A Quantum Model for Autonomous Learning Automata

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The idea of information encoding on quantum bearers and its quantum-mechanical processing has revolutionized our world and brought mankind on the verge of enigmatic era of quantum technologies. Inspired by this idea, in present paper we search for advantages of quantum information processing in the field of machine learning. Exploiting only basic properties of the Hilbert space, superposition principle of quantum mechanics and quantum measurements, we construct a quantum analog for Rosenblatt’s perceptron, which is the simplest learning machine. We demonstrate that the quantum perceptron is superior its classical counterpart in learning capabilities. In particular, we show that the quantum perceptron is able to learn an arbitrary (Boolean) logical function, perform the classification on previously unseen classes and even recognize the superpositions of learned classes – the task of high importance in applied medical engineering.

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I. INTRODUCTION

During last few decades, we have been witnessing unification of quantum physics and classical information science that resulted in constitution of new disciplines – quantum information and quantum computation [1,2]. While processing of information, which is encoded in systems exhibiting quantum properties suggests, for example, unconditionally secure quantum communication [3] and superdense coding [4], computers that operate according to the laws of quantum mechanics offer efficient solving of problems that are intractable on conventional computers [5]. Having paramount practical importance, these announced technological benefits have indicated the main directions of the research in the field of quantum information and quantum computation, somehow leaving aside other potential applications of quantum physics in information science. So far, for instance, very little attention has been paid on possible advantages of quantum information processing in such areas of modern information science as machine learning [6] and artificial intelligence [7]. Using standard quantum computation formalism, it has been shown that machine learning governed by quantum mechanics has certain advantages over classical learning [8,12]. These advantages, however, are strongly coupled with more sophisticated optimization procedure than in the classical case, and thus require an efficiently working quantum computer [13] to handle the optimization. This paper, in contrast, presents a new approach to machine learning, which, in the simplest case, does not require any optimization at all.

Our focus is on perceptron, which is the simplest learning machine. Perceptron is a model of neuron that was originally introduced by Rosenblatt [12] to perform visual perception tasks, which, in mathematical terms, result in solution of the linear classification problem. There are two essential stages of the perceptron functioning: supervised learning session and new data classification. During the first stage, the perceptron is given a labeled set of examples. Its task is of inferring weights of a linear function according to some error-correcting rule. Subsequently, this function is utilized for classification of new previously unseen data.

In spite of its very simple internal structure and learning rule, the perceptron’s capabilities are seriously limited [10]. Perceptron can not provide the classification, if there is an overlap in the data or if the data can not be linearly separated. It is also incapable of learning complex logical functions, such as XOR function. Moreover, by its design, the perceptron can distinguish only between previously seen classes and, therefore, can not resolve the situation when the input belongs to none of the learned classes, or represents a superposition of seen classes.

In this paper we show that all the mentioned problems can be, in principle, overcome by a quantum analog for perceptron. There are also two operational stages for the quantum perceptron. During the learning stage all the data are formally represented through quantum states of physical systems. This representation allows expanding the data space to a physical Hilbert space. It is important to note, that there is no need to involve real physical systems during this stage. Thus, the learning is essentially a classical procedure. The subject of the learning is a set of positive operator valued measurements (POVM) [1]. The set is constructed by making superpositions of the training data in a way that each operator is responsible for detection of one particular class. This procedure is linear and does not require solving equations or optimizing parameters. When the learning is over, there are two possibilities to achieve the required classification of new data. First, new data are encoded into the states of real quantum systems, which are measured by detectors adjusted in accordance with the learned POVM. Second, new data may be formally encoded into the states of quantum systems and processed with the POVM. Both

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mentions ways allow to achieve the classification.

This paper is organized as follows. In the next section, we first overview the classical perceptron and discuss the origin of the restrictions on its learning capabilities. After this, in Section III we introduce the quantum perceptron and show its properties. We demonstrate, in Section IV three examples of how the quantum perceptron is superior its classical counterpart in learning capabilities: complex logical function learning, classification of new data on previously unseen classes and recognition of superpositions of classes. We conclude in Section V.

