Shannon entropy as a measure of structure in optical beams

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While information is ubiquitously generated, shared, and analyzed in a modern-day life, there is still some controversy around the ways to assess the amount and quality of information inside a noisy channel. A number of theoretical approaches based on, e.g., conditional Shannon entropy and Fisher information have been developed, along with some experimental validations. Some of these approaches are limited to a certain alphabet, while others tend to fall short when considering optical beams with non-trivial wavefront topology, such as orbital momentum laser modes. Here, we introduce the concept of expressing information as a measure of structure in a laser beam. We propose a new definition of classical Shannon information via the Wigner distribution function, while respecting the Heisenberg inequality. Following this, we calculate the amount of information in a Gaussian, Hermite-Gaussian, and Laguerre-Gaussian laser modes in juxtaposition and experimentally validate it by reconstruction of the Wigner distribution function from the intensity distribution of structured laser beams. We experimentally demonstrate measuring structure of the laser beams in singular optics to assess the amount of contained information. Given the generality of this approach of defining information via analyzing the beam complexity is applicable to laser modes of any topology that can be described by ‘well-behaved’ functions. Classical Shannon information defined in this way is detached from a particular alphabet, i.e. communication scheme, and scales with the structural complexity of the system. Such a synergy between the Wigner distribution function encompassing the information in both real and reciprocal space, and information being a measure of disorder, can contribute into future coherent detection algorithms and remote sensing.

I. INTRODUCTION

An electromagnetic field is a fundamental physical carrier of information. It is capable of reliably transmitting a modulated signal and collecting information about the propagation channel itself. With relevance to this IT-driven age, the two longstanding goals in information processing are: (i) achieving higher channel capacity (i.e. throughput), and (ii) (pre)processing of collected information. However, the fundamental challenge of rigorous qualification and quantification of information of EM waves still remains a topic of debate including both physical and even semi-philosophical notions. Information theory it stands out from most of other approaches in physics. Being a higher level of abstraction, information theory focuses on a configuration of a system under consideration in the context of its prehistory, similarly to Thermodynamics, without attachment to a particular class of objects under study in its axiomatics. Out of the broad scope of studies where information theory has a potential to contribute, at this point we exemplify explore its applications to Electrical Engineering, and Signal processing.

One way of defining information is associating it with the presence of distinctive features. For instance, human speech can carry up to 16,000 distinctive features, only a small subset of which is realized in any particular known language. The amount of information a human can transmit per unit sentence containing a fixed number of words, is indirectly correlated with the amount of distinctive features the language can handle (i.e. language capacity). Translating this into optics, it has been understood that monochromatic plane-wave photon in free space can carry a rather limited amount of distinctive features (i.e. polarization and wavelength), providing up to one bit of information per photon. Naturally, if one generates photons with up to one bit of information, one is also able to detect only 1-bit information per unit carrier. To use an analogy; consider an marine biologist casting a fishing net with two inches wide meshes for exploring the life on the ocean, naturally one should not be surprised finding only sea-creature larger than two inches long [1]. This stumbling block has been shifted with the seminal work by Allen et.al [2] where it was experimentally confirmed that laser beams are capable of carrying a well-defined orbital angular momentum (OAM). An ability of such laser modes to carry theoretically unbounded amount of information per photon, e.g. [3], dramatically expands the EM-field’s “language capacity”.

There are several approaches to assess a signal’s information capacity developed in modern information theory, e.g. [4, 5]. In many cases, information is defined with respect to a particular alphabet, giving up the generality offered by statistics in the foundation of information theory. Several groups used conditional information approach to qualify the signal capacity [5].

Here, we introduce the concept of expressing information as a measure of structure in a physical system by applying the Shannon information theory to singular optical beams. We show how the Wigner distribution function (WDF) can be taken as a corresponding
probability distribution function accounting for partial quantumness of the topologically shaped photon source. The important synergy between a comprehensive description of physical systems in their phase-space, delivered by the WDF, and the generalized axiomatics of the information theory, have the potential to conduct to a cumulative approach to high-information-density telecommunications and adaptive signal processing techniques. We validate this theoretical framework by experimentally showing how increased structural complexity of wavefront-shaped optical beams, such as Hermite-Gauss (HG) modes and optical vortices [7,8], can be analyzed using wavefront sensors.

Due to the wide scope of this topic, we refer the reader to a short literature review, offered in the supplementary materials.

