On Reconciling Gottfried Sum Rule Violation with Cabibbo Theory†

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Abstract

We discuss the seemingly contradictory constraints of simultaneously preserving the SU(3)-symmetric Cabibbo description of the weak vector baryon matrix elements, accounting for SU(3) flavor symmetry breaking and describing the observed violation of the Gottfried Sum Rule. We try to construct a simple model that will satisfy these constraints and use it to explain the generic difficulties and tradeoffs.

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1 Introduction

So far there has been no successful model for the flavor structure of the proton which is simultaneously consistent with three established experimental results:

1. The agreement of the data on weak semileptonic baryon decays produced by the conserved vector current with the predictions of Cabibbo theory. The vector matrix element is uniquely determined by Cabibbo theory in the $SU(3)$ symmetry limit.

2. The necessity for flavor $SU(3)$ breaking implied by the $K - \pi$ mass difference and the mass difference between strange and nonstrange quarks and confirmed experimentally by the observation of a suppression of the strange component in the sea of $q\bar{q}$ pairs in the nucleon. The strange quark contribution to the proton sea is already known from experiment to be reduced roughly by a factor of two from that of a flavor-symmetric sea [2].

3. The observed violation of the Gottfried sum rule (see [3] for a review). This requires an isovector component in the sea. Isospin invariance then immediately dictates that the proton wave function contains a valence neutron and a positively charged sea ($u\bar{d}$). A well-known implementation of such a fluctuation is $p \leftrightarrow n\pi^+$. Had $SU(3)$ been unbroken, it would in turn require, in analogy with the isovector invariance, that the proton wave function contains a valence hyperon and a sea with net strangeness, like in $p \leftrightarrow \Lambda K^+$. There is little experimental support for the presence of such a component in the proton wave function. We thus see that the violation of the Gottfried sum rule provides additional substantial evidence of a significant $SU(3)$ breaking in the nucleon wave function.

There are models which are consistent with Cabibbo theory and $SU(3)$ breaking in the sea by assuming that only valence quarks contribute to hyperon decays and that the $SU(3)$ breaking occurs only in the sea and is the same for all baryons. One such model [4] for breaking $SU(3)$ via the mechanism (2) keeps all the good results of Cabibbo theory by introducing a flavor asymmetric sea with no net flavor quantum numbers into a baryon wave function whose valence quarks satisfy $SU(6)$ symmetry and whose sea is the same
for all baryons. In ref. [4] it was shown that in this model all charged current matrix elements are given by the valence quarks. This provides an explicit justification for the hand-waving argument [1] in a proposed toy model that in the hyperon decay the sea behaves as a spectator. In particular, for the strangeness-changing vector charge producing the $\Sigma^- \rightarrow n$ decay. But such models fail to account for the violation of the Gottfried sum rule.

Other models introduce the violation of the Gottfried sum rule by introducing a sea which is not isoscalar. These can incorporate the observed strange quark suppression in the sea, but so far only at the price of violating the predictions of Cabibbo theory for hyperon decays. They can be made consistent with Cabibbo theory by retaining full $SU(3)$ symmetry. But this cannot be consistent with the observed $SU(3)$ breaking effects. For example, if one postulates a pion cloud around the nucleon to explain the violation of the Gottfried sum rule, one is required by $SU(3)$ symmetry to also include a kaon cloud around a valence hyperon in the same wave function, and cannot explain the observed suppression of strange quarks in the sea.

