Inspection of new physics in $B_s^0 \to K^+K^-$ decay mode

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Abstract

We scrutinize the penguin dominated $B_s \to K^+K^-$ decay mode involving $b \to s$ quark level transition in family non-universal $Z'$ and vector-like down quark model. There is discrepancy in the standard model branching ratio value of this mode with the experimental results reported by Belle, CDF and LHCb Collaborations. Additionally, the measured values of CP-violating asymmetries $C_{K^+K^-}$ and $S_{K^+K^-}$ deviate from the SM results. We constrain the new parameter space by using the existing experimental limits on leptonic $B_s \to \ell\ell$ ($\ell = e, \mu, \tau$) processes. We then check the effects of new physics on the branching ratio and CP violating parameters of the $B_s \to K^+K^-$ process.

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I. INTRODUCTION

The in-depth search for physics beyond the standard model (SM) plays an important role in the area of particle physics. It is known that the CP asymmetry, the symmetry violation of combination of charge conjugation (C) and parity (P), is the main source for matter-antimatter asymmetry that is observed in our present universe. In the sector of quarks, the Cabbibo–Kobayasi–Maskawa (CKM) matrix indicates a message for an insight to the gateway of CP violation particularly in B and K meson decays in the SM. However, it is not sufficient to understand the observed baryon asymmetry. Recently, various experimental hunts are going on to probe the physics beyond the SM. In this regard, B meson system provides an important role to study prominent observables like branching ratio, and CP violating parameters such as direct and mixing induced CP asymmetry to probe new physics.

We would like to study the $b \to s$ penguin dominated $B^0_s \to K^+K^-$ decay mode which appears to have discrepancies in standard model values of CP-averaged branching ratio and CP violating parameters with the corresponding observed values. The theoretical result for the observables are given in the TABLE-I. Additionally, the TABLE-II shows the results from Belle, CDF, LHCb Collaborations along with world averages. Thus these discrepancies between observed and predicted values could lead to probe the physics beyond the SM.

In addition to this, the leptonic decays of pseudo-scalar meson $B^0_s$ sector plays a vital role and enthusiastically makes more attention to explore the physics beyond the SM. In particular, we study $B^0_s \to \mu^+\mu^-$ decay mode because of careful observation of decay constant of neutral $B^0_s$ meson from lattice. On the other side, the study of $B^0_s \to \ell\ell$ (where $\ell = e, \tau$) put a less mark on the board as they have upper bounds. The former one has branching ratio with an upper limit of $2.8 \times 10^{-7}$ (90% C.L) [1], reported by LHCb where as $<6.8 \times 10^{-3}$ (95% C.L) reported by CDF Collaboration [2] for later decay mode. The SM values of branching ratio of $B^0_s \to \tau^+\tau^-$ and $B^0_s \to e^+e^-$ decays have $\mathcal{O}(10^{-7})$ and $\mathcal{O}(10^{-14})$ respectively where as for $B^0_s \to \mu^+\mu^-$, it is of the order of $10^{-9}$ [3]. Thus there is a possibility of contribution to both decays along with $B^0_s \to \mu^+\mu^-$ mode in the new physics scenario.

Inspired by these discrepancies of the $B^0_s \to K^+K^-$ decay mode, we would like to investigate, in QCD factorisation approach, the NP effect on CP-averaged branching ratio as well as the CP violation parameters arising due to $Z'$ model where an extra $U(1)'$ gauge boson $Z'$ takes part in the play. Several studies [4–7] have been done in the scenario of FCNC
TABLE I: The theoretical predictions of CP-averaged branching ratio ($\mathcal{B}$)(in the units of $10^{-6}$), CP-violating asymmetries such as direct ($C_{KK}$ in %) and mixing-induced CP asymmetry ($S_{KK}$).

| $B_s^0 \rightarrow K^+K^-$ | QCDF [11] | PQCD [12, 13] | SCET [14] |
|---------------------------|----------|----------------|-----------|
| $\mathcal{BR}$            | 25.2 $^{+12.7+12.5}_{-7.2-9.1}$ | 13.6 $^{19.7+6.6}_{-5.7}$ | 18.2 |
| $C_{KK}$                  | -7.7 $^{+1.6+4.0}_{-1.2-5.1}$ | -23.3 | -6 |
| $S_{KK}$                  | 0.22 $^{+0.04+0.05}_{-0.05-0.03}$ | 28 $^{20.6}_{19}$ |

TABLE II: Measured values of branching ratio ($\mathcal{B}$)(in the units of $10^{-6}$), CP-violating asymmetries such as direct ($C_{KK}$) and mixing-induced ($S_{KK}$) reported by Belle, CDF, LHCb collaborations and world averages.

| $B_s^0 \rightarrow K^+K^-$ | Belle [15] | CDF [16] | LHCb [17–19] | HFLAV [20] | PDG [21] |
|---------------------------|-----------|--------|-------------|----------|--------|
| $\mathcal{BR}$            | 38$^{+10}_{-09}$ $\pm 7$ | 23.9 $\pm 1.4 \pm 3.6$ | 23.0 $\pm 0.7 \pm 2.3$ | 24.8 $\pm 1.7$ | 25.9 $\pm 1.7$ |
| $C_{KK}$                  | --        | --     | 0.14 $\pm 0.11 \pm 0.03$ | 0.20 $\pm 0.06 \pm 0.02$ | -- |
| $S_{KK}$                  | --        | --     | 0.30 $\pm 0.12 \pm 0.04$ | 0.18 $\pm 0.06 \pm 0.02$ | -- |

mediated by $Z'$ boson. And the new physics impact due to vector-like down quark (VLDQ) model [7–10] where an extra $SU(2)_L$ singlet down type quark has been added to SM quark sector which include a CP and flavor violating FCNC mediated by $Z$ boson at tree level. The new coupling $Z' - b - s (Z - b - s)$ associated with $Z'$ (VLDQ) model can be constrained by using the experimental limit for all leptonic modes, and using the allowed parameter space, we examine the new physics impact on $B_s^0 \rightarrow K^+K^-$ decay mode observables.

