Unruh effect in a real scalar field with the Higgs potential on a dynamically variable background space-time

Shingo Takeuchi

The Institute for Fundamental Study “The Tah Poe Academia Institute”
Naresuan University Phitsanulok 65000, Thailand

shingo(at)nu.ac.th

Abstract

It is predicted in Refs. [1, 2, 3, 4, 5] that an accelerated electron performs the Brownian motion at the inertial frame. This Brownian motion at the inertial frame roots in the interaction with the thermal excitation given by the Unruh effect at the accelerated frame. If such a prediction is possible, correspondingly we propose a prediction in this study that the thermal radiation is emitted at the inertial frame from an electron warmed due to the Unruh effect at the accelerated frame. The point in our prediction is although the Unruh effect is the one only at the accelerated frame, as well as what the Brownian motion rooted in the Unruh effect appears at the inertial frame, the warm of the particle appears at the inertial frame. In this paper based on such a prediction we investigate phenomena at the neighborhood of an accelerated electron in the inertial frame. The model we consider is the four-dimensional Klein-Gordon real scalar field model with the Higgs potential term at the finite temperature identified with the Unruh temperature on a variable de sitter space-time. Why we take the Higgs potential term is that since the Higgs particle has been detected recently, it can be thought that the potential terms in the realistic models are given by the Higgs potential term. We compute the one-loop effective potential in the inertial frame with the corrections by the thermal radiation rooted in the Unruh effect at the accelerated frame. In this computation we take into account that the background space-time is dynamically varied due to the field theory’s corrected one-loop effective potential. Based on such an analysis, we illustrate the restoration of the spontaneous symmetry breaking and the dynamical variation of the background-space time. Further we examine the accelerated particle’s world-line and the amount of the energy to grow the acceleration at each acceleration.
1 Introduction

The ultra high intensity lasers have been making significant developments recently. Those developments are said to make the experimental confirmations for the theoretically predicted strongly quantum effects possible \[1\]. One of them is the Unruh effect \[6, 7\]. It is a prediction that one moving in the Minkowski space-time with a linear constant acceleration experiences the space-time as a thermal-bath given by the canonical ensemble with the Unruh temperature, \[ T_U = \frac{\hbar a}{2\pi c k_B} \approx 4 \times 10^{-23} \frac{a}{(1 \text{ cm/s}^2)} \text{[K]}, \]
where \( a \) is the acceleration.

In the above situation we think that it is timely and meaningful to investigate the theoretical aspect of the phenomena on the Unruh effect. There are already considerations and attempts to detect the Unruh effect \[1, 2, 3, 4, 5\]. It is the Larmor radiation an accelerated electron emits. Here the Larmor radiation they aim is additional Larmor radiation emitted by the Brownian motion rooted in the thermal-bath given by the Unruh effect at the accelerated frame in addition to the normal Larmor radiation. Such an additional Larmor radiation is called Unruh radiation in those papers.

One of the interesting points in the Unruh radiation is that although the Unruh effect itself is the phenomenon only at the accelerated frame, the Brownian motion rooted in the thermal-bath given by the Unruh effect at the accelerated frame is predicted at the inertial frame, and as a result the Unruh radiation is emitted at the inertial frame. If such a Brownian motion can be predicted, the thermal radiation at the inertial frame that a particle warmed by the Unruh effect at the accelerated frame emits could also be predicted as well.

So we propose such a prediction in this study. The point in our prediction is although the Unruh effect is the one only at the accelerated frame, as well as what the Brownian motion rooted in the Unruh effect appears at the inertial frame, the warm of the particle appears at the inertial frame. We call such a thermal radiation Unruh thermal radiation, and perform this study based on such a Unruh thermal radiation.

Here let us turn to the recent detection of the Higgs particle \[8, 9\]. This leads us to the prediction that the realistic field’s vacuum energies are given by some Higgs potentials. Since the spontaneous symmetry breaking described by the Higgs potentials is restored by the thermal effect \[10\], the Unruh thermal radiation can give rise the restoration of the spontaneous symmetry breaking as well. At that time the vacuum energies with such a thermal collections have an effect on the structure of the space-time. Because the vacuum energies can play the role as the cosmological constant.

So the purpose of this study is to compute the thermal field theory’s vacuum energy with the corrections describing the restoration of the spontaneous symmetry breaking based on our prediction for the Unruh thermal radiation. We perform this computation with taking into account that the background space-time varies dynamically due to the field theory’s corrected vacuum energy. Then based on such an analysis we investigate the phenomena.

