Dynamic simulation model of Fiber Optical Current Transformer considering error factors

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Abstract. The paper studies and establishes the dynamic model of fiber optical current transformer by MATLAB. The equivalent model of FOCT’s digital closed loop control system is created by reasonably simplifying deviation and feedback of FOCT. By analyzing the four components of FOCT’s main sources of error, 10 factors that cause FOCT measurement errors are obtained. Then the variable 10 error parameters are introduced into the model to establish a dynamic model of fiber optical current transformer considering the error factor. The model can evaluate the performance of the fiber optical current transformer by using the variable FOCT’s error parameters. Also, model dynamically combines the FOCT’s related parameters and environmental factors with the transformer's transfer characteristics and reflects the influence of FOCT error factors on FOCT performance.

1. Introduction

Fiber-Optical Current Transformer (FOCT) is an electronic current transformer based on Faraday magneto-optical effect. It has much advantages such as high measurement accuracy, large dynamic range, simple insulation, good electromagnetic compatibility, flexible installation, FOCT can measure AC and DC current at the same time, and satisfy the developing needs of power energy metering, power quality monitoring, protection control, fault recording and power grid dynamic observation, and it is used more and more widely in the smart grid [1-4]. However, due to the time of FOCT running in the grid is short, the research on FOCT mainly focuses on solving application problems, but the research on the FOCT model construction is still not deep though it is basic and important. Establishing a dynamic model of FOCT will help. In-depth study of FOCT performance, It is of great significance to optimize the parameters and analyze the influencing factors of FOCT. At the same time, a complete FOCT dynamic model also helps to study the impact of FOCT's parameters in transient system analysis. A detailed and deep study has been made in [5] shows the method of establishing the complete model of fiber optical current transformer. The Jones matrix is used to simulate the optical system. The transfer function of the electronic part of FOCT is obtained by using the frequency response data of FOCT through the network analysis method. The simulation of the optical part in this paper does not take into account the influence of optical device error and environmental factors. A first-order linear model of FOCT is established by linear simplification in [6], but this paper only analyzed the noise signal of FOCT. Based on Jones matrix of each component, a mathematical model of FOCT components is established in [7]. However, the model only considers the performance of FOCT under ideal conditions. A digital closed-loop control system model considering temperature is established by
RTDS in [8], but this model only considering the influence of Verdet constant and $\lambda/4$ plate in different temperature. The influence of birefringence which is induced by temperature is neglected, and other factors of FOCT are also neglected. Generally speaking, the current research on FOCT modeling is focused on the ideal closed-loop control model. In fact, environmental factors such as temperature and the performance parameters of FOCT components will have a great impact on the transfer characteristics of FOCT. Introducing error factors of FOCT and simulating the impact of each factor can dynamically evaluate the influence of error parameters and propose requirements for the FOCT's performance index.

This paper firstly established the closed-loop control system model of fiber optical current transformer in MATLAB. The mechanism of FOCT's error influencing factors is analyzed, and the influence of environmental factors and FOCT components on FOCT transfer results is also studied. The error factors are introduced into the FOCT model and analyzed the influence of temperature, initial phase angle of $\lambda/4$ plate and phase modulator error.

2. Principles of FOCT

Fiber-Optical Current Transformer (FOCT) is a new type of electronic current transformer based on Faraday magneto-optical effect and Ampere loop theorem. The system structure consists of three parts: optical fiber sensing ring, polarization-maintaining optical fiber delay loop and signal processing unit, as shown in Figure 1.

The working principle of fiber optical current transformer is as follows [9]. The light emitted by the source is changed into linearly polarized light by the coupler and polarizer. The linearly polarized light is injected into the delay line of polarization-maintaining fiber at 45 degrees and propagates along the X and Y axes of polarization-maintaining fiber respectively. After $\lambda/4$ wave plate, the linearly polarized light turns into left-handed and right-handed circularly polarized light respectively and enters the optical fiber sensing ring. Faraday magneto-optical effect produced by measured current in current-carrying conductor changes the phase difference between two circularly polarized beams and transmits them at different velocities. After two circularly polarized beams are reflected by a mirror, the polarization modes are exchanged and passed through the sensing fiber again. The phase difference is doubled by Faraday Effect. After passing $\lambda/4$ wave plate, the circularly polarized light returns to the linearly polarized light and interferes at the polarizer. The light carrying phase difference signal enters the photoelectric detector, and is finally output after photoelectric conversion, filtering and amplification, A/D conversion and demodulation. According to Faraday magneto-optic effect and Ampere’s loop theorem, the rotation angle of polarization plane of light wave is

$$\theta = \int V H dl = V N_i \int H dl = V N N_i I$$

The measured current is

$$I = \frac{\theta}{V N N_i}$$

V-Verdet Constant; N-Number of sensing Fiber Rings; $N_i$-Number of Current-carrying Conductors through Fiber Loops.

