Optimal lockdowns for COVID-19 pandemics: Analyzing the efficiency of sanitary policies in Europe

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Abstract
Two main nonpharmaceutical policy strategies have been used in Europe in response to the COVID-19 epidemic: one aimed at natural herd immunity and the other at avoiding saturation of hospital capacity by crushing the curve. The two strategies lead to different results in terms of the number of lives saved on the one hand and production loss on the other hand. Using a susceptible–infected–recovered–dead model, we investigate and compare these two strategies. As the results are sensitive to the initial reproduction number, we estimate the latter for 10 European countries for each wave from January 2020 till March 2021 using a double sigmoid statistical model and the Oxford COVID-19 Government Response Tracker data set. Our results show that Denmark, which opted for crushing the curve, managed to minimize both economic and human losses. Natural herd immunity, sought by Sweden and the Netherlands does not appear to have been a particularly effective strategy, especially for Sweden, both in economic terms and in terms of lives saved. The results are more mixed for other countries, but with no evident trade-off between deaths and production losses.
1 INTRODUCTION

Epidemiologic models such as the SIR model of Kermack and McKendrick (1927) and its variants are well designed to explore different counterfactual policy scenarios, which in the absence of a vaccine or a therapy, are all based on lockdowns with nontrivial psychological and economic consequences. Epidemiologists (Ferguson et al., 2020; Pathak et al., 2020; or Salje et al., 2020 to quote a few) are looking for lockdowns which are optimal for either reaching herd immunity, limiting the diffusion of the disease, preserving hospital capacity, avoiding a second wave, or a rebound in the epidemic.

Economists have different objectives. They imbedded variants of the SIR epidemiologic model into a more or less complex general equilibrium model to explore the trade-off between saved lives and production losses, introducing the controversial value of a statistical life (see, e.g., Acemoglu et al., 2020; Alvarez et al., 2020; Gollier, 2020; Rachel, 2020; or Gori, 2021). They rely on a calibration of their model, limiting thus their use to the exploration of general situations. As a matter of fact, sanitary policies can be quite different depending on the strength of the epidemic and on the value of the now famous reproduction number $R_0$ as we detail in Section 2. Moreover, as shown in Haug et al. (2020), social distancing measures and sanitary policies can be extremely different over the world, the most extreme measures not being necessarily the most efficient ones to reduce the effective reproduction number.

The strategies adopted by European governments during the early stages of the pandemic were mainly designed by epidemiologists (e.g., Neil Ferguson in Great Britain) and not by economists. They can be grouped into two classes. One aims at achieving natural herd immunity. This is the strategy which was adopted by Sweden and the Netherlands, at least during the first wave of the epidemic. These choices were well documented in the press. The other European countries have opted for a quite different approach, with the objective to crush down the epidemic curve to avoid hospital congestion in intensive care units (ICU). In this paper, we build a susceptible–infected–recovered–dead (SIRD) model to study and compare these two strategies. We have chosen to leave aside a third possible strategy which opts for zero COVID as in Australia, China, New Zealand, or Iceland at the cost of severe tracing and isolation. Our model includes an ICU capacity limit which has an impact on the death rate, in a very similar way to that of Gollier (2020), but without heterogeneity based on age classes and in continuous time. As a matter of fact, with the progress of the pandemic and of the vaccination, young age groups that were rather spared during the first wave are now more and more concerned by the pandemic. Starting from this model, we introduce two loss functions corresponding to the two classes of sanitary policies. The loss function corresponding to natural herd immunity seeks to minimize the overshooting of the SIR model, that is, stopping the epidemic by a confinement when the infection rate has reached its maximum value $1/R_0$. The loss function corresponding to ICU capacity constraint seeks to limit the infection rate to the ICU capacity. For a given starting point of confinement (which is a political decision), these two loss functions define an optimal couple length-severity for a lockdown. Corresponding to these values, we compute the production loss due to both the confinement of the working population and to the number of deaths. With various simulations, we show that it is not obvious to point out a trade-off between production losses and life losses as it may highly depend on the value of the $R_0$.

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1 Anna Holligan (April 4, 2020) on BBC News: Coronavirus: Why Dutch lockdown may be a high-risk strategy. Maddy Savage (July 23, 2020) on BBC news: Did Swedenaposs coronavirus strategy succeed or fail?
The second aim of the paper is to confront these simulation results to real data because sanitary situations were quite different among European countries. In a way, this attempt is not new. For instance, Pesaran and Yang (2020) propose an estimation method for the effective reproduction number in the case of underreporting and apply their results to six European countries from March to October 2020, using a stochastic multigroup SIR model. In a counterfactual exercise, they explore the consequences of early or late confinement in Germany and Great Britain. Another example is Chernozhukov et al. (2021) who investigate the causal relation between policy, behavior, and information and their impact on cases for the United States, but without considering explicitly a SIR model.

We have chosen to consider a data set of 10 European countries which cover two complete waves from January 2020 till March 2021. This data set provides information on daily cases and deaths on the one side and on the other side information on the severity of the sanitary measures which were adopted by these countries during two complete waves of the epidemic. Because of the close relation between a SIR model and phenomenological models coming from the biological literature on the growth of species (see, e.g., Wang et al., 2012), we opted for a double sigmoid statistical model to make inference on the dynamics of the epidemic in the presence of sanitary policies. From these inference results, we derive an estimate of the $R_0$ for each wave and predict the final number of deaths. To deal with the question of underreporting of cases, we adjust our model on registered deaths. We then evaluate the production loss due to observed confinement, using the estimated $R_0$ plugged into our initial SIRD model. With all these elements, we rank the 10 European countries with respect to predicted deaths, psychological costs, and production losses.

We reach the following conclusions. With our theoretical model, we show that natural herd immunity policy is feasible only with a low $R_0$. And anyway this policy is very costly in terms of human lives. The economic cost of the ICU policy is higher, but saves more lives. When looking at empirical results, Denmark has managed to minimize both the economic cost of confinement and the number of deaths. It has applied an ICU policy. With the same policy, France has apparently chosen to minimize the economic cost while Ireland managed to minimize the number of deaths at huge economic cost. Sweden and the Netherlands managed to reach very average results both in terms of economic cost and life costs. But the Netherlands, which changed its policy between the two waves got a better result.

The paper is organized as follows. In Section 2, we show how to introduce lockdowns in a calibrated SIR model and how to characterize optimality using two loss functions, one for herd immunity, the other for ICU constraint. We compare their implied trade-off between production loss and deaths. Section 3 is devoted to our data set and its stylized facts. Section 4 details our statistical model and how it can be used to make inference on the effective reproduction number. Section 5 presents empirical results with a prediction of the number of deaths at the end of the first two waves, prediction which is confronted to an evaluation of the economic and psychological cost of the adopted confinement. Section 6 concludes.

