Hadron Structure

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Outline

Nucleon charges:

\[ g_A \]: benchmark quantity.

\[ g_S, g_T \] → searching for BSM signals in precision \( \beta \) decay experiments.

Isovector nucleon form factors:

Electromagnetic: \( G_E(Q^2), G_M(Q^2) \) → proton radius puzzle, large \( Q^2 \) behaviour.

Axial: \( G_A(Q^2) \) → neutrino oscillation experiments.

Isoscalar nucleon form factors:

\[ G_{E,M}^s(Q^2), G_A^s(Q^2) \] → contributions of strangeness to properties of nucleon.

Pion form factor: electromagnetic, \( f_+(Q^2) \) → large \( Q^2 \) behaviour to be studied at JLab Hall C to 6 GeV\(^2\).

Nucleon sigma terms: \( \sigma_{\pi N}, \sigma_s, \sigma_{c,b,t} \) → predicting dark matter scattering cross-sections.

Nucleon quark momentum fraction. EM transition form factors of resonances.

Summary/Outlook. Many results are preliminary and can change!
General considerations: $\langle N|\bar{q}\Gamma q|N\rangle$

(Isospin symmetric limit) Isovector combinations only connected. Isoscalar also disconnected.

Systematics:

- **Excited state pollution.**
- **Renormalisation+ improvement:** for $\bar{p} = \bar{p}' = 0$ some operators/actions $c_\mathcal{O} = 0$ or $b_\mathcal{O} = 0$
  \[ \mathcal{O}^{\overline{\text{MS}}} (\mu) = Z^{\overline{\text{MS}},\text{latt}} (a\mu) \left[ (1 + b_\mathcal{O} a m_q) \mathcal{O}^{\text{latt}} + a c_\mathcal{O} \mathcal{O}^{1\text{latt}} \right] \]
- **Volume:** exponentially suppressed $\sim e^{-L m_\pi}$, $L m_\pi > 4$.
- **Discretisation effects:** $\mathcal{O}(a)$ or $\mathcal{O}(a^2)$.
- **Physical point extrapolation:** chiral pert. (inspired) $m_\pi \to m_\pi^{\text{phys}}$. 
Ensembles used for hadron structure 3pt functions

\[ N_f = 2: \text{RQCD (RQCD/QCDSF), ETMC, Mainz (CLS), QCDSF} \]

\[ N_f = 2 + 1: \text{LHPC (BMW-c), RBC/UKQCD, Mainz (CLS), RQCD (CLS), JLQCD, PACS, } \chi \text{QCD (RBC/UKQCD), NME (JLab/W&M), QCDSF/UKQCD/CSSM} \]

\[ N_f = 2 + 1 + 1: \text{PNDME (MILC), HPQCD (MILC), ETMC} \]
Nucleon isovector charges and form factors

Charged currents ($n \rightarrow p$): $\bar{u} \Gamma d$

Neutral currents ($p \rightarrow p$): $\bar{u} \Gamma u$, $\bar{d} \Gamma d$, $\bar{s} \Gamma s$

Isospin limit: $\langle p | \bar{u} \Gamma d | n \rangle = \langle p | \bar{u} \Gamma u - \bar{d} \Gamma d | p \rangle = \langle n | \bar{d} \Gamma d - \bar{u} \Gamma u | n \rangle$

Also: $\Gamma = \gamma_\mu$, $\langle p | \bar{u} \gamma_\mu d | n \rangle = \langle p | J_{em} | p \rangle - \langle n | J_{em} | n \rangle$ etc.

Isovector form factors:

$\Gamma = \gamma_\mu$

$F_1^\nu(Q^2) \gamma_\mu + \frac{F_2^\nu(Q^2)}{2m_N} \sigma_{\mu\nu} Q^\nu \quad Q^2 \rightarrow 0 \quad 1$

$\Gamma = \gamma_\mu \gamma_5$

$G_A^\nu(Q^2) \gamma_\mu \gamma_5 - i \gamma_5 \frac{\tilde{G}_P^\nu(Q^2)}{2m_N} Q_\mu \quad \rightarrow g_A$

$\Gamma = \sigma_{\mu\nu}$

$G_T^\nu(Q^2) \sigma_{\mu\nu} \quad \rightarrow g_T$

$\Gamma = 1$

$G_S^\nu(Q^2) 1 \quad \rightarrow g_S$

$\Gamma = \gamma_5$

$G_P^\nu(Q^2) \gamma_5 \quad \rightarrow g_P$
Isovector charges $g_A = \Delta u - \Delta d$

$\beta$-decay, $g_A/g_V = 1.2723(23)$ PDG 2015.

