Photon gas thermodynamics in dS and AdS momentum spaces

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Abstract. In this paper, we study thermostatistical properties of a photon gas in the framework of two deformed special relativity models defined by the cosmological coordinatizations of the de Sitter (dS) and anti-de Sitter (AdS) momentum spaces. The dS model is a doubly special relativity theory in which an ultraviolet length scale is invariant under the deformed Lorentz transformations. For the case of the AdS model, however, the Lorentz symmetry breaks at the high energy regime. We show that the existence of a maximal momentum in dS momentum space leads to maximal pressure and temperature at the thermodynamical level, while maximal internal energy and entropy arise for the case of the AdS momentum space due to the existence of a maximal kinematical energy. These results show that the thermodynamical duality of these models is very similar to their well-known kinematical duality.

Keywords: quantum fluids, quantum gases
1. Introduction

The first attempts in formulating quantum field theory revealed that the divergences are the integral part of the setup. In order to resolve this problem, Heisenberg suggested the noncommutativity between the spacetime coordinates as $[\hat{x}^\mu, \hat{x}^\nu] \sim \hbar$, which results in a natural ultraviolet (UV) cutoff, of the order of the Planck scale, for the system under consideration [1]. This suggestion immediately implies a fuzzy structure for spacetime at short distances (UV regime) such that the position of particles cannot be determined with zero uncertainty. The appearance of the Planck scale shows that this setup will emerge from the flat (non-gravitational) limit of a fundamental quantum theory of gravity. Therefore, it seems that incorporating gravity in quantum physics can naturally remove the divergences in quantum field theory on one side, and also may resolve the singularities in general relativity on the other side. The noncommutativity between the spacetime coordinates signals a nonzero curvature for the space of the corresponding conjugate variables that is the momentum space. This is the Born reciprocity conjecture which states that a quantum theory of gravity should be formulated on curved momentum space [2]. The first attempt in this direction was taken by Snyder in 1947 who has formulated a noncommutative Lorentz invariant spacetime [3]. The space of momenta then turns out to be curved with de Sitter (dS) geometry and, interestingly, it is shown that the quantum field theories are naturally UV-regularized in this setup [4]. Recently, the relation between the seminal work of Snyder and the noncommutativity of spacetime coordinate becomes clear in the context of doubly (deformed) special relativity (DSR) theories. Indeed, any quantum theory of gravity such as string theory and loop quantum gravity suggest the existence of a minimal observable length scale [5, 6]. It is therefore natural to expect that a non-gravitational theory which supports the existence of minimal length scale (as a natural UV cutoff for the system) would emerge at the flat limit of quantum gravity proposal [7]. In the absence of a full quantum theory of gravity one may proceed in reverse: one starts...
with special relativity and deforms it in such a way that it supports the existence of a minimal observer-independent length scale. This is the main idea of the DSR theories which was suggested by Amelino-Camelia [8]. It was then shown that there are many DSR models [9] and any model can be understood as a different coordinatization of dS [10] and also recently proposed AdS momentum spaces [11, 12]. The Snyder model was then realized as a DSR model determined by a particular basis of dS momentum space [13]. Evidently, the DSR models have very different behaviors at the UV regime. For instance, some of them predict dynamical dimensional reduction at the UV regime while others do not [14]. In this paper, we consider two deformed special relativity models defined by the same coordinatizations of dS and AdS momentum spaces from the thermostatistical point of view. The model defined on dS momentum space is indeed a DSR theory in the sense that a UV length scale is invariant under the associated deformed Lorentz transformations, while the Lorentz symmetry breaks at the UV regime in the AdS case. This consideration may open a new window to compare the deformed special relativity theories such as the DSR theories from a thermodynamical point of view.