The layout of this paper is structured as follows. In section II, we discuss the effective Hamiltonian responsible for the non-leptonic $b \rightarrow sq\bar{q}$ processes. We have also presented the framework for $B_s \rightarrow K^+K^-$ observables such as branching ratio and CP-violating parameters in the standard model. We constrain the new parameter space of $Z'$ model from the branching ratios of leptonic $B_s$ modes in section III, and address the footprint of this
model on $B_s \to K^+K^-$ process by using the new couplings. In section IV, we draw an
attention to the interactions of the VLDQ model and check the impact on the aforementioned
observables for $B_s \to K^+K^-$ decay mode. In the end, our results are summarized in section
V.

II. $B_s \to K^+K^-$ PROCESS IN THE STANDARD MODEL

In the standard model, the penguin dominated $B_s \to K^+K^-$ decay mode receives con-
tribution from quark level transition $b \to s$ where the weak effective Hamiltonian describing
the decay $b \to s q \bar{q}$ is given as \cite{22}

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ V_{ub}^* V_{us} \left[ C_1(\mu) O_1^u(\mu) + C_2(\mu) O_2^u(\mu) \right] - V_{tb}^* V_{ts} \left[ \sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right] \right] + \text{h.c} \quad (1)$$

where $G_F = 1.16639 \times 10^{-5} \text{ GeV}^{-2}$ is the Fermi coupling constant, $V_{\alpha\beta}$ are the CKM matrix
element ($\alpha, \beta = u, b, s, t$). Here $O_{1,2}$ are the current-current operators, $O_{3,\ldots,6}$ are QCD
penguin operators, $O_{7,\ldots,10}$ are electroweak penguin operators, and $C_i(\mu)(i = 1, \ldots, 10)$ are the
corresponding Wilson coefficients evaluated at renormalization scale $\mu = m_b$ scale.

Using the framework of QCD factorization approach \cite{23}, the decay amplitude can be
written in the form as

$$\langle K^+K^-|O_i|B_s^0 \rangle = \langle K^+K^-|O_i|B_s^0 \rangle_{\text{fact}} \left[ 1 + \sum r_n \alpha_n^p + O(\Lambda_{QCD} / m_b) \right], \quad (2)$$

where $\langle K^+K^-|O_i|B_s^0 \rangle_{\text{fact}}$ represents the hadronic factorized matrix element, the second and
third terms in the square bracket are higher order corrections. $\alpha_s$ is the strong coupling
constant, $\Lambda_{QCD} = 0.225 \text{ GeV}$ is the QCD scale.

In the heavy quark limit, the amplitude of this decay mode can be represented as \cite{23},

$$A_{B_s^0 \to K^+K^-} = A_{KK} \left[ \delta_{pu} \alpha_1 + \alpha_4^p + \alpha_{4,EW}^p + \beta_3^p + \beta_4^p - \frac{1}{2} \beta_{3,EW}^p - \frac{1}{2} \beta_{4,EW}^p \right] + B_{KK} \left[ \delta_{pu} b_1 + b_4^p + b_{4,EW}^p \right], \quad (3)$$

where

$$A_{KK} = i \frac{G_F}{\sqrt{2} m_B^2} F_0^{B_s \to K}(0) f_K, \quad B_{KK} = i \frac{G_F}{\sqrt{2}} f_{B_s} f_K f_K, \quad (4)$$
which includes the form factor $F_{B_s^0 \to K}(0)$ at zero recoil momentum, and decay constants $f_{B_s}$ and $f_K$. The expressions of coefficients $\alpha_i$ and $\beta_i(b_i)$ are given in detail in ref. [23], which include factorisable along with non factorisable contributions to the above decay amplitude.

The CP-averaged branching ratio can be obtained using the following formula,

$$\mathcal{BR}(B_s^0 \to K^+K^-) = \frac{\tau_{B_s}}{8\pi m_{B_s}^2} \frac{p_c}{2} \left[ |A_{B_s^0 \to K^+K^-}|^2 + |A_{\bar{B}_s^0 \to K^+K^-}|^2 \right],$$

(5)

where $\tau_{B_s}$ ($m_{B_s}$) are the life time (mass) of $B_s$ meson and the center of mass (c.o.m) momentum in the rest frame of $B$ meson is given as

$$p_c = \sqrt{\left(m_{B_s}^2 - (m_{K^+} + m_{K^-})^2\right)\left(m_{B_s}^2 - (m_{K^+} - m_{K^-})^2\right)}.$$  

(6)

The time dependent CP asymmetry of $B_s^0$ meson decaying to final CP eigenstate $K^+K^-$ can be written as [24]

$$A_{K^+K^-}(t) = C_{KK} \cos(\Delta M_{B^0_s} t) + S_{KK} \sin(\Delta M_{B^0_s} t),$$

