On the mixing strength in the two lowest $0^-$ states in $^{208}\text{Pb}$

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With a resolution of 3 keV, the two lowest $0^-$ states in $^{208}\text{Pb}$ are identified by measurements of the reaction $^{207}\text{Pb}(d, p)$ with the München Q3D magnetic spectrograph in a region where the average level spacing is 6 keV. Precise relative spectroscopic factors are determined. Matrix elements of the residual interaction among one-particle one-hole configurations in a two-level scheme are derived for the two lowest $0^-$ states in $^{208}\text{Pb}$. The off-diagonal mixing strength is determined as $105 \pm 10$ (experimental) $\pm 40$ (systematic) keV. Measurements of the reaction $^{208}\text{Pb}(p, p')$ via isobaric analog resonances in $^{209}\text{Bi}$ support the structure information obtained.

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I. INTRODUCTION

The nucleus $^{208}\text{Pb}$ offers the singular chance to study a two-level scheme in the space of shell model configurations. Below $E_x = 6.1$ MeV, only two $0^-$ states among about 120 one-particle one-hole configurations are expected from shell model calculations [1,2]. They are identified [3] but their structure is not known in detail. With the average residual interaction known from experiment [4,5] they are predicted to consist essentially of the two lowest configurations $s_{1/2}p_{1/2}$ and $d_{5/2}f_{5/2}$, since the next particle-hole configuration is ten times more distant than an average matrix element of the residual interaction among one-particle one-hole configurations (m.e.).

We took spectra of the reaction $^{207}\text{Pb}(d, p)$ at a resolution of 3 keV [6] up to $E_x = 8$ MeV and identified the two $0^-$ states in the region $E_x = 5.2 - 5.7$ MeV where the mean level distance is 6 keV.

Most of the low-lying states in $^{208}\text{Pb}$ are considered as excited states created by the coupling of exactly one particle and one hole to the ground state. We postulate that each particle-hole state is completely described as a mixture of a few particle-hole configurations. The ground state of $^{207}\text{Pb}$ is assumed to be a pure $p_{1/2}$ neutron hole state in relation to the ground state of $^{208}\text{Pb}$. In the $^{207}\text{Pb}(d, p)$ reaction, the particle-hole states in $^{208}\text{Pb}$ with spin $0^-$ are populated by $L = 0$ transfer only, whereas the $1^-$ states are populated by both $L = 0$ and $L = 2$ transfer.

For two spin $0^-$ and nine $1^-$ states below $E_x = 6.5$ MeV, relative spectroscopic factors are measured. Using the method of Ref. [4] and assuming the two lowest configurations to be almost completely contained in the two lowest $0^-$ states, matrix elements of the residual interaction between the $0^-$ configurations $s_{1/2}p_{1/2}$ and $d_{5/2}f_{5/2}$ are deduced.

Results of the inelastic proton scattering on $^{208}\text{Pb}$ via isobaric analog resonances (IAR) in $^{209}\text{Bi}$ populating the two $0^-$ states and some $1^-$ states [6,7] are discussed.

II. EXPERIMENTAL DATA

A. Experiments with the Q3D magnetic spectrograph

Using the Q3D magnetic spectrograph of the tandem accelerator of the Maier-Leibnitz laboratory at München, experiments of the reactions $^{207}\text{Pb}(d, p)$ and $^{208}\text{Pb}(p, p')$ via isobaric analog resonances in $^{209}\text{Bi}$ (IAR-pp') are performed. They are described in detail in Ref. [6]. The resolution of about 3 keV, the low background (up to 1:5000) and reliable identification of contamination lines from light nuclei (by the kinematic broadening proportional to different slit openings), and a sophisticated fit of the spectra by the computer code GASPAN [8], allow to resolve nearby levels and to detect weakly excited states. Here we refer to data obtained from the $^{207}\text{Pb}(d, p)$ experiment in the region $E_x = 5.2 - 5.7$ MeV. Compared to earlier work with a resolution of 18 keV from the Heidelberg multi-gap magnetic spectrograph [9] and following work [6,10,11,12], the resolution has been improved and the background lowered.
FIG. 1: (online: color) $^{207}$Pb($d, p$) spectrum taken at $\Theta = 30^\circ$ for $E_x = 5.23 - 5.36$ MeV. The 5280 $0^-$ state (marked •) is resolved from the two neighbors in 4-7 keV distance. It is displayed on a logarithmic scale since the background is $1/2000$ of the maximum peak, but many levels with 1% of the maximum are clearly resolved. The drawn curves show the fit by the computer code GASPAN [8], where the energies are taken from Table I and only the centroid of all energies together and the peak heights are varied. The widths and tails are interpolated from a table generated by inspection of several strong, rather isolated peaks in the whole spectrum covering about 1.2 MeV. A weak contamination line from $^{23}$Na is identified near $E_x = 5.31$ MeV.

