Electromagnetic game modeling through Tensor Analysis of Networks and Game Theory

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Abstract. A complex system involves events coming from natural behaviors. Whatever is
the complicated face of machines, they are still far from the complexity of natural systems.
Currently, economy is one of the rare science trying to find out some ways to model human
behavior. These attempts involve game theory and psychology. Our purpose is to develop
a formalism able to take in charge both game and hardware modeling. We first present the
Tensorial Analysis of Networks, used for the material part of the system. Then, we detail the
mathematical objects defined in order to describe the evolution of the system and its gaming
side. To illustrate the discussion we consider the case of a drone whose electronic can be
disturbed by a radar field, but this drone must fly as near as possible close to this radar.

1. Introduction
Complex problems cannot be approached through classical techniques. One reason comes from
the diversity of elements that compose this complexity. Deterministic tools can be used once
a first understanding of the problems is covered by analysis and physical looking. Moreover,
psychological side of complex systems like human ones must incorporate notions of choices,
payoff, etc.; these notions that doesn’t belong to numerical tools. Anyway, reality has to be
symbolized to dispose of some representation of the global problem. In order to realize this task,
networks give an efficient tool. Once can refer to numerous works on heat science, mechanics,
computational fluid dynamics, biochemistry [1] . . . , and of course in electricity where they are
usually employed. These networks are made of nodes and edges, they can be extended using
meshes and chords. The chords allow to support any kind of mathematical interaction between
two nodes, edges or two meshes. We propose here for example using chord to take in charge the
electromagnetic interaction between the radar and a moving target.

2. Physical modeling
As explained in the introduction, a global problem can be decomposed into two part, the first
one that consists in the modeling of the physical problem using the network theory. The systems
under study consists in a fixed radar and a moving target. Each previous subsystem and their
interactions should be modeled. Sir G. Kron was the first to understand the benefit of using
tensorial algebra in such networks study [2]. As the systems may change over time, an extension
of the method has been proposed by the authors [3]: it consists in the definition of tenfolds as
explained later that will be able to follow an evolution using gamma matrix [4].
2.1. Physical issue: the Kron’s method
The system under study includes two main objects: the radar and the target. Each one can be represented by a so-called primitive network under the Tensorial Analysis of Network (TAN) formalism so they can exist independently of the system. To create our scene the operation consists in coupling the radar and the target through an electromagnetic interaction, this being made on an operational theater. So the system modeling is realized in three parts.

First, the radar consists in an amplifier, source of the electromagnetic energy and an antenna. Each element can be associated with an edge: one for the antenna and the other for the amplifier. Each edge is associated respectively to an impedance: $Z_1$ and $Z_2$. The models construction requires various operations. The entire system impedance is obtained from a direct summation of the objects: $g_1 = \begin{bmatrix} Z_1 & 0 \\ 0 & Z_2 \end{bmatrix}$. Next step consists in connecting the two edges (connectivity): an incidence from edges to meshes. The target modeling is similar to the radar: the receiver is depicted by 2 edges / impedances ($Z_3$ and $Z_4$) and the system impedance is named $g_2$. The whole system needs to gather radar and target ($g_1 \oplus g_2$) and include near and far field interactions. The impedances tensor is $g$, which gives the equations of the problem: $E = g k$, or tensorially writing: $E_{\nu} = g_{\nu\sigma} k^{\sigma}$ where $k$ are the unknown mesh currents to solve and $E$ stands for sources.

2.2. Tenfold
Assuming any system may be represented by a graph of networks, they are defined through their incidence and connectivity (giving the relations between $N$ nodes, $B$ edges and $M$ meshes).

The set (topology, $T$, impedance tensor, $g$, called the metrics and sources, $E$) gives all the information to describe a network or a system. These 3 previous objects are gathered in a list named “tenfold” (symbol $\tilde{u}$, Fig.1(b)) associated with the set $(T, g, E)$. Linking transformations to the tenfold enables transient topology evolutions; we may define transformers: $\tilde{u}' = t \cdot \tilde{u}$ with $t = (\Lambda, M, N)$. Mathematically, the evolution may be represented via gamma matrix [5].

3. Human factor
A system evolution can be entirely represented using a “tree model” (Fig.1 (b)), standing for the whole transformations. At this stage, one may wonder what is the reason to chose one particular transformation? Actually the result is the consequence of a decision depending of the so-called Human Factor (HF). So it is natural to define the probability of making one choice compared to the others. Probabilities weights have to be affected to the different branches. This offers an introduction of HF. Game theory [6] details the interpretation of the evolution tree and may be convenient for communication networks [7, 8]. The information vector can be separate into two parts: first part still devoted to the state (tenfold), and second one is devoted to ways probability. It is automatically obtained through the gamma matrix also separated into two parts (transformations and probabilities of evolution). In this issue, thinking a solution is better to another may lead people to act differently. That is why the definition of a valuable payoff is crucial (the higher the gain is, the better the strategy of the gamer is). Once an output is computed at each step of evolution, the product of the probability to reach this state by the related gain provides useful information. The addition of all possible trajectories gives the average gain function of one player.

