Cosmic Strings and Black Holes†

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Abstract

In the first part of this talk, I consider some exact string solutions in curved spacetimes. In curved spacetimes with a Killing vector (timelike or spacelike), the string equations of motion and constraints are reduced to the Hamilton equations of a relativistic point-particle in a scalar potential, by imposing a particular ansatz. As special examples I consider circular strings in axially symmetric spacetimes, as well as stationary strings in stationary spacetimes.

In the second part of the talk, I then consider in more detail the stationary strings in the Kerr-Newman geometry. It is shown that the world-sheet of a stationary string, that passes the static limit of the 4-D Kerr-Newman black hole, describes a 2-D black hole. Mathematical results for 2-D black holes can therefore be applied to physical objects; (say) cosmic strings in the vicinity of Kerr black holes. As an immediate general result, it follows that the string modes are thermally excited.

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1 Introduction

In a generic curved spacetime, the classical equations of motion of a bosonic string take the simplest form in the orthonormal gauge:

\[ \ddot{x}^\mu - x'^\mu + \Gamma^\mu_{\rho\sigma}(\dot{x}^\rho \dot{x}^\sigma - x'^\rho x'^\sigma) = 0, \]

supplemented by the constraints:

\[ g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu = g_{\mu\nu}(\dot{x}^\mu \dot{x}^\nu + x'^\mu x'^\nu) = 0. \]

Here dot and prime denote differentiation with respect to the world-sheet coordinates \( \tau \) and \( \sigma \), respectively. \( \Gamma^\mu_{\rho\sigma} \) is the Christoffel symbol corresponding to the spacetime metric \( g_{\mu\nu} \).

The main complication, as compared to the case of flat Minkowski spacetime, is related to the non-linearity of the equations of motion (1.1). It makes it possible to obtain the complete analytic solution only in a very few special cases like conical spacetime [1] and plane-wave/shock-wave backgrounds [2]. There are however also very general results concerning integrability and solvability for maximally symmetric spacetimes [3, 4] and gauged WZW models [5, 6]. These are the exceptional cases; generally the string equations of motion in curved spacetimes are not integrable and even if they are, it is usually an extremely difficult task to actually separate the equations, integrate them and finally write down the complete solution in closed form. Fortunately, there are several different ways to ”attack” a system of coupled non-linear partial differential equations, using either numerical, approximative or exact methods (for some recent reviews on strings in curved spacetimes, see for instance [7, 8]).

In this paper, we consider some exact string solutions in curved spacetimes, obtained by imposing a particular ansatz. In Section 2, it is shown
that in curved spacetimes with a Killing vector (timelike or spacelike), the string equations of motion and constraints (1.1)-(1.2) reduce to the Hamilton equations of a relativistic point-particle in a very simple scalar potential. As special examples we consider circular strings in axially symmetric spacetimes as well as stationary strings in stationary spacetimes. In Section 3, we consider in more detail the stationary strings in the Kerr-Newman geometry. It is shown that the world-sheet of a stationary string, that passes the static limit of the 4-D Kerr-Newman black hole in the equatorial plane, describes a 2-D Reissner-Nordström black hole. Mathematical results for 2-D black holes can therefore be applied to physical objects; (say) cosmic strings in the vicinity of Kerr black holes. An immediate general result is that the string modes are thermally excited.

2 Exact String Solutions

In this section we shall consider some exact string solutions in curved spacetimes. We first consider, as special examples, circular strings and stationary strings. Eventually we then give a more general description valid in curved spacetimes with an arbitrary Killing vector.

A. Circular Strings

The condition for a curved spacetime to allow for circular strings, is that the spacetime is axially symmetric. We can thus use a coordinate system $x^\mu = (t, r, \theta, \phi)$ where the metric is explicitly independent of the azimuthal angle $\phi$:

$$ds^2 = g_{\mu\nu}(t, r, \theta)dx^\mu dx^\nu. \quad (2.1)$$

We consider for simplicity a 4-D spacetime, since we have mainly cosmic strings in mind. The ansatz which describes a dynamical circular string is
provided by:
\[ t = t(\tau), \quad r = r(\tau), \quad \theta = \theta(\tau), \quad \phi = \sigma + f(\tau). \quad (2.2) \]
That is, we identify the spatial world-sheet coordinate \( \sigma \) with the azimuthal angle \( \phi \) up to the addition of a function of the world-sheet coordinate \( \tau \), and let the other coordinates depend only on \( \tau \); this guarantees that the string is circular. The four functions \( t(\tau), r(\tau), \theta(\tau) \) and \( f(\tau) \) are now to be determined by the equations of motion and constraints (1.1)-(1.2).

The invariant definition of a circular string in an axially symmetric spacetime is that the Killing vector corresponding to the axial symmetry is tangent to the string world-sheet. It is then straightforward to show that the ansatz (2.2) is in fact the most general ansatz describing circular strings, at least up to residual gauge transformations and constant rescalings of \( \sigma \).

By inserting the ansatz (2.2) into eqs.(1.1), it can be shown that the equations of motion reduce to the Hamilton equations of the point-particle Hamiltonian:
\[ H = \frac{1}{2} g^{\mu\nu} \mathcal{P}_\mu \mathcal{P}_\nu + \frac{1}{2} g_{\phi\phi}, \quad (2.3) \]
while the two constraints eqs.(1.2) become:
\[ \mathcal{H} = 0, \quad \mathcal{P}_\phi = 0. \quad (2.4) \]
The dynamics of a circular string in an axially symmetric curved spacetime is thus mathematically equivalent to the dynamics of a \textit{massless} zero-angular-momentum point-particle, moving in the same curved spacetime as well as in a scalar potential given simply by the \((\phi\phi)\)-component of the metric. By a conformal transformation, this system is also equivalent to the system of a \textit{massive} point-particle moving in some unphysical spacetime (see [9] for a discussion of this point in the more general case of charge-current carrying circular strings, as well as the original references [10, 11] for the similar
discussion for stationary strings. See also Section 3); however, the latter approach will not be considered in this section. We find it more convenient to work directly with the physical metric \( g_{\mu\nu} \).

