Multistage Voting Model with Alternative Elimination

Oleg A. Malafeyev\textsuperscript{1,b)}, Denis Rylow\textsuperscript{1,a)}, Irina Zaitseva\textsuperscript{2,3,c)}, Anna Ermakova\textsuperscript{2,d)} and Dmitry Shlaev\textsuperscript{2,e)}

\textsuperscript{1}St. Petersburg State University, 7/9 Universitetskaya nab., St. Petersburg, 199034, Russia.
\textsuperscript{2}Stavropol State Agrarian University, Zootekhnicheskiy lane 12, Stavropol, 355017, Russia.
\textsuperscript{3}Stavropol branch of the Moscow Pedagogical State University, Dovatortsev str. 66 g, Stavropol, 355042, Russia.

\textsuperscript{a)}Corresponding author: denisrylow@gmail.com
\textsuperscript{b)}malafeyevoa@mail.ru
\textsuperscript{c)}ziki@mail.ru
\textsuperscript{d)}dannar@list.ru
\textsuperscript{e)}shl-dmitrij@yandex.ru

Abstract. The voting process is formalized as a multistage voting model with successive alternative elimination. A finite number of agents vote for one of the alternatives each round subject to their preferences. If the number of votes given to the alternative is less than a threshold, it gets eliminated from the game. A special subclass of repeated games that always stop after a finite number of stages is considered. Threshold updating rule is proposed. A computer simulation is used to illustrate two properties of these voting games.

INTRODUCTION

Voting is a procedure for consensus determination among group members with different opinions concerning alternatives. It plays an important role in modern societies: national assemblies and presidents are elected through voting process, chief executive officers of public companies are appointed by shareholders through a voting procedure. Voting itself can be viewed as an aggregating function of individual choices into collective compromise. The simplest forms of voting have been used since ancient times but the first formal analysis was given by Jean-Antoine the marquis de Condorcet and Jean-Charles de Borda. Both considered plurality voting and proposed different voting methods which became known as Condorcet method and Borda count [1]. Their works inspired further research and lead to such notable results as Dogson’s method [2], Kemeny rule [3, 4], Condorcet polling [5]. Borda count and simple plurality voting are special cases of the voting model with scoring methods (see p. 133, [6]). The widely discussed approval voting is a special case of grading voting or voting system with grading methods [7, 8]. Grading voting is a nonranked voting procedure. Voters grade each alternative by assigning it an admissible number (usually from a finite set of consecutive natural numbers). Different alternatives may receive same grades from one agent. The winning alternative is selected subject to the grades and some rule.

All models mentioned above are one stage models, the single alternative is chosen after only one cast of ballouts. Voting procedures that involve several rounds of voting are called multistage. Two round elections are multistage voting procedures and usually involve elimination of the least popular alternatives from the voting process before proceeding to the next round. A good overview of the literature is given in [9].

In this paper the voting process is formalized as a multistage voting model. Unlike runoff voting, alternatives are eliminated after each round of voting if they do not receive a sufficient number of votes, i. e. do not reach a certain threshold. Agents vote again after each elimination of alternatives. The process stops when there is only one alternative left. Methods of the game theory are potent tools for modeling and analysis of socio-economic interaction [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23]. These methods can be applied to the voting process as well, especially when agents vote strategically. We formalize the voting process as a repeated voting game but do not
perform game-theoretic analysis as the simplest case of agents voting sincerely according to their preferences is considered. The problems of stability and numerical analysis are presented in [24, 25, 26, 27, 28, 29, 30].

The paper is organized as follows. The voting process is described in section “Non-Formal Voting Process Description” and formalized in the next section. The model is studied in section “Analysis”: a subclass of games that stop after a finite number of stages is defined, a threshold updating rule is proposed. In section “Computer Simulation” the results of computer program are used to demonstrate two properties of the subclass of repeated games that stop after a finite number of stages as defined in the preceding sections. Conclusions are drawn in the last section.

Non-Formal Voting Process Description

Consider a voting process lasting several rounds with a finite number of alternatives that a finite number of voters participate in. Each voter has a finite number of votes which he/she assigns to one of the alternatives each round. Each alternative has a threshold — a minimum number of votes necessary for the alternative to be passed to the next stage of the voting process. Once an alternative is eliminated, it cannot be voted for again in any following rounds. The voting process stops when there is only one non-eliminated alternative left or less, i.e., zero. All voters are assumed to be rational and sincere. They vote according to their preferences. Preferences do not change during the voting process.

Formalization

Consider a voting process lasting several rounds with a finite number of alternatives that a finite number of voters participate in. Each voter has a finite number of votes which he/she assigns to one of the alternatives each round. Each alternative has a threshold — a minimum number of votes necessary for the alternative to be passed to the next stage of the voting process. Once an alternative is eliminated, it cannot be voted for again in any following rounds. The voting process stops when there is only one non-eliminated alternative left or less, i.e., zero. All voters are assumed to be rational and sincere. They vote according to their preferences. Preferences do not change during the voting process.

Example 1. Let \( n \geq 1 \) be rational and sincere. They vote according to their preferences. Preferences do not change during the voting process.

After a finite number of stages is defined, a threshold updating rule is proposed. In section “Computer Simulation” the results of computer program are used to demonstrate two properties of the subclass of repeated games that stop after a finite number of stages as defined in the preceding sections. Conclusions are drawn in the last section.

