Methodology of assessing the regulated crossing throughput with a dedicated lane for ground public transport based on a probabilistic model

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Abstract. One of the problems of traffic management in the allocation of lanes for ground public transport (GPT) is the organization of the right turn at regulated and unregulated intersections. High traffic intensity at a regulated intersection on the main carriageway and a significant proportion of right-turn traffic leads to numerous violations of the rules of passage when making a right turn. A method for assessing the throughput of a controlled intersection with a dedicated lane before joining is presented on the basis of a deterministic stochastic model.

1. Introduction
The scheme of intersection planning and traffic management of a transport intersection provides, first of all, to ensure minimization of the number of road accidents and temporary delays of vehicles [2]. A decrease in the speed of traffic at regulated intersections with a high flow density occurs due to the prolonged phase of the prohibitory signal of the traffic light and the organization of right-turn traffic. At regulated intersections, the intensity of traffic of cars turning to the right makes it difficult for the NOT vehicles to move along the designated lane and causes a queue to form in the second lane. Cars turning from the second lane create a queue, which leads to the impossibility of performing a maneuver for several cycles and the formation of a congestion at the intersection. Vehicle infiltration into the designated lane can also be difficult due to the high traffic volume of vehicles in the designated lane. The queue of GPT funds on the designated lane also increases the time delays for public transport vehicles passing through the intersection [1].

2. Description of traffic at the intersection
To assess the characteristics of the traffic flow at a regulated intersection with a dedicated lane for GPT, field observations were carried out on the road network of Moscow. Figure 1 shows the intersection - the object of study, located at the address: Leningradskiy Prospect, 40.
The main carriageway of the intersection (Leningradskiy Prospect) has four lanes, including:

- the first (rightmost lane) lane is a dedicated lane for the movement of GPT funds. The lane is designed for vehicles to move only in the forward direction and make a right turn.
- there is a forward movement in the second lane, but a small part of the traffic flow slows down the forward direction while waiting for the possibility of changing lanes to the right to perform a right turn.
- on the third lane there is movement only in the forward direction.
- from the fourth lane, a left-turn manoeuvre is being carried out, and only a small part of the traffic flow is moving in the forward direction.

The intensity of traffic in the fourth lane does not have any effect on the formation of the features of traffic in the dedicated GTP lane. This simple consideration leads to the conclusion that the main component of the traffic flow is formed on the 1st, 2nd and 3rd lanes.

Traffic intensity studies were carried out on weekdays at different times of the day during the busiest days and hours to obtain the characteristics of the traffic flow at different traffic intensities, with a dry surface. The results of measuring the traffic intensity on the lanes that affect the movement of the GTP on the designated lane are presented in Table 1.

### Table 1. Traffic intensity on crossing lanes

| Lane parts | Vehicle type | Traffic intensity (vehicle / hour) by hours of the day |
|------------|--------------|------------------------------------------------------|
|            |              | 05.00-07.00 | 07.00-09.00 | 14.00-16.00 | 19.00-21.00 |
| 1          | Bus          | 11          | 29          | 29          | 36          |
|            | Truck        | 02          | 02          | 12          | 08          |
|            | Car          | 37          | 120         | 178         | 597         |
| 2          | Bus          | 06          | 17          | 17          | 13          |
|            | Truck        | 11          | 21          | 26          | 29          |
|            | Car          | 78          | 245         | 388         | 617         |
| 3          | Bus          | 0           | 0           | 0           | 0           |
|            | Truck        | 08          | 12          | 19          | 27          |
|            | Car          | 135         | 413         | 627         | 961         |

3. Analysis of the traffic intensity data obtained as a result of measurements

Each vehicle passing through the intersection was assigned to one of three types: buses (type 1), trucks (type 2) and cars (type 3). Here are the measurement results for the following four-time intervals with a duration of 2 hours: from 5 a.m. to 7 (interval 1), from 7 a.m. to 9 (interval 2), from 2 p.m. to 16 (interval 3), from 19 hours to 21 (interval 4). Let us denote by $q_{ijk}$ - the intensity of movement of means of the $i$-th type on the $j$-th lane for the $k$-th measurement time interval.
As a result of measurements, the following values were obtained:

\[ q_{111} = 11; q_{112} = 29; q_{113} = 29; q_{114} = 36; q_{211} = 2; q_{212} = 2; q_{213} = 12; q_{214} = 8; \]
\[ q_{311} = 37; q_{312} = 120; q_{313} = 178; q_{314} = 597; q_{121} = 6; q_{122} = 17; q_{123} = 17; q_{124} = 13; \]
\[ q_{221} = 11; q_{222} = 21; q_{223} = 26; q_{224} = 29; q_{321} = 78; q_{322} = 245; q_{323} = 388; q_{324} = 617; \]
\[ q_{331} = 135; q_{332} = 413; q_{333} = 627; q_{334} = 961; \]

The resulting measurements can be used as input data and settings for the parameters of the mathematical model of movement on a multi-lane carriageway of various types of vehicles [3].

