High-capacity quantum secure direct communication based on quantum hyperdense coding with hyperentanglement*

Tie-Jun Wang¹²³, Tao Li¹, Fang-Fang Du¹, and Fu-Guo Deng¹†
¹ Department of Physics, Applied Optics Beijing Area Major Laboratory, Beijing Normal University, Beijing 100875
² College of Nuclear Science and Technology, Beijing Normal University, Beijing 100875
³ Beijing Radiation Center, Beijing 100875
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We present a quantum hyperdense coding protocol with hyperentanglement in polarization and spatial-mode degrees of freedom of photons first and then give the details for a quantum secure direct communication (QSDC) protocol based on this quantum hyperdense coding protocol. This QSDC protocol has the advantage of having a higher capacity than the quantum communication protocols with a qubit system. Compared with the QSDC protocol based on superdense coding with $d$-dimensional systems, this QSDC protocol is more feasible as the preparation of a high-dimension quantum system is more difficult than that of a two-level quantum system at present.

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Quantum mechanics provides some good ways for secure quantum communication which is originally used to create a private key between two remote legitimate users, say the sender Alice and the receiver Bob. The noncloning theorem provides a physical principle for the fact that an eavesdropper can not steal the information transmitted from the sender to the receiver fully and freely as an unknown quantum state can not be copied perfectly. In 1984, with four nonorthogonal single-photon polarization states, Bennett and Brassard [1] proposed the first quantum key distribution (QKD) protocol. In this QKD protocol, Alice prepares her single-photon states by choosing randomly one of two measuring bases (MBs), and Bob measures his single-photon states by choosing randomly these two MBs after he receives the photons from Alice. In this way, Alice and Bob will obtain some correlated outcomes with the success probability 50%. In 1992, Bennett [2] simplified this protocol with two nonorthogonal states and decreased the success probability to 25% in principle. The nonlocality feature of an entangled system also provides a good tool for designing secure quantum communication. For example, in 1991, Ekert [3] designed a QKD protocol with Einstein-Podolsky-Rosen (EPR) pairs, resorting to the Bell inequality for checking eavesdropping, and its success probability is also 50%. Subsequently, Bennett, Brassard, and Mermin [4] simplified its process for checking eavesdropping. Now, QKD has attracted much attention [5–7].

Different from QKD, quantum secure direct communication (QSDC) is used to transmit the secret message securely and directly, not resorting to a private key. In 2002, Long and Liu [7] introduced the concept of quantum data block into quantum communication and proposed the first QSDC scheme based on entangled quantum systems and it is detailed in the two-step QSDC protocol [8] in 2003. Of course, in 2002 Beige et al. [9] proposed another deterministic quantum communication scheme based on single-photon two-qubit states, in which the message can be read out only after a transmission of an additional classical information for each qubit, i.e., the cryptographic key of the sender [10]. In 2002, Boström and Felbinger [11] presented a quasi-secure direct communication scheme with an EPR pair, call it the ping-pong scheme. However, the ping-pong scheme is insecure as the title suggests, and it can be attacked fully if the transmission efficiencies are lower than 60% as the two parties of quantum communication only exploit one basis to check eavesdropping [12]. In 2004, Cai et al. [13] improved the ping-pong scheme with the similar way to the two-step protocol [8] and proposed another deterministic secure communication protocol based on single photons [14]. The quantum one-time pad protocol [15] is the first QSDC scheme based on a sequence of single photons with a single-qubit quantum privacy amplification protocol [16]. In 2005, Wang et al. [17] proposed a QSDC protocol with high-dimension superdense coding. Now, there are many deterministic quantum communication protocols, including QSDC protocols [3, 8, 13, 17, 19] and deterministic secure quantum communication protocols [3, 20, 31].

Hyperentanglement is a state of being simultaneously entangled in multiple degrees of freedom and it plays an important role in quantum information processing recently. For example, it is used to analyze the four Bell states in polarization degree of freedom of photons [32, 34]. In 2008, with the help of the hyperentangled state in both polarization and orbit angular momentum, Barreiro et al. [33] beat the channel capacity limit for linear photonic superdense coding. In 2002, Simon

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†Corresponding author: fgdeng@bnu.edu.cn
and Pan \[36\] exploited the hyperentanglement in spatial-mode and polarization degrees of freedom of photons to purify the entanglement in polarization degree of freedom. In 2009, Sheng, Deng and Zhou \[37\] exploited the hyperentanglement in spatial and polarization degrees of freedom to purify a parametric down-conversion entanglement source. In 2010, Sheng and Deng \[38\] proposed the concept of deterministic entanglement purification and presented the first protocol with hyperentanglement in polarization, spatial, and frequency degrees for freedom. In 2010, an one-step deterministic polarization entanglement purification protocol by using spatial entanglement was proposed \[39, 40\] with hyperentanglement in polarization and spatial-mode degrees of freedom. More recently, a complete hyperentangled-Bell-state analysis (CHBSA) protocol for quantum communication was proposed with the help of cross-Kerr nonlinearity \[41\]. With CHBSA, some important processes can be accomplished such as teleportation and entanglement swapping in two degrees of freedom, which will improve the capacity of quantum communication largely.

