Spin echo in spinor dipolar Bose–Einstein condensates

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We theoretically propose and numerically realize spin echo in a spinor Bose–Einstein condensate (BEC). We investigate the influence on the spin echo of phase separation of the condensate. The equation of motion of the spin density exhibits two relaxation times. We use two methods to separate the relaxation times and hence demonstrate a technique to reveal magnetic dipole–dipole interactions in spinor BECs.

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NMR and ESR have revealed states of magnetic spin systems [1]. Especially the spectrum broadening contains informations of spin relaxation given by spin-orbit interaction, spin-spin interaction and inhomogeneous magnetic fields. Usually the inhomogeneity hides the other characteristic effects in the system. Spin echo technique, however, is able to remove effects of the inhomogeneous diffusing. Thus the technique has been used in experiments of the magnetic spin system.

The discovery of spin echo in 1950 by Hahn was momentous in the field of magnetic resonance [2]. Hahn observed the recovery of a free induction decay signal of spins by using the combination of three $\pi/2$ pulses, naming the phenomenon “spin echo”. Carr and Purcell later proposed other methods of producing spin echo. The first, called “Method A”, uses the $\pi/2$–$\pi$ pulse sequence, while in “Method B” the echoes are obtained by applying many $\pi$ pulses after a $\pi/2$ pulse [3]. Spin echo has contributed to the development of magnetic resonance, especially as a method of measuring the relaxation time for magnetic resonance imaging (MRI). MRI is used to visualize the structure of human bodies for medical examinations in hospitals. Spin echo is thus a field of physics with practical applications.

In low temperature physics, particularly for superfluid $^3$He, NMR has made significant contributions in determining superfluid phases and revealing vortices and textures [4]. For example, spin supercurrent [5, 6] and Bose–Einstein condensation of magnons [7] have been still under investigation. Spin echo also was found in A and B phases in the superfluid $^3$He [8]. Thus magnetic resonance in superfluid $^3$He has been developing.

Recently, BECs with a magnetic dipole–dipole interaction (MDDI) have received considerable attention [9, 10, 11, 12, 13]. The MDDI may be closely related to the relaxation mechanism of the spins. $^{52}$Cr BECs have been realized by Griesmaier et al. [14]. A $^{52}$Cr atom has a magnetic moment six times larger than that of an alkali atom. Hence, a $^{52}$Cr condensate clearly shows the effects of the MDDI as an anisotropic free expansion which depends on the orientation of the atomic dipole moments [15]. The magnitude of the MDDI can be controlled by modulating the s-wave scattering length [16].

Although the MDDI of alkali atomic BECs is quite small, it would be possible to observe the MDDI in the systems as well as in a $^{52}$Cr BEC [17].

In this letter, we propose spin echo in a spinor BEC for the first time, studying effect of Stern-Gerlach separation and of MDDI by using Carr–Purcell–Meiboom–Gill sequence. Investigating spin echo will provide information on the MDDI, which is exhibited even by $^{87}$Rb BECs.

First, we briefly review relaxation in magnetic resonance and spin echo. A spin $\mathbf{S}$ with gyro-magnetic ratio $\gamma$ precesses with Larmor frequency $\omega_L = \gamma H_0$ under a homogeneous magnetic field $\mathbf{H} = H_0 \hat{z}$, which is described by the equation of motion of the spin $d\mathbf{S}/dt = \gamma [\mathbf{S} \times \mathbf{H}]$. In many body systems of spin, if spins do not have an interaction with others, they precess with same Larmor frequency around a homogeneous magnetic field. On the other hand, The interaction between spins would make gradually the inhomogeneous precession through the local magnetic field. The decay signal, therefore, contains informations about spin-spin and spin-orbit interactions. However, if the magnetic field is inhomogeneous, the decay signal depends on also the inhomogeneity. Therefore it is difficult to choose effects of the interaction from the diffusion. Spin echo technique can remove the effect of inhomogeneity.