It must be stressed that the reduction of the system (1.1)-(1.2) to the system (2.3)-(2.4) of course does not mean that we have solved anything. In fact, Hamiltonian systems of the form (2.3), with or without the scalar potential \( g_{\phi\phi} \), are generally not separable, and even if the ”standard” point-particle part \( g^{\mu\nu}P_\mu P_\nu \) of the Hamiltonian is separable, the separability is generally destroyed by the scalar potential \( g_{\phi\phi} \). This is precisely what happens in the case of the Kerr-Newman black hole: the \( g^{\mu\nu}P_\mu P_\nu \)-part of the Hamiltonian (2.3) is separable, as is well known, but the \( g_{\phi\phi} \)-part is not of separable type \( 1, 2 \).

In most cases it is therefore necessary to make further reductions. Consider for instance 4-D spacetimes of the form:

\[
d s^2 = g_{tt}(r, \theta)dt^2 + g_{rr}(r, \theta)dr^2 + g_{\theta\theta}(r, \theta)d\theta^2 + g_{\phi\phi}(r, \theta)d\phi^2 + 2g_{t\phi}(r, \theta)dtd\phi,
\]

which covers the black hole spacetimes, some cosmological spacetimes (in static coordinates) the 2 + 1 black hole and black string spacetimes (taking \( \theta = \text{const.} \)) etc. If we further consider circular strings in the equatorial plane \( \theta = \pi/2 \), then it can be shown that the equation determining the radius of the string-loop, as obtained from the Hamiltonian (2.3), takes the form:

\[
\dot{r}^2 + V(r) = 0,
\]

where:

\[
V(r) = g^{rr}(E^2 g^{tt} + g_{\phi\phi}); \quad E = \text{const.}
\]

That is, everything is solved in quadratures, and the dynamics of the circular string is exactly known in closed form.
Using this approach, it is now a relatively easy task to describe the circular string dynamics in any curved spacetime of the form (2.5). Simply by looking at the line-element, one can immediately read off the potential $V(r)$, which determines the dynamics of the circular string. Next, one can then solve the equations exactly and the physical properties (energy-density, pressure,..) can be extracted.

Circular strings were first discussed in Minkowski spacetime in [13]. More recently the dynamics of circular strings has been investigated in detail in both cosmological and black hole spacetimes [14-26] (charged circular strings in curved spacetimes were discussed in [12] and references given therein). We refer the interested reader to these publications and only give a couple of examples here. In general the dynamics of a circular string is governed by the tension, which always tries to contract the string, and the local gravitational field which can be either attractive or repulsive.

In the absence of a repulsive gravitational field, the circular string will always collapse from some maximal size. This is illustrated in Fig.1 in the cases of (a) Minkowski, (b) Schwarzschild, (c) anti de Sitter and (d) Schwarzschild-anti de Sitter. In all these cases the string collapses to zero size. There are however differences due to the causal structure of the spacetimes. In Minkowski and anti de Sitter spacetimes, the string dynamics is truly oscillatory (we neglect the gravitational backreaction), while in the black hole spacetimes the dynamics stops when the string falls into the spacetime singularity.

In the presence of a repulsive gravitational field, the situation is more complicated. A circular string can be oscillating between a maximal size and zero size, oscillating between a maximal size and a non-zero minimal size, it can be expanding forever or it can even be of constant size. This is illustrated in Fig.2 in the cases of (a) de Sitter and (b) Kerr black hole.
The potential shown for de Sitter spacetime corresponds to the special case where \( H^2E^2 < 1/4 \), where \( H \) is the Hubble constant and \( E \) is the integration constant introduced in (2.7); the more general analysis for arbitrary values of \( H^2E^2 \) is presented in [22]. From the potential (a) follows that small strings will be oscillating between a maximal size and zero size, while large strings will bounce on the barrier and re-expand forever. The other types of circular string dynamics in de Sitter spacetime are described in detail in [22].

The potential shown for the Kerr Black hole corresponds to the special case where \( a^2 > E^2 \), where \( a \) is the angular momentum of the black hole; the more general potential for arbitrary values of \( a \) is given in [23]. From the potential (b) follows that a circular string contracts from its maximal size to a non-zero minimal size. When the string reaches its minimal size, it is actually inside the event horizon and thus cannot re-expand because of the causal structure. However, such solution can be interpreted as a string contracting in one spacetime, expanding in another etc [20, 23].

B. Stationary Strings

As another example of a family of exact string solutions in curved spacetimes, we now consider stationary strings. It is well known that in flat Minkowski spacetime there is only one stationary string configuration, namely the straight string; a string with any other shape will start vibrating. However, we shall now show that the situation in curved spacetimes is completely different. In fact, in some curved spacetimes stationary strings can have infinitely many completely different shapes, and there can be very interesting physics associated with them.

The condition for a curved spacetime to allow for stationary strings, is that the spacetime itself is stationary. We can thus use a coordinate system \( x^\mu = (t, r, \theta, \phi) \) where the metric is explicitly independent of the time-
coordinate $t$:

$$ds^2 = g_{\mu\nu}(r, \theta, \phi)dx^\mu dx^\nu. \quad (2.8)$$

The ansatz which describes a stationary string is:

$$t = \tau + f(\sigma), \quad r = r(\sigma), \quad \theta = \theta(\sigma), \quad \phi = \phi(\sigma). \quad (2.9)$$

That is, we identify the temporal world-sheet coordinate $\tau$ with the time-coordinate $t$ up to the addition of a function of the world-sheet coordinate $\sigma$, and let the other coordinates depend only on $\sigma$; this guarantees that the string is stationary. The four functions $f(\sigma)$, $r(\sigma)$, $\theta(\sigma)$ and $\phi(\sigma)$ are now to be determined by the equations of motion and constraints (1.1)-(1.2).

