Abstract

We investigate the production and the decays of pseudo-Goldstone bosons (PGBs) predicted by technicolor theories with the GIM mechanism (TC-GIM). The TC-GIM models contain exotic fermion families that do not interact under weak $SU(2)$, but they do have color and hypercharge interactions. These fermions form PGBs, which are the lightest exotic particles in the TC-GIM models. The spectrum of PGBs consists of color octets, leptoquarks and neutral particles. The masses of leptoquarks and color octets depend on a free parameter—the scale of confining interactions. Characteristic for TC-GIM models is a very light ($\approx 1$ GeV) neutral particle with anomalous couplings to gauge boson pairs. We show how current experiments constrain the free parameters of the models. The best tests are provided by the $pp \rightarrow TT$ and $e^+e^- \rightarrow P^0\gamma$ reactions. Experiments at LHC and NLC can find PGBs of TC-GIM models in a wide range of parameter space. However, TC-GIM models can be distinguished from other TC models only if several PGBs are discovered.

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1 Introduction

Models of weak interactions based on the technicolor idea [1] need a mechanism which communicates the symmetry breaking to the quarks and leptons. One way of coupling the technifermions to the quarks and leptons is the introduction of extended-technicolor (ETC) interactions [2], which generate fermion masses. However, simple extended-technicolor models suffer unacceptable flavor-changing neutral currents. More limitations on viable technicolor theories are imposed by measurements of the electroweak parameters [4].

An interesting solution to these problems are models which incorporate the Glashow-Iliopoulous-Maiani (GIM) mechanism [5]. The first technicolor models that used GIM mechanism to avoid unacceptably large flavor-changing neutral currents were the composite technicolor standard models [6]. These models realize the GIM mechanism by separating the ETC interactions into several ETC groups. There are separate ETC groups for the left-handed fermion fields, the right-handed up quarks, and the right-handed down quarks. Such construction introduces a large global symmetry associated with quark flavor. This flavor symmetry is the essence of the GIM mechanism. Breaking of the global symmetry is responsible for the fermion masses and the quark mixing—the existence of the Kobayashi-Maskawa matrix. However, the composite technicolor models presented in Ref. [6] were toy models of weak interactions, since the models did not incorporate leptons.

Realistic technicolor models with the GIM mechanism were described in Refs. [7] and [8]. We will refer to these models as technicolor-GIM models (TC-GIM). Not only do the TC-GIM models avoid trouble with the flavor-changing neutral currents, but they also limit the number of technifermion doublets to one, thereby avoiding conflict with precise electroweak measurements [4]. A noticeable feature of these models is the presence of exotic light fermions. From the point of view of a model builder, the most difficult task is to create a model with an appropriate pattern of breaking flavor and gauge symmetries. The symmetry breaking is achieved by introducing numerous heavy fermion fields and gauge bosons. The light fermions that were mentioned before exist in the TC-GIM models only to cancel certain anomalies. In QCD-like models the light fermions seem to be a necessary ingredient. Since the light fermions are a necessary feature of TC-GIM models, their signatures are the best place to test and study this kind of models.

In this paper we explore the phenomenological consequences of the light fermion sector. The light fermions transform under the ETC groups and also some additional confining interactions. The scale of confining interactions, depending on a particular model, can be from tens to hundreds of GeV. Below the confinement scale, there are pseudo-Goldstone bosons (PGBs) in the particle spectrum whose constituents are the light fermions. The PGBs are the lightest exotic particles in the spectrum of the TC-GIM models. While present experiments put a lower bound on the scale of the confining interactions, future experiments
may find signatures of the PGBs. The light fermions, constituents of the PGBs, do not transform under the ordinary $SU(2)_L$ gauge group. Their interactions with the quarks and leptons are mediated by the ETC gauge bosons.

PGBs in TC-GIM have a different origin than PGBs in other types of TC models. Usually, PGBs are associated with chiral symmetry breaking by the technicolor group. In TC-GIM models, PGBs arise from dynamical breaking of symmetry by some new gauge interactions. Of course, the TC group also breaks chiral symmetries of technifermions. However, in TC-GIM, there is only one doublet of technifermions and the Goldstone bosons become longitudinal degrees of freedom of the $W^\pm$ and the $Z$. TC-GIM models, like any other TC models, have techni-$\rho$ mesons, but these are heavier than the technicolor scale and more difficult to observe than PGBs. Therefore, the PGBs are the best test of TC-GIM models. If PGBs are discovered and some of their properties are known, it will be possible to distinguish between different TC scenarios.

In the next section, we begin with a short introduction to the TC-GIM models and explain various possible realizations of the light fermion sectors. In Section 3, we present the spectrum of the PGBs and their couplings to ordinary particles. Section 4 contains the discussion of the PGB phenomenology. We leave some remarks about the scales of the interactions that confine the light fermions until Section 5.

## 2 TC-GIM models

The basic building blocks of the TC-GIM models are $SU(N)$ gauge groups and chiral fermions. Because many gauge groups are necessary, a special notation “moose notation” is helpful in describing these models 4. An $SU(N)$ gauge group is represented by a circle. Fermions in the (anti-)fundamental representation are depicted by an (in-)out-going line. A fermion line connecting two circles represents fermions transforming under both gauge groups depicted by the circles. A line whose one end is not connected to any circle indicates that the fermions transform under a gauge group and a global symmetry group. A graphic illustration of these ideas is presented in Fig. 1, where a simple moose diagram represents three fermion fields transforming under two global groups and two gauge groups. When referring to fermion fields, we will label the fermions $[NM]$, where $N$ and $M$ stand for $SU(N)$ and $SU(M)$ groups under which the fermions transform.

All gauge groups are guaranteed to be anomaly-free by having the same number of ingoing and outgoing fermion lines. This way, all fermions transforming under a given gauge group form a vector representation, which is anomaly-free. The requirement of anomaly cancellation is how the light fermions find their way into TC-GIM models. The ETC gauge groups would not be anomaly-free without these additional fermions.
Figure 1: An example of moose diagram. The circles represent \( SU(N) \) and \( SU(M) \) gauge groups, the lines represent fermion fields. Line endings without circles represent global symmetry groups — \( SU(M) \) and \( SU(N) \).

Figure 2: The full TC-GIM model with a low-scale PGB sector.

The structure of a TC-GIM model described in Ref. [7] is illustrated in Fig. 2. The gauge groups are ordered by their scale, with the lowest scale groups at the bottom of the moose. There are two \( SU(S - 1) \) groups labeled U and D. The fermions transforming under these groups are the light fermions forming PGBs. The single \( SU(2) \) group is the familiar group of weak interactions. The model contains three separate ETC groups: one for the left-handed quarks and leptons, one for the right-handed down quarks and charged leptons, and one for the right-handed up quarks and neutrinos. These are the three \( SU(N + 12) \) groups. \( N \) is the number of techni-colors, while 12 is the number of left-handed doublets of quarks and leptons. The fact that there are right-handed neutrinos in the model does not present any problem.

There exist several plausible mechanisms to ensure small masses of the neutrinos [7]. The
two $SU(S)$’s together with the $SU(2S)$ and $SU(N)$ groups at the top of the moose are the highest scale groups. They break the ETC groups and merge several $SU(N)$ subgroups into one technicolor $SU(N)$. As usual, the technifermion condensates break the weak $SU(2)$. We will not elaborate on the details of the TC-GIM model building, instead we refer the reader to Refs. [7] and [8]. The $[n + 12_L, 2]$ fermions include all left-handed quarks and leptons, and also one technifermion doublet. The right-handed quarks and leptons are contained in the $[n + 12_U, 1]$ and $[n + 12_D, 1]$ fermion lines.

We will focus on the PGBs formed from fermions transforming under $SU(S - 1)$ groups. The $SU(S - 1)$ groups have to become strongly interacting at some scale, otherwise there would be massless or very light fermions (with masses comparable to those of leptons and quarks) present in the particle spectrum. The scale of the $SU(S - 1)$ interactions is not related to the technicolor scale. In most technicolor models, the lightest exotic particles are PGBs formed by technifermions. Such PGBs form when the TC group dynamically breaks chiral symmetries of the technifermions. The same process gives masses to the electroweak gauge bosons, thus the scale of these PGBs is related to the scale of electroweak symmetry breaking. The situation is different in TC-GIM models. PGBs are created when the $SU(S - 1)$ interactions form fermion condensates. Therefore, the scale at which PGBs form in TC-GIM models is a free parameter but there are both upper and lower bounds on its value. It cannot be too low in order to avoid conflict with experiments. On the other hand, too large a scale would give too large contributions to lepton and quark masses, or be so high as to upset the hierarchy of symmetry breaking. We postpone the discussion of these problems to Sec. 5, after we discuss the phenomenological constraints.

There are several possibilities for constructing the light-fermion sectors. The one illustrated in Fig. 2 is characterized by a relatively low scale of the $SU(S - 1)$ interactions. We will later show that this specific model cannot accommodate heavy PGBs. In particular, the lightest leptoquarks have to be lighter than approximately 50 GeV, therefore the low scale model is ruled out. From now on, we will suppress the structure of the models irrelevant to PGB study, showing only parts of the moose we are interested in: the ETC groups, the ordinary fermions and the light-fermion sector. The up and down sectors of the moose are identical so it is enough to describe any one of the two sectors.

