Soret and Dufour features in peristaltic motion of chemically reactive fluid in a tapered asymmetric channel in the presence of Hall current

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Abstract
The present work examines heat and mass transfer characteristics of peristaltic motion of Johnson-Segalman fluid in a tapered asymmetric channel along with chemical reaction, by taking into account the Soret and Dufour effects. Effects of Hall current have also been discussed in mathematical modeling and analysis. Following the peristaltic wave procedure, the tapered asymmetric channel is based on the non uniform boundaries having diverse phases and amplitudes. The channel walls show excellent agreement with more realistic convective conditions. The modeled flow problem is directed into ordinary differential equations set with proper utilization of similarity quantities. The estimation of high wavelength as well as small Reynolds number are acknowledged to deduce the equations of Johnson-Segalman liquid model. The adopted solution procedure is constructed via homotopic algorithm. The results have been analyzed for various parameters of interest and sketched for better understanding. The velocity profile reveals decreasing behavior for increasing values of Weissenberg number and Hartman number while converse behavior is found for mean flow rate and Hall parameter. The temperature profile falloffs for heat transfer Biot number and Hartman number whereas it increases for Prandtl number, Brinkman number, Dufour number and Hall parameter. The concentration profile tends to decrease for mass transfer Biot number and increase for Schmidt constant.

Nomenclature

| Symbol | Description |
|--------|-------------|
| $\lambda$ | wavelength |
| $(a_1, a_2)$ | wave amplitudes |
| $(p)$ | pressure |
| $I$ | identity tensor |
| $e$ | slip parameter |
| $d/dt'$ | material time derivative |
| $T_m$ | mean temperature |
| $C$ | concentration |
| $D_1$ | mass diffusivity coefficient |
| $\sigma_0$ | electrical conductivity |

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1. Introduction

The idea of peristalsis is deduced from a Greek word ‘περιστάλημα’ which represents the phenomenon of condensing as well as embracing. The novel implementation of this phenomenon is to evaluate the revolutionary wave contraction associated with a non-uniform cross sectional area of tube or channel. In physiology, it is used with the aid of the physique to drive or blend the tube contents as in gastro-intestinal tract, ureter, bile and different glandular ducts. The mechanism of peristaltic movement is being abused for industrial purposes including transport of sanitary and corrosive fluids where the contact of the fluid with the apparatus components is denied. The mechanism of peristalsis contains wave like series of muscle contractions and relaxations that causes the motion of bio-fluid in diverse processes. Peristalsis is involved in various phenomenon’s such as movement of urine from kidney to gallbladder, transport of chime in small intestine, food swallowing through esophagus, bile duct applications, vasomotion of small blood vessels, spermatozoa transport, locomotion of warms, pumping blood in dialysis etc. In medical and industrial systems, peristalsis has its numerous bio-medical applications like kidney to the intestine, tube pumps, dialysis heart lung equipments and open-heart surgery etc. Latham’s [1] first attempt was conducted to figure out the peristaltic action of different fluids under various conditions along with various assumptions of long wave length, Reynolds number, material parameters of the fluid and wave amplitude etc. with reference to mechanical and physical situations. Later on Shapiro et al [2] used the concept of low Reynolds and long wavelength assumptions to explore the peristaltic pumping configured by an asymmetric channel by using linear fluid model. Now abundant information is present on peristalsis in the existing literature. The magnetohydrodynamic flow of blood under long wave-length estimation is made by Agrawal et al [3]. Mekheimer [4] worked on the utilization of magnetic field impact for mechanism of nonlinear peristaltic in inclined channel. In literature, various of theoretical
investigations are found in which the blood and other biological fluids have been assumed to perform alike Newtonian fluid.

There is abundant literature, comprising experimental, analytical and numerical measurements on peristaltic flow in various geometries. In prior studies much attention was given to Newtonian fluids’ peristaltic movement but now non-Newtonian motion is dragging the consideration. The peristaltic aspects regarding the involvement of nonlinear fluid models also trashed out the scientists attention as it encountered priceless human body, medical and physical significance. The above argued phenomenon becomes quite interesting when the complex effects of viscoelasticity introduced. The viscoelastic materials encountered the classical viscous as well as elastic features referred to the non-Newtonian fluids. Many biological fluids like blood, tissues cells, human tissue and structural proteins are classified to this category. Shear stress and shear rate’s relation in non-Newtonian motion is found to be nonlinear character wise. Following to the literature survey, it is noticed that some investigations regarding the peristaltic aspects of non-Newtonian fluids are available but in limited way. Due to diverse prospective of non-Newtonian liquids, different flow model models are imposed having complex rheological properties. Among these non-Newtonian fluids, Johnson–Segalman and Giesekus nonlinear fluid models are those which possessed a non-monotone shear stress and deformation behavior. Our present continuation also divulge the diverse Johnson–Segalman liquid characteristics. It is named after two mathematicians Johnson and Segalman [5], who derived its constitutive equations symmetrically with the help of molecular theory of Gaussian networks as well as molecular bead spring model with Hookean springs. It falls in the list of viscoelastic fluids. This model can also describe the ‘spurt’ procedure. The expression ‘spurt’ is helpful in describing enormous rise in volume to a slight rise in driving pressure gradient. Several investigators tracked the fluid model to evaluate and explore the spurt phenomenon. The study of McLeish and Ball [6] explore the spurt features in the polymer melts. The analysis regarding shear aspects in non–Newtonian liquids has been thrashed out by Malkus et al [7]. Hayat et al [8], investigated the effects of MHD on peristaltic movement of Johnson–Segalman fluid in a channel, the walls of which are compliant. Since then, in various geometries with specific assumptions such as lesser wave number, slight amplitude ratio, low Reynolds number and long wavelength. On this end, a considerable investigations have been carried out to examine the peristalsis. Yet, it is obviously seen that in such investigations greater consideration is paid to the peristaltic motion of fluids in symmetric channels and tubes. However, these analyses are observed to be narrowed down when non-Newtonian fluids incorporated. The fundamental interest towards the utilization non-Newtonian fluids are justified as most of biological and physiological liquids encountered the viscoelastic properties. The peristaltic phenomenon induced by an asymmetric channel was originated by Eytan and Elad [9]. The main motivation to perform such analysis is based on its prestigious applications intra uterine flow in a non-pregnant uterus. Afterwards, peristaltic transport in an asymmetric channel became the source of attraction for researchers and some attempts have been made in this regard. Sobh et al [10], studied the effects of heat transfer in peristaltic flow of viscoelastic fluid in an asymmetric channel. Mekheimer et al [11], investigated the effects of heat transfer and space porosity on peristaltic flow in a vertical asymmetric channel. Hayat et al [12], investigated the slip effects on peristaltic flow of Maxwell fluid with heat and mass transfer.

