Sliding mode Control using 3D-SVM for Three-phase Four-Leg Shunt Active Filter

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ABSTRACT

This paper proposes a sliding mode control strategy for a three-phase shunt active power filter. The SAPF consists of four-leg voltage source inverter bridge. The SAPF ensures full compensation for harmonic phase currents, harmonic neutral current, reactive power compensation and unbalanced nonlinear load currents. The modulation task can be carried out with three dimensional space vector modulation, which operate under a constant switching frequency. The simulation results show that the performance of the four-leg SAPF with the proposed control algorithm – compared with PI controller – is found considerably effective and adequate to compensate harmonics, reactive power, neutral current and balance load currents.

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1. INTRODUCTION

One of the most important problems to be solved in power systems, in recent years, is the line-current harmonics problem. It is well known that harmonic currents, in the case of sinusoidal line voltage, do not contribute to active power [1]. The same holds for the reactive current. The undesired current components cause stress to the power system, generating disturbed fundamental and harmonic voltage drops in the network impedances. The active power filter (APF) is a common approach to eliminate the undesired current components by injection of compensation currents in opposition to them [1].

The distribution systems are inherently unbalanced because of untransposed distribution lines and unbalanced loads. Therefore maintaining the distribution voltage within certain limits of the perfectly balanced waveform is quite difficult. In addition, if the phases are unequally loaded, they produce undesired negative and zero sequence currents. Under nonlinear voltage and current, the harmonic current have contribution to active power [2].

The negative sequence will cause excessive heating in machines and low-frequency ripples in rectifiers [2, 3]. The zero sequence currents cause not only excessive power losses in neutral lines, but also degrade the circuit protection [3].

For full compensation capability a four-leg inverter connected as a shunt APF (SAPF) can be used [4]. In this configuration, as shown in Fig. 1, the compensated neutral current is provided through a fourth leg allowing a better controllability than the three-leg with split-capacitor configuration [5]. The main advantage of the four-leg configuration is the ability to suppress the neutral current from the source without any drawback in the filtering performance [5].
A large number of control strategies applied to APF have been reported; however the sliding mode control (SMC) seems to be the most appropriate one, because the time varying nature of an APF makes it suitable to be controlled by a variable structure approach such as the SMC [6]-[9].

Furthermore, the robustness characteristic and the simplicity of the implementation make the sliding mode control particularly attractive. In addition to the abovementioned features, the chattering caused by the switching operation, which is commonly considered a weakness of sliding mode control, is an intrinsic characteristic of power electronic converters, and hence unrelated to the control methodology.

In this work a three-phase four-leg SAPF is considered to compensate harmonic distortion, reactive power and current unbalance caused by the nonlinear load. The state space model of the SAPF, developed in the stationary $\alpha\beta\theta$ frame, is used to obtain the desired closed loop dynamics and to generate the reference inverter voltage by applying a sliding mode control strategy.

2. CONTROL DESIGN

The basic operation of the proposed control method is shown in Fig. 1. It consists of a four-leg inverter connected at the PCC to a three-phase four-wire grid through the interface inductances. The switch control signals are derived from a 3D-SVM. The voltage references for the 3D-SVM are derived from a sliding mode current controller. The reference currents are computed by using the instantaneous $p$-$q$ theory. The compensation objective is to compensate for load current harmonics, reactive power compensation, neutral current elimination, and to regulate the dc bus during bidirectional active power exchange between the DC side load/source and the power system grid.

2.1. Mathematical Model of Four-leg PWM inverter

The differential equations describing the dynamic model of the inverter are defined in $\alpha$-$\beta$-$\theta$ reference frame, as given in equation (1).

$$\begin{align*}
\frac{di_{Fa}}{dt} &= \frac{1}{L_F} (v_{Fa} - v_a - R_F i_{Fa}) \\
\frac{di_{F\beta}}{dt} &= \frac{1}{L_F} (v_{F\beta} - v_\beta - R_F i_{F\beta}) \\
\frac{di_{Fo}}{dt} &= \frac{1}{L_F} (v_{Fo} - v_\theta - R_F i_{Fo}) \\
\frac{dv_{\alpha}}{dt} &= \frac{P_{dc}}{Cv_{dc}}
\end{align*}$$

Where, $v_a$, $v_\beta$ and $v_\theta$ are voltages of (PCC) in the $\alpha$-$\beta$-$\theta$ coordinates, $i_{Fa}$, $i_{F\beta}$ and $i_{Fo}$ are the $\alpha$-$\beta$-$\theta$ axis AC current of SAPF, and $v_{Fa}$, $v_{F\beta}$ and $v_{Fo}$ are the AC side voltages of four-leg SAPF.

