ALMA Super-resolution Imaging of T Tau: \( r = 12 \) au Gap in the Compact Dust Disk around T Tau N

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Received 2021 April 29; revised 2021 September 29; accepted 2021 September 29; published 2021 December 15

Abstract

Based on Atacama Large Millimeter/submillimeter Array (ALMA) observations, compact protoplanetary disks with dust radii of \( r \lesssim 20–40 \) au were found to be dominant in nearby low-mass star formation regions. However, their substructures have not been investigated because of the limited spatial resolution achieved so far. We apply a newly developed super-resolution imaging technique utilizing sparse modeling (SpM) to explore several au-scale structures in such compact disks. SpM imaging can directly solve for the incomplete sampling of visibilities in the spatial frequency and potentially improve the fidelity and effective spatial resolution of ALMA images. Here we present the results of the application to the T Tau system. We use the ALMA 1.3 mm continuum data and achieve an effective spatial resolution of \( \sim 30\% \) (5 au) compared with the conventional CLEAN beam size at a resolution of 17 au. The reconstructed image reveals a new annular gap structure at \( r = 12 \) au in the T Tau N compact disk, with a dust radius of 24 au, and resolves the T Tau Sa and Sb binary into two sources. If the observed gap structure in the T Tau N disk is caused by an embedded planet, we estimate a Saturn-mass planet when the viscous parameter of the disk is \( 10^{-3} \). Ultimately, ALMA observations with enough angular resolution and sensitivity should be able to verify the consistency of the super-resolution imaging and definitely confirm the existence of this disk substructure.

Unified Astronomy Thesaurus concepts: Circumstellar disks (235); Planet formation (1241); Radio interferometry (1346)

1. Introduction

Planets are formed in protoplanetary disks (PPDs) around young stars, which are composed of gas and dust (e.g., Hayashi et al. 1985). The structure and evolution of PPDs are thought to be closely linked to the formation process of planets for both core accretion and disk instability models (e.g., Johansen et al. 2007; Ida et al. 2013). Protoplanets with sufficiently large mass can induce the formation of a gap in the disks (e.g., Lin & Papaloizou 1986; Takeuchi et al. 1996; Pinilla et al. 2012; Zhu et al. 2012). The minimum gap-opening mass depends on the viscosity and scale height of the disk, and ideally super-Earth mass planets \( (\sim 10M_\oplus) \) can produce detectable gaps in the disk submillimeter regime (Rosotti et al. 2016; Zhang et al. 2018). Once the gap is spatially resolved, its width and depth can be closely linked to the formation process of planets for both core accretion and disk instability models (e.g., Papaloizou 1986; Takeuchi et al. 1996; Pinilla et al. 2012; Zhu et al. 2012). The minimum gap-opening mass depends on the viscosity and scale height of the disk, and ideally super-Earth mass planets \( (\sim 10M_\oplus) \) can produce detectable gaps in the disk submillimeter regime (Rosotti et al. 2016; Zhang et al. 2018).

The advent of the Atacama Large Millimeter/submillimeter Array (ALMA) has enabled us to observe PPDs with high spatial resolution, and transformational images or analysis (e.g., interferometric modeling) of disks have been produced. For instance, the disk substructures at the High Angular Resolution Project (SHARP) and the Ophiuchus Disc Survey Employing ALMA (ODISEA) provided ALMA images for bright and large disks with radii of \( r = 50–260 \) au with angular resolutions down to \( 2–5 \) au (for consistency, we always refer to “dust disk” as “disk”; Andrews et al. 2018a; Cieza et al. 2021). These results reveal an annular gap structure, which is likely carved by a planet with Neptune to Jupiter mass (Zhang et al. 2018). However, disks of small sizes \( (r \lesssim 20–40 \) au) of PPDs were found to be dominant in the fraction \( (\sim 70\%–90\% \) in low-mass star-forming regions (Cieza et al. 2019; Long et al. 2019; Ansdell et al. 2016), but their substructures have not been well investigated. Small disks are typically less massive in terms of the disk mass compared with large disks, and they will be key to investigating the missing link between PPD substructures such as gaps and their locations. There is the need for extensive research on such a major PPD population to investigate the inner \( r = 5–40 \) au region in such PPDs as a possible location.
for the formation of giant planets (e.g., Bate 2018; Lodato et al. 2019).

To explore a few au-scale gap structures in such compact disks in nearby low-mass star formation regions, a high spatial resolution of $\lesssim0^\prime.035$ is required to resolve, e.g., a gap formed by a Jupiter-mass planet orbiting around a low-mass star (0.5 $M_\odot$, $d = 140$ pc), where the gap width is assumed to be roughly 5 au and calculated to be 5.5 times the Hill radius at $r = 10$ au (Lodato et al. 2019). The highest angular resolution in ALMA Band 6 observations achieved thus far is $\sim0^\prime.02–0^\prime.005$ (e.g., DSHARP and ODISEA). ALMA high-resolution observations potentially resolve the compact disk’s substructure as well with sufficient $uv$-coverage by longer observing time (e.g., SR 4, DoAr 33, and WSB 52; Huang et al. 2018). Sparse modeling (SpM) is another approach, i.e., a promising technique that can achieve such a high spatial resolution, even in the lower-frequency ALMA Bands 4 and 6. This technique has already been applied to the imaging of the Event Horizon Telescope (EHT; Event Horizon Telescope Collaboration et al. 2019) and ALMA (Aizawa et al. 2020; Yamaguchi et al. 2020). To date, the use of EHT mock observational data and ALMA actual observational data has confirmed that this technique achieves a higher-fidelity image than the conventional CLEAN algorithm at the angular scale of 30%–40% of the CLEAN beam (i.e., super-resolution; Honma et al. 2014; Akiyama et al. 2017a, 2017b; Kuramochi et al. 2018; Yamaguchi et al. 2020). Furthermore, with an emphasis on improving the fidelity even in super-resolution regimes and at the calculation speeds, a new SpM imaging software intended for ALMA observational data has been developed over the past several years (Nakazato & Ikeda 2020).

Here, we focus on the PPD around the T Tau triple-star system. T Tau is a triple star that became an eponymous member of the class of low-mass, pre-main-sequence stars (Joy 1945). This system consists of a star (T Tau N) in the north and a close binary (T Tau Sa and Sb) in the south (Dyck et al. 1982; Koaresko 2000), located in the Taurus star-forming region at a distance of 143.7 ± 1.2 pc, as measured by Gaia DR2 (Gaia Collaboration et al. 2016, 2018). Both T Tau N and T Tau S (Sa + Sb) are embedded in an infalling envelope (Momose et al. 1996); jets have been found to be associated with both sources (Beck et al. 2020). T Tau N is classified as Class II, while T Tau S is a Class I system (Furlan et al. 2006; Luhrman et al. 2010). The masses of T Tau Sa and Sb are 2.1 and 0.4 $M_\odot$, respectively (Schaefer et al. 2020). T Tau N is one of the brightest classical T Tauri stars in Taurus. The stellar properties of T Tau N have been calculated using optical spectral types combined with stellar evolutionary models in several studies, and we adopted a stellar bolometric luminosity of $6.82 L_\odot$ and a stellar mass of 2.19 $M_\odot$ (Herczeg & Hillenbrand 2014).

Previous ALMA continuum observations targeted the dust disk around the T Tau system but were unable to spatially resolve them on the CLEAN image because of the insufficient spatial resolution of $0^\prime.12$ (or 17 au; Long et al. 2019). The T Tau Sa/Sb disk is only seen as a single Gaussian-like distribution. The T Tau N disk was found to be a bright disk with a total flux of $\sim180$ mJy at 1.3 mm, but it is only seen as a flat compact disk with a radius of $\sim20$ au. Similarly, neither ground-based near-infrared adaptive optics observations nor space-based optical observations can resolve the disk around T Tau N and T Tau Sa/Sb well with a resolution of $\sim0^\prime.07$ or 10 au (e.g., Yang et al. 2018). Intriguingly, Manara et al. (2019) pointed out that significance residuals ($\sim3\sigma$) were found at both T Tau N and T Tau S after subtracting axisymmetric models of the two sources from the ALMA continuum image. This can be interpreted as tentative evidence of disk substructures around the T Tau system. Such bright and compact disks around the T Tau system would be most suitable for exploring several au-scale structures using SpM imaging.

In this study, by using super-resolution imaging with SpM, which is an approach that has been proven in previous studies, we present a high-resolution (5 au or 0′.03) image of the T Tau system. We find an annular gap at $r = 12$ au in the disk around T Tau N and two separate point-like dust continuum emissions, which are located at positions corresponding to T Tau Sa and Sb. In Section 2, we describe the data reduction and imaging with both CLEAN and SpM. In Section 3, we show the resulting images of a 1.3 mm continuum emission and present the findings of the substructure of the T Tau N disk and the two disks originating from T Tau Sa and Sb on the SpM image. In Section 4, we discuss the expected origins of the annular gap found in the T Tau N disk.

2. Data Reduction and Imaging

2.1. Data Reduction and Imaging with CLEAN

We reanalyzed the ALMA archival data obtained for T Tau on 2017 August 18, as part of the project 2016.1.01164.S (PI: Herczeg), including the continuum at 225.5 GHz and $^{13}$CO ($J = 2–1$) and C$^{18}$O ($J = 2–1$) line data. Continuum data have already been published in Long et al. (2019), Manara et al. (2019), and Beck et al. (2020). The observations were performed with a 12 m array consisting of 43 12 m antennas (C40-7 antenna configuration with the baseline length extending from 21.0 to 3637.7 m), and the on-source time of the target source was 8 minutes.