According to Faraday's magneto-optical effect and Ampere's loop law, the magnitude of the current transmitted in the current-carrying wire is proportional to the phase difference caused by Faraday's effect. Therefore, the measured current can be calculated by detecting the optical phase difference signal.
3. Closed-loop control system model of FOCT

According to the principle of FOCT, without introducing square wave offset and feedback phase shift, the current transmitted in the current-carrying wire is proportional to the phase difference caused by Faraday magneto-optic effect, so the current value to be measured can be calculated by detecting the optical phase difference signal.

Due to the strong anti-interference ability of the optical system of FOCT, the signal entering the photoelectric detector only carries the phase difference signal. The expression of the signal after photoelectric conversion by photoelectric detector is as follows:

\[ I_{\text{out}} = 0.5 \cdot K_p \cdot P_0 (1 + \cos \phi_F) \]  

(3)

\( K_p \) is the photoelectric conversion coefficient of the detector, \( P_0 \) is the light source output intensity, \( \phi_F = 4VNI \) is the phase difference of magneto-optical Faraday effect (where \( N \) is the number of turns of the sensing fiber, \( V \) is the Verdet constant of the sensing fiber, \( I \) is the value of the current in the wire.)

Because the Faraday phase shift is a small value, the working point of FOCT is near the origin. According to the nature of cosine function, in the working area of FOCT, the change rate of output signal is close to zero. The sensitivity of FOCT in detecting small signal is low, and it is difficult to extract Faraday phase shift information. The error becomes large under small current. At the same time, because cosine function is even function, the directivity of Faraday phase shift can not be measured. Therefore, it is necessary to move the working point to the working area with high change rate, and at the same time, the directivity of Faraday phase shift can also be detected. Because of the properties of sinusoidal function, phase modulation of \( \pm \pi/2 \) can be added to FOCT system to make cosine function become sinusoidal function. According to the working principle of the phase modulator, the optical phase of the fiber optic sensor head in and out of the two half cycles of square wave modulation will be biased by \(+\pi/2\) and \(-\pi/2\) respectively.

The intensity of light modulated by \( \pi/2 \) square wave is

\[ I_{\text{out}} = 0.5 K_p \cdot P_0 \cdot (1 + \cos(\phi_F \pm \phi_{\text{mod}})) \]

\[ = 0.5 K_p \cdot P_0 \cdot \left(1 + \cos(\phi_F \pm \pi/2)\right) \]

(4)

\( \phi_{\text{mod}} \) is the modulation angle of the phase modulator, the ideal condition is 90 degrees. Because of the periodicity of sinusoidal function, in order to obtain the unique solution of current, it is necessary to limit the phase shift between \(+\pi/2\) and \(-\pi/2\). the measurement range of transformer is severely limited. Because of the nonlinearity of sinusoidal function, small angle approximation \( \sin \phi_F = \phi_F \) can be used when the Faraday phase shift is small, and large error will be caused when the phase shift is large. The operation of inverse trigonometric function will greatly increase the amount of operation. In order to reduce the system output non-linear error and increase the dynamic measurement range, FOCT is designed as a closed-loop detection system. In the optical system, a
phase difference $\phi_x$ is added to the propagation of two identical polarized light beams to satisfy the following relationship with Faraday phase shift $\phi_F$, and $\phi_{\phi} = -\phi_F$.

