Remarks on the mixed Ramond -Ramond, open string scattering amplitudes of BPS, non-BPS and brane-anti brane

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Abstract

From the world-sheet point of view we compute all three, four and five point BPS and non-BPS scattering amplitudes of type IIA and IIB superstring theory. All these mixed S-matrix elements including a closed string Ramond-Ramond (RR) in the bulk and a scalar/gauge or tachyons (in both non-BPS and brane anti brane formalism) in their all different pictures with RR in the asymmetric and symmetric pictures have been carried out.

We have also shown that in asymmetric pictures various equations must be kept fixed. More importantly, by direct calculations on upper half plane, it is realized that some of the equations (that must be held) of BPS branes can not be necessarily applied to non-BPS amplitudes. We also derive the S-Matrix elements of $<V_{C}^{-2}V_{\phi}^{0}V_{A}^{0}V_{T}^{0}>$ and clarify the fact that in the presence of the scalar field and RR, the terms carrying momentum of RR in transverse directions play important role in the entire form of the S-matrix and their presence is needed in order to have gauge invariance for the entire S-matrix elements of type IIA (IIB) superstring theory.
1 Introduction

In a basic paper [1] it has been extensively clarified that the sources for all different kinds of D-branes are known to be Ramond-Ramond fields. It is worthwhile looking at some other concrete references regarding Ramond-Ramond (RR) fields [2, 3], besides them, RR fields play a very crucial effect in understanding the phenomenon of Dielectric branes which was first demonstrated by Myers in [4]. Note that one of the applications of Dielectric effect would be finding out and interpreting $N^3$ entropy of M5 branes from this effect by employing several RR couplings [5].

It is known that if one wants to work with the effective actions of type IIA, IIB string theory, then one needs to deal with just DBI and Chern-Simons effective actions which can be accordingly found in [6], [7], [8] and [9].

By making use of the scattering theory of D-branes in the world volume of BPS branes in type II string theory we have also explored various new Chern-Simons couplings including their all order $\alpha'$ corrections to the low energy effective actions of D-branes. In fact it is in detail explained that for BPS (non-BPS) branes all the corrections to D-brane effective actions can just be derived by having the S-matrix formalism and not by any other tools like T-duality transformation. The reason for this sharp conclusion is that with S-matrix we are able to fix all the coefficients of couplings and their higher order $\alpha'$ corrections without any ambiguities (see [11, 12] for BPS and [13] for non-BPS branes), where all the three different ways of obtaining the couplings in the effective field theory involving pull-back, Taylor expansions have been explained in [11] which is the so called the generalization to Myers action.

It is emphasized in [14, 15, 16] that to get to the effective theory of non supersymmetric cases or non-BPS branes one has to integrate out all the massive modes and needs to effectively deal with just the massless strings such as scalar, gauge fields and also the real components of tachyon fields.

There are various motivations to perform scattering theory of all BPS and non-BPS branes, basically one of the main reasons to employ it is to actually have the entire form of S-matrix elements which are physical quantities and the other would be to deal with its strong potential of gaining new terms including their corrections (of course without ny

1Some of the new curvature corrections of type II have been recently obtained in [10].
ambiguity) in string theory effective actions.

Having set this formalism, one may end up obtaining several new couplings for all BPS branes in RNS formalism [17, 18] where eventually one could investigate to get to a proposal for the corrections to some of the couplings [19] and in particular it is shown that some of these couplings must be employed to be able to work with some of the applications of either M-theory [20] or gauge-gravity duality [21].

One might be interested in looking at various applications in the world volume of non-BPS branes as were comprehensively pointed out in [13], but for the concreteness we highlight some of them.

As an instance in [22] it has been illustrated that as long as the effective field theory description holds, in the large volume case despite having the fact that we are dealing with non supersymmetric cases, Ads minima are indeed stable vacua. Not only the phenomena of the production of branes are investigated in [23, 24] but also Inflation in the language of D-brane and anti D-brane systems in string theory was revealed [25, 26, 27, 28] and finally it is worth mentioning that tachyons of type IIA(IIB) superstring theory (with odd-parity) have been taken into account to make various remarks on some holographic QCD models [29, 30].

In [31] various issues on the scattering amplitudes have been extensively discussed, however, one has to concern about some other issues on the mixed amplitudes involving a closed string Ramond-Ramond (RR) and some other open strings which have been empirically addressed in [19]. The content of this paper is beyond what has been appeared in those both references. Indeed we would like to understand in the presence of symmetric and asymmetric picture of RR and a scalar field and some other open strings what happens to the terms that carrying momentum of RR in transverse directions, arguing that the S-matrices that satisfy Ward identity and do involve all the contact interactions are definitely the correct S-matrices.

The paper organized as follows. In the next section, first we try to come up with the entire details of the scattering computations of a closed string Ramond-Ramond (RR) in asymmetric picture or in \((C^{-2})\)-picture (in terms of its potential not its field strength) and naturally a scalar field in zero picture, which is a three point function from the world sheet point of view and two point function from the space time prospective. Then we carry out
its S-matrix in the other picture of RR, namely in its symmetric picture or in \((C^{-1})\)-picture (in terms of RR’s field strength) and scalar picture in \((-1)\) picture, and finally we compare both answers and make remarks on a Bianchi identity that must be held to get to picture independence result. For the completeness we also talk about \(CT, CA\) amplitudes as well.

The four point correlation function of \(< T^{(0)} T^{(0)} C^{(-2)} >\) in the world volume of D-brane anti D-brane system has just been carried out to show that the result is the same as \(< T^{(-1)} T^{(0)} C^{(-1)} >\) \([32]\) where it clearly confirms that there is no any issue of picture dependence of the mixed closed string RR and strings that move on the world volume space such as gauge fields or tachyons, but not scalar field, due to its polarization in the bulk and its non-zero correlator with RR.

Hence due to momentum conservation along the world volume of branes and as long as we are dealing with world volume gauge fields, tachyons in the presence of RR, there is no any issue about choosing the picture of RR and in fact in order to get to the final answer for the S-matrix as fast as it is possible one could put RR and a gauge field in \((-1)\) picture and the other tachyons/gauges on zero picture. However because of the non-zero correlation function between RR and scalar in zero picture and due to the fact that the terms carrying momentum of RR in transverse directions \(p^i, p^j\) can not be derived by any duality transformations \([11, 13, 33]\) one has to be concerned about various issues. In the following we release various subtleties that have to be considered for mixed amplitudes involving a closed string RR, a scalar and some arbitrary number of gauges or tachyons in both world volume of non-BPS, BPS and brane and anti brane systems.

We then calculate all the four point non-BPS functions of type II in their all different pictures \(< T^{(-1)} \phi^{(0)} C^{(-1)} >, < T^{(0)} \phi^{(0)} C^{(-2)} >\). Although in this case for \(< T^{(-1)} \phi^{(0)} C^{(-1)} >\) we will see that the term that carries momentum of RR in transverse direction disappears after having applied an equation but one has to be careful about these terms in higher point functions as their presence plays crucial role in the gauge invariance of the mixed amplitudes. The motivation for this calculation is to find the terms that carrying momentum of RR in transverse (bulk direction). Since there is a non-zero correlation function between RR and the first part of the vertex operator of scalar field in zero picture, basically one has to think about those terms that carry momentum of RR in transverse direction \((p^i, p^j\) ) as they can not be derived by any duality transformation \([11]\), where we observe several equations must hold for BPS cases, however some of the equations may not nec-
necessarily hold for non-BPS branes otherwise the whole amplitude of a non supersymmetric case vanishes. For the completeness we perform \(< \phi^{(0)} A^{(0)} C^{(-2)} >\) and \(< \phi^{(-1)} A^{(0)} C^{(-1)} >\) and \(< \phi^{(0)} A^{(-1)} C^{(-1)} >\) as well.

We would like to go over to the five point functions of non-BPS branes to see what happens, if we carry them out in both symmetric \((C^{-1})\), asymmetric picture of RR \((C^{-2})\), in the presence of just one scalar , two tachyons in the world volume of brane anti brane systems. Basically we have three different choices. Indeed for these particular amplitudes there is no Ward identity and a-priori one does not know which picture gives us the correct S-matrix as naturally one needs to know the proper Bianchi identities to be able to produce all the effective field theory couplings to be produced with the correct S-matrix with all its infinite contact interactions. Most importantly here the terms of \(p^i, p^j\) remain after taking integrations properly on the upper half plane. We also determine the fact that if we apply some of the equations of BPS branes to non-BPS amplitudes including a RR and for example the most simplest one three tachyons then the whole S-matrix disappears, this clearly shows that those equations do not hold for non-BPS cases as there are non zero field theory couplings in the world volume of non-BPS branes. We will also mention several subtleties that might have potentially something to do with some of the issues on the perturbative string theory that are released and have been pointed out in a series of papers appeared by Witten \cite{34}.

The five point correlations of \(< V_C^{-1} V_{\phi}^{-1} V_T^0 V_A >\) has been computed in \cite{35}. In order to get to know what happens to the gauge invariance of the amplitudes we come over to the same amplitude but with different picture of scalar as follows:

\[< V_C^{-1} V_{\phi}^0 V_T^0 V_A^{-1} >\] (1)

Given the fact that scalar field in zero picture carries two different terms and in particular its first part has non zero correlation function with closed string RR, we see that here the terms carrying momentum of RR (all \(p, \xi\) terms) survive after taking integration on upper half plane and indeed since momentum is conserved just on the world volume directions of brane \((k_1 + k_2 + k_3 + p)_a = 0\), due to all non vanishing \(p, \xi\) terms the final form of the S-matrix does not satisfy Ward identity associated to gauge field. Since we can not give up

\[< \phi^{(-1)} T^{(0)} T^{(0)} C^{(-1)} >, < \phi^{(0)} T^{(-1)} T^{(0)} C^{(-1)} >\) and finally in asymmetric picture of RR which is \(< \phi^{(0)} T^{(0)} T^{(0)} C^{(-2)} >\).

\footnote{We may wonder why we can not see this result in four point function, say for one RR, one scalar and}
or overlook having gauge invariance of the S-matrix, we need to come up with some ideas. That is why we look at it in an asymmetric picture of RR and have to embed all the open strings in zero picture $< V_C^{-2} V_\phi^0 V_T^0 V_A^0 >$ where in this picture upon considering the known Bianchi identities not only will we observe that the amplitude respects the Ward identity associated to the gauge field, but also we are able to get to the whole contact interactions of the related S-matrix properly.

