On possible composite structure of scalar fields in expanding universe

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Abstract Scalar fields in curved backgrounds are assumed to be composite objects. As an example realizing such a possibility we consider a model of the massless tensor field $l_{\mu\nu}(x)$ in a 4-dim. background $g_{\mu\nu}(x)$ with spontaneously broken Weyl and scale symmetries. It is shown that the potential of $l_{\mu\nu}$, represented by a scalar quartic polynomial, has the degenerate extremal described by the composite Nambu–Goldstone scalar boson $\phi(x) := g^\mu\nu l_{\mu\nu}$. Removal of the degeneracy shows that $\phi$ acquires a non-zero vev $\langle \phi \rangle_0 = \mu$ which, together with the free parameters of the potential, defines the cosmological constant. The latter is zero for a certain choice of the parameters.

Our universe expands and this process has two certain periods. The inflation period was very short and lasted from $10^{-36}$ to $10^{-32}$ s after the Big Bang [1–4]. The period of accelerated expansion began eight billion years later and is continuing on now. Both are conjectured as caused by hypothetic quantum fields with special properties of their vacua. Inflation posits inflaton – a scalar field with the specific shape of its potential which provides a non-zero vacuum expectation value for inflaton. At the end of the slow-rolling regime inflaton reaches the bottom of its potential. This evolution picture matches the exponentially expanding de Sitter universe and the Standard Model [5], ([6] and refs. there). The assumed slow-rolling constraint requires fine-tuning of the initial data with the postulates of quantum field theory and general relativity. Although the inflation predicts and explains many observational data it is not known yet what powered it.

The period of accelerated expansion is explained by the presence of dark energy which is homogeneously distributed across all space-time. In the period of early universe the effect of dark energy was negligibly small but its effect becomes dominant at very long distances [7–10]. Dark energy creates a repulsive force which dominates the attractive gravitational force of visible and dark matter responsible for decelerated expansion of the universe. It is believed that dark energy is bound up with the cosmological constant encoding vacuum energy of the space-time [11–13]. A hard problem here is a huge discrepancy between the experimental value of the cosmological constant and vacuum energy predicted by quantum field theory [14–16]. It was proposed to treat dark energy as a vacuum energy called quintessence that originated from potential energy of very light dynamical particles [17–19]. They are associated with a scalar field creating the fifth repulsive force that causes the accelerated expansion [20–22]. The latter is generally predicted to be slightly less than the one given by the cosmological constant. A special case is phantom energy which is the quintessence energy characterized by the density increasing in time. Such a case may cause the expansion that exceeds the speed of light resulting in improbable scenario in which all interactions vanish through a finite time [23]. Modified gravity models suppose alternative way which does not need to use the dark energy image [24–28]. It is amazing that for each of the complicated evolutional periods on different energy scales, the use of scalar fields such as inflaton or quintessence boson has been successful in getting closer to understanding of the dynamics of expanded universe. A key element in this picture seems to be connected with the conception of spontaneously broken symmetries caused by degeneration of vacua that trigger creation of the Nambu–Goldstone (N–G) bosons. Scalar bosons were also predicted by the Standard Model and string theory, but the Higgs boson was the only fundamental scalar field observed at the current energies of the LHC. This boson is responsible for the electroweak symmetry breaking. So, it was natural to identify the Higgs boson with the inflaton [29,30]. However, it soon became clear that this assumption violated the perturbative unitarity condition [31–33]. Attempts to overcome this problem showed [34] that the Higgs boson alone was not able...
to yield the dynamics mediated by the inflaton without adding new degrees of freedom [35]. Such scalar DOF arise in the supergravity model unifying the Volkov–Akulov [36] and a chiral superfield [37]. A wider set of new DOF described by the massless chiral $N' = 1 \ D = 4$ and $N' = 1 \ D = 10$ multiplets ($s, s + 1/2$) with spins ($s = 0, 1/2, 1, 3/2, 2, \ldots$) emerges from the $\theta$-twistor description [38–42]. Spontaneous breaking of the local supersymmetry points to degeneration among the extremals of the effective quartic Higgs potential in supergravity models [43]. A hierarchy between the Planck mass, cosmological constant and electroweak scale was explained by this degeneration. Detection of new DOF in accelerators and cosmic experiments on TEV scales may lead to new physics outside the SM. Scalar fields which arise in phenomenological models of systems with spontaneously broken symmetries identified with the (pseudo)N–G bosons [44–50]. Axion and $\pi$-mesons give well-known examples of such bosons arising due to spontaneous breaking of the approximate $U(1)_{\text{axial}}$ and chiral symmetries, where $\pi$-mesons are considered as composed by $q\bar{q}$ pairs. This hints that scalar fields in cosmology may also have a composite structure formed by the metric field $g_{\mu\nu}(x)$ of a curved background and hypothetical tensor fields. The latter may originate from quark–gluon plasma and have their own internal multiquark structure described a global symmetry $G$. Study of tensor models invariant under the group $G$ and diffeomorphisms may help, in particular, to understand the nature of the inflaton and dark matter. On this way it’s interesting to find such nonlinear interaction potentials of the tensors that would have nontrivial extremals. The latter together with the equations of motion (EOM) could indicate on the presence of spontaneous breaking of the symmetry group $G$. We suppose that the extremals of the potential are expressed in terms of covariant effective fields composed of the metric and tensor fields. Then the corresponding condensates of the tensor fields will encode effective modes of excitations of quark–gluon plasma. Cooling of plasma rebuilds the tensors of the previous stage due to changes of thermodynamic scales. This deforms the interaction potential and its extremals defining non-zero vev of the composite fields. If these fields appear in the form of diffeomorphism-invariant combinations of $g_{\mu\nu}$ and tensors, then they can be seen as composite scalars. An example which sustains the above scenario in a 4-dim. background is proposed here. For this purpose we use the known idea to treat the global scale symmetry of our universe as spontaneously broken symmetry [51–55]. It is in agreement with the experimental data of Planck [9] and other high precision experiments.

