Multiplierless and Sparse Machine Learning
based on Margin Propagation Networks

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Abstract—The new generation of machine learning processors have evolved from multi-core and parallel architectures (for example graphical processing units) that were designed to efficiently implement matrix-vector-multiplications (MVMs). This is because at the fundamental level, neural network and machine learning operations extensively use MVM operations and hardware compilers exploit the inherent parallelism in MVM operations to achieve hardware acceleration on GPUs, TPUs and FPGAs. A natural question to ask is whether MVM operations are even necessary to implement ML algorithms and whether simpler hardware primitives can be used to implement an ultra-energy-efficient ML processor/architecture. In this paper we propose an alternate hardware-software codesign of ML and neural network architectures where instead of using MVM operations and non-linear activation functions, the architecture only uses simple addition and thresholding operations to implement inference and learning. At the core of the proposed approach is margin-propagation based computation that maps multiplications into additions and additions into a dynamic rectifying-linear-unit (ReLU) operations. This mapping results in significant improvement in computational and hence energy cost. The training of a margin-propagation (MP) network involves optimizing an $L_1$ cost function, which in conjunction with ReLU operations leads to network sparsity and weight updates using only Boolean predicates. In this paper, we show how the MP network formulation can be applied for designing linear classifiers, multi-layer perceptrons and for designing support vector networks.

Index Terms—Margin Propagation, Low Power, Machine learning, Multi-layer Perceptron, Support Vector Machine, Approximate Computing.

1 INTRODUCTION

Reducing the energy footprint is one of the major goals in the design of current and future machine learning (ML) systems. This is not only applicable for deep-learning platforms that run on data servers, consuming mega-watts of power [1], but is also applicable for Internet-of-things and edge computing platforms that are highly energy-constrained [2]. Computation in most of these ML systems are highly regular and involve repeated use of matrix-vector-multiplication (MVM) and non-linear activation and pooling operations. Therefore, current hardware compilers achieve performance acceleration and energy-efficiency by optimizing these fundamental operations on parallel hardware like the Graphical Processing Units (GPUs) or the Tensor Processing Units (TPUs). This mapping onto hardware accelerators can be viewed as a top-down approach where the goal from the perspective of a hardware designers to efficiently but faithfully map well-established ML algorithms without modifying the basic MVM or the activation functions. However, if the MVMs and the non-linear activation functions could be combined in a manner that the resulting architecture becomes multiplier-less and uses much simpler computational primitives, then significant energy-efficiency could be potentially achieved at the system-level. In this paper we argue that a margin-propagation (MP) based computation can achieve this simplification by mapping multiplications into additions and additions into a dynamic rectifying-linear-unit (ReLU) operations.

The consequence of this mapping is a significant reduction in the complexity of inference and training which in turn leads to significant improvement in system energy-efficiency. To illustrate this, consider a very simple example as shown in Fig.1(a) and (b) for a comprising of a single training parameter $w$ and a one-dimensional input $x$. In a conventional architecture minimizing a loss-function $E(.)$ in Fig. 1(a) results in a learning/parameter update step that requires modulating the gradient with the input. In the equivalent margin-approximation, as shown in Fig. 1(b), the absence of multiplication implies that each parameter update is independent and the use of ReLU operations leads to learning update that involves only Boolean predicates. Rather than modulating the gradient with the input (as shown in Fig.1(a)), the new updates are based on comparing the sum of $w$ and $x$ with respect to a dynamic threshold $z$, as shown in Fig. 1(b). This significantly simplifies the learning phase, and the storage of the parameters $w$. This is illustrated in Fig.1(c) using a single-layer network with three-dimensional input/parameters. The margin nodes not only implement the forward computation but also provide a continuous feedback to updates parameters $w_{11} - w_{13}$. For a digital implementation, this could be a simple up/down flag; for an analog implementation this could be equivalent to charging or discharging a capacitor storing the values of $w_{11} - w_{13}$.

