Constraining Bag constant for Hybrid Neutron stars

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Received Day Month Year
Revised Day Month Year

We study the star matter properties for Hybrid equation of state (EoS) by varying the bag constant. We use the Effective-Field-Theory motivated Relativistic Mean-Field model (E-RMF) for hadron phase with recently reported FSUGarne t, G3 and IOPB-I parameter sets. The result of NL3 set is also shown for comparison. The simple MIT Bag model is applied for the quark phase to construct the hybrid EoS. The hybrid neutron star mass and radius are calculated by varying with $B_1/4$ to constrain the $B_1/4$ values. It is found that $B_1/4=130-160$ MeV is suitable for explaining the quark matter in neutron stars.

Keywords: Equation of State; Bag constant; Hybrid stars.

PACS numbers:

1. Introduction

The existence of quark matter in the interior of hot dense objects like neutron stars was first pointed out by Witten [1]. This assumption of quark matter being completely stable may turn out to be the true ground state of hadronic matter. This quark matter can exist either as a pure phase in the central regions or in mixed phase with hadronic matter [2]. The presence of quarks inside the neutron stars will change the star matter properties.

In the outer part of the neutron star (NS), where the density is low, the equation of state (EoS) is described by hadronic matter. With the increasing density, there is a possible phase transition from hadron matter to quark matter. The hadron-quark mixed phase is formed over a density range, thereafter a pure quark phase exists. The density range over which a mixed phases exists is determined by the charge neutrality condition. The information about the EoS of such stars, called
Hybrid stars, can be studied and analyzed using the measurements from the recent gravitational wave constraints and the massive NS (PSR J0740+6620) $M=2.14^{+0.20}_{-0.18}$ $M_\odot$.

To study hybrid star EoS, we employ Effective-field-theory motivated Relativistic Mean-Field (E-RMF) model for hadronic matter. This model has been very successful in determining the properties of finite\(^5\)\(^6\)\(^7\)\(^9\) as well as infinite nuclear matter\(^10\)\(^11\). For the quark phase, we use the simple MIT bag model\(^2\)\(^12\)\(^13\) to describe the quark matter in hybrid stars. In this model, the quarks are assumed to be confined in a bag of finite dimensions. The quarks are assumed to have very low mass inside the bag as compared to those outside, where the mass is very high. To balance the behavior of the bag and to find its size, a bag constant $B$ is introduced as a constant energy density in the system. At the surface of bag, the outward pressure produced by the quarks is balanced by the inward pressure $B$. Thus the quark pressure decreases with the increasing value of $B$ thereby influencing the structure of the star. With very low mass of $u$ and $d$ quarks, the value of $B$ depends on the mass of the strange quark. The value of $B$ varies from $B^{1/4} \approx 145$-160 MeV for massless strange quark\(^14\). This range narrows down with the increase in the mass of strange quark. However, different bag values have been used in the literature. The bag value $B^{1/4} \approx 200$ MeV is used in the QCD calculations by H. Satz\(^15\). Also, in accordance with the CERN-SPS and RHIC data, the bag constants are allowed to have a wider range\(^16\)\(^17\). So, we can consider the bag constant as an effective free parameter.

The value of bag constant plays an important role in determining the structure and properties of hybrid stars. The EoS parameters like pressure, energy density and maximum mass of hybrid star are affected by varying the bag value. Although the standard value of $B$ in the MIT bag model is taken as $B^{1/4} \approx 140$ MeV\(^18\)\(^19\), a wide range of values varying from small to large have been used\(^20\)\(^23\). But the definite range of bag values for hybrid stars is yet to be obtained. It is important to obtain a definite range of bag values for hybrid stars that will correspond to its stable configuration. The proper choice of bag constant can explain the hybrid stars with constrains imposed from recent gravitational wave observation using the simple MIT bag model.

In the present study, we combine the hadron phase and quark phase to build a single hybrid EoS. We determine the EoS for hybrid stars and calculate the mass and radius for different EoS’s to constrain the bag constant.

