A primary method for the complex calibration of a hydrophone from 1 Hz to 2 kHz

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Abstract

A primary calibration method is demonstrated to obtain the magnitude and phase of the complex sensitivity for a hydrophone at frequencies between 1 Hz and 2 kHz. The measurement is performed in a coupler reciprocity chamber (‘coupler’); a closed test chamber where time harmonic oscillations in pressure can be achieved and the reciprocity conditions required for a primary calibration can be realized. Relevant theory is reviewed and the reciprocity parameter updated for the complex measurement. Systematic errors and corrections for magnitude are reviewed and more added for phase. The combined expanded uncertainties of the magnitude and phase of the complex sensitivity at 1 Hz were 0.1 dB re 1 V µPa−1 and ±1°, respectively. Complex sensitivity, sensitivity magnitude, and phase measurements are presented on an example primary reference hydrophone.

Keywords: coupler, reciprocity, phase, calibration, complex, uncertainty, hydrophone

(Some figures may appear in colour only in the online journal)
phase errors in the calibration where the type H48 phase response is not constant.

Driven by the needs of the ultrasound community, calibrations can now include phase measurement using non-linear acoustic wave propagation [5, 6], time-delay spectrometry [7, 8], and pulse excitation [9, 10] methods. Lower frequency, underwater calibrations are also using phase [11]. To supply these calibrations with traceable phase-calibrated references, metrology laboratories developed methods for primary phase calibrations. These include self-reciprocity covering 1-7 MHz [12], heterodyne interferometer measurement with pulse excitation covering 1–70 MHz (with extension to 100 MHz) [13], and three-transducer, spherical-wave reciprocity covering 20–140kHz [14], extended to 10–400kHz along with a comparison to heterodyne interferometer results [15]. The method in this paper adds a primary phase calibration in the low frequency range, 1 Hz to 2kHz, that can cover temperatures between 0 and 40 °C and hydrostatic pressures between 0 and 13.8 MPa, important ranges for underwater applications.

This paper describes a primary method for the complex calibration of the type H48 reference hydrophone in the USRD coupler reciprocity system. It is arranged as follows. Section 2 describes the coupler and coupler reciprocity. Section 3 describes the USRD type H48 experimental setup and method. Section 4 describes systematic errors and corrections in the experimental setup. In section 5, we calculate the type A and B uncertainty in the sensitivity. In section 6, we present sensitivity data from two example type H48 hydrophones, serial numbers 2 and 4. Section 7 concludes.

2. Theory

The coupler is a fluid-filled cavity in which three transducers are installed to perform a reciprocity calibration to calculate the low-frequency, pressure sensitivity of an unknown hydrophone. Figure 1 shows the cross section of an example coupler. Identical transducers are installed on the left and right sides of the coupler and a reference hydrophone (to be calibrated) in the center. A fluid fills the sealed cavity.

The pressure sensitivity is obtained via the reciprocity method by measuring the current to the projector and the voltages at the receiving transducer and unknown hydrophone. The reciprocity parameter relates these measurements to the sensitivity of the unknown hydrophone. The calibration method invokes a set of assumptions that, when satisfied, ensure the pressure and free-field sensitivities are equal, allowing the more useful free-field sensitivity measurement to be calculated during the calibration.

Throughout this paper, complex quantities are shown in bold font.

2.1. Coupler calibration assumptions

Several assumptions are made when performing a calibration in a coupler to enforce the condition that the sound pressure is the same everywhere in the coupler at a given instant in time, as described in [4]:

(i) The wavelength of sound must be much larger than the largest dimension of the coupler, its length.

(ii) The walls of the coupler and transducer elements must be rigid—they must have a much higher impedance than the acoustic medium.

(iii) No air or any other low impedance material can be present in the coupler.

While the free-field sensitivity is the desired parameter, it is the pressure sensitivity that is measured in the coupler. Under the test conditions of the coupler, however, the pressure sensitivity and free-field sensitivity are equivalent. Figure 2 shows a hydrophone with an open circuit voltage, \(e\), and an acoustical impedance, \(Z_a\). The applied pressure at the acoustic input in the coupler is \(p_e\) and an acoustical Thevenin generator represents the effects of a free field: the radiation impedance, \(Z_r\), and the blocked and free-field pressures, \(p_b\) and \(p_f\). The ratio \(\frac{p_f}{p_b}\) is a frequency dependent diffraction constant, \(D\).

