Gravity–capillary wave interactions generated by moving disturbances: Euler equations framework

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Abstract The aim of this work is to investigate the interaction of gravity–capillary waves resonantly excited by two moving disturbances along the free surface. The problem is formulated using the full Euler equations, and numerical computations are performed in a simplified domain through the use of a conformal mapping which flattens the free surface. We focus on nearly-critical flows with intermediate capillary effects and characterise their main features. In the supercritical regime, the wave interaction can be strongly nonlinear leading to the onset of wave breaking. In the subcritical regime, depression solitary waves are generated remaining trapped between the disturbances bouncing back and forth. In addition, we notice a dependence of the number of trapped waves on the distance of the disturbances. Furthermore, differently from when only one disturbance is considered, we find that the critical regime captures features from the subcritical and supercritical regime simultaneously.

Keywords Euler equations · Gravity–capillary waves · Solitary Waves · Trapped waves

1 Introduction

Waves excited by a moving disturbance have been studied using different mathematical models. The main framework used is the full Euler equations [1–3]. However, due to intrinsic difficulties present in the Euler equations such as nonlinearity and free boundary conditions, reduced models based on asymptotic theory have been applied as an alternative to describe this phenomenon. The most notorious are the forced Korteweg–de Vries equation (fKdV) [4–9] and the fifth-order fKdV equation [10–12].

The flow is governed by three parameters: the magnitude of the applied pressure, the Froude number ($F$) and the Bond number ($B$) defined as

$$F = \frac{U_0}{\sqrt{gh_0}}, \quad B = \frac{\sigma}{\rho gh_0^2}.$$
Here, $U_0$ is the speed of the applied pressure forcing, $g$ is the acceleration of gravity, $h_0$ is the undisturbed depth of a channel, $\sigma$ is the coefficient of the surface tension and $\rho$ is the constant density of the fluid. The Froude number and the Bond number are called critical when $F = 1$ and $B = 1/3$. In the weakly nonlinear regime, the fKdV equation is applied to study nearly-critical flows ($F \approx 1$) with external forcings of small amplitudes [4,6–9], however, it fails when $B = 1/3$ since the higher order dispersive term vanishes and the fKdV approaches asymptotically to a Burgers’ equation [13]. When the flow is nearly critical and the capillary effects are intermediate ($B \approx 1/3$) higher order fKdV models arise. Under these conditions, Milewski and Vanden-Broeck [11] derived a fifth-order fKdV for an obstacle of small amplitude and showed that this equation has unsteady solitary wave solutions with small oscillating tails. More recently, Hanazaki et al. [3] used the body-fitted curvilinear coordinates to solve Euler’s equations numerically in the presence of an obstacle with a uniform flow and compared the results with the fKdV and the fifth-order fKdV for the critical flow and intermediate capillary effects ($B \approx 1/3$). They showed that the fifth-order fKdV qualitatively captures the main features of the flow, however, it overestimates the wavelengths.

Considering two well-separated localised obstacles in the absence of surface tension, Grimshaw and Malewoong [14] used the fKdV model to investigate the interaction of the waves generated in the nearly-critical regime. According to their results the flow is classified into three steps. In the first step, which occurs at early times, the flow is featured by the formation of an undular bore above each obstacle independently. In the second step, the waves generated interact between the obstacles, and in the third one happens the controlling of the dynamic by the larger obstacle. Later, these authors revisited the interaction of waves generated over two obstacles (bumps and holes) in the nearly-critical regime and detailed the wave interactions [15]. Flamarion and Ribeiro-Jr [10] investigated gravity–capillary flows over obstacles and showed that different from gravity waves the flow is not necessarily governed by the obstacle with larger amplitude. Moreover, they found evidences that in the subcritical regime ($F < 1$) waves may be trapped between the obstacles.

In the Euler equations framework, Wang [16] studied numerically gravity flows past two separated obstacles and compared his results with the linear and weakly nonlinear theory. The results presented in his article complement the numerical findings reported by Grimshaw and Malewoong [14,15]. Considering capillary–gravity waves in infinity depth, Wang [17] showed numerically that stable elevation waves can be excited by moving two fully localised well-separated pressures on the free surface with the speed close to the phase speed minimum and switching off the pressures simultaneously after a certain period of time.