(7)

where $C_{KK} = \frac{|A|^2 - 1}{1 + |A|^2}$ and $S_{KK} = 2 \frac{\operatorname{Im}(\lambda)}{1 + |A|^2}$ are the direct and the mixing- induced CP asymmetries, respectively. The parameter $\lambda$ is given as

$$\lambda = \frac{q}{p} \frac{A_{B_s^0 \to K^+K^-}}{A_{\bar{B}_s^0 \to K^+K^-}},$$

(8)

where, $q$ and $p$ are mixing parameters which are connected to the standard model CKM elements as

$$\frac{q}{p} = \frac{V_{tb}^* V_{ts}}{V_{tb} V_{ts}^*}.$$  

(9)

Symbolically, the amplitude of the $B_s \to K^+K^-$ decay mode can be written as

$$A_{B_s^0 \to K^+K^-} = \zeta_u A_u + \zeta_c A_c$$

$$= \zeta_c A_c \left[ 1 + \varphi a e^{i(\delta_1 - \gamma)} \right]$$

(10)

where $\zeta_q = V_{qs}^* V_{qs} (q = u, c)$, $a = \frac{|\zeta_u|}{|\zeta_c|}$, $\varphi = \frac{|A_u|}{|A_c|}$, $\gamma$ is the weak phase of CKM element $V_{ub}$, and $\delta_1$ is the relative weak phase between $A_u$ and $A_c$. From the amplitude given in 10, the parameters $\mathcal{BR}$, $C_{KK}$ and $S_{KK}$ can be obtained respectively as

$$\mathcal{BR} = \frac{\tau_{B_s} p_c}{8\pi m_{B_s}^2} \left| \frac{\zeta_c A_c}{2} \right|^2 \left\{ 1 + (\varphi a)^2 + 2 \varphi a \cos \delta_1 \cos \gamma \right\},$$

(11)

$$C_{KK} = -\frac{2 \varphi a \sin \delta_1 \sin \gamma}{1 + (\varphi a)^2 + 2 \varphi a \cos \delta_1 \cos \gamma},$$

(12)

$$S_{KK} = \frac{\sin 2\beta + 2 \varphi a \sin \delta_1 \sin(2\beta - \gamma) + (\varphi a)^2 \sin(2\beta - 2\gamma)}{1 + (\varphi a)^2 + 2 \varphi a \cos \delta_1 \cos \gamma}.$$  

(13)
For numerical computation of these observables in the SM, the CKM matrix elements along
with the weak angle $\gamma$, all particle masses and the life time of $B_s$ meson are taken from
[21]. We use the value of $\beta_s$ angle from [25]. The calculated Wilson coefficients in naive
dimensional regularization scheme at NLO are taken from [22] at $m_b$ scale. The value of
form factor $F_{B_s}(0)$ at zero recoil momentum and the decay constant $f_{B_s}$ are taken from [26].
In addition to this the decay constant $f_K$ is taken from [27]. Using these input values, the
predicted SM values of the $B_s \to K^+K^-$ observables are given as

$$ BR = (34.37^{+7.99}_{-5.61}) \times 10^{-6}, $$ \hfill (14) \\
$$ C_{KK} = -0.11^{+0.0168}_{-0.0151}, $$ \hfill (15) \\
$$ S_{KK} = 0.32^{+0.042}_{-0.035}. $$ \hfill (16) \\

Here, the theoretical uncertainties for the above observables mainly arise from form factor,
decay constants [26, 27], CKM matrix elements [21].

III. Z’ MODEL

In this section we discuss the effects of new physics associated with $Z'$ model on the
observables of $B_s \to K^+K^-$ decay process. We constrain the $Z'$ new couplings by using
the experimental limits on $B_s \to \ell\ell$ (where $\ell$ is any charged leptons), mediated by the
FCNC transitions $b \to s\ell\ell$. These are the theoretically cleanest $B$ decays as the only non-
perturbative quantity involved in the description of these processes is the $B_s$ meson decay
constant.

A. $B_s \to \ell^+\ell^- (\ell = e, \mu, \tau)$ processes

In the SM, the effective Hamiltonian for quark level transitions $b \to s\ell^+\ell^-$ is given by
[28, 29]

$$ H_{\text{eff}} = -\frac{G_F}{\sqrt{2}} [\lambda_{\ell}^{(q)} H_{\text{eff}}^{(t)} + \lambda_{u}^{(q)} H_{\text{eff}}^{(u)}] + h.c., $$ \hfill (17) \\

where

$$ H_{\text{eff}}^{(u)} = C_1(O_1^c - O_1^u) + C_2(O_2^c - O_2^u), $$ \\
$$ H_{\text{eff}}^{(t)} = C_1 O_1^c + C_2 O_1^u + \sum_{i=3}^{10} C_i O_i, $$ \hfill (18)
\[ \chi_k^{(q)} = V_{kb}V_{kq}^* \] and \( C_i \)'s are the Wilson coefficients. Using effective Hamiltonian 17, the transition amplitude for this process is given as

\[ \mathcal{M}(B_s \rightarrow \ell^+\ell^-) = i \frac{G_F \alpha}{\sqrt{2\pi}} V_{tb}V_{ts}^* f_{B_s} C_{10} m_\ell (\bar{\ell}\gamma_5 \ell), \quad (19) \]

where \( \alpha \) is the fine structure constant. Here, we have used the vacuum insertion method to define the decay constant in the matrix element as

\[ <0|\bar{s}\gamma^\mu\gamma_5 b|B_s^0> = i f_{B_s} p_B^\mu, \quad (20) \]

where \( p_B^\mu = p_\ell^+ + p_\ell^- \). In general, from equation 19, the associated branching ratio is given as 30

\[ \mathcal{B}_R(B_s \rightarrow \ell^+\ell^-) = \frac{G_F^2 \tau_{B_s}}{16\pi^3} \alpha^2 f_{B_s}^2 m_{B_s} m_\ell^2 |V_{tb}V_{ts}^*|^2 |C_{10}|^2 \sqrt{1 - \frac{4m_\ell^2}{m_{B_s}^2}} \quad (21) \]

