An Approach for Measurement of Young’s Modulus of Glass in the Form of a Slab

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Abstract

Objectives: The study of elastic properties of material is very important for commercial use as well as for research work. But especially for this purpose we are not always very much interested in study with materials other than metals. Now, there will be a study of elastic property of glass, mainly to determine the value of Young’s Modulus experimentally.

Methods/Statistical Analysis: In the present study the process of flexure of a thin beam of glass is used. When external force in the form of weight is applied on the middle point of glass slab, the change in length of different layers of the slab occurs. As a result finally total Internal Bending Moment (IBM) comes to play. Findings: Directly, IBM cannot be measured but at equilibrium it can be treated equal in magnitude to Applied (external) Bending Moment (ABM). Theoretically a relation between ABM and Young’s modulus can be generated, from this study; Young’s modulus can be estimated. The result got from this study is very close to the well-established value of Young’s modulus of glass. Though in the process, the changes in length for glass is very small, breaking load is also very low and regular, symmetric shape of glass is essential, still it is very easy and interesting to determine the value of Young’s modulus of glass. This process not only useful for glass or metals, but can be used for other non-metals.

Application/Improvements: The flexure process may be the quickest and the result very close to the actual result for glass.

Keywords: Applied Bending Moment, Breaking Load, Elastic, Internal Bending Moment, Modulus

1. Introduction

For metals, different well established processes are available for the measurement of Young’s modulus. Main advantages for metals are, their higher breaking loads and higher elastic limits compared to glass, as a result heavy loads are successively applied for deformations. But for glass higher loads are necessary for moderate deformation, at the same time their breaking load is low. In the present experiment, long, thin, flat, straight, uniform, and homogeneous glass slab is placed horizontally near the ends on knife points of two stands and load in the form of weight is applied in the exactly middle portion of the glass slab with a hanger. The horizontal glass slab which is placed on the knife edges may be considered that it consists of by large number of horizontal straight layers having equal lengths. When concentrated load is applied on the middle point of the beam, the straight beam converts into a curve with very large radius of curvature. The layers which are outside the curvature get elongated and

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inside layers get contracted with respect to the neutral layer. For different successive loads different deformations of layers are generated. Elongated and contracted lengths cannot be measured experimentally, but resisting couples forms and finally internal bending moment comes into play. At equilibrium internal bending moment is equal in magnitude to external bending moment. Using this concept, in the present case Young’s modulus of glass is determined.

2. Experiment

Where AB is a thin, long, uniform, straight glass slab, placed horizontally near the ends on two knife edges of the stands (SS). Load (W) is applied at the exact middle position of the slab with a hanger (H). For the present case, when load is applied the slab becomes bend by the applied torque as a result tensile and compressional forces act on different layers. Nearest layers outside the curve are extended and nearest layers inside the curve get compressed, as shown in Figure 1. This is the reason of developing internal forces to counteract the effect of bending. Where is the angle that makes the two ends faces of the curved slab. XY is neutral layer. It is original and its length remains constant during bending. If R is the radius of curvature of the neutral layer, R is the length of the neutral layer. Dotted is elongated and it is at a distance Z from the neutral layer. So due to bending, elongated length is (R+Z) Θ. So the extension of the layer is ZΘ and strain is Z/R. Longitudinal force is \( F = \frac{YZA}{R} \). This force resists the extension and arises out of the elastic properties. Similarly for a layer at a distance Z opposite to the elongated layer, same resisting force comes into play. Two equal and opposite forces constitute a couple. Hence the moment of force is FZ and for whole slab it is equals \( \Sigma FZ \), i.e. total internal bending moment of the slab \( = \frac{Y}{R} \Sigma A Z^2 = \frac{YAK^2}{R} \) ---- (1). Where A is cross sectional area of the glass slab and it is, A = breadth x thickness (a is breadth and b is thickness of the glass bar) and K is called radius of gyration and the value of K depends on the geometry of bar used. For present case, as rectangular bar is used, \( K = \frac{b^2}{12} \). For the larger value of R, it is equals to \( \frac{1}{d^2} \). At equilibrium internal bending moment is equal to external bending moment, details shown in Figure 2.

![Figure 1. Elongation and Contraction of Different layers with Respect to Neutral Axis.](image-url)
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Integrating the equation (2) by using the boundary conditions, the value of depression can be determined and it is

\[ \frac{YAK^2}{R} = \left[w\left(\frac{l}{2} - x\right) - \frac{w}{2}(l - x)\right] \]

And \[ \frac{d^2x}{d^2y} = \left[w\left(\frac{l}{2} - x\right) - \frac{w}{2}(l - x)\right] \quad \text{........ (2)} \]

Integrating the equation (2) by using the boundary conditions, the value of depression can be determined and it is

\[ Y_k = \frac{wL^3}{4ab^3} \quad 1.258.5 \]

where \( Y \) is Young's modulus of glass and

\( W \) is weight hanged at the middle point i.e. \( W = mg \).