2.2. The $p$–$q$ Theory Based Control Strategy

Instantaneous active and reactive powers of the nonlinear load are calculated as:

$$\begin{bmatrix}
p_L \\
q_L
\end{bmatrix} = 
\begin{bmatrix}
v_a & v_\beta & 1
\end{bmatrix}
\begin{bmatrix}
i_{La} \\
i_{L\beta}
\end{bmatrix}$$

(2)
The instantaneous active and reactive powers include AC and DC values and can be expressed as follows:

\[
p_L = \bar{p}_L + \tilde{p}_L \\
q_L = \bar{q}_L + \tilde{q}_L
\]  

(3)

DC values \((\bar{p}_L, \bar{q}_L)\) of the \(p_L\) and \(q_L\) are the average active and reactive power originating from the positive-sequence component of the nonlinear load current. AC values \((\tilde{p}_L, \tilde{q}_L)\) of the \(p_L\) and \(q_L\) are the ripple active and reactive powers.

For harmonic, reactive power compensation and balancing of unbalanced 3-phase load currents, all of the reactive power \((\tilde{q}_L\) and \(\tilde{q}_L\) components) and harmonic component \((\tilde{p}_L)\) of active power are selected as compensation power references and the compensation currents reference are calculated as (4).

\[
\begin{bmatrix}
i_{ref}^a \\ i_{ref}^b \\
i_{ref}^c
\end{bmatrix} = \frac{1}{v_a^2 + v_b^2 + v_c^2} \begin{bmatrix}
v_a \\ v_b \\ v_c
\end{bmatrix} \begin{bmatrix}
\tilde{p}_L - p_L \\ \tilde{q}_L\end{bmatrix}
\]  

(4)

The signal \(p_d\) is used as an average real power and is obtained from the DC voltage controller. Since the zero-sequence current must be compensated, the reference of homopolar current is given as:

\[
i_{ref}^0 = i_{L0}
\]  

(5)
2.3. Sliding Mode Controller Synthesis

From equations (1), it is obvious that the converter is a nonlinear and coupled system. So a nonlinear controller based on the sliding mode method is developed in this section.

The system (1) is subdivided in four subsystems as follows:

**Subsystem 1:**
The first subsystem is characterized by only one state $x = v_{dc}$ and only one control input $u = p_{dc}$.

$$\frac{dv_{dc}}{dt} = \frac{p_{dc}}{Cv_{dc}}$$  \hfill (6)

The equation (6) can be written as follow:

$$\begin{align*}
\dot{x}_1 &= L_f h_1 + L_d h_2 u_1 \\
y_1 &= h_1(x) = v_{dc}, \quad y_{id} = v_{dc}^* 
\end{align*}$$  \hfill (7)

Where:

$$x_1 = v_{dc}, \quad u = p_{dc}, \quad L_f h_1 = 0, \quad L_d h_2 = \frac{1}{Cv_{dc}}$$

**Subsystem 2:**
The second subsystem is also characterized by only one state $x = i_{Fa}$ and only one control input $u = v_{Fa}$.

$$\frac{di_{Fa}}{dt} = \frac{1}{L_f} (v_{Fa} - v_a - R_f i_{Fa})$$  \hfill (8)

The equation (8) can also be written as follow:

$$\begin{align*}
\dot{x}_2 &= L_f h_2 + L_d h_2 u_2 \\
y_2 &= h_2(x) = i_{Fa}, \quad y_{id} = i_{Fa}^* 
\end{align*}$$  \hfill (9)

Where:

$$x_2 = i_{Fa}, \quad L_f h_2(x) = \frac{1}{L_f} (-v_a - R_f i_{Fa}), \quad L_d h_2 = \frac{1}{L_f}, \quad u_2 = v_{Fa}$$