In this Letter, we will first introduce a quantum hyperdense coding protocol with hyperentanglement in polarization and spatial-mode degrees of freedom, and give the way for encoding the information with linear optical elements. Then we will give a QSDC protocol and discuss its security against Trojan horse attack strategies. Compared with the QSDC protocols with a qubit system \[8, 18\], this QSDC protocol has the advantage of having a higher capacity as each photon can carry 4 bits of information from the sender to the receiver. Compared with the QSDC protocol based on superdense coding with d-dimension systems \[17\], this QSDC protocol is more feasible as the preparation of a high-dimension quantum system is more difficult than that of a two-level quantum system at present.

A hyperentangled state in polarization and spatial-mode degrees of freedom can be described as follows:

\[
|\Psi_{AB}\rangle_{PS} = \frac{1}{2}(|HH\rangle + |VV\rangle)_{AB} \otimes (|a_1 b_1\rangle + |a_2 b_2\rangle)_{AB},
\]

where $H$ and $V$ are the horizontal and the vertical polarizations of photons, respectively. The subscripts $A$ and $B$ represent the two photons in the hyperentangled state. $a_1(b_1)$ and $a_2(b_2)$ are the different spatial modes for photon $A$ ($B$), shown in Fig.1. The subscript $P$ denotes the polarization degree of freedom and $S$ is the spatial-mode degree of freedom. As pointed out in Refs. \[36, 37\], a pump pulse of ultraviolet light passes through a beta barium borate (BBO) crystal and produces correlated pairs of photons into the modes $a_1$ and $b_1$. Then it is reflected and traverses the crystal a second time, and produces correlated pairs of photons into the modes $a_2$ and $b_2$.

The Hamiltonian can be approximately described as

\[
H_{PDC} = \gamma [(a_{1H}^+ b_{1H}^+ + a_{1V}^+ b_{1V}^+) + r e^{i\phi} (a_{2H}^+ b_{2H}^+ + a_{2V}^+ b_{2V}^+)] + H.c.,
\]

where $r$ denotes the relative probability of emission of photons into the lower modes compared to the upper modes, and $\phi$ is the phase between these two possibilities \[36, 37\]. As the same as Refs. \[36, 37\], in a simple case we assume $r = 1$ and $\phi = 0$. Thus the state of the photon pair can be described by $(a_{1H}^+ b_{1H}^+ + a_{1V}^+ b_{1V}^+ + a_{2H}^+ b_{2H}^+ + a_{2V}^+ b_{2V}^+)|0\rangle$. It also can be written as $(|a_1 b_1\rangle + |a_2 b_2\rangle)(|H_a\rangle|H_b\rangle + |V_a\rangle|V_b\rangle)$. If the pump pulse is faint, the four-photon state produced by this PDC source is negligible.

For a quantum system with two photons hyperentangled in polarization and spatial-mode degrees of freedom, there are 16 generalized Bell states, shown as

\[
|\Psi_{AB}\rangle_{PS} = |\theta\rangle_P \otimes |\xi\rangle_S,
\]

where $|\theta\rangle_P$ is one of the four Bell states in polarization degree of freedom as

\[
|\phi^\pm\rangle_P = \frac{1}{\sqrt{2}}(|HH\rangle \pm |VV\rangle)_{AB},
\]

\[
|\psi^\pm\rangle_P = \frac{1}{\sqrt{2}}(|HV\rangle \pm |VH\rangle)_{AB},
\]

and $|\xi\rangle_S$ is one of the four Bell states in spatial-mode degree of freedom as

\[
|\phi^\pm\rangle_S = \frac{1}{\sqrt{2}}(|a_1 b_1\rangle \pm |a_2 b_2\rangle)_{AB},
\]

\[
|\psi^\pm\rangle_S = \frac{1}{\sqrt{2}}(|a_1 b_2\rangle \pm |a_2 b_1\rangle)_{AB}.
\]

With CHBSA \[41\], one can in principle distinguish these 16 hyperentangled Bell states $|\Psi_{AB}\rangle_{PS}$.

Now, we first construct a quantum hyperdense coding protocol with the hyperentanglement in polarization and spatial-mode degrees of freedom and then apply it into QSDC for obtaining a high channel capacity.