In previous known models the scale symmetry breaking is achieved by using a scalar field $\phi$ of the Brans–Dicke type that is understood to represent an elementary particle with zero spin. The corresponding potential $V \propto \lambda \phi^4$ which is added in a gravity action quadratic in curvature. Here we study an action invariant under the global Weyl and scale symmetries that includes a symmetric massless tensor $l_{\mu\nu}$ instead of the scalar $\phi$. In the particle physics such a tensor is understood as an elementary particle carrying any spin from the set $s = 2, 1, 0$. The proposed model reveals that $l_{\mu\nu}$ encodes a composite scalar field carrying spin zero. It is notable that this result holds for the general polynomial potential admissible with the invariance of the action under diffeomorphisms and scale transformations.

Therefore, the presence in the proposed model of two massless symmetric tensors $l_{\mu\nu}$ and $g_{\mu\nu}$ does not break the well known no-go theorem in the theory of gravity that forbids the presence of multiple massless particles with spins equal to 2. A hint of this result follows also from the existence of a natural geometric scalar $\phi = g^{\mu\nu}l_{\mu\nu}$, where the tensor field encodes the degrees of freedom of an evolutionary period of the expanding universe. The potential of $l_{\mu\nu}$ consistent with the symmetries of the studied action is given by the general homogenous quartic polynomial containing a few dimensionless parameters, that generalizes the $\lambda\phi^4$ potential. Study of critical points of the potential of $l_{\mu\nu}$ reveals its non-trivial extremal $l_{0\mu\nu}(x) = (\phi_0(x)/4)g_{\mu\nu}$, where $\phi_0 = l_{0\mu\nu}g^{\mu\nu}$. This extremal preserves all symmetries of the action and forms an infinitely degenerate vacuum manifold.1 Substitution of $l_{0\mu\nu}$ into the EOM for $l_{\mu\nu}$ shows the presence of the particular solution $\phi_0 = \mu = \text{constant}$. This solution removes infinite degeneration of the vacuum manifold described by the order parameter $\phi_0(x)$ forming a linear representation of the Weyl and scale groups. The removal of this degeneration is realized by choosing a fixed value of the function $\phi_0(x)$ equal to $\phi_0$. This choice is interpreted as fixing the vacuum state. Pictorially this fixing is represented by using two different subscripts “0” and “o”. Thereby it is realized spontaneous breakdown of the scale symmetry, where $\phi(x)$ turns out to be the composite N–G boson having its vev $\langle \phi(x) \rangle_0 = \mu$. The free constant $\mu$ introduces the characteristic mass scale $\mu$ which also defines the vev of $l_{\mu\nu}$ given by the formula $\langle l_{\mu\nu}(x) \rangle_0 = (\mu/4)g_{\mu\nu}(x)$.

At the broken vacuum state marked by $\phi_0$ the potential becomes a constant and we identify it with the cosmological constant. As a result, we get the opportunity to control the value of the cosmological constant by varying free dimensionless parameters of the potential. Using this possibility we find such two- and four-parametric potentials for which the cosmological constant becomes equal to zero.

The sections of the paper are distributed as follows. In Sect. 1 we present the action, the potential of $l_{\mu\nu}(x)$ in the background with a metric $g_{\mu\nu}(x)$ and define the action symmetries. In order not to overload the paper we restrict ourselves to the case, where $g_{\mu\nu}$ is considered as an external

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1 Assumption that the parameters of spontaneously symmetry breaking can be higher rank tensors was previously pointed out in [56, 57].
gravitational field. In Sect. 2 the EOM and its representation including the Riemann tensor $R_{\mu\nu\rho\sigma}$ of the background are derived. In Sect. 3 spontaneous emergence of the composite scalar field $\phi(x)$ is detected. In Sect. 4 the non-trivial extremal of the two-parametric potential is found out. It is shown that on this extremal the action reduces to the scale and diff invariant action of the massless $\phi^4$-theory in the gravitational background with the coupling constant equal to $\frac{\sqrt{2}}{2\pi}(\alpha - \beta)$, where $\alpha$ and $\beta$ are free parameters. Revealed in Sect. 5 is spontaneous breaking of the Weyl and scale symmetries resulting in the cosmological constant equal to $\lambda = \frac{\sqrt{2}}{2\pi}(\phi_0)^4$. This value is equal to the potential of the vacuum solution breaking the Weyl and scale symmetries. In Sect. 6 it is shown that the extremal of a four-parametric potential coincides with the found extremal of the two-parametric potential. This result shows that spontaneous breakdown of the Weyl and scale symmetries takes place for the general polynomial potential in four-dimensional gravitational backgrounds. The obtained results and further prospects are discussed in Summary.

1 Weyl and scale invariant action in a background

Let us consider the generally covariant action for a massless symmetric tensor field $l_{\mu\nu}(x^\rho)$ in a 4-dim. curved background with the metric tensor $g_{\mu\nu}(x)$

$$S = \frac{1}{k^2} \int d^4x \sqrt{|g|} \left(\frac{1}{2} \nabla_\mu l_{\nu\rho} \nabla^{\mu} l^{\nu\rho} - \nabla^{\mu} l^{\nu\rho} + U(g, l)\right), \tag{1}$$

where the brackets $[\mu\nu]$, $\{\mu\nu\}$ imply the symmetrization and antisymmetrization ($\mu\nu \pm \nu\mu$), respectively [58]. As already noted in the Introduction and will be proved below, the presence of two massless tensor fields in (1) will not violate the no-go theorem discussed there. One can choose the scalar potential $U$ as the sum of homogenous quartic polynomials invariant under diffeomorphisms with free dimensionless phenomenological constants $\alpha, \beta, b_2, b_4, b_4'$

$$U = \frac{2}{3} \alpha Tr l Tr(l^2) - \frac{1}{2} \beta (Tr(l^2))^2 + b_2 Tr(l^2) (Trl)^2 + b_4 (Trl)^4 + b_4' Tr(l^4). \tag{2}$$

