The Power of Non-Superpowers*

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Abstract

We propose a game-theoretic model to investigate how non-superpowers with heterogeneous preferences and endowments shape the superpower competition for a sphere of influence. Two superpowers play a Stackelberg game by providing club goods. Their utility depends on non-superpowers who form coalitions to join a club in the presence of externality. The coalition formation, which depends on the characteristics of non-superpowers, influences the behavior of superpowers and thus the size of their clubs. Our data-based simulations of the subgame perfect equilibrium capture how the US-China competition depends on other countries.

Keywords: superpowers; club goods; deterrence; coalition formation; subgame perfect equilibrium

JEL Classification: C7; D6; D7; F5

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“Economic primacy, at its best, involves less dominance or hegemony than the public good of leadership of the world economy, not ordering others to behave as the leader directs, but pointing the way and convincing others of the desirability of following.” (Kindleberger, 1996, p.13)

1 Introduction

The Russian invasion of Ukraine has prompted a response by the West to limit the access of seven Russian banks to the Society for Worldwide Interbank Financial Telecommunication (SWIFT). Excluded from the messaging system for cross-border payments, the banks became unable to transact with their Western counterparts in the US dollar or Euro and some started to make use of China’s alternative, the Cross-Border Interbank Payment System (CIPS). The number of participating banks in CIPS has risen to 1341 banks as of June 2022 since its establishment in 2015.

When banks join CIPS, they affect the benefit for others in the system. China considers this externality when providing CIPS to banks under the sanctions by the West. In the presence of such externality, how the behavior of countries shape the superpower competition for a sphere of influence is analyzed in a game-theoretic model we propose. We are particularly interested in how non-superpowers - heterogeneous in preferences and endowments - form coalitions and shape the competition between two superpowers who provide club goods sequentially. The clubs have an externality such that when a country joins a club, it reduces the cost for everyone because members share the cost. Our main findings can be summarized in three points:

1. The status-quo power can form a unipolar world even when the circumstances are symmetric to the challenger. This is because the status-quo superpower can compromise with non-superpowers to deter the club formation by the challenger. Non-superpowers institutionally close to the status-quo power are essential for this deterrence. The result explains the importance of close allies for the predominance of the US club.

2. The superpower compromises with non-superpowers to establish a club, but when other countries depend more on it, it compromises less while its club size increases. This is
because when a country depends more on a superpower, the benefit it derives from a club increases. The results explain how China becomes institutionally pro-active in 2013 but later less so, while its sphere of influence expands when other countries trade more with China.

3. Growth of a superpower’s endowment can attract new members. This is because when a superpower’s endowment increases, the cost of joining its club decreases for new members. The result explains why China’s growth leads to its growing sphere of influence today. On the other hand, the endowment growth of a non-superpower member can cause it to leave the club due to its rising cost share, making others leave too. This result predicts a radical shift in power balance between the US and China when a third power such as India continues to grow and leaves the US club in the future.

In our model, two superpowers play a Stackelberg game and attract members by providing a club good. Everyone has a fixed location and derives utility that is decreasing in the distance to the location of the good as in Hotelling (1929). There is a strand of literature in industrial organization that studies the leader-follower competition in a location model (see Drezner, 1982; Plastria and Vanhaverbeke, 2008; Elena Sáiz et al., 2009; Drezner et al., 2015). In those models the utility of customers is not affected by the market share of firms, and thus the choices of customers are independent of each other. In our model, non-superpowers form coalitions in the presence of an externality, which is similar to the network externality in the platform competition in two-sided markets such as Rochet and Tirole (2003); Jullien (2011); Lee (2014). However, the end users in those models do not internalize the externality and do not form coalitions to shape the platform competition as non-superpowers do in our model.

Non-superpowers in our model are like citizens who pick a community that best matches their preference. This principle of mobile citizens “voting with their feet” goes back to the idea by Tiebout (1956) and has been used to analyze various problems such as voluntary work and club size in Barham et al. (1997), the number and size of nations in Alesina and Spolaore (1997), the choice of jurisdictions in Casella and Feinstein (2002), and inequality and community formation in Jaramillo et al. (2003). In Alesina and Spolaore (1997) the utility of citizens depends on their distance to the public good in each nation and the location of the public good is determined mechanically by a median voter. The social planner decides
the size and number of nations. Instead, superpowers in our model compete strategically by choosing the location of their club good, while non-superpowers decide jointly to join a club. In two-stage games by Barham et al. (1997), Casella and Feinstein (2002) and Jaramillo et al. (2003), individuals form coalitions in the first stage and decide how much to contribute to the coalitions in the second stage. In other words, individual choices made in the second stage do not affect the club formation in the first stage, or there is a social planner that decides on the optimal coalition size. In our two-stage game, superpowers locate the public goods strategically to maximize their payoffs, which affect the coalition formation in each stage. Moreover, unlike citizens who must belong to a community, non-superpowers can choose not to join any club. To the best of our knowledge, we are the first to model an interaction between strategically competing players and the formation of coalitions. This interaction allows us to investigate the power of cooperative non-superpowers over competing superpowers, the understanding of which is essential to explain the dynamics of international relations.

Naturally, the status-quo power has a first-mover advantage as it can compromise with non-superpowers to form a club by anticipating the response of the challenger. For this deterrence by the status-quo power non-superpowers close to the status-quo power are essential. This illustrates the importance of allies that share institutional similarity with the status-quo power as we interpret the location as a position in the spectrum of institutional orientation. The location of club goods may then be interpreted as the institutional orientation of policies regarding the rule of law, governance structure, transparency, human rights and free speech. Our model structure captures the prevalence of the US-led world order challenged by the emergence of China.

Despite having a first-mover advantage, the US is challenged at least in two dimensions today. First, the average trade ratio of countries with the US is declining, while that with China is increasing. In fact, China has surpassed the US as the dominant trading partner on average in 2019 (see Figure 1(a)). The trade ratio of countries with the US and China may be a proxy for the dependency of non-superpowers on superpowers, which is a parameter of the utility function for non-superpowers in our model. Second, the gap of GDP between full democracies and authoritarian regimes is narrowing (see Figure 1(b)). We use the Democ-
(a) Trade ratios with the US and China
(b) Total GDP of Full Democracies and Authoritarian Regimes

Figure 1: The dependency and GDP of non-superpowers

Source: World Development Indicators, UN Comtrade and The Economists Intelligent Unit. Trade ratio with the US = Trade (export + import) with the US/Total trade (in USD). The same for China. We take 5-year moving averages over 189 countries. Each country is given a score between 0 and 10 and categorized into authoritarian regimes (0-4), hybrid regimes (4.01-6), flawed democracies (6.01-8) and full democracies (8.01,10).

We show how the democracy score, the trade ratio, and the GDP of non-superpowers shape the behavior of superpowers and the distribution of their power - how the world changes from a unipolar to a bipolar world. The simulation of our stylized model captures how the US and China formed their clubs from 2006 to 2019. Based on long-run GDP projections up to 2100, we show that the GDP growth of India relative to others changes the power balance between the US and China dramatically by 2030.

The rest of the paper is organized as follows. Section 2 introduces our model and presents our solution concepts. Section 3 analyzes comparative statics for the special cases of our model. Section 4 presents simulation results of the the US-China competition. Section 5 concludes. We relegate the proofs to the appendix.
2 The two-stage sequential game

This section introduces the general framework of the game (Section 2.1), describes the choice of non-superpowers at each stage given the choice of two superpowers (Section 2.2), then solves for the equilibrium of the two-stage sequential game between two superpowers given the behavior of non-superpowers (Section 2.3).

2.1 The general framework

The world consists of a set of two superpowers $H = \{a, b\}$ and a finite set of non-superpowers $I$. Only superpowers can provide a club good at a fixed cost to form a club. We call a superpower who forms a club a hegemon. Each country has a fixed location in a finite metric space $L$. The two superpowers compete for a sphere of influence by choosing a location for their club goods. The club good is non-rivalrous as its supply is not diminished by consumption, but excludable as only members benefit from the club good. Non-superpowers can join only one club at a time. The utility that countries derive from a club good depends on their distance to the club good. Members of a club (including the superpower) share the cost of providing the club good. We assume that a superpower cannot become a member of another superpower’s club. The two superpowers play a two-stage sequential Stackelberg game. In stage one, the status-quo power $a$ chooses a location for its club good (the choice is irreversible). In stage two, the challenger $b$ chooses a location for its club good. Non-superpowers decide whether to join $a$’s club or not in stage one and decide whether to join either $a$ or $b$’s club or no club in stage two. We assume there is no cost for switching from $a$ to $b$’s club. We will see later that it is essential for $a$’s first-mover advantage that non-superpowers make their choice in stage one and $b$ takes the choices as given in stage two.

Below we define the utility of non-superpowers in each stage separately. In stage one, the

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1 When $I$ is an infinite (countable or uncountable) set, the subgame perfect equilibrium we define later may not exist.

2 If we allowed non-superpowers to simultaneously be a member of both clubs, the superpowers would no longer play a sequential game; $a$ would lose its first-mover advantage as it could no longer deter non-superpowers from joining $b$’s club.

3 We assume that non-superpowers are not forward-looking. This can be justified by the information
choices of non-superpowers can be represented as an element in \( \{a, 0\}^I \) where \( a \) represents “join \( a \)” and \( 0 \) “not join \( a \)”.

Given the location of \( a \)'s club good \( \ell_a \in L \) and the choice of non-superpowers \( c = (c_i)_{i \in I} \in \{a, 0\}^I \), the utility of non-superpower \( i \in I \) in stage one is given by

\[
   u_i(\ell_a, c) = \begin{cases} 
   g_{ia}(1 - d_{i,\ell_a}) - \rho_{ia}(I_a(c)) & \text{if } c_i = a \\
   0 & \text{if } c_i = 0
   \end{cases}
\]

(SU1)

where \( g_{ia} \) is a measure of dependency of \( i \) on \( a \); \( d_{i,\ell_a} : L \times L \to [0, 1] \) is the distance from country \( i \) to \( \ell_a \); \( I_a(c) := \{ j \in I : c_j = a \} \) is the set of non-superpowers joining \( a \)'s club; and \( \rho_{ia}(E) \) is a cost share strictly decreasing with respect to \( E \), i.e., for \( E \subset E' \subseteq I \), \( \rho_{ia}(E') < \rho_{ia}(E) \). The benefit of a country \( i \) depends on its distance to the club good \( d_{i,\ell_a} \) and its dependency on \( a \), i.e., \( g_{ia} \). Everything else equal, the more dependent a country is on a superpower or the closer a country is to a club good, the higher is the utility it derives from the good.