Margin-propagation (MP) was originally proposed in [3] and then was used in [4], [5] in the context of approximate computing and synthesis of piece-wise linear circuits. In [4], [5], [6], [7], [8] the MP formulation was used to synthesize ML algorithms, by replacing the MVM oper-
In order to learn complex functions, a group of perceptrons can be stacked up in multiple layers to form a multilayer perceptron (MLP). A three layer MLP for a two class problem is shown in Fig. 3. The weighted sum of the input vector $\vec{x} = \{x_i\}; 1 \leq i \leq N$ with the weights $\vec{w} = \{w_i\}; 1 \leq i \leq N; 1 \leq j \leq M$ of the hidden layer is the input to the activation functions in the hidden layer. In the figure, $b_j, 1 \leq j \leq M$ and $b_k, k = 1$ indicates the input bias to each node in the hidden layer and output layer, respectively. The activation function in the hidden layer is used as a linear binary classifier as shown in Fig. 2. Let input vector to a perceptron be $\vec{x} = \{x_i\}; 0 \leq i \leq N; x_0$ is the bias. The weighted sum of these inputs and the bias with the weights $\vec{w} = \{w_i\}; 0 \leq i \leq N$ is taken which is then fed into the activation function which maps the input into one of the two classes. For learning the perceptron weights standard gradient descent can be used with sum of squared errors as our cost function as given below;

$$E(\vec{w}) = \frac{1}{2} \sum_n [y_n - \hat{y}_n]^2$$

where $y_n$ is the actual output for sample $n$ and $\hat{y}_n$ is the estimated output.

Support Vector Machine (SVM) is a supervised machine learning algorithm which is used mostly for classification problems. Given labeled training data, SVM outputs an optimal hyperplane which categorizes any new test input into one of the classes. Given a test input $x_i; 1 \leq i \leq N$ where $x_i \in \mathbb{R}$, the decision function for SVM is given as,

$$f(x_i) = \sum_s w_s K(x_s, x_i) = \sum_s (w_s^+ - w_s^-)(K_s^+ - K_s^-)$$

where $K$ is the kernel function, $x_s$ is the $s^{th}$ support vector and $x_i$ is the $i^{th}$ sample of the input vector.

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where $K$ is the kernel function, $x_s$ is the $s^{th}$ support vector and $x_i$ is the $i^{th}$ sample of the input vector.
learned using the backpropagation algorithm. In this case also a squared error cost function is used.

$$E(\overline{w}) = \frac{1}{2} \sum_{n} \sum_{k} [y_{nk} - \hat{y}_{nk}]^2$$  \hspace{1cm} (3)

where \( k \in \text{output} = 1 \), in this case.

2 Margin Propagation Computation and Complexity

MP algorithm is based on the reverse water filling procedure \([4, 5]\) as shown in Fig. 4. The algorithm computes the normalization factor \( z \), given a set of scores \( L_i \in \mathbb{R}, 1 \leq i \leq N \) using the constraint;

$$\sum_{i=1}^{N} [L_i - z]_+ = \gamma$$  \hspace{1cm} (4)

where \([.]_+ = \max(., 0)\) is the rectification operation and \( \gamma \) is the algorithm parameter. This is a recursive algorithm which computes \( z \) such that the net balance of score \( L_i \) in excess to \( z \), is \( \gamma \) \([4, 5]\). Thus given a set of input scores \( L_i \), we can obtain the factor \( z \) as;

$$z = \text{MP}(L, \gamma)$$  \hspace{1cm} (5)

where \( L = \{L_i\}; 1 \leq i \leq N \)

2.1 Complexity

As mentioned before replacing the MVM operations in the perceptron, SVM and MLP into simple addition and thresholding operations in the log-likelihood domain using MP algorithm during inference and learning, significantly reduces the complexity. If \( N \) is the dimension of the input vector \( \overline{x} \), then the overall complexity for an MVM operation,

$$z = \sum_{i=1}^{N} w_i x_i$$  \hspace{1cm} (6)

is

$$C_{\text{MVM}} = N \times C_{M} + N \times C_{A}$$  \hspace{1cm} (7)

$$= N \times O(d^2) + N \times O(d)$$  \hspace{1cm} (8)

where \( C_{\text{MVM}}, C_{M} \) and \( C_{A} \) are the complexity of MVM, multiplication and addition and \( d \) is the number of digits.

