Influence of traction conditions on the power balance in the vehicle 4wd system

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Abstract. Vehicles with the 4WD system of different construction and functional solutions are offered by various manufacturing companies. Users of such vehicles expect continuous improvement of the efficiency in the field conditions while meeting at the same time the ecological requirements. Due to the high reliability and high efficiency in the field conditions the purely mechanical solution systems are often used. The article presents an analysis of the 4WD system operation which uses mechanical solution for torque distribution between drive axes. The tests were carried out on the MAHA LPS 3000 dual-axle chassis dynamometer using the registration of front and rear axle wheels driving forces and rotational speed values. During the experimental tests the vertical wheel loads as well as the tire pressures and the rolling circumferences were being changed. Analysis of the obtained results has served to make an efficiency assessment of the vehicle 4WD transmission system.

1. Introduction

The origin of the systems driving all four wheels of the vehicle [1] goes back to the beginnings of motorization. One of the first vehicles using a 4WD drive was a Porsche powered by four electric motors. However, due to the large mass of battery batteries from this project has been abandoned. In the following years, the 4WD drive was not very popular, due to the difficulty in transmitting the drive torque from one source, which is the vehicle's engine, to all four wheels. It was only the period of the Second World War that contributed to the use of 4WD drive on a wider scale in military vehicles. The next step was to introduce all four wheels to the serial passenger car in the 1970s. However, like in military constructions, it was a simple system using a constant drive of one axis and a second axis drive hand-joined by the driver. This solution worked well in the field, however, due to the lack of the ability to vary the speed between the wheels of the front and rear axles, it was not suitable for driving the vehicle under good grip conditions. The crucial moment was the introduction by AUDI in the 1980s of a passenger car with a fixed drive for both axles with an inter-axle differential.

Among the many designs used to distribute the torque between the drive axes, the Torsen differential (TORque SENsing) deserves particular attention. This mechanism was patented in 1958 by Verna Gleasman [2]. Torsen is used in AUDI cars as an inter-axle differential [3], as well as in sports versions of Toyota, Mazda or Alfa Romeo as a differential on the drive axle. It characterizes the operation of this mechanism, the blocking factor (TBR) varies between 2.5 and 5 [1, 4 - 6].

The principle of the Torsen mechanism (Fig. 1) is based on the properties of a self-locking worm gear, which allows differentiation of the rotational speeds of the axle drive shafts. At the same time, the generated frictional moment on the helical gearbox, when encountering different traction
conditions of the wheels of both axes, causes that the driving torque is directed to the wheels moving on the surface with better grip. Thus, without additional control, the Torsen mechanism provides functionality comparable to constructions using clutches or locks in combination with a symmetrical differential.

![Figure 1. Torsen differential circuit diagram [2, 3]](image)

Research on the influence of constructional and geometric parameters of the transmission on characteristics Torsen is discussed in [4-7]. At the same time, the literature analysis showed that the impact of traction conditions on the power balance in the considered mechanism and the drive of the vehicle as a whole has not been sufficiently examined. The results of such a study, based on mathematical [8-11] and physical modeling, as well as research in real operating conditions, are the basis for the further development of 4WD vehicle and the introduction of electronic torque control systems [1, 12]. It is this work that is devoted to such research using a mathematical model of a 4WD vehicle.

2. 4WD vehicle system model
In this article, the diagram of the car's drive with the mid-axle differential was considered (fig. 2). According to the direction of the flow of torque from the engine to the wheels of the vehicle, attention should primarily be paid to the issue of the balance of the torque and the moment of resistance to motion.
The basic of equations describing this process are:

