Optimization of hybrid beamforming for multiuser massive MIMO systems with individual SINR and SLL constraints

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Abstract: This paper considers designing hybrid beamformer (HBF) to enhance communication performance and reduce inter-cell interference for massive MIMO system. The goal is minimizing the transmission power under individual signal-to-interference-plus-noise ratio (SINR) and sidelobe level (SLL) constraints, which is a non-convex optimization problem. To tackle the SLL over continuous intervals, we convert it into a finite number of constraints. Then we give the optimal solution of the problem for different RF chain to user number ratio. Simulation results show proposed algorithm not only effectively reduce inter-cell interference but also exhibits high numerical stability in multiuser massive MIMO system.

Keywords: sidelobe level, hybrid beamformer, individual SINR, inter-cell interference

Classification: Wireless Communication Technologies

References

[1] G. Zang, Y. Cui, H.V. Cheng, F. Yang, L. Ding, and H. Liu, “Optimal hybrid beamforming for multiuser massive MIMO systems with individual SINR constraints,” IEEE Wireless Commun. Lett., vol. 8, no. 2, pp. 532–535, April 2019. DOI: 10.1109/LWC.2018.2878766

[2] M. Zhou, X. Ma, P. Shen, and W. Sheng, “Weighted subspace-constrained adaptive beamforming for sidelobe control,” IEEE Commun. Lett., vol. 23, no. 3, pp. 458–461, March 2019. DOI: 10.1109/LCOMM.2019.2896578

[3] N.D. Sidiropoulos, T.N. Davidson, and Z.-Q. Luo, “Transmit beamforming for physical-layer multicasting,” IEEE Trans. Signal Process., vol. 54, no. 6,
1 Introduction

Hybrid beamformer (HBF) consists of an analog beamformer (ABF) in radio frequency (RF) domain and a digital beamformer (DBF) in baseband domain. It can achieve high transmission rate and reduce hardware complexity. Almost all the existing hybrid beamformer design just focus on the sum data rate or the inner cell users SINR [1], while take little consideration of sidelobe level. Too high sidelobe level may increase the inter-cell interference.

To fill this gap, sidelobe level control is required. Unfortunately, due to the special hardware structure of hybrid beamformer, traditional fully-digital beamformer (such as Chebyshev, Olen, QSC beamformer) for sidelobe level suppression are not suitable for the hybrid beamformer design. The WS-MVDR algorithm [2] does not well denote the characteristics of the sidelobe angle region so that it needs more transmission power to suppress sidelobe level in the multiuser system. Furthermore, using the inappropriate approximation method may render WS-MVDR infeasible in the hybrid beamformer.

In this letter, we consider designing a hybrid beamformer to minimize the transmission power with individual SINR and SLL constraints in multiuser downlink system. To tackle the infinitely many SLL constraints, we convert it into a finite number of constraints. We solve the hybrid beamformer design problem in two cases: the number of users is less than or equal to the number of RF chains and the else. Simulation results show the proposed algorithm requires less transmission power and exhibits higher numerical stability compared with the WS-MVDR algorithm.

2 System model and problem statement

Consider the downlink transmission of a multiuser mmWave MIMO system, which comprising a base station (BS) with multi-antenna and K single-antenna users. These users is denoted by $\mathcal{K} \triangleq \{1, \ldots, K\}$. The BS is equipped with a uniform linear array (ULA) of $M$ ($M \geq K$) antennas and $N$ ($N < M$) RF chains. Fig. 1(a) shows the fully-connected hybrid structure we adopted, where each antenna is connected to all $N$ RF chains. The analog beamformer is denoted by $V \in \mathbb{C}^{M \times N}$. The digital beamformer is denoted by $W \triangleq [w_1, \ldots, w_K] \in \mathbb{C}^{N \times K}$, where $w_k \in \mathbb{C}^{N \times 1}$ denotes the digital beamforming vector for user $k$.