2 The $\phi^{(0)} - C^{(-2)}$ amplitude

In this section we are going to derive the full S-matrix elements of one scalar field and one closed string Ramond-Ramond (RR) in type IIA (IIB) String theory, where for some reasons we would like to keep track of RR in asymmetric picture. That is, we consider its vertex operator in terms of its potential (not its field strength) so we deal with RR with $C^{-3/2, -1/2}$ picture. Thus, this four point function from world-sheet (three point function, from the space-time) point of view of one scalar and one RR closed string is given by its following correlation function:

$$\mathcal{A}^{C^{(-2)} \phi^{(0)}} \sim \int dx dz d\bar{z} \langle V^{(-2)}_{RR}(z, \bar{z}) V^{(0)}_{\phi}(x) \rangle$$

Note that their vertices can be read off as follows:

$$V^{(0)}_{\phi}(x) = \xi_1 (\partial^i X(x) + i \alpha' k \psi \psi^i(x)) e^{i \alpha' k X(x)}$$

$$V^{(-2)}_{RR}(z, \bar{z}) = (P_{\mathcal{C}(n)} M_p)^{\alpha \beta} e^{-3\phi(z)/2} S_\alpha(z) e^{i \frac{p}{2} \cdot X(z)} e^{-\phi(\bar{z})/2} S_\beta(\bar{z}) e^{i \frac{p}{2} \cdot D \cdot X(\bar{z})}$$

one gauge/tachyon. The answer is that in this four point function we do have that extra term (carrying momentum of RR in transverse direction), however after gauge fixing the integral should be taken on the whole space time, namely it should be taken from $-\infty$ to $\infty$ and since the integrand (including $p^i$ term) is odd but the intervals of the integral are symmetric, the final result naturally is zero. However for five and higher point functions after fixing SL(2,R) invariant we need to take the integrals on the position of closed string and the remaining terms are not vanished and at the end of the day, they might not satisfy Ward identity, where the resolution for this problem is to either calculate all of them in asymmetric picture or introduce some new Bianchi identities.

Motivation for doing this computation in both pictures is that, there is no Ward identity for the scalar field and one might wonder what happens in higher point functions of all mixed amplitudes including RR, scalar field and tachyons (but not gauge field), or which picture is going to give us the complete answer including all the infinite contact interactions of string theory.

Notations are $\mu, \nu, \ldots$ run over the whole space-time and $a, b, c, \ldots$ and $i, j, k, \ldots$ are world volume and transverse directions accordingly.
We are considering the disk level amplitude so the RR has to be put in its middle while scalar field has to be replaced just in its boundary.

On-shell conditions for scalar and RR are accordingly given\(^6\)

\[ k^2 = p^2 = 0, \quad k \cdot \xi_1 = 0 \]

For simplicity it is really useful to just make use of World-sheet’s holomorphic elements, which means that we employ some valuable change of variables in below

\[ \tilde{X}^\mu(z) \rightarrow D_\mu X^\nu(z), \quad \tilde{\psi}^\mu(z) \rightarrow D_\mu \psi^\nu(z), \quad \tilde{\phi}(z) \rightarrow \phi(z), \quad \text{and} \quad \tilde{S}_\alpha(\bar{z}) \rightarrow M_\alpha^\beta S_\beta(\bar{z}), \]

whereas, the following definitions might be important to highlight.

Applying those vertex operators and Wick theorem, our desired S-matrix can be written by

\[ \int dx_1 dx_4 dx_5 (P_- \mathcal{C}'_n) M_p)^{\alpha\beta} (x_{45})^{-3/4} \xi_{11} (I_1 + (2ik_{1a})I_2) I \]

so that \( x_4 = z = x + iy, x_5 = \bar{z} = x - iy \)

\[ I = |x_{14} x_{15}|^{\frac{\alpha'}{2} k_1.p} |x_{45}|^{\frac{\alpha'}{4} p.D.p}, \quad (4) \]

\(^6\) The definition of projector and the field strength of closed string is

\[ P_- = \frac{1}{2} (1 - \gamma^{11}), \quad H_{(n)} = \frac{a_n}{n!} H_{\mu_1...\mu_n} \gamma^{\mu_1} ... \gamma^{\mu_n} \]

where for type IIA (type IIB) \( n = 2, 4, a_n = i \ (n = 1, 3, 5, a_n = 1) \) with \((P_- H_{(n)})^{\alpha\beta} = C^{\alpha i} (P_- H_{(n)})_{\delta^i}^{\beta}\) notation for spinor.

\[ D = \left( \begin{array}{cc} -1_{g-p} & 0 \\ 0 & 1_{p+1} \end{array} \right), \quad \text{and} \quad M_p = \left\{ \begin{array}{c} \frac{\gamma_{i_1} \gamma_{i_2} ... \gamma_{i_{p+1}} \epsilon_{i_1 ... i_{p+1}}}{(p+1)!} \gamma_{i_1} \gamma_{i_2} ... \gamma_{i_{p+1}} \epsilon_{i_1 ... i_{p+1}} \quad \text{for } p \text{ even} \\ \frac{\gamma_{i_1} \gamma_{i_2} ... \gamma_{i_{p+1}} \epsilon_{i_1 ... i_{p+1}}}{(p+1)!} \gamma_{i_1} \gamma_{i_2} ... \gamma_{i_{p+1}} \epsilon_{i_1 ... i_{p+1}} \quad \text{for } p \text{ odd} \end{array} \right. \]

whereby now the propagators for all world-sheet fields can be written as

\[ \langle X^\mu(z) X^\nu(w) \rangle = - \frac{\alpha'}{2} \eta^{\mu\nu} \log(z - w), \]

\[ \langle \psi^\mu(z) \psi^\nu(w) \rangle = - \frac{\alpha'}{2} \eta^{\mu\nu} (z - w)^{-1}, \]

\[ \langle \phi(z) \phi(w) \rangle = - \log(z - w). \quad (3) \]
with

\[
I_1 = ip^i(x_{45})^{-5/4}C_{\alpha\beta}^{-1}\frac{x_{45}}{x_{14}x_{15}} \quad (5)
\]

It is also important to talk about the following correlator which is obtained by generalizing the Wick-like rule \[36, 37\]

\[
I_2 = <\alpha \beta : S(x_{4}) : S_{\beta}(x_{5}) : \psi^a_1 : \psi^i_1 > = 2^{-1}(x_{14}x_{15})^{-1}(x_{43})^{-1/4}(\Gamma_{i\alpha}C_{\gamma}^{-1})_{\alpha\beta}
\]

If we would replace the above correlators inside the amplitude we would see that our S-matrix does respect the SL(2, R) invariance, and eventually we gauge fix it as \((x_{1}, z, \bar{z}) = (\infty, i, -i)\) so that the final result for our S-matrix in this certain picture is

\[
\mathcal{A}_{\phi^0, C_{-2}} = \left[ ip^i \text{Tr} (P \gamma (n) M_p) + ik_1 \text{Tr} (P \gamma (n) M_p \Gamma_{ab}) \right] \xi_{1a} \quad (6)
\]

As it can be seen from the above S-matrix, it sounds to have two different terms in our amplitude while in below we show that some crucial subtleties are needed. Indeed after the derivation of the S-matrix, one could start writing all its field theory couplings to be compared with the amplitude while before doing so, we claim that one has to know the correct form of the S-matrix. Hence let us carry out this amplitude in the other picture in the next section and back to the subtlety associated to \((6)\) afterwards.

It is worth deriving the S-matrix of one RR and a gauge field in the asymmetric picture of RR.

\[8\] Let us just mention the final answer

\[
\mathcal{A}_{\phi^0, C_{-2}} = \xi_{1a} \left[ - \text{ip}^a \text{Tr} (P \gamma (n) M_p) + ik_1 \text{Tr} (P \gamma (n) M_p \Gamma_{ab}) \right]
\]

Note that the first term in above S-matrix has definitely no contribution because if we apply momentum conservation along the world volume of brane \((k_1 + p)^a = 0\) and on-shell condition for the gauge field gives rise the first part of the S-matrix to be vanished. Indeed the second part of the S-matrix can be produced by \(2\pi \alpha' \int C_{p-1} \wedge F\) as the three point function of one closed string RR and one gauge field in both (-1) picture should be given by

\[
V_A^{-1}(x) = e^{-\phi(x)} \xi_{1a} \psi^a(x) e^{i\phi(x)}
\]

\[
\mathcal{A}_{C^{-1}A^{-1}} \sim 2^{-1/2} \xi_{1a} \text{Tr} (P \gamma (n) M_p \gamma^a)
\]
3 The $C^{-1} - \phi^{-1}$ amplitude

The three point function from the world-sheet prospective (or two point function from the space time point of view) of one closed string RR and a real tachyon of type II super string in all their different pictures can been done.

This three point function from the world-sheet prospective with both RR and scalar field in $(-1)$ picture is given by

$$\mathcal{A}_{\phi,RR} \sim \int dx dz \langle V_{\phi}^{(-1)}(x)V_{RR}^{(-1)}(z,\bar{z}) \rangle$$

(7)

Now the vertices are changed so that RR should be written in terms of its field strength in symmetric picture. They are presented as follows

$$V_{\phi}^{(-1)}(x) = e^{-\phi(x)}\xi_i \psi^i(x)e^{2i\alpha X(x)}$$

$$V_{RR}^{(-1)}(z,\bar{z}) = (\mathcal{P} - \mathcal{H} / n) e^{-\phi(z)/2}S_\alpha(z)e^{i\mathcal{D}_p \cdot X(z)}e^{-\phi(\bar{z})/2}S_\beta(\bar{z})e^{i\mathcal{D}_p \cdot D \cdot X(\bar{z})}$$

Obviously all the previous definitions of the first section for projector, holomorphic components and the other field contents must hold here as well. Once more we substitute the defined vertex operators into (7) and it reduces to

$$\int dx_1 dx_4 dx_5 (\mathcal{P} - \mathcal{H} / n) e^{-\phi(z)/2}S_\alpha(z)e^{i\mathcal{D}_p \cdot X(z)}e^{-\phi(\bar{z})/2}S_\beta(\bar{z})e^{i\mathcal{D}_p \cdot D \cdot X(\bar{z})}$$

where the result for the following correlation function is necessary, that is,

$$\langle S_\alpha(x_4) : S_\beta(x_5) : \psi^i(x_1) \rangle = 2^{-1/2}(x_{14}x_{15})^{-1/2}(x_{45})^{-3/4}(\gamma^i C^{-1})_{\alpha\beta}$$

and also

$$\mathcal{A}^{C^{-1}T^{-1}} \sim -2iTr (\mathcal{P} - \mathcal{H} / n)$$

$$\mathcal{A}^{C^{-2}T^0} \sim 2^{1/2}(2ik_{\alpha} \gamma^\alpha) Tr (\mathcal{P} - \mathcal{H} / n)$$

So if we apply momentum conservation along the world volume of brane $(k_1 + p)^a = 0$ and extract the trace and use $p\mathcal{C} = \mathcal{H}$ up to normalisation constant we get the same answer in both pictures, whereby this S-Matrix can be generated with $2i\pi \alpha' \beta^\prime \mu_p \oint C_p \wedge DT$.
The $SL(2,R)$ invariance of the S-matrix can be readily checked and we did gauge fixing as before. The final result of the S-matrix of one scalar and one RR closed string in this picture has got to be

$$A^{C^{-1} \phi^{-1}} = 2^{-1/2} \text{Tr} \left( P_- H_{(n)} M_p \gamma^i \right) \xi_{1i}. \quad (8)$$

One should pay particular attention to the conservation of momentum along the world volume of brane as $k_1^a + p^a = 0$.