Analysis

The formal definition of the repeated voting game given in the previous section does not guarantee that the number of stages \( K \) is finite. Consider a simple example.

Example 1. Let \( n \geq 1 \), \( N = \{1, 2, 3\} \), \( X^1 = \{x_1^1, x_2^1, x_3^1\} \), \( f^k_1 = f^k_2 = f^k_3 = 2 \), for any \( k \). Assume that \( x_1^1 \succ x_2^1 \succ x_3^1 \), and \( x_1^2 \succ x_2^2 \succ x_3^2 \), for any \( k \). The game goes on ad infinitum.

Proposition 1. Let \( \Gamma \) be a repeated voting game as formalized in section “Formalization”. If \( \sum_{i=1}^n f^k_i > \sum_{i=1}^n v_i \) for any \( k \) then

1. at least one alternative is eliminated at every stage of the repeated voting game;
2. the length of the repeated voting game \( K \leq n - 1 \).
The second part of proposition immediately follows from the first part, as there are only \( m \) alternatives and the game stops when there is only one alternative left or less.

The proof of the first part is straightforward. Assume there exists such \( k \) that no alternatives are eliminated from the game at the stage \( k \). Then \( r^k_i \geq f^k_i \), \( i = 1, \ldots, m_k \), where \( r^k_i \) is the total number of votes given to the alternative \( x^k_i \) by agents at the stage of game \( k \). The sum \( \sum_{i=1}^{m_k} r^k_i = \sum_{i=1}^{n} v_i \) because all agents vote for one of the alternatives. But then \( \sum_{i=1}^{n} v_i = \sum_{i=1}^{m_k} f_i \geq \sum_{i=1}^{m_k} f_i \) which contradicts the conditions of the proposition.

Proposition 1 allows us to consider a subclass of repeated voting games which stop after a finite number of stages regardless of the way agents vote. The proposition provides only sufficient conditions. It is possible to find other subclasses of repeated voting games that stop after a finite number of stages. Notice that the number of terms in \( \sum_{i=1}^{m_k} f^k_i \) decreases as \( k \) increases while the sum \( \sum_{i=1}^{n} v_i \) stays constant. It means that in general case some terms of the sum \( \sum_{i=1}^{m_k} f^k_i \) have to be increased as the game progresses for \( \sum_{i=1}^{m_k} r^k_i < \sum_{i=1}^{m_k} f^k_i \) to hold. One way to solve this problem is to define a rule that will guarantee \( \sum_{i=1}^{m_k} r^k_i < \sum_{i=1}^{m_k} f^k_i \) for every \( m_k \).

Let \( d_i^k = r^k_i - f^k_i \) be the relative popularity of the alternative \( x^k_i \in X^k \) among agents, \( i = 1, \ldots, m_k \) at the stage of the game \( k \). Denote the previous lower index \( i \) of the alternative \( x^k_i \) by \( l(i) \), so that \( x^k_i = x^{k-1}_{l(i)}, f^k_i = f^{k-1}_{l(i)} \).

Now we can define the threshold updating rule to recursively find \( f^k_i \) knowing \( f^{k-1}_{l(i)} \), \( a^{k-1}_{l(i)} \) and \( f^{k-1}_{j}, f^k_j = f^{k-1}_{j} + \frac{a^{k-1}_{j}}{\sum_{j=1}^{m_k} a^{k-1}_{j}} \left( \sum_{j=1}^{m_k} f^{k-1}_{j} - \sum_{j=1}^{m_k} f^{k-1}_{l(j)} \right) \). This equation has clear interpretation. The threshold of each non-eliminated alternative \( f^{k-1}_{l(i)} \) is increased at each stage of the repeated voting game by a fraction of the sum of thresholds corresponding to the eliminated alternatives. The increases of \( f^{k-1}_{l(i)} \) are proportional to the relative popularity \( a^{k-1}_{l(i)} \) of the corresponding alternative \( x^{k-1}_{l(i)} \). Notice that the sum \( \sum_{j=1}^{m_k} f^{k-1}_{j} \) stays constant: \( \sum_{i=1}^{m_k} f^k_i = \sum_{i=1}^{m_k} f^{k-1}_i \), for any \( k, j \in \{1, \ldots, K\} \).

So, if \( \sum_{j=1}^{m_k} f^1_j > \sum_{j=1}^{m_k} v_j \). The repeated voting game always stops after a finite amount of stages according to the proposition 1. Note however, that some games may stop after a finite number of stages with all alternatives being eliminated.

**Computer Simulation**

Consider a voting process with a fixed number of participating agents and a fixed number of alternatives. Assume that every agent has one vote. Take \( f^1_i = 2 \times \frac{\text{number of agents}}{\text{number of alternatives}} = \frac{2}{m_i}, i = 1, m_1 \). All other \( f^k_i, k = 2, 3, \ldots \), are recursively found. Then it can be demonstrated that for any distribution of agents’ preferences:

1. increments in the number of participating agents at first increase and then decrease the length of games on average if the initial number of agents is small;
2. decrements in the number of available alternatives increase and then decrease the length of games on average if the number of agents is sufficiently large.