We thus see that in any model which includes a pion cloud in the proton wave function, $SU(3)$ breaking must reduce the kaon cloud relative to the pion cloud from the value in the symmetry limit. This necessary breaking seems to have a serious effect on the matrix elements of the strangeness-changing current responsible for hyperon decays. The nature of this inconsistency is illuminated by noting that production of a state of strangeness $+1$ by the action of the $SU(3)$ generator $V_+$ ($s \rightarrow u$ and $\bar{u} \rightarrow \bar{s}$) when acting on a “nucleon+pion” proton model wave function, indicates that this proton wave function is not a pure $SU(3)$ octet but contains a $27$ admixture:

$$|N\pi\rangle \ni |p\pi^0\rangle : \quad V_+ |p\pi^0\rangle = \ldots + |pK^+\rangle$$

When $SU(3)$ symmetry is restored in this model wave function by adding the correct admixture of $\Lambda K$, $\Sigma K$ and $p\eta_8$ states to the wavefunction with the pion cloud, the action of the operator $V_+$ on these components produces the $pK^+$ state with just the right phase to cancel the $pK^+$ state produced on the nucleon-pion state.

We now wish to generalize the treatment for the case of the pion cloud to the general case of a physical proton wave function with a valence nucleon and a sea of quark-antiquark pairs in which the numbers of $\bar{u}$ and $\bar{d}$ are different and the difference is adjusted to fit the violation of the Gottfried
sum rule. However, we wish to preserve isospin symmetry, which requires that a proton wave function with a valence proton and a sea that is not isoscalar must also have a component with a valence neutron and a sea which carries a positive electric charge to give the electric charge of the physical proton. We therefore include such components in our proton wave function. Similarly flavor SU(3) symmetry would require including components with a valence hyperon and with an excess of strange $\bar{s}$ antiquarks in the sea to balance the valence strangeness of the hyperon. We do not wish to include such components because SU(3) is experimentally known to be broken, and there is a deficiency of strange $\bar{s}$ antiquarks in the sea and not an excess. We therefore assume that our baryon wave functions have a sea which can contain $s\bar{s}$ pairs but no net strangeness; e.g. they can contain pion clouds but no kaon clouds.

We shall now show that such baryon wave functions are not consistent with the predictions of Cabibbo theory for hyperon decays. The transition matrix elements for the weak vector semileptonic decays of the $\Lambda$ and $\Sigma^o$ hyperons to this proton wave function cannot give the results predicted by Cabibbo theory and which agree with experiment.

Let us write the general wave function for the physical proton as

$$|p_{\text{phys}}\rangle = \cos \xi \cdot |p_{\text{val}} \cdot s\rangle + \sin \xi \cdot \left( |p_{\text{val}} \cdot v_o\rangle - \sqrt{2} |n_{\text{val}} \cdot v_+\rangle \right)$$

(2)

where $p_{\text{val}}$ and $n_{\text{val}}$ denote respectively valence proton and neutron, $s$ denotes an isoscalar sea of $q\bar{q}$ pairs and $v_o$ and $v_+$ denote the three components of an isovector sea. The parameter $\xi$ determines the isospin asymmetry in the nucleon sea.

A basic ambiguity arises in any formulation which separates quarks into valence and sea, as there is no physical label on a given quark to specify whether it is a valence quark or a sea quark. We assume here that the valence and sea quarks have very different momentum distributions, with the valence quarks being “hard” and the sea quarks “soft”, and that the overlap region between the two momentum distributions is negligible. This classification can break down in matrix elements describing processes with high momentum transfer, where an initial quark with soft momentum can turn into a final quark with hard momentum and vice versa. However we are concerned here only with matrix elements having essentially zero momentum
transfer and only require that the overlap region between valence and sea quark momentum distributions be negligibly small.

For a specific example we can write

$$\langle p_{\text{phys}} | = \frac{\cos \xi}{\sqrt{2}} \cdot [ | p_o \cdot (d \bar{d})_s \rangle + | p_o \cdot (u \bar{u})_s \rangle ] +$$

$$+ \frac{\sin \xi}{\sqrt{6}} \cdot [ | p_o \cdot (d \bar{d})_v \rangle - | p_o \cdot (u \bar{u})_v \rangle - 2 | n_o \cdot (u \bar{d})_v \rangle ] \quad (3)$$

where $p_o$ denotes a valence proton surrounded by an isoscalar sea of $q \bar{q}$ pairs which are inert and do not contribute to the violation of the Gottfried sum rule, nor to the weak strangeness-changing transitions, and similarly $n_o$ denotes a valence neutron surrounded by an isoscalar sea. The additional isoscalar and isovector $q \bar{q}$ pairs are expressed explicitly in states labeled by the subscripts $s$ and $v$.