The data consisted of four spectral windows (SPWs). Two of the SPWs were used for the continuum observations and had center frequencies of 218 and 233 GHz. The average observation frequency was 225.5 GHz (wavelength of 1.3 mm). The other SPWs were used to cover $^{13}$CO and C$^{18}$O with a velocity resolution of 0.16 km s$^{-1}$. In this study, we used continuum SPWs to reconstruct images by employing two different techniques, namely, CLEAN and SpM. The $^{13}$CO and C$^{18}$O data were analyzed, but emissions associated with T Tau S and N were not identified in the two lines.

The raw data were calibrated using the Common Astronomy Software Applications package (CASA; McMullin et al. 2007), version 5.1.1. The initial calibration was performed using the ALMA pipeline on CASA. In the pipeline, J0423–0120 was used for the flux and bandpass calibration, and J0431+1731 was used for phase calibration. The positional offset between the phase (map) center and the emission peak of the T Tau N disk was adjusted using the CASA task fixvis.

The data were first imaged with the tclean task (hereafter CLEAN) by adopting Briggs weighting (robust=0.5). CLEAN is the most standard image reconstruction algorithm and also one of the nonlinear deconvolution techniques (e.g., Högbom 1974; Clark 1980; Schwab 1984; Cornwell 2008; Zhang et al. 2020). The technique iteratively determines the point source on the image domain that best fits the observed visibilities, starting from a dirty image, which is obtained by
the Fourier transform of the observed visibility with non-observed data filled with zero. This process is repeated until some convergence requirement is met. The final image is obtained by convolving the point-source model (CLEAN model) with an idealized CLEAN beam (usually an elliptical Gaussian fitted to a synthesized beam). We note that the beam convolution in the image domain corresponds to multiplication in the visibility domain, which causes a loss in spatial resolution in the visibility domain via an underestimate of the observed visibility amplitudes (see Appendix A).

Next, to improve the signal-to-noise ratio (S/N) of the image by correcting a systematic gain error (e.g., antenna-based and baseline-based errors), we performed two rounds of phase (longer at the first (98 s) and down to the integration time at the second (49 s) with calmode = p) and one round of amplitude and phase (integration time of 98 s with calmode = ap) self-calibrations. We obtained the final CLEAN image (=CLEAN model convolved with CLEAN beam + residual map) after self-calibration with the S/N improved by a factor of 3.8, compared with the initial one. The CLEAN beam size $\theta_{\text{CLEAN}}$ was 0″14 × 0″10 at PA of 34°1, and its peak intensity and r.m.s. noise level (collected noise values for $r = 3"$0 from the phase center) were 63.81 mJy beam$^{-1}$ and 41 μJy beam$^{-1}$, respectively. These values are in relatively good agreement with those reported previously in Long et al. (2019) (i.e., peak intensity = 64.56 mJy beam$^{-1}$, r.m.s. noise = 52 μJy beam$^{-1}$).

### 2.2. Imaging with Sparse Modeling

We performed the SpM imaging (Yamaguchi et al. 2020). Here we briefly describe the outline of the SpM imaging and the cross-validation (CV), which were used for the imaging.

The self-calibrated visibility data were adopted for the image reconstruction with the latest SpM imaging task, PRIISM, ver. 0.3.0$^8$ (Nakazono & Ikeda 2020) working with CASA. The image is reconstructed by minimizing a cost function in which two convex regularization terms of the brightness distribution, $\ell_1$-norm and total squared variation (TSV), were utilized with the chi-squared error term (Kuramochi et al. 2018). The two regularizers adjust the sparsity and smoothness of the reconstructed image. We minimize the cost function to obtain the optimum image, which is formulated as

$$I = \text{argmin} \left( ||W(V - FI)||_2^2 + \sum_{i} \sum_{j} ||I_{i,j}||^2 + \sum_{i} \sum_{j} \left( |I_{i+1,j} - I_{i,j}|^2 + |I_{i,j+1} - I_{i,j}|^2 \right) \right),$$

where $I = \{I_{i,j}\}$ is a two-dimensional (2D) image to be reconstructed, where $i$ and $j$ represent the pixel indices, $V$ is the observed visibility (i.e., the self-calibrated visibilities), $F$ is the Fourier matrix, and $W$ is a diagonal weight matrix (each diagonal element is $1/\sigma_i^2$, where $\sigma_i$ is the observational error of each visibility point indexed by $s$, normalizing the residual visibility ($V - FI$) on the chi-squared term. The two regularization terms are controlled by the positive variables $\Lambda_1$ and $\Lambda_{\text{TSV}}$, respectively.

CV is a statistical method that is employed to choose the optimal values of regularization parameters (see Akiyama et al. 2017a, 2017b, for details). In this study, we used the 10-fold CV implemented in PRIISM and searched for the optimal parameter set with $5 \times 5$ sets of regularization parameters, which are $\Lambda_1 = (10^5, 10^6, \ldots, 10^9)$ and $\Lambda_{\text{TSV}} = (10^2, 10^3, \ldots, 10^9)$. In the imaging using PRIISM, we used nonuniform fast Fourier transform (NuFFT) algorithms to compute the Fourier transform and perform iterative fitting of the model to visibility data ($N_{\text{iter}} = 1000$) until the iteration algorithm converges.

In the process of the N-fold CV, the data set $V$ is randomly divided into $N$ subsets, and $N - 1$ sets are used for image reconstruction by employing the SpM imaging method with a fixed $(\Lambda_1, \Lambda_{\text{TSV}})$. The reconstructed image is then Fourier transformed, and the weighted chi-squared error, which is defined below, is computed for the remaining subset:

$$\text{MSE} = \frac{||W(V - FI)||^2}{\text{tr}W},$$

where $\text{tr}W$ is the trace of matrix $W$. This process is iterated $N$ times by taking different subsets, deriving the CV error (CVE) formulated in the mean squared error (MSE) ($\text{MSE} = \sum_{i=1}^{N} \text{MSE}_i/N$), as well as the standard deviation ($\text{MSE}_N^{1/2}/(\text{MSE}_N - \text{CVE})^{1/2}/N^{1/2} - 1$).

We obtained 25 images corresponding to 25 different sets of $(\Lambda_1, \Lambda_{\text{TSV}})$. The wide range of parameter space is selected via pre-tuning so that we do not miss the optimal image and so that it is possible to find it near the center of the image matrix. In this pre-tuning, $\Lambda_1$ is first fixed, and an optimal $\Lambda_{\text{TSV}}$ with the minimum CVE is searched in a wide range via SpM imaging. Next, the obtained optimal $\Lambda_{\text{TSV}}$ is fixed, and an optimal $\Lambda_1$ is similarly searched in a wide range for $\Lambda_1$. The values and ranges can be tuned according to the target source properties in PRIISM. Figure B1 shows the reconstructed images, together with the calculated values of CVE. An image with the minimum CVE can be regarded as the optimal image (see Appendix B; Akiyama et al. 2017a, 2017b; Kuramochi et al. 2018; Yamaguchi et al. 2020). In other words, the image of $(\Lambda_1, \Lambda_{\text{TSV}}) = (10^5, 10^3)$ is selected as the optimal one.

In order to quantify the effective resolution $\theta_{\text{eff}}$ of the technique, we have performed the following evaluation. We injected an artificial point source to the observed data in the visibility domain. We then performed the SpM imaging with the same regularization parameters of the optimal image, as well as other sets of parameters. In the SpM images, effective resolution was evaluated with an elliptical Gaussian fit to the point source (see Figure C1). In these simulations, we refer to an evaluation of an effective spatial resolution from the nonparametric image modeling with the maximum entropy method (MEM; Cárcamo et al. 2018; Pérez et al. 2020). The input flux density of the point source is 7.1 mJy, which is comparable to that of the emission around T Tau Sa (see Table 1). The reconstructed image for the optimal parameter provides the FWHM size (i.e., effective spatial resolution) of the point source, $\theta_{\text{eff}} = 0"038 \times 0"027$ (or 5 × 4 au) with a PA of 45°3, and recovers a total flux of 7.9 mJy ($\sim$10% higher than the input value). The obtained effective resolution is roughly consistent with the empirical values of $\sim$30% of CLEAN beams $\theta_{\text{CLEAN}}$ (Akiyama et al. 2017a, 2017b; Kuramochi et al. 2018; Yamaguchi et al. 2020), which is $\theta_{\text{CLEAN}} = 0"14 \times 0"10$ with a PA of 34°1. The derived $\theta_{\text{eff}}$ is comparable to those in high-resolution observations ($\theta_{\text{CLEAN}} \sim 0"02-0"05$) such as DSHARP and ODISEA even though the maximum baseline length of our data ($\sim$4 km) is

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$^8$ PRIISM (Python Module for Radio Interferometry Imaging with Sparse Modeling) is an imaging tool for ALMA based on the SpM technique and is publicly available at https://github.com/tnakazono/priism.
Table 1
Properties of Dust Disks in T Tau System

| Source       | CLEAN | Sparse Modeling (SpM) |
|--------------|-------|-----------------------|
|              | $F_{\nu} > 5\sigma$ (mJy) | Peak $I_{\nu}$ (Jy ascc$^{-2}$) | $F_{\nu} > I_{DT}$ (mJy) | Peak $I_{\nu}$ (Jy ascc$^{-2}$) | Peak $I_{\nu}$ Position (RA, Decl.) |
| T Tau N      | 175.0 | 4.0                   | 174.4                    | 9.1                     | (04°21′59″4475, +19°32′06″1731) |
| T Tau S (Sa+Sb) | 8.0      | 0.4                   | 7.9                     | ...                     | ...                                |
| T Tau Sa     | ...     | ...                   | 7.1                     | 5.1                     | (04°21′59″4362, +19°32′05″5131) |
| T Tau Sb     | ...     | ...                   | 0.8                     | 0.8                     | (04°21′59″4365, +19°32′05″6131) |

Note. The first column lists the total flux ($F_{\nu}$) above the 5$\sigma$ (13 mJy ascc$^{-2}$) level and peak intensity (Peak $I_{\nu}$) on the CLEAN image. The last three columns list the total flux ($F_{\nu}$) above the $I_{DT}$ (272 mJy ascc$^{-2}$) level, peak intensity (Peak $I_{\nu}$), and its position (R.A., decl.) on the SpM image.