Let us first reproduce the field theory of above S-Matrix. The amplitude might be normalized by a coefficient of $(2^{1/2} \pi \mu_p/8)$ such that $\mu_p$ is Ramond-Ramond charge of brane, and also the trace is done as follows:

$$\text{Tr} \left( H_{(n)} M_p \gamma^i \right) \delta_{p+2,n} = \pm \frac{32}{(p+2)!} \epsilon^{a_0 \cdots a_p} H_{i a_0 \cdots a_p} \delta_{p+2,n}$$

Eventually this S-matrix can be precisely produced with the following field theory coupling

$$\mu_p (2 \pi \alpha') \int_{\Sigma_{p+1}} \left( \text{Tr} \left( \partial_i C_{p+1} \phi^i \right) \right) \quad (9)$$

Indeed we have used what is the so called Taylor expansion of a scalar field. Meanwhile in the other picture, it was found to be

$$C^{-2} = \left[ i p^i \text{Tr} \left( P_- \phi_{(n)} M_p \right) + i k_{1a} \text{Tr} \left( P_- \phi_{(n)} M_p \Gamma^{ia} \right) \right] \xi_{1i}$$

Let us compare (6) with (8). Since we know that $p^i C = H^i$ and up to normalisation constant the first term of (6) does exactly produce the same answer of (8) therefore we claim that the second term of (6) has no contribution to the S-matrix of one scalar and one RR at all. Hence the prescription for getting rid of the second term of (6) is as follows.

We first apply the momentum conservation along the world volume of brane to the second term ($k_1^a + p^a = 0$) and then extract the trace as follows:

$$\text{Tr} \left( \phi_{(n)} M_p \Gamma^{ia} \right) \delta_{p+1,n} = \pm \frac{32}{(p+1)!} \epsilon^{a_0 \cdots a_{p-1} a} C_{i a_0 \cdots a_{p-1}} \delta_{p+1,n}$$

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10 we set $\alpha' = 2$
11 The trace with $\gamma^{11}$ can be shown that all above results keep held even for the following case

$$p > 3, H_n = *H_{10-n}, n \geq 5$$
more importantly in order to get to the same answer as $A^{\phi^{-1}C^{-1}}$, we get to the point that the following Bianchi identity must hold

$$p^a \epsilon^{a_0 \ldots a_{p-1} a} = 0 \quad (10)$$

However, this is not the full story as we will see in the next sections for the higher point functions, one has to generalize all these identities and in particular in order not to miss any contact interactions, one might need to look at the answers for the other pictures of higher point functions as well.

4 The $T^{-1} - \phi^0 - C^{-1}$ amplitude

The four point function from the world-sheet prospective (or three point function from the space time point of view) of one closed string RR and two real tachyons of type II superstring in all their different pictures can been done.\textsuperscript{12}

The four point function of one tachyon, a scalar and one RR from the world-sheet point of view has been performed in detail in [35]. Indeed both the S-matrix and its field theory part in the following picture $T^{-1} - \phi^0 - C^{-1}$ have been computed. Truly, after carrying out the gauge fixing as $(x_1, x_2, z, \bar{z}) = (x, -x, i, -i)$ with $u = -\frac{k'_1}{2}(k_1 + k_2)^2$ the S-matrix is

$$V_T^{(-1)}(y) = e^{-\phi(y)} e^{2ik \cdot X(y)} \otimes \sigma_2$$

$$A^{-1}T^{-1}T^a \sim 2^{3/2} \pi \frac{\Gamma[-2u]}{\Gamma[1/2 - u]^2} \text{Tr}(P_+ H_{(n)} M_p \gamma^a) k_{2a} \quad (11)$$

where we can use momentum conservation along the brane $-k^1 - p^a = k^a_2$ and in particular the identity $p^a \epsilon^{a_0 \ldots a_{p-1} a} = 0$ must be used to show that the S-matrix is antisymmetric with respect to interchanging the two tachyons whereas

$$A^{-2}T^0 T^a \sim 4k_{1a}k_{2b} \int_{-\infty}^{\infty} dx (2x)^{-2u-1} \left(1 + x^2\right)^{2u} \text{Tr}(P_+ \Phi_{(n)} M_p \Gamma^{ba}) - 2\eta^{ab} \frac{1 - x^2}{4x^4} \text{Tr}(P_+ \Phi_{(n)} M_p \Gamma^{ba}) \right],$$

where evidently the second term has no contribution to the S-matrix, because the integration must be taken over the whole space and the integrand is zero. If we apply momentum conservation, in order to make sense of non-zero S-matrix in this asymmetric picture we get to know the following equation

$$p_0 \epsilon^{a_0 \ldots a_{p-1} a}$$

must be non-zero otherwise the whole S-matrix vanishes while in the last section we have shown that $p_0 \epsilon^{a_0 \ldots a_{p-1} a} = 0$ to make sense of another non supersymmetric amplitude. Notice that all u-channel poles with an infinite higher derivative corrections to $(2\pi \alpha')^2 \int C_{p-1} \wedge DT \wedge DT$ can be derived.

12 Indeed after performing the gauge fixing as $(x_1, x_2, z, \bar{z}) = (x, -x, i, -i)$ with $u = -\frac{k'_1}{2}(k_1 + k_2)^2$ the S-matrix is

$$V_T^{(-1)}(y) = e^{-\phi(y)} e^{2ik \cdot X(y)} \otimes \sigma_2$$

$$A^{-1}T^{-1}T^a \sim 2^{3/2} \pi \frac{\Gamma[-2u]}{\Gamma[1/2 - u]^2} \text{Tr}(P_+ H_{(n)} M_p \gamma^a) k_{2a} \quad (11)$$

where we can use momentum conservation along the brane $-k^1 - p^a = k^a_2$ and in particular the identity $p^a \epsilon^{a_0 \ldots a_{p-1} a} = 0$ must be used to show that the S-matrix is antisymmetric with respect to interchanging the two tachyons whereas

$$A^{-2}T^0 T^a \sim 4k_{1a}k_{2b} \int_{-\infty}^{\infty} dx (2x)^{-2u-1} \left(1 + x^2\right)^{2u} \text{Tr}(P_+ \Phi_{(n)} M_p \Gamma^{ba}) - 2\eta^{ab} \frac{1 - x^2}{4x^4} \text{Tr}(P_+ \Phi_{(n)} M_p \Gamma^{ba}) \right],$$

where evidently the second term has no contribution to the S-matrix, because the integration must be taken over the whole space and the integrand is zero. If we apply momentum conservation, in order to make sense of non-zero S-matrix in this asymmetric picture we get to know the following equation

$$p_0 \epsilon^{a_0 \ldots a_{p-1} a}$$

must be non-zero otherwise the whole S-matrix vanishes while in the last section we have shown that $p_0 \epsilon^{a_0 \ldots a_{p-1} a} = 0$ to make sense of another non supersymmetric amplitude. Notice that all u-channel poles with an infinite higher derivative corrections to $(2\pi \alpha')^2 \int C_{p-1} \wedge DT \wedge DT$ can be derived.
the gauge fixing as \((x_1, x_2, z, \bar{z}) = (x, -x, i, -i)\) the S-matrix was given by

\[
4k_1 \xi_i \int_{-\infty}^{\infty} dx (2x)^{-2u-1/2} \left(1 + x^2 \right)^{-1/2+2u} \left( \text{Tr} \left( P_+ H_{(n)} M_p \Gamma^i \right) \right),
\]

The result of this non super symmetric amplitude was read off as

\[
\mathcal{A}^{T_0, \phi^{-1}, C^{-1}} = (\pi' \mu'_p)^2 \sqrt{\pi} \frac{\Gamma[-u + 1/4]}{\Gamma[3/4 - u]} \text{Tr} \left( P_+ H_{(n)} M_p \Gamma^i \right) k_{1a} \xi_i \text{Tr} (\lambda_1 \lambda_2). \tag{12}
\]

with \((\pi' \mu'_p/2) \beta' \) and \(\mu'_p \) defined as normalisation constant, WZ and the RR charge of brane. It was shown that , this S-matrix can be precisely reproduced by the following coupling of type II string theory (with odd parity):

\[
2i \beta' \mu'_p (2\pi \alpha')^2 \int_{\Sigma_{p+1}} \left( \text{Tr} \left( \partial_i C_p \wedge DT^i \phi \right) \right), \tag{13}
\]

Finally all its infinite corrections were derived. \[14\]

If we use the above vertex operators and perform all the correlators with same techniques that have been explained in the previous section then one explores the final form of the S-matrix in this picture as

\[
\mathcal{A}^{C^{-1}, \phi^{0}, T^{-1}} \sim 4 \xi_i \int_{-\infty}^{\infty} dx (2x)^{-2u-1/2} \left(1 + x^2 \right)^{2u-1/2} \times \left[ k_{1a} \text{Tr} \left( P_+ H_{(n)} M_p \Gamma^a \right) - p^i \text{Tr} \left( P_- H_{(n)} M_p \right) \right]. \tag{15}
\]

Now one has to apply the momentum conservation \((k_1 + k_2 + p)^a = 0\) and keep in mind the following Bianchi identity

\[
p^i \epsilon^{a_0 \ldots a_p} H_{a_0 \ldots a_p} + p^a \epsilon^{a_0 \ldots a_p-1 \alpha} H^{i}_{a_0 \ldots a_p-1} = 0 \tag{16}
\]

to get to the same S-matrix for this four point world sheet amplitude as appeared in \(\tag{12} \).

\[13\] with \(u = -\frac{\alpha'}{2} (k_1 + k_2)^2\) and \((k_1^a + k_2^a + p^a = 0)\).

\[14\]
Let us calculate it in asymmetric picture of RR and zero picture of scalar field and make some essential comments.

Notice that there is no coupling between two gauge fields and one RR in the world volume of BPS branes of type II string theory.

\[ 15 \]

5 The \( T^0 - \phi^0 - C^{-2} \) amplitude

This four point function in asymmetric picture of RR and all the scalar/tachyon strings in zero pictures can be investigated by the following correlation function:

\[ \mathcal{A}^{T^0 \phi^0 C^{-2}} \sim \int dx_1 dx_2 d^2z \langle V_T^{(0)}(x_1)V_T^{(0)}(x_2)V_{RR}^{(-2)}(z, \bar{z}) \rangle \]  \hspace{1cm} (17)

where the tachyon, scalar field and RR vertex operators are given as \[ 16 \]

\[
V_T^{(0)}(y) = \alpha' ik_1 \psi(y) e^{\alpha' ik_1 \cdot X(y)} \lambda \otimes \sigma_1 \\
V_\phi^{(0)}(x) = \xi_{i1} (\partial^i X(x) + i \alpha' k_1 \psi^i(x)) e^{\alpha' ik \cdot X(x)} \lambda \otimes I \\
V_{RR}^{(-2)}(z, \bar{z}) = (P_- \mathcal{C}_{(n)} M_p)^{\alpha \beta} e^{-3 \phi(z)/2} S_\alpha(z) e^{\alpha' 2p \cdot X(z)} e^{-\phi(z)/2} S_\beta(\bar{z}) e^{i \alpha' 2p \cdot \bar{D} \cdot \bar{X}(\bar{z})} \lambda \otimes \sigma_3 \sigma_1
\]

