Deformation and strength of inclined RC isolated columns using experimental and three-dimensional finite element analyses

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Abstract: This paper shows how the inclination angle affects the stiffness and strength of RC columns. Experimental work of two scaled-down vertical columns with a length of 1000 and 1250mm was achieved to provide data for subsequent validation of analytical and numerical finite element solutions. The analytical solution is based on current design assumptions while the numerical solution adopts a sophisticated FE simulation with three-dimensional elements for the concrete mass and link element for the rebars. The rebars are assumed to be fully bonded in the FE model. Hooke’s law and Damaged Plasticity models are respectively used to simulate the elastic and inelastic behavior of the concrete, whereas the elastic perfectly plastic model is proposed for the rebars’ behavior. Subsequently, the validated FE model is used to investigate the response of two inclined columns with a length of 1000mm and 1250mm. Three inclination angles of $5^\circ$, $7.5^\circ$ and $10^\circ$ are considered for each length. FE results indicate that column inclination reduces its axial stiffness and strength. A strength reduction ratio of about 0.8\% up to 3.4\% is noted for inclination angles of $5^\circ$ to $10^\circ$. It is also noted that the longer column is more sensitive to the inclination angles.

1. Introduction

Even though there are no specific criteria for the inclined columns, they have an increased application in high-rise constructions for the last few years. For example, Capital Gate in Abu Dhabi and Evolution Tower in Moscow were designed using inclined columns in their structures. In addition to aesthetical aspects, the inclined columns are useful for planning and functional design aspects.

At the time being, there are few studies about the structural behavior of the inclined columns. The inclination may produce additional eccentricity in comparing with the vertical column, cause a horizontal component for the applied loads, and result in different behavior for slender columns.

Jenkins and Ryan \cite{1} determined the procedure to prove that the short-term behavior of slender columns is better understood than long-term behavior. Design methods were developed primarily from the results of experimental tests.

Chang \cite{2} used an asymmetrically malformation conical shell with rigid edges to represent the inclined columns. The adopted variational formulation was integrated with Lagrange’s multiplier to account for the boundary conditions and to improve a set of non-linear equations. The exact solutions for these equations were gained for inclined columns with various end conditions. He found that the effective length concept, which has been mainly developed for the straight columns, is not clear when applied for the inclined slender columns.
Allouzi [3] developed numerical non-linear FE models to imitate the performance of RC slender columns. FE models were calibrated based on the experimental work of Jenkins and Ryan [1] to serve a further study of the slenderness aspects. The analytical second-order analysis was performed based on the force equilibrium in the malformed shape. She concluded that the effective flexural rigidity of inclined RC columns depends on the slenderness ratio at the buckling threshold.

In contrast to Allouzi [3] work, this paper aims to study how the inclination angle can affect the stiffness and strength of circular columns in general. The vertical loads are applied to the cross-sectional area of the inclined columns instead of the horizontal projection in the Allouzi [3] work. In the author’s opinion, this simulation is more representative of the practical cases.

Two scaled-down vertical reinforced concrete columns were tested and their load-deformation results are used to develop an analytical solution and to validate a sophisticated finite element simulation. The validate finite element model is adopted to achieve the analysis for two circular inclined columns with a diameter of 100mm and lengths of 1000mm and 1250mm. Three inclination angles 5°, 7.5° and 10° are considered for each length.

ABAQUS CAE/2016 software with Concrete Damaged Plasticity, CDP, model is used in the finite element simulation. Parameters of CDP were determined based on the experimental results of this work with the aid of the recommendation of Kmiecik and Kaminski [4]. Tensile strength and deformations of concrete are simulated based on extensive tests of Belarbi [5]. The flow chart presented in Figure 1 illustrates the steps adopted in this study.

![Figure 1. Plan flow chart of the study.](image)

2. Experimental Work

Two vertical scaled-down reinforced concrete column models are tested to provide a basis for subsequent validation for the finite element modeling. Both models are tied and subjected to a vertical load. This section gives details of the experimental work including the material properties, mixes, casting, curing, attaching measurement devices, and the test procedure.

