Collaborative Filtering via Group-Structured Dictionary Learning

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Abstract—Structured sparse coding and the related structured dictionary learning problems are novel research areas in machine learning. In this paper we present a new application of structured dictionary learning for collaborative filtering based recommender systems. Our extensive numerical experiments demonstrate that the presented technique outperforms its state-of-the-art competitors and has several advantages over approaches that do not put structured constraints on the dictionary elements.

Keywords—collaborative filtering, structured dictionary learning

I. INTRODUCTION

The proliferation of online services and the thriving electronic commerce overwhelms us with alternatives in our daily lives. To handle this information overload and to help users in efficient decision making, recommender systems (RS) have been designed. The goal of RSs is to recommend personalized items for online users when they need to choose among several items. Typical problems include recommendations for which items to watch, which jokes/books/news to read, which hotel to stay at, or which songs to listen to.

One of the most popular approaches in the field of recommender systems is collaborative filtering (CF). The underlying idea of CF is very simple: Users generally express their tastes in an explicit way by rating the items. CF tries to estimate the users’ preferences based on the ratings they have already made on items and based on the ratings of other, similar users. For a recent review on recommender systems and collaborative filtering, see e.g., [1].

Novel advances on CF show that dictionary learning based approaches can be efficient for making predictions about users’ preferences [2]. The dictionary learning based approach assumes that (i) there is a latent, unstructured feature space (hidden representation) behind the users’ ratings, and (ii) a rating of an item is equal to the product of the item and the user’s feature. To increase the generalization capability, usually ℓ2 regularization is introduced both for the dictionary and for the users’ representation.

There are several problems that belong to the task of dictionary learning [3], a.k.a. matrix factorization [4]. This set of problems includes, for example, (sparse) principal component analysis [5], independent component analysis [6], independent subspace analysis [7], non-negative matrix factorization [8], and structured dictionary learning, which will be the target of our paper.

One predecessor of the structured dictionary learning problem is the sparse coding task [9], which is a considerably simpler problem. Here the dictionary is already given, and we assume that the observations can be approximated well enough using only a few dictionary elements. Although finding the solution that uses the minimal number of dictionary elements is NP hard in general [10], there exist efficient approximations. One prominent example is the Lasso approach [11], which applies convex ℓ1 relaxation to the code words. Lasso does not enforce any group structure on the components of the representation (covariates).

However, using structured sparsity, that is, forcing different kind of structures (e.g., disjunct groups, trees) on the sparse codes can lead to increased performances in several applications. Indeed, as it has been theoretically proved recently structured sparsity can ease feature selection [12], [13], and makes possible robust compressed sensing with substantially decreased observation number [14]. Many other real life applications also confirm the benefits of structured sparsity, for example (i) automatic image annotation [15], (ii) group-structured feature selection for micro array data processing [16]–[19], (iii) multi-task learning problems (a.k.a. transfer learning) [20]–[22], (iv) multiple kernel learning [23], [24], (v) face recognition [25], and (vi) structure learning in graphical models [26], [27]. For an excellent review on structured sparsity, see [28].

All the above mentioned examples only consider the structured sparse coding problem, where we assume that the dictionary is already given and available to us. A more interesting (and challenging) problem is the combination of these two tasks, i.e., learning the best structured dictionary and structured representation. This is the structured dictionary learning (SDL) problem. SDL is more difficult; one can find only few solutions in the literature [29]–[34]. This novel field is appealing for (i) transformation invariant feature extraction [33], (ii) image denoising/ inpainting [29], [31], [34], (iii) background subtraction [31], (iv) analysis of text corpora [29], and (v) face recognition [30].

Our goal is to extend the application domain of SDL in the direction of collaborative filtering. With respect to CF,
further constraints appear for SDL since (i) online learning is desired and (ii) missing information is typical. There are good reasons for them: novel items/users may appear and user preferences may change over time. Adaptation to users also motivate online methods. Online methods have the additional advantage with respect to offline ones that they can process more instances in the same amount of time, and in many cases this can lead to increased performance. For a theoretical proof of this claim, see [35]. Furthermore, users can evaluate only a small portion of the available items, which leads to incomplete observations, missing rating values. In order to cope with these constraints of the collaborative filtering problem, we will use a novel extension of the structured dictionary learning problem, the so-called online group-structured dictionary learning (OSDL) [36]. OSDL allows (i) overlapping group structures with (ii) non-convex sparsity inducing regularization, (iii) partial observation (iv) in an online framework.

Our paper is structured as follows: We briefly review the OSDL problem, its cost function, and optimization method in Section II. We cast the CF problem as an OSDL task in Section III. Numerical results are presented in Section IV. Conclusions are drawn in Section V.

