Proximity effect in superconductor/conical magnet/ferromagnet heterostructures

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Abstract
At the interface between a superconductor and a ferromagnetic metal spin-singlet Cooper pairs can penetrate into the ferromagnetic part of the heterostructure with an oscillating and decaying spin-singlet Cooper pair density. However, if the interface allows for a spin-mixing effect, equal-spin spin-triplet Cooper pairs can be generated that can penetrate much further into the ferromagnetic part of the heterostructure, known as the long-range proximity effect. Here, we present results of spin-mixing based on self-consistent solutions of the microscopic Bogoliubov–de Gennes equations in the clean limit incorporating a tight-binding model. In particular, we include a conical magnet into our model heterostructure to generate the spin-triplet Cooper pairs and analyse the influence of conical and ferromagnetic layer thickness on the unequal-spin and equal-spin spin-triplet pairing correlations. It will be shown that, in agreement with experimental observations, a minimum thickness of the conical magnet is necessary to generate a sufficient amount of equal-spin spin-triplet Cooper pairs allowing for the long-range proximity effect.

Keywords: proximity effect, Bogoliubov–de Gennes equations, spin-triplet Cooper pairs, odd-frequency pairing, ferromagnet–superconductor interface
1. Introduction

At the interface between a normal metal and a superconductor (SC) an incoming electron with an energy above the Fermi energy (or chemical potential) $\mu$ can be reflected back into the metal as a hole with opposite spin orientation. This phenomenon, known as Andreev reflection [1], gives rise to the well-known proximity effect resulting in superconducting properties decaying into the normal metal part of the heterostructure. Replacing the normal metal by a ferromagnet (FM) drastically changes the behaviour at the interface. At first sight the phenomena of ferromagnetism and superconductivity appear to be mutually exclusive due to the specific requirements concerning spin orientations. In ferromagnetic materials the Pauli principle requires the spins to orient parallel whereas superconducting spin-singlet Cooper pairs require an antiparallel orientation. Based on these intrinsic spin orientations there are interesting phenomena to be expected at the interface between an FM and an SC. The exchange interaction in the FM leads to different Fermi velocities of electrons in the spin-up and spin-down channel. Therefore, the centre of mass motion is modulated and superconducting correlations in the FM show an oscillating behaviour [2–4]. These are essentially the FFLO oscillations named after Fulde and Ferrell [5] and Larkin and Ovchinnikov [6], respectively. The penetration of these spin-singlet superconducting correlations are strongly suppressed by the ferromagnetic exchange field and are only short-range. In addition, the exchange field also generates $S_z = 0$ (unequal-spin) components of spin-triplet correlations, which also oscillate and decay. Similar to the spin-singlet correlations these are short-range as well.

However, it has been suggested theoretically by Bergeret et al [7], that due to spin–flip processes at the interface equal-spin spin-triplet Cooper pairs can form which should be unaffected by the ferromagnetic exchange field thereby allowing much larger penetration depths. This phenomenon is called the long-range proximity effect and has triggered a lot of experimental and theoretical work. The proximity effect in SC-FM heterostructures is reviewed by Buzdin [8], whereas Bergeret et al [9] review the physics behind this type of ‘odd-triplet’ superconductivity. The symmetry relations between the different pairing correlations within the SC-FM heterostructures is discussed in detail by Eschrig et al [10].

From the experimental side several multilayer setups have been suggested to observe spin-triplet proximity effect including supercurrents in Josephson junctions [11, 12] or spin-valves [13]. These results can only be understood theoretically assuming the generation of equal-spin spin-triplet pairing correlations within these structures. Typical examples of multilayer systems to generate spin-triplet Cooper pairs experimentally involve noncollinear magnetizations within the different ferromagnetic layers [11–13], or helical (or conical) magnets in the multilayer setup [14–16]. This is accompanied by respective theoretical investigations of those systems containing noncollinear magnetizations [17, 18] and helical (or conical) magnetic material in the multilayer setup [19]. Additionally, also the effects of Bloch [20] or Néel [21] domain walls, spin–orbit coupling [22] or specific interface potentials [23–26] at the SC/FM interface on the generation of spin-triplet Cooper pairs have been investigated theoretically. Another route towards generating spin-triplet Cooper pairs leads to the inclusion of half-metallic FM’s such as CrO$_2$ into the heterostructures [27–29] which would pave the way for a marriage between supercurrents and spintronics applications [30].

