How to Distinguish Dark Energy and Modified Gravity?

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ABSTRACT

The current accelerated expansion of our universe could be due to an unknown energy component (dark energy) or a modification to general relativity (modified gravity). In the literature, it has been proposed that combining the probes of the cosmic expansion history and growth history can distinguish between dark energy and modified gravity. In this work, without invoking non-trivial dark energy clustering, we show that the possible interaction between dark energy and dark matter could make the interacting dark energy model and the modified gravity model indistinguishable. An explicit example is also given. Therefore, it is required to seek some complementary probes beyond the ones of cosmic expansion history and growth history.

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I. INTRODUCTION

The current accelerated expansion of our universe [1, 2, 3, 4, 5, 6, 7, 8, 9, 10] has been one of the most active fields in modern cosmology. There are very strong model-independent evidences [11] (see also e.g. [12]) for the accelerated expansion. Many cosmological models have been proposed to interpret this mysterious phenomenon, see e.g. [1] for a recent review.

In the flood of various cosmological models, one of the most important tasks is to distinguish between them. The current accelerated expansion of our universe could be due to an unknown energy component (dark energy) or a modification to general relativity (modified gravity) [1, 13]. Recently, some efforts have focused on differentiating dark energy and modified gravity with the growth function \( \delta(z) \equiv \delta_{\rho_m}/\rho_m \) of the linear matter density contrast as a function of redshift \( z \). Up until now, most cosmological observations merely probe the expansion history of our universe [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. As is well known, it is very easy to build models which share the same cosmic expansion history by means of reconstruction between models. Therefore, to distinguish various models, some independent and complementary probes are required. Recently, it was argued that the measurement of growth function \( \delta(z) \) might be competent, see e.g. [13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 47, 48, 49, 50]. If the dark energy model and modified gravity model share the same cosmic expansion history, they might have different growth histories. Thus, they might be distinguished from each other.

However, the approach mentioned above has been challenged by some authors. For instance, in [26], Kunz and Sapone explicitly demonstrated that the dark energy models with non-vanishing anisotropic stress cannot be distinguished from modified gravity models (e.g. DGP model) using growth function. In [27], Bertschinger and Zukin found that if dark energy is generalized to include both entropy and shear stress perturbations, and the dynamics of dark energy is unknown a priori, then modified gravity [e.g. \( f(R) \) theories] cannot in general be distinguished from dark energy using cosmological linear perturbations.

Here, we investigate this issue in another way. Instead of invoking non-trivial dark energy clustering (e.g. non-vanishing anisotropic stress), it is of interest to see whether the interaction between dark energy and cold dark matter can make dark energy and modified gravity indistinguishable. In fact, since the nature of both dark energy and dark matter are still unknown, there is no physical argument to exclude the possible interaction between them. On the contrary, some observational evidences of this interaction have been found recently. For example, in a series of papers by Bertolami et al. [28], they show that the Abell Cluster A586 exhibits evidence of the interaction between dark energy and dark matter, and they argue that this interaction might imply a violation of the equivalence principle. On the other hand, in [29], Abdalla et al. found the signature of interaction between dark energy and dark matter by using optical, X-ray and weak lensing data from 33 relaxed galaxy clusters. In [32], Ichiki and Keum discussed the cosmological signatures of interaction between dark energy and massive neutrino (which is also a candidate for dark matter) by using CMB power spectra and matter power spectrum. Therefore, it is reasonable to consider the interaction between dark energy and dark matter in cosmology. Since dark energy can decay into cold dark matter (and vice versa) through interaction, both expansion history and growth history can be affected simultaneously, similar to the case of modified gravity. Thus, it is natural to ask whether the combined probes of expansion history and growth history can distinguish between interacting dark energy models and modified gravity models. The answer might be no. So, it is required to seek some complementary probes beyond the ones of cosmic expansion history and growth history.