Notations. Vectors (a) and matrices (A) are denoted by bold letters. diag(a) represents the diagonal matrix with coordinates of vector a in its diagonal. The ith coordinate of vector a is ai. Notation |·| means the number of elements of a set and the absolute value for a real number. For set O ⊆ {1, . . . , d}, aO ∈ R|O| denotes the coordinates of vector a ∈ Rd in O. For matrix A ∈ Rd×D, A⊙ ∈ R|O|×D stands for the restriction of matrix A to the rows O. I and 0 denote the identity and the null matrices, respectively. AT is the transposed form of A.

A. Cost Function

The online group-structured dictionary learning (OSDL) task is defined with the following quantities. Let the dimension of the observations be denoted by dO. Assume that in each time instant (i = 1, 2, . . .) a set Oi ⊆ {1, . . . , dO} is given, that is, we know which coordinates are observable at time i, and the observation is xo,i. Our goal is to find a dictionary D ∈ RdO×dO that can approximate the observations xo,i well from the linear combination of its columns. The columns of D are assumed to belong to a closed, convex, and bounded set D = x∈dO D. To formulate the cost of dictionary D, first a fixed time instant i, observation xo,i, dictionary D is considered, and the hidden representation αi associated to this (xo,i, D, Oi) triple is defined. Representation αi is allowed to belong to a closed, convex set A ⊆ RdO (αi ∈ A) with certain structural constraints. The structural constraint on αi are expressed by making use of a given G group structure, which is a set system (also called hypergraph) on {1, . . . , dO}.

It is also assumed that weight vectors dG ∈ RdO (G ∈ G) are available for us and that they are positive on G and 0 otherwise. Representation α belonging to a triple (xo,i, D, O) is defined as the solution of the structured sparse coding task

\[ l(x_o, D) = l_{A,x,G}((a^G)_{G∈G}^y(x_o, D)) \]

\[ = \min_{\alpha \in A} \left[ \frac{1}{2} \|x_o - D_o \alpha\|_2^2 + \kappa \Omega(\alpha) \right], \]

where \( l(x_o, D) \) denotes the loss, \( \kappa > 0 \), and

\[ \Omega(\gamma) = |G| \{a^G)_{G∈G}^y(\gamma) = \|((d^G \circ \gamma)_2)_{G∈G}\|_2 \]

is the structured regularizer associated to \( G \) and \( (d^G)_{G∈G} \), \( \eta \in (0, 2) \). Here, the first term of (2) is responsible for the quality of approximation on the observed coordinates, whereas for \( \eta \leq 1 \) the other term [3] constrains the solution according to the group structure \( G \) similarly to the sparsity inducing regularizer \( \Omega \) in [30]: it eliminates the terms \( \|d^G \circ \gamma\|_2 \) \( (G \in G) \) by means of \( \|\gamma\|_2 \). The OSDL problem is defined as the minimization of the cost function:

\[ \min_{D \in \mathbb{D}} f_i(D) := \frac{1}{\sum_{j=1}^{t}(j/t)^\rho} \sum_{i=1}^{t} \left( \frac{1}{T} \right)^\rho l(x_o,i, D_o,i), \]

that is, the goal is to minimize the average loss belonging to the dictionary, where \( \rho \) is a non-negative forgetting factor. If \( \rho = 0 \), the classical average \( f_i(D) = \frac{1}{T} \sum_{i=1}^{t} l(x_o,i, D_o,i) \) is recovered.

As an example, let \( D_i = S_{dO}^{dO} (\gamma_i), A = \mathbb{R}^{dO} \). In this case, columns of D are restricted to the Euclidean unit sphere and we have no constraints for \( \alpha \). Now, let \( |S| = d_o \) and \( G = \{desc_1, . . . , desc_{d_o} \}, \) where \( desc_i \) represents the \( i^{th} \) node and its children in a fixed tree. Then the coordinates \( \alpha_i \) are searched in a hierarchical tree structure and the hierarchical dictionary D is optimized accordingly.

B. Optimization

Optimization of cost function (4) is equivalent to the joint optimization of dictionary D and representation \( \{\alpha_i\}_{i=1}^T \):

\[ \arg \min_{D \in \mathbb{D}, \{\alpha_i\}_{i=1}^T} f_i(D, \{\alpha_i\}_{i=1}^T), \]

where

\[ f_i = \frac{1}{\sum_{j=1}^{t}(j/t)^\rho} \sum_{i=1}^{t} \left( \frac{1}{T} \right)^\rho \frac{1}{2} \|x_o,i - D_o,i \alpha_i\|_2^2 + \kappa \Omega(\alpha_i) \].

D is optimized by using the sequential observations xo,i online in an alternating manner.
1) The actual dictionary estimation \( \mathbf{D}_{t-1} \) and sample \( \mathbf{x}_t \) is used to optimize (2) for representation \( \alpha_t \).