Heat and mass transfer phenomena have numerous of applications in all branches of science and technology and can be seen obviously everywhere in nature. This simultaneous transportation phenomenon signify diverse applications in engineering, environmental sciences, industrial applications, oceanography, food industries, bio–logical significances, manufacturing processes, aerodynamics, thermal engineering applications, cooling systems, energy transport systems etc. The joint features of heat/mass transportation are always been the center of interest for scientists and engineers for their varied practical importance in various fields. In order to report the complete and deep mechanism of heat transportation, a deep understanding is quite essential to design the economical heat storage devices, combustion engines and boilers. The role of heat transportation in nuclear reactors cannot be denied for which heat is generated instantly in fluid after energy is released. In field of Aeronautics, the heat transfer process particulate a primary contribution for the durability and apposite performance of space vehicles to evade structural letdown and for safety causes. Moreover, some specified applications of heat transfer include cooling systems, heat processes, chemical industries, technological applications. The study of heat transfer helps in food processing and to develop a better variety of seeds. Ahmed [13] examined the heat transfer consequences in flow of Al2O3–Cu hybrid nanoparticles induced by a porous wavy surface. In another investigation Ahmed [14] inspected the enhancement of heat transfer in natural convection flow of nanofluid within trapezoidal enclosures. Rashad et al [15] analyzed the convective heat transfer aspects in square enclosure filled with porous space. The applications of heat transfer in flow of hybrid nanoparticles configured by convergent/divergent cavities in presence of external heat source were analyzed by Ahmed [16]. In another useful contribution, Ahmed et al [17] focused on heat transfer analysis with utilization of nanofluid subject to inclined magnetic field. Hayat et al [18] investigated the heat transfer phenomenon in flow of copper and silver nanoparticles. Qayyum et al [19] presented entropy generation features in flow of
Williamson nanofluid between rotating disks. The swirling flow in presence of entropy generation, Soret and Dufour consequences was analyzed by Qayyum et al. [20].

The peristaltic flows with heat transfer are of great importance in the processes like hemodialysis and oxygenation. Such motivation dragged the attention of many researchers to examine the interaction of heat transfer with peristalsis. Radhakrishnamacharya and Srinivasulu [21], made their efforts to see the influence of wall properties on peristaltic flow with heat transfer. Vajravelu et al. [22], studied peristaltic flow with heat transfer in a vertical annulus with long wave approximation. The assessment of wall features and heat transport in peristaltic investigation having different wall features was worked out by Kothandapani and Srinivas [23]. Mekheimer and AbdElmaboud [24], studied the influence of heat transfer and magnetic field on peristaltic transport of Newtonian fluid in a vertical annulus. Srinivas and Kothandapani [25], involved heat features on peristaltic prospective in asymmetric tube. Nadeem and Akbar [26], made findings on the influence of heat transfer on peristaltic flow of Johnson–Segalman fluid in a non-uniform tube. Hayat et al. [27], examined the simultaneous effects of slip and heat transfer on the peristaltic flow. A relatively intricate relationship occurs between fluxes and driving potentials during heat/mass process. The energy flux is appeared because of join utilization of heat and mass gradients. The existing literature comprises few other attempts on peristaltic flow with heat and mass transfer.

The utilization of chemical reaction in combined heat/mass transportation systems is another diligent feature which specified valuable applications in the manufacturing and chemical technologies. The chemical reaction reports many physical applications in the petroleum recovery, nuclear reaction cooling, fission and fusion processes, thermal insulation, drying and desert cooler applications. In various practical diffusive operations, the molecular diffusive species with associated chemical reactions play vital role [28–33].

It is often observed that during the process of joint heat and mass transportation, the temperature gradient is altered not only from temperature flux but also due to concentration flux. Similarly, the concentration gradient is affected due to impact of concentration and temperature fluxes. The energy flux has been induced due to composition gradient termed as Dufour effects. Beside this, the temperature gradient also deduced mass flux namely Soret features. It is emphasized that these thermo-diffusion features are of lesser order magnitude as compared to the features prearranged is case of Fourier’s or Fick’s theories and are consistently disregard to perform combined heat/mass analysis. The Soret features can be efficient to distinguish the isotopes separation and in gases mixture when light molecular weight (H₂, He) becomes extremely high. The Dufour effects play a significant role in medium molecular weight (N₂, air) havinga desirable magnitude. It is noticed that these thermo-diffusion features are not carried out properly in most of recent investigations. In area of thermal field flow fractioning, the different size of molecules is split up from solvent by using these thermo features. This useful application can be seen in the biological samples like DNA, cells and proteins for which higher temperature difference can disintegrate these samples. Many physical applications of these thermo-diffusion characteristics include the Haber process which involves the nitrogen binding from air to prepare the ammonia. Similarly, the process of disinfection in involves the killing of bacteria and other viruses are also leads to these applications. Hayat et al. [34] intended the Soret and Dufour attribute for convectively heated curved channel in peristaltic flow of Jeffrey liquid.

There are different analytical methods like differential transformation method (DTM), homotopy perturbation method (HPM), adomian decomposition method (ADM), homotopy analysis method (HAM) for solving the physical and engineering problems. The utmost competent method in solving different type of nonlinear equations such as homogeneous, non-homogeneous and coupled systems is homotopic procedure. The disadvantage of many other analytic methods is that they have some limitations for solving nonlinear equations. Unlike other analytical methods, HAM is independent of any small or large parameter. The proper utilization of initial guesses and auxiliary parameters improve the convergence procedure in contrast to other analytical techniques. Primarily, it is developed by Liao [35]. He further modified with a non-zero auxiliary parameter which is also acknowledge as convergence control parameter $h$. It is a non-physical variable which shows a convenient way to ensure the convergence of approximation series. This method is being used by many researchers and observed to be very operative in deriving an analytic solution particularly for non-linear differential equations [36–43].