To illustrate these two points we provide the results of computer simulation. One hundred experiments are performed and the average length of games is computed for different numbers of alternatives and different numbers of voting agents. Agent’s preferences are uniformly distributed. The results are listed in the table 1; rows correspond to the number of alternatives, columns correspond to the number of voting agents.

**Table 1. Average game length**

| Alternatives | 10   | 20   | 40   | 80   | 160  | 320  | 640  | 1280 | 2560 | 5120 |
|--------------|------|------|------|------|------|------|------|------|------|------|
|   2          | 2.89 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 |
|   4          | 2.66 | 2.99 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 |
|   8          | 2.01 | 2.20 | 2.49 | 2.90 | 2.49 | 2.16 | 2.00 | 2.00 | 2.00 | 2.00 |
|   16         | 2.00 | 2.01 | 2.01 | 2.01 | 2.49 | 2.16 | 2.00 | 2.00 | 2.00 | 2.00 |
|   32         | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 |
|   64         | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 |
|   128        | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 |
|   256        | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 |
|   512        | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 | 3.00 |
Conclusion

Voting process is formalized as a $n$-persons non-cooperative multistage repeated voting game. Sufficient conditions that allow us to consider repeated voting games that stop after a finite number of stages are given. Threshold updating rule is formulated and applied recursively to the threshold of every alternative at every stage of the repeated voting game except the first stage. A computer simulation is used to illustrate two properties of repeated voting games that satisfy sufficient conditions.

REFERENCES

[1] McLean Iain, Urken Arnold. Classics of Social Choice. — University of Michigan, 1995.
[2] Bartholdi J. III, Tovey C. A., Trick M. A. Voting schemes for which it can be difficult to tell who won the election // Social Choice and Welfare. — 1989. — Vol. 6, no. 2. — P. 157–165.
[3] Ratliff Thomas C. A Comparison of Dodgson’s Method and the Borda Count // Economic Theory. — 2002. — Vol. 20, no. 2. — P. 357–372.
[4] Kemeny John G. Mathematics without Numbers // Daedalus. — 1959. — Vol. 88, no. 4. — P. 577–591.
[5] Potthoff Richard F. Condorcet Polling // Public Choice. — 2011. — Vol. 148, no. 1/2. — P. 67–86.
[6] Moulin Herve J. Fair division and collective allocation. — MIT Press, 2003.
[7] Brams Steven J., Potthoff Richard F. // Public Choice. — 2015. — Vol. 165, no. 3. — P. 193–210.
[8] Brams S.J., Fishburn P.C. Approval voting. — Springer, 2007.
[9] Kenneth J. Arrow A.K. Sen Kotaro Suzumura. Handbook of Social Choice and Welfare, Volume 1 (Handbooks in Economics). — 1 edition. — North Holland, 2002. — Vol. 1.
[10] Alferov G.V., Malafeyev O.A., Malteva A.S. // AIP Conference Proceedings. — 2015. — Vol. 1648, no. 450009.
[11] Neverova E.G., Malafeyef O.A. // AIP Conference Proceedings. — 2015. — Vol. 1648, no. 450012.
[12] M.A. Gureva, S.V. Lyubimtseva, T.N. Tukhkanen et al. // International Journal of Economics and Financial Issues. — 2016. — Vol. 6, no. 8Special Issue. — P. 115–120.
[13] A.N. Nepp, O.I. Nikonov, P.V. Kryuchkova, A.N. Semin // AIP Conference Proceedings. — 2015. — Vol. 1648, no. 070004.
[14] L. Wang, H.W. Gao, L. Petrosyan et al. // Science China Mathematics. — 2016. — Vol. 59, no. 5. — P. 1015–1028.
[15] Model of interaction between anticorruption authorities and corruption groups / O.A. Malafeyev, E.G. Neverova, G.V. Alferov, T.E. Smirnova // 2015 International Conference on “Stability and Control Processes” in Memory of V.I. Zubov, SCP 2015 - Proceedings. — New Jersey : Institute of Electrical and Electronics Engineers Inc., 2015. — P. 488–490.
[16] Malafeyev O.A., Redinskikh N.D., Alferov G.V. Electric circuits analogies in economics modeling: Corruption networks // 2014 2nd International Conference on Emission Electronics, ICEE 2014. — Institute of Electrical and Electronics Engineers Inc., 2014. — P. 28–32.
[17] Malafeev O., Grigorieva X. // Applied Mathematical Sciences. — 2014. — Vol. 8, no. 146. — P. 7249–7258.
[18] Malafeyev O.A., Nemnyugin S.A., Ivaniukovich G.A. Stochastic models of social-economic dynamics // 2015 International Conference on “Stability and Control Processes” in Memory of V.I. Zubov, SCP 2015 - Proceedings. — New Jersey : Institute of Electrical and Electronics Engineers Inc., 2015. — P. 483–485.
[19] Kolokoltsov V.N., Malafeyev O.A. // Dynamic Games and Applications. — 2017. — Vol. 7, no. 1. — P. 34–47.
[20] Malafeyev O., Pichugin Y., Alferov G. // Contemporary Engineering Sciences. — 2016. — Vol. 9, no. 1-4. — P. 175–185.
[21] Malafeyev O.A., Kolokolsov V.N. Understanding game theory: Introduction to the analysis of many agent systems with competition and cooperation. — New Jersey : World Scientific Publishing Co., 2010.
[22] Pichugin Y.A., Malafeyev O.A. // Applied Mathematical Sciences. — 2016. — Vol. 10, no. 41-44. — P. 2065–2073.
[23] Malafeyev O.A., Drozdov G.D., Nemnyugin S.A. Multicomponent dynamics of competitive single-sector economy development // 2015 International Conference on “Stability and Control Processes” in Memory of V.I. Zubov, SCP 2015 - Proceedings. — New Jersey : Institute of Electrical and Electronics Engineers Inc., 2015. — P. 457–459.
[24] Kvitko A. // AIP Conference Proceedings. — 2015. — Vol. 1648, no. 450002.

