Optimal Estimation Using Deep Neural Networks Applied to Navigation and Motion Control

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Abstract. The critical analysis is given concerning the current state of using deep neural networks with convolutional and recurrent layers, a recurrent network of Long Short-Term Memory, Gated Recurrent Units for estimation tasks in relation to navigation and motion control. A comparison of neural network and traditional methods is given for understanding and explaining their functioning. The differences, advantages and disadvantages of deep neural networks in relation to solving estimation problems are revealed. The possibility of machine training with reinforcement is analyzed for estimation tasks in navigation and motion control in real time. The prospects of using neural networks in the processing of navigation data, as well as for the tasks of adaptive estimation and trajectory tracking, are formulated.

Introduction

Artificial neural networks (NN) have been widely applied in various areas. At the moment the deep neural networks are rapidly developing, that are based on convolutional [1] and recurrent [2, 3] layers and used for processing video images in computer vision systems and time series. In recent years, good results have been obtained for face, technical objects, dynamic situations recognition, information security, time series forecasting [4]. To a lesser extent, this is applied to the processing of navigation information, although recently there appeared the works devoted to the use of traditional and deep NN in the tasks of navigation information processing [5]. In this case, however, only the fact of NN efficiency is most often illustrated.

However, in these applications, and particularly in navigation, the results obtained when using neural network methods require understanding and explanation. In accordance with the current trend of the Explainable Artificial Intelligence (XAI) development [6] this must be done for neural networks, as well as for any intelligent technology, to avoid speculation in the application of intelligent technologies when solving the problems of state estimation and complex technical systems control.

In this regard, the purpose of this work is the critical analysis of the optimal estimation method based on the use of deep neural networks in relation to the problems of navigation information processing with the formulation of advantages, significant shortcomings and qualitative limitations of their use for navigation and motion control.
1. The statement of the estimation task
It is necessary to find the estimate of the \( n \)-dimensional state vector of a dynamic system, the stochastic process of which is not necessarily the first-order Markov process, according to the \( m \)-dimensional measurement vector, based on the condition of minimizing the given criterion. It should be noted that in some cases the connection between the measurements and the state can be defined with a linear or nonlinear equation, which additively or multiplicatively includes a random vector that conveys the presence of measurement errors. The estimation task is presented as a scheme (Fig. 1).

![Fig. 1. The estimation task](image)

Specific applied problems of non-recurrent and recurrent estimation of a random vector and random sequences associated with navigation and motion control can be reduced to such a formulation [5, 7, 8].

2. The solution of the estimation task
Traditional optimal and suboptimal algorithms are briefly considered for the solution: non-recurrent Bayesian algorithms, Kalman type filters, Monte-Carlo method.

The networks with convolutional and recurrent layers, a Long Short-Term Memory recurrent network (LSTM), and Gated Recurrent Units (GRU) are considered as deep neural networks for estimating the state of dynamic systems.

The traditional and neural network methods are compared. As a result of comparing these methods, two statements on the interpretation of convolutional NN are presented.

Statement 1. The filter with growing memory. This statement allows us to interpret a convolutional NN, trained in accordance with the given criterion as an optimal linear non-recurrent Bayesian algorithm.

If there is a dataset (training sample)

\[
\{(y^{(j)}, x^{(j)})\}, \quad j = 1 \ldots n_0,
\]

in which the pairs \( y^{(j)}, x^{(j)} \), \( j = 1 \ldots n_0 \) represent the independent of each other implementations of the composite random vector \( z = [x^T, y^T]^T \) with the probability density function (p.d.f.) \( f(x, y) \), \( x = x_i, \quad y = Y_i = [y_1^T, \ldots, y_{i-1}^T, y_i^T]^T \), then the convolutional NN with the dimension of the kernel \( c \) equals to the dimension of the measurement vector \( k \)

\[
\tilde{x}^{CNN}(y, \hat{W}) = w_0 + W_y,
\]

when choosing to train it with a supervised learning criterion, the function
provides the estimations when the number of implementations \( n_0 \) is used for training increase. These estimations are similar in their properties to the estimations obtained using the Bayesian algorithm, which is optimal in the class of linear algorithms:

\[
\hat{x}_{\text{CNN}}(y, \hat{W}) = \hat{x} + P_{xy}^* (P_{yy}^*)^{-1} [y - \hat{y}^*],
\]

where \( \hat{x} = m_x^* ; \hat{y} = m_y^* ; P_{xy}^* ; P_{yy}^* \) represent sample values of mathematical expectations and corresponding covariance matrices.

**Statement 2.** The filter with limited memory. This statement allows us to treat the convolutional NN trained in accordance with the given criterion as a suboptimal linear non-recurrent Bayesian algorithm using \( c \)-last measurements.

