The $\Sigma\Sigma$ interactions in finite-density QCD sum rules

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The properties of $\Sigma$-hyperons in pure $\Sigma$ matter are studied with the finite-density quantum chromo-dynamics sum rule (QCDSR) approach. The $\Sigma\Sigma$ nuclear potential $U_{\Sigma\Sigma}$ is most likely strongly attractive, it could be about $-50\text{ MeV}$ or even more attractive at normal nuclear density. If this prediction is the case, the interactions between $\Sigma$-hyperons should play crucial roles in the strange nuclear matter, when there are multi-$\Sigma$ hyperons. The bound state of double-$\Sigma$ maybe exist.

I. INTRODUCTION

The baryon-baryon interactions are basic issues in nuclear physics. In the past years, there has been a lot of work on the nucleon-nucleon interactions ($NN$), for there exists much experimental information from nucleon-nucleon scattering. However, the interactions of hyperon-nucleon ($YN$) are much less known than these of $NN$ due to the difficulties in performing scattering experiments with the unstable hyperons. In these $YN$ interactions, only the $\Lambda N$ interaction is known for us by studying the hyper-nuclei. The hyperon-hyperon ($YY$) interactions are the least known ones in baryon-baryon interactions for the very scarce information in experiments. More studies of $YN$, $YY$ interactions are needed not only by the development of the nuclear physics, but also by the application in the other fields, such as in astrophysics.

There have been some typical models for the study of $YN$ and $YY$ interactions. Such as the $SU(6)$ quark model [1, 2, 3], the $SU(3)$ chiral quark model [4, 5], the chiral effective field theory [6, 7, 8], the lattice QCD [9], the chiral unitary approach [10], the meson-exchange model [11, 12] and the QCDSR [13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23]. With this approach, the properties of nucleons, $\Lambda$- and $\Sigma$-hyperons in nucleonic nuclear matter have been reasonably described. Based on the sum rules for $\Sigma N$ interactions [18], the $\Sigma\Sigma$ interactions can be described by substituting the in-medium condensates in nucleonic nuclear matter with pure $\Sigma$ matter, this approach has been adopted in our previous work, in which the $\Lambda\Lambda$ interactions were discussed [23].

The finite-density QCDSR approach focuses on a correlation function of interpolating fields, made up of quark fields, which carry the quantum numbers of the hadron of interest. Unlike usual ones, the correlation function is evaluated in the ground state of the nuclear matter rather than in vacuum. The correlation function can represent in a simple phenomenological ansatz for these spectral densities on the one hand. On the other hand, the correlation function can be evaluated at large space-like momenta using an operator product expansion (OPE). Finally, one can deduce the sum-rules by equating these two different representations using appropriately weighted integrals. The baryon self-energies in medium matter can be related to QCD Lagrangian parameters and finite-density condensates.

For simplicity, only the leading order of the in-medium condensates are taken into account in this work, which is a reasonable approximation at low nuclear densities [24, 25]. In the OPE for $\Sigma$ correlation function, we consider all condensates up to dimension 4, and the terms up to the first order in the strange quark mass $m_s$. In addition, the contributions from the dimension-6 four-quark condensate are included for their importance. And the leading order in-medium gluon condensates, $\langle \bar{q}q \rangle_\rho$, $\langle \bar{s}s \rangle_\rho$, $\langle \bar{u}u [ (u' \cdot G)^2 + (u' \cdot G')^2 ] \rangle_\rho$, $\langle \bar{G}G \rangle_\rho$, $\langle \bar{q}iD_0q \rangle_\rho$ and $\langle \bar{s}iD_0s \rangle_\rho$ are derived from the chiral perturbation theory (ChPT). Following the Ref. [14], to deal with the determined scalar-scalar four-quark condensate $\langle \bar{q}q \rangle_\rho^2$, we introduce an arbitrary parameter $f$ to describe its density dependence.

The paper is organized as follows. In the subsequent section, the sum rules and the condensates are given. The calculations and analysis are presented in Sec. III. Section IV is a summary.