II. BASIC CONSTRUCTIONS

A. Rosenblatt’s Perceptron

Operational structure of the classical perceptron is simple. Given an input vector \(x\) (which is usually called a feature vector) consisting of \(n\) features, perceptron computes a weighted sum of its components \(f(x) = \sum a_i x_i\), where weights \(a_i\) have been previously learned. The output from a perceptron is given by \(o = \text{sign}(f(x))\), where \(\text{sign}(\ldots)\) is the Heaviside function

\[
\text{sign}(y) = \begin{cases} 
+1 & y > 0 \\
-1 & y \leq 0 
\end{cases}
\]

Depending on the binary output signal \(o \in \{+1, -1\}\), the input feature vector \(x\) is classified between two feature classes, one of which is associated with output \(o = +1\) and the other with output \(o = -1\).

As we have mentioned above, the perceptron needs to be trained before its autonomous operation. During the training, a set of \(P\) training data pairs \(\{x_i, d_i, i = 1, ..., P\}\) is given, where \(x_i\) are the \(n\)-dimensional feature vectors and \(d_i\) are desired binary outputs. Typically, at the beginning of the learning procedure the initial weights \(a_i\) of the linear function are generated randomly. When a data pair is chosen from the training set, the output \(a_i = \text{sign}(f(x_i))\) is computed from the input feature vector \(x_i\) and is compared to the desired output \(d_i\). If the actual and the desired outputs match \(a_i = d_i\), the weights \(a_i\) are left without change and the next pair from the data set is taken for the analysis. If \(a_i \neq d_i\), the weights \(a_i\) of the linear function are to be changed according to the error-correcting rule \(a' = a + c a = a + (d_i - a_i)x_i\), which is applied hereafter and until the condition \(a_i = d_i\) is met.

The training procedure has clear geometric interpretation. The weights \(a_i\) of the linear function define a \((n-1)\)-dimensional hyperplane in the \(n\)-dimensional feature space. The training procedure results in a hyperplane that divides the feature space on two subspaces, so that each feature class occupies one of the subspaces. Due to this interpretation, the origin of the restrictions on learning capabilities of the classical perceptron becomes visible: a hyperplane that separates the two classes may not exist. The simplest example of two classes that can not be linearly separated is XOR logical function of two variables, which is given by the truth table

\[
\begin{array}{cccc}
   x_1 & 0 & 0 & 1 & 1 \\
   x_2 & 0 & 1 & 0 & 1 \\
   f & 0 & 1 & 1 & 0 \\
   o & -1 & +1 & +1 & -1
\end{array}
\]

A schematic representation of this function in the two-dimensional feature space is shown in Fig. I.

There are, however, limitations on the learning capabilities of the perceptron even in the case when the separating hyperplane exists. As we mentioned above the hyperplane divides the feature space on two subspaces, in spite of the fact that the feature classes occupy two particular hypervolumes. This enforces the classification on the two learned classes even so the given feature is essentially different from the classes, i.e. form a new class.

It is very important to note that certain tasks undoable by Rosenblatt’s perceptron, such as complex logical functions learning and classifying data with an overlap, can be performed in the framework of more sophisticated classical learning models, for example, by support vector machines [6]. However, these classical implementations always demand nonlinear optimization, which complicates rapidly with growth of the feature space. This effect is known as the curse of dimensionality of the classical learning models [6]. In the next section, we present a new model for the learning machine, which, however, is linear, but is superior Rosenblatt’s perceptron in its learning capabilities.

