Electron and electron-hole quasiparticle states in a driven quantum contact

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(Dated: January 12, 2016)

We study the many-body electronic state created by a time-dependent drive of a mesoscopic contact. The many-body state is expressed manifestly in terms of single-electron and electron-hole quasiparticle excitations with the amplitudes and probabilities of creation which depend on the details of the applied voltage. We experimentally probe the time dependence of the constituent electronic states by using an analog of the optical Hong-Ou-Mandel correlation experiment where electrons emitted from the terminals with a relative time delay collide at the contact. The electron wave packet overlap is directly related to the current noise power in the contact. We have confirmed the time dependence of the electronic states predicted theoretically by measurements of the current noise power in a tunnel junction under harmonic excitation.

PACS numbers: 72.70.+m, 72.10.Bg, 73.23.-b, 05.40.-a

Recent years have seen a tremendous experimental and theoretical progress in the emerging field of electron quantum optics.1 Following the example of optics, the quantum nature of electronic transport has been demonstrated in electronic Mach-Zehnder interferometers2 and Hanbury Brown-Twiss3,4 and Hong-Ou-Mandel5,6 intensity correlation experiments. Although quantum optics with electrons is in general analogous to the one with photons, there are important distinctions between the two due to differences in particle statistics, vacuum state photons, there are important distinctions between the Fermi sea vs. photonic vacuum), interaction between electrons, decoherence, etc. In particular, a simple constant voltage source can act as a single-electron turnstile due to the Fermi statistics which is responsible for regular emission of electrons on a time scale $h/eV$, where $e$ is the electron charge, $h$ is the Planck constant, and $V$ is the dc voltage drop over the conductor.

A step forward towards electron quantum optics has been made recently with the realization of on-demand electron sources7,8,9,10,11 which can create single- to few-particle excitations11,12,13. This facilitates the full control of the quantum state of electrons in mesoscopic conductors and the dynamical control of elementary excitations using suitably tailored voltage pulses.14 In particular, time-dependent drive creates quasiparticle excitations in the Fermi sea that are single-electron and electron-hole pairs whose number and probability of creation depend on the shape and the amplitude of the applied voltage15 Lorentzian pulses $V(t)$ of a quantized area $\int eV(t)dt/h = N$ ($N$ is an integer) are special as they create N electrons above the Fermi level leaving the rest of the Fermi sea unperturbed.16 Experimentally, the presence of electron-hole pairs in the system can be seen in the zero-frequency photon-assisted current noise power which is increased with respect to the dc noise level20,21. More recently, quantum noise oscillations have been observed in a driven tunnel junction22 and noise spectroscopy using a more complex biharmonic voltage drive has been carried out approximating Lorentzian pulses.17 The creation of single-electron excitations has been realized experimentally18 and the resulting quantum states have been reconstructed using the quantum state tomography.19

Even though the progress has been exceptional, a fundamental question remains: What is the many-body electronic state created in the Fermi sea by a voltage drive? In this article we find that the many-body state is:

$$|\Psi\rangle = \hat{\mathcal{C}}^\dagger \prod_k \left( \sqrt{1 - \hat{p}_k} + i\sqrt{\hat{p}_k} \hat{A}_k^\dagger \hat{B}_k \right) |F\rangle.$$  \hspace{1cm} (1)

