Group decision making-based TODIM under Linguistic Aggregation Majority Additive operator

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Abstract. This paper proposes an extension of TODIM (interactive multi-criteria decision making) under group decision making (GDM) using the Linguistic Aggregation Majority Additive (LAMA) operator. TODIM is an effective method in modelling experts’ psychological behaviour in the decision-making process. However, under the GDM, the method is based solely on the average of all experts’ judgments without any consideration to soft aggregation processes that include majority and/or minority concepts. In this work, the LAMA operator is used to be integrated with TODIM-GDM to aggregate the experts’ opinions with respect to the majority concept under the linguistic domain. This is to provide a greater flexibility in reaching a consensus instead of only considering equally average using the classical averaging operators. Two linguistic representations, namely, symbolic approach and 2-tuple linguistic approach for LAMA operator are proposed to be utilised in the method. A numerical example in investment selection problem is provided to illustrate the applicability of the proposed method. Finally, the comparison of these two linguistic approaches is presented. The results show that LAMA under the 2-tuple linguistic approach is preferable to the symbolic approach in case of there is a tie between alternatives in the final ranking.

1. Introduction

Multi-criteria group decision making (MCGDM) is dealing with the ranking of alternatives. It is based on the preference judgment of experts over a multiple set of criteria. In this article, we focused on the indirect approach (IA) of group decision making (GDM) [1] where the knowledge of each one of experts is combined to form a collective judgment. In real-life, experts often express their opinions in natural language which are fuzzy and vague. After all, the fuzzy multi-criteria group decision making (FMCGDM) was introduced to deal with a possible range of linguistic judgment instead of setting a fixed numerical value. Most of the time, experts consider it more convenient and valid to define their subjective judgments in linguistic or interval scale judgments. There are various methods under fuzzy environment that have been proposed in the literature, see for examples [2-3]. Since then, the FMCGDM methods have gaining tremendous attention in many diverse research fields such as science and technology, engineering, etc. [4-5].

TODIM is one of the well-known decision-making methods and was proposed by Gomez and Lima in 1992 [6]. This method is based on the prospect theory (PT) developed by Kahneman and Tversky [7], where its value function is defined in terms of gains and losses relative to a psychologically neutral reference point. The value function (or known as prospect value function) is S-shaped; concave
in the region of gains above the reference point, and convex in the region of losses [8]. This theory was originated from the behavioural decision theory which formally begun with Edwards [9] and later was improved by Eisenführ et al. [10]. It is considered as having a greater consistency than expected utility theory (EUT) [11] as a psychological theory of preferences under risk. Based on this principal idea, TODIM was developed by considering individual behaviour to calculate the dominance of one alternative over another by establishing the S-shaped value function. Due to its popularity of how people make judgment when facing risk, a lot of research on TODIM method have been done recently [12-13].

In the standard formulation, the TODIM method only deals with crisp numbers. However, it was extended to include uncertainty theories, such as fuzzy numbers [14], probabilistic linguistic [15] and interval-valued intuitionistic fuzzy information [16], to mention a few. One of the sources of uncertainty in decision-making problems is the user provided information. This leads to the use of fuzzy linguistic approach in dealing with inherent imprecision concepts in human reasoning. Under this approach, variables can be quantified by linguistic values. Each linguistic value is characterized by a label and a meaning, where the label is a sentence of a language and the meaning is a fuzzy subset of a universe of discourse. There are several well-known fuzzy linguistic approaches in the literature, such as the symbolic approach [17], the 2-tuple linguistic approach [18], the virtual linguistic approach [19], among others.

In TODIM-GDM methods, the arithmetic mean (AM) and the weighted arithmetic mean (WAM) are among the technically used aggregation operators in determining the representative value of experts as a collective decision. However, these operators are essentially taken as the average of all experts’ judgments without any consideration to the soft aggregation processes that include the majority and/or the minority concept(s). Some studies on the extension of these aggregation operators in GDM have been conducted in [20, 21]. In this work, one of the families of ordered weighted average (OWA) operator under linguistic domain called linguistic aggregation majority additive (LAMA) operator [22] is proposed to be integrated with TODIM-GDM. The main motivation is to provide a greater flexibility in aggregating experts’ judgments with the inclusion of the majority and/or minority concepts, inclusively.

The objectives of this work are two-fold: i) to extend TODIM-GDM under the indirect approach (IA) using LAMA operator, i.e., based on symbolic approach and 2-tuple linguistic approach; and ii) to compare the results of TODIM-GDM method under these two linguistic approaches as well as different values of risk behaviour and convexity/concavity S-shaped function. The proposed method not only provides a flexibility in determining the overall GDM outcome based on the majority/minority concept, but also the inclusion of risk behaviour of experts with respect to gains and losses.

