A domain wall description of brane inflation and observational aspects

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We consider a brane cosmology scenario by taking an inflating 3D domain wall immersed in a five-dimensional Minkowski space in the presence of a stack of $N$ parallel domain walls. They are static BPS solutions of the bosonic sector of a 5D supergravity theory. However, one can move towards each other due to an attractive force in between driven by bulk particle collisions and resonant tunneling effect. The accelerating domain wall is a 3-brane that is assumed to be our inflating early Universe. We analyze this inflationary phase governed by the inflaton potential induced on the brane. We compute the slow-roll parameters and show that the spectral index and the tensor-to-scalar ratio are within the recent observational data.

I. INTRODUCTION

Inflationary cosmological scenarios were proposed by Guth and Linde [1, 2]. This phase of the Universe has been supported by Cosmic Microwave Background (CMB), discovered by Penzias and Wilson in 1964 [3] and verified accurately by COBE (Cosmic Background Explorer), WMAP (Wilkinson Microwave Anisotropy Probe) and PLANCK. The observations of the CMB have shown to develop enormous importance in modern cosmology concerning constrain several models that have emerged as an attempt to explain the expansion of the Universe [4–7]. Thus, the inflationary scenarios have been severely constrained by the recent data from the Planck collaboration [8–10]. An interesting possibility is to associate the models that describe the acceleration of the Universe, such as during inflation and dark energy phases, to the scenario known as the Dvali-Tye brane inflation [11]. In this way, we consider our Universe as a 3-brane embedded in a 5D Minkowski spacetime [11, 12] that undergoes an accelerated expansion due to the presence of an induced scalar potential for a scalar field that corresponds to inter-brane distance. This is the direction of looking from the inflationary scenario in the realm of fundamental theories such as string theories and effective limits, namely supergravity — for a recent study on this issue see [13]. In the latter case, the bulk is asymptotically AdS$_5$ space-time. The effective inflaton potential in such theories is induced as the radion potential which comprises the inter brane potential as a function of the distance in between. The induced four-dimensional radion/inflaton appears naturally in AdS$_5$ bulk space due to modes that are integrated out [14–16]. In the limit of Minkowski bulk space, however, it is also possible to find an inter brane potential in five-dimensional bulk that can induce a four-dimensional inflaton potential on the brane. In this scenario, one can take into account several forces among the branes [11]. In the present study, we investigate the realization of the Dvali-Tye scenario in the context of domain wall solutions in a 5D gravity coupled to scalar fields in a way that can be viewed as the bosonic sector of a particular supergravity theory in five dimensions. We shall consider a particular force due to elastic collisions of bulk particles with the branes. Besides, one can also consider other forces of gravitational and electromagnetic nature. We shall focus on the electromagnetic case to address issues in our setup. Due to the resonant tunneling effect placed between the branes, favoring the transmission rate in contrast to the reflection rate, it is expected to exist an attractive force in between whose magnitude increases (decreases) as the inter distance increases (decreases) — Fig. 1. An analogous behavior in optical systems can be found in the optical spring phenomenon in a Fabry-Perot cavity [17].

This paper is organized as follows. In Sec. II, we introduce supergravity inspired model from which we will find 3D domain wall solutions that represent the 3-branes. Sec. III, we present a brane scenario in which we explore forces acting to the brane due to elastic collisions of bulk particles that can produce an acceleration in our Universe. We also discuss the presence of an electric field between parallel branes. After considering these forces, we deduce our potential induced on the brane. Sec. IV, we consider such a model to determine the main parameters that govern the inflationary phase and discuss the results by making comparisons with the recent Planck data. Finally in Sec. V, we summarize the main results.

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II. THE MODEL

We consider the scalar bosonic sector of a supergravity theory in 5D, which can be thought of as a theory that comes from another fundamental theory via compactification which Lagrangian is given by [12, 18–21]:

\[ e^{-1} L_{\text{sugra}} = - \frac{1}{4} M_3^2 R(5) + G_{AB} \partial_{\mu} \phi^A \partial^{\mu} \phi^B - \frac{1}{4} G^{AB} \frac{\partial W(\phi)}{\partial \phi^A} \frac{\partial W(\phi)}{\partial \phi^B} + \frac{1}{3} \frac{1}{M_3^2} W(\phi)^2, \]

where \( G_{AB} \) is the metric one the scalar field space and \( e = |\det g_{\mu\nu}|^{1/2} \). \( R(5) \) is the Ricci scalar and \( 1/M_3 \) represents the five-dimensional Planck length. Below we shall consider the limit, \( W/M_3 \ll 1 \), in which the 3D domain walls are solutions embedded in 5D Minkowski space. The four-dimensional gravity can be induced on the brane via DGP mechanism [22, 23], i.e., through quantum loops of matter fields localized on the brane. In the following analysis we will also restrict the scalar field manifold to two dimensions, that is, \( \phi_A = (\phi/\sqrt{2}, \chi/\sqrt{2}) \).