Ultimately the value of young's modulus

\[ Y = \frac{g^3}{4ab^3} \quad 1.2.4.7.5.10 \]

In this experiment the value of \( Y \) is estimated from the reciprocal of the gradient of best fitted straight line, which are drawn by plotting mass (in terms of loads) along X-axis and corresponding depression along Y-axis.

### 3. Experimental Procedure

For the present experiment at room temperature faint glass slab is used in the form of a bar. Here, a glass slab of length nearly 50 cm, different breadth and thickness is chosen. The glass slab is placed near the ends on the stands (SS). The slab made completely horizontal with adjusting the levelling screws of the stands and weight pan is hanged with the hanger at the exactly middle position of the slab. The distance between the contact points of the slab and the stands are accurately measured. The hanger is attached with a pointer as shown (P) in the Figure 3.

A travelling microscope (resolution 0.001cm.) is placed at a suitable distance in front of the slab and adjusting the main scale and also vernier scale, the reading of the pointer is taken thrice and obtained the mean value. It is considered as zero load reading.

Now the load 100gm. is applied on the weight pan and after waiting 15 min. Microscopes is adjusted and estimates the actual reading. This reading when subtracted from the zero load reading gives depression of the slab for 100gm load. This way load is increasing up to 1600gm and for every load the microscopic reading is noted down.

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**Figure 2.** Internal Bending Moment is equal to External Bending Moment at Equilibrium.
After applying total loads, now load is decreased in equal steps, like at the time of increasing, and the reading for a particular load is recorded. Mean reading for a particular load at the time of increasing and decreasing, when subtracted from zero load reading estimates the depression of the glass slab for that particular load. This way for a particular length, breadth, and thickness of a glass slab, the load and corresponding depressions are estimated several times (20 times) and the grand mean of depressions are used for drawing graph, from the graph calculating the reciprocal of the gradient, the final value of Young's modulus of glass can be estimated out.

4. Results

Considering first a glass slab of breadth (a) = 2.4633 cm, thickness (b) = 0.3755 cm, and two different effective lengths (40 cm and 35 cm) are used separately for microscopic reading of depression against the various applied loads. Each experiment has done several times and mean values of depressions against loads are shown in Table 1. Secondly another glass slab having breadth(a)=2.4454 cm, thickness(b)=0.5929 cm, are chosen for two different lengths(40cm and 35 cm) and grant mean values of depressions are tabled in Table 2.
### Table 1. Load and depression

| Load in gm. | Micros reading in cm. | Depression ($y_1$) in cm. | Load in gm. | Micros reading in cm. | Depression ($y_1$) in cm. |
|-------------|-----------------------|---------------------------|-------------|-----------------------|---------------------------|
| 0           | 9.9535                | 0                         | 0           | 9.9465                | 0                         |
| 100         | 9.9335                | 0.0200                    | 100         | 9.933                 | 0.0135                    |
| 200         | 9.9175                | 0.0360                    | 200         | 9.9185                | 0.028                     |
| 300         | 9.9030                | 0.0505                    | 300         | 9.9040                | 0.0425                    |
| 400         | 9.8810                | 0.0725                    | 400         | 9.896                 | 0.0505                    |
| 500         | 9.8635                | 0.090                     | 500         | 9.8835                | 0.0630                    |
| 600         | 9.8450                | 0.1085                    | 600         | 9.8695                | 0.0770                    |
| 700         | 9.8225                | 0.1310                    | 700         | 9.858                 | 0.0885                    |
| 800         | 9.8115                | 0.142                     | 800         | 9.8475                | 0.099                     |
| 900         | 9.7930                | 0.1612                    | 900         | 9.839                 | 0.1075                    |
| 1000        | 9.778                 | 0.1755                    | 1000        | 9.8299                | 0.117                     |
| 1100        | 9.7605                | 0.193                     | 1100        | 9.8195                | 0.127                     |
| 1200        | 9.740                 | 0.2135                    | 1200        | 9.808                 | 0.1385                    |
Figure 4. Depression with load (mass) for length of glass slab 40cm, breadth 2.4633cm and thickness 0.3755 cm.

