Charmonia decay widths in magnetized matter using a model for composite hadrons

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Abstract

The decay widths of the charmonium states to $D\bar{D}$ in isospin asymmetric nuclear matter in the presence of a magnetic field are studied, using a field theoretical model for composite hadrons with quark/antiquark constituents. The medium modifications of these partial decay widths arise due to the changes in the masses of the decaying charmonium state and the produced $D$ and $\bar{D}$ mesons in the magnetized hadronic matter, calculated within a chiral effective model. The decay widths are computed using the light quark–antiquark pair creation term of the free Dirac Hamiltonian in terms of the constituent quark field operators. The results of the present investigation are compared with the in-medium decay widths obtained within the $^3P_0$ model. Within the $^3P_0$ model, the charmonium decay widths are calculated using the creation of a light quark–antiquark pair in the $^3P_0$ state. In the presence of a magnetic field, the Landau level contributions give rise to positive shifts in the masses of the charged $D$ and $\bar{D}$ mesons. This leads to the decay of charmonium to the charged $D^+D^-$ to be suppressed as compared to the neutral $D\bar{D}$ pair in symmetric nuclear matter, whereas in asymmetric nuclear matter, the larger mass drop of the $D^+D^-$ pair, as compared to the $D^0\bar{D}^0$ pair leads to the production of charged open charm meson pairs to be enhanced as compared to the charmonium decay channel to $D^0\bar{D}^0$.
I. INTRODUCTION

The properties of hadrons at high temperatures and/or densities comprise an important area of research, due to their relevance in the context of the ongoing and future ultra relativistic heavy ion collision experiments at various high energy particle accelerators, as well as in the study of the bulk matter of astrophysical objects, e.g., neutron stars. The estimated huge magnetic fields at Relativistic Heavy Ion Collider (RHIC) at BNL and Large Hadron Collider (LHC) at CERN [1], have recently initiated a lot of work on the study of the hadrons in the presence of strong magnetic fields. The time evolution of the magnetic field [1], however, needs the solutions of the magnetohydrodynamic equations, with a proper estimate of the electrical conductivity of the medium, and is still an open question. Recently, there has been a lot of work on the heavy flavour hadrons [2], and, due to the attractive interaction of the $J/\psi$ in nuclear matter [3–5], the possibility of the $J/\psi$ forming bound states with nuclei have been predicted [6]. The heavy quarkonium (charmonium and bottomonium) states have been studied using the potential models [7–11]. The heavy quarkonium state, as a bound state of the heavy quark, $Q$ and heavy antiquark ($\bar{Q}$) interacting by color Coulomb potential, has been studied in the presence of gluonic field [12–14]. Assuming the separation between $Q$ and $\bar{Q}$ to be small compared to the scale of gluonic fluctuations, in the leading order, the mass of the quarkonium state is observed to be proportional to the gluon condensate [12–14]. The heavy flavour mesons have been studied in the literature using the quark meson coupling (QMC) model, the QCD sum rule approach the coupled channel approach, a chiral effective model, using heavy quark symmetry and interaction of these mesons with nucleons via pion exchange, using heavy meson effective theory [2]. The open charm mesons [15–18] as well as the charmonium states [19–21] have also been studied in the presence of strong magnetic fields. The mixing of the pseudoscalar charmonium states with the longitudinal components of the vector charmonium states in the presence of strong magnetic fields, is observed to increase (decrease) the masses of the vector (pseudoscalar) charmonium states [22, 23]. In Ref. [22], a study of the effects of the spin mixing on the formation times of the charmonium states, is observed to lead to delayed (earlier) formation times for the vector (pseudoscalar) charmonium states. If the formation of $J/\psi$ is delayed, then the heavy quarks might pass through the medium before thermalization and the $J/\psi$, being unaffected by the thermal medium, can
have a higher survival probability 22.

The chiral SU(3) model based on a non-linear realization of chiral symmetry and broken scale invariance 24, generalized to SU(4) to include the charm sector, is used to study the masses of the charmonium and open charm mesons in a hadronic medium in the presence of a magnetic field. The charmonium masses are calculated from the medium modification of a scalar dilaton field which mimics the gluon condensates of QCD 4, 5, 21 and the in-medium masses of the open charm (D, \bar{D}) mesons, are computed from their interactions with the baryons and scalar mesons 4, 5, 18 in the (magnetized) hadronic medium. The in-medium decay widths of the charmonium states to D\bar{D} at zero magnetic field, have been studied using a model with light quark pair created in the \(^3\)P\(_0\) state 5, 25–27, as well as, using a field theoretical model with composite hadrons 28. The results of the present investigation of the charmonium decay widths in magnetized nuclear matter calculated using the model for composite hadrons are compared with the results obtained using the \(^3\)P\(_0\) model 29.

The outline of the paper is as follows: In section II, we describe briefly the field theoretical model with composite hadrons with quark/antiquark constituents, used in the present work to compute the partial decay widths of the charmonium states to D\bar{D} in magnetized hadronic matter. These in-medium decay widths are computed from the mass modifications of the decaying charmonium state and the produced D and \bar{D} mesons, calculated within a chiral effective model. In section III, we discuss the results obtained in the present investigation of these in-medium charmonium decay widths. In section IV, we summarize the findings of the present study. The salient features of the field theoretic model for composite hadrons are presented in Appendix A, and the chiral effective model for study of the charmed mesons is briefly described in Appendix B.

II. DECAY WIDTH OF CHARMONIUM STATE TO D\bar{D} WITHIN A MODEL FOR COMPOSITE HADRONS

We use a field theoretical model for composite hadrons with quark/antiquark constituents 30, 32 to study the charmonium decay widths to D\bar{D} in isospin asymmetric nuclear matter in the presence of a magnetic field. The model has been used to investigate these charmonium decay widths in hadronic matter in the absence of a magnetic field 28. With explicit construc-
tions of the charmonium state and the open charm mesons, the decay width is calculated using the quark antiquark pair creation term of the free Dirac Hamiltonian for constituent quark field \[28\]. The model for the composite hadrons with quark constituents has been described in Appendix A.

The relevant part of the quark pair creation term is through the \( \bar{d}d(u \bar{u}) \) creation for decay of the charmonium state, \( \Psi \), to the final state \( D^+D^-(D^0\bar{D}^0) \).

For \( \Psi \rightarrow D^+(p)D^-(p') \), this pair creation term is given as

\[
\mathcal{H}_{\bar{d}d}(\mathbf{x}, t = 0) = Q_\bar{d}(\mathbf{x})^\dagger(-i\alpha \cdot \nabla + \beta M_d)\tilde{Q}_d(\mathbf{x})
\]

(1)

where, \( M_d \) is the constituent mass of the \( d \) quark. The subscript \( d \) of the field operators in equation (1) refers to the fact that the \( \bar{d} \) and \( d \) are the constituents of the \( D^+ \) and \( D^- \) mesons with momenta \( \mathbf{p} \) and \( \mathbf{p}' \) respectively in the final state of the decay of the charmonium state, \( \Psi \).