Using the \( B_s \) decay constant from 26 and remaining input parameters from PDG 21, the predicted SM branching ratio values are presented below. The errors in the SM results are coming mainly from decay constants and CKM matrix elements. Here we also show the corresponding experimental limits for all leptonic decay modes 21.

\[ \mathcal{B}_R^{SM}_{B_s \rightarrow \mu^+\mu^-} = (3.47^{+1.21}_{-0.85}) \times 10^{-9}, \quad \mathcal{B}_R^{Exp}_{B_s \rightarrow \mu^+\mu^-} = (2.7^{+0.6}_{-0.5}) \times 10^{-9}, \]
\[ \mathcal{B}_R^{SM}_{B_s \rightarrow \tau^+\tau^-} = (7.44^{+2.6}_{-1.82}) \times 10^{-7}, \quad \mathcal{B}_R^{Exp}_{B_s \rightarrow \tau^+\tau^-} < 6.8 \times 10^{-3}, \]
\[ \mathcal{B}_R^{SM}_{B_s \rightarrow e^+e^-} = (8.19^{+2.86}_{-2.01}) \times 10^{-14}, \quad \mathcal{B}_R^{Exp}_{B_s \rightarrow e^+e^-} < 2.8 \times 10^{-7}. \quad (22) \]

Though \( B_s \rightarrow \ell^+\ell^- \) decays occur only at one-loop level in the SM, these processes can occur at tree level in the presence of new Z' gauge boson arising due to the \( U(1)' \) gauge extension of the SM. The effective Hamiltonian corresponding to the transition \( b \rightarrow s\ell^+\ell^- \) process is given by 31, 32

\[ \mathcal{H}_{eff}^{Z'} = -\frac{2G_F}{\sqrt{2}} V_{tb}V_{ts}^* \left( \frac{g'M_Z}{g_1 M_Z'} \right)^2 \left[ \frac{U_{b\ell}^L U_{\ell\ell}^L}{V_{tb}V_{ts}^*} (\bar{s}b)_{V-A}(\bar{\ell}\ell)_{V-A} \right] \]
\[ - \frac{U_{b\ell}^L U_{\ell\ell}^R}{V_{tb}V_{ts}^*} (\bar{s}b)_{V-A}(\bar{\ell}\ell)_{V+A} + h.c., \quad (23) \]

where \( g_1(g') \) is the coupling constant of \( Z(Z') \) boson. According to the SM effective Hamiltonian 17, the Hamiltonian in \( Z' \) can be written as

\[ \mathcal{H}_{eff}^{Z'} = -\frac{G_F}{\sqrt{2}} V_{tb}V_{ts}^* \left[ C_{9}^{Z'} O_9 + C_{10}^{Z'} O_{10} \right] + h.c., \quad (24) \]
where the new Wilson coefficients are given as

\[ C_{Z'}^9 = -2 \left( \frac{g'M_Z}{g_1 M_{Z'}} \right)^2 \frac{U_{bs}^L}{V_{tb} V_{ts}^*} (U_{\ell\ell}^L + U_{\ell\ell}^R), \]

\[ C_{Z'}^{10} = 2 \left( \frac{g'M_Z}{g_1 M_{Z'}} \right)^2 \frac{U_{bs}^L}{V_{tb} V_{ts}^*} (U_{\ell\ell}^L - U_{\ell\ell}^R) = 2 \left( \frac{g'M_Z}{g_1 M_{Z'}} \right)^2 |U_{bs}| e^{i\phi_s} (U_{\ell\ell}^L - U_{\ell\ell}^R), \]

with \( \phi_s \) is the associated weak phase of \( U_{bs} \). We consider \( \frac{g'}{g_1} \sim 1 \) with the assumption that both the \( U(1) \) groups have same origin from some grand unified theory. For a TeV-scale \( Z' \) boson, their ratio of masses \( M_Z/M_{Z'} \) will be \( \sim 10^{-1} \). In this analysis, the coupling of \( Z' \) boson to leptons \( U_{\ell\ell}^{L(R)} \) are considered to be SM-like. Now, comparing the theoretical values of \( B_s \to \ell\ell \) branching ratios with their corresponding 1\( \sigma \) range of experimental data, we constrain the new \( Z' - b - s \) coupling \( (U_{bs}) \) and weak phase \( (\phi_s) \) as shown in FIG. 1. From this figure, the allowed range of \( U_{bs} \) and \( \phi_s \) parameters of \( Z' \) model are given as

\[ 1.2 \times 10^{-5} \leq |U_{bs}| \leq 0.99, \quad 0^\circ \leq \phi_s \leq 360^\circ. \]

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{constraints.png}
\caption{Constraints on new coupling parameter from the branching ratios of leptonic \( B_s \to \ell^+\ell^- (\ell = e, \mu, \tau) \) processes in \( Z' \) model.}
\end{figure}

