Realization of Bound States in the Continuum in Anti-PT-Symmetric Optical Systems: A Proposal and Analysis

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Novel physical concepts that originate from quantum mechanics, such as non-Hermitian systems (dealing mostly with PT and anti-PT symmetry) and bound states in the continuum (BICs), have attracted great interest in the optics and photonics community. To date, BICs and anti-PT symmetry seem to be two independent topics. Here, a parallel cascaded-resonator system is proposed for the first time to achieve BICs and anti-PT symmetry simultaneously. It is found that the requirements for the Fabry–Perot BIC and anti-PT symmetry can both be satisfied when the phase shift between any two adjacent resonators is an integer multiple of \( \pi \). The cascaded-resonator systems consisting of different numbers of resonators are further analyzed and their robustness to fabrication imperfections is demonstrated. The proposed structure can readily be realized on an integrated photonic platform, and can enable many applications that benefit from the advantages of both BICs and anti-PT symmetry, such as optical delay and storage, all-optical nonlinear processing, high-sensitivity sensing, and chiral mode switching.

1. Introduction

Non-Hermitian physics, which exhibits properties significantly different from that of a closed system, has attracted increasing interest.\(^1,2\) In 1998, PT symmetry was proposed by Bender and Boettcher in quantum mechanics to investigate non-Hermitian systems with real eigenvalues.\(^3\) PT-symmetric systems are invariant under the combined action of parity reversal (P) and time-reversal (T) operations. The exceptional point separates the PT-symmetric phase (with real eigenvalues) from the PT-broken phase (with complex eigenvalues). PT-symmetric systems have been investigated and demonstrated in various areas.\(^4-10\)

Among them, optical systems can provide ideal platforms for non-Hermitian physics with tunability and accessibility. Optical realizations of PT-symmetric systems not only advance the theoretical studies of non-Hermitian physics, but also give rise to various applications in photonics, including PT-symmetric lasers,\(^11-13\) high-sensitivity sensing at exceptional points,\(^14-16\) and nonreciprocal light propagation.\(^17,18\)

Anti-PT (APT) symmetry, the counterpart of PT-symmetry, imposes that the Hamiltonian anticommute with the joint parity and time operator. Unlike the PT-symmetric systems,\(^19\) APT-symmetric systems can be realized in the absence of a gain medium, which makes such systems much more accessible.\(^20,21\) To date, APT symmetry has been observed and investigated in cold atoms,\(^22,23\) microwaves,\(^24,25\) nonlinear optical systems,\(^26-28\) and integrated photonic systems.\(^20,21\) In integrated photonics, there are some easy-to-implement configurations for realizing APT symmetry, including optical waveguides and microcavities.\(^20,21\)

Bound states in the continuum (BICs) were initially proposed in quantum mechanics by von Neumann and Wigner. BIC refers to a wave state which is perfectly confined without any radiation loss even though it exists in a continuous spectrum.\(^29\) BICs have been investigated and demonstrated in electromagnetic, acoustic, and water waves.\(^30\) Recently, the development of nanofabrication technologies has enabled realizations of BICs in photonics.\(^31-36\) leading to many new applications such as enhanced optical nonlinearity,\(^37\) lasers,\(^38,39\) filters,\(^40\) and sensors.\(^39,41\) In integrated photonics, BICs can be achieved by coupling resonators to the radiation reservoir with perfectly destructive interference among the dissipation channels.\(^39\)

To date, BICs and APT symmetry seem to be two independent topics. Here, we propose a parallel cascaded-resonator system to achieve BICs and APT symmetry simultaneously. For APT-symmetric optical systems consisting of \( N > 2 \) resonators, we theoretically studied the evolution of eigenfrequencies, as well as the transmission characteristics. Then, we numerically simulated these structures with a finite-element method, and the simulated results agree well with the analytical results. We also compared the cascaded-resonator systems consisting of different numbers of resonators, and analyzed the effects of experimental imperfections. The proposed structures can be easily realized on an integrated photonic platform and will enable many applications that benefit from the advantages of both BICs and anti-PT symmetry.
2. Results

