Fractional Quantum Hall Effect in the Second Landau Level

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We present activation gap measurements of the fractional quantum Hall effect (FQHE) in the second Landau level. Signatures for 14 (5) distinct incompressible FQHE states are seen in a high (low) mobility sample with the enigmatic 5/2 even denominator FQHE having a large activation gap of ~600 (~300mK) in the high (low) mobility sample. Our measured large relative gaps for 5/2, 7/3, and 8/3 FQHE indicate emergence of exotic FQHE correlations in the second Landau level, possibly quite different from the well-known lowest Landau level Laughlin correlations. Our measured 5/2 gap is found to be in reasonable agreement with the theoretical gap once finite width and disorder broadening corrections are taken into account.

The clean (i.e. high mobility) two-dimensional electron system (2DES) at low temperatures and high magnetic fields exhibits a rich array of exotic, highly correlated incompressible ground states. In the lowest Landau level (LLL), the physics is dominated by the sequence of FQHE states at odd denominator filling fractions with more than 50 FQHE states with odd denominators as large as 19 observed so far. The Laughlin wave function describes the primary FQHE states at LLL filling fractions \( \nu = 1/m, m = 3, 5, 7 \ldots \) as an incompressible quantum fluid of electrons [1]. The Laughlin states feature a gap in the energy spectrum with fractionally charged quasiparticles with charge \( q = \pm e/n \) as the lowest energy excitation. The sequence of hierarchical \( \nu = p/(2p \pm 1), p = 1, 2, 3 \ldots \) higher order FQHE states is described by the composite fermion model [2, 3]. The odd denominator constraint arises from the antisymmetry of the many-body wave functions required under the exchange of two electrons [1]. To date no even denominator FQHE state has been observed in the LLL for a single-layered 2DES, although certain anomalies have been observed at \( \nu = 3/8 \) [4].

The startling exception to the odd denominator rule is the even denominator FQHE at \( \nu = 5/2 = (2 + 1/2) \) in the second Landau level (SLL). Early experiments [5] showed a weakly formed quantized Hall plateau with a finite longitudinal resistance at low temperatures. Improvement in the sample quality led to the formation of a fully quantized Hall plateau along with a vanishing longitudinal resistance at low temperatures [6, 7, 8]. In contrast to the LLL, the SLL features an array of competing ground states including odd denominator FQHE, reentrant insulating states, and even denominator FQHE at \( \nu = 5/2 \) and \( 19/8 = (2 + 3/8) \) [7].

The theoretical understanding of the even-denominator FQHE at \( \nu = 5/2 \) is based on the \( p \)-wave pairing of composite fermions, similar to the pairing in a chiral \( p \)-wave BCS superconductor [9, 10]. The variational wave function for the paired Hall states is modified by a Pfaffian that creates a FQHE state. Numerical diagonalization calculations provide strong support for the Pfaffian state as the ground state at \( \nu = 5/2 \) [11, 12]. The non-Abelian quasiparticle statistics of the Pfaffian 5/2 state has received much attention recently for the prospect of realizing topologically protected qubits [13]. The existence of the even denominator FQHE state and the general paucity of a large number of odd denominator fractions clearly differentiate the FQHE physics of the SLL from that in the LLL. In particular, as we demonstrate in this paper, the standard composite fermion LLL hierarchy states seem to be strongly suppressed in the SLL. The nature of SLL interaction and correlation are not well-understood and the 5/2 state, although it is an even denominator state with no analogy in the LLL, is both the best understood and the strongest FQHE state in the SLL. In fact, theoretical work [14] indicates that only \( \nu < 2 + 1/3 \) Laughlin states would be stable in the SLL!

In this paper, we report on the observation of a large (~14) number of possible incompressible states and their energy gaps in the second Landau level. We find that the energy gap of the \( \nu = 5/2 \) FQHE states exceeds 500 mK in the high mobility samples. Comparing results from two samples with "high" and "low" mobilities, we conclude that in general the \( \nu = 5/2 \) state is the strongest FQHE in the SLL, followed by the 7/3 and 8/3 states with all other fractions being far weaker. The fact that an even denominator fraction, considered to be a \( p \)-wave paired Hall state, is the strongest FQHE state in the SLL provides a sharp contrast between the FQHE physics in the LLL and the SLL. We find the 7/3 and 8/3 states to be much stronger than the other odd denominator FQHE states in the SLL.

