Control of self-excited vibrations in face milling with two-rim mill

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Abstract. The paper provides an overview of self-excited vibration damping methods for face milling. The main source to maintain and develop self-excited vibrations is the mechanism of their regeneration. To suppress it, two methods are currently actively used: tools with variable tooth pitch and cutting speed modulation. A combination of these methods was used to develop a vibration-resistant face mill with two-rims provided with alternating teeth and connected with adjustable stiffness by spring. The spring also allows a variable pitch of adjacent teeth to be adjusted. During cutter operation, the elastic connection of the rims creates difference in the cutting speed of adjacent teeth, which suppresses regeneration of self-excited vibrations and decreases their amplitude. The self-excited vibration damping efficiency of the proposed mill was determined by comparing its operation with that of a conventional mill in the same conditions of roughing a steel workpiece. The dynamics of the milling machine technological system during operation of both tools was studied using a simulation method. It was found that the conventional mill cannot be used under cutting conditions accepted in the study because of increased vibrations of the technological system elements. On the contrary, a two-rim mill provides almost quiescent processing conditions at appropriate values of the elastic connection stiffness of rims and their relative angular displacement. The settings of the two-rim mill were determined with respect to a moving rim and a spindle with another rim as a dual-mass system with two degrees of freedom.

1. Introduction

Face milling is widely used in modern engineering showing high technological performance. However, intensive cutting conditions or insufficient stiffness of the technological system (TS) can cause self-excited vibrations that dramatically worsen the accuracy and quality of surface treatment, tool life, equipment durability, ergonomic indicators of operating conditions, productivity and cost of treatment. Conventional methods used to suppress self-excited vibrations, such as increased stiffness of weak TS elements, changed blade geometry, scheme and cutting mode parameters cannot be used in some cases or can decrease machining performance. Therefore, development of efficient techniques to control self-excited vibrations in order to suppress them is the challenge of modern mechanical engineering.

To reduce the amplitude of self-excited vibrations, dampers embedded in the carrier system of the machine [1] or face mills are sometimes used [2, 3]. However, this technique does not always provide satisfactory results.

The vibration-free cutting conditions are searched based on the study of the dynamic stability of
cutting processes, which has been rapidly developing since the middle of the last century [4–10, etc.]. Among the types of mechanical processing, face and end milling is characterized by the greatest complexity of dynamic processes [11, 12]. Therefore, for its study, a simulation method with petal stability diagrams constructed in the "speed – cut depth" coordinates is widely used [13, 14, 15]. For the given machining conditions, these diagrams help to find the cutting speed that corresponds to the maximum TS dynamic stability.

The most energetically powerful mechanism to maintain and develop self-excited vibrations is their regeneration when cutting off a wavy trace on the cutting surface made by the previous tooth [7, 9, 16, 17]. Therefore, to suppress self-excited vibrations, it is primarily advisable to suppress regeneration, which is currently actively implemented by using a variable pitch of teeth and a periodically changed (modulated) cutting speed [18].

Although the efficiency of tools with variable tooth pitch is proved theoretically and experimentally [19, 20], they need an individual choice of the pitch for a particular value of the cutting speed. With regard to face mills operating at different values of cutting speed, there is a need to create structures that enable smooth control of the teeth pitch.

Suppression of self-excited vibrations using modulated cutting speed has been applied since the 70s of the last century in turning, and since the 90s it was used in milling, mainly end milling [12]. A variable cutting speed is created using both special tooling that converts uniform rotation of the spindle into non-uniform rotation of the workpiece or tool [21], and programmatic techniques using CNC machines [22].

The first path typically reduces the machine working space. The second technique is hampered by high inertia of the spindle with the workpiece or tool attached to it, especially in face milling. In the latter case, this can be eliminated by transferring the modulated angular rotation speed not to the entire face mill, but only to its light rim with even teeth installed coaxially in the mill body, where odd teeth are located [23]. During operation, the spindle with the mill body rotates uniformly, and the light rim rotates non-uniformly being initiated by the spindle through the built-in modulating mechanism. As a result, the adjacent tool teeth pass through the same area of the cutting surface at different speeds, which weakens or suppresses the self-excited vibration regeneration mechanism. However, the complexity and bulkiness of the kinematic connection of the moving rim with the mill body is a significant practical drawback of the proposed design.