2.1. Theoretical Analysis

Figure 1 shows the schematic of our proposed structure, which consists of N parallel cascaded-microresonators with their intrinsic resonant frequencies \( \omega_0, \omega_1, \ldots, \omega_N \). For the sake of simplicity, the distances between any two adjacent resonators are assumed to be equal and sufficiently large, such that any resonator is coupled with the others indirectly through the bus waveguides. Assuming that the coupling rates between the resonators and bus waveguides are \( \gamma / 2 \) and the intrinsic loss rates of the resonators are all \( \gamma / 2 \), the effective Hamiltonian of the system is expressed as (see the Supporting Information)

\[
H = \begin{bmatrix}
\Delta_1 - i\gamma_c & -i\gamma_c e^{i\theta} & \cdots & -i\gamma_c e^{(N-1)i\theta} \\
-i\gamma_c e^{i\theta} & \Delta_2 - i\gamma_c & \cdots & -i\gamma_c e^{(N-2)i\theta} \\
\vdots & \vdots & \ddots & \vdots \\
-i\gamma_c e^{(N-1)i\theta} & -i\gamma_c e^{(N-2)i\theta} & \cdots & \Delta_N - i\gamma_c
\end{bmatrix}
\]  

(1)

where \( \Delta_m = \omega_m - \omega_0 \) is the frequency detuning of the \( m \)-th resonator from the average frequency of \( N \) resonators with \( \omega_0 \) equal to \( (\omega_0 + \omega_1 + \ldots + \omega_N) / N \) and \( \theta \) is the phase shift between any two adjacent resonators. This system satisfies the APT-symmetry requirement when \( \theta = n\pi \) and \( \Delta_m + \Delta_{N+1-m} = 0 \) with \( n \) being an integer. When \( \theta = n\pi \) and \( \exp(i\theta) = (-1)^n \), the effective Hamiltonian can be simplified as

\[
H = \begin{bmatrix}
\Delta_1 - i\gamma_c & (-1)^{n+1}i\gamma_c & \cdots & (-1)^{N-n+1}i\gamma_c \\
(-1)^{n+1}i\gamma_c & \Delta_2 - i\gamma_c & \cdots & (-1)^{N-n+1}i\gamma_c \\
\vdots & \vdots & \ddots & \vdots \\
(-1)^{N-n+1}i\gamma_c & (-1)^{N-n+1}i\gamma_c & \cdots & \Delta_N - i\gamma_c
\end{bmatrix}
\]  

(2)

Next, we analyzed the eigenfrequencies of such an APT-symmetric system. With different numbers (\( N \)) of resonators, the eigenvalues \( (\sigma) \) of the Hamiltonian behave differently. Eigenfrequencies \( (\omega) \) of such an APT-symmetric system are \( \omega_0 = -i\gamma_c / 2 + \sigma \). When \( N = 2k \) is an even number, the eigenvalues of the Hamiltonian satisfy

\[
\left( \sigma^2 - \Delta_1^2 \right) \cdots \left( \sigma^2 - \Delta_k^2 \right) \left[ 1 + 2i\gamma_c \sigma \left( \frac{1}{\sigma^2 - \Delta_1^2} + \frac{1}{\sigma^2 - \Delta_2^2} + \cdots \right)
\right. 
\left. + \frac{1}{\sigma^2 - \Delta_k^2} + \frac{1}{2\sigma^2} \right] = 0
\]  

(3)

It shows that the eigenvalues of the system are determined by the frequency detuning of each resonator. When \( \Delta_1 = \Delta_2 = \ldots = \Delta_k = \Delta \), the eigenvalues satisfy

\[
(\sigma - \Delta)^{k-1}(\sigma + \Delta)^{k-1}(\sigma^2 + 2ik\gamma_c \sigma - \Delta^2) = 0
\]  

