Electrons as quasi-bosons in magnetic white dwarfs

Jerzy Dryzek \textsuperscript{a}
Institute of Nuclear Physics, PL-31-342 Kraków, ul.Radzikowskiego 152, Poland

Akira Kato \textsuperscript{b}, Gerardo Muñoz \textsuperscript{c}, Douglas Singleton \textsuperscript{d}
Dept. of Physics, CSU Fresno, Fresno, CA 93740-8031, USA

Abstract
A white dwarf star achieves its equilibrium from the balancing of the gravitational compression against the Fermi degeneracy pressure of the electron gas. In field theory there are examples (e.g. the monopole-charge system) where a strong magnetic field can transform a boson into a fermion or a fermion into a boson. In some condensed matter systems (e.g. fractional quantum Hall systems) a strong magnetic field can transform electrons into effective fermions, or effective anyons. Based on these examples we investigate the possibility that the strong magnetic fields of some white dwarfs may transform some fraction of the electrons into effective bosons. This could have consequences for the structure of highly magnetized white dwarfs. It would alter the mass-radius relationship, and in certain instances one could envision a scenario where a white dwarf below the Chandrasekhar limit could nevertheless collapse into a neutron star due to a weakening of the electron degeneracy pressure. In addition the transformation of electrons into effective bosons could result in the electrons Bose condensing, which could speed up the cooling rate of white dwarfs.

1 Introduction
It has been known for a long time\textsuperscript{1} that certain stars at the end of their life cycle reach an equilibrium where the gravitational compression is balanced by the Fermi degeneracy pressure of the electron gas. These white dwarf stars are theoretically interesting since understanding their stability requires an understanding of gravity, and of a quantum Fermi gas. The stability argument for a white dwarf can be framed in terms of the Fermi energy of the electrons versus their gravitational binding energy. The Fermi energy for a relativistic electron is approximately

\begin{equation}
E_F \approx \frac{\hbar c N^{1/3}}{R} \tag{1}
\end{equation}

\textsuperscript{a}e-mail: jerzy.dryzek@ifj.edu.pl
\textsuperscript{b}e-mail: ak086@csufresno.edu
\textsuperscript{c}e-mail: gerardom@csufresno.edu
\textsuperscript{d}e-mail: dougs@csufresno.edu
where $N$ is the number of electrons in the object, and $R$ is the radius of the object. The gravitational energy per fermion is approximately

$$E_G \approx -\frac{GMm_n}{R} \quad (2)$$

where $m_n \approx 1.67 \times 10^{-24}$ g is the nucleon mass, and $M = N m_n$ is roughly the total mass of the star. The total energy is then $E_{tot} = E_F + E_G$. If the physical constants in $E_F$ and $E_G$ are such that $E_{tot} > 0$ then $E_{tot}$ can be decreased by increasing $R$ and a stable situation is eventually reached where the star is supported by its Fermi degeneracy pressure. If the physical constants in $E_F$ and $E_G$ are such that $E_{tot} < 0$ then $E_{tot}$ decreases without bound by decreasing $R$ and no equilibrium exists. The boundary between these two situations occurs when $\hbar c N^{1/3} = GMm_n^2$ which implies a maximum baryon number of $N_{max} \approx (\hbar c/Gm_n^2)^{3/2} \approx 2 \times 10^{57}$ and a maximum total mass of $M_{max} \approx N_{max} m_n \approx 1.5 M_\odot$. This simple argument gives an approximation of the Chandrasekhar mass limit for white dwarfs. Crucial to this argument (or more rigorous versions) is that the electrons should behave as fermions in order to give rise to the Fermi degeneracy pressure. In lower dimensional field theories there are examples, such as the sine-Gordon model in one space and one time dimension, where bosonic and fermionic degrees of freedom can be taken as dual or interchangeable. These lower dimensional examples can be extended to $3 + 1$ dimensions. In Abelian and non-Abelian field theories there are configurations where, through the action of a magnetic field, the statistics of the system can be transformed (i.e., the system can be fermionic even though all the fields involved in its construction are bosonic). There are also certain condensed matter systems, where fermions can be converted into effective fermions or effective bosons. The fractional quantum Hall effect offers one such example, where electrons in 2D systems in the presence of a large magnetic flux can act as effective fermions or effective bosons, depending on which picture/approach one uses. Here we look at the possibility that in highly magnetized white dwarfs a similar transformation may occur for some fraction of the electrons of the star. Converting some fraction of the electrons of the star into quasi-bosons would mean that they would no longer be involved in giving rise to the Fermi degeneracy pressure: the $N$ in Eq. (1) would be reduced by the fraction of the electrons which are converted to effective bosons. This weakening of the Fermi degeneracy pressure could result in a white dwarf with a mass below the Chandrasekhar limit collapsing into a neutron star. Further, this collapse of an otherwise stable white dwarf to a neutron star, might occur without a supernova. This could offer an explanation for certain pulsar systems which have planets orbiting them. If the neutron star formed via a supernova then the original planets of the progenitor star should have been blown out of the system. Also
transforming electrons into quasi-bosons could result in these electrons Bose condensing, which could speed up the cooling rate of the white dwarf.