Specifically mentioning what we do, first we consider a particle moving with a linear constant acceleration in the four-dimensional Minkowski space-time. Then as mentioned earlier, if the prediction for the Brownian motion rooted in the Unruh effect is possible at
the inertial frame, we propose a prediction in this paper for the Unruh thermal radiation as well. As a result, at the inertial frame the field in the neighborhood at the particle gets warmed and the field’s vacuum energy gets the thermal corrections. The field we consider in this study is a real scalar field described by the Klein-Gordon equation with the Higgs potential term at finite temperature, where the temperature is brought into according to the imaginary time formalism, which is identified with the Unruh temperature. Then we compute the one-loop effective potential with the corrections given by the Unruh thermal radiation, where the corrected one-loop effective potential describes the restoration of the spontaneous symmetry breaking. We perform this computation with taking into account that the background space-time is dynamically varied due to the corrected one-loop effective potential playing a role of the cosmological constant. From the consideration that the space-time should be flat at the zero-temperature, we define the corrected one-loop effective potential so as to vanish at the zero-temperature. Based on such a corrected one-loop effective potential we examine the amount of the energy to grow the acceleration and the accelerated particle’s world-line at each acceleration.

We take up a few basic papers on the spontaneous symmetry breaking and its restoration caused by the thermal effect and Unruh effect. In Ref. [10] the symmetry breaking and restoration in the flat space at finite temperature has been investigated. The restoration of the spontaneous symmetry breaking in a real scalar field in the Rindler space has been investigated [11]. Further the chiral condensation and the quark and diquark condensation in Nambu-Jona-Lasinio model in the Rindler space have been investigated in Ref. [12] and Ref. [13], respectively. There are also papers concluding that the Unruh effect does not give arise the spontaneous symmetry breaking restoration [14, 15] at the Rindler space.

One may notice that the symmetry restorations in Refs. [11, 12, 13] are the ones at the accelerated frame. On the other hand the symmetry restoration in this paper is the one at the inertial frame. Further the phenomena under the situation that the spontaneous symmetry breaking is restoring with dynamically variable background space-times due to the field’s vacuum energy with corrections given by the Unruh thermal radiation have never been investigated so far, and we investigate those in this paper. As for Refs. [14, 15], although opinions on their conclusion seem to vary, their conclusion does not apply to this study. Because the thermal source of ours is not the Unruh effect itself but the warm of the particle at the inertial frame rooted in the Unruh effect.

By the reason that the effective potentials in the realistic fields can be predicted to be some Higgs potentials, there is probability that the phenomena similar with what we investigate in this paper are occurring actually in our world. For this reason this study could be thought to be important and intriguing.

We mention the organization of this paper. In Chapter 2 we give the background space-time and the model in the field theory in this study. In Chapter 3 introducing the finite temperature according to the imaginary formalism, we calculate the one-loop effective potential in the field theory at the finite temperature, and then define it such that it can vanish at zero temperature. In Chapter 4 by regarding the one-loop effective potential obtained in Chapter 3 as the cosmological constant, we give the relation to fix
the background space-time. In Chapter 5, we show the results in this study: the Hubble constant characterizing the variation of the de Sitter space and the variation of the shape of the one-loop effective potential against the temperature, and after that the amount of the energy to grow the temperature at each temperature and the accelerated particle’s world-line with the corrections. In Chapter 6, we remark on this study.

2 The model

We start with the following action:

\[
S^M = \int d^4x \sqrt{-g^M} \left( R^M + \frac{1}{2} \partial_\mu \phi \partial^{\mu} \phi + \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4!} \phi^4 \right)
\]

\[
\equiv \int d^4x \sqrt{-g^M} (R^M + \mathcal{L}^M),
\]

(1)

where \( R \) and \( \phi \) are the scalar curvature and the real scalar field, respectively. \( m^2 \) is some negative constant to have a Higgs potential. In this paper we attach “\( M \)” to the quantities given in the Minkowski space-time, On the other hand we denote the quantities given in the Euclid space as it is without “\( E \)”, when distinguishing is thought to be needed.

It is turned out later that the field’s effective potential plays a role of the positive cosmological constant. Hence we take the four-dimensional de sitter space-time\(^1\)

\[
ds^2_M = dt^2 - e^{2H_M t} \left( dx^2 + dy^2 + dz^2 \right),
\]

(2)

as our background space-time, where the Hubble constant \( H_M \) is not fixed at this stage.

Now we consider a particle with a linear constant acceleration \( a \). Following the Unruh effect’s prediction, the particle experiences the space-time where the particle exists as the thermal-bath with the Unruh temperature,

\[
T_U = \frac{a}{2\pi},
\]

(3)

at the accelerated frame (We use the natural units.). Following our prediction for the Unruh thermal radiation mentioned in Chapter \(^1\), the particle is thought to have some temperature rooted in the Unruh temperature and emit the thermal radiation. The thermal radiation gradually dumps as getting away from the particle due to thermal dissipation in nature, and the radiation range is finite. However in this paper we take the radiation range from zero to infinity for simplicity in the analysis. Further we regard the temperature distribution as uniform in the entire region.