A model similar to the one presented in Fig. 2 was also presented in Ref. [7]. We illustrate the relevant part in Fig. 3. This model is characterized by a higher scale of the $SU(S - 1)$ interactions. The difference between this model and the one introduced before is the addition of $[12_A, S - 1]$ and $[S - 1, 12_A]$ fermion fields. When the $SU(S - 1)$ interactions become strong, we assume that the $[12_A, S - 1]$ fermions form condensates with the $[S - 1, n + 12_D]$ fermions. Likewise $[n + 12_L, S - 1]$ fermions form condensates with the $[S - 1, 12_A]$. This is clearly not the only possibility. If the $[n + 12_L, S - 1]$ fermions formed a condensate with the $[S - 1, n + 12_D]$ fermions, this model would not be very different from the model depicted
It is possible to construct models whose light fermion sectors have unambiguous vacuum state. Such a model was introduced in Ref. [8], whose relevant part we reproduce in Fig. 4a. A variation of such a model is illustrated in Fig. 4b. There are two separate $SU(S - 1)$ groups in these models. For simplicity, we assume that both groups are characterized by the same scale, but it would not present any difficulties to deal with different scales. In these models, the $[12_A, S - 1_2]$ fermions form condensates with the $[S - 1_2, n + 12_D]$ fermions, and similarly $[n + 12_L, S - 1_1]$ with $[S - 1_1, 12_A]$. We will refer to the model illustrated in Fig. 4a as
the low-scale model, because the scale of $SU(S-1)$ interactions in this model has a strong upper bound. All other models presented here avoid that upper limit, so we will refer to them as the high-scale models.

In the limit where there is no lepton mixing and the K-M matrix is diagonal, there are separate conserved lepton and quark numbers for each flavor of quarks and leptons. The exotic fermions transform under the global $U(1)$ symmetries associated with the flavor numbers. Therefore, we can attribute flavor to the exotic fermions and, consequently, refer to them as ‘exotic electron’ or ‘exotic top quark’, etc. The exotic and usual fermions carry the same quantum numbers of color and electric charge, however the exotic fermions are always singlets of the weak $SU(2)_L$.

3 Spectrum and couplings

We will now describe the PGBs formed below the scale of the $SU(S-1)$ interactions. We first enumerate the PGBs and estimate their masses. We then derive the couplings of the PGBs to ordinary quarks, leptons and gauge bosons. In what follows, we use chiral Lagrangian techniques. Such a description is valid for the momenta of the PGBs smaller than the chiral symmetry breaking scale of the $SU(S-1)$ interactions.

In the low-scale model, the PGBs are associated with the breaking of the $SU(12) \times SU(12)$ flavor symmetry, where the $SU(12)$ groups are subgroups of $SU(n+12)$ ETC groups. The $SU(12)^2$ global symmetry breaks down to $SU(12)_L$, so there will be $12^2 - 1 = 143$ PGBs. The symmetry group of the high-scale models is $SU(12)^4$. This group is dynamically broken to $SU(12)^2$, doubling the number of the PGBs. Not all bosons remain in the particle spectrum, some may be eaten by the Higgs mechanism. The $SU(12)^2$ or $SU(12)^4$ are not exact symmetries. Once we take into account symmetry breaking interactions, the PGBs will acquire masses.

We describe here the PGBs formed in the sector associated with the down quarks. Precisely the same analysis applies to the up sector. The $SU(12)$ symmetries contain the ordinary color $SU(3)$ gauge group. The exotic quarks and leptons have the same charge assignment as the quarks and leptons. Therefore, there exist two flavor $SU(3)$ symmetries embedded in the $SU(12)$. These symmetries are associated with independent rotations in the $(d,s,b)$ and $(e,\mu,\tau)$ spaces, where we use the same letters to describe the ordinary or exotic fermions.

Let $Q_c$ and $L$ denote flavor $SU(3)$ triplets:

$$Q_c = \begin{pmatrix} d_c \\ s_c \\ b_c \end{pmatrix}, \quad L = \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}$$
and $\lambda^i$ denote the Gell-Mann matrices, normalized such that $Tr(\lambda^i\lambda^j) = \frac{1}{2}\delta^{ij}$. The upper index refers to flavor space; the lower one to color space.

We classify the PGBs according to the embedding of the $SU(3)_{\text{color}} \times SU(3)_Q \times SU(3)_L$ symmetry in the $SU(12)$ group. The spectrum of PGBs is a straightforward generalization of the spectrum in the old one-family technicolor model \cite{10}. The 143 PGBs can be written as the following combinations of fields

\begin{align}
\theta^i_a &\sim \sqrt{2}\bar{Q}_a \gamma_5^i \lambda^i_Q, \\
\theta_a &\sim \frac{1}{\sqrt{2}}\bar{Q}_a \lambda^a_Q, \\
T^i_c &\sim \sqrt{2}\bar{Q}_c \gamma_5^i \lambda^i_L, \\
\bar{T}^i_c &\sim \frac{1}{\sqrt{2}}\bar{L}_c \lambda^i_L, \\
\Pi^i &\sim \frac{1}{2} \left( \bar{Q}_c \gamma_5^i \lambda^i_Q + \bar{L}_c \gamma_5^i \lambda^i_L \right), \\
P^i &\sim \frac{1}{\sqrt{2}} \left( \bar{Q}_c \gamma_5^i \lambda^i_Q + 3\bar{L}_c \gamma_5^i \lambda^i_L \right), \\
P^0 &\sim \frac{1}{6\sqrt{2}} \left( \bar{Q}_c \gamma_5^i \lambda^i_Q - 3\bar{L}_c \gamma_5^i \lambda^i_L \right),
\end{align}

where $i, a = 1, \ldots, 8$ and $c = 1, 2, 3$. Whenever a flavor or color matrix is omitted, it should be understood that the identity matrix is present in the relevant space.

The 64 $\theta^i_a$ bosons and 8 $\theta_a$ are color octets. The 48 $T^i_c, \bar{T}^i_c$ and the 6 $T_c, T_c$ are color triplets. The $T$'s carry both quark and lepton numbers. We will refer to them as leptoquarks. There are also 17 color singlet states, which are the $\Pi^i, P^i$ and $P^0$. Color singlet and octet states do not carry electric charges, the leptoquarks have charge $-\frac{2}{3}e$ ($+\frac{2}{3}e$ for the leptoquarks in the up sector). The spectrum of the high-scale models is a simple replication of the spectrum just described.

In order to estimate masses of the PGBs, we need to itemize terms that break the $SU(12)$ global symmetries. In the low-scale model there is an explicit mass term for the exotic fermions. The mass term originates from multi-fermion operators \cite{7}. Let

$$\Sigma = \exp\left(2iT_\alpha \pi^a / f_{S-1}\right)$$

be the nonlinear representation of the PGBs where $T_\alpha$ are the $SU(12)$ matrices. Let $L$ and $D$ be the $SU(12)$ subgroups of the ETC groups $SU(N + 12)_L$ and $SU(N + 12)_D$ under which ordinary fermions transform linearly

$$\psi_L \rightarrow L\psi_L \text{ and } d_R \rightarrow Dd_R.$$  

Then $\Sigma$ transforms in the following manner

$$\Sigma \rightarrow L^\dagger \Sigma D.$$
The mass matrix for the exotic fermions is related to the mass matrix for the quarks and leptons: 

\[ M = m \left( \frac{f_{ETC}}{v_{TC}} \right)^2, \]

where \( f_{ETC} \) is the scale of the ETC interactions, \( v_{TC} \) is the value of the technicolor condensate, which is approximately 250 GeV while \( m \) is the mass matrix for the down quarks and charged leptons. The lowest-order contribution to the PGBs masses comes from the term

\[ v_{S-1}^3 tr(\Sigma^1 M) + h.c., \]

which gives the following mass-squared matrix:

\[ (\Delta m_{ab})^2 = 8 v_{S-1}^3 \cdot tr\left(T_a T_b M\right). \]

Color and electromagnetic interactions also break the \( SU(12) \times SU(12) \) symmetry. The standard approach in computing the color and electromagnetic contributions is to rescale the electromagnetic mass splitting in the \( \pi^\pm - \pi^0 \) system \[11\]. For the charged pions the leading effect comes from a one-photon exchange. An exchange of one gluon is similar in structure, except for different coupling constant and some \( SU(3) \) group factors. Therefore, the contribution to the color octet (triplet) masses can be related to the pion mass difference

\[ \frac{(\Delta m_\theta)^2}{m_{\pi^\pm}^2 - m_{\pi^0}^2} = \left( \frac{f_{S-1}}{f_{\pi}} \right)^2 \frac{\alpha_{QCD}(f_{S-1})}{\alpha_{em}} 3 \left( \frac{4}{3} \right), \]

where the factor of 3 applies to the octet pseudos and \( \frac{4}{3} \) to the triplets. The numerical value of this contribution to the masses of the color octet PGBs is

\[ \Delta m_\theta \approx \left( \frac{f_{S-1}}{15\text{GeV}} \right) 45 \text{ GeV}. \]

Electromagnetic contributions to the leptoquark masses can be computed in the same manner. Of course, instead of group theory factors there is a factor of \( \frac{4}{3} \), which is the charge squared.