In this paper, we use the full Euler equations to investigate numerically the interaction of gravity–capillary waves excited by two disturbances moving along the free surface with nearly-critical speed ($F \approx 1$) in a finite depth channel under intermediate capillary effects ($B = 1/3$). The problem is formulated in the disturbances moving frame and computations are performed in a simplified domain through the use of the conformal mapping technique. We find that in the supercritical regime ($F > 1$), the wave interaction is strongly nonlinear and numerical evidences show that the wave may break. In the subcritical regime ($F < 1$), we observe the generation of depression solitary waves which bounce back and forth between the disturbances remaining trapped for large times. We notice that the larger is the distance of the disturbances the larger is the number of trapped waves. Besides, in the critical regime ($F = 1$), we observe a formation of an undular bore where the pressure is applied and the arising of depression solitary waves for large times which does not occur when only one disturbance is considered. It is worthy to mention that all these features have not been reported in previous works.

This article is organised as follows. In Sect. 2, we present the mathematical formulation of the nondimensional Euler equations. The conformal mapping formulation of the problem and numerical methods are presented in Sect. 3, results in Sect. 4 and conclusion in Sect. 5.

2 Mathematical formulation

We consider an inviscid fluid with constant density ($\rho$) in a two-dimensional channel with finite depth ($h_0$) under the force of gravity ($g$), with surface tension ($\sigma$), and in the presence of a left-going pressure distribution ($P$) which travels with constant speed ($U_0$) along the free surface. The flow is assumed to be incompressible and irrotational.
Moreover, we denote the velocity potential in the bulk fluid by $\tilde{\phi}(x, y, t)$ and the free surface by $\tilde{\zeta}(x, t)$. Using the typical wavelength $h_0$ as the horizontal and vertical length, $(g h_0)^{1/2}$ as the velocity potential scale, $(h_0/g)^{1/2}$ as the time scale and $\rho gh_0$ as the pressure scale, we obtain the dimensionless Euler equations

$$\Delta \tilde{\phi} = 0 \text{ for } -1 < y < \tilde{\zeta}(x, t),$$
$$\tilde{\phi}_y = 0 \text{ at } y = -1,$$
$$\tilde{\zeta}_t + \tilde{\phi}_x \tilde{\zeta}_x - \tilde{\phi}_y = 0 \text{ at } y = \tilde{\zeta}(x, t),$$

$$\tilde{\phi}_t + \frac{1}{2} (\tilde{\phi}_{xx}^2 + \tilde{\phi}_{yy}^2) + \tilde{\zeta} - B \frac{\tilde{\zeta}_{xx}}{(1 + \tilde{\zeta}_x^2)^{3/2}} = -P(x + Ft) \text{ at } y = \tilde{\zeta}(x, t),$$

where $F = U_0/(gh_0)^{1/2}$ is the Froude number and $B = \sigma/\rho gh_0^2$ is the Bond number. Now, we rewrite Eqs. (1) in the moving frame $x \to x + Ft$. To this end, we define

$$\tilde{\zeta}(x - Ft, t) = \tilde{\zeta}(x, t), \quad \tilde{\phi}(x - Ft, y, t) = \tilde{\phi}(x, y, t).$$

Substituting (2) in (1) yields the system

$$\Delta \tilde{\phi} = 0 \text{ for } -1 < y < \tilde{\zeta}(x, t),$$
$$\tilde{\phi}_y = 0 \text{ at } y = -1,$$
$$\tilde{\zeta}_t + F \tilde{\phi}_x + \tilde{\phi}_x \tilde{\zeta}_x - \tilde{\phi}_y = 0 \text{ at } y = \tilde{\zeta}(x, t),$$

$$\tilde{\phi}_t + F \tilde{\phi}_x + \frac{1}{2} (\tilde{\phi}_{xx}^2 + \tilde{\phi}_{yy}^2) + \tilde{\zeta} - B \frac{\tilde{\zeta}_{xx}}{(1 + \tilde{\zeta}_x^2)^{3/2}} = -P(x) \text{ at } y = \tilde{\zeta}(x, t).$$

In the next section we write Eqs. (3) in a simplified domain using a conformal mapping and present the numerical method to solve them (Fig. 1).

### 3 Conformal mapping and numerical methods

We solve the system (3) using the method introduced by Dyachenko et al. [18]. Here, we summarise only the main steps. First, we construct a time-dependent conformal mapping

$$z(\xi, \eta, t) = x(\xi, \eta, t) + iy(\xi, \eta, t),$$

which flattens the free surface and maps a strip of width $D(t)$ onto the fluid domain and satisfies the boundary conditions

$$y(\xi, 0, t) = \tilde{\zeta}(x(\xi, 0, t), t) \quad \text{and} \quad y(\xi, -D(t), t) = -1.$$

