Determining the theoretical trajectory of the centre of tractor’s mass when turning in a combined way

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Abstract. One of the ways to increase the stability of movement and improve the manoeuvrability and controllability of a wheeled universal row-crop tractor in curvilinear motion is the choice of a rational turning method. To solve the tasks, the authors proposed a combined method of turning the tractor, which consists in crabbing in the entry into the rotation, after which the rear wheels automatically return to the neutral position and then the direction of movement in the established rotation and exit from the rotation uses front steering wheels. The article developed a methodology for determining the theoretical trajectory of the centre of mass of a wheeled tractor with a combined method of rotation for the three distinct sections of the movement included in it. Based on the conditions of smoothness and continuity, we glued together three characteristic sections of the curvilinear trajectory of the centre of mass and obtained a continuous non-linear piecewise smooth function of the investigated combined rotation method. It is possible to apply the research results for theoretical calculations of the parameters for the curvilinear motion of tractors, as well as other wheeled vehicles, that allows choosing rational laws to change the speed and control modes of turning the wheels, to identify the patterns of change for ensuring the necessary stability, manoeuvrability and controllability and confirming effectiveness in comparison with known and previously investigated laws.

1. Introduction
As a rule, a wheeled universal row-crop tractor rotates by deflecting the front steered wheels by an angle relative to its skeleton. At the same time, ensuring a minimum turning radius requires a maximum angle of rotation of the steered wheels. However, its increase during rotation on loose soils does not give the desired result, since the resistance to rotation increases due to slipping, simultaneously with rolling, in the direction of the tractor axis due to the bulldozer effect. This negative phenomenon is especially intense at the entrance to the turn when there is a gradual transition from an infinitely large radius to a minimum [1].

2. Problem statement
Since one of the promising directions for the development of agricultural tractor construction is to increase the fleet of universal row-crop tractors with all steered wheels, we examined a combined turning method, namely the front and rear steered wheels synchronously rotate in the same direction
relative to the skeleton; when they reach the maximum angle, the rear wheels automatically return to
the neutral position and further rotation uses the front wheels (Fig. 1).

Currently, known methods for calculating the trajectories of the curved motion of wheeled vehicles
and evaluating their results [1-7] focuses mainly on traditional rotation methods, so we set the task of
determining the trajectory of a wheeled tractor with a combined rotation method since this version of
curved movement has not been investigated.

3. Materials and methods

Analysing the trajectories of the centre of mass at the entrance to the turn by crabbing (section I in Fig.
1) at constant translational speeds $v$ and angular speeds of rotation of the wheels of the front and rear
axles $\omega_1 = \omega_2 = \omega$ in the transverse plane, we found that the tractor makes not plane-parallel but
translational motion, which we investigated as the motion of a point around a circle with a radius
$R_1 = \frac{v}{\omega}$ and received the law of its motion in the form of the current coordinates of the theoretical
trajectory [8-10]

$$x(t) = \frac{v}{\omega} (1 - \cos \omega t);$$

$$y(t) = \frac{v}{\omega} \sin \omega t. \tag{1}$$

At the turn-in time $t_1$, the coordinates of the end-point, when the wheels are turned at the
maximum angles $\alpha_{1 \text{ max}} = \alpha_{2 \text{ max}}$, are equal according to equations (1) (Fig. 1)
At this point, the angle of inclination of the tangent to the trajectory is determined by the formula

$$
\varphi_1 = \arctg(y'(x_{\text{max}1}))
$$

where \( y' \) is the derivative of the function \( y(x) \).

From this moment, the rear wheels begin to turn in the opposite direction and come to the neutral position during time \( t_2 \). We assume that the return of the rear wheels to the neutral position begins at time \( t_2 = 0 \). In this case, the current coordinates for the theoretical trajectory of the centre of tractor’s mass in section II are described by the formulas [1]

$$
\begin{align*}
{x_{\text{max}1}} &= \frac{v}{\omega} \left(1 - \cos \omega t_1\right); \\
{y_{\text{max}1}} &= \frac{v}{\omega} \sin \omega t_1.
\end{align*}
$$

where \( \omega \) is the tractor base, m; \( B \) is tractor track width, m.

