Mathematical modelling of thermolabile solutions concentration in vortex chamber

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Abstract. The study represents a computation scheme for a fixed cylindrical chamber with an air tangential swirler, used for the thermal concentration of liquids. During the evaporation, the drops are not in direct contact with hot surfaces, which is critically important for thermosensitive substances. This is achieved by adjusting droplets motion in the form of a suspended rotatory drop layer. For this purpose, a detailed mathematical model was constructed, and a numerical study of heat-and-mass transfer was performed in terms of the real flow mechanics. It has been found that the time needed to change the drop volume during the process of concentration to the desired final volume uniquely depends on the construction of the chamber and operating regime in the workspace. So, numerical calculation of evaporating drops motion in a swirling gas flow has been performed over a wide range of different parameters such as heating gas temperature, inlet gas velocity, liquid-gas mass-flow ratio, inclination angle of the swirler blades and circulation ratio.

1. Introduction and mathematical model description
Thermal concentration processes are of widest use for producing commercial and intermediate products in chemical, pharmaceutical, food and other industries. Traditional techniques and equipment for concentrating and drying thermolabile liquids are bounded by their strongly marked temperature-sensitive properties, i.e. depend critically on the physicochemical properties of the starting materials. Even under low heat, irreversible changes in the chemical structure of the product occur. For instance, it is the decrease in the concentration of the desired substance because of thermal decomposition processes. The problem has been explored by using a vortex chamber which design and technological operating regime completely account for material thermosensitivity to achieve required quality of the finished products.

Despite the principles of designing vortex apparatuses have been studied for a long time, wide industrial use is complicated by the absence of valid and well-grounded methods for calculating the processes in the chamber [1–2]. So, the purpose of this paper is to present a mathematical model and perform a numerical study of heat-and-mass transfer in terms of using a vortex chamber for concentrating thermolabile liquids. In order to achieve this, the real flow mechanics principles in the workspace of the chamber should be considered.

The vortex chamber operates as follows (figure 1). The heating agent, in particular, hot air, is inserted through the gas distribution device into the vortex chamber. Gas in passing through the tangential blade swirler generates a rotary motion, and swirling gas flow is formed in the workspace of
the chamber. Liquid enters the chamber through holes in the tubular distributing device and instantly breaks up into discrete droplets to produce a suspended rotatory gas-liquid layer between the swirler blades and the discharge radius. The entry of the droplets into the inner surface of the chamber with a temperature close to that of the heating gas may be dangerous, because it causes the decomposition of thermosensitive substances and burning.

Moving droplets gradually come out and revolve in the equilibrium radii which match associated diameters and operating parameters of the chamber. This forms suspended droplets which have no direct contact with hot surfaces. Drops move down under gravity. Droplet size and equilibrium radius decrease as evaporation proceeds; that is because the radial position of the drop layer tends to the axis. The radius of gas discharge is governed by the equilibrium radius of drops with the desired content of solids. The cooling-water jacket is mounted in the lower part of the bottom, where drops tend to settle, in order to prevent the chemical decomposition of the desired product. Concentrated liquid crosses the separation gap device and flows out from the chamber. Gas is removed through the lower axial discharge.

When the vortex chamber operates without recirculation, the entire liquid phase interacts with heating gas once. By contrast, in operation using a circulation circuit, the liquid phase outflow is returned to the chamber after adding fresh flow. Therefore, the solids content of the incoming fluid is higher in case of the circulation scheme than the single-entry one under comparable conditions. Liquid recirculation can help to augment the concentration and increase the final solids content of the product.

Figure 1. A schematic of the vortex chamber:
1 — body; 2 — cover; 3 — bottom; 4 — tangential swirler; 5 — blade of the swirler; 6 — gas distributing device; 7 — heating gas intake; 8 — liquid intake; 9 — tubular liquid distributing device; 10 — gas discharge; 11 — separation gap device; 12 — concentrated liquid discharge; 13 — cooling-water jacket; 14, 15 — input-output technological pipes.

