On uniqueness of static spacetimes with non-trivial conformal scalar field

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Abstract
We discuss the uniqueness of static spacetimes with non-trivial conformal scalar field. We can show that the spacetime is unique to be the Bocharova–Bronnikov–Melnikov–Bekenstein solution outside the surface composed of the unstable circular orbit of a photon (photon surface). In addition, we see that multiple disconnected photon surfaces having the same values of the scalar field do not exist.

Keywords: black holes, uniqueness of static spacetimes, scalar hair

1. Introduction

The uniqueness theorem for static/stationary vacuum black holes is one of the honorable consequences in general relativity [1]. This theorem guarantees comprehensive and definite predictions in black hole astrophysics/astronomy.

Black holes in string theory are interesting objects. One may want to consider the Einstein–scalar system as a simple model because scalar fields often appear in string theory. It is well known that there is a no-hair theorem by Bekenstein [2] (see also [3]), that is, asymptotically flat and static black holes do not have regular scalar hair with non-negative potential. Interestingly, the Bocharova–Bronnikov–Melnikov–Bekenstein (BBMB) black hole solution [4, 5] exists in the Einstein–conformal scalar field system. The metric of the spacetime is the same as that of the extreme Reissner–Nordström black hole. This solution avoids Bekenstein’s no-hair theorem because the scalar field is not regular at the event horizon. It is natural to ask whether the uniqueness of this black hole solution holds. In our previous work [6], we addressed this issue and then what we could prove was the uniqueness of the photon surface of the BBMB solution, not a black hole. That is to say, the static spacetimes have to be the BBMB solution outside the photon surface. The photon surface is defined as a generalization
of the surface composed of the unstable circular orbit of a photon in spherically symmetric black hole spacetimes (see [8] for the precise definition of the photon surface). A nontrivial point here is that we do not need the assumption of the existence of photon surface a priori. The requirement of the spacetime regularity gives the inner boundary of the region where we can discuss the uniqueness in the Einstein-conformal scalar field system. Eventually, the inner boundary turns out to be the photon surface.

The uniqueness of photon surface itself is also interesting. This is because the photon surface is outside the black hole and then it can be observed. Therefore, if the uniqueness theorem holds for the region outside the photon surface, it would be ideal. The study for this direction was initiated by Cederbaum [9] (see also [10] for other cases), but the argument is restricted to the photon surface with the constant time lapse function (called photon sphere). Although a perturbative study gives us a positive result for the uniqueness [11], it is hard to prove that without the additional requirement for the lapse function at the non-perturbative level. Thus, our previous work for the Einstein-conformal scalar field system [6] may give a hint to removing such an assumption in general cases. In [6], we employed the Israel-type proof [12], that is, the first version of the proof of the uniqueness theorem for static and vacuum black hole. Since the connectedness of the horizon is essential in the Israel-type proof, our previous proof is also restricted to cases corresponding to a single object. That is, we cannot remove the possibility of multiple disconnected photon surfaces. Here we recall that, in a completely different way without the assumption of the connectedness of the horizon, the uniqueness of the black hole has also been proven by Bunting and Masood-ul-Alam [13]. This means non-existence of multiple disconnected black holes in static vacuum spacetimes. Employing the Bunting and Masood-ul-Alam type proof, the non-existence of multiple disconnected photon spheres has been proven for vacuum spacetimes [14].

In this paper, employing Bunting and Masood-ul-Alam’s way [13], we discuss the uniqueness of the BBMB photon surface without the assumption of connectedness of the photon surface. There exist two kinds of photon surfaces characterized by the value of scalar field on each surface. We prove that if only one of two kinds exists, the solution outside the surface is isometric to the BBMB solution outside the photon surface. Thus, static multiple disconnected photon surfaces with the same scalar field values do not exist in the Einstein-conformal scalar field system.

The rest of this paper is organized as follows. In section 2, we will describe the set-up and the BBMB solution. In section 3, we will review a part of our previous work [6] that we will use in this paper; the relation between the scalar field and time lapse function, and the regularity conditions at the inner boundaries. In section 4, we will complete our proof. Finally, we will give the summary and discussion in section 5.

