Ultra-narrow spin wave metasurface for focusing application

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In this paper we show that the phase shift of the spin waves can be controlled in transmission through metasurface represented as an ultra-narrow non-magnetic spacer separating two ferromagnetic films. We design this metasurface to present the focusing of spin waves in an Co thin film. For this purpose we exploit the strength of the interlayer exchange coupling interactions of RKKY type which allows to control the phase of the transmitted and reflected spin waves in the wide range of angles $[-\pi/2;\pi/2]$. We combined this phase-shift dependency with the lens equation to demonstrate numerically the lens for spin waves based on ultra-narrow metasurface.

Spin waves (SWs) are promising data carriers for future logic devices, information processing and communication with low-energy dissipation, relatively high and flexible frequency operation and possible miniaturization down to nanoscale. However, still many basic units, such as SW generators and detectors, SW waveguides, SW amplifiers or SW modulators have to be further developed to demonstrate their useful performance. Control of the phase of propagating SWs is expected to be one of the key element in future magnonics like it is in microwave technology, electronics, and photonics. To fulfill the requirements of miniaturization, the expected magnonic component shall be small, ideally shorter than the wavelength. An interesting idea to fulfill this condition comes from photonics, where the use of ultra-thin slabs of sub-wavelength thickness, called metasurfaces (MS) have been introduced. They promise valuable advantages in controlling the waves by spatial manipulation of the wavefront characteristics.

The MS concept is based on the phase gradient introduced by the specially arranged tiny elements having a sub-wavelength size as compared with the wavelength $\lambda$ of the incident wave. In optics, these tiny phase-shifting elements are mostly designed from metallic materials using plasmonic phenomena or dielectrics. They rely on subwavelength gratings, resonators, waveguides, and they introduce geometric phase differences to tune the phase of the reflected or transmitted wave. Their working principle is well described with the generalized Snell’s law which takes into account an inhomogeneous phase change on the two-dimensional plane of the MS. Such structures are widely investigated for possible applications in steering propagation of electromagnetic waves and also acoustic waves. Originally designed at radio-wave frequencies for radar and space communication, MS have been implemented to design many new planar devices at infrared and optical frequencies, such as an ultra-thin lens, vortex plates, a polarization converter, color filters, holograms. Let’s first consider the system of two semi-infinite ferromagnetic thin films separated by an ultra-narrow interface along the $y$-axis with the uniform static external magnetic field $H_0$ which is parallel to the interface, and saturates the system. We assume that the magnetizations in both ferromagnets are exchange coupled, with the strength of the coupling described by the parameter $A_{12}$, which depends on the interlayer width. Different models of the oscillatory interlayer exchange coupling between ferromagnetic layers separated by a non-magnetic metal spacer have a common physical mechanism: due to contact of the spacer with the first ferromagnetic layer, the spacer conduction electrons experience spin polarization. The latter extends throughout the spacer and interacts with the second ferromagnet and results in the effective exchange interaction. Bruno describes the interlayer coupling in terms of the bulk band structure of the spacer material and of spin-dependent reflection amplitudes of electrons. Depending on the assumptions, various models can be explored (RKKY, tight-binding, hole-confinement, free-electron, s-d mixing etc. According to the model developed in Ref. the RKKY
coupling energy per unit area with zero angles between the magnetizations in the ferromagnets is proportional to the interlayer coupling constant \( J_{12} \):

\[
A_{12} = J_{12}/(M_{01}M_{02}),
\]

where \( M_{01} \) and \( M_{02} \) are the saturation magnetizations of the first and the second material, respectively.

To relate variation of \( A_{12} \) with respect the interface width \( D \) we use the model developed in Ref. [20]. For the interlayer exchange coupling between the two semi-infinite fcc Co layers with Cu(001) spacer, Bruno found a good agreement between the theoretical model and the experimental observations [27,28] (see, Fig. 3). According to the theory, the interlayer coupling constant dependence on the spacer thickness \( D \) (in atomic layers AL units) can be modeled with a simplified formula [20]

\[
J_{12} = \frac{I_1}{D^2} \sin \left( \frac{2\pi}{A_1} + \phi_1 \right) + \frac{I_2}{D^2} \sin \left( \frac{2\pi}{A_2} + \phi_2 \right)
\]

where \( I_1 \) and \( I_2 \) are the coefficients describing the coupling strengths between ferromagnets, \( A_1 \) and \( A_2 \) are the oscillation periods, \( \phi_1 \) and \( \phi_2 \) are the phases of the oscillatory coupling, Eq. (1) reproduces well the experimental data of Co/Cu(001)/Co multilayers with \( A_1 = 2.6 \) AL and \( A_2 = 8.0 \) AL [25,26] coupling strengths \( I_1 = 17.7 \) erg/cm\(^2\) and \( I_2 = 4.3 \) erg/cm\(^2\) [29] and phases \( \phi_1 = \pi/2 \) and \( \phi_2 = \pi/2 \) [20].