The mean level spacing is about 6 keV in the regions near the two $0^-$ states. Peaks are identified by comparison to the known data [3, 11, 12, 13, 14, 15], see Table I. A comparison to the preliminary analysis of the $^{208}$Pb($p, p'$) data on seven IAR in $^{209}$Bi with similar resolution [6] allows to verify the identifications.

Figs. 1 and 2 show two extracts of $^{207}$Pb($d, p$) spectra, each covering 1.2 MeV totally. Whereas the neighbors of the 5599 $0^-$ state are 12-15 keV away, the 5280 $0^-$ state is surrounded by two levels in 4-7 keV distance. At scattering angles of $\Theta = 20^\circ - 30^\circ$, the 5276 and the 5287 state are excited with cross sections of 1-20% of the 5280 state.

Peaks from light contaminations ($^{12}$C, $^{14}$N, $^{16}$O, $^{23}$Na and more) are identified in the whole spectra by the kinematic shift in a series of spectra taken at scattering angles $\Theta = 20^\circ - 30^\circ$ and the kinematic broadening for different openings of the entrance slit to the Q3D magnetic spectrograph, see Ref. [9]. In the region of $E_x = 5.5 - 5.7$ MeV, contamination lines from $^{14}$N with cross sections of a few $\mu$b/sr are detected at scattering angles $\Theta = 20^\circ$ and $30^\circ$.

B. Extraction of relative spectroscopic factors

By use of the GASPAN code [8] with the option of fixed energy distances, and the excitation energies from Table I, the cross sections are precisely determined. Figs. 1 and 2 shows spectra for the regions around the 5280 $0^-$ and the 5599 $0^-$ levels. Fig. 3 shows the angular distributions for the 5280 $0^-$, 5292 $1^-$ and 5599 $0^-$ levels. For scattering angles $\Theta = 20^\circ - 30^\circ$, the cross sections differ by a constant factor (0.32 and 0.05 for the two $0^-$ states in relation to 5292 $1^-$ state) within the errors. For $\Theta = 20^\circ - 30^\circ$, DWBA calculations yield the steep slope observed for $L = 0$ in contrast to a rather flat angular distribution for $L = 2$ [11, 12].

In view of the weak cross sections at $\Theta = 20^\circ$, especially for the 5599 $0^-$ state, we determine relative spectroscopic factors by first calculating a mean angular distribution of the three states,

$$\frac{d\sigma}{d\Omega}(\Theta) = \sum_{E_x} \left\{ \frac{d\sigma}{d\Omega}(E_x, \Theta)/\sum_{\theta} \frac{d\sigma}{d\Omega}(E_x, \Theta) \right\}. \quad (1)$$

The energy dependence of the cross section is neglected because of the small energy range. In a least squares fit we then obtain the mean cross section

$$\left\langle \frac{d\sigma}{d\Omega}(E_x) \right\rangle = \sum_{\theta} \left\{ \frac{d\sigma}{d\Omega}(E_x, \Theta)/\sum_{\theta} \frac{d\sigma}{d\Omega}(\Theta) \right\} \quad (2)$$

as a measure of the relative spectroscopic factors. In Table II we adjust the mean values to the cross section of the 5292 state at the scattering angle $\Theta = 25^\circ$. 

