Subharmonic Gap Structure in Superconductor/Ferromagnet/Superconductor Junctions

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The behavior of dc subgap current in magnetic quantum point contact is discussed for the case of low-transparency junction with different tunnel probabilities for spin-up (\(D_{\uparrow}\)) and spin-down (\(D_{\downarrow}\)) electrons. Due to the presence of Andreev bound states \(\pm \varepsilon_0\) in the system, the positions of subgap electric current steps \(eV_n = (\Delta \pm \varepsilon_0)/n\) are split at temperature \(T \neq 0\) with respect to the nonmagnetic result \(eV_n = 2\Delta/n\). It is found that under the condition \(D_{\uparrow} \neq D_{\downarrow}\) the spin current also manifests subgap structure, but only for odd values of \(n\). The split steps corresponding to \(n = 1, 2\) in subgap electric and spin currents are analytically calculated and the following steps are described qualitatively.

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It is well established now that subharmonic gap structure in superconducting weak links can be described in terms of multiple Andreev reflection (MAR). The concept of MAR was first introduced for SNS (superconductor - normal metal - superconductor) junctions and then extended to include the effect of resistance of SN interface (OTBK theory). Then it was shown that this mechanism is the reason for subharmonic gap structures in junctions of any type, and specifically in superconducting point contacts. As long as the junction is short on the scale of the coherence length \(\xi_0\), the microscopic details of the junction are irrelevant and for the nonmagnetic junction the only parameter is the transmission probability of the barrier \(D\). An exact theory of MAR for short nonmagnetic junctions between clean superconductors has been developed in Refs.\(^2\) on the basis of scattering amplitudes method and in Ref.\(^5\) making use of nonequilibrium Green function techniques. It was obtained that for this case MAR manifests itself in dc current as current steps at voltages \(eV_n = 2\Delta/n\), \(n = 1, 2, \ldots\). The magnitude of the current onset at \(V_n\) is proportional to \(D^0\). The theoretical results are consistent with the experimental situation in atomic-size superconducting point contacts.\(^7\) The subharmonic gap structure due to photon-assisted MAR in the presence of microwave radiation has also been studied and theoretical predictions are confirmed by the experimental observations.\(^12\)

Arnold\(^1\) was the first to take into account the phase coherence of subsequent Andreev reflections, which were absent in the OTBK theory. The quantum coherence in short ballistic junctions lead to ac Josephson effect.\(^13\) In addition, in long ballistic SNS junctions coherence effects give rise to resonant structures in the dc current due to Andreev quantization.\(^14\)

MAR in diffusive nonmagnetic junctions has also been investigated in details. Short diffusive junctions with Thouless energy \(E_{Th} \gg \Delta\) can be described by coherent MAR theory and show subharmonic gap structures at \(eV_n = 2\Delta/n\).\(^14\) In long diffusive junctions with \(E_{Th} \lesssim \Delta\) additional peaks in the differential conductance at \(eV_n \approx 2(\Delta + E_{Th})/2n\) are predicted.\(^14\) This result is in qualitative agreement with the experimental data.\(^17\)

The incoherent MAR regime with \(eV, \Delta \gg E_{Th}\) has also been studied and it was found that subharmonic gap structure shows qualitatively different behavior for even and odd subharmonic gap structures. Magnetic interfaces can not be characterized by the only parameter \(D\) and Andreev bound states are known to take place on the magnetic surface of s-wave superconductors.\(^20\) So it is natural that new qualitative features with respect to nonmagnetic case appear in dc current even in the simplest models of a short magnetic junction.\(^21\) In Ref.\(^22\) the dc-current-voltage characteristic of magnetic quantum point contact between two superconductors is studied theoretically for the model where magnetic quantum dot is characterized by two parameters: transparency \(D\) and the spin-mixing angle \(\Theta\). It is found that subgap current steps take place at voltages \(eV_n = \Delta(1 + \cos \Theta/2)/n\).

