( P \) so that \( F^L P/4 \sim 1 \) (in normalized units).

Some of the main results of our investigation are presented in Fig. \( \mathbb{5} \) and \( \mathbb{6} \). Fig. \( \mathbb{5} \) clearly pictures the healing of defects that results from the oscillatory motion of vortices. We start with a perfectly ordered VL. Then we apply a steady force \( F^L = 0.0118 f_0 \) in the horizontal direction. The left panel shows a snapshot of the configuration of the VL after reaching a stationary state in which the average velocity of vortices and the average concentration of defects remained constant. This state presents a relatively high concentration of defects. We then turn to an alternating driving force which reverses periodically but keeps the same strength \( (0.0118 f_0) \). This alternating force is applied to the disordered state in the left panel. After 500 cycles, it obtained the configuration of the VL shown in the right panel. The corresponding structure factors of both configurations are also shown. The Delaunay triangulation and the structure factor reveal an important reduction in \( n_{\text{def}} \) when the oscillatory force is applied. It is fundamental to emphasize that the initial state was obtained with a steady force with the same strength starting with a perfectly ordered VL. This implies that, for certain values of the applied force, the flow of vortices can be plastic and disordered introducing defects in the VL but, if the same force is applied in an alternating way, the defects heal and the VL reordered.

In the Fig. \( \mathbb{5} b \), we show the evolution of this reordering. We plot the concentration of defects, \( n_{\text{def}} \), the average value of the absolute velocity, \( V \), and the average quadratic displacements of vortices per cycle, \( (\Delta^2 x_N) \) and \( (\Delta^2 y_N) \), as a function of the number of cycles, \( N \). We define \( (\Delta^2 x_N) = \frac{1}{x_N} \sum_{i=1}^{x_N} [(x_i(N) - \bar{x}_i(N-1))^2 \) and \( (\Delta^2 y_N) = \frac{1}{x_N} \sum_{i=1}^{x_N} [(y_i(N) - \bar{y}_i(N-1))^2 \), where \( \bar{x}_i(N-1) = x_i(N-1) + \Delta X_{cm}(N) \) and \( \bar{y}_i(N-1) = y_i(N-1) + \Delta Y_{cm}(N) \) with \( (\Delta X_{cm}(N), \Delta Y_{cm}(N)) \) a correction due to the displacement of the center of mass in the cycle \( N \). The results shown correspond to the average over 4 different random distributions of the pinning centers. We see that, for an increasing number of cycles, \( n_{\text{def}} \) clearly decreases and the vortex mobility is enhanced (as reflected by a 40% increase in the average vortex velocity). We also note that \( (\Delta^2 x_N, y_N) \) diminish and that all magnitudes vary in an approximately logarithmic way as observed experimentally \( \mathbb{4} \). The combination of these observations indicates that, as the number of defects decreases, vortices are clearly more mobile and that, after completing one oscillation, they return closer to their original positions \( (\Delta^2 x_N) \) and \( (\Delta^2 y_N) \) decrease. This suggests that vortices organize and move more coherently as they are forced to oscillate.

In Fig. \( \mathbb{5} b) \), we show what happens when we vary \( P(F^L) \) while keeping \( F^L(P) \) fixed. We plot \( n_{\text{def}} \) as a function of the parameter \( F^L/P \) after applying 100 cycles of the square force to an initial disordered state as in Fig. \( \mathbb{5} \) and Fig. \( \mathbb{5} b) \). An important result is that, if the period \( P \) is increased so that the excursion of vortices greatly exceeds the lattice constant, the VL distorts and does not reorder. This is not surprising because if the
amplitude of the oscillation is high enough, the system should behave in the same way as when driven by steady forces. This is a prediction that should be observed in experiments as those in Ref. [3]. For low enough \( P \), vortices oscillate in their pinning sites and the VL does not reorder either.

Notably, if there is a tiny asymmetry in the amplitude of the square force (\( \sim 5\% \)) the VL quickly disorders and, after a few cycles, both \( n_{\text{def}} \) and the vortex mobility reach values close to those found with steady forces. This observation is reminiscent to the experimental results reported in Ref. [4] where the vortex lattice is observed to quickly become less-mobile when shaken by a sawtooth magnetic field. The reduction in the mobility of the VL has been attributed to disorder produced by a ratchet-like tearing of the VL due to the different Lorentz forces involved in the ramp up and down of the field [3]. We should note, however, that in our simulations there is a net displacement of vortices after completing one cycle of the asymmetric waveform that is not expected to occur in the experiments.

An interesting issue surges in relation to the orientation of the VL. At high-enough steady forces (\( F^L > F_T \)), a VL’s principal vector usually aligns with the direction of motion [4]. Though the details of this phenomenon are not completely understood, Schmid and Hauger have argued that such an orientation is preferred because it minimizes power dissipation [5]. They focused on the vortex motion at high velocities so that the effect of the pinning potential can be handled as a small perturbation.