2 Electron field angular momentum in a magnetic white dwarf

In this section we will give a simple picture for how an electron inside a highly magnetized white dwarf can be transformed into an effective boson. We will frame our arguments for this transformation in terms of the field angular momentum of the system rather than the additional phase factor that arises in the exchange of the charge/magnetic flux composites. Various 2D and 3D examples have found that both approaches lead to an equivalent understanding of the change in statistics of these composite systems.

The idea that a fermion can be converted to a boson, or a boson into a fermion via a magnetic field, can be found in certain non-Abelian field theories. These examples involve placing a charged particle in the vicinity of one of the finite energy magnetic monopole solutions which exist in these field theories. This composite of particle plus monopole possesses a field angular momentum which gives the system statistics the opposite of what is naively expected. In the original example of Refs. both the monopole and charged particle were bosons, while the bound state composite was a fermion. A related phenomenon in 2D was discussed by where the electric charge combined with a magnetic flux tube quantum to give a composite which had different statistics from naive expectations. For the 2D case one can have objects (anyons) which have statistics that fall anywhere between fermions and bosons. In three spatial dimensions, however, one can show that on topological grounds only bosons or fermions occur. These arguments leave open the possibility of boson to fermion, or fermion to boson transformations in three spatial dimensions (it is the latter case that we are interested in). Recently this transformation of fermions into composite bosons, or bosons into composite fermions has been experimentally observed in the effectively 2D fractional quantum Hall systems. Since magnetic white dwarfs are 3D objects the fermion to quasi-boson transformation that we are proposing here has more in common with the 3D monopole/charge systems.

Based on these field theory and condensed matter examples we propose that electrons in a highly magnetic white dwarf could be transformed into quasi-bosons via the star’s magnetic field. Specifically the magnetic field could combine with the electric field of the electron to give rise to a field angular momentum. If this field angular momentum is of the correct magnitude (i.e. some half-integer multiple of ) then the combination of the electrons’ spin plus the field angular momentum will
result in an effective boson which will not contribute to the Fermi degeneracy pressure. This would affect the equilibrium of the white dwarf. It would change its mass-radius relationship, so that for a given mass one would have a smaller radius than for a white dwarf with a smaller magnetic field. It might even be possible for such a magnetic white dwarf to collapse into a neutron star despite being below the Chandrasekhar limit.

The angular momentum carried in the electric and magnetic fields can be written as

$$L_{\text{em}} = \frac{1}{4\pi c} \int r \times (E \times B) d^3 r$$  \hspace{1cm} (3)

Now we consider an electron located at the origin in a uniform magnetic field whose direction is taken to define the $z$-axis. We will assume that near the location of the electron the magnetic field $B = B_0 \hat{z}$, where $B_0$ is the magnitude of the magnetic field. In a white dwarf the electrons form a conducting gas with a background lattice of the positive nuclei. The electric field of the electron will be screened past a certain distance, $R_{\text{sc}}$. This screening effect is taken into account by using a Yukawa potential for the electron’s electric field

$$\phi(r) = \frac{e \exp(-r/R_{\text{sc}})}{r}$$  \hspace{1cm} (4)

