Abstract

In relativistic Hamiltonians the two-nucleon interaction is expressed as a sum of $\tilde{v}_{ij}$, the interaction in the $P_{ij} = 0$ rest frame, and the “boost interaction” $\delta v(P_{ij})$ which depends upon the total momentum $P_{ij}$ and vanishes in the rest frame. The $\delta v$ can be regarded as a sum of four terms: $\delta v_{RE}$, $\delta v_{LC}$, $\delta v_{TP}$ and $\delta v_{QM}$; the first three originate from the relativistic energy-momentum relation, Lorentz contraction and Thomas precession, while the last is purely quantum. The contributions of $\delta v_{RE}$ and $\delta v_{LC}$ have been previously calculated with the variational Monte Carlo method for $^3$H and $^4$He. In this brief note we report the results of similar calculations for the contributions of $\delta v_{TP}$.
and $\delta v_{QM}$. These are found to be rather small.

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Recently we reported [1] results of variational Monte Carlo calculations of $^3\text{H}$ and $^4\text{He}$ with a relativistic Hamiltonian based on the work of Foldy [2], Krajcik and Foldy [3] and Friar [4]. This Hamiltonian has the form:

$$H = \sum_i \left[ (m_i^2 + p_i^2)^{1/2} - m \right] + \sum_{i<j} [\tilde{v}_{ij} + \delta v(P_{ij})] + \sum_{i<j<k} V_{ijk},$$  

(1)

where $p_i$ label momenta of particles, and $P_{ij} = p_i + p_j$ is the total momentum of the pair $ij$. The two-nucleon interaction $\tilde{v}_{ij}$ is obtained by fitting the scattering data in the $P_{ij} = 0$ frame. The boost interaction $\delta v(P_{ij})$ is zero when $P_{ij} = 0$, and is generally given by:

$$\delta v(P_{ij}) = -\frac{P_{ij}^2}{8m^2} \tilde{v}_{ij} + \frac{1}{8m^2}[P_{ij} \cdot P_{ij} \cdot \nabla_{ij} \tilde{v}_{ij}] + \frac{1}{8m^2} [(\sigma_i - \sigma_j) \times P_{ij} \cdot \nabla_{ij} \tilde{v}_{ij}]$$

(2)

up to order $P_{ij}^2/m^2$. Only the first two terms of this $\delta v(P_{ij})$ were considered in ref. [1]. The last term, having $(\sigma_i - \sigma_j)$, does not have diagonal matrix elements in eigenstates of $S^2 = (\sigma_i + \sigma_j)^2$. Hence it was neglected in [1]. The Urbana model VII of $V_{ijk}$ is used, and its boost correction $\delta V_{ijk}(P_{ijk})$ is neglected. This correction is zero for $^3\text{H}$ in its rest frame, and in $^4\text{He}$ it is expected to contribute much less than the $\delta v(P_{ij})$.

In the present work we calculate the expectation value of the $(\sigma_i - \sigma_j)$ term in $\delta v(P_{ij})$. This term can couple the dominant two-nucleon $T, S = 1, 0$ and $0, 1$ waves in the wave function of $^3\text{H}$ and $^4\text{He}$ to the small P-waves having $T, S = 1, 1$ and $0, 0$ respectively. The $\tilde{v}_{ij}$ has fourteen terms like those of the Urbana $v_{14}$ interaction [5]. The first six of these have operators $(1, \sigma_i \cdot \sigma_j, S_{ij}) \otimes (1, \tau_i \cdot \tau_j)$, and are denoted by $\tilde{v}_{6,ij}$:

$$\tilde{v}_{6,ij} = v_c(r_{ij}) + v_\sigma(r_{ij})\sigma_i \cdot \sigma_j + v_t(r_{ij})S_{ij}$$

$$+ [v_\tau(r_{ij}) + v_\sigma\tau(r_{ij})\sigma_i \cdot \sigma_j + v_{t\tau}(r_{ij})S_{ij}] \tau_i \cdot \tau_j.$$  

(3)

The $\tilde{v}_{6,ij}$ gives $> 98\%$ of the $\langle \tilde{v}_{ij} \rangle$ in $^3\text{H}$ and $^4\text{He}$, therefore we approximate the $\tilde{v}_{ij}$ in the $(\sigma_i - \sigma_j)$ term of $\delta v(P_{ij})$ by $\tilde{v}_{6,ij}$.