Applying Wick theorem, the amplitude can be got

\[
\int dx_1 dx_2 dx_4 dx_5 (P_- \mathcal{C}_{(n)} M_p)^{\alpha \beta} (x_{45})^{-3/4} (4i k_{2a}) \xi_{i1} \left( I_3 + (2ik_1e) I_4 \right) I_5
\]

\[ I_5 = |x_{12}|^{\alpha' k_1 \cdot k_2} |x_{14} x_{15}|^{\alpha' 2k_1 \cdot p} |x_{24} x_{25}|^{\alpha' 2k_2 \cdot p} |x_{45}|^{\alpha' 2p \cdot D \cdot p}, \]  \hspace{1cm} (18)

\[ 15 \] In \[ 38 \] it is shown that

\[
\mathcal{A}^{-1 \mathcal{A}^{-1}} \sim 2^{-3/2} \xi_{a1} \xi_{a2} \int_{-\infty}^{\infty} dx (2x)^{-2a} \left( 1 + x^2 \right)^{2a-1} \left( \frac{1 - x^2}{x} \right) \\
\times \left[ k_{1b} \text{Tr} (P_- H_{(n)} M_p \gamma^a) + k_{2c} \left( - \eta^{ac} \text{Tr} (P_- H_{(n)} M_p \gamma^b) + \eta^{ab} \text{Tr} (P_- H_{(n)} M_p \gamma^c) \right) \right]
\]

where obviously the entire S-matrix is zero because the integration must be taken over the whole space and the integrand is odd.

\[ 16 \] To see the Chan-Paton factors see \[ 35 \]
Now using Wick-like rule one can get to the following correlator

\[ I_4 = \langle :S_\alpha(x_4) : S_\beta(x_5) : \psi^c(x_1) : \psi^a(x_2) : \rangle = 2^{-3/2}(x_{24} x_{25})^{-1/2}(x_{14} x_{15})^{-1}(x_{45})^{1/4} \]

\[ \left[ (\Gamma^{aic} C^{-1})_{\alpha\beta} + 2 R e \frac{[x_{14} x_{25}]}{x_{12} x_{45}} \eta^{ac} (\gamma^i C^{-1})_{\alpha\beta} \right] \]

By applying the above correlators into this four point amplitude we can easily observe that the integrand or S-matrix is SL(2, R) invariant. We do the proper gauge fixing as \((x_1, x_2, z, \bar{z}) = (x, -x, i, -i)\), taking \(t = -\frac{\alpha'}{2}(k_1 + k_2)^2\) we obtain the S-matrix as

\[
\mathcal{A}^{T_0 \phi C^{-2}} = \mathcal{A}_1 + \mathcal{A}_2
\]

such that

\[
\mathcal{A}_1^{T_0 \phi C^{-2}} = -2^{3/2} \xi_{1i} k_{2a} p^i \text{Tr} \left( C_{(n)} M_p \gamma^a \right) \int_{-\infty}^{\infty} (2x)^{-2(1/2)(x^2 + 1)^{1/2}}
\]

\[
= -2^{3/2} \xi_{1i} k_{2a} p^i \text{Tr} \left( C_{(n)} M_p \gamma^a \right) \sqrt{\frac{\pi}{\Gamma[3/4 - u]}}
\]

\[ \mathcal{A}_2^{T_0 \phi C^{-2}} = 2^{3/2} \xi_{1i} k_{1c} k_{2a} \int_{-\infty}^{\infty} (2x)^{-2(1/2)(x^2 + 1)^{1/2}} \left[ 2\eta^{ac} \text{Tr} \left( C_{(n)} M_p \gamma^i \right) \right]
\]

\[ + \text{Tr} \left( C_{(n)} M_p \Gamma^{aic} \right) \]

where the first term in \((21)\) is indeed zero because, the integration is taken over the whole space while integrand is odd and so the answer naturally for the first term of \((21)\) is zero\(17\)

\[
2^{3/2} \xi_{1i} (-t - 1/4) \text{Tr} \left( C_{(n)} M_p \gamma^i \right) \int_{-\infty}^{\infty} \left( \frac{(1 + x^2)^2}{(4x^2)} \right)^{1/4+t} \frac{1 - x^2}{(x^2 + 1)(4xt)} = 0
\]
But as it is clear the result for the second term of (21) is non-zero, that is
\[ A_2^T \tilde{\phi} \phi C^{-2} = \frac{3}{2} \xi_{1i}^4 k_{1a} k_{2a} \text{Tr} \left( \mathcal{G} \left( n \right) M_p \Gamma_{aic} \right) \sqrt{\frac{\Gamma[-t + 1/4]}{\Gamma[3/4 - t]}} \] (23)

We might think of this term as the contact interaction, however, after having applied momentum conservation along the world volume of brane and using Bianchi identities\(^\text{18}\), we believe that this term has no contribution for this part of the S-matrix.\(^\text{19}\)

6 The $\phi^0 - A^0 - C^{-2}$ amplitude

This four point function in asymmetric picture of a RR, a scalar and a gauge field in zero pictures can be done by the following correlation function,

\[ \mathcal{A}\phi^0 A^0 C^{-2} \sim \int dx_1 dx_2 d^2z \langle V_\phi (0) (x_1) V_A (0) (x_2) V_{RR} (-2) (z, \bar{z}) \rangle \] (24)

where the scalar field and RR vertex operators are given in the previous sections, and for the gauge field we have

\[ V_A (0) (x) = \xi_{2a} (\partial^{\alpha} X (x) + i \alpha' k. \psi \gamma^a \psi (x)) e^{\alpha' ik. X (x)} \]

Having set the Wick theorem, the amplitude may have been written as

\[ \int dx_1 dx_2 dx_4 dx_5 (P_n \mathcal{G} \left( n \right) M_p)^{\alpha \beta} (x_{45})^{-3/4} \xi_{1i} \xi_{2a} \left( J_1 + J_2 + J_3 + J_4 \right) I_5 \]

\[ I_5 = |x_{12}|^{\alpha^2 k_{1a} k_{2a}} |x_{14} x_{15}|^{\alpha^2 k_{1a} p} |x_{24} x_{25}|^{\alpha^2 k_{2a} p} |x_{45}|^{\alpha^2 p.D.p}, \] (25)

where by applying the generalization of Wick-like rule one can obtain all the correlators

\(^{18}\) $\rho^{a_0 \cdots a_{n-2} ca} = \rho\epsilon^{a_0 \cdots a_{n-2} ca} = 0$

\(^{19}\) It is important to point out that this term by itself without applying any equation could be meant to be non-zero and might have been confused to play the role of the whole infinite contact interactions/surface terms or total derivatives that obtained after taking integrations by parts.
\[ J_1 = \frac{i \tilde{p}^i}{x_{14} x_{15}} \left( l_a(x_{45}) \right)^{-5/4} (C^{-1})_{\alpha\beta} \]

\[ l_a = -i k_{1a} \left[ \frac{x_{14}}{x_{12} x_{24}} + \frac{x_{15}}{x_{12} x_{25}} \right] \]

\[ J_2 = -p^i k_{2b} \frac{x_{54}}{x_{14} x_{15}} (x_{24} x_{25})^{-1} (x_{45})^{-1/4} (\Gamma^{ac} C^{-1})_{\alpha\beta} \]

\[ J_3 = (l_a) i k_{1b} (x_{14} x_{15})^{-1} (x_{45})^{-1/4} (\Gamma^{ib} C^{-1})_{\alpha\beta} \]

\[ J_4 = -k_{1b} k_{2c} (x_{14} x_{15} x_{24} x_{25})^{-1} (x_{45})^{3/4} \]

\[ \times \left[ (\Gamma^{acib} C^{-1})_{\alpha\beta} + (2\eta^{bc} (\Gamma^{ai} C^{-1})_{\alpha\beta} - 2\eta^{ib} (\Gamma^{ci} C^{-1})_{\alpha\beta}) \right] \]

\[ \text{Re} \left[ x_{14} x_{25} \right] \right] \quad (26) \]

By applying (26) into this four point amplitude we can easily determine that the S-matrix is SL(2, R) invariant. We do the proper gauge fixing as \((x_{1}, x_{2}, z, \bar{z}) = (x, -x, i, -i)\), taking \( t = -\frac{q^2}{2}(k_1 + k_2)^2 \) to get to the S-matrix as

\[ \mathcal{A}^{\phi_0 \phi' C^{-2}} = -\xi_1 \xi_2 \int_{-\infty}^{\infty} dx (1 + x^2)^{2t-1} (2x)^{-2t} \left[ \frac{1 - x^2}{x} \right] \left( -i p^i k_{1a} \text{Tr} (P_- \phi'_{(n)} M_p) \right. \]

\[ + k_{1b} k_{1a} \text{Tr} (P_- \phi'_{(n)} M_p \Gamma^{ib}) + \eta^{bc} \text{Tr} (P_- \phi'_{(n)} M_p \Gamma^{ai}) - \eta^{abd} \text{Tr} (P_- \phi'_{(n)} M_p \Gamma^{ci}) \right) \]

\[ + 2i k_{2c} p^i \text{Tr} (P_- \phi'_{(n)} M_p \Gamma^{ac}) - 2i k_{1b} k_{2c} \text{Tr} (P_- \phi'_{(n)} M_p \Gamma^{acib}) \]

\[ \right) \quad (27) \]

where the first, second, third and fourth term do not have any contribution to the S-matrix because the integration is taken on the whole space and the integrand is odd. This so happens for the last term as well, if we use the momentum conservation \((k_1 + k_2 + p)^a = 0\) and the Bianchi identity \( p_\mu \epsilon^{\alpha_0 \alpha_1 \ldots \alpha_{m-3} \beta} = 0 \), then we come to the point only the fifth term has non-zero contribution to the S-matrix of a RR, a scalar and a gauge field.

So the final result is

\[ \mathcal{A}^{\phi_0 \phi' C^{-2}} = -\xi_1 \xi_2 2i k_{2c} p^i \text{Tr} (P_- \phi'_{(n)} M_p \Gamma^{ac}) \left[ 1/2 \right] \frac{\Gamma[-t + 1/2]}{\Gamma[1 - t]} \quad (28) \]

where the expansion of the amplitude is non-zero for \( p = n \) case and it does not include any poles as it is clear from the answer, because the low energy expansion is \( t \rightarrow 0 \) limit and all the infinite contact interactions have been derived in [33]. Given the closed form of the above correlation functions one can get to the following answer as well.
\[ A^{\phi^{-1}A^{-1}C^{-1}} = 2^{-3/2}\xi_1\xi_2a \int_{-\infty}^{\infty} dx (1 + x^2)^{2t-1}(2x)^{-2t} \left[ \frac{1 - x^2}{x} \left( k_{1a} \text{Tr} \left( P - \mathbb{H}_n \right) M_p \gamma^i \right) \right] \]

where again the first term has no contribution and the second term precisely produces \(2^{3/2}\) up to a coefficient of \(2^{3/2}\). Finally in its last picture it can be given as

\[ A^{\phi^{-1}A^{-1}C^{-1}} = 2^{1/2}\xi_1\xi_2 \int_{-\infty}^{\infty} dx (1 + x^2)^{2t-1}(2x)^{-2t} \left[ k_{1b} \text{Tr} \left( P - \mathbb{H}_n \right) M_p \Gamma^{bai} \right] \]

where in \(29\), one has to apply momentum conservation so that the following equation can be derived from S-matrix:

\[ p_b\epsilon^{a_0\cdots a_{p-2}b_{a}}H_{a_0\cdots a_{p-2}} + p^i\epsilon^{a_0\cdots a_{p-1}a}H_{a_0\cdots a_{p-1}} = 0 \]  

So again we are left over just with one term for the final answer of the a RR, a gauge and a scalar field and this term is necessary because it has to be produced in field theory by \( (2\pi\alpha')^2\mu_p \int_{H^+} \partial_i C_{a_0\cdots a_{p-2}} F_{a_{p-1}a_p} \phi^i \) where the scalar field comes from Taylor expansion and note that by comparing this S-matrix we come to understand that there should not be any other term coming from the pull-back.

