Leading-Order Gluon-Pair Production from a Space-Time Dependent Chromofield

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Abstract

We describe gluon-pair production from a space-time dependent chromofield via vacuum polarization within the framework of the background field method of QCD. The processes we consider are first order in the action. We derive the corresponding source terms for gluon pair production for situations that can be described by a 1+1 dimensional approach. Especially, we observe that within the range of applicability of our approach the principal contribution to gluon production is included. Gluon production from a space-time dependent chromofield will play an important role in the production and evolution of the quark-gluon plasma in ultra relativistic heavy-ion collisions at RHIC and LHC.

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I. INTRODUCTION

The quark-gluon plasma (QGP), a deconfined state of matter, is predicted to exist at high temperatures around 200 MeV and/or high energy densities around 2 GeV/fm$^3$ [1]. Apart from situations shortly after the big bang or inside a neutron star, heavy-ion colliders could produce the necessary conditions for its existence [2]. Even if this state of matter is then available in the laboratory, it will live for only such minute timespans and inside so small volumes that its direct detection is rendered impossible. This is the reason, why one is forced to look for indirect signatures like $J/\Psi$ suppression [3], strangeness enhancement [4,5], and electromagnetic probes such as dileptons and photons [6–9]. But, due to many uncertainties in the theoretical predictions and difficulties in the experiments, it is up to now still not possible to conclude about the existence of the QGP based on observations of these indicators. That fact is mostly due to the deficient knowledge about the space-time evolution of the parton distribution in these reactions. Initially, they are in pronounced non-equilibrium. The way, the QGP relaxes towards equilibrium has got to be studied carefully. This can in principle be achieved through the solution of a relativistic non-abelian transport equation for quarks and gluons. Herefore, it is necessary to understand the production of partons in ultra-relativistic heavy-ion collisions. There, the two colliding nuclei travel almost at the speed of light and are thus highly Lorentz contracted. When they pass through each other, a chromofield is formed between them due to the exchange of soft gluons [10–15]. This picture is the extension of the color flux-tube or string model widely used in high energy $pp$, $e^+e^-$, and $pA$ collisions [16,17]. The chromofield so formed polarizes the QCD vacuum and produces $q\bar{q}$-pairs and gluons. These partons collide with each other and proceed towards equilibrium.

In all previous studies [11,14,15], the production of the QGP is investigated by taking parton production from a constant chromofield into account (which has been studied in [18,19]). However, as seen in numerical studies [14,15], the chromofield acquires a strong space-time dependence due to a combination of such effects as expansion, background acceleration, color rotation, parton collision and parton production. In situations like these, parton production from a constant chromofield is not applicable and one has got to find the corresponding expression for a general space-time dependent field.

$e^+e^-$ production from a space-time dependent chromofield is studied by Schwinger [18]. This is equivalent to computing the probability from the amplitude $<k_1,k_2|S(1)|0>$ (denoted by $A_{el} \rightarrow e^+(k_1)e^-(k_2)$) with $S(1)$ obtained from the interaction lagrangian density of a classical field and quantized Dirac fields [20]. Due to the same structure of the interaction lagrangian density, the production of a $q\bar{q}$ pair is similar to the $e^+e^-$ case except for color factors [21]. However, the computation of the probability for the production of gluons from a space-time dependend chromofield is greater than that for the production of fermionic, massive quarks and antiquarks. That is due to the larger phase space at the disposal for particles described by the adjoint representation. Hence, for a correct study of the production and equilibration of the QGP, it is necessary to describe, how gluons are produced from a strongly variable chromofield.

In [22] we have studied for the first time, the production of gluon pairs from a space-time dependent chromofield via vacuum polarization. There, we derived corresponding source terms, discussed their range of applicability, and explicitly evaluated them for a time-dependent model field. However, the
chromofield formed at RHIC and LHC is not only time dependent, but inhomogeneous in space and time. In the pre-equilibrium stage of the evolution of the QGP it is a good approximation to consider a 1+1-dimensionally expanding system, i.e. \( A \) depends on the time \( t \) and the longitudinal coordinate \( z \). In this paper, we derive the source term for a \( t \) and \( z \) dependent chromofield. Also, for a comparison to Schwinger’s formula for fermion-pair production (see Eq. 6.33 in [18]), we evaluate the total probability for gluon-pair production from a space-time dependent chromofield. In future, we hope to use the source terms derived in this paper for a \( t \) and \( z \) dependent chromofield in a non-abelian relativistic transport equation to study the production and evolution of the QGP for situations similar to those found at RHIC and LHC.

