Constraining $H_0$ in General Dark Energy Models from Sunyaev-Zeldovich/X-ray Technique and Complementary Probes

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In accelerating dark energy models, the estimates of $H_0$ from Sunyaev-Zeldovich effect (SZE) and X-ray surface brightness of galaxy clusters may depend on the matter content ($\Omega_M$), the curvature ($\Omega_K$) and the equation of state parameter ($\omega$). In this article, by using a sample of 25 angular diameter distances from galaxy clusters obtained through SZE/X-ray technique, we constrain $H_0$ in the framework of a general $\Lambda$CDM models (free curvature) and a flat XCDM model with equation of state parameter $\omega = p/\rho$ (\omega=constant). In order to broke the degeneracy on the cosmological parameters, we apply a joint analysis involving the baryon acoustic oscillations (BAO) and the CMB Shift Parameter signature. By neglecting systematic uncertainties, for nonflat $\Lambda$CDM cosmologies we obtain $H_0 = 73.2^{+3.7}_{-3.3} \text{ km s}^{-1} \text{ Mpc}^{-1} (1\sigma)$ whereas for a flat universe with constant equation of state parameter we find $H_0 = 71.4^{+4.4}_{-4.1} \text{ km s}^{-1} \text{ Mpc}^{-1} (1\sigma)$. Such results are also in good agreement with independent studies from the Hubble Space Telescope key project and recent estimates based on Wilkinson Microwave Anisotropy Probe, thereby suggesting that the combination of these three independent phenomena provides an interesting method to constrain the Hubble constant. In particular, comparing these results with a recent determination for a flat $\Lambda$CDM model using only the SZE technique and BAO [Cunha et al. MNRAS 379, L1 2007], we see that the geometry has a very weak influence on $H_0$ estimates for this combination of data.

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I. INTRODUCTION

Clusters of galaxies are the largest gravitationally bound structures in the universe and they can be regarded as being representative of the universe as a whole. An important phenomenon occurring in clusters is the Sunyaev-Zeldovich effect (SZE), a small distortion of the Cosmic Microwave Background (CMB) spectrum, provoked by the inverse Compton scattering of the CMB photons passing through a population of hot electrons [3–7]. The SZE is quite independent of the redshift, so it provides an useful tool for studies of intermediate and high redshift galaxy clusters, where the cosmological model adopted plays an important role (for a reviews see [8,9]).

When the X-ray emission of the intracluster medium (ICM) is combined with the SZE, it is possible to estimate the angular diameter distance (ADD) $D_A$. In other words, the SZE/X-ray method provides distances to the clusters and consequently a measure of the Hubble parameter $H_0$. The main advantage of this method for estimating $H_0$ is that it does not rely on the extragalactic distance ladder, being fully independent of any local calibrator [10].

It should be stressed that the determination of Hubble parameter has a practical and theoretical importance to many astrophysical properties and cosmological observations [11]. Komatsu et al. have shown that CMB studies can not supply strong constraints to the value of $H_0$ on their own [12]. This problem occurs due to the degeneracy on the parameter space and may be circumvented only by using independent measurements of $H_0$. In this connection, Sandage and collaborators [13] announced the results from their HST programme, $H_0 = 62 \pm 5 \text{ km/s/Mpc}$, whereas Van Leeuwen et al. [14] revised Hipparcos parallaxes for Cepheid distance and obtained higher values than previous results advocated by Sandage et al. [13] and Freedman et al. [15] groups.

Later on, Riess et al. [16] reported results from a program to determine the Hubble parameter to $\approx 5\%$ precision from a refurbished distance ladder based on extensive use of differential measurements. They obtained $H_0 = 74.2 \pm 3.6 \text{ km/s/Mpc}$, a 4.8% uncertainty including both statistical and systematic errors. Indeed, some estimates of $H_0$ have yielded $\approx 74 \text{ km/s/Mpc}$ [14,16,17].