The remaining part of this paper is structured as follows: Section 2 includes the description of the related concepts, definitions, and procedures of TODIM method and LAMA operator. In Section 3, the proposed method TODIM-GDM under LAMA operator for indirect approach is discussed. In Section 4, a numerical example is given to explain the proposed method. Finally, outcome and discussion are given in Section 5 and conclusion in Section 6.

2. Preliminaries
2.1. Generalized TODIM method
The main idea of TODIM is in measuring the dominance degree of each alternative over the remaining ones by the pairwise comparison. This is done with respect to each criterion in accordance with the prospect value function. The formal definition of classical TODIM and its step-by-step algorithm can be referred in [6]. Here, we summarize only the basic feature (i.e., function of dominance degree) of the generalized TODIM as proposed by Llamazares [13]. To do so, we review first the fundamental idea of the discrete decision-making problem.
Assume that the performance of all alternatives with respect to all criteria are known. Given that \( \mathcal{A} = (A_1, A_2, ..., A_m) \) be a finite set of alternatives and \( \mathcal{C} = (C_1, C_2, ..., C_n) \) be a finite set of criteria where \( M \) and \( N \) represent the sets of indices, respectively. Denote \( z_{ij} \in [0,1] \) as the normalized performance value of alternative \( A_i \) with respect to criterion \( C_j \), for all \( i \in M \) and \( j \in N \). The matrix \( Z = [z_{ij}]_{m \times n} \) be the decision matrix. Likewise, denote \( w_j \in [0,1] \) is the weight of criterion \( C_j \), such that \( \mathbf{w} = (w_1, w_2, ..., w_n) \) is a weight vector and \( \sum_{j=1}^{n} w_j = 1 \), for all \( j \in N \). The function of dominance degree of the generalized TODIM method can be defined as the following:

**Definition 1** [13]: Let \( \Phi \) is a function of dominance degree in the generalized TODIM method. For each \( i, h \in M \) and \( j \in N \), the dominance degree of alternative \( A_i \) over alternative \( A_h \) with respect to criterion \( C_j \) is given by the following expression:

\[
\Phi_j(A_i, A_h) = \begin{cases} 
  g_1(w_j) f_1(z_{ij} - z_{jh}) & \text{if } z_{ij} \geq z_{jh} \\
  -g_2(w_j) f_2(z_{jh} - z_{ij}) & \text{if } z_{ij} < z_{jh} 
\end{cases}
\]

\( g_1, g_2 : (0,1) \to (0, +\infty), f_1, f_2 : [0,1] \to [0, +\infty) \) and \( f_1(0) = f_2(0) = 0 \).

Notice that, the general expression is obtained when \( g_1(x) = f_1(x) = f_2(x) = x^\alpha \) and \( g_2(x) = \lambda / x^\alpha \). The parameter \( \alpha \), with \( \alpha \in (0,1) \), is an estimable coefficient determining the convexity/concavity of the function. Correspondingly, the dominance degree of the classical TODIM [6] can be recovered if \( g_1(x) = f_1(x) = f_2(x) = \sqrt{x} \) and \( g_2(x) = \lambda / \sqrt{x} \), such that:

\[
\Phi_j(A_i, A_h) = \begin{cases} 
  \sqrt{w_j(z_{ij} - z_{jh})} & \text{if } z_{ij} \geq z_{jh} \\
  -\lambda \sqrt{w_j(z_{jh} - z_{ij})} & \text{if } z_{ij} < z_{jh} 
\end{cases}
\]

where \( \lambda = 1/\theta \), and \( \theta > 0 \). The parameter \( \theta \) determines the effect of losses. So, this parameter allows experts to rank the alternatives according to gains and losses: for large values of \( \theta \) the best alternatives are those that provide more gains while for small values of \( \theta \) the best alternatives are those provides small losses. Moreover, \( \lambda \) is called the loss aversion/risk behaviour of expert.