Finally, we discuss the validity of our approach to parton production, especially in view of an application to situations found at RHIC and LHC. As our approach is perturbative, it is only capable of describing the production of particles with a momentum \( p \) larger than the product of the coupling constant \( g \) and the gauge field \( A \). This is the principal contribution to the production of particles as long as \( gA > \Lambda_{QCD} \) is satisfied. That means also that our approach breaks down as soon as the decaying field \( A \) reaches the value \( \Lambda_{QCD}/g \) from above. After this time \( t_{end} \), a non-perturbative treatment of particle production is needed which is beyond the scope of this paper. In this paper, we investigate, whether the stage of particle production is already almost over before we reach \( t_{end} \) or whether any significant contribution in the semi-hard sector is still to be expected.

The paper is organized as follows: In chapter II, we explain the setup of our method by deriving the probability for the production of gluon pairs to the lowest order in the effective action. In section III, we give the source terms for a chromofield variable in 1+1 dimensions. Chapter IV contains a brief summary.

II. PROBABILITY FOR THE PRODUCTION OF GLUON PAIRS

In this section, we will compute the probability for the production of gluon pairs from a space-time dependent chromofield via vacuum polarization. The process which contributes to the gluon pair production in leading order of the action is diagrammatically represented in Fig.(1). To evaluate these diagrams, we follow the background field method of QCD which, in a gauge invariant manner contains a classical background field and a quantum gluonic field simultaneously.

In the background field method of QCD [23,24], the gauge field \( A_\mu \) is split into two parts: \( A_\mu \rightarrow A_\mu + Q_\mu \). Here, \( A_\mu \) is a classical background field which will not be quantized and \( Q_\mu \) is a quantum field representing the gluons. As the \( A \)-field is not quantized it does not occur in the functional integral and not in the coupling to the external source \([dA] \rightarrow [dQ] \) and \( JA \rightarrow JQ \).

In this method, a so called background field gauge:

\[
G^a = \partial^\mu Q^a_\mu + gf^{abc} A_\mu^b Q^c_\mu = (D^\mu [A] Q_\mu)^a \tag{1}
\]

is chosen in a way as to make the total lagrangian \( \mathcal{L} = \mathcal{L}_G + \mathcal{L}_{GF} + \mathcal{L}_{FP} \) with the lagrangian density for the gauge fields:

\[
\mathcal{L}_G = -\frac{1}{4} F^a_{\mu\nu} [A + Q] F^{a\mu\nu} [A + Q], \tag{2}
\]

with the contribution to the total lagrangian due to gauge fixing [23]

\[
\mathcal{L}_{GF} = -\frac{1}{2} (G^a)^2, \tag{3}
\]
and the corresponding lagrangian density for the ghost fields

\[ \mathcal{L}_{FP} = - (D_{\mu} [A] \chi_{a}) (D^{\mu} [A + Q] \chi_{a}) \quad (4) \]

gauge invariant with respect to type I gauge transformations \[26\]. We evaluate the vacuum-polarization matrix element \( M = \langle k_{1} k_{2} | S^{(1)} | 0 \rangle \) where \( S^{(1)} \) contains all interaction terms of the Lagrangian density involving two \( Q \)-fields. For the diagram for the production of two gluons by coupling to the \( A \)-field once see Fig.(1a). The Feynman rule for this vertex can be obtained from that part of the lagrangian density which involves one \( A \) field and two \( Q \) fields in the corresponding part of the lagrangian density and is given by:

\[ (V_{1A})^{abcd}_{\mu \nu \lambda \rho} = = g f^{abcd} [-2g_{\mu \nu}K_{\rho} - g_{\nu \lambda}(k_{1} - k_{2})_{\mu} + 2g_{\mu \rho}K_{\nu}] \quad (5) \]