In stage two, \( b \) comes into play and decides whether to form a club or not and where to locate the club good. When \( b \) chooses a location to form a club, each non-superpower \( i \in I \) needs to recalibrate its choice in stage one and decides whether to remain in \( a \)'s club, join \( b \)'s club, or join no club. Hence, we obtain a vector of choices of all non-superpowers \( c = (c_i)_{i \in I} \in \{a, b, 0\}^I \). Given \( a \) choosing \( \ell_a \), \( b \) choosing \( \ell_b \), and the choice of non-superpowers \( c \), the utility of non-superpower \( i \in I \) in stage two is given by

\[
   u_i(\ell_a, \ell_b, c) = \begin{cases} 
   g_{i,c_i}(1 - d_{i,\ell_{ci}}) - \rho_{i,c_i}(I_{ci}(c)) & \text{if } c_i \in H \\
   0 & \text{if } c_i = 0.
   \end{cases}
\]

(SU2)

Lastly, the utility of superpower \( e \in H \) is given by

\[
   u_e(\ell_e, c) = \begin{cases} 
   1 - d_{e,\ell_e} - \rho_e(I_e(c)) & \text{if } \ell_e \in L \\
   0 & \text{if no club.}
   \end{cases}
\]

(HU)

Each superpower maximizes the utility by choosing a location of its club good. Note that when a superpower places the club good at its own location (at home), it yields the maximum asymmetry between the status-quo superpower and non-superpowers that prevents non-superpowers to exploit the strategic space whereas the superpower \( a \) anticipates that \( b \) comes into play in stage two.
benefit from the club good. On the other hand, it might want to move the club good away from home to attract more members. We call this behavior “compromise” as it increases the benefit for non-superpowers at the expense of the benefit to the superpower. The utility function (HU) implies that superpowers locate the club good further away from home only when more non-superpowers join the club and share the cost.

The two superpowers maximize their utility (HU) by playing a sequential game while the cost for each country to join a club good depends on others who join the club. To determine the choice of non-superpowers, we may require it to be a Nash equilibrium. In general, there are multiple Nash equilibria in each stage, making it hard to uniquely determine the club size and the utility of superpowers. In international relations, sovereign states write legally binding treaties and agreements. Hence, it is reasonable to assume that non-superpowers coordinate their behavior when it is in their interests to do so. We introduce a coalition-formation process that leads to a core outcome and a strong Nash equilibrium (or a coalition-proof Nash equilibrium). Based on the coalition formation in each stage, the payoff of the superpowers can be uniquely determined for any location choice of the club good. This allows us to analyze the subgame perfect equilibrium of the two-stage sequential Stackelberg game between a and b.

2.2 The coalition-formation process

This section describes the behavior of non-superpowers as a process of coordination given the choice of superpowers. We show that this process leads to a unique core outcome in each stage.

2.2.1 Stage one

This section describes the process in stage one. Given a’s choice $\ell_a$, the 1st stage coalition-formation process yields a finite sequence $I_1, ..., I_m$ satisfying\(^4\)

1. Join step-wise: For each $k \in \{1, ..., m\}$, $I_k \subseteq I$, and $I_k \cap I_{k'} = \emptyset$ if $k \neq k'$.

\(^4\)The sequence is finite as the utility function (SU2) of non-superpowers is increasing in the club size.
2. \textit{Everyone is better off}: For each $k \in \{1, \ldots, m\}$,

$$g_{ia}(1 - d_{i,\ell_a}) - \rho_{ia}(\bigcup_{t=1}^{k} I_t) \geq 0 \text{ for each } i \in I_k. \quad (1)$$

3. \textit{Maximality}: There is no nonempty subset $I_{m+1} \subseteq I \setminus \bigcup_{k=1}^{m} I_k$ such that (1) holds for $k = m + 1$.

Given $a$’s choice $\ell_a$, the sequence describes the process of non-superpowers joining $a$’s club. To begin, there might be a group of non-superpowers who join $a$’s club $I_1$ as they get a positive payoff. This might attract another group of countries $I_2$ to join the club, which might attract yet another group $I_3$ to join the club, and so on. This might happen as the club good is non-rivalrous and the payoff of members increases as new members join and share the cost. The process stops at step $m$ when no outside group can benefit from joining the club. Let us define the core (see Aumann and Peleg, 1960).\footnote{Given $a$’s choice $\ell_a$, the behavior of non-superpowers can be formulated as a coalitional game with nontransferable utility.}

\textbf{Definition 1}. Given $\ell_a$, a profile of choices by non-superpowers $(c_i)_{i \in I} \in \{a, 0\}^I$ is in the core if there is no coalition $T \subseteq I$ and $c_T' = (c'_i)_{i \in T}$ such that for each $i \in T$, $u_i(\ell_a, (c_T', c_{I \setminus T})) > u_i(\ell_a, c)$.

Given $a$’s choice $\ell_a$ and a coalition-formation process $I_1, \ldots, I_m$, we define $I^*(\ell_a) = \bigcup_{k=1}^{m} I_k$ as the maximum sphere of influence for $a$. We have the following lemma.

\textbf{Lemma 1}. Given $\ell_a$, the 1st stage coalition-formation process

1. is order-independent, and the shortest one has a length one.

2. leads to the profile of choices $c^{I^*(\ell_a)}$ which is

   (a) in the core.

   (b) the largest core outcome (the core is a lattice).

Lemma 1 implies that $I^*(\ell_a)$ is well defined. We have that $I^*(\ell_a) \cup \{a\}$ is the club of $a$ when $a$ chooses $\ell_a$. The coalition-formation process leads to a core outcome. This means when a club is formed, there is no other subgroup that can make everyone in that group
better off. We would like to keep track of the choice of non-superpowers for stage two. Therefore, for each \( E \subseteq I \) we define \( c^E \in \{0, a\}^I \) such that the \( i \)-th entry is \( a \) if and only if \( i \in E \). It turns out that \( c^{I^*}(\ell_a) \) is a unique core outcome except for degenerate cases\(^6\) and \( I^*(\ell_a) \) is the maximum sphere of influence based on the core outcomes.

### 2.2.2 Stage two

Given \( \ell_a \) and the club \( I^*(\ell_a) \) in stage one, if \( b \) chooses \( \ell_b \) to locate its club good in stage two, non-superpowers need to recalibrate their choice in stage one. This can be described as a process of coordination. Countries in \( I^*(\ell_a) \) evaluate if they can achieve a higher payoff in \( b \)'s club than staying in \( a \)'s club. Therefore, given \( \ell_b \) in stage two, the 2nd stage coalition-formation process yields a finite sequence \( (\tilde{I}_1, I_1), ..., (\tilde{I}_p, I_p) \) satisfying

1. **Shift step-wise:** For each \( k \in \{1, ..., p\} \), \( \tilde{I}_k \subseteq I_k \subseteq I \), and \( \tilde{I}_k \cap \tilde{I}_{k'} = \emptyset \) if \( k \neq k' \).

2. **Everyone is better off:** For each \( k \in \{1, ..., p\} \),

   2.1 each \( i \in \tilde{I}_k \) satisfies
   
   \[
   g_{ib}(1 - d_{i, \ell_b}) - \rho_{ib}(\bigcup_{t=1}^k \tilde{I}_t) \geq 0 \quad \text{if } i \notin I^*(\ell_a) \tag{2}
   
   g_{ia}(1 - d_{i, \ell_a}) - \rho_{ia}(I^*(\ell_a) \setminus \bigcup_{t=1}^{k-1} I_t) < g_{ib}(1 - d_{i, \ell_b}) - \rho_{ib}(\bigcup_{t=1}^k \tilde{I}_t) \quad \text{if } i \in I^*(\ell_a)
   
   \]
   
   where \( g_{ib}(1 - d_{i, \ell_b}) - \rho_{ib}(\bigcup_{t=1}^k \tilde{I}_t) \geq 0 \); 

   2.2 each \( i \in I_k \setminus \tilde{I}_k \) satisfies \( g_{ia}(1 - d_{i, \ell_a}) - \rho_{ia}(I^*(\ell_a) \setminus \bigcup_{t=1}^{k-1} I_t) < g_{ib}(1 - d_{i, \ell_b}) - \rho_{ib}(\bigcup_{t=1}^k \tilde{I}_t) \) for \( i \in I^*(\ell_a) \); 

   where we stipulate that \( \bigcup_{t=1}^p I_t = \emptyset \).

3. **Maximality:** There is no nonempty \( \tilde{I}_{p+1} \subseteq I \setminus \bigcup_{k=1}^p \tilde{I}_k \) and \( I_{p+1} \subseteq I \setminus \bigcup_{k=1}^p I_k \) such that the conditions in 2 hold.

For each \( k \), \( \tilde{I}_k \) is the set of non-superpowers moving to \( b \)'s club, and \( I_k \) is the set of non-superpowers changing their behaviors from the previous step in the sequence. The first line in (2) is for the countries that stayed out of \( a \)'s club in stage one but now join \( b \)'s club. The second line in (2) is for the countries that joined \( a \)'s club in stage one but now shift to

\(^6\)There are cases when the non-superpower is indifferent between the choices \( a \) and 0. In this case we assume that it joins \( a \). This makes sense since countries can always negotiate a binding contract to keep the indifferent countries (for example, by providing a side-payment).
b’s club. The second line contains two cases: the ones that leave a’s club and immediately join b’s club, and the others that have left a’s club in a previous step and now join b’s club (formally, there is $k' < k$ such that $i \in I_{k'} \setminus \tilde{I}_{k'}$).\footnote{When $b$ comes into play in stage two, non-superpowers have an additional option to join b’s club. However, this does not necessarily make everyone better off because when a country leaves a club, it directly affects the utility of the others remaining in the club by increasing the cost each member pays for the club good. This is in contrast to the results in a competitive market for partners where more options on one side of the market benefit the other side as in Gale and Shapley (1962).} Similar to Lemma 1, the following statement holds.