Benchmark quantity sensitive to systematics.

Presented 2016:
PNDME, NME, Mainz, RQCD, ETMC, PACS, $\chi$QCD, QCDSF, ...
Excited state contamination.
Mainz: [von Hippel, 1605.00564]

Vary interpolator

CSSM [Dragos, 1606.03195]

Finite volume

RQCD [Bali, 1412.7336]

Vary interpolator

PNDME [Bhattacharya, 1606.07049].
Isovector charges \( g_A \)

Several \( m_\pi < 165 \text{ MeV} \) results.

Impose \( Lm_\pi > 4, \ a < 0.1 \text{ fm} \)

ETMC: [Alexandrou, Mon, 15:15] \( N_f = 2 \) twisted mass fermions, \( Lm_\pi = 3 \). Increased statistics on 1507.04936, 579 configs \( \times \) 16 measurements, \( g_A = 1.22(3)(2) \) - systematics from fitting.

PACS: [Kuramashi, Thu, 16:30] \( N_f = 2 + 1 \) NP clover, stout smeared links, \( m_\pi = 145 \text{ MeV}, \ a = 0.085 \text{ fm} \), 146 configs \( \times \) 64 measurements, \( t_f - t_i = 1.3 \text{ fm}, \ Lm_\pi = 6 \)

PNDME: [Gupta, Thu, 17:50]
Isovector charges $g_A$

**PNDME:** 1606.07049: $N_f = 2 + 1 + 1$ MILC HISQ, $a = 0.06 - 0.12$ fm Valence clover (tree-level, tadpole improved) $m_\pi = 135 - 315$ MeV

**Final result:** $g_A = 1.195(33)(20)$

**NME:** [Gupta, Thu, 17:50] $N_f = 2 + 1$ JLab/W&M tadpole clover, $a = 0.114$ and 0.080 fm, $m_\pi = 200, 315$ MeV.
Isovector charges \( g_A \)

**CLS:** \( N_f = 2 + 1 \) NP clover

Physical point along:

\( 2m_l + m_s = \text{const.} \) and \( m_s = \text{const.} \)

\( a = 0.039 - 0.085 \) fm

**RQCD:** [Bali,Fri,16:50] Had. Spec. & Int.  

**Mainz:** [Harris,Thu,16:50]

Improved axial vector currents: QCDSF [Perlt,Tue,17:50], \( \chi QCD \) [Yi-Bo,Mon,14:35]  
Feynman-Hellmann theorem: [Walker-Loud,Mon,16:45].
Isovector charges: \( g_S \) and \( g_T \)

BSM contributions to \( \beta \) decay.

\[
\mathcal{L}_{CC} = -\frac{G_F^{(0)} V_{ud}}{\sqrt{2}} \left[ \epsilon_S \bar{e}(1 - \gamma_5)\nu_\ell \cdot \bar{u}d + \epsilon_T \bar{e}\sigma_{\mu\nu}(1 - \gamma_5)\nu_\ell \cdot \bar{u}\sigma^{\mu\nu}(1 - \gamma_5)d \right]
\]

Studied in planned precision \( \beta \) decay expts. + LHC \( pp \to e\nu + X \)

Estimate of \( g_T \):

Transverse spin \( g_T = \int dx [\delta u(x) - \delta d(x)] \)

Phenomenological estimates from fits to SIDIS data \( g_T \approx 1, \Delta g_T/g_T \gtrsim 25\% \).

Estimate of \( g_S \):

CVC relation \( \partial_\mu(\bar{u}\gamma_\mu d) = -i(m_u - m_d)\bar{u}d \) applied to \( \langle p(p_\ell)|\bar{u}\gamma_\mu d|n(p_i)\rangle \)

Forward limit: \( (m_u - m_d)g_S = (m_p - m_n)^{QCD} \).

[Gonzalez-Alonso,1309.4434]:

Lattice estimates of \( m_u - m_d \) and \( (m_p - m_n)^{QCD} \to g_S = 1.02(11) \).
Isovector charges: $g_S$ and $g_T$

PNDME: [Gupta, Thu, 17:50], NME: [Gupta, Thu, 17:50], Mainz: [von Hippel, Wed, 9:40], ETMC: [Alexandrou, Mon, 15:15]
Isovector charges: $g_S$ and $g_T$

PNDME
[Bhattacharya,1606.07049]:
continuum + chiral extrap
$g_{S}^{u-d} = 0.97(12)(6)$
$g_{T}^{u-d} = 0.987(51)(20)$
Aim for precision: improving statistics

- Increase sampling per configuration.
- More sources/volume averaging/low modes.
- Cost mitigated by using e.g. All-mode-averaging (AMA) techniques [Blum,1208.4349].

