Trajectory portraits for the two perturbed centrally symmetric systems of point vortices

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Abstract. We describe the types of motion for two centrally symmetric systems of point vortices (in one vortices are located in the corners of a regular hexagon, in the other – in the corners of a regular octagon), caused by spatially periodic perturbation: in which the vortices are in the corners of two regular triangles or two squares with a common center.

1. Introduction
The relevance of the study of vortex structures is due to the practical benefits: vorticity present in various natural phenomena (example: tornadoes, tornados); manifested not only on our planet, but also throughout the solar system; in the human heart there are vortices of liquid; occur when moving aircraft and ships. Point vortices, in themselves, do not occur in nature, but the structures of point vortices can be simulated real vortices. Phenomena that are associated with vortex rings are destructive and at the moment there are no effective methods of counteraction and prediction of these phenomena, respectively, a deeper understanding of the processes of formation and growth of vortices will contribute to the creation of methods for solving problems. Interest point vortices lately resumed this subject are devoted [1]-[6].

The scientific novelty of the work is that detailed trajectory portraits even for similar vortex structures have not yet been obtained.

The result of this work was the construction of detailed trajectory portraits for both declared point vortex systems. These results can be used at the Institute of Oceanology, the Institute of mechanics and other institutions that deal with the problem of vortex motion.

2. Solution
Let the vortices be at the vertices of a regular hexagon or octagon with the distance between the opposite vertices 2R.
The self-induced motion of such a system is a motion along a circle of radius $R$ with a constant velocity. Note that the speed of rotation the greater the smaller $R$.

Suppose the next outrage: vortices using a single shift value ($\delta r, \theta$). Then three (or four) vortices are at the vertices of an equilateral triangle (or square) inscribed in a circle of radius $R$, we call them the main vortices, and the other three (or four) vortices will be at the vertices of a regular triangle (or square), inscribed in a circle of radius $r$ with the same center as the first circle, and rotated relative to the first triangle (square) at an angle $\theta$.

The system as a whole rotates around a common center, and the mutual arrangement of vortices in the system changes. For a better understanding of the mutual motion, we assume that three (or four) vortices are located on a circle of radius $R = 1$, and the position of three (or four) other vortices will be calculated. The complete picture of the lines of motion of some vortices relative to others is called a trajectory portrait.

It is required to construct trajectory portraits for these two systems of point vortices.

The formula for calculating trajectory portraits is taken from the article by Boyarintseva T. E. and Savin A. S. [7]

$$C(r, \theta) = (r \sqrt{1 - r^2})^{n-1} \left( (1 + r^2) - 2r^2 \sqrt{1 - r^2} \cos(3\theta) \right)$$

Trajectory portrait is obtained as a set of a large number of points for each value of $C$. For the practical calculation of trajectory points was written in C++, implemented on a computer DELL Inspiron 7577 - 5990.
The program for solving the problem implements the following algorithm:

Set the initial value \( C = 0 \).

In nested loops we iterate over all variants of \( r \) values from 0 to 1.1 with a given step and all variants of \( \theta \) values from 0 to 6.4 (up to \( 2\pi \)) with a given step. The range of radius values is set by selection, based on the fact that there are no points of interest for values of radius greater than the specified maximum.

For all pairs of values \( r \) and \( \theta \) calculate the result according to the given formula and compare the result with the selected value \( C \). If it fits into the permissible error (\( \pm 0.01\% \)), then this pair \( r \) and \( \theta \) is the point of the graph.

Increase the value of \( C \) by a given step. Go to 2.

Repeat step 4 until \( C \) reaches the pre-selected value \( 2\pi \).

140 billion points have been tested.

Results

Trajectory portrait for a system of three pairs of point vortices: this system has three types of mutual vortex motion:

1) Movement in the area close to the common center of the system.

2) the offset Rotation of the vortices around unbiased. This type of motion occurs if the displaced vortices are close to the unbiased ones.

3) the Rotation of the displaced vortices near the points of their original position. This type of movement occurs not only with a relatively small initial displacement, its zone is a buffer between the zones of the first and second types of movement.

The points of stable equilibrium are on the same circle as the main vortices. For pic. 3 these points are clearly visible. The locations of the unbiased vortices in pic. 3 not visible, since only the motion of displaced vortices was studied, but it is seen that these points are also points of stable equilibrium. Points of unstable equilibrium are located at the joints of two different zones of movement of the third type, these points are also adjacent to the zone of the first and second types.
Consider a trajectory portrait for four pairs of point vortices. In this system, there are five types of reciprocal movements of the vortices.

1) Movement in the area close to the common center of the system.
2) The offset Rotation of the vortices around unbiased. Also similar to the second type for the system of their three pairs of vortices.
3) No change of the relative position of vortices, that is, the rotation of the system as a single rigid whole point of stable equilibrium are displaced, they are not on the same circle that the main vortices. In those places where there were vortices before the beginning of the perturbation of the system, there are points of unstable equilibrium. And for each such point, two points of stable equilibrium are formed, located on the same radial direction, but a little closer to the center of the system and a little further from it. In addition to them, there are also points of unstable equilibrium on the segments between the center of the system and the points of location of the main, that is, unbiased vortices.
4) The rotation of the displaced vortices around the points of stable equilibrium.
5) The Rotation of the displaced vortices near the points of their initial position, but skirting both zones of oscillation around the corresponding points of stable equilibrium.

3. Conclusion

It is worth noting a paradoxical fact - in both cases near the center of the system (zone of the first type of movement) there are no trajectories going around
this center. There is a large number of different trajectories, which in General can be considered as an area of stochastic motion.

References

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