The output optical information can be expressed as follows

$$I_{out} = 0.5K_p \cdot P_o \cdot \left(1 \mp \sin(\phi_x + \phi_{\phi})\right)$$

(5)

The measured current is induced into the optical signal by Faraday magneto-optic effect, and the optical signal containing the measured current value is transmitted to the signal processing unit in the form of output light intensity for corresponding demodulation processing. The output signal of FOCT is obtained by double optical path detection method.

$$u_p = I_{in}^+ - I_{in}^-$$

(6)

In the Formula, $I_{in}^+ = 0.5K_p \cdot P_o \cdot \left(1 + \sin(\phi_x + \phi_{\phi})\right) \quad I_{in}^- = 0.5K_p \cdot P_o \cdot \left(1 - \sin(\phi_x + \phi_{\phi})\right)$

The output signal of FOCT is

$$u_p = K_p \cdot P_o \cdot \sin(\phi_x + \phi_{\phi}) \approx K_p \cdot P_o \cdot (\phi_x + \phi_{\phi})$$

(7)

The stepped wave is obtained by integrating $u_p$ with the method of infinitesimal element method. The height of the stepped wave is $S_{out}$. On the one hand, $S_{out}$ forms an analog step-wave to drive phase modulator by D/A conversion and its auxiliary circuit which introduce compensating phase difference between two beams. At the same time, $S_{out}$ makes FOCT digital output to reflect the magnitude and direction of the input current of the transformer. Secondary integration of the phase step will produce digital step-wave, which is then input to the phase modulator after D/A conversion and amplification. Then, from the digital ramp generator to the phase modulator to the feedback phase shift on the optical path, the complex implementation process can be equivalent to a proportional link, and the proportional coefficient is equal to $k_2$. FOCT closed-loop control system can be equivalent to Figure 2.

**Figure 2.** FOCT closed-loop control system.

Because the closed-loop processing technology of FOG is mature, the noise of electronic devices such as photoelectric converters and the errors of A/D and D/A converters are neglected in the modeling of closed-loop control model. The closed-loop control model of FOCT is obtained in MATLAB is in Figure 3.

**Figure 3.** FOCT closed-loop control system in MATLAB.

4. FOCT dynamic model considering error factor

The error of FOCT refers to the difference between the input and the measured value caused by the internal structure defect of transformer or the change of external factors such as components and environment. According to the working principle and structure of FOCT, the error of FOCT mainly comes from the following devices:

1. Polarizer; 2. Phase modulator; 3. $\lambda/4$ wave plate; 4. Sensing optical fiber

4.1. Polarizer extinction ratio
In practical application, the performance of polarizer has a certain influence on the measurement error of FOCT. The main performance indexes of polarizer are extinction ratio and fusion angle [10]. In optical system, the current technology can limit the welding angle error of polarizer to within ±1°. Therefore, the measurement error caused by welding angle is small and can be neglected. The extinction ratio of polarizer has a great influence on the output of FOCT, large extinction will cause large measurement error.

If the extinction ratio coefficient of the polarizer is \( \varepsilon \), the output signal of FOCT considering the extinction ratio of the polarizer is

\[
I_{\text{out}} = 0.5K_p \cdot P_o \left[ \left( 1 + \varepsilon^2 \right)^2 + \left( 1 - \varepsilon^2 \right)^2 \cos (\varphi_p + \phi_{\text{mod}}) \right]
\]  

(8)

Considering the extinction ratio of polarizer, the output signal of FOCT is as follows:

\[
u_o = \frac{1 - \varepsilon^2}{1 + \varepsilon^2} \cdot 4\text{VNI}
\]  

(9)

### 4.2. Phase modulator error

When the light passes through phase modulation, the interference output of optical system becomes:

\[
I_{\text{out}} = 0.5K_p \cdot P_o \cdot [1 + \cos (\varphi_f + \phi_{\text{mod}})]
\]  

(10)

\( \phi_{\text{mod}} \) is Modulation Phase Shift for Phase Modulator.

When the fast and slow axis modulation phase shift errors of the phase modulator are inconsistent, an additional modulation phase shift error will be added to the interference output of the optical system. The interference output of the optical system is as follows:

\[
I_{\text{out}} = 0.5K_p \cdot P_o \cdot [1 + \cos (\varphi_f + \phi_{\text{mod}} + \Delta\varphi)]
\]  

(11)

\( \Delta\varphi = \varphi_f - \varphi_s \), \( \varphi_f \) is Fast Axis Modulation Phase Shift Error, \( \varphi_s \) is Slow Axis Modulation Phase Shift Error.

