Collapsar Gamma-Ray Bursts: how the luminosity function dictates the duration distribution

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ABSTRACT
Jets in long-duration γ-ray bursts (GRBs) have to drill through the collapsing star in order to break out of it and produce the γ-ray signal while the central engine is still active. If the breakout time is shorter for more powerful engines, then the jet-collapsar interaction acts as a filter of less luminous jets. We show that the observed broken power-law GRB luminosity function is a natural outcome of this process. For a theoretically motivated breakout time that scales with jet luminosity as $L^{-\chi}$ with $\chi \sim 1/3 - 1/2$, we show that the shape of the γ-ray duration distribution can be uniquely determined by the GRB luminosity function and matches the observed one. This analysis has also interesting implications about the supernova-central engine connection. We show that not only successful jets can deposit sufficient energy in the stellar envelope to power the GRB-associated supernovae, but also failed jets may operate in all Type Ib/c supernovae.

Key words: gamma-ray burst: general

1 INTRODUCTION

The central engine of γ-ray bursts (GRBs, see, e.g., Kumar & Zhang 2015 for a review) is hidden to direct observation. However, its workings may be imprinted in observational signals of this phenomenon. Bromberg et al. (2012, 2013) proposed that the prompt γ-ray duration distribution of collapsar GRBs exhibits a plateau, indicating the fact that GRB jets launched at the core of a collapsing star must drill their way out of the star and break out from its surface before producing the observed γ-rays (for the collapsar model, see, e.g. Paczyński 1998; MacFadyen & Woosley 1999). This claim can be extended to understand low-luminosity GRBs as jets that barely failed to break out (Bromberg et al. 2011a). Recently, these arguments have been used to suggest that failed jets may operate in all Type Ib/c supernovae (SNe) (Sobacchi et al. 2017).

Previous works have focused on the γ-ray duration distribution and assumed a single breakout time for all collapsar GRBs (e.g., Sobacchi et al. 2017). However, differences in the properties of long-duration GRB engines should yield different breakout times. In particular, one expects that more powerful jets will propagate more easily through the star and will break out from it much quicker than weaker jets (e.g., Zhang et al. 2003; Morsony et al. 2007; Mizuta & Aloy 2009; Lazzati et al. 2012). Both analytical estimates (Bromberg et al. 2011a,b) and numerical simulations (see Lazzati et al. 2012, and references therein) suggest that the jet’s breakout time depends upon the engine’s isotropic luminosity as $L^{-\chi}$: the power-law index lies between 1/3 and 1/2 depending on properties of the stellar envelope (e.g., density profile, radius and mass) and/or the properties of the jet (e.g., jet composition).

In this paper, we consider the luminosity dependence of the breakout time in a scenario where the jet needs to drill through the collapsing star before it can power a GRB. An extended distribution in engine luminosities is motivated by the GRB luminosity function itself, which extends several orders of magnitude in isotropic luminosity (e.g., Wanderman & Piran 2010). We show that a broken power-law GRB luminosity function is an expected outcome of the jet-envelope interaction for central engines having a single power-law luminosity distribution. After matching the parameters of the model-predicted GRB luminosity function to the observed one, we derive a mono-parametric γ-ray duration distribution. The maximum breakout time (i.e., its single tunable parameter) can be inferred by comparison to the observed GRB collapsar duration distribution. This framework is quite powerful as it connects observable quantities with the properties of the GRB central engine and of the collapsing star. It also makes a tantalizing connection between GRBs and jet-driven core-collapse supernovae (e.g., Soderberg et al. 2010; Lazzati et al. 2012; Margutti et al. 2014; Sobacchi et al. 2017; Piran et al. 2017; Bear et al. 2017, for a review, see, e.g., Soker 2016 and references therein).

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2 MODEL SETUP

We consider a similar scenario to that presented in Bromberg et al. (2012). A long-duration GRB central engine that launches a jet \(^1\) for a duration \(t_b\) must be active for a duration longer than the time \(t_b\) it takes the jet to break out of the stellar envelope in order to produce a \(\gamma\)-ray signal. The duration of the prompt \(\gamma\)-ray emission is thus \(t_e = t_b - t_0\). For \(t_e < t_b\), the jet is unable to break out from the star and “fails”, but still injects its energy to the stellar envelope, possibly powering the supernova explosion (see section 4). Contrary to previous studies (Bromberg et al. 2012; Sobacchi et al. 2017), we consider a distribution of engine luminosities which, in turn, translates into (i) a distribution of observed GRB luminosities and (ii) a distribution of breakout times. In this section we describe our assumptions regarding the distribution of engine luminosities and their duration.

