Heterogeneous Scaling of Research Institutions

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Research institutions provide the infrastructure for scientific discovery, yet their role in the production of knowledge is not well understood. To address this gap, we analyze activity of researchers and their institutions from millions of scientific papers. Our analysis reveals statistical regularities in the growth of institutions, and how collaborations, research output and its impact scale with institution size. We find that scaling is heterogeneous and time-independent. Significantly, this result is missed by cross-sectional analysis, which measures complex systems at a point in time. To help explain the findings, we create a simple statistical model and show that it reproduces the scaling patterns of collaboration and institution growth, including heterogeneous densification within a collaboration network. Our work provides policy insights to facilitate innovation and methods to infer statistical patterns of complex systems.

Despite a long tradition of bibliometric research and “science of science” (I–9), the focus has only recently shifted from individual scientists (9, 10) and teams (11–13) to how institutions affect researcher productivity and impact (14, 15). Still, the role played by research institutions — which include universities, government and industrial labs, as well as national academies
— in science is not fully understood. Improving this understanding is an important goal, as institutions not only facilitate collaboration by supporting disciplinary and interdisciplinary interactions, but also provide loci for testing new policies to improve the practice of research.

To address this knowledge gap, we analyze a large bibliographic database spanning many decades and multiple scientific disciplines. The database contains almost ten million scientific publications from which the names of authors and their affiliations have been extracted. The data allow us to study scaling laws of research collaborations, output, and impact of growing institutions over time.

We find that the relationship between all three metrics and institution size when measured longitudinally, i.e., over time for each institution, follows heterogeneous scaling laws. For example, while most institutions have a superlinear scaling law with the number of collaborations, i.e., collaborations densify (16–18), the scaling laws are different for each institution. Therefore we find that at the meso-scale (institution level), parts of the collaboration network densify at different rates. Collaborations are increasingly important to scientific innovation: they produce transformative research (13, 19) that garners more citations and appears in higher-impact venues (12). Superlinear collaboration scaling therefore suggests that increasing the number of researchers at an institution can broadly benefit its impact. This is further corroborated by our analysis of impact scaling, and find that, within each institution, impact improves as institutions grow, although some institutions benefit from their size more than others. In comparison, output is roughly linear, which implies that output per researcher is roughly a constant for each institution.

Surprisingly, we would have reached a substantially different (and incorrect) conclusion using cross-sectional analysis, a common approach to studying scaling laws that compares an outcome variable to the size of individual systems at a single point in time (20–24). Cross-sectional analysis implies that the collaboration scaling law is often sublinear, suggesting that
collaborations sparsify, i.e., the number of collaborations per researcher decreases as institutions grow. Surprisingly, exponents can even be negative, which suggests that institutions lose collaborations as they grow. This is not physically meaningful in our data.

In addition, while we uncover positive scaling laws between impact and institution size, we expect that impact is mediated by other factors. We therefore create a mixed-effect model (where the random effect is the institution) and discover that impact is directly enhanced by institution size (in agreement with (14)), but output and past collaborations, especially external collaborations, have substantially larger effects. We hypothesize that institution size indirectly improves impact by enhancing collaborations and directly improves impact via enhancing venue prestige (14, 15). The additional benefit of external collaborations strengthens previous findings (25, 26) that cross-institution teams improve impact.

Finally, we describe a statistical model that explains how institutions and collaborations grow. Despite its simplicity, the model reproduces a range of empirical observations, including the number and size of research institutions, and how pockets of increasingly dense structures form in collaboration networks. Moreover, the model shows that the disagreement between cross-sectional and longitudinal analysis arises as a consequence of different institutions following different scaling laws.

Creating an efficient and robust research system is necessary for accelerating scientific innovation (27) and educating the technical workforce (24). Our results highlight the importance of cross-institutional collaboration in producing impactful research, and identify institution size as one of the determinants of collaboration. Our work thus elucidates some of the factors that make research institutions successful, providing insights both for the macro science policy aimed at improving the research system as a whole, and also for the micro policy of individual institutions attempting to improve their prestige.
Materials and Methods

Data

We use bibliographic data from Microsoft Academic Graph (MAG)\(^1\), from which researcher names (authors), their institutional affiliations, and references made to other papers have been extracted (28, 29). A collaboration includes all coauthors of the same paper. The data enable us to measure institution size (the number of published authors affiliated with the institution), productivity (number of papers written), and collaborations (co-authors of the same paper), both within and among institutions. As a proxy for paper quality (7) we measure cumulative citations over five years, which we call impact. We gather data from papers published in four fields of study between 1800 and 2018: computer science (14,666,855 papers), physics (8,428,923 papers), math (6,192,706 papers), and sociology (4,407,288 papers). Because the metadata for MAG are extracted automatically, many papers have some missing values among extracted names, references, institution, or year published. As part of the data cleaning process, we remove papers with missing fields, and also papers with more than 25 authors. These many-authored papers only represent 0.70% of all physics papers, and < 0.036% of papers in other fields but are removed because they may be too large to constitute a meaningful collaboration between any individuals. This leaves 3,916,332 computer science papers, 2,494,000 physics papers, 2,370,712 math papers, and 1,115,841 sociology papers. Supplementary Figure 1 shows the descriptive statistics of the data, including the growth of the number of researchers, institutions, and papers published in the four disciplines, and the five largest institutions in each field. Notably, while the largest Physics, Sociology, and Math institutions are universities, the largest computer science institutions are often companies.

\(^1\)https://www.microsoft.com/en-us/research/project/microsoft-academic-graph/
Measuring Institutions and Collaborations

We define *institution size* in a given year as the number of authors who have been affiliated with that institution up until that year. Similarly, *collaborations* are defined as two researchers who have co-authored a paper up until that year. We distinguish between *internal collaborations* between co-authors at the same institution, from *external collaborations* between co-authors affiliated with different institutions.

We use cumulative statistics to reduce statistical variations and to better compare our results to a simple statistical model in which researchers do not leave an institution (allowing researchers to leave institutions adds significant complexity). To check the robustness of results, we compare to an alternate dynamic definition of institution size and collaborations in the SI. We find all qualitative results are the same, in part because both definitions are highly correlated.

Cross-sectional vs Longitudinal Analysis

We explore the relationship between collaborations and institution size using both cross-sectional and longitudinal analysis. In cross-sectional analysis, we measure the outcome variable as a function of the size of all institutions in a given year. In contrast, in longitudinal analysis we track the outcome variable as function of institution size separately for each institution over time.

Homogeneous Densification (Null) Model

To show that longitudinal scaling laws are heterogeneous, we create a null model in which the heterogeneity is only due to statistical fluctuations and compare it to data. To make this null model, we first fit each of the scaling laws for each institution. We keep the residual values, \( r_i \), and their \( x \) positions, \( x_i \), creating a set of pairs \( \{x_i, r_i\} \). The null model homogeneous scal-
null, is the average scaling laws, $\beta_i$, across all institutions, weighted by the inverse of the standard error squared, $1/\sigma^2_{\beta_i}$. For each institution, we randomly permute the residuals: 
$\{x_i, r_i\} \rightarrow \{x_i, r_j\}$, and create new data: $\{x_i, y_{null}\} = \{x_i, x_i\beta_{null} + r_j\}$. Because we randomly permute residuals, we assume the data are homoscedastic, but make no other assumptions (not even whether the residuals are normally distributed). Finally, we refit each new set of points $\{x_i, y_{null}\}$ for each institution. Due to statistical fluctuations, we should expect the new null model coefficient for each institution will fluctuate around $\beta_{null}$. To see whether the distribution of null model coefficients differs from the empirically derived coefficients, we use the Kolmogorov-Smirnov test on these two distributions (30).