Above all, the major attention of this study is to explore the impacts of heat and mass transfer with chemical reaction on peristaltic flow of Johnson–Segalman fluid in a tapered asymmetric channel. Additionally, Soret and Dufour effects are considered here along with Hall current. Although some studies regarding flow of non-Newtonian fluids in tapered asymmetric channel are already reported in the literature. However, investigation regarding heat and mass transfer characteristics in presence of Soret and Dufour effects has not been investigated yet. The Soret and Dufour features involve diverse applications in chemical engineering and geo-sciences. Moreover, the analysis is performed by using convective boundary conditions. The results are obtained and analyzed by using homotopy analysis method under the assumptions of long wavelength and low Reynolds number.
2. Mathematical formulation

The motion of an incompressible Johnson-Segalman fluid in two dimensional infinite tapered asymmetric channel is intended in this researcher. The constituted coordinate system for current analysis has been imposed such that \( X \) is taken in parallel direction while \( Y \) is imposed normally. The walls of channel satisfy the convective conditions of heat and mass transfer. Moreover, along the channel walls an infinite peristaltic wave train travelling with velocity \( c \) has been considered which generates the flow. It is assumed a peristaltic wave in a channel walls in order to induce asymmetry in the channel which results in different amplitudes and phase as shown in figure 1. Soret and Dufour effects have also been analyzed in mathematical modeling and analysis. The Hall current impact are also incorporated in current investigation \[26, 34\]:

\[
\begin{align*}
H_2(X, t') &= d + k'X + a_1 \sin \left( \frac{2\pi X}{\lambda} - \frac{ct'}{\lambda} \right), \\
H_1(X, t') &= -d - k'X - a_1 \sin \left( \frac{2\pi X}{\lambda} - \frac{ct'}{\lambda} + \varphi \right).
\end{align*}
\]

Equation (1) reports the upper wave in channel flow while mathematical expression given in equation (2) are associated with lower waves of the tapered configuration.

In above expressions, \( \lambda, k', (a_1, a_2) \) and \( \varphi \in [0, \pi] \) stands for wavelength, non-uniform parameter, wave amplitudes and phase difference, respectively. It is remarked that channel waves in phase and out of phase are generated for \( \varphi = 0 \) and \( \varphi = \pi \), respectively. Furthermore the following inequality is satisfied by \( a_1, a_2, d_0, d_1 \) and \( \varphi \) at the outlet of convergent or inlet of divergent channel, otherwise collision is set between both the walls.

\[
a_1^2 + a_2^2 + 2a_1a_2 \cos \varphi \leq (2d)^2.
\]

The equations representing the present flow analysis are

\[
\text{div} V = 0, \quad \text{div} T = \rho \frac{dV}{dt}.
\]

For Johnson-Segalman fluid, the Cauchy stress tensor is given as \[26\]

\[
T = -\rho I + \sigma, \quad \sigma = 2\mu D + S,
\]

\[
S + m \left[ \frac{dS}{dt} + S(W - eD) + (W - eD)^T S \right] = 2\eta D,
\]

\[
D = \frac{1}{2} [L + L^T], \quad W = \frac{1}{2} [L - L^T], \quad L = \nabla V.
\]

Physical quantities in above expressions are pressure \( (p) \), relaxation time \( (m) \), identity tensor \( (I) \), slip parameter \( (e) \), dynamic viscosities \( (\mu, \eta) \), symmetric and skew symmetric parts of the velocity gradient \( (D, W) \).
It is obvious that flow model in equation (1) returns to the Maxwell fluid for \( c = 1 \) and \( \mu = 0 \), we get the classical Navier-Stokes fluid model.

The flow equations for heat transfer, mass transfer along with first order chemical reaction and Soret and Dufour features and Johnson-Segalman fluid’s extra stress tensor \( S \) are given as \([34]\)

\[
\frac{\rho c_p}{d t'} d T = k^* \nabla^2 T + S(\text{grad} U) + \frac{D_J}{c_e} (\nabla^2 C),
\]

\[
\frac{dC}{dt} = D_1 \nabla^2 C + \frac{D_J}{T_m} \nabla^2 T - k_i (C - C_0).
\]

where \( d/dt' \) represents the material time derivative, \( k^* \) the thermal conductivity, \( T \) is temperature, \( S \) the extra stress tensor, \( C \) is concentration, \( c_e \) reflects the concentration susceptibility, \( D_1 \) mass diffusivity coefficient, \( T_m \) is for mean temperature, \( K_r \) thermal diffusion rate and \( k_i \) the chemical reaction parameter.

We consider the uniform magnetic field of the form

\[
B_0 = (0, 0, B_0).
\]

From the application of generalized Ohm’s law, we have

\[
J = \sigma_0 \left[ E + V \times B_0 - \frac{1}{\varepsilon_0 \mu} (J \times B_0) \right].
\]

where \( \sigma_0 \), \( V \), \( \varepsilon_0 \), \( \mu \) and \( J \) are respectively symbolized the electrical conductivity, velocity vector, number density of electrons, electric charge and current density. Since the electric field features are neglected so \( E = 0 \).

For the present flow analysis, the velocity is same as defined in previous chapter. i.e.

\[
V = [U(X, Y, t'), \ V(X, Y, t'), 0].
\]

Under the above velocity field, the equations (11) and (12) give

\[
J \times B_0 = \frac{-\sigma_0 B_0^2}{1 + m^2} [(U - mV), (V + mU), 0].
\]

Where \( m = \frac{\gamma_0 B_0}{\varepsilon_0 \mu} \), serves as Hall parameter.

The corresponding equations for the present flow analysis are as follows \([26, 34]\):

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0,
\]

\[
\rho \left( \frac{\partial}{\partial t'} + U \frac{\partial}{\partial X} + V \frac{\partial}{\partial Y} \right) U = \frac{-\partial P}{\partial X} + \frac{\mu}{\partial X^2} + \frac{\mu}{\partial Y^2} \right) U + \frac{\partial S_{XX}}{\partial X} + \frac{\partial S_{XY}}{\partial Y} - \frac{\sigma B_0^2}{1 + m^2} (U - mV),
\]

\[
\rho \left( \frac{\partial}{\partial t'} + U \frac{\partial}{\partial X} + V \frac{\partial}{\partial Y} \right) V = \frac{-\partial P}{\partial Y} + \frac{\mu}{\partial X^2} + \frac{\mu}{\partial Y^2} \right) V + \frac{\partial S_{XY}}{\partial X} + \frac{\partial S_{YY}}{\partial Y} - \frac{\sigma B_0^2}{1 + m^2} (V + mU),
\]

\[
2\eta \frac{\partial U}{\partial X} = S_{XX} + m_e \left[ \frac{\partial}{\partial t'} + U \frac{\partial}{\partial X} + V \frac{\partial}{\partial Y} \right] S_{XX} - 2\eta m_s \frac{\partial U}{\partial X} \frac{\partial U}{\partial X} + m_e \left[ (1 - e) \frac{\partial V}{\partial X} - (1 + e) \frac{\partial U}{\partial X} \right] S_{XY},
\]