In this note, we propose a general approach in Sec. II to build an interacting quintessence model which shares both the same expansion history and growth history with the modified gravity model. In Sec. III, following this prescription, as an example, we explicitly demonstrate the interacting quintessence model which is indistinguishable with the DGP model in this sense. A brief discussion is given in Sec. IV, in which some suggestions to break this degeneracy are also discussed.

II. GENERAL FORMALISM

Quintessence [30, 31] is a well-known dark energy candidate. In this section, we consider the interacting quintessence model [32, 33, 34, 35, 36, 37] and show how it can share both the same expansion history and growth history with modified gravity models, without invoking non-trivial dark energy clustering.
We consider a flat Friedmann-Robertson-Walker (FRW) universe. As is well known, the quintessence is described by a canonical scalar field with a Lagrangian density $L = \frac{1}{2} (\partial_\mu \phi)^2 - V(\phi)$. Assuming the scalar field $\phi$ is homogeneous, one obtains the pressure and energy density for quintessence

$$p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi), \quad \rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi),$$

(1)

where a dot denotes the derivative with respect to cosmic time $t$. The Friedmann equation reads

$$3H^2 = \kappa^2 (\rho_m + \rho_\phi),$$

(2)

where $H \equiv \dot{a}/a$ is the Hubble parameter; $a = (1 + z)^{-1}$ is the scale factor (we have set $a_0 = 1$; the subscript “0” indicates the present value of corresponding quantity; $z$ is the redshift); $\rho_m$ is the energy density of cold dark matter (we assume the baryon component to be negligible); $\kappa^2 \equiv 8\pi G$. We assume that quintessence and cold dark matter interact through $\ddot{\rho}_m + 3H \rho_m = -\kappa Q \rho_m \dot{\phi}$,

\begin{equation}
\dot{\rho}_m + 3H \rho_m = -\kappa Q \rho_m \dot{\phi},
\end{equation}

(3)

and

\begin{equation}
\dot{\rho}_\phi + 3H (\rho_\phi + p_\phi) = \kappa Q \rho_m \dot{\phi},
\end{equation}

(4)

which preserves the total energy conservation equation $\dot{\rho}_{\text{tot}} + 3H (\rho_{\text{tot}} + p_{\text{tot}}) = 0$. The dimensionless coupling coefficient $Q = Q(\phi)$ is an arbitrary function of $\phi$. Eq. (4) is equivalent to

$$\ddot{\phi} + 3H \dot{\phi} + \frac{dV}{d\phi} = \kappa Q \rho_m.$$

(5)

Using Eqs. (2)—(4), one can obtain the Raychaudhuri equation

$$\dot{H} = -\frac{\kappa^2}{2} (\rho_m + \rho_\phi + p_\phi) = -\frac{\kappa^2}{2} (\rho_m + \dot{\phi}^2).$$

(6)

It is worth noting that due to the non-vanishing interaction, $\rho_m$ does not scale as $a^{-3}$. The above equations are associated with the expansion history. On the side of growth history, as shown in [33], the perturbation equation in the sub-horizon regime is

$$\ddot{\delta} + \left(2 + \frac{H'}{H} - \kappa Q \psi\right) \dot{\delta} = \frac{3}{2} (1 + 2Q^2) \Omega_m \delta,$$

(7)

where $\delta \equiv \delta \rho_m/\rho_m$ is the linear matter density contrast; $\Omega_m \equiv \kappa^2 \rho_m/(3H^2)$ is the fractional energy density of cold dark matter; and a prime denotes a derivative with respect to $\ln a$. Note that in [33] the absence of anisotropic stress has been assumed, namely, in longitudinal (conformal Newtonian) gauge the metric perturbations $\Phi = \Psi$. Obviously, when $Q = 0$, Eq. (7) reduces to the standard form in general relativity [14, 15, 16, 17, 18, 24, 38]

$$\ddot{\delta} + 2H \dot{\delta} = 4\pi G \rho_m \delta.$$

(8)