2.1 Description of Specimens

Two reinforced scale-down columns were designed with a circular section of 100 mm diameter, 6 mm diameter rebars for longitudinal reinforcement, and 4 mm diameter ties. The scale-down and rebar sizes have been selected based on the capacity of the loading frame and the available materials. A
A concrete cover of 10mm was provided on all sides. The designation names indicated in Table 1 are used to describe the columns in the subsequent work.

| Column Code | Meaning |
|-------------|---------|
| Vr-1000     | Vr: Vertical column 1000: column length (mm) |
| Vr-1250     | Vr: Vertical column 1250: column length (mm) |

Length of 1000mm and 1250mm was used respectively for the first and second columns to provide slenderness ratios of:

\[
\left( \frac{kl}{r} \right)_{Vr-1000} = \frac{1.0 \times 1000}{0.25 \times 100} = 40, \quad \left( \frac{kl}{r} \right)_{Vr-1250} = \frac{1.0 \times 1250}{0.25 \times 100} = 50
\]

Details for columns model Vr-1000 is presented in Figure 2.

2.2 Material Properties
A concrete mix of (1:2.4:1.9) was adopted to cast the columns and cylinder specimens with \( f_c = 25 \text{MPa} \).
2.3. Reinforcement Cage
The fabrication of steel cages for columns is presented in Figure 3.

(a) Plan view. (b) Longitudinal view.

Figure 3. Fabrication cage of the reinforcement.

2.4. Casting and Curing
A six 150×300 mm concrete cylinders were cured after 12 hours for 28 days (according to ASTM C31/C31M-12) and tested to get the concrete compressive strength (according to ASTM C39/C39M) and the tensile spilling strength (according to ASTM C496/C496M-11) see Figure 4. By following the formula of splitting tensile strength

\[ f_s = \frac{2P}{\pi DL} \]  

where:

- \( f_s \): Splitting tensile strength (MPa)
- \( P \): Maximum applied load (N)
- \( D \): Diameter of the cylinder (mm)
- \( L \): Length of the cylinder (mm)

(a) Compressive test. (b) Splitting tensile test

Figure 4. Specimens for compressive and tensile tests of concrete.
2.5. **RC column Specimens Test**

A compression system with a capacity of 1300 kN used to apply a monotonic compression load to the column specimens. One dial gauge on the top of the specimen was used to measure the longitudinal deflection and tow LVDT (one at the middle height and the other at the top of the specimen) were used to measure the lateral deflection.

Strain gauges type (FLAB-5-11-3LJC-F), which installed on the rebar, measured the strain of steel while strain gauges type (PL-60-11-3LJC-F), which installed on the surface of the specimen, measured the strain of concrete. All strain gauges and LVDT were connected to the strain indicator which connected to the computer to show all results during the test as indicated in Figure 5. All specimens tested under a constant loading step of (6 kN/second.)

![Compression system](image)

![Strain indicator](image)

**Figure 5.** Tests system and connecting tools

The two columns have been tested until reaching the failure load, and all strain reading was computerized. The load gauge and dial gauge reads were documented. Shapes of failure are shown in Figure 6.

![Failure of (Vr-1250)](image)

![Failure of (Vr-10000)](image)

**Figure 6.** Shape of failure in columns.
3. Analytical $P - \Delta$ Curve

This section presents a traditional analytical solution to determine the deformation, strength, and $P - \Delta$ curve for an axially loaded vertical column. This analysis is used in subsequent sections to check the accuracy and integrity of the numerical model.

The traditional kinematic assumption of a plane section before loading remains plane after loading is adopted in the analytical solution to determine the strain distribution. Todeschini stress-strain curve presented in Figure 7 was used to deduce the stress distribution from the corresponding strain distribution. According to Wight [6], the reduced compressive strength, $f''_c$, has been used in the Todeschini model to reflect the difference in loading rate between the specimen and the structural member.

$$f_c = \frac{2f_c'' \left( \frac{\varepsilon}{\varepsilon_0} \right)}{1 + \left( \frac{\varepsilon}{\varepsilon_0} \right)^2}$$  \hspace{1cm} (2)

Regarding steel, the stress is related to strain based on the elastic-perfectly plastic assumption.

$$f_s = \begin{cases} E_s \varepsilon & \text{if } f_s \leq f_y \\ f_y & \text{if } f_s > f_y \end{cases}$$  \hspace{1cm} (3)

The equilibrium equation is applied to determine the total applied load, $P$, from the corresponding stresses.

$$P = f_c \left( A_g - A_n \right) + f_s A_n$$  \hspace{1cm} (4)

where $A_g$ is the gross area of the column section and $A_n$ is the total area of the longitudinal rebars.

Finally, axial deformation, $\Delta$, is related to the strain according to the following relation.

$$\Delta = \varepsilon L$$  \hspace{1cm} (5)

where $L$ is the total length or height of the column.

