Soil-water characteristic curves – Determination, estimation and application

Eng-Choon Leong

Associate Professor, School of Civil & Environmental Engineering, Nanyang Technological University, Singapore.

ABSTRACT

The soil-water characteristic curve (SWCC) is a relationship between water content in a soil and suction. The SWCC was first plotted by Edgar Buckingham, a soil physicist, in 1907 for six soils ranging in texture from sand to clay. It was adopted for use in unsaturated soil mechanics by the geotechnical engineering community. The SWCC is now almost treated as the index property of unsaturated soils. It has been used as a proxy for permeability and shear strength of unsaturated soil. Most soils have a sigmoidal SWCC, otherwise known as a unimodal SWCC as opposed to a bimodal SWCC which has been found for some soils. Although determining the SWCC is easier than determining permeability or shear strength for unsaturated soil, the test is still time-consuming and it is not easy to determine the entire SWCC. Incomplete or insufficient SWCC data may lead to an incorrect SWCC and hence inaccurate determination of permeability and shear strength. Progress has been made to expedite the experimental determination of SWCC as well as estimating the SWCC from basic soil properties using pedotransfer functions. In addition, SWCC has been represented using volumetric water content, gravimetric water content or degree of saturation. Different representations may have dire consequences on its application. Determining the SWCC using volumetric water content or degree of saturation presents challenges in estimating the instantaneous volume during the experiment. In this paper, the current state-of-the-art in determining, representing and estimating SWCC and its applications are described and critically examined.

Keywords: Soil-water characteristic curve, unsaturated soils, soil properties.

1 INTRODUCTION

The development of unsaturated soil mechanics has benefitted from the work done in agriculture-related disciplines. One important relationship that has become a mainstay in unsaturated soil mechanics is the soil-water characteristic curve (SWCC) which expresses the relationship between the water content in a soil and suction. The water content can be expressed in terms of volumetric water content (θw), gravimetric water content (w) or degree of saturation (S) and the corresponding SWCCs are denoted as θw-SWCC, w-SWCC and S-SWCC, respectively, after Fredlund (2018). This relationship was first plotted in soil physics by Edgar Buckingham in 1907 for six soils ranging in texture from sand to clay. The SWCC is also known as water retention curve, soil-moisture retention curve, soil-water release curve and soil-moisture characteristic curve in other disciplines. Most SWCCs are sigmoidal or unimodal. However, some soils were found to possess bimodal SWCCs. An illustration of unimodal and bimodal SWCCs is shown in Fig. 1. The closest analogy to determining SWCC in the study of unsaturated soils is the determination of compaction curve for compacted soils in classical soil mechanics. Some have treated the SWCC as an index property when investigating unsaturated soils.

There has been a lot of research in SWCC from determination to modelling. The two main applications of SWCC is modelling water movement in unsaturated soils and indirect determination of unsaturated soil properties such as permeability and shear strength. These will form the major discussions in this paper.

![Unimodal and bimodal SWCCs](image-url)

Fig. 1. Unimodal and bimodal SWCCs.

2 DETERMINATION OF SWCC

2.1 Standard methods

The determination of the SWCC has been standardized in ASTM D6836-16. In the standard, there are five methods, Method A to E, to determine the SWCC. Method A makes use of a hanging column and is valid only for suctions in the range of 0 to 80 kPa. Methods B and C use pressure plate extractors and differ in terms of the measurements made. Method B
measures the volume of water outflow from the specimen (volumetric) whereas Method C measures the weight of the specimen (gravimetric). Methods B and C are used for suctions in the range of 0 to 1500 kPa. The upper suction limit is imposed by the ceramic plate or porous membrane. Method D makes use of the chilled mirror hygrometer and is suitable for determining SWCC for suction range of 1000 kPa to 100 MPa. Method E makes use of a laboratory bench centrifuge and is able to measure SWCC in the suction range of 0 to 120 kPa by varying the angular velocity of the centrifuge. The ASTM D6836-16 further suggests Method A to be used typically for coarse soils, Methods B and C to be typically used for finer soils, Method D to be used when SWCC near saturation is not required and Method E to be used for coarser soils compared to those tested by Method A. Typical SWCCs shown in ASTM D6836-16 are unimodal. ASTM D6836-16 also suggests that the methods may be combined to form a more detailed SWCC. A summary of the suggested suctions in ASTM D6836-16 to determine the SWCC for each method is shown in Table 1.

| Method | A | B & C | D | E* |
|--------|---|-------|---|----|
| 0      | 10|       | 0 |    |
| 0.05   | 50|       | 0.5|    |
| 0.2    | 100|      | 2 |    |
| 0.4    | 300|      | 8.5|   |
| 1      | 500|     | 34|  |
| 2      | 1000|   | 120|   |
| 4      | 1500| Not specified |
| 6      |  |
| 10     |  |
| 15     |  |
| 20     |  |
| 80     |  |

Note: * Suctions were calculated from suggested angular velocity in ASTM D6836-16 based on a typical laboratory bench centrifuge.