B. A quantum analog for Rosenblatt’s perceptron

As its classical counterpart, quantum perceptron is to be trained to perform the classification task. Suppose, we are given a set of \(K\) training data pairs consisting of feature vectors \(\{x_k, d_k, k = 1, ..., K\}\) with the desired binary outputs \(d \in \{+1, -1\}\); and each feature vector consists of \(n\) features \(x = \{x_1, x_2, ..., x_n\}\). Let us suppose that each
feature is restricted in a certain interval, so that all features can be normalized to the unit interval $x_k' \in [0, 1]$ for $k = 1, \ldots, n$. This allows us to represent the input feature vectors through the states of a (discrete) $2^n$-dimensional quantum system, so that $|x\rangle = |x'_1, x'_2, \ldots, x'_n\rangle$. With this quantum representation we have extended the classical $n$-dimensional feature space to $2^n$-dimensional Hilbert space of the quantum system. We shall drop "primes" hereafter assuming that the features are normalized.

Let us construct a projection operator $|x\rangle \langle x|$ for each given feature vector $|x\rangle$. With the help of these projectors, let us define two operators

$$P_{-1} = \frac{1}{N_{-1}} \sum_{d=-1} |x\rangle \langle x|,$$

$$P_{+1} = \frac{1}{N_{+1}} \sum_{d=+1} |x\rangle \langle x|,$$

where $N_{-1}$ and $N_{+1}$ are normalization factors. All feature vectors that correspond to the output $d = -1$ are summed in the operator $P_{-1}$, while all feature vectors corresponding $d = +1$ are collected in $P_{+1}$. The construction of these operators concludes the learning procedure.

There are only four possibilities of how the operators $P_{-1}$ and $P_{+1}$ may be related:

A. Operators $P_{-1}$ and $P_{+1}$ are orthogonal $P_{-1} P_{+1} = 0$ and form a complete set $P_{-1} + P_{+1} = I$, where $I$ is the identity operator. This means that there was no overlap between the training data, and the two classes $P_{-1}$ and $P_{+1}$ occupy the whole feature space. As the result any input feature vector can be classified between the two classes with no mistake. This situation can be simulated in principle by the classical perceptron.

B. Operators $P_{-1}$ and $P_{+1}$ are orthogonal $P_{-1} P_{+1} = 0$, but do not form a complete set $P_{-1} + P_{+1} \neq I$. This is an extremely interesting case. The third operator must be defined as $P_0 = I - P_{-1} - P_{+1}$ to fulfill the POVM competence requirement. The operator $P_0$ is, moreover, orthogonal to $P_{-1}$ and $P_{+1}$, because $P_{-1} P_{+1} = 0$. When operating autonomously, the quantum perceptron generates three outputs $d \in \{+1, 0, -1\}$, namely that the feature vector belongs to the one of the previously seen classes $d \in \{+1, -1\}$ or it is essentially different from the learned classes $d = 0$ – it belongs to a new previously unseen class. The classification on previously unseen classes is an extremely hard learning problem, which can not be done by classical perceptron neither by the most of the classical perceptron networks [3]. Quantum perceptron is capable of performing this task. Moreover, there will be no mistake in the classification between the three classes because of the orthogonality of the operators $P_{-1}, P_{+1}$ and $P_0$.

C. Operators $P_{-1}$ and $P_{+1}$ are not orthogonal $P_{-1} P_{+1} \neq 0$, but form a complete set $P_{-1} + P_{+1} = I$. In this case all the input data can be classified between the two classes with some nonzero probability of mistake. This is the case of probabilistic classification, which can not be done by the classical perceptron, although can be performed by more sophisticated classical learning models.

D. The most general case is when operators $P_{-1}$ and $P_{+1}$ are not orthogonal $P_{-1} P_{+1} \neq 0$ and do not form a complete set $P_{-1} + P_{+1} \neq I$. One again defines the third operator $P_0 = I - P_{-1} - P_{+1}$, which this time is not orthogonal to $P_{-1}$ and $P_{+1}$. In this situation, quantum perceptron classifies all the input feature vectors on three classes, one of which is a new class, with some nonzero probability of mistake. This situation cannot be simulated by the classical perceptron.