If there is a dataset (training sample)

\[
\{(Y_i^{(j)}, x^{(j)}) \}, \quad j = \overline{1,n_0},
\]

in which the pairs \( Y_i^{(j)}, x^{(j)} \), \( j = \overline{1,n_0} \) represent the independent of each other implementations of the composite random vector \( z=[x^T, Y_i^{(j)^T}]^T \) with p.d.f. \( f(x, Y_i^{(j)}) \), \( x = x_i, Y_i^{(j)} = [Y_{i-1}^T, Y_i^T]^T \), \( i_1 = i - c + 1, c < i \), then the convolutional NN with the dimension of the kernel \( c \), is less than the dimension of the measurement vector \( k \)

\[
\hat{x}_{\text{CNN}}(Y_i^{(j)}, \hat{W}) = w_0 + W_{Y_i^{(j)}},
\]

when choosing to train it with a supervised learning criterion, the function

\[
\hat{J}^*(\hat{W}) = \frac{1}{n_0} \sum_{j=1}^{n_0} \left\| x^{(j)} - \hat{x}_{\text{CNN}}(Y_i^{(j)}, \hat{W}) \right\|^2
\]

provides the estimations when the number of implementations \( n_0 \) is used for training increase. These estimations are similar in their properties to the estimations obtained using the Bayesian algorithm, in which not the entire set of measurements is used from 1 to \( i \), but the set or a measurement window from \( i_1 = i - c + 1, c < i \), to \( i \), including \( c \)-last measurements:

\[
\hat{x}_{\text{CNN}}(Y_i^{(j)}, \hat{W}) = \hat{x} + P_{xY_i^{(j)}}^* (P_{Y_i^{(j)}Y_i^{(j)}}^*)^{-1} [y_i^{(j)} - \hat{y}_{i1}^*], \quad x = x_i, \quad Y_i^{(j)} = [Y_{i-1}^T, Y_i^T]^T,
\]

where \( \hat{x} = m_x^* ; \hat{y}_{i1}^* = m_{Y_i^{(j)}}^* ; P_{xY_i^{(j)}}^* ; P_{Y_i^{(j)}Y_i^{(j)}}^* \) represent sample values of mathematical expectations and corresponding covariance matrices.

In practice there are some situations when the navigation system or the mobile objects control system operates in conditions of environmental change and some a priori uncertainty appears. Meanwhile, the possibility of using machine training with reinforcement is considered to be promising for the adaptation of neural network algorithms.

3. **The results of modeling**

Two examples have been considered. The first example presents the comparison of a non-recurrent optimal linear filtering algorithm and a convolutional NN. The second example illustrates the capabilities of NN with recurrent layers in comparison with Kalman-type recurrent algorithms.
4. Conclusion
The critical analysis of the current state of using deep neural networks with convolutional and recurrent layers, a recurrent Long Short-Term Memory network, Gated Recurrent Units for estimation tasks is given in relation to navigation and motion control.

The comparison of neural network methods with traditional ones is given for understanding and explaining their functioning. The differences, advantages and disadvantages of deep neural networks in relation to the estimation problems solution are revealed:

1) The use of a convolutional network for solving a linear problem of non-recurrent estimation is analyzed. The case of estimating one scalar sequence over another is considered. When the size of the convolution kernel (filter) coincides with the dimension of the input dimension vector for the NN, the formula for estimation using a single convolutional layer of the NN coincides with the formula for the estimation found using the optimal linear non-recurrent Bayesian algorithm.

In the case when the convolution kernel dimension is less than the dimension of the input NN, not the entire set of dimensions is used, but only the set of the most recent dimensions. The formula for the estimation using a single convolutional NN layer coincides with the formula for the estimation found using an optimal linear non-recurrent Bayesian algorithm that uses only the last dimensions. The result is generalized to the multidimensional case.

2) It is shown that for the Gaussian case and linear measurements in the formula for calculating the optimal estimation using the Monte-Carlo method, the integrals in the numerator and denominator are a convolution.

3) The analysis of the machine training possibility is given with reinforcement for estimation the tasks in trajectory tracking. The opportunities for using the neural networks with recurrent layers LSTM и GRU are considered for tracking maneuvering objects.

4) The opportunities for using the neural networks in the processing of navigation data, including the problems of adaptive estimation and trajectory tracking, are considered.

References
[1] LeCun Y, Bottou L, Bengio Y, Haffner P 1998 Gradient based learning applied to document recognition IEEE Intelligent Signal Processing 306–351
[2] Hochreiter S, Schmidhuber J 1997 Long short-term memory J Neur Comp 9(8) 1735–1780
[3] Gal Y and Ghahramani Z 2016 A theoretically grounded application of dropout in recurrent neural networks Proceedings of the 30th International Conference on Neural Information Processing Systems 1027–1035
[4] Amosov O S, Amosova S G, Ivanov Y S, Zhiganov S V 2020 Using the deep neural networks for normal and abnormal situation recognition in the automatic access monitoring and control system of vehicles J Neur Comp & Appl
[5] Stepanov O A and Amosov O S 2007 The Comparison of the Monte-Carlo Method and Neural Networks Algorithms in Nonlinear Estimation Problems J IFAC-PapersOnLine 9 392–397
[6] Edwards L, Veale M 2017 Slave to the Algorithm? Why a 'Right to an Explanation' Is Probably Not the Remedy You Are Looking For 16 Duke Law & Technology Review 18
[7] Amosov O S, Malashevskaya E A, Baena S G 2009 High-speed neurofuzzy algorithms for filtering the mobile object trajectory parameters 23rd Saint Petersburg Intern Conf on Integrated Navigation Systems 3(21) 101–109
[8] Amosov O S and Baena S G 2015 Decomposition Synthetic Approach for Optimum Nonlinear Estimation J IFAC-PapersOnLine 48(11) 819–824