Note that, for each \( i \in M \), the value \( \Phi_j(A_i, A_h) \) can be congregated in \( m \times n \) matrix of the form:

\[
\Phi(A_i) = \begin{bmatrix}
\Phi_1(A_i, A_1) & \Phi_2(A_i, A_1) & \cdots & \Phi_n(A_i, A_1) \\
\Phi_1(A_i, A_2) & \Phi_2(A_i, A_2) & \cdots & \Phi_n(A_i, A_2) \\
\vdots & \vdots & \ddots & \vdots \\
\Phi_1(A_i, A_m) & \Phi_2(A_i, A_m) & \cdots & \Phi_n(A_i, A_m)
\end{bmatrix}
\]

Then, for each \( i \in M \), the overall performance of alternative \( A_i \) can be determined with respect to the degrees of dominance \( \Phi_j(A_i, A_h) \) as follows:

\[
\Phi(A_i) = \sum_{h=1}^{m} \sum_{j=1}^{n} \Phi_j(A_i, A_h),
\]

which corresponds to the sum of all the elements of the matrix \( \Phi(A_i) \).
Llamazares [13] has shown that, the generalized TODIM method satisfies the properties of weight consistent and weight monotonicity if \(g_1, g_2, f_1\) and \(f_2\) are nondecreasing functions.

**Property 1:** Let \(F\) be a generalized TODIM method. \(F\) is weight consistent if for each decision problem \((Z, w)\) where there exist two alternatives \(A_p\) and \(A_q\) and two criteria \(C_r\) and \(C_s\) such that \(z_{pr} = z_{qs} \geq z_{ps} = z_{qr}, z_{pj} = z_{qj}\) for all \(j \neq r, s\) and \(z_{hr} = z_{hs}\) for all \(h \neq p, q\), the following condition is satisfied:

\[
w_r \geq w_s \Rightarrow A_p \succeq A_q.
\]  

**Property 2:** Let \(F\) be a generalized TODIM method. \(F\) satisfies weight monotonicity if for each decision problem \((Z, w)\), for each pair of alternatives \(A_p\) and \(A_q\) and each pair of criteria \(C_r\) and \(C_s\) such that \(z_{pr} \geq z_{qr}\) and \(z_{ps} \leq z_{qs}\) and for each normalized weight vector \(w'\) such that \(w'_j = w_j (j \neq r, s), w'_r = w_r + \varepsilon,\) and \(w'_s = w_s - \varepsilon,\) where \(0 < \varepsilon < w_s\), the following condition is satisfied:

\[
A_p \succeq^w A_q \Rightarrow A_p \succeq^{w'} A_q.
\]

2.2. **Linguistic Aggregation Majority Additive (LAMA)**

Ordered weighted average (OWA) operator is a parameterized family of mean-type aggregation operators [23]. It provides a flexibility to aggregate the argument values between two extreme cases, i.e., minimum to maximum. In [24], the majority additive-OWA which is a new family of OWA (i.e., neat-OWA) has been proposed. Then, based on MA-OWA, its linguistic version called as linguistic aggregation MA-OWA (LAMA) has been introduced [22]. This LAMA operator is proposed with respect to two linguistic approaches, namely, the symbolic approach [25] and the 2-tuple linguistic approach [26].

2.2.1 **LAMA operator under the symbolic approach**

The LAMA operator under the symbolic approach can be defined as follows.

**Definition 2** [22]: Let \(p_1, p_2, ..., p_n \in \mathbb{P}\) be a set of linguistic labels such that \(t > 0\) and let \(m_1, m_2, ..., m_n \in \mathbb{N}\) be the frequency or cardinality of the linguistic labels, where \(m_s \leq m_{s+1}\) for all \(1 \leq s \leq p - 1\). The LAMA operator under the symbolic approach is the label \(p_m\) given by:

\[
p_m = LAMA(p_1, m_1, p_2, m_2, ..., p_n, m_n),
\]

\[
p_1 \oplus \kappa_1 \oplus p_2 \oplus \kappa_2 \oplus ... \oplus p_n \oplus \kappa_n
\]

where

\[
\kappa_i = \begin{cases} 
\frac{1}{d_i}, & \text{if } i = 1 \\
\frac{1 - n m_2}{d_i - 1 - n}, & \text{if } i = 2 \\
\frac{1 - n (m_i - 1)}{d_{i-1} - 1 - n}, & \text{if } i > 2 
\end{cases}
\]

with

\[
d_i = \frac{1 - n m_i - 1}{1 - n (i - 1)}
\]
where $\Theta$ is the sum of labels and $\otimes$ is the product of labels by a positive real defined in [25].

2.2.2 LAMA operator under the 2-tuple linguistic approach

The LAMA operator under the 2-tuple linguistic approach can be defined as follows.

