A Robust and Constrained Multi-Agent Reinforcement Learning Framework for Electric Vehicle AMoD Systems

Sihong He, Yue Wang, Shuo Han, Shaofeng Zou, Fei Miao

Abstract—Electric vehicles (EVs) play critical roles in autonomous mobility-on-demand (AMoD) systems, but their unique charging patterns increase the model uncertainties in AMoD systems (e.g., state transition probability). Since there usually exists a mismatch between the training and test (true) environments, incorporating model uncertainty into system design is of critical importance in real-world applications. However, model uncertainties have not been considered explicitly in EV AMoD systems rebalancing by existing literature yet and remain an urgent and challenging task. In this work, we design a robust and constrained multi-agent reinforcement learning (MARL) framework with transition kernel uncertainty for the EV rebalancing and charging problem. We then propose a robust and constrained MARL algorithm (ROCOMA) that trains a robust EV rebalancing policy to balance the supply-demand ratio and the charging utilization rate across the whole city under state transition uncertainty. Experiments show that the ROCOMA can learn an effective and robust rebalancing policy. It outperforms non-robust MARL methods when there are model uncertainties. It increases the system fairness by 19.6% and decreases the rebalancing costs by 75.8%.

I. INTRODUCTION

Autonomous mobility-on-demand (AMoD) system is one of the most promising energy-efficient transportation solutions as it provides people one-way rides from their origins to destinations [1]–[3]. Electric vehicles (EVs) are being adopted worldwide for environmental and economical benefits [4], and AMoD systems embrace this trend without exception. However, the trips sporadically appear, and the origins and destinations are asymmetrically distributed and hard to predict in AMoD systems [3], [5]. Such spatial-temporal nature of urban mobility increases the management difficulty of a large-scale vehicle fleet and makes the system sensitive to disturbances [5]–[7]. Moreover, EVs’ unique charging patterns (long charging time, high charging frequency, and unpredictable charging behaviors) increase the complexity and uncertainty of the EV AMoD system dynamics [8]–[10].

In real-world applications, we usually do not have perfectly accurate knowledge of the true system model, e.g., the state transition probability of the AMoD systems, and therefore, there usually exists a model mismatch between the simulator (training environment) and the real-world application (test environment). Thus, existing EV AMoD vehicle coordination methods [6], [8], [9], [11]–[14] may have significant performance degradation in the test (true) environment. Despite model-based methods considering prediction errors in mobility demand or vehicle supply [15]–[18], uncertainty in system state transition remains largely unexplored in AMoD systems. More importantly, practical deployment usually needs to satisfy constraints on, e.g., safety and fairness, while maximizing the performance. In this work, we propose a robust and constrained multi-agent reinforcement learning (MARL) framework for EV AMoD systems. The goal is to achieve AMoD system fairness by finding robust rebalancing policies for idle and low-battery EVs with minimal rebalancing cost, under model uncertainty. The advantages of our formulation are two-folds: (i) fairness constraints can still be satisfied even if there is model mismatch; and (ii) the rebalancing cost is still optimized when there is model mismatch.

Our Key Contributions are as follows:

(1) To the best of our knowledge, this work is the first to formulate EV AMoD system vehicle rebalancing as a robust and constrained multi-agent reinforcement learning problem under model uncertainty. Via a proper design of the state, action, reward, cost constraints, and uncertainty set, we set our goal as minimizing the rebalancing cost while balancing the city’s charging utilization and service quality, under model uncertainty.

(2) We design a robust and constrained MARL algorithm (ROCOMA) to efficiently train robust policies. The proposed algorithm adopts the centralized training and decentralized execution (CTDE) framework and develops the first robust natural policy gradient (NPG) to improve the efficiency of policy training.

(3) We run experiments based on real-world E-taxi system data. We show that our proposed algorithm per-
forms better in terms of reward and fairness, which are increased by 19.6%, and 75.8%, respectively, compared with a non-robust MARL-based method when model uncertainty is present.

II. RELATED WORK

AMoD system vehicle rebalancing algorithms re-allocate idle vehicles, sometimes considering charging constraints [3]. Heuristics can lead to sub-optimal rebalancing solutions [19]. Other major categories of AMoD system rebalancing methods include optimization-based algorithms [17], Model Predictive Control (MPC) [20] and Reinforcement Learning (RL) [21].

Optimization and MPC-based approaches usually formulate the AMoD system vehicle rebalancing problem as a convex optimization or mixed-integer programming problem, where the objective is to improve service quality [22], [23] or maximize the number of served passengers with fewer vehicles [2], [7], [15]. These model-based approaches usually rely on precise knowledge of the probability transition model of the complex dynamics of AMoD systems and, consequently, are sensitive to model uncertainties. Though robust and distributionally robust optimization-based methods have been designed to consider uncertainties caused by mobility demand, supply, or covariates predictions [16]–[18], the probability transition error or uncertainty in system dynamics has not been addressed yet. RL-based approaches relax the dependency on system dynamic models. Various RL-based methods, including DQN, A2C and their variants [6], [12]–[14], [24]–[27] have been proposed to solve the vehicle rebalancing problem. However, RL suffers from the sim-to-real gap; that is, the gap between the simulator and the real world often leads to unsuccessful implementation if the learned policy is not robust to model uncertainties [28], [29]. None of the above RL-based rebalancing strategies consider this gap.

