Design and Error Analysis of The Linkage Type Steering Mechanism for Multiple Wheel Heavy Vehicle

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Abstract—In this paper, the steering system dimension of multiple wheel heavy vehicle which is composed of center-arm linkage type steering mechanism has been designed. The steering mechanism of the vehicle is composed of five axles of Watt-II center-arm six-bar mechanism. The vehicle is designed to meet the steering requirements. Based on the Ackerman principle of vehicle steering, the mathematical model of steering mechanism is established, and the precise steering angle is determined. Then the corresponding calculation program has been written by using MATLAB software, and the Broyden's iterative algorithm has been used. The each bar dimensions of the center-arm six-bar mechanism has been obtained through cyclic approximation. Finally, the kinematics simulation and error analysis of the steering mechanism are carried out, and the results show that the steering mechanism can meet the steering requirements of the vehicle.

Keywords—heavy vehicle; six bar mechanism; dimension design; Ackerman principle

I. INTRODUCTION

With the development of our economy, special vehicles and engineering vehicles gradually to the development direction of large-scale and heavy-duty, most of these vehicles have more than two steering axles. The more the number of axles, the more difficult for vehicle steering and getting through in complex curve road, its steering performance have directly effect on vehicle agility, manipulate stability and use economy, so it has very real significance to research multi-shaft vehicles. Optimal steering process requires all wheels to be in a pure rolling state without sliding, or only little slippage; or else large slippage can aggravate wheel’s wear and tear. Reasonable steering mechanism can reduce steering resistance and achieve the purpose of light steering.

This article aim at the research on the multi-axle Watt-II center-arm six-bar steering mechanism for heavy vehicle with multiple wheel[1,2], which aims to promote a kinematic point of view to improve the dimension of the center-arm-linkage-type six bar mechanism in order to improve the steering performance of the heavy vehicle with multiple wheel.

II. THE DESIGN OF CENTER-ARM-LINKAGE-TYPE STEERING MECHANISM

A. Steering Principle and Composition of Multi-wheel Steering Mechanism

As heavy-duty vehicles with multiple wheel and large load, want to make the move when the steering wheel to keep rolling contact with the ground without sliding, wheel deflection must satisfy the Ackerman Principle[3,4] (as Figure I). In other words, The front wheel alignment angle of

FIGURE I. THE CORNER RELATION OF INSIDE AND OUTSIDE CAR WHEEL

1- toggle arm  2- center-arm  3- linkage  4- tension rod  K- body width

FIGURE II. THE FIVE SHAFTS OF WATT-II CENTER-ARM SIX BAR MECHANISM
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In this paper, a steering device composed of a five-axle Watt-II center-arm six-bar mechanism is taken as an object of study (as Figure II). The six-bar mechanism used in the steering device is Watt-II, the five six-bar mechanisms connected by five rods, the first rod is connected to steering power to realize multi-wheel steering of heavy-duty vehicles [5, 6]. When the vehicle is steering, tension rod 4 driven by steering rocker, through the linkage 3 drive toggle vehicles [5, 6]. When the vehicle is steering, tension rod 4 driven by steering rocker, through the linkage 3 drive toggle vehicles [5, 6].

B. Mathematical Model of Center-arm Six-bar Steering Mechanism

Establishment of two dimensional coordinate system for the center-arm six-bar mechanism (as Figure III). Take $a_{0}$ as the origin, assume that the body width is unit length [1]"[7], then the coordinate points of the six bar mechanism are $a_{0}(0, 0), a_{1}(0, x_{0}), b_{3}(b_{13}, b_{10}), c_{0}(0.5, c_{0}), c_{1}(-b_{1}, b_{10}), d_{1}(1, a_{10}), b_{4}(1, 0)$, if we can know $a_{1}, b_{3}, b_{10}, c_{0}, d_{1}, b_{5}$ have been fixed, so the dimension of the Watt-II center-arm-linkage-type six-bar mechanism can be fixed. As $a_{1}$ is fixed-length which has been determined in advance, so just solve the value about $b_{13}, b_{10}, c_{0}$.

