One-instanton test of the exact prepotential for $\mathcal{N} = 2$ SQCD coupled to a symmetric tensor hypermultiplet

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Abstract
Using the ADHM instanton calculus, we evaluate the one-instanton contribution to the low-energy effective prepotential of $\mathcal{N} = 2$ supersymmetric $SU(N)$ Yang-Mills theory with $N_F$ flavors of hypermultiplets in the fundamental representation and a hypermultiplet in the symmetric rank two tensor representation. For $N_F < N - 2$, when the theory is asymptotically free, our result is compared with the exact solution that was obtained using M-theory and we find complete agreement.

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An important development in the study of four-dimensional Yang-Mills theory has been the prediction of exact results for models with $\mathcal{N} = 2$ supersymmetry. The pioneering work was that of Seiberg and Witten, who investigated the pure $SU(2)$ Yang-Mills theory [1] and also $SU(2)$ models with matter transforming in the fundamental or the adjoint representation [2]. Subsequently, their results have been generalized to a wide range of models with simple or direct product gauge group and matter in various representations.

The general prescription for the exact results involves an algebraic curve, whose form depends on the model being considered, and a particular one-form defined on the curve. The one-form is to be integrated over one-cycles that form a canonical basis on the curve. This yields an exact solution for the effective prepotential, the holomorphic function that describes the low-energy Coulomb branch physics.

A general property of the effective prepotential is that at weak-coupling it has an expansion consisting of a one-loop perturbative term plus an infinite series of non-perturbative terms that correspond to instanton effects. In principle, it is possible to directly calculate these effects from first principles in the weakly-coupled quantum field theory. This is an important exercise, since by comparing the results of first-principles instanton calculations with the predictions extracted from the curves one can directly test the exact results and the means by which they have been derived. Over the last few years there has arisen a program of research concerned with carrying out precisely such tests of the exact results [3–10].

In this Letter, we continue this program of instanton tests. We focus on a model which has gauge group $SU(N)$ and $N_F$ flavors of hypermultiplets in the fundamental representation as well as a hypermultiplet transforming in the symmetric rank two tensor representation. In other words, we investigate $\mathcal{N} = 2$ supersymmetric QCD ($\mathcal{N} = 2$ SQCD) coupled to a symmetric tensor hypermultiplet. The algebraic curve corresponding to this model was derived by Landsteiner, Lopez and Lowe by studying brane configurations in M-theory [11]. The one-instanton contribution to the effective prepotential predicted by the curve has been explicitly determined by Ennes et al. [12]. To test the curve, we calculate the same contribution from first principles. Our result completely agrees with the curve prediction.
thus verifying the exact solution and the M-theory method at the one-instanton level.²

To perform the instanton calculation, we employ the instanton calculus, based on the construction of Atiyah, Drinfeld, Hitchin and Manin (ADHM) [14], that has been developed in [4, 5, 13, 11]. Whilst this calculus is important mainly for the study of multi-instanton effects, it is useful in the present context because it greatly facilitates the collective coordinate integration at the heart of the instanton calculation. (In particular, it simplifies an otherwise highly non-trivial integration over the group space collective coordinates of the instanton.)

To begin, we will briefly summarize the essential features of the ADHM instanton calculus for the specific case of $\mathcal{N} = 2$ SQCD [4, 13, 10], borrowing from the results of Ref. [10] in particular. We will subsequently describe how this calculus is extended when $\mathcal{N} = 2$ SQCD is coupled to a symmetric tensor hypermultiplet. We specialize to the one-instanton case throughout, this being sufficient for our purpose.