Given that designing an efficient and robust EV rebalancing method under model uncertainties is still an unsolved challenge, Robust RL aims to find a policy that maximizes the worst-case cumulative reward over an uncertainty set of MDPs [29]–[32]. To achieve high system fairness while minimizing rebalancing cost under model uncertainty, we put the fairness constraints in our RL formulation, which is known as Constrained RL that aims to find a policy that maximizes an objective function while satisfying certain cost constraints [33], [34]. However, applying robust and constrained RL to AMoD rebalancing is still a challenge due to the high-dimensional state and action spaces commonly present in transportation systems. And the problem of robust constrained RL itself is very challenging even in the simple tabular case. Our proposed robust and constrained MARL formulation and algorithm explicitly consider model uncertainties and system fairness to learn robust rebalancing solutions for AMoD systems. And we derive a natural policy gradient for robust and constrained MARL to improve the efficiency of policy training.

III. ROBUST AND CONSTRAINED MARL FRAMEWORK FOR EV REBALANCING

A. Problem Statement

We consider the problem of managing a large-scale EV fleet to provide fair and robust AMoD service. The goal is to (i) rebalance idle and energy-efficient EVs (we denote them as vacant EVs for notation convenience) among different regions to provide fair mobility service on the passenger’s side; (ii) allocate low-battery EVs to charging stations for fair charging service on the EVs’ side; (iii) minimize the managing cost of (i) and (ii); (iv) find rebalancing policies robust to model uncertainty, i.e. uncertainty in transition kernel of the MDP.

We assume that a city is divided into $N$ regions according to a pre-defined partition method [8], [16], [35]. A day is divided into equal-length time intervals. In each time interval $[t, t + 1)$, customers’ ride requests and EVs’ charging need are aggregated in each region. After the location and status of each EV are observed, a local trip and charging assignment algorithm matches available EVs with passengers and low-battery EVs with charging stations, using existing methods in the literature [36]–[38]. Then the state information of each region is updated, including the numbers of vacant EVs and available charging spots in each region. Each region then rebalances both vacant and low-battery EVs according to the trained MARL policy. This work focuses on a robust EV rebalancing algorithm design under model uncertainties to maximize the worst-case expected reward of the system while satisfying fairness constraints. For notational convenience, the parameters and variables defined in the following parts of this section omit the time index $t$ when there is no confusion.

B. Preliminary: Multi-Agent Reinforcement Learning

We denote a Multi-Agent Reinforcement Learning (MARL) problem by a tuple $G = \langle \mathcal{N}, S, A, r, p, \gamma \rangle$, in which $\mathcal{N}$ is the set of $N$ agents. Each agent is associated with an action $a^i \in A^i$ and a state $s^i \in S$. We use $A = A^1 \times \cdots \times A^N$ to denote the joint action space, and $S = S^1 \times \cdots \times S^N$ the joint state space. At time $t$, each agent chooses an action $a^i_t$ according to a policy $\pi^i : \Delta(A^i) \rightarrow \Delta(A^i)$, where $\Delta(A^i)$ represents the set of probability distributions over the action set $A^i$. We use $\pi = \prod_{i=1}^{N} \pi^i : S \rightarrow \Delta(A)$ to denote the joint policy.
After executing the joint action, the next state follows the state transition probability which depends on the current state and the joint action, i.e. $p: S \times A \rightarrow \Delta(S)$. And each agent receives a reward according to the reward function $r^i: S \times A \rightarrow \mathbb{R}$. Each agent aims to learn a policy $\pi^i$ to maximize its expected total discounted reward, i.e. $\max_{\pi^i} v^i_\pi(s) \text{ for all } s \in S$, where $v^i_\pi(s) = \mathbb{E}[\sum_{t=1}^{\infty} \gamma^{t-1} r^i_t(s_t, a_t)|a_t \sim \pi(\cdot|s_t), s_1 = s]$ which is also known as the state value function for agent $i$. $\gamma \in (0, 1)$ is the discounted rate. If these agents belong to a team, the objective of all agents is to collaboratively maximize the average expected total discounted reward over all agents, i.e. $\max_{\pi} v^i_\pi(s) \text{ for all } s \in S$, where $v^i_\pi(s) = \mathbb{E}_{\pi}[\sum_{t=1}^{\infty} \gamma^{t-1} \sum_{i \in N} r^i_t(s_t, a_t)/N|s_1 = s]$.

**C. Robust and Constrained Multi-Agent Reinforcement Learning Formulation for EV Rebalancing**

We formulate the EV rebalancing problem as a robust and constrained MARL problem $G_{rc} = \langle N, S, A, P, r, c, d, \gamma \rangle$, and we define the agent, state, action, probability transition kernel uncertainty set, reward, and cost and fairness constraints as follows.

- **a) Agent:** We define for each region a region agent, who determines the rebalancing of vacant and low-battery EVs at every time step. This distributed agent setting is more tractable for large-scale fleet management than a single agent setting because the action space can be prohibitively large if we use a single system-wide agent [39].

- **b) State:** A state $s^i$ of a region agent $i$ consists of two parts that indicate its spatiotemporal status from both the local view and global view of the city. We define the state $s^i = \{s^i_{loc}, s^i_{glow}\}$, where $s^i_{loc} = (V_i, L_i, D_i, E_i, C_i)$ is the state of region $i$ from the local view, denoting the number/amount of vacant EVs, low-battery EVs, mobility demand, empty charging spots, and total charging spots in region $i$, respectively. And $s^i_{glow} = (t, pos_i)$, where $t$ is the time index (which time interval), $pos_i$ is region location information (longitudes, latitudes, region index). The initial state distribution is $\rho$.