The aim of the paper is as follows. A heterostructural setup similar to those used in the experiments of Robinson et al [15] consisting of SC, conical magnet (CM) and ferromagnet will be investigated using self-consistent solutions to the spin-dependent microscopic Bogoliubov–de Gennes (BdG) equations [31] in the clean limit. One focus of the work lies on
the influence of the CM’s opening and turning angles on the induced spin-triplet pairing correlations for which a detailed symmetry analysis will be provided. Secondly, we focus on the influence of conical magnetic layer thickness on the spin-triplet correlations. It will be shown that a minimum number of conical magnetic layers are necessary to efficiently generate equal-spin spin-triplet correlations, in agreement with experimental observations.

The paper is organized as follows. Section 2 starts with a description of the theoretical method, namely the self-consistent solution of the spin-dependent BdG equations. This is followed by a detailed description of the multilayer structure used in the calculations, a setup for the conical magnetic structure, and finally the spin-dependent pairing correlations. Results are presented in section 3, where first a symmetry analysis of different CM orientations is presented followed by the CM’s and FM’s thickness dependence of the spin-dependent triplet pairing correlations. A concluding summary and an outlook will be given in section 4.

2. Theoretical background and computational details

2.1. BdG equations and tight-binding Hamiltonian

All our calculations are based on self-consistent solutions of the microscopic BdG equations in the clean limit which for the spin-dependent case read [31–33]

\[
\begin{pmatrix}
\mathcal{H}_0 - h_z & -h_x + ih_y & \Delta_{\uparrow\downarrow} & \Delta_{\downarrow\uparrow} \\
-h_y & \mathcal{H}_0 + h_z & \Delta_{\downarrow\uparrow} & \Delta_{\uparrow\downarrow} \\
\Delta_{\uparrow\downarrow}^* & \Delta_{\downarrow\uparrow}^* & -\mathcal{H}_0 + h_z & h_x + ih_y \\
\Delta_{\downarrow\uparrow}^* & \Delta_{\uparrow\downarrow}^* & h_x - ih_y & -\mathcal{H}_0 - h_z
\end{pmatrix}
\begin{pmatrix}
u_{n\uparrow} \\ u_{n\uparrow} \\ v_{n\downarrow} \\ u_{n\downarrow}
\end{pmatrix}
= \epsilon_n
\begin{pmatrix}
u_{n\uparrow} \\ u_{n\uparrow} \\ v_{n\downarrow} \\ u_{n\downarrow}
\end{pmatrix}
\] (1)

where \(\epsilon_n\) denote the eigenvalues of the matrix equation, and \(u_{n\sigma}\) and \(v_{n\sigma}\) are the quasiparticle and quasihole amplitudes for spin \(\sigma\), respectively. \(\mathcal{H}_0\) is the tight-binding Hamiltonian, which for a system of two-dimensional layers can be written as

\[
\mathcal{H}(k) = -t \sum_{nk} (c_{nk}c_{n+1,k} + c_{n+1,k}^\dagger c_{nk}) + \sum_{nk} (\epsilon_{nk} - \mu) c_{nk}^\dagger c_{nk},
\] (2)

with \(t\) being the next-nearest neighbour hopping parameter setting the energy scale with \(t = 1\), and \(\mu = 0\) being the chemical potential (Fermi energy) set to half-filling. \(c_{nk}\) and \(c_{nk}^\dagger\) are electronic creation and destruction operators at multilayer index \(n\) with momentum \(\hbar k\) within the layers, respectively. Since the main focus of the present work lies on the presence of an interface within the multilayer or heterostructure, the only valid \(k\) values in equation (2) are to be found within the interface plane. For each of these \(k\) values the BdG equations (1) lead to a one-dimensional inhomogeneous problem in the multilayer index \(n\) [34]. For the sake of simplicity and since this would only lead to a parametrical dependence of the Hamiltonian on a discretized \(k\) mesh in the present work we neglect this \(k\) dependence. Equation (2) then simplifies to

\[
\mathcal{H}_0 = -t \sum_{n} (c_{n+1}^\dagger c_n + c_n^\dagger c_{n+1}) + \sum_{n} (\epsilon_n - \mu) c_n^\dagger c_n. 
\] (3)