We consider a normalized power-law distribution of isotropic engine luminosities \(L_e\) as

\[
p(L_e) \equiv \frac{dN}{dL_e} = \frac{a - 1}{L_{e,\text{min}}} \left( \frac{L_e}{L_{e,\text{min}}} \right)^{-a}, \quad a > 1,\]

where \(L_{e,\text{min}}\) is the engine’s minimum isotropic luminosity. Whether or not the jet breaks out from the star depends on the breakout time \(t_b\), which is related to \(L_e\) as:

\[
t_b = t_0 \left( \frac{L_e}{L_{e,0}} \right)^{-\chi}. \tag{2}
\]

where \(L_{e,0} = 10^{51}\) erg s\(^{-1}\). Henceforth, we consider a narrow range of possible power-law indices that are motivated by relativistic hydrodynamic simulations of jet propagation in collapsars, namely \(1/3 \leq \chi \leq 1/2\) (e.g. Bromberg et al. 2011a; Lazzati et al. 2012; Nakar 2015). The normalization \(t_0\) encodes information about the jet’s collimation and the properties of the stellar envelope. In our analysis, we will treat \(t_0\) as a free parameter and assume it is the same among all GRB collapsars.

The distribution of breakout times, \(p_b(t_b)\), can be determined by the condition \(p_b(t_b) dt_b = p(L_e) dL_e\) using equations (1) and (2). The resulting distribution can be written as:

\[
p_b(t_b) = A_b \left( \frac{t_b}{t_0} \right)^s H[t_b - t_{b,\text{max}} - t_b] \tag{3}
\]

where \(H[x]\) is the Heaviside step function, \(s = (a - 1)/\chi\), and

\[
A_b = \frac{a - 1}{\chi t_0} \left( \frac{L_{e,0}}{L_{e,\text{min}}} \right)^{-a + 1}. \tag{4}
\]

The maximum breakout time \(t_{b,\text{max}}\) corresponds to an engine with the minimum luminosity and is given by:

\[
t_{b,\text{max}} = t_0 \left( \frac{L_{e,\text{min}}}{L_{e,0}} \right)^{-\chi}. \tag{5}
\]

The distribution of engine durations \(t_e\) is assumed to be unrelated to the distribution of engine luminosities \(L_e\) and to follow a power law, similar to previous studies (e.g., Bromberg et al. 2012; Sobacchi et al. 2017):

\[
p_e(t_e) = A_e \left( \frac{t_e}{t_{e,\text{min}}} \right)^{-\beta} H[t_e - t_{e,\text{min}}, \beta > 1,\]

where \(t_e = t_b - t_0\) and \(t_{e,\text{min}}\) is the minimum duration of the central engine. The normalization is

\[
A_e = \frac{\beta - 1}{t_{e,\text{min}}} \tag{7}
\]

and ensures that \(\int_{t_e}^{\infty} dt_e p_e(t_e) = 1\).

3 RESULTS

In this section we connect the engine distributions described previously with the observed distributions of luminosities and durations of long (collapsar) GRBs.

Of all possible central engines, only those with \(t_e > t_b\), can lead to “successful” GRBs, that is, jets that can break out from the star and produce a \(\gamma\)-ray signal while the engine is still active. The fraction of such successful jets is given by:

\[
f(t_b) = \int_{t_b}^{\infty} dt_e p_e(t_e) \tag{8}
\]

and can be written as:

\[
f(t_b; L_e) = \left( \frac{t_b}{t_{e,\text{min}}} \right)^{-\beta + 1} H[t_b - t_{e,\text{min}} + H[t_{e,\text{min}} - t_b], \tag{9}
\]

where the dependence upon \(L_e\) comes through \(t_b\). The last term in the right-hand-side of the equation shows that all engines with breakout times shorter than the minimum duration \(t_{e,\text{min}}\) will be successful in producing GRBs.

