Optimal Combined Heat and Power Economic Dispatch Using Stochastic Fractal Search Algorithm

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Abstract—Combined heat and power (CHP) generation is a valuable scheme for concurrent generation of electrical and thermal energies. The interdependency of power and heat productions in CHP units introduces complications and non-convexities in their modeling and optimization. This paper uses the stochastic fractal search (SFS) optimization technique to treat the highly non-linear CHP economic dispatch (CHPED) problem, where the objective is to minimize the total operation cost of both power and heat from generation units while fulfilling several operation interdependent limits and constraints. The CHPED problem has bounded feasible operation regions and many local minima. The SFS, which is a recent metaheuristic global optimization solver, outperforms many current reputable solvers. Handling constraints of the CHPED is achieved by employing external penalty parameters, which penalize infeasible solution during the iterative process. To confirm the strength of this algorithm, it has been tested on two different test systems that are regularly used. The obtained outcomes are compared with former outcomes achieved by many different methods reported in literature of CHPED. The results of this work affirm that the SFS algorithm can achieve improved near-global solution and compare favorably with other commonly used global optimization techniques in terms of the quality of solution, handling of constraints and computation time.

Index Terms—Combined heat and power (CHP), economic dispatch, global optimization, metaheuristic algorithms, non-convex optimization problem, power systems, stochastic fractal search.

I. INTRODUCTION

ECONOMIC dispatch (ED) of electric power generation units is among the most imperative operational optimization issues in power systems. This ED-constrained non-linear problem must be solved to minimize the total operation cost of committed units while satisfying the load demand and obeying physical limits of units. It can also be augmented to consider other concerns such as losses in transmission network and pollution quantity made by generation units.

The combined heat and power (CHP) generation ( cogeneration) refers to the joint generation of electrical/mechanical power and advantageous thermal energy from the same source of energy for heating and cooling intentions. The CHP generation is the most effective model for concurrent production of both electrical energy and thermal energy [1]. As an environmentally friendly system, cogeneration offers considerable savings of the generation cost compared to the heat-only boilers and traditional thermal units. The production of CHP is restricted by the feasible operation region (FOR), as productions of heat and power in these units are jointly connected.

Nowadays, optimal dispatch of generation mix involving CHP units is an attractive and crucial optimization issue in power system operation. The major worldwide shift within the energy sector caused by launching microgrid initiatives and technologies has increased the interest in CHP units. Generation units in CHP ED (CHPED) comprise classical thermal power-only supplying units, CHP generation units, and heat-only generation units (boilers). The complexity of this dispatch originates from the FORs of CHP units, which indicates dual non-linear dependency between heat and power productions. CHPED aims to find an optimal schedule of heat and power generation while respecting various electrical and operational constraints and limits of the generation units. The optimization problem is non-linear and non-convex which requires global solvers.

Solution techniques of the CHPED are broadly classified as conventional mathematical methods and recent methods. Conventional methods include Lagrangian relaxation (LR) [2], Benders decomposition [3], [4], mixed-integer non-linear programming [5], and branch and bound (B&B) algorithm [6]. The classical techniques cannot perform effectively for solving CHPED problems as they have shortcomings such as sensitivity to initial estimates, convergence into local optimal solution, computational complexity, and difficulties in handling discontinuities and non-smooth functions, especially when the dispatch problem is highly non-linear.

In the past decade, various recent optimization solvers have been disclosed in the literature to reach global or near-global solutions of non-linear optimization problems. Recent methods include nature-inspired metaheuristic optimization algorithms. Application of these algorithms for CHPED problems has received research attention in the last few years, as they can deal with discontinuities, high non-linearities, and
non-convexities in objective functions and constraints. Many metaheuristic techniques have been positively utilized to treat CHPED problems such as genetic algorithm (GA), harmony search (HS), particle swarm optimization (PSO), firefly algorithm (FA), cuckoo search algorithm (CSA), artificial bee colony (ABC), gravitational search algorithm (GSA), group search optimization (GSO), grey wolf optimization (GWO), bat algorithm (BA), differential evolution (DE), ant swarm optimization (ASO), invasive weed optimization (IWO), teaching learning based optimization (TLBO), artificial immune system (AIS), krill herd (KH), and evolutionary programming (EP). The most recent applied heuristic optimization methods to attain optimum results of CHPED problem were reported in [1].

Reference [7] presented a general review of modeling, planning and energy management in a microgrid that used combined cooling, heating and power (CCHP). Reference [8] provided an ample review of recent trend in CCHP schemes and optimization techniques used to improve their performance. Reference [9] presented a review on definition, benefits, characteristics, and various configurations of CCHP systems. Reference [10] presented the development, benefits and analysis of CCHP schemes, and reviewed sizing, control, optimization procedure, and management of these systems. Reference [11] reviewed the current and future trends in micro CHP systems to improve system efficiencies and reduce gas emissions, and investigated such systems for residential applications including modeling and simulation.

In [12], the self-adaptive real-coded genetic algorithm (SARGA) was utilized to solve the non-convex CHPED optimization problem considering the inequality and equality constraints, and penalty technique was suggested to handle the constraints. In [13], an improved GA technique utilizing a multiplier updating was presented to solve the CHPED with a small population size. In [14], CHPED with losses in transmission network and valve-point effects was considered, where a mutation operator of real coded GA was used to improve the convergence time and optimal operation cost. In [15], the researcher presented a solution of a multi-objective economic-environmental CHPED problem using non-dominated sorting real coded GA.

In [16], PSO was employed to deal with a multi-objective CHPED problem considering operation cost, gas emissions, and wind power resources. A PSO using time-varying acceleration factors was used in [17] to obtain the optimal productions of power and heat units in CHPED considering valve-point effects and network losses. An improved PSO (IPSO) technique was employed to solve a stochastic model of CHPED problem in [18], where both demands of heat and power in the system were treated as random variables.