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II. THE METHOD

A. QCDSR for Σ hyperons in Σ matter

The finite-density QCDSR approach has been well devolved in the series lectures \cite{13, 14, 15, 16, 17, 18, 19, 20, 21}. As done in \cite{23}, we can easily extend the Σ sum rules in nuclear matter \cite{14} to describe the ΣΣ interactions in the pure Σ matter by changing the quark and gluon condensates in nuclear matter to those in pure Σ matter (the sum rules are listed in appendix A). Using the obtained sum rules, the baryon scalar self-energy Σ\(s\) and the vector self-energy Σ\(v\) and the effective mass \(M_\Sigma^*\) can be related to the in-medium quark and gluon condensates at finite-density. Then, the ΣΣ nuclear potential \(U_\Sigma\) can be valued by the formula \(U_\Sigma = \Sigma_s + \Sigma_v\). The essential quark and gluon condensates are calculated in the subsequent section.

B. In-medium condensates

To obtain the predictions for the ΣΣ interactions in pure Σ matter from the sum rules described above, we need to know the condensates in pure Σ matter. The first order of the condensates in the nuclear matter can be written as

\[
\langle \hat{O} \rangle_\rho = \langle \hat{O} \rangle_0 + \langle \hat{O} \rangle_\Sigma \rho + \ldots,
\]

where the ellipsis denote the corrections of higher order density, and \(\langle \hat{O} \rangle_\Sigma\) is the spin averaged Σ matrix element.

As we know, in the QCD Hamiltonian density \(H_{QCD}\), chiral symmetry is explicitly broken by the current quark mass terms. Neglecting the isospin breaking effects, one has the Hamiltonian \cite{21}:

\[
H_{\text{mass}} \equiv 2m_q\bar{q}q + m_s\bar{s}s + \ldots,
\]

where \(m_s\) and \(m_q\) are the strange and light \(u, d\) current quark masses, respectively; \(q\) and \(s\) stand for the \(u, d\) quark and strange quark fields, respectively. Taking the Hamiltonian \(H_{\text{mass}}\) as a function of \(m_q\), in the Hellmann-Feynman theorem, one obtains

\[
2m_q\langle \Psi(m_q) | \int \! dx^3 \bar{q}q | \Psi(m_q) \rangle = m_q \frac{d}{dm_q} \langle \Psi(m_q) | \int \! dx^3 \! H_{\text{mass}} | \Psi(m_q) \rangle,
\]

In the above equation, we consider the cases of \(| \Psi(m_q) \rangle = |\text{vac}\rangle\) and \(| \Psi(m_q) \rangle = |\rho\rangle\), where \(| \Psi(m_q) \rangle = |\rho\rangle\) denotes the ground state of Σ matter with Σ density \(\rho\) and \(| \Psi(m_q) \rangle = |\text{vac}\rangle\) denotes the vacuum state. Taking the difference of these two cases, and taking into account the uniformity of the system yields

\[
2m_q\langle \bar{q}q \rangle_\rho - \langle \bar{q}q \rangle_0 = m_q \frac{d\mathcal{E}}{dm_q},
\]

where \(\mathcal{E}\) is the energy density of the Σ matter, which is given by

\[
\mathcal{E} = M_\Sigma \rho + \delta\mathcal{E},
\]

where \(\delta\mathcal{E}\) is of the higher order term. Recently, the in-medium condensates are studied in Refs. \cite{24, 25}, from their analysis it is found that the contributions of the higher order term \(\delta\mathcal{E}\) to the in-medium condensates are small at low density \(\rho \leq \rho_0\). Thus, the contributions of the higher order term \(\delta\mathcal{E}\) are neglected in the calculations. In the chiral perturbation theory (see the Appendix B of \cite{23}), the Σ mass is given by

\[
M_\Sigma = M_N + 4(b_D - b_F)B_0m_s
- 4(b_D - b_F)B_0m_q,
\]

where \(b_D\) and \(b_F\) are real parameters in the chiral Lagrangian, which can be seen in many references for example \cite{23}. Then, from the Eq. \ref{eq:1}, we obtain

\[
\langle \bar{q}q \rangle_\rho = \langle \bar{q}q \rangle_0 + \frac{1}{2m_q} \left[ \sigma_{\pi N} - 4m_q(b_D - b_F)B_0 \right] \rho,
\]
where $\sigma_{\pi N}$ is the $\pi N$ sigma term, which is given as

$$\sigma_{\pi N} = m_q \frac{dM_N}{dm_q}.$$  \hfill (8)