2) For the estimated representations \( \{\alpha_i\}_{i=1}^I \), the dictionary estimation \( \mathbf{D}_t \) is derived from the quadratic optimization problem

\[
\hat{f}_t(\mathbf{D}_t) = \min_{\mathbf{D}_t \in \mathcal{D}} f_t(\mathbf{D}_t, \{\alpha_i\}_{i=1}^I).
\]  

1) Representation optimization (\( \alpha \)): Note that (2) is a non-convex optimization problem with respect to \( \alpha \). The variational properties of norm \( \| \cdot \|_2 \) can be overcome this problem. One can show, alike to [30], that by introducing an auxiliary variable \( z \in \mathbb{R}^{|\mathcal{O}^0|} \), the solution \( \alpha \) of the optimization task (9) is equal to the solution of (2):

\[
J(\alpha, z) = \arg \min_{\alpha \in \mathcal{A}, z \in \mathbb{R}^{|\mathcal{O}^0|}} J(\alpha, z), \quad \text{where}
\]

\[
J(\alpha, z) = \frac{1}{2} \| x_0 - (\mathbf{D}_{t-1})_0 \alpha \|_2^2 + \frac{1}{2} \alpha^T \text{diag}(\zeta) \alpha + \| z \|_2^2, \quad \zeta = \zeta(z) \in \mathbb{R}^{d_\alpha} \quad \text{and} \quad \zeta_j = \sum_{G \in \mathcal{G}, G \ni j} (\hat{d}_G)^2 / z_G.
\]

The optimization of (9) can be carried out by iterative alternating steps. One can minimize the quadratic cost function on the convex set \( \mathcal{A} \) for a given \( z \) with standard solvers [37]. Then, one can use the variation principle and find solution \( z = (z_G)_{G \in \mathcal{G}} \) for a fixed \( \alpha \) by means of the explicit expression

\[
z_G = \| d_G \circ \alpha \|_2^{-2} \| (d_G \circ \alpha) G \in \mathcal{G} \|_2^{-1}.
\]

Note that for numerical stability, smoothing \( z = \max(z, \varepsilon) \) (0 < \( \varepsilon \) << 1) is suggested in practice.

2) Dictionary optimization (\( \mathbf{D} \)): The block-coordinate descent (BCD) method [37] is used for the optimization of \( \mathbf{D} \): columns \( \mathbf{d}_i \) in \( \mathbf{D} \) are optimized one-by-one by keeping the other columns \( \{\mathbf{d}_i, i \neq j\} \) fixed. For a given \( j \), \( \hat{f}_t \) is quadratic in \( \mathbf{d}_j \). The minimum is found by solving \( \partial f_t(\mathbf{d}_j) = 0 \), and then this solution is projected to the constraint set \( \mathcal{D}_j \) (\( \mathbf{d}_j \leftarrow \Pi_{\mathcal{D}_j}(\mathbf{d}_j) \)). One can show by executing the differentiation that \( \mathbf{u}_j \) satisfies the linear equation system

\[
\mathbf{C}_{j,t} \mathbf{u}_j = \mathbf{b}_{j,t} - \mathbf{e}_{j,t} + \mathbf{C}_{j,t} \mathbf{d}_j,
\]

where

\[
\mathbf{C}_{j,t} = \sum_{i=1}^t \left( \frac{i}{t} \right)^\rho \Delta_i \mathbf{d}_{i,j} \in \mathbb{R}^{d_x \times d_x},
\]

\[
\mathbf{e}_{j,t} = \sum_{i=1}^t \left( \frac{i}{t} \right)^\rho \Delta_i \mathbf{d} \alpha_i \mathbf{e}_{i,t} \in \mathbb{R}^{d_x},
\]

\[
\mathbf{B}_t = \sum_{i=1}^t \left( \frac{i}{t} \right)^\rho \Delta_i \mathbf{d} \alpha_i^{\mathbf{r}_t} = [\mathbf{b}_{1,t}, \ldots, \mathbf{b}_{d_{\alpha,t+1}}],
\]

matrices \( \mathbf{C}_{j,t} \) are diagonal, \( \mathbf{B}_t \in \mathbb{R}^{d_x \times d_{\alpha}} \), and \( \Delta_i \in \mathbb{R}^{d_x \times d_x} \) is the diagonal matrix representation of the \( O_i \) set (for \( j \in O_i \) the \( j \)-th diagonal is 1 and is 0 otherwise). It is sufficient to update statistics \( \{\{\mathbf{C}_{j,t}\}_{j=1}^d, \{\mathbf{B}_t, \{\mathbf{e}_{i,t}\}_{i=1}^d\} \} \) online for the optimization of \( \hat{f}_t \), which can be done exactly for \( \mathbf{C}_{j,t} \) and \( \mathbf{B}_t \):