In order for \(p_f\) and \(p_b\) to be equal, the diffraction constant \(D\) must equal 1. Analyzing the circuit diagram, \(p_f\) and \(p_a\) are approximately equal if \(Z_r\) is much smaller than \(Z_a\). In situations where the hydrophone’s physical dimensions are much smaller than a wavelength, as in the coupler, \(D = 1\), \(Z_r\) will be small, and the \(Z_r \ll Z_a\) criteria will be met by a piezoelectric
transducer operated at frequencies that are well below the first resonance. Given these criteria, the free-field pressure is equivalent to the applied pressure at the acoustic input [4].

2.2. Reciprocity calculation for the complex voltage sensitivity

The primary calibration performed using the USRD coupler relies on the principle of reciprocity to calculate the complex voltage sensitivity of an unknown hydrophone. In a reciprocity calculation, the voltage sensitivity of a hydrophone can be calculated through the measurement of six electrical parameters made on three transducers (reciprocal transducers X and Y, and a hydrophone) as shown in table 1. Two measurement setups are required to obtain all six electrical parameters: (a) reciprocal transducer X transmitting and reciprocal transducer Y and hydrophone receiving, and (b) reciprocal transducer Y transmitting and reciprocal transducer X and hydrophone receiving. The electrical parameters are: \( i_X \), the input current to transducer X when it is transmitting, \( e_{HX} \), the open-circuit output voltage from the hydrophone when transducer X is transmitting, \( e_{XY} \), the open-circuit output voltage from transducer Y when transducer X is transmitting, \( i_Y \), the input current to transducer Y when it is transmitting, \( e_{HY} \), the open-circuit output voltage from the hydrophone when transducer Y is transmitting, and \( e_{XY} \), the open-circuit output voltage from transducer X when transducer Y is transmitting.

The output voltage measurements can be expressed in terms of a voltage sensitivity and the pressure generated by the transmitting transducer. They are manipulated with the goal of expressing the sensitivity of the unknown hydrophone while eliminating any acoustical parameters of the other transducers in order to produce a primary calibration as shown in [4]. For configuration (a) this expression is

\[
e_{HX} = M_H i_X S_X,
\]

where \( M_H \) is the open-circuit receiving voltage sensitivity of the hydrophone and \( S_X \) is the transmit current response of reciprocal transducer X. Unlike the free-field form of this expression, there is no dependence on separation distances, and the transmit current response is the pressure inside the cavity for a given current—there is no reference distance factor. Similarly, the open-circuit output voltage of the transducer Y is

\[
e_{XY} = M_Y i_Y S_X,
\]

where \( M_Y \) is the open-circuit receiving voltage sensitivity of reciprocal transducer Y. The ratio of the output voltages in configuration (a) is

\[
\frac{e_{HX}}{e_{XY}} = \frac{M_H}{M_Y}.
\]

From configuration (b)

\[
e_{HY} = M_H i_Y S_Y,
\]

where \( S_Y \) is the transmit current response of reciprocal transducer Y. Using the property of the reciprocal transducer that the ratio of the receive sensitivity and transmit response is equal to the reciprocity parameter,

\[
J = \frac{M_Y}{S_Y}.
\]

Equations (3)–(5) can be solved for the hydrophone sensitivity as shown in (6).

\[
M_H = \left( \frac{e_{HX} e_{HY}}{e_{XY} e_{CY}} \right)^{1/2}.
\]

In the USRD coupler reciprocity procedure, redundant measurements using reciprocal transducers X and Y as both a projector and receiver reduce measurement uncertainty and create convenient symmetry to simplify phase measurement. Measurement (b) in table 1 is collected when the cables driving the projecting transducer and recording from the receiving transducer are interchanged from configuration (a), and the measurement is repeated. The extra output voltage measurement, \( e_{XY} \), is incorporated into the denominator of (6) where the geometric mean of both receiving transducer measurements gives

\[
M_H = \left( \frac{e_{HX} e_{HY}}{\sqrt{e_{XY} e_{CX} e_{CY} j \omega C_p}} \right)^{1/2}.
\]