The invariants $Tr(l^2)$ are formed by covariant $g$-contractions of n symmetric tensors $l_{\mu
u}$

$$Tr l = l_{\mu\nu} g^{\alpha\beta}, \quad Tr(l^2) = l_{\mu\nu} l^{\mu\nu}, \quad Tr(l^3) = l_{\mu\rho l^{\nu\rho} l_\nu^{\mu}}, \quad Tr(l^4) = l_{\mu\rho l^{\nu\rho} l_\nu^{\mu}}. \tag{3}$$

This action has a dimensionless coupling $k$ and can be treated as a generally covariant extension of $\phi^4$ theory with $\phi \to l_{\mu\nu}$. The functional (1) is invariant under the global Weyl transformations $l_{\mu\nu} = \lambda^{\alpha\beta} l_{\mu\nu}$, then

$$g_{\mu\nu}(x') = e^{2\alpha} g_{\mu\nu}(x), \quad l_{\mu\nu}'(x') = e^{2\alpha} l_{\mu\nu}(x). \tag{4}$$

and scaling transformations

$$x_{\mu}' = e^{-\beta} x_{\mu}, \quad g_{\mu\nu}(x') = g_{\mu\nu}(x), \quad l_{\mu\nu}'(x') = e^{\beta} l_{\mu\nu}(x). \tag{5}$$

Our goal is to study the shapes of the interaction potential encoding self-interaction of the field $l_{\mu\nu}$ in the gravitational background with the metric $g_{\mu\nu}(x)$. The knowledge of the extremal of $U$ has to give an information about the possibility for spontaneous breaking of the above discussed global symmetries.

2 Equation of motion and its diverse formulations

The covariant equation of motion for the dynamical field $l_{\mu\nu}$ following from (1) is

$$\frac{1}{2} \nabla_\mu \nabla^{\nu}[l^{\rho\nu}] = - \frac{1}{2} [\nabla_\mu, \nabla^{\nu}] l^{\rho\nu} = \frac{\partial U}{\partial l^{\rho\nu}}, \tag{6}$$

where the notation $\nabla_\mu F^{\star \beta \ldots \lambda}$ encodes the Einstein’s summing rule: $F^{\star \beta \ldots \lambda} = \epsilon^{\alpha \beta \ldots \lambda} F_\alpha$. Equation (6) hides the Riemann tensor $R_{\mu\nu\rho\sigma}$ of the background metric $g_{\mu\nu}$. To show the presence of this tensor in (6) we extract the commutator in its l.h.s.

$$\frac{1}{2} \nabla_\mu \nabla^{\nu}[l^{\rho\nu}] + [\nabla_\nu, \nabla^{\nu}] l^{\rho\nu} = \frac{\partial U}{\partial l^{\rho\nu}} \tag{7}$$

and write (7) in the form containing the D’Alembertian $\Box \equiv \nabla_\mu \nabla^{\mu}$ in its l.h.s.

$$\nabla_\mu \nabla^{\nu} l^{\rho\nu} - \frac{1}{2} [\nabla_\nu, \nabla^{\nu}] l^{\rho\nu} = - \frac{1}{2} [\nabla_\nu, \nabla^{\nu}] l^{\rho\nu} + \frac{\partial U}{\partial l^{\rho\nu}}. \tag{8}$$

Then the Bianchi identity

$$[\nabla_\mu, \nabla_\nu] l^{\rho\nu} = R_{\mu\nu} l^{\lambda\rho} + R_{\mu\nu} l^{\rho\lambda} \tag{9}$$

allows to write the $\nu\rho$-symmetrized commutator in (8) in the desired form

$$[\nabla_\nu, \nabla^{\nu}] l^{\rho\nu} = R^{\nu\rho} l^{\mu\nu} + R^{\nu\rho} l^{\mu\nu} \tag{10}$$

For compactness, we write this equation in a condensed form

$$[\nabla_\nu, \nabla^{\nu}] l^{\rho\nu} = K^{\nu\rho} l^{\mu\nu}, \tag{11}$$

introducing the tensor $K^{\nu\rho}$ which denotes the r.h.s. of (10)

$$K^{\nu\rho} l^{\mu\nu} := R^{\nu\rho} l^{\mu\nu} + R^{\nu\rho} l^{\mu\nu} \tag{12}$$

It is easy to check that $K^{\nu\rho}$ obeys the $g$-traceless condition

$$K^{\nu\rho} l^{\mu\nu} := g_{\nu\rho} K^{\nu\rho} = 0. \tag{13}$$
As a result, we obtain the representation of EOM (6) including the D’Alembertian □ in the curved space-time together with its Riemann and Ricci tensors
\[
\nabla_\mu \nabla^\nu \rho - \frac{1}{2} \nabla [\nabla_\mu \nabla_\nu \rho] = - \frac{1}{2} K (\nabla_\rho \nabla) + \frac{\partial U}{\partial I_\rho \mu}.
\]
(14)

Alternatively, one can extract the total covariant divergence in the l.h.s. of EOM (6)
\[
\nabla_\mu \left[ \nabla (\nabla_\nu I^\rho \rho) \right] = - \left[ \nabla_\mu, \nabla (\nabla_\nu I^\rho \rho) \right] + \frac{\partial U}{\partial I_\rho \mu}.
\]
(15)
and using (11) find another interesting representation for EOM (6)
\[
\nabla_\mu \left[ \nabla (\nabla_\nu I^\rho \rho) \right] = - K (\nabla_\rho \nabla) + \frac{\partial U}{\partial I_\rho \mu}.
\]
(16)
This representation reveals the presence of \( \nabla_\mu I_\rho \mu \) in EOM in the special combination
\[
H^{\mu \nu} := \nabla (\nabla_\nu I^\rho \rho)
\]
(17)
Thus, we have the two equivalent formulations (14) and (16) for the EOM (6) that will help to investigate the proposed model (1).