### 2 | OPTIMAL LOCKDOWN POLICIES WITHIN A SIMULATED SIRD MODEL

The epidemiologic literature (from Kermack & McKendrick, 1927 to Ferguson et al., 2020 or Salje et al., 2020) divides the total population of size $N$ into exclusive compartments. The susceptible groups $S$ are those who remain to be infected, roughly the whole population at the
start of the epidemic. An already infected person (group \( I \)) can infect several susceptible persons and then can either recover (group \( R \)) or die (group \( D \)), leading to the short-term conservation identity:

\[
S + I + R + D = N,
\]

as there is no birth or emigration in the model. It is customary to normalize the total population to 1. The passage between the compartments is governed by a set of differential equations:

\[
\frac{dS}{dt} = -\beta I S,
\]
\[
\frac{dI}{dt} = \beta I S - \gamma I,
\]
\[
\frac{dR}{dt} = (1 - \pi) \gamma I,
\]
\[
\frac{dD}{dt} = \pi \gamma I.
\]

A person can be infected by meeting an already infected person and the risk of contagion corresponds to \( \beta t \). Once infected a person remains infected for a period corresponding to \( \frac{1}{\gamma} \), \( \gamma \) being the fixed quitting rate. A person who is infected can either die at rate \( \pi \) or recover at rate \( (1 - \pi) \). When she has recovered, the person is supposed to be immune and cannot be reinfected.

The main parameters of the model are \( \gamma \) (disease dependent), \( \beta \) a social parameter that we suppose time varying to introduce the possibility of a lockdown. The fatality rate \( \pi \) is usually fixed in a normal situation. But it can increase a lot if hospitals become overcrowded because \( I_t \) is growing too fast. So in many models (see, e.g., Acemoglu et al., 2020; Alvarez et al., 2020; Gollier, 2020) \( \pi \) is a function of \( I \). Gollier (2020) assumes that \( \pi \) follows a step function depending on the sign of \( \xi - I_t \) where \( \xi \) is the proportion of ICU available for the total population. Because the dynamic of the model can be very sensitive to the shape of this step function, we prefer to use a progressive switching mechanism, which avoids a jump in the death rate:

\[
\pi_t = \pi + \begin{cases} 
0 & \text{if } I_t < \xi, \\
\alpha (I_t - \xi) & \text{otherwise},
\end{cases}
\]

where \( \alpha > 0 \). An extra parameter is often overlooked in empirical studies, the initial condition \( I_0 \) with

\[
S_0 = 1 - I_0, \quad I_0 = 0, \quad R_0 = 0, \quad D_0 = 0.
\]

\(^2\)This model is expressed in continuous time. For simulation purposes, it is discretized using an Euler scheme. If its parameters have a correspondence in terms of days, we can recover daily values, which are indicated with the subscript \( t \), in \( I_t \) for instance. When writing \( \pi_t \) or \( \beta_t \), we refer to a time-varying parameter with a correspondence to a day frequency.
If $I_0 = 0$, the epidemic cannot start. The value chosen for $I_0$ determines the date of the peak, and is thus essential to calibrate the model on real data.

In the absence of a vaccine, only a nonpharmaceutical policy can be adopted. It aims at isolating people to stop contagion by reducing the value of $\beta_t$ and thus obtaining an $R_0 < 1$. When the planner decides to lock down a fraction $\ell_t$ of the susceptible and of the infected, the value of a fixed $\beta$ is changed to

$$\beta_t = \beta (1 - \theta \ell_t), \quad 0 \leq \ell_t \leq 1,$$

where $\theta \leq 1$ corresponds to the efficiency of confinement for stopping disease transmission by the infected.

### 2.1 Herd immunity and overshooting

The model assumes that an infected person recovers or dies, but can never be reinfected (contrary to SIS models, see, e.g., Boucekkine et al., 2021 for references). Because of the conservation identity $S + I + R + D = 1$, the number of susceptible decreases while the number of recovered or dead increases. The speed at which this dynamics operates depends on the reproduction number $R_0 = \beta/\gamma$. It corresponds to the number of persons that one infected person will contaminate at the start of the epidemic. The effective reproduction number $\mathcal{R}_t = \beta_t/\gamma \times S_t$ evolves over time. When $\mathcal{R}_t > 1$, the epidemics expand exponentially. It decreases exponentially for $\mathcal{R}_t < 1$.

In the long run $I_t$ tends to 0, but the number of susceptible does not decrease to zero, because herd immunity is reached when a sufficient proportion of individuals $R^*$ have become immune to the virus. This proportion of immune people depends on the contagiousness of the disease with

$$R^* = 1 - 1/R_0,$$

and corresponds to the equilibrium proportion of remaining people in the susceptible group:

$$S^* = 1/R_0.$$

This proportion is reached at the peak of the epidemic and is lower than the limiting value $S_\infty$. So the model is overshooting by a nonnegligible percentage given by the difference $S_\infty - S^*$ as illustrated in Figure 1, using the calibration of Table 1.

In this phase diagram, the peak is reached at $S_t = S^*$. But despite the fact that herd immunity is reached, the epidemic does not stop until $S_t = S_\infty$, inducing a large overshooting gap $S_\infty - S^*$. The aim of a sanitary policy is to cut this overshooting (see, e.g., Rachel, 2020). Note that when $R_0$ is low (like in Sweden or Denmark), the maximum infection rate is not too high and herd immunity is reached quickly. With a higher $R_0$, the situation is totally different.

### 2.2 Economic loss due to the pandemic

An economist is interested in the trade-off between the production loss due to a lockdown and its rewards in terms of saved lives. In this case, the epidemic model is imbedded into a more or
less complex macro model as in Eichenbaum et al. (2021), Garriga et al. (2020), or Jones et al. (2020) to quote a few. However, one can simply report the production loss due to confinement and the value of a statistical life as in Acemoglu et al. (2020), Alvarez et al. (2020), or Gollier (2020).