The potential in Eq. (4) gives an E-field of

$$E = -\frac{e \exp(-r/R_{\text{sc}})}{r^2} \left[ 1 + \frac{r}{R_{\text{sc}}} \right] \hat{r}$$  \hspace{1cm} (5)

Carrying out the calculation of the field angular momentum which results from this combination of magnetic and electric fields yields

$$L_{\text{em}} = \frac{2eB_0 R_{\text{sc}}^2}{c} \hat{z}$$  \hspace{1cm} (6)

The direction of field angular momentum is in the $\hat{z}$ direction, which is determined by the direction of the external magnetic field. Despite the singularity in the Coulomb field of the electron, the net field angular momentum does not diverge. The $B_0 R_{\text{sc}}^2$ part of this expression is proportional to the magnetic flux, $\Phi$ “trapped” by the electron. As will be discussed below $R_{\text{sc}}$ for a typical white dwarf is in general small – on the order of $10^{-10}$ cm. Both the spin of the electron and the field angular momentum will be localized within a sphere of radius $10^{-10}$ cm. Thus it is physically reasonable to view this screened electron plus trapped magnetic flux as a composite system whose total angular momentum is a combination of the electron’s spin plus the field angular momentum –
\[ L_{\text{tot}} = \text{Spin} + \text{Field}. \] (One can contrast this with the monopole/charge system where the field angular momentum density is not necessarily well localized around the charge and yet is still taken as being part of the total angular momentum of the composite system). If \( L^e_m \) in Eq. (6) takes on some half-integer multiple of \( \hbar/2 \) then the total angular momentum should be that of a boson (such arguments were originally used in \cite{20,21} to arrive at the Dirac quantization condition for electric charge). As in the monopole/charge system of Refs. \cite{6,7}, the electron will behave as an effective boson and not contribute to the degeneracy pressure. A similar picture (i.e. the combining of a field angular momentum with the spin of the electron) was proposed\cite{22} to give an alternative, complementary picture of the fractional quantum Hall effect where the electrons are transformed into effective fermions by the external field. Here, in contrast, we want the magnetic field magnitude, \( B_0 \), and/or screening distance, \( R_{sc} \), to take on values which yield \( L^e_m = (n + \frac{1}{2})\hbar \) so that \( L_{\text{tot}}^z = L_{\text{spin}}^z + L^e_m = m\hbar \) (\( n, m \) are integers) thus making effective bosons. Highly magnetized white dwarfs can have surface magnetic field strengths up to \( 10^8 \) or \( 10^9 \) gauss. We are interested in the magnetic field strengths in the interior for which unfortunately there is no direct experimental determination. However, as an estimate we will take the interior field to be two orders of magnitude greater than the surface fields (i.e. interior fields of \( 10^{10} \) or \( 10^{11} \) G). It is not uncommon to postulate interior magnetic fields as large as \( 10^{13} \) G\cite{23}).

The screening length of the electric field inside a white dwarf can be estimated as

\[ R_{sc} = \sqrt{\frac{E_F}{6\pi e^2 Z n_0}} \]  

(7)

where \( E_F \) is the Fermi energy, \( n_0 \) the number density of the ions that form the lattice, and \( Z \) is the number of positive charges of each ion. For these quantities we take the following values which are typical for carbon white dwarfs - \( Z = 6, E_F = 0.6 \times 10^{-6} \) ergs, \( n_0 = 1.7 \times 10^{29} \text{ cm}^{-3} \) (this corresponds to an electron number density of \( 1.0 \times 10^{30} \text{ cm}^{-3} \)) - which leads to \( R_{sc} = 3.7 \times 10^{-10} \text{ cm} \). Given this screening length and requiring that \( L^e_m \) from Eq. (6) equal \( \hbar/2 \) yields \( B_0 = 3.6 \times 10^{13} \) G. This magnetic field strength is possible in the interiors of highly magnetized white dwarfs\cite{23}. Thus some fraction of the electrons within a highly magnetized white dwarf could be transformed into quasi-bosons in analogy to what occurs in magnetic charge/electric charge composites, or to the quasi-bosons in certain condensed matter systems with strong magnetic fields.

