Radiative Lepton Masses with Heavy Lepton Seed

Gwo-Guang Wong and Wei-Shu Hou

Department of Physics, National Taiwan University, Taipei, Taiwan 10764, R.O.C.

We construct a $Z_8$ model for leptons where all Yukawa couplings are of order unity, but known lepton masses are generated radiatively, order by order. The seed is provided by fourth generation leptons $E$ and $N$, which receive (Dirac) mass in usual way. Two additional Higgs doublets with nontrivial $Z_8$ charge are introduced to give nearest neighbor Yukawa couplings. Hence, nonstandard Higgs bosons are flavor changing in an unusual way. Loop masses are generated when $Z_8$ is softly broken down to $Z_2$. However, $e$ and $\mu$ mass generation require new Higgs bosons to be at weak scale. Neutral scalar mixing underlies $m_\mu/m_\tau \gg m_e/m_\mu$, $m_\tau/m_E$. The $Z_2$ symmetry forbids $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$. The most stringent bound comes from $\tau \rightarrow \mu\mu^\pm e^\mp$. The model has interesting implications for $\tau \rightarrow e\gamma$, $\mu\bar{e} \rightarrow \bar{\mu}e$ conversion, $\mu \rightarrow e\nu\bar{\nu}_\mu$, and FCNC decays of $E$ and $N$ (such as $E \rightarrow \tau e^\pm \mu^\mp$).

PACS numbers: 12.15.Ff, 12.60.Fr, 14.80.Cp, 13.35.-r
A major mystery regarding fermion flavor is the very wide range of its mass spectrum. Neutrinos appear to be massless, while known masses range from the electron’s 0.511 MeV, to over 120 GeV \[^{[1]}\] for the top quark. In the standard SU\(_{(2)}_L \times U(1)\) electroweak model (SM), a single Higgs doublet is responsible for generating all masses. The natural scale is 
\[ v = (\sqrt{2} G_F)^{-1/2} \approx 246 \text{ GeV}, \]
where \( G_F \) is the Fermi coupling. The puzzle is then two fold. On one hand, the dimensionless Yukawa couplings \( f \) are scattered over a wide range, and there is an empirical family hierarchy, e.g. \( f_e \ll f_\mu \ll f_\tau \), for each type of fermion. On the other hand, we have \( f \ll 1 \) for all known fermions, except for the top quark where \( f_t \sim 1 \). This is in contrast with the gauge couplings (at W scale) \( g_1 \approx 0.36, g_2 \approx 0.65 \) and \( g_3 \approx 1.2 \) for strong SU\(_{(3)}\). It was suggested a long time ago \[^{[2]}\] that the observed fermion mass hierarchy may be due to radiative mechanisms. It is desirable to have Yukawa couplings \( f \sim 1 \), just like gauge couplings. Given that \( \sqrt{2} m_\tau / v \approx 0.01 \) is itself rather small, we would naturally need new leptons to provide the “seed” for mass generation in the lepton sector.

Neutrino counting \( N_\nu = 2.99 \pm 0.04 \) and direct search limits \( m_E, m_N > M_Z/2 \[^{[3]}\] \) imply that new sequential leptons \( E \) and \( N \) are at scale \( v \), deviating sharply from earlier patterns \[^{[1]}\]. In this Letter, we construct a simple model where \( E \) and \( N \) receive mass at tree level, but all lower generation lepton masses are generated by loop processes.

We start with the lepton sector of minimal “3+1” generations \[^{[1]}\], where there is only one right-handed neutral lepton \( N_R \). Consider a discrete \( Z_8 \) symmetry \((\omega^8 = 1)\). We assign both \( \bar{\ell}_iL = (\bar{\nu}_iL, \bar{e}_iL) \) and \( e_iR \) to transform as \( \omega^3, \omega^2, \omega^1, \omega^4 \) for \( i = 1 - 4 \), respectively, while \( N_R \) transforms as \( \omega^4 \). The scalar sector consists of three doublets, \( \Phi_0, \Phi_3 \) and \( \Phi_5 \), transforming as \( 1, \omega^3 \) and \( \omega^5 \), respectively. Thus, aside from \( E \simeq e_4 \) and \( N \), only nearest-neighbor Yukawa couplings are allowed,

