Two-body hadronic weak decays of antitriplet charmed baryons

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We study Cabibbo-favored (CF) and singly Cabibbo-suppressed (SCS) two-body hadronic weak decays of the antitriplet charmed baryons $\Lambda^+_c$, $\Xi^0_c$, and $\Xi^+_c$ with more focus on the last two. Both factorizable and nonfactorizable contributions are considered in the topologic diagram approach. The estimation of nonfactorizable contributions from $W$-exchange and inner $W$-emission diagrams relies on the pole model and current algebra. The non-perturbative parameters in both factorizable and nonfactorizable parts are calculated in the MIT bag model. Branching fractions and up-down decay asymmetries for all the CF and SCS decays of antitriplet charmed baryons are presented. The prediction of $B(\Xi^+_c \rightarrow \Xi^0 \pi^+)$ agrees well with the measurements inferred from Belle and CLEO, while the calculated $B(\Xi^0_c \rightarrow \Xi^- \pi^+)$ is too large compared to the recent Belle measurement. We conclude that these two $\Xi_c \rightarrow \Xi \pi^+$ modes cannot be simultaneously explained within the current-algebra framework for $S$-wave amplitudes. This issue needs to be resolved in future study. The long-standing puzzle with the branching fraction and decay asymmetry of $\Lambda^+_c \rightarrow \Xi^0 K^+$ is resolved by noting that only type-II $W$-exchange diagram will contribute to this mode. We find that not only the calculated rate agrees with experiment but also the predicted decay asymmetry is consistent with the SU(3)-flavor symmetry approach in sign and magnitude. Likewise, the CF mode $\Xi^0_c \rightarrow \Sigma^+ K^-$ and the SCS decays $\Xi^0_c \rightarrow p K^-, \Sigma^+ \pi^-$ proceed only through type-II $W$-exchange. They are predicted to have large and positive decay asymmetries. These features can be tested in the near future.

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I. INTRODUCTION

Recently, there has been a significant progress in the experimental study of charm physics. In the meson sector, LHCb measured the $CP$ asymmetry difference between $D^{0} \to K^{-}K^{+}$ and $D^{0} \to \pi^{-}\pi^{+}$, giving $\Delta A_{CP} = (-15.4 \pm 2.9) \times 10^{-4}$ \cite{1}, which is the first observation of $CP$ violation in the charm sector. The progress in charmed baryon physics is also impressive. The long-quested doubly charmed baryon was first observed through the process $Ξ_{cc}^{++} \to Λ_{c}^{+}K^{-}\pi^{+}\pi^{+}$ at LHCb in 2017 \cite{2}. Later in 2018, the lifetime of $Ξ_{cc}^{++}$ \cite{3}, its mass and the two-body weak decay channel $Ξ_{cc}^{++} \to Ξ_{c}^{+}\pi^{+}$ \cite{4} were measured by LHCb. Some breakthrough has also been made in singly charmed baryons as well, especially the lightest one $Λ_{c}^{+}$. Both Belle \cite{5} and BESIII \cite{6} have measured the absolute branching fraction of the decay $Λ_{c}^{+} \to pK^{-}\pi^{+}$. A new average of $(6.28 \pm 0.32)\%$ for this benchmark mode is quoted by the Particle Data Group (PDG) \cite{7}.

In addition to $Λ_{c}^{+}$, there have been some new developments in the study of $Ξ_{c}^{0}$ and $Ξ_{c}^{+}$, the two other singly charmed baryons in the antitriplet. By using a data set comprising $(772 \pm 11) \times 10^{6}$ $B\bar{B}$ pairs collected at $\Upsilon(4S)$ resonance, Belle was able to measure the absolute branching fraction for $B^{-} \to \bar{Λ}_{c}^{+}Ξ_{c}^{0}$ \cite{8}. Combining the subsequently measured product branching fractions such as $B(B^{-} \to \bar{Λ}_{c}^{+}Ξ_{c}^{0})B(Ξ_{c}^{0} \to Ξ^{-}\pi^{+})$, Belle reported the first weak decay of $Ξ_{c}^{0}$ \cite{9}.

\[ B(Ξ_{c}^{0} \to Ξ^{-}\pi^{+}) = (1.80 \pm 0.50 \pm 0.14) \times 10^{-2}. \] (1)

Using the same technique, a channel of two-body weak decay with a vector meson in final state was also measured, $B(Ξ_{c}^{+} \to pK^{0}(892)) = (0.25 \pm 0.16 \pm 0.04) \times 10^{-2}$ \cite{10}. It is worth pointing out that the absolute branching fraction for three-body decay was obtained by Belle \cite{10} to be $B(Ξ_{c}^{+} \to Ξ^{-}\pi^{+}\pi^{+}) = (2.86 \pm 1.21 \pm 0.38) \times 10^{-2}$, from which we can read

\[ B(Ξ_{c}^{+} \to Ξ^{0}\pi^{+}) = (1.57 \pm 0.83)\%. \] (2)

where use of $\Gamma(Ξ_{c}^{+} \to Ξ^{0}\pi^{+})/\Gamma(Ξ_{c}^{+} \to Ξ^{-}\pi^{+}\pi^{+}) = (0.55 \pm 0.13 \pm 0.09)$ obtained by the CLEO \cite{11} has been made.

Inspired by latest experimental results of $Ξ_{c}$ decays, there have been some efforts from theorists \cite{12-17}. Indeed, the study of charmed baryon weak decays, including the charged and neutral $Ξ_{c}$ baryons, is an old subject. To understand the underlying dynamical mechanism in hadronic weak decays, one may draw the topological diagrams according to the hadron’s content \cite{18}. In charmed baryon decays, nonfactorizable contributions from $W$-exchange or inner $W$-emission diagrams play an essential role and they cannot be neglected, in contrast with the negligible effects in heavy meson decays. The fact that all the decays of $Ξ_{c}^{+,0}$ receive nonfactorizable contributions, especially some decays such as $Ξ_{c}^{0} \to Σ^{0}K^{-},Ξ_{c}^{0}\pi^{0}$ proceed only through purely nonfactorizable diagrams, will allow us to check the importance and necessity of nonfactorizable contributions. However, we still do not have a reliable phenomenological model to calculate charmed baryon hadronic decays so far. In the 1990s various techniques were developed, including relativistic quark model (RQM) \cite{19, 20}, pole model \cite{21, 22} and current algebra \cite{22, 23}, to estimate the nonfactorizable effects in Cabibbo-favored $Ξ_{c}^{+,0}$ decays. The predicted branching fractions and decay asymmetries in various early model calculations are summarized in Table I \cite{19, 20, 21, 22, 23}.

Now with more experimental data accumulated, there are some updated studies in theory \cite{12-17, 24}. In these works except \cite{17}, the experimental results are taken as input and global fitting analyses are carried out.

\footnote{1 For early model calculations of Cabibbo-allowed $Λ_{c}^{+} \to B + P$ decays, see Table I of $Ξ_{c}$.}
TABLE I. Branching fractions (upper entry) and up-down decay asymmetries $\alpha$ (lower entry) of Cabibbo-allowed $\Xi_c^{\pm,0} \to B + P$ decays in various early model calculations. All the model results for branching fractions (in percent) have been normalized using the current world averages of $\tau(\Xi_c^0)$ and $\tau(\Xi_c^0)$ (see Eq. (6) below).

| Decay                      | Körner, Krämer [19] | Xu, Kamal [21] | Cheng, Tseng [22] | Ivanov et al [20] | Zenczykowski, Sharma, Expt. [7, 9] | CA | Pole | Expt. |
|---------------------------|---------------------|----------------|------------------|------------------|---------------------------------|----|------|-------|
| $\Xi_c^+ \to \Sigma^+ K^0$| 6.66                | 0.46           | 0.05             | 0.87             | 1.57 ± 0.83                     |    |      |       |
| $\Xi_c^0 \to \Sigma^0 \pi^+$| 3.65                | 3.47           | 0.87             | 4.06             | 1.57 ± 0.83                     |    |      |       |
| $\Xi_c^0 \to \Lambda K^0$ | 0.17                | 0.50           | 1.36             | 0.37             | 0.55                            |    |      |       |
| $\Xi_c^0 \to \Sigma^0 K^0$| 1.61                | 0.14           | 0.03             | 0.18             | 0.26                            |    |      |       |
| $\Xi_c^0 \to \Sigma^+ K^-$ | 0.17                | 0.17           | 0.35             |                 |                                 |    |      |       |
| $\Xi_c^0 \to \Sigma^0 \pi^0$| 0.05                | 0.77           | 1.71             | 0.38             | 0.05                            |    |      |       |
| $\Xi_c^+ \to \Xi^+ \pi^+$ | 1.42                | 2.37           | 1.13             | 1.71             | 1.60                            |    |      |       |
| $\Xi_c^+ \to \Xi^0 \eta$  | 0.32                | 0.37           |                  |                 |                                 |    |      |       |
| $\Xi_c^+ \to \Xi^0 \eta'$ | 1.16                | 0.41           |                  |                 |                                 |    |      |       |
| $\Xi_c^+ \to \Sigma^+ K^0$| −1.0                | 0.24           | 0.43             | −0.09            | −0.99                           |    | 1.0  | 0.54  |
| $\Xi_c^+ \to \Sigma^0 \pi^+$| −0.78               | −0.81          | −0.77            | −0.77            | −0.97                           |    | 1.0  | −0.27 |
| $\Xi_c^+ \to \Lambda K^0$ | −0.76               | 1.00           | −0.88            | −0.73            | −0.75                           |    | −0.29| −0.79 |
| $\Xi_c^+ \to \Sigma^0 K^0$| −0.96               | −0.99          | 0.85             | −0.59            | −0.55                           |    | −0.50| 0.48  |
| $\Xi_c^0 \to \Sigma^+ K^-$ | 0.92                | 0.92           | −0.78            | −0.54            | 0.94                            | 0.21| −0.80|       |
| $\Xi_c^0 \to \Sigma^0 \pi^+$| −0.38               | −0.38          | −0.47            | −0.99            | −0.84                           | −0.79| −0.97| −0.6 ± 0.4 |
| $\Xi_c^0 \to \Xi^0 \eta$  | −0.92               | −0.92          | −1.0             | −0.21            | −0.37                           |    |      |       |
| $\Xi_c^0 \to \Xi^0 \eta'$ | −0.38               | −0.32          | −0.04            | 0.56             |                                 |    |      |       |

at the hadron level based on SU(3) flavor symmetry without resorting to the detailed dynamics. Apparently, a reconsideration of charmed baryon weak decays, revealing the dynamics at the quark level, is timely and necessary. Pole model is one of the choices.

In the pole model, important low-lying $1/2^+$ and $1/2^-$ states are usually considered under the pole approximation. In the decay with a pseudoscalar in the final state, $B_c \to B^+ + P$, the nonfactorizable $S$- and $P$-wave amplitudes are dominated by $1/2^-$ low-lying baryon resonances and $1/2^+$ ground state baryons, respectively. The $S$-wave amplitude can be further reduced to current algebra in the soft-pseudoscalar limit. That is, the evaluation of the $S$-wave amplitude does not require the information of the troublesome negative-parity baryon resonances which are not well understood in the quark model. The methodology was developed and applied in the earlier work [22]. It turns out if the $S$-wave amplitude is evaluated in the pole model or in the covariant quark model and its variant, the decay asymmetries for both $\Lambda_c^+ \to \Sigma^+ \pi^0$ and $\Sigma_c^0 \pi^+$ were always predicted to be positive, while it was measured to be $-0.45 \pm 0.31 \pm 0.06$ for $\Sigma_c^+ \pi^0$ by CLEO [24]. In contrast, current algebra always leads to a negative decay asymmetry for aforementioned two modes: $-0.49$ in [22], $-0.31$ in [24], $-0.76$ in [27] and $-0.47$ in [28]. The issue with the sign of $\alpha(\Lambda_c^+ \to \Sigma^+ \pi^0)$ was finally resolved by BESIII. The decay asymmetry parameters of $\Lambda_c^+ \to \Lambda \pi^+$, $\Sigma_c^0 \pi^+$, $\Sigma_c^+ \pi^0$ and $pK_S$ were recently measured by BESIII [29] (see Table III below), for example, $\alpha(\Lambda_c^+ \to \Sigma^+ \pi^0) = -0.57 \pm 0.12$ was obtained. Hence, the negative sign of $\alpha(\Lambda_c^+ \to \Sigma^+ \pi^0)$ measured by CLEO is nicely confirmed by BESIII. This is one of the strong reasons why we adapt current algebra to work out parity-violating amplitudes.

It is well known that there is a long-standing puzzle with the branching fraction and decay asymmetry of $\Lambda_c^+ \to \Xi^0 K^+$. The calculated branching fraction turns out to be too small compared to experiment and the decay asymmetry is predicted to be zero owing to the vanishing $S$-wave amplitude. We shall examine this issue in this work and point out a solution to this puzzle. This has important implications to the $\Xi_c^0$ sector where the CF mode $\Xi_c^0 \to \Sigma^+ K^-$ and the SCS decays $\Xi_c^0 \to pK^-, \Sigma^+ \pi^-$ will encounter similar problems in naive calculations.
Recently, we have followed this approach to calculate singly Cabibbo-suppressed (SCS) decays of $\Lambda_c^+$, in which the predictions of $\Lambda_c^+ \rightarrow p\pi^0, p\eta$ are in good agreement with the BESIII measurement. In this work, we shall continue working in the pole model together with current algebra to compute both CF and SCS two-body weak decays of $\Xi_c$ baryons.

This paper is organized as follows. In Sec. II we will set up the formalism for computing branching fractions and up-down decay asymmetries, including contributions from both factorizable and nonfactorizable terms. Numerical results are presented in Sec. III. A conclusion will be given in Sec. IV. In Appendix A, we write down the baryon wave functions to fix our convention and then examine their behavior under $U$-, $V$-, and $I$-spin in Appendix B. Appendix C is devoted to the form factors for $\Lambda_c^+ \rightarrow B$ transitions evaluated in the MIT bag model. The expressions of baryon matrix elements and axial-vector form factor calculated in MIT bag model will be presented in Appendix D.