2. DSR theories

In their modern formulation, DSR theories are defined on curved momentum spaces in the context of relative locality principle [15]. The observer-independence of the minimum length scale implies constant curvature for the corresponding momentum space and therefore the dS and AdS spaces are the appropriate candidates. From the global point of view dS and AdS spaces have $\mathbb{R} \times S^3$ and $S^1 \times \mathbb{R}^3$ topologies respectively. Therefore, a maximal momentum and a maximal energy will arise by the relevant identification of compact $S^3$ topology with the space of momenta and $S^1$ topology with the energy space in dS and AdS momentum spaces respectively [16]. The other identifications lead to the non-isotropic speed of light [12]. From the local point of view, different coordinatizations of these curved momentum spaces lead to the different deformed special relativity models. Depending on which local coordinatization is employed, the Lorentz symmetry may be preserved or broken at the UV regime. The DSR theories however are defined by those coordinatizations which preserve the Lorentz symmetry even in the UV regime. Among all possible coordinatizations of dS and AdS momentum spaces, the natural coordinate system on dS momentum space is inspired by the bi-cross product basis of $\kappa$-Poincaré algebra, which is known as the cosmological coordinate since it corresponds to the cosmological rendition of dS space in position space. In this model, the Lorentz transformations are deformed such that the Lorentz symmetry preserves even at the UV regime. Thus, this is a DSR theory. On the other hand, its counterpart on AdS momentum space, i.e. the model that is defined by the cosmological coordinatization of AdS momentum space, suggests the Lorentz violation at the UV regime and therefore it is not a DSR theory. Both of these models predict dynamical dimensional reduction from 4 to 3 at the UV regime. We restrict ourselves to these two models which are the same coordinatizations of the different dS and AdS momentum spaces in order to explore the thermodynamical properties of each.
2.1. dS momentum space

For the case of the free particle, with which we are interested in this paper, DSR theories on curved momentum spaces are completely defined by the metric of momentum space and also the mass-shell (modified dispersion relation) condition (see also [17] for a more general case). In the case of cosmological coordinization of dS momentum space, the metric is given by (see [12] for details)

\[ ds^2 = -dE^2 + \exp(2lE) \sum_{i=1}^{3} dp_i^2. \] (1)

The above line element gives the invariant integration measure\(^3\)

\[ \frac{d\mu(E, \vec{p})}{4\pi} = \exp(3lE) dE p^2 dp, \] (2)

on the momentum space. For the massless particles with which we are interested in this paper, the deformed mass-shell relation is given by

\[ C \left( 1 - \frac{l^2C}{4} \right) = 0, \] (3)

in which

\[ C = -\frac{4}{l^2} \sinh^2(lE/2) + p^2 \exp(lE). \] (4)

The mass-shell condition (3) provides two possibilities: \( C = 0 \) and \( 1 - \frac{p^2C}{4} = 0 \). The latter case leads to the dispersion relations \( E_{\pm} = -l^{-1}\ln(-1 \pm lp) \) which gives \( E_{\pm} \approx 2l^{-1} \pm p \) in the low energy limit \( lE \to 0 \). Therefore, these models do not respect the correspondence principle and we abandon them. For the first case with \( C = 0 \), the modified dispersion relations are given by

\[ E_{\pm} = \mp l^{-1}\ln(1 - lp). \] (5)

In the low energy limit \( lE \to 0 \), the above relation gives \( E_{\pm} \approx \pm p \) which shows that the constraint \( C = 0 \) is the relevant constraint for the massless particles in this setup. The appearance of the maximal momentum \( p \lesssim l^{-1} \) is another feature of this model, which is the consequence of compact \( S^3 \) topology of the space of momenta [16]. Moreover, equation (5) implies \( E \in (-\infty, 0] \) if we deal with \( E_- \) and \( E \in [0, \infty) \) for the case of \( E_+ \), which shows that the solution \( E_+ \) is positive definite and thus is physically relevant.

In the flat low energy limit \( lE \propto E/E_{Pl} \ll 1 \), the line element (1) reduces to the flat case \( ds^2 \approx -dE^2 + \sum_{i=1}^{3} dp_i^2 \), the invariant measure (2) reduces to the standard well-known measure \( d\mu(E, \vec{p}) = 4\pi dE p^2 dp \), and the deformed mass-shell condition (4) leads to the standard Einsteinian dispersion relation \( C = -E^2 + p^2 \) with \( E, p \in [0, \infty) \).

\(^3\) The numerical factor \( 4\pi \) is considered to recover the standard thermodynamical results at the low energy regime.
2.2. AdS momentum space

In terms of physical energy and momenta \((E, p)\), the line element of AdS momentum space in the cosmological coordinate is given by

\[
ds^2 = -dE^2 + \cos^2(lE)\left(\frac{dp^2}{1 + l^2p^2} + p^2d\Omega^2\right),
\]

and the invariant measure on the momentum space will be

\[
\frac{d\mu(E, \vec{p})}{4\pi} = \cos^3(lE)dE\frac{p^2dp}{\sqrt{1 + l^2p^2}}.
\]

