Masses of excited heavy baryons in the relativistic quark-diquark picture

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The mass spectra of the excited heavy baryons consisting of two light (u, d, s) and one heavy (c, b) quarks are calculated in the heavy-quark–light-diquark approximation within the constituent quark model. The light quarks, forming the diquark, and the light diquark in the baryon are treated completely relativistically. The expansion in $v/c$ up to the second order is used only for the heavy (b and c) quarks. The internal structure of the diquark is taken into account by inserting the diquark-gluon interaction form factor. An overall good agreement of the obtained predictions with available experimental data is found.

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the quark-diquark approximation should work even better than for the ground state heavy baryons.

In the quasipotential approach and quark-diquark picture of heavy baryons the interaction of two light quarks in a diquark and the heavy quark interaction with a light diquark in a baryon are described by the diquark wave function $\Psi_d$ of the bound quark-quark state and by the baryon wave function $\Psi_B$ of the bound quark-diquark state respectively, which satisfy the quasipotential equation \[ \text{[6]} \] of the Schrödinger type \[ \text{[7]} \]

\[
\left( \frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R} \right) \Psi_{d,B}(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p}, \mathbf{q}; M) \Psi_{d,B}(\mathbf{q}),
\]

where the relativistic reduced mass is

\[
\mu_R = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3},
\]

and $E_1$, $E_2$ are given by

\[
E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M}, \quad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}.
\]

Here $M = E_1 + E_2$ is the bound state mass (diquark or baryon), $m_{1,2}$ are the masses of light quarks ($q_1$ and $q_2$) which form the diquark or of the light diquark ($d$) and heavy quark ($Q$) which form the heavy baryon ($B$), and $\mathbf{p}$ is their relative momentum. In the center of mass system the relative momentum squared on mass shell reads

\[
b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}.
\]

The kernel $V(\mathbf{p}, \mathbf{q}; M)$ in Eq. \[ \text{[1]} \] is the quasipotential operator of the quark-quark or quark-diquark interaction. It is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive energy states. In the following analysis we closely follow the similar construction of the quark-antiquark interaction in mesons which were extensively studied in our relativistic quark model \[ \text{[8]} \]. For the quark-quark interaction in a diquark we use the relation $V_{qq} = V_{q\bar{q}}/2$ arising under the assumption about the octet structure of the interaction from the difference of the $qq$ and $q\bar{q}$ colour states. An important role in this construction is played by the Lorentz-structure of the nonperturbative confining interaction. In our analysis of mesons, while constructing the quasipotential of the quark-antiquark interaction, we adopted that the effective interaction is the sum of the usual one-gluon exchange term with the mixture of long-range vector and scalar linear confining potentials, where the vector confining potential contains the Pauli terms. We use the same conventions for the construction of the quark-quark and quark-diquark interactions in the baryon. The quasipotential is then defined by the following expressions \[ \text{[8]} \] \[ \text{[9]} \]

(a) for the quark-quark ($qq$) interaction

\[
V(\mathbf{p}, \mathbf{q}; M) = \bar{u}_1(p)\bar{u}_2(-p)V(\mathbf{p}, \mathbf{q}; M)u_1(q)u_2(-q),
\]

with

\[
V(\mathbf{p}, \mathbf{q}; M) = \frac{1}{2} \left[ \frac{4}{3} \alpha_s D_{\mu\nu}(\mathbf{k})\gamma^\mu_1\gamma^\nu_2 + V^{V}_{\text{conf}}(\mathbf{k})\Gamma_{1\mu}(\mathbf{k})\Gamma_{2\nu}(\mathbf{k}) + V^{S}_{\text{conf}}(\mathbf{k}) \right],
\]
(b) for quark-diquark ($Qd$) interaction

\[
V(p, q; M) = \frac{\langle d(P)|J_{\mu}|d(Q)\rangle}{2\sqrt{E_d(p)E_d(q)}} \bar{u}_Q(p) \frac{4}{3} \alpha_s \gamma^\nu u_Q(q) \\
+ \psi^*_d(P)\bar{u}_Q(p)J_{d\mu}^\nu(k)V^V_{conf}(k)u_Q(q)\psi_d(Q) \\
+ \psi^*_d(P)\bar{u}_Q(p)V^S_{conf}(k)u_Q(q)\psi_d(Q),
\]

(6)

where $\alpha_s$ is the QCD coupling constant, $\langle d(P)|J_{\mu}|d(Q)\rangle$ is the vertex of the diquark-gluon interaction which takes into account the diquark internal structure. $D_{\mu\nu}$ is the gluon propagator in the Coulomb gauge, $k = p - q$; $\gamma^\mu$ and $u(p)$ are the Dirac matrices and spinors.