This paper is organized as follows: in section 2 we discuss the theoretical approaches employed to study the equation of state of different phases. For hadronic matter, we employ Effective-field-theory motivated Relativistic Mean-Field (E-RMF) model\(^5\) using recently proposed different parameter sets. In the Quark matter, we employ the MIT Bag Model to describe the quark matter in neutron stars. We then combine the two phases to form a mixed hadron-quark phase. In section 3, we discuss the nuclear matter (NM) properties for different parameter sets used. The EoS obtained for hybrid star is also discussed. In section 4, we calculate the star
matter properties like mass and radius for the obtained hybrid EoSs by varying the bag constant. All the calculated results are discussed in this section. Finally the summary and conclusions are given in section 5.

2. Formalism

The basic relativistic Lagrangian of the Effective Field Theory (EFT) for strong interaction\(^{10,24,25}\) at low energies contains the contribution from \(\sigma, \omega\) and \(\rho\) mesons without any self-coupling terms which is basically the original Walecka model\(^6\). The addition of self-coupling terms by Boguta and Bodmer minimized the value of determined parameters to an acceptable range. Further, the contribution from \(\delta\) meson explained certain properties of high dense matter\(^{26,27}\). Although the \(\delta\) meson contribution to finite nuclei are nominal, its contribution to high dense matter is large and hence the contribution from \(\delta\) meson should be considered. Since, the inclusion of cross-couplings have a large impact on neutron-skin thickness, symmetry energy and radius of neutron star, a systematic formalism based on naturalness and Naive Dimensional Analysis (NDA), the effective field theory motivated relativistic mean field (E-RMF) lagrangian is constructed.

The E-RMF Lagrangian with \(\delta\) meson inclusion, exchange mesons (\(\sigma, \omega \) and \(\rho\)) and all other self- and cross-couplings is given by ref\(^{5,28,29}\). Using the variational principle, the field equations for mesons and baryons are solved.

The EoS is obtained using the energy-momentum tensor expression

\[
T_{\mu\nu} = \sum_i \partial_\nu \phi_i \frac{\partial L}{\partial (\partial_\mu \phi_i)} - g_{\mu\nu} L, \tag{1}
\]

The energy density and pressure for the hadronic phase are

\[
\mathcal{E}_H = \langle 0 | T_{00} | 0 \rangle = \frac{2}{(2\pi)^3} \int d^3k E^*_i(k) + \rho W + \frac{m^2 \Phi^2}{g_s^2} \left( \frac{1}{2} + \frac{k_3}{3!} \frac{\Phi}{M} + \frac{k_4}{4!} \frac{\Phi^2}{M^2} \right) - \frac{1}{4!} \frac{\zeta_0 W^4}{g_\omega^4} + \frac{1}{2} \rho_3 R - \frac{1}{2} m_\omega \frac{W^2}{g_\omega^2} \left( 1 + \frac{\eta_1}{M} + \frac{\eta_2}{2} \frac{\Phi^2}{M^2} \right) - \frac{1}{2} \left( 1 + \frac{\eta_4}{M} \right) \frac{m^2}{g_\rho^2} R^2 - \Lambda_\omega (R^2 W^2) + \frac{1}{2} \frac{m_5^2}{g_5} (D^2) + \sum_l \mathcal{E}_l, \tag{2}
\]

and
where,

\[ E_l = 1 \sum_{i=1}^{3} < 0|T_{ii}|0 > = \frac{2}{3(2\pi)^3} \int d^3k E_i^*(k) - \frac{m_i^2 \Phi^2}{g_s^2} \left( \frac{1}{2} + \frac{k_i \Phi}{3! M} + \frac{k_i \Phi^2}{4! M^2} \right) \]

\[ + \frac{1}{4!} \zeta_0 W^4 + \frac{1}{2} m_{\omega^2} W^2 \left( 1 + \eta_1 \Phi M + \eta_2 \Phi^2 M^2 \right) + \frac{1}{2} \left( 1 + \frac{\eta_1 \Phi M}{2} \right) \frac{m_i^2 R^2}{g_s^2} \]

\[ + \Lambda_\omega (R^2 W^2) - \frac{1}{2} \frac{m_i^2}{g_s^2} (D^2) + \sum_l P_l, \]