Here, $\hat{\mathcal{C}}^\dagger = \int d\varepsilon \gamma^\dagger(\varepsilon)\hat{c}^\dagger(\varepsilon)$ is the creation operator of a single-electron quasiparticle state, $\hat{A}_k^\dagger = \int d\varepsilon \gamma^\dagger(\varepsilon)\hat{c}_k^\dagger(\varepsilon)$ and $\hat{B}_k = \int d\varepsilon \gamma(\varepsilon)\hat{c}_k(\varepsilon)$ are the operators that create electron and hole from the electron-hole pair, $\hat{c}_k^\dagger(\varepsilon)$ is the electron creation (annihilation) operators, and $|F\rangle$ is the filled Fermi sea. [We have assumed, for simplicity, that there is one single-electron quasiparticle created per period, which is the case for $eV_{dc}/\hbar\omega = 1$ where $V_{dc}$ is the dc voltage component and $\omega$ is the frequency of the drive.] In addition to a single-electron excitation, there is a number of electron-hole pairs created in the system (labelled by $k = 1, 2, \ldots$) due to the ac voltage component. The probabilities of the electron-hole pair creations $\hat{p}_k$ and the single-electron and electron-hole quasiparticle amplitudes $\gamma(\varepsilon)$ and $u_k^\dagger(\varepsilon)$ depend on the properties of the applied voltage. For optimal Lorentzian drive, there are no electron-hole pairs created ($\hat{p}_k = 0$) and the state has only single-electron excitations, as expected.

Quasiparticle amplitudes $\gamma$ and $u_k^\dagger$ in Eq. (1) also give the time-dependent probabilities of single-electron and electron-hole pair creations, $|\gamma(t)|^2$ and $|u_k^\dagger(t)|^2$. The time dependence of the wave functions that constitute the many-body state in Eq. (1) can be probed by an elec-
tronic analog of the optical Hong-Ou-Mandel correlation experiment, where electrons emitted from two terminals with a relative time delay collide at the contact. When the wave packets arrive at the contact simultaneously, their transmission in the same output channel is blocked by the Pauli principle, which suppresses the current fluctuations. The magnitude of the noise suppression is proportional to the wave-packet overlap at the contact. In the present paper, we have performed this experiment and measured current noise power in a tunnel junction driven by harmonic time-dependent voltage. We have found that the correlation noise as a function of a time delay is in agreement with the theoretically predicted quasiparticle amplitudes of electrons and electron-hole pairs in Eq. 1.

Let us consider a generic quantum contact with spin-degenerate transmission eigenvalues \( T_n \) that are independent of energy. The contact is driven by a voltage \( V(t) = V_{dc} + V_{ac}(t) \), where \( V_{dc} \) is a constant dc offset and \( V_{ac}(t) \) is a periodic ac voltage component with zero average and the period \( T = 2\pi/\omega \). The cumulant generating function of the charge transfer statistic is given by

\[
S(\chi) = 2\sum_n \text{Tr} \ln [f_L(1 - f_R)T_n(e^{i\chi X} - 1) + (1 - f_L)f_R T_n(e^{-i\chi X} - 1)].
\]

Here, \( f_L(\alpha) \) are generalized electronic distribution functions in the left (right) terminal which depend on two or more energy arguments:

\[
f_L = e^{-i\phi(t)} f_{ac}(t') e^{i\phi(t')} f_{dc}, \quad f_R = f(t' - t').
\]

Let us consider a Fermi sea of the right lead we have \( f = \sum_k f_k \) where \( f_k = f \tilde{P}_k \). The single-electron state \( \Psi \) is given by \( f v = \Psi \) and \( f v = 0 \).

The first term \( |\Psi| \langle \Psi | \) in Eq. 2 describes a single-electron state injected to the contact while \( f_k \) describe the electron-hole pairs. By taking diagonal in time components of \( |\Psi| \langle \Psi | \) we gain information on the time independence of single-electron and electron-hole pair wave functions. Similarly, by taking diagonal in energy components we gain information on the single-electron and electron-hole pair contributions in the overall distribution function. Indeed, for Lorentzian pulses \( V_{lor}(t) \) that carry a single charge quantum per cycle, there are no additional electron-hole excitations and the time-dependent probability of the single-electron injection is proportional to the drive, \( |v(t)|^2 = e V_{lor}(t)/\omega \). This is no longer true for a general time-dependent drive where \( |v(t)|^2 \neq e V(t)/\omega \) due to the presence of electron-hole pairs.