For the production of two gluons by coupling to the background field twice see Fig.(1b). The Feynman rule for this vertex can be extracted from that part of the lagrangian density which involves two \( A \) fields and two \( Q \) fields in the corresponding part of the lagrangian density and is given by:

\[ (V_{2A})^{abcd}_{\mu \nu \lambda \rho} = = -ig^{2}[f^{abc} f^{cde}(g_{\mu \lambda}g_{\nu \rho} - g_{\mu \rho}g_{\nu \lambda} + g_{\mu \nu}g_{\rho \lambda}) + f^{ade} f^{bce}(g_{\mu \nu}g_{\lambda \rho} - g_{\mu \rho}g_{\nu \lambda} - g_{\mu \lambda}g_{\nu \rho}) + f^{ace} f^{abd}(g_{\mu \nu}g_{\lambda \rho} - g_{\mu \rho}g_{\nu \lambda})] \quad (6) \]

These Feynman rules coincide with those derived in \[28\]. The amplitude for the production of gluon-pairs from the processes shown in Fig.(1) is given by:

\[ M = M_{1A} + M_{2A} \quad (7) \]

where

\[ M_{1A} = \frac{(2\pi)^{2}}{2} \int d^{4}K \delta^{(4)}(K - k_{1} - k_{2}) A^{\mu}(K) e^{b\nu}(k_{1}) e^{\rho}(k_{2})(V_{1A})^{abcd}_{\mu \nu \lambda \rho} \quad (8) \]

for the three-vertex and:

\[ M_{2A} = \frac{1}{4} \int d^{4}k_{3} d^{4}k_{4} \delta^{(4)}(k_{1} + k_{2} - k_{3} - k_{4}) A^{\mu}(k_{3}) A^{\lambda}(k_{4}) e^{b\nu}(k_{1}) e^{d\rho}(k_{2})(V_{2A})^{abcd}_{\mu \nu \lambda \rho} \quad (9) \]

for the four-vertex. Here \( A(K) \) is the Fourier transformation of the classical field \( A(x) \) given by:

\[ A(K) = \frac{1}{(2\pi)^{2}} \int d^{4}xe^{iK \cdot x} A(x). \quad (10) \]

Note that the above amplitudes include all the weight factors needed in order to retrieve the correct lagrangian density. The probability is obtained from the amplitudes by the relation:

\[ W = \sum_{\text{spin}} \int \frac{d^{3}k_{1}}{(2\pi)^{3}2K_{1}} \frac{d^{3}k_{2}}{(2\pi)^{3}2K_{2}} MM^{*}. \quad (11) \]

To obtain the correct physical gluon polarizations in the final state we use:

\[ \sum_{\text{spin}} \epsilon^{\nu}(k_{1}) \epsilon^{*\nu}(k_{1}) = \sum_{\text{spin}} \epsilon^{\nu}(k_{2}) \epsilon^{*\nu}(k_{2}) = -g^{\nu \nu} \quad (12) \]

for the spin-sum of the gluons and afterwards deduct the corresponding ghost contributions which are shown in Fig.(2). For the integration over the phase space of the gluons there is an additional factor of \( 1/2 \) because these particles (not the ghosts) in the final state are identical. Evaluating Eq.(14) for the gluons, not yet including the ghost corrections, we find:

\[ W^{A} = W_{1A,1A} + W_{1A,2A} + W_{2A,1A} + W_{2A,2A} \quad (13) \]

with:

\[ W_{1A,1A} = \frac{10}{8} \alpha_{s} \int d^{4}K \times \left[ (A^{a}(K) \cdot A^{a}(K)) K^{2} - (A^{a}(K) \cdot K)(A^{a}(K) \cdot K) \right], \quad (14) \]
\[ W_{1A,2A} = W_{2A,1A} = \frac{3i g \alpha_s}{4} \int d^4K \frac{d^4k_3}{(2\pi)^2} f^{a'd'}(\{A^a(K) \cdot A^{a'}(k_3)\}(A^{*d'}(K-k_3) \cdot K)) \] 

\[ (M^{FP})^{bd}_2 = \frac{1}{4} \int d^4k_3 d^4k_1 \times \delta^{(4)}(k_1 + k_2 - k_3 - k_4) \times A^{a\mu}(k_3) A^{\lambda\nu}(k_4) (V^{FP}_{2A})_{\mu\lambda \nu} \] 