Then

\[ q_{ij} = \frac{1}{3} \sum_{k=1}^{3} q_{ijk} \]  

\( q_{ij} \) is an average over measurement intervals intensity of vehicles of the \( i \)-th type on the \( j \)-th lane, \( i; j = 1; 2; 3; \)

\[ q_i = \sum_{j=1}^{3} q_{ij} ; \quad q_j = \sum_{i=1}^{3} q_{ij} \]  

\( q_i \) - the average over the intervals of measurements, the total intensity of vehicles of the \( i \)-th type on all lanes, \( i = 1; 2; 3; \)

\( q_j \) is the total intensity of all types of vehicles on lane \( j \) averaged over the measurement intervals; \( j = 1; 2; 3; \)

\[ q = \sum_{i=1}^{3} q_i = \sum_{j=1}^{3} q_j \]  

\( q \) is an average over measurement intervals intensity of vehicles of all types on the lanes;

\[ \alpha(j) = \frac{q_j}{q} ; \quad \beta(i) = \frac{q_i}{q} \]  

\( \alpha(j) \) is the ratio of the traffic intensity of all types of vehicles on lane \( j \) averaged over the measurement intervals to the traffic intensity averaged over the measurement intervals of the vehicles of types on all lanes, \( j = 1; 2; 3; \)

\( \beta(i) \) is the ratio of the flow rate of vehicles of the \( i \)-th type averaged over the measurement intervals in all lanes to the rate averaged over the measurement intervals of the flow rate of vehicles of all types in all lanes \( j = 1; 2; 3; \)

\[ \alpha_{ik}(j) = \frac{q_{ijk}}{\sum_{k=1}^{3} q_{ijk}} ; \quad \beta_{jk}(i) = \frac{q_{ijk}}{\sum_{i=1}^{3} q_{ijk}} \]  

\( \alpha_{ik}(j) \) is the ratio of the traffic intensity of vehicles of the \( i \)-th type on the \( j \)-th lane for the \( k \)-th time interval to the intensity of the flow of vehicles of the \( i \)-type in all lanes for the \( k \)-th time interval, \( i; j = 1; 2; 3; k = 1; 2; 3; 4; \)

\( \beta_{jk}(i) \) is the ratio of the flow rate of vehicles of type \( i \) on lane \( j \) to the flow rate of all types of vehicles on lane \( j \) for interval \( k; i; j = 1; 2; 3; k = 1; 2; 3; 4; \)
\begin{equation}
\alpha_i(j) = \frac{\sum_{j=1}^{4} q_{ijk}}{\sum_{i=1}^{4} \sum_{k=1}^{4} q_{iuk}} \quad ; \quad \beta_j(i) = \frac{\sum_{j=1}^{4} q_{ijk}}{\sum_{j=1}^{4} \sum_{k=1}^{4} q_{ijk}}
\end{equation}

\(\alpha_i(j)\) is the ratio of the intensity of the flow of vehicles of type \(i\) averaged over measurement intervals in lane \(j\) to the intensity of the total flow of vehicles of type \(i\) averaged over measurement intervals in all lanes; \(s; i; j = 1; 2; 3; k = 1; 2; 3; 4\);

\(\beta_j(i)\) is the ratio of the intensity averaged over measurement intervals of the total flow of vehicles of type \(i\) in lane \(j\) to the intensity averaged over measurement intervals of the total flow of vehicles of all types in lane \(j\); \(s; i; j = 1; 2; 3; k = 1; 2; 3; 4\).

There are some calculated values:
\[\alpha_1(1) = 0.665, \alpha_11(1) = 0.647; \alpha_12(1) = 0.630; \alpha_13(1) = 0.630; \alpha_114(1) = 0.735; \]
\[\alpha_2(1) = 0.135; \alpha_22(2) = 0.492; \alpha_23(3) = 0.373; \]
\[\alpha(1) = 0.22; \alpha(2) = 0.31; \alpha(3) = 0.47; \beta(1) = 0.03; \beta(2) = 0.04; \beta(1) = 0.93; \]

Based on the results of measurements and calculations performed, the following conclusions can be drawn:

- GPT funds move mainly along lane 1, \(\alpha_1(1) = 0.665\); 66.5\% of the total traffic intensity of GPT vehicles corresponds to traffic on lane 1, and the value of this share at two-hour intervals varies from 63.0\% in the time intervals of 7.00 - 9.00 hours and 14.00 - 16.00 hours to 73.5\% in the time interval of 19.00 - 21.00. Of the measurement time intervals, the interval 19.00 - 21.00 corresponds to the highest traffic intensity. With such an intensity, GPT vehicles are less likely to enter lane 2 and this can explain the fact that in this time interval the share of buses on lane 1 is maximum;