Results

Figure 1 shows the network of scientific collaborations in sociology for 2017. Figure 1a shows external collaborations between researchers at different institutions, and Figure 1b shows the largest connected component of internal collaborations between researchers within one institution (Harvard). The figures hints at the richness, complexity and heterogeneity of the data. External collaborations form a giant connected component punctuated by dense clusters. The growth in institution size and number of collaborations is diverse: between 2012 and 2017 some institutions formed many new collaborations (dark links) while others barely any (light links). Similarly, some institutions grew rapidly (dark nodes) while others were more sluggish (lighter nodes). In the rest of the paper we explore institution-specific patterns that reveal how collaborations form.

Heterogeneous Scaling of Collaborations

As the first step towards characterizing the complexity of institution scaling, we measure how collaborations scale with institution size. Figure 13a shows the number of internal and exter-
Figure 1: Collaborations in sociology in 2017. (a) External collaborations and institution size. Each node represents a research institution. Institutions with more researchers are represented by larger nodes, and more collaborations are represented by thicker lines (edge weights). Darker nodes represent faster-growing institutions, and darker links represent faster-growing collaborations. Links with fewer than 10 collaborations are removed, as are isolated nodes. (b) The largest connected component of internal collaborations within Harvard University. Each node represents a researcher.

These results would seemingly imply that smaller institutions are better able to facilitate collaborations, and researchers add few new collaborations in larger institutions. The result is
misleading, however, and an artifact of cross-sectional analysis. Though it is often used in scaling analysis of cities (20), institutions (15, 24, 31), and allometry (22), cross-sectional analysis combines entities of different ages, and often gets the scaling law wrong. A similar discrepancy was observed in city scaling (32, 33), and in allometry (34). For example, combining traffic data across many cities (i.e., cross-sectional analysis) yields a qualitatively different relationship for the city size-dependent scaling of traffic delay than does looking at the evolution of traffic delay for any specific city (i.e., longitudinal analysis) (32).

To address the problem with cross-sectional analysis, we perform longitudinal analysis, tracking the growth of collaborations within each institution over time. Figure 3a–b show how the number of internal and external collaborations changes over time for several institutions from different disciplines. While each institution follows a scaling law ($R^2$ is consistently around 1, see SI), the exponents differ substantially between institutions.

To show that the scaling exponents of all institutions are different, we create a null model
Figure 3: Collaboration scaling laws for internal, and external collaborations. (a) Internal and (b) external collaborations versus institution size for 3 large institutions in each field and simulation. Insets: standard deviation of exponents are much larger than expected by a null model. Standard errors are smaller than plot markers. Exponents for (c) internal and (d) external collaborations versus institution size among all institutions. Plot points are mean scaling exponents, while shaded regions are 50% quantiles.
(see Methods) in which all institutions follow the same scaling law. The exponent of this scaling law is given by the weighted mean of all scaling exponents, with noise generated by randomly reshuffling the residuals. If this null model were accurate, the variance in the exponent would be explained purely from noise. Instead, the variance of the scaling laws across all institutions, shown in the inset, is much higher, and the distributions are substantially different (two-sample Kolmogorov-Smirnov statistic p-value is typically $<0.001$). We therefore reject the hypothesis that all the exponents are the same within statistical error.

We explore the dependence of these exponents on institution size in Fig. 3c–d. The figures show the mean scaling exponent, along with 50% quantiles for all institutions as a function of institution size (as of 2017). Unlike cross-sectional analysis, the internal and external collaboration scaling exponents are approximately constant for institutions of different size. Moreover, almost all scaling exponents are superlinear, consistent with densification of networks (16). This means that as institutions grow larger, the number of collaborations that they facilitate increases even faster. We believe these results are unlikely to be a finite size effect as the variance in exponents do not shrink considerably with institution size. The shrinkage of the exponent variance can exist because small institutions may have different “scaling laws” that settle into a single scaling law as they grow. We show an example of this in the SI. In comparison, we discover that the variance of city scaling exponents shrink with city size (32, 33), therefore while cities may well have different scaling laws, the scaling laws may be more universal as cities become larger.

**Heterogeneous Scaling of Research Output and Impact**

Are larger institutions more productive? Figure 4 shows the scaling law exponents of paper output versus institution size. The scaling exponents are centered around 1.0, although these scaling laws differ between institutions. This suggests, surprisingly, that paper output per re-
Figure 4: Probability distribution of output scaling laws for computer science, physics, math, and sociology. Gray dashed lines are the PDF of the null model in which all institutions within that field have the same scaling law.

Table 1: Mixed Effect Model of \( \log(\text{Mean Paper Impact Per Institution}) \)

| All papers | CS            | Physics       | Math          | Sociology     |
|------------|---------------|---------------|---------------|---------------|
| Const.     | 0.34 ± 0.01   | 0.28 ± 0.01   | 0.35 ± 0.01   | 0.29 ± 0.02   |
| log(Size)  | 0.069 ± 0.005 | 0.062 ± 0.004 | 0.085 ± 0.004 | 0.10 ± 0.01   |
| log(Int. Collab.) | 0.31 ± 0.01 | 0.15 ± 0.01   | 0.14 ± 0.01   | 0.18 ± 0.01   |
| log(Ext. Collab.) | 0.243 ± 0.006 | 0.244 ± 0.004 | 0.333 ± 0.005 | 0.49 ± 0.01   |
| log(Output) | 0.16 ± 0.02   | 0.26 ± 0.02   | 0.23 ± 0.02   | 0.13 ± 0.05   |
searcher is approximately independent of institution size. That being said, some institutions become more productive with size and some less, which requires further exploration in the future. Cross-sectional scaling, meanwhile, hides this rich diversity, and instead shows approximately linear scaling (we show results in more detail in the SI).

We also find that the mean paper impact scales positively with institution size, shown in the SI. We might expect this is chiefly because institution size creates more prestigious venues for conducting research (14, 15). We recall, however, that impact is affected by collaborations (12, 13, 19, 25, 26), and output (9), so to what degree are these scaling laws a direct consequence of institution size, versus a consequence of other mediator variables?

To answer this question, we created a mixed effect model for mean paper impact for each institution, shown in Table 1. The fixed effects were mean output, mean internal collaborations, and mean external collaborations per researcher as well as institution size. This corroborates previous work that finds inter-institution (external) teams may offer a greater overall benefit (25, 26). The random effect was the institution. We discover that while the coefficients for institution size are statistically significant and positive, output and collaborations, especially external collaborations, have a significantly greater relationship with impact. Overall, we hypothesize that institution size offers a small direct benefit, but indirectly improves impact by improving the ability to form collaborations.

### Institution Size Growth and Distribution

We now uncover patterns in institution growth that will help us model how collaborations form. We find that the total number of institutions grows sublinearly with the total number of researchers (Fig. 5a): as new researchers start their careers, new institutions eventually form. The number of institutions, therefore, follows Heaps’ law (35). In contrast, the distribution of institution sizes in a given year follows Zipf’s law (Fig. 5b). This is similar to the power-law-like
Figure 5: How institutions grow in time. (a) Number of institutions versus total number of researchers, and (b) the total number of researchers who were ever in each institution as of 2017 (cumulative institution size) for computer science, physics, math, and sociology.

| Field       | Heaps’ Law Exponent | Zipf’s Law Exponent |
|-------------|---------------------|---------------------|
| Comp. Sci.  | 0.554 ± 0.004       | −1.470 ± 0.005      |
| Physics     | 0.501 ± 0.007       | −1.474 ± 0.006      |
| Math        | 0.549 ± 0.008       | −1.516 ± 0.006      |
| Sociology   | 0.622 ± 0.005       | −1.603 ± 0.009      |
| Simulation  | 0.5                 | −1.5                |

Table 2: Zipf’s Law and Heaps’ Law Exponents

distribution of city sizes ([36, 37]) among other things. Exact scaling law values for each field can be found in Table 2 where we let the number of researchers, \( N > 20 \) when calculating Heaps’ law, and institution size, \( n > 10 \) when calculating Zipf’s law.