3 Spontaneous emergence of composite scalar field

It is hard to solve nonlinear Eqs. (14) or (16). But it is easier to find classical vacua defined by extremals of \( U \). To this end we multiply (14) by \( g_{\nu \rho} \) that gives
\[
\nabla_\mu \nabla_\nu I_\rho \mu = g_{\nu \rho} \frac{\partial U}{\partial I_\rho \mu}
\]
(18)
after using (13). It shows that for potentials obeying the condition
\[
\frac{\partial U}{\partial I_\rho \mu} = 0
\]
(19)
Equation (18) transforms into the continuity equation
\[
\nabla_\mu \left[ \nabla_\rho I_\nu \mu \right] = 0.
\]
(20)
The latter is presented in the form of the momentum conservation
\[
\frac{1}{\sqrt{|g|}} \partial_\mu (\sqrt{|g|} (g^{\mu \nu} \partial_\nu \phi - \nabla_\nu I^\nu \mu)) = 0
\]
(21)
of the spontaneously emerging scalar field \( \phi \) accompanied with other components of \( I_\mu \nu \)
\[
\phi := I_\rho \mu \equiv g^{\mu \nu} I_\mu \nu.
\]
(22)

\footnote{We impose this condition in order to simplify the analysis of possible solutions of nonlinear EOM (14). In the next Section we show that (19) leads to the constraints (28) that reduce the number of independent parameters of the general potential \( U \) (2) to two parameters \( \alpha \) and \( \beta \).}

Isolating the D’Alembertian □ from (21) gives the generalized Klein–Gordon equation for \( \phi \)
\[
\square \phi = \frac{1}{\sqrt{|g|}} \partial_\mu (\sqrt{|g|} \nabla_\nu I^\nu \mu),
\]
(23)
where the vector field \( \nabla_\nu I^\nu \mu \) might be treated as an effective source. If the source is absent, e.g. when \( \nabla_\nu I^\nu \mu = 0 \), Eq. (23) acquires the particular solution
\[
\nabla_\nu I^\nu \mu = 0 \rightarrow I_{\nu \mu \rho} = \frac{\mu}{4} g_{\nu \rho} \rightarrow \phi_0 = \mu.
\]
(24)

4 Extremals of interaction potential

To answer this question we study the derivative of \( U \)
\[
\frac{1}{2} \frac{\partial U}{\partial I_\rho \mu} = (\alpha (I^2)^\rho \mu - \beta I^{\rho \mu} T r (I^2))
\]
\[+b_2 I^{\rho \mu} (T r I)^2 + 2b_4 (I^2)^\rho \mu
\]
\[+\left[ \frac{\alpha}{3} T r (I^3) + b_2 T r I T r (I^2) + 2b_4 (T r I)^3 \right] \frac{\partial T r I}{\partial I_\rho \mu}.
\]
(25)
Subsequent substitution of \( g \)-contracted Eq. (25) in Eq. (19) gives the equation
\[
\frac{1}{2} g_{\nu \rho} \frac{\partial U}{\partial I_\rho \mu} = (\alpha - \beta) T r I T r (I^2) + b_2 T r (T r I)^2
\]
\[+2b_4 T r (I^3)
\]
\[+\left[ \frac{\alpha}{3} T r (I^3) + b_2 T r I T r (I^2) + 2b_4 (T r I)^3 \right] \left( g_{\nu \rho} \frac{\partial I}{\partial I_\rho \mu} \right) = 0.
\]
(26)
Then the substitution of \( \frac{\partial T r I}{\partial I_\rho \mu} = g^{\nu \rho} \) in (26) yields the algebraic equation
\[
\left( \frac{4}{3} \frac{\alpha}{\beta} + 2b_4 \right) T r (I^3) + [(\alpha - \beta) + 4b_2] T r I T r (I^2)
\]
\[+(b_2 + 8b_4) (T r I)^3 = 0
\]
(27)
which is solved by the choice of the three phenomenological constants
\[
b_4 = - \frac{2}{3} \alpha, \quad b_2 = \frac{1}{4} (\beta - \alpha), \quad b_4 = - \frac{1}{8} b_2.
\]
(28)
Thus, we obtain the two-parametric homogenous potential

\[ U_{II} = \frac{2}{3} \alpha [Trl Tr(l^3) - Tr(l^4)] - \frac{1}{2} \beta (Tr(l^2))^2 \]
\[ + \frac{1}{4} (\beta - \alpha) [Tr(l^2)(Trl) - \frac{1}{8} (Trl)^3] \tag{29} \]

and its partial derivative with respect to \( l_{\nu\rho} \)

\[ \frac{\partial U_{II}}{\partial l_{\nu\rho}} = \frac{2}{3} \alpha [3(l^2)_{\nu\rho} Trl + g_{\nu\rho} Tr l^3] - 4(l^3)_{\nu\rho} \]
\[ = -2\beta l_{\nu\rho} Trl + \frac{1}{2} (\beta - \alpha) [l_{\nu\rho} (Trl)^2 + g_{\nu\rho} Trl Trl(l^2) \]
\[ - \frac{1}{2} g^{\nu\rho} (Trl)^3]. \tag{30} \]