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**FIG. 1:** Schematic diagram of a quantum hyperentanglement source in polarization and spatial-mode degrees of freedom, which can be produced with a beta barium borate (BBO) crystal and a pump pulse of ultraviolet light.
Similar to the quantum dense coding protocols [42–44] with entanglement in polarization degree of freedom, a quantum hyperdense coding protocol can be accomplished with the following steps:

1. The receiver Bob prepares a hyperentangled photon pair \( AB \) in the state \( |Φ_{AB}^+⟩_{PS} \), as shown in Fig. 1

2. Bob sends the photon \( A \) to the sender Alice with two fiber channels.

3. After receiving the photon \( A \), Alice encodes her information on the photon \( A \) with the unitary operation \( U_{ij} = U_P^i ⊗ U_P^j \) (\( i, j = 1, 2, 3, 4 \)). Here

   \[
   \begin{align*}
   U_P^1 &= |H⟩⟨H| + |V⟩⟨V|, \\
   U_P^2 &= |H⟩⟨H| - |V⟩⟨V|, \\
   U_P^3 &= |V⟩⟨V| + |H⟩⟨H|, \\
   U_P^4 &= |V⟩⟨V| - |H⟩⟨H|,
   \end{align*}
   \]

4. After performing one of the 16 unitary operations on the photon \( A \), Alice sends it to Bob.

5. Bob distinguishes the state of the hyperentangled photon pair \( AB \) with CHBSA [41] and can obtain 4 bits of information from Alice.

With these five steps, Alice and Bob can in principle accomplish their quantum hyperdense coding. In fact, this quantum hyperdense coding protocol is just the generalization of the original quantum dense coding protocol from a two-level system to a hyperentangled system with two degrees of freedom. Also, it can be viewed as an experimental example for the quantum superdense coding protocol with a \( d \)-level quantum system [17, 43]. From the view of quantum entanglement source, this hyperdense coding protocol may be a feasible one with current techniques as the hyperentanglement in polarization and spatial degrees of freedom of photons can be prepared with a BBO crystal at experiment. The four unitary operations in the polarization degree of freedom of photons can be implemented with linear optical wave plates. The bit-flip operation \( U^S \) and the phase-flip operation \( U^Z \) in the spatial-mode degree of freedom of photons can be accomplished by exchanging the two fiber channels \( a_1 \) and \( a_2 \) and by adding a \( \lambda/2 \) wave plate on the spatial mode \( a_2 \) respectively.

In quantum communication protocols, Alice and Bob should exploit at least two uncommutative MBs to check eavesdropping [26]. The two MBs in polarization degree of freedom can be chosen as \( Z^P = \{ |H⟩, |V⟩ \} \) and \( X^P = \{ |+⟩_P = \frac{1}{\sqrt{2}}(|H⟩ + |V⟩), |−⟩_P = \frac{1}{\sqrt{2}}(|H⟩ − |V⟩) \} \). These two MBs can be transferred into each other with a \( \lambda/4 \) wave plate which acts as a Hadamard operation. The two MBs in spatial-mode degree of freedom can be chosen as \( Z^S = \{ |a_1⟩, |a_2⟩ \} \) and \( X^S = \{ |+⟩_S = \frac{1}{\sqrt{2}}(|a_1⟩ + |a_2⟩), |−⟩_S = \frac{1}{\sqrt{2}}(|a_1⟩ − |a_2⟩) \} \). The Hadamard operation between these two MBs can be completed with a 50:50 beam splitter, shown in Fig. 2. In order to distinguish the four orthogonal states \( \{ |H⟩|a_1⟩, |V⟩|a_1⟩, |H⟩|a_2⟩, |V⟩|a_2⟩ \} \) in polarization and spatial-mode degrees of freedom of each photon, Alice should add a polarization beam splitter (PBS) at each of the two paths \( a_1 \) and \( a_2 \).

With this quantum hyperdense coding protocol, the principle of our high-capacity QSDC protocol with hyperentanglement can be described as follows.

1. The sender Alice and the receiver Bob agree that the 16 unitary operations \( U_{ij} = U_P^i ⊗ U_P^j \) are encoded into 16 different binary strings.

2. Bob prepares a sequence of photon pairs in the hyperentangled state \( |Φ_{AB}^+⟩_{PS} \). He takes the photon \( A \) from each hyperentangled photon pair \( AB \) for making up of an ordered partner particle sequence \( S_A \) and remaining partner particles compose another particle sequence \( S_B \), similar to Refs. [8, 17].