2. Set-up and BBMB solution

The theory that we consider is the Einstein-conformal scalar field system, whose action is written in

\[
S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} R - \int d^4x \sqrt{-g} \left( \frac{1}{2} (\nabla \phi)^2 + \frac{1}{12} R \phi^2 \right). \tag{1}
\]

At a classical level, the Einstein-conformal scalar field system (say, Jordan frame) is equivalent to the Einstein-massless scalar field system (say, Einstein frame) via a conformal transformation. In [7], the uniqueness of the photon sphere (see the next paragraph) in the Einstein-massless scalar field system has been discussed. There, the existence of a photon sphere together with the constancy of the scalar field was assumed. In contrast, we will not assume the existence of such photon spheres a priori. Moreover, the spacetime in the Einstein frame has the curvature singularity at the locus of the photon sphere in the Jordan frame.
Here, $\kappa := 8\pi G$. $\phi$ is the conformal scalar field and $R$ is the Ricci scalar.

In this paper, we consider static spacetimes. The metric of static spacetime is written as

$$\text{d}s^2 = -V^2(x')\text{d}t^2 + g_{ij}(x')\text{d}x^i\text{d}x^j,$$

(2)

where the Latin indices stand for the spatial components. We consider asymptotically flat spacetimes. As seen from linear perturbations around the Minkowski spacetime, we see that the metric near the spatial infinity behaves as

$$V = 1 - m/r + O(1/r^2), \quad g_{ij} = (1 + 2m/r)\delta_{ij} + O(1/r^2),$$

(3)

where $r := \sqrt{\delta_{ij}x^ix^j}^{1/2}$ and $m$ is the Arnowitt–Deser–Misner (ADM) mass. $\{x^i\}$ is the asymptotically Cartesian coordinate near the spatial infinity. For the scalar field, we see the following asymptotic behavior:

$$\phi = O(1/r),$$

(4)

The above asymptotic behaviors for the metric and scalar field give us the boundary condition at the spatial infinity.

The equation of motion for the scalar field is written in

$$D_i(VD^i\phi) = 0,$$

(5)

where $D_i$ is the covariant derivative with respect to the metric $g_{ij}$ of $\Sigma$. The Einstein equation becomes

$$\left(1 - \frac{\kappa}{6}\phi^2\right)D^2V = \frac{\kappa}{6} \left[V(D\phi)^2 + 2\phi D^iD_i\phi\right]$$

(6)

and

$$\left(1 - \frac{\kappa}{6}\phi^2\right)\left(\frac{1}{3}R_{ij} - V^{-1}D_iD_jV\right) = \frac{\kappa}{6} \left[4D_i\phi D_j\phi - g_{ij}(D\phi)^2 - 2\phi D_iD_j\phi\right],$$

(7)

where $\frac{1}{3}R_{ij}$ is the Ricci tensor with respect to the metric $g_{ij}$ of $t = \text{constant}$ space-like hypersurface. As seen from the above equations, one has to impose the regularity condition at the surface $S_{p\pm}$ specified by $\phi = \phi_{p\pm} := \pm \sqrt{6/\kappa}$. In general, $S_{p\pm}$ can have the disconnected multi-components. Even for such cases, we simply write $S_{p\pm}$. Here we note that the expression of the field equations in the Einstein frame is regular and then the requirement of the regularity in Einstein frame does not appear automatically.

It is known that there is a static and spherically symmetric black hole solution for the current theory [4, 5]. The metric and scalar field are given by

$$\text{d}s^2 = -f(r)\text{d}t^2 + f^{-1}(r)\text{d}r^2 + r^2\text{d}\Omega_2^2$$

(8)

The theory indeed has a nontrivial symmetry. The action (1) is invariant under

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = \frac{[(1 + \varphi) + \epsilon(1 - \varphi)]^2}{4\epsilon}g_{\mu\nu},$$

$$\varphi \to \tilde{\varphi} = \frac{(1 + \varphi) - \epsilon(1 - \varphi)}{(1 + \varphi) + \epsilon(1 - \varphi)},$$

where $\epsilon$ is a constant and $\varphi := \sqrt{6/\kappa}\phi$. This transformation actually corresponds to the constant shift of massless scalar field in the Einstein frame. By this transformation, the solution with another asymptotic condition where the scalar field has a nonzero constant $\tilde{\varphi}_\infty = (1 - \epsilon)/(1 + \epsilon)$ at the spatial infinity can be related to one with the asymptotic condition (4), if the transformation is regular. Thus, we simply consider the asymptotic condition (4).
and
\[ \phi = \pm \sqrt{\frac{6}{\kappa}} \frac{m}{r - m}, \]  
where \( f(r) = (1 - m/r)^2 \), \( m \) is the mass of black hole and \( d\Omega_2^2 \) is the metric of the unit 2-sphere, that is, \( d\Omega_2^2 = d\theta^2 + \sin^2 \theta d\varphi^2 \). The event horizon is located at \( r = m \) and the scalar field diverges there. This solution is called the BBMB solution. \( S_{p\pm} \) is located at \( r = 2m \) and coincides with the locus of the unstable circular orbit of photon.