Before we proceed with the specific examples, let us bring \( f_k \) in a more transparent form. States \( \Psi_0 \) and \( \Psi_{-\alpha} \) in general possess both positive and negative energy components. We can make a rotation of the basis \( u_\alpha = (\Psi_0 \pm \Psi_{-\alpha})/\sqrt{2} \) in the subspace \( (\Psi_0, \Psi_{-\alpha}) \) such that new basis vectors \( u_\pm \) and \( u_0 \) possess non-zero components only for \( \alpha > 0 \) and \( \alpha < 0 \), respectively. Using electron-hole contributions only for \( \alpha > 0 \) and \( \alpha < 0 \), we gain information on the single-electron and electron-hole pair contributions in the overall distribution function.

Next we obtain a decomposition of \( f \) into single-electron and electron-hole states. As shown in Ref. 23, the notion of single-electron and electron-hole pair excitations is related to the eigenproblem of \( \{ h, \tilde{h} \} \equiv hh + hh \), where \( f = (1 - h)/2 \) and \( \tilde{f} = (1 - \tilde{h})/2 \). For integer \( e V_{dc}/\omega = N \), there is a \( N \)-dimensional subspace of \( \{ h, \tilde{h} \} \) spanned by \( N \) special vectors that are eigenvectors of both \( h \) and \( \tilde{h} \):

\[
hv = -v, \quad hv = v.
\]

This subspace corresponds to \( N \) electrons injected to the contact per voltage cycle. In addition, the operator \( \{ h, \tilde{h} \} \) has a series of two-dimensional subspaces that are spanned by vectors \( v_\alpha \) and \( v_{-\alpha} \equiv hv_\alpha \) which are given by \( hhv_\alpha = e^{i\alpha}v_\alpha \). The spaces \( (v_\alpha, v_{-\alpha}) \) correspond to the electron-hole pairs (labelled by \( k = 1, 2, \ldots \)) created per voltage cycle with the probabilities \( p_k = \sin^2(\alpha_k/2) \).

At zero temperature, \( f \) and \( \tilde{f} \) commute with \( \{ h, \tilde{h} \} \), that is, they reduce in the single-electron and electron-hole pair subspaces of \( \{ h, \tilde{h} \} \). For simplicity, we restrict our consideration to the case \( N = 1 \) in which there is only one electron injected per voltage cycle. The eigenproblem of \( \{ h, \tilde{h} \} \) defines a resolution of the identity

\[
\sum_k \tilde{P}_k = 1, \quad \text{where } |\Psi \rangle \langle \Psi | = \sum_k \tilde{P}_k = |\Psi_{\alpha_k} \rangle \langle \Psi_{\alpha_k} | + |\Psi_{-\alpha_k} \rangle \langle \Psi_{-\alpha_k} |.
\]

This defines a decomposition of \( f \) into single-electron and electron-hole contributions,

\[
\tilde{f} = |\Psi \rangle \langle \Psi | + \sum_k \tilde{f}_k,
\]

where \( \tilde{f}_k \equiv \tilde{P}_k/\sum_k \tilde{P}_k \).

Finally, we can analyze the meaning of the amplitudes \( u_{\pm} \) is manifest in Ref. 1 and can further be elaborated by taking diagonal in energy components of \( \tilde{f}_k \). For \( \alpha > 0 \) we find that \( \tilde{f}_k(\alpha) = p_k|u_{\pm}(\alpha)|^2 \). For \( \alpha < 0 \) it is more convenient to consider the distribution of holes \( \tilde{f}_k(h) \equiv \tilde{P}_k - \tilde{f}_k \).
time components \( \tilde{f}_k(t) = |u^k(t)|^2 \) and \( \tilde{f}^{(b)}_k(t) = |u^k(t)|^2 \). Apart from electron and hole states on the diagonal, \( \tilde{f}_k \) contains also the off-diagonal terms proportional to \( \sqrt{p_k(1-p_k)} \) that are responsible for mixing of the two, see Eq. (3). Electron-hole pairs with \( p_k \approx 0 \) give no contribution to the transport. On the other hand, for \( p_k \approx 1 \) the electron and the hole from a pair are practically decoupled from each other (off-diagonal mixing terms vanish), cf. Eq. (1). This can also be seen in the cumulant generating function which becomes a sum of independent electron and hole contributions.22