The charmonium state, \( \Psi (J/\psi, \psi' \equiv \psi(3686), \psi'' \equiv \psi(3770)) \) with spin projection \( m \), at rest is written as

\[
|\Psi_m(0)\rangle = \int d\mathbf{k} c_r^i(\mathbf{k})^\dagger u_r a_m(\Psi, \mathbf{k}) c_s^i(-\mathbf{k}) v_s |\text{vac}\rangle,
\]

(2)

where, \( i \) is the color index of the quark/antiquark operators. The wave functions for the charmonium states, \( J/\psi, \psi(3686) \) and \( \psi(3770) \) are assumed to be harmonic oscillator wave functions, corresponding to the 1S, 2S and 1D states respectively. With this assumption, the expressions for \( a_m(\Psi, \mathbf{k}) \) are given as

\[
a_m(\Psi, \mathbf{k}) = \sigma_m \frac{1}{\sqrt{6}} \left( \frac{R_\Psi^2}{\pi} \right)^{3/4} \exp \left( - \frac{R_\Psi^2 k^2}{2} \right),
\]

(3)

for \( \Psi \equiv J/\psi \),

\[
a_m(\Psi, \mathbf{k}) = \sigma_m \frac{1}{2} \left( \frac{R_\Psi^2}{\pi} \right)^{3/4} \left( \frac{2}{3} R_\Psi^2 k^2 - 1 \right) \exp \left( - \frac{R_\Psi^2 k^2}{2} \right).
\]

(4)

for \( \Psi \equiv \psi(3686) \), and,

\[
a_m(\Psi, \mathbf{k}) = \frac{1}{3\sqrt{5\pi}} \pi^{-1/4} (R_\Psi^2)^{7/4} k^2 \left( \sigma_m - 3(\mathbf{\sigma} \cdot \mathbf{k}) \hat{k}^m \right)
\]

\times \exp \left( - \frac{1}{2} R_\Psi^2 k^2 \right),
\]

(5)

for \( \Psi \equiv \psi(3770) \). In equations (3), (4) and (5), \( R_\Psi \) corresponds to the strength of the harmonic oscillator potential for the charmonium state, \( \Psi \). The parameters \( R_{J/\psi}, R_{\psi'}, R_{\psi''} \), are fitted to
their rms radii to be \((0.47 \text{ fm})^2\), \((0.96 \text{ fm})^2\) and \(1 \text{ fm}^2\) respectively, which yield their values as \((520 \text{ MeV})^{-1}\), \((390 \text{ MeV})^{-1}\) and \((370 \text{ MeV})^{-1}\). The \(D(D^+, D^0)\) and \(\bar{D}(D^-, \bar{D}^0)\) states, with finite momenta are contracted in terms of the constituent quark field operators obtained from the quark field operators of these mesons at rest through a Lorentz boosting [32] (see Appendix A). These states are explicitly given as

\[
|D(\mathbf{p})\rangle = \int u_D(k_2) c_r^{i_1}(k_2 + \lambda_2 \mathbf{p})^\dagger q_s^{i_1}(-k_2 + \lambda_1 \mathbf{p}) v_s d\mathbf{k}_2
\]

and

\[
|\bar{D}(\mathbf{p'})\rangle = \int u_{\bar{D}}(k_3) q_r^{i_2}(k_3 + \lambda_1 \mathbf{p}')^\dagger c_s^{i_2}(-k_3 + \lambda_2 \mathbf{p}') v_s d\mathbf{k}_3.
\]

In the above, \(q = (d, u)\) for \((D^+, D^-)\) and \((D^0, \bar{D}^0)\) respectively, and, we assume harmonic oscillator wave functions for the \(D\) and \(\bar{D}\) mesons given as

\[
u_{D(\bar{D})}(\mathbf{k}) = \frac{1}{\sqrt{6}} \left(\frac{R_D^2}{\pi}\right)^{3/4} \exp \left(-\frac{R_D^2 k^2}{2}\right),
\]

where the harmonic oscillator strength parameter for \(D(\bar{D})\) meson is taken as \(R_D = (310 \text{ MeV})^{-1}\) so as to give the experimental values of the vacuum decay widths of \(\psi(3770) \rightarrow D\bar{D}\), and, \(\psi(4040) \rightarrow D\bar{D}, D\bar{D}^*, D^*\bar{D}\) and \(D^*\bar{D}^*\) [3].

In equations (6) and (7), \(\lambda_1\) and \(\lambda_2\) are the fractions of the mass (energy) of the \(D(\bar{D})\) meson at rest (in motion), carried by the constituent light \((d, u)\) antiquark (quark) and the constituent heavy charm quark (antiquark), with \(\lambda_1 + \lambda_2 = 1\). These are calculated by assuming the binding energy of the hadron as shared by the quark(antiquark) are inversely proportional to the quark (antiquark) masses [31]. The energies of the the light antiquark (quark) and heavy charm quark (antiquark), \(\omega_i = \lambda_i m_D (i = 1, 2)\), are assumed to be [28, 31]

\[
\omega_1 = M_q + \frac{\mu}{M_q} \times BE, \quad \omega_2 = M_c + \frac{\mu}{M_c} \times BE,
\]

where \(BE = (m_D - M_c - M_q)\) is the binding energy of \(D(\bar{D})\) meson, with \(M_c\) and \(M_q\) as the masses of the constituent charm and light quark (antiquark), and, \(\mu\) is the reduced mass of the heavy-light quark-antiquark system (the \(D(\bar{D})\) meson), defined by \(1/\mu = 1/M_q + 1/M_c\). The reason for making this assumption comes from the example of hydrogen atom, which is the bound state of the proton and the electron. As the mass of proton is much larger as
compared to the mass of the electron, the binding energy contribution from the electron is 
\[ \frac{\mu}{m_e} \times BE \simeq BE \] of hydrogen atom, and the contribution from the proton is 
\[ \frac{\mu}{m_p} \times BE \], which is negligible as compared to the total binding energy of hydrogen atom, since \( m_p \gg m_e \). With this assumption, the binding energies of the heavy-light mesons, e.g., \( D \) and \( (\bar{D}) \) mesons [28] and, \( B \bar{B} \) mesons [34], mostly arise from the contribution from the light quark (antiquark).

The expression of the decay width of the charmonium state, \( \Psi \) to \( D\bar{D} \), as calculated in the present model for composite hadrons is given by equation (A.13). The parameter, \( \gamma_\Psi \), in the expression for the charmonium decay width, is a measure of the coupling strength for the creation of the light quark antiquark pair, to produce the \( D\bar{D} \) final state. This parameter is adjusted to reproduce the vacuum decay widths of \( \psi(3770) \) to \( D^+D^- \) and \( D^0\bar{D}^0 \) [28]. The decay width of the charmonium state is observed to have the dependence on the magnitude of the 3-momentum of the produced \( D(\bar{D}) \) meson, \( |p| \), as a polynomial part multiplied by an exponential term. The medium modification of the charmonium decay width is studied due to the mass modifications of the charmonium state, the \( D \) and \( \bar{D} \) mesons through \( |p| \), which is given as,

\[ |p| = \left( \frac{m_\psi^2}{4} - \frac{m_D^2 + m_{\bar{D}}^2}{2} + \frac{(m_D^2 - m_{\bar{D}}^2)^2}{4m_\psi^2} \right)^{1/2}. \]  

In equation (10), the masses of the charmonium and open charm mesons are the effective masses in the hadronic matter in the presence of a magnetic field. These in-medium masses are calculated using a chiral effective model, which is briefly described in Appendix B.