3–4 times shorter than that of those high-resolution data (~13–16 km; Andrews et al. 2018a; Cieza et al. 2021).

The effective resolution depends on the regularization parameters, especially on the TSV term. The simulation results for other sets of regularization parameters (mainly for different $\Lambda_{\text{ns}}$) are given in Appendix C; the smaller $\Lambda_{\text{ns}}$ is, the higher spatial resolution is. In Appendix C, we also add an interferometric theory-based probable explanation on why the SpM imaging can achieve roughly three times better spatial resolution.

3. Results

3.1. SpM Image and Evaluation of Its Noise Levels

We evaluated the noise and significance levels of the optimal SpM image. As described in Yamaguchi et al. (2020), owing to both thermal and systematic noise, an SpM image suffers from (unexpected) positive emissions in its off-source area (i.e., outside the target source area). This is because nonnegative constraints have been adopted in the SpM imaging algorithm, and artificial emissions with positive intensity may be present in the off-source area. Here, we define a detection threshold (DT) in the target source area as the maximum intensity ($I_{\text{DT}}$) of such artificial emissions (note that $I_{\text{DT}}$ is the same definition as $I_{100}$ in Yamaguchi et al. 2020). $I_{\text{DT}}$ was found to be 272 mJy ascc$^{-2}$ by analyzing noise statistics at the pixel scale (0′′05) outside the source ($\sigma > 0′′8$).

For direct comparison with the noise level in the CLEAN image, we convolved the optimal SpM image with the same beam size as that used for the rms noise estimate of the CLEAN image (robust=0.5). We found that the beam-convolved $I_{\text{DT}}$ is 152 mJy beam$^{-1}$ for the optimal SpM image and was approximately 3.7 times higher than the rms noise of the CLEAN image (see Figure C2).

Another way of estimating the detection threshold is the usage of image simulation of an injected artificial point source, as described in Appendix C. We changed the flux density of the input point source from 1000 to 100 mJy in increments of 100 mJy in the SpM simulation and judged the detection of the point source in the image. In the optimal image case ($\Lambda_{\text{ns}}= \{10^{5} , 10^{6}\}$), the detection threshold is 300 mJy beam$^{-1}$ (see Figure C2), which is two times higher than $I_{\text{DT}}$. This value would not be accurate but would provide some reference to the threshold if we consider that the increment of the flux density is rough and the selection of the position source is not so optimized for evaluating the detection threshold precisely.

It would also be possible to estimate the noise levels by measuring the rms noise of a residual map, which can be obtained by performing the 2D Fourier transform of residual visibilities between the SpM model data (which can be obtained by the inverse 2D Fourier transform of the SpM image) and the observed data. The residual map was reconstructed from the residual visibilities using DIFMAP (Shepherd et al. 1994). The residual map was created by adopting a natural $uv$ weighting, providing synthesized beams of $0′′17 \times 0′′12$ with a PA = 36°8 and an rms noise of 31 mJy beam$^{-1}$. This is smaller than that in the CLEAN image, and this could be because the SpM model image retrieves positive noises. For comparison, we convolved the optimal SpM image with the same beam size as that used for the rms noise estimate. We found that the beam-convolved $I_{\text{DT}}$ (=152 mJy beam$^{-1}$) was approximately five times higher than the rms. Based on the above evaluations, emission features above $I_{\text{DT}}$ are considered significant in the SpM image.

Figure 1 shows the SpM and CLEAN images of the T Tau system. The SpM image spatially resolves the disk structure around T Tau N, and an annular gap structure has been newly found. The emission around the T Tau S system is spatially resolved into two sources, although the CLEAN image does not resolve them. Table 1 shows that the total fluxes of these sources obtained from the SpM image above the $I_{\text{DT}}$ level are generally consistent with the values obtained from the CLEAN image above 5$\sigma$ levels. SpM reproduces a high-fidelity image that better fits the observed visibilities than the CLEAN image, but it provides similar results in the visibility domain to the CLEAN model (see Appendix A). We consider that the SpM image better reconstructs the disk surface brightness distribution while the CLEAN model reconstructs an image with a sum of a number of point sources as shown in Figure 1(d), which do not reflect the disk structures precisely. Therefore, in the following sections we adopt the SpM image to derive the physical properties of the T Tau system.

3.2. Dust Emissions from T Tau Sa and Sb

As described in Section 3.1, two separate emissions were found around T Tau Sa and Sb. Figure 2 presents a close-up view of the Tau Sa and Sb regions. 2D Gaussian fitting was applied to each of them using the CASA task imfit. The results are listed in Table 2.

To confirm whether each of the two emissions originated from T Tau Sa or Sb, Figure 2 compares the image with stellar positions predicted by the stellar orbit model of the T Tau S binary in Köhler et al. (2016), based on observational data spanning approximately 18 yr. The coordinate systems of the binary were derived for the date of the ALMA observation (2017 August 18). The offsets between the emission peaks and predicted stellar positions are calculated to be 9.3 mas (1.3 au)
and 14.6 mas (2.1 au) for Sa and Sb, respectively and will be roughly within errors involved in the calculations, e.g., the uncertainties of the model, a few mas (Köhler et al. 2016, private communication), and 1σ positional errors for T Tau Sa and Sb in the SpM image, ~1 and 5 mas, respectively. In addition, each total flux roughly fits each spectral energy distribution (SED) predicted by an accretion disk model (Ratzka et al. 2009). Hence, it can explain that the two emissions originate from T Tau Sa or Sb.

As shown in Table 2, the best-fit sizes (i.e., the FWHM of the semimajor/semiminor axes from the Gaussian fitting) of T Tau Sa and Sb were found to be $6 \times 4$ au and $7 \times 3$ au, respectively. These disk sizes are slightly larger than the effective spatial resolution of the SpM image ($\theta_{\text{eff}} = 5 \times 4$ au) and not resolved sufficiently. Hence, these sizes should be considered to be the conservative upper limits. The total flux density of T Tau Sb is a factor of seven smaller than T Tau Sa, which is in good agreement with the factor of eight given in

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**Figure 1.** Gallery of ALMA continuum images at 1.3 mm (Band 6) of the PPD T Tau system. The same color scale given by a power law with a scaling exponent of $\gamma = 0.6$ was adopted, except for the CLEAN model image ($\gamma = 0.3$). A white bar of $0''1 (=14.4$ au) is provided for reference to the angular scales. (a) SpM image. The filled white ellipse denotes the effective spatial resolution with a size of $0''038 \times 0''027$ for a PA of 45°3 in the lower left corner. The resolution is estimated from an artificial point-source simulation. The contour corresponds to $I_{\text{DET}}$, where $I_{\text{DET}}$ is the detection threshold of 272 mJy asec$^{-2}$. Note that the SpM image is not processed by a synthesized beam convolution as a CLEAN image is, and the unit of the SpM image is not Jy beam$^{-1}$. The unit of the SpM image that was initially obtained from the imaging is Jy pixel$^{-2}$, and we convert it to Jy arcsec$^{-2}$. (b) Close-up view centered on T Tau N of the SpM image. A field of view of $0''5 \times 0''5$ is adopted. (c) CLEAN image with Briggs weighting with a robust parameter of 0.5. The filled white ellipse denotes the synthesized beam with a size of $0''14 \times 0''10$ for a PA of 34°1 in the lower left corner. The contour corresponds to 20$\sigma_I$, where $\sigma_I$ is the rms noise of 2.58 mJy asec$^{-2}$ ($= 41$ μJy beam$^{-1}$). (d) CLEAN model image before convolution with the CLEAN beam.
Table 2

Results of 2D Gaussian Fits to T Tau Sa and Sb

| Source | $\theta_{maj}$ (mas) | $\theta_{maj}$ (au) | $\theta_{min}$ (mas) | $\theta_{min}$ (au) | PA (deg) | Inclination (deg) | Peak $I_C$ (Jy arcsec$^{-2}$) | $F_\nu$ (mJy) |
|--------|----------------------|---------------------|----------------------|---------------------|----------|------------------|-----------------------------|--------------|
| T Tau Sa | 44.7 ± 0.7 | 6.4 ± 0.1 | 27.1 ± 0.4 | 3.9 ± 0.1 | 25.2 ± 1.1 | 52.8 ± 0.6 | 5.6 ± 0.1 | 7.7 ± 0.1 |
| T Tau Sb | 49.7 ± 5.0 | 7.2 ± 0.7 | 22.4 ± 2.3 | 3.2 ± 0.3 | 64.9 ± 4.6 | 63.2 ± 2.9 | 0.9 ± 0.1 | 11.1 ± 0.1 |

Note. Disk properties for T Tau Sa and Sb obtained using the $\text{imfit}$ task in CASA to fit a 2D Gaussian on the SpM image. The task returns the total flux density ($F_\nu$) of the source along with the statistical uncertainty, the FWHM along the semimajor ($\theta_{maj}$) and semiminor ($\theta_{min}$) axes, and the position angle (PA). The inclination is derived from the $\theta_{maj}$-to-$\theta_{min}$ ratio, assuming a perfect-circle disk. Estimates of the uncertainties derived from $\text{imfit}$ are based on Condon (1997), and that of the inclination is derived from error propagation. Note that $\theta_{maj}$ and $\theta_{min}$ give upper limits, and these uncertainties in astronomical units do not account for the uncertainty in the distance to the source.