This technical solution was improved when kinematic connection was replaced with an elastic one in the form of a spring with adjustable stiffness, which allowed a more effective design of the vibration-resistant two-rim face mill [24]. It also provides adjustment of the mutual angular position of the rims to create a variable pitch of adjacent teeth. Thus, the proposed mill combines both techniques of suppression of self-excited vibration regeneration: a variable tooth pitch and modulation of the cutting speed.

The paper aims to study the dynamics of the cutting process with a two-rim mill to establish the physical pattern of its operation and to evaluate the efficiency of self-excited vibration damping.

2. Equipment and research technique
The mill used in the study was upgraded with respect to its basic version [24] to increase the efficiency by replacing the flat plain bearing with rolling bearings (Fig. 1). The mill body includes flange 1 and rim 2 with odd teeth fastened together by two pins 3 and four screws 4. Rim 6 with even cutting teeth is mounted on rim 2 through two angular-resistant rolling bearings 5. The tension in rolling bearings is created using four screws 7 secured by screws 8. Spring 9 made in the form of a split ring, thrust 10 and axles 11, 12 and 13 elastically connect rims. Threaded connection of thrust 10 and axle head 13 allows adjustment of the variable pitch of cutting teeth during mill assembling. Mobile supports 14 installed in a circular groove of the wedge-shaped profile on rim 6 are used to change the stiffness of the elastic connection of rims 2 and 6. The cutter teeth are equipped with tangentially located replaceable carbide blades 15. Casing 16 fixed on flange 1 protects spring 9 from outgoing chips.
Simulation was used to study the dynamics of milling [25]. The coordinate directions of the TS machine were as follows: X – along the longitudinal table, Y – vertical direction, Z – along the spindle axle. Experimentally obtained parameters of the mathematical model of the machine elastic system are given in Table 1, where: \( n \) is spindle speed, \( f \) is self-excited vibration frequency, \( C \) is stiffness, \( m \) is reduced mass, \( I \) is inertia, \( h \) is the resistance force coefficient. The nature and structure of vibrations in TS were identified based on the analysis of vibration records and the vibration spectra of the contours of the tool and workpiece subsystems and the spindle torsion contour. The vibrations of the TS components were modeled for processing conditions of the workpiece made from steel 45 (HB=235) without cooling lubricant using a horizontal milling machine mode 6M82. The workpiece was compared with conventional TU 2-035-618-78 and a special two-rim face mills with equal diameter of 160 mm, the number of carbide teeth 10 and the geometry of the cutting blades: \( \varphi=85^\circ, \varphi_1=5^\circ, \gamma_{rad}=-11^\circ, \gamma_{ax}=-10^\circ \). To guarantee excitation of self-excited vibrations in TS, the mode of rough symmetrical milling was used: \( B=120 \text{ mm}, t=6 \text{ mm}, S_z=0.1 \text{ mm / tooth}, v=158 \text{ m / min (n=315 rpm)}. \)
Table 1. Parameters of the mathematical model of elastic machine and tool system