\[
-\mathcal{L}_Y = f_{44} \bar{\ell}_{4L} e_{4R} \Phi_0 + \bar{f}_{44} \bar{\ell}_{4L} N_R \Phi_0 \\
+ f_{43} \bar{\ell}_{4L} e_{3R} \Phi_3 + f_{34} \bar{\ell}_{3L} e_{4R} \Phi_3 + \bar{f}_{34} \bar{\ell}_{3L} N_R \Phi_5 \\
+ f_{32} \bar{\ell}_{3L} e_{2R} \Phi_5 + f_{23} \bar{\ell}_{2L} e_{3R} \Phi_5 \\
+ f_{21} \bar{\ell}_{2L} e_{1R} \Phi_3 + f_{12} \bar{\ell}_{1L} e_{2R} \Phi_3 + H.c. \quad (1)
\]


where $\tilde{\Phi} \equiv i\sigma_2 \Phi^* = (\phi^0, -\phi^-)$ as usual. We assume $CP$ invariance for simplicity.

If only $\langle \phi_0^0 \rangle = v/\sqrt{2}$, $E$ and $N$ become massive and are naturally at $v$ scale if $f_{44}, \tilde{f}_{44} \sim 1$. The lower generation leptons remain massless at this stage, protected by the $Z_8$ symmetry. To allow for radiative mass generation, the $Z_8$ symmetry is softly broken down to $Z_2$ in the Higgs potential by $\Phi_3, \Phi_5$ mixing. Explicitly,

$$V = \sum_i \mu_i^2 \Phi_i^\dagger \Phi_i + \sum_{i,j} \lambda_{ij} (\Phi_i^\dagger \Phi_i)(\Phi_j^\dagger \Phi_j) + \sum_{i \neq j} \eta_{ij} (\Phi_i^\dagger \Phi_j)(\Phi_j^\dagger \Phi_i)$$

$$+ [\tilde{\mu}^2 \Phi_3^\dagger \Phi_5 + \zeta (\Phi_0^\dagger \Phi_3)(\Phi_0^\dagger \Phi_5) + H.c.] .$$

(2)

Note that the $\zeta$ term is $Z_8$ invariant, while the gauge invariant “mass” $\tilde{\mu}^2$ transforms as $\omega^2$. Since only $\mu_0^2 < 0$, while $\mu_3^2$ and $\mu_5^2 > 0$, $\phi_0^0 \rightarrow (v + H_0 + i\chi_0)/\sqrt{2}$, and $\phi_i^0 \rightarrow (h_i + i\chi_i)/\sqrt{2}$ for $i = 3, 5$. The quadratic part of $V$ is

$$V^{(2)} = \lambda_{00} v^2 H_0^2 + \sum_{i \neq 0} \left( \frac{1}{2} (\mu_i^2 + \lambda_{0i} v^2 + \eta_{0i} v^2)(h_i^2 + \chi_i^2) + (\mu_i^2 + \lambda_{0i} v^2)|\phi_i^0|^2 \right)$$

$$+ \tilde{\mu}^2 (h_3 h_5 + \chi_3 \chi_5 + \phi_3^+ \phi_3^- + \phi_5^+ \phi_5^-) + \frac{1}{2} \zeta v^2 (h_3 h_5 - \chi_3 \chi_5) .$$

(3)

The standard Higgs boson $H_0$ couples only diagonally to heavy particles. The nonstandard scalars $(\phi_3^+, \phi_5^+), (h_3, h_5)$ and $(\chi_3, \chi_5)$ mix via $\tilde{\mu}^2$ and $\zeta$ terms. Rotating by $\theta_+, \theta_H$ and $\theta_A$, we obtain the mass basis $(H_1^+, H_2^+), (H_1, H_2)$ and $(A_1, A_2)$, respectively. It is clear that $\sin \theta_+ \rightarrow 0, (\theta_A, m_{A_1}, m_{A_2}) \rightarrow (-\theta_H, m_{H_1}, m_{H_2})$ as $\tilde{\mu}^2 \rightarrow 0$, while in the limit $\zeta \rightarrow 0, (\theta_A, m_{A_1}, m_{A_2}) \rightarrow (+\theta_H, m_{H_1}, m_{H_2})$. These two limits restore the two extra $U(1)$ symmetries of the doublets $\Phi_3$ and $\Phi_5$. As we shall see, fermion mass generation is due to mixing and nondegeneracy of the two charged scalars, and especially the four real scalar fields.

