On the Reduction of CP Violation Phases

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Abstract

We describe a method that is used to reduce, significantly, the number of CP violating complex phases in the Yukawa parameters. With this Reduction of Complex Phases (RCP) we obtain only one CP violating complex phase in the case where the neutrinos have an (effective) $3 \times 3$ Majorana mass matrix. For the See Saw extension of the SM with three righthanded neutrinos, and in connection with CP violation in leptogenesis, we reduce the usual 6 complex phases to only 2.

Understanding the origin of CP violation is one of the outstanding problems of particle physics. In many extensions of the Standard Model (SM), e.g. in the See Saw mechanism and leptogenesis theories in neutrino physics [1] [2] or in supersymmetric theories [3], one has a large number of complex Yukawa coupling parameters which lead to CP violation [4]. However, a large number of (complex) parameters may also obscure some fundamental structure leading to CP violation. In contrast, a specific choice of parameters used to describe CP violation may play an important and crucial rôle in solving the mechanisms involved.

In this letter, we describe a simple method that can reduce, significantly, the number of CP violating complex phases in the Yukawa parameters, and which can be applied to many extensions of the SM and theories. Here, the Reduction of Complex Phases (RCP) is used to minimize the number of complex phases in neutrino physics. In particular, we reduce (by a factor of 3) the number of CP violating complex phases in the case where the neutrinos have an (effective) $3 \times 3$ Majorana mass matrix, and in the case of the See Saw extension of the SM with three righthanded neutrinos. We also study briefly some implications of RCP to CP violation invariants relevant for leptogenesis.

Reduction of Complex Phases

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RCP is based on the following, rather simple, transformation: it is always possible to rotate any pair of complex parameters \((a + ix, b + iy)\) with an orthogonal matrix such that one (say the first parameter) becomes real, i.e.

\[
\begin{pmatrix}
  a + ix \\
  b + iy
\end{pmatrix}
\rightarrow
\begin{pmatrix}
  a' \\
  b' + iy'
\end{pmatrix}
= \begin{pmatrix}
  \cos(\theta) & -\sin(\theta) \\
  \sin(\theta) & \cos(\theta)
\end{pmatrix}
\cdot
\begin{pmatrix}
  a + ix \\
  b + iy
\end{pmatrix}
\quad (1)
\]

Clearly, the first parameter will have no imaginary part if one chooses \(\tan(\theta) = x/y\).

Next, we prove that any Hermitian matrix \(H\) can be written as

\[
H = (OK_\alpha \hat{O})^\dagger \cdot D \cdot OK_\alpha \hat{O}
\quad (2)
\]

where \(D\) is a real diagonal matrix, \(O, \hat{O}\) are orthogonal matrices and \(K_\alpha\) is a diagonal unitary matrix with only one complex phase, \(K_\alpha = \text{diag} \left[1, 1, e^{i\alpha}\right]\).

Taking a general Hermitian matrix

\[
H = \begin{bmatrix}
  r_1 & c_4 & c_6 \\
  c_4^* & r_2 & c_5 \\
  c_6^* & c_5^* & r_3
\end{bmatrix}
\quad (3)
\]

where the \(r_i\) are real and the \(c_i\) are complex entries and rotating the 2-3 position with and orthogonal matrix \(O_{23}\), i.e.

\[
H \rightarrow H' = O^T_{23} \cdot H \cdot O_{23}
\quad (4)
\]

where

\[
O_{23} = \begin{bmatrix}
  1 & 0 & 0 \\
  0 & \cos(\theta_{23}) & \sin(\theta_{23}) \\
  0 & -\sin(\theta_{23}) & \cos(\theta_{23})
\end{bmatrix}
\]

and applying the RCP method described in Eq. (1), one obtains \(c_4 \rightarrow c'_4 = r_4\) real. Then, by rotating the 1-2 position, we find \(c_6 = r_6\) real, and subsequently by the rotating again the 2-3 position we can even have \(r_6 = 0\). Thus, any
Hermitian matrix can be written in the following way:

$$H = \hat{O}^T \cdot \begin{bmatrix} r_1 & r_4 & 0 \\ r_4 & r_2 & r_5 e^{i\alpha} \\ 0 & r_5 e^{-i\alpha} & r_3 \end{bmatrix} \cdot \hat{O}$$  \hspace{1cm} (5)$$