- **c) Action:** The rebalancing action for vacant EVs is denoted as $a^i_v = \{a^i_{v,j}\} \in \text{Neb}_v$, the charging action for low-battery EVs as $a^i_l = \{a^i_{l,j}\} \in \text{Neb}_l$, where $a^i_{v,j}, a^i_{l,j} \in [0, 1]$ is the percentage of currently vacant EVs and low-battery EVs to be assigned to region $j$ from region $i$, respectively. And $\text{Neb}_v, \text{Neb}_l$ is the set consisting of region $i$ and its adjacent regions as defined by the given partition. Therefore $\sum_{j \in \text{Neb}_v} a^i_{v,j} = 1$ and $\sum_{j \in \text{Neb}_l} a^i_{l,j} = 1$ for all $i$. We denote $m^i_{v,j} = h(a^i_{v,j} v^i)$ the actual number of vacant EVs assigned from region $i$ to region $j$, $m^i_{l,j} = h(a^i_{l,j} l^i)$ the actual number of low-battery EVs in region $i$ assigned to region $j$. The function $h(\cdot)$ is used to ensure that the numbers remain as integers and the constraints $\sum_j m^i_{v,j} = v^i, \sum_j m^i_{l,j} = l^i$ hold for all $i$.

- **d) Transition Kernel Uncertainty Set:** We restrict the transition kernel $p$ to a $\delta$-contamination uncertainty set $P[40]–[42]$, in which the state transition could be arbitrarily perturbed by a small probability $\delta$. Specifically, let $\tilde{p} = \{\tilde{p}_s^i \mid s \in S, a \in A\}$ be the centroid transition kernel, from which training samples are generated. The $\delta$-contamination uncertainty set centered at $\tilde{p}$ is defined as $P := \bigotimes_{s \in S, a \in A} P_s^a$, where $P_s^a := \{(1 - \delta)\tilde{p}_s^a + \delta q \mid q \in \Delta(S)\}, s \in S, a \in A$.

- **e) Reward:** Since one of our goals is to minimize the rebalancing cost, we define the shared reward as the negative value of the total rebalancing cost after EVs execute the decisions: $r(s, a) := -[c_v(s, a) + \alpha c_l(s, a)]$, where $\alpha$ is a positive coefficient, and $c_v(s, a), c_l(s, a)$ are moving distances of all vacant and low-battery EVs under the joint state $s$ and action $a$, respectively. We then define the worst-case value function of a joint policy $\pi$ as the worst-case expected total discounted reward under joint policy $\pi$ over $P$: $v^T_\pi(s) = \min_{p \in P} \mathbb{E}_{\pi}[\sum_{t=1}^{\infty} \gamma^{t-1} r^i_t(s_t, a_t)|s_1 = s]$. The notation is the same as MARL without considering uncertainty. By maximizing the shared worst-case value function, region agents are cooperating for the same goal.

- **f) Cost and Fairness Constraints:** Another goal is to achieve the system-level benefit, i.e., balanced charging utilization and fair service. We define the charging fairness $u_c$ and mobility fairness $u_m$ in Subsection [III-D]. If the values of these fairness metrics are higher than some thresholds by applying a rebalancing policy $\pi$, we say the policy $\pi$ provides fair mobility and charging services among the city. We then augment the MARL problem $G$ with an auxiliary cost function $c$, and a limit $d$. The function $c : S \times A \rightarrow \mathbb{R}$ maps transition tuples to cost, like the usual reward. Similarly, we let $v^c_\pi(s)$ denote the worst-case state value function of policy $c$ with respect to cost function $c$: $v^c_\pi(s) = \min_{p \in P} \mathbb{E}_{\pi}[\sum_{t=1}^{\infty} \gamma^{t-1} c(s_t, a_t)|s_1 = s]$. The cost function $c$ is defined as the system fairness (a weighted sum of city’s charging fairness $u_c$ and mobility fairness $u_m$), i.e., $c(s, a) := u_c(s, a) + \beta u_m(s, a)$, where $\beta$ is a positive coefficient. Then the set of feasible joint policies for our robust and constrained MARL EV rebalancing problem is $\Pi_C = \{\pi : \forall s \in S, v^c_\pi(s) \geq d\}$.

- **g) Goal:** The goal of our robust and constrained MARL EV rebalancing problem is to find an optimal joint policy $\pi^*$ that maximizes the worst-case value function subject to constraints on the worst-case expected cost $\pi^* = \arg\max_{\pi \in \Pi_C} v^c_\pi(p)$, where $v^c_\pi(p) = \mathbb{E}_{s,a} [v^c_\pi(s), t \in \{r, c\}$ We consider policies $\pi(\cdot|\theta)$ parameterized by $\theta$ and consider the following equiva-
lent max-min problem based on the Lagrangian [43]:
\[
\max_{\theta} \min_{\lambda \geq 0} J(\theta, \lambda) := v^\pi_{\theta}(\rho) + \lambda (v^\pi_{\theta}(\rho) - d),
\]

(1)

**D. Fairness Definition**

We consider both the mobility supply-demand ratio [17], [23], [44] and the charging utilization rate [8], [9], [16], [45] in each region as service quality metrics for the EV AMoD system. However, with limited supply volume in a city, achieving high supply-demand ratios in all regions may not be possible. Keeping the supply-demand ratio of each region at a similar level allows passengers in the city to receive fair service [1], [15]. Similarly, given a limited number of charging stations and spots, to improve the charging service quality and charging efficiency with limited infrastructure, balancing the charging utilization rate of all regions across the entire city is usually one objective in the scheduling of EV charging [9], [11], [45].

