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To cite this article: A Gorbunov et al 2017 J. Phys.: Conf. Ser. 891 012057

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Characteristic features of heat transfer in inhomogeneous supercritical fluid

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Abstract. “Speeding up” heating (so called “piston-effect”) of a closed domain of supercritical fluid is studied using 1D Navier-Stokes equations and van der Waals equation of state. Time dependences of temperature, density, pressure and velocity distributions are calculated for various initial conditions on temperature, density and gravity. It is shown that for some threshold value ε of the distance from critical point density stratification caused by gravity does not affect the characteristic time of the piston-effect τPE. The time τPE rises more rapid as a function of characteristic density µ when fluid has the step temperature-density stratification in comparison with that for the homogeneous density. The results are compared with the analytical solutions obtained by Onuki.

1. Introduction

Application of supercritical fluid in industry including rocket and space engineering demands knowledge on heat transfer in a SCF cell with density inhomogeneities caused by gravity and g-loading. The heat exchange in a closed domain of SCF was intensively studied in the last two decades primarily owing to the space experiments carried out onboard MIR station with ALICE-1/2 instruments [1-5] and ground-based preparation of the KRIT space experiment on the ISS [6]. It was found that when fluid in a closed domain is heated from the boundaries the temperature of the fluid bulk rises more rapid as one approaches the critical point. As the fluid near the heated wall expands and the fluid bulk compresses, the process looks like a compression of the bulk with the near wall fluid. So the process was named as a “piston effect”.

An important characteristic of the process is the time of bulk heating, so called time of the piston effect. In [7] the author found an analytical solution of the problem of heat transfer from the immediately heated wall to the bulk of the fluid in weightlessness (g=0). It is assumed that the temperature of the wall is constant, the near wall layer is thin with respect to the bulk and is heated only due to the thermal conductivity. In addition, the near wall layer expands isobarically, and the bulk compresses adiabatically. The solution gives a formula for the characteristic time of the piston effect:

$$\tau_{PE} \propto \frac{L^2}{\left(y\right)^2 D_T},$$

(1)
where L is the characteristic length of the domain, \( \gamma = c_p/c_v \), \( c_p \) is the specific heat at constant pressure, \( c_v \) is the specific heat at constant volume, \( D_r = \lambda/\rho c_p \), \( \lambda \) is the thermal conductivity coefficient, and \( \rho \) is the density.

If we know the dependences of the parameters in equation (1) on \( \varepsilon \), where \( \varepsilon = (T_\text{in} - T_c)/T_c \). \( T_\text{in} \) is the initial temperature and \( T_c \) is the critical temperature, it is possible to estimate how the characteristic piston-effect time varies as we approach the critical temperature. In particular, in [9] the authors propose to use \( D_r \sim \varepsilon^{0.66} \) and \( \gamma \sim \varepsilon^{1.13} \) that gives \( \tau_{PE} \sim \varepsilon^{1.6} \).

Under gravity or \( g \)-loading conditions the supercritical fluid becomes inhomogeneous. The mass force causes the fluid to be stratified in density in order to provide mechanical equilibrium. This means that when \( g \neq 0 \) the properties of the fluid vary with height and in addition to the parameter \( \varepsilon \) we should introduce a parameter \( \mu = (\rho_\text{in} - \rho_c)/\rho_c \), where \( \rho_\text{in} \) is the initial density and \( \rho_c \) is the critical density. Like the parameter \( \varepsilon \), the parameter \( \mu \) characterizes the relative deviation of the density from the critical one and in inhomogeneous fluid varies with height. Accordingly, in case of inhomogeneous fluid the equation for the characteristic piston-effect time (1) should be modified. An analysis similar to that presented in [7] was carried out in [8] for the case of a fluid under the action of gravity. The authors derive a modified equation for the characteristic time of the piston effect in the form:

\[
\tau_{PE} \sim \frac{L^2}{\left( \langle c_p \rangle_b \right)^2 \left( D_r \right)_b},
\]

Here \( L \) is the characteristic length of the domain, \( \langle c_v \rangle \) is the space averaged \( c_v \), and index “b” denotes boundary, i.e. this means that we deal with a value of the parameter near the heated boundary.

Thus, in such a case, it is assumed that the characteristic time \( \tau_{PE} \) is a function of two parameters: \( \varepsilon \) and \( \mu \).

