Correlation-driven multigap $d$-wave superconductivity in Anderson lattice model

Marcin M. Wysokiński,1,2 Jan Kaczmarczyk,1,2 and Jozef Spalek1,2

1 Marian Smoluchowski Institute of Physics, Jagiellonian University, ulica prof. S. Lojasiewicza 11, PL-30-348 Kraków, Poland
2 Institute of Science and Technology Austria, Am Campus 1, A-3400 Klosterneuburg, Austria

(Dated: October 2, 2015)

We present the full Gutzwiller-wave-function solution of the Anderson lattice model in two dimensions that leads to the correlation-driven multigap superconducting (SC) ground state with the dominant $d_{x^2-y^2}$-wave symmetry. The results are consistent with the principal properties of the heavy-fermion superconductor CeCoIn$_5$. We regard the pairing mechanism as universal and thus applicable to other Ce-based heavy-fermion compounds. Additionally, a gain in kinetic energy in the SC state takes place, as is also the case for high-temperature superconductors.

PACS numbers: 74.70.Tx, 74.20.-z, 71.27.+a, 74.20.Mn

Introduction. Starting from the first observation of superconductivity (SC) in the heavy fermion system (HFS) CeCu$_2$Si$_2$[1,2], there is an ongoing discussion concerning the microscopic mechanism(s) of pairing in this class of unconventional superconductors [3,6]. At present, over thirty of HFS are known to be superconducting [7] and majority of them exhibit universal features of the electronic structure, that has their source in the strong inter-electronic correlations. Thus the assumption, that also the pairing may have a common, non-phononic microscopic origin is widely accepted [8,9,11,12]. Associated with this is the fundamental question whether the strong correlations can also be the source of pairing in those systems, in the same fashion, as they are for the appearance of heavy quasiparticle states and non-trivial magnetic properties [13,14]. Such a single approach might be regarded as providing a class of universality defined by the type of many-particle theoretical model. The purpose of this work is to present an affirmative answer to such a program by solving Anderson lattice model (ALM) with the full Gutzwiller wave function for the SC state. The quasiparticle properties in the normal state have been accounted for in the zeroth order of our expansion and shown statistically consistent elaboration (SGA) [50–52], are reproduced in the zeroth order of our expansion and shown statistically consistent elaboration (SGA) [50–52], are reproduced in the zeroth order of our expansion and shown statistically consistent elaboration (SGA) [50–52].

Model and method. Our starting point is the Anderson lattice model

$$\hat{H} = \sum_{i,j,\sigma} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} - \mu \sum_{i,\sigma} n_{i\sigma}^f + (\epsilon_f - \mu) \sum_{i,\sigma} \hat{n}_i^f + U \sum_i \hat{n}_{i \uparrow}^f \hat{n}_{i \downarrow}^f + \sum_{i,j,\sigma} (V_{ij} \hat{c}_{i\uparrow}^\dagger \hat{c}_{j\downarrow} + \text{H.c.}),$$

on a two-dimensional (2D), translationally invariant square lattice, with the chemical potential $\mu$, and with the usual notation [15].

Ground-state properties of (1) have been obtained variationally by means of a novel Diagrammatic Expansion technique for the Gutzwiller Wave Function (DE-GWF) [16,17,18]. The main advance introduced by this method is an accurate evaluation of the expectation values with the full Gutzwiller wave function (GWF). The standard Gutzwiller Approximation (GA) [19], and its subsequent statistically consistent elaboration (SGA) [50,51,52], are reproduced in the zeroth order of our expansion and shown to include only the local correlations which do not lead to any SC solution. DE-GWF method accounts for nonlo-
cral correlations being the essential factor leading to the appearance of the SC state. This particular feature of our approach is shared with the variational Monte Carlo (VMC) approach \cite{52,54}. However, our technique does not suffer from the system finite-size limitation and is computationally efficient. In general, DE-GWF reproduces the VMC results with an improved accuracy, as has been shown previously on the example of SC solutions for both the Hubbard \cite{45} and the t-J \cite{46} models.