The fairness metrics of the charging utilization rate \( u_c \) and supply-demand ratio \( u_{sv} \) are designed based on the difference between the local and global quantities:
\[
u_c(s, a) = - \sum_{i=1}^{N} \frac{E_i}{C_i} - \frac{\sum_{j=1}^{N} E_j}{\sum_{j=1}^{N} C_j},
\]

\[
u_{sv}(s, a) = - \sum_{i=1}^{N} \frac{D_i}{V_i^{ava}} - \frac{\sum_{j=1}^{N} D_j}{\sum_{j=1}^{N} V_j^{ava}},
\]

where \( V_i^{ava} \) is the number of available EVs in region \( i \). The fairness metrics \( u_c(s, a) \) and \( u_{sv}(s, a) \) are calculated given the EVs rebalancing action \( a \), and the larger the better. One advantage of the proposed robust and constrained MARL formulation is that the form of the reward/cost function does not need to satisfy the requirements as those of the robust optimization methods [17], [35], e.g., the objective/constraints do not need to be convex of the decision variable or concave of the uncertain parameters.

**IV. ALGORITHM**

In this section, we derive robust natural policy gradient for robust and constrained MARL to efficiently train policies and alleviate overshooting/undershooting and high variance which results in slow convergence [46]. Then we present a robust and constrained MARL algorithm (ROCOMA) to solve the EV recharging problem under model uncertainties and fairness constraints.

**A. Robust Policy Gradient Descent Ascent**

The problem (1) can be solved by Gradient Descent Ascent (GDA) [47], which currently is a widely used algorithm for solving minimax optimization problems. At each iteration, GDA simultaneously performs gradient descent update on the variable \( \lambda \) and gradient ascent update on the variable \( \theta \) with step sizes \( \alpha_t \) and \( \beta_t \):
\[
\theta_{t+1} = \theta_t + \alpha_t (\nabla_{\theta} v^\pi_{\theta}(\rho) + \lambda_t \nabla_{\lambda} v^\pi_{\lambda}(\rho)), \quad \lambda_{t+1} = \text{proj}_{\lambda \geq 0} [\lambda_t - \beta_t (v^\pi_{\lambda}(\rho) - d)].
\]

(2)

For notation convenience, we omit the subscripts \( r \) and \( c \) in the value functions when there is no confusion.

The robust policy gradient of the value function is given by \( \nabla_{\theta} v^\pi_{\theta}(s_1) = \sum_{s,a} d^\pi_{s,a} s_1(s) \nabla_{\theta} \pi(s) \phi^\pi(\tau) + b^\pi \propto E_{s_1}[\phi^\pi(\tau) \nabla_{\theta} \log \pi(s) + b^\pi] \) [34], where \( d^\pi_{s,a} s_1 = \sum_k \gamma^k (1-\delta)^k p^\pi(s_k = s | s_1) \) is the discounted visitation distribution of \( s_k = s \) when initial state is \( s_1 \) and policy \( \pi \) is used; \( \tau \) denotes a trajectory \((s, a, r, c, s')\); \( \phi^\pi(\tau) := r + \gamma \delta \min_s v^\pi(s) + (1-\delta) v^\pi(s') - v^\pi(s) \) is the TD residual; \( b^\pi := \gamma \delta (1-\gamma - \gamma \delta) \partial_\theta \min_s v^\pi(s) \). Similar to the vanilla policy gradient in the non-robust RL setting, robust policy gradient suffers from over-shooting or undershooting and high variance, which results in slow convergence [46]. Hence we propose a robust natural policy gradient (RNPG) method as follows to update the policy along the steepest ascent direction in the policy space [48], [49].

**B. Robust Natural Policy Gradient (RNPG)**

Natural policy gradient (NPG) [24], [50], [51] applies a preconditioning matrix to the gradient, and updates the policy along the steepest descent direction in the policy space [48], [49]. It has been proved that NPG moves toward choosing a greedy optimal action rather than just a better action in the literature [49]. Generally, for a function \( L \) defined on a Riemannian manifold \( \Theta \) with a metric \( M \), the steepest descent direction of \( L \) at \( \theta \) is given by \(-M^{-1}(\theta) \nabla L(\theta)\), which is called the natural gradient of \( L \) [52]. In the policy parameter space \([\pi_\theta]\), the natural gradient of \( L \) at \( \theta \) is given by \( \nabla L(\theta) = F(\theta)^{-1} \nabla L(\theta) \), where \( F(\theta) := E_s[F_s(\theta)] \) is the Fisher information matrix at \( \theta \) and \( F_s(\theta) = E_s[\frac{\partial \log \pi(s | a, \theta)}{\partial a}] \) [49].

Although the natural gradient method has been studied in non-robust RL, it is not straightforward to efficiently find the NPG for a robust and constrained MARL problem. We show the natural policy gradient for robust and constrained MARL in the following Theorem 4.1.