**FIGURE 1** Overshooting after natural herd immunity. Notes: The vertical dashed lines represent both the peak of the epidemic and the level of herd immunity $S^*$. The dots indicate the end of the epidemic and the remaining proportion of susceptible $S_\infty$. Overshooting corresponds to the distance between $S_\infty$ and $S^*$. Euler’s method was used to solve the system of differential equations with a discretization step $\Delta t = 0.01$

**TABLE 1** Calibration parameters

| Par. | Meaning | Sources |
|------|---------|---------|
| $\gamma$ | The inverse of the average recovery time | From 7 to 14 days, so 1/10 on average (Park et al., 2020) |
| $\beta$ | The inverse of time delay between contacts | Can be stated in reference to an $\mathcal{R}_0 = \beta/\gamma$ between 2.4 and 3.3 for Great Britain (Ferguson et al., 2020) |
| $\theta$ | Efficiency of confinement to stop virus transmission | Set to 1.0 for the simulations |
| $\pi$ | Probability of dying when infected | 0.9% (Ferguson et al., 2020) |
| $\xi$ | Hospital capacity in percentage of the population | 0.05 (see, e.g., Pathak et al., 2020) |
| $\alpha$ | Parameter of the death function | 0.2 so as to obtain the value $\pi_t = 5 \times \pi$ for $I_t = 0.23$ (the maximum value of $\pi_t$ in Gollier, 2020) |
| $w$ | Daily individual production | Normalized to 1 |
| $\chi$ | Value of a statistical life | $20 \times w$ (Alvarez et al., 2020) |
| $I_0$ | Initial condition | Computed as a function of the chosen peak date and of $\mathcal{R}_0$ |
We suppose that the \( N \) individuals are producing \( w \times N \) every day. When the pandemic starts, in the absence of a lockdown, those who are infected can no longer work as well as those who are dead. So at a given date, the production is limited to \( w(S_t + R_t) \). A lockdown prevents susceptible from being infected but also from working, thus reducing production to

\[
w((1 - \ell_t) \times S_t + R_t).
\]  

The daily economic loss corresponds to

\[
W_t = w \times N - w((1 - \ell_t) \times S_t + R_t) = w(N - (1 - \ell_t) \times S_t - R_t).
\]  

The total economic loss \( W \) over the period when the infection is still running is obtained by summing all the \( W_t \) (neglecting discounting as interest rates were near zero over the period).

This total economic loss is usually confronted to the value of a statistical life. Acemoglu et al. (2020) decided to avoid fixing a value to life and explore the Pareto frontier between economic loss and the loss of lives. Fixing a value to life (20 years of gross domestic product [GDP] per capita), Alvarez et al. (2020) define an optimal lockdown as the balance between these two costs. Gollier (2020) simply explores various scenarios reporting these two costs. We believe that an optimal lockdown is built around epidemiologic considerations, even if it is necessary to report the associated economic cost and value of life losses. Let us measure the statistical cost of total lives lost as

\[
L = \chi \sum_{t=0}^{T} D_t,
\]

where \( \chi \) is a linear function of GDP per capita and \( T \) is the length of the pandemic (or the length of the simulation period).

### 2.3 Calibration

For calibrating the model, we use the standard values collected in the literature as reproduced in Table 1.

With the COVID-19 pandemic, the average number of days to recover in most nonsevere cases is between 7 and 14 days (see, e.g., Park et al., 2020). This parameter is most of the time taken as fixed in applied work, with values ranging from \( \gamma = 1/7 \) (Moll, 2020) to \( \gamma = 1/18 \) (Wang et al., 2020). A middle range choice of \( \gamma = 0.1 \) was adopted in Toda (2020).

The contact rate \( \beta \) is fundamentally a social parameter because it depends on the habits of the population (shaking hands, wearing masks, population density, ...). It can vary a lot between countries and is the main object of inference.

Most of the controversies reported in the literature (see, e.g., Adam, 2020) concern the predicted number of deaths. But we should keep in mind that these simulation models are designed to explore counterfactual situations. The fatality rate \( \pi \) is very hard to estimate because cases are underreported by lack of tests at the outbreak of the epidemic while the number of deaths is reported more accurately. By early December 2020, the empirical fatality rate varied between 0.9% (Denmark) and 3.5% (UK and Italy). The value adopted in Ferguson et al. (2020) was 0.9%. We made that parameter dependent of \( I_t \) in the case of ICU capacity saturation.
Sanitary policies in Europe were designed by epidemiologists, leading to two main types of policy. The first policy aims at reaching *natural herd immunity* as soon as possible. It was adopted in Sweden and at least partly in the Netherlands, arguing on a low value of $\mathcal{R}_0$. The second policy aims at *crushing the curve* to avoid hospital overcrowding. This strategy was adopted more or less stringently in many European countries. A third policy, that we shall not explore, is the zero case policy. It consists of massive testing and isolating the infected so as to maintain the number of cases near zero. It entails a total closure of frontiers. It was adopted for instance by Australia, New Zealand, China, and only Iceland in Europe. In this case, herd immunity is never reached before massive vaccination.

### 2.4 Lockdown for herd immunity

Three parameters are at work when designing a lockdown: its starting date $l_s$, its severity $\ell$, and its duration $l_t$ with $\lambda^* = (l_s, \ell, l_t)$. If only herd immunity is looked for as it was the case in Sweden and partially in the Netherlands, the objective of a confinement is to cut the overshooting so as to constraint $S_\infty$ to be equal to $S^*$, leading to the following minimization problem:

$$\min_{\lambda} (S_\infty(\lambda) - 1/\mathcal{R}_0)^2 + ((I_\infty(\lambda)) - I_0)^2. \quad (14)$$

At a point $t^* < T$ ($T$ being the length of the simulation period), we reach $S_{t^*} = S^*$ and the number of new infected $I_{t^*}$ returns to its initial value. However, we should note that the minimization of this loss function leads to a corner solution with a very strong confinement starting as close as possible to the peak (see, e.g., Rachel, 2020 and the last line of Table 2). As this strategy is politically unfeasible, the starting date $l_s$ becomes a political decision variable. In this case, we minimize the loss function (14) only with respect to severity $\ell$ and length $l_t$. Table 2 is based on $\mathcal{R}_0 = 2.5$, a horizon $T = 300$ and a calibration of the initial condition so that the theoretical peak would be located at $t = 65$ (65 days after the start of the epidemic). Figure 2 illustrates the corresponding dynamics.

Choosing when to start has nontrivial consequences. Starting confinement late minimizes overshooting, but maximizes the cost in terms of lives and is not necessarily optimal from an economic point of view. In Table 2, minimizing the economic cost implies starting confinement just 20 days before the peak, which seems a rather long delay. If one chooses to minimize the number of deaths, a very quick start would be needed. Note also that the later the start, the higher the severity of confinement, but the shorter the confinement duration. Anyway, the durations found are very important, from 3 to 4 months, well beyond observed ones. From Table 2, there is no apparent trade-off between production loss $W$ and life cost $L$.

In fact relying on herd immunity is not feasible when $\mathcal{R}_0$ is large because it leads to too many deaths. Let us alternatively consider a lower $\mathcal{R}_0 = 1.6$ which is close to what happened in Sweden or in Denmark during the first wave. Even in the absence of a lockdown, life cost $L$ is much lower as seen in the first line of Table 3.

