Weakened Constraints from $b \to s\gamma$ on Supersymmetry Flavor Mixing Due to Next–To–Leading–Order Corrections

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We examine the process $B \to X_s\gamma$ in minimal supersymmetry (SUSY) with general squark flavor mixings. We include all relevant next–to–leading order (NLO) QCD corrections and dominant NLO SUSY effects from the gluino. We find that gluino–squark corrections to down–type quark masses induce large NLO corrections to the dominant Wilson coefficients whose size is often similar to those at LO, especially at large $\tan \beta$. For $\mu > 0$, destructive interference, and suppression by the renormalization group running lead to a “focusing effect” of reducing the size of gluino corrections to the branching ratio, and also of reducing the LO sensitivity to flavor mixings among squarks. Constraints from $B(B \to X_s\gamma)$ on the SUSY–breaking scale can become significantly weakened relative to the minimal flavor violation case, even, at large $\tan \beta$, for small flavor mixings. The case of $\mu < 0$ can also become allowed.

Introduction.— The radiative decay $B \to X_s\gamma$ provides a powerful tool for testing various extensions of the standard model (SM), such as supersymmetry. This is because in $b \to s\gamma$ “new physics” effects can appear at a one–loop level, which is the same as the lowest–order SM flavor changing neutral current (FCNC), and can therefore be of comparable size$^3$. However, as both experimental results$^2$ and a SM prediction at the full next–to–leading–order (NLO) QCD level$^2$ have reached an error of $\sim 10\%$, 

$$B(B \to X_s\gamma)_{\text{exp}} = (3.34 \pm 0.38) \times 10^{-4}, \quad (1)$$

$$B(B \to X_s\gamma)_{\text{SM}} = (3.70 \pm 0.30) \times 10^{-4}, \quad (2)$$

it has become clear that little room is now left for new physics contributions. This has in turn been used to impose severe constraints on flavor–violating (FV) interactions beyond the SM. These are usually related to some scale $\Lambda$ at which new physics appears. In the case of low–energy supersymmetry (SUSY), $\Lambda$ is set by the SUSY–breaking scale $M_{\text{SUSY}}$, which is expected to remain within $\mathcal{O}(1 \text{ TeV})$ on the grounds of naturalness.

The SM contribution to $b \to s\gamma$ involves a one–loop exchange of the top quark and the $W^–$. In 2–Higgs doublet models (2HDM), there is, in addition, a constructive contribution from the charged Higgs $H^–$. A complete NLO analysis in the SM has been performed in several stages and recently completed in$^4$, and, likewise, in the 2HDM$^5$. In SUSY, additional one–loop diagrams come from an exchange of the charged Higgs–top $H^–t$ and the chargino–stop $\chi^–t$. The latter contributes constructively (destructively), depending on whether the Higgs/higgsino mass parameter $\mu > 0$ or $< 0$, respectively. Detailed studies of SUSY contributions to $B(B \to X_s\gamma)$ have been performed beyond the LO$^6$, $^7$, $^8$, $^9$, $^{10}$, $^{11}$. In particular, dominant NLO contributions have been calculated$^9$, $^{10}$, $^{11}$ and shown to be important. They are enhanced by factors involving large tan $\beta = \langle H^0_2 \rangle / \langle H^0_1 \rangle$ [the ratio of the neutral Higgs VEVs] and $\log (M_{\text{SUSY}}/m_W)$.

The good agreement between the ranges$^1$ and$^2$ has also been used to derive stringent constraints on the mass spectra of the superpartners in specific popular models, like the minimal supersymmetric SM (MSSM), or the constrained MSSM (CMSSM). For example, in the CMSSM, the case of $\mu < 0$ has been shown to be ruled out, except for the very large common gaugino $m_{1/2}$ and scalar $m_0$ masses ($\sim$ few TeV), where SUSY contributions become tiny. For $\mu > 0$, stringent lower bounds of a few hundred GeV have also been derived on $m_{1/2}$ and $m_0$, especially at large $\tan \beta$$^{12}$.

Such bounds, even if model dependent, have clear implications for searches for SUSY in accelerators, for the neutralino as a dark matter weakly interacting massive particle (WIMP), and in other nonaccelerator processes. For example, in dark matter searches the constraint from $b \to s\gamma$ often forbids larger values of the spin–independent scattering cross section of WIMPs on proton $\sigma^S_{p}\,^t$ that would otherwise be accessible to today’s experimental sensitivity$^{13}$.