The question that is not addressed in the above analysis is what fraction of the electrons are transformed into quasi-bosons? In the present scenario the field angular momentum needs to take on odd-integer multi-
amples of $\hbar/2$ in order to turn the electrons into quasi-bosons. The field angular momentum depends on both the background magnetic field, $B_0\hat{z}$, and the screening distance, $R_{sc}$. If one takes these two quantities as varying with position inside the white dwarf, then as one moves around the interior of the white dwarf some regions will have the correct values to $B_0$ and $R_{sc}$ in order for the transformation to occur. As an illustrative example consider a magnetic white dwarf where it is assumed that $R_{sc}$ is constant throughout the star and takes on the value given after Eq. (7) – $R_{sc} \approx 3.7 \times 10^{-10}$ cm. Take the magnetic field at the surface to be on the order of $10^{9}$ G and let its magnitude increase linearly with radius, from the surface of the star, to an interior, central value of $10^{13}$ G. Taking the radius of the white dwarf to be $5 \times 10^{8}$ cm then implies that at a radius of $4.8 \times 10^{8}$ cm (i.e. just below the surface) the magnetic field will take on the correct value in order to generate a field angular momentum of $\hbar/2$. All the electrons in the spherical shell at this radius will be transformed into quasi-bosons. As one continues further into the star a new radius will be reached where the magnetic field strength will increase to the point that the field angular momentum will become $3\hbar/2$ again transforming the electrons into quasi-bosons. Going all the way into the center of the white dwarf one encounters a series of spherical shells where the electrons are changed into effective bosons. The result being that the electron degeneracy pressure is reduced, altering the mass-radius relationship for the star.

Since the magnetic fields discussed here are large relative to those that can be found in a laboratory, one should also ask if quantum modification of the magnetic field strength alter any of the above analysis. If the quantum corrections reduced the magnetic field strength this would tend to increase the point at which the transformation of electrons into quasi-bosons took place. However, for magnetic field strengths on the order of $\simeq 10^{15}$ G one can show that the quantum corrections are on the order of $\simeq 1\%$. For the magnetic field strengths considered here the quantum corrections can be safely ignored.

3 Physical consequences of quasi-bosonic electrons

There are two main physical consequences which could result from the transformation of electrons into bosons. First, as mentioned in the previous sections, the mass-radius relationship would be altered in such a way that the radius would be smaller than what would normally be expected. This decreasing of the radius, due to the transformation of the electrons into effective bosons, is in the opposite direction of another effect of a strong magnetic field. In Ref. [4] it is shown that for magnetic white dwarfs the pressure of the electron gas is increased through a strong magnetic field, tending to make the radius for a given mass larger.
than without the magnetic field. However, this effect only starts to become important for magnetic field strengths in the range of $10^{12} - 10^{13}$ G. The magnetic field strengths that transform electrons into effective bosons are one or two orders of magnitude less than this, so that the decrease in radius due to the transformation of the electrons into bosons should begin before the increase in radius due to the mechanism of Ref. 23. One interesting possible consequence of this weakening of the electron degeneracy pressure through the fermion $\rightarrow$ boson mechanism, is that it may allow magnetic white dwarfs which are slightly below the Chandrasekhar limit to collapse, without a supernova, into a neutron star. Consider a magnetic white dwarf which is just below the Chandrasekhar limit, and should therefore be stable against further collapse. Take the interior magnetic field of the white dwarf to be in the range of $10^{12} - 10^{13}$ G so that the fermion $\rightarrow$ bosons transformation could occur in the manner described at the end of the last section (i.e. the magnetic field magnitude varies linearly from its surface value to its interior value so that there will be a series of concentric shells where the electrons become quasi-bosons). The Fermi degeneracy pressure could be reduced enough so that the star’s inward gravitational pressure would be slightly larger, allowing the white dwarf to slowly collapse, without a supernova explosion, into a neutron star. This collapse scenario may offer an explanation of the planet-pulsar systems 12. In these systems one has up to three planets orbiting a pulsar. If this pulsar formed in a supernova collapse then the initial planets should have been blown out of the system, which is taken to imply that these planets must have formed after the supernova. However, if the original star collapsed slowly due to the fermion $\rightarrow$ boson transformation, then the current planets might be the original planets of the star.