The chiral effective model incorporates the broken scale invariance of QCD, as has already been mentioned, through a scalar dilaton field, which mimics the gluon condensates of QCD [24]. The charmonium masses are calculated from the medium modifications of the gluon condensates, which is obtained from the medium changes of the scalar dilaton field [5, 21]. The mass modifications of the \( D \) and \( \bar{D} \) mesons are calculated from their interactions with the nucleons and the scalar mesons in the magnetized nuclear medium [5, 18], within the chiral effective model. The proton, which is the charged nucleon, has contributions from the Landau energy levels. The anomalous magnetic moments of the nucleons are considered in the present study. The charged \( D^\pm \) mesons have positive contributions in the masses in the presence of a magnetic field, and we account for the lowest Landau level contribution to the masses of these charged open charm mesons [18].
III. RESULTS AND DISCUSSIONS

In the present work, the charmonium decay widths to $D\bar{D}$ in magnetized nuclear matter are investigated within a field theoretical model for composite hadrons with quark/antiquark constituents. The medium modifications of these decay widths are computed from the changes in the masses of the charmonium state, $\Psi$, and the open charm mesons, calculated within a chiral effective model \cite{18, 21, 29}. The charmonium decay width, $\Gamma(\Psi \rightarrow D\bar{D})$ is calculated using the light quark pair creation term of the free Dirac Hamiltonian expressed in terms of the constituent quark operators, using explicit constructions for the charmonium and the open charm ($D, \bar{D}$) mesons. The matrix element for the calculation of the decay width is multiplied with a factor $\gamma_\psi$, which gives the strength of the light quark pair creation leading to the decay of the charmonium state to $D\bar{D}$ in the magnetized hadronic medium \cite{28}. The value of $\gamma_\psi$ to be 1.35, is chosen so as to reproduce the decay widths of $\psi(3770) \rightarrow D^+D^-$ and $\psi(3770) \rightarrow D^0\bar{D}^0$ in vacuum, to be around 12 MeV and 16 MeV respectively \cite{28}. The constituent quark masses for the light quarks ($u$ and $d$) are taken to be 330 MeV and for the charm quark, the value is taken to be $M_c = 1600$ MeV \cite{28}.

The effects of isospin asymmetry, density and magnetic field on the partial decay width of $\Psi \rightarrow D\bar{D}$ are investigated within the composite model for the hadrons. The charmonium masses are studied in the magnetized nuclear matter within the chiral effective model, due to the medium changes in the scalar dilaton field, $\chi$, which mimics the gluon condensates of QCD \cite{21}. The masses of the open charm mesons in the magnetized nuclear matter are modified due to their interactions with the nucleons as well as the scalar mesons, $\sigma$, $\zeta$, and $\delta$ \cite{18}. These scalar fields and the dilaton field $\chi$ are solved from their coupled equations of motion. It might be noted here that the $D$ and $\bar{D}$ mesons in Ref. \cite{18} as well as the kaons and antikaons \cite{35} were studied in the frozen glueball approximation, i.e. the value of $\chi$ was fixed at its vacuum value. This approximation was chosen as the modifications of the open charm (strange) mesons arise due to the scalar mesons, $\sigma$, $\zeta$ and $\delta$, which do not have large modifications, when the medium dependence of the scalar dilaton field, $\chi$ is taken into consideration as well. The $D$ and $\bar{D}$ masses in the magnetized nuclear matter, including the variation of the dilaton field has been considered in Ref. \cite{21} to study the mass modifications of the charmonium states (which are calculated from the medium changes of the $\chi$ field), as well as in Ref. \cite{29}, where the partial...
FIG. 1: (Color online) In-medium masses of $D^+$ (in panels (a), (c) and (e)) and $D^0$ (in panels (b), (d) and (f)), as calculated using the chiral effective model, are plotted as functions of $\rho_B/\rho_0$, for given values of the magnetic field for symmetric ($\eta=0$) and asymmetric (with $\eta=0.5$) nuclear matter.

decay widths of charmonium states to $D\bar{D}$ have been studied using the $^3P_0$ model. The present study also uses the medium dependence of the dilaton field to calculate the charmonium decay widths to $D\bar{D}$ using a field theoretical model for composite hadrons. For given values of the baryon density, $\rho_B$, isospin asymmetry parameter, $\eta = (\rho_n - \rho_0)/(2\rho_B)$, and magnetic field, the values of the scalar fields are solved from their equations of motion, within the chiral effective model, which are then used to calculate the in-medium masses of the charmonium states and the open charm mesons.

The charmonium decay widths are modified in the medium due to the mass modifications of the charmonium state and the open charm mesons, calculated using a chiral effective model
FIG. 2: (Color online) Same as fig. 1 for $D^-$ and $\bar{D}^0$.

In the absence of a magnetic field, the charmonium decay widths to $D\bar{D}$ have been studied using the model for composite hadrons as used in the present work [28]. It was observed that the decay of $J/\psi$ to $D\bar{D}$ is possible, for densities higher than $4-4.5\rho_0$, for zero magnetic field. However, in the presence of a magnetic field, for $eB = 4m^2_\pi$ and $eB = 8m^2_\pi$ as considered in the present work, the decay of $J/\psi$ to $D\bar{D}$ does not become kinematically possible.

For the sake of completeness, we show the effects of the magnetic field, baryon density as well as isospin asymmetry on the masses of the $D(D^+, D^0)$, $\bar{D}(D^-, \bar{D}^0)$ mesons and the charmonium states ($\psi(3686)$ and $\psi(3770)$) in figures 1, 2 and 3, respectively. The masses obtained in presence of a magnetic field are compared to the case of zero magnetic field, shown in panels, (a) and (b) in these figures, for the symmetric and asymmetric nuclear matter. The
charmonium masses are calculated from the in-medium values of the scalar gluon condensate, obtained from the medium changes of the dilaton field within the chiral effective model, as has already mentioned. The values of $-9.3$, $-126.4$ and $-167.5$ MeV for the mass shifts for the charmonium states, $J/\psi$, $\psi(3686)$ and $\psi(3770)$ at $\rho_B = \rho_0$, may be compared to the values of $-8$, $-100$ and $-140$ MeV calculated using the linear density approximation [3].

The model for composite hadrons with quark/antiquark constituents, used in the present work of calculation of the charmonium decay widths is described briefly in Appendix A. The expression for the charmonium decay width as given by equation (A.13), has dependence on the in-medium masses of the charmonium state and the open charm mesons, through the magnitude of the 3-momentum of the $D(\bar{D})$ meson, given by equation (10). The dependence of the decay width on $|p|$ is a polynomial part multiplied by an exponential function in $|p|$. 