The goal of our study is to obtain knowledge on piston effect in inhomogeneous fluid near the critical point by direct calculation on 1D Navier-Stokes equations and van der Waals equation of state. In particular we study the behavior of characteristic time of the piston effect in the vicinity of the critical point for various \( \varepsilon \) and \( \mu \).

2. Formulation of the problem

The dimensionless set of equation of state, balance of energy, motion and conservation of mass is presented in [10].

The calculations are performed for vertical 1D domain of height \( L = 1 \text{cm} \) in the field of gravity force directed downward. The working fluid is the sulfur hexafluoride (SF\(_6\)).

The temperature of the top boundary is immediately rises by the value of \( 1 \text{mK} \) and keeps constant during the calculations. The bottom boundary is assumed to be adiabatic.

There are three types of initial conditions for the calculations.

(i) The average density in the domain is \( \rho_c \), the distribution of pressure and density corresponds to a given value of acceleration of gravity, the temperature is a constant over the height and corresponds to a given supercritical temperature \( T_{in} \).

(ii) The average density in the domain is also \( \rho_c \), \( g=0 \), but the density and temperature in the top half, \( z \in (0,0.5) \), are \( \rho_c + \Delta \rho \) and \( T_{in} + \Delta T \) whereas in the bottom half, \( z \in (0.5,1) \), they are respectively \( \rho_c - \Delta \rho \) and \( T_{in} - \Delta T \). Here the values \( \Delta \rho \) and \( \Delta T \) are set so to provide the constant pressure over the height.

(iii) The density in the domain is less than \( \rho_c \) and fluid is homogeneous (\( g=0 \)).

To calculate the dimensionless parameters in the equations we used the constants for SF\(_6\): \( c_v = 1000 \text{ J/kg-K}, \lambda = 1.2 \text{ Wt/m-K}, \mu = 40-10^6 \text{ Pa-s}, R = 56.9 \text{ J/kg-K}, \gamma_0 = 1.0569 \) and \( \rho_c = 744 \text{ kg/m}^3 \), \( T_c = 45.7^\circ \text{ C} \).
For the calculations we use an explicit finite differences scheme. The space step is $h=10^{-3}$, the time step is $h(h)^{0.5}$.

In the calculations we determine time dependencies of distributions of temperature, pressure, density and velocity over the height for various initial values of temperature, density and acceleration of gravity.

The calculations are performed for $g=0$ (weightlessness), $g=g_0=9.81\text{m/s}^2$ (gravity), $g=5g_0$ and $g=10g_0$ (overload) for the case (i) as well as for $\mu=-0.005; -0.01; -0.015; -0.02$ and $-0.025$ for the case (ii) and (iii).

The initial vertical profiles of density for $g\neq0$ at, e.g., $\Delta T=T_{\text{in}}-T_c=10\text{mK}$ look like a plot of uneven exponent function with characteristic inflection at $z=0.5$ and $\rho=1$, whereas just not far from the critical point ($\Delta T=500\text{mK}$) the density distributions are essentially linear. The density distribution for the case (ii) looks like a step with discontinuity at $z=0.5$ and in the case (iii) the density is homogeneous.

3. Results

Figure 1 shows time dependencies of deviation of dimensionless temperature $\delta T=(T-T_{\text{in}})/T_c$ at the center of the calculation domain $(z=0.5)$ for various values of acceleration of gravity. The difference between the initial and critical temperature is $\Delta T=10\text{mK}$.

![Figure 1. Deviation of dimensionless temperature $\delta T$ at the center of calculation domain ($z=0.5$) as a function of time for various values of acceleration of gravity.](image-url)
The value of the characteristic piston effect $\tau_{PE}$ is determined from the temperature plots. As pointed out in [8], $\tau_{PE}$ is a time period for which the temperature in the bulk reaches the value of 0.572 of the difference between the initial temperature and the final temperature. In our case the difference is $T_{fin} - T_{in} = 1mK$, and dimensionless value of the deviation of temperature in the bulk that corresponds to the values of 0.572($T_{fin} - T_{in}$) is $\delta T_{PE} = 1.7910^{-6}$. Then, for example, for $\Delta T = 10mK$ and $g = g_0$ (see figure 1), $\tau_{PE} = 0.25c$

![Figure 2](image)

**Figure 2.** Characteristic time of the piston effect $\tau_{PE}$ as a function of $\varepsilon$ for various values of $g$.