The implications of this simplification will be taken into account when discussing the obtained results in section 3.
The pairing matrix can be rewritten according to the Balian–Werthamer transformation [35, 36] utilizing the Pauli matrices \( \sigma \),

\[
\begin{pmatrix}
\Delta_{11} & \Delta_{12} \\
\Delta_{21} & \Delta_{22}
\end{pmatrix}
= (\Delta + \sigma \mathbf{d}) i \sigma_2 = \begin{pmatrix}
-d_z + i d_y & \Delta + d_z \\
-\Delta + d_z & d_z + i d_y
\end{pmatrix},
\]

(4)

which effectively describes the superconducting order parameter comprising of a singlet (scalar) part \( \Delta \) and a triplet (vector) part \( \mathbf{d} \), respectively. In the present work \( \Delta \) is restricted to the \( s \)-wave singlet pairing potential in the SC sides of the heterostructure, to be determined self-consistently from the condition

\[
\Delta(\mathbf{r}) = \frac{g(\mathbf{r})}{2} \sum_n (u_{n_1}(\mathbf{r}) v_{n_1}^*(\mathbf{r}) \left[ 1 - f(\varepsilon_n) \right] + u_{n_1}(\mathbf{r}) v_{n_1}^*(\mathbf{r}) f(\varepsilon_n)),
\]

(5)

where the summation is performed over the positive eigenvalues \( \varepsilon_n \). \( f(\varepsilon_n) \) is the Fermi distribution function evaluated as a step function for zero temperature and \( g(\mathbf{r}) \) the effective superconducting coupling set to 1 in our calculations. It is assumed to be constant within the SC and to vanish elsewhere.

Finally, \( h_x, h_y, \) and \( h_z \) generally describe the vector components of a noncollinear exchange field to be added to the tight-binding Hamiltonian in the form \( \mathbf{h} \sigma \), with the vector components of \( \sigma \) being the Pauli matrices, respectively. In section 2.2 \( \mathbf{h} \) will be defined to describe the conical magnetic structure within the multilayer setup.

2.2. Multilayer structural setup

The multilayer setup used in the present work is schematically shown in figure 1(a). It consists of a spin-singlet \( s \)-wave SC of \( n_{SC} = 250 \) layers, a CM of \( n_{CM} = 0 \cdots 25 \) layers, a ferromagnetic metal of up to \( n_{FM} = 500 \) layers, followed by the same number of layers of CM \( n_{CM} \) and spin-singlet \( s \)-wave SC \( n_{SC} \) to the right, respectively. The description of the CM is chosen according to Wu et al [19]

\[
\mathbf{h} = h_0 \left\{ \cos \alpha \mathbf{y} + \sin \alpha \left[ \sin \left( \frac{\beta y}{a} \right) \mathbf{x} + \cos \left( \frac{\beta y}{a} \right) \mathbf{z} \right] \right\},
\]

(6)
with $h_0 = 0.1$ being the strength of the CM’s exchange field and $a$ being the lattice constant (set to unity $a = 1$). As can be seen from equation (6) and figure 1(b), the opening angle $\alpha$ is measured from the positive $y$ axis towards the positive $z$ axis, whereas the turning angle $\beta$ is measured from the positive $z$ axis towards the positive $x$ axis. Here these angles have been kept fixed to the values $\alpha = 80^\circ$ and $\beta = 30^\circ$ to represent the CM Holmium, a transition metal routinely used in similar experimental investigations [15]. Since the experimental geometry of how the conical structure is oriented with respect to the ferromagnetic region is unknown [37] our first set of calculations will examine the effects of different orientations and turning angle directions of the conical structure with respect to the two different ferromagnetic interfaces in section 3.1.