AMA (truncating the solver [Bali,0910.3970]):

\[
\langle C \rangle^{AMA} = \frac{1}{n} \sum_{source\ i=1}^{n} \langle C_{x_i}^{approx} \rangle + \langle C_{x_0}^{exact} - C_{x_0}^{approx} \rangle
\]

Efficiency depends on the solver + smearing/contraction overhead.

NME: [Yoon,1602.07737]: Multigrid solver [Babich,1005.3043] \(\rightarrow\) factor of \(\sqrt{2}\) reduction in errors for fixed cost at \(m_\pi = 312\) MeV.

Cost effectiveness improves as \(m_\pi\) decreases [Bhattacharya,1606.07049].

Mainz: [von Hippel,1605.00564]: Locally deflated SAP-preconditioned solver (OpenQCD) \(\rightarrow\) factor of 2 reduction in errors.
Aim for precision: excited state contamination

Keeping statistical errors small $\rightarrow$ small sink-source separation $\rightarrow$ excited state contamination.

$m_\pi \sim 280 - 320$ MeV

More smearing, pro: smaller $t_f - t_i$, con: more expensive ($a \rightarrow 0$), larger errors.

Strategies:
(A) multiple $t_f - t_i$, single smearing $\rightarrow$ 2-state fits, summation method, ...
(B) single $t_f - t_i$, multiple smearings $\rightarrow$ variational method.

Investigated in CSSM: [Dragos,1606.03195] and NME: [Yoon,1602.07737].

Also on excited states: [Hansen,Fri,14:20], [Walker-Loud,Mon,16:45]
Nucleon isovector form factors

\[ \langle p(p_f) | V_{\mu}^{u-d} | p(p_i) \rangle = \bar{u}_p(p_f) \left[ F_1^\gamma(Q^2) \gamma_\mu + \frac{F_2^\gamma(Q^2)}{2m_N} \sigma_{\mu\nu} Q^\nu \right] u_p(p_i) \]

\[ \langle p(p_f) | A_{\mu}^{u-d} | p(p_i) \rangle = \bar{u}_p(p_f) \left[ G_A^\gamma(Q^2) \gamma_\mu - i \frac{\tilde{G}_P^\gamma(Q^2)}{2m_N} Q_\mu \right] \gamma_5 u_p(p_i) \]

Sachs ff.:

\[ G_E^\gamma(Q^2) = F_1^\gamma(Q^2) - \frac{Q^2}{4m_N^2} F_2^\gamma(Q^2), \quad G_M^\gamma(Q^2) = F_1^\gamma(Q^2) + F_2^\gamma(Q^2) \]

Forward limit:

\[ G_E^\gamma(0) = F_1^\gamma(0) = 1, \quad G_A^\gamma(0) = g_A, \quad G_M^\gamma(0) = 1 + F_2^\gamma(0) = \mu^{p-n} = 1 + \kappa^{p-n} = 4.79 \]

Shape at low \( Q^2 \), \( \langle r_X^2 \rangle = -6 \frac{dG_X(Q^2)}{dQ^2} \): different probe \( \rightarrow \) different radius.

\[ G_X(Q^2) = G_X(0) \left[ 1 - \frac{1}{6} \langle r_X^2 \rangle Q^2 + \ldots \right] \]
Nucleon isovector form factors

Challenges for the lattice:

Achieving low $Q^2$, (conventionally) $\rightarrow$ large $L$, $ap = (2\pi n/L)$.

Very sensitive to $m_\pi$: radii diverge as $m_\pi \rightarrow 0$.

Parameterising $\rightarrow$ dipole form, $z$ expansion etc $\rightarrow \langle r^2_E \rangle$, $\langle r^2_A \rangle$.

Extrapolation $\rightarrow \tilde{G}^\nu_P(0)$, $G^\nu_M(0)$, $\langle r^2_M \rangle$. 
Electromagnetic form factors

Proton charge radius:

Radius: would need $< 2\%$ error with all systematics included.