Next we study single-electron and electron-hole pair states for different voltage drives, see Fig. 1. Let us consider a harmonic drive \( V(t) = V_{dc} + V_0 \cos(\omega t) \) where the dc offset \( N = 1 \) is kept fixed while the amplitude \( V_0 \) of the ac component is varied. The drive is characterized by the coefficients \( a_n = J_n(eV_0/\omega) \) where \( J_n \) are the Bessel functions of the first kind. In addition to a single-electron state, there are also electron-hole pairs created in the system and they become more relevant for transport as the amplitude \( V_0 \) is increased. For \( eV_0/\omega \ll 2 \) there is only one electron-hole pair with \( p_1 = (1/2)\sum_{n=\infty}^{n=0} |a_n|^2 / N \) in addition to the single-electron state injected. For \( eV_0/\omega \ll 1 \), the probability \( p_1 \) practically vanishes and only a single-electron state remains. Time dependence of the single-electron wave packet \( |v(t)|^2 \) is shown in Figs. 1(a–c) in comparison to the voltage drive \( V(t) \). We find that for ac drive amplitudes much smaller than dc voltage bias, the electron-hole pair creation is not effective and the single-electron wave packet coincides with the drive, \( |v(t)|^2 \approx eV(t)/\omega \), see Fig. 1(a). For ac drive amplitudes comparable or larger than dc offset, the single-electron wave packet differs from the drive, see Figs. 1(b,c). In that case the electron-hole pairs become important and their wave packets together with the single-electron one ensure \( I(t) = GV(t) \), where \( G = (e^2/\pi) \sum_n T_n \). The wave functions \( |u_\pm(t)|^2 \) of an electron-hole pair are shown in Fig. 1(d) for the drive without dc bias \( (N = 0) \) which does not create single-electron states. The nonequilibrium distribution functions \( f(\varepsilon) \) for the voltage drives at hand are shown in Figs. 1(e–h) (solid curves) together with the approximations (dash-dotted curves) computed using the most important single-electron or electron-hole pair states in Figs. 1(a–d). From Figs. 1(b,c,e,f,g) we find that electron-hole pairs give a more significant contribution in time domain than in \( f(\varepsilon) \). Indeed, while the single-electron wave packets clearly differ from the voltage drive, the distribution function is nevertheless to a good accuracy given by a single-electron state, \( f(\varepsilon) \approx |v(\varepsilon)|^2 + \theta(-\varepsilon) \). This is because for small \( p_k \), the electron-hole pair functions \( f_k \) in Eq. (3) have dominant off-diagonal electron-hole mixing components (proportional to \( \sqrt{p_k} \)) which do not contribute to diagonal in energy distribution \( \tilde{f}_k(\varepsilon) \), while they do contribute to diagonal in time distribution.

The time dependence of the electronic wave functions can be accessed experimentally by studying current noise power in a setup where two voltage drives with time shift \( \tau \) are applied to the terminals, \( V_L(t) = V_{dc} + V_0 \cos(\omega t) \) and \( V_R(t) = V_L(t - \tau) \). This can be viewed as the electronic analog of the optical Hong-Ou-Mandel (HOM) experiment, in which electron wave packets emitted from the terminals with time delay \( \tau \) collide at the contact.29-30 In the analog HOM experiment, \( V_{dc} \) is the static bias voltage between the input and output ports, which is in our case only defined with respect to virtual output terminals. Because of the gauge invariance, our two-terminal setup is formally equivalent to the case of the voltage \( \delta V(t) = V_L(t) - V_R(t) \) applied only to the left lead with the right lead unperturbed. The current noise power as a function of the time delay \( \tau \) then reads \( S_2(\tau) = S_0 |\sum_{n=\infty}^{n=0} J_n^2 |(2eV_0/\omega) \sin(\omega \tau/2)|^2 \) where \( S_0 = (e^2/\pi) \sum_n T_n R_n \) with \( R_n = 1 - T_n \). To express \( S_2(\tau) \) in terms of the overlap of the wave packets, we proceed as follows. From the cumulant generating function \( S \) we find \( S_2(\tau) \propto \text{Tr}[(f_L - f_R)^2] = \text{Tr}[(f_L - f)^2 + (f_R - f)^2 - 2(f_L - f)(f_R - f)] \) where we have introduced the distribution function \( f \) of the unperturbed Fermi sea. Here, the first two terms on
the right-hand side give the noise when the voltage is applied to one lead only while the other lead is unperturbed. Both terms give the same contribution to the noise $S_L = S_R = S_0 \sum_{n=\infty}^{\infty} |n + N| |J_n^2(eV_0/\omega)|$ independent of $\tau$. The term $-\text{Tr}[2(f_L - f)(f_R - f)]$ gives the noise suppression due to wave packet overlap at the contact. The noise reads

