Force Analysis of Fork Type Connecting Structure Pin Roll of Aircraft Vertical Tail in Comprehensive State

Tianle Xing¹

¹School of Aeronautic Science and Engineering, Beihang University, Beijing, 100191, China
*TianleXing@hotmail.com

Abstract. Aiming at the stress problem of pin roll connection at the vertical tail of aircraft, the thesis establishes a simplified simulation model of vertical tail. From the mechanical perspective, it takes into account the gravity, assembly force of interference fit and the influence of air resistance on the fork force. Compared with the yield stress of titanium alloy material used in the actual fork, it is concluded that actual stress is much smaller than the yield stress of the material, which verifies the rationality of pin roll design.

1. Introduction
The empennage is a stabilizing and maneuvering device installed at the rear of the aircraft. The vertical tail is the empennage in the vertical direction, which is composed of a fixed vertical stabilizer and a movable rudder. The vertical stabilizer plays the role of directional stability, while the rudder plays the role of directional manipulation.

Aerodynamic and mass loads act on the vertical empennage, among which the aerodynamic load consists of a balancing load, maneuver load, and side force increment under the gusts during a turbulent air flow. The fork connection, also known as the clamping groove connection or lug connection, is a structural form that is widely used in the empennage connection. The fork structure generally consists of lugs, rod pieces and pin rolls. The fork connection structure is simply constructed and easily reassembled, with clear force transferring. However, it is not capable of bearing excessive wing plane load. In this thesis, a simplified vertical tail model is established, in which both sides are connected with the rear fuselage by three fork structures, and titanium alloy (Ti-6Al-4V) is selected as the material. The gravity and interference assembly stress are respectively obtained via mechanical calculation, and the AnsysWorkbench finite element simulation is used to analyze the influence of air resistance on fork stress. The rationality of pin roll design is verified by synthesizing the above factors.

2. The Effect of Gravity on Pin Roll
There are three connection points between the vertical tail and the fuselage bulkhead, which are statically indeterminate structures. It is impossible to directly solve the component force of gravity on six (three on each side) pin rolls.

As is measured by Solidworks, the center of gravity position for the vertical tail model is located at the place of (3972.09,2204.97, -202.10), with the very front end of the vertical tail as the origin. The coordinates of six connection points can also be directly measured. Considering one side, the vertical tail is a symmetric model, whose center of gravity position falls after the position of three pin rolls.
Figure 1. Gravity component analysis of vertical tail

The four points in Figure 1 are schematic diagrams where pin roll fixed point and action of gravity point are projected on the same line. The direction of component force at point C is upward. If only point B exists, the direction of component force at point B is downward. Adding a point A will not change the direction of component force at point B, which increases the stability of the structure, whereas the component force at point A is downward. The equilibrium equation can be obtained as:

\[
\begin{align*}
F_1 + F_2 + G &= F_3 \\
F_1 x_1 + F_2 x_2 + G x_4 - F_3 x_3 &= 0
\end{align*}
\]

This equation cannot be solved directly, and there are no other conditions. However, it can be figured out that point C is under the biggest force among the three points of application. When there are only points B and C, the effects of gravity are all borne by points B and C, and the component force of point B reaches the ultimate maximum. In this case, the equilibrium equation can be solved:

\[
\begin{align*}
F_2 + G &= F_3 \\
F_2 x_2 + G x_4 - F_3 x_3 &= 0
\end{align*}
\]

Where: \(G = 1500\text{kg}, x_2 = 1579.44\text{mm}, x_3 = 2169.90\text{mm}, x_1 = 1005.35\text{mm}\).