The Gutzwiller wave function, \( \langle \psi_G \rangle \), is constructed from its uncorrelated counterpart, \( \langle \psi_0 \rangle \), by projecting out fraction of the local double \( f \)-occupancies by means of the Gutzwiller operator, i.e., \( \langle \psi_G \rangle \equiv \hat{P}_G \langle \psi_0 \rangle \equiv \prod_i \hat{P}_{G,i} \langle \psi_0 \rangle \). The operator \( \hat{P}_{G,i} \) is defined by \( \hat{P}_{G,i} \equiv 1 + x \hat{d}_{i_h}^{HF} \), where \( x \) is a variational parameter and \( \hat{d}_{i_h}^{HF} \equiv \hat{a}_{i_h}^{HF} \hat{a}_{i_h}^{HF} = (\hat{n}_{i_h}^f - n_{o_f}) (\hat{n}_{i_h}^f - n_{o_f}) \) is relative to the Hartree-Fock (HF) double occupancy operator, with \( n_{o_f} \equiv \langle \hat{n}_{i_f}^f \rangle \). Hereafter we refer a shortened notation \( \langle \ldots \rangle_0 \equiv \langle \bar{\psi}_0 | \ldots | \bar{\psi}_0 \rangle \).

Formally, the expectation value with GWF for any product operator, \( \hat{O}_{ij} \), acting on site \( i \) and \( j \), can be expanded in a power series in 

\[
\langle \hat{O}_{ij} \rangle_G = \left( \langle \hat{O}_{ij} \rangle_0 \prod_{p \neq (i,j)} \hat{P}_{G,p} \right)_0 = \sum_{k=0}^{\infty} \frac{x^k}{k!} \sum_{i_1, \ldots, i_k} \langle \hat{O}_{ij} \rangle_0 \hat{d}_{i_1}^{HF} \cdots \hat{d}_{i_k}^{HF}.
\]

(2)

where we have defined \( \langle \hat{O}_{ij} \rangle_0 = \hat{P}_{G,i} \hat{P}_{G,j} \langle \hat{O}_{ij} \rangle_0 \), and \( \hat{d}_{i_1}^{HF} \cdots \hat{d}_{i_k}^{HF} \). The primed summation denotes the following restrictions: \( I_p \neq I_i, j \), and \( I_p \neq i, j \) for all \( p \) and \( p' \). Power series in \( x \) allows for a systematic incorporation of the long-range correlations among \( k \) non-local sites (\( I_1, \ldots, I_k \)) and the local ones (\( I_i, j \)). For \( k = 0 \) we reproduce the results of GA, where only local sites are projected. In this manner, the number of non-local correlated sites taken into account is the expansion parameter. The resulting expectation values with \( \langle \psi_0 \rangle \) can be evaluated by applying the Wick’s theorem. The products of two-operator contractions can be visualized as diagrams, with sites and two-operator contractions playing the role of vertices and lines, respectively. \cite{12,13,16}. Here we allow for both the paramagnetic (PM) and SC contractions (lines) defined respectively as

\[
P_{ij}^{\alpha, \beta} \equiv \langle \hat{d}_{i}^\dagger \hat{d}_{j}^\dagger \rangle_0 - \delta_{ij} \delta_{\alpha f} \delta_{\beta f} n_{o_f}, \quad S_{ij}^{\alpha, \beta} \equiv \langle \hat{a}_{i}^\dagger \hat{a}_{j}^\dagger \rangle_0.
\]

where \( \alpha, \beta \in \{c, f\} \) and \( I, I' \) are the lattice indices. We have accounted for SC contractions, \( S_{ij}^{\alpha, \beta} \), which lead to the gap of \( d \)-wave symmetry, with the nodal lines along the diagonal directions. For an infinite lattice, we introduce a real space cutoff. Namely, we consider only the lines limited by the distance \( |l_x - l'_x|^2 + |l_y - l'_y|^2 \leq 10 \) (measured in lattice constants).

The resulting expectation value of the Hamiltonian can be expressed by the diagrammatic sums (for details see Ref. \cite{13}), and in this manner, it depends explicitly on the variational parameter \( x \), the correlation functions (lines) \cite{53} and on \( n_{o_f} \).

The iterative procedure for obtaining the physical ground state of \( \langle \psi_0 \rangle \) is as follows:
1. \( \langle \mathcal{H} \rangle_G \) is evaluated diagrammatically for selected \( \langle \psi_0 \rangle \).
2. \( \langle \mathcal{H} \rangle_G \) is minimized with respect to \( x \).
3. The effective single particle Hamiltonian \( \hat{H}^{\text{eff}} \) for the uncorrelated wave function \( \langle \psi_0 \rangle \) is determined.
4. New trial \( \langle \psi_{G} \rangle \) is obtained as the ground state of \( \hat{H}^{\text{eff}} \). The points 1-4 are repeated in a self-consistent loop until a satisfactory convergence, i.e., the condition \( \langle \psi_0 \rangle = \langle \psi_G \rangle \) is reached to a desired accuracy, typically \( 10^{-6} \). The details of the method for the normal state of ALM are thoroughly discussed in Ref. \cite{13}.