The quantum perceptron learning rule may have the following geometric interpretation. In contrast to the classical perceptron, which constructs a hyperplane separating the feature space on two subspaces, quantum perceptron constructs two (hyper-)volumes in the physical Hilbert space. These volumes are defined by the POVM operators [3]. During the autonomous functioning, the POVM operators project the given feature vector $|\psi\rangle$ to one of the volumes (or to the space unoccupied by them) allowing us to perform the desired classification. For example, if $\langle \psi | P_{-1} | \psi \rangle \neq 0$, while $\langle \psi | P_{+1} | \psi \rangle = 0$ and $\langle \psi | P_0 | \psi \rangle = 0$, the feature vector $|\psi\rangle$ belongs to the class $d = -1$, and the probability of misclassification equals zero. If, in contrast, $\langle \psi | P_{-1} | \psi \rangle \neq 0$, $\langle \psi | P_{+1} | \psi \rangle \neq 0$ and $\langle \psi | P_0 | \psi \rangle = 0$, the feature vector belongs to the two classes with degrees defined by the corresponding expectation values $\langle \psi | P_{-1} | \psi \rangle$ and $\langle \psi | P_{+1} | \psi \rangle$. In the latter situation, one may perform a probabilistic classification according to the expectation values.

We would like to stress that the construction of the operators [3] is no way unique. There may be more sophisticated ways to construct the POVM set in order to ensure a better performance of the learning model for a classification problem at hand. In fact, our construction is the simplest linear model for a quantum learning machine. Only in this sense the presented quantum perceptron is the analog for Rosenblatt’s perceptron, while their learning rules are essentially different.

As we mentioned in the Introduction, there are two ways to achieve the desired classification with the POVM. One may get real physical systems involved or use the POVM operators as purely mathematical instrument. In order of clarity, the advantages of the first of these approaches will be discussed in Section III A on particular examples, while in the rest of the next section we use the quantum perceptron as pure mathematical tool.

III. APPLICATIONS

In spite of the extreme simplicity of its learning rule, quantum perceptron may perform a number of tasks infeasible for classical (Rosenblatt) perceptron. In this section we give three examples of such tasks. We start with logical function learning. Historically, the fact that classical perceptron can not learn an arbitrary logical func-
tion was the main limitation on the learning capabilities of this linear model. We show that quantum perceptron, in contrast, is able of learning an arbitrary logical function irrespective of its kind and order. In Section III.B we show that quantum perceptron can, in certain cases, perform the classification without previous training, the so-called unsupervised learning task. Classical perceptron, in contrast, can not perform this task by construction. Finally, in Section III.C we show that quantum perceptron may recognize superpositions of previously learned classes. This task is of particular interests in applied medical engineering, where simultaneous and proportional myoelectric control of artificial limb is a long desired goal.

A. Logical Function Learning

Let us consider a particular example of logical function – XOR, which is given by the truth table. During the learning session, we are given a set of four training data pairs \( \{x_i, d_i\}_{i=1,...,4} \), where the feature vector consists of two features \( x \in \{x_1, x_2\} \), and the desired output \( d \in \{-1,1\} \) is a binary function. Let us represent the input features through the states of a two-dimensional quantum system – qubit, so that each feature is given by one of the basis states \( |x_i\rangle \in \{|0\},|1\rangle\) for \( i = 1, 2 \), where \( \{|0\},|1\rangle \) denotes the computational basis for each feature. In the above representation, the feature vector \( x \) is given by one of the four two-qubit states \( |x_1, x_2\rangle \). Following the procedure, which is described in Section III.B, the POVM operators are constructed as

\[
P_{-1} = |0, 0\rangle \langle 0, 0| + |1, 1\rangle \langle 1, 1|,
\]

\[
P_{+1} = |0, 1\rangle \langle 0, 1| + |1, 0\rangle \langle 1, 0|.
\]