Two symmetrically \( \delta \)-doped 30 nm wide quantum well samples with identical structures were studied. The mobility for sample A (high-mobility) is \( \mu = 28.3 \times 10^{5} \text{cm}^2/\text{Vs} \) with the electron density of \( n = 3.2 \times 10^{11} \text{cm}^{-2} \). The mobility for sample B (low-mobility) is \( \mu = 10.5 \times 10^{5} \text{cm}^2/\text{Vs} \) with the electron density of \( n = 2.8 \times 10^{11} \text{cm}^{-2} \). The samples in a van der Pauw geometry were attached to the cold finger of a dilution refrigerator. Magnetotransport studies were made after illuminating the specimens with a red light emitting diode at 4K. Measurement was made using a low-frequency AC
The most prominent FQHE states are found at fillings \( \nu \) immediately makes it obvious that the strongest SLL FQHE states occur at \( \nu = 5/2, 7/3, 8/3, 16/7, \) and \( 11/5 \) in sample A show a demonstrably weaker activation at same fillings compared with sample A.

The energy gap \( \Delta \) for the various FQHE states can be determined from the Arrhenius plot using the activated resistance \( R_{xx} \propto \exp(-\Delta/kT) \). While a greater range of activation behavior is desired for sample B and the higher order fractions in sample A, the data for sample A with strong activation behavior allows us to constraint the energy gap values for the states that exhibit limited activation. For sample A at \( \nu = 8/3 \), an anomalous change in the slope was observed near \( \sim 80 \text{mK} \). The origin of this puzzling feature is unclear. No energy gaps

lock-in technique at low temperatures down to \( \sim 30 \text{mK} \).

Figs. 1a and 1b show the low temperature magnetoresistance for the sample A and sample B. In the higher mobility sample A, a remarkable array of 14 different FQHE states are observed, as reflected in well-defined \( R_{xx} \) minima even at a relatively moderate temperature of 36 mK. The most prominent FQHE states are found at fillings \( \nu = 5/2, 7/3, 8/3, 14/5, 11/5, 12/5, 16/7, \) and \( 19/7 \). Additional features may be attributed to \( \nu = 13/5, 17/7, 22/9, \) and \( 23/9 \) states. Finally, resistance minima can be seen at even denominator fillings \( \nu = 19/8 \) and \( 21/8 \). In addition to these FQHE states, signatures of reentrant insulating states of varying strengths can be detected at \( \nu = 2.69, 2.58, 2.44, \) and \( 2.31 \). The large resistance arising from the reentrant insulating states weakens the FQHE states found at \( \nu = 13/5 \) and \( 17/7 \).

Fig. 1b shows that only five of the 14 states seen in sample A survive in sample B: the FQHE states at \( \nu = 5/2, 7/3, 8/3, 11/5, \) and \( 14/5 \). This is presumably a direct suppression due to disorder since the two samples are very similar except for a factor of \( \sim 3 \) difference in mobility. Along with the absence of many higher order FQHE states, the magnetoresistance minima of the principal FQHE states have become much weaker in sample B compared with sample A. A cursory look at our Fig.
I. The energy gaps for the strongest FQHE states are summarized in Table I. The gaps were obtained for sample B at \( \nu = 16/7 \) and 11/5. Results of our activation analysis are summarized in Table I. The energy gaps for \( \nu = 8/3, 5/2, \) and 7/3 in sample A are the largest with their magnitudes exceeding 500mK. In sample B the 5/2 FQHE state is by far the strongest state in the second Landau level in the units of Coulomb energy \( e^2/\epsilon \ell \), where \( \epsilon \) is the dielectric constant and \( \ell = \sqrt{\hbar/eB} \) is the magnetic length. Solid dots (squares) represent the energy gaps for sample A (B).

![Graph](image_url)

**FIG. 3:** Energy gaps for the fractional quantum Hall effect states in the second Landau level in the units of Coulomb energy \( e^2/\epsilon \ell \), where \( \epsilon \) is the dielectric constant and \( \ell = \sqrt{\hbar/eB} \) is the magnetic length. Solid dots (squares) represent the energy gaps for sample A (B).