In fact, Eq. (7) from [33] is valid for any $Q = Q(\phi)$ and generalizes the one of [34] which is only valid for constant $Q$. On the other hand, in modified gravity, the perturbation equation (8) has been modified to [18, 21, 22, 38, 39]

$$\ddot{\tilde{\delta}} + 2\tilde{H} \dot{\tilde{\delta}} = 4\pi G_{\text{eff}} \rho_m \tilde{\delta},$$

(9)

where the quantities in modified gravity are labeled by a tilde “≈”; $G_{\text{eff}}$ is the effective local gravitational “constant” measured by Cavendish-type experiment, which is time-dependent. In general, $G_{\text{eff}}$ can be written as

$$G_{\text{eff}} = G \left(1 + \frac{1}{3\beta}\right),$$

(10)
where \( \beta \) is determined once we specify the modified gravity theory. Eq. (9) can be rewritten as

\[
\ddot{\delta} + \left(2 + \frac{\dot{H}'}{H}\right) \dot{\delta} = \frac{3}{2} \left(1 + \frac{1}{3\beta}\right) \dot{\Omega}_m \dot{\delta}.
\]

Now, we require that the interacting quintessence model shares both the same expansion history and growth history with the modified gravity model. That is, we identify

\[
H = \bar{H} \quad \text{and} \quad \delta = \bar{\delta}.
\]

Comparing Eq. (7) with Eq. (11), we find that

\[
\kappa Q \phi' \delta' = \frac{3}{2} \left(1 + \frac{1}{3\beta}\right) \dot{\Omega}_m - \left(1 + 2Q^2\right) \Omega_m.
\]

Note that \( \Omega_m \neq \bar{\Omega}_m \) in general. From Eq. (6), we have

\[
(\kappa \phi')^2 = -3\Omega_m - 2\frac{H'}{H}.
\]

We can recast Eq. (3) to

\[
\Omega'_m = -\left(3 + 2\frac{H'}{H} + \kappa Q \phi'\right) \Omega_m.
\]

It turns out

\[
\kappa Q \phi' = -3 - 2\frac{H'}{H} - \frac{\Omega'_m}{\Omega_m}.
\]

From Eqs. (13) and (16), we obtain

\[
Q^2 = \frac{(\kappa Q \phi')^2}{(\kappa \phi')^2} = \left(\frac{3 + 2\frac{H'}{H} + \Omega'_m}{\Omega_m}\right)^2.
\]

Noting that \( \delta = \bar{\delta} \), we can find \( \delta \) in Eq. (13) from Eq. (11). Once \( \bar{\Omega}_m, \beta, \bar{H}, \) and corresponding \( \bar{\delta} \) in the modified gravity are given, substituting Eqs. (16) and (17) into Eq. (13) and noting Eq. (12), we obtain a differential equation for \( \bar{\Omega}_m \) with respect to \( \ln a \). After we find \( \bar{\Omega}_m(\ln a) \) from this differential equation, \( \kappa \phi'(\ln a) \) can be obtained from Eq. (13), while \( H = \bar{H} \). Then, by using Eqs. (14) and (10), \( Q(\ln a) = (\kappa Q \phi') / (\kappa \phi') \) is in hand. From Eq. (2), \( \Omega_\phi \equiv \kappa^2 \rho_\phi / (3H^2) = 1 - \Omega_m \). Noting Eq. (11) and using Eq. (14), we find the dimensionless potential of quintessence

\[
U \equiv \frac{\kappa^2 V}{H_0^2} = 3E^2 \left(1 - \frac{\Omega_m}{2} + \frac{1}{3} \frac{H'}{H}\right),
\]

where \( E \equiv H/H_0 \). Notice that \( H'/H = E'/E \). By integrating \( \kappa \phi'(\ln a) \), we find \( \kappa \phi \) as a function of \( \ln a \). Therefore, we can finally obtain \( Q, U \) as functions of \( \kappa \phi \).

