STUDY OF THE $K^+K^-$ FINAL STATE INTERACTION IN PROTON–PROTON AND ELECTRON–POSITRON COLLISIONS*

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The strength of the kaon–antikaon interaction is a crucial quantity for many physics topics. It is, for example, an important parameter in the discussion on the nature of the scalar resonances $a_0(980)$ and $f_0(980)$, in particular, for their interpretation as $K\bar{K}$ molecules. So far, one of the few possibilities to study this interaction is the kaon pair production in multi-particle exit channels such as $pp \to ppK^+K^-$. In this article, we present the latest results of the $K^+K^-$ interaction performed based on near threshold data gathered at the Cooler Synchrotron COSY. We discuss also shortly perspectives for a new measurement of the kaon–antikaon scattering length in the $e^+e^-$ collisions.

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1. Introduction

The motivation for investigating the low energy $K^+K^-$ interaction is closely connected with understanding of the nature of scalar resonances $f_0(980)$ and $a_0(980)$. Besides the interpretation as $q\bar{q}$ mesons [1], these particles were also proposed to be $qq\bar{q}\bar{q}$ tetraquark states [2], hybrid $q\bar{q}$/meson–meson systems [3] or even quark-less gluonic hadrons [4]. Since both $f_0(980)$ and $a_0(980)$ masses are very close to the sum of the $K^+$ and $K^-$ masses, they are considered also as $K\bar{K}$ bound states [5, 6]. The strength of the $K\bar{K}$ interaction is a crucial quantity regarding the formation of such molecules.

The $K^+K^-$ interaction was studied experimentally inter alia in the $pp \to ppK^+K^-$ reaction with COSY-11 and ANKE detectors operating at the COSY synchrotron in Jülich in Germany. The experimental data collected systematically below [7–9] and above [10–12] the $\phi$ meson threshold

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revealed a significant enhancement in the shape of the excitation function near the kinematical threshold. On the other hand, despite the search done by the COSY-11 experiment [8, 13] and analysis based on big data samples collected by ANKE and WASA-at-COSY experiments, there is no clear evidence of the $K^+K^-$ pairs production through the $f_0(980)$ or $a_0(980)$ resonances. The enhancement of the excitation function near the threshold may be due to the final state interaction (FSI) in the $ppK^+K^-$ system. Indeed, the differential spectra obtained by the COSY-11 [9, 14] and ANKE [10] groups indicate a strong interaction in the $pK^-$ and $ppK^-$ subsystems. The phenomenological model proposed by the ANKE Collaboration based on the factorization of the final state interaction into interactions in the $pp$ and $pK^-$ subsystems allowed to describe the experimental $pK^-$ and $ppK^-$ invariant mass distributions assuming an effective $pK^-$ scattering length $a_{pK^-} = 1.5i$ fm [10, 14]. However, the data very close to the kinematical threshold remain underestimated, which indicates that in the low energy region the influence of the $K^+K^-$ final state interaction may be significant [10, 14, 15]. Motivated by this observation, the COSY-11 Collaboration has estimated the scattering length of the $K^+K^-$ interaction based for the first time on the low energy $pp \rightarrow ppK^+K^-$ Goldhaber plot distributions measured at excess energies of $Q = 10$ MeV and 28 MeV [14].

In this article, we present preliminary results of the $K^+K^-$–FSI studies combining the Goldhaber plot distributions established by the COSY-11 group with the experimental excitation function near threshold.

2. Parametrization of the interaction in the $ppK^+K^-$ system

The final state interaction model used in the presented analysis is based on the factorization ansatz mentioned before, with an additional term describing the interaction of the $K^+K^-$ pair (the $pK^+$ interaction was neglected since it was found to be weak [10]). We have assumed that the overall enhancement factor originating from final state interaction can be factorized into enhancements in the proton–proton, the two $pK^-$ and the $K^+K^-$ subsystems

$$F_{\text{FSI}} = F_{pp}(k_1) \times F_{p_1K^-}(k_2) \times F_{p_2K^-}(k_3) \times F_{K^+K^-}(k_4),$$