In [19], DE was used to find the optimal schedule of CHP units considering valve-point effects and losses in transmission network. In [20], a DE utilizing a Gaussian mutation operator was employed to solve the CHPED considering valve-point effects and network losses. In [21], the DE was integrated with the sequential quadratic programming to solve a short-term scheduling of CHP generating units. A combination of continuous grasp algorithm and DE for solving non-smooth non-convex CHPED problem was introduced in [22] to enhance the global search ability and avoid convergence to local minima. Reference [23] introduced a hybrid DE with multiplier updating to obtain the optimal solution of a CHPED within the FOR of CHP units.

The GSO was presented in [24] to deal with the non-convex non-smooth CHPED considering transmission losses, valve-point effects and prohibited operating zones of classical thermal generation units. A modified GSO utilizing the B-Spline wavelet theory was introduced in [25] to solve the CHPED problem to avoid premature convergence of the solution. The GSA was used in [26] to solve the CHPED including transmission network losses and valve-point effects.

The CSA was used in [27], [28] to solicit the optimal schedule of generation in CHP units considering transmission losses and valve-point effects. A CSA technique utilizing an external penalty function was used in [29] for the CHPED. Reference [30] used the GWO for different formulations of CHPED optimizations considering ramp-rate limits, transmission losses, valve-point effects, and spinning reserve.

In [33], a hybrid HS-GA was used for the CHPED problem, where GA features were used to handle the difficulties of non-linearity and non-convergence, and HS features were used to increase the probability of global optimal solution.

The IWO algorithm was used in [34] for solving CHPED problem. The bee colony optimization (BCO) was used in [35] for solving non-convex CHPED problem considering power transmission losses. The TLBO integrated with opposition-based learning (OBL) for improved convergence characteristics was used in [36] to solve the non-linear CHPED optimization problem, and the AIS algorithm was suggested in [37]. The FA was proposed in [38] to treat the CHPED problem with the two objective functions of the total fuel cost and gas emission, considering the spinning reserves, where the FA was used to attain a series of non-dominated solutions utilizing chaotic mechanism and new mutation procedures. In [39], the FA was used for the CHPED problem, with an improved random search process. The KH algorithm was used in [40] to obtain the power and heat generation scheduling in the CHPED, considering transmission network losses and valve-point effects. In [41], the crisscross optimization (CSO) algorithm was used for solving CHPED problem. The algorithm was very competent in both accuracy and convergence rate compared to other algorithms.

The metaheuristic global optimizer, stochastic fractal search (SFS), was a nature-inspired algorithm proposed in [42]. Based on diffusion and fractal properties, this algorithm overcame the shortcomings of the other commonly used metaheuristic solvers. By employing uncomplicated operations, it can realize a global or a better near-global solution with fewer iterations, larger accuracy, and less convergence time [42]. The SFS replicated the natural growth phenomenon by using the fractal mathematic concept [42].

The work in this paper utilizes the SFS technique to deal with the CHPED problem. Handling equality and inequality constraints is achieved in this paper using penalty parameters that penalize infeasible solution during the iterative pro-
cess. Through these parameters, the constrained CHPED problem is transformed into an unconstrained optimization. To confirm its efficient performance, the algorithm has been used for two distinct systems, which are commonly used as test cases in CHPED literature. Comparisons of results presented in this paper disclose that the optimal SFS-based solution can decrease the production costs with a short computation time. It also reveals that the optimal SFS-based solutions are superior to some of the frequently employed global optimization solutions.

The remainder of this paper is organized as follows. Section II presents the mathematical formulation of CHPED. Section III presents a detailed description and the mathematical modeling of the SFS. Section IV presents the test systems, optimization results, and discussion of results. Section V presents summary of the main findings, concluding statements, and recommendations for future effort.

II. MATHEMATICAL FORMULATION OF CHPED

OPTIMIZATION PROBLEM

The optimization problem of CHPED is non-linear and non-convex, whose intent is to achieve the optimal mix of heat and power generations from three different types of resources: conventional power-only units, co-generation (CHP) units, and heat-only units. Accordingly, the objective function, which is the total production cost, is a combination of three different cost functions with equality and inequality constraints. This optimization problem has two equality constraints. The first is that the total electrical power produced from all power generation units meets the total power demand \( p_d \), and the second is that the total heat produced from all heat generation units meets the total heat demand \( h_d \). The inequality constraints relate to the CHP units, which require that optimal productions of CHP units should lie within the FORs. The limits indicate the upper and lower limits of all units participating in the CHPED. Figure 1 shows three possible types of heat-power plane of a CHP unit, which represent possible FOR of any CHP unit \([3, 12], [17, 23, 29, 32]-[34, 39]\), where \( P \) and \( H \) indicate real power production and heat production of a CHP unit, respectively.

The CHPED optimization problem can be mathematically formulated as follows \([3, 12]-[14], [23, 27, 29, 32]-[34, 39]\):

\[
\begin{align*}
\min C &= \sum_{i=1}^{N_p} C_i(p_i) + \sum_{j=1}^{N_c} C_j(p_j, h_j) + \sum_{k=1}^{N_h} C_k(h_k) \\
\text{s.t.} & \\
\sum_{i=1}^{N_p} p_i &= p_d \quad (2) \\
\sum_{j=1}^{N_c} h_j + \sum_{k=1}^{N_h} h_k &= h_d \quad (3) \\
p_{i\min} \leq p_i \leq p_{i\max} & \quad i = 1, 2, \ldots, N_p \quad (4) \\
p_{j\min} \leq p_j \leq p_{j\max} & \quad j = 1, 2, \ldots, N_c \quad (5) \\
h_{j\min} \leq h_j \leq h_{j\max} & \quad j = 1, 2, \ldots, N_c \quad (6) \\
h_{k\min} \leq h_k \leq h_{k\max} & \quad k = 1, 2, \ldots, N_h \quad (7)
\end{align*}
\]