More recently, by studying time delay from gravitational lens, two groups obtained $H_0$ estimates in a flat $\Lambda$CDM framework, Fadely et al. [18] adding constraints from stellar population synthesis models obtained $H_0 = 79.3^{+6.5}_{-8.8} \text{ km/s/Mpc}$ (1$\sigma$, without systematic errors), and Suyu et al. [19], in combination with WMAP obtained $H_0 = 69.7^{+4.9}_{-3.0} \text{ km/s/Mpc}$ (1$\sigma$, without systematic errors). The importance to access the distance scale by different methods, and, more important, in a manner independent of any distance calibrator has been discussed in the review paper by Jackson [20].

A couple of years ago, Cunha, Marassi and Lima [21] (henceforth CML), derived new constraints on the matter...
density and Hubble parameters \((\Omega_m, H_0)\), by using the SZE/X-ray technique in the framework of a flat ΛCDM model. By considering a sample of 25 galaxy clusters compiled by De Filippis et al. (2005) [22], the degeneracy on the \(\Omega_m\) parameter was broken through a joint analysis combining the SZE/X-ray data with the recent measurements of the baryon acoustic oscillation (BAO) signature from SDSS catalog [23, 24]. The main advantage of the method is that we do not need to adopt a fixed cosmological concordance model in our analysis, as usually done in the literature [10, 23, 32]. For a flat ΛCDM our joint analysis yielded \(\Omega_m = 0.27_{-0.02}^{+0.03}\) and \(H_0 = 73.8_{-4.2}^{+4.2}\) km/s/Mpc (1σ, neglecting systematic uncertainties).

On the other hand, astronomical observations in the last decade have suggested that our world behaves like a spatially flat scenario, dominated by cold dark matter (CDM) plus an exotic component endowed with large negative pressure, usually named dark energy [33–36]. In the framework of general relativity, besides the cosmological constant, there are several candidates for dark energy, among them: a vacuum decaying energy density, or a time varying \(\Lambda(x)\) [37], the so-called “X-matter” [38], a relic scalar field [39], a Chaplygin gas [40, 41], and cosmologies proposed to reduce the dark sector, among them, models with creation of cold dark matter particles [42]. For a scalar field component and “X-matter” scenarios, the equation of state parameter may be a function of the redshift (see, for example, [43]) or still, as has been discussed by many authors, it may violate the null energy condition [44].

In this work, we relax the flat geometry condition of the cosmic concordance model (ΛCDM). This procedure will prove the robustness of the previous \(H_0\) estimate using the SZE/X-ray technique. In addition, in order to test the real dependence of the method with the equation of state parameter, we compare the predictions of the general ΛCDM model and “X-matter” cosmologies [38]. For the “X-matter” model we assume a flat XCDM cosmology with constant equation of state (EoS) parameter. We use the 25 ellipsoidal clusters from De Filippis et al. (2005) [22]. To broke the degeneracy on the basic cosmological parameters, we apply a joint analysis using BAO [23, 24] and the CMB probe known as shift parameter [45, 48].

The paper is organized as follows. In section II, we present the basic equations to angular distance and models studied. In section III, we give a short description of the observational data we have used. The corresponding constraints on the cosmological parameters are investigated in section IV. The article is ended with a summary of the main results in the conclusion section.

II. BASIC EQUATIONS AND MODELS

Let us now assume that the Universe is well described by an homogeneous and isotropic geometry

\[
ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right),
\]

where \(a(t)\) is the scale factor and \(k = 0, \pm 1\) is the curvature parameter. Throughout we use units such that \(c = 1\).

In such a background, the angular diameter distance \(D_A\) reads

\[
D_A = \frac{3000h^{-1}}{(1 + z) \sqrt{|\Omega_k|}} S_k \left[ \sqrt{|\Omega_k|} \int_0^z dz' E(z') \right] \text{Mpc},
\]

where \(h = H_0/100\) km s^{-1} Mpc^{-1} (henceforth we use this notation for our Hubble parameter estimates), the function \(E(z) = H(z)/H_0\) is the normalized Hubble parameter which defined by the specific cosmology adopted, \(\Omega_k\) is the curvature parameter, and \(S_k(x) = \sin x, x, \sinh x\) for \(k = +1, 0, -1\), respectively.