**Definition 3** [22]: Let $(\varphi_1, \zeta_1), (\varphi_2, \zeta_2), \ldots, (\varphi_n, \zeta_n)$ be 2-tuple linguistic labels to be aggregated such that $\zeta_j \in [-0.5, 0.5]$, and let $m_1, m_2, \ldots, m_n \in \mathbb{N}$ be the frequency or cardinality of the 2-tuple linguistic labels, where $m_s \leq m_{s+1}$ for all $1 \leq s \leq p - 1$. The LAMA operator under the 2-tuple linguistic approach (denoted as $LAMA^\varphi$) is the label $\varphi_m$ given by:

$$
\varphi_m = LAMA^\varphi((\varphi_1, \zeta_1), (\varphi_2, \zeta_2), \ldots, (\varphi_n, \zeta_n)) = \Delta(\Delta^{-1}(\varphi_1, \zeta_1), \kappa_1 + \Delta^{-1}(\varphi_2, \zeta_2), \kappa_2 + \Delta^{-1}(\varphi_n, \zeta_n), \kappa_n),
$$

where the value of $\kappa_i$ is calculated as in equation (8) and equation (9) using the cardinality or frequency of each label $\varphi_m$.

3. Group Decision Making-based TODIM under LAMA Operator

In this section, the extension of TODIM-GDM based on LAMA operator is presented. As highlighted in the previous section, the indirect approach (IA) of GDM will be implemented here [1]. The LAMA operator is proposed to be used in the experts’ aggregation-stage to provide a flexibility in generating the final group decision. The integration of TODIM-GDM method with LAMA operator under symbolic approach and 2-tuple linguistic approach are presented in following sub-sections.

3.1. TODIM-GDM based on LAMA operator under symbolic approach

Recall that a set of alternatives and a set of criteria are presented as $\mathcal{A} = \{A_1, A_2, \ldots, A_m\}$ and $\mathcal{C} = \{C_1, C_2, \ldots, C_n\}$, respectively. Moreover, a group of experts is given as $\mathcal{E} = \{E_1, E_2, \ldots, E_s\}$. Let $M$, $N$ and $S$ represent the sets of indices, respectively. Step-by-step procedure of this method can be given as follows:

**Step 1**: Each expert, $E_k$, $k \in S$ provides his/her preferences for all alternatives, $A_i$, $i \in M$ with respect to all criteria, $C_j$, $j \in N$ in the form of a decision matrix $Z^k = [z^k_{ij}]_{m \times n}$. The preferences of experts are mainly based on the linguistic labels $\varphi_1, \varphi_2, \ldots, \varphi_n \in \mathbb{P}$.

**Step 2**: Form the majority-group decision matrix, $Z^{maj} = [z^{maj}_{ij}]_{m \times n}$ by aggregating the preferences of all experts, $z^k_{ij}$ $(k = 1, 2, \ldots, s)$ using the LAMA operator as in equations (7) – (9). Note that, here we use $p^{maj}$ notation instead of $\varphi_m$ to represent the majority value in majority-group decision matrix or collective group decision.

**Step 3**: Afterwards, compute the dominance degree of alternatives with respect to each criterion using equation (1) for $g_1(x) = f_1(x) = f_2(x) = x^\alpha$ and $g_2(x) = \lambda/x^\alpha$:

$$
d_i = \begin{cases} 
1 & \text{if } i = 1, n = 1 \\
\frac{n^{m_2}}{m_2 \prod_{j=1}^{n-2}(n-j)^{m_{j+1}}} & \text{if } i = 1, n = 2 \\
\frac{n^{m_2} \prod_{j=1}^{n-2}(n-j)^{m_{j+1}}} & \text{if } i = 1, n > 1, \\
\prod_{j=1}^{n-2}(n-j)^{m_{j+1}} & \text{if } i > 1
\end{cases}
$$
\[ \Phi_j(A_i, A_h) = \begin{cases} 
(w_j(z_{ij}^{maj} - z_{hj}^{maj})^\alpha & \text{if } z_{ij} \geq z_{hj} \\
-\lambda \left(\frac{(z_{hj}^{maj} - z_{ij}^{maj})}{w_j}\right)^\alpha & \text{if } z_{ij} < z_{hj} 
\end{cases} \]  

(11)

where \( \alpha \) represents estimated coefficient for the convexity/concavity of the function and \( \lambda = 1/\theta \) characterizes risk behaviour as \( \lambda < 1 \) (risk averse), \( \lambda = 1 \) (neutral) and \( \lambda > 1 \) (risk taker). Then, construct the decision matrix of dominance degrees as shown in equation (3).