Following the steps above and those in Ref. [23], the other dimension 3 and 4 quark and gluon condensates are easily obtained. The results are

$$\langle q^i q \rangle, \langle u^i u \rangle, \langle d^i d \rangle, \langle s^i s \rangle = \rho,$$  \hfill (9)

$$\langle \bar{s}s \rangle = \langle \bar{s}s \rangle_0 + \frac{1}{m_s} \left[ S - 4m_s(b_F - b_D)B_0 \right] \rho,$$  \hfill (10)

$$\langle q^i i D_0 q \rangle = \frac{m_q}{4} \langle \bar{q}q \rangle_0 + \frac{3}{8} M_{\Sigma}[A_2^q(\mu^2) + A_2^d(\mu^2)] \rho,$$  \hfill (11)

$$\langle s^i i D_0 s \rangle = \frac{m_q}{4} \langle \bar{s}s \rangle_0 + \frac{3}{4} M_{\Sigma}[A_2^s(\mu^2)] \rho,$$  \hfill (12)

$$\langle \frac{\alpha_s}{\pi} G^2 \rangle_L = \frac{8}{9} \left( M_{\Sigma} - \Sigma_{\pi N} + S + K \right) \rho,$$  \hfill (13)

$$\left\langle \frac{\alpha_s}{\pi} \left[ (u' \cdot G)^2 + (u' \cdot \tilde{G})^2 \right] \right\rangle = -\frac{3}{2\pi} M_{\Sigma} C(\mu^2) \rho,$$  \hfill (14)

where $S = m_s \frac{dM_N}{dm_q} = \frac{1}{2} m_q \sigma_{\pi N} y$ \hfill [21] is the strangeness content of nucleon with a dimensionless quantity $y \equiv \langle \bar{s}s \rangle_N / \langle \bar{q}q \rangle_N$, the moments of parton distribution functions $A_2^q(\mu^2)$ in $\Sigma$ hyperon matter are $A_2^q + A_2^d \simeq A_2^s \simeq 0.3 \hfill [23]$, and the value of the $\sigma(\mu^2) = \alpha_s(\mu^2) A_2^q(\mu^2)$ is about 0.22 \hfill [14], and the parameters $K$ express as $K = 4(m_s - m_q)(b_D - b_F)B_0$. The other parameters, such as the vacuum condensates and the current quark masses, are adopted the same as those in our previous work \hfill [23].

Finally, the in-medium four-quark condensate, $\langle \bar{q}q \rangle_\rho$, should be considered justly, because they are numerically important in the finite density sum rules. As pointed out in Refs. \hfill [15] \hfill [16], the in-medium four-quark condensates in the $\Sigma$ sum rules are their factorized forms, which may not be justified in nuclear matter because the four-quark condensates are sensitive to the nuclear density, one might suspect that this is an artifact of the factorization. Thus, as done in Refs. \hfill [15] \hfill [16] we choose to parameterize the scalar-scalar four quark condensates so that they interpolate between their factorized form in free space and their factorized form in $\Sigma$ matter. That is, in the calculations we need replace $\langle \bar{q}q \rangle_\rho$ in Eqs. \hfill [A1] \hfill [A2] by modified form $\langle \bar{q}q \rangle_\rho^2$:

$$\langle \bar{q}q \rangle_\rho^2 = (1 - f)\langle \bar{q}q \rangle_0^2 + f\langle \bar{q}q \rangle_\rho^2,$$  \hfill (15)

where $f$ is the real parameter. The predictions in Refs. \hfill [14] \hfill [15] \hfill [16] \hfill [17] \hfill [22] \hfill [23] suggest that the four-quark condensate, $\langle \bar{q}q \rangle_\rho^2$, should depend weakly on the nuclear density. That is, the artificial parameter $f$ is most possibly in the range of $0 \leq f \leq 0.5$.

