Error estimation of the multi-angle phase triangulation method for measuring the three-dimensional geometry of convex and extended objects

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Abstract. The work describes the estimation of the error of the multi-angle phase triangulation method for measuring the three-dimensional geometry of convex and extended objects. Theoretical estimates of the measurement error are made and the results of mathematical modeling are presented. It is shown that if the measurement error by the phase triangulation method has normal distribution, the results of stitching a three-dimensional surface will also have a normal distribution with similar characteristics. The multi-angle phase triangulation method for measuring the three-dimensional geometry of convex and extended objects provides the necessary level of measurement accuracy and can be used to obtain the three-dimensional geometry of complex-shaped convex and extended objects in industrial conditions.

1. Introduction

Improving the methods of three-dimensional diagnostics of complex objects is relevant for energy technologies because of the increasing requirements for the geometric accuracy of energy system elements [1-3].

When measuring the three-dimensional geometry of complex-shaped convex and extended objects by phase triangulation methods [4-5], it is difficult to measure the entire surface of the object. To solve this problem, multi-angle methods of measuring the three-dimensional geometry are used. Basically, all known solutions can be reduced to the methods involving the use of special markers for linking various measured fragments in the final three-dimensional model [6]; methods involving the rotation of the measured object at a predetermined angle [7]; methods involving rotation of the elements of the optical system [8]; methods using spatial cross-correlation algorithms to determine the parameters of combining fragments of the measured surface [9].

Methods using markers have a rather low accuracy of combining the measured fragments, limited by the number of markers used and the quality of their manufacture. In addition, the experimenters are not able to use a very large number of markers, since with an increase in their number on the measured surface, the quality of the measured three-dimensional model decreases. Nevertheless, the advantage of this method is its invariance to the shape and texture of the surface of the measured object.

The development of the method using markers is the multi-angle measurement method using light markers [10]. The essence of the method is to combine the measured fragments of the surface of the measured object with markers that are not directly applied to the surface of the object, but illuminate local areas of the surface that should be clearly distinguishable from both measurement angles. This
approach is more universal than the use of classical markers; however, it requires additional recognition of the coordinates highlighted by a marker on the surface of the object.

In the framework of this work, a multi-angle method of phase triangulation was used, which allows stitching of the measured regions based on information about the local illumination parameters on the surface of the measured object. The essence of the method is as follows. Two photodetectors, whose position relative to each other is fixed, and one source of structured illumination are used. Measurement of three-dimensional geometry is carried out sequentially, while at each step a fragment of the surface is measured. The measurement is carried out on the basis of the principle of a stereo pair with the help of two photodetectors, and the structured illumination generated by the optical radiation source is used as the analyzed texture. As a result, the highest resolution of the texture is ensured and the measurement error is minimized.

The combination of the measured fragments is performed similarly to the method of reference markers on the measured surface [10]. The difference lies in the fact that the markers are the parameters of structured illumination observed by photodetectors at each point on the measured surface. This approach minimizes the alignment error when measuring the three-dimensional geometry of extended and convex objects. At the moment, there are no works aimed at estimating the error of the multi-angle phase triangulation method.

The purpose of this work is to evaluate the error of the multi-angle phase triangulation method when implementing the procedure for stitching fragments measured from different angles.

2. Method description

When combining fragments of the surface, the error estimation can be performed based on the assessment of the maximum deviation of the point position after combining the measured fragments, or by the deviation of the spatial position of the geometric center of the cloud of measured fragments.

The theoretical error estimation is based on the assumption that the measured fragments have deviations with a normal distribution.

Let $C_{l1}$ and $C_{l2}$ be 2 clouds of points defining a fragment measured from different angles. The relationship of the measured points will be as follows:

$$p_{i}^{c_{l2}} = [(p_{i}^{c_{l1}} + E_{i}) \cdot M],$$

where $p_{i}^{c_{l1}}$ is the vector defining the i-th measured point of the cloud $C_{l1}$, $p_{i}^{c_{l2}}$ is the vector defining the i-th measured point of the cloud $C_{l2}$, $E_{i}$ is the deviation of the measurement result of the i-th point of the cloud $C_{l1}$ relative to the i-th point of the cloud $C_{l2}$, $M$ is the transformation matrix of the coordinate system of the cloud $C_{l1}$ to the coordinate system of the cloud $C_{l2}$.