We consider that spins without interaction are polarized to the $z$ axis along an inhomogeneous magnetic field whose the spatial magnitude is almost distributed around $H_0$. Then by applying a $\pi/2$ pulse resonant with $\gamma H_0$, the spins tilt to the $y$ axis with precessing. After the pulse, the spins precess on the $x-y$ plane, losing the coherence gradually. Subsequently applying a $\pi$ pulse at $\tau$ after the first pulse reverses the direction of spins, recovering the coherence gradually. The coherence recovers perfectly at $\tau$ after the second pulse. This method is called “spin echo”. On the other hand, the procedure can not give a perfect recovery when there is interaction between spins. The imperfect recovery comes from not the inhomogeneous magnetic field but the interaction.

To obtain spin echo in a $^{87}$Rb BEC trapped by a harmonic potential of frequency $\omega_\perp$, we study the two-dimensional spin-1 Gross–Pitaevskii (GP) equations with...
the MDDI:

\[
\frac{i\hbar}{\partial t} \psi_a = \left( -\frac{\hbar^2}{2M} \nabla^2 + V_{\text{trap}} - \mu \right) \psi_a - g\mu_B H_i S^{i}_{\alpha\beta} \psi_\beta + c_0 \psi_\alpha^* \psi_\beta \psi_\alpha + c_2 S^{i}_{\alpha\beta} \psi_\beta + c_{\text{dd}} \int \frac{\delta \theta_j - 3c\epsilon_j}{|\mathbf{r} - \mathbf{r}'|^3} S_i(\mathbf{r}') S^{i}_{\alpha\beta} \psi_\beta. \tag{1}
\]

Here \( V_{\text{trap}} = M\omega^2(x^2 + y^2)/2 \) is the trapping potential, \( \mu \) the chemical potential, \( g \) the Landé’s g-factor, \( \mu_B \) the Bohr magneton, and

\[
S_i = \sum_{\alpha, \beta} \psi_\alpha^* S^{i}_{\alpha\beta} \psi_\beta \tag{2}
\]

is the component of the spin density vector \( \mathbf{S} \) represented by the spin matrices \( S^{i}_{\alpha\beta} \). The short range interaction constants are \( c_0 = 4\pi\hbar^2(a_0 + 2a_2)/3M \) and \( c_2 = 4\pi\hbar^2(a_2 - a_0)/3M \) given by the s-wave scattering lengths \( a_0 \) and \( a_2 \). The long range interaction constant is \( c_{\text{dd}} = \mu_0 g^2 \mu_B^2/4\pi \) with vacuum magnetic permeability \( \mu_0 \). For \(^{87}\text{Rb} \), the parameters satisfy the relation \( c_{\text{dd}} < -c_0 \ll c_2 \) \(^{12} \). We investigate the spin echo under a gradient magnetic field \( \mathbf{H} = (Gx + H_0)\hat{z} \) with constant \( G \).

The dynamics start from the stationary states with \( |c_{\text{dd}}/c_2| = 0 \) and 0.02 in a gradient magnetic field of magnitude \( 0.98H_0 < |\mathbf{H}| < 1.02H_0 \). Applying a \( \mathbf{H}_{\text{rot}} = H_1 \{ \cos(\gamma H_0\hat{t})\hat{x} - \sin(\gamma H_0\hat{t})\hat{y} \} \) with \( H_1 = 0.1H_0 \), we calculate the GP equation using the Crank–Nicolson method, obtaining the spin echoes and effects of dephasing in the echo.