whereas for the margin propagation algorithm

$$\sum_{i=1}^{N} [w_i + x_i - z]_+ = \gamma$$  \hspace{1cm} (9)

the overall complexity is given as,

$$C_{\text{MP}} = N \times C_{A} + K \times \log(N) \times C_{C}$$  \hspace{1cm} (10)

$$= N \times O(d) + K \times \log(N) \times O(d)$$  \hspace{1cm} (11)

where \( K \) is the sparsity factor of the thresholding operation determined by \( \gamma \). This will also result in significant improvement in energy cost, as energy per multiplication is more than energy per addition operation as explored in \([12]\). In \([12]\), they show that for an 8 bit integer multiplication the rough energy cost is 0.2\( pJ \) with a relative area cost of 282\( \mu m^2 \) whereas for an 8 bit addition it is only 0.03\( pJ \) and 36\( \mu m^2 \). For 32 bit integer case, the energy cost is 3.1\( pJ \) and area cost is 3495\( \mu m^2 \) for multiplication and 0.1\( pJ \) and 137\( \mu m^2 \) for addition. The \( L_1 \) cost function used in conjunction with the ReLU operation ensures network sparsity.

3 Perceptron using MP algorithm

A single layer perceptron using MP algorithm is shown in Fig. 5. We minimize the \( L_1 \) norm given in eq. 16 as the cost function to learn the network parameters. The inputs and weights are in the log-likelihood domain so that the network can be implemented using MP algorithm as mentioned in \([6]\).

3.1 Inference

Let the input vector to the perceptron in the log-likelihood domain be \( x = \{x_i\}; 1 \leq i \leq N \) and let \( \{w_i\} \) be the learned weights.

From Fig 5 the perceptron output in differential form is,

$$p(x) = p^+ - p^-$$  \hspace{1cm} (12)

For the output node;

$$p^+ = [z^+ - z]_+$$

$$p^- = [z^- - z]_-$$  \hspace{1cm} (13)

where \( z \) is estimated such that \( p^+ + p^- = 1 \implies z = \text{MP}([z^+, z^-], 1) \). \( z^+ \) and \( z^- \) are computed using the reverse water-filling constraints as;
\[\sum_i [w_i^+ + x_i^+ - z_i^+]_+ + [w_i^- + x_i^- - z_i^-]_+ + [b^+ - z_i^+]_+ = \gamma \]  
\[\sum_i [w_i^+ + x_i^+ - z_i^-]_+ + [w_i^- + x_i^- - z_i^+]_+ + [b^- - z_i^-]_+ = \gamma \]

where \(x_i\) is the input sample and \(w_i\) is the corresponding weight in the log-likelihood domain.

### 3.2 Training: evaluation of error-function derivatives

Considering a two class problem class+ and class−, the error function can be written as:

\[E = \sum_n [y_i^+ - p^+] + [y_n^- - p^-] \]  
\[\text{from eq. (16)} \]

where

\[y_i^+\] : label for class+ for \(n\)th sample

\[y_n^-\] : label for class− for \(n\)th sample

\[y_n^+ + y_n^- = 1 \]

Using the error gradient obtained from above, the weight and bias are updated during each iteration as follows;

\[\frac{\partial E}{\partial w_i} = \sum_n \text{sign}(p^+ - y_i^+) \frac{\partial p^+}{\partial w_i} + \text{sign}(p^- - y_n^-) \frac{\partial p^-}{\partial w_i} \]  
\[\text{from eq. (17)} \]

\[\frac{\partial E}{\partial b_i} = \sum_n \text{sign}(p^+ - y_i^+) \frac{\partial p^+}{\partial b_i} + \text{sign}(p^- - y_n^-) \frac{\partial p^-}{\partial b_i} \]

As \(\frac{\partial p^+}{\partial b_i} = 0\)

Similarly using (13), (14), (18) and (19)

\[\frac{\partial p^+}{\partial w_i} = \left(1 - \frac{1}{\#\text{active}}\right) \frac{\partial E}{\partial w_i} (z^+ > z) \]  
\[\text{from eq. (20)} \]

\[\frac{\partial p^-}{\partial w_i} = \left(1 - \frac{1}{\#\text{active}}\right) \frac{\partial E}{\partial w_i} (z^- > z) \]  
\[\text{from eq. (21)} \]

Substituting (20) and (21) in (17) we get, \(\frac{\partial E}{\partial w_i}\)