[25] Kvitko A. Syntheses of terminal control for nonlinear stationary controlled system under incomplete information // AIP Conference Proceedings. — 2016. — Vol. 1738, no. 160002.

[26] L.A. Bondarenko, A.V. Zubov, V.B. Orlov et al. // Journal of Theoretical and Applied Information Technology. — 2016. — Vol. 85, no. 3. — P. 305–308.

[27] Zubov A.V., Dikusar V.V., Zubov N.V. // Doklady Mathematics. — 2010. — Vol. 81, no. 1. — P. 6–7.

[28] Smirnov N.V., Smirnova T.E., Shakhov Y.A. // Journal of Computer and Systems Sciences International. — 2012. — Vol. 51, no. 2. — P. 169–175.

[29] Papadopoulos D.F., Kosmas O.T., Simos T.E. // AIP Conference Proceedings. — 2012. — Vol. 1479, no. 1407.

[30] Kalogiratou Z., Monovasilis Th., Simos T.E. // AIP Conference Proceedings. — 2012. — Vol. 1479, no. 1387.

[31] Malafeyev O.A., Alferov G.V., Maltseva A.S. // AIP Conference Proceedings. — 2015. — Vol. 1648, no. 450009. — P. 1–3.

[32] Multi-criteria model of laser radiation control / O.A. Malafeyev, E.G. Neverova, S.A. Nemnyugin, G.V. Alferov // 2014 2nd International Conference on Emission Electronics, ICIEE 2014. — New Jersey : Institute of Electrical and Electronics Engineers Inc., 2014. — P. 33–37.

[33] Malafeyev O., Alferov G., Andreyeva M. Group strategy of robots in game-theoretic model of interception with incomplete information // 2015 International Conference on Mechanics - Seventh Polyakhov's Reading. — New Jersey : Institute of Electrical and Electronics Engineers Inc., 2015. — P. 1–3.

[34] Smith C. D., Jones E. F. Load-cycling in cubic press // Shock Compression of Condensed Matter-2001 / Ed. by M. D. Furnish. — AIP Conference Proceedings no. 620. — Melville, NY : American Institute of Physics, 2002. — P. 651–654.

[35] Malafeyev O.A., Alferov G.V., Maltseva A.S. Programming the robot in tasks of inspection and interception // 2015 International Conference on Mechanics - Seventh Polyakhov’s Reading. — New Jersey : Institute of Electrical and Electronics Engineers Inc., 2015. — P. 1–3.

[36] Malafeyev O.A., Nemnyugin S.A., Alferov G.V. Charged particles beam focusing with uncontrollable changing parameters // 2014 2nd International Conference on Emission Electronics, ICIEE 2014. — New Jersey : Institute of Electrical and Electronics Engineers Inc., 2014. — P. 25–27.

[37] Malafeyev O.A., Alferov G.V. The robot control strategy in a domain with dynamical obstacles // Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics). — 1996. — Vol. 1093. — P. 211–217.

[38] Malafeev O.A., Nemnyugin S.A. Generalized dynamic model of a system moving in an external field with stochastic components // Theoretical and Mathematical Physics. — 1996. — Vol. 107, no. 3. — P. 770–774.

[39] EXPERIMENTAL STUDY OF AN ELECTRON BEAM IN DRIFT SPACE // Soviet journal of communications technology & electronics. — 1986. — Vol. 31, no. 3. — P. 145–149.

[40] Malafeev O.A. The existence of situations of -equilibrium in dynamic games with dependent movements // USSR Computational Mathematics and Mathematical Physics. — 1974. — Vol. 14, no. 1. — P. 88–99.

[41] Malafeev O.A. Stationary strategies in differential games // USSR Computational Mathematics and Mathematical Physics. — 1977. — Vol. 17, no. 1. — P. 37–46.

[42] Malafeev O.A. Equilibrium situations in dynamic games // Cybernetics. — 1974. — Vol. 10, no. 3. — P. 504–513.

[43] Malafeev O.A., Troeva M.S. A weak equilibrium solution for multicriteria optimization problem // Control Applications of Optimization 2000: Proceedings of the 11th Ifac Workshop. — Amsterdam : Elsevier science & technology, 2000. — P. 363–368.

[44] Malafeyev O.A., Redinskikh N.D. Stochastic analysis of the dynamics of corrupt hybrid networks // Proceedings of 2016 International Conference "Stability and Oscillations of Nonlinear Control Systems" (Pyatnitskiy’s Conference), STAB 2016. — New Jersey : Institute of Electrical and Electronics Engineers Inc., 2016. — P. 1–4.

