1. Introduction

Low Energy Theorem (LET) has been successfully studied in the case of pion photoproduction at the threshold region [1]. The expansion of the transition amplitude was performed by means of the pseudo-vector theory in terms of the ratio between pion and nucleon masses. The theorem was first put forward by Kroll and Ruderman [3] and is based on the assumption that the pion mass is significantly smaller than the nucleon mass as well as on the hypothesis of the partial conservation of axial current [2]. The three-momentum of pion at the threshold is found to be small. The photon and pion four-momenta have been considered to be small.

For kaon photoproduction LET has not been considered before, since kaon mass is clearly much heavier. Therefore, in this paper we propose to derive LET for the threshold cross-section of kaon photoproduction. The cross-section will be derived by expanding the transition amplitude of kaon photoproduction in the pseudovector theory. The amplitudes are extracted from the appropriate Feynman diagrams. The background amplitude in the pseudo-vector theory is given in Ref. [4].

As widely known, kaon photoproduction uses the nucleon as a target and produces a Λ-hyperon as a recoil. Since the nucleon and Λ-hyperon have different masses, we propose to expand the cross-section of kaon photoproduction in terms of the ratio between kaon mass and the averaged mass of nucleon and Λ. Besides that, we already know that the mass of kaon is heavier than the mass of pion and, furthermore, the threshold of kaon photoproduction is significantly higher than that of the pion. Clearly, we need to expand the amplitude to the much higher order than in the case of pion. In this work, we perform the expansion up to the 3th order.

2. FORMALISM AND METHOD

The Feynman diagrams for kaon photoproduction are shown in Fig. 1. The background amplitude is constructed by using these Feynman diagrams in the pseudo-vector theory [4], i.e.,

\[ M_B = i \frac{g_{K\Lambda N}}{(m_\Lambda + m_\pi)} \bar{u}_\Lambda \left[ \gamma_5 \frac{\not{p} + \not{k} + m_\pi}{s - m_\pi^2} \left( \frac{\not{F}_1^p + i \sigma^{\mu\nu} \epsilon_\mu k_\nu F_2^\rho}{e F_1^p + \frac{k_\rho e F_1^p}{k^2}} \right) \right] (1) \]
Figure 1. Feynman diagrams of kaon photoproduction for the s-, t-, and u-channel, along with the contact diagram to preserve the gauge invariance. [4]

\[ +i\sigma^{\mu\nu}e_\mu k_\nu \mu A F^2 \frac{p_\Lambda - \vec{k} + m_\Lambda}{u - m_\Lambda^2} \gamma_5 q + \gamma_5 (q - \vec{k}) \left\{ \frac{2q - k}{t - m_\Lambda^2} + \frac{k.\epsilon}{k^2} \right\} e F^K u_p \]

\[ +j \frac{g_{K\Sigma N}}{(m_\Lambda + m_p)} \bar{u}_\Lambda \left[ i\sigma^{\mu\nu}e_\mu k_\nu \mu T F^2 \frac{p_\Sigma - \vec{k} + m_\Sigma}{u - m_\Sigma^2} \gamma_5 q \right] u_p , \]

where \( k \) and \( q \) are momenta of kaon and photon, \( \epsilon \) is the photon polarization, \( F \) is the corresponding electromagnetic form factors (equals to 1 in the case of photoproduction), while \( \mu_p \), \( \mu_\Lambda \) and \( \mu_T \) represent the magnetic moments of proton, \( \Lambda \)-hyperon, and \( \Sigma^0 - \Lambda \) transition [4]. For the \( K^+ \) and \( K_1 \) t-channel vector mesons the amplitudes can be written as

\[ M_{fi} = \bar{u}(p_\Lambda) \sum_{j=1}^{6} A_j(s, t, k^2) M_j u(p_p) \]  

(2)

where \( s \) and \( t \) are the Mandelstam variables, while \( M_j \) are the gauge and Lorentz invariant matrices given by

\[ M_1 = \frac{1}{2} \gamma_5 (q \cdot \vec{k} - \vec{k} \cdot \ell) \]

\[ M_2 = \gamma_5 [(2q - k) \cdot \epsilon P \cdot k - (2q - k) \cdot k P \cdot \epsilon] \]

\[ M_3 = \gamma_5 (q \cdot k - q \cdot \epsilon \vec{k}) \]

\[ M_4 = i\epsilon_{\mu\nu\rho\sigma} \gamma^{\mu} q^{\nu} \epsilon^{\rho} k^{\sigma} \]

\[ M_5 = \gamma_5 [q \cdot \epsilon k^2 - q \cdot k \cdot \epsilon] \]

\[ M_6 = \gamma_5 (k \cdot \epsilon \vec{k} - k \cdot \ell) \]

(3)

with \( P = (p_p + p_\Lambda)/2 \) and \( \epsilon_{\mu\nu\rho\sigma} \) is Levi-Civita tensor.

