Assessment of damage localization based on spatial filters using numerical crack propagation models

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Abstract. This paper is concerned with vibration based structural health monitoring with a focus on non-model based damage localization. The type of damage investigated is cracking of concrete structures due to the loss of prestress. In previous works, an automated method based on spatial filtering techniques applied to large dynamic strain sensor networks has been proposed and tested using data from numerical simulations. In the simulations, simplified representations of cracks (such as a reduced Young’s modulus) have been used. While this gives the general trend for global properties such as eigen frequencies, the change of more local features, such as strains, is not adequately represented. Instead, crack propagation models should be used. In this study, a first attempt is made in this direction for concrete structures (quasi brittle material with softening laws) using crack-band models implemented in the commercial software DIANA. The strategy consists in performing a non-linear computation which leads to cracking of the concrete, followed by a dynamic analysis. The dynamic response is then used as the input to the previously designed damage localization system in order to assess its performances. The approach is illustrated on a simply supported beam modeled with 2D plane stress elements.

1. Introduction
With the aging of critical infrastructures, there is a need for the development of tools to assess in real time their structural condition. As an alternative to the local inspection techniques, global techniques based on vibration measurements have been developed for many years [1]. The final goal is to design robust and fully automated structural health monitoring (SHM) systems based on the dynamic response under operational conditions. The present study focuses on one given type of damage which is thought to be the most common problem encountered in modern concrete structures: local cracks due to prestress losses [2]. An important step in the design of SHM systems is their validation which is usually done, in a first step, using numerical simulations. In a previous study, we have shown the interest of using dynamic strain measurements rather than acceleration measurements for automated data based damage localization [7]. This was illustrated using a simplified model for the cracked region (a reduced Young’s modulus in a given area). While this gives the general trend for global properties such as eigen frequencies, the change of more local features is not accurately represented and these models should not be used with confidence for the design of SHM systems based on dynamic strain measurements. Instead, crack propagation models developed in other scientific communities [3, 4, 5, 6] should be used.
In this paper, we propose to use crack-band type models available in the commercial code TNO DIANA \cite{8} in order to better capture the effect of the crack on the dynamic response. The strategy consists first in computing in DIANA the non-linear response of the progressively cracked structure under increasing quasi-static load followed by a total unloading. The second step is to compute the linear dynamic response under small levels of vibrations (typical of ambient excitations) which do not lead to further damage in the structure. By computing the dynamic signature in such a way, it will be more representative of the dynamic response of a cracked concrete structure than the dynamic signature computed using simplified damage models such as introducing a reduced Young’s modulus in a given area.

The strategy is illustrated for a simply supported concrete beam equipped with a large network of dynamic strain sensors which is gradually damaged using a quasi-static load applied in the middle of the span. A modal analysis is performed in DIANA after the unloading and the results are imported in Matlab. The dynamic response under ambient vibrations is then computed using an in-house code developed under Matlab and used to design an automated damage localization procedure based on local modal filters \cite{10}. A more accurate modeling of the impact of damage on the dynamic signature is thought to be necessary in order to design adequately the SHM system. In particular, sensor placement is of great importance when trying to develop data based methods relying on dynamic strain measurements.

2. Numerical crack propagation model

In this study, concrete cracking is considered in the form of a smeared cracking model \cite{12}. The total strain at each point \( \varepsilon \) is decomposed into the elastic strain \( \varepsilon^e \) and the crack strain \( \varepsilon^{cr} \):

\[
\varepsilon = \varepsilon^e + \varepsilon^{cr}
\]

For the undamaged structure, the strains are elastic (\( \varepsilon^{cr} = 0 \)). Cracking starts when the maximum elastic strain, defined by \( \sigma_{tt}/E \) (where \( \sigma_{tt} \) is the tension cut-off and \( E \) is the Young’s modulus) is exceeded. The cracking initiates in the direction perpendicular to the principal direction corresponding to the maximum principal stress. From this point, the normal \( n \) and tangential \( t \) components of stress and strain are described with respect to the crack orientation (Figure 1). The constitutive relation in the cracked region is given by:

\[
\begin{bmatrix}
\sigma_{nn}^{cr}

\tau_{nt}^{cr}
\end{bmatrix} =
\begin{bmatrix}
D^I_{secant} & 0 \\
0 & D^II
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{nn}^{cr}

\gamma_{nt}^{cr}
\end{bmatrix}
\]

In this study, we adopt a linear tension softening law for \( \sigma_{nn}^{cr} \) (Figure 2) and a constant shear retention law given by:

\[
\tau_{nt}^{cr} = D^II \gamma_{nt}^{cr} = G \frac{\beta}{1 - \beta} \gamma_{nt}^{cr}
\]

Figure 1: Definition of the local coordinates to describe the constitutive relation in the cracked region
In addition to the elastic parameters for the constitutive law, two parameters are needed for the linear tension softening: the tension cut-off $f_t$ and the ultimate normal crack strain $\varepsilon_{nn,u}^r$, and a single parameter $\beta$ is needed for the constant shear retention.