Compute isovector form factors: comparing with $\langle r_E^2 \rangle^v = \langle r_E^2 \rangle^{p-n}$. 
\( G^V_E(Q^2) \) and \( G^V_M(Q^2) \)

\[
\langle r^2_{E,M} \rangle \quad \text{increase as} \quad m_\pi \to 0, \quad \langle r^2_E \rangle \quad \text{decreases as} \quad a \to 0
\]

\textbf{PNDME}: \( N_f = 2 + 1 + 1 \) MILC HISQ, \( a = 0.06 - 0.12 \) fm Valence clover (tree-level, tadpole improved) \( m_\pi = 135 - 315 \) MeV, AMA

\textbf{NME}: \( N_f = 2 + 1 \) JLab/W&M tadpole clover, \( a = 0.114 \) and \( 0.080 \) fm, \( m_\pi = 200, 315 \) MeV, AMA. \textit{Consistency between clover+HISQ and clover-clover results.}
$G_E(Q^2)$ and $G_M(Q^2)$

![Graphs showing $G_E(Q^2)$ and $G_M(Q^2)$ vs $Q^2$]

Mainz: [Harris, Thu, 16:50] $N_f = 2 + 1$, NP clover CLS, $m_\pi = 200 - 350$ MeV, $Lm_\pi \gtrsim 4$, $a = 0.084, 0.06$ fm, 8000 – 30000 measurements (AMA), $t_f - t_i = 0.7 - 1.3$ fm.
$G_E^V(Q^2)$ and $G_M^V(Q^2)$

ETMC: [Koutsou, Thu, 17:10] $m_\pi = 131$ MeV, $a = 0.093$ fm, $Lm_\pi = 3$, $L = 4.5$ fm
$N_f = 2$ twisted mass with clover term, $t_f - t_i = 1.1 - 1.7$ fm, 6816-69784 measurements.

PNDME: [Jang, Thu, 15:00] $m_\pi = 138$ MeV, $a = 0.09$ fm, $Lm_\pi = 3.9$, $L = 5.6$ fm
$G_E(Q^2)$ and $G_M(Q^2)$

PACS: [Kuramashi, Thu, 16:30], $m_\pi = 145$ MeV, $a = 0.085$ fm, $Lm_\pi = 6$, $L = 8.1$ fm
$N_f = 2 + 1$ NP clover, stout smeared links, 146 configs $\times$ 64 measurements, $t_f - t_i = 1.3$ fm

ETMC: [Koutsou, Thu, 17:10] $m_\pi = 131$ MeV, $a = 0.093$ fm, $Lm_\pi = 3$, $L = 4.5$ fm
$N_f = 2$ twisted mass with clover term, $t_f - t_i = 1.1 - 1.7$ fm, 6816-69784 measurements.

PNDME: [Jang, Thu, 15:00] $m_\pi = 138$ MeV, $a = 0.09$ fm, $Lm_\pi = 3.9$, $L = 5.6$ fm
Direct extraction of radii and anomalous magnetic moment

Applying similar methods to those used in hadronic vacuum polarisation studies.

E.g. \( C_{3pt}^i(t, \vec{q}, \Gamma_k) = \Gamma_k \langle \mathcal{N}(\vec{p}_f, t_f) J_{i}^{em,u-d}(\vec{q}, t) \mathcal{N}(\vec{0}, 0) \rangle \propto \epsilon_{ijk} q_j G_M(Q^2) \)

Extract \( G_M(0) \) using \( \lim_{q^2 \to 0} \frac{\partial}{\partial q_j} C_{3pt}^i(t, \vec{q}, \Gamma_k) \)

**ETMC:** [Koutsou, Thu, 17:10]:

Position space method: lattice version of \( \int d^3x \, i x_j \, C_{3pt}^i(t, \vec{x}, \Gamma_k) \)

Tested with \( C_{3pt}^i(t, \vec{q}, \Gamma_i) \propto q_i G_E(Q^2) \)

\[ [\text{Alexandrou}, 1605.07327] \]
\[ N_f = 2 + 1 + 1, \]
\[ m_\pi = 373 \text{ MeV}, \]
\[ a = 0.08 \text{ fm}. \]
\[ G_M^v(0) = 4.45(17)(7), \]
previously 3.93(12).
Direct extraction of radii and anomalous magnetic moment

[Chang, Thu, 14:40]

\( N_f = 2 + 1 \) clover with \( a = 0.12 \) fm, 
\( m_\pi = 400 \) MeV

Also: LHPC [Hasan, Thu, 15:20] using “Rome method” [de Divitiis, 1208.5914]:

Expand correlation functions with respect to the spatial components of external momenta.

\[
C[\vec{p}; U] = C^{(0)}[U] + p_k C_k^{(1)}[U] + \frac{p_h p_k}{2} C_{hk}^{(2)}[U] + \cdots .
\]
Isovector form factors

Achieving large $Q^2$: momentum smearing RQCD [Bali,1602.05525]

Poster: B. Lang (RQCD) for pion. Talk: [Syritsyn,Mon,15:55]

Achieving large $Q^2$: Feynman-Hellmann approach CSSM/QCDSF/UKQCD [Chambers,Mon,17:05]

Also:

Variational method for form factors including parity projection for $\vec{p} \neq \vec{0}$ [Stokes,Phys.Rev.D92,114506] and [Stokes,Fri,15:40] → extract EM form factors for first three states of $P = \pm$ nucleon at $m_\pi = 515$ MeV.
Nucleon axial form factor $G_A(Q^2)$

Previously, [Lin,0802.0863], [Yamazaki,0904.2039], [Bratt,1001.3620], [Bali,1412.7336]

Needed for neutrino oscillation experiments:

Charged current quasielastic (CCQE) neutrino-nucleus interaction must be known to high precision.