A Model of Institution Growth

Next, we create a simple statistical model of institution growth that elucidates how institutions and collaborations jointly grow. Results from the previous section hint at how collaboration scaling laws arise: researchers are hired with some preferential attachment to larger institutions (thus explaining Zipf’s law), but are hired with some probability to brand new institutions (thus explaining Heaps’ law). New researchers arrive and collaborate with existing researchers.
Larger institutions may create more internal collaborations because new workers have more researchers to collaborate with. External collaboration superscaling similarly arises because as an institution grows, all of its neighboring institutions are growing too. Each researcher has more opportunities to create external collaborations when institutions are larger compared to when they were smaller.

We model institution formation and growth with a Polya’s urn-like model described in (35) and the evolution of collaborations with a variant of a network densification model (17, 18). By combining these two models, we reproduce some of our main empirical findings. Unlike existing models of network densification (16–18), our model reproduces the heterogeneous densification of internal and external collaborations, as well as institution formation and growth with few parameters.

Adapting the model proposed by Tria et al. (35), we imagine an urn containing balls of different colors, with each color representing a different institution, as shown in Fig. 6a. We pick balls with replacement and record those colors in a sequence. A new researcher is hired by an institution when a ball with its color is chosen (uniformly at random) from the urn. Following the “reinforcement” step, the chosen ball and \( \rho \) balls of the same color are added to the urn to represent the additional resources and prestige given to a larger institution (left panel of Fig. 6a). If a new (never before seen) color is chosen, then \( \nu + 1 \) uniquely-colored balls are placed into the urn following the “triggering” step (right panel of Fig. 6a). The new colors represent the “adjacent possible” (35), which in this context represents institutions that might form now that new institutions have already formed. For example, UC Davis was spun out of UC Berkeley, and USC’s Information Sciences Institute was founded by researchers from the Rand Corporation.

This mechanism can create a broad range of scaling laws. Heaps’ law is \( N^{\nu/\rho} \) while Zipf’s law is \( n^{-(1+\nu/\rho)} \). For simplicity, we chose \( \rho = 4 \) and \( \nu = 2 \), which creates Heaps’ law equal to \( N^{1/2} \) and Zipf’s law equal to \( n^{-3/2} \), in qualitative agreement with data (Fig. 5). We also show in the
Figure 6: Schematic of our institution growth model. (a) New researchers are hired by an institution following a Polya’s Urn-like model (35). In this model, a new researcher is hired by an institution, denoted by a colored ball, picked uniformly at random from an urn. A new institution, where no researcher has been hired before, triggers $\nu + 1$ new colors to enter the urn, increasing the likelihood of more new institutions to hire a researcher. Both new and old institutions experience reinforcement, where $\rho$ balls of the same color enter the urn. This creates a rich-get-richer effect where large institutions are more likely to hire a new researcher. (b) Each institution is composed of both internal collaborators (green lines) and external collaborators (purple lines). Once a researcher is hired, they choose one random internal and one random external collaborator. New collaborations are formed independently with probability $p_A$, if hired by institution A, and $p_B$ if hired by institution B. These new connections form triangles.

SI that this model predicts the rate of institution growth is proportional to institution size (i.e., follows a preferential attachment law), which we find is approximately correct.

Other plausible mechanisms for Zipf’s law previously applied to cities (38, 39) and companies (40) could be used. Similarly, alternative mechanisms for Heaps’ law exist, although Zipf’s law needs to be assumed (41). Our simple model can explain both Heaps’ and Zipf’s laws.

Next, we explain heterogenous and superlinear scaling of collaboration through a model of network densification. In a simple model of densification (17, 18), a node enters a network and connects to an existing node uniformly at random. With an independent probability $p$,
the new node will form ties with any of their neighbor’s neighbors, thus forming triangles. We make two changes to this simple model to explain the behavior we see in institutions (see Fig. 6b). First, the newly hired researcher, represented as a node, will make a connection to a random researcher within the same institution, as well as an external researcher at a different institution picked uniformly at random (left panel of Fig. 6b). New collaborators are then chosen independently from neighbors of neighbors with probability \( p_i \), where \( p_i \) is a probability unique to each institution that hired the new researcher (right panel of Fig. 6b). We let \( p_i \) be a Gaussian distributed random variable with mean 0.6, and standard deviation 0.1 (values below zero are set to 0 and values above one are set to 1). Previous research found that for institutions of size \( n \), collaborations scale as \( n^{2p} \) across all network realizations (17, 18). We numerically find that this well approximates the scaling law for individual realizations when \( n \) is large, while scaling laws are above this limit for small \( n \) (see SI). In comparison, we prove that the parameter \( p_i \) produces different heterogenous scaling laws for external collaborations that only weakly correlates internal collaborations.

To summarize, our model has four parameters: \( \rho = 4 \), \( \nu = 2 \), mean \( p_i (\mu_p = 0.6) \), and standard deviation of \( p_i (\sigma_p = 0.1) \). Simulating the growth of institutions using the model reproduces the Heaps’ and Zipf’s laws describing the scaling of the number of institutions and their size distribution (Fig. 5 and Table 2). Simulations also produce heterogenous scaling of internal and external collaborations shown in Fig. 3. This adds more nuance to a more complex model created previously (16), becaue the collaboration model that creates pockets within the collaboration network with higher densification.

We find that this model can also produce a qualitative explanation for the cross-sectional scaling. Specifically, we show in the SI that cross-sectional internal collaboration scaling exponents produced by the model vary in time and are higher than external collaboration scaling exponents. Unlike what we see in Fig. 13 however, scaling exponents decrease in time.
Discussion

We identify strong statistical patterns of research institution growth, including scaling of collaborations with institution size. Cross-sectional analysis would suggest that the scaling exponents vary in time, that networks sparsify, and that internal and external collaborations scale very differently. We show that these patterns are incorrect. Longitudinal analysis instead allows us to correctly infer scaling laws that are heterogeneous across institutions. When patterns of institution formation and growth are incorporated into a minimal statistical model, we are able to reproduce the surprising regularity of institution formation, and the heterogeneity of collaboration densification.

Our work compliments previous studies that show cross-sectional analysis of cities scaling laws do not describe longitudinal scaling laws of individual cities (32, 33). Despite these concerns, researchers continue to use cross-sectional analysis, including to study scaling of revenue and impact of universities (15, 24, 31). Our large-scale study of scaling laws in research institutions reveals the potential harms of cross-sectional analysis in these types of datasets too. Moreover, we apply these findings to uncover and model heterogeneity in network densification, that was unobserved in previous work (16–18).

Our results uncovering patterns in institution growth also parallels previous work on the growth and death of companies (42–45). We therefore believe future work should resolve similarities and differences between companies and institutions through studying how their size and endowments fluctuate over time. Other future work includes understanding the spatial distribution of institutions, and how it relates to the population of researchers and society at large, similar to previous work on the spatial distribution of cities (41).
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**Supplementary Information**

**Cumulative vs Year-to-year Measures**

While the main text measured the cumulative size of institutions and collaborations, the findings are qualitatively the same if the growth of research institutions was measured on a year-to-year basis. Institution size is therefore the number of active authors affiliated with that institution who published in that particular year. Collaborations were similarly based on papers published that year, etc.