During its autonomous operation, quantum perceptron may be given four basis states \( |x_1, x_2\rangle \in \{|0, 0\},|0, 1\rangle,|1, 0\rangle,|1, 1\rangle\) as inputs. Since \( \langle x_1, x_2|P_{-1}|x_1, x_2\rangle \neq 0 \) only for \( |x_1, x_2\rangle \in \{|0, 0\},|1, 1\rangle\), these states are classified to \( d = -1 \), while the other two states \( \{|0, 1\},|1, 0\rangle\) are classified to \( d = +1 \). The fact that the operators \( P_{-1} \) and \( P_{+1} \) are orthogonal ensures zero probability of misclassification, while the completeness of the set of operators guarantees classification of any input. Conclusively, the quantum perceptron has learned XOR function.

The successful XOR function learning by quantum perceptron is the consequence of the representation of the classical feature vector \( x \) through the two-qubit states. In the classical representation, the feature vectors can not be linearly separated on a plane, see Fig. 1. In the quantum representation, four mutually orthogonal states \( |x_1, x_2\rangle \) in the four-dimensional Hilbert space can be separated on two classes in an arbitrary fashion. This implies that an arbitrary logical function of two variables can be learned by quantum perceptron. For example, learning of logical AND function leads to the construction of operators \( P_{-1} = |0, 0\rangle \langle 0, 0| + |0, 1\rangle \langle 0, 1| + |1, 0\rangle \langle 1, 0| \) and \( P_{+1} = |1, 1\rangle \langle 1, 1| \). Moreover, an arbitrary logical function of an arbitrary number of inputs (arbitrary order) also can be learned by quantum perceptron, because the number of inputs of such a function growth exponentially as \( 2^n \) with the order of the function \( n \) and exactly as fast as dimensionality of the Hilbert space that is needed to represent the logical function.

In the above discussion the need to use real quantum systems has not emerged. Let us now consider a situation, when one can benefit from utilizing real quantum systems. Let us slightly modify the problem of XOR learning. In real-life learning tasks the training data may be corrupted by noise \( \mathcal{F} \). In some cases, noise may lead to overlapping of the training data, which result in misclassification of feature vectors during the training stage and during further autonomous functioning. For example, if, during the XOR learning, there is a finite small probability \( \delta \) that feature \( x_1 \) takes a wrong binary value, but the other feature and the desired output are not affected by noise, after a big number of trainings (which are usually required in case of learning from noisy data), the POVM operators are given by

\[
P'_{-1} = (1 - \delta) (|0, 0\rangle \langle 0, 0| + |1, 1\rangle \langle 1, 1|) + \delta (|0, 1\rangle \langle 0, 1| + |1, 0\rangle \langle 1, 0|),
\]

\[
P'_{+1} = (1 - \delta) (|0, 1\rangle \langle 0, 1| + |1, 0\rangle \langle 1, 0|) + \delta (|0, 0\rangle \langle 0, 0| + |1, 1\rangle \langle 1, 1|).
\]

Operators \( P'_{-1} \) and \( P'_{+1} \) are not orthogonal \( P'_{-1}P'_{+1} \neq 0 \) in contrast to operators (4), but still form a complete set. This means that during the autonomous operation of the quantum perceptron, the input feature vectors can be misclassified. Nevertheless, each feature is classified between the two classes and, on average, most of the feature vectors are classified correctly. This means that quantum perceptron simulates XOR function with a degree of accuracy given by \( 1 - \delta \).

If we use real physical systems to encode feature vectors during autonomous functioning of the perceptron and measure the states of the systems with experimental setup adjusted in accordance with the POVM (4), we can perform a probabilistic classification. Moreover, we can exactly (in probabilistic sense) reproduce fluctuations that have been observed during the training. In certain sense such learning is too accurate and may be of use in some cases. Anyway, classical perceptron can not do any similar task.