2.3. (Spin-triplet) Pairing correlations

The general expression for the on-site superconducting pairing correlation of spins $\alpha$ and $\beta$ for times $\tau = \tau$ and $\tau' = 0$ reads

$$ f_{\alpha\beta} (r, \tau, 0) = \frac{1}{2} \left\langle \hat{\Psi}_\alpha (r, \tau) \hat{\Psi}_\beta (r, 0) \right\rangle. \quad (7) $$

Therein, $\hat{\Psi}_\sigma (r, \tau)$ denotes the many-body field operator for spin $\sigma$ at time $\tau$, and the time-dependence is governed by the Heisenberg equation of motion. Notice that this pairing correlation is local in space and so the spin-triplet contributions vanish automatically in the case $\tau = 0$ according to the Pauli principle [17]. Therefore, such a pairing field is only non-zero at finite times $\tau$, an example of odd-frequency triplet pairing [9]. Substituting the field operators valid for our setup and phase convention the spin-dependent triplet pairing correlations read

$$ f_{\uparrow\downarrow} (y, \tau) + f_{\downarrow\uparrow} (y, \tau) = \frac{1}{2} \sum_n \left( u_{n\uparrow} (y) v_{n\downarrow}^* (y) + u_{n\downarrow} (y) v_{n\uparrow}^* (y) \right) \zeta_n (\tau) $$
$$ f_{\downarrow\downarrow} (y, \tau) = \frac{1}{2} \sum_n \left( u_{n\downarrow} (y) v_{n\downarrow}^* (y) \right) \zeta_n (\tau) $$
$$ f_{\uparrow\uparrow} (y, \tau) = \frac{1}{2} \sum_n \left( u_{n\uparrow} (y) v_{n\uparrow}^* (y) \right) \zeta_n (\tau) \quad (8) $$

depending on the time parameter $\tau$ (fixed to $\tau = 10$) and with $\zeta_n (\tau)$ given by

$$ \zeta_n (\tau) = \cos (e_n \tau) - i \sin (e_n \tau) \left[ 1 - 2f (e_n) \right]. \quad (9) $$

These spin-triplet pairing correlations correspond to $S_z = 0 \left( f_{\downarrow\uparrow} + f_{\uparrow\downarrow} \right), +1 \left( f_{\uparrow\uparrow} \right)$ and $-1 \left( f_{\downarrow\downarrow} \right)$, respectively.

3. Results and discussion

3.1. Influence of conical magnet

As soon as a conical magnetic structure is included in the multilayer setup there are several ways to orient the magnetic moments with respect to the direction perpendicular to the interface layer (being the $y$ axis in our multilayer setup). As mentioned earlier, the opening angle $\alpha = 80^\circ$
and turning angle $\beta = 30^\circ$ of the CM are chosen to represent the magnetic structure of Holmium, routinely used in experimentally available multilayer structures. Experimental evidence shows that the magnetic coupling at the CM/FM interface is most likely antiferromagnetic [37]. Looking for the moment at the right FM/CM interface (figure 1(a)) and assuming the ferromagnetic moments to orient along the $+z$ axis, the conical magnetic moment closest to the interface can have two different antiferromagnetic-like orientations, namely pointing slightly towards the FM side ($\alpha = 260^\circ$ case, with the cone opening into the FM layer) or slightly towards the CM side of the interface ($\alpha = 280^\circ$ case, with the cone opening away from the FM layer). These angles are reversed at the left CM/FM interface, respectively.

Furthermore, the handedness of the respective cone is determined not only by the turning angle $\beta$ ($30^\circ$ and $-30^\circ$) but also influenced by the respective opening angle. Looking again at the right FM/CM interface and an opening angle $\alpha = 280^\circ$ (cone opening away from the FM layer) a turning angle $\beta = 30^\circ$ ($\beta = -30^\circ$) describes a clockwise (counterclockwise) rotation of the conical magnetic structure. If, however, the opening angle amounts to $\alpha = 260^\circ$ (cone opening into the FM layer), a turning angle $\beta = 30^\circ$ ($\beta = -30^\circ$) again describes a clockwise (counterclockwise) rotation but now viewed along the $-y$ direction.