II. FORMALISM

A. Kinematics

Without loss of generality, the amplitude for the two-body weak decay $B_i \rightarrow B_f P$ can be parameterized as

$$M(B_i \rightarrow B_f P) = i\bar{u}_f(A - B\gamma^5)u_i,$$

where $P$ denotes a pseudoscalar meson. Based on the $S$- and $P$-wave amplitudes, $A$ and $B$, the decay width and up-down spin asymmetry are given by

$$\Gamma = \frac{p_c}{8\pi} \left[ \frac{(m_i + m_f)^2 - m_P^2}{m_i^2} |A|^2 + \frac{(m_i - m_f)^2 - m_P^2}{m_i^2} |B|^2 \right],$$

$$\alpha = \frac{\kappa \text{Re}(A^*B)}{|A|^2 + \kappa^2|B|^2},$$

with $\kappa = p_c/(E_f + m_f) = \sqrt{(E_f - m_f)/(E_f + m_f)}$ and $p_c$ is the three-momentum in the rest frame of mother particle. To proceed, we need the lifetimes of the relevant charmed baryons which are quoted as the new world averages

$$\tau(\Xi_c^+) = (4.56 \pm 0.05) \times 10^{-13} \text{ s}, \quad \tau(\Xi_c^0) = (1.53 \pm 0.02) \times 10^{-13} \text{ s},$$

dominated by the most recent lifetime measurements by the LHCb. Note that the measured $\Xi_c^0$ lifetime by the LHCb is approximately 3.3 standard deviations larger than the old world average value.

The $S$- and $P$-wave amplitudes of the two-body decay generically receive both factorizable and nonfactorizable contributions

$$A = A^{\text{fac}} + A^{\text{nf}}, \quad B = B^{\text{fac}} + B^{\text{nf}}.$$
B. Factorizable contribution

The description of the factorizable contribution of the charmed baryon decay $B_c \rightarrow BP$ is based on the effective Hamiltonian approach.

1. General expression of factorizable amplitudes

The effective Hamiltonian for CF process is

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{\text{us}} V_{\text{ud}}^* (c_1 O_1 + c_2 O_2) + \text{h.c.},$$

where the four-quark operators are given by

$$O_1 = (\bar{u}c)(\bar{d}d), \quad O_2 = (\bar{u}c)(\bar{d}d),$$

with $(q_1 q_2) \equiv q_1 \gamma_\mu (1 - \gamma_5) q_2$. The Wilson coefficients to the leading order are given as $c_1 = 1.346$ and $c_2 = -0.636$ at $\mu = 1.25 \text{ GeV}$ and $\Lambda_{\overline{MS}}^{(4)} = 325 \text{ MeV}$ [31]. Under naive factorization the amplitude can be written down as

$$M = \langle B | \mathcal{H}_{\text{eff}} | B_c \rangle = \left\{ \begin{array}{l} \frac{G_F}{\sqrt{2}} V_{\text{us}} V_{\text{ud}}^* a_2 \langle P | (\bar{u}d) | 0 \rangle \langle B | (\bar{c}u) | B_c \rangle, \quad P = \overline{K}^0, \\ \frac{G_F}{\sqrt{2}} V_{\text{us}} V_{\text{ud}}^* a_1 \langle P | (\bar{u}d) | 0 \rangle \langle B | (\bar{c}u) | B_c \rangle, \quad P = \pi^+. \end{array} \right.$$  

(10)

where $a_1 = c_1 + \frac{q_1}{p_1}$ and $a_2 = c_2 + \frac{q_2}{p_2}$. In terms of the decay constants [2]

$$\langle K(q) | \overline{c} \gamma_\mu (1 - \gamma_5) d | 0 \rangle = i f_K q_\mu, \quad \langle \pi(q) | \overline{u} \gamma_\mu (1 - \gamma_5) d | 0 \rangle = i f_\pi q_\mu,$$

and the form factors defined by

$$\langle B(p_2) | \overline{c} \gamma_\mu (1 - \gamma_5) u | B_c (p_1) \rangle = \bar{u}_2 \left[ f_1 (q^2) \gamma_\mu - f_2 (q^2) i \sigma_{\mu\nu} \frac{q^\nu}{M} + f_3 (q^2) \frac{q_\mu}{M} \right] + \left( g_1 (q^2) \gamma_\mu - g_2 (q^2) i \sigma_{\mu\nu} \frac{q^\nu}{M} + g_3 (q^2) \frac{q_\mu}{M} \right) \gamma_5 u_1,$$

with the momentum transfer $q = p_1 - p_2$, we obtain the amplitude

$$M(B_c \rightarrow BP) = \frac{G_F}{\sqrt{2}} a_{1,2} V_{\text{us}}^* V_{\text{us}} f_P \bar{u}_2 (p_2) \left[ (m_1 - m_2) f_1 (q^2) + (m_1 + m_2) g_1 (q^2) \gamma_5 \right] u_1 (p_1),$$

(13)

where contributions from the form factors $f_3$ and $g_3$ can be neglected. The factorizable contributions to $A$ and $B$ terms finally read

$$A_{\text{fac}}^{1,2} |_{\text{CP}} = \frac{G_F}{\sqrt{2}} a_{1,2} V_{\text{us}}^* V_{\text{us}} f_P (m_{B_c} - m_B) f_1 (q^2),$$

$$B_{\text{fac}}^{1,2} |_{\text{CP}} = -\frac{G_F}{\sqrt{2}} a_{1,2} V_{\text{us}}^* V_{\text{us}} f_P (m_{B_c} + m_B) g_1 (q^2),$$

(14)

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2 Here we follow the PDG convention $\langle 0 | A_\mu (0) | P(q) \rangle = i f_P q_\mu$ for the decay constant. This differs from the sign convention used in [23].
where the choice of $a_i$ can be referred to Eq. (10).

Likewise, the $S$- and $P$-wave amplitudes for SCS processes are given by

$$A^{\text{fac}}_{\text{SCS}} = \frac{G_F}{\sqrt{2}} a_{1,2} V_{uq}^+ V_{cq} f_P(m_{B_c} - m_S) f_1(q^2),$$

$$B^{\text{fac}}_{\text{SCS}} = -\frac{G_F}{\sqrt{2}} a_{1,2} V_{uq}^+ V_{cq} f_P(m_{B_c} + m_B) g_1(q^2),$$

where the flavor of the down-type quark $q, d$ or $s$, depends on the process. If $P = \eta_S$, both flavors contribute, for example,

$$A^{\text{fac}}(\Lambda^+_c \rightarrow \eta \gamma) = \frac{G_F}{\sqrt{2}} a_2 \left( V_{c\eta} f_0 + \frac{1}{\sqrt{2}} V_{cd} f_0^* \right) (m_{\Lambda_c} - m_p) f_1^\Lambda_{\eta\gamma}(m_\eta^2),$$

$$B^{\text{fac}}(\Lambda^+_c \rightarrow \eta \gamma) = -\frac{G_F}{\sqrt{2}} a_2 \left( V_{c\eta} f_0 + \frac{1}{\sqrt{2}} V_{cd} f_0^* \right) (m_{\Lambda_c} + m_p) g_1^\Lambda_{\eta\gamma}(m_\eta^2),$$

where the decay constants are defined by

$$\langle \eta | \hat{q} \gamma_{\mu} (1 - \gamma_5) q | 0 \rangle = i \frac{1}{\sqrt{2}} f_3^\eta q_{\mu}, \quad \langle \eta | \bar{s} \gamma_{\mu} (1 - \gamma_5) s | 0 \rangle = i f_4^\eta q_{\mu}. \quad (17)$$

We shall follow [32] to use $f_3^\eta = 107$ MeV and $f_4^\eta = -112$ MeV. Notice that in the case of $\pi^0$ production in the final state, one should replace $a_2$ by $-a_2/\sqrt{2}$ in the factorizable amplitude, where the extra factor of $-1/\sqrt{2}$ comes from the wave function of the $\pi^0$, $\pi^0 = (u\bar{u} - d\bar{d})/\sqrt{2}$.

2. The parameterization of form factors

There are two different non-perturbative parameters in factorizable amplitudes, the decay constant and the form factor (FF). There exist some efforts for estimating the FFs for $\Xi_c \rightarrow B$ transition [16, 33–35]. In this work we prefer to work out FFs for $\Xi_c - B$ transition and baryonic matrix elements all within the MIT bag model [30]. Since the decay rates and decay asymmetries are sensitive to the relative sign between factorizable and non-factorizable amplitudes, it is also desired to have an estimation of FFs in a globally consistent convention.

In this work we follow [34] to write the $q^2$ dependence of FF as

$$f_i(q^2) = \frac{f_i(0)}{(1 - q^2/m_{i}^2)^2}, \quad g_i(q^2) = \frac{g_i(0)}{(1 - q^2/m_{i}^2)^2}, \quad (18)$$

where $m_V = 2.01$ GeV, $m_A = 2.42$ GeV for the $(cd)$ quark content, and $m_V = 2.11$ GeV, $m_A = 2.54$ GeV for $(cs)$ quark content. In the zero recoil limit where $q_{\text{max}}^2 = (m_i - m_f)^2$, FFs can be expressed within the MIT bag model as [22]

$$f_i^{B_iB_i}(q_{\text{max}}^2) = \langle B_i^+ | b_{i1} b_{i2} | B_i^+ \rangle \int d^3r \left( u_{q_1}(r) u_{q_2}(r) + v_{q_1}(r) v_{q_2}(r) \right),$$

$$g_i^{B_iB_i}(q_{\text{max}}^2) = \langle B_i^+ | b_{i1} b_{i2} \sigma_z | B_i^+ \rangle \int d^3r \left( u_{q_1}(r) u_{q_2}(r) - \frac{1}{3} v_{q_1}(r) v_{q_2}(r) \right), \quad (19)$$

where $u(r)$ and $v(r)$ are the large and small components, respectively, of the quark wave function in the bag model. FFs at different $q^2$ are related via

$$f_i(q_2^2) = \left( \frac{1 - q_2^2/m_{i}^2}{1 - q_1^2/m_{i}^2} \right)^2 f_i(q_1^2), \quad g_i(q_2^2) = \left( \frac{1 - q_2^2/m_{i}^2}{1 - q_1^2/m_{i}^2} \right)^2 g_i(q_1^2). \quad (20)$$
This allows us to obtain the physical FF at $q^2 = m_{1/2}^2$.

It is obvious that the FF at $q_{\text{max}}^2$ is determined only by the baryons in initial and final states. However, its evolution with $q^2$ is governed by both the final-state meson and relevant quark content. Such dependence is reflected in Table II in which the quark contents are shown in the second column. In the zero recoil limit, the FFs at $q_{\text{max}}^2$ calculated from Eq. (18) are presented in the third and fifth columns. And then in the fourth and sixth columns, the evolution of FFs from $q^2 = q_{\text{max}}^2$ to $q^2 = m_{1/2}^2$ are derived according to Eq. (20). The auxiliary quantities $Y_{1/2}^{(s)}$ are defined in terms of

$$
Y_1 = 4\pi \int r^2 dr (u_u u_c + v_u v_c), \quad Y_1^s = 4\pi \int r^2 dr (u_u u_c + v_u v_c), \\
Y_2 = 4\pi \int r^2 dr (u_u u_c - \frac{1}{3} v_u v_c), \quad Y_2^s = 4\pi \int r^2 dr (u_u u_c - \frac{1}{3} v_u v_c).
$$

The model parameters are adopted from [25] and references therein. Numerically, we have $Y_1 = 0.88, Y_1^s = 0.95, Y_2 = 0.77, Y_2^s = 0.86$, which are consistent with the corresponding numbers in [22].

In the second column of Table V in [16], we see that the sign of the FFs $f_1^{\Xi_0^0 \Xi^-}$ and $g_1^{\Xi_0^0 \Xi^-}$ is positive, which differs from our result shown in Table I. As mentioned before, such a sign difference will affect the decay rates and asymmetries for processes involving both factorizable and nonfactorizable terms.
C. Nonfactorizable contribution

We work in the framework of the pole model to estimate nonfactorizable contributions. It is known that the S-wave amplitude is dominated by the low-lying \(1/2^-\) resonances, while the P-wave one governed by the ground-state \(1/2^+\) pole. The general formulas for \(A\) (S-wave) and \(B\) (P-wave) terms in the pole model are given by

\[
A_{\text{pole}} = \sum_{\Delta S=1/2^-} \left[ b_n r \frac{A S P B_n P}{m_f - m_n^*} + \frac{b_{n^*} A S P B_n P}{m_f - m_n^*} \right],
\]

\[
B_{\text{pole}} = \sum_{\Delta S=1/2^-} \left[ b_n r \frac{A S P B_n P}{m_f - m_n^*} + \frac{b_{n^*} A S P B_n P}{m_f - m_n^*} \right],
\]

where \(a_{ij}, b_{ij}\) are the baryon matrix elements defined by

\[
\langle B_n | H | B_i \rangle = \bar{u}_n (a_{ni} - b_{ni} \gamma_5) u_i, \quad \langle B_i^*(1/2^-) | H | B_j \rangle = \bar{u}_i b_{i^* j} u_j.
\]  

(23)

In the soft-meson limit, the intermediate excited \(1/2^-\) states in the S-wave amplitude can be summed up and reduced to a commutator term

\[
A_{\text{com}} = -\frac{\sqrt{2}}{f_{P^a}} \langle B_j | (Q^a_{5}, H_{PV}^{\text{eff}}) | B_i \rangle = \frac{\sqrt{2}}{f_{P^a}} \langle B_j | (Q^a, H_{PC}^{\text{eff}}) | B_i \rangle
\]

(24)

with

\[
Q^a = \int d^3 \bar{q} \gamma^0 \frac{\lambda^a}{2} q, \quad Q^a_5 = \int d^3 \bar{q} \gamma^5 \gamma^0 \frac{\lambda^a}{2} q.
\]

(25)

By applying the generalized Goldberger-Treiman relation

\[
g_{B' B} = \sqrt{2} \left( m_B + m_{B'} \right) g_{B'B}^A,
\]

(26)

the P-wave amplitude can be simplified to

\[
B_{\text{pole}} = \frac{\sqrt{2}}{f_{P^a}} \sum_{\Delta S=1/2^-} \left[ g_{B' B}^A \frac{m_f + m_n^*}{m_i - m_n^*} a_{ni} + \frac{m_i + m_n}{m_f - m_n^*} g_{B' B}^A \right].
\]

(27)

Therefore, the two master equations Eq. (24) and Eq. (27) for the nonfactorizable contributions in the pole model rely on the commutator terms and the axial-vector form factor \(g_{B'B}^A\) which will be calculated in the MIT bag model in this work.