| TS component | Coordinate or n, min⁻¹ | f, Hz | C, MN/m or N·m/rad | m, kg or I, kg·m² | h, N·s/m or N·s·m |
|--------------|------------------------|-------|---------------------|-------------------|-------------------|
| Table        | X                      | 94    | 44                  | 126               | 5742              |
|              | Y                      | 97    | 132                 | 354               | 23598             |
|              | Z                      | 479   | 787                 | 87                | 22887             |
|              | X                      | 185   | 304                 | 225               | 25752             |
| Apron        | Y                      | 110   | 519                 | 1083              | 78480             |
|              | Z                      | 641   | 1556                | 96                | 30597             |
| Console      | X                      | 92    | 106                 | 321               | 17558             |
|              | Y                      | 267   | 206                 | 73                | 21719             |
|              | Z                      | 105   | 1094                | 2507              | 147064            |
| Spindle      | X                      | 516   | 107                 | 10                | 4651              |
| with face mill| Y                    | 431   | 175                 | 24                | 10635             |
| Spindle      | X                      | 487   | 102                 | 11                | 5023              |
| with two-rim mill| Y          | 379   | 162                 | 29                | 9237              |
| Moving rim of the two-rim mill| X          | 1226  | 161                 | 2.70              | 609               |
|              | Z                      | 1190  | 165                 | 2.95              | 523               |
| Main drive with face mill| X          | 516   | 106                 | 10                | 4769              |
| Main drive with two-rim mill| X          | 315   | 86                  | 11959             | 0.12              |
|              | φ                      | 370   | 2804                | 0.01              | 0.03              |
|              |                       |       |                     |                   |                   |

3. Results and discussion

Treatment with conventional mill produces strong vibrations of the tool in the X and Z directions and those of the workpiece in the Z direction. Table 2 shows the frequencies and amplitudes of the vibration harmonics of subsystems of the spindle main motion drive, and the tool and workpiece in all coordinate directions. Vibration records of tool and workpiece vibrations in the direction of feed and torsional vibrations of the spindle are presented in Fig. 2. The spectrum of tool vibrations in the X direction is shown in Fig. 3.

Table 2. Range R and spectral composition of vibrations of ts subsystems in coordinate directions

| n, min⁻¹ | Torsion subsystem | Contours of the tool subsystem | Contours of the workpiece subsystem |
|----------|-------------------|-------------------------------|-----------------------------------|
|          | X                 | Y                             | Z                                |
|          | X                 | Y                             | Z                                |
|          | Vibration range (deg, µm) |                             |                                   |
|          | Frequencies (Hz) and amplitudes (deg, µm) of vibration harmonics |                             |                                   |

| f, Hz | A | f | A | f | A | f | A | f | A | f | A | f | A | f | A | f | A |
|-------|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 315   | 0.35 | 50 | 50 | 50 | 1.4 | 50 | 4 | 52 | 75 | 50 | 26 | 50 | 1 |
|       | 72  | 0.23 | 472 | 14 | 420 | 2.5 | 558 | 70 | 58 | 11 | 102 | 2 | 102 | 2 |
|       | 86  | 0.25 | 558 | 39 | 470 | 3.1 | 102 | 13 | 378 | 4 |
|       | 92  | 0.23 | 556 | 3.6 | 558 | 2 |     |   |     |   |     |   |     |   |     |   |
| 85.7  | 102 | 0.21 |     |   |     |   |     |   |     |   |     |   |     |   |     |   |   |
Intense self-excited vibrations with amplitudes of the dominant harmonics $AX=39$ μm and $AZ=70$ μm are found in the X and Z contours of the bending tool subsystem, respectively (Fig. 3). These harmonics with a frequency of 558 Hz are caused by natural frequencies of the tool subsystem in the X and Z directions (516 and 545 Hz, respectively), with increased cutting stiffness (Table 1). Self-excited vibrations a range of 574 μm in the X direction and that of 771 μm in the Z direction. The range of the almost harmonic forced vibrations of the workpiece subsystem in the X direction is 366 μm, and that of the torsional spindle subsystem is 3.5 degrees.