\[
\mathbf{C}_{j,t} = \gamma_t \mathbf{C}_{j,t-1} + \Delta_i \alpha_i^2, \quad \mathbf{B}_t = \gamma_t \mathbf{B}_{t-1} + \Delta_i \mathbf{x}_t \alpha_i^{T},
\]

where \( \gamma_t = (1 - \frac{1}{t})^\rho \) and the recursions are initialized by (i) \( C_{j,0} = 0 \), \( B_0 = 0 \) for \( \rho = 0 \) and (ii) in an arbitrary way for \( \rho > 0 \). According to numerical experiences,

\[
e_{j,t} = \gamma_t e_{j,t-1} + \Delta_i \mathbf{D}_i \alpha_i \alpha_{i,j}.
\]

is a good approximation for \( e_{j,t} \) with the actual estimation \( \mathbf{D}_t \) and with initialization \( e_{j,0} = 0 \). It may be worth noting that the convergence speed is often improved if statistics are updated in mini-batches \( \{\mathbf{x}_{O_t,1}, \ldots, \mathbf{x}_{O_t,\rho}\} \).

III. OSDL Based Collaborative Filtering

We formulate the CF task as an OSDL optimization problem in Section III-A. According to the CF literature, oftentimes neighbor-based corrections improve the precision of the estimation. We also use this technique (Section III-B) to improve the OSDL estimations.

A. CF Casted as an OSDL Problem

Below, we transform the CF task into an OSDL problem. Consider the \( t \)-th user’s known ratings as OSDL observations \( \mathbf{x}_{O_t} \). Let the optimized group-structured dictionary on these observations be \( \mathbf{D} \). Now, assume that we have a test user and his/her ratings, i.e., \( \mathbf{x}_0 \in \mathbb{R}^{O} \). The task is to estimate \( \mathbf{x}_{\{1, \ldots, d_x\}\setminus O} \), that is, the missing coordinates of \( \mathbf{x} \) (the missing ratings of the user) that can be accomplished as follows:

1) Remove the rows of the non-observed \( \{1, \ldots, d_x\}\setminus O \) coordinates from \( \mathbf{D} \). The obtained \( O \times d_x \) sized matrix \( \mathbf{D}_O \) and \( \mathbf{x}_O \) can be used to estimate \( \alpha \) by solving the structured sparse coding problem (2).

2) Using the estimated representation \( \alpha \), estimate \( \mathbf{x} \) as

\[
\hat{\mathbf{x}} = \hat{\mathbf{D}} \hat{\alpha}.
\]

B. Neighbor Based Correction

According to the CF literature, neighbor based correction schemes may further improve the precision of the estimations [1]. This neighbor correction approach

- relies on the assumption that similar items (e.g., jokes/movies) are rated similarly and
- can be adapted to OSDL-based CF estimation in a natural fashion.

Here, we detail the idea. Let us assume that the similarities \( s_{ij} \in \mathbb{R} (i, j \in \{1, \ldots, d_x\}) \) between individual items are given. We shall provide similarity forms in Section IV-B. Let \( \mathbf{d}_k \alpha_k \in \mathbb{R} \) be the OSDL estimation for the rating of the \( k \)-th non-observed item of the \( t \)-th user (\( k \notin O_t \)), where \( \mathbf{d}_k \in \mathbb{R}^{1 \times d_x} \) is the \( k \)-th row of matrix \( \mathbf{D} \in \mathbb{R}^{d_x \times d_x} \), and \( \alpha_k \in \mathbb{R}^{d_x} \) is computed according to Section III-A.

Let the prediction error on the observable item neighbors (\( j \)) of the \( k \)-th item of the \( t \)-th user (\( j \in O_t \setminus \{k\} \)) be \( \mathbf{d}_k \alpha_k - x_{jt} \in \mathbb{R} \). These prediction errors can be used for the correction of

\[1\] The Matlab code of the OSDL method is available at http://nipg.inf.elte.hu/szzoli.
the OSDL estimation \((d_k \alpha_t)\) by taking into account the \(s_{ij}\) similarities:

\[
\hat{x}_{kt} = d_k \alpha_t + \gamma_1 \left[ \frac{\sum_{j \in O_i \setminus \{k\}} s_{kj} (d_j \alpha_t - x_{jt})}{\sum_{j \in O_i \setminus \{k\}} s_{kj}} \right], \quad \text{or (19)}
\]

\[
\hat{x}_{kt} = \gamma_0 (d_k \alpha_t) + \gamma_1 \left[ \frac{\sum_{j \in O_i \setminus \{k\}} s_{kj} (d_j \alpha_t - x_{jt})}{\sum_{j \in O_i \setminus \{k\}} s_{kj}} \right], \quad \text{(20)}
\]

where \(k \notin O_i\). Here, (19) is analogous to the form of [2], (20) is a simple modification: it modulates the first term with a separate \(\gamma_0\) weight.