The output signal of FOCT is

\[
u_o = 4\text{VNI} + \Delta\varphi
\]  

(12)

### 4.3. The effect of temperature on \( \lambda/4 \) wave plate

The errors of \( \lambda/4 \) wave plate are mainly manifested in two aspects: one is the influence of the fabrication process of \( \lambda/4 \) wave plate on the axis angle, which can be accurately controlled in the range of ±1° by optical fiber fusing machine, so the influence of the axis angle error on the output light intensity of FOCT is very small; the other is the influence of temperature on the \( \lambda/4 \) wave plate.

The phase delay angle can be expressed as

\[
\Delta\delta_{\text{plane}} = \Delta\beta_0 L_\varnothing C (T - T_0)
\]  

(13)

\( \Delta\beta_0 \) is the difference of the propagation constants of X-axis and Y-axis of polarization-maintaining fiber with \( \lambda/4 \) wave plate, \( L_\varnothing \) is the length of the optical fibre wave plate, \( C \) is the variation coefficient of the phase delay angle of PMF with temperature, \( T \) is the environment temperature; \( T_0 \) is the temperature when making wave plates, which generally 25°C. \( \Delta\beta_0 = 2\pi / L_\varnothing \), \( L_\varnothing = L_b / 4 \), \( L_b \) is the beat length of \( \lambda/4 \) wave plate, \( C = -0.0126/\circ\text{C} \).

The output signal of FOCT is

\[
u_o = 4\text{VNI} \cdot \sin (\delta_\varnothing + 0.09(25 - T))
\]  

(14)

### 4.4. Error of optical fiber sensing ring

The error of the sensing ring is the main component of FOCT error. The error caused by the sensing ring mainly comes from the error caused by the influence of temperature on the Verdet constant and the error caused by the linear birefringence effect of the sensing ring.
4.4.1. Effect of temperature on Verdet constant. Since SiO₂ is the main component of the optical fibers that make up the FOCT optical path, the relationship between the change of Verdet constant and temperature is as follows.

\[
\frac{1}{V_0} \frac{\partial V}{\partial T} = 7.0 \times 10^{-5} / ^\circ C
\]  

(15)

\(V_0\) is the Verdet constant at 25°C, \(T\) is the environment temperature, \(V_0 = 0.999 \times 10^{-6} \text{ rad/A} \).

The output signal of FOCT is

\[
u_o = 4VNI = 4V_0 \times \left[1 + 7.0 \times 10^{-5} (T - 25)\right] NI
\]

(16)

4.4.2. Linear birefringence of sensing optical fibers. There are inherent linear birefringence and linear birefringence caused by external factors. The linear birefringence error can be divided into three parts: inherent birefringence, bending birefringence and temperature-induced birefringence.

Inherent linear birefringence depends on the level of optical fiber manufacturing technology, and does not change with the fluctuation of external factors.

Bending-induced linear birefringence caused by bending stress on the sensing fiber when the sensing ring is bent into a ring. It can be expressed as

\[
\delta_w = \frac{\pi n^3}{2 \lambda} (1 + \nu) (p_{12} - p_{11}) \left(\frac{r}{R}\right)^2
\]

(17)

\(p_{11}, p_{12}\) is Photoelastic Tensor of Fiber Core, \(\nu\) is Poisson ratio, \(\lambda\) is Wavelength of incident light, \(n\) is core refractive index, \(R\) is the outer radius of the core.

Temperature-induced birefringence is caused by the stress produced by the different thermal expansion coefficients of the core and cladding of the sensing fiber when the ambient temperature changes.

The phase difference of temperature-induced linear birefringence caused by temperature change of unit length is as follows

\[
\delta_T = \frac{\pi n^3}{6 \lambda (1 - 2\nu)} (p_{12} - p_{11}) \Delta\alpha (T - T_0)
\]

(18)

\(\Delta\alpha\) is Linear birefringence phase difference caused by temperature change, \(\lambda\) is Wavelength of incident light, \(p_{12}\) and \(p_{11}\) are the photoelastic tensors of the core, \(n\) is the refractive index of the core, and \(E\) is the Young's modulus of the core, \(E = 3E_0 (1 - 2\nu)\).