As a result of the algorithm for stitching the measured fragments of the surface, the matrix $M'$ transforms the coordinate system of the cloud $C_{l1}$ to the coordinate system of the cloud $C_{l2}$. That is, the coordinates of the i-th point after the coordinate transformation of the cloud $C_{l1}$ in the coordinate system of the cloud $C_{l2}$ are:

$$p_{i}^{c_{l2}'} = p_{i}^{c_{l1}} \cdot M'.$$

Cloud alignment error can be expressed as a random variable by analogy with $E_{i}$:

$$E_{i}' = 
\begin{bmatrix}
X(p_{i}^{c_{l2}'}) - X(p_{i}^{c_{l2}}) \\
Y(p_{i}^{c_{l2}'}) - Y(p_{i}^{c_{l2}}) \\
Z(p_{i}^{c_{l2}'}) - Z(p_{i}^{c_{l2}})
\end{bmatrix},$$

where $X$, $Y$, $Z$ are the corresponding Cartesian coordinates of the point in the report system. The cloud alignment error can be estimated as the maximum deviation of the coordinates of the point $p_{i}^{c_{l2}'}$ from the coordinates $p_{i}^{c_{l2}}$.
where \( N \) is the number of points in the clouds Cl1 and Cl2. Considering that the Cl2 point cloud was originally built by adding a random vector \( E_i \), then with the correct operation of the algorithm for stitching the measured surface fragments, the condition can be considered true:

\[
Err_{\text{max}} = \max(\{|E'_i|\}_{i=1,N}),
\]

If the random variable \( E_i \) has a normal distribution, then the random variable that determines the deviation of the coordinates of the points in different clouds after stitching will have a normal distribution with similar parameters.

3. Experimental results
To test the performance, mathematical modeling of the operation of the cloud stitching algorithm was performed. A flat cloud \( C_1 \) of randomly distributed points on the same plane was generated in a limited volume. The second cloud \( C_2 \) is generated as cloud \( C_1 \) with the addition of a random variable to each coordinate of each point of the set shifted by a random vector value and rotated by 2 random angles relative to the X and Z axes. Spatial transformations can be described by a linear matrix \( M_0 \):

\[
C_2 = (C_1 + E) \cdot M_0
\]

After that, we apply the algorithm for stitching surface fragments described in [5], and we obtain a transformed point cloud \( C_3 \) obtained using the calculated transformation matrix \( M_1 \). We estimate the maximum deviation of the geometric center of the cloud \( C_3 \) relative to \( C_1 \). Figure 1 shows clouds \( C1 \) and \( C2 \) in the coordinate system of cloud \( C1 \).

![Figure 1. The distribution of clouds C1 (blue dots) and C2 (red dots) in the coordinate system of cloud C1.](image)

As a result of the cross linking algorithm, the distribution of \( C3 \) clouds was treated (Fig. 2). The value of \( Err_{\text{max}} \) is 3.63 mm with a standard deviation of the set \( E \) of 1 mm.
Figure 2. The distribution of clouds $C1$ (blue dots) and $C3$ (green dots) in the coordinate system of cloud $C1$ after crosslinking of clouds $C1$ and $C2$.

The dynamics of $Err_{\text{max}}$ versus the experiment number for the same parameters is placed below: standard deviation of $E$ is 1 mm, the number of points in the cloud is 1000, the size of the flat area on where points measured is 100x100 mm.

Figure 3. The dynamics of $Err_{\text{max}}$ vs the experiment number at standard deviation of $E = 1$ mm.
The average value of $Err_{max}$ was 3.54 mm. Figure 4 shows the dependence of the average $Err_{max}$ on the standard deviation of E. As a result, the dependence $Err_{max}$ is very close to linear.

Conclusions
The theoretical estimates of the measurement error are made and the results of mathematical modeling of the operation of the cloud stitching algorithm are presented. Theoretical estimates and results of mathematical modeling are presented, which make it possible to estimate the error in measuring three-dimensional geometry by the proposed method. It is shown that if the measurement error by the phase triangulation method has normal distribution, the results of stitching a three-dimensional surface will also have a normal distribution with similar characteristics. Presented method of multi-angle phase triangulation for measuring the three-dimensional geometry of convex and extended objects provides the necessary level of measurement accuracy and can be used to obtain three-dimensional geometry of complex-shaped convex and extended objects in industrial conditions.

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