Density with Cubes

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Abstract. The article deals with an inquiry-based approach to teaching density to the age group 12-13. The whole methodological sequence is presented, including necessary equipment, worksheet for pupils, an example of results measured by pupils, reflection of the lesson, etc. Also, the experience of the implementation of the lessons is mentioned.

1. Introduction

Density is considered a difficult concept for students to learn because it is abstract and because it is derived from the concepts of mass and volume. There are different approaches for teaching this topic. Hitt [1] uses the concepts of density at three different conceptual levels: macroscopic, particle or modelling, and symbolic. More often the teaching-learning sequence of density starts by experimenting with floating and sinking [2]. Dawkins [3] also uses experiments with floating and sinking and summarizes this issue as follows: References to mathematically based scientific understandings refer to applications of precise numerical relationships. In practice, instruction of children often focuses on the notions of floating and sinking or heaviness without attention to the finer points of equal masses and different volumes or vice versa, ideas that could easily extend children’s inferences about density without involving calculations.

Xu [4] describes the research based on the theory of Distributed Cognition. Studies employing Distributed Cognition are concerned with how cognitive activity is distributed across individual minds, artefacts external to the individual, as well as how cognition is distributed across space and time. During the research two lessons on density were prepared. Teacher used two metal blocks (lead and aluminium) to show different mass of objects with the same volume. In the second part of activity students compared the density of a candle and a marble. As a result, Xu shows four dominant views on density which were identified during his lessons (Figure 1).

Figure 1. Schematic diagram of the four views of density [4]
A very interesting method of teaching density is used by Abud [5]. Using the inquiry process, students observe some regularly shaped solid objects, as well as water, and determine the relationship between their mass and volume. In the last activity of his teaching-learning sequence students measure the density of a person. The measuring of mass is not a problem, but measuring the volume of their schoolmate (in the classroom) is a challenge. Abud shows, how students use a 44 gallon can or bucket to determine the human density. In my classroom, I was not able to prepare such fascinating activity, but previous steps of the sequence are similar to the approach described in this article. It is visible, that many teachers solve the problem how to teach the topic density in a similar way, because both activities (Abud’s and mine) were developed independently.

Similarly to other countries, the topic Density is one of the least popular topics in physics in the 6th grade of junior secondary school, where it is usually taught in Czech schools. Density is one of the most important physical quantities that describe the basic properties of substances, but it is very abstract concept for pupils, because they have no experience with it. Besides, it is necessary to work with fractions that are often not well understood by pupils of the age about 13.

The teaching-learning sequence described in this paper is based on active learning, or inquiry-based learning method. It includes several research elements. Pupils have the opportunity to discover this physical quantity based on the experiments and problems presented by the teacher. This method allows them to understand deeper this physical quantity.

The methodology is elaborated in three lessons (it does not include any further training, testing, etc.). However, it is possible that in some classes pupils will not be able to proceed as quickly and they will need more time to solve problems. In this case, I recommend that the teacher gives them enough time to investigate, doesn’t speed up teaching by changing it to a standard lesson. I am convinced that it is better to let the pupils examine the physical phenomena at their pace rather than simply "go through" the topic.

I am aware of the fact that the methodology is also demanding for teachers, that it is necessary to think about the steps, to prepare aids, etc. However, I believe that the effect the pupils begin to gain this way is worth it. (I use the term "begin to gain" intentionally, because one activity does not change their thinking much. However, if the teacher uses a similar method consistently, the scientific thinking of the pupils evolves gradually. [6])

2. Preparation

I think that the tools I have prepared play a key role in this teaching-learning sequence. I do not use "classic" aids that are used for measuring density in many schools, i.e. small cylinders of different materials (they have a volume of about 16 cm³, they are made of iron, aluminium, brass and plastic). I see a big disadvantage of these bodies, they have the shape of a cylinder, so the pupils have to determine their volume only with a graduated cylinder (the formula for calculating the volume of the cylinder in the 6th class is not yet known), moreover the bodies are small and not interesting for the pupils.

As the basis for the new set of tools, I chose a cube of 5 cm edge of different materials. These cubes are large enough and very interesting (not only) for children (Figure 2). After about five years of collecting the cubes, I have 19 cubes from different materials – some of them I bought, some of them I got from colleagues, two cubes were given to me by pupils from my class as a present (sugar and sandstone). In addition to cubes, it is also advisable to prepare small plastic boxes for different liquids or loose materials – e.g. water, saltwater, ethanol, sand, salt, etc. (and one more box which stays empty). I decided to choose boxes with volume of 150 ml because they look similar to cubes. Pupils know the volume of boxes; they have to measure only the mass of material inside the cube. They measure the mass of empty box and then subtract this value from the mass of full box.
Figure 2. Equipment for the experiments

It is necessary to prepare also digital scales for measuring mass of all the bodies (it usually should be two scales with different range; pupils will measure the masses from several grams to more than one kilogram), pupils can use calculators. For the first part of the lesson, the teacher needs also a larger and smaller piece of iron (large screw and nail), larger and smaller piece of polystyrene (the weight of a larger piece of polystyrene must be greater than the weight of the nail).

