DUAL WAVES

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ABSTRACT

We study the gravitational waves in the 10-dimensional target space of the superstring theory. Some of these waves have unbroken supersymmetries. They consist of Brinkmann metric and of a 2-form field. Sigma-model duality is applied to such waves. The corresponding solutions we call dual partners of gravitational waves, or dual waves. Some of these dual waves upon Kaluza-Klein dimensional reduction to 4 dimensions become equivalent to the conformo-stationary solutions of axion-dilaton gravity. Such solutions include dilaton extreme black holes, axion-dilaton Israel-Werner-Perjes-type spacetimes and extreme charged axion-dilaton Taub-Nut solutions. The unbroken supersymmetry of the gravitational waves transfers to the unbroken supersymmetry of axion-dilaton IWP solutions. More general supersymmetric 4-dimensional configurations derivable from 10-dimensional waves are described.

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1 Introduction

The fundamental reason for the existence of any supersymmetric non-renormalization theorem is related to the fact that supersymmetric theories have a natural description in superspace, i.e. in a space with both bosonic and fermionic coordinates. The famous rules of integration over anticommuting variables were discovered by Berezin. These rules form the basis for the proof of the generic supersymmetric non-renormalization theorems, which state that some semiclassical properties of supersymmetric theories are not changed when all quantum corrections are taken into account.

In recent years an active field of research has been the search for and also the study of non-perturbative solutions to the classical equations of motion of superstring effective field theories and the corresponding sigma-models. Many bosonic solutions of such theories have been discovered. Some special solutions turned out to have a highly non-trivial property: although bosonic, they have some unbroken supersymmetries. This means that when embedded into the right supersymmetric theory they admit Killing spinors. Bosonic solutions with unbroken supersymmetries in theories of quantum gravity are very special because they have some kind of supersymmetric non-renormalization property. Arguments that explain these properties were given for supersymmetric string solitons [1], [2] and for extremal black holes [3].

A particularly interesting kind of metric is provided by the so called pp-waves\(^2\). Recently we have found the metrics in this class which, together with appropriate dilaton, axion and gauge fields, provide solutions of the lowest order superstring effective action and have unbroken supersymmetries [4]. They have been shown also to be free of stringy corrections.

There exists an extensive literature on gravitational waves. We start by reviewing some results relevant for our purposes. Pp-wave geometries are space-times admitting a covariantly constant null vector field

\[ \nabla_\mu l_\nu = 0 , \quad l^\nu l_\nu = 0 . \tag{1} \]

Spacetimes with this property were first discovered by Brinkmann in 1923 [5]. In four dimensions the metrics of these spaces can be written in the general form

\[ ds^2 = 2dudv + K(u, \xi, \bar{\xi})du^2 - d\xi d\bar{\xi} , \tag{2} \]

where \( u \) and \( v \) are light-cone coordinates defined by

\[ l_\mu = \partial_\mu u , \quad l^\mu \partial_\mu v = 1 . \tag{3} \]

\(^2\)Here pp-waves stands for plane fronted waves with parallel rays.
(thus the metric does not depend on \(v\)) and \(\xi = x + iy\) and \(\bar{\xi} = x - iy\) are complex transverse coordinates. These metrics are classified and described in detail in [6]. Different pp-wave spaces are characterized by different choices of the function \(K\) in eq. (2). For example, when \(K\) is quadratic in \(\xi\) and \(\bar{\xi}\),

\[
K(u, \xi, \bar{\xi}) = f(u)\xi^2 + \bar{f}(u)\bar{\xi}^2 + g(u)\xi\bar{\xi} .
\]

they are called exact plane waves. Plane waves (2) with \(K\) of the form

\[
K(u, \xi, \bar{\xi}) = \delta(u)f(\xi, \bar{\xi}) .
\]

are called shock waves. A specific example of shock waves is given by the Aichelburg-Sexl geometry [7]

\[
K(u, \xi, \bar{\xi}) = \delta(u)\ln(\xi\bar{\xi}) .
\]

which describes the gravitational field of a point-like particle boosted to the speed of light.

Güven established in 1987 [8] that a solution to the lowest order superstring effective action, consisting of a generalization of the four dimensional exact plane waves to \(d = 10\) with dilaton, axion and Yang-Mills fields, has half of the \(N = 1, d = 10\) supersymmetries unbroken and is also a solution of the equations of motion of the superstring effective action including all the \(\alpha'\)-corrections. Investigations on the supersymmetry of plane-fronted waves in general relativity were made even earlier. In [9] there is a reference to unpublished work of J. Richer who found that pp-waves are supersymmetric. Very general classes of pp-waves were found by Tod to be supersymmetric in the context of \(d = 4, \bar{N} = 2\) ungauged supergravity in 1983 [10]. Different aspects of plane wave solutions in string theory have been investigated in the last few years [11], [12].

The existence of a covariantly constant null vector field has dramatic consequences. For instance, for the class of \(d\)-dimensional pp-waves with metrics of the form

\[
ds^2 = 2dudv + K(u, x^i)du^2 - dx^i dx^i ,
\]

where \(i, j = 1, 2, ..., d - 2\), the Riemann curvature is [12]

\[
R_{\mu\nu\rho\sigma} = -2l_{[\mu}(\partial_{\nu]}\partial_{\rho]}K)l_{\sigma]} .
\]

The curvature is orthogonal to \(l_\mu\) in all its indices. This fact is of crucial importance in establishing that all higher order in \(\alpha'\) terms in the equations of motion are zero due to the vanishing of all the possible contractions of curvature tensors. Güven proved that the corrections to the supersymmetry transformations vanish.

In arbitrary dimension \(d\) the most general metrics admitting a covariantly constant null vector ([1]), were discovered by Brinkmann in 1925 [13]:

\[
ds^2 = 2dudv + A_\mu(u, x^i)dx^\mu du - g_{ij}(u, x^i)dx^i dx^j , \quad l^\mu A_\mu = 0 ,
\]

3
where $\mu, \nu = 0, 1, \ldots, d - 1$ and $i, j = 1, 2, \ldots, d - 2$. Note that the general Brinkmann metric (9) in $d = 10$ has 8 functions $A_i(u, x^i)$ and 28 functions $g_{ij}(u, x^i)$ more than the metric (7) investigated by Güven, where only the $uu$-component of the metric $A_u = \frac{1}{2}K$ was present and was quadratic in $x^i$.