### III. CALCULATIONS AND ANALYSIS

In the calculations, to quantify the fit of the left- and right- sides of the $\Sigma$ sum rules, we use the logarithmic measure

$$\delta(M^2) = \ln \left[ \frac{\max \left\{ \lambda^2 e^{-\left( E_{\pi}^v - q^2 \right)/M^2}, \Pi_\rho'(M^2)/M_{\Sigma_v}^v, \Pi_\rho(M^2)/M_{\Sigma_v}^v \right\}}{\min \left\{ \lambda^2 e^{-\left( E_{\pi}^v - q^2 \right)/M^2}, \Pi_\rho'(M^2)/M_{\Sigma_v}^v, \Pi_\rho(M^2)/M_{\Sigma_v}^v \right\}} \right].$$  \hfill (16)

Here $\Pi_\rho'(M^2)$, $\Pi_\rho(M^2)$ and $\Pi_\rho'(M^2)$ denote the right-hand sides of the Eqs.\hfill [A1] \hfill [A2], respectively. In principle, this three terms are equal to $\lambda^2 e^{-\left( E_{\pi}^v - q^2 \right)/M^2}$. The predictions for $\lambda^2$, $s_0^v$, $M_{\Sigma_v}^v$, $\Sigma_v$ are obtained by minimizing the measure $\delta$. In the zero-density density, we can obtain the $\Sigma$ mass in vacuum applying the same procedure to the sum rules.
A. Borel mass

Firstly, we should choose a proper Borel mass $M^2$ in the calculation. In principle the predictions should be independent of the Borel mass $M^2$. However, in practice one has to truncate the operator product expansion and use a simple phenomenological ansatz for the spectral density, which cause the sum rules to overlap only in some limited range of $M^2$. The previous studies for the octet baryons show that the sum rules do not provide a particular convincing plateau. Nevertheless, we can assume that the sum rules actually has a region of overlap, although it is imperfect. In order to compensate for at least some of the limitations of the truncated sum rules, we normalize the finite-density predictions for all self-energies to the zero-density prediction for the mass. In Refs. [14, 15, 16, 23], the optimization region of $M^2$ is suggested as $0.8 \leq M^2 \leq 1.4 \text{ GeV}^2$, thus, in this work we choose the proper Borel mass $M^2$ around this region.

To find an optimization region for $M^2$ (in this region the predictions should be less sensitive to $M^2$ than those in other regions), we plot the $\Sigma$ masses in vacuum and in nuclear medium as a function of Borel mass $M^2$ in its possible range $0.8 \leq M^2 \leq 1.7 \text{ GeV}^2$ in Fig. 1 and Fig. 2, respectively. It is found that by normalizing the finite-density predictions in the calculation, a good plateau appears in the range of $1.1 \leq M^2 \leq 1.6 \text{ GeV}^2$. This optimal Borel mass predicted by us consists with the previous predictions in Refs. [14, 15, 16, 23]. In our later calculations, we choose the medium value $M^2 = 1.4 \text{ GeV}^2$.

![FIG. 1: Σ mass in vacuum as a function of the auxiliary parameter $M^2$](image)

B. Sensitivity to the $f$ and $|q|$ 

Then, the sensitivity of the predictions to the $f$ is illustrated in Fig. 3 where $\sigma_{\pi N}$, $y$ and $|q|$ are fixed at 56 MeV, 0.5 and 270 MeV, respectively, as done in [23]. The Fig. 3 is about the optimum results as a function of the momentum $|q|$ at the normal nuclear density $\rho = \rho_0 = (110 \text{ MeV})^3$ with three different values of $f$. From the figures, it is seen that the predictions for $M^*_\Sigma/M_\Sigma$ and $U_\Sigma/M_\Sigma$ are sensitive to $f$ (i.e. the four-quark condensate) but slightly dependent on $|q|$, they monotonously increase with the increment of the $f$. The $U_\Sigma/M_\Sigma$ is insensitive to both the $f$ and $|q|$.