TABLE I: Energy gap measured for the fractional quantum Hall states in the second Landau level.

| \( \nu \)  | \( \Delta \) (mK) | \( \Delta (e^2/\epsilon \ell) \) |
|----------|------------------|-------------------------------|
| sample A |                  |                               |
| \( 14/5 \) | 252              | 0.0023                        |
| \( 19/7 \) | 108              | 0.0010                        |
| \( 8/3 \)  | 562              | 0.0050                        |
| \( 5/2 \)  | 544              | 0.0047                        |
| \( 7/3 \)  | 584              | 0.0049                        |
| \( 16/7 \) | 94               | 0.0008                        |
| \( 11/5 \) | 160              | 0.0013                        |

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One of the more intriguing features of the data is that energy gap of the \( \nu = 7/3 \) and 8/3 states are disproportionately larger than what may be expected under the standard model of FQHE in the LLL. Within the composite fermion model, the FQHE states at \( \nu = 2/5 \) and 3/5 are the strongest after \( \nu = 1/3 \) and 2/3 FQHE states. The LLL energy gaps for the \( \nu = 2\) and 3/5 states are approximately half of the gaps for the \( \nu = 1/3 \) and 2/3 states. The gaps for the higher order FQHE states decrease proportionally with their position in the sequence \( \nu = 7/3 \) and 8/3 states are anomalously enhanced compared to what may be expected under the composite fermion model. The enhanced gaps of the 7/3 and 8/3 states may be related to their anomalous angular dependence.

Our finding that the strongest SLL incompressible state are \( 5/2, 7/3, 8/3, 14/5, 11/5 \) should be contrasted with the strongest LLL incompressible states: \( 1/3, 2/3, 2/5, 3/5, 2/7 \). Apart from the enigmatic 5/2 even denominator states, which has no analog in the LLL, there are several additional features of the SLL FQHE states worth emphasizing: (1) the weakness of hierarchical (e.g. \( 2+2/5 \) and \( 2+2/3 \) states); (2) the dominance of primary fractions, e.g. \( 2+1/3, 2+1/5, 2+2/3, 2+4/5 \); (3) the relative strength, \( \Delta^{7/3,8/3} \approx \Delta^{11/5,14/5} \approx 2-3 \), of the 1/3 SLL state compared with the 1/5 SLL state. All three of these SLL features are in sharp contrast with the LLL, where the hierarchy states (i.e. \( 2/5, 3/5 \), etc.) are the strongest fractions after the 1/3 state and the observed \( \Delta^{11/5,14/5} \leq \Delta^{7/3,8/3} \) whereas the observed \( \Delta^{11/5,14/5} \approx 2 \). We find \( \Delta^{7/3,11/5} \approx 2 \) whereas \( \Delta^{7/3,12/5} \approx 10 \) using the activation gap of 70 mK quoted in ref. 7 for the 12/5 state. Thus our observed 7/3 and 8/3 states are anomalously enhanced compared to what may be expected under the composite fermion model.

Based on the above observations, we conclude that the 7/3, 8/3 states are unlikely to be the SLL analogs of the 1/3, 2/3 LLL Laughlin correlated states whereas our observed 11/5, 14/5 states are likely to be Laughlin states. This conclusion is consistent with several theoretical predictions where the Laughlin states are found to be the unstable for Coulomb interaction in the SLL for...
\[ \nu = 2 + 1/3 \]. We therefore believe it to be likely that all three strongly incompressible SLL states (i.e. 5/2, 7/3, 8/3) are exotic non-Laughlin FQHE states. This conclusion is also consistent with the proposal of the weak 12/5, 13/5 SLL states being parafermionic Read-Rezayi states\(^{18}\) rather than the garden variety hierarchy states. It seems, therefore, that the SLL correlations are much more subtle than the LLL correlations.

The best current theoretical estimate for the infinite system extrapolated excitation gap for the 5/2 incompressible state is \( \Delta_{ex} \approx 0.025 \) in the Coulomb energy unit\(^{13, 20}\). This ideal gap value requires (at least) three corrections due to the finite width\(^{19}\) of the quasi-2D system, disorder\(^{19}\), and Landau level coupling\(^{21}\), before any comparison with experiment can be made. All three corrections suppress the theoretical gap with the suppression due to the finite width effect, which softens the Coulomb interaction, being the easiest to calculate. The finite width correction depends\(^{19}\) on the parameter \( w/\ell \) where \( w \) is the quantum well width (\( w = 30 \text{ nm} \) for both samples A and B). Using the applied magnetic field values (5.3T for A and 4.6T for B) we find \( w/\ell \approx 2.7 \) (sample A) and 2.50 (sample B). Such large values of \( w/\ell \) imply rather strong finite width corrections\(^{19}\) reducing \( \Delta_{ex} \) at \( \nu = 5/2 \) by a factor of 2 or more to about 0.013, which corresponds to a gap of 1.5K (sample A) and 1.4K (sample B). Our observed 5/2 activation gaps \( \Delta = 0.54 \text{K (sample A)}; 0.27 \text{K (sample B)} \) are substantially below the ideal gap values because of disorder (and, possibly, Landau level mixing), effects which are difficult to treat theoretically. We ignore Landau level coupling effects, although it may very well not be negligible in reality, based on the argument that \( (e^2/\ell \epsilon_1)/\hbar \omega_c \approx 0.4 \) is small, where \( \ell_1 = \sqrt{(2n + 1)\ell} \) is the Landau radius in the \( n=1 \) Landau level.