The probability in this case is:

\[ W^{FP} = \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} (M^{FP})^{bd}_2 (M^{FP})^{*bd}_2. \] 

Evaluating Eq. (24) yields:

\[ W^{FP} = W_{1A,1A} + W^{FP}_{2A,2A} \] 

with:

\[ W^{FP}_{1A,1A} = -\frac{\alpha_s}{8} \int d^4K \times \] 

\[ \times [\{A^a(K) \cdot A^{a'}(K)\} K^2 - \{A^{a}(K) \cdot K\}(A^{a'}(K) \cdot K)] \] 

and:

\[ W^{FP}_{2A,2A} = \frac{\alpha_s g^2}{32(2\pi)^4} \int d^4K \times \] 

\[ \times (f^{a'b,c'}(K) \cdot f^{a'd,c'}(K)) \times \] 

\[ \times (f^{a'b,c'}(K) \cdot f^{a'd,c'}(K)) \times \] 

\[ \times (f^{a'b,c'}(K) \cdot f^{a'd,c'}(K)). \] 

Note that \( W^{FP}_{1A,2A} = W^{FP}_{2A,1A} = 0 \). To obtain the probability \( W \) for the production of a real \( gg \) pair from a space-time dependent classical chromofield \( A \), the result for the ghosts from Eq.(23) has to be subtracted from the result for the gluons given in Eq.(13).

III. CHROMOFIELD IN 1+1 DIMENSIONS

We now give the source term for gluon production for a \( t \) and \( z \) dependent chromofield. This 1+1 dimensional approach is likely to properly describe the very early phase of a heavy ion-collision at RHIC and LHC.
The probability for particle production and the source term are connected by:

\[
W = \int d^4x d^3k \frac{dW}{d^4x d^3k}.
\]  

(28)

Note that of any expression given for a source term throughout the paper only the real part has got to be taken into account. The source term for the production of gluon pairs already corrected with the respective ghost contribution again can be composed analogously to Eq.(13). The general expressions for the gluon source terms given in [22] in 1+1 dimensions specialize to:

\[
\begin{align*}
\frac{dW_{gg}^{A,1,1}}{d^4x d^3k} &= \frac{3g^2}{16(2\pi)^4} k^0 A^{\mu \nu}(\vec{x}_L) \times \\
\times &\int dx' e^{i\vec{k}_L(\vec{x}_L-\vec{x}_L')} A^{\alpha \beta}(\vec{x}_L') \times \\
\times &\left[ 8g_{\mu \nu} - 3(k_{\mu} k_{\nu} + k_{\mu}' k_{\nu}') - \\
&- 5(k_{\mu} k_{\nu}' + k_{\mu}' k_{\nu}) \right] E_{L}^{-\rightarrow -k_T} \times \\
&\times K_0(|\vec{k}_T| \zeta)
\end{align*}
\]

(29)

where the transverse momentum components of \(k'\) are replaced by derivatives with respect to \(i\vec{x}_T\). In principle, they can be calculated analytically, but the result is rather lengthy. Vectors with index \(L\) or \(T\) are always defined as follows:

\[
\vec{x}_L = (t, 0, 0, z) \text{ and } \vec{k}_T = (0, k_x, k_y, 0)
\]

(30)

Additionally, we define the distance \(\zeta\) as:

\[
\zeta = (- (x_L - x_L')^2)^{1/2}.
\]

(31)

