Sudakov effects in electroweak corrections

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Abstract

In perturbation theory the infrared structure of the electroweak interactions produces large corrections proportional to double logarithms \(\log^2 \frac{s}{m^2}\), similar to Sudakov logarithms in QED, when the scale \(s\) is much larger than the typical mass \(m\) of the particles running in the loops. These energy growing corrections can be particularly relevant for the planned Next Linear Colliders. We study these effects in the Standard Model for the process \(e^+ e^- \rightarrow f \bar{f}\) and we compare them with similar corrections coming from SUSY loops.
1 IR divergences: qualitative discussion

Infrared (IR) divergences arise in perturbative calculations from regions of integration over the momentum $k$ where $k$ is small compared to the typical scales of the process. This is a well known fact in QED for instance where the problem of an unphysical divergence is solved by giving the photon a fictitious mass which acts as a cutoff for the IR divergent integral. When real (bremsstrahlung) and virtual contributions are summed, the dependence on this mass cancels and the final result is finite. The (double) logarithms coming from these contributions are large and, growing with the scale, can spoil perturbation theory and need to be resumed. They are usually called Sudakov double logarithms. In the case of electroweak corrections, similar logarithms arise when the typical scale of the process considered is much larger than the mass of the particles running in the loops, typically the $W(Z)$ mass. The expansion parameter results then $\frac{\log^2 M}{M}$, which is already 10% for energies of the order of 1 TeV. This kind of corrections becomes therefore particularly relevant for next generation of linear colliders (NLC). In the case of corrections coming from loops with $W(Z)$, there is no equivalent of “bremsstrahlung” like in QED or QCD: the $W(Z)$, unlike the photon, has a definite nonzero mass and is experimentally detected like a separate particle. In this way the full dependence on the $W(Z)$ mass is retained in the corrections. Other singularities arise in perturbation theory, namely those coming from the ultraviolet (UV) region. These divergences can be treated with the usual renormalization procedure and can be resummed through RGE equations. However they produce single logs and we expect them to be asymptotically subdominant with respect to the double logs of IR origin.

We consider here the process $e^+e^- \to f\bar{f}$ in the limit of massless external fermions. Our notation is that $p_1$ ($p_2$) is the momentum of the incoming $e^-$ ($e^+$) and $p_3$ ($p_4$) is the momentum of the outgoing $f$ ($\bar{f}$). Furthermore, we define the Mandelstam variables: $s = (p_1 + p_2)^2 = 2p_1p_2$, $t = (p_1 - p_3)^2 = -\frac{s}{4}(1 - \cos \theta)$, $u = (p_1 - p_4)^2 = -\frac{s}{4}(1 + \cos \theta)$. In the following we consider only the dominant double logs corrections of IR and collinear origin coming from one loop perturbation theory and we neglect systematically single logs (IR, collinear or UV) and “finite” contributions that do not grow with energy. We discuss the kind of diagrams where we expect these corrections to be present, and evaluate them in the asymptotic regime $s \gg M_W^2$.

2 Sudakov logarithms in the vertices

We will consider first as an example, to have a grasp over the effect of the IR double logs, the “SM-like case” in which a “W boson” having mass $M$ and coupling with fermions like the photon is exchanged. We take the Born QED amplitude as the reference tree level amplitude. Then we denote the tree level photon exchange amplitude with $M_0 = i\frac{e^2}{4} \bar{\gamma}_{\mu}(p_1)\gamma_{\mu} u_c(p_2)\bar{u}_f(p_3)\gamma_\nu v_f(p_4)$ and the tree level photon vertex with $V_0 = -ie\bar{\gamma}_\mu(p_1)\gamma_{\mu} u_c(p_2)$; $e$ is the electron charge.

Let us first consider IR divergences coming from vertex corrections. Since we work in the limit of massless fermions, there is no coupling to the Higgs sector. Moreover, by power counting arguments, it is easy to see that the vertex correction where the trilinear gauge boson coupling appears is not IR divergent. The only potentially IR divergent diagram is then the one of fig. 1, where a gauge boson is exchanged in the $t$-channel. It is convenient to choose the momentum of integration $k$ to be the one of the exchanged particle, the boson in this case. Then, by simple power counting arguments it is easy to see that the IR divergence