We will prove that if the spacetime has either \( S_{p+} \) or \( S_{p-} \) and no horizon exists outside \( S_{p\pm} \), the asymptotically flat solution is unique as the BBMB solution. Then, it will turn out that \( S_p(S_{p+} \text{ or } S_{p-}) \) should have a single component.

3. Relation between scalar field and lapse function

In this section we prove that if a solution has either \( S_{p+} \) or \( S_{p-} \), there is a relation between the scalar field \( \phi \) and lapse function \( V \). Then we will examine the regularity on \( S_{p+} \) (or \( S_{p-} \)). The discussion basically follows our previous work \([6]\).

The assumption here is that the static solution has either \( S_{p+} \) or \( S_{p-} \). Without loss of generality, the solution has only \( S_{p+} \) because the theory is invariant under the flip of the sign of the scalar field \( \phi \) (\( \phi \leftrightarrow -\phi \)). Hereafter, for simplicity, we write \( S_p \) for \( S_{p+} \). We also assume that no horizon exists outside \( S_p \).

From equations (5) and (6), we can have an equation
\[ D_i \left[ (1 - \phi)D^i \left( (1 + \phi)V \right) \right] = 0, \]  
where \( \phi := \sqrt{\kappa/6}\phi \).

We consider the region \( \Sigma \) satisfying \(|\phi| \leq 1 \). Then, \( \Sigma \) has two kinds of boundaries; the spatial infinity \( S_\infty \) and the surfaces \( S_p(\phi = 1) \). In general, \( S_p \) is composed of multi-components. Integrating equation (10) over \( \Sigma \), we have
\[ 0 = \int_{\Sigma} D_i \left[ (1 - \phi)D^i \left( (1 + \phi)V \right) \right] d\Sigma \]
\[ = \int_{S_\infty} (1 - \phi)D^i \left( (1 + \phi)V \right) dS' - \int_{S_p} (1 - \phi)D^i \left( (1 + \phi)V \right) dS' \]
\[ = \int_{S_\infty} D_i \left( (1 + \phi)V \right) dS'. \]

7 If the event horizon \( (V = 0) \) encloses \( S_p \), one can perform the conformal transformation so that the system becomes the Einstein-massless scalar field system and then one can realize that non-trivial scalar fields cannot exist due to Bekenstein’s no-hair theorem \([2]\). Thus, the spacetimes should be the Schwarzschild spacetime \([12, 13]\). There is still a room for the possibility that the event horizon and \( S_p \) coexist, and that both \( S_{p+} \) and \( S_{p-} \) exist. These cases are outside the scope of this paper.

Next, we consider the following volume integration over \( \Sigma \),
\[ 0 = \int_\Sigma (1 + \phi) V D_i [((1 + \phi)V)]^\Sigma \]
\[ = - \int_\Sigma (1 - \phi) [D((1 + \phi)V)]^2 d\Sigma + \int_\infty (1 - \phi^2) V D_i ((1 + \phi)V) dS - \int_\infty (1 - \phi^2) V D_i ((1 + \phi)V) dS' \]
\[ = - \int_\Sigma (1 - \phi) [D((1 + \phi)V)]^2 d\Sigma + \int_\infty D_i ((1 + \phi)V) dS' \]
\[ = - \int_\Sigma (1 - \phi) [D((1 + \phi)V)]^2 d\Sigma. \tag{12} \]

In the third equality, we used the fact from the direct calculation that the third term in the second line vanishes. In the forth equality, we used equation (11). Then, we see that \((1 + \phi)V\) is constant in \(\Sigma\). The value of \((1 + \phi)V\) can be fixed by the asymptotic condition \(((1 + \phi)V)_{r, r, \infty} = 1\), and then we have
\[ \phi = V^{-1} - 1. \tag{13} \]
Note that \(\phi = \phi_0 (\phi = 1)\) corresponds to \(V = 1/2\). As stressed, we assumed that \(S_p\) is either \(S_{p_1}\) or \(S_{p_2}\). If not, the third term in the second line of equation (12) does not vanish and then equation (13) does not hold.