Similarly,

\[\frac{\partial E}{\partial w_i} = \sum_n \text{sign}(p^+ - y_i^+) \frac{\partial p^+}{\partial w_i} + \text{sign}(p^- - y_n^-) \frac{\partial p^-}{\partial w_i} \]  
\[\text{from eq. (22)} \]

where,

\[\frac{\partial p^+}{\partial w_i} = \left(1 - \frac{1}{\#\text{active}}\right) \frac{\partial E}{\partial w_i} (x_i^+ + w_i^+ > z^+) \]  
\[\text{from eq. (23)} \]

\[\frac{\partial p^-}{\partial w_i} = \left(1 - \frac{1}{\#\text{active}}\right) \frac{\partial E}{\partial w_i} (x_i^- + w_i^- > z^-) \]  
\[\text{from eq. (24)} \]

### 3.3 Parameter update rule

Using the error gradient obtained from above, the weight and bias are updated during each iteration as follows;

\[(\tau)w_i^+ = (\tau-1)w_i^+ - \epsilon \frac{\partial E}{\partial w_i} \]  
\[\text{from eq. (29)} \]

\[(\tau)w_i^- = (\tau-1)w_i^- - \epsilon \frac{\partial E}{\partial w_i} \]  
\[\text{from eq. (30)} \]

\[(\tau)b^+ = (\tau-1)b^+ - \epsilon \frac{\partial E}{\partial b^+} \]  
\[\text{from eq. (31)} \]

\[(\tau)b^- = (\tau-1)b^- - \epsilon \frac{\partial E}{\partial b^-} \]  
\[\text{from eq. (32)} \]

where \(\epsilon\) is the learning rate and \(\tau\) indicates the iteration step.

### 3.4 Implementation and results

|       | Overall | Class 1 | Class 2 | Overall | Class 1 | Class 2 |
|-------|---------|---------|---------|---------|---------|---------|
| Train | 99      | 99      | 99      | 100     | 100     | 100     |
| Test  | 99      | 100     | 100     | 100     | 100     | 100     |

**Table 1:** Perceptron classification accuracy for the synthetic train and test data

The formulation is sec. 3 is implemented and results are evaluated using MATLAB. A linearly separable Markovian data is simulated using MATLAB functions for training and testing. We use 100 data samples as train set and 100 samples as test set.

#### 3.4.1 Results and discussion

Figure 6 shows the scatter plot of the linearly separable two class training and test data. The training curve is shown in Fig. 6a which shows that the cost function value reduces during each iteration. The algorithm performs really well as can be seen from the classification results in Table 1. The contour plot of the inference results are also shown in Fig. 6c
4 MULTILAYER PERCEPTRON BASED ON MP ALGORITHM

Figure 7 shows an MLP synthesized using MP algorithm. The network consists of an input layer $I$, a hidden layer $J$ and an output layer $K$ with 2 nodes in the hidden layer. The network parameters are learned by minimizing the $l_1$ norm cost function as shown in (40). We use an algorithm similar to backpropagation to evaluate the error gradient in order to update the network parameters. The red arrows indicate the backward propagation of error information w.r.t the weights $w_{11}$ and $w_{11}$.

4.1 Inference

Let the input vector in the log-likelihood domain be $x = \{x_i\}; 1 \leq i \leq N$. Let $\{w_{ij}\}$ and $\{w_{jk}\}$ be the set of learned weights from node $i$ of layer $I$ to the node $j$ in layer $J$ and node $j$ of layer $J$ to the node in output layer $K$ respectively.

From Fig. 7 the output in differential form is,

$$ p(x) = p_k^+ - p_k^- $$

(33)

For the output layer $K$,

$$ p_k^+ = [z_k^+ - z_k^-] $$

$$ p_k^- = [z_k^- - z_k^-] $$

(34)

where $z_k$ is estimated such that $p_k^+ + p_k^- = 1 \Rightarrow z_k = MP(\{z_k^+, z_k^-\}, 1)$ and $z_k^+$ and $z_k^-$ are computed using

$$ \sum_i [w_{jk}^+ p_j^+ - z_k^+] + [w_{jk}^- p_j^- - z_k^-] + [b_j^- - z_k^-] = \gamma_k $$