Figure [7] shows the growth of four academic disciplines, including (a) the number of published papers, (b) the number of institutions and (b) the number of researchers each year all increase exponentially, regardless of whether these are measured on the cumulative (all) or year-to-year basis.
Figure 7: Growth of academic disciplines. (a) Number of papers per decade and number of valid papers that contain author, references, institution, and year. (b) Number of new institutions over time. (c) Number of new researchers over time. (d) Largest institutions in 2017. C.N.R.S. stands for Centre national de la recherche and U.F.R.G.S. stands for Universidade Federal do Rio Grande do Sul. scientifique

Figure 8: Cumulative versus year-to-year data. (a) Cumulative versus year-to-year institution size, (b) cumulative versus year-to-year internal collaborations, and (c) cumulative versus year-to-year external collaborations.

| Table 3: Cumulative Vs. year-to-year Spearman Correlations |
|-----------------|-----------------|-----------------|-----------------|
| Data            | Size            | Internal Collab. | External Collab. |
| CS              | 0.85            | 0.83             | 0.83             |
| Physics         | 0.85            | 0.84             | 0.84             |
| Math            | 0.84            | 0.82             | 0.82             |
| Sociology       | 0.81            | 0.71             | 0.71             |

Figure 8 further demonstrates the robustness of our results, regardless of how they are measured. This figure shows that the cumulative institution size, cumulative number of internal and external collaborations are well correlated with their year-to-year values. The correlations are
in Table 3, where Spearman correlations are 0.7–0.8 or higher. Comparison between yearly and cumulative results can also be seen in Fig. 9 where we show cross-sectional collaboration scaling for researchers active in 2017 as well as cumulative collaboration scaling for cumulative institution size.

Figure 9: Cross-sectional analysis of the scaling of collaborations. (a) Cumulative internal collaborators and (b) cumulative external collaborators versus final institution size, \( n \), for physics in 2017.

Figure 10 reproduces Fig. 4 in the main text, except we calculate the number of researchers, institutions, and institution size per year. Figure 10a shows the number of institutions in a given year versus the number of researchers in a given year. We see, much like in the main text, a sub-linear scaling between the number of institutions and researchers. Figure 10b shows the institution size distribution. Importantly, the institution size distribution might change over time, therefore we plotted the institution size per year for 1970–1972, 1990–1992, and 2010–2012, and found the distribution was extremely stable in time and across fields.

Figure 11 shows the year-to-year collaboration scaling exponents versus the largest institution size, as well as the quality of the linear model fits (\( R^2 \)). We see, much like the main text, a large variance in the exponent values, but that they do not significantly change with institution size. That said, the scaling law \( R^2 \) is higher for large institutions in agreement with the expectation that the scaling law works best in the large-\( n \) limit of institution sizes. This is also similar to what was found in the main text for cumulative sizes.
Figure 10: Institution growth statistics by year. (a) Number of institutions versus number of researchers each year for computer science, physics, math, and sociology. (b) Institution size distribution for 1970–1972, 1990–1992, and 2010–2012 for the same fields.

Figure 11: Parameter fits versus final institution size per year. (a) Internal and (b) external collaboration scaling exponents versus final institution size for collaborations per year. $R^2$ is lower for smaller institutions but quickly approaches 1.0 for (c) internal and (d) external collaboration per year. Compare to Fig. 3 in the main text.
Finally, we show null models, described in the main text, versus the largest institution size for both cumulative and year-to-year collaborative scaling (Fig. 12). We find that scaling exponents vary much more than the null model would predict. This is consistent with our simulations, shown in Fig. 26. In this figure, we run the simulation 15 separate times, which creates 15 different null-model exponents, shown as stratified flat lines in the figure. In a given run, exponents would vary significantly, while the null model will have almost no variance due to the lack of noise in our simulations.
Comparison Between Data and Simulations

For the rest of the SI, we will just discuss cumulative results. Figure 13a is similar to the main text by showing that cumulative cross-scaling exponents vary, but here the x-axis is the total number of researchers who have authored a paper up until that date. This slightly unusual x-axis allows us to compare these results to cross-sectional scaling in simulations shown in Fig. 13b. Parameters in some of these simulations are the same as the main text, with $\nu = 2$, $\rho = 4$, $\mu_p = 0.6$, and $\sigma_p = 0.1$. For these parameters, we discover that, much like in the data, internal scaling laws are higher than external scaling laws, even though, for each institution, both should be centered around the horizontal lines labeled “longitudinal” (which corresponds to the mean values in the longitudinal analysis. We also notice that, like the data, the exponents vary as a function of the total number of researchers. These simulations do not just allow us to reproduce results, however, but we can make contrapositive hypotheses. For example, what would the statistics look like if there was no statistical variation in the longitudinal scaling laws? To better understand this, we let $\sigma_p = 0.0$, and discover that the results are quantitatively almost exactly the same. If institutions had the exact same scaling laws and the exact same constant coefficients, then the cross-sectional and longitudinal scaling laws would be the same. These discrepancies point to either finite size effects or different constant coefficients are dominant factors in explaining why cross-sectional scaling and longitudinal scaling laws differ and vary in time, at least in simulations. We hypothesize similar effects in empirical data as well, although there are qualitative differences in data, such as scaling laws increasing rather than decreasing which point to limitations of the simulations.

Focusing on longitudinal scaling, however, we observe in Fig. 14 that the data and simulations are both well-characterized by linear relations in log-log space. Namely, the figure shows that a linear fit of log(collaborations) versus log(institution size) have an $R^2$ value of nearly 1 for each institute. If their final size as of 2017 is large, then $R^2$ is even closer to 1, in agreement
Figure 13: Cross-sectional analysis of the scaling of collaborations. (a) Internal collaboration and external collaboration scaling exponents versus the cumulative number of researchers. Error bars are standard errors. (b) Simulation cross-institution collaboration scaling versus the number of researchers. Black lines: internal collaboration, red dashed lines: external collaboration. Shaded regions are 95% confidence intervals in the mean across different simulation realizations.

with what we should expect in the thermodynamic limit (where finite size effects are negligible). The data is therefore well-characterized as a power law, but these power law values vary between institutions, as shown in Fig. 3 of the main text.

Figure 14: Quality of scaling law fits of (a) internal and (b) external collaborations. The $R^2$ metric of log-log fits averaged for all institutions of size $n$ in 2017 quickly approaches 1.0 as $n$ grows. Error bars are standard errors.

**Impact Scaling**

We analyze how impact scales with institution size. Alike to collaboration scaling results, we consistently find that impact scaling differs qualitatively between cross-sectional and longitudinal analysis. While size is measured cumulatively, impact is not; impact is always the mean
Figure 15: Cross-sectional impact versus size. (a) We appear to find impact scales negatively with institution size. This is in contrast to longitudinal impact scaling (Fig. 16). (b) Partly, this is because we cannot determine scaling laws when impact is 0, which creates a U-like shape in the impact versus size plot (shown here for computer science in 2017).

cumulative number of citations among all papers published that year by authors affiliated with a particular institution. This helps explain the wider variance. Figure 15a shows that the cross-sectional scaling laws vary in time for all fields studied, and they are often negative, which suggests that small institutes have higher impact papers than larger institutes. The bizarre behavior can be better understood in Fig. 15b where we find that, for CS in 2017, there is no real scaling law to begin with. The impact is higher for small sizes in part because we have to remove zero-impact institutions, whose values in log-scale are undefined.

In contrast, we show longitudinal scaling in Fig. 16. We see that impact consistently increases with institution size among the many institutions we observe, as seen in Fig. 16. This figure shows that, broadly speaking impact scales positively with institution size. That being said, the data is very noisy, so we cannot be absolutely certain that the scaling relation is a power law.