3.1 Intrinsic GRB luminosity function

We argue that the (normalized) isotropic GRB luminosity function is the product of the fraction \(f\) of successful jets and the engine luminosity function \(p(L_e)\):

\[
\frac{dN_{\text{GRB}}}{dL} = \frac{f(t_b; L_e)}{\eta} \frac{dN}{dL_e}, \tag{10}
\]

where we assumed that the isotropic GRB (radiated) luminosity \(L\) is a constant fraction of the isotropic engine luminosity (i.e., \(L = \eta L_e\)), as motivated by the narrow distribution of the \(\gamma\)-ray efficiency of GRBs (e.g. Fan & Piran 2006; Beniamini et al. 2016). Depending on whether \(L\) denotes the peak or average burst luminosity, the numerical value of \(\eta\) will differ by a factor of \(~3\).

Using equations (1), (2), and (9) we find:

\[
\frac{dN_{\text{GRB}}}{dL} = N_0 \left( \frac{L}{L_{\text{min}}} \right)^{-a} \left\{ \begin{array}{ll}
(\frac{L}{L_{br}})^{-w}, & L \leq L_{br} \\
1, & L > L_{br}.
\end{array} \right. \tag{11}
\]

where \(N_0 \equiv (a - 1)/L_{\text{min}}, w = \chi(-\beta + 1) < 0\) and the break luminosity is defined as:

\[
L_{br} \equiv L_0 \left( \frac{t_{e,\text{min}}}{t_0} \right)^{-1/\chi}. \tag{12}
\]

Therefore, a power-law distribution of the central engine luminosity results in a broken power-law distribution of the...
GRB luminosity function. The break at $L_{\text{br}}$ is due to the fact that an increasing fraction of engines with lower luminosities are not able to produce successful GRBs. In contrast, the GRB luminosity function reflects the luminosity function of the central engine for $L > L_{\text{br}}$.

Observationally, the GRB luminosity function is, indeed, best described by a broken power law (e.g. Wanderman & Piran 2010; Salvaterra et al. 2012; Pescalli et al. 2016):

$$\phi(L) \equiv \frac{dN_{\text{GRB}}}{d\log L} = \left\{ \begin{array}{ll}
\left( \frac{L}{L_\star} \right)^{-\alpha L} & L \leq L_\star \\
\left( \frac{L}{L_\star} \right)^{-\beta L} & L > L_\star.
\end{array} \right.$$  \hspace{1cm} (13)

Comparison of the GRB luminosity function with the predicted one, equation (11), allows us to uniquely determine the critical parameters of our model that describe the engine luminosity distribution and its duration:

$$L_{\text{br}} = L_\star$$  \hspace{1cm} (14)

$$a = \beta_L + 1$$  \hspace{1cm} (15)

$$\beta = \frac{\beta_L - \alpha L}{\chi} + 1$$  \hspace{1cm} (16)

$$t_{e,\text{min}} = t_0 \left( \frac{10 L_\star}{Q_x (\eta - 1) L_{\text{br}}} \right)^{-\chi},$$  \hspace{1cm} (17)

where we use the standard $Q_x = 10^{-3}Q$ notation. We present a graphical view of our comparison in Fig. 1 and summarize our results for the two indicative $\chi$ values in Table 1. Interestingly, the minimum engine activity timescale cannot be arbitrarily short for a given $t_0$.

### 3.2 Distribution of $\gamma$-ray durations

The distribution of rest-frame $\gamma$-ray durations of GRBs $t_\gamma = t_\gamma - t_0$ can be calculated using the procedure outlined in Sobacchi et al. (2017). However, we relax their assumption of a fixed $t_\gamma$ value by considering a distribution of breakout times as determined by the jet luminosity. In this case, $p_\gamma$ is calculated as:

$$p_\gamma(t_\gamma) = \int_0^\infty dt_b p_b(t_b + t_\gamma) \frac{p_b(t_b + t_\gamma)}{f(t_b; L_e)}.$$  \hspace{1cm} (18)

The above equation can be recast in the form:

$$p_\gamma(t_\gamma) = A_{\eta, 1} A_{\alpha_L} \left( \frac{t_\gamma}{t_{e,\text{min}}} \right)^{-\beta} [I_1(t_\gamma) + I_2(t_\gamma)]$$  \hspace{1cm} (19)

$$I_1(t_\gamma) = \int_{t_{e,\text{min}}}^{t_{e,\text{max}}} dt_b \left( \frac{t_b}{t_{e,\text{min}}} \right)^{-\beta}$$  \hspace{1cm} (20)

$$I_2(t_\gamma) = t_{e,\text{min}}^{-\beta} \int_{t_{e,\text{min}}}^{t_{e,\text{max}}} dt_b \left[ t_b + \left( \frac{t_b}{t_{e,\text{min}}} \right)^{-\beta} \right]$$  \hspace{1cm} (21)

where $t_{b,\text{max}}$ corresponds to the breakout time of the minimum isotropic engine luminosity $L_{e,\text{min}}$, see equation (5). The distribution of $\gamma$-ray durations can be analytically derived in two limiting regimes (see also Bromberg et al. 2012):