Let us now introduce a supersymmetric model of two interacting real scalar fields developing a \( Z_2 \times Z_2 \) symmetry [24, 25], described by the following Lagrangian

\[ L = \frac{1}{2} \partial_M \phi \partial^M \phi + \frac{1}{2} \partial_M \chi \partial^M \chi - V(\phi, \chi), \]

whose equations of motion are

\[ \Box \phi + \frac{\partial V}{\partial \phi} = 0, \]
\[ \Box \chi + \frac{\partial V}{\partial \chi} = 0. \]

(3)

Since the model is supersymmetric, the potential can be given in terms of a superpotential [25], i.e.,

\[ V(\phi, \chi) = \frac{1}{2} \left( \frac{\partial W}{\partial \phi} \right)^2 + \frac{1}{2} \left( \frac{\partial W}{\partial \chi} \right)^2. \]

(4)

The most general form of the superpotential that generates scalar potential with a \( Z_2 \times Z_2 \)-symmetry [26] is given by

\[ W = \lambda \left( \frac{\phi^3}{3} - a^2 \phi \right) + \mu \phi \chi^2. \]

(5)

Substituting Eq.(5) into Eq.(4), we can write the scalar potential as

\[ V(\phi, \chi) = \frac{1}{2} \lambda^2 (\phi^2 - a^2)^2 + (2\mu^2 + \lambda \mu) \phi^2 \chi^2 - \lambda a^2 \mu \chi^2 + \frac{1}{2} \mu^2 \chi^4. \]

(6)

For scalar fields depending only on the fifth dimension \( x_5 \) that represents the coordinate transverse to the brane and using the fact that the equations of motion of such systems can be reduced to first-order equations then we can write

\[ \frac{d\phi}{dx_5} = W_\phi, \]
\[ \frac{d\chi}{dx_5} = W_\chi \]

(7)

where the subscripts \( \phi, \chi \) stand for derivatives with respect to these fields. The Bogomol’nyi-Prasad-Sommerfield (BPS) solutions of the first-order differential equations are of type I

\[ \phi = -a \tanh(\lambda ax_5) \]
\[ \chi = 0 \]

(8)

and type II

\[ \phi = -a \tanh(2\mu x_5) \]
\[ \chi = \pm a \sqrt{2 \mu - \frac{1}{\mu}} \sech(2\mu x_5) \]

(9)
where $\pm a$ are the minima of the potential. The type I solution is not interesting for our proposal since it produces reflectionless domain walls — see below.

Now performing small perturbations around a particular solution, say, $\bar{\phi}$ and $\bar{\chi}$, that is

$$
\chi = \bar{\chi} + \zeta
$$

$$
\phi = \bar{\phi},
$$

in the equations of motion (3), then we obtain a linear equation for the fluctuations

$$
\partial_\mu \partial^\mu \zeta + \bar{V}_{\chi\chi} \zeta = 0.
$$

For Type II solution above [12, 26] we have

$$
\bar{V}_{\chi\chi}(x_5) = m_\chi^2 - m_\chi^2 \left(4 - \frac{\lambda}{\mu}\right) \text{sech}^2(2\mu a x_5)
$$

$$
\text{(12)}
$$

with $m_\chi^2 = 4\mu^2 a^2$ being the mass squared of the elementary excitations of the scalar field $\chi$. The Ansatz for the perturbation around a three-dimensional domain wall can be chosen as

$$
\zeta = \zeta(x_5) e^{-i(\omega t - k_x x - k_y y - k_z z)}.
$$

$$
\text{(13)}
$$

Substituting this into equation (11) we find the Schroedinger-like equation

$$
-\frac{d^2 \zeta}{dx_5^2} + U_{II}(x_5) \zeta = k_5^2 \zeta
$$

$$
\text{(14)}
$$

with $-k_5^2 = -\omega^2 + k_x^2 + k_y^2 + k_z^2 + m_\chi^2$. Here $k_5$ is the fifth-component bulk particles momentum and