II. THE WDF AND CLASSICAL INFORMATION IN OPTICS

The WDF belongs to the generalized Cohen’s class of dual-domain distributions. It is simultaneously the most complete analytical description of an optical beam, and an observable that can be experimentally measured. It provides access to the spatial beam profile and its Fourier transform. The WDF can be defined as

$$W(x,k) = \frac{1}{2\pi} \int dy \ u(x + \frac{y}{2}) u^*(x - \frac{y}{2}) e^{-ikyx}$$  \hspace{1cm} (1)

where $x$ is the variable in the coordinate space, and $k$ is the corresponding coordinate in reciprocal space. The following integrals have a probabilistic interpretation:

$$|u(k)|^2 = \int dx \ W(x,k);$$  \hspace{1cm} (2)

$$|u(x)|^2 = \int dk \ W(x,k);$$  \hspace{1cm} (3)

$$U_{tot} = \int dx \ dk \ W(x,k);$$  \hspace{1cm} (4)

where $|u(k)|^2$ is the momentum distribution; $|u(x)|^2$ is the intensity distribution, and $U_{tot}$ is the total energy of the incoming signal. For a fully coherent light source, the WDF’s Fourier-transformed function $\Gamma(x) = u(x + a)u^*(x - a)$ is known as the mutual intensity used in wavefront sensing for turbulence analysis and adaptive detection techniques. For brevity, a list of useful optical properties of the WDF can be found elsewhere, e.g. [9, 10].

Now, let us consider a Gaussian beam, expressed in the following form [7]:

$$u_G(\rho) = \frac{A}{w} e^{-\frac{\rho^2}{w^2}} e^{i\frac{k_R^2}{4}}$$  \hspace{1cm} (5)

where $\rho > 0$ is the position-vector in the beam profile $\rho = \sqrt{x^2 + y^2}$, $w = w(z)$ is a beam waist, $R = R(z)$ is the radius of curvature of the beam’s wavefront, and $A$ is the normalization constant. The wave-number $k = \{k_x, k_y, k_z\}$ is distinctive from the Fourier transform parameter $k = \{k_x, k_y\}$ in [1]. The corresponding 1D WDF is

$$W^{(G)}(\xi, k_x) = \frac{A}{\sqrt{2\pi}} e^{-\frac{x^2 - k_x^2}{2}}$$  \hspace{1cm} (6)

Here $\xi = \sqrt{2}x/w$ and $k_x$ is the redefined momentum:

$$k_x = \frac{w}{\sqrt{2}} \left( \frac{k_x - k_x}{R} \right)$$  \hspace{1cm} (7)

This Wigner distribution is properly normalized, delivering the beam intensity distribution when integrated over the entire momentum space.

Classical Shannon information, see [12], represents the amount of structure in the corresponding system. Its analog for an optical mode, characterised by its WDF, is:

$$S = -\int_{\mathbb{R}^2} dr \ dp \ W(r,p) \cdot \ln[W(r,p)] \hspace{1cm} (8)$$

This definition of information applied to characterizing optical fields, while having been suggested in the field of optics earlier [ref], has yet to be applied to topological optical beams, to the best of the authors knowledge as introduced here. The main problem with this definition, eqn. (8), is that the WDF is not a positive-semidefinite function and, hence, does not represent a proper distribution. Negativity of the WDF can be interpreted as a marker of a phase-space interference and non-classicality [13]. However, in quantum descriptions interference is associated with a local violation of Heisenberg inequality, e.g. [14]. For these reasons, and to keep the formalism applicable in our experiment, we seek a definition that would comply with the Heisenberg inequality.

Here we propose to replace the regular product in the definition of information [8] with the Groenewold associative product, and hereby call it Shannon-Groenewold...
FIG. 2: The Shannon-Groenewold information $S_{WG}$ as a function of total energy $U_{tot}$ for HG$_0$/Gauss mode (black), HG$_1$ (dashed-yellow) and HG$_2$ (dotted-green). One can see the overall tendency for the amount of information to increase with growth of the overall complexity of the corresponding optical signal.