where the cost functions are given by:

\[
\begin{align*}
C_i(p_i) &= a_i + b_i p_i + c_i p_i^2 \\
C_j(p_j, h_j) &= a_j + b_j p_j + c_j p_j^2 + d_j h_j + e_j h_j^2 + f_j p_j h_j \\
C_k(h_k) &= a_k + b_k h_k + c_k h_k^2
\end{align*}
\]

where \( C \) is the total fuel (production) cost; \( C_p, C_c, C_h \) are the fuel costs of the conventional power-only unit, co-generation unit, and heat-only unit, respectively; \( a_i, b_i, c_i \) are the fuel cost coefficients of the \( i^{th} \) conventional power-only unit; \( a_j, b_j, c_j, d_j, e_j, f_j \) are the fuel cost coefficients of the \( j^{th} \) co-genera-
tion unit; \( a_k, b_k, c_k \) are the fuel cost coefficients of the \( k^{th} \) heat-only unit; \( p_i \) and \( p_j \) are the power productions of conventional power and co-generation units, respectively; \( h_j \) and \( h_k \) are the heat productions of co-generation and heat-alone units, respectively; \( h_d \) and \( p_d \) are the heat and power demands, respectively; \( N_p, N_c, N_h \) are the numbers of conventional power units, co-generation units and heat-alone units, respectively; \( p_{i\min} \) and \( p_{i\max} \) are the minimum and maximum power generation limits of the \( i^{th} \) conventional unit, respectively; \( p_{j\min} \) and \( p_{j\max} \) are the minimum and maximum power generation limits of the \( j^{th} \) co-generation unit, respectively; \( h_{j\min} \) and \( h_{j\max} \) are the minimum and maximum heat generation limits of the \( j^{th} \) co-generation unit, respectively; and \( h_{k\min} \) and \( h_{k\max} \) are the minimum and maximum heat generation limits of the \( k^{th} \) heat-only unit, respectively.

The constrained CHPED problem is transformed into an unconstrained one by handling equality and inequality constraints utilizing the approach of penalty parameters \([12, 29]\), which penalizes the infeasible solution during the iterative process. The proper settings of the penalty factors must be cautiously selected after some trials towards improved optimal solutions while respecting the constraints \([29]\). Because various constraints have various orders of magnitude, it is more appropriate to initially normalize each equality and inequality constraint. The \( i^{th} \) equality constraint, \( g_i(x) - a_i = 0 \), and the \( j^{th} \) inequality constraint, \( h_j(x) - b_j \leq 0 \),
can be normalized to be $g_i(x) = g_i'(x)/a_i - 1 = 0$ and $h_j(x) = h_j'(x)/b_j - 1 = 0$, respectively. After normalizing the constraints, the optimization problem can be expressed in compact form as follows:

$$\min C(x_1, x_2, \ldots, x_n) \quad \text{(11)}$$

s.t.

$$g_i(x_1, x_2, \ldots, x_n) = 0, \quad i = 1, 2, \ldots, N_e$$
$$h_j(x_1, x_2, \ldots, x_n) \leq 0, \quad j = 1, 2, \ldots, N_c$$
$$x_k^\min \leq x_k \leq x_k^\max, \quad k = 1, 2, \ldots, n \quad \text{(14)}$$

where $n$, $N_e$, and $N_c$ are the number of state variables (unknown), number of equality constraints and number of inequality constraints, respectively. The panelized objective function becomes:

$$f = C(x_1, x_2, \ldots, x_n) + r \left[ \sum_{i=1}^{N_e} g_i(x_1, x_2, \ldots, x_n) \right]^2 + \sum_{j=1}^{N_c} \max \left(0, h_j(x_1, x_2, \ldots, x_n)\right)^2 \quad \text{(15)}$$

where $r$ is the penalty factor.

### III. SFS Algorithm

The nature-inspired SFS technique is a new metaheuristic global optimization algorithm, which uses the concept of fractals to imitate the natural growth. The diffusion property frequently used in random fractals is utilized by the particles in this technique to efficiently explore the search space [42].

Random fractals can be produced by adjusting the iteration process using stochastic rules. The SFS algorithm uses a random walk to model the diffusion process, where the diffusing particle stays connected with the seed particle which produces it. This process is repeated until a cluster is established [42].

The diffusion and the updating are the two major processes that occur in the SFS technique. Figure 2 presents the flowchart of the SFS and summarizes the diffusion phase and the updating process which involve two updating processes. The two processes will be detailed next. In this figure, $GW_1$ and $GW_2$ represent the Gaussian walks participating in the diffusion process, $BP$ is the best point location in the group, and $P_i$ represents the $i^{th}$ point in a group, which will be discussed later in the paper. In the diffusion process, every particle diffuses around its current position to guarantee exploitation characteristic. The diffusion process prevents entrapment in local minima and improves the opportunity of achieving the global solution. In the updating process, the SFS shows how a point among a group revises its position based on the positions of other points in the same group. The SFS adopts a static diffusion process, where the best produced particle achieved from the diffusing process is the only particle taken into consideration, while the other particles are neglected. The SFS utilizes random methods as the updating processes.