FIG. 3: (Color online) Same as fig. 1 for $\psi(3686)$ and $\psi(3770)$. 
FIG. 4: (Color online) Decay widths of $\psi(3686)$ to (i) $D^+D^-$, (ii) $D^0\bar{D}^0$, and the total of these two channels ((i)+(ii)), as calculated using the field theoretic model for composite hadrons, are plotted as functions of $\rho_B/\rho_0$, for given values of the magnetic field for symmetric ($\eta=0$) and asymmetric (with $\eta=0.5$) nuclear matter.

The $^3P_0$ model describes the charmonium decay through a light quark antiquark pair creation in the $^3P_0$ state. The created quark (antiquark) combine with the charm antiquark (quark) to form the $D$ and $\bar{D}$ mesons. The matrix element for a general decay $A \rightarrow BC$ is taken as $M_{A\rightarrow BC} = \langle BC|\gamma(q_s\bar{q}_s)^3 R_0|A\rangle$, where $\gamma$ is a coupling parameter, which characterizes the probability of creating a quark-antiquark pair. The parameter $\gamma$ is fitted to the experimentally observed decay width of $A \rightarrow BC$. The decay amplitudes for a variety of decay processes have been listed in Ref. 26. A formulation which is equivalent to the $^3P_0$ model is due to an interaction Hamiltonian involving quark fields given as $H_I = g \int d\mathbf{x} \bar{\psi}(\mathbf{x})\psi(\mathbf{x})$. The
quark-antiquark pair creation term of this interaction Hamiltonian, in the non-relativistic limit, gives the decay amplitudes to be same as obtained using the $^3P_0$ model, with the identification $\gamma = \frac{g}{2m_q}$, where $m_q$ is the mass of the light quark field. A typical value of $m_q$ is 330 MeV \cite{26}, which is also constituent quark mass of the light quark (u,d) chosen in the present work of charmonium decay widths using the model for the composite hadrons with quark/antiquarks. The $^3P_0$ model does not account for the color. However, taking the color of the quark into account will lead to a multiplying factor, which would simply lead to a redefinition of the parameter, $\gamma$ (or equivalently $g$).

The changes in the decay widths of the charmonium states to the $DD$ in nuclear matter were studied in Ref. \cite{27} using the $^3P_0$ model, arising due to mass drop of the $D$ meson in the medium. These were studied as functions of the mass of the $D$ (assuming the mass of

FIG. 5: (Color online) Same as fig. 4 for decay widths of $\psi(3770)$. 

FIG. 6: (Color online) Same as fig. 4 within $^{3}P_{0}$ model.

the $\bar{D}$ meson to be identical with the $D$ meson mass). The decay widths of $\psi(3686)$, $\psi(3770)$ and $\chi_{c0}$ to $D\bar{D}$ were observed to increase initially with drop in the mass of the $D$ meson, and then decreased with further drop in the $D$ meson mass reaching a value of zero (so called nodes). This was followed by an increase with further drop in the $D$ meson mass. This kind of behaviour for the charmonium decay widths were observed using the $^{3}P_{0}$ model, when the masses of the $D$ and $\bar{D}$ mesons are calculated using the chiral effective model [5], and the mass modifications of the charmonium states are neglected. However, the vanishing of the decay widths were no longer observed when the mass modifications of the charmonium states were also considered [5]. The behaviour of the decay width has a form of a polynomial function multiplied by an exponential function of $|p|$, given by equation (10), in terms of the masses of the charmonium state and the open charm mesons. The effects of magnetic fields on the
charmonium decay widths calculated using the $^3P_0$ model due to the mass modifications of the charmonium, $D$ and $\bar{D}$ mesons were also studied in Ref. [29]. The results of the decay widths calculated using the model for composite hadrons (as shown in figures 4 and 5) as used in the present work, are compared with the results obtained using the the $^3P_0$ model (shown in figures 6 and 7). In both the $^3P_0$ model, as well as, in the model for the composite hadrons as in the present work, the medium dependence of the charmonium decay width is through $|\mathbf{p}|$, and the expression for the charmonium decay width turns out to be the form of a polynomial function multiplied by an exponential part, calculated in the respective models.

The effects of magnetic field, isospin asymmetry and density on the decay widths of $\psi(3686)$ to $DD$ and $\psi(3770)$ to $\bar{D}D$, as calculated in the present work, are plotted in figures 4 and 5 respectively. These decay widths are shown as functions of the baryon density in units of $14$.

FIG. 7: (Color online) Same as fig. 5, within $^3P_0$ model.
nuclear matter density, for the channels (i) \( D^+ D^- \), (ii) \( D^0 \bar{D}^0 \) as well as (iii) the total of these two channels. For \( eB = 4m^2_\pi \) and \( eB = 8m^2_\pi \), these are plotted for the symmetric nuclear matter (\( \eta = 0 \)) and asymmetric nuclear matter (with \( \eta = 0.5 \)), which may be compared with the results for the zero magnetic field shown in (a) and (b). For \( \eta = 0 \) (symmetric nuclear matter) and \( eB = 4m^2_\pi \), the decays \( \psi(3686) \) to \( D^+ D^- \) and \( \psi(3686) \) to \( D^0 \bar{D}^0 \) are observed to be possible for densities above \( 4.3\rho_0 \) and \( 2.5\rho_0 \) respectively, as can be seen in panel (c) of figure 4. The masses of \( D^+ \) and \( D^- \) mesons have positive shifts due to the contributions from the lowest Landau levels \([18]\) in the presence of magnetic field. This leads to the threshold density for the decay to \( D^+ D^- \) to be larger as compared to the decay to \( D^0 \bar{D}^0 \) in symmetric nuclear matter (\( \eta = 0 \)). For the larger value of the magnetic field (\( eB = 8m^2_\pi \) and for \( \eta = 0 \), as shown in panel (e), the decay of \( \psi(3686) \) to the neutral \( D^0 \bar{D}^0 \) mesons becomes possible at a density of \( 2.3\rho_0 \), whereas the decay to the charged \( D^+ D^- \) is not observed even upto a density of \( 6\rho_0 \). The in-medium decay widths in asymmetric nuclear matter (with \( \eta = 0.5 \)) are plotted in the panels (d) and (f) for magnetic fields, \( eB = 4m^2_\pi \) and \( eB = 8m^2_\pi \), which are compared to the case of zero magnetic field, shown in (b). In the presence of asymmetry in the medium, the mass of the \( D^0 \bar{D}^0 \) pair is observed to be larger as compared to the mass of the \( D^+ D^- \) pair, as can be seen from the masses of the \( D \) and \( \bar{D} \) mesons plotted in figures 1 and 2. This leads to the production of the neutral open charm mesons to be suppressed as compared to the \( D^+ D^- \) pair.

The threshold density above which the production of \( D^0 \bar{D}^0 \) from decay of the charmonium state \( \psi(3686) \) becomes possible (\( \sim 3\rho_0 \)) is observed to be larger than the production of \( D^+ D^- \) for isospin asymmetric nuclear matter, in the absence of a magnetic field, as can be seen from panel (b) of the figure 4. In the presence of a magnetic field, as can be seen from the panels (d) and (f) of the same figures, the threshold density is shifted to higher values for the production of \( D^+ D^- \) and the decay to the neutral charm meson pair is not kinematically possible.