Even if the length of the optimal lockdown is huge, its severity is very low so that the economic loss $W$ does not vary too much with the starting date $l_s$. There is an interest in

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3This is just a scaling choice for obtaining nice plots, given the value of $\mathcal{R}_0$. 
starting as soon as possible because in this case we remain below the hospital capacity as can be seen in Figure 3. Taking advantage of its low $R_0$, Sweden decided to follow this type of policy.

### 2.5 Optimal lockdown with an ICU constraint

When hospital capacity matters and in particular ICU, we have to modify our loss function, because the objective is now to crush the curve of infections. Maximum capacity is defined as a
percentage $\xi$ to be compared with the maximum proportion of infected people over the period, leading to the following alternative loss function, using the same terminal condition as before:

$$\min \lambda \frac{100}{t} \max (I_t(\lambda) - \xi)^2 + (I_\infty(\lambda) - I_0)^2.$$  

Herd immunity is no longer looked for, leading to a different confinement profile. It is equal to 0.0 before the starting date, then goes up to $\ell$ for a period equal to $l$. These are the two parameters to be found by optimization. But instead of returning to zero, there is a follow-up period with mild conservatory sanitary measures of severity parameter that we fixed equal to

### TABLE 3  Optimal lockdown for herd immunity with $R_0 = 1.6$

| $I_s$ | $\ell$ | $l$ | Overshooting | Max $I_t$ | $W$ | $L$ |
|-------|--------|-----|-------------|----------|----|----|
| None  | 0.000  | 0.000 | 0.352       | 0.086    | 8.29 | 36.27 |
| 40    | 0.25   | 223.27| 0.004       | 0.034    | 44.45 | 15.05 |
| 30    | 0.28   | 212.16| 0.002       | 0.046    | 45.87 | 15.69 |
| 20    | 0.35   | 178.21| 0.001       | 0.065    | 46.27 | 17.20 |
| 15    | 0.41   | 166.65| 0.001       | 0.074    | 49.41 | 18.42 |

**Notes:** The loss function was minimized using the derivative-free method of Nelder-Mead in `optim` of R, using a discretization step of 0.01 to solve the model. Overshooting is $S_w - S^*$. $W$ is the total economic loss and $L$ the total cost of lost lives. The first line corresponds to the no confinement case. The last line corresponds to the case when optimization is done on the three parameters. Start is the distance to the peak which was calibrated at $t = 65$.  

### FIGURE 3  Optimal lockdown when $R_0$ is low. **Notes:** Herd immunity corresponds to the dashed vertical line. The black thick line corresponds to the original trajectory of the epidemic. The red horizontal point line indicates the ICU capacity limit. Starting date is the distance to the peak. ICU, intensive care unit
This corresponds to a waiting time for the appearance of a vaccine. We obtain a totally different optimal policy as reported in Table 4 and Figure 4. This alternative policy is characterized by a long and soft confinement, followed by permanent but mild conservatory sanitary measures. For minimizing both overshooting and the number of deaths, it is optimal to start as soon as possible. Following strictly an economic objective would mean starting late, but in this case, the constraint of ICU capacity would be violated. This is an illustration of how difficult it is to conciliate economic and sanitary objectives.

### 3 | THE DIVERSITY OF SITUATIONS IN 10 EUROPEAN COUNTRIES

To summarize our findings, the design of an optimal policy depends very much on the value of $R_0$. Looking for herd immunity seems feasible only for a low $R_0$. Taking into account ICU capacity has a higher economic cost, but leads to a much lower cost in terms of lives. In light of these results, how did European countries react with which consequences in terms of efficiency?

We focus our attention on 10 European countries that we think are representative of both the pandemic diversity and of the different sanitary policies that were applied to contain national epidemics. For ease of presentation, these countries are divided into two groups according to their population: France, Germany, Italy, Spain, and the United Kingdom for large

| $l_s$ | $\ell$ | $l_l$ | Overshooting | Max $I_t$ | $W$ | $L$ |
|-------|-------|-------|--------------|----------|-----|-----|
| 40    | 0.43  | 115.27| 0.32         | 0.050    | 66.11| 19.18|
| 35    | 0.43  | 97.97 | 0.33         | 0.050    | 60.36| 20.52|
| 30    | 0.45  | 80.80 | 0.33         | 0.050    | 55.26| 21.74|
| 20    | 0.54  | 36.49 | 0.43         | 0.065    | 44.39| 28.61|

Notes: The loss function was minimized using the derivative-free method of Nelder–Mead in `optim` of R, using a discretization step of 0.01 to solve the model. Overshooting is $S_s - S^*$. $W$ is the total economic loss and $L$ the total cost of lost lives. The first line corresponds to the no confinement case. The last line corresponds to the case when optimization is done on the three parameters. Start is the distance to the peak which was calibrated at $t = 65$.

**FIGURE 4** Waiting for a vaccine with a very long confinement. ICU, intensive care unit

0.25 and which lasts till the end of the simulation horizon $T = 300$ (see Pathak et al., 2020). This corresponds to a waiting time for the appearance of a vaccine. We obtain a totally different optimal policy as reported in Table 4 and Figure 4.

This alternative policy is characterized by a long and soft confinement, followed by permanent but mild conservatory sanitary measures. For minimizing both overshooting and the number of deaths, it is optimal to start as soon as possible. Following strictly an economic objective would mean starting late, but in this case, the constraint of ICU capacity would be violated. This is an illustration of how difficult it is to conciliate economic and sanitary objectives.
countries, and Belgium, Denmark, Ireland, the Netherlands, and Sweden for smaller countries. Our sample corresponds to the recordings of the epidemic in Europe from January 22, 2020 to March 31, 2021 to analyze the effect of sanitary policies on two complete waves.

3.1 The Oxford data set

We use the Oxford COVID-19 Government Response Tracker data, because they contain both the usual epidemic data and data on the implemented sanitary policies.\(^4\) Several other data sets exist which report policy data, see the data section in Haug et al. (2020). However and up to our knowledge, the Oxford data set is the only one that reports a severity index for sanitary restrictions. More precisely, the Oxford Blavatnik School of Government collected data on lockdown policies for more than 180 countries and derived various government response indices which are helpful to appraise the severity of these policies. These indices are constructed from various ordinal measures of the severity of policies undertaken by governments. Each ordinal scale represents one aspect of the severity of closures and containment (school and workplace closures, canceling public events, bans on private gatherings, closing of public transport, stay at home requirements, and restrictions on internal movement), economic measures (income support and freezing financial obligations), or health measures (public information campaigns, testing policy, contact tracing, and vaccination). The three global indices vary between 0 and 100, 0 indicating a “laissez faire” policy, and 100 indicating the highest level of severity of government responses. Details on the composition of the indices are provided in Hale et al. (2021). Among those indices, we have chosen the Stringency Index which accounts for nine items related to confinement and health information.