The influence of different opening angles of the conical magnetic layers on both sides of the ferromagnetic region on the spin-triplet pairing correlations $f_{\uparrow\uparrow}$ and $f_{\downarrow\downarrow}$ are shown in the left, middle, and right panels of figure 2, whereas upper and lower panels depict the influence of equal and opposite handedness of the two conical magnetic structures, respectively. The relation between the spin-triplet pairing correlations $f_{\uparrow\uparrow}$ and $f_{\downarrow\downarrow}$ depending on different choices of $\alpha$ and $\beta$ are given in table 1. Looking for the moment only at case 1 in figure 2 and table 1 ($\alpha = 260^\circ$, $\alpha = 280^\circ$) with $\beta_L = \beta_R = 30^\circ$. From the symmetry discussion of the two cones it’s apparent that the conical magnetic structure left of the FM interface is opening away from the interface towards the $-y$ direction with a clockwise rotation. The same holds for the right side of the interface; the cone is opening away from the interface towards the $y$ direction with a clockwise rotation. For this setup both conical magnetic structures seem to be identical; they both open away from the FM interface into the CM layers with a clockwise rotation of the conical magnetization. But the results for this setup show a sign change between the left and right side CM/FM interfaces (figure 2 and table 1) with $f_{\uparrow\uparrow}^L = -f_{\uparrow\uparrow}^R$ and $f_{\downarrow\downarrow}^L = -f_{\downarrow\downarrow}^R$. At first this looks like a discrepancy, but in fact this stems from the underlying symmetry of the $d$-vector describing the spin-triplet pairing correlations which will be discussed now. Although in case 1 and $\beta_L = \beta_R = 30^\circ$ both conical magnetic structures seem to be identical, in fact they can be transformed into one another by a $C_2$ rotation about the $z$ axis located in the middle of the FM layers. According to Tinkham [38] the transformation of an arbitrary vector $r$ under a symmetry operation described by a transformation matrix $R(u)$ reads

$$r' = \sum_j \left[ R(u) \right]_{ij} r_j. \quad (10)$$
Applying the transformation matrix for a $C_2$ rotation given by

$$ R \left( C_2 \right) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} $$

(11)

to the $d$-vector written as $(d_x, d_y, d_z)^T$ where $T$ denotes the transpose of the vector yields

$$ R \left( C_2 \right) (d_x, d_y, d_z)^T = (-d_x - d_z, d_y)^T. $$

(12)

Keeping in mind the Balian–Werthamer transformation of the superconducting order parameter as of equation (4), the sign change in $d_x$ and $d_y$ leads to a sign change in $\Delta_i$ and $\Delta_i$ and

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**Figure 2.** Influence of different opening angles of the left ($\alpha_L$) and right ($\alpha_R$) CM structure ($n_{CM} = 13$) adjacent to the middle FM layer ($n_{FM} = 100$) on the real (green) and imaginary (orange) parts of the equal-spin spin-triplet pairing correlations $f_{i\pi}$ and $f_{i\pi}$. Upper and lower panels depict the influence of equal ($\beta_L = \beta_R = 30^\circ$) and opposite ($\beta_L = -\beta_R = 30^\circ$) handedness of the two CM structures. The definition of $\alpha$ and $\beta$ is given according to equation (6) and figure 1(b). Zero multilayer index $n$ lies in the centre of the ferromagnetic layer and the vertical dashed lines represent the FM/CM and CM/SC interfaces, respectively.
expresses exactly what is displayed in figure 2 and given in table 1, namely $f_{\uparrow\uparrow}^L$ and $f_{\uparrow\uparrow}^R$, respectively. As can also be seen from figure 2 the given dependencies apply equally for the real and imaginary part.

| case 1: | case 2: | case 3: |
|---------|---------|---------|
| $\alpha_L = 260^\circ$, $\alpha_R = 280^\circ$ | $\alpha_L = 280^\circ$, $\alpha_R = 260^\circ$ | $\alpha_L = 280^\circ$, $\alpha_R = 280^\circ$ |
| $\beta_L = 30^\circ$, $\beta_R = 30^\circ$ | $\beta_L = 30^\circ$, $\beta_R = 30^\circ$ | $\beta_L = 30^\circ$, $\beta_R = 30^\circ$ |
| $f_{\uparrow\uparrow}^L = -f_{\uparrow\uparrow}^R$ | $f_{\uparrow\uparrow}^L = -f_{\uparrow\uparrow}^R$ | $f_{\uparrow\uparrow}^L = -f_{\uparrow\uparrow}^R$ |
| $f_{\downarrow\downarrow}^L = -f_{\downarrow\downarrow}^R$ | $f_{\downarrow\downarrow}^L = -f_{\downarrow\downarrow}^R$ | $f_{\downarrow\downarrow}^L = -f_{\downarrow\downarrow}^R$ |