**B. Impact on non-leptonic \( B_s \to KK \) decay mode**

Now, we will discuss the impact of family non-universal \( Z' \) gauge boson on the \( B_s \to K^+K^- \) decay mode. The effective Hamiltonian describing the \( b \to sq\bar{q} \) (q=u,d) transition
for $B_s \to K^+K^-$ decay mode is given as \[5\]

$$
\mathcal{H}_{\text{eff}}^{Z'} = \frac{2G_F}{\sqrt{2}} \left( \frac{g' M_Z}{g_1 M_{Z'}} \right)^2 U_{bs}^L \sum_q [U_{qq}^L(qq)_{V-A} + U_{qq}^R(qq)_{V+A}] + h.c.
$$

(27)

Due to hemiticy of effective Hamiltonian, the diagonal elements of $U_{qq}^{L(R)}$ are real whereas the non-diagonal elements of $U_{bs}^{L(R)}$ might be complex along with a phase $\phi_s$. Now, comparing the effective Hamiltonian of $Z'$ model \[27\] with the general effective Hamiltonian \[\text{(28)}\]

$$
\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_q [C_9' O_9 + C'_7 O_7],
$$

we obtain

$$
C_{9(7)}' = -\frac{4}{3} \left( \frac{g' M_Z}{g_1 M_{Z'}} \right)^2 U_{bs}^L \left( U_{uu}^{L(R)} - U_{dd}^{L(R)} \right). \quad \text{(29)}
$$

where $C_{9}', C'_7$ are the new Wilson coefficients arising due to $Z'$ gauge boson. Many studies have been done in \[5\, 33\, 37\] with the manifestation of electroweak contribution assuming $U_{uu}^{L(R)} \approx -2U_{dd}^{L(R)}$. Thus,

$$
C_{9(7)}' = 4 \left( \frac{g' M_Z}{g_1 M_{Z'}} \right)^2 \frac{U_{bs}^L U_{dd}^{L(R)}}{V_{tb} V_{ts}^*}. \quad \text{(30)}
$$

Now for convenience, these coefficients can be written in the following parametric form as

$$
C_{9(7)}' = 4 \left( \frac{g' M_Z}{g_1 M_{Z'}} \right)^2 U_{bs}^L = 4 \left( \frac{g' M_Z}{g_1 M_{Z'}} \right)^2 \frac{|U_{bs}^L| e^{i\phi_s}}{V_{tb} V_{ts}^*},
$$

(31)

where the assumption of $U_{qq}^{L(R)} \sim 1$ has been taken out from experimental data of $B_s$ meson \[4\]. The decay amplitude in presence of additional $Z'$ boson can be written as

$$
\mathcal{A}_{B_s^0 \to K^+K^-} = A_{KK} [\delta_{pu} \alpha_1 + \alpha_4 + \alpha_{4,EW}^p + \beta_3 + \beta_4 - \frac{1}{2} \beta_{3,EW}^p - \frac{1}{2} \beta_{4,EW}^p] + B_{KK} [\delta_{pu} b_1 + b_4 + b_{4,EW}^p] + \zeta_t \left[ \tilde{A}_{KK} (\tilde{\alpha}_4 + \tilde{\alpha}_{4,EW}^p + \tilde{\beta}_3 - \frac{1}{2} \tilde{\beta}_{3,EW}^p) \right], \quad \text{(32)}
$$

where terms $\tilde{\alpha}$ and $\tilde{\beta}$ arise due to new physics contributions. We can represent the above transition amplitude in the parametrized form as

$$
\mathcal{A} = \mathcal{A}^{SM} - \zeta_t \mathcal{A}^{NP} = \zeta_t A_c \left[ 1 + \varphi e^{i(\delta_1 - \gamma)} - \varphi' e^{i(\delta_2 + \phi)} \right]. \quad \text{(33)}
$$

In addition to $\varphi$ and $a$, given in the previous section, the new parameters in the above amplitude are defined as $b = |\bar{G}| \zeta_t^2$, $\varphi' = |\mathcal{A}^{NP}|$, and $\delta_2$ is the relative strong phase.
The CP-averaged branching ratio can be written as

\[
\langle \text{BR} \rangle = \frac{\tau_{B_s} P_c}{8 \pi m_{B_s}^2} |\zeta_c A_c|^2 \left[ G + 2 \phi a \cos \delta_1 \cos \gamma - 2 \phi' b \cos \delta_2 \cos \phi_s - 2 \phi \phi' ab \cos(\delta_1 - \delta_2) \cos(\gamma + \phi_s) \right].
\]  

(34)

The direct CP asymmetry is given as

\[
C_{KK} = -\frac{2 \left[ r a \sin \delta_1 \sin \gamma + \phi' b \sin \delta_2 \sin \phi_s + \phi \phi' ab \sin(\delta_2 - \delta_1) \sin(\gamma + \phi_s) \right]}{\left[ G + 2 \left( \phi a \cos \delta_1 \cos \gamma - 2 \phi' b \cos \phi_s \delta_2 - 2 \phi \phi' bb' \cos(\gamma + \phi_s) \cos(\delta_2 - \delta_1) \right) \right]},
\]  

(35)

The mixing induced CP asymmetry can be represented as

\[
S_{KK} = \frac{M}{G + 2 \left( \phi a \cos \delta_1 \cos \gamma - 2 \phi' b \cos \phi_s \delta_2 - 2 \phi \phi' bb' \cos(\gamma + \phi_s) \cos(\delta_2 - \delta_1) \right)},
\]  

(36)

where

\[
G = 1 + (\phi a)^2 + (\phi' b)^2
\]  

and

\[
M = \sin 2\beta + 2 \phi a \cos \delta_1 \sin(2\beta + \gamma) - 2 \phi' b \cos \delta_2 \sin(2\beta - \phi_s) + (\phi a)^2 \sin(2\beta + 2\gamma)
\]  

