Mechanical analysis of flexible bending of tubes

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Abstract. Flexible bending successfully solves the problem of multi-curvature and small batch of bending tubes. In the flexible bending process of tubes, the stress state and deformation of bent tubes are complex. In order to better understand and analyze the bending deformation of tubes, the bending process of tubes was analyzed mechanically and the formula of bending moment was deduced. The residual stress in the forming process of tubes was analyzed by finite element numerical simulation. The results show that the bending moment is closely related to the tube specifications, material properties and bending curvature. In the process of flexible bending, the distribution of circumferential residual stress has some regularity; the thrust of pusher has stability and increases with the decrease of bending radius.

1. Introduction
The tube has a series of advantages, such as high strength and light weight. It attracts more and more attention and is widely used in aerospace, automobile and other industries. Flexible bending is a relatively new method of bending tube. Murata [1-2] proposed a flexible bending method called MOS and carried out a series of studies. Gantner et al. [3-4] put forward a forming method called free-bending. The principle is to feed the tube through a die cavity which can rotate freely in a certain range, and then form tubes with different radii. Li Pengfei et al. [5-6] studied the flexible bending process of profiles and tubes according to the new flexible bending equipment.

The bending process of tubes involves material nonlinearity, geometric nonlinearity and boundary condition nonlinearity, which makes the stress and deformation in the bending process complex. In this paper, the bending moment is analyzed and the corresponding formula is deduced according to the stress state of bent tubes during bending process. The stress in flexible bending process of tubes was studied by finite element numerical simulation.

2. Analysis of bending moment
Bending moment has an important influence on the deformation. In this paper, we take the bending moment as the research object and carry out a series of analysis to find out the factors affecting the bending moment. In order to deduce the formula of bending moment conveniently, we should analyze the stress state, neutral layer and material model.
Figure 1. Analysis diagram of the bent tubes

In the process of bending tubes, there are some deformation phenomena such as the tangential elongation and thinning of the outer-wall, the tangential shortening and thickening of the inner-wall and the circumferential distortion of the cross-section. Considering these deformation behaviors, we can reasonably define the tangential direction of bent tubes, radial and circumferential directions of cross-section as three principal stress directions in mechanical analysis. In this way, it is convenient for explaining the deformation behavior and beneficial for us to carry out mechanical analysis and calculation. Figure 1 shows the analysis diagram of the bent tubes. The tangential tensile and compressive deformations are the main reasons of straight tubes becoming bent tubes, while the radial and circumferential strains are smaller. In order to simplify the analysis, the effects of radial stress and circumferential stress can be neglected and only the tangential stress is considered. As shown in Figure 1 (b), it is a diagram of tangential stress.

When analyzing the bending moment, the effect of unchanged area on deformation area is neglected, and it is considered that the deformation of longitudinal metal layer with the same radius is identical. For bending tubes, there will be tangential tension stress and tangential compression stress at any moment of deformation. There is a boundary layer between which is called instantaneous stress neutral layer. There will also be a neutral layer with instantaneous strain. The stress neutral layer and the strain neutral layer all move with the change of bending degree and do not coincide with each other. In order to simplify analysis, it is assumed that the stress neutral layer and the strain neutral layer merge with each other and coincide with the original central layer of the bent tube.

Generally speaking, material model is a functional expression describing the stress-strain relationship of material during deformation. Different material models have different stress-strain relationship. In this paper, 304 stainless steel is used. Considering the problems of elastic deformation and springback, we adopt the common linear hardening elastic-plastic material model. The stress-strain relationship is shown as follows.

\[ \sigma = \sigma_s + H(\varepsilon - \varepsilon_s) = \sigma_s \left(1 - \frac{H}{E}\right) + H\varepsilon \]  \hspace{1cm} (1)

In the formula, \( E \)-elasticity modulus, \( H \)-plastic modulus, \( \sigma_s \)-Yield stress, \( \varepsilon_s \)-Ultimate elastic strain corresponding to yield stress

In order to simplify the calculation, the strain is taken as engineering strain. Because the circumferential and radial stress are neglected, it can be treated as the uniaxial stress state. Then the engineering tangential strain at this time can be expressed as

\[ \varepsilon_\theta = \frac{r_0 \cdot \sin \phi}{\rho_0} \approx \frac{r_0 \cdot \sin \phi}{R_0} \]  \hspace{1cm} (2)

In the formula, \( r_0 \)-The tube radius of any point on the cross section, \( \rho_0 \)-Curvature radius of
instantaneous strain neutral layer, $R_0$ -The radius of curvature of the original central layer, $\phi$ -The angle between the radius line of any point on the cross-section and the original central layer.

2.1 Basic assumptions
Based on the above analysis, we can put forward the following reasonable assumptions.

1. Ignoring the influence of the unchanged area on the deformation area, it is considered that the stress and deformation of each point on the longitudinal metal layer with the same radius are identical.

2. Ignoring the change of stress neutral layer and strain neutral layer, it is considered that it coincides with the original central layer of bent tubes.

3. In bending process, tangential direction of bent tubes, radial and circumferential direction of cross-section are the three principal stress directions in mechanical analysis. The influence of radial and circumferential stresses is neglected.

4. We take the plane hypothesis holding that the cross-section of the bent tube is still plane and orthogonal to the axis of the bend tubes.

5. We take the pure bending hypothesis holding that there is only bending moment on the cross-section of the bent tube, meaning that there is only vertical stress on the cross section.

