Lorentz violation and neutrinos∗

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Abstract

Neutrino oscillations provide an opportunity for sensitive tests of Lorentz invariance. This talk reviews some aspects of Lorentz violation in neutrinos and the prospect of testing Lorentz invariance in neutrino-oscillation experiments. A general Lorentz-violating theory for neutrinos is discussed, and some signals of Lorentz violation are identified.

1 Introduction

Neutrinos offer a promising avenue for the detection of new physics. Evidence for neutrino oscillations already indicates that the minimal Standard Model (SM) of particle physics needs modification.1 The experiments providing this evidence are in an excellent position to detect tiny violations of Lorentz invariance2 that may exist as the low-energy remnants of Planck-scale physics.3 Here we discuss a general theoretical framework describing the free propagation of neutrinos in the presence of Lorentz violation. We examine the effects of Lorentz violation on neutrino oscillations and identify unconventional behavior and experimental signals.

At attainable energies, violations of Lorentz invariance are described by a framework called the Standard-Model Extension (SME).4 While the SME was originally motivated by string theory,5 it also encompasses other origins for Lorentz violation such as spacetime varying couplings.5 The SME provides the basis for a large number of experiments.6 In neutrinos, it gives a consistent theoretical framework for the study of Lorentz violation in oscillations and other phenomena. Neutrino-oscillation experiments provide sensitivity to Lorentz-violating effects2,7–10 that rival the best tests in any other sector of the SME.11,12

∗presented at the Third Meeting on CPT and Lorentz Symmetry, Bloomington, Indiana, August, 2004.
Remarkably, the current evidence for neutrino oscillations lies at levels where Planck-suppressed effects might be expected to appear. Furthermore, the possibility remains that Lorentz violation may be responsible at least in part for the observed oscillations. Further analysis and experimentation is needed to determine the extent to which Lorentz violation may play a role in neutrino oscillations.

2 Framework

In the SME, the propagation of neutrinos is governed by a modified multigeneration Dirac equation:

\[(i\Gamma^\nu_{AB}\partial_\nu - M_{AB})\nu_B = 0\]  \hspace{1cm} (1)

where three neutrino fields and their charge conjugates are included in order to allow for general Dirac- and Majorana-type terms; \(\nu_A = \{\nu_e, \nu_\mu, \nu_\tau, \nu^C_e, \nu^C_\mu, \nu^C_\tau\}\). Each of the quantities \(\Gamma^\nu_{AB}\) and \(M_{AB}\) are 4×4 constant matrices in spinor space.

Here we have included all terms arising from operators of renormalizable dimension, but in general, higher derivative terms can occur and may be important. 12 13 It is also straightforward to include additional generations in order to accommodate sterile neutrinos. Common Lorentz-conserving scenarios exist as subsets of the general case.

The matrices \(\Gamma^a_{AB}\) and \(M_{AB}\) can be decomposed using the basis of \(\gamma\) matrices. Following standard conventions, we define

\[
\Gamma^\nu_{AB} := \gamma^\nu \delta_{AB} + c_{AB}^\nu \gamma_\mu + d_{AB}^\mu \gamma^5 \gamma_\mu + e_{AB}^\nu + i f_{AB}^\nu \gamma^5 + \frac{1}{2} g_{AB}^\mu \sigma_\mu, \\
M_{AB} := m_{AB} + i m_5_{AB} \gamma^5 + a_{AB}^\mu \gamma_\mu + b_{AB}^\mu \gamma^5 \gamma_\mu + \frac{1}{2} H_{AB}^\mu \sigma_\mu. 
\]  \hspace{1cm} (2)

In these equations, the masses \(m\) and \(m_5\) are Lorentz and CPT conserving. The coefficients \(c, d, H\) are CPT conserving but Lorentz violating, while \(a, b, e, f, g\) are both CPT and Lorentz violating. Requiring hermiticity of the theory imposes the conditions \(\Gamma^\nu_{AB} = \gamma^0 (\Gamma^\nu_{BA})^\dagger \gamma^0\) and \(M_{AB} = \gamma^0 (M_{BA})^\dagger \gamma^0\), which implies all coefficients are hermitian in generation space.