The effective single-particle Hamiltonian \( \hat{H}^{\text{eff}} \) for the uncorrelated wave function \( \langle \psi_0 \rangle \) is determined from the condition that its optimal expectation value coincides with \( \langle \mathcal{H} \rangle_G \). This leads to the condition

\[
\delta \langle H \rangle_G (P_{ij}^{\alpha, \beta}, S_{ij}^{\alpha, \beta}, n_{o_f}) = \delta \langle \mathcal{H} \rangle_G (P_{ij}^{\alpha, \beta}, S_{ij}^{\alpha, \beta}, n_{o_f})
\]

\[
= \sum_{I, I'} \left( \frac{\partial \langle \mathcal{H} \rangle_G}{\partial P_{I, I'}^{\alpha, \beta}} \delta P_{I, I'}^{\alpha, \beta} + \frac{\partial \langle \mathcal{H} \rangle_G}{\partial S_{I, I'}^{\alpha, \beta}} \delta S_{I, I'}^{\alpha, \beta} + \frac{\partial \langle \mathcal{H} \rangle_G}{\partial n_{o_f}} \delta n_{o_f} \right).
\]

(4)

Explicitly, the effective single-particle Hamiltonian reads

\[
\hat{H}^{\text{eff}} = \sum_{I, I'} \left[ t_{ij}^{\alpha} \hat{c}_{I, \alpha}^\dagger \hat{c}_{I'}^\dagger \hat{c}_{I'} \hat{c}_{I, \alpha} + t_{ij}^{\beta} \hat{c}_{I, \beta}^\dagger \hat{c}_{I'}^\dagger \hat{c}_{I'} \hat{c}_{I, \beta} + H.c. \right] + \sum_{I, I'} \Delta_{ij}^{\alpha, \beta} (\hat{c}_{I, \alpha}^\dagger \hat{c}_{I, \beta} + H.c.) + \sum_{I, I'} \Delta_{ij}^{\beta, \alpha} (\hat{c}_{I, \beta}^\dagger \hat{c}_{I, \alpha} + H.c.).
\]

(5)

with the effective microscopic parameters determined by

\[
t_{ij}^{\alpha} = \frac{\partial \langle H \rangle_G}{\partial P_{I, I'}^{\alpha, \beta}}, \quad \Delta_{ij}^{\alpha, \beta} = \frac{\partial \langle H \rangle_G}{\partial S_{I, I'}^{\alpha, \beta}}, \quad t_{ij}^{\beta} = \frac{\partial \langle H \rangle_G}{\partial n_{o_f}}.
\]

(6)

Parenthetically, as GWF introduces correlations within the \( f \) orbital only, there is no effective pairing between \( c \)-electrons, since there are no \( S_{ij}^{c, c} \) lines in the diagrams visualizing the Wick’s contractions in \( \hat{H}^{\text{eff}} \).

In the momentum space, the effective Hamiltonian can be reformulated in the Bogoliubov - de Gennes - Nambu form \cite{56}

\[
\hat{H}^{\text{eff}} = \sum_k \Psi_k^\dagger \left( \begin{array}{cccc}
\epsilon_{c_k} & 0 & 0 & \Delta_{fc}^{L, \alpha} \\
0 & \epsilon_{f_k} & -\epsilon_{c_k} & \Delta_{fc}^{L, \beta} \\
0 & -\epsilon_{c_k} & \epsilon_{f_k} & \Delta_{fc}^{L, \alpha} \\
\Delta_{fc}^{L, \beta} & \Delta_{fc}^{L, \alpha} & \Delta_{fc}^{L, \beta} & \epsilon_{f_k}
\end{array} \right) \Psi_k,
\]