(3)

\[ \xi_l \text{ and } P_l \text{ are the energy density and pressure for leptons. } \]

\[ M_p^* \text{ and } M_n^* \text{ are the effective masses of proton and neutron. For neutron star matter, the } \beta\text{-equilibrium and charge neutrality are two important conditions to be satisfied to determine the composition of the system. These are} \]

\[ \mu_p = \mu_n - \mu_e. \]

(4)

and

\[ q_{\text{total}} = \sum_{i=n,p} q_i k_i^3 / (3\pi^2) + \sum_l q_l k_l^3 / (3\pi^2) = 0, \]

(5)

For the quark phase, we employ MIT Bag model for the unpaired quark matter. It assumes the quarks to be confined in a large colorless region where the quarks are free to move. The quark masses are taken as \( m_u = m_d = 5.0 \text{ MeV} \) and \( m_s = 150 \text{ MeV} \). We ignore the one gluon exchange inside the gas. The equilibrium condition in the quark matter is given as

\[ \mu_d = \mu_s = \mu_n + \mu_e. \]

(6)

The chemical potential of the quarks follow from the two independent chemical potentials \( \mu_n \) and \( \mu_e \) as:

\[ \mu_u = \frac{1}{3} \mu_n - \frac{2}{3} \mu_e, \]

(7)

\[ \mu_d = \frac{1}{3} \mu_n + \frac{1}{3} \mu_e, \]

(8)

and

\[ \mu_s = \frac{1}{3} \mu_n + \frac{1}{3} \mu_e. \]

(9)

The charge neutrality condition is satisfied by

\[ \frac{2}{3} n_u - \frac{1}{3} n_d - \frac{1}{3} n_s - n_e = 0, \]

(10)
where, \( n_q(q = u, d, s, e) \) is the number density of quarks. The total quark density is
\[
 n_Q = \frac{1}{3}(n_u + n_d + n_s). \tag{11}
\]
The pressure of the quarks \((q = u, d, s)\) is given by
\[
 P_Q = \frac{1}{4\pi^2} \sum_q \left\{ \mu_q k_q \left( \mu_q^2 - \frac{5}{2} m_q^2 \right) + \frac{3}{2} m_q^4 \ln \left( \frac{\mu_q + k_q}{m_q} \right) \right\}. \tag{12}
\]
The total pressure due to quarks and leptons is given by
\[
 P = P_Q + P_l - B, \tag{13}
\]
where, \( B \) is the Bag constant.

At high density, the boundary between the pure HM and QM is supposed to coexist, but it is not well defined\(^3\). The density range over which a mixed phase can exist is governed by the \( \beta \)-equilibrium and charge neutrality conditions. Various methods have been used in the literature concerning the quark-hadron phase transition in neutron stars\(^3\),\(^{10,31-33}\). The Gibbs construction (GC)\(^3\) and the Maxwell construction (MC)\(^34\) are commonly used to describe the quark-hadron phase transition. It has been shown that beyond a limiting value of surface tension, the mixed phase becomes mechanically unstable and hence Maxwell construction provides a more relevant way in describing the Hybrid star properties\(^35,36\). However, the value of surface tension at the mixed phase is still not known properly and hence the formation of hadron-quark mixed phase by Gibbs construction is also possible even if the surface tension is too small. In the present work, we assume that the surface tension is not too high to ensure the formation of mixed phase by GC.

The Gibbs conditions for the mixed phase are given as:
\[
 P_{HP}(\mu_{HP}) = P_{QP}(\mu_{QP}) = P_M, \tag{14}
\]
and
\[
 \mu_{HP,i} = \mu_{QP,i} = \mu_i, \quad i = n, e. \tag{15}
\]

