Flavour symmetries at the EW scale

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Abstract. We study the possibility of extending the Standard Model symmetries with a flavour symmetry at the electroweak scale. This requires the existence of a flavour-triplet of Higgses. We study the related scalar potential and its possible minima. It turns out that the new Higgses lead to extra contributions to the oblique corrections, Z and W decay and flavour violating processes. These can be used to constrain the viability of flavour models.

1. Introduction
After the discovery of neutrino oscillations and the determination of the related mixing angles, it was observed that the data may show interesting structures. The observations show two large mixing angles and one angle that is either very small or vanishing. Particularly interesting is that the measured values are reasonably close to the so-called tri-bimaximal mixing pattern, specified by $\sin^2 \theta_{13} = 0$, $\sin^2 \theta_{23} = 1/2$ and $\sin^2 \theta_{12} = 1/3$. See figures 1 and 2 for a comparison of the data and the tri-bimaximal mixing pattern.

![Figure 1](image1.png) ![Figure 2](image2.png)

**Figure 1.** Pie charts showing the flavor content of the neutrino mass eigenstates according to the central values of the mixing angles [1].

**Figure 2.** Pie charts showing the flavor content of the neutrino mass eigenstates according to the tri-bimaximal prediction.

Obviously, the similarity of the data and this mixing pattern can be purely coincidental. Most of the parameters in the Standard Model (SM) are related to the fermion mass sector and it is well possible that their values are not restricted to satisfy some underlying principle, but just happen to be close to some special values. However, it might be more interesting to see if the patterns can really be explained. The answer is that they can, but the price to pay is the introduction of quite a large sector of new physics.

2. The pros and cons of flavons
Many models have been constructed to reproduce the tri-bimaximal mixing pattern; for a review, see e.g. [2]. In these models, it is assumed that there is a horizontal symmetry next to the...
four stable structures. Two of them have all vacuum expectation values real:

\[ \Phi = (\Phi_1, \Phi_2, \Phi_3) \quad (1) \]

In this case, the potential for the Higgses is relatively simple and depends on one dimensionful parameter \( \lambda_3 \) and one phase \( \epsilon \).

\[
V = \mu^2 (\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 + \Phi_3^\dagger \Phi_3) + \lambda_3 (\Phi_1^\dagger \Phi_1 \Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 + \Phi_3^\dagger \Phi_3)^2 + \lambda_4 (\Phi_1^\dagger \Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 \Phi_1^\dagger \Phi_1 + \Phi_3^\dagger \Phi_3 \Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_3 \Phi_1^\dagger \Phi_1 + \Phi_3^\dagger \Phi_2 \Phi_1^\dagger \Phi_1)
\]

In [3] we performed a systematic analysis of the possible minima of this potential. We found four stable structures. Two of them have all vacuum expectation values real:

- \((v, v, v)\),
• \((v, 0, 0)\).

Two others have complex vevs and thus provide an extra possibility for CP violation
• \((ve^{i\omega}, rv, 0)\)
• \((ve^{i\omega}, ve^{-i\omega}, rv)\).

4. The physical Higgs fields
Our set-up is a three Higgs doublet model. This implies that there are more Higgs bosons than the one of the Standard Model. Of the three charged components of the Higgs fields, one linear combination becomes the (complex) Goldstone boson responsible for the \(W^{\pm}\) masses, while two others are charged Higgs fields \(H_{1,2}^{\pm}\). In the neutral sector, one linear combination of the six real components is a Goldstone boson related to the mass of the \(Z\) boson. Five others are physical Higgs fields \(H_{1-5}^{(0)}\). In the CP conserving case, three of them are scalars and the two others are pseudoscalars. In the CP violating case, it is not possible to identify separate scalars and pseudoscalars.

5. Constraining minima of the potential
Not all choices of parameters that lead to the vev structures of the previous section give realistic models. In [3] we consider a number of ‘tests’ for the viability of a model that are model independent in the sense that they depend on the \(A_4\) representation of the Higgs fields (namely a triplet), but not on the representations of the fermions in the theory.

Firstly, we demand \(m^2\) to be positive for all two charged and five neutral Higgses. Secondly, we consider the tree level unitarity constraints coming from the additional scalars present in the theory. Thirdly, we study the decay of \(Z\) and \(W^{\pm}\) bosons. Additional Higgses open up new decay channels that cannot give too large contributions. Lastly, we consider the effect of the extra Higgses on the oblique parameters.

In [4] we continue the analysis in a model dependent way. Once the \(A_4\) representations of the fermions are given, it is possible to study rare decays. The additional Higgses need not be flavour conserving and thus rare decays like \(\mu^- \rightarrow e^- e^- e^+\) or \(\tau^- \rightarrow \mu^- \mu^- e^+\) are possible at tree level, while they are very constrained in the Standard Model. The strong experimental bounds on these processes can be used to restrict the parameter space. In the case of quarks, the additional Higgs fields also mediate meson-antimeson oscillations via tree diagrams instead of box diagrams as in the SM. See also figure 3.

![Feynman diagrams](image)

**Figure 3.** Feynman diagrams for charged lepton decays (left: \(\mu^- \rightarrow e^- e^- e^+\); middle: \(\tau^- \rightarrow \mu^- \mu^- e^+\)) and meson oscillations (right: \(B \leftrightarrow \bar{B}\)) that are possible at tree level due to the extra Higgs bosons.