The maximum of the solution is: \(F_{2\text{max}} = 4578.27\text{N}, F_{3\text{max}} = 6078.27\text{N}\)

Under the action of gravity, the pin is in direct contact with the fork hole, and the local stress on the contact surface becomes the extrusion stress. When the extrusion stress is too large, significant plastic deformation will occur in the local area of contact between the hole and pin, thus affecting the normal fit. The thickness of the lug is \(\delta\), and the diameter of the pin or hole is \(d\). According to the test and analysis results, the maximum extrusion stress is:

\[
\sigma_{\text{max}} \approx \frac{F_{3\text{max}}}{\delta d} = \frac{6078.27\text{N}}{15\text{mm} \times 50\text{mm}} = 8.104 \times 10^6\text{Pa}
\]

3. Calculation of Interference Fit Stress

The vertical tail is equipped with the hole-axis assembly, of which the basic dimension is 30mm. The fit tolerance of pin roll is \(\varnothing30t6(\pm0.054)\), while that of the assembled hole is \(\varnothing30H7(\pm0.021)\). \[1\] Its maximum magnitude of interference is \(Y_{\text{max}} = E_1 - es = 0.054\text{mm}\).

By adopting the Harold method for calculating interference fit stress, the hole-axis fit is simplified to an interference fit model with two equal-length components. As is demonstrated in the following figure (Figure 2), the common interference amount after the assembly of two components is: \(\delta_1 = a_2 - b_2 = Y_{\text{max}} = 54\mu\text{m}.\) \[2\]
The equation applicable to the interference fit of two components is: 

\[ \delta = -p_0 k_{3,1} + p_1 \left(k_{4,1} + k_{3,2}\right) - p_2 k_{2,2} \]

where \( p_0 = 0 \), \( p_2 = 0 \), conversion factor \( k_{4,1} = \frac{b_1}{E} \left(\frac{a_1^2 + b_1^2}{a_1^2 - b_1^2} - \mu_1\right) \), \( k_{4,2} = \frac{a_2}{E} \left(\frac{a_2^2 + b_2^2}{a_2^2 - b_2^2} - \mu_1\right) \). Among them, titanium alloy (Ti-6AL-4V) is selected for the vertical tail assembly structure: elastic modulus \( E = 1.078 \times 10^{11} \text{Pa} \), Poisson’s ratio \( \mu = 0.31 \). Size of outer ring component \( a_1 = 23.34 \text{mm} \), \( b_1 = 15 \text{mm} \), size of solid shaft \( a_2 = 15 \text{mm} \), \( b_2 = 0 \text{mm} \)

Substitute into the formula for calculation:

\[
\begin{align*}
\bullet \quad k_{4,1} &= \frac{b_1}{E} \left(\frac{a_1^2 + b_1^2}{a_1^2 - b_1^2} - \mu_1\right) \\
&= \frac{15 \times 10^{-3}}{1.078 \times 10^{11}} \left(\frac{(23.34 \times 10^{-3})^2 + (15 \times 10^{-3})^2}{(23.34 \times 10^{-3})^2 - (15 \times 10^{-3})^2} - 0.31\right) = 2.918 \times 10^{-13} \\
\bullet \quad k_{4,2} &= \frac{a_2}{E} \left(\frac{a_2^2 + b_2^2}{a_2^2 - b_2^2} - \mu_1\right) \\
&= \frac{15 \times 10^{-3}}{1.078 \times 10^{11}} \left(\frac{(15 \times 10^{-3})^2 + 0}{(15 \times 10^{-3})^2 - 0} - 0.31\right) \\
&= 9.601 \times 10^{-14}
\end{align*}
\]

The stress of interference fit is: 

\[ P_1 = \frac{\delta_1}{k_{4,1} + k_{4,2}} = \frac{54 \times 10^{-6}}{2.918 \times 10^{-13} + 9.601 \times 10^{-14}} = 1.392 \times 10^8 \text{Pa} \]

4. Estimation of Interference Fit Assembly Force

For vertical tail, the hole-axis assembly is adopted, of which the basic dimension is 30mm. The fit tolerance of pin roll is \( \varnothing30t6^{(+0.054)} \), while that of the assembled hole is \( \varnothing30H7^{(+0.021)} \). Its maximum magnitude of interference is \( Y_{max} = EI - es = 0.054 \text{mm} \).