The diquark state in the confining part of the quark-diquark quasipotential (6) is described by the wave functions

\[
\psi_d(p) = \begin{cases} 1 & \text{for scalar diquark} \\ \varepsilon_d(p) & \text{for axial vector diquark} \end{cases}
\]

(7)

where $\varepsilon_d$ is the polarization vector of the axial vector diquark. The effective long-range vector vertex of the diquark can be presented in the form

\[
J_{d\mu} = \begin{cases} \frac{(P + Q)_{\mu}}{2\sqrt{E_d(p)E_d(q)}} & \text{for scalar diquark} \\ \frac{(P + Q)_{\mu}}{2\sqrt{E_d(p)E_d(q)}} - \frac{i\mu}{2M_d} \sum_{\nu} \xi_{\mu\nu} \tilde{k}^\nu & \text{for axial vector diquark} \end{cases}
\]

(8)

where $\tilde{k} = (0, k)$. Here the $\Sigma_{\mu\nu}$ is the antisymmetric tensor

\[
(\Sigma_{\rho\sigma})_{\mu}^\nu = -i(g_{\mu\rho}\delta_{\sigma}^\nu - g_{\mu\sigma}\delta_{\rho}^\nu),
\]

(9)

and the axial vector diquark spin $S_d$ is given by $(S_{d\mu})_{il} = -i\varepsilon_{kil}$. We choose the total chromomagnetic moment of the axial vector diquark $\mu_d = 0$ [10].

The effective long-range vector vertex of the quark is defined by [8, 11]

\[
\Gamma_{\mu}(k) = \gamma_{\mu} + \frac{i\kappa}{2m} \sigma_{\mu\nu} \tilde{k}^\nu, \quad \tilde{k} = (0, k),
\]

(10)

where $\kappa$ is the Pauli interaction constant characterizing the anomalous chromomagnetic moment of quarks. In the configuration space the vector and scalar confining potentials in the nonrelativistic limit reduce to

\[
V^V_{conf}(r) = (1 - \varepsilon)V_{conf}(r), \\
V^S_{conf}(r) = \varepsilon V_{conf}(r),
\]

(11)

with

\[
V_{conf}(r) = V^S_{conf}(r) + V^V_{conf}(r) = Ar + B,
\]

(12)

where $\varepsilon$ is the mixing coefficient.

The constituent quark masses $m_b = 4.88$ GeV, $m_c = 1.55$ GeV, $m_u = m_d = 0.33$ GeV, $m_s = 0.5$ GeV and the parameters of the linear potential $A = 0.18$ GeV$^2$ and $B = -0.3$ GeV have the usual values of quark models. The value of the mixing coefficient of vector and
TABLE I: Masses of light ground state diquarks (in MeV). S and A denotes scalar and axial vector diquarks antisymmetric \([q, q']\) and symmetric \(\{q, q\}\) in flavour, respectively.

| Quark content | Diquark type | Mass          |
|---------------|-------------|---------------|
| \([u, d]\)    | S           | [1] 710       |
|               |             | [16] 705      |
|               |             | [17] 737      |
|               |             | [18] 820      |
|               |             | [19] 694(22)  |
| \(\{u, d\}\) | A           | [1] 909       |
|               |             | [16] 875      |
|               |             | [17] 949      |
|               |             | [18] 1020     |
|               |             | [19] 806(50)  |
| \([u, s]\)    | S           | [1] 948       |
|               |             | [16] 895      |
|               |             | [17] 882      |
|               |             | [18] 1100     |
| \(\{u, s\}\) | A           | [1] 1069      |
|               |             | [16] 1050     |
|               |             | [17] 1050     |
|               |             | [18] 1300     |
| \(\{s, s\}\) | A           | [1] 1203      |
|               |             | [16] 1215     |
|               |             | [17] 1130     |
|               |             | [18] 1440     |

Scalar confining potentials \(\varepsilon = -1\) has been determined from the consideration of charmonium radiative decays \([12]\) and the heavy quark expansion \([13]\). Finally, the universal Pauli interaction constant \(\kappa = -1\) has been fixed from the analysis of the fine splitting of heavy quarkonia \(3P_J\) states \([12]\). In the literature the ‘t Hooft-like interaction between quarks induced by instantons \([14]\) is widely discussed. This interaction can be effectively described by introducing the quark anomalous chromomagnetic moment having an approximate value \(\kappa \approx -0.75\) (Diakonov \([14]\)). This value is of the same sign and order of magnitude as the Pauli constant \(\kappa = -1\) in our model. Thus the Pauli term incorporates at least partly the instanton contribution to the \(q\bar{q}\) interaction. Note that the long-range chromomagnetic contribution to the potential in our model is proportional to \((1 + \kappa)\) and thus vanishes for the chosen value of \(\kappa = -1\).

At the first step, we calculate the masses and form factors of the light diquark. As it is well known, the light quarks are highly relativistic, which makes the \(v/c\) expansion inapplicable and thus, a completely relativistic treatment is required. To achieve this goal in describing light diquarks, we closely follow our recent consideration of the spectra of light mesons \([15]\) and adopt the same procedure to make the relativistic quark potential local by replacing \(\epsilon_1(p) = \sqrt{m_1^2 + p^2} \rightarrow E_{1,2}\) (see \([3]\) and discussion in Ref. \([15]\)).

The quasipotential equation \([11]\) is solved numerically for the complete relativistic potential which depends on the diquark mass in a complicated highly nonlinear way \([1]\). The obtained ground state masses of scalar and axial vector light diquarks are presented in Table I. These masses are in good agreement with values found within the Nambu–Jona-Lasinio model \([16]\), by solving the Bethe-Salpeter equation with different types of kernel \([17, 18]\) and in quenched lattice calculations \([19]\). It follows from Table I that the mass difference between the scalar and vector diquark decreases from \(~200\) to \(~120\) MeV, when one of the \(u, d\) quarks is replaced by the \(s\) quark in accord with the statement of Ref. \([20]\).