We consider that the channel between the BS and $k$th ($k \in \mathcal{K}$) user, denoted by $h_k^H \in \mathbb{C}^{1 \times M}$, is a LoS (Line of Sight) path. In particular, $h_k^H = \sqrt{\beta_k} a^H(\theta_k)$, where $\beta_k$ is the distance-dependent path loss from the $k$th user to the BS, $\theta_k$ is the angle of departure (AOD) in azimuth dimension, $a(\bullet) \in \mathbb{C}^{M \times 1}$ denotes the array response vector, it can be denoted as:

$$a(\theta) = [1, e^{j \frac{2\pi}{4} d \sin \theta}, e^{j \frac{2\pi}{4} \cdot 2 d \sin \theta}, \ldots, e^{j \frac{2\pi}{4} \cdot (M-1) d \sin \theta}]^T$$

(1)
where $\lambda$ is the signal wavelength and $d$ is the antenna separation. The received signal of $k$th user is given by

$$y_k = h_k^H w_k s_k + \sum_{i \in \mathcal{K}, i \neq k} h_i^H w_i s_i + n_k,$$

where $n_k \sim \mathcal{CN}(0, \sigma_k^2)$ and $s_k$ denote the additive Gaussian noise of user $k$ and the transmitted information symbol, respectively. For user $k \in \mathcal{K}$, to capture the quality of service requirement, the SINR of user $k$ is required to be above a threshold $\eta_k$, i.e.,

$$\frac{|h_k^H w_k|^2}{\sum_{i \in \mathcal{K}, i \neq k} |h_i^H w_i|^2 + \sigma_k^2} \geq \eta_k, \quad k \in \mathcal{K}. \quad (2)$$

However, in cellular networks as shown in Fig. 1(b), due to inter-cell interference, the communication quality of users in adjacent cells is degraded if we just focus on the SINR of user $k \in \mathcal{K}$. Let $\Theta_{SL} \subseteq [-90^{\circ}, 90^{\circ})$ denote the adjacent cells angle range to be considered. To reduce inter-cell interference, we require that all inner users sidelobe level in $\Theta_{SL}$ is below a threshold $\mu$

$$\frac{|a^H(\theta) w_k|^2}{|a^H(\theta_k) w_k|^2} \leq \mu, \quad \theta \in \Theta_{SL}, \quad k \in \mathcal{K} \quad (3)$$

Let $\| \cdot \|_F$ denote the Frobenius norm, the total transmission power is given by $\|VW\|_F^2$. Our hybrid beamformer design problem can be formulated as

$$\mathcal{P}0 : \min_{V,W} \|VW\|_F^2 \quad \text{s.t.} \quad (2)(3).$$

3 HBF design

Note that Problem $\mathcal{P}0$ has been shown to be NP-hard [3] without SLL constraints in (3). Due to the infinitely many constraints in (3), problem $\mathcal{P}0$ is more challenging. Specifically, to tackle the challenge caused by (3), we first transform the SLL constraints in (3) into a finite number of constraints.

**Theorem 1.**

$$|a^H(\theta) w|^2 \leq M^2 \max_{i \in Q} |q_i^H w|^2, \quad \theta \in \Theta_{SL} \quad (4)$$

Where $w \in \mathbb{C}^{M \times 1}$ is transmission beamformer, $q_i \in \mathbb{C}^{M \times 1}$ is the eigenvector corresponding to the $i$th eigenvalue of matrix $\int_{\Theta_{SL}} a(\theta) a^H(\theta) d\theta$ in a descending order.

Let $\mathcal{M} = \{1, 2, \ldots, M\}$, $Q$ is given by $Q = \{i \mid i \in \mathcal{M}, \max_{\theta \in \Theta_{SL}} |a^H(\theta) q_i| \geq \varepsilon\}$, $\varepsilon \to 0$ is a parameter controlling the accuracy, $\varepsilon = 10^{-3}$ in this letter.

