Equation of lightning return current in tropical zones

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Abstract. The purpose of this study is to model mathematically the equation of the lightning return current in tropical zones and to validate it with data obtained from direct measurements in Colombia and Brazil. This study is carried out because the lightning return current in tropical zones is greater than in other latitudes of the planet. For this reason, it is very important for different applications in electrical engineering. Nowadays, the Heidler equation is used for the simulation of faults in electrical systems. This equation will serve as a reference to make a comparison between the data of the equation obtained in this article with those from the equation of Heidler.

1. Introduction

Colombia and Brazil are countries located in the tropical zone of the planet[1]. This zone has an elevated ground flash density and a lightning peak current higher than in other latitudes[2]. This atmospheric condition in tropical zones can induce potentially elevated problems to people and electrical systems.

Colombia has a high activity by lightning because of the aforementioned. For this reason, this phenomenon is studied in the country, since approximately 67 people die by lightning in the country per year, according to data of the Departamento Administrativo Nacional de Estadística (DANE) or in English National Administrative Department of Statistics, the entity in charge of the statistics of the country[3]. In terms of faults in the Colombian electrical system, the most common damages in distribution transformers is by an electric atmospheric discharge[4], due to lightning. Thus, it is important to study and search for alternatives to lower the indices of deaths and damages in electrical systems in the country.

Colombia and Brazil have direct lightning measurement stations because of their high activity by lightning, in which direct lightning signals have been recorded and measured. Therefore, this article proposes to develop and verify a mathematical model of the lightning return current for tropical zones. In addition, the data obtained from the model of this article will be compared with the equation of Professor Heidler[5], which is currently used for engineering designs and software simulations such as EMTP-ATP, that brings the Heidler type font in its library of elements.
2. Lightning peak current in the world

The first measurements of the lightning peak current were made in temperate zones. The first results were presented by Berger in Monte San Salvatore in Switzerland, with a median of 30kA[6][2], comparable with those of countries in non-tropical zones. The first data of the magnitude of lightning return current obtained in tropical zone was recorded by Lee in Malaysia, which showed a median of 36kA[2].

In the tropical zone of South America, measurements were made in Brazil and Colombia, in the stations of Morro Cachimbo and Ilyapa, respectively. Professor Torres in Colombia obtained a median peak current of 42.9kA[2]. Table 1 shows the median current in different countries.

3. Ground flash density in Colombia and Brazil

According to IEC 62305-2, the Ground Flash Density (GFD) is the number of lightning flashes per \( \text{km}^2 \) per year. In many areas of the world, this value can be derived from data provided by Lighting Location Systems (LLS)[7]. The GFD was initially studied in Colombia by the Research Program on Acquisition and Analysis of Signals of the National University of Colombia (PAAS-UN)[4]. Currently, the company Keraunos S.A.S. is responsible for supplying the data corresponding to the GFD. The company use the network LINET, that has 10 magnetic field antennas located in different points of the country. This network has an efficiency over 90 percent[8]. The map of GFD in Colombia, published by the company Keraunos S.A.S. for the year 2014, is shown in figure 1.

![Figure 1. Ground Flash Density in Colombia 2014. [4]](image1)

Colombia has areas with a GFD greater than 80 \( \frac{\text{strokes}}{\text{km}^2 \text{year}} \). This values are higher than in other countries like Brazil [10], the highest GFD in Brazil is about 8 \( \frac{\text{strokes}}{\text{km}^2 \text{year}} \) and it is in areas of Sao Paulo and Rio de Janeiro, data of the GFD in Brazil are obtained from the Integrated Lightning Detection Network (RINDAT), which is used to determine Brazilian standards for lightning protection [11], the map of GFD to Brazil is shown in Figure 2.