The in-medium decay widths of \( \psi(3770) \) to \( D \bar{D} \) are plotted in figure 5 as functions of the baryon density for given values of the magnetic field for the symmetric as well as asymmetric nuclear matter. As has already been mentioned the decay of \( \psi(3770) \) to \( D \bar{D} \) takes place in vacuum with the values of the decay widths to \( D^+ D^- \) and \( D^0 \bar{D}^0 \) to be around 12 MeV and 16 MeV respectively. At zero magnetic field \([2]\) and for symmetric nuclear matter, there is observed to be an increase in the decay width in both the channels \( \psi(3770) \rightarrow D^+ D^- \) as well as \( \psi(3770) \)}
to $D^0\bar{D}^0$, reaching a maximum value at around 0.5$\rho_0$, followed by a drop up to around 2$\rho_0$, and a slow increase with further increase in density. In asymmetric nuclear matter, the decay width in the charged $D\bar{D}$ channel shows a monotonic increase whereas the decay width to neutral $D\bar{D}$ mesons shows a drop and then vanishing above densities of around 2.5 $\rho_0$. The difference in the behaviours of the decay widths of the charged and neutral $D\bar{D}$ channels should have modify the production of the open charm mesons in the compressed baryonic matter experiments at the future facility at GSI, which should have enhanced production of the $D^+D^-$ as compared to the neutral $D\bar{D}$ mesons.

In the presence of magnetic field, the masses of the charged mesons ($D^+$ and $D^-$) have a positive shift due to the lowest Landau level contribution and in symmetric nuclear matter, the decay of $\psi(3770)$ to $D^+D^-$ is not observed up to a density (in units of $\rho_0$) of around 4.4 for $eB = 4m^2\pi$, as can be seen in (c) of figure 5. The decay of $\psi(3770)$ to $D^0\bar{D}^0$ is observed to decrease initially when the density is raised, up to a density of around 1.2 $\rho_0$, followed by a rise with further increase in the density. The behaviour remains the same for the case of the higher magnetic field, for the decay to $D^0\bar{D}^0$, as can be seen in panel (e) of figure 5. The decay to $D^+D^-$ is not observed for $eB = 8m^2\pi$ for the symmetric nuclear matter even up to a density of 6$\rho_0$.

The behaviour of the decay widths of $\psi(3686)$ to $D^+D^-$ ($D^0\bar{D}^0$) are very similar to those calculated within the $^3P_0$ model [29] as shown in figure 6 but the values for the decay widths within the present model for composite hadrons, are observed to be much smaller as compared to the results obtained using the $^3P_0$ model [29].

The decay widths of $\psi(3770)$ to $D^+D^-$ ($D^0\bar{D}^0$) as calculated in the present work are compared with the results obtained using the the $^3P_0$ model [29], as shown in figure 7. The behaviour of these decay widths with density are observed to be similar in both the models, however, the values of the present model are observed to be larger than those in the $^3P_0$ model.

The effect of isospin asymmetry is observed to be important for both the cases of zero as well as finite magnetic fields, and the production of the $D^+D^-$ ($D^0\bar{D}^0$) are observed to be enhanced (suppressed) in the symmetric (asymmetric) nuclear matter. This should have consequences on the $D^+D^-$ production to be abundant as compared to $D^0\bar{D}^0$ from the asymmetric heavy ion collisions in compressed baryonic matter (CBM) experiments at FAIR, GSI. At very small
densities, the decay width of $\psi(3770)$ to $D^+D^-$ is observed to have a sharp increase (decrease) with density from the vacuum value for asymmetric nuclear matter in the absence (presence) of a magnetic field. Also, in the presence of a magnetic field, the decay of $\psi(3770)$ to $D^0\bar{D}^0$ is not kinematically possible for small densities in the asymmetric matter. These might have consequences on the production of the open charm mesons from the charmonium state $\psi(3770)$ at the asymmetric heavy ion collision experiments at RHIC.

IV. SUMMARY

The effects of magnetic field, isospin asymmetry and baryon density on the partial decay widths of the charmonium states ($J/\psi$, $\psi(3686)$, $\psi(3770)$) to $D\bar{D}$ are studied using a field theoretical model for composite hadrons with quark (antiquark) constituents. The matrix element for the calculation of the decay width is calculated from the free Dirac Hamiltonian of the constituent quarks, using the explicit constructions of the charmonium state, the $D$ and the $\bar{D}$ mesons. The in-medium charmonium partial decay widths are computed from the medium modifications of the charmonium and the open charm mesons calculated using a chiral effective model. The results of the present work are observed to have similar behaviour to the results obtained using the $^3P_0$ model, but the values of the decay widths obtained for decay of $\psi(3686)$ are seen to be much smaller in magnitude than those obtained in the $^3P_0$ model. The charmonium decay to $D^+D^-$ and $D^0\bar{D}^0$ are observed to be very different in the presence of magnetic field even in symmetric nuclear matter, due to the masses of the charged mesons ($D^+, D^-$) which have a positive shift due to the Landau levels, whereas there is no such contribution to the masses of the neutral $D^0$ and $\bar{D}^0$ mesons. The effects of isospin asymmetry give rise to different modifications of the masses of the mesons within the $D$ and $\bar{D}$ doublets. The striking difference in the charmonium partial decay widths for the charged and neutral $D\bar{D}$ pair channels in the isospin symmetric (asymmetric) nuclear matter will have consequences in the asymmetric heavy ion collisions in the CBM experiments at FAIR, GSI as a enhancement of the charged $D^\pm$ mesons as compared to the neutral $D\bar{D}$ mesons. At subnuclear densities, there is observed to be a sharp drop for the decay width of $\psi(3770)$ to $D^\pm D^-$ from its vacuum value in the presence of a magnetic field for asymmetric nuclear matter, and the decay to neutral $D\bar{D}$ is not kinematically possible. These might have consequences on the production of the
charm state, $\psi(3770)$ as well as open charm mesons in asymmetric heavy ion collisions at RHIC.

One of the authors (AM) is grateful to ITP, University of Frankfurt, for warm hospitality and acknowledges financial support from Alexander von Humboldt Stiftung when this work was initiated.