![Figure 7. Todeschini analytical approximations to the compressive stress-strain curve for concrete. The $P - \Delta$ curves for Vr-1000 and Vr-1250 specimens are presented in Figure 8 and Figure 9.](image)
4. **Finite Element Modeling**

The three-dimensional finite element model is used to simulate the vertical columns of the experimental work for this study. After validation of the finite element model, it is used in the subsequent case studies of the inclined column presented in (Section 5 and 6).
Ten-node tetrahedron elements and truss elements indicate in Figure 10 are used respectively to discretize the concrete mass and rebars (longitudinal and ties). With the truss element, the dowel action of the longitudinal rebars is implicitly neglected. A mesh size of 25mm has been adopted for the whole model as presented in Figure 11.

In the finite element model, the rebars have been assumed to be perfectly bonded to the surrounding concrete mass. In columns that are dominantly subjected to compressive forces, this assumption seems justified [7]. From a finite element point of view, a tie constraint has been used to achieve this perfect bond [8].

As discussed in (Section 1.2), all concretes have cylinder compressive strength of 25 MPa. Concrete elastic modulus, $E_c$, was determined according to [9]. Poisson’s ratio of 0.2 was used [7].

For the compression behavior of concrete, the stress-strain relation of equation (2) has been adopted. The hardening part where the total stress is related to the plastic strain is generated based on the following relation that implicitly assumes the unloading path has a stiffness equal to the initial stiffness.

$$
\varepsilon_{Plastic} = \varepsilon_{Total} - \varepsilon_{Elastic} = \varepsilon_{Total} - \frac{f_c}{E_c}
$$

(6)

The following stress-strain relation proposed by [1] has been used to simulate the tensile behavior of the concrete.

$$
f_t = f_{ct} \left( \frac{\varepsilon_{ct}}{\varepsilon} \right)^{0.4}
$$

(7)

where $f_t$ is the tensile stress of concrete, $\varepsilon_{ct}$ is the corresponding concrete tensile strain, $f_{ct}$ is the modulus of rapture that has been determined based on the formula of [2], and $\varepsilon_{ct}$ is the concrete cracking strain of $80 \times 10^{-6}$.
For the plasticity aspects, Concrete Damaged Plasticity, CDP, has been used. As discussed below, it is based on parameters with an explicit physical interpretation.

It is a modified of the Drucker–Prager strength model. Its failure surface can be adjusted using the parameter $K_c$ as indicated in Figure 12. Physically, this parameter can be interpreted as a ratio of the distances between the hydrostatic axis with the meridians of tension and compression in the deviatoric cross-section. Its value is usually larger than 0.5. The deviatoric cross-section of the failure surface would be circular when a value of 1 is assumed. In this study, a value of $2/3$ has been assumed according to experimental results [11]. With this value, the shape is a combination of three mutually tangent ellipses of [10].

![Figure 12. Deviatoric cross-section of failure surface in the CDP model.](image)

In the CDP, the plane meridians are set to be hyperbolic in shape to match the experimental results. The shape can be modified using eccentricity (plastic potential eccentricity) that has a small positive value to express the approaching rate for the plastic potential hyperbola to its asymptote. The potential eccentricity is a length taken along the hydrostatic axis between the vertex of the hyperbola and the intersection of the asymptotes. It can be determined as a ratio of tensile strength to compressive strength. A value of $\varepsilon = 0.1$ has been recommended by the CDP for the potential eccentricity in compare with the classic Drucker-Prager hypothesis that using a straight-line meridional plane of $\varepsilon = 0$, see Figure 13 below.

The ratio $\sigma_{bb} / \sigma_{bc} (f_{bb} / f_{bc})$ has to be defined to describe the state of when the concrete experiences failure under biaxial compression. Based on the approximation of Kupler's (1969) experimental results with an elliptic equation, a value of 1.16 has been concluded for the ration of $f_{bb} / f_{bc}$, see Figure 14. Finally, the internal friction of concrete has been simulated through using a dilation angle, $\psi$, of 36° to 40°.

![Figure 13. Hyperbolic surface of plastic potential in meridional plane 0.](image)
In summary, the parameters of the CDP model have been presented in Table 2.

