Evidence of dark energy in different cosmological observations

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Abstract The idea of a negative pressure dark energy component in the Universe which causes an accelerated expansion in the late Universe has deep implications in models of field theory and general relativity. In this article, we survey the evidence for dark energy from cosmological observations which started from the compilation of distance-luminosity plots of Type Ia supernovae. This turned out to be consistent with the dark energy inferred from the CMB observations and large scale surveys and gave rise to the concordance $\Lambda$CDM model of cosmology. In this article, we discuss the observational evidence for dark energy from Type Ia supernovae, CMB, galaxy surveys, observations of the Sunyaev–Zeldovich effect from clusters, and lensing by clusters. We also discuss the observational discrepancy in the values of $H_0$ and $\sigma_8$ between CMB and large scale structures and discuss if varying dark energy models are able to resolve these tensions between different observations.

1 Introduction

The idea of a cosmological constant was introduced by Einstein to counteract the expansion in the Universe caused by normal matter as 'Einstein's Universe' was designed to be static. Later the concept of the cosmological constant evolved to the idea that it was a form of vacuum energy with positive energy density and negative pressure whose effect at cosmological scales would be to cause an accelerated expansion of the Universe. The idea of vacuum energy has been known since the prediction and observation of the ‘Casimir effect’ which arises from the virtual electron-positron pairs produced from the vacuum. From the point of quantum field theory, the existence of the cosmological vacuum energy would be natural, however, the problem is that the scale of vacuum energy expected in particle physics would be many orders of magnitude different from the energy density of the Universe, and this problem is usually called the ‘cosmological constant problem’ [1]. Another problem from the purely cosmological perspective is to explain why the cosmological constant which does not change with the expansion of the Universe happens to be of the same order of magnitude as the matter in the present Universe since the two vary differently during the expansion history of the Universe. This ‘why now’ problem is explained by coupling dark matter to normal matter so that they track each other over the cosmological history of the Universe.

From the observational perspective, the first confirmation of an accelerated expanding Universe came from the measurement of luminosity distance plots of Type Ia supernovae by Riess et. al [2] and Perlmutter et al. [3], which prompted the rebirth of the idea of a dark energy dominated Universe in the present epoch. Subsequently, the observations of CMB anisotropy and large scale structures confirmed the evidence of an accelerating phase for the low redshift Universe. The observations of supernovae luminosity, CMB and large scale structures give a concordance model called $\Lambda$CDM cosmology where the present Universe comprises about 68\% dark energy, 27\% dark matter and 5\% baryonic matter. However not all is well with the $\Lambda$CDM model, for example, there is a 4-\sigma discrepancy between the observation of the Hubble expansion rate at present epoch ($H_0$) derived from the CMB and those from the local measurements. These discrepancies may be addressed by the evolving dark energy models.

In this article, we will very briefly review the different ways of measuring dark energy parameters and the current status of individual measurements as well as joint analyses.

The Hubble parameter which determines the rate of the expansion in an FLRW Universe depends on different components of the Universe in the following way

$$H(z) = H_0\sqrt{\Omega_m(1+z)^3 + \Omega_{\Lambda}(1+z)^2 + \Omega_k(1+z)^2}, \quad (1)$$
where $z$ represents the redshift, $\Omega_m$ is the matter density fraction, $\Omega_r$ is the radiation density fraction, $\Omega_k$ is the energy density fraction corresponds to spatial curvature of the FLRW metric and $\Omega_{DE}$ is the energy density fraction of dark energy at present with

$$f(z) = \exp \left[ 3 \int_0^z \frac{1 + w(z')}{1 + z'} \, dz' \right]. \tag{2}$$

Here $w(z)$ defines the equation of state the dark energy. We will consider three different types of dark energy scenarios; namely, the cosmological constant when $\Omega_{DE} = \Omega_\Lambda$ and $w(z) = -1$; dark energy with a fixed non-zero equation of state parameter when $w(z) = w$ (known as $\omega$CDM model); and dark energy with varying equation of state parameter. For the last case, we consider the equation of state of dark energy to be characterized by the CPL parameterization \cite{4,5}

$$w(a) = w_0 + w_a (1 - a), \tag{3}$$

where $a$ is the scale factor of the FLRW metric of the Universe.