In fact, this general approach proposed here is just a normal reconstruction method. For any given modified gravity model, we can always construct an interacting quintessence model in the framework of general relativity which shares both the same expansion history and growth history with this given modified gravity model. As is well known, an interacting quintessence model can be completely described by its potential \( V(\phi) \) and the coupling \( Q(\phi) \). For a given modified gravity model, its \( \bar{\Omega}_m, \beta, \) and \( \bar{H} \) are known, whereas its corresponding \( \bar{\delta} \) can be obtained from Eq. (11). Following the procedure described in the text below Eq. (17), one can easily reconstruct the dimensionless potential \( U(\kappa \phi) \) [which is equivalent to the potential \( V(\phi) \) obviously] and the coupling \( Q(\kappa \phi) \) [which is \( Q(\phi) \) in fact] for the interacting quintessence model. Up until now, the desired interacting quintessence model which shares both the same expansion history and growth history with the given modified gravity model has been constructed. Therefore, the cosmological observations might be unable to distinguish between them, unless other complementary probes beyond the ones of cosmic expansion history and growth history are used.
FIG. 1: $\delta = \bar{\delta}$, $\Omega_m$, $\kappa \phi'$, $Q$, $U \equiv \kappa^2 V / H_0^2$, and $\kappa \phi$ as functions of $\ln a$. See text for details.
III. EXPLICIT EXAMPLE

In this section, we give an explicit example following the prescription proposed in Sec. II. Here, we consider the DGP braneworld model \[40\] (see also e.g. \[18, 21, 22, 11\]), which is the simplest modified gravity model. Assuming the flatness of our universe, in the DGP model (here we only consider the self-accelerating branch), \(\tilde{E} \equiv \tilde{H}/H_0\) is given by \[18, 21, 22\]

\[
\tilde{E} = \sqrt{\tilde{\Omega}_{m0}(1 + z)^3 + \tilde{\Omega}_{r_e} + \sqrt{\tilde{\Omega}_{r_e}}} = \sqrt{\tilde{\Omega}_{m0} e^{-3 \ln a} + \tilde{\Omega}_{r_e} + \sqrt{\tilde{\Omega}_{r_e}}}, \tag{19}
\]

where \(\tilde{\Omega}_{r_e}\) is constant. \(\tilde{E}(z = 0) = 1\) requires

\[
\tilde{\Omega}_{m0} = 1 - 2\sqrt{\tilde{\Omega}_{r_e}}. \tag{20}
\]

Therefore, the DGP model has only one independent model parameter. The fractional energy density of matter in the DGP model reads

\[
\tilde{\Omega}_m = \frac{\tilde{\Omega}_{m0}(1 + z)^3}{E^2(z)} = \frac{\tilde{\Omega}_{m0} e^{-3 \ln a}}{E^2(\ln a)}. \tag{21}
\]

In addition, the \(\beta\) in Eq. \[10\] for the flat DGP model is given by \[18, 21, 22\]

\[
\beta = \frac{1 + \tilde{\Omega}_m}{1 - \tilde{\Omega}_m}. \tag{22}
\]