In this paper, we consider that the Universe is driven by cold dark matter \((\Omega_M)\) plus a dark energy component \((\Omega_x)\), with constant EoS parameter \((\omega \equiv p/\rho)\). Below we will summarize the function \(E(z)\) of the cosmological models adopted in this paper.

(i) ΛCDM model. By allowing deviations from flatness, the normalized Hubble parameter is given by

\[
E^2(z) = \Omega_M (1 + z)^3 + \Omega_\Lambda + \Omega_k (1 + z)^2,
\]

where \(\Omega_k = 1 - \Omega_\Lambda - \Omega_M\).

(ii) Parametric Dark Energy model (XCDM). In this case, the normalized Hubble parameter reads:

\[
E^2(z) = \Omega_M (1 + z)^3 + \Omega_x (1 + z)^{3(1 + \omega)} + \Omega_k (1 + z)^2.
\]

where \(\Omega_k = 1 - \Omega_x - \Omega_M\). For \(\omega = -1\), the above XCDM expression reduces to the previous ΛCDM case \((\Omega_x = \Omega_\Lambda)\). When the EoS parameter of dark energy is restricted on the interval \(-1 \leq \omega < 0\), it may be represented by a scalar field (quintessence), and whether \(\omega < -1\) thereby violating the null energy condition is the case of phantom dark energy [44].

Given the above expressions, we see clearly that \(D_A\) is a function of \(z, h\) and depending on the model adopted of \(\omega, \Omega_M, \Omega_x\). Due to the excessive number of free parameters in XCDM models, in our analysis of this dark energy model we will concentrate our attention to the flat case.

III. THEORETICAL METHOD AND SAMPLES

It should be recalled that the basic aim here is to constrain the Hubble and other cosmological parameters
of the above models. The method adopted is primarily based on the angular diameter distances, $D_A$, furnished by the SZE/X-ray technique. The degeneracies on the remaining cosmological parameters will be broken by a joint analysis involving BAO and CMB signature (shift parameter).

A. SZE/X-ray

By using an elliptical 2-Dimensional $\beta$-model to describe the galaxy clusters geometry, De Filippis and coworkers \cite{22} derived $D_A$ measurements for 25 clusters from two previous compilations: one set of data compiled by Reese et al. \cite{28}, composed by a selection of 18 galaxy clusters distributed over the redshift interval $0.14 < z < 0.8$, and the sample of Mason et al. \cite{27}, which has 7 clusters from the X-ray limited flux sample of Ebeling et al. (1996) \cite{51}. These two previous compilations used a spherical isothermal $\beta$ model to describe the clusters geometry.

In the CML paper \cite{21}, the Abell 773 cluster was excluded from the statistical analysis due to its large contribution to the $\chi^2$. Now, in order to preserve the completeness of the original data set, these statistical cut-offs arguments will be neglected, and, as such, all the 25 clusters from the original De Fillipis et al. \cite{22} sample will be considered.

B. Baryon Acoustic Oscillations (BAO)

The detection of a peak in the large-scale correlation function of luminous red galaxies selected from the Sloan Digital Sky Survey (SDSS) Main Sample gave rise to a powerful cosmological probe, often referred to as BAO scale.

Basically, the peak detected at the scale of 100 $h^{-1}$ Mpc, happens due to the baryon acoustic oscillations in the primordial baryon-photon plasma prior to recombination, and, such a result, provides a suitable “standard ruler” for constraining dark energy models \cite{23,24}.