(35)

$$ \sum_i [w_{jk}^+ p_j^+ - z_k^-] + [w_{jk}^- p_j^- + z_k^-] + [b_j^+ - z_k^-] = \gamma_k $$

(36)

Similarly

For the hidden layer $J$,

$$ p_j^+ = [z_j^+ - z_j^-] $$

$$ p_j^- = [z_j^- - z_j^-] $$

(37)

where $z_j$ is estimated such that $p_j^+ + p_j^- = 1 \Rightarrow z_j = MP(\{z_j^+, z_j^-\}, 1)$

where,

$$ \sum_j [w_{ij}^+ p_i^+ - x_j^+] + [w_{ij}^- p_i^- - x_j^-] + [b_i^+ - x_j^-] = \gamma_j $$

(38)

$$ \sum_i [w_{ij}^+ x_j^+ - z_j^-] + [w_{ij}^- x_j^- - z_j^-] + [b_j^- - z_j^-] = \gamma_j $$

(39)

4.2 Training: evaluation of error-function derivatives

Considering a two class problem class+ and class-, the error function can be written as;

$$ E = \sum_n |y_{nk}^+ - p_k^+| + |y_{nk}^- - p_k^-| $$

(40)

where

$y_{nk}^+$: label for class+ for $n^{th}$ sample

$y_{nk}^-$: label for class- for $n^{th}$ sample

$y_{nk}^+ + y_{nk}^- = 1$

Output layer $K$

From eq. (40)

$$ \frac{dE}{dw_{jk}} = \sum_n \text{sign}(y_{nk}^+ - y_{nk}^-) \frac{d p_k^+}{dw_{jk}} + \text{sign}(p_k^- - y_{nk}^-) \frac{d p_k^-}{dw_{jk}} $$

(41)

Using equations (34), (35), (18) and (19)

$$ \frac{d p_k^+}{dw_{jk}} = \left(1 - \frac{1}{\#active_{k}}\right) \mathbb{1}(z_k^+ > z_k) \frac{1}{\#active_{p_k}} \mathbb{1}(p_j^+ + w_{jk}^+ > z_k^+) $$

(42)

Similarly using (34), (36), (18) and (19)

$$ \frac{d p_k^-}{dw_{jk}} = \left(1 - \frac{1}{\#active_{k}}\right) \mathbb{1}(z_k^- > z_k) \frac{1}{\#active_{p_k}} \mathbb{1}(p_j^- + w_{jk}^- > z_k^-) $$

(43)

Substituting (42) and (43) in (41) we get, $\frac{dE}{dw_{jk}}$

Similarly,

$$ \frac{dE}{dw_{jk}} = \sum_n \text{sign}(p_k^- - y_{nk}^-) \frac{d p_k^+}{dw_{jk}} + \text{sign}(p_k^+ - y_{nk}^+) \frac{d p_k^-}{dw_{jk}} $$

(44)

where

$$ \frac{d p_k^-}{dw_{jk}} = \left(1 - \frac{1}{\#active_{k}}\right) \mathbb{1}(z_k^+ > z_k) \frac{1}{\#active_{p_k}} \mathbb{1}(p_j^- + w_{jk}^- > z_k^+) $$

(45)
\[
\frac{\partial p_k^-}{\partial w_{jk}} = \left(1 - \frac{1}{\text{active}}\right) \text{I}(z_k^- > z_k) \frac{1}{\text{active}} \text{I}(p_j^+ + w_{jk} > z_k^-)
\]

(46)

4.2.1 Derivatives with respect to bias

From eq. (40)

\[\frac{\partial E}{\partial b_k} = \sum_n \text{sign}(p_k^+ - y_{nk}) \frac{\partial p_k^+}{\partial b_k}\]

(47)

As \(\frac{\partial p_k^+}{\partial b_k} = 0\)

Using equations (34), (35), (18) and (19)

\[\frac{\partial p_k^+}{\partial b_k} = \left(1 - \frac{1}{\text{active}}\right) \text{I}(z_k^+ > z_k) \frac{1}{\text{active}} \text{I}(b_k^+ > z_k^+)\]

(48)

Similarly,

\[\frac{\partial E}{\partial b_k} = \sum_n \text{sign}(p_k^- - y_{nk}) \frac{\partial p_k^-}{\partial b_k}\]