In this paper the dc subgap current due to MAR processes in short ballistic quantum point contact for the most general case of symmetric magnetic interface including different transparencies for spin-up and spin-down quasiparticles \((D_{\uparrow} \neq D_{\downarrow})\) is studied theoretically. I consider the case of small \(D_{\uparrow}\) and \(D_{\downarrow}\) of the same order of magnitude and find that at \(T \neq 0\) current steps occur at voltages \(eV_n = \Delta(1 \pm \cos \Theta/2)/n\). This splitting disappears in the limit \(T \rightarrow 0\) and the result obtained in Ref.\(^22\) for the current step positions is recovered. Also I found that the difference between \(D_{\uparrow}\) and \(D_{\downarrow}\) is very important. It leads to non-zero spin current which has subharmonic gap structure different from that one for the dc electric current.

The theoretical analysis is based on the non-equilibrium quasiclassical theory of superconductivity in terms of Riccati amplitudes developed by Eschrig and generalized for the case of magnetic interfaces in Refs.\(^23\) Now I briefly outline this formalism. The fundamental quantity in non-equilibrium quasiclassical theory of superconductivity is the quasiclassical Green’s function \(g = g(p_f, R, \epsilon, t)\). It is a \(8 \times 8\) matrix form in the product space of Keldysh, particle-hole and spin
variables. In general the quasiclassical Green’s functions are
depend on space $R$, time $t$, variables, the direction of quasi-particle
Fermi momentum $p_f$ and the excitation energy $\epsilon$. In our case of one-mode quantum point contact
the problem is effectively one-dimensional and $R \equiv x$, where $x$ - is the coordinate measured along the
normal to the junction. The momentum $p_f$ has only two values,
which correspond to incoming and outgoing trajectories.

The electric and spin currents should be calculated via
Keldysh part of the quasiclassical Green’s function. For
the one-mode quantum point contact the corresponding
expression for the charge current reads as follows

$$I_{ch}^{\epsilon} = \frac{\text{sgn} p_f}{2\pi} \int_{-\infty}^{+\infty} \frac{d\epsilon}{4\pi i} \times \Tr_{\gamma} \left[ \hat{\gamma}_3 \hat{\sigma}_0 \left( \hat{g}^K(p_f, x, \epsilon, t) - \hat{g}^K(p_f^*, x, \epsilon, t) \right) \right],$$

(1)

where $e$ is the electron charge and $\hbar = 1$ throughout the

Here subscript 1 means that the corresponding functions
should be taken at $x = -0$, argument $p_f$ of all the Riccati
functions is omitted for brevity. The product $\otimes$ of two
functions of energy and time is defined by the noncommu-
tative convolution $A \otimes B = e^{i\epsilon A B - \sum_{a \alpha} a^B A(\epsilon, t) B(\epsilon, t)}$. Keldysh Green’s function $\hat{g}_1(p_f)$ for the outgoing
trajectory can be obtained from Eqs. (2), (3) by the
substitution $(\hat{\gamma}_1^R, \hat{\bar{\gamma}}_1^R, \hat{x}_1^K)(p_f) \rightarrow (\hat{\Gamma}_1^R, \hat{\bar{\Gamma}}_1^R, \hat{X}_1^K)(p_f)$ and
$(\hat{\Gamma}_1^R, \hat{\Gamma}_1^A, \hat{X}_1^K)(p_f) \rightarrow (\hat{\gamma}_1^R, \hat{\bar{\gamma}}_1^R, \hat{x}_1^K)(p_f)$.

Riccati coherence and distribution functions obey
Riccati-type transport equations. The equations are
given in Refs. [2], [3], so I do not write them here. The
quantities $(\hat{\gamma}_1^R, \hat{\bar{\gamma}}_1^R, \hat{x}_1^K, \hat{X}_1^K)$, denoted by lower case symbols,
are obtained by solving the Riccati equations for the appropriate trajectory with the asymptotic
conditions, which are for spin-singlet s-wave superconductor as follows

$$\hat{x}_{l,r}^A(\epsilon, t) = \begin{cases} \frac{\Delta e^{-2ieV_{l,r}}}{\epsilon \pm i \sqrt{\Delta^2 - \epsilon^2} \hat{\sigma}_2}, & |\epsilon| < \Delta, \\ \frac{\Delta e^{-2ieV_{l,r}}}{\epsilon + \text{sgn} \sqrt{\epsilon^2 - \Delta^2} \hat{\sigma}_2}, & |\epsilon| > \Delta. \end{cases}$$