Connecting quark - nucleon level: $G_A(Q^2)$ form factor.

nucleon - nucleus level: nuclear model.

Traditionally: information on $G_A(Q^2)$ extracted from expt. using dipole fit:

$$G_A(Q^2) = \frac{g_A}{(1 + \frac{Q^2}{M_A^2})^2}$$

$$\langle r_A^2 \rangle = \frac{12}{M_A^2}$$

World average (pre 1990) from $\nu$ scattering $M_A = 1.026(21) \text{ GeV}$.

Overconstrained form: different measurements, different $M_A$.

Lower energy expts: e.g. MiniBooNE: $M_A = 1.35(17) \text{ GeV}$ [Aguilar-Arevalo,1002.2680]

Systematics being explored including new analysis of old expt data:

$$\langle r_A^2 \rangle = 0.46(22) \text{ fm}^2 \rightarrow M_A = 1.01(24) \text{ GeV} \text{ from z-expansion} \ [Meyer,1603.03048]$$
Nucleon axial form factor $G_A(Q^2)$

Mainz: [von Hippel, Wed, 9:40], SM params & renorm.

$N_f = 2$ CLS NP clover, $m_\pi = 195 - 450$ MeV, $Lm_\pi \gtrsim 4$, $t_f - t_i \sim 0.6 - 1.4$ fm

$\langle r_A^2 \rangle$ extracted using z-expansion.

Expt: dipole form with $M_A = 1.02$. 
Nucleon axial form factor $G_A(Q^2)$

![Graph showing the dependence of $G_A/Q_A$ on $Q^2$ with different lattice spacings and masses.

[Jang, Thu, 15:00]

Small dependence of $\langle r_A^2 \rangle$ on lattice spacing (dipole fit).

**PNDME:** $N_f = 2 + 1 + 1$ MILC HISQ, $a = 0.06 - 0.12$ fm Valence clover (tree-level, tadpole improved) $m_\pi = 135 - 315$ MeV

**NME:** $N_f = 2 + 1$ JLab/W&M tadpole clover, $a = 0.114$ and $0.080$ fm, $m_\pi = 200, 315$ MeV. Consistency between clover+HISQ and clover-clover results.
Nucleon axial form factor $G_A(Q^2)$

Also:

RBC/UKQCD: [Ohta, Mon, 17:25] \( N_f = 2 + 1 \) domain wall

PACS: [Kuramashi, Thu, 16:30] \( N_f = 2 + 1 \) NP clover, stout smeared links

FERMILAB: [Meyer, Thu, 18:10] \( N_f = 2 + 1 + 1 \) HISQ

ETMC: [Koutsou, Thu, 17:10] \( N_f = 2 \) twisted mass
Nucleon strangeness electric and magnetic form factors

Expt: extracted from parity violating $e - N$ scattering, interference between $\gamma$ and $Z$ exchange.

Last year: LHPC [Green,1505.01803], $N_f = 2 + 1$ BMW-c clover ensembles, clover valence fermions, $m_\pi = 317$ MeV, $a = 0.11$ fm.

This year: $\chi$QCD [Sufian,1606.07075], $N_f = 2 + 1$, RBC/UKQCD domain wall ensembles, overlap valence fermions, $a = 0.08, 0.11$ fm, $m_\pi = 139, 300$ and $330$ MeV + non-unitary values.

Shown $m_\pi = 330$ MeV

[Liu,Mon,17:45]

Expt: e.g. HAPPEX [1107.0913]

$G_M^s(0.62 \text{GeV}^2) = -0.070(67)$
Nucleon strangeness electric and magnetic form factors

LHPC [Green,1505.01803]

\[ \langle r_E^2 \rangle_s = -0.0067(10)(17)(15) \text{fm}^2 \]
\[ \langle r_M^2 \rangle_s = -0.018(6)(5)(5) \text{fm}^2 \]
\[ \mu_s = -0.022(4)(4)(6) \mu_N \]

\[ \chi_{QCD} \] [Sufian,1606.07075]

\[ \langle r_E^2 \rangle_s = -0.0046(21)(02)(09)(04) \text{fm}^2 \]
\[ \mu_s = -0.073(17)(04)(06)(04) \mu_N \]
Isoscalar axial form factor

Renormalisation: non-singlet currents: \( Z^{ns} = Z^{ns}(a) \).