Appendix A: Model for composite hadrons

The model describes hadrons comprising of quark and/or antiquark constituents. The field operator for a constituent quark for a hadron at rest at time, $t=0$, is written as

$$\psi(x, t = 0) = (2\pi)^{-3/2} \int \left[ U(k) u_r(k) \exp(i k \cdot x) + V(k) v_s(\tilde{k}) \exp(-i \tilde{k} \cdot x) \right] dk$$

$$\equiv Q(x) + \tilde{Q}(x), \quad (A.1)$$

where, $U(k)$ and $V(k)$ are given as

$$U(k) = \begin{pmatrix} f(|k|) \\ \sigma \cdot k g(|k|) \end{pmatrix}, \quad V(k) = \begin{pmatrix} \sigma \cdot k g(|k|) \\ f(|k|) \end{pmatrix}, \quad (A.2)$$

The functions $f(|k|)$ and $g(|k|)$ satisfy the constraint $f^2 + g^2 k^2 = 1$, as obtained from the equal time anticommutation relation for the four-component Dirac field operators. These functions, for the case of free Dirac field of mass $M$, are given as,

$$f(|k|) = \left( \frac{k_0 + M}{2k_0} \right)^{1/2}, \quad g(|k|) = \left( \frac{1}{2k_0(k_0 + M)} \right)^{1/2}, \quad (A.3)$$

where $k_0 = (|k|^2 + M^2)^{1/2}$. In the above, $M$ is the constituent quark/antiquark mass. In equation (A.1), $u_r$ and $v_s$ are the two component spinors for the quark and antiquark respectively, satisfying the relations $u^\dagger_r u_s = v^\dagger_s v_s = \delta_{rs}$. The operator $q_r(k)$ annihilates a quark with spin $r$ and momentum $k$, whereas, $\tilde{q}_s(\tilde{k})$ creates an antiquark with spin $s$ and momentum $\tilde{k}$, and these operators satisfy the usual anticommutation relations

$$\{ q_r(k), q_s(k')^\dagger \} = \{ \tilde{q}_r(k), \tilde{q}_s(k')^\dagger \} = \delta_{rs} \delta(k - k'). \quad (A.4)$$

The field operator for the constituent quark of hadron with finite momentum is obtained by Lorentz boosting the field operator of the constituent quark of hadron at rest, which requires
the time dependence of the quark field operators. Similar to the MIT bag model \cite{36}, where the quarks (antiquarks) occupy specific energy levels inside the hadron, it is assumed in the present model for the composite hadrons that the quark/antiquark constituents carry fractions of the mass (energy) of the hadron at rest (in motion) \cite{30,31}. The time dependence for the \(i\)-th quark/antiquark of a hadron of mass \(m_H\) at rest is given as

\[
Q_i(x) = Q_i(x)e^{-i\lambda_im_Ht}, \quad \tilde{Q}_i(x) = \tilde{Q}_i(x)e^{i\lambda_im_Ht},
\] (A.5)

where \(\lambda_i\) is the fraction of the energy (mass) of the hadron carried by the quark (antiquark), with \(\sum_i \lambda_i = 1\). For a hadron in motion with four momentum \(p\), the field operators for quark annihilation and antiquark creation, for \(t=0\), are obtained by Lorentz boosting the field operator of the hadron at rest, and are given as \cite{32}

\[
Q(p)(x,t) = \int \frac{d\mathbf{k}}{(2\pi)^{3/2}} S(L(p))U(\mathbf{k})Q(\mathbf{k} + \lambda p)\exp[i(\mathbf{k} + \lambda \mathbf{p}) \cdot \mathbf{x} - i\lambda p^0 t]
\] (A.6)

and,

\[
\tilde{Q}(p)(x,t) = \int \frac{d\mathbf{k}}{(2\pi)^{3/2}} S(L(p))V(-\mathbf{k})\tilde{Q}(-\mathbf{k} + \lambda \mathbf{p})\exp[-i(-\mathbf{k} + \lambda \mathbf{p}) \cdot \mathbf{x} + i\lambda p^0 t].
\] (A.7)

In the above, \(\lambda\) is the fraction of the energy of the hadron, carried by the constituent quark (antiquark). In equations (A.6) and (A.7), \(L(p)\) is the Lorentz transformation matrix, which yields the hadron at finite four-momentum \(p\) from the hadron at rest, and is given as \cite{31}

\[
L_{\mu\nu} = L_{0\mu} = \frac{p^\mu}{m_H}; \quad L_{ij} = \delta_{ij} + \frac{p^i p^j}{m_H(p^0 + m_H)},
\] (A.8)

where, \(\mu = 0,1,2,3\) and \(i = 1,2,3\), and the Lorentz boosting factor \(S(L(p))\) is given as

\[
S(L(p)) = \left[\frac{(p^0 + m_H)}{2m_H}\right]^{1/2} + \left[\frac{1}{2m_H(p^0 + m_H)}\right]^{1/2} \vec{\alpha} \cdot \vec{p},
\] (A.9)

where, \(\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}\), are the Dirac matrices. The Lorentz transformations used to obtain the constituent quark and antiquark operators for hadron at rest to hadron with momentum, \(p\), as given by equations (A.6) and (A.7) have the effect of addition of the hadron fractional momentum, \(\lambda p\), as a translation to the constituent quark (antiquark) momentum, \(\mathbf{k}(-\mathbf{k})\) \cite{32}. This is similar to the quasipotential approach, where the Lorentz transformation plays the
role of a translation [37]. Using the composite model picture with Lorentz transformations as considered in the present work, the various properties of hadrons, e.g., charge radii of the proton and pion, the nucleon magnetic moments [30, 31], and, the diffraction slopes as well as total cattering cross-sections for the baryon-baryon and meson-baryon scattering have been studied which have reasonable agreement with experiments [32].

The pair creation term of the Dirac Hamiltonian density

$$H_{Q\bar{Q}}(x) = Q(x)^\dagger(-i\alpha \cdot \nabla + \beta M)\bar{Q}(x)$$ (A.10)

is used to describe the strong decay of the hadron, A at rest to B(p) and C(p'). The operators for the quark and antiquark creation in the above term, thus belong to different hadrons, B and C with 4-momenta p and p' respectively. The light quark pair creation term of the Hamiltonian density, is used to describe the decay of a heavy charmonium state (\bar{c}c) to D and \bar{D} states, which are bound states of c\bar{q} and \bar{c}q repectively, with light (u, d) quark antiquark pair creation. The present description of the decay of \Psi \rightarrow D\bar{D} using the pair creation term of the Hamiltonian, as well as using an interaction term \mathcal{H}_I \sim \int d\bar{\psi}(x)\psi(x) for the light quark pair creation term used in the \(^3P_0\) model [25], are consistent with the Okubo-Zweig-Iizuka (OZI) rule [38–40].

The composite model of hadrons with quark (antiquark) constituents, using the pair creation term of the free Dirac Hamiltonian, as used in the present work, has been applied to study the strong decay processes \rho \rightarrow 2\pi, \Delta \rightarrow N\pi, \phi \rightarrow K\bar{K}, K^* \rightarrow K\pi, which have good agreement with the experimental values of these decay widths [31].