The remaining contributions are:

\[
\begin{align*}
\frac{dW_{gg}^{2A,1A}}{d^4x d^3k} &= \frac{3g^3}{8(2\pi)^4} k^0 A^{\mu \nu}(\vec{x}_L) \times \\
\times &\int \frac{d^3\vec{x}_L}{\zeta} A^{\mu \nu}(\vec{x}_L) A^{\alpha \beta}(\vec{x}_L) f^{a' ac} g_{\mu \nu} \times \\
\times &e^{i\vec{k}_L(\vec{x}_L-\vec{x}_L')} K_1(|\vec{k}_T| \zeta) (\vec{k}_L - i\vec{x}_L) L
\end{align*}
\]

(32)

and

\[
\begin{align*}
\frac{dW_{gg}^{2A,2A}}{d^4x d^3k} &= \frac{g^4}{64(2\pi)^2} A^{\mu \nu}(\vec{x}_L) A^{\alpha \beta}(\vec{x}_L) \times \\
\times &\int \frac{d^2\vec{x}_L}{\zeta} A^{\mu \nu}(\vec{x}_L) A^{\alpha \beta}(\vec{x}_L) f^{a' ac} g_{\mu \nu} \times \\
\times &e^{i\vec{k}_L(\vec{x}_L-\vec{x}_L')} K_1(|\vec{k}_T| \zeta) (\vec{k}_L - i\vec{x}_L) L
\end{align*}
\]

(33)

where \(\vec{T}\) differs from \(T\) by an additional factor of 2 in the second summand. The total source term is the sum of Eq.(29), twice Eq.(32), and Eq.(33). Now, we ask the question, if the processes depicted in Figs. 1 and 2 mainly only contribute to the particle multiplicities inside the range of applicability of our approach. For this purpose we choose a special form of the field:

\[
A^{\mu}(x) = A^{a(3)}_{in} e^{-t/\tau_0} \theta(t + z) \theta(t - \tau_0) \theta(t),
\]

(34)

which is confined to the inside of the light-cone. All other components are equal to zero. Many other forms of the field could have been chosen, but this choice is inspired by a numerical study presented in [15]. We define the time \(t_f\) which \(gA\) needs to reach \(\Lambda_{QCD}\):

\[
t_f = t_0 \ln \frac{gA_{in}}{\Lambda_{QCD}}
\]

(35)

For \(t_0 = 0.5 \text{ fm}, g = 1.5, A_{in} = 1.5 \text{ GeV}, and } \Lambda_{QCD} = 150 \text{ MeV} \), \(t_f\) is approximately equal to 1.3 \text{ fm}. The values for the decay time \(t_0\), the coupling constant \(g\), and the initial magnitude of the gauge field \(A_{in}\) are rough estimates for an ultra-relativistic heavy-ion collision at LHC (see [24]). Now let us regard the accumulated density of produced particles up to a time \(t\):

\[
w = \int_0^t dt' \frac{dW}{d^3x d^3p}. \]

(36)

In our case this quantity is given by:

\[
w = \frac{g^2 A_{in}^2}{16 \pi t_0} [11(e^{-2\tau_0} - e^{-2\tau_0^2}) + \\
+ 36g^2 A_{in}^2 \tau_0^2 (e^{-4\tau_0^2} - e^{-4\tau_0^2}) \times \\
\times \theta(t + z) \theta(t - z) \theta(t)].
\]

(37)
In Fig.(3) we have plotted the accumulated density \( w \) up to the time \( t_f \) for the parameters given above and for different values of the longitudinal coordinate \( z \). One observes right away that the principal contribution to the production of particles is from times significantly smaller than \( t_f \). To quantify this, let us concentrate on the accumulated number-density \( w \) at \( t_f \) in the central region \( z = 0 \). We find:

\[
\begin{align*}
    w &= \frac{1}{16\pi t_0} [11(g^2 A_m^2 - \Lambda_{QCD}^2) + \\
        &+ 36\alpha_s^2(g^4 A_m^4 - \Lambda_{QCD}^4)].
\end{align*}
\]  

(38)

From Eq.(37) one sees that for \( gA_m >\Lambda_{QCD} \) the typical time to approach this end value of the density is of the order \( t_0 \). Which is by a factor of \( \ln(gA_m/\Lambda_{QCD}) \) smaller than the time \( t_f \). So for the set of parameters chosen here, this factor is approximately 2.7. Again regarding Fig.(3), we see that this argumentation is well justified. From these facts also arises that the produced particle density is sharply peaked in the central region (see Fig.(4)). This is because the size of the expanding system is still comparably small when most of the particles are produced.