(4)

Among all the eigenvalues, two of them satisfy \( \sigma^2 + 2ik\gamma_c \sigma - \Delta^2 = 0 \). Therefore, the two eigenfrequencies are \( \omega = \omega_0 - i\gamma_c / 2 - i\gamma_c \pm (\Delta^2 - k^2\gamma_c^2)^{1/2} \), which behave similarly as those of APT-symmetric systems with two resonators. The spontaneous phase transition occurs at \( \Delta = k\gamma_c \). The two eigenmodes take the same resonant frequency but different decay rates in the APT-symmetric phase \( (\Delta < k\gamma_c) \), while taking different resonant frequencies but the same decay rate in the APT-broken phase \( (\Delta > k\gamma_c) \). System performance can also be enhanced by increasing the number of resonators. For example, non-Hermitian systems can be used for high-sensitivity sensing. The sensitivity of such a system working at the exceptional point is defined as the change in eigenfrequencies when the frequency detuning \( \Delta \) deviates from \( k\gamma_c \) by \( \epsilon \). Increasing the number of resonators can enhance the sensitivity, which is approximately proportional to \( (k\gamma_c)^{1/2} \). The other eigenfrequencies \( (\omega = \omega_0 - i\gamma_c / 2 \pm \Delta) \) belong to the Fabry–Pérot (FP) BICs. However, when \( N = 2k + 1 \) is an odd number, the eigenvalues of the Hamiltonian satisfy

\[
\sigma(\sigma^2 - \Delta_1^2) \cdots (\sigma^2 - \Delta_k^2) \left[ 1 + 2i\gamma_c \sigma \left( \frac{1}{\sigma^2 - \Delta_1^2} + \frac{1}{\sigma^2 - \Delta_2^2} + \cdots \right)
\right. 
\left. + \frac{1}{\sigma^2 - \Delta_k^2} + \frac{1}{2\sigma^2} \right] = 0
\]  

(5)

Similar to the discussion above, when \( \Delta_1 = \Delta_2 = \ldots = \Delta_k = \Delta \), Equation (5) reduces to

\[
(\sigma - \Delta)^{k-1}(\sigma + \Delta)^{k-1}(\sigma^2 + (2k + 1)i\gamma_c \sigma - \Delta^2) = 0
\]  

(6)

where exceptional points do not exist.
In this case, the quality factor of this cascaded FP BIC (bus waveguides) interferes destructively with each other. In radiation into the continuum through the radiation channels light is trapped between the resonator systems. When the intrinsic resonant frequencies of the four resonators are different, there are four dips at the resonant frequencies and three peaks sandwiched by the resonant dips. When two resonant frequencies are equal ($\omega_1 = \omega_2$, $\omega_3 = \omega_4$), the cascaded FP BIC exists, and the transmission is 0 at the resonant frequency. Such transmission spectra are similar to those of electromagnetically induced transparency (EIT), which is a quantum interference effect originating from atomic physics. Indeed, there have been many investigations on the transmission characteristics of similar systems, including all-optical EIT in coupled-resonator systems.

Figure 2 shows the numerically calculated transmission spectra for a system consisting of four cascaded resonators. The resonant wavelengths are set to $\lambda_1 = 1548.2$ nm, $\lambda_2 = 1548.4$ nm, $\lambda_3 = 1548.6$ nm, $\lambda_4 = 1548.8$ nm, and $\lambda_1 = 1548.2$ nm, $\lambda_2 = 1548.5$ nm, $\lambda_4 = 1548.8$ nm, and c) $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 1548.5$ nm. The distances between any two adjacent resonators are equal and sufficiently large.