where $\hat{O}$ is some general (appropriate) orthogonal matrix. Finally, one concludes:

$$H = \hat{O}^T \cdot K^* \cdot \begin{bmatrix} r_1 & r_4 & 0 \\ r_4 & r_2 & r_5 \\ 0 & r_5 & r_3 \end{bmatrix} \cdot K_\alpha \cdot \hat{O} = (OK_\alpha \hat{O})^\dagger \cdot D \cdot OK_\alpha \hat{O}$$  \hspace{1cm} (6)$$

where $O$ is just another orthogonal matrix, $D$ real diagonal and $K_\alpha = \text{diag} [1, 1, e^{i\alpha}]$. Furthermore, taking into account that any orthogonal matrix results from three rotations $O = O_{12}O_{23}O_{12}$, and that any $O_{12}$ commutes with $K_\alpha$, one finds that the combination $OK_\alpha \hat{O}$ is a product of just 5 different rotations and the diagonal unitary matrix $K_\alpha$ with only one complex phase:

$$OK_\alpha \hat{O} = O_{12}O_{23}O_{12}' K_\alpha \hat{O}_{23} \hat{O}_{12}$$  \hspace{1cm} (7)$$

Thus, any Hermitian matrix is a combination of its 3 eigenvalues, 5 rotations and only one complex phase. This gives a total of 9 parameters, which corresponds exactly with the same number of parameters as in the general case for any Hermitian matrix (as in Eq. (3)), where we have 6 real parameters and 3 complex phases.

We shall now apply the reduction of complex phases to several cases and scenarios.

1 - Majorana neutrinos

Consider the case where we have (only) 3 lefthanded Majorana type neutrinos with some Majorana mass matrix. This may also be an effective Majorana mass matrix. Without loss of generality, one can find a weak-basis where the the Majorana neutrino mass matrix is real and diagonal while the charged lepton mass matrix $M_e$ is Hermitic. Then, applying RCP in this weak-basis, one obtains

$$M_\nu = D_\nu \equiv \text{diag} [m_{\nu_1}, m_{\nu_2}, m_{\nu_3}] ; \quad M_e = U^\dagger \cdot D \cdot U$$  \hspace{1cm} (8)$$
where \( U = O K \hat{O} \), the combination described in Eq. (7), is a unitary matrix with only one complex phase and five angle rotations:

\[
U = O_{12} O_{23} O'_{12} K_\alpha \hat{O}_{23} \hat{O}_{12} : K_\alpha = \text{diag} [1, 1, e^{i\alpha}] \quad (9)
\]

\( U \) corresponds, of course, to the neutrino mixing matrix which, for the case of Majorana neutrinos, will have \( 3 + 1 + 2 = 6 \) physical parameters: 3 angles, one CP violating phase of the CKM-type and 2 CP violating complex phases of the Majorana type. In order to find these, we write \( U \) also in the following combination:

\[
U = K_\alpha \cdot V \cdot K_M \quad (10)
\]

where \( K_\alpha = \text{diag} [e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}], K_M = \text{diag} [1, e^{i\alpha M}, e^{i\beta M}] \) and \( V \) is a (CKM-type) unitary matrix where the first line and row are real. As can be seen, by substituting this combination for \( U \) in \( M_e \) of Eq. (8), the diagonal unitary matrix \( K_\alpha \) is unphysical and does not contribute to \( M_e \). The two physical unitary matrices are \( V \) and \( K_M \). The matrix \( K_M = \text{diag} [1, e^{i\alpha M}, e^{i\beta M}] \) contains the two Majorana type CP violating complex phases \((\alpha_M, \beta_M)\), and \( V \) contains the CKM-type CP violating phase \( \delta_{KM} \). All 3 phases \((\alpha_M, \beta_M, \delta_{KM})\) are functions of the initial 5 angles \((\theta_{12}, \theta_{23}, \theta'_{12}, \hat{\theta}_{23}, \hat{\theta}_{12})\) contained in \( O_{12}, O_{23}, O'_{12}, \hat{O}_{23}, \hat{O}_{12} \), and the unique complex phase \( \alpha \) contained in \( K_\alpha \). However, comparing the two formulas for \( U \) in Eq. (9) and Eq. (10), it is quite clear that all CP violation, be it of the CKM-type or the Majorana type, is controlled by the one single complex phase \( \alpha \) and that \((\alpha_M, \beta_M, \delta_{KM}) = 0 \) if \( \alpha = 0 \). Furthermore, \( \delta_{KM} = 0 \) if \( \theta_{23} = 0 \) or \( \hat{\theta}_{23} = 0 \). In this case, the matrix \( K \) commutes with the remaining matrices \( O_{12}, O'_{12} \) or \( \hat{O}_{12} \). For the Majorana type CP violating phases, we obtain \((\alpha_M, \beta_M) = 0 \) if \( \theta_{23} = 0 \) or if \( \theta_{12} = 0 \). We find also \((\alpha_M, \beta_M) = (0, \alpha) \) if \( \theta_{23} = 0 \) (for \( \theta_{12} \) and \( \theta_{23} \neq 0 \)). Explicitly, one has

