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To cite this article: Chen Xianghong et al 2017 IOP Conf. Ser.: Earth Environ. Sci. 61 012029

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Double-parameter foundation beam modal of overlying thin rock stratum under loose layers grouting mining

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Abstract. Strata movement of drilling grouting is distinctly different from that of traditional caving mining, especially for overlying thin rock stratum. Considering the horizontal shear transfer of foundation, a mechanical model of overlying strata movement was established using the theory of Pasternak double-parameter foundation beam. The deflection and internal force equations of rock beam were derived, and the influence mechanisms of grouting material parameter were discussed. The results showed that the thickness of layer grouting was the key influential factor than the foundation modulus coefficient and grout diffusion radius. The comparison with the theory of Winkler elastic foundation beam also showed that double-parameter foundation beam method is more applicable to reinforced strata.

1. Introduction
Thin rock stratum can often be found widely scattered in some coalfields in China, and its sudden breakage has significant impact on mining activities, even catastrophic losses, so controlling the roof rock’s deformation has remained universally technical challenge facing scientific researchers of mining engineering. The strata movement and its control theory have been introduced and promoted, i.e. pressure arch theory [1, 2], hinged rock block theory [3], voussoir beam theory [4-6], elastic foundation beam theory [7, 8] and plate theory [9]. As to thin rock stratum above coal seam, its related mechanisms have also been studied by researchers, For example, based on the theory of elastic thin plate, Li et al. [10] studied the relationship between the elastic foundation coefficient and the roof deformation; Huang [11] obtained the critical unbroken conditions of main roof using the principle of virtual work. Using numerical simulation, theoretical analysis and in-site measurement, Fang et al. [12] investigated the strata movement rules that bedrock thickness and mechanical properties are the key influential elements for the stable structure of “stacked layer of blocks”; Du et al. [13] analysed the deformation and breakage law of semi-arch structural model of immediate roof in Sima Coal Mine of Lu’an; Li et al. [14] analysed the regularity of strata behaviour in mining face under different ratios of rock and loading. The aforementioned investigations are mainly for untreated rock stratum, as to the behaviour of reinforced strata, its correlative theory still need to be discussed.

In this paper we take the grouting technology and site strata in Ordos Coal Field in China. The roof rock’s behaviour is outlined based on the mathematical model and analytical solution, in which a Pasternak elastic foundation beam model is proposed for the deformation assessment of rock beam and the transfer matrix method is used for the solution. Finally, the influential effects of grouting parameters are studied.
2. Site description
The Stone GeTai Coal Mine lies in Ordos, Inner Mongolia of China, and coal mine 22303 is in the fourth panel of this mine. According to the field drilling investigation and adjacent mining experience, the overlying strata profile revealed: the cover depth of coal seam is about 60-100 m with 10-50 m roof rock mainly composed of fine-grained sandstone. Under such geological conditions, the fracture in roof rock grows quickly during coal mining, in order to improve the mining safety and efficiency, grouting reinforcement measures are proposed to stabilize the overlying strata. The schematic graph of 10 m thick roof rock is illustrated in Figure 1.

![Figure 1. Schematic graph of layer grouting](image)

3. Pasternak double-parameter beam model

3.1. Analytical model and assumptions
To analyse the mechanism of the roof rock, an Euler-Bernoulli beam is provided, and the overlying grouted loose layers are simulated as Pasternak double-parameter elastic foundation considering the shear transfer mechanism along horizontal, then the solutions for the beam deflection and internal force can then be obtained using the beam differential equation. The simplified analytical model is illustrated in Figure 2.

![Figure 2. Sketch map of simplified strata model](image)

In Cartesian coordinate $xyz$, the beam is assumed to be homogeneous material with thickness $h$, breadth $B$ and length $L$; $F_z$ denotes the vertical supporting force at both ends of the rock beam; $H$ denotes the depth of loose layers; $k_v$, $G_v$ denote the equivalent foundation reaction coefficient and shear transfer coefficient.

Before analyses, we introduce the following assumptions:

1. The deformation of rock beam accords with the plane section assumption.
(2) Without considering the influence of other external loadings, the vertical supporting force is:

\[ F_i = \frac{1}{2} \sum_{i=1}^{n} \rho_i g H_i \]  

where \( \rho_i, H_i \) are the density and thickness of layer \( i \); \( g \) is the acceleration due to gravity.

(3) According to Vlazov theory [15], the foundation coefficient \( k_i, G_i \) can be expressed as:

\[ k_i = \sum_{i=1}^{n} E_i \int_0^H \left( \frac{d\xi}{d\xi} \right)^2 d\xi_i \quad G_i = \sum_{i=1}^{n} E_i \int_0^H \left( h_i(\xi) \right)^2 d\xi_i \]  

where \( h_i(\xi) \) is the vertical displacement distribution function of layer \( i \); \( \xi_i \) is the local coordinates along the coordinate \( z \) direction; \( G_i \) is the shearing modulus.