The $\tau$ lepton acquires mass via the one-loop diagram shown in Fig. 1,

$$m_{33} = \left( \frac{\tilde{f}_{43} f_{43}}{32 \pi^2} \right) \sin 2\theta_+ [G(m_{H_1^+}/m_N) - G(m_{H_2^+}/m_N)] m_N$$

$$+ \left( \frac{f_{43} f_{43}}{32 \pi^2} \right) \left[ \cos^2 \theta_H G(m_{H_1}/m_E) + \sin^2 \theta_H G(m_{H_2}/m_E) - \cos^2 \theta_A G(m_{A_1}/m_E) - \sin^2 \theta_A G(m_{A_2}/m_E) \right] m_E ,$$

(4)

where $G(x) = (x^2 \ln x^2)/(x^2 - 1)$, while $m_{34} = m_{43} = 0$. As a numerical exercise, let $f_{43} = \tilde{f}_{34} = f_{44} = \tilde{f}_{44} \sim \sqrt{2}$ so $\tilde{f}_{43} f_{43}/4\pi \simeq 1/2\pi$ and $m_N, m_E \simeq 246$ GeV. Then $\sin 2\theta_+ (G(x_1) -
$G(x_2) \simeq 0.6$ would make $m_\tau = m_{33} \simeq 1.8$ GeV, if the $m_E$ term contributes as much as the $m_N$ term, which is usually the case. There are separate GIM-like cancellation mechanisms rooted in charged and neutral scalar mixing. In general $G(x_1) - G(x_2)$ is regulated by $\sin 2\theta_+$, hence typically $\sin 2\theta_+ (G(x_1) - G(x_2)) \lesssim 1$. Similar statements can be made for the neutral scalar contribution. This implies that $f_{43}, f_{34}, f_{44}, \tilde{f}_{44} \sim f \gtrsim 1$ is natural in our model.

If $\tilde{\mu}_2 \to 0$, both terms would vanish and $m_\tau = 0$. It is interesting to note that even if $\tilde{\mu}_2 \neq 0$, the neutral scalar contribution would vanish if $\zeta = 0$. This is of crucial importance for muon and electron mass generation, for they receive radiative masses at two- and three-loop order, respectively, via neutral scalar loops that are similar to Fig. 1(b). These “nested” diagrams are illustrated in Fig. 2. The upshot then is that $\zeta v^2/2$ should be of similar order of magnitude as $\tilde{\mu}_2$, which in turn implies that $\mu_3^2$ and $\mu_5^2$ should also be of order $v^2$. Hence, nonstandard Higgs boson masses cannot be too far above the electroweak scale!

Since off-diagonal mass terms $m_{24}$ and $m_{13}$ are also at two- and three-loop order, respectively, we have the mass hierarchy $m_E : m_\tau : m_\mu : m_e \sim 1 : \lambda : \lambda^2 : \lambda^3$, where $\lambda$ is the loop expansion parameter. That is, $m_i/m_{i+1} \sim f_{i,i+1}f_{i+1,i}/32\pi^2$ or more. If $\sqrt{f_{i,i+1}f_{i+1,i}} \sim 1$ for all $i = 1 - 3$, the mass hierarchy of order $10^{-1} - 10^{-2}$ can be realized. Together with $f_{44}, \tilde{f}_{44} \sim 1$, we see that Yukuwa couplings could be generation blind. It is amusing that in our model, all dimensionless couplings seem to be of order one, and all scale parameters are of order $v$.

It is intriguing that the model could account for $m_e/m_\mu$ ($\sim m_\tau/m_E$) $\sim 1/200 \ll m_\mu/m_\tau \sim 1/20$. Concentrating on neutral scalar loops, from Figs. 1 and 2 we see that $m_e, m_\tau$ come from $m_\mu$, $m_E$ seeds via “$\phi_3^0$” loop, while $m_\mu$ arises from $m_\tau$ seed via “$\phi_5^0$” loop. With obvious notation, we estimate $m_e/m_\mu \simeq \lambda \left[ s_H^2 \ln(m_{H_2}^2/m_{H_1}^2) - s_A^2 \ln(m_{A_2}^2/m_{A_1}^2) + \ln(m_{H_2}^2/m_{A_1}^2) \right]$, while for $m_\mu/m_\tau$ one has $s_{H,A}^2 \leftrightarrow c_{H,A}^2$. As an example, we could have $m_{H_1} \sim m_{A_1} \sim m_{A_2}$, then $(m_\mu/m_\tau)/(m_e/m_\mu) \simeq \cot^2 \theta_H \simeq 12$, and $\sin \theta_H \simeq 0.28$ which is rather reasonable.