The scale \( f_{S-1} \) cannot exceed 15 GeV in case of the low-scale model. We will later explain how this bound is obtained. Consequently, the lightest PGB in this model is the color singlet boson associated with symmetry breaking by the electron mass and its mass is about 1 GeV. The lightest leptoquarks in this model have masses approximately 50 GeV. Such a low-scale model is therefore ruled out, as we will show in the next chapter when we discuss leptoquark searches. However, in the high-scale models, the scale of the \( SU(S - 1) \) interactions can be much larger. Of course, the same formula holds for the contributions to the masses from color and electromagnetic interactions. Therefore, the leptoquarks and the octet particles can be quite heavy with masses of the order of several hundred GeV’s.
We now estimate the masses of the PGBs in the high-scale models presented in Figs. 3 and 4a. We describe contributions arising from breaking of the $SU(12)_L$ and $SU(12)_D$ symmetries by fermion mass terms. These contributions are different from the ones in the low-scale model. The high-scale models do not contain multi-fermion operators that could give explicit mass terms for the light fermions. Instead, the flavor dynamics is generated at a high scale, and its low-energy manifestation is the mixing among the ETC gauge bosons in different ETC groups. Such a mixing generates the same masses for the ordinary fermions and the light ones.

As mentioned before, we assume that in the model of Fig. 3 certain fermion condensates form. The $[12_A, S-1]$ fermions form condensates with the $[S-1, n+12_D]$, and $[n+12_L, S-1]$ with $[S-1, 12_A]$. In the model presented in Fig. 4a this assumption is fulfilled automatically. As before, we describe the PGBs in terms of their nonlinear representations $\Sigma_1$ and $\Sigma_2$. $\Sigma_1$ refers to the condensate of $[12_A, S-1]$ and $[S-1, n+12_D]$, and $\Sigma_2$ to the condensate of $[n+12_L, S-1]$ and $[S-1, 12_A]$.

The $\Sigma_i$ matrices have the following transformation properties

$$\Sigma_1 \rightarrow A_1^\dagger \Sigma_1 D \quad \text{and} \quad \Sigma_2 \rightarrow L^\dagger \Sigma_2 A_2.$$  \hspace{1cm} (9)

The gauge group $A$ is weakly gauged, so the two sets of fermion fields $[12_A, S-1]$ and $[S-1, 12_A]$ can transform independently. The $SU(12) \times SU(12)$ group of $A_1 \times A_2$ transformations is broken by small terms proportional to the gauge coupling of $A$. The mass term for the quarks, leptons and the exotic fermions transforms as

$$m \rightarrow L^\dagger m D,$$  \hspace{1cm} (10)

and the generators of the $SU(S-1)_A$ group transform both under the $A_1$ and $A_2$ matrices:

$$T^a_A \rightarrow A_1^\dagger T^a_A A_1 \quad \text{and} \quad T^a_A \rightarrow A_2^\dagger T^a_A A_2.$$  \hspace{1cm} (11)

Having written all the symmetry properties, we are ready to estimate the masses of the PGBs. The lowest-order term contributing to the masses has the form

$$\frac{\alpha_{S-1}}{4\pi} f_{S-1}^2 tr\left(\Sigma_2 T^a_A \Sigma^\dagger_2 m \Sigma^\dagger_1 T^a_A \Sigma_1 m^\dagger\right).$$  \hspace{1cm} (12)

Such a contribution arises from the diagram illustrated in Fig. 5a. This term gives masses to the linear combination of PGBs: $\pi_+^a = \frac{1}{\sqrt{2}}(\pi_1^a + \pi_2^a)$, where the mass matrix squared is

$$\Delta m^2_{ab} = \frac{\alpha_{S-1}}{2\pi} [tr(m) tr(T_a T_b m) - tr(T_a m) tr(T_b m)].$$  \hspace{1cm} (13)

The above equation reveals an interesting feature of the PGBs spectrum in this model. The contribution to masses of the PGBs from the term in Eq. 12 does not depend on the scale
of $SU(S - 1)$ interactions. Thus, the masses of the neutral PGBs do not depend on the scale $f_{S-1}$ in the lowest order. The estimate for the masses depends on the value of $\alpha_{S-1}$ and an unknown coefficient of order one. For instance, the mass squared of the $P^0$ boson is about 1 GeV$^2$ times $\alpha_{S-1}$. Of course, there are also higher-order contributions to the PGBs masses. These can arise from the exchange of the ETC gauge bosons. Such terms will be proportional to additional powers of $f_{ETC}^2$ which are significant only for a very large $f_{S-1}$. The masses of color octet and triplet states will be much larger due to the gluon exchange contributions described in Eq. [4]. The orthogonal combination $\pi^a = \frac{1}{\sqrt{2}}(\pi_1^a - \pi_2^a)$ remains massless. The $\pi^a$ bosons are the would-be Goldstone bosons which are eaten when the group $A$ gets broken. $A$ is completely broken below the scale of the $SU(S - 1)$ interactions. The only unbroken gauge symmetry at low energies are color $SU(3)$ and hypercharge $U(1)$, color group is a linear combination of $SU(3)$ subgroups of the $A$ group and the ETC groups. Thus, the spectrum consists of 143 PGBs, which is the same number as in the low-scale model.

![Diagram](image_url)

Figure 5: a) Diagram contributing to the masses of PGBs in the high-scale models.
b) Diagram contributing to the PGB-fermion couplings in the high-scale models.

A qualitatively different mass spectrum might be the feature of the high-scale model described in Fig. [3b]. In this model, the masses of the fermions transforming under $SU(S - 1)$ are unrelated to the masses of quarks and leptons. The masses of the neutral PGBs can arise only from explicit mass terms for the $[12^1, S - 1^1]$ and $[S - 1^1, n + 12^1_0]$ fermions, and mass
terms for the \([n + 12_L, S - 1]\) and \([S - 1, 12_L]\). Such mass terms can result from four-fermion operators created at some very high scale.

Multi-fermion operators are inevitable in the TC-GIM models. Without their presence it is impossible to generate a nontrivial Kobayashi-Maskawa mixing matrix. A non-diagonal form of the mixing matrix implies that independent flavor symmetries for right-handed and left-handed fermions are completely broken. One cannot achieve such breaking by mass terms only, as such terms always leave several \(U(1)\) symmetries \([3]\). The multi-fermion operators are assumed to be generated at some high scale, perhaps as a result of fermion compositeness or some other mechanism. Specific operators needed for generating the Kobayashi-Maskawa matrix are listed in Refs. \([7]\) and \([8]\). It is plausible that those are not the only multi-fermion operators present in this model. Additional operators may generate masses for the light fermions. For instance, the mass term for the \([12_1, S - 1]\) and \([S - 1, n + 12_D]\) fermions can be generated by the operator

\[
\frac{1}{f^2_F} [12_1, S - 1] [S - 1, n + 12_D] [n + 12_D, S_D] [S_D, 12_2],
\]

where we have used the notation introduced in Fig. \(2\) and \(f_F\) is the scale at which such operators arise. When \(SU(S_D)\) interactions become strong, this operator gives a mass term with magnitude proportional to \(4\pi f_S^3/f_F^2\).

Although we do not know the particular form of the mass matrix, we can estimate an upper bound on the magnitude of its elements. The scale of \(SU(S)\) and \(SU(2S)\) interactions is the same as the scale at which the ETC groups get broken, since the mechanism that triggers breaking of the ETC groups is the dynamical breaking of \(SU(S)\) and \(SU(2S)\) symmetries. Therefore, the exotic fermions have masses not larger than

\[
0.04 \text{GeV} (\frac{f_{ETC}}{1.5 \text{TeV}})^3 (\frac{1000 \text{TeV}}{f_F})^2.
\]  

(14)
The form of the term contributing to the masses of the PGBs is identical to the one in Eq. \(3\); the mass matrix should be replaced with the mass matrix for the relevant exotic fermions. As a result, the contributions to the masses cannot exceed approximately \(8 \text{ GeV} \sqrt{\frac{f_{ETC}}{f_F}}\). For the neutral PGBs this is the only contribution, and it can be treated as an upper bound on their masses. Of course, in order to obtain masses of colored PGBs, one should add independent contributions to the square of PGBs masses. In this case, the above contribution from explicit breaking terms has to be added to the gluon exchange contribution described in Eq. \(7\).

We now describe the couplings of the PGBs to the quarks and leptons. The symmetry properties of the quarks, leptons and the low-scale PGBs allow very simple invariant couplings:

\[
\frac{4\pi f_{S-1}^3 v^2_T}{f_{ETC}^4} (\bar{\psi}_L \Sigma d_R + \text{h.c.}),
\]  

(15)
where the coefficient $\frac{v^2}{f_{ETC}}$ reflects the fact that such operator is created by ETC interactions. The lowest-dimension term in the above equation is the contribution of the $S-1$ condensate to the masses of the ordinary fermions. Such a contribution cannot be larger than the electron mass, which limits the scale $f_{S-1}$ to be less than 15 GeV. Given this constraint, the masses of PGBs are small, such that the Tevatron experiments should see signals of leptoquarks. We want to make leptoquarks as heavy as possible, saturating the bound, and then see if such PGBs can evade detection. Therefore, we assume that the factor $\frac{4\pi f_{S-1}^2v^2}{f_{ETC}}$ is numerically equal to the electron mass.

The PGBs in the high-scale models exhibit dramatically different couplings to the quarks and leptons. Such couplings are generated by diagrams of the type presented in Fig. 5b. The relevant terms are:

$$\left(\frac{f_{S-1}}{f_{ETC}}\right)^2 \bar{\psi}_L \gamma^\mu (\partial_\mu \Sigma_2) \Sigma_1^\dagger \psi_L$$

and

$$\left(\frac{f_{S-1}}{f_{ETC}}\right)^2 \bar{d}_R \gamma^\mu (\partial_\mu \Sigma_1) \Sigma_1^\dagger d_R$$

Thus, the PGBs couple derivatively to the quarks and leptons. We should mention here that these couplings are quite different from couplings predicted for PGBs in other technicolor models [15, 13]. In generic TC models, the couplings are non-derivative and proportional to the fermion masses. Typically, they are of the form

$$\frac{m_q}{f} \Pi \bar{q} \gamma q.$$  

Such terms have a large magnitude for the coupling to the top quark. In TC-GIM models the magnitudes of couplings are much smaller and are also suppressed by the momentum of PGBs. This difference in couplings invalidates bounds on PGBs and leptoquarks obtained by the studies of flavor-changing neutral currents mediated by those particles [16, 17].