FIG. 2: (online: color) $^{207}$Pb($d, p$) spectrum taken at $\Theta = 25^\circ$ for $E_x = 5.54 - 5.65$ MeV. The 5599 $0^-$ state (marked •) is well isolated. For other details see Fig. 1.
C. Determination of mixing amplitudes

The lowest negative parity states in $^{208}$Pb are assumed to be well described by the shell model as particle-hole states in relation to the ground state of $^{208}$Pb. Especially, the two lowest $0^-$ states $|E_x, I^>\rangle$ are excited by the admixtures of higher configurations $|s_{1/2p_{1/2}}^>\rangle$ and $|d_{5/2f_{5/2}}^>\rangle$ with admixtures of higher configurations $|C_q^>\rangle$,

$$|5280, 0^-> = t_{11} |s_{1/2p_{1/2}}^>\rangle + t_{12} |d_{5/2f_{5/2}}^>\rangle + \sum_q t_{1q} |C_q^>\rangle,$$

$$|5599, 0^-> = t_{21} |s_{1/2p_{1/2}}^>\rangle + t_{22} |d_{5/2f_{5/2}}^>\rangle + \sum_q t_{2q} |C_q^>\rangle.$$  \hspace{1cm} (3)

The $^{207}$Pb(d, p) reaction populates the $s_{1/2p_{1/2}}$ component only.

In contrast to spin $0^-$, for spin $1^-$ the shell model predicts eight states below $E_x = 6.5$ MeV. Two configurations, $s_{1/2p_{1/2}}$ and $d_{3/2p_{1/2}}$, of the identified $1^-$ states (Table III) are excited by the $^{208}$Pb(d, p) reaction. Hence the $n 1^-$ states are described by

$$|n, 1^-> = t_{11n1} |s_{1/2p_{1/2}}^>\rangle + t_{12n1} |d_{3/2p_{1/2}}^>\rangle + \sum_q t_{1qn} |C_q^>\rangle.$$  \hspace{1cm} (4)

We want to determine the matrix elements of the residual interaction between the two lowest $0^-$ configurations in $^{208}$Pb. In the truncated two-level configuration space of one-particle one-hole configurations, the matrix $t$ is only approximately unitary,

$$tt^\dagger = \begin{pmatrix} 1 - d_{11} & d_{12} \\ d_{21} & 1 - d_{22} \end{pmatrix} \approx \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$  \hspace{1cm} (5)

We postulate the deviation from unitarity to be small,

$$d = \begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix} \approx 0.$$  \hspace{1cm} (6)

Each element of the deviation matrix contains only products of the amplitudes $t_{1q}t_{2q}$ of higher configurations assumed to be weak [Eq. (3)] and the amplitudes $t_{q1}, t_{q2}$ of the configurations $s_{1/2p_{1/2}}$, $d_{3/2f_{5/2}}$ in higher excited states assumed to be weak, too.

According to the shell model without residual interaction, the two configurations $s_{1/2p_{1/2}}$ and $d_{3/2f_{5/2}}$ have the lowest excitation energies for the $1^-$ states, too. For the $1^-$ states a similar deviation matrix can be defined with elements $d_{1n1}, d_{1n2}, n = 1, 9$ referring to these two configurations.

An essential assumption is the proportionality of the sum of the strengths of the configuration $s_{1/2p_{1/2}}$ in all states for the spins $I^> = 0^+, 1^-$ to the spin factor (2$I$+1),

$$\sum_n t_{1n1}^2 = 3(t_{11}^2 + t_{21}^2 + d_{11}).$$  \hspace{1cm} (7)

We then use the observation that the configurations $s_{1/2p_{1/2}}$ and $d_{3/2f_{5/2}}$ produce angular distributions which are easily distinguished, to derive upper and lower limits of the complete $s_{1/2p_{1/2}}$ strength $\sum_n t_{1n1}^2$ in the $1^-$ states and thus derive an upper limit for the deviation matrix $|d|$ by use of Eq. (4).