To calculate the phase shift \( \varphi_T \) and the transmission coefficient \( T \) of the SW transmitted through interface connecting the two ferromagnetic thin films we use an analytical approach for the SW propagation in the system of two ferromagnetic films separated by an infinitely thin non-magnetic interface. Based on the Landau-Lifshitz equations [20] in Supplemental Materials) in linear approximation, assuming the plane wave solutions with wavenumbers \( k_1 \) and \( k_2 \) homogeneous across the film thickness we solve the system of equations obtained from the exchange boundary conditions at the interface \( x = 0 \) [20,29]

\[
\begin{align*}
\left\{ A_{12}(m_{2n} - \xi m_{1n}) + \alpha_1 \frac{\partial m_{1n}}{\partial x} \right\} & \bigg|_{x=0} = 0, \\
\left\{ A_{12}(m_{1n} - \frac{m_{2n}}{\xi}) - \alpha_2 \frac{\partial m_{2n}}{\partial x} \right\} & \bigg|_{x=0} = 0,
\end{align*}
\]

where \( \xi = M_{02}/M_{01} \), \( n = x, z \), \( m_{jn} \) is a dynamic magnetization component and \( \alpha_j \) is the exchange parameter in the \( j \)-th ferromagnet, \( j = 1, 2 \): \( \alpha_j = A_{exj}/M^2_{0j} \), where \( A_{exj} \) is the \( j \)-th ferromagnet exchange stiffness constant.

We end with the following formulas:

\[
\varphi_T = \arctan \left( \frac{A_{12} \xi \alpha_2 k_2 + \alpha_1 k_1/\xi}{\alpha_1 \alpha_2 k_1 k_2} \right) - \begin{cases} 
\pi, & A_{12} < 0 \\
0, & A_{12} > 0
\end{cases}
\]

\[
T = \frac{4 \alpha_1^2 k_1^2}{(\alpha_1 \alpha_2 k_1 k_2/A_{12})^2 + (\xi \alpha_2 k_2 + \alpha_1 k_1/\xi)^2}.
\]

Based on Eqs. (1) we will obtain the phase shift and intensity of the transmitted SW in dependence on \( A_{12} \), which through Eq. (2) depends also on \( D \). The assumption of the infinitely narrow interface is justified whenever \( \lambda \ll D \), as we will see the condition fulfilled to the large extent in presented in the Letter results. The analytical results from Eq. (4) will be directly compared to micromagnetic simulations results.

Fig. 1. Schematic of the system design with a focusing metasurface. The system involves the two ferromagnetic materials with a non-magnetic material of varying width along the interface between them to focus a plane wave at a single point.

To demonstrate the MS for SWs we perform the finite-difference time-domain micromagnetic simulations with Mumax\(^3\) solver, taking into account exchange and dipolar interactions with damping neglected [31]. We assume homogeneous Co film of 10 nm thickness, magnetised to saturation by the external magnetic field 1 kOe oriented along the \( y \) axis. We excite the harmonic SW plane waves of 80 GHz frequency (which relate to the 28 nm of the wavelength) on the left side of the interface (1 \( \mu \)m from the interface, see Fig. 1) and record the data from the whole structure in the following time steps until the waves reach the opposite side of the simulation area where the absorbing boundary has been implemented [32]. The interface has been implemented as the unit cell of the discretized mesh with the artificially introduced exchange interaction \( A_{12ex} \) scaled suitably to use the \( A_{12} \) parameter from the analytical model [33].

\( A_{12ex} = \Delta A_{12} M^2_{0}/2A_{ex} \), where \( \Delta \) is the lateral size of the unit cell in micromagnetic simulations. For the simulations the following parameters, which are related to Co, have been taken: \( M_{01} = M_{02} = M_0 = 1422 \) kA/m, \( A_{ex1} = A_{ex2} = A_{ex} = 3 \times 10^{-11} \) J/m [32] uniaxial magnetocrystalline anisotropy constant \( K_1 = K_2 = K = 4.5 \times 10^6 \) erg/cm\(^3\) [34] and \( \Delta = 0.75 \) nm.