Table 3

Physical Properties of T Tau N Disk

| Parameters | Measurements |
|------------|--------------|
| Position angle (deg) | 91.4 ± 3.0 |
| Inclination (deg) | 25.2 ± 1.1 |
| Disk radius, $r_d$ (mas, au) | 166 ± 25, 24 ± 4 |
| Outer ring peak, $r_{peak}$ (mas, au) | 109 ± 1, 15.7 ± 0.1 |
| Gap location, $r_{gap}$ (mas, au) | 81 ± 2, 11.6 ± 0.3 |
| Gap width, $\Delta r$ | 0.28 ± 0.02 |
| Gap depth, $b_1$ | 1.22 ± 0.06 |

Note. The physical parameters of the T Tau N disk are calculated from the SpM image and the radial intensity profile. These uncertainties in astronomical units do not account for the uncertainty in the distance to the source.

Next, we derive a disk radius $r_d$ using a curve-of-growth method similar to that described in Ansdell et al. (2016). The disk radius is measured with successively larger photometric apertures on a deprojected image until the measured flux reaches 95% of the total flux. As a result, $r_d$ was calculated to be 24 ± 4 au. The error on $r_d$ is calculated by taking the range of radii within the uncertainties of the 95% flux measurement. The obtained effective radius is in good agreement with the one ($r_d \approx 21$ au) from Manara et al. (2019) in the same definition of the measurement. Most populated Taurus disks (spectral type earlier than M3) are known to be faint and compact (total flux of <100 mJy at 1.3 mm, dust radii of <40 au; Long et al. 2019). Thus, the T Tau N disk can be regarded as a bright compact disk.

3.4. Gap Structure in the T Tau N Disk

Figure 3 shows the deprojected and azimuthally averaged radial intensity profile $I_r(r)$, where $r = 0$ au is set to the peak intensity of the T Tau N disk. The uncertainty of the radial profile is evaluated as the error of the mean at each radius, where we consider the effective spatial resolution of the major axis ($\theta_{eff,maj}$) as the smallest independent unit. That is, the error is the standard deviation of each elliptical bin divided by the square root of the number of $\theta_{eff,maj}$ spanning the whole azimuthal angle at each radial bin. For comparison, a standard deviation at each radius is also plotted in the radial profile in Figure 3.

We identify an annular gap (local minimum in $I_r(r)$ at $r_{gap} = 11.6 \pm 0.3$ au) and an outer peak (the local maximum in $I_r(r)$ at $r_{peak} = 15.7 \pm 0.1$ au) in the radial intensity profile. We then adopt the same approach as in Zhang et al. (2018) to measure the gap depth $\delta_1$ and the gap width $\Delta r$. The gap depth is defined as $\delta_1 = I_r(r_{peak})/I_r(r_{gap})$. The gap width is defined as

Figure 2. Comparison of positions of T Tau Sa and Sb between the peak emission on the SpM image and the stellar orbit model on the date of the ALMA observations (2017 August 18 UTC; Köhler et al. 2016). The positions of the submillimeter emission peaks are marked with stars, and the predicted positions of T Tau Sa and Sb are marked with crosses. These marked positions were superimposed on the SpM image. The image contours correspond to (1, 2, 3, 6, 9, 12, 15) $\times I_{16p}$. Black bars are provided for reference to angular scales.

Beck et al. (2020). This implies that the actual disk size of T Tau Sb would be about three times smaller than that of T Tau Sa when we consider the scaling relation between the millimeter-continuum disk radii $R_{mm}$ and luminosities $L_{mm}$: $L_{mm} \propto R_{mm}^2$ (Tripathi et al. 2017; Andrews et al. 2018b; Hendler et al. 2020).

3.3. Disk Structure of T Tau N

Here we investigate the global disk properties of T Tau N derived from the SpM image and compare them with previous studies based on mid-infrared and millimeter observations. The T Tau N disk is known to be viewed as nearly face-on (Akeson et al. 1998; Ratzka et al. 2009; Long et al. 2019; Manara et al. 2019; Beck et al. 2020). We derived the inclination and PA of the disk on the image by fitting an ellipse to the outer ring, as described in Appendix D. As shown in Table 3, the measured inclination of 25° ± 1° agrees well with $<30°$ derived from mid-infrared interferometric observations with very large telescope interferometer and SED simulations (Ratzka et al. 2009), as well as with $\approx 28°$ from visibility fitting using the same ALMA data (Manara et al. 2019; Beck et al. 2020).
\[ \Delta_I = (r_{\text{out}} - r_{\text{in}})/r_{\text{out}}, \] where \( r_{\text{out}} \) and \( r_{\text{in}} \) are the inner edges of the outer ring and the outer edge of the inner disk, respectively. The relationship between \( I_{\text{edge}} = 0.5 \{ I_1(r_{\text{peak}}) + I_1(r_{\text{gap}}) \} \) defines the edge locations. The edge location \( r_{\text{in}} \) is defined as the smallest value \( r \) satisfying the criteria \( I_{\text{edge}} = I_1(r_{\text{in}}) \) and \( r < r_{\text{gap}} \). Another edge location \( r_{\text{out}} \) is defined as the largest value satisfying the criteria \( I_{\text{edge}} = I_1(r_{\text{out}}) \) and \( r_{\text{gap}} < r < r_{\text{peak}} \). The measured parameters are listed in Table 3. In Section 4.2, the measured \( \Delta I = (0.28 \pm 0.02) \) and \( \Delta I_{12} = (1.22 \pm 0.06) \) are used to estimate the planetary mass under the hypothesis of planet-induced gap.

### 3.5. Physical Properties of T Tau N Disk

Here we derive the disk temperature \( T_d(r) \), optical depth \( \tau_v(r) \), and dust surface density \( \Sigma_d(r) \) of the T Tau N disk based on the SpM image to characterize the disk and substructure. We employ the radiative transfer equation expressed as

\[ I_1(r) = B_v(T_d(r))(1 - e^{-\kappa_v \Sigma_d(r)}), \tag{3} \]

where \( B_v(T) \) and \( T_d(r) \) denote the full Planck function and the dust temperature, respectively, and \( \tau_v(r) \) is the optical depth expressed as \( \tau_v(r) = \kappa_v \Sigma_d(r) \). Here \( \kappa_v \) and \( \Sigma_d(r) \) denote the absorption dust opacity and the dust surface density, respectively. The brightness temperature \( T_{br}(r) \) can be calculated from

\[ T_{br}(r) = \frac{h\nu}{k} \left[ \ln \left( \frac{2h\nu^3}{e^2I_1(r)} + 1 \right) \right]^{-1}, \tag{4} \]

where \( c, h \), and \( k \) denote the speed of light, Planck’s constant, and the Boltzmann constant, respectively. Figure 4 (top panel) shows that \( T_{br}(r) \) reaches \( 257 \pm 1 \, \text{K} \) at the peak, and the average \( T_{br}(r) \) over the disk \( = \frac{\int T_{br}(r)rdr}{\int rdr} \), where \( r \leq 24 \, \text{au} \) is calculated to be \( 97 \pm 1 \, \text{K} \). The average \( T_{br}(r) \) predominantly exceeds that predicted from the dust temperature model \( T_d(\sim 30–40 \, \text{K}) \), which is simply scaled using the stellar luminosity (Andrews et al. 2013; van der Plas et al. 2016). We should point out that the peak \( T_{br} \) is much higher than standard peak values (\( \sim 20–100 \, \text{K} \)) of other PPDs in the same observational wavelength and similar resolutions (see Figure 4 in...
Facchini et al. 2019). Moreover, the average spectral index over the disk is estimated to be $\alpha_{\text{mm}} = 1.9 \pm 0.1$ (see Appendix E).

From the high brightness temperature and the low spectral index described above, the disk tends to be optically thick overall ($r_\nu \geq 1$), and the measured $T_b(r)$ should represent the temperature of the emitting layer from the disk atmosphere. The innermost region ($r < 5$ au) seems to be thicker than the outer ring, and the brightness temperature should be close to the dust temperature near the disk surface in such a case. Therefore, we assume that $T_d(r)$ is equal to $T_b(r)$ at the optically thick region with $r_\nu \geq 1$. The disk temperature profile can be obtained as $T_d(r) = 360(r/1 \text{ au})^{-0.5}$ [K] by assuming that $T_d(r)$ has a power-law form, such as $T_d(r) \propto r^{-0.5}$ (Kenyon & Hartmann 1987). In this fitting, $T_d(r)$ is smoothed with $\theta_{\text{eff}}$ to match the $T_b(r)$ profile. It should be noted that a disk midplane temperature will generally be lower than $T_b(r)$ at optically thick regions. Thus, dust surface densities (and dust masses) estimated in what follows would be lower limits in such a case.

We have estimated $\tau_d(r)$ and $\Sigma_d(r)$ by adopting $T_d(r)$ derived above as the disk temperature. The optical depth $\tau_d(r)$ is calculated using the radiative transfer calculation of Equation (3) as

$$\tau_d(r) = \ln \left(1 - \frac{L(r)}{B_\nu(T_d(r))}\right).$$

Figure 4 shows the derived $\tau_d(r)$ profile. $\Sigma_d(r)$ is also expressed as

$$\Sigma_d(r) = \frac{\tau_d(r)}{\kappa_d(r)}.$$  

If we fix the disk temperature, another uncertainty in $\Sigma_d(r)$ comes from assumption of the dust opacity $\kappa_d(r)$ which usually depends on the grain size and many other factors. Here we consider two independent dust opacity models (but do not indicate which opacity model reproduces a “better” nature of T Tau N). One is the DSHARP opacity model $\kappa_{\nu,1}$ (=0.43 cm$^2$ g$^{-1}$; Bismiel et al. 2018) assuming a maximum grain size of 0.1 mm supported by recent (sub)millimeter polarization measurements of other Class II PPDs in the Taurus region (Bacciotti et al. 2018). The model value is constrained by dust size distribution with reference to its measurements from (sub)millimeter observations. Another is a conventional model $\kappa_{\nu,II}$ (=2.3 cm$^2$ g$^{-1}$; Beckwith & Sargent 1991), which can be expressed as $\kappa_{\nu,II} = 2.3(\nu/230 \text{ GHz})^{-0.8}[\text{cm}^2 \text{ g}^{-1}]$ and is simply parameterized because of the large uncertainties in the opacity. $\kappa_{\nu,II}$ has been widely used for PPDs (e.g., Williams & Cieza 2011) and is supported by spatially resolved multi-wavelength continuum observations of other PPDs (Lin et al. 2021).