**Correlates of Impact**

Given the scaling laws discussed before, we see that impact is positively correlated with institution size. Why is that? Is it directly because of institution size or are there mediator variables? To answer these questions we look at four variables that mediate scientific impact across all
Figure 16: Example plots of longitudinal impact versus size for computer science, physics, math, and sociology.

Figure 17: Longitudinal impact versus size. We find impact increases with institution size for (a) computer science, (b) physics, (c) math, and (d) sociology. Compare to Fig. 15.
four fields studied: output, institution size, internal and external collaborations. In all our results, impact is defined as the mean cumulative number of citations five years after publication for each paper with an author in the institution. Output and collaboration are defined as the mean cumulative number of papers or authors, respectively, for each researcher within that institution.

We explore the relative effects of size, output, and collaboration through non-parametric relationships and a mixed-effect linear model: \( \log(\text{Impact}) \sim \log(\text{Size}) + \log(\text{Int.Collab.}) + \log(\text{Ext. Collab.}) + \log(\text{Output}) + \text{Institution} \) The fixed effects are size, output and collaborations while institution is a random effect. We fit the model for each institution in each respective field (see Table for the number of datapoints and groups), where we remove data for which there is only one datapoint or all data is after 2012 in order to have 5 years cumulative impact for each datapoint. Because the number of researchers working on a given paper can significantly boost the paper’s impact (12, 26), we separately create a model for one-author papers, which removes this confounder. Within each field (Tables S1, S2, S3, & S4), impact correlates positively with institution size alike to (15), but collaboration and output are substantially larger correlates. The relative impact of these correlates has not been tested previously. Generally, except for computer science, external collaboration benefits institution impact more than internal collaboration, consistent with previous work (26), although these findings imply that collaborating in the past, even when writing one-author papers substantially improves collaborations. Thus, past interactions have longer-term benefits than the paper being written. Similarly, the relationship between output and impact is consistent with what has been observed before (9). Overall, we find that size does not, in and of itself, strongly correlate with impact. Instead, it is the mediator variables that strongly correlate.

Because impact, size, collaborations, and output all have a heavy tail, we are forced to make the model proportional to the log of each feature, which meant discarding many data points in which values are zero (because \( \log(0) \) is undefined).
To address this concern, we separately determine the Spearman rank correlation between impact and internal collaborators and external collaborators for each researcher, binned by institution size (Fig. 23). In almost all cases, $P < 0.01$. In agreement with the linear model, there is a weak, although positive, correlation between internal collaboration and impact, especially for small institutions (Fig. 23a, c). In comparison, there is a much stronger correlation between external collaboration and impact (Fig. 23b, d). That said, the correlation falls slightly or asymptotes while the correlation between internal collaboration and impact generally increases with institution size. In fact, for CS, we find a higher correlation between impact and internal collaboration than impact and external collaboration. This is not captured by the linear model, because there are relatively few institutions that are large.

We also explore the correlation between institution size and impact, controlling for the total number of collaborators for each researcher. As before, for almost all points, $P < 0.01$. In agreement with the mixed effect model, we find that size alone has a weak effect on impact. While this effect is mostly positive for researchers with few collaborations, this can be a negligible or negative effect for researchers with many collaborations. The benefit of large institutions is therefore the collaborations they provide, rather than the size itself, although the size usually has some beneficial impact (15). Researchers with many collaborations, who tend to be successful already, are unlikely to benefit significantly from institution sizes. In comparison, young faculty, or those with few collaborators in a field, find a strong correlation, and potentially a strong benefit from larger institutions.

**Output Scaling**

Although we see strong scaling laws for collaboration with respect to institution size, we wondered if similar results exist for output. Previous work found a relationship between output and impact (9), therefore we may better understand the benefit of institution size if we study
### Table 4: Impact Mixed Effect Models (CS)

| All papers | Estimate | Standard Error | Z-Score | P-Value |
|------------|----------|----------------|---------|---------|
| Const.     | 0.343    | 0.010          | 33.38   | < 0.001 |
| log(Size)  | 0.069    | 0.005          | 14.36   | < 0.001 |
| log(Int. Collab.) | 0.306 | 0.010          | 31.23   | < 0.001 |
| log(Ext. Collab.) | 0.243 | 0.006          | 40.06   | < 0.001 |
| log(Output) | 0.164   | 0.020          | 8.14    | < 0.001 |

| One-author | Estimate | Standard Error | t-Statistic | P-Value |
|------------|----------|----------------|-------------|---------|
| Const.     | 0.419    | 0.011          | 38.10       | < 0.001 |
| log(Size)  | 0.022    | 0.005          | 4.31        | < 0.001 |
| log(Int. Collab.) | 0.336 | 0.011          | 31.65       | < 0.001 |
| log(Ext. Collab.) | 0.250 | 0.007          | 38.08       | < 0.001 |
| log(Output) | 0.162   | 0.022          | 7.40        | < 0.001 |

### Table 5: Impact Mixed Effect Models (Physics)

| All papers | Estimate | Standard Error | Z-Score | P-Value |
|------------|----------|----------------|---------|---------|
| Const.     | 0.281    | 0.009          | 32.79   | < 0.001 |
| log(Size)  | 0.062    | 0.004          | 14.90   | < 0.001 |
| log(Int. Collab.) | 0.152 | 0.009          | 17.16   | < 0.001 |
| log(Ext. Collab.) | 0.244 | 0.004          | 57.51   | < 0.001 |
| log(Output) | 0.264   | 0.015          | 17.39   | < 0.001 |

| One-author | Estimate | Standard Error | Z-Score | P-Value |
|------------|----------|----------------|---------|---------|
| Const.     | 0.305    | 0.009          | 32.94   | < 0.001 |
| 3 log(Size) | 0.048  | 0.005          | 10.64   | < 0.001 |
| log(Int. Collab.) | 0.152 | 0.010          | 15.50   | < 0.001 |
| log(Ext. Collab.) | 0.265 | 0.005          | 56.70   | < 0.001 |
| log(Output) | 0.281   | 0.017          | 16.83   | < 0.001 |
### Table 6: Impact Mixed Effect Models (Math)

|                      | All papers |          |          |          |          |
|----------------------|------------|----------|----------|----------|----------|
|                      | Estimate   | Standard Error | Z-Score | P-Value  |
| Const.               | 0.349      | 0.009    | 39.74    | < 0.001  |
| log(Size)            | 0.085      | 0.004    | 21.31    | < 0.001  |
| log(Int. Collab.)    | 0.138      | 0.008    | 18.25    | < 0.001  |
| log(Ext. Collab.)    | 0.333      | 0.005    | 62.04    | < 0.001  |
| log(Output)          | 0.233      | 0.016    | 14.61    | < 0.001  |

|                      | One-author |          |          |          |          |
|----------------------|------------|----------|----------|----------|----------|
|                      | Estimate   | Standard Error | Z-Score | P-Value  |
| Const.               | 0.407      | 0.009    | 44.79    | < 0.001  |
| log(Size)            | 0.067      | 0.004    | 16.24    | < 0.001  |
| log(Int. Collab.)    | 0.172      | 0.008    | 21.71    | < 0.001  |
| log(Ext. Collab.)    | 0.343      | 0.006    | 61.30    | < 0.001  |
| log(Output)          | 0.191      | 0.017    | 11.57    | < 0.001  |

