Fringe spacing and phase of interfering matter waves

O. Vainio,1,2 C. J. Vale,1 M. J. Davis,1 N. R. Heckenberg,1 and H. Rubinsztein-Dunlop1

1School of Physical Sciences, University of Queensland, St Lucia, Qld 4072, Australia
2Department of Physics, University of Turku, FIN-20014, Turku, Finland

(Dated: September 21, 2018)

We experimentally investigate the outcoupling of atoms from Bose-Einstein condensates using two radio-frequency (rf) fields in the presence of gravity. We show that the fringe separation in the resulting interference pattern derives entirely from the energy difference between the two rf fields and not the gravitational potential difference. We subsequently demonstrate how the phase and polarisation of the rf radiation directly control the phase of the matter wave interference and provide a semi-classical interpretation of the results.

PACS numbers: 03.75.Hh, 03.75.Be, 39.20.+q, 03.75.Pp

Ever since the first realisations of Bose-Einstein condensates (BECs) in dilute atomic gases, their coherence properties have been the subject of much investigation. The first clear demonstration that BECs possess long range phase coherence was through the interference of two spatially separated condensates [1]. Since then, other experiments have studied the coherence properties using atom laser output from an array of tunnel coupled condensates in an optical standing wave [2], Bragg spectroscopy [3], density fluctuations [4] and interference [5, 6]. An elegant scheme to probe condensate coherence, based on interferating atom laser beams, was reported by Bloch et al. [7, 8]. This used two radio frequency (rf) fields to outcouple atoms from different locations within a condensate. A high contrast matter wave interference pattern was observed at temperatures well below the BEC transition temperature, confirming the phase coherence of the condensate. A numerical model of two outcoupled modes agreed with the experimental observations [9]. More recently, the atom-by-atom build up of a matter wave interference pattern has been observed using single atom detection [10]. To date, experiments have primarily focussed on the visibility of the interference patterns. In this paper, we describe experiments which address the fringe spacing, phase and nature of the interference.

Outcoupling atoms from Bose-Einstein condensates with rf fields has been used extensively to produce beams of atoms, generally referred to as “atom lasers” [11, 12, 13]. The rf radiation drives resonant (stimulated) transitions from a trapped Zeeman sublevel to an untrapped state in which the atom falls under gravity. Outcoupling occurs at locations where the total energy difference between the trapped and untrapped states is equal to $\hbar \omega_{rf}$. This is usually determined by the Zeeman potential so that the resonant condition may be written

$$\hbar \omega_{rf} = \mu_B g_F |B(r)|$$

where $\mu_B$ is the Bohr magneton, $g_F$ is the Landé g-factor and $B(r)$ is the magnetic field.

Consider a condensate trapped in a cigar-shaped magnetic potential of the form $U(r) = m \omega_0^2 (\kappa_x^2 x^2 + y^2 + z^2)/2$ where $m$ is the mass of the atom, $\omega_0 = \omega_y$ is the trapping frequency in the tight directions of the trap and $\kappa = \omega_z/\omega_y$. Atoms can be outcoupled from the surface of an ellipsoid of the magnetic equipotential which satisfies the resonance condition (1). However, gravity will cause a displacement of the minimum of the total potential from the magnetic field minimum. This gravitational sag means a harmonically trapped condensate will be displaced from the magnetic field minimum by a distance, $z_0 = -g/\omega_z^2$, where $\omega_z$ is the trapping frequency in the direction of gravity. This displacement is typically greater than the size of the condensate, so that the ellipsoidal equipotential surfaces can be approximated by planes which intersect the condensate at different heights, $z$. In this situation, the dependence on the $x$ and $y$ coordinates can be neglected for many quantitative purposes and only the $z$ dimension need be considered.

In previous work [7, 8], two rf fields of frequencies, $\omega_1$ and $\omega_2$, were used to outcouple atoms from a condensate. The two spatially separated resonances were interpreted as creating two slits from which atoms were extracted from the condensate. The outcoupled atoms formed two matter waves which interfered, in close analogy with a Young’s double slit experiment. The visibility of the interference pattern provided a measure of the first order phase coherence of the condensate.

The outcoupling points, $z_1$ and $z_2$, used in [7, 8], were chosen to be centred around the middle of the condensate, located at $z_0$, the minimum of the combined magnetic (harmonic) and gravitational (linear) potential. Under this condition, the gravitational energy difference between the two outcoupling points, determined by the slit separation, $\Delta z = z_1 - z_2$, is exactly equal to the difference in energy between the two applied rf fields. This can easily be seen from the derivative of the magnetic potential, where $\Delta E \approx m \omega_0^2 z \Delta z$. At the central position, $z_0$, we find $\Delta E \approx \hbar (\omega_1 - \omega_2) = m g \Delta z$.