Table 2. Adopted parameters of the CDP model under compound stress.

| Parameter name   | Value  |
|------------------|--------|
| Dilatation angle | 36°    |
| Eccentricity     | 0.1    |
| $f_{b0} / f_{c0}$| 1.16   |
| $K$              | 0.667  |
| Viscosity        | 0      |

It is common to use a viscoplastic regularization of the concrete damaged plasticity model to overcome some of the convergence difficulties for material models exhibiting softening behavior and stiffness degradation by permitting stresses to be outside of the yield surface. Use a generalization of the Duvaut-Lions regularization, the viscoplastic strain rate tensor, $\dot{\varepsilon}^{pl}_v$, presented below:

$$\dot{\varepsilon}^{pl}_v = \frac{1}{\mu} \left( \varepsilon^{pl}_v - \varepsilon^{pl}_o \right)$$  (8)

where $\mu$ is the viscosity parameter representing the relaxation time of the viscoplastic system, and $\varepsilon^{pl}$ is the plastic strain evaluated in the inviscid backbone model.

In the same manner, a viscous stiffness degradation variable, $d^v_v$, for the viscoplastic system is defined as:

$$d^v_v = \frac{1}{\mu} \left( d - d^v_v \right)$$  (9)

where $d$ is the degradation variable evaluated in the inviscid backbone model.

Finally, the stress-strain relation of the viscoplastic model can be expressed as:

$$\sigma = (1-d_o) D^{el}_o \left( \varepsilon - \varepsilon^{pl}_o \right)$$  (10)

With a small value for the viscosity parameter, the viscoplastic regularization can help to enhance the rate of convergence in the softening region. As $t / \mu \to \infty$, where $t$ represents time, the solution of the viscoplastic system will relax to that of the inviscid system. In this study, it has been found that the default zero value of the viscosity parameter is sufficient to simulate the load-deformation curves.
Hinge boundary conditions that restrained all translation degrees of freedom have been assigned to the lower end of the columns. On the upper end, lateral translation degrees of freedom were restrained. Loads were applied as a pressure act on the upper face of the columns. Finally, a Newton-Raphson static solver has been used to solve the resulting simultaneous equilibrium equations.

5. Validation
In this section, the results of experimental work for (Section 2) have been used to assess and validate the output for analytical and numerical solutions of (Section 3 and 4). Load-deflection curves for different solution approaches are presented in Figure 15 and Figure 16. These curves show that the analytical and finite element solutions are in good agreement with the experimental work. In spite of finite element accuracy in simulation of the elastic and inelastic parts, it is incapable to trace the softening region of the curve. This may be explained in terms of the solver that implicit in nature. Due to its generality and applicability for different boundary conditions, the finite element model instead of the analytical model is used in the subsequent analysis for the inclined columns of (Section 6).

![Figure 15: Load-deflection curves for Vr-1000](image1)

![Figure 16: Load-deflection curves for Vr-1250](image2)
6. The response of Inclined Columns
For the inclined columns, six case studies have been considered. Three cases with a length of 1000 mm and designation name of In-1000 while the others have a length of 1250 mm and designation name of In-1250. For each length, three angles of $5^\circ$, $7.5^\circ$, and $10^\circ$ have been considered. The angles are measured from the vertical. To be more representative of the practical cases, load and the boundary conditions are applied as shown in Figure 17. The axial deformation has been measured as illustrated in Figure 17 to exclude the movement due to the rigid body radiation of the column.

![Figure 17. Load, boundary conditions, and deformations for the inclined columns.](image)

7. Results and Discussions
Depending on the finite element model, the $P-\Delta$ curves for the vertical columns and the corresponding inclined columns have been determined and presented in Figure 18 and Figure 19. These figures indicate a more inclined column with less axial stiffness. Reduction in the axial stiffness is more notable in the long column of 1250 mm. The initial stiffness seems less sensitive for the inclination comparing with the tangent stiffness near the ultimate load.

How column strength reduces versus an increase in the inclination angle is presented in Figure 20 where the strength ratio is defined as in equation (11). It shows the non-linear nature of the relationship where the column strength is more sensitive for large inclination angle and larger column length.

\[
\text{strength ratio} = \frac{P_{\text{Inclined}} - P_{\text{vertical}}}{P_{\text{vertical}}} \times 100\%
\]  

![Figure 18. Load-deformation curves for Vr-1000, In1000-5, In1000-7.5 and In1000-10.](image)
Figure 19. Load-deformation curves for Vr-1250, In1250-5, In1250-7.5 and In1250-10.

Figure 20. Strength ratio versus inclination angle.

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