To examine the regularity at \(S_{p_0}\), we look at the squares of the four-dimensional Ricci and Riemann tensors. For the current case, we have [6]
\[ R_{\mu \nu} R^{\mu \nu} = \frac{1}{\rho^2} + \frac{1}{(2V - 1)^2 \rho^2} \left(2(1 - V)k - \frac{1}{\rho} h_{ij} \right)^2 \]
\[ + \left(-2(1 - V)k + \frac{1 + 2V}{\rho} \right)^2 + \frac{8(1 - V)^2}{\rho^2} (D\rho)^2, \tag{14} \]
\[ R_{\mu \nu \rho \sigma} R^{\mu \nu \rho \sigma} = \frac{4}{V^2} D_i D_j V D_i D_j V + 4(3) R_{ij}^2 (3) R^{ij} - (3) R \]
\[ = - \frac{4}{\rho^2} + \frac{4}{\rho^2} \left[k + \frac{2}{\rho^2} (D\rho)^2 + \left(k - \frac{1}{\rho} \right)^2 \right] \]
\[ + \frac{4}{(2V - 1)^2 \rho^2} \left[\left(k + \frac{1}{\rho} h_{ij} \right)^2 + \frac{2}{\rho^2} (D\rho)^2 + \left(k - \frac{4V}{\rho} \right)^2 \right], \tag{15} \]
where \(h_{ij}\) and \(D_i\) are the induced metric and the covariant derivative of \(S_{p_0}\), respectively. Moreover, \(k_{ij}\) is the extrinsic curvature of \(S_p\) and \(\rho := |D \ln V|^{-1}\). In the above, we used the facts that, with the relation (13), equation (5) becomes
\[ D^2 V = \frac{1}{V} (DV)^2 \tag{16} \]
and then the \((i,j)\)-component of Einstein equation (7) becomes
\[ \frac{2V - 1}{V^2} (3) R_{ij} - \frac{1}{V} D_i D_j V = \frac{4}{V^2} D_i V D_j V \frac{1}{V^4} - \frac{2}{V^2} (DV)^2 - \frac{1}{V^2} (1) D_i D_j V^{-1}. \tag{17} \]
We would remind the reader that the trace of equation (17) gives
\[ (3) R = \frac{2}{V^2} (DV)^2. \tag{18} \]

\(^8\)In the current theory, the equations of motion give the vanishing of the four-dimensional Ricci scalar, \(R = 0\).
From equations (14) and (15), we see that the spacetime is singular at $S_p(V = 1/2)$ if the numerators do not vanish at $S_p$. Then, the regularity at $S_p$ implies

$$k_{ij}|_{S_p} = \frac{1}{\rho_p} h_{ij}|_{S_p}$$  \hspace{1cm} (19)$$

and

$$D_i \rho|_{S_p} = 0.$$  \hspace{1cm} (20)

The index 'p' indicates the estimation at $S_p$. At each connected component of $S_p$, $\rho_p$ can have a different constant value.

4. Uniqueness of BBMB spacetime

Now we are ready to discuss the uniqueness of the BBMB spacetime. Firstly, we consider the following two conformal transformations for $\Sigma$

$$\tilde{g}_{ij}^\pm = \Omega_{\pm}^2 g_{ij},$$  \hspace{1cm} (21)$$

where

$$\Omega_+ = V$$  \hspace{1cm} (22)$$

and

$$\Omega_- = (1 - V)^2 / V.$$  \hspace{1cm} (23)

Then, we have the two manifolds $(\tilde{\Sigma}^\pm, \tilde{g}^\pm)$.

Using equation (18), it is easy to see that the Ricci scalar for $\tilde{g}^\pm_{ij}$ vanishes as

$$\Omega_{\pm}^2 (3)^R = 2 \left[ \left( \frac{1}{V} - \frac{\Omega'_\pm}{\Omega_{\pm}} \right)^2 - 2 \frac{\Omega''_{\pm}}{\Omega_{\pm}} \right] (DV)^2 = 0,$$  \hspace{1cm} (24)$$

where the prime means the ordinary derivative with respect to $V$.

Let $\tilde{S}_p^\pm$ be the inner boundaries of $\tilde{\Sigma}^\pm$ corresponding to $S_p$ in $\Sigma$. Then, we see that the metric $\tilde{g}^\pm$ and the extrinsic curvature $\tilde{k}^\pm_{ij}$ of $\tilde{S}_p^\pm$ become

$$\tilde{g}_{ij}^\pm|_{\tilde{S}_p^\pm} = \tilde{g}_{ij}|_{\tilde{S}_p^\pm} = \frac{1}{4} g_{ij}|_{S_p}$$  \hspace{1cm} (25)$$

and

$$\tilde{k}^\pm_{ij}|_{\tilde{S}_p^\pm} = \pm \frac{1}{\rho_p} h_{ij}|_{S_p}.$$  \hspace{1cm} (26)$$

respectively. In the above, we used equations (19) and (20). Thus, we can glue $(\tilde{\Sigma}^\pm, \tilde{g}^\pm)$ at $\tilde{S}_p^\pm$ without a jump of the extrinsic curvature$^9$ and then we have $\tilde{\Sigma} = \tilde{\Sigma}^+ \cup \tilde{\Sigma}^-$ such that the Ricci scalar is zero even at $\tilde{S}_p^\pm$.