1. S-wave amplitude

We have deduced that the S-wave amplitude is determined by the commutator terms of conserving charge \(Q^a\) and the parity-conserving part of the Hamiltonian. In the following we list the expressions of

---

3. Note we have corrected the sign of the \(B\) term in [23].

4. The applied relation \([Q_5, H_{\text{eff}}^{\text{PV}}] = -[Q, H_{\text{eff}}^{\text{PC}}]\) differs from that in [23] in sign.
$A^{\text{com}}$ according to Eq. (24):

$$A^{\text{com}}(B_i \to B_f \pi^+) = \frac{1}{f_\pi} \langle B_f | [I_{\pi}, H_{\text{eff}}^{PC}] | B_i \rangle,$$

$$A^{\text{com}}(B_i \to B_f \pi^0) = \frac{\sqrt{2}}{f_\pi} \langle B_f | I_{\pi} | B_i \rangle,$$

$$A^{\text{com}}(B_i \to B_f \eta_8) = \sqrt{\frac{3}{2}} \frac{1}{f_{\eta_8}} \langle B_f | [Y, H_{\text{eff}}^{PC}] | B_i \rangle,$$

$$A^{\text{com}}(B_i \to B_f K^+) = \frac{1}{f_K} \langle B_f | [V_+, H_{\text{eff}}^{PC}] | B_i \rangle,$$

$$A^{\text{com}}(B_i \to B_f \bar{K}^0) = \frac{1}{f_K} \langle B_f | [U_+, H_{\text{eff}}^{PC}] | B_i \rangle,$$

$$A^{\text{com}}(B_i \to B_f K^0) = \frac{1}{f_K} \langle B_f | [U_-, H_{\text{eff}}^{PC}] | B_i \rangle,$$

where we have introduced the isospin, $U$-spin and $V$-spin ladder operators with

$$I_+ |d\rangle = |u\rangle, \quad I_- |u\rangle = |d\rangle, \quad U_+ |s\rangle = |d\rangle, \quad U_- |d\rangle = |s\rangle, \quad V_+ |s\rangle = |u\rangle, \quad V_- |u\rangle = |s\rangle.$$  \hfill (29)

In Eq. (28), $\eta_8$ is the octet component of the $\eta$ and $\eta'$

$$\eta = \cos \theta_{\eta_8} - \sin \theta_{\eta_8}, \quad \eta' = \sin \theta_{\eta_8} + \cos \theta_{\eta_8},$$  \hfill (30)

with $\theta = -15.4^\circ$. For the decay constant $f_{\eta_8}$, we shall follow [32] to use $f_{\eta_8} = f_8 \cos \theta$ with $f_8 = 1.26 f_{\pi}$. Hypercharge $Y$, the conserving charge for processes involving $\eta_8$ in the final state, is taken to be $Y = B + S - C$ as argued in [28]. The baryon matrix elements of commutators in Eq. (28), after the action of the ladder operators on baryon wave functions shown in Appendix B can be further reduced to pure matrix elements of effective Hamiltonian, denoted by $a_{BG} \equiv \langle B'| H_{\text{eff}}^{PC} | B \rangle$. Then in terms of $a_{BG}$, nonfactorizable contributions to $S$-wave amplitudes for charmed baryon decays are calculable.

For the Cabibbo-favored processes, we have

$$A^{\text{com}}(\Lambda_c^+ \to p K^0) = \frac{1}{f_K} a_{\Sigma^+ \Lambda_c^+}, \quad A^{\text{com}}(\Lambda_c^+ \to \Lambda \pi^+) = 0,$$

$$A^{\text{com}}(\Lambda_c^+ \to \Sigma^0 \pi^+) = -\frac{\sqrt{2}}{f_\pi} a_{\Sigma^+ \Lambda_c^+}, \quad A^{\text{com}}(\Lambda_c^+ \to \Sigma^+ \pi^0) = \frac{\sqrt{2}}{f_\pi} a_{\Sigma^+ \Lambda_c^+},$$

$$A^{\text{com}}(\Lambda_c^+ \to \Xi^0 K^0) = \frac{1}{f_K} a_{\Sigma^+ \Lambda_c^+}, \quad A^{\text{com}}(\Lambda_c^+ \to \Sigma^+ \eta_8) = \frac{\sqrt{2}}{\sqrt{3} f_{\eta_8}} a_{\Sigma^+ \Sigma^+},$$  \hfill (31)

and

$$A^{\text{com}}(\Xi_c^+ \to \Sigma^+ K^0) = \frac{1}{f_K} a_{\Sigma^+ \Lambda_c^+}, \quad A^{\text{com}}(\Xi_c^+ \to \Xi^0 \pi^+) = -\frac{1}{f_\pi} a_{\Xi^0 \Xi^0},$$

$$A^{\text{com}}(\Xi_c^0 \to \Lambda K^0) = \frac{\sqrt{6}}{f_K} a_{\Xi^0 \Xi^0}, \quad A^{\text{com}}(\Xi_c^0 \to \Sigma^0 K^0) = -\frac{\sqrt{2}}{2 f_\pi} a_{\Xi^0 \Xi^0},$$

$$A^{\text{com}}(\Xi_c^0 \to \Sigma^+ K^-) = \frac{1}{f_K} a_{\Xi^0 \Xi^0}, \quad A^{\text{com}}(\Xi_c^0 \to \Sigma^0 \eta_8) = \frac{\sqrt{2}}{f_\pi} a_{\Xi^0 \Xi^0},$$

$$A^{\text{com}}(\Xi_c^0 \to \Xi^0 \eta_8) = \frac{\sqrt{6}}{f_{\eta_8}} a_{\Xi^0 \Xi^0}, \quad A^{\text{com}}(\Xi_c^0 \to \Xi^0 \pi^+) = \frac{1}{f_\pi} a_{\Xi^0 \Xi^0}. \hfill (32)$$
For singly Cabibbo-suppressed processes we have

\[
A^{\text{com}}(\Xi_c^+ \to \Sigma^0 \pi^+) = -\frac{1}{f_\pi} \left( \sqrt{2} g_{\Sigma^0 \Xi^+} + a_{\Sigma^0 \Xi^+} \right)
\]

\[
A^{\text{com}}(\Xi_c^+ \to pK^0) = -\frac{1}{f_K} \left( a_{\Sigma^0 \Xi^+} - a_{\Lambda \Xi^+} \right)
\]

\[
A^{\text{com}}(\Xi_c^+ \to \Sigma^+ \eta_8) = \frac{\sqrt{6}}{2} \frac{1}{f_{\eta_8}} a_{\Sigma^+ \Xi^+}
\]

\[
A^{\text{com}}(\Xi_c^+ \to \Xi^0 K^0) = \frac{1}{f_K} \left( -\frac{\sqrt{2}}{2} a_{\Sigma^0 \Xi^+} + \frac{\sqrt{6}}{2} a_{\Lambda \Xi^+} \right)
\]

\[
A^{\text{com}}(\Xi_c^+ \to \Sigma^0 \pi^0) = \frac{1}{f_\pi} \left( -\frac{\sqrt{2}}{2} a_{\Sigma^0 \Xi^+} + \frac{\sqrt{6}}{2} a_{\Lambda \Xi^+} \right)
\]

\[
A^{\text{com}}(\Xi_c^+ \to \Lambda \pi^+) = -\frac{1}{f_\pi} \left( a_{\Lambda \Xi^+} - a_{\Lambda \Xi^+} \right)
\]

\[
A^{\text{com}}(\Xi_c^+ \to \Sigma^+ \pi^0) = -\frac{\sqrt{2}}{f_K} \left( a_{\Sigma^0 \Xi^+} + \frac{\sqrt{6}}{2} a_{\Lambda \Xi^+} \right)
\]

\[
A^{\text{com}}(\Xi_c^+ \to \Xi^0 K^+ + \Lambda^0 \pi^-) = \frac{\sqrt{6}}{2} \frac{1}{f_{\eta_8}} a_{\Lambda \Xi^+}
\]

\[
A^{\text{com}}(\Xi_c^+ \to \Sigma^0 K^+) = \frac{1}{f_K} \left( \frac{\sqrt{2}}{2} a_{\Sigma^0 \Xi^+} - \frac{\sqrt{6}}{2} a_{\Lambda \Xi^+} \right)
\]

\[
A^{\text{com}}(\Xi_c^+ \to \Lambda \eta_8) = \frac{\sqrt{6}}{2} \frac{1}{f_{\eta_8}} a_{\Lambda \Xi^+}
\]

and

\[
A^{\text{com}}(\Xi_c^0 \to \Xi^0 K^0) = \frac{1}{f_K} \left( \frac{\sqrt{2}}{2} a_{\Sigma^0 \Xi^+} + \frac{\sqrt{6}}{2} a_{\Lambda \Xi^+} \right)
\]

\[
A^{\text{com}}(\Xi_c^0 \to \Xi^0 \eta_8) = \frac{\sqrt{6}}{2} \frac{1}{f_{\eta_8}} a_{\Lambda \Xi^+}
\]

\[
A^{\text{com}}(\Xi_c^0 \to \Lambda^0 \pi^-) = \frac{1}{f_\pi} \left( -\frac{\sqrt{2}}{2} a_{\Sigma^0 \Xi^+} + \frac{\sqrt{6}}{2} a_{\Lambda \Xi^+} \right)
\]

\[
A^{\text{com}}(\Xi_c^0 \to \Sigma^0 \eta_8) = \frac{\sqrt{6}}{2} \frac{1}{f_{\eta_8}} a_{\Lambda \Xi^+}
\]

The nonfactorizable S-wave amplitudes for SCS decays of \(\Lambda_c^+\) can be found in [23]. The evaluation of the baryon matrix elements \(a_{B'g}\) in the MIT bag model and results are presented in Appendix D 1.

\[
2. \quad P\text{-wave amplitude}
\]

Through the generalized Goldberger-Treiman relation Eq. (26), the strong coupling of \(\mathcal{B}'\mathcal{M}\) can be expressed in terms of the axial-vector form factor \(g_{A^0}^B\). Based on Eq. (27), P-wave amplitudes are given as follows. For Cabibbo-favored processes we have

\[
B^{\text{ca}}(\Lambda_c^+ \to pK^0) = \frac{1}{f_K} \left( g_{p\Sigma^+} m_{\rho^+} + m_{\Sigma^+} a_{\Sigma^+ \Lambda^+} \right)
\]

\[
B^{\text{ca}}(\Lambda_c^+ \to \Lambda \pi^+) = \frac{1}{f_\pi} \left( a_{\Lambda \Sigma^+} m_{\rho^+} + m_{\Sigma^+} g_{A^0}^E \right)
\]

\[
B^{\text{ca}}(\Lambda_c^+ \to \Sigma^0 \eta_8) = \frac{\sqrt{6}}{2} \frac{1}{f_{\eta_8}} a_{\Lambda \Xi^+}
\]

\[
B^{\text{ca}}(\Lambda_c^+ \to \Sigma^0 K^+) = \frac{1}{f_K} \left( g_{\Sigma^0 \Xi^+} m_{\rho^+} + m_{\Sigma^+} a_{\Sigma^+ \Lambda^+} \right)
\]

\[
B^{\text{ca}}(\Lambda_c^+ \to \Sigma^0 \pi^+) = \frac{\sqrt{2}}{f_\pi} \left( a_{\Sigma^0 \Xi^+} + a_{\Lambda \Xi^+} \right)
\]

\[
B^{\text{ca}}(\Lambda_c^+ \to \Sigma^0 \eta_8) = \frac{\sqrt{6}}{2} \frac{1}{f_{\eta_8}} a_{\Lambda \Xi^+}
\]

\[
B^{\text{ca}}(\Lambda_c^+ \to \Xi^0 K^+) = \frac{1}{f_K} \left( g_{\Xi^0 \Xi^+} m_{\rho^+} + m_{\Sigma^+} a_{\Sigma^+ \Lambda^+} \right)
\]

\[
B^{\text{ca}}(\Lambda_c^+ \to \Lambda \eta_8) = \frac{\sqrt{6}}{2} \frac{1}{f_{\eta_8}} a_{\Lambda \Xi^+}
\]