For all the above-mentioned cases we will study the recent observational status from different ways of measuring dark energy. So far there are three different ways to measure the dark energy. The first one is through the measuring the expansion rate of the Universe directly. This type of measurement provides the Baryon Acoustic Oscillation (BAO) scale as the standard ruler. This method requires information about the velocity of the combined system. This can help to infer the expansion rate of the Universe directly. However, this method would require the next few decades of observation to constrain dark energy parameters efficiently.

The second type of measurement is using the Baryon Acoustic Oscillation (BAO) scale as the standard ruler. This type of measurement provides the integrated history of $H(z)$ evolution. The third type of observations measures the growth rate of the dark matter density perturbations at late time. These types of observations are the large scale structure surveys like lensing surveys, Sunyaev-Zeldovich (SZ) surveys, galaxy surveys and, in future, the 21-cm surveys.

We organise the article in the following way. In Sects. 2, 3 and 4 we describe the above-mentioned three different methods of determining dark energy parameters. In these sections we presented the recent results of the observations in tabulated format (Tables 1 and 2). Then in two small sections, Sects. 5 and 6 we discuss the current issues in reconstructing dark energy equation of state from the observations and various attempts to resolve the $H_0$ tension by modifying the dark energy sector. Then we summarise the current status of all the observations in Sect. 8.

## 2 Measurement from the observations of type Ia supernovae

In a binary system, the mass of a white dwarf increases either due to accretion from the other star or due to merger. A type Ia supernovae (SNIa) occur when a white dwarf in a binary system explodes as its mass reaches the Chandrasekhar limit. These supernovae are the brightest of all the supernovae and they follow quite a similar light curve with a consistent peak luminosity. That is why we can use these supernovae as the standard candles to measure the distances. However, there are a few difficulties and problems with SNIa observations. For example, after the supernovae explosion, SNIa reaches the peak luminosity in a few weeks after that these SNIa fade away within a few months. Also, it is very difficult to predict a supernovae explosion event and these events take place a few times per millennium in a galaxy. Therefore, this makes it very difficult to track all the events. In addition, although most SNIa has quite a similar light curve, a few SNIa is either a little bit fainter or brighter \cite{6}. Moreover, the SNIa are observed in a particular band filter depending on their peak luminosity. However, some part of the spectrum, other than the observing filter, comes from the filter during observations. Therefore, we need to correct this difference in the spectrum to get the accurate results \cite{6}. Finally, the distance modulus from the observation ($\mu_{obs}$) of SNIa can be obtained as \cite{7,8}

$$\mu_{obs} = m_B + \alpha x_1 - \beta C + M_0 + \gamma G_{host} + \Delta \mu_{bias}, \tag{4}$$

where $M_0$ is the absolute magnitude of SNIa and $m_B = -2.5 \log(x_0)$ with $x_0$ being the amplitude of the light curve. Here, $x_1$ and $C$ represent the light curve width and color for each SNIa, respectively. Moreover, $\alpha$ characterize the relation between SNIa luminosity and width of SNIa light curve, and $\beta$ describes the correlation between color and SNIa luminosity. These parameters are obtained by fitting the light curve for each SNIa. Furthermore, $\gamma$ accounts for the correction due to host-galaxy stellar mass and $G_{host} = +1/2$ if host-galaxy stellar mass is larger than $10^{10}$ solar mass, and $G_{host} = -1/2$ if host-galaxy stellar mass is smaller than $10^{10}$ solar mass. At last, $\Delta \mu_{bias}$ is determined from simulation and accounts for the selection bias.

Cosmology with supernovae depends on the luminosity distance measurement as a function of redshift for a number of SNIa and comparing the observed results with the theoretical prediction of distances in different cosmological models. Given a cosmological model, luminosity distance $d_L(z)$ to a source at redshift $z$ can be calculated by the following relation

$$d_L(z) = \frac{c}{H_0} (1 + z) \times \begin{cases} \frac{1}{\sqrt{|\Omega_k|}} \sinh(\sqrt{|\Omega_k|} \int_0^z \frac{dz'}{\sqrt{|\Omega_k| - 1}}) & \Omega_k > 0 \\ \frac{1}{\int_0^z \frac{dz'}{\sqrt{|\Omega_k| - 1}}} & \Omega_k = 0 \\ \frac{1}{\sqrt{|\Omega_k|}} \sin(\sqrt{|\Omega_k|} \int_0^z \frac{dz'}{\sqrt{|\Omega_k|} - 1}) & \Omega_k < 0 \end{cases} \tag{5}$$
Table 1 The values of dark energy related quantities from the SNIa data by pantheon [7] and DES-SN3YR [8] samples and from the combinations of SNIa, CMB [9] and BAO [10–13] data are shown in this table.