Next, we look at the asymptotic behaviors on $(\tilde{\Sigma}^\pm, \tilde{g}^\pm)$:

$$\tilde{g}_{ij}^\pm = \delta_{ij} + O(1/r^2)$$  \hspace{1cm} (27)$$

$^9$ There is a similar study on the photon sphere of the Schwarzschild spacetime [14] (see also [10]). However, the two manifolds constructed via conformal transformation could not be glued at the photon sphere. To glue them continuously, the additional part made from a part of the Schwarzschild spacetime was essential.
and
\[ \bar{g}_{ij}\,dx^i\,dx^j = \frac{m^4}{r^4} \left( 1 + O(1/r) \right) \left( dr^2 + r^2 d\Omega^2_2 \right) = \left( 1 + O(\bar{r}) \right) \left( d\bar{r}^2 + \bar{r}^2 d\Omega^2_2 \right), \]
(28)

where \( \bar{r} := \frac{m^2}{r} \). The spatial infinity in \( \Sigma \) corresponds to a point \( q \) in \( \bar{\Sigma}^- \), i.e. adding the point \( q \), we can have \( \bar{\Sigma} \cup \{ q \} \) with the zero Ricci scalar. Meanwhile, the ADM mass in \( \bar{\Sigma} \cup \{ q \} \) vanishes. Since the Ricci scalar of \( \bar{\Sigma} \cup \{ q \} \) is zero, the positive mass theorem [15] tells us that \( \bar{\Sigma} \cup \{ q \} \) is flat. Thus, \( (\Sigma, g) \) is conformally flat.

It is easy to see that \( V^{-1} \) is the harmonic function in the flat space \( (\bar{\Sigma}^+, \delta) \)

\[ \Delta_\delta V^{-1} = 0, \]
(29)

where \( \Delta_\delta \) is the flat Laplacian. Since, as seen in equation (26), \( \bar{\Sigma}_p \) is a totally umbilic surface in the flat space and \( V^{-1} \) satisfying the flat Laplace equation (29) is constant on \( \bar{\Sigma}_p \), we see that \( \bar{\Sigma}_p \) is a spherically symmetric surface. Then, because of equation (29) with the spherically symmetric boundary, all \( V = \) constant surfaces are spherically symmetric in \( \bar{\Sigma}_p \). Therefore, \( (\Sigma, g) \) is also spherically symmetric because the conformal factor \( \Omega \) depends only on \( V \). Now, there is no room for the possibility of multiple disconnected photon spheres. It is known that the spherically symmetric solution is uniquely the BBMB solution [16]. Moreover, \( S_p \) corresponds to the unstable circular orbit of photon at \( r = 2m \). Finally, we can conclude that \( \Sigma \) is isometric to the outside region of the photon sphere of the BBMB solution.

We summarize the above argument with the following theorem:

**Theorem 1.** Let us consider the static and asymptotically flat spacetimes in the Einstein-conformal scalar field system satisfying the asymptotic conditions (3) and (4). Then, the regular spacetime with the inner boundary \( S_p \), specified by \( \varphi := \sqrt{\kappa/6} \phi = 1 \) or \(-1 \) is uniquely the BBMB solution and \( S_p \) is connected.

5. Summary and discussion

In this paper, we proved that the outside region of \( S_p \) is uniquely the BBMB solution in the Einstein gravity with the conformal scalar field. The proof can be applied to cases where no horizon exists outside \( S_p \), and \( S_p \) is either \( S_{p+} \) or \( S_{p-} \). As a consequence, \( S_p \) is the surface composed of the unstable circular orbit of photon (photon sphere) in the BBMB solution. We also saw that no static multiple disconnected photon surfaces exist. We would stress that we did not assume the existence of the photon sphere and connectedness of object a priori.

We could not address the geometry of the inside region of the photon sphere. It is also interesting to consider the uniqueness for systems with electric charge and/or other asymptotic boundary conditions [17]. They remain a future issue.

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