It is common to find that the suction levels in Table 1 are not strictly followed as seen for a typical bimodal SWCC in Fig. 2. The implication of using only one method to obtain the SWCC according to the suctions in Table 1 is also illustrated in Fig. 2. Determination of SWCC frequently requires more than one method and may need three methods to obtain a complete SWCC. This shows that determining SWCC can be a costly and time-consuming affair. A further observation from Fig. 2 is the suction levels suggested in ASTM D6836-16 for each method’s suction range may be too excessive. The water contents measured in ASTM D6836-16 are volumetric and gravimetric water contents. Although provision is made in ASTM D6836-16 to convert $\theta_{w}$ to $S$, it is implicit in the procedures that the volume of the soil specimen remains constant during the test, i.e., dry density of the soil specimen remains constant. If the volume of the soil specimen changes during the test, the SWCC plotted with either $\theta_{w}$ or $w$ ($0-SWCC$ or $w-SWCC$) will be different from the SWCC plotted with $S$ ($S-SWCC$). Finally, ASTM D6836-16 only describes the procedures for determining the drying SWCC. The SWCC shows hysteresis on drying and wetting as illustrated in Fig. 3. A closed-system for Methods B and C need to be used to determine the wetting SWCC (see Leong et al. 2004).

2.2 Drying methods

Besides the methods listed in ASTM 6836-16, there are methods based on drying of the soil samples. These include vapour equilibrium technique (Agus et al. 2001; Tang and Cui 2005), HYPROP© measurement system (Schindler et al., 2010; Schindler and Müller, 2017; Bezerra-Coelho et al., 2018) and isotherm systems (Likos and Lu 2001, Likos et al. 2011).

The principles behind these methods are similar. A soil sample is left to dry (HYPROP© measurement system) or equilibrate in a closed chamber of fixed relative humidity (vapour equilibrium technique and...
isotherm systems) and the suction is either measured or inferred from the relative humidity generated by the salt solution. For the HYPROP® and isotherm systems, it is possible to measure both the drying and wetting SWCCs.

2.3 Consideration of volume change

When a soil specimen experiences volume change during a SWCC test, the volume change will affect \( \theta_w \) and \( S \) which are defined, respectively, as

\[
\theta_w = \frac{V_w}{V} \quad (1)
\]

\[
S = \frac{V_v}{V} = \frac{\theta_w}{1 - \frac{\rho_d}{G_s \rho_w}} \quad (2)
\]

where \( V \) is total volume, \( V_w \) is volume of water, \( V_v \) is volume of voids, \( \rho_d \) is dry density, \( \rho_w \) is density of water, \( G_s \) is specific gravity of soil solids. From Eqs. 1 and 2, the effect of volume change on \( \theta_w \) and \( S \) are not the same. The only SWCC not affected by volume change is w-SWCC.

To account for volume change, the volume of the soil specimen at suction equilibrium has to be determined. Two methods for determining volume of a soil specimen, Methods A and B, are described in ASTM D7263 –09(2018)e2. Method A determines the volume of a wax coated specimen using the water displacement method and is only applicable to soil specimens in which the wax will not penetrate the soil specimen’s outer surface. It is simpler to use Method B in ASTM D7263-09(2018)e2 where the volume of the soil specimen is determined from physical measurements of the soil specimen, typically cylindrical, and assuming that the soil specimen remains prismatic at each suction equilibrium. However, physically measuring the volume of a soil specimen is challenging, especially when the soil specimen distorts on drying.

By plotting the void ratio instead of volume of the soil specimen with gravimetric water content gives the more familiar shrinkage curve. Advances have been made in the measurement of shrinkage curves using laser triangulation (Jain et al. 2015), photogrammetry (Upreti and Leong, 2017; Li and Zhang, 2018), and 3D scanner (Wong et al. 2018). Essentially these measurements enable 3D models of the soil specimen as it dries to be constructed for volume determination. The shrinkage curve could then be used together with w-SWCC to produce either \( \theta \)-SWCC or S-SWCC.

The SWCC is dependent on the initial dry density or initial void ratio (Tarantino, 2009; Zhou et al, 2014; Wijaya and Leong, 2017). However, the effect of initial dry density on w-SWCC, \( \theta \)-SWCC and S-SWCC is different. Initial dry density only affects the initial portion of the w-SWCC (Wijaya and Leong, 2017).

Hence, the effect of dry density on w-SWCC can be estimated using only one w-SWCC. This is illustrated in Fig. 4 for a clayey silty sand.

![Fig. 4. Estimating w-SWCC for different dry densities using one w-SWCC for a clayey silty sand (from Wijaya and Leong, 2017).](image)

2.4 Parameters of SWCC

The use of the SWCC depends on the discipline. In agronomy, the water contents at suctions of 10, 33, 1500 kPa are important where they represent the field capacity for sandy soils, field capacity for other soils and wilting point, respectively (e.g. Grewal et al. 1990).

In unsaturated soil mechanics, the air-entry value (AEV) and residual suction are important as these points separate unsaturated soil behaviour into three distinct zones: boundary effect, transition and residual. These three zones have also been described using air saturation: insular, fuzzy and pendular as shown in Fig. 5. To obtain the correct AEV, S-SWCC should be used in order to determine the suction at which air enters the largest pores (Wijaya et al. 2015). If volume change is negligible, \( \theta \)-SWCC and w-SWCC will give the same AEV.

![Fig. 5. SWCC and parameters (modified from Kohgo, 2003).](image)
3 ESTIMATION OF SWCC

As measurement of SWCC is costly and time-consuming, many methods have been developed to estimate the SWCC. Many of these methods use the grain size distribution (GSD) of the soil. However, the discovery that not all SWCCs are unimodal necessitates that separate methods be used for estimation of unimodal and bimodal SWCCs. To apply the separate methods, it is needed to know a priori if a soil has a unimodal or bimodal SWCC.