$$
U_{II}(x_5) = -m_\chi^2 \left(4 - \frac{\lambda}{\mu}\right) \text{sech}^2(2\mu a x_5)
$$

$$
\text{(15)}
$$

is the Schroedinger-like potential. The reflection coefficient [27] for the barrier potential described in Eq. (15) is

$$
R = \frac{\cos^2\left(\frac{\pi}{2} \sqrt{17 - 4\lambda \mu}\right)}{\sinh^2\left(\frac{\pi k_5}{2\mu a}\right) + \cos^2\left(\frac{\pi}{2} \sqrt{17 - 4\lambda \mu}\right)}
$$

$$
\text{(16)}
$$

Notice that for $\lambda = 2\mu$, that reduces the solution type II (9) to type I (8), the reflection coefficient becomes zero.

III. THE INDUCED INFLATON POTENTIAL

So by considering the transverse force along the inter distance $r$ in the fifth coordinate due to elastic collisions we have [27]

$$
F_r = M_{\text{wall}} \ddot{r}(t) \approx K R = -\frac{\partial U}{\partial r},
$$

$$
\text{(17)}
$$

where $K$ depends on the density of colliding bulk particles and their incoming momenta $k_5$, that we shall assume to be time independent. This is expected because of the conservation of momentum of both colliding particles and the brane. We can compare this to a similar computation such as the recoil of nuclei in alpha decay due to a specific barrier potential. The essential difference concerns to the fact that, while in the latter case the recoil is usually disregarded since one assumes heavy nuclei, in the former case we shall assume that the conservation of momentum involves motion of both particles and branes. We can obtain the potential $U(r)$ from equation (17) as long as we are able to find the reflection coefficient as a function of the inter distance $r$, i.e., $R(r)$. This is indeed the case if we take in consideration a second parallel brane put near the first brane — see Fig. 1 (top). As such the reflection coefficient changes because of the resonant tunneling effect [28, 29], Fig. 1 (bottom), where the transmission coefficient is given by [29]

$$
T = \frac{4}{(4\theta^2 + \frac{1}{4\theta^2}) \cos^2 L + 4 \sin^2 L},
$$

$$
\text{(18)}
$$
FIG. 1: Reflecting particles in multiple barrier potentials. (Top panel) The left barrier is sufficiently distant from the right barrier and does not experiment the resonant tunneling effect \cite{28, 29} which would diminish the reflection rate. (Bottom panel) The left barrier is sufficiently near, at a distance \( r \), to an array of barrier on the right. Now the resonant tunneling effect takes place in between favoring the transmission rate in contrast to the reflection rate. Then it is expected to exist an attractive force between two branes whose magnitude increases (decreases) as the inter distance \( r \) increases (decreases).

where \cite{33}

\[
\theta = \exp \left( \int_a^b \kappa(x_5) dx_5 \right), \quad \kappa(x_5) = |\mathcal{V}(x_5) - E|^{1/2}, \quad E < \mathcal{V}(x_5),
\]

(19)
gives the height and thickness of the barrier Fig. 1 in terms of the energy and

\[
L = \int_{-r}^{r} k(x_5), \quad k(x_5) = |E - \mathcal{V}(x_5)|^{1/2}, \quad E > \mathcal{V}(x_5).
\]

(20)

In the limit of pronounced resonances \((\theta \gg 1)\) one may assume around the resonances the approximation \( \cos L \approx \mp (\partial L/\partial E)_{E=E_0}(E - E_0) \) and \( \sin L \approx 1 \). Then we can still write the transmission coefficient as

\[
T = \frac{(\Gamma/2)^2}{(E - E_0)^2 + (\Gamma/2)^2}
\]

(21)
where by definition $\Gamma = (\theta^2(\partial L/\partial E)_{E=E_0})^{-1}$. Now by assuming the Schroedinger-like potential $\mathcal{V}(x_5)$ in between the barriers is sufficiently small (which is true for branes sufficiently far from each other) we can find the reflection coefficient $R = 1 - T$, given in the form

$$R = \frac{(E - E_0)^2}{(E - E_0)^2 + (\Gamma/2)^2} \rightarrow R(r) = \frac{1}{1 + \frac{r^2}{\Delta^2}}$$

(22)