3.2. Differential equation and solution

The vertical displacement along the Euler-Bernoulli beam is denoted by \( w(x) \), the bending moment \( M(x) \) and shearing force \( Q(x) \) for unit rock beam element can be expressed as:

\[ \begin{align*}
\frac{dQ(x)}{dx} &= p_i(x)B' \\
\frac{dM(x)}{dx} &= Q(x)
\end{align*} \]  

where \( p_i(x) = k_iw(x) - G_iw'(x) \) is the vertical reaction of beam foundation, \( B' = B + \sqrt{G_i/k_i} \) is the equivalent calculation width [16].

Based on the theory of material mechanics, we get the differential equation:

\[ \frac{d^4w}{dx^4} - \frac{G_i B'}{EI} \frac{d^2w}{dx^2} + \frac{k_i B'}{EI} w = 0 \]  

where \( E \) denotes the Young’s modulus of beam and \( I \) is the moment of inertia about y-axis.

As to equation (4), the general solution is:

\[ w(x) = C_1 \sinh(\alpha x) \cos(\beta x) + C_2 \cosh(\alpha x) \cos(\beta x) + C_3 \sinh(\alpha x) \sin(\beta x) + C_4 \cosh(\alpha x) \sin(\beta x) \]  

where,

\[ \alpha = \sqrt{\frac{k_i B'}{4EI} + \frac{G_i B'}{4EI}}, \quad \beta = \sqrt{\frac{k_i B'}{4EI} - \frac{G_i B'}{4EI}} \]

Then the equivalent internal forces are shown below:

\[ \begin{align*}
\{M(x)\} &= \begin{bmatrix} A_1 \\ D_1 \\ A_2 \\ D_2 \\ A_3 \\ D_3 \\ A_4 \\ D_4 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} \\
\{V(x)\} &= \begin{bmatrix} A_1 \\ D_1 \\ A_2 \\ D_2 \\ A_3 \\ D_3 \\ A_4 \\ D_4 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix}
\end{align*} \]

in which,

\[ \begin{align*}
A_1 &= (\alpha^2 - \beta^2)X_1 + 2\alpha\beta X_4 \\
A_2 &= -\frac{EI}{(\alpha^2 - \beta^2)}X_2 + 2\alpha\beta X_4 \\
A_3 &= (\alpha^2 - \beta^2)X_3 + 2\alpha\beta X_2 \\
A_4 &= (\alpha^2 - \beta^2)X_4 + 2\alpha\beta X_2
\end{align*} \]

\[ \begin{align*}
D_1 &= (\beta^2 - 3\alpha^2 \beta)X_1 + (\alpha^2 - 3\alpha^2 \beta)X_2 \\
D_2 &= (\beta^2 - 3\alpha^2 \beta)X_2 + (\alpha^2 - 3\alpha^2 \beta)X_3 \\
D_3 &= (\beta^2 - 3\alpha^2 \beta)X_3 + (\alpha^2 - 3\alpha^2 \beta)X_4 \\
D_4 &= (\beta^2 - 3\alpha^2 \beta)X_4 + (\alpha^2 - 3\alpha^2 \beta)X_2
\end{align*} \]

\[ \begin{align*}
X_1 &= \sinh(\alpha x) \cos(\beta x) \\
X_2 &= \cosh(\alpha x) \cos(\beta x) \\
X_3 &= \sinh(\alpha x) \sin(\beta x) \\
X_4 &= \cosh(\alpha x) \sin(\beta x)
\end{align*} \]

Using the internal force equilibrium and deformation coordination conditions, the beam is divided into \( m \) sections according to the grouting influence domains, the mathematical relationship in the interval of unit rock beam \( x_i \) and \( x_{i+1} \) is
in which,

\[
\hat{T}_i = \begin{bmatrix}
X_1 & X'_1 & -EI\phi_1 & -EI\phi_1 + G\beta\phi_2 & X'_2 & X_2 \\
X'_1 & X'_2 & -EI\phi_2 & -EI\phi_2 + G\beta\phi_2 & X'_3 & X_3 \\
X_3 & X'_3 & -EI\phi_3 & -EI\phi_3 + G\beta\phi_4 & X'_4 & X_4
\end{bmatrix}
\]

Based on the transfer matrix method, let \( S = [w \ \theta \ M \ V]^T \) we get

\[
S(x = L) = \prod_{i=1}^{m} \hat{T}_i S(x = 0)
\]

In order to get the constants of \( C_1-C_4 \), two boundary conditions are proposed

\[
\begin{align*}
C-1: & \quad M(x_0) = M(x_L) = 0 \\
& \quad V(x_0) = V(x_L) = 0 \\
& \quad V(x_0) = -F_z \\
& \quad V(x_L) = F_z \\
C-2: & \quad \theta(x_0) = \theta(x_L) = 0 \\
& \quad V(x_0) = -F_z \\
& \quad V(x_L) = F_z
\end{align*}
\]