In this coupler setup, current measurements are obtained by measuring the voltage across a standard capacitor placed in series with the input current. Using the relationships \( i_X = e_{CX} j \omega C_p \) and \( i_Y = e_{CY} j \omega C_p \), where \( e_{CX} \) and \( e_{CY} \) are the voltages across the standard capacitor and \( C_p \) is the capacitance of the standard capacitor, the currents in (7) can be substituted for voltages, producing

\[
M_H = \left( \frac{e_{HX} e_{HY}}{\sqrt{e_{XY} e_{CX} e_{CY} j \omega C_p}} J \right)^{1/2}.
\]

2.3. Reciprocity parameter

The complex pressure reciprocity parameter, \( J \), must be calculated for a small rigid-walled cavity in order to obtain the hydrophone sensitivity from the voltage and current measurements in (8). The crux of the primary calibration is that the reciprocity parameter is derived in terms of the acoustic properties of the fluid medium. No electrical properties of any of the transducers must be known. To derive this parameter, the
The pressure and volume velocity can be related by the acoustical radiation impedance seen by one transducer when it alone is radiating; and the ratio of the blocked pressure at one transducer to the volume velocity at the other.

The first term in each impedance shown in (17) and (18) contains the bulk modulus in the numerator, a term that represents the elasticity of the coupler volume under pressure, and the second term in each is the ratio of the acoustic pressure at transducer when it alone is radiating; and the acoustical transfer impedance [16]. Using the first term of their Taylor series expansion, we get the following long-wavelength approximations for the self-radiation impedance

\[ \frac{Z_{11}}{Z_{12}} = \frac{p_1}{p_2} \text{ at } U_1 = 0 = \frac{p_1}{U_2} \frac{p_2}{U_1} \]

and for the acoustical transfer impedance,

\[ \frac{Z_{12}}{Z_{21}} = \frac{p_1}{p_2} \frac{U_1}{U_2} = \frac{1}{j\omega} \frac{kL}{S} \]

Taking the first two terms of their Taylor series expansion, we get the following long-wavelength approximations for the self-radiation impedance

\[ \frac{Z_{11}}{Z_{12}} \approx -\frac{j\rho c}{S} \left( \frac{1}{kL} + \frac{kL}{3} \right) = \frac{B_{ad}}{j\omega S L} + \frac{j\omega p L}{3} \]

and the acoustical transfer impedance

\[ \frac{Z_{12}}{Z_{21}} \approx -\frac{j\rho c}{S} \left( \frac{1}{kL} + \frac{kL}{6} \right) = \frac{B_{ad}}{j\omega S L} - \frac{j\omega p L}{6}, \]

where \( B_{ad} = \rho c^2 \) is the adiabatic bulk modulus of the fluid.

The first term in each impedance shown in (17) and (18) contains the bulk modulus in the numerator, a term that represents the elasticity of the coupler volume under pressure, and the frequency and volume in the denominator. This term represents the acoustical compliance of the fluid. The second term in each impedance contains the density and length in the numerator and the surface area in the denominator, forming an inerter term that describes the pressure difference required to cause a change in flow rate with time.

The reciprocity parameter, \( J \), is equal to the reciprocal of the acoustical transfer impedance [16]. Using the first term of the Taylor series expansion in powers of \( kL \) for \( Z_{12} \), from (18), the commonly accepted reciprocity parameter for the coupler is
where \( C \) is the acoustical compliance of the coupler fluid medium as long as the fluid compliance is much greater than the compliance of the coupler walls and transducers. The coupler is compliance (or stiffness) dominated as long as 
\[
\frac{1}{\kappa L} \gg \frac{1}{k L}.
\]
For the worst case conditions at the highest temperature, lowest pressure, and highest frequency, 
\[
\frac{1}{\kappa L} = 1.32 \quad \text{and} \quad \frac{1}{k L} = 0.126.
\]
From (8), substituting the complex pressure reciprocity calibration parameter from (19), we get
\[
M_H = \sqrt{e_{HX}e_{HY}} \sqrt{e_{YX}e_{CY}} \left( \frac{V}{C_p \rho c^2} \right) \text{ V Pa}^{-1},
\]
where \( V \) is the volume of the coupler cavity. The voltage sensitivity in dB re 1 V \( \mu \text{Pa}^{-1} \) is given by
\[
M_H = 20 \log_{10} |M_H| - 120 \text{ dB re 1 V } \mu \text{Pa}^{-1}.
\]
The factor of 120 is used to convert the dB reference to \( \mu \text{Pa} \).