\[
\begin{align*}
\tan(\delta_{KM}) &= \sin(\alpha) \sin(\theta_{23}) \sin(\hat{\theta}_{23}) \ A_1 \\
\tan(\alpha_M) &= \sin(\alpha) \sin(\theta_{12}) \sin(\theta_{23}) \sin(\hat{\theta}_{23}) \ A_2 \\
\tan(\beta_M) &= \tan(\alpha) \sin(\theta_{12}) \sin(\theta_{23}) \ A_3
\end{align*}
\quad (11)
\]

where \( A_1, A_2 \) and \( A_3 \) are functions of sinuses and cosinuses of \((\alpha, \theta_{12}, \theta_{23}, \theta'_{12}, \hat{\theta}_{23}, \hat{\theta}_{12})\); \( A_1 \) is well defined if \( \alpha \) or \( \theta_{23} \) or \( \hat{\theta}_{23} \) are zero, \( A_2 \) is also well defined if \( \alpha \) or \( \theta_{12} \) or \( \theta_{23} \) or \( \hat{\theta}_{23} \) are zero, and the same applies to \( A_3 \) if \( \alpha \) or \( \theta_{12} \) or \( \theta_{23} \) are zero, furthermore, \( A_3 = 1/ \sin(\theta_{12}) \sin(\theta_{23}) \) if \( \theta_{23} = 0 \) (for \( \theta_{12} \) and \( \theta_{23} \neq 0 \)). A special case occurs for \( \delta_{KM} \) when \((\theta_{12}, \theta'_{12}) = 0 \). It can be readily verified from Eq. (9) that, in this case, \( U_{13} = 0 \) and that \( A_1 = 0 \); thus, \( \delta_{KM} = 0 \).
2 - See Saw

Let us now consider the see saw scenario with 3 lefthanded and 3 righthanded neutrinos, and where the neutrinos have only a Dirac mass matrix $M_D$ and a righthanded Majorana mass matrix $M_R$. As argued in Ref. [5], one can always find a weak-basis where the righthanded Majorana neutrino mass matrix and charged lepton mass matrix are diagonal and real, while the Dirac neutrino mass matrix $M_D$ has 6 complex phases, i.e.

$$M_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & D_R \end{pmatrix} ; \quad M_e = D_e$$  \hspace{1cm} (12)

where $D_R \equiv \text{diag}[M_1, M_2, M_3] = M_R$, $D_e \equiv \text{diag}[m_e, m_\mu, m_\tau]$ and the $(M_D)_{i1}$ are real. Thus, we have a total of $3+3+9+6=21$ parameters.

Next, we apply RCP to the see saw mechanism. To do this, one must first consider a different weak-basis from the one in Eq. (12). Transforming (only) the lefthanded neutrinos with a unitary matrix $U$:

$$\nu_L_i \rightarrow U_{ij} \nu_L_j$$  \hspace{1cm} (13)

and keeping the charged lepton mass matrix Hermitic, Eq. (12) becomes

$$M_\nu = \begin{pmatrix} 0 & U^T M_D \\ M_D^T U & D_R \end{pmatrix} ; \quad M_e = U^\dagger D_e U$$  \hspace{1cm} (14)