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1 only vertex and box corrections will be analyzed, since vacuum polarization corrections give only single logs, both of ultraviolet and infrared origin.
can only be produced by regions of integration where \( k \approx 0 \). The only potentially IR divergent integral is then the scalar integral, usually called \( C_0 \) in the literature. Any other integral with \( k_\mu, k_\nu, k_\rho \) in the numerator cannot, again by power counting, be IR divergent. To understand the origin of the divergences, let us consider the diagram of fig.1 with all the masses set to zero. For \( k \approx 0 \) the leading term of the vertex amplitude is given by:

\[
\mathcal{V} \approx -\frac{\alpha}{4\pi} V_0 \int \frac{d^4 k}{i\pi^2} \frac{(p_1 p_2)}{k^2(kp_1)(kp_2)} \approx -\frac{\alpha}{2\pi} V_0 \int_0^1 \frac{dx}{x} \int_0^{1-x} \frac{dy}{y}
\]

(1)

We can see here the two logarithmic divergences that arise from the integration over the \( x, y \) Feynmann parameters. As is well known, one of them is of collinear origin and the other one is a proper IR divergence. When we take some of the external squared momenta and/or masses different from zero, they serve as cutoffs for the divergences. Let us consider now some simple cases that will be useful in the following, where the cutoff is given by a single scale \( M \). The behavior for \( C_0 \) in the asymptotic region \( s \equiv (p_1^2 + p_2^2) \gg M^2 \) is as follows:

\[
C_0(m_1, m_2, M, p_1^2, p_2^2, s) \equiv \frac{1}{i\pi^2} \int \frac{d^4 k}{[(k + p_1)^2 - m_1^2][(k - p_2)^2 - m_2^2]}
\]

(2)

\[
\text{Re}\{C_0(0, 0, M, 0, 0, 0, s)\} \to \frac{1}{2s} \log^2 \left(\frac{s}{M^2}\right) \quad \text{for } s \gg M^2
\]

(3)

\[
\text{Re}\{C_0(M, M, 0, 0, 0, s)\} \to \frac{1}{2s} \log^2 \left(\frac{s}{M^2}\right) \quad \text{for } s \gg M^2
\]

(4)

Then we can use (3) for the vertex of fig. 1 in the asymptotic region finding:

\[
\mathcal{V} \approx -\frac{\alpha}{4\pi} V_0 \log^2 \left(\frac{s}{M^2}\right) \quad \text{for } s \gg M^2
\]

(5)

where we can see the double logarithm behavior of the vertex correction for \( s \gg M^2 \).

The dependence on the IR logs simply factorizes for the cross section:

\[
\sigma \propto \frac{1}{s} \int_0^s \frac{dt}{s} |\mathcal{M}|^2 \left[1 - 2\frac{\alpha}{4\pi} \log^2 \frac{s}{M^2}\right] = \sigma_0 \left[1 - 2\frac{\alpha}{4\pi} \log^2 \frac{s}{M^2}\right]
\]

Now let us consider the “susy-like” case in which a fermion is exchanged and a scalar couples to the external gauge boson (fig. 1). In supersymmetry the internal fermion and scalar, for instance a neutralino and a selectron, have masses of the same order and can, for our purposes, be taken to have the same mass \( M \). In fact the distinction between the two masses is irrelevant as long as they are of the same order, since \( \log(\frac{s}{\sqrt{M}}) \log(\frac{m^2}{\sqrt{M}}) = \log^2(\frac{s}{\sqrt{M}}) + \log(\frac{s}{\sqrt{M}}) \log(\frac{m^2}{\sqrt{M}}) \approx \log^2(\frac{s}{\sqrt{M}}) \) and we are interested only in double logs (single logs are neglected). Expression (5) is in this case substituted by:

\[
\mathcal{V} \approx -\frac{\alpha}{4\pi} V_0 \int \frac{d^4 k}{i\pi^2} \frac{M^2}{k^2(kp_1)(kp_2)} \approx -\frac{M^2}{s} \frac{\alpha}{4\pi} V_0 \log^2 \left(\frac{s}{M^2}\right)
\]

