Completion of the universal $I$-Love-$Q$ relations in compact stars including the mass

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Accepted XXX. Received YYY; in original form ZZZ

ABSTRACT
In a recent paper we applied a rigorous perturbed matching framework to show the amendment of the mass of rotating stars in Hartle’s model. Here, we apply this framework to the tidal problem in binary systems. Our approach fully accounts for the correction to the Love numbers needed to obtain the universal $I$-Love-$Q$ relations. We compute the corrected mass vs radius configurations of rotating quark stars, revisiting a classical paper on the subject. These corrections allow us to find a universal relation involving the second-order contribution to the mass $\delta M$. We thus complete the set of universal relations for the tidal problem in binary systems, involving four perturbation parameters, namely $I$, Love, $Q$, and $\delta M$. These relations can be used to obtain the perturbation parameters directly from observational data.

Key words: binaries: general – gravitational waves – stars: neutron – stars: rotation

1 INTRODUCTION

The construction of analytic models of astrophysical compact bodies in general relativity (GR) relies on the matching of spacetimes theory. The idea is to consider two different bounded regions, namely an interior fluid and a vacuum exterior, and to impose appropriate matching conditions on a timelike hypersurface $\Sigma$ separating them. Therefore, a global model is constructed by joining the common boundary data on $\Sigma$. While the search for an exact global model for a rotating compact body is a major challenge, the situation becomes tractable when one resorts to approximate methods such as perturbation theory. In this context, models that describe rotating stars (Hartle 1967), tidal effects (Damour & Nagar 2009), or collapsing stars (Brizuela et al. 2010) have been developed.

Although the matching of spacetimes in the exact case has been well understood for decades, the matching in a perturbative scheme has needed a much longer concoction. The first fully general and consistent perturbation theory of hyper-surfaces to second order is due to Mars (2005). On top of a background spacetime where two regions are matched on some $\Sigma_0$, first and second-order problems for the corresponding regions are developed. The theory provides the common boundary data on $\Sigma_0$ for such problems and the gauge-independent equations for the quantities that describe the deformation of the surface. Perturbed matching is commonly treated in the literature by prescribing some extension of the exact matching conditions to the perturbative scheme, or assuming the continuity of the functions driving the perturbations across $\Sigma_0$ (see Mars et al. (2007)). However, this is not ensured a priori, and assuming explicit choices of coordinates (and gauges) in which the perturbations satisfy certain continuity and differentiability conditions may subtract generality to the model. Even worse, it may lead to wrong outcomes. We discuss two relevant examples next.

The first example has to do with slowly-rotating relativistic stars. In their pioneering work, Hartle and Thorne presented the general relativistic treatment of isolated rotating compact stars in equilibrium (Hartle & Thorne 1968), known as “Hartle’s model” in short. This stands as the basis to construct analytical models in axial symmetry (see Stergioulas (2003) and references therein). A perturbative scheme is built upon a spherical non-rotating configuration, on top of which stationary (rotating) and axially symmetric perturbations are taken to second order. Models are built assuming a perfect fluid interior with barotropic equation of state (EOS) that rotates rigidly and with no convective motions. Under these assumptions the perturbations are described by four functions, whose values at the surface of the star determine the dragging of inertial frames, the deformation, and the total mass of the star in terms of the central density. At each order those values are computed (i) integrating from the regular centre given an EOS, (ii) solving

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the asymptotically flat vacuum exterior, and (iii) assuming the continuity of the functions across the surface choosing an explicit coordinate system.

Assumption (iii) has significant implications on the properties of the stellar models. As shown by Reina & Vera (2015) using the framework of Mars (2005), a relevant perturbative function presents a discontinuity proportional to the (background) energy density in the stellar surface. This function enters the computation of the mass of the star at second order \( \delta M \) and, therefore, the original expression for the total mass of the rotating star given in Hartle (1967) has to be amended. Idealised, constant-density stars, originally studied by Chandrasekhar & Miller (1974), were subsequently analyzed in Reina (2016), showing that the deviations in the mass-radius diagrams are far from negligible.