3.1 Identification of soils with bimodal SWCC

Bimodal SWCC is attributed to dual porosity in soils. Dual porosity in soils can arise due to bimodal GSD, compaction or other features such as cracks in the soil (Li and Zhang, 2009; Satyanaga et al., 2013; Li et al., 2014). Pores in dual-porosity soils are largely governed by the arrangement of coarse grains and fine grains, which cause large pores (macro-pores) and small pores (micro-pores), respectively (Burger and Shackelford, 2001; Zhang and Chen, 2005). Hence, most bimodal SWCCs belong to soils with bimodal GSD. Simple criteria to identify bimodal GSD soils which have bimodal SWCC have been proposed by Satyanaga et al. (2013), Li et al. (2014) and Zou and Leong (2019).

The classification tree proposed by Zou and Leong (2019) to identify bimodal GSD soils which have bimodal SWCC was shown to perform better than the others and it is shown in Fig. 6. In Fig. 6, the definition of Y and percentage of grains of major peak grain size (MaP) are shown in Fig. 7 and e is void ratio.

3.2 Unimodal SWCC

3.2.1 One-point method

The most successful methods to estimate unimodal SWCC are those using one point measurement of the SWCC together with basic soil properties. Vanapalli and Catana (2005) presented a method to estimate the SWCC of coarse-grained soils using one SWCC measurement point in the suction range of 0.1 to 10 kPa and basic soil parameters (dominant grain size diameter d, grain size at 10% passing D10, grain size at 60% passing D60, and void ratio e). For fine-grained soils, the Catana et al. (2006) proposed to estimate the SWCC using one SWCC measurement point in the range of 50-500 kPa and suction capacity, C, which is correlated with the product of liquid limit and clay fraction. Houston et al. (2006) proposes a one-point method to estimate SWCC of non-plastic soils using grain sizes at 10, 20, 30, 60 and 90% passing (D10, D20, D30, D60, and D90 respectively) and fines content less than 200 μm (P200) with one SWCC measurement point. For plastic soils, Houston et al. (2006) proposed using a product of plasticity index (PI) and P200 with one SWCC measurement point. The product of PI and P200 is adjusted until the SWCC passes through the SWCC measurement point. Chin et al. (2010) proposed a one-point method for coarse-grained and fine-grained soils and recommend that the SWCC be measured at 10 kPa and 500 kPa (or 100 kPa) for coarse-grained and fine-grained soils, respectively. Chin et al. (2010) defined coarse-grained soils as soils where P200 is less than 30% and fine-grained soils as soils where P200 is more or equal to 30%.

\[ \theta_w = C(\psi) \frac{\theta_s}{\left\{ \ln \left[ \exp(1) + \left( \frac{\psi}{\psi_c} \right)^a \right] \right\}^m} \]  

(3a)

\[ C(\psi) = 1 - \frac{\left\{ \ln \left[ 1 + \frac{\psi}{\psi_c} \right] \ln \left[ 1 + \frac{1000000}{\psi_c} \right] \right\}}{\ln \left[ 1 + \frac{1000000}{\psi_c} \right]} \]  

(3b)
where $\theta_s$ is saturated volumetric water content; $a$, $n$ and $m$ are the curve-fitting parameters; and $\psi_r$ is the matric suction corresponding to the residual volumetric water content, $\theta_r$. For coarse-grained soils, Chin et al. (2010) recommend $a$, $n$, $m$ and $\psi_r$ as follows:

$$a = 0.53D_0^{0.96} \text{ kPa}$$ (4a)

$$n = x$$ (4b)

$$m = -0.23 \ln(x) + 1.13$$ (4c)

$$\psi_r = 100 \text{ kPa}$$ (4d)

For fine-grained soils, Chin et al. (2010) recommend $a$, $n$, $m$ and $\psi_r$ as follows:

$$a = 2.4x + 722 \text{ kPa}$$ (5a)

$$n = 0.07x^{0.4}$$ (5b)

$$m = 0.015x^{0.7}$$ (5c)

$$\psi_r = 914 \exp(-0.002x) \text{ kPa}$$ (5d)

In Eqs. 4 and 5, $x$ is adjusted such that the curve passes through the SWCC measurement point.

### 3.2.2 Zero experimental point method

Attempts have been made to replace the one SWCC measurement point with an estimated SWCC point instead. Such a point can be obtained using point pedotransfer functions (PTFs) which have been proposed in agriculture-related disciplines to estimate water contents at suctions of 3 or 4, 10, 33, 100 and 1500 kPa. Pedotransfer functions are predictive functions of soil properties using more easily measurable soil properties. It is observed that point PTFs for 10 kPa and 100 kPa may be suitable in Chin et al. (2010) one-point method to estimate coarse-grained and fine-grained soils SWCCs, respectively. However, Zou (2018) found that two point PTFs gave better estimate of the SWCC than using a single point PTF. For coarse-grained soils, Gupta and Larson (1979) point PTFs at 4 and 10 kPa suctions are recommended. For fine-grained soils, Gupta and Larson (1979) point PTF at 100 kPa suction and Rawls et al. (1982) point PTF at 1500 kPa suction are recommended.

Following the discussion above, it is not too difficult to realize that point PTFs can be used to estimate the complete unimodal SWCC. This can be achieved by considering an ensemble of point PTFs (Guber et al., 2009; Cichota et al., 2013; Zou and Leong, 2019) to provide points of the SWCC at suctions of 4, 10, 33, 100 and 1500 kPa. Figure 8 shows the attempt from Zou and Leong (2019).