Suppose $\varepsilon$ and $\varepsilon'$ are two random numbers which are distributed uniformly in the range $[0, 1]$. A series of Gaussian walks ($GW_1$ and $GW_2$) engaged in the diffusion course are defined by [42]:

$$GW_1 = \text{Gaussian}\left(\mu, \sigma\right) + \varepsilon \cdot BP - \varepsilon'P_i \quad \text{(16)}$$

$$GW_2 = \text{Gaussian}\left(\mu, \sigma\right) \quad \text{(17)}$$

where $\mu$, $\mu'$ and $\sigma$ are the Gaussian means and standard deviation, respectively; $\mu = |BP|$ in (16), and $\mu = |P_i|$ in (17). If $g$ refers to the generation (iteration) number, $\sigma$ in (16) and (17) is determined as follows [42]:

$$\sigma = \frac{P_i - BP}{\log g} \quad \text{(18)}$$

For an optimization problem with dimension $D$, every spe-
cific particle assumed to solve the problem has been made based on a D-dimensional vector. In the initialization phase as shown in Fig. 2, each point is randomly initialized based on its maximum and minimum bounds. The initial value of the \( f \)th point \( (P_f) \) is determined as follows [42]:

\[
P_f = LB + \varepsilon(UB - LB)
\]

where \( LB \) and \( UB \) refer to the upper and the lower bounds of problem variables, respectively.

After the initialization of all particles, the fitness value of each particle is evaluated to reach the best point \( BP \) among all particles. For consistency with the exploitation ability in the diffusion process, all points roam around their current location to exploit the problem search space [42].

Due to the exploration property, SFS uses two statistical actions to improve the better space exploration. The first one is applied for each individual vector index, while the other one is then performed for all points. Initially, the first action ranks all points based on fitness values. Each point \( i \) in the group is then designated a probability value \( (P_{ai}) \) which obeys a uniform distribution determined by [42]:

\[
P_{ai} = \text{rank } (P_i)/N
\]

where \( \text{rank } (P_i) \) is the rank of point \( P_i \) among other points in the group; and \( N \) is the number of all points in the group. Equation (20) indicates that the better the rank of point, the higher the probability to be selected. As illustrated in Fig. 2, for each point \( P_i \) in a group, if \( P_{ai} < \varepsilon \) is met, the \( f \)th component of \( P_i \) is revised based on the following relation; and if it is not met, it stays unaltered [42].

\[
P'_i(j) = P_i(j) - \varepsilon(P_i(j) - P_{ai}(j))
\]

where \( P'_i(j) \) is the new updated location of point \( P'_i \) and \( P_a \) and \( P_i \) are the points selected randomly in the group.

The preceding discussion indicates that the first statistical process is performed for the components of the points. As shown in Fig. 2, the other statistical process adjusts the location of a point considering the position of other points in the group to enhance the quality of exploration and to fulfill the diversification property. Ahead of starting the second statistical process, all points achieved from the first statistical process are ranked based on (20). As in the first process, if \( P_{ai} < \varepsilon \) is satisfied for a new point \( P'_i \), the existing position of \( P'_i \) is revised confirming to (22) and (23); and if \( P_{ai} < \varepsilon \) is not satisfied, no update takes place [42].

\[
P''_i = P'_i - \beta(P'_i - BP) \quad \beta \leq 0.5
\]

\[
P''_i = P'_i + \beta(P'_i - P_i) \quad \beta > 0.5
\]

where \( P'_i \) and \( P_i \) are the randomly chosen points achieved from the first statistical process; and \( \beta \) is the number produced randomly from the Gaussian normal distribution. \( P'_i \) replaces the new point \( P''_i \) if the fitness value of \( P''_i \) is superior to that of \( P'_i \) [42].

IV. RESULTS AND DISCUSSION

A. Test System 1

This system is the well-known four-unit test system in the literature of CHPED, which is presented in [1]–[3], [12]–[14], [17], [23], [27], [29], [31]–[34], [39] and many other related references. It comprises one conventional power-only unit (unit 1), two CHP units (units 2 and 3), and one heat-only unit (unit 4). The minimum and maximum limits of the conventional power unit are 0 and 150 MW, respectively. The minimum and maximum limits of the heat-only unit are 0 and 2690 MWth, respectively. The FORs of the two CHP units are illustrated in Fig. 3. The system power demand \( p_d \) and the heat demand \( h_d \) are 200 MW and 115 MWth, respectively.

Fig. 3. FORs of CHP units of test system 1. (a) First CHP unit (unit 2). (b) Second CHP unit (unit 3).

The four units has the following cost functions [1]–[3], [12]–[14], [17], [23], [27], [29], [31]–[34], [39]:

\[
C_1(p_1) = 50p_1,
\]

\[
C_2(p_2, h_2) = 2650 + 14.5p_2 + 0.0345p_2^2 + 4.2h_2 + 0.030h_2^3 + 0.031p_2h_2
\]

\[
C_3(p_3, h_3) = 1250 + 36.0p_3 + 0.0435p_3^2 + 0.6h_3 + 0.027h_3^2 + 0.011p_3h_3
\]

\[
C_4(h_4) = 23.4h_4
\]

The optimization problem of CHPED of this system is formulated as:

\[
\min C = C_1(p_1) + C_2(p_2, h_2) + C_3(p_3, h_3) + C_4(h_4)
\]

The constraints are as follows:

1) Equality constraints:

\[
\begin{align*}
    p_1 + p_2 + p_3 & = 20 \\
    h_2 + h_3 + h_4 & = 115
\end{align*}
\]

2) Inequality constraints which represent the FOR of the two CHP units:
\[
\begin{align*}
1.781914894h_2 - p_2 - 105.7446809 & \leq 0 \\
0.17777778h_2 + p_2 - 247.0000000 & \leq 0 \\
-0.169847328h_2 - p_2 + 98.8000000 & \leq 0 \\
1.15841584h_1 - p_3 - 46.88118818 & \leq 0 \\
0.151162791h_1 + p_3 - 130.6976744 & \leq 0 \\
-0.067681895h_3 - p_3 + 45.07614213 & \leq 0 \\
44 - p_3 & \leq 0 \\
\end{align*}
\]
(26)

3) The limits:
\[
\begin{align*}
0 \leq p_1 & \leq 150 \\
81 \leq p_2 & \leq 247 \\
0 \leq h_1 & \leq 180 \\
40 \leq p_3 & \leq 125.8 \\
0 \leq h_3 & \leq 135.6 \\
0 \leq h_4 & \leq 2695.2 \\
\end{align*}
\]
(27)

For this system, the results achieved by the proposed SFS method for the above CHPED equations will be put in comparison with the previously obtained results reported in [1], [2], [12]-[14], [17], [27], [29], [31]-[34], [39], [41] using LR, B&B, improved ant colony search (IACS) algorithm, GA-based penalty function (GAPF) method, PSO, EP, FA, improved genetic algorithm with multiplier updating (IGAMU), HS, self-adaptive real-coded genetic algorithm (SARGA), artificial bee colony (ABC), DE, mesh adaptive direct search and particle swarm optimization (MADS-PSO), mesh adaptive direct search and Latin hypercube sampling (MADS-LHS), IWO, GSA, CSA, and CSO.