\[
\begin{align*}
(J_{k} + J_{s} \cdot i_{s}^{2} + J_{s} \cdot i_{sb} \cdot i_{sr}^{2}) \cdot \dot{\phi}_{k} &= M_{s} \cdot i_{sb}^{2} \cdot i_{sr}^{2} - c_{wnp} \cdot (\phi_{s} - \phi_{wnp} \cdot i_{sb} \cdot i_{sr}) + \\
&- d_{wnp} \cdot (\dot{\phi}_{s} - \dot{\phi}_{wnp} \cdot i_{sb} \cdot i_{sr}) - c_{wnt} \cdot (\phi_{s} - \phi_{wnt} \cdot i_{sb} \cdot i_{sr}) + \\
&- d_{wnt} \cdot (\dot{\phi}_{s} - \dot{\phi}_{wnt} \cdot i_{sb} \cdot i_{sr}) \\
J_{mri} \cdot \dot{\phi}_{wnt} &= i_{g}^{2} \cdot \left[ c_{wnt} \cdot \left( \frac{\phi_{s}}{i_{sb} \cdot i_{sr}} - \phi_{wnt} \right) + d_{wnt} \cdot \left( \frac{\dot{\phi}_{s}}{i_{sb} \cdot i_{sr}} - \dot{\phi}_{wnt} \right) \right] + \\
&- i_{g} \cdot \left[ c_{wji} \cdot \left( \frac{\phi_{wnt}}{i_{g}} - \phi_{k} \right) + d_{wji} \cdot \left( \frac{\dot{\phi}_{wnt}}{i_{g}} - \dot{\phi}_{k} \right) \right] \\
(4J_{k} + m \cdot r_{d}^{2}) \cdot \ddot{\phi}_{k} &= c_{wji} \cdot \left( \frac{\phi_{wnt}}{i_{g}} - \phi_{k} \right) + d_{wji} \cdot \left( \frac{\dot{\phi}_{wnt}}{i_{g}} - \dot{\phi}_{k} \right) + \\
&- \frac{1}{2} C_{x} \cdot \rho \cdot A \cdot r_{d}^{3} \cdot \dot{\phi}_{k} - m \cdot g \cdot r_{d} \cdot (f \cdot \cos \alpha - \sin \alpha)
\end{align*}
\]  

where: 
- \( A \) - frontal area of the vehicle, 
- \( C_{x} \) - air penetration coefficient, 
- \( f \) - rolling resistance coefficient, 
- \( i_{c} \) - total transmission in drive transmission, 
- \( i_{g} \) - final drive ratio, 
- \( i_{sb} \) - gearbox ratio, 
- \( i_{sr} \) - transmission ratio, 
- \( J_{k} \) - mass moment of inertia of the wheel, 
- \( J_{mri} \) - mass moment of inertia of the axle differential, 
- \( J_{s} \) - mass moment of inertia of the engine, 
- \( J_{sb} \) - mass moment of inertia of the gearbox, 
- \( J_{sr} \) - mass moment of inertia of the distribution box with mid-axis differential, 
- \( m \) - vehicle mass, 
- \( M_{s} \) - torque engine, 
- \( r_{d} \) - wheel dynamic radius, 
- \( \alpha \) - road inclination angle with respect to the level, 
- \( \rho \) - air density, 
- \( \phi_{k} \) - wheel rotation angle, 
- \( \phi_{s} \) - engine crankshaft rotation angle, 
- \( \phi_{wnt} \) - the rotation angle of the front axle drive shaft / rear axle.

Changes in traction conditions mainly concern the issue of cooperation between the vehicle's driving wheels and the road surface (Figure 3). Therefore, the equation of motion of one of the drive wheels was considered:

![Diagram](image_url)
\[
\frac{dv}{dt} = \frac{1}{m_k} (P_t - P_W - P_b) \\
\frac{d\omega}{dt} = \frac{1}{J_{kt}} (M_k - P_tr_d - M_r)
\]  \hspace{1cm} (2)

Equations (2) apply to each of the other drive wheels.

![Figure 3. The system of forces and moments acting on one of the drive wheels](image)

Normal force of the ground reaction on the wheels of the considered axis:

\[
Q_i = Q_{oi} + m_k g \cos \alpha , \hspace{1cm} (3)
\]

where: \(\omega_k\) - angular velocity of the drive wheel, \(v\) - linear velocity of the drive wheel axis, \(m_k\) - wheel mass, \(J_k\) - mass moment of inertia of the wheel, \(P_t\) - driving force of the wheel in question, \(P_b\) - inertia resistance force acting on the wheel axis, \(M_k\) - torque on the wheel axis, \(M_r\) - wheel rolling resistance moment, \(Q_i\) - ground reaction force on the wheel, \(Q_{oi}\) - normal force per wheel, \(r_d\) - wheel dynamic radius, \(\alpha\) - road slope angle in relation to the level.