The above conclusions have often been reached by assuming, sometimes implicitly, that the mixings among the mass eigenstates of squarks closely resemble the Cabibbo–Kobayashi–Maskawa (CKM) structure of the quark sector. This scenario, often called minimal flavor violation (MFV), is perhaps the simplest way of addressing the nagging flavor problem of SUSY. FV interactions are in general not forbidden by gauge symmetry and, without assuming any organizing principle, can lead to exceeding experimental bounds by many orders of magnitude$^{14}$. The mixings of the first two generations of squarks are strongly constrained by the $K^0 – \bar{K}^0$ system but bounds on mixings with the third generation are much weaker. In the case of $b \to s\gamma$, rather stringent constraints have been derived, in the LO approximation, on the magnitude of possible FV terms beyond MFV in specific scenarios for flavor structure, but shown to be highly
model dependent \cite{15,16}. The question thus arises about the robustness of constraints on SUSY derived in MFV or other specific scenarios.

In this Letter, we examine the process $b \to s \gamma$ in the MSSM with general flavor mixing (GFM) in the down-type squark sector. In addition to the full NLO QCD corrections from the SM+2HDM \cite{12,13,14,15}, we introduce NLO QCD resummation above $m_W$ and threshold corrections from the heavy gluino field $\tilde{g}$ in the presence of all mixings in the down–squark sector. It is reasonable to expect a hierarchy among the squarks.) It is reasonable to expect a hierarchy among the squarks. It is reasonable to expect a hierarchy among the squarks. (We assume no large mass splittings for the other states. (We assume no large mass splittings for the other states. (We assume no large mass splittings for the other states. (We assume no large mass splittings for the other states.

In SUSY, the new mass scale $M_{\text{SUSY}}$ is set by $m_{\tilde{g}}$ and the squark mass $m_{\tilde{q}}$, which tend to be heavier than the other states. (We assume no large mass splittings among the squarks.) It is reasonable to expect a hierarchy $M_{\text{SUSY}} > m_{\tilde{g}}$ just like one has $m_{\tilde{W}} > m_{\tilde{g}}$. In order for QCD corrections beyond the LO to match the underlying SUSY theory and the effective theory at $m_{\tilde{g}}$, we extend the treatment of \cite{8,11} to the case of GFM. The usual procedure, which we follow here, is to assume that $m_{\tilde{g}} \sim m_{\tilde{q}} \sim m_{\text{SUSY}}$, while all the other SUSY states play a role at $m_{\tilde{g}}$ \cite{11}. We thus do not resum the logarithms of $m_{H}/m_{\tilde{W}}$ and $m_{\tilde{g}(3)}/m_{\text{SUSY}}$. The resummation of QCD correction between $m_{\text{SUSY}}$ and $m_{\tilde{W}}$ is performed with the NLO anomalous dimensions with six quark flavors \cite{20}. We compute two–loop gluon corrections to the $H^-$ contributions to $C_{4,7,8}$ at $m_W$, and those from the $\chi^-$, $\chi^0$ and $\tilde{g}$ fields to $C_{7,8}$ at $m_{\text{SUSY}}$ as in \cite{8}. We have verified that their contributions to $C_{1-6}$ and their mixing to $C_{10}$ are numerically negligible when using available five–flavor anomalous dimensions \cite{21}. We also neglect operators outside of the SM basis. At LO they are numerically subdominant \cite{17}, and we could not identify any enhancement mechanism beyond the LO. At the NLO–level, SUSY QCD contributions come from $\mathcal{O}(\alpha_s)$ corrections involving the gluino. For the states at $m_{\tilde{W}}$ ($W$, $\phi^-$ and $H^-$), these are absorbed into the effective vertices by integrating out the $\tilde{g}$ field \cite{8}. We have calculated these vertices with GFM in the down–squark sector \cite{22}.

For the fields at $m_{\text{SUSY}}$, this effective approach cannot be applied. Instead of calculating a full set of two–loop diagrams (a rather formidable task), we include finite threshold corrections to Yukawa matrices, which become important at large $\tan \beta$ \cite{10,11,23}. As described in detail in \cite{22}, we first integrate out the gluino and the squarks to calculate effective quark mass matrices in the super-CKM (SCKM) basis. By identifying these with physical ones, we extract the Yukawa couplings that appear in the higgsino–quark–squark vertices and in the F–term contributions to squark mass matrices, and further perturbatively include “$\tan \beta$ resummation” \cite{10,11}, generalized to GFM \cite{22} for self–consistency.