The PGBs also couple to gauge bosons. We describe both the minimal couplings of charged or colored particles and couplings arising from the anomalies. The constituents of the PGBs are singlets of the weak $SU(2)$ interactions, thus the PGBs do not couple to the $W^\pm$ bosons. Couplings to the photon and the $Z^0$ originate from the hypercharge interactions. The lowest-order couplings are contained in the kinetic energy term for the PGBs

$$L_{kin} = \frac{f_{S-1}}{4} \text{tr} \left[(D^\mu \Sigma)\Sigma^\dagger (D_\mu \Sigma)\right],$$

$$D_\mu \Sigma = \partial_\mu \Sigma + i e A_\mu (Q, \Sigma) + i e \tan(\theta_W) Z_\mu (Q, \Sigma) + i g_3 G^a_\mu [\lambda_a, \Sigma],$$

where $\theta_W$ is the Weinberg angle and $Q$ is the fermion charge matrix. Only the leptoquarks couple to the photon and the $Z^0$, while other PGBs are neutral.

The anomalous couplings are restricted to particles that are singlets with respect to flavor symmetries. Thus, the $P^0$ is the only particle that couples to a photon pair. Color octet
bosons $\theta_a$ couple both to a pair of gluons and to a photon and a gluon. A general coupling of a PGB to a pair of gauge bosons can be written \[18\] as

$$(S - 1)A \frac{g_1 g_2}{\pi^2 f_{S-1}} \epsilon_{\mu \nu \rho \sigma} k_1^\mu k_2^\nu \epsilon_1^\rho \epsilon_2^\sigma,$$

where $k_i$ and $\epsilon_i$ are the momenta and polarizations of the gauge bosons, $g_i$ are the coupling constants. Below, we list all the coefficients of non-vanishing anomalous couplings:

$$
A_{\rho_\gamma \gamma} = \left( \frac{-2}{\sqrt{2}}, 1 \right),
$$

$$
A_{\rho_\gamma g} = \frac{1}{16 \sqrt{2}} \delta_{ab},
$$

$$
A_{\theta_a g_a g} = \frac{\sqrt{3}}{8} d_{abc},
$$

$$
A_{\theta_a g_a \gamma} = \left( \frac{-1}{\sqrt{3}}, 2 \right),
$$

The coefficient $d_{abc}$ is the $SU(3)$ symmetric structure constant. The constants $A_{\rho_\gamma \gamma}$ and $A_{\theta_a g_a \gamma}$ are different for the up and down sectors because the charge matrices in those sectors are not identical. The first number in parenthesis refers to the down sectors and the second one to the up sector. Obviously, the $Z^0$ coupling has the same structure as the photon coupling. Therefore, every photon field in the above equations can be replaced by $Z^0$, while the electric charge is replaced by $e \tan \theta_W$.

4 Decays, production rates and signatures

In this section we describe decays of the PGBs and discuss various production mechanisms at both electron and hadron colliders. We present predictions for the operating machines – LEP, HERA and Tevatron, and also for the planned colliders: upgraded Tevatron, LHC and NLC. The section ends with a comparison of PGBs’ features in TC-GIM models with other technicolor scenarios.

The PGBs decay dominantly into fermion-antifermion pairs, but the decay widths are model dependent. In the low-scale model, decays are governed by the couplings described in Eq. [18] and the corresponding decay widths are

$$
\Gamma_{\text{low}}(\Pi^a \rightarrow f_i \bar{f}_j) = \frac{2m_e^2 |T_{\Pi^a}^{\Pi^a}|^2}{\pi M_{\Pi^a}^2 f_{S-1}^2} k \left( \sqrt{k^2 + m_i^2} \sqrt{k^2 + m_j^2} + k^2 + m_i m_j \right)
$$

$$
\approx \begin{cases} 
\frac{m_e^2 |T_{\Pi^a}^{\Pi^a}|^2}{2\pi} M_{\Pi^a}^2 f_{S-1}^2 \sqrt{1 - \frac{4m^2}{M^2}} & \text{for } m_i \approx m_j = m \\
\frac{m_e^2 |T_{\Pi^a}^{\Pi^a}|^2}{2\pi} M_{\Pi^a}^2 f_{S-1}^2 (1 - \frac{m_i^2}{M^2})^2 & \text{for } m_i \ll m_j = m
\end{cases}
$$

where $k^2 = \frac{(M-m_i-m_j)(M-m_i+m_j)(M+m_i-m_j)(M+m_i+m_j)}{4M^4}$, and $T_{\Pi^a}$ are $SU(12)$ matrices described in Eq. [11]. These decay widths do not depend, except for the kinematical factors, on fermion
masses. For decays into two fermions much lighter than a $\Pi$, the decay width equals approximately $0.18\text{ eV}|T_{ij}^{\Pi a}|^2\left(\frac{15\text{ GeV}}{f_{S-1}}\right)^2\frac{M_{\Pi a}}{1\text{ GeV}}$. The scale of the $SU(S-1)$ interactions is restricted in low-scale model to be less than 15 GeV, so all PGBs are very short-lived and decay inside a detector.

The high-scale PGBs couple derivatively to the quarks and leptons, as described in Eq. 16. The resulting decay widths are

$$\Gamma_{\text{high}}(\Pi^a \to f_i \bar{f}_j) = \frac{f_{S-1}^2}{f_{\text{ETC}}^2} \frac{|T_{ij}^{\Pi a}|^2 k^2}{2\pi M_{\Pi a}^2} [(m_i^2 + m_j^2)(M_{\Pi a}^2 - m_i^2 - m_j^2) + 4m_i^2 m_j^2]$$

$$\approx \begin{cases} f_{S-1}^2 \frac{|T_{ij}^{\Pi a}|^2}{f_{\text{ETC}}^2} m^2 M \sqrt{1 - \frac{4m^2}{M^2}} & \text{for } m_i \approx m_j = m \\ f_{S-1}^{k} \frac{|T_{ij}^{\Pi a}|^2}{f_{\text{ETC}}^2} m^2 M \left(1 - \frac{m^2}{M^2}\right)^2 & \text{for } m_i \ll m_j = m \end{cases}.$$  

These decay widths are proportional to the masses squared of the fermions in the final state. This is a result of chiral suppression, similar to the familiar $\pi^+ \to \mu^+\nu_\mu$ decay. PGBs decays into fermions are caused by the ETC interactions, which preserve lepton and quark flavor. Therefore, the PGBs decay into quarks and leptons of the same flavor as PGBs constituents. The partial width of the $P^0$ decay into a $\mu^+\mu^-$ pair equals approximately $1.5 \cdot 10^{-5}\text{ eV} \frac{M}{1\text{ GeV}} \left(\frac{f_{S-1}}{15\text{ GeV}}\right)^2 \left(\frac{f_{\text{ETC}}}{15\text{ TeV}}\right)^4$. We frequently use $P^0$ as an illustration because this particle couples to pairs of photons and gluons, and therefore its properties will be very important later on. It is very interesting that PGBs in the high-scale models are also very narrow resonances, even the heaviest scalars have widths smaller than 1 GeV.

Few PGBs have anomalous couplings to gluons and photons. The couplings are described in Eqs. 18 and 19. The resulting widths of PGBs decays into a pair of massless vector bosons are

$$\Gamma(\Pi^a \to V_i V_j) = |A_{\Pi^aV_iV_j}|^2 \frac{g_i^2 g_j^2}{32\pi^3} (S - 1)^2 \frac{M^3}{f_{S-1}^2} \frac{1}{1 + \delta_{V_iV_j}}.$$  

For example, the width of the decay $P^0 \to \gamma\gamma$ equals $0.16\text{ eV} (S - 1)^2 \left(\frac{M}{1\text{ GeV}}\right)^3 \left(\frac{f_{S-1}}{15\text{ GeV}}\right)^2$.

We present a summary of the particle spectrum of the high-scale models in Table 1. We list the dimension of $SU(3)_C$ representation the PGBs belong to, their masses and major decay modes. Which decay modes dominate depends on the scale of $SU(S-1)$ interactions and particle masses. The heavier the particle the more important the decays into vector bosons are, since their widths grow with the mass cubed, while the widths of fermionic decays are linearly proportional to PGBs masses. The scale $f_{S-1}$ suppresses decays into vector bosons, while it enhances fermionic decay modes, compare Eqs. 21 and 22.
| particle | $SU(3)$ | mass [GeV] | decay modes |
|-----------------|-----------|-----------|-------------|
| $\theta_a^i$    | 8         | 45 (f/15) | $q\bar{q}$  |
| $\theta_a^a$    | 8         | 45 (f/15) | $gg, \gamma g, q\bar{q}$ |
| $T_i^c, T_c^c$  | 3         | 31 (f/15) | $ql$        |
| $P^i_l, P^{i_1}$| 1         | 0.1-5     | $q\bar{q}, ll$ |
| $P^0$           | 1         | 1         | $gg, q\bar{q}, ll$ |

Table 1: The particle spectrum in the high-scale models.