\[
2\eta \frac{\partial V}{\partial Y} = S_{XY} + m_e \left[ \frac{\partial}{\partial t'} + U \frac{\partial}{\partial X} + V \frac{\partial}{\partial Y} \right] S_{XY} + \frac{m_e}{2} \left[ (1 - e) \frac{\partial V}{\partial Y} - (1 + e) \frac{\partial U}{\partial Y} \right] S_{XX}
\]

\[
2\eta \frac{\partial V}{\partial Y} = S_{YY} + m_e \left[ \frac{\partial}{\partial t'} + U \frac{\partial}{\partial X} + V \frac{\partial}{\partial Y} \right] S_{YY} - 2\eta m_s \frac{\partial V}{\partial Y} \frac{\partial V}{\partial Y} + m_e \left[ (1 - e) \frac{\partial U}{\partial Y} - (1 + e) \frac{\partial V}{\partial Y} \right] S_{XY}
\]

\[
\rho c_p \left( \frac{\partial}{\partial t'} + U \frac{\partial}{\partial X} + V \frac{\partial}{\partial Y} \right) T = k^* \left( \frac{\partial T}{\partial X^2} + \frac{\partial T}{\partial Y^2} \right) + \frac{D_J}{c_e} \left( \frac{\partial C}{\partial X^2} + \frac{\partial C}{\partial Y^2} \right) + S_{XX} \frac{\partial U}{\partial X} + S_{XY} \frac{\partial V}{\partial Y} + S_{YY} \frac{\partial V}{\partial Y},
\]
\[
\left( \frac{\partial}{\partial t'} + U \frac{\partial}{\partial x} + V \frac{\partial}{\partial y} \right) C = \frac{D_T}{\lambda^2} \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) + \frac{D_T K_T}{\lambda T_m} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) - k_c (C - C_0). \tag{22}
\]

The corresponding boundary conditions for the present flow analysis of tapered asymmetric channel are given below:

\[
\begin{align*}
\psi &= \frac{F}{2} \frac{\partial \psi}{\partial y} = 0 \text{ at } Y = H_2 = d + k'x + a_1 \sin \left( \frac{2\pi X}{\lambda} - \frac{ct'}{\lambda} \right) \\
\psi &= -\frac{F}{2} \frac{\partial \psi}{\partial y} = 0 \text{ at } Y = H_1 = -d - k'x - a_1 \sin \left( \frac{2\pi X}{\lambda} - \frac{ct'}{\lambda} + \varphi \right)
\end{align*}
\tag{23}
\]

The boundary assumptions in both walls channel in convective forms are peculated as:

\[
\begin{align*}
-k_c \frac{\partial T}{\partial Y} &= h_o(T_0 - T), \quad \text{at } Y = H_o, \\
-k_c \frac{\partial T}{\partial Y} &= h_h(T - T_i), \quad \text{at } Y = H_h, \\
-D_T \frac{\partial C}{\partial Y} &= h_m(C_0 - C), \quad \text{at } Y = H_m, \\
-D_T \frac{\partial C}{\partial Y} &= h_m(C - C_h), \quad \text{at } Y = H_h,
\end{align*}
\tag{24}
\]

where \(h_m\) and \(h_h\) designate the convective mass and heat transfer coefficients respectively. We define the following dimensionless quantities

\[
x = \frac{X}{\lambda}, \ y = \frac{Y}{d}, \ t = \frac{ct'}{\lambda}, \ u = \frac{U}{c}, \ v = \frac{V}{c}, \ \delta = \frac{d}{\lambda}, \ h_1 = \frac{H_1}{d}, \ h_2 = \frac{H_2}{d}, \\
p = \frac{d^2p}{\lambda^2 \mu}, \ Re = \frac{pdcd}{\mu}, \ We = \frac{m_s c}{d}, \ S = \frac{d}{\mu c} S(x), \ a_1 = \frac{a_1}{d}, \ b_1 = \frac{a_2}{d}, \\
k = \frac{\lambda^2}{d}, \ \theta = \frac{T - T_0}{T_1 - T_0}, \ \sigma = \frac{C - C_0}{C_1 - C_0}, \ Br = \frac{\mu c^2}{k_s(T_1 - T_0)} = Pr Ec, \ Pr = \frac{\mu e^2}{k^2}, \\
Ec = \frac{c^2}{c_p(T_1 - T_0)}, \ Sr = \frac{D_k(T_1 - T_0)}{\nu T_m(C_1 - C_0)}, \ D_h = \frac{D_k(T_1 - T_0)}{\mu c_p(T_1 - T_0)}, \ Sc = \frac{\nu}{D_1}, \\
\gamma = \frac{k d^2}{\nu}, \ B_h = \frac{h_o d}{k^2}, \ B_m = \frac{h_m d}{D_1}, \ m_1^2 = \frac{\sigma d^2 B_0^2}{\nu}.
\tag{25}
\]

In the above expression, Br denotes the Brinkman number, Ec the Eckert number, Sc the Soret number, \(D_h\) relates Dufour number, \(Pr\) is for Prandtl number, \(Sc\) reports the Schmidt number, \(\gamma\) is reaction constant, \(\delta\) determine the wave number, \(B_h\) and \(B_m\) the heat and mass transfer Biot numbers respectively, \(\theta\) the dimensionless temperature and \(\sigma\) the dimensionless concentration.

After using above dimensionless quantities and \(u = \frac{\partial}{\partial y}, \ v = -\frac{\partial}{\partial x}\) for the stream function \(\psi(x, y)\), equation (15) given above is satisfied identically and equations (16)–(22) deduce to

\[
\begin{align*}
\text{Re } \delta (\psi_y \psi_{xy} - \psi_x \psi_{yy}) &= -\frac{\partial p}{\partial x} \left( \frac{\mu + \eta}{\mu} \right) + \delta^2 (\psi_{xy} + \psi_{yy}) + \delta \frac{\partial}{\partial x} (S_{xx}) + \frac{\partial}{\partial y} (S_{yy}) \\
&\quad - \frac{m_1^2}{1 + m^2} (\psi_y + m \delta \psi_x), \\
-\text{Re } \delta^3 (\psi_y \psi_{xx} - \psi_x \psi_{xy}) &= -\frac{\partial p}{\partial y} \left( \frac{\mu + \eta}{\mu} \right) - \delta^2 (\psi_{xx} + \psi_{yy}) + \delta \frac{\partial}{\partial x} (S_{yy}) \\
&\quad + \delta \frac{\partial}{\partial y} (S_{yy}) - \frac{m_1^2}{1 + m^2} (m \psi_y - \delta \psi_x), \\
\left( \frac{2\delta p}{\mu} \right) \psi_y &= S_{xx} + \delta We \left[ \psi_x \frac{\partial}{\partial x} (S_{xx}) - \psi_y \frac{\partial}{\partial y} (S_{xx}) \right] - 2e We S_{xx} \psi_{xy} \\
&\quad - We \left[ (1 - e) \delta^2 \psi_{xx} + (1 + e) \psi_{yy} \right] S_{yy},
\end{align*}
\tag{28}
\]