This voltage sensitivity is equivalent to the free-field voltage sensitivity while the assumptions in section 2.1 are valid.

3. Experimental setup and methods

A method to perform a reciprocity calibration, including phase, was developed for the USRD H48 coupler reciprocity system. The procedure mimicked a standard USRD coupler reciprocity calibration to provide data at similar temperatures, pressures, and frequency ranges for comparison with results provided by the existing measurement method.

3.1. Physical setup

A diagram of the physical setup is shown in figure 5, with component details provided in table 2. A signal is generated at the function generator, amplified, and passed through a standard capacitor wired in series with the projector. A custom isolated differential amplifier measures the voltage across the standard capacitor, which is proportional to the current supplied to the projector. A hand operated pressure ram controls the hydrostatic pressure in the coupler fluid, castor oil. Castor oil is used in place of distilled water to reduce corrosion and provide a dielectric between the transducers. Chamber temperature is controlled by heating or cooling a distilled water and propylene glycol mixture from a temperature bath, which flows through channels in the coupler walls.

Within the coupler, three transducers participate in the calibration: two reciprocal lithium sulfate spheres and the type H48 reference hydrophone positioned in between. The wiring to and from reciprocal transducers X and Y can be interchanged in order to record signals generated by either reciprocal transducer. The hydrophone has a built in preamplifier and is connected directly to the data acquisition equipment. A computer records the time series data and controls the function generator.

3.2. Procedure

A complex-valued coupler reciprocity calibration was performed across frequency, temperature, and pressure in accordance with [17]. Pressure was varied from 345 kPa to 6.9 MPa. Temperature was varied from 0 °C to 40 °C. For this paper, at each temperature and pressure, a logarithmic frequency sweep was performed from 1 to 2000 Hz. At each frequency, 20 cycles of the time series signals were recorded at a sample rate five times the signal frequency. Each frequency sweep was repeated ten times.

4. Systematic errors and corrections

The magnitude of systematic errors in the coupler measurement apparatus and method exceeded the uncertainties in the measurements and their relevant constants. These errors arise from simplifying theoretical assumptions not fully borne out in the measurement setup and phase errors introduced from the phase responses of the measurement equipment.

4.1. Phase corrections

To isolate the phase response of the hydrophone, phase change contributions by the other components were measured and corrected. The block diagram in figure 6 illustrates the coupler setup, which can be considered a linear electro-acoustic system. Significant measurement node locations are shown as dots and the signal path as solid black lines. Dashed lines indicate the different measurement pathways through the system.
Each component in the diagram has an associated linear complex gain element.

Three measurement pathways through the electro-acoustic system, shown in figure 6 are of interest: (a), (b), and (c). Each one begins at measurement node 1 in the fluid in the coupler cavity at the face of the hydrophone and ends at measurement node 4 where the hydrophone output voltage is recorded at the data acquisition system. Pathway (a) leads from the fluid in the coupler cavity through the receiving reciprocal transducer (X in figure 6), preamplifier, and data acquisition system. Pathway (b) leads from the fluid in the coupler cavity through the transmitting reciprocal transducer (Y in figure 6), capacitor/isolated amplifier, and data acquisition system. The complex gain through pathway (a) or (b) will satisfy

\[ G_{\text{tot}a} = G_{1-2m} \cdot G_{2-3m} \cdot G_{3-4m} \]  

where subscript numbers signify the nodes the complex gains act between and m can signify pathway (a) or (b). The magnitudes satisfy

\[ G_{\text{tot}a} = G_{1-2m} \cdot G_{2-3m} \cdot G_{3-4m} \]  

and phases satisfy

\[ \phi_{\text{tot}a} = \phi_{1-2m} + \phi_{2-3m} + \phi_{3-4m} \]  