where $k_j$ stands for the relative momentum of particles in the corresponding subsystem. The proton–proton scattering amplitude was taken into account using the following parametrization

$$F_{pp} = \frac{e^{i\delta_{pp}(1S_0)} \sin\delta_{pp}(1S_0)}{C k_1},$$
where \( C \) stands for the square root of the Coulomb penetration factor [16]. The parameter \( \delta_{pp}(^1S_0) \) denotes the phase shift calculated according to the modified Cini–Fubini–Stanghellini formula with the Wong–Noyes Coulomb correction [17–19]. Factors describing the enhancement originating from the \( pK^- \) and \( K^+K^- – FSI \) were instead parametrized using the scattering length approximation

\[
F_{pK^-} = \frac{1}{1 - i k a_{pK^-}}, \quad F_{K^+K^-} = \frac{1}{1 - i k_4 a_{K^+K^-}},
\]

(2)

where \( a_{K^+K^-} \) is the scattering length of the \( K^+K^- \) interaction treated as a free parameter in the analysis. Since the \( pK^- \) scattering length estimated by the ANKE group should be rather treated as an effective parameter [10], in the analysis we have used more realistic \( a_{pK^-} \) value estimated independently as a mean of all values summarized in Refs. [20, 21]: \( a_{pK^-} = (-0.65 + 0.78i) \) fm.

It has to be stressed, that within this simple model we neglect the charge-exchange interaction allowing for the \( K^0K^0 \Rightarrow K^+K^- \) transitions, and generating a significant cusp effect in the \( K^+K^- \) invariant mass spectrum near the \( K^0K^0 \) threshold [22]. However, the ANKE data can be described well without introducing the cusp effect [22], thus we neglect it in this analysis. We also cannot distinguish between the isospin \( I = 0 \) and \( I = 1 \) states of the \( K^+K^- \) system. However, as pointed out in Ref. [22], the production with \( I = 0 \) is dominant in the \( pp \rightarrow ppK^+K^- \) reaction independent of the exact values of the scattering lengths.

### 3. Fit to the experimental data

In order to estimate the strength of the \( K^+K^- \) interaction the experimental Goldhaber plots, determined at excess energies of \( Q = 10 \) MeV and \( Q = 28 \) MeV [14], were compared together with the total cross sections to the results of the Monte Carlo simulations treating the \( K^+K^- \) scattering length \( a_{K^+K^-} \) as an unknown parameter. We have constructed the following \( \chi^2 \) statistics

\[
\chi^2 (a_{K^+K^-}, \alpha) = \sum_{i=1}^{8} \frac{(\sigma^\text{exp}_i - \alpha \sigma^m_i)^2}{(\Delta \sigma^\text{exp}_i)^2} + 2 \sum_{j=1}^{2} \sum_{k=1}^{10} \left[ \beta_j N^s_{jk} - N^e_{jk} + N^e_{jk} \ln \left( \frac{N^e_{jk}}{\beta_j N^s_{jk}} \right) \right],
\]

(3)

where the first term was defined following the Neyman’s \( \chi^2 \) statistics, and accounts for the excitation function near threshold for the \( pp \rightarrow ppK^+K^- \)
reaction. $\sigma_i^{\exp}$ denotes the $i^{th}$ experimental total cross section measured with uncertainty $\Delta \sigma_i^{\exp}$ and $\sigma_i^m$ stands for the calculated total cross section normalized with a factor $\alpha$ treated as an additional parameter of the fit. $\sigma_i^m$ was calculated for each excess energy $Q$ as a phase space integral over five independent invariant masses [23]. The second term of Eq. (3) corresponds to the Poisson likelihood $\chi^2$ [24] describing goodness of the fit to the Goldhaber plots determined at excess energies $Q = 10$ MeV ($j = 1$) and $Q = 28$ MeV ($j = 2$) using COSY-11 data [14]. $N_{jk}$ denotes the number of events in the $k^{th}$ bin of the $j^{th}$ experimental Goldhaber plot, and $N_{jk}^s$ stands for the content of the same bin in the simulated distributions. The simulations were normalized with a factor defined for the $j^{th}$ excess energy as: $\beta_j = L_j\alpha\sigma_j^m/N_{j}^\text{gen}$. Here, $L_j$ stands for the total luminosity [9] and $N_{j}^\text{gen}$ denotes the total number of simulated $pp \to ppK^+K^-$ events. The $\chi^2$ distribution obtained after subtraction of its minimum value is presented in Fig. 1 as a function of the real and imaginary part of the $K^+K^-$ scattering length. The best fit to the experimental data corresponds to