[45] Woldemariam K., Kemmerer R. A., Villafiorita A. Formal Analysis of Attacks for E-voting System // 2009 Fourth International Conference on Risks and Security of Internet and Systems (CRiSIS 2009). — Institute of Electrical and Electronics Engineers Inc., 2009. — Oct. — P. 26–34.
Malafeev O. A., Zenovich O. S., Sevek V. K. Mnogoagentnoe vzaimodejstvie v dinamicheskoj zadache upravleniya venchurny'yi stroitel'ny'yi proektami // E'konomicheskoe vozrozhdenie Rossi. — 2012. — no. 1. — P. 124–131.

Malafeev O. A., Drozdova I. V., Parshina L. G. E'ffektivnost' variantov rekonstrukcii gorodskoj zhiloj zastrojki // E'konomicheskoe vozrozhdenie Rossi. — 2008. — no. 3. — P. 63–67.

Malafeev O. A., Paxar O. V. Dinamicheskaya nestacjionnaya zadacha investirovaniya proektov v usloviyax konkurencii // Problemy' mexaniki i upravleniya: Nelinejn'ye dinamicheskie sistemy'. — 2009. — no. 41. — P. 103–108.

Malafeev O. A., Gordeev D. A., Titova N. D. Probabilistic and deterministic model of the influence factors on the activities of the organization to innovate // E'konomicheskoe vozrozhdenie Rossi. — 2011. — no. 1. — P. 73–82.

Malafeev O. A., Drozdova I. V., Ivanov A. S. Statisticheskaya koalitsionnaya model' investirovaniya innovacziy'x proektov // E'konomicheskoe vozrozhdenie Rossi. — 2011. — no. 4. — P. 90–98.

Malafeev O. A., Cherny'x K. S. Matematicheskoe modelirovanie razvitiya kompanii // E'konomicheskoe vozrozhdenie Rossi. — 2004. — no. 1. — P. 60–66.

Malafeev O. A., Drozdova I. V., Titova N. D. Stoxastiches kaya model' prinyatiya resheniya o vy'vode na ry'nok innovacziy'x proektov // Vestnik grazhdanskix inzhenerov. — 2007. — no. 2. — P. 161–166.

Malafeev O. A., Kolokol'czov V. N. Matematichesko modelirovanie mnogoagentny'x sistem konkurencii i kooperacii (teoriya igr dlya vsekh). — Sankt-Peterburg : Lan', 2012.

Malafeev O. A., Akulenkova I. V., Drozdov G. D. Problemy' rekonstrukcii zhilishhhno-kommunal'nogo xozyajstva megapolisa. — Sankt-Peterburg : Sankt-Peterburgskij gosudarstvenny'j universitet servisa i e'konomiki, 2007.

Malafeev O. A., Parfenov A. P. Ravnovesie i kompromissnoe upravlenie v setev'yx modelyax mnogoagentnogo vzaimodejstviya // Problemy' mexaniki i upravleniya: Nelinejn'ye dinamicheskie sistemy'. — 2007. — no. 39. — P. 36–45.

Malafeev O. A., Trueva M. S. A weak solution of Hamilton-Jacobi equation for a differential two-person zero-sum game // Preprints of the Eighth International Symposium on Differential Games and Applications. — Maastricht : Rijksuniversiteit te Utrecht, Universiteit Maastricht, Rijksuniversiteit te Groningen, 1998. — P. 366–369.

Malafeev O. A., Drozdova I. V., Drozdov G. D. Modelirovanie procecssov rekonstrukcii zhilishhhkomunal'nogo xozyajstva megapolisa v usloviyax konkurentnoj sredy'. — Sankt-Peterburg : Sankt-Peterburgskij gosudarstvenny'j universitet servisa i e'konomiki, 2008.

Kompromiss i ravnovesie v modelyax mnogoagentnogo vzaimodejstviya v korrupcziy'x seti socziuma / O. A. Malafeev, D. S. Bojczov, N. D. Redinskix, E. G. Neverova. — Molodoj ucheny'j. — 2014. — no. 10 (69). — P. 14–17.

Malafeev O. A., Ry'lov D. S. Formalizaciya grazhdanskogo procecssa pri anglijskom i amerikanskom sudebnom pravile // Vestnik Sankt-Peterburgskogo universiteta. Seriya 1. Matematika. Mexanika. Astronomiya. — 1972. — no. 4. — P. 41–46.

Malafeev O. A., Murav'ev A. I. Matematicheskoe modeli konfliktny'x situacii i ix razreshenie. — Sankt-Peterburg : Sankt-Peterburgskij gosudarstvenny'j universitet servisa i e'konomiki i finansov, 2001. — Vol. 2.
Malafeev O. A., Murav’ev A. I. Matematicheskie modeli konfliktny’x situacij i ix razreshenie. — Sankt-Peterburg : Sankt-Peterburgskij gosudarstvenny’j universitet e’konomiki i finansov, 2000. — Vol. 1.

Malafeev O. A., Drozdov G. D. Modelirovanie processov v sisteme upravleniya gorodskim stroitel’stvom. — Sankt-Peterburg : Sankt-Peterburgskij gosudarstvenny’j arkhitekturno-stroiteln’yj universitet, 2001. — Vol. 1.

