Effect of spin fluctuations on tunneling conductance in diffusive normal metal/conventional superconductor junctions

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Abstract. Transport property in diffusive normal metal/conventional superconductor (DN/CS) junctions is studied for the effect of spin fluctuations under various situations by solving the Usadel equation with Nazarov’s generalized boundary condition. Tunneling conductance of the DN/CS junctions is calculated by changing the magnitude of the resistance in DN, Thouless energy in DN, and the transparency of the insulating barrier, together with the magnitude of spin fluctuations. A zero-bias conductance dip (ZBCD) and a zero-bias conductance peak (ZBCP) with the width given by Thouless energy occur in line shapes by way of a coherent Andreev reflection (CAR) around zero energy in the system of DN/CS junctions. We have found that both of the ZBCD and the ZBCP sharpen with increasing the magnitude of spin fluctuations in the region of the relatively large resistance in DN.

1. Introduction

The superconducting proximity effect and the Andreev reflection play important roles in the low energy transport for diffusive normal metal/conventional superconductor (DN/CS) junctions. One of the striking experimental manifestations is a zero-bias conductance peak (ZBCP) \cite{1}. Volkov, Zaitsev and Klapwijk (VZK) solved the Usadel equation \cite{2}, and showed that this ZBCP is due to the enhancement of the pair amplitude in DN by the superconducting proximity effect, which is induced to the coherent Andreev reflection (CAR) \cite{3}. The VZK theory was applied the Kupriyanov and Lukichev (KL) boundary condition for the Keldysh-Nambu Green’s function \cite{4}. Recently, Tanaka \textit{et al.} developed the VZK theory for \textit{s}-wave superconductors using more general boundary conditions provided by the circuit theory of Nazarov \cite{5}. The extended VZK theory reveals that a crossover from a zero-bias conductance peak (ZBCP) to a zero-bias conductance dip (ZBCD) is produced in tunneling conductance of DN/CS junctions.

Here, we will focus how spin fluctuations influences line shapes of tunneling conductance $\sigma_T(eV)$ in the extended VZK theory. Therefore, we will calculate $\sigma_T(eV)$ as a function of the bias voltage $V$ in DN/CS junctions for various parameters, such as the height $Z$ of the insulating barrier at the junction interface, the resistance $R_d$ in DN, the Thouless energy $E_{Th}$ in DN, and spin fluctuations $\lambda_{sf}$. In the present paper, we confine ourselves to zero temperature and put $k_B = h = 1$. 
2. Formulation

We consider a junction consisting of normal and superconducting reservoirs connected by a quasi-one-dimensional diffusive normal metal (DN) conductor with a length $l$ much larger than the mean-free path $l$ in the diffusive region. The interface between the DN conductor and the CS electrode has a resistance $R_b$. The position of the N/DN interface and the DN/CS interface is denoted as $x = 0$ and $x = l$, respectively. We model infinitely narrow insulating barrier by the δ-function $U(x) = H\delta(x - L)$. The resulting transparency of the junction $T_m$ is given by $T_m = 4\cos^2\phi/(4\cos^2\phi + Z^2)$, where $Z = 2H/v_F$ is dimensionless constant and $\phi$ is the injection angle measured from the interface normal to the junction and $v_F$ is Fermi velocity.

We apply the quasiclassical Keldysh formalism in the following calculation of the tunneling conductance. The $4 \times 4$ Green’s function in N, DN and CS are denoted by $\hat{G}_0(x)$, $\hat{G}_1(x)$ and $\hat{G}_2(x)$, where the Keldysh components $\hat{K}_0$, $\hat{K}_1$ and $\hat{K}_2$ are given by $\hat{K}_i(x) = \hat{R}_i(x)f_i(x) - \hat{f}_i(x)\hat{A}_i(x)$ with retarded component $\hat{R}_i(x)$, advanced component $\hat{A}_i(x) = -\hat{R}_i^*(x)$ using distribution function $\hat{f}_i$ ($i = 0, 1, 2$). In the above, $\hat{R}_0(x)$ is expressed by $\hat{R}_0(x) = \hat{\tau}_3$ and $\hat{f}_0(x) = f_{\delta 0} + \hat{\tau}_3f_{\delta 0}$. $\hat{R}_2(x)$ is expressed by $\hat{R}_2(x) = g\hat{\tau}_3 + f\hat{\tau}_2$ with $g = \varepsilon/\sqrt{\varepsilon^2 - \Delta_0^2}$ and $f = \Delta_0/\sqrt{\Delta_0^2 - \varepsilon^2}$, where $\varepsilon$ denotes the quasiparticle energy measured from the Fermi energy, $\hat{f}_2(x) = \tanh[\varepsilon/(2T)]$ in thermal equilibrium with temperature $T$, and $f_{\delta 0} = 1/2\{\tanh[(\varepsilon + eV)/(2T)] - \tanh[(\varepsilon - eV)/(2T)]\}$. We put the electrical potential zero in the CS electrode. In this case, the spatial dependence of $\hat{G}_1(x)$ in DN is determined by the static Usadel equation [2].