Let $\phi(\xi, \eta, t) = \tilde{\phi}(x(\xi, \eta, t), y(\xi, \eta, t), t)$ and $\psi(\xi, \eta, t) = \tilde{\psi}(x(\xi, \eta, t), y(\xi, \eta, t), t)$ be its harmonic conjugate, and denote by $\Phi(\xi, t)$ and $\Psi(\xi, t)$ their traces along $\eta = 0$, respectively. Considering $X(\xi, t)$ and $Y(\xi, t)$ as the horizontal and vertical coordinates at $\eta = 0$, Kinematic and Bernoulli conditions (3)\textsubscript{3,4} are read as

$$Y_t = Y_\xi C \left[ \frac{\Theta_\xi}{J} \right] - X_\xi \frac{\Theta_\xi}{J},$$
$$\Phi_t = -Y - \frac{1}{2J} (\Phi_\xi^2 - \Psi_\xi^2) + \Phi_\xi C \left[ \frac{\Theta_\xi}{J} \right] - \frac{1}{J} F X_\xi \Phi_\xi + B \frac{X_\xi Y_\xi - Y_\xi X_\xi}{J^{3/2}} - P(X),$$

$$\Psi_t = Y_\xi C \left[ \frac{\Theta_\xi}{J} \right] - X_\xi \frac{\Theta_\xi}{J},$$
$$\phi_t = -Y - \frac{1}{2J} (\phi_\xi^2 - \psi_\xi^2) + \phi_\xi C \left[ \frac{\Theta_\xi}{J} \right] - \frac{1}{J} F X_\xi \phi_\xi + B \frac{X_\xi Y_\xi - Y_\xi X_\xi}{J^{3/2}} - P(X),$$

$$\psi_t = Y_\xi C \left[ \frac{\Theta_\xi}{J} \right] - X_\xi \frac{\Theta_\xi}{J},$$
where $\Theta_\xi(\xi, t) = \Psi_\xi + FY_\xi$, $J = X_\xi^2 + Y_\xi^2$ is the Jacobian of the conformal mapping evaluated at $\eta = 0$, $X_\xi = 1 - C[Y_\xi]$, $\Phi_\xi = -C[\Psi_\xi]$, and $C$ is the operator
$$C = F_{k\neq0}^{-1}\coth(k_j D)F_{k\neq0},$$
where $F$ denotes the Fourier modes
$$F_{k_j}[g(\xi)] = \hat{g}(k_j) = \frac{1}{2L} \int_{-L}^{L} g(\xi)e^{-ik_j \xi} d\xi,$$
$$F_{k_j}^{-1}[\hat{g}(k_j)](\xi) = g(\xi) = \sum_{j=-\infty}^{\infty} \hat{g}(k_j)e^{ik_j \xi},$$
where $k_j = (\pi/L) j$, $j \in \mathbb{Z}$. Computationally, it is interesting to impose the canonical domain and physical domain to have same length. To this end, we define
$$D = 1 + \frac{1}{2L} \int_{-L}^{L} Y(\xi, t) d\xi.$$ 
More details of this numerical method can be found in [1].

The dynamic of the waves generated is found integrating in time the family of ordinary differential equations (4) with the fourth-order Runge–Kutta method and the derivatives in $\xi$ are performed using the Fast Fourier Transform (FFT) [19]. In our simulations, we take the computational domain $[-L, L]$, with a uniform grid with $N$ points where $L$ is taken large enough to avoid the effects of spatial periodicity. For the interested reader, a study of the resolution of the numerical method is presented in Appendix A.

4 Numerical results

The disturbances over the free surface are modelled by two well-separated gaussians
$$P(x) = \epsilon_1 \exp\left(-\frac{(x - x_a)^2}{w}\right) + \epsilon_2 \exp\left(-\frac{(x - x_b)^2}{w}\right), \quad (5)$$
where $\epsilon_1$ and $\epsilon_2$ are the amplitudes of each disturbance, $w$ is their width, and $x_a$ and $x_b$ are their locations. In order to see the contribution of each disturbance to the dynamic independently, we take the disturbances to be far apart, thus wave interactions only occur for large times. For this purpose, we fix $x_a = -150$, $x_b = 150$ and $w = 100$. In this paper, we do not attempt an exhaustive study of the wave interactions. Instead, we present interesting regimes.
uncovered in the literature. In this section, we present our results in the nearly-critical regimes with critical Bond number \(B = 1/3\). Simulations were carried out using different meshgrids and the results are independent of resolution.