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\[
\left( \frac{\eta}{\mu} \right) \psi_{yy} - \delta^2 \psi_{xx} = S_{xy} + We \left[ \psi_y \frac{\partial}{\partial x} (S_{xy}) - \psi_x \frac{\partial}{\partial y} (S_{xy}) \right] + \frac{We}{2} \left[ (1 - e) \psi_{yy} + \delta^2 (1 + e) \psi_{xx} - \frac{We}{2} \delta^2 (1 + e) \psi_{xy} + \frac{We}{2} (1 - e) \psi_{yy} \right] S_{yy},
\]
\[(29)\]
\[
\left( -\frac{2\eta^2}{\mu} \right) \psi_{xy} = S_{yy} + We \left[ \psi_y \frac{\partial}{\partial x} (S_{xy}) - \psi_x \frac{\partial}{\partial y} (S_{xy}) \right] + 2e\delta We S_{xy} \psi_{xy}
+ We \left[ (1 - e) \psi_{yy} + \delta^2 (1 + e) \psi_{xx} \right] S_{xy},
\]
\[(30)\]
\[\delta Pr \ Re \ \frac{\partial \theta}{\partial t} = \delta^2 \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + Ec Pr (\delta S_{xx} \psi_{xx} - \delta^2 S_{xy} \psi_{xx} + S_{xy} \psi_{yy} + \delta S_{yy} \psi_{yy})
+ Du Pr \left[ \delta^2 \frac{\partial^2 \sigma}{\partial x^2} + \frac{\partial^2 \sigma}{\partial y^2} \right],
\]
\[(31)\]
\[\delta Sc \ Re \ \left( \psi_y \frac{\partial \sigma}{\partial x} - \psi_x \frac{\partial \sigma}{\partial y} \right) = \delta^2 \frac{\partial^2 \sigma}{\partial x^2} + \frac{\partial^2 \sigma}{\partial y^2} + Sr Sc \left[ \delta^2 \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right] - Sc \gamma \sigma.
\]
\[(32)\]

Now by employing low Reynolds number and long wavelength assumptions, equations (26)–(32) result as
\[
\left( \frac{\mu + \eta}{\mu} \right) \frac{\partial p}{\partial x} = \frac{\partial}{\partial y} (S_{xy}) + \psi_{yy} = \frac{-m_1^2}{1 + m^2} \psi_{yy},
\]
\[(33)\]
\[
\frac{\partial p}{\partial y} = 0,
\]
\[(34)\]
\[S_{xx} = We (1 + e) \psi_{yy} S_{xy},
\]
\[(35)\]
\[
\left( \frac{\eta}{\mu} \right) \psi_{yy} = S_{xy} + \frac{We}{2} (1 - e) \psi_{yy} S_{xx} - \frac{We}{2} (1 + e) \psi_{yy} S_{yy},
\]
\[(36)\]
\[S_{yy} = -We (1 - e) \psi_{yy} S_{xy},
\]
\[(37)\]
\[
\frac{\partial^2 \theta}{\partial y^2} + Br S_{xy} \psi_{yy} + Du Pr \left[ \frac{\partial^2 \sigma}{\partial y^2} \right] = 0,
\]
\[(38)\]
\[
\frac{\partial^2 \sigma}{\partial y^2} + Sr Sc \left[ \frac{\partial \theta}{\partial y^2} \right] = Sc \gamma \sigma = 0.
\]
\[(39)\]

It is clear from equation (34) that \( p = p(y) \). By eliminating \( p \) from equations (33) and (34), we have the resulting equation as
\[
\frac{\partial^2}{\partial y^2} S_{xy} + \psi_{yyy} = \frac{-m_1^2}{1 + m^2} \psi_{yy} = 0.
\]
\[(40)\]

With the help of equations (35) and (37), equation (36) results as
\[
\left( \frac{\eta}{\mu} \right) \psi_{yy} = S_{xy} + We^2 (1 - e^2) (\psi_{yy})^2 S_{xy},
\]
\[(41)\]

by simplifying equation (41), we get
\[
S_{xy} = \left( \frac{\eta/\mu}{1 + We^2 (1 - e^2) (\psi_{yy})^2} \right) \psi_{yy}.
\]
\[(42)\]

With the substitution of equation (42) into equation (40), we obtain
\[
\frac{\partial^2}{\partial y^2} \left[ \left( \frac{\eta}{\mu} + 1 \right) \psi_{yy} + We^2 (1 - e^2) (\psi_{yy})^3 \right] \frac{1 + m^2}{1 + m^2} \psi_{yy} = 0,
\]
\[(43)\]
\[
\frac{\partial^2}{\partial y^2} \left[ \left( \frac{\eta}{\mu} + 1 \right) \psi_{yy} + We^2 (1 - e^2) (\psi_{yy})^3 \right] [1 + We^2 (1 - e^2) (\psi_{yy})^2]^{-1} \frac{1 + m^2}{1 + m^2} \psi_{yy} = 0.
\]
\[(44)\]
Now by applying Binomial expansion on equation (44) for small \( \text{We}^2 \) and simplifying, we have
\[
\frac{\partial^2}{\partial y^2}(\psi_{yy}) + \text{We}^2\alpha_1(\psi_{yy})^3 + \text{We}^4\alpha_2(\psi_{yy})^5 - \frac{m_i^2}{1 + m^2}\left(\frac{\mu}{\mu + \eta}\right)\psi_{yy} = 0.
\] (45)

Now by putting equation (42) into (33), we have
\[
\left(\frac{\mu + \eta}{\mu}\right)\frac{\partial p}{\partial x} = \frac{\partial}{\partial y}\left[ \frac{(\eta/\mu)\psi_{yy}}{1 + \text{We}^2(1 - e^2)(\psi_{yy})^2} \right] + \psi_{yyy} - \frac{m_i^2}{1 + m^2}\psi_y = 0,
\] (46)

by applying binomial expansion for small \( \text{We}^2 \) and omitting \( \text{We}^6 \) and the higher terms, equation (46) can be written in simplified form as
\[
\frac{\partial p}{\partial x} = \psi_{yyy} + \text{We}^2\alpha_1\left[ (\psi_{yy})^3 \right] + \text{We}^4\alpha_2\left[ (\psi_{yy})^5 \right] - \frac{m_i^2}{1 + m^2}\left(\frac{\mu}{\mu + \eta}\right)\psi_y.
\] (47)