In $\mathcal{N} = 2$ SQCD, the supersymmetric ADHM one-instanton background is parameterized by: i) an $(N + 2) \times 2$ complex-valued matrix $a$ (and its conjugate $\bar{a}$), associated with the gauge field, ii) a pair of $(N + 2)$-dimensional Grassmann-valued vectors $\{\mathcal{M}, \mathcal{N}\}$ (and the conjugate pair $\{\bar{\mathcal{M}}, \bar{\mathcal{N}}\}$), associated with the two adjoint Weyl fermions, and iii) $N_F$ pairs of Grassmann parameters $\{\mathcal{K}_f, \bar{\mathcal{K}}_f\}$ ($f = 1, \ldots, N_F$), associated with the $N_F$ flavors of fundamental and conjugate-fundamental Weyl fermions. Whereas $\mathcal{K}_f$ and $\bar{\mathcal{K}}_f$ are free parameters, the collective coordinates associated with the adjoint fields, $a, \mathcal{M}$ and $\mathcal{N}$, are required to satisfy certain constraint equations. To write these equations, it is first useful to decompose $a$, $\mathcal{M}$ and $\mathcal{N}$ (and their conjugates) as follows:

$$a_{\lambda \dot{\alpha}} = \begin{pmatrix} w_{u\dot{\alpha}} \\ \bar{a}'_{\alpha \dot{\alpha}} \end{pmatrix}, \quad \bar{a}^{\dot{\alpha}}_{\lambda} = \begin{pmatrix} \bar{w}^{\dot{\alpha}}_u \\ -\bar{a}'_{\alpha \dot{\alpha}} \end{pmatrix},$$

$$\mathcal{M}_\lambda = \begin{pmatrix} \mu_u \\ \mathcal{M}'_{\alpha} \end{pmatrix}, \quad \bar{\mathcal{M}}_\lambda = \begin{pmatrix} \bar{\mu}_u \\ \bar{\mathcal{M}}'_{\alpha} \end{pmatrix}.$$

²Our attention is restricted to $N_F < N - 2$, when the theory is asymptotically free; we are unable to provide a test in the vanishing $\beta$-function case $N_F = N - 2$ since here the curve of [11] has not been explicitly parameterized. Instanton tests of the exact results for $\mathcal{N} = 2$ SQCD have revealed important discrepancies in the vanishing $\beta$-function case that — at least for $N > 3$ — have yet to be completely resolved [3, 13, 11].
\[ N_\lambda = \begin{pmatrix} \nu_u \\ N'_\alpha \end{pmatrix}, \quad \bar{N}_\lambda = \begin{pmatrix} \bar{\nu}_u \\ \bar{N}'^\alpha \end{pmatrix}, \]

where the ranges of the various indices are \( 1 \leq \lambda \leq N + 2, 1 \leq u \leq N \) and \( \alpha, \bar{\alpha} = 1, 2 \).

Correspondingly, the constraints on \( a \) read

\[ a'_m = \bar{a}'_m, \quad (\tau^c)^{\bar{\alpha}}_\beta \bar{w}_u^{\beta} w_{u\dot{\alpha}} = 0, \]

where the \( \tau^c (c = 1, 2, 3) \) are the Pauli matrices and \( a'_m \) and \( \bar{a}'_m \) are defined by the quaternionic expansions \( a'_{\dot{a}a} = a'_m \sigma^m_{\dot{a}a} \) and \( \bar{a}'^{\dot{\alpha}\alpha} = \bar{a}'^m \bar{\sigma}^m_{\dot{\alpha}\alpha} \). The constraints on \( M \) and \( N \) read

\[ M'_\alpha = \bar{M}'_\alpha, \quad N'_\alpha = \bar{N}'_\alpha, \]

\[ \bar{\mu}_u w_{u\dot{\alpha}} = -\bar{w}_{\dot{\alpha}u} \mu_u, \quad \bar{\nu}_u w_{u\dot{\alpha}} = -\bar{w}_{\dot{\alpha}u} \nu_u. \]

Besides these collective coordinates, the ADHM one-instanton background in \( \mathcal{N} = 2 \) SQCD is further described by a parameter \( \mathcal{A}_{\text{tot}} \) associated with the adjoint scalar field \( A \).