(37)

\[
+ (\phi' b)^2 \sin(2\beta - 2\phi_s) - 2 \phi \phi' ab \cos(\delta - \delta') \sin(2\beta + \gamma - \phi_s).
\]  

(38)

After collecting the detailed expression for CP-averaged branching ratio and the CP violating parameters in the presence of new \(Z'\) gauge boson, we now proceed for numerical analysis for these observables. Using the allowed parameter space from equation(26) we show the variation of CP-averaged branching ratio (top-left panel), direct CP violation (top-right panel) and mixing induced CP asymmetry (bottom panel) with respect to mixing weak phase \(\phi_s\) with some benchmark entries of \(U_{bs}\) (color) as 0.1(Green), 0.3(Blue) and 0.5(Orange) in FIG. 2. As we see from the top-left one, for both blue and green colored \(U_{bs}\) values, the NP effect of branching ratios lie in 1\(\sigma\) range where as the one with 0.5 value, is not accommodating in the same error limit. On the other side, if we observe the CP violating parameters in the new physics scenario, both \(C_{KK}\) and \(S_{KK}\) lie within 1\(\sigma\) experimental data. FIG.3 depicts the correlation among all the discussed observables. In this figure, the top-left (right) panel displays \(BR - C_{KK}\) (\(S_{KK}\)) correlations and the plot in the bottom panel shows a relationship between \(C_{KK}\) and \(S_{KK}\). We have shown the predicted results of branching ratios, CP violating observables for different values of \(U_{bs}\) and \(\phi_s\) in the top-section of Table III.
FIG. 2: Variation of CP-averaged branching ratio (in the units of $10^{-5}$) (top-left panel), direct CP asymmetry (top-right panel) and mixing induced CP asymmetry (bottom panel) (in %) with the new weak phase $\phi_s$ for different $|U_{bs}|$ entries. The black color horizontal dotted line represents the central experimental value while the magenta dotted lines denote the $1\sigma$ error limit.

IV. VECTOR LIKE DOWN QUARK (VLDQ) MODEL

Here we study the minimal extension of SM where the quark sector is expanded by an extra vector-like down quark. Because of this, we obtain a $4 \times 4$ matrix $V_{i\alpha} \ (i = u, c, t, t'$ and $\alpha = d, s, b, b')$ from which the interaction of this extra down-type quark with the SM quarks could be obtainable and scrutinize the deviations of the unitarity relation of the CKM matrix. This mixing provides a remarkable study of flavor changing neutral current (FCNC) interaction where $Z$ particle is mediated through tree level contribution. In general, this model include the following Lagrangian [10].

$$\mathcal{L}_Z = \frac{g}{2\cos\theta_W} \left[ \bar{u}_{Li} \gamma^\mu u_{Li} - \bar{d}_{L\alpha} Q_{\alpha\beta} \gamma^\mu d_{L\alpha} - 2 \sin^2\theta_W J_{\text{em}}^\mu \right] Z_\mu$$

where $L$ denote the left handed chiral particles, $i$ and $\alpha, \beta$ denote the generation indices for up-type and down-type quarks respectively. The second term in the above Lagrangian corre-
FIG. 3: Correlation plots of CP-averaged branching ratio (in the units of $10^{-4}$) with direct CP asymmetry (top-left panel) and mixing induced CP asymmetry (top-right panel) (in %), and direct CP asymmetry with mixing induced CP asymmetry (bottom panel).

The matrix $Q_{\alpha\beta}$ can be represented as

$$Q_{\alpha\beta} = \sum_{i=\mu, c, t} V_{\alpha i}^* V_{i\beta} = \delta_{\alpha\beta} - V_{4\alpha}^* V_{4\beta}$$

(39)

Here $V$ is not unitary as an extra down-type vector like quark of charge $(-\frac{1}{3})$ has been added to the SM. It provides a new signal to probe the physics beyond the SM and modify the CP asymmetries and branching ratio predictions. We constrain the new parameters from the $\text{Br}(B_s \to \ell^+\ell^-)$, to be presented in the subsequent section.

**A. $B_s \to \ell^+\ell^- (\ell = e, \mu, \tau)$ processes**

Though $B_s \to \ell^+\ell^-$ process are suppressed in the SM, but can be significant in the presence of extra vector like down quark particle where $Z$ is mediated at tree level whose contribution provides the physics beyond the SM. The branching ratio of $B_s \to \ell^+\ell^-$ in $Z$
mediated VLDQ model is given by \[38\]

\[
\mathcal{BR}(B_s \to \ell^+\ell^-) = \frac{G_F^2\alpha^2 m_{B_s} f_{B_s}^2 \tau_{B_s}}{16\pi^3} |V_{tb}^*V_{ts}|^2 \sqrt{1 - 4\left(\frac{m_\ell^2}{m_{B_s}^2}\right)} |C_{10}^{tot}|^2, \tag{40}
\]

where

\[
C_{10}^{tot} = C_{10} - \frac{\pi}{\alpha} \frac{Q_{bs}}{V_{tb}^*V_{ts}}. \tag{41}
\]

Here the second term defines the new parameter when b quark couples to s quark at tree level. Using the theoretical and experimental values of \( B_s \to \ell\ell \) \[22\], the constraint on \( Q_{bs} - \phi_s \) parameters are presented in FIG. 4. The ranges obtained from the constrained plot are given as

\[
0 \leq |Q_{bs}| \leq 8.52 \times 10^{-4}, \quad 0^\circ \leq \phi_s \leq 360^\circ. \tag{42}
\]