- the intensity of the flow of trucks averaged over the measurement intervals is: 13.5\% in lane 1, 49.2\% in lane 2, 37.3\% in lane 3;

- Distribution of vehicles by lanes: the number of moving vehicles in lane 3 amounted to almost half (47\%) of the total traffic intensity. The corresponding figures for the second and first bands are 31\% and 22\%, respectively. This can be explained by the fact that most cars will continue to move in the forward direction when passing the intersection, while part of the traffic intensity in lane 1 and lane 2 will turn to the right. Therefore, in order to minimize the influence of cars turning to the right, drivers prefer to change lanes in advance in the third lane of the carriageway. The intensity of traffic on the first lane outside rush hour is relatively low (from 50 to 250 vehicles / h), since this lane is intended for the movement of GPT vehicles. Its value increases only during peak hours, when the traffic intensity of the remaining lanes is close to the capacity.

- The results of the analysis also show that in the composition of the traffic flow, cars make up an absolute majority - 93\%, and trucks and GPT vehicles are quite a small share (4\% and 3\%). In the course of the analysis, we can assume that in the composition of the traffic flow during modelling, it is possible to assert with a high degree of reliability only cars and GPT vehicles within the range of permissible error.

4. Model of movement on two lanes of the carriageway of two types of vehicles, differing in speed and size

Suppose there are two types of particles moving along a two-lane carriageway. Particle types differ in the direction of movement at the end of the section under consideration. Particles of the 1st type move along one strip and do not tend to move to another strip. Each type 2 particle tends to move to the far-right lane. The length of the section and the intensity of traffic flows on the lanes at the beginning of the section are given. (Fig. 2).
The position of the particle is characterized by the point \((k, y)\): where \(k\) is the traffic lane index, \(k = 1, 2\). The points with the values of the abscissa \(k = 1\) will be called the points of the left (first) strip, and the points with the values of the abscissa \(k = 2\) - the points of the right (second) strip; \(y\) - zone index; \(y = 1, 2\).

The site is divided into \(M\) segments. Each segment contains “\(l\)” cells. The size (length) of each cell located on the “\(m\)”, segment is \(d_m, m=1,2…M\). The \(d_m\) value corresponds to the dynamic dimensions of the vehicle [3,5,6]:

\[
d_m = 5.7 + 0.5v_m + 0.03v_m^2
\]  

The rules for individual movement of a particle in the \(m\)-th zone, \(m = 1, 2\), are as follows [4]:

- If a particle of the 1st type is on the strip \(k\), then it will continue to move along the section on this strip. In one quantum of time, a particle of the 1-st type located on the \(k\)-th strip and the \(m\)-th section, with the probability \(\lambda_{1,m}\), tends to move one cell in the direction of motion.
- If a particle of the 2-nd type is on the cell \((1, j)\), and the neighboring cell \((2, j)\) is free, then with probability \(\lambda_{2,m}\) the particle goes to the cell \((2, j)\).
- If a particle of the second type is on the 1-st lane, and the cell located in front of this particle is free, and the neighbouring cell on the 2-nd lane is occupied, then the particle does not move.

Let us make the following assumptions: in each cell located in the \(m\)-th zone on the \(k\)-th strip, with the probability \(r(k, m)\) there is a particle of the \(i\)-th type, \(i, k=1, 2; m=1, 2\). With probability \(1 - r(k, m)\), this cell is free, where \(r(k, m) = r_i(k, m) + r_{\bar{i}}(k, m)\). The probabilities of the states of points do not depend on each other.

Let a particle of the \(i\)-th type be in the \(m\)-th zone on the \(I\)-st lane in the cell \((1, ld_m)\) at time \(s\). Then we denote by \(p(I, m)\) the probability that the particle at the time instant \((s+1)\) will be in the cell \((2, ld)\), \(m = 1, 2\):

\[
p(1, m) = [1 - r(2, m)]\lambda_{2,m}\Delta t
\]  

Let \(r_i(k, m)\) be the flux densities of a particle of the \(i\)-th type on the \(k\)-th strip of the \(m\)-th zone:

\[
r_i(k, m) = d_m \rho_i(k, m)
\]  

where \(r_i(k, 1)\) are flux densities of a particle of the \(i\)-th type on the \(k\)-th strip of the \(I\)-st zone, are given.