The second example has to do with the so-called I-Love-\( Q \) relations (Yagi & Yunes 2013b), where "I" and "Q" refer to the moment of inertia and the quadrupole moment of the star respectively, and the Love numbers "\( k \)" are associated with the tidal field due to the presence of a companion star. The most basic treatment of this problem fits in Hartle's scheme and can be solved in the regime of stationary objects. However, using a correction identified in Yagi & Yunes (2014) obtained an amended expression for the Love numbers that leads to universal relations (Yagi & Yunes 2013b), where "\( k \)" refer to the moment of inertia and the quadrupole moment of stars and quark stars. However, using a correction identified with the tidal problem is indeed related: the result of Yagi & Yunes (2014) gives the corrections in the context of Hartle's model to quark stars. Let us note that the jumps in the tidal perturbation tensors in each region (let us drop \( \epsilon \) and \( \delta \)) can be studied independently. In the exterior, \( \tilde{\Omega} \) either \( \pm \) the corrections in the context of Hartle's model to quark stars.

2.1 Perturbative scheme for rotating stars

Hartle's model is based upon the following forms for the perturbation tensors in each region (let us drop \( \pm \) for clarity) to first and second order respectively \(^1\)

\[
K_0^H = -2r^2 \omega(r) \sin^2 \theta d\theta d\phi, \quad (1)
\]

\[
K_2^H = \left( -4 \xi(r) h(r, \theta) + 2r^2 \sin^2 \theta \omega^2(r) \right) dr^2 + 4e^{2k}r^2(1-m(r, \theta))dr^2 + 2m_0 \omega(r) \sin \theta d\theta d\phi,
\]

where \( h(r, \theta) = h_0(r) + \xi_1(r)P_2(\cos \theta), \ m(r, \theta) = m_0(r) + m_1(r)P_2(\cos \theta) \) and \( k(r, \theta) = k_0(r) + k_1(r)P_2(\cos \theta) \). The perturbed (unit) fluid flow with rigid rotation and no convection reads \( \vec{u} = \Omega \vec{\theta} \) and \( \vec{u} = e^{3\nu} \vec{\Omega} \vec{\phi} + 2\Omega \vec{\phi} + K_2^H / 2 \vec{\theta} \), for a constant \( \Omega \). Hence, the energy density and pressure perturbations only enter the second order as \( E_2(r, \theta) = \bar{E}_2(r, \theta) = \bar{P}_2(\omega(r), \theta) = P_0^2 + P_2(\omega(r), \theta) \), \( P_2(\omega(r), \theta) = P_0(\omega(r), \theta) + P_2(\omega(r), \theta) \). It is convenient to introduce a rescaled pressure defined by \( \bar{P} = \bar{P}_2 / 2 \bar{\theta} = 0 / 2 \bar{\theta} = 0 / 2 \bar{\theta} \). The first and second order quantities rescale under a perturbation parameter \( \epsilon \), so that any functions \( A^{(1)} \) and \( A^{(2)} \) at first and second order respectively, enter the model through the scale invariants \( \epsilon A^{(1)} \) and \( \epsilon A^{(2)} \).

A convenient substitution to the function \( \omega = \bar{\omega} \) in (1) is \( \omega(r) := \bar{\Omega} - \bar{\omega}(r) \), that satisfies the single ODE (43) in Hartle (1967). It is integrated from the origin \( \bar{\omega}(0) := \tilde{\omega}_0 \) outwards. Given that the relevant quantity is \( \epsilon \tilde{\omega}_0 \) one is free to fix either \( \tilde{\omega}_0 = 1 \) or \( \varepsilon = 1 \) (as in Hartle (1967)). For convenience we choose the former. In the exterior, \( \bar{\omega} \) \( \Omega = 2r^{-3} \) for some constant \( \mu \).