Let us consider one of the most simplest five point functions and deal with more subtleties.

7 The five point world-sheet S-matrix of brane-anti brane system

It is known that the world volume of brane-anti brane system must have two real tachyon fields\[20\] where the complete form of the amplitude of a gauge ,two real tachyons and a closed string RR for brane anti brane for various \(p, n\) cases in the symmetric picture of RR \( \langle V_{A}^{(-1)}(x_1)V_{T}^{(0)}(x_2)V_{T}^{(0)}(x_3)V_{RR}^{(-\frac{1}{2}, -\frac{1}{2})}(z, \bar{z}) \rangle \) has been derived in \[40\].

\[20\] In \[39\] it is discussed in detail that even brane anti brane should be investigated by means of the effective field theory techniques which seems to be the proper way of realizing classical or even for loop divergences in which anti brane dynamics in the presence of some of background fields may play a major role.
For the completeness we have done it in the asymmetric picture of RR
\[ \langle V^{(-1)}_A(x) V^{(0)}_T(x_2) V^{(0)}_T(x_3) V^{(-1)}_{RR}(z, \bar{z}) \rangle \] and in the following picture
\[ \langle V^{(-1)}_A(x) V^{(-1)}_T(x_2) V^{(0)}_T(x_3) V^{(-1)}_{RR}(z, \bar{z}) \rangle \] as well.

The final result for the amplitude is exactly the same as appeared in [40] and it does
satisfy the Ward identity associated to the gauge field.

The S-matrix elements of one closed string RR field, one scalar field and two tachyons
on the world volume brane anti brane system in the following picture has been computed
\[ ^{21} \] in detail in [41].

\[ \langle V^{(-1)}_\phi(x_1) V^{(0)}_T(x_2) V^{(0)}_T(x_3) V^{(-1)}_{RR}(z, \bar{z}) \rangle \] (31)

so that \[ ^{22} \]

The final form of this S-matrix in this particular picture was given as
\[ \mathcal{A}^{C(-1)\phi(-1)T(0)T(0)} = \mathcal{A}_1 + \mathcal{A}_2 \] (34)
where
\[ \mathcal{A}_1 \sim -8\xi_1k_{20}k_{30}2^{-3/2}Tr(P_\pm H^{(n)}_m M_p \Gamma^{(n)})L_1, \]
\[ \mathcal{A}_2 \sim 8\xi_12^{-3/2}\left\{ Tr(P_\pm H^{(n)}_m M_p \gamma^i) \right\} L_2 \] (35)
where \( L_1, L_2 \) are written down in below just in terms of Gamma functions (no hypergeometric function is needed)
\[ L_1 = (2)^{-2(t+s+u)-1}\pi \frac{\Gamma(-u)\Gamma(-s+\frac{3}{2})\Gamma(-t+\frac{1}{2})\Gamma(-t-s-u)}{\Gamma(-u-t+\frac{1}{4})\Gamma(-t-s+\frac{3}{4})\Gamma(-s-u+\frac{1}{4})}, \] (36)
\[ L_2 = (2)^{-2(t+s+u+1)}\pi \frac{\Gamma(-u+\frac{1}{2})\Gamma(-s+\frac{3}{2})\Gamma(-t+\frac{3}{2})\Gamma(-t-s-u-\frac{1}{2})}{\Gamma(-u-t+\frac{3}{4})\Gamma(-t-s+\frac{7}{4})\Gamma(-s-u+\frac{3}{4})}. \]

\[ ^{21} \] With the following vertices and with their correct Chan-Paton factors for D-brane anti D-brane as follows
\[ ^{22} \]
\[ V^{(-1)}_\phi(x) = \xi_i \psi^i(x) e^{2ik \cdot X(x)} e^{-\phi(x)} \otimes \sigma_3 \]
\[ V^{(0)}_T(y) = 2ik \psi(y) \frac{e^{2ik \cdot X(y)}}{\sigma_1} \]
\[ V^{(-\frac{1}{2}, -\frac{1}{2})}_{RR}(z, \bar{z}) = (P_\pm H^{(n)}_m M_p) \alpha \beta e^{-\phi(z)/2} S_{\alpha}(z) e^{i p \cdot X(z)} e^{-\phi(\bar{z})/2} S_{\beta}(\bar{z}) e^{i p \cdot D \cdot X(\bar{z})} \otimes \sigma_3, \]
with \( k^2 = 1/4 \) the condition for tachyons in type II string theory and the following definitions for Mandelstam variables
\[ s = -\frac{\alpha'}{2}(k_1 + k_3)^2, t = -\frac{\alpha'}{2}(k_1 + k_2)^2, u = -\frac{\alpha'}{2}(k_2 + k_3)^2, \] (33)
where all the infinite \( u \)-channel gauge poles of \( L_1 \) and \( t + s' + u' \)-channel scalar poles of this S-matrix of have been precisely produced, in addition to that all the infinite higher derivative corrections to two scalars-two tachyons of world volume of brane anti brane have been derived without any ambiguity.

Now let us deal with the other pictures of both closed-open strings to get to the complete form of the S-matrix with all its contact interactions.

8 \( C^{(-2)} \phi^{(0)} T^{(0)} T^{(0)} \)

The S-matrix element of one closed string RR field, one scalar field and two tachyons on the world volume of brane anti brane system in asymmetric picture of closed string RR can be found as follows:

\[
\mathcal{A}^{C^{(-2)} \phi^{(0)} T^{(0)} T^{(0)}} \sim \int dx_1 dx_2 dx_3 d^2 z (V_\phi^{(0)}(x_1) V_T^{(0)}(x_2) V_T^{(0)}(x_3)) V_{RR}^{(-\frac{\Delta}{2}, -\frac{i}{2})}(z, \bar{z})
\]

so that \( |x| \) Replacing the vertex operators inside (37) and performing all the correlators by Wick theorem one can get to the whole amplitude as below

\[
\mathcal{A}^{C^{(-2)} \phi^{(0)} T^{(0)} T^{(0)}} \sim \int dx_1 dx_2 dx_3 dx_4 dx_5 (P_\phi'(n) M_p)^{\alpha\beta} \xi_1 x_4^{3/4} (-8 k_{28} k_{36}) I
\]

\[
\times \left( (ip \frac{x_{14}}{x_{15}}) I_{1_{12}} + 2ik_1a I_{1_{23}} \right)
\]

where \( x_{ij} = x_i - x_j \),

\[
I = |x_{12}|^{4k_1 k_2} |x_{13}|^{4k_1 k_3} |x_{14} x_{15}|^{2k_1 p} |x_{23}|^{4k_2 k_3} |x_{24} x_{25}|^{2k_2 p} |x_{34} x_{35}|^{2k_3 p} |x_{45}|^{p, D, p}
\]

We have already generalized the Wick theorem to get to the fermionic correlations in the presence of currents so that one can find the correlators as

\[
I_{1_{12}} = \langle :S_\alpha(x_4) S_\beta(x_5) :\psi^b(x_2) :\psi^c(x_3) :\rangle - 2^{-1} x_{45}^{-1/4} (x_{24} x_{25} x_{34} x_{35})^{-1/2}
\]

\(23 \text{ With the following vertices and with their correct Chan-Paton factor for D-brane-anti D-brane as below,} \)

\[
\begin{align*}
V^{(0)}_\phi(x) &= \xi_{1i}(\partial X(x) + i\alpha' k_x \psi^4 (x)) e^{\alpha' ik_x X(x)} \lambda \otimes I \\
V^{(0)}_T(y) &= 2ik_x \psi(y) e^{2ik_x X(y)} \otimes \sigma_1 \\
V^{(-\frac{\Delta}{2}, -\frac{i}{2})}_{RR}(z, \bar{z}) &= (P_\phi' n) M_p)^{\alpha\beta} e^{-3\phi(z)/2} S_\alpha(z) e^{ip \cdot X(z)} e^{-\phi(z)/2} S_\beta(z) e^{ip \cdot D \cdot X(z)} \otimes I,
\end{align*}
\]

with the same definitions for Mandelstam variables kept held.
\[
\times \left( (\Gamma^{cb} C^{-1})_{\alpha \beta} - 2\eta^{be} \frac{Re[x_{24}x_{35}]}{x_{23}x_{45}} (C^{-1})_{\alpha \beta} \right)
\]

also

\[
I_2^{cia} = < S_{\alpha}(x_4) : S_{\beta}(x_5) : \psi^a(x_1) : \psi^b(x_2) : \psi^c(x_3) : > = 2^{-2}x_{45}^{3/4}(x_{24}x_{25}x_{34}x_{35})^{-1/2}(x_{14}x_{15})^{-1}\left\{ (\Gamma^{cia} C^{-1})_{\alpha \beta} + 2\eta^{ab} \frac{Re[x_{14}x_{25}]}{x_{12}x_{45}} (\Gamma^{ci} C^{-1})_{\alpha \beta} - 2\eta^{ac} \frac{Re[x_{14}x_{35}]}{x_{13}x_{45}} (\Gamma^{bi} C^{-1})_{\alpha \beta} \\
- 2\eta^{bc} \frac{Re[x_{24}x_{35}]}{x_{23}x_{45}} (\Gamma^{ia} C^{-1})_{\alpha \beta} \right\}
\]

If we substitute all the above correlations inside the amplitude, then one can show the property of SL(2,R) invariance is being investigated. Here we are working with five point function and to our knowledge the best gauge fixing of this symmetry is to use the locations of all open strings

\[x_1 = 0, \quad x_2 = 1, \quad x_3 \to \infty,\]

If we do so, then we get the entire form of the S-matrix in terms of some integrations done on the upper half plane so that the following integrations for various cases need to be come over

\[
\int d^2z |1 - z|^a |z|^b (z - \bar{z})^c(z + \bar{z})^d, \quad (41)
\]

with \(d = 0, 1, 2\) and \(a, b, c\) must be just given in terms of the Mandelstam variables, notice to the point that the result of the above integrations for \(d = 0, 1\) shall be got from [42, 40] while for for \(d = 2\) the result is given in [13].

If we do gauge fixing, make use of various pure algebraic simplifications and most particularly use of the above integrals, we can write down the final result for the amplitude (39) in this asymmetric picture as follows

\[
A^{-2} = A_1 + A_2 + A_3 + A_4
\]

where

\[
A_1 \sim ip^i \xi_{1i} (-4k_{26}k_{3c}) \text{Tr} (P_- \mathcal{C} G M \Gamma^{cb}) L_1,
\]
\[ A_2 \sim -4i\xi \left\{ \text{Tr} \left( P - C M_p \right) \right\} L_2 \]
\[ A_3 \sim -4i\xi_{kk} k_{2b} k_{3c} \left\{ \text{Tr} \left( P - C \Gamma^{c} \right) \right\} L_1 \]
\[ A_4 \sim 4i\xi_{kk} \left\{ \text{Tr} \left( P - C \Gamma^{c} \right) \right\} L_2 (k_{1b} + k_{2b} + k_{3b}) \tag{43} \]

where the functions \( L_1, L_2 \) are given in (36).