Since $E - E_0 \ll 1$, in the last step above we have recast the formula in terms of the fixed distance scale $r_0 = k_0/[2\theta^2(E - E_0)]$ and also used $(\partial L/\partial E)_{E=E_0} = r/k_0$. Now substituting (22) into equation (17), we can integrate $R(r)$ to find the potential that acts in between the parallel branes

$$U(r) = Kr_0 \arctan \left( \frac{r}{r_0} \right) - Kr$$

(23)

However, due to the existence of a linear potential contribution as a consequence of constant electric and gravitational fields between the branes [11], the total potential governing the motion of such branes is indeed

$$U_{\text{eff}}(r) = U(r) + \mathcal{E}_0 r$$

(24)

Thus for $\mathcal{E}_0 = K$, we can write the effective potential

$$U_{\text{eff}}(r) = Kr_0 \arctan \left( \frac{r}{r_0} \right)$$

(25)

In the following we shall show how the electric field enters in the present scenario. This can be well justified by introducing gauge fields contribution into the Lagrangian (2) as follows [30, 31]:

$$\mathcal{L} = -\frac{1}{4} F_{MN} F^{MN} + J_M A^M + \frac{1}{2} \partial M \phi \partial^M \phi + \frac{1}{2} \partial M \chi^* \partial^M \chi - V(\phi, |\chi|).$$

(26)

Here we have promoted the scalar field $\chi$ to be a complex field $\chi(x_5,x_\mu) = \chi(x_5) \exp(i\theta_\mu x^\mu)$ in order to describe charged domain walls with the current $J_M = i q (\chi \partial_\mu \chi^* - \chi^* \partial_\mu \chi) = (J_\mu, 0)$, where $J_\mu = -q \theta_\mu \chi(x_5)^2$, $q$ is the electric charge and $\mu = 0, 1, 2, 3$ labels the brane world-volume coordinates. For static gauge fields with translational symmetry along the brane embedded in 5D Minkowski space, the Gauss law simply reduces to

$$\frac{d^2 U}{dx_5^2} = \rho(x_5),$$

(27)

where $A^0 = U, \vec{A} = 0$, and $\rho = J_0$. Now by using equation (9) for the solution $\chi(x_5)$ we find the charge density on two parallel domain walls located at $\pm r/2$ as

$$\rho(x_5) = \frac{\sigma}{2\Delta} \text{sech}^2 \left( \frac{x_5 \pm r/2}{\Delta} \right),$$

(28)

where $\Delta \sim 1/(2\mu a)$ is the domain wall thickness and $\sigma = q/\Delta^2$. We also have eliminated the parameter $\theta_0$ in terms of the parameters $\lambda, \mu, a$ that appear in the amplitude of the solution for $\chi$ (9). It is not difficult to show that the electric field between two parallel domain walls with opposite charge is

$$\frac{dU}{dx_5} = \int dx_5 \left( \rho(x_5 + r/2) - \rho(x_5 - r/2) \right) = \frac{\sigma}{2} \left( \tanh \frac{x_5 + r/2}{\Delta} - \tanh \frac{x_5 - r/2}{\Delta} \right),$$

(29)

where it clearly approaches to a constant electric field for large distance $r$ and becomes zero as they overlap, i.e., at $r = 0$, as expected. The potential is obtained by integrating the electric field in the interval $(-r, r)$ to find

$$U = \frac{\sigma}{2} \Delta \ln \left( \cosh \frac{x_5 + r/2}{\Delta} \text{sech} \frac{x_5 - r/2}{\Delta} \right) \bigg|_{-r/2}^{r/2} \to U = \mathcal{E}_0 r,$$

(30)

where in the last step we have taken the large distance limite $r \gg \Delta$ and defined $\mathcal{E}_0 = \sigma$. This is precisely the linear term added to the ‘effective potential’ given in (24).
Now considering the total energy of the brane per unit volume, we find
\[ \frac{E}{V^{(3)}} = \frac{1}{2} M_{wall} \dot{r}^2 + \frac{U_{eff}}{V^{(3)}} \]  
where \( \rho_{wall} = \frac{E}{V^{(3)}} \) is the energy density of the 3-brane, \( T_{wall} = \frac{M_{wall}}{V^{(3)}} \) is the tension and \( V(r) = \frac{U_{eff}}{V^{(3)}} \) is the potential density in four dimensions. Then
\[ \rho_{wall} = \frac{1}{2} T_{wall} \dot{r}^2 + V(r) \]  
and admitting that \( \sqrt{T_{wall}} \dot{r}(t) \leftrightarrow \phi(t) \) we find the total energy density
\[ \rho_{wall} = \frac{1}{2} \dot{\phi}^2 + V(\phi), \]  
where \( \phi = \phi(t) \) is denominated inflaton whose associated potential is
\[ V(\phi) = K \beta \arctan \left( \frac{\phi}{\beta} \right), \]  
with \( \beta = \sqrt{T_{wall}} r_0 \) describes the scale of energy. This induced potential will drive the inflationary scenario discussed in the next section.