The relevant distance measure is the so-called dilation scale that can be modeled as the cube root of the radial dilation times the square of the transverse dilation, at the typical redshift of the galaxy sample, $z = 0.35$ \cite{23}:

$$D_V(z) = \left[D_A(z)^2 z / H(z)\right]^{1/3},$$

(5)

Recalling that the comoving size of the sound horizon at $z_{ls}$ is $\sim 1/\sqrt{\Omega_M H_0^2}$, it was pointed out that the combination $A(z) = D_V(z) \sqrt{\Omega_M H_0^2} / z$ is independent of $H_0$, and this dimensionless combination is also well constrained by the BAO data

$$A(0.35) = D_V(0.35) \frac{\sqrt{\Omega_M H_0^2}}{0.35} = 0.469 \pm 0.017.$$

(6)

The BAO quantity $A(0.35)$ is exactly what we add to the $\chi^2$, in the joint statistical analysis of the cosmological models studied here.

C. CMB - Shift Parameter

Another interesting cosmological probe to dark energy models is the so-called CMB shift parameter. Such a quantity is encoded in the location $l_{1}^{TT}$ of the first peak of the angular (CMB) power spectrum \cite{43,46}:

$$\theta_A = \frac{r_s(z_{ls})}{D_A(z_{ls})},$$

(7)

where $z_{ls}$ is the redshift of the last scattering surface and $D_A(z_{ls})$ is the angular distance to the last-scattering surface. The quantity, $r_s(z_{ls}) \sim 1/\sqrt{\Omega_M H_0^2}$, is the comoving size of the sound horizon at $z_{ls}$.

By using the WMAP three years result \cite{12} Davis et al. \cite{47} converted the location of the first peak in a reduced distance to the last-scattering surface

$$R = \sqrt{\frac{\Omega_M}{\Omega_k}} S_k \left[\frac{\sqrt{\Omega_k}}{E(z)} \int_0^{z_{ls}} dz \right] = 1.71 \pm 0.03.$$

(8)

The robustness of the shift parameter has been tested by Elgaroy & Multamäki \cite{48} and compared to fits of the full CMB power spectrum. As a result, it is now widely believed that the shift parameter is an accurate geometric measure even for non-standard cosmologies. It is weakly dependent on $h$, since $z_{ls} = z_{ls}(h, \Omega_M, \Omega_b)$ is a smooth function of $h$, and the degeneracies that arise from using $R$ rather than fitting the full CMB power spectrum are well constrained by other data such as BAO and SZE/X-ray.

In the following computations for the theoretical shift parameter we will apply the correction for $z_{ls}$ suggested in Refs. \cite{48,52}. In addition, independent of the adopted dark energy model, we keep the value $R = 1.70 \pm 0.03$ for a flat universe and $R = 1.71 \pm 0.03$ for nonzero cosmic curvature \cite{52}.

IV. ANALYSIS AND RESULTS

In our statistical procedure we apply a maximum likelihood analysis determined by a $\chi^2$ statistics

$$\chi^2(z|p) = \sum_i \frac{(D_A(z_i|p) - D_{Ao,i})^2}{\sigma_{D_{Ao,i}}},$$

(9)

where $D_{Ao,i}$ is the observational angular diameter distance, $\sigma_{D_{Ao,i}}$ is the uncertainty in the individual distance and $p$ is the complete set of parameters. For the $\Lambda$CDM model $p \equiv (h, \Omega_M, \Omega_\Lambda)$ and $p \equiv (h, \omega, \Omega_M)$ for the flat XCDM model.

All the systematic effects still need to be considered. The common errors are: SZ calibration $\pm 8\%$, X-ray flux...
independent from the geometry of the Universe. This result has been derived by marginalizing over $\Omega_M$. b) Combining different probes. Contours at 68.3%, 95.4% and 99.7% c.l. in the $(h, \Omega_k)$ plane trough a joint analysis involving SZE/X-ray + BAO + Shift Parameter. c) Probability of the $h$ parameter. The horizontal lines represent cuts of 68.3% and 95.4% of statistical confidence. The best-fit result is $h = 0.73$ with a reduced $\chi^2_{red} = 1.12$.