(35)
and

\[
B^{ca}(\Xi^+_c \rightarrow \Sigma^+ K^0) = \frac{f_K}{f} \left( \frac{m_{\Xi^0} + m_{\Lambda^+}}{m_{\Sigma^+} - m_{\Lambda^+}} g_{\Lambda(\Sigma^+)} + \frac{m_{\Xi^0} + m_{\Sigma^+}}{m_{\Sigma^+} - m_{\Xi^0}} g_{\Xi(\Sigma^0)} \right),
\]

\[
B^{ca}(\Xi^+_c \rightarrow \Xi^0 \pi^+) = \frac{f_K}{f} \left( \frac{m_{\Xi^0} + m_{\Sigma^0}}{m_{\Sigma^+} - m_{\Xi^0}} g_{\Lambda(\Sigma^0)} + \frac{m_{\Xi^0} + m_{\Sigma^0}}{m_{\Sigma^0} - m_{\Xi^0}} g_{\Xi(\Sigma^0)} \right),
\]

\[
B^{ca}(\Xi^+_c \rightarrow \Lambda K^0) = \frac{f_K}{f} \left( \frac{m_{\Xi^0} + m_{\Sigma^0}}{m_{\Lambda} - m_{\Sigma^0}} g_{\Lambda(\Sigma^0)} + \frac{m_{\Xi^0} + m_{\Sigma^0}}{m_{\Sigma^0} - m_{\Xi^0}} g_{\Xi(\Sigma^0)} \right),
\]

\[
B^{ca}(\Xi^+_c \rightarrow \Sigma^0 K^+) = \frac{f_K}{f} \left( \frac{m_{\Xi^0} + m_{\Sigma^+}}{m_{\Sigma^0} - m_{\Xi^0}} g_{\Lambda(\Sigma^0)} + \frac{m_{\Xi^0} + m_{\Sigma^+}}{m_{\Sigma^0} - m_{\Xi^0}} g_{\Xi(\Sigma^0)} \right),
\]

\[
B^{ca}(\Xi^+_c \rightarrow \Sigma^+ K^-) = \frac{f_K}{f} \left( \frac{m_{\Xi^0} + m_{\Sigma^+}}{m_{\Sigma^0} - m_{\Xi^0}} g_{\Lambda(\Sigma^0)} + \frac{m_{\Xi^0} + m_{\Sigma^+}}{m_{\Sigma^0} - m_{\Xi^0}} g_{\Xi(\Sigma^0)} \right),
\]

\[
B^{ca}(\Xi^+_c \rightarrow \Xi^0 \eta_8) = \frac{f_K}{f} \left( \frac{m_{\Xi^0} + m_{\Sigma^0}}{m_{\Xi^0} - m_{\Xi^0}} g_{\Lambda(\Sigma^0)} + \frac{m_{\Xi^0} + m_{\Sigma^0}}{m_{\Xi^0} - m_{\Xi^0}} g_{\Xi(\Sigma^0)} \right),
\]

\[
B^{ca}(\Xi^+_c \rightarrow \Xi^- \pi^+) = \frac{f_K}{f} \left( \frac{m_{\Xi^0} + m_{\Sigma^0}}{m_{\Xi^0} - m_{\Xi^0}} g_{\Lambda(\Sigma^0)} + \frac{m_{\Xi^0} + m_{\Sigma^0}}{m_{\Xi^0} - m_{\Xi^0}} g_{\Xi(\Sigma^0)} \right),
\]

The P-wave amplitudes for singly Cabibbo-suppressed processes read

\[
B^{ca}(\Xi^+_c \rightarrow \Lambda \pi^+) = \frac{f_K}{f} \left( g_{\Lambda(\pi^+)} \frac{m_{\Lambda} + m_{\Sigma^+}}{m_{\Lambda} - m_{\Sigma^+}} a_{\Sigma^+ \pi^+} + \frac{m_{\Xi^0} + m_{\Sigma^0}}{m_{\Lambda} - m_{\Sigma^+}} g_{\Lambda(\Sigma^0)} \right),
\]

\[
B^{ca}(\Xi^+_c \rightarrow \Sigma^0 \pi^+) = \frac{f_K}{f} \left( g_{\Sigma^0(\pi^+)} \frac{m_{\Xi^0} + m_{\Sigma^0}}{m_{\Sigma^0} - m_{\Xi^0}} a_{\Sigma^0 \pi^+} + \frac{m_{\Xi^0} + m_{\Sigma^0}}{m_{\Sigma^0} - m_{\Xi^0}} g_{\Sigma^0(\Sigma^0)} \right),
\]

\[
B^{ca}(\Xi^+_c \rightarrow \Sigma^+ \pi^0) = \frac{\sqrt{2}}{f} \left( g_{\Sigma^0(\pi^0)} \frac{m_{\Xi^0} + m_{\Sigma^0}}{m_{\Sigma^0} - m_{\Xi^0}} a_{\Sigma^0 \pi^0} + \frac{m_{\Xi^0} + m_{\Sigma^0}}{m_{\Sigma^0} - m_{\Xi^0}} g_{\Sigma^0(\Sigma^0)} \right),
\]

\[
B^{ca}(\Xi^+_c \rightarrow \Xi^0 \eta_8) = \frac{\sqrt{2}}{f_{hs}} \left( g_{\Lambda(\eta_8)} \frac{m_{\Xi^0} + m_{\Sigma^+}}{m_{\Xi^0} - m_{\Xi^0}} a_{\Xi^0 \eta_8} + \frac{m_{\Xi^0} + m_{\Sigma^+}}{m_{\Xi^0} - m_{\Xi^0}} g_{\Xi^0(\eta_8)} \right),
\]

\[
B^{ca}(\Xi^+_c \rightarrow \Xi^- \eta_8) = \frac{\sqrt{2}}{f_{hs}} \left( g_{\Xi^0(\eta_8)} \frac{m_{\Xi^0} + m_{\Sigma^+}}{m_{\Xi^0} - m_{\Xi^0}} a_{\Xi^0 \eta_8} + \frac{m_{\Xi^0} + m_{\Sigma^+}}{m_{\Xi^0} - m_{\Xi^0}} g_{\Xi^0(\eta_8)} \right),
\]

\[
B^{ca}(\Xi^+_c \rightarrow \Xi^0 K^+) = \frac{f_K}{f} \left( g_{\Xi^0(K^+)} \frac{m_{\Xi^0} + m_{\Sigma^0}}{m_{\Xi^0} - m_{\Xi^0}} a_{\Xi^0 K^+} + \frac{m_{\Xi^0} + m_{\Sigma^0}}{m_{\Xi^0} - m_{\Xi^0}} g_{\Xi^0(K^+)} \right),
\]

\[
B^{ca}(\Xi^+_c \rightarrow \Xi^- K^+) = \frac{f_K}{f} \left( g_{\Xi^0(K^+)} \frac{m_{\Xi^0} + m_{\Sigma^0}}{m_{\Xi^0} - m_{\Xi^0}} a_{\Xi^0 K^+} + \frac{m_{\Xi^0} + m_{\Sigma^0}}{m_{\Xi^0} - m_{\Xi^0}} g_{\Xi^0(K^+)} \right),
\]
and

\[ B^{\sigma a} (\Xi^0 \rightarrow \Lambda \pi^0) = \sqrt{\frac{3}{2}} \left( g_{A(\sigma a)}^a \frac{m_\Lambda + m_{\Sigma^0}}{m_{\Xi^0} - m_{\Sigma^0}} a_{\Sigma^0} + g_{A(\sigma a)}^a \frac{m_\Lambda + m_{\Lambda}}{m_{\Xi^0} - m_{\Lambda}} a_{\Lambda^0} \right), \]

\[ B^{\sigma a} (\Xi^0 \rightarrow \Lambda \eta_b) = \sqrt{\frac{3}{2}} \left( g_{A(\Lambda \eta_b)}^a \frac{m_\Lambda + m_{\eta_b}}{m_{\Xi^0} - m_{\eta_b}} a_{\eta_b} + g_{A(\Lambda \eta_b)}^a \frac{m_\Lambda + m_{\Lambda}}{m_{\Xi^0} - m_{\Lambda}} a_{\Lambda^0} \right), \]

Before proceeding to the \( \Xi^c \) sector, we first discuss \( \Lambda^+_c \) decays as the measurements of branching fractions and decay asymmetries are well established for many of the channels. The goal is to see what we can learn.
from the $\Lambda_c^+$ physics. We show in Table III the results of calculations for CF and SCS $\Lambda_c^+$ decays. For the form factors $f_1$ and $g_1$, we follow (37) to use
\begin{equation}
\begin{align*}
f_1^{\Lambda_c^p}(0) &= -0.470, \\
g_1^{\Lambda_c^p}(0) &= -0.414,
\end{align*}
\end{equation}
for $\Lambda_c^-p$ transition and rescale the form factors for $\Lambda_c^-\Lambda$ transition to fit the decay $\Lambda_c^+ \rightarrow \Lambda \pi^+$ so that $f_1^{\Lambda_c^+}(0) = 0.406$ and $g_1^{\Lambda_c^+}(0) = 0.370$. We see from Table III that the calculated branching fractions and decay asymmetries are in general consistent with experiment except for the decay asymmetry in the decay $\Lambda_c^+ \rightarrow pK^0_S$. While all the predictions of $\alpha(\Lambda_c^+ \rightarrow pK^0_S)$ in the literature are all negative except [21], the measured asymmetry by BESIII turns out to be positive with a large uncertainty, $0.18 \pm 0.45$ [20]. This issue needs to be resolved in future study.

We next turn to the mode $\Lambda_c^+ \rightarrow \Xi^0 K^+$ which deserves a special attention. It has been shown that its $S$- and $P$-wave amplitudes are very small due to strong cancellation between various contributions. More specifically (see e.g. [22]),
\begin{equation}
\begin{align*}
A^{\text{com}}(\Lambda_c^+ \rightarrow \Xi^0 K^+) &= \frac{1}{f_K} \left( a_{\Sigma^+ \Lambda_c^+} - a_{\Xi^0 \Xi^0} \right), \\
B^{\text{ca}}(\Lambda_c^+ \rightarrow \Xi^0 K^+) &= \frac{1}{f_K} \left( g_{\Sigma^0 \Lambda_c^+} \frac{m_{\Xi^0} + m_{\Xi^+}}{m_{\Sigma^0} + m_{\Sigma^+}} a_{\Sigma^+ \Lambda_c^+} + a_{\Xi^0 \Xi^0} g_{\Lambda_c^0 \Lambda_c^+} \frac{m_{\Xi^0} + m_{\Lambda_c^+}}{m_{\Xi^0} - m_{\Xi^+}} \right) \\
&\quad + a_{\Xi^0 \Xi^0} \frac{m_{\Xi^0} + m_{\Lambda_c^+}}{m_{\Xi^0} - m_{\Xi^+}} g_{\Xi^0 \Xi^0} \Lambda_c^+.
\end{align*}
\end{equation}

Since the matrix elements $a_{\Sigma^+ \Lambda_c^+}$ and $a_{\Xi^0 \Xi^0}$ are identical in the SU(3) limit and since there is a large cancellation between the first and third terms in $B^{\text{ca}}$ (no contribution from the second term due to the vanishing $g_{\Xi^0 \Xi^0 \Lambda_c^+}$; for details see [22]), the calculated branching fraction turns out to be too small compared to experiment and the decay asymmetry is predicted to be zero owing to the vanishing $S$-wave amplitude [13, 21, 23, 24]. However, a recent BESIII measurement leads to $\alpha(\Lambda_c^+ \rightarrow \Xi^0 K^+) = 0.77 \pm 0.78$ [33], though it is still dominated by the statistic uncertainty. This is a long-standing puzzle.

To solve the above-mentioned puzzle, we notice that one of the $W$-exchange diagrams depicted in the left panel of Fig. 1(a) can be described by two distinct pole diagrams at the hadron level shown in the right panel of the diagram 1(a). These two pole diagrams are called type-III diagrams in [19] and (d1) and (d2) in [22]. As first pointed out by Körner and Krämmer [19], type-III diagram contributes only to the $P$-wave amplitude. Moreover, they showed that this diagram is empirically observed to be strongly suppressed. It was argued by Żenczykowski [23] that contributions from diagrams (d1) and (d2) cancel due to the spin-flavor structure. Hence, its $S$- and $P$-wave amplitudes vanish. The smallness of type-III $W$-exchange diagram also can be numerically checked through Eq. (40). In other words, the conventional expression of parity-violating and -conserving amplitudes given in Eq. (40) is actually for the type-III $W$-exchange diagram in Fig. 1(a). As a result, non-vanishing nonfactorizable $S$- and $P$-wave amplitudes arise solely from the $W$-exchange diagram depicted in Fig. 1(b) (called type-II $W$-exchange diagram in [19] and (b)-type diagram in [23]). The nonfactorizable amplitudes induced from type-II $W$-exchange now read
\begin{equation}
\begin{align*}
A^{\text{com}}(\Lambda_c^+ \rightarrow \Xi^0 K^+) &= \frac{1}{f_K} a_{\Sigma^+ \Lambda_c^+}, \\
B^{\text{ca}}(\Lambda_c^+ \rightarrow \Xi^0 K^+) &= \frac{1}{f_K} \left( g_{\Sigma^0 \Lambda_c^+} \frac{m_{\Xi^0} + m_{\Xi^+}}{m_{\Sigma^0} - m_{\Sigma^+}} a_{\Sigma^+ \Lambda_c^+} \right). \tag{41}
\end{align*}
\end{equation}

5 The sign of the form factors is fixed by Eq. (19).
6 We have checked if the form factors for $\Lambda_c^+\Lambda$ and $\Lambda_c^+\Lambda$ transitions given in Appendix C are used, the resulting decay asymmetries will remain stable, but the calculated branching fractions are not as good as those shown in Table III but within a factor of 2.
are taken from [29] except the modes Λ⁺⁺[7]. Consequently, both partial wave amplitudes are not subject to large cancellations.