Equation (1) provides a basis for a general Lorentz- and CPT-violating relativistic quantum mechanics for freely propagating neutrinos. Construction of the relativistic hamiltonian is complicated by the unconventional time-derivative term, but this difficulty may be overcome in a manner similar to that employed in the QED extension. \(^{14}\) The result is

\[
\mathcal{H} = \mathcal{H}_0 - \frac{1}{2} (\gamma^0 \delta \Gamma^0 \mathcal{H}_0 + \mathcal{H}_0 \gamma^0 \delta \Gamma^0) - \gamma^0 (i \delta \Gamma^j \partial_j - \delta M) \hspace{1cm} (3)
\]

where \(\mathcal{H}_0 = -\gamma^0 (i \gamma^j \partial_j - M_0)\) is the general Lorentz-conserving hamiltonian, \(M_0\) is the Lorentz-conserving part of \(M\), and \(\delta \Gamma, \delta M\) are the Lorentz-violating parts of \(\Gamma, M\).
A general treatment is possible but rather cumbersome and beyond the intended scope of this work. Therefore, we consider a simple physically reasonable case where oscillation between left- and right-handed neutrinos is highly suppressed. The resulting theory describes oscillations between three flavors of left-handed neutrinos due to mass or coefficients Lorentz violation. Within this restriction, a calculation gives a $6 \times 6$ effective hamiltonian describing the time evolution of active neutrinos and antineutrinos with momentum $\vec{p}$:

$$
\begin{pmatrix}
\nu_a(t; \vec{p}) \\
\bar{\nu}_a(t; \vec{p})
\end{pmatrix} = \exp(-i h_{\text{eff}} t)ab
\begin{pmatrix}
\nu_b(0; \vec{p}) \\
\bar{\nu}_b(0; \vec{p})
\end{pmatrix},
$$

(4)

where $\nu_a$ and $\bar{\nu}_a$ represent active neutrino (negative helicity) and antineutrino (positive helicity) states, and indices $a, b$ range over $\{e, \mu, \tau\}$. The effective hamiltonian is given by

$$
(h_{\text{eff}})_{ab} = |\vec{p}| \delta_{ab} + \frac{1}{2|\vec{p}|} \begin{pmatrix}
(m^2)_{ab} & 0 \\
0 & (\tilde{m}^2)^*_{ab}
\end{pmatrix}
$$

$$
+ \frac{1}{|\vec{p}|} \begin{pmatrix}
[(a_L)_{ab}^\mu p_\mu - (c_L)_{ab}^\mu p_\mu p_\nu]_{ab} & -i\sqrt{2}p_\mu (\epsilon_+)_{ab} \left[ (g_{\mu\sigma} p_\sigma + H_{\mu\nu}) C_{ab} \right] \\
i\sqrt{2}p_\mu (\epsilon_+)_{ab} \left[ (g_{\mu\sigma} p_\sigma + H_{\mu\nu}) C_{ab}^* \right] & [(a_L)_{ab}^\mu p_\mu - (c_L)_{ab}^\mu p_\mu p_\nu]_{ab}^*
\end{pmatrix}.
$$

(5)

This result assumes relativistic neutrinos with momentum $|\vec{p}|$ much larger than both mass and Lorentz-violating contributions. At leading order, the four momentum $p_\mu$ may be taken as $p_\mu = (|\vec{p}|; -\vec{p})$, and a suitable choice for $(\epsilon_+)^*_{ab} = (\epsilon_{ab})^* = \frac{1}{\sqrt{2}} (0; \hat{\epsilon}_1 + i\hat{\epsilon}_2)$, where $\hat{\epsilon}_1, \hat{\epsilon}_2$ are real and $\{\vec{p}/|\vec{p}|, \hat{\epsilon}_1, \hat{\epsilon}_2\}$ form a right-handed orthonormal triad.

The above hamiltonian is consistent with the standard seesaw mechanism, where the right-handed Majorana masses are much larger than Dirac or left-handed Majorana masses. However, the above equations apply to any situation where only left-handed neutrinos are allowed to propagate or intermix.

Only the first term in Eq. (5) arises from the minimal Standard Model. The second term corresponds to the usual massive-neutrino case without sterile neutrinos. The leading-order Lorentz-violating contributions are given by the last term. Lorentz-violating $\nu \leftrightarrow \nu$ mixing is controlled by the coefficient combinations $(c_L)_{ab}^\mu \equiv (c + d)_{ab}^\mu$ and $(a_L)_{ab} \equiv (a + b)_{ab}^\mu$. The remaining coefficients, $(g_{\mu\sigma} C)_{ab}$ and $(H_{\mu\nu} C)_{ab}$, arise from gauge-violating Majorana-like couplings and generate Lorentz-violating $\nu \leftrightarrow \bar{\nu}$ mixing resulting in lepton-number violations. Note that some combinations of coefficients are unobservable, either because of symmetries or because they can be removed through field redefinitions.4,14–16

Although this theory is observer independent and therefore independent of choice of coordinates, it is important to specify a frame for reporting experimental results. By convention this frame is taken as a Sun-centered celestial equatorial frame with coordinates $\{T, X, Y, Z\}$.
3 Features

A complete analysis of this construction is hampered by its generality and lies outside our present scope. Two lines of attack have been initiated in order to understand the theoretical and experimental implications of Lorentz violation. The first involves the construction of simple models that illustrate the various unconventional features and their potential to explain experimental data. Some possibilities are considered in the next section. An alternative strategy is to search for ‘smoking-gun’ signals that are indicators of Lorentz violation.