$$S_2(\tau)/S_0 = (S_L + S_R)/S_0 - 2C(\tau),$$

where $C(\tau)$ is the overlap that we compute using Eqs. (2-3). For $p_k \ll 1$, the dominant contribution in $C(\tau)$ is the overlap of the single-electron wave functions $v(t)$ and $v'(t) \equiv v(t-\tau)$ injected from the leads,

$$C(\tau) \approx |\langle v|v' \rangle|^2 = \left| \int_0^T \frac{dt}{T} v^*(t)v(t-\tau) \right|^2.$$

The noise $S_2(\tau)$ is shown in Fig. 2(a) together with the noise computed using the most important single-electron wave packet overlap in Eq. (5). The corresponding voltage drives and the electron wave functions are shown in Figs. 2(a–c).

So far we have analyzed the single-electron wave packets. To probe the electron-hole states we can use the voltages $V_L(t) = V_{dc} + V_{OL} \cos(\omega t)$ and $V_R(t) = V_{OR} \cos(\omega(t-\tau))$ which create single-electron wave packets at the left lead and electron-hole pairs at the right lead. The current noise power in this case is given by $S_2(\tau) = S_0 \sum_{n=\infty}^{\infty} |n + N| |J_n^2(eV_0/\omega)|$, where $V_0(\tau) = [V_{OL}^2 + V_{OR}^2 - 2V_{OL}V_{OR} \cos(\omega t)]^{1/2}$. In terms of the overlap, the noise is given by Eq. (4) where $S_L = S_0 \sum_{n=0}^{\infty} |n + N| J_n^2(eV_0/\omega)$, $S_R = S_0 \sum_{n=0}^{\infty} |n| J_n^2(eV_0/\omega)$, and $C(\tau) \approx p_R^2|\langle v|u'_n \rangle|^2$ is the overlap between the single-electron state $v$ at the left lead and the electron part $u'_n$ of the most dominant electron-hole pair $u_n$ generated in the right lead ($p_R = 0.630$). The noise $S_2(\tau)$ in this case is shown in Figs. 2(b–c); the corresponding single-electron and electron-hole wave functions are shown in Figs. 2(a,b,d).

To verify this picture, we have measured $S_2(\tau)$ in a tunnel junction under harmonic excitation with $\omega/2\pi = 20$ GHz (see Supplemental material). Experimental results are shown in Figs. 2(d–f), in agreement with the current noise power obtained theoretically. This proves that single-electron and electron-hole excitations in Eq. (1) can be created by time-dependent voltage and accessed experimentally in a noise correlation experiment which measures the overlap of the electronic wave functions.

In conclusion, we have obtained the many-body electronic state created by a time dependent drive of a quantum contact in terms of single-electron and electron-hole quasiparticle excitations. We have confirmed our theoretical predictions by probing the constituent quasiparticle states in a HOM-type experiment on a tunnel junction. The knowledge of the many-body state opens a way of engineering the required time profile or energy distribution of single-electron and electron-hole excitations. Since the electronic state in a conductor determines the electromagnetic field it generates, our work can be used to produce non-classical states of electromagnetic field, such as squeezed or entangled photonic states that have been observed recently.