Considering the linear birefringence effect and the Jones matrix of the sensing ring, the output signal of FOCT considering linear birefringence is

\[
u_o = 2VNI \frac{\sin 4\pi RN \delta}{2\pi R N \delta} = 2VNI \frac{\sin 4\pi RN (\delta_w + \delta_v + \delta_T)}{2\pi R N (\delta_w + \delta_v + \delta_T)}
\]

(19)

\(\delta_w\) is the Intrinsic linear birefringence, \(\delta_v\) is the Bending-induced linear birefringence, \(\delta_T\) is Linear birefringence phase difference caused by temperature change.

4.5. FOCT dynamic model considering error factor

The error factors are introduced into the closed-loop control model of FOCT, and the dynamic model of FOCT about the error factors is obtained as shown in the Figure 4. It can dynamically combine transformer parameters and environmental factors with transformer characteristics to reflect the influence of FOCT error factors on FOCT performance.
The dynamic model of Fiber-Optical Current Transformer consists of seven modules.  
1. Module of the influence of temperature on Wilde constant;  
2. Module of the influence of temperature on λ/4 wave plate;  
3. Module of birefringence (divided into fixed birefringence, bending birefringence and temperature birefringence);  
4. Module of polarizer extinction ratio error;  
5. Module of phase modulation;  
6. Module of phase modulation;  
7. Module of closed-loop control system.  
The first five modules consist of error control module as shown in the Figure 5. The linear birefringence error consists of fixed birefringence, bending birefringence and temperature birefringence.

Through the error dynamic model of FOCT in the Figure 5, the following parameters of fiber optical current transformer can be used as input to evaluate the performance of fiber optical current transformer:  
1. Temperature;  
2. Poisson’s ratio of optical fibers;  
3. Central wavelength of optical fibers;  
4. Core radius;  
5. Fiber ring radius;  
6. Fiber ring turns;  
7. Polarizer extinction ratio;  
8. Intrinsic birefringence of sensing optical fibers;  
9. Phase modulator error;  
10. λ/4 initial phase angle of glass.  
The model can dynamically combine the transformer parameters and environmental factors with the transformer characteristics, reflecting the influence of FOCT error factors on FOCT performance.

5. Performance evaluation of transformer and conclusion
Table 1. Transformer parameter setting.

| Parametric                              | numerical | Parametric                              | numerical |
|-----------------------------------------|-----------|-----------------------------------------|-----------|
| Turn Number of Transformer             | $N = 4$  | Wavelength of incident light            | $\lambda = 1300$ nm |
| Core refractive index                   | $n = 1.456$ | Poisson ratio                           | $\nu = 0.17$ |
| Core diameter                           | $r = 4.5 \mu m$ | Reference Verdet Constant              | $V_0 = 9.999 \times 10^7$ rad/A |
| Bending radius of sensing ring          | $R = 0.1585m$ | Thermal expansion coefficient difference | $\Delta \alpha = 10^{10}$ |
| Fiber Core Photoelastic Tensor          | $\mu_{11} = 0.121$ | Extinction ratio coefficient            | $\varepsilon = 0.0076$ |
|                                        | $\mu_{12} = 0.27$ |                                           |           |

**Figure 6.** Temperature-difference curve.
Set the FOCT parameters as shown in Table 1, and draw the curve of FOCT ratio variation with temperature in the Figure 6. Initial phase angle of $\lambda/4$ plate is 90°. From Figure 6, we can easily get the ratio of transformers at the corresponding temperature, so as to evaluate the performance of FOCT changing with temperature.

**Figure 7.** Initial phase angle of $\lambda/4$ plate-difference curve.
The variation curve of FOCT ratio difference with the initial phase angle of $\lambda/4$ plate at 15°C is obtained as shown in the Figure 7. Changing the initial phase angle of $\lambda/4$ wave plate can compensate the variation of Verdet constant with temperature to a certain extent and reduce the error of FOCT. FOCT’s error is smallest at 15°C when the initial phase angle of $\lambda/4$ plate is 84.3 degree.

**Figure 8.** Phase modulator error-difference curve.
The variation curve of FOCT ratio difference with phase modulator error is obtained as shown in the Figure 8. Positive phase modulator error can also compensate FOCT error. Through this model, the relationship between other FOCT parameters and the difference of transformer ratio can be easily obtained, which is helpful for evaluating the performance of FOCT and optimizing the parameters of transformer.

6. References
Acknowledgments

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