Note that $N_R$ is introduced solely for the purpose of satisfying LEP bound [4]. Having just a massive $E$ would have been sufficient for charged lepton mass generation. However,
could in principle have Majorana mass $m_R$, which could serve as seed for radiatively generating Majorana mass for the three left-handed neutrinos. We emphasize that $m_R \gg m_N$ is not allowed [4], for then the seesaw mechanism [5] would drive $N_L$ mass effectively to zero, violating LEP bound. Rough estimates of loop induced Majorana neutrino masses indicate that $m_R$ should be rather small, and we set $m_R = 0$ in the present work. However, our model provides interesting, new mechanisms, details of which will be reported elsewhere.

We turn to phenomenological prospects. These depend on the lowest allowed mass(es) for the charged or FCNC neutral Higgs bosons. Mixing effects in the charged current [4] can be ignored since they are radiatively generated. As discussed earlier, radiative mass generation for $\mu$ and $e$ suggest that nonstandard Higgs boson masses should not be too far above the electroweak scale $v$. Hence, one might worry about low energy FCNC effects. Very stringent limits exist for $\mu \rightarrow e\gamma$ [3]. Interestingly, our model has a dichotomy [6] of leptons: $E$, $N$, $\mu$, and $\nu_\mu$ are even under $Z_2$, whereas $\tau$, $e$, $\nu_\tau$, and $\nu_e$ are odd. For scalars, $\Phi_0$ is even, while $\Phi_3$ and $\Phi_5$ are odd. Hence, $\mu \rightarrow e\gamma$ is forbidden in our model, since the photon is $Z_2$ invariant. Similarly, $\tau \rightarrow \mu\gamma$ is forbidden, but $\tau \rightarrow e\gamma$ is allowed, as we shall discuss later.

The present experimental errors [3] on $g - 2$ for $e$ and $\mu$ imply [7] lower bounds of a few hundred GeV on the effective mass of the exchanged scalar bosons. Standard $\mu$ and $\tau$ decay modes such as $\mu \rightarrow e\nu\nu$ and $\tau \rightarrow \mu\nu\nu$ do not give better constraints because of uncertainty in $M_W$ measurement [6]. The most stringent constraint on our model turns out to be from leptonic FCNC $\tau$ decays: $\tau \rightarrow \mu\mu\mp e^\pm$ (the decay modes $\tau \rightarrow ee^\pm\mu^\mp$ are forbidden). Each has four contributions, two of which are shown in Fig. 3 for $\tau^- \rightarrow \mu^-\mu^- e^+$. The inverse effective mass squared $1/m^2(\phi^0_{3,5})$ corresponds to a sum over neutral (pseudo)scalars $\sum_{i=1}^4 a_i/m_i^2$ where we now order according to mass $m_i$, while $a_i$ are mixing factors that should satisfy $0 < |a_i| \leq 1$, $\sum_i a_i = 0$. They are nothing but $\pm s_{H,A} c_{H,A}$. Thus, in general $m^2(\phi^0_{3,5}) > m_1^2$, the lightest (pseudo)scalar mass, and could be much larger than $v^2$. Assuming single (lightest) channel dominance, we find

$$B(\tau^- \rightarrow \mu^-\mu^- e^+) \simeq \frac{1}{2} \left( \frac{f_{e\mu} f_{\mu\nu} a_1/m_1^2}{g^2/M_W^2} \right)^2 \frac{\alpha}{\pi} B(\tau \rightarrow \mu\nu\bar{\nu}),$$

(5)
where \( 2f_{\mu\tau}^2f_{\mu}^2 \equiv \{(f_{23}f_{12})^2 + (f_{32}f_{21})^2\} \). We show in Fig. 4 \( B(\tau^- \rightarrow \mu^-\mu^-e^+) \) vs. \( \sqrt{f_{\mu\tau}f_{\tau\mu}} \vert a_1 \vert \) for \( m_1 = (0.5, 1, 2, 4) \) \( v \) \( (\equiv 125, 250, 500, 1000 \text{ GeV}) \). The present experimental bound of \( B(\tau^- \rightarrow \mu^-\mu^-e^+) < 1.6 \times 10^{-5} \) is also shown. We see that, because of \( |a_1| < 1 \) and cancellation effects, it is possible to have all Yukawa couplings of order unity while physical nonstandard scalar masses are of order \( v \) or greater. Our model could naturally account for \( B(\tau^- \rightarrow \mu^-\mu^-e^+) \) just below \( 10^{-5} \). Similar results are obtained from \( B(\tau^- \rightarrow \mu^-\mu^+e^-) < 2.7 \times 10^{-5} \). These decays are exceptionally clean, and should be searched for vigorously.