Fitting the DGP model to the 192 SNIa data compiled by Davis et al. \[10\] which are joint data from ESSENCE \[3\] and Gold07 \[3\], we find that the best-fit parameter \(\tilde{\Omega}_{r_e} = 0.170\) with \(\chi^2_{min} = 196.128\) while the corresponding \(\tilde{\Omega}_{m0}\) is given by Eq. \[20\]. Substituting this \(\tilde{\Omega}_{m0}\) into Eqs. \[21\], \[22\] and \[19\], \(\tilde{\Omega}_m, \beta, \) and \(E\) as functions of \(\ln a\) are known (since our main aim is to demonstrate the prescription proposed in Sec. II we do not consider the errors, even one can use any \(\tilde{\Omega}_{m0}\) here for demonstration). Noting that \(H'/H = E'/E\), following the prescription proposed in Sec. II we can easily construct the desired interacting quintessence model which shares both the same expansion history and growth history with the corresponding DGP model. At the first step, we obtain \(\delta = \tilde{\delta}\) from Eq. \[11\]. As is well known, \(\delta' = \tilde{\delta} = a\) at \(z \gg 1\) (see e.g. \[14, 15\]). Thus, we use the initial condition \(\delta' = \tilde{\delta} = a_{ini}\) at \(z_{ini} = 1000\) for the differential equation \[11\]. The resulting \(\delta = \tilde{\delta}\) as a function of \(\ln a\) is shown in Fig. \[1\]. At the second step, substituting Eqs. \[10\] and \[11\] into Eq. \[13\], we obtain \(\tilde{\Omega}_m\) as a function of \(\ln a\) from the resulting differential equation. Note that cold dark matter can decay to quintessence through interaction, \(\tilde{\Omega}_m\) is unnecessary to be 1 at high redshift. So, for demonstration, we choose the initial condition \(\tilde{\Omega}_m(z = z_{ini}) = 0.995\) at \(z_{ini} = 1000\) for the differential equation of \(\tilde{\Omega}_m\). Different values of \(\tilde{\Omega}_m(z = z_{ini})\) only mean different displacements of \(\kappa\phi'\) and the dimensionless potential \(U \equiv \kappa^2 V/H_0^2\) at \(z_{ini}\). The resulting \(\tilde{\Omega}_m\) as a function of \(\ln a\) is also shown in Fig. \[1\]. Then, following the prescription proposed in Sec. II it is straightforward to obtain \(\kappa\phi'\), \(Q, U \equiv \kappa^2 V/H_0^2\), and \(\kappa\phi\) as functions of \(\ln a\), while for demonstration we choose the negative branch for \(\kappa\phi'\), and choose \(\phi_0 = 0\) when we get \(\kappa\phi\). They are also shown in Fig. \[1\]. Once we obtain \(Q, U\) and \(\kappa\phi\) as functions of \(\ln a\), it is easy to find \(Q\) and \(U\) as functions of \(\kappa\phi\). The results are shown in Fig. \[2\].

In short, here we have faithfully followed the reconstruction method proposed in Sec. II and successfully constructed an interacting quintessence model which shares both the same expansion history and growth history with the modified gravity model. Therefore, the

IV. CONCLUSION AND DISCUSSION

In summary, we proposed a general approach to build an interacting quintessence model which shares both the same expansion history and growth history with the modified gravity model. Therefore, the
cosmological observations might be unable to distinguish between them, unless other complementary probes beyond the ones of cosmic expansion history and growth history are used. As an example, we also explicitly demonstrated the interacting quintessence model which is indistinguishable with the DGP model in this sense. In fact, this proposed prescription is also valid for other modified gravity models, such as \( f(R) \) theories, braneworld-type models, scalar-tensor theories (including Brans-Dicke theory), TeVeS/MOND models, and so on [43, 44, 45]. Of course, one can also extend the interacting quintessence model to other interacting dark energy models.

\[ Q \equiv \kappa^2 V / H_0^2 \]

\[ U \text{ in units of } 10^6 \]