The difference between the numbers of $\bar{d}$ and $\bar{u}$ antiquarks in the proton is given by

$$\langle p_{\text{phys}} | \delta N(\bar{q}) | p_{\text{phys}} \rangle = \frac{\sin 2\xi}{\sqrt{3}} \cdot Re \langle p_{\text{val}} \cdot s | \delta N(\bar{q}) | p_{\text{val}} \cdot v_o \rangle +$$

$$+ \frac{2}{3} \cdot \sin^2 \xi \cdot \langle n_{\text{val}} \cdot v_+ | \delta N(\bar{q}) | n_{\text{val}} \cdot v_+ \rangle \quad (4)$$

where $\delta N(\bar{q}) \equiv N(\bar{d}) - N(\bar{u})$. For the specific example (3)

$$N(\bar{d}) - N(\bar{u}) = \frac{\sin 2\xi \cos \phi}{\sqrt{6}} + \frac{2}{3} \cdot \sin^2 \xi \quad (5)$$

where $\cos \phi$ denotes the overlap between the isoscalar and isovector states,

$$\cos \phi \equiv \langle p_o \cdot (d \bar{d})_s | p_o \cdot (d \bar{d})_v \rangle \quad (6)$$

We note here that the difference between the number of $\bar{d}$ and $\bar{u}$ antiquarks in the proton has two contributions, one linear in the isospin asymmetry parameter $\sin \xi$ and one quadratic. The linear term may be crucial in allowing a small value of $\sin \xi$ to produce a violation of the Gottfried sum rule, while the violations of Cabibbo theory will be shown below to be quadratic in $\sin \xi$ and can therefore be much smaller. However, the linear term depends upon the overlap between an isoscalar sea and an isovector sea. This overlap
vanishes if the isovector sea is due to a pion cloud, since there is no isoscalar partner to the pion. Therefore the relative magnitudes of the violations of the Gottfried sum rule and Cabibbo theory are model dependent and depend upon the relative magnitudes of the linear and quadratic terms.

An example of a model which is probably not realistic and would give such a linear term has a sea due to a vector meson cloud, rather than to a pion cloud. Here the isoscalar sea is due to the $\omega$ and the isovector to the $\rho$. Interference between the $\rho$ and $\omega$ amplitudes can produce the linear term that violates the Gottfried sum rule.

We now examine the symmetry properties of this wave function to check that it indeed satisfies isospin symmetry and whether its manifest $SU(3)$ symmetry breaking must necessarily lead to violation of the predictions of Cabibbo theory.

We first construct the wave function for the physical neutron and check that these wave functions are a doublet of isospin $1/2$ by applying the isospin raising and lowering operators $I^\pm$ to these wave functions,

$$|n_{phys}\rangle = I^- |p_{phys}\rangle = \cos \xi \cdot |n_{val} \cdot s\rangle + \frac{\sin \xi}{\sqrt{3}} \cdot \left[ |n_{val} \cdot v_o\rangle - \sqrt{2} |p_{val} \cdot v_-\rangle \right]$$

For the specific example (3),

$$|n_{phys}\rangle = \frac{\cos \xi}{\sqrt{2}} \cdot \left[ |n_o \cdot (dd)s\rangle + |n_o \cdot (u\bar{u})s\rangle \right] + \frac{\sin \xi}{\sqrt{6}} \cdot \left[ |n_o \cdot (u\bar{d})v\rangle - |n_o \cdot (d\bar{d})v\rangle - 2 |p_o \cdot (d\bar{u})v\rangle \right]$$

$$I^+ |p_{phys}\rangle = \frac{\sin \xi}{\sqrt{6}} \cdot 2\left[ |p_o \cdot (u\bar{d})v\rangle - |p_o \cdot (u\bar{d})v\rangle \right] = 0$$

$$I^- |n_{phys}\rangle = \frac{\sin \xi}{\sqrt{6}} \cdot 2\left[ |n_o \cdot (d\bar{u})v\rangle - |n_o \cdot (d\bar{u})v\rangle \right] = 0$$

These wave functions satisfy the constraints of isospin, and the parameter $\xi$ can be fixed to satisfy the Gottfried sum rule.