In [4] we have found the 10-dimensional supersymmetric generalization of Brinkmann metrics. We called this solution supersymmetric string waves (SSW). The various properties of these solutions have been studied recently [14], [15], [16], [17]. In this paper we present a review of the recent results related to the dual partners of supersymmetric string waves, which we call dual waves. In addition, we describe shortly more general supersymmetric 4-dimensional configurations, which may be obtained from dual waves. Duality transformations of string theory [18], which we are using here is not a standard electromagnetic duality which has been used in General Relativity often. This is the reason why the gravitational waves are usually not related to to black-hole-type solutions. From the point of view of General Relativity those are very different geometries. However the string theory brings in a completely different concept of “equivalent” background geometries. It was understood some time ago [13] that the pp-waves are dual to the fundamental strings [1]. The corresponding duality transformation, which is known as sigma-model duality transformation [18], has a very particular property: it changes the value of the dilaton field $e^{2\phi}$ by the $g_{xx}$-component of the metric, where $x$ is some direction on which all fields are independent. In our recent paper [14] we have established an analogous dual relation between more general solutions of the effective equations of the critical ($d = 10$) superstring theories.

A remarkable property of a class of SSW has been discovered recently [13]: their dual partners are the extreme 4-dimensional dilaton black holes [20] upon Kaluza-Klein dimensional reduction. And vice versa, after $d = 4$ extreme black holes are lifted up to $d = 10$, the corresponding configuration became a Brinkmann-type wave upon dual rotation.

In [16] we have found a general class of supersymmetric solutions of a 4-dimensional axion-dilaton gravity. Such solutions include dilaton extreme black holes and axion-dilaton Israel-Werner-Perjes-type spacetimes, which include also the extreme charged axion-dilaton Taub-Nut solutions. Here we will present the detailed derivation of these solutions from SSW in $d = 10$. This will require us to to investigate a more general class of SSW and to establish their relation to a very general class of solutions of 4-dimensional dilaton-axion gravity.

In this paper we will limit ourselves only to the zero slope limit of the superstring effective action ($N = 1, d = 10$ supergravity). The coupling to Yang-Mills multiplet will enter via $\alpha'$ corrections and will not be discussed here. It will be studied later along the lines of refs. [4] and [14].

The paper is organized as follows. Sec. 2 we will describe supersymmetric string waves
and their dual partners. We identify some of the dual waves as the fundamental strings [1] and indicate the relation between some waves and 4-dimensional black holes. Sec. 3 will describes some details of the supersymmetric Kaluza-Klein dimensional reduction of the bosonic part of the superstring effective action. Under such supersymmetric truncation the unbroken supersymmetry of 10-dimensional configuration is transferred automatically to unbroken supersymmetry of the 4-dimensional configuration. In Sec. 4 we will perform the supersymmetric Kaluza-Klein dimensional reduction of special case of dual wave and we will compare it with 4-dimensional extreme black holes. In Sec. 5 we consider more general dual waves and compare it with 4-dimensional stationary solutions. In Conclusion we will discuss this new method of generating non-trivial solutions in gravitational theories and display new possibilities of deriving 4-dimensional supersymmetric solutions of $N = 4$ supergravity interacting with $N = 4$ supersymmetric multiplets.

2 Supersymmetric String Waves and the Dual Waves

We will use the sigma-model duality of the string theory and relate solutions of 4-dimensional and 10-dimensional effective actions of string theory. We will limit ourselves by keeping only one scalar field, the fundamental dilaton. The pseudoscalar axion field will appear in $d = 4$ from the 3-form field strength $H$. The method which we develop here may give many other interesting relations for the class of solutions which will include more fields of the string theory. Consider first the pp-waves [3] in some dimension $d$. The metric is:

$$ds^2 = 2dudv + 2A_u dudu - \sum_{i=1}^{d-2} dx^i dx^i,$$

where the function $A_u$ depends only on transverse directions $x^i, i = 1, \ldots, d - 2$. The equation that $A_u(x^i)$ has to satisfy is:

$$\Delta A_u = 0,$$

where the Laplacian is taken over the transverse directions only. Sigma-model duality transformation [8] defines the changes in the metric, 2-form field $B_{\mu\nu}$ and in the dilaton field $e^{2\phi}$.

$$g'_{xx} = 1/g_{xx}, \quad g'_{x\alpha} = B_{x\alpha}/g_{xx},$$
$$g'_{\alpha\beta} = g_{\alpha\beta} - (g_{x\alpha}g_{x\beta} - B_{x\alpha}B_{x\beta})/g_{xx},$$
$$B'_{x\alpha} = g_{x\alpha}/g_{xx}, \quad B'_{\alpha\beta} = B_{\alpha\beta} + 2g_{x[\alpha}B_{\beta]x}/g_{xx},$$
$$\phi' = \phi - \frac{1}{2} \log |g_{xx}|.$$

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Our notation are that of [14], [4] and [3].
This transformation is defined for configurations with a non-null Killing vector in the $x$-direction. The string theory considers such configurations as equivalent under the condition that the $x$-direction is compact. Leaving aside this issue we will start by presenting the upshot of the dual relation between pp-waves ([1]) and fundamental strings ([1]).

Since we have chosen the metric of our pp-waves to be $u$-independent and they are $v$-independent by the definition of pp-waves, the metric is also independent on $x = \frac{1}{\sqrt{2}}(u - v)$ and $t = \frac{1}{\sqrt{2}}(u + v)$. A straightforward application of the sigma-model duality transformations given in ([1]) on pp-waves given in eq. ([1]) leads to the following new solution.

$$ds^2 = 2e^{2\phi} du dv - \sum_{i=1}^{d-2} dx^i dx^i, \quad B = 2(1 - e^{2\phi}) du \wedge dv, \quad e^{-2\phi} = 1 - A_u.$$  \hspace{1cm} (13)

Let the function $A_u$ be spherically symmetric and depend only on $r^2 = \sum_{i=1}^{d-2} x^i x^i$. The choice

$$A_u = -\frac{\mu}{r^{d-4}}$$  \hspace{1cm} (14)

solves the harmonic equation ([1]) at $r \neq 0$ if $\mu$ is a constant. With this choice of the function $A_u$ eq. ([1]) is the solution found in [1] describing the field outside of a fundamental string. For $d = 10$, which is the critical dimension of the superstring theory, this solutions looks as follows

$$ds^2 = (1 + \frac{\mu}{r^6})^{-1}(dt^2 - dx^2) - \sum_{i=1}^{i=8} dx^i dx^i, \quad B_{xt} = \frac{\mu}{r^6 + \mu} \quad e^{-2\phi} = 1 + \frac{\mu}{r^6}.$$  \hspace{1cm} (15)

The dual partner of the fundamental string is the following pp-wave

$$ds^2 = 2du dv - \frac{2\mu}{r^6} du du - \sum_{i=1}^{i=8} dx^i dx^i,$$  \hspace{1cm} (16)

and the 2-form field and the dilaton are absent (or could be equal to some constants). Note that in $d = 10$ both the pp-waves as well as the fundamental strings have unbroken supersymmetries [8], [1]. With this reminder of the duality between the fundamental string and pp-wave we may proceed to display the duality between supersymmetric string waves and the “lifted” extreme black holes.