C.f. the previous discussion of circular strings in axially symmetric space-times, the invariant definition of a stationary string in a stationary spacetime is that the timelike Killing vector $\partial_t$ is tangent to the string world-sheet. It is then straightforward to show that the ansatz (2.9) is the most general ansatz describing stationary strings up to residual gauge transformations and constant rescalings of $\tau$.

By inserting the ansatz (2.9) into eqs.(1.1), it can be shown that the equations of motion reduce to the Hamilton equations of the point-particle Hamiltonian:

$$\mathcal{H} = \frac{1}{2}g^{\mu\nu}\mathcal{P}_\mu\mathcal{P}_\nu + \frac{1}{2}g_{tt}, \quad (2.10)$$

while the two constraints eqs.(1.2) become:

$$\mathcal{H} = 0, \quad \mathcal{P}_t = 0, \quad (2.11)$$

compare with eqs.(2.3)-(2.4). Thus mathematically the problem of finding the stationary string configurations looks very similar to the problem of describing the dynamical circular strings. It is however a very interesting result that in the case of the Kerr-Newman black hole, the Hamiltonian system
(2.10) is actually separable \[10\]. Thus contrary to the case of the circular strings in the Kerr-Newman background, the \( g_{tt} \)-part of the Hamiltonian (2.10) does not destroy the separability of the system, and therefore the stationary string configurations can be described completely and explicitly \[11\].

In most other cases it is however still necessary to make further reductions to solve the system. If we consider again the 4-D spacetimes of the form (2.5), and restrict ourselves to describing only stationary strings in the equatorial plane \( \theta = \pi/2 \), then it can be shown that the equation determining the radial coordinate of the stationary string, as obtained from the Hamiltonian (2.10), takes the form:

\[ r'^2 + U(r) = 0, \quad (2.12) \]

where:

\[ U(r) = g^{rr}(L^2 g^{\phi\phi} + g_{tt}); \quad L = \text{const.} \quad (2.13) \]

That is, everything is solved in quadratures, and the stationary string configuration is exactly known in closed form.

Using this approach, it is now a relatively easy task to describe the stationary strings in any curved spacetime of the form (2.5). Everything goes exactly as in the previous case of the circular strings: simply by looking at the line-element, one can immediately read off the potential \( U(r) \), which determines the location of the stationary string. Next, one can then solve the equations exactly and the physical properties (energy-density, pressure,..) can be extracted.

As already mentioned, the only stationary string in Minkowski spacetime is the straight string. In stationary spacetimes with an attractive gravitational field, the stationary strings typically extend out to spatial infinity. This is illustrated in Fig.3 in the cases of (a) Minkowski, (b) Schwarzschild
and (c) anti de Sitter spacetimes. For the potentials shown, the two ends of the stationary string are at spatial infinity, and the string passes $r = 0$ with some impact parameter related to the integration constant $L$. For a more complete discussion of these stationary strings we refer the reader to [11, 27].

If the spacetime has a repulsive gravitational field it is also possible to have stationary strings in a small compact region. This is illustrated in Fig.4 in the case of de Sitter spacetime. In this case the stationary string is located somewhere between two finite values of the radial coordinate (both being smaller than the horizon size). It turns out that there are infinitely many completely different stationary string configurations in de Sitter spacetime classified, using a simple underlying mathematical structure, in terms of two integers [27]. An example of a stationary string in de Sitter spacetime is presented in Fig.5. The complete discussion is given in [27].

C. General Formalism

It is apparent from the previous discussions that the mathematics of the circular strings is very similar to the mathematics of the stationary strings, although the physics is of course completely different. By comparing the formulas (2.1)-(2.7) with (2.8)-(2.13) we notice that there is actually a kind of ”duality” between circular strings in axially symmetric spacetimes and stationary strings in stationary spacetimes [9, 27]; the ”duality-transformation” being simply:

$t \leftrightarrow \phi , \quad \tau \leftrightarrow \sigma$ \hspace{1cm} (2.14)

It must be stressed, however, that there is absolutely nothing mysterious about this ”duality”. In fact, the previous discussions of circular strings and stationary strings are just special examples of a more general approach for obtaining special exact solutions of the string equations of motion and constraints in curved spacetimes with a Killing vector:
Consider an arbitrary curved spacetime with an arbitrary Killing vector $\xi$. Then take a coordinate system such that the Killing vector has the standard form:

$$\xi^\mu = \delta_i^\mu,$$

for some $i$. (2.15)

Now make the following ansatz for the string:

$$x^i = \sigma^1 + f(\sigma^2), \quad x^j = x^j(\sigma^2); \quad j \neq i$$

(2.16)

where $(\sigma^1, \sigma^2)$ are the two world-sheet coordinates. That is, the targetspace coordinate corresponding to the Killing vector is identified with one of the world-sheet coordinates plus a function of the other, while the other targetspace coordinates are only functions of the other world-sheet coordinate.

In that case it is easy to show that the string equations of motion and constraints (1.1)-(1.2) reduce to the Hamiltonian system:

$$H = \frac{1}{2} g^{\mu \nu} P_\mu P_\nu + \frac{1}{2} g_{ii},$$

$$H = 0, \quad P_i = 0.$$  

(2.17)  

(2.18)

This is just the Hamiltonian of a massless point-particle moving in the curved spacetime as well as in a scalar potential given simply by the $(ii)$-component of the metric. If the Killing vector is timelike, this system describes stationary strings, while if it is spacelike, it describes dynamical strings of some non-trivial shape.