| Experiment                      | Model      | $\Omega_A$      | $w$       | $w_0$       | $w_a$       |
|---------------------------------|------------|------------------|-----------|-------------|-------------|
| Pantheon-SN-stat                | $\Lambda$CDM | 0.716 ± 0.012  | –         | –           | –           |
| Pantheon-SN-stat                | $w$CDM    | –                | –1.251 ± 0.144 | –           | –           |
| Pantheon-SN                     | $\Lambda$CDM | 0.702 ± 0.022  | –         | –           | –           |
| Pantheon-SN                     | $w$CDM    | –                | –1.090 ± 0.220 | –           | –           |
| Pantheon-SN+CMB                 | $w$CDM    | –                | –1.026 ± 0.041 | –           | –           |
| Pantheon-SN+BAO                 | $w$CDM    | –                | –1.014 ± 0.040 | –           | –           |
| Pantheon-SN+BAO                 | $w$CDM    | –                | –993 ± 0.087  | –1.026 ± 0.384 | –           |
| DES-SN3YR                       | $\Lambda$CDM | 0.669 ± 0.038  | –         | –           | –           |
| DES-SN3YR+CB                   | $\Lambda$CDM | 0.670 ± 0.032  | –         | –           | –           |
| DES-SN3YR+CB                   | $w$CDM    | –                | –0.978 ± 0.059 | –           | –           |
| DES-SN3YR+CB+BAO               | $w$CDM    | –                | –0.977 ± 0.047 | –           | –           |
| DES-SN3YR+CB+BAO               | $w$CDM    | –                | –0.885 ± 0.114 | –0.387 ± 0.430 | –           |

Table 2 The values of dark energy related quantities from the observations of baryon acoustic oscillation, RSD, SZ and lensing. In the second and third column “Planck” means “Planck TT, TE, EE+lowE+lensing”

| Experiment                      | $\Omega_A$      | $w$       | $w_0$       | $w_a$       |
|---------------------------------|------------------|-----------|-------------|-------------|
| Planck (TT,TE,EE, lowE)         | 0.6847 ± 0.0073  | –         | –           | –           |
| Planck+SNE+BAO                  | –                | –1.028 ± 0.031 | –0.957 ± 0.080 | –0.29 ±0.32 |
| Planck+BAO/RSD+WL               | –                | –         | –0.76 ± 0.20 | –0.72 ±0.62 |
| Planck+JLA+WiggleZ              | –                | –         | –0.96 ± 0.10 | –0.12 ±0.32 |
| +CFHTLens+SDSS-DR12 [52]        |                  |           |             |             |
| Planck+SDSS-DR12 [52]           | –                |           | –1.2 ± 0.32 | –0.33 ± 0.75 |
| Planck-SZ + BAO                 | –                | –         | 1.01 ± 0.18 | –           |
| CFHTLens+WMAP7                  | 0.729 ± 0.010    | –         | –1.02 ± 0.09 | –           |
| +BOSS+HST [53] (for flat Universe) |                  |           |             |             |
| DES Y1                          | 0.73 ±0.0307    | 0.82      | –           | –           |
| DES+Planck+BAO+SNe             | 0.702 ± 0.0077  | –1 ±0.005 | –           | –           |

where $E(z) = \frac{H(z)}{H_0}$ and $H(z)$ is given by eq. (1) and $c$ is the speed of light. Once we know the luminosity distance, we can calculate the distance modulus ($\mu_{th}$) for a given theoretical model as

$$\mu_{th} = 5 \log_{10}(d_L(z)/10pc).$$

Now, after comparing the theoretical and observational predictions for the distance modulus, we can put constraints on the cosmological parameters.