In the next two sections, we will consider two of the allowed minima in more detail. These relate to three models in the literature, [5] for the vacuum \((v, v, v)\) and [6] (focussing on leptons) and [7] (focussing on quarks) for the vacuum \((ve^{i\omega}, ve^{-i\omega}, rv)\) and permutations.
6. The vacuum \((v, v, v)\)
We first study the CP conserving vacuum \((v, v, v)\) \([5]\), in which case \(A_4\) is not completely broken, but leaves a residual \(Z_3\) symmetry. In the neutral Higgs sector, there is one boson that is like the SM Higgs in the sense that it is flavour conserving. The four other neutral bosons come as a pair of degenerate scalars and a pair of degenerate pseudoscalars. As can be seen in figure 4 there are quite a lot of points in parameter space that satisfy the model independent bounds. Yellow points indicate that all masses are positive, but that subsequent tests fail. If also unitarity constraints are met, points are blue. Green points indicate that also the \(Z\) and \(W\) decay tests are passed and points that satisfy all constraints, including those from the oblique parameters are red.

![Figure 4](image-url)

*Figure 4.* A slice of parameter space for the model of section 6. \(m_1\) and \(m_2\) are respectively the lightest and the second lightest Higgs masses. All masses \(\geq 0\): yellow points; unitarity OK: blue points; \(Z\) and \(W\) decay OK: green points; oblique parameters OK: red points.

Due to the residual symmetry, many rare decays are still forbidden in this model. Of the decays we studied, only \(\tau^- \to \mu^- \bar{\mu} e^+\) is allowed. Its branching ratio is below experimental bounds for most values of the lightest Higgs mass as can be seen in figure 5.

We conclude that this set-up provides some interesting physics and is not too much constrained by low-energy observables.

7. The vacuum \((ve^{i\omega}, ve^{-i\omega}, rv)\)
We continue to study the CP violating vacuum solution \((ve^{i\omega}, ve^{-i\omega}, rv)\), that was used in \([6]\) and \([7]\) to model respectively the lepton sector and the quark sector. As opposed to the previous case, there is no residual symmetry as \(A_4\) is fully broken. Furthermore, the fact that two of the vevs are complex, shows that CP is broken in the Higgs sector.

We find that, although the minimum is a genuine minimum of the potential \((2)\), there are very few points in parameter space that have all masses non-negative. See figure 6, where we
**Figure 5.** The branching ratio for the tau decay of figure 3 as a function of the lightest Higgs mass. The dashed line gives the experimental upper bound.

**Figure 6.** A slice of parameter space for the model 7 showing the $r$ parameter versus the lightest Higgs mass. The colour coding is the same as in figure 4.

We can partially solve this by adding a small term to the potential that softly breaks the $A_4$ symmetry

$$V_{A_4soft} = v_w^2 m_0^2 (\phi_1^4 \phi_2 + \phi_2^4 \phi_1) + v_w^2 n_0^2 (\phi_2^4 \phi_3 + \phi_3^4 \phi_2) + v_w^2 k (\phi_1^4 \phi_3 + \phi_3^4 \phi_1), \quad (3)$$
With these terms added, more points are in the phenomenologically viable range as shown in figures 7, although they remain rather few.

**Figure 7.** A slice of parameter space for the model of section 7 showing the lightest Higgs mass versus the next-to lightest Higgs mass. The colour coding is as in figure 4

As there are no remaining symmetries, many fermion number violating processes can indeed occur at tree level. We consider for instance the process $\mu^- \rightarrow e^- e^- e^+$ in the model of [6] or $\Delta M_{B_d}$ in $B_d - \bar{B}_d$ oscillations in the model of [7]. As is shown in figures 8 and 9, the current experimental bounds strongly limit the validity of these models.

8. Conclusions

In conclusion, we state that the use of flavour symmetries is an interesting tool to explain the apparent structures in the fermion mass sector. The flavour symmetries can be realized with or without the use of separate flavon fields. If we assume these to be absent, relatively minimal models can be constructed. These models have multiple copies of the Standard Model Higgs field, where the copies are related by the flavour symmetry. We analysed the case where the flavour symmetry is $\mathbb{A}_4$ and only a few minima configurations are allowed by the potential. We showed that low energy observables can strongly restrict some of the possible models.

References

[1] M. C. Gonzalez-Garcia, Nucl. Phys. A827 (2009) 5C-14C. [arXiv:0901.2505 [hep-ph]].
[2] G. Altarelli and F. Feruglio, Rev. Mod. Phys. 82 (2010) 2701 [arXiv:1002.0211 [hep-ph]].
[3] R. de Adelhart Toorop, F. Bazzocchi, L. Merlo and A. Paris, [arXiv:1012.1791 [hep-ph]].
[4] R. de Adelhart Toorop, F. Bazzocchi, L. Merlo and A. Paris, [arXiv:1012.2091 [hep-ph]].
[5] E. Ma, G. Rajasekaran, Phys. Rev. D64 (2001) 113012. [hep-ph/0106291].
[6] S. Morisi, E. Peinado, Phys. Rev. D80 (2009) 113011. [arXiv:0910.4389 [hep-ph]].
[7] L. Lavoura, H. Kuhbock, Eur. Phys. J. C55 (2008) 309-308. [arXiv:0711.0670 [hep-ph]].
Figure 8. The branching ratio for the muon decay into two electrons and a positron in the model of [6] versus the lightest Higgs mass. The dashed line shows the experimental upper bound.

Figure 9. $\Delta M_{B_d}$ in the model of [7] versus the lightest Higgs mass. The dashed line shows the experimental value.