The \( l = 0 \) sector of the second-order perturbations can be studied independently. In the \( l = 0 \) sector a gauge fixing allows to set \( k_0 = 0 \) (see Reina & Vera (2015)), so that the only functions involved are \( [m_0, \tilde{h}_0] \). In the interior, a first integral (see (64) in Reina & Vera (2015), or (90) in

\[ \frac{\epsilon^k}{d\tau^2} + \frac{r^2}{\sigma^2} \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \], that are matched across the boundaries, \( \Sigma_0 := \Sigma_0^+ = \{ r = a_+ \} \) with constants \( a_+ > 0 \). The interior region (+) radial coordinate ranges in \( r' \in (0, a') \) and the exterior (-) in \( r' \in (a', \infty) \). The matching conditions for this setting are \( a := a' = a \), \( |v| = |v'| = 0 \), where \( r' := d/dr \) and \( f \) is the difference of the function \( f \) evaluated on \( \Sigma_0 \) from the (+) and (-) sides, i.e. \( f := f|_{\Sigma_0} + f|_{\Sigma_0} \).

The perfect fluid interior is described by its unit fluid configuration. Consider two static spherically symmetric space-times \( (\gamma^+, g^+, \Sigma_0^+) \) and \( (\gamma^-, g^-, \Sigma_0^-) \), with \( g^+ = -e^{2\nu} \, dt^2_+ + e^{k} \, dr^2_+ + r^2_+ (d\theta^2_+ + \sin^2 \theta_+ d\phi^2_+), \) that are matched across the boundaries, \( \Sigma_0 := \Sigma_0^+ = \{ r = a_+ \} \) with constants \( a_+ > 0 \). The interior region (+) radial coordinate ranges in \( r' \in (0, a') \) and the exterior (-) in \( r' \in (a', \infty) \). The matching conditions for this setting are \( a := a' = a \), \( |v| = |v'| = 0 \), where \( r' := d/dr \) and \( f \) is the difference of the function \( f \) evaluated on \( \Sigma_0 \) from the (+) and (-) sides, i.e. \( f := f|_{\Sigma_0} + f|_{\Sigma_0} \).

The perfect fluid interior is described by its unit fluid flow \( \bar{u} \), whose energy density \( E \geq 0 \) and pressure \( P \geq 0 \) are related by a barotropic EOS \( E(P) \). The mass function is defined by \( e^{-k} = 1 - 2M(r) / r \). The TOV equations hold (Hartle 1967) and determine the interior configuration given the central value of the energy density \( E(0) := E_0 \). Function \( v \) is determined up to an additive constant. The asymptotically flat vacuum exterior is Schwarzschild, determined by the total mass \( M \), explicitly \( e^{-k}(r) = 1 - 2M(r) / r \).

Given this matter content, the matching conditions are interpreted as follows: \( |v| = 0 \) fixes the constant \( M \) to be the mass of the fluid, \( M = M^0(u) \), \( |v| = 0 \) fixes the value of \( v' \) at the origin, and \( |v'| = 0 \) is just \( P(u) = 0 \), which determines \( a \).

1 The function \( m(r, \theta) \) in Eq. (2) corresponds to the \( m \) used in Hartle (1967), whereas \( m \) in Reina & Vera (2015) is \( r^{-k} m \) here.

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Hartle (1967)) is used to substitute \( h_0^\nu \) by \( P_0^{(2)} \) and the rest of the field equations provide an inhomogeneous system of first-order ODEs for the set \( \{m_0^2, P_0^{(2)}\} \) (see Eqs. (61)-(62) in Reina & Vera (2015), or (97), (100) in Hartle (1967)).

The equations are integrated from a regular origin taking \( P_0^{(2)}(0) = 0 \) to obtain the perturbed configuration quantities in terms of \( E_\epsilon \) (see below). For the exterior we have

\[
m_0 (r_\epsilon) = \delta M - \frac{J^2}{r_\epsilon^2}, \quad h_0 (r_\epsilon) = \frac{1}{1 - 2M} \left( -\delta M + \frac{J^2}{r_\epsilon^2} \right),
\]

for some arbitrary constant \( \delta M \) (Hartle 1967).