\(^1\) We consider the finite temperature according to the imaginary time formalism. At that time the de Sitter space in the static coordinate, \( ds^2_M = (1 - r^2/\alpha^2) dt^2 - (1 - r^2/\alpha^2)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \), is convenient. Because this is the time-independent metric. (\( \alpha \) is the radius of the de Sitter space in a dimensionally one higher Minkowski space-time in which the de sitter space-time is embedded.) However for a convenience in the actual analysis, we take physically equivalent another coordinate, the flat slicing coordinate \(^2\). Even in this coordinate, we can get a time-independent effective potential as mentioned under eq. (21).
3 One-loop effective potential at finite temperature

We calculate the one-loop effective potential of the real scalar field at finite temperature. To introduce the temperature we perform the Wick rotation $t \to -i \tau$ and put a periodicity with the period $\beta = 1/T$ to the imaginary time direction. Here this $T$ is identified in this study to the Unruh temperature $T_U$ given in eq. (3) as mentioned in the last paragraph of Chapter 2. Then the background geometry (2) is transformed to

$$ds^2_M \to ds^2 = -d\tau^2 - e^{2H\tau} (dx^2 + dy^2 + dz^2)$$

(4)

with $H^M \to iH$, where we have considered that the Hubble constant is described by the usual form using scale factor and its time-derivative. Correspondingly the probability amplitude with the action (1) is changed to the partition function as

$$Z = \int D\phi \exp \left(i \int d^4x \sqrt{-g^M} (R^M + L^M) \right)$$

$\to \int D\phi \exp \left(\int_0^\beta d\tau \int d^3x \sqrt{g} (R + L) \right)$

$$= \exp \left(\int_0^\beta d\tau \int d^3x \sqrt{g} R \right) \cdot \int D\phi \exp \left(\int_0^\beta d\tau \int d^3x \sqrt{g} L \right)$$

$$\equiv \exp \left(-\int_0^\beta d\tau \int d^3x \sqrt{g} (-R) \right) \cdot Z_\phi. \tag{6}$$

We describe the determinant of the metric and the Lagrangian density after the transformation as $g$ and $L$, respectively.

Now we consider the quantum fluctuation of $\phi$ as $\phi = \phi_0 + \delta \phi$, where $\phi_0$ is the scalar field’s condensation independent of the space-time at the symmetry breaking phase and $\delta \phi$ is the quantum fluctuation. Then $Z_\phi$ can be written as

$$Z_\phi = \int D\phi \exp \left(\int_0^\beta d\tau \int d^3x \sqrt{g} L_0 \right)$$

$$= \exp \left(-\int_0^\beta d\tau \int d^3x \sqrt{g} L_0 \right) \cdot \int D(\delta \phi) \exp \left(-\frac{1}{2} \int_0^\beta d\tau \int d^3x \delta \phi G^{-1} \delta \phi \right), \tag{7}$$

where

$$L_0 = \frac{m^2}{2} \phi_0^2 + \frac{\lambda}{4!} \phi_0^4, \tag{8}$$

$$G^{-1} = e^{H\tau} \partial_\tau^2 + \sqrt{g} (M^2 - 3H \partial_\tau - \partial_\tau^2). \tag{9}$$

Here

$$M^2 \equiv m^2 + \frac{\lambda}{2} \phi_0^2. \tag{10}$$
Then integrating out the $\delta \phi$, we obtain

$$Z_{\phi} = \exp \left( - \int_0^\beta d\tau \int d^3 x \sqrt{g} L_0^E \left( - \frac{1}{2} \log \text{Det} G^{-1} \right) \right) \equiv \exp(-\Gamma_E). \quad (11)$$

Performing the Fourier transformation and evaluating the functional determinant, we obtain

$$\Gamma_E = \int_0^\beta d\tau \int d^3 x \sqrt{g} L_0$$

$$+ \frac{1}{2\beta (2\pi)^3} \sum_{n=-\infty}^\infty \int d\tau d^3 k \log \left( e^{H_\tau k^2 + \sqrt{g} (M^2 - 3iH\omega_n + \omega_n^2)} \right)$$

$$\equiv \int d^3 x \cdot V. \quad (12)$$

where $\omega_n \equiv 2\pi n / \beta$ ($n$ is integers). We have one comment. Now we have factored out $\int d^3 x$. If factoring out the volume factor, $\int d^3 x \sqrt{g}$ should be factored out. However our integrand function depends on $\tau$. Because of this, factoring out cannot be done entirely at this moment. However we factor out the rest part, $\int d\tau \sqrt{g}$, under eq.(21).