Now by substituting equation (42) into equation (38), we have
\[
\frac{\partial^2\theta}{\partial y^2} + \text{Pr}\left(\frac{\eta}{\mu + \eta}\right)\frac{\psi_{yy}}{1 + \text{We}^2(1 - e^2)(\psi_{yy})^2} + \text{DuPr}\left(\frac{\partial^2\sigma}{\partial y^2}\right) = 0,
\] (48)

which after using Binomial expansion and simplification gives
\[
\frac{\partial^2\theta}{\partial y^2} + \text{Pr}\left(\psi_{yy}\right)\left(\frac{\eta}{\mu + \eta}\right) + \alpha_1\text{We}^2(\psi_{yy})^2 + \alpha_2\text{We}^4(\psi_{yy})^4 + \text{DuPr}\left(\frac{\partial^2\sigma}{\partial y^2}\right) = 0.
\] (49)

where \( \alpha_1 = \frac{e - \eta}{\eta + \mu} \) and \( \alpha_2 = (e^2 - 1)\alpha_1 \).

Followings are the reduced boundary assumptions.
\[
\psi = \frac{F}{2} \frac{\partial \psi}{\partial y} = 0, \text{ at } y = h_2 = 1 + kx + b \sin(2\pi(x - t)),
\] (50)
\[
\psi = -\frac{F}{2} \frac{\partial \psi}{\partial y} = 0, \text{ at } y = h_1 = -1 - kx - a \sin(2\pi(x - t) + \varphi),
\] (51)

and the convective boundary conditions take the form
\[
\frac{\partial \theta}{\partial y} = \theta B_{h_1}, \text{ at } y = h_2 = 1 + kx + b \sin(2\pi(x - t)),
\] (52)
\[
\frac{\partial \theta}{\partial y} = (1 - \theta) B_{h_1}, \text{ at } y = h_1 = -1 - kx - a \sin(2\pi(x - t) + \varphi),
\] (53)
\[
\frac{\partial \sigma}{\partial y} = \sigma B_{m_1}, \text{ at } y = h_2 = 1 + kx + b \sin(2\pi(x - t)),
\] (54)
\[
\frac{\partial \sigma}{\partial y} = (1 - \sigma) B_{m_1}, \text{ at } y = h_1 = -1 - kx - a \sin(2\pi(x - t) + \varphi).
\] (55)

3. HAM solution

Let us find out the solution of constructed flow equations via homotopy analysis procedure by assuming following initial guesses for \( \psi(y), \theta(y), \) and \( \sigma(y) \)
\[
\psi_0(y) = \frac{F(h_i^3 - 3h_i^2h_2 + h_2^3 - 6h_2y^2 + 4y^3 - 3h_2(h_2^2 - 4h_2y + 2y^2))}{2(h_1 - h_2)^3},
\]
\[
\theta_0(y) = \frac{1 - B_{h_1} + B_{h_2}^y}{2 - B_{h_1}h_1 + B_{h_2}h_2},
\]
\[
\sigma_0(y) = \frac{1 - B_{m_1}h_2 + B_{m_2}y}{2 + B_{m_1}h_1 - B_{m_2}h_2}.
\]
(56)

Where \( h_1 = -1 - kx - a \sin(2\pi(x - t) + \varphi) \) and \( h_2 = 1 + kx + b \sin(2\pi(x - t)) \).

The forms of auxiliary linear differential operators for the problem under consideration are
\[
L_\psi[\psi] = \frac{d^4\psi}{dy^4}, \quad L_\theta[\theta] = \frac{d^2\theta}{dy^2}, \quad L_\sigma[\sigma] = \frac{d^2\sigma}{dy^2}.
\] (59)
Let's construct the zeroth order deformation expressions

\[
(1 - q)L_v[\psi(y; q) - \psi_0(y)] = q h_v N_v[\psi(y; q)],
\]
\[
(1 - q)L_\theta[\theta(y; q) - \theta_0(y)] = q h_\theta N_\theta[\theta(y; q)],
\]
\[
(1 - q)L_\sigma[\sigma(y; q) - \sigma_0(y)] = q h_\sigma N_\sigma[\sigma(y; q)],
\]
along with boundary conditions

\[
\psi(h_1, q) = -\frac{F}{2}, \psi(h_2, q) = \frac{F}{2}, \psi'(h_1, q) = \psi'(h_2, q) = 0, \tag{63}
\]
\[
\theta'(h_1) = B_\theta \theta(h_1), \theta'(h_2) = (1 - \theta(h_2)) B_\theta, \tag{64}
\]
\[
\sigma'(h_1) = B_m \sigma(h_1), \sigma'(h_2) = B_m(1 - \sigma(h_2)). \tag{65}
\]

Where \( h_v, h_\theta \) and \( h_\sigma \) are auxiliary parameters and \( q \in [0, 1] \) is embedding parameter.

\[
N_v[\psi(y, q)] = \frac{\partial^4 \psi(y, q)}{\partial y^4} + 3 We^2 \alpha_1 \left( \frac{2 \left( \frac{\partial^3 \psi(y, q)}{\partial y^3} \right)^2 \left( \frac{\partial^2 \psi(y, q)}{\partial y^2} \right)}{\left( \frac{\partial \psi(y, q)}{\partial y} \right)^2} \right) + 5 We^2 \alpha_2 \left( \frac{\partial^4 \psi(y, q)}{\partial y^4} \right)^2 \left( \frac{\partial^2 \psi(y, q)}{\partial y^2} \right) + \left( \frac{\partial^4 \psi(y, q)}{\partial y^4} \right)^3
\]
\[
- m_1^2 \left( \frac{\mu}{\mu + \eta} \right) \frac{\partial^2 \psi(y, q)}{\partial y^2}, \tag{66}
\]

\[
N_\theta[\theta(y; q), \sigma(y; q), \psi(y; q)] = \frac{\partial^2 \theta(y, q)}{\partial y^2} + \left( \frac{\mu + \eta}{\mu} \right) Br \left( \frac{\partial^2 \psi(y, q)}{\partial y^2} \right) + \frac{\eta}{\mu + \eta} + \alpha_1 We^3 \left( \frac{\partial^2 \psi(y, q)}{\partial y^2} \right)^2 + \alpha_2 We^4 \left( \frac{\partial^2 \psi(y, q)}{\partial y^2} \right)^3 + Du Pr \left( \frac{\partial^2 \sigma(y, q)}{\partial y^2} \right), \tag{67}
\]

\[
N_\sigma[\sigma(y; q), \theta(y; q)] = \frac{\partial^2 \sigma(y, q)}{\partial y^2} + Sr Sc \left( \frac{\partial^2 \theta(y, q)}{\partial y^2} \right) - Sc \gamma \sigma. \tag{68}
\]

For \( q = 0 \), we develop the initial guess and for \( q = 1 \), we meet with the solution of the problem under consideration.