The many coefficients for Lorentz violation that appear in the effective Hamiltonian (5) introduce a plethora of new effects, including unusual energy dependence, dynamics dependent on the direction of propagation, and neutrino-antineutrino mixing. Below we list six classes of model-independent features that represent characteristic signals of Lorentz violation in neutrino-oscillation experiments. A positive signal in any one of these classes would suggest the presence of Lorentz violation.

Spectral anomalies. Each of the coefficients for Lorentz violation introduces energy dependence differing from the usual mass case. In the conventional massive-neutrino case, oscillations of neutrinos in the vacuum are determined by the energy-independent mixing angles $\theta_{12}$, $\theta_{13}$, $\theta_{23}$, phase $\delta$, and mass-squared differences $\delta m$, $\Delta m$. In this case, energy dependence enters the oscillation probabilities through the oscillation lengths $L_0 \propto E/\delta m^2, E/\Delta m^2$. In contrast, coefficients for Lorentz violation can cause oscillation lengths that are either constant or decrease linearly with energy. For example, a simple model with only $c_L$ coefficients has much of the same structure as the mass case except that it has oscillation lengths $L_0 \propto (E\delta c_L)^{-1}, (E\Delta c_L)^{-1}$. Combinations of coefficients with different dimension can lead to very complex energy dependence in both the oscillation lengths and the mixing angles. Detection of a vacuum oscillation length that differs from the usual $\propto E$ dependence or of energy dependence in the vacuum mixing angles would constitute a clear signal of Lorentz violation.

$L-E$ conflicts. This class of signal refers to a set of null and positive measurements that conflict in any scenarios based on mass-squared differences. In the usual case, baseline and energy dependence enter through the ratio $L/L_0 \propto L/E$. So experiments that measure the same oscillation mode at similar ranges in $L/E$ will have comparable sensitivity to neutrino oscillations. Because of the unusual energy dependence, in Lorentz-violating scenarios this may no longer be the case. If oscillations are caused by coefficients for Lorentz violation, it is possible that experiments operating in the same region of $L/E$ space could see drastically different oscillation probabilities. A measurement of this effect would indicate physics beyond the simple mass case and would
constitute a possible signal of Lorentz violation.

*Periodic variations.* This signal indicates a violation of rotation invariance and would commonly manifest itself as either sidereal or annual variations in neutrino flux. The appearance of $p$ in the effective Hamiltonian (5) implies that oscillations can depend on the direction of the propagation. In terrestrial experiments, where both the detector and the source are fixed relative to the Earth, the direction of the neutrino propagation changes as the Earth rotates. This can lead to periodic variations at the sidereal frequency $\omega_{\oplus} \approx 2\pi/(23\ h\ 56\ min)$. For solar neutrinos, the variation in propagation of the detected neutrinos is due to the orbital motion of the Earth and can cause annual variations.

*Compass asymmetries.* This class includes time-independent effects of rotation-invariance violations. They consist of unexplained directional asymmetries in the observed neutrino flux. For terrestrial experiments, averaging over time eliminates any sidereal variations, but may leave a dependence on the direction of propagation as seen from the laboratory. This can result in asymmetries between the compass directions north, south, east, and west.

*Neutrino-antineutrino mixing.* This class includes any measurement that can be traced to $\nu \leftrightarrow \bar{\nu}$ oscillations. This would indicate lepton-number violation that could be due to $g$ and $H$ coefficients. All of these coefficients introduce rotation violation, so this signal may be accompanied by direction-dependent signals.

*Classic CPT test.* This is the traditional test of CPT involving searches for violations of the relationship $P_{\nu_b \rightarrow \nu_a}(t) = P_{\bar{\nu}_a \rightarrow \bar{\nu}_b}(t)$. This equation holds provided CPT is unbroken. An additional result holds in the event of lepton-number violation: $P_{\nu_b \leftrightarrow \nu_a}(t) = P_{\bar{\nu}_a \leftrightarrow \bar{\nu}_b}(t)$, if CPT is unbroken. A measurement that contradicts either of these relations is a signal of CPT violation and would therefore imply Lorentz violation.