We also analyzed the FP BICs in the APT-symmetric system in detail. When $N = 2$, the FP BIC can be realized if the intrinsic frequencies of resonators are equal and the phase shift is an integer multiple of $\pi$. The phase shift naturally satisfies the FP resonance requirement since APT-symmetric systems require that $\theta = n\pi$. When $N > 2$, we can also obtain the cascaded FP BIC if there is no resonance detuning ($\Delta = 0$). The eigenfrequencies of the resonator system thus become $\omega_0 - i\gamma/2$ and $\omega_0 - i\gamma/2 - iN\gamma_2$. The eigenmodes with frequency $\omega_{\text{BIC}} (= \omega_0 - i\gamma/2)$ are FP BICs and the degeneracy of $\omega_{\text{BIC}}$ is $N - 1$. For the FP BIC, the light is trapped between the $N$ resonators, where the resonant radiation into the continuum through the radiation channels (bus waveguides) interferes destructively with each other. In this case, the quality factor of this cascaded FP BIC ($\omega_0/\gamma$) is relatively high, and the photon lifetime becomes infinitely long when the intrinsic loss rate $\gamma/2$ is negligible.

We further analyzed the transmission characteristics for the parallel cascaded-resonator system based on the temporal coupled-mode theory (see the Supporting Information)

$$t = e^{i\theta} \frac{(i\delta\omega_1 - \gamma/2)(i\delta\omega_2 - \gamma/2) + 2i\gamma_2 \sin \theta + \kappa^2}{(i\delta\omega_1 - \gamma/2)(i\delta\omega_2 - \gamma/2) - \gamma_2 [i(\delta\omega_1 + \delta\omega_2) - \gamma] - 2i\gamma_2 e^{i\theta} (\kappa + \gamma_2 \sin \theta) + \kappa^2}$$

$$t = \left[1 - \gamma_2 \left[1/(i\delta\omega_1 - \gamma/2) + 1/(i\delta\omega_2 - \gamma/2) + \cdots \right]ight]^{-1}
+ 1/(i\delta\omega_N - \gamma/2)]^{-1}$$

where $T (= |t|^2)$ is the fraction of power transmission and $\delta\omega_j (= \omega_j - \omega_0)$ is the difference between the probe frequency $\omega_0$ and the resonant frequency of the $j$th resonator ($\omega_j$). If $\gamma_2 >> \delta\omega_j >> \gamma$, we can find that $T$ approaches 0 when the probe frequency coincides with any one of the intrinsic resonant frequencies. There also exists a peak (T approaching 1) between any two adjacent resonant frequencies in the transmission spectrum. Figure 2 shows the numerically calculated transmission spectra for a system consisting of four cascaded resonators. When the intrinsic resonant frequencies of the four resonators are different, there are four dips at the resonant frequencies and three peaks sandwiched by the resonant dips. When two resonant frequencies are equal ($\omega_1 = \omega_2$, the second and third dips merge. When all the resonant frequencies are equal ($\omega_1 = \omega_2 = \omega_3 = \omega_4$), the cascaded FP BIC exists, and the transmission is 0 at the resonant frequency.

Such transmission spectra are similar to those of electromagnetically induced transparency (EIT), which is a quantum interference effect originating from atomic physics. Indeed, there have been many investigations on the transmission characteristics of similar systems, including all-optical EIT in coupled-resonator systems. These phenomena have found applications in optical delay and storage, high-sensitivity sensing, and optical force enhancement. Our results also prove that the artificial transparency windows, which are the peaks in the transmission spectra, can be dynamically controlled by finely tuning the intrinsic resonant frequencies of the resonators. The quality factor and position of transmission peaks can also be tuned by changing the intrinsic resonant frequencies of the resonators.