The maximum driving force \(P_{t\mu_{\text{max}}}\), depending on the product of the strength of the ground reaction on the wheel and its adhesion to the ground, is described by the formula:

\[
P_{t\mu_{\text{max}}} = \mu \cdot Q_i , \hspace{1cm} (4)
\]

where: \(\mu\) - coefficient of wheels friction to the substrate [1].

The dependence of the \(\mu\) coefficient in the function of normal force on the circle \(Q_{oi}\) and slip \(s\), in the literature is called the "magic formula" [1]. This model describes the value of tangential force due to friction as a function of slip. In order to determine the driving force \(P_t\) using the "magic formula" the following dependence was obtained, where \(B_p\), \(C_p\), \(D_p\), \(E_p\) are the empirical coefficients characterizing the construction of a given tire and its operating conditions:

\[
P_{t\mu} = Q_i \cdot D_p \cdot \sin[C_p \cdot \arctan\left(B_p \cdot s - E_p \cdot \left(B_p \cdot s - \arctan(B_p \cdot s)\right)\right)] , \hspace{1cm} (5)
\]
where: \( s = 1 - \frac{v}{\omega k r_d} \) - wheel slip, \( B_p \) - stiffness factor, \( C_p \) - shape factor, \( D_p \) - peak factor, \( E_p \) - curvature factor.

In the above equations for a change in traction conditions correspond to the size of the coefficient of adhesion \( \mu \), the dynamic wheel radius \( r_d \), the normal force per wheel \( Q_i \).

To determine the power balance between the drive axes, such values as angular velocity of individual drive shafts (\( \omega_{wnp}, \omega_{wnl} \)) and wheels (\( \omega_{kp}, \omega_{kt} \)), torques of drive shafts (\( M_{wnp}, M_{wnl} \)) and wheels (\( M_{kp}, M_{kt} \)) were used as well as the moment \( M_T \) friction in the Torsen differential between axial.

Using the AmeSIM program [13, 14], a simulation model was developed using the above relationships. The possibilities of using the developed model are wide, due to the possibility of changing not only the initial conditions of the conducted simulation, but also conducting research on various structures separating the torque between the drive axes by changing the values of the internal \( M_T \) friction torque:

\[
M_T = \begin{cases} 
0 & \text{if } \text{const} > 0 \\
\text{const} & \text{if } \text{other condition}
\end{cases}
\]

where: \( k \) - coefficient of coupling torque between mid-axle differential [15], \( M_{dod} \) - residual friction torque in inter-axial differential [15].

The simulation program enables testing of 4WD system of various construction solutions and for different driving conditions.

### 3. Examination of the influence of traction conditions on the vehicle power balance

#### 3.1. Starting conditions

The greatest demand for power supplied to the drive wheels usually takes place during acceleration of the vehicle. It is limited by the engine torque or the traction of the wheels. During the simulation tests, an inter-axial symmetrical differential mechanism and the Torsen mechanism have been used. For each of the above mentioned mechanisms the different inter-axial differential mechanism torque coefficient \( k \) and the residual friction torque \( M_{dod} \) have been used. Both mechanisms have not been kinematically limited (blocked). Simulation tests have been carried out for two substrates of different longitudinal wheels friction coefficient \( \mu \) values equal to 0.9 and 0.1.