### 4 Illustrative models

In this section, we discuss some simple subsets of the general case (5) that exhibit some of the unconventional effects. While in most cases these models are not expected to agree with all existing data, they do provide useful insight into the novel behavior that Lorentz violation can introduce. An interesting open challenge is to identify general classes of realistic models that could be compared to experiment. The bicycle model\(^7\) and its variants offer possibilities that have no mass-squared differences and few degrees of freedom.
4.1 Fried-chicken models

One simple class of models are those dubbed ‘fried-chicken’ (FC) models. The idea behind these is to restrict attention to direction-independent behavior by only considering isotropic coefficients. This restriction reduces the effective hamiltonian to

\[
(h_{\text{eff}})^{\text{FC}}_{ab} = \text{diag} \left[ \left( \tilde{m}^2 / (2E) + (a_L)^T - \frac{4}{3} (c_L)^{T T} E \right)_{ab} , \right. \\
\left. \left( \tilde{m}^2 / (2E) - (a_L)^T - \frac{4}{3} (c_L)^{T T} E \right)^*_{ab} \right].
\]

(6)

A majority of the Lorentz-violating models considered in the literature are subsets of this general FC model.\(^9\)

The differences in energy dependence between the various types of coefficients and mass is apparent in Eq. (6). FC models provide a workable context for studying the unconventional energy dependence without the complication of direction-dependent effects. However, it should be noted that Eq. (6) is a highly frame dependent. Isotropy in a given frame necessarily implies anisotropy in other frames boosted with respect to the isotropic one. While it may be appealing to impose isotropy in a frame such as the cosmic-microwave-background frame, it is difficult to motivate theoretically.

4.2 Vector models

In contrast to FC models, vector models are designed to study the effects of rotation-symmetry violation. These models contain coefficients that can be viewed as three-dimensional vectors that point in given directions. They are particularly useful in determining the types of signals that a given experiment might expect to see if rotation symmetry is violated.

As an example consider a model where only the coefficients \((a_L)^X_{\mu \tau}, (a_L)^Y_{\mu \tau}, (c_L)^{TX}_{\mu \tau}, \text{ and } (c_L)^{TY}_{\mu \tau}\) are nonzero. Each of these can be viewed as vectors lying in the Earth’s equatorial plane. They are chosen to illustrate the periodic signals discussed in the previous section. With the above choice, we would see maximal mixing between \(\nu_\mu \leftrightarrow \nu_\tau\) and \(\bar{\nu}_\mu \leftrightarrow \bar{\nu}_\tau\), which are relevant oscillation modes for atmospheric neutrinos. So, this simple special case may serve as a test model for searches for sidereal variations in atmospheric neutrinos.

The vacuum oscillation probability for a terrestrial experiment is

\[
P_{\nu_\mu \leftrightarrow \nu_\tau} = \sin^2 L \left( (A_s)_{\mu \tau} \sin \omega \Theta T_\oplus + (A_c)_{\mu \tau} \cos \omega \Theta T_\oplus \right),
\]

(7)

where

\[
(A_s)_{\mu \tau} = \hat{N}^X \left( (a_L)^X_{\mu \tau} - 2E(c_L)^{TX}_{\mu \tau} \right) - \hat{N}^X \left( (a_L)^Y_{\mu \tau} - 2E(c_L)^{TY}_{\mu \tau} \right),
\]

(8)

\[
(A_c)_{\mu \tau} = -\hat{N}^X \left( (a_L)^X_{\mu \tau} - 2E(c_L)^{TX}_{\mu \tau} \right) - \hat{N}^Y \left( (a_L)^Y_{\mu \tau} - 2E(c_L)^{TY}_{\mu \tau} \right).
\]

(9)
Here \( N^X \) and \( N^Y \) are factors that are determined by the direction of the neutrino propagation as seen in the laboratory. In this example, both the unusual energy dependence and the sidereal variations are readily apparent. The dependence on beam direction through \( N^X \) and \( N^Y \) implies that a time average in this model also gives rise to compass asymmetries. These could be sought in atmospheric experiments and other experiments where neutrinos originate from different compass directions.

### 4.3 The bicycle model

One class of interesting special cases are those that involve a Lorentz-violating seesaw mechanism. The resulting dynamics can be dramatically different than what is naively expected from the effective Hamiltonian (5). One such model is the bicycle model.\(^7\) This model is also interesting because it crudely matches the basic features seen in solar and atmospheric neutrinos using only two degrees of freedom.