### 3 Stationary Strings and 2-D Black Holes

(This section is essentially a reproduction of [28])

The aim of this section is to attract the attention to the remarkable fact, that the study of propagation of perturbations along a stationary cosmic string located in the gravitational field of a stationary 4-dimensional black hole, is
directly related with physics of 2-dimensional black holes. If one neglects
the gravitational effects of a cosmic string and assumes that its thickness is
zero, then a string configuration in a given gravitational field is a minimal
surface, and its equations of motion can be obtained by variation of the
Goto-Nambu action; these equations are of course equivalent to eqs.(1.1)-
(1.2). An important case is when both the gravitational field and a string
configuration are stationary. This case corresponds to a physical situation
of a string which is in equilibrium in the gravitational field. The problem of
finding the equilibrium string configurations in a stationary spacetime can
be reduced to the problem of solving geodesic equations in a 3-dimensional
unphysical space \[10\]. The remarkable property of the gravitational field
of a stationary black hole is that these geodesic equations can be solved in
quadratures \[10, 11\]. Consider a 2-dimensional time-like world-sheet \(\Sigma\) of
the string defined by the equations \(x^\mu = x^\mu(\sigma^A)\) \((x^\mu (\mu = 0, \ldots, 4)\) denote
spacetime coordinates and \(\sigma^A (A = 0, 1)\) are the coordinates on the world-
sheet). The metric \(G_{AB}\) induced on \(\Sigma\) reads:

\[
G_{AB} = g_{\mu\nu}x^\mu_{;A}x^\nu_{;B}. \tag{3.1}
\]

where \(g_{\mu\nu}\) is the spacetime metric. Consider a stationary spacetime and
let \(\xi^\mu\) be the corresponding 4-dimensional Killing vector. We call a string
stationary if \(\xi^\mu\) is tangent to its world sheet \(\Sigma\). Denote by \(\eta^A\) a 2-dimensional
vector \(\eta^A = G^{AB}x^\mu_{;B}\xi^\mu\). It is easy to show that:

\[
\xi^\mu = \eta^A x^\mu_{;A}, \tag{3.2}
\]

\[
\eta_{A|B} = x^\mu_{;A}x^\nu_{;B}\xi_{;\mu\nu}, \tag{3.3}
\]

where semicolon and vertical line denote covariant derivatives with respect
to 4 and 2-dimensional metrics, respectively. The last relation implies that
\( \eta \) is a Killing vector for the induced metric \( G_{AB} \). Denote:

\[
F = -g_{\mu\nu} \xi^\mu \xi^\nu = -G_{AB} \eta^A \eta^B.
\]  

(3.4)

Then the induced line-element \( dl^2 = G_{AB} d\sigma^A d\sigma^B \) on \( \Sigma \) can be written as:

\[
dl^2 = -F \hat{\tau}^2 + F^{-1} d\hat{\sigma}^2,
\]

(3.5)

where \( \hat{\tau} = \hat{\tau}(\sigma^A) \), \( \hat{\sigma} = \hat{\sigma}(\sigma^A) \). This representation is valid in the regions where \( F \neq 0 \), and hence \( \xi \) is either time-like or space-like. We assume that the 4-dimensional spacetime is asymptotically flat and contains a black hole. We assume also that \( \hat{\sigma} = \infty \) corresponds to the points of the string located in the asymptotically flat region of the physical spacetime, so that \( F(\hat{\sigma} = \infty) = 1 \).

The metric, eq.(3.5), describes a 2-dimensional black hole if \( F = 0 \) at finite value of \( \hat{\sigma} \). At this point the Killing 2-vector \( \eta \) is null. It happens at the points where the string world-sheet crosses the infinite-red-shift surface (the static limit), i.e. the surface where \( \xi^2 = 0 \). For a static black hole this surface coincides with the event horizon. For stationary (rotating) black holes it is lying outside the horizon. The region located between infinite-red-shift surface and the event horizon of a stationary black hole is known as the ergosphere. Points of the string located inside the ergosphere thus correspond to the interior of the 2-dimensional black hole.

The main observation we would like to make now is that perturbations propagating along the string can be described by a coupled system of a pair of scalar field equations in the 2-dimensional metric \( G_{AB} \). For this reason, if a stationary string is passing through the ergosphere of a 4-dimensional black hole, the physics of string excitations is effectively reduced to the physics of 2-dimensional black holes.

We shall first derive the differential equations describing the perturbations propagating along a stationary string configuration embedded in an arbitrary
stationary 4-dimensional spacetime. We then consider, as a special example, stationary strings in the background of a Kerr-Newman black hole.

To be more specific, we write the metric of a generic 4-dimensional stationary spacetime in the form:

$$g_{\mu\nu} = \begin{pmatrix} -F & -FA_i \\ -FA_i & -FA_iA_j + H_{ij}/F \end{pmatrix},$$

(3.6)

where $\partial_t F = 0$, $\partial_t A_i = 0$, $\partial_t H_{ij} = 0$. That is to say, the Killing vector is given explicitly by:

$$\xi^\mu = (1, 0, 0, 0), \quad \xi^\mu = (-F, -FA_i),$$

(3.7)

consistent with the notation of eq.(3.4). A stationary string configuration is parametrized in the following way:

$$t = x^0 = \tau, \quad x^i = x^i(\sigma); \quad (\tau \equiv \sigma^0, \sigma \equiv \sigma^1).$$

(3.8)

Then the equations of motion corresponding to the Goto-Nambu action reduce to [10]:

$$x^{i\prime\prime} + \tilde{\Gamma}_{jk}^i x^{j\prime} x^{k\prime} = 0, \quad H_{ij} x^{i\prime} x^{j\prime} = 1,$$