The difference between the initial and critical temperature $\Delta T = T_{in} - T_c$ varies from 10 mK to 1000 mK, that corresponds to the variation of $\varepsilon$ within $3 \cdot 10^{-5} < \varepsilon < 3 \cdot 10^{-3}$. The characteristic time of the piston effect $\tau_{PE}$ as a function of $\varepsilon$ for various values of $g$ is shown in figure 2. As is seen from the figure the dependencies of $\tau_{PE}$ on $\varepsilon$ for all values of acceleration of gravity ($g=g_0$, $5g_0$, $10g_0$) and for weightless ($g=0$) are almost in coincidence for $\varepsilon > 10^{-3}$. The inclination of the line is close to the unity, i.e. $\tau_{PE} \sim \varepsilon$.

Such an inclination is in agreement with that one given by the equation (1.1). Indeed, for $c_v = \text{const}$, $\lambda = \text{const}$ (that is assumed in our model) and for $\gamma >> 1$

$$\tau_{PE} \propto \frac{1}{c_p} \propto \left(\frac{\partial \rho}{\partial T}\right)^{-1}_p$$

and in accordance with the van der Waals equation of state used in the model $\left(\frac{\partial \rho}{\partial T}\right)_p \propto \varepsilon^{-1}$, i.e.

$$\tau_{PE} \propto \varepsilon.$$
When $\varepsilon < 10^{-3}$ the inclination of the plot for $g=0$ is practically a constant, whereas in gravity the time of the piston effect essentially increases. The difference is a maximum for the maximum value of gravity acceleration. For example, for $\varepsilon=10^{-4}$ the characteristic time of the piston effect for $g=10g_0$ is more than that in weightlessness by an order of value. When one approaches the critical point, the larger $g$, the larger value of $\varepsilon$ at which the characteristic time of the piston effect begins to deviate from its value in weightlessness.

Figure 3 shows a comparison of the results of the calculations for the density step (case (ii)) and for the homogeneous density (case (iii)). As is seen from the figure, the characteristic time of the piston effect slightly increases with $\mu$ close to the critical temperature (e.g. for $\varepsilon=3.13\times10^{-3}$ and $\varepsilon=9.4\times10^{-5}$). As $\varepsilon$ increases, the rate of the $\tau_{PE}$ rise with $\mu$ increases showing an exponential rise for the maximum calculation value of $\varepsilon$ ($3.13\times10^{-3}$). In contrast to the previous case $\tau_{PE}$ for the homogeneous density demonstrates slight linear rise with $\mu$ for all values of $\varepsilon$. It is interesting to compare the results with equation (2). To this purpose we use Eq. (4.10) from [11] for the derivation $(\partial \rho/\partial T)_P$ as a function of $\mu$. It follows that

$$
\tau_{PE} \propto \left( \frac{\partial \rho}{\partial T}_P \right)^{-1} \propto 4(\varepsilon + 1)(2 - \mu)^{-1} - (\mu + 1)(2 - \mu)
$$

A plot of this function for $\varepsilon=3.13\times10^{-3}$ is shown in figure 3. As is seen from the figure equation (2) well describes the homogeneous density case and dramatically differs from the density step case.

![Figure 3](image)

Figure 3. The characteristic time of the piston effect as a function of $|\mu|$ for various $\varepsilon$ for the case of density step and homogeneous density. The line is a plot of equation (2) for $\varepsilon=3.13\times10^{-3}$ and $g=0$. 


4. Conclusions

Characteristic features of the “speeding up” heating of supercritical sulfur hexafluoride are studied numerically with the use of the Navier-Stokes equations and Van der Waals equation of state. The calculations are carried out for 1D case within \[3 \times 10^{-5} < \varepsilon < 3 \times 10^{-3}\] for various gravity conditions \([g=0, \, g=g_0=9.81\text{m/s}^2\text{ and overload } g=5g_0 \text{ and } g=10g_0]\) as well as for the step density stratification and homogeneous density \((g=0)\) within \(-0.025<\mu<-0.005\). It is obtained that as the critical point approaches, the characteristic time of the piston effect, \(\tau_{PE}\), essentially increases with respect to that for weightlessness conditions for \(\varepsilon<10^{-3}\). In so doing, the greater acceleration of gravity, the larger the value of \(\varepsilon\) at which the difference becomes significant. In case of the step density stratification in the weightlessness \(\tau_{PE}\) rises essentially more rapid than that is predicted by the Onuki’s analysis (Eq. (2)) whereas the equation describes well the case of homogeneous density.

The study is supported in part by the Russian Foundation for Basic Research (grant № 15-01-02012).

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