![Figure 2. Ground Flash Density in Brazil. [11]](image2)

4. Types of lightning discharges

The literature of atmospheric electric discharges concludes that there are four types of lightning discharges, which are presented in next figure [12], where about 80 % of the total lightning are discharges that occur intracloud or cloud-to-cloud and only 20 % of the total discharges are cloud-to-ground discharges. [13].
With respect to cloud-to-ground discharges, we can say that there are different types of cloud-to-ground lightning discharges, which are downward negative lightning, upward negative lightning, downward positive lightning and upward positive lightning over cloud-to-ground discharges [14], the following figure shows each of the Types of cloud-to-ground Lightning

According to studies and statistics it is said that 90% of the total Cloud-to-Ground Lightning Discharges are negative and only 10% positive [12]. Another thing that has been demonstrated with measurements and studies is that downward positive lightning current magnitudes are greater and destructive than those presented by downward negative lightning [2, 15]

5. Direct measurement of the lightning return current

A direct lightning measuring station must withstand the nominal values of an electric discharge, this considering the location of the station and the values that can be presented. The elements that must have a direct measurement station should be a derivative instrument, Rogowski coil, Shunt resistor, Pearson transformer and recording instruments as oscilloscopes [4].

In Colombia, a direct lightning metering station was implemented, which was called “experimental lightning direct measurement station, Ilyapa”. This station was located in Samaná (Caldas), a point of high atmospheric electrical activity that was determined from the Information obtained from indirect measurements through Colombian LLS systems. This station was implemented with three objectives: to obtain direct measurements of lightning...
parameters, to compare the magnitudes of the lightning parameters with other latitudes and to improve lightning protection systems [4].

In Brazil, as in Colombia, a direct lightning metering station was implemented, this station was located in Morro do Cachimbo, this station was instrumented in 1985 at 15 km of Belo Horizonte [16], with this station has been able to measure more than 157 strokes in more than 13 years of operation [17].

6. **Heidler’s equation of lightning return current**

Professor Heidler proposed the following equation that is currently used in IEC 62305:

\[
i(t) = \frac{i_{\text{max}}}{k} \cdot \left(\frac{t}{\tau_1}\right)^n \cdot e^{-t/\tau_2}
\]

(1)

Where, \(i_{\text{max}}\) is the peak current, \(t\) is the time, \(k\) is the correction factor of the peak current, the coefficients \(y\) are the front time constant and the tail time constant respectively [7] [18].

![Figure 5. Definition of the short strike current where T1 is the front time and T2 is the tail time.](image)

For a short-stroke current, the signal has a front time \(T_1\) and a tail time \(T_2\), for the particular case of this article, the constants \(T_1\), \(T_2\), \(k\) and \(n\) will be found mathematically in order to adjust and minimize the error with respect to the direct lightning signals in Colombia and Brazil. Then, the behavior of Heidler equation will be observed with respect to measurements taken directly and compared with those obtained with the formula found in this article.

7. **Direct measurement signals of the lightning return current for verifying the model**

For the development of this article, three signs of verification were considered. Two of them were taken in Colombia and one in Brazil, with different parameters.

The parameters of each signal are totally different in front time, decay time and peak current, which makes them suitable to make the comparison of the formula of Heidler and the one presented in this article in different conditions.
Figure 6. First lightning signal from Ilyapa, front time 0.00000074 [s], decay time 0.0001622 [s] and peak current 125714 [A].

Figure 7. Second lightning signal from Ilyapa, front time 0.0000057 [s], decay time 0.000013 [s] and peak current 17190 [A].

Figure 8. Lightning signal from Morro Cachimbo, front time 0.0000057 [s], decay time 0.0000189 [s] and peak current 42000 [A].

8. Mathematical modeling
The model is analyzed in three parts: signal rise, decay and amplitude.

8.1. Signal rise
The initial part of the typical graph of a lightning corresponds to the rise from t=0 to the peak current time. We use a pulse type function with stabilization to model it, as shown below:
The previous function is analyzed considering that \( t_u \) is the ascendant time, which is the time from \( t = 0 \) to the time it takes the signal to reach 90\% of the peak current as shown in the next figure.