Self-excited vibrations in TS were analyzed in the framework of the theory of their regeneration described in [18]. During dynamically unstable blade machining, several complete self-excited vibration waves and their fractional remainder are located on the cutting surface that corresponds to the contact length of the adjacent tool teeth and the workpiece. At cutting speeds, when this balance is 0.75 of the wavelength, at the instant when the tool blade meets with the chatter mark of the preceeding tooth, the current TS self-excited vibrations are ahead of chatter marks by 0.25 waves, which corresponds to the phase shift angle of $+90^\circ$. In this case, TS does not need to adapt to chatter marks since it always spontaneously tends to this value of the phase shift. If the phase shift initial values are different, TS rearranges (self-organizes) its first vibrations after meeting by stretching or compressing them to adapt to chatter marks a quarter of the wave ahead. TS tuning is required to minimize the energy costs of its vibration motion. Simultaneously, a phase shift of $+90^\circ$ ensures the portion of energy supplied in TS at each vibration since when the tool moves from the workpiece, the slice thickness is greater than that during its penetration. If the initial phase shift is $-90^\circ$ (0.25 wave lag of chatter marks), the pattern changes the other way around and a portion of energy is removed from TS at each vibration. Phase shifts of 0° and 180° (cophasal and antiphase vibrations, respectively) do not change the amount of energy stored in TS and do not change its dynamic stability limit. Thus, the initial phase shift of $+90^\circ$ creates conditions for free regeneration of self-excited vibrations, at which the TS dynamic stability is minimal. In the stability diagram, these conditions correspond to the lowest points of the lobe zone boundaries that correspond to the minimum depth of cutting, the excess of which excites self-excited vibrations. In this case, the self-excited vibration frequency slightly exceeds the natural frequency of the TS limiting component due to the effect of cutting stiffness.

At a spindle rotation frequency of 315 rpm, its current torsional vibrations are behind the chatter mark by 0.37 of the wavelengths (determined by the ratio of the cutting path length between adjacent teeth and the vibration wavelength), which contributes to their suppression. In addition, in this case, the stiffness of the drive subsystem of the main motion of the machine is sufficiently high, and the volume of excitation energy generated by cutting is insufficient for torsional self-excited vibrations to occur. Therefore, it sets the mode of forced vibrations, the first and weaker second harmonics of which with frequencies of 50 and 102 Hz, respectively, are caused by the "tooth" frequency of the mill (52.5 Hz).

![Figure 3. Vibration spectrum of the instrument subsystem in the feed direction](image-url)
In addition to these, the spectrum has three more harmonics, one of which with a frequency of 86 Hz is a resonant response of the torsional subsystem at its natural frequency. An insignificant relative depth of modulation of the cutting speed (6% for the fundamental harmonic) cannot have a strong impact on cutting. Therefore, intensive regenerative self-excited vibrations with a frequency of 556 ... 558 Hz occur in all contours of the tool bending subsystem. The main reason for their occurrence is the advance of chatter marks by 0.31 of the wavelengths. Self-excited vibrations occur in the form of beats due to the weak harmonic in their spectrum with a "tooth" frequency of the mill (50 Hz). This frequency causes forced vibrations of the X and Y contours of the workpiece subsystem, which occur at two multiple harmonics. In the Z contour of the workpiece subsystem, self-excited vibrations of the table occur with a frequency of 378 Hz (Table 2) modulated by the "tooth" frequency of the mill.

When using a conventional face mill with a rotational speed of 315 rpm, the torsional spindle subsystem makes forced vibrations, the most powerful first harmonic of which with a frequency of 50 Hz is caused by the frequency of rotation of the teeth (52.5 Hz). The ratio of the length of the cutting path between the adjacent teeth of the mill and the wavelength of torsional vibrations shows that the fractional remainder of the latter is 0.37 of the wavelength. This prevents their transformation into self-excited vibrations. Torsional vibrations of the spindle modulate the cutting speed with a depth of 6%, which is not sufficient to suppress intense self-excited vibrations of the bending tool subsystem in the X and Z directions occurring at 558 Hz. The main reason for their occurrence is the advance of chatter marks by 0.31 of the wavelengths. The subsystem procurement also does not remain quiescent. Strong forced vibrations occur in the direction of the longitudinal feed, and those three times weaker occur in the vertical direction at the “tooth” frequency of the mill.

The pattern of vibrations of the TS components shows that the rough cutting mode used in the study does not allow workpiece treatment using a conventional face mill to intense vibrations. This processing requires either reduced cutting conditions that decreases productivity, or suppressed self-excited vibrations by means of the two-rim face mill, as an example.