IV. SUMMARY

To summarize, we have derived the probability for the production of a gluon pair from an arbitrary space-time dependent chromofield to the first order in the effective action within the framework of the background field method of QCD. In order to make a connection to the experimental situation, we have derived a source term for the production of gluons from a \( t \) and \( z \) dependent external chromofield. For this 1+1 dimensional description of the plasma in the pre-equilibrium stage of an ultra-relativistic heavy-ion collision, we have observed that the principal contribution to the production of gluons is included within the range of applicability of our approach. Also, for a comparison with Schwinger’s result for the total probability for the production of fermion anti-fermion pairs, we have evaluated the total probability for the production of real gluon pairs from an arbitrary space-time dependent chromofield. We plan to include the source terms derived in this paper in a non-abelian transport equation with collisions between the partons taken into account. A selfconsistent solution of this equation would serve to study the production and equilibration of the quark-gluon plasma at RHIC and LHC.

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REFERENCES

[1] See, e.g., L. McLerran and B. Svetitsky, Phys. Rev. D24 (1981) 450; L. McLerran, Phys. Rev. D36 (1987) 3291; R.V. Gavai, in Quantum Fields on the Computer, ed. M. Creutz, (World Scientific, 1992), p. 51; F. Karsch and E. Laermann, Rep. Prog. Phys. 56 (1993) 1347; M. Oevers, F. Karsch, E. Laermann and P. Schmidt, in Proc. of Lattice '97: Nucl. Phys. Proc. Suppl. 63 (1998) 394.

[2] Proceedings of 14th International Conference on Ultrarelativistic Nucleus-Nucleus Collisions (Quark Matter 99), Torino, Italy, 10-15 May 1999, Nucl. Phys. A 661 (1999).

[3] T. Matsui and H. Satz, Phys. Lett. B 178 (1986) 416.

[4] J. Rafelski and B. Mueller, Phys. Rev. Lett. 48 (1981) 1066.

[5] R. Baier and R. R"uckl, Z. Phys. C 19 (1983) 251; R. Gastmans, W. Troost and T. T. Wu, Nucl. Phys. B 291 (1987) 731.

[6] M.T. Strickland, Phys. Lett. B 331 (1994) 245.

[7] E.V. Shuryak and L. Xiong, Phys. Rev. Lett. 70 (1993) 2274.

[8] Jan-e Alam, Sibaji Raha and Bikash Sinha, Phys. Rep. 273 (1996) 243.

[9] Gouranga C. Nayak, Phys. Lett. B 442 (1995) 427.

[10] F. E. Low, Phys. Rev. D12 (1975) 163; S. Nussinov, Phys. Rev. Lett. 34 (1975) 1286.

[11] K. Kajantie and T. Matsui, Phys. Lett. B164 (1985) 373; A. Karman, T. Matsui and B. Svetitsky, Phys. Rev. Lett. 56, 219 (1986); G. Gatoff, A. K. Karman and T. Matsui, Phys. Rev. D36 (1987) 114; A. Bialas, W. Czyz, A. Dyrek and W. Florkowski, Nucl. Phys. B296 (1988) 611; B. Banerjee, R. S. Bhalerao and V. Ravishankar, Phys. Lett. B224 (1989) 16; M. Asakawa and T. Matsui, Phys. Rev. D43 (1991) 2871; K. J. Eskola and M. Gyulassy, Phys. Rev. C47 (1993) 2329; J. M. Eisenberg, Found. Phys. 27 (1997) 1213.

[12] C. D. Roberts and S. M. Schmidt, Prog. Part. Nucl. Phys. 45 Suppl. 1, (1999) 103, and references therein; D.F. Litim and C. Manuel, Phys. Rev. Lett. 82, 4981 (1999); Nucl. Phys. B562 (1999) 237; Phys. Rev. D61, 125004 (2000).