Information:

$$\tilde{S} = -\frac{1}{(\pi\hbar)^2} \int_{\mathbb{R}^2} dr \, dp \, W(r, p) \ast \ln[W(r, p)]$$

(9)

where $W(r, p)$ is the four-dimensional (4D) WDF and $\ast$ is the Groenewold’s associative product [15]. A manifestation of this definition is the Weyl quantization of a classical observable in phase-space [16]. It respects the Heisenberg uncertainty principle and is normalized to the phase space volume. As an entropic observable, it measures the structure present in the optical field. In the context of Information Theory, it may be interpreted as a measure of the amount of classical information that can be extracted from the laser mode if all the uncertainty is removed by an appropriate experiment.

Using the Shannon-Groenewold information (9) we obtain the following expression for the information in the Gaussian beam:

$$\tilde{S} \simeq -U_{tot} \left[ \ln \left( \frac{U_{tot}}{\pi} \right) - 1 \right]$$

(10)

where $U_{tot} = A\sqrt{\pi/2}$ is the total EM-energy of the beam per spatial degree of freedom. We assume that the radius of curvature of the beam wavefront is always larger than the physical dimensions of the beam spot size: $R \gg \{x, y\}$. It is important to note that for the Gaussian mode both definitions (8) and (9) produce the same result. This is what one would expect since the WDF of a pure Gaussian source is positive-semidefinite and can be interpreted as a classical proper distribution function [17].

III. HIGHER ORDER MODES AND SHANNON-GROENEWOLD INFORMATION

The HG mode is a higher-order solution of the Gaussian family of beam-like solutions of the paraxial equation. It carries a non-trivial beam geometry with singularities, stable on propagation [18] due to being topologically protected. Hence, it is only logical to expect that the amount of information for this family of modes is higher than for the zero-order Gauss modes $\tilde{S}_G < \tilde{S}_{HG}$.

Let us investigate this statement next.

The general solution of the 1D paraxial wave equation in cylindrical coordinates is given by

$$u_{HG}(x, z) = \sqrt{\frac{A}{w}} H_m \left( \frac{\sqrt{2} x}{w} \right) e^{-x^2/w^2} d^{k_x} x^2/2R$$

(11)

The normalization is consistent with the one in eqn. (5). The WDF for this mode is known, e.g. [19]. In our case, to keep the normalization consistent, we obtain:

$$W^{(HG)}_m(x, k_x) = \frac{A\sqrt{\pi}}{(2\pi)^{3/2}} e^{-x^2} \times$$

$$\times \int_{-\infty}^{\infty} dy e^{ik_x y} \frac{H_m(x + \frac{y}{2}) H_m(x - \frac{y}{2})}{x^2/2R}$$

(12)

As expected, for $m = 0$, the WDF of HG signals is exactly equal to the Gaussian WDF, eqn. (5), and so is the
corresponding information in eqn. (10). Then, the first two higher order WDFs have ‘elegant’ analytical expressions:

\[
W_1^{(HG)}(x,k_x) = \frac{2A}{\sqrt{2\pi}} e^{-x^2-\kappa^2} H_1(\bar{x}^2 + \kappa^2 - \frac{1}{8}); \tag{13}
\]
\[
W_2^{(HG)}(x,k_x) = \frac{4A}{\sqrt{2\pi}} e^{-x^2-\kappa^2} H_2(\bar{x}^2 + \kappa^2 - 1). \tag{14}
\]

In a similar manner one can work out higher order modes using the integral form in eqn. (12).

Next, we explore these WDF expressions further; we find that these WDFs take negative values in the central region of the phase-space (Fig. 1). The fundamental statement of classical information theory that “the more we know about a system’s parameter space, the less is its uncertainty” is inevitably broken in the quantum context when considering correlated (conjugate) variables, such as position \(x\) and momentum \(p\). If such “quantumness” is present in the corresponding PDF, it pushes the distribution into the negative domain. This makes coherent detection (simultaneous intensity and phase) for the purpose of the WDF reconstruction problematic, since negative values are non-detectable. Fortunately, the problem can be worked around by introducing Weyl-Wigner quantization in the definition of information (9):

\[
\bar{S}_1^{(HG)} \simeq -2U_{1tot} \left[ \ln \left( \frac{2U_{1tot}}{\pi} \right) - 3 \right]; \tag{15}
\]
\[
\bar{S}_2^{(HG)} \simeq -8U_{2tot} \left[ \ln \left( \frac{4U_{2tot}}{\pi} \right) - 5 \right]. \tag{16}
\]

The classical assessment of the amount of order in the optical mode is clearly increasing with the increasing complexity of the beam profile, see Fig. 2 as one would expect from general considerations. It is important to recall that in the context of physical meaning of Shannon unconditional information, the WDF is normalised to \(U_{1tot} = \text{total energy}\). Consequently, constant \(A\) is bounded from above by \(\sqrt{2/\pi}\).