The latter expression is the cubic polynomial in \( l_{\nu\rho} \)

\[ \frac{\partial U_{II}}{\partial l_{\nu\rho}} = \left\{ - \frac{8}{3} \alpha (l^2)_{\nu\rho} + 2\alpha Trl (l^2)_{\nu\rho} \right\} \]
\[ + \left[ \frac{1}{2} (\beta - \alpha) (Trl)^2 - 2\beta Trl(l^2) \right] l_{\nu\rho} \]
\[ + \left\{ \frac{2}{3} \alpha Tr(l^3) + \frac{1}{2} (\beta - \alpha) [Tr(l^2) - \frac{1}{2} g_{\alpha\beta}] \phi \right\} g^{\nu\rho} = 0. \tag{31} \]

spanned by the tensors \( g_{\nu\rho}, l_{\nu\rho} \) and their products. It is easy to check that (31) obeys the constraint (19) and the homogeneity condition

\[ g_{\nu\rho} \frac{\partial U_{II}}{\partial l_{\nu\rho}} = 0, \quad l_{\nu\rho} \frac{\partial U_{II}}{\partial l_{\nu\rho}} = 4U_{II}. \tag{32} \]

Then the extremals of the potential (29) are given by zeroes of Eq. (31)

\[ \left\{ - \frac{8}{3} \alpha (l^2)_{\nu\rho} + 2\alpha Trl (l^2)_{\nu\rho} \right\} \]
\[ + \left[ \frac{1}{2} (\beta - \alpha) (Trl)^2 - 2\beta Trl(l^2) \right] l_{\nu\rho} \]
\[ + \left\{ \frac{2}{3} \alpha Tr(l^3) + \frac{1}{2} (\beta - \alpha) [Tr(l^2) - \frac{1}{4} g_{\alpha\beta}] \phi \right\} g^{\nu\rho} = 0, \tag{33} \]

where \( \phi = Trl \) (22). For the case \( \alpha = 0 \) Eq. (33) factorises in the product

\[ \frac{\partial U_{II}}{\partial l_{\alpha\beta}} |_{\alpha=0} = -2\beta l_{\alpha\beta} \left( l_{\mu\nu} - \frac{\phi}{4} g_{\mu\nu} \right) \left( l_{\nu\rho} - \frac{\phi}{4} g_{\nu\rho} \right) = 0 \tag{34} \]

which shows that we have three roots. The first of them is the zero root

\[ l_{\mu\nu} = \phi_1 = 0. \tag{35} \]

but the second double degenerate root is

\[ l_{0\mu\nu}(x) = \frac{Trl_0(x)}{4} g_{\mu\nu} = \frac{\phi_0(x)}{4} g_{\mu\nu}. \tag{36} \]

So, we obtain the reduced one-parametric potential

\[ U_{II}|_{\alpha=0} = -\frac{1}{2} \beta \left( (Tr(l^2))^2 - \frac{1}{2} (Tr(l^2)) (Trl) - \frac{1}{8} (Trl)^3 \right) \tag{37} \]

with the one non-trivial extremal (36). The exclusive property of this matrix is that its nth power is equal to itself multiplied by \((\frac{2}{3} l_{\mu\nu})^{n-1}\)

\[ (l^n_{0\mu\nu})_{\mu\nu} = \left( \frac{\phi_0(x)}{4} \right)^n g_{\mu\nu} = \left( \frac{\phi_0(x)}{4} \right)^{n-1} l_{0\mu\nu}, \; Tr(l^n_{0\mu\nu}) = 4 \left( \frac{\phi_0(x)}{4} \right)^n \frac{\phi_0(x)}{4} l_{0\mu\nu}. \tag{38} \]

Due to this, \( l_{0\mu\nu} \) turns out to be a root of Eq. (33) with \( \beta = 0 \) as seen from the representation

\[ \frac{\partial U_{II}}{\partial l_{\nu\rho}} |_{\beta=0} = -\alpha \left\{ \frac{8}{3} \left( (l^3)_{\nu\rho} - \frac{1}{4} Tr(l^3) g^{\nu\rho} \right) \right\} \]
\[ - 2Trl \left[ (l^2)_{\nu\rho} - \frac{1}{4} Tr(l^2) l_{\nu\rho} \right] + \frac{1}{2} Trl \left[ Tr(l^2) \right] \]
\[ - \frac{1}{4} (Trl)^2 g^{\nu\rho} \] \tag{39} \]

obtained by regrouping of the terms in (33). Indeed, each term in square brackets in (39) vanishes after substitution of \( l_{0\mu\nu} \) there. So, we obtain another one-parametric potential

\[ U_{II}|_{\beta=0} = -\alpha \left\{ \frac{2}{3} (Tr(l^4) - Trl Tr(l^3)) \right\} \]
\[ + \frac{1}{4} \left[ Tr(l^2)(Trl)^2 - \frac{1}{8} (Trl)^4 \right]. \tag{40} \]

Thus, we see that the two-parametric potential (29)

\[ U_{II} = \beta U_{II}|_{\alpha=0} + \alpha U_{II}|_{\beta=0} \tag{41} \]

has the same extremals as \( U_{II}|_{\alpha=0} \) and \( U_{II}|_{\beta=0} \). At the extremal (36) \( U_{II} \) takes the form

\[ U_{II}|_{x=0} = \frac{1}{32} (\alpha - \beta) \phi_0^4 \tag{42} \]

due to the relations between the polynomial values at \( l_{0\mu\nu} \)

\[ \frac{2}{3} Trl_0 Tp(l^n_{0\mu\nu}) - \frac{1}{24} (Trl_0)^4 \]
\[ = \frac{1}{2} (Tr(l^2)_0)^2 - \frac{1}{32} (Trl_0)^4 = 0, \]
\[ Tr(l^2)_0 (Trl_0)^2 - \frac{1}{8} (Trl_0)^4 = \frac{1}{8} (Trl_0)^4, \]
\[ Tp(l^n_{0\mu\nu}) - \frac{1}{64} (Trl_0)^4 = 0. \] \tag{43}
When $\alpha = \beta$ the potential (41) transforms to the polynomial

$$U_{II}|_{\alpha=\beta} = \alpha \left\{ \frac{2}{3} Tr l Tr (l^3) - \frac{1}{2} (Tr (l^2))^2 - \frac{2}{3} Tr (l^4) \right\}$$