**Lemma 2.** Given $\ell_a$, the 2nd stage coalition-formation process

1. is order-independent for any $\ell_b$.

2. leads to profile of choices $c(I_a(\ell_a, \ell_b), I_b(\ell_a, \ell_b))$ that is

   (a) in the core.

   (b) a core outcome that favors $a$. Moreover, among all the core outcomes that favor $a$, it favors $b$ most.

While the 1st stage coalition-formation process can be done in one step, the 2nd stage coalition-formation process may require multiple steps even for the “fastest” one. Given $\ell_a$ in stage one, $\ell_b$ in stage two and a shifting sequence $I_1, ..., I_p$, we define $I_a(\ell_a, \ell_b) = I^*(\ell_a) \setminus (\cup_{k=1}^p I_k)$ and $I_b(\ell_a, \ell_b) = \cup_{k=1}^p \tilde{I}_p$. Lemma 2 guarantees that $I_a(\ell_a, \ell_b)$ and $I_b(\ell_a, \ell_b)$ are well defined. Then, at the end of the game, a’s club is $I_a(\ell_a, \ell_b) \cup \{a\}$ and b’s club is $I_b(\ell_a, \ell_b) \cup \{b\}$, which determine $a$ and $b$’s payoff. We can define the core in stage two. For each disjoint $E, F \subseteq I$, we define $c(E, F) \in \{a, b, 0\}^I$ such that the $i$-th entry is $a$ if $i \in E$, $b$ if $i \in F$, and $0$ if $i \in I \setminus (E \cup F)$. The core in stage two is a lattice.\footnote{Here, for each $c_1 = c(E, F)$ and $c_2 = c(E', F')$, $c_1 \wedge c_2 := c(E \cap E', F \cup F')$ and $c_1 \lor c_2 := c(E \cup E', F \cap F')$.} The 2nd stage coalition-formation process selects a “saddle” core outcome. The club formed by $a$ in stage one is a fait accompli, based on which non-superpowers behave in stage two. This means that $a$ has the first-mover advantage. For $b$ to gain more members than $a$ in stage two, $b$’s club must yield higher utility than $a$’s club for all members.

Figure 2 shows an example where the game structure is perfectly symmetric but $a$ has
the first-mover advantage \( (g_{1a} = g_{2a} = g_{3b} = g_{4b} = 0.5, g_{1b} = g_{2b} = g_{3a} = g_{4a} = 0.7, \text{ and } m_i = 1 \text{ for all } i). \) The figure shows that given \( \ell_a \) and \( \ell_b \) the core in stage two contains both cases when all non-superpowers join either \( a \) or \( b \)’s club. However, non-superpowers have already made their choices in stage one and do not agree to move jointly to \( b \)'s club in stage two; countries 3 and 4 would get a higher payoff by joining \( b \)'s club but only when 1 and 2 join too, yet 1 and 2 are better off staying in \( a \)'s club. This highlights the importance of non-superpowers close to the first mover for deterring the club formation by the challenger. Finally, all four countries stay in \( a \)'s club.

Given the choice of superpowers, the game among non-superpowers is essentially a voting game in the sense of Peleg (2002). Moreover, the coalition-formation process allows sequential deviations until no subset of non-superpowers deviates. Therefore, it leads by definition to a Nash, a strong Nash, and a coalition-proof Nash equilibrium.\(^9\)

### 2.3 The subgame perfect equilibrium

Section 2.2 shows that the clubs in each stage are uniquely formed given \( a \) and \( b \)'s choice. Based on the results, this section defines a two-stage sequential game between the two superpowers with perfect information and shows that the game has a subgame perfect equilibrium.

First, \( a \) chooses either a location of its club good in \( L \) to establish a club or not to establish

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\(^9\)Given the choice of superpowers, the behavior of non-superpowers can be formulated as a supermodular game. The supermodular game has the smallest Nash equilibrium, which can be reached by Topkis (1979)’s algorithms. Except in cases when some non-superpowers are indifferent between joining and staying out of a club, the smallest Nash equilibrium is our core outcome at stage two when we apply Topkis (1979)’s algorithms with all non-superpowers in \( a \)'’ club at the start. We thank Satoru Takahashi for pointing out this equivalence to us.
a club. Given $\ell_a$, non-superpowers in $I^*(\ell_a)$ choose to join $a$’s club. Observing this, $b$ chooses either a location in $L$ for its club good or not to establish a club. Given $\ell_b$, the choice of non-superpowers is represented by the vector $c(I_a(\ell_a, \ell_b), I_b(\ell_a, \ell_b))$, i.e., countries in $I_a(\ell_a, \ell_b)$ are in $a$’s club, those in $I_b(\ell_a, \ell_b)$ are in $b$’s club, and the remaining countries do not belong to any club. Now, we can rewrite (HU) to express the payoff for each superpower $e \in \{a, b\}$ as $U_e(\ell_a, \ell_b) = u_e(\ell_e, c(I_a(\ell_a, \ell_b), I_b(\ell_a, \ell_b)))$. Given the choice of $a$, $b$ maximizes its payoff by choosing a best response, whereas $a$ takes $b$’s responses into account when it makes its choice to maximize its payoff. This backward induction leads to a subgame perfect equilibrium. Kuhn’s Theorem (Kuhn, 1953) guarantees that the game has at least one subgame perfect equilibrium.

3 Why non-superpowers matter

This section analyzes the comparative statics of club formation, i.e., how the size and location of the clubs change, when the dependency of non-superpowers on superpowers and the endowment of non-superpowers change. While the general model presented above allows for maximum flexibility regarding the location of countries and club goods, the endowment of countries, and the cost share of each country, it is hopeless to characterize the subgame perfect equilibrium in a general way. Therefore, this section treats special cases of the model in Section 2.

![Figure 3: The structure of the game in the special cases](image)

Figure 3 illustrates the structure of the game where the line segment $[0, 1]$ is divided by $n \in \mathbb{N}$ and the set of locations is $L = \{0, 1/n, \ldots, (n - 1)/n, 1\}$. We assume that the two superpowers are at the opposite ends of the segment ($a$ at 0 and $b$ at 1) and the non-
superpowers are located evenly-spaced in between with each \(i \in I = \{1, \ldots, n - 1\}\) at \(i/n\). For simplicity we assume \(n\) to be an odd number without loss of generality.\(^{10}\) We assume that the cost share of country \(j\) in club \(e\) takes the form

\[
\rho_{je}(I_e(c)) = \frac{\omega_j}{\omega_e + \sum_{i \in I_e(c)} \omega_i}
\]  

(3)

where \(\omega_j > 0\) is the endowment for each \(j \in I \cup H\), \(c = (c_i)_{i \in I} \in \{a, b, 0\}^I\), and \(\rho_{je} = \rho_e\) when \(j = e \in H\). Hence, the cost share is proportional to its endowment in the club. We denote \(\delta_e\) as the number of member countries in the club established by \(e\).

The following assumptions on the behavior of superpowers yield a unique subgame perfect equilibrium.

**A1:** When the payoffs are the same for multiple locations of the club good, the superpowers choose the closest one to home.

**A2:** When the payoff of forming a club is 0, no club is formed.

Both assumptions highlight the cases when superpowers are indifferent between choices. A1 implies that superpowers form the smallest club that maximizes its payoff. A2 implies that the payoff of superpowers is positive when they form a club. We have the following statement.

**Lemma 3.** Suppose that A1 and A2 are satisfied. The game has a unique subgame perfect equilibrium.

Figure 4 illustrates the subgame perfect equilibrium where there are three non-superpowers located at \(1/4, 1/2,\) and \(3/4\), \(a\) at 0 and \(b\) at 1, and \(g_{ia} = g_{ib} = 0.5\) for any non-superpower \(i\), and the endowment of countries are set to one. Note that the structure of the game is symmetric, i.e., the only difference between \(a\) and \(b\) is the timing of entry into the game.

Without \(b\), if \(a\) chooses 0 (i.e., home) or 1, no non-superpower would join its club; if \(a\) chooses \(1/4, 1/2,\) or \(3/4\), all non-superpowers would join its club. Therefore, \(a\)'s choice that maximizes its payoff is \(1/4\). However, \(b\) can observe \(a\)'s choice and choose its optimal reaction. Hence, if \(a\) chose \(1/4\), \(b\)'s best response would be \(1/2\), which would attract all non-superpowers to \(b\)'s club, i.e., the blue branches of the game tree. Therefore, \(a\) chooses

\(^{10}\)We obtain analogous results when \(n\) is even.
1/2 (compromises) to deter \( b \) who gives up establishing a club as it can not attract any member. This is the subgame perfect equilibrium highlighted in red in Figure 4. The first-mover advantage of \( a \) is that it can observe the full game tree (perfect information) when it chooses the location of the club good in stage one. In other words, \( a \) can make decisions in stage one anticipating \( b \)'s response in stage two but \( b \) can only make decisions in stage two given \( a \)'s choice in stage one. Moreover, non-superpowers form a coalition in stage one. This means that \( b \) needs to break the already formed coalition to attract countries from \( a \)'s club in stage two.

In the following, we analyze how the dependency of non-superpowers on superpowers (Section 3.1) and the endowment of non-superpowers (Section 3.2) shape the behavior of superpowers and the distribution of power. Note that we can only obtain the full set of results when there are sufficiently many non-superpowers, which we assume for the remainder of this paper.\(^{11}\)

### 3.1 The dependency of non-superpowers

This section assumes that \( \omega_j \) is the same for all \( j \in I \cup H \) and analyzes how the behavior of superpowers and the distribution of power are shaped by \((g_{ia}, g_{ib}) = (g_a, g_b) \in [0, 1]^2\). Figure 5 shows simulation results where each dot is an outcome of a game. The red dots indicate

\(^{11}\)Intuitively, when there are too few non-superpowers whose locations are dispersed, the results become “unstable” with overlapping regions of club formations given \( g_{ie} \). The sufficient number of non-superpowers depend on the utility functions (SU1), (SU2) and (HU) as well as our assumptions on \( g_{ie} \).
that $b$ is the only hegemon who forms a club, and the blue dots indicate that $a$ is the only hegemon. The purple dots indicate that both $a$ and $b$ form a club. We can see that some dots are missing around $(0,0)$. This is because $(g_a, g_b)$ are so low that neither $a$ nor $b$ forms a club. Lastly, the circled dots, where only $a$ forms a club, indicate that the results are not completely symmetric because of $a$’s first-mover advantage. We will now analyze the symmetric case and the asymmetric case that correspond to the diagonals in Figure 5.