IV. INFLATIONARY COSMOLOGY ON THE BRANE

The central idea of scalar field inflation models is to consider that the energy of the early Universe has been dominated by the potential energy of scalar fields. The parameters that characterize the slow-roll
\[ \epsilon = \frac{M_{Pl}^2}{16\pi} \left( \frac{V'(\phi)}{V(\phi)} \right)^2 \]  
and
\[ \eta = \frac{M_{Pl}^2}{8\pi} \left( \frac{V''(\phi)}{V(\phi)} \right) \]  
are valid as long as both are small \( (\epsilon \ll 1, \eta \ll 1) \) \[32\]. The spectral index \( n_s \) and the tensor-to-scalar ratio \( r \) are given in terms of these parameters as follows
\[ n_s = 1 + 2\eta - 6\epsilon \]  
and
\[ r = 16\epsilon \]  
The tensor-to-scalar ratio \( r \) measures how much the tensor perturbations change with the scale and according to more recent data has the upper bound \( r < 0.07 \) \[9, 10\] and \( r < 0.02 \) \[8\].

The main results can be summarized as follows. To work with Eqs.(37) and (38) we should analyze the behavior of the scalar field \( \phi \) and cosmological parameters in the slow-roll regime, that is, \( \phi \gg \beta \). This precisely happens in the flat region of the potential (34). The inflationary phase is maintained as long as \( \epsilon(\phi) \ll 1 \) and ends as \( \epsilon(\phi) \sim 1 \). The number of \( e \)-folds for the slow-roll approximation can be obtained as a function of the scalar field and is found by using
\[ N = \frac{8\pi}{M_{Pl}^2} \int_{\phi_{end}}^{\phi} \frac{V(\phi)}{V(\phi)} d\phi. \]  
The Tab. IV A shows the values of the cosmological parameters obtained from the analysis of the model for \( \beta \) given in \( M_{Pl} \). The parameter \( K \) of the potential (34) does not enters in the analysis since it is canceled out. The first column shows constraints on the \( \Lambda \)CDM reference model (TT + lowE 68% confidence) verified from the observations of the Planck and Collaborations 2018 \[8\], the second and third columns respectively show the values of \( n_s \) and \( r \) for \( N = 50 \) and \( N = 60 \) obtained in our model.
TABLE IV A: 68% confidence limits for the cosmological parameters using the TT+lowE Planck (2018).

| Parameter       | $\Lambda$ CDM     | brane-infla. $N = 50$ | brane-infla. $N = 60$ |
|-----------------|-------------------|-----------------------|-----------------------|
| $n_s$           | 0.9626 ± 0.0057   | 0.9600 ± 0.0027       | 0.9774 ± 0.0015       |
| $r_{0.002}$     | < 0.102           | 0.0141 ± 0.0012       | 0.0068 ± 0.0013       |
| $\beta$         | -                 | 0.032 ± 0.001         | 0.032 ± 0.001         |

V. CONCLUSIONS

In this work, we use field theory to construct a cosmological model obtained through the brane inflation scenario, constructed by assuming the possibility of the Universe is living on a thick brane that moves toward a stack of branes due to collision of particles that live in a five-dimensional bulk. From this model, we analyze some aspects of the theory of inflation governed by a scalar field that assumes the role of inflaton. We find some cosmological parameters to compare with the last data of Planck and Collaborations [8]. All the values of parameters we find are in agreement with the currently expected data for the inflationary era.

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[33] Here we have adapted dimensions in the Schroedinger-like equation (14) by dividing each term by a scale of mass $2m$. This implies on $V(x_5) = U_{11}(x_5)/2m$ and $E = k_5^2/2m$. This also happens with the resonance energy $E_0 = k_5^2/2m$ — see below. For the sake of convenience, however, we also have made use of the convention $\hbar = 1$ and $2m = 1$. 