3. Results and Discussions

In this section, we discuss our results for hybrid EoS. For hadron phase we used different parameter sets NL3\(^37\), FSUGarnet\(^38\), G3\(^39\) and IOPB-I\(^29\). The NM parameters for the EoS at saturation density \( J, L, K_{sym} \) and \( Q_{sym} \) for all parameter sets are listed in Table I. For NL3 set, the symmetry energy \( J = 37.43 \text{ MeV} \) and slope parameter \( L = 118.65 \text{ MeV} \) are little higher than the empirical value \( J = 31.6 \pm 2.66 \text{ MeV} \) and \( L = 58.9 \pm 16 \text{ MeV} \). The \( J \) and \( L \) for other parameter sets lie well within the given range. The value of incompressibility \( K \) for different parameter sets are compatible with the observational data from various experiments\(^31,22\).
Table 1: Parameter sets and the corresponding nuclear matter properties for hadron matter. For all the sets, the nucleon mass is $M = 939.0$ MeV. All the coupling constants are dimensionless except $k_3$ which has the dimensions of fm$^{-1}$.

|                | NL3  | FSUGarnet | G3   | IOPB-1 |
|----------------|------|-----------|------|--------|
| $m_s/M$        | 0.541| 0.529     | 0.559| 0.533  |
| $m_\omega/M$   | 0.833| 0.833     | 0.832| 0.833  |
| $m_\rho/M$     | 0.812| 0.812     | 0.820| 0.812  |
| $m_\delta/M$   | 0.0  | 0.0       | 1.043| 0.0    |
| $g_s/4\pi$     | 0.813| 0.837     | 0.782| 0.827  |
| $g_\omega/4\pi$| 1.024| 1.091     | 0.923| 1.062  |
| $g_\rho/4\pi$  | 0.712| 1.105     | 0.962| 0.885  |
| $g_\delta/4\pi$| 0.0  | 0.0       | 0.160| 0.0    |
| $k_3$          | 1.465| 1.368     | 2.606| 1.496  |
| $k_4$          | -5.688| -1.397    | 1.694| -2.932 |
| $\zeta_0$      | 0.0  | 4.410     | 1.010| 3.103  |
| $\eta_1$       | 0.0  | 0.0       | 0.424| 0.0    |
| $\eta_2$       | 0.0  | 0.0       | 0.114| 0.0    |
| $\eta_\rho$    | 0.0  | 0.0       | 0.645| 0.0    |
| $\Lambda_\omega$| 0.0 | 0.043     | 0.038| 0.024  |
| $\alpha_1$     | 0.0  | 0.0       | 2.000| 0.0    |
| $\alpha_2$     | 0.0  | 0.0       | -1.468| 0.0    |
| $f_\omega/4$   | 0.0  | 0.0       | 0.220| 0.0    |
| $f_\rho/4$     | 0.0  | 0.0       | 1.239| 0.0    |
| $\beta_\sigma$ | 0.0  | 0.0       | -0.087| 0.0    |
| $\beta_\omega$ | 0.0  | 0.0       | -0.484| 0.0    |

Fig. 1 shows the variation of pressure with the baryon density for pure neutron matter (PNM). The results are compared with the experimental flow data obtained from the analysis of heavy ion collisions, where upper one (stiff-expt.) corresponds to the strong density dependence of $S(\rho)$ and the lower one (soft-expt.) corresponds to the weak dependence. It is clear from Fig. 1 that the PNM EoS for G3 set is compatible with the experimental data. The NL3 set produces stiffer results than the other forces at high densities. The IOPB-I and FSUGarnet EoS is also compatible with the data.

The pressure vs energy density for $\beta$-equilibrated charge neutral neutron star matter for parameter sets NL3, FSUGarnet, IOPB-I and G3 is also displayed. The NL3 parameter set yields a stiffer EoS. FSUGarnet and IOPB-I have similar EoSs at high density but they differ at low density. FSUGarnet has soft EoS at low density but becomes stiff at higher density as compared to IOPB-I and G3. The G3 set provides the soft EoS.
Fig. 1: a) Pressure vs baryon density for Pure Neutron Matter (PNM) and b) EoS for NS matter in $\beta$-equilibrium and charge neutrality condition with different E-RMF parameters.

Fig. 2: Hybrid EoS for different bag constants for NL3, IOPB-I and G3 parameter sets.