### Table 7: Impact Mixed Effect Models (Sociology)

|                      | All papers |          |          |          |          |
|----------------------|------------|----------|----------|----------|----------|
|                      | Estimate   | Standard Error | Z-Score | P-Value  |
| Const.               | 0.293      | 0.019    | 15.44    | < 0.001  |
| log(Size)            | 0.102      | 0.009    | 11.91    | < 0.001  |
| log(Int. Collab.)    | 0.181      | 0.011    | 16.64    | < 0.001  |
| log(Ext. Collab.)    | 0.494      | 0.010    | 49.85    | < 0.001  |
| log(Output)          | 0.132      | 0.045    | 2.98     | 0.003    |

|                      | One-author |          |          |          |          |
|----------------------|------------|----------|----------|----------|----------|
|                      | Estimate   | Standard Error | Z-Score | P-Value  |
| Const.               | 0.331      | 0.019    | 17.36    | < 0.001  |
| log(Size)            | 0.078      | 0.009    | 8.96     | < 0.001  |
| log(Int. Collab.)    | 0.176      | 0.011    | 15.93    | < 0.001  |
| log(Ext. Collab.)    | 0.500      | 0.010    | 49.76    | < 0.001  |
| log(Output)          | 0.085      | 0.045    | 1.88     | 0.060    |

### Table 8: Statistics of the Impact Model Fit

| Data     | Datapoints | Groups | Group Var (All) $\times 10^{-2}$ | Group Var (One-Author) $\times 10^{-2}$ |
|----------|------------|--------|----------------------------------|------------------------------------------|
| CS       | 68550      | 7598   | 9.8 ± 0.6                        | 9.4 ± 0.6                                 |
| Physics  | 73180      | 5681   | 4.8 ± 0.4                        | 4.2 ± 0.3                                 |
| Math     | 72886      | 6186   | 3.8 ± 0.3                        | 3.5 ± 0.3                                 |
| Sociology| 30648      | 2944   | 6.8 ± 0.8                        | 6.4 ± 0.7                                 |
how output scales with size. We define output to be the cumulative number of papers written. Cross-sectional scaling laws were found to be surprisingly stable in time, with a value of almost exactly 1.0, as shown in Fig. 18. This means that the output per person is independent of institution size. This holds true when we look at different the different fields we study, as well as if we look at output per year or cumulative output. This is in strong contrast to cross-sectional scaling laws of collaboration. We explore the scaling laws in longitudinal data as well in Fig. 19. These results also show approximately linear scaling relationships. That said, we compare the longitudinal data to a homogeneous scaling null model. We find that institutions have a greater variance in their scaling laws than the null models would predict. This means that some institutions create slightly more papers per person as the institution grows, while others show a reduction in output. The overall effect, however, appears to be subtle. Overall, institution size appears to affect collaborations much more than output.

Figure 18: Paper output scaling over time across institutions. (a) Paper output per year and (b) cumulative paper output. We see that the number of papers scales linearly with institution size regardless of the field.

Collaboration Scaling and Correlations

We show correlations between internal and external collaboration exponents in cross-section data and longitudinal data. In Fig. 20, we see a strong correlation between the internal and external collaboration exponents in cross-sectional data. Each field has Spearman rank correla-
Figure 19: Histogram of paper output scaling laws for each institution for (a–b) computer science, (c–d) physics, (e–f) math, and (g–h) sociology. (a, c, e, and g) paper output per year scaling. (b, d, f, and h) Cumulative paper output scaling. Green bars are null models, while data are red bars.

...tions with \( s > 0.88 \) (\( P < 10^{-6} \)), suggesting strong correlations between these two exponents. This is in contrast to simulations, where among 10 simulations, the average value of \( s \) is -0.31 (range is between -0.83 and 0.28, \( P < 10^{-6} \)). An example of a plot of internal and external collaboration exponents is seen in Fig. 21. That said, both simulations and data show strong differences between cross-sectional and longitudinal data exponents.

For cumulative collaboration, scaling exponents in longitudinal data, computer science has \( s = -0.04 \) (\( P = 0.77 \)); physics, \( s = 0.271 \) (\( P < 10^{-5} \)); math, \( s = 0.11 \) (\( P = 0.22 \)); and sociology, \( s = 0.367 \) (\( P = 0.004 \)). For year-to-year collaboration scaling exponents in longitudinal data, computer science has \( s = -0.04 \) (\( P = 0.92 \)); physics, \( s = 0.05 \) (\( P = 0.57 \));
Figure 20: Correlation between internal and external collaboration exponents for computer science, physics, math, and sociology. (a) Internal collaboration per year exponents versus external collaboration per year exponents, and (b) cumulative internal collaboration exponents versus cumulative external collaboration exponents.

Figure 21: Example of simulated cross-institute correlation between internal and external collaboration scaling coefficient. Spearman correlation, s, is -0.342.

math, $s = 0.397$ ($P = 0.022$); sociology, $s = 0.639$ ($P = 0.01$). Correlations in real data are either negligible or comparable to the simulations shown in Fig. 22. This figure shows $s = 0.33$ (95% confidence interval [0.27–0.39]), while the null model shows $s = 0.01$ (95% confidence interval [-0.03–0.05]). Overall, longitudinal scaling shows correlations are much lower than cross-sectional scaling correlations would suggest.

**Limitations of Cross-sectional Analysis**

To give some intuition about the limitations of cross-sectional analysis, let’s assume we have new institutions that appear at each time step, $t$, and grow at rate 1 ($n \rightarrow n + 1$). Furthermore, let’s assume each institution, $i$, has some quantity $y_i$ such that $y_i = \alpha(t_i)n^\beta$. The constant factor $\alpha$ depends on when the institution first formed ($t_i$), but the scaling exponent, $\beta$, is the same across institutions. What happens, however, if we look at cross-sectional data? If $\alpha(t_i)$
Figure 22: Simulation collaboration correlations over time. We find the Spearman rank correlation in our simulations are significantly higher than null models. Mean correlation for the simulation is 0.33 (95% confidence interval [0.27–0.39]). Mean correlation for the null model is 0.01 (95% confidence interval [-0.03–0.05]).

were a constant, then cross-sectional and longitudinal analysis would reveal the same pattern. If, instead, \( \alpha(t_i) = \alpha t_i = \alpha(t-n) \), the ergodicity assumption is broken. In that case, \( y_i(t) = \alpha(t-n)n^\beta = \alpha tn^\beta - \alpha n^{\beta+1} \). While each institution really scales as \( \sim n^b \), cross-sectional analysis suggests an entirely different —and wrong—relationship that depends on time, \( \sim (t-n)n^\beta \).

While there might be motivations to record institution size and collaborations on an annual basis rather than cumulatively over time, we find that all results in our paper are qualitatively and quantitatively similar. Because we measure collaborations using paper coauthorship, we checked if the number of papers written per researcher changes substantially with institution size. We find, however, that the number of papers written is approximately constant (see Output Exponents above), therefore changes in collaborations have more to do with interactions rather than the number of papers written. This suggests researchers have a cognitive or time constraint that limits the number of papers they can feasibly write regardless of institution size.

**Simulation Theory**

How are the scaling laws for internal and external collaborators in our model (see main text for model description) expected to relate to institution size? And why are the scaling laws poorly correlated with each other? We begin with the internal collaborations. Because our
mechanism to form internal collaborations ignores all nodes and links besides those within the institution itself, we can consider the institution’s internal collaboration network as an isolated network. The mechanism to make and increase collaborations within this network can therefore be reduced to that of a previous set of papers (17, 18). The number of internal collaborations, \( L_{\text{int}} \) increases with institution size, \( n \) via the following formula

\[
L_{\text{int}}(n + 1) = L_{\text{int}}(n) + 1 + p(k_{\text{int}}) \tag{1}
\]

\[
= L_{\text{int}}(n) + 1 + 2pL_{\text{int}}(n)/n \tag{2}
\]
Figure 24: Mean degree of external institutions versus the cumulative number of researchers for several simulations. Solid black line are simulations with $\sigma_p = 0.1$, dashed black line are simulations with $\sigma_p = 0.0$. Solid red line is finite $\sigma_p = 0.1$ theory, and dashed red line is $\sigma_p = 0.0$ theory, shown in Eq. 18. Shaded areas are 95% confidence intervals of the mean.

where $\langle k_{\text{int}} \rangle$ is the mean number of internal collaborations per researcher, equal to $2L_{\text{int}}(n)/n$. Intuitively, we add an edge by default, plus $p\langle k_{\text{int}} \rangle$ edges through additional collaborations. Using the results from previous papers (17, 18), we find that

$$L_{\text{int}}(n) = \begin{cases} \frac{n}{1-2p} & p < 1/2 \\ n\ln(n) & p = 1/2 \\ A(p)n^{2p} & p > 1/2 \end{cases}$$

where $A(p) = [(2p - 1)\Gamma(1 + 2p)]^{-1}$. The scaling constants and exponents in this theory are taken across all realizations. In practice, however, the exponent works well for when large institutes, and underestimates the exponent for small institutes, most likely because of finite size effects.