Step 4: For each \( i \in M \), calculate the overall performance of alternative \( A_i \) using equation (4).

3.2. TODIM-GDM based on LAMA operator under 2-tuple linguistic approach

Step-by-step procedure of this method can be given as follows:

Step 1: This step is similar as in the previous sub-section.

Step 2: In this step, the majority-group decision matrix, \( Z^{maj} = [z_{ij}^{maj}]_{m \times n} \) of all experts \( z_{ij}^k \) \((k = 1,2,\ldots,s)\), is derived using the LAMA operator based on 2-tuple linguistic approach. This can be done using the equation (10) and the computation processes are based on equation (8) and equation (9).

Step 3: Compute the dominance degree using the following expression:

\[ \Phi_j^f(A_i, A_h) = \begin{cases} 
(w_j(z_{ij}^{maj} - z_{hj}^{maj})^\alpha & \text{if } z_{ij} \geq z_{hj} \\
-\lambda \left(\frac{(z_{hj}^{maj} - z_{ij}^{maj})}{w_j}\right)^\alpha & \text{if } z_{ij} < z_{hj} 
\end{cases} \]  

(12)

where the operation of \( \Phi_j^f(A_i, A_h) \) is based on the 2-tuple linguistic approach.

Step 4: Similarly, in this step, for each \( i \in M \), calculate the overall performance of alternative \( A_i \) using equation (4).

4. Numerical Example

The proposed method is illustrated in this section by a numerical example. For this purpose, the hypothetical data in [20] on the investment strategy selection problem is used in this work. The aim here is to guide an investor in analysing and selecting the best investment strategy with respect to the judgment provided by a group of experts. The potential alternatives include hedge funds, investment funds, bonds, stocks, and equity derivatives. Five experts are involved to evaluate the alternatives with respect to five criteria: benefits in the short term, benefits in the long term, risk of the investment, socially responsible investment, and difficulty of the investment. To enable the experts in formulating their judgements in a natural way, a set \( \mathbb{P} \) of linguistic labels is supplied. One possibility is by directly supplying the term set by considering all terms distributed on a scale on which a total order is defined. For example, a set of seven terms could be given as follows:

\[ \mathbb{P} = \{p_0 = \text{none}, p_1 = \text{very low}, p_2 = \text{low}, p_3 = \text{medium}, p_4 = \text{high}, p_5 = \text{very high}, p_6 = \text{perfect}\} \]
The related data or judgment provided by all experts is shown in Table 1. Given the weighting vector of criteria as \( \mathbf{w} = (0.2, 0.5, 0.05, 0.10, 0.10) \), then, step-by-step procedure for analysing and deriving the final ranking of alternatives can be presented as the following.

### 4.1. TODIM-GDM based on LAMA under symbolic approach

Step 1: Each expert provides the preferences as in Table 1. At this stage, the preference of each expert is represented in a decision matrix \( \mathbf{Z}^k = [z_{ij}^k]_{5 \times 5} \), where \( k = 1, 2, \ldots, 5 \).

Step 2: Then, form the collective decision matrix by using the LAMA operator. At this stage, the decision matrix \( \mathbf{Z}^{maj} = [z_{ij}^{maj}] \) is derived by aggregating all the individual decision matrix of experts, \( \mathbf{Z}^k = [z_{ij}^k] \). For example, consider the argument values provided by experts \( E_k, k \in S, k = 1, 2, \ldots, 5 \) for alternative \( A_4 \) with respect to criterion \( C_1 \) as follows:

\[
Z_{11}^k = \{p_3, p_2, p_1, p_4, p_4\},
\]

and the number of different labels is 3 and the cardinality is the following:

\[
m_1 = 1(\text{high}), m_2 = 1(\text{medium}), m_3 = 3(\text{low}).
\]

Then, calculate the cardinality for \( C_1 \) with respect to \( A_4 \) using equations (7)-(9) and the collective decision matrix of majority of experts is given in Table 2.