For the massless case, the associated mass-shell condition reads

\[
C = -\frac{1}{l^2} \sin^2(lE) + p^2\cos^2(lE) = 0,
\]

by solving of which one can easily find the modified dispersion relations

\[
E_{\pm} = \pm l^{-1}\tan^{-1}(lp).
\]

At the low energy regime \(lE \to 0\), we have \(E_{\pm} \approx \pm p\) which shows that the setup respects the correspondence principle. Also, \(E \in (-\infty, 0]\) for the case of \(E_-\) and \(E \in [0, \infty)\) for \(E_+\). We therefore again consider \(E_+\) to be the appropriate solution for the mass-shell condition. From the above relation it is clear that there exists a maximal energy \(E \leq (\pi/2)l^{-1}\) in this setup. The relations (6)–(8) reduce to their standard counterparts in the flat low energy limit \(lE \propto E/E_{Pl} \ll 1\).

3. Statistical mechanics

In this section we are going to consider the statistical mechanics of a photon gas in the framework of two models that were presented in the previous section. In [18], the statistical mechanics of such theories with different coordinizations of dS and AdS momentum spaces was generally formulated. Here, we briefly review the main results and then use the setup to find the canonical partition function for the photon gas.

In standard statistical mechanics one is dealing with an invariant measure on a six-dimensional phase space \(\Gamma = \Gamma(\vec{x}; \vec{p})\) corresponding to a nonrelativistic particle. This measure determines the number of microstates for the system by means of which one may study the statistical mechanics in any ensemble. The deformed special relativity theories such as the DSR theories are, however, formulated on an eight-dimensional extended phase space \(\Gamma = \Gamma(t, \vec{x}; E, p)\) for a particle, while the number of physically distinct microstates is determined by the measure on nonrelativistic phase space. Therefore, one should impose the mass-shell condition and also the gauge transformation generated by it (time evolution of the system) to obtain the invariant measure on the space of physically distinct microstates. This can be easily deduced by using the disintegration theorem which leads to the following invariant measure (see [18] for details)
where $d\mu(t, \vec{x}) = dtd^3x$ is the standard Lebesgue measure on the spacetime sector and $d\mu(E, \vec{p}) = \sqrt{-g}dEd^3p$ is the invariant measure on the momentum sector of the extended phase space $\Gamma = \Gamma(t, \vec{x}; E, \vec{p})$ with $g = g(E, \vec{p})$ being the determinant of the metric of the momentum space. Although the measure (10) restrict the eight-dimensional extended phase space $\Gamma$ to a six-dimensional nonrelativistic phase space, it is indeed not uniquely defined. For instance, one could also consider $\delta(C^2)$ instead of $\delta(C)$ which is again consistent but leads to the different statistical results! At first glance, one could see that substituting even the standard Einsteinian dispersion relation $C = -E^2 + p^2$ in (10), the delta function decomposes to two separate branches and one of them is indeed corresponding to the negative energies which are irrelevant. More generally, $\delta(C)$ in (10) decomposes as $\delta(C) = \frac{\delta(E - E_+)}{\left| \frac{dC}{dE} \right|_{E=E_+}} + \frac{\delta(E - E_-)}{\left| \frac{dC}{dE} \right|_{E=E_-}}$. Note that only the positive definite energies determined by the solution $E_+$ are physically relevant. In order to restrict ourselves to the positive energies, we thus consider the step function $\theta(E)$. The other important issue is the correspondence principle according to which we should recover the well-known results of standard statistical mechanics at the low energy (or temperature) regime. In order to do so, we should replace $\delta(C)$ with $\left| \frac{dC}{dE} \right|_{C=0} \delta(C)$. The correct measure which only includes the positive energies and respects the correspondence principle, then uniquely determined and is given by

$$\mu_p = \int d\mu_p = \int \theta(E) \left| \frac{dC}{dE} \right|_{C=0} \delta(C)\delta(t - t_0)d\mu(t, \vec{x})d\mu(E, \vec{p}).$$

(11)

Having the measure (11) at hand, we are adequately equipped to study the statistical mechanics for deformed special relativity theories in any ensemble. In canonical ensemble, the system is supposed to be in a thermal bath with temperature $T$ and the ensemble density is given by the Boltzmann factor. The associated single-particle partition function is then given by

$$Z_1 = \frac{1}{\hbar^3} \int d\mu_p \exp(-E/T),$$

(12)

where $d\mu_p$ is defined in (11). The total partition function $Z_N$ of a $N$-particles system can be written as $Z_N = Z_N^N/N!$, when particles are assumed to be kinematically and dynamically decoupled. All the thermodynamical quantities can then be derived from $Z_N$ by the standard definitions.