**Theorem 4.1 (Robust Natural Policy Gradient):** Let \( \hat{\gamma}^* \) minimizes the objective \( J(\hat{\gamma}, \pi_\theta) \) defined as follows:
\[
\sum_{s,a} d^\pi_{\gamma, s_1} \pi(a | s) \hat{\gamma}^T \psi^\pi(s,a) - \phi^\pi(\tau) - b^\pi)^2.
\]

Then \( \hat{\gamma}^* = F(\theta)^{-1} \nabla_{\theta} v^\pi_{\theta}(s_1) \) being the natural policy gradient of the objective function \( v^\pi_{\theta}(s_1) \).
\textbf{Proof:} Since \( \tilde{g}^* \) minimizes \( \mathcal{J} \), it satisfies the condition \( \partial \mathcal{J} / \partial \tilde{g} = 0 \), which implies: 
\[
\sum_{s,a} d^n_{s,a}(s,a) \pi(a|s) \times \psi^a(s,a)[\psi^a(s,a) \psi^a(s,a) ^T \tilde{g}^* - \phi^a(\tau) - b^a] = 0 \text{ or equivalently:}
\]
\[
\sum_{s,a} d^n_{s,a}(s,a) \pi(a|s) \psi^a(s,a) \psi^a(s,a) ^T \tilde{g}^* = \sum_{s,a} d^n_{s,a}(s,a) \pi(a|s) \phi^a(\tau) + b^a.
\]
By the definition of Fisher information: LHS = \( F(\theta) \tilde{g}^* \) and RHS = \( \nabla_\theta \psi^a(s_1) \), which lead to: \( F(\theta) \tilde{g}^* = \nabla_\theta \psi^a(s_1) \). Solving for \( \tilde{g}^* \) gives \( \tilde{g}^* = F(\theta)^{-1} \nabla_\theta \psi^a(s_1) \) which follows from the definition of the NPG.

\section{C. Robust and Constrained Multi-Agent Reinforcement Learning Algorithm (ROCOMA)}

We then propose a robust and constrained MARL (ROCOMA) algorithm to solve the problem \( \mathcal{I} \) and train a robust joint policy \( \pi \) using GDA and RNPG. The proposed algorithm is shown in Algorithm 1.

\begin{algorithm}
\caption{Robust and Constrained Multi-Agent Reinforcement Learning Algorithm (ROCOMA)}
\begin{algorithmic}[1]
\State Input \( \zeta, \alpha, \beta, \gamma, \delta \). Initialize \( \theta_0, \lambda_0 \).
\For {\( t = 0 \) to \( T \)}
\State \( \text{Estimate } v_{\theta}, v_{\theta}^* \) using Algorithm 3 in [34].
\For {\( j = 1 \) to \( M \)}
\State Sample \( T_j \sim \text{Geom}(1 - \gamma + \gamma \delta) \), \( s_j^1 \sim \rho \).
\State \( \text{Sample trajectory from } s_j^1: (s_j^1, a_j^1, \cdots, s_j^T) \).
\For {agent \( i = 1 \) to \( N \)}
\State \( \text{for } k = 1 \) to \( W \) \do
\State \( \bar{g}_{t,k}^i(i) = \arg\min_{\tilde{g}} \| \tilde{g}_{t,k}^i(i) \tilde{g} - \zeta \nabla_\tilde{g} L(\tilde{g}_{t,k}^i(i), \theta_t) \| \), \( L \) is defined in (4).
\EndFor
\State \( \tilde{g}_{t,k}^i(i) = \sum_{i=1}^N \bar{g}_{t,k}^i(i) / N \).
\EndFor
\State \( \bar{g}_t = \sum_{k=1}^W \bar{g}_{t,k} / MW \).
\State \( \theta_{t+1} = \theta_t + \alpha \delta (\tilde{g}_t - \lambda_t \tilde{g}_t \cdot \lambda_t \tilde{g}_t) \).
\State \( \lambda_{t+1} = \max\{\lambda_t - \beta_t (\sum_j v_{\theta}^c(s_j^1) / (M - d)), 0\} \).
\EndFor
\State \( \text{Estimate the RNPG for } v_{\theta}^a, v_{\theta}^c \).
\State \( \text{Output } \theta_T \).
\end{algorithmic}
\end{algorithm}

We first randomly initialize the actor neural network parameter \( \theta_0 \) and the Lagrange multiplier parameter \( \lambda_0 \).

At each iteration \( t \), we first estimate the critic neural networks \( v_{\theta}^a, v_{\theta}^c \) under policy \( \pi_{\theta} \) using Algorithm 3 in [34]. Line 2 to line 14 in Algorithm 1 estimate the RNPG for \( v_{\theta}^a \) and \( v_{\theta}^c \).

We then sample an initial state \( s_j^1 \) following the initial distribution \( \rho \) and a time horizon \( T_j \) from the geometric distribution \( \text{Geom}(1 - \gamma + \gamma \delta) \) at iteration \( j = 1, \cdots, M \). These samples are used to estimate the RNPG.

As shown in Theorem 5.1 we can get the RNPG of \( v_{\theta}^a(s_1) \) by minimizing the objective defined in (2).

To minimize (2) and get the minimizer, we initialize \( \bar{g}_{t,0} = 0 \) and use the following stochastic gradient descent (SGD) steps with \( \lambda \)-projection: \( \bar{g}_{t,k+1} = \arg\min_{\tilde{g}} \| \bar{g}_{t,k} - \zeta \nabla_\tilde{g} L(\bar{g}_{t,k}, \theta_t) - \tilde{g} \| \), where \( \zeta \) is the learning rate and \( L \) is defined in (4) as follows.

\[ L(\bar{g}, \theta) = \sum_{\mathcal{D}(s_{t, j}^1)} | | g \psi^a(s, a) - \phi^a(\tau) - \beta^a |^2 / D, \]

where \( \mathcal{D}(s_{t, j}^1) \) is a set of trajectories \( \tau \) starting at \( s_{t, j}^1 \) using policy \( \pi_{\theta} \), i.e. \( (s_{t, j}^1, a, r, c, s') \), \( D = \mathcal{D}(s_{t, j}^1) \). After \( W \) steps of SGD iterations, the robust natural policy gradient for \( \psi_{\theta}(s_1) \) is estimated as \( \sum_{k=1}^W \bar{g}_{t,k} / W \).