Supersymmetric String Waves (SSW) ([1]) are the plane-wave-type solutions of the superstring effective action which have unbroken space-time supersymmetries. They describe
dilaton, axion and gauge fields in a stringy generalization of the Brinkmann metric. Some conspiracy between the metric, the axion field and gauge fields is required. We will consider here only the zero slope limit of the effective string action. This limit corresponds to 10-dimensional $N = 1$ supergravity. The Yang-Mills multiplet will appear in the first order of $\alpha'$ string corrections. The conspiracy between the metric and the axion field is the characteristic property of the SSW and is necessary to provide unbroken supersymmetry even in the zero slope limit.

The SSW in $d = 10$ are given by the Brinkmann metric and the following 2-form

$$\begin{align*}
\text{ds}^2 &= 2d\tilde{u}d\tilde{v} + 2A_Md\tilde{x}^Md\tilde{u} - \sum_{i=1}^{i=8}d\tilde{x}^id\tilde{x}^i, \\
B &= 2A_Md\tilde{x}^M \wedge d\tilde{u}, \quad A_v = 0,
\end{align*}$$

where $i = 1, \ldots, 8$, $M = 0, 1, \ldots, 8, 9$ and we are using the following notation for the 10-dimensional coordinates $x^M = \{\tilde{u}, \tilde{v}, \tilde{x}^i\}$. We have put the tilde over the 10-dimensional coordinates in this section, since we will have to compare the original 10-dimensional configuration with the 4-dimensional one, embedded into the 10-dimensional space. A rather non-trivial identification of coordinates, describing these solutions will be required later.

Note that our SSW have a particular conspiracy between the metric and the axion field

$$g_{i\tilde{u}} = A_i = B_{i\tilde{u}}.$$  \hfill (18)

It is interesting that the solution in which the vector function in the metric is related to the one in the axion was mentioned by Tseytlin \cite{21} as the most natural one from the point of view of the sigma model equations. In the work of Duval, Horváth and Horváthy \cite{22} it was found that such conspiracy between the metric and the axion field leads to the absence of conformal anomaly at the two-loop level.

The equations that $A_u(\tilde{x}^i)$ and $A_i(\tilde{x}^j)$ have to satisfy are:

$$\begin{align*}
\triangle A_u &= 0, \\
\triangle \partial^{i}\partial^{j}A_i &= 0,
\end{align*}$$

where the Laplacian is taken over the transverse directions only. This solution has 8 functions $A_i$ more than that of pp-waves (10) where only the function $A_u$ is non-vanishing.

A straightforward application of the sigma-model duality transformations given in (12) on the SSW solution given in eq. (17) leads to the following new supersymmetric solution of the zero slope limit equations of motion:

$$\begin{align*}
ds^2 &= 2e^{2\phi}\{d\tilde{u}d\tilde{v} + A_i d\tilde{u}d\tilde{x}^i\} - \sum_{i=1}^{i=8}d\tilde{x}^id\tilde{x}^i,
\end{align*}$$

7
\[ B = -2 e^{2\phi} \{ A_u d\tilde{u} \wedge d\tilde{v} + A_i d\tilde{u} \wedge d\tilde{x}^i \}, \quad (20) \]
\[ e^{-2\phi} = 1 - A_u, \]

where as before, the functions \( A_M = \{ A_u = A_u(\tilde{x}^j), A_v = 0, A_i = A_i(\tilde{x}^j) \} \) satisfy equations (19). We call these solutions dual partners of the waves, or for simplicity, dual waves.

We can make the following particular choice of the vector function \( A_M \). First of all these functions will depend only on 3 of the transverse coordinates, \( \tilde{x}^1, \tilde{x}^2, \tilde{x}^3 \), corresponding to our 3-dimensional space. Secondly, we choose one of \( A_i \), e. g. \( A_4 \) to be related to \( A_u \).

\[ A_u = -\frac{\mu}{\rho}, \quad A_4 = \xi A_u, \quad A_5 = \ldots = A_8 = 0, \quad (21) \]

where \( \rho^2 = \sum_{i=1}^{i=3} \tilde{x}^i \tilde{x}^i \equiv \tilde{x}^2 \), and \( \mu \) is a constant. We will specify the constant \( \xi \) later. Note that eqs. (19) are solved outside \( \rho = 0 \)\(^4\). We get

\[ ds^2 = 2 e^{2\phi} \{ d\tilde{u} d\tilde{v} + \xi (1 - e^{-2\phi}) d\tilde{x}^4 d\tilde{u} \} - \sum_{i=1}^{i=8} d\tilde{x}^i d\tilde{x}^i, \]
\[ B = -2 e^{2\phi} \{ (1 - e^{-2\phi}) \{ d\tilde{u} \wedge d\tilde{v} + \xi d\tilde{u} \wedge d\tilde{x}^4 \} \}, \]
\[ e^{-2\phi} = 1 + \frac{\mu}{\rho}. \quad (22) \]

The solution given in (22) is different from the solution of [1] corresponding to the field outside a fundamental string and from our generalized FS solution, since it depends only on 3 coordinates and not on 8. Also we take a particular case of our solution with relations between \( A_4 \) and \( A_u \). We perform the coordinate change

\[ \hat{x} = \tilde{x}^4 + \xi \tilde{u}, \quad \hat{v} = \tilde{v} + \xi \tilde{x}^4. \quad (23) \]

We also shift \( B \) on a constant value, since equations of motion depend on \( H = dB \) only.

The dual wave solution (22) takes the form

\[ ds^2 = 2 e^{2\phi} d\hat{u} d\hat{v} + \xi^2 d\hat{u}^2 - d\hat{x}^2 - \sum_{i=1}^{i=3} d\tilde{x}^i d\tilde{x}^i - \sum_{i=5}^{i=8} d\tilde{x}^i d\tilde{x}^i, \]
\[ B = 2 e^{2\phi} d\hat{v} \wedge d\hat{u}, \]
\[ e^{-2\phi} = 1 + \frac{\mu}{\rho}. \quad (24) \]

\(^4\)In order to solve the equations (13) everywhere, it is understood that a source term at \( \rho = 0 \), representing an unknown object, perhaps a six-brane, has to be added to these equations. We hope that it can be worked out in an analogy with the combined action for the macroscopic fundamental string, where the source term comes from the sigma-model action, see eqs. (3.1) - (3.3) in [1].
When \( \xi^2 = -1 \) we have

\[
\begin{align*}
 ds^2 &= 2e^{2\phi}d\tilde{v}d\tilde{u} - d\tilde{u}^2 - d\tilde{x}^2 - \sum_{i=1}^{3} d\tilde{x}^i d\tilde{x}^i - \sum_{i=5}^{8} d\tilde{x}^i d\tilde{x}^i, \\
 B &= 2e^{2\phi}d\tilde{v} \wedge d\tilde{u}, \\
 e^{-2\phi} &= 1 + \frac{\mu}{\rho}.
\end{align*}
\]

