Condition Absolute Stability Control System of Electromagnetoelastic Actuator for Communication Equipment

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ABSTRACT

We obtained the condition absolute stability on the derivative for the control system of electromagnetoelastic actuator for communication equipment. We applied the frequency methods for Lyapunov stable control system to calculate the condition absolute stability control system of electromagnetoelastic actuator. We used Yakubovich criterion absolute stability system with the condition on the derivative. The aim of this work is to determine the condition of the absolute stability on the derivative for the control system of electromagnetoelastic actuator. We received the stationary set of the control system of the hysteresis deformation of the electromagnetoelastic actuator. The stationary set is the segment of the straight line.

Keywords: Electromagnetoelastic actuator; Piezoactuator; Hysteresis; Control system; Condition absolute stability; Stationary set.

1 Introduction

The application of the electromagnetoelastic actuator based on the electromagnetoelasticity as the piezoelectric, piezomagnetic, electrostriction, magnetostriction effects is promising for the control system of electromagnetoelastic actuator for communication equipment. The piezoactuator solves problems of the precise matching in the nanotechnology, the compensation of the temperature and gravitational deformations, the atmospheric turbulence by the wave front correction in adaptive optics. The piezoactuator for the nanotechnology is used in the scanning tunneling microscopes, the scanning force microscopes, the atomic force microscopes [1–15].

Piezoactuator is piezomechanical device intended for actuation of mechanisms, systems or management based on the piezoelectric effect, converts the electrical signals into the mechanical movement or the force. The piezoactuators for the drives of nano- and micrometric movements provide the movement range from nanometers to hundred of micrometers, the loading capacity of up to 1000 N, and the transmission band of up to 100 Hz. The piezoactuators are used in the majority of nanomanipulators for adaptive optics and nanomechatronics. The nanorobotic manipulator of nano- and microdisplacements with the electromagnetoelastic actuator is the key component in nanorobotic systems. The main requirement for nanomanipulator with the electromagnetoelastic actuator is to guarantee the positioning accurate to nanometers in communication equipment [14–26].
The problems of using condition of absolute stability the control system of the electromagnetoelastic actuator nano- and microdisplacements are discussed [2, 3]. The stationary set of the control system of the electromagnetoelastic actuator is determined.

We obtained condition of the absolute stability on the derivative for the control system with the electromagnetoelastic actuator. The condition of the Yakubovich absolute stability on the derivative for the control system of electromagnetoelastic actuator for communication equipment are determined. The stationary set of the control system of the deformation of the electromagnetoelastic actuator is the segment of the straight line. The condition absolute stability on the derivative for the control systems of the piezoactuator in the case of the longitudinal, transverse and shift piezoeffect for the hysteresis characteristic of the deformation of the piezoactuator are obtained.

2 Stationary set control system of electromagnetoelastic actuator

The analytical expression for the sufficient absolute stability condition of the system with the hysteresis nonlinearity of the electromagnetoelastic actuators is written using the Yakubovich absolute stability criterion with the condition on the derivative [2, 3]. The Yakubovich absolute stability criterion is next development of the Popov absolute stability criterion. For the Lyapunov-stable control system the Yakubovich absolute stability criterion for the systems with the single hysteresis nonlinearity is provided the simplest and pictorial representation of results of the investigation of the stability of the system. We are used the correcting devices of the system ensuring the stability of the strain control systems with the electromagnetoelastic actuators. The aim of this work is to receive the condition of the absolute stability on the derivative for the control system of electromagnetoelastic actuator for communication equipment.

The measurements of the end face movement of the piezoactuator were made using Model 214 electronic measuring system of Caliber plant. The experimental static strain characteristic of the piezoactuator from PZT for longitudinal piezoeffect is shown in Figure 1 with the main hysteresis loop with the symmetrical voltage change and with partial cycle with the asymmetric voltage change on the plates of the piezoactuator.