The values \(q_i(k, l)\) of the flow rates of particles of the \(i\)-th type on the \(k\)-th strip at the beginning of the \(I\)-st zone are calculated by the formulas:

\[
q_i(i, 1) = r_i(i, 1)[v_i + \{1 - r(i, 1)\}\lambda_{i,m}\Delta t]
\]  

\[
q_i(k, 1) = r_i(k, 1)v_i, \ i \neq k
\]  

\[
q_i(k, 2) = q_i(k, 1)
\]
Let $\beta(1, m)$ be the probability that a particle of the 2nd type, which began to move in the $m$-th zone on the 1st lane, will have time to move to the 2nd lane during its movement in this zone, $m = 1, 2$. Then the intensities $q_2(k, 2)$ the flux of particles of the 2nd type on the $k$-th band in the 2nd zone are calculated by the formulas:

\[
q_2(1, 2) = q_2(1, 1)[1 - \beta(1, 1)] \\
q_2(2, 2) = q_2(1, 1)\beta(1, 1) + q_2(2, 1)
\]  

(4.7)

(4.8)

The probabilities that the points of the 2nd zone are occupied by particles of the corresponding types at fixed $k = 1, 2$ are found from the relations:

\[
q_i(i, 2) = r_i(i, 2)[v_2 + \{1 - r_i(i, 2)\}\lambda_{i, 2}\Delta t] \\
q_i(k, 2) = r_i(k, 2)v_2, \ i \neq k
\]  

(4.9)

(4.10)

\[
r(k, m) = r_i(k, m) + r_i(k, m) \leq 1; \ r_i(k, m) > 0, \ r_i(k, m) > 0
\]  

(4.11)

This system of equations for each value $k = 1, 2$ is reduced to a quadratic equation. If the system has more than one solution, then we take the smallest values $r_1(k, 2)$ and $r_2(k, 2)$.

Let the random variable $\eta(1, m)$ be the increment in the stationary system of the ordinate of a particle of the 2nd type, which began to move along the $m$-th zone in the 1st lane, until the moment the particle transitions to the 2nd lane under the assumption that $L_m = \infty$, $m = 1, 2$.

We estimate the required probability $\beta(1, m)$, assuming that the random variable $\eta(1, m)$ can be considered distributed according to the normal law with the corresponding parameters, $m = 1.2$:

\[
\beta(1, m) = P[\eta(1, m) < L_m] = \frac{1}{2} + \Phi \left[ \frac{L_m - a(1, m)}{\sigma(1, m)} \right]
\]  

(4.12)

Where $\Phi(x)$ is the Laplace function: $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-z^2} dz$

\[
a(1, m) = \frac{v_m\Delta t}{p(1, m)}; \ \sigma^2(1, m) = \left[1 - p(1, m)\right][v_m\Delta t]^2 \left[p(1, m)\right]^2
\]  

(4.13)

Suppose that the $\alpha$ probability that a particle of the 2nd type passes into the right lane at the end of the section under consideration is:

\[
\alpha = \frac{q_2(2, 2)}{\sum_{i=1}^3 q_i(k, 1)}
\]  

(4.14)

5. Numerical examples

We consider the movement of cars with three different values of the parameters in the far zone and with the same values of the other parameters of the model. The values of the input parameters are as follows:

$\Delta t = 0.8$ (s), $\lambda_{1, 1} = \lambda_{2, 2} = 0.5$ (1/s), $\lambda_{2, 1} = \lambda_{2, 2} = 1.0$ (1/s),

$\rho_1(1, 1) = \rho_1(2, 1) = \rho_2(1, 1) = \rho_2(2, 1) = 0.01$ (1/m), $L_1=L_2=30$ (m),

Let’s carry out calculations for three values $v_m$: $v_m = 8$ (m/s), $v_m = 10$ (m/s), $v_m = 12$ (m/s). The calculation results are given in Table 2:
Table 2. $\alpha$ vs $v_m$ dependencies

| $v_m$ (m/s) | $\alpha$   |
|------------|------------|
| 8          | 0.999      |
| 10         | 0.986      |
| 12         | 0.939      |

Thus, in the considered speed range, with a decrease in the deterministic component of the flow rate, the probability of a timely transition of the car to the right lane increases.

6. Conclusion
The presented methodology for assessing the throughput of a regulated intersection section on an element of a road network in the presence of a dedicated lane for GPT on the roadway on the basis of a stochastic segregation model and the results of field studies can be used to determine the throughput of traffic segregation sections, which will increase the validity of design decisions in the design regulated intersections and will help improve the quality of transport services.

References
[1] Babkov V F 1967 Road conditions and modes of movement of vehicles (Moscow: Transport)
[2] Lobanov E M 1990 Transportation planning of cities (Moscow: Transport)
[3] Inosse H, Tanaka T 1983 The road traffic control (Moscow: Transport)
[4] GOST 33100-2014 Automobile roads of general use. Rules of road projecting (Moscow: Informavtodor [Federal Agency for Technical Regulation and Metrology]), 2014
[5] Barnes H A 1955 Operation Research 3(4) 536-544
[6] Nagel K. Wolf D E, Wagner P, Simon P M 1998 Physical Review E. 58(2) 1425-1437