(3.9)

where $\tilde{\Gamma}_{jk}^i$ is the Christoffel connection for the metric $H_{ij}$ and a prime denotes differentiation with respect to $\sigma$. The induced metric on the world-sheet now takes the form:

$$G_{AB} = \begin{pmatrix} -F & -FA \\ -FA & -FA^2 + 1/F \end{pmatrix}; \quad A \equiv A_i x^{i\prime},$$

(3.10)

so that $\det G = -1$. The following coordinate transformation on the world-sheet:

$$\hat{\tau} = \tau + \int^\sigma A \, d\sigma, \quad \hat{\sigma} = \sigma,$$

(3.11)
brings the induced line element into the form of eq.(3.5), that is:

\[ \hat{G}_{AB} = \begin{pmatrix} -F & 0 \\ 0 & 1/F \end{pmatrix}. \]  

(3.12)

A covariant approach describing the propagation of perturbations along an arbitrary string configuration embedded in an arbitrary spacetime, was developed in Ref.[29] (see also [30, 31]). The general transverse (physical) perturbation around a background Goto-Nambu string configuration is written as:

\[ \delta x^\mu = \Phi R n^\mu_R, \quad (R = 2, 3), \]  

(3.13)

where the two vectors \( n^\mu_R \), normal to the string world-sheet, fulfill:

\[ g_{\mu\nu} n^\mu_R n^\nu_S = \delta_{RS}, \quad g_{\mu\nu} x^\mu_A n^\nu_R = 0, \]  

(3.14)

as well as the completeness relation:

\[ g^{\mu\nu} = G^{AB} x^\mu_A x^\nu_B + \delta^{RS} n^\mu_R n^\nu_S. \]  

(3.15)

It can be shown [29] that the perturbations \( \Phi^R \) are determined by the following effective action:

\[ S_{\text{eff.}} = \int d^2 \zeta \sqrt{-G} \Phi^R \left\{ G^{AB} \left( \delta^T R A + \mu^T R A \right) \left( \delta T S B + \mu T S B \right) + \mathcal{V}_{RS} \right\} \Phi^S, \]  

(3.16)

where \( \mathcal{V}_{RS} = \mathcal{V}_{(RS)} \) are scalar potentials and \( \mu_{RS}^A = \mu_{[RS]}^A \) are vector potentials on \( \Sigma \), determined by its embedding into the 4-dimensional spacetime, and \( \nabla_A \) is the covariant derivative with respect to the metric \( G_{AB} \). The vector potentials \( \mu_{RSA} \) coincide with the normal fundamental form:

\[ \mu_{RSA} = g_{\mu\nu} n^\mu_R x^\nu_A \nabla_\rho n^\rho_S, \]  

(3.17)

where \( \nabla_\rho \) is the covariant derivative with respect to the metric \( g_{\mu\nu} \). The scalar potentials \( \mathcal{V}_{RS} \) are defined as:

\[ \mathcal{V}_{RS} \equiv \Omega_{RAB} \Omega^A_S - G^{AB} x^\mu_A x^\nu_B R_{\mu\rho\nu\sigma} n^\rho_R n^\sigma_S, \]  

(3.18)
where $\Omega_{RAB}$ is the second fundamental form:

$$\Omega_{RAB} = g_{\mu\nu} n^\mu_R x'^\alpha_A \nabla_\rho x'^\nu_B, \quad (3.19)$$

and $R_{\mu\rho\sigma\nu}$ is the Riemann tensor corresponding to the metric $g_{\mu\nu}$. The equation describing the propagation of perturbations along the string world-sheet is:

$$\left\{ G^{AB} (\delta_T \nabla_A + \mu_T \nabla_A) (\delta_S \nabla_B + \mu_S \nabla_B) + \mathcal{V}_{RS} \right\} \Phi^S = 0. \quad (3.20)$$

We obtain now explicit expressions for vector $\mu_{RS}^A$ and scalar $\mathcal{V}_{RS}$ potentials which enter the propagation equations for a stationary string obeying eqs.(3.9), in the background (3.6). Using eqs.(3.14), the normal vectors $n^\mu_R$ take the form:

$$n^\mu_R = (-A_i n^i_R, n^i_R), \quad n_{\mu R} = (0, F^{-1} H_{ij} n^j_R) \equiv (0, n_i R). \quad (3.21)$$

It is convenient to introduce also 3-dimensional vectors

$$\tilde{n}^i_R = |F|^{-1/2} n^i_R, \quad (3.22)$$

which together with $x'^i$ form an orthonormal system $(x'^i, \tilde{n}^2_R, \tilde{n}^3_R)$ in the 3-dimensional space with metric $H_{ij}$. We note that there is an ambiguity $\tilde{n}^i_R \rightarrow \tilde{n}^i_R + \delta(\tilde{n}^i_R)$ in the choice of pair of normal vectors $\tilde{n}^i_R$:

$$\delta(\tilde{n}^i_R) = \Lambda_R S \tilde{n}^i_S; \quad \Lambda_{RS}(\zeta) = -\Lambda_{SR}(\zeta). \quad (3.23)$$