It is less clear what would happen if the effect of pinning were more important and the plastic distortions or the elastic energy involved during the vortex motion increased. The perturbative solution would fail to correctly describe the vortex behavior and the elastic energy of the VL might play a preponderant role. Indeed, in our simulations we find that, during the dynamical reordering induced by symmetrical forces, a VL’s principal vector is aligned in most cases perpendicular to the direction of motion (see Fig. [1]). In order to show that this phenomenon is not due to the boundary conditions or to a particular distribution of the randomly located pinning centers, we performed a series of simulations where we slowly cooled the VL from \( T \approx T_m \) down to \( T = 0 \) while the VL was subjected to the square force in different fixed directions. The temperature was decreased in small steps after each cycle. After 600 cycles, \( T = 0 \). The resulting structure factor of the VL at \( T = 0 \) is shown in Fig. [1b] for three different directions of the applied force (indicated by the white line). The distribution of defects is the same for the three cases. The clear tendency of the lattice to choose a transverse orientation (note that the structure factor is rotated 90° in relation to the real lattice) strongly suggests that this phenomenon is not the result of either the boundary conditions or an orientation favored by the particular distribution of defects. In fact, the preferred orientation of the VL can be understood with a simple energetic argument [5] [4]. In our calculations, the vortex motion tends to be disordered and neighboring vortices move at different average velocities. Some of the vortices can be pinned and might not even move during a complete oscillation. The oscillatory force produces a back-and-forth movement of vortices and a consequent repeated interaction between neighbors. If the overlap of vortices is avoided, the average elastic energy during one period of the oscillation will be small.
This favors an ordered structure. Moreover, the most favorable condition will be the one that minimizes the elastic energy. Because of that, we believe that the elastic energy is a good quantity to estimate the stability of the different orientations of the VL. To this end, we consider one single vortex moving a distance $\delta x$ relative to its neighbors, or vice versa, and we calculate the restitutive force for different orientations [Fig. 3b)]. From this calculation, it is straightforwardly seen that the lower elastic energy will correspond to the lattice with the principal axis transversal to the direction of motion, as observed in simulations. The same conclusion is drawn if we consider the distortions that are introduced in a nonrigid lattice.

Similar arguments have been used to explain the collective motion of colloidal particles in oscillatory shear flow [15]. The main and very important qualitative difference between the colloidal and vortex systems is the presence of pinning centers in the latter. However, the pinning centers favor an oscillatory shear even when the applied force is uniform and, as a consequence, the underlying mechanism of ordering is probably the same in both systems.

In conclusion, we performed numerical simulations on the dynamics of the vortex lattice that emphasize the intrinsic properties of the oscillatory motion. We show that the vortex lattice may order when forced to move with an oscillating force, even when the motion that produces a steady force with the same absolute value is plastic and disordered. As the vortex lattice orders, its mobility increases and it moves in an increasingly coherent way. The number of defects in the lattice and its mobility vary as the logarithm of the number of cycles of the oscillating force. The lattice orders because an ordered structure avoids the overlap of neighboring vortices. For the same reason, the lattice orients with one of its principal vectors perpendicular to the direction of the force.

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[1] A.I. Larkin and Y.N. Ovchinnikov, J. Low. Temp. Phys. 34, 409 (1979).
[2] H. Jensen et al., Phys. Rev. Lett. 60, 1676 (1988); A. Brass, et al., Phys. Rev. B 39, 102 (1989).
[3] A.E. Koshelev, Physica C 198, 371 (1992).
[4] P. Thorel et al., J. Phys. (Paris) 34, 447 (1973); S. Bhattacharya and M.J. Higgins, Phys. Rev. Lett. 70, 2617 (1993); U. Yaron et al., Nature 376, 753 (1995); M.C. Hellerqvist, et al., Phys. Rev. Lett. 76, 4022 (1996); W. Henderson et al., Phys. Rev. Lett. 77, 2077 (1996); A. Duarte et al., Phys. Rev. B 53, 11336 (1996).
[5] A.E. Koshelev and V.M. Vinokur, Phys. Rev. Lett. 73, 3580 (1994); M.C. Faleski et al., Phys. Rev. B 54, 12427 (1996); K. Moon et al., Phys. Rev. Lett. 77, 2778 (1996); S. Ryu et al., Phys. Rev. Lett. 77, 5114 (1996); S. Scheid and V.M. Vinokur, Phys. Rev. B 57, 13800 (1998). See also T. Giamarchi and P. Le Doussal, Phys. Rev. Lett. 76, 3408 (1996); P. Le Doussal and T. Giamarchi, Phys. Rev. B 57, 11356 (1998); L. Balents et al., Phys. Rev. Lett. 78, 751 (1997) and Phys. Rev. B 57, 7705 (1998); D. Domínguez, Phys. Rev. Lett. 82, 181 (1999); A.B. Kolton et al., Phys. Rev. Lett. 83, 3061 (1999).
[6] C.J. Olson et al., Phys. Rev. Lett. 81, 3757 (1998).
[7] W. Henderson et al., Phys. Rev. Lett. 81, 2352 (1998); E.Y. Andrei et al., J. Phys. IV 10, 5 (1999); Y. Paltiel et al., Nature 403, 398 (2000).
[8] S.N. Gordeev et al., Nature 385, 324 (1997); Z.L. Xiao et al., Phys. Rev. Lett. 83, 1664 (1999); S.O. Valenzuela and V. Bekeris, Phys. Rev. Lett. 84, 4200 (2000).
[9] S.O. Valenzuela and V. Bekeris, Phys. Rev. Lett. 86, 504 (2001); Phys. Rev. B 65, 134513 (2002).
[10] A. Schmid and W. Hauger, J. Low. Temp. Phys. 11, 667 (1973). See also A.T. Fiory, Phys. Rev. Lett. 27, 501 (1971).
[11] X.S. Ling et al., Phys. Rev. Lett. 86, 712 (2001); H. Safar et al. (unpublished).
[12] This value of $A_s$ is chosen to keep the correlation length of the VL smaller than the size of the system.
[13] Here $T_m$ is the melting temperature of the undisturbed VL.
[14] M.J. Rozenberg, private communication.
[15] B.J. Ackerson and P.N. Pusey, Phys. Rev. Lett. 61, 1033 (1989); W. Xue and G.S. Grest, Phys. Rev. Lett. 64, 419 (1990); H. Komatsugawa and S. Nosé, Phys. Rev. E 51, 5944 (1995).