**Proof.** We have
\[ |a^H(\theta)w| \leq \sum_{i=1}^{M} |a^H(\theta)q_iq_i^Hw| \leq \sum_{i=1}^{M} |a^H(\theta)q_i||q_i^Hw| = \sum_{i \in Q} |a^H(\theta)q_i||q_i^Hw| + \sum_{i \in M, i \notin Q} |a^H(\theta)q_i||q_i^Hw| \leq \max_{i \in Q} |q_i^Hw| \sum_{i \in Q} |a^H(\theta)q_i| + \varepsilon \sum_{i \in M, i \notin Q} |q_i^Hw| = \max_{i \in Q} |q_i^Hw| \sum_{i \in Q} |a^H(\theta)q_i|, \quad \theta \in \Theta_{SL} \] 

Let \( R = \max_{\theta \in \Theta_{SL}} \sum_{i \in Q} |a^H(\theta)q_i| \), we can conclude
\[ |a^H(\theta)w| \leq R^2 \max_{i \in Q} |q_i^Hw|^2, \quad \theta \in \Theta_{SL} \] 

The equality holds when \( a^H(\theta)q_m = a^H(\theta)q_n, q_mw = q_nw \) for \( m \in Q, n \in Q \). We complete the proof.

By using Theorem 1, we can rewrite the SLL constraints in (3) as follows:
\[ \frac{R^2|q_i^HVw_k|^2}{|a^H(\theta_k)Vw_k|^2} \leq \mu, \ k \in \mathcal{K}, i \in Q \] 

Thus, the hybrid beformer design problem \( \mathcal{P}0 \) is rewritten as
\[ \mathcal{P}1 : \min_{V,W} \| VW \|_F^2 \quad \text{s.t. (2)(7).} \]

In the following section (3.1) and (3.2), we give different solutions to solve this challenging non-convex problem \( \mathcal{P}1 \) in the cases of \( K \leq N \) and \( K > N \).

### 3.1 Case i: \( K \leq N \)

\[ \mathcal{P}2 : \min_{W_D} \| W_D \|_F^2 \]
\[ \text{s.t.} \quad \frac{\| [k^H W_D]_k \|^2}{\sum_{i \in \mathcal{K}, i \neq k} \| [k^H W_D]_i \|^2} \geq \eta_k, \ k \in \mathcal{K} \] 
\[ R^2 \frac{\| d_k^H W_D \|_k^2}{\| a^H(\theta_k)W_D \|_k^2} \leq \mu, \ k \in \mathcal{K}, i \in Q \]

In this case, we transform problem \( \mathcal{P}1 \) into the fully-digital beamformer design problem and obtain a globally optimal solution. First, we transform problem \( \mathcal{P}1 \) into problem \( \mathcal{P}2 \) by letting \( W_D = VW \in \mathbb{C}^{M \times K} \), where \( [ \cdot ]_k \) denotes the \( k \)th element of the argument and \( W_D \) denotes the fully-digital beamformer. Problem \( \mathcal{P}2 \) can be solved optimally using SOCP method proposed in [4] with low computational complexity. By letting \( W_D^* \) denotes the globally optimal solution of problem \( \mathcal{P}2 \), we can use \( V^* = W_D^* ((W_D^*)^H W_D^*)^{-1} (W_D^*)^H \) to obtain \( V^* \in \mathbb{C}^{M \times N} \), where \( W^* \in \mathbb{C}^{N \times K} \) is randomly generated with linearly independent columns. Following similar proof method in [1], \( (V^*, W^*) \) can be proved to be a globally optimal solution of problem \( \mathcal{P}1 \).

### 3.2 Case ii: \( K > N \)

We consider the following problem.
Solve the problem

\[ \mathcal{P} : \min_Y \| G_v Y G_w \|_F^2 \]

s.t. \[ \left\| \left( h_k^H G_v Y G_w \right)^H \right\|_{2} \leq \sqrt{\frac{1 + \eta_k}{\eta_k}} h_k^H G_v Y b_k, \ k \in \mathcal{K} \] (10)

\[ R_d^i Y G_v b_k \leq \sqrt{\mu a^H (\theta_k) G_v Y b_k}, \ k \in \mathcal{K}, \ i \in \mathcal{Q} \] (11)

\[ Y \succeq 0 \] (12)

\[ \text{rank}(Y) \leq N \] (13)