The objectives in the calculation of the vortex chamber are to determine the design parameters and operating conditions based on the mathematical model in terms of the real flow mechanics and heat-and-mass transfer principles in the workspace of the chamber. This takes into account the following assumptions:

- polydisperse liquid spray is replaced with monodisperse one. The Sauter mean diameter is taken as an averaging parameter;
- potential flow has been used to describe the velocity distribution of gas swirling flow;
- heat-and-mass transfer is quasistationary;
all physical properties of gas, namely, air-vapor mixture flowing around a drop, must be estimated at average temperature in the thermal boundary layer and at average concentration of vapor in the diffusion boundary layer;

- it is assumed to be the ideal stirred model in the bulk of the chamber;
- liquid phase has wet-bulb temperature at initial time.

The system of equations for the unit drop motion in terms of the cylindrical fixed coordinate system (axis of the chamber extends in a z-axis direction) can be written as follows [3]:

\[
\begin{align*}
\frac{dV_l}{d\tau} &= \frac{V_l^2}{r} + \frac{3}{4} \frac{\rho_{liq}}{\rho_{liq}} \frac{V_{ref}}{a} (W_l - V_l) \\
\frac{dV_g}{d\tau} &= -\frac{V_g V_l}{r} + \frac{3}{4} \frac{\rho_g}{\rho_{liq}} \frac{V_{ref}}{a} (W_g - V_g) \\
\frac{dV_s}{d\tau} &= g + \frac{3}{4} \frac{\rho_g}{\rho_{liq}} \frac{V_{ref}}{a} (W_s - V_s) \\
\frac{dr}{d\tau} &= V_r, \quad \frac{d\varphi}{d\tau} = V_{\varphi}, \quad \frac{dz}{d\tau} = V_z \\
\frac{da}{d\tau} &= -2 \frac{\beta_j}{\rho_{liq}} (y_1 - y)
\end{align*}
\]  

(1)

The numerical solution of system (1) using four-order Runge-Kutta method with initial conditions is as follows:

\[
\tau = 0, \quad r = r_1, \quad \varphi = 0, \quad z = z_1, \quad V_r = V_{r1}, \quad V_{\varphi} = V_{\varphi1}, \quad V_z = 0, \quad a = a_1,
\]  

(2)

where \( r_1 \) is the radius of liquid input; \( z_1 \) is the distance from chamber cover; and \( V_1 \) is the feeder velocity.

The radial component of the drop velocity can be found from the constant gas mass flow condition in the chamber in terms of its geometry, whereas the tangential component — from the law of circulation conservation. The presence of liquid is accounted for an empirical coefficient of effective velocity reduction in the chamber [4, 5].

The formed calculation algorithm has the following features:

1) The temperature at the drop surface \( t_s \) is determined by bisection method from a balance between the rate of heat and mass transfer at interface — \( \beta_j r_s (y_s - y) = \alpha (t - t_s) \).

Experimental data on mass-transfer rates for spheres may be correlated by the following empirical equation of the form used by Froessling [6]:

\[
Nu = 2 + 0,6 Re^{1/2} Pr^{1/3}, \quad Sh = 2 + 0,6 Re^{1/2} Sc^{1/3};
\]  

(3)

2) The moisture content of the gas mixture \( X \) should be corrected at each time step by the expression derived from mass balance:

\[
X = X_i + \Lambda (1 + X_i) \left[ 1 - \frac{\rho_{liq}}{\rho_{liq1}} \left( \frac{a_s}{a_t} \right)^3 \right],
\]  

(4)

where \( \Lambda \) is the input liquid-gas mass-flow ratio, \( L_{ml} / G_m \); 

3) Using the same procedure, temperature of the gas mixture \( t \) should be corrected at each time step by the expression derived from heat balance:
\[
t = \frac{t_1 \left( \bar{\rho}_p + X \bar{\rho}_v \right) - \Lambda (1 + X) \left[ 1 - \frac{\rho_{liq2}}{\rho_{liq1}} \left( \frac{a_2}{a_1} \right)^3 \right] \left( r_s - \bar{\rho}_p t_1 \right)}{\Lambda (1 + X) \left[ 1 - \frac{\rho_{liq2}}{\rho_{liq1}} \left( \frac{a_2}{a_1} \right)^3 \right] \bar{\rho}_p + \bar{\rho}_v + X \bar{\rho}_p}.
\]

Computations have been made by using an air-vapor mixture, thermolabile solution with known physical properties, as an example of a liquid-gas system. The mean heat capacity of gas was estimated using the rule of mass additivity, and the mean density was estimated according to volume additivity. The heat capacity at constant pressure for heating gas components and the vaporization heat of pure water were taken as an approximation by Beard from reference data [7].