In order to determine the diquark interaction with the gluon field, which takes into account the diquark structure, it is necessary to calculate the corresponding matrix element of the quark current between diquark states. Such calculation leads to the emergence of the form factor \(F(r)\) entering the vertex of the diquark-gluon interaction \([1]\). This form factor is expressed through the overlap integral of the diquark wave functions. Using the numerical diquark wave functions we find that \(F(r)\) can be approximated with a high accuracy by the expression \([1]\)

\[
F(r) = 1 - e^{-\xi r - \zeta r^2}.
\]
TABLE II: Parameters $\xi$ and $\zeta$ for ground state light diquarks.

| Quark content | Diquark type | $\xi$ (GeV) | $\zeta$ (GeV$^2$) |
|---------------|-------------|-------------|------------------|
| $[u, d]$      | S           | 1.09        | 0.185            |
| $\{u, d\}$   | A           | 1.185       | 0.365            |
| $[u, s]$      | S           | 1.23        | 0.225            |
| $\{u, s\}$   | A           | 1.15        | 0.325            |
| $\{s, s\}$   | A           | 1.13        | 0.280            |

The values of the parameters $\xi$ and $\zeta$ for the ground states of the scalar $[q, q']$ and axial vector $\{q, q'\}$ light diquarks are given in Table II.

At the second step, we calculate the masses of heavy baryons as the bound states of a heavy quark and light diquark. For the potential of the heavy-quark–light-diquark interaction (6) we use the expansion in $p/m_Q$. Since the light diquark is not heavy enough for the applicability of an $p/m_d$ expansion, it should be treated fully relativistically. To simplify the potential we follow the same procedure, which was used for light quarks in a diquark, and replace the diquark energies $E_d(p) = \sqrt{p^2 + M_d^2}$ in Eqs. (6), (8). This substitution makes the Fourier transform of the potential (6) local. At leading order in $p/m_Q$ the resulting potential can be presented in the form:

for the scalar diquark

$$V^{(0)}(r) = \hat{V}_{\text{Coul}}(r) + V_{\text{conf}}(r),$$

and for the axial vector diquark

$$V^{(0)}(r) = \hat{V}_{\text{Coul}}(r) + V_{\text{conf}}(r) + \frac{1}{M_d(E_d + M_d)} \left[ \frac{M_d}{E_d} \hat{V}'_{\text{Coul}}(r) \right] L S_d,$$

$$\hat{V}_{\text{Coul}}(r) = -\frac{4}{3} \alpha_s \frac{F(r)}{r}, \quad V_{\text{conf}}(r) = V_{\text{conf}}^S(r) + V_{\text{conf}}^V(r) = Ar + B,$$

where $\hat{V}_{\text{Coul}}(r)$ is the smeared Coulomb potential (which accounts for the diquark structure). Note that both the one-gluon exchange and confining potential contribute to the diquark spin-orbit interaction. In this limit the heavy baryon levels are degenerate doublets with respect to the heavy quark spin, since the heavy quark spin-orbit and spin-spin interactions arise only at first order in $p/m_Q$. Solving Eq. (1) numerically we get the spin-independent part of the baryon wave function $\Psi_B$. Then the total baryon wave function is a product of $\Psi_B$ and the spin-dependent part $U_B$ (for details see Eq. (43) of Ref. [21]).

The leading order degeneracy of heavy baryon states is broken by $p/m_Q$ corrections. The ground-state quark-diquark potential (6) up to the second order of the $p/m_Q$ expansion is given by the following expressions:

(a) scalar diquark

$$\delta V(r) = \frac{1}{E_d m_Q} \left\{ p \left[ \hat{V}_{\text{Coul}}(r) + V_{\text{conf}}^V(r) \right] p + \hat{V}'_{\text{Coul}}(r) \frac{L^2}{2r} - \frac{1}{4} \Delta V_{\text{conf}}^V(r) \right\}.$$
to (1 + \kappa) V_{\text{conf}}^\nabla(r) \right) \mathbf{LS}_Q \right) \right.
+ \frac{1}{2} \mathbf{p} V_{\text{conf}}^S(r) \mathbf{p}
+ \frac{1}{2r} \left( \hat{V}_{\text{Coul}}'(r) - V_{\text{conf}}'(r) + 2(1 + \kappa) V_{\text{conf}}^\nabla(r) \right) \mathbf{LS}_Q, \tag{16}