The SFS algorithm is coded in MATLAB and executed using a 1.8 GHz, 8 GB RAM Pentium Core i5 PC. The results of the other methods are taken from literature. The user-supplied parameter setting of the SFS are population size, maximum generation, and MDN, which are selected to be 120, 1000, and 4, respectively.

The characteristics curve of the SFS convergence for test system 1 is shown in Fig. 4, which illustrates a fast convergence.

The state variables, fuel costs, and average CPU times are selected to examine the performance of the SFS with the other techniques. Table I summarizes the results obtained for this system using SFS method and the other methods. As can be seen from Table I, the SFS algorithm obtains the lowest minimum cost with less computation time. The computation time is among the shortest compared to that of other algorithms.

| Algorithm   | \( p_1 \) (MW) | \( p_2 \) (MW) | \( p_3 \) (MW) | \( h_2 \) (MWh) | \( h_3 \) (MWh) | \( h_4 \) (MWh) | Total cost ($/h) | Computation time (s) |
|-------------|----------------|----------------|----------------|----------------|----------------|----------------|-------------------|---------------------|
| IACS [14]   | 0.0800         | 150.9300       | 49.0000        | 48.8400        | 65.7900        | 0.3700         | 9452.20           | 5.26                |
| MADS-PSO [27]| 0.0092         | 157.9392       | 42.0516        | 42.4459        | 72.5522        | 0.0019         | 9301.38           | 7.56                |
| MADS-LHS [27]| 0.0017         | 159.8000       | 40.2014        | 42.4042        | 72.3904        | 0.2054         | 9277.13           | 7.04                |
| ABC [29]    | 0.2400         | 158.7800       | 40.9600        | 39.5800        | 75.2300        | 0.1800         | 9276.70           | NA                  |
| GPF [12]    | 0.0500         | 159.4300       | 40.5700        | 39.9700        | 75.0300        | 0              | 9265.10           | 3.09                |
| PSO [12]    | 0.0200         | 159.9400       | 39.9300        | 40.0200        | 74.9900        | 0.0600         | 9258.90           | NA                  |
| DE [29]     | 0.0014         | 159.9986       | 40.0000        | 40.0000        | 75.0000        | 0              | 9257.10           | 3.98                |
| LR [12]     | 0.0014         | 160.0000       | 40.0000        | 40.0000        | 75.0000        | 0              | 9257.09           | 5.53                |
| B&B [12]    | 0.0014         | 160.0000       | 40.0000        | 40.0000        | 75.0000        | 0              | 9257.07           | 4.21                |
| EP [12]     | 0.0014         | 160.0000       | 40.0000        | 40.0000        | 75.0000        | 0              | 9257.07           | 3.76                |
| FA [39]     | 0.0014         | 160.0000       | 40.0000        | 40.0000        | 75.0000        | 0              | 9257.07           | 1.18                |
| IGAMU [12]  | 0.0014         | 160.0000       | 40.0000        | 40.0000        | 75.0000        | 0              | 9257.07           | 4.21                |
| HS [12]     | 0.0014         | 160.0000       | 40.0000        | 40.0000        | 75.0000        | 0              | 9257.07           | 3.76                |
| SARGA [12]  | 0.0014         | 160.0000       | 40.0000        | 40.0000        | 75.0000        | 0              | 9257.07           | 1.18                |
| CSO [41]    | 0.0014         | 160.0000       | 40.0000        | 40.0000        | 75.0000        | 0              | 9257.07           | 4.21                |
| IWO         | 0.0014         | 160.0000       | 40.0000        | 40.0000        | 75.0000        | 0              | 9257.07           | 3.76                |
| GSA         | 0.0003         | 159.4494       | 40.5494        | 38.8850        | 75.4736        | 0.6414         | 9269.14           | 7.26                |
| CSA         | 0.0003         | 160.0000       | 40.0000        | 40.0000        | 75.0000        | 0              | 9257.07           | 5.62                |
| SFS         | 0.0003         | 160.0000       | 40.0000        | 40.0000        | 75.0000        | 0              | 9257.07           | 3.78                |

Fig. 4. Convergence characteristics of SFS algorithm for test system 1.
Figure 5 illustrates comparisons of the convergence characteristics of SFS and other nine algorithms used in this test system and the next test system. These comparisons demonstrate the convergence speed and efficiency of the SFS to reach the optimal solution.

The computation time of the SFS algorithm can be reduced by selecting proper values of the SFS parameters including population size, maximum generation, and MND. Table II shows improvements in calculation time of test system I for selected sets of SFS parameters. Note that most of the computation time is much better than those of all algorithms compared in Table I.