The impact of the two-rim face mill on the regeneration of self-excited vibrations is realized by means of spontaneously arising modulation of the angular speed of a moving rim, which is an additional tangent contour in the tool body. The effect of a moving rim with even teeth is determined by stiffness of the elastic relationship between $C_{rim}$ with the mill body with odd teeth and its initial circumferential displacement $\alpha$ relative to the body. The first parameter determines the depth of the cutting speed modulation, and the second one determines the difference in pitches of the mill teeth. Displacement of the moving rim is considered positive direction of the cutting torque. Consider the dynamics of performance of the two-rim face mill in the conditions similar to those for a conventional mill.

At the beginning of the study it was assumed that the spindle with the mill body would perform almost uniform rotation, and the moving rim would perform torsional vibrations to create the modulation of the cutting speed of even teeth to suppress regenerative self-excited vibrations of the TS bend subsystems. However, the preliminary study showed that both rims exhibit torsional vibrations of the same order. Therefore, it is effective to produce synchronous anti-phase torsional vibrations of the rims to ensure the difference in the cutting speeds of adjacent teeth. Assume that the torsion subsystem "spindle with the mill body – moving rim" is dual-mass and double-degree-of-freedom and neglect damping to simplify calculations. Fig. 4 shows the dynamic scheme of this subsystem, with the following designations: $C_{sp-mill}$, $C_{rim}$ are torsional stiffnesses of the spindle kinematic chain and elastic connection of the moving rim; $I_{sp-mill}$, $I_{rim}$ are inertia moments of the spindle with a mill and the moving rim; $\phi_1$, $\phi_2$ are angular coordinates of vibrations of the spindle with the mill body and moving rim.

We believe that cutting is performed uniformly by the teeth of both rims. The cutting torque $M_p$ acting on these rims are equal in absolute value, but their variable components are anti-phase, since the teeth of both rims alternate and start working sequentially. The frequency of change of these variable components of the cutting torque is equal to half of the "tooth" frequency of the mill.
The vibrational properties of dual-mass systems with two degrees of freedom are well known [26]. They have two natural frequencies of vibration. The ratio of the natural $\omega$ and partial $\omega'$ circular frequencies of the system vibrations for the first lower mass (moving rim) and for the second higher mass (spindle with the mill body) is $\omega_1 < \omega'_1 < \omega'_2 < \omega_2$. Natural vibrations of both masses occur simultaneously at one of the natural frequencies. In this case, at the first (lower) frequency the masses vibrate cophasally, and at the second (higher) frequency they vibrate antiphasally. If an external periodic force acts on one of the masses, they produce forced vibration: in the frequency range of the first natural frequency, the masses vibrate antiphasally, and in the frequency range of the second natural frequency, the masses vibrate cophasally. The smaller mass shows an increased amplitude of vibrations, both natural and forced ones, as compared to the greater one.

The vibration of the torsional subsystem "spindle with the mill body – moving rim" under the action of antiphasally changing variable components of the cutting torque is of a complex nature. At different values of $C_{rim}$ and $a$ parameters. vibration records of the subsystem vibrations show the possibility of the dominance of both cophasal and antiphase forced vibrations of both rims at frequencies of the first and second harmonics of changes in the cutting torque, antiphase self-excited vibrations of the rims at the second natural frequency, and mixed cases. Self-excited vibrations of the rims at the first natural frequency occur at a significantly smaller amplitude compared to the second frequency, since the vibrations require much higher energy costs of TS that agrees well with Hamilton-Ostrogradsky principle of the least action [27]. Since self-excited vibrations of the rims can be interpreted as steady-state natural vibrations of TS components, their phase relations will be similar to natural vibrations.