However, this result is only true when $z = z_0$, (where $z = (z_1 + z_2)/2$, is the distance from the magnetic field minimum to the centre of the two slits). If the
 gravitational energy difference

outcouple atoms from the BEC at different locations. The lines represent two pairs of rf fields, with equal ∆m,

however, in dual rf outcoupling experiments with a fixed

in the untrapped state is included, the total energy difference between the two outcoupled matter waves is always equal to ℏω.

The fringe spacing of the interference pattern depends on the total energy difference between the two indistin-

gual and mean field energies. The untrapped (outcou-

The fringe spacing, λ, of the interference pattern is proportional to ∆ω and may change significantly across the width of a condensate. For a harmonic potential, the slit separation is approximately

In a Young’s double slit experiment, the fringe spacing, λ, of the interference pattern is proportional to ∆z. However, in dual rf outcoupling experiments with a fixed ∆ω, λ is independent of ∆z. While the gravitational energy difference, mg∆z, between the two resonant locations can change, this is not the only energy to consider. The fringe spacing of the interference pattern depends on the total energy difference between the two indistinguishable outcoupling paths and must always equal ℏω to satisfy energy conservation. This is a general result which we discuss below, for a BEC in the Thomas-Fermi (TF) regime.

Consider the specific case of an F = 1 87Rb TF condensate. The total energy of the trapped state |F = 1, mF = −1⟩ consists of the sum of its magnetic, gravitational and mean field energies. The untrapped (outcou-

pled) state |1, 0⟩ experiences negligible magnetic potential, but, while still within the condensate, experiences both the mean field and gravitational potentials. In 1D, the total energy of a particular substate can be written as

\[ E_{m_F}(z) = -m_F\frac{1}{2}m\omega_z^2z^2 + \mu_B g_FB_0 - mgz + g_{1d}|\psi(z)|^2 \]

(3)

where \( B_0 \) is the magnetic field at the minimum of the trap, \( g_{1d} \) is the 1D effective interaction strength (assumed to be the same for all \( m_F \), which is approximately true but not an essential point in this discussion) and \( |\psi(z)|^2 = \sum_{m_F} |\psi_{m_F}(z)|^2 \) is the total atomic density.

The energies of the trapped \(|1, -1⟩\) state and the untrapped \(|1, 0⟩\) state are plotted (solid lines) in Fig. 1a for the parameters used in our experiments. The shaded regions indicate the mean field contribution to the total energy. Also shown are two pairs of rf fields, dashed and dashed-dotted lines, with the same ∆ω, chosen to lie within the width of the condensate. As the two pairs are centred around different \( z \), the resulting ∆f for each pair is different, but the total energy difference is \( ℏ\Delta ω \).

In the TF limit the interaction energy between the condensate and the outcoupled state exactly compensates for the difference in gravitational potential at different slit locations. The density profile, \( |\psi(z)|^2 \), mirrors the shape of the magnetic trapping potential so that the energy splitting between the two states is always given by the difference in their magnetic potentials. Additionally, the energy of trapped atoms within the condensate is independent of \( z \) so that only the final energies on the \( m_F = 0 \) curve determine the energy difference between the two outcoupled beams.

Having established that the fringe spacing, \( \lambda \), depends only on \( \Delta ω \), it can easily be shown that

\[ \lambda(z) = \sqrt{2g(z - z_0)} \frac{\Delta f}{\Delta f} \]

(4)

where \( \Delta f = \Delta ω/2\pi \), \( \lambda \) varies with \( z \) because the outcoupled atoms accelerate in the \( z \)-direction under gravity, as can be seen in Fig. 1b.

We have performed a range of experiments to verify this for several values of \( \Delta ω \) with the resonant points centred around various \( z \) positions. Our experimental procedure for producing condensates has been described elsewhere [14] and was used here with only slight modifications. An atom chip is used to produce near pure condensates containing \( 2 \times 10^5 \) 87Rb atoms in the \( |1, -1⟩ \) ground state. Our chip design facilitates the production of relatively large condensates in highly stable trapping fields. The final trapping frequencies are 160 Hz in the tight direction and 6.7 Hz in the weak direction. The elongated geometry of the trap means we must cool well below the 3D critical temperature \( T_c \) to produce fully phase coherent condensates (typically \( T_c \) for our parameters is less than \( T_c/2 \) [17]). Outcoupling is induced by turning on two rf fields of the same amplitude, with frequencies \( \omega_1 \) and \( \omega_2 \) tuned to be resonant with atoms in
the condensate, and Rabi frequencies, $\Omega = \mu_B g_F |B|/2\hbar$, of 50 Hz for each rf source. After outcoupling for 10 ms, the trap is left on for a further 3 ms before being turned off abruptly. An absorption image is taken after 5.3 ms of free expansion.