The database provides also as complementary information, the usual count epidemic variables: confirmed cases and number of deaths. It is well-known that cases were under-evaluated during the first wave just by lack of adequate tests (see, e.g., Wu et al., 2020). This was less the case for the second wave. Deaths were much easier to identify, and thus the reported number of deaths provides a better vision of the epidemic. Consequently, we shall analyze only those data, as anyway one of the objectives of a sanitary policy is to minimize the number of deaths.

3.2 Epidemic dynamics and policy reactions

We first present in Figure 5 the smoothed number of deaths per day for large and small countries, normalized by the population.

From Figure 5, the first wave was sharp and single peaked. The second wave usually evolves at a reduced pace with often a rebound. Table 5 helps to date precisely the start, peak, and end of each wave while Table 6 provides the details of policy reactions to the epidemic.

The epidemic started earlier in Italy (February 24, 2020) and by the beginning of March 2020 for the other countries. The peak was quickly reached by early or mid-April 2020, except for Sweden and Ireland where it occurred slightly later. The first wave ended around the third week of July, but earlier in Spain. The second wave started between August and November with

\(^4\)The database can be accessed at https://github.com/OxCGRT/.
a peak between November 2020 and February 2021. It ended by the end of March 2021, with the exception of Italy where it ended 1 month earlier.

Table 6 details the policy reactions. On average, countries had their first sanitary reaction well before the epidemic peak. During the first wave, their strong reaction leading to a

FIGURE 5  Deaths and policy reactions for large and small countries. Notes: The solid lines represent the smoothed normalized number of daily new deaths, using a loess regression. The dashed lines represent the severity index of governments’ responses. Decimal dates correspond to month.day, while consecutive years 2020 and 2021 are not indicated. Source: University of Oxford, Blavatnik School of Government and authors’ calculations. BEL, Belgium; DNK, Denmark; FRA, France; GBR, United Kingdom; GER, Germany; IRL, Ireland; ITA, Italy; NLD, the Netherlands; SPA, Spain; SWE, Sweden
### Table 5  Main points of the two waves

| Country | **First wave** | | | **Second wave** | | |
| --- | --- | --- | --- | --- | --- | --- |
| | Start | Peak | End | Start | Peak | End |
| GBR | Mar 10 | Apr 15 | Jul 21 | Sep 27 | Jan 22 | Mar 31 |
| SPA | Mar 06 | Apr 10 | Jun 08 | Aug 04 | Feb 06 | Mar 31 |
| ITA | Feb 24 | Apr 02 | Jul 20 | Oct 06 | Dec 01 | Feb 24 |
| GER | Mar 14 | Apr 14 | Jul 21 | Oct 05 | Jan 14 | Mar 30 |
| FRA | Mar 05 | Apr 12 | Jul 21 | Sep 02 | Nov 17 | Mar 18 |
| SWE | Mar 12 | Apr 26 | Jul 21 | Nov 11 | Jan 19 | Mar 16 |
| BEL | Mar 11 | Apr 14 | Jul 11 | Sep 25 | Nov 15 | Mar 09 |
| NLD | Mar 08 | Apr 12 | Jul 21 | Aug 04 | Jan 09 | Mar 31 |
| IRL | Mar 11 | Apr 24 | Jul 21 | Sep 30 | Jan 30 | Mar 31 |
| DNK | Mar 14 | Apr 12 | Jul 08 | Sep 05 | Jan 10 | Mar 15 |

**Notes:** Deaths are smoothed by a loess regression with parameter 0.2. The start of the first wave is when the number of deaths is greater than 1 for 100,000. This wave ends when the local minimum is reached. The second wave starts when the number of deaths is twice the previous minimum. It ends when the number of deaths is minimum before March 31.

**Abbreviations:** BEL, Belgium; DNK, Denmark; FRA, France; GBR, United Kingdom; GER, Germany; IRL, Ireland; ITA, Italy; NLD, the Netherlands; SPA, Spain; SWE, Sweden.

**Source:** University of Oxford, Blavatnik School of Government and authors’ calculations.

### Table 6  Dating policy interventions

| Country | **First wave** | | | **Second wave** | | |
| --- | --- | --- | --- | --- | --- | --- |
| | Start | Lock start | $l_t$ | Max $\ell_t$ | Start | Lock start | $l_t$ | Max $\ell_t$ |
| GBR | Mar 17 | Mar 23 | 58 | 79.63 | Oct 12 | Dec 25 | 80 | 87.96 |
| SPA | Mar 09 | Mar 30 | 53 | 85.19 | Oct 22 | Oct 26 | 117 | 78.70 |
| ITA | Feb 21 | Mar 11 | 54 | 93.52 | Oct 06 | Oct 24 | 125 | 84.26 |
| GER | Feb 29 | Mar 22 | 45 | 76.85 | Oct 05 | Dec 17 | 94 | 85.19 |
| FRA | Feb 29 | Mar 17 | 55 | 87.96 | Oct 10 | Oct 31 | 46 | 78.70 |
| SWE | Mar 12 | Mar 29 | 115 | 64.81 | Nov 11 | Nov 25 | 113 | 69.44 |
| BEL | Mar 13 | Mar 20 | 77 | 81.48 | Sep 30 | Nov 03 | 128 | 65.74 |
| NLD | Mar 10 | Mar 23 | 70 | 78.70 | Aug 04 | Dec 16 | 107 | 82.41 |
| IRL | Mar 12 | Mar 28 | 69 | 90.74 | Sep 30 | Oct 22 | 139 | 87.96 |
| DNK | Mar 03 | Mar 14 | 69 | 72.22 | Oct 21 | Jan 05 | 47 | 70.37 |

**Notes:** The first start is the date when the severity index was greater than 20. Lock start is the date when the severity index reaches 90% of its maximum. Length corresponds to the period when severity was at least 90% of the maximum severity. Strength is the value of the severity index at its maximum during the first and the second wave. The second start is when the severity index increases again after the first wave.

**Abbreviations:** BEL, Belgium; DNK, Denmark; FRA, France; GBR, United Kingdom; GER, Germany; IRL, Ireland; ITA, Italy; NLD, the Netherlands; SPA, Spain; SWE, Sweden.

**Source:** University of Oxford, Blavatnik School of Government and authors’ calculations.
confinement or something similar occurred very few days after. This delay could be much more important for the second wave (UK, Germany, the Netherlands, and Denmark) with first measures well before the second peak. With a hypothetical $R_0 = 2.5$, the theoretical model predicted in this case a severity of 0.42 and a length of 141 days when herd immunity was looked for and severity 0.55 with a length of 37 days in the case of ICU policy. For all countries, the measured severity was much stronger than these predictions, which means that the efficiency of a lockdown to limit the propagation of the virus ($\theta$ in Equation 8) was lower than 1.