Note that since we have been dealing with the five point world sheet scattering of branes and using the above mentioned gauge fixing the terms carrying momentum of RR in transverse directions (after taking integration on the upper half plane) are no longer vanished and these terms potentially have something to do with some of the issues on the perturbative string theory that are related to taking integration on different moduli space that have been pointed out in series of papers appeared in [34].

Now if we compare the S-matrix in this asymmetric picture (43) with (35), then we might think of the fact that the last two terms of (43) are extra terms. Indeed if we do not apply the Bianchi identity and momentum conservation along the brane to (43) as a matter of fact these two terms are extra terms by themselves. However, if we extract the trace in the last term and make use of the momentum conservation along the world volume of brane as below

\[(k_1 + k_2 + k_3)^a = -p^a \tag{44} \]

we come to the conclusion that in order to get the same answer as (35), the extra term should be removed or in the other words the following equation must hold

\[ p^b \epsilon^{a_{0}...a_{p-1}b} C_{a_{0}...a_{p-1}} = 0 \tag{45} \]

But in below we will show that the following equations should not be vanished for the correlators \( \langle V^0_T(x_1)V^0_T(x_2)V^0_T(x_3)V^R_T(z, \bar{z}) \rangle \).

What about the third term of (43)? One might add it with the first term of (43) and come to the point that

\[ A^{-T^a T^a} = -16i(2^{-3/2} k_{1a} k_{2b} k_{3c}) (P - C M_p) \alpha \beta \int d^2z |1 - z|^{2t + 2u |z|^2 + 2s} (z - \bar{z})^{-2(t + s + u + 1)} \]
\[ \times \left[ (\Gamma^{c} C^{-1})_{a\beta} + \frac{(z + \bar{z})}{2(z - \bar{z})} \right] \left( 2\eta^{ac} (\gamma^{b} C^{-1})_{a\beta} - (2\eta^{ab} (\gamma^{c} C^{-1})_{a\beta} - (2\eta^{bc} (\gamma^{a} C^{-1})_{a\beta} \right) \right] \]
\[ k_{2b}k_{3c}\xi_{11}\left(\rho^i\epsilon^{a_0\cdots a_{p-2}cb}C_{a_0\cdots a_{p-2}} + p^a\epsilon^{a_0\cdots a_{p-3}cba}C_{a_0\cdots a_{p-3}}\right) \]

should be vanished, however from (35) we know that the first term of (43) does hold and plays the crucial role in effective field theory so the only way of getting rid of the third term of (43) is to actually apply momentum conservation and because of the antisymmetric property of \( \epsilon \) we conclude that the equation \( p^a\epsilon^{a_0\cdots a_{p-3}cba}C_{a_0\cdots a_{p-3}} \) must be vanished for

\[ +\left(2\eta^{bc}(\gamma^aC^{-1})_{\alpha\beta}\right)\frac{1}{(z - \bar{z})} + \left(2\eta^{ab}(\gamma^cC^{-1})_{\alpha\beta}\right)\frac{|z|^2}{(z - \bar{z})}. \]

We make use of various pure algebraic simplifications to write down the final result for the above amplitude in this asymmetric picture as follows

\[ \mathcal{A}^{C-2p^aq^ap} = \mathcal{A}_1 + \mathcal{A}_2 \]

where

\[ \mathcal{A}_1 \sim -16i(2^{-3/2}k_{1a}k_{2b}k_{3c})\mathrm{Tr}\left(P_{-}\Phi^{(n)}M_p\gamma^a\right)N_1 \]
\[ \mathcal{A}_2 \sim -16i(2^{-3/2})\mathrm{Tr}\left(P_{-}\Phi^{(n)}M_p\gamma^a\right)N_2(k_{1a} + k_{2a} + k_{3a}) \]

where the functions \( N_1, N_2 \) are given as

\[
N_1 = (2)^{-2(t+s+u+1)}\frac{\Gamma(-u)\Gamma(-s)\Gamma(-t)\Gamma(-t-s-u-\frac{1}{2})}{\Gamma(-u-t)\Gamma(-t-s)\Gamma(-s-u)}, \tag{46}
\]

\[
N_2 = (2)^{-2(t+s+u+3)}\frac{\Gamma(-u+\frac{1}{2})\Gamma(-s+\frac{1}{2})\Gamma(-t+\frac{1}{2})\Gamma(-t-s-u-1)}{\Gamma(-u-t)\Gamma(-t-s)\Gamma(-s-u)}. \]

Note that if we use momentum conservation \( p^a = -(k_1 + k_2 + k_3)^a \) for both first and second part of the amplitude we get \( p^a\epsilon^{a_0\cdots a_{p-3}cba}C_{a_0\cdots a_{p-3}}, p^a\epsilon^{a_0\cdots a_{p-3}cba}C_{a_0\cdots a_{p-3}}, p^a\epsilon^{a_0\cdots a_{p-1}a}C_{a_0\cdots a_{p-1}} \) are not vanished and in particular all the following equations

\[ p^a\epsilon^{a_0\cdots a_{p-3}cba}, \quad p^a\epsilon^{a_0\cdots a_{p-1}a} \quad \tag{47} \]

must be non vanished to make sense of non supersymmetric amplitudes in the world volume of both brane -antibrane and non-BPS branes.

Hence for non super symmetric amplitudes first we must extract the traces and keep track of the above points, that is, not to apply the equations that we found for some of the BPS branes, including the Bianchi identities that have been obtained for the scalar field in the presence of closed string Ramond-Ramond. Indeed we expect to see that behaviour because the equations that hold for BPS cases not necessary can be held for non supersymmetric cases whereas for BPS branes the equations seem to be more manifest while this may have been changed after symmetry breaking. The lesson is that for scattering of the mixed scalars- tachyons in the presence of RR ( in its asymmetric picture), one has to break several identities that necessary hold for BPS branes.
non-BPS amplitudes.

Let us go over the same S-matrix in its last way of strings’ s pictures.

\[ C^{(-1)} \phi^{(0)} T^{(-1)} T^{(0)} \]

Finally this S-matrix element of one RR field, one scalar and two tachyons on the world volume of brane anti brane system in its last picture can be written down in the symmetric picture as follows:

\[ A C^{(-1)} \phi^{(0)} T^{(-1)} T^{(0)} \sim \int dx_1 dx_2 dx_3 d^2 z \langle V^{(0)}_\phi (x_1) V^{(-1)}_T (x_2) V^{(0)}_T (x_3) V^{(-\frac{1}{2}, -\frac{1}{2})}_{RR} (z, \bar{z}) \rangle \]

with the following vertices to begin with

\[ V^{(-1)}_T (y) = e^{-\phi(y)} e^{2ik \cdot X(y)} \otimes \sigma_2 \]
\[ V^{(-\frac{1}{2}, -\frac{1}{2})}_{RR} (z, \bar{z}) = (P - H_{(n)} M_p)_{\alpha \beta} e^{-\phi(z)/2} S_a (z) e^{i p \cdot X(z)} e^{-\phi(\bar{z})/2} S_b (\bar{z}) e^{i p \cdot D \cdot X(\bar{z})} \otimes \sigma_3, \]

Lets us just write down the amplitude in its compact form as

\[ A C^{(-1)} \phi^{(0)} T^{(-1)} T^{(0)} \sim \int dx_1 dx_2 dx_3 dx_4 dx_5 (P - H_{(n)} M_p)_{\alpha \beta} \xi_{1i} (-4k_{3b}) x^{1/4}_{45} (x_{24} x_{25})^{-1/2} I \]
\[ \times \left( (ip \frac{x_{15}}{x_{14} x_{15}}) I^b_1 + 2i k_{1a} I^{bia} \right) \]

where \( x_{ij} = x_i - x_j \),

\[ I = |x_{12}|^{4k_1} |x_{13}|^{4k_1, k_2} |x_{14} x_{15}|^{2k_1} |x_{23}|^{4k_2} |x_{24} x_{25}|^{2k_2} |x_{34} x_{35}|^{2k_3} |x_{45}|^{p \cdot D \cdot p} \]

with the given correlators to be replaced inside the amplitude

\[ I^b_1 = < S_\alpha (x_4) : S_\beta (x_5) : \psi^b (x_3) : > = 2^{-1/2} x^{3/4}_{45} (x_{34} x_{35})^{-1/2} (\gamma^b C^{-1})_{\alpha \beta} \]

also

\[ 25 \] Therefore, the lesson is that for a RR in asymmetric picture, a scalar and some other tachyon amplitudes one may need to find out new Bianchi identities to get to the same answer as obtained by RR (in its symmetric picture), a scalar and some tachyons. The reason is that we have no gauge field to check the gauge invariance of the amplitude and more accurately there is no Ward identity for the scalar field.
\[ I_{b_2}^{\text{via}} = \langle S_\alpha(x_4) : S_\beta(x_5) : \psi^\alpha \psi^\beta(x_1) : \psi^\beta(x_3) : \rangle = 2^{-3/2} x_{45}^{1/4} (x_{34} x_{35})^{-1/2} (x_{14} x_{15})^{-1} \]

\[ \left\{ (\Gamma^{\text{via}} C^{-1})_{\alpha\beta} + 2 \eta^{ab} \Re \left[ \frac{\epsilon_{x_{14} x_{35}}}{x_{13} x_{45}} (\gamma^i C^{-1})_{\alpha\beta} \right] \right\} \]

Notice that with the given correlations, one may easily probe the SL(2,R) transformation of the S-matrix. We performed gauge fixing by just fixing the location of open strings so that the final integration needs to be done over the closed string RR's position on the upper half complex plane.

Once more one finds the same sort of the integration as we discussed earlier on.