2.2 Calculation of bending moment
There is micro-internal force $\sigma da$ vertical to the cross-section which acts on any micro-element of the cross-section of the bent tubes. The micro-internal force is tension on the outside of the stress neutral layer and pressure on the inside. When all the micro-internal forces take the moments from the stress neutral layer and sum the moments together, the bending moment $M$ can be obtained.

$$M = M_x = \int_y \sigma y da$$  \hspace{1cm} (3)

In the formula, $da$-micro elements on cross-section; $y$- distance from micro element to neutral layer.

As shown in Figure 1(c), the cross-section is divided into innumerable layers along the radial direction and the thickness of the layers is $t$. The cross section is divided into innumerable micro-angles along the circumferential direction and the angle of micro-angle is $d\phi$. Then the micro-element $da$ can be obtained. The normal stress on the cross-section is actually the tangential stress of the bent tube, which is also vertical to the bending radius of the point. We first derive the moment of any micro-internal force towards the neutral layer, and then we can get the bending moment by integrating the circumferential direction and the radial direction of the cross-section in turn.

From formulas (1) and (2), the moment of any micro-internal force toward the neutral layer is

$$\sigma da \cdot y = \left[ \sigma \frac{1 - H}{E} + H \cdot \frac{r_0 \cdot \sin \phi}{R_0} \right] \cdot \Delta t \cdot r_0^2 \cdot \sin \phi \cdot d\phi$$ \hspace{1cm} (4)

By integrating $\sigma da \cdot y$ along the circumference direction, the moment $M_D$ of tangential force of material of each layer to the neutral layer can be obtained

$$M_D = 2 \int_0^{\pi} \sigma da \cdot y d\phi = \left[ 4\sigma \left( 1 - \frac{H}{E} \right) + \frac{H \cdot r_0 \cdot \pi}{R_0} \right] \cdot \Delta t \cdot r_0^2$$ \hspace{1cm} (5)

In order to simplify the analysis, the tube radius of any micro-element can be obtained $r_0 = r + i\Delta t \approx r$.

By integrating $M_D$ along the radial direction, the sum of the moments of all micro-internal forces to the stress neutral layer can be obtained.

$$M_x = \int_0^r M_D dr = \int_0^r \left[ 4\sigma \left( 1 - \frac{H}{E} \right) + \frac{H \cdot r \cdot \pi}{R_0} \right] \cdot r^2 dt$$ \hspace{1cm} (6)

So the bending moment can be obtained
Formula (7) shows that bending moment is closely related to the specific size, material properties and curvature of bent tube. The bending moment is inversely proportional to the radius of curvature under given tube specifications and material parameters.

\[ M = M_s = 4r^3 \sigma \left( 1 - \frac{H}{E} \right) + tr^3 \frac{H \cdot \pi}{R_0} \]  

(7)

3. The principle of flexible Bending

The flexible bending is shown in figure 2. The guide is fixed on the workbench and plays the role of stabilizing the tubes. The bend die can move on the plane vertical to workbench (XY plane) and play the role of bending tubes. The pusher can feed tubes along the direction of worktable (Z direction) to realize continuous forming. When bending tubes, the bend die is moved a certain distance on the XY plane, and then the tube is pushed forward to realize continuous forming. The forming radii of bent tubes depend on the offset of the die. Different forming radii can be realized by changing the position of the bend die.

4. Finite element model

ABAQUS/explicit is used to simulate the flexible bending process of tubes. The finite element model is shown in figure 3. The pusher, the guide and the bend die are set as rigid bodies and divided into R3D4 rigid body elements. The tube is set as a deformable body and divided into C3D8R solid elements. The material parameters of the tube are as follows: density is 7930 Kg/m$^3$, elastic modulus is 204 Gpa, yield strength is 205 Mpa and Poisson's ratio is 0.25. Considering friction, the friction coefficient at contact surfaces is 0.2.

5. Analysis of numerical simulation results

The residual stress in the stable forming section affects the forming quality and service life of bent tubes, so it is necessary to explore the distribution law of residual stress. As shown in figure 4, the AB is the stable forming area. According to the characteristics of continuous forming of flexible bending, the forming condition of each longitudinal metal layer in the stable forming area is identical. Therefore, the circumferential stress on the outer surface of bent tubes is analyzed in this paper.
radii is obtained when diameter \( D = 12 \text{mm} \) and wall thickness \( t = 1 \text{mm} \). It can be seen that the circumferential residual stress presents the same distribution law under different radii. There are two peaks of stress which appear on both sides of bent tubes. The maximum residual stress increases with the decrease of bending radius. It is because that the degree of plastic deformation deepens with the increase of bending degree and the corresponding maximum residual stress increases.

The thrust of the pusher in the bending process can reflect the magnitude of external force and forming stability. Figure 7 shows the changing trend of the thrust along with time under different bending radii when \( D = 12 \text{mm} \) and \( T = 1 \text{mm} \). It can be seen that in the bending process, the thrust of the pusher increases rapidly from zero to a certain value, then falls back and fluctuates near a certain stable value. The changing trend shows the stability of external forces which confirms the stability of the bending process. As the radius decreases, the thrust of the pusher increases. This is because the smaller the radius is, the more difficult to form and more outside force is needed. At the same time, the reliability of the bending moment formula is indirectly verified.

![Figure 6. The thrust of pusher](image)

![Figure 7. Thrust under different bending radii](image)

**6. Conclusion**

The stress and deformation of tubes in bending process are complicated. The bending moment is affected by the size, material properties and curvature of bent tube.

The stress state of bent tubes in flexible bending process is analyzed. The distribution of circumferential residual stress shows some regularity. Its maximum value is related to the bending radius. The smaller the bending radius is, the larger the maximum residual stress is.

The thrust of the pusher fluctuates in a certain stable value during the bending process, which shows the stability of flexible bending. With the decrease of the bending radius, the thrust of pusher increases and more external force is needed.

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