Now we evaluate the summation of $n$. To this purpose we rewrite the concerned part as

$$\sum_{n=-\infty}^\infty \log \left( e^{H_\tau k^2 + \sqrt{g} (M^2 - 3iH\omega_n + \omega_n^2)} \right)$$

$$= \lim_{n \to \infty} 3H_\tau (2n + 1) + \sum_{n=-\infty}^\infty \log \left( e^{-2H_\tau k^2 + M^2 - 3iH\omega_n + \omega_n^2} \right). \quad (13)$$

Then once performing the derivative as $\partial_\chi \log (\chi^2 - 3iH\omega_n + \omega_n^2)$, where $\chi^2 \equiv e^{-2H_\tau k^2 + M^2}$, we perform the summation. After that we integrate it. As a result we can obtain

$$V = V_0 + V_1, \quad (14)$$

where

$$V_0 = \int_0^\beta d\tau e^{3H_\tau} L_0; \quad (15)$$

$$V_1 = \frac{1}{(2\pi)^2 \beta} \int d\tau dk \sqrt{g} k^2 \log \left( - \sinh \left( \frac{\beta}{4} \left( \sqrt{9H^2 + 4 (e^{-2H_\tau k^2 + M^2}) - 3H} \right) \right) \right.$$

$$\times \sinh \left( \frac{\beta}{4} \left( \sqrt{9H^2 + 4 (e^{-2H_\tau k^2 + M^2}) + 3H} \right) \right)$$

$$\left. + \frac{1}{2(2\pi)^3 \beta} \int d\tau dk^3 \lim_{n \to \infty} 3H_\tau (2n + 1). \quad (16) \right)$$
Performing the rescaling $k \rightarrow e^{H\tau} k$, we can write $V_1$ as
\begin{equation}
V_1 = \frac{1}{(2\pi)^2 \beta} \int d\tau \, dk \sqrt{g} \, k^2 \log \left[ -\sinh\left( \frac{\beta}{4} \left( \sqrt{9H^2 + 4(k^2 + M^2)} - 3H \right) \right) \right.
\times \sinh\left( \frac{\beta}{4} \left( \sqrt{9H^2 + 4(k^2 + M^2)} + 3H \right) \right) \left. \right] 
+ \frac{1}{2\beta (2\pi)^3} \int d\tau \, dk^3 \lim_{n \to \infty} 3H \tau (2n + 1). 
\end{equation}

Push forwarding the calculation further, we can write $V_1$ as
\begin{equation}
V_1 = V_{1,(T=0)} + V_{1,(T>0)},
\end{equation}
where $V_{1,(T=0)}$ and $V_{1,(T>0)}$ are the contributions independent of the temperature and depending on the finite temperature as
\begin{align}
V_{1,(T=0)} &= \frac{1}{4} \int_{0}^{\beta} d\tau \, \sqrt{g} \int \frac{d^3k}{(2\pi)^3} \sqrt{9H^2 + 4(k^2 + M^2)} \label{eq:V1T0} \\
V_{1,(T>0)} &= \frac{1}{(2\pi)^2 \beta} \int_{0}^{\beta} d\tau \, \sqrt{g} \int dk \, k^2 \log \left[ \left( 1 - e^{-\frac{\beta}{2} \left( \sqrt{9H^2 + 4(k^2 + M^2)} - 3H \right) } \right) \right.
\times \left. \left( 1 - e^{-\frac{\beta}{2} \left( \sqrt{9H^2 + 4(k^2 + M^2)} + 3H \right) } \right) \right] + \Omega \label{eq:V1T>0}
\end{align}
and
\begin{equation}
\Omega = \frac{1}{2\beta (2\pi)^3} \int d\tau \, dk^3 \left( \lim_{n \to \infty} 3H \tau (2n + 1) + e^{3H\tau} (-2 \log(2) + i\pi) \right). \label{eq:Omega}
\end{equation}

Now it can be seen that we can factor out as $V \rightarrow \int d\tau \sqrt{g} \cdot V$. By combining this with the factoring out of the space part performed at eq.(12), now we can think that the factoring out of the whole volume factor has been done.

Next let us turn to the squared mass in the effective potential, which is the coefficient of $\phi_0^2$ and currently being given as $m^2$. It is assumed that it should be originally written as $m_{\text{eff}}^2 = m^2 + \delta m^2$. $m_{\text{eff}}^2$ is generally called the squared effective mass and $\delta m^2$ formally represents the contribution from all the thermal loops. (Our expression at the one-loop order is eq.(28).) At this time $M^2$ given at eq.(10) would be changed to $M_{\text{eff}}^2 \equiv m_{\text{eff}}^2 + \frac{3}{2} \phi_0^2$. Correspondingly regarding $M^2$ as $M_{\text{eff}}^2$ in eq.(20), we expand $V_{1,(T>0)}$ around $M_{\text{eff}}^2 = 0$ to the linear order, where both $m_{\text{eff}}^2$ and $\phi_0^2$ composing $M_{\text{eff}}^2$ take small values around the symmetry breaking restoration.