### 3.3 Bimodal SWCC

Literature on soils with bimodal SWCC are limited. Generally, there are three approaches to obtain bimodal SWCC equation (Wijaya and Leong, 2016): piecewise, fraction volume and unique parameter. In the piecewise approach, the bimodal SWCC is separated into two unimodal segments connected at a point. The location of the point has a significant effect on the parameters of the SWCC equation and the parameters are highly non-unique (Wijaya and Leong, 2016). This approach may not give a smooth transition between the two unimodal segments. The fraction volume approach avoids this problem by dividing the porosity into macro- and micro-pores fractions described by a volume factor. In both piecewise and fraction volume approaches, the same unimodal SWCC equation is used to describe the macro- and micro-pores. Fitting of the parameters in the SWCC equations is done concurrently. As the number of parameters in the bimodal SWCC equation is more than that in the unimodal SWCC equation, determining of the parameters is not easy. The unique parameter approach incorporates some or all graphically determined parameters to reduce the number of the parameters needed in the bimodal SWCC equation. The third approach is more attractive as less parameters need to be determined (Wijaya and Leong, 2016). Wijaya and Leong (2016) propose the following equation which can be used to describe unimodal and bimodal SWCCs:

$$w = w_{sat} - m_1(x - x_1) - \sum_{i=2}^{n} R_i(x)(m_i - m_{i-1})$$ (6)

where $w$ is the gravimetric water content; $w_{sat}$ is the saturated gravimetric water content; $m_1$, $m_n$, $m_{i-1}$ are the slopes of the linear segments 1, i and i-1 of the SWCC; $x$, $x_1$ are log base 10 of the suction; $R_i$ is the ramp function for linear segment i given by Eq. 7.

$$R_i(x) = \frac{1}{2} \left[ (x - x_i) + \ln \left( \cosh \left[ c_i (x - x_i) \right] \right) \right]$$ (7)

where $c_i$ is the curvature at the intersection point between linear segments i and i-1. In Eq. 6, $n = 3$ for unimodal SWCC and $n = 5$ for bimodal SWCC.

The discussion on bimodal SWCC equations above does not cover the estimation of bimodal SWCC. Satyanaga et al. (2013) and Li et al. (2014) have attempted to estimate the bimodal SWCC. Satyanaga et
al. (2013) proposed that the bimodal SWCC be estimated according to the flowchart shown in Fig. 9. To use the model, the percent coarse (>P200) W1 and fine (<P200) W2 and GSD must first be curve fitted (Fig. 10) to obtain the parameters dmax1, dmax2, d1, d2, s1, and s2 shown in the flowchart. Curve fitting of the GSD may not be easy without proper constraints on the fitted parameters.

Li et al. (2014) used a modified form of Brutsaert (1966) SWCC equation to develop a bimodal SWCC equation as shown in Eq. 8.

$$w = \left(0.75w_s - 2w_r\right)\sqrt{\frac{2 \log \frac{w_s}{w_r}}{\psi}} + \left(0.25w_s - w_r\right)\left(4\psi_r\right)^{0.8} + \psi_r^{0.8} + (4\psi_r)^{0.8}$$

$$\psi = \frac{3w_s\sqrt{\psi_{s1}\psi_r}}{\psi_{s2}} + \frac{w_r(4\psi_r)^{0.8}}{\psi_r^{0.8} + (4\psi_r)^{0.8}}$$

where w is the gravimetric water content; $w_s$ is the saturated gravimetric water content; $w_r$ is the residual water content of the micropores; $\psi$ is suction, with subscripts a, t, a2, and r representing the air-entry value of the macropores, residual suction of the macropores, air-entry value of the micropores and residual suction of the micropores, respectively. For estimating bimodal SWCC, Li et al. (2014) suggest:

$$w_s = 0.03e + 0.005 \log C_u \quad (9a)$$

$$\psi_a = \frac{1.4}{3.6d_{50}C_u^{0.25}} \quad (9b)$$

$$\psi_r = \frac{4C_u^{0.4}}{a_{10}} \quad (9c)$$

$$\psi_t = 1.7C_u^{0.59} \quad (9d)$$

$$\psi_{a2} = \frac{0.11d_{10}^{0.7}}{d_{30}^{1.2}} \psi_r \quad (9e)$$

where e is void ratio, C_u is coefficient of uniformity, d_{10} is grain size at 10% passing and d_{30} is grain size at 30% passing. However, the estimated bimodal SWCC only works well for granular soils such as loam soils, sandy soils and gravel soils (Li et al., 2014).

Further research is needed to develop more robust and simpler estimation models for bimodal SWCC.

3.4 Estimation of wetting SWCC

The SWCC determined experimentally is usually the drying SWCC. A number of hysteresis models has been proposed to estimate the wetting SWCC from the drying SWCC but these models are only limited to unimodal SWCC. Pham et al. (2005) summarized 28 of these models. Two hysteresis models to estimate the wetting SWCC from the drying unimodal SWCC are described below.

Pham et al. (2005) developed a hysteresis model based on Feng and Fredlund (1999) water characteristic equation for thermal conductivity sensor and the correction factor C($\psi$) in Eq. 3b to give Eq. 10a. Equation 10a has four parameters $b$, $c$, $d$ and $\psi_r$. Pham et al. (2005) correlated the parameters $b$, $c$, $d$ and $\psi_r$ (with subscript w) for the wetting SWCC with the corresponding parameters (with subscript d) for the drying SWCC shown in Eqs. 10b to 10e.

$$\theta = C(\psi)\frac{\theta_d + c\psi^d}{b + \psi^d} \quad (10a)$$

$$c_w = c_d \quad (10b)$$
\[ b_w = \left( \frac{b_d}{(10^D)_{\kappa}} \right)^{\frac{1}{r}} \]  \hspace{1cm} (10c) \\
\[ d_w = \frac{d_e}{R} \]  \hspace{1cm} (10d) \\
\[ \psi_{r,w} = \left( 2.7 b_w \right)^{\frac{1}{r}} \]  \hspace{1cm} (10e)

where \( \theta \) is volumetric water content, \( \theta_k \) is saturated volumetric water content, \( \psi \) is matric suction and \( \psi_r \) is the residual matric suction. Parameters, \( D \) and \( R \), are the distance and slope ratio, respectively, between the wetting SWCC and the drying SWCC on a semi-logarithmic SWCC plot. Values of \( D \) and \( R \) are suggested for different soil types as summarized in Table 2.