Sweden, which claimed to have opted for natural herd immunity conforms to the theoretical model as it adopted the longest measures (115 days) and the softer reaction (64.81) during the first wave. The Netherlands who also claimed to look for natural herd immunity adopted much shorter (70 days) and more severe (78.70) restrictive measures. Large countries mostly opted for the ICU constraint and behaved in a coherent way regarding the theoretical model with on average 55 days of severe confinement. Remaining smaller countries opted for a slightly longer average confinement of 70 days together with a comparable severity. Note that two countries had a regular and very progressive decrease in the severity of their sanitary measures: Sweden and Denmark.

Sweden maintained the same policy during the second wave, but the Netherlands changed it for a much longer and more severe sanitary measure. Denmark and France were the only countries to soften their sanitary policy during the second wave.

A key policy variable is the starting date of the lockdown. Natural herd immunity means starting late, close to the peak, while ICU policy means starting early allowing less deaths. During the first wave, countries on average started their strongest measures 23 days before the peak with a standard deviation of 5 days. For the second wave, countries anticipated their lockdown with 41 days before the peak on average, but France, Belgium, and Denmark did just the reverse with a smaller distance to the peak. So the standard deviation of that anticipation was much larger with 35 days. The Netherlands had a confinement close to the first peak and very early from the second peak, illustrating a change of policy, contrary to Sweden that maintained its initial policy.

So very few countries seem to have designed optimally their sanitary policy. One element is missing for appraising their actual efficiency, the value of $R_0$. The natural herd immunity policy is feasible only if $R_0$ is low enough. And a change of $R_0$ between the two waves may lead to a change of policy. We need a statistical model to estimate the effective reproduction number and to predict the number of deaths resulting from the implementation of a particular policy. In Section 4, we explain how phenomenological and double sigmoid models are a nice solution.

4 | PHENOMENOLOGICAL MODELS FOR MODELING WAVES

Epidemiologic models are convenient for designing policies, but they are difficult to fit to actual data because of their built-in constraints due to their very small number of parameters. Phenomenological models coming from the biological literature on the growth of species (see Ma, 2020; Tsoularis & Wallace, 2002; or Turner et al., 1976 for a survey) are less constrained while as explained in Wang et al. (2012), they can encompass the main features of a SIR model. Moreover, they can be combined into a double sigmoid function to account for a second wave.
4.1 Richards and Gompertz models

The starting point is a differential equation describing the growth of confirmed cases $C(t)$:

$$\frac{dC}{dt} = rC \left[ 1 - \left( \frac{C}{K} \right)^\delta \right], \quad (15)$$

with $r$, $\delta$, and $K$ being positive real numbers. This equation covers two mechanisms. A growth rate with the term $rC$ that corresponds to the epidemic at its beginning, equivalent to an exponential model. A reversion mechanism (or self-regulating) where the term $\left( \frac{C}{K} \right)^\delta$ says that the epidemic $C$ will eventually reach a maximum $K$. This differential equation corresponds to Richards’ model (Richards, 1959) with solution:

$$C(t) = \frac{K}{[1 + \delta \exp(-r(t - \tau))]^{1/\delta}}, \quad (16)$$

where $\lim_{t \to \infty} C(t) = K$. This solution has introduced a new parameter $\tau$ which monitors the date of occurrence of the peak at $t = \tau$ with value:

$$C_{\text{inf}} = \frac{K}{(1 + \delta)^{1/\delta}}.$$

So $\delta > 0$ monitors the value of the curve at the inflection point and the asymmetry around $t = \tau$. This point is of particular importance because it corresponds to the period when the epidemic starts to regress, or equivalently when the effective reproduction number $R_t$ starts to be below 1 in a SIR model.

The first-order derivative $C'(t)$ of this function provides an estimate of the number of infected at each point of time:

$$C'(t) = I_t = \frac{K r [\delta \exp(r(\tau - t)) + 1]^{-1/\delta}}{\delta + \exp(r(t - \tau))}. \quad (17)$$

When imposing a restriction on $\delta$ in (16), we get two of the usual models of the literature. The logistic model (Verhulst, 1845) is obtained for $\delta = 1$. Its inflection point is $K/2$, just midway between the start and the end of the epidemic, a restriction which is unrealistic for the COVID-19. The model of Gompertz (Gompertz, 1825) is obtained by taking the limit of (16) for $\delta \to 0$ which gives

$$C(t) = K \exp[-\exp(-r(t - \tau))]. \quad (18)$$

The corresponding inflection point is

$$C_{\text{inf}} = Ke^{-1}.$$
So the Gompertz model is a very parsimonious way of obtaining an inflection point lower than \( K/2 \). The first-order derivative of this function provides an estimate of the number of infected \( I_t \) at each point in time:

\[
C'(t) = I_t = \mathcal{R} \exp[r(\tau - t) - \exp(r(\tau - t))].
\]  

(19)

4.2 Double sigmoid functions to account for a second wave

A double sigmoid model, necessary for modeling a second wave, is obtained by adding two single peak models:

\[
C(t) = C_1(t) + C_2(t),
\]

(20)

where \( C_1(t) \) and \( C_2(t) \) are the first and second sigmoid functions. This type of model appeared quite early in the literature with Bock et al. (1973) or Thissen et al. (1976) and led to some developments with Lipovetsky (2010) or Oswald et al. (2012). If we consider two Gompertz curves, we have

\[
C(t) = K_1 \exp[-\exp(-r_1(t - \tau_1))] + (K_2 - K_1) \exp[-\exp(-r_2(t - \tau_2))].
\]

(21)

\( K_1 \) and \( K_2 \) are the intermediate and final plateau of saturation so that \( \lim_{t \to \infty} C(t) = K_2 \). \( \tau_1 \) and \( \tau_2 \) monitor the position of the peak of each phase while the growth rate of the process is determined by \( r_1 \) and \( r_2 \). However there is no analytical formula to determine the value and position of the two peaks.

The first-order derivative of \( C(t) \) provides the number of infected at any point in time. For the double Gompertz model, we have

\[
C'(t) = I_t = K_1 r_1 \exp[-\exp(-r_1(t - \tau_1))] - \exp(-r_1(t - \tau_1))]
+ (K_2 - K_1) r_2 \exp[-r_2(t - \tau_2) - \exp(-r_2(t - \tau_2))].
\]

(21)

It helps also to locate numerically the two peaks.