The dynamic viscosity and thermal conductivity of the gas mixture were calculated from the Wilke equation [8] as functions of the molecular weights and appropriate characteristics of the pure components determined from Sutherland type equation [9]. The calculation of the diffusion coefficient for binary gas mixture followed the method given by Fuller [10].

2. Results and discussion

The system of equations (1) under initial conditions (2) was used to establish the behavior of the evaporating droplets in time, depending on drop diameters, and circulation ratio of liquid and gas temperature for the algorithm given above. A necessary condition of the concentration completion is to achieve droplet diameters \( a_2 \) that are in proportion to the desired concentration.

A numerical calculation of the drop trajectory in the vortex chamber with part-load recirculation indicated that both the cylindrical height \( H \) and the radius of the discharge \( R_0 \) were determined by the circulation ratio \( K_{liq} \), closely associated with the final diameter of the drop with coordinates \((h, r)\), moreover \( h = H, r = R_0 \).

Figure 2 presents data for the lifetime of the drops evaporated to the final concentration of 70 % in the vortex chamber under changes in the liquid circulation ratio. In the absence of the circulation circuit \( (K_{liq} = 0) \) the lifetime of drops with a diameter of 0.25 mm in the chamber with 0.35 m radius is 3.56 s, but when the circulation ratio \( K_{liq} \) lies in the range from 0 to 5, the duration of stay decreases 7.91 times. For drops with a diameter of 0.30 mm in the chamber with 0.35 m radius the decrease is 9.07 times, and for drops with a diameter of 0.37 mm in the chamber with a diameter of 0.65 m it is 9.37 times.

**Figure 2.** Effect of circulation ratio on lifetime for evaporating droplets with different diameters \( a_t \), mm: 1 — 0.25 \((R = 0.35 \text{ m}, r_t = 0.34 \text{ m})\); 2 — 0.30 \((R = 0.45 \text{ m}, r_t = 0.44 \text{ m})\); 3 — 0.37 \((R = 0.65 \text{ m}, r_t = 0.63 \text{ m})\);

\( a = 35^\circ; n = 12 \text{ pcs}; W_m = 13 \text{ m/s}; L_m / G_m = 0.2; X_1 = 0.03 \text{ kg/kg}; t_i = 110 \text{ °C}, z_1 = 0.05 \text{ m}; V_{sl} = 0; V_{ql} = 9 \text{ m/s}. \)
There are several parameters that have a direct and substantial impact on drop trajectory during evaporation in the vortex chamber. The system (1) was computed under a wide range of variables, such as averaged gas velocity in the effective cross-section of the swirler $W_{in}$ — from 10 to 20 m/s (figure 3a), liquid-gas mass-flow ratio $L_m/G_m$ — from 0.1 to 0.5 (figure 3b) and inclination angle of the swirler blades $\square$ — from 25 to 45° (figure 3c) for drops with initial diameter $a_i = 0.25, 0.30, 0.37$ mm (at concentration degree of 70 % and $K_{liq} = 3$ the values of the final droplet diameters are $a_z = 0.225, 0.270$ and 0.333 mm respectively).

Figure 3. Dependence of the relative displacement of drops in $z$-direction on:

a) averaged gas velocity in the effective cross-section of the swirler ($\alpha = 35^\circ$; $n = 12$ pcs; $L_m/G_m = 0.2$);
b) inclination angle of the swirler blades ($n = 12$ pcs; $L_m/G_m = 0.2$; $W_{in} = 15$ m/s);
c) liquid-gas mass-flow ratio ($\alpha = 35^\circ$; $n = 12$ pcs.; $W_{in} = 15$ m/s);

$a_i$, mm: 1 — 0.25; 2 — 0.30; 3 — 0.37.

Figure 4 shows results of computations for the relative displacement of drops in $z$-direction under changes of gas temperature for different liquid circulation ratio.