3. Preparatory activity – Playing with cubes
   If possible, let the pupils play with the cubes before they have to measure, weigh, etc. The cubes are so nice in themselves that it would be a pity to depriving the pupils of the experience to investigate them. Offer pupils to get a good look at the different cubes, to weight them in their hands (for example, metal cubes are surprising for children as they are heavy), try to immerse them in water (if it is possible, of course, sugar cube does not survive the dipping). If a given cube floats, pupils can notice what part of the cube is below the surface.

4. First lesson – Realization of the inquiry-based learning cycle

4.1. Objectives of the 1st lesson:
   - Pupils propose and implement an experiment in which they will be able to compare which of the investigated materials is heavier (has a higher density).
   - Pupils describe the basic elements of the process they have undergone during their examination.

4.2. Introduction, motivation, plan of experiment
   Tell the students that today they will explore a whole new feature of the substances they may not have heard of so far and that they will work as scientists who discover something new. They will propose an experiment to a given problem, implement the experiment, record its results and draw conclusions from it.
   Show the pupils a large screw and a small piece of polystyrene and ask them what is heavier, whether polystyrene or iron. The pupils may not have a problem answering that the iron is heavier. Then show them a small nail and a larger piece of polystyrene, and ask the same question. This time, polystyrene is heavier. So what is heavier – polystyrene or iron? Let the pupils talk in pairs for a moment and make suggestions. Pupils should find that they have to take the same piece of polystyrene and iron and compare their weight. So they conclude that if we want to compare which substance (which material) is heavier, we have to take the same big piece of both substances and consider it.
   Note: The term “heavy” is often used rather vaguely in common talk. It may denote either object of large mass or materials of high density. In this activity we intentionally create cognitive conflict in students’ minds to realize this difference. This way we help students to distinguish these meanings and to arrive to better understanding of the concept of density.
4.3. Experimenting, writing the measurement results

Show prepared cubes and boxes to pupils and tell them both the goal of the experiment and the instructions for further work. This experiment aims to compare and write down the mass of one or more pairs of bodies of the same volume. (Note that the number of body pairs examined may vary for different pupils depending on their abilities – this may lead to greater individualization of the teaching.) They will work in pairs or groups (arbitrarily, up to 4 pupils per group), each group will choose a pair of bodies of the same volume, determine the volume of the bodies (by measuring the length of the edges and calculating) and their mass. Pupils write their results to both the workbook and the table on the computer (table 1, at the beginning the table has only the columns Substance, Volume, Mass).

**Table 1. Data measured by pupils**

| Substance      | Volume (cm$^3$) | Mass (g) | Mass of 1 cm$^3$(g) |
|----------------|-----------------|----------|---------------------|
| iron           | 125             | 968      | 7.74                |
| lead           | 125             | 1488     | 11.90               |
| balsa          | 125             | 14       | 0.11                |
| wax            | 125             | 114.3    | 0.91                |
| linden wood    | 125             | 61.6     | 0.49                |
| polystyrene    | 83.1            | 1.4      | 0.02                |
| cork           | 125             | 38.1     | 0.30                |
| plastic        | 118.65          | 43.2     | 0.37                |
| foam           | 125             | 3        | 0.02                |
| ebony          | 125             | 149.8    | 1.20                |
| soap           | 83.5            | 108.4    | 1.31                |
| brass          | 125             | 1075     | 8.60                |
| iron in a box  | 150             | 681.7    | 4.54                |
| sand           | 150             | 153.5    | 1.02                |
| water          | 150             | 150      | 1.00                |
| salt           | 150             | 199.5    | 1.33                |
| salt water     | 150             | 158      | 1.05                |
| glass          | 125             | 285      | 2.28                |
| aluminium      | 125             | 356      | 2.85                |
| granite        | 216             | 563      | 2.61                |

4.4. Application of results, comparison of different substances

When you have completed the measurement and entered the results into the table, you can discuss with pupils whether the right number of significant figures is stated in their data (for example the measuring of the volume of plastic has too many figures). Then ask the pupils whether ebony and wax, salt or sand, etc., are heavier. The next question is “What is heavier – glass or granite”? In my experience, students quickly suggest that it is necessary to calculate how much 1 cm$^3$ of glass and granite weighs. Teacher adds a column labelled Mass of 1 cm$^3$ to the right side of the table; calculate the values and pupils can compare weight of all substances.