Second, the cooling period of these magnetic white dwarfs could be accelerated, since the quasi-bosonic electrons could Bose condense. In Ref. 26 the idea was advanced that certain white dwarfs might be Bose condensed systems, with the positively charged ions being the bosons which are condensing. For an ideal Bose gas the condensation temperature is

$$T_0 = \frac{2\pi h^2}{mk} \left( \frac{n}{2.612} \right)^{2/3}$$  \hspace{1cm} (8)

$k$ is Boltzmann’s constant and $n$ is the number density. When an ideal Bose gas drops below this critical temperature it will Bose condense. Assuming that this is a first order transition 25 implies that an energy of

$$\Delta E = LM$$  \hspace{1cm} (9)

is released by the condensation. $L$ is the latent heat of condensation and $M$ is the total mass of the condensing system. This increases the cooling rate of the star. In Ref. 26 the condensing particles are the positively
charged ions of the white dwarf. This resulted in a condensation temperature from Eq. (8) in the range $10^4 \sim 10^6$ K. Using the fermion to boson transformation proposed above one can think to apply this Bose condensation idea to electrons which sit in an appropriately strong magnetic field. Because of the inverse relationship between $T_0$ and the mass of the condensing particle in Eq. (9), and since the electron is three to four orders of magnitude lighter than the ions, the critical temperature of these quasi-bosonic electrons to Bose condense is in the range of $10^7 \sim 10^9$ K. The Bose condensation of these electrons would occur at much higher temperatures as compared to the condensation of the ions. One might think that the energy released by the electron condensation would be smaller since in this case $M$ from Eq. (9) would be the mass of all the electrons which were transformed into quasi-bosons. However, the smaller mass of the electrons relative to the ions is compensated for by the fact that the latent heat of condensation is proportional to $T_0$ : $L \propto T_0$ (see Ref. 26). Since the $T_0$ for the quasi-bosonic electrons is increased by the same factor that $M$ is decreased, these two effects cancel in the expression for $\Delta E$. One final difference between a Bose condensation of ions versus a Bose condensation of quasi-bosonic electrons, is that for the electrons only those which are transformed will Bose condense, while for the ions they all condense. However the number of electrons is larger than the number of ions (e.g. for a carbon white dwarf there are 6 electrons for every ion), thus even if only some fraction of the electrons are transformed they could nevertheless give a similar contribution to $\Delta E$ as in the ion condensation scheme of Ref. 26. For the carbon white dwarf example if 15% of the electrons are transformed into bosons then the number of quasi-bosonic electrons will be only a little less than the number of ions. Thus the Bose condensation mechanism of Ref. 26 could be applied to the transformed electrons, but the critical temperature at which Bose condensation occurs would be much higher, because of the smaller electron mass. This would have the effect of accelerating the cooling time for such highly magnetic white dwarfs.

4 Discussion

In this paper we have proposed a speculative mechanism whereby some fraction of the electrons within a highly magnetized white dwarf can be transformed by a sufficiently strong magnetic field into effective bosons. The system of an electron placed in a uniform magnetic field has a field angular momentum, whose magnitude is proportional to the strength of the magnetic field and the square of the screening distance of the electric field of the electron. For certain values of the magnetic field and screening distance this field angular momentum can take on half-integer values, which combined with the half-integer spin of the electron leads
to the combined object of electron plus trapped magnetic flux having an integer angular momentum. It is postulated that this transforms the electron into a quasi-boson. Similar transformations of fermions into bosons or bosons into fermions via the action of a magnetic field can be found in the charge/monopole systems studied in particle physics and in the fractional quantum Hall systems in condensed matter physics. There are two physical consequences that could result from turning some of the electrons inside a highly magnetized white dwarf into effective bosons: First, the Fermi degeneracy pressure would be decreased, altering the mass-radius relationship of the white dwarf so that for a given mass the radius would be smaller than if the magnetic field were absent. Such highly magnetic white dwarfs, with quasi-bosonic electrons, might be able to collapse into neutron stars despite being below the Chandrasekhar limit. The second possible physical consequence is a faster cooling rate of such magnetic white dwarfs, via Bose condensation. This is similar to the idea of Ref. 26 where an accelerated cooling rate was proposed through the Bose condensation of the integer spin nuclei of the white dwarf. The present work differs from this in that the objects which are taken as Bose condensing are the transformed quasi-bosonic electrons.