External scaling laws are much more nuanced. Note that we have two goals. First, we want to theoretically show that internal and external collaboration exponents are superlinear. Second, we want to understand why internal and external collaboration exponents are poorly correlated. To this end, we start with a similar equation as before, but this time for external collaborations,
$L_{\text{ext}}$:

\[
L_{\text{ext}}(n + 1) = L_{\text{ext}}(n) + 1 + p\langle k_{\text{ext}}\rangle
\]  

Our goal is to first find $\langle k_{\text{ext}}\rangle$, mean number of internal collaborations per researcher at external institutions. This value is surprisingly non-trivial compared to internal collaborations. First, we note that the first researcher is chosen at random among all researchers, meaning there is a preference to attach to researchers in larger institutes. While the institution size follows Zipf’s law (35),

\[
p(n) = \frac{\nu}{\rho} n^{-(1+\nu/\rho)},
\]

where we take the discrete size $n$ to be continuous, which works well for large institution sizes. The preference to attach to larger institutes means that we choose an institute of size $n_{\text{ext}}$ with probability

\[
q(n) = \frac{np(n)}{\langle n \rangle}
\]

where

\[
\langle n \rangle = \int_{1}^{N} dn \frac{\nu}{\rho} n^{-(1+\nu/\rho)}
\]

\[
\approx \frac{\nu}{\rho - \nu} N^{1-\nu/\rho}
\]

Because $\nu/\rho < 1$, we discover that $\langle n \rangle$ diverges. Therefore, we set of cut-off equal to the total number of researchers, $N$. In full form, $q(n)$ is:

\[
q(n) = \frac{(\rho - \nu)n^{-\nu/\rho}}{\rho N^{1-\nu/\rho}}
\]

Moreover, by construct, we have the probability of $p$, $f(p)$, be Gaussian distributed, with mean $\mu_p$ and variance $\sigma_p^2$. Finally, $k_{\text{ext}}$ for an arbitrary institution is $2L_{\text{int}}(n)/n$. Putting all this together, we discover that
\langle k_{\text{ext}} \rangle = \frac{2(\rho - \nu)}{\sigma \sqrt{2\pi \rho N^{1-\nu/\rho}}} \int_1^N dn \left\{ n^{-\nu/\rho} \int_0^{1/2} dp \frac{\exp[-(p - \mu_p)^2/(2\sigma_p^2)]}{1 - 2p} + \int_1^{1/2} dp \frac{n^{2p} \exp[-(p - \mu_p)^2/(2\sigma_p^2)]}{(2p - 1)\Gamma(1 + 2p)} \right\} 

(10)

\left\{ \int_0^{1/2} dp \frac{n^{2p} \exp[-(p - \mu_p)^2/(2\sigma_p^2)]}{(2p - 1)\Gamma(1 + 2p)} \right\} 

(11)

Sadly, this equation is not simple to solve. First, of course, it diverges near \( p = 1/2 \). At this special point, the scaling law approaches \( L_{\text{int}}(n) \sim n \ln(n) \), which is why the assumptions around \( p \approx 1/2 \) break down. If we take the ends of the integrals to be 0 to \( 1/2 - \epsilon \) and \( 1/2 + \epsilon \) to 1, then \( \langle k_{\text{ext}} \rangle \) becomes a constant proportional to \( \ln(1/\epsilon) \). If this is small compared to \( N \) then one can prove that \( L_{\text{ext}}(n) \sim n \), which does not agree with our findings. On the other hand, \( \ln(1/\epsilon) \) cannot be larger than \( N \) (i.e., we can only connect to as many as nodes as there are in the network). If we assume \( \langle k_{\text{ext}} \rangle \sim N \), then it can be shown that \( L_{\text{ext}}(n) \sim n^2 \). This demonstrates a breakdown in the assumptions of this theory. That being said, we can make perturbative expansions around \( \mu_p \) assuming \( \sigma_p \) is small. In this limit, \( \exp[-(p - \mu_p)^2/(2\sigma_p^2)] \) approaches zero faster than \( 1/(2p - 1) \) approaches infinity, therefore we can integrate around \( \mu_p \).

If \( \sigma_p \) is small, we can focus on \( p > 1/2 \) (assuming \( \mu_p > 1/2 \)) and note that \( \exp[-(p - \mu_p)^2/(2\sigma_p^2)] \) varies much more than the denominator, which we can approximate as \( (2\mu_p - 1)\Gamma(1 + 2\mu_p) \). On the other hand, because \( n \) is assumed to be large, a small variation in \( p \) could significantly change the numerator, therefore we have no reason to set \( n^{2p} \) to \( n^{2\mu_p} \) unless \( \sigma \to 0 \), and the Gaussian distribution becomes a Dirac delta function.

\begin{align*}
\langle k_{\text{ext}} \rangle &= \frac{2(\rho - \nu)}{\sigma_p \sqrt{2 \pi \rho N^{1-\nu/\rho}}} \int_1^N dn \frac{n^{(1+\nu/\rho)}}{\int_1^{1/2} dp \frac{\exp[2p\ln(n) - (p - \mu_p)^2/(2\sigma_p^2)]}{(2\mu_p - 1)\Gamma(1 + 2\mu_p)}}
\end{align*}

(12)

because the PDF quickly approaches 0 around \( p = \mu_p \), we can extend the integral of \( p \) to \( \pm \infty \).
Once we integrate, the result becomes

\[
\langle k_{\text{ext}} \rangle = \frac{2(\rho - \nu)}{(2\mu_p - 1)\Gamma(1 + 2\mu_p)\rho N^{1-\nu/\rho}} \int_1^N dn \ n^{2\mu_p-(1+\nu/\rho)} \exp[2\sigma_p^2 \ln(n)^2] \tag{13}
\]

after integrating over \( dn \), the result become

\[
\langle k_{\text{ext}} \rangle = \frac{2(\rho - \nu)}{\sqrt{2\sigma(2\mu_p - 1)\Gamma(1 + 2\mu_p)\rho N^{1-\nu/\rho}}}
\left\{ F \left[ \frac{\nu - 2\mu_p \rho}{2\sqrt{2\rho \sigma_p}} \right] + N^{-\nu/\rho} + 2\mu_p + 2\sigma_p^2 \log(N) \right\}, \tag{14}
\]

where \( F \) is the Dawson function (46). If, on the other hand, we have a delta function, in which \( \sigma_p = 0 \), then, after replacing the Gaussian distribution with a Dirac delta, the equation becomes

\[
\langle k_{\text{ext}} \rangle_{\sigma_p=0} = \frac{2(\rho - \nu)}{(2\mu_p - 1)\Gamma(1 + 2\mu_p)\rho N^{1-\nu/\rho}} \int_1^N dn \ n^{2\mu_p-(1+\nu/\rho)} \tag{16}
\]

\[
\langle k_{\text{ext}} \rangle_{\sigma_p=0} = \frac{2(\rho - \nu)}{(2\mu_p - 1)\Gamma(1 + 2\mu_p)\rho(2\mu_p - \nu/\rho)}(N^{2\mu_p-1} - 1) \tag{17}
\]