| \( E_1 \) | \( C_1 \) | \( C_2 \) | \( C_3 \) | \( C_4 \) | \( C_5 \) |
|---|---|---|---|---|---|
| \( A_1 \) | \( p_3 \) | \( p_2 \) | \( p_3 \) | \( p_2 \) | \( p_5 \) |
| \( A_2 \) | \( p_4 \) | \( p_6 \) | \( p_1 \) | \( p_6 \) | \( p_2 \) |
| \( A_3 \) | \( p_2 \) | \( p_3 \) | \( p_2 \) | \( p_1 \) | \( p_1 \) |
| \( A_4 \) | \( p_6 \) | \( p_2 \) | \( p_4 \) | \( p_6 \) | \( p_5 \) |
| \( A_5 \) | \( p_2 \) | \( p_6 \) | \( p_3 \) | \( p_4 \) | \( p_5 \) |

| \( E_2 \) | \( C_1 \) | \( C_2 \) | \( C_3 \) | \( C_4 \) | \( C_5 \) |
|---|---|---|---|---|---|
| \( p_2 \) | \( p_5 \) | \( p_6 \) | \( p_5 \) | \( p_5 \) |
| \( p_6 \) | \( p_3 \) | \( p_1 \) | \( p_6 \) | \( p_4 \) |
| \( p_1 \) | \( p_5 \) | \( p_4 \) | \( p_3 \) | \( p_2 \) |
| \( p_5 \) | \( p_1 \) | \( p_3 \) | \( p_6 \) | \( p_5 \) |
| \( p_3 \) | \( p_3 \) | \( p_5 \) | \( p_5 \) | \( p_5 \) |

| \( E_3 \) | \( C_1 \) | \( C_2 \) | \( C_3 \) | \( C_4 \) | \( C_5 \) |
|---|---|---|---|---|---|
| \( A_1 \) | \( p_1 \) | \( p_3 \) | \( p_5 \) | \( p_4 \) | \( p_5 \) |
| \( A_2 \) | \( p_5 \) | \( p_5 \) | \( p_1 \) | \( p_6 \) | \( p_3 \) |
| \( A_3 \) | \( p_4 \) | \( p_4 \) | \( p_3 \) | \( p_3 \) | \( p_2 \) |
| \( A_4 \) | \( p_6 \) | \( p_1 \) | \( p_4 \) | \( p_6 \) | \( p_3 \) |
| \( A_5 \) | \( p_4 \) | \( p_3 \) | \( p_4 \) | \( p_5 \) | \( p_4 \) |

| \( E_4 \) | \( C_1 \) | \( C_2 \) | \( C_3 \) | \( C_4 \) | \( C_5 \) |
|---|---|---|---|---|---|
| \( p_1 \) | \( p_3 \) | \( p_5 \) | \( p_4 \) | \( p_4 \) |
| \( p_5 \) | \( p_3 \) | \( p_2 \) | \( p_5 \) | \( p_2 \) |
| \( p_2 \) | \( p_2 \) | \( p_1 \) | \( p_4 \) | \( p_1 \) |
| \( p_3 \) | \( p_1 \) | \( p_3 \) | \( p_3 \) | \( p_5 \) |
| \( p_2 \) | \( p_2 \) | \( p_3 \) | \( p_4 \) | \( p_5 \) |

| \( E_5 \) | \( C_1 \) | \( C_2 \) | \( C_3 \) | \( C_4 \) | \( C_5 \) |
|---|---|---|---|---|---|
| \( A_1 \) | \( p_1 \) | \( p_2 \) | \( p_3 \) | \( p_2 \) | \( p_4 \) |
| \( A_2 \) | \( p_2 \) | \( p_4 \) | \( p_1 \) | \( p_5 \) | \( p_1 \) |
| \( A_3 \) | \( p_2 \) | \( p_2 \) | \( p_1 \) | \( p_4 \) | \( p_3 \) |
| \( A_4 \) | \( p_4 \) | \( p_4 \) | \( p_2 \) | \( p_5 \) | \( p_4 \) |
| \( A_5 \) | \( p_1 \) | \( p_2 \) | \( p_4 \) | \( p_2 \) | \( p_4 \) |
Table 2. The collective decision matrix of majority of experts

|      | $C_1$  | $C_2$  | $C_3$  | $C_4$  | $C_5$  |
|------|--------|--------|--------|--------|--------|
| $A_1$ | $p_1$  | $p_3$  | $p_4$  | $p_3$  | $p_5$  |
| $A_2$ | $p_5$  | $p_4$  | $p_1$  | $p_6$  | $p_2$  |
| $A_3$ | $p_2$  | $p_3$  | $p_2$  | $p_3$  | $p_2$  |
| $A_4$ | $p_5$  | $p_3$  | $p_3$  | $p_6$  | $p_5$  |
| $A_5$ | $p_2$  | $p_3$  | $p_4$  | $p_4$  | $p_5$  |

Step 3: Calculate the partial dominance of each alternative $A_i$ over each alternative $A_h$, such that $\Phi_{ij}^{maj}(A_i, A_h), i, h = 1, 2, ..., 5$. For example, the partial dominance of the alternative $A_1$ with respect to all criteria $C_j$ is shown in Table 3.