The commutator can be written as:

$$\frac{1}{8m^2} [(\sigma_i - \sigma_j) \times P_{ij} \cdot \nabla_{ij} \tilde{v}_{6,ij}] = \delta v_{TP}(P_{ij}) + \delta v_{QM}(P_{ij}),$$  

(4)
where
\[ \delta v_{TP}(P_{ij}) = \frac{1}{8m^2}(\sigma_i - \sigma_j) \times P_{ij} \cdot (\nabla_{ij} \mathring{v}_{6,ij}), \]  
(5)
and \( \delta v_{QM}(P_{ij}) \) contains terms that come from the commutator of \( (\sigma_i - \sigma_j) \) with the spin operators in \( \mathring{v}_{6,ij} \). The \( \delta v_{TP}(P_{ij}) \) originates from the classical Thomas precession \[6,7\]. The precession of the spin \( s_i \) in the frame moving with velocity \( P_{ij}/2m \) is given by
\[ -\frac{1}{2} \sigma_i \cdot \nabla_{ij} \mathring{v}_{ij} \times P_{ij}/4m^2 \]
up to order \( 1/m^2 \). Thus the Thomas precession potential for particle \( i \) is:
\[ -\frac{1}{2} \sigma_i \cdot \nabla_{ij} \mathring{v}_{ij} \times P_{ij}/4m^2 = \frac{1}{8m^2} \sigma_i \times P_{ij} \cdot (\nabla_{ij} \mathring{v}_{ij}). \]  
(6)
Both particles have same velocity due to their center of mass motion, but their accelerations due to \( \mathring{v}_{ij} \) are equal and opposite. Therefore the Thomas precession potential for the particle \( j \) is
\[ -\sigma_j \times P_{ij} \cdot (\nabla_{ij} \mathring{v}_{ij})/8m^2, \]
and together with (6) it makes up the \( \delta v_{TP}(P_{ij}) \). After some algebra we obtain:
\[ \delta v_{TP}(P_{ij}) = \frac{1}{8m^2 r} \left[ \left( v_c' - v_s' + v_t' + 3\frac{v_t}{r} \right) P \cdot r \times (\sigma_i - \sigma_j) - i \left( 2v_s' + v_t' + 3\frac{v_t}{r} \right) (P \cdot \sigma_i r \cdot \sigma_j - P \cdot \sigma_j r \cdot \sigma_i) \right] + \tau_i \cdot \tau_j \text{ term}, \]  
(7)
where \( v_x' \) denotes \( \partial v_x/\partial r \), the \( ij \) subscripts of \( r, P \) and \( v_x \) are omitted for brevity, and the \( \tau_i \cdot \tau_j \) term has \( v_r, v_{\sigma r} \) and \( v_{tr} \) in place of \( v_c, v_s \) and \( v_t \).

The \( \delta v_{QM}(P_{ij}) \) does not have a classical analogue; it is found to be:
\[ \delta v_{QM}(P_{ij}) = \frac{i}{2m^2}(v_t - v_s)(P \cdot \sigma_i \sigma_j \cdot \nabla - P \cdot \sigma_j \sigma_i \cdot \nabla) - \frac{3i}{4m^2} \frac{v_t}{r^2} P \cdot r (\sigma_i \cdot r \sigma_j \cdot \nabla - \sigma_j \cdot r \sigma_i \cdot \nabla) - \frac{3i}{4m^2} \frac{v_t}{r^2} (P \cdot \sigma_i r \cdot \sigma_j - P \cdot \sigma_j r \cdot \sigma_i) r \cdot \nabla \]
\[ + \tau_i \cdot \tau_j \text{ terms} \]  
(8)
from eq. (4).

