Ability to damp traffic wave when controlling every car on the road by FollowerStopper controller

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Abstract. A small fluctuation in velocities of the vehicles in a traffic system can cause a traffic jam which travels through the road as a stop-and-go wave. FollowerStopper controller is a strategy used in self-driving cars, designed for damping stop-and-go-wave. The previous study shows that controlling only one car can damp the wave in the whole circuit road. Interestingly, in some values of car density, we have found that using FollowerStopper on every car on the road cannot damp the stop-and-go wave. In this article, we consider the effect of the density on the ability to damp the wave when using the FollowerStopper on every car. To control the density, we run simulations in a circuit road. The ability to damp the wave is reflected as the standard deviation and average velocity of every car. We carried out the simulations of 24 cars running on a circuit road in different circumferences. Our simulation shows that when desired speed is at 7.00 m/s, traffic wave is completely dissipated while density is under 0.06 cars per meter.

1. Introduction

Stop-and-go waves or traffic waves occur and travel as a wave through the road, which, on the way, disturbs the value of car density. These waves can cause traffic jam. Principally, these waves can be emerged by the high car density and human behaviours. Human tends to perform over acceleration which perturbs the traffic system and leads to traffic jam [1]. Raphael Stern et al. designed the controllers to damp traffic waves, including the FollowerStopper controller. This controller can make stop-and-go-wave dissipate by controlling only one car for 22 cars on a closed-circuit road [2]. This represents the possibility to design a controller for damping stop-and-go wave in the real-world scenarios.

In this article, we investigate the ability to damp traffic wave when every car on a closed-circuit road is controlled by FollowerStopper controller. It is hypothesized that the ability has some correlation with the car density.
Figure 1. The simulations repeat these step with different road length. (a) placing 24 cars on the road uniformly. (b) waiting for traffic jam emergence. (c) turning the self driving mode on.

2. Traffic system simulation
We simulate the experiments of 24 cars running on the closed-circular track. Figure 1 is a scheme of our experiment. In each experiment, the length of circumference of circuit road is randomly picked from uniform distribution in range 144 meters to 600 meters. In the beginning, every car on the road is placed uniformly on the circuit road. Experiment is started when every car starts running. After that, the traffic system goes into the steady state when the standard deviation of velocity and average velocity of all car at that particular time becomes quite steady through the time. We start controlling every car with FollowerStopper when the system is in a steady state. In this case, we turn the controller on at the 600th second after starting.

2.1. Human-driven cars simulation
To simulate the human-driven cars, Intelligent Driver Model (IDM) is used for calculating the acceleration. The acceleration is turned into the position of the cars on the road at each time step. IDM is stated as the following equation [3, 4].

\[ \frac{dv}{dt} = a \left(1 - \left(\frac{v}{v_0}\right)^\delta - \left(\frac{s^*}{s}\right)^2\right) + \xi \]  

This model considers each vehicle acceleration as the function of its maximum acceleration \(a\) and velocity \(v_0\), its instantaneous velocity \(v\), the acceleration exponent \(\delta\), desired gap distance \(s^*\), and actual gap distance \(s\) between itself and the leading one. The term \(\xi \sim \mathcal{N}(0, 0.2)\) is human acceleration noise [4]. The desired gap distance is a function of its velocity \(v\), driver’s reaction time \(T\), comfortable deceleration \(b\) and the leading vehicle velocity \(v_{\text{lead}}\).

\[ s^*(v, \Delta v) = s_0 + \max(0, vT + \frac{v\Delta v}{2\sqrt{ab}}) \]  

For IDM, \(\Delta v = v_{\text{lead}} - v\). We set these parameters as in [3], which is set to be fitted with empirical data from highway scenarios. The parameters are set as following \(v_0 = 120 \text{ km/h}\), \(T = 1.0 \text{ s}\), \(s_0 = 2 \text{ m}\), \(\delta = 4\), \(a = 1.00 \text{ m/s}^2\), and \(b = 1.5 \text{ m/s}^2\).