Under these 'gauge' transformations, which leave invariant the perturbations (3.13) and the effective action (3.16), the quantities $\Phi^S$, $\mu_{RSA}$, and $\mathcal{V}_{RS}$ are transformed as follows:

$$\delta(\Phi^R) = \Lambda^R S \Phi^S, \quad \delta(\mu_{RSA}) = \Lambda_R T \mu_{TSA} - \Lambda_S T \mu_{TRA} - \Lambda_{RSA}, \quad (3.24)$$

$$\delta(\mathcal{V}_{RS}) = \Lambda_R T \mathcal{V}_{TS} + \Lambda_S T \mathcal{V}_{RT}. \quad (3.25)$$
We fix the freedom of ‘gauge’ by choosing the vectors $\hat{n}_R$ to be covariantly constant in the 3-dimensional space [29]. After straightforward but tedious calculations we get:

$$\mu_{RS} A = \eta^A \frac{A_{ij}}{2} \hat{n}_R^i \hat{n}_S^j = \eta^A \frac{1}{2F^2} \xi_{[0,i} \xi_{j]} \hat{n}_R^i \hat{n}_S^j, \quad (3.25)$$

$$\nu_{RS} = -x'^i x'^j \tilde{R}_{iklj} n_R^k n_S^l + \frac{\delta_{RS}}{2} x'^i x'^j (\tilde{\nabla}_i \tilde{\nabla}_j F)$$

$$- \frac{\delta_{RS}}{4F} x'^i x'^j (\tilde{\nabla}_i F)(\tilde{\nabla}_j F) + \frac{F}{4} \delta^{TU} A_{il} A_{kj} n_U^i n_T^j n_R^k n_S^l. \quad (3.26)$$

Here the "field-strength" $A_{ij}$ is defined by:

$$A_{ij} = A_{i,j} - A_{j,i} = \tilde{\nabla}_j A_i - \tilde{\nabla}_i A_j, \quad (3.27)$$

where $\tilde{\nabla}_i$ is the covariant derivative with respect to the metric $H_{ij}$ and $\tilde{R}_{iklj}$ is the Riemann tensor corresponding to the metric $H_{ij}$. It is easy to verify that for our choice of 'gauge' the vector potentials $\mu_{RS} A$ obey the analog of the Lorentz gauge conditions $\nabla_A \mu_{RS} A = 0$. The anti-symmetric products of normal vectors, appearing in the scalar and vector potentials, eqs.(3.25)-(3.26), can be eliminated using the identity:

$$\epsilon^{RS} \hat{n}_R^i \hat{n}_S^j = e^{ijk} H_{kl} x'^k, \quad e^{ijk} = (H)^{-1/2} \epsilon^{ijk}. \quad (3.28)$$

In particular we have:

$$x'^i x'^j \tilde{R}_{iklj} n_R^k n_S^l = F \delta_{RS} (\frac{1}{2} \tilde{R} - \tilde{R}_{ij} x'^i x'^j) - \tilde{R}_{ij} \hat{n}_R^i \hat{n}_S^j, \quad (3.29)$$

$$\frac{F}{4} \delta^{TU} A_{il} A_{kj} n_U^i n_T^j n_R^k n_S^l = \frac{F}{4} \delta_{RS} (A_{ik} A_{j}^k x'^i x'^j - \frac{1}{2} A_{ij} A^{ij}). \quad (3.30)$$

By using the equations (3.29)-(3.30) we get:

$$\mu_{RS} A = \mu \epsilon_{RS} \eta^A ; \quad \mu = \frac{F}{4} A_{ij} \tilde{\epsilon}^{ijk} H_{kl} x'^l, \quad (3.31)$$
\[ \mathcal{V}_{RS} = \delta_{RS} \frac{x'^i x'^j}{2} (\tilde{\nabla}_i \tilde{\nabla}_j F) - \frac{1}{4F} x'^i x'^j (\tilde{\nabla}_i F)(\tilde{\nabla}_j F) - F \left( \frac{1}{2} \tilde{R} - \tilde{R}_{ij} x'^i x'^j \right) \]
\[ + \frac{F^3}{4} (A_{ik} A_{j}^{\ i} x'^i x'^j - \frac{1}{2} A_{ij} A^{ij}) + \tilde{R}_{ij} n^i_n n^j. \] (3.32)

As a special application of the above formalism, we now consider perturbations propagating along a stationary string in the Kerr-Newman black hole background. The Kerr-Newman metric in Boyer-Lindquist coordinates [32] reads:

\[ ds^2 = -\frac{\Delta}{\rho^2} [dt - a \sin^2 \theta d\phi]^2 + \frac{\sin^2 \theta}{\rho^2} [(r^2 + a^2) d\phi - a dt]^2 + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2, \] (3.33)

where \( \Delta = r^2 - 2Mr + Q^2 + a^2 \) and \( \rho^2 = r^2 + a^2 \cos^2 \theta \). This metric is of the form (3.6) with:

\[ H_{rr} = \frac{\Delta - a^2 \sin^2 \theta}{\Delta}, \quad H_{\theta \theta} = \Delta - a^2 \sin^2 \theta, \quad H_{\phi \phi} = \Delta \sin^2 \theta, \] (3.34)

\[ F = \frac{\Delta - a^2 \sin^2 \theta}{r^2 + a^2 \cos^2 \theta}, \quad A_{\phi} = a \sin^2 \theta \frac{2Mr - Q^2}{\Delta - a^2 \sin^2 \theta}. \] (3.35)

The unperturbed stationary string configurations are obtained by solving eqs.(3.9), [10]:

\[ (H_{rr} r')^2 = \frac{a^2 b^2}{\Delta^2} - \frac{q^2}{\Delta} + 1, \]
\[ (H_{\theta \theta} \theta')^2 = q^2 - \frac{b^2}{\sin^2 \theta} - a^2 \sin^2 \theta \equiv q^2 - U(\theta), \] (3.36)
\[ (H_{\phi \phi} \phi')^2 = b^2, \]

where \( b \) and \( q \) are integration constants. Equations (3.36) are evidently invariant with respect to the reflection \( \phi \rightarrow -\phi \). It means that if \( (r(\sigma), \theta(\sigma), \phi(\sigma)) \) is a solution of eqs.(3.36) then \( (r(\sigma), \theta(\sigma), -\phi(\sigma)) \) is also a solution.