![Figure 5: Distribution of power: $n = 31$](image)

### 3.1.1 The symmetric case

This section analyzes the symmetric case, where the endowment of all countries is the same and the dependency of non-superpowers is the same, i.e., when $g = g_a = g_b \in [0, 1]$, which corresponds to the diagonal from $(0, 0)$ to $(1, 1)$ in Figure 5. Here, the model is symmetric for two superpowers $a$ and $b$ except their timing to enter the game: $a$ in stage one and $b$ in stage two. Below, we identify thresholds $0 < g_U(n) < g_B(n) < 1$ and show that there are three cases. In the first case, when $g \in [0, g_U(n))$, no hegemon emerges. In the second case, when $g \in [g_U(n), g_B(n)]$, a unipolar world emerges where the status-quo power $a$ deters the challenger $b$. In the third case, when $g \in (g_B(n), 1]$, a bipolar world emerges where each superpower forms a club of an equal size. For $g \in (g_B^h(n), 1]$ ($g_B^h(n) > g_B(n)$) both superpowers locate the club good at home. Figure 6(a) illustrates the distribution of power as a function of $g$ and Figure 6(b) shows the choices of the club good location by superpower.
When superpower $b$ exists and when it hypothetically does not.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig6.png}
\caption{The symmetric case}
\end{figure}

The following proposition shows how the dependency of non-superpowers on superpowers shapes the behavior of superpowers and the distribution of power.

**Proposition 1** (The symmetric dependency of non-superpowers).

1. **No hegemon:** No club is formed if $g \in [0, g_U(n))$.

2. **Unipolar world:** Only $a$ forms a club if $g \in [g_U(n), g_B(n)]$. Moreover, $\ell_a = \frac{n-1}{2n}$ when $g = g_U(n)$.

3. **Bipolar world:** The world is divided into two clubs of an equal size if $g \in (g_B(n), 1]$. Moreover, $\ell_a = \frac{1}{n}$ and $\ell_b = \frac{n-1}{n}$ if $g \in (g_B(n), g_B^h(n))$, and $\ell_a = 0$ and $\ell_b = 1$ if $g \in [g_B^h(n), 1]$.

When $g \leq g_U(n)$, the dependency of non-superpowers on superpowers is so low that there are not enough countries who are willing to form a club (Proposition 1.1). Proposition 1.2 highlights the first-mover advantage of $a$, the single hegemon forming a club with the only difference between $a$ and $b$ being the timing of entry into the game in the two-stage game.

**Emergence of a club (the status-quo power):** The club that $a$ forms when $g = g_U(n)$ is large covering almost all non-superpowers. This means beyond the point $g = g_U(n)$, $a$ suddenly forms a large club, even though the change in $g$ is continuous. This happens because the cost reduction can be large (discrete) as the cost is shared by a discrete number.
of member countries, even when a change in the dependency is small (infinitesimal). In other words, an infinitesimal increase in the dependency can entice many to form a club because they share the cost of the club.

**Deterrence**: The first-mover $a$ locates the club good away from home at the expense of its own benefit from the club good. There are two reasons why $a$ might want to do this. First, by moving the club good away from home, $a$ can gain more members as the club good gets closer to more countries. Second, the move deters some members from switching to $b$’s club and thus blocks the formation of $b$’s club. Figure 6(b) shows that $a$ places its club good closer to $1/2$ with $b$ than without $b$ for $g \in [g_U(n), g_B(n)]$.\(^{12}\)

Proposition 1.3 shows that the outcome in the bipolar world is completely symmetric: $a$ and $b$ locate the club goods either at one location away from home or at home. This indicates that non-superpowers are so dependent on two superpowers that the first-mover $a$ can no longer deter the emergence of $b$’s club.

**Emergence of a club (the challenger)**: The bipolar world emerges suddenly. Beyond the point $g = g_B(n)$, $b$ suddenly forms a club by attracting half of non-superpowers, and the situation remains the same for $g \in (g_B(n), 1]$. This means that it is difficult to foresee a bipolar world as we do not see a big jump in $g$.

Let us now use the model to interpret the evolution of the post-war international monetary system. The establishment of the Bretton-Woods system led by the US in the aftermath of the World War II is a club by the first-mover compromising with non-superpowers. The system kick-started a new world order with 44 members initially but gained more members over time, as the IMF and the World Bank maintained the gold standard by providing development assistance and helping countries in balance of payment crises. The core members of the system continue to support the US-led system, even after the collapse of the gold standard. With the rise of emerging economies, who would like to see their interests being better accommodated, many have criticized the slow reform process of the IMF and the World Bank. Later, some reforms were implemented to reflect the increasing economic power of emerging economies. This is described in our model by the second case where the

\(^{12}\)This means at least some non-superpowers must be better off, while others worse off, with $b$ entering the game.
status-quo power first moves the club good closer to home, before moving it away to deter the challenger. The deterrence, however, can not prevent the formation of a new club by the challenger and the status-quo power moves the club good home accommodating the challenger in the third case of our model. In fact, China took a lead to announce the Belt and Road Initiative (BRI) in 2014 to better accommodate the interests of emerging economies. The BRI aims to build infrastructures that connect China with the rest of the world. The number of countries that signed up for the BRI has increased dramatically in a short period of time with 147 countries having BRI-related infrastructure projects as of March 2022.

3.1.2 The asymmetric case

This section analyzes the case where the endowment of all countries is the same for each \( j \in I \cup H \), but the dependency of countries on one superpower has an inverse linear relationship with that on the other, i.e., \( g = g_{ia} = 1 - g_{ib} \in [0, 1] \) for all \( i \in I \), which corresponds to the diagonal from \((0, 1)\) to \((1, 0)\) in Figure 5. This means the environment is no longer symmetric. When \( g > 1/2 \), countries are more dependent on \( a \) than \( b \), while the opposite is true when \( g < 1/2 \). When \( g \in [0, 1] \) changes, the incentives of non-superpowers change too. Given the location of club goods, when \( g > 1/2 \), countries gain higher utility from joining \( a \)'s club. The opposite is true when \( g < 1/2 \). Hence, we obtain an almost symmetric outcome; the superpower on which non-superpowers are more dependent becomes the single hegemon, whereas two hegemons emerge when the dependency of non-superpowers on superpower is similar.

The following proposition shows how the dependency of non-superpowers on superpowers shapes the behavior of superpowers and the distribution of power. In particular, we identify thresholds \( 0 < g_B < 2g_B^h < g(n) < \bar{g}(n) < \bar{g}_B < \bar{g}_B^h < 1 \) and show that \( a \) forms a unipolar world when \( g \in [\bar{g}_B(n), 1] \), \( a \) and \( b \) form a bipolar world when \( g \in [g_B(n), \bar{g}_B(n)] \), and \( b \) forms a unipolar world when \( g \in [0, g_B(n)] \) (see Figure 7(a)).\(^{13}\) Moreover, we show when one club size is larger than the other \( g \in [0, g(n)] \) or \( g \in [\bar{g}(n), 1] \) (see Figure 7(a)) and when

\(^{13}\)The world with no hegemon does not arise in the asymmetric case. This is because when \( g_a \) is small \( g_b = 1 - g_a \) is large so that \( b \) forms a club and vice versa when \( g_a \) is large. It follows that the benefit from at least one club good is always high enough so that at least one club is formed.
superpowers compromise with non-superpowers $g \in [g_B(n), g^b_B(n))$ or $g \in [g^h_B(n), g_B(n))$ (see Figure 7(b)).

**Proposition 2** (The asymmetric dependency of non-superpowers).

1. **Unipolar world:** Only $b$ forms a club with $\ell_b = 1$, and all but country 1 join its club if $g \in (0, g_B(n))$. Only $a$ forms a club with $\ell_a = 0$, and all but country $n-1$ join its club if $g \in [g_B(n), 1)$. When $g \in \{0,1\}$, all join the club of the single hegemon.

2. **Bipolar world:** The world is divided into two clubs

   (a) with $\delta_a = \delta_b = \frac{n+1}{2}$ if $g \in [g(n), \bar{g}(n))$, with $\delta_a < \delta_b$ if $g \in [g_B(n), g(n))$, and with $\delta_a > \delta_b$ if $g \in [\bar{g}(n), g_B(n)]$.

   (b) with $\ell_a = 0$ and $\ell_b = 1$ if $g \in [g^h_B(n), \bar{g}_B(n))$, with $\ell_a \in (0, 1/2)$ and $\ell_b = 1$ if $g \in [g_B(n), g^h_B(n))$, and with $\ell_b \in (1/2, 1)$ and $\ell_a = 0$ if $g \in [\bar{g}_B(n), g_B(n))$.

Proposition 2.1 analyzes the situation when a unipolar world emerges. The hegemon that forms a club is $a$ when $g$ is close to 1 and $b$ when $g$ is close to 0. Moreover, each hegemon places the club good at home, and all but the most far away non-superpower become a member of the club.

*The single hegemon:* Unlike in the symmetric case where the single hegemon locates the club good closer to 1/2, the single hegemon in the asymmetric case always locates the club good at home. This suggests that if $g \in [\bar{g}_B(n), 1)$ non-superpowers are so dependent on $a
that all but one country join its club without a having to compromise. Analogously for b if \( g \in [0, g_B^h(n)) \).