(4)

where $\Delta$ is the bulk absolute value of supercon-
ducting order parameter for a given temperature, which
has only two values, $\hat{\sigma}_2$ are Pauli matrices in
space variable and coincide with the asymptotic condi-
tions in the corresponding superconductor. $\hat{\Gamma}_3$ and $\hat{\sigma}_0$ are Pauli matrices in
space variable and spin spaces, respectively. The spin cur-
rent $j^\sigma_v/s^\sigma$ can be calculated making use of Eq. (4) with
the substitution $\hat{\sigma}_n$. $s^\sigma = 1/2$ is the electron spin.

Quasiclassical Green’s function $\hat{g}$ can be expressed in
terms of Riccati coherence functions $\hat{\gamma}_1^R, \hat{\bar{\gamma}}_1^R$, and $\hat{x}_1^K, \hat{X}_1^K$, which
are the relative amplitudes for normal-state quasiparticle
devices and quasihole excitations, and distribution functions $\hat{x}_1^K$ and $\hat{X}_1^K$. All these functions are $2 \times 2$
matrices in spin space and depend on $(p_f, x, \epsilon, t)$.

In this paper I consider a short junction with the char-
acteristic size $d \ll \xi_0$. But it is assumed that despite the
small size of the grain charging effects can be neglected.
For definiteness the current is calculated on the left side
of the interface. Keldysh Green’s function for incoming
trajectory is parameterized by

$$\hat{g}_1^K(p_f) = -2i\pi N^R \otimes \left( \begin{array}{ccc} (\hat{x}_1^K - \hat{\gamma}_1^R \otimes \hat{X}_1^K) & - (\hat{\gamma}_1^R \otimes \hat{x}_1^K) \\ (\hat{\Gamma}_1^R \otimes \hat{x}_1^K - \hat{\gamma}_1^R \otimes \hat{X}_1^K) & -(\hat{\gamma}_1^R \otimes \hat{x}_1^K) \end{array} \right) \otimes \hat{N}_A,$$

(2)

$$\hat{N}_A = \left( \begin{array}{cc} 1 - \hat{\gamma}_1^R(\hat{\Gamma}_1^R) \otimes \hat{\Gamma}_1^R(\hat{\gamma}_1^R) & 0 \\ 0 & 1 - \hat{\gamma}_1^R(\hat{\Gamma}_1^R) \otimes \hat{\Gamma}_1^R(\hat{\gamma}_1^R) \end{array} \right).$$

(3)

$$\hat{x}_{l,r}^K(\epsilon) = (1 - |\gamma_1^{R,l}(\epsilon - eV_{l,r})| t)^2 \tanh \frac{\epsilon - eV_{l,r}}{2T},$$

(5)

where the subscript $l, r$ denotes that the appropriate Riccati
correlation function belongs to the bulk of the left (right) superconductor. $\Delta$ is the bulk absolute value of supercon-
ducting order parameter for a given temperature, which
is assumed to be the same in the both superconductors. $V_{l,r}$ is the electric potential in the bulk of left (right)
superconductor, so $V = V_r - V_l$ is the voltage bias
applied to the junction. Quantities $\hat{\gamma}_1^{R,l}$ and $\hat{x}_{l,r}^K$ are
obtained from Eqs. (2), (3) respectively by the operation
$\hat{a}(\epsilon, t) = a(-\epsilon, \epsilon)^*$.

For analytical consideration superconducting order pa-
rameter and electric potential are assumed to be spa-

