SUPERNOVAE CONSTRAINTS ON COSMOLOGICAL DENSITY PARAMETERS AND COSMIC TOPOLOGY

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We illustrate the constraints that a possible detection of a non-trivial spatial topology may place on the cosmological density parameters by considering the ΛCDM model Poincaré dodecahedal space (PDS) topology as a circles-in-the-sky detectable topology. To this end we reanalyze the type Ia supernovae constraints on the density parameter plane Ω_k – Ω_Λ and show that a circles-in-the-sky detectable PDS topology gives rise to important constraints on this parameters plane.

1. Introduction

Let us begin by stating our basic context. In the light of the current observations, we assume that the current matter content of the Universe is well approximated by a dust of density ρ_m, (baryonic plus dark matter) plus a cosmological constant Λ (p = −ρ_Λ). In this ΛCDM context, we additionally assume that the Universe is modelled by a space-time manifold M_4 = R x M_3 with a locally (spatially) homogeneous and isotropic metric

\[ ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right], \]

where, depending on the spatial curvature k, the geometry of the 3–space M_3 is either Euclidean (k = 0), spherical (k = 1), or hyperbolic (k = −1).

The metric (1) only expresses the principle of spatial homogeneity and isotropy along with the existence of a cosmic time t, it does not specify the underlying space-time manifold M_4 nor the spatial section M, which are often taken to be the (simply-connected) spaces: Euclidean R^3, spherical S^3, or hyperbolic space H^3. This has led to a common misconception that the Gaussian curvature k of M_3 is all one needs to determine the topology of the spatial sections M_3. However, it is a mathematical result that the great majority of constant curvature 3–spaces, M_3, are multiply-connected quotient manifolds of the form R^3/Γ, S^3/Γ, and H^3/Γ, where Γ is a fixed-point free group of isometries of the corresponding covering space (see Ref. 1 for details).

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In the general relativity (GR) framework the Friedmann written in the form \( \Omega_k = \Omega_m + \Omega_\Lambda - 1 \) makes apparent that the chief point in the search for the spatial curvature (and the associated geometry) is to constrain the total density \( \Omega_{\text{tot}} = \Omega_m + \Omega_\Lambda \) from observations. This amounts to determining regions in the parametric planes \( \Omega_m - \Omega_\Lambda \) (or \( \Omega_m - \Omega_k \)), which consistently account for the observations, and from which one expects to deduce the geometry of the Universe.

Now given that the spatial geometry can be probed but its knowledge does not determine the topology of the 3–space \( M_3 \), the question arises as to whether the spatial topology is an observable that can be used to set constraints on the density parameters \( \Omega_m \) and \( \Omega_k \). An important observational consequence of a observable nontrivial topology\(^2\) of \( M_3 \) is the existence of the circles-in-the-sky,\(^3\) i.e., pairs of correlated circles with the same fluctuation of temperature distribution \( \delta T \) will be imprinted on the CMBR anisotropy sky maps.\(^3,4\) Hence, to observationally probe a putative a nontrivial topology of \( M_3 \), one ought to examine the full-sky CMBR maps in order to extract the pairs of correlated circles and determine the spatial topology.

Here we briefly illustrate the constraints that a possible detection of a non-trivial spatial topology may place on the cosmological density parameters in the \( \Lambda \)CDM, i.e., by assuming a Poincaré dodecahedral (PDS) topology as a circles-in-the-sky observable spatial topology we reanalyze the type Ia supernovae constraints on the density parameter plane \( \Omega_k - \Omega_m \) that arise from gold set of 157 SNe Ia, as compiled by Riess \textit{et al.},\(^5\) and show that a circles-in-the-sky detection of the PDS topology gives rise to important additional constraints on parameters of this plane.