To reduce the computational complexity, we adopt the centralized training and decentralized execution (CTDE) framework [53] in ROCOMA and assume all agents share the same policy \( \pi_{\theta}(a_1^1|s_1^1) \), where \( \theta^1 = \cdots = \theta^N = \theta \). Then we have \( \nabla \pi(a|s) = \sum_{i=1}^N \pi_{\theta}^i (a, s) \) where \( \pi_{\theta}^i(a, s) := \pi^{-i}(a^{-i}|s^{-i}) \nabla \pi^i(a_1^1|s_1^1), \pi^{-i}(a^{-i}|s^{-i}) := \prod_{j \neq i} \pi^j(a_j^1|s_j^1) \). Therefore, in lines 7 to 12 we address the high-dimensional action and state space issue in computing RNPG by using \( \psi_{\theta}^i(s, a) \) instead of \( \psi_{\theta}(s, a) \) in (4). Finally, we update \( \theta_{t+1} \) and \( \lambda_{t+1} \) using GDA.

\section{V. EXPERIMENT}

\subsection{A. Experiment Setup}

Three different data sets [16], [54] including E-taxi GPS data, transaction data and charging station data are used to build an EV AMoD system simulator as the training and testing environment. The simulated map is set as a grid city. The policy networks and critic networks are two-layer fully-connected networks, both with 32 nodes. We use Softplus as activations to ensure the output is positive. The output of policy networks is used to be the concentration parameters of the Dirichlet distribution to satisfy the action constraints (sum to one). We set the maximal training episode number = 20000, the maximal policy/critic estimation number = 2000, the RNPG SGD iteration number = 500, the discount rate \( \gamma = 0.99 \), the perturbed rate \( \delta = 0.05 \), the coefficients \( \alpha = \beta = 1 \), the fairness constraint limit \( d = -20 \) for one simulation step, and use AdamOptimizer with a learning rate of 0.001 for both policy/critic networks.

\subsection{B. Experiment Results}

Our goal of the experiments is to validate the following hypothesis: (1) The proposed ROCOMA can learn effective rebalancing policies; (2) Our proposed method is more robust than a non-robust MARL algorithm through our robust and constrained MARL design. Other than \textbf{Rebalancing cost}: the total moving distance of
and order response rate by about 93.2% and 32.9%, respectively. Besides, ROCOMA achieves a higher system fairness and order response rate using less rebalancing cost than EDP and RDP. Though ROCOMA takes more rebalancing costs than COP, it has a better system fairness and order response rate. It is within expectation since the constrained optimization method is a centralized method that aims to optimize the rebalancing cost and it does not consider any uncertainties.

b) ROCOMA is robust: In Tables II and III we compare ROCOMA with (1) Non-constrain MARL algorithm: Instead of considering fairness constraints in MARL, the reward is designed as a weighted sum of negative rebalancing cost and system fairness. The coefficient is 1. And model uncertainty is considered; (2) Non-robust MARL algorithm: The model uncertainty is not considered but the fairness constraint is considered in MARL. They use the same network structures and other hyper-parameters as that in ROCOMA.

In Table III we test well-trained robust and non-robust methods in a perturbed environment to show the robustness of the ROCOMA policy. We modify the parameters of the simulator model such that the testing environment is different from the training environment. We can see ROCOMA policy achieves better performance in terms of all metrics. Specifically, ROCOMA decreases the rebalancing cost and increases the system fairness by about 19.6% and 75.8%, respectively, when model uncertainty exists, compared to the non-robust method.

In Table III ROCOMA achieves 83.9% higher in fairness compared to the non-constrained MARL algorithm with just 4% extra rebalancing cost. Without the fairness constraint design, the non-constrained MARL method falls into a pit that sacrifices fairness to achieve a lower rebalancing cost since its objective is a weighted sum of them. It would take a lot of effort to tune the hyper-parameter to find a policy that performs well in both rebalancing cost and fairness. The constrained MARL design of ROCOMA avoids such extra tuning efforts.

VI. CONCLUSION

It remains challenging to address AMoD system model uncertainties caused by EVs’ unique charging patterns and AMoD systems’ mobility dynamics in algorithm design. In this work, we design a robust and constrained multi-agent reinforcement learning framework to balance mobility supply-demand ratio and charging utilization rate, and minimize the rebalancing cost for EV AMoD systems under probability transition uncertainties. We then design a robust and constrained MARL algorithm (ROCOMA) to train robust policies. Experiments show that our proposed robust algorithm can learn effective and robust rebalancing policies.
REFERENCES

[1] R. Iglesias, F. Rossi, R. Zhang, and M. Pavone, “A bcnp network approach to modeling and controlling autonomous mobility-on-demand systems,” The International Journal of Robotics Research, vol. 38, no. 2-3, pp. 357–374, 2019. [Online]. Available: https://doi.org/10.1177/0278364918780353

[2] R. Iglesias, F. Rossi, K. Wang, D. Hallac, J. Leskovec, and M. Pavone, “Data-driven model predictive control of autonomous mobility-on-demand systems.” in IEEE International Conference on Robotics and Automation, vol. abs/1709.07032, 2018.

[3] G. Zardini, N. Lanzetti, M. Pavone, and E. Frazzoli, “Analysis and control of autonomous mobility-on-demand systems: A review,” arXiv preprint arXiv:2106.14827, 2021.

[4] IEA, “Iea (2020), global ev outlook 2020, iea, paris, “ 2020.

[5] D. Gammelli, K. Yang, J. Harrison, F. Rodrigues, F. C. Pereira, and M. Pavone, “Graph neural network reinforcement learning for autonomous mobility-on-demand systems,” in 2021 60th IEEE Conference on Decision and Control (CDC). IEEE, 2021, pp. 2996–3003.