A. $\Lambda CD M$ Model

Allowing for deviations from flatness, we first consider the general $\Lambda$CDM model as described by Eq. (3). In Figure 1a, we display the contours (68.3%, 95.4% and 99.7% c.l.) in the $(h, \Omega_k)$ plane provided by the distance angular distances from SZE/X-ray technique (we have marginalized over $\Omega_M$). We see clearly a degeneracy between $\Omega_k \equiv h$, and, therefore, the possible values for $h$ are very weakly constrained from SZE/X-ray alone. Note that the $h$ parameter lies on the interval $0.63 < h < 0.93$ at 99.7% c.l. (1 free parameter).

In Figure 1b, we show the results for a joint analysis involving SZE/X-ray + BAO + Shift Parameter. In this case we obtain $h = 0.733^{+0.042}_{-0.037}$, $\Omega_k = -0.010^{+0.012}_{-0.013}$ and $\chi^2_{min} = 28.12$ at 68.3% (c.l.). Its reduced value, that is, taking into account the associated degrees of freedom is $\chi^2_{red} = 1.12$ thereby showing that the fit is very good.

In Figure 1c, we display the likelihood function of the $h$ parameter by using the SZE/X-ray data + BAO + Shift Parameter. To obtain this graph we have marginalized over $\Omega_M$ and $\Omega_\Lambda$ parameters. The horizontal lines are cuts in the probability regions of 68.3% and 95.4%. This plot is very similar to the Fig. (4) of the previous CML paper [21] for a flat $\Lambda$CDM model. Therefore, it is safe to conclude that the constraints on $h$ derived here are independent from the geometry of the Universe.

B. Flat XCDM Model

As we know, some dark energy candidates are phenomenologically described by an equation of state of the form, $p = \omega \rho$, where $\omega$ is a constant parameter. In the flat case, the normalized Hubble parameter is readily obtained by taking $\Omega_k = 0$ in Eq. (4). In this context, by relax the usual imposition $\omega \geq -1$ we investigate some implications for the so-called phantom dark energy. The basic idea here is to test the sensibility of the SZE/X-ray data + BAO + Shift Parameter with respect to the $\omega$ parameter. In addition, we also study the influence of XCDM on the Hubble constant determination, thereby performing also a direct comparison to the $\Lambda$CDM model ($\omega = -1$).

In Figure 2a, we display the $(h, \omega)$ plane (marginalizing over $\Omega_M$) using the 25 angular diameter distances from galaxy clusters. The limits on $h$ in the $(h, \omega)$ plane is wider than in the $\Lambda$CDM models, but the $h$ value has a very weak dependence on $\omega$ parameter. We stress that we have explored a large range of this parameter ($-3 < \omega < 0.5$).

In Figure 2b, we show the results when a joint analysis involving the SZE/X-ray data set + BAO + Shift Parameter is performed. We find $h = 0.714^{+0.044}_{-0.034}$, $\omega = -0.76^{+0.19}_{-0.28}$ and $\chi^2_{min} = 28.35$ at 68.3% confidence level, whereas its reduced value is $\chi^2_{red} = 1.13$. Again, we have an excellent fit.

In Figure 2c, we display the $(\omega, h)$ plane for the joint analysis involving SZE/X-ray + BAO + Shift, and marginalizing over the possible values of $\Omega_M$. In this figure, one may see the observational limits to the phantom behavior of the dark energy in our analysis for any $\Omega_M$ parameter. In spite of the phantom dark energy to have a large region permitted for $2\sigma$ and $3\sigma$, right of the phantom barrier with $\omega \geq -1$. However, the best-fit value is left of the phantom barrier $\omega = -0.76$ outside of the phantom energy zone. In accord with recent results from WMAP plus others tests [12].