In work we use the transfer function of the linear part of the system \( W_{ij}(s) \) with operator \( s = j\omega \), for \( s \) we have \( j \) the imaginary unity and \( \omega \) the frequency, and the hysteresis function of the relative deformation

**Figure 1. Hysteresis type characteristic deformation of piezoactuator for longitudinal piezoeffect**
$S_j$ of the electromagnetoelastic actuator, $i$ and $j$ are axis. We use the transfer function of the linear part of the system $W_{ij}(s)$ and the function of the relative deformation $S_j$ of the electromagnetoelastic actuator for description of the system [3]. The equation of the hysteresis nonlinearity of the electromagnetoelastic actuator has the form

$$S_j = F[\Psi_i^0, t, S_j(0), \text{sign}(d\Psi_i/dt)]$$  

where $S_j$ is the relative displacement of the cross section of the actuator along $j$ axis, $\Psi_i$ is the control parameter of the actuator along $i$ axis. The hysteresis function $S_j$ at each time instant $t$ depends on the behavior of the function $\Psi_i = E_i$ or $\Psi_i = H_i$, where $E_i$ and $H_i$ are the electric field strength and the magnetic field strength on the interval $[0, t]$, the value of $t$, the initial value $S_j(0)$, and the sign of the rate $d\Psi_i/dt$ of the field strength variation. We consider the alternating-sign hysteresis characteristic of the piezoactuator on Figure 1 and Figure 2. We have for the piezoactuator the set $L[E_i(0)]$ in form the vertical segment $[S_j^0, -S_j^0]$ bounded by the points of intersection of the ordinate axis with the hysteresis loop at the maximum admissible field strength in the piezoactuator. For the stable linear part of the control system of the piezoactuator we obtain the straight line $D$ with the equation

$$E_i + k_{ij}S_j = 0$$

where $k_{ij} = W_{ij}(0)$ is the transmission coefficient of the linear part of the control system, accordingly $W_{ij}(0)$ is the value of the transfer function of the linear part of the control system of the piezoactuator for $\omega = 0$.

The set of points $M$ for intersection of this straight line $D$ with the hysteresis characteristic represents the segment of the straight line marked in Figure 2 for $E_{i0} = E_i(0)$ and $S_{j0} = S_j(0)$ we have the stationary solution to the control system. The stationary set $M$ of the system is the marked segment of straight line $D$ in Figure 1 with the set of pairs $(E_{i0}, S_{j0})$. Each point of intersection of the hysteresis nonlinearity with the partial loops and the straight line $D$ corresponds to one equilibrium position with the coordinates $(E_{i0}, S_{j0})$.

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The equation of the main hysteresis loop [3] of the piezoactuator on Figure 2 has the form

\[
S_j = d_{ij}E_i - \gamma_{ij}E_{im}\left(1 - \frac{E_i^2}{E_{im}^2}\right)^{n_{ij}}\text{sign}(dE_i/dt),
\]

(3)

where \(S_j\) is the relative displacement of the piezoactuator, \(E_i\) is the electric field strength, \(d_{ij}\) is the piezomodule, \(E_{im}\) is the electric field strength amplitude, \(\gamma_{ij}\) is the hysteresis coefficient, \(n_{ij}\) is the power for piezoceramics based on lead zirconate titanate PZT we have \(n_{ij} = 1\) and \(dE_i/dt\) is the rate of the electric field strength variation. From (2) and (3) we have for the piezoactuator the equation

\[
\Delta E_{im} + k_{ij}E_{im}\left[d_{ij}\Delta - \gamma_{ij}(1 - \Delta^2)\right] = 0
\]

(4)

where \(\Delta\) is the relative electric field strength at the extreme equilibrium point for the control system of the deformation of the piezoactuator. From (4) after conversion we have the equation

\[
\Delta + k_{ij}\left[d_{ij}\Delta - \gamma_{ij}(1 - \Delta^2)\right] = 0
\]

(5)

We obtain the quadratic equation for determination of the width of the rest domain \(2\Delta\) of the piezoactuator for the longitudinal piezoeffect in the form

\[
\Delta^2 + \frac{(1+k_{ij}d_{ij})}{k_{ij}\gamma_{ij}}\Delta - 1 = 0
\]

(6)

The width of the rest domain \(2\Delta\) of the piezoactuator has the form

\[
2\Delta = -\frac{(1+k_{ij}d_{ij})}{k_{ij}\gamma_{ij}} + \sqrt{\frac{(1+k_{ij}d_{ij})^2}{k_{ij}\gamma_{ij}^2} + 4}
\]

(7)

The characteristic of the electromagnetoelastic actuators has the alternating-sign hysteresis type for the piezoactuator on Figure 2 or the constant-sign butterfly type for the electrostriction actuator on Figure 3. In the magnetostriction or electrostriction actuator initial operating point is chosen on one wing of the butterfly in the first quadrant. The deformation range has symmetric at both sides of initial point. The initial working point is displaced from zero by the half deformation range.