Table 2. Suggested \( R \) and \( D \) values for different soil types from Pham et al. (2005)

| Soil Type                | \( R \) | \( D \) (log-scale) |
|-------------------------|--------|---------------------|
| Sand                    | 2.0    | 0.20                |
| Sandy loam              | 2.5    | 0.25                |
| Silt loam and clay loam| 1.5    | 0.50                |
| Compacted silt and compacted sand | 1.0    | 0.35                |

4 APPLICATIONS

For applications, the SWCC is used either directly (as an input) or indirectly (to obtain other parameters). Direct use of the SWCC appears in modelling flow and estimation of volume change in unsaturated soils. Indirect uses of the SWCC includes estimation of permeability, shear strength, and aqueous diffusion functions of unsaturated soils. Only flow and estimation of permeability and shear strength are discussed below.

4.1 Flow

The hydraulic properties that are needed to model flow in unsaturated soils are the SWCC and the permeability function. The SWCC is needed to satisfy the mass balance equation in the model.

Most flow problems encountered involved a wetting process. Hence, the wetting SWCC and the scanning curves (Fig. 3) are more appropriate in modelling flow in unsaturated soils. Currently, most modelling programs do not have the option to account for hysteresis and scanning curves of the SWCC.

4.2 Estimation of permeability

Mualem (1986) has classified permeability functions for unsaturated soils into three categories: empirical equations, macroscopic models and statistical models. The statistical models are the most rigorous and require the SWCC. Hence if the SWCC is available, the statistical model is the natural choice for obtaining the permeability functions.

The three main statistical models are Childs and Collis-George (1950), Burdine (1953), and Mualem (1976) models. Agus et al. (2001) found that the statistical models performed better if soils are divided into sands and clays. The form that is most well-used is Childs and Collis-George as modified by Kunze (1968) given in Eq. 12.

\[
k_r = \frac{1}{\psi} \int_0^{\theta_s} \left( \frac{\theta_s - \xi}{\psi} \right) d\xi - \int_0^{\theta_r} \left( \frac{\theta_r - \xi}{\psi} \right) d\xi
\]

where \( k_r \) is the relative water coefficient of permeability determined as \( k_r/k_s \); \( k_w \) and \( k_s \) are the unsaturated and saturated water coefficients of permeability, respectively; \( \theta_s \) and \( \theta_r \) are the volumetric and saturated volumetric water contents, respectively; \( \psi \) denotes suction whereas \( \xi \) is a dummy integration variable.

To determine the permeability function, the SWCC is divided into several equal volumetric-water-content increments. The relative coefficient of permeability, \( k_r \) at a specific volumetric water content, \( \theta_{wi} \) is computed by summing the suction values corresponding to volumetric water contents at and below \( \theta_{wi} \). Recently, it was...
highlighted that the lower limit of integration in Eq. 12 may lead to a lower or erroneous estimate of the permeability function (Zhang et al., 2018). The problem is related to soils with high volume change on drying and the type of SWCC used. It has been highlighted earlier that the AEV determined from θ−SWCC, w-SWCC and S-SWCC are different for soils with volume change. The correct AEV is given by S-SWCC and this AEV should be used as the upper bound of the lower limit of integration (sL) when determining the permeability function from either θ−SWCC or w-SWCC (Wijaya and Leong, 2018). If S-SWCC is used, the difference in kr if sL values from less than AEV to AEV is only about 0.5 order in magnitude (Wijaya and Leong, 2018) as illustrated for oil tailing sands in Fig. 11.

![Fig. 11. Effect of sL on kr function (from Wijaya and Leong, 2018).](image)

### 4.3 Estimation of shear strength

In the literature, shear strength of unsaturated soils can be expressed in terms of either a single stress state variable or two stress state variables. The most well-known single stress state variable was proposed by Bishop (1959), similar to the effective stress in saturated soils where

\[ \sigma' = (\sigma - u_a) + \chi \psi \]  

(12)

where \( \sigma \) is normal stress, \( u_a \) is pore-air pressure and \( \psi \) is matric suction given by difference of pore-air and pore-water pressures and \( \chi \) is a parameter related to degree of saturation and typically ranges from 0 to 1. The terms \( \sigma - u_a \) and \( \psi \) are considered independent in the two stress state variables. Variations of \( \chi \psi \) exist. For example, Karube et al. (1996) defined suction stress as a product of effective degree of saturation and suction, \( S_d \psi \) while Lu and Likos (2006) defined a suction stress characteristic curve. Using the most commonly used Mohr-Coulomb shear strength equation for saturated soils, the equivalent form for unsaturated soils using single and two stress state variables are, respectively,

\[ \tau = c' + (\sigma - u_a) \tan \phi' \]  

(13)

\[ \tau = c' + \psi \tan \phi'b \]  