4.3 Estimating the effective reproduction number

Cori et al. (2013) noted that the expectation of the number of infected people at \( t \) is \( E(I_t) = \mathcal{R}_t \sum_s I_{t-s} \omega(s) \) where \( \omega(s) \) is the infection profile and \( \mathcal{R}_t \) the effective reproduction number. They deduce a general formulation for estimating the effective reproduction number:

\[
\mathcal{R}_t = \frac{I_t}{\sum_{s=1}^{h} I_{t-s} \omega(s)}.
\]

(22)

It corresponds to the number of infected at \( t \) divided by the number of past infected weighted by their infection profile, taking into account only a finite past horizon of \( h \). However, with COVID-19, the time of infection is not observed, only symptoms are observed, as during the period of incubation there are no symptoms. So \( \omega(s) \) is approximated by the serial interval
distribution which has to be estimated using individual pairs. The serial interval distribution found in Li et al. (2020) is a gamma density with a shape parameter of 4.87 and a scale parameter of 1.54. With \( h = 18 \), we cover 99% of that gamma density. Because observed data of daily cases or deaths have a too strong variability, we can replace \( I_t \) in (22) by its predicted value according to the first-order derivative of a properly selected double sigmoid model. For the first wave, the \( R_0 \) corresponds to \( R_{n+1} \). For the second wave, \( R_0 \) is \( R_t \) computed at the start of the second wave.

5 | EMPIRICAL RESULTS

We first estimate the \( R_0 \) at the start of each wave and then predict the cumulated number of deaths at the end of the second wave together with the implied losses due to the successive confinements. We can thus compare the efficiency of each national sanitary policy.

5.1 | Inference for \( R_t \)

We have adjusted two models, a double Richards and a double Gompertz and selected the best one according to a Bayesian information criterion (BIC). The start and the end of the sample were chosen following Table 5. Deaths are expressed for 100,000 habitants.

The value of \( R_0 \) varies a lot between countries for the first wave, ranging from 1.45 (Denmark) to 4.82 (France), showing that the epidemic did not hit the 10 European countries in the same way. In all the countries, the first confinement managed to reduce the \( R_t \) below 0.60, except in Sweden and marginally in Italy. The second wave was much milder, except for Italy and Belgium. The sanitary policies implemented on that occasion were in general comparable to those of the first wave. But they were less efficient, because the obtained minimum \( R_t \) were in general higher than those of the first wave. There is evidently a decrease in the value of \( \theta \), the efficiency parameter in (8).

5.2 | Comparing policies

There are several ways to measure the success of a sanitary policy. A first criterion is the avoidance of a second wave. No country has managed to avoid it. A second and widely used criterion (see, e.g., Haug et al., 2020) is the ability to reduce the value of the effective reproduction number \( R_t \), which means as a consequence reducing the number of deaths. We provide the predicted number of total deaths at the end of the second wave in the last column of Table 7. It represents the total cost of the epidemic in terms of lives. We can confront it with two other types of costs. The first cost, which is often overlooked, is the psychological cost of a lockdown. Remember that a lockdown means social distancing and that human beings are fundamentally social beings. We can measure the psychological cost as \( \ell \times l_t \). In this way, this is an easy measure. The production loss entailed by a confinement is less easy to measure in our context. The theoretical measure of this loss given in (12) requires the evaluation of \( N_t - D_t = S_t + R_t + I_t \). Our statistical model provides \( D_t \), the predicted number of deaths while the total population is given. But in this case only the sum \( S_t + R_t + I_t \) is identified. We cannot recover \( S_t \) and \( R_t \) separately without the help of a SIRD model. We have calibrated our previous
theoretical model using the estimated $R_0$ found in Table 7 for each country and each wave separately, adjusting the value of $I_0$ so as to calibrate the peak on the date indicated in Table 5. We then compute the production loss, using the simulated values of $S_t$ and $R_t$ and the policy indications of Table 6, assuming that there is no effective lockdown before and after each policy intervention. This is of course a coarse method for evaluating the production loss. It assumes first that the $R_0$ estimated on death data is also valid for the case data and second that the SIRD model adequately describes the dynamics of the epidemic. A look at Figure 5 shows that this might not be the case.

TABLE 7  Inference for effective reproduction numbers

| Country | Model | First wave | Second wave |
|---------|-------|------------|-------------|
|         | $R_0$ | $\min R_t$ | $R_0$ | $\min R_t$ | Total deaths |
| GBR     | 1.47  | 0.36       | 1.21  | 0.61       | 19,336       |
| SPA     | 2.63  | 0.56       | 1.29  | 0.98       | 16,270       |
| ITA     | 2.52  | 0.66       | 3.21  | 0.66       | 15,941       |
| GER     | 1.79  | 0.50       | 1.46  | 0.74       | 9071         |
| FRA     | 4.82  | 0.56       | 2.19  | 0.90       | 13,204       |
| SWE     | 1.65  | 0.74       | 1.86  | 0.74       | 13,455       |
| BEL     | 4.20  | 0.55       | 6.08  | 0.55       | 19,710       |
| NLD     | 2.66  | 0.60       | 1.95  | 0.91       | 9850         |
| IRL     | 1.66  | 0.27       | 1.31  | 0.55       | 9267         |
| DNK     | 1.45  | 0.35       | 1.32  | 0.47       | 3979         |

Notes: The double sigmoid model was estimated using the R package nls.1m with positivity constraints. The optimal model was selected according to a BIC. For the first wave $R_0$ corresponds to the value of $R_t$ at the first observation +18 days (99% of the serial interval). For the second wave $R_0$ corresponds to the maximum value of $R_t$ on that part of the sample. The predicted number of deaths is expressed per 100,000 inhabitants at the end of the second wave.

Abbreviations: BEL, Belgium; BIC, Bayesian information criterion; DNK, Denmark; FRA, France; GBR, United Kingdom; GER, Germany; IRL, Ireland; ITA, Italy; NLD, the Netherlands; SPA, Spain; SWE, Sweden.

5.3  Conventional or unconventional sanitary policies?

Two political decisions were amply discussed in the press about the choice made for containing the epidemic: Sweden and the Netherlands. To summarize, Sweden has relied on voluntary social distancing, limiting people gathering, and the activity in bars and restaurants, leading to the smallest severity indices of 64.81 and 69.44 for the two waves. But the government said also that fighting against the pandemic was a marathon and not a sprint, which explains why the Swedish severity index kept its highest value for so long (115 and 113 days). The aim was to reach the peak and herd immunity as soon as possible. In fact, because of a low $R_0$, the peak occurred later than in the neighboring countries. But Sweden experienced the highest number of deaths compared with the other Nordic countries. This policy was condemned by its neighbors, Norway, Denmark, and Finland which excluded Swedish tourists when reopening their borders for summer 2020. Sweden maintained its policy for the second wave.
The Netherlands have also promoted natural herd immunity to implement a targeted lockdown, closing only activities that required close physical interactions, including schools and universities. There was no tight lockdown, people were advised to stay at home, but could go out, provided they respected social distancing. In its implementation, this policy led to a higher but still in a way mild severity index of 78.70, however coupled with a shorter length of 70 days. Here again, this strategy was not appreciated by the neighboring countries of the Netherlands. However, the Netherlands seem to have changed their policy for the second wave.