The change in phase in pathway (c), the unknown hydrophone phase response, is the phase of the complex sensitivity of the hydrophone calculated in (20) and the signals recorded at measurement nodes 3a, 3b, and 4 are \( e_{XY} \), \( e_{CY} \), and \( e_{HY} \), respectively. The other parameters in (20) are recorded separately when the measurements are repeated with the preamplifier connected to transducer Y and the capacitor/isolated amplifier connected to transducer X (not shown in figure 6). The phase change, \( \phi_{\text{tot}a} \) for pathways (a) and (b) must be calculated and applied as a systematic phase correction to the signal measured at nodes 3a and 3b before they are used in (20).

Systematic phase corrections are applied to (20) as given by

\[ M_{\text{PhCor}} = \left( \frac{e_{XY} e_{HY}}{\sqrt{(G_{2-3a} e_{XY}) (G_{2-3b} e_{CY}) (G_{2-3a} e_{CY})}} \right)^{1/2} \]  

The magnitude of each of these complex gains is unity. The following subsections discuss each of the complex gain term.

4.1.1. \( G_{1-2m} \). As the signal passes from measurement node 1 in the coupler fluid to measurement node 2a or 2b, the voltage output and current input to transducer X and Y, respectively, is modified by the complex gain represented by the receiving sensitivity and transmitting current response of reciprocal transducers X and Y. These gain and phase changes will not affect the hydrophone sensitivity calculation because the reciprocity method allows the complex receiving sensitivities and transmitting current responses of the reciprocal transducers to be rewritten in terms of the reciprocity parameter, which is equal to purely acoustic properties of the coupler, as shown in (19).

4.1.2. \( G_{2-3m} \). As the signal passes from measurement nodes 2a/2b, the electrical output/input to the reciprocal transducers, to 3a/3b, the outputs of signal amplification equipment, a complex gain is applied equal to the complex response of the amplifying equipment. The preamplifier and isolated amplifier are both unity gain, but they do shift the phase of the signals they amplify. To measure phase change \( \phi_{2-3a} \), the phase response of the preamplifier was measured and is nearly flat within the ±0.5° accuracy of the HP35565A used to make the measurement, shown in figure 7. The same procedure is applied to the standard capacitor/differential amplifier package to measure phase change \( \phi_{2-3b} \) and is shown in figure 8.

4.1.3. \( G_{3-4m} \). As the signals pass from measurement nodes (3a)/(3b), each at a different channel of the data acquisition system, to node (4), they are subject to a complex gain that represents magnitude and phase changes that occur due to unequal treatment of the signals at different channels. It is assumed that there is no change applied to the magnitude of the recorded signals from one channel to the next, but the same is not true for phase as the channels are not recorded simultaneously in this setup. The NI-6259 used for data acquisition scans through all active channels on the device consecutively in the space between each sample. As the
sample rate allows, constant padding is used between each channel scan. This padding can be measured and the resulting phase error between channels corrected. Inspecting (20), however, shows that any pair of equal and opposite phase shifts (e^{i\theta} terms) made to signals from the reciprocal transducers in the denominator will cancel. Arranging the three input channels with the reciprocal transducers symmetrically ahead of and behind the hydrophone channel allows $\phi_{3-4a}$ and $\phi_{3-4b}$ to cancel.

4.2. Magnitude corrections

Systematic magnitude errors in the coupler reciprocity measurement procedure are treated by Zalesak [18] and summarized in sections 4.2.1 and 4.2.2. Correction factors $F_A$, $F_H$, and $F_T$ are applied after the phase corrections in (25) producing

$$|M_{H\text{PhCor}}|_{\text{MagCor}} = \frac{V}{\sqrt{F_T}} |M_{H\text{PhCor}}|.$$  \hspace{1cm} (26)

$F_A$ is applied to the compliance term $V/\rho c^2$; $F_H$ is applied once to $e_{HX}$ and once to $e_{HY}$; and $F_T$ is applied once to $e_{YX}$ and once to $e_{XY}$.