Competing hegemons: Proposition 2.2a identifies the range of \( g \) around 1/2 where each superpower has a club of an equal size. Moreover, it shows that b’s club size is larger than a’s for \( g \in [g_B^h(n), 1/2) \), and the opposite is true for \( g \in [1/2, g_B^h(n)) \). For \( g \in [g_B^h(n), g_B^h(n)) \), a’s club size is monotonically increasing in \( g \), while b’s club size is monotonically decreasing in \( g \) (see Figure 7(a)). Proposition 2.2b identifies the range of \( g \) around 1/2, for which both superpowers place their club good at home diving the world into two clubs, and the ranges of \( g \) for which the superpower, on which non-superpowers are more dependent, places the club good at home, while the other places it closer to the center (1/2). There are ranges of \( g \) for which the rising hegemon places the club good away from home but gradually bring the club good back home when it becomes more dominant. We obtain a structure that looks symmetric around 1/2, but we can show that the unipolar range is larger for a than b, i.e., \( g_B^h(n) < 1 - g_B^h(n) \), and non-compromising range is larger for a than b, i.e., \( g_B^h(n) < 1 - g_B^h(n) \) indicating a’s first-mover advantage.

| \( g \) | symmetric case | asymmetric case |
|-------|----------------|----------------|
| low   | 1.1 (no hegemon) | 2.1 (unipolar, a dominates) |
| mid   | 1.2 (unipolar, a compromises) | 2.2a and 2.2b (bipolar. a or b compromises) |
| high  | 1.3 (bipolar) | 2.1 (unipolar, b dominates) |

Table 1: Main results in Sections 3.1.1 and 3.1.2

Table 1 summarizes the main results of the propositions in Section 3.1.1 and 3.1.2. In the symmetric case, the first-mover must compromise with non-superpowers when it forms a unipolar world. This highlights the power of a coalition. In the asymmetric case, the superpower, on which non-superpowers depend less, compromises with non-superpowers and the other forms a larger club in a bipolar world.

3.2 The endowment of non-superpowers

So far we have assumed that the endowments of all countries are the same. This section analyzes the case when the endowments of countries are not identical. To focus on the
effects of the endowments, we assume that 1) $g = g_{ia} = g_{ib} \in [0, 1]$ as in the symmetric case in Section 3.1.1 and 2) $g > g^h_B(n)$ such that superpowers form a bipolar world without compromising with non-superpowers.

**Proposition 3** (The endowment of members and the club size). Let $I' = \{1, \ldots, \frac{n-1}{2}\} \cup \{a\}$.

1. Suppose $\omega_a \in [1, \infty)$ and $\omega_j = 1$ for each $j \in I \cup \{b\}$. For any $g \in (g^h_B(n), 1]$, $\delta_a$ is weakly increasing in $\omega_a$.

2. Suppose $\omega_i = \omega \in [1, \infty)$ for all $i \in I'$ and $\omega_j = 1$ for all $j \in (I \cup \{a, b\}) \setminus I'$. There exists $g^* \in (g^h_B(n), 1]$ so that for any $g \in (g^*, 1]$, $\delta_a$ is weakly increasing in $\omega$ with $\ell_a = 0$ and $\ell_b = 1$.

3. Suppose $\omega_i = \omega \in [1, \infty)$ for a member $i \in I'$ and $\omega_j = 1$ for all $j \in (I \cup \{a, b\}) \setminus i$. For any $g \in (g^h_B(n), 1)$, $i$ will choose not to belong to any club if $\omega \to \infty$. It follows that $\delta_a < \delta_b$ and $\omega_a + \sum_{h \in I_a} \omega_h < \omega_b + \sum_{h \in I_b} \omega_h$ as $\omega \to \infty$.

Analogous results hold for b’s club.

Proposition 3.1 and Proposition 3.2 show that a superpower expands its sphere of influence - given a sufficiently high dependency on superpowers - when its endowment or that of its members grows. Intuitively, non-superpowers are more likely to join a club when the original club members bear a higher cost share. On the other hand, Proposition 3.3 shows when a single member decides to leave a club due to its growing endowment, making the club smaller than the other.

4 **The US-China competition**

This section takes the US as the status-quo power and China as the challenger and simulates how they form clubs, given the exogenous variables such as the location and the endowment of non-superpowers and the dependency of non-superpowers on both superpowers.\textsuperscript{14} Endogenous are the location choice of the US and China for their club goods and the choice\textsuperscript{14}The simulation presents the equilibrium in each year. Hence, we do not interpret the two stages in the game as if they follow a historical timeline. Rather, the two stages are a conceptual devise to describe the situation, in which the status-quo power has a first-mover advantage because of its established reputation.
of non-superpowers whether to join either club. The simulation includes 145 countries for which the DI and bilateral trade data are available from 2006 to 2019. We take 5-year moving averages for all exogenous variables. The DI is normalized to locate countries in the range of $[0,1]$, where the country with the highest democracy score is at 0 and the one with the lowest score is at 1. \(^\text{15}\) The US is located around 0.2 and China around 0.8. We take GDP as a proxy for the endowment of countries implying that countries with a larger GDP pays a higher cost share for the club good. \(^\text{16}\) The trade ratio of each country with the US and China (the ratio of trade volume with the US and China to the total trade) is used as a proxy for the dependency of each country on the US and China, $g_{i,US}$ and $g_{i,China}$ for country $i$. \(^\text{17}\) Lastly, the US and China can choose any integer $x$ in range $[0, 500]$ to locate their club goods at $\frac{x}{500}$.

### 4.1 Simulation from 2006 to 2019

Figure 1(a) shows that the average trade volume of 189 countries with the US accounted for about 20 percent of the total trade volume in 2006. Since 2019, however, it has been surpassed by the average trade volume with China which accounts for about 15 percent of the total trade volume of countries. Given each country’s trade ratio with the US and China, the DI and GDP for each year between 2006 and 2019, Figures 8(a) and 8(b) show the club formation of the US and China, while 8(c) shows the location choices of the club good by the US and China. China starts to gain significantly more members after 2013 when it compromises (see Proposition 2.2b). It gains more members than the US in 2017, although it compromises less and less because of growing trade of countries with China and its growing GDP (see Propositions 2.2a and 3.1). However, the total GDP of its members is

\[^{15}\text{We normalize the score } \ell_i \text{ for each country } i \text{ to be between 0 and 1 setting } \ell_i = \frac{\text{maxDI} - \text{DI}_i}{\text{maxDI} - \text{minDI}}.\]

\[^{16}\text{The World Bank and the OECD: https://data.worldbank.org/indicator/NY.GDP.MKTP.CD}\]

\[^{17}\text{The trade data for many countries before 2015, especially those with China are missing. In cases where only the trade data with the US are available, we let } g_{i,China} = 0. \text{ This may over-represent the dependency of countries on the US, but it helps not to exclude too many countries that are located closer to China. In cases where the trade data with both the US and China are missing, we let } g_{i,US} = g_{i,China} = 0, \text{ hence such countries would not join any club.}\]
Figure 8: The choice of the US and China and their club formation with 145 countries

still much smaller than that of the US club members even in 2019. Moreover, although the number of the US club members has started falling significantly since 2013, the total GDP of its club members has not changed much. In other words, China has attracted countries with a relatively low GDP implying that it bears most of the cost to provide the club good.

Figure 9 shows the location choice of the US and China and the club choice of other countries. The average trade ratio of countries with the US and the ratio with China change from 0.13 and 0.02 in 2006 to 0.10 and 0.14 in 2019. In 2006, GDP of the US is much higher than that of China and the majority of countries trade more with the US. Hence, the US attracts most countries even without compromising. In 2019, China has more club members than the US as its GDP approaches that of the US and the majority of countries trade more with China than with the US.
Figure 9: The choices of the US and China and 145 countries

Note: For expositional purpose the bar depicts ln(x + 1.03) where x is GDP in trillion current USD. The top 30 GDP countries are captioned.

4.2 Projection from 2030 to 2100

We simulate the US-China competition based on the long-run GDP projection from 2030 to 2100 provided by the International Institute for Applied Systems Analysis (see Riahi et al., 2017).\(^{18}\) While we obtain GDP projections, there is no reliable projection for the DI and the

\(^{18}\)The GDP projections are on a PPP basis. There are five different scenarios. We adopt the Shared Socioeconomic Pathway 2, which is a scenario with medium challenges that “the world follows a path in which social, economic, and technological trends do not shift markedly from historical patterns.” The choices
trade ratio with the US and China. Therefore, we take the data in 2019 (5-year averages) for those variables to simulate our model from 2030 to 2100. China’s club is larger than the US club in terms of the number of club members already in 2017 and continues to be so up to 2100.

Figure 10: China’s club dominates the US club in terms of GDP of members by 2035.

Note: There are 145 non-superpowers. For expositional purpose the bar depicts \( \ln(x + 1.03) \) where \( x \) is GDP in trillion current USD. The top 30 GDP countries are captioned.

The total GDP of China’s club members (excluding China) surpasses that of the US club members (excluding the US) between 2030 and 2035 (see Figure 10). The major contributing of non-superpowers and superpowers remain robust for different scenarios.
factor for China’s dominance is India’s departure from the US club by 2030 as its GDP and thus its cost share grows significantly (see Proposition 3.3). In response, China compromises and moves its club good closer to the center making countries such as Korea, Brazil, Argentina and the Philippines switch over to its club by 2035 (see Proposition 2.2b). With India leaving the US club, China has an advantage of being the dominant trade partner for most countries (see Proposition 2.2a) outpowering the locational advantage of the US having countries with a large GDP in its proximity.

Simulations based on the long-run GDP projection highlight limitations of our model. First, we used the data for 2019 for the trade ratio and the DI up to 2100. Second, the number of superpowers is exogenous. The projection shows that India will be the largest economy surpassing the US and China by 2080. Hence, the natural question follows if India should compete with China or the US for hegemony. Moreover, our model does not allow for a multipolar world nor does it allow for a group of countries, such as the EU explored in Alesina et al. (2005), to behave like a superpower collectively. Third, the location of countries is exogenous. Many countries that have a low democracy score are projected to grow fast over the next decades. How their score will change as their GDP grows will affect the outcome of the competition in a fundamental way. Lastly, the capacity of superpowers in our model depends solely on their GDP. Dalio (2020) observes that superpowers historically ascended to primacy in phases by first expanding international trade, next establishing the world’s leading financial centers, then possessing the world’s leading reserve currency. We observe today that the US is in the last phase while China is in the first. From this viewpoint, China needs to liberalize its capital account and establish the world’s leading financial centers to ascend to primacy.

4.3 Position towards the US-led liberal order

This section shows how our model performs in matching the data. In particular, we see how the choice of non-superpowers to join either the US or China’s club in our simulation aligns with the estimate of state positions toward the US-led liberal order provided by Bailey et al.