### IV. Numerical Results

We have chosen the Jester dataset (Section IV-A) for the illustration of the OSDL based CF approach. It is a standard benchmark for CF. We detail our preferred item similarities in Section IV-B. To evaluate the CF based estimation, we use the performance measures given in Section IV-C. Section IV-D is about our numerical experiences.

#### A. The Jester Dataset

The dataset [38] contains 4,136,360 ratings from 73,421 users to 100 jokes on a continuous \([-10, 10]\) range. The worst and best possible gradings are \(-10\) and \(+10\), respectively. A fixed 10 element subset of the jokes is called gauge set and it was evaluated by all users. Two thirds of the users have rated at least 36 jokes, and the remaining ones have rated between 15 and 35 jokes. The average number of user ratings per joke is 46.

#### B. Item Similarities

In the neighbor correction step (19) or (20) we need the \(s_{ij}\) values representing the similarities of the \(i^{th}\) and \(j^{th}\) items. We define this value as the similarity of the \(i^{th}\) and \(j^{th}\) rows \((d_i\) and \(d_j\)\) of the optimized OSDL dictionary \(D\) [2]:

\[
S_1 : \quad s_{ij} = s_{ij}(d_i, d_j) = \left( \frac{\max(0, (d_i, d_j))}{\|d_i\|_2 \|d_j\|_2} \right)^{\beta}, \quad \text{or (21)}
\]

\[
S_2 : \quad s_{ij} = s_{ij}(d_i, d_j) = \left( \frac{\|d_i - d_j\|_2^2}{\|d_i\|_2 \|d_j\|_2} \right)^{-\beta}, \quad \text{(22)}
\]

where \(\beta > 0\) is the parameter of the similarity measure. Quantities \(s_{ij}\) are non-negative; if the value of \(s_{ij}\) is close to zero (large) then the \(i^{th}\) and \(j^{th}\) items are very different (very similar).

#### C. Performance Measure

In our numerical experiments we used the RMSE (root mean square error) and the MAE (mean absolute error) measure for the evaluation of the quality of the estimation, since these are the most popular measures in the CF literature. The RMSE and MAE measure is the average squared/absolute difference of the true and the estimated rating values, respectively:

\[
RMSE = \sqrt{\frac{1}{|S|} \sum_{(i,t) \in S} (x_{it} - \hat{x}_{it})^2}, \quad \text{(23)}
\]

\[
MAE = \frac{1}{|S|} \sum_{(i,t) \in S} |x_{it} - \hat{x}_{it}|, \quad \text{(24)}
\]

where \(S\) denotes either the validation or the test set.

#### D. Evaluation

Here we illustrate the efficiency of the OSDL-based CF estimation on the Jester dataset (Section IV-A) using the RMSE and MAE performance measures (Section IV-C). We start our discussion with the RMSE results. The MAE performance measure led to similar results; for the sake of completeness we report these results at the end of this section. To the best of our knowledge, the top results on this database are RMSE = 4.1123 [39] and RMSE = 4.1229 [2]. Both works are from the same authors. The method in the first paper is called item neighbor and it makes use of only neighbor information. In [2], the authors used a bridge regression based unstructured dictionary learning model—with a neighbor correction scheme—the optimized the dictionary by gradient descent and set \(d_\alpha\) to 100. These are our performance baselines.

To study the capability of the OSDL approach in CF, we focused on the following issues:

- Is structured dictionary \(D\) beneficial for prediction purposes, and how does it compare to the dictionary of classical (unstructured) sparse dictionary?
- How does the OSDL parameters and the similarity/neighbor correction applied affect the efficiency of the prediction?
- How do different group structures \(\mathcal{G}\) fit to the CF task?

In our numerical studies we chose the Euclidean unit sphere for \(\mathcal{D}_1 = S_d^2\) (\(\forall i\)), and \(\mathcal{A} = \mathbb{R}^{d_\alpha}\) and no additional weighting was applied (\(d^G = \chi_G, \forall G \in \mathcal{G}\), where \(\chi\) is the indicator function). We set \(\eta\) of the group-structured regularizer \(\Omega\) to 0.5. Group structure \(\mathcal{G}\) of vector \(\alpha\) was realized on

- a \(d \times d\) toroid \((d_\alpha = d^2)\) with \(|\mathcal{G}| = d_\alpha\) applying \(r \geq 0\) neighbors to define \(\mathcal{G}\). For \(r = 0\) \((\mathcal{G} = \{1\})\) the classical sparse representation based dictionary is recovered.
- a hierarchy with a complete binary tree structure. In this case:

  \(- |\mathcal{G}| = d_\alpha\), and group \(G\) of \(\alpha_i\) contains the \(i^{th}\) node and its descendants on the tree, and
  \(- the size of the tree is determined by the number of levels \(l\). The dimension of the hidden representation is then \(d_\alpha = 2^l - 1\).