Since the reaction $^{207}$Pb(d, p) excites only the $s_{1/2p_{1/2}}$ component of the $0^-$ states [Eq. (3)], the ratio of the measured mean cross sections (Table III)

$$t_{21}^2/t_{11}^2 = \left(\frac{d\sigma}{d\Omega}(5599)\right)/\left(\frac{d\sigma}{d\Omega}(5280)\right)$$  \hspace{1cm} (8)

is used to derive the amplitudes $t_{11}, t_{12}, t_{21}, t_{22}$ as

$$|t_{11}| = |t_{22}| = 0.928 \pm 0.012,$$

$$|t_{12}| = |t_{21}| = 0.37 \pm 0.04.$$  \hspace{1cm} (9)
Here the deviation matrix $d$ [Eq. (3)] is assumed to vanish.

FIG. 4: The two lowest $0^-$ configurations in $^{208}$Pb are separated from the next higher configurations by a large gap $\Delta$ allowing to discuss the simple case of a two-level configuration mixing in the $[52800^- > | 55990^- >$ states. The residual interaction is decomposed into the m.e. $v_{11}$ and $v_{22}$ describing the shift of the two levels, and the m.e. $v_{12} = v_{21}$ describing the level repulsion.

D. Completeness of the strength in the truncated configuration space

Higher $0^-$ states are not known, but they should have energies above $E_x \approx 6.8$ MeV, see Fig. 1. In contrast, nine $1^-$ states are known as predicted by the shell model.

The cross sections $\left( \frac{d\sigma}{dE_{\gamma}}(E_x) \right)$ (Table II) for the two $0^-$ states and all $1^-$ states up to $E_x = 6.5$ MeV are consistent with the data of Refs. [11, 12] within the errors. The ratios agree also with the population strengths of Ref. [3] but they are more precise.

The reaction $^{207}$Pb$(d, p)$ excites the two configurations $s_{1/2}p_{1/2}$ and $d_{5/2}p_{1/2}$ in all $1^-$ states, but only the configuration $s_{1/2}p_{1/2}$ in the $0^-$ states. The two lowest $0^-$ states contain almost the complete $s_{1/2}p_{1/2}$ $0^-$ strength by comparison to DWBA calculations [11, 12]. Because higher configurations admix little due to the gap $\Delta$ between the second and third $0^-$ configurations, $d_{5/2}f_{5/2}$ and $g_{9/2}h_{9/2}$, being larger than ten times the mean m.e., the deviation matrix $d$ almost vanishes. By comparing the detected strength of the $0^-$ and $1^-$ $s_{1/2}p_{1/2}$ configurations, we deduce an upper limit for $|d|$

The $5292 1^-$ state contains less than 90% of the $s_{1/2}p_{1/2}$ strength, since the ratio of its cross section to the sum of the two $0^-$ states is less than the ratio 3:1 expected from the spin factor $(2I + 1)$ [Eq. (7)]. Other $1^-$ states contain the remaining $s_{1/2}p_{1/2}$ strength, but the $5292 1^-$ state contains also some of the $d_{3/2}p_{1/2}$ strength (besides other configurations not detected by $^{207}$Pb$(d, p)$) but by IAR-pp'). The missing $s_{1/2}p_{1/2}$ strength is contained in the other eight $1^-$ states.

(a) All $1^-$ states except for the $5292 1^-$ state listed in Table II have rather flat angular distributions for $\Theta = 20^\circ - 30^\circ$. For the states considered, the dependence of the cross section on the energy $E_x$ for states with the same configuration mixture is negligible [11, 12]. (b) For the $5294 2^-$ and $5947 1^-$ states, the angular distribution for $\Theta = 20^\circ - 30^\circ$ is flat (similarly as for states with $d_{5/2}p_{1/2}$ strength) in contrast to the steep rise for the $s_{1/2}p_{1/2}$ configuration [11, 12]. The $5294 2^-$ and $5947 1^-$ states contain most of the $d_{3/2}p_{1/2}$ strength [11, 12] and the spin assignments are firm [3]. (c) In the $5947$ state, the comparison of the shape of the angular distribution to the $5294 2^-$ state allows to deduce an upper limit for the $s_{1/2}p_{1/2}$ strength of about 8% or a ratio $r_{2,0} = t_{2,0}^2/t_{1,1}^2 > 12$ [Eq. (11)]. (d) The deviation of the slope of the cross section for the $5292 1^-$ state in comparison to the two $0^-$ states implies up to 10% $d_{5/2}p_{1/2}$ admixture (Fig. 3). (e) For the other $1^-$ states besides the $5292$ and $5947$ states, from the comparison of the shape of the angular distribution to the $5292 1^-$ and $5294 2^-$ states the ratio $r_{2,0}$ is derived, see Table II.