### Table II

| Population size | Maximum generation | MDN | Total cost ($/h) | Computation time (s) |
|-----------------|--------------------|-----|----------------|---------------------|
| 16              | 500                | 1   | 9257.07        | 0.897198            |
| 16              | 600                | 1   | 9257.07        | 0.968269            |
| 16              | 700                | 1   | 9257.07        | 1.112184            |
| 16              | 800                | 1   | 9257.07        | 1.272459            |
| 18              | 500                | 1   | 9257.07        | 0.894139            |
| 18              | 600                | 1   | 9257.07        | 1.067141            |
| 18              | 700                | 1   | 9257.07        | 1.241997            |
| 18              | 800                | 1   | 9257.07        | 1.417746            |
| 20              | 500                | 1   | 9257.07        | 0.987465            |
| 20              | 600                | 1   | 9257.07        | 1.181672            |
| 20              | 700                | 1   | 9257.07        | 1.383194            |
| 20              | 800                | 1   | 9257.07        | 1.570533            |
| 22              | 500                | 1   | 9257.07        | 1.079808            |
| 22              | 600                | 1   | 9257.07        | 1.290776            |
| 22              | 700                | 1   | 9257.07        | 1.512136            |
| 22              | 800                | 1   | 9257.07        | 1.731690            |

B. Test System 2

This system is a five-unit system, which is also a well-known system available in many references such as [1], [3], [23], [25], [29], [31]-[34], [39]. It consists of one conventional power-only unit (unit 1), three CHP units (units 2, 3 and 4), and one heat-only unit (unit 5). The maximum and minimum limits of the conventional power unit are 135 MW and 35 MW, respectively. The minimum and maximum limits of the heat-only unit are 0 and 60 MWth, respectively. The FORs of the three CHP units are illustrated in Fig. 6.

This test system will be examined for three different cases of power and heat demands:

1) Case 1: $p_d = 300$ MW, $h_d = 150$ MWth.
2) Case 2: $p_d = 250$ MW, $h_d = 175$ MWth.
3) Case 3: $p_d = 150$ MW, $h_d = 220$ MWth.

The cost functions of the five units are given by [1], [3], [23], [25], [29], [31]-[34], [39]:

\[
\begin{align*}
C_1(p_1) &= 254.8863 + 7.6997p_1 + 0.00172p_1^2 + 0.00115p_1^3 \\
C_2(p_2, h_2) &= 1250 + 36.0p_2 + 0.0435p_2^2 + 0.6h_2 + 0.027h_2^2 + 0.011p_2h_2 \\
C_3(p_3, h_3) &= 2650 + 34.5p_3 + 0.1035p_3^2 + 2.203h_3 + 0.025h_3^2 + 0.051p_3h_3 \\
C_4(p_4, h_4) &= 1565 + 20p_4 + 0.072p_4^2 + 2.3h_4 + 0.02h_4^2 + 0.04p_4h_4 \\
C_5(h_5) &= 950 + 2.0109h_5 + 0.038h_5^2 
\end{align*}
\]

The optimization problem of the CHPED of this system is formulated as:
The computation time to solve the above optimization problem using SFS is 10.76 s, 10.74 s, and 10.83 s for cases 1, 2, and 3, respectively, which is considered reasonable. The characteristic curve of the SFS convergence for case 1 of test system 2 is shown in Fig. 7. For the second test system, the results achieved by the proposed SFS technique will be compared with those results obtained using GA [32], RCGA [15], HS [32], classic PSO (CPSO [17], TVAC-PSO [17], COA [28], GSA [26], RCGA-IMM [14], CSA [27], IWO [34], FA [39], DE, ABC, and SARGA. The results obtained for this system using SFS and the other techniques (as reported in literature), are summarized in Tables III-V for the three cases, respectively. Again, as can be observed from these tables, the SFS technique is able to obtain the lowest possible minimum cost with small computation time.

![Figure 7. Convergence characteristics of SFS algorithm for case 1 of test system 2.](image)

**TABLE III**

| Algorithm  | $p_1$ (MW) | $p_2$ (MW) | $p_3$ (MW) | $p_4$ (MW) | $h_1$ (MWth) | $h_2$ (MWth) | $h_3$ (MWth) | $h_4$ (MWth) | Total cost ($/h) |
|------------|------------|------------|------------|------------|--------------|--------------|--------------|--------------|-----------------|
| GA [32]    | 135.0000   | 70.8100    | 10.8400    | 83.2800    | 80.5400      | 39.8100      | 0.0000       | 29.6400      | 13779.50        |
| RCGA [15]  | 134.9904   | 49.9525    | 25.0827    | 89.9744    | 73.5089      | 35.8519      | 1.2916       | 39.3476      | 13776.14        |
| HS [32]    | 134.7400   | 48.2000    | 16.2300    | 100.8500   | 81.0900      | 23.9200      | 6.2900       | 38.7000      | 13723.20        |
| CPSO [17]  | 135.0000   | 40.7309    | 19.2728    | 105.0000   | 64.4003      | 26.4119      | 0.0000       | 59.1955      | 13692.52        |
| IWO [34]   | 134.7300   | 40.0000    | 20.8600    | 104.4100   | 75.0000      | 37.6000      | 0.0000       | 37.4000      | 13683.65        |
| FA [39]    | 134.7400   | 40.0000    | 20.2500    | 105.0000   | 75.0000      | 27.8700      | 0.0000       | 47.1200      | 13683.22        |
| TVAC-PSO [17]| 135.0000  | 41.4019    | 18.5981    | 105.0000   | 73.3562      | 37.4295      | 0.0000       | 39.2143      | 13672.89        |
| COA [28]   | 135.0000   | 40.7687    | 19.2313    | 105.0000   | 73.5955      | 36.7760      | 0.0000       | 39.6279      | 13672.83        |
| SFS        | 135.0000   | 40.7689    | 19.2311    | 105.0000   | 73.5955      | 36.7760      | 0.0000       | 39.6279      | 13672.83        |

The results summarized in Tables III-V show that the SFS solver outperforms many other methods in terms of the total cost $C_T$, but presents the same best values using the rest of the methods presented in the tables.