The size \(R\) of mini-batches was set either to 8, or to 16 and the forgetting factor \(\rho\) was chosen from set \([0, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]\). The \(\kappa\) weight of structure inducing regularizer \(\Omega\) was chosen from the set \([\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \frac{1}{128}, \frac{1}{256}, \frac{1}{512}, \frac{1}{1024}, \frac{1}{2048}, \frac{1}{4096}, \frac{1}{8192}, \frac{1}{16384}, \frac{1}{32768}, \frac{1}{65536}, \frac{1}{131072}]\). We studied similarities \(S_1\), \(S_2\) [see (21)-(22)] with both neighbor correction schemes.
In what follows, corrections based on (19) and (20) will be called $S_1$, $S_2$ and $S_0^1$, $S_0^2$, respectively. Similarity parameter $\beta$ was chosen from the set $\{0.2, 1, 1.8, 2.6, \ldots, 14.6\}$. In the BCD step of the optimization of $D$, 5 iterations were applied. In the $\alpha$ optimization step, we set 5 iterations, whereas smoothing parameter $\epsilon$ was $10^{-5}$.

We used a $90\% - 10\%$ random split for the observable ratings in our experiments, similarly to [2]:

- training set ($90\%$) was further divided into 2 parts:
  - we chose the $80\%$ observation set $\{O_i\}$ randomly, and optimized $D$ according to the corresponding $x_{O_i}$ observations,
  - we used the remaining $10\%$ for validation, that is for choosing the optimal OSDL parameters ($r$ or $l$, $\kappa$, $\beta$), BCD optimization parameter ($R$), neighbor correction ($S_1$, $S_2$, $S_0^1$, $S_0^2$), similarity parameter ($\beta$), and correction weights ($\gamma$) in (19) or (20).
- we used the remaining $10\%$ of the data for testing.

The optimal parameters were estimated on the validation set, and then used on the test set. The resulting RMSE/MAE score was the performance of the estimation.

1) Toroid Group Structure. In this section we provide results using toroid group structure. We set $d = 10$. The size of the toroid was $10 \times 10$, and thus the dimension of the representation was $d_o = 100$.

In the first experiment we study how the size of neighborhood ($r$) affects the results. This parameter corresponds to the “smoothness” imposed on the group structure: when $r = 0$, then there is no relation between the $d^j \in \mathbb{R}^{d_o}$ columns in $D$ (no structure). As we increase $r$, the $d^j$ feature vectors will be more and more aligned in a smooth way. To this end, we set the neighborhood size to $r = 0$ (no structure), and then increased it to 1, 2, 3, 4, and 5. For each $(\kappa, \rho, \beta)$, we calculated the RMSE of our estimation, and then for each fixed $(\kappa, \rho)$ pair, we minimized these RMSE values in $\beta$. The resulting validation and test surfaces are shown in Fig. 1. For the best $(\kappa, \rho)$ pair, we also present the RMSE values as a function of $\beta$ (Fig. 2).

In this illustration we used $S_1^0$ neighbor correction and $R = 8$ mini-batch size. We note that we got similar results using $R = 16$ too. Our results can be summarized as follows.

- For a fixed neighborhood parameter $r$, we have that:
  - The validation and test surfaces are very similar (see Fig. 1(e)-(f)). It implies that the validation surfaces are good indicators for the test errors. For the best $r$, $\kappa$ and $\rho$ parameters, we can observe that the validation and test curves (as functions of $\beta$) are very similar. This is demonstrated in Fig. 2, where we used $r = 4$ neighborhood size and $S_1^0$ neighbor correction. We can also notice that (i) both curves have only one local minimum, and (ii) these minimum points are close to each other.
  - The quality of the estimation depends mostly on the $\kappa$ regularization parameter. As we increase $r$, the best $\kappa$ value is decreasing.
  - The estimation is robust to the different choices of forgetting factors (see Fig. 1(a)-(e)). In other words, this parameter $\rho$ can help in fine-tuning the results.
- Structured dictionaries ($r > 0$) are advantageous over those methods that do not impose structure on the dictionary elements ($r = 0$). For $S_0^2$ and $S_0^2$ neighbor corrections, we summarize the RMSE results in Table I. Based on this table we can conclude that in the studied parameter domain
  - the estimation is robust to the selection of the mini-batch size ($R$). We got the best results using $R = 8$. Similarly to the role of parameter $\rho$, adjusting $R$ can be used for fine-tuning.
  - the $S_0^2$ neighbor correction lead to the smallest RMSE value.
  - When we increase $r$ up to $r = 4$, the results improve. However, for $r = 5$, the RMSE values do not improve anymore; they are about the same that we have using $r = 4$.
  - The smallest RMSE we could achieve was 4.0774, and the best known result so far was RMSE = 4.1123 [39]. This proves the efficiency of our OSDL based collaborative filtering algorithm.