(14)

where \( \tau \) is shear strength, \( c' \) is effective cohesion, \( \phi' = \) effective friction angle, and \( \phi'b \) is angle indicating a change in shear strength due to matric suction. Fredlund et al. (1978) presented Eq. 14 as the Mohr-Coulomb extended failure criterion. Mathematically, it can easily be shown that \( \chi \) and \( \tan \phi'b \) can be related as

\[ \chi = \frac{\tan \phi'b}{\tan \phi'} \]  

(15)

The SWCC has been used to provide estimates of \( \chi \) and \( \tan \phi'b \) and many forms of shear strength equation exists. A summary of shear strength equations can be found in Goh et al. (2010). Most of the proposed shear strength equations uses the drying SWCC. The shear strength equation proposed by Goh et al. (2010) can be used for unsaturated soils under drying and wetting conditions and takes the form of Eq. 14 where

\[ \tan \phi' = \tan \phi' \text{ if } \psi < \text{AEV} \]  

(16a)

\[ \tan \phi'b = \left[ \frac{AEV}{\psi} (1 - b\Theta^d) + b\Theta^d \right] \tan \phi' \text{ if } \psi \geq \text{AEV} \]  

(16b)

and

\[ \kappa = (\log \psi - \log \text{AEV})^\gamma \]  

(16c)

where \( b \) and \( y \) are parameters that takes different values for drying and wetting given below:

\[ y_d = 0.502 \ln (I_p + 2.7) - 0.387 \]  

(17a)

\[ b_d = -0.245 \left\{ \ln \left[ n_d (I_p + 4.4) \right] \right\}^2 
+ 2.114 \left\{ \ln \left[ n_d (I_p + 4.4) \right] \right\} - 3.522 \]  

(17b)

\[ y_w = 3.55 y_d - 3.0 \]  

(17c)

\[ b_w = 0.542 b_d \left( \frac{n_w}{n_d} \right) + 0.389 \]  

(17d)

where subscripts \( d \) and \( w \) represent drying and wetting, respectively, \( n \) is parameter in Fredlund and Xing (1994) SWCC equation and \( I_p \) is plasticity index. The Goh et al. (2010) shear strength model has only been tested on a limited number of soils.

The use of the SWCC to estimate shear strength has led to confusion on the contribution of salt in the pore water to shear strength. Beyond 1000 kPa, salt solutions are sometimes used to obtain the SWCC. The relative humidity in the air space above the salt solution in a closed chamber is used to infer the total suction. In this regard, osmotic and total suction are used interchangeably. Blight (1983) and Katte and Blight (2012), and more recently Leong and Abuel-Naga (2018) have demonstrated that concentration of salts in
the pore water do not contribute to the increase in shear strength.

5 CONCLUSION

The work done to date on the SWCC has been voluminous. This allows a number of ingenious methods to be developed and continue to be developed to obtain SWCC by experiments and estimation. The SWCC has also been applied in modeling flow in unsaturated soils, and obtaining other unsaturated soil properties such as shear strength and permeability.

While research on SWCC will still continue, more effort should be concentrated on application of unsaturated soil mechanics to engineering practice. One possible avenue for greater application unsaturated soil mechanics is through standardization of the estimation methods for SWCC and its use in estimation of permeability and shear strength of unsaturated soils.

ACKNOWLEDGEMENTS

First, I would like to acknowledge my colleague and friend, Prof. Harianto Rahardjo, who introduced unsaturated soil mechanics to me and remains my research collaborator till today. I am also fortunate to have the opportunity to work with Prof. Del Fredlund and the late Prof. Geoffrey Blight. I would like to acknowledge my past and present students from whom I learn and gain my insights into unsaturated soil mechanics. For this paper, I have drawn from the works of my past graduate students, especially Dr Chin Kheng Boon, Dr Martin Wijaya and Dr Zou Lei. Finally, I thank the organizing committee of APUNSAT 2019 for giving me the opportunity to present my bias and opinion on the subject.