[6] G. Wang, S. Zhong, S. Wang, F. Miao, Z. Dong, and D. Zhang, “Data-driven fairness-aware vehicle displacement for large-scale electric taxi fleets,” in 2021 IEEE 37th International Conference on Data Engineering (ICDE). IEEE, 2021, pp. 1200–1211.

[7] A. Wallar, M. Van Der Zee, J. Alonso-Mora, and D. Rus, “Vehicle rebalancing for mobility-on-demand systems with ride-sharing,” in 2018 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), Oct 2018, pp. 4539–4546.

[8] B. Turan, R. Pedarsani, and M. Alishaeb, “Dynamic pricing and fleet management for electric autonomous mobility on demand systems,” Transportation Research Part C: Emerging Technologies, vol. 121, p. 102829, 2020.

[9] Y. Yuan, D. Zhang, F. Miao, J. Chen, T. He, and S. Lin, “p2charging proactive partial charging for electric taxi systems,” in IEEE International Conference on Distributed Computing Systems, ser. ICDCS’19, 2019.

[10] E. Wang, R. Ding, Z. Yang, H. Jin, C. Miao, L. Su, F. Zhang, C. Qiao, and X. Wang, “Joint charging and relocation recommendation for e-taxi drivers via multi-agent mean field hierarchical reinforcement learning,” IEEE Transactions on Mobile Computing, pp. 1–1, 2020.

[11] N. Sadeghianpourhamami, J. Deleu, and C. Develder, “Definition and evaluation of model-free coordination of electrical vehicle charging with reinforcement learning,” IEEE Transactions on Smart Grid, vol. 11, no. 1, pp. 203–214, 2020.

[12] J. Wen, J. Zhao, and P. Jaillet, “Rebalancing shared mobility-on-demand systems: A reinforcement learning approach,” in 2017 IEEE 20th international conference on intelligent transportation systems (ITSC). Ieee, 2017, pp. 220–225.

[13] M. Gueriu and I. Dusparic, “Samod: Shared autonomous mobility-on-demand using decentralized reinforcement learning,” in 2018 21st International Conference on Intelligent Transportation Systems (ITSC). IEEE, 2018, pp. 1558–1563.

[14] J. Holler, R.Vuorio, Z. Qin, X. Tang, Y. Jiao, T. Jin, S. Singh, C. Wang, and J. Ye, “Deep reinforcement learning for multi-driver vehicle dispatching and repositioning problem,” in 2019 IEEE International Conference on Data Mining (ICDM). IEEE, 2019, pp. 1090–1095.

[15] R. Zhang, F. Rossi, and M. Pavone, “Model predictive control of autonomous mobility-on-demand systems,” in 2016 IEEE International Conference on Robotics and Automation (ICRA). IEEE, 2016, pp. 1382–1389.

[16] S. He, L. Pepin, G. Wang, D. Zhang, and F. Miao, “Data-driven distributionally robust vehicle balancing for mobility-on-demand systems under demand and supply uncertainties,” in 2020 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS). IEEE, 2020, pp. 2165–2172.

[17] F. Miao, S. He, L. Pepin, S. Han, A. Hendawi, M. E. Khalefa, J. A. Stankovic, and G. Pappas, “Data-driven distributionally robust optimization for vehicle balancing of mobility-on-demand systems,” ACM Transactions on Cyber-Physical Systems, 2021.

[18] Z. Hao, L. He, Z. Hu, and J. Jiang, “Robust vehicle pre-allocation with uncertain covariates,” Production and Operations Management, vol. 29, no. 4, pp. 955–972, 2020.

[19] Z. Liu, T. Miwa, W. Zeng, M. G. Bell, and T. Morikawa, “Dynamic shared autonomous taxi system considering on-time arrival reliability,” Transportation Research Part C: Emerging Technologies, vol. 103, pp. 281–297, 2019.

[20] E. F. Camacho and C. B. Alba, Model predictive control. Springer science & business media, 2013.

[21] R. S. Sutton and A. G. Barto, Reinforcement learning: An introduction. MIT press, 2018.

[22] J. Miller and J. P. How, “Predictive positioning and quality of service ridesharing for campus mobility on demand systems,” in 2017 IEEE International Conference on Robotics and Automation (ICRA), May 2017, pp. 1402–1408.

[23] P. Frazier, J. Waller, G. Pappas, M. Pavone, “Dynamic vehicle redistribution and online price incentives in shared mobility systems,” IEEE Transactions on Intelligent Transportation Systems, vol. 15, no. 4, pp. 1567–1578, Aug 2014.

[24] V. Mnih, K. Kavukcuoglu, D. Silver, A. A. Rusu, J. Veness, M. G. Bellemare, A. Graves, M. Riedmiller, A. K. Fidjeland, G. Ostrovski et al., “Human-level control through deep reinforcement learning,” nature, vol. 518, no. 7540, pp. 529–533, 2015.

[25] V. Konda and J. Tsitsiklis, “Actor-critic algorithms,” Advances in neural information processing systems, vol. 12, 1999.

[26] K. Lin, R. Zhao, Z. Xu, and J. Zhou, “Efficient large-scale fleet management via multi-agent deep reinforcement learning,” in Proceedings of the 24th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining, 2018, pp. 1774–1783.

[27] S. He and K. G. Shin, “Spatio-temporal capsule-based reinforcement learning for mobility-on-demand coordination,” IEEE Transactions on Knowledge and Data Engineering, 2020.