In equation 2, the time \( t_u \) of different signals is replaced and the coefficient \( Cc \) is changed with different values. Then, the results with a wave of 1.2 us of rising time and 50 us of decay time are shown with different coefficients \( Cc \).

From the above graphs, it can be determined that when \( Cc \) increases, the signal stabilization is faster. Now, the problem is that, when evaluating equation 2 with each value of \( t_u \), we did not find the value 0.9 of the magnitude. In the case of \( Cc=1 \) and \( Cc=10 \) in a time of 1.2 us, we found that in both cases \( f(1.2\mu s)=0.5 \), which is only a 50\% of the signal.

Therefore, we propose to modify equation 2 and find a new term, as shown in equation 3. Evaluating each value of \( Cc \) at time \( t_u \) of the different signals, the error can be minimized and closer to 90\%.

\[
\frac{\left( \frac{t}{t_u} \right)^{Cc}}{1 + \left( \frac{t}{t_u} \right)^{Cc}} = 0.9
\]

The solution of equation 3 was performed using Newton’s numerical method and statistical methods, which determined that the proper value of \( X \) is 0.2338. This solution has an error of 0.0000001\% . Thus, the first part of the equation will be:
Equation 4 ensured that when \( C_c \) is greater than 10, the function reaches 90\% as predicted. Consequently, when the values of \( C_c \) are less than 10, the function does not reach 90\%, but the error is minimum compared to the results obtained in equation 2. For instance, even when \( C_c = 1 \), the function reaches 57\%, that can be considered the worst case, and improves the 7\% of the equation 2. Nevertheless, this condition will be compensated during the later development of the model. Now we present the graphs obtained with equation 4:

**Figure 12.** Ascendant time with \( C_c = 1 \), \( y(t) \) is a equation 4.

**Figure 13.** Ascendant time with \( C_c = 10 \), \( y(t) \) is a equation 4.

The next step for the development of the model is to transfer the function to zero. Thus, we evaluate the function at zero for each value of \( C_c \) to have a reference point:

**Figure 14.** Vertical offset based on \( C_c \), \( y(t) \) is a equation 4 \( \times X \), where \( X \) is a new term for transfer this function to zero. Equation 5, have two terms, the left part is a equation 4 and right equation is a term determined in this graph like offset.

The previous Figure, 0.7773 \( e^{-1.37C_c} \) was determined as the needed term to transfer the signal to zero. The term slightly affects the signal. 90\% is kept when it is evaluated in \( t_u \), with \( C_c = 10 \). In contrast, the signal is decreased to 51\% when \( C_c = 1 \). The function with the term previously found is presented below:

\[
\frac{\left(\frac{t+(0.2338 t_u)}{t_u}\right)^{C_c}}{1 + \left(\frac{t+(0.2338 t_u)}{t_u}\right)^{C_c}} - 0.7773 \times e^{-1.37 \times C_c}
\]
The behavior of the above function is shown below with values of Cc=1 and 10, and an ascendant time of 1.2us.

![Figure 15. Front signal in equation 5 with Cc=10.](image1)

![Figure 16. Front signal in equation 5 with Cc=1.](image2)

The equation found for the rise of the signal works properly with values greater than Cc=4. With the different values calculated in tu, the signal is greater than 80 %. We will try to increase this value with the next procedures.

### 8.2. Decrease in signal

Once the approach of the front of the model has been completed, the decay of the model is proceeded. The decreasing exponential is used as shown in equation 6:

$$e^{-\frac{t}{td}}$$ (6)

Equation 6 presents an ideal behavior of a decrease. However, the function must be transferred on the horizontal axis, so that it starts in the value of the ascendant time, and ensure the 90 % obtained in the rise of the curve. The graph of the equation 7 was done using the decay time and the ascendant time with 1.2us and 50 us respectively. The modification of the equation 6 is shown in the following function:

$$e^{-\left(\frac{t-t_s}{td}\right)}$$ (7)

The graph of next figure shows that the equivalent percentage of the signal in the decay time td is 39 %.