TABLE III. The predicted S- and P-wave amplitudes of Cabibbo-favored (upper entry) and singly Cabibbo-suppressed (lower entry) Λ⁺⁺ → B + P decays in units of 10⁻² GeV². Branching fractions and the asymmetry parameter α are shown in the last four columns. Experimental results for decay asymmetries are taken from [29] except the modes Λπ⁺ and Σ⁺π⁰ where the world averages are obtained from [28] and [30].

| Channel         | Λ⁺⁺ → pK⁺ | Λ⁺⁺ → Λπ⁺ | Λ⁺⁺ → Λ⁺π⁺ | Λ⁺⁺ → Σ⁺⁺π⁺ | Λ⁺⁺ → Σ⁺π⁰ | Λ⁺⁺ → Σ⁺⁺π⁰ | Λ⁺⁺ → Σ⁺⁺π⁰ | Λ⁺⁺ → Σ⁺⁺π⁰ | Λ⁺⁺ → Σ⁺⁺π⁰ | Λ⁺⁺ → Σ⁺⁺π⁰ | Λ⁺⁺ → Σ⁺⁺π⁰ | Λ⁺⁺ → Σ⁺⁺π⁰ | Λ⁺⁺ → Σ⁺⁺π⁰ | Λ⁺⁺ → Σ⁺⁺π⁰ | Λ⁺⁺ → Σ⁺⁺π⁰ | Λ⁺⁺ → Σ⁺⁺π⁰ |
|-----------------|-----------|-----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| Λ⁺⁺ → pK⁺       | 3.45      | 4.48      | 7.93        | -6.98       | -2.06       | -9.04       | -2.11×10⁻² | (3.18 ± 0.16)×10⁻² | -0.75       | 0.18 ± 0.45 |
| Λ⁺⁺ → Λπ⁺       | 5.34      | 0         | 5.34        | -14.11      | 3.60        | -10.51      | 1.30×10⁻²  | (1.30 ± 0.07)×10⁻²  | -0.93       | -0.84 ± 0.09|
| Λ⁺⁺ → Λ⁺π⁺      | 0         | 7.68      | 7.68        | 0           | -11.38      | -11.38      | 2.24×10⁻²  | (1.29 ± 0.07)×10⁻²  | -0.76       | -0.73 ± 0.18 |
| Λ⁺⁺ → Σ⁺⁺π⁺     | 0         | -7.68     | -7.68       | 0           | 11.34       | 11.34       | 2.24×10⁻²  | (1.25 ± 0.10)×10⁻²  | -0.76       | -0.55 ± 0.11 |
| Λ⁺⁺ → Σ⁺⁺π⁺     | 0         | -4.48     | -4.48       | 0           | -12.10      | -12.10      | 0.73×10⁻²  | (0.55 ± 0.07)×10⁻²  | 0.90        | 0.77 ± 0.78  |
| Λ⁺⁺ → Σ⁺⁺π⁺     | 0         | 3.10      | 3.10        | 0           | -15.54      | -15.54      | 0.74×10⁻²  | (0.53 ± 0.15)×10⁻²  | -0.95       | -0.95        |

Consequently, both partial wave amplitudes are not subject to large cancellations. Note that the pole diagram induced by type-II W-exchange is the same as the second pole diagram (i.e. a weak transition of Λ⁺⁺ → Σ⁺⁺ followed by a strong emission of K⁺) in Fig. 1(a), but it is no longer canceled by the first pole diagram. A vanishing S-wave amplitude was often claimed in the literature. We wish to stress again that the parity-violating amplitude can be induced from type-II W-exchange through...
current algebra. Eq. (11) leads to \(B(\Lambda_c^+ \to \Xi^0 K^+) = 0.71\%\), which is consistent with the data of \((0.55 \pm 0.07)\%\) [28]. Moreover, the predicted positive decay asymmetry of order 0.90 is consistent with the BESIII’s measurement of 0.77 \pm 0.78 [38]. It is interesting to notice that \(\alpha\) is also predicted to be \(0.94^{+0.06}_{-0.11}\) in the SU(3)-flavor approach [14]. Therefore, the long-standing puzzle with the branching fraction and the decay asymmetry of \(\Lambda_c^+ \to \Xi^0 K^+\) is resolved.

In the \(\Xi_c\) sector, vanishing type-III \(W\)-exchange contributions also occur in the CF decay \(\Xi_c^0 \to \Sigma^+ K^-\) and the SCS modes \(\Xi_c^0 \to pK^-, \Sigma^+ \pi^-\). We will come to this point later.

Comparing Table III with Table II of [25] for SCS \(\Lambda_c^+\) decays, we see some changes in the \(P\)-wave amplitudes of \(\Lambda_c^+ \to p\pi^0, p\eta, n\pi^+\). This is because the first equation in (C2) of [25] should read

\[
g_{\alpha p}^{A(\pi^+)} = 2g_{\alpha p}^{A(\pi^0)} = 10 \sqrt{3} g_{\alpha p}^{A(\eta)} = \frac{5}{3}(4\pi Z_1).
\]

Consequently, we find \(B(\Lambda_c^+ \to p\pi^0)\) is modified from \(0.75 \times 10^{-4}\) [25] to the current value of \(1.26 \times 10^{-4}\). As for \(\Lambda_c^+ \to n\pi^+\), after correcting the error with the axial-vector form factor \(g_{\alpha n}^{A(\pi^+)}\), we find large cancellation in both \(S\)- and \(P\)-wave amplitudes, resulting very small branching fraction of order \(0.9 \times 10^{-4}\).

### B. \(\Xi_c\) decays

Branching fractions and up-down decay asymmetries of CF and SCS \(\Xi_c^{\pm,0}\) weak decays are calculated according to Eqs. (9), (13) and (17), yielding the numerical results shown in Tables IV and V respectively. One interesting point is that there does not exist any decay mode which proceeds only through the factorizable diagram. Among all the processes, the three modes \(\Xi_c^0 \to \Sigma^+ K^-, \Xi_c^0 \Sigma^0, \Xi_c^0 \eta \) in CF processes and the five SCS modes \(\Xi_c^0 \to pK^-, \Xi_c^0 \to \Xi_c^0 K^0, pK^-, nK^+, \Sigma^+ \pi^-\) proceed only through the nonfactorizable diagrams, while all the other channels receive contributions from both factorizable and nonfactorizable terms. The relative sign between factorizable and nonfactorizable is crucial governing whether the interference term is destructive or constructive. For example, factorizable and nonfactorizable terms in both the \(S\)- and \(P\)-wave amplitudes of \(\Lambda_c^+ \to p\pi^0\), \(p\eta\), \(n\pi^+\) and \(\Xi_c^0 \to \Xi_c^0 K^0\) interfere destructively, leading to small branching fractions, especially for the last mode. On the contrary, interference in the channels \(\Xi_c^0 \to \Lambda K^0, \Xi^\mp \pi^\pm\) is found to be constructive.

The CF decay \(\Xi_c^0 \to \Sigma^+ K^-\) and the SCS modes \(\Xi_c^0 \to pK^-, \Sigma^+ \pi^-\) are of special interest among all the \(\Xi_c\) weak decays. Their naive \(S\)-wave amplitudes are given by

\[
A^{\text{com}}(\Xi_c^0 \to \Sigma^+ K^-) = \frac{1}{f_K} \left( a_{\Sigma^0 K^+} - a_{\Sigma^+ \Lambda_c^+} \right),
\]

\[
A^{\text{com}}(\Xi_c^0 \to pK^-) = -\frac{1}{f_K} \left( \frac{\sqrt{3}}{2} a_{\Sigma^0 g^0} + \frac{\sqrt{3}}{2} a_{\Sigma^0 g^0} + a_{p\Lambda_c^+} \right),
\]

\[
A^{\text{com}}(\Xi_c^0 \to \Sigma^+ \pi^-) = -\frac{1}{f_{\pi}} \left( \sqrt{2} a_{\Sigma^0 g^0} + a_{\Sigma^+ \Xi_c^0} \right).
\]

From Eqs. (12) and (13) for baryon matrix elements, it is easily seen that they all vanish in the SU(3) limit. Likewise, their \(P\)-wave amplitudes are also subject to large cancellations. Just as the decay \(\Lambda_c^+ \to \Xi^0 K^+\) discussed in Sec. III.A, we should neglect the contributions from type-III \(W\)-exchange diagrams and focus on SU(4) symmetry

\[\text{[14][25].}\]

This is no longer true in the presence of SU(4)-symmetry breaking.

\[\text{[7]}\]

It had been argued that the contribution from type-II diagrams to the \(S\)-wave amplitude of \(\Lambda_c^+ \to \Xi^0 K^+\) vanishes based on SU(4) symmetry \[14][25].
Since the measured branching fractions are (1.57 ± 0.83)% and (1.80 ± 0.52)%, respectively, this implies that there should be a large destructive interference between factorizable and nonfactorizable terms in the former and a smaller destructive interference in the latter. We notice that the factorizable amplitudes of these two modes are very similar.\footnote{We have confirmed that the sign of the factorizable contribution in the earlier work of~\cite{22} has to be flipped due to the sign convention with the form factors $f_1$ and $g_1$.} From Eq. (32), it is clear that the commutator terms of both type-II $W$-exchange ones. The resulting amplitudes for these three modes are given by

$$A_{\text{com}}^0(\Xi_c^+ \rightarrow \Sigma^+ K^-) = \frac{1}{f_K} a_{\Sigma^0 \Xi_c^+},$$

$$A_{\text{com}}^0(\Xi_c^+ \rightarrow pK^-) = -\frac{1}{f_K} \left( \frac{\sqrt{2}}{2} a_{\Sigma^0 \Xi_c^+} + \frac{\sqrt{5}}{2} a_{\Lambda \Xi_c^0} \right),$$

$$A_{\text{com}}^0(\Xi_c^0 \rightarrow \Sigma^+ \pi^-) = -\frac{\sqrt{7}}{f_K} a_{\Sigma^0 \Xi_c^+},$$

for S-wave (see Eqs. (32) and (34)) and Eqs. (36), (38) for $P$-wave. From Tables XV and VI we see that

$$\alpha(\Xi_c^0 \rightarrow \Sigma^+ K^-) \approx 0.98, \quad \alpha(\Xi_c^0 \rightarrow pK^-) \approx 0.99, \quad \alpha(\Xi_c^0 \rightarrow \Sigma^+ \pi^-) \approx 0.89.$$  \hspace{1cm} (44)

Hence, their decay asymmetries are all positive and close to unity. It is interesting to notice that the decay asymmetries of these three modes are also predicted to be positive and large in the SU(3) approach of~\cite{14}.

Besides the above-mentioned three modes of $\Xi_c^0$, type-III $W$-exchange diagram also exists in the following channels: $\Xi_c^+ \rightarrow \Xi^0 K^+, \Sigma^+(\pi^0, \eta)$ and $\Xi_c^0 \rightarrow (\Lambda, \Sigma^0)(\Sigma^0, \pi^0)$. However, the effects of vanishing type-III $W$-exchange can be seen only in the $P$-wave amplitudes of $\Xi_c^+ \rightarrow \Sigma^+ \pi^0$ and $\Xi_c^0 \rightarrow (\Lambda, \Sigma^0)\pi^0$. In Eqs. (37) and (38) for these three modes we have explicitly dropped the pole contributions with the strong $\pi^0$ emission from $\Xi_c^0$, followed by a weak transition.

As for the two modes $\Xi_c^+ \rightarrow \Xi^0 \pi^+$ and $\Xi_c^0 \rightarrow \Xi^- \pi^+$, we see from Table XV that our prediction is in good agreement with experiment for the former, but it is too large compared to the experimental measurement for the latter. This is mainly due to the relative sign between factorizable and nonfactorizable terms. In the absence of nonfactorizable contributions, we find $B(\Xi_c^+ \rightarrow \Xi^0 \pi^+) \approx 9.9\%$ and $B(\Xi_c^0 \rightarrow \Xi^- \pi^+) \approx 3.3\%$. Since the measured branching fractions are (1.57 ± 0.83)% and (1.80 ± 0.52)%, respectively, this implies that there should be a large destructive interference between factorizable and nonfactorizable terms in the former and a smaller destructive interference in the latter. We notice that the factorizable amplitudes of these two modes are very similar. From Eq. (32), it is clear that the commutator terms of both

| Channel     | $A_{\text{fac}}$ | $A_{\text{com}}$ | $A_{\text{tot}}$ | $B_{\text{fac}}$ | $B_{\text{com}}$ | $B_{\text{tot}}$ | $\alpha_{\text{theo}}$ | $\alpha_{\text{exp}}$ |
|-------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\Xi_c^+ \rightarrow \Sigma^+ K^+$ | 2.98            | -4.48           | -1.50           | -9.95           | 12.28           | 2.32            | 0.20            | -0.80           |
| $\Xi_c^+ \rightarrow \Xi^0 \pi^+$ | -7.41           | 5.36            | -2.05           | 28.07           | -14.03          | 14.04           | 1.72            | 1.57 ± 0.83     |
| $\Xi_c^0 \rightarrow \Xi^0 K^+$ | -1.11           | -5.41           | -6.52           | 3.66            | 6.87            | 10.52           | 1.33            | -0.86           |
| $\Xi_c^0 \rightarrow \Sigma^+ K^+$ | -2.11           | 3.12            | 1.02            | 7.05            | -9.39           | -2.33           | 0.04            | -0.96           |
| $\Xi_c^0 \rightarrow \Xi^0 \pi^+$ | -7.58           | -7.58           | 0               | 11.79           | 11.79           | 1.82            | -0.77           | -0.77           |
| $\Xi_c^0 \rightarrow \Xi^0 \eta$ | -10.80          | -10.80          | 0               | -6.17           | -6.17           | 2.67            | -0.30           | -0.30           |
| $\Xi_c^0 \rightarrow \Xi^- \pi^+$ | -7.42           | -5.36           | -12.78          | 28.24           | 2.65            | 39.89           | 6.47            | 1.80 ± 0.52     |
modes denoted by $A^\text{com}$ are the same in magnitude but opposite in sign. Consequently, the interference between $A^\text{fact}$ and $A^\text{com}$ is destructive in $\Xi_c^+ \to \Lambda \pi^+$ but constructive in $\Xi_c^0 \to \Sigma^- \pi^+$ (see also [21]). As a result, the predicted branching fraction of order 6.5% for the latter is too large. If we use the form factors $f_{\Xi_c^+}^{B}(0) = -0.590$ and $g_{\Xi_c^0}^{B}(0) = -0.582$ [35] in conjunction with the $q^2$ dependence given by Eq. (48), the branching fraction will be reduced only slightly from 6.5% to 6.2%. Hence, we conclude that these two modes cannot be simultaneously explained within the framework of current algebra for $S$-wave amplitudes.