In Fig. 2 we show the phase-shift of the SW transmitted through the interface in dependence on the interlayer exchange coupling constant \( A_{12} \). The positive and negative values of the parameter \( A_{12} \) relate to ferromagnetic and antiferromagnetic coupling between the ferromagnetic materials, respectively. We see, that the strength of the \( A_{12} \) determines the value of the phase shift, and also the intensity of the SW transmission (superimposed in
the figure with the color map). $T$ decreases with $|A_{12}|$ approaching 0, i.e., a limit which indicates exchange decoupled materials. Interestingly, at small $A_{12}$ the most significant changes of the $\varphi_T$ with $A_{12}$ are observed, indicating the region suitable for exploitation. Micromagnetic simulation results are in very good agreement with the analytical model (green line).

According to Eq. (2), the interlayer exchange constant depends on $D$. This dependence for Co/Cu/Co multilayer is shown in Fig. 3 (a), where very good agreement with experimental results and perfect with micromagnetic simulations are demonstrated. The SW phase shift and transmittance related to the dependence of $A_{12}$ on $D$ are shown in Fig. 3 (b) and (c), respectively. The largest change of the phase shift and significant transmission variations are observed in the widths of the spacer up to 2 AL, where $A_{12}$ also changes significantly. These results clearly show that the spacers of the few atomic layers thickness can determine the phase acquired by the transmitted SW in the range from -90° up to 90°. This range of $A_{12}(D)$ will be further exploited to design lens for SWs.

The phase gradient introduced along the line of the MS will provide an effective wave vector along the interface that is imparted to the transmitted and reflected waves. This effect is described by the generalized Snell law[2] which can be derived by considering the conservation of the wave vector along the interface:

$$\sin(\theta_i)n_i - \sin(\theta_t)n_t = \frac{\lambda}{2\pi} \frac{d\varphi_T}{dy},$$  \hspace{1cm} (5)

where $\theta_i$ and $\theta_t$ are angles of incident and refracted waves with respect to the normal to the surface, and $n_i$, $n_t$ are refractive indexes of the materials. This additional momentum defined in the phase gradient term $\frac{d\varphi_T}{dy}$ is realized in photonic MS by nanoantennas placed on the surface. The Snell’s law describes also the refraction of SWs, therefore, the phase discontinuity approach used in photonics can be expected to give similar effects for SWs. We propose to use the exchange coupling introduced above to provide a phase discontinuity for SWs.

The generalized Snell law indicates, that the transmitted and reflected waves can be bent into the arbitrary direction in the receptive half space, depending on the direction and the magnitude of the phase gradient on the MS, as well as the refractive indices of the surrounding media[29]. Thus, the phase-shift on MS can be used to design surfaces offering different functionalities, including focusing.

The flat interface will work as a meta-lens, if for a given focal length $f$, the phase shift $\varphi_T(y)$ imposed on every point of the interface along the $y$ axis satisfy the following equation:

$$\varphi_T(y) = \frac{2\pi}{\lambda} \sqrt{(y-y_0)^2 + f^2 - f}. \hspace{1cm} (6)$$

To impose this flat lens phase profile $\varphi_T(y)$ along the interface we assume, that the width of the spacer $D$ will be dependent on $y$. In micromagnetic simulations instead of $D(y)$ we used a discrete change of the exchange scale parameter $|A_{12}(y)|$, with the step of 1 nm[29] which is directly related to $D(y)$ according to Eqs. (1) and (2).

Fig. 2. The color scaled dots represent the simulated dependence of the phase shift between the transmitted and incident SWs on the $A_{12}$ exchange parameter for 80 GHz waves. Color and size of the dots corresponds to the intensity of the transmitted SWs. Results of the analytical model have been presented as a green line.