C. Sensitivity to $\sigma_{\pi N}$ and $y$

There are large uncertainties of the $\pi N$ sigma term $\sigma_{\pi N}$ and the strangeness content of the nucleon $y$. The recent determinations suggest large values for $\sigma_{\pi N} = 64 \pm 8 \text{ MeV}$, and hence a large strangeness content of the nucleon ,i.e., $y = 0.5$ are obtained. The $\Sigma N$ sum rule study suggests large strangeness content $y = 0.5$, which is also in agreement with our recent predictions $y = 0.5$ and $\sigma_{\pi N} = 56 \text{ MeV}$ in the study of the $\Lambda\Lambda$ interaction with QCDSR. While the usual adopted values of $\sigma_{\pi N}$ and $y$ is $\sigma_{\pi N} = 45 \text{ MeV}$ and $y \simeq 0.2$. To study the effect of the parameters $y$ and $\sigma_{\pi N}$, we plot $M^*_\Sigma/M_\Sigma$ and $\Sigma V/M_\Sigma$ as functions of $y$ and $\sigma_{\pi N}$, respectively, in Fig. 5 and Fig. 6.
FIG. 2: $M_\Sigma^*/M_\Sigma$ and $\Sigma_V/M_\Sigma$ as functions of Borel mass $M^2$, where $y = 0.5$, $\sigma_{\pi N} = 56$ MeV and $\rho = \rho_0 = (110$ MeV$)^3$. The three curves correspond to $f = 0.0$ (diamond), $f = 0.5$ (circle), and $f = 1.0$ (square), respectively.

From the two figures, it can be seen that $\Sigma_V/M_\Sigma$ is insensitive to both $y$ and $\sigma_{\pi N}$. However, the scalar self-energy is sensitive to the strange quark content $y$, and slightly depends on the $\sigma_{\pi N}$. $M_\Sigma^*/M_\Sigma$ increases with the increment of the $f$, however, decreases with the increment of the $y$.

D. The predictions versus density

Finally, to study the in-medium properties of the $\Sigma$-hyperon, the effective mass $M_\Sigma^*/M_\Sigma$, the vector self-energy $\Sigma_V/M_\Sigma$ and the potential $U_M/M_\Sigma$ as functions of densities $\rho$ are plotted. For the uncertainties of the $\sigma_{\pi N}$ and $y$, two sets of the $\sigma_{\pi N}$ and $y$ are adopted in this work. One set is the new determinations $\sigma_{\pi N} = 56$ MeV and $y = 0.5$; and the other set is the usual values $\sigma_{\pi N} = 45$ MeV and $y = 0.2$.

From the figures 7 and 8, we see the effective mass $M_\Sigma^*/M_\Sigma$ decreases, whereas the vector self-energy $\Sigma_v/M_\Sigma$ increases monotonously with the increment of the $\Sigma$ density. The differences of the effective mass $M_\Sigma^*/M_\Sigma$ between the parameter $f = 0.0$ and $f = 0.5$ are more and more obvious with the increment of the density $\rho$, while the vector self-energy $\Sigma_v/M_\Sigma$ is insensitive to the parameter $f$ at different densities.
FIG. 4: $M^*_\Sigma/M_\Sigma$ and $\Sigma_V/M_\Sigma$ as functions of three momentum $|q|$. The other input parameters are the same as in Fig. 2.

FIG. 5: $M^*_\Sigma/M_\Sigma$ and $\Sigma_V/M_\Sigma$ as functions of $y$ with $\sigma_{\pi N} = 45$ MeV. The other input parameters are the same as in Fig. 2.

From the figure, we also find that the potential $U_\Sigma$ has strong parameter dependence in the whole density region $0 \leq \rho \leq \rho_0$. The differences between the two sets $f = 0.0$ and $f = 0.5$ become more and more obvious with the increment of the density.