4.2.1. Systematic error from compliant transducers and coupler walls. The acoustic compliance found in (19) was derived assuming that the coupler walls and transducers are rigid. In actuality, coupler wall and transducer compliance add a small amount to the acoustic compliance term. Zalesak [18] has calculated these compliances, $1.236 \times 10^{-15}$ m$^3$ Pa$^{-1}$ and $2.658 \times 10^{-15}$ m$^3$ Pa$^{-1}$, respectively, and corrected (20), which assumes only the castor oil has a compliance, by multiplying by the factor

$$F_A = \sqrt{\frac{V/\rho c^2}{V/\rho c^2 + C_T}}.$$ \hspace{1cm} (27)

where $C_T$ is the compliance of the transducers and coupler walls. This correction amounts to between 0.25 and 0.33 dB over the temperature and pressure range of the calibration.

4.2.2. Wavelength correction factors. In developing the sensitivity equation for the coupler, it was assumed that pressure was constant everywhere in the coupler. This statement is a good approximation, but at higher frequencies there will be a non-negligible pressure variation across the coupler. Zalesak [18] derives a correction, F, to remove most of the error from this effect:

$$F = \frac{\sin(kL)}{kL \cos(kd) \cos(kx)}.$$ \hspace{1cm} (28)

where $k$ is the wavenumber of sound in the coupler, $L$ is the length of the coupler, $d$ is the distance of the reciprocal transducers from the walls at each end of the coupler, and $x$ is the distance from the end of the coupler where the origin is defined to the point in the coupler cavity where the sound is measured. The factor $F$ is used to develop the two voltage correction factors $F_H$ and $F_T$, as follows.

Two voltage measurements must be corrected in (26): the hydrophone voltage and the receiving transducer voltage. The hydrophone voltage correction is given by (28) using $x = L/2$:

$$F_H = \frac{\sin(kL)}{kL \cos(kd) \cos(kL/2)} = \frac{2 \sin(kL/2)}{kL \cos kd}.$$ \hspace{1cm} (29)

The receiving transducer voltage correction is also given by (28) using $x = d$:

$$F_T = \frac{\sin(kL)}{kL \cos^2(kd)}.$$ \hspace{1cm} (30)

5. Uncertainty estimation

The new complex coupler reciprocity calibration procedure must account for the uncertainties treated by Zalesak [18] along with uncertainty in the phase response of the coupler components, the electronic equipment in the signal paths, the NI-6259 data acquisition hardware, and the effect of random error in calculating the complex sensitivity. The combined uncertainty is composed of type A and type B as described in [19].
5.1. Estimation of type A uncertainty

Uncertainty in the complex sensitivity includes magnitude and phase. Unlike measurements that fall onto the set of real numbers, magnitudes must be greater than zero and phases fall in the range of \(-\pi\) to \(\pi\), which is problematic for the math involved in uncertainty calculations. Analyzing the uncertainty in the complex plane, where the real and imaginary axes extend to positive and negative infinity avoids these problems [20]. Only after the uncertainties of the real and imaginary parts of the complex sensitivity are calculated are they converted to magnitude and phase.

The random error in the complex sensitivity is estimated using a bivariate normal distribution that accounts for the uncertainty in each of the real and imaginary parts as well as the correlation between the two, represented by a covariance matrix. Using singular value decomposition, the eigenvalues and eigenvectors of the covariance matrix were used to define an elliptical region of uncertainty in the complex plane [20]. An example from a type H48 data set is shown in figure 9. The plot shows the real and imaginary parts of ten measurements of the complex sensitivity recorded at a single frequency. This data group and confidence region represent one of the data groups and confidence regions found in figure 12, a data set recorded across the frequency range 5 Hz to 2 kHz. The lengths of the major and minor semiaxes are related to the eigenvalues of the covariance matrix, and the rotation angle is determined by the correlation of the real and imaginary parts. The length of the major semiaxis, \(a\), is given by the largest eigenvalue and the minor semiaxis, \(b\), by the smallest, as shown in (31) and (32):

\[
a = \sqrt{s\lambda_{\text{max}}},
\]