It is, however, important to note that practical simulation of quantum perceptron with real physical systems may not be always possible. In Section III.B we discussed situations when operators \( P_{-1} \) and \( P_{+1} \) do not form a compete set, and constructed the third operator \( P_0 = I - P_{-1} - P_{+1} \). It is possible in principle that the constructed operator \( P_0 \) is negative, i.e. unphysical. This means that the classification problem at hand can not be physically simulated with our linear model, although the
problem may be treated mathematically with the quantum perceptron approach.

In this section we have seen how quantum representation and quantum measurements contribute to advanced learning abilities of the quantum perceptron. Even without these features, however, quantum perceptron is superior its classical counterpart in learning capabilities due to specific algebraic structure of the POVM operators. In the following sections we provide two examples, where advanced learning relays only on the structure of the POVM set.

**B. Unsupervised Learning**

The (supervised) learning stage, has been embedded into quantum perceptron by analogy with classical perceptron. Surprisingly, however, that the learning rule of the quantum perceptron allows to perform learning tasks beyond supervised learning paradigm. Suppose, for example, that we are given an unlabeled set of feature vectors and need to find a possible structure of this set, i.e. we need to answer whether there are any feature classes in the set. The following protocol allows us to resolve such an unsupervised learning task under certain conditions.

Being given the first feature vector $|x_1\rangle$ from the set, let us define two classes with the POVM operators

$$P_{-1}^{(0)} = |x_1\rangle \langle x_1| ,$$

$$P_{+1}^{(0)} = I - P_{-1}^{(0)} .$$

(6)

where $I$ is the identity operator. Here, the class $d = +1$ is formally defined as "not $d = -1". The next given feature vector $|x_2\rangle$ is tested to belong to one of these classes. If $\langle x_2| P_{-1}^{(0)} | x_2\rangle > \langle x_2| P_{+1}^{(0)} | x_2\rangle$, the feature vector $|x_2\rangle$ is close enough to $|x_1\rangle$ and thus belongs to class $d = -1$. In this case the POVM operators (6) are updated to

$$P_{-1}^{(1)} = |x_1\rangle \langle x_1| + |x_2\rangle \langle x_2| ,$$

$$P_{+1}^{(1)} = I - P_{-1} .$$

(7)

If, in contrast, $\langle x_2| P_{-1}^{(0)} | x_2\rangle \geq \langle x_2| P_{+1}^{(0)} | x_2\rangle$, the feature vector $|x_2\rangle$ is distant sufficiently from $|x_1\rangle$ and therefore can be assigned a new class $d = +1$. Due to the first representative of the $d = +1$ class, we may update the formal definition of the $P_{+1}^{(0)}$ introducing a new POVM set

$$P_{-1}^{(1)} = |x_1\rangle \langle x_1| ,$$

$$P_{+1}^{(1)} = |x_2\rangle \langle x_2| .$$

(8)

This procedure is repeated iteratively until all the feature vectors are classified between the two classes $d = -1$ and $d = +1$.

The above protocol will work if only there are at least two feature vectors $|x\rangle$ and $|y\rangle$ in the given feature set such as $\langle x| (I - 2P) | x\rangle \geq 0$, where $P = |y\rangle \langle y|$. In the opposite case, unsupervised learning within the protocol is not possible. Moreover, the classification crucially depends on order of examples, because first seen feature vectors define the classes. This situation is, however, typical for unsupervised learning models [8]. To reduce the dependence of the classification on the order of the feature vectors appearance, it is possible to repeat the learning many times taking different order of the input feature vectors, and compare the results of the classification. In spite of the above limitations, the unsupervised classification can be in principle performed by the quantum perceptron, while this task is undoable for the classical perceptron.

