Wave Function Mismatches and Coulomb Drag

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In this paper I study the topological excitations in a pairing state in double layer systems at \( \nu = 1/2 \) in the presence of disorders. Due to mismatches between single particle wave functions of composite Fermions in different layers, the sensitivity of the Chern number of the pairing composite Fermion state, with respect to changes of impurities, is infinity. Consequently, Goldstone mode in this pairing state is strongly damped at low temperature. This leads to a unique temperature dependence of the drag resistance at low temperature.

\[
\frac{\kappa^c(q)}{\kappa^cF(q = 0)} = (1 + \frac{k_F^2 N_{se}}{4q^2 N_e})^{-1}.
\]  

Here \( \kappa^c, CF \) are the irreducible compressibility of electrons and composite Fermions respectively at wave vector \( q \); \( N_{se}, e \) are the composite Fermion superfluid density and electron number density. Following Eqs.1,2, \( R_{11} = R_{12} \) when composite Fermions have an off-diagonal long range order \( \langle \sigma_{cond} = \infty \rangle \) and the double layer system becomes incompressible, i.e. \( \kappa^*(0) = 0 \).

In the following I consider the topological excitations in a pairing state of composite Fermions in a disordered double layer system. Due to mismatches between single particle wave functions for composite Fermions in different layers, the pairing state wave function in the presence of different impurity configurations is topologically different. Furthermore, topological excitations have a continuous energy spectrum. These low lying topological excitations lead to a novel coupled double layer liquid, characterized by

\[
R_{11} - R_{12} = \frac{1}{\sigma_{cond} \propto T^2}
\]  

at low temperature limit. Eq. 3 is valid only in the strongly coupled limit when \( \sigma_{cond} \gg \sigma_{quasi} \).

To obtain these results, let me first consider a double layer 2DEG where the impurity scattering potential in one layer \( V_1(r) \) is different from the impurity potential in another layer \( V_2(r) \). The correlation of these two sets of impurity potentials is determined by

\[
\theta_{12} = 1 - \frac{\langle V_1(r) V_2(r) \rangle}{\langle V_2^2(r) \rangle}.
\]  

Furthmore, I assume short range impurity potential \( s^{-1} < V_1(r) V_1(r') > = s^{-1} < V_1(r) V_2(r') > = \delta(r-r') V_0^2, s \) is the area of the sample. When \( V_1, V_2 \) are completely (un)correlated, \( \theta_{12} = (1)0 \).

Let me introduce the composite Fermion pairing wave function in a general form: \( \Delta_1(r, R) = \psi_1(R + r/2) \psi_2(R - r/2) > \chi(1,2), \psi_{1,2} \) are the composite Fermion operators in layer 1, 2 and \( \chi(1,2) \) is the pair wave function in layer index space. It satisfies the gap equation generalized to a double layer system with an interlayer attractive interaction constant \( \lambda_{12} \).

Coulomb drag in double layer systems with Landau level filling factor \( \nu = 1/2 \) in each layer has attracted much attention for the past few years. [5] [6] [7]. When the distance between layers \( d \) is less than the magnetic length \( L_H \), numerical result of [5] shows that the ground state can be approximated by a \{111\} state proposed in [1]; when \( d \gg L_H \), it was argued that the interlayer attractive interaction mediated by the Chern Simons field leads to an S-wave pairing BCS state of composite Fermions at half filled Landau levels [8]. This is a generalization of the idea of having an incompressible quantum Hall liquid with even-denominator filling factor discussed some years ago [9-12]. Attempts have been made to identify these incompressible states proposed in [9] with pairing composite Fermion states. As pointed out in [3], \{331\} state in [1], and Pfaffian states in [11,12] can be expressed in terms of \( p \)-wave pairing composite Fermions states with total spin (in the layer index space in the present case) equal to zero along different directions; while the hollow-core state in [10] can be connected with a singlet \( d \)-wave pairing.