(31)

\[
b = \sqrt{s\lambda_{\text{min}}},
\]

(32)

where \(s\) is a coverage factor given by

\[
s = \left(\frac{2(n - 1)}{n - 2}F_{2,n-2}(\alpha)\right)^{1/2},
\]

(33)
determined by the \(F_{2,n-2}\) inverse cumulative distribution function with two degrees of freedom and \(n\) observations at a level of confidence \(\alpha\) [21]. The \(F_{2,n-2}\) distribution is used instead of the standard normal distribution used for scalar measurements, resulting in coverage factors different than those used for scalar measurements, (i.e. \(k = 1.96\) for 95% confidence). The ellipse in figure 9 is scaled using (33) with a 95% confidence interval resulting in a coverage factor of 2.45 for 1000 observations.

The uncertainty in the magnitude and phase can be calculated from the magnitude and phase boundary points on the covariance ellipse shown in figure 9. The magnitude boundary points are selected to give the longest and shortest phasors (from the origin to the covariance ellipse boundary) and the phase boundary points, to give the smallest and largest possible angles from a phasor on the covariance ellipse boundary to the real axis. The magnitude and phase errors plotted across the frequency range are shown in figure 10.

5.2. Estimation of type B uncertainty

Random error from the complex voltage measurements contributes to uncertainty in the magnitude and phase of the sensitivity, but other factors will contribute additional uncertainty. The total phase uncertainty is that calculated from the covariance matrix depicted in figure 10, combined with the uncertainty in the phase measurements taken with the HP35565 signal analyzer, \(±0.5°\), used to correct for the phase shift in the differential amplifier,

\[
\delta \phi_{\text{tot}} = \sqrt{\delta \phi_{\text{rand}}^2 + 0.5^2}.
\]

(34)

The final magnitude uncertainty is a combination of uncertainties calculated from the covariance matrix and additional uncertainties from measured quantities in (20), as shown in (35) (complex values assumed here):

\[
\frac{\delta |M_{\text{H}}|}{|M_{\text{H}}|_{\text{total}}} = \sqrt{\left(\frac{\delta |M_{\text{H}}|}{|M_{\text{H}}|}ight)_{\text{cov}}^2 + \left(\frac{\delta |M_{\text{H}}|}{|M_{\text{H}}|}\right)_{\text{add}}^2}.
\]

(35)

The additional uncertainty consists of electrical and acoustic compliance related terms, with a term assigned for each parameter in (20). These additional fractional uncertainty terms can be combined in quadrature as shown in (36):
\[
\frac{\delta|M_H|}{M_H}_{\text{add}} = \frac{1}{2} \left[ \left( \frac{\delta c}{c} \right)^2 + \left( \frac{\delta V}{V} \right)^2 + \left( \frac{\delta e_{CX}}{e_{CX}} \right)^2 + \left( \frac{\delta e_{CY}}{e_{CY}} \right)^2 + \left( \frac{\delta e_{XY}}{e_{XY}} \right)^2 + \left( \frac{\delta C_p}{C_p} \right)^2 + \left( \frac{\delta \rho}{\rho} \right)^2 \right]^{1/2}.
\]

An example set of these terms are shown in figure 11. They are calculated from, in descending order:

(i) The uncertainty in the speed of sound in castor oil, \( \delta c \), derived from uncertainties in the estimate of that value [22] and the uncertainty in the temperature and pressure inside the coupler cavity.

(ii) The uncertainty in the volume of the coupler cavity, \( \delta V \), associated with the method used to measure that volume.

(iii) The uncertainties in voltage measurements across the standard capacitor, \( \delta e_{CX} \) and \( \delta e_{CY} \), derived from the accuracy limitation of the NI-6259 and the custom differential amplifier.

(iv) The uncertainties in voltage measurements from the receiving reciprocal transducers, \( \delta e_{XY} \) and \( \delta e_{YX} \), derived from the accuracy limitation of the NI-6259 and the Ithaco 1201 preamplifier.

