Predictions for the Spatial Distribution of Gluons in the Initial Nuclear State

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Abstract

We make predictions for the $t$-differential cross section of exclusive vector meson production (EVMP) in electron-ion collisions, with the aim of comparing DGLAP evolution to CGC models. In the current picture for the high-energy nucleus, nonlinear effects need to be understood in terms of low-$x$ gluon radiation and recombination as well as how this leads to saturation. EVMP grants experimental access to the edge region of the highly-boosted nuclear wavefunction, where the saturation scale for CGC calculations becomes inaccessible to pQCD. On the other hand, DGLAP evolution requires careful consideration of unitarity effects. The existing $J/\psi$ photoproduction data in $ep$ collisions provides a baseline for these theoretical calculations. Under different small-$x$ frameworks we obtain a measurable distinction in both the shape and normalization of the differential cross section predictions. These considerations are relevant for heavy ion collisions because the initial state may be further constrained, thus aiding in quantitative study of the quark-gluon plasma.

Keywords: color glass condensate, photoproduction, saturation, exclusive vector meson production

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1. Introduction

Precise measurements of the strong force responsible for holding the nucleus together test the basic properties of matter as predicted by QCD physics. The gluonic structure of the nucleus can be scanned by a quark-antiquark pair produced from a virtual photon. Such is the case in exclusive vector meson production (EVMP), where a particular final state of the dipole is produced. Data from HERA is compatible with existing theoretical models, and has indicated the existence of a saturation scale [1]. The nonlinear effects arise from gluon radiation and recombination which becomes increasingly important at low-$x$.

What concerns us here, is whether different underlying physical assumptions for the evolution of gluon densities in nuclei could be used to arrive at testable predictions with measurable differences. This adds to the case for future electron-ion collider facilities, as EVMP provides a fertile testing ground for the gluon distribution [2]. We shall begin with an analysis of $ep$ collisions, using this to confirm that the momentum distributions are evolving correctly. Following this, we present results for nuclear scattering. Traditional DGLAP evolution will be compared to novel CGC physics based on the running coupling BK equation [3] (rcBK). The DGLAP models feature impact parameter dependence based on work in [4]. CGC based calculations include explicit dependence on the number of overlapping nucleons at a given impact parameter [5].
2. The nucleon as a baseline

Consider the electron-proton interaction \( e + p \rightarrow e + V + p \), which proceeds through the exchange of a virtual photon between the proton and electron. It is standard to focus on the QCD contribution, namely \( \gamma' + p \rightarrow V + p \). When \( V \) is a vector meson, the momentum fraction is given by

\[
x = \frac{Q^2 + M_V^2}{Q^2 + W^2}.
\]

Therefore the squared invariant mass \( W^2 \), of the produced hadronic matter, may be used as a proxy for the \( x \) evolution. We shall be using data from [6, 7] in order to fix the parameters in our models. The observation that \( d\sigma/dt \sim e^{R_p} \) gives the total cross section as a proportion of the forward scattering of the differential cross section,

\[
\sigma_{tot}(W) = \frac{1}{B_p} \left. \frac{d\sigma^{\gamma p \rightarrow Vp}}{dt} \right|_{t=0}.
\]

A formal description of EVMP in the dipole picture may be found in [8]. For our purpose, the elastic diffractive cross section may be written in terms of the squared amplitude,

\[
\frac{d\sigma^{\gamma p \rightarrow Vp}_{T,L}}{dt} = \frac{1}{16\pi} |\mathcal{A}^{\gamma p \rightarrow Vp}(x, Q^2, \Delta)|^2.
\]

Where the \( Q^2 \) is the virtuality of the incoming photon, and \( \Delta \) is the momentum imparted to the target (i.e. \( t = -\Delta^2 \)). The subscripts \( T \) and \( L \) refer to the transverse and longitudinally polarised photons respectively. By considering the timescales involved [9], it is justified to write the amplitude as

\[
\mathcal{A}(x, Q^2, \Delta) = \int d^2r \int \frac{dz}{4\pi} (\Psi_V^* \Psi)(r, Q^2, z) \frac{d\sigma^p}{dt}.
\]

Information pertaining to the gluonic interaction is encoded in the dipole term. The vector-meson photon overlap \((\Psi_V^* \Psi)\) gives the amplitude for the incoming photon to split into a \( q\bar{q} \) pair and then recombine into a vector-meson. Kinematic choices for \( b \) set it to be Fourier conjugate to the momentum transfer \( \Delta \).