(7)

where we have defined \( \Psi_k \equiv \left( \hat{c}_{k, \alpha}^\dagger, \hat{c}_{-k, \beta}, \hat{f}_{k, \beta}, \hat{f}_{-k, \beta} \right) \), and \( \epsilon_{\alpha}^{\beta}(\Delta_{\alpha}^{\beta}) = \left( 1/L \right) \sum_{i, i'} \epsilon_{i, \alpha}(\Delta_{i, \alpha}) e^{i(i-j)k} \), where \( L \) is the number of lattice sites. Hamiltonian \( \hat{H}^{\text{eff}} \) can be easily diagonalized by the Bogoliubov-type of transformation and thus the new lines and \( n_{o_f} \) determining the ground state of \( \hat{H}^{\text{eff}} \) can be obtained, i.e.,

\[
P_{ij}^{\alpha, \beta}(S_{ij}^{\alpha, \beta}) = \frac{1}{L} \sum_k \langle \hat{g}_{k}^\dagger \hat{g}_{k} \rangle_0 \left( \langle \hat{d}_{i, \alpha}^\dagger \hat{d}_{j}^\dagger \rangle_0 \right) e^{i(i-j)k},
\]

\[
n_{o_f} = \frac{1}{L} \sum_k \langle \hat{g}_{k}^\dagger \hat{g}_{k} \rangle_0.
\]

(8)
up to the third order, the expansion, if not stated otherwise, has been carried out restricted our analysis to the electrons with nearest-neighbor hopping parameters realistic for the Ce-based HFS, namely \( c_{\text{parameters}} \neq 0 \).

Our variational scheme enables us to determine both \( \hat{H}_{\text{eff}} \) and \( |\psi_0| \) and in turn, the renormalized SC order parameters

\[
\Delta_{G}^{\alpha\beta} \equiv \langle \hat{a}_{i+}^\dagger \hat{a}_{j+} \rangle_{G} = \langle \mathcal{P}_{G} \hat{a}_{i+}^\dagger \hat{a}_{j+} \mathcal{P}_{G} \rangle_{0}.
\]

Note that although, there is no pairing term between the \( c \) electrons in \( \hat{H}_{\text{eff}} \), the corresponding SC order parameters are nonzero.

**Results.** We have selected the starting microscopic parameters realistic for the Ce-based HFS, namely \( c \) electrons with nearest-neighbor hopping \( t \) and the second nearest-neighbor hopping, \( t' = 0.25|t| \), \( f \) electron atomic level position \( \epsilon_f = -3|t| \), and \( f-f \) Coulomb repulsion, \( U = 10|t| \). The bare \( c-f \) hybridization with the amplitude \( V \) is considered as a variable and of the nearest-neighbor origin. Physical energies are obtained by assuming \( |t| = 50 \text{ meV} \) (see e.g. [11]). We have restricted our analysis to the \( f \) orbital filling not exceeding unity or slightly larger, \( n_f \lesssim 1.05 \), to include possible local \( f^{3+}, f^{4+} \), and \( f^{2+} \) configurations. The diagrammatic expansion, if not stated otherwise, has been carried out up to the third order, \( k = 3 \).

![FIG. 1. Phase diagram on total band filling – hybridization magnitude plane, with the dominant, \( d_{2-y^2} \)-wave superconducting order parameter for \( f \) electrons, \( \Delta_{G}^{ff} \), see main text. The dashed lines mark the selected \( f \)-orbital isovalents. The dotted line singles out the non-BCS region defined by the gain in the kinetic energy, \( \Delta E_{\text{kin}} > 0 \) in SC state. For the selected point \( (n = 1.8, |V|/|t| = 0.65) \) we show in the inset that SC appears with the increasing \( f-f \) interaction.](image1)

![FIG. 2. Angle dependence of the SC gaps at the Fermi surface for selected parameters \( n = 1.8 \) and \( |V|/|t| = 0.65 \). The larger gap, \( \Delta_1 \) follows \( d_{2-y^2} \) dependence. The inset shows the quarter of the Brillouin zone with explicitly drawn absolute values of the SC gaps.](image2)

In Fig. 1 we present the phase diagram characterized by the leading value of the \( f-f \) electron \( d_{2-y^2} \)-component of the order parameter, \( \Delta_{G}^{ff} \), on the total filling, \( n-|V|/|t| \), plane, with the marked isovalents (dashed lines) by the \( f \) orbital occupation number, \( n_f \). It can be inferred that correlations for \( f \)-orbital filling in the range \( n_f \lesssim 0.8 - 0.85 \) are too weak to lead to robust SC state. In the inset to Fig. 1 (for the selected point on the diagram) we show explicitly the appearance of \( \Delta_{G}^{ff} \) and the associated condensation energy \( \Delta E = E_{\text{PM}} - E_{\text{SC}} \) emerging with the increasing interaction amplitude, \( U \). Here, we have defined \( E_{\text{PM}} \) and \( E_{\text{SC}} \) as the ground state energies for the normal and SC states, respectively.