Fig. 3. Dependence of the selected parameters on the number of atomic layers on the interface: (a) the strength of the interlayer coupling constant $J$ (the exchange coupling parameter $A_{12}$ used in micromagnetic simulations), (b) spin wave phase shift and (c) transmittance as a function of $D$. In (a) the $J(D)$ dependence is based on the theoretical estimation for RKKY-coupling parameter proposed by P. Bruno in Ref. [26]. Bright dots correspond to the micromagnetic simulation results, while violet dots correspond to the experimentally measured values[29]. Bold lines in the shadow rectangle in the panels (b) and (c) indicate the values used to design lensing metasurface.
To design meta-lens based on the interlayer coupling at Co/Cu/Co interface shown in Fig. 3(a), we select its width in the range up to 3 ALs. This allows to shape the phase of the transmitted wave in a relatively wide range with still valuable transmission, see, Fig. 3(b) and (c), although, the angles of the phase shift from 0 to 65° are not accessible. To increase the intensity of the focused SWs we use 14 repetitions of the available angles along the interface, distributed symmetrically on both sides of the MS center. The designed profile of the $\varphi(y)$ to focus the wave along the bisector line of the MS at 150 nm distance from MS is shown in Fig. 4. The spatial dependence of the transmission of SWs through the MS is indicated by the color map. Interestingly, the whole transmittance is about 40% and this low value is related to the large area of the zero-transmission regions marked with the dark horizontal lines (at $\varphi(y) = 0$).

The numerical demonstration of our focusing MS with spatially designed $A_{12}(y)$ to obtain a localized and intense focal spot of SWs at $y = 0$, $x = 200$ nm (according to the phase profile shown in Fig. 4) is shown in Fig. 5 (a). These results clearly show that focusing of SWs is possible and the intensity at the focal spot significantly stands out of the SW landscape. In the focal plane, the SW intensity at the spot features a 7 times raised amplitude in comparison to the whole interference pattern at the right side of the sample. The half-width at full maximum of the spot size is 20 and 10 nm along the $x$ and $y$ axis, respectively, which is below the wavelength of the SWs concentration is dependent of the design, available angles of the phase shift, and is also limited by the loss of transmission through the interface. Those parameters can be optimized in the each case of the metalens.

In conclusion, we have shown in micromagnetic simulations the focusing of the exchange SWs in thin Co film by ultra-narrow flat phase-shifting metasurface. The proposed metasurface exploits the RKKY exchange coupling through Cu spacer between Co thin ferromagnetic films to achieve required phase shift profile along the interface axis. We show this for the RKKY exchange coupling estimated on the experimental data points and interpolated to a continuous variation. We have also shown, that the substantial focusing of SWs can be achieved at the arbitrary selected point on the films, by proper design the phase profile. The effectiveness of the focusing depends on the correct representation of the phase profile along the interface. Moreover, based on the analytical model we conclude that the other mechanism of the magnetization coupling between ferromagnets can be also exploited for designing the metasurfaces for SWs.

**I. ACKNOWLEDGMENTS**

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We used a uniformly discretizing grid with the cell size 0.5-1.0 nm $\times$ 0.5-1.0 nm $\times$ 10 nm along the x, y, and z axis, respectively.
Supplemental Materials: Ultra-narrow spin wave metasurface for focusing application

II. ANALYTICAL MODEL

We consider the system of two semi-infinite ferromagnetic media FM-1 and FM-2, separated with an ultra thin interface in the $yz$-plane in the uniform static external magnetic field $H_0$, which is parallel to the $y$-axis. To calculate the phase shifts and intensities of the SW refracted from the interface we use the Landau-Lifshitz (LL) equation that describes magnetization vector dynamics in the effective magnetic field $H_{\text{eff}}$:

$$\frac{\partial M_j}{\partial t} = \gamma |(M_j \times H_{\text{eff}})|,$$ (S1)

where $\gamma$ is a gyromagnetic ratio, $M_j$ indicates the magnetization vector, a damping term is neglected in our derivations. $H_{\text{eff}}$ denotes the effective magnetic field of the $j$-th ferromagnet and is determined as the functional derivative of the system total magnetic energy with respect to the magnetization vector: $H_{\text{eff}} = -\delta w/\delta M_j$.

The total magnetic energy of the system is written as follows:

$$W = \int_V \left[ A_{12} \delta(x) (M_1 \cdot M_2) + \sum_{j=1}^{2} \theta((-1)^j x) w_j \right] dV,$$ (S2)

where the first term denotes the surface energy density at the interface with the interlayer exchange constant $A_{12}$, $\delta(x)$ corresponds to the Dirac $\delta$-function and $\theta(x)$ to the Heaviside step function, $w_j$ corresponds to the bulk energy density of the $j$-th ferromagnet:

$$w_j = \frac{\alpha_j}{2} \left( \frac{\partial M_j}{\partial x} \right)^2 - \frac{1}{2} \beta_j (M_j \cdot n_j) - (H_0 \cdot M_j).$$ (S3)

Eq. (S3) includes the non-uniform exchange energy density term for $j$-th ferromagnet with the $\alpha_j = A_{xx,j}/M_0^2$, where $A_{xx,j}$ being the $j$-th ferromagnet exchange stiffness constant and $M_0$ is the saturation magnetization of the $j$-th ferromagnet. Anisotropy energy density term contains $\beta_j = K_j/M_0^2$, where $K_j$ is the uniaxial magnetic anisotropy constant of $j$-th ferromagnet and the unit vector of the easy axis is $n_j$. The last term in Eq. (S3) represents the $j$-th ferromagnet Zeeman energy density.

