Simulation of ultra-relativistic dynamics of electrons in the interaction of a short-intense laser pulse with plasma

N.O. Ramirez-Soto¹ and R. Ondarza-Rovira²

¹Universidad de Sonora, Depto. Física, C.P. 83000, Hermosillo, Sonora, Mexico
²Instituto Nacional de Investigaciones Nucleares, A.P. 18-1027, México 11801, Distrito Federal, Mexico

E-mail: ¹nahari.ramirez@gmail.com , ²ricardo.ondarza@inin.gob.mx

Abstract. This paper presents numerical simulations of the interaction of an ultra-intense laser field with a single electron and studies the nonlinear plasma effects produced when an ambient harmonic restoring force is considered during the relativistic acceleration of the particle. The motion of a charged particle in a strong laser pulse of varying wave form and polarization is explored. Mainly, the objective is to study how linear polarization leads to different energy distributions for electrons stripped from atoms by the laser field. The numerical solutions that make use of the variable-step Runge-Kutta technique are presented and describe the relativistic dynamics of a single charged particle in plasma, through the relativistic equations of motion. A harmonic restoring force is used as a simple model to simulate charge separation effects and the generation of electrostatic forces during the laser field interaction in a plasma. The motion of the relativistic electron is discussed at length considering different initial conditions for time-varying pulse intensities. The dynamics of particles at rest are studied before the interaction with the laser pulse. In addition, the time dependent longitudinal momentum for various pulse intensities is compared after the interaction with the laser for a range of optical cycles. Specifically, we focus on the time dependence of the longitudinal momentum with linear polarization for different laser intensities. In addition to the longitudinal position and momentum, the effect of the residual momentum and energy of the electrons in the plasma is carefully studied. It was found a strong dependence of the residual momentum on plasma density and laser intensity for a number of ultra-relativistic laser-plasma scenarios considered

1. Introduction
A highly intense laser pulse that interacts with matter can stimulate the atoms leading to ionization [1-2]. Those electrons—the so-called photoelectrons—are due to a number of light ionization mechanisms. The incident light can provide the photoelectrons with a high energy that can be measured in the wake of a laser field [3]. It is possible to analyze the energy spectrum of such particles that after being detached from the atom attain different energy states and interact with the laser [4]. Depending on the polarization, the photoelectrons will have a wide range of energies [5]. Also, the polarization leads to
preferential ionization in which the electrons in the atoms are separated from the atom at points in which the electromagnetic radiation is highly intense [6]. For circular polarization it is found no preferential points of ionization in the optical cycles of the light [7]. The different polarizations of light lead to differing dynamics for the charged particles in the wake of the plasma. Another factor of consideration is the intensity of the light pulse which clearly has an effect on the dynamics of photoelectrons. Although the relativistic dynamics of charged particles in plane-wave laser fields can be solved exactly (as demonstrated by Sarachik and Schappert [8]) for particles at rest prior to the arrival of the pulse, in this paper, the general solution is found by means of numerical simulation. In this paper, the electrons are considered to be at rest before the arrival of the pulse, but studies have been conducted in which the velocity is non-zero [9]. Thus, it would be possible to study the relativistic dynamics of photoelectrons after being stripped from the atoms by a strong laser pulse.

2. Exact solution for a single electron in a strong plane wave
The relativistic equations of motion can be solved exactly for a vector field of the form [7]

\[ \mathbf{A}(\eta = \eta_0) = \mathbf{A}_0. \]  

Specifically, the form of the pulse is given by

\[ \mathbf{A}(\eta) = A_p P(\eta) \left[ x \delta \cos \eta + y \sqrt{1 - \delta^2} \sin \eta \right]. \]  

Here, the phase is given by

\[ \eta = \omega t - kz, \]  

and \( \delta \) is the polarization factor and values \( \delta = 0, \pm 1 \) stand for linear polarization. The kinetic momentum of a charged particle in this case is given by

\[ p = p_0 = p_{0T} + p_{0L} \hat{z}. \]  

The canonical momentum on the other hand is

\[ \Pi = \Pi_0 = \left( p_{0T} - eA_0 \right) + p_{0L} \hat{z}. \]  

with the initial energy given by

\[ E_0 = \sqrt{p_{0T}^2 + p_{0L}^2 + m^2c^4}. \]  