In this note, without invoking non-trivial dark energy clustering (e.g. non-vanishing anisotropic stress), we find that the interaction between dark energy and dark matter could also make dark energy model and modified gravity model indistinguishable. How can we break this degeneracy? Firstly, we should carefully check the evidences of the interaction between dark energy and dark matter [28, 29, 42]. If this interaction does not exist, the combined probes of cosmic expansion history and growth history might be enough to distinguish between dark energy and modified gravity. It is worth noting that the current observational constraints on the interaction between dark energy and dark matter in the literature (e.g. [28, 29, 42, 46]) considered other interaction forms (e.g. \( \propto H \rho_m \), \( H \rho_{de} \) or \( H \rho_{tot} \)) which are different from the one of the present work [cf. Eqs. (3) and (4)], and hence cannot be used to compare with the interaction considered here. Therefore, it is of interest to consider the observational constraints on the type of interactions \( Q(\phi) \rho_m \dot{\phi} \) in future works. Secondly, to break the degeneracy, the other complementary probes beyond the ones of cosmic expansion history and growth history are desirable. For instance, these complementary probes might include local tests of gravity, high energy phenomenology, and non-linear structure formation. Thirdly, in addition to the linear matter density contrast \( \delta(z) \), one can also test the metric perturbations \( \Phi \) and \( \Psi \) by using the relationship between gravitational lensing and matter overdensity [50]. For the interacting quintessence model \( \Phi = \Psi \), whereas for the DGP model \( \Phi \neq \Psi \). Thus, it is possible to distinguish between them by using delicate measurements. We consider that this issue deserves further investigations and believe that a promising future is awaiting us.