These considerations suggest that low energy transport in double layer systems at \( \nu = 1/2 \), in general, is determined by the properties of a pairing composite Fermion state. Especially, in the absence of tunneling between two layers and at zero temperature, the Coulomb drag in a pairing state is nonzero [5]. In contrast, the drag between two Fermi liquids vanishes as the temperature \( T \) goes to zero. The in-plane resistance \( R_{11} \) and the interlayer drag resistance \( R_{12} \) in double layers in a pairing state of composite Fermions can be expressed as [5]

\[
R_{11,12} = \frac{1}{2\sigma_{quasi}} \pm \frac{1}{2\sigma_{cond}}.
\]  

For an S-wave pairing state, \( \sigma_{quasi} = g \exp(-\Delta/kT) \) is the conductivity from thermally excited quasiparticles; \( g = k_F^2 e^2 / h \) and \( \Delta \) is the quasiparticle energy gap of the pairing state, or the mobility gap when the quasiparticles are localized. \( \lambda \) is the elastic mean free path of the composite Fermions in the presence of disorders, much longer than the Fermi wave length \( k_F^{-1} \). \( \sigma_{cond} \) is from the composite Fermion condensate, as defined in Eq.12.

Meanwhile,
\[ \Delta(r, R) = \int dr' dr'' dR' \lambda_{12}(r, r'') \int de [1 - 2n_F(e)] \Delta(r', R') \]
\[ [G^{1R}_{d}(R + r''/2, R' + r''/2)G^{2A}_{d}(R - r''/2, R' - r''/2)] + G^{00}_{d}(R + r''/2, R' + r''/2)G^{1A}_{d}(R - r''/2, R' - r''/2)] \]

Here \( n_F(e) \) is the Fermi distribution function; \( G^{1R,2R}_{d}(r, r') = G^{1A,2A}_{d}(r, r') \) are the retarded(advanced) exact Green function in layer 1 and 2 in the absence(presence) of pairing potential. Depending on the form of \( \lambda_{12} \), the order parameter would have different symmetries. In a clean limit, when \( \lambda_{12}(r, r') = \lambda_p \delta(r) \delta(r') \), \( \Delta(r, R) = \delta(r) \Delta_a(R) \chi^A(1, 2) \) corresponding to an S-wave state. When \( \lambda_{12}(r, r') = \lambda_p k_F^{-2} \nabla_r \delta(r) \nabla_r \delta(r') \), \( \Delta(r, R) = \nabla_r \delta(r) \Delta_m(R) \chi^S(1, 2) \) representing a p-wave state, \( i = x, y, \chi^S(1, 2) = \chi^S(2, 1) \) and \( \chi^A(1, 2) = -\chi^A(2, 1) \). Without losing generality, I will restrict myself to the S-wave pairing composite Fermion state. The final conclusion can be extended to a p-wave pairing state.

Consider the exact Green function at \( \epsilon = 0 \) and \( \delta a = 0 \) in 2D. \( G_{00}^{1R,2R}(r, r') = \cos(k_F r - r' - \pi/2 + \phi_{12}(r)) \times m/\sqrt{k_F \|r - r'\|} \) with \( \phi_{12}(r) \) as the random phase shift at point \( r \) in the presence of impurity potential \( V_{12}(r) \). By the same token, in a given sample, \( G_{00}^{1R}(r, r')G_{00}^{2A}(r, r') \propto |r - r'|^{-1} \cos(\phi_1(r) - \phi_2(r)) \), with the fast oscillating phase factor \( k_F \|r - r'\| \) cancelled out. When \( V_{12}(r) \) is not completely correlated with \( V_2(r) \), \( \phi_1(r) - \phi_2(r) \) is finite. \( G_{00}^{1R}G_{00}^{2A} \) consists a series of nodes at points \( r \) where \( \phi_1(r) - \phi_2(r) = (n + 1/2)\pi \), due to mismatches of two sets of single particle wave functions at the Fermi energy in a double layer system. However, the presence of these nodes only reveals itself via an exponential decay of Green functions with a given sign when the ensemble averaged is taken \([12]\). For example, \( \int d\epsilon < G_{00}^{1R}(r, r')G_{00}^{2A}(r, r') > = n2k_F^{-2} |r - r'|^{-1} \exp(-\theta_{12} \|r - r'\|/l), < > \) represents the impurity average. Consequently, following Eq.5, at \( T = 0 \) the self consistent equation doesn’t have a homogeneous solution with \( \delta a = 0 \) and \( \Delta_a(r) \) a constant when \( \theta_{12} \geq \theta_c = \tau \Delta_0(\ll 1) \); \( \Delta_0 \) is the order parameter in the absence of disorders and \( \tau \) is the elastic scattering time.