When we set $\sigma_{\pi N} = 56$ MeV, $y = 0.5$, and $|q| = 270$ MeV, the effective mass and vector self-energy at $\rho_0$ are

$$M^*_\Sigma \simeq (0.551 - 0.658)M_\Sigma, \tag{17}$$

$$\Sigma_v \simeq (0.153 - 0.173)M_\Sigma, \tag{18}$$

in the range $0 \leq f \leq 0.5$ (see Fig. 7); and the potential $U_\Sigma$ is strongly attractive, the strength increases monotonously with the increment of the $\Sigma$ density $\rho$. At $\rho = \rho_0$, the potential can reach to

$$U_\Sigma \simeq -(296 \sim 174) \text{ MeV}, \tag{19}$$

which is much stronger than the nuclear potential of nucleon at normal density. Similarly, the attractive $\Sigma \Sigma$ potential is also predicted in $[3, 12]$, which is even stronger than the $NN$ one.

If we set $\sigma_{\pi N} = 45$ MeV, $y = 0.2$ (see Fig. 8), it is seen that the effective mass $M^*_\Sigma/M_\Sigma$, the vector self-energy $\Sigma_v/M_\Sigma$ at $\rho_0$ are

$$M^*_\Sigma \simeq (0.763 \sim 0.863)M_\Sigma, \tag{20}$$

$$\Sigma_v \simeq (0.140 \sim 0.146)M_\Sigma, \tag{21}$$
FIG. 6: $M_{N}^{*}/M_{N}$ and $\Sigma_{V}/M_{N}$ as functions of the $\pi N$ sigma term $\sigma_{\pi N}$ with $y = 0.2$. The other input parameters are the same as in Fig. [2].

FIG. 7: $M_{N}^{*}/M_{N}$, $\Sigma_{V}/M_{N}$ and $U_{\Sigma}/M_{N}$ as functions of the $\Sigma$ density $\rho$ with $f = 0.0$ (square) and $f = 0.5$ (circle), respectively, where $y = 0.5, \sigma_{\pi N} = 56$ MeV. The other input parameters are the same as in the Fig. [2].

and the potential is

$$U_{\Sigma} \simeq -(50 \pm 70) \text{ MeV.}$$

(22)

In this case, the medium value of the potential $U_{\Sigma} \simeq -50$ MeV is also strongly attractive. Comparing with the predictions of the two parameter sets, we find the vector self-energies of them are almost equal, however, the nuclear potential with $y = 0.2, \sigma_{\pi N} = 45$ MeV are much weaker than that with $y = 0.5, \sigma_{\pi N} = 56$ MeV.

Although the sum-rule predictions for the scalar self-energy are quite sensitive to the four-quark condensates in nuclear medium and parameter $y$, according to the analysis of the four-quark condensates in the series papers, we could predict that the $\Sigma\Sigma$ potential is most likely strongly attractive. This potential is much stronger than the $\Lambda\Lambda$ potential [23] in the same conditions. Thus, when we deal with the strange nuclear matter, if many $\Sigma$ hyperons appear, the interactions between $\Sigma$ hyperons should play crucial roles. According to our predictions, the bound state of double-$\Sigma$ may exist.
FIG. 8: $M \Sigma^*/M \Sigma$, $\Sigma V/M \Sigma$ and $U \Sigma/M \Sigma$ as functions of the $\Sigma$ density $\rho$ with $f = 0.0$ (square) and $f = 0.5$ (circle), respectively, where $y = 0.2, \sigma_{m} = 45$ MeV. The other input parameters are the same as in the Fig. 2.

IV. SUMMARY

In this paper, the $\Sigma\Sigma$ interactions are analyzed carefully with the finite-density QCDSR approach. The sum-rule analysis indicates that the vector self-energy $\Sigma V$ is insensitive to the sum rule parameters. However, the potential $U \Sigma$ and the scalar self-energy $\Sigma S$ have strong parameter dependence, especially, they are very sensitive to the four quark condensates. Although the predictions strongly depend on the undetermined parameters $f$ and $y$, it can predict that the $\Sigma\Sigma$ potential $U \Sigma$ is most likely strongly attractive, which could be $-50$ MeV or even more attractive at normal nuclear density. If this prediction is the case, the interactions between $\Sigma$ hyperons should play crucial roles in the strange nuclear matter, when there are multi-$\Sigma$ hyperons. The bound state of double-$\Sigma$ maybe exist.