![FIG. 4: Constraints on new coupling parameter from all the branching ratios of leptonic \( B_s \to \ell^+\ell^- (\ell = e, \mu, \tau) \) processes in VLDQ model.](image)

B. Effect on \( B_s \to KK \) mode

The effective Hamiltonian corresponding to new interaction describing \( b \to sq\bar{q} \) can be represented as

\[
\mathcal{H}_{\text{eff}}^Z = -\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} [\tilde{C}_3 O_3 + \tilde{C}_7 O_7 + \tilde{C}_9 O_9],
\]
where the new Wilson coefficients at the $M_Z$ scale are given as \[9, 39\]

\[
\begin{align*}
\tilde{C}_3(M_Z) &= \frac{1}{6} \frac{Q_{bs}}{V_{tb}V_{ts}^*}, \\
\tilde{C}_7(M_Z) &= \frac{2}{3} \frac{Q_{bs}}{V_{tb}V_{ts}^*} \sin^2 \theta_W, \\
\tilde{C}_9(M_Z) &= -\frac{2}{3} \frac{Q_{bs}}{V_{tb}V_{ts}^*} (1 - \sin^2 \theta_W). 
\end{align*}
\] (43)

Here $Q_{bs} = |Q_{bs}| e^{i \phi_s}$ and $\sin^2 \theta_W = 0.231$. As the new couplings are in $M_Z$ scale, so these can be evolved to $m_b$ scale employing renormalization group equation \[22\]. By using RGE, these three couplings can be generated and we consider the above values from the ref. \[40\].

Now using the unitarity condition from equation \[39\], we get

\[
\xi_u + \xi_c + \xi_t = Q_{bs}. 
\] (44)

To this relation, we can express the decay amplitude along with the new physics contributions as

\[
A_{B_0^s \to K^+K^-} = A_{KK} \left[ \delta_{pu} \alpha_1 + \alpha_4^p + \alpha_{4,EW}^p + \beta_3^p - \frac{1}{2} \beta_{3,EW}^p - \frac{1}{2} \beta_{4,EW}^p \right]
\]
\[
+ B_{KK} \left[ \delta_{pu} b_1 + b_4^p + b_{4,EW}^p \right]
\]
\[
- Q_{bs} \left[ A_{KK} \left( \tilde{\alpha}_4^p + \tilde{\alpha}_{4,EW}^p + \tilde{\beta}_3^p - \frac{1}{2} \tilde{\beta}_{3,EW}^p \right) \right]. 
\] (45)

Here, $\tilde{\alpha}^p$ and $\tilde{\beta}^p$ provides the dominant contributions to NP amplitude which contain all the above three new couplings as given in the equation \[40\]. Symbolically, the full amplitude can be written as

\[
A = \xi_u A_u + \xi_c A_c - Q_{bs} A_{NP}
\]
\[
= \xi_c A_c \left[ 1 + \varrho a' e^{i(\delta'_1 - \gamma)} - \varrho' b' e^{i(\delta'_2 + \phi_s)} \right], 
\] (46)

where

\[
a' = \left| \frac{\xi_u}{\xi_c} \right|, \quad b' = \left| \frac{Q_{bs}}{\xi_c} \right|, \quad \varrho = \left| \frac{A_u}{A_c} \right|, \quad \varrho' = \left| \frac{A_{NP}}{A_c} \right|. 
\] (47)

Here, $\gamma$ is the weak phase of $V_{ub}$, $\phi_s$ is the weak phase of $Q_{bs}$ and $\delta'_1 (\delta'_2)$ is the relative strong phase between $A_u$ and $A_c (A_{NP} \text{ and } A_c)$. From the parametrized amplitude, the CP-averaged branching ratio can be written as

\[
\langle BR \rangle = \frac{\tau_{B_s} \rho_c}{8 \pi m_{B_s}^2} |\xi_c A_c|^2 \left[ G' + 2 \varrho a' \cos \delta'_1 \cos \gamma - 2 \varrho' b' \cos \delta'_2 \cos \phi_s 
\right. \\
- \left. 2 \varrho \varrho' a' b' \cos (\delta'_1 - \delta'_2) \cos (\gamma + \phi_s) \right], 
\] (48)
On the other hand, the direct CP asymmetry can be written as

\[ C_{KK} = -\frac{2[\rho a' \sin \delta'_1 \sin \gamma + \rho' b' \sin \delta'_2 \sin \phi_s + \rho \rho' a b' \sin(\delta_2 - \delta_1) \sin(\gamma + \phi_s)]}{[G' + 2(\rho a' \cos \delta'_1 \cos \gamma - 2\rho' b' \cos \phi_s \delta'_2 - 2\rho \rho' a b' \cos(\gamma + \phi_s) \cos(\delta'_2 - \delta'_1))]}, \tag{49} \]