(25)

We can identify this particular dual partner of the SSW solution with the uplifted dilaton black hole if we make the following identification of coordinates

\[
\begin{align*}
 t &= \hat{v} = \tilde{v} + \xi \tilde{x}^4, \\
x^4 &= \tilde{u}, \\
x^9 &= \hat{x} = \tilde{x}^4 + \xi \tilde{u}, \\
x^{1,2,3,5,...,8} &= \tilde{x}^{1,2,3,5,...,8}.
\end{align*}
\]

(26)

Our dual wave becomes

\[
\begin{align*}
 ds^2 &= 2e^{2\phi}dt d\tilde{x}^4 - \sum_{i=4}^{9} dx^i dx^i - d\tilde{x}^2, \\
 B &= 2e^{2\phi}dt \wedge d\tilde{x}^4, \\
 e^{-2\phi} &= 1 + \frac{\mu}{\rho}.
\end{align*}
\]

(27)

This is an extreme electrically charged 4-dimensional black hole \cite{20}, which is embedded into 10-dimensional geometry in stringy frame, as we are going to explain in the next section.

### 3 Supersymmetric dimensional reduction

The embedding of the 4-dimensional bosonic solutions of the effective superstring action into 10-dimensional geometry is not unique, in general \footnote{We are grateful to E. Witten for attracting our attention to this problem.}. There are different ways to identify the vector field of the charged black hole in 4 with the non-diagonal component of the metric in the extra dimensions as well as with the 2-form field. Also the identification of the 4-dimensional dilaton with the fundamental 10-dimensional dilaton and/or with some components of the metric in the extra dimension is possible.

However the identification of the 4-dimensional solution with the 10-dimensional one becomes unique under the conditions that the supersymmetric embedding for both solutions...
is identified. Dimensional reduction of $N = 1$ supergravity down to $d = 4$ has been studied by Chamseddine \cite{23} in canonical geometry. We are working in stringy metric and also in slightly different notation. In a subsequent publication we will present a detailed derivation of the compactification of the bosonic part of the effective action of the 10-dimensional string theory which is consistent with supersymmetry \cite{15}. Here we are interested in the relation between the extreme dilaton charged black holes, which have unbroken supersymmetry \cite{3} when imbedded into $d = 4, N = 4$ supergravity\footnote{We do not know at present, whether the embedding of these black holes into other theories, including the Abelian part of Yang-Mills multiplet, will also correspond to some unbroken supersymmetries.} and the corresponding 10-dimensional supersymmetric configuration. The stationary IWP solutions of 4-dimensional axion-dilaton gravity also happen to be supersymmetric when embedded into $d = 4, N = 4$ supergravity. For more general 4-dimensional supersymmetric configurations, which have to be embedded into $d = 4, N = 4$ supergravity interacting with $d = 4, N = 4$ supersymmetric matter multiplets, a more general formulae will be required.

We start with the zero slope limit of the effective 10-dimensional superstring action. The bosonic part of the action is

\begin{equation}
S = \frac{1}{2} \int d^{10}x e^{-2\phi} \sqrt{-g} \left[ -R + 4(\partial \phi)^2 - \frac{3}{4} H^2 \right],
\end{equation}

where the 10-dimensional fields are the metric, the axion and the dilaton.

We want to make connection with the bosonic part of $N = 4, d = 4$ action. In this particular case we are interested in compactifying 6 space-like coordinates. All fields are assumed to be independent of six compactified dimensions. According to Chamseddine \cite{23} dimensional reduction of $N = 1, d = 10$ supergravity to $d = 4$ gives $N = 4$ supergravity coupled to 6 matter multiplets. We are interested here only in dimensional reduction to $N = 4$ supergravity without matter multiplets.

Let us first reduce from $d = 10$ to $d = 5$ by trivial dimensional reduction, when we do not keep the non-diagonal components of the metric and 2-form field. We denote the 10-dimensional fields by un upper index \((10)\) and the 5-dimensional fields by a hat. The 10-dimensional indices are capital letters $M, N = 0, \ldots, 9$, the 5-dimensional indices will carry a hat $\hat{\mu}, \hat{\nu} = 0, \ldots, 4$, and the compactified dimensions will be denoted by capital $I$'s and $J$'s, $I, J = 5, \ldots, 9$. We take the $d = 10$ fields to be related to the $d = 5$ ones by

\begin{align*}
g_{\hat{\mu}\hat{\nu}}^{(10)} &= \hat{g}_{\hat{\mu}\hat{\nu}}, \\
g_{\hat{\mu}\nu}^{(10)} &= 0, \\
g_{IJ}^{(10)} &= \eta_{IJ} = -\delta_{IJ}, \\
B_{\hat{\mu}\hat{\nu}}^{(10)} &= \hat{B}_{\hat{\mu}\hat{\nu}},
\end{align*}

where $\eta_{IJ}$ is the Minkowski metric.
B^{(10)}_{1\rho} = 0,
B^{(10)}_{IJ} = 0,
\phi^{(10)} = \hat{\phi}.

(29)

We get

\[ S = \frac{1}{2} \int d^5 x e^{-2\phi} \sqrt{-\hat{g}} \left[ -\hat{R} + 4(\partial \hat{\phi})^2 - \frac{3}{4} \hat{H}^2 \right]. \]

(30)

As a second step we reduce from \( d = 5 \) to \( d = 4 \), keeping the non-diagonal components of the metric and 2-form field. Since we are interested also in supersymmetry, we will work with the 5-beins at this stage. The 4-dimensional indices do not carry a hat. We parametrize the 5-beins as follows

\[
(\hat{e}_\mu^a) = \left( e_\mu^a A_\mu \right), \quad (\hat{e}_a^\mu) = \left( e_a^\mu - A_a \right),
\]

(31)

where \( A_a = e_a^\mu A_\mu \). With this parametrization, the 5-dimensional fields decompose as follows

\[
\hat{g}_{44} = \hat{g}_{44} = -1,
\hat{g}_{4\mu} = -A_\mu,
\hat{g}_{\mu\nu} = g_{\mu\nu} - A_\mu A_\nu,
\hat{B}_{4\mu} = B_\mu,
\hat{B}_{\mu\nu} = B_{\mu\nu} + A_{[\mu} B_{\nu]} ,
\hat{\phi} = \phi,
\]

(32)

where \( \{g_{\mu\nu}, B_{\mu\nu}, \phi, A_\mu, B_\mu\} \) are the 4-dimensional fields.