### 4.1 Electron Colliders

Charged PGBs can be pair-produced in $e^+e^-$ collisions. Charged bosons, which are only leptoquarks, couple both to the photon and to the $Z^0$ with coupling described in Eq. 17. Thus, the leptoquarks can be produced at the $Z^0$ peak and also at energies above the $Z^0$ mass, where both photon and $Z^0$ exchanges contribute to the production rate. The decay width of a $Z^0$ into a pair of leptoquarks is

$$\Gamma(Z^0 \rightarrow T\bar{T}) = \alpha \left( \frac{2}{3} \tan \theta_W \right)^2 \frac{M_Z}{4} \left( 1 - \frac{4m_T^2}{M_Z^2} \right)^{\frac{3}{2}}.$$  \hspace{1cm} (23)

Signatures of such events are quite distinct, each leptoquark decays into a hadronic jet and an isolated lepton. The events would have two opposite-sign leptons, two hadronic jets and no missing energy. The main background comes from the $Z^0 \rightarrow b\bar{b}$ decays, followed by semileptonic decays of both $b$ quarks. However, the event shape is different and the background can be efficiently rejected. Searches performed at LEP exclude pair-produced leptoquarks up to 45.5 GeV \cite{19}, which is almost the kinematic limit. The limit reported in Ref. \cite{19} does apply to leptoquarks in our model, even though both the coupling to the $Z^0$ and decay modes differ. The couplings of the $Z^0$ to a pair of leptoquarks in our model and leptoquarks in superstring-inspired models \cite{19} result in cross sections that are numerically very close. Leptoquarks in our model have larger charge, but they do not interact weakly, and the two effects roughly compensate. The decay mode $T \rightarrow de^+$ is experimentally indistinguishable from the mode $T \rightarrow \bar{u}e^+$, which was searched for at LEP.

There are two production mechanisms of single PGBs at the $Z^0$ pole. A $Z^0$ can decay into a fermion pair, and subsequently a PGB is radiated off a fermion line. Such a mechanism is model dependent, since the magnitudes of couplings of PGBs to fermions are not dictated.
by the gauge invariance. Corresponding decay widths in the low-scale model are

\[
\Gamma_{\text{low}}(Z^0 \rightarrow f_i \bar{f}_j \Pi^a) = \left( \frac{g m_{\Pi^a}}{\cos \theta_W f_{S-1}} \right)^2 |T_{ij}^{P^a}|^2 \frac{(g^2 + g_A^2) M_Z}{56 \pi^3} (-17 + 9 r + 9 r^2 - r^3 - 6 \log r - 18 \log r),
\]

where \( r = (\frac{m_{\Pi^a}}{M_Z})^2 \). Meanwhile, \( g_V = \frac{1}{2} T_3 - Q \sin^2 \theta_W \) and \( g_A = \frac{1}{2} T_3 \) are vector and axial couplings of the \( Z^0 \) to an \( f_i \bar{f}_j \) pair. We assumed that fermion masses are negligible compared to the \( Z^0 \) mass. The decay rate diverges as the mass of the scalar particle approaches zero, which is a result of divergent fermion propagator when a light scalar is being emitted collinearly with the fermion. However, for reasonable values of PGB masses, even as light as 1 GeV, this decay width is orders of magnitude too small for such a process to be observed.

PGBs couplings to fermions are different in the high-scale models, so the decay width has a different form

\[
\Gamma_{\text{high}}(Z^0 \rightarrow f_i \bar{f}_j \Pi^a) = \left( \frac{g_{\text{EFT}} M_Z}{\cos \theta_W f_{\text{EFT}}} \right)^2 |T_{ij}^{P^a}|^2 \frac{(g_V + g_A)^2 M_Z}{1152 \pi^3} (1 + 9 r - 9 r^2 - r^3 + 6 r \log r + 6 r^2 \log r),
\]

where \( g_V - g_A \) applies to the PGBs that couple to right-handed fermions, and \( g_V + g_A \) to left handed ones. PGBs in the model described in Figs. 3 and 4a are linear combinations of both types of PGBs, they couple to the left and right-handed fermions. This decay width is not divergent for small masses of the scalar, the coupling of the scalar is proportional to its momentum, which annihilates divergence of the fermion propagator. For light PGBs the decay width can be approximated as \( \Gamma(Z^0 \rightarrow f_i \bar{f}_j \Pi^a) \approx 0.13 \cdot 10^{-7}\text{GeV} \ (\frac{f_{S-1}}{150 \text{GeV}})^2 (\frac{1.5 \text{TeV}}{f_{\text{EFT}}})^4 |T_{ij}^{P^a}|^2 \), which is too small to be observed at LEP. In principle, this process could provide an upper bound on the scale \( f_{S-1} \). If the scale \( f_{S-1} \) is very large, of the order of 1 TeV, and at the same time the scale of the ETC interactions is also around 1 TeV, some events could be observed at LEP. However, such a case is not too interesting, because the hierarchy of symmetry breaking in the model would not work as expected. Such a small decay width of the \( Z^0 \) into a PGB and a fermion pair makes this process impossible to observe at LEP. Mass limits on singly-produced leptoquarks presented in Ref. [19] assume a different form of PGBs’ couplings to fermions. Therefore, those limits are not valid in the TC-GIM models.

Other sources of PGBs production are anomalous couplings to a \( Z^0 \) and a photon. The width of the \( Z^0 \) decay into a PGB and a photon has been calculated in Ref. [20]. We use their results together with anomaly factors from Eq. [19] and obtain the decay width

\[
\Gamma(Z^0 \rightarrow P^0 \gamma) = \left( \frac{A_{\gamma \gamma}^{P^0}}{f_{S-1}} \right)^2 \frac{\alpha^2 \tan^2 \theta_W}{6 \pi^3} M_Z^2 \left( 1 - \frac{m_{P^0}^2}{M_Z^2} \right)^3 (S - 1)^2 \left( \frac{15 \text{GeV}}{f_{S-1}} \right)^2 \left( 1 - \frac{m_Z^2}{M_Z^2} \right)^3 \times (4, 1),
\]

by using the formula

\[
\frac{1}{4} (\frac{1}{2} \theta - \pi) |V_{ij}|^2 |V_{k\bar{k}}|^2 \left( \frac{g^2 + g_A^2}{56 \pi^3} \right)^2 (S - 1)^2 \left( \frac{15 \text{GeV}}{f_{S-1}} \right)^2 \left( 1 - \frac{m_{P^0}^2}{M_Z^2} \right)^3 \times (4, 1),
\]
where 4 refers to the down-type $P^0$ and 1 to the up-type. This decay rate is large enough to constrain the scale of the $SU(S - 1)$ interactions. For numerical estimates we will assume that $S = 4$. The signature of such events is quite unique – an isolated monoenergetic photon and $P^0$ decay products. A one GeV $P^0$ boson in the high-scale models decays most likely into a small number of pions or a pair of $K$ mesons, therefore the events are characterized by very low hadron multiplicity. Such a signature is similar to the signature of the $Z^0 \rightarrow \eta' \gamma$ decays, where the $\eta'$ decay products contain a $\pi^+ \pi^-$ pair. The ALEPH Collaboration reported that the $BR(Z^0 \rightarrow \eta' \gamma)$ is less than $4.2 \cdot 10^{-5}$ [21]. Assuming that the branching ratio of the decays $P^0 \rightarrow \pi^+ \pi^- X$ is at least 50%, same bound can be placed on the branching ratio of the $Z^0 \rightarrow P^0 \gamma$ decays. Using Eq. 26 this branching ratio translates to a lower limit of 38 GeV on $f_{S-1}$. At the time of the analysis the ALEPH collaboration collected only 8.5 pb$^{-1}$ of integrated luminosity. Currently, the LEP experiments have data from over 100 pb$^{-1}$. If all the data were analyzed one could place a limit of $10^{-5}$ on the $BR(Z^0 \rightarrow P^0 \gamma)$. Such a limit corresponds to exploring the scale $f_{S-1}$ up to 80 GeV.

We now turn our attention to future $e^+e^-$ colliders — LEP2 operating above the $W^+W^-$ threshold and a collider with the CM energy of 500 GeV, which is one of the options for the proposed Next Linear Collider. We assume that LEP2 will collect 500 pb$^{-1}$ of integrated luminosity per year of running [22] and the NLC will achieve its design luminosity of 50 fb$^{-1}$ per year [23, 24].

The most obvious process to look for is pair production of leptoquarks. The cross section for this process is

$$\sigma(e^+e^- \rightarrow T\bar{T}) = \xi \left( \frac{3}{2} \right)^2 \frac{\alpha^2}{s} \left( 1 - \frac{4m^2}{s} \right)^{3/2}$$

$$\xi = \left( 1 + \frac{g_V}{\cos^2 \theta_W (1-y)} \right)^2 + \left( \frac{g_A}{\cos^2 \theta_W (1-y)} \right)^2,$$  

(27)

where $\frac{3}{2}$ is the leptoquark charge, $y = \frac{m^2}{s}$, $g_V$ and $g_A$ are the vector and axial couplings of the $Z^0$ to an $e^+e^-$ pair. The reaction is mediated by both photon and $Z^0$ exchanges, which are both included in the derivation of Eq. 27 and manifest as the factor $\xi$. Such a simple result for the two diagrams and their interference is caused by the simplicity of the $Z^0$ couplings to the exotic particles. The $Z^0$ couples exactly the way photon does with the coupling multiplied by $\tan \theta_W$. The signatures of such events are relatively easy to disentangle from the backgrounds. Therefore, leptoquarks can be discovered if their masses are only few GeV smaller than the kinematic limits. The potential for leptoquark discovery can be translated into limits on the scale $f_{S-1}$ using the leptoquark mass estimate from Eq. 7. LEP2 can probe the scale $f_{S-1}$ up to 45 GeV while NLC up to 120 GeV.