One can obtain the mixing induced CP asymmetry parameter as

\[ S_{KK} = \frac{M'}{G' + 2[\rho a' \cos \delta'_1 \cos \gamma - 2\rho' b' \cos \phi_s \delta'_2 - 2\rho \rho' a b' \cos(\gamma + \phi_s) \cos(\delta'_2 - \delta'_1)]}, \tag{50} \]

where \( G' = 1 + (\rho a')^2 + (\rho' b')^2 \) and

\[ M = \sin 2\beta + 2\rho a \cos \delta_1 \sin(2\beta + \gamma) - 2r' b \cos \delta_2 \sin(2\beta - \phi_s) + (ra)^2 \sin(2\beta + 2\gamma) \]
\[ + (r' b)^2 \sin(2\beta - 2\phi_s) - 2rr' ab \cos(\delta - \delta') \sin(2\beta + \gamma - \phi_s). \tag{51} \]

\[ M' = \sin 2\beta + 2\rho a \cos \delta_1 \sin(2\beta + \gamma) - 2r' b \cos \delta_2 \sin(2\beta - \phi_s) + (ra)^2 \sin(2\beta + 2\gamma) \]
\[ + (r' b)^2 \sin(2\beta - 2\phi_s) - 2rr' ab \cos(\delta - \delta') \sin(2\beta + \gamma - \phi_s). \tag{52} \]

\[ Q_{bs} = 5 \times 10^{-4} \]
\[ Q_{bs} = 7 \times 10^{-4} \]
\[ Q_{bs} = 8.51 \times 10^{-4} \]

**FIG. 5:** Variation of CP-averaged branching ratio (in the units of \(10^{-5}\)) (top-left panel), direct CP asymmetry (top-right panel) and mixing induced CP asymmetry (bottom panel) with respect to the weak phase \(\phi_s\) for different \(|U_{bs}|\) entries. The horizontal solid line (black color) represents the central experimental value whereas the dotted lines (magenta color) denote the 1σ error limit.

In **FIG. 5**, we present the variation of CP-averaged branching ratios (top-left panel), direct (top-right panel) and mixing induced CP asymmetry (bottom panel) with respect
FIG. 6: Correlation plots of CP-averaged branching ratio (in the units of $10^{-4}$) with direct CP asymmetry (top-left panel) and mixing induced CP asymmetry (top-right panel) (in %), and direct CP asymmetry with mixing induced CP asymmetry (bottom panel).

to the weak phase $\phi_s$ for three benchmark $Q_{bs}$ values. Here, green, blue and orange colors represent the predictions obtained by using $Q_{bs} = 5 \times 10^{-4}$, $7 \times 10^{-4}$ and $8.51 \times 10^{-4}$, respectively. For $\phi$ below $50^\circ$ (above $290^\circ$) and all the discussed benchmark points of $Q_{bs}$, the predicted branching of $B_s \to KK$ are accommodating within 1$\sigma$ range of experimental central value. We also notice that, the CP violating parameters are also explained within 1$\sigma$ data. The correlation plots among all the observables are shown in FIG. 6 for the same inputs of $Q_{bs}$. The predicted results of branching ratio, CP violating observables for different values of $Q_{bs}$ and $\phi_s$ are shown in the bottom section of Table III.

V. CONCLUSION

We have investigated the observables of $B^0_s \to K^+K^-$, a penguin induced decay mode occurring at quark level transition $b \to s$, both in standard model as well as beyond the SM scenarios. In the new physics scenario, we consider both the $Z'$ and vector-like down quark
TABLE III: Predicted values of CP-averaged branching ratio and CP violating observables for different benchmark values of $U_{bs}$ ($Q_{bs}$) and $\phi_s$ parameters in the $Z'$ (VLDQ) model.

| Model  | $U_{bs}(Q_{bs})$ | $\phi_s$ | $BR$        | $C_{KK}$ | $S_{KK}$ |
|--------|------------------|-----------|-------------|----------|----------|
| $Z'$ Model | 0.1          | 0°        | $54.06 \times 10^{-6}$ | -0.04    | 0.24     |
|         | 180°        |           | $21.76 \times 10^{-6}$ | -0.25    | 0.42     |
|         | 0.3          | 0°        | $11.46 \times 10^{-5}$ | 0.007    | 0.15     |
|         | 180°        |           | $17.75 \times 10^{-6}$ | -0.49    | 0.32     |
|         | 0.5          | 0°        | $20.35 \times 10^{-5}$ | 0.02     | 0.11     |
|         | 180°        |           | $4.2 \times 10^{-5}$    | -0.28    | 0.07     |
| VLDQ Model | $5 \times 10^{-4}$ | 0°        | $4.41 \times 10^{-6}$ | -1.14    | 0.50     |
|         | 180°        |           | $10.17 \times 10^{-5}$ | -0.02    | 0.20     |
|         | $7 \times 10^{-4}$ | 0°        | $2.89 \times 10^{-6}$ | -1.90    | 0.39     |
|         | 180°        |           | $13.91 \times 10^{-5}$ | -0.01    | 0.17     |
|         | $8.51 \times 10^{-4}$ | 0°        | $5.7 \times 10^{-6}$ | -1.02    | -0.63    |
|         | 180°        |           | $17.13 \times 10^{-5}$ | -0.012   | 0.164    |

model, where the consequence of the former one is nothing but the minimal extension of SM having $U(1)'$ gauge group added to it and the later one provides the interaction of $Z$ mediated FCNC at the tree level. We have constrained the NP parameter associated with "$Z'^{-} - b - s$" interactions from the branching ratios of all leptonic $B_s$ decay modes and mainly checked whether the new physics coupling has impact on the physical observables of non-leptonic $B_s \to K^+K^-$ decay mode. We have found that the CP-averaged branching ratios have deviated significantly from the SM results for sizable new physics parameters $U_{bs}(Q_{bs})$ of $Z'(VLDQ)$ model. Furthermore, the CP violating parameters such as direct and mixing-induced also have profound deviation in the presence of new physics. To conclude, we have noticed that the observables of $B_s \to KK$ mode can be explained in both $Z'$ and VLDQ models.
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