Fermion masses and the symmetry breaking; Strong interactions spin quantum number as the unifier of the strong and electroweak interactions

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Abstract

The origin of the fermion generations is discussed. A strong interactions spin $I_S$ is introduced which unifies the quarks and leptons as two multiplets of this spin. The electroweak vector bosons and gluons emerge as the fused states of these fermion multiplets.

We discussed the possibility of existence of the electroweak spin quantum number $I_W = \frac{1}{2}$ for all fermions besides their space - time spin quantum number $s$ in $\mathbb{H}$. This allowed to regard the electroweak vector bosons as the fused and condensed states of the fermions:

$$ |s_L = \frac{1}{2}, I_W = \frac{1}{2}, m_W = \pm \frac{1}{2} > \times |s_L = \frac{1}{2}, I_W = \frac{1}{2}, m_W = \pm \frac{1}{2} > =
$$

$$|s_L = 1, I_W = 1 > + |s_L = 1, I_W = 0 > \quad (1)$$

Here the index $L$ indicates the left - handedness of the fermions. The electroweak spin triplet states are represented by the triplet of the vector gauge bosons $(W^\mu)_L$ and the singlet state is represented by the vector boson $(B^\mu)_L$. The fusion and condensation of the right - handed fermions leads to the similar set of the electroweak vector bosons.

The states with $s_L = 1$, $(W^\mu)_L$ and $(B^\mu)_L$ couple to the left-handed fermions only and the states with $s_R = 1$, $(W^\mu)_R$ and $(B^\mu)_R$ couple to the right - handed fermions only.

The $SU(2)_L \times SU(2)_R$ symmetry of the fermions breaks when the $(B^\mu)_L$ also couples to the right - handed fermions. The assumption of the proportionality of the $W^\mu$ and $B^\mu$ masses\(^1\) induced by the symmetry breaking to their corresponding coupling strengths

$$\frac{M_W}{M_B} = \frac{g}{g'} \quad (2)$$

leads to the usual definition of the $Z$ boson and to the well - known $M_W - M_Z$ mass relation \([1]\). The fermions gain the masses at this stage also within this model.

The three generations of the fermions could be the result of symmetry breaking in the form of the second order phase transitions. The most general form of the potential energy in $3 + 1$ dimensions is

\(^1\)We will often drop the indices $L$ and $R$ whenever their presence is not important
\[ V(E) = \alpha E^4 + \beta E^3 + \gamma E^2 + \delta E + \eta \quad (3) \]

where \(\alpha, \beta, \gamma, \delta\) and \(\eta\) are the coefficients characterizing the medium. \(E\), which is a scalar quantity, represents the fundamental field mentioned earlier (see [1]). Below the critical point of the second order phase transitions, after the symmetry breaking, this function will have three minima corresponding to three generations of fermions. The further is the minimum from the minimum of the unbroken symmetry (at \(E = 0\)), the more prominent is its broken symmetry nature. Therefore the fermion masses can be considered as the measure of the brokenness of the symmetry of the original fermion states. This possible fermion mass generation mechanism also explains why the fermion masses are not so essential as their space-time spin and the electroweak spin quantum numbers in the fermion-vector boson coupling patterns.

It is natural to assume that, similar to the case with the left and right-handed fermions, matter and antimatter make up two irreducible representations of the fundamental field (each with its own left and right handed fermion representations). Therefore the evenness of \(V(E)\), \(V(E) = V(-E)\) is not required.

Interestingly, this treatment of the symmetry breaking also leads to the possibility of the existence of the second and third generations of the superheavy electroweak vector bosons, provided that they are the fused states of the fermions.

Could the leptons and quarks be different multiplets of another type of spin quantum number? Indeed, if we assign an additional strong interactions spin quantum number \(I_S\) to the fermions, the quarks can be regarded as the triplet \(I_S = 1\) states (quark color states) of this spin and the leptons will correspond to the singlet \(I_S = 0\) state. It is also plausible to assume that in addition to the space-time and the electroweak spin components, fermions also have two additional components, each of which has \(I_S = \frac{1}{2}\) besides other possible quantum numbers\(^2\). The states \(|m_{S1} = \pm \frac{1}{2} > \times |m_{S2} = \pm \frac{1}{2} >\) (with all other quantum numbers the same) can be separated into the strong interactions (color) triplet quarks and the strong interactions singlet leptons. The fact that there are approximately three times more quarks than leptons in nature (in the original state of the stars) is consistent with this construction.

At the stage, when \(W^\mu\) triplet are not coupled to the fermions, the hyper-charge \(Y\) is the only charge of the fermions and their sum is zero for the upper members as well as for the lower members of the electroweak doublets [1]:

\[
Y(\nu) + Y(u) \times 3 = -1 + \frac{1}{3} \times 3 = 0
\]

\[
Y(e) + Y(d) \times 3 = -1 + \frac{1}{3} \times 3 = 0 \quad (4)
\]