In 1998, it was first discovered by Riess et al [2] using the SNIa data from Hubble space telescope (HST) observation that the Universe is expanding at an accelerating rate. In 1999, using the data of 42 SNIa, Perlmutter et al. [3] had also found that the expansion rate of the Universe is accelerating. Since then the use of SNIa as standard candles has been of critical importance and has attracted great attention in cosmology. Over the last two decades, there have been a number of supernovae surveys by different groups which probed a large redshift range. There have been many surveys which search for SNIa in the low redshift range (0.01 < z < 0.1) e.g. CfA1-CfA4 [14–18], the Carnegie Supernova Project (CSP) [19–21] and the Lick Observatory Supernova Search (LOSS) [22] etc. Moreover, some surveys like the ESSENCE supernova survey [23–25], SuperNovA Legacy Survey (SNLS) [26,27], Sloan Digital Sky Survey (SDSS) [28–30] and Pan-STARRS survey (PS1) [31,32] have assembled SNIa data in the redshift range $z > 0.1$. There are also some other surveys like SCP [33], GOODS [34,35] and CANDELS/CLASH [36–38] survey which look for SNIa in high-z range ($z > 1.0$). Data from these surveys had been used to constrain the cosmological parameters. Recently, Scolnic et al. [7] have assembled the data of 1042 SNIa in the redshift range from $z \sim 0.01$ to $z \sim 2.0$ from PS1, CfA1-A4, CSP, SDSS, SNLS, SCP, GOODS and CANDELS/CLASH surveys and called it pantheon sample. They did analysis for different cosmological models with just pantheon-SN data and with the combination of pantheon-SN data and data from other cosmological probes such as CMB and BAO. The results for differ-
cent cosmological models are shown in Table 1. In addition
to these, recently Dark Energy Survey Supernova Program
(DES-SN) have also reported 207 SNIa in the redshift range 0.015 < z < 0.7 [8]. They did the analysis
with a total of 329 SNIa in which they have included 122 low redshift SNIa from the literature and called
this sample DES-SN3YR. The results for different cos-

mological models with DES-SN3YR data and data from
other probes are also shown in Table 1.

Recently the authors of ref [39] have argued that the
cosmic acceleration inferred from the Type Ia
supernovae (more particularly the JLA data) has a
scale-dependent dipolar modulation. This effect can be
attributed to the bulk flow in the local Universe in
which the observer is located. Therefore, the direct “evidence”
of dark energy can be an artifact of the inho-
mogeneous nature of the Universe at present time.

3 Measurement from the imprints of
baryon acoustic oscillation

The primordial perturbations reenter the horizon at the
time of radiation dominated and the matter-dominated
era. The photon-baryon fluid in which baryon and photon
are strongly coupled by the Thompson scattering
undergoes the acoustic oscillations due to the pri-

mordial density perturbations encountered during the
 expansion of the Universe. These acoustic waves set the
pattern in the CMB and the galaxy distributions of the
Universe. The characteristic length scale of these oscil-
ations patterns, known as the baryon acoustic oscillation
(bao) scale, works as the standard ruler in the
Universe in measuring the Hubble parameter and its
evolution. It is because the length scale of BAO at the
redshift of recombination can be expressed as [40,41]

\[ r_s = \int_{z_{rec}}^{\infty} \frac{c_s}{H(z)} dz \]

\[ = \frac{1}{\Omega_m H_0^2 \sqrt{\Omega_b \Omega_m}} \frac{2c}{\sqrt{3} \Omega_b H_0} \ln \left( \frac{\sqrt{1 + R_{rec}} + \sqrt{R_{eq}} + \sqrt{R_{rec}}}{1 + \sqrt{R_{rec}}} \right) \]  

(7)

where \( R_{rec} \) and \( R_{eq} \) are baryon-photon ratio (3\( \rho_b / 4 \rho_c \))
at the time of recombination and radiation matter
equality, respectively. Here, \( c_s \) is the sound speed in
baryon-photon fluid, \( z_{rec} \) and \( z_{eq} \) are the redshift cor-
responding to the recombination and radiation matter
equality epoch. This \( r_s \) can be estimated accurately if
the redshift at recombination and the baryon density of the Universe is known properly. Therefore this scale
serves the purpose of the standard ruler in estimating
the size of the BAO patterns in CMB. Similarly, for
the galaxy surveys, the BAO scale is measured at \( z_{drag} \)
which is the value of redshift when baryons and photons
decouple dynamically.

Planck measurement of CMB provides the most accu-
rate measurement of the baryon acoustic oscillation
peaks. The CMB power-spectrum is calculated using
the first order cosmological perturbation theory where
the interaction between the baryon and photon are
accounted for (see the equations in ref [43]). The \( \Delta \)
which, are the Legendre coefficients of the tempera-
ture fluctuations in CMB is calculated by solving these
Boltzmann equations. The solution of \( \Delta \) has an oscil-
latory part as well as a damping part. Further, the \( \Delta \)’s
are decomposed in terms of spherical harmonics and the
two-point correlation between the coefficients of spherical
harmonics, which are written as \( C_\ell \), are calculated.
The oscillatory part in \( C_\ell \) corresponds to the BAO and
the damping is known as the Silk damping which arises
from the viscosity in photon-baryon fluid (see Fig. 1).
The oscillatory part of the CMB can be approximated
as \( \cos(k r_s + \phi) \) where \( r_s \) is defined in eq. (7), \( \phi \) is the
phase factor which can depend on the effects of the
other components of the Universe (dark matter, neutrinos)
on CMB. The peak multipoles shown in Fig. 1 is
related to the \( k \) as