(i) $t_\gamma \ll t_b$, where $t_b \approx t_0$. Using also $t_{b,\text{max}} \gg t_{e,\text{min}}$, equation (19) can be cast in the form:

$$p_\gamma(t_\gamma) \approx \frac{\beta_L (\beta_L - \alpha_L)}{\chi (\beta_L - \chi)} t_{b,\text{max}}^{-\chi}.$$  \hspace{1cm} (22)

which is independent of $t_\gamma$. For short enough $\gamma$-ray durations, $p_\gamma$ is set by the maximum breakout timescale and the power-law indices of the GRB luminosity function.

(ii) $t_\gamma \gg t_b$, where $t_b \approx t_0$. Equation (18) then results in:

$$p_\gamma(t_\gamma) \approx \frac{\beta_L (\beta_L - \alpha_L)}{\chi (\beta_L - \alpha_L) t_{\gamma,\text{max}}} \left( \frac{t_\gamma}{t_{\gamma,\text{max}}} \right)^{-\beta}.$$  \hspace{1cm} (23)

In this regime, the distribution of durations reflects the distribution of engine timescales ($p_\gamma \propto t_\gamma^{-\beta}$), in agreement with previous works (Bromberg et al. 2012; Sobacchi et al. 2017). However, in our study, the power-law slope of the engine time distribution is not a free parameter to be determined by comparison to the GRB duration distribution. It is instead uniquely determined by the power-law slopes of the GRB luminosity function (i.e. $\alpha_L, \beta_L$) and $\chi$ and can be used as a test of the model against the observed duration distribution of GRBs.

For a specific jet propagation theory ($\chi$ value) and GRB luminosity function, the only free parameter which enters the calculation of $p_\gamma(t_\gamma)$ is the maximum breakout time. This affects both the “plateau” and the turnover of the distribution $p_\gamma$, see equations (22) and (23). Thus, $t_{b,\text{max}}$ can be determined by fitting the model, equation (19), to the duration distribution of collapsar GRBs.

To derive the observed GRB duration distribution, we

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Table 1. Model parameters for jet propagation through the stellar envelope as determined by the GRB luminosity function and the collapsar GRB duration distribution. Results for two theoretically motivated $\chi$ values are shown. The power-law indices are defined in equations (1) and (6) $t_{b,\text{max}}$ is obtained from the observed $\gamma$-ray duration distribution, see Fig. 2. The parameter values of the GRB luminosity function are fixed to the best-fit values of Wanderman & Piran (2010).

| $\chi$ | $a$ | $\beta$ | $t_{e,\text{min}}$ | $t_{b,\text{max}}$ |
|-------|-----|--------|------------------|------------------|
| 1/3   | 2.4 | 4.6    | $t_0/\tau$       | 69 s             |
| 1/2   | 2.4 | 3.5    | $t_0/18$         | 47 s             |

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Figure 1. The distribution of engine luminosities, $dN/dL = (1/\eta) dN/dL_e$, assumed to be a power-law (blue dashed line), experiences a break as we consider only jets that can break out of the star to produce a distribution of successful GRBs, $dN_{\text{GRB}}/dL$ (red solid line). We match this distribution to that found by Wanderman & Piran (2010). A similar matching can be done for other luminosity functions. The inset shows the fraction of successful jets as a function of luminosity, see equation (11).
used the $T_{90}$ value of 1066 GRBs detected by the Swift satellite during the period of 2004–2017. Redshift information is available only for $\sim 1/3$ of the sample, with a median redshift of $z \approx 1.7$. This was adopted as the typical redshift of the entire Swift sample in order to construct the histogram of rest-frame $\gamma$-ray durations as $t_{90} = T_{90}/(1+z)$.