At \( m \)-th order the current problem takes the form

\[
L_v[\psi_m(y)] - \chi_m \psi_{m-1}(y)] = h_v R_m(y), \tag{69}
\]
\[
L_\theta[\theta_m(y)] - \chi_m \theta_{m-1}(y)] = h_\theta R_m(y), \tag{70}
\]
\[
L_\sigma[\sigma_m(y)] - \chi_m \sigma_{m-1}(y)] = h_\sigma R_m(y), \tag{71}
\]

and the boundary at this order are given as

\[
\psi_m(h_1) = \psi_m(h_2) = 0, \psi_m'(h_1) = \psi_m'(h_2) = 0, \tag{72}
\]
\[
\theta_m'(h_1) = B_\theta \theta(h_1), \theta_m'(h_2) = - B_\theta \theta(h_2), \tag{73}
\]
\[
\sigma_m'(h_1) = B_m \sigma(h_1), \sigma_m'(h_2) = - B_m \sigma(h_2). \tag{74}
\]

where

\[
\mathcal{A}_m = \begin{cases} 0, & m \leq 1 \\ 1, & m > 0. \end{cases}
\]

We reached at the solution of the considered problem iteratively for \( m = 1, 2, 3,... \) using Mathematica.

**4. Convergence exploration**

Homotopy analysis method plays a vivacious role in order to get convergent series solution. Auxiliary parameters \( h_v, h_\theta \) and \( h_\sigma \) have been used here to tackle and settle the required convergent region. The ranges of these auxiliary parameters detect primary contribution to achieve convergent series solution. The homotopic
convergent is achieved from ranges of $\xi(0 \leq \xi \leq \infty)$ for $h_\psi = h_\theta = h_\sigma = -1$. The tolerable ranges of values for concerned auxiliary parameters are $-1.9 \leq h_\psi \leq 0.2$, $-2.4 \leq h_\theta \leq 0.2$ and $-2 \leq h_\sigma \leq 0.4$.

Table 1 depicts the convergence of stream function, temperature and concentration. It is noticed that 14th order of approximation is sufficient for $\psi^r(h_1)$, $\theta^q(h_1)$ and $\sigma^s(h_1)$ which have been sketched graphically in the figure 2 for stream function, temperature and concentration profiles respectively.

### 5. Graphical discussion of results

The main aim of presenting this section to explore the physical output of parameters when varied against axial velocity, temperature, pressure and concentration profiles. We used homotopy analysis method to acquire the required solutions of axial velocity, pressure gradient, temperature and concentration. HAM is found to be the best convergent method. Intended for such determination, figures 3–17 have been plotted and their preliminary performances are evaluated.

#### 5.1. Velocity profile

Figure 3(a) reflects the altered profile of axial velocity due to amplitude of lower wall $\alpha$. A axial turn down velocity component is associated with maximum variation of $\alpha$. Figure 3(b) reveals the influence of amplitude of the upper wall $b$ on the axial velocity. It can be obviously noticed that the axial velocity profile increases on increasing the value of amplitude of the upper wall. The impact of non-uniform parameter $k$ on velocity distribution of the fluid has been plotted in figure 4(a). The observations perceived that an increasing trend is noticed in the lower parts and upper region of tapered asymmetric channel. Conversely, opposite behavior is find out in the core trapped channel region. Figure 4(b) illustrates the dependence of axial velocity on the phase difference $\varphi$. A decrease in velocity profile is observed when phase difference is enhanced. The effect of
Weissenberg number $We$ on the velocity distribution of the fluid has been displayed in figure 5(a). It reveals that an increase in the value of $We$ results as a decrease in the axial velocity. The physical justification behind such trend is associated with the dominant of viscous forces. Figure 5(b) deliberates the physical consequences of mean flow rate $Q$ on component of axial velocity. An arise axial velocity profile has been reported with leading values of $Q$. Figure 6(a) has been displayed to study the outcome of velocity distribution in the presence of Hartman number $m_1$. The results claimed that maximum variation in $m_1$ is associated with lower axial velocity.
component in the center region while it gets upshot level near the channel walls. Figure 6(b) aim to highlight the axial velocity change when Hall parameter $m$ assigns maximum variation. The careful observation reported that middle channel region shows increasing behavior due to $m$.

**Figure 6.** Effect of Hartman number (a) and Hall parameter (b) on axial velocity for $a = 0.01, x = 0.1, k = 1.2, t = 0.1, \varphi = \frac{\pi}{2}, \epsilon = 0.5, \mu = 0.4, \eta = 0.5, We = 0.4, Q = 1.5, b = 0.3$ and $m = 1.2$.

**Figure 7.** Effect of phase difference (a) and mean flow rate (b) on temperature for $a = 0.2, b = 0.5, x = 0.1, k = 0.1, t = 0.1, \varphi = \frac{\pi}{2}, \epsilon = 0.6, \mu = 0.4, \eta = 0.5, We = 0.4, Q = 1.5, m_1 = 0.1, m = 2, \Gamma = 0.1, B = 0.2, B = 0.3, Du = 0.4, Pr = 0.5, Sc = 0.6$ and $\gamma = 0.2$.

**Figure 8.** Effect of Brinkman number (a) and Prandtl number (b) on temperature for $a = 0.7, b = 0.5, x = 0.1, k = 0.6, t = 0.1, \varphi = \frac{\pi}{2}, \epsilon = 0.2, \mu = 0.4, \eta = 0.5, We = 0.5, Q = 2.3, m_1 = 0.1, m = 0.2, B = 4.2, B = 2.3, Du = 2.4, Pr = 0.5, Sc = 0.1$ and $\gamma = 1.2$.
5.2. Temperature profile

Figures 7–10 address the influences of various evolving parameters on temperature profile. The impacts of phase difference $\varphi$ and mean flow rate $Q$ on temperature have been exposed in figure 7. It is prominent to note that an increase in the values of $\varphi$ and $Q$ results in an increase in the temperature. The explanation of Brinkman number $Br$ and Prandtl number $Pr$ on temperature have been conspired in figure 8. The internal resistance between the
fluid particles is enhanced by means of $Br$ which results an improved distribution of temperature. Analogous behavior is examined towards $Pr$. The effects of heat transfer Biot numbers $Bh$ and Dufour number $Du$ on temperature have been inspected in figure 9. The temperature of fluid depressed for leading numerical variation of $Bh$ and reported in figure 9(a). In contrast to this figure, a reverse trend (declining behavior) is found out for $Du$ (figure 9(b)). Now we reported the change in temperature due to Hartman number $m_1$ by sketching figure 10(a). It can be realized that huge estimation of Hartman number reveals a decline in temperature.
Figure 10(b) depicts the upshot of Hall parameter $m$ on temperature. It demonstrates that fluid temperature enhances by enhancing the values of $m$. This is caused by the fact that electrical conductivity increases by increasing the value of $m$.