The 4-dimensional action for the 4-dimensional fields becomes

\[
S = \frac{1}{2} \int d^4 x e^{-2\phi} \sqrt{-g} \left[ -R + 4(\partial \phi)^2 - \frac{3}{4} H^2 \right] + \frac{1}{4} F^2(A) + \frac{1}{4} F^2(B),
\]

(33)

where

\[
F_{\mu\nu}(A) = 2\partial_{[\mu} A_{\nu]},
F_{\mu\nu}(B) = 2\partial_{[\mu} B_{\nu]},
H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]} + \frac{1}{2} \{ A_{[\mu} F_{\nu\rho]}(B) + B_{[\mu} F_{\nu\rho]}(A) \}.
\]

(34)

Now, we study the dimensional reduction of gravitino. We are specifically interested in the supersymmetry transformation rule of gravitino in \( d = 4 \) supergravity without matter. This
leads to identification of the matter vector fields $D_\mu$ and the supergravity vector fields $V_\mu$.

$$D_\mu = \frac{1}{2}(A_\mu - B_\mu),$$

$$V_\mu = \frac{1}{2}(A_\mu + B_\mu),$$

respectively. Now we want to truncate the theory keeping only the supergravity vector field $V_\mu$. We have then

$$V_\mu = A_\mu = B_\mu, \quad D_\mu = 0.$$ (36)

The truncated action is

$$S = \frac{1}{2} \int d^4x e^{-2\phi} \sqrt{-g}[-R + 4(\partial\phi)^2 - \frac{3}{4}H^2 + \frac{1}{2}F^2(V)],$$ (37)

where

$$F_{\mu\nu}(V) = 2\partial_{[\mu}V_{\nu]},$$

$$H_{\mu\nu\rho} = \partial_{[\mu}B_{\nu\rho]} + V_{[\mu}F_{\nu\rho]}(V).$$ (38)

The embedding of the 4-dimensional fields in this action in $d = 10$ is the following:

$$g_{\mu\nu}^{(10)} = g_{\mu\nu} - V_\mu V_\nu,$$

$$g_{4\nu}^{(10)} = -V_\nu,$$

$$g_{44}^{(10)} = -1,$$

$$g_{IJ}^{(10)} = \eta_{IJ} = -\delta_{IJ},$$

$$B_{\mu\nu}^{(10)} = B_{\mu\nu},$$

$$B_{4\nu}^{(10)} = V_\nu,$$

$$\phi^{(10)} = \phi.$$ (39)

This formulae can be used to uplift any $U(1)$ 4-dimensional field configurations, including dilaton and axion, to a 10-dimensional field configurations in a way consistent with supersymmetry.

The conclusion of this supersymmetric dimensional reduction is the following.

i) The dilaton of the supersymmetric 4-dimensional extreme black holes is identified as a fundamental dilaton of string theory (and not one of the modulus fields).

---

7This action, which came from the 10-dimensional theory is slightly different from the corresponding 4-dimensional action in our previous papers, e.g. in [3], due to the difference in notation. The detailed explanation of this difference will be given in [3].
ii) Dimensional reduction of $d = 10$ supergravity to $d = 4$ gives $N = 4$ supergravity without 6 matter multiplets under condition that $g_{4\mu} = -B_{4\mu}$. Therefore the vector field of the 4-dimensional configuration is actually a non-diagonal component of the metric in the extra dimension as well as the 2-form field. This works in our case since we have according to \([27]\)
\[
g_{4t}^{(10)} = -B_{4t}^{(10)} = -V_t = e^{2\phi} . \tag{40}
\]

iii) The supersymmetric truncation, presented above has the following important property by construction: if the 4-dimensional configuration has unbroken supersymmetries, the uplifted one also has them and vice versa, if one starts with the supersymmetric configuration in $d = 10$ one ends up with the supersymmetric configuration in $d = 4$. The reason for this is simple. The rules, given in eq. \([39]\) have been derived in such a way that the equations $\Psi = 0$, $\delta \Psi = 0$ on all fermions remain correct and still have solutions with non-vanishing Killing spinors.

## 4 Uplifting the Black Hole

We will use the formulae from the section above to uplift the dilaton black hole with one vector field. The electrically charged extreme 4d black hole is given by \([20]\)
\[
\begin{align*}
\mathcal{d}s_{\text{str}}^2 & = e^{4\phi} dt^2 - dx^2 , \\
V & = -e^{2\phi} dt , \\
B & = 0 , \\
e^{-2\phi} & = 1 + \frac{2M}{\rho} . \tag{41}
\end{align*}
\]

The uplifted configuration, according to eq. \([39]\) is:
\[
\begin{align*}
ds^2 & = 2e^{2\phi} dt dx^4 - dx^2 - (dx^4)^2 - dx^I dx^I , \\
B^{(10)} & \equiv B_{MN} dx^M \wedge dx^N = -2e^{2\phi} dx^4 \wedge dt , \\
\phi^{(10)} & = \phi . \tag{42}
\end{align*}
\]

Let us choose the parameter $\mu$ in the dual partner to the wave, given in eq. \([23]\) equal to the double mass of the black hole.

\[
\mu = 2M . \tag{43}
\]

\[8\text{There is a difference of a } 1/\sqrt{2} \text{ factor in the vector field with respect to the one given in } [3].\]
This makes the uplifted black hole (42) identical to the dual partner to the wave, given in eq. (23).

For better understanding of black-hole-wave relation it is useful to do the following. By adding and subtracting from the metric the term $e^{4\phi}dt^2$ we can rewrite the dual wave in $d = 10$, given in eq. (27) as follows.

$$
\begin{align*}
 ds^2 &= e^{4\phi}dt^2 - d\mathbf{x}^2 - (dx^4 - e^{2\phi}dt)^2 - dx^I dx^I, \\
 B &= -2e^{2\phi}dx^4 \wedge dt, \\
 e^{-2\phi} &= 1 + \frac{2M}{\rho}.
\end{align*}
$$

Now it is easy to recognize in the first 2 terms in the metric the 4-dimensional metric and in the third term the non-diagonal component of the 10-dimensional metric which together with the non-diagonal component of the 2-form plays the role of the vector field in the 4-dimensional geometry.