![Image](image)

**FIGURE III.** THE CENTER-ARM SIX BAR MECHANISM TURN RIGHT

Suppose that when the vehicle turns, the rotation angle of $a_{0}a_{1}, a_{1}a_{3}$ and the center-arm are $g_{0}, b_{3}, b_{10}$, when vehicle turn a corner, make $a_{1}, b_{3}, c_{1}, d_{1}$ turn to $a_{1}, b_{3}, c_{0}, d_{1}$. The design equation of the mechanism is equal length constraint equation of $a_{1}, b_{3}, c_{0}, d_{1}$ [8]:

$$\begin{align*}
(a_{j} - b_{j})^2 + (c_{j} - b_{j})^2 &= (a_{j} - c_{j})^2 + (c_{j} - d_{j})^2 \\
(j &= 2, 3, 4, 5 \ldots) \tag{2}
\end{align*}$$

$$\begin{align*}
(a_{j} - b_{j})^2 + (a_{j} - c_{j})^2 &= (a_{j} - d_{j})^2 + (c_{j} - d_{j})^2 \\
(a_{j} - b_{j})^2 + (c_{j} - d_{j})^2 &= (c_{j} - d_{j})^2 + (c_{j} - d_{j})^2
\end{align*}$$

in this type:

$$\begin{align*}
\begin{bmatrix}
    a_{j} \\
    b_{j} \\
    c_{j} \\
    d_{j}
\end{bmatrix} &= \begin{bmatrix}
    D_{1,j} & a_{1} \\
    1 & b_{1} \\
    1 & c_{1} \\
    1 & d_{1}
\end{bmatrix}
\end{align*}$$

$$\begin{align*}
\begin{bmatrix}
    a_{j} \\
    b_{j} \\
    c_{j} \\
    d_{j}
\end{bmatrix} &= \begin{bmatrix}
    D_{1,j} & a_{1} \\
    1 & b_{1} \\
    1 & c_{1} \\
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\end{bmatrix}
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    a_{j} \\
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    c_{j} \\
    d_{j}
\end{bmatrix} &= \begin{bmatrix}
    D_{1,j} & a_{1} \\
    1 & b_{1} \\
    1 & c_{1} \\
    1 & d_{1}
\end{bmatrix}
\end{align*}$$

To solve the value of $b_{13}, b_{10}, c_{0}$, put the above equations into type(2), obtained:

$$\begin{align*}
E_{1,j} \cos(\theta_{j}) + F_{1,j} \sin(\theta_{j}) &= G_{1,j} \\
E_{2,j} \cos(\theta_{j}) + G_{2,j} \sin(\theta_{j}) &= G_{2,j}
\end{align*}$$

$$\begin{align*}
\cos(\theta_{j}) &= \frac{G_{1,j} F_{1,j} - G_{2,j} F_{2,j}}{E_{1,j} F_{1,j} - E_{2,j} F_{2,j}} \\
\sin(\theta_{j}) &= \frac{E_{1,j} G_{2,j} - E_{2,j} G_{1,j}}{E_{1,j} F_{1,j} - E_{2,j} F_{2,j}} \tag{3}
\end{align*}$$

in type(3), according to $\cos^{2}(\theta_{j}) + \sin^{2}(\theta_{j}) = 1$, obtained:

$$\begin{align*}
(E_{1,j} G_{2,j} - E_{2,j} G_{1,j})^2 + (G_{1,j} F_{1,j} - G_{2,j} F_{2,j})^2 - (E_{1,j} F_{1,j} - E_{2,j} F_{2,j})^2 &= 0 \tag{4}
\end{align*}$$

in this type:

$$\begin{align*}
E_{1,j} &= 2[M_{1,j}(c_{0,j} - b_{1,j}) - N_{1,j}(c_{0,j} - b_{1,j})] \\
F_{1,j} &= 2[-N_{1,j}(c_{0,j} - b_{1,j}) - M_{1,j}(c_{0,j} - b_{1,j})] \\
E_{2,j} &= 2[-M_{2,j}(c_{0,j} - c_{1,j}) - N_{2,j}(c_{0,j} - c_{1,j})] \\
F_{2,j} &= 2[-N_{2,j}(c_{0,j} - c_{1,j}) + M_{2,j}(c_{0,j} - c_{1,j})]
\end{align*}$$