Figure 3. Butterfly type characteristic of electrostrictive actuator
3 Condition absolute stability of control system of electromagnetoelastic actuator

The analytical expression for the sufficient absolute stability condition of the system with the hysteresis nonlinearity of the electromagnetoelastic actuator is written using the Yakubovich absolute stability criterion with the condition on the derivative. The Yakubovich criterion is the development of the Popov absolute stability criterion. For the Lyapunov-stable control system the Yakubovich absolute stability criterion for the system with the single hysteresis nonlinearity provides the simplest and pictorial representation of results of the investigation of the stability [3]. The correcting devices are chosen for providing the high quality the control system of the deformation of the piezoactuator. The function $S(E_i)$ of the hysteresis nonlinearity of the piezoactuator is continuous, then we have quantities

$$\nu_{1ij}, \quad \nu_{2ij} \in [0, \nu_{ij}], \quad \nu_{ij} = \max(dS_i/dE_i)$$

(8)

where the quantities $\nu_{1ij}$ and $\nu_{2ij}$ are calculated using the hysteresis static characteristic on Figure 1 measured at the maximum admissible electric field strength in the piezoactuator. The quantities $\nu_{1ij} = 0$ and $\nu_{2ij} = \nu_{ij}$ are the minimum and the maximum values of the tangent of the inclination angle of the tangent line to the hysteresis nonlinearity of the piezoactuator. We obtain the following equation

$$\nu_{33} \cdot \nu_{31} \cdot \nu_{15} = d_{33} : d_{31} : d_{15}$$

(9)

where the ratios of the tangents of the inclination angle of the tangent line to the hysteresis nonlinearity of the piezoactuator for longitudinal, transverse and shift piezoeffects are proportional to the ratios of the piezomodules. The sufficient Yakubovich absolute stability condition of the control system of the deformation [4–7] of the actuator have the form

$$\text{Re } \nu_{ij} W_{ij}(j\omega) \geq -1$$

(10)

where in brackets $j$ is the imaginary unity and $\omega$ is the frequency. The amplitude-phase characteristic of the open-loop system $\nu_{ij} W_{ij}(j\omega)$ should be situated on Figure 4 to right of the straight line

$$\text{Re } \nu_{ij} W_{ij}(j\omega) = -1$$

(11)

for all $\omega \geq 0$ and the corrected amplitude frequency characteristic is to right of this straight line.

![Figure 4. Condition absolute stability control system of electromagnetoelastic actuator for communication equipment is met for shaded corrected amplitude frequency characteristic](http://dx.doi.org/10.14738/tnc.81.7775)
The Yakubovich absolute stability criterion (11) on Figure 4 is simple and convenient for synthesis of correcting devices for the control system.

We have the absolute stability criterion for the system on Figure 5 in the plane of the logarithmic amplitude frequency characteristic and the phase frequency characteristic

\[ L(\omega) = Q[\varphi(\omega)] , \quad L(\omega) = 20\log|v_{ij}W_{ij}(j\omega)| \]  

where the corrected logarithmic amplitude frequency characteristic is below the boundary curve in the following form

\[ L(\omega) = 20\log|1/\cos\varphi| \]

![Figure 5. Condition absolute stability control system of electromagnetoelastic actuator for communication equipment is met for shaded corrected logarithmic amplitude frequency characteristic](image)

For the piezoactuator from PZT the value of the maximum tangent of the inclination angle of the tangent line to the nonlinearity is about 1 nm/V for longitudinal piezoeffect and about 0.6 nm/V for transverse piezoeffect. The characteristics of the correcting device for the control system are determined from the condition of the absolute stability of control system.

### 4 Conclusion

We have the stationary set of the control system of the deformation of the electromagnetoelastic actuator in the form of the segment of the straight line. We used Yakubovich criterion absolute stability system and the frequency methods for Lyapunov stable control system to calculate the condition absolute stability control system. We received condition of the absolute stability on the derivative for the control system with the electromagnetoelastic actuator for communication equipment.

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