The last diagram shown in Fig. 1 is the contact term required in the pseudo-vector theory to maintain the gauge invariance of the amplitude. In this work, we use the contact term based on the previous work [5],

\[ M^c = \frac{F^c g_{K\Lambda N}}{k^2 (m_p + m_\Lambda)} \bar{u}_\Lambda \left[ -\gamma_5 (k^2 \ell - k \cdot \ell) \right] \]  

(4)

Equation (2) presents the amplitude of kaon photo- and electroproduction. In the case of photoproduction, i.e., \( k^2 = 0 \), the matrices \( M_5 \) and \( M_6 \) do not exist and the formulation is significantly simplified.
LET assumes that kaon has a small mass compared to other baryon masses. To obtain the cross section at threshold, we approximate the three-momentum of kaon to zero, i.e., $|q| \approx 0$. The expansion of amplitude will be performed by using this approximation.

At $q \approx 0$, by using Eqs. (1), (2) and (3) it is obvious that $A_1(kq = m_K^2 = 0) = 0$. Therefore, we have to expand the $A_1$ in terms of $m_K$ up to 2nd order. The function $A_i$ can obtained from the previous study [4].

$$
A_1 = - \frac{G_{K^*}^T F^{K^*}}{M(t - m_{K^*}^2 + im_{K^*} \Gamma_{K^*})(m_p + m_\Lambda)}
$$

$$
A_2 = \frac{G_{K^*}^T F^{K^*}}{M(t - m_{K^*}^2 + im_{K^*} \Gamma_{K^*})(m_p + m_\Lambda)} - \frac{G_{K^1}^T F^{K^1}}{M(t - m_{K^1}^2 + im_{K^1} \Gamma_{K^1})(m_p + m_\Lambda)}
$$

$$
A_3 = - \frac{G_{K^*}^V F^{K^1}}{M(t - m_{K^1}^2 + im_{K^1} \Gamma_{K^1})}
$$

$$
A_4 = \frac{G_{K^1}^V F^{K^*}}{M(t - m_{K^*}^2 + im_{K^*} \Gamma_{K^*})}
$$

$M = 1$GeV is introduced to make coupling constants of $K_1$ and $K^*$ dimensionless. $\Gamma_i$ is width of each particle. The imaginary part of $A_i$ will be ignored to simplify the calculation.

3. GENERAL RESULTS AND DISCUSSION

The analytic form of cross-section is calculated from the expansion of Eqs. (1), (2), (3) and (4) in terms of the ratio of kaon mass (493.677 MeV) and the average of nucleon (938.272 MeV) and $\Lambda$ (1115.683 MeV) masses [7] ($m \approx m_p \approx m_\Lambda \approx m_\Sigma$).

$$
|M_{fi}|^2 = \left| -6 g_{K^*N}^2 + (6 e g_{K^*N}^2 + 4 A_1 g_{K^*N} m^2) x + (- g_{K^*N}^2 - 2 e g_{K^*N}^2 + e^2 g_{K^*N}^2 + 4 g_{K^*N}^2 \mu_\Lambda m - 12 e g_{K^*N}^2 \mu_\Sigma m - 4 g_{K^*N} \mu_\Sigma m \mu_\tau m + 24 A_3 g_{K^*N} m^3 - 16 A_3^2 m^4) x^2 + (g_{K^*N}^2 + 2 e g_{K^*N}^2 + 128 A_4 g_{K^*N} m + 2 g_{K^*N}^2 \mu_\Lambda m + 4 e g_{K^*N}^2 \mu_\Lambda m + 6 e g_{K^*N} \mu_\Sigma m + 4 e g_{K^*N} \mu_\Sigma m + 2 g_{K^*N} \mu_\Sigma m \mu_\tau m + 6 A_1 g_{K^*N} m^2 + 4 A_1 e g_{K^*N} m^2 - 8 A_3 g_{K^*N} m^3 - 8 A_3 e g_{K^*N} m^3 + 16 A_1 g_{K^*N} \mu_\Lambda m^3 + 16 A_1 e g_{K^*N} \mu_\Sigma m^3 + 16 A_1 \mu_\Sigma m^3) x^3 + O(x^4) \right|
$$

where $x = m_K/m$, with $m$ the average mass of nucleon and $\Lambda$, and $O(x^4)$ indicates the neglected higher orders.

The threshold cross-section of kaon photoproduction $\gamma p \rightarrow K^+\Lambda$ is given by

$$
\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \frac{1}{s} \frac{|k|}{|q|} |M_{fi}|^2
$$

From the above result it is obvious that the contribution of $A_4$ is very small. Since the cross-section does not depend on $A_2$, only the $A_1$ and $A_3$ appear in the formula.

4. SUMMARY AND CONCLUSIONS

We have successfully expanded the cross-section of kaon photoproduction at the threshold region up to the 3th order. As the follow up of this work, we will calculate the cross-section up to 5th order or more to refine the calculation. We will also try to compare the cross-section with experimental data near the threshold by using the particles properties listed in the particle listings of Particle Data Group (PDG) [7].
5. ACKNOWLEDGMENTS
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