The final results of $\Sigma_d(r)$ using $\kappa_{\nu,1}$ and $\kappa_{\nu,II}$ are plotted in Figure 4, together with that of the minimum-mass solar nebula (MMSN; $\Sigma_d(r) = 30(r/1 \text{ au})^{-1.5}[\text{g cm}^{-2}]$; Weidenschilling 1977; Hayashi 1981). We found that the surface density profiles of the T Tau N disk are locally more massive than the MMSN by a factor of 15 for $\kappa_{\nu,1}$ and a factor of 3 for $\kappa_{\nu,II}$ around the outer ring, but it sharply decreases at the disk edge. For comparison, the dust surface density profiles of the disks in Ophiuchus, Taurus-Auriga (Andrews 2015), and Lupus (Tazzari et al. 2017) generally appear less massive than the MMSN, while only a few of them have a comparable or larger mass. The T Tau N disk, despite being a small dust disk, would be regarded as more massive than typical disks in the low-mass star-forming regions.

The millimeter-dust mass $M_{\text{dust}}$ of the outer ring ($r_{\text{gap}} \leq r \leq r_d$) can be computed using the obtained $\Sigma_d(r)$, which is defined as

$$M_{\text{dust}} = \int^{r_d}_{r_{\text{gap}}} \Sigma_d(r)2\pi rd r.$$

The dust ring mass results in a wide range of values depending on the dust opacity: $\sim 104 M_\oplus$ for $\kappa_{\nu,1}$ and $\sim 20 M_\oplus$ for $\kappa_{\nu,II}$. We note that these dust ring masses should be considered as a lower limit owing to the uncertainty of the midplane temperature. In the range of the inferred dust masses, the outer ring of T Tau N is roughly as massive as the outer ring (at a location of $\sim 100$ au, $M_{\text{dust}} \sim 67 M_\oplus$) in Herbig Ae star MWC 480, located in the Taurus region (Liu et al. 2019).

To check whether the T Tau N disk is gravitationally stable, Toomre Q (Toomre 1964) was calculated using the formula

$$Q = c_s \Omega_K / \pi G \Sigma_{\text{gas}},$$

where $c_s$ is sound speed, $\Omega_K$ is angular velocity, $G$ is the gravitational constant, and $\Sigma_{\text{gas}}$ is gas surface density. If the disk follows the criterion $Q \lesssim 1.5$, the disk is gravitationally unstable and grows spiral arms (Laughlin & Bodenheimer 1994). Here we employed the Toomre Q under the standard assumption of $\Sigma_{\text{gas}}/\Sigma_{\text{dust}} = 100$ (Bohlin et al. 1978). In both $\kappa_{\nu,1}$ and $\kappa_{\nu,II}$, the Toomre Q values exceed unity around the outer ring area; $Q \gtrsim 5$ for $\kappa_{\nu,1}$ and $Q \gtrsim 10$ for $\kappa_{\nu,II}$. The T Tau N disk thus appears to be gravitationally stable. It should be noted that a secular gravitational instability (requiring high gas-to-dust ratios <100 and low viscous parameter $\alpha \lesssim 10^{-3}$) can generate a ring-like structure in the disk (Takahashi & Inutsuka 2014, 2016). That being said, the required physical parameters remain highly uncertain at this stage, and the possibility of secular gravitational instability as the origin of the rings cannot be concluded yet.

4. Discussion

4.1. Origins of Gap in the T Tau N Disk

Recent high-resolution observations have revealed multiple annular gap structures in bright giant disks (e.g., Andrews et al. 2018a; Cieza et al. 2021). Several detections of gaps or cavities in compact disks have also been reported so far: the transitional disks around XZ Tau B (disk radius of 3 au, cavity radius of 1.3 au; Osorio et al. 2016) and around several candidates (see Pinilla et al. 2018, for details), and the annular gap structure in the disks around a single star SR 4 ($r_{\text{gap}} = 11$ au, $r_d = 31$ au; Huang et al. 2018), DoAr 33 ($r_{\text{gap}} = 9$ au, $r_d = 27$ au; Huang et al. 2018), WSB 52 ($r_{\text{gap}} = 21$ au, $r_d = 32$ au; Huang et al. 2018), CIDA 1 ($r_{\text{gap}} = 8$ au, $r_d = 40$ au; Pinilla et al. 2021), J0433 ($r_{\text{gap}} = 15$ au, $r_d = 46$ au; Kurtovic et al. 2021), and one member of a binary system GQ Lup A ($r_{\text{gap}} = 8$ au, $r_d = 20$ au; Long et al. 2020). The T Tau N case is very similar to SR 4, DoAr 33, and GQ Lup A in terms of the radius of the disk and gap location. In previous studies on gap origins in disks (e.g., Huang et al. 2018; Long et al. 2018), two main possibilities have been investigated: the snow line and planet origins.

The snow line, which is also referred to as an ice sublimation front, is the location in the disk midplane where dust opacity and collisional growth are expected to change, producing features such as ring-like substructures seen in continuum images (Zhang et al. 2015; Okuzumi et al. 2016). An estimate of the snow-line location inferred from disk midplane
temperature models (e.g., Dullemont et al. 2001) should be calculated and is confirmed by comparing the gap location. However, the brightness temperature of the T Tau N disk is much higher than that of the regular disk, and this disk appears optically thick at 1.3 mm. It can thus be challenging to find a reasonable disk midplane temperature model that matches observations. This problem would be solved by observing the optically thin disk at lower wavelengths to determine an adequate model.

4.2. Planetary Origin and Planet Mass Estimates

Another possible origin of this gap is the planet clearing of the disk material. Below, we estimate planetary masses by applying two different methods that connect the planetary mass and gap size. We first apply the relationship between the planet mass and gap size using the constant opacity models proposed by Kanagawa et al. 2015, 2016; Zhang et al. 2018.

We first apply the relationship between the planet mass and the width and depth of the gaseous gap according to the theory proposed by Kanagawa et al. 2015, 2016. The gap width and depth are defined as the difference between the initial and gap-formed surface density profiles in the theory, but this definition cannot be adopted in our case, as the initial model cannot be set because of a lack of complete information of $\Sigma_d(r)$. Instead of the original definition, we adopt the gap width $\Delta \Sigma$ and depth $\delta \Sigma$ given in the dust surface density profile $\Sigma_d(r)$. As shown in Equation (6), $\Sigma_d(r)$ is simply calculated by dividing the optical depth $\tau_d(r)$ by the constant opacity models ($\kappa_{v,d}$ or $\kappa_{v,gh}$).

Therefore, the gap width and depth of the dust surface density profile would not change regardless of which of the two opacities is used. Here, we apply a Gaussian fit to $\Sigma_d(r)$ at the gap area ($r = 8–16$ au) in a similar manner to Segura-Cox et al. 2020, by using the least-squares method implemented in `optimize.leastsq` from SciPy (Virtanen et al. 2020). The uncertainties are the statistical uncertainties from the Gaussian fitting. The FWHM gap width $\Delta \Sigma$ is derived as $8.3 \pm 0.1$ au at a gap location of $r_{\Sigma \text{gap}} = 10.89 \pm 0.02$ au. We define the gap depth $\delta \Sigma = \Sigma_{\text{peak}}/\Sigma_{\text{gap}}$, where $\Sigma_{\text{peak}}$ is the local peak of the outer ring and $\Sigma_{\text{gap}}$ is the local minimum at $r_{\Sigma \text{gap}}$. The gap depth was calculated to be $\delta \Sigma = 2.1 \pm 0.3$.

Here we assume that the dust is well coupled to the gas content of the disk and the radial location of the planet is at $r_{\Sigma \text{gap}}$. Note that our defined gap depth may be underestimated because it may be shallower than the original one. We use the relationship between the planetary mass $M_p$ and the gap depth $\delta \Sigma$ as follows (Equation (7) in Kanagawa et al. 2015):

$$\frac{M_p}{M_\oplus} = 0.16(\delta \Sigma - 1)0.5 \left( \frac{h_{\Sigma \text{gap}}}{r_{\Sigma \text{gap}}} \right)^{2.5} \left( \frac{\alpha_{\text{vis}}}{10^{-3}} \right)^{0.5},$$ (7)

where $h_{\Sigma \text{gap}}$ is the scale height at $r_{\Sigma \text{gap}}$ and $\alpha_{\text{vis}}$ is the viscous parameter (Shakura & Sunyaev 1973). We also use the relation with the gap width $\Delta \Sigma$ as follows (Equation (5) in Kanagawa et al. 2016):

$$\frac{M_p}{M_\oplus} = 0.19\left( \frac{\Delta \Sigma}{r_{\Sigma \text{gap}}} \right)^{3} \left( \frac{h_{\Sigma \text{gap}}}{r_{\Sigma \text{gap}}} \right)^{1.5} \left( \frac{\alpha_{\text{vis}}}{10^{-3}} \right)^{0.5}.$$(8)