Single PGB production via anomaly coupling is a process whose importance grows with
energy. The anomaly coupling of a PGB and two vector bosons is proportional to vector boson momentum, so the production cross section does not decrease with $\sqrt{s}$. The cross section for producing a $P^0$ and a photon equals

$$
\sigma(e^+e^- \rightarrow \gamma P^0) = \frac{2\alpha^3(S-1)^2}{3\pi^2f_{S-1}^2}(A_{\gamma\gamma}^{P^0})^2 \left(1 - \frac{m_{P^0}^2}{s}\right)^{\frac{3}{2}}
$$

(28)

$$
\approx 3.7 \text{ fb} (\frac{15 \text{ GeV}}{f_{S-1}})^2 \left(1 - \frac{m_{P^0}^2}{s}\right)^{\frac{3}{2}} \times (4,1),
$$

where, again, 4 refers to the down-type $P^0$ and 1 to the up-type one. The study of this process at LEP2 will not provide any new information beyond what we already know from the $Z^0$ decays, the luminosity will be too small to produce any events. If $f_{S-1}$ is smaller than about 200 GeV dominant decay modes of $P^0$ are hadronic. For larger values of $f_{S-1}$ the $P^0 \rightarrow \mu^+\mu^-$ will dominate. This is a great help in the detection of $P^0$ as $\mu^+\mu^-$ pairs are measured with large efficiency and good angular resolution. The signature is then a monoenergetic photon and a $\mu^+\mu^-$ pair with the invariant mass about 1 GeV. Such events have very little background, so we assume that as few as 10 produced $P^0$ bosons are enough to be detected. Consequently, the NLC is likely to probe the scale of $SU(S-1)$ interactions up to 390 GeV. As before, limits on the $f_{S-1}$ scale in the model depicted in Fig. 5b are at most few GeV lower than the limits in other high-scale models. One can also look for $P^0$ produced together with a $Z^0$. The corresponding cross section

$$
\sigma(e^+e^- \rightarrow Z^0 P^0) = \tan^2\theta_W \xi \frac{2\alpha^3(S-1)^2}{3\pi^2f_{S-1}^2}(A_{\gamma\gamma}^{P^0})^2 \left(1 - \frac{(m_Z + m_{P^0})^2}{s}\right)^{\frac{3}{2}}(1 - \frac{(m_Z - m_{P^0})^2}{s})^{\frac{3}{2}}
$$

(29)

is about 28% of that for $e^+e^- \rightarrow \gamma P^0$. This process can be useful only for relatively small scales $f_{S-1}$, not larger than 100 GeV. A large number of events is needed because the $Z^0$ can be measured precisely only in leptonic channels, whose branching ratios are small.

### 4.2 Hadron Colliders

There are variety of processes in which the PGBs can be produced in hadron colliders. Gluon-gluon and quark anti-quark annihilations are sources of PGB pair production. Quark-gluon fusion produces single PGBs. The production of single PGBs via anomalous couplings to two gluons is also a possibility. Unfortunately, neutral PGBs, with the exception of $P^0$, do not have large enough production rates to be observed in hadron collisions. PGBs’ couplings to fermions are too small to give significant cross section. For this reason, HERA does not provide any information about PGBs in the TC-GIM models.
The cross section for the pair production of PGBs has been calculated in Ref. [25] for
the general case of scalar particles in the D-dimensional representation of color SU(3). The
quark anti-quark fusion cross section is
\[ \frac{d\sigma}{dt}(q\bar{q} \rightarrow \Pi \Pi) = \frac{2\pi\alpha_s^2}{9\hat{s}^2} k_D \beta^2(1 - z^2) \] (30)
and for gluon-gluon annihilation
\[ \frac{d\sigma}{dt}(gg \rightarrow \Pi \Pi) = \frac{2\pi\alpha_s^2}{\hat{s}^2} k_D \left( \frac{k_D}{D} - \frac{3}{32}(1 - \beta^2 z^2) \right) \left( 1 - 2V + 2V^2 \right). \] (31)
In the above formulas, \( k_D \) is the Dynkin index of the D-dimensional representation \( (k_3 = \frac{1}{2}, k_8 = 3) \), \( z \) is the cosine of parton scattering angle in the center of mass,
\[ V = 1 - \frac{1 - \beta^2}{1 - \beta^2 z^2} \] and \( \beta^2 = 1 - \frac{4m_{\Pi}^2}{\hat{s}} \),
while \( \hat{s} \) and \( \hat{t} \) are Mandelstam variables for the annihilating partons. Using these formulas and parton distributions from Ref. [26] (set 1), we obtain production rates for leptoquarks and octet particles. The cross sections are presented in Figs. 6 and 7, which agree with the results of Refs. [25, 28].

![Figure 6: The cross section for pair production of leptoquarks in pp collisions. The three curves correspond to \( \sqrt{s} = 1.8, 4 \) and 16 TeV.](image-url)
Figure 7: The cross section for pair production of color-octet PGBs in pp collisions. Different curves correspond to the CM energy at the Tevatron, upgraded Tevatron and the LHC.

Leptoquarks are a feature of many extensions of the Standard Model [27]. Several conclusions about leptoquark searches do apply to TC-GIM models. For instance, pair production of leptoquarks is almost model independent. Since gluon-gluon annihilation dominates over quark anti-quark annihilation, the production rates do not depend on leptoquarks’ couplings to quark pairs. There is a difference, however: in most models leptoquarks decay into a quark and a lepton of the same generation. In the TC-GIM models, the leptoquarks carry lepton and quark numbers of any generation. There are single-generation leptoquarks, but there exist leptoquarks that mix different generations as well.

A leptoquark decaying into an electron and a d quark has the same signature as the so-called first generation leptoquark. The first generation leptoquarks decay into an electron and a first generation quark with branching ratio $\beta$ or into a neutrino and a quark with branching ratio $1 - \beta$. The experimental limits depend on the unknown ratio $\beta$. The down-type leptoquarks in the TC-GIM models have $\beta = 1$, while the up-type ones have $\beta = 0$. The strongest limit on the leptoquark masses has been obtained by the D0 Collaboration [29]. Their results exclude down-type leptoquarks up to 130 GeV. The second generation leptoquarks are excluded up to 133 GeV [30]. These results limit the scale $f_{S-1}$ to be larger than approximately 65 GeV. That is why, as we previously claimed, the low-scale model is excluded.

The signatures of pair-produced leptoquarks are quite unique. Hundred $T\bar{T}$ pairs should suffice to discover leptoquarks at the $\sqrt{s} = 4$ TeV upgraded Tevatron. Using the cross
sections from Fig. 6, we estimate that the upgraded Tevatron can push the leptoquark mass limit up to 440 GeV, when 10 fb$^{-1}$ is collected in one year of running [31]. At the LHC, one expects CM energy of 16 TeV and the integrated luminosity of 100 fb$^{-1}$ per one year of running [32], so the LHC can discover leptoquarks up to approximately 1160 GeV. Thus, the $f_{S-1}$ scale can be probed up to 215 GeV at the upgraded Tevatron and up to 560 GeV at the LHC.

We now turn our attention to color-octet particles. The production cross sections for octets are about an order of magnitude larger than those for leptoquarks due to color factors. Unfortunately, the detection of octet particles is difficult. Octet PGBs decay into two hadronic jets, so pair-produced octets yield four-jet signals. The QCD four-jet production is the main source of background, the QCD resulting rate has been estimated [33], and it is quite large. The authors of Ref. [34] have studied four-jet processes as a probe of new physics signals. They propose certain kinematic variables designed to study such events. First, out of the three possible groupings of four jets into pairs the one that gives most equal invariant masses is chosen. The average of the two invariant masses, called balanced doublet mass, is an important parameter in the study. Then, a strong cut on the transverse jet momentum is imposed. The value of the transverse momentum cut depends on the mass of the particle one looks for. The QCD background peaks at approximately $3p_T^{min}$, so particles lighter than $3p_T^{min}$ can be observed by using such a cut. An excess of events on the balanced-doublet mass plot would be a signal of pair-produced octet particles. The authors conclude that a 375 GeV octet PGBs can be detected at the LHC.

This is a rather modest discovery potential given the fact that octet particles are about 1.5 times heavier than the leptoquarks. TC-GIM models have 18 different color-octet particles, nine in each sector. If these octet particles are nearly degenerate in mass they may give a stronger signal. However, if the mass splittings are comparable to the invariant mass resolution of four-jet signals, the situation might be more complicated since a wider peak is more difficult to disentangle from the background. The search for the octet particles will be interesting only if the upgraded Tevatron discovers leptoquarks lighter than 250 GeV. Then, one can expect to see signals of color octet particles at the LHC.