Looking now at case 1 but for the two cones having different handednesses (lower left panels of figure 2 and table 1) one notices a mixture between $\uparrow\uparrow$ and $\downarrow\downarrow$ contributions, namely $f_{\uparrow\uparrow}^L = -f_{\downarrow\downarrow}^R$ and $f_{\downarrow\downarrow}^L = -f_{\uparrow\uparrow}^R$, respectively. In this case the transformation between the left and right conical magnetic structure is realized by a $\sigma_{xz}$ mirror plane again located in the middle of the FM layers with the respective transformation matrix

$$R(\sigma_{xz}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$  

Applying $R(\sigma_{xz})$ to the $d$-vector yields

$$R(\sigma_{xz}) (d_{\downarrow} d_{\uparrow})^T = (d_{\uparrow} - d_{\downarrow})^T,$$  

in agreement with results displayed in figure 2 and given in table 1. Again, the sign changes and swapping of contributions for $\uparrow\uparrow$ and $\downarrow\downarrow$ between the left and right conical magnetic structures reveal the underlying symmetry properties of the $d$-vector for the chosen multilayer setup. Now similar arguments along those lines explain the results for case 2 shown in the middle panels of figure 2 and table 1, respectively. Summarizing this, opposite opening angles on both interfaces just give a sign change, whereas a different handedness in addition mixes the $\uparrow\uparrow$ and $\downarrow\downarrow$ contributions. Although there are no symmetry arguments available for case 3 shown in the right panels of figure 2 and table 1 one can understand the results on the basis of symmetry arguments provided from case 1 and 2 above. The most striking difference in case 3 is the nodeless behaviour of the $\uparrow\uparrow$ and $\downarrow\downarrow$ contributions.
There is nearly no influence of different $\alpha$s and $\beta$s on the spin-triplet pairing correlation corresponding to the $S_z = 0$ case (as depicted in the upper panel of figure 3).

3.2. Influence of time parameter $\tau$ on spin-triplet pairing correlations

Chosing a specific fixed setup (case 1 with $\beta_L = -\beta_R = 30^\circ$ as mentioned in section 3.1) this section deals with the influence of the time parameter $\tau$ entering the evaluations of the spin-dependent triplet pairing correlations in equation (8) utilizing equation (9). In addition to figure 2 the real and imaginary parts of $f_{\uparrow\downarrow} + f_{\downarrow\uparrow}$ (upper panels), $f_{\uparrow\downarrow}$ (middle panels) and $f_{\downarrow\uparrow}$ (lower panels), respectively. The definition of $\alpha$ and $\beta$ is given according to equation (6) and figure 1(b). Zero multilayer index $n$ lies in the centre of the ferromagnetic layer and the vertical dashed lines represent the FM/CM and CM/SC interfaces, respectively.

There is nearly no influence of different $\alpha$s and $\beta$s on the $f_{\uparrow\downarrow} + f_{\downarrow\uparrow}$ spin-triplet pairing correlation corresponding to the $S_z = 0$ case (as depicted in the upper panel of figure 3).

Figure 3. Influence of the time parameter $\tau$ entering equation (9) on the real (green) and imaginary (orange) parts of the spin-triplet pairing correlations $f_{\uparrow\downarrow} + f_{\downarrow\uparrow}$ (upper panels), $f_{\uparrow\downarrow}$ (middle panels) and $f_{\downarrow\uparrow}$ (lower panels), respectively. The definition of $\alpha$ and $\beta$ is given according to equation (6) and figure 1(b). Zero multilayer index $n$ lies in the centre of the ferromagnetic layer and the vertical dashed lines represent the FM/CM and CM/SC interfaces, respectively.

Concentrating for the moment on the middle panels of figure 3 for $\tau = 10$ one immediately recognizes a change by a factor of 1/2 (left panels) and 2 (right panels) in the spin-triplet pairing correlations when comparing with the left and right panels showing results obtained for times $\tau$ which are also changed by a factor of 1/2 and 2, respectively. Recognizing this essentially linear dependence on the time factor $\tau$ in the present regime $\tau \times \Delta \ll 1$ entering the calculation of the spin-triplet pairing correlations via equation (9), further calculations are restricted to a time parameter $\tau = 10$. A more detailed investigation of the influence of $\tau$ on the spin-triplet pairing correlations will be part of a later work. But here we simply note again that this spin-triplet
pairing correlation which is spatially local but retarded in time, vanishes at $\tau = 0$, corresponding to the ‘odd-triplet’ pairing state derived by quasiclassical arguments by Bergeret et al [9] and Eschrig et al [10].