In Figure 2c, we plot the likelihood function for the $h$ parameter by marginalizing in $\omega$ and $\Omega_m$. The horizontal
and obtained compiled distance determinations to 26 galaxy clusters, son et al. (2001), using 5 clusters, gives $h = 0.66^{+0.11}_{-0.10}$; Reese and coauthors (2002), using 18 clusters, found $h = 0.60 \pm 0.04$; Reese (2004), with 41 clusters, obtains $h \approx 0.61 \pm 0.03$; Jones et al. (2005), using a sample of 5 clusters, obtained $h = 0.66^{+0.11}_{-0.10}$; and Schmidt et al. (2004) obtain a best-fit $h = 0.69 \pm 0.08$

The mean value of $h$ from the SZE/X-ray measurements above appears systematically lower than those estimated with other methods: e.g. $h = 0.72 \pm 0.08$ from the Hubble Space Telescope (HST) Project [10], and $h = 0.73 \pm 0.03$ from the CMB anisotropy [12]. On the other hand, the recent work of Sandage et al. (2006) use type Ia Supernovae, predicts $h = 0.62 \pm 0.013$ (random) $\pm 0.05$ (systematics). Yet, we must to point out the work of Bonamente et al. (2006) using 38 clusters and the SZE/X-ray method (with the cosmic concordance model), which obtains $h = 0.769^{+0.039}_{-0.034}$.

In this concern, we also recall that Alcaniz 2005 used 17 data between 0.14 $< z < 0.78$ from SZE/X-Ray angular distances to constrain the $\omega$ parameter. Performing a joint analysis involving SZE/X-Ray + SNe Ia + CMB data and fixing $\Omega_M = 0.27$, he obtained $\omega = -1.2^{+0.11}_{-0.18}$ at 68.3% c.l. (1 free parameter). In a point of fact, our main interest here is to determine constraints on the Hubble parameter independent of SNe Ia data because such observations already provide a very precise determination of the Hubble parameter (see [17] and Refs. there in).

On the other hand, since the assumed cluster shape affects considerably the SZE/X-ray distances, and, therefore, the $H_0$ estimates, we also compare our results with the ones of Holanda et al. [53]. These authors used 38 angular diameter distances from galaxy clusters, where a spherical $\beta$ model was assumed to describe the clusters + BAO in a flat $\Lambda$CDM model. They found $h = 0.765 \pm 0.035$, a value slightly larger than the ones found in this work. We also stress that the constraints on the Hubble parameter and $\Omega_M$ derived here using the $\Lambda$CDM (free geometry) and the flat XCDM model, are in agreement with the independent measurements provided by the WMAP team [12], the HST Project [10], the work of Bonamente et al. (2006) [31] and, more recently, Riess et al. [17].

C. Comparing Results

In the last few years, several measurements of $h = H_0/100$ were presented in the literature, obtained using the SZE/X-ray method and fixing the cosmology (using the cosmic concordance model): Carlstrom et al. (2002) compiled distance determinations to 26 galaxy clusters, and obtained $h = 0.60 \pm 0.03$ km s$^{-1}$ Mpc$^{-1}$ [26, 53]; Mason et al. (2001), using 5 clusters, gives $h = 0.66^{+0.11}_{-0.10}$ [27]; Reese and coauthors (2002), using 18 clusters, found $h = 0.60 \pm 0.04$; Reese (2004), with 41 clusters, obtains $h \approx 0.61 \pm 0.03$; Jones et al. (2005), using a sample of 5 clusters, obtained $h = 0.66^{+0.11}_{-0.10}$; and Schmidt et al. (2004) obtain a best-fit $h = 0.69 \pm 0.08$.