[13] Y. Kluger, J. M. Eisenberg, B. Svetitsky, F. Cooper and E. Mottola, Phys. Rev. Lett. 67 (1991) 2427; F. Cooper, J. M. Eisenberg, Y. Kluger, E. Mottola and B. Svetitsky, Phys. Rev. D 48 (1993) 190; J. M. Eisenberg, Phys. Rev. D 51 (1995) 1938; F. Cooper and E. Mottola, Phys. Rev. D 36 (1987) 3114; ibid D 40 (1989) 456; T. S. Biro, H. B. Nielsen and J. Knoll, Nucl. Phys. B 245 (1984) 449; M. Herrmann and J. Knoll, Phys. Lett. B 234 (1990) 437; D. Boyanovsky, H. J. de Vega, R. Holman, D. S. Lee and A. Singh, Phys. Rev. D 51 (1995) 4419; H. Gies, Phys. Rev. D 61 (2000) 085021.

[14] G. C. Nayak and V. Ravishankar, Phys. Rev. D55 (1997) 6877; G. C. Nayak and V. Ravishankar, Phys. Rev. C58 (1998) 356.

[15] R. S. Bhalerao and G. C. Nayak, Phys. Rev. C61, 054907 (2000).

[16] V. D. Barger and R. J. N. Phillips, Collider Physics (Addison-Wesley Publishing Company, 1987).

[17] B. Andersson, G. Gustafson, G. Ingelman and T. Sjostrand, Phys. Rept. 97 (1983) 31; B. Andersson, G. Gustafson and B. Nilsson-Almqvist, Nucl. Phys. B281 (1987) 289.

[18] J. Schwinger, Phys. Rev. 82, 664 (1951).

[19] A. Casher, H. Neuberger and S. Nussinov, Phys. Rev. D 20, 179 (1979).

[20] C. Itzykson and J. Zuber, Quantum Field Theory (McGraw-Hill Inc., 1980), R. S. Bhalerao and V. Ravishankar,
Phys. Lett. **B409**, 38 (1997).

[21] G. C. Nayak and W. Greiner, *hep-th/0001009*

[22] D. D. Dietrich, G. C. Nayak and W. Greiner, *hep-th/0007139*, accepted for publication in PRD; G. C. Nayak, D. D. Dietrich and W. Greiner, in ”Exploring Quark-Matter” proceedings of the workshops on Quark Matter in Astro-and Particle Physics Rostock (Germany) Nov 2000 and Dynamical Aspects of the QCD Phase transition, Trento (Italy) March 2001, Edited by Burau et al., *hep-ph/0104030*.

[23] B. S. DeWitt, Phys. Rev. **162**, 1195 and 1239 (1967); in Dynamic theory of groups and fields (Gordon and Breach, 1965).

[24] G. ’t Hooft, Nucl. Phys. **B62**, 444 (1973).

[25] B. W. Lee and J. Zinn-Justin, Phys. Rev. D7, 1049 (1973); H. Kluberg-Stern and J. B. Zuber, Phys. Rev. D12, 482 (1975).

[26] L. F. Abbott, Nucl. Phys. **B185**, 189 (1981).
Fig. 1 Feynman diagrams for the production of two gluons by coupling to the field $A$ once or twice.

Fig. 2 Feynman diagrams for the ghosts, corresponding to the gluon vertices.
Fig. 3 Accumulated density of particles $w$ in $GeV^3$ versus the elapsed time $t$ in $fm$ for different values of the longitudinal coordinate $z$. From top to bottom these are: $z = 0.00 fm$, $z = 0.05 fm$, $z = 0.10 fm$, $z = 0.15 fm$, $z = 0.20 fm$, and $z = 0.25 fm$. 
Fig. 4 Accumulated density of particles $w$ in GeV$^3$ versus the longitudinal coordinate $z$ in fm for different values of the elapsed time $t$ in fm. From top to bottom these are: $t = 1.3 fm$, $t = 0.2 fm$, and $t = 0.1 fm$. 