We use Nazarov’s boundary condition for $\hat{G}_1(x)$ at the DN/CS interface:

$$\frac{L}{R_d}\left(\hat{G}_1(x)\frac{\partial \hat{G}_1(x)}{\partial x}\right)_{x=L_} = \frac{\langle B\rangle}{R_b} = B = \frac{2T_m[\hat{G}_1(L_+), \hat{G}_2(L_+)]}{4 + T_m[\hat{G}_1(L_+), \hat{G}_2(L_+)] - 2}. \tag{1}$$

The average over the various angles of injected particles at the interface is defined as

$$\langle B(\phi)\rangle = \int_{-\pi/2}^{\pi/2} d\phi \ B(\phi) \cos \phi / \int_{-\pi/2}^{\pi/2} d\phi \ T(\phi) \cos \phi, \tag{2}$$

with $B(\phi) = B$ and $T(\phi) = T_m$. The resistance $R_b$ of the DN/CS interface is given by

$$R_b = R_0 \frac{2}{\int_{-\pi/2}^{\pi/2} d\phi \ T(\phi) \cos \phi}. \tag{3}$$

Here, $R_0$ is the Sharvin resistance [6]. In the actual calculation, it is convenient to use the standard $\theta$ parameterization when function $\hat{R}_1(x)$ is expressed as $\hat{R}_1(x) = \hat{\tau}_3 \cos \theta(x) + \hat{\tau}_2 \sin \theta(x)$. The parameter $\theta(x)$ is a measure of the superconducting proximity effect in DN. From the retarded or advanced component of the Usadel equation, the spatial dependence of $\theta(x)$ is determined by the following equation, including the effect of spin fluctuations

$$D\frac{\partial^2 \theta(x)}{\partial x^2} + 2i(1 + \lambda_{sf})\varepsilon \sin \theta(x) = 0, \tag{4}$$

where $D$ is the diffusive constant in DN and $\lambda_{sf}$ is the electron-spin fluctuations renormalization constant [7].

Next, we focus on the boundary condition at the DN/CS interface. Taking the retarded part of equation (1), we obtain

$$\frac{L}{R_d} \left. \frac{\partial \theta(x)}{\partial x} \right|_{x=L_} = \frac{\langle F\rangle}{R_b}, \quad F = \frac{2(f \cos \theta_L - g \sin \theta_L)T_m}{(2 - T_m) + T_m[g \cos \theta_L + f \sin \theta_L]}, \tag{5}$$

$$\frac{\partial \hat{G}_1(x)}{\partial x} \left|_{x=L_} = \frac{\langle G(\phi)\rangle}{R_b}, \quad G(\phi) = \int_{-\pi/2}^{\pi/2} d\phi \ \hat{G}_1(x, \phi) \cos \phi. \tag{6}$$
with $\theta_L = \theta(L_-)$. After some calculations, we obtain the following final result for the electric current

$$I_{el} = \frac{1}{2e} \int_0^\infty dx \frac{R_b}{\langle I_{b0} \rangle} + \frac{R_d}{L} \int_0^L dx \frac{f_{l0}}{\cosh^2 \theta_{im}(x)}. \tag{6}$$

Then differential resistance $R$ at zero temperature is given by

$$R = \frac{2R_b}{\langle I_{b0} \rangle} + \frac{2R_d}{L} \int_0^L dx \frac{dx}{\cosh^2 \theta_{im}(x)}, \tag{7}$$

with

$$I_{b0} = \frac{T_m^2 A_1 + 2T_m(2 - T_m)A_2}{2[(2 - T_m) + T_m(g \cos \theta_L + f \sin \theta_L)]^2}, \tag{8}$$

$$A_1 = (1 + |\cos \theta_L|^2 + |\sin \theta_L|^2)g^2 + (|f|^2 + 1) + 4\Im(fg^*)\Im(\cos \theta_L \sin \theta^*_L), \tag{9}$$

$$A_2 = \Re\{g(\cos \theta_L + \cos \theta^*_L) + f(\sin \theta_L + \sin \theta^*_L)\}. \tag{10}$$

This is an extended version of the VZK formula [3]. In the above, $\theta_{im}(x)$ denotes the imaginary part of $\theta(x)$.