On the Coulomb branch, this field satisfies

\[ \langle A \rangle = \text{diag}(v_1, v_2, \ldots, v_N), \]

where the \( v_u \) are arbitrary complex numbers that sum to zero. The parameter \( \mathcal{A}_{\text{tot}} \) is determined in terms of these vacuum expectation values (VEVs) and the collective coordinates \( a, M \) and \( N \) by the following equation:

\[ L \cdot \mathcal{A}_{\text{tot}} = \Lambda + \Lambda_f, \]

where \( L = \bar{w}^{\dot{\alpha}u} w_{u\dot{\alpha}} \) and

\[ \Lambda = i \bar{w}^{\dot{\alpha}u} v_u w_{u\dot{\alpha}}, \]

\[ \Lambda_f = \frac{1}{2\sqrt{2}} (\bar{\mu}_u \nu_u - \bar{\nu}_u \mu_u). \]

The ADHM instanton calculus provides expressions for both the action and the collective coordinate integration measure corresponding to the supersymmetric ADHM instanton
In terms of the above quantities, the one-instanton action for $\mathcal{N} = 2$ SQCD is given by \[5, 10\]

\[
S_{SQCD}^{1\text{-}1} = \frac{8\pi^2}{g^2} + 8\pi^2|\nu_u|^2 \bar{w}^\alpha_u w_{u\alpha} - 2\sqrt{2} \pi^2 i (\bar{\mu}_u \bar{v}_u \nu_u - \bar{\nu}_u \bar{v}_u \mu_u) - 8\pi^2 (\bar{\Lambda} + \Lambda_{hyp}) A_{tot} + \pi^2 \sum_{f=1}^{N_F} m_f \bar{K}_f K_f,
\]

(10)

where the $m_f$ are the bare masses of the hypermultiplets and

\[
\Lambda_{hyp} = \frac{i}{4\sqrt{2}} \sum_{f=1}^{N_F} K_f \bar{K}_f.
\]

(11)

This fermion bilinear is associated with the conjugate adjoint scalar field $A^\dagger$ in the same way that $\Lambda_f$ is associated with $A$; it reflects a Yukawa source term involving the fundamental fermions in the Euler-Lagrange equation for this field \[5, 10\].

The one-instanton collective coordinate integration measure for $\mathcal{N} = 2$ SQCD takes the form of a flat measure with respect to the parameters $\{a, M, \mathcal{N}, A_{tot}, K, \bar{K}\}$, with the constraint equations (3), (5) and (7) imposed by means of $\delta$-functions under the integral sign \[15, 10\]:

\[
\int d\mu_{SQCD}^{1\text{-}1} = \frac{C_1'}{2\pi \pi^2 N_F} \int dA_{tot} d^{2N} w d^{2N} \bar{w} d^N \mu d^N \bar{\mu} d^N \nu d^N \bar{\nu} d^4 a' d^2 M' d^2 N' d^{N_F} K d^{N_F} \bar{K}
\times \delta (L \cdot A_{tot} - (\Lambda + \Lambda_f)) \left[ \prod_{c=1}^{3} \delta \left( \frac{1}{2} (\tau^c)_{\bar{\alpha} \beta} \bar{w}^\beta_u w_{u\alpha} \right) \right]
\times \left[ \prod_{\bar{\alpha}=1,2} \delta (\bar{\mu}_u w_{u\bar{\alpha}} + \bar{w}_{\bar{\alpha}u} \mu_u) \delta (\bar{\nu}_u w_{u\bar{\alpha}} + \bar{w}_{\bar{\alpha}u} \nu_u) \right].
\]

(12)

The constant prefactor $C_1'$ can be determined by comparing with the standard ’t Hooft-Bernard one-instanton measure \[16, 17\]. Thus, in the Pauli-Villars regularization scheme, one finds $C_1' = 2^{2\pi - 2N} M^{2N - N_F}$, where $M$ is the regularization mass.

Having summarized the ADHM instanton calculus for $\mathcal{N} = 2$ SQCD, we now describe its extension when this theory is coupled to a symmetric tensor hypermultiplet. The supersymmetric instanton background for the resulting theory will include configurations corresponding to the component fields of the extra hypermultiplet. The relevant fermionic field
configurations are given by the “zero-mode” solutions to the massless Dirac equation in the symmetric tensor or conjugate-symmetric tensor representation. At the one-instanton level, these solutions are parameterized by a pair of \((N + 2)\)-dimensional Grassmann-valued vectors, \(\mathcal{R}\) and \(\tilde{\mathcal{R}}\); these can be decomposed as

\[
\mathcal{R}_\lambda = \begin{pmatrix} \rho_u \\ \mathcal{R}'_\alpha \end{pmatrix}, \quad \tilde{\mathcal{R}}_\lambda = \begin{pmatrix} \tilde{\rho}_u \\ \tilde{\mathcal{R}}'_{\tilde{\alpha}} \end{pmatrix}.
\]

(13)

A useful simplification is that there are no constraints on these collective coordinates \([18]\); the number of parameters in each of \(\mathcal{R}\) and \(\tilde{\mathcal{R}}\) precisely equals the number of zero-modes of the Dirac operator in the symmetric tensor representation.