To circumvent the difficulty with $\Xi_c^0 \to \Sigma^- \pi^+$, one possibility is to consider the correction to the current-algebra calculation of the parity-violating amplitude by writing

$$A = A^{CA} + (A - A^{CA}),$$

where the term $(A - A^{CA})$ can be regarded as an on-shell correction to the current-algebra result. It turns out that in the existing pole model calculations [21,22,34], the on-shell correction $(A - A^{CA})$ always has a sign opposite to that of $A^{CA}$. Moreover, the on-shell correction is sometimes large enough to flip the sign of the parity-violating amplitudes. It is conceivable that on-shell corrections could be large for $\Xi^- \pi^+$ but small for $\Xi^0 \pi^+$. This issue needs to be clarified in the future. Nevertheless, we have learned from Table III that current algebra generally works well in $\Lambda_c^+ \to B + P$ decays.

For the up-down decay asymmetry, there is only one measurement thus far. In 2001, CLEO collaboration measured $\Xi_c^+ \to \Xi^- \pi^+$ and found $\alpha(\Xi_c^+ \to \Xi^- \pi^+) = -0.6 \pm 0.4$ [34]. Our prediction is consistent with the CLEO’s value. Decay asymmetries are usually negative in most of the channels. However, besides the three modes $\Xi_c^+ \to \Sigma^+ K^-, pK^-, \Sigma^+ \pi^-$ as discussed before, the following channels $\Xi_c^0 \to \Sigma^0 \eta, \Sigma^0 \pi^0$ and $\Xi_c^0 \to \Sigma^+ \pi^0, \Sigma^+ \eta$ in the $\Xi_c^0$ sector are also predicted to have positive decay asymmetries (see Tables IV and V). We hope that these predictions could be tested in the near future by Belle/Belle II.

### Table V

| Channel | $A^{\text{fact}}$ | $A^{\text{com}}$ | $A^{\text{tot}}$ | $B^{\text{fact}}$ | $B^{\text{tot}}$ | $B_{\text{theo}}$ | $\alpha_{\text{theo}}$ |
|---------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\Xi_c^+ \to \Lambda \pi^+$ | 0.46 | -1.50 | -1.04 | -1.69 | 2.16 | 0.47 | 0.85 | -0.33 |
| $\Xi_c^+ \to \Sigma^0 \pi^+$ | -0.90 | -1.00 | -1.90 | 3.29 | 0.74 | 4.03 | 4.30 | -0.95 |
| $\Xi_c^+ \to \Sigma^+ \pi^0$ | 0.32 | 0.99 | 1.32 | -1.16 | 1.61 | 0.44 | 1.36 | 0.23 |
| $\Xi_c^+ \to \Sigma^+ \eta$ | -0.74 | 1.42 | 0.68 | 2.58 | -2.19 | 0.39 | 0.32 | 0.36 |
| $\Xi_c^+ \to pK^0$ | 0 | -2.10 | -2.10 | 0 | 2.64 | 2.64 | 3.96 | -0.83 |
| $\Xi_c^+ \to \Xi^0 K^+$ | -2.30 | 1.16 | -1.14 | 8.43 | -3.46 | 4.97 | 2.20 | -0.98 |
| $\Xi_c^0 \to \Lambda \pi^0$ | -0.12 | 1.06 | 0.95 | 0.42 | -0.96 | -0.53 | 0.24 | -0.44 |
| $\Xi_c^0 \to \Lambda \eta$ | 0.27 | 1.51 | 1.78 | -0.94 | -0.27 | -1.20 | 0.77 | -0.45 |
| $\Xi_c^0 \to \Sigma^0 \pi^+$ | -0.23 | -0.70 | -0.93 | 0.82 | 1.36 | 2.18 | 0.38 | -0.98 |
| $\Xi_c^0 \to \Sigma^0 \eta$ | 0.53 | -1.00 | -0.48 | -1.83 | 1.55 | -0.28 | 0.05 | 0.36 |
| $\Xi_c^0 \to \Sigma^- \pi^+$ | -1.28 | -1.41 | -2.69 | 4.67 | 0.22 | 4.89 | 2.62 | -0.90 |
| $\Xi_c^0 \to \Sigma^- \pi^-$ | 0 | 1.41 | 1.41 | 0 | 2.49 | 2.49 | 0.71 | 0.89 |
| $\Xi_c^0 \to \Sigma^- \eta$ | 0 | -0.94 | -0.94 | 0 | -1.86 | -1.86 | 0.35 | 0.99 |
| $\Xi_c^0 \to pK^0$ | 0 | -2.10 | -2.10 | 0 | 2.96 | 2.96 | 1.40 | -0.89 |
| $\Xi_c^0 \to K^0$ | 0 | 2.10 | 2.10 | 0 | -4.17 | -4.17 | 1.32 | -0.85 |
| $\Xi_c^0 \to \Xi^- K^+$ | -2.31 | -0.94 | -3.24 | 8.49 | 0.71 | 9.20 | 3.90 | -0.97 |
TABLE VI. Comparison of this work with [14, 46] for the branching fractions in units of $10^{-2}$ for Cabibbo-favored $\Lambda_c^+$ decays (upper entry) and $10^{-3}$ for singly Cabibbo-suppressed ones (lower entry). Decay asymmetries are shown in parentheses.

| Modes                  | This work         | Geng et al. [14, 46] | Expt.          |
|------------------------|-------------------|----------------------|----------------|
| $\Lambda_c^+ \to \Lambda \pi^+$  | 1.30 (–0.93)      | 1.27 ± 0.07 (–0.77 ± 0.07) | 1.30 ± 0.07 (–0.84 ± 0.09) |
| $\Lambda_c^+ \to \Sigma^0 \pi^+$ | 2.24 (–0.76)     | 1.26 ± 0.06 (–0.58 ± 0.10) | 1.29 ± 0.07 (–0.73 ± 0.18) |
| $\Lambda_c^+ \to \Sigma^+ \pi^0$ | 2.24 (–0.76)     | 1.26 ± 0.06 (–0.58 ± 0.10) | 1.25 ± 0.10 (–0.55 ± 0.11) |
| $\Lambda_c^+ \to \Sigma^+ \eta$ | 0.74 (–0.95)     | 0.29 ± 0.12 (–0.70 ± 0.59) | 0.53 ± 0.15 |
| $\Lambda_c^+ \to pK^0$  | 2.11 (–0.75)      | 3.14 ± 0.15 (–0.99 ± 0.09) | 3.18 ± 0.16 (0.18 ± 0.45) |
| $\Lambda_c^+ \to \Xi^0 K^+$ | 0.73 (0.90)     | 0.57 ± 0.09 (1.00 ± 0.06) | 0.55 ± 0.07 (0.77 ± 0.78) |
| $\Lambda_c^+ \to \pi^0\pi^0$ | 0.13 (–0.97)     | 0.11 ± 0.11 (0.24 ± 0.68) | < 0.27 |
| $\Lambda_c^+ \to p\eta$  | 1.28 (–0.55)      | 1.12 ± 0.28 (1.00 ± 0.06) | 1.24 ± 0.29 |
| $\Lambda_c^+ \to n\pi^+$ | 0.09 (–0.73)      | 0.76 ± 0.11 (0.27 ± 0.11) | |
| $\Lambda_c^+ \to \Lambda K^+$ | 1.07 (–0.96)     | 0.66 ± 0.09 (0.09 ± 0.26) | 0.61 ± 0.12 |
| $\Lambda_c^+ \to \Sigma^0 K^+$ | 0.72 (–0.73)     | 0.52 ± 0.07 (0.98 ± 0.02) | 0.52 ± 0.08 |
| $\Lambda_c^+ \to \Xi^0 K^+$ | 1.44 (–0.73)     | 1.05 ± 0.14 (0.98 ± 0.02) | |

C. Comparison with the SU(3) approach

Besides dynamical model calculations, two-body nonleptonic decays of charmed baryons have been analyzed in terms of SU(3)-irreducible-representation amplitudes [40, 41]. There are two distinct approaches to implement this idea. One is to write down the SU(3)-irreducible-representation amplitudes by decomposing the effective Hamiltonian through the Wigner-Eckart theorem. The other is to use the topological quark diagrams which are related in different decay channels via SU(3) flavor symmetry. Each approach has its own advantage. A general formulation of the quark-diagram scheme for charmed baryons is given in [42] (see also [43]). Analysis of Cabibbo-suppressed decays using SU(3) flavor symmetry was first carried out in [44]. This approach became very popular recently [12, 14, 44]. Although SU(3) flavor symmetry is approximate, it does provide very useful information. In Tables VI and VII we compare our results for $\Lambda_c^+$ and $\Xi_c^{+0}$ decays, respectively, with the $SU(3)_F$ approach in [14, 46], in which the parameters for both $S$- and $P$-wave amplitudes are obtained by fitting to the data.

We see from Table VII that it appears the SU(3) approach gives a better description of the measured branching fractions because it fits to the data. However, it is worth of mentioning that in the beginning the SU(3) practitioners tended to make the assumption of the sextet dominance over $\Xi_c^{+0}$. Under this hypothesis, one will lead to $B(\Lambda_c^+ \to p\pi^0) \sim 5 \times 10^{-4}$ [12, 42], which exceeds the current experimental limit of $2.7 \times 10^{-4}$ [17]. Our dynamic calculation in [23] predicted $B(\Lambda_c^+ \to p\pi^0) \sim 1 \times 10^{-4}$. As far as the branching fraction is concerned, it is important to measure the mode $\Lambda_c^+ \to n\pi^+$ to distinguish our prediction from the SU(3) approach. As for decay asymmetries, while we agree on the sign and magnitude of $\alpha(\Xi_c^{+0} K^+)$, we disagree on the signs of $\alpha$ in $\Lambda_c^+ \to n\pi^+$ and $\Lambda K^+$. Hopefully, these can be tested in the future.

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9 Many early studies in the $SU(3)_F$ approach have overlooked the fact that charmed baryon decays are governed by several different partial-wave amplitudes which have distinct kinematic and dynamic effects.
TABLE VII. Comparison of this work with [14, 46] for the branching fractions in units of $10^{-2}$ for Cabibbo-favored $\Xi^{+5,0}$ decays (upper entry) and $10^{-3}$ for singly Cabibbo-suppressed ones (lower entry). Decay asymmetries are shown in parentheses. Experimental results are taken from [9–11] for branching fractions and [39] for decay asymmetry.

| Modes                  | This work     | Geng et al. [14, 46] | Expt.    |
|------------------------|---------------|---------------------|----------|
| $\Xi^{+}_{c} \rightarrow \Sigma^{+} K^{0}$ | 0.20 (−0.80) | 0.78$^{+0.52}_{-0.78}$ (0.93$^{+0.07}_{-0.14}$) |          |
| $\Xi^{+}_{c} \rightarrow \Xi^{0} \pi^{+}$ | 1.72 (−0.78) | 0.42 ± 0.17 (−0.43 ± 0.57) | 1.57 ± 0.83 |
| $\Xi^{+}_{c} \rightarrow \Sigma^{0} K^{0}$ | 1.33 (−0.86) | 1.42 ± 0.09 (−0.85 ± 0.15) |          |
| $\Xi^{+}_{c} \rightarrow \Xi^{0} K^{0}$ | 0.78 (−0.80) | 0.76 ± 0.14 (0.93 ± 0.08) |          |
| $\Xi^{+}_{c} \rightarrow \Xi^{+} K^{+}$ | 1.82 (−0.77) | 1.00 ± 0.14 (−0.96 ± 0.05) |          |
| $\Xi^{+}_{c} \rightarrow \Sigma^{0} \eta$ | 2.67 (0.30)  | 1.30 ± 0.23 (0.80 ± 0.16) |          |
| $\Xi^{+}_{c} \rightarrow \Xi^{+} \pi^{+}$ | 0.67 (−0.95) | 2.95 ± 0.14 (1.00 ± 0.01) | 1.80 ± 0.52 (−0.6 ± 0.4) |
| $\Xi^{+}_{c} \rightarrow \Lambda \pi^{+}$ | 0.85 (−0.33) | 1.23 ± 0.42 (0.03 ± 0.18) |          |
| $\Xi^{+}_{c} \rightarrow \Sigma^{0} \pi^{+}$ | 4.30 (−0.95) | 2.65 ± 0.25 (−0.61 ± 0.12) |          |
| $\Xi^{+}_{c} \rightarrow \Sigma^{0} \pi^{0}$ | 1.36 (0.23)  | 2.61 ± 0.67 (−0.18 ± 0.36) |          |
| $\Xi^{+}_{c} \rightarrow \Lambda \pi^{0}$ | 0.32 (0.36)  | 1.50 ± 1.06 (0.30 ± 0.60) |          |
| $\Xi^{+}_{c} \rightarrow \Xi^{0} K^{+}$ | 2.20 (−0.98) | 0.76 ± 0.12 (0.39 ± 0.16) |          |
| $\Xi^{+}_{c} \rightarrow \Xi^{0} \eta$ | 0.24 (−0.41) | 0.31 ± 0.11 (0.08 ± 0.22) |          |
| $\Xi^{+}_{c} \rightarrow \Sigma^{0} \pi^{+}$ | 0.76 (−0.45) | 0.79 ± 0.27 (−0.17 ± 0.26) |          |
| $\Xi^{+}_{c} \rightarrow \Sigma^{0} \pi^{0}$ | 0.38 (−0.98) | 0.50 ± 0.09 (−0.74 ± 0.25) |          |
| $\Xi^{+}_{c} \rightarrow \Sigma^{0} \eta$ | 0.05 (0.36)  | 0.18 ± 0.11 (−0.20 ± 0.76) |          |
| $\Xi^{+}_{c} \rightarrow \Sigma^{+} \pi^{+}$ | 2.62 (−0.90) | 1.83 ± 0.09 (−0.99 ± 0.01) |          |
| $\Xi^{+}_{c} \rightarrow \Sigma^{+} \pi^{0}$ | 0.71 (0.89)  | 0.49 ± 0.09 (0.91 ± 0.09) |          |
| $\Xi^{+}_{c} \rightarrow \Sigma^{0} K^{+}$ | 0.35 (0.99)  | 0.60 ± 0.13 (0.82 ± 0.11) |          |
| $\Xi^{+}_{c} \rightarrow \Xi^{0} K^{0}$ | 1.40 (−0.89) | 1.07 ± 0.06 (−0.74 ± 0.12) |          |
| $\Xi^{+}_{c} \rightarrow \Xi^0 K^+$ | 1.32 (−0.85) | 0.96 ± 0.04 (−0.53 ± 0.09) |          |
| $\Xi^{+}_{c} \rightarrow \Xi^{0} K^+$ | 3.90 (−0.97) | 1.28 ± 0.06 (−1.00 ± 0.01) |          |

It is clear from Table VII that except $\Xi^{+}_{c} \rightarrow \Sigma^{0} K^{0}, \Xi^{0} \pi^{+}, \Xi^{0} K^{+}$ and $\Xi^{0}_{c} \rightarrow \Xi^{+}, \Xi^{+} K^{+}$ all the branching fractions of $\Xi^{+0}$ decays in this work and in the SU(3) approach are consistent with each other within a factor of 2. Furthermore, we agree on the signs of decay asymmetries except $\Xi^{+}_{c} \rightarrow \Sigma^{0} K^{0}$ and $\Xi^{+}_{c} \rightarrow \Xi^{0} K^{+}$. Notice that both approaches lead to $B(\Xi^{0}_{c} \rightarrow \Xi^{0} K^{+}) \gg B(\Xi^{0}_{c} \rightarrow \Xi^{0} \pi^{+})$, contrary to the current data. Hence, it is of great importance to measure the branching fractions of them more accurately in order to test their underlying mechanism.