By eliminating the planetary mass in Equations (7) and (8), the relationship between $\delta \Sigma$ and $\Delta \Sigma$ can be obtained as follows:

$$\Delta \Sigma = 0.92(\delta \Sigma - 1)^{0.25} \left( \frac{r_{\Sigma \text{gap}}}{1 \text{ au}} \right) \left( \frac{h_{\Sigma \text{gap}}}{r_{\Sigma \text{gap}}} \right)^{0.5} \text{ au.}$$ (9)

Using $T_d(r)$, the aspect ratio $h_{\Sigma \text{gap}}/r_{\Sigma \text{gap}}$ was calculated to be 0.05, and the viscous parameter is set to be $10^{-3}$ for the T Tau N disk, as in Kanagawa et al. 2015. We found that the derived depth and width are too shallow and too wide compared to the theoretical curve. Nomura et al. 2016 reported that, due to beam smearing, the derived measurements $\delta \Sigma$ and $\Delta \Sigma$ give a lower limit and upper limit, respectively. Following the discussion by Nomura et al. 2016, we assume that the gap depth times the gap width conserves the value derived from the observations (i.e., $\Delta \Sigma \propto \delta \Sigma$) and $r_p$ remains fixed, as illustrated in Figure 5. Thus, the relation can be plotted in the left panel of Figure 6. The crossing point between the two curves is located at the width and depth of $\Delta \Sigma = 3.2 \pm 0.3$ au and $\delta \Sigma = 5.4 \pm 1.3$ under the condition of $\Delta \Sigma = 17.4 \pm 2.5 \delta \Sigma$ au. The crossing point and the use of Equations (7) and (8) give the planetary mass of 1.4 ± 0.2 $M_{\text{Saturn}}$. The error range of the planetary mass results from the uncertainty of the product, which can change the location of the crossing point.

Next, we consider another relationship in Zhang et al. 2018. This approach defines the gap depth $\delta_1$ and width $\Delta_1$ in $L_s(r)$ in Section 3.4 without assuming a functional form for the substructures or an initial surface density; it has the relationships to derive the planet mass from the measured $\delta_1$ and $\Delta_1$ in Section 3.4. We now use the relationship between the planet
mass and the gap depth $\delta_1$ (Equation (24) in Zhang et al. 2018):

$$\frac{M_p}{M_\text{M*}} = 0.073 \left( \frac{\delta_1 - 1}{C} \right)^{1/B} \frac{h_{\text{gap}}}{r_{\text{gap}}} \frac{0.38}{10^{-3}}. \quad (10)$$

We also used the one with the gap width $\Delta_1$ (Equation (22) in Zhang et al. 2018):

$$\frac{M_p}{M_\text{M*}} = 0.115 \left( \frac{\Delta_1}{A} \right)^{1/B} \frac{h_{\text{gap}}}{r_{\text{gap}}} \frac{\alpha_{\text{vis}}}{10^{-3}}. \quad (11)$$

By eliminating the planetary mass in Equations (10) and (11), the relationship between $\Delta_1$ and $\delta_1$ can be obtained as follows:

$$\Delta_1 = A \left[ 0.635 \left( \frac{\delta_1 - 1}{C} \right)^{1/B} \right. \left. \times \left( \frac{h_{\text{gap}}}{r_{\text{gap}}} \right)^{2.63} \frac{\alpha_{\text{vis}}}{10^{-3}} \right]^{0.07} \frac{1}{D}, \quad (12)$$

where $A$, $B$, $C$, and $D$ are constant parameters introduced by Zhang et al. (2018) and depend on the gas surface density $\Sigma_g$ and the maximum grain size ($s_{\text{max}} = 0.1 \sim 10$ mm). Figure 18 of Zhang et al. (2018) shows the relationship between a gas surface density and an averaged dust surface density $\Sigma_d$ at an outer disk (or ring) for hydrodynamical simulations. We can then use their Figure 18 to estimate the gas surface density $\Sigma_g$ based on the average $\Sigma_d$ ($\sim 3.4$ g cm$^{-2}$) at the outer ring of T Tau N and the aspect ratio $h_{\text{gap}}/r_{\text{gap}}$ ($\approx 0.05$). Finally, the four parameters ($A = 1.11, B = 0.29, C = 0.0478, \text{and } D = 1.23$) are selected from Tables 1 and 2 of Zhang et al. (2018), when considering the estimated $\Sigma_d$ ($> 100$ g cm$^{-2}$) and the maximum dust particle size ($s_{\text{max}} = 0.1$ mm; Bacciotti et al. 2018) in the disk.

$\Delta_1$ is calculated as a function of $\delta_1$, as shown in Figure 6, where we used the obtained parameters for $(A, B, C, \text{and } D)$, the aspect ratio $h_{\text{gap}}/r_{\text{gap}}$ of 0.05, and the viscous parameter of $\alpha_{\text{vis}} = 10^{-3}$. A conservation relation derived from the measured $\Delta_1$ and $\delta_1$ can also be obtained by assuming that the product of the gap depth and width is conserved in the radial intensity profile (i.e., $\Delta_1 \approx \delta_1^{-1}$), as shown in the right panel of Figure 6. We obtain the relation $\Delta_1 = 0.34 \pm 0.03 \delta_1$ for $\Delta_1 = 0.19 \pm 0.01$ and $\delta_1 = 1.78 \pm 0.11$. The crossing point between the two curves gives a planetary mass of $1.2 \pm 0.1 M_{\text{Saturn}}$, which agrees well with $1.4 \pm 0.2 M_{\text{Saturn}}$ derived from the analytic formula by Kanagawa et al. (2015, 2016) within the uncertainties involved.

We assumed the viscous parameter $\alpha_{\text{vis}} = 10^{-3}$ for the above estimate. If we take $\alpha_{\text{vis}}$ over a wide range of $\alpha_{\text{vis}} = 10^{-2} - 10^{-4}$, the derived planetary masses vary by a factor of $\sim 3$ and are calculated to be $0.5 - 4.5 M_{\text{Saturn}}$ for the analytic formula by Kanagawa et al. (2015, 2016) and $0.5 - 2.7 M_{\text{Saturn}}$ for the analytic formula by Zhang et al. (2018). Even considering the wide range of $\alpha_{\text{vis}}$, the planet mass is still similar to Saturn’s mass. In addition, the gap location ($r = 12$ au) is close to Saturn’s orbit, and T Tau N is an interesting example that is analogous to the solar planetary system.

There are two other cases of gaps at $r \approx 10$ au in the compact disks, indicating the presence of planets: SR 4 (Zhang et al. 2018) and GQ Lup A (Long et al. 2020). Both cases indicate upper limits of planet masses of $M_p \lesssim 7.2 M_{\text{Saturn}}$ for SR 4 ($F_\nu = 69$ mJy at 1.3 mm; Andrews et al. 2018b) and $M_p \lesssim 0.1 M_{\text{Saturn}}$ for GQ Lup A ($F_\nu = 28$ mJy at 1.3 mm; Wu et al. 2017), by using the same manner as Zhang et al. (2018) taken from a gap width alone for $\alpha_{\text{vis}} = 10^{-3}$ and $s_{\text{max}} = 0.1$ mm. While there are a few samples of the inferred planet mass for the compact disks at this stage, it could be a correlation between planet mass and (sub)millimeter disk flux (or disk mass) in such disks, suggesting that more massive disks tend to produce more massive planets (Lodato et al. 2019). Planets inferred to be forming in the larger DSHARP disks are roughly in a Neptune-mass group at the outer disk ($r > 10$ au) and in a Saturn-to-Jupiter-mass group at the inner disk ($r \approx 10$ au; see Figure 21 in Zhang et al. 2018), i.e., the planet mass could be higher at smaller radii. Thus, investigating
further the tendency for compact disks versus large disks would lead to an intriguing study for understanding planet mass induced by disk size.

According to the core accretion model of giant planets in an MMSN, the optimal formation site is believed to be $r = 5\sim10$ au (Helled et al. 2014), although the distance at which a giant gas planet can form could be greater than 10 au on the assumption of pebble accretion (Lambrechts & Johansen 2012) or the maximum rate of planetesimal accretion (Rafikov 2011). Investigations of such an inner 5\sim10 au region for compact disk sources, including our target with higher spatial resolution, is also of great interest and could be very valuable for comparing theoretical and observational studies.

5. Conclusions

By using the super-resolution imaging with SpM (Yamaguchi et al. 2020), we investigated a young triple system T Tau using ALMA 1.3 mm archival data. A summary of our findings is as follows:

1. The imaging drastically improves the spatial resolution on the continuum image of the T Tau system. We then find an annular emission gap in the T Tau N disk and two new emissions around T Tau Sa and Sb.

2. The effective spatial resolution of the image achieves $\sim30\%$ ($38 \times 27$ mas or $5 \times 4$ au) compared with the CLEAN beam size confirmed by tests evaluating the response to artificial point-source injections. This result is in good agreement with the prediction that interferometric imaging can use visibility amplitudes at maximum baselines for deriving source structures by $\sim30\%$ (or $1/3$) of the synthesized beam size.

3. Each position of the separated two emissions around T Tau Sa and Sb is in good agreement within their uncertainties, with each one predicted by the stellar orbital model in Köhler et al. (2016). In addition, each total flux roughly fits each SED predicted by an accretion disk model (Ratzka et al. 2009). The two emissions can thus be regarded as dust emissions originating from the circumstellar disks of T Tau Sa and Sb. The dust disk sizes of T Tau Sa and Sb are smaller than $6 \times 4$ au ($45 \times 27$ mas) and $7 \times 3$ au ($50 \times 22$ mas), respectively. The total flux density of T Tau Sb is about seven times lower than that of T Tau Sa. This ratio implies that the actual disk size of T Tau Sb would be smaller than that of T Tau Sa when considering general scaling relations between disk properties (Tripathi et al. 2017; Andrews et al. 2018b; Hendler et al. 2020).