![Figure 17. Signal decay, y(t) is a equation 7.](image3)

The possibility of transferring the model to the ascendant time in the equation 5 and compacting it with the equation 7 ensures that a front signal between 62 % and 90 % in the ascendant time. 62 % was obtained with Cc=1 and 90 % with Cc=10. The compact equation of ascendant and decay times (8) and its correspondent next 2 graphs are shown below.
The function found to model the decay time behaves properly between a range of 38.5% and 50% , where the approach value increases when Cc is lower.

8.3. Relationship between ascendant time and decay time, Cc
So far, the rise of the signal and its decay have been modeled independently. Now we are going to propose a relation between both through the term Cc , which is going to be called coefficient of curvature. This coefficient Cc relates the rise of the signal with the decay by means of a new term called curvature time tc , which is the time it takes the 50% of the rise wave to reach the 50% of the decay signal, as shown in the next figure.

When the term Cc was found to relate the rise and the decay signal as represented in equation 9, a correction had to be made at the decay signal term.

\[ C_c = \frac{2t_c}{1*10^{-5}} \]  

Considering the coefficient of curvature, the adaptation of the decay signal was made, as shown in the following figure.

The coefficient found for the correction is 0.1952 ln(Cc) + 0.3274. It was coupled in equation 10 as follows:
Figure 21. Decay signal correction, Y axis is Cc and X axis is tc.

\[
e^{\frac{-(t-t_u)}{0.3274+0.1952\ln(Cc)}} \times \left( \frac{\left(t+0.2338t_u\right)Cc}{t_u} \right)^{Cc} \left(1 + \frac{t+0.2338t_u}{t_u}\right)^{-0.7773} \times e^{-1.37t_u Cc} \right) (10)
\]

In the implementation of the equation, the term Cc can be used with real signals as shown in equation 9. However, the values of Cc= 10 and Cc = 1 are assumed for testing previous equation.

Figure 22. Graph of equation 10 with Cc=1.Percentage of the signal at time tu= 71.5 % , Percentage of the signal at time td= 9.5 %

Figure 23. Graph of equation 10 with Cc=10.Percentage of the signal at time tu= 90 % , Percentage of the signal at time td= 36 %

Now, equation 10 is solved and its results are compared with the results obtained in equation 8 with a signal of 1.2 / 50 us and Cc = 1.

Results went from 61 % in equation 8, to 71.5 % in equation 10, evaluating the ascendant time. The signal had an improvement of 10.5 % . However, when the decay time was evaluated, it went from 50 % in equation 8, to 9.5 % in equation 10, which means a loss of 40.5 % . Alternatively, when the signal is evaluated with Cc= 10, the ascendant time kept 90 % in both equations, but the decay time went from 38.5 % in equation 8, to 36 % in equation 10, which is a loss of 2.5 %.

According to this, it is possible to determine that Equation 10 improved the percentages of the ascendant time in cases where Cc was small, and maintain them when Cc had large values. On the contrary, the decay time was lower in all cases. Nevertheless, the signal front is more important in lightning signals. Therefore, this aspect was improved and an expression for the coefficient Cc could be found.
8.4. Signal amplitude

For the signal amplitude, the maximum peak of a signal to be analyzed is used, but an additional coefficient must be applied to adapt the signal as a function of $C_c$. The graph obtained from the adaptation data as a function of $C_c$ is shown below:

\[
\begin{align*}
  i(t) &= \left( \frac{A}{0.4545 + 0.1847 \ln(C_c)} \right) \times \left( e^{t_d(0.3274 + 0.1952 \ln(C_c))} \right) \times \left( \frac{(t + 0.2338 \times t_u)}{t_u} \right)^{C_c} \times \left( 1 + \left( \frac{t + 0.2338 \times t_u}{t_u} \right)^{C_c} \right) - 0.7773 \times e^{-1.37 \times C_c} \\
  &\quad \left(11\right)
\end{align*}
\]

Remembering that, $C_c = \frac{2 \times t_c}{1 \times 10^{-5}}$

9. Mathematical model verification

The verification of the model found is made, contrasting it with the Heidler equation, for which both equations are applied to the three signals presented in section 7. Making the contrast of Heidler’s equation with that found in this model will determine the advantages and disadvantages of the model found.