Let us now compare in Table 8 the performance of these two types of policies, herd immunity and ICU constraint among our 10 European countries. We have ranked countries in terms of total predicted deaths, but also indicated ranks corresponding to psychological costs and production losses, the upper the rank, the lower the cost.

Denmark, applying an ICU policy, obtained the best results according to all criteria, probably thanks to an initial small $R_0$ and an early reaction. Germany also managed to limit the number of deaths thanks to an initial moderate $R_0$ and an early reaction. It managed to limit the psychological cost, but not the economic cost. Ireland managed to minimize the number of deaths, but at a huge economic cost, the largest of the list. France privileged the psychological cost, but is roughly equally ranked for deaths and economic cost. When comparing sanitary efficiency within this group of countries that have applied similar policies, there does not seem to be a trade-off between lives and economic cost. For instance, Great Britain had a very high death rate together with a high economic cost as well as Spain. The case of Belgium is difficult to interpret as this country had an extensive counting method for the origin of deaths.

Sweden, which maintained its herd immunity policy over the two waves, does not seem to have been particularly efficient as it cumulates high death rates, high economic and

| Country | Policy    | Pred. deaths | Death rank | Psycho. cost | Psycho. rank | Prod. loss in % | Eco. rank |
|---------|-----------|--------------|------------|--------------|--------------|----------------|----------|
| DNK     | ICU       | 3979         | 1          | 82.91        | 1            | 6.69           | 1        |
| GER     | ICU       | 9071         | 2          | 114.66       | 3            | 8.43           | 6        |
| IRL     | ICU       | 9267         | 3          | 184.88       | 10           | 14.44          | 10       |
| NLD     | Herd im.  | 9850         | 4          | 143.27       | 6            | 7.57           | 3        |
| FRA     | ICU       | 13,204       | 5          | 84.58        | 2            | 7.61           | 4        |
| SWE     | Herd im.  | 13,455       | 6          | 153.00       | 8            | 9.74           | 8        |
| ITA     | ICU       | 15,941       | 7          | 155.83       | 9            | 8.07           | 5        |
| SPA     | ICU       | 16,270       | 8          | 137.23       | 5            | 10.83          | 9        |
| GBR     | ICU       | 19,336       | 9          | 116.55       | 4            | 9.57           | 7        |
| BEL     | ICU       | 19,710       | 10         | 146.89       | 7            | 6.81           | 2        |

Notes: The double sigmoid model was used to predict deaths for 100,000 inhabitants at the end of the second wave. Economic cost is the loss in percentage of annual GDP, computed inside a SIRD model using the estimated $R_0$ for each wave and the observed value of the severity index. Psychological cost is approximated by the product of confinement severity and length of confinement.

Abbreviations: BEL, Belgium; DNK, Denmark; FRA, France; GBR, United Kingdom; GDP, gross domestic product; GER, Germany; ICU, intensive care unit; IRL, Ireland; ITA, Italy; NLD, the Netherlands; SIRD, susceptible–infected–recovered–dead; SPA, Spain; SWE, Sweden.
psychological costs. This is not in favor of the natural herd immunity policy. The Netherlands apparently decided to change their policy between the two waves with more severe measures despite a lower $R_0$. Moreover, they had a quicker reaction at the beginning of the first wave, contrary to Sweden which was late. They were apparently right because at the end they were better than Sweden for deaths despite a higher $R_0$ and they also manage to minimize the economic cost.

Should we question the validity of SIR models to design an optimal sanitary policy? Natural herd immunity policy means a confinement as close to the peak as possible when we have seen that countries with a quick sanitary policy were much more successful. Pesaran and Yang (2020) also concluded that countries with a quick reaction had managed to save many lives.

6 CONCLUSION

The United Kingdom sanitary policy was decided after the release of dreadful death predictions coming from a SIR-like model with no confinement. The predictive accuracy of these epidemiologic models was highly questioned in the academic press (see, e.g., Chin et al., 2020 or Jewell et al., 2020 among many others). Actually, simple epidemiologic models are useful to explore various sanitary policy scenarios. They should not be asked to do what they are not designed for. On the contrary statistical models are essential to explore empirically the consequences of a sanitary policy because they take into account the interplay between the epidemic dynamics and the impact of confinements. Moreover, they allow easily for the presence of several waves. They are thus the ideal instruments for making predictions.

Using our theoretical model, we found that natural herd immunity policies were feasible only in the case of a low $R_0$. When confinement was started early enough, the resulting infection rate could naturally be maintained below ICU capacity. With a higher $R_0$ ICU policies are more costly in terms of production losses, but they manage to contain the number of deaths to lower values. When confronting these results to the data, we find that the two countries which had decided to rely on natural herd immunity experienced a large number of deaths, and that only the Netherlands managed to reach a rather low production loss. Denmark, with a more traditional policy managed to limit both the number of deaths and the economic cost. Consequently, it is rather difficult to observe a trade-off between production losses and number of deaths in real life such as the one explored in Acemoglu et al. (2020).

Our paper is not without limitations. There is first the evident data problem. We could not use efficiently the number of cases because they were totally underestimated during the first wave, either because valid tests were slow to be implemented or because sanitary authorities had an interest in minimizing the impact of the epidemic (e.g., Brazil, China, and Iran). Finding a way to cope with this question becomes an essential empirical question. The second limitation comes from the fact that we have treated countries as if they were independent because sanitary policies are usually defined at the national level. One item of a lockdown policy is to close borders because viruses circulate over borders. So the interaction between countries can be an important factor when modeling the spreading of an epidemic. More generally speaking, it is important to take into account geographical features. For instance, the epidemic has been more severe in the east part of France or in the north part of Italy, with no apparent reason. Not many papers include a spatial dimension. Cacciapaglia and Sannino (2020) adapted a model from high-energy physics to take into account interactions between different regions. One of their conclusions is that closing the border is efficient only if done
before the peak. Hafner (2020) has the same type of concern in a statistical framework using a spatial autoregressive model. He identifies a high degree of spillover between countries. The final limitation comes from the period we have chosen. If we had limited our attention only to the first wave, the country ranking would have been different because the two waves that we studied did not hit all the countries in the same way. And some countries have changed their sanitary policies meanwhile. Our conclusions are thus limited, as the pandemic is not finished anyway.

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CONFLICT OF INTERESTS
The authors declare that there are no conflict of interests.

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