The \( I = 2 \) sector involves the functions \( h_2, k_2 \) and \( m_2 \). The field equations provide a quadrature for \( m_2 \), a first integral that relates \( h_2 \) and \( P_2^{(2)} \), and a system of first-order ODEs (see Eqs. (67)-(68) in Reina & Vera (2015) or (125)-(126) in Hartle (1967)) for the pair \( (h_2, \nu := h_2 + k_2) \). For a regular origin, the set \( \{h_2, \nu^+\} \) is determined up to one arbitrary constant. The exterior is found in terms of an arbitrary constant \( K \) and the Legendre functions of the second kind \( (P_{\ell m}^{(2)}) \) (Hartle & Thorne 1968)

\[
h_2 (r_\epsilon) = K Q^\ell_2 \left( \frac{r_\epsilon}{M} - 1 \right) + P^\ell_2 \left( \frac{r_\epsilon}{M} \right), \quad \nu^+ (r_\epsilon) = K \frac{2M}{\sqrt{r_\epsilon^2 - 2Mr_\epsilon}} Q^\ell_2 \left( \frac{r_\epsilon}{M} - 1 \right) - \frac{J^2}{r_\epsilon^2},
\]

where \( Q^\ell_2, P^\ell_2 \) are determined using the framework developed by Reina & Vera (2015), or (97), (100) in Hartle (1967).

\[
\frac{\delta M}{M} = 2\frac{\pi}{\omega} (a - 2M) E(a)^2/2 M (a - 2M), \quad \frac{\delta M}{M} = 2\frac{\pi}{\omega} (a - 2M) E(a)^2/2 M (a - 2M), \quad \frac{\delta M}{M} = 2\frac{\pi}{\omega} (a - 2M) E(a)^2/2 M (a - 2M).
\]

For the second order, in the \( I = 0 \) sector we only need for our purposes here (Theorem 1 in Reina & Vera (2015))

\[
J = \frac{1}{\delta M} \omega^+ (a) \quad \Omega = \omega^+ (a) + \frac{2J}{\delta M}. \tag{10}
\]

\[
E(a) = \frac{M}{a^2} e^{\lambda(a)/2} \Xi_0 = 0. \tag{12}
\]

Given the set \( \{m_0^2 (r_\epsilon), P_0^{(2)} (r_\epsilon)\} \) has been determined in the interior, (11) fixes \( \delta M \) in (3) as,

\[
\delta M = m_0^2 (a) + \frac{J^2}{\delta M} + 4\frac{\pi}{\omega} (a - 2M) E(a) P_0^{(2)} (a). \tag{13}
\]

The total mass, in terms of a fixed \( E_\epsilon \), reads

\[
M_T (E_\epsilon) = M(E_\epsilon) + e^2 \delta M (E_\epsilon). \tag{14}
\]

The second order correction to the mass \( \delta M^S := e^2 \delta M \) is usually called the change in mass. The correction to the original Hartle’s model comes from the discontinuity (11), yielding the last term in (13). Eq. (12) provides \( \Xi_0 \) if \( E(a) \neq 0 \) (see below), which describes the star \( l = 0 \) deformation (in the gauge used) through the average radius of the rotating star \( R_T = a - e^2 \alpha (a - 2M) \Xi_0 /2 \) (Reina & Vera 2015), producing the usual \( R_T = a - e^2 \alpha (a - 2M) \Xi_0 /2 \) (Reina & Vera 1967).

The matching conditions for the \( I = 2 \) sector can be split into two sets. The first one contains two purely geometrical (independent of Einstein’s equations) relations

\[
h_0 (r_\epsilon) = a_\nu P^\ell_2 \left( \frac{r_\epsilon}{M} - 1 \right) + a_Q Q^\ell_2 \left( \frac{r_\epsilon}{M} - 1 \right), \tag{9}
\]

for arbitrary constants \( a_\nu \) and \( a_Q \) (Damour & Nagar 2009).

\[
P_2^\ell := \left( \frac{2}{\pi} \frac{\Gamma((\ell+2)/2)}{\Gamma(\ell/2)} \right)^{-1} P^\ell_2 \quad \text{and} \quad Q^\ell_2 := \left( \frac{3}{4\pi} \frac{\Gamma((\ell+3)/2)}{\Gamma(\ell/2)} \right)^{-1} Q^\ell_2.
\]