In this Letter we relate the parameter $A_{12}$ to the RKKY interactions between the two ferromagnetic materials through the interface $\delta$.[S22–S24\textsuperscript{-}] According to the one dimensional model developed in Ref. [S25\textsuperscript{-}] the coupling energy per unit area with zero angle between the magnetizations of coupling monolayers equals to the interlayer coupling constant $J_{12}$ for the exchange RKKY coupling between two ferromagnetic layers:

$$A_{12} = J_{12}/(M_{01} M_{02}).$$ (S4)

Different models of the oscillatory interlayer exchange coupling between ferromagnetic layers separated by a non-magnetic metal spacer have a common physical mechanism: due to contact of the spacer with the first ferromagnetic layer, the spacer conduction electrons experience spin polarization.[S21\textsuperscript{-}] The latter extends throughout the spacer and interacts with the second ferromagnet and results in effective exchange interaction. Bruno describes the interlayer coupling in terms of the bulk band structure of the spacer material and of spin-dependent reflection amplitudes. However, depending on the kind of assumptions, various models can be explored, i.e., RKKY, tight-binding, hole-confinement, free-electron, s-d mixing etc.[S21\textsuperscript{-}]

For the interlayer exchange coupling between two semi-infinite fcc Co layers with Cu(001) spacer Bruno found a good agreement between the theoretical predictions and the experimental observations (See Fig. 2a in the Letter)[S22\textsuperscript{-}], and the interlayer coupling constant could be presented in a simplified form:

$$J_{12} = \frac{I_1}{D^2} \sin \left( -\frac{2\pi}{\Lambda_1} + \phi_1 \right) + \frac{I_2}{D^2} \sin \left( \frac{2\pi}{\Lambda_2} + \phi_2 \right)$$ (S5)
where $I_1$ and $I_2$ are the coupling strengths of the first and second ferromagnets, $D$ is the spacer thickness, expressed in atomic layers (AL), $\Lambda_1$ and $\Lambda_2$ are the oscillation periods, $\phi_1$ and $\phi_2$ are the phases of the oscillatory coupling.

The experimental results of Johnson et al.\textsuperscript{227} for Co/Cu/Co (001) for two oscillation periods are in agreement with the predictions of the RKKY theory.\textsuperscript{224}

We treat the magnetization dynamics excitation as the small deviations of the magnetization vector $\mathbf{M}_j$ from the ground state $\mathbf{M}_{j,0}$ in the form: $\mathbf{M}_j = \mathbf{M}_{j,0} + \mathbf{m}_j$, where $\mathbf{m}_j$ denotes the magnetization vector dynamical component in the $j$-th ferromagnet. As the magnetization vector is preserved, then the following equality holds $m_{j,x}^2 + M_{j,y}^2 + m_{j,z}^2 = M_{j,0}^2$. Thus $M_{j,y} = \sqrt{M_{j,0}^2 - m_{j,x}^2 - m_{j,z}^2} \approx M_{j,0} - (m_{j,x}^2 + m_{j,z}^2)/2M_{j,0}$, which in the linear approximation with respect of $\mathbf{m}$ results in the magnetization vector $y$-component coincidence with the saturation magnetization value $M_{j,y} \approx M_{j,0}$.

Solutions of the LL-equations in the homogeneous media could be found in the form of plane waves, therefore for FM-1 and FM-2 correspondingly:

$$m_{1x} + im_{1z} = I_1 \exp(i(k_1 x - \omega t) + I_1 R \exp(i(k_1 x - \omega t + \varphi_R)),$$

$$m_{2x} + im_{2z} = I_T \exp(i(k_2 x - \omega t + \varphi_T)),$$

where $I_1, I_{1R}, I_T$ are the amplitudes of the incident, reflected and refracted waves and $\varphi_R, \varphi_T$ are the phase shifts of the reflected and refracted waves, respectively.