The normalized histogram of $t_{90}$ of all Swift GRBs is plotted in Fig. 2 with a black line. The corresponding histogram of Swift GRBs with measured redshifts is also shown for comparison (orange line). Overall, the two histograms are qualitatively similar with the main differences appearing at short durations. Regardless, GRBs with $t_{90} \lesssim 0.3 \text{ s}$ (i.e., short GRBs) do not have a collapsar origin and are neglected in this study. Overplotted as coloured curves are our model predictions with $t_{b,\text{max}} \approx 69 \text{ s}$ for $\chi = 1/3$ and $t_{b,\text{max}} \approx 47 \text{ s}$ for $\chi = 1/2$ (see also Table 1). Both provide very good quantitative descriptions to the collapsar GRB duration distribution. In fact, the plateau of the duration distribution is expected to be similar for models with the same product of $\chi$ and $t_{b,\text{max}}$, as long as $\beta_t \gg \chi$. More specifically, $p_t \propto (\chi t_{b,\text{max}})^{-1}$ at short enough durations, see equation (22). From the “fit” to the duration distributions with $\chi = 1/3$ and $\chi = 1/2$ we find $\chi t_{b,\text{max}} \approx T_{\text{max}}$ where $T_{\text{max}} \approx 23 \text{ s}$.

A timescale of $\sim 60 \text{ s}$ – beyond which the duration distribution turns into a steep power law – is also found by Sobachchi et al. (2017). The authors associate this timescale with a single breakout time assumed for all GRBs, whereas in our framework it is clearly related to the maximum breakout time. In addition, Sobachchi et al. (2017) fit their data in order to derive $\beta$, whereas in our model $\beta$ is uniquely determined by the GRB luminosity function and $\chi$ – see equation (23). A fit to the data is not guaranteed in our model. For instance, the observed $p_t$ distribution cannot be explained by luminosity functions with $\beta_t - \alpha_L < 1$ for $\chi$ in the range $1/3 - 1/2$, as motivated by theory (Bromberg et al. 2011a; Lazzati et al. 2012).

The maximum breakout time inferred from the observed GRB duration distribution can also be translated to a constraint between $t_0$ and a minimum luminosity for a given $\chi$ value. Using $\chi t_{b,\text{max}} \approx T_{\text{max}}$ together with equation (5), we find:

$$T_{\text{max}} \approx \chi t_{b,\text{max}} = \chi t_0 \left( \frac{L_{\text{min}}}{L_{\text{iso},0}} \right)^{-\chi}. \quad (24)$$

### 4 DISCUSSION

The shape of the $\gamma$-ray duration distribution of collapsar GRBs (i.e., plateau followed by a power-law) is generic and the result of: (i) the relation between the three timescales, i.e., $t_s = t_b - t_b$, which stems from the jet-stellar envelope interaction, (ii) at least one power-law distribution (either of $t_b$ or $t_s$), and (iii) a characteristic timescale set by either $t_b$ or $t_s$. The plateau in $p_t$ at short durations is an even more general result of the jet-envelope interaction model, as it only requires that $p_t$ is a smooth function of $t_b$ (Bromberg et al. 2012).

#### 4.1 Completeness of the GRB sample

In our analysis we assumed that the Swift GRB sample is complete with respect to the observed burst duration. In other words, for a fixed luminosity the detection of the burst does not depend on its duration. This is a simplifying assumption, for the minimum detectable flux of a GRB depends upon the exposure time $T$ as $T^{-1/2}$ (Baumgartner et al. 2013; Lien et al. 2016). For GRBs, the maximum exposure time is set by their duration. For simplicity, let us identify $T$ as the observed duration $T_{90}$. This implies that the flux limit for detection is higher for GRBs with shorter duration. Indeed, there is lack of luminous and short bursts (i.e., $L_{\text{iso}} > 10^{52} \text{ erg s}^{-1}$ and $T_{90} < 2 \text{ s}$) in the third Swift/BAT GRB catalog (see Fig. 25 in Lien et al. 2016). This can be the result of a genuine lack of GRBs with these properties or of unavailable redshift measurements. Regardless, these results suggest that the Swift/BAT sample is complete with respect to burst duration for long GRBs. Therefore, we do not expect our main conclusions to depend strongly on this assumption.