\[ \ell_{peak} = \left( m \pi - \phi \right) \frac{D_A}{r_s} = \left( m \pi - \phi \right) \frac{D_A}{\theta_s} \]  

where, \( D_A \) is the angular diameter distance which is
given by

\[ D_A = \frac{1}{(1 + z_{rec})} \int_{0}^{z_{rec}} \frac{cdz}{H(z)} \]  

(9)

The quantity \( \theta_s = r_s / D_A \) is known as the angular size
of BAO. Planck can measure seven BAO peaks in
their CMB pattern and determines the \( \theta_s \) with an 0.1% accu-

racy. The information about dark energy, or back-
ground cosmology at late time in general, is inferred
from CMB through this angular diameter distance.
Whereas the Planck CMB data is extremely accurate in
predicting the values of \( \Omega_A \) and \( w \) of \( \omega \)CDM model,
the parameters of varying dark energy show high degen-

eracy with Planck data alone. To break the degener-
acy, complementary observations like BAO and red-
shift space distortions are also taken into account. The results corresponding to these analyses are shown in Table 2.

Apart from the space based surveys like WMAP or Planck, there are some ground based surveys like SPT and ACT survey which also measures the CMB temperature fluctuations in a particular portion of the sky. These experiments can also observe the BAO scales in the CMB and provide the value of the Hubble parameter. For example, the latest value of $H_0$ inferred from the ACT data alone is $67.9 \pm 1.5$ km/sec/Mpc$^{-1}$ and $\Omega_A$ is equal to $0.696 \pm 0.022$ [44], which are in good agreement with the Planck measurement. It has been reported that SPT data alone provides the value of $H_0$ to be $73.5 \pm 5.2$ km/sec/Mpc$^{-1}$ and $\Omega_A$ is equal to $0.726 \pm 0.028$ with varying number of relativistic degrees of freedom ($N_{\text{eff}}$) [45].

### 3.2 Galaxy surveys

The galaxy surveys estimate the values of the position of the galaxies in redshift space from which the galaxy distribution in real space is calculated. The power spectrum of the galaxy field is

$$P_g(k) = b_1^2 P(k),$$  \hspace{1cm} (10)

where $b_1$ is the linear bias factor and $P(k)$ is the cold dark matter power spectrum. In Fig. 2b the matter power spectrum of the observed galaxy field is plotted after subtracting the smooth theoretical power spectrum from it. The left-over power spectrum shows the oscillation pattern of BAO.

From these BAO patterns, the galaxy surveys measure

$$d_z = \frac{r_s(z)}{D_V(z)},$$  \hspace{1cm} (11)

where,

$$D_V(z) = \left[ (1+z)^2 D_A(z)^2 \frac{c z}{H(z)} \right]^{1/3}. \hspace{1cm} (12)$$

Here, $D_A(z)$, the angular-diameter-distance whose expression is given in eq. (9) in which $z_{\text{rec}}$ has to be substituted by the redshift of the galaxies.

The experiments that measure BAO in galaxy power-spectrum are 2dF Galaxy survey [47], SDSS and BOSS [13, 46, 48], Wiggle-Z [49], 6dF galaxy survey [10]. The upcoming galaxy surveys are Euclid [50] and DESI [51]. These galaxy surveys not only provide the BAO pattern but also provides the information of growth factor from redshift space distortion which we will be discussing later in this article.