(b) axial vector diquark

\delta V(r) = \frac{1}{E_d m_Q} \left\{ \mathbf{p} \left[ \hat{V}_{\text{Coul}}(r) + V_{\text{conf}}^\nabla(r) \right] \mathbf{p} + \hat{V}_{\text{Coul}}(r) \frac{L^2}{2r} - \frac{1}{4} \Delta V_{\text{conf}}^\nabla(r)
+ \frac{1}{r} \left( \hat{V}_{\text{Coul}}'(r) + \frac{\mu_d}{2} V_{\text{conf}}^\nabla(r) \right) \mathbf{LS}_d + \frac{1}{r} \left( \hat{V}_{\text{Coul}}'(r) + (1 + \kappa) V_{\text{conf}}^\nabla(r) \right) \mathbf{LS}_Q
+ \frac{1}{3} \left[ \hat{V}_{\text{Coul}}'(r) - \hat{V}_{\text{Coul}}''(r) + \frac{\mu_d}{2} (1 + \kappa) \left( \frac{1}{r} V_{\text{conf}}^\nabla(r) - V_{\text{conf}}^\nabla(r) \right) \right]
\times \left[ - S_d S_Q + \frac{3}{2r} (S_d r)(S_Q r) \right] + \frac{2}{3} \left[ \Delta \hat{V}_{\text{Coul}}(r) + \frac{\mu_d}{2} (1 + \kappa) \Delta V_{\text{conf}}^\nabla(r) \right] S_d S_Q
+ \frac{1}{m_Q^2} \left\{ \frac{1}{8} \Delta \left( \hat{V}_{\text{Coul}}(r) + V_{\text{conf}}^S(r) - [1 - 2(1 + \kappa)] V_{\text{conf}}^\nabla(r) \right) - \frac{1}{2} \mathbf{p} V_{\text{conf}}^S(r) \mathbf{p}
+ \frac{1}{2r} \left( \hat{V}_{\text{Coul}}'(r) - V_{\text{conf}}'(r) + 2(1 + \kappa) V_{\text{conf}}^\nabla(r) \right) \mathbf{LS}_Q \right\}, \tag{17}

where \mathbf{L} is the orbital momentum, \mathbf{S}_d and \mathbf{S}_Q are the light diquark and heavy quark spins, respectively. It is necessary to note that the confining vector interaction gives a contribution to the spin-dependent part at first order of the heavy quark expansion which is proportional to (1 + \kappa) or \mu_d. Thus this vanishes for the chosen values of \kappa = -1 and \mu_d = 0, while the confining vector contribution to the spin-independent part is nonzero at this order. The first nonvanishing contribution of the confining interaction to the heavy quark spin-orbit part arises only at second order of the heavy quark expansion.

Now we can calculate the mass spectra of heavy baryons with the account of all corrections of order \(p^2/m_Q^2\). For this purpose we consider Eq. (1) with the quasipotential which is the sum of the leading order potentials \(V^{(0)}(r)\) \(\tag{14}\) or \(\tag{15}\) and the corrections \(\delta V(r)\) \(\tag{16}\), \(\tag{17}\), respectively. We average the resulting equation over the wave functions of Eq. (1) calculated with the leading order potential \(V^{(0)}(r)\). In this way we obtain the mass equation

$$\frac{b^2(M)}{2\mu_R} = \frac{\langle \mathbf{p}^2 \rangle}{2\mu_R} + \langle V^{(0)}(r) \rangle + \langle \delta V(r) \rangle. \tag{18}$$

It is important to note that the presence of the spin-orbit interaction \(\mathbf{LS}_Q\) and of the tensor interaction in the quark-diquark potential \(\tag{16}-\tag{17}\) results in a mixing of states which have the same total angular momentum \(J\) and parity \(P\) but different light diquark total angular momentum \((\mathbf{L} + \mathbf{S}_d)\). Such mixing is considered along the same lines as in our previous calculations of the mass spectra of doubly heavy baryons \(\tag{9}\).

The calculated values of the ground state and excited baryon masses are given in Tables III-VII in comparison with available experimental data \(\tag{2}, \tag{4}, \tag{22}, \tag{23}, \tag{24}, \tag{25}, \tag{26}\). In the first two columns we put the baryon quantum numbers and the state of the heavy-quark–light-diquark bound system (in usual notations \(nL\)), while in the rest columns our predictions for the masses and experimental data are shown.
### TABLE III: Masses of the $\Lambda_Q$ ($Q = c, b$) heavy baryons (in MeV).

| $I(J^P)$ | $Qd$ state | $Q = c$ | $Q = b$ |
|----------|------------|--------|--------|
|          |            | $M$    | $M_{\text{exp} \ [2]}$ | $M$    | $M_{\text{exp} \ [2]}$ | $M_{\text{exp} \ [22]}$ |
| $0(\frac{1}{2}^-)$ | $1S$ | 2297 | 2286.46(14) | 5622 | 5624(9) | 5619.7(2.4) |
| $0(\frac{1}{2}^-)$ | $1P$ | 2598 | 2595.4(6) | 5930 |
| $0(\frac{3}{2}^-)$ | $1P$ | 2628 | 2628.1(6) | 5947 |
| $0(\frac{1}{2}^+)$ | $2S$ | 2772 | 2766.6(2.4) | 6086 |
| $0(\frac{3}{2}^-)$ | $1D$ | 2874 | 6189 |  |
| $0(\frac{3}{2}^-)$ | $1D$ | 2883 | 2882.5(2.2) | 6197 |
| $0(\frac{1}{2}^-)$ | $2P$ | 3017 | 6328 |  |
| $0(\frac{3}{2}^-)$ | $2P$ | 3034 | 6337 |  |
| $0(\frac{1}{2}^-)$ | $1F$ | 3061 | 6401 |  |
| $0(\frac{1}{2}^-)$ | $1F$ | 3057 | 6405 |  |
| $0(\frac{3}{2}^-)$ | $3S$ | 3150 | 6465 |  |
| $0(\frac{3}{2}^-)$ | $2D$ | 3262 | 6540 |  |
| $0(\frac{2}{2}^-)$ | $2D$ | 3268 | 6548 |  |