For the plane waves with in-plane $k_j$ the dispersion relation in the $j$-th ferromagnetic film of the thickness $L_j$ could be represented by the well-known dispersion relation: \textsuperscript{7}

$$\omega_j^2(k_j) = \left(\omega_{0j} + 4\pi |\gamma| M_{0j}(1 - \psi_j(k_j L_j))\right) \right) \right) \right) \right) \right) \right) \right) \left(\omega_{0j} + 4\pi |\gamma| M_{0j}\psi_j(k_j L_j)\sin^2(\theta),$$

where $\omega_{0j} = |\gamma| \left( H_0 + M_{0j}(\beta_j + \alpha_j k_j^2) \right)$.

\psi(k_j L_j) = 1 - (1 - e^{-k_j L_j}) / k_j L_j, $\theta$ is an angle between SW direction of propagation ($k_j$) and magnetization orientation ($\mathbf{M}_j$) and for the monochromatic SWs \textsuperscript{6} $\omega_1 = \omega_2 = \omega$. Regarding the limit of the exchange SWs, i.e. $k_j L_j >> 1$ and taking into account the Damon-Eshbach geometry ($\theta = \pi/2$) we derive wave vector value in the $j$-th ferromagnet:

$$k_j = \left( \frac{1}{\alpha_j} \left( \sqrt{\frac{\omega_j^2}{\gamma^2 M_{0j}^2} + 4\pi^2 - \frac{H_0}{M_{0j}} - \left(\frac{2\pi + \beta_j}{\omega_{0j}}\right) \right) \right) \right)^{\frac{1}{2}}.$$

Implementation of the total energy \textsuperscript{22} to the LL-equations \textsuperscript{1} in the linear approximation and integration in the infinitely small neighborhood of a point $x = 0$ gives the boundary conditions for the magnetization dynamical components \textsuperscript{22,29}:

$$\begin{cases}
A_{12}(m_{2n} - \xi m_{1n}) + \alpha_1 \frac{\partial m_{1n}}{\partial x} \bigg|_{x=0} = 0,
A_{12}(m_{1n} - \frac{m_{2n}}{\xi}) - \alpha_2 \frac{\partial m_{2n}}{\partial x} \bigg|_{x=0} = 0,
\end{cases}$$

where $\xi = M_{02}/M_{01}$ and $n = x, z$.

To find the transmitted SW phase shift $\varphi_T$ and intensity $T = (I_T/I_0)^2$ using the dynamic magnetization components \textsuperscript{6} and wave vector modulus \textsuperscript{8} we solve the system of equations \textsuperscript{9} for the infinitely thin interface and derive:

$$\varphi_T = \arctan \left( A_{12} \frac{\xi \alpha_2 k_2 + \alpha_1 k_1/\xi}{\alpha_1 \alpha_2 k_1 k_2} \right) - \left\{ \begin{array}{ll}
\pi, & A_{12} < 0 \\
0, & A_{12} > 0 
\end{array} \right.,$$

$$T = \left( \frac{4\alpha_1^2 k_1^2}{(\alpha_1 \alpha_2 k_1 k_2/A_{12})^2 + (\xi \alpha_2 k_2 + \alpha_1 k_1/\xi)^2} \right).$$

Let us introduce the interlayer exchange constant $A_{12}$ as a normalized parameter $A_{ex12}$, which is suitable for further comparison of analytical results with ones from numerical experiment. For that we made an evaluation of the energies from both models in Gaussian and SI units, i.e. $\int_V [A_{12}\delta(x)(\mathbf{M}_1 \cdot \mathbf{M}_2)] dV$ and $\int_V [A_{ex}(\partial \mathbf{m}_j/\partial x_i)^2] dV$.
where $\mathbf{m}_1 = M_1/M_0$ is reduced magnetization. To get dimensionless exchange parameter $A_{ex,12}$ it is convenient to make normalization of uniform exchange constant $A_{12}$ with its limit values $A_{12,c}$ (i.e. uniform exchange constant for Co): $A_{ex,12} = A_{12}/A_{12,c}$. With the few steps of easy calculations one can receive equality: $A_{ex,12} = A_{12}\left(\frac{\Delta \cdot M_{ex,12}^2}{2A_{ex,c}}\right)$, where $A_{ex,c}$ is the CO exchange stiffness constant, $\Delta$ is the unit cell size in micromagnetic simulations.