Having calculated all the desired integrals, one goes over to the eventual answer for the S-matrix in its particular picture with

\[ \mathcal{A}^{C(-1)\phi(0)T(-1)T(0)} = \mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_3 \]

in such a way that all its components are given by

\[ \mathcal{A}_1 \sim -2i 2^{1/2} \xi_{11} k_{36} \text{Tr} (P_\gamma (H_{(n)} M_\rho \gamma^b) ) L_1, \]

\[ \mathcal{A}_2 \sim -2i 2^{1/2} \xi_{11} \left\{ \text{Tr} (P_\gamma (H_{(n)} M_\rho \Gamma^{\text{via}}) ) \right\} k_{1a} k_{36} L_1 \]

\[ \mathcal{A}_3 \sim -2i 2^{1/2} \xi_{11} \left\{ \text{Tr} (P_\gamma (H_{(n)} M_\rho \gamma^i) ) \right\} L_2 \]

so that \( L_1, L_2 \) functions are already given in (36). If we compare (51) with (35), then one might wonder whether the first term of \( \mathcal{A}^{C(-1)\phi(0)T(-1)T(0)} \) is extra term, because it also related to all infinite singularities. However, we have already produced all the infinite u-channel gauge poles by taking into account the second term of (51). Therefore one has to

\[ x_1 = 0, \quad x_2 = 1, \quad x_3 \to \infty, \quad dx_1 dx_2 dx_3 \to x_3^2. \]

\[ \int d^2 z |1 - z|^a |z|^b (z - \bar{z})^c (z + \bar{z})^d, \]

with the same definitions for Mandelstam ones.
end up having extra Bianchi identities to get rid of the first term of (51). Hence we apply momentum conservation $k_{1a} = -k_{2a} - k_{3a} - p_a$; extract the traces, use the antisymmetric property of $\epsilon$ tensor and eventually add the first and second components of the amplitude in this special picture to derive the following identity

$$\xi_{1i} k^{3b} \left( p_a \epsilon^{a_0 \cdots a_{p-2} b a} H_{a_0 \cdots a_{p-2}}^i + p_i \epsilon^{a_0 \cdots a_{p-1} b} H_{a_0 \cdots a_{p-1}} \right) = 0 \quad (52)$$

So upon holding (52) and keeping in mind momentum conservation along the world volume of brane we precisely get the same answer as appeared in (35) so we come to the crucial fact that if we consider closed string RR in the asymmetric picture $C^{-3/2,-1/2}$ and scalar field in zero picture then one has to find out all new Bianchi identities to get rid of the extra singularities.

In the next section we will reveal some remarks to respect/restore Ward identity associated (gauge invariance) to the gauge field and derive all the precise contact interactions of mixed RR, scalar/ gauge and tachyon string amplitudes. Namely if we consider the scalar field in zero picture and the RR in asymmetric picture $(C^{-3/2,-1/2})$ then in this particular case there is no need to apply new Bianchi identities to the S-matrix and more importantly it does respect the Ward identity associated to the gauge field.

10 \quad $C^{-1}\phi^0A^{-1}T^0$ S-matrix

In (73) the five point world-sheet amplitude of a closed string RR with one scalar, a gauge field and a tachyon in the world volume of non-BPS branes of type II string theory with odd parity with the following picture $C^{-1}\phi^{-1}A^0T^0$ was achieved and all the correlators were
The final result in this very particular picture was read off as

\[ \mathcal{A}^{C^{-1}\phi^{-1}A^{0}{T^{0}}} = A_1 + A_2 \]  

where

\[ A_1 \sim 2\xi_1\xi_2a k_3c k_2d \text{Tr} (P(H_{(n)}M_{p}\Gamma^{cadi}) L'_1, \]

\[ A_2 \sim (2L'_3) \left\{ -t\text{Tr} (P(H_{(n)}M_{p}\gamma_2\gamma_1) u' - 2tk_3\xi_2 \text{Tr} (P(H_{(n)}M_{p}\gamma_2\gamma_1), \right. \]

\[ + \text{Tr} (P(H_{(n)}M_{p}\gamma_2\gamma_3\gamma_1) \left( -2tk_3\xi_2 + 2u'k_1\xi_2 \right) \right\} \]  

with

\[ L'_1 = (2t + s + u) - p^a p_a - \frac{1}{4} \]

and also apply \( t \rightarrow 0, \quad s \rightarrow -\frac{1}{4}, \quad u \rightarrow -\frac{1}{4} \) to the S-matrix to be able to derive all infinite \( u', t \) channel tachyon and scalar poles of the non super symmetric amplitudes accordingly. Note that \( L'_1 \) has involved just infinite contact interactions.

It is of high importance to note that this particular \( \mathcal{A}^{C^{-1}\phi^{-1}A^{0}{T^{0}}} \) does respect all the symmetries and most importantly it does satisfy Ward identity associated to the gauge field. Indeed if we replace \( \xi_2a \rightarrow k_2a \) inside \( (53) \) the first term is automatically zero because

\[ \mathcal{A}^{C^{-1}\phi^{-1}A^{0}{T^{0}}} \sim \int dx_1 dx_2 dx_3 dx_4 \langle V_{\phi}^{(-1)}(x_1) V_A^{(0)}(x_2) V_T^{(0)}(x_3) V_{RR}^{(-\frac{1}{4} - \frac{3}{4})}(z, \bar{z}) \rangle, \]

with the rest of the vertex operators:

\[ V_{\phi}^{(-1)}(y) = \xi_{1i} \psi_i(y) e^{\alpha'_i k \cdot x(y)} e^{-\phi(y)} \otimes \sigma_3 \]

\[ V_A^{(0)}(x) = \xi_{2a} (\partial^a X(x) + i\alpha' q \cdot \psi \psi^a(x)) e^{\alpha' q \cdot x} \otimes I \]

\[ u' = (-u - \frac{1}{4}) \] and also

\[ L'_1 = (2)^{-(t+s+u)} \pi \frac{\Gamma(-u + \frac{1}{4}) \Gamma(-s + \frac{1}{4}) \Gamma(-t + \frac{1}{4}) \Gamma(-t - s - u + \frac{1}{4})}{\Gamma(-u - t + \frac{1}{4}) \Gamma(-t - s + \frac{1}{4}) \Gamma(-s - u + \frac{1}{4})}, \]

\[ L'_3 = (2)^{-(t+s+u)} \pi \frac{\Gamma(-u - \frac{1}{4}) \Gamma(-s + \frac{3}{4}) \Gamma(-t) \Gamma(-t - s - u)}{\Gamma(-u - t + \frac{1}{4}) \Gamma(-t - s + \frac{3}{4}) \Gamma(-s - u + \frac{1}{4})} \]
and upon replacing $\xi_{2a} \to k_{2a}$ inside the second/third , fourth and fifth term of (53) appropriately we get zero result so gauge invariance is satisfied so that based on replacing $\xi_{2a} \to k_{2a}$ for just the gauge field the whole S-matrix gives rise zero result.

Having explained all the needed ingredients of the S-matrices , below we would like to see what happens to the gauge invariance of a mixture of five point world-sheet amplitude of a closed string RR with one scalar, a gauge field and a tachyon in the world volume of non-BPS branes of type II string theory with odd parity in their different picture. This $C^{-1} \phi^{0} A^{-1} T^{0}$ amplitude is given by looking for the following correlation function:

$$ A^{C^{-1} \phi^{0} A^{-1} T^{0}} \sim \int dx_{1} dx_{2} dx_{3} dz d\bar{z} \langle V^{(0)}_{\phi}(x_{1}) V^{(-1)}_{A}(x_{2}) V^{(0)}_{T}(x_{3}) V^{(-1/2, -1/2)}_{RR}(z, \bar{z}) \rangle, \quad (56) $$

Let us write down the rest of the vertex operators including their CP factors:

$$ V^{(-1)}_{A}(y) = \xi_{2a} \psi^{a}(y) e^{\alpha' q.X(y)} e^{-\phi(y)} \otimes \sigma_{3} $$

$$ V^{(0)}_{\phi}(x) = \xi_{1i}(\partial^{i} X(x) + i\alpha' k.\psi \psi^{i}(x)) e^{\alpha' k.X(x)} \otimes I \quad (57) $$

with the following on-shell conditions for all scalar, gauge RR and tachyon

Once more we deal with just the holomorphic elements of the all fields involving $X^{\mu} \psi^{\mu}, \phi$, such that the S-matrix is given by

$$ k_{2a} \xi_{1i}(u - u') \text{Tr} (P_{-} H_{(n)} M_{\gamma}^{\alpha} \gamma^{i}) = 0 $$

$$ k_{3a} \xi_{1i}(tu - tu') \text{Tr} (P_{-} H_{(n)} M_{\gamma}^{\alpha} \gamma^{i}) = 0 $$

$$ k^{2} = q^{2} = p^{2} = 0, \quad k_{1}^{2} = 1/4, \quad q.\xi_{1} = q.\xi_{2} = 0 $$

26
\[ \mathcal{A}^{-1} \phi^{(-1)} T^{0} \sim \int dx_{1} dx_{2} dx_{3} dx_{4} dx_{5} \left( \mathcal{F}(\xi_{1} \xi_{2} (\alpha') \lambda k_{3}) x^{-1/4}_{45} (x_{24} x_{25})^{-1/2} \times (I_{1} + I_{2}) \right) \]

where \( x_{ij} = x_{i} - x_{j} \), Let us find out all the fermionic and bosonic correlators as

\[
I_{1} = \langle \partial X^{\dagger}(x_{1}) e^{\alpha' i k_{1} \cdot X(x_{1})} : e^{\alpha' i k_{2} \cdot X(x_{2})} : e^{\alpha' i k_{3} \cdot X(x_{3})} : e^{\alpha' i p \cdot D \cdot X(x_{5})} : \rangle \times \langle S_{\alpha}(x_{4}) : S_{\beta}(x_{5}) : \psi^{\dagger}(x_{2}) : \psi^{\dagger}(x_{3}) : \rangle,
\]

\[
I_{2} = \langle e^{\alpha' i k_{1} \cdot X(x_{1})} : e^{\alpha' i k_{2} \cdot X(x_{2})} : e^{\alpha' i k_{3} \cdot X(x_{3})} : e^{\alpha' i p \cdot D \cdot X(x_{5})} : \rangle \alpha' i k_{1 d} \langle S_{\alpha}(x_{4}) : S_{\beta}(x_{5}) : \psi^{\dagger}(x_{1}) : \psi^{\dagger}(x_{2}) : \psi^{\dagger}(x_{3}) : \rangle
\]

We should use the Wick-like rule \([36]\) to get to all the generalizations of the correlation functions of two spin and two fermion operators such as

\[
I^{a}_{5} = \langle S_{\alpha}(x_{4}) : S_{\beta}(x_{5}) : \psi^{\dagger}(x_{2}) : \psi^{\dagger}(x_{3}) : \rangle = 2^{-1} x^{-1/4}_{45} (x_{24} x_{25} x_{34} x_{35})^{-1/2} \times \left\{ (\Gamma^{a c} C^{-1})_{\alpha \beta} - 2 Re \left[ x_{24} x_{35} \right] \eta^{ac} (C^{-1})_{\alpha \beta} \right\}
\]

Now we need to make use of \([12]\) to go over the final correlations in ten dimensions

\[
I^{caid}_{6} = \langle S_{\alpha}(x_{4}) : S_{\beta}(x_{5}) : \psi^{\dagger}(x_{1}) : \psi^{\dagger}(x_{2}) : \psi^{\dagger}(x_{3}) : \rangle = \left\{ (\Gamma^{caid} C^{-1})_{\alpha \beta} + 2 Re \left[ x_{14} x_{25} \right] (2 \eta^{da} (\Gamma^{ci} C^{-1})_{\alpha \beta}) - 2 Re \left[ x_{14} x_{35} \right] \eta^{dc} (\Gamma^{ai} C^{-1})_{\alpha \beta} \right\} 2^{-2} x^{3/4}_{45} (x_{24} x_{25} x_{34} x_{35})^{-1/2} (x_{14} x_{15})^{-1}
\]