(44)

and vanishes on the extremal $l_{\mu \nu}$

$$U_{II}|_{\alpha=\beta; l=l_0} = 0.$$  

(45)

Going back to the expression for the action (1) at $l_{\mu \nu}(x)$ we find its kinetic term

$$\left( \frac{1}{2} \nabla_{\rho} l_{\nu \rho} \nabla^{\mu} l^{\nu} - \nabla_{\mu} l^{\nu} \nabla_{\nu} l_{\mu} \right) |_{l=l_0}$$

$$= \frac{1}{4} \partial_{\mu} \phi_0 \partial^{\mu} \phi_0.$$  

(46)

Then $S (1)$ transforms in the scale and diff-invariant action of the massless $\phi^4$ theory

$$S_{II0} = \int d^4 x \sqrt{|g|} \left( \partial_{\mu} \phi_0 \partial^{\mu} \phi_0 + \frac{k^2}{2} (\alpha - \beta) \phi_0^4 \right).$$  

(47)

in the background, where we substitute $U_{II}|=l_0$ (42) and redefine $\phi_0$ in $\phi_0 := (2k)^{-1} \phi_0$.

Thus, we build the potential function $U_{II}$ with the extremals (35) and (36). The former extremal is a particular solution of EOM (14) and (16), but this property should be verified for $l_{0 \mu \nu}(x)$ to confirm the assumption made in Sect. 3. Let us note that the potential term in (47) vanishes for the case when the free parameters $\alpha$ and $\beta$ are equal. Below this fact will be used in the proof of the vanishing of the cosmological constant under spontaneous breaking of the scale symmetry.

5 Breaking of the Weyl and scale symmetries

To check whether the extremal (36) is a solution of EOM (14) we note that the expression under the covariant derivative in Eq. (20) reduces to the derivative of $\phi_0$

$$\nabla_{\rho} T r l_0 - \nabla^{\rho} l_0^* = \frac{3}{4} \partial_{\rho} \phi_0$$

(48)

due to the relation

$$\nabla^{\rho} l_0^* = \frac{1}{4} g^{\rho \nu} \partial_{\nu} \phi_0.$$  

(49)

Then Eq. (20) itself is transformed into the D’Alembert equation in the curve space

$$\nabla^{\rho} \{ \nabla_{\rho} T r l_0 - \nabla^{\rho} l_0^* \} = \frac{3}{4} \Box \phi_0(x) = 0$$

$$\rightarrow \Box l_{0 \mu \nu}(x) = 0,$$  

(50)

since the second term in the l.h.s. of (50) becomes proportional to $\Box \phi_0(x)$

$$\nabla^{\rho} \nabla^{\nu} l_0^* = \frac{1}{4} \Box \phi_0(x).$$

(51)

The operator $\Box$ is obtained from $\partial_{\rho} \partial^{\rho}$ by the replacement of the flat metric $\eta_{\mu \nu}$ in $g_{\mu \nu}(x)$

$$\Box \phi_0 := \nabla^{\rho} \nabla_{\rho} \phi_0 = g^{\nu \rho} \partial_{\nu} \partial_{\rho} \phi_0 - g^{\nu \rho} \Gamma_{\nu \rho}^\lambda \partial_\lambda \phi_0.$$  

(52)

On the extremals of $U_{II}$ EOM (14) reduces to

$$\nabla^{\nu} \nabla_{\nu} l_0^* = K^{(\nu \rho)} l_0.$$  

(53)

due to Eq. (50). Inserting (49) in (53) and using (11), that defines $K^{(\nu \rho)} l_0$ gives

$$g^{\nu \rho} \nabla_{\nu} \partial_{\rho} \phi_0 = K^{(\nu \rho)} \phi_0 = (R^{(\nu \rho)} + G^{\nu \rho}) \phi_0.$$  

(54)

Then taking into account the identity

$$R^{(\nu \rho)} - R^{\nu \rho} \rightarrow (R^{(\nu \rho)} + R^{\nu \rho}) = 0$$  

(55)

the r.h.s. of Eq. (54) vanishes, because

$$K^{(\nu \rho)} l_0 \rightarrow 0.$$  

(56)

So, EOM (54) on the extremal (36) is simplified to the equation

$$g^{\nu \rho} \nabla_{\nu} \partial_{\rho} \phi_0 = 0.$$  

(57)

and its multiplication by $g_{\nu \rho}$ brings us back to the Klein–Gordon equation (50)

$$g_{\nu \rho} g^{\nu \rho} \partial_{\nu} \partial_{\rho} \phi_0 = 2 \Box \phi_0 = 0.$$  

(58)

In the explicit form Eq. (57) is written as

$$g^{\nu \rho} [\partial_{\nu} \partial_{\rho} \phi_0 - \Gamma^\alpha_{\nu \rho} \partial_\alpha \phi_0] = 0 \rightarrow \partial_{\nu} \partial_{\rho} \phi_0$$

$$- \Gamma^\alpha_{\nu \rho} \partial_\alpha \phi_0 = 0.$$  

(59)

The simplest covariant solution of both Eq. (57) and the Klein–Gordon Eq. (58) is

$$\phi_0(x) \rightarrow \phi_0 = \mu = constant.$$  

(60)