4. The T Tau N disk has a radius of $24\pm4$ au enclosing 95\% of the total flux and has an annular gap at $r = 11.6 \pm 0.3$ au. Its total flux is as large as 174 mJy, comparable to that of much larger disks. The disk then shows the high brightness temperature of $257 \pm 1$ K at the peak and the low spectral index of $1.9 \pm 0.1$, suggesting that this disk is optically thick at $1.3$ mm. The lower-limited dust surface density appears higher than the MMSN case locally at the outer ring, even though a majority of disks in the low-mass star-forming regions generally appear less massive than the MMSN (Andrews 2015; Tazzari et al. 2017). The T Tau N disk, despite being a small dust disk, would be regarded as more massive than regular disks. Meanwhile, given the relatively high values of the Toomre $Q$ parameter ($Q > 3$) at the outer ring, it appears to be gravitationally stable.

5. We considered a possibility for the origin of the gap in the T Tau N disk by using two different methods that connect the planetary mass and gap shape. If we take a viscous parameter over a wide range of $10^{-2} \sim 10^{3}$, the derived planetary masses are similar to Saturn’s mass: $0.5\sim4.5 M_{\text{Saturn}}$ for the analytic formula by Kanagawa et al. (2015, 2016) and $0.5\sim2.7 M_{\text{Saturn}}$ for the analytic formula by Zhang et al. (2018).

Our super-resolution imaging is potentially impressive, but the reliability of the resolution and the findings for the T Tau system would depend on the confirmation of the substructure by future observations with better angular resolution and sensitivity. Ultimately, ALMA observations with a higher spatial resolution comparable to the effective resolution, at least $30\,$mas, can assess the consistency of the super-resolution imaging and confirm the existence of the substructure.

We thank the anonymous referee for all of the comments and advice that helped improve the manuscript and contents of this study. We also thank all of the East Asian ALMA staff members at NAOJ for their kind support and Sai Jinshi for the helpful conversation. We are grateful to R. Köhler for providing us with the predicted coordinate data of the target source. We finally thank Editage (https://www.editage.com) for English language editing. M.Y. was financially supported by the Public Trust Iwai Hisao Memorial Tokyo Scholarship Fund and the Sasakawa Scientific Research Grant from the Japan Science Society. This work was financially supported in part by Japan Society for the Promotion of Science (JSPS) KAKENHI grant Nos. 17H01103, 18H05441, and 19K03932 (T.M.); Nos. JP17K14244 and JP20K0417 (T.T.); Nos. 18H05441, 19K03910, and 20H00182 (H.N.), No. 20H01951 (S.I.), and Nos. 18H05442, 15H02063, and 22000005 (M.T.). This study uses the following ALMA data: ADS/JAO.ALMA/##2016.1.01164.S. ALMA is a partnership of ESO (representing its member states), NSF (USA) and NINS (Japan), together with NRC (Canada), MOST and ASIAA (Taiwan), and KASI (Republic of Korea), in cooperation with the Republic of Chile. The Joint ALMA Observatory is operated by ESO, AUI/NRAO, and NAOJ. Data analysis was in part carried out on the multiwavelength data analysis system operated by the Astronomy Data Center (ADC), National Astronomical Observatory of Japan. This study used data from the European Space Agency (ESA) mission Gaia (https://www.cosmos.esa.int/gaia), processed by the Gaia Data Processing and Analysis Consortium (DPAC, https://www.cosmos.esa.int/web/gaia/dpac/consortium). Funding for the DPAC has been provided by national institutions, in particular the institutions participating in the Gaia Multilateral Agreement.

Software: AnalysisUtilities (https://casaguides.nrao.edu/index.php?title=Analysis_Utilities), Astropy (Astropy Collaboration et al. 2013), CASA (McMullin et al. 2007), DIFMAP (Shepherd et al. 1994), matplotlib (Hunter 2007), PRIISM (Nakazato & Ikeda 2020), SciPy (Virtanen et al. 2020), least-squares fitting (Hammel & Sullivan-Molina 2020).

Appendix A

Comparison between CLEAN and SpM Images

As shown in Figure 1, we have three images for the T Tau system: the beam-convolved CLEAN image (Figure 1(c)), the
CLEAN components or CLEAN model (Figure 1(d)), and SpM (Figure 1(b)). Here we compare the three images in the visibility domain and discuss which image can be best used for image analyses.

Figure A1 shows radial visibility profiles of the T Tau system calculated from the SpM image, the CLEAN model, and the beam-convolved CLEAN image, together with the real part of observed visibilities. Here the visibility of the beam-convolved CLEAN image is obtained from the Fourier transform of the final CLEAN image (see Figure 1(c)) by extracting the Fourier component that corresponds to the observed visibility. The observed visibilities and modeled M are deprojected in the uv-plane using the derived PA and inclination of the T Tau N disk in Appendix D.

To evaluate the goodness of fits between the models and the observation, we calculated the reduced $\chi^2$. The formula is given by $\chi^2 = N^{-1}\sum_{i=1}^{N} |W_i(O_i - M_i)|^2$, where $N$ is the total number of visibilities and $W_i$ is the weight of the $i$th observed visibility $O_i$. The values of $W_i$ are obtained in the measurement set of ALMA data. The factor $f$ is the ratio between the weight and the standard deviation (stddev) of the visibility ($f = \text{stddev}^2$/weight), which is reported to be $\sim 0.2$–0.3 in other disk observations (Hashimoto et al. 2021a, 2021b). To estimate $f$ of the T Tau data, we calculated the standard deviation of the real part of visibility in every $3\lambda$ bin along the uv-distance. We have obtained $f=0.29$. Finally, all the visibility models corresponding to the three images (CLEAN image, CLEAN model, and SpM) have resulted in the reduced $\chi^2$ of around unity: 1.31 for the CLEAN image, 1.18 for the CLEAN model, and 1.18 for the SpM. Therefore, all three images equally reproduce the visibility distribution in the 2D uv-plane.

However, the situation changes when we consider azimuthally averaged visibility profiles. We have binned the visibility data every $3\lambda$ of the uv-distance and have taken the average in each bin (Figures A1(b) and (c)). The noise of the azimuthally averaged visibility is much smaller than the original 2D visibility. As a result, we found the following two features: (1) the CLEAN model and the SpM image reproduce the observed visibility even after azimuthal average, and (2) the visibility profile obtained from the CLEAN image significantly deviates from the observed visibility at 0.2–1.1 $\lambda$ and at 1.5–2.2 $\lambda$. We expect that the deviation of the CLEAN image is caused by the convolution by the restoring beam. We therefore consider that either the CLEAN model or the SpM image better reproduces observations compared to the CLEAN image.

It is not possible to distinguish the CLEAN model and SpM image from the goodness of fit measured by the reduced $\chi^2$ values. However, we consider that the SpM image better reconstructs the disk surface brightness distribution. The CLEAN model reconstructs an image with a sum of a number of point sources (CLEAN components; Högbom 1974; Clark 1980). As a result, we see a patchy pattern in the CLEAN model image, which we consider irrelevant for disk structures. The SpM image shows more smooth structures than the CLEAN model and therefore seems more reasonable. Therefore, in this paper we mainly use the SpM image for image analyses. We do not yet have more quantitative measurements that can distinguish the SpM image from the CLEAN model image, and the bias that the SpM image may have is still an open question.

Appendix B
Selection of Optimum Image in SpM Imaging of T Tau N Disk

Figure B1 shows 25 SpM images ($1''6 \times 1''6$) of the T Tau system and a close-up of T Tau N ($0''5 \times 0''5$), each of which corresponds to two sets of 25 SpM images of the T Tau system corresponding to 25 combinations of regularization parameters ($\Lambda_r, \Lambda_{\text{tv}}$). We observe that the reconstructed disk structures change depending on the combination. We also show the visibility (real and imaginary parts) plots and calculated CVE with $1\sigma$ uncertainty. The optimal image is selected as one for ($\Lambda_r = 10^5$, $\Lambda_{\text{tv}} = 10^9$), giving the minimal CVE (100.0000% ± 0.13%). It is clearly seen that images with larger CVEs, i.e., ($\Lambda_r = 10^7$, $\Lambda_{\text{tv}} = 10^7$.. $10^{11}$) and...
also show large deviations of model visibilities compared with observed data, especially at higher spatial frequencies. It should be noted that the annular gap structure of the T Tau N disk and the two separated emissions around T Tau Sa/Sb are commonly seen in images with a CVE of approximately 100.0%, for example, \((\Lambda_r = 10^3, \Lambda_{tsv} = 10^6)\) and \((\Lambda_r = 10^3, \Lambda_{tsv} = 10^6)\). This indicates that the presence of these structures is quite robust. In contrast, the image for \((\Lambda_r = 10^3, \Lambda_{tsv} = 10^6)\) also yields a low CVE (100.012\% \pm 0.13\%), and the inner disk is likely to be more resolved than other images. As described in Section 2.2, the effective spatial resolution \(\theta_{\text{eff}}\) is 0\".03. The ratio of the T Tau Sa disk size to the CLEAN beam was obtained as \(\sim 19\%\) for the image with \((\Lambda_r = 10^5, \Lambda_{tsv} = 10^6)\), which is smaller than \(\theta_{\text{eff}}\) (\(\sim 30\%\) of the CLEAN beam) for the optimal image with \((\Lambda_r = 10^5, \Lambda_{tsv} = 10^6)\). In a previous study, the SpM image allowed us to achieve a smaller beam size, that is, typically \(\sim 30\%–40\%\) (Akiyama et al. 2017a, 2017b; Kuramochi et al. 2018; Yamaguchi et al. 2020) compared to the corresponding CLEAN image. The SpM image with such a “hyper” spatial resolution (i.e., beam smaller than 20\% of the CLEAN beam) may reflect the presence of a small substructure in the inner disk, but it appears difficult to evaluate the feasibility. Therefore, we conclude that the optimal

**Figure B1.** SpM imaging of the PPD T Tau system. Each panel corresponds to a gallery of 20 images with a combination of \(\Lambda_r\) and \(\Lambda_{tsv}\). Top left: SpM images. A wide field of view of 1\"6 x 1\"6 is adopted. The dashed line box indicates the optimal image \((\Lambda_r = 10^5, \Lambda_{tsv} = 10^6)\) selected by CV. Top right: close-up view of SpM images. A field of view of 0\"5 x 0\"5 is adopted. Bottom left: radial visibility profile. The upper and lower panels indicate the real and imaginary parts obtained from the observed data (gray color) and the Fourier transform of the SpM images (purple color). Bottom right: CVEs and 1\(\sigma\) uncertainties. The CVEs are the residual values between the observed data and SpM data using the mean squared error. Outputs denote the CVEs normalized to the minimum value.
image selected from the CV would be the best among the images in Figure B1 in terms of spatial resolution improvement, and it is also the best for quantitative analysis.