The inclusion of disorder in the theory of FQHE gap is problematic in the absence of a true transport theory. A simple procedure, used extensively if somewhat unjustifiably, is to write the disorder-induced gap as \( \Delta = \Delta_{ex} - \Gamma \) where \( \Gamma \) is the calculated level broadening. We can theoretically estimate the zero-field level broadening of samples A and B by using the sample structures (\( w = 30 \text{ nm} \) with a spacer layer of \( d = 80 \text{ nm} \)) and the mobilities to get \( \Gamma_A \approx 0.38 \text{K}, \Gamma_B \approx 0.63 \text{K} \), where the level broadening \( \Gamma \) corresponds to the so-called quantum single-particle impurity broadening \( (\Gamma_s = h/2\tau_s) \) rather than the transport mobility broadening \( (\Gamma_t = h/2\tau_t) \). It is well-known\(^{22}\) that in high-mobility modulation-doped structures \( \tau_t/\tau_s \gg 1 \), and in fact, for the high-mobility structures used in our experiments \( \Gamma_s \approx 200\Gamma_t \) due to the very large values of \( q_s d \approx 20 \), where \( q_s \) is the screening wave vector.

Incorporating disorder (and finite width) correction in the theoretical gap values we arrive at the following predictions for the activation gaps: \( \Delta_A = 1.5 \text{K} - 0.8 \text{K} \approx 0.7 \text{K}; \Delta_B = 1.4 \text{K} - 1.2 \text{K} \approx 0.2 \text{K} \), which are comparable with our experimentally measured gaps of 0.54K and 0.27K, respectively. We note that much of the suppression (a factor of 2) of the measured 5/2 gap \( (\sim 0.005) \) compared with the theoretical 5/2 gap \( (\sim 0.025) \) arises from the large effective well width value \( (\omega/\ell \approx 3) \) in our sample, which differs somewhat from earlier theoretical works in the literature where disorder broadening\(^{19}\) or Landau level mixing\(^{21}\) were taken to be the dominant mechanisms suppressing the experimental gap. We also note that further improvement (above the \( \mu_A = 28 \times 10^6 \text{cm}^2/\text{Vs} \) value of sample A) in the sample quality could enhance the gap at most by a factor of 2 provided mobilities above 50 millions could be achieved.

Our theoretical consideration actually suggests two alternative (and perhaps simpler) techniques for enhancing the 5/2 gap: (i) Use thinner quantum well samples so that the finite width correction is smaller; (ii) Use higher carrier density so that the 5/2 FQHE state occurs at higher magnetic field values.

Based on our extensive FQHE activation measurements in the second Landau level, we conclude that (1) the even-denominator 5/2 state, which has no analog in the LLL, is the strongest incompressible state in the SLL; and (2) the \( 2+1/3 \) (and the related \( 2+2/3 \)) SLL states are unlikely to be Laughlin-like states similar to the corresponding \( 1+1/3 \) or \( 2+3/3 \) states in the LLL. Our measured activation energy for the \( 7/3 \) state is an order of magnitude larger than the \( 12/5 \) activation energy, but is within a factor of 2 of the \( 11/5 \) activation energy. For the LLL Laughlin-like states the situation is precisely reversed with the \( 1+1/3 \) state having an activation energy an order of magnitude (only a factor of 2) larger than the \( 1/5 \) \((2/5) \) state. The SLL, in contrast to the LLL where the Laughlin correlation dominates except at the smallest filling factors, possesses many competing ground states of comparable energies for all fillings, considerably complicating the task of understanding its unique and rich quantum phase diagram.

We thank R.H. Morf for careful reading of the manuscript, W. Pan for useful discussions, and the University of Chicago MRSEC for the use of shared facilities. The work is supported by the Microsoft Q Project.

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