Malafeev O. A., Koroleva O. A. Model’ korrupcii pri zaklyucheni kontraktov // Procssesy’ upravleniya i ustojchivost’ Trudy’ XXXIX mezhdunarodnoj nauchnoj konferenciji aspirantov i studentov pod redakcijej N. V. Smirnova, G. SH. Tamasyana. — Sankt-Peterburg : Sankt-Peterburgskij gosudarstvenny’j universitet, 2008. — P. 446–449.

Malafeev O. A., Murav’ev A. I. Modelirovanie konfliktny’x situacij v soczial’no-e’konomicheskix sistemax. — Sankt-Peterburg : Sankt-Peterburgskij gosudarstvenny’j universitet e’konomiki i finansov, 1998.

Malafeev O. A., Drozdov G. D. Modelirovanie mnogoagentnogo vzaimodejstviya processov straxovaniya. — Sankt-Peterburg : Sankt-Peterburgskij gosudarstvennyj universitet servisa i e’konomiki, 2010.

Malafeev O. A., Zubova A. F. Ustojchivost’ po Lyapunovu i kolebatel’nost’ v e’konomicheskix modelyax. — Sankt-Peterburg : Sankt-Peterburgskij gosudarstvenny’j universitet, 2001.

Malafeev O. A., Bure V. M. Soglasovannaya strategiya v povtoryayushhixsya konechny’x igax // Vestnik Sankt-Peterburgskogo universiteta. Seriya 1. Matematika. Mexanika. Astronomiya. — 1995. — no. 1. — P. 120–122.

Malafeev O. A. O sushhestovanii znacheniya igry’ pres ledovaniya // Sibirskij zhurnal issledovaniya operaczij. — 1970. — no. 5. — P. 25–36.

Malafeev O. A. Konfliktno upravlyaemy’e processy’ so mnogimi uchastnikami : Dissertacziya na soiskanie uchenoj stepeni doktora fiziko-matematicheskix nauk / O. A. Malafeev ; Leningradskij gosudarstvenny’j universitet. — Leningrad, 1987.

Malafeev O. A. Ustojchivost’ reshenij zadach mnogokri terial’noj optimizaczii i konfliktno upravlyaemy’e dinamicheskie processy’. — Sankt-Peterburg : Sankt-Peterburgskij gosudarstvenny’j universitet, 1990.

Korrupcziya v modelyax aukcziion pervoj czeny’ / O. A. Malafeev, N. D. Redinskix, G. V. Alfyorov, T. E. Smirnova // Upravlenie v morskix i ae’rokosmicheskix sistemax (UMAS-2014) 7-ya Rossiijskaya mul’tikonferencziya po problemam upravleniya: materialy’ konferenczii. GNCZ RF OAO “CZentral’ny’j nauchno-issledovatel’skij institut “E’lektropribor”. — Sankt-Peterburg : Konczern “CZentral’ny’j nauchno-issledovatel’skij institut “E’lektropribor”, 2014. — P. 141–146.

Malafeev O. A., Redinskix N. D., Smirnova T. E. Setevaya model’ investirovaniya proektov s korrupcziej // Procssesy’ upravleniya i ustojchivost’ Trudy’ XLVI mezhdunarodnoj nauchnoj konferenczii aspirantov i studentov. — Sankt-Peterburg : Sankt-Peterburgskij gosudarstvenny’j universitet, 2015. — P. 659–664.

Malafeev O. A., Pichugin YU. A. Ob ocezenke riska bankrotstva firmy’ // Tezisy’ dokladov VI Mezhdunar. konf. “Dinamicheskie sistemy’: ustojchivost’, upravlenie, optimizacziya” (DSSCO13). — Minsk : Belorussskij gosudarstvennyj universitet, 2013. — P. 204–206.

Malafeev O. A., Redinskix N. D., Gerchiu A. L. Optimizacziyana model’ razmeshheniya korruptzionerov v seti // Stroitel’stvo i e’kspluatacziiya e’nergoe’fektivny’x zdanij (teoriya i praktika s uchetom korrupcionnogo faktora) / L. M. Kolchedanczev, I. N. Legalov, G. M. Bad’in et al. — Borovichi : NP “NTO strojindustrii Sankt-Peterburga”, 2015. — P. 128–140.

Malafeev O. A., Koroleva O. A., Vasil’ev YU. G. Kompromissnoe reshenie v auczione pervoj czeny’ s korrumpirovannym aukcziionistom // Stroitel’stvo i e’kspluatacziiya e’nergoe’fektivny’x zdanij (teoriya i praktika s uchetom korrupcionnogo faktora) / L. M. Kolchedanczev, I. N. Legalov, G. M. Bad’in et al. — Borovichi : NP “NTO strojindustrii Sankt-Peterburga”, 2015. — P. 119–127.

Malafeev O. A., Redinskix N. D., Smirnova T. E. Model’ investirovaniya proektov s korrupcziej // Stroitel’stvo i e’kspluatacziiya e’nergoe’fektivny’x zdanij (teoriya i praktika s uchetom korrupcionnogo faktora) / L. M. Kolchedanczev, I. N. Legalov, G. M. Bad’in et al. — Borovichi : NP “NTO strojindustrii Sankt-Peterburga”, 2015. — P. 140–146.