General flavor mixings.— In the absence of an underlying theory of flavor in the squark sector, all the entries of the $6 \times 6$ mass matrix square of down–type squarks are in general nonzero (and likewise for the up–type squarks). It is natural to break it into four $3 \times 3$ submatrices of the LL, LR, RL and RR sectors. For the 2nd to 3rd generation mixing of relevance to $b \to s \gamma$, the departure from the MFV case can be parameterized by introducing $\delta^d_{LL} = (m_{d,LL}^2)^{23}/(m_{d,LL}^2)^{23}(m_{d,LL}^{23})^{33}$ and $\delta^d_{LR} = (m_{d,LR}^2)^{23}/(m_{d,LL}^2)^{23}(m_{d,RR}^{23})^{22}$, and analogously for $\delta^d_{RL}$ and $\delta^d_{RR}$, where $m_{d,LL}^{23}$ and $m_{d,RR}^{23}$ are, respectively, the $3 \times 3$ soft mass matrices $m_{Q(D)}^{23}$ and the soft trilinear term matrix $A^d_{23}$ rotated into the SCKM basis \cite{22}.

In the case of GFM, these mixings induce new 1–loop contributions to $b \to s \gamma$, with one internal line involving $\tilde{g}$ (or $\chi^0$) and the other from $\tilde{b}$ turning into $\tilde{s}$ due to the 2nd to 3rd generation mixing. We insert the $\delta^d$s at $m_{\text{SUSY}}$ and all the corrections described above are generalized to the case of GFM. The $6 \times 6$ mass matrices are diagonalized numerically, instead of using the less accurate mass–insertion approximation.

Focusing effect.— We find that, relative to the LO MC
at $\mu_W$. NLO corrections generally reduce SUSY contributions (mostly from the gluino) to $B(B \to X_s\gamma)$. As a result, constraints on $m_{\tilde{g}}$ and related SUSY parameters (like the $\chi^\pm$ and $\chi^0$ soft mass parameters), and on the FV couplings become considerably relaxed. This focusing effect becomes particularly strong at large $\tan \beta$. This is illustrated in Fig. 1. We present ranges of the $B(B \to X_s\gamma)$ as a function of the common squark mass, $m_{\tilde{q}} \equiv \sqrt{(m_{\tilde{Q},U,D})_i^2}$ in the case of MFV ($\delta_{LR}^d = 0$) and for $\delta_{LR}^d = \pm 0.02$. We scan over the ranges of $1 < m_{\tilde{q}}^2/m_{\tilde{g}}^2 < 2$ and apply collider bounds on superpartner masses. The brown (darker) bands are obtained by using our expressions for dominant NLO–level contributions, while the green (light) bands correspond to applying the approximation of the LO matching at $\mu_W$. One can clearly see a strong suppression of the SUSY contribution at the NLO–level. While some focusing is already present in the MFV case, the effect becomes strongly enhanced in the case of GFM.

Focusing comes from two sources. First, RGE evolution between $m_{\text{SUSY}}$ and $\mu_W$ generally reduces SUSY contributions. For example, for $m_{\text{SUSY}} \sim 1$ TeV, one finds $\alpha_s(m_{\text{SUSY}})/\alpha_s(\mu_W) \sim 0.8$, $C_7(\mu_W) \sim 0.8C_7(m_{\text{SUSY}}) + 0.06C_8(m_{\text{SUSY}})$ and $C_8(\mu_W) \sim 0.8C_8(m_{\text{SUSY}})$. (The NLO QCD matching condition at $m_{\text{SUSY}}$ causes some additional reduction.) Second, at $\mu > 0$ we find a remarkable tendency of “alignment” between the effective $b$–quark mass and the corresponding penguin operator at large $\tan \beta$. This reduces the gluino contribution to $C_{7,8}$ and, as a result, the SUSY contribution to $B(B \to X_s\gamma)$ in the case of GFM. The essential point is that flavor mixing in the soft SUSY–breaking terms induces flavor off–diagonal elements in the effective quark mass matrices through gluino–squark loops. In order to rediagonalize them, one also rotates the Yukawa couplings that appear in the higgsino vertices and squark mass matrices, thus inducing $O(\alpha_s)$ corrections to them in SCKM basis. The resultant $C_{7,8}$ is also “rotated”. The effect is enhanced by large $\tan \beta$. For example, $\delta_{RR}^d$ (or $\delta_{RL}^d$) induces a correction proportional to $\delta m_{\tilde{\chi}^0_i}/\cos \beta$ in the higgsino vertex. This generates a diagram with an exchange of $\tilde{c}_L$ and $\chi^-$, and another one with the $\tilde{b}_R \tilde{s}_L$ (with mass insertion $\delta m_{\tilde{b}_R}^\mu \tan \beta$ ) and the gluino in the loop, as shown in Fig. 2. Both are proportional to $\tan \beta$ and the rotation occurs in the direction that reduces the LO effect at $\mu > 0$.