(49)

Using (34), (36), (18) and (19)

\[\frac{\partial p_k^-}{\partial b_k} = \left(1 - \frac{1}{\text{active}}\right) \text{I}(z_k^- > z_k) \frac{1}{\text{active}} \text{I}(b_k^- > z_k^-)\]

(50)

Hidden layer \(J\)

From (40)

\[\frac{\partial E}{\partial w_{ij}} = \sum_n \text{sign}(p_k^+ - y_{nk}) \frac{\partial p_k^+}{\partial w_{ij}} + \text{sign}(p_k^- - y_{nk}) \frac{\partial p_k^-}{\partial w_{ij}}\]

(51)

Using equations (34), (35), (37), (38) and (39) we get,

\[\frac{\partial p_k^+}{\partial w_{ij}} = \frac{\partial p_k^+}{\partial z_k^+} \frac{\partial z_k^+}{\partial w_{ij}} + \frac{\partial p_k^+}{\partial z_k^-} \frac{\partial z_k^-}{\partial w_{ij}}\]

(52)

Using equations (34), (36), (37), (38) and (39) we get,

\[\frac{\partial p_k^-}{\partial w_{ij}} = \left(1 - \frac{1}{\text{active}}\right) \text{I}(z_k^+ > z_k) \frac{1}{\text{active}} \text{I}(p_j^+ + w_{jk} > z_k^-)\]

(53)

Similarly,

\[\frac{\partial E}{\partial w_{ij}} = \sum_n \text{sign}(p_k^+ - y_{nk}) \frac{\partial p_k^+}{\partial w_{ij}} + \text{sign}(p_k^- - y_{nk}) \frac{\partial p_k^-}{\partial w_{ij}}\]

(54)

where,

\[\frac{\partial p_k^+}{\partial w_{ij}} = \frac{\partial p_k^+}{\partial z_k^+} \frac{\partial z_k^+}{\partial w_{ij}} + \frac{\partial p_k^+}{\partial z_k^-} \frac{\partial z_k^-}{\partial w_{ij}}\]
4.2.2 Derivatives with respect to bias

\[
(1 - \frac{1}{\#active_k}) \mathbb{I}(z_k^+ > z_k) \frac{1}{\#active_p_k} \mathbb{I}(p_j^+ + w_j^k > z_k^+)
\]
\[
(1 - \frac{1}{\#active_j}) \mathbb{I}(z_j^+ > z_j) \frac{1}{\#active_p_j} \mathbb{I}(x_j^+ + w_j^k > z_j^+)
\]
\[
+ (1 - \frac{1}{\#active_k}) \mathbb{I}(z_k^- > z_k) \frac{1}{\#active_p_k} \mathbb{I}(p_j^- + w_j^k > z_k^-)
\]
\[
(1 - \frac{1}{\#active_j}) \mathbb{I}(z_j^- > z_j) \frac{1}{\#active_p_j} \mathbb{I}(x_j^- + w_j^k > z_j^-)
\]

(55)

\[
\frac{\partial p_k^-}{\partial w_{ij}} =
\]
\[
(1 - \frac{1}{\#active_k}) \mathbb{I}(z_k^- > z_k) \frac{1}{\#active_p_k} \mathbb{I}(p_j^- + w_j^k > z_k^-)
\]
\[
(1 - \frac{1}{\#active_j}) \mathbb{I}(z_j^- > z_j) \frac{1}{\#active_p_j} \mathbb{I}(x_j^- + w_j^k > z_j^-)
\]
\[
+ (1 - \frac{1}{\#active_k}) \mathbb{I}(z_k^- > z_k) \frac{1}{\#active_p_k} \mathbb{I}(p_j^- + w_j^k > z_k^-)
\]
\[
(1 - \frac{1}{\#active_j}) \mathbb{I}(z_j^- > z_j) \frac{1}{\#active_p_j} \mathbb{I}(x_j^- + w_j^k > z_j^-)
\]

(56)

4.3 Parameter update rule

The weight and bias are updated using the obtained error gradient during each iteration as follows;

\[
(w_j^-)_\tau = (w_j^-)_{\tau-1} - \epsilon \frac{\partial E}{\partial w_{ij}^-}
\]