At present the best experimentally studied quantities are the mass spectra of the $\Lambda_Q$ and $\Sigma_Q$ baryons, which contain the light scalar or axial vector diquarks, respectively. They are presented in Tables III, IV. Masses of the ground states are measured both for charmed and bottom $\Lambda_Q$ and $\Sigma_Q$ baryons. Note that the masses of the ground state $\Sigma_b$ and $\Sigma_b^*$ baryons were first reported very recently by CDF \[4\]: $M_{\Sigma_b} = 5807.5^{+1.2}_{-2.2} \pm 1.7$ MeV, $M_{\Sigma_b^*} = 5815.2^{+1.0}_{-0.9} \pm 1.7$ MeV, $M_{\Sigma_b^{*+}} = 5829.0^{+1.6}_{-1.7} \pm 1.7$ MeV, $M_{\Sigma_b^{*-}} = 5836.7^{+2.0}_{-1.8} \pm 1.7$ MeV. CDF also significantly improved the precision of the $\Lambda_b$ mass \[22\]. For charmed baryons the masses of several excited states are also known. It is important to emphasize that the $J^P$ quantum numbers for most excited heavy baryons have not been determined experimentally, but are assigned by PDG on the basis of quark model predictions. For some excited charm baryons such as the $\Lambda_c(2765)$, $\Lambda_c(2880)$ and $\Lambda_c(2940)$ it is even not known if they are excitations of the $\Lambda_c$ or $\Sigma_c$. \(^1\) Our calculations show that the $\Lambda_c(2765)$ can be either the first radial $(2S)$ excitation of the $\Lambda_c$ with $J^P = \frac{1}{2}^+$ containing the light scalar diquark or the first orbital excitation $(1P)$ of the $\Sigma_c$ with $J^P = \frac{3}{2}^-$ containing the light axial vector diquark. The $\Lambda_c(2880)$ baryon in our model is well described by the second orbital $(1D)$ excitation of the $\Lambda_c$ with $J^P = \frac{5}{2}^+$ in agreement with the recent spin assignment \[24\] based on the analysis of angular distributions in the decays $\Lambda_c(2880)^+ \rightarrow \Sigma_c(2455)^0 + \pi^+ + \pi^-$. Our model suggests that the charmed baryon $\Lambda_c(2940)$, recently discovered by BaBar \[23\] and then also confirmed by Belle \[24\], could be the first radial $(2S)$ excitation of the $\Sigma_c$ with $J^P = \frac{3}{2}^+$ which mass is predicted slightly below the experimental value. If this state proves to be an excited $\Lambda_c$, for which we have no candidates around 2940 MeV, then it will indicate that excitations inside the diquark should be also considered. \(^2\) The $\Sigma_c(2800)$ baryon can be identified in our model

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\(^1\) In Tables III, IV, VII we mark with ? the states which interpretation is ambiguous.

\(^2\) The $\Lambda_c$ baryon with the first orbital excitation of the diquark is expected to have a mass in this region.
TABLE IV: Masses of the $\Sigma_Q$ $(Q = c, b)$ heavy baryons (in MeV).

| $I(J^P)$ | $Qd$ state | $M$ | $M^{\text{exp}} [2]$ | $M^{\text{exp}} [23]$ | $M^{\text{exp}} [24]$ | $Q = c$ | $M^{\text{exp}} [4]$ | $M^{\text{exp}} [4]$ |
|----------|------------|-----|-----------------------|------------------------|------------------------|----------|-----------------------|------------------------|
| $1(\frac{1}{2}^+)$ | $1S$ | 2439 | 2453.76(18) | | | | 5805 | 5807.5$^*$ | 5815.2$^\dagger$ |
| $1(\frac{3}{2}^+)$ | $1S$ | 2518 | 2518.0(5) | | | | 5834 | 5829.0$^*$ | 5836.7$^\dagger$ |
| $1(\frac{1}{2}^-)$ | $1P$ | 2805 | | | | | 6122 | | |
| $1(\frac{3}{2}^-)$ | $1P$ | 2795 | | | | | 6108 | | |
| $1(\frac{5}{2}^-)$ | $1P$ | 2799 | 2802(2) | | | | 6106 | | |
| $1(\frac{3}{2}^-)$ | $1P$ | 2761 | 2766.6(2.4)? | | | | 6076 | | |
| $1(\frac{5}{2}^-)$ | $1P$ | 2790 | | | | | 6083 | | |
| $1(\frac{7}{2}^-)$ | $2S$ | 2864 | | | | | 6202 | | |
| $1(\frac{5}{2}^-)$ | $2S$ | 2912 | 2939.8(2.3)? | 2938(3)$^\dagger$ | | | 6222 | | |
| $1(\frac{1}{2}^-)$ | $1D$ | 3014 | | | | | 6300 | | |
| $1(\frac{3}{2}^-)$ | $1D$ | 3005 | | | | | 6287 | | |
| $1(\frac{5}{2}^-)$ | $1D$ | 3010 | | | | | 6291 | | |
| $1(\frac{7}{2}^-)$ | $1D$ | 3001 | | | | | 6279 | | |
| $1(\frac{9}{2}^-)$ | $1D$ | 2960 | | | | | 6248 | | |
| $1(\frac{1}{2}^-)$ | $1D$ | 3015 | | | | | 6262 | | |
| $1(\frac{3}{2}^-)$ | $2P$ | 3186 | | | | | 6411 | | |
| $1(\frac{5}{2}^-)$ | $2P$ | 3176 | | | | | 6401 | | |
| $1(\frac{7}{2}^-)$ | $2P$ | 3180 | | | | | 6400 | | |
| $1(\frac{9}{2}^-)$ | $2P$ | 3147 | | | | | 6379 | | |
| $1(\frac{11}{2}^-)$ | $2P$ | 3167 | | | | | 6383 | | |

* data for $\Sigma_b^{(*)+}$, experimental errors are given in the text
$^\dagger$ data for $\Sigma_b^{(*)-}$, experimental errors are given in the text

with one of the orbital $(1P)$ excitations of the $\Sigma_c$ with $J^P = \frac{1}{2}^-, \frac{3}{2}^-$ or $\frac{5}{2}^-$ which predicted mass differences are less than 15 MeV. Thus masses of all these states are compatible with the experimental value within errors.