We consider at first the case when a stationary string is located in the equatorial plane \( \theta = \pi/2 \), corresponding to \( |b| \geq |a| \) and \( q^2 = a^2 + b^2 \). In that
case, the two covariantly constant normal vectors $\tilde{n}_R^i$, introduced in eq.(3.22), are given by:

$$\tilde{n}_2^i \equiv \tilde{n}_\perp^i = \frac{1}{\sqrt{|r^2 - 2Mr + Q^2|}}(0, 1, 0),$$

(3.37)

$$\tilde{n}_3^i \equiv \tilde{n}_\parallel^i = \frac{1}{\sqrt{|r^2 - 2Mr + Q^2|}}(-b, 0, H_{rr})_r.$$  

(3.38)

It is now straightforward to compute the vector and scalar potentials given by eqs.(3.31),(3.32):

$$\mu = 0,$$

(3.39)

$$V_{\perp \perp} = V_{\parallel \parallel} + \frac{2(M^2 - Q^2)(a^2 - b^2)}{r^2(\Delta - a^2)^2}$$

$$= \frac{M^2 - Q^2}{r^2(\Delta - a^2)^2} [1 + \frac{2(a^2 - b^2)}{\Delta - a^2}] + \frac{(r - M)(b^2 - a^2)}{(\Delta - a^2)^2} \left[ \frac{M}{r^2} - \frac{Q^2}{r^3} \right]$$

$$+ \frac{\Delta - b^2}{(\Delta - a^2)^2} \left[ -\frac{2M}{r} + \frac{3Q^2}{r^2} + \frac{3M^2}{r^2} - \frac{6MQ^2}{r^3} + \frac{2Q^4}{r^4} \right].$$

(3.40)

In the generic case, $|b| > |a|$, the stationary configuration in the equatorial plane describes an infinitely long open string with two "arms" in the asymptotically flat regions and a turning point outside the ergosphere [10]. The perturbation equations (3.20) with the potentials (3.39)-(3.40) then determine the reflection and transmission of waves ('phonons') travelling along the string between the asymptotically flat regions [29, 33]. However, in the special case $|b| = |a|$, the string passes the static limit and spirals inside the ergosphere towards the horizon. This is the case we are interested in, c.f. the discussion after eq.(3.5). Notice that the potential (3.40), in general being divergent at the static limit ($\Delta = a^2$), is finite for $|b| = |a|$, i.e. the divergences precisely cancel out in the particular case where the string actually crosses the static limit.
Let us now consider the case $|b| = |a|$ in more detail. We use the notations $\Sigma_\pm$ for a pair of string configurations connected by the reflection transformation, $\phi \to -\phi$, discussed after eq.(3.36). In order to make this prescription unique we choose $dr/d\phi < 0$ for $\Sigma_+$ and $dr/d\phi > 0$ for $\Sigma_-$. Then we find from eq.(3.11):

$$\hat{\sigma} = r, \quad \hat{\tau} = t \pm a^2 \int^r 2Mr - Q^2 \Delta(\Delta - a^2) \, dr,$$

(3.41)

where the signs $\pm$ correspond to the string configuration $\Sigma_\pm$. For both configurations $\Sigma_\pm$:

$$F = 1 - \frac{2M}{r} + \frac{Q^2}{r^2},$$

(3.42)

thus the world-sheet line element (3.5) takes the form of a 2-dimensional Reissner-Nordström black hole. The parameters of mass $M$ and charge $Q$ of this 2-dimensional black hole are the same as the corresponding parameters of the original 4-dimensional Kerr-Newman metric. The world-sheet spatial coordinate $\hat{\sigma}$ equals the Boyer-Lindquist radial coordinate $r$, while the world-sheet time $\hat{\tau}$ approaches the Boyer-Lindquist time $t$ in the asymptotically flat region, $r \to \infty$. The coordinates $(\hat{\tau}, r)$ cover only the exterior region of the string hole. One can easily obtain the string metric valid in a wider region. For this purpose it is convenient to introduce the Eddington-Finkelstein coordinates $(u_\pm, \varphi_\pm)$:

$$dt = du_\pm \mp \Delta^{-1}(r^2 + a^2)dr, \quad d\phi = d\varphi_\pm \mp \Delta^{-1}adr,$$

(3.43)

and to rewrite the Boyer-Lindquist metric (3.33) as (34):

$$ds^2 = -\frac{\Delta}{\rho^2}[du_\pm - a\sin^2\theta d\varphi_\pm]^2 + \frac{\sin^2\theta}{\rho^2}[(r^2 + a^2)d\varphi_\pm - adu_\pm]^2 + \rho^2 d\theta^2 \pm 2dr[du_\pm - a\sin^2\theta d\varphi_\pm].$$

(3.44)
In these coordinates, the strings $\Sigma_{\pm}$ are described by equations $\theta = \pi/2$, $\varphi_{\pm} = \text{const.}$, so that the induced metric on $\Sigma_{\pm}$ is:

$$ds^2 = -F du^2_{\pm} \pm 2dr du_{\pm}. \quad (3.45)$$

This metric for $\Sigma_{+}$ describes a black hole, while for $\Sigma_{-}$ it describes a white hole. In both cases the perturbation equations (3.20) on $\Sigma_{\pm}$ reduce to:

$$\Box \Phi^R = \frac{\Box r}{r} \Phi^R; \quad R = \perp, \parallel \quad (3.46)$$

where $\Box$ is the d’Alambertian on the world-sheet, $\Box = G^{AB} \nabla_A \nabla_B$. We also have $\Box r = F_r$.