To show that our 10-dimensional solution is the embedding of the extreme 4d black hole we may also present the 10-dimensional metric in Kaluza-Klein parametrization, where we have a 5d metric $g_{\hat{\mu}\hat{\nu}} \times x^I$-flat space ($\hat{\mu} = 0, 1, 2, 3, 4, I = 5, \ldots, 9$).

$$
 g_{\hat{\mu}\hat{\nu}} = \begin{pmatrix}
 0 & 0 & 0 & 0 & e^{2\phi} \\
 0 & -1 & 0 & 0 & 0 \\
 0 & 0 & -1 & 0 & 0 \\
 0 & 0 & 0 & -1 & 0 \\
 e^{2\phi} & 0 & 0 & 0 & -1
\end{pmatrix} = \begin{pmatrix}
 g_{00} + A_0A_0g_{44} & 0 & 0 & 0 & A_0g_{44} \\
 0 & -1 & 0 & 0 & 0 \\
 0 & 0 & -1 & 0 & 0 \\
 0 & 0 & 0 & -1 & 0 \\
 A_0g_{44} & 0 & 0 & 0 & g_{44}
\end{pmatrix}
$$

(45)

where $g_{00} = e^{4\phi} = (A_0)^2$, $g_{11} = -1$, $g_{22} = -1$, $g_{33} = -1$, $g_{44} = -1$. Thus the 10-dimensional non-diagonal component of the metric $g_{04} = A_0g_{44}$ together with the part of $B_{04}$ forms the 4-dimensional vector field, as usual in Kaluza-Klein theory. The relation is, see eq. (35)

$$
 A_0 = -V_t = e^{2\phi} = \frac{1}{2}(g_{04}^{(10)} + B_{04}^{(10)}).
$$

(46)

The left hand side of this equation supplies the nice simple form of the dual partner of the wave metric

$$
 ds^2 = 2e^{2\phi}dt dx^4 - d\mathbf{x}^2 - (dx^4)^2 - dx^I dx^I,
$$

(47)

the right-hand side shows that if this metric is rewritten in a more complicated form by replacing the zero in the upper right corner of the matrix by $g_{00} + A_0A_0g_{44}$, we can recognize the $g_{00}$-piece of the 4-dimensional black hole.

The case $\xi^2 = -1$ which gives the 4-dimensional black hole in Minkowski space with the signature (1, 3) times the compact 6-dimensional space with the signature (0, 6) corresponds...
to a complex 10-dimensional wave in the space with the signature (1, 9).

\[ ds^2 = 2d\bar{u}d\bar{v} - \frac{4M}{\rho} d\bar{u}(d\bar{u} - i\bar{d}x^4) - \sum_{i=1}^{8} d\bar{x}^i d\bar{x}^i, \]

\[ B = -i \frac{4M}{\rho} d\bar{u} \wedge d\bar{x}^4. \] \hspace{1cm} (48)

By performing a rotation \( i\bar{x}^4 = \tilde{\tau} \) one can get

\[ ds^2 = 2d\bar{u}d\bar{v} - \frac{4M}{\rho} d\bar{u}(d\bar{u} - d\tilde{\tau}) + d\tilde{\tau}^2 - \sum_{i=1}^{7} d\bar{x}^i d\bar{x}^i, \]

\[ B = - \frac{4M}{\rho} d\bar{u} \wedge d\tilde{\tau}. \] \hspace{1cm} (49)

This makes the wave real but with the signature of the space (2, 8).

Thus we may conclude that string theory considers as dual partners the extreme 4d electrically charged dilaton black hole embedded into 10-dimensional geometry, as given in eq. (44) or (42), and Brinkmann-type 10-dimensional wave (48), (49).

If we choose \( \xi^2 = 1 \) case we get the stringy equivalence between Brinkmann-type 10-dimensional wave

\[ ds^2 = 2d\bar{u}d\bar{v} - \frac{4M}{\rho} d\bar{u}(d\bar{u} - d\bar{x}^4) - \sum_{i=1}^{8} d\bar{x}^i d\bar{x}^i, \]

\[ B = - \frac{4M}{\rho} d\bar{u} \wedge d\bar{x}^4, \] \hspace{1cm} (50)

and lifted Euclidean 4-dimensional electrically charged dilaton black hole with the signature (0, 4) and the 6-dimensional space has the signature (1, 5),

\[ ds^2 = -e^{4\phi} dt^2 - dx^2 + (dx^4 + e^{2\phi} dt)^2 - dx^I dx^I, \]

\[ B = -2e^{2\phi} dx^4 \wedge dt, \]

\[ e^{-2\phi} = 1 + \frac{2M}{\rho}. \] \hspace{1cm} (51)

With such choice of the signature the gravitational wave does not have imaginary components. However, the fact that the metric as well as the 2-form field of the gravitational wave in \( d = 10 \) have an imaginary component to be dual to the lifted black hole in Minkowski space is strange. Note that this is necessary only if one insists that the \( d = 10 \) space as well as the \( d = 4 \) space are both Minkowski spaces. One can avoid imaginary components by allowing the changes in the signature of the space-time when performing duality and dimensional reduction as explained above. Still this remains a puzzle.
5 From Waves to IWP and Vice Versa

We consider again only the zero slope limit of supersymmetric string waves (SSW) in $d = 10$. To generate a dual-wave, which upon dimensional reduction gives the 4-dimensional IWP solutions, we make the following choices for the vector $A_M$ in the 10-dimensional SSW (17). The components $A_M$ are taken to depend only on three of the transverse coordinates, $\tilde{x}^1, \tilde{x}^2, \tilde{x}^3$, which will ultimately correspond to our 3-dimensional space. We choose one component, e.g., $A_4$ to be related to according to $A_4 = \xi A_u$, where $\xi^2 = \pm 1$ depending on the signature of spacetime. Recall that from (20) $A_u$ is related to the dilaton by $A_u = 1 - e^{-2\phi}$.

For the remaining components, we take only $A_1, A_2, A_3$ to be non-vanishing, and we relabel these as $\omega_1, \omega_2, \omega_3$ for obvious reasons. To summarize, we then have

$$A_4 = \xi A_u, \quad A_1 \equiv \omega_1, \quad A_2 \equiv \omega_2, \quad A_3 \equiv \omega_3, \quad A_5 = \ldots = A_8 = 0. \quad (52)$$

We get the following wave:

$$ds^2 = 2e^{2\phi}\{d\tilde{u}d\tilde{v} + \xi(1 - e^{-2\phi})d\tilde{x}^4d\tilde{u} + \omega_i d\tilde{u}d\tilde{x}^i\} - \sum_{i=1}^{i=8} d\tilde{x}^i d\tilde{x}^i, \quad B = -2e^{2\phi}\{(1 - e^{-2\phi})\{d\tilde{u} \wedge d\tilde{v} + \xi d\tilde{u} \wedge d\tilde{x}^4 + \omega_i d\tilde{u} \wedge d\tilde{x}^i\}, \quad e^{-2\phi} = 1 + \frac{\mu}{\rho}. \quad (53)$$

The equations which the functions $e^{-2\phi}$ and $\omega_i$ have to satisfy for the gravitational wave to be a solution of equations of motion, following from the action (28), are:

$$\triangle e^{-2\phi} = 0, \quad \triangle \bar{\partial}^{[i} \omega^{j]} = 0, \quad (54)$$

where again the Laplacian is taken over the transverse directions only. These equations follow from eqs. (19).