We now investigate the action of the strangeness-changing components of the charged weak vector current on the proton wave function (3). At zero-momentum transfer, these are just the $V$-spin raising and lowering operators,
denoted by $V_\pm$, which generate $u \leftrightarrow s$ and $\bar{s} \leftrightarrow \bar{u}$ transitions at the quark level \[4\]. The requirement that the proton and $\Lambda$ are members of the same $SU(3)$ octet gives the two conditions:

$$V_+ |p_{phys}\rangle = 0 \quad (11)$$

$$P(I = 0) \cdot V_- |p_{phys}\rangle = \frac{\sqrt{6}}{2} |\Lambda_{phys}\rangle \quad (12)$$

where $P(I = 0)$ denotes a projection operator which projects out the $I = 0$ component of the wave function and $|\Lambda_{phys}\rangle$ denotes the normalized physical $\Lambda$ wave function. These two conditions required by Cabibbo theory were shown in ref. \[4\] to be manifestly violated by the proton wave function with a pion cloud.

The condition \(11\) is also seen to be violated by the physical proton wave function \(3\) used here, since

$$V_+ |u\bar{u}\rangle = - |u\bar{s}\rangle; \quad V_+ |s\bar{s}\rangle = |u\bar{s}\rangle \quad (13)$$

When the sea is $SU(3)$ symmetric, the numbers of $u\bar{u}$ and $s\bar{s}$ pairs are equal and the condition \(11\) is satisfied because the two terms in eq. \(13\) cancel. This cancellation no longer occurs when the numbers of $u\bar{u}$ and $s\bar{s}$ are unequal, as is required to fit the known suppression of the strange component in the nucleon sea \[2\].

We now attempt to bypass the $SU(3)$ breaking effects indicated by the violation of the condition \(11\) by constructing baryon wave functions such that the transition matrix elements between the physical baryon states satisfy $SU(3)$, even though the physical baryon wave functions are no longer members of the same $SU(3)$ octet; i.e. they contain admixtures of other representations.

We assume that all the baryon wave functions are constructed like $|p_{phys}\rangle$ having no valence hyperons and a sea which can contain $s\bar{s}$ pairs but no net strangeness; e.g. they can contain pion clouds but no kaon clouds.

In order to examine closer the action of $V_\pm$ on the sea, we now define modified $V$-spin raising and lowering operators $V_{+\text{val}}$ and $V_{-\text{val}}$, which by definition act only on the valence baryon component of the wave function and not on the sea quarks, therefore leaving the sea unchanged.
We now note that the operation of the strangeness-changing operators on a sea with no net strangeness creates a sea with nonvanishing strangeness. Since our baryon wave functions are constructed to have no net strangeness in the sea,

\[ \langle \Lambda_{phys} | [V_ - - V_-^{val}] | p_{phys} \rangle = \langle \Sigma^o_{phys} | [V_ - - V_-^{val}] | p_{phys} \rangle = 0. \quad (14) \]

This is seen in specific example where

\[ [V_ - - V_-^{val}] | p_{phys} \rangle = \cos \xi \sqrt{2} \cdot | p_o \cdot (s\bar{u})_s \rangle + \sin \xi \sqrt{6} \cdot | | p_o \cdot (s\bar{u})_v \rangle - 2 | n_o \cdot (s\bar{d})_v \rangle \]

(15)

The RHS of eq (15) has a sea with nonvanishing strangeness.