To account random signs of Green functions in the presence of mismatches of wave function, one has to take into account higher order contributions in term of \( (k_F l)^{-1} \); this is beyond the conventional approximation used in the theory of disordered superconductors \([13]\).

Performing an expansion of the self consistent equation around \( \theta_c \) diagrammatically (See Fig.1), I obtain for \( \Delta_a \),
\[ [\xi_0^2 \nabla^2 + \delta \theta_{12}/\theta_c] \Delta_a(r) + \int dr K_s(r, r') \Delta_a(r') = |\Delta_0^2| \Delta_a(r), \]
\[ < K_s(r, r') > = 0, \]
\[ < K_s(r_1, r_1')K_s(r_2, r_2') > = \frac{1}{16g^2} |\delta(r_1 - r_2)\delta(r_1' - r_2')/|r_1 - r_1'|^3| \]
\[ + \frac{\xi_0^2}{|r_1 - r_1'|^3}. \]

Here \( \nabla = \nabla - i\delta a, \delta \theta_{12} = \theta_{12} - \theta_c, \xi_0 = \sqrt{D/\Delta_0}. \)

Meanwhile, \( \delta a = a + A \) is the Chern Simons vector potential and \( A \) is the vector potential of the external magnetic field; \( \nabla \times \delta a = |\Delta_a^2| - |\Delta_a^2| \), which shows that the variation of pairing wave function results in \( \delta a \). \( (\Delta_a^2) \) is the uniform solution of Eq.6.) When \( \theta_c = 0 \), Eq.6 yields a vortex solution with the size \( \sqrt{\theta_c - \theta_{12}} \). The energy cost of a vortex excitation is \( g \Delta_0(\theta_c - \theta_{12}) \) for the paired composite Fermion state coupled with a Chern-Simons field. Coulomb energies associated with these topological excitations are negligible because they extend over a distance much larger than the magnetic length.

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**Fig.1.** Solid lines are electron Green functions and dashed lines are impurity scatterings. 1, 2 are layer indices. Diagrams in b) stand for a Hikami Box. Diagrams in c) represent the Landau Ginsburg expansion while the diagram in d) is the typical contribution of \( K_s(r, r') \).

Eq.6 is a stochastic Landau Ginzburg equation with \( K_s \) counting spatial random oscillations of \( G_{00}^{1R}G_{00}^{2A} \) at a length scale much larger than \( 1/\theta_{12}. \) \( 1/g^2 \) factor in the variance of \( K_s(r, r') \) in Eq.6 reflects the Wigner-Dyson level statistics; namely, the fluctuation of number of levels within an energy band of Thouless energy of size of coherence length is unity, regardless of the corresponding average number of levels \( g \) \([10]\).

The statistical property of solutions of this equation was discussed previously in the context of a dirty superconducting film in Clogston limit \([17,18]\). At \( \theta_{12} - \theta_c \geq \theta_c/g \), pairing takes place at a length scale \( L_d = \)
A typical topological excitation. 0, π, 2π stand for the lift of the condensate phase in those regions.

First let me point out that \( \mathcal{N}_T \) of the ground state is a nonzero sample specific quantity. Consider a condensate wave function, which is topologically different from the ground state with \( \mathcal{N}_T' \neq \mathcal{N}_T \). Physically, this new condensate wave function can be obtained by adding a phase \( \pi \) to the condensate wave function in the region enclosed by contour \( C_1 \), 2π phase factor to the region enclosed by contour \( C_2 \) while keeping the phase in the region enclosed by \( C_3 \) unchanged. The remaining region is enclosed by a contour \( C_4 \), which has three extremely narrow stripes, \( s_1, s_2, s_3 \). \( \mathcal{N}_T' \) is the summations of four contour integrals defined in Eq.7 along \( C_1, C_2, C_3, C_4 \).