**Subsystem 3:**
The third subsystem is characterized by one state $x = i_{F\beta}$ and one control input $u = v_{F\beta}$.

$$\frac{di_{F\beta}}{dt} = \frac{1}{L_f} (v_{F\beta} - v_{\beta} - R_f i_{F\beta})$$  \hfill (10)

The equation (10) can also be written as follow:

$$\begin{align*}
\dot{x}_3 &= L_f h_3 + L_d h_3 u_3 \\
y_3 &= h_3(x) = i_{F\beta}, \quad y_{id} = i_{F\beta}^* 
\end{align*}$$  \hfill (11)

Where:

$$x_3 = i_{F\beta}, \quad L_f h_3(x) = \frac{1}{L_f} (-v_{\beta} - R_f i_{F\beta}), \quad L_d h_3 = \frac{1}{L_f}, \quad u_3 = v_{F\beta}$$

**Subsystem 4:**
The fourth subsystem is characterized by one state $x = i_{fo}$ and one control input $u = v_{fo}$.
The fourth subsystem (12) can also be written as follow:

\[ \begin{align*}
\dot{x}_4 &= L_f h_4 + L_f h_5 u_4 \\
y_4 &= h_4 = i_{f0}, y_{4d} = i_{f0}^*
\end{align*} \tag{13} \]

Where

\[ x_4 = i_{f0}, \quad L_f h_4(x) = \frac{1}{L_f} (-v_0 - R_f i_{f0}), L_f h_5 = \frac{1}{L_f}, u_4 = v_{f0} \]

For \( v_{dc}, \; i_a, \; i_p \) and \( i_o \), the surfaces \( S_1, S_2, S_3 \) and \( S_4 \) are given by the following expression:

\[ S_i = k_i (v_{dc} - v_{dc}^*) + k_{i1} \int (v_{dc} - v_{dc}^*) dt \]

\[ S_2 = k_2 (i_{fa} - i_{fa}^*) + k_{i2} \int (i_{fa} - i_{fa}^*) dt \]

\[ S_3 = k_3 (i_{fb} - i_{fb}^*) + k_{i3} \int (i_{fb} - i_{fb}^*) dt \]

\[ S_4 = k_4 (i_{fo} - i_{fo}^*) + k_{i4} \int (i_{fo} - i_{fo}^*) dt \tag{14} \]

And consequently, their temporal derivatives are given by:

\[ \dot{S}_1 = k_1 \frac{d}{dt}(v_{dc} - v_{dc}^*) + k_{i1} \int (v_{dc} - v_{dc}^*) dt \]

\[ \dot{S}_2 = k_2 \frac{d}{dt}(i_{fa} - i_{fa}^*) + k_{i2} \int (i_{fa} - i_{fa}^*) dt \]

\[ \dot{S}_3 = k_3 \frac{d}{dt}(i_{fb} - i_{fb}^*) + k_{i3} \int (i_{fb} - i_{fb}^*) dt \]

\[ \dot{S}_4 = k_4 \frac{d}{dt}(i_{fo} - i_{fo}^*) + k_{i4} \int (i_{fo} - i_{fo}^*) dt \tag{15} \]

The equivalent control can be calculated from the formula \( \dot{S} = 0 \), and the stabilizing control is given to guarantee the convergence condition [6-9]. Finally, the control law is given by:

\[ p_{dc} = \frac{1}{L_f h_1} \left( -L_f h_1 + \frac{dv_{dc}^*}{dt} - \frac{k_{i1}}{k_1} (v_{dc} - v_{dc}^*) \right) - k_a \text{sign}(S_1) \]

\[ v_{f0}^* = \frac{1}{L_f h_3} \left( -L_f h_3 + \frac{di_{fa}^*}{dt} - \frac{k_{i2}}{k_2} (i_{fa} - i_{fa}^*) \right) - k_a \text{sign}(S_2) \tag{16} \]

\[ v_{fb}^* = \frac{1}{L_f h_3} \left( -L_f h_3 + \frac{di_{fb}^*}{dt} - \frac{k_{i3}}{k_3} (i_{fb} - i_{fb}^*) \right) - k_a \text{sign}(S_3) \]

\[ v_{fo}^* = \frac{1}{L_f h_3} \left( -L_f h_3 + \frac{di_{fo}^*}{dt} - \frac{k_{i4}}{k_4} (i_{fo} - i_{fo}^*) \right) - k_a \text{sign}(S_4) \]

Where: \( k_a, k_{a}, k_p, k_{a}, k_{a}, k_{a}, k_{a}, k_{a}, k_{a}, k_{a}, k_{a}, k_{a} \) and \( k_a \) are positive constants.