In the shallow water limit and for a single moving loading with nearly-critical speed, the problem has a steady solution with an upstream horizontal free surface of higher elevation which is smoothly connected over the obstacle to a downstream horizontal free surface of lower elevation \([6,11]\). In the following simulations, when the two disturbances are considered, we will see the formation of four distinct level at early times, each of these in a neighbourhood of where the pressure is exerted. As the wave interaction occurs, these levels might be reduced to only two. Thus, we can say that the mean free-surface level is controlled by the one-disturbance hydraulic problem associated with the stronger disturbance.

Supercritical regime

When only a single localised disturbance is considered, the supercritical flow has as its main feature the formation of an undular bore headed by a wave train with a train of waves propagating downstream \([11]\).

We start out our discussion considering the case in which the left disturbance has larger amplitude. For this purpose, we fix \(\epsilon_1 = 0.01\) and \(\epsilon_2 = 0.005\). The dynamic resembles qualitatively the one reported by Flamarion and Ribeiro-Jr \([10,\text{Fig. 4}]\). We observe that waves are generated at each disturbances with depression solitary waves
Fig. 4 The $L^\infty$-norm of the gradient of the free surface as a function of time with $F = 1.11$, $\epsilon_1 = 0.005$ and $\epsilon_2 = 0.01$.

Fig. 5 Supercritical near-critical regime free-surface evolution with $F = 1.11$, $\epsilon_1 = 0.01$ and $\epsilon_2 = 0.01$.

Fig. 6 Subcritical near-critical regime free-surface evolution with $F = 0.92$, $\epsilon_1 = 0.005$ and $\epsilon_2 = 0.01$.

Being emitted downstream from each one. While passing by the disturbance region, these waves radiate short waves with small amplitudes upstream. The radiation disturbs the elevation wave at the right disturbance and it is no longer a steady wave. This dynamic is shown in detail in Fig. 2.

Swapping the position of the disturbances, at early times the dynamic is qualitatively similar to the one exhibited in Fig. 2. However, for larger times the interaction in the neighbourhood where the weaker pressure is applied is somehow more nonlinear and we find numerical evidences that the wave may break during this interaction. This onset of wave breaking is the same as reported in the works of Flamarion et al. [1] and Grimshaw and Maleewong [2]. The amplitude of the upstream wave grows towards a maximum value with a sharp gradient (see upper wave profile in Fig. 3(left) and in Fig. 4 the $L^\infty$-norm of the gradient of the free surface as a function of time). In order to verify that this possible wave breaking is due to the presence of the second disturbance, we turn off the weaker pressure and analyse the behaviour of the solution. Figure 3 (right) describes this scenario. As it can be seen, when only one pressure is applied on the free surface the wave generated does not break at any time. Although the forced fifth-order Korteweg–de Vries captures qualitatively the pattern of the generated waves [10] it fails to predict wave breaking, this phenomenon can only be captured using the full nonlinear model.

In the same spirit, we maintain $\epsilon_1 = 0.01$ and increase the amplitude of the right pressure, namely, $\epsilon_2 = 0.01$. As a consequence of that, wave breaking seems to occur at earlier times in a neighbourhood of where the left pressure is applied. This scenario is displayed in Fig. 5. 
Subcritical regime

Considering a single localised disturbance in the fifth-order fKdV model, Milewski and Vanden-Broeck [11] showed that the main feature of subcritical flows is the periodic generation of depression solitary waves which propagate downstream. Later, Flamarion and Ribeiro-Jr [10] found numerical evidences that when two localised obstacles are considered depression solitary waves may bounce back and forth in the region between them remaining trapped for large times. Here, we confirm their prediction for the full nonlinear model.

For disturbances with different amplitudes, for instance $\epsilon_1 = 0.005$ and $\epsilon_2 = 0.01$, two trapped depression solitary waves can be seen in Fig. 6 as well as their interactions. Although the left obstacle is very small, it still changes drastically the flow by producing a nonlinear phenomenon, namely, trapping of waves.

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**Fig. 7** Trapped wave solutions between the disturbances with $F = 0.92$, $\epsilon_1 = 0.005$ and $\epsilon_2 = 0.01$. From top to bottom and left to right $d = 100, 200, 300, 400$
Next, we investigate the trapping mechanism by changing the distance of the disturbances. When they are closer the number of trapped waves between them decreases and when they are farther apart it increases. Figure 7 shows the dependence of the generation of trapped waves on the distance of the disturbances \(d = |x_a - x_b|\). When \(d = 0\) it is well known that there are not trapped waves. However, when we increase the distance between the disturbances trapped waves arise.

Critical regime

In the one-disturbance problem, the critical flow is mainly described by the formation of an undular bore where the pressure is applied and a wave train propagating downstream [11].