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[1] E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D 15, 1753 (2006) [hep-th/0603057].
[2] A. G. Riess et al. [Supernova Search Team Collaboration], Astron. J. 116, 1009 (1998) [astro-ph/9805201].
S. Perlmutter et al. [Supernova Cosmology Project Collaboration], Astrophys. J. 517, 565 (1999) [astro-ph/9812133].
J. L. Tonry et al. [Supernova Search Team Collaboration], Astrophys. J. 594, 1 (2003) [astro-ph/0305008].
R. A. Knop et al. [Supernova Cosmology Project Collaboration], Astrophys. J. 598, 102 (2003) [astro-ph/0309368].
A. G. Riess et al. [Supernova Search Team Collaboration], Astrophys. J. 607, 665 (2004) [astro-ph/0402512].
[3] A. G. Riess et al. [Supernova Search Team Collaboration], Astrophys. J. 659, 98 (2007) [astro-ph/0611572].
[4] P. Astier et al. [SNLS Collaboration], Astron. Astrophys. 447, 31 (2006) [astro-ph/0510447].
[5] D. N. Spergel et al. [WMAP Collaboration], Astrophys. J. Suppl. 170, 377 (2007) [astro-ph/0603449].
L. Page et al. [WMAP Collaboration], Astrophys. J. Suppl. 170, 335 (2007) [astro-ph/0603450].
G. Hinshaw et al. [WMAP Collaboration], Astrophys. J. Suppl. 170, 288 (2007) [astro-ph/0603451].
N. Jarosik et al. [WMAP Collaboration], Astrophys. J. Suppl. 170, 263 (2007) [astro-ph/0603452].
[6] E. Komatsu et al. [WMAP Collaboration], arXiv:0803.0547 [astro-ph].
R. S. Hill et al. [WMAP Collaboration], arXiv:0803.0570 [astro-ph].
E. L. Wright et al. [WMAP Collaboration], arXiv:0803.0577 [astro-ph].
J. Dunkley et al. [WMAP Collaboration], arXiv:0803.0586 [astro-ph].
M. R. Nolta et al. [WMAP Collaboration], arXiv:0803.0593 [astro-ph].
B. Gold et al. [WMAP Collaboration], arXiv:0803.0715 [astro-ph].
G. Hinshaw et al. [WMAP Collaboration], arXiv:0803.0732 [astro-ph].
[7] M. Tegmark et al. [SDSS Collaboration], Phys. Rev. D 69, 103501 (2004) [astro-ph/0310723].
M. Tegmark et al. [SDSS Collaboration], Astrophys. J. 606, 702 (2004) [astro-ph/0310725].
U. Seljak et al., Phys. Rev. D 71, 103515 (2005) [astro-ph/0407372].
M. Tegmark et al. [SDSS Collaboration], Phys. Rev. D 74, 123507 (2006) [astro-ph/0608632].
[8] S. W. Allen et al., Mon. Not. Roy. Astron. Soc. 353, 457 (2004) [astro-ph/0403401].
S. W. Allen et al., arXiv:0706.0033 [astro-ph].
A. Mantz, S. W. Allen, H. Ebeling and D. Rapetti, arXiv:0709.4294 [astro-ph].
[9] W. M. Wood-Vasey et al. [ESSENCE Collaboration], Astrophys. J. 666, 694 (2007) [astro-ph/0701041].
G. Miknaitis et al. [ESSENCE Collaboration], Astrophys. J. 666, 674 (2007) [astro-ph/0701043].
[10] T. M. Davis et al., Astrophys. J. 666, 716 (2007) [astro-ph/0701510].
It compiled the joint 192 SNIa data from ESSENCE [4] and Gold07 [3].
The numerical data of the full sample are available at http://www.ctio.noao.edu/essence or http://braeburn.pha.jhu.edu/~ariess/R06
[11] C. Shapiro and M. S. Turner, Astrophys. J. 649, 563 (2006) [astro-ph/0512586].
[12] M. Seikel and D. J. Schwarz, JCAP 0802, 007 (2008) [arXiv:0711.3180].
Y. Gong, A. Wang, Q. Wu and Y. Z. Zhang, JCAP 0708, 018 (2007) [astro-ph/0703583].
Y. Gong and A. Wang, Phys. Rev. D 73, 083506 (2006) [astro-ph/0601453].
M. P. Lima, S. Vitenti and M. J. Reboucas, Phys. Rev. D 77, 083518 (2008) [arXiv:0802.0706].
[13] Y. Wang, arXiv:0710.3885 [astro-ph].
Y. Wang, arXiv:0712.0041 [astro-ph].
[14] E. V. Linder and R. N. Cahn, Astropart. Phys. 28, 481 (2007) [astro-ph/0701317].
E. V. Linder, Phys. Rev. D 72, 043529 (2005) [astro-ph/0507263].
[15] D. Huterer and E. V. Linder, Phys. Rev. D 75, 023519 (2007) [astro-ph/0608081].
[16] L. M. Wang and P. J. Steinhardt, Astrophys. J. 508, 483 (1998) [astro-ph/9804015].
[18] H. Wei, Phys. Lett. B 664, 1 (2008) [arXiv:0802.4122].
[19] S. Wang, L. Hui, M. May and Z. Haiman, Phys. Rev. D 76, 063503 (2007) [arXiv:0705.0165].
[20] Y. S. Song and K. Koyama, arXiv:0802.3897 [astro-ph];
A. Cardoso, K. Koyama, S. S. Seahra and F. P. Silva, arXiv:0711.2563 [astro-ph];
K. Koyama, Gen. Rel. Grav. 40, 421 (2008) [arXiv:0706.1557].
[21] A. Lue, R. Scoccimarro and G. D. Starkman, Phys. Rev. D 69, 124015 (2004) [astro-ph/0401515].
[22] A. Lue, Phys. Rept. 423, 1 (2006) [astro-ph/0510068].
[23] C. Di Porto and L. Amendola, Phys. Rev. D 77, 083508 (2008) [arXiv:0707.2686];
L. Amendola, M. Kunz and D. Sapone, JCAP 0804, 013 (2008) [arXiv:0704.2421];
D. Sapone and L. Amendola, arXiv:0709.2792 [astro-ph].
[24] S. Nesseris and L. Perivolaropoulos, Phys. Rev. D 77, 023504 (2008) [arXiv:0710.1092].
[25] D. Polarski and R. Gannouji, Phys. Rev. D 69, 124015 (2004) [astro-ph/0401515].
[26] A. Lue, R. Scoccimarro and G. D. Starkman, Phys. Rev. D 76, 063503 (2007) [arXiv:0705.0165].
[27] A. Lue, Phys. Rept. 423, 1 (2006) [astro-ph/0510068].
[28] C. Di Porto and L. Amendola, Phys. Rev. D 77, 083508 (2008) [arXiv:0707.2686];
L. Amendola, M. Kunz and D. Sapone, JCAP 0804, 013 (2008) [arXiv:0704.2421];
D. Sapone and L. Amendola, arXiv:0709.2792 [astro-ph].
[29] S. Nesseris and L. Perivolaropoulos, Phys. Rev. D 77, 023504 (2008) [arXiv:0710.1092].
[30] D. Polarski and R. Gannouji, Phys. Lett. B 660, 439 (2008) [arXiv:0711.2563 [astro-ph]].
[31] A. Cardoso, K. Koyama, S. S. Seahra and F. P. Silva, arXiv:0711.2563 [astro-ph];
K. Koyama, Gen. Rel. Grav. 40, 421 (2008) [arXiv:0706.1557].
[32] A. Lue, R. Scoccimarro and G. D. Starkman, Phys. Rev. D 69, 124015 (2004) [astro-ph/0401515].
[33] A. Lue, Phys. Rept. 423, 1 (2006) [astro-ph/0510068].
[34] C. Di Porto and L. Amendola, Phys. Rev. D 77, 083508 (2008) [arXiv:0707.2686];
L. Amendola, M. Kunz and D. Sapone, JCAP 0804, 013 (2008) [arXiv:0704.2421];
D. Sapone and L. Amendola, arXiv:0709.2792 [astro-ph].
[35] S. Nesseris and L. Perivolaropoulos, Phys. Rev. D 77, 023508 (2008) [arXiv:0710.1092].
[36] D. Polarski and R. Gannouji, Phys. Lett. B 660, 439 (2008) [arXiv:0710.1510];
R. Gannouji and D. Polarski, arXiv:0802.4196 [astro-ph].
[37] M. Kunz and D. Sapone, Phys. Rev. Lett. 98, 121301 (2007) [astro-ph/0612452].
[38] E. Bertschinger and P. Zukin, arXiv:0801.2431 [astro-ph].
F. P. Schuller and M. N. R. Wohlfarth, JCAP 0712, 013 (2007) [arXiv:0705.4656].
R. Punzi, F. P. Schuller and M. N. R. Wohlfarth, Phys. Rev. D 76, 101501 (2007) [hep-th/0612133].