Figure 4. Dependence of the relative displacement of drops in $z$-direction on heating gas temperature at varying circulation ratio $K_{liq}$: • — 0; ▼ — 1; × — 2; ▲ — 3; ♦ — 4; ■ — 5:

$R = 0.35$ m; $a_i = 0.25$ mm; $\alpha = 35^\circ$; $n = 12$ pcs; $W_{in} = 13$ m/s; $L_m/G_m = 0.2$; $X_i = 0.03$ kg/kg; $t_i = 110^\circ$C; $z_i = 0.05$ m; $r_i = 0.34$ m; $V_{ri} = 0$; $V_{oi} = 8.5$ m/s.
The value of \( h/R \) decreases with a rise in temperature from 80 to 130 °C for drops of 0.25 mm in diameter. Furthermore, there is more substantial impact on the relative height of the chamber at \( K_{\text{liq}} = 0 \) and \( K_{\text{liq}} = 1 \) than in the other cases. For example, temperature on the drop surface \( t_s = 52.12 \) °C (\( t_s = 105 \) °C, \( K_{\text{liq}} = 3 \)) is sufficient to prevent chemical decomposition processes of thermolabile substances.

As a result of the numerical experiment the following conclusions can be made:

1) the lifetime of the evaporating drops with different diameters tend to decrease on average of 8.8 times when the circulation ratio of the concentrated liquid increases from 0 to 5;
2) the reduction of the relative displacement of drops in z-direction has been established under an increase in the averaged gas velocity in the effective cross-section of the swirler and a decrease of inclination angle of the swirler blades and liquid-gas mass-flow ratio for drops with different initial diameters;
3) the gas temperature within \( 105 \pm 25 \) °C has no significant effect on chamber geometry at the circulation ratio \( K_{\text{liq}} \geq 2 \).

**Notation**

- \( a \) = diameter of drop;
- \( c_a \) = aerodynamic drag coefficient;
- \( \bar{c}_p, \bar{c}_g, \bar{\rho}_g \) = averaged isobaric heat capacity of the gas mixture, dry air and water vapor at constant pressure, respectively;
- \( g \) = acceleration of gravity;
- \( K_{\text{liq}} \) = liquid circulation ratio;
- \( \alpha \) = heat transfer coefficient;
- \( \beta_g \) = mass transfer coefficient;
- \( \lambda_g \) = thermal conductivity of gas;
- \( \mu_g \) = dynamic and kinematic viscosity of gas;
- \( \rho_g \) = density of gas;
- \( \rho_{\text{liq}} \) = density of liquid in drop;
- \( \nu \) = time;
- \( \phi \) = constant pressure;
- \( \rho \) = density;
- \( r \) = radial component;
- \( s \) = on the drop surface;
- \( z \) = axial component;
- \( \theta \) = angular component;
- \( \text{st}, \text{yy} \) = subscript;
- \( \text{rel}, \text{ac} \) = subscript;
- \( \text{gg}, \text{liq} \) = subscript;
- \( \text{D} \) = constant pressure;
- \( \text{Pr} \) = Prandtl number;
- \( \text{Sc} \) = Schmidt number;
- \( \text{Sh} \) = Sherwood number;
- \( \text{Re} \) = Reynolds number;
- \( \text{Nu} \) = Nusselt number;
- \( \text{X} \) = moisture content of the gas mixture;
- \( \gamma_s, \gamma \) = water vapor concentration on the drop surface and outside the boundary layer, respectively;
- \( \alpha \) = heat transfer coefficient;
- \( \lambda_g \) = thermal conductivity of gas;
- \( \nu_g \) = dynamic and kinematic viscosity of gas;
- \( \rho_g \) = density of gas;
- \( \rho_{\text{liq}} \) = density of liquid in drop;
- \( t \) = time;
- \( \nu_l \) = water vapor concentration on the drop surface; components of vector fields for the drop velocity; \( V_{\text{at}} \) = droplet velocity related to the gas; \( V_x, V_y, V_z \) = components of vector fields for the drop flow velocity; \( X \) = moisture content of the gas mixture; \( \gamma_s, \gamma \) = water vapor concentration on the drop surface and outside the boundary layer, respectively;
- \( \alpha \) = heat transfer coefficient;
- \( \beta_g \) = mass transfer coefficient;
- \( \lambda_g \) = thermal conductivity of gas;
- \( \mu_g \) = dynamic and kinematic viscosity of gas;
- \( \rho_g \) = density of gas;
- \( \rho_{\text{liq}} \) = density of liquid in drop;
- \( \nu_l \) = water vapor concentration on the drop surface; components of vector fields for the drop flow velocity; \( X \) = moisture content of the gas mixture; \( \gamma_s, \gamma \) = water vapor concentration on the drop surface and outside the boundary layer, respectively;
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