There are several sources of single PGB production in hadron colliders. A process that gives quite a large ratio is gluon-gluon annihilation into $P^0$. We compute the production cross section using the narrow width approximation:

$$\frac{d\sigma}{dy}(pp \rightarrow P^0 X) = \frac{\pi^2}{8s} \frac{\Gamma (P^0 \rightarrow gg)}{m_{P^0}} f_g(\sqrt{\tau e^y}) f_g(\sqrt{\tau e^{-y}}),$$

(32)

where $\tau = \frac{m^2_{P^0}}{s}$. The cross sections of $P^0$ productions are depicted in Fig. 8. Since the width of the $P^0 \rightarrow gg$ decay is inversely proportional to $f^2_{S-1}$, so is the production rate. Despite the fact that the cross section is quite large, a light $P^0$ cannot be detected in hadron colliders.
Figure 8: The cross section for $P^0$ production in pp collisions as a function of the $f_{S-1}$ scale. The curves correspond to $\sqrt{s} = 1.8$, 4 and 16 TeV.

A one GeV $P^0$ decays predominantly into pions. Such a process does not stand out from QCD background. The branching ratio for the $P^0 \rightarrow \gamma \gamma$ decay is about 2%, which still does not help much. A one GeV $P^0$ would decay into two almost collinear photons, which cannot be distinguished. Such a light $P^0$ cannot be observed at high energy hadron colliders.

PGBs can also be produced by the quark-gluon fusion. However, the relevant Feynman diagrams always involve PGBs coupling to fermion pairs. Such couplings are too small to give significant cross sections.

4.3 PGBs in TC-GIM versus other technicolor models

In this subsection we summarize the properties of PGBs in the high-scale models presented in Figs. 3 and 4a. We list their masses, dominant decay modes and recall the most suitable reactions for the detection. We compare the results with generic one-family, QCD-like model [10, 13, 14, 25] and walking technicolor [15]. This is by no means an exhaustive survey of PGBs in TC scenarios. We take two examples to show the similarities and stress the differences of TC-GIM models.

Color octet particles in TC-GIM models receive the dominant contribution to their masses from the one-gluon exchange. The masses of color octet particles can range from 190 GeV to 2 TeV if the scale $f_{S-1}$ is large, where the dependence of the mass on the scale $f_{S-1}$ is
described by Eqs. 7 and 8. The lower bound comes from the fact that leptoquarks lighter than 130 GeV are excluded, which implies that $f_{S-1}$ is larger than 65 GeV. The octet particles decay into two jets. Their production cross sections in $pp$ collisions are depicted in Fig. 7, the cross sections are large. However, the signature of pair-produced octets is a four-jet signal that has a large QCD background. The LHC will be able to observe color octet PGBs if they are lighter than 375 GeV, while lower energy hadron colliders have no chance of discovering octet PGBs.

Color octet particles are also present in other TC models. Their signatures are identical to the ones of TC-GIM. However, the expected mass range is not as large as in TC-GIM models because the scale of TC interactions that create PGBs is related to the scale of the electroweak symmetry breaking. In the one-family model, one expects the octet particles to have masses between 200 GeV and 400 GeV. In walking TC models, all PGBs receive large masses from ETC interactions. These interactions are characterized by a large scale due to walking. The octet particles in walking TC models are expected to have masses in the 200-500 GeV range. The production cross section for octet particles is dominated by the gluon-gluon fusion. Such process is governed entirely by couplings that are restricted by the gauge invariance (Eq. 17). Consequently, the production rate is almost model independent, so the observation of color-octet PGBs does not distinguish the models. For some choices of parameters, in walking TC models color octet PGBs can be produced more copiously than in the TC-GIM models due to techni-$\rho$ meson decays into color octets. If color-octet particles are discovered one expects the observation of leptoquarks, whose masses are 1.5 times smaller than that of octet PGBs.

Color triplet particles in TC-GIM models also receive dominant mass contribution from the one-gluon and one-photon exchanges. Their masses are bounded by Tevatron experiments to be larger than 130 GeV, while the upper limit is as high as 2 TeV. The masses of leptoquarks are described by Eq. 4. Leptoquarks can be pair-produced in $e^+e^-$ colliders and discovered up to the kinematic limit. However, since they are expected to be heavy, the best place for their discovery is a hadron collider. The production cross section in $pp$ collisions is presented in Fig. 6. The upgraded Tevatron will be able to discover leptoquarks up to 440 GeV and the LHC up to 1160 GeV.

As we have already described, the right-handed parts of the down quarks and the charged leptons transform under different ETC group than the right-handed parts of up quarks and neutrinos. Similarly, there are separate copies of the light fermions, one copy for the up sector and one for the down sector. This separations of the sectors is visible from the moose diagram in Fig. 2. Consequently, there are two types of PGBs. The up-type bosons decay into charge 2/3 quarks and/or neutrinos, the down-type into charge 1/3 quarks and/or charged leptons. This feature is important in case of leptoquarks. TC-GIM models have two types of leptoquarks, one whose decay products always contain a charged lepton and the other type
with a neutrino among its decay products. All our remarks about leptoquarks that we made so far apply to the down-type leptoquarks. The up-type leptoquarks are more difficult to observe, since their signatures are a hadronic jet and missing energy. Leptoquarks in other technicolor models can have various branching ratio for decays into neutrinos and charged leptons [10, 14].

The one-family model predicts leptoquarks in the range of 150–350 GeV, while walking TC models between 200 and 500 GeV. The discovery limit depends on the details of the model—the branching ratio of the decays into a charged lepton and a jet—and that limit is generally smaller than in TC-GIM models. Like in the octet case, the rates for pair production in hadron collisions are almost model independent. If leptoquarks are discovered, the measurements of the branching ratio into charged leptons and the masses of leptoquarks can give some hints about viable models. Moreover, in most TC models, the couplings of PGBs to the ordinary fermions have much larger magnitudes such that the leptoquarks can be singly-produced with an associated fermion pair. The expected rates of the production of single PGBs in TC-GIM models are negligibly small.

Color neutral particles are very light in TC-GIM models. Their masses range from 0.1 to 5 GeV as described in Eq. 12 and Table 1. In the lowest order chiral perturbation theory, their masses do not depend on the scale \( f_{S-1} \). In QCD-like models one expects the masses of color-singlet PGBs to be in the range of 4 to 40 GeV [14]. The ETC interactions in walking TC models have large contributions to the masses of PGBs, which are between 100 and 350 GeV [15]. The majority of neutral PGBs in the TC-GIM models are unobservable in any existing or planned experiment because of the very weak couplings to ordinary fermions. The only neutral particle with anomalous couplings to gauge boson pairs is the \( P^0 \). Its mass is about 1 GeV and it decays most likely into a small number of pions or a \( \mu^+\mu^- \) pair if \( f_{S-1} \) is larger than 200 GeV. The \( P^0 \) can be produced in hadron collisions, but it does not stand out from the hadronic background. The best environment for the discovery of \( P^0 \) are \( e^+e^- \) colliders using the \( e^+e^- \rightarrow P^0\gamma \) reaction. The production cross sections for this process are given by Eqs. 26 and 28. Since the \( P^0 \) mass is so small and lies in a narrow range around 1 GeV, the discovery potential depends not on the mass but on the strength of the anomalous coupling. The magnitude of the anomalous coupling is inversely proportional to \( f_{S-1} \) (Eq. 18), thus the production rates are proportional to \( \frac{1}{f_{S-1}^2} \). The higher the collider’s energy and luminosity, the larger are the values of \( f_{S-1} \) that can be probed. The LEP collaborations have collected enough data to probe \( f_{S-1} \) up to 80 GeV (not all the data has been analyzed), while the NLC will be able to probe the scale of \( SU(S-1) \) interactions up to 390 GeV.

In QCD-like and walking TC models, there is usually a larger number of neutral PGBs that exhibit anomalous couplings [13, 14, 15]. These PGBs can be searched for in the same channels as the TC-GIM models: \( e^+e^- \rightarrow P\gamma \) and \( e^+e^- \rightarrow PZ^0 \). QCD-like models generally
predict low production cross sections \cite{20}. The production rates depend on the number of technicolors, the technicolor scale and the anomaly coefficient. The rates are very close to limits LEP can place. For some models, PGBs might escape detection at LEP due to small rates. Only the NLC will provide sufficient energy and luminosity to observe neutral PGBs in the whole range of expected masses. Compared to the one-family model, the walking TC models are characterized by several low scales which greatly enhance the production rates. LEP2 has a chance of observing walking-TC bosons if their masses are about 100 GeV. The NLC with $\sqrt{s} = 500$ GeV can discover neutral PGBs as heavy as 350 GeV \cite{15}. Obviously, the decay modes of PGBs in QCD-like and walking TC models depend on their masses. Bosons lighter than 100-150 GeV decay predominantly into $b\bar{b}$ pairs, while heavier particles into $\gamma\gamma$ pairs.

Another feature that might help distinguish the different TC scenarios are PGBs that are color neutral but carry the electric charge. Such PGBs do not exist in TC-GIM models, although many other TC models predict them. Due to the coupling to the photon, they can be pair-produced in $e^+e^-$ colliders and discovered almost up to the kinematic limit. Future experiments may easily exclude models which predict such particles with masses that are too low.

A strong support for the TC-GIM models would be the observation of several types of PGBs predicted by the model. One could then test the ratio of the octet PGBs masses to the masses of the leptoquarks. This ratio should be very close to 1.5, which reflects the fact that the dominant contribution to the masses comes from the one-gluon exchange diagrams. The ratio of octet to triplet masses provides an indirect estimate of the size of ETC contributions to the masses of PGBs. Smaller ratio indicates large ETC contributions. For instance, the walking TC scenarios \cite{13} predict this ratio to be around one. A ratio smaller than 1.4 would rule out TC-GIM models in their present form. Once the masses of the leptoquarks are measured, next goal would be the measurement of the $P^0$ production rate in $e^+e^-$ collisions. Both the masses and the production rates depend on $f_{S-1}$, so it would be possible to check if they yield consistent values of $f_{S-1}$.