Finally the one-loop effective potential we obtain by the factor out of the above factor out of the volume factor and the linear expansion of $M_{\text{eff}}$ are
\begin{equation}
V = V_0 + V_{1,(T=0)} + V_{1,(T>0)},
\end{equation}
where
\begin{align}
V_0 &= \frac{1}{(2\pi)^2 \beta} \int d\tau \, \sqrt{g} \int \frac{d^3k}{(2\pi)^3} \sqrt{9H^2 + 4(k^2 + M^2)} \label{eq:V0} \\
V_{1,(T=0)} &= \frac{1}{4} \int_{0}^{\beta} d\tau \, \sqrt{g} \int \frac{d^3k}{(2\pi)^3} \sqrt{9H^2 + 4(k^2 + M^2)} \label{eq:V1T0} \\
V_{1,(T>0)} &= \frac{1}{(2\pi)^2 \beta} \int_{0}^{\beta} d\tau \, \sqrt{g} \int \frac{d^3k}{(2\pi)^3} \int d\tau \, dk^3 \lim_{n \to \infty} 3H \tau (2n + 1) + e^{3H\tau} (-2 \log(2) + i\pi) \label{eq:V1T>0}
\end{align}
where now \( Z_\phi = \exp \left( - \int_0^\beta d\tau \int d^3x \sqrt{g} \cdot V \right) \) and

\[
V_0 = \mathcal{L}_0,
\]

\[
V_{1, (T=0)} = \frac{1}{4} \int \frac{d^3k}{(2\pi)^3} \sqrt{9H^2 + 4 (k^2 + M_{\text{eff}}^2)},
\]

\[
V_{1, (T>0)} = \frac{1}{(2\pi)^2} \int dk \ k^2 \left\{ \frac{1}{\beta} \log \left[ \left( 1 - e^{-\frac{2}{\beta} (\sqrt{9H^2 + 4k^2} - 3H)} \right) \left( 1 - e^{-\frac{2}{\beta} (\sqrt{9H^2 + 4k^2} + 3H)} \right) \right] + \frac{1}{\sqrt{9H^2 + 4k^2}} \left( e_{\frac{2}{\beta} (\sqrt{9H^2 + 4k^2} + 3H)} - 1\right) \right\} + \Omega.
\]

and

\[
\Omega = \frac{1}{2\beta (2\pi)^3} \int dk^3 \left( \frac{1}{V_\tau} \int_0^\beta d\tau \lim_{n \to \infty} 3H \tau (2n + 1) - 2 \log(2) + i\pi \right).
\]

Here \( V_\tau \equiv \int_0^\beta d\tau \sqrt{g} = (-1 + e^{H \beta}) / 3H \). The above can agree to the one in the flat space at \( H = 0 \). It can be seen from the order-counting that eq. (24) diverges at the ultraviolet region. On the other hand eq. (25) does not diverge.

Now we fix the \( m_{\text{eff}}^2 = m^2 + \delta m^2 \) at the one-loop order. \( \delta m^2 \) is fixed such that it cancels out the actually calculated loop contributions in the effective potentials such that the coefficient of \( \phi_0^2 \) can be written as \( m_{\text{eff}}^2 \) finally. Specifically it is turned out that \( \delta m^2 \) is given using the coefficient of \( M_{\text{eff}}^2 \) in \( V_{1, (T>0)} \) given at eq. (27), and the result is

\[
m_{\text{eff}}^2 = m^2 + \frac{\lambda}{(2\pi)^2} \int dk \ k^2 \sqrt{9H^2 + 4k^2} \left( 1 - e_{\frac{2}{\beta} (\sqrt{9H^2 + 4k^2} + 3H)} \right) + \frac{1}{e_{\frac{2}{\beta} (\sqrt{9H^2 + 4k^2} - 3H)} - 1}.
\]

Now we define the effective potential for our study. From the phenomenological’s point of view we think that the space-time should be flat at \( T = 0 \). For this reason we set our one-loop effective potential such that it can be zero when \( T = 0 \) at the symmetry breaking phase. Writing such an effective potential as \( \Delta V(\beta) \), we set \( \Delta V(\beta) \) as

\[
\Delta V(\beta) \equiv V(\beta) - C = V_0(\phi) - V_0(\phi_0) + V_{1, (T>0)},
\]

by subtracting a constant \( C \equiv V_0(\phi_0) + V_{1, (T=0)} + \Omega \) from \( V(\beta) \). Correspondingly \( Z_\phi \) is changed as follows:

\[
Z_\phi = \exp \left( - \int_0^\beta d\tau \int d^3x \sqrt{g} \cdot V(\beta) \right) \rightarrow \exp \left( - \int_0^\beta d\tau \int d^3x \sqrt{g} \cdot \Delta V(\beta) \right).
\]

Eq. (25) can be regarded as the result in the high temperature expansion by the condition \( |M_{\text{eff}}/T| \ll 1 \). Eq. (25) at \( H = 0 \) can be written as

\[
V_{1, (T>0)}|_{H=0} = -\frac{\pi^2}{90 \beta^4} + \frac{M_{\text{eff}}^2}{24 \beta^2}.
\]

This is the well-known result in the high temperature expansion in the flat space.
We can see that $\Omega$ given in eq. (26) is diverging. In this paper we just ignore $\Omega$ in $\Delta V(\beta)$.