Malafeev O. A., Axmady’shina A. R., Demidova D. A. Model’ tendera na ry’nke rie’lterskix uslug s uchetom korrupcii // Stroitel’stvo i e’kspluatacziiya e’nergoe’fektivny’x zdanij (teoriya i praktika s uchetom korrupcionnogo faktora) / L. M. Kolchedanczev, I. N. Legalov, G. M. Bad’in et al. — Borovichi : NP “NTO strojindustrii Sankt-Peterburga”, 2015. — P. 161–168.
[103] Malafeev O. A., Kefeli I. F. Matematicheskie nachala global'noj geopolitiki. — Sankt-Peterburgskij gosudarstvennyj’ politteknicheskij universitet, 2013.

[104] Malafeev O. A., Maraxov V. G. E’volyuczionnyj’ mekanizm dejstviya istochnikov i dvizhushhixsya sil grazhdanskogo obshestva v sfere finansovoy i e’konomichekoy komponenty’ XXI veka // K. Marks i budushhee filosofi Rossi. — S. V. Busov, S. I. Dudnik, K. YU. Zhirkov et al. — Sankt-Peterburg : OOO “Izdatel’stvo VVM”, 2016. — P. 112–135.

[105] Malafeev O. A., Nemnyugin S. A. Stoxasticheskaya mode l’ soczial’no-e’konomicheskoy dinamiki // Ustojchivost’ i proczessy’ upravleniya Materialy’ III mezhdunarodnoj konferenciz. — Sankt-Peterburg : Izdatel’skij dom Fedorovoj G. V., 2015. — P. 433–434.

[106] Malafeev O. A., Redinskix N. D. Stoxasticheskoy oczenivanie i prognoz e’ffektivnosti strategii razvitiya firmy’ v usloviyax korruzionnogo vozdejstviya // Ustojchivost’ i proczessy’ upravleniya Materialy’ III mezhdunarodnoj konferenciz. — Sankt-Peterburg : Izdatel’skij dom Fedorovoj G. V., 2015. — P. 437–438.

[107] Filosofskie strategii soczial’ny’x preobrazovanij XXI veka / O. A. Malafeev, V. N. Vovlich, T. A. Delieva et al. — Sankt-Peterburg : Sankt-Peterburgskij gosudarstvennyj’ universitet, 2014.

[108] Malafeev O. A., Dejnega L. A., Andreeva M. A. Model’ vzaimodejstviya korrumpirovannogo predpriyatiya i federal’nogo otdela po bor’be s korruippieij // Molodoj uchenyj. — 2015. — no. 12 (92). — P. 15–20.

[109] Malafeev O. A., Drozdov G. D. Modelirovanie tamozhennogo dela. — Sankt-Peterburgskij gosudarstvennyj’ universitet servisa i e’konomiki, 2013.

[110] Malafeev O. A., Ivashov L. G., Kefeli I. F. Global’ naya arkticheskaya igra i ee uchastniki // Geopolitika i bezopasnost’. — 2014. — no. 1 (25). — P. 34–49.

[111] Evrazijskaya duga nestabil’nosti i problemy’ region al’noj bezopasnosti ot Vostochnoj Azii do Severnoj Afriki / O. A. Malafeev, M. L. Titarenko, L. G. Ivashov et al. — Sankt-Peterburg : Studiya NP-Print, 2013.

[112] Malafeev O. A., Kefeli I. F. O matematicheskix modelyax global’ny’x geopoliticheskh procezsov mnogoagentnogo vzaimodejstviya // Geopolitika i bezopasnost’. — 2013. — no. 2. — P. 44–57.

[113] Malafeev O. A., Kefeli I. F. Necotory’e zadachi obespecheniya oboronnoj bezopasnosti // Geopolitika i bezopasnost’. — 2013. — no. 3. — P. 84–92.

[114] Linejnaya algebra s prilozheniyami k modelirovaniyu korruzionn’x sistem i procezsov / O. A. Malafeev, N.N. Sotnikova, I.V. Zajczeva et al. — Stavropol’ : OOO “Izdatel’skij dom “TE’SE’RA’”, 2016.

[115] Malafeev O. A. Upravlenie v konfliktny’x dinamicheskix sistemax. — Sankt-Peterburg : SPbGU, 1993.

[116] Neverova E.G., Malafeyef O.A. A model of interaction between anticorruption authority and corruption groups // AIP Conference Proceedings. — 2015. — Vol. 1648, no. 450012.

[117] Gelyofand I. M., Yaglom A. M. Computation of the amount of information about a stochastic function contained in another such function // Uspehi Mat. Nauk (N.S.). — 1957. — Vol. 12, no. 1(73). — P. 352.