ACKNOWLEDGMENTS

We are grateful to Lafe Spieitz for providing us with the sample. We acknowledge Z. Radović and M. Aprili for valuable discussions. The research was supported by the Serbian Ministry of Science Project No. 171027, the bilateral project CNRS – MESTD, ANR-10-LABX-0039-PALM, ANR-11-JS04-006-01, DFG through SFB 767, the German Excellence Initiative through CAP, and the Canada Excellence Research Chairs program.

FIG. 2. Noise $S_2$ as a function of a time delay $\tau$ between two harmonic signals applied at the leads: (a) $N = 1$ and $eV_0/\omega = 1.5, 1, 0.5$ (solid lines, top to bottom); (b) left lead: $N = 1, eV_0/\omega = 0.5$; right lead: $N = 0, eV_0/\omega = 2$. (c) left lead: $N = 1, eV_0/\omega = 1$; right lead: $N = 0, eV_0/\omega = 2$. Approximations for $S_2$ calculated using the overlaps $C(\tau)$ of the wave functions depicted in Fig. 1 are shown for comparison (dash-dotted lines). (d)–(f) $S_2(\tau)$ measured in a tunnel junction at $\omega/2\pi = 20$ GHz (symbols). Theoretical results are shown for temperatures $T_c = 0$ (solid lines) and $T_c = 0.1\omega$ (dashed lines).
Supplemental Material for
“Electron and electron-hole quasiparticle states in a driven quantum contact”

I. EXPERIMENTAL SETUP

In order to generate a single electron state by applying a periodic voltage bias \( V(t) = V_{dc} + V_0 \cos(\omega t) \) on a quantum contact, the experiment has to be performed in the quantum regime, that is, \( \hbar \omega \gg k_B T \) at low temperature \( T \sim 100 \text{ mK} \) and high frequency \( \omega / 2\pi = 20 \text{ GHz} \). To provide a good matching to the coaxial cable and avoid reflection of the ac excitation, one use a tunnel junction as a quantum contact. Indeed, the time dependence of single-electron and electron-hole wave functions in a driven quantum contact does not depend either on the number nor on the transmission of its conduction channels. The experiment is performed in a dilution refrigerator where a 0.1 T perpendicular magnetic field is applied to turn the Al normal. A bias tee, sketched in Fig. 3 by an inductor and a capacitor, allows to separate the dc bias voltage applied on the junction from the high frequency part of the setup. The ac excitation is imposed through a directional coupler and the current fluctuations emerging from the sample are amplified by a low noise cryogenic amplifier (noise temperature \( T_N \sim 7 \text{ K} \)). Current fluctuations are band-pass filtered between \( \Delta f = 1.2 - 3 \text{ GHz} \) and the noise power density \( S_2 \) integrated over the bandwidth of the filter is measured with a broadband square law detectors as a function of dc bias voltage for different ac excitations. The derivative of the noise \( \partial S_2 / \partial (eV_{dc}) \) is measured with an additional 77 Hz, small voltage modulation and a usual lock-in detection. The base temperature \( T_{ph} \) of the dilution refrigerator significantly increases with increasing ac voltage. It implies an increase of the electronic temperature \( T_e \) which is measured by fitting the data of \( \partial S_2 / \partial (eV_{dc}) \) (see inset in Fig. 3).