Appendix C
Effective Spatial Resolution and Detection Threshold of SpM Image

We performed two kinds of SpM imaging simulations. One is for estimating effective spatial resolution in the SpM imaging, and the other is for evaluating the detection threshold. For both, we injected an artificial point source at 0°.4 north in the observed data. We made the SpM images for $\Lambda_{tsv} = 10^7, 10^8, 10^9, 10^{10},$ and $10^{11}$ and fixed $\Lambda_l = 10^5$. For the effective spatial resolution purpose, we injected the point source with a flux density of 7.1 mJy (which corresponds to the total flux of the T Tau Sa disk). We fitted a 2D Gaussian to the retrieved image of the point source. The obtained geometric mean of the source size of major and minor axis for each regularization parameter is plotted in Figure C1. For the detection threshold estimate, we change the flux density of the point sources from 100 to 1000 $\mu$Jy in increments of 100 $\mu$Jy. We judged the detection in the image according to the criterion that the point source can be located at the injected position and be reconstructed as a single source with roughly more than 90% of the input flux density. The results are summarized in Figure C2, together with $I_{DT}$ estimated from

![Figure C1. Evaluation of effective spatial resolutions of the SpM images. The effective spatial resolution of the SpM image is evaluated in the way of an elliptical Gaussian fit to an artificial point source injected into 0°.4 north in the observed data. Each resolution is tuned by the regularization parameter of $\Lambda_{tsv}$, while the regularization parameter of $\Lambda_l$ is fixed to be log $\Lambda_l = 5$.](image)

![Figure C2. Comparison of spatial resolutions and noise levels on the SpM (left side) and CLEAN (right side) images. Each resolution is plotted in the top panel as the geometric mean of the major and minor axes of the beam (or point source; see Figure C1) normalized by the diffraction-limited resolution ($\lambda/D_{\text{max}} = 0.11$ arcsec) given by the maximum baseline length $D_{\text{max}}$. The bottom left panel indicates the point-source sensitivity (gray color) in the SpM image set to be the threshold that the point source can be located at the injected position and be reconstructed as a single source with roughly more than 90% of the input flux density. The results are summarized in the panel together with $I_{DT}$ (purple color) estimated from the beam-convolved image (mJy beam$^{-1}$; beam size: $\theta = 0°.14 \times 0°.10$). The bottom right panel shows the rms noise $\sigma$ of the CLEAN image for each Briggs robust parameter.](image)
the beam-convolved image (Jy beam\(^{-1}\); beam size: \(\theta = 0''14 \times 0''10\)). The rms noise \(\sigma\) and CLEAN beam size for each robust parameter are summarized for comparison (see Figure C2). The values of the detection threshold for simulation in different \(\lambda_{\text{rms}}\) or \(I_{\text{DT}}\) can be compared to CLEAN cases, and we found that those correspond to roughly 4\(\sigma\) of the CLEAN image for \text{robust}=0.5 and are lower than the 4\(\sigma\) level for \text{robust}=-2 or -1. These results show that the SpM imaging can achieve super-resolution without significant degradation of point-source sensitivity.

We then try to explain why the SpM imaging can achieve roughly three times better spatial resolution. The usual diffraction-limited resolution is roughly wavelength \(\lambda\) divided by maximum baselines \((\lambda/D_{\text{max}})\), where \(\lambda\) is observing wavelength and \(D_{\text{max}}\) is an aperture size or the longest baseline length in the interferometer). In the interferometric synthesis observations, the synthesized beam (i.e., response to a point-source or point-spread function) can be expressed as a summation of each visibility pattern given by a two-element interferometer. The (one-dimensional) visibility pattern produced by the longest baseline has a fringe spacing of \(\sim \lambda/D_{\text{max}}\) (Thompson et al. 2017). It then has a sharper spatial amplitude response with an FWHM of \(1/3 \times \lambda/D_{\text{max}}\) if we consider only its positive side contributing to forming the final beam. It is roughly three times smaller than the synthesized beam with a size of \(\lambda/D_{\text{max}}\) (see inset in Figure C2). We have confirmed the FWHM size of \(\sim 0''04\) for the visibility pattern for the longest baseline (\(\sim 4\) km) in the ALMA configurations, DV23 and DA42. The SpM imaging is a regularized least-squares method where observed visibilities are directly used to retrieve the image, and the long baselines can be exploited as much as possible in super-resolution imaging. On the other hand, visibility amplitude distribution as a function of \(uv\)-distance can be used for estimating the source sizes of compact objects. For a source with Gaussian spatial distribution, the FWHM source size \(\theta_{\text{FWHM}}\) and the \(uv\)-distance giving the half visibility amplitude \(UV_{1/2}\) have a relation such as \((UV_{1/2}/100 \text{ kA}) \times (\theta_{\text{FWHM}}/1''0) = 0.91\) (Kawabe et al. 2018). If the \(UV_{1/2}\) is equal to \(D_{\text{max}}\), \(\theta_{\text{FWHM}}\) in units of radians is equal to 0.44 \(\times \lambda/D_{\text{max}}\). It should be roughly \(1/3 \times \theta_{\text{synth}}\) if \(\theta_{\text{synth}} \approx 1.22\lambda/D_{\text{max}}\) (corresponding to the Rayleigh criterion to resolve two-point sources; see ALMA technical handbook). This expression means that interferometric imaging could exploit visibility amplitudes measured with high S/N even at long baselines for deriving source structures much smaller than the synthesized beam.

### Appendix D

#### Disk Inclination and Position Angle of T Tau N

To derive the position angle (PA) and inclination of the T Tau N disk using the SpM image, we performed an ellipse fit to the outer ring of the disk using least-squares fitting (Hammel & Sullivan-Molina 2020), together with the Monte Carlo routine, as shown in Figure D1. Here we assume that the outer ring is a perfect circle face-on, and we estimated the inclination angle from the aspect ratio of the ellipse.

First, we sampled the radial peak position of the outer ring on the PA profile every 1° in the azimuthal angle \(\theta\). We averaged the peak position \(r_{\text{peak}}(\theta)\) and derived its standard deviation \(\sigma_{r}(\theta)\) from the measurements within the azimuthal angle spacing \(\Delta \theta\), and we used them for the ellipse fit. \(\Delta \theta\) is set to be of the order of the major axis of the spatial resolution element \(\theta_{\text{eff, maj}}\) (= 0''038) in Section 3.1. Given that the radial peak positions are roughly located at a radius of \(r = 0''1\) (estimated from visual inspection), the azimuthal angle spacing in degrees is thus derived as \(\Delta \theta \approx \theta_{\text{eff, maj}} \times 360°/2\pi r \approx 20°\).

Because the radial peak positions in the range of \(PA = 165°-220°\) cannot be identified owing to an insufficient angular resolution or low S/N, these samples are excluded from the use of the ellipse fit. To calculate the error of the fit, a Monte Carlo routine was performed by randomly sampling the \(\sigma_{r}(\theta)\) deviation, and then \(r_{\text{peak}}(\theta)\) were added to them. A total of 5000 iterative calculations were performed, and the best-fit ellipse can be obtained from the average values of the iterations. Uncertainties in each parameter of the ellipse fit were calculated by taking the standard deviation found with the iterations. The best-fit results of the PA and inclination are \(91°4 \pm 3°0\) and \(25°2 \pm 1°1\), respectively, as summarized in Table 3.

### Appendix E

#### Spectral Index of T Tau N Disk

The spectral index \(\alpha\) of the T Tau N disk was estimated using two SPWs that were used for the continuum observations at Band 6, with center frequencies at \(\nu_1 = 218\) GHz and \(\nu_2 = 233\) GHz. Two CLEAN images were restored with the same beam size identical to that obtained at a lower frequency. Using the CLEAN maps, we obtained the total flux densities \(F_1\) and \(F_2\) for \(\nu_1 = 218\) GHz and \(\nu_2 = 233\) GHz, respectively. The spectral index was calculated as \(\alpha = \ln(F_2/F_1)/\ln(\nu_2/\nu_1)\) and was obtained as \(\alpha = 1.9 \pm 0.1\). For the optically thick emission, the Rayleigh–Jeans limit gives \(\alpha \approx 2\). For other PPDs in the Taurus, the typical \(\alpha_{\text{mm}}\) values are below 3.0, and several of them are even below 2.0, as in the case of the T Tau N disk.
(e.g., Ricci et al. 2010; Akeson & Jensen 2014; Pinilla et al. 2014; Ribas et al. 2017; Zagaria et al. 2021). The obtained spectral index lower than the limit can be explained by considering the additional effect of dust self-scattering (Liu 2019; Zhu et al. 2019).

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