[45] M. Szydlowski and W. Godlowski, Phys. Lett. B 633, 427 (2006) [astro-ph/0509415];
M. Szydlowski, A. Kurek and A. Krawiec, Phys. Lett. B 642, 171 (2006) [astro-ph/0604327];
A. Kurek and M. Szydlowski, Astrophys. J. 675, 1 (2008).

[46] C. Feng, B. Wang, E. Abdalla and R. K. Su, [arXiv:0804.0110] [astro-ph];
J. H. He and B. Wang, [arXiv:0801.4233] [astro-ph].

[47] C. Schimd, J. P. Uzan and A. Riazuelo, Phys. Rev. D 71, 083512 (2005) [astro-ph/0412120];
J. P. Uzan, Gen. Rel. Grav. 39, 307 (2007) [astro-ph/0605313];
C. Schimd et al., Astron. Astrophys. 463, 405 (2007) [astro-ph/0603158].

[48] M. Manera and D. F. Mota, Mon. Not. Roy. Astron. Soc. 371, 1373 (2006) [astro-ph/0504519].

[49] V. Acquaviva, A. Hajian, D. N. Spergel and S. Das, arXiv:0803.2236 [astro-ph];
O. Dore et al., arXiv:0712.1599 [astro-ph];
S. F. Daniel, R. R. Caldwell, A. Cooray and A. Melchiorri, arXiv:0802.1068 [astro-ph].

[50] P. Zhang, M. Liguori, R. Bean and S. Dodelson, Phys. Rev. Lett. 99, 141302 (2007) [arXiv:0704.1932];
B. Jain and P. Zhang, arXiv:0709.2375 [astro-ph].