2.4. 3D-SVM for the four-leg inverter

In 3D-SVM, there are 16 possible switching vectors: fourteen active nonzero vectors and two null vectors. These are shown in Fig. 2. The entire 3-D Space is divided in 6 prisms and 24 tetrahedrons. Each prism consists of four tetrahedrons. An instantaneous reference input vector may lie in any of these tetrahedrons at any point of time [10].
The 3D-SVM algorithm is composed by four essential steps as follows:
- Identification of prism;
- Identification of tetrahedrons;
- Duty cycle calculation;
- Generation of PWM sequence for the switches.

![Switching vectors of a three-phase four legged inverter](image)

Figure 2. Switching vectors of a three-phase four legged inverter

3. SIMULATION RESULTS

Harmonic current filtering, reactive power compensation, load current balancing and neutral current elimination performance of the four leg-SAPF with the proposed control have been examined under balanced and unbalanced nonlinear load. The parameters used are shown in Table 1.

| Table 1. System parameters                                      |       |
|----------------------------------------------------------------|-------|
| RMS value of phase voltage                                     | 220 V |
| DC-link capacitor C                                            | 5 mF  |
| Source impedance $R_s, L_s$                                    | 1 mΩ, 1 mH |
| Filter impedance $R_f, L_f$                                    | 0.1 mΩ, 0.1 mH |
| Line impedance $R_L, L_L$                                      | 1 mΩ, 1 mH |
| DC-link voltage reference $v_{dc\,\text{ref}}$                 | 800 V |
| Diode rectifier load $R_d, L_d$                               | 5 Ω, 10 mH |
| Switching frequency $f_s$                                      | 10 kHz |
| Sampling frequency $f_s$                                       | 1 MHz |
| $k_{d}, k_{q}, k_{f}$, $k_{s}$ constants                       | 120, $10^4$ |
| $k_f=k_a=k_b=k_c=k_d=k_e=$ $k_{a}=k_{c}=k_e=k_c$ constants     | $4.10^4$, $5.10^4$, $10^4$ |
Figure 3. Simulation results of the proposed sliding mode controller. (a) Source current, (b) Load current, (c) Source voltage and source current of a-phase, (d) Neutral current. (e) DC-link voltage \( v_{dc} \).

Figure 4. Simulation results with PI controller. (a) Source current, (b) Load current, (c) Source voltage and source current of a-phase, (d) Neutral current, (e) DC-link voltage \( v_{dc} \).
The harmonic spectrums of AC supply current before and after compensation are illustrated in Fig. 5. It results that the active filter decreases the total harmonic distortion (THD) in the supply currents from 16.94% to 2.18% with PI controller. However, with sliding mode controller, the THD is increased to 0.09% which proves the effectiveness of the proposed nonlinear controller.

The dynamic behavior under a step change of the load in phase b at t = 0.3s (to make unbalanced of the nonlinear load) is presented in Figs. 3 and 4. It can be observed that the network current become perfectly sinusoidal after the control application, and unity power factor operation is successfully achieved, even in this transient state.

We also observe that the neutral current is almost canceled with a low ripple in case where the sliding mode control is applied (2.86% for the sliding mode controller and 18.57% for the PI controller).

The absence of an overshoot in dc voltage response during load change, low rise time and low source current THD, demonstrates the superiority of the sliding mode controller compared to its counterpart traditional PI controller.

4. CONCLUSION

This work presented a sliding mode control technique applied to a three-phase four-leg SAPF. The developed controller was aimed to compensate for harmonics and unbalance in the case of distorted nonlinear load currents, by making the source currents in phase with their corresponding phase voltages. Also, the controller was capable of eliminating the current flowing in the neutral line. Finally, the simulation results validated both the steady state and dynamic behavior of the proposed controller.

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