The contact stress \( P_1 \) of the fitting surface is generated due to the magnitude of interference, which can be calculated according to the referencing formula[2]:

\[
\begin{align*}
\bullet \quad P_1 &= \frac{E \delta}{4r_2} \times \frac{r_3^2 - r_2^2}{r_2^2} = \frac{E \delta}{4r_2} \times \frac{(\frac{r_3}{r_2})^2 - 1}{(\frac{r_3}{r_2})^2} \\
&= \frac{(\frac{r_3}{r_2})^2 - 1}{r_2^2}. \quad \text{Set} \quad K = \frac{(\frac{r_3}{r_2})^2 - 1}{(\frac{r_3}{r_2})^2}. \quad \text{Take} \quad K = 1. \quad \text{According to the maximum magnitude of interference} \quad Y_{max}, \quad \text{the estimated value of contact pressure is:}
\end{align*}
\]
Titanium alloy (TI-6AL-4V) is selected for the vertical tail assembly structure: elastic modulus \( E = 1.078 \times 10^{11} \text{Pa} \), radius of contact surface \( r_2 = 30 \text{mm} \).

Substitute it into the formula for estimation:

\[ P_1 = 4.851 \times 10^7 \text{N/m}^2. \]

The interference fit stress is proportional to the pressed area of fitting surface and friction coefficient. Its formula is as follows:

\[ P = 2P_1 \pi r_2 l f \]

\( L \) is the length of fitting surface, while \( f \) is the static friction coefficient.

Aircraft assembly is usually carried out below 20 °C, and the temperature during service does not exceed 50 °C. Referring to the figure, the estimated friction coefficient of titanium alloy (TC4) is \( f = 0.6 \), and the length of fitting surface is \( l = 40 \text{mm} \).

Substitute it into the formula for estimation:

\[ P = 2.195 \times 10^5 \text{N} \]

This estimated value is larger than the exact value. Consequently, a reasonable assembly structure can be designed, and the size of components can be determined, so as to smoothly realize the hole-axis assembly.

5. The Impact of Air Resistance on Pin Roll

The workbench is used to establish the model of vertical tail under the vertical side force (Figure 3). The analysis results are mainly influenced by fluid analysis, while structural analysis plays a small role in the deformation results.[3] The unidirectional fluid-solid coupling analysis is adopted.[4] Unidirectional fluid-solid coupling analysis refers to one-way data transmission at the interface. The CFD analysis results are transferred to the solid structure analysis, but no solid structure analysis results are transmitted to the fluid analysis. Among which, fluid analysis is performed in CFX, whereas structural analysis is conducted in Transient Structural. Because it is a unidirectional fluid-solid coupling analysis, only the extracted fluid calculation results are required to be applied to the solid calculation.

![Figure 3. Internal surface model of lug hole](image)

In the fluid analysis, the wind speed is set as 900m/s, and the fixed constraint of vertical tail is the internal surface model of the lug hole. As is shown in Figure 4:
It can be seen from Figure 5 that when subjected to the side force, stress concentration point is at the pin, and the pin end of the tail bears the most force. The maximum stress is $3.83 \times 10^7$ Pa.

6. Conclusion

In this paper, the force of the pin shaft connecting the vertical tail and the rear fuselage of the aircraft is studied, and the allowable stress of the pin shaft is compared with the actual stress.

This paper analyzes the actual working condition of the pin shaft, and finds three aspects that affect the force on the pin shaft: the gravity of the vertical tail, the assembly force of interference fit and the air resistance of the vertical tail during flight. The gravity of the vertical tail and the assembly stress are analyzed by numerical calculation method, and the influence of vertical tail air resistance on the pin shaft is analyzed by finite element analysis method.

After the above analysis, the pin junction end at vertical tail is the most stressed. Counting the influence of gravity, interference assembly force and flight resistance, the maximum force is:

$$σ_m = σ_G + σ_A + σ_f = 8.104\text{MPa} + 139.2\text{MPa} + 38.3\text{MPa} = 185.604\text{MPa}$$

Taking into account the high reliability requirements of aircraft parts, the safety factor of pin design is selected as 3.5 according to experiences, and the maximum force after considering the safety factor is:

$$σ_m = 3.5 \times 185.604\text{MPa} = 649.614\text{MPa}$$

The yield limit of the titanium alloy (Ti-6Al-4V) used at the pin is 827MPa, and the applied stress exceeds its yield stress. Therefore, the pin roll design is reasonable, and it can meet the requirements of the complex environment of flight.
References

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