It is convenient \[4\] to express \( \delta v(P_{ij}) \) given by eq. (2) as:
\[ \delta v(P_{ij}) = \delta v_{RE}(P_{ij}) + \delta v_{LC}(P_{ij}) + \delta v_{TP}(P_{ij}) + \delta v_{QM}(P_{ij}). \]  
(9)
Its first term:

\[
\delta v_{RE}(P_{ij}) = -\frac{P_{ij}^2 \tilde{v}_{ij}}{8m^2}
\]  

(10)

comes from the relativistic energy, and the second:

\[
\delta v_{LC}(P_{ij}) = \frac{1}{8m^2} P_{ij} \cdot r_{ij} P_{ij} \cdot (\nabla_{ij} \tilde{v}_{ij})
\]  

(11)

from Lorentz contraction. The \([P_{ij} \cdot r_{ij} P_{ij} \cdot \nabla_{ij}, \tilde{v}_{ij}]\) can have terms in addition to those in \(\delta v_{LC}\) when \(\tilde{v}_{ij}\) depends upon the relative momentum \(p_{ij}\). These terms are to be regarded as a part of \(\delta v_{QM}\). However, they vanish when \(\tilde{v}_{ij}\) is approximated with \(\tilde{v}_{6,ij}\).

The expectation values of \(\delta v_{TP}(P_{ij})\) and \(\delta v_{QM}(P_{ij})\) are calculated with the variational wave function of ref. [1] using the Monte Carlo methods described in [1]. The results are tabulated in table I along with others of interest from [1]. The contributions of \(\delta v_{TP}\) and \(\delta v_{QM}\) are much smaller than those of \(\delta v_{RE}\) and \(\delta v_{LC}\) as expected. These contributions would be exactly zero if there were no two-nucleon P-waves in these nuclei.

Stadler and Gross [8] have also estimated these contributions in \(^3\)H with a different method and obtained similar results.

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### TABLE I. Expectation values in MeV

| Expression | $^3\text{H}$      | $^4\text{He}$ |
|------------|------------------|----------------|
| $\langle \sum_i (m_i^2 + p_i^2)^{1/2} - m \rangle$ | 48.7(2) | 105.0(6) |
| $\langle \sum_{i<j} \tilde{v}_{ij} \rangle$ | -55.9(2) | -127.4(5) |
| $\langle \sum_{i<j<k} \tilde{v}_{ijk} \rangle$ | -1.21(2) | -5.43(15) |
| $\langle \sum_{i<j} \delta v_{RE}(P_{ij}) \rangle$ | 0.23(2) | 1.17(3) |
| $\langle \sum_{i<j} \delta v_{LC}(P_{ij}) \rangle$ | 0.10(1) | 0.53(1) |
| $\langle \sum_{i<j} \delta v_{TP}(P_{ij}) \rangle$ | 0.016(2) | 0.074(4) |
| $\langle \sum_{i<j} \delta v_{QM}(P_{ij}) \rangle$ | -0.004(2) | -0.014(4) |
| $\langle H \rangle$ | -8.07(3) | -25.90(8) |
REFERENCES

[1] J. Carlson, V.R. Pandharipande and R. Schiavilla, Phys. Rev. C 47, 484 (1993).

[2] L.L. Foldy, Phys. Rev. 122, 275 (1961).

[3] R.A. Krajcik and L.L. Foldy, Phys. Rev. D 10, 1777 (1974).

[4] J.L. Friar, Phys. Rev. C 12, 695 (1975).

[5] I.E. Lagaris and V.R. Pandharipande, Nucl. Phys. A359, 331 (1981).

[6] J.D. Jackson, *Classical Electrodynamics*, 2nd edition, chapter 11, John Wiley (1975).

[7] J.L. Forest, V.R. Pandharipande and J.L. Friar, to be published, (1994).

[8] A. Stadler and F. Gross, Contribution to the 14th International Conference on Few body problems in Physics, Williamsburg, Virginia (1994), p. 922.