The order parameter \( \Delta_{G}^{ff} \) is representative as it dominates practically always. In particular, in the singled out region, marked as non-BCS, the absolute values of the remaining order parameters \( \Delta_{G}^{\alpha\beta} \) are less than 35% of \( \Delta_{G}^{ff} \). In the non-BCS region, in distinction to that of BCS, a gain in the kinetic energy appears with respect to PM state, \( \Delta E_{\text{kin}} = E_{\text{kin}} - E_{\text{PM}} \). The kinetic energy comprises all the contributions to the ground state energy except the potential part, i.e., \( E_{\text{kin}} = E_{G} - U \sum_{i,j} \hat{n}_{i+} \hat{n}_{j+} \mathcal{P}_{G} \) in the non-BCS regime the density of states at the Fermi level (not shown explicitly) in PM state is significantly enhanced signaling heavy quasiparticle masses, and suggesting that this parameter range is appropriate for the HFS description. On this basis, we predict that the gain in the kinetic energy in the SC state for the Ce-based heavy fermion superconductors, and for specifically addressed here CeCoIn5, is the next feature shared with the high-temperature superconductors, in addition to the d-wave symmetry of the order parameter, the competition of SC with antiferromagnetism, and emergence of pseudogap [12].
CeCoIn₅ represents a thoroughly studied SC system among Ce-based superconductors. Unprecedentedly among HFS, for this compound the gap function was directly observed and characterized. To compare our results to those of CeCoIn₅ we have selected the representative point on the phase diagram (Fig. 1), determined by the values $n = 1.8$ and $|V| \approx 0.65$. The total filling $n$ has been chosen so that the Fermi level is placed in the energy below the middle point of the hybridization gap. In turn, hybridization amplitude $V$ has been adjusted to obtain the value of the condensation energy ($\Delta E \approx 2.3K$ - cf. Fig. 1 inset) corresponding to the SC critical temperature of CeCoIn₅.

We obtain a nodal pairing with the two distinct gaps at the Fermi surface, in accord with the findings for CeCoIn₅. In the Fig. 2 we have shown explicitly their angular dependence and shape (cf. inset) in the quarter of the first Brillouin zone. The larger gap, $\Delta_1$ follows pure $d_{x^2-y^2}$-symmetry dependence as suggested for CeCoIn₅ by various experiments cf. e.g. Refs.   

To track the leading contribution to the pairing, in Fig. 3, we display the effective SC pairing components (lines) along the selected direction of constant bond filling, $n = 1.8$. The dominant pairing amplitude, $\Delta_{1,0}^{ff}$ describes the $f-f$ pairing part and is of the $d_{x^2-y^2}$-wave symmetry. The remaining $f-f$ and $f-c$ pairing components constitute less than 15% of $\Delta_{1,0}^{ff}$. Such circumstance explains in a natural manner the common origin of magnetic and SC orderings in CeCoIn₅ as the former is associated mostly with $f$-electrons. Additionally, in Fig. 3, we present the convergence of our results with the order $k$ of the expansion on example of the condensation energy for $n = 1.8$. Absence of any considerable difference between the $k = 3$ and $k = 4$ plots proves that the convergence is already reached for $k = 3$.

In Fig. 4b we plot the condensation energy, $\Delta E$ for different $n$ values. The energy which should be of the order of the transition temperature is reasonable, especially in the limit of the total filling, $n < 2$. Nonetheless, $\Delta E$ in our model can be suppressed by introducing an onsite contribution to hybridization, here considered of a purely intersite form. In Fig. 4b we plot the gain in kinetic energy in SC state. The $non-BCS$ state appears for lower values of the hybridization amplitude $|V|$.

Summary. The Anderson lattice model has been solved diagrammatically with a full Gutzwiller wave function. This leads to the generic correlation-driven unconventional superconductivity. The SC state exhibits principal properties detected in a clear manner in CeCoIn₅, as well as can be expected to appear also in other Ce-based superconductors: (i) superconductivity is of multigap ($f-f$ and $f-c$ pairings) character; (ii) the leading gap component is due to $f-f$ pairing and of $d_{x^2-y^2}$-wave symmetry; and (iii) the condensation energy of a reasonable value, i.e. of the order of critical temperature. Additionally, we also show that in a direct analogy to high-temperature superconductors, heavy fermion systems can be characterized by the presence of non-BCS regime.