Spin echo is clearly observed in Fig. 1, showing the dynamics of \( \mathbf{S} \) projected on the \( x-y \) plane for \( c_{\text{dd}} = 0 \) and \( \omega_\perp \tau = 5 \). After the \( \pi/2 \) pulse, the spins are oriented in the -y axis (Fig. 1(a)) and start precessing on the plane. Then, the precessions gradually diffuse because the Larmor frequency is spatially dependent due to the gradient magnetic field (Fig. 1(b), (c)). At \( t = \tau + t_{\pi/2} \) (Fig. 1(d)), the \( \pi \) pulse is applied, which reverses the direction of the spin (Fig. 1(e)). The spins then gradually become coherent (Fig. 1(f), (g)), eventually refocusing at \( t_{\text{peak}} = 2\tau + \pi/2 + \tau \) (Fig. 1(h)). After the echo, the spins start to defocus again. These features are represented in the time development of the expectation values \( \langle S_y \rangle = \int d\mathbf{r}' S^{i}_{\alpha\beta} \psi_\beta \) in Fig. 2. The signal of \( \langle S_y \rangle \) gradually decays from \( t_{\pi/2} \) (a) to \( \tau + t_{\pi/2} \) (d) and then increases from \( \tau + t_{\pi/2} + \pi/2 \) (e) to \( 2\tau + 3t_{\pi/2} \) (h).

To show that this is genuine spin echo, we investigate \( t_{\text{peak}} \) by changing \( \tau \), thus confirming the spin echo for \( |c_{\text{dd}}/c_2| = 0 \) and 0.02. Figure 3 indicates that the relation is satisfied until \( \omega_\perp \tau = 6.5 \) for \( c_{\text{dd}} = 0 \) and 4.5 for \( |c_{\text{dd}}/c_2| = 0.02 \). These deviations from the relation \( |c_{\text{dd}}/c_2| = 2\tau + 3t_{\pi/2} \) can be understood by investigating the equation of motion of spin based on the GP equations. From Eqs. (1) and (2), we obtain the equation of motion

\[
\frac{\partial S_i}{\partial t} = \nabla \cdot \mathbf{J}_i + \gamma \left[ (\mathbf{S} \times \mathbf{H})_i + \frac{c_{\text{dd}}}{\hbar} R^{i}_{\text{dd}} S_\sharp (i,j) \right], \tag{3}
\]

where \( \mathbf{J}_i = \hbar/(2M) S^{i}_{\alpha\beta}(\psi_\beta^* \nabla \psi_\alpha - \psi_\alpha \nabla \psi_\beta^*) \) is term of spin current, which is similar to spin current in superfluid \(^{4}\text{He} \), and \( R^{i}_{\text{dd}} = \int d\mathbf{r}' \frac{\delta \theta_j - 3c\epsilon_j}{|\mathbf{r} - \mathbf{r}'|^3} S_k(\mathbf{r}') \). The first, second, and third terms of Eq. (3) are derived from the kinetic, Zeeman, and MDDI terms of the GP equation. The differences between the circles and triangles in Fig. 3 are due to the dipole term, while their deviations from \( t_{\text{peak}} = 2\tau + 3t_{\pi/2} \) are caused by the spin current given by separation of the condensates due to the gradient magnetic field.

The magnetic resonance of this system is greatly affected by separation of spinor condensates. As shown in Fig. 4 (a), (b), (c), the condensates separate under the gradient field after the \( \pi/2 \) pulse, keeping approximate symmetry between \( x > 0 \) and \( x < 0 \). Applying the \( \pi \) pulse changes the \( \psi_1 \) and \( \psi_{-1} \) components to \( \psi_{-1} \) and \( \psi_1 \) respectively, which reverses \( \mathbf{S} \). The \( \pi \) pulse breaks the symmetry of the distribution of the condensates. After
and the other is the dipolar relaxation time $T_{\text{relaxation time}}$ is characterized by two relaxation times. One is the re-
times, to spinor condensates. The dynamics of Eq. (3)
sequences in NMR and ESR for measuring two relaxation
the echo without the MDDI, as shown in Fig. 3.
ψ
other hand, the MDDI operates as an attraction between
because phase separation is not yet significant. On the
other hand, the MDDI operates as an attraction between
ψ1 and ψ−1 and makes the echo peaks appear earlier than
the echo without the MDDI, as shown in Fig. 3.