We have a comment. We have performed the expansion of $M^2_{\text{eff}}$ to the linear order in the above. Generally the high temperature expansion is the expansion with regard to the ratio of $M_{\text{eff}}/T$. Since our situation that $M_{\text{eff}}$ is smaller can be regarded as the situation that $T$ is higher in the ratio of $M_{\text{eff}}/T$. Because of this our expansion is equal to the high temperature expansion.

Now we consider the contribution from the functional integral measure. It can be known that the functional integral measure can be considered to be written as $D\phi = c\beta d\phi$, where $c$ is some dimensionless number.\(^3\) As a result the contribution from the functional integral measure to the effective potential can be written by the following way,

$$Z_\phi = \int D\phi \exp \left( - \int_0^\beta d\tau \int d^3 x \sqrt{g} \cdot \mathcal{L} \right)$$

$$= \int d\phi c\beta \exp \left( - \int_0^\beta d\tau \int d^3 x \sqrt{g} \cdot \mathcal{L} \right)$$

$$= \exp \left( - \int_0^\beta d\tau \int d^3 x \sqrt{g} \cdot \left( V(\beta) - \frac{1}{\text{Vol.}} (\log \beta + \log c) \right) \right), \quad (30)$$

where $\text{Vol.} \equiv \int_0^\beta d\tau \int d^3 x \sqrt{g}$. We can see that the contribution from the measure does not contribute due to the infinite contribution of $\text{Vol}$. Consequently it is turned out that we do not need to take into account the contribution from the functional integral measure.

Finally the effective potential we take is eq. (28) and our $Z_\phi$ can be written as

$$Z_\phi = \exp \left( - \int_0^\beta d\tau \int d^3 x \sqrt{g} \cdot \Delta V(\beta) \right). \quad (31)$$

4 Fixing the background space-time

In this chapter we obtain the equation to determine the background geometry by regarding the effective potential as the positive cosmological constant. Now our total partition function can be written as

$$Z = \exp \left( - \int_0^\beta d\tau \int d^3 x \sqrt{g} \cdot \left( - R + \Delta V(\beta) \right) \right). \quad (32)$$

\(^3\) Generally the dimension of the real scalar fields counted by the mass dimension is $\frac{(n-2)}{2}$, where $n$ is the space-time’s dimension. Since $n$ is 4 in this study, it can be seen that $[\phi] = [M^1]$. Integral measures in path-integrals for partition functions are dimensionless as $\text{dim}[D\phi] = \text{dim}[M d\phi] = 0$, where $D\phi$ is functional integral measures and $M$ is factor parts. The dimension of inverse temperatures is $[\beta] = [M^{-1}]$. Hence the functional integral measure can be written as $D\phi = c\beta d\phi$, where $c$ is some dimensionless number.
Aside from our story, we describe the Wick rotation to a Einstein-Hilbert action as

$$iS^M = \int d^4x \sqrt{-g^M} \left( R^M - 2\Lambda^M \right)$$

$$\rightarrow \int d\tau \int d^3x \sqrt{g} \left( R - 2\Lambda \right)$$

$$= - \int d\tau \int d^3x \sqrt{g} \left( -R + 2\Lambda \right) = -S,$$  \hspace{1cm} (33)

Further we can confirm that the Einstein equation in the Euclid de Sitter space \([1]\) can be satisfied when $$\Lambda = -3H^2$$.

Hence comparing eq. (32) and eq. (33), we can have the following equation:

$$6H^2 = -\Delta V(\beta)\big|_{\phi=\phi_0},$$

with $$\partial (\Delta V(\beta))/\partial \phi = 0$$ to determine the condensation $$\phi_0$$ at the spontaneous symmetry breaking phase. So actually we have two unknown variables, $$H$$ and $$\phi_0$$, and we have two equations. Solving those simultaneously, we determine the Hubble constant $$H$$ which characterizes the background space-time metric \([1]\).

5 Space-time, energy for growing acceleration and particle’s world-line

Solving eq. (35) numerically, we obtain the results: The effective potential with the background space’s contribution and the Hubble constant $$H$$, against the temperature. Based on those results we perform evaluation of the accelerated particle’s world-line and the amount of the energy to grow the temperature at the each temperature. All the temperature in the results are thought as the Unruh temperature \([3]\).

In all the results the red results represent the results obtained by solving for the two variables $$H$$ and $$\phi_0$$. On the other hand the blue results represent the results obtained by solving only for $$\phi_0$$ with fixing $$H$$ to zero. So the blue points are the results in the flat space.

In all the results the parameters $$(m^2, \lambda)$$ are always taken to be $$(−1, 1)$$. At that time it is turned out that the critical temperature $$T_c$$ for the symmetry breaking restoration is given as $$T_c \approx 4.402482673$$. Here we determine $$T_c$$ from whether the values of the squared effective mass $$m_{\text{eff}}^2$$ \([28]\) and the condensation $$\phi$$ are numerically zero or not.