We compare this to simulation data in Fig. 24 and find similar scaling behavior, although the values are off by a factor of 10, possibly due to the finite size of most institutions, where the scaling laws assumed above might not hold. To understand the long-term behavior, however, we can take the limit that \( N \to \infty \)

\[
\langle k_{\text{ext}} \rangle \approx \begin{cases} 
C_1 N^{2\mu_p-1} & \sigma_p = 0 \ (p = \mu_p) \\
C_2 \frac{N^{2\mu_p-1+2\sigma_p^2 \log(N)}}{\ln(N)} & \sigma_p \ll 1
\end{cases} \tag{18}
\]

where

\[
C_1 = \frac{2(\rho - \nu)}{(2\mu_p - \nu/\rho)(2\mu_p - 1)\Gamma(2\mu_p + 1)}, \tag{19}
\]

and

\[
C_2 = \frac{\rho - \nu}{\rho \sigma_p^2(2 - \mu_p - 1)\Gamma(2\mu_p + 1)}, \tag{20}
\]

We notice that variance increases the mean degree, but also that that, for finite \( \sigma_p \), the scaling relation is not a power law. Of course, we do not want \( \langle k_{\text{ext}} \rangle \) to depend on \( N \) but instead on \( n \),
i.e., the institution size. Note, however, that previous work shows, to first order, that \( n = N/N_i \), where \( N_i \) is the number of researchers when the first institute formed \((35)\). Substituting this into Eq. \([18]\) we get \( \langle k_{\text{ext}} \rangle \) as a function of \( n \), while acknowledging that this implies a dependence on \( N_i \). We substitute \( \langle k_{\text{ext}}(n) \rangle \) into Eq. \([4] \) and notice that \( \langle k_{\text{ext}} \rangle \) does not depend on \( L_{\text{ext}} \), in contrast to internal collaborations. Knowing that \( L_{\text{ext}}(1) = 0 \), this iterative equation can be solved in the form of a series:

\[
L_{\text{ext}}(n) = \sum_{j=1}^{n-1} 1 + p \langle k_{\text{ext}} \rangle(j)
\]

\[
= n - 1 + p \sum_{j=1}^{n-1} \langle k_{\text{ext}} \rangle(j)
\]

(sadly, there is in general no simple formula for this series, although if \( \sigma_p = 0 \)

\[
L_{\text{ext}}(n)_{\sigma_p=0} \sim p \sum_{j=1}^{n-1} j^{2\mu_p - 1} = H(n - 1, 1 - 2\mu_p)
\]

where \( H \) is the harmonic function. The asymptotics of the harmonic function tell us that \( L_{\text{ext}}(n) \sim n^{2\mu_p} \), therefore, if \( \sigma_p = 0 \), the external collaboration is super-scaling with roughly the same exponent values as internal collaboration scaling. Sadly, when \( \sigma_p > 0 \) the formula cannot be written more compactly, although it is not a power law. To make this formula numerically easier to compute, however, we note that we can approximate this sum as an integral, which does not affect the results quantitatively (values are effectively the same, but now much easier to compute). In either case, it is approximately a powerlaw, as shown in Fig. \([25] \). Because the cumulative number of institutes grows as \( N^{\nu/\rho} = N^{1/2} \), the number of new institutes scales as \( N^{-1/2} \). Let \( N_i \) will be the value of \( N \) when that institute first appears. In this figure, we notice that, while the \( \sigma_p = 0 \) theory produces no dependence on \( N_i \), there is a surprising dependence on \( N_i \) for the \( \sigma_p > 0 \) theory. Based on the numerical relation between the scaling exponent and \( N_i \), and the rate of new institutes as a function of \( N \), we can create a histogram of the external
collaboration exponents up to $N = 10^6$. We notice a significant variance in these exponents, between 1 and 2, in agreement with what we find in simulations (see main text Fig. 3 and Fig. 28). That said, the reason for the relation is a dependence on $N_i$, which does not agree with what we find in simulations. Namely, the final size, $n = N/N_i$, where $N$ is fixed. This implies that the external collaboration exponent should depend on the final institution size, but that is not what we find in simulations or in data (see main text Fig. 3d). Moreover, the variance Fig. 25b is much smaller than what we find in simulations, shown in Fig. 28. We therefore have to conclude that this theory is too simplistic, but at least begins to explain the relevant phenomena that we see.

**Alternative Simulations**

We might wonder whether our model is sensitive to stochastic variations in how the model behaves. For example, we might ask whether changing the number of initial collaborators from 1 to a range of values will affect results. To this end, we made an additional model in which the number of initial internal and external collaborators was Poisson distributed, with $\lambda = 1$ (i.e., on average one internal and one external collaborator). This will not affect the institution
Figure 26: Simulations with different scaling laws ($\sigma_p = 0.1$) and constant collaboration scaling laws ($\sigma_p = 0.0$). (a) Internal collaboration scaling exponents versus final institution size, (b) $R^2$ versus final institution size for internal collaboration scaling, (c) external collaboration scaling exponents versus final institution size, (d) $R^2$ versus final institution size for external collaboration scaling. In all cases $\mu_p = 0.6$. Comparison between the cross-sectional scaling laws are in the main text (Fig. ).

Figure 27: Internal and external longitudinal collaboration exponents for alternative simulation models. (a) Internal and external exponents for simulations with $\lambda = 1$ Poisson distributed numbers of initial collaborators (on average one internal collaborator, and one external collaborator). (b) The same histograms for the current simulation with exactly one internal and one external collaborator.
Figure 28: Internal and external cross-sectional collaboration exponents for alternative simulation models. (a) Internal and external exponents versus the cumulative number of researchers for simulations with $\lambda = 1$ Poisson distributed numbers of initial collaborators (on average one internal collaborator, and one external collaborator). (b) The same figure for the current simulation with exactly one internal and one external collaborator.

formation, but it might affect the institution growth, e.g., the longitudinal collaboration scaling exponents. Importantly, Bhat et al. and Lambiotte et al. shows that number of links over time are not self-averaging (17, 18), therefore initial conditions greatly affect the final number of links. Figure 28 shows our results. We find that, while there are slightly more outliers in the scaling exponent distribution, results are quantitatively very similar.

Comparison Institution Data and Growth Mechanism

We compare the rate institutions grow in the four different fields, to predictions of the growth mechanism in the main text (Fig. 29). In the growth model, the probability an institution hires a researcher is proportional to the number of balls of that institution in an urn. Because the urn increases in size by $\rho$ whenever a ball is picked, the probability an institution hires a researcher is proportional to its size, $n$ (black line). Instead, we see a slight deviation with growth proportional to $n^\alpha$ (dashed line), where $\alpha$ is equal to $0.88 \pm 0.02$, $0.80 \pm 0.02$, $0.91 \pm 0.02$, and $0.84 \pm 0.01$ for computer science, physics, math, and sociology, respectively. This is alike to previous findings in preferential attachment (47, 48), but demonstrates the mechanism approximately captures the relationship between size and growth.
Figure 29: Rich-get-richer effect in institutions. The y-axis is the mean increase in institution size the next year as a function of its size in the current year for the fields of computer science, physics, math, and sociology. The model in the main text predicts the rate of institution growth is proportional to its size (black line), which is approximate agreement with our data, in which, institution growth follows a power-law $\sim n^\alpha$ with $\alpha \approx 0.9$. 