[28] Z. Ding and H. Dong, “Challenges of reinforcement learning,” in Deep Reinforcement Learning. Springer, 2020, pp. 249–272.

[29] L. Pinto, J. Davidson, R. Sukthankar, and A. Gupta, “Robust adversarial reinforcement learning,” in International Conference on Machine Learning, PMLR, 2017, pp. 2817–2826.

[30] J. A. Bagnell, A. Y. Ng, and J. G. Schneider, “Solving uncertain markov decision processes,” 2001.

[31] A. N. Lym and L. Ghaoui, “Robustness in markov decision processes,” 2001.

[32] Z. Ding and H. Dong, “Challenges of reinforcement learning,” in Deep Reinforcement Learning. Springer, 2020, pp. 249–272.

[33] L. Pinto, J. Davidson, R. Sukthankar, and A. Gupta, “Robust adversarial reinforcement learning,” in International Conference on Machine Learning, PMLR, 2017, pp. 2817–2826.

[34] J. A. Bagnell, A. Y. Ng, and J. G. Schneider, “Solving uncertain markov decision processes,” 2001.

[35] A. N. Lym and L. Ghaoui, “Robustness in markov decision processes,” 2001.

[36] Z. Ding and H. Dong, “Challenges of reinforcement learning,” in Deep Reinforcement Learning. Springer, 2020, pp. 249–272.

[37] L. Pinto, J. Davidson, R. Sukthankar, and A. Gupta, “Robust adversarial reinforcement learning,” in International Conference on Machine Learning, PMLR, 2017, pp. 2817–2826.

[38] J. A. Bagnell, A. Y. Ng, and J. G. Schneider, “Solving uncertain markov decision processes,” 2001.
[38] X. Chen, F. Miao, G. Pappas, and V. Preciado, “Hierarchical data-driven vehicle dispatch and ride-sharing,” in Proceedings of the IEEE 56th Conference on Decision and Control, ser. CDC’17, 2017, pp. 4458–4463.

[39] K. Lin, R. Zhao, Z. Xu, and J. Zhou, “Efficient large-scale fleet management via multi-agent deep reinforcement learning,” in Proceedings of the 24th ACM SIGKDD International Conference on Knowledge Discovery: Data Mining, ser. KDD’18. Association for Computing Machinery, 2018. [Online]. Available: [https://doi.org/10.1145/3219819.3219993](https://doi.org/10.1145/3219819.3219993)

[40] E. M. Ronchetti and P. J. Huber, Robust statistics. John Wiley & Sons, 2009.

[41] K. G. Nishimura and H. Ozaki, “An axiomatic approach to $\epsilon$-contamination,” Economic Theory, vol. 27, no. 2, pp. 333–340, 2006.

[42] A. Prasad, V. Srinivasan, S. Balakrishnan, and P. Ravikumar, “On learning ising models under huber’s contamination model,” Advances in neural information processing systems, vol. 33, pp. 16327–16338, 2020.

[43] S. Boyd and L. Vandenberghe, Convex Optimization. USA: Cambridge University Press, 2004.

[44] J. Wen, J. Zhao, and P. Jaillet, “Rebalancing shared mobility-on-demand systems: A reinforcement learning approach,” in 2017 IEEE 20th International Conference on Intelligent Transportation Systems (ITSC), 2017, pp. 220–225.

[45] Z. Wan, H. Li, H. He, and D. Prokhorov, “Model-free real-time ev charging scheduling based on deep reinforcement learning,” IEEE Transactions on Smart Grid, vol. 10, no. 5, pp. 5246–5257, 2019.

[46] Y. Liu, K. Zhang, T. Basar, and W. Yin, “An improved analysis of (variance-reduced) policy gradient and natural policy gradient methods,” Advances in Neural Information Processing Systems, vol. 33, pp. 7624–7636, 2020.

[47] T. Lin, C. Jin, and M. Jordan, “On gradient descent ascent for nonconvex-concave minimax problems,” in International Conference on Machine Learning, PMLR, 2020, pp. 6083–6093.

[48] D. Ding, K. Zhang, T. Basar, and M. Jovanovic, “Natural policy gradient primal-dual method for constrained markov decision processes,” Advances in Neural Information Processing Systems, vol. 33, pp. 8378–8390, 2020.

[49] S. M. Kakade, “A natural policy gradient,” Advances in neural information processing systems, vol. 14, 2001.

[50] J. Schulman, S. Levine, P. Abbeel, M. Jordan, and P. Moritz, “Trust region policy optimization,” in International conference on machine learning, PMLR, 2015, pp. 1889–1897.

[51] T. P. Lillicrap, J. J. Hunt, A. Pritzel, N. Heess, T. Erez, Y. Tassa, D. Silver, and D. Wierstra, “Continuous control with deep reinforcement learning,” arXiv preprint arXiv:1509.02971, 2015.

[52] S.-I. Amari, “Natural gradient works efficiently in learning,” Neural computation, vol. 10, no. 2, pp. 251–276, 1998.

[53] R. Lowe and Y. I. Wu, “Multi-agent actor-critic for mixed cooperative-competitive environments,” in NeurIPS, 2017, pp. 6379–6390.

[54] G. Wang, W. Li, J. Zhang, Y. Ge, Z. Fu, F. Zhang, Y. Wang, and D. Zhang, “Sharedcharging: Data-driven shared charging for large-scale heterogeneous electric vehicle fleets,” Proceedings of the ACM on Interactive, Mobile, Wearable and Ubiquitous Technologies (IMWUT), vol. 3, no. 3, p. 108, 2019.