3.3. Influence of ferromagnetic layer thickness $n_{FM}$ on spin-triplet pairing correlations

Using the same geometries as above (case 1 with $\beta_L = \beta_R = 30^\circ$, case 1 with $\beta_L = -\beta_R = 30^\circ$, and case 3 with $\beta_L = -\beta_R = 30^\circ$ of section 3.1) and one full conical magnetic structure on either side of the ferromagnetic layer now the influence of the FM’s layer thickness on the spin-dependent spin-triplet pairing correlations shall be investigated. Figure 4 displays the magnitude of the spin-triplet pairing correlations $f_{\uparrow\downarrow} + f_{\downarrow\uparrow}$ in the left and $f_{\uparrow\uparrow}$ contributions in the right panels for conical magnetic orientations and multilayer setups according to case 1 with $\beta_L = \beta_R = 30^\circ$ (upper panels), case 1 with $\beta_L = -\beta_R = 30^\circ$ (middle panels), and case 3 with $\beta_L = -\beta_R = 30^\circ$ (lower panels) as discussed in section 3.1 and depicted in figure 2, respectively. The $f_{\downarrow\downarrow}$ contributions are identical to the $f_{\uparrow\uparrow}$ contributions and are not shown here. Concentrating for a moment only on the left panels of figure 4, one notices the influence of increasing ferromagnetic layer thickness as more and more oscillations are appearing in this region. This $f_{\uparrow\downarrow} + f_{\downarrow\uparrow}$ contributions are unaffected by the specific multilayer setup. It should be noted at this point that the very slowly decaying $f_{\uparrow\downarrow} + f_{\downarrow\uparrow}$ spin-triplet pairing correlations are an artefact of the simplified linear chain model used in these calculations and decay much faster once the fully $k$ dependent Hamiltonian in equation (2) is used in the calculations. However, the interest of the present work lies only on the equal-spin spin-triplet correlations and how effectively they can be generated at an interface containing a conical magnetic structure. Looking now at the right panels of figure 4 one notices the zeros along the line belonging to the multilayer index $n = 0$ in the upper two panels showing results for multilayer setup case 1 with different handednesses. This is in line with figure 2 and shows that an increasing ferromagnetic layer thickness $(n_{FM})$ does not give rise to more zeros in the ferromagnetic region. The nonvanishing contributions present in multilayer setup case 3 as shown in figure 2 (right panels) are also present for increasing ferromagnetic thickness (lower right panel of figure 4). All in all there is no influence of the ferromagnetic layer thickness on the behaviour of the equal-spin spin-triplet correlations with respect to showing additional or less zeros. For the next investigations the number of ferromagnetic layers will be fixed to $n_{FM} = 100$ layers, respectively.

3.4. Influence of conical magnetic layer thickness $n_{CM}$ on spin-triplet pairing correlations

The results presented in this section allow for a deeper understanding of the influence of the conical magnetic layer thickness on the spin-triplet pairing correlations. Since the influence of the overall conical magnetic layer orientation and number of ferromagnetic layers can be understood from the results already presented in section 3.2 and section 3.3 the multilayer setup will now be fixed to case 1 with $n_{FM} = 100$ ferromagnetic layers, but with a conical magnetic layer thickness ranging from $n_{CM} = 0$ to $n_{CM} = 25$ layers (representing two full turns of the CM along the growth direction). Figure 5 shows the influence of the conical magnetic layer thickness $n_{CM}$ on the real (left panels) and imaginary parts (right panels) of the spin-triplet correlations for $f_{\uparrow\downarrow} + f_{\downarrow\uparrow}$ (upper panels), $f_{\downarrow\uparrow}$ (middle panels), and $f_{\uparrow\uparrow}$ contributions (lower
panels), respectively. In addition to the features already observed in the previous sections (figure 2) new features develop due to the increasing conical magnetic layer thickness. For all three contributions to the spin-triplet correlations there is an oscillating behaviour in the real and imaginary parts depending on the conical layer thickness, in the case of the imaginary part of the $f_{||1}$ contribution superimposed to the oscillations within the ferromagnetic region. Apparently, the maximum and minimum values observed in figure 2 for the spin-triplet correlations on either side of the ferromagnetic middle layer are strongly affected by the conical