In case that the cascaded resonators are close to each other causing direct coupling ($\kappa$) between them, the effective Hamiltonian for an $N = 2$ system can be expressed as

$$H = \begin{bmatrix} 0 & \kappa \\ -\kappa & 0 \end{bmatrix} + \frac{\Delta - i\gamma_2}{(1 + i\gamma_2)(1 - i\gamma_2)}$$

$$\frac{\Delta - i\gamma_2}{(1 + i\gamma_2)(1 - i\gamma_2)}$$

which has the eigenvalues $\sigma = -i\gamma_2 \pm (|\Delta|^2 + (\kappa - |\gamma|^2)^2)^{1/2}$. When $\Delta = 0$, the eigenfrequencies are $\omega_0 - i\gamma_2/2 - (1)^i \kappa$ and $\omega_0 - i\gamma_2/2 + (1)^i \kappa - 2i\gamma_2$. The FP BIC still exists with $\omega_{\text{BIC}} = \omega_0 - i\gamma_2/2 - (1)^i \kappa$. The two eigenmodes no longer have the same resonant frequency because the direct coupling breaks the APT symmetry. The eigenfrequencies are not degenerate at the exceptional point ($\Delta = \gamma_2$) but are separated with $\Delta\omega = 2\kappa^2 (1 - (1)^2 (2i\gamma_2)^2)^{1/2}$. The transmission spectra have also changed significantly (see the Supporting Information).

Figure 3 plots the transmission spectra for different phase shifts $\theta$ and direct coupling rates $\kappa$. For a system consisting of two identical resonators, FP BIC exists when $\theta = 2n\pi$, giving rise to only one dip in the transmission spectra (Figure 3a). The resonant frequency of the FP BIC shifts by $\kappa$ when the direct coupling $\kappa$ is introduced. However, for $\theta = 2n\pi + \pi/2$, the dip splits into two separate dips as $\kappa$ increases (Figure 3b). When $\theta = 2n\pi$, and the resonant frequencies of the two resonators are different, an optical analog of the EIT phenomenon appears with $\kappa = 0$. The
two dips are further separated with the increase of $\kappa$, evolving into a high-$Q$ dip and a low-$Q$ dip (Figure 3c). For $\theta = 2\pi + \pi/2$, the two dips are also further separated as $\kappa$ increases but with the same linewidth (Figure 3d). These phenomena as shown in the transmission spectra in Figure 3 enable versatile and flexible applications in optical switching and filtering. With such a parallel cascaded-resonator system, we can control the transmission (or the “on/off” state in optical switching) at a certain wavelength by tuning any one of these parameters: intrinsic resonant frequencies, phase shift $\theta$, and coupling rate $\kappa$.

2.2. Numerical Simulation

Based on the theoretical calculation, we adopted a 2D finite-element method in COMSOL to simulate the APT-symmetric optical system. Generalizations of our 2D simulation to 3D structures are straightforward. Without loss of generality, we designed the structures based on a silicon integrated photonic platform. All the microring resonators have a radius of 3.1 $\mu$m and a width of 0.4 $\mu$m. The width of the bus waveguides is 0.4 $\mu$m and the gaps between one bus waveguide and the ring resonators are 0.1 $\mu$m. Silicon has a negligible intrinsic absorption rate ($\gamma_i = 0$) in the communication band ($\approx 1550$ nm). The resonant frequency is tuned by slightly changing the microring’s inner radius. We analyzed the FP BICs and the evolution of the eigenfrequencies with $N (= 2, 3, 4)$ cascaded resonators.

**Figure 4a,b** plots the eigenfrequencies of the APT-symmetric system with two resonators. The requirement of FP BIC is satisfied at $\Delta = 0$ ($\omega_1 = \omega_2$). In this case, the light is trapped between the two resonators, as shown in an inset of Figure 4b. The exceptional point is located at $\Delta/2\pi = \gamma_c/2\pi = 5.15$ GHz. The system works in the APT-symmetric phase at $\Delta < \gamma_c$, as the two eigenmodes share the same resonant frequency but have different loss rates. The system works in the APT-broken phase at $\Delta > \gamma_c$ as the two eigenmodes share the same loss rate but have different resonant frequencies and asymmetric optical field distributions. The two eigenmodes coalesce at the exceptional point ($\Delta = \gamma_c$) where their eigenfrequencies are degenerate.