We can directly associate a collective coordinate integration measure with the extra fermion zero-modes. Since \(\mathcal{R}\) and \(\tilde{\mathcal{R}}\) are free parameters, it is simply the (suitably normalized) flat measure

\[
\int d\mu^{1-I}_{\text{hyp}} = \frac{M^{-(N+2)}}{2^{2N} \pi^{2(N+2)}} \int d^N \rho d^N \tilde{\rho} d^2 \mathcal{R}' d^2 \tilde{\mathcal{R}}'.
\]

(14)

The full one-instanton integration measure for the theory is now given by the product of this and the full \(\mathcal{N} = 2\) SQCD measure \([12]\).

The one-instanton action for the theory is straightforward to determine using the methods of \([4, 5, 10]\). It is the sum of the \(\mathcal{N} = 2\) SQCD action \((10)\) and the hypermultiplet-induced contribution

\[
\Delta S^\text{1-I}_{\text{hyp}} = -4 \sqrt{2} \pi^2 \tilde{\rho}_u \nu_u \rho_u - 8 \pi^2 \Lambda^\prime_{\text{hyp}} A_{\text{tot}} + 2 m \pi^2 (2 \tilde{\rho}_u \rho_u + \tilde{\mathcal{R}}'_{\tilde{\alpha}} \mathcal{R}'_{\alpha}),
\]

(15)

where \(m\) is the bare mass of the symmetric tensor hypermultiplet and

\[
\Lambda^\prime_{\text{hyp}} = -\frac{i}{\sqrt{2}} \tilde{\mathcal{R}}_\lambda \mathcal{R}_\lambda.
\]

(16)

The origin of this fermion bilinear is similar to that of \(\Lambda_{\text{hyp}}\); it reflects a Yukawa source term involving the symmetric tensor fermions that appears in the Euler-Lagrange equation for \(A^\dagger\).

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3This is true only at the one-instanton level; at higher order instanton levels the collective coordinate matrices that parameterize the symmetric tensor fermion zero-mode solutions must indeed satisfy constraint equations \([18]\).
In terms of the instanton action and collective coordinate integration measure, one can derive the following expression for the one-instanton contribution to the effective prepotential of the theory \[19\]:

\[
F_1 = 8\pi i \int d\bar{\mu}_{SQCD}^1 d\mu_{hyp}^1 \exp(- (S_{SQCD}^1 + \Delta S_{hyp}^1)).
\] (17)

Here \( \int d\bar{\mu}_{SQCD}^1 \) is a reduced version of the full \( \mathcal{N} = 2 \) SQCD measure, which excludes the integration over the global position of the instanton in \( \mathcal{N} = 2 \) superspace, as represented by \((x_0, \xi_1, \xi_2) = (a', \mathcal{M}'/4, \mathcal{N}'/4)\). Our task is now to explicitly evaluate this integral expression. To do this, we will follow closely the one-instanton calculation for \( \mathcal{N} = 2 \) SQCD performed in Sec. 8 of Ref. \[10\].

The first step is to exponentiate the \( \delta \)-function constraints in the measure \( \int d\bar{\mu}_{SQCD}^1 \) by introducing a set of auxiliary (Lagrange multiplier) integration variables. Thus, for the “spin-1” constraints on \( a \), we write

\[
\prod_{c=1}^3 \delta \left( \frac{1}{2} (\tau^c)_{\dot{\alpha}} \dot{\beta} \bar{w}_{u\dot{\alpha}} w_{u\dot{\beta}} \right) = \frac{1}{\pi^3} \int d^3 p \exp \left( ip^c (\tau^c)_{\dot{\alpha}} \dot{\beta} \bar{w}_{u\dot{\alpha}} w_{u\dot{\beta}} \right),
\] (18)