Here, the subscripts \( T \) and \( L \) refer to the transverse and longitudinal components, respectively. We assume the propagation of the laser wave to be along the \( z \) direction. Initially at rest at the origin, the electron is strongly accelerated by the action of an incident laser field. The laser pulse is assumed to be transverse, plane and arbitrarily polarized. Throughout the simulations the laser frequency is taken as \( \omega = 0.18 \), value that corresponds to a Kr:F laser. A pulse-shape factor \( P(\eta) \) emulating a Gaussian shape given by a sine-squared envelope is used. For solving the relativistic equations of motion we consider the Hamilton-Jacobi equation
and assume a solution form for the Hamilton principal function \( S(r, t) \) of the form
\[
S(r, t) = \alpha \cdot r + \beta ct + \Phi(\eta),
\]
here \( \alpha \) and \( \beta \) are constants determined by the initial conditions, and \( \Phi(\eta) \) is determined by the Hamilton-Jacobi equation. In our particular case, \( \alpha = \Pi_0 \) and \( \beta = \frac{E_0}{c} - p_{0L} \). By imposing these boundary conditions on the particle and differentiating the principal function with respect to the constant \( \alpha \) and equating the result to the initial coordinate, the complete solutions can be obtained. Therefore, the kinetic momentum of the electron is
\[
\mathbf{p} - p_0 = \sigma + \frac{\sigma \cdot (\sigma + 2p_{0r})}{2\beta},
\]
where
\[
\sigma = \frac{e}{c}[A(\eta) - A_0].
\]
Using this the energy of the electron attains the form
\[
E^2 = m^2c^4 + p^2c^2.
\]
The momentum of the electron after the laser interaction is obtained by setting \( A = 0 \) in the kinetic momentum, equation (9). Thus
\[
\mathbf{p}_f = \left( p_0 - \frac{e}{c}A_0 \right) - \frac{eA_0}{2\beta c} \left( 2p_{0r} - \frac{e}{c}A_0 \right).
\]

3. Numerical simulations
To study the dynamics of a single electron, in these calculations space charge forces were modeled by adding a harmonic restoring force of the form
\[
\mathbf{F} = -m \omega_p^2 \mathbf{z}.
\]
Here \( \omega_p \) is the plasma frequency. We solved the relativistic Newton-Lorentz equation,
\[
\frac{d}{dt}\mathbf{P} = -e\mathbf{E} - \frac{e}{c}\mathbf{v} \times \mathbf{B} - m \omega_p^2 \mathbf{z},
\]
using a Runge-Kutta variable-step method. Furthermore, the time-varying laser intensity is defined by the sine-squared envelope
\[
E_\lambda(\eta) = E_0 \sin \eta \sin^2(\eta/N).
\]
In figure 1, the residual longitudinal momentum of the electron is relatively large when compared to the amplitude of the longitudinal momentum during the interaction with the pulse. In figure 2, the plasma frequency is one-tenth the size of the plasma frequency in figure 1. Here, we see that the residual longitudinal momentum is much less than in the previous case.
In figure 3, we observe another possible resonance region for the specified plasma frequency, which is close to the natural frequency of the laser. We note that the longitudinal residual momentum is comparable to the pulse interaction momentum and on average larger after the passage of the pulse when compared to figure 1.

In figure 4 we observe the dependence of the maximum residual momentum (momentum obtained by the electron after the passage of the pulse) for a fixed value of the plasma frequency and pulse length. For small values of the field strength, the maximum residual longitudinal momentum increases.
greatly. As the field strength increases, however, saturation seems to be reached for the maximum residual momentum.

![Graphs showing the maximum residual momentum for different field strengths](image)

**Figure 5.** A comparison of various values of the maximum residual momentum for a fixed frequency. As for other graphs of this form saturation seems to be reached for small values of the field strength.
Figure 6. Same as in figure 5, except the plasma frequency is significantly higher. The maximum residual momentum for higher values of the plasma frequency appears to decrease slightly.

Figure 7. Residual momentum of the electron for a 100 pulse length laser as a function of the plasma frequency. Here the intensity of the field is held constant at various values.
Figure 8. Residual momentum of the electron for a 100 pulse length laser as a function of the plasma frequency. The intensity of the field is held constant at various values.

In figure 7 and figure 8 we track the dependence of the maximum residual momentum on the plasma frequency itself while keeping the laser intensity fixed. We notice that for higher values of the field intensity, the maximum residual momentum increases. The behavior of the electron as the plasma frequency increases has no discernible pattern. It is evident that for low values of the field strength, the resonance regions are far more pronounced than that for fields in the ultra-relativistic regime.

4. Conclusions
Although the analytic methods for solving the electron motion are important for the understanding of the dynamics of charged particles in intense laser fields, it is necessary to use detailed numerical calculations to find solutions to more complex and thereby realistic interactions. In order to consider the effect of the plasma, we considered a simple model that allowed us to simulate the effects of different laser and plasma parameters in the highly irregular and nonlinear behavior of the plasma electrons. We found that in the regime of relativistic dynamics the pattern of electron residual momentum gain completely decayed into unpredictable electron motion. At higher field intensities the irregular behavior of the electron becomes conspicuous, independent of the plasma density. The evident increase in the maximum and average residual momentum of electrons for certain parameter conditions suggests that there should exist alternative methods for heating plasmas. It is possible that the dynamics for different residual momenta when an electron emerges from an ion corresponds when the laser amplitude is close to its maximum, leading to a mismatch between the phase of the electromagnetic field and the subsequent oscillations of the photoelectrons.

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