The simulation assumed a straight accelerated motion of the vehicle on the dry asphalt surface (\( \mu_{\text{max}} = 0.9 \)). Acceleration was conditioned by the external characteristics of the engine and took place in one gear. The acceleration of the vehicle took place until the speed reached 20 m/s. The vehicle mass as well as the coordinates of the centre of mass of the vehicle, the mass distribution between the drive axes, the values of the dynamic radius and the course of changes in the coefficient of adhesion \( \mu \) in the slip function according to the Pacejko equation were introduced to the simulation program. During the simulation a disturbance was introduced in the form of an instantaneous change in the value of the adhesion coefficient \( \mu \) exclusively for the wheels of the front axle, in order to examine the system response in the disturbance of the power balance between the drive axles. The described situation took place in the seventh second of the duration of the simulation and consisted in the change of the coefficient \( \mu \) to the value corresponding to the movement of the “ice wheels” \( \mu = 0.1 \). After another two seconds of simulation, the value of the \( \mu \) coefficient returned to the initial value. Thanks to this, one can observe how the system reacts to such dynamic changes and how they affect the instantaneous distribution of torque between the drive axes.

#### 3.2. Modeling results

Selected simulation results presented in Figs. 4, 5, allow analysing the influence of traction conditions on the power balance in inter-axial differential mechanisms of 4WD vehicles.
The results of the simulation show that when driving on ice, a symmetrical differential causes a sharp increase in the angular speed of the wheels sliding. For example, during the simulation on the 9 s (when the vehicle was moving with one axis on an icy surface), the angular velocities of the front and rear axle wheels were 1100 rad/s and 200 rad/s, respectively. In an analogous situation for a vehicle equipped with the Torsen mechanism respectively: 200,3 rad/s and 200,2 rad/s. It follows that the Torsen inter-axle differential did not allow the wheels of the drive axles to slip even when the vehicle is moving on ice.

Fig. 4 shows how the torque distribution between the drive axes occurs. In the case of a symmetrical mechanism, the division is 50/50, since the same torque value is directed to the front and rear axle drive shafts. The lower value of the torque at the symmetrical mechanism resulted from the cooperation of the front axle wheels with the surface with the coefficient of adhesion $\mu = 0.1$. Under these conditions, the front axle wheels slid (Fig.5), which resulted in increased energy losses. At the moment of adhesion recovery (in the 9th second of the simulation), a significant increase in the torque value can be observed. This is the result of the accumulation of rotational energy of the sliding wheels and, due to the higher rotational speed of the crankshaft of the engine, operation at another point of the external characteristic. It should be noted that in an analogous situation, the inter-axial differential Torsen showed completely different properties. In the initial acceleration phase, due to differences in the values of vertical forces on the wheels of the front and rear axles, as well as small differences in the dynamic values of the wheels of both axles, small differences in the torque values for the front and rear axles can be observed. When the front wheels encounter an icy surface (on the 7 s of simulation) there is an increase in the value of the drive torque per rear axle and a decrease in the torque value attributable to the front axle. The torque was directed to the axle, the wheels of which had better grip to the road surface. In the 9th second of simulation, the values of drive torques delivered to the front and rear axle wheels were 310 and 1250 Nm, respectively. The change in the value of the slip of the front axle wheels is negligible (Fig.5). At the moment when the two axles move again in the conditions of adhesion corresponding to the asphalt surface (value of $\mu = 0.9$), the distribution of torque between the drive axles returns to the ratio from the beginning of the simulation.

Figure 4. Trends of torque changes delivered to the front and rear axles at a symmetrical inter-axial differential (dashed line), Torsen (solid line)
4. Conclusion

A simulation model of the 4WD drive system was developed using the AmeSIM program, in which the values and dependencies describing this model were used. The possibilities of using the developed model are wide, due to the possibility of changing not only the initial conditions of the conducted simulation, but also conducting research on various structures separating the torque between the drive axes through changes in the internal friction torque MT. When comparing changes in traction parameters for systems with a symmetrical mechanism and the Torsen mechanism, it should be noted that on a surface with a lower coefficient of grip, a vehicle equipped with a symmetrical inter-axle differential accelerates less well than a vehicle equipped with a Torsen inter-axial mechanism. This demonstrates the better suitability of the Torsen mechanism in conditions of deteriorated adhesion. At the same time, it should be noted that on dry asphalt road slight disturbances in the form of the difference of pressure forces on the wheels of a given axis and small changes of dynamic radii, the Torsen mechanism changes the proportions of torque distribution between drive axles. Such a situation may cause faster wear of vehicle tires and increased resistance of vehicle movement due to existing kinematic incompatibilities in the drive system.

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