The second set is obtained by combining the rest of the geometrical matching conditions with the field equations.
We use the MIT bag model with a linear EOS of the type 

can be solved numerically, for the cases of static and rotating stars, with the goal of obtaining accurate models of strange stars. In Figure 1 we show the total mass vs radius diagram for strange stars. For the rotating models (red and blue curves) significant differences appear when the total mass is correctly computed. In particular, the maximum mass is \( 2M_e \), \( \sim 11\% \) larger than that attained in Hartle’s approach. In addition, Table 1 compares the numerical values of relevant model parameters (compare with Table 1 of Colpi & Miller (1992)). We find that the maximum mass difference is \( \sim 0.5M_e \), achieved for a density \( E_c = 6.34 \times 10^{14} \text{ g cm}^{-3} \).

4 TIDAL LINEARIZED MATCHING

A similar analysis can be carried out for the perturbations describing the full tidal problem, Eq. (6), generalising the matching conditions from Reina & Vera (2015) to a non-axisymmetric setup. Such study will be presented elsewhere. However, under the assumption of staticity made in Section 2.2, the matching for the axisymmetric \( \{l \geq 2, m = 0\} \) sector of the tidal problem becomes a subcase of the matching given by Eqs. (1)-(2) after \( l \) setting \( \omega = 0 \) and \( ii \) identifying \( h(r, \theta) = -\mathcal{H}_l(r, \theta)/4, m(r, \theta) = \mathcal{H}_l(r, \theta)/4 \) and \( k(r, \theta) = K(r, \theta)/4 \). Proposition 2 in Reina & Vera (2015) ensures then that the corresponding spherical-harmonic decomposition coefficients for all \( l \geq 2 \) satisfy equations equivalent to (15) and (16)-(18), leading to

\[
\left[ \mathcal{H}_{l} \right] = \left[ K_{l} \right] = 0,
\]

\[
\left[ H_{0} \right] = \left[ K_{l} \right] = \frac{4\pi a^{2}}{M} E(a) H_{0}(a), \quad \left[ H_{2l} \right] = 0,
\]

while the analogous to (19) for all \( l \geq 2 \) leads to

\[
E(a) \left[ \mathcal{H}_{l}(a) + \mathcal{V}(a) e^{-\lambda(a)/2} \mathcal{Z}_{2} + \frac{1}{3} \mathcal{V}^{2}(a) e^{-\lambda(a)/2} \mathcal{Z}_{2} \right] = 0.
\]

Two remarks are in order. First, conditions (21) are independent of the field equations and therefore \( H_{0} \) and \( K_{l} \) will be continuous irrespective of the theory used. However, the continuity of \( H_{2l} \) (as well as conditions (24)) is a consequence of both the geometrical matching plus Einstein’s equations for a perfect fluid. For other matter content or theory of gravity, the geometric matching conditions from Proposition 2 in Reina & Vera (2015) must be conveniently combined with the corresponding field equations. Of course, if \( E \) presents a jump at \( r = a \) (and \( H_{0}(a) \neq 0 \), so will \( H_{0} \) and \( K_{l} \)).

Second, the deformation of the star (in the gauge used) due to the tidal field is encoded in \( \mathcal{Z}_{0}^{\text{tid}} \). It is remarkable that the perturbed matching procedure allows its determination only when \( E(a) \neq 0 \), through (23). This is equivalent to what happens to \( \mathcal{Z}_{0} \) and \( \mathcal{Z}_{2} \) in the rotating star setting, Eqs. (12)
and (19). However, as shown in Reina & Vera (2015), \( \xi \) satisfy the vanishing of the second factor in (23) even when \( E(\alpha) = 0 \) whenever a solution of the problem for all orders of the perturbative expansion exists – this is, in fact, the argument implicitly used in the literature, e.g. Hartle (1967).