We follow previous work [2], and assume that the momentum scale of the interaction depends on the magnitude of the dipole separation, \( \mu^2(r) = C/r^2 + \mu_0^2 \). There are existing best-fit values for \( C \) and \( \mu_0 \) in the literature, however we shall be obtaining our own values with relevance to the \( x \) evolution. A more precise derivation of this scale is left for future work. Identifying the opacity as

\[
\Omega = r^2 F(x, r) T(b), \quad \text{where} \quad F(x, r) = \frac{\pi^2}{2N_c} \alpha_s(\mu^2) x g(x, \mu^2),
\]

assuming that the \( b \) dependence of \( x g \) can be neglected. For a thick target, the dipole cross section has the form

\[
\frac{d\sigma^p}{dt}(b, r, x) = 2 \left[ 1 - \exp(-\Omega) \right],
\]

in accordance with the Glauber-Mueller formula, (henceforth abbreviated “GM”). We shall also study a linearised version of this expression with a unitary cut-off, viz.

\[
\frac{d\sigma^p}{dt}(b, r, x) = \Omega \theta (2 - \Omega) + 2\theta (\Omega - 2),
\]

which clearly respects the large \( r \) behaviour. This we name “\( \theta \)-CK” [2]. Comparison between (2) and (3) will indicate the importance of the saturation scale.

Our results depicting the total cross section (1) as a function of \( W^2 \) are reproduced in Fig. 1. The parameters \( \mu_0 \) and \( C \) have been left as free, with best-fit values presented in Fig. 1. The “MNRT” parametrisation is a simplification for the evolutions of the gluon pdfs [10]. Proper DGLAP evolution is applied using “MSTW” code [11]. The MSTW produces a run-away result at large \( W \) (small-\( x \)), due to the fact that the gluon pdfs are unconstrained below \( x = 10^{-5} \).
In configuration space, we now treat the nucleus in the transverse plane as a collection of nucleons with coordinates \( \{b_1, b_2, \ldots, b_A\} \). The DIS event is characterised by the scattering matrix \( S_A(r, b, x) \), which may be expressed in terms of a product over the nucleons with the independent scattering approximation. Positions of the nucleons must be averaged over, in order to talk about positions of the nucleons with the independent scattering approximation. Positions of the nucleons must be averaged over, in order to talk about an average over nucleon coordinates in a simplified manner and thus describe a “smooth nucleus”. In addition, we shall consider convolution of Woods-Saxon distributions, by modifying the opacity according to \( \langle |A| \rangle \) is calculated through the variance \( \langle (|A|)^2 \rangle - \langle |A| \rangle^2 \). The DIS event is characterised by the scattering matrix \( T(b) \) as a normalised transverse Woods-Saxon distribution, denoted by \( T_A \). Thus, in this way, the nucleus also inherits model (3). Both of these models average over nucleon coordinates in a simplified manner and thus describe a “smooth nucleus”. In addition, we shall approximate the “lumpy nucleus” in the following way. Averaging the impact parameter cross section involves a convolution of \( A \) Woods-Saxon distributions, by modifying the opacity according to \( T(b) \rightarrow \sum_{i=1}^{A} T_p(b - b_i) \).

\[
\left( \frac{d\sigma}{d^2b} \right) = 2 \int \prod_{i=1}^{A} d^2b_i T_A(b_i) \left[ 1 - \exp \left( -r^2 F(x, r) \sum_{i=1}^{A} T_p(b - b_i) \right) \right] = 2 \left[ 1 - (1 - I(b))^A \right].
\]

by factoring the \( A \) integrals, where

\[
I(b) = \int d^2b' T_A(b + b') \left[ 1 - \exp \left( -r^2 F(x, r) T_p(b') \right) \right].
\]

Observing that the integral over \( b' \) gains most contribution over the size of the proton, which is small compared to the nucleus. Pursuing this, we suppose \( T_A(b + b') \approx T_A(b) \).

\[
I(b) \approx T_A(b) 2\pi B_p \left[ 1 - \text{Ei} \left( -r^2 F(x, r) / 2\pi B_p \right) + \ln \left( r^2 F(x, r) / 2\pi B_p \right) \right].
\]

Here \( \gamma \) is the Euler-Mascheroni constant and \( \text{Ei} \) is the exponential integral. Fig. 2 shows the differential cross sections produced using these various models. The size of the nucleus grows (in apparent physical size) as \( x \) shrinks, visible from the dip and peak positions. Within the GM models, a more sophisticated lumpy calculation is almost indistinguishable from the smooth approximation. The \( \theta \)-CK models overestimates the cross section, as expected. The GM models reproduce an, on average, lower normalisations than rcBK.
4. Conclusion

Fig. 2 is our main result. The two approaches, namely DGLAP and rcBK evolution, were checked against existing data for electron-hadron collisions. They both gave reasonable agreement in the relevant $x$ range. However, when tested with electron-ion collisions the predictions appear different, both in normalisation and peak-position.

This work is ongoing, and many improvements are to be made. In particular, it is a priority to include “nuclear shadowing” effects by making use of newer nPDF software. Another avenue to explore would be to repeat the above for $\phi$-meson scattering, as the larger dipole size $r$ makes for a more sensitive probe of the saturation region.

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