4.5. Conclusion of the first lesson, reflection

As part of the content reflection, go back to the beginning of the lesson and ask the pupils what new property they have studied during the lesson and how they would call it. Probably you have to tell the pupils the name of the new physical quantity; they usually do not know the word density. Then ask the pupils to describe in pairs how they proceeded at this lesson, how they started, how they continued.
problem is difficult for pupils, as they usually do not think about how to learn, but still try to formulate a basic cognitive process together with them: OBSERVATION - QUESTION - EXPERIMENT PLAN - EXPERIMENT EXECUTION - CONCLUSION. Let the pupils describe how they did this "scientific work", what they had problems with, what they would do differently next time.

5. Second lesson – Calculation of density, density units

5.1. Objectives of the 2nd lesson:
- Pupils derive the basic density unit and how it is calculated; derive other density units and transfers between them.

5.2. Repetition
At the beginning of the second lesson let the pupils in pairs remember what they did last hour, what they discovered. This reflective activity is preferable than the method when the teacher repeats with the whole class at once, because now all pupils get to talk, everyone is thinking about the problem. Then someone briefly tells others what he remembered, others can add.

5.3. Finding the units of density
The next step is working with worksheets. Every pupil gets a worksheet of paper (Figure 3); pupils can work with a classmate to fill it out.
I recommend stopping pupils after completing point 2, letting them suggest what unit density can have, commenting on their suggestions, and finally telling the students that the physicists have agreed on a "gram per cm$^3$" unit written by "g/cm$^3$".
Then let the pupils work independently again, browse through the classroom, answer any questions, watch them work.
Worksheet - Density

1. Fill in the blanks with the values you measured in the last lesson:
   We found that \( \ldots \ldots \text{cm}^3 \text{ of } \ldots \ldots \ldots \text{ (Add some solid)} \) weighs \( \ldots \ldots \ldots \text{ grams. From this we can calculate that } 1 \text{ cm}^3 \text{ of } \ldots \ldots \ldots \ldots \text{ weighs } \ldots \ldots \ldots \text{ grams.} \)

2. We can reformulate the previous sentence. Try to fill the sentence correctly.
   If \( 1 \text{ cm}^3 \ldots \ldots \text{ weighs } \ldots \ldots \ldots \text{ grams, it means that the density of } \ldots \ldots \ldots \text{ is } \ldots \ldots \ldots \).

3. Continue filling-in:
   We also found that \( \ldots \ldots \text{ cm}^3 \text{ of WATER weighs } \ldots \ldots \ldots \text{ grams, so } 1 \text{ cm}^3 \text{ of water weighs } \ldots \ldots \ldots \text{ g. So we can say that the density of water is } \ldots \ldots \ldots \).

4. Remember what the original definition of kilogram was, and write what you know about it.
   Can you now write down the density of water using such a unit in which there is a kilogram instead of gram?
   \( \text{One } \ldots \ldots \ldots \text{ of water weighs one } \ldots \ldots \ldots \text{, that is, water density is } \ldots \ldots \ldots \).

5. Calculate how much would \( 1 \text{ m}^3 \) of water weigh and write down the water density using the unit where \( m^3 \) occurs.
   \( 1 \text{ m}^3 \text{ of water weighs } \ldots \ldots \ldots \text{ kg, so we can also write down the water density as } \ldots \ldots \ldots \).

Worksheet – Density – Completed worksheet (real data)

1. Fill in the blanks with the values you measured in the last hour:
   We found that \( 125 \text{ cm}^3 \text{ of iron weighs } 968 \text{ grams. From this we can calculate that } 1 \text{ cm}^3 \text{ of iron weighs } 7.74 \text{ grams.} \)

2. We can reformulate the previous sentence. Try to fill the sentence correctly.
   If \( 1 \text{ cm}^3 \text{ of iron weighs } 7.74 \text{ grams, it means that density of iron is } 7.74 \text{ grams per cm}^3, \text{ i.e. } 7.74 \text{ g/cm}^3. \)

3. Continue filling-in:
   We also found that \( 150 \text{ cm}^3 \text{ of WATER weighs } 150 \text{ grams, so } 1 \text{ cm}^3 \text{ of water weighs } 1 \text{ g. So we can say that the density of water is } 1 \text{ g/cm}^3. \)