This structure for the neutrinos as well as for the charged leptons has the same physical content as the one in Eq. (12). However, we may now choose $U$ such that the new Dirac neutrino mass matrix $U^T M_D$ is also Hermitic\footnote{This is clear. Every matrix $M_D$ can be diagonalized by a biunitary transformation: $W M_D V = D$. Thus, $M_D = W^\dagger D V^\dagger$ and we find that by multiplying $M_D$ on the left by a suitable unitary matrix $(VW)$ $M_D = V D V^\dagger$, we obtain an Hermitic matrix.}. Thus, there exists a weak-basis where both the Dirac neutrino and charged lepton mass matrices $M_D$ and $M_e$ are Hermitic, while the (righthanded) Majorana neutrino mass matrix $M_R = D_R$ is diagonal.

We are now ready to apply the reduction of complex phases, and may conclude,
as in Eq. (2), that there is a weakbasis where

\[ M_D = (O_D K_D \hat{O}_D) \dagger D O_D K_D \hat{O}_D \]
\[ M_e = (O_e K_e \hat{O}_e) \dagger D_e O_e K_e \hat{O}_e \]
\[ M_R = D_R \]

where the \( O \)'s are orthogonal, the \( D \)'s are real and diagonal and the \( K \)'s are unitary diagonal matrices with, each, only one complex phase. By transforming again the lefthanded neutrinos, as in Eqs. (13, 14), one can even find a weakbasis where \( M_D \) is somewhat simplified:

\[ M_D = D O_D K_D \hat{O}_D \]
\[ M_e = (O_e K_e \hat{O}_e) \dagger D_e O_e K_e \hat{O}_e \]
\[ M_R = D_R \]

(15)

and where the \( O_e \)'s and \( K_e \)'s have been redefined.

Thus, we find, in addition to the result obtained in [5], a weak-basis where the total number of complex phases in the see-saw model is (not 6) but only 2. However, it must be kept in mind that the total number of parameters is still the same as in [5], i.e. 3 masses, 5 angles and 1 phase in \( M_D \) and \( M_e \), and 3 masses in \( M_R \): \( 2 \times (3 + 5 + 1) + 3 = 21 \).

With respect to CP violation, it is clear that all weak-basis invariants measuring CP violation, dependent on the lepton mass matrices, will be zero if both complex phases \( \alpha_D, \alpha_e \) in \( K_D \) and \( K_e \) are zero. E.g.

\[ I_\nu = Tr[M_D^\dagger M_D, M_R^\dagger M_R]^3 = 0 \quad if \quad \alpha_D = 0 \]
\[ I_{e\nu} = Tr[M_D^T M_D, M_e^T M_e]^3 = 0 \quad if \quad \alpha_e = 0 \]

(16)

Of course, \( (I_\nu, I_{e\nu}) = 0 \) does not imply that only \( (\alpha_D, \alpha_e) = 0 \). To study under which conditions \( (I_\nu, I_{e\nu}) = 0 \), it is useful to write \( O_D K_D \hat{O}_D \) in \( M_D \) and \( O_e K_e \hat{O}_e \) in \( M_e \) as in Eq. (7):

\[ O_D K_D \hat{O}_D = O_{12}^D O_{23}^D O_{12}^{D'} K_D \hat{O}_{23}^D \hat{O}_{12}^D \]
\[ O_e K_e \hat{O}_e = O_{12}^e O_{23}^e O_{12}^{e'} K_e \hat{O}_{23}^e \hat{O}_{12}^e \]

(17)
Clearly, as explained in the previous paragraph we find also

\[
I_\nu = 0 \text{ if } \alpha_D = 0 \text{ or } \theta_{23}^D = 0 \text{ or } \hat{\theta}_{23}^D = 0 \text{ or } (\theta_{12}^D, \theta_{12}^{D'}) = 0 \\
I_{e\nu} = 0 \text{ if } \alpha_e = 0 \text{ or } \theta_{23}^e = 0 \text{ or } \hat{\theta}_{23}^e = 0 \text{ or } (\theta_{12}^e, \theta_{12}^{e'}) = 0
\]  \hspace{1cm} (18)