Mass spectra of the $\Xi_Q$ baryons with the scalar and axial vector light ($qs$) diquarks are given in Tables V, VI. Experimental data here are available only for charm-strange baryons. We can identify the $\Xi_c(2790)$ and $\Xi_c(2815)$ with the first orbital $(1P)$ excitations of the $\Xi_c$ with $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^-$, respectively, containing the light scalar diquark, which is in agreement with the PDG [2] assignment. Recently Belle [25] reported the first observation of two baryons $\Xi_{cs}(2980)$ and $\Xi_{cs}(3077)$, which existence was also confirmed by BaBar [26]. The $\Xi_{cs}(2980)$ can be interpreted in our model as the first radial $(2S)$ excitation of the $\Xi_c$ with $J^P = \frac{1}{2}^+$ containing the light axial vector diquark. On the other hand the $\Xi_{cs}(3077)$ corresponds to the second orbital $(1D)$ excitation in this system with $J^P = \frac{5}{2}^+$. For the $\Omega_Q$ baryons only masses of the ground-state charmed baryons are known. The $\Omega_c^*$ baryon was very recently discovered by BaBar [3]. The measured mass difference of the $\Omega_c^*$ and $\Omega_c$ baryons of $(70.8 \pm 1.0 \pm 1.1)$ MeV is in very good agreement with the prediction of our model 70 MeV [1].
### TABLE V: Masses of the $\Xi_Q$ ($Q = c, b$) heavy baryons with scalar diquark (in MeV).

| $I(J^P)$ | $Qd$ state | $Q = c$ | $Q^\exp$ [2] | $Q = b$ |
|----------|-------------|---------|---------------|---------|
| $\frac{1}{2}(\frac{1}{2}^+)$ | $1S$ | 2481 | 2471.0(4) | 5812 |
| $\frac{1}{2}(\frac{1}{2}^-)$ | $1P$ | 2801 | 2791.9(3.3) | 6119 |
| $\frac{1}{2}(\frac{3}{2}^-)$ | $1P$ | 2820 | 2818.2(2.1) | 6130 |
| $\frac{1}{2}(\frac{3}{2}^+)$ | $2S$ | 2923 | | 6264 |
| $\frac{1}{2}(\frac{1}{2}^-)$ | $1D$ | 3030 | | 6359 |
| $\frac{1}{2}(\frac{3}{2}^-)$ | $2P$ | 3199 | | 6494 |
| $\frac{1}{2}(\frac{3}{2}^-)$ | $1F$ | 3208 | | 6558 |
| $\frac{1}{2}(\frac{1}{2}^-)$ | $3S$ | 3313 | | 6618 |
| $\frac{1}{2}(\frac{1}{2}^-)$ | $2D$ | 3411 | | 6688 |

### TABLE VI: Masses of the $\Xi_Q$ ($Q = c, b$) heavy baryons with axial vector diquark (in MeV).

| $I(J^P)$ | $Qd$ state | $Q = c$ | $Q^\exp$ [2] | $Q^\exp$ [25] | $Q^\exp$ [26] | $Q = b$ |
|----------|-------------|---------|---------------|---------------|---------------|---------|
| $\frac{1}{2}(\frac{1}{2}^+)$ | $1S$ | 2578 | 2578.0(2.9) | | | 5937 |
| $\frac{1}{2}(\frac{1}{2}^-)$ | $1S$ | 2654 | 2646.1(1.2) | | | 5963 |
| $\frac{1}{2}(\frac{1}{2}^-)$ | $1P$ | 2934 | | | | 6249 |
| $\frac{1}{2}(\frac{1}{2}^-)$ | $1P$ | 2928 | | | | 6238 |
| $\frac{1}{2}(\frac{1}{2}^-)$ | $1P$ | 2931 | | | | 6237 |
| $\frac{1}{2}(\frac{1}{2}^-)$ | $1P$ | 2900 | | | | 6212 |
| $\frac{1}{2}(\frac{1}{2}^-)$ | $1P$ | 2921 | | | | 6218 |
| $\frac{1}{2}(\frac{1}{2}^-)$ | $2S$ | 2984 | 2978.5(4.1) | 2967.1(2.9) | | 6327 |
| $\frac{1}{2}(\frac{1}{2}^-)$ | $2S$ | 3035 | | | | 6341 |
| $\frac{1}{2}(\frac{1}{2}^-)$ | $1D$ | 3132 | | | | 6420 |
| $\frac{1}{2}(\frac{1}{2}^-)$ | $1D$ | 3127 | | | | 6410 |
| $\frac{1}{2}(\frac{1}{2}^-)$ | $1D$ | 3131 | | | | 6412 |
| $\frac{1}{2}(\frac{1}{2}^-)$ | $1D$ | 3123 | | | | 6403 |
| $\frac{1}{2}(\frac{1}{2}^-)$ | $1D$ | 3087 | 3082.8(3.3) | 3076.4(1.0) | | 6377 |
| $\frac{1}{2}(\frac{1}{2}^-)$ | $1D$ | 3136 | | | | 6390 |
| $\frac{1}{2}(\frac{1}{2}^-)$ | $2P$ | 3300 | | | | 6527 |
| $\frac{1}{2}(\frac{1}{2}^-)$ | $2P$ | 3294 | | | | 6519 |
| $\frac{1}{2}(\frac{1}{2}^-)$ | $2P$ | 3296 | | | | 6518 |
| $\frac{1}{2}(\frac{1}{2}^-)$ | $2P$ | 3269 | | | | 6500 |
| $\frac{1}{2}(\frac{1}{2}^-)$ | $2P$ | 3282 | | | | 6504 |
### TABLE VII: Masses of the Ω_Q (Q = c, b) heavy baryons (in MeV).