4. Results discussion

To verify the applicability of the proposed model, we define the material parameters as: rock beam thickness is chosen as \( h=10 \) m, calculation span is \( L=60 \) m, the overlying loose layer thickness is 50 m. The main material properties are shown in Table 1.

| Properties                      | Rock stratum | Loose layer | Grouting section |
|---------------------------------|--------------|-------------|------------------|
| Elastic modulus (Mpa)           | 5 000        | 10          | 100              |
| Density (kg/m³)                 | 2 600        | 1 600       | 1 600            |
| Poisson ratio                   | 0.2          | 0.3         | 0.25             |

According to grouting technology, the grouting holes spacing is 6 m, diffusion radius is 1.0 m, grouting thickness is 10 m. The calculation results for boundary condition C-1 are shown in Figure 3 whether the shear transfer coefficient is adopted.

![Figure 3](image-url)

(a) Relative deformation   (b) Vertical reaction

**Figure 3.** Calculation result comparisons between Pasternak and Winkler model (C-1)

It can be seen from Figure 3 that, the maximum errors calculated between Pasternak and Winkler method were about 5.0%; and Pasternak double parameter foundation model considering the horizontal shear force is more suitable than Winkler foundation model.
5. Parameter analyses
To analyze the effects of drilling grouting on beam deformation, using the Pasternak model proposed above, we introduce the non-dimensional variables $\Delta E$, $\Delta H$ and $\Delta w$:

$$
\Delta E = \frac{E_2}{E_1}, \quad \Delta H = \frac{h^*}{h}, \quad \Delta w = \frac{W_2 - W_1}{W_1} \times 100\%
$$

where, $E_1$ and $E_2$ denote the elastic modulus of no grouting and grouting region, accordingly $W_1$ and $W_2$ denote the maximum relative displacement of rock beam; $h^*$ is the thickness of layer grouting.

In the next paragraph, we will take $L=60$ m and $120$ m for example and investigate the influential mechanisms of grouting material and geometric factors on the deformation of rock beam.

5.1. Elastic modulus ratio $\Delta E$
Figure 4 shows the non-dimensional displacement $\Delta w$ for different $\Delta E$ in the range of 1-15. It can be seen from Figure 4 that, the non-dimensional displacement $\Delta w$ increases nonlinearly with $\Delta E$ increasing from 1 to 15; and the displacement of rock beam decreases sharply firstly and then slows down. On the other hand, according to the comparisons of two boundary conditions and beam lengths, the results show that, the effects of grouting is more obvious with the increase of beam length especially for boundary condition C-1.

5.2. Poisson’s ratio $\mu$.
Figure 5 shows the effects of Poisson’s ratio of grouting material in the range of 0.3-0.05 on the non-dimensional displacement $\Delta w$. From Figure 5, we conclude that the deformation changes non-obviously with Poisson’s ratio.

5.3. Grout diffusion radius $r$.
Figure 6 shows the effects of diffusion radius $r$ on the non-dimensional displacement $\Delta w$. As can be seen from Figure 6, the deformation increment $\Delta w$ increase near-linearly by 6.62% (C-1) and 6.48% (C-2) as $L=60$ m, by 9.49% (C-1) and 7.19% (C-2) as $L=120$ m with the diffusion radius.

5.4. Grouting thickness ratio $\Delta H$.
Figure 7 shows the non-dimensional displacement $\Delta w$ for different $\Delta H$ in the range of 10-50 m. In Figure 7, the non-dimensional displacement $\Delta w$ decreases nonlinearly with the increase of the thickness ratio $\Delta H$. So, optimal design of grouting parameters is crucial in mine grouting project.
Figure 6. Effect of grouting diffusion radius r on the non-dimensional deformation $\Delta w$ ($\Delta E = 2$)

Figure 7. Effect of grouting thickness ratio $\Delta H$ on the non-dimensional deformation $\Delta w$ ($\Delta E = 2$)

6. Conclusions

A simplified mathematical model of the deformation behaviour of thin rock roof during the extraction of coal seam has been proposed that accounts for the compressive stresses created by the overlying strata. Based on Pasternak foundation beam and transfer matrix theory, the rock deformation is investigated and the influence parameters of drilling grouting have been discussed: (1) Pasternak double parameter foundation model is more reasonable to simulate the shear transfer behaviour of reinforced stratum; (2) Different boundary condition assumption has important influence on the deformation characteristics of different calculation length of rock beam; (3) The key factors affecting the characteristics and development regularity of overlying rock beam deformation depends mainly on the elastic modulus, diffusion radius and the thickness of grouting.

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