The \( \tau^- \rightarrow \mu^-\mu^+e^- \) decay leads to \( \tau \rightarrow e\gamma \) at one-loop order. We find

\[
\frac{B(\tau \rightarrow e\gamma)}{B(\tau \rightarrow \mu\bar{\nu}e)} \lesssim 24\frac{\alpha}{\pi} \left( \frac{m_\mu}{m_\tau} \right)^2 \left( \ln \frac{m_\mu^2}{m_\tau^2} \right)^2,
\]

(6)

where we assume same channel dominance, and drop a constant term accompanying the large logarithm. As an estimate, the ratio is less than \( 1/21 - 1/15 \) for \( m_1 \approx 250 \text{ GeV} \). Hence \( \tau \rightarrow e\gamma \) is typically just one order of magnitude below \( \tau \rightarrow \mu\bar{\nu}e \), i.e. at \( 10^{-6} \) or lower. The present experimental bound is \( \sim 10^{-4} \).

In our model neutral scalars couple to \( \mu \bar{e} \) and \( \bar{\mu}e \) simultaneously, therefore they mediate muonium-antimuonium conversion \( \text{[8]} \) (without doubly charged Higgs!). Unlike \( \tau \rightarrow \mu\bar{\mu}e^\pm \) which is mediated by \( \phi_3^0\phi_3^{0(s)} \) mixing, here we have a \( \phi_3^0 \) mediated process. The scalar mixing factors \( a_i \) are of same sign i.e. of the form \(+c_{H,A}^2, +s_{H,A}^2 \) and \( \sum_i a_i = 2 \). Assuming that the dominating scalar has mass \( \sim v \) and the mixing factor ranges from \( 0.01 - 1 \), we estimate that the effective four Fermi coupling is of order \( (0.001 - 0.1) \) \( G_F \), compared with the present bound of \( 0.16 \) \( G_F \). The limit may be improved to \( 10^{-3} \) \( G_F \) soon \( \text{[9]} \). Note that we have an unusual effective interaction \( \bar{\mu}(1 - \gamma_5)e \bar{\mu}(1 + \gamma_5)e \). In the same vein, the process \( \mu \rightarrow e\nu_{\mu}\bar{\nu}_\mu \) is mediated by \( \phi_3^+ \), and has a four Fermi coupling of similar order. The present bound is \( 0.14 \) \( G_F \), but may be pushed down to \( 10^{-2} \) \( G_F \) \( \text{[10]} \) in near future.

Consider now the decays of the heavy lepton \( E \). If \( m_E \leq m_N \), since charged current mixing is loop suppressed, \( \phi_{3,5}^\pm \)-induced \( E \rightarrow \nu_\tau(e\bar{\nu}_\mu, \mu\bar{\nu}_e, \mu\bar{\nu}_\tau, \tau\bar{\nu}_\mu) \) and \( \phi_{3,5}^0 \)-induced FCNC \( E \rightarrow \tau(e^\pm\mu^\pm, \mu^\pm\tau^\pm) \) decays could be dominant. They could still be prominent for \( m_E \geq m_N \).
since $W$-induced transitions such as $E \to N\nu_e$ are kinematically suppressed until $m_E - m_N$ approaches $M_W$. However, for $m_E \gtrsim m_N$, new scalar induced decays such as $N \to \nu_\tau \mu^\pm e^\mp$ would dominate $N$ decay. Since the lightest scalar might be lighter than $m_E$ or $m_N$, it may even be produced directly in $E$, $N$ decay. These decays would provide dramatic signatures at future colliders.