Obviously, the energy cost of the new configuration is from singular narrow strips \( s_1, s_2 \), along which the phases jump by ±π. Energy of such a configuration is \( |J|L^{1/2} \), with \( L^{1/2} \) originating from the summation of random exchange interaction \( \{J\} \) along the stripes. \(|J|\) is a random positive quantity. Let the probability distribution function be \( P(|J|) \), which is continuous at \( J = 0 \). For a given \( L, LP(J_{min})J_{min} \approx 1 \). Therefore \( J_{min} \approx L^{-\gamma} \) with \( \gamma = 1 \). The typical energy of topological excitations is proportional to \( L^{1/2-\gamma} \). \( \gamma \geq 1/2 \) means excitations are of finite energies, as suggested in [13].

Now let me introduce \( \{J' = J + \delta J\} \). The energy cost of a configuration with \( \mathcal{N}_T \) considered above acquires an additional contribution of a random sign from the contour \( C_1 \), of order of \( \delta J L^{1/2} \). For a given change of \( \delta J \), we can always choose a \( L \) such that \( J_{min}(L) \approx L^{-\gamma} \ll \delta J \). In this case, the new ground state belongs to a homotopy class different from that of \( \mathcal{N}_T \). Excitations of different \( \mathcal{N}_T \) with minima energies of a given homotopy class correspond to topologically distinct metal stable states. The existence of these low energy metal stable states suggests possible hysteresis.

Let \( \theta \) be \( 1 - V_1(r)V_1'(r') + V_2(r)V_2'(r') \). Excitations of different \( \mathcal{N}_T \) with minima energies of a given homotopy class correspond to topologically distinct metal stable states. The existence of these low energy metal stable states suggests possible hysteresis.

\[
\lim_{\theta \to 0} < \frac{\mathcal{N}_T(V_1, V_2) - \mathcal{N}_T(V'_1, V'_2)}{\theta^2} > = \infty. \tag{8}
\]

By contrast, the sensitivity of the Chern number defined in Eq.8, is zero for a normal mesoscopic sample, a BCS superconductor or a quantum Hall state.

Infinity sensitivity implies that there are gapless topological excitations. To account this, we introduce

\[
\frac{\partial \mathcal{N}_T}{\partial t} = -\frac{\mathcal{N}_T}{\tau_{\kappa}}, \tag{9}
\]

with \( \tau_{\kappa}^{-1} \) as the thermal activation rate of all the possible topologically distinct excitations. The thermal activation rate for a given barrier \( E \) is \( c T \exp(-E/kT) \), \( c \) is a number of order unity; the distribution of the finite energy barrier \( E \) is assumed to be \( P_B(E) \). Naturally, in the presence of states nearly degenerating with the ground state, \( P_B(0) > 0 \). As a result,

\[
\frac{1}{\tau_{\kappa}} = c T \int dV \exp(-\frac{E}{kT}) P_B(E) \approx c T^2 P_B(0), \tag{10}
\]

which is a power law function of \( T \) instead of an exponential function at low temperature limit. As far as \( \tau_{\kappa} \) is longer than any microscopic time scale, Eq.9 represents the low energy dynamics.

Consequently, collective modes become strongly damped because of thermally excited topologically different excitations. The situation here is similar to the gauge theory in the hydrodynamic limit proposed for Heisenberg spin glass [20]. The density-fluctuation and current-current fluctuation can be calculated as [21]

\[
\pi_{dd} = \frac{me^2}{\hbar^2} \frac{v_s^2 q^2}{\omega^4 + i \omega \tau_{\kappa}} - \omega^2, \quad \pi_{cc} = \frac{N_s e^2}{m} i \omega + \omega^2. \tag{11}
\]

which agree with \( \pi_{dd}, \pi_{cc} \) obtained in [1] when topological excitations are absent. \( N_s \) and \( v_s \) are the superfluid
density and the zero sound velocity respectively in the absence of topological excitations. The conductivity of this state is given as

$$\sigma_{\text{cond}} = \lim_{\omega \to 0, \mathbf{q} \to 0} \frac{i \omega}{g^2} \pi_{dd}(i \omega, \mathbf{q}) = \frac{e^2 m}{\hbar^2} \nu_0^2 \tau_e(T). \quad (12)$$

Substituting the results in Eqs.10, 12 into Eq.1, we obtain Eq.3. Strictly speaking, at zero temperature $\tau_e^{-1}$ is nonzero, determined by the quantum tunneling; it is however exponentially small when $q \gg 1$.