5.1 Effective potential and space-time

First let us consider to obtain $$H$$ against $$T$$. Then the points in Fig. 1 are the calculated results for the Hubble constant $$H$$, and dotted line is a fitting result. The fitting function we have adopted is $$c_1 T + c_2 T^2$$, where $$c_1$$ and $$c_2$$ are the fitting parameters. These are fixed to $$(c_1, c_2) = (-0.0943597, 0.173624)$$ in our fitting.

Now we think that we have obtained the Hubble constant depending on the temperature. Further we can see that as $$T$$ gets higher, $$H$$ grows in Fig. 1. Hence we can see that as the temperature gets higher the background space transforms to the de Sitter space.
It turned out that the scalar curvature $R$ is given by $-12H^2$ on the background space eq.(4). Substituting $H$ into it we can evaluate the background’s contribution in the effective potential. By this we can obtain Fig.2 in which the effective potential with the background space’s contribution is plotted. We can see from Fig.3 that the background space's effect lowers the critical temperature. We show Fig.3 (the numerical results of the squared effective mass $m_{\text{eff}}^2$ and the field’s condensation $|\phi_0|$) to support this.

Here it would be noteworthy to note that the Lagrangian density $L$ is expanded originally as $L \sim \sum_{n=0}^{\infty} \delta\phi^{2n}$ before integrating out $\delta\phi$. As the temperature increases, since the concaves in the effective potential’s profile become locally shallow, the quantum fluctuation can grow. As a result the higher order terms in the expansion becomes less able to be disregarded at higher temperatures. The profiles shown in Fig.2 are the results obtained from the expansion to the one-loop order. For this reason, properly speaking, there might be some corrections in the shape as getting away from the vacuum (expansion point) in higher temperature results. Furthermore we should notice that we have performed the expansion equivalent to the high temperature expansion as mentioned under eq.(28). So one of things we can say is that our analysis at the neighborhood of the vacuum in the higher temperature region satisfying $|M_{\text{eff}}/T| \ll 1$ is precise.

![Figure 1: The Hubble constant $H$ defined in eq.(2) against the temperature $T$. The dotted line is the fitting result. We describe the fitting function and the fitting result at the beginning of Chapter 5.1](image)

5.2 Energy for growing acceleration and particle’s world-line

The effective potential we have computed can be regarded as a free energy. Hence we can read out the entropy density according to the general relation $S = -\partial F/\partial T$ ($S$ and $F$ are general entropy densities and free energies). Actually we have done it by computing the temperature difference numerically at several temperatures. Those results are shown in Fig.4.

The entropy density in Fig.4 can be interpreted as the amount of the energy to grow the unit temperature at each temperature. The temperature in this study are thought to be proportional to the acceleration through the Unruh effect (3).
Figure 2: The effective potential $\Delta V$ defined at eq. (28) against the condensation value $\phi$ at various $T$. The red lines represent the results with the background space’s contribution ($H \neq 0$), and the blue lines represent the results without the background space’s contribution ($H = 0$). The same kind of line is used for the results computed at same temperatures.

Figure 3: The squared effective mass $m_{\text{eff}}^2$ in eq. (28) and the condensation $|\phi_0|$ against the temperature $T$.

Hence we get a conclusion that the amount of the energy to further grow the acceleration at each acceleration is proportional to the entropy density at each temperature that the acceleration corresponds. Further we can see from Fig. 4 that the higher the acceleration gets, the more the amount of the energy is required.

Next we turn to the issue of the accelerated particle’s world-line. In our study the space-time at the neighborhood of the accelerated particle is predicted to be the de sitter. Hence it is considered that the accelerated particle’s world-line gets the correction compared with the case of the flat space.

To investigate these, representing the position of the particle as $r^\mu(\tau)$ (\tau is the proper
Figure 4: The entropy density against the temperature $T$. These are obtained by calculating $\partial F/\partial T$ numerically where $F = -R + \Delta V$ in Fig. 2.

(time), we solve the geodesic equation on the Minkowski de sitter (2) with a force added for the uniform acceleration of the particle, $F^\mu = ma(r^{1'}(\tau), r^{0'}(\tau), 0, 0)$. Concretely,

$$e^{2H \tau} H (r^{1'})^2 + r^{0''} = F^0,$$
$$2H r^{0'} + r^{1''} = F^1,$$

(36) (37)

where we have confined the motion of the particle to $(0, 1)$-plane. $H$ is assumed to have the temperature dependence obtained from the fitting in the previous chapter 5.1 and $a$ is proportional to $T$ through the Unruh effect (3). We solve the above numerically with the boundary condition:

$$(r^0(0), r^{0'}(0)) = (0, 1) \quad \text{and} \quad (r^1(0), r^{1'}(0)) = (0, 0)$$

(38)

at several $T$. As a result we obtain the results shown in Fig. 5.

We can see that the Unruh thermal radiation has the effect to diminish the coming particle’s acceleration while grow the separating particle’s acceleration. (The particle’s acceleration can be read out from the gradients in Fig. 5)

6 Remark

In this paper corresponding to the prediction in Refs. [1, 2, 3, 4, 5] that an accelerated particle performs the Brownian motion at the inertial frame rooted in the Unruh effect at the accelerated frame, we have proposed a prediction that thermal radiation rooted in the Unruh effect is emitted from an accelerated particle at the inertial frame. We have named such a radiation Unruh thermal radiation.