#### 4.2 Progenitors of collapsar GRBs

For a distribution of breakout times we showed that the only free parameter that controls $p_t(t_s)$ (both the plateau and position of the break) is the maximum breakout time. This, in turn, depends upon $t_0$ and $L_{\text{min}}$, see equations (5) and (24). Here, $L_{\text{min}}$ should not be interpreted as the intrinsic GRB minimum luminosity. In fact, the GRB sample becomes progressively less complete for lower isotropic GRB luminosities (e.g., Wanderman & Piran 2010). Thus,
the duration distribution probes GRBs with isotropic luminosities down to an “effective” minimum luminosity \( L_{\text{eff}} \). Substitution of \( L_{\text{eff}} \simeq 10^{51} \text{ erg s}^{-1} \) to equation (24) leads to a conservative constraint:

\[
\frac{t_0}{t_{\text{max}}} \approx \frac{10^5}{\chi} \left( \frac{L_{\text{eff},51}}{\eta^{-1}} \right)^{3/2},
\]

which translates to \( t_0 \sim 150 \text{ s} \) for all \( \chi \) values in the range \( 1/3 - 1/2 \) and \( t_{\text{max}} \sim 23 \text{ s} \). Values of \( t_0 \gg 20 \text{ s} \) cannot be easily reconciled with the scenario of jet propagation through a compact progenitor, because of the weak dependence that \( t_0 \) has on stellar properties (see Bromberg et al. 2011a, 2012). Our analysis suggests the existence of a low-mass envelope surrounding the GRB progenitor, as concluded independently by Sobacchi et al. (2017). The weak dependence of the breakout time on \( L_{\text{eff}} \) makes this conclusion quite robust.

Assuming a common \( t_0 \sim 150 \text{ s} \) for all GRB collapsars, we can further predict that the duration distribution will extend towards longer durations (i.e., larger \( t_{\text{max}} \)) as the sample becomes more complete at lower luminosities thanks to more sensitive future missions. Falsification of this prediction would provide indirect evidence for a distribution of \( t_0 \) among GRB progenitors, which lies beyond the scope of this paper.

4.3 Energetics of central engines

We next examine the energetics of central engines and their potential in powering a supernova explosion. All engines with \( L_e > L_{e,\ast} \equiv \eta^{-1} L_\ast \) produce successful GRBs. The associated energy density deposited to the stellar envelope by successful engines is

\[
E_{e,\ast} \approx f_b L_e t_b(L_e) \approx E_{e,\ast} \left( \frac{L_e}{L_{e,\ast}} \right)^{1-\chi},
\]

where \( f_b = 0.01 \) is the jet beaming fraction and

\[
E_{e,\ast} \equiv f_b L_{e,\ast} t_{e,\ast,\min}.
\]

The minimum engine timescale \( t_{e,\ast,\min} \) is given by equation (26). Failed jets can also inject an amount of energy to the stellar envelope. For \( L_{e,\min} \leq L_e \leq L_{e,\ast} \), the engine activity timescale \( t_e \) lies between \( t_{b,\min} \) and \( t_{e,\min} \). The latter is still a good proxy for the average \( t_e \) in the range \([t_{e,\min}, t_{b,\max}] \) because of the steepness of \( p_e(t_e) \). The energy of a failed GRB engine is then given by

\[
E_{e,t} \approx E_{e,\ast} \frac{L_e}{L_{e,\ast}},
\]

Figure 3 presents the beaming-corrected energy, \( E_{e,t} \), of successful and failed engines (see equations (26) and (28), respectively) as a function of \( L_e \) for \( \chi = 1/3 - 1/2 \) and \( t_0 = 150 \text{ s} \). For a typical GRB engine (i.e., \( L_e = L_{e,\ast} \)), we find

\[
E_{e,\ast} \approx (2 - 7) \times 10^{52} f_{b,\ast} \left( \frac{t_{b,\max}}{\eta^{-1}} \right)^{1-\chi} \text{ erg}
\]

for \( \chi = 1/3 - 1/2 \). This is comparable to the kinetic energy inferred in SNe associated to long GRBs (e.g., Hjorth & Bloom 2012) and provides further support that GRB-SNe are jet-driven (e.g., Hjorth 2013). Failed jets that inject \( \sim 10^{51} \text{ erg} \) (1 foe) to the stellar envelope – an energy comparable to the binding energy of the stellar envelope and the typical kinetic energy of core-collapse SNe – could facilitate the explosion mechanism in these SNe (Sobacchi et al. 2017; Piran et al. 2017), but they are not expected to drive mildly relativistic SNe ejecta (see e.g. Soderberg et al. 2010). The isotropic engine luminosity in this case is \( L_{e,\ast} = 10^{52} E_{e,51} f_{b,\ast}^{-1} t_{b,\min,1}^{-1} \text{ erg s}^{-1} \).