Here our assumption neglecting the overlap region between the valence and sea quark momentum distributions is seen to be essential in any model which attempts to preserve the SU(3) relations of Cabibbo theory despite the large SU(3) breaking in the sea. The operator \( V_- \) acting on the proton sea can create a strange quark in a final hyperon, both via the \( u\bar{u} \rightarrow s\bar{u} \) and the \( s\bar{s} \rightarrow s\bar{u} \) transitions. Since the \( s\bar{s} \) component in the proton sea is suppressed by SU(3) breaking, any contribution from these transitions to the matrix elements between physical nucleon and hyperon states will violate SU(3) in the physical matrix elements and violate Cabibbo theory. Thus it is necessary to assume that the SU(3) breaking remains in the sea and affects only the magnitude of a sea with nonvanishing strangeness which has no overlap with the baryon wave functions. Thus only valence quarks contribute to hyperon decays as in a previous toy model[1].

Under this assumption the transition matrix elements for the \( \Lambda \rightarrow p \) and \( \Sigma^o \rightarrow p \) decays are respectively \( \langle p | V_+^{val} | \Lambda \rangle \) and \( \langle p | V_+^{val} | \Sigma^o \rangle \). We now note that

\[ | \langle p | V_+^{val} | \Lambda \rangle |^2 + | \langle p | V_+^{val} | \Sigma^o \rangle |^2 \leq \sum_i | \langle p | V_+^{val} | i \rangle |^2 = \langle p | V_+^{val} V_-^{val} | p \rangle \quad (16) \]

where the sum over \( i \) denotes a complete set of states.

Since \( V_+^{val} | p \rangle = 0 \), we can replace the product \( V_+^{val} V_-^{val} \) in (16) by the commutator,

\[ | \langle p | V_+^{val} | \Lambda \rangle |^2 + | \langle p | V_+^{val} | \Sigma^o \rangle |^2 \leq \langle p | V_+^{val} V_-^{val} | p \rangle = 2 \langle p | V_+^{val} | p \rangle \quad (17) \]
Substituting the expression (3) for the physical proton wave function and noting that the eigenvalues of $V_{val}^{+}$ for the proton and neutron are respectively $+1$ and $+(1/2)$, we obtain

$$
| \langle p | V_{val}^{+} | \Lambda_o \rangle |^2 + | \langle p | V_{val}^{+} | \Sigma_o \rangle |^2 \leq 2 \cos^2 \xi + \frac{4}{3} \cdot \sin^2 \xi = 2 - \frac{2}{3} \cdot \sin^2 \xi \quad (18)
$$

Thus the sum of the semileptonic vector decay rates for the $\Lambda \rightarrow p$ and $\Sigma^o \rightarrow p$ decays agrees with the value 2 predicted by Cabibbo theory only when $\sin^2 \xi = 0$, i.e. when there is no flavor asymmetry in the sea and the Gottfried sum rule is not violated.

We have shown that explaining the observed violation of the Gottfried sum rule while keeping the good results of the Cabibbo theory requires the introduction of net strangeness in the nucleon sea. The latter seems to be in conflict with experiment. There are two possible directions for avoiding this conflict.

1. It may not be a real conflict with the real data. This requires a quantitative analysis of how much violation of Cabibbo theory is allowed by the real data with real errors. The question remains of whether violations of both the Gottfried sum rule and Cabibbo theory can be consistent with present data. In that case better data to reduce the errors will be of interest. The answer to this question is model-dependent, since it depends upon the overlap (6) between isoscalar and isovector seas.

2. The second logical possibility, as unlikely as it may sound is that there may be a small component of “valence like” strange quarks in the proton. By “valence like” we mean quarks with large values of $x$, usually associated with valence components of the nucleon wave function. Of course the total number of strange plus anti-strange quarks remains zero, but the respective $x$ distributions might be very different. This can be checked by better measurements of the $x$-dependence of the strangeness in the proton. Indeed, there have been suggestions in the literature (see for example [6]-[9]), that the strange sea might exhibit a considerable degree of asymmetry. However, as far as we are aware, until now there has been no discussion of whether such an asymmetric strange sea is compatible with the successful Cabibbo theory. Here again quantitative limits are needed.
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