5 The $f_{S-1}$ scale

In this section we discuss the scale of $SU(S - 1)$ interactions. We describe theoretical constraints on that scale, summarize current experimental limits and the discovery reach of future colliders. We also comment on the possibility that the exotic fermions are heavier than $f_{S-1}$. In such a case fermions do not condense. Instead of forming PGBs they form mesons resembling heavy-quark systems.

The scale of $SU(S - 1)$ interactions is to a large extent a free parameter of the TC-GIM
models. As long as this scale is somewhat below the scale of ETC interactions, it does not affect the pattern of symmetry breaking. Thus, all one can expect is that $f_{S-1}$ is smaller than about 1000 GeV \[7, 8\]. In some models, $f_{S-1}$ can be limited to a much smaller value. An example being the low-scale model, where condensates of fermions transforming under $SU(S-1)$ group contribute to the masses of ordinary fermions. The ETC interactions create four-fermion operators of the form

$$\frac{v_{ETC}^2}{f_{ETC}^4} (\bar{q}_L q_R) (\bar{Q}_L Q_R),$$

which involve ordinary fermions $q$ and exotic fermions $Q$. When $Q_L$ and $Q_R$ form condensates, such operators contribute to the masses of quarks and leptons by $\frac{4\pi f_{S-1}^3 v_{ETC}^2}{f_{ETC}^4}$. In the low scale model, mass contribution from the $SU(S-1)$ condensates is identical for all down-type and all up-type fermions, thus it should not exceed electron mass. This means that $f_{S-1}$ cannot be larger than 15 GeV [7].

The high-scale models avoid this limitation by arranging the fermion condensates such that they do not contribute to ordinary fermion masses. The high-scale model depicted in Fig. 3 leaves the vacuum alignment to be determined by the strong dynamics. It is not impossible that the way condensates form depends on fermion flavor. Some flavors may form condensates of the form $[n+12_L, S-1]$ with $[S-1, A]$ and $[A, S-1]$ with $[S-1, n+12_D]$, these do not contribute to the masses of ordinary fermions. Other flavors may form condensates $[n+12_L, S-1]$ with $[S-1, n+12_D]$ and $[A, S-1]$ with $[S-1, A]$. It is not a disaster if such condensates form for the top quark; such condensates do not limit $f_{S-1}$ because the top quark is so heavy. Large mass contributions originating from $SU(S-1)$ condensates could explain the fermion mass hierarchy and at the same time make it possible to have a larger scale of ETC interactions. Such contributions are proportional to the cube of $f_{S-1}$ and are of the order of the top quark mass only if $f_{S-1} \sim 1$ TeV. Numerically, the contribution equals $150 \text{GeV}(\frac{f_{S-1}}{1 \text{TeV}})^3$.

Lack of experimental evidence for new fermions or light PGBs imposes lower limits on $f_{S-1}$. We summarize the reach of various experiments in Fig. 4. Presented results apply to the high-scale models of Figs. 3 and 4a. Currently, the best limits come from the Tevatron experiments, where one places a lower bound of 130 GeV on leptoquark masses. Thus, $f_{S-1}$ must be larger than 65 GeV. The LEP experiments have not observed $P^0$, which places a limit of 38 GeV on $f_{S-1}$. However, this result has not been updated yet with all the data collected up to know. If all the data were analyzed, LEP could probe $f_{S_1}$ up to 80 GeV, which would be the most competitive result available at present. The future proton colliders can greatly enhance leptoquark mass limits. The $\sqrt{s} = 4$ TeV Tevatron will probe $f_{S-1}$ up to 215 GeV, and the LHC up to 560 GeV. LEP2 is not likely to provide any interesting information about $SU(S-1)$ interactions. One cannot fully take advantage of having energy
Figure 9: The potential of probing the scale of $SU(S-1)$ interactions at present and future colliders (the high-scale models). The most important reaction(s) for each collider is(are) indicated inside the bars.

larger than the $Z^0$ mass due to the small cross sections outside the $Z^0$ resonance peak. Limits comparable to the LHC discovery reach can be obtained by a high-energy $e^+e^-$ collider. The NLC will be able to probe $f_{S-1}$ up to 390 GeV by searching for the process $e^+e^- \rightarrow \gamma P^0$.

The current limits and discovery potential are not very different for the high-scale model of Fig. 4b. Usually the limits are at most a few GeV lower than the limits presented in Fig. 9. However, there is an exception. The process $pp \rightarrow P^0 X$ can be a very sensitive probe, depending on the value of $P^0$ mass. In this model, $P^0$ can be much heavier than 1 GeV, and consequently, easier to detect. Irrespective of the mass, the $P^0$ should be distinguishable from the SM Higgs boson. $P^0$ has large branching ratio of the decays $P^0 \rightarrow \gamma\gamma$ of about 2%. Unlike the Higgs boson, it decays mostly into light mesons, due to large $P^0 \rightarrow gg$ decay width.

It is also possible that at least some of the fermions are heavier than the scale of $SU(S-1)$ interactions. This might happen in the model of Fig. 3 in case of misaligned exotic top quark.
Also, in the model of Fig. 4b, such a possibility exists if the operators providing masses to the exotic fermions are much larger than expected. In such a case, fermion condensates do not form, and observable particles are no longer Goldstone bosons of spontaneously broken symmetry. Mass terms are large enough, so that the chiral symmetry is explicitly broken at the scale where SU(S – 1) interactions become confining.

This situation essentially resembles heavy quark case in QCD. Heavy exotic fermions decay via four-fermion interactions connecting ordinary and exotic fermions. The operators responsible for their decays have the familiar form of current-current interaction

$$\frac{1}{f_{ETC}^2} \left( \bar{q}_1 \gamma^\mu (1 \pm \gamma^5) q_2 \right) \left( \bar{Q}_2 \gamma_\mu (1 \pm \gamma^5) Q_1 \right),$$

where $q_i$ are ordinary fermions and $Q_i$ the exotic ones. A heavy exotic fermion $Q_1$ decays into a lighter exotic fermion $Q_2$ and a pair of ordinary fermions. The lightest exotic fermions cannot decay this way. Depending on their masses they either form PGBs, as we have described in detail, or heavy meson states that are singlets under $SU(S – 1)$ interactions. Heavy mesons decays are mediated by the same four-fermion operators; here two exotic quarks annihilate into two ordinary fermions. Such heavy fermions can be searched for by means similar to searches for the fourth generation quarks and leptons. The lack of weak $SU(2)$ interactions does not play any important role in hadronic experiments. Exotic quarks are excluded up to masses comparable to the mass of the top quark. How do we know that the recently discovered top quark is not an exotic quark? The top quark decays into a real $W^\pm$, which would not be the case of exotic quarks. LEP experiments exclude both exotic quarks and leptons up to half of the $Z^0$ mass.

6 Conclusions

We investigated the phenomenology of realistic technicolor models that incorporate the GIM mechanism. The PGBs of the TC-GIM models are not formed by the technifermions, unlike in old technicolor theories or more realistic walking technicolor scenarios. Anomaly-free models require additional fermions at low-energy scales, below the scale of ETC interactions. These extra fermions form PGBs, which are the lightest new states in the TC-GIM models.

We have described and studied several possible realizations of the light-fermions sectors. The spectrum of the PGBs is very rich. In all cases, among the PGBs there are leptoquarks, color octet particles and color and charge neutral states. Despite the fact that couplings to fermions are relatively small, all PGBs are short-lived particles, with lifetimes small enough not to escape detection.

Hadron colliders are most suitable for studies of leptoquarks. Color octet particles are much more difficult to observe due to too large QCD background, even though they are
produced more copiously than leptoquarks. By studying the processes $pp \rightarrow \theta\theta$ and $pp \rightarrow TT$, the LHC experiments can discover octet particles lighter than 375 GeV and leptoquarks lighter than 1160 GeV. Currently, the best limits are placed by experiments at Tevatron, which exclude leptoquarks lighter than 130 GeV.

Electron colliders are capable of studying both leptoquarks and $P^0$ production. However, electron colliders do not have large enough energy to contribute significantly to leptoquark searches. In TC-GIM, cross sections for single production of leptoquarks are too small to yield observable rates. This limits the discovery reach of $e^+e^-$ colliders to half of the CM energy. $P^0$ is the only neutral particle that can be produced with large enough rates, which result from the anomalous couplings of $P^0$. The rate for the process $e^+e^- \rightarrow P^0\gamma$ depends on the strength of the anomalous coupling, so the rate is sensitive to $f_{S-1}$. At present, LEP excludes $f_{S-1}$ smaller than 38 GeV, but after analyzing all the data collected so far it can probe this scale up to 80 GeV. Significant improvement can be achieved at NLC, which can test $f_{S-1}$ up to 390 GeV.

A new feature of TC-GIM models is a very light $P^0$, with mass around 1 GeV. The mass of $P^0$ does not depend on the scale of $SU(S-1)$ interactions. This is a consequence of the flavor symmetries of the high-scale models. Another prediction of the TC-GIM models is the fact that masses and interactions of the PGBs depend on very few parameters—scales of confining interactions. Various colliders can independently probe those scales. Once a new signal is discovered it will be easy to check if that signal supports TC-GIM models, since such a large number of particles is predicted. Planned colliders, from upgraded Tevatron to LHC and NLC have a chance of exploring large range of the allowed parameter region, and quite big chances of finding signs of PGBs.

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