- We note that our RMSE result seems to be significantly better than the that of the competitors: we repeated this experiment 5 more times with different randomly selected training, test, and validation sets, and our RMSE results have never been worse than 4.08.

In the second experiment we studied how the different neighbor corrections ($S_1$, $S_2$, $S_0^1$, $S_0^2$) affect the performance of the proposed algorithm. To this end, we set the neighborhood parameter to $r = 4$ because it proved to be optimal in the previous experiment. Our results are summarized in Table II. From these results we can observe that

- our method is robust to the selection of correction methods. Similarly to the $\rho$ and $R$ parameters, the neighbor correction scheme can help in fine-tuning the results.
- The introduction of $\gamma_0$ in (20) with the application of $S_1^0$ and $S_2^0$ instead of $S_1$ and $S_2$ proved to be advantageous in the neighbor correction phase.
- For the studied CF problem, the $S_0^2$ neighbor correction method (with $R = 8$) lead to the smallest RMSE value,
For a fixed $(\kappa, \rho)$ parameter pair, the surfaces show the best RMSE values optimized in the $\beta$ similarity parameter. The group structure (5) is toroid. The applied neighbor correction was $S_0^0$. (a): $r = 0$ (no structure). (b): $r = 1$. (c): $r = 2$. (d): $r = 3$. (e)-(f): $r = 4$, on the same scale.

The $R \in \{8, 16\}$ setting yielded us similarly good results. Even with $R = 16$, the RMSE value was 4.0777.

2) Hierarchical Group Structure: In this section we provide results using hierarchical $\alpha$ representation. The group structure $G$ was chosen to represent a complete binary tree.

In our third experiment we study how the number of levels $(l)$ of the tree affects the results. To this end, we set the number of levels to $l = 3, 4, 5, \text{and } 6$. Since $d_\alpha$, the dimension of the hidden representation $\alpha$, equals to $2^l - 1$, these $l$ values give rise to dimensions $d_\alpha = 7, 15, 31, \text{and } 63$. Validation and test surfaces are provided in Fig. 3(a)-(c) and (e)-(f), respectively. The surfaces show for each $(\kappa, \rho)$ pair, the minimum RMSE values taken in the similarity parameter $\beta$. For the best $(\kappa, \rho)$ parameter pair, the dependence of RMSE on $\beta$ is presented in Fig. 3(d). In this illustration we used $S_0^0$ neighbor correction, and the mini-batch size was set to $R = 8$. Our results are summarized below. We note that we obtained similar results with mini-batch size $R = 16$.

- For fixed number of levels $l$, similarly to the toroid group structure (where the size $r$ of the neighborhood was fixed),
  - validation and test surfaces are very similar, see Fig. 3(b)-(c). Validation and test curves as a function of $\beta$ behave alike, see Fig. 3(d).
Our results are summarized in Table IV. We found that neighbor corrections ($R = 16$). Odd rows: $S_1^0$, even rows: $S_2^0$ neighbor correction. For fixed $R$, the best performance is highlighted with boldface typesetting.

|          | $t = 3$ | $t = 4$ | $t = 5$ | $t = 6$ |
|----------|---------|---------|---------|---------|
| $R = 8$  | $S_1^0$ | 4.1572  | 4.1220  | 4.1241  | 4.1374  |
|          | $S_2^0$ | 4.1609  | 4.1285  | 4.1298  | 4.1362  |
| $R = 16$ | $S_1^0$ | 4.1578  | 4.1261  | 4.1249  | 4.1373  |
|          | $S_2^0$ | 4.1638  | 4.1332  | 4.1303  | 4.1383  |

The obtained RMSE values are summarized in Table III for $S_1^0$ and $S_2^0$ neighbor corrections. According to the table, the quality of estimation is about the same for mini-batch size $R = 8$ and $R = 16$; the $R = 8$ based estimation seems somewhat more precise. Considering the neighbor correction schemes $S_1^0$ and $S_2^0$, $S_1^0$ provided better predictions.

- As a function of the number of levels, we got the best result for $l = 4$, RMSE = 4.1220; RMSE values decrease until $l = 4$ and then increase for $l > 4$.
- Our best obtained RMSE value is 4.1220; it was achieved for dimension only $d_n = 15$. We note that this small dimensional, hierarchical group structure based result is also better than that of [2] with RMSE = 4.1229, which makes use of unstructured dictionaries with $d_n = 100$. The result is also competitive with the RMSE = 4.1123 value of [39].