(6)

where we still have the double logarithm behavior coming from the integration over the region \( k \approx 0 \) (remember that always \( s \gg M^2 \)). In this case however, \( 2p_1 p_2 = s \) is substituted by \( M^2 \) in the numerator, so that the we have \( \log^2 \frac{s}{\sqrt{M^2}} \rightarrow \frac{M^2}{s} \log^2 \frac{s}{\sqrt{M^2}} \). In the end the double logarithm behavior is strongly suppressed by a factor \( \frac{M^2}{s} \) for the SUSY vertex with respect to the SM case. This is due to the different couplings that appear in the vertex corrections: fermion-gauge boson coupling in the “SM like” case and fermion-scalar in the “susy like” case. In the first case the coupling is, at high energy, proportional to \( p_i^\mu \) where \( i \) is the label of the external fermion the exchanged boson couples to. Then we have a factor \( p_i \cdot p_j \) where \( i \) and \( j \) are the fermions connected by the exchanged boson. In the “susy like” case where scalars and fermions are exchanged, no such factor is present and \( p_i \cdot p_j \) gets substituted by \( M^2 \), generally subdominant at high energies.
3 Sudakov logarithms in the boxes

Let us consider the exchange of a vector boson of mass M in the s-channel (see fig. 2). In the limit $M \to 0$ and in the IR region, the amplitude is given by:

$$\mathcal{M} \approx \frac{\alpha}{4\pi} \mathcal{M}_0 \int \frac{d^4 k}{i\pi^2} \left\{ \frac{(p_1 p_3)}{k^2 (p_1 k)(p_3 k)} + \frac{(p_2 p_4)}{k^2 (p_2 k)(p_4 k)} \right\}$$  \(7\)

As is shown schematically in fig. 2, the two terms in this equation come from two different region of integration. When $k \approx 0$, then $(k + p_1 + p_2)^2 \approx s$ and we can think the “upper” boson line to be shrunk, like shown in the figure. The mirror situation is $k + p_1 + p_2 \approx 0$, $k^2 \approx s$. This makes evident the fact that the IR structure of the box is the same of the vertex. Expression (7) is identical with (1) but with the difference that $2p_1 p_2 = s$ gets substituted by $-2p_1 p_3 = -2p_2 p_4 = t$. So for SM boxes we have an exchange of $s$ and $t$ variables with respect to SM vertices. In the end for the box contribution in the IR region we can write:

$$\mathcal{M} \approx -\frac{\alpha}{2\pi} \mathcal{M}_0 \log^2 \left( \frac{t}{M^2} \right)$$  \(8\)

It must be stressed however that this expression is valid only in the asymptotic region $t \gg M^2$ where the double log behavior is generated, while we assumed $s \gg M^2$.

Let us now consider the “susy like” box where a scalar particle is exchanged in the t-channel (see figure 3). In this case the amplitude is:

$$\mathcal{M} \approx \frac{\alpha}{4\pi} \mathcal{M}_0 \int \frac{d^4 k}{i\pi^2} \left\{ \frac{M^2}{k^2 (p_1 k)(p_3 k)} + \frac{M^2}{k^2 (p_2 k)(p_4 k)} \right\}$$  \(9\)

Comparing eqs. (7) and (9) we note that the susy amplitude has a factor $M^2$ with respect to the SM one. In the IR region $t \gg M^2$ we have, using (7):

$$\mathcal{M} \approx \frac{\alpha}{4\pi} \mathcal{M}_0 \frac{M^2}{t} \log^2 \left( \frac{t}{M^2} \right)$$  \(10\)

Care must be taken when we compute cross sections since, as noted above, eqs. (7) and (10) are valid only when $t \gg M^2$. Let us then consider a region of the phase space from a certain fixed value of $t$ of order $s$ on, let’s say $-s < t < -\frac{s}{2}$. Then, if $s \gg M^2$, we can use the expressions valid for $t \gg M^2$. Neglecting unessential factors, the leading box corrections to the tree level cross sections are given by:

$$SM \quad \Delta \sigma \approx \frac{\alpha}{s} \int_{-s}^{-\frac{s}{2}} \frac{dt}{t} \log^2 \left( \frac{t}{M^2} \right) \approx \sigma_0 \alpha \log^2 \left( \frac{s}{M^2} \right)$$

$$SUSY \quad \Delta \sigma \approx \frac{\alpha}{s} \int_{-s}^{-\frac{s}{2}} \frac{dt}{t} \frac{M^2}{t} \log^2 \left( \frac{t}{M^2} \right) \approx \sigma_0 \alpha \frac{M^2}{s} \log^2 \left( \frac{s}{M^2} \right)$$

Again, SUSY boxes are depressed by a power factor with respect to SM ones.

To conclude, we expect double logs of IR and collinear origin to give at high energies large one loop corrections to observables in the SM. This is true both for box and vertex corrections. On the other hand, in a susy theory, due to the different spins of the particles exchanged in the loops, these double logs are expected to be power suppressed. For this reason, in the following we will consider in detail only SM electroweak corrections.