| I(J^P)   | Qd state | M   | M^{exp} [2] | M^{exp} [3] | Q = c | Q = b |
|----------|----------|-----|-------------|-------------|-------|-------|
| 0(\frac{1}{2}^+) | 1S | 2698 | 2697.5(2.6) | 6065 |
| 0(\frac{3}{2}^+) | 1S | 2768 | 2768.3(3.0) | 6088 |
| 0(\frac{1}{2}^-) | 1P | 3025 |            | 6361 |
| 0(\frac{3}{2}^-) | 1P | 3020 |            | 6352 |
| 0(\frac{3}{2}^-) | 1P | 3026 |            | 6351 |
| 0(\frac{3}{2}^-) | 1P | 2998 |            | 6330 |
| 0(\frac{4}{2}^-) | 2S | 3065 |            | 6440 |
| 0(\frac{4}{2}^-) | 2S | 3119 |            | 6454 |
| 0(\frac{5}{2}^-) | 1D | 3222 |            | 6526 |
| 0(\frac{5}{2}^-) | 1D | 3215 |            | 6518 |
| 0(\frac{5}{2}^-) | 1D | 3217 |            | 6520 |
| 0(\frac{5}{2}^-) | 1D | 3218 |            | 6512 |
| 0(\frac{5}{2}^-) | 1D | 3187 |            | 6490 |
| 0(\frac{7}{2}^-) | 1D | 3237 |            | 6502 |
| 0(\frac{7}{2}^-) | 2P | 3376 |            | 6630 |
| 0(\frac{7}{2}^-) | 2P | 3371 |            | 6624 |
| 0(\frac{7}{2}^-) | 2P | 3374 |            | 6623 |
| 0(\frac{7}{2}^-) | 2P | 3350 |            | 6608 |
| 0(\frac{7}{2}^-) | 2P | 3365 |            | 6611 |

Our predictions for the heavy baryon mass spectra can be also compared with results of other calculations, e.g. [27, 28, 29]. In Ref. [27] the variational approach is used to solve the three-body problem in the relativized quark model with the QCD motivated quark potential. Authors of Ref. [28] calculate the mass spectra of charmed baryons within a relativistic quark model based on the Salpeter equation with a potential containing both the confining potential and instanton induced interactions. In Ref. [29] the three-quark problem is solved by means of the Faddeev method in momentum space with the quark-quark interaction consisting of the one-gluon exchange, confinement and boson exchange potentials. All these approaches are three-body ones and thus they predict the mass spectra of excited heavy baryons with significantly more levels than we get in our model, since we use the quark-diquark approximation. The comparison given in Table VIII shows that our predictions agree with experiment in most cases better than the results of the above mentioned approaches. The most clear example is our prediction [1] for the masses of the Ω_c^* and Σ_b, Σ_b^*, which agree with experiment with high accuracy. The accurate predictions for the Σ_b and Σ_b^* masses are also given in Ref. [30].

In conclusion we emphasize that, in calculating the heavy baryon masses, we do not use any free adjustable parameters, thus all obtained results are pure predictions. Indeed, the values of all parameters of the model (including quark masses and parameters of the quark potential) were fixed in our previous considerations of meson properties. Note that the light
TABLE VIII: Comparison of theoretical predictions for masses (in MeV) of heavy baryons (for $J = \frac{1}{2}, \frac{3}{2}$) with experimental data.