The hybrid EoS for different hadronic matter parameter sets (NL3, IOPB-I and G3) and for different quark matter bag values ($B^{1/4} = 100, 130, 160, 180$ and $200$ MeV) are shown in Fig. 2. It is clear that the energy density increases with the bag values and hence the pressure will decrease. This means that the hybrid EoS becomes softer as the bag values increase. The importance of hybrid EoS lies in the formation of mixed phase. The transition from HM to QM using Gibbs condition determines the stiffness or softness of the mixed phase.

The transition density at which the hadron-quark phase transition takes place varies with the bag constant. For very small bag constants, the phase transition takes place at density lower than the normal nuclear density ($\rho_0=0.16fm^{-3}$). For higher values, the hadron-quark mixed phase exists in the density range $\rho=(2-7)\rho_0$. 
4. Structure and Properties of hybrid star

With the EoS obtained, we can calculate the properties of hybrid stars. Assuming the star to be spherical and stationary, the Tolman-Oppenheimer-Volkoff (TOV) equations\textsuperscript{44,45} are used to evaluate the structure of the star.

\[
\frac{dP(r)}{dr} = -G\frac{[\mathcal{E}(r) + P(r)][M(r) + 4\pi r^3 P(r)]}{r^2(1 - 2M(r)/r)}
\]

(16)

and

\[
\frac{dM(r)}{dr} = 4\pi r^2 \mathcal{E}(r)
\]

(17)

where, \(G\) is the gravitational constant and \(M(r)\) is the gravitational mass. For a given EoS, the above equations can be solved for the given boundary conditions \(P(0) = P_c, M(0) = 0\), where \(P_c\) is the central pressure. The value of radius \((r = R)\) at which the pressure vanishes defines the surface of the star.

Fig. 3 shows the mass radius profile obtained for pure hadronic matter using different parameter sets. The NL3 set predicts a large radius and mass for neutron star. The NS mass is found to be \(2.81M_\odot\) and the corresponding radius is 13.20 km. For IOPB-I parameter set, the maximum mass is around \(2.15M_\odot\) and the radius is 12.27 km. The G3 set predicts a maximum mass of \(2.03M_\odot\) and the corresponding radius 11.06 km\textsuperscript{29}. The maximum mass of a non-rotating NS is in the range \(2.01\pm0.04 \leq M(M_\odot) \leq 2.16\pm0.03\). This range was obtained by Rezzolla \textit{et al}\textsuperscript{48} by combining the recent gravitational wave observation of a binary neutron star merger (GW170817)\textsuperscript{10} with the quasi-universal relation between rotating and
Fig. 4: Mass Radius profile of hybrid star for a) NL3, b) IOPB-I and c) G3 parameter sets

non-rotating neutron star maximum mass.

The mass radius profile for hybrid EoS obtained for different bag constants is shown in Fig 4. The green band represents the maximum mass range obtained for a non-rotating star. This band also satisfies the precisely measured mass of PSR J0348+0432 and PSR J1614-2230 with mass \((2.01 \pm 0.04)M_\odot\) and \((1.97 \pm 0.04)M_\odot\) respectively. These measurements imply that the maximum mass of any NS predicted by any theoretical model should reach the limit \(\approx 2.0\ M_\odot\). It is clear that the maximum mass of hybrid star decreases as the bag constant increases. For lower bag values, the maximum mass produced is very high \(\approx 2.5\ M_\odot\) for IOPB-I and G3 sets and for higher values of bag, the mass is reduced to \(\approx 1.8\ M_\odot\). The mass radius profile of pure hadronic matter is also shown (dotted curve) again for all parameter sets for comparison. The mass of pure hadronic matter for IOPB-I
and G3 sets lie well within the bag values \( B^{1/4} = 130-160 \text{ MeV} \).