where the \( p^c \) are a triplet of standard bosonic Lagrange multipliers, and for the “spin-1/2” constraints on \( \mathcal{M} \) and \( \mathcal{N} \), we write

\[
\prod_{\dot{\alpha}=1,2} \delta \left( \bar{\mu}_u w_{u\dot{\alpha}} + \bar{w}_{\dot{\alpha}u} \mu_u \right) = 2 \int d^2 \xi \exp \left( \xi^{\dot{\alpha}} (\bar{\mu}_u w_{u\dot{\alpha}} + \bar{w}_{\dot{\alpha}u} \mu_u) \right),
\]

\[
\prod_{\dot{\alpha}=1,2} \delta \left( \bar{\nu}_u w_{u\dot{\alpha}} + \bar{w}_{\dot{\alpha}u} \nu_u \right) = 2 \int d^2 \eta \exp \left( \eta^{\dot{\alpha}} (\bar{\nu}_u w_{u\dot{\alpha}} + \bar{w}_{\dot{\alpha}u} \nu_u) \right),
\] (19)

where the Grassmann spinors \( \xi^{\dot{\alpha}} \) and \( \eta^{\dot{\alpha}} \) serve as fermionic Lagrange multipliers. The exponentiation of the “spin-0” \( \delta \)-function constraint on \( A_{tot} \) is accomplished in a more subtle way involving the \( A_{tot} \)-dependent terms in the instanton action. We write

\[
\int dA_{tot} \delta \left( L \cdot A_{tot} - (\Lambda + \Lambda_f) \right) \exp \left( 8\pi^2 (\bar{\Lambda} + \Lambda_{hyp} + \Lambda'_{hyp}) A_{tot} \right) \equiv \frac{1}{L} \exp \left( 8\pi^2 (\bar{\Lambda} + \Lambda_{hyp} + \Lambda'_{hyp}) \cdot L^{-1} \cdot (\Lambda + \Lambda_f) \right)
\]

\[
= 8\pi \int d(\text{Re} z) d(\text{Im} z) \exp \left( -8\pi^2 (\bar{z}Lz - (\bar{\Lambda} + \Lambda_{hyp} + \Lambda'_{hyp})z - \bar{z}(\Lambda + \Lambda_f)) \right),
\] (20)
where we have introduced a complex auxiliary integration variable $z$.

After the exponentiation of the $\delta$-function constraints, the next step is to integrate over the collective coordinates $\{w, \mu, \nu, \mathcal{K}, \tilde{\mathcal{K}}, \mathcal{R}, \tilde{\mathcal{R}}\}$. This is a fairly straightforward procedure since all the relevant integrals are Gaussian; the result is

$$
\mathcal{F}_1 = \frac{iC'_1}{2\pi^2} e^{-(8\pi^2/g^2)} \int d^3p \, d^2\xi \, d^2\eta \, d(\text{Re} \, z) \, d(\text{Im} \, z) \left[ \prod_{u=1}^{N_F} \frac{32\pi^6 \alpha_u^2}{(2\pi^2|\alpha_u|^2)^2 + \sum_{c=1,2,3}(p^c + \Xi_u^c)^2} \right] 
\times \left[ \prod_{f=1}^{N_F} (m_f + i\sqrt{2}z) \right] F(z),
$$

(21)

where $\alpha_u = v_u + i\varepsilon$ and $\Xi_u^\alpha = (\xi_{\alpha}^\beta \tau^\beta \eta^\gamma)^2/2\sqrt{2}\pi^2 \alpha_u$ and the factor $F(z)$, which is induced by the symmetric tensor hypermultiplet, is given by

$$
F(z) = \int d\mu_{hyp}^{1+4} \exp \left( 4\sqrt{2}\pi^2 \rho_u \nu_u \rho_u + 8\pi^2 \Lambda_{hyp}^\alpha z - 2m\pi^2 (2\tilde{\rho}_u \nu_u + \tilde{\mathcal{R}}^\alpha \mathcal{R}_\alpha) \right) 
= M^{-(N+2)} \left[ \prod_{u=1}^{N} (m + \sqrt{2}iz - \sqrt{2}v_u) \right] \left( m + 2\sqrt{2}iz \right)^2.
$$