Summing up thus derived upper limits of $s_{1/2}p_{1/2}$ admixtures $t_{2,0}^2/t_{1,1}^2$ to all other $1^-$ states, we derive a lower limit 80% of the $s_{1/2}p_{1/2}$ configuration in the $5292 1^-$ state.

Together with the upper limit of 90% derived before, from Eq. (7) we conclude the sum of the $s_{1/2}p_{1/2}$ strength in the $5280 0^-$ and $5599 0^-$ states to be complete within better than 97%. This yields an upper limit for the deviation matrix [Eq. (6)],

$$ d_{11} \approx d_{22} < 0.03, $$

$$ |d_{12}| \approx |d_{21}| < 0.02. $$

E. Excitation energies

From the known single particle and single hole states in the lead region [10], the lowest one-particle one-hole configurations in $^{208}$Pb with spin 0$^-$ are predicted as $\nu s_{1/2}p_{1/2}, \nu d_{5/2}f_{5/2}, \nu g_{9/2}h_{9/2}, \nu d_{5/2}p_{1/2}, \pi d_{3/2}f_{3/2}$ (the lowest proton particle-hole configuration) at $E_x = 5463, 5568, 6844, 6866, 7383$ keV, respectively, see Fig. 4. The gap $\Delta$ described by Ref. [4] between the two lowest $s_{1/2}p_{1/2}$ and $d_{5/2}f_{5/2}$ and the next configurations is 1276 keV. Since it is more than ten times higher
than the mean m.e. the mixing of the two lowest 0\(^-\) configurations in \(^{208}\)Pb represents an excellent example of a two-level scheme. The energies of the shell model configurations are derived from the single particle and single hole states in the four neighboring nuclei \([10]\), \(e^0 = \begin{pmatrix} 5463 & 0 \\ 0 & 5568 \end{pmatrix}\) keV. The experimental data yield the excitation energies of the two states, \(E = \begin{pmatrix} 5280 & 0 \\ 0 & 5599 \end{pmatrix}\) keV.

### III. RESULTS AND DISCUSSION

#### A. Determination of matrix elements of the residual interaction

\[
v = tE t^\dagger - \frac{1}{2} (tt^\dagger e^0 + e^0 tt^\dagger) + r,
\]  

(11)

The matrix elements of the residual interaction between the two lowest 0\(^-\) configurations are derived in the truncated space of the first two configurations by the method described in Ref. \([4]\),