This solution confirms the conjecture of Sect. 3 on the solution (24) to be an extremal of $U$ which in its turn proves spontaneous breakdown of both the Weyl and scale symmetries. Indeed, the extremal (36) of $U_{II}$, realized by the propagating field $l_{0 \mu \nu}(x)$, transforms as

$$\phi_0^0(x') = e^{-\alpha} \phi_0(x) \rightarrow l^0_{0 \mu \nu}(x') = e^{\alpha} l_{0 \mu \nu}(x)$$  

(61)

under the Weyl transformation (4) and

$$\phi_0^0(x') = e^{\alpha} \phi_0(x) \rightarrow l^0_{0 \mu \nu}(x') = e^{-\alpha} l_{0 \mu \nu}(x)$$  

(62)

under the scale transformations (5). The laws (61–62) coincide with those of $l_{0 \mu \nu}(x)$ itself and hence realize the action symmetries on the extremal $l_{0 \mu \nu}(x)$ which is their linear representations. Therefore, the orbit swept by $l_{0 \mu \nu}(x)$ under the action of the Weyl and scale transformations gives rise to
an infinitely degenerate vacuum manifold. The requirement for $l_{\mu \nu}(x)$ to obey EOM (57) removes this degeneracy by choosing the vacuum solution (60)

$$\phi_o = \mu \rightarrow \phi_{\nu p}(x) = \frac{\mu}{4} g_{\nu p}(x)$$

(63)
covariant under diffeomorphisms, but breaking the symmetries (61–62). Thus, one can treat $\phi(x)$ as the Nambu–Goldstone boson generated by the simultaneous breakdown of the Weyl and scale symmetries and having the non-zero vacuum expectation value $\langle \phi \rangle_0$

$$\langle \phi \rangle_0 \equiv Tr l_o = \mu.$$ 

(64)
It brings the mass scale $\mu$ in the considered model (1) with the potential term $U = U_{I I}$.

On the solution (63) the kinetic term (46) vanishes and $U_{I I}$ is converted to the constant

$$U_{I I} \mid = l_o = \frac{1}{32}(\alpha - \beta) \mu^4.$$ 

(65)
The resulting vacuum action takes the form

$$S_{I I o} = \frac{\mu^4}{32 \kappa^2} (\alpha - \beta) \int d^4 x \sqrt{|g|}$$

$$\left[ \frac{1}{k^2} \int d^4 x \sqrt{|g|} U_{I I} \right] \mid = l_o$$

(66)
corresponding to the contribution of the cosmological constant

$$\lambda_{I I} = \frac{\alpha - \beta}{32 \kappa^2} \langle \phi \rangle_0.$$ 

(67)
This expression shows that on the classical level the cosmological constant is proportional to the potential of the classical vacuum state and vanishes when $\alpha = \beta$.

6 Four-parametric potential with broken symmetry

The results of Sect. 4 show that the potential $U$ (2) has the same extremals (35), (36) as the potential (41) through the relations (38). The latter substituted in Eq. (25) yield the condition for the phenomenological constants necessary for $l_{\nu p}(x)$ to be an extremal of $U$

$$\frac{1}{2} \frac{\partial U}{\partial l_{\nu p} \mid = l_o} = \left( \frac{12}{16} \alpha - \frac{1}{16} \beta + \frac{1}{2} b_2 \right)$$

$$+ 2 b_4 + \frac{1}{3} b' \phi_0^3(x) g_{\nu p}(x) = 0.$$ 

(68)
Equation (68) is satisfied if $\phi_0 = 0$ or the constants obey the relation

$$\frac{1}{12} \alpha - \frac{1}{16} \beta + \frac{1}{32} b'_4 + \frac{1}{2} b_2 + 2 b_4 = 0$$

(69)
which leaves freedom in the choice of four parameters. One can solve (69) expressing, for example, $b_4$ in terms of the remaining constants

$$b_4 = - \frac{1}{24} \alpha + \frac{1}{32} \beta - \frac{1}{64} b'_4 - \frac{1}{4} b_2$$

(70)
that results in the following four-parametric potential

$$U_I = \frac{2}{3} \alpha Tr l Tr (l^3) - \frac{1}{2} \beta (Tr (l^2))^2$$

$$+ b_2 Tr (l^2) (Tr l)^2$$

$$+ b'_4 Tr (l^4) - \left[ \frac{1}{24} \alpha - \frac{1}{32} \beta + \frac{1}{64} b'_4 \right]$$

$$+ \frac{1}{4} b_2] (Tr l)^4.$$ 

(71)
This expression can be rewritten in a more suitable form

$$U_I = \alpha \left[ \frac{2}{3} Tr l Tr (l^3) - \frac{1}{24} (Tr l)^4 \right]$$

$$- \beta \left[ \frac{1}{2} (Tr (l^2))^2 - \frac{1}{32} (Tr l)^4 \right]$$

$$+ b'_4 \left[ Tr (l^4) - \frac{1}{64} (Tr l)^4 \right] + b_2 \left[ Tr (l^2) (Tr l)^2 \right]$$

$$- \frac{1}{4} (Tr l)^4$$

(72)
which gives the zero value of the cosmological constant $\lambda$

$$U_I \mid = l_o = 0 \rightarrow \lambda_I = 0$$

(73)
for any choice of $\alpha$, $\beta$, $b_2$ and $b'_4$ in contrast to (67) given by the potential $U_{I I} \mid = l_o$ (65) which was derived using the constraint (19).

So for example choosing $b_2 = b'_4 = 0$ in (71) we get the two-parametric potential

$$U_{I'} = \frac{2}{3} \alpha Tr l Tr (l^3) - \frac{1}{2} \beta (Tr (l^2))^2$$

$$- \left[ \frac{\alpha}{24} - \frac{\beta}{32} \right] (Tr l)^4$$

(74)
which unlike the potential (29) does not contain the monomials $Tr (l^4)$ and $Tr (l^2) (Tr l)^2$. It is explained by using (19) under the derivation of $U_{I I}$. If $\alpha = \beta$ then (74) reduces to

$$U_{I'} \mid = \alpha = \beta = 0 \rightarrow \lambda_{I'} = 0$$

(76)
like $U_{I I} \mid = \alpha = \beta = 0$ in (45) and $\lambda_{I I}$ in (67).