$$\begin{align*}
G_{1,j} &= (a_{j} - b_{j})^2 + (a_{j} - c_{j})^2 - (c_{j} - d_{j})^2 - (b_{j} - c_{j})^2 - N_{1,j}^2 - M_{1,j}^2 \\
G_{2,j} &= (c_{j} - d_{j})^2 + (c_{j} - d_{j})^2 - (c_{j} - c_{j})^2 - (c_{j} - c_{j})^2 - N_{2,j}^2 - M_{2,j}^2
\end{align*}$$

which:
\[ M_{1j} = a_{1j} \cos(g_j) + a_{1j} \sin(g_j) - c_{0x} \]
\[ N_{1j} = a_{1j} \sin(g_j) - a_{1j} \cos(g_j) + c_{0y} \]
\[ M_{2j} = -d_{1j} \cos(h_j) - d_{1j} \sin(h_j) - 1 + \cos(h_j) + c_{0x} \]
\[ N_{2j} = d_{1j} \sin(h_j) - d_{1j} \cos(h_j) - \sin(h_j) + c_{0y} \]

Type(4) is non-linear equations set without unknown trigonometric functions, which can be solved with selecting the appropriate initial value.

The unknown number \( \theta_j \), which is the angular displacement of the member \( c_0b_1c_1 \) is included in the displacement matrix \([D_{1j}]\), its number is \((j-1)\). In Figure III coordinate system, \( a_0b_0 \) as unit length, then \( b_0 = (1,0), a_1 = (0,a_1y), b_1 = (b_1x,b_1y), d_1 = (1,d_1y), c_0 = (0.5,c_0y) \).

Then in type (2), the unknown number include \( b_1x, b_1y, c_1x, c_1y, d_1y, c_0y \) and \( \theta_j \), the number of unknown number is \( U = 6 + (j-1) = j + 5 \), the number of design equation is \( V = 2(j-1) \). Let \( U = V \), so \( j = 7 \). That is to say seven exact points of given function can be realized by the Watt-II center-arm six-bar mechanism designed in this paper[7].

III. BROYDEN ITERATIVE METHOD BASED ON MATLAB

The value of \((g_j,h_j)\) in equations set(4) is determine by Chebyshev theorem[9]. As the heavy-duty vehicle at low speeds are usually for large angle, assuming the wheel angle between \(10^\circ\) and \(40^\circ\)[10,11], namely: \( p = 40^\circ, q = 10^\circ \).

\[ g_2 = q + 0.067 \times (p - q) \]
\[ g_1 = q + 0.500 \times (p - q) \]
\[ g_4 = q + 0.933 \times (p - q) \]

By Ackerman principle type(1), obtained:

\[ h_5 = \arccot(\cot(g_5) - (k/l)) \]
\[ h_4 = \arccot(\cot(g_4) - (k/l)) \]

\[ F_j = (E_jG_j - E_jG_j)^2 + (G_jF_j - G_jF_j)^2 - (E_jF_j - E_jF_j)^2 \] (5)

By type(4) obtained:

\[ F_j = 0, \quad j = 2, 3, 4, \ldots \] (6)

Type(6) is non-linear equations set without unknown trigonometric functions, convert the type (6) to the following system of linear equations:

\[ \begin{align*}
\begin{bmatrix}
\delta F_j / \partial c_{1x} \\
\delta F_j / \partial c_{1y} \\
\delta F_j / \partial b_{1x} \\
\delta F_j / \partial b_{1y} \\
\delta F_j / \partial h_{1x} \\
\delta F_j / \partial h_{1y}
\end{bmatrix} =
\begin{bmatrix}
\Delta c_{1x} \\
\Delta c_{1y} \\
\Delta b_{1x} \\
\Delta b_{1y} \\
\Delta h_{1x} \\
\Delta h_{1y}
\end{bmatrix}
\end{align*} \]

(7)

Express the above as: \( A \delta = F \), \( A \) is Jacobian matrix. The Broyden iterative method[12,13] is used to obtain the exact value of the above calculation, the calculation of the above process can be implement through the preparation of the corresponding MATLAB programs, the flow chart of MATLAB programming is as follows(as Figure IV).