| energy label | \(E_x\) keV | \(E_x\) keV | \(E_x\) keV | \(E_x\) keV | \(I\) | Ref. |
|--------------|------------|------------|------------|------------|-----|-----|
| this work    |            |            |            |            |     |     |
| region near \(5280 \ 0^-\) and \(5292 \ 1^-\) |
| \(5239\)     | 5239.5 ± 0.8 | 5239.35 ± 0.36 | 4\(^-\) | [6] |
| \(5241\)     | 5241 | 5240.8 ± 1.5 | 0\(^+\) | [15] |
| \(5245\)     | 5245.4 ± 0.3 | 5245.28 ± 0.06 | 5245.2 ± 0.1 | 5244.6 ± 1.0 | 3\(^-\) | [3] |
| \(5254\)     | 5254.2 ± 0.8 | 5254.16 ± 0.15 | | | |
| \(5261\)     | 5261.2 ± 0.8 | | | | |
| \(5266\)     | 5266.6 ± 0.9 | | | | |
| \(5276\)     | 5276.3 ± 0.4 | 5277.1 ± 1.5 | 4\(^-\) | [6] |
| \(5280\)     | 5280.5 ± 0.1 | 5280.32 ± 0.08 | 5280.5 ± 0.1 | 5281.3 ± 1.5 | 0\(^+\) | [3] |
| \(5287\)     | 5287.8 ± 1.9 | | | | |
| \(5292\)     | 5292.2 ± 0.1 | 5292.00 ± 0.20 | 5292.1 ± 0.1 | 5292.6 ± 1.5 | 1\(^-\) | [3] |
| \(5307\)     | 5307.6 ± 1.5 | | | | |
| \(5316\)     | 5313.0 ± 1.0 | 5317.00 ± 0.20 | | | |
| \(5317\)     | 5316.9 ± 1.5 | 5317.30 ± 0.06 | 5317.7 ± 0.6 | | | |
| \(5326\)     | 5326.9 ± 0.6 | | | | | |
| \(5339\)     | 5340.0 ± 0.9 | 5339.46 ± 0.16 | 5340.1 ± 1.5 | 8\(^+\) | [3] |
| \(5347\)     | 5347.4 ± 0.2 | 5347.15 ± 0.25 | 5348.4 ± 0.6 | 3\(^-\) | [3] |
| region near \(5599 \ 0^-\) |
| \(5548\)     | 5548.5 ± 0.4 | 5548.08 ± 0.20 | 5548.2 ± 0.1 | 5547.5 ± 1.5 | 2\(^-\) | [3] |
| \(5557\)     | 5557.2 ± 1.0 | | | | | |
| \(5565\)     | 5563.9 ± 0.3 | 5563.58 ± 0.14 | 5563.6 ± 0.1 | 5564.7 ± 0.6 | 3\(^-\), 4\(^-\) | [3] |
| \(5596\)     | 5566.00 ± 0.60 | | | | | |
| \(5572\)     | 5572.0 ± 0.8 | | | | | |
| \(5577\)     | 5579.0 ± 0.9 | | | | | |
| \(5587\)     | 5587.4 ± 1.0 | | | | | |
| \(5599\)     | 5599.8 ± 0.5 | 5599.40 ± 0.08 | 5601.7 ± 0.1 | 5599.6 ± 0.4 | 0\(^-\) | [3] |
| \(5614\)     | 5614.4 ± 1.7 | | | | | |
| \(5641\)     | 5640.7 ± 0.6 | 5641.10 ± 0.50 | 5641.4 ± 0.5 | 5649.9 ± 1.5 | (1\(^-\), 2\(^+\)) | [13, 14] |
| \(5643\)     | | | | | | |
| \(5649\)     | 5648.7 ± 0.5 | 5649.70 ± 0.28 | 5649.8 ± 0.9 | (5\(^-\)) | | |

\([10]\)\([13, 14]\)
Explicitly we have

\[ v_{11} = t_{11}^2 E_{11} + t_{12}^2 E_{22} - (t_{11}^2 + t_{12}^2) e^0_{11} + r_{11}, \]
\[ v_{22} = t_{21}^2 E_{11} + t_{22}^2 E_{22} - (t_{21}^2 + t_{22}^2) e^0_{22} + r_{22}, \]
\[ v_{12} = t_{11}t_{21} E_{11} + t_{12}t_{22} E_{22} - \frac{1}{2} (t_{11}t_{21} + t_{12}t_{22}) (e^0_{11} + e^0_{22}) + r_{12}, \]
\[ v_{21} = t_{21}t_{11} E_{11} + t_{22}t_{12} E_{22} - \frac{1}{2} (t_{21}t_{11} + t_{22}t_{12}) (e^0_{11} + e^0_{22}) + r_{21}. \]

(12)

Using Eqs. [9 10 12] we obtain the m.e.

\[ v_{11} = -140 \pm 10 \text{ (exp.)} \pm 40 \text{ (syst.)} \text{keV}; \]
\[ v_{22} = -5 \pm 10 \text{ (exp.)} \pm 40 \text{ (syst.)} \text{keV}; \]
\[ v_{12} = v_{21} = \pm (105 \pm 10) \text{ (exp.)} \pm 40 \text{ (syst.)} \text{keV}. \]

(13)
The sign of the off-diagonal terms \( v_{12}, v_{21} \) cannot be determined from our data. The diagonal terms \( v_{11}, v_{22} \) describe the level shift, the off-diagonal terms \( v_{12}, v_{21} \) the level repulsion, see Fig. [4].