4 Sudakov logarithms in the Standard model

We study the purely electroweak double logarithmic corrections in the Standard Model coming from the exchange of the $W$ and $Z$ gauge bosons to the process $e^+ e^- \to \bar{f} f$ in the massless case.
For the moment we consider only the massless external fermions $\mu$ for leptons, $u$, $c$ and $d$, $s$ for quarks, and we neglect, for the moment, the bottom quark whose corrections contain a non trivial flavor dependence on the top mass (future analyses).

This kind of contributions, as explained before, come from only vertex corrections in which one gauge boson it is exchanged and from the boxes (direct and crossed) with two $Z$s or two $W$s.

The effective vertices $\gamma(Z)\bar{f}f$ including tree level and dominant double logs are given by $\bar{v}_e(p_1)\gamma_{\mu}(V_{fL}^\gamma(Z)P_L+V_{fR}^\gamma(Z)P_R)u_{e}(p_2)$ with

$$V_{fL}^\gamma = ig \bar{w} Q_f(1 - \frac{1}{16\pi^2} \frac{g^2}{c_w} g_{fL}^2 \log^2 \frac{s}{m_Z^2} - \frac{1}{16\pi^2} \frac{g^2}{2} Q_{f'} \log^2 \frac{s}{m_W^2})$$ \hspace{1cm} (11)

$$V_{fR}^\gamma = ig \bar{w} Q_f(1 - \frac{1}{16\pi^2} \frac{g^2}{c_w} g_{fR}^2 \log^2 \frac{s}{m_Z^2})$$ \hspace{1cm} (12)

and

$$V_{fL}^Z = ig \frac{g}{c_w} g_{fL}(1 - \frac{1}{16\pi^2} \frac{g^2}{c_w} g_{fL}^2 \log^2 \frac{s}{m_Z^2} - \frac{1}{16\pi^2} \frac{g^2}{2} g_{fL} \log^2 \frac{s}{m_W^2})$$ \hspace{1cm} (13)

$$V_{fR}^Z = ig \frac{g}{c_w} g_{fR}(1 - \frac{1}{16\pi^2} \frac{g^2}{c_w} g_{fR}^2 \log^2 \frac{s}{m_Z^2})$$ \hspace{1cm} (14)

Here $f$ is the external fermion and $f'$ its isospin partner. Moreover, $g_{f(f')L} = -Q_{f(f')} s_W^2$ and $g_{f(f')R} = T^J_{\bar{f}f}$.

Defining

$$\bar{v}_e(p_1)\gamma_{\mu}P_{L,R}u_{e}(p_2)\bar{u}_f(p_3)\gamma_{\mu}P_{L,R}v_f(p_4) \equiv P_{L,R} \otimes P_{L,R}$$ \hspace{1cm} (15)

the corrections from box diagrams come from direct and crossed diagrams as a sum of projected amplitudes on the left-right chiral basis:

$$B_{LL} \gamma_{\mu}P_{L} \otimes \gamma_{\mu}P_{L} + B_{LR} \gamma_{\mu}P_{L} \otimes \gamma_{\mu}P_{R} +$$

$$B_{RL} \gamma_{\mu}P_{R} \otimes \gamma_{\mu}P_{L} + B_{RR} \gamma_{\mu}P_{R} \otimes \gamma_{\mu}P_{R}$$

where

$$B_{LL} = \frac{i g^4}{8 \pi^2} \frac{g_{TL}^2 g_{TZ}^2}{c_w} \left( \log^2 \frac{s + t}{m_Z^2} - \log^2 t \right) + \frac{1}{4} (\theta_{1f} \log^2 \frac{s + t}{m_W^2} - \theta_{1f} \log^2 t \right)$$

$$B_{LR} = \frac{i g^4}{8 \pi^2} \frac{g_{TL}^2 g_{TZ}^2}{c_w} \left( \log^2 \frac{s + t}{m_Z^2} - \log^2 t \right)$$

$$B_{RL} = \frac{i g^4}{8 \pi^2} \frac{g_{TL}^2 g_{TZ}^2}{c_w} \left( \log^2 \frac{s + t}{m_Z^2} - \log^2 t \right)$$

$$B_{RR} = \frac{i g^4}{8 \pi^2} \frac{g_{TL}^2 g_{TZ}^2}{c_w} \left( \log^2 \frac{s + t}{m_Z^2} - \log^2 t \right)$$

with the above expressions obtained in the limit $s, t \gg M_{Z,W}^2$ and

$\theta_{1f} = 1$ for $f = \mu, d$ and zero otherwise;

$\theta_{2f} = 1$ for $f = \nu, u$ and zero otherwise;
The positive double-log contributions come from the crossed box, while the negative ones from the direct diagrams.