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10 Those predictions of $\alpha$ with the uncertainty greater than the central value are not taken into account.
IV. CONCLUSION

In this work we have systematically studied the branching fractions and up-down decay asymmetries of CF and SCS decays of antitriplet charmed baryons. To estimate the nonfactorizable contributions, we work in the pole model for the \( P \)-wave amplitudes and current algebra for \( S \)-wave ones. Throughout the whole calculations, all the non-perturbative parameters, including form factors, baryon matrix elements and axial-vector form factors are evaluated using the MIT bag model.

We draw some conclusions from our analysis:

- The long-standing puzzle with the branching fraction and decay asymmetry of \( \Lambda_c^+ \to \Xi^0 K^+ \) is resolved by realizing that only type-II \( W \)-exchange diagram will contribute to this mode. We find that not only the predicted rate agrees with experiment but also the decay asymmetry is consistent in sign and magnitude with the SU(3) flavor approach and the recent BESIII measurement, though the latter is still dominated by the statistic uncertainty. Hence, it is most likely that \( \alpha(\Lambda_c^+ \to \Xi^0 K^+) \) is large and positive.

- In analog to \( \Lambda_c^+ \to \Xi^0 K^+ \), the CF mode \( \Xi_c^0 \to \Sigma^+ K^- \) and the SCS decays \( \Xi_c^0 \to p K^- \), \( \Sigma^+ \pi^- \) proceed only through type-II \( W \)-exchange. They are predicted to have large and positive decay asymmetries. This can be tested in the near future.

- The predicted \( B(\Xi_c^+ \to \Xi^0 \pi^+) \) agrees well with the measurement inferred from Belle and CLEO, while the calculated \( B(\Xi_c^0 \to \Xi^- \pi^+) \) is too large compared to the recent Belle measurement. We find \( B(\Xi_c^0 \to \Xi^- \pi^+) \gg B(\Xi_c^+ \to \Xi^0 \pi^+) \) and conclude that these two modes cannot be simultaneously explained within the current-algebra framework for \( S \)-wave amplitudes. More accurate measurements of them are called for to set the issue.

- Owing to large cancellation between factorizable and nonfactorizable contributions, the rate of \( \Lambda_c^+ \to n \pi^+ \) is found to be of the same order as that of \( \Lambda_c^+ \to p \pi^0 \). It is important to measure both SCS modes to understand their underlying mechanism.

- Although \( \Xi_c^0 \to \Sigma^0 \bar{K}^0 \) and \( \Xi_c^+ \to \Sigma^+ \bar{K}^0 \) are Cabibbo-favored decays, their branching fractions are small especially for the former due to large destructive interference between factorizable and nonfactorizable amplitudes.

- We have compared our results with the approach of SU(3) flavor symmetry. Excluding those predictions of \( \alpha \) with the uncertainty greater than the central value, we find that both approaches agree on the signs of decay asymmetries except the three modes: \( \Lambda_c^+ \to n \pi^+ \), \( \Xi_c^+ \to \Sigma^+ \bar{K}^0 \) and \( \Xi_c^+ \to \Xi^0 K^+ \). We also agree on the hierarchy \( B(\Xi_c^0 \to \Xi^- \pi^+) \gg B(\Xi_c^+ \to \Xi^0 \pi^+) \).

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Throughout this paper, we follow the convention in [25] for the wave functions of baryons with $S_z = 1/2$:

\[
p = \frac{1}{\sqrt{3}} [uud_x + (13) + (23)], \quad n = -\frac{1}{\sqrt{3}} [ddu_x + (13) + (23)],
\]

\[
\Sigma^+ = -\frac{1}{\sqrt{3}} [uus_x + (13) + (23)], \quad \Sigma^0 = \frac{1}{\sqrt{6}} [(uds + dus) x_S + (13) + (23)],
\]

\[
\Xi^0 = \frac{1}{\sqrt{3}} [ssu_x + (13) + (23)], \quad \Xi^- = \frac{1}{\sqrt{6}} [ssd_x + (13) + (23)],
\]

\[
\Lambda = -\frac{1}{\sqrt{6}} [(uds - dus) x_A + (13) + (23)], \quad \Lambda^+_c = -\frac{1}{\sqrt{6}} [(ucd - duc) x_A + (13) + (23)], \quad \Lambda^+_c = -\frac{1}{\sqrt{6}} [(ucd - duc) x_A + (13) + (23)],
\]

\[
\Sigma^+_c = \frac{1}{\sqrt{6}} [(ucd + duc) x_S + (13) + (23)], \quad \Sigma^0_c = \frac{1}{\sqrt{3}} [dsc x_S + (13) + (23)],
\]

\[
\Xi^+_c = \frac{1}{\sqrt{6}} [(usc - suc) x_A + (13) + (23)], \quad \Xi^0_c = \frac{1}{\sqrt{6}} [(dsc - sdc) x_A + (13) + (23)],
\]

\[
\Omega^+_c = \frac{1}{\sqrt{3}} [ssc x_S + (13) + (23)], \quad \Sigma^- = \frac{1}{\sqrt{3}} [dsc x_S + (13) + (23)],
\]

where $abc x_S = (2a^1 b^1 c^1 - a^1 b^2 c^2 - a^2 b^1 c^1) / \sqrt{6}$ and $abc x_A = (a^1 b^1 c^1 - a^2 b^2 c^2) / \sqrt{\sqrt{3}}$. There are two useful relations under the $U^-, V^-$, and $I^-$-spin:

\[
abc x_S + \sqrt{3} abc x_A = -\left(\frac{\sqrt{2}}{3}\right) (2a^1 b^1 c^1 - a^1 b^2 c^2 - a^2 b^1 c^1),
\]

\[
abc x_S - \sqrt{3} abc x_A = -\left(\frac{\sqrt{2}}{3}\right) (2a^1 b^1 c^1 - a^1 b^2 c^2 - a^2 b^1 c^1). \quad (A2)
\]

**Appendix B: Baryons under $U^-, V^-$ and $I^-$-spin**

In practical calculations, we need to specify the behaviors of baryon wave functions under the isospin, $U^-$-spin and $V^-$-spin ladder operators. Based on the wave functions given by Eq. (A1), we have the following relations:

\[
U_+ \left| \Sigma^+ \right> = - \left| p \right>, \quad U_+ \left| \Xi^- \right> = - \left| \Sigma^- \right>,
\]

\[
U_+ \left| \Sigma^0 \right> = \frac{\sqrt{2}}{2} \left| n \right>, \quad U_+ \left| \Lambda \right> = - \frac{\sqrt{6}}{2} \left| n \right>,
\]

\[
U_+ \left| \Xi^0 \right> = - \frac{\sqrt{2}}{2} \left| \Sigma^0 \right> + \frac{\sqrt{6}}{2} \left| \Lambda \right>, \quad U_+ \left| \Xi^+ \right> = - \left| \Lambda^+_c \right>, \quad (B1)
\]

\[
U_- \left| \Sigma^0 \right> = - \frac{\sqrt{2}}{2} \left| \Xi^0 \right>, \quad U_- \left| \Lambda \right> = \frac{\sqrt{6}}{2} \left| \Xi^0 \right>,
\]

\[
U_- \left| n \right> = \frac{\sqrt{2}}{2} \left| \Sigma^0 \right> - \frac{\sqrt{6}}{2} \left| \Lambda \right>, \quad U_- \left| p \right> = - \left| \Sigma^+ \right>.
\]
for $U$-spin ladder operators,

\[ V_+ |\Sigma^+\rangle = \frac{\sqrt{6}}{2} |p\rangle, \quad V_+ |\Sigma^-\rangle = -|n\rangle, \]
\[ V_+ |\Lambda\rangle = -\frac{\sqrt{6}}{2} |p\rangle, \quad V_+ |\Xi^0\rangle = -\frac{\sqrt{2}}{2} |p\rangle, \]
\[ V_+ |\Xi^-\rangle = -\frac{\sqrt{6}}{2} |\Sigma^0\rangle - \frac{\sqrt{6}}{2} |\Lambda\rangle, \quad V_+ |\Xi^0\rangle = |\Lambda^+_c\rangle, \]
\[ V_- |p\rangle = -\frac{\sqrt{2}}{2} |\Sigma^0\rangle - \frac{\sqrt{6}}{2} |\Lambda\rangle, \quad V_- |\Sigma^+\rangle = |\Xi^0\rangle, \]

for $V$-spin ladder operators, and

\[ I_+ |n\rangle = |p\rangle, \quad I_+ |\Xi^-\rangle = |\Xi^0\rangle, \]
\[ I_+ |\Sigma^-\rangle = \sqrt{2} |\Sigma^0\rangle, \quad I_+ |\Sigma^0\rangle = -\sqrt{2} |\Sigma^+\rangle, \]
\[ I_+ |\Lambda\rangle = 0, \quad I_+ |\Xi^0\rangle = |\Xi^+_c\rangle, \]
\[ I_- |\Xi^+_c\rangle = |\Xi^0\rangle, \quad I_- |\Sigma^+\rangle = -\sqrt{2} |\Sigma^0\rangle, \]

for isospin ladder operators. Note some of the relations may have signs different from the textbook due to our wave function convention. The ladder operators satisfy the commutator relations

\[ [U_+, U_-] = 2U_3, \quad [V_+, V_-] = 2V_3, \quad [I_+, I_-] = 2I_3. \]

**Appendix C: Form factors for $\Lambda_c^+$ decays**

Form factors for $\Lambda_c^+ \to B$ transitions evaluated in the MIT bag model are shown in Table VIII. For $\Lambda_c^+ \to p\eta_8$, we have assumed that form factors are dominated by the $(cd)$ quark content.

| Modes \( \to B \) | (cq) \( f_1(q_{\text{max}}^2) f_1(m_B^2)/f_1(q_{\text{max}}^2) f_1(m_B^2) \) | \( g_1(q_{\text{max}}^2) g_1(m_B^2)/g_1(q_{\text{max}}^2) g_1(m_B^2) \) |
|-----------------|---------------------------------|---------------------------------|
| $\Lambda_c^+ \to p\eta_8$ | (cd) \( \frac{2}{\sqrt{6}} Y_1 \) | \( \frac{2}{\sqrt{6}} Y_2 \) | \( \frac{2}{\sqrt{6}} Y_2 \) |
| $\Lambda_c^+ \to \Lambda\pi^+$ | (cs) \( Y_1^a \) | \( Y_1^a \) | \( Y_1^a \) |
| $\Lambda_c^+ \to p\eta$ | (cd) \( \frac{2}{\sqrt{6}} Y_1 \) | \( \frac{2}{\sqrt{6}} Y_2 \) | \( \frac{2}{\sqrt{6}} Y_2 \) |
| $\Lambda_c^+ \to p\pi$ | (cd) \( \frac{2}{\sqrt{6}} Y_1 \) | \( \frac{2}{\sqrt{6}} Y_2 \) | \( \frac{2}{\sqrt{6}} Y_2 \) |
| $\Lambda_c^+ \to \pi\eta$ | (cd) \( \frac{2}{\sqrt{6}} Y_1 \) | \( \frac{2}{\sqrt{6}} Y_2 \) | \( \frac{2}{\sqrt{6}} Y_2 \) |
| $\Lambda_c^+ \to \Lambda\pi$ | (cs) \( Y_1^a \) | \( Y_1^a \) | \( Y_1^a \) |

**Appendix D: Hadronic matrix elements and axial-vector form factors**

We use the MIT bag model to evaluate the baryon matrix elements and the axial-vector form factors (see e.g. [34] for details).
1. Baryon matrix elements

The hadronic matrix elements \( a_{\text{B}} \) play an essential role both in \( S \)-wave and \( P \)-wave amplitudes. The general expressions are given by

\[
a_{\text{B}} \equiv \langle B' | H_{\text{eff}}^{\text{pc}} | B \rangle = \begin{cases} \frac{G_F}{2\sqrt{2}} V_{ud} V_{us}^* c_\text{C} \langle B' | O_\text{SCS}^- | B \rangle, & \text{CF} \\ \frac{G_F}{2\sqrt{2}} V_{us} V_{us}^* c_\text{C} \langle B' | O_\text{SCS}^+ | B \rangle, & \text{SCS} \end{cases}
\]

for CF and SCS processes, respectively, where \( q = d, s \). Note in SCS process there are in general two operators. For the definition of operators and Wilson coefficients, taking CF process as an example, we have \( O_\text{C} = \langle s\bar{c} | \bar{u}d \rangle | d\bar{u} \rangle \), \( c_\text{C} = c_1 - c_2 \) and then we have the relation \( c_\text{C} O_\text{C} + c_\text{C} O_\text{C} = 2c_1 O_1 + c_2 O_2 \).