(22)

It remains to integrate over the auxiliary integration variables $\{p, \xi, \eta, z\}$. Since the hypermultiplet-induced factor $F(z)$ has no dependence on $p$, $\xi$, or $\eta$, the integration over these variables is exactly as described in Sec. 8 of Ref. [10]. Thus we arrive at the expression

$$
\mathcal{F}_1 = \frac{i\Lambda_{PV}^{b_0}}{2^{7/2}\pi^2} \left( \sum_{u=1}^{N} \frac{\partial}{\partial \nu_u} \right)^2 \int d(\text{Re} \, z) \, d(\text{Im} \, z) \left\{ \sum_{u=1}^{N_F} \frac{\tilde{\alpha}_u}{\alpha_u} \prod_{v \neq u} \frac{\tilde{\alpha}_v^2}{|\alpha_v|^4 - |\alpha_u|^4} \right\} 
\times \left[ \prod_{f=1}^{N_F} (m_f + \sqrt{2}iz) \right] \left[ \prod_{w=1}^{N} (m - \sqrt{2}v_w + \sqrt{2}iz) \right] \left( m + 2\sqrt{2}iz \right)^2.
$$

(23)

(Here we have introduced the dynamical scale $\Lambda_{PV}$, defined by $\Lambda_{PV}^{b_0} = M^{b_0} \exp(-8\pi^2/g^2)$, where $b_0 = N - 2 - N_F$ is the first $\beta$-function coefficient.) As in [10], we evaluate this final integral by rewriting the operator $(\sum_u \partial/\partial \nu_u)^2$ as $-\partial^2/\partial z^2$ acting inside the integral and then working out the residues of the various poles. The result is

$$
\mathcal{F}_1 = \frac{\Lambda_{PV}^{b_0}}{2^{7/2}\pi i} \left\{ \sum_{u=1}^{N} \sum_{v \neq u} \frac{1}{(v_u - v_v)^2} \left[ \prod_{w \neq u,v} \frac{1}{(v_w - v_u)(v_w - v_v)} \right] \right\} 
\times \left[ \prod_{f=1}^{N_F} \left( m_f - \frac{1}{\sqrt{2}}(v_u + v_v) \right) \right] 
\times \left[ \prod_{w=1}^{N} \left( m_w - \frac{1}{\sqrt{2}}(v_u + v_v) \right) \right] 
\times \left[ \prod_{f=1}^{N_F} (m_f + \sqrt{2}iz) \right] \left[ \prod_{w=1}^{N} (m - \sqrt{2}v_w + \sqrt{2}iz) \right] \left( m + 2\sqrt{2}iz \right)^2.
$$

(24)
\[ \times \left[ \prod_{w=1}^{N} \left( m - \sqrt{2} v_w - \frac{1}{\sqrt{2}} (v_u + v_v) \right) \right] \left( m - \sqrt{2} (v_u + v_v) \right)^2 \right\}. \quad (24) \]

Now we compare our first-principles result with the exact solution derived in [11]. The prediction for \( F_1 \) extracted from this solution is [12]

\[ F_1 = \frac{\Lambda_{PV}^{b_0}}{2^N \pi i} \left\{ \sum_{u=1}^{N} \left[ \prod_{v \neq u} \left( \frac{1}{v_v - v_u} \right) \right] \left[ \prod_{f=1}^{N_f} \left( m_f - \sqrt{2} v_u \right) \right] \right\} \times \left[ \prod_{w=1}^{N} \left( m - \sqrt{2} (v_u + v_w) \right) \right] \left( m - 2 \sqrt{2} v_u \right)^2 \right\}. \quad (25) \]

Whilst this is not identical to the expression (24) for general \( N_F \), it is equivalent to that expression for \( N_F < N - 2 \), which is the range of validity of the curve presented in [11]. One can show this by checking that the singularity structure of the two expressions is the same at the points \( v_u - v_v = 0 \). Hence the difference between the two expressions must be given by a regular function of the VEVs; by dimensional analysis, this function must be zero for \( N_F < N - 4 \), equal to a numerical constant in the case \( N_F = N - 4 \), and given by a linear combination of the masses, \( A m + B \sum_f m_f \), where \( A \) and \( B \) are numerical constants, in the case \( N_F = N - 3 \). (In this last case, there can be no linear dependence on the VEVs since \( \sum_u v_u \equiv 0 \).) Since information about the low-energy physics is obtained by differentiating the prepotential at least once with respect to the VEVs, it follows that the two expressions are indeed physically equivalent for \( N_F < N - 2 \). Thus our first-principles one-instanton calculation has provided a successful test of the exact solution and the M-theory method that was used to derive it.