By programming, some of the calculated results are obtained(as Table I).

TABLE I. THE DESIGN PARAMETER OF CENTER-ARM\((a_0y = 0.3)\)

| L/K | \( c_{0x} \) | \( b_{1x} \) | \( b_{1y} \) |
|-----|--------------|--------------|--------------|
| 0.5 | 0.1458       | 0.4355       | -0.0878      |
| 0.9 | 0.1655       | 0.4354       | -0.0870      |
| 1.0 | 0.1854       | 0.4395       | -0.0880      |
| 1.2 | 0.2142       | 0.4491       | -0.0906      |
| 1.4 | 0.2312       | 0.4565       | -0.0924      |
| 1.6 | 0.2414       | 0.4616       | -0.0934      |
| 1.9 | 0.2499       | 0.4663       | -0.0942      |
| 2.0 | 0.2517       | 0.4673       | -0.0943      |
IV. DESIGN EXAMPLE

Suppose the vehicle is composed of five-axle center-arm six bar mechanism. The first axle is longitudinal center, and the distance from remaining four axle to the first axle are: $L_1=1400\text{mm}$, $L_2=2800\text{mm}$, $L_3=4200\text{mm}$, $L_4=5600\text{mm}$, the width of body is $K=3000\text{mm}$, namely $L_1/K=0.5$, $L_2=2800\text{mm}$, $L_3=4200\text{mm}$, $L_4=5600\text{mm}$, the width of body is $K=3000\text{mm}$, namely $L_1/K=0.5$, $L_2/K=0.9$, $L_3/K=1.4$, $L_4/K=1.9$. Put the above data into the MATLAB program that can be separately obtained the dimension of every bar in the five-axle center-arm six bar mechanism (as Table II).

|       | One  | Two  | Three | Four  | Five  |
|-------|------|------|-------|-------|-------|
| $S_1$ | 639  | 639  | 639   | 639   | 639   |
| $S_2$ | 1242.1 | 1242.1 | 1240.8 | 1282.2 | 1300.6 |
| $S_3$ | 516.2 | 516.2 | 555.1 | 695.5 | 756.4 |
| $S_4$ | 274.8 | 274.8 | 275.2 | 185.3 | 143.6 |
| $S_5$ | 1089.4 | 310.6 | 1752.5 | 3292.5 | 4732.3 |

Which: $S_1$ is the length of the toggle arm, $S_2$ is the length of the linkage, $S_3$ is the length of center-arm tripod’s waist, $S_4$ is the length of center-arm tripod’s bottom, $S_5$ is the distance between the center-arm tripod’s vertices to longitudinal center. The position of every dimension is as follows (as Figure V).

Thus available the dimension of the center-arm six bar mechanism which satisfy the steering requirement of multi-wheel vehicle. It provides reference for further analysis and practical design.

V. SIMULATION AND ERROR ANALYSIS OF STEERING DEVICE

According to rod length of the five-axle center-arm six bar mechanism and $L_1-L_4$, establish a 3D solid model for its steering by Pro/E (as Figure VI).
VI. THE MODEL OF CENTER-ARM SIX BAR MECHANISM STEERING SYSTEM

It is very important to reasonably match the bar system of Multi-wheel steering vehicles, not only the rotation angles of two wheels on the same axle should match each other, but also different steering axle wheels should match well with each other. In this section, the angle error of different axle wheels that constitute steering system has been analyzed.

Multi-wheel heavy vehicles steering, suppose the vehicle turns to the right, the wheel rotation angle not on the same axle should satisfy the following relationships[3,4]:

The rotation angle relationship of the same side toggle arm(wheel) satisfy the following formula:

\[ L_i \times \cot(\beta_i) = L_j \times \cot(\beta_j) \]

\[ L_i \times \cot(\alpha_i) = L_j \times \cot(\alpha_j) \]

then:

\[ \beta_i = \arccot \left( \frac{L_i \times \cot(\beta_j)}{L_j} \right) \]

\[ \alpha_i = \arccot \left( \frac{L_j \times \cot(\alpha_j)}{L_i} \right) \]

in this type:

\( \beta_i \)—the left wheel(outer wheel) angle of the center arm mechanism of the \( i \) axle of the vehicle

\( \alpha_i \)—the right wheel(Inner wheel) angle of the center arm mechanism of the \( i \) axle of the vehicle

\( L_i \)—Distance from longitudinal center of vehicle \( i \) axle center-arm mechanism

\( i, j \) are natural number.