In summary, we have presented a realistic model for radiative lepton mass generation. The model has $Z_8$ symmetry and three Higgs doublets with nearest-neighbor Yukawa couplings. Only $E$ and $N$ receive tree level masses upon spontaneous symmetry breaking. When $Z_8$ is softly broken down to $Z_2$, they provide the seed for generating, order by order, loop masses for $\tau$, $\mu$ and $e$. Yukawa couplings are naturally of order unity, and the mass pattern $m_E : m_\tau : m_\mu : m_e \simeq 1 : \lambda : \lambda^2 : \lambda^3$ emerges, which suggests the possibility of universal Yukawa couplings. The model could account for $m_\mu/m_\tau \gg m_e/m_\mu$, $m_\tau/m_E$ as a consequence of mixing effects among nonstandard Higgs bosons. The residual $Z_2$ symmetry forbids $\mu \to e\gamma$ and $\tau \to \mu\gamma$ type of transitions. The charged and FCNC neutral Higgs bosons have weak scale masses. However, due to GIM-like cancellations among themselves, they could mimic TeV scale physics. The most promising channels seem to be FCNC tau decays $\tau \to \mu\mu^\pm e^\mp$ (but not $\tau \to e e^\pm \mu^\mp$) and $\tau \to e\gamma$ at just below $10^{-5}$ and $10^{-6}$, respectively. Muonium-antimuonium conversion and $\mu \to e\nu_\mu P_\mu$ could have strength $(0.001 - 0.1)G_F$.

FCNC leptonic decays of the fourth generation $E$ and $N$ are likely to be quite prominent. We have consistently assumed that the Majorana mass $m_R = 0$ for $N_R$. A small $m_R$ could induce Majorana masses for left-handed neutrinos via loop processes. The quark sector is clearly richer but more difficult. Work is in progress, and will be reported elsewhere.

ACKNOWLEDGMENTS

We thank E. Ma, D. Chang and C.-Q. Geng for useful discussions. The work of GGW is supported in part by grant NSC 83-0208-M-002-025-Y, and WSH by NSC 82-0208-M-002-151 of the Republic of China.
REFERENCES

[1] Talks by T. Chikamatsu (CDF Collaboration) and P. Grannis (D0 Collaboration), presented at 9th International Topical Workshop on $p\bar{p}$ Collider Physics, Tsukuba, Japan (October 1993).

[2] A complete list is impossible. Some earlier works are: S. Weinberg, Phys. Rev. Lett. 29, 388 (1972); H. Georgi and S. L. Glashow, Phys. Rev. D6, 2977 (1972); D7, 2457 (1973); R. N. Mohapatra, Phys. Rev. D9, 3461 (1974); S. M. Barr and A. Zee, Phys. Rev. D15, 2652 (1977); D17, 1854 (1978).

[3] Review of Particle Properties, Phys. Rev. D 45, S1 (1992).

[4] W. S. Hou and G.-G. Wong, preprint NTUTH-93-10, to appear in Phys. Rev. D; S. F. King, Phys. Lett. B281, 295 (1992); Phys. Rev. D46, R4804 (1992).

[5] M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity, edited by P. van Nieuwenhuizen and D. Freedman (North Holland, Amsterdam, 1979) p. 315; T. Yanagida, in Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe, edited by O. Sawada and A. Sugamoto (KEK Report No. 79-18, Tsukuba, Japan, 1979), p. 95.

[6] E. Ma and G.-G. Wong, Phys. Rev. D41, 992 (1990).

[7] E. Ma, D. Ng, and G.-G. Wong, Z. Phys. C47, 431 (1990).

[8] P. Herczeg and R. N. Mohapatra, Phys. Rev. lett. 69, 2475 (1992).

[9] K. Jungmann et al., PSI Experiment No. R-89-06.1.

[10] X. Q. Lu et al., LAMPF Proposal No. LA-11842-P, 1990.
FIGURES

FIG. 1. Mechanism for $m_\tau$.

FIG. 2. Mechanisms for $m_\mu$ and $m_e$ via “nested” neutral scalar diagrams.

FIG. 3. Some diagrams contributing to $\tau \to \mu \mu^- e^+$.

FIG. 4. $B(\tau^- \to \mu^- \mu^- e^+)$ vs. $\sqrt{f_{e\mu} f_{\tau\mu}} |a_1|$ for (top to bottom) $m_1 = 125, 250, 500, 1000$ GeV. Straight line indicates present limit.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9401251v1
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9401251v1
This figure "fig1-3.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9401251v1