Singlet currents: (cont.) \( U_{A}(1) \) anomaly means \( Z^{s} = Z^{s}(\mu, a) \).

Deviation \( z(\mu, a) = Z^{s}(\mu, a) - Z^{ns}(a) \) is \( O(\alpha^{2}) \) in pert. theory.

Leads to mixing matrix (no \( O(a) \)):

\[
\begin{pmatrix}
\Delta u(\mu) + \Delta d(\mu) \\
\Delta s(\mu)
\end{pmatrix}
= \begin{pmatrix}
Z^{ns} + \frac{1}{3}z & \frac{1}{3}z \\
\frac{1}{3}z & Z^{ns} + \frac{1}{3}z
\end{pmatrix}
\begin{pmatrix}
\Delta u(a) + \Delta d(a) \\
\Delta s(a)
\end{pmatrix}
\]

LHPC: non-perturbative determination via Rome-Southampton method. [Green, Thu, 15:40]

\( m_{\pi} = 317 \text{ MeV, } a = 0.11 \text{ fm} \).

Also RQCD \( N_{f} = 2 \) [Piemonte, Wed, 9:20], SM params & renorm.
Isoscalar axial and induced pseudoscalar form factors

LHPC: $N_f = 2 + 1$ BMW-c clover ensembles, clover valence fermions, $m_\pi = 317$ MeV, $a = 0.11$ fm.

- Data points: statistical errors only. Fits using z-expansion, stat.+sys. error shown.
- $G_A^s$ is small.
Pion electromagnetic form factors

Expt: JLab Hall C Fpi12, charged pion form factor to 6 GeV$^2$.

HPQCD [Koponen,1511.07382]: $N_f = 2 + 1 + 1$ MILC HISQ, $a = 0.088 - 0.15$ fm, $m_\pi \sim 133$ MeV, $Lm_\pi = 3.3 - 3.9$, $t_f - t_i = 1.4 - 2.3$ fm.

Twisted b.c.: $q^2$ down to $-0.006$ GeV$^2$ $\langle r_E^2 \rangle = 0.403(18)(6)$ fm$^2$

Fit:

$$f_+(q^2) = \frac{1}{(1 - \langle r^2 \rangle q^2/6)}$$

modified for disc. effects
+ mistuning $m_q^{sea}$,
$m_\pi \neq m_\pi^{phys}$.
Pion electromagnetic form factors

ETMC: [Kostrzewa,Fri,15:00] \( N_f = 2 \) clover twisted mass, \( m_\pi \sim 131 \) MeV, \( a = 0.093 \) fm, \( Lm_\pi = 3 \) and 4.

Twisted b.c.: \( Q^2 \) down to 0.0078 GeV^2.

\[
\langle r_E^2 \rangle_{\text{eff}} = 6 \left( 1 - f_+(Q^2) \right) \frac{Q^2}{Q^2}
\]

Fits to \( f_+(Q^2) \): \( 1 - \langle r^2 \rangle/6Q^2 + cQ^4 \), \( c \approx 0 \)

\[
L=48 \quad \langle r_E^2 \rangle = 0.328(29) \text{ fm}^2
\]

\[
L=64 \quad \langle r_E^2 \rangle = 0.437(42) \text{ fm}^2
\]
Pion electromagnetic form factors

**PACS:** [Kakazu,Fri,14:40] \( N_f = 2 + 1 \) NP clover, stout smeared links, \( m_\pi = 145 \) MeV, \( a = 0.085 \) fm, 40 configs \( \times 32 \) measurements, \( t_f - t_i = 3 \) fm, \( Lm_\pi = 6 \).

Large volume: \( L \sim 8.1 \) fm

\( Q^2_{\text{min}} \sim 0.019 \) GeV\(^2\)

\[
\langle r_E^2 \rangle^\text{eff} = 6 \left( 1 - f_+(Q^2) \right) \frac{Q^2}{Q^2}
\]

Fit NLO SU(2) ChPT expression for \( f_+(Q^2) \rightarrow \) l.e.c. \( l_6 = -0.01234(72) \) (consistent with FLAG 2015)

Adjust to \( m_\pi^{\text{phys}} \), \( \langle r_E^2 \rangle = 0.412(21) - 0.415(20) \) fm\(^2\).
Pion electromagnetic form factors

PACS: [Kakazu,Fri,14:40]

Also Mainz [Brandt,1306.2916] $\langle r_E^2 \rangle = 0.481(33)(13) \text{ fm}^2$ in chiral limit.
Sigma terms: \( \sigma_q = m_q \left( \langle N | \bar{q}q | N \rangle - \langle 0 | \bar{q}q | 0 \rangle \right) \)

Scattering of DM candidates off nuclei (near zero recoil).