9.1. Verification with Lightning Ilyapa signal

The Ilyapa signal parameters for the model found are shown below:

\[
\begin{align*}
  C_c &= \left. \frac{2 \times t_c}{1 \times 10^{-5}} \right. \\
  t_c &= 145, 0 \times 10^{-6} s \\
  C_c &= \left. \frac{2 \times 145, 0 \times 10^{-6}}{1 \times 10^{-5}} \right. = 29 \\
  C_c &= 29 \\
  t_u &= 4 \times 10^{-5} [s] \\
  t_d &= 1,626 \times 10^{-4} [s]
\end{align*}
\]

With the defined parameters, the following three signals are plotted:
9.2. Verification with Lightning Ilyapa signal 2
The Ilyapa signal 2 parameters for the model found are shown below:

\[
C_c = \frac{2 \times t_c}{1 \times 10^{-5}}
\]

\[
t_c = 2,09 \times 10^{-6} \text{s}
\]

\[
C_c = \frac{2 \times 2,09 \times 10^{-6}}{1 \times 10^{-5}}
\]

\[
C_c = 4,18
\]

\[
t_u = 9 \times 10^{-6} \text{[s]}
\]

\[
t_d = 2,0004 \times 10^{-5} \text{[s]}
\]

With the defined parameters, the following three signals are plotted:

9.3. Verification with Lightning Morro Cachimbo signal
The Morro Cachimbo signal parameters for the model found are shown below:

\[
C_c = \frac{2 \times t_c}{1 \times 10^{-5}}
\]

\[
t_c = 70,02 \times 10^{-6} \text{s}
\]

\[
C_c = \frac{2 \times 70,02 \times 10^{-6}}{1 \times 10^{-5}}
\]

\[
C_c = 14
\]

\[
t_u = 6,474 \times 10^{-6} \text{[s]}
\]

\[
t_d = 9,6102 \times 10^{-6} \text{[s]}
\]

With the defined parameters, the following three signals are plotted:
Figure 26. Graph of the Ilyapa signal 2, comparison, between Heidler and model found.

Figure 27. Graph of the Morro Cachimbo, comparison, between Heidler and model found.

10. Correlation
Here we use the Spearman’s correlation coefficient, since it is non-parametric and allows measuring relation between non-normal samples, in this case the Pearson’s correlation coefficient is not applicable because since data does not accomplish assumptions. Bellow we show the Spearman’s correlation for each one of the signals:
Table 1. Correlation Ilyapa signal 1.

| Method | Value  |
|--------|--------|
| Heidler| 0.8580 |
| Model  | 0.9677 |

Table 2. Correlation Ilyapa signal 2.

| Method | Value  |
|--------|--------|
| Heidler| 0.9730 |
| Model  | 0.9878 |

Table 3. Correlation Morro Cachimbo signal.

| Method | Value  |
|--------|--------|
| Heidler| 0.9500 |
| Model  | 0.9174 |

The correlation for the first signal, the equation presented in this paper obtains a better correlation respect to real signal, in the second correlation for the Ilyapa signal 2, the equation presented in this paper obtains a better correlation respect to real signal, respecto to correlation of Morro do Cachimbo the signal of the Heidler model presents a proper behavior as particularly can be seen in the second subsequent discharge, which if had not been present the model obtained in this paper would have had an ideal performance.

11. Conclusion
With respect to the first approximation of the mathematical model found for the return current applied to real signals, it can be said that it fits well to the curves of the different signals, especially in the front of the signal.

The coefficient C correlates well in the model found and can be used in the Heidler equation, this value specifically might to be helpful for replicate a real signal.

The curvature time t is an easy parameter to find and important to recreate the signal.

The variables of the equation are A, tu, td and tc, all these variables are dates of a direct signal.