Where \( G_v \triangleq [I_{M \times M}, 0_{M \times K}] \in \mathbb{C}^{M \times (K+M)}, G_w \triangleq [0_{K \times M}, I_{K \times K}]^T \in \mathbb{C}^{(K+M) \times K} \) and \( b_k \in \mathbb{C}^{(K+M) \times 1} \) denotes the vector whose \((k+M)\)th element is 1 and the others are 0. Let \( Y = U U^H \) denote any feasible solution \( Y \) of problem \( \mathcal{P} \). \( U \) can be rewritten as \( U = [V^H, W]^H \), so we obtain \( V = G_v U \) and \( W = U^H G_w \). By introducing intermediate variables \( Z \), following the similar iterative algorithm in [1], we can obtain a stationary point by Algorithm 1. By letting \( Y^{(t)} \) denote \( Y \) in the \( t \)th iteration, \( Y^{(t+1)} \) can be updated as

\[ \mathcal{P} \text{.3.1} : Y^{(t+1)} = \arg \min_Y \left( \| G_v Y G_w \|_F^2 + \tau \text{trace}(Z^{(t+1)} (Z^{(t+1)})^H Y) \right) \] s.t. (10), (11), (12).

where \( Z^{(t+1)} \in \mathbb{C}^{(K+M) \times (K+M-N)} \) consists of the \( K+M-N \) eigenvectors corresponding to the smallest \( K+M-N \) eigenvalues of \( Y^{(t)} \). The problem \( \mathcal{P} \text{.3.1} \) is a convex SDP problem, which can be efficiently solved using the standard interior-point toolboxes (e.g. CVX) with complexity \( \mathcal{O}((M + KQ)^{4.5}) \), where \( Q (Q \leq M) \) denote the total number of elements in \( Q \).

### 4 Simulation results

In this section, we provide numerical results to illustrate the performance of our proposed algorithm. We choose \( d = 0.5 \lambda, \eta_k = 3 \sigma, \sigma_K^2 = 1 \). Fig. 2(a) shows the beam patterns at \( K = 1, \mu = -40^\circ, \Theta_{SL} = [-70^\circ, -50^\circ] \). We see that the proposed algorithm effectively reduces sidelobe level below \( \mu \), and has almost no effect outside \( \Theta_{SL} \). The WS-MVDR algorithm increases the transmission power outside \( \Theta_{SL} \). Fig. 2(b) shows the transmission power required, where \( \mu = -20^\circ, \Theta_{SL} = [-90^\circ, -30^\circ] \) and the AOD of \( K \) users is randomly given outside \( \Theta_{SL} \). From Fig. 2(b), we see that the proposed algorithm just makes the transmission power
Fig. 2. Simulation results

increase a little and it can still obtain a feasible solution when the WS-MVDR algorithm is invalid. Fig. 2(c) shows that the computational time of the proposed algorithm is obviously short in case i. Though the computational time increases in case ii due to the iteration, it is still acceptable.

We consider a 3-cell network in Fig. 1(b). The cell radius is 500 m and the base station height is 40 m. We assume K users are uniformly-randomly distributed in each cell, K users in cell B are distributed on the edge of the cell B and receive the interference from BS in adjacent cells. Fig. 2(d) shows the sum rate of K users in cell B, where \( \mu = -20db, \eta_k = 3db \). We see that the proposed algorithm can effectively reduce inter-cell interference especially when \( K \) is large. Thus, the simulation results show the effectiveness, practicality, and stability of the proposed algorithm.

5 Conclusion

In this letter, under individual SINR constraints and SLL constraints, we consider designing the optimal hybrid beamformer to minimize the total transmission power in the multiuser massive MIMO system. According to whether the covariance matrix of the DBF matrix is invertible, we propose algorithmic solutions in two cases. Simulation results show that proposed algorithm requires less transmission power and exhibits higher numerical stability compared with the WS-MVDR algorithm.

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