19The total GDP of China’s club members starts to dominate that of the US club members when GDP of countries such as Russia, Turkey and Iran, who are relatively close to China in terms of the DI, grows.
Figure 11: State positions based on UN General Assembly votes

Figure 11 shows histograms of the state positions and the choices of countries based on simulation in 2006 and in 2019. We can see a higher concentration of countries around China in 2019 than in 2006. In fact, the average distance to China’s voting position decreased by around 21%, from 0.86 in 2006 to 0.68 in 2019, while the average distance to the US’s voting position decreased only slightly (7%) from 2.91 in 2006 to 2.71 in 2009. This tells us that there are more countries whose voting behavior has become more aligned with that of China over the period, whereas the change concerning the US is minor. This change in the estimates of state positions is consistent with the growth of China’s club over the same period in our simulation.

5 Conclusion

We build a model where two superpowers play a Stackelberg game by choosing a location for their club good that has an externality. The cost each member pays depends on others who join the club, as members share the cost, while the benefit is decreasing in distance to the good. This presents a trade-off to the superpowers as they might want to attract more
members by locating their club good away from home at the expense of their own benefit from the good. We analyze how the choice of the superpowers can result in a world with no hegemon, a single hegemon or two hegemons when the dependency of non-superpowers on superpowers and the endowment of superpowers change.

The general model presented in Section 2 contains the specifications for simulations in Section 4. The comparative statics in Section 3 is derived for special cases, where countries are located evenly-spaced on \([0, 1]\), have the same endowment, and equally depend on each superpower (except for Section 3.2 where we analyze the effect of a change in endowments). The simulation in Section 4 uses the trade ratio of each country with the US and China as a proxy for the dependency of a country on the US and China. This means that the simulation allows for heterogeneous dependency on the US and China across countries.

We obtain the following insights regarding how non-superpowers influence the outcome of the game through their characteristics \((i/n, g_{i,a}, g_{i,b}, \omega_i)\) based on Propositions 1, 2 and 3:

1. The status-quo power compromises with non-superpowers to deter the challenger and forms a unipolar world - even in the symmetric case - when all non-superpowers depend equally on two superpowers. Non-superpowers \((i/n)\), who are close to the status-quo power, make the coalition hard to break and the status-quo power be the single hegemon. This captures the importance of close US allies for the predominance of the US-led institutions up to today.

2. The superpower compromises with non-superpowers to establish a club, but it does so less as non-superpowers depend more on it (higher \(g_{i,a}\) or \(g_{i,b}\)) and its club size grows. Our simulation shows that China moves its club good away from home in 2013 - the year when the BRI was announced. However, it compromises less as its trade dominance grows. The increasing trade ratio of countries with China attracts more countries to its club, eventually gaining more members than the US in 2017.

3. The club size increases when the endowment of a superpower \((\omega_e)\) grows. Our simulation shows that China’s GDP growth - over the period 2006-2019 - lowers the cost to join its club and increases its club size. On the other hand, when the endowment of a non-superpower member \((\omega_i)\) grows, it may leave the club as its cost share grows. Some others may follow, breaking the power balance between the two superpowers. Our simulation shows
that India’s departure from the US club by 2030 makes other countries switch over to China’s club, which then dominates the US club even for the total GDP of members.

Our analysis and simulation highlight how the characteristics of non-superpowers shape the US-China competition for a sphere of influence. China’s growing influence between 2006 and 2019 shown by our simulation is consistent with the change in the voting behavior of countries in the UN General Assembly during the same period. The influence of the US and thus its club size depend on countries with a relatively large GDP and a similar institution. On the other hand, the influence of China depends on countries trading increasingly more with China than the US and the growing GDP of China. The case of India described above highlights how even a single non-superpower might affect coalition formation and cause a shift in power balance between the US and China in the future because of the externality presented by our model.

A Proofs

A.1 Preliminary results

An abstract reduction system is a pair \((X, \to)\) where \(X\) is a non-empty set and \(\to\) is a binary relation on \(X\). An element \(x \in X\) is called an endpoint in \((X, \to)\) iff there is no \(x' \in X\) such that \(x \to x'\). We say that \(\{x_n : n = 0, 1, \ldots\}\) (finite or infinite) in \(X\) is a \(\to\)-sequence iff \(x_n \to x_{n+1}\) (as far as \(x_{n+1}\) is defined). We use \(\to^*\) to denote the reflexive and transitive closure of \(\to\). We say that \((X, \to)\) is weakly confluent iff for each \(x, y, z \in X\), if \(x \to y\) and \(x \to z\), then \(y \to^* x'\) and \(z \to^* x'\) for some \(x' \in X\).

Lemma A.1. (Newman, 1942) If an abstract reduction system \((X, \to)\) satisfies (N1) each \(\to\)-sequence is finite and (N2) \((X, \to)\) is weakly confluent, then for each \(x \in X\) there is a unique endpoint \(x' \in X\) such that \(x \to^* x'\).

The proof of following two lemmas can be seen in any textbook of discrete convex analysis, e.g., Murota (2003).

Lemma A.2. Let \(f : \mathbb{Z} \to \mathbb{R}\) be a discrete convex function. A minimum solution is in \(\{m, m+1\}\) if and only if \(f(m) \leq f(m-1)\) and \(f(m+1) \leq f(m+2)\).
Lemma A.3. Let $f : \mathbb{Z} \to \mathbb{R}$ be a discrete concave function. Integer $m$ is a maximum solution if and only if $f(m) \geq f(m - 1)$ and $f(m) \geq f(m + 1)$.

Lemma A.4. Let $\sigma_n : [1, \frac{n}{2}] \to \mathbb{R}$ by $\sigma_n(x) = \frac{x}{n} + \frac{1}{x + 1} - \frac{1}{n + 1 - x}$. Then, \[ \arg\min_{k \in \{1, \ldots, \frac{n}{2}\}} \sigma_n^D(k) \subseteq \left\{ \frac{1 + 4n - 1}{2} \right\} \cap \left( \frac{1 + 4n - 1}{2}, 1 \right]. \]

Proof. It is clear that $\sigma_n(x)$ is strictly convex, and henceforth its restriction $\sigma_n^D := \sigma_n|_{\mathbb{N} \cup \{1, \frac{n}{2}\}}$ on integers is also convex. Let $m = \left\lfloor \frac{1 + 4n - 1}{2} \right\rfloor$. By Lemma A.2, we have to show that $\sigma_n^D(m - 1) \geq \sigma_n^D(m)$ and $\sigma_n^D(m + 2) \geq \sigma_n^D(m + 1)$. First, $m = \left\lfloor \frac{1 + 4n - 1}{2} \right\rfloor$ implies that $\frac{1 + 4n - 3}{2} < m \leq \frac{1 + 4n - 1}{2}$. Let $y(m) := \sigma_n^D(m) - \sigma_n^D(m) = -\frac{1}{n} + \frac{1}{m + 1} + \frac{1}{n + 1 - m}(n - m)$. It is straightforward to see that $y(m)$ is a decreasing function. Since $y\left(\frac{1 + 4n - 1}{2}\right) = -\frac{1}{n} + \frac{1}{n + 1 - m}(n - m) > 0$, it follows that $\sigma_n^D(m - 1) \geq \sigma_n^D(m)$. To show that $\sigma_n^D(m + 2) - \sigma_n^D(m + 1) = \frac{1}{n} - \frac{1}{m + 1} - \frac{1}{n - m} \geq 0$, since the right-hand side is increasing in $m$, we need to prove that $\frac{1}{n} - \frac{1}{x + 1} - \frac{1}{n - x} \geq 0$ when $x = \frac{1 + 4n - 3}{2}$, i.e., $\frac{1}{n} - \frac{1}{(1 + 4n + 1)(1 + 4n + 3)} - \frac{4}{(1 + 4n - 2 - 2n)(1 + 4n - 3 - 2n)} \geq 0$. The left-hand side of the inequality is decreasing in $n$ and converges to 0 when $n \to \infty$. Therefore, we have shown that $\sigma_n^D(m + 2) - \sigma_n^D(m + 1) > 0$. \hfill\Box

Lemma A.5. Let $\lambda_n : [1, \frac{n}{2}] \to \mathbb{R}$ by $\lambda_n(x) = 1 - \frac{x}{n} - \frac{1}{x + 1} + \frac{1}{n - 1}$. Then $\left\lfloor \frac{1 + 4n - 1}{2} \right\rfloor$ is a maximum solution of $\lambda_n^D(x)$.

Proof. It is clear that $\lambda_n$ is a strictly concave function. Therefore, its restriction $\lambda_n^D := \lambda_n|_{\mathbb{N} \cup \{1, \frac{n}{2}\}}$ on integers is also concave. Let $m = \left\lfloor \frac{1 + 4n - 1}{2} \right\rfloor$. By Lemma A.3, we show that $\lambda_n^D(m) > \lambda_n^D(m - 1)$ and $\lambda_n^D(m) > \lambda_n^D(m + 1)$. Still, $\frac{1 + 4n - 3}{2} < m \leq \frac{1 + 4n - 1}{2}$, and consequently $m(m + 1) \leq n$ and $(m + 1)(m + 2) > n$. For the former, if we consider the quadratic function $x(x + 1) - n$, its negative interval is $\left(\frac{-1 - \sqrt{1 + 4m}}{2}, \frac{-1 + \sqrt{1 + 4m}}{2}\right)$, and $m \leq \frac{1 + 4m - 1}{2}$. Similarly, for the latter, if we consider $(x + 1)(x + 2) - n$, its positive interval on $\mathbb{R}_+$ is $\left(\frac{\sqrt{1 + 4m - 3}}{2}, \infty\right)$ and $m > \frac{1 + 4m - 3}{2}$. Therefore, $\frac{\lambda_n^D(m + 1) - \lambda_n^D(m)}{m + 1} - \frac{\lambda_n^D(m - 1) - \lambda_n^D(m)}{m} = \frac{1}{m + 1} - \frac{1}{m}$. It follows that $\lambda_n^D(m) \geq \lambda_n^D(m - 1)$ and $\lambda_n^D(m) \geq \lambda_n^D(m + 1)$. \hfill\Box

Lemma A.6. Consider the setting in Section 3.2. Superpower $a$ will choose $0$ as the location of its club good as long as its club contains at least $(n - 1)/2$ non-superpowers.
Proof. Let \( m \geq 1 \) and \( \delta \) be the number of non-superpowers in \( \{ \frac{n+1}{2}, \ldots, n-1 \} \) joining \( a \)'s club when the endowment of countries in \( I' \) is \( m \) if \( a \) chooses 0 and \( \delta' \) be that if \( a \) chooses \( 1/n \). Let \( M_a = m_a + \sum_{i=1}^{n-1} m_i \). If the choice \( 1/n \) benefits \( a \), it holds that \( 1 - \frac{m_a}{M_a + \delta} < 1 - \frac{1}{n} - \frac{m_a}{M_a + \delta'} \), i.e., \( \frac{1}{n} - \frac{\delta}{M_a + \delta} < \frac{\delta' - \delta}{(M_a + \delta)(M_a + \delta')} \). Since \( M_a \geq m_a + \frac{n-1}{2} \) and \( n \) is large enough, \( \delta' - \delta \leq 1 \). Hence, the inequality is never satisfied, that is, \( a \) will choose 0.