Table 3. The partial dominance of the alternatives $A_1$ with respect to criteria $C_j$

| $\Phi_{1}^{maj}$ | $C_1$  | $C_2$  | $C_3$  | $C_4$  | $C_5$  |
|-------------------|--------|--------|--------|--------|--------|
| $A_1, A_2$        | -0.50  | -0.44  | -0.50  | -0.50  | -0.45  |
| $A_1, A_3$        | -0.24  | 0.07   | -0.44  | -0.10  | -0.50  |
| $A_1, A_4$        | -0.48  | 0.24   | -0.27  | -0.48  | -0.10  |
| $A_1, A_5$        | -0.18  | -0.14  | -0.21  | -0.31  | 0.00   |

Step 4: Calculate the overall dominance degree of matrix $\Phi(A_1)$. In the similar way we can compute the overall dominance degree for the rest of alternatives. Finally, rank the alternatives based on descending order: $A_2 = p_6 (1.00), A_4 = p_3 (0.37), A_3 = p_5 (0.33), A_5 = p_2 (0.19)$ and $A_1 = p_1 (0.00)$.

4.2. TODIM-GDM based on LAMA under 2-tuple linguistic approach

Step 1: For 2-tuple linguistic approach, we need to transform the given data into 2-tuple linguistic representation. For example, $p_3$ is represented as $(p_3, 0)$ and form the decision matrix.

Step 2: Next, aggregate the collective decision matrix by using the LAMA$^2$ operator. We can prevent the losing of information at this stage since the 2-tuple linguistic representation gives the exact aggregation value. The collective decision matrix of majority of experts is given in Table 4.

Table 4. The collective decision matrix of majority of experts

|      | $C_1$  | $C_2$  | $C_3$  | $C_4$  | $C_5$  |
|------|--------|--------|--------|--------|--------|
| $A_1$ | $(p_1, 0.25)$ | $(p_3, -0.22)$ | $(p_4, 0.22)$ | $(p_5, 0.22)$ | $(p_5, -0.13)$ |
| $A_2$ | $(p_5, 0.00)$ | $(p_4, -0.25)$ | $(p_1, 0.06)$ | $(p_6, -0.13)$ | $(p_2, 0.25)$ |
| $A_3$ | $(p_2, 0.08)$ | $(p_3, -0.25)$ | $(p_2, -0.25)$ | $(p_3, 0.33)$ | $(p_2, -0.33)$ |
| $A_4$ | $(p_5, -0.25)$ | $(p_3, -0.50)$ | $(p_3, 0.33)$ | $(p_6, -0.33)$ | $(p_5, -0.25)$ |
| $A_5$ | $(p_2, -0.25)$ | $(p_3, -0.13)$ | $(p_4, -0.33)$ | $(p_4, 0.22)$ | $(p_5, -0.13)$ |
Step 3: Calculate the partial dominance degree of each alternative \(A_i\) over each alternative \(A_h\), \[\Phi^m_{ij}(A_i, A_h), i, h = 1, 2, ..., 5\]. The partial dominance degree of alternatives \(A_i\) based on criteria \(C_j\) can be derived similarly as in Table 3 above.

Step 4: Calculate the overall dominance degree of matrix. Finally, the overall value of each alternative can be obtained and rank the alternative based on the descending order: \(A_2 = (p_6, 0.00); 1.00, A_4 = (p_3, 0.01); 0.28, A_3 = (p_3, 0.02); 0.27, A_5 = (p_2, 0.07); 0.21\) and \(A_1 = (p_1, 0.00); 0.00\).

5. Results and Discussion

The above finding is focused primarily on the risk neutrality of the majority of experts as defined by the value \(\lambda = 1\) and the convexity/concavity of function is set as \(\alpha = 0.5\). Table 5 below shows the results in two different linguistic representations.

| Table 5. The comparison of the proposed method with two linguistic representations |
|-----------------------------------|------------------|------------------|------------------|
|                                   | Symbolic approach | 2-tuple linguistic | Ranking          |
| \(A_1\)                           | \(p_1\)           | \((p_1, 0.00)\)    | 5                |
| \(A_2\)                           | \(p_6\)           | \((p_6, 0.00)\)    | 1                |
| \(A_3\)                           | \(p_3\)           | \((p_3, 0.02)\)    | 3                |
| \(A_4\)                           | \(p_3\)           | \((p_3, 0.01)\)    | 2                |
| \(A_5\)                           | \(p_2\)           | \((p_2, 0.07)\)    | 4                |

Based on the Table 5 above we can observe that the ranking is the same for the two linguistic approaches. \(A_2\) is ranked as the best strategy for investment, which is fund investment, while \(A_4\) is the worst strategy. However, for the 2-tuple linguistic representation, the result is preferable compared to the symbolic approach since it reduces the loss of information in it. Moreover, it can segregate between alternatives in case of tie. For instance, \(A_3\) is preferred than \(A_4\) due to \((p_3, 0.02) > (p_3, 0.01)\).