The cases are different for systems with a larger number of resonators. Figure 4c,d shows the simulated results for an APT-symmetric system with $N = 3$. Cascaded FP BIC also exists at $\Delta = 0$ ($\omega_1 = \omega_2 = \omega_3$). There are two high-$Q$ modes with asymmetric field distributions and one low-$Q$ mode at $\Delta \neq 0$. The evolution behavior of the eigenfrequencies indicates that the eigenmodes of the parallel cascaded-resonator system with an odd $N$ do not merge and thus no exceptional point exists, which agrees with the analytical results. Figure 4e,f shows the simulated results for an APT-symmetric system with $N = 4$. When all the resonators share the same resonant frequency, the cascaded FP BIC is formed where the light is confined to the middle two resonators, as shown in an inset of Figure 4f. Similar to the case of $N = 2$, two eigenmodes take the same resonant frequency but different loss rates in the APT-symmetric phase ($\Delta_1 = \Delta_2 < k\gamma_c$) while taking the same loss rate and exhibiting different resonant frequencies and asymmetric field distributions in the APT-broken phase ($\Delta_1 = \Delta_2 > k\gamma_c$). The other two eigenmodes with purely real eigenfrequencies are the FP BICs formed between the first and
Figure 4. Simulated evolution of the eigenfrequencies for the parallel cascaded-resonator system. a,b) Real and imaginary part of the eigenfrequencies for the $N = 2$ system. The insets in (a) show the field distributions (|$E$| component) of the two eigenmodes in the APT-broken phase ($\Delta = 1.9185 \gamma_c$, $\gamma_c/2 \pi = 5.15$ GHz), while the insets in (b) show those in the APT-symmetric phase ($\Delta = 0.5232 \gamma_c$) as well as that of the FP BIC. c,d) Real and imaginary part of the eigenfrequencies for the $N = 3$ system. The insets in (c) show the field distributions (|$E$| component) of the three eigenmodes at $\Delta = 4.5882 \gamma_c$ ($\gamma_c/2 \pi = 2.48$ GHz), while the insets in (d) show that of a low-$Q$ state ($\Delta = 1.0587 \gamma_c$) as well as that of the FP BIC. There is no exceptional point in this case. e,f) Real and imaginary part of the eigenfrequencies for the $N = 4$ system. The insets in (e) show the field distributions (|$E$| component) of two eigenmodes in the APT-broken phase ($\Delta_{1,2} = \Delta = 1.8188 \gamma_c, \gamma_c/2 \pi = 3.21$ GHz), while the insets in (f) show those in the APT-symmetric phase ($\Delta_{1,2} = \Delta = 0.5968 \gamma_c$) as well as that of the cascaded FP BIC.

Last resonators with $\Delta_1 = \Delta_2$. See the Supporting Information for the simulated results of systems with more than four resonators.

2.3. Effects of Experimental Imperfections

The proposed structure can be realized on an integrated photonic platform. In real systems, the phase shift between adjacent resonators and the intrinsic resonant frequencies can be tuned precisely by the electro-optic or thermo-optic effect. However, experimental imperfections are unavoidable. Here, we investigated the effects of imperfections in our cascaded-resonator systems. For a system consisting of two resonators where the phase shift $\theta$ is not equal to $n \pi$ and the coupling rates between the bus waveguides and the two resonators are different, the effective Hamiltonian of this system becomes

$$H = \begin{bmatrix} \Delta - i\gamma_{c1} & -i\sqrt{\gamma_{c1}\gamma_{c2}} e^{i\theta} \\ -i\sqrt{\gamma_{c1}\gamma_{c2}} e^{-i\theta} & -\Delta - i\gamma_{c2} \end{bmatrix}$$

(10)

The eigenfrequencies of this system are $\omega_\pm = \omega_0 - i\gamma_c/2 - i\gamma_{c1} + \gamma_{c2}/2 \pm [(\gamma_{c1} - \gamma_{c2})^2/4 + \Delta^2 - i\Delta(\gamma_{c1} - \gamma_{c2}) - \gamma_{c1}\gamma_{c2}\exp(2i\theta)]^{1/2}$.