Before obtaining the deformed partition functions for the photon gas in deformed special relativity models with dS and AdS momentum spaces, it is useful to obtain the

$^4$ Note that the spacetime sector does not have the standard Minkowski metric, but it is indeed the noncommutative $\kappa$-Minkowski spacetime ($\kappa \sim l^{-1}$ in our notation) [19]. Defining an appropriate measure which respects all the desired symmetries on this noncommutative spacetime is not an easy task (see for instance [20] where it is shown that the standard Lebesgue measure leads to the noncyclic action). The standard Lebesgue measure however respects the $\kappa$-Poincaré symmetries and also the correspondence principle which is necessary for our aim in this paper. Therefore, we work with the Lebesgue measure on the spacetime sector of the extended phase space $\Gamma$. 

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well-known partition function of the photon gas in standard special relativity within the constructed setup. In the framework of standard special relativity, the measures are the standard Lebesgue measures \( d\mu_E = dt d^3x \) and \( d\mu_{\bar{E}} = dE d^3p \), and the mass-shell condition for the massless particles is given by the standard Einsteinian dispersion relation \(-E^2 + p^2 = 0\) which has the solutions \( E_\pm = \pm p \). Substituting in relation (12), the partition function for the photon gas will be

\[
Z_{1}^{\text{SR}} = \frac{1}{h^3} \int dt dE d^3p \exp(-E/T) = \frac{1}{h^3} \int dE d^3p \theta(E) \left[ \frac{dC}{dE} \bigg|_{c=0} \delta(C) \delta(t - t_0) \exp(-E/T) \right]
\]

\[
= \frac{4\pi V}{h^3} \int dE d^3p \exp(-E/T) \left[ \frac{\delta(E - p)}{2E|_{E=p}} + \frac{\delta(E + p)}{2E|_{E=-p}} \right] \exp(-E/T)
\]

\[
= \frac{4\pi V}{h^3} \int dE d^3p \exp(-E/T) = \frac{4\pi V}{h^3} \int_0^\infty E^2 dE \exp(-E/T) = \frac{8\pi V T^3}{h^3},
\]

which is nothing other than the result in the standard statistical mechanics which leads to the usual thermodynamics of a photon gas. The nontrivial effects may occur when the measure \( d\mu_X \) or/and the constraint \( C \) is deformed. In theories with minimal length that formulated on the reduced (non-relativistic) phase space, such as the generalized uncertainty principle [21], noncommutative reduced phase spaces [22] and polymerized phase spaces [23], always one of the phase space measure or the dispersion relation is modified (it depends on what one prefers to work, in canonical (Darboux) or noncanonical charts on the reduced phase space [16, 24, 25]). In deformed special relativity theories such as the DSR models, depending on the coordinate that one implements, both the measure and the dispersion relation can be simultaneously modified [26, 27].

Using the deformed Hamiltonian constraint (4) and substituting the invariant measure (2) into the definition (12), the canonical partition function for the photon gas in cosmological coordinatization of dS momentum space takes the form

\[
Z_{1}^{\text{DSR} - \text{dS}} = \frac{4\pi V}{l^2 h^3} \int dE d^3p \exp(3lE)p^2 d\theta(E) \left[ \frac{dC}{dE} \bigg|_{c=0} \delta(C) \exp(-E/T) \right]
\]

\[
= \frac{16\pi V}{l^2 h^3} \int_0^\infty dE \sinh^2(lE/2) \exp\left(-\frac{(1 - 2lT)E}{T}\right)
\]

\[
= \frac{8\pi V T^3}{h^3} \frac{1}{(1 - 6lT + 11l^2 T^2 - 6l^3 T^3)^{-1}},
\]

where we have substituted \( C \) from (4). The integral in the above relation is evaluated over the allowed domain \( T^{-1} - 2l > 0 \) which shows that there is a maximal temperature

\[
T < T_{\text{max}} = \frac{1}{2l} \propto T_{\text{Pl}},
\]

for the photon gas in this setup. This result (with a different approach) is also obtained in [26] where the first time the statistical mechanics for DSR theories was studied. In the same manner, using the constraint (8) and substituting the associated invariant
measure (7) into the definition (12), the canonical partition function for the photon gas in cosmological coordinatization of AdS momentum space turns out to be

$$Z_{1}^{DSR-AdS} = \frac{4\pi}{h^3} \int \int dt d^3x \delta(t - t_0) \int \int dE \frac{\cos^2(\theta(E))}{\sqrt{1 + \tilde{p}^2}} p^2 dp \times \theta(E) \left| \frac{dC}{dE} \right| \delta(C) \exp(-E/T)$$

$$= \frac{\pi V}{l^2 h^3} \int_0^\pi dE \sin^2(2lE) \exp(-E/T) = \frac{8\pi}{h^3} \frac{VT^3}{1 + 16lT^2}$$