So, we have constructed the four-parametric potential (71), which gives the zero cosmological constant on its extremal $l_{\mu \nu}(x)$ (36) and on the vacuum extremal $l_{\mu \nu}(x)$, respectively. As a result, we have two- and four-parametric potentials for which the cosmological constant becomes

$$\lambda.$$
equal to zero. These potentials, together with the above built one-parameter potentials, make it possible to control the value of the cosmological constant.

7 Summary

Studied is an attempt to understand the amazing efficiency of scalar fields such as the inflaton in describing the expanding universe. Our approach is based on the assumption about a composite structure of such scalar particles. The latters is considered. It is assumed that such type tensor fields could describe some degrees of freedom of the quark–gluon plasma associated with higher spins.

To explain the proposed scenario, we analyze the critical points of the potential of \( l_{\mu \nu} \), represented by general quartic polynomial, and find a classical vacuum that points to spontaneous breaking of both the Weyl and scale symmetries. The vacuum-related extremal is characterized by the emerging composite Nambu–Goldstone boson \( \phi(x) \) := \( g^{\mu \nu} l_{\mu \nu}(x) \) which acquires a non-zero vev \( \langle \phi \rangle_0 = \mu \). The arbitrary constant \( \mu \) introduces a characteristic mass scale which defines the vev \( l_{\mu \nu}(x) \) = \( (\mu/4)g_{\mu \nu}(x) \). We show that \( \mu \), together with the free dimensionless parameters of the potential, defines the cosmological constant. As a result, it becomes possible to control the value of the cosmological constant by varying these free parameters. We find, in particular, that the cosmological constant vanishes for a certain choice of the free parameters. In our example the composite structure of the tensor \( l_{\mu \nu} \) confirms the known no-go theorem which forbids the presence of multiple massless particles with spins equal to 2 in any general covariant action consistent with the perturbative unitarity condition. Nevertheless, the theorem allows the field \( l_{\mu \nu} \) to carry spin(s) 1 and 0. However, spontaneous breaking of the global symmetries, preserving the diff-invariance of the action, demands the extremal \( l_{0\mu \nu}(x) \) to be aligned along the external field \( g_{\mu \nu}(x) \) similarly to the case of (anti)ferromagnetics in magnetic field. Therefore the polarization DOF of \( l_{0\mu \nu}(x) \) are determined by the metric background tensor \( g_{\mu \nu}(x) \). As a result, the \( g \)-trace of the extremal \( l_{0\mu \nu} \) remains its only degree of freedom identified with the composite scalar field. This fact points to the possible distinguished role of composite massless scalar fields in the dynamics of the expanding universe.

It was noted in the Introduction that \( l_{\mu \nu}(x) \) is treated as belonging to tensor modes generated by the quark–gluon plasma. The scale invariant potential \( U \) captures the dynamics of this mode in the approximation of massless quarks that results in creation of the composite N–G boson \( \phi \). In QCD and Standard Model a finite radius of action of forces binding quarks and gluons is taken into account by introducing the energy scale parameter \( \Lambda_{QCD} \). The experimentally observed quark masses are expressed in terms of \( \Lambda_{QCD} \). The pion mass, in particular, is defined by the relation \( m_\pi \propto \sqrt{2m_u\Lambda_{QCD}} \), where \( m_u \) is the mass of the light current u-quark. The relatively small mass of pion in comparison with other mesons is explained by the observation that pion is N–G boson of spontaneously broken chiral symmetry.

Taking into account that in our universe conformal symmetry is broken, one can try to implement a similar mechanism for understanding the emergence of mass for the discussed N–G fields. To do this, note that the expression \( \Lambda_{QCD} = \langle \bar{u}u \rangle_0/f^2 \) contains two dimensional parameters which are the quark condensate and the pion decay constant \( f \). The role of the quark condensate in our model plays \( \langle \phi \rangle_0 \equiv Trl_0 = \mu \). Therefore, we need to have a new dimensional constant \( f \) encoding an effective mass of \( l_{\mu \nu} \). Using \( f \) allows to extend the scale invariant potential \( U \) by the additional term \( U_m \) quadratic in \( l_{\mu \nu} \):

\[
U_{\text{broken}} = af^2 Tr(l^2) + \frac{b}{4} f^2 (Tr l)^2
\]

which explicitly breaks the scale symmetry and contains two dimensionless parameters. On the extremal \( U_{\text{broken}} \) takes the form of the mass term \( U_m \) for the N–G boson \( \phi_0(x) \):

\[
U_{\text{broken}} \rightarrow U_m = \frac{(a+b) f^2}{4} \phi_0^2.
\]

The sum of \( U \) and \( U_{\text{broken}} \) yields an effective potential \( U^* \):

\[
U^* = U + U_{\text{broken}}
\]

that explicitly violates the Weyl/scale symmetries but retains the memory of the N–G mechanism of the appearance of the composite scalar \( \phi(x) \).

Another important generalization of the model is the construction of the kinetic term for the gravitational field. In the theory of gravity the kinetic term for \( g_{\mu \nu} \) is given by the Hilbert-Einstein action including the scalar curvature \( R \). In the studied model, the requirement that the action be invariant under diffeomorphisms is extended to its invariance under scaling transformations. These requirements are satisfied by the terms linear in \( R_{\mu \nu \rho \sigma} \) and quadratic in \( l_{\mu \nu} \). Such terms naturally generalize the term \( R\phi^2 \) used in the previous gravitational models involving the elementary scalar \( \phi(x) \). These extensions of the model will be studied in more detail in a different place.

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