Spin-independent effective interaction
\( \sim \bar{\chi} \chi \bar{q}q \)
\( \sigma_{SI}^{N} \propto |f_N|^2 \)
\( N = p, n. \)

\[
\frac{f_N}{m_N} = \sum_{q \in \{u,d,s,c,b,t\}} f_{T_q}^N \frac{\alpha_q}{m_q}, \quad f_{T_q}^N = \frac{\sigma_q^N}{m_N}
\]

Higgs example: \( \alpha_q \propto m_q / m_W \)

For heavy flavours \( \langle \bar{h}h \rangle_N \propto 1/m_h, m_h \rightarrow \infty \).

If \( \alpha_q \propto m_q, t, b \) and \( c \) matter.

If \( \alpha_q \) insensitive to \( m_q \), only \( u, d, (s) \) play a role.
Sigma terms: experiment

21st July 2016, referring to the Lux expt.

PDG 2015
Direct determinations of $\sigma_{\pi N} = \sigma_u + \sigma_d$

RQCD [Bali,1603.00827], ETMC [Abdel-Rehim,1601.01624], ETMC [Dinter,1202.1480], χQCD [Yang,1511.09089]

Note: $a c_{\text{FF}} \langle FF \rangle$ neglected $\rightarrow \mathcal{O}(a)$ errors for all actions.

ETMC: [Alexandrou,Mon,15:15], [Vaquero,Thu,17:30]: update with increased statistics, $\sigma_{\pi N} = 36(2) \text{ MeV}$ - statistical errors only.
Strangeness: \( f_{Ts} = \frac{\sigma_s}{m_N} \) and \( y = \frac{2\langle \bar{s}s \rangle}{\langle \bar{u}u + \bar{d}d \rangle} \)

**ETMC:** [Alexandrou, Mon, 15:15], [Vaquero, Thu, 17:30]: update with increased statistics, \( \sigma_s = 37(8) \) MeV \( \rightarrow f_{Ts} = 0.039(9) \) - statistical errors only.

Also: JLQCD [Ohki, 1208.4185] \( N_f = 2 + 1 \) overlap, \( a = 0.11 \) fm, \( m_\pi = 300 - 540 \) MeV, \( f_{Ts} = 0.009(22) \).

RQCD [Bali, 1603.00827] \( N_f = 2 \), RQCD/QCDSF NP clover, \( a = 0.06 - 0.08 \) fm, \( f_{Ts} = 0.037(13) \).

Hybrid method: [Freeman and Toussaint, 1204.3866] \( N_f = 2 + 1 \) MILC Asqtad \( f_{Ts} = 0.061(9) \), \( N_f = 2 + 1 + 1 \) MILC HISQ, \( a = 0.06 - 0.15 \) fm, \( f_{Ts} = 0.044(1) \).
Nucleon sigma terms from Feynman-Hellmann theorem

\[ \sigma_{\pi N} = \sigma_u^N + \sigma_d^N = m_u \frac{\partial m_N}{\partial m_u} + m_d \frac{\partial m_N}{\partial m_d} \approx m_\pi^2 \frac{\partial m_N}{\partial m_\pi^2}, \]

\[ \sigma_s^N = m_s \frac{\partial m_N}{\partial m_s} \approx m_{ss}^2 \frac{\partial m_N}{\partial m_{ss}^2}. \]

**BMW-c [Dürr,1510.08013]:**

\( N_f = 2 + 1, \) 2HEX-Clover,

\( a = 0.054 - 0.116 \) fm,

\( m_\pi \geq 120 \) MeV.

**Figure:** [Dürr,1011.2711]
Nucleon sigma terms from Feynman-Hellmann theorem

BMW-c [Dürr,1510.08013]

\[ M_N = M_N^{(\Phi)} (1 + g_N^a(a)) (1 + g_N^{FV}(M_\pi, L)) \left( 1 + c_N^{a,ud}(a) \tilde{m}_{ud} + c_N^{a,s}(a) \tilde{m}_s + \ldots \right) \]

\[ \sigma_{\pi N} = 38(3)(3) \text{ MeV} \quad \sigma_s = 105(41)(37) \text{ MeV} \]

Also: PACS [Ishikawa,1511.09222]

\[ m_{ud}^{RGI} = 40 \text{ MeV} \rightarrow m_\pi \approx 400 \text{ MeV}. \quad \text{Update: [Lellouch, Thu, 14:20]} \quad N_f = 1 + 1 + 1 + 1, \]

Also: PACS [Ishikawa,1511.09222]
Summary of sigma terms and $f_{Tq}$

Values from phenomenological analyses of $\pi$-N scattering tend to be higher.