[118] Broomhead D.S., King G.P. Extracting qualitative dyn amics from experimental data // Physica D. — 1986. — Vol. 20. — P. 21–236.
[128] Karhunen Kari. Uber lineare Methoden in der Wahrscheinlichkeitsrechnung // Annales Academiae scientiarum Fennicae. Series A. 1, Mathematica-physica. — 1947. — Vol. 37. — P. 1–79.
[129] Pearson K. On lines and planes of closest fit to systems of points in space // Philosophical Magazine. — 1901. — Vol. 2. — P. 559–572.
[130] Numerical integration of Chaplain and stuart model / P. Petrakis, Z. Kalogiratou, Th. Monovasilis, T.E. Simos // AIP Conference Proceedings. — 2016. — Vol. 1738, no. 480131.
[131] Alexandridis Alex, Famelis Ioannis Th., Tsitouras Charalampos. Particle swarm optimization for complex nonlinear optimization problems // AIP Conference Proceedings. — 2016. — Vol. 1738, no. 480120.
[132] Meirong Wu, Shaochen Cao, Huazhen Zhu. On axiomatizations of the Shapley value for bi-cooperative games // AIP Conference Proceedings. — 2016. — Vol. 1738, no. 080002.
[133] e Silva Eliana Costa, Correia Aldina, Lopes Isabel Cristina. Optimization in generalized linear models: A case study // AIP Conference Proceedings. — 2016. — Vol. 1738, no. 300002.
[134] Kvitko Alexander. A method for solving a local boundary problem for nonlinear controlled system // AIP Conference Proceedings. — 2015. — Vol. 1648, no. 450002.
[135] An optimized two-step hybrid block method for solving general second order initial-value problems of the form $y = f(x, y, y')$ / Higinio Ramos, Z. Kalogiratou, Th. Monovasilis, T.E. Simos // AIP Conference Proceedings. — 2015. — Vol. 1648, no. 810006.
[136] A trigonometrically fitted optimized two-step hybrid block method for solving initial-value problems of the form $y = f(x, y, y)$ with oscillatory solutions / Higinio Ramos, Z. Kalogiratou, Th. Monovasilis, T.E. Simos // AIP Conference Proceedings. — 2015. — Vol. 1648, no. 810007.
[137] Kabrits S.A., Kolpak E.P. Finding bifurcation branches in nonlinear problems of statics of shells numerically // 2015 International Conference on "Stability and Control Processes" in Memory of V.I. Zubov, SCP 2015 - Proceedings. — 2015. — P. 389–391.
[138] Kolpak E.P., Ivanov S.E. On the three-dimensional Klein-Gordon equation with a cubic nonlinearity // International Journal of Mathematical Analysis. — 2016. — Vol. 10, no. 13-16. — P. 611–622.
[139] Cooperation in two-stage games on undirected networks / H. Gao, L. Petrosyan, H. Qiao, A. Sedakov // Journal of Systems Science and Complexity. — 2017. — Vol. 30, no. 3. — P. 680–693.
[140] Yeung D.W.K., Petrosyan L.A. Subgame consistent cooperative solution for NTU dynamic games via variable weights // Automatica. — 2015. — Vol. 59. — P. 84–89.
[141] Application in practice and optimization of industrial information systems / L.A. Bondarenko, A.V. Zubov, V.B. Orlov et al. // Journal of Theoretical and Applied Information Technology. — 2016. — Vol. 85, no. 3. — P. 305–308.
[142] Borisenkov E. P., Pichugin YU. A. POSSIBLE NEGATIVE SCENARIOS OF BIOSPHERE DYNAMICS AS A RESULT OF ANTHROPOGENIC ACTIVITY // DOKLADY EARTH SCIENCES. — 2001. — Vol. 379. — P. 581–583.
[143] Pichugin YU. A. POSSIBLE NEGATIVE SCENARIOS OF BIOSPHERE DYNAMICS AS A RESULT OF ANTHROPOGENIC ACTIVITY // RUSSIAN METEOROLOGY AND HYDROLOGY. — 2001. — no. 10. — P. 24–28.
[144] Pichugin YU. A. THE PROBLEM OF STATISTICAL CONTROL OF OBSERVATION DATA ON SURFACE TEMPERATURE AT DISTANT STATIONS (PART 2) // RUSSIAN METEOROLOGY AND HYDROLOGY. — 2001. — no. 11. — P. 13–16.
[145] Pichugin YU. A. THE PROBLEM OF STATISTICAL CONTROL OF OBSERVATION DATA ON SURFACE TEMPERATURE AT DISTANT STATIONS // RUSSIAN METEOROLOGY AND HYDROLOGY. — 2001. — no. 10. — P. 12–17.
[146] Pichugin YU. A. CLASSIFICATION OF SUMMER WEATHER REGIMES IN ST. PETERSBURG // RUSSIAN METEOROLOGY AND HYDROLOGY. — 2000. — no. 5. — P. 21–27.
[147] Pichugin YU. A. SAMPLING PRINCIPAL COMPONENTS OF MOVING INTERVAL IN THE ANALYSIS OF TIME SERIES OF METEOROLOGICAL DATA // RUSSIAN METEOROLOGY AND HYDROLOGY. — 1999. — no. 8. — P. 22–27.
[148] Pichugin YU. A. HYDRODYNAMIC LONG-TERM ENSEMBLE WEATHER PREDICTION // RUSSIAN METEOROLOGY AND HYDROLOGY. — 1998. — no. 2. — P. 1–9.
[149] Pichugin YU. A. SEASONAL FEATURES OF AUTOCORRELATIONS OF SURFACE AIR TEMPERATURE // RUSSIAN METEOROLOGY AND HYDROLOGY. — 1998. — no. 9. — P. 50–56.