3 - Leptogenesis

Next we consider, as in Ref. [5], another class of weak-basis invariants which measure CP-violation and are also relevant for leptogenesis:

\[
I_1 = \text{ImTr}[h_D H_R M_R^* h_D^* M_R] \\
I_2 = \text{ImTr}[h_D H_R^2 M_R^* h_D^* M_R] \\
I_3 = \text{ImTr}[h_D H_R^2 M_R^* h_D^* M_R H_R]
\]  \hspace{1cm} (19)

where \( h_D = M_D^\dagger M_D \) and \( H_R = M_R^\dagger M_R \). Using the parametrization defined in Eqs. (15, 17), one obtains the following relations for these weak-basis invariants

\[
I_i = \sin(\alpha_D) \sin(\theta_{23}^D) \, B_i
\]  \hspace{1cm} (20)

where the \( B_i \) are polynomials in sinus and cosinus of \( \theta_{ij}, \theta_{12}^D, \theta_{23}^D \) and \( \alpha_D \) and the eigenvalues of \( M_D \) and \( M_R \). Clearly, \( I_i = 0 \) if \( \alpha_D = 0 \) or \( \theta_{23}^D = 0 \).

4 - CP violation and low energy effective theory.

Our parametrization is useful to study all kinds of conditions under which CP-violation related to leptogenesis will occur. E.g., it can be verified that the invariants \( I_i \neq 0 \), (i.e. CP-violation will still occur) even when all forms CP-violation are absent in the approximate effective low energy theory with an effective neutrino mass matrix

\[
m_{e\text{ff}} = -M_D \, M_R^{-1} \, M_D^T
\]  \hspace{1cm} (21)

Thus, the two effective low energy Majorana phases and CKM-type phase, resulting from an (appropriate) effective neutrino mass matrix and the charged leptons mass matrix, can be zero, while the (total) high energy theory violates CP[6]. In order to see this, take, in the parametrization of Eqs. (15, 17) for \( M_D \) and \( M_e \), all 1-2 angles zero. This will guarantee a zero CKM-type phase, for the effective low energy theory. In addition, for this simple case, we construct a class of matrices (resulting in \( I_i \neq 0 \) but) with no CP violation Majorana phases for the approximate effective low energy theory. This class depends on
two parameters $\theta, \alpha$:

\[
M_D = D F^T K_\beta \\
M_e = K_\alpha F^T D_e F K_\alpha^* ; \quad F = \begin{bmatrix} 1 & 0 & 0 \\
0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
0 & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}
\]

(22)

where $D = m \text{ diag}[\lambda, \sin(\theta), \cos(\theta)]$, $D_R = M \text{ diag}[\mu, b^+, b^-]$, with $b^\pm = (1 \pm \sin(2\theta) \cos(\alpha))$ and $\lambda, \mu, m$ real. $K_\alpha = \text{ diag}[1, 1, e^{i\alpha}]$, $K_\beta = \text{ diag}[1, 1, ie^{i\beta}]$ with $\tan(\beta) = -\tan(2\theta) \sin(\alpha)$. $D_e$ may be any real diagonal matrix. For this specific choice of parametrization for the neutrino Dirac, Majorana and charged lepton mass matrices and in a weak-basis where the charged lepton mass matrix is diagonal, the low energy effective neutrino mass matrix has the special form,

\[
m_{\text{eff}} = -\tilde{M}_D D_R^{-1} \tilde{M}_D^T = \begin{bmatrix} r & 0 & 0 \\
0 & 0 & c \\
0 & c & c' \end{bmatrix}
\]

(23)

where $\tilde{M}_D = F K_\alpha D F^T K_\beta$ and where $r$ is real and the $c$'s are complex. As a result (note that $(m_{\text{eff}})_{22} = 0$) all the phases in the $c$’s are irrelevant and can be absorbed through (or into) the righthanded charged lepton fields. Thus, not only the CKM-type phase but also the Majorana phases are zero and there is no form of CP-violation in the low energy theory. However, for $M_D^T M_D$ we obtain

\[
M_D^T M_D = \frac{m^2}{2} \begin{bmatrix} 2\lambda^2 & 0 & 0 \\
0 & 1 & \cos(2\theta) e^{i\beta} \\
0 & \cos(2\theta) e^{-i\beta} & 1 \end{bmatrix}
\]