REFERENCES

1) Agus, S.S., Leong, E.C., and Rahardjo, H. (2001): Soil-water characteristic curves of Singapore residual soils. Geotechnical and Geological Engineering, 19(3-4): 284 - 389.
2) ASTM D6836-16, Standard Test Methods for Determination of the Soil Water Characteristic Curve for Desorption Using Hanging Column, Pressure Extractor, Chilled Mirror Hygrometer, or Centrifuge, ASTM International, West Conshohocken, PA, 2016, www.astm.org.
3) ASTM D7263-09(2018)e2, Standard Test Methods for Laboratory Determination of Density (Unit Weight) of Soil Specimens, ASTM International, West Conshohocken, PA, 2018. www.astm.org.
4) Bezerra-Coelho, C. R., Zhuang, L., Barbosa, M. C., Soto, M., and van Genuchten, M.Th. (2018): Further tests of the HYPROP evaporation method for estimating the unsaturated soil hydraulic properties, Journal of Hydrology and Hydromechanics, 66(2), 161-169. doi: https://doi.org/10.1515/johh-2017-0046.
5) Bishop, A. (1959): The principle of effective stress, Tek. Ukebl., 39, 859-863.
6) Blight, G.E. (1983): Aspects of the capillary model for unsaturated soils. Proc. 7th Asian Conf. on Soil Mech. and Found. Eng., Haifa, Israel, Vol. 1, pp. 3-7.
7) Brutsaert, W (1966): Probability laws for pore size distributions. Soil Sci, 101, 85–92.
8) Burdine, N.T. (1953): Relative permeability calculations from pore-size distribution data, Journal of Petroleum Technology, 198, 71-78.
9) Burger, C.A. and Shackelford, C.D. (2001): Soil-water characteristic curves and dual porosity of sand-diaceous earth mixtures, Journal of Geotechnical and Geoenvironmental Engineering, 127, 790-800.
10) Catana, M.C., Vanapalli, S.K., and Garga, V.K. (2006): The water retention characteristics of compacted clays. In Proceedings of 4th International Conference of Unsaturated Soils, Carefree, Ariz., 2-6 Apr 2006. Unsaturated Soil 2006. Edited by Miller, G.A., Zapata, C.E., Houston, S.L., and Fredlund, D.G Published by ASCE Reston, VA, USA. 1348-1359.
11) Childs, E.C., and Collis-George, G.N. (1950): The permeability of porous materials. Proceedings of the Royal Society of London, Series A, London, U.K., 201, 392-405.
12) Chin, K.B., Leong E.C., and Rahardjo, H. (2010): A simplified method to estimate the soil-water characteristic curve. Can. Geotech. J., 47, 1382-1400.
13) Cichota, R., Vogeler, I., Snow, V.O., Webb, T.H. (2013): Ensemble pedotransfer functions to derive hydraulic properties for New Zealand soils. Soil Research, 51, 94-111.
14) Feng, M., and Fredlund, D.G. (1999): Hysteretic influence associated with thermal conductivity sensor measurements. In Proceedings of the 52nd Canadian Geotechnical Conference, Regina, Sask., Canada, 23-24 Oct. 1999. 651-657.
15) Fredlund, D.G. (2018): State of practice for use of the soil-water characteristic curve in geotechnical engineering, Can. Geotech. J., doi.org/10.1139/cgj-2018-0434.
16) Fredlund, D.G., Morgenstern, N.R., and Wdigter, R.A., (1978): The shear strength of unsaturated soils, Can. Geotech. J., 15, 313-321.
17) Fredlund, D.G., and Xing A. (1994): Equations for the soil-water characteristic curve, Can. Geotech. J., 31, 521-532.
18) Grewal, K.S., Buchan, G.D. and Tonkin, P.J. (1990): Estimation of field capacity and wilting point of some New Zealand soils from their saturation percentages, New Zealand Journal of Crop and Horticultural Science, 18(4), 241-246.
19) Goh, S.G., Rahardjo, H., and Leong, E.C. (2010): Shear strength equations for unsaturated soil under drying and wetting, Journal of Geotechnical and Geoenvironmental Engineering, 136, 594–606.
20) Guber, A.K.; Pachepsky, Y.A.; van Genuchten, M.Th.; Simunek, J.; Jacques, D.; Nemes, A.; Nicholson, T.J. and Cady, R.E. (2009): Multimodel simulation of water flow in a field soil using pedotransfer functions. Vadose Zone J., 8:1-10, 2009.
21) Gupta, S.C., and Larson, W.E. (1979): Estimating soil water retention characteristics from particle size distribution, organic matter percent, and bulk density. Water Resources Research, 15, 1633-1635.
22) Houston, W. N., Dye, H. B., Zapata, C. E., Perera, Y. Y., and Harraz, A. (2006): Determination of SWCC using one-point suction measurement and standard curves. In Proceedings of 4th International Conference of Unsaturated Soils, Carefree, Ariz., 2-6 Apr 2006. Unsaturated Soil 2006. Edited by Miller, G.A., Zapata, C.E., Houston, S.L., and Fredlund, D.G Published by ASCE Reston, VA, USA. 1482-1493.
23) Jain, S., Wang, Y.H., and Fredlund, D.G., (2015): Non-Contact Sensing System to Measure Specimen Volume During Shrinkage Test, Geotech. Test. J., 38(6), 1–14.
24) Katte, V., and Blight, G. (2012): The roles of solute suction and surface tension in the strength of unsaturated soil. In, Unsaturated Soils: Research and Applications, Mancuso C., Jommi C., D’Onza F. (Eds). Springer, Berlin, Heidelberg: 2012.
25) Kraube, D., Kato, S., Hamada, K. and Honda, M. (1996): The relationship between the medical behaviour and the state of pore-water in unsaturated soil. J. of Japanese Society of Civil Eng., no. 535/111-34, pp. 83-92 (in Japanese).
26) Kohgo, Y. (2003): Review of constitutive models for unsaturated soils and initial-boundary value analysis. In Proc. of the 2nd Asian Conference on Unsaturated Soils. (Karube, D., Iizuka, A., Kato, S., Kawai, K., and Tateyama, K. (eds)) UNSAT-ASIA 2003, Osaka-Japan.
27) Leong, E.-C. and Abuel-Naga, H.M. (2018): Contribution of osmotic suction to shear strength of unsaturated high plasticity silty soil. Geomechanics for Energy and the Environment, https://doi.org/10.1016/j.geete.2017.11.002
28) Li, X. Li, J.H. and Zhang, L.M. (2014): Predicting bimodal soil-water characteristic curves and permeability functions using physically based parameters, Computers and Geotechnics, 57, 85-96.