The change of the barrier $\delta E$ in the presence of a transport current $I$ can be estimated according to minimal coupling principle. $\delta E = \hbar/e \int d\mathbf{x} \cdot \nabla \chi \sim \hbar I/e$, with $j$ the current density and $\nabla \chi$ the phase gradient of the condensate. When $\delta E$ is of the order of $kT$, the typical energy of topological excitations, the nonlinear effect becomes important. This yields a characteristic current $I_{\text{on}} \propto kT \hbar/e^2$, at which $\sigma_{\text{cond}}$ is reduced by a factor of $1/2$ of that given in Eq.12. It implies that $R_{12}(I > I_{\text{on}}) \ll R_{12}(I = 0)$. On the other hand, at this current, the raise of quasiparticle temperature due to Joule heat, $\delta T$, can be estimated as $\sigma_{\text{ques}} E^2 \tau_{\text{ph}}/C_e$, with $E$ as the applied electric field, $\tau_{\text{ph}}$ the inelastic scattering time, $C_e$ the electronic specific heat. It is of order of $T^2 g^2 \times L_{\text{ph}}/L^2$, vanishing in the limit when $L_{\text{ph}} \ll L$. Here $L_{\text{ph}}$ is the inelastic mean free path, and $L$ is the sample size.

Modulations of local chemical potentials due to random impurity potentials lead to pinning of vortices as well. For short range random impurity potentials with the correlation length $r_c \ll \xi_0$, the pinning potential of a vortex is $\int d\mathbf{r} \rho V_0(\mathbf{r}) \propto g \Delta_0 / \epsilon_F \times r_c / \xi_0$, while the vortex excitation energy in a paired composite Fermion state is $g \Delta$. When $\Delta_0 / \epsilon_F, r_c / \xi_0 \ll 1$, the pinning effect is rather insignificant. For the modulation doped GaAs/AlGaAs 2DEG, the correlation length of random impurity potentials $r_c \sim 1000 \text{A}$ can be much longer than the Fermi wave length $k_F^{-1} \sim 200 \text{A}$) and $\xi_0^{-1} \sim 200 \text{A}$). In this case, pinning can be substantial and the coefficient $c$ in Eq. 10 should be modified. However, the $T^2$ law in Eq.3 remains valid.

Skyrmions represent a spin texture which is a nontrivial mapping from a sphere onto itself [24]. This type excitation has an energy gap due to Zeeman splitting and Coulomb interaction, which is barely affected by disorder in the present limit. At low temperatures, its contribution to $\sigma_{\text{cond}}$ is negligible.

For a $p$-wave pairing, $\Delta_{pi}$ is suppressed in the presence of disorders even when $\theta_{12} = 0$ and becomes zero when $\tau = \tau_e \sim 1/\Delta_0$. Nonetheless, one can make the following substitutions in Eq.6 to obtain an equation for $\Delta_{pi}$ when $\tau$ is close to $\tau_e$: $\delta \theta_{22}/\tau_e \to (1 - \tau/\tau_e) \delta \theta_{22}/\tau_e$, $\Delta_{p} \to \Delta_{pi} \nabla^2 \to \nabla^2 \delta \theta_{22}/\tau_e$, with $< K_{pij} K_{pj'j'} > \equiv [\delta_{ii'} \delta_{jj'} + \delta_{ij} \delta_{j'i'}] < K_{p} K_{p} >$. Again, $K_{pij}$ is due to Friedel oscillations with random phase shifts, i.e. "mismatches" of wave functions in different oscillation periods.

Though a complete microscopic theory for the pairing quantum Hall state is still absent, it is conceivable that $\mathbf{E}$ is qualitatively correct for other pairing states such as $\{111\}$ state, $\{331\}$ state, or Pfaffian states in the presence of disorders. Particularly, the stochastic nature of Eq.6 is generic to a pairing state involving wave function mismatches. In weakly coupled limits the drag conductance is related to the topology of the ground state wave function, similar to the Hall conductance [23]. Recently, I. Aleiner showed that the Coulomb drag between two disordered Fermi liquids can be of a random sign in a mesoscopic limit [24].

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