2.2. Self-driving cars simulation
The self-driving cars used in the simulations are controlled by the FollowerStopper controller. This controller is designed for damping the traffic wave. Generally, how this controller command the velocity depends on the regions on \(\Delta x - \Delta v\) phase space. Those regions are divided by 3 lines in equation (3) and the \(\Delta x_k\) will be used in equation (4).
\[ \Delta x_k = \Delta x_k^0 + \frac{1}{2d_k} (\Delta v_-)^2, \text{ for } k = 1, 2, 3. \]

\( \Delta x_k \) are defined by the parameter \( \Delta x_k^0 \) (the value of \( \Delta x_k \) at \( \Delta v_- = 0 \)), deceleration rate \( d_k \), and the different velocity of leading vehicle and itself in negative region \( \Delta v_- = \min(\Delta v, 0) \). For the FollowerStopper, \( \Delta v = v - v_{\text{lead}} \).

To determine command velocity, FollowerStopper considers \( v \) as the positive velocity of leading vehicle \( v_{\text{lead}} \) or 0 (if negative) which does not exceed the desired velocity \( U \), \( v \) defined as \( v = \min(\max(0, v_{\text{lead}}), U) \). Command velocity \( v_{\text{cmd}} \) is defined as

\[
\begin{align*}
v_{\text{cmd}} &= \begin{cases} 
0 & \text{if } \Delta x \leq \Delta x_1 \\
v \frac{\Delta x - \Delta x_1}{\Delta x_2 - \Delta x_1} & \text{if } \Delta x_1 < \Delta x \leq \Delta x_2 \\
v + (U - v) \frac{\Delta x - \Delta x_1}{\Delta x_2 - \Delta x_1} & \text{if } \Delta x_2 < \Delta x \leq \Delta x_3 \\
U & \text{if } \Delta x_3 < \Delta x
\end{cases}
\end{align*}
\]

These parameters are set as in [2], which \( \Delta x_0 = 4.5 \text{ m}, \Delta x_0^0 = 5.25 \text{ m}, \Delta x_0^3 = 6.0 \text{ m}, d_1 = 1.5 \text{ m/s}^2, d_2 = 1.0 \text{ m/s}^2 d_3 = 0.5 \text{ m/s}^2 \) and the desired velocity is set at \( U = 13.00 \text{ m/s} \).

*Figure 2.* Relation of density of cars on the road with standard deviation of velocities (upper figure) and average velocities (lower figure). Light colour represents the values 800 seconds after turning on the self-driving cars. Black colour represents the values before turning self-driving cars on.
3. Results and discussions
The presence of traffic wave can be indicated by higher standard deviation of velocity and lower
average velocity. The traffic wave disturbs the car density in the system. This variation of car
density will result in the higher variation of car velocities. We collect velocities of every car
every 0.1 sec. At the 600th and the 1400th second, standard deviation and average velocity at
that particular time are calculated and plotted with the density, as can be seen in figure 2.

As shown in the figure 2, at about the density of 0.06 cars per meter, the standard deviation
at the 1400th second dramatically and discontinuity increases from nearly zero to about 3.0.
In contrast, average velocity suddenly drops from 7.0 m/s to about 4.0 m/s. This represents
the phase transition and the presence of traffic wave. The traffic system which is controlled
by FollowerStopper is in a jam state when the density has exceeded 0.06 cars per meter. And
when the density has exceeded about 0.14 cars/meters FollowerStopper happens to make the
cars completely stopped.

Interestingly, there are some densities that can give 2 different stable phases. When
FollowerStopper is turned on, about the density of 0.06 cars per meter can give the result
in both jam and free-flow phase. These 2 possible phases might be caused by the randomness
of the system.

From both diagram, FollowerStopper happen to make every car drive more uniformly, which
can be seen in lower standard deviation. But, FollowerStopper also make the average velocity
slower in the most part. These show that FollowerStopper is quite capable for making system
more uniform, but not capable for increasing average velocity. This capability and incapability
might be optimized by changing the constant parameters of FollowerStopper specificity to the
situation.

4. Conclusion
In this work, we have investigated the effects of different car density and the ability to damp the
wave when every car is controlled by FollowerStopper. We found that when the desired speed
of FollowerStopper is set at 7.0 m/s the wave is only dissipated when the car density is lower
than 0.06 cars per meter. Overall, FollowerStopper makes traffic flow more uniformly, although
traffic wave is still exist. Furthermore, in some density, the cars can be either in 2 different
states. This point is interesting to be investigated in further studies.

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