### 2. Constraints and Concluding Remarks

Using the three-year data the WMAP team\(^6\) reports six different values for the total density \( \Omega_{\text{tot}} \) ranging from a very nearly flat \( \Omega_{\text{tot}} = 1.003^{+0.017}_{-0.015} \) to positively curved \( \Omega_{\text{tot}} = 1.037^{+0.021}_{-0.021} \) depending on the combination of data set used to resolve the geometrical degeneracy. The Poincaré dodecahedral space (PDS), \( D = S^3/I^* \), fits both this latter density and the suppression of power of the low multipoles observed by the WMAP team.\(^8\) Attempts to find antipodal or nearly-antipodal circles-in-the-sky in the WMAP data have failed.\(^9,10\) On the other hand, hints of matching circles\(^11\) in first year ILC WMAP maps have be found, but although a second group has confirmed the circles,\(^10\) they have also shown that the circle detection lies below the false positive threshold.\(^10\) Even if one adopts the result that pairs of antipodal (or nearly antipodal) circles of radius \( \gamma \geq 5^{\circ} \) are undetectable in the current CMBR maps the question arises as to whether the circles are not there or are simply hidden or destroyed by various sources of contamination or even due to the angular resolution of the current CMBR maps.\(^13\) The answer to these questions

\(^a\)Other spherical topologies, notably \( O = S^3/O^* \), where \( O^* \) is the binary octahedral group, also explain these data analysis.\(^8\)
requires great care, among other things, because the level of contamination depends on both the choice of the cosmological models (parameters) and on the topology. Results so far remain non-conclusive, i.e., one group finds their negative outcome to be robust for globally homogeneous topologies, including the dodecahedral space, in spite of contamination, while another group finds the contamination strong enough to hide the possible correlated circles in the current CMBR maps. Thus, it is conceivable that the correlated circles may have been overlooked in the CMBR sky maps searches.

Fig. 1. A schematic illustration of two antipodal matching circles in the LSS.

In the Poincaré dodecahedral space $D$, which we shall assume here, the pairs of matching circles are necessarily antipodal as shown in Fig. 1. Clearly the distance between the centers of each pair of the correlated circles is twice the injectivity radius of the smallest sphere inscribable $D$. A straightforward use of trigonometric relations for the right-angled spherical triangle shown in Fig. 1 yields

$$
\chi_{lss} = \frac{d_{lss}}{a_0} = \sqrt{\Omega_k} \int_1^{1+z_{lss}} \frac{dx}{\sqrt{\Omega_m x^3 + \Omega_k x^2 - (\Omega_m + \Omega_k) + 1}} = \tan^{-1} \left[ \tan \frac{r_{inj}}{\cos \alpha} \right],
$$

where $d_{lss}$ is the radius of the LSS, $x = 1 + z$ is an integration variable, $\Omega_k = 1 - \Omega_{tot}$, $r_{inj}$ is a topological invariant that is equal to $\pi/10$ for $D$, the distance $\chi_{lss}$ is measured in units of the curvature radius, $a_0 = a(t_0) = (H_0 \sqrt{1 - \Omega_{tot}})^{-1}$, and $z_{lss} = 1089$. Equation (2) makes apparent that $\chi_{lss}$ depends on the cosmological scenario, which we have taken to be $\Lambda$CDM.

Equation (2) give the relation between the angular radius $\alpha$ and the parameters of the $\Lambda$CDM model, and thus can be used to set constraints on these parameters. For a detailed analysis of topological constraints in the context of other models and data sets, including braneworld inspired models, we refer the readers to Refs. 16 and Refs. 17.

To illustrate the role of the cosmic topology in constraining the density parameter in the context of $\Lambda$CDM model, we consider the $D$ spatial topology, and assume the angular radius $\alpha = 50^\circ$ and uncertainty $\delta \alpha \simeq 6^\circ$. Figure 2 shows the results...
of our joint SNe Ia gold sample plus cosmic topology analysis. There we display the confidence regions in the parametric plane $\Omega_k - \Omega_m$ and also the regions from the conventional analysis. The comparison between these regions makes clear that the effect of the $D$ topology is to reduce considerably the area corresponding to the confidence intervals in the parametric plane as well as to break degeneracies arising from the current SNe Ia measurements. The best-fit parameters for this joint analysis are $\Omega_m = 0.32$ and $\Omega_k = -0.022$.

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