When two disturbances are considered our numerical simulations show that in certain regimes depression solitary waves arise propagating downstream for large times. Figure 8 displays the free surface evolution in the critical regime for \(\epsilon_1 = 0.01\) and \(\epsilon_2 = 0.005\). At first we observe a dynamic similar to the one-disturbance problem, i.e. the formation of undular bores in the region where the pressures are applied with wave trains propagating downstream from each region. Once the waves start to interact, we notice that the flow is mainly controlled by the stronger disturbance, in the sense that the role of the weaker pressure cannot be noticed in the motion. The novelty here is that well-defined depression solitary waves emerge from the propagating downstream wave train for larger times. Under these circumstances, we infer that the critical regime captures features from the subcritical and supercritical regime simultaneously, i.e. the flow has depression solitary waves propagating downstream and an undular bore at the disturbance headed by an upstream wave train.

5 Conclusion

In this article, we have investigated capillary–gravity flows generated by the passage of two localised disturbances. Through the use of the conformal mapping technique, we find new features that were uncovered and not captured by fifth-order fKdV. Our numerical simulations show that in the supercritical regime the wave interaction is strongly nonlinear which leads to an onset of wave breaking—even when one of the disturbances has very small amplitude. In the subcritical regime, depression solitary waves remain trapped bouncing back and forth between the disturbances. In addition, we notice that more waves are trapped as we increase the distance between the forcings. In the critical regime, the presence of a second disturbance may generate depression solitary waves which propagate downstream. Besides, in this scenario we find that the critical regime carries features of the supercritical and subcritical flows simultaneously.

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Appendix A: Numerical resolution

In this appendix, we present results from a resolution study with the Euler equations similar to the one reported by Flamarion et al. [1]. Our goal is to verify whether the wave profiles are accurately captured by different meshgrids. Although we had carried out simulations of flows with two disturbances, since the width of the disturbance controls the wavelength of the waves generated, for the resolution study it is enough to carry out simulations with a single disturbance. To this end, we fix the amplitudes of the disturbances as $\epsilon_1 = 0$, $\epsilon_2 = 0.01$ and compute the wave solution at $t = 1000$. This choice of parameters assures that the wave solutions are well developed. In order to compute the relative error (in $\ell^2$), we fix the length of the computational domain $2L = 1000$ and use the wave elevation $Y$ computed on our finest grid with $\Delta\xi = 2L/N = 0.03$ in the canonical domain as references.

Results for the supercritical, subcritical and critical regime are displayed in Tables 1, 2 and 3, respectively. The relative error (in $\ell^2$) decays as the mesh size is reduced. This lead us to conclude that the numerical solutions presented in the article do not depend on the resolution of the spatial grid.

| Table 1 | Supercritical regime with $F = 1.11$. The reference solution $Y$ is for the finest grid with $N = 32768$ and $\Delta\xi = 0.03$ |
|---------|-------------------------------------------------|
| $N$     | $\Delta\xi$ | $||Y - Y_{\Delta\xi}||_2/||Y||_2$ |
| 2048    | 0.48         | $1.00 \times 10^{-4}$ |
| 4096    | 0.24         | $6.14 \times 10^{-8}$ |
| 8192    | 0.12         | $3.21 \times 10^{-13}$ |
| 16384   | 0.06         | $9.22 \times 10^{-14}$ |

| Table 2 | Subcritical regime with $F = 0.92$. The reference solution $Y$ is for the finest grid with $N = 32768$ and $\Delta\xi = 0.03$ |
|---------|-------------------------------------------------|
| $N$     | $\Delta\xi$ | $||Y - Y_{\Delta\xi}||_2/||Y||_2$ |
| 2048    | 0.48         | $1.6 \times 10^{-4}$ |
| 4096    | 0.24         | $2.27 \times 10^{-9}$ |
| 8192    | 0.12         | $6.82 \times 10^{-13}$ |
| 16384   | 0.06         | $1.87 \times 10^{-14}$ |

| Table 3 | Critical regime with $F = 1$. The reference solution $Y$ is for the finest grid with $N = 32768$ and $\Delta\xi = 0.03$ |
|---------|-------------------------------------------------|
| $N$     | $\Delta\xi$ | $||Y - Y_{\Delta\xi}||_2/||Y||_2$ |
| 2048    | 0.48         | $4.64 \times 10^{-6}$ |
| 4096    | 0.24         | $4.86 \times 10^{-11}$ |
| 8192    | 0.12         | $2.90 \times 10^{-13}$ |
| 16384   | 0.06         | $7.24 \times 10^{-14}$ |
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