Regenerative self-excited vibrations both normal and tangent to the cutting surface (torsional) are of similar physical nature. If the speed of the mill rotation at a constant feed rate varies according to a harmonic law, a wavy trace is formed on the surface after each tooth. The mark vibration lead by about a quarter of the wave, i.e. initial phase shift $+90^\circ$ is required to maintain self-excited vibrations of both types. This is relevant for conventional mills with a constant tooth pitch. However, for a two-rim mill with a moving flexibly fixed rim, this condition changes the other way around: to maintain self-excited vibrations, the initial phase shift of torsional vibrations of the teeth of at least one (preferably moving) rim should be $-90^\circ$. In other words, these vibrations should lag behind the chatter marks by a quarter of the wave.

Figure 4. Simplified dynamic scheme of a dual-mass torsional subsystem "spindle with the mill body – moving rim" with two degrees of freedom
Figure 5. Torsional vibration records $\phi$ of a double-rim mill (a, d, g), vibrations of its rotational speed $\dot{\phi}$ (b, e, h), bending vibrations $X_c$ of the spindle with the mill and workpiece in the feed direction (c, f, i) with: a, b, c – $\alpha_{rim} = -5^\circ$, d, e, f – $\alpha_{rim} = -2^\circ$, $C_{rim} = 3000$ N·m/rad; g, h, i – $\alpha_{rim} = 0^\circ$, $C_{rim} = 7000$ N·m/rad; 1 – spindle with the mill body, 2 – moving rim, 3 – workpiece.
The rim vibrations in antiphase can occur both in the self-excited vibration mode and in the forced vibration mode. The first option is implemented in cases where the fractional balance of vibratory waves on the cutting surface between adjacent teeth takes a value of 0.25. Similar excitation conditions for self-excited vibrations of even and odd teeth are provided by preliminary rotation of the moving rim by angle $\alpha$ to compensate for the constant component of the cutting torque. The second option is associated with the creation of a resonance of any (preferably smaller) order in the torsion subsystem at a "tooth" frequency of the mill. In this case, the rims make forced resonant antiphase vibrations, since the variable components of the cutting torque cause them are antiphase. In both cases, the natural frequency of vibrations of the moving rim is chosen by changing $C_{rim}$ stiffness. The correctness of the above considerations is confirmed by the analysis of vibration processes in TS at various values of the parameters $C_{rim}$ and $\alpha$.

The initiation of antiphase vibrations of the rims in the first embodiment requires the choice of $C_{rim}$ stiffness to provide 1.25 length $L_\varphi$ of the wave of self-excited vibrations between adjacent teeth with their uniform arrangement along the length $L_{nom}$:

$$L_\varphi = \frac{L_{nom}}{1.25} = \frac{50.265}{1.25} = 40.2 \text{ mm}.$$  

In this case, the second natural frequency of the torsional subsystem is:

$$f_2 = \frac{v}{L_\varphi} = \frac{2.639 \cdot 10^3}{40.212} \approx 66 \text{ Hz}.$$  

The first and second natural frequencies of vibrations of the dual-mass system with two degrees of freedom with no regard to damping and cutting rigidity are determined by the equation [26]:

$$(2\pi f_{1,2})^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$  

where: $a = L_{sp-mill} \cdot I_{rim}$; $c = C_{sp} \cdot C_{rim}$; $b = -[L_{sp-mill} \cdot C_{rim} + I_{rim} \cdot (C_{sp} + C_{rim})]$.

Table 1 presents experimentally obtained parameters of the torsion subsystem of the spindle of a horizontal milling machine 6M82: $L_{sp-mill}=0.12 \text{ kg m}^2$, $I_{rim}=0.0206 \text{ kg m}^2$, and $C_{sp}=11959 \text{ N m rad}$. Let $C_{rim}=3000 \text{ N m rad}$. Then the natural frequencies of the system will be: $f_1=44 \text{ Hz}$ and $f_2=70 \text{ Hz}$. Two teeth from each rim of the mill of 160 mm in diameter are simultaneously involved in cutting during symmetrical milling of a 120 mm wide workpiece. When feeding $S_f=0.1 \text{ mm/tooth}$, the average cut thickness along the contact arch with the workpiece is 0.08 mm, and the cutting torque acting on each rim is 252 N m. For $C_{rim}=3000 \text{ N m rad}$, the average rotation angle of the moving rim under the action of the cutting torque is $\alpha_m=4.8^\circ$ (6.7 mm circumference displacement). To compensate for this rotation and to create similar conditions for excitation of self-excited vibrations of both rims, we take its initial displacement angle $\alpha=-5^\circ$.