3. Cracking of a simply supported beam using crack propagation models

3.1. Beam cracking due to quasi-static loading

We consider a simply supported beam represented in Figure 3. The beam is made of concrete. The elastic properties are $E = 31\, GPa$, $\nu = 0.15$ and $\rho = 2400\, kg/m^3$. For the tension softening and the shear retention, the parameters are $f_t = 2.4\, MPa$, $\varepsilon_{nn,u}^r = 0.013$ and $\beta = 0.001$. The computations are performed using two dimensional plane stress elements in the finite element code DIANA. The mesh used for the computations is shown in Figure 4.
The beam is loaded with a vertical quasi-static load $F_{\text{stat}}$ from 0 to 250 N, located at the mid span (point A). In order to localize the initiation of the cracking, the two elements at the bottom and at the mid-span of the beam have a reduced tension cut-off ($f_t = 0.24 MPA$, Figure 4). After loading and crack initiation and propagation, the beam is unloaded. The loading applied results in a smeared cracking pattern which develops over six elements in the mesh (Figure 5a). Figure 5b shows the vertical displacement of the point where the force is applied (point A) as a function of the magnitude of the applied load. The graph shows a very weak non-linearity during the loading phase due to the cracking.

3.2. Dynamic response of the cracked beam

In order to test the efficiency of a previously developed methodology for damage localization using large strain sensor networks [10] on the cracked beam described above, it is necessary to compute the dynamic response before and after damage. The beam is excited with a band limited white noise (0-4 kHz) at point B (Figure 3) exciting the first four modes (200 Hz, 790 Hz, 1735 Hz, 2994 Hz) of the structure and the response is measured using fiber optic strain sensors. One optical fiber contains two hundred sensors (one sensor per element in the finite element mesh) which measure the strain in the direction of the fiber. We consider different possible positions for the fiber ($h$) as shown in Figure 6.
The time-domain response is computed using a numerical simulator developed under *Matlab*. The simulator requires the eigen frequencies and mode shapes of the structure in order to compute the time domain dynamic response. A modal analysis is performed in *DIANA* before and after the loading/unloading phase and the results are exported to *Matlab*. The sampling frequency of the measured responses is 10 kHz and the length of each measurement sample is 1 second. Noise is added in the form of an independent white noise on each sensor response:

$$y_n(t) = y_n^0(t) + \beta \left[ \max_t |y_n^0(t)| \right] N(0, 1),$$

where $N(0, 1)$ is a random Gaussian variable with zero-mean and unitary standard deviation, $y_n^0(t)$ is the time-domain response at sensor $n$ without any added noise. In this example, the level of noise is $\beta = 0.05$ (5% noise). The dynamic response projected on the sensors is computed one hundred times for the undamaged structure using different realizations of the input signal (samples 1 to 100) and one hundred times for the damaged structure using other different realizations of the input signal (samples 101 to 200).

As shown in [7], damage localization using strain measurements is more effective when considering the low order mode shapes. In particular, it has been shown, based on a beam model and a simplified modeling of damage, that the first mode shape is very useful for damage localization. Figure 7 shows the first mode shape projected on the strain sensor networks for values of $h = 2.43cm$ (top of the beam), and $h = 0.32cm$ (bottom of the beam). From the figure, it is clear that the effect of damage on the first mode shape is very local, although it is not strictly limited to the damaged area. It is also clear that the damage effect is larger when putting the optical fiber at the bottom of the beam where the damage occurs. The effect is however still easily visible even when the strain sensors are not measuring directly in the damaged area ($h = 2.43cm$). This result is of very important interest for sensor placement when designing an SHM system based on dynamic strain measurements.
4. Spatial filters for damage localization

Based on the measured time-domain responses, a technique for damage localization using local spatial filters is implemented. For more details on this technique, the reader can refer to [9, 10]. The main idea of the technique is described here. First, the network of 200 sensors is divided in six local networks of 33 sensors (Figure 8). For each network, a single modal filter is setup. The principle of modal filtering is to perform a linear combination of the sensor outputs in such a way that some of the modes are filtered out. The idea here is to filter out mode 1 and keep mode 2. When the shape of mode 1 changes, it is not correctly filtered and appears in the output of the modal filter. Based on the power spectral density (PSD) of the output of the modal filter tuned to mode 2, a peak indicator is computed around the frequency of mode 1 in order to detect the reappearance of mode 1. This processing is performed in each local filter resulting in six peak indicators used as features for damage localization. Each feature is monitored independently using $x$-bar type control charts [11].