Figure 8 illustrates the comparisons of convergence characteristics of SFS and other nine algorithms for case 2 of test system 2. It shows that the SFS has fast convergence, which is one of the best algorithms used in the comparison.
TABLE IV

Comparison of CHPED Results of Test System 2 for Case 2

| Algorithm | $p_1$ (MW) | $p_2$ (MW) | $p_3$ (MW) | $p_4$ (MW) | $h_1$ (MW/h) | $h_2$ (MW/h) | $h_3$ (MW/h) | $h_4$ (MW/h) | Total cost ($/h) |
|-----------|-----------|-----------|-----------|-----------|-------------|-------------|-------------|-------------|----------------|
| GA [32]   | 119.2200  | 45.1200   | 15.8200   | 69.8900   | 78.9400     | 22.6300     | 18.4000     | 54.9900     | 12327.37      |
| HS [32]   | 134.6700  | 52.9900   | 10.1100   | 52.2300   | 85.6900     | 39.7300     | 4.1800      | 45.4000     | 12284.45      |
| IWO [34]  | 134.5900  | 40.0000   | 10.9400   | 64.4700   | 75.0000     | 38.9800     | 8.8100      | 52.2100     | 12134.33      |
| CPSO [17] | 135.0000  | 40.3446   | 10.0506   | 64.6060   | 70.9318     | 39.9918     | 4.0773      | 60.0000     | 12132.86      |
| FA [39]   | 134.8100  | 40.0000   | 10.0000   | 65.1800   | 75.0000     | 40.0000     | 16.9700     | 43.0200     | 12119.86      |
| TVAC-PSO  | 135.0000  | 40.0118   | 10.0391   | 64.9491   | 74.8263     | 39.8443     | 16.1867     | 44.1428     | 12117.39      |
| GSA [26]  | 135.0000  | 39.9998   | 10.0000   | 64.9807   | 74.9844     | 40.0000     | 17.8939     | 42.1095     | 12117.37      |
| COA [28]  | 135.0000  | 40.0000   | 10.0000   | 64.9910   | 75.0000     | 40.0000     | 14.4001     | 45.6000     | 12116.60      |
| CSA [27]  | 135.0000  | 40.0000   | 10.0000   | 65.0000   | 75.0000     | 40.0000     | 14.4004     | 45.5954     | 12116.60      |
| DE        | 135.0000  | 40.0488   | 16.2528   | 58.7008   | 74.3491     | 42.6795     | 18.7334     | 39.2381     | 12178.49      |
| ABC       | 135.0000  | 40.2096   | 10.2792   | 64.5112   | 71.9024     | 38.8834     | 16.0870     | 48.1273     | 12123.81      |
| SFS       | 135.0000  | 40.0000   | 10.0000   | 65.0000   | 75.0000     | 40.0000     | 14.4043     | 45.5957     | 12116.60      |

TABLE V

Comparison of CHPED Results of Test System 2 for Case 3

| Algorithm | $p_1$ (MW) | $p_2$ (MW) | $p_3$ (MW) | $p_4$ (MW) | $h_1$ (MW/h) | $h_2$ (MW/h) | $h_3$ (MW/h) | $h_4$ (MW/h) | Total cost ($/h) |
|-----------|-----------|-----------|-----------|-----------|-------------|-------------|-------------|-------------|----------------|
| GA [32]   | 37.9800   | 75.3900   | 10.4100   | 35.0300   | 106.0000    | 38.3700     | 15.8400     | 59.9700     | 11837.40      |
| HS [32]   | 41.4100   | 66.6100   | 10.5900   | 41.3900   | 97.7300     | 40.2300     | 22.8300     | 59.2100     | 11810.88      |
| CPSO [35] | 35.5972   | 57.3554   | 10.0070   | 57.0587   | 89.9767     | 40.0255     | 30.0232     | 60.0000     | 11781.37      |
| RCGA-IMM [14] | 42.1660 | 64.6523   | 10.0000   | 43.1817   | 96.2810     | 40.0000     | 23.7190     | 60.0000     | 11758.64      |
| TVAC-PSO  | 42.1433   | 64.6271   | 10.0001   | 43.2295   | 96.2593     | 40.0001     | 23.7404     | 60.0000     | 11758.06      |
| COA [28]  | 42.1497   | 64.6342   | 10.0000   | 43.2161   | 96.2654     | 40.0000     | 23.7346     | 60.0000     | 11758.06      |
| CSA [27]  | 42.2652   | 64.7630   | 10.0112   | 42.9706   | 96.3766     | 40.0005     | 23.6230     | 59.9990     | 11758.09      |
| SFS       | 42.1454   | 64.6294   | 10.0000   | 43.2252   | 96.2613     | 40.0000     | 23.7387     | 60.0000     | 11758.06      |

The advantages of SFS can be summarized as follows:

1) From comparisons with the reported best cost results obtained by many other solvers, SFS can reach all best solutions reported in literature for all test cases investigated in the paper. This indicates that the algorithm is capable of overcoming the local minima of the problem found by some other algorithms.

2) The algorithm demonstrates robust behavior when its user-supplied parameters are changed or when the number of decision variables changes.

3) Shorter computation time for the same best solution can be achieved using the SFS by the proper selection of its parameters.

4) The algorithm guarantees fast convergence and accuracy in smaller numbers of iterations compared to many other solvers.

5) The SFS has uncomplicated mathematical operations.