[Hoferichter,1506.04142] $\sigma_{\pi N} = 59.1(3.5)$ MeV.

[Hoferichter,1602.07688] request for a lattice computation of $\pi N$ scattering lengths.

Also interesting: $\sigma_{\pi N}(Q^2 = -2m_{\pi}^2) - \sigma_{\pi N}(0) \rightarrow$ scalar radius from form factor.
Heavy quark sigma terms

HQET relates $\sigma_{h=c,b,t}^N$ to $(2/27)(m_N - \sum_{q=u,d,s}^{q=H} \sigma_q^H)$ at leading order in $1/m_h$ and $\alpha$ [Shifman,1978].

Taking:
$\sigma_{\pi N} = 35(6)\text{ MeV}$, $\sigma_s = 35(12)\text{ MeV}$
gives
$f_{T_c} = 0.075(4)$,
$f_{T_b} = 0.072(2)$,
$f_{T_t} = 0.070(1)$,
using $N^3$LO:[Chetyrkin,hep-ph/9708255] + others

ETMC: [Alexandrou,Mon,15:15], [Vaquero,Thu,17:30]: update with increased statistics, $\sigma_c = 87(17)\text{ MeV}$ - statistical errors only.
Quark and gluon momentum fraction

First moment of $q/g$ parton distribution function: $\langle x \rangle_{q/g} = \int dx \, x \, F_{q/g}(x)$.

Connected insertion: $u, d$.
Disconnected insertion: $u, d, s, g$

ETMC: [Vaquero, Thu, 17:30] $N_f = 2$ twisted mass with clover term,

$m_\pi = 131$ MeV, $Lm_\pi = 3$,
$a = 0.093$ fm.
Stout smearing to reduce noise.
Approx: 2000 (cfgs) $\times$ 100 (sources)

Renormalisation: mixing between $\sum_q \langle x \rangle_q$ and $\langle x \rangle_g$: 1-loop to $\overline{\text{MS}}$ at 2 GeV.

$\langle x \rangle_{g, \text{bare}} = 0.318(24) \rightarrow \langle x \rangle_{g, \overline{\text{MS}}} = 0.320(24), \quad (\langle x \rangle_u + \langle x \rangle_d + \langle x \rangle_s)_{\overline{\text{MS}}} = 0.72(11)$

Momentum sum satisfied: $\sum_q \langle x \rangle_q + \langle x \rangle_g = 1.04(11)$

Consistent with $\chi_{\text{QCD}}$ quenched calculation [Deka, 1312.4816].

Also computed: $g_A^{u,d}$, $g_T^{u,d}$, $g_S^{u,d}$.
**P wave** $\pi \pi \rightarrow \pi \gamma$

Extracting $\rho \rightarrow \pi \gamma$ form factor first step towards e.g. $N\gamma^* \rightarrow \Delta \rightarrow N\pi$

HSC: using formalism developed in [Briceño,1406.5965] for $1 \rightarrow 2$ particle transition amplitudes.

Infinite volume amplitude $A_{\pi \pi,\pi,\gamma^*}(E_{\pi \pi}^*, Q^2)$ related to finite volume $\tilde{A}(E_{\pi \pi}^*, Q^2; L)$

Anisotropic $N_f = 2+1$ clover fermions $a_s/a_t \approx 3.5$, $a_s \approx 0.12$ fm, $m_\pi \approx 400$ MeV..

See also talk by David Wilson and for another study [Leskovec,Fri,17:50].
Summary

Benchmark quantities:

- First calculations with main systematics considered (continuum limit, finite $V$, physical point results, excited states, ...). More in progress.
- The same effort needs to be applied to $\langle x \rangle_{u-d}$.

Overall agreement on charges: $g_S$, $g_T$.

Efficient methods for achieving statistical precision + $m^{\text{phys}}_\pi$ simulations.

Impact on:

- Form factor determinations, $f_+^{\pi}(Q^2)$, $G_{E,M}(Q^2)$, $G_A(Q^2)$.
- $\sigma_{\pi N}$.
- Systematics needs to be explored.

Progress in disconnected techniques:

- Enable calculations of strangeness form factors.
- Estimates of disconnected contributions also at the physical point $\rightarrow G_M^s(Q^2)$, $\langle x \rangle_q$, $g_{A,S,T}^q$. 

$\Box$
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