### 4 Measurement from the growth rate of large scale structures

The growth rate of density perturbations of cold dark matter in the late time of the evolution of the Universe provides a robust probe to the existence of dark energy and its equation of state. The linear growth factor of the dark matter density perturbations can be written as [54–56]

$$G(z) = 5 \frac{\Omega_m E(z)}{2} \int_0^\infty \frac{(1+z) dz}{E(z')}^3. \hspace{1cm} (13)$$

where $E(z) = H(z)/H_0$ and $E(z)$ contains the by dark matter density ($\Omega_m$) and dark energy density ($\Omega_A$). The 5/2 factor in the above equation is a normalization factor to make $G$ equals to one at $a = 1$. In the case of the matter dominated Universe, the $G(z)$ becomes equal to
(1/1 + z) or $a$. However, in the case of dark energy dominated Universe the situation changes and the growth factor is often denoted by a quantity

$$f = \frac{d \ln G}{d \ln a}.$$  \hspace{1cm} (14)

Therefore, the deviation of $f$ from the value of one provides the information about dark energy. It is customary to fit the $G(z)$ numerically and express it in terms of a monomial function of dark matter density fraction $\Omega_m(z)$ as

$$G(z) = \Omega_m(z)^\alpha.$$ \hspace{1cm} (15)

The reason behind it is it makes the calculation of higher order perturbation analytically possible.

### 4.1 Redshift space distortion

Redshift space distortion imprints unique patterns on the distribution of the tracer field (galaxy or atomic hydrogen) which changes the position of the tracer field in redshift space depending on their peculiar velocities. Within the fully virialized objects, like the large halos, the peculiar velocities of the galaxies are much higher and random. However, on the larger scales the peculiar velocities are correlated to the density contrast and the line of sight. To quantify this distortion it is customary to decompose the anisotropy in terms of spherical harmonics. In that case the monopole of the redshift space power spectrum can be written as

$$P_g^0(k, \mu) = b_1^2 \left(1 + \beta \mu^2\right)^2 P(k),$$ \hspace{1cm} (16)

where $\mu$ is the cosine of the angle between the $k$-vector and the line of sight. To quantify this distortion it is customary to decompose the anisotropy in terms of spherical harmonics. In that case the monopole of the redshift space power-spectrum is amplified by the Kaiser factor which can be written as [57]

$$P_g^0(k) = b_1^2 \left(1 + \frac{2}{3} \beta + \frac{1}{5} \beta^2\right) P(k).$$ \hspace{1cm} (17)

Here, $\beta = f/b_1$. Similarly, the quadrupole and hexadecapoles can also be calculated. However, the model power-spectrum considered by BOSS-DR12 is even more complicated which contains not only the extension of the Kaiser model but also the higher order terms[58],

$$P_g^0(k, \mu) = \exp \left[-(f k \mu \sigma_v)^2 \right] \{P_{g,88}(k) + 2 f^2 \mu^2 P_{g,86}(k) + \text{other correction terms}\}. \hspace{1cm} (18)$$

The exponential term defines the “Finger of God” effect due to the random velocities of the galaxies on the small scales. Here, $\theta$ is the divergence of the velocity fields of the galaxies, $\sigma_v$ is the standard deviation of the velocity distribution of galaxies. Measurement of the multipoles of this power spectrum provides the value of $f(z)\sigma_8(z)$. For example the latest BOSS result provides the value of $f\sigma_8 = 0.289^{+0.085}_{-0.090}$ for $\Omega_m = 0.76$. Therefore, if $\sigma_8$ is measured from other complementary observation, then RSD can constrain the growth quite effectively. So, when Planck CMB data alone cannot constrain dark energy parameters effectively, RSD is incorporated in the analysis which provide the value of $w_a$ and $w_0$ with reasonable accuracy (see Table 2). In an earlier analysis of BOSS-DR 11 power spectrum along with Planck provided the value of CPL parameters [60] to be $w_0 = -0.87^{+15}_{-16}$ and $w_a = -0.61^{+0.76}_{-0.61}$.

However, the degeneracy between $\sigma_8$ and $f$ cannot be broken with the observation of the redshift space power spectrum alone. For that purpose, it is essential to measure the bi-spectrum in the redshift space. The multipole moments of redshift space bi-spectrum for the second order perturbation theory has been studied in ref [61,62]. However, there do not exist any constraints on dark energy parameters using redshift space bispectrum as of now.

### 4.2 Lensing surveys

These surveys observe the lensing features, mainly the B-modes, produced by the large scale structures on CMB and traces back the lensing potential($\phi$) from that. The power spectrum of lensing potential ($C_{\ell}^{\phi \phi}$) can be theoretically obtained from the power-spectrum of the gravitational potentials ($P_\Psi$) [63,64].

$$C_{\ell}^{\phi \phi} = \frac{8 \pi^2}{\ell^3} \int_0^{z_{\text{rec}}} \frac{dz}{H(z)} \frac{-\chi(z)(\chi(z_{\text{rec}}))}{\chi(z)^2} P_\Psi(z, k = \ell/\chi(z))$$ \hspace{1cm} (19)