(16)

in which we have used the modified dispersion relation (8). The deformed partition functions (14) and (16) can be rewritten in a compact form as

$$Z_1^{DSR}(l; V, T) = Z_1^{SR}(V, T) f(lT),$$

(17)

where $Z_1^{SR}(V, T) = \frac{8\pi VT^3}{h^3}$ is the partition function of the photon gas in the standard special relativity (see (13)) and

$$f(lT) = \begin{cases} (1 - 6lT + 11l^2T^2 - 6l^3T^3)^{-1}, & \text{dS} \\ (1 - \exp(-\pi/2lT))(1 + 16l^2T^2)^{-1}, & \text{AdS} \end{cases}$$

(18)

It is seen that all quantum gravity effects are summarized in the function (18). In the low temperature limit $lT \propto T/T_H \ll 1$, where these effects are negligible, the function (18) tends to unity, $\lim_{lT \to 0} f(lT) = 1$, and relation (17) gives $Z_1^{DSR}(l; V, T) = Z_1^{SR}(V, T)$, which shows that the result of standard special relativity is recovered in this limit.

In the absence of quantum correlations, the total partition function for the $N$-particles system in Maxwell–Boltzmann statistics will be

$$Z_N^{DSR} = \frac{1}{N!} (Z_1^{SR})^N f^N = Z_N^{SR} f^N,$$

(19)

where $Z_N^{SR} = (Z_1^{SR})^N/N!$ is the total partition function in the standard special relativity.

4. Thermodynamics

From the total partition function (19), one can derive all the thermodynamical quantities. The internal energy $U = T^2 \frac{\partial \ln Z_N^{DSR}}{\partial T}$ works out to be

$$U^{DSR} = U^{SR} + NT^2 \langle \ln f \rangle',$$

(20)

where a prime denotes derivative with respect to the temperature. The first term in the right hand side, $U^{SR} = 3NT$, is nothing but the internal energy of the photon gas in standard special relativity and the second term comes from the quantum gravity corrections. The specific heat $C_v = \left( \frac{\partial U}{\partial T} \right)_V$ in this setup is obtained as

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where again $C_v^{SR} = 3N$ is the well-known specific heat in the standard special relativity and the two other terms in the right hand side of the above relation show the quantum gravity effects. In figures 1 and 2 we have plotted the internal energy and the specific heat versus temperature for both the dS and AdS momentum spaces. A comparison with their standard counterparts in special relativity is also shown in these figures. The quantum gravity effects will only become important at the high temperature regime. However, one can obtain a bound on the quantum gravity scale $l$ or estimate the magnitude of these corrections at the accessible temperatures \[28\]. The quantum gravity corrections to the heat capacity of the photon gas in this setup are

$$\frac{\Delta C_v}{C_v^{SR}} = \frac{C_v^{DSR} - C_v^{SR}}{C_v^{SR}} = \frac{1}{3} (T^2(\ln f)' + N T^2(\ln f)'^2),$$

where

$$C_v^{DSR} = C_v^{SR} + 2NT(\ln f)' + NT^2(\ln f)'^2,$$

which are too small to be detected by the accessible energy scales.

Using the standard definition $P = T \frac{\partial \ln Z_v}{\partial V}$, the equation of state can be obtained as

$$PV = NT,$$

which shows that the form of equation of state preserves in deformed special relativity framework. In Maxwell–Boltzmann statistics, the form of equation of state in all phenomenological approaches to the minimal length is also preserved and it seems that this is a general feature (see for instance \[21–24\]). An interesting result in this setup is that there is a maximal pressure for the photon gas for the case of dS momentum...
The existence of a maximal temperature given by the relation (15) immediately implies a maximal pressure as

\[ P_{\text{max}} = \left( \frac{N}{V} \right) T_{\text{max}}, \]

for the photon gas (note that while \( N \) and \( V \) are extensive quantities, \( P \) and \( T \) are intensive). As far as we know, among the other phenomenological approaches to the issue of minimal length scale, the existence of a maximal temperature and consequently an upper bound (24) on the pressure are only suggested by the deformed special relativity models which are defined on the dS momentum space.