We now apply Methods A and B, which are major se-
quen
cquences in NMR and ESR for measuring two relaxation
times, to spinor condensates. The dynamics of Eq. (3)
is characterized by two relaxation times. One is the re-
combination of Methods A and B, shown in Fig. 5. Figure 5 (a)
depicts the result of Eq. (3) for various values of $\tau$. For Method A, the echo peaks decay with $\tau$. When
the condensates separate for large $\tau$, the decay does not
satisfy the exponential behavior. Thus, we fit the ex-
ponential formula to the echo peaks until $\omega_1 \tau = 7.5$, obtaining a diffusion time of $\omega_1 \tau = 40$ when $\langle S_y \rangle/\hbar$ decays to $e^{-1}$. This diffusion time should be equal to $T_{nu}$. Figure 5(b) shows a single exposure of the echo peaks for Method B with $\omega_1 \tau = 3.5$ and $n = 17$. In the simulation, rather than applying the original $\pi/2−\pi−\pi\ldots$ sequence, we apply a $\pi/2−\pi−(-\pi)−(-\pi)\ldots$ sequence, considered by Meiboom and Gill [19], to cancel deviation due to
repeated $\pi$ pulses. In the case of no MDDI, the echo peaks
do not decay, showing that the Method B sequence re-
moves the effect of phase separation. While the signal
with the MDDI decays, the relaxation time should be
$\omega_1 T_{dd} \sim 500$. These results show $T_{nu} < T_{dd}$. Method B enables us to reveal the MDDI through $T_{dd}$
in spinor dipolar BECs even if the MDDI is too weak to be
easily detected.

In conclusion, we have numerically realized spin echo
in a trapped $^{87}$Rb BEC by calculating the spin-1 two-
dimensional GP equation. We have investigated how the
spin echoes are affected by the phase separation and the
dipole–dipole interaction. The equation of motion of the
spin density (Eq. (3)) derived from the GP equation shows two relaxation times $T_{nu}$ and $T_{dd}$. In order to
distinguish $T_{dd}$ from $T_{nu}$ by using spin echo, we introduce
Methods A and B, obtaining the relation $T_{nu} > T_{dd}$.

The study of spin echo of spinor BECs will help develop
the field of magnetic resonance in cold atomic BECs sys-
tem and may be applicable to MRI in these systems.
FIG. 4: Density profiles $|\tilde{\psi}_\alpha(y = 0)|^2 = a^2_\alpha |\psi_\alpha(y = 0)|^2/N$ with $a_\alpha = \sqrt{\hbar/M\omega_\perp}$ and $\tilde{x} = x/a_\alpha$ without the MDDI just before the $\pi$ pulse ((a), (b), (c)) and just after the $\pi$ pulse ((d), (e), (f)) for $\omega_\perp \tau = 5, 7.5,$ and $12.5$.

FIG. 5: (a) Results of Method A, showing multiple exposures of $\langle \hat{S}_y \rangle /\hbar$ with (circles) and without (triangles) the MDDI at $t_{\text{peak}}$ for various $\tau$, where $\omega_\perp \tau = 3.5, 4.0, \ldots, 7.5, 10, 12.5, 15, 17.5$. The black line shows the Carr–Purcell form $\exp(-bt - c^2t^3)$ with $b/c = 0.072$. (b) Results of Method B, showing a single exposure of the signal of peaks with $\omega_\perp \tau = 3.5$ and $n = 17$. The line $\exp(-bt)$ is fit to the early peaks, which are little affected by $\exp(-c^2t^3/17^2)$. We obtain $\omega_\perp T_{dd} \sim 500$.

Although this work is limited to the two-dimensional case, three-dimensional spinor BECs, especially with the MDDI, should show interesting features depending on geometry. A study of three-dimensional spinor BECs will be reported soon elsewhere. We also would like to study spin wave and spin transport as analogy to magnetic resonance of superfluid $^3$He.

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