In this section, an analysis on various values of \(\alpha\) and \(\lambda\) is conducted to observe how these parameters affect the final results and rankings of the proposed method.

| Table 6. The overall results for different values of \(\lambda\) when \(\alpha = 0.5\) |
|---------------------------------|------------|------------|------------|------------|
| Parameter \(\lambda\)           | \(A_1\)   | \(A_2\)   | \(A_3\)   | \(A_4\)   | \(A_5\)   |
| 0.0001                           | 5          | 1          | 2          | 3          | 4          |
| 0.1                              | 5          | 1          | 2          | 3          | 4          |
| 5                                | 5          | 1          | 2          | 3          | 4          |
| 10                               | 5          | 1          | 2          | 3          | 4          |
Table 7. The overall results for different values of $\alpha$ when $\lambda$

| Parameter | 5 | 1 | 2 | 3 | 4 |
|-----------|---|---|---|---|---|
| $\alpha=0.0001$ | | | | | |
| $\alpha=0.5$ | | | | | |
| $\alpha=0.88$ | | | | | |
| $\alpha=1$ | | | | | |

In Table 6, the results of the model for different values of $\lambda$ with respect to a fixed value of $\alpha$ are presented. The ranking is remain the same when value of $\alpha = 0.5$ and $\lambda = 0.88$. The fund investment, $A_2$ is the best strategy for investment. The value of loss-aversion coefficient, means the individuals are more sensitive to losses than gains and a risk taker person. In the positive domain, for a risk averse person, $\lambda \lambda \lambda \lambda \lambda \lambda \lambda \lambda \lambda \lambda \lambda$ will choose for a sure gain over a larger gain that is merely probable. However, in the negative domain, the same effect leads to a risk seeking preference for a loss that is merely probable over a smaller loss that is certain. The same psychological principle also found if the overweighting of risk aversion coefficient in the domain of gains and risk seeking in the domain of losses. Moreover, when we set the $\lambda \lambda \lambda \lambda \lambda \lambda \lambda \lambda \lambda \lambda \lambda$ (linear function in gain and loss region) and $\lambda \lambda \lambda \lambda \lambda \lambda \lambda \lambda \lambda \lambda \lambda$ the ranking slightly changes for the second and third places.

Table 7 above also shown the same ranking for all different values of $\lambda$ when $\lambda$ is fixed. The ranking when value $\lambda \lambda \lambda \lambda \lambda \lambda \lambda \lambda \lambda \lambda \lambda \lambda \lambda \lambda \lambda \lambda \lambda \lambda \lambda \lambda \lambda$ are also remain the same for the best and worst strategy of investment. The value function models psychological processes: the concavity for gains describes a risk aversion attitude, the convexity describes a risk seeking attitude. Based on prospect theory graph, the losses carry more weight than gains are represented by a steeper negative function side. Experimental studies carried out by Tversky and Kahneman [9] reveals median values of $\alpha = 0.88$.

6. Conclusion

This paper proposed the extension of TODIM-GDM based on the LAMA operator. It generalizes the group-stage aggregation process to include the majority-based consensus measure. The integration of TODIM-GDM with LAMA operator under two linguistic approaches, namely, the symbolic approach and the 2-tuple linguistic approach were proposed. The advantages of the proposed method are the flexibility in the aggregation of final consensus measure, besides the inclusion of risk behaviour of group of experts. The analysis has been conducted and the results show the effectiveness of the proposed method. The comparison of the two linguistic approaches has been analysed. Even though both provides the same ranking, however, 2-tuple linguistic approach provided more specificity and precise results compared to symbolic approach. Moreover, analysis on different values of $\lambda$ and showed that the flexibility of expert in making the final decision based on their risk behaviour or risk appetite. Nevertheless, we just limit the scope of this work to the case of equal degree of importance of each expert. For future research, weighted LAMA can be proposed to be integrated with TODIM-GDM model. Besides, further analysis can be conducted with respect to cardinality relevant factor as studied in [20].

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