The entropy can also be obtained from its standard definition

\[ S_{\text{SR}} = \frac{U}{T} + \ln Z_N \]

which gives

\[ S_{\text{DSR}} = S_{\text{SR}} + N \ln f + NT(\ln f)', \]

in which \( S_{\text{SR}} \) denotes the standard entropy of the photon gas in special relativity and the other terms are modifications due to the quantum effects of gravity. In figure 3 we have plotted the entropy versus temperature. As this figure shows, the entropy, in standard special relativity, increases with decreasing rate when the temperature increases. However, it increases with increasing rate in the case of DSR with dS momentum space and finally it diverges when the temperature approaches the maximal temperature (15). For the case of the AdS momentum space, it increases with decreasing rate lower than the standard one at the high temperature regime and finally it approaches a maximal value when the temperature goes to infinity.

**Figure 2.** The figure shows the variation of the specific heat in terms of temperature. While this is a temperature-independent quantity in special relativity, it significantly changes with temperature at the high temperature regime in deformed special relativity framework. For the dS momentum space, it diverges at the maximal temperature (15) and it tends to zero when the temperature goes to infinity in the case of AdS momentum space.
All the thermodynamical features of the photon gas in the two deformed special relativity models with dS and AdS momentum spaces are summarized in table 1 and they are compared with each other and also with the standard special relativity results.

5. Summary and conclusions

The DSR theories are the most well-known candidates for the flat limit of the quantum gravity proposal. There is an observer-independent length scale (preferably of the order of the Planck length) in these setups which leads to a natural UV cutoff for the system under consideration. These theories are generally formulated on curved dS and AdS momentum spaces with $\mathbb{R} \times S^3$ and $S^1 \times \mathbb{R}^3$ topologies, respectively. The various DSR theories can then be realized from the different coordinatizations of these curved momentum spaces. At the kinematical level, a maximal momentum and maximal energy arise in dS and AdS momentum spaces, respectively, through the relevant identification of compact $S^3$ topology with the space of momenta in dS space and compact $S^1$ topology with the energy space in AdS case. In this respect, these two spaces are kinematically dual to each other. In this paper, we have studied the thermodynamical properties of a photon gas in the framework of two different deformed special relativity models defined by cosmological coordinatizations of dS and AdS momentum spaces. The model defined on the dS momentum space is a DSR theory in the sense that it preserves the Lorentz symmetry even at the UV regime. The AdS model is however a deformed special
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**Table 1.** Thermodynamical results of photon gas in the deformed special relativity models defined by cosmological coordinatization on dS and AdS momentum spaces are presented and they are compared with each other and also with the standard results of the special relativity.

| Topology of momentum space | Maximal momentum | Maximal energy | Lorentz invariance | Maximal internal energy | Maximal entropy | Maximal pressure | Maximal temperature |
|----------------------------|------------------|----------------|--------------------|-------------------------|----------------|-----------------|-------------------|
| SR $\mathbb{R}^4$          | No               | No             | Yes                | No                      | No             | No              | No                |
| dS Model $\mathbb{R} \times S^3$ | Yes               | No             | Yes                | No                      | No             | Yes             | Yes               |
| AdS Model $S^1 \times \mathbb{R}^3$ | No               | Yes            | No                 | Yes                     | Yes            | No              | No                |

relativity model which supports the existence of a UV length scale but cannot preserve the Lorentz symmetry at the UV regime. The results show that the thermodynamical properties of the photon gas are significantly modified at the high temperature regime such that all thermodynamical quantities enhance in the case of dS momentum space and saturate for the AdS case. We found that the existence of maximal momentum in dS momentum space leads to the maximal pressure and maximal temperature at the thermodynamical level while maximal internal energy and maximal entropy emerged in the AdS momentum space due to the existence of maximal kinematical energy. In this respect these spaces are thermodynamically dual to each other, very similar to their well-known kinematical duality. All of these kinematical and thermodynamical UV cutoffs are originated from the compact topologies $S^3$ and $S^1$ associated to the dS and AdS momentum spaces. Therefore, although, we have considered a particular case of photon gas in canonical ensemble, it seems that these results are a common feature of dS and AdS momentum spaces. Moreover, there are many deformed special relativity models defined by different coordinatizations of dS and AdS momentum spaces and there is no clear reason to prefer one from the other. Our consideration, however, opens a new window to compare these models from a thermodynamical point of view.

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