We first checked the reproducibility of the fringe spacing for a fixed $\Delta \omega$ of $2\pi \times 1000 \text{s}^{-1}$ as $\Delta z$ was varied over the range 390 nm to 560 nm by varying $\dot{z}$. While the visibility of the interference pattern decreased near the edges of the condensate, the wavelength was consistent with the value predicted by equation (4) to within 1%. Experimental values of $\lambda$ were determined by taking an image of the outcoupled atoms, similar to that shown in Fig. 1(b), converting the $z$ spatial axis into a time axis through the relation $t = \sqrt{2(z - z_0)/g}$, integrating the output over $x$ and fitting a $\cos^2$ function to the data. The uncertainty in $\lambda$ is determined by the uncertainty in the fitted frequency of the $\cos^2$ function. Variations at the level of 1% are within our experimental uncertainties and not significant when compared to what would be expected if $\lambda$ was determined by $\Delta z^{-1}$. This would lead to a variation of more than 30% across the range of values we measured.

A semi-classical interpretation of these dual rf outcoupling experiments, based on the interference of the applied rf fields, may also be used to understand these experiments. Defining the mean frequency, $\bar{\omega} = (\omega_1 + \omega_2)/2$, and the beat frequency, $\delta = (\omega_1 - \omega_2)/2$, and recalling the standard trigonometric identity

$$\sin \omega_1 t + \sin \omega_2 t = 2 \sin \bar{\omega} t \cos \delta t$$

we see that the sum of two oscillating fields is equivalent to an amplitude modulated carrier wave. In our case, the carrier frequency, $\bar{\omega}$, is typically three orders of magnitude higher than the beat frequency, $\delta$. Adding a phase, $\phi$, to one of the rf fields shifts the phase of both the carrier and beating terms by half this amount.

We may now consider the condensate interacting with a single rf field at the carrier frequency, which is amplitude modulated in time. The number of atoms outcoupled is proportional to the Rabi frequency squared (ie. proportional to the amplitude squared of the rf field at time $t$) and is modulated at $2\delta = \Delta \omega$. Once outcoupled, the atoms fall under gravity and, provided they have a low spread of initial momenta, the outcoupled density will be modulated in time.

To demonstrate this, a sequence of dual rf outcoupling experiments was performed using fixed values $\omega_1$ and $\omega_2$ ($\Delta \omega = 2\pi \times 500 \text{s}^{-1}$) but varying the relative phase of the two rf fields. This has the effect of shifting the phase of the rf beat note. All other experimental parameters were kept fixed. Two examples of the data obtained are shown in Fig. 2. Absorption images of the outcoupled atoms appear on the left and the beat note of the corresponding rf fields used for outcoupling (measured on an oscilloscope) are shown on the right. The $z$ axis of the absorption images has been rescaled by $\sqrt{2(z - z_0)/g}$ to linearise the time axis for ease of comparison with the rf.

It is clear from these images that the outcoupled atoms correspond to the largest amplitude of the rf beat note.

In order to quantify this, we have analysed a series of similar data in which the relative phase of the rf fields was allowed to vary randomly over the range 0 to $2\pi$. The fitting procedure described earlier was applied to all of the absorption images to determine the phase of the outcoupled beam. A similar fit was applied to the square of the measured rf beat note and the two phases are plotted against each other (filled circles) in Fig. 3. The dashed line through this data is a plot of $y = x$. The phase of the modulated atom beam matches very well the phase of the applied rf field.

In these experiments both rf fields were provided by passing the two rf currents through the same coil. This means the rf field in the vicinity of the BEC was linearly polarised, perpendicular to the quantisation axis $z$. We have also performed experiments using separate, orthogonally mounted coils where each rf current was sent through a different coil. The rf interference is no longer linearly polarised but rather a field whose polarisation varies from vertical linear, to left hand circular, to horizontal linear, to right hand circular and back to vertical linear in a single beat period ($T = 1/\Delta \omega$). This is analogous to the optical field used in lin $\perp$ lin sub-Doppler (Sisyphus) laser cooling [16] but the field is periodic in time rather than space.