Then we have computed the one-loop effective potential at finite temperature with taking into account that the background space-time is dynamically varied from the flat to the de Sitter due to the field theory’s corrected one-loop effective potential. The temperature is thought to be proportional to the Unruh temperature. In this paper we
Figure 5: The world-Lines of the uniformly accelerated particle at various $T$. The red lines represent the results with the background space's contribution ($H \neq 0$), and the blue lines represent the results without the background space's contribution ($H = 0$). The same kind of line is used for the results computed at same temperatures.

have simply thought that the temperature is the Unruh temperature. The model we have considered has been a real scalar field described by the four-dimensional Klein-Gordon model with the Higgs potential term at the finite temperature.

As for the technical things, we touch on a problem thought to be noticed particularly. It is the region where the Unruh thermal radiation can reach and the temperature distribution in that region. We have taken the radial direction in that region from zero to infinity, and regarded that the temperature distribution is uniform in the entire region. However it should be finite and the temperature distribution dumps as getting away from the particle in nature. But if these were taken in, it is expected that the factoring out of the volume factor would become less able to be performed. As a result the analysis would become very complicated. In our future work we are going to treat these issues more precisely with taking into account the thermal dissipation.

As for our results, when the dynamically variable background space-time is taken, it has been turned out that the critical temperature becomes smaller than the case that the background space-time is fixed to the static flat. Further we have revealed that the amount of the energy for growing the acceleration at each acceleration increases as the acceleration gets higher and the correction in the world-line of the accelerated particle.

It would be intriguing to examine how such results turn out in the future process that we develop the model to more realistic ones, and to confirm whether the obtained tendencies are consistent or not with the experimental observations. Furthermore the Unruh effect has close relation with the Hawking temperature. The Hawking temperature at the moment is a result in the semi-classical gravity, and it is no surprise if it gets some corrections in the full quantum gravity. Also in this sense the future development of this study would be intriguing.
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References

[1] P.G. Thirolf, D. Habs, A. Henig, D. Jung, D. Kiefer, C. Lang, J. Schreiber, C. Maia, G. Schaller, R. Schutzhold, and T. Tajima “Signatures of the Unruh effect via high-power, short-pulse lasers,” Eur. Phys. J. D 55, 379-389 (2009).

[2] D. J. Raine, D. W. Sciama and P. G. Grove, “Does a uniformly accelerated quantum oscillator radiate?,” Proc. R. Soc. Lond. A (1991) 435, 205-215

[3] A. Raval, B. L. Hu and J. Anglin, “Stochastic theory of accelerated detectors in a quantum field,” Phys. Rev. D 53, 7003 (1996) [gr-qc/9510002].

[4] P. Chen and T. Tajima, “Testing Unruh radiation with ultraintense lasers,” Phys. Rev. Lett. 83, 256 (1999).

[5] S. Iso, Y. Yamamoto and S. Zhang, “Stochastic Analysis of an Accelerated Charged Particle - Transverse Fluctuations -,” Phys. Rev. D 84, 025005 (2011) [arXiv:1011.4191 [hep-th]].

[6] P. C. W. Davies, “Scalar particle production in Schwarzschild and Rindler metrics,” J. Phys. A 8, 609 (1975).

[7] W. G. Unruh, “Notes on black hole evaporation,” Phys. Rev. D 14, 870 (1976).

[8] G. Aad et al. [ATLAS Collaboration], “Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC,” Phys. Lett. B 716, 1 (2012) [arXiv:1207.7214 [hep-ex]].

[9] S. Chatrchyan et al. [CMS Collaboration], “Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC,” Phys. Lett. B 716, 30 (2012) [arXiv:1207.7235 [hep-ex]].

[10] L. Dolan and R. Jackiw, “Symmetry Behavior at Finite Temperature,” Phys. Rev. D 9, 3320 (1974).

[11] P. Castorina and M. Finocchiaro, “Symmetry Restoration By Acceleration,” Physics 3, 1703 (2012) [arXiv:1207.3677 [hep-th]].

[12] T. Ohsaku, “Dynamical chiral symmetry breaking and its restoration for an accelerated observer,” Phys. Lett. B 599, 102 (2004) [hep-th/0407067].
[13] D. Ebert and V. C. Zhukovsky, “Restoration of Dynamically Broken Chiral and Color Symmetries for an Accelerated Observer,” Phys. Lett. B 645, 267 (2007) [hep-th/0612009].

[14] W. G. Unruh and N. Weiss, “Acceleration Radiation in Interacting Field Theories,” Phys. Rev. D 29, 1656 (1984).

[15] C. T. Hill, “Can the Hawking Effect Thaw a Broken Symmetry?,” Phys. Lett. B 155, 343 (1985).