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In Table II we summarize the $H_0$ constraints based on the SZE/X-ray technique. All estimates, except the last five lines, are obtained in the framework of the flat $\Lambda$CDM model ($\Omega_m = 0.3, \Omega_\Lambda = 0.7$ - known as “cosmic concordance”). In the 8th and 9th lines, we see the results of CML paper [21]. They used a flat $\Lambda$CDM model and 24 galaxy clusters, with and without BAO (here labeled as “Cunha et al. 2007” and “Cunha et al. 2007 + BAO”, respectively). In the 10th line, we display the results of Holanda et al. using 38 galaxy clusters (spherical symmetry) + BAO in a flat $\Lambda$CDM [53]. In the last two lines we present our results derived through a joint analysis involving SZE/X-ray, BAO and CMB shift parameter.
TABLE I: Constraints on the “little” Hubble parameter $h$ based on SZE/X-ray technique applied to galaxy clusters data.

| Reference          | $\omega_m$ | $h$ (1σ) | $\chi^2_{\text{red}}$ |
|--------------------|------------|----------|---------------------|
| Mason et al. 2001  | 0.3        | 0.66±0.05 | 0.35                |
| Carlstrom et al. 2002 | 0.3    | 0.60±0.03 |                    |
| Reese et al. 2002  | 0.3        | 0.60±0.04 | 0.97                |
| Reese 2004         | 0.3        | 0.61±0.03 |                    |
| Jones et al. 2005  | 0.3        | 0.66±0.04 | 0.96                |
| Schmidt et al. 2004| 0.3        | 0.69±0.08 |                    |
| Bonamente et al. 2006 | 0.3       | 0.77±0.03 | 0.83                |
| Cunha et al. 2007  | 0.15±0.05  | 0.75±0.07 | 1.06                |
| Cunha et al. 2007 + BAO | 0.27±0.03 | 0.74±0.03 | 1.06                |
| Holanda et al. 2007 + BAO | 0.27±0.03 | 0.765±0.035 | 0.96                |
| This Work (ΛCDM ) | 0.272±0.005 | 0.732±0.032 | 1.12                |
| This Work (flat-XCDM) | 0.30±0.05 | 0.714±0.044 | 1.13                |

V. CONCLUSIONS

In this work we have discussed a determination of the Hubble parameter and other relevant cosmological quantities based on the SZE/X-ray distance technique for a sample of 25 clusters compiled by De Filippis et al. In order to prove the robustness of the $H_0$ parameter we relaxed the flatness condition of the ΛCDM cosmology. The degeneracy on the cosmological parameters was broken using BAO and shift parameter. While the former test is independent of $H_0$ the last one is weakly dependent. By comparing the results of this work with the ones of Cunha et al. (see Table I), which uses only a flat ΛCDM model and BAO, we clearly see that the present estimates of $H_0$ are virtually independent of the geometry of the Universe. For a general ΛCDM cosmology we obtain $H_0 = 73.2^{+4.3}_{-3.7}$ km s$^{-1}$ Mpc$^{-1}$ (1σ - without systematic errors).

In order to test the real dependence of the method with the adopted cosmology, we have compared the constraints from ΛCDM model with the flat XCDM. In the same way, we also conclude that the $H_0$ estimates presents a negligible dependence on dark energy models with constant $\omega$. For a flat XCDM we obtain $H_0 = 71.4^{+4.4}_{-3.4}$ km s$^{-1}$ Mpc$^{-1}$ (1σ - without systematic errors). We study also possible observational limits to the phantom behavior of the dark energy for these SZE/X-Ray data plus BAO and Shift Parameter. We results indicate a large region permitted by phantom energy ($\omega \leq -1$), but the best-fit value is $\omega = -0.76$ outside of this phantom energy zone.

The constraints on the Hubble parameter derived here are also consistent with some recent cosmological observations like the WMAP and the HST Key Project. Implicitly, such an agreement suggests that the elliptical morphology describing the cluster sample and the associated isothermal β-model is quite realistic.

Finally, we stress that the combination of these four independent phenomena (SZE, X-Ray, BAO and Shift) provides an interesting method to constrain the Hubble constant, and more important, it is independent of any calibrator usually adopted in the determinations of the distance scale.

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