This is a preliminary attempt to study the $\Sigma\Sigma$ interactions in finite $\Sigma$ density. More studies are needed to describe the details of the potential. The four quark condensate in medium should be studied further also.

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APPENDIX A: THE $\Sigma$ SUM RULES

The sum rules for the $\Sigma$ hyperon in the nuclear matter had been deduced by Xueimin Jin and Marina Nielsen[14], which are given by

$$\lambda_{\Sigma}^{2} M_{\Sigma}^{2} e^{-E_{1}^{2} q^{2}}/M_{\Sigma}^{2} = \frac{m_{s}}{16 \pi^{2}} M^{4} E_{2} L^{-8/9} - \frac{M^{4}}{4 \pi^{2}} E_{1} \langle \bar{\Sigma} s \rangle_{\rho} + \frac{m_{s}}{2 \pi^{2}} q \cdot M^{2} E_{0} \langle q \bar{q} \rangle_{\rho} - \frac{4}{3} m_{s}^{3} q \cdot (\bar{q} q)_{\rho} - \frac{4}{3} E_{q} (\bar{q} q)_{\rho} \langle \bar{q} q \rangle_{\rho},$$

(A1)
\[ \lambda_\Sigma^2 e^{-(E_\Sigma^2 - q^2)/M^2} = \frac{M^6}{32\pi^4} E_2 L^{-4/9} + \frac{M^2}{144\pi^2} (E_0 - 4 \frac{q^2}{M^2}) \times \left< \frac{\alpha_s}{\pi} \left[ (u' \cdot G)^2 + (u' \cdot \tilde{G})^2 \right] \right> \rho L^{-4/9} \\
+ \frac{m_s}{18\pi^2} M^2 (5E_0 - 2 \frac{q^2}{M^2}) \langle \bar{s}s \rangle \rho L^{-4/9} + \frac{M^2}{32\pi^2} \times \left< \frac{\alpha_s}{\pi} (G^2) \right> \rho E_0 L^{-4/9} \\
- \frac{4M^2}{9\pi^2} (E_0 - \frac{q^2}{M^2}) \times \left< q^4 iD_0 q \right> \rho L^{-4/9} - \frac{M^2}{9\pi^2} (E_0 - 4 \frac{q^2}{M^2}) \left< (s^4 iD_0 s) \right> \rho L^{-4/9} \\
+ \frac{E_q}{6\pi^2} M^2 E_0 (\langle q^1 q \rangle \rho + \langle s^1 s \rangle \rho) L^{-4/9} + \frac{4}{3} \langle q^4 q \rangle \rho (\langle s^1 s \rangle \rho L^{-4/9} + \frac{2}{3} \langle \bar{q}q \rangle \rho L^{-4/9}, \right. \] (A2)

\[ \lambda_\Sigma^2 \Sigma e^{-(E_\Sigma^2 - q^2)/M^2} = \frac{1}{12\pi^2} M^4 E_1 (7 \langle q^1 q \rangle \rho + \langle s^1 s \rangle \rho) L^{-4/9} - \frac{E_q}{9\pi^2} M^2 E_0 (m_s \langle \bar{s}s \rangle \rho \\
- 16 \langle q^4 iD_0 q \rangle \rho - 4 \langle s^4 iD_0 s \rangle \rho) L^{-4/9} + \langle s^1 s \rangle \rho) L^{-4/9} \\
- \frac{E_q}{36\pi^2} M^2 E_0 \left< \frac{\alpha_s}{\pi} \left[ (u' \cdot G)^2 + (u' \cdot \tilde{G})^2 \right] \right> \rho L^{-4/9} + \frac{4E_q}{3} \langle q^1 q \rangle \rho \langle q^4 q \rangle \rho. \quad (A3) \]