(v) The uncertainty in the capacitance of the standard capacitor as manufactured, \( \delta C_p \).

(vi) The uncertainty in the density of castor oil, \( \delta \rho \), derived from the uncertainty in the temperature and pressure inside the coupler cavity.

(vii) The uncertainties in voltage measurements from the hydrophone, \( \delta e_{HX} \) and \( \delta e_{HY} \) due to the accuracy limitation of the NI-6259.

These terms are explained in more detail in [18] and [23].

6. Results and discussion

The results of two complex hydrophone calibrations with systematic magnitude and phase corrections and type A and B uncertainties are presented. The complex sensitivity is plotted in the complex plane, and the magnitude and phase are plotted with respect to frequency.
The complex free-field voltage sensitivity (FFVS) of H48 serial number two with the associated covariance ellipses are shown in figure 12. Each ellipse represents a confidence region around the average sensitivity measurement representing the type A error at a particular frequency between 5 Hz and 2000 Hz (spaced logarithmically), calculated as shown in section 5.1. The frequencies begin at 5 Hz at the smallest values of the real and imaginary parts of the sensitivity in figure 12. As frequency increases, both the real and imaginary parts of the sensitivity increase. As the imaginary part of the sensitivity approaches zero above 100 Hz, the real part decreases until the frequency reaches 2000 Hz.

The magnitude portion of the FFVS is calculated by taking the magnitude of average sensitivity values from figure 12 and converting to dB re 1 V μPa⁻¹. The FFVS magnitude of H48 serial number 4 is shown in figure 13. These data can be verified by comparison with the FFVS magnitude calculated from RMS voltage measurements made with the multimeters in a standard coupler calibration [17]. The FFVS magnitudes measured from each method agree within measurement uncertainty above 15 Hz. Below 15 Hz, the signal is below the minimum frequency the HP 3478A multimeters are certified to measure, which accounts for the disagreement in the data and the upward trend in the FFVS calculated from multimeter data at low frequency. This upward trend is not supported by the H48 design document that specifies the typical FFVS for an H48 hydrophone [2].

The phase response is calculated by taking the angle of the complex values in figure 12. At 1 Hz, the measured phase is 20° ± 1°. It should be noted for low frequency calibrations that the zero phase assumption breaks down between 10 Hz and 20 Hz and gets increasingly worse as frequency continues to decrease.

7. Conclusion

A determination of the phase as well as the magnitude of a hydrophone’s free-field voltage sensitivity was incorporated into the USRD coupler reciprocity calibration procedure by using the hydrophone sensitivity equation for a reciprocity calibration, as shown in (6), with complex voltage inputs and a complex reciprocity parameter, given in (19).

The physical setup remains the same as the previous, magnitude only calibration, with data acquisition equipment added to record time series data in place of the RMS multimeters. The electronics introduce various phase errors to the system, which must be separately measured and corrected to isolate the phase response of the hydrophone under test.

Data sets were recorded on two H48 hydrophones using the standard RMS multimeters and new data acquisition equipment to record time series measurements. The magnitude calculated from these data sets agreed within the expanded measurement uncertainty.

Both magnitude and phase response began to roll off as frequencies approached 1 Hz from above, but the trend is easier to see in the phase data, which begins to roll off a decade higher than the magnitude data for a given cutoff frequency. This roll-off could lead to inaccuracies when measuring phase with an H48 hydrophone or associated secondary references under the assumption that phase is zero down to 1 Hz.

Errors and uncertainties in the calibration were similar to that calculated previously [18] for the magnitude-only system, with the exception of new errors for the new data acquisition equipment. Additional errors and uncertainties were accounted for in the phase measurements, the majority of which come from the electronic equipment. Uncertainty from random error was calculated from the covariance matrix of the complex sensitivity data.

Additional acoustic sources of error come into play higher in the H48 frequency range, and should be modeled in future studies of this coupler. The irregularities in the shape of the coupler, the size of the reciprocal transducers compared to the coupler and their position within, and viscous effects can be estimated with a lumped parameter model, but would be best addressed through finite element analysis.

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