It is clear that the interference between the two amplitudes, for the exchange of $Z$ bosons, leads to a depression of the full contribution due to the fact that
\[
\log^2 \frac{s + t}{m_Z^2} - \log^2 \frac{t}{m_Z^2} = 2 \log \frac{s}{m_Z^2} \log \frac{1 + \cos \theta}{1 - \cos \theta} + \text{finite}
\] (16)

where finite means contributions not increasing as $\log s$. In such a way we lose the leading $\log^2 s$ factor and we remain with a single log that we neglect. So in leading approximation, box diagram contributions come only from $W$ exchange.

To obtain the physical observables we must square the full amplitude:
\[
M = M_{LL} \gamma \mu P_L \otimes \gamma \mu P_L + M_{LR} \gamma \mu P_L \otimes \gamma \mu P_R + M_{RL} \gamma \mu P_R \otimes \gamma \mu P_L + M_{RR} \gamma \mu P_R \otimes \gamma \mu P_R
\] (17)

where
\[
\begin{align*}
M_{LL} &= \frac{i}{s} (V_{eL}^c V_{fL}^\gamma + V_{eL}^Z V_{fL}^Z) + B_{LL} \\
M_{RL} &= \frac{i}{s} (V_{eR}^c V_{fL}^\gamma + V_{eR}^Z V_{fL}^Z) + B_{RL} \\
M_{LR} &= \frac{i}{s} (V_{eL}^c V_{fR}^\gamma + V_{eL}^Z V_{fR}^Z) + B_{LR} \\
M_{RR} &= \frac{i}{s} (V_{eR}^c V_{fR}^\gamma + V_{eR}^Z V_{fR}^Z) + B_{RR}
\end{align*}
\]

and compute the differential cross section
\[
\frac{d\sigma}{d\Omega} = \frac{s}{256\pi^2} N_f \left[ (|M_{LL}|^2 + |M_{RR}|^2)(1 + \cos \theta)^2 + (|M_{RL}|^2 + |M_{LR}|^2)(1 - \cos \theta)^2 \right]
\] (18)

with $N_f = 1(3)$ for final state leptons (quarks) and $-1 + 2 \frac{m_2^2}{s} < \cos \theta < 1 - 2 \frac{m_2^2}{s}$ to be consistent with the above approximations ($t \gg -m_Z^2$). In any case we can extend the integration region to the full $\pm 1$ range without modifying the leading results.

5 Sudakov logs in the cross section and in the forward backward asymmetry for $e^+ e^- \rightarrow f \bar{f}$

We define $\sigma_B$ and $\sigma_T$ respectively as the tree level (Born) cross section and as the total cross section containing only the one loop double logarithms. The explicit expressions for different fermionic final states are given by:
\[
\begin{align*}
\sigma_T / \sigma_B (e^+ e^- \rightarrow \mu \bar{\mu}) &= 1 + (-1.345_{\text{Box}} + 0.282) \alpha_W - 0.330 \alpha_Z \\
\sigma_T / \sigma_B (e^+ e^- \rightarrow u \bar{u}) &= 1 + (-2.139_{\text{Box}} + 0.864) \alpha_W - 0.385 \alpha_Z \\
\sigma_T / \sigma_B (e^+ e^- \rightarrow d \bar{d}) &= 1 + (-3.423_{\text{Box}} + 1.807) \alpha_W - 0.557 \alpha_Z
\end{align*}
\] (20-22)

where $\alpha_{W,Z} = \frac{g^2}{16\pi^2} \log^2 \frac{s}{m_{W,Z}^2} \simeq 2.7 \cdot 10^{-3} \log^2 \frac{s}{m_{W,Z}^2}$. With the underline “Box” we give the contributions coming from box diagrams, the rest is from vertex corrections.
For the forward-backward asymmetry \( A_{FB}(e^+ e^- \rightarrow f \bar{f}) \) the analytic expressions are:

\[
\frac{A^T_{FB}}{A^B_{FB}}(e^+ e^- \rightarrow \mu \bar{\mu}) = 1 + (-0.807_{Box} + 0.770) \alpha_W - 0.002 \alpha_Z \\
\frac{A^T_{FB}}{A^B_{FB}}(e^+ e^- \rightarrow u \bar{u}) = 1 + (-0.521_{Box} + 0.454) \alpha_W - 0.023 \alpha_Z \\
\frac{A^T_{FB}}{A^B_{FB}}(e^+ e^- \rightarrow d \bar{d}) = 1 + (-0.620_{Box} + 0.508) \alpha_W - 0.029 \alpha_Z
\]