3. Bob sends the sequence \( S_A \) to Alice and he keeps the sequence \( S_B \).

4. After receiving the sequence \( S_A \), Alice and Bob check eavesdropping for the security of their transmission.

The procedure for eavesdropping check can be similar to that in the two-step QSDC scheme [8]: (a) Alice chooses randomly some sample photons from the sequence \( S_A \) and uses one of two MBs in each degree of freedom to measure the sample photons randomly. (b) Alice tells Bob the positions of the sample photons and the information of the measurements including the outcomes and the MBs. (c) Bob takes a suitable measurement on each sample photon with the same MBs as those of Alice’s. (d) Bob compares his outcomes with Alice’s to determine whether there is an eavesdropper, say Eve monitoring the transmission over the two fiber channels. It is the first eavesdropping check. If their outcomes are correlated, they can continue the QSDC to next step, otherwise they abort the quantum communication.

5. If Alice and Bob confirm that their transmission is secure, Alice encodes the secret message on the sequence \( S_A \) with the unitary operations \( U_{ij} = U_P^i ⊗ U_P^j \) and then transmits the sequence \( S_A \) to Bob.

Similar to Refs. [8, 17], Alice has to add a small trick in the procedure of encoding the secret message for the second eavesdropping check. She will select randomly some photons in the sequence \( S_A \), called sample pairs (which are composed of the sample photons in the sequence \( S_A \) and the photons correlated in the sequence \( S_B \) and perform on them one of the 16 unitary opera-
tions $U_{ij}$ randomly. It equals to the fact that Alice adds some redundancy in the coding for checking the security of the transmission of the sequence $S_A$ from Alice to Bob. She keeps the secret including the positions of the sample photons and the operations performed on them until Bob receives the sequence $S_A$.

(6) Bob can read out the message encoded on each hyperentangled photon pair with CHBSA [11].

(7) Alice tells Bob the positions of the sample pairs and the unitary operations on them. Bob completes the second eavesdropping-check analysis.

(8) If the transmission is secure, Bob will obtain the secret message encoded by Alice. Otherwise, Alice and Bob abandon the results of the transmission and repeat the procedures from the beginning.

If this protocol is secure for Trojan horse attack strategies, Alice and Bob can exploit the quantum correlation between the two photons in a hyperentangled state to analyze the error rate of the qubits transmitted and determine whether this protocol is secure for quantum direct communication or not, similar to the two-step QSDC protocol [8] whose security is in principle the same as the BBM92 QKD protocol [4, 47, 48].

As discussed in Ref. [26], in order to prevent Eve from eavesdropping the quantum communication with Trojan horse attack strategies, Alice should not only estimate the number of photons in each legitimate quantum signal in the sequence $S_A$ transmitted from Bob but also filter out the illegitimate photon signal with some special filters. The primary Trojan horse attack strategies include a multi-photon-signal attack [13], an invisible-photon attack [46], or a delay-photon attack [20]. At least, these three kinds of Trojan horse attack strategies in principle do not increase the error rate in the outcomes obtained by Alice and Bob, and can not be detected with common eavesdropping check strategies. Certainly, Alice can also exploit the ways discussed in Ref. [20] to prevent Eve from eavesdropping the transmission with these three kinds of Trojan horse attack strategies. In detail, after receiving the sequence $S_A$, Alice first adds a filter before the devices with which she operates the photons including performing unitary operations and measurements. The filters can only let the wavelengths close to the operating one in the devices. In this way, the invisible photons in each quantum signal will be filtered out. Second, Alice exploits a photon number splitter (PNS) in each path to divide each signal into two pieces. With the PNS and two single-photon measurements for each original signal, the users can distinguish whether there exists a multi-photon signal (including the delay-photon signal and the invisible photon whose wavelength is so close to the legitimate one that it cannot be filtered out with the filter) or not. At present, an ideal PNS is not feasible and the users can use a photon beam splitter (50/50) to replace a PNS for preventing Eve from stealing the secret message with multi-photon signal attacks. That is to say, this QSDC protocol based on hyperdense coding is in principle secure.

Compared with the two-step QSDC protocol [8], each hyperentangled photon pair in this QSDC protocol can carry 4 bits of information with one photon traveling forth and back from Bob to Alice, which means the capacity of this protocol is double of that in the two-step protocol. Moreover, this QSDC protocol only exploits an entangled two-level photon system to carry out a high-capacity quantum communication, not a 4-level photon system. It will make this QSDC protocol more feasible than the QSDC protocol based superdense coding [17] as the preparation of a high-dimension quantum signal is difficult at present [19].

In summary, we have introduced a quantum hyperdense coding protocol with hypertanglement in polarization and spatial-mode degrees of freedom of photons and then given the details for a quantum secure direct communication protocol based on this quantum hyperdense coding protocol. This QSDC protocol has the advantage of having a higher capacity than the quantum communication protocols with a qubit system [8,18] as each photon can carry 4 bits of information from the sender to the receiver. Compared with the QSDC protocol based on superdense coding with $d$-dimension systems [17], this QSDC protocol is more feasible as the preparation of a high-dimension quantum system is more difficult than that of a two-level system at present. We also discuss this QSDC protocol under Trojan horse attack strategies.

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