II. CALIBRATION PROCEDURE

The current noise power associated to the collision of two single-electron wave packets generated by two voltage drives \( V_L(t) = V_{dc} + V_0 \cos(\omega t) \) and \( V_R(t) = V_L(t-\tau) \) is simply given by the harmonic photon-assisted noise with \( V_{ac} = 2eV_0 \sin(\omega \tau / 2) \):

\[
S_2(eV_{dc}, eV_{ac}) = \sum_{n=-\infty}^{\infty} J_n^2 \left( \frac{eV_{ac}}{\hbar \omega} \right) R_0 (eV_{dc} + n\hbar \omega) \frac{\theta \left( \frac{eV_{ac} + n\hbar \omega}{2k_B T_e} \right)}{2}\]

where \( S_0 = \hbar \omega / R_0 \) and \( T_e \) is the temperature of electrons. Since we measure the amplified current noise power density \( G(S_2 + 2kBT_N)\Delta f \) and we want to extract its absolute value \( S_2 \), we need to calibrate the total gain \( G \) of the measurement setup (including the amplification and the cable attenuation), the noise added by the amplifier \( T_N \) and the coupling \( eV_{ac} / \hbar \omega \) between the sample and the excitation line. This calibration has been done in two stages: (i) \( G \Delta f \) and \( T_N \) are deduced from the high voltage dependence of the noise power which is simply given by the classical shot noise:

\[
G \Delta f = \frac{S_0}{T_N} \frac{\theta \left( \frac{eV_{ac} + n\hbar \omega}{2k_B T_e} \right)}{2}.
\]
$S_2(eV_{dc} \gg eV_{ac}, k_B T_e) = eV_{dc}/R_0$ (see red dash line in Fig. [3]). (ii) Figure [5] shows the calculated (a) and measured (b) second derivative of the photon-assisted noise $\partial^2 (S_2/S_0)/\partial(eV_{dc})^2$. One observes maxima corresponding to maxima of $J_1^2(eV_{ac}/\hbar\omega)$ which are approximately aligned with $eV_{ac} = (0.79\pm0.05)\times\hbar\omega+(1.06\pm0.01)\times eV_{dc}$ (black dash line on Fig. [4]). These remarkable points are used to calibrate ac coupling and our data are in very good agreement with the theoretical predictions for $eV_{dc}/\hbar\omega \geq 0.5$. However, we notice a discrepancy for $eV_{dc}/\hbar\omega < 0.5$ (red circle on Fig. [5]) which reveals a smaller coupling at low dc bias voltage. It is attributed to non-linearities due to Coulomb blockade effects appearing at low temperature and low bias voltage (see inset on Fig. [4]). Indeed, in spite of a small effect on the resistance ($\delta R/R_0 \sim 1\%$) the impedance mismatch can be emphasized by interference effects. To properly calibrate the ac coupling, we have plotted on Fig. [6] the derivative of the photon-assisted noise $\partial^2 (S_2/S_0)/\partial(eV_{dc})^2$ as a function of normalized ac bias $eV_{ac}/\hbar\omega$. The fit gives an ac coupling 6% lower at low bias voltage ($eV_{dc}/\hbar\omega < 1$) than high bias voltage ($eV_{dc}/\hbar\omega \geq 1$).

FIG. 4. (a) Measured noise power density $S_2$ and (b) differential noise power density $dS_2/d(eV_{dc})$ for various levels of excitation $eV_{dc}/\hbar\omega$. Arrows correspond to the sense of $eV_{dc}/\hbar\omega$ increasing. Inset: Dynamic resistance measured at 80 and 155 mK. Non-linearities disappear for $T_e > 400$ mK and the resistance of the sample is $R_0 = 50.4\Omega$.

FIG. 5. (a) Calculated and (b) measured second derivative of the photon-assisted noise $\partial^2 (S_2/S_0)/\partial(eV_{dc})^2$ as a function of normalized dc bias $eV_{dc}/\hbar\omega$ and normalized ac bias $eV_{ac}/\hbar\omega$. The black dots correspond to the maximum of $J_1^2(eV_{ac}/\hbar\omega)$ and $J_2^2(eV_{ac}/\hbar\omega)$.

FIG. 6. Calculated (markers) and fitted (solid lines) derivative of the photon-assisted noise $\partial^2 (S_2/S_0)/\partial(eV_{dc})^2$ as a function of normalized ac bias $eV_{ac}/\hbar\omega$ at different dc bias: $eV_{dc}/\hbar\omega = 0.061, 1.04, 2.02$ and 2.99.