(63)

\[
(w_j^+)_\tau = (w_j^+)_{\tau-1} - \epsilon \frac{\partial E}{\partial w_{ij}^+}
\]

(64)

\[
(w_k^-)_\tau = (w_k^-)_{\tau-1} - \epsilon \frac{\partial E}{\partial w_{jk}^-}
\]

(65)

\[
(w_k^+)_\tau = (w_k^+)_{\tau-1} - \epsilon \frac{\partial E}{\partial w_{jk}^+}
\]

(66)

\[
(b_j^-)_\tau = (b_j^-)_{\tau-1} - \epsilon \frac{\partial E}{\partial b_j^-}
\]

(67)

\[
(b_j^+)_\tau = (b_j^+)_{\tau-1} - \epsilon \frac{\partial E}{\partial b_j^+}
\]

(68)

\[
(b_k^-)_\tau = (b_k^-)_{\tau-1} - \epsilon \frac{\partial E}{\partial b_k^-}
\]

(69)

\[
(b_k^+)_\tau = (b_k^+)_{\tau-1} - \epsilon \frac{\partial E}{\partial b_k^+}
\]

(70)

where \(\epsilon\) is the learning rate and \(\tau\) indicates the iteration step.
4.4 Implementation and results

We use a synthetic non-linearly separable xor data for evaluating our MLP formulation as well. The train and test set consists of 100 samples each. The network consists of a single hidden layer with 2 neurons and an input and output layer.

4.4.1 Results and discussion

The scatter plot of the training and test set is shown in Fig. 8a. The training curve in Fig. 8b shows a decreasing cost function per iteration and converging after a point which ensures the learning of the network. Figure 8c and the table shows the classification accuracies of our MLP algorithm on the xor dataset. The algorithm proves to be effective as a non-linear binary classifier as can be seen from the results.

|        | Overall | Class 1 | Class 2 | Overall | Class 1 | Class 2 |
|--------|---------|---------|---------|---------|---------|---------|
| Accuracy (%) | 98      | 100     | 97      | 97      | 97      | 98      |

Table 2: MLP classification accuracies for the synthetic train and test data

5 SVM based on MP algorithm

We also implement a support vector machine (SVM) using the MP algorithm for a two class non linearly separable problem. We use a Cauchy kernel and by choosing the appropriate normalization, parameters are converted into positive to apply MP approximation.

The formulation is as follows;

For a given input  \( \hat{x} \),

\[
    f(\hat{x}) = \sum_s \hat{w}_s \hat{K}(\hat{x}_s, \hat{x}) = \sum_s (\hat{w}_s^+ - \hat{w}_s^-) (\hat{K}_s^+ - \hat{K}_s^-) \quad (71)
\]

where  \( \hat{K} \) is the kernel function,  \( \hat{x}_s \) is the  \( s \)th support vector and  \( \hat{x} \) is the input sample (Here  \( \hat{\cdot} \) indicates that the parameters are not in the log likelihood domain).

\[
    f^+ - f^- = \left( \frac{\hat{w}_s^+ \hat{K}_s^+ + \hat{w}_s^- \hat{K}_s^-}{\hat{K}_s^-} \right) - \left( \frac{\hat{w}_s^+ \hat{K}_s^+ + \hat{w}_s^- \hat{K}_s^-}{\hat{K}_s^+} \right) \quad (72)
\]

Converting into log likelihood domain:

\[
    L_f^+ - L_f^- = \log \left( \sum_s e^{\hat{w}_s^+ + \hat{K}_s^+} + e^{\hat{w}_s^- + \hat{K}_s^-} \right) - \log \left( \sum_s e^{\hat{w}_s^+ + \hat{K}_s^+} + e^{\hat{w}_s^- + \hat{K}_s^-} \right) \quad (73)
\]

The above can be approximated using MP algorithm as:

\[
    L_f^+ - L_f^- = MP \left( \{ w_s^+ + K_s^+, w_s^- + K_s^- \}, \gamma \right) - MP \left( \{ w_s^+ + K_s^+, w_s^- + K_s^- \}, \gamma \right) \quad (74)
\]

The formulation in eq. (74) is similar to that in eqs. (14) and (15). Hence the parameter update rules are similar to that of perceptron defined in sec. 3.2 using  \( l_1 \) norm (16) as the cost function.