$$|\text{Re} (a_{K^+K^-})| = 0.0 \pm 0.1_{\text{stat}} \pm 0.6_{\text{sys}} \text{ fm}, \quad \text{Im} (a_{K^+K^-}) = 1.1 \pm 0.3_{\text{stat}} \pm 0.9_{\text{sys}} \text{ fm}$$  \quad (4)$$

with $\chi^2/\text{ndof} = 1.87$. The statistical uncertainties were determined at the 70% C.L., taking into account that the number of fit parameters is equal to three [25]. Systematic errors due to the uncertainties in the assumed $pK^-$ scattering length were instead estimated as a maximal difference between the obtained result and the $K^+K^-$ scattering length determined using different $a_{pK^-}$ values quoted in Refs. [20, 21]. In the case of the $|\text{Re}(a_{K^+K^-})|$, the differences were negligible. The final state interaction enhancement factor

Fig. 1. $\chi^2-\chi^2_{\text{min}}$ distribution as a function of $|\text{Re}(a_{K^+K^-})|$ and $\text{Im}(a_{K^+K^-})$ (left). In the middle and right plots $\chi^2_{\text{min}}$ denotes the minimum of $\chi^2$ with respect to $|\text{Re}(a_{K^+K^-})|$ and $\text{Im}(a_{K^+K^-})$, respectively. In the figure on the left, the area of the squares is proportional to the $\chi^2-\chi^2_{\text{min}}$ value.
$|F_{K^+K^-}|^2$ in the scattering length approximation is symmetrical with respect to the sign of $\text{Re}(a_{K^+K^-})$, therefore, we have determined only its absolute value. The result of the analysis with inclusion of the interaction in the $K^+K^-$ system described in this article is shown as the solid curve in Fig. 2. One can see that it describes the experimental data over the whole energy range quite well.

Fig. 2. Excitation function for the $pp \to ppK^+K^-$ reaction. The triangle and circles represent the DISTO and ANKE measurements, respectively [10, 12]. The squares are results of the COSY-11 [7, 8, 14] measurements. The dash-dotted, dashed and solid curves represent the energy dependence obtained assuming that there is no interaction between particles, assuming the $pp$ and $pK^-$–FSI and taking into account $pp$, $pK$ and $K^+K^-$ interaction, respectively. The dashed and dash-dotted curves are normalized to the DISTO data point at $Q = 114$ MeV.

4. Summary and outlook

A combined analysis of both total and differential cross section distributions for the $pp \to ppK^+K^-$ reaction in the framework of a simple factorization ansatz allowed to estimate the $K^+K^-$ scattering length by a factor five more precise than the previous one [14]. However, the determined $a_{K^+K^-}$ value is still consistent with zero, which indicates that in the $ppK^+K^-$ system the interaction between protons and the $K^-$ meson is dominant. All studies of the $pp \to ppK^+K^-$ reaction suggest also that the resonant $K^+K^-$ pair production near threshold proceeds rather through the $\Lambda(1405)$ resonance than through scalar $a_0(980)/f_0(980)$ mesons [10].
Therefore, precise determination of the $K^+K^-$ scattering length requires less complicated final states like $K^+K^-\gamma$, where only kaons interact strongly. This final state can be studied, for example, via the $e^+e^- \rightarrow K^+K^-\gamma$ reactions with the KLOE-2 detector operating at the DAΦNE $\phi$-factory [26]. Analysis of the invariant mass distributions obtained in this reaction would allow detailed studies of the $K^+K^-\text{FSI}$, including the contribution from the production through scalar resonances. Thus, it would be a continuation of the $a_0(980)$ and $f_0(980)$ studies done so far by the KLOE Collaboration [27–30].

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