For $C_{rim}=3000 \text{ N m rad}$ and $\alpha=-5^\circ$, Figs 5a, 5b and 5c present torsional vibration records and angular speeds of both rims, as well as bending vibrations of the spindle with a mill and the workpiece in the feed direction. Figs 6a, 6b and 6c show the spectra of these vibrations. As expected, the chosen $C_{rim}$ values and $\alpha$ parameters for the given cutting mode provided dominant and almost antiphase self-excited vibrations of both rims at a frequency of 66 Hz, which is close to the second natural frequency of the torsional subsystem. The vibration spectra of the rims also exhibit weaker harmonics of their cophasal self-excited vibrations at 40 Hz, which is close to the first natural frequency. The range of angular vibrations was $7^\circ$ for the fixed rim and $15^\circ$ for the moving one. Anti-phase modulation of the cutting speed of adjacent teeth at a frequency of 66 Hz resulted in almost complete suppression of bending self-excited vibrations of the "spindle – mill" subsystem (line 1, Fig. 5c). The vibrations were transformed into damped forced ones at the first, second and third harmonics of this frequency and natural frequency of bending spindle vibrations with an amplitude of 13 $\mu$m. However, a large range of torsional vibrations of the mill rims caused a significant range of forced vibrations of the workpiece.
(590 μm) and the spindle with the tool (225 μm) in the feed direction.

There are two ways to reduce the level of forced vibrations of the workpiece and tool subsystems without losing the effect of damping the bending self-excited vibrations of the spindle with the mill. The first way is to choose a higher value of $C_{\text{rim}}$ stiffness to increase the second natural frequency of vibrations of the torsion subsystem so that the wavelength of its self-excited vibrations becomes \(2.25\)-fold less than the distance between adjacent teeth. However, this weakens the regenerative effect and reduces the possibility of exciting torsional self-excited vibrations of the mill rims. Therefore, we will choose the second way to reduce the intensity of these vibrations with similar $C_{\text{rim}}$ stiffness through weakened conditions of their excitation when the angle $\alpha$ is changed from $–5^\circ$ to $–2^\circ$. In this case, the average length of the cutting surface in front of the teeth of the moving and fixed rims will be:

$$L_{\text{mov}} = L_{\text{nom}} + \frac{\alpha_{\text{rot}} - \alpha_{\text{rim}}}{57,3} \cdot R_{\text{mill}} = 50,265 + \frac{4,8 - 2}{57,3} \cdot 80 = 54,2 \text{ mm};$$

$$L_f = L_{\text{nom}} - \frac{\alpha_{\text{rot}} - \alpha_{\text{rim}}}{57,3} \cdot R_{\text{mill}} = 50,265 - \frac{4,8 - 2}{57,3} \cdot 80 = 46,4 \text{ mm}.$$

The initial phase shift of the tooth vibrations of moving and fixed rims at a frequency of 70 Hz:

$$\phi_{\text{mov}} = \frac{L_{\text{mov}} - L_{\phi}}{L_{\phi}} = \frac{54,174 - 40,212}{40,212} \approx 0.35; \phi_f = \frac{L_f - L_{\phi}}{L_{\phi}} = \frac{46,356 - 40,212}{40,212} \approx 0.15.$$

As can be seen, changes in the angle $\alpha$ from $–5^\circ$ to $–2^\circ$ lead to changes in the initial phase shifts of the rim torsional vibrations from the most favorable value of 0.25 to less favorable values of 0.35 and 0.15. This should weaken the regenerative excitation of torsional self-excited vibrations of the mill rims.