Table 2: Mass and Radius of hybrid stars for different bag constants.

| \( B^{1/4} \) (MeV) | 100 | 130 | 160 | 180 | 200 |
|---------------------|-----|-----|-----|-----|-----|
| NL3                 |     |     |     |     |     |
| \( M(M_\odot) \)    | 2.98| 2.43| 2.08| 1.97| 1.81|
| \( R(\text{km}) \)  | 16.64| 14.72| 14.39| 14.13| 14.65|
| IOPB-I              |     |     |     |     |     |
| \( M(M_\odot) \)    | 2.46| 2.32| 2.07| 1.95| 1.90|
| \( R(\text{km}) \)  | 13.75| 13.50| 12.91| 12.64| 13.00|
| G3                  |     |     |     |     |     |
| \( M(M_\odot) \)    | 2.40| 2.15| 1.88| 1.82| 1.69|
| \( R(\text{km}) \)  | 13.40| 12.59| 12.24| 11.88| 11.57|

The maximum mass along with the corresponding radius for different parameter sets is shown in Table 2 for different bag constants.

Fig. 5 shows the variation of maximum mass and radius with bag constants. The maximum mass for NL3 set at bag value \( B^{1/4} = 100 \text{ MeV} \) is 2.98 \( M_\odot \) while for
pure hadronic matter, it predicts a mass of $2.81\, M_\odot$. As the bag constant increases, the maximum mass decreases from 2.98 to 1.81 $M_\odot$. So, while the GW170817 data rules out the pure NL3 EoSs, the addition of quarks softens the EoS and hence reduces the maximum mass satisfying the GW170817 mass constraints. Similarly, the maximum mass of IOPB-I and G3 set decreases from 2.46 to 1.90 $M_\odot$ and 2.40 to 1.69 $M_\odot$ respectively.

From the gravitational wave observation of the maximum mass of NS in the range $2.01\pm0.04 \leq M(M_\odot) \leq 2.16\pm0.03$, one can see that the bag constant $B^{1/4} = 130$ MeV produces a maximum mass of $2.15\, M_\odot$ for G3 set, while the same bag constant gives $2.43\, M_\odot$ for NL3 set and $2.32\, M_\odot$ for IOPB-I set. For $B^{1/4} = 160$ MeV, the maximum mass is 2.08, 2.07 and 1.82 $M_\odot$ for NL3, IOPB-I and G3 respectively. The radius of the canonical mass for the bag constants $B^{1/4} = 130-160$ MeV is found to be in the range 12.5-13.2 km which is well within the range of $R_{1.4} \leq 13.76$ km as extracted from the neutron star tidal deformability of GW170817 event. Further, the recently gravitational-wave detected GW190425 constrains the NS mass in the range 1.12 to 2.52 $M_\odot$. Thus we see that the most probable values of bag constant for the obtained EoSs lies in the range $130\, \text{MeV} \leq B^{1/4} \leq 160$ MeV. This range of bag constants determines the properties of hybrid stars that agree with the recent gravitational-wave observations GW170817 and GW190425. A. Aziz et al. has constrained the value of bag constant in the range $150\, \text{MeV} \leq B^{1/4} \leq 180$ MeV. The precisely measured mass of $1.97\pm0.04\, M_\odot$ for binary millisecond PSR J1614-2230 by Demorest et al. supports the presence of quarks in the NS.

5. Summary and Conclusion

We studied the hybrid EoS by mixing hadron matter and quark matter using Gibbs conditions. We employed the E-RMF model for hadron matter with recently reported parameter sets and MIT bag model for quark matter with different bag constants. We calculated the star matter properties like mass and radius for different bag constants. We found that the value of bag constant in the range $130\, \text{MeV} \leq B^{1/4} \leq 160$ MeV is suitable for explaining the quark matter in neutron stars. The increase in the bag value softens the EoS and hence reduces the maximum mass of the star. Other important quantities like tidal deformability will also be affected by the presence of quarks and hence the study of quark matter in neutron stars is of great importance to constrain the EoS.

Since the bag constant has a huge effect on the EoS, a more dominant theoretical approach is required to constrain the value of bag constant. The presence of exotic phases like kaons, hyperons etc in the neutron stars will further modify the EoS. The presence of quarks will provide a new insight into the physics of neutron stars and other high dense objects.
Acknowledgment

Ishfaq A. Rather is thankful to the Institute of Physics, Bhubaneswar for providing the hospitality during the work.

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