It would appear that the above mechanism is not applicable to pure, ideal neutron stars. Even though the magnetic field strength of a neutron star can be several orders of magnitude larger than that of a white dwarf, a neutron carries no charge, and therefore no field angular momentum would be generated by placing the neutron in a magnetic field. This of course assumes the approximation of treating the neutron as a fundamental, chargeless object. As one goes to smaller distance/larger energy scales (i.e. as one considers neutron stars with increasing densities) there may be a transition where one needs to describe the matter, not in terms of neutrons, but in terms of a gas of charged quarks. At this point one might again consider applying the mechanism discussed in this paper, but now the magnetic field would be transforming the fermionic quarks into quasi-bosons.

Acknowledgment

We wish to thank Dr. Fred Ringwald for comments and discussions.

References

1. S. Chandrasekhar, Phil. Mag., 11, 592 (1931); ApJ, 74, 81 (1931)
2. L.D. Landau, Phys. Z. Sowjetunion, 1, 285 (1932)
3. S. Coleman, Phys. Rev. D11, 2088 (1975)
4. S. Mandelstam, Phys. Rev. D11, 3026 (1975)
5. A. Luther, Phys. Rept. 49, 261 (1979)
6. R. Jackiw and C. Rebbi, Phys. Rev. Lett., 36, 1116 (1976)
7. P. Hasenfratz, and G. ’t Hooft, Phys. Rev. Lett., 36, 1119 (1976)
8. R.E. Prange and S.M. Girvin, The Quantum Hall Effect, 1990 2nd Ed. (Springer-Verlag, New York)
9. J.K. Jain, Phys. Rev. Lett., 63, 199 (1989)
10. S.M. Girvin and A.H. MacDonald, Phys. Rev. Lett., 58, 1252 (1988)
11. D.H. Lee, and C.L. Kane, Phys. Rev. Lett., 64, 1313 (1990)
12. A. Wolszczan and D.A. Frail, Nature, 355, 145 (1992)
13. G. ’t Hooft, Nucl. Phys. B79, 276 (1974)
14. A.M. Polyakov, JETP Lett., 20, 194 (1974)
15. F. Wilczek, Phys. Rev. Lett., 48, 1144 (1982)
16. A. Goldhaber, 1976, Phys. Rev. Lett., 36, 1122 (1976)
17. G.S. Canright and S.M. Girvin, Science, 247, 1197 (1990)
18. D.C. Tsui, H.L. Stormer and A.C. Gossard, Phys. Rev. Lett., 48, 1559 (1982).
19. J.D. Jackson, 1975, Classical Electrodynamics (Wiley, New York), 2nd Ed., p. 251
20. M.N. Saha, Indian J. Phys., 10, 145 (1936); M.N. Saha, Phys. Rev. 75, 1968 (1949)
21. H.A. Wilson, Phys. Rev. 75, 309 (1949)
22. D. Singleton, and J. Dryzek, Phys. Rev. B62, 13070 (2000)
23. I.S. Suh, and G.J. Mathews, ApJ , 530, 949 (2000)
24. S.L. Shapiro and S.A. Teukolsky, 1983, Black Holes, White Dwarfs, and Neutron Stars, (John Wiley and Sons, New York)
25. G. Muñoz, Am. J. Phys., 64, 1285 (1996)
26. N. Nag, and S. Chakrabarty, astro-ph/0008477