Figs 5d, 5e and 5f present vibration records of the same ST subsystems, and Figs 6d, 6e and 6f show their spectra for $C_{\text{rim}} = 3000 \text{ N•m/}rad$ and $\alpha = –2^\circ$. The range of torsional self-excited vibrations of the fixed rim decreased to $3^\circ$, and that of the moving rim decreased to $10^\circ$ (lines 1 and 2, Fig. 5d) with an increase in their frequency up to 70 Hz. The vibrations of the angular speed of both rims of the mill (lines 1 and 2, Fig. 5e) slightly decreased and became almost antiphase. This decreased the amplitudes of self-excited vibrations to 9 μm, the range of forced vibrations of the workpiece to 150 μm, and the spindle with the tool to 160 μm.

To produce antiphase vibrations of the mill rims based on the forced ones, we will choose the second natural frequency of the torsion subsystem, which is almost multiply "toothed" frequency of the mill – 102 Hz. This frequency corresponds to the stiffness of $C_{\text{rim}} = 7000 \text{ N•m/}rad$, at which the calculated average rotation angle of the moving rim under the action of the cutting torque is $2^\circ$. The wavelength of possible torsional self-excited vibrations at a frequency of 102 Hz is equal to 25.9 mm. At the initial rim displacement $\alpha = 0^\circ$, the initial phase shifts of vibrations of the teeth of the moving and fixed rims are: $\phi_{\text{mov}} = 0.05$ and $\phi_f = 0.83$. This shows that the regenerative effect does not affect excitation of torsional self-excited vibrations of the moving rim and prevents their occurrence in the fixed rim. Since there are no conditions for excitation of self-excited vibrations, the rim vibrations are forced. The fixed rim produces forced vibrations mainly at the first harmonic of the "toothed" frequency of the mill (50 Hz), and the moving rim vibrates at the second harmonic (100 Hz). Compared to the previous case, the ranges of torsional vibrations of the rims and their angular speeds are almost similar (Fig. 5g vs. Fig. 5d, Fig. 5h vs. Fig. 5e), and the range of forced vibrations in the X direction increased to 370 μm for the workpiece (line 3, Figs 5i and 5f) and decreased to 120 μm for the spindle with the tool (line 1, Figs 5i and 5f). In this case, the amplitude of self-excited vibrations decreased to 7 μm (Fig. 6i).
Figure 6. Spectra of torsional vibrations of a two-rim mill (a, d, f), vibrations of its rotational speed (b, e, g), bending vibrations of a spindle with a mill in the feed direction (c, f, i) with: a, b, c - $\alpha_{\text{rim}}$ = -5°; d, e, f - $\alpha_{\text{rim}}$ = -2°, $C_{\text{rim}}$ = 3000 N·m/rad; g, h, i - $\alpha_{\text{rim}}$ = 0°, $C_{\text{rim}}$ = 7000 N·m/rad; 1 - spindle with a mill body, 2 - a moving rim.

Thus, the use of the two-rim mill in the last two setting options instead of the conventional one reduced the range of forced vibrations of the workpiece 4 ... 5-fold, the spindle with the tool 2 ... 3-fold, and the amplitude of self-excited vibrations of the tool from 39 to 7 ... 9 $\mu$m, that is 4 ... 5-fold. In other words, we reported the suppression of self-excited vibrations to an acceptable level. According to prof. I.G. Zharkova, this level for carbide tools is equal to 17 ... 20 $\mu$m at a self-excited vibration frequency of 500 ... 600 Hz [28].

4. Conclusion
The creation of antiphase torsional vibrations of the mill rims based on both self-excited and forced vibrations is almost equally efficient for increasing the dynamic stability of face milling. Therefore, the option to create the cutting speed modulation should be chosen on a competitive basis for specific
machining conditions.

The study proved high efficiency of self-excited vibration damping by the proposed two-rim face mill, which combines two modern techniques for suppressing regeneration. The idea embodied in the mill design can be implemented for other types of tools, for example, countersinks and multi-tooth boring heads.

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