**Figure 4.** Influence of ferromagnetic layer thickness $n_{FM}$ on the magnitude of spin-triplet pairing correlations. Shown are the $f_{||1} + f_{||1}$ (left panels) and $f_{||1}$ contributions (right panels) for multilayer setups case 1 with $\beta_L = \beta_R = 30^\circ$ (upper panels), case 1 with $\beta_L = -\beta_R = 30^\circ$ (middle panels), and case 3 with $\beta_L = -\beta_R = 30^\circ$ (lower panels) as discussed in section 3.1 and depicted in figure 2, respectively. Zero multilayer index $n$ lies in the centre of the ferromagnetic layer.
magnetic layer thickness and even change sign. However, for a fixed number \( n_{CM} \) the symmetry relations observed in section 3.1 are still valid. To get more insight into the results figure 6 now shows the magnitudes of the \( f_{↑↓} + f_{↓↑} \) (upper panels) and \( f_{↑↑} \) (lower panels) spin-triplet correlations in full view (left panels) and as a top view (right panels), respectively. From the magnitude of the \( f_{↑↓} + f_{↓↑} \) spin-triplet correlation one again notices the FFLO oscillations within the ferromagnetic region (upper panels), whereas the lower panels clearly show that the conical magnetic layer thickness strongly affects the strength of the \( f_{↑↑} \) spin-triplet correlation. To get

**Figure 5.** Influence of increasing conical magnetic layer thickness \( n_{CM} \) on the real (left) and imaginary (right) parts of spin-triplet pairing correlations. The top row shows the unequal-spin contributions corresponding to \( f_{↑↓} + f_{↓↑} \), whereas the lower two rows show contributions for \( f_{↑↑} \) and \( f_{↓↓} \), respectively. Please note the different scales between the top and the lower two rows. Zero multilayer index \( n \) lies in the centre of the ferromagnetic layer.
even more insight figure 7 finally shows sideviews of the real part (left panel) and the magnitude (right panel) of the \( f_{↑↓} \) spin-triplet correlations. It is apparent that a minimum number of conical magnetic layers are necessary to generate equal-spin spin-triplet correlations with the first maximum in the magnitude (right panel) appearing for a thickness corresponding to half a turn of a full conical magnetic structure. This is in agreement with experimental observations [15] where peak values in the spin-triplet supercurrents have also been observed at CM thicknesses corresponding to half a turn of the conical magnetic structure. Keeping in mind that the multilayer setup is always starting with an antiferromagnetic-like coupling between the CM and the ferromagnetic layer, an increasing number of conical magnetic layers as displayed in the results of figures 5–7 includes a different orientation of the conical magnetic structure at the SC/CM interface. This detail requires more investigations as to whether the specific orientation of the CM at the SC/CM interface is partly responsible for the oscillating behaviour shown in the results with increasing conical magnetic layer thickness.

4. Summary and outlook

In summary, we presented a detailed analysis of spin-triplet pairing correlations within a SC/CM/FM/CM/SC heterostructure, similar to the ones investigated experimentally by Robinson
et al [15]. The results have been obtained by self-consistent solutions to the microscopic spin-dependent BdG equations in the clean limit which easily incorporate noncollinear exchange fields required to model the conical magnetic structure of Holmium also used in the experimental multilayers. While using a similar approach as Wu et al [19] we extended their CM/SC bilayer investigation to cover the whole heterostructure mentioned above. A detailed symmetry analysis of the equal-spin spin-triplet correlations from both, the left and the right hand side conical magnetic structure in the heterostructure, revealed at first sight surprising relations. These relations have been traced back to the specific underlying symmetry of our heterostructure setup. In addition, it has been shown that, in agreement with experimental observations, a certain minimum number of conical magnetic layers is necessary to sufficiently generate equal-spin spin-triplet Cooper pairs required for the long-range triplet proximity effect.

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Figure 7. Sideviews indicating a minimum number of conical magnetic layers to generate equal-spin spin-triplet pairing correlations. Shown is the real part (left panel) and the magnitude (right panel) of the $f_{11}$ contribution depending on the conical magnetic layer thickness $n_{CM}$. 
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