If we take $\delta_{RR}^d$ (or $\delta_{RL}^d$) instead, analogous diagrams are induced for the $b_L \to s_R$ transition, which reduces the gluino contribution at $\mu > 0$ in a similar manner. The first diagram can be even larger than the LO gluino effect if the “diagonal part” of the chargino penguin is arbitrarily large. For more details see [22]. At $\mu < 0$, the rotation described above occurs in the direction which increases the LO effect. This competes with the reduction caused by the RGE evolution. As $m_{\text{SUSY}}$ increases, the RGE effect increases, while the quark mass correction decreases. At $m_{\text{SUSY}} \sim 1$ TeV, the two effects have similar size for both signs of $\mu$.

**Relaxation of constraints on SUSY and FV terms**—The constraints obtained in the MFV scenario [10, 11] are no longer valid with GFM, and their robustness is of particular phenomenological interest. We illustrate this by including the above large NLO–level corrections. In the three panels of Fig. 3 we show the contours of $B(B \to X_s\gamma)$ in the plane of $m_\tilde{g}$ and $\delta_{LR}^d$ (left panel) and $\delta_{LL}^d$ (middle and right panels) for $\tan \beta = 45$, $m_{\tilde{q}}^2/m_{\tilde{g}}^2 = 2$, $A_U = -m_{\tilde{q}}$, and $\mu > 0$ (left and right panels) and $\mu < 0$ (middle panel). The light green (light grey) and dark yellow (dark grey) bands agree with the experimental range [11] at the $1\sigma$ (2$\sigma$) C.L. Larger departures are denoted as “excluded”. In the case of $\delta_{LR}^d$, in MFV one finds $m_{\tilde{g}} \gtrsim 800$ GeV at the $2\sigma$ C.L. for $\mu > 0$.

In contrast, even relatively small nonzero values of $|\delta_{LR}^d| \simeq -0.04$ remove the constraint from $b \to s\gamma$ altogether. The effect is even more striking for $\delta_{LL}^d$ and $\mu < 0$ where relatively light squark becomes again allowed. We have checked numerically that, at $\mu > 0$, constraints on

\[ \begin{align*}
&\tilde{c}_L \quad \mu \langle H_1^0 \rangle M_2 \quad \tilde{s}_L \\
&-\delta m_{\tilde{b}_R}^d \langle H_1^0 \rangle^{-1} \chi^- \\
&\tilde{b}_R \quad M_2 \quad \tilde{s}_L \quad \mu \langle H_1^0 \rangle M_2 \\
&\delta m_{\tilde{b}_R}^d \langle H_1^0 \rangle^{-1} \chi^- \\
\end{align*} \]

**FIG. 2:** Examples of $\tan \beta$–enhancement in GFM.
FV terms at NLO become also considerably relaxed. For example, in Fig. 3 at \( m_\tilde{g} = 1000 \text{ GeV} \) and \( \mu > 0 \), we find \( \delta_{LR}^4 \text{(NLO)} / \delta_{LL}^4 \text{(LO)} \approx 7 \). Similar GFM deconstraining can also be obtained by introducing nonzero \( \delta_{LL}^4 \) instead, for both \( \mu > 0 \) and \( \mu < 0 \) but with a milder NLO effect. It is interesting to note the existence of two branches of allowed solutions in Fig. 3. By simultaneously allowing for more than one \( \delta^4 \) to become nonzero, one can relax the constraint from \( b \to s\gamma \) even further, even at LO. In addition, in general FV contributions can also come from the up-squark sector. We have checked that this effect is numerically less important.

Finally, we must reemphasize that in our analysis we assume \( M_{\text{SUSY}}/m_W \) larger than a few and compute only the leading NLO effects, which are enhanced by \( M_{\text{SUSY}}/m_W \), large \( \tan \beta \), or flavor mixing. We also remain in the SM basis of operators, instead of considering the full operator basis of the MSSM. Despite this, we do believe that the results presented here should remain as the dominant effect in a more complete study.

**Conclusions and outlook.**— Contributions from new physics to rare processes like \( b \to s\gamma \), do not necessarily have to be suppressed by the largeness of the related effective mass scale beyond the SM. Instead, this can be caused by new effects beyond the LO. In the case of SUSY with GFM, at the NLO level we have pointed out the existence of one such suppressing effect, which we call focusing. Despite this, \( b \to s\gamma \) still maintains strong sensitivity to even small deviations from MFV (although less so than at LO) of especially the flavor mixing terms \( \delta_{LL}^4 \) and \( \delta_{LR}^4 \). This puts into question the robustness of the commonly assumed constraints on \( M_{\text{SUSY}} \) in MFV. It appears that the alignment mechanism is generic to processes involving chirality flip. We are now exploring its role in other FCNC processes.

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