The importance of applying the Cc coefficient either in the model found in this paper or in Heidler’s equation is to be able to simulate real signals.

Each variable necessary for the equation finding in this paper are easily calculates for replicate real signals.

References
[1] J. Galvin and C. Jones, *The weather and climate of the tropics: Part 10 – Tropical agriculture*, Weather, vol 64, pp. 156-161, June, 2009.
[2] H. Torres, *Protección contra rayos*, National University of Colombia and Icontec, second edition, pp. 35-50, Bogotá, October, 2010.
[3] J. Rubiano, *Análisis estadístico de dos parámetros de rayo en zona tropical*, Francisco José de Caldas District University, Thesis, pp. 26-27, Bogotá, June, 2016.
[4] H. Torres, *El Rayo. Mitos, leyendas, ciencia y tecnología*, National University of Colombia, Second edition, Chapter 4, pp. 111-171. , Bogotá, 2002.

[5] F. Heidler, *Parameters of lightning current given in IEC 62305 - background, experience and outlook*, 29th International Conference on lightning protection, pp. 1-22. , Uppsala, Sweden, June, 2008.

[6] M. Uman, *The Lightning Discharge*, Academic Press INC., vol 39, pp. 32-54. , Orlando San Diego, EE.UU., 1987.

[7] IEC, *IEC 62305, Protection against lightning – Risk management*, International Electrotechnical Commission, part 2, Ed 3., Geneva, Switzerland, 2005.

[8] M. Uman, *K Services*, Keraunos science of lightning, Keraunos S.A.S., http://keraunos.co/index.php/es-co/. , Bogotá, Colombia, March, 2017.

[9] NASA, *National Aeronautics and Space Administration*, https://ghrc.nasa.gov/lightning/images/browse/mission.png, Department of Meteorology. , 2015.

[10] G. Huffines and R. Orville, *Lightning Ground Flash Density and Thunderstorm Duration in the Continental United States: 1989-96*, Cooperative Institute for Applied Meteorological Studies, Department of Meteorology. Texas, pp. 7. , 1998.

[11] I. Pinto and O. Pinto and K. Naccarato, *HOW GROUND FLASH DENSITY OBTAINED BY LIGHTNING LOCATION NETWORKS CAN BE USED IN LIGHTNING PROTECTION STANDARDS: A CASE STUDY IN BRAZIL*, 19th International Lightning Detection Conference, pp. 1-3. , 2006.

[12] V. Rakov, *Fundamentals of Lightning*, International Symposium on Lightning Protection, pp. 4-7. , Kathmandu, Nepal, October, 2011.

[13] B.Kucienska , *Los misterios de la nube de tormenta*, Seminario de divulgacion del centro de ciencias de la atmosfera, Universidad Nacional Autonoma de Mexico, pp. 18-21. , City of Mexico, Mexico,13, October, 2013.

[14] V. Rakov, *Lightning Parameters of engineering interest: Application of lightning detection technologies*, International Symposium on Lightning Protection, pp. 10-13. , Bangkok, Thailand,7, November, 2012.

[15] O. Pinto and L. Pinto, *Tormentas Positivas : Sorpresa en los Cielos Brasileños*, Revista de Divulgación Científica y Tecnológica de la Asociación Ciencia Hoy, Vol 8, No 44. , Argentina, January, 1998.

[16] M. Rubinstein and M. Paolone and C. Romero and F. Rachidi, *Instrumentation of the Santis Tower for Lightning Current Measurements*, ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE. , January, 2012.

[17] S. Visacro and A. Soares, *Statistical analysis of lightning current parameters: Measurements at Morro do Cachimbo Station*, JOURNAL OF GEOPHYSICAL RESEARCH, vol 109,pp. 5-12. , Argentina,10, January, 2004.

[18] F. Heidler and Z. Filisowski, *Parameters of lightning current given in IEC 62305 - background, experience and outlook*, 29th International Conference on lightning protection, pp. 4-20. , Upssala, Sweden,23-26, June, 2008.