4. Numerical application
For demonstration purposes, the game mesh is sampled considering at maximum 10 steps (3 choices are proposed each time, see Fig.1 (a)).
The equation of the problem is

\[ GE = \left( \tilde{\gamma}_n \ldots \tilde{\gamma}_0 \tilde{I}_0 \right)^n/e^{\text{Distance} \left( \gamma_n \ldots \gamma_0 \tilde{u}_0 \right)}, \]

where \( GE \) is the gain expectation, \( n_e \) the ending iteration of the game, and the \( \tilde{\gamma} \) part of the propagator \( \tilde{\gamma} \) [4] is given from player’s “psychological” profile.

4.1. Description

A radiation pattern is affected to each antenna, as antenna gains. The Green’s function coupling electronics is

\[ G(\theta, \xi) = G_1 G_2 \cos(\theta) \left\{ e^{-j k R} + e^{-j k R'} \cos\xi \right\}. \]

\( G_1 \) and \( G_2 \) stand for antenna gains. \( R \) and \( R' \) are respectively the distances, direct interaction and via ground reflection, separating moving target and RADAR. The angles \( \theta \) and \( \xi \) are related to the target radiation pattern and to its angle with the ground; thus \( G_1 \) is fixed and \( G_2 \) depends on \( \theta \) angle (given \( G_2 = \cos(\theta)^2 + 0.001 \)).

Three profiles are considered in this study: “go-getter” gamers, ones who are “undecided” and those whose path is entirely “random”. For each kind of psychological profile a sequence is randomly or deterministically defined: it is a list of letters: “d” for right, “b” for down and “h” for up, see Fig.1 (a). Each time the player chooses one step beyond: he may go one step up, down or go on straightforward (right). After achieving a movement, one may compute the distance and angles between him and the radar and so compute the interaction between them. To this end, the tensorial equation detailed before \((E = gk)\) is solved. Assuming the electronic device susceptibility, it is straightforward to compute its potential disturbance; depending on the result, the game continues or ends. The potential player’s choices allow defining 2 different “classes”: “straight” displacement (“bbb...” for instance) and “change of direction” (including “undecided” and “random” profiles). The “profiles” definitions rely on a priori 3 profiles: “go-getter” (3 paths straight along “d”, “b” or “h” axis), “undecided” (3 different paths ups and downs through “b” and “h” axis) and purely “random”. Each step of the game brings crucial information: the PG graph offers a current view of the system state via payoff computation. The probabilistic graphs obtained for each profile enables defining the best strategy to be as close as possible to the radar. The probabilities for the player to chose either “d”, “b” or “h” rely on the given profile and are: “go-getter” \( \rightarrow (0.8, 0.1, 0.1) \) (putting the focus on “d” trajectory), “undecided” \( \rightarrow (0.25, 0.25, 0.5) \) (emphasizing path oscillating around \( y = 0m \)) and “random”.
→ (1/3, 1/3, 1/3). The payoff gain is linked with the inverse of the distance to the radar when the game comes to an end.

4.2. Results and discussion
A profile always following the same group of direction (PG graph) is called “border trajectory” (case of “go-getter” gamers). “Undecided” and “random” trajectories are both characterized by “straight” and “change of direction” groups of transformations.

Figure 2. PG graphs from (a) “go-getter”, (b) “undecided” and (c) “random” profiles.

First, from “go-getter” profile, the gamer follows a straight direction all the time and reach a highest payoff around 0.47. Figure 2 (a) shows the trajectories in the PG graph for the three possible sequences: “bbb...”, “hhh...” and “ddd...”. The expected gain is $GE = 0.07$ with a major part dedicated to third trajectory (payoff is 0.33) whereas first and second paths offer highest payoffs. Similarly to “go-getter” profile, the Fig.2 (b) depicts the paths from “undecided” case. It leads in the best case to a payoff of 0.3 with $GE = 0.00264$. Finally, three trajectories were randomly simulated and are depicted in Fig.2 (c). Since no a priori behavior is expected (random), $GE$ is smaller than in previous cases ($GE = 0.0005$) but the payoff is very similar to “go-getter” and “undecided” strategies.

5. Conclusion
As depicted previously, gain expectations give information about the optimum decision. Among the defined strategy (“go-getter”, “undecided”, “random”), “go-getter” profile was the more interesting: the proposed methodology allowed to characterize the highest payoff with the smallest distance to the radar (2.15m). This cannot be intuitively obtained (radiation patterns influence and complexity of the Green’s function could have favored “ups and downs” approach). Future works should improve the formalism (for instance partial payoff and reliability testing).

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