It is convenient to define the function \( y := a H_0 / H_0 \) in order to compute the Love numbers. The boundary conditions are directly obtained from (21) and (22) and read

\[
[y] = 4\pi a^2 E(\alpha)/M. \tag{24}
\]

This expression recovers the correction addressed in Damour & Nagar (2009) for homogeneous stars, which is used in Hinderer et al. (2010) for other EOS with nonvanishing energy density at the boundary. The constant ratio \( a_1 := aQ/aP \) of the exterior solution (9) is thus determined from the interior, using (24), by

\[
\begin{aligned}
[a_1] &= -\frac{\partial_r \tilde{P} - (y/a) \tilde{P}}{\partial_r \tilde{Q} - (y/a) \tilde{Q}} \bigg|_{r=a} \\
&= -\frac{\partial_r \tilde{P} - (y/a) \tilde{P} + (4\pi a^2 E(\alpha)/M) \tilde{P}}{\partial_r \tilde{Q} - (y/a) \tilde{Q} + (4\pi a^2 E(\alpha)/M) \tilde{Q}}. \tag{25}
\end{aligned}
\]

We compare the exterior solution (9) with the internally and externally generated parts of the gravitational potential \( W \) defined in the DSX approach (see section IV.C in Damour & Nagar (2009)) in order to relate the constant \( a_1 \) to the tidal Love numbers \( k_1 = \frac{1}{2} \left( \frac{y}{a} \right)^2 \). In the numerical analysis we concentrate on \( l = 2 \) and we shall use instead the quantity \( \lambda_2 := a_2/3 \) (see Yagi & Yunes (2013a)).

5 UNIVERSALITY OF I-LOVE-Q RELATIONS

We turn next to discuss the implications our approach has in the universality of I-Love-Q relations. Let us first define the rotation-independent (and dimensionless) quantity \( \overline{\Delta M} := M^3 \Delta M / \tilde{P}^2 = M^3 \Delta M / \tilde{Q}^2 \). Using the correct (amended) expressions for \( \overline{\Delta M} \) and \( \lambda_2 \) we compute their relation for six different EOS configurations, including two neutron stars and four quark stars. We choose a range of \( \lambda_2 \) between 10\(^{0.7} \) and 10\(^4 \), which comprises a range of TOV configurations with mass \( M_0 \leq M \leq 2.3 M_0 \) (the actual range depends on the particular EOS). The numerical results are summarised in Fig. 2. We find a strong indication of a universal relation between \( \overline{\Delta M} \) and \( \lambda_2 \), as the fit of the numerical data shows (the relative error is displayed in the bottom panel). Such a universal relation is not found for strange stars when using the original (incorrect) version of \( \overline{\Delta M} \), as shown in the inset of the top panel.

Therefore, the right use of the perturbed matching yields the corrections used in Yagi & Yunes (2014) to find universal I-Love-Q relations. Our results show that the second order \( l = 0 \) parameter \( \overline{\Delta M} \) completes the universal relations that involve the first order parameter \( \overline{\tilde{Q}} \), the second order \( l = 2 \) parameter \( \overline{\tilde{Q}} \), and the tidal number \( k_2 \). This can be used to fix the problems inherent, precisely, to the relations involving (only) the latter three parameters. As discussed in Yagi & Yunes (2013a), \( \overline{\tilde{Q}} \) is defined from \( M \), but the relevant observational quantity is the mass \( M_T \) (14). From observables one would need to calculate the corresponding static configuration to find \( M \), so as to make the universal I-Love-Q relations truly useful in observational astrophysics (at least for weak magnetic fields (Haskell et al. 2014)). That procedure is model-dependent and, in this regard, the use of \( M_T \) instead of \( M \) in e.g. \( \overline{\tilde{Q}} \) is claimed in Yagi & Yunes (2013a) to be of little numerical importance. In this work we have shown that the inclusion of \( \overline{\Delta M} \) in the I-Love-Q relations provides, however, a complete set of relations between all perturbation quantities (to this order), which allows to obtain any such quantity from observational input alone.

ACKNOWLEDGEMENTS

We thank Emanuele Berti for suggesting this investigation and Nikolaos Stergioulas for providing the exact data. Work supported by the Spanish MINECO and FEDER (AYA2013-40979-P, AYA2015-66899-C2-1-P, FIS2014-57956-P), the Generalitat Valenciana (PROMETEOII-2014-069, ACIF/2015/216), and the Basque Government (IT-956-16, POS-2016-I-0075).

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