In our fourth experiment we investigate how the different neighbor corrections ($S_1$, $S_2$, $S_1^0$, $S_2^0$) affect the precision of the estimations. We fixed the number of levels to $l = 4$, since it proved to be the optimal choice in our previous experiment. Our results are summarized in Table IV. We found that:

- the precision of the estimation depends mostly on the regularization parameter $\kappa$; forgetting factor $\rho$ enables fine-tuning.
- As a function of the number of levels, we got the best result for $l = 4$, RMSE = 4.1220; RMSE values decrease until $l = 4$ and then increase for $l > 4$.
- Our best obtained RMSE value is 4.1220; it was achieved for dimension only $d_n = 15$. We note that this small dimensional, hierarchical group structure based result is also better than that of [2] with RMSE = 4.1229, which makes use of unstructured dictionaries with $d_n = 100$. The result is also competitive with the RMSE = 4.1123 value of [39].

V. CONCLUSIONS

We have dealt with collaborative filtering (CF) based recommender systems and extended the application domain of structured dictionaries to CF. We used online group-structured dictionary learning (OSDL) to solve the CF problem; we were similar to those of the RMSE. We got the best results using toroid group structure, thus we present more details for this case.

- With the usage of structured dictionaries we can get better results: the estimation errors were decreasing when we increased the neighbor size $r$ up to 4. (Table V). The validation and test surfaces/curves are very similar, see Fig. 4(e)-(f), Fig. 5.
- The quality of the estimation depends mostly on the $\kappa$ regularization parameter (Fig. 4(a)-(e)). The applied $\rho$ forgetting factor, $R$ mini-batch size and neighbor correction method can help in fine-tuning the results, see Fig. 4(a)-(e), Table V and Table VI, respectively.
- The smallest MAE we could achieve was 3.1544, using $r = 4$ neighbor size, $S_1^0$ neighbor correction and $R = 8$ mini-batch size. The baseline methods led to [39] MAE = 3.1616, [2] MAE = 3.1606 results. Our approach outperformed both of the state-of-the-art competitors. We also repeated this experiment 5 more times with different randomly selected training, test, and validation sets, and our MAE results have never been worse than 3.155. This demonstrates the efficiency of our approach.
the best performance is highlighted with boldface typesetting.

Columns: applied neighbor corrections. Rows: mini-batch size \( R = 8 \).

Fig. 3: RMSE validation surfaces [(a)-(b), (e)-(f)] and test surfaces (c) as a function of forgetting factor \((\kappa)\). (d): validation and test curve using the optimal number of levels \( l = 4 \), regularization weight \( \kappa = \frac{1}{2} \), forgetting factor \( \rho = 0 \), mini-batch size \( R = 8 \), similarity parameter \( \beta = 1.8 \). Group structure (5): complete binary tree. Neighbor correction: \( S_1^0 \).

(a)-(c), (e)-(f): for fixed \((\kappa, \rho)\) parameter pair, the surfaces show the best RMSE values optimized in the \( \beta \) similarity parameter. (a): \( l = 3 \). (b)-(c): \( l = 4 \), on the same scale. (e): \( l = 5 \). (f): \( l = 6 \).

TABLE VI: Performance (MAE) of the OSDL prediction for different neighbor corrections using toroid group structure (5). Columns: applied neighbor corrections. Rows: mini-batch size \( R = 8 \) and 16. The neighbor size was set to \( r = 4 \). For fixed \( R \), the best performance is highlighted with boldface typesetting.

| \( R \) | \( S_1 \) | \( S_2 \) | \( S_1^0 \) | \( S_2^0 \) |
|---|---|---|---|---|
| 8 | 3.1719 | 3.1779 | \textbf{3.1544} | 3.1511 |
| 16 | 3.1726 | 3.1778 | \textbf{3.1546} | 3.1568 |

casted the CF estimation task as an OSDL problem. We demonstrated the applicability of our novel approach on joke recommendations. Our extensive numerical experiments show that structured dictionaries have several advantages over the state-of-the-art CF methods: more precise estimation can be obtained, and smaller dimensional feature representation can be sufficient by applying group structured dictionaries. Moreover, the estimation behaves robustly as a function of the OSDL parameters and the applied group structure.

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Fig. 4: MAE validation surfaces [(a)-(e)] and test surfaces (f) as a function of forgetting factor (ρ) and regularization (κ). For a fixed (κ, ρ) parameter pair, the surfaces show the best MAE values optimized in the β similarity parameter. The group structure (5) is toroid. The applied neighbor correction was \( S_0^r \). (a): \( r = 0 \) (no structure). (b): \( r = 1 \). (c): \( r = 2 \). (d): \( r = 3 \). (e)-(f): \( r = 4 \), on the same scale.

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