In what follows we repeat exactly the same steps which have allowed to show that in absence of $\omega_i$ the dual partner of a wave is an extreme black hole upon dimensional reduction. We redefine $\tilde{v}$

$$\tilde{v} = \tilde{v} + \xi \tilde{x}^4, \quad (55)$$

Solution takes the form

$$ds^2 = e^{2\phi}2d\tilde{u}(d\tilde{v} + \omega_i dx^i) - 2\xi d\tilde{u}d\tilde{x}^4 - \sum_{i=1}^{i=8} (d\tilde{x}^i)^2, \quad B = 2(1 - e^{2\phi})d\tilde{u} \wedge d\tilde{v} - 2e^{2\phi} \omega_i d\tilde{u} \wedge d\tilde{x}^i, \quad (56)$$

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The dilaton-axion field \( \lambda \) satisfy the harmonic equation (71). We may bring the metric to the form

\[
 ds^2 = 2e^{2\phi} d\hat{u}(d\hat{v} + \omega_i d\tilde{x}^i) + \xi^2 d\hat{u}^2 - \sum_{i=3}^{i=8} d\tilde{x}^i d\tilde{x}^i - (d\hat{x})^2 - \sum_{i=5}^{i=8} d\tilde{x}^i d\tilde{x}^i ,
\]

(57)

where

\[
 \hat{x} = \tilde{x}^4 + \xi \tilde{u} ,
\]

(58)

One can shift \( B \) on a constant value, since equations of motion depend on \( H = dB \) only. We get

\[
 B = 2e^{2\phi}(d\hat{v} + \omega_i d\tilde{x}^i) \wedge \hat{u} ,
\]

(59)

When \( \xi^2 = -1 \) we have

\[
 ds^2 = 2e^{2\phi} d\hat{u}(d\hat{v} + \omega_i d\tilde{x}^i) - d\hat{u}^2 - \sum_{i=1}^{i=3} d\tilde{x}^i d\tilde{x}^i - (d\hat{x})^2 - \sum_{i=5}^{i=8} d\tilde{x}^i d\tilde{x}^i ,
\]

\[
 B = 2e^{2\phi}(d\hat{v} + \omega_i d\tilde{x}^i) \wedge \hat{u} ,
\]

\[
 \triangle e^{-2\phi} = 0 , \quad \triangle \bar{\partial}^i \omega^j = 0 .
\]

(60)

A particular identification of the coordinates in the dual wave solution with those in the uplifted IWP solution can be performed,

\[
 t = \hat{v} = \tilde{v} + \xi \tilde{x}^4 , \\
 x^4 = \tilde{u} , \\
 x^9 = \hat{x} = \tilde{x}^4 + \xi \tilde{u} , \\
 x^{1,2,3,5,\ldots,8} = \tilde{x}^{1,2,3,5,\ldots,8} .
\]

(61)

After all these steps our 10-dimensional dual wave becomes

\[
 ds^2 = 2e^{2\phi} dx^4(dt + \bar{\omega} \cdot d\tilde{x}) - \sum_{i=1}^{i=9} dx^i dx^i - d\tilde{x}^2 , \\
 B = -2e^{2\phi} dx^4 \wedge (dt + \bar{\omega} \cdot d\tilde{x}) .
\]

(62)

This allows to identify the stationary supersymmetric axion-dilaton IWP solution [16], embedded into 10d geometry in stringy metric. All arguments, given in [15] about the uniqueness of the embedding into higher dimension, once the supersymmetry is identified, are valid in this case.
To recognize this as the lifted IWP solution, add and subtract from the metric the term $e^{4\phi}(dt + \vec{\omega} \cdot d\vec{x})^2$. We can then rewrite the dual wave metric (61) as

$$ds^2 = e^{4\phi}(dt + \vec{\omega} \cdot d\vec{x})^2 - dx^2 - \left(dx^4 - e^{2\phi}(dt + \vec{\omega} \cdot d\vec{x})\right)^2 - \sum_5^9 dx^i dx^i. \quad (63)$$

The first two terms now give the string metric for the 4-dimensional IWP solutions. The non-diagonal components $g_{\mu 4}$, in the third term, are interpreted as the 4-dimensional gauge field components, showing the Kaluza-Klein origin of the gauge field in this construction. Note that the 4-dimensional vector field components are also equal to the off-diagonal components of the 2-form gauge field, giving the overall identifications

$$g_{t4} = B_{t4} = e^{2\phi} = -V_4, \quad g_{i4} = B_{i4} = e^{2\phi}\omega_i = -V_i. \quad (64)$$

The dilaton of the IWP solution, is identified with the fundamental dilaton of string theory, rather than with one of the modulus fields. The axion is identified with the 4-dimensional part of the 3-form field strength $H$ given in eq. (38). Note that these components of $H$ come totally from the second term in (38), since the 4-dimensional $B_{\mu \nu}$ vanishes.

To explain this relation better let us remind that the stationary supersymmetric axion-dilaton IWP solution [16] is

$$ds^2_{str} = e^{4\phi}(dt + \omega_i dx^i)^2 - dx^2, \quad \partial [i \omega_j] = -\frac{1}{2} \epsilon_{ijk} \partial_k a, \quad A_\mu = \frac{1}{\sqrt{2}} e^{2\phi} (1, \omega_i),$$

$$\triangle e^{-2\phi} = 0, \quad \triangle \partial [i \omega_j] = 0. \quad (65)$$

The Kaluza-Klein parametrization of the metric means the following relation between the higher-dimensional metric $g_{MN}$ and the reduced one $g_{\mu \nu}$:

$$g_{MN} = \begin{pmatrix} g_{\mu \nu} + A_\mu^k g_{kl} A_\nu^l & A_\mu^k g_{ik} \\ A_\nu^k g_{kj} & g_{ij} \end{pmatrix}. \quad (66)$$

To show that our 10d solution is the embedding of the axion-dilaton 4d IWP metric we may present the 10d metric in K.K. parametrization, where we have a 5d metric $g_{\hat{\mu} \hat{\nu}} \times x^I$-flat space ($\hat{\mu}, \hat{\nu} = 0, 1, 2, 3, 4) \quad I, J = 5, \ldots, 9.$

$$g_{\hat{\mu} \hat{\nu}} = \begin{pmatrix} 0 & 0 & 0 & 0 & e^{2\phi} \\ 0 & -1 & 0 & 0 & e^{2\phi}\omega_1 \\ 0 & 0 & -1 & 0 & e^{2\phi}\omega_2 \\ 0 & 0 & 0 & -1 & e^{2\phi}\omega_3 \\ e^{2\phi} & e^{2\phi}\omega_1 & e^{2\phi}\omega_2 & e^{2\phi}\omega_3 & -1 \end{pmatrix} =$$