It would be desirable to perform instanton tests of the exact results for \( SU(N) \) models with matter hypermultiplets in other representations such as the antisymmetric and the adjoint. The instanton calculation is more of a challenge here since the collective coordinates associated with the hypermultiplets are not unconstrained (even at the one-instanton level).
and one must introduce extra Lagrange multipliers to deal with the extra δ-functions in the integration measure. We aim to address this challenge elsewhere.

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References

[1] N. Seiberg and E. Witten, Nucl. Phys. B426 (1994) 19, ibid (E) B430 (1994) 485, hep-th/9407087.
[2] N. Seiberg and E. Witten, Nucl. Phys. B431 (1994) 484, hep-th/9408099.
[3] D. Finnell and P. Pouliot, Nucl. Phys. B453 (1995) 225, hep-th/9503115.
[4] N. Dorey, V.V. Khoze and M.P. Mattis, Phys. Rev. D54 (1996) 2921, hep-th/9603136.
[5] N. Dorey, V.V. Khoze and M.P. Mattis, Phys. Lett. B388 (1996) 324, hep-th/9607066; Phys. Rev. D54 (1996) 7832, hep-th/9607202.
[6] H. Aoyama, T. Harano, M. Sato and S. Wada, Phys. Lett. B388 (1996) 331, hep-th/9607076; T. Harano and M. Sato, Nucl. Phys. B484 (1997) 167, hep-th/9608060.
[7] N. Dorey, V.V. Khoze and M.P. Mattis, Phys. Lett. B396 (1997) 141, hep-th/9612231.
[8] K. Ito and N. Sasakura, Phys. Lett. B382 (1996) 95, hep-th/9602073; Nucl. Phys. B484 (1997) 141, hep-th/9608054; Mod. Phys. Lett. A12 (1997) 205, hep-th/9609104.
[9] M.J. Slater, Phys. Lett. B403 (1997) 57, hep-th/9701170.
[10] V. V. Khoze, M. P. Mattis and M. J. Slater, Nucl. Phys. B536 (1998) 69, hep-th/9804009.
[11] K. Landsteiner, E. Lopez, D.A. Lowe, JHEP 9807 (1998) 011, hep-th/9805158.
    K. Landsteiner, E. Lopez, Nucl. Phys. B516 (1998) 273, hep-th/9708118.
[12] I.P. Ennes, S.G. Naculich, H. Rhedin, H.J. Schnitzer, Nucl. Phys. B536 (1998) 245, hep-th/9806144; Int. J. Mod. Phys. A14 (1999) 301, hep-th/9804151.

[13] N. Dorey, V.V. Khoze and M.P. Mattis, Nucl. Phys. B492 (1997) 607, hep-th/9611016.

[14] M. Atiyah, V. Drinfeld, N. Hitchin and Yu. Manin, Phys. Lett. A65 (1978) 185.

[15] N. Dorey, V.V. Khoze and M.P. Mattis, Nucl. Phys. B513 (1998) 681, hep-th/9708036; N. Dorey, T. Hollowood, V.V. Khoze and M.P. Mattis, Nucl. Phys. B519 (1998) 470, hep-th/9709072.

[16] G. 't Hooft, Phys. Rev. D14 (1976) 3432, ibid (E) D18 (1978) 2199.

[17] C. Bernard, Phys. Rev. D19 (1979) 3013.

[18] E. Corrigan, P. Goddard and S. Templeton, Nucl. Phys. B151 (1979) 93.

[19] N. Dorey, V.V. Khoze and M.P. Mattis, Phys. Lett. B390 (1997) 205, hep-th/9606199.