Based on the Hamiltonian, we first considered the fabrication imperfections affecting the gaps between the bus waveguides and the two resonators rendering $\gamma_{c1} \neq \gamma_{c2}$. For $\theta = n \pi$, the
Figure 5. Numerical results of potential experimental imperfections. a) Numerically simulated deviation $\text{Im}[\omega_{\text{BIC}} - \omega_{\text{NBIC}}/2\pi \gamma]$ for systems consisting of $N = 2, 3, \ldots, 8$ resonators. b) Numerically simulated effect of $\delta \theta$ on the APT symmetry for a system consisting of four resonators. c) Numerically simulated deviation $\text{Re}[\Delta \omega/2\pi \gamma]$ at the exceptional point for systems consisting of $N = 2, 4, 6, 8$ resonators.

eigenfrequencies become $\omega_\pm = \omega_0 - i\gamma/2 - i[(\gamma_{c1} + \gamma_{c2})/2 \pm \sqrt{(\gamma_{c1} + \gamma_{c2})^2/4 + \Delta^2 - i\Delta(\gamma_{c1} - \gamma_{c2})}]$. The FP BIC ($\omega_{\text{BIC}} = \omega_0 - i\gamma/2$) still exists at $\Delta = 0$. The FP BIC also exists for a system consisting of $N$ resonators with $\omega_{\text{BIC}} = \omega_0 - i\gamma/2$, which indicates that the FP BIC exists irrespective of the fabrication imperfections about the gaps between the bus waveguides and the resonators. However, the eigenfrequencies are not degenerate at the exceptional point (at $\Delta \neq 0$) in the presence of imperfections. The separation of the eigenfrequencies $\Delta \omega (=\omega_+ - \omega_-)$ at the exceptional point, which vanishes in the ideal case, increases as the degree of imperfections ($\gamma_{c1} - \gamma_{c2}$) increases.

We also analyzed the effect of a deviation $\delta \theta$ of $\theta$ from n.r. With $\gamma_{c1} = \gamma_{c2}$ and $\Delta = 0$, the eigenfrequencies become $\omega_\pm = \omega_0 - i\gamma/2 - i\gamma_{c1} [1 \pm \exp(i\delta \theta)]$. Deviating from the FP BIC, the eigenmode transits to a state near the BIC with the eigenfrequency $\omega_{\text{BIC}}$ equal to $\omega_0 - i\gamma/2 - i\gamma_{c1} [1 \pm \exp(i\delta \theta)]$. In this case, the FP resonance condition is broken. Figure 5a plots the numerically calculated deviation of the imaginary part of the eigenfrequencies as a function of $\delta \theta$ for systems consisting of $N$ resonators. It indicates that the quality factors of the eigenmodes in a system consisting of more resonators have a higher tolerance to the $\theta$ deviation. With $\delta \theta \neq 0$, the evolution of the eigenfrequencies also deviates from that of the ideal APT-symmetric system. For $\delta \theta = \theta - n\pi << 1$, the eigenfrequencies can be approximated as $\omega_\pm = \omega_0 - i\gamma/2 - i\gamma_{c1} [1 \pm (1 - 1)\delta \theta]/2$ at the exceptional point ($\Delta = \gamma_{c1}$). $\Delta \omega$ (=$\omega_+ - \omega_-)$ at the exceptional point, which consists of both real and imaginary parts, increases as $\delta \theta$ increases. Figure 5b plots the effect of $\delta \theta$ on the imaginary part of the eigenfrequencies. Figure 5c plots the calculated real part of $\Delta \omega/2\pi \gamma_{c1}$ as a function of $\delta \theta$ and $N$. It indicates that the evolution of the eigenfrequencies of a system consisting of more resonators deviates more from its ideal case with the same degree of $\theta$ deviation.