As HG modes form a complete orthonormal set, they can be used as an expansion basis [20]. Hence, this approach can be straightforwardly applied to OAM modes, such as Laguerre-Gauss (LG), e.g. [7]. Let us express the LG mode as follows:

\[
\mu_{LC}(\rho, z) = \frac{A}{w} \left( \sqrt{2\rho} \right) L_p^{(1)}(\sqrt{2\rho^2}) e^{-\rho^2/w^2} e^{j\rho^2/2R} e^{j\phi} \tag{17}
\]

where \(\phi\) is the vorticity of the twisted mode; \(\phi\) is the azimuthal angle in cylindrical coordinates, where the remaining terms follow the definitions of Gauss [5] and HG [11] modes. Supplied the results from [12], we can express LG-modes in terms of HG-modes with a straightforward calculation; for instance, for 1D-LG WDF with \(p = 0\) and \(\ell = 1\):

\[
W_1^{(LG)}(x,k_x) = \frac{A^2}{4\pi} e^{-x^2-\kappa^2} \left( 2H_1(\bar{x}^2 + \kappa^2 - \frac{1}{8}) - 1 \right) \tag{18}
\]

where \(p\) and \(\ell\) are correspondingly the order- and degree - numbers of the generalized Laguerre polynomial \(L_p^{(\ell)}(\cdot)\). The corresponding entropy can be calculated in a closed analytical form. With this theoretical framework, we are ready to perform an experiment to explore the possibility of obtaining the amount of information by measuring the structure of in optical laser beams of various topology.

![FIG. 4: Measuring the amount of information in optical beams.](image-url)
Among the tools of adaptive optics, Shack-Hartman sensors (SHS) \(^{21}\) occupy a unique place as a fast, affordable, compact off-the-shelf tool for simultaneous intensity and angular distribution measurements. Advanced techniques for SHS state tomography \(^{22}\) and WDF reconstruction \(^{23}\) have been suggested alongside with conventional aberration correction techniques. Measurements of the wavefront distortions in EM beams with non-trivial topology are also of interest for both communication and sensing purposes, e.g. \(^{24}\).

For the aim of this work, we use the SHS to reconstruct the WDF for the purpose of discriminating between the modes, assessing the beam quality and, ultimately, information. In our experiment, see Fig. 3, we measure the Gaussian intensity distribution in the superpixel array of the SHS and compare it to the modelled intensity distribution based on the theoretical WDF calculation, whose approximation can be modelled as \(^{25}\):

\[
I(r) = \frac{1}{\lambda f} \sum_{m=-M}^{L=M} \text{SWDF}[W_b, W_a](r'_{\ell,m}, u'_{\ell,m}) \text{rect}(r'_{\ell,m})
\]

(19)

The smooth WDF is defined following \(^{25}\):

\[
\text{SWDF}[W_b, W_a](r'_{\ell,m}, u'_{\ell,m}) = \iint d^2R d^2U \times W_b(R, U)W_a(R - r'_{\ell,m}, U - u'_{\ell,m})
\]

(20)

where the coordinate shift is defined as

\[
r'_{\ell,m} = \{R_x - \ell \omega, R_y - mw\}
\]

(21)

\[
u'_{\ell,m} = \{U_x - \frac{x - \ell \omega}{\lambda f}, U_y - \frac{y - mw}{\lambda f}\}
\]

(22)

The functions \(W_b\) and \(W_a\) are the WDF of the incoming signal, e.g. \(^{5,13,14}\) or \(^{18}\), and the transmission function of a single lens aperture correspondingly. The parameters \(f\) - focal length, \(w\) - width of a single lens in a lenset array are the parameters specific to the detector and defining the angles in the local wavefront of the field.

Using this model, we simulated the synthetic data sets for Gauss and HG modes, and compare and contrast the models with the collected real data from the SHS. In the model we considered the WDF \(W_b\) in the following general form:

\[
W(\tilde{x}, \kappa_x) = \frac{A}{\sqrt{2\pi}} e^{-\frac{x^2 - \kappa_x^2}{2}} P(\tilde{x}^2 + \kappa_x^2)
\]

(23)

where \(P(\alpha)\) is the polynomial of \(\alpha\). This model includes all the orders of the HG-mode \(^{12}\).