A.2 Main proofs

Proof of Lemma 1. Part 1: Let \( X = 2^I \) and \( E \to F \) if \( E \subset F \), all countries in \( E \) are in \( a \)'s club (the club good located at \( \ell_a \)), and all countries in \( F \setminus E \) obtain non-negative payoffs by joining \( a \)'s club together. Clearly, \((X, \to)\) describes a coalition-formation process and is a reduction system satisfying (N1) and (N2) in Lemma A.1. Hence it is order-independent. Let \( E^* \) be the final term of all coalition-formation processes. Since countries in \( E^* \) can join \( a \) and obtain non-negative payoffs, the shortest sequence has length one.

Part 2a: No non-superpower outside of \( a \)'s club has an incentive to join \( a \)'s club since all non-superpowers who can benefit from joining the club have already joined the club. Moreover, no country in \( a \)'s club has an incentive to leave the club because members benefit from other countries joining the club. Therefore, no group of countries can strictly improve their payoff by altering their behavior, and thus \( c^{I^*}(\ell_a) \) is a core outcome.

Part 2b: From \( c^{I^*}(\ell) \) we can remove “indifferent” countries and obtain all possible core outcomes. Given \( a \)'s choice \( \ell_a \) and a nonempty set \( E \subseteq I, i \in E \) is called an indifferent country iff \( g_{ia}(1 - d_{i,\ell}) - \rho_{ia}(E) = 0 \). We use \( M(\ell_a, E) \) to denote the set of all indifferent countries in \( E \) for \( a \)’s club good located at \( \ell_a \). By removing those indifferent countries some others may have to leave since their payoffs now become negative. By removing all those non-superpowers we obtain a smaller core outcome. Clearly, the core is a lattice with \( c^{I^*}(\ell) \) as the largest.

Proof of Lemma 2. Part 1: Fix \( \ell_a \) and \( \ell_b \). Let \( X = \{0, a, b\}^I \). For each \( c, c' \in X \), we define \( c \Delta c' = \{ i \in I : c_i \neq c'_i \} \) and a binary relation \( \to \) on \( X \) as follows: For each \( c, c' \in X \), \( c \to c' \) iff (1) for each \( i \in c \Delta c' \), \( u_i(c') \geq u_i(c) \) and (2) for some \( i \in c \Delta c' \), \( u_i(c') > u_i(c) \). Define another binary relation \( \Rightarrow \) on \( \{0, a, b\}^I \) such that \( c \Rightarrow c' \) iff (1) \( c \to c' \) and (2) for
each $i \in c \Delta c'$, $c_i > c'_i$ (here, we stipulate that $a > 0 > b$). It can be seen that $\Rightarrow$-sequences characterize the coalition-formation process in the second stage. Now we show that when $I$ is finite, ($\{0, a, b\}^I, \Rightarrow$) satisfies (N1) and (N2) in Lemma A.1. First, it is clear that (N1) holds since every $\Rightarrow$-sequence is monotonic with respect to the club of $a$, i.e., for each $t \in \mathbb{N}$, $I_a(c^{t+1}) \subseteq I_a(c^t)$. For (N2), let $c, c', c'' \in \{0, a, b\}^I$ with $c \Rightarrow c'$ and $c \Rightarrow c''$. Let $E' = \{i \in c \Delta c' : c'_i = b\}$ and $E'' = \{i \in c \Delta c'' : c''_i = b\}$. It is clear that $E' \cup E''$ can move as a whole to $b$ which increases everyone’s payoff. For those who leaves $a$’s club and joins no club, the same argument holds. Hence, there is a $\hat{c}$ such that $c' \Rightarrow \hat{c}$ and $c'' \Rightarrow \hat{c}$. Therefore, (N2) is also satisfied. By Lemma A.1, the process is order-independent.

Part 2a: Similar to Proposition 1.2a and omitted.

Part 2b: Suppose that there were a core outcome that had more non-superpowers in $a$’s club than the core outcome of the coalition-formation process. Then, the extra countries could improve their payoffs by choosing $b$’s club. This means that the supposed outcome is not a core outcome. Therefore, the coalition-formation process leads to a core outcome that favors $a$. Since we let countries outside $a$’s club which obtain 0 by joining $b$’s club join $b$’s club, among all core outcomes that favor $a$, the core outcome we choose favors $b$ most.

Proof of Lemma 3. By the way of backward induction we start from $b$’s choice. Given $a$’s choice, suppose a tie (of a maximum payoff) between $b$’s choices. There are two possibilities. First, all locations might be a choice. There must be one location closest to $b$’s location, and $b$ chooses the closest one by A1. Second, one choice among the choices might be “not to establish a club”. Then, $b$ chooses not to establish a club by A2. Analogously, $a$’s choice is unique. Hence, the result in Lemma 3 holds.

Proof of Proposition 1. Part 1: Since $I$ is symmetric, the farthest that $a$ would locate the club good is $\frac{n-1}{2n}$. Hence, $a$ would not establish a club if $a$ chooses $\frac{n-1}{2n}$ but can not obtain any member. This holds if the country $\frac{n-1}{2}$ does not join the club only with two members, the countries $\frac{n-3}{2}$ and $\frac{n+1}{2}$ do not join the club only with four members, and so on, implying $g < g_U(n) := \frac{2n}{(n+3)(n-1)}$. Hence, $a$ (and $b$) can not form a club when $g < g_U(n)$.

Parts 2 and 3: It can be shown that when $g$ is relatively close to 1, $a$ and $b$ locate their club good at home and divide the non-superpowers into two clubs of the same size. Yet,
when \( g \) is lower than and close to a threshold \( g_B^h(n) \), \( a \) moves its club good to \( \frac{1}{n} \) and \( b \) moves its club good to \( \frac{n-1}{n} \) to form a club (it can be shown that \( a \) as well as \( b \) either can capture half of countries in \( I \) or none). Similar to the proof of Proposition 1.1, for both superpowers to move their club good it should be that \( g < \frac{n}{(n-k)(k+1)} \) for each \( k \in \{1, \ldots, \frac{n-1}{n}\} \), the minimum value of which implies \( g < g_B^h(n) := \frac{4n}{(n+1)^2} \).

This breaks down when \( b \) can no longer take away countries from \( a \) and sustain \( \{\frac{n+1}{2}, \ldots, \frac{n-1}{n}\} \) as a club. By solving the optimization problem, one can see that the boundary case is when at stage one, \( a \) takes \( \{1, \ldots, \frac{n-1}{2}, \frac{n+1}{2}\} \) and it is not beneficial for \( \frac{n+1}{2} \) to join \( b \)’s club even if \( \{\frac{n+3}{2}, \ldots, \frac{n-1}{n}\} \) all join \( b \)’s club, i.e., \( g < g_B(n) := \frac{4n}{(n+3)(n+1)} \). For \( g \in \{g_U(n), g_b(n)\} \) there is only \( a \)’s club because \( a \)’s best response is to move its club good further away than \( b \) does and take everyone for any choice of \( b \). It follows that \( a \) chooses a location close to the center and takes most countries to deter \( b \).

Proof of Proposition 2. There exists a minimum club size \( \delta^*(n) \) that \( a \) tries to defend by moving the club good away from home if \( g \in \left[ g_B^h(n), g_B^b(n) \right) \). If \( g \in [0, g_B(n)) \), \( a \) can no longer defend \( \delta^*(n) \) and gives up establishing its club. Analogously, \( b \) tries to defend its last stronghold by moving the club good away from home if \( g \in \left[ g_B^h(n), g_B(n) \right) \) but gives up establishing its club if \( g \in [g_B(n), 1) \). We start by deriving \( \delta^*(n) \). Suppose that \( a \)’s club contains \( \delta_a \) countries (including itself) with the club good at 0 and \( \delta'_a \) countries with the club good at \( 1/n \). It is beneficial for \( a \) to move the club good from 0 to \( 1/n \) if \( 1 - \frac{1}{n} - \frac{1}{\delta'_a} > 1 - \frac{1}{\delta_a} \) or \( \frac{\delta_a - \delta'_a}{\delta'_a \delta_a} > \frac{1}{n} \). Since \( n \) is large enough, \( \delta'_a - \delta_a = 1 \). Hence, the inequality implies \( \delta_a^2 + \delta_a - n < 0 \).

Since \( \delta_a \) is a non-negative integer, \( \delta_a \leq \delta^*(n) := \lfloor \sqrt{\frac{4n^4-1}{2n}} \rfloor \). Note that \( \delta^*(n) \) does not depend on \( a \)’s location, so \( a \) moves its club good toward the center whenever it cannot hold \( \delta^*(n) \) when \( g_b \) increases (i.e., \( g_a \) decreases). When \( a \) moves its club good to the last stronghold but still cannot hold the club, it dissolves a club (due to the discreteness, depending on the numerical value of \( \sqrt{\frac{4n^4-1}{2n}} \), for some \( n \), \( a \) moves its club good to \( \delta^*(n) + 1 \) and keeps all the non-superpowers to the left).