Several interesting remarks are now in order. First notice that the perturbation equations for the two transverse polarizations $\Phi^R$ are decoupled and identical. Secondly, eq.(3.46) is precisely the $s$-wave scalar field equation in the 4-dimensional Reissner-Nordström black hole background:

$$\Box^{(4)} \phi = 0, \quad g^{(4)}_{\mu\nu} = \text{diag}(\!-F, \!1/F, \!r^2, \!r^2 \sin^2 \theta), \quad (3.47)$$

with $F$ given by eq.(3.42). The decomposition $\phi = \sum r^{-1} \Phi_l(r,t) \ Y_{lm}(\theta,\varphi)$ yields:

$$- \frac{1}{F} \partial_t^2 \Phi_l + \partial_r(F \partial_r \Phi_l) = \frac{F_r}{r} \Phi_l + \frac{l(l+1)}{r^2} \Phi_l, \quad (3.48)$$

which is identical to equation (3.46) for $l = 0$, as is most easily seen by writing eq.(3.46) in the $(\hat{\tau},r)$ world-sheet coordinate system. (Equation (3.48) has been studied in detail in the literature, see for instance [35] and references given therein.) The string exitations for a stationary string passing through the ergosphere in the equatorial plane of a Kerr-Newman black hole are therefore described mathematically by the $l = 0$ scalar waves in the background of a 4-dimensional Reissner-Nordström black hole.

The above results allow generalization to the case when a stationary string is located not in the equatorial plane but on the cone $\theta = \theta_0 \neq \pi/2$. For this
case the parameters which enter the equations (3.36) are related as $q^2 = 2|ab|$. The corresponding $\theta_0$ is determined as the minimum of the potential $U(\theta)$ and is, $\sin^2 \theta_0 = |b/a|$. This relation implies that $|b| < |a|$. The remarkable fact is that such a string allows a simple geometrical description. The Kerr-Newman metric possesses two principal null geodesic congruences, one of them is formed by incoming and the other by outgoing principal null rays \[34\]. Take one of these null geodesics $\gamma_\pm$ ($-$ stands for an outgoing and $+$ stands for an incoming ray). Consider two-dimensional surfaces $\Sigma_\pm$ formed by Killing trajectories passing through $\gamma_\pm$. It is possible to prove that $\Sigma_\pm$ is a minimal surface and it describes an "equilibrium" string configuration with the parameter $q^2 = 2|ab|$. Two string configurations $\Sigma_\pm$ differ by signs of $r'$ in (3.36). The metric induced on both surfaces $\Sigma_\pm$ possesses the Killing horizon. In case of $\Sigma_+$ it describes a black hole, while in case of $\Sigma_-$ it describes a white hole (for a white hole a future directed timelike curve crossing the Killing horizon enters the black hole exterior). The above considered case of a stationary string located on the equatorial plane and crossing the infinite-red-shift surface is a special example of stationary cone strings. The perturbation equations for cone strings have been considered in \[36\].

To summarize, we have shown that the metric induced on a stationary string crossing the infinite-red-shift surface describes a two-dimensional geometry of a black or white hole. It opens remarkable possibilities to apply results of mathematical study of two-dimensional black holes to physical objects (cosmic strings in the vicinity of a black hole), which at least in principle allow experimental observations. In particular, in the presence of the horizon on $\Sigma_+$ (for the two-dimensional string black hole) the conditions that the quantum state is regular near the horizon implies that the string perturbations $\Phi^R$ are to be thermally excited. It means that there will be a thermal flux of the string excitations ('phonons') propagating to infinity which forms
the corresponding Hawking radiation. For a radial string crossing the event horizon of the Schwarzschild black hole this effect was considered in [37]. We would like to stress that the analogous radiation will also be present when a stationary string crosses the infinite-red-shift surface (which for a rotating black hole is located outside the horizon). String perturbations ('phonons') generated in the region lying inside the 2-horizon and propagating along the string $\Sigma_+$ cannot escape to infinity. But it is well known that the causal signals emitted in the ergosphere and propagating in the 4-dimensional space-time can reach a distant observer. One can use such signals ('photons') in order to get information from the interior of the two-dimensional string black hole. This situation is similar to one which happens in a 'dumb' hole considered by Unruh [38]. In order to define a 'dumb' hole one uses the causal structure connected with the propagation of phonons. The photons propagating with supersonic velocity can escape a 'dumb' hole interior. The nice feature which differs our model is that it is constructed in the framework of completely relativistic theory.

Possibility of getting information from a black hole interior was also discussed in [39], where a gedanken experiment was proposed in which a traversable wormhole is used. The mechanism discusses in the present paper which makes it possible to extract information from the interior of string black holes is different. It is connected with the presence of extra-dimensions and does not require non-trivial topology. The possibility of the information extraction from the interior of a string black hole might give new insight to the problem of information loss in black holes. In particular the arguments of Ref. [40] being applied to string black holes indicate that black hole complementarity may be inconsistent, at least for these black hole models.
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Figure Captions

Figure 1. The potential $V(r)$, eq.(2.7), for a circular string in the equatorial plane of the four 3 + 1 dimensional spacetimes: (a) Minkowski, (b) Schwarzschild black hole, (c) anti de Sitter and (d) Schwarzschild anti de Sitter. The details are explained in the text.

Figure 2. The potential $V(r)$, eq.(2.7), for a circular string in the equatorial plane of the two 3 + 1 dimensional spacetimes: (a) de Sitter and (b) Kerr black hole. The details are explained in the text.

Figure 3. The potential $U(r)$, eq.(2.13), for a stationary string in the equatorial plane of the three 3 + 1 dimensional spacetimes: (a) Minkowski, (b) anti de Sitter and (c) Schwarzschild black hole. The details are explained in the text.

Figure 4. The potential $U(r)$, eq.(2.13), for a stationary string in the equatorial plane of de Sitter spacetime. The details are explained in the text.

Figure 5. The so-called $(N, M) = (3, 2)$ stationary string solution inside the horizon of de Sitter spacetime. Besides the circular string, this is the simplest stationary closed string configuration in de Sitter spacetime.