Here $\chi(z)$ is the comoving distance at $z$. The power spectrum for gravitational potential is related to the matter power spectrum through the Poisson’s equation as

$$P_\Psi(z, k) = \frac{9 \Omega_m(z)^2 H(z)^4 \left\{P(z, k) / k \right\}}{8 \pi^2}$$ \hspace{1cm} (20)

The non-linear matter power-spectrum is calculated using the HaloFit [65] extrapolation and linear matter power-spectrum as the input. The dependence of the linear matter spectrum on the growth function is...
discussed in the last subsection. Therefore, the dependence of the lensing potential with the growth factor is evident. The most important lensing surveys are CFHTLens [53,66], KiDs [67], Planck Lensing survey [68] and DES [69]. The constraints on the equation of state in $w_{\text{CDM}}$ model from all the lensing surveys are shown in Fig. 3. The values of dark energy parameters constrained using lensing surveys are listed in Table 2.

4.3 Sunyaev Zeldovich surveys

These surveys aim at counting the number of galaxy clusters from their SZ effect on CMB. In SZ effect [70–72] the black-body spectrum of the CMB gets distorted due to the inverse Compton scattering of CMB photon by the free electrons present in the intergalactic medium of the halo containing a galaxy cluster. Among the two types of SZ distortion known as $\mu$ distortion and $y$ distortion, it is the $y$ distortion that is mainly used to count the number of the clusters in the CMB surveys like space based Planck and ground based SPT and ACT survey. The masses of the observed clusters in SZ survey are assigned from the lensing survey. In this way, SZ surveys provide the number of halos in a given mass range and redshift range in the Universe. Theoretically the number density of halos of a given mass range can be calculated from the halo mass function. Although the simplest form of halo mass function, known as the Press-Schechter mass function can be calculated analytically, more accurate halo mass function requires $N$-body simulation. The most popular numerically fitted halo mass function is the Tinker [73] halo mass function which considers dark energy as cosmological constant. Whether different dark energy models will change the halo mass function or not is still an unresolved matter in cosmology. In general, it is assumed that halo mass function should depend on $\Omega_m$ and $\sigma_8$ only, which is known as the universality of halo mass function. However, it has been reported that different models of dark energy break the universality [74]. Recently it has been again claimed that universality can be restored by rescaling some variable [75]. Therefore, SZ surveys provide mainly the values of $\Omega_m$ and $\sigma_8$ and it cannot distinguish between the different dark energy models in an efficient way. However, when the measurements of $\sigma_8$ and $\Omega_m$ from an SZ survey is combined with other experiments it can provide reasonably good constraints on dark energy parameters. Planck SZ and BAO joint analysis provides $w = -1.01 \pm 0.18$ for $w_{\text{CDM}}$ model [76]. A recent joint analysis of DES and SPT has provided $w = -1.76^{+0.33}_{-0.46}$ for $w_{\text{CDM}}$ model along with varying neutrino mass [77].

5 Reconstruction of dark-energy equation of state from observations:

From the above discussions, we find that the most accurate determination of dark energy parameters come from the BAO and the measurement of the expansion of the Universe through the standard candles. Different galaxy surveys can provide BAO scales at different $z$ values. As well as CMB can provide the estimation of angular scale at $z_{\text{rec}}$. Therefore, in recent years there has been series of attempts to reconstruct the $H(z)$ from the BAO values of different observations. The reconstruction of $H(z)$ also provides the reconstruction of the dark energy equation of state ($w(z)$) [78–81]. The reconstruction depends highly on the parametrization of the $w(z)$. However, there is a common finding among most of the studies. For some values of $z$, $w(z)$ becomes less than $-1$. Any field which has an equation of state less than $-1$ is called the phantom field and therefore this type of dark energy is known as phantom dark energy. Some studies even show an oscillating feature in the dark energy equation of state (see Fig. 4).