Acknowledgements. We are very grateful for stimulating discussions with J. Bünemann. The work has been supported by the National Science Centre (NCN) under the Grant MAESTRO, No. DEC-2012/04/A/ST3/00342. JK acknowledges support from the People Programme (Marie Curie Actions) of the European Union’s Seventh Framework Programme (FP7/2007-2013) under REA grant agreement n° [291734].
metal/heavy-fermion superconductor CeCoin$_5$ interface,” Phys. Rev. B 72, 052509 (2005)

[33] C. Stock, C. Broholm, J. Hudis, H. J. Kang, and C. Petrovic, “Spin resonance in the d-wave superconductor ceceo,” Phys. Rev. Lett. 100, 087001 (2008)

[34] K. Izawa, H. Yamaguchi, Yuji Matsuda, H. Shishido, R. Settai, and Y. Onuki, “Angular position of nodes in the superconducting gap of quasi-2d heavy-fermion superconductor ceceo,” Phys. Rev. Lett. 87, 057002 (2001)

[35] Morten Ring Eskildsen, Charles D. Dewhurst, Bart W. Hoogenboom, Cedoriz Petrovic, and Paul C. Canfield, “Hexagonal and square flux line lattices in ceceo,” Phys. Rev. Lett. 90, 187001 (2003)

[36] Franziska Weickert, Philipp Gegenwart, Hyekyung Won, David Parker, and Kazumi Maki, “In-plane angular dependence of the upper critical field in ceceo,” Phys. Rev. Lett. 90, 057005 (2003)

[37] J. Bunemann, T. Schickling, and F. Gebhard, “Variational study of fermi surface deformation in hubbard models,” Eur. Phys. Lett. 98, 27006 (2012).

[38] Jan Kaczmarczyk, Jozef Spalek, Tobias Schickling, and Jörg Bünnemann, “Superconductivity in the two-dimensional hubbard model: Gutzwiller wave function solution,” Phys. Rev. B 88, 115127 (2013).

[39] J. Kaczmarczyk, J. Bünnemann, and J. Spalek, “High-temperature superconductivity in the two-dimensional $t-J$ model: Gutzwiller wave function solution,” New J. Phys. 16, 073018 (2014).

[40] J. Kaczmarczyk, “Comparison of two approaches for the treatment of gutzwiller variational wave functions,” Phil. Mag. 95, 563–573 (2015).

[41] J. Kaczmarczyk, T. Schickling, and J. Bünnemann, “Evaluation techniques for gutzwiller wave functions in finite dimensions,” Phys. Status Solidi b 252, 2059 (2015).

[42] T. M. Rice and K. Ueda, “Gutzwiller variational approximation to the heavy-fermion ground state of the periodic anderson model,” Phys. Rev. Lett. 55, 995–998 (1985).

[43] Marcin M Wysokiński and Józef Spak, “Properties of an almost localized fermi liquid in an applied magnetic field revisited: a statistically consistent gutzwiller approach,” J. Phys.: Condens. Matter 26, 055601 (2014).

[44] Marcin M. Wysokiński, Marcin Abram, and Józef Spalek, “Ferromagnetism in uge:$t$: A microscopic model,” Phys. Rev. B 90, 081114(R) (2014).

[45] Marcin M. Wysokiński, Marcin Abram, and Józef Spalek, “Criticalities in the itinerant ferromagnet uge$_2$,” Phys. Rev. B 91, 081108(R) (2015).

[46] B. Edegger, V. N. Muthukumar, and C. Gros, “Gutzwiller-rvb theory of high-temperature superconductivity: Results from renormalized mean-field theory and variational monte carlo calculations,” Advances in Physics 56, 927–1033 (2007).

[47] Hiroshi Watanabe, Kazuhiro Seki, and Seiji Yunoki, “Charge-density wave induced by combined electron-electron and electron-phonon interactions in $1t - \text{tise}_2$: A variational monte carlo study,” Phys. Rev. B 91, 205135 (2015).

[48] Florian Gebhard, “Gutzwiller correlated wave functions in finite dimensions $d$: A systematic expansion in $1/d$,” Phys. Rev. B 41, 9452–9473 (1990).

[49] P. G. de Gennes, Superconductivity in Metals and Alloys (W. A. Benjamin, New York, 1966).