To compute the decay width of the charmonium state, \Psi to D\bar{D}, we evaluate the matrix element of the quark-antiquark pair creation part of the Hamiltonian, between the initial charmonium state and the final state for the reaction \Psi \rightarrow D(p) + \bar{D}(p'). The relevant part of the quark pair creation term is through the d\bar{d}\bar(u\bar{u}) creation for decay to the final state \(D^+D^- (D^0\bar{D}^0)\). Using this pair creation term and the explicit expressions for the states \(|\Psi_m(0)\rangle\), \(|D(p)\rangle\) and \(|\bar{D}(p')\rangle\) in terms of the quark and antiquark constituents, we can evaluate

$$\langle D(p)\rangle|\bar{D}(p')\rangle\int \mathcal{H}_{d\bar{d}}(x, t = 0)d\bar{x}|\Psi_m(\bar{0})\rangle = \delta(p + p')A^\Psi(|p|)p_m.$$ (A.11)

With \langle f|S|i\rangle = \delta_4(P_f - P_i)M_{fi}, we have

$$M_{fi} = 2\pi(-iA^\Psi(|p|))p_m.$$ (A.12)
For evaluation of the matrix element of the quark-antiquark pair creation part of the Hamiltonian, between the initial charmonium state and the final state $D\bar{D}$ state as given by equation (A.11). As the $D$ and $\bar{D}$ mesons are nonrelativistic, we shall assume $S(L(p))$ and $S(L(p'))$ to be unity. We shall also take the approximate forms (with a small momentum expansion) for the functions $f(|k|)$ and $g(|k|)$ of the field operator as given by $g(|k|) = 1/(2k_0(k_0 + M))^{1/2} \approx \sqrt{2}k_0(k_0 + M)$, and

$$f(|k|) = (1 - g^2k^2)^{-1/2} \approx 1 - ((g^2k^2)/2)$$

The expression for the decay width is obtained as

$$\Gamma(\Psi \to D(p)\bar{D}(-p)) = \gamma^2 \frac{1}{2\pi} \int \delta(m_\Psi - p_0^D - p_0^\bar{D})|M_f|_D^2 \cdot 4\pi|p_D|^2|p_D|$$

$$= \gamma^2 \frac{8\pi^2}{3} |p_0^Dp_0^\bar{D}|^2 A^\Psi(|p|)^2 \frac{A^\Psi(|p|)^2}{m_\Psi}$$

(A.13)

In the above, $p_0^D(p_0^\bar{D}) = (m_\Psi^2 + p^2)^{1/2}$, and, $|p|$ is the magnitude of the momentum of the outgoing $D(\bar{D})$ mesons. The expression for $A^\Psi(|p|)$ in the above equation is given as

$$A^\Psi(|p|) = 6c_\Psi \exp[(a_\Psi b_\Psi^2 - R_D^2\lambda_2^2)p^2] \cdot \left(\frac{\pi}{a_\Psi}\right)^{3/2} \left[F_0^\Psi + \frac{F_1^\Psi}{2a_\Psi} + \frac{F_2^\Psi}{4a_\Psi^2}\right],$$

(A.14)

where $a_\Psi, b_\Psi$ are given as

$$a_\Psi = \frac{1}{2}R_\Psi^2 + R_D^2; \quad b_\Psi = R_D^2\lambda_2/a_\Psi,$$

(A.15)

and $c_\Psi, \Psi = J/\psi, \psi', \psi''$, are given by the expressions

$$c_{J/\psi} = \frac{1}{\sqrt{6}} \cdot \left(\frac{R_\Psi^2}{\pi}\right)^{3/4} \cdot \frac{1}{6} \cdot \left(\frac{R_D^2}{\pi}\right)^{3/2},$$

(A.16)

$$c_{\psi'} = \frac{1}{\sqrt{6}} \left(\frac{3}{2}\right)^{1/2} \left(\frac{R_\Psi^2}{\pi}\right)^{3/4} \cdot \frac{1}{6} \cdot \left(\frac{R_D^2}{\pi}\right)^{3/2},$$

(A.17)

$$c_{\psi''} = \frac{1}{4\sqrt{3}\pi} \left(\frac{16}{15}\right)^{1/2} \cdot \pi^{-1/4} \cdot (R_\Psi)^7/4 \cdot \frac{1}{6} \cdot \left(\frac{R_D^2}{\pi}\right)^{3/2}.$$

(A.18)

In the above expressions, $R_\Psi$ and $R_D$ refer to the strengths of the harmonic oscillator wave functions for the charmonium state, $\Psi(J/\psi, \psi(3686), \psi(3770))$ and the $D(\bar{D})$ mesons, whose explicit constructions are given by equations ([2], [6] and [7]). The expressions for $F_0^\Psi, F_1^\Psi, F_2^\Psi$, for $\Psi \equiv J/\psi, \psi', \psi''$, are given as

$$F_0^{J/\psi} = (\lambda_2 - 1) - 2g^2p^2(b_{J/\psi} - \lambda_2)$$
\[ \times \left( \frac{3}{4} b_{J/\psi}^2 - \left(1 + \frac{1}{2} \lambda_2\right) b_{J/\psi} + \lambda_2 - \frac{1}{4} \lambda_2^2 \right) \],
\[ F_{1J/\psi} = g^2 \left[ -\frac{5}{2} b_{J/\psi} + \frac{2}{3} + \frac{11}{6} \lambda_2 \right], \]
\[ F_{2J/\psi} = 0, \]
\[ (A.19) \]
\[ F_0^{\psi'} = \left( \frac{2}{3} R_{\psi'}^2 b_{\psi'} g^2 \mathbf{p}^2 - 1 \right) F_0^{J/\psi}, \]
\[ F_1^{\psi'} = \frac{2}{3} R_{\psi'}^2 b_{\psi'} g^2 \mathbf{p}^2 + \left( \frac{2}{3} R_{\psi'}^2 b_{\psi'} g^2 \mathbf{p}^2 - 1 \right) F_1^{J/\psi} - \frac{8}{9} R_{\psi'}^2 b_{\psi'} g^2 \mathbf{p}^2 \left[ \frac{9}{4} b_{\psi'}^2 - b_{\psi'} \left(2 + \frac{5}{2} \lambda_2\right) + 2 \lambda_2 + \frac{1}{4} \lambda_2^2 \right], \]
\[ F_2^{\psi'} = \frac{2}{3} R_{\psi'}^2 g^2 \left[ -\frac{7}{2} b_{\psi'} + \frac{2}{3} + \frac{11}{6} \lambda_2 \right], \]
\[ (A.20) \]

and,
\[ F_0^{\psi''} = 2 b_{\psi''}^2 (1 - \lambda_2) \mathbf{p}^2 + 2 b_{\psi''} g^2 (\mathbf{p}^2)^2 (\mathbf{p}^2 - \lambda_2) ((3/2) b_{\psi''}^2 - (2 + \lambda_2) b_{\psi''} + 2 \lambda_2 - (1/2) \lambda_2^2), \]
\[ F_1^{\psi''} = g^2 \mathbf{p}^2 \left[ 14 b_{\psi''}^3 - b_{\psi''}^2 ((32/3) \lambda_2 + (37/3) \lambda_2) + b_{\psi''} ((28/3) \lambda_2 - (1/3) \lambda_2^2) \right], \]
\[ F_2^{\psi''} = g^2 \left[ 7 b_{\psi''} - (2/3) \lambda_2 - (4/3) \right]. \]
\[ (A.21) \]

In the expressions for the decay widths of the charmonium state, Ψ decaying to \( D^+ D^- (D^0 \bar{D}^0) \), the parameter, \( \gamma_\Psi \) is introduced, which refers to the production strength of \( \bar{D}D \) from decay of charmonium Ψ through light quark pair creation. In the \( ^3P_0 \) model, such a light quark-antiquark pair creation strength parameter, \( \gamma \) has been introduced \[5, 25, 27\]. The parameter, \( \gamma \) in the \( ^3P_0 \) model, as well as the parameter \( \gamma_\Psi \) in the model for composite hadrons \[28\] used in the present work, are chosen so as to reproduce the vacuum decay widths for the decay channels \( \psi'' \rightarrow D^+ D^- \) and \( \psi'' \rightarrow D^0 \bar{D}^0 \).