The m.e. (especially the off-diagonal m.e.) agree with the mean m.e. of about 100 keV obtained from the analysis of the lowest 20 particle-hole configurations in \(^{208}\text{Pb}\), see [4 8]. The values \( v \) are compatible with theoretical calculations [1 2], but more precise.

The systematic error is well estimated for the diagonal m.e. [4] by use of the deviation matrix \( d \) [Eq. (10)]. The systematic error for the off-diagonal m.e. is estimated from the residual matrix element

\[ r_{12} = \sum_q (t_{11} E_{11} t_{1q} + t_{1q} E_{qq} t_{11}). \]

(14)

From Eqs. [5 10] we derive contributions from higher states and higher configurations to be small, \(|t_{1q}| < 0.14, |t_{q1}| < 0.14\). Shell model calculations support the assumption of statistically distributed signs for the amplitudes \( t_{1q}, t_{q1} \). So, a systematic error of the off-diagonal m.e. of about 40 keV may be assumed.

### B. Data from IAR-pp'

A preliminary analysis of the IAR-pp' data [6] is consistent with the spin assignments given in Table [1]. Especially the 5292 \( 1^{-} \), 5947 \( 1^{-} \) states are selectively excited by the \( s_{1/2}, d_{3/2}, d_{5/2} \) IAR, respectively.

In early IAR-pp' experiments [2] excitation function were measured for several multiplets with a resolution of 26 keV. The energies given by Ref. [2] derive from the calibration of IAR-pp' spectra taken with the Enge split-pole magnetic spectrograph [16]. They are about 0.13% too low [6].

Measurements of the excitation function for the unresolved \( 5280 0^{-}, 5929 1^{-} \) doublet ("5.284 MeV") show a strong excitation by the \( s_{1/2} \) IAR. A weak excitation by the \( d_{5/2} \) IAR is explained by the \( d_{5/2} f_{5/2} \) component in the \( 5280 0^{-} \) state [Eqs. (6 9)] and \( d_{5/2} f_{5/2}, d_{5/2} p_{3/2} \) components in the \( 5922 1^{-} \) state [Eq. 4].

Similarly the resolved 5924 2\(^{-}\), 5947 1\(^{-}\) doublet ("5.914 + 5.936 MeV") is dominantly excited by the \( d_{3/2} \) IAR proving the presence of about equal \( d_{3/2} p_{1/2} \) components in both states in agreement with the results from \(^{207}\text{Pb}(d, p)\). Whereas the 5924 state clearly resonates on the \( s_{1/2} \) IAR (which is explained by weak \( s_{1/2} f_{5/2} \) and \( s_{1/2} p_{3/2} \) components), the decay curve of 5947 state near the \( s_{1/2} \) IAR is smooth.

The \( d_{3/2} \) and \( s_{1/2} \) IAR are not well isolated, \( E_{res} = 16.496, 16.965 \text{ MeV}\) and \( E_{tot} = 45 \pm 5, 45 \pm 8\), respectively. Assuming isolated IAR and using the amplitudes of Eq. [13], a calculation of the cross sections for the 5280 0\(^{-}\) and 5599 0\(^{-}\) states on the \( d_{5/2} \) and \( s_{1/2} \) IAR (using the IAR parameters of Ref. [6]) roughly agrees with the measured data. An essay following Ref. [17] to describe the angular distributions by interfering IAR did not yield conclusive results essentially because of missing data at scattering angles \( \Theta < 40^\circ \).

### IV. SUMMARY

Up to \( E_x = 6.1 \text{ MeV}\), the shell model predicts about 120 one-particle one-hole states in \(^{208}\text{Pb}\) but only two states with spin 0\(^{-}\). From a measurement of the reaction \(^{207}\text{Pb}(d, p)\) at a resolution of 3 keV, we identify the two known states with spin 0\(^{-}\) among about 150 states in a region where the mean level spacing is 6 keV. Spectroscopic information for the two 0\(^{-}\) states is used to determine their structure.