It is time to replace all the correlators inside \([58]\) to get to

\[
\mathcal{A}^{-1} \phi^{(-1)} T^{0} \sim \int dx_{1} dx_{2} dx_{3} dx_{4} dx_{5} (P_{-} \mathcal{H}(n) M_{p})^{\alpha \beta} \xi_{1} \xi_{2} (\alpha' i k_{3}) x^{-1/4}_{45} (x_{24} x_{25})^{-1/2} \times \left( a^{i}_{2} I^{a}_{5} + \alpha' i k_{1 d} I^{caid}_{6} \right) \text{Tr} (\lambda_{1} \lambda_{2} \lambda_{3}),
\]

\[
I = \left[ x_{12} \right]^{\alpha' k_{1}, k_{2}} \left[ x_{13} \right]^{\alpha' k_{1}, k_{2}} \left[ x_{14} x_{15} \right] \left[ x_{23} \right]^{\alpha' k_{2}, k_{3}} \left[ x_{24} x_{25} \right] \left[ x_{34} x_{35} \right] \left[ x_{45} \right] \left[ x_{5} \right]^{\alpha' p D p} \quad \text{(59)}
\]

\[I = \left| x_{12} \right|^{\alpha' k_{1}, k_{2}} \left| x_{13} \right|^{\alpha' k_{1}, k_{2}} \left| x_{14} x_{15} \right| \left| x_{23} \right|^{\alpha' k_{2}, k_{3}} \left| x_{24} x_{25} \right| \left| x_{34} x_{35} \right| \left| x_{45} \right| \left| x_{5} \right|^{\alpha' p D p} \quad \text{(59)}
\]
so that

\[ a_2^i = i p^i \left( \frac{x_{54}}{x_{14} x_{15}} \right) \]  \hspace{1cm} (63)

Having found these correlators we can investigate that the amplitude has satisfied the \( SL(2, R) \) invariance so we did gauge fixing by just fixing the positions of open strings.\footnote{33}

The solutions for all the integrals on upper half plane have been revealed so that the ultimate result will be obtained as

\[ \mathcal{A}^{g_0 A^0 T^0} = \mathcal{A}_1 + \mathcal{A}_2, \]  \hspace{1cm} (66)

where

\[
\mathcal{A}_1 \sim \left( 2\xi_1 \xi_2 a k_3 k_1 d \text{Tr} \left( P_{-} H_n M_p \Gamma^{caid} \right) - \xi_1 p (2k_3 \xi_2 a) \text{Tr} \left( P_{-} H_n M_p \Gamma^{ca} \right) \right) L_1',
\]

\[
\mathcal{A}_2 \sim \left\{ t \xi_1 p (4k_3 \xi_2) \text{Tr} \left( P_{-} H_n M_p \right) + 4(u + \frac{1}{4}) k_3 \xi_1 i \text{Tr} \left( P_{-} H_n M_p \Gamma^{ib} \right) k_1 \xi_2 
- 4tk_3 \xi_2 k_1 i \xi_1 i \text{Tr} \left( P_{-} H_n M_p \Gamma^{ib} \right) - 2t(u + \frac{1}{4}) \xi_1 i \xi_2 a \text{Tr} \left( P_{-} H_n M_p \Gamma^{ai} \right) \right\} L_3'. \]  \hspace{1cm} (67)

We have already analyzed all infinite \( u' \) tachyon and massless \( t \) channel scalar poles of the amplitude.

In this picture after replacing \( \xi_2 a \rightarrow k_2 a \) (due to the terms \( \xi_1 p \)) the amplitude does not satisfy Ward identity associated to the gauge field and indeed the second and third terms would remain whereas \( L_3' \) can not be removed by \( L_1' \).

Now we would like to compare the result of the amplitude in two different pictures to make a statement on mixed closed and open string amplitudes including one scalar and one

\[ x_1 = 0, \quad x_2 = 1, \quad x_3 \rightarrow \infty, \]  \hspace{1cm} (64)

we lead to

\[ \int d^2 z |1 - z|^a |\bar{z}|^b (z - \bar{z})^c (z + \bar{z})^d, \]  \hspace{1cm} (65)

with following definitions

\[ s = -\frac{\alpha'}{2} (k_1 + k_3)^2, \quad t = -\frac{\alpha'}{2} (k_1 + k_2)^2, \quad u = -\frac{\alpha'}{2} (k_2 + k_3)^2. \]

see \footnote{42}
RR and some other open strings. The last term in (67) is exactly the second term of (55), the fourth term in (67) is exactly the last term of (55). Note also that if we add the third and fourth term of (55) and use the momentum conservation along the world volume of brane ($\alpha'(k_1 + k_2 + k_3 + p)^a = 0$) then the result is precisely equivalent with the fifth term of (67). Once more by using momentum conservation in world volume direction the first term of (67) is exactly equivalent with the first term in (55).

However the second and the third terms of (67) are extra terms and in particular if we use anti commutator relation of $\gamma$ matrices these two terms can not cancel each other due to the fact that $L'_3$ is different from $L'_1$.

Indeed if we replace $\xi_{2a} \rightarrow k_{2a}$ inside (67) and use momentum conservation along the brane, the first term is automatically zero because

$$k_{2a}k_{3c}k_{1d}\epsilon^{a_0...a_p-2cad} = 0$$

and upon replacing $\xi_{2a} \rightarrow k_{2a}$ inside the fourth/fifth and sixth terms of (67) appropriately we get zero result.

$$-2tu'\xi_{1i}(k_{2a} + k_{1a} + k_{3a})\epsilon^{a_0...a_p-1a} = 0$$

Thus the second and third terms which are extra terms give rise the amplitude not to be gauge invariant unless one finds out some new Bianchi identities. $^{34}$

10.1 $C^{-2}\phi^0A^0T^0$ S-matrix

One can do the same CFT methods to actually derive the entire S-matrix of the above strings in the asymmetric picture. Hence the final answer for the five point world-sheet

$^{34}$The resolution for this problem to get satisfied gauge invariance of the above S-matrix is to write down the third term of the above S-matrix and add it up with the the other terms in $A_2$ also to do the same formalism to the first and second term of (67) to actually get to the so called new identities accordingly as below

$$\xi_{1i}(p^i\epsilon^{a_0...a_p}H_{a_0...a_p} - p_c\epsilon^{a_0...a_p-1}H_{a_0...a_p-1}) = 0$$
$$\xi_{1i}k_{3c}k_{2a}(-p_d\epsilon^{a_0...a_p-3cad}H_{a_0...a_p-3} + p'\epsilon^{a_0...a_p-2ca}H_{a_0...a_p-2}) = 0$$

(68)
amplitude of a closed string RR (in asymmetric picture) with one scalar, a gauge field and a tachyon in the world volume of non-BPS branes of type II string theory with odd parity with the following picture $C^{-2} \phi^0 A^0 T^0$ is

\[ \mathcal{A}^{C^{-2} \phi^0 A^0 T^0} = \mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_3 + \mathcal{A}_4 \]  

(69)

where

\[ \mathcal{A}_1 \sim 2^{3/2} i \xi_{1i} \xi_{2a} k_{3c} k_{2b} L_1 (p^i \text{Tr} (P_- \Phi' (n) M_p \Gamma^{cab}) - k_{1d} \text{Tr} (P_- \Phi' (n) M_p \Gamma^{cabid})) \]

\[ \mathcal{A}_2 \sim 2^{3/2} i \xi_{1i} p L_3' \text{Tr} (P_- \Phi' (n) M_p \gamma^c) \left( 2tk_3 \xi_2 [-k_{3c} - k_{2c}] + 2k_1 \xi_2 u' k_3 c - tu' \xi_2 c \right) \]

\[ \mathcal{A}_3 \sim 2^{3/2} i \xi_{1i} L_3' \text{Tr} (P_- \Phi' (n) M_p \Gamma^{cijd}) \left[ -2k_1 \xi_2 u' k_3 c (k_1 d + k_2 d) + 2tk_3 \xi_2 k_1 (k_3 c + k_2 c) \right] \]

\[ \mathcal{A}_4 \sim 2^{3/2} i \xi_{1i} L_3' u' \xi_2 a \text{Tr} (P_- \Phi' (n) M_p \Gamma^{cai}) (k_3 c + k_1 c + k_2 c) \]

(70)

with \( u' = (-u - \frac{1}{4}) \) and the same \( L_1', L_3' \), where \( L_1' \) has infinite contact interactions and \( L_3' \) has infinite \( t, u' \) scalar-tachyon channel poles accordingly.

The nice thing about this amplitude and in particular about this asymmetric picture is that it automatically does satisfy the Ward identity associated to the gauge field, which means that if we replace \( \xi_{2a} \to k_{2a} \) the whole S-matrix vanishes where the following points are needed. In the first term of \( \mathcal{A}_2 \) one has to apply momentum conservation along the world volume of brane \(-k_{3c} - k_{2c} = p_c + k_{1c} \) and apply the following identity

\[ p_c \epsilon^{a_0 \ldots a_{p-1} c} = 0. \]

We have to apply momentum conservation in \( \mathcal{A}_3 \)'s first term, that is \(-k_{1d} - k_{2d} = p_d + k_{3d} \) and then draw particular attention to the fact that this part of the S-matrix involves \( k_{3d} k_{3c} \epsilon^{a_0 \ldots a_{p-2} d e} C_{a_0 \ldots a_{p-2} i} \) which is zero because of the antisymmetric property of the \( \epsilon \) tensor. Likewise, we need to replace \( k_{3c} + k_{2c} = -p_c - k_{1c} \) and \( k_{1d} k_{1e} \epsilon^{a_0 \ldots a_{p-2} d e} C_{a_0 \ldots a_{p-2} i} = 0 \) and finally apply momentum conservation for the last term and noting to the point that \( p_c \epsilon^{a_0 \ldots a_{p-2} c a} C_{a_0 \ldots a_{p-2} i} \) plays the crucial rule in checking the Ward identity.

The last remark about the asymmetric picture of the S-matrices is that, one finds out all the entire contact interactions of string theory amplitudes, properly. As an instance this amplitude does include several contact interaction terms like the first term and the last terms of \( (70) \) which could be missed in its symmetric picture \( (67) \).
11 Conclusion

We have derived scattering amplitudes of all three, four and five point BPS and non-BPS mixture of a closed string Ramond-Ramond, a scalar field, gauge and tachyons in all their different pictures of both world volume and transverse directions (for general $p, n$ cases) of the type IIA (IIB) String theory.

In particular we have shown that if we carry out the calculations in asymmetric picture of Ramond-Ramond (taking its vertex operator in terms of its potential $C^{-2}$) and scalar field in zero picture then, various equations must be kept fixed, the entire contact interactions can be definitely obtained and most importantly Ward identity associated to the gauge field is satisfied. More accurately, we have also observed by direct calculations on upper half plane that some of the Bianchi identities (that must be held) of BPS branes can not be necessarily applied to non-BPS or non supersymmetric amplitudes, otherwise the whole S-matrix might be vanished. Indeed the in the presence of the scalar field and RR, the terms carrying momentum of RR in transverse directions ($p^i, p^j$) play important rule in the entire form of the S-matrix and one has to keep track of them in five (from the world sheet point of view) point functions.

We expect it to be true for higher point functions of string theory amplitudes and it would be nice to check it. It would also be nice to deal with some other subtleties of the perturbative string theory [31].

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