The bicycle model consists of an isotropic \( c_L \) with nonzero element \( \frac{1}{2}(c_L)_{zz}^T \equiv 2\hat{c} > 0 \) and an anisotropic \( a_L \) with degenerate nonzero real elements \( (a_L)_{e\mu} \equiv (a_L)_{e\tau}^Z \equiv \hat{a}/\sqrt{2} \). The vacuum oscillation probabilities are

\[
\begin{align*}
P_{\nu_e \to \nu_e} &= 1 - 4 \sin^2 \theta \cos^2 \theta \sin^2(\Delta_{31} L/2) , \\
P_{\nu_e \leftrightarrow \nu_\mu} &= P_{\nu_e \leftrightarrow \nu_\tau} = 2 \sin^2 \theta \cos^2 \theta \sin^2(\Delta_{31} L/2) , \\
\nonumber
P_{\nu_\mu \to \nu_\mu} &= P_{\nu_\tau \to \nu_\tau} = 1 - \sin^2 \theta \sin^2(\Delta_{21} L/2) \\
&\quad - \sin^2 \theta \cos^2 \theta \sin^2(\Delta_{31} L/2) - \cos^2 \theta \sin^2(\Delta_{32} L/2) , \\
\nonumber
P_{\nu_\mu \leftrightarrow \nu_\tau} &= \sin^2 \theta \sin^2(\Delta_{21} L/2) \\
&\quad - \sin^2 \theta \cos^2 \theta \sin^2(\Delta_{31} L/2) + \cos^2 \theta \sin^2(\Delta_{32} L/2) , \quad (10)
\end{align*}
\]

where

\[
\begin{align*}
\Delta_{21} &= \sqrt{(\hat{c} E)^2 + (\hat{a} \cos \Theta)^2} + \hat{c} E , \\
\Delta_{31} &= 2\sqrt{(\hat{c} E)^2 + (\hat{a} \cos \Theta)^2} , \\
\Delta_{32} &= \sqrt{(\hat{c} E)^2 + (\hat{a} \cos \Theta)^2} - \hat{c} E , \\
\sin^2 \theta &= \frac{1}{2} \left[ 1 - \hat{c} E/\sqrt{(\hat{c} E)^2 + (\hat{a} \cos \Theta)^2} \right] , \quad (11)
\end{align*}
\]

and where \( \Theta \) is defined as the angle between the celestial north pole and the direction of propagation. These probabilities also hold for antineutrinos, which implies that it is possible to violate \( CPT \) and not produce the last signal discussed in Sec. 3.

An important feature of this model is that at high energies, \( E \gg |\hat{a}|/\hat{c} \), a seesaw mechanism takes effect and oscillations reduce to two-generation mixing with \( P_{\nu_\mu \leftrightarrow \nu_\tau} \approx \sin^2(\Delta_{32} L/2) \), \( \Delta_{32} \approx \hat{a}^2 \cos^2 \Theta / 2\hat{c} E \). The energy dependence in
this regime mimics exactly that of the usual mass case. However, the quantity that takes the place of mass, the pseudomass \( \Delta m^2 = \bar{a}^2 \cos^2 \Theta / \bar{c} \), is dependent on the direction of propagation. So it is possible to construct models with conventional energy dependence but unconventional direction dependence.

5 Short baseline experiments

Some circumstances are amenable to more general analyses. One case where this is true is when the baseline of an experiment is short compared to the oscillation lengths given by the Hamiltonian (5). In this situation, the transition amplitudes can be linearized, which results in leading order probabilities given by

\[
P_{\nu_a \rightarrow \nu_a} \simeq \begin{cases} 
1 - \sum_{c,c \neq a} P_{\nu_a \rightarrow \nu_c}, & a = b, \\
|(h_{\text{eff}})_{ab}|^2 L^2 / (\hbar c)^2, & a \neq b.
\end{cases}
\]

This approximation allows direct access to the coefficients for Lorentz violation without the complication of diagonalizing the Hamiltonian. This makes an analysis of the general Hamiltonian (5) more practical.

This type of analysis may be relevant for the LSND experiment, which is consistent with a small oscillation probability \( P_{\bar{\nu}_\mu \rightarrow \bar{\nu}_e} \simeq 0.26 \) over a short baseline of about 30 m. This result is of particular interest because it does not seem to fit into the simple three-generation solution to solar and atmospheric data. The possibility exists that Lorentz violation may provide a solution.

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