Jets can deposit a significant energy into the stellar envelope on timescales of tens of seconds (i.e., \( t \gtrsim t_{e,\min} \)), but it is unlikely that they can produce the necessary mass of radioactive \( ^{54}\text{Ni} \) to partially power the optical light curve of core-collapse SNe (i.e., \( t \sim 0.02 M_\odot - 0.2 M_\odot \)) (Hamuy 2003; Müller et al. 2017). Being a by-product of the explosive silicon burning process, \( ^{54}\text{Ni} \) can be synthesized under specific temperature and density conditions (e.g. Woosley et al. 1973; Woosley & Weaver 1995; Jerkestrand et al. 2015), which cannot be easily met once the ejecta has expanded far from the central engine.

Although failed jets cannot be the sole mechanism that drives the SN explosion, a picture emerges where they may be an important component of this process (e.g. Lazzati et al. 2012; Sobacchi et al. 2017; Piran et al. 2017). The limitation on jet-driven explosions stems from the energetic requirement of injecting \( \sim 10^{51} \text{ erg} \) and not from a lack of engines at these lower luminosities (see dash-dotted line in Fig. 3). The number of failed GRB engines with \( L_e < L_{e,\ast} \) can be written as \( N_{\text{f}}(L_e) \approx (L_e/L_{e,\ast})^{-\beta_e} \), where a normalization of one engine at luminosity \( L_{e,\ast} \) was assumed. The true rate of failed engines capable of injecting \( \sim 10^{51} \text{ erg} \) at typical \( \tilde{z} = 2 \) can be then estimated as:

\[
R_{\text{f}}(\tilde{z}) \approx N_{\text{f}}(L_{e,\ast}) R_{\text{GRB}}(\tilde{z}) f_{b,\ast}^{-1} \approx 10^5 \tilde{z}^{-4} \left( \frac{\eta^{-1} E_{e,51}}{t_{e,\min,1}^{-1} L_{e,52.5}} \right)^{-1.4} \text{ Gpc}^{-3} \text{ yr}^{-1}
\]

where \( R_{\text{GRB}}(\tilde{z}) \approx 10 \text{ Gpc}^{-3} \text{ yr}^{-1} \) is the observed (i.e., uncorrected for beaming) collapsar GRB rate (Wanderman & Piran 2010; Sobacchi et al. 2017). We also used \( \beta_e = 1.4 \) and assumed a redshift independent engine luminosity function. Interestingly, this order-of-magnitude
prediction is consistent with inferred rates of Type Ib/c SNe at $z \sim 2$ (e.g., Dahlen et al. 2004, 2012; Cappellaro et al. 2015; Strolger et al. 2015) suggesting an active role of the central engine in all Ib/c SNe (see also Sobacchi et al. 2017; Bear et al. 2017).

Our results, although consistent with those by Sobacchi et al. (2017), are derived with a very different method. The number and energetics of our failed jets, as well as the minimum engine timescale are naturally obtained by matching the distribution of successful engines with the GRB luminosity function, instead of relying on an extrapolation of the central engine duration distribution to an (almost) arbitrary value.

Our analysis was performed independently of the nature of the GRB central engine, which can be either a newly born black hole or a magnetar (i.e., a rapidly spinning highly magnetized neutron star), see, e.g., Kumar & Zhang (2015) for a review on GRB central engines. Our predictions for the energetics of central engines are summarized in Fig. 3 and can be used to constrain their nature. We showed that the beaming-corrected energy of a typical GRB engine that is active for $t_{\text{min}} \sim 10$ s is a few $10^{52}$ erg, see also equation (29). This might be already in tension with the magnetar scenario, although recent studies cast doubt on previous estimates on the maximum energy extractable in the magnetar model (Metzger et al. 2015). In addition, the distributions of the engine duration and luminosity are expected to differ between models of the central engine. In the magnetar scenario, for example, where the luminosity and duration of the engine depend both on the magnetic field and spin period of the magnetar (e.g., Usov 1992), the distributions of $L_{\text{env}}$ and $t_e$ will be correlated. A careful comparison of our model predictions against several observational constraints, such as the GRB distribution on the $L - T_{\text{90}}$ plane, will be the subject of a follow-up study.

5 CONCLUSIONS

The propagation of the jet through the envelope of a collapsing star, which acts as a filter of less powerful jets, may explain the observed GRB luminosity function and the GRB duration distribution. Our work helps to better understand the properties of long GRB central engines, while it provides interesting hints for the jet-driven SN scenario.

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