Figure 8: Definition of the five independent sensor networks used to perform modal filtering

Figure 9 shows the control charts for local filters 1 through 6 ($h = 2.43\text{cm}$). The control charts show a deviation from normal conditions (the feature goes below the lower control limit) in local filters 3 and 4 where the damage is present, demonstrating the efficiency of the SHM system. Similar results have been found for other positions of the optical fiber (different values of $h$). In particular, when the fiber is close to damage, the deviation from normal condition in the control charts is stronger. As stated earlier, it is interesting to note that even when the fiber is not measuring directly in the damaged area (which is the case when $h = 2.43\text{cm}$), damage localization is possible because the strain maps are affected at a certain distance from the damage location. This shows the usefulness of such computations which allow to better design the sensor placement for optimal damage localization.
Figure 9: Control charts for the six local filters showing the correct localization of damage in local filters 3 and 4

5. Conclusion
Many current research activities are focused on the development of vibration-based SHM systems. Before going to costly test campaigns, the proposed systems are usually tested using numerical simulations in which the damage is represented by very simple models, such as a reduced Young’s modulus in a given area in the structure. This modeling of damage is not very representative of the physical reality, especially when the system is relying on dynamic strain measurements. In order to produce numerical results which are closer to the physical reality, we propose to introduce the damage in the structure using non-linear crack propagation models. A quasi-static high-level load is applied to the structure resulting in small progressive damage. The dynamic response is then computed, taking into account the damage accumulated in the structure. This approach is illustrated on a simple example of a concrete beam. Damage is introduced in the mid-span of the beam using a quasi-static load and the dynamic response under ambient vibrations is computed and recorded in a set of 100 samples for the undamaged structure and 100 samples for the damaged structure. An automated damage localization technique based on local spatial filters and a large network of dynamic strain sensors is then applied successfully to locate the damage. This simple example illustrates the interest of precisely modeling the impact of a real crack pattern on the dynamics strain sensors response, especially with respect to the sensor placement issue. The intent is to extend this type of computations to more complicated three-dimensional structures in order to build a data set of virtual measurement campaigns on damaged (cracked) concrete structures which are representative of measurement campaigns.
performed on real damaged concrete structures. The underlying idea is to replace as much as possible costly experimental validation campaigns by much cheaper virtual measurement campaigns.

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References
[1] S.W. Doebling, C.R. Farrar and M.B. Prime 1998, A Summary Review of Vibration-Based Damage Identification Methods, Shock and Vibration Digest 30(2), p91-105
[2] B. Espion, J.-P.Elinck, O.Germain, P.-M.Dubois and R.Dekeyser 2007, On site estimation of the 30 years old composite prebended railway bridge, ridge Engineering Journal (Proceedings of the Institution of Civil Engineers) 160(BE2), p89-98.
[3] Z.P. Bazant and J. Planas 1998, Fracture and Size Effect in Concrete and Other Quasibrittle Materials, CRC Press, Boca Raton and London
[4] G. Pijaudier-Cabot and Z.P. Bazant 1987, Nonlocal Damage Theory, Journal of Engineering Mechanics, ASCE 113(10), p 1512-1533
[5] N. Sukumar, D. L. Chopp, E. Bechet and N. Moes 2008, Three-Dimensional Non-Planar Crack Growth by a Coupled Extended Finite Element and Fast Marching Method, International Journal for Numerical Methods in Engineering 76(5), p 727-748
[6] M. Geers, R. de Borst and R Peerlings 2000, Damage and crack modeling in single-edge and double-edge notched concrete beams, Engineering Fracture Mechanics 65, p 247-261
[7] A. Deraemaeker 2010, On the use of dynamic strains and curvatures for vibration based damage localization, Proc EWSHM 2010, Sorento, Italy.
[8] TNO DIANA http://tnodiana.com/
[9] G. Tondreau and A. Deraemaeker 2010, Vibration based damage localization using multi-scale filters and large strain sensor networks Proc EWSHM 2010, Sorento, Italy.
[10] G. Tondreau and A. Deraemaeker 2010, Damage localization in bridges using multi-scale filters and large strain sensor networks, Proc ISMA 2010, Leuven, Belgium.
[11] D.C. Montgomery 2009, Statistical quality control: a modern introduction, John Wiley and Sons, Inc
[12] R. de Borst, J. J. C. Remmers, A. Needleman and M.A. Abellan 2004, Discrete vs smeared crack models for concrete fracture: bridging the gap Int. J. Num. and Analyt. Meth. in Geomechanics 28, p 583-607