4. Remember what the original definition of kilogram was, and write what you know about it.
   \( 1 \text{ kilogram was originally defined as the mass of } 1 \text{ dm}^3 \text{ of water.} \)

Can you now write down the density of water using such a unit in which there is a kilogram instead of gram?
   \( \text{One dm}^3 \text{ of water weighs one kg, that is, water density is } 1 \text{ kg/dm}^3. \)

5. Calculate how much would \( 1 \text{ m}^3 \) of water weigh and write down the water density using the unit where \( m^3 \) occurs.
   \( 1 \text{ m}^3 \text{ of water weighs } 1000 \text{ kg, so we can also write down the water density as } 1000 \text{ kg/m}^3. \)
Children stick a worksheet to a workbook and write down how density has been calculated and how its units are converted.

5.4. Reflection the 2nd lesson
As part of the reflection, you can ask students how difficult it was for them, where they see the benefits of what they have learned today, where the density of different substances is reflected in practice, when and where it can be useful everywhere, etc.

6. Third lesson – Determination of the density of an irregular body, work with tables

6.1. Objectives of the 3rd lesson:
- Pupils determine the density of an irregular body.
- Pupils compare the results of their experiment with the table value, explaining the difference of the result.

6.2. Determination of irregular shape body density
Show pupils an irregularly shaped stone or other body and ask them how they will find out its density. Let them create an experiment plan in groups and write:
- what quantities to measure
- how they plan to do it, what aids they will use
- how to determine the density of the body from the measured values (verbal expression of density calculation from a known volume and mass of a body)

I recommend that pupils do not see volumetric cylinders prepared in their class so that they have to figure out the measuring of the volume of an irregular body using the volumetric cylinder themselves, without help. After designing the experiment plan, give them the necessary aids (stones or other bodies, cylinders, threads for hanging bodies, digital scales with suitable ranges) and let them determine the density of the unknown substance.

6.3. Working with tables, comparison of own result with table value
Let pupils find the density values of various materials in the Physics tables. Let them compare the density of different materials they found in the first lesson with the "official" values in tables. Let them also estimate by density what substance probably was the irregular body whose density had been determined just recently (maybe it will be necessary to help them, pupils' results may be quite inaccurate, and volume measurement using volumetric cylinders will probably be burdened by a large error). Ask pupils to try to explain in groups why the tabular values of the body density and the density they determine differ.

Warn them that it is normal for real measurements in physics (as science) that measurement is not entirely accurate and measurement inaccuracies are foreseen. I recommend using the current term "measurement inaccuracy" or "measurement uncertainty" instead of the former term "measurement errors" (which leads to the assumption that they can be somehow eliminated).

Then assign different tasks yourself or let the students formulate them in pairs. Let pupils solve them - for example: “Compare the table density of granite with what we measured at the 1st lesson.”, “Find the element with the highest density”, “Is the density of gold greater or less than that of silver?” etc.

6.4. Derivation of the formula, conclusion of the lesson
Let pupils remind how they determined the density of cubes in the first lesson. Show pupils that also the density unit itself shows that the weight needs to be divided by volume in the calculation. Tell them the physical density symbol and remind the base unit. Up to now pupils solved different problems concerned density just by reasoning, without any formulas. If your pupils know mathematics quite well (especially fractions) you can also show them a formula for calculating density. However, in the normal classroom (in the Czech Republic), it is not expected that pupils about 12 years old will be able to work with the
formula and use it for calculation of the volume or weight. In the sixth grade, I recommend to solve only the simplest tasks for calculating density or weight and to let the more difficult tasks for higher grades. (Note: I have experience that pupils who did not master density at all in grade 6 then asked me in grade 8 to explain this topic again because they needed it in chemistry.)

6.5. Reflection of the activity
Remind the students that you told them in the first lesson that they would work as scientists. Let them recall in groups in which parts of the last three lessons they really have worked as scientists, what they have learned. Summarize the conclusions of this group discussion and let pupils write it in the workbook.

7. Experience with this teaching approach
I have gained a lot of experience over the five years I have been using the set of cubes at school. Pupils like the cubes very much and their motivation to learn density is increased. So, the above-described activity enables pupils to better understand the physical quantity of density, but also to become aware and begin to practice the principles of scientific exploration of the world. We have shown this way of teaching to the teachers at the seminars (Figure 4) in the Heureka project [6] and we know that many of them have already started to create a collection of cubes and to use this approach. Teachers particularly appreciate the increased interest of pupils in learning this topic, which is so difficult for them.

Figure 4. Teachers examining the cubes

8. References
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