The matrix element of \( O_{\text{C}} \) vanishes since this operator is symmetric in color indices. Below, we show the results of \( \langle B' | O^{(q)} | B \rangle \) in the MIT bag model.

The relevant matrix elements for Cabibbo-favored processes are

\[
\langle \Sigma^+ | O_\text{C}^- | \Lambda_\text{C}^+ \rangle = -\frac{2\sqrt{6}}{3} (X_1 + 3X_2)(4\pi), \quad \langle \Xi^0 | O_\text{C}^- | \Xi_\text{C}^0 \rangle = \frac{2\sqrt{6}}{3} (X_1 - 3X_2)(4\pi),
\]
\[
\langle \Xi^0 | O_\text{C}^- | \Xi_\text{C}^0 \rangle = -\frac{2\sqrt{6}}{3} (X_1 - 9X_2)(4\pi), \quad \langle \Sigma^0 | O_\text{C}^- | \Sigma_\text{C}^0 \rangle = -\frac{2\sqrt{6}}{3} (X_1 - 9X_2)(4\pi),
\]
\[
\langle \Lambda | O_\text{C}^- | \Sigma_\text{C}^0 \rangle = \frac{2\sqrt{6}}{3} (X_1 - 3X_2)(4\pi), \quad \langle \Lambda | O_\text{C}^- | \Xi_\text{C}^0 \rangle = \frac{2\sqrt{6}}{3} (X_1 + 3X_2)(4\pi),
\]
\[
\langle \Xi^0 | O_\text{C}^- | \Xi_\text{C}^0 \rangle = \frac{60}{\sqrt{2}} (X_1^q + 9X_2^q)(4\pi), \quad \langle \Sigma^0 | O_\text{C}^- | \Xi_\text{C}^0 \rangle = -\frac{2\sqrt{6}}{3} (X_1^q - 9X_2^q)(4\pi),
\]
\[
\langle \Sigma^0 | O_\text{C}^- | \Xi_\text{C}^0 \rangle = -\frac{2\sqrt{6}}{3} (X_1^q - 9X_2^q)(4\pi), \quad \langle \Lambda | O_\text{C}^- | \Xi_\text{C}^0 \rangle = -\frac{2\sqrt{6}}{3} (X_1^q + 3X_2^q)(4\pi),
\]
\[
\langle \Xi^0 | O_\text{C}^- | \Xi_\text{C}^0 \rangle = \frac{60}{\sqrt{2}} (X_1^q + 9X_2^q)(4\pi), \quad \langle \Sigma^0 | O_\text{C}^- | \Xi_\text{C}^0 \rangle = -\frac{2\sqrt{6}}{3} (X_1^q - 9X_2^q)(4\pi),
\]
\[
\langle \Sigma^0 | O_\text{C}^- | \Xi_\text{C}^0 \rangle = -\frac{2\sqrt{6}}{3} (X_1^q - 9X_2^q)(4\pi), \quad \langle \Lambda | O_\text{C}^- | \Xi_\text{C}^0 \rangle = -\frac{2\sqrt{6}}{3} (X_1^q + 3X_2^q)(4\pi),
\]

where we have introduced

\[
X_1 = \int_0^R r^2 dr (u_s u_u - v_s v_u)(u_u v_u - v_u u_u), \quad X_2 = \int_0^R r^2 dr (u_s u_u + v_s v_u)(u_u u_u + v_u v_u),
\]
\[
X_1^q = \int_0^R r^2 dr (u_q u_u - v_q v_u)(u_u v_u - v_u u_u), \quad X_2^q = \int_0^R r^2 dr (u_q u_u + v_q v_u)(u_u v_u + v_u u_u),
\]

with \( q = d, s \). Numerically we obtain \( X_1^d = 0, X_2^d = 1.60 \times 10^{-4}, X_1^s = 2.60 \times 10^{-6}, X_2^s = 1.96 \times 10^{-4} \), and \( X_1 = 3.56 \times 10^{-6}, X_2 = 1.74 \times 10^{-4} \).
2. Axial-vector form factors

In the MIT bag model the axial form factor in the static limit can be expressed as

$$g_A^{(F)} = \langle B' \uparrow | b_i^\dagger b_{q2} \sigma_3 | B \uparrow \rangle \int d^3r \left( u_{q1} u_{q2} - \frac{1}{3} v_{q1} v_{q2} \right).$$  \hspace{1cm} (D5)

Based on Eq. (D5), the axial-vector form factors related to CF processes are

$$g_A^{(π^+)} = \frac{2}{3} g_{8\Sigma^0\Xi^0} = g_{8\Sigma^0\Xi^0} = g_{8\Sigma^0\Omega^-} = 0,$$

$$g_A^{(π^0)} = \frac{2}{3} g_{8\Sigma^0\Xi^0} = g_{8\Sigma^0\Xi^0} = g_{8\Sigma^0\Omega^-} = 0,$$  \hspace{1cm} (D6)

$$g_A^{(π^0)} = \frac{2}{3} g_{8\Sigma^0\Xi^0} = g_{8\Sigma^0\Xi^0} = g_{8\Sigma^0\Omega^-} = 0,$$

$$g_A^{(π^0)} = \frac{2}{3} g_{8\Sigma^0\Xi^0} = g_{8\Sigma^0\Xi^0} = g_{8\Sigma^0\Omega^-} = 0,$$  \hspace{1cm} (D7)

and

$$g_A^{(π^0)} = \frac{2}{3} g_{8\Sigma^0\Xi^0} = g_{8\Sigma^0\Xi^0} = g_{8\Sigma^0\Omega^-} = 0,$$

$$g_A^{(π^0)} = \frac{2}{3} g_{8\Sigma^0\Xi^0} = g_{8\Sigma^0\Xi^0} = g_{8\Sigma^0\Omega^-} = 0,$$  \hspace{1cm} (D10)

$$g_A^{(π^0)} = \frac{2}{3} g_{8\Sigma^0\Xi^0} = g_{8\Sigma^0\Xi^0} = g_{8\Sigma^0\Omega^-} = 0,$$

$$g_A^{(π^0)} = \frac{2}{3} g_{8\Sigma^0\Xi^0} = g_{8\Sigma^0\Xi^0} = g_{8\Sigma^0\Omega^-} = 0,$$  \hspace{1cm} (D11)

for SCS processes, where the auxiliary parameters are introduced

$$Z_1 = \int r^2 dr \left( u_u^2 - \frac{1}{3} u_u^2 \right), \quad Z_2 = \int r^2 dr \left( u_u u_s - \frac{1}{3} u_u u_s \right)$$  \hspace{1cm} (D12)

in the bag model. The numerical results for $Z_1, Z_2$ are $(4\pi)Z_1 = 0.65$ and $(4\pi)Z_2 = 0.71.$

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[1] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 122, 211803 (2019) [arXiv:1903.08726 [hep-ex]].
[2] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 119, 112001 (2017) [arXiv:1707.01621 [hep-ex]].

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11 Recall that the axial-vector current is $(\bar{u}\gamma_\mu \gamma_5 u - \bar{d}\gamma_\mu \gamma_5 d)/2$ for $\pi^0$ and $(\bar{u}\gamma_\mu \gamma_5 u + \bar{d}\gamma_\mu \gamma_5 d - 2\bar{s}\gamma_\mu \gamma_5 s)/(2\sqrt{3})$ for $\eta_8$ in our convention.
[3] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 121, 052002 (2018) [arXiv:1806.02744 [hep-ex]].
[4] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 121, 162002 (2018) [arXiv:1807.01919 [hep-ex]].
[5] A. Zupanc et al. [Belle Collaboration], Phys. Rev. Lett. 113, 042002 (2014) [arXiv:1312.7826 [hep-ex]].
[6] M. Ablikim et al. [BESIII Collaboration], Phys. Rev. Lett. 116, 052001 (2016) [arXiv:1511.08380 [hep-ex]].
[7] M. Tanabashi et al. [Particle Data Group], Phys. Rev. D 98, 030001 (2018).
[8] M. Ablikim et al. [BESIII Collaboration], Phys. Rev. D 100, 031102 (2019) [arXiv:1702.05279 [hep-ex]].
[9] Y. B. Li et al. [Belle Collaboration], Phys. Rev. Lett. 122, 082001 (2019) [arXiv:1811.09738 [hep-ex]].
[10] Y. B. Li et al. [Belle Collaboration], Phys. Rev. D 100, 031101 (2019) [arXiv:1901.01666 [hep-ph]].
[11] K. W. Edwards et al. [CLEO Collaboration], Phys. Lett. B 373, 261 (1996).
[12] C. Q. Geng, Y. K. Hsiao, C. W. Liu and T. H. Tsai, Eur. Phys. J. C 78, 593 (2018) [arXiv:1804.01666 [hep-ph]].
[13] C. Q. Geng, Y. K. Hsiao, C. W. Liu and T. H. Tsai, Phys. Rev. D 97, 074028 (2018) [arXiv:1801.08625 [hep-ph]].
[14] H. J. Zhao, Y. K. Hsiao and Y. Yao, arXiv:1901.03276 [hep-ph].
[15] Z. X. Zhao, Chin. Phys. C 42, 093101 (2018) [arXiv:1803.02292 [hep-ph]].
[16] C. Q. Geng, Y. K. Hsiao, C. W. Liu and T. H. Tsai, Phys. Rev. Lett. B 794, 19 (2019) [arXiv:1902.06189 [hep-ph]].
[17] H. Y. Cheng and B. Tseng, Phys. Rev. D 48, 4188 (1993) [hep-ph/9304286].
[18] P. ˙Zenczykowski, Phys. Rev. D 50, 5787 (1994).
[19] K. K. Sharma and R. C. Verma, Eur. Phys. J. C 7, 217 (1999) [hep-ph/9803302].
[20] H. Y. Cheng, X. W. Kang and F. Xu, Phys. Rev. D 57, 1697 (1998) [hep-ph/9709372].
[21] Q. P. Xu and A. N. Kamal, Phys. Rev. D 46, 270 (1992).
[22] H. Y. Cheng and B. Tseng, Phys. Rev. D 48, 4188 (1993) [hep-ph/9304286].
[23] P. ˙Zenczykowski, Phys. Rev. D 50, 5787 (1994).
[24] K. K. Sharma and R. C. Verma, Eur. Phys. J. C 7, 217 (1999) [hep-ph/9803302].
[25] H. Y. Cheng, X. W. Kang and F. Xu, Phys. Rev. D 57, 1697 (1998) [hep-ph/9709372].
[26] M. Ablikim et al. [BESIII Collaboration], Phys. Rev. D 100, 032001 (2019) [arXiv:1906.08350 [hep-ex]].
[27] G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. 58, 1125 (1996) [hep-ph/9512380].
[28] T. Feldmann, P. Kroll and B. Stech, Phys. Lett. B 449, 339 (1999) [hep-ph/9812269]; Phys. Rev. D 58, 114006 (1998) [hep-ph/9804249].
[29] M. Ablikim et al. [BESIII Collaboration], Phys. Rev. D 100, 032001 (2019) [arXiv:1906.08350 [hep-ex]].
[30] R. Aaij et al. [LHCb Collaboration], Phys. Rev. D 100, 032001 (2019) [arXiv:1906.08350 [hep-ex]].
[31] G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. 58, 1125 (1996) [hep-ph/9512380].
[32] T. Feldmann, P. Kroll and B. Stech, Phys. Lett. B 449, 339 (1999) [hep-ph/9812269]; Phys. Rev. D 58, 114006 (1998) [hep-ph/9804249].
[33] R. Perez-Marcial, R. Huerta, A. Garcia and M. Avila-Aoki, Phys. Rev. D 40, 2955 (1989) Erratum: [Phys. Rev. D 44, 2203 (1991)].
[34] H. Y. Cheng and B. Tseng, Phys. Rev. D 46, 1042 (1992) [hep-ph/9207302].
[35] N. R. Faustov and V. O. Galkin, Eur. Phys. J. C 79, 703 (2019) [arXiv:1905.08652 [hep-ph]].
[36] A. Chodos, R. L. Jaffe, K. Johnson and C. B. Thorn, “Baryon Structure in the Bag Theory”, Phys. Rev. D 10, 2599 (1974); T. A. DeGrand, R. L. Jaffe, K. Johnson and J. E. Kiskis, “Masses and Other Parameters of the Light Hadrons”, Phys. Rev. D 12, 2060 (1975).
[37] T. Gutsche, M. A. Ivanov, J. G. Krner, V. E. Lyubovitskij and A. G. Rusetsky, Phys. Rev. D 57, 5632 (1998) [hep-ph].
[38] M. Ablikim et al. [BESIII Collaboration], Phys. Rev. D 100, 072004 (2019) [arXiv:1905.04707 [hep-ex]].
[39] R. Aaij et al. [LHCb Collaboration], Phys. Rev. D 100, 032001 (2019) [arXiv:1906.08350 [hep-ex]].
[41] S.M. Sheikholeslami, M.P. Khanna, and R.C. Verma, Phys. Rev. D 43, 170 (1991); R.C. Verma and M.P. Khanna, Phys. Rev. D 53, 3723 (1996).
[42] L. L. Chau, H. Y. Cheng and B. Tseng, Phys. Rev. D 54, 2132 (1996) [hep-ph/9508382].
[43] Y. Kohara, Phys. Rev. D 44, 2799 (1991).
[44] K. K. Sharma and R. C. Verma, Phys. Rev. D 55, 7067 (1997) [hep-ph/9704391].
[45] C. D. Lu, W. Wang and F. S. Yu, Phys. Rev. D 93, 056008 (2016) [arXiv:1601.04241 [hep-ph]].
[46] C. Q. Geng, privite communication.
[47] M. Ablikim et al. [BESIII Collaboration], Phys. Rev. D 95, 111102 (2017) [arXiv:1702.05279 [hep-ex]].