We started by testing two cases, namely, a Gauss mode \(^{6}\) and a LG of order 1 \(^{18}\). The polynomial fit in eqn. \(^{23}\) was taken to be

\[
P(\tilde{x}^2 + \kappa_x^2) = a + b(\tilde{x}^2 + \kappa_x^2) - 1/2
\]

(24)

with \(a\) and \(b\) being the fitting parameters (Fig. 4). One can see that when \(a = 1.5\) and \(b = 0\) the fit corresponds to a Gaussian profile \(^{6}\) and when \(b/a = 2\) the mode is LG$_1$-like \(^{18}\). Hence, this fit can discriminate between the two modes and, hence, provide the corresponding WDF. To assess the quality of the model, besides estimating the \(\chi^2\) per each fit, we run a simulation with 1000 fits to synthetic data with, e.g. \(^{19}\), obtaining a histogram of the deviation between the supplied fitting parameters and those from the best-fit (Fig. 5). The resulting data show that the parameters are centered near the “true” values, showing the satisfactory quality of the fit.

These results can be used to assess the information stored in a physical channel and to compare them to theoretical curves (Fig. 2). As our simulation software is under development, we see assessing the information being one of the future goals of this project. As for applications, the full 2D model can also provide information about the medium the beam interacted with, that can be useful in remote sensing. Based on the results of Sec. \[\[\]\] in the course of future research, we expect topological beams to outperform the modes with planar phase structure for two main reasons: 1. greater library of non-trivial signatures in the original beam profile; 2. reported robustness and self-healing properties of vortex modes.

V. DISCUSSION AND SUMMARY

Interestingly, while information technologies have seen outstanding progress over the last century which
lead to the digital revolution and created flourishing businesses, the field of information theory has remained being relatively under-explored. We believe, that a universal technique to assess both the quality and quantity of information in a received signal, if proven, could provide a conceptually novel tool to physicists and engineers alike. The approach described in this work is by far not the first attempt, neither is it the most general. However, in this approach, classical information does not require early choice of a communication scheme (i.e. alphabet). It is rather based on a fundamental assessment of an optical system’s capability to carry information, based on its overall complexity. The WDF is uniquely used here as a probability density function for Shannon information in optics. The constraints of probability theory on the definition of information and of quantum mechanics on conjugate observables are satisfied working around the properties of the WDF – a pseudo-probability distribution.

We foresee the relevance of this formalism in the context of recent developments for: (i) free-space information processing optics [26]; (ii) integrated photonics-based information processing [27] such as neural network-based accelerators [28] and photonic tensor cores [29]; (iii) adaptive sensing [30]; and (iv) analog optical and photonic processors [31–33]. As the data compression coefficient is naturally bounded by Shannon information, carried by the beam [34], this work indirectly points towards higher information capacity in beams with non-trivial structure, like HG, LG, Bessel-Gauss modes etc. Furthermore, due to the intrinsic connection between optical information theory and computer vision [35–36], our approach may end up being a source of new applications and tools in engineering designs of future generations of artificial intelligence.

Due to the WDF’s relation to the EM-field correlation function, we foresee our approach to be extremely useful in adaptive optics. The reconstruction algorithm, when fully developed, has the potential to characterize the effects of decoherence in turbulent media, the 2D ambiguity function, time-resolved frequency distribution, alongside with commonly available corrections for aberration, astigmatism, peak-valley and rms deformation provided by the SHS measurements. The WDF formalism uniquely gives access to such characteristics as mutual intensity of stochastic wave fields, which is of high importance when describing partially coherent sources. These also have a potential to contribute in the fields of artificial intelligence through holography, optical encryption [37], and free-space communication [38].

In perspective, as the demand on high-speed data transfer and streaming grows exponentially, ADSL and fiber-to-home technologies are less and less likely to satisfy even an average consumer’s data hunger, not to mention business and government agencies calls. These, together with the recent advents in optical processing [26], micro- and nanofabrication [39], and OAM communications [40] put forward the mid-20th century’s excitement around free-space communications in a new light. The new-generation free-space links will require coherent detection techniques to realise their potential to the fullest. We believe that our approach may result in a better understanding of which types of measurements and device architectures are needed to efficiently mine information from a free-space link.

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