Figure 5. Depression with load (mass) for length of glass slab 35cm, breadth 2.4633cm and thickness 0.3755 cm.
Table 2. Load and depression

| Load in gm. | Micros reading in cm. | Depression \( y_i \) in cm. | Load in gm. | Micros reading in cm. | Depression \( y_i \) in cm. |
|------------|------------------------|----------------------------|------------|------------------------|----------------------------|
| 0          | 10.1745                | 0                          | 0          | 10.178                 | 0                          |
| 200        | 10.1680                | 0.0065                     | 300        | 10.170                 | 0.008                      |
| 400        | 10.1600                | 0.0145                     | 600        | 10.163                 | 0.015                      |
| 600        | 10.1490                | 0.0255                     | 900        | 10.153                 | 0.025                      |
| 800        | 10.1400                | 0.0345                     | 1200       | 10.145                 | 0.033                      |
| 1000       | 10.128                 | 0.0465                     | 1500       | 10.138                 | 0.040                      |
| 1200       | 10.120                 | 0.0545                     | 1800       | 10.129                 | 0.049                      |
| 1400       | 10.114                 | 0.0605                     | 1600       | 10.108                 | 0.0665                     |
| 1600       |                        |                            |            |                        |                            |
Figure 6. Depression with the load (mass) for length of glass slab 40cm, breadth 2.4454cm and thickness 0.5929 cm.

Figure 7. Depression with the load (mass) for length of glass slab 35cm, breadth 2.4454cm and thickness 0.5929 cm.
Figure 4 shows the variation of depression with the load (mass) when, length of glass slab = 40cm, breadth= 2.4633cm and thickness=0.3755 cm (1st part of the table 1 and gradient of the best fitted straight line i.e.\( \frac{y}{m} = 0.0001767 \), using this obtained value of Young’s modulus is 68.03x10^10 dynes/cm^2.

Figure 5 shows the variation of depression with the load (mass) when, length of glass slab = 35cm, breadth= 2.4633cm and thickness=0.3755 cm (2nd part of the table 1 and gradient of the best fitted straight line i.e.\( \frac{y}{m} = 0.00011384 \), using this obtained value of Young’s modulus is 70.68x10^10 dynes/cm^2.

Figure 6 shows the variation of depression with the load (mass) when, length of glass slab = 40cm, breadth= 2.4454cm and thickness=0.5929 cm (1st part of the table 2 and gradient of the best fitted straight line i.e.\( \frac{y}{m} = 0.00004408 \), using this obtained value of Young’s modulus is 69.78x10^10 dynes/cm^2.

Figure 7 shows the variation of depression with the load (mass) when, length of glass slab = 35cm, breadth= 2.4454cm and thickness=0.5929 cm (2nd part of the table 2 and gradient of the best fitted straight line i.e.\( \frac{y}{m} = 0.00003 \), using, obtained value of Young’s modulus is 68.69x10^10 dynes/cm^2.

5. Discussion and Conclusion

It is found from the experiment, that the mean value of Young’s modulus of glass is 69.29x10^10 dynes/cm^2, which is very close to the standard value. This experiment is comparatively easy, minimum apparatus needed and it is suitable for a long slab but less breadth and thickness, though simultaneously applied loads are very small. In this way instead of using a rectangular bar, circular bar can be used. To get the accurate result more and more trials are required for slabs having different lengths, breadth and thickness. While taking the readings for depression at least 15 minutes have to wait after applying load because of ‘elastic after effect’ of glass. To apply the method the temperature should be kept constant as Y depends on temperature\(^{11}\).

6. References

1. Chatterjee H, Sengupta RA. Treatise on G.P.M, New Central Book Agency (P) Ltd. 6th Edition. 2004; p. 475-554.
2. Maiti SN, Chaudhuri DP. Classical Mechanics and G.P.M. New Age (p) Ltd. Revised 2nd Edition. 2006; p. 461-521.
3. Mathur DS. Elements of Properties of Matter (11e), Shayam Lal Charitable Trust, 11th Edition, New Delhi -110055. 1992.
4. He H. Thope MF. Elastic properties of Glass. Physical Review Letters 54. May 1985; 54(19):1-4.
5. Yavorsky BM, PinskyAA. Mir Publishers: Moscow: Fundamentals of Physics. 1975; 1.
6. Phillips CJ. Glass the Miracle Maker (pitman) Publishing Corporation. New York. 1948; p. 152-53. PMid:18871486
7. Baker TC, Preston FW. Journal of Applied Physics. 1946; 17:170. CrossRef.
8. Timoshenko S. Theory of Elasticity (McGraw-Hill book Company), Inc, New York. 1934; p. 290.
9. Murgatroyed JB. Journal of Society Glass Technoly, XXVIII. 1944; 21(5):1-2.
10. Sinclair D. A Bending Method for Measurement of the Tensile strength and Young’s Modulus of Glass Fibers, Journal of Applied Physics. Published online. 2004 April; 21(5):1-6.
11. McGraw DA. A method for determining Young’s modulus of Glass at Elevated temperatures. Journal of the American Ceramic Society, version of Record online. 2006; 35(1):22-27.