cially constant in the superconductors. Under this as-
sumption the voltage drop only occurs at the junction
region. These simplifications seem to be reason-
able for quantum point contact. Under the assump-
tions above the solutions of Riccati equations for
$(\hat{\gamma}_1^{R,l}(x), \hat{x}_{l,r}^K(x))$ do not depend on the
space variable and coincide with the asymptotic condi-
tions in the corresponding superconductor.
The quantities \( \langle \hat{R}^{R,A}, \hat{z}^{R,A}, \hat{X}^K, \hat{X}^K \rangle \), denoted by upper case symbols, are expressed via
\( (\hat{x}_i^{R,A}, \hat{z}_i^{R,A}, \hat{X}_i^K, \hat{X}_i^K) = (\langle \hat{x}_i^{R,A}, \hat{z}_i^{R,A}, \hat{X}_i^K, \hat{X}_i^K \rangle) \) and the elements of the interface scattering matrix \( S \) for the normal-state electrons and holes with the energies at the Fermi surface. The interface \( S \)-matrix is a unitary \( 8 \times 8 \) matrix in the combined spin, particle-hole and directional spaces. The explicit structure of \( S \)-matrix in directional spaces is
\[
S = \begin{pmatrix}
\hat{S}_{11} & \hat{S}_{12} \\
\hat{S}_{21} & \hat{S}_{22}
\end{pmatrix},
\]
where matrix \( \hat{S}_{ij} \) contains spin-dependent reflection amplitudes of normal-state quasiparticles from the interface in \( i \)-th half-space, while \( \hat{S}_{ij} \) with \( i \neq j \) incorporates spin-dependent transmission amplitudes of normal-state quasiparticles from side \( j \). Each element \( \hat{S}_{ij} \) is a diagonal matrix in particle-hole space \( \hat{S}_{ij} = \hat{S}_{ij}(1 + \hat{\gamma}_j)/2 + \hat{S}_{ij}(1 - \hat{\gamma}_j)/2 \). The most general form of \( S \)-matrix for a symmetric magnetic interface without spin-orbit interaction can be written as:
\[
\hat{S}_{11} = \hat{S}_{22} = \begin{pmatrix}
\sqrt{R_{11}}e^{i\theta_{11}/2} & 0 \\
0 & \sqrt{R_{22}}e^{-i\theta_{22}/2}
\end{pmatrix},
\]
\[
\hat{S}_{12} = \hat{S}_{21} = \pm i \begin{pmatrix}
\sqrt{R_{12}}e^{i\theta_{12}/2} & 0 \\
0 & -\alpha \sqrt{R_{21}}e^{-i\theta_{21}/2}
\end{pmatrix},
\]
where \( R_{ij} \) are the transmission coefficients.

The particular expressions for \( \langle \hat{R}^{R,A}, \hat{z}^{R,A}, \hat{X}^K, \hat{X}^K \rangle \) in terms of \( (\hat{x}_i^{R,A}, \hat{z}_i^{R,A}, \hat{X}_i^K, \hat{X}_i^K) \) and \( S \)-matrix elements can be found in Ref.\(^2\), so I do not write them explicitly.

Now we can proceed with the current. Substituting Keldysh Green’s function \(^2\) into Eq.\(^1\) it can be seen that the current is expressed as a sum over harmonics
\[
j(t) = \sum_N J_N e^{2iNeVt}.
\]
I focus on the dc component and only calculate the term corresponding to \( N = 0 \). I-V characteristics in charge and spin currents exhibit steps. In the tunnel limit one can extract the elementary processes giving rise to each of these steps. In this paper the positions, height and shape of several first steps in subgap I-V characteristic are calculated analytically. The linear in transparency thermally-activated term in the dc current, which is not related to the steps, has also been calculated, but it is not pronounced in the subgap region for the considered problem and is not discussed below. The part of the dc current corresponding to the steps takes the form:
\[
\begin{align*}
    J_{1+}^{ch} &= \frac{\Delta |\sin \Theta/2|}{4}(D_{1+} \pm D_{1+}) \frac{|\varepsilon_0 - eV|}{|\varepsilon_0 - eV|^2 - \Delta^2} \frac{1 + |\gamma(\varepsilon_0 - eV)|^2}{|\varepsilon_0 - 2eV|e^{\Theta \varepsilon_0}/2T - \tanh \frac{\varepsilon_0}{2T}} , \\
    J_{2+}^{ch} &= \frac{\Delta |\sin \Theta/2|}{4}D_{1+} \frac{|\varepsilon_0 - 2eV|}{|\varepsilon_0 - 2eV|^2 - \Delta^2} \frac{1 + |\gamma(\varepsilon_0 - eV)|^2}{|\varepsilon_0 - 2eV|e^{\Theta \varepsilon_0}/2T - \tanh \frac{\varepsilon_0}{2T}}.
\end{align*}
\]

for the case of nonmagnetic junction between two d-wave superconductors\(^2\).