Matrix elements of the residual interaction for the unique case of a two-level mixing between the two lowest 0\(^{-}\) configurations in \(^{208}\text{Pb}\) are derived with higher precision than current shell model calculations. Spectroscopic information for the nine lowest 1\(^{-}\) states is used to quantify the systematic uncertainty.

Additional data from inelastic proton scattering via IAR in \(^{209}\text{Bi}\) support the structure information obtained.

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TABLE II: Up to $E_x = 6.5$ MeV, for the two states with spin 0$^-$ (marked •) and nine states with spin 1$^-$, the mean cross section $\langle \frac{d\sigma}{d\Omega} (E_x) \rangle$ [see Eq. (2)] adjusted to reproduce the cross section at $\Theta = 25^\circ$ for the 5924 1$^-$ state is shown. Within 1–2 keV, the energy label reflects the energies $E_x$ from Refs. 3, 11, 12, 13, 14 or this work. The reaction $^{207}$Pb$(d, p)$ was measured with the same deuteron energy $E_d = 22.000$ MeV as Refs. 11, 12. In the states with spin 1$^-$, the $L = 0$ and $L = 2$ transfer excites the $s_{1/2}p_{1/2}$ and $d_{3/2}p_{1/2}$ configurations, respectively, but in the two 0$^-$ states only the $s_{1/2}p_{1/2}$ component is excited by the $L = 0$ transfer [Eqs. (3), (4)]. From the measured angular distributions, we derive the ratio $r_{2,0}$ of the strength $I^2$ for the configurations $d_{3/2}p_{1/2}$ ($L = 2$) and $s_{1/2}p_{1/2}$ ($L = 0$). Namely, the angular distribution for $L = 2$ is flat in contrast to the steep slope for $L = 0$. For the same S.F. the relative cross section at $\Theta = 25^\circ$ $\langle \frac{d\sigma}{d\Omega} (E_x) \rangle$ rates as about 1 : 0.5 for $L = 2$ to $L = 0$ 11, 12.

| $n$ | Energy label $I^\pi$ | $S_{(d,p\gamma)}$ $\times$1000 | $S_F$ $\times$1000 | $r_{2,0}$ $\langle \frac{d\sigma}{d\Omega} (E_x) \rangle$ $\mu b/sr$ |
|-----|------------------|------------------|------------------|------------------|
| 1   | 4841 1$^-$        | 11 ± 4           | >0.5             | 22 ± 5           |
| 2   | 5280 0$^-$        | 0 377 ± 32       | 0 650            | 0 250 ± 10       |
| 3   | 5512 1$^-$        | 0 1071 ± 325     | 0 1550           | <0.1 785 ± 30    |
| •   | 5599 0$^-$        | 0 60 ± 6         | >0.8 160 ± 15    |
| 4   | 5641 1$^{-1}$     | 4 2              | >0.7 22 ± 3      |
| 5   | 5947 1$^{-2}$     | 2 1266 ± 488     | 2 1390           | >12$^3$ 1300 ± 80$^3$ |
| 6   | 6263 1$^{-3}$     | 2 55 ± 23        | 2 7              | >0.6 25 ± 10     |
| 7   | 6314 1$^{-4}$     | 2 88 ± 38        | 0 113            | >0.7 38 ± 12     |
| 8   | 6360 1$^{-5}$     | 2 29 ± 13        | 2 13             | >0.7 9 ± 3       |
| 9   | 6486 1$^{-6}$     | 30$^2$           | 2 38             | >0.8 30 ± 5      |

1 $I^\pi = (1^-, 2^+)$ from Refs. 13, 14. The preliminary analysis of our data excludes spin 2$^+$.
2 Derived from the relative population strength ($S_{expt}$).
3 By comparison to the 5924 2$^-$ state with $L = 2$ only.
4 The error includes the variation of the angular distribution with $\Theta$.
5 $I^\pi = 1^-$ from Refs. 13, 14.
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