**C. Simultaneous and Proportional Myoelectric Control**

The problem of signal classification has found remarkable applications in medical engineering. It is known that muscle contraction in human body is governed by electrical neural signals. These signals can be acquired by different means [17], but are typically summarized into so-called electromyogram (EMG). In principle, processing the EMG, one may predict muscular response to the neural signals and subsequent respond of the body. This idea is widely used in many applications, including myoelectric-controlled artificial limb, where the surface EMG is recorded from the remnant muscles of the stump and used, after processing, for activating certain prosthetic functions of the artificial limb, such as hand open/close [18].

Despite decades of research and development, however, none of the commercial prostheses is using pattern classification based controller [18]. The main limitation on successful practical application of pattern classification for myoelectric control is that it leads to very unnatural control scheme. While natural movements are continuous and require activations of several degrees of freedom (DOF) simultaneously and proportionally, classical schemes for pattern recognition allow only sequential control, i.e. activation of only one class that corresponds to a particular action in one decision [18]. Simultaneous activation of two DOFs is thus recognized as a new class of action, but not as a combination of known actions. Moreover, all these classes as well as their superpositions must be previously learned. This leads to higher rehabilitation cost and more frustration of the user, who must spend hours in a lab to learn the artificial limb control.

Recently, we have taken quantum perceptron approach to the problem of simultaneous and proportional myoelectric control [19]. We considered a very simple control scheme, where two quantum perceptrons were controlling two degrees of freedom of the wrist prosthesis. We took EMG signals with corresponding angles of the wrist
position from an able-bodied subject who performs wrist contractions. For the training we used only those EMG that correspond to the activation of a single DOF. During the test, the control scheme was given EMG activating multiple DOFs. We found that in 45 of 55 data blocks of the actions were recognized correctly with accuracy exceeding 73%, which is comparable to the accuracy of the classical schemes for classification.

In the above example, we used a specific representation of the feature vectors. Since the features (i.e. the neural signals) are real and positive numbers there was no need to expand the feature space. Moreover, in general it is not possible to scale a given feature on the unit interval, because the neural signals observed during the learning and autonomous functioning may differ significantly in amplitude, and a priori scaling may lead to misapplication of the artificial limb. Therefore, the amplitude of a signal was normalized over amplitudes from all the channels to ensure proportional control of the prosthesis. In fact, the specific structure of the POVM set was the only feature of the quantum perceptron that we used. With this feature alone we were able to recognize 4 original classes observed during the training and 4 new (previously unseen) classes that correspond to simultaneous activation of two DOF. In general, within the above control scheme, \( n \) quantum perceptrons are able to recognize \( 2n \) original classes with \( (2n)!/[2(2n−2)!] \) \( n \) additional two-class superpositions of these classes. In contrast, \( n \) classical perceptrons may recognize only \( 2n \) classes, which were seen during the learning. The advantage of the quantum perceptron over the classical perceptron can be understood from the geometric interpretation discussed in Section [III]. While \( n \) classical perceptrons construct \( n \) hyperplanes in the feature space, which separate the feature space on \( 2n \) non-overlapping classes, \( n \) quantum perceptrons build \( n \) hypervolumes, which may not fill the whole feature space and may overlap.

IV. CONCLUSION

Bridging between quantum information science and machine learning theory, we showed that the capabilities of an autonomous learning automata can be dramatically increased using the quantum information formalism. We have constructed the simplest linear quantum model for learning machine, which, however, is superior its classical counterpart in learning capabilities. Due to the quantum representation of the feature vectors, the probabilistic nature of quantum measurements and the specific structure of the POVM set, the quantum perceptron is capable of learning an arbitrary logical function, performing probabilistic classification, recognizing superpositions of previously seen classes and even classifying on previously unseen classes. Since all classical learning models track back to Rosenblatt’s perceptron, we hope that the linear quantum perceptron will serve as a basis for future development of practically powerful quantum learning models, and especially in the domain of nonlinear classification problems.

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