3. Applications

Our proposal for the realization of BICs in an APT-symmetric system not only connects the two independent topics but also brings new opportunities and applications to APT-symmetric systems that may not be possible without BICs.

First, BICs can help to achieve all-optical nonlinearity (such as harmonic generation, four-wave mixing) in APT-symmetric systems. In the APT-symmetric phase, the field of eigenmodes is distributed in all the components (in our system, microring resonators), while in the APT-broken phase, the field is localized in half of the components. It means that the field localization in the APT-broken phase is normally stronger than that in the APT-symmetric phase. Therefore, the strong field localization in the APT-broken phase can help to obtain the nonlinear response at much lower power than the APT-symmetric phase.$^{[15]}$ However, if one can achieve BIC in the APT-symmetric phase in an APT-symmetric system, as proposed in this work, it would be much easier to achieve optical nonlinearity in both the APT-symmetric and APT-broken phases, because the BIC can naturally provide strong light confinement for enhancing the nonlinear response.

In addition, BIC can serve as an intermediate state while dynamically encircling an exceptional point in APT-symmetric systems. It has been shown that dynamically encircling an exceptional point in PT- and APT-symmetric systems exhibits
interesting chiral dynamics and can be exploited for switching between eigenmodes. For example, APT-broken modes can be used for asymmetric mode switching in APT-symmetric waveguide systems.\textsuperscript{22} In our coupled-resonator APT-symmetric system, dynamically encircling an exceptional point can be realized easily through tuning $\Delta$ (intrinsic resonant frequency) and $\theta$ (phase shift). More specifically, the exceptional point is achieved at $\Delta = \gamma_c$, in a two-resonator system according to our theoretical results. Let us consider a loop enclosing the exceptional point parameterized by $\Delta(t) = \gamma_c[1 + \rho \cos(\gamma t)]$ and $\theta(t) = 2\pi \rho + \rho \sin(\gamma t)$, where $\rho$ is the loop radius and $\gamma$ is the angular velocity of the encircling process. At $t = 0$, the starting point is $\Delta = (1 + \rho)\gamma_c$ and $\theta = 2\pi \rho$, indicating that the system is in the APT-broken phase and the eigenstates are localized in one of the two resonators. At $t = \pi / \gamma$, an intermediate state can be reached at $\Delta = (1 - \rho)\gamma_c$ and $\theta = 2\pi \rho$. With $\rho = 1$ ($\Delta = 0$ and $\theta = 2\pi \rho$), this intermediate state will turn into the FP BIC. At the end point, $t = 2\pi / \gamma$, the eigenstate will switch to the one that is antisymmetric to the initial state. This phenomenon, combining FP BIC with asymmetric mode switching for symmetry-broken modes, would be highly desirable for light manipulation in on-chip optical systems.

4. Discussion

In summary, we have theoretically shown that Fabry–Pérot BICs and anti-PT symmetry can be realized simultaneously in a parallel cascaded-resonator system with interesting features. First, the characteristic transmission of such systems, such as the optical analog of electromagnetically induced transparency, enables versatile applications in optical filtering and switching. Second, the sensitivity at the exceptional points is enhanced in a system with more cascaded resonators. Last, taking potential imperfections into consideration, the Fabry–Pérot BICs can be realized regardless of the fabrication imperfections causing variations of the gaps between the bus waveguides and the resonators, and a system consisting of more resonators is more tolerant to deviations of the phase shift between resonators. Our proposed structures that simultaneously achieve BICs and anti-PT symmetry on an integrated photonic platform will lead to new applications that benefit from the advantages of both BICs and anti-PT symmetry, such as optical delay and storage, all-optical nonlinear processing, high-sensitivity sensing, and chiral mode switching.

Supporting Information

Supporting Information is available from the Wiley Online Library or from the author.

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Conflict of Interest

The authors declare no conflict of interest.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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