The outcoupling transition from $|1, -1\rangle$ to $|1, 0\rangle$ requires $\sigma^+$ radiation. When the rf is derived from a single coil, it is easy to see that the maximum amplitude of

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure2.png}
\caption{The phase of the interfering matter wave beams is determined by the phase of the beating rf fields used in to drive the outcoupling. (a) and (b) represent different runs of the experiment under identical conditions apart from different phases of the applied rf field. On the left are absorption images of the condensate (top) and outcoupled atoms and on the right is the beat note of the corresponding rf used to drive the outcoupling, measured on an oscilloscope. The vertical axis indicates the time before the image was taken and was obtained for the atom images through the relationship, $t = \sqrt{2(z - z_0)/g}$.}
\end{figure}
rf fields performed with (near) perpendicularly oriented phase of the atom laser versus the phase of the beating out of phase. The dashed line is a plot of against the phase of the beating rf field. Filled circles represent experimentally measured data using a single coil and the dotted line is a plot of against the phase of the beat note. This happens when the rf fields are counter rotating circular fields and it is the component of which couples to the atom. With perpendicularly oriented coils however, the maximum amplitude of the field occurs during the circular polarised phase of the beat note. This happens when the rf fields are out of phase. Also shown in Fig. (open squares) is a plot of the phase of the atom laser versus the phase of the beating rf fields performed with (near) perpendicularly oriented coils. The atomic output is phase shifted by approximately from the rf as expected. The slight mismatch between the measured shift of and the expected shift of due to imperfect alignment of the rf coils (precise perpendicular alignment would have impeded optical access in our setup). For coils mounted antiparallel maximum outcoupling would occur when the two rf fields are out of phase.

In conclusion, we have studied the origins of the fringe spacing and phase of matter wave interference patterns, produced by outcoupling atoms from a BEC with two rf fields. We have shown that the energy difference between the two rf fields determines the spacing of the interference pattern, not the gravitational potential difference determined from the classical slit separation. Semi-classical arguments based on interfering rf fields correctly predict the experimental observations. These also show how the phase and polarisation of the rf field determine the phase of the observed matter wave interference pattern. This extends previous work which looked at the fringe visibility for the specific case where , Our findings do not contradict the phase coherence studies reported in ref. Indeed, any random phase gradients within the BEC would lead to random initial velocities that would degrade the observed interference patterns. Finally, we note that we have also performed experiments with cold thermal atoms and see (as in ) that the visibility of interference pattern diminishes, due to the thermal spread of velocities in the trapped gas.

We acknowledge valuable discussions with Craig Savage, Ashton Bradley and Murray Olsen and technical assistance from Evan Jones. O. V. acknowledges financial support from the Jenny and Antti Wihuri Foundation and the Academy of Finland (project 206108). This work was supported by the Australian Research Council.

[1] M. R. Andrews, C. G. Townsend, H. J. Miesner, D. S. Durfee, D. M. Kurn, and W. Ketterle, Science 275, 637 (1997).
[2] B. Anderson and M. Kasevich, Science 283, 1686 (1998).
[3] J. Stenger, S. Inouye, A. P. Chikkatur, D. M. Stamper-Kurn, D. E. Pritchard, and W. Ketterle, Phys. Rev. Lett. 82, 4569 (1999).
[4] S. Dettmer, D. Hellweg, P. Ryttty, J. J. Arlt, W. Ertmer, and K. Sengstock, D. S. Petrov, G. V. Shlyapnikov, H. Kreutzmann, L. Santos and M. Lewenstein, Phys. Rev. Lett. 87, 160406 (2001).
[5] D. Hellweg, L. Cacciapuoti, M. Kottke, T. Schulte, K. Sengstock, W. Ertmer, and J.J. Arlt, Phys. Rev. Lett. 91, 010406 (2003).
[6] M. Hubgart, J. A. Retter, F. Gerbier, A. Varon, S. Richard, J. H. Thywissen, D. Clement, P. Bouyer and A. Aspect, Eur. Phys. J. D 35, 155 (2005).
[7] I. Bloch, T. W. Hänsch and T. Esslinger, Phys. Rev. Lett. 82, 3008 (1999).
[8] Y. Le Coq, J. H. Thywissen, S. A. Rangwala, F. Gerbier, S. Richard, G. Delannoy, P. Bouyer and A. Aspect, Phys. Rev. Lett. 87, 170403 (2001).
[9] N. P. Robins, C. M. Savage, J. J. Hope, J. E. Lye, C. S. Fletcher, S. A. Haine and J. D. Close, Phys. Rev. A 69, 051602(R) (2004).