6 $H_0$ tension and dark energy:

The value of Hubble parameter inferred from the CMB and BAO measurements using the $\Lambda$CDM cosmology strongly disagrees with the Hubble value obtained from
the direct measurement of HST [82]. This tension has opened up many possibilities of modifying the dark energy sector to resolve this tension. An incomplete list of such models includes early dark energy [83–86], interacting dark energy [87], dynamical dark energy [88] etc. However, it has been also argued that no model can resolve the $H_0$ tension by just modifying the dark energy dynamics in late time [89]. Here, we briefly review some of the features these models

- **Early dark energy** This solution proposes that some component of dark energy in the early times ($z \geq 3000$) behaves as a cosmological constant and makes a significant contribution to the energy density of the Universe. Then it decays down to radiation or some other component. This kind of models essentially modifies the growth rate of perturbations in the early times by modifying the background expansion rate for a certain period. However, there remain some issues with the early dark energy as it eases the tension between CMB and direct measurement of Hubble value but cannot resolve the tension between the inferred values from galaxy surveys and direct observation [90,91]. To resolve these issues further “new early dark energy” models are also proposed [92,93].

- **Interacting dark energy** In these models interaction between dark matter and dark energy are considered where the energy exchange is proportional to the four-velocity of the dark matter [94–96]. These models ease the tension between Planck and HST data. However, these models fail to resolve the tension between the BAO data and HST data [97].

- **Dynamical dark energy** As discussed earlier these types of model consider the dark energy equation of state to vary with time and CPL parameterization is the most popular way to of quantifying that variation. It has been shown that a joint analysis of Planck+HST data provides [88] $w_0 = -1.39^{+0.39}_{-0.32}$ and $w_a = -0.2^{+0.8}_{-1.0}$ and $H_0$ to be $73.9 \pm 2.0$. However, when BAO+Planck data are considered the analysis provides a very low value of $H_0$. Therefore, it cannot be claimed as a solution to the $H_0$ tension.

In spite of all these attempts, the very essence of $H_0$ tension is still intact. No other cosmological model except $\Lambda$CDM can fit the Planck CMB data alone with a better chi-squared value. The tension in between the different data-sets are still there and any solution which reduces the tension in $H_0$ increases the tension in $\sigma_8$ or other parameters.

7 $\sigma_8$ tension and dark energy:

Moreover, it has also been reported that there is a mismatch between the value of $\sigma_8$, the r.m.s fluctuations of density fluctuations at $8 \ h^{-1}\text{Mpc}^{-1}$, inferred from CMB fitted parameters under the $\Lambda$CDM framework and LSS observations [98–100]. This is commonly known as $\sigma_8$ tension. There have been a few attempts to resolve this tension by modifying the dark energy physics which includes interacting dark energy, dynamical dark energy etc [97,101–109]. In refs. [97,107], interaction between the dark energy and dark matter have been explored and they show that $\sigma_8$ tension significantly reduces in such models. In ref. [108], minimally and non-minimally coupled scalar field, which can act as the possible alternatives for dark energy, have been proposed to ease the $\sigma_8$ tension. In ref. [106], dynamical dark energy (CPL parameterization [4,5]) and $f(R)$ gravity model for dark energy have been analyzed and they find that $\sigma_8$ tension slightly decreases in the case of dynamical dark energy, whereas it worsens in case of $f(R)$ gravity model for dark energy. Furthermore, there are a few works that explore the running vacuum models as a resolution to the $\sigma_8$ tension [104,105]. The interested reader can see ref. [100] for a brief review on the current status of $\sigma_8$ tension.

8 Summary

In this article, we have briefly reviewed the methods and the current status of measuring dark energy parameters from different observations. The best measurement comes from the baryon acoustic oscillation scales of Planck CMB data. For the case of the $\Lambda$CDM model experiments like the recent lensing observations (DES) or Planck data can provide significantly tight constraint by themselves. However, for the case of the $w$CDM model or dynamically varying dark energy models still, no single observation can constrain the parameters. In these cases, the joint analyses help to resolve the degeneracy between the parameters. For example, the RSD data of BOSS-DR12 can provide only the $f\sigma_8$ combination in a particular $z_{\text{eff}}$. SZ observations can provide a contour in the $\Omega_m-\sigma_8$ plane. Lensing observations also provide the best constraints on the $\Omega_m-\sigma_8$ plane. BAO from the galaxy surveys or Planck shows huge degeneracy in CPL parameters when analysed alone. But When Planck data is combined with RSD and lensing data it narrows down $w_0$ to $-1$ and $w_a$ to 0.

Different observations, although helps to break the degeneracy in the more complicated models, create tensions for the simplest models. The BAO data at different $z$ values from different observations does not favor simple $\Lambda$CDM cosmology. Rather the recent reconstruction of the dark energy equation of state from these data sets has shown some hint of phantom dark energy for some particular ranges of redshift values. Similarly, the Hubble tension between the CMB and the direct detection of $H_0$ also opened up the scope of exploring different dark energy models.

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