We note that the internal integral

$$
\int_0^{t_2} \frac{\sin(\alpha_{\text{max}1} - (\alpha_{\text{max}2} - \omega_2 \tau))}{B \left(\cos(\alpha_{\text{max}1} - \omega_1 \tau) + L \right) \cos \alpha_{\text{max}1} \cos(\alpha_{\text{max}2} - \omega_2 \tau)} d\tau
$$

with a variable upper integration limit \( t_2 \) may have a singularity of the integrand, and therefore, computer mathematics programs do not consider it [11]. We have proposed its polynomial approximation [11]. For example, when \( \alpha_{\text{max}1} = 0.56 \) rad, \( \alpha_{\text{max}2} = 0.56 \) rad, \( \omega_1 = 0 \) s\(^{-1}\), \( \omega_2 = -0.56 \) s\(^{-1}\), \( B = 1.8 \) m, \( L = 2.6 \) m, it has the following form:

$$
{v_2}(\tau) = 0.3\tau - 0.17\tau^2 + 0.095\tau^3 - 0.024\tau^4.
$$

Determining the continuous trajectory of the centre of tractor’s mass in the second and third sections requires a coordinate transformation consisting in the parallel transfer of the coordinate axes and their rotation by a certain angle [9].

For the second investigated motion section with a time interval \( t \in [0, t_2] \), at \( t_2 = 1 \) s, using formulas (2), we compose a two-dimensional array of points \( (x_2i, y_2i) \). For smooth glueing of the first and second sections of the trajectory, we rotate the coordinate system by an angle \( \varphi_2 = \pi / 2 - \varphi_1 \) according to the formulas

$$
\begin{align*}
{x_2i} &= x_2i \cos \varphi_2 + y_2i \sin \varphi_2; \\
y_2i &= -x_2i \sin \varphi_2 + y_2i \cos \varphi_2.
\end{align*}
$$

The resulting array is approximated by the function \( q_2(x) \) [11] passing through a point with coordinates \((0, 0)\), after which it is transferred in parallel to a point with coordinates \((x_{\text{max}1}, y_{\text{max}1})\)

$$
q_2(x) = q_2(x - x_{\text{max}1}) + y_{\text{max}1}.
$$

For the case in question

$$
q_2(x) = (94.84 - (x - 8.868)^2)^{1/2} - 3.16.
$$

The piecewise smooth function of the first and second sections of motion has the form [11]

$$
f_{1,2} = y(x), x \leq x_{\text{max}1}, \quad q_3(x), x > x_{\text{max}1}.
$$

The coordinates of the end-point of the second section \((x_{\text{max}2}, y_{\text{max}2})\) are determined by formulas (2), and the angle of inclination of the tangent to the trajectory at this point

$$
\varphi_3 = \arctg(f'_{1,2}(x_{\text{max}2})).
$$
Since the movement in section III occurs at a constant angle of rotation of the front wheels characterized by a constant radius of curvature $R_1$ (with the above parameters it is a circle of radius $R_1 = 5.061\, m$), at this stage the trajectory is constructed as the equation of a circle $q_4(x)$ of the specified radius $R_1$ passing through the point $(x_{\text{max2}}, y_{\text{max2}})$, which has a tangent at an angle $\phi_3$ in it.

And finally, we have a continuous piecewise smooth function for all three studied sections of the motion

$$f_{1,2,3}(x) = \begin{cases} f_{1,2}(x), & x \leq x_{\text{max2}}, \\ q_4(x), & x > x_{\text{max2}}. \end{cases}$$

(3)

Figure 2. The trajectory of the centre of tractor’s mass with a combined method of rotation

4. Conclusion

Figure 2 presents the results of calculations by formulas (3). With the accepted initial data, we obtained a maximum turn abscissa $x_{\text{max}} = 9.21\, m$, a maximum turn ordinate $y_{\text{max}} = 4.69\, m$ and a turn path length $S = 14.82\, m$.

For comparison, we present the results of calculations of the same values for a tractor with the same structural and kinematic characteristics as in the present studies, when turning the front steered wheels: $x_{\text{max}} = 10.2\, m$, $y_{\text{max}} = 5.93\, m$, $S = 17.84\, m$.

The comparison of the results showed that when turning the front wheels, relative to the combined method, the abscissa, the ordinate and the length of the turning path increase, respectively, by 9.7%, 20.9% and 16.9%.

Thus, we found that the most rational in terms of ensuring better stability, controllability and manoeuvrability is the combined method of rotation.

The obtained formulas for evaluating the tractor’s movement in a combined way allow theoretical calculations of the characteristics of its curved path to select rational laws of change in speed and control the rotation of the wheels, to identify patterns of change for ensuring the required parameters and confirming effectiveness in comparison with the known and previously investigated laws.
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