5.1 Kernel function

\[
    K(\hat{x}_s, \hat{x}) = K_s^+ - K_s^- \quad (75)
\]

We use a Cauchy kernel function given as,

\[
    \hat{K}(\hat{x}_s, \hat{x}) = \frac{1}{c + ||\hat{x}_s - \hat{x}||^2} \quad (76)
\]

Ensuring  \( ||\hat{x}|| < 1 \) or  \( \hat{x} = \hat{x}^+ - \hat{x}^- \) such that  \( \hat{x}^+ + \hat{x}^- = 1 \) we get,

\[
    \sum_i (\hat{x}_s^i - \hat{x}_i)^2 = \sum_i (\hat{x}_s^+ + \hat{x}_s^- - \hat{x}_s^+ - \hat{x}_s^-)^2 = \sum_i (\hat{x}_s^+ + \hat{x}_s^- - \hat{x}_s^+ - \hat{x}_s^-)^2 = -2\hat{x}_s^+ \hat{x}_s^- + 2\hat{x}_s^+ \hat{x}_s^- + 2\hat{x}_s^+ \hat{x}_s^+ - 2\hat{x}_s^- \hat{x}_s^- \quad (77)
\]

Here  \( \hat{x}_s^i \) indicates the  \( i \)th sample of the support vector  \( \hat{x}_s \)

Each of the terms in eq. (77) is added with a constant  \( c \) as per eq. (76). By choosing appropriate value for  \( c \), parameters can be converted to positive values to apply MP approximation as given below;

Consider the negative term

\[
    -2\hat{x}_s^+ \hat{x}_s^- + 2 \implies -2\hat{x}_s^+ \hat{x}_s^- + 2(\hat{x}_s^+ + \hat{x}_s^-) \implies 2\hat{x}_s^+ + 2\hat{x}_s^- (1 - \hat{x}_s^+) \implies 2\hat{x}_s^- + 2\hat{x}_s^+ \quad (78)
\]
which ensures all such terms to be positive and hence MP approximation can be applied.

Converting into the log-likelihood domain

$$\log \left[ \frac{1}{C + \sum_i (\hat{x}_i - \bar{x})^2} \right] = -\log \left[ \sum_i \hat{x}_i^+ \hat{x}_i^- + \hat{x}_i^+ \hat{x}_i^+ \cdots \right]$$

$$-\log \left[ \sum_i e^{\hat{x}_i^+} + e^{\hat{x}_i^+} + e^{\hat{x}_i^-} + \cdots \right]$$

(79)

Hence

$$K_s = MP \left\{ \{x_i^+, x_i^+, x_i^+, x_i^+ \cdots \} , \gamma_2 \right\}$$

(80)

5.2 Implementation and results

For evaluating our SVM formulation explained in sec 5 we use a synthetic non-linearly separable xor data for training and inference using MATLAB. In this case also we use 100 train and test samples each.

5.2.1 Results and discussion

The scatter plot of the dataset and the training curve during learning is shown in Figs. and . It can be observed from the training curve that the cost function value reduces during each iteration and converges afterwards ensuring the learning of the network. The classification accuracies for the train and test data and the contour plot of the inference results shown in Table and Fig. shows the effectiveness of the algorithm as a non-linear classifier.

|          | Train | Test |
|----------|-------|------|
|          | Overall | Class 1 | Class 2 | Overall | Class 1 | Class 2 |
| Accuracy (%) | 97 | 99 | 96 | 100 | 100 | 100 |

TABLE 3: SVM classification accuracies for the synthetic train and test data

6 CONCLUSION

In this paper we proposed an alternate hardware-software codesign of ML and neural network architectures. The architecture only uses simple addition and thresholding operations to implement inference and learning instead of using MVM operations and non-linear activation functions. The margin-propagation based computation maps multiplications into additions and additions into a dynamic rectifying-linear-unit (ReLU) operation which results in significant improvement in computational and hence energy cost. The formulation also enables network sparsity. We showed the application of MP formulation for the design of linear classifiers, multi-layer perceptrons as well as support vector machines and evaluated the performance of the same on some synthetic data.

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