We see that already at \( \sqrt{s} = 1 \) (0.5) TeV the parameter \( \alpha_{Z,W} \simeq 6 \) (3) \( 10^{-2} \) so that the above corrections can exceed the ten (six) percent for the cross sections and a resummation technique (which is under study) is needed.

In the limit \( \alpha_Z \simeq \alpha_W \) we can summarize the above results in:

\[
\frac{\sigma_T}{\sigma_B}(\mu \bar{\mu}) \simeq 1 - 1.39 \alpha_{Z,W}; \quad \frac{A^T_{FB}}{A^B_{FB}}(\mu \bar{\mu}) \simeq 1 - 0.04 \alpha_{Z,W};
\]

\[
\frac{\sigma_T}{\sigma_B}(u \bar{u}) \simeq 1 - 1.66 \alpha_{Z,W}; \quad \frac{A^T_{FB}}{A^B_{FB}}(u \bar{u}) \simeq 1 - 0.09 \alpha_{Z,W};
\]

\[
\frac{\sigma_T}{\sigma_B}(d \bar{d}) \simeq 1 - 2.17 \alpha_{Z,W}; \quad \frac{A^T_{FB}}{A^B_{FB}}(d \bar{d}) \simeq 1 - 0.11 \alpha_{Z,W};
\]

We can make several comments to these results:

- **Z boson exchange** is negative (photon-like) in the vertex corrections: it decreases both left and right effective vertices. In the boxes, Z exchange contribution does not give a double log behavior due to a cancellation between direct and crossed diagrams.

- **W boson exchange**, due to his chiral structure, affects only the left gamma vertex proportionally to \(-\frac{\alpha' L}{\alpha L}\) and the left Z vertex to \(-\frac{\alpha' L}{\gamma L}\), giving always contributions that are positive with respect to the tree level values. Also box diagrams are peculiar because they affect only the left-left structure of the amplitude and they always give a negative contribution.

- In \( \frac{\sigma_T}{\sigma_B} \) box corrections are dominant (more than three times the vertex ones). Since, as noted above, box corrections are given only by W exchange, the e.w. Sudakov corrections are a peculiar signature of the left-left structure of the full amplitude.

- In \( \frac{A^T_{FB}}{A^B_{FB}} \) Z corrections almost cancel. W contributions from vertex are accidently almost equal and opposite to the box’s ones leaving a negligible contribution. As a result, the double logs relative effect is more than one order of magnitude smaller than for the full cross sections.

- The total effect from virtual double logs is negative both for the cross sections and for the asymmetries.

## 6 Conclusions

We have investigated, in one loop electroweak corrections, the IR origin of double logs that we denote as e.w. Sudakov corrections. These Sudakov effects can be important for next generation of colliders running at TeV energies since they grow with energy like the square of a logarithm. In supersymmetric models, loops containing the supersymmetric partners of the usual particles do not have double log asymptotical behavior (i.e., the double logs are
present but power suppressed). In the SM the e.w. Sudakov corrections are present with a peculiar chiral structure due to $W$ boson exchange dominance; it should be possible to test the different chiral contributions with colliders with polarized beams. In any case, already for TeV machines, proper resummation of such large contributions seems to be needed; in fact for the various cross sections we find that contributions of order 5-8% are present for the planned 500 GeV $e^+e^-$ NLC [6]. The corrections to the asymmetries considered in this paper, due to the accidental cancellation between box and vertices contributions, are almost negligible (one order of magnitude smaller with respect to the cross sections relative corrections). Sudakov effects in other kind of asymmetries (for instance polarized asymmetries) and in general in other observables, are currently under study.

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Figure 1: Vertex diagram in SM (left) and SUSY (right) generating a \( \log^2 \frac{s}{m^2} \). \( p_1 \) and \( p_2 \) are ingoing.

Figure 2: Box contribution for the SM and effective Feynman diagrams in the IR region

Figure 3: Box contribution for supersymmetry (the crossed diagram is also shown)