Appendix B: Charm mesons in the chiral effective model

We study the medium modifications of the masses of the open charm (\( D \) and \( \bar{D} \)) mesons and the charmonium states in the presence of a magnetic field, within a chiral effective model \[5, 18, 21\]. The model is a generalization of a chiral SU(3) model to SU(4) so as to include the interactions of the charmed mesons with the light hadronic sector.
The Lagrangian density of the chiral SU(3) model, in the presence of magnetic field, is given as

\[ \mathcal{L} = \mathcal{L}_{\text{kin}} + \sum_{W} \mathcal{L}_{BW} + \mathcal{L}_{\text{vec}} + \mathcal{L}_{0} + \mathcal{L}_{\text{scalebreak}} + \mathcal{L}_{SB} + \mathcal{L}_{\text{mag}}, \]  

where, \( \mathcal{L}_{\text{kin}} \) corresponds to the kinetic energy terms of the baryons and the mesons, \( \mathcal{L}_{BW} \) contains the baryons with the meson, \( W \) (scalar, pseudoscalar, vector, axialvector meson), \( \mathcal{L}_{\text{vec}} \) describes the dynamical mass generation of the vector mesons via couplings to the scalar fields and contains additionally quartic self-interactions of the vector fields, \( \mathcal{L}_{0} \) contains the meson-meson interaction terms \( \mathcal{L}_{\text{scalebreak}} \) is a scale invariance breaking logarithmic potential and \( \mathcal{L}_{SB} \) describes the explicit chiral symmetry breaking. The term \( \mathcal{L}_{\text{mag}}^{B\gamma} \), describing the interaction of the baryons with the electromagnetic field is given as

\[ \mathcal{L}_{\text{mag}}^{B\gamma} = -\bar{\psi}_{i} q_{i} \gamma^{\mu} A_{\mu} \psi_{i} - \frac{1}{4} \kappa_{i} \mu_{N} \bar{\psi}_{i} \sigma^{\mu\nu} F_{\mu\nu} \psi_{i}, \]  

where, \( \psi_{i} \) corresponds to the \( i \)-th baryon, with electric charge \( q_{i} \), \( A_{\mu} \) is the electromagnetic field, \( F^{\mu\nu} \) is the electromagnetic field tensor and \( \mu_{N} \) is the nuclear magneton. The tensorial interaction of baryons with the electromagnetic field given by the second term of the above equation accounts for the anomalous magnetic moments of the baryons (through the values of \( \kappa_{i} \)). The charged baryons have contributions from the Landau energy levels.

The concept of broken scale invariance of QCD is simulated in the effective Lagrangian model through the introduction of the scale breaking term

\[ \mathcal{L}_{\text{scalebreak}} = -\frac{1}{4} \chi^{4} \ln \left( \frac{\chi^{4}}{\chi_{0}^{4}} \right) + \frac{d}{3} \chi^{4} \ln \left( \frac{I_{3}}{\det \langle X \rangle_{0}} \right), \]  

where \( I_{3} = \det \langle X \rangle \), with \( X \) as the multiplet for the scalar mesons. Equating the trace of the energy momentum tensor of QCD in the massless quarks limit to that of the chiral effective model, and, using the one loop beta function for \( N_{c}=3 \) and \( N_{f}=3 \), leads to the relation of the scalar gluon condensate to the dilaton field as given by

\[ \left\langle \frac{\alpha_{s}}{\pi} G_{\mu\nu}^{a} G^{\mu\nu a} \right\rangle = \frac{8}{9} (1 - d) \chi^{4}. \]  

The scalar fields (non-strange scalar-isoscalar, field \( \sigma \), non-strange scalar isovector, \( \delta \), the strange field \( \zeta \)) and the scalar dilaton field, \( \chi \) are solved from their equations of motion in
isospin asymmetric nuclear matter in the presence of magnetic fields. For given values of
the baryon density, $\rho_B$, the isospin asymmetry parameter, $\eta = (\rho_n - \rho_p)/(2\rho_B)$ (where $\rho_n$ and $\rho_p$ are
the number densities of the neutron and the proton respectively) and magnetic field, the values
of the scalar fields are obtained. We neglect the small isospin effect arising from the difference
in the current quark masses of the u and d quarks. The calculations for the scalar fields are
carried out in the mean field approximation, where the meson fields are replaced by their
expectation values. In addition, we also use the approximations that $\bar{\psi}_i \psi_j = \delta_{ij} \langle \bar{\psi}_i \psi_i \rangle$ and
$\bar{\psi}_i \gamma^\mu \psi_j = \delta_{ij} \delta^{\mu 0} \langle \bar{\psi}_i \gamma^0 \psi_i \rangle$, where, $\rho^s_i$ and $\rho_i$ are the scalar and number density
of $i$-th baryon. The values of the scalar fields in the (magnetized) hadronic medium are used
to obtain the in-medium masses of the $D$ and $\bar{D}$ mesons as well as the charmonium states.

The open charm $D$ and $\bar{D}$ mesons are studied within the chiral model by generalizing chiral
SU(3) to chiral SU(4) to obtain the interactions of the charmed mesons with the light hadrons.
As the baryons belong to 20-plet and mesons to the 16-plet representation in SU(4), the baryons
are represented by the tensor $B^{ijk}$ (which are antisymmetric in the first two indices), to derive
the interactions of the pseudoscalar $D$ and $\bar{D}$ mesons with the baryons. The in-medium
masses of these open charm mesons are computed from their interactions with the baryons and
the scalar mesons, arising from the leading Weinberg-Tomozawa vectorial interaction with the
baryons, the scalar exchange interactions and the range terms. The dispersion relations for
the $D$ and $\bar{D}$ mesons are obtained from the Fourier transformations of the equations of motion
of these mesons. In the presence of the magnetic field, the charged $D^{\pm}$ mesons have positive
contributions to their masses from the Landau levels, and we retain the lowest Landau level
contribution in the masses of the charged $D$ and $\bar{D}$ mesons.

The mass shift of the charmonium state, $\Psi (J/\psi, \psi(3686) \text{ and } \psi(3770))$ is obtained from
the medium modification of the scalar gluon condensate, which is computed from the medium
change of the dilaton field calculated within the chiral effective model, using the equation (B.4).
The shift in the charmonium mass is given as

$$\Delta m_\Psi = \frac{4}{81} (1 - d) \int dk^2 \langle \left| \frac{\partial \psi(\vec{k})}{\partial \vec{k}} \right|^2 \rangle \frac{k}{k^2/m_c + \epsilon} \left( \chi^4 - \chi^0 \right),$$

(B.5)
where

$$\langle |\frac{\partial \psi(\vec{k})}{\partial \vec{k}}|^2 \rangle = \frac{1}{4\pi} \int |\frac{\partial \psi(\vec{k})}{\partial \vec{k}}|^2 d\Omega,$$

(B.6)

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