(24)

where, remember, $\tan(\beta) = -\tan(2\theta) \sin(\alpha)$. Therefore, in contrast with the approximate effective low energy theory, CP-violation does indeed occur for the (total) high energy theory and the CP violation leptogenesis invariants $I_i$,
computed with Eq. (19), are different from zero:

\[ I_1 = \frac{m^4 M^4 \sin^2(2\theta) \cos^2(\alpha)-1}{\tan^2(2\theta) \sin^2(\alpha)+1} \sin(2\theta) \sin(4\theta) \sin(2\alpha) \]

\[ I_2 = m^4 M^6 \frac{\sin^4(2\theta) \cos^4(\alpha)-1}{\tan^4(2\theta) \sin^4(\alpha)+1} \sin(2\theta) \sin(4\theta) \sin(2\alpha) \] (25)

\[ I_3 = \frac{m^4 M^8 \frac{\sin(2\theta) \cos(\alpha)-1}{\tan(2\theta) \sin(\alpha)+1}^3}{\tan^8(2\theta) \sin^8(\alpha)+1} \sin(2\theta) \sin(4\theta) \sin(2\alpha) \]

Conclusions

Using RCP we reduce, significantly, the number of CP violating complex phases in the Yukawa parameters in two important cases. In the case of an (effective) 3 × 3 Majorana neutrino mass matrix, we obtain only one complex CP violating phase, in contrast with the general structure, where there is one CKM-type and 2 Majorana type complex phases. For the See Saw extension of the SM with three righthanded neutrinos, and in connection with CP violation in leptogenesis, we reduce the usual 6 complex phases to only 2. We also show that RCP is useful in the analysis of some specific scenarios. In particular, we give an example where, in the case of the see saw mechanism, the (total) high energy theory violates CP even when all forms CP-violation are absent in the approximate effective low energy theory.

References

[1] M. Fukugita and T. Yanagida, Phys. Lett. B 174 (1986) 45; for a review and references see W. Buchmüller and M. Plümacher, Int. J. Mod. Phys. A 15 (2000) 5047.

[2] R. N. Mohapatra and X. Zhang, Phys. Rev. D 46 (1992) 5331; C. W. Chiang, Phys. Rev. D 63 (2001) 076009 [hep-ph/0011193]; A. S. Joshipura, E. A. Paschos and W. Rodejohann, JHEP 0108 (2001) 029 [hep-ph/0105173]; F. Buccella, D. Falcone and F. Tramontano, Phys. Lett. B 524 (2002) 241 [hep-ph/0108172]; W. Buchmüller and D. Wyler, Phys. Lett. B 521 (2001) 291 [hep-ph/0108216]; M. S. Berger and K. Siyeon, Phys. Rev. D 65 (2002) 053019 [hep-ph/0110001]; H. B. Nielsen and Y. Takanishi, Nucl. Phys. B 636 (2002) 305 [hep-ph/0204027]; J. Ellis and M. Raidal, hep-ph/0206173; Z. z. Xing, hep-ph/0206245; J. Ellis, M. Raidal and T. Yanagida, hep-ph/0206300; S. Davidson and A. Ibarra, hep-ph/0206304; W. Rodejohann, hep-ph/0207053; G.C. Branco, R. Gonzalez Felipe, F.R. Joaquim, I. Masina, M.N. Rebelo, C.A. Savoy, hep-ph/0211001.

[3] See for example T. Dent and J.I. Silva-Marcos, hep-ph/0206086; T. Ibrahim P. Nath, hep-ph/0210251; A.M. Teixeira, G.C. Branco, M.E. Gomez S. Khalil,
[4] See e.g. M. Frigerio, A.Yu. Smirnov, Nucl. Phys. B 640 (2002) 233 [hep-ph/0202247]; I.I. Bigi, hep-ph/0206261; A. Arhrib, W.-S. Hou, hep-ph/0211267; J.W.F. Valle, hep-ph/0209047; G.C. Branco, M.N. Rebelo and L. Lavoura, Phys. Lett. B 180 (1986) 264.

[5] G.C. Branco, T. Morozumi, B.M. Nobre and M.N. Rebelo, Nucl. Phys. B 617 (2001) 475 [hep-ph/0107164].

[6] M.N. Rebelo, hep-ph/0207236.