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\[
\begin{pmatrix}
g_{00} + A_0^4 A_0^4 g_{44} & g_{10} + A_1^4 A_0^4 g_{44} & g_{20} + A_2^4 A_0^4 g_{44} & g_{01} + A_3^4 A_0^4 g_{44} & A_0^4 g_{44} \\
g_{01} + A_0^4 A_1^4 g_{44} & -1 & 0 & 0 & A_1^4 g_{44} \\
g_{02} + A_0^4 A_2^4 g_{44} & 0 & -1 & 0 & A_2^4 g_{44} \\
g_{03} + A_0^4 A_3^4 g_{44} & 0 & 0 & -1 & A_3^4 g_{44} \\
A_0^4 g_{44} & A_1^4 g_{44} & A_2^4 g_{44} & A_3^4 g_{44} & g_{44}
\end{pmatrix}
\]

(67)

where \( g_{00} = e^{4\phi} = (A_0^4)^2 \), \( g_{ii} = -1 \), \( g_{0i} = e^{4\phi} \omega_i \). The 10-dimensional non-diagonal components of the metric \( g_{04} = A_{04}^4 g_{44} \), \( g_{0i} = A_{i4}^4 g_{44} \) together with the part of \( B_{04}, B_{0i} \) form the 4d vector field, as usual in K.K. The relation is, see eq. (65)

\[
A_0 = -V_t = e^{2\phi} = \frac{1}{2} (g_{04} + B_{04}) ,
\]

\[
A_i = -V_i = e^{2\phi} \omega_i = \frac{1}{2} (g_{i4} + B_{i4}) .
\]

The left hand side of equation (67) supplies the nice simple form of the dual partner of the wave metric

\[
ds^2 = 2e^{2\phi} (dt + \omega_i dx^i) dx^4 - dx^i dx^i - (dx^4)^2 - dx^I dx^I.
\]

(69)

The right-hand side shows that if this metric is rewritten in a much more complicated form by replacing many zero’s in the upper right corner of the matrix by the terms like \( g_{00} + A_{04}^4 g_{44} \), we can recognize the \( g_{00} \)-piece and other pieces of the 4-dimensional axion-dilaton IWP metrics.

We may use the 3-dimensional antisymmetric tensor \( \epsilon_{ijk} \) to express \( \partial_{[i} \omega_{j]} \) through the axion field \( a(x^i) \).

\[
\partial_{[i} \omega_{j]} = -\frac{1}{2} \epsilon_{ijk} \partial_k a .
\]

(70)

By introducing the standard dilaton-axion complex field \( \lambda = a + ie^{-2\phi} \) we may rewrite equations (11) as follows:

\[
\triangle \lambda = 0 .
\]

(71)

This accomplishes the derivation of the 4-dimensional IWP axion-dilaton solutions from the 10-dimensional gravitational waves.
Could we actually consider the dual relation between waves and lifted black holes as something more than pure algebraic curiosity? We believe that the answer to this question is “yes”. The dual relation displayed above was established at the zero slope limit of the effective action of the superstring theory. The issue of $\alpha'$-corrections in string theory has been studied extensively for the waves [4], [14]. The pp-waves have the best known properties of absence of such quantum corrections [8]. The SSW are known to have special property of the absence of $\alpha'$-corrections under the condition that the non-abelian Yang-Mills fields is added to the configuration which at the zero slope limit $\alpha' = 0$ consists only of the metric and 2-form [4].

It was explained in [14] that the importance of sigma-model duality between supersymmetric configurations is in the fact that the structure of $\alpha'$ corrections is under control for the dual solution if it was under control for the original solution. In this way we have found that the nice properties of the pp-waves [8] are carried over to the fundamental string solutions. The present investigation shows that the electrically charged extreme black hole embedded into ten-dimensional geometry may require to be supplemented by some non-abelian Yang-Mills configuration, to avoid the possible $\alpha'$-corrections. In this respect we would like to stress that the study of the properties of quantum corrections established via duality may become a powerful mechanism of the investigation of quantum theory despite the strange imaginary factors in the waves, which are dual partners of the uplifted black holes.

At the very minimal level one can consider the method developed above, which consists of stringy duality combined with Kaluza-Klein dimensional reduction, as the solution generating method. This method has the advantage of generating new supersymmetric solutions from the original ones. If we would not know that extreme 4-dimensional black holes are supersymmetric, we would discover this via the supersymmetric properties of 10-dimensional waves. If we would not found the IWP axion-dilaton spaces in the 4-dimensional world, we could have found them from SSW via dual rotation and dimensional reduction. Simultaneously we would establish the existence of unbroken supersymmetry of IWP geometries.

The SSW solutions found in [4] are not the most general supersymmetric wave solutions, more general ones may exist. However, if we look only on those SSW solutions, about which we know that they are supersymmetric, could we still use them to produce more general 4-dimensional configurations with unbroken supersymmetries? The answer to this question is

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9 We have found in [4] that under natural assumptions the fundamental strings are exact solutions of string theory due to unbroken supersymmetry and dual relations to gravitational waves. The exactness of fundamental string solutions was confirmed recently by Horowitz and Tseytlin [24] by the analysis of the $\beta$-functions and specific renormalization scheme in conformal field theory.
positive. Indeed, we may relax few conditions, used in eq. (52): we may consider $A_4$ unrelated to $A_u$ and choose $A_5, \ldots, A_9$ all non-vanishing. The net result of this is the following. The fields which become part of the six vector matter supermultiplet upon dimensional reduction of dual waves are non-vanishing for new solutions. Indeed, by choosing $A_4$ to be proportional to $A_u$ we have achieved that the metric in $x^4$ direction is flat. Otherwise, $g_{44}$ would not be equal to a constant. The choice $A_5 \neq 0$ will make the 6-dimensional space non-flat, $g_{45} = -B_{45} = e^{2\phi}A_5$, etc. The combination of vector fields which enter the matter multiplet will be non-vanishing for these solutions, since the definition of vector fields will include above mentioned $g_{4I}$-component of the metric and $B_{4I}$-component of the 2-form. The corresponding 4-dimensional theory upon dimensional reduction will contain additional vectors and scalars as well as pseudoscalars. By construction, we will have a bosonic action and configurations which solve its equation of motion. These configurations will have half of unbroken supersymmetries when embedded into $N = 4$ supergravity with six abelian vector multiplets. We can also take into account the Yang-Mills part of the 10-dimensional gravitational wave solution SSW. This will lead to the 4-dimensional solutions of equations of motion which have unbroken supersymmetry when embedded into $N = 4$ supergravity with six abelian vector supermultiplets and non-abelian Yang-Mills supermultiplet. The action and the corresponding solutions will be presented in an explicit form in the future publication.

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