Let us describe all the steps for \( n > 2 \) qualitatively. The estimate for the magnitude of \( J_n^{ch(s)p} \) is easily obtained if one considers physical processes of MAR leading to subgap current steps. For the spin-up band the bound state energy is \( \varepsilon_B = +\varepsilon_0 \) and at \( T = 0 \) the following process gives spin-up current at \( eV_{n+}^+ \): a spin-up electron tunnels into the bound state from the continuum states below \( -\Delta \) undergoing \( n-1 \) Andreev reflections inside the gap. An electron converts into a hole and vice versa under each Andreev reflection event. In every passing of the electron (hole) through the boundary the amplitude of its wave function should be multiplied by \( \sqrt{D_{1+}/D_{1+}} \). For the spin-down band with \( \varepsilon_B = -\varepsilon_0 \) the same process takes place with the modification that
FIG. 1: (a) The subgap structure of dc electric current (in units of $\Delta_0 e/h$) is plotted in accordance with Eq. (10) for $T = 0, 0.5T_c$. (b) The same for the spin current. $\Theta = \pi/2$, $D_\uparrow = 0.2$, $D_\downarrow = 0.1$. $\Delta_0 \equiv \Delta(T = 0)$. 

a spin-down electron starts in the bound state and goes up into the continuum states at $+\Delta$. So the current $j_{\uparrow,2k}$ is $\propto (D_\uparrow D_\downarrow)^k$ and $j_{\downarrow,2k+1} \propto (D_\uparrow D_\downarrow)^k D_\downarrow$. Finally, we obtain for $j_{n}^{ch(sp)} = j_{n\uparrow} \pm j_{n\downarrow}$ the following result: $j_{2k+1}^{ch(sp)} \propto (D_\uparrow D_\downarrow)^k (D_\uparrow \pm D_\downarrow)$ and $j_{2k}^{sp} \propto 2(D_\uparrow D_\downarrow)^k$, $j_{2k}^{sp} = 0$.

For $T \neq 0$ there is a non-zero probability to find a spin-up electron at $-\varepsilon_0$, and a vacant place for a spin-down electron at $-\varepsilon_0$. So the processes leading to the additional steps at $e\varepsilon_n^-$ arise: (i) a spin-up electron tunneling from $\varepsilon_B = \varepsilon_0$ to the continuum spectrum at $+\Delta$ and (ii) a spin-down electron tunneling from the continuum spectrum below $-\Delta$ into the bound state $-\varepsilon_0$. Consequently, for $T \neq 0$ the splitting of current step positions occurs and there are all the steps at $e\varepsilon_n^- = (\Delta \pm \varepsilon_0)/n$ in the charge current and only odd split steps in the spin current.

Of course, the processes of inelastic scattering lead to impossibility of observing the subgap structures corresponding to large $n$ due to the loss of coherence if the time $\tau = n\varepsilon_0/e\gamma$, which a quasiparticle spends in the junction region, exceeds the average time of inelastic scattering $\tau_{in}$. So the steps with $n$ more than $\tau_{in} \Delta_0 (\varepsilon_0 / \varepsilon_{eff})$ can not be observed. For the case under consideration this estimate seems not to be restrictive because $\varepsilon_0 / \varepsilon_{eff} \sim 1$ for a short junction and $\tau_{in} \Delta_0$ due to electron-phonon scattering $\propto (\omega_D/\varepsilon_0)^2(\Delta_0/T)$ and considerably exceeds unity for temperatures up to $T_c$.

In conclusion, I have presented a theoretical study of the dc electric and spin current subgap structure in magnetic point contact. At $T \neq 0$ electric current step positions $e\varepsilon_n^- = (\varepsilon_0 \pm \Delta)/n$ are split compared to the case of nonmagnetic junction and at $T = 0$ they are just shifted: $e\varepsilon_n^- = (\varepsilon_0 + \Delta)/n$. The subgap spin current, taking place at $D_\uparrow \neq D_\downarrow$ manifests steps at the same voltages but only for odd values of $n$. The current for steps with $n = 1, 2$ is calculated analytically and a qualitative description of the following steps is presented.

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