Part 1: First, we define \( g_B^h(n) \), which is the threshold of \( g \) below which \( a \) gives up establishing a club. Based on the argument above, we assume that \( a \) tries to keep \( \{1, \ldots, k\} \) for some \( k \in [\delta^*(n), n] \) (since if \( k < \delta^*(n) \), \( a \) will not establish a club). The highest compromise \( a \) would make is to move the club good to \( k/n \), and keep it there until \( k \) leaves the club.
Given that $b$ places its club good at 1 (because now $b$’s club has more than half of the non-superpowers and it does not need to move its club), it is when $g - \frac{1}{k+1} < (1 - g) \left( 1 - \frac{n-k}{n} \right) - \frac{1}{n+1-k}$ or $g < \frac{n}{n+k} \left( \frac{k}{n} + \frac{1}{k+1} - \frac{1}{n+1-k} \right)$. If we take the right-hand side as a function of $k$, using convex optimization methods similar to those in Lemmas A.4 and A.5, it can be shown that $\arg \min_k \left\{ \frac{n}{n+k} \left( \frac{k}{n} + \frac{1}{k+1} - \frac{1}{n+1-k} \right) \right\} = \frac{\sqrt{1+4n} - 1}{2} + 1 = \delta^*(n) + 1$. Substituting $\delta^*(n) + 1$ into the right-hand side we obtain $g_B(n)$. It follows that all but country 1 joins $b$’s club if $g \in (0, g_B(n))$. When $g = 0$, all countries join $b$.

We now define $g_B(n)$, which is the threshold of $g$, above which $b$ gives up establishing a club. The last stronghold of $b$, i.e., $n - k$, switches from $b$ to $a$’s club when $(1 - g) - \frac{1}{k+1} < g \left( 1 - \frac{n-k}{n} \right) - \frac{1}{n-1}$ or $g > \frac{n}{n+k} \left( \frac{k}{n} + \frac{1}{k+1} + \frac{1}{n-1} \right)$. Given $a$ stays at 0 (due to the same reason as $b$ stays at 1 above), if we take the right-hand side as a function of $k$, using convex optimization methods similar to those in Lemmas A.4 and A.5, it can be shown that $\arg \max_k \left\{ \frac{n}{n+k} \left( \frac{k}{n} + \frac{1}{k+1} + \frac{1}{n-1} \right) \right\} = \delta^*(n)$ or $\delta^*(n) + 1$. Denote the value by $\delta^*(n)$. Substituting $\delta^*(n)$ into the right-hand side, we obtain $g_B(n)$. It follows that all but country $n-1$ joins $a$’s club if $g \in \left[ g_B(n), 1 \right)$. When $g = 1$, all countries join $a$.

Part 2a: In the proof of Proposition 2 we show that superpowers move their club good away from home when their club size is smaller than $\left\lfloor \frac{\sqrt{1+4n} - 1}{2} \right\rfloor$ or $\left\lceil \frac{\sqrt{1+4n} - 1}{2} \right\rceil + 1$. For $n \geq 12$ and odd, the threshold is smaller than $(n-1)/2$. Hence, $a$ locates its club at 0 and $b$ at 1 when their club size is around $(n-1)/2$. The size of $a$’s club exceeds that of $b$’s club when the non-superpower at $(n+1)/2$ chooses $a$’s club, i.e., $g_a \left( 1 - \frac{n+1}{2n} \right) - \frac{2}{n+3} \geq g_b \left( 1 - \frac{n-1}{2n} \right) - \frac{2}{n+1}$ or $\bar{g}(n) := \frac{n+1}{2n} + \frac{2}{n+3} - \frac{2}{n+1}$ since $g_a = g$ and $a + g_a = 1$. Similarly, $b$’s club exceeds that of $a$’s club when the non-superpower $\frac{n-1}{2n}$ chooses to join $b$’s club, i.e., $g_a \left( 1 - \frac{n-1}{2n} \right) - \frac{2}{n+1} < g_b \left( 1 - \frac{n+1}{2n} \right) - \frac{2}{n+3}$ or $\underline{g}(n) := \frac{n-1}{2n} - \frac{2}{n+3} + \frac{2}{n+1}$. It follows that both clubs are of equal size $\frac{n+1}{2}$ when $g(n) \leq \bar{g}(n)$. The two values are symmetric to $1/2$ since $\bar{g}(n) - \frac{1}{2} = \frac{1}{2} - \underline{g}(n)$.

Part 2b: We first consider $g_B^h(n)$, above which $a$ keeps its club without compromise. Suppose that $a$’s club is $\{1, \ldots, k\}$ where $k \in \{1, \ldots, n-1\}$. Superpower $a$ can hold this club without compromise when its last stronghold is not taken away by $b$, which requires $g \left( 1 - \frac{k}{n} \right) - \frac{1}{k+1} > (1 - g) \left( 1 - \frac{n-k}{n} \right) - \frac{1}{n+1-k}$ or $g > \frac{k}{n} + \frac{1}{k+1} - \frac{1}{n+1-k}$. Taking the right-hand side as a function of $k$ and by Lemma A.4, we can find an integer $\delta^*(n) := \arg \max_k \left\{ \frac{k}{n} + \frac{1}{k+1} - \frac{1}{n+1-k} \right\}$, which is either $\delta^*(n)$ or $\delta^*(n) + 1$ due to the discreteness of $I$. Substituting $\delta^*(n)$ into the
right-hand side we obtain $\mathcal{g}^h(n)$.

Now we consider $\mathcal{g}^b_B(n)$, below which $b$ keeps its club without compromise. Note that $\mathcal{g}^b_B > \frac{1}{2}$ since, as shown in Proposition 1.3, when $g = \frac{1}{2}$, $a$ and $b$ divide $I$ by placing the club good at their respective home, 0 and 1. For $g \in (\frac{1}{2}, 1)$, in stage one, $a$’s club is $\{1, \ldots, n-2\}$. Let $k \in \{1, \ldots, n-2\}$. For $n \geq 12$, establishing a club of size $\{n-k, \ldots, n-1\}$ is equivalent to solving $(1-g) \left(1 - \frac{k}{n}\right) - \frac{1}{k+1} > g \left(1 - \frac{n-k}{n}\right) - \frac{1}{n-1}$ or $g < 1 - \frac{k}{n} - \frac{1}{k+1} - \frac{1}{n-1}$. Taking the right-hand side as a function of $k$ and by Lemma A.5, we can find an integer $\delta^b(n) := \arg \max_k \{1 - \frac{k}{n} - \frac{1}{k+1} - \frac{1}{n-1}\}$. Substituting $\delta^b(n)$ into the right-hand side we obtain $\mathcal{g}^b_B(n)$.

**Proof of Proposition 3.** Part 1: Since $g \in (g^b_B(n), 1]$, when $\omega_a = 1$, $a$ and $b$ locate their club good at 0 and 1, and they equally divide $I$. For a higher $\omega_a$, since the cost for each non-superpower to join $a$’s club decreases, $a$’s club is weakly increasing in stage 1. Given each non-superpower $k$ (with location $\frac{k}{n}$), one can see that the boundary condition for $k$ to be the last country in $a$’s club (i.e., countries 1, ..., $k$ join $a$’s club and $(k+1)$, ..., $(n-1)$ join $b$’s club) is $\frac{g(n-k)}{n} > \frac{gk}{n} - \frac{1}{n-k+1}$. It is easy to see that the maximum value of $k$ for the inequality to hold is $\lfloor \frac{n}{2} \rfloor + 1$, which is smaller than $n - \delta^b + 1$, a threshold we have calculated in Proposition 2. Hence, $b$ does not move its club good away from home and it follows that $\delta_a$ is weakly increasing in $\omega_a$.

Part 2: By Lemma A.6, as long as $a$’s club expands, $a$ locates its club good at home 0. We identify the range of $g$ such that $b$ forms a club without compromise no matter how large $\omega$ is. The benefit for a non-superpower $k$ to stay in $a$’s club approaches to $g \left(1 - \frac{k}{n}\right)$ as $\omega \to \infty$. Here, $b$ can keep a club if and only if for some $k$, $\frac{gk}{n} - \frac{1}{n+1-k} > g \left(1 - \frac{k}{n}\right)$, i.e., $(n+1-k)(2k-n) > \frac{n}{g}$ (note that $n+1-k$ is the size of $b$’s club when everyone from $k$, i.e., $k, k+1, \ldots, n-1$ and $b$, are in $b$’s club). Since $k$ is an integer, the maximum value of the left-hand side of the inequality is obtained when $k = \lfloor \frac{3n+2}{4} \rfloor$ ($[\cdot]$ is the rounding function). Therefore, when $g > g^b(n) := \frac{\lfloor \frac{3n+2}{4} \rfloor}{(n+1-k)(2\lfloor \frac{3n+2}{4} \rfloor-k-n)}$, $b$ can always form a club. Note that the smallest size of the club is $n - \lfloor \frac{3n+2}{4} \rfloor + 1 \approx n \approx \frac{n}{4} \approx \delta^b(n)$ ($\gg$ means much larger than). Hence, by Proposition 2 $b$ does not need to compromise and no matter how large $\omega$ is, $\ell_a = 0$ and $\ell_b = 1$. Therefore, when $g > g^b(n)$, as $\omega$ increases, the size of $a$’s club weakly increases with an upper bound $\lfloor \frac{3n+2}{4} \rfloor$.

36
Part 3: We prove this statement for \( i = 1 \). The others \( i \in I' \) are further from 0 and benefit less from joining \( a \)'s club while they have to pay the same cost. Therefore, if the statement holds for 1, it holds for all other \( i \in I' \). We use \( I_a(\omega) \) to denote the set of countries in \( a \)'s club when \( \omega_1 = \omega \). First, by joining \( a \)'s club, country 1’s benefit is \( g \times \frac{n-1}{n} \), which is strictly less than 1. In contrast, as \( \omega \to \infty \), its cost share \( \rho_{1a}(I_a(\omega)) \) goes to 1. Therefore, by continuity, at some \( \omega^* \), \( g \times \frac{n-1}{n} < \rho_{1a}(I_a(\omega)) \), i.e., country 1’s cost exceeds its benefit from joining \( a \)'s club, and consequently it leaves the club and does not join any club. When country 1 leaves, \( a \) has less members than \( b \) since now at least 1 is not in \( a \)'s club anymore, i.e., \( \delta_a < \delta_b \) and \( \omega_a + \sum_{h \in I_a} \omega_h < \omega_b + \sum_{h \in I_b} \omega_h \).

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