\(^2\)Closeness of the space-time and the electroweak spin components is clear from [1] and will be further supported by the discussions in this work, although they are not so close that to have two the same quantum numbers as in the case of the strong interactions components.
In other words, \((\nu, u_r, u_b, u_g)\) and \((e, d_r, d_b, d_g)\) manifest themselves as two different representations (not irreducible) of the strong interactions spin \(I_S\), as expected (the quark indices \(r, b\) and \(g\) stand for the red, blue and green colors). Naturally, this explains the original hypercharge values of the quarks and leptons. It is important to mention that the right-handed fermions and the left-handed fermions have their own individual sets of these representations. At the stage of the \(SU(2)_L \times SU(2)_R\) symmetry breaking, the charges of the left-handed fermions gain a \(\pm \frac{1}{2}\) \((x e)\) contribution due to the coupling to the triplet of the \(W^\mu\) bosons and the charges of the right-handed fermions gain the same contribution due to the \(Y = \pm 1\) couplings of the \((B^\mu)_L\) to these fermions ('+' contributions for the upper members of the doublets and '-' contributions for the lower members of the doublets). With no right-handed fermion doublets around (in terms of the hypercharges the left-handed doublets still exist after the symmetry breaking), the representations of the strong interactions spin now become all four fermions of one generation together, up to the masses of the particles:

\[
Q(\nu) + Q(e) + Q(u) \times 3 + Q(d) \times 3 = 0 - 1 + \frac{2}{3} \times 3 - \frac{1}{3} \times 3 = 0
\]

The representations of the strong interactions spin change after the breaking of the \(SU(2)_L \times SU(2)_R\) symmetry, the strong interactions also experience the consequences of the symmetry breaking.

The lepton-lepton type fusion leads to the electroweak gauge bosons discussed in Eq. (1). The quark-quark type fusion leads to the following states (we omit non-essential indices in this equation, see Eq. (1)):

\[
|m_W = \pm \frac{1}{2}, I_S = 1 > \times |m_W = \pm \frac{1}{2}, I_S = 1 > =

|I_W singlet, I_S singlet > + |I_W triplet, I_S singlet > +

|I_W singlet, I_S octet > + |I_W triplet, I_S octet >
\]

(electroweak spin singlet + electroweak spin triplet) \times (strong interactions spin singlet) states are represented by 4 electroweak interactions vector bosons, (electroweak spin singlet, strong interactions spin octet) states are represented by 8 gluons. As one might expect, the strong interactions octet states, the gluons, do not couple to the strong interactions singlet states, to the leptons. (electroweak spin triplet, strong interactions spin octet) states, the last term in Eq. (1), either do not couple to the fermions (similar to the \((W^\mu)_R\)) or they couple very weakly (in the latter case quite exotic particles, color changing photons would exist).

For the electroweak spin singlet gluons the disappearance of the right-handed doublets after the symmetry breaking is not an obstacle and they couple with equal strength to both left-handed fermions and right-handed fermions. Different from the \(B^\mu\), the strong interactions spin octet gluons also couple with
the equal strength to the upper and lower members of the fermion doublets. These equal gluon couplings, especially the equal left-right couplings, must be the reason for their masslessness after the breaking of the $SU(2)_L \times SU(2)_R$ symmetry. The masslessness of the gluons could well be connected also to the fact that the left-handed and the right-handed fermions still constitute two unmixed representations of the strong interactions spin even after the symmetry breaking. The electroweak interactions bosons couple differently to the left-handed and right-handed fermions and gain mass as a result of the symmetry breaking.

Interestingly, all the vector bosons with $m_W = 0$, gluons, the $Z$ bosons and the photons carry the interactions between the fermions of the same type, between the particles with the same handedness, $m_W$ and generation, whereas the vector bosons with $m_W = \pm 1$, $W^\pm_\mu$ couple the upper and lower members of the fermion doublets which have different values of $m_W$. $W^\pm_\mu$ also induce the intergenerational couplings between the quarks with the differing values of $m_W$. Thus it is the $m_W = 0$ values of the corresponding intermediaries that leads to the flavor conservation in the strong interactions and the absence of the flavor changing neutral electroweak currents.

The fermions of the different values of $m_W$ also have different masses. This indicates to the existence of the connection between the electroweak spin quantum numbers of the particles and their masses. Also, the quarks with the larger $Y_R$ (the index $R$ stands for the right-handedness), $u, c, t$ make up heavier sequence of masses compared to the quarks of the smaller $Y_R$, $d, s, b$ and the leptons with $Y_R = 0$, the neutrinos are massless.

$W^\pm_\mu$ carry the interactions between the fermions of the different $m_W$ and consequently of the different mass. It is the capacity of the vector bosons with $m_W \neq 0$, $W^\pm_\mu$, to couple the fermions of the different masses that produces the intergenerational couplings of the quarks, the Cabibbo mixing of quarks. $W^\pm_\mu$ do not induce intergenerational couplings for the leptons. Perhaps, it is outside their capacity to couple the massive fermion of one generation to the massless fermion of another generation.

**Conclusion**

The three fermion generations could be the reflection of the three minima of the potential energy built out of the fundamental field, emerging after the symmetry breaking. The introduction of the strong interactions spin quantum number allows to consider the strong interactions and the electroweak interactions in a unified form. The electroweak interactions, the strong interactions and the space-time properties of the particles are intricately connected to each other. The separation of these attributes of the particles is always conditional. The masses of the particles, their space-time related quantum number are generated due to the electroweak symmetry breaking. The leptons and quarks can't generate masses through any direct interaction with the strong interactions.

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\textsuperscript{3}The incompatibility of the tendency of the strong interactions spin octet states to couple symmetrically to the fermions with the absence of the right-handed fermion-electroweak triplet state couplings might well be an obstacle for the coupling of the (electroweak spin triplet, strong interactions spin octet) states to the fermions.
be considered as two multiplets of the strong interactions spin. The electroweak
triplet of the vector bosons, $W^\mu$ induce the intergenerational couplings only
between the strongly interacting particles, between the quarks. The particles
with $m_W = 0$, the gluons, the $Z$ bosons and the photons do not couple fermions
of the different generations as well as of the different values of $m_W$.

References

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