In the following section, we will discuss the normalized tunneling conductance $\sigma_T(eV) = \sigma_S(eV)/\sigma_N(eV)$, where $\sigma_S(eV)$ is the total tunneling conductance in the superconducting state given by $\sigma_S(eV) = 1/R$ and $\sigma_N(eV)$ is the total tunneling conductance in the normal state given by $\sigma_N(eV) = \sigma_N = 1/(R_d + R_b)$. It is important to note that in the present circuit theory, $R_d/R_b$ can be varied independently of $T_m$, i.e., independently of $Z$, because one can change the magnitude of the constriction area $S_c$ independently [6].

3. Results

Let us first choose relatively low transparency of tunneling barrier, $Z = 5$. In this case, the ZBCP occurs due to the CAR, which induces the superconducting proximity effect in DN. Figure 1 represents the tunneling conductance $\sigma_T(eV)$ for $Z = 5$, $R_d/R_b = 1$, $E_{Th}/\Delta_0 = 0.01$ and various $\lambda_{sf}$. The ZBCP gradually sharpens with increasing $\lambda_{sf}$. Figure 2 shows $\sigma_T(eV)$ for $Z = 5$, $R_d/R_b = 1$, $E_{Th}/\Delta_0 = 0.1$ and various $\lambda_{sf}$. The ZBCP also appears in figure 2, as well as in figure 1, but the ZBCP width becomes much broader than that in figure 1. For all of $\sigma_T(eV)$’s, the order of magnitude of the ZBCP width is given by $E_{Th}$ and the tunneling conductance $\sigma_T(0)$ at zero energy does not change even when the magnitude of spin fluctuations $\lambda_{sf}$ varies. Figure 3 represents $\sigma_T(eV)$ for $Z = 0$, $R_d/R_b = 1$, $E_{Th}/\Delta_0 = 0.1$ and various $\lambda_{sf}$. The coherent peak structure smears and the ZBCD sharpens with decreasing $\lambda_{sf}$. On the other hand, in the case of an intermediate barrier strength, $Z = 1$, the line shape of $\sigma_T(eV)$ becomes rather complex. Figure 4 displays $\sigma_T(eV)$ for $Z = 1$, $R_d/R_b = 1$, $E_{Th}/\Delta_0 = 0.1$ and various $\lambda_{sf}$. $\sigma_T(eV)$ has a shallow gap structure together with a small ZBCD at zero energy. With the increase of $\lambda_{sf}$, the coherent peak structure at $eV = \pm \Delta_0$ is slightly smeared out, the width and the depth of the ZBCD become small, and the hump structure edges outside the ZBCD. As shown in figures 2–4, the line shapes of $\sigma_T(eV)$ changes from the ZBCP into the ZBCD with the increase of $Z$.

Contrastively, the behaviors of the line shapes of $\sigma_T(eV)$ become quite different in the relatively large Thouless energy with $E_{Th}/\Delta_0 = 1$. In this case, there are no remarkable changes in line shapes of $\sigma_T(eV)$. Hence, we can conclude that spin fluctuations effectively sharpen the ZBCP and the ZBCD at approximately $E_{Th}/\Delta_0 = 0.1$, and smear the coherent peak structure at $eV = \pm \Delta_0$ in the region of relatively small $Z$. 


4. Conclusions
In the present paper, based on Nazarov’s boundary condition, we have calculated tunneling conductance of DN/CS junctions. When the magnitude of spin fluctuations increases, the conductance value at zero energy for the ZBCP and the ZBCD does not change, but ZBCP and ZBCD widths sharpen. We have found that the spin fluctuations influence effectively on the line shapes of tunneling conductance around zero energy at approximately $E_{Th}/\Delta_0 = 0.1$.

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