5.3. Concentration profile
The effects of several embedding parameters on concentration profile have been displayed in figures 11–14. The response of non-uniform parameter $k$ and phase difference $\varphi$ on concentration are elaborated via figure 11.
Figure 11(a) discloses the effect of non-uniform parameter $k$ on concentration. It is noticed that an increase in the value of $k$ leads to the increase in concentration. The effect of phase difference $\varphi$ on concentration is drafted in figure 11(b). Figure 12(a) is exhibited to study the influence of mean flow rate $Q$ on concentration. It shows that as the value of $Q$ enhances the concentration also enhances. Figure 12(b) is made to view the effect of Brinkman number $Br$ on concentration. Increasing value of $Br$ causes an increase in concentration. In order to check variation in concentration due to mass transfer Biot number $B_{m}$, figure 13(a) is organized. The fall in amount of concentration is censored when $B_{m}$ enhances. The effect of Schmidt number $Sc$ on concentration is depicted in figure 13(b). The progress in magnitude of concentration is regarded by increasing the value of $Sc$. The effect Soret number on concentration is articulated in figure 14(a). By increasing the value of $Sr$ the concentration gets increased. The appliance of $\gamma$ on concentration is thrashed out in figure 14(b). The observations reveal that by expanding the value of $\gamma$ the concentration distribution also expands.

### 5.4. Pumping characteristics

The effects of amplitude of lower wall $a$, upper wall $b$, non-uniform parameter $k$, Weissenberg number $We$, Hartman number $m_{1}$ and Hall parameter $m$ on distribution of pressure are reported in figures 15–17. Figure 15(a) briefs the impact of amplitude of lower wall $a$ on pressure gradient. It is seen that the pressure gradient tends to enhance as we enhance the values of $a$. The observations explored in figure 15(b) justify the input of amplitude of upper wall $b$ with pressure gradient graphically. The pressure gradient increases as value of $b$ goes up. Figure 16(a) depicts the effect of non-uniform parameter $k$ on pressure gradient. It is captured that pressure gradient attains minimum value as we increase the values of $k$. Figure 16(a) represents the effect of Weissenberg number $We$ on pressure gradient. It is pointed out that pressure shows declining trend in the core channel region when we increase the value of $We$. The impact of Hartman number $m_{1}$ on pressure gradient is evoked in figure 17(a). It is interesting to perceive that pressure gradient goes up by increasing the value of $m_{1}$. Figure 17(b) exposes the effect of Hall parameter $m$ on pressure gradient. It shows that pressure gradient declines on increasing the value of $m$. Table 2 present the numerical values of Nusselt number at upper and walls of disks. It is noted that Nusselt number at upper wall increases with Pr and Br.

### 6. Concluding remarks

In contemporary work, a mathematical model is developed to observe heat and mass transfer characteristics in the peristaltic movement of Johnson-Segalman fluid. The motion takes place in tapered asymmetric channel in the presence of Hall current. Furthermore, Sore and Dufour effects have also been discussed and analyzed in mathematical modeling and analysis. Instead of diverse phase and amplitudes on non-uniform channel boundaries, we have considered here the peristaltic wave train which gives rise to tapered asymmetric channel. We have treated the model with the assumptions of peak wavelength and smaller Reynolds theories. The resulting differential equations have been solved for stream function, axial velocity, temperature, concentration and pressure gradient with the aid of convergence procedure. Various parameters effects are graphically underlined for respective profiles. The outcomes have been summarized as follows:

| $Br$ | Pr | $B_{m}$ | $Du$ | Lowe wall | Upper wall |
|------|----|--------|------|-----------|------------|
| 1.0  | 0.5| 0.2    | 0.4  | -4.844 95 | 4.4668     |
| 1.3  | 0.5| 0.2    | 0.4  | -6.259 99 | 5.819 51   |
| 1.6  | 0.5| 0.2    | 0.4  | -7.675 03 | 7.172 23   |
| 1.9  | 0.5| 0.2    | 0.4  | -9.090 07 | 8.524 95   |
| 0.5  | 1.0| 0.2    | 0.4  | -2.534 74 | 2.248 29   |
| 0.5  | 1.3| 0.2    | 0.4  | -2.563 65 | 2.269 91   |
| 0.5  | 1.6| 0.2    | 0.4  | -2.592 57 | 2.291 52   |
| 0.5  | 1.9| 0.2    | 0.4  | -2.621 48 | 2.313 14   |
| 0.5  | 0.2| 1.0    | 0.4  | -2.591 56 | 2.056 74   |
| 0.5  | 0.2| 1.3    | 0.4  | -2.614 86 | 2.033 44   |
| 0.5  | 0.2| 1.6    | 0.4  | -2.632 22 | 2.016 08   |
| 0.5  | 0.2| 1.9    | 0.4  | -2.645 65 | 2.002 64   |
| 0.5  | 0.2| 0.4    | 1.0  | -2.536 45 | 2.162 38   |
| 0.5  | 0.2| 0.4    | 1.3  | -2.530 4 | 2.173 69   |
| 0.5  | 0.2| 0.4    | 1.6  | -2.564 35 | 2.185 04   |
| 0.5  | 0.2| 0.4    | 1.9  | -2.578 29 | 2.196 32   |
The velocity profile exhibits decreasing behavior for increasing values of amplitude of lower wall, non-uniform parameter, Weissenberg parameter and Hartman number.

The fluid velocity increases for amplitude of upper wall, mean flow rate and Hall parameter.

The temperature profile decreases for heat transfer Biot number and Hartman number.

Phase difference, mean flow rate, Prandtl number, Brinkman number, Dufour number, and Hall parameter enhance the temperature profile.

The concentration profile tends to decrease for phase difference and mass transfer Biot number.

The concentration profile enhances for non-uniform parameter, mean flow rate, Brinkman number, Schmidt number, Soret number and chemical reaction parameter.

The pressure gradient shows decreasing behavior for non-uniform parameter, Hall parameter and Weissenberg number.

The pressure gradient enlarges Hartman number.

The transfer Biot number variation results decreasing heat transpiration are lower wall as compared to upper wall channel.

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