| $J^P$ | exp. | our | [27] | [28] | [29] | exp. | our | [27] | [28] | [29] |
|-------|------|-----|------|------|------|------|-----|------|------|------|
|       |      | $\Lambda_c$ |      |      |      |      | $\Sigma_c$ |      |      |      |
| $\frac{1}{2}^+$ | 2286 | 2297 | 2265 | 2272 | 2292 | 2454 | 2439 | 2440 | 2459 | 2448 |
| $\frac{3}{2}^+$ | 2766? | 2772 | 2775 | 2769 | 2669 | 2864 | 2890 | 2947 | 2793 |      |
| $\frac{1}{2}^+$ | 2874 | 2910 | 2848 | 2906 |      | 2518 | 2518 | 2495 | 2539 | 2505 |
| $\frac{3}{2}^+$ | 3262 | 3035 | 3100 | 3061 |      | 2912 | 2985 | 3010 | 2825 |      |
| $\frac{1}{2}^-$ | 2595 | 2598 | 2630 | 2594 | 2559 | 2802? | 2795 | 2765 | 2769 | 2706 |
| $\frac{3}{2}^-$ | 3017 | 2780 | 2853 | 2779 |      | 2802? | 2805 | 2770 | 2817 | 2791 |
| $\frac{1}{2}^+$ | 2628 | 2628 | 2640 | 2586 | 2559 | 2766? | 2761 | 2770 | 2799 | 2706 |
| $\frac{3}{2}^-$ | 3034 | 2840 | 2874 | 2779 |      | 2802? | 2799 | 2805 | 2815 | 2791 |
|       |      | $\Xi_c$ |      |      |      |      | $\Omega_c$ |      |      |      |
| $\frac{1}{2}^+$ | 2471 | 2481 | 2469 | 2496 |      | 2698 | 2698 | 2668 | 2701 |      |
| $\frac{3}{2}^+$ | 2578 | 2578 | 2595 | 2574 |      | 3065 | 3169 | 3044 |      |      |
| $\frac{1}{2}^+$ | 2646 | 2654 | 2651 | 2633 |      | 2768 | 2768 | 2721 | 2759 |      |
| $\frac{3}{2}^+$ | 3030 |      | 2951 |      |      | 3119 |      |      | 3080 |      |
| $\frac{1}{2}^-$ | 2792 | 2801 | 2769 | 2749 |      | 3020 |      |      | 2959 |      |
| $\frac{3}{2}^-$ | 2928 |      | 2829 |      |      | 3025 |      |      | 3029 |      |
| $\frac{1}{2}^+$ | 2818 | 2820 | 2771 | 2749 |      | 2998 |      |      | 2959 |      |
| $\frac{3}{2}^-$ | 2900 |      | 2829 |      |      | 3026 |      |      | 3029 |      |
|       |      | $\Lambda_b$ |      |      |      |      | $\Sigma_b$ |      |      |      |
| $\frac{1}{2}^+$ | 5620 | 5622 | 5585 | 5624 |      | 5808 | 5805 | 5795 | 5789 |      |
| $\frac{3}{2}^+$ | 6189 | 6145 | 6246 |      |      | 5829 | 5834 | 5805 | 5844 |      |
| $\frac{1}{2}^-$ | 5930 | 5912 | 5890 |      |      | 6108 | 6070 | 6070 | 6039 |      |
| $\frac{3}{2}^-$ | 5947 | 5920 | 5890 |      |      | 6076 | 6070 | 6070 | 6039 |      |
|       |      | $\Xi_b$ |      |      |      |      | $\Omega_b$ |      |      |      |
| $\frac{1}{2}^+$ | 5812 | 5825 |      |      |      | 6065 |      |      | 6037 |      |
| $\frac{3}{2}^+$ | 5963 | 5967 |      |      |      | 6088 |      |      | 6090 |      |
| $\frac{1}{2}^-$ | 6119 | 6076 |      |      |      | 6352 |      |      | 6278 |      |
| $\frac{3}{2}^-$ | 6130 | 6076 |      |      |      | 6330 |      |      | 6278 |      |

*a Only central values of measured masses are given. Experimental errors can be found in Tables III-VII.

diquark in our approach is not considered as a point-like object. Instead we use its wave functions to calculate diquark-gluon interaction form factors and thus take into account the finite (and relatively large) size of the light diquark. The other important advantage of our model is the completely relativistic treatment of the light quarks in the diquark and of the light diquark in the heavy baryon. We use the $v/c$ expansion only for heavy ($b$ and $c$) quarks.

We find that all presently available experimental data for the ground and excited states of heavy baryons can be accommodated in the picture treating a heavy baryon as the bound system of the light diquark and heavy quark, experiencing orbital and radial excitations only between these constituents.
The obtained wave functions of the ground-state and excited heavy baryons can be used for calculations of the semileptonic and nonleptonic weak decays and of the one-pion transitions between excited and ground states. The heavy-to-heavy semileptonic decays of bottom baryons to charmed baryons were already studied by us in Ref. [31]. For the calculation of the heavy-to-light semileptonic decays the light baryon wave functions are necessary. The application of a simple quark-diaquark approximation for light baryons is controversial and thus more sophisticated methods should be used.

Note added: After this letter was submitted for publication the D0 Collaboration [32] reported the discovery of the $\Xi_b^-$ baryon with the mass $M_{\Xi_b^-} = 5774 \pm 11 \pm 15$ MeV. The CDF Collaboration [33] confirmed this observation and gave the more precise value $M_{\Xi_b^-} = 5792.9 \pm 2.5 \pm 1.7$ MeV. Our model prediction $M_{\Xi_b^-} = 5812$ MeV is in a reasonable agreement with these new data. The BaBar Collaboration [34] announced observation of two new charmed baryons $\Xi_c(3055)$ with the mass $M = 3054.2 \pm 1.2 \pm 0.5$ MeV and $\Xi_c(3123)$ with the mass $M = 3122.9 \pm 1.3 \pm 0.3$ MeV. These states can be interpreted in our model as the second orbital (1$D$) excitations of the $\Xi_c$ with $J^P = \frac{3}{2}^+$ containing scalar and axial vector diquarks, respectively. Their predicted masses are 3042 MeV and 3123 MeV.

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