The steering device composed of five Watt-II center-arm six-bar mechanism was introduced into ADAMS for kinematics simulation. To deflect the steering device to the right, import simulation angle relation data from ADAMS into MATLAB, compared the results with computing result of the angle theoretical formula (8), (9) and (10), rotation Angle error Analysis of toggle arm in other position based on the angle of toggle arm at \( L/H = 1.9 \). Due to the limitations of the length of the paper, only analyze error about the rotation angle of the left toggle arm of each axle.

Angle error analysis of the left toggle arm (outer wheel) of each axle at \( L/K = 0.5, L/K = 0.9, L/K = 1.4, L/K = 1.9 \) relative to the left toggle arm (outer wheel) at \( L/H = 1.9 \), in Figure VI, Figure VIII, Figure IX and Figure X, the left picture (a) is the angle curve of the left toggle arm (outer wheel) of each axle at \( L/K = 0.5, L/K = 0.9, L/K = 1.4, L/K = 1.9 \) relative to the left toggle arm (outer wheel) at \( L/H = 1.9 \), the right picture (b) is the corresponding angle error curve. Abscissa in groups of (a) diagrams is the left toggle arm rotation angle at \( L/K = 0.9 \), ordinate is the left toggle arm rotation angle at other position. Dotted line is the left toggle arm rotation angle by ADAMS, full line is the left toggle arm rotation angle by the angle theoretical formula.
FIGURE VII. THE STEERING GEOMETRY OF LEFT TOGGLE ARM ($L/K=0.5$)

(a) correlation curve

(b) error curve

FIGURE VIII. THE STEERING GEOMETRY OF LEFT TOGGLE ARM ($L/K=0.9$)

(a) correlation curve

(b) error curve

FIGURE IX. THE STEERING GEOMETRY OF LEFT TOGGLE ARM ($L/K=1.4$)

(a) correlation curve

(b) error curve
It can be seen from the curves in (a) in Figure VII ~ X, in the process of increasing the longitudinal center distance of each wheelbase from \( L_1 = 1400 \text{mm} \) to \( L_4 = 5600 \text{mm} \), the angle of the left toggle arm increased from 16° to 20°. It can be seen from the curves in (b) in above picture, the maximum error angle of the left toggle arm (outer wheel) of each axle at \( L/K = 0.5, L/K = 0.9, \) and \( L/K = 1.4 \) relative to the left toggle arm (outer wheel) at \( L/K = 1.9 \) is -6.5° ~ -4°. The angle of rotation error of the left toggle arm is small enough to be negligible at \( L/K = 1.9 \). It can be seen from the error analysis that the wheel steering error is small, the maximum error angle is within the permitted range, the results show that the steering device of Watt-II center-arm six-bar mechanism is basically satisfied Ackerman principle and can meet the steering requirements of multi-wheel heavy vehicle.

VII. CONCLUSIONS

The center-arm six-bar steering mechanism of multi-wheel heavy-duty vehicle has the advantages of reliable structure, convenient control, fast response speed and low cost compared with the Multi-wheel hydraulic steering system. This paper is based on the Ackerman principle when the multi-wheel heavy-duty vehicle wheel is deflected, the Watt-II center-arm six-bar mechanism is studied, the corresponding mathematical model is established, programmed with MATLAB, the dimensions of each bar of the six-bar mechanism is calculated, the simulation and error analysis of the steering device are carried out. Design method proposed in this paper will provide a theoretical basis for the active design and dynamic analysis of the steering mechanism of this kind of heavy-duty vehicle in the future.

ACKNOWLEDGMENT

The Project Supported by Natural Science Basic Research Plan in Shaanxi Province of China (Program No. 2018JM5045).

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