29) Li, X., and Zhang L.M. (2009): Characterization of dual-structure pore-size distribution of soil. Can. Geotech. J., 46, 129-141.
30) Li, L. and Zhang, X. (2018): A new approach to measure soil shrinkage curve, Geotech. Testing J., https://doi.org/10.1520/GTJ20150237.
31) Likos, W. J., and Lu, N. (2001): Automated measurement of total suction characteristics in high-suction range: Application to assessment of swelling potential, Transportation Research Record, 1755(1), 119–128.
32) Likos, W. J., Lu, N. and Godt, J. (2014): Hysteresis and uncertainty in soil water-retention curve parameters, Journal of Geotechnical and Geoenvironmental Engineering, 140(4): 04013050.
33) Likos, W. J., Lu, N., and Wenzel, W. (2011): Performance of a dynamic dew point method for moisture isotherms of clays. Geotech. Test. J., 34(4), 373–382.
34) Maqsoud, A., Bussière, B., Mbonimpa, M. and Aubertin, M. (2017): Comparison between the predictive modified Kovács model and a simplified one-point method measurement to estimate the water retention curve, Archives of Agronomy and Soil Science, 63(4), 443-454.
35) Mendes, R.M. (2008): Estudo das propriedades geotécnicas de solos residuais tropicais não saturados de ubatuba (SP). Ph.D. thesis, Escola Politécnica da Universidade de São Paulo, São Paulo (In Portuguese).
36) Mualem, Y. (1976): A new model for predicting the hydraulic conductivity of unsaturated porous media, Water Resources Research, 12, 513-522.
37) Mualem, Y. (1986): Hydraulic conductivity of unsaturated soils: prediction and formulas. Methods of Soil Analysis: Part I—Physical and Mineralogical Methods. 799-823.
38) Pham, H.Q., Fredlund, D.G. and Barbour, S.L. (2005): A study of hysteresis models for soil-water characteristic curves, Can. Geotech. J., 42, 1548-1568.
39) Rawls, W.J., Brakensiek, D.L., and Saxton, K.E. (1982): Estimation of soil water properties. Transactions of the American Society of Agricultural Engineers, 25, 1316-1320.
40) Satyanaga, A., Rahardjo, H., Leong E.C., and Wang, J. (2013): Water characteristic curve of soil with bimodal grain-size distribution, Computers and Geotechnics. 48, 51-61.
41) Schindler, U., Durner, W., von Unold, G., Müller, L. and Wieland, R. (2010): The evaporation method – Extending the measurement range of soil hydraulic properties using the air-entry pressure of the ceramic cup. J. Plant Nutr. Soil Sci., 173(4), 563–572.
42) Schindler, U. and Müller, L. (2017): Soil hydraulic functions of international soils measured with the Extended Evaporation Method (EEM) and the HYPROP device, Open Data Journal for Agricultural Research, 3, 10-16.
43) Tang, A.-M., Cui, Y.-J. (2005): Controlling suction by the vapour equilibrium technique at different temperatures and its application in determining the water retention properties of MX80 clay, Can. Geotech. J., 2005, 42:287-296.
44) Tarantino, A. (2009): A water retention model for deformable soils, Géotechnique, 59, 751-762.
45) Tarantino, A., and El Mountassir, G. (2013): Making un saturated soil mechanics accessible for engineers: preliminary hydraulic-mechanical characterization and stability assessment, Engineering Geology, 165, 89-104.
46) Upreti, K. and Leong, E.C. (2018): Measurement of soil shrinkage curve using photogrammetry. Proc. 2nd Pan American Conf. Unsaturated Soils, Dallas, Texas, USA, Issue 303, pp. 71-90.
47) van Genuchten, M.T., (1980): A closed-form equation for predicting the hydraulic conductivity of unsaturated soils, Soil Science Society American Journal, 44, 892-898.
48) Vanapalli, S.K. and Catana, M.C. (2005): Estimation of the soil-water characteristic curve of coarse-grained soils using one-point measurement and simple properties. In Proceedings of the International Symposium on Advanced Experimental Unsaturated Soil Mechanics, Trento, Italy, 27-29 Jun 2005. Edited by A. Tarantino, E. Romero, and Y., Cui. Published by Taylor & Francis, London, UK. 401-407.
49) Wijaya, M., and Leong, E.C. (2016): Equation for unimodal and bimodal soil-water characteristic curves, Soils and Foundations, 56(2), 291-300.
50) Wijaya M., Leong, E.C. (2017): Modelling the effect of density on the unimodal soil-water characteristic curve, Géotechnique 0 0:0, 1-9. DOI: http://dx.doi.org/10.1680/jgeot.15.P270
51) Wijaya, M. and Leong, E.C. (2018): Discussion of “Permeability function for oil sands tailings undergoing volume change during drying” Canadian Geotechnical Journal, 55(1), 191-207, https://doi.org/10.1139/cgj-2016-0486”, Can. Geotech. J., https://doi.org/10.1139/cgj-2018-0136
52) Wong, J.M., Elwood, D.E.Y., and Fredlund, D.G (2018): Use of a 3D scanner for shrinkage curve tests. Can. Geotech. J., https://doi.org/10.1139/cgj-2017-0700
53) Zhang, L.M., and Chen, Q. (2005): Predicting bimodal soil-water characteristic curves. Journal of Geotechnical and Geoenvironmental Engineering, 131, 666-670.
54) Zhang, F., Wilson, G.W. and Fredlund, D.G. (2017): Permeability function for oil sands tailings undergoing volume change during drying. Can. Geotech. J., 55(2), 191-207.
55) Zhou, W.-H., Yuen, K.-V., and Tan, F. (2014): Estimation of soil–water characteristic curve and relative permeability for granular soils with different initial dry densities. Engineering Geology, 179(0):1-9.
56) Zou, L. (2018): Effects of grain-size distribution and hysteresis on soil-water characteristic curve (SWCC), PhD thesis, Nanyang Technological University, Singapore.
57) Zou, L. and Leong, E.C. (2019): A classification tree guide to soil-water characteristic curve test for soils with bimodal grain-size distribution. Geotechnical Engineering Journal of the SEAGS & AGSSEA, 50(1) (In Press).