V. CONCLUSION AND RECOMMENDATION

This paper presents the application of the SFS algorithm to solve the CHPED optimization problem which is currently a crucial issue in power system operations. The SFS technique is among the promising and powerful global optimization solvers and can outperform some present well-known metaheuristic optimization techniques. The CHPED formulation is a non-convex non-linear optimization problem that models concurrent production of both electrical power and
thermal energy, whose objective is minimizing the production cost of heat and power generation units while fulfilling various inequality and equality constraints and interdependent limits. The equality and inequality constraints are handled in this paper by employing penalty parameters, which are able to penalize infeasible solution during the iterative process, where the constrained CHPED problem is transformed into an unconstrained one. The algorithm has been tested on two different well-known test systems used in the literature of CHPED. The SFS-based results are compared with those obtained by many other commonly used and efficient global optimization techniques. The results have verified that the optimal solutions obtained using the SFS algorithm perform better than many of these frequently used methods. It is also revealed that the optimal SFS-based solution has lowered the system operation costs and achieves the best feasible solution obtained by the rest of other optimization techniques reported in literature. The proposed algorithm is robust in obtaining the best reported feasible solutions for different systems and case studies, and has accomplished improved near-global optimal solutions with very reasonable computation time.

For further work, a variety of ideas can be recommended such as study cases with higher dimension, using SFS to solve optimal power flow (OPF) incorporating CHP units, renewable energy resources in the CHPED problem, application of SFS to the CHPED problem with valve-point effects of thermal power units, gas emission levels in the objective function for environmental concerns, and multi-objective CHPED problems, etc.

REFERENCES

[1] M. Nazari-Heris, B. Mohammadi-Ivatloo, and G. B. Gharahpetian, “A comprehensive review of heuristic optimization algorithms for optimal combined heat and power dispatch from economic and environmental perspectives,” Renewable and Sustainable Energy Reviews, vol. 81, pp. 2128-2143, Jan. 2018.

[2] A. Sadrirokh, J. Pasqapuliet, N. Moin et al., “Combined heat and power (CHP) economic dispatch solved using Lagrangian relaxation with surrogate subgradient multiplier updates,” International Journal of Electrical Power & Energy Systems, vol. 44, no. 1, pp. 421-430, Jan. 2013.

[3] H. R. Abdolmohammadi and A. Kazemi, “A benders decomposition approach for a combined heat and power economic dispatch,” Energy Conversion & Management, vol. 71, pp. 21-31, Jul. 2013.

[4] H. Sadeghian and M. Ardehali, “A novel approach for optimal economic dispatch scheduling of integrated combined heat and power systems for maximum economic profit and minimum environmental emissions based on Benders decomposition,” Energy, vol. 102, pp. 10-23, May 2016.

[5] J. S. Kim and T. F. Edgar, “Optimal scheduling of combined heat and power plants using mixed-integer nonlinear programming,” Energy, vol. 77, pp. 675-690, Dec. 2014.

[6] R. Aong and R. Lahdelma, “An efficient envelope-based branch and bound algorithm for non-convex combined heat and power production planning,” European Journal of Operational Research, vol. 183, no. 1, pp. 412-431, Nov. 2007.

[7] W. Gu, Z. Wu, R. Bo et al., “Modeling, planning and optimal energy management of combined cooling, heating and power microgrid: a review,” International Journal of Electrical Power & Energy Systems, vol. 54, pp. 26-37, Jan. 2014.

[8] H. Cho, A. D. Smith, and P. Mago, “Combined cooling, heating and power: a review of performance improvement and optimization,” Applied Energy, vol. 136, pp. 168-185, Dec. 2014.

[9] D. Wu and R. Wang, “Combined cooling, heating and power: a review,” Progress in Energy and Combustion Science, vol. 32, no. 5-6, pp. 459-495, Sept.-Nov. 2006.

[10] M. Liu, Y. Shi, and F. Fang, “Combined cooling, heating and power systems: a survey,” Renewable and Sustainable Energy Reviews, vol. 35, pp. 1-22, Jul. 2014.

[11] S. Murugan and B. Horák, “A review of micro combined heat and power systems for residential applications,” Renewable and Sustainable Energy Reviews, vol. 64, pp. 144-162, Oct. 2016.

[12] P. Subbaraj, R. Rengaraj, and S. Salivahanan, “Enhancement of combined heat and power economic dispatch using self adaptive real-coded genetic algorithm,” Applied Energy, vol. 86, no. 6, pp. 915-921, Jun. 2009.

[13] C. T. Su and C. L. Chiang, “An incorporated algorithm for combined heat and power economic dispatch,” Electric Power System Research, vol. 69, no. 2-3, pp. 187-195, May 2004.

[14] A. Haghhigh, M. Nazari-Heris, and B. Mohammadi-Ivatloo, “Solving combined heat and power economic dispatch problem using real coded genetic algorithm with improved Mühlenbein mutation,” Applied Thermal Engineering, vol. 99, pp. 465-475, Apr. 2016.

[15] M. Basu, “Combined heat and power economic emission dispatch using nondominated sorting genetic algorithm-II,” International Journal of Electrical Power & Energy Systems, vol. 53, pp. 135-141, Dec. 2013.

[16] G. Piperagkas, A. Anastasiadis, and N. Hatzigryrou, “Stochastic PSO-based heat and power dispatch under environmental constraints incorporating CHP and wind power units,” Electric Power System Research, vol. 81, no. 1, pp. 209-218, Jan. 2011.

[17] B. Mohammadi-Ivatloo, M. Moradi-Dulvand, and A. Rabiee, “Combined heat and power economic dispatch problem solution using particle swarm optimization with time varying acceleration coefficients,” Electric Power System Research, vol. 95, pp. 9-18, Feb. 2013.

[18] L. Wu and C. Singh, “Stochastic combined heat and power dispatch based on multiobjective particle swarm optimization,” International Journal of Electrical Power & Energy Systems, vol. 30, no. 3, pp. 226-234, Mar. 2008.

[19] M. Basu, “Combined heat and power economic dispatch by using differential evolution,” Electric Power Components and Systems, vol. 38, no. 8, pp. 996-1004, May 2010.

[20] G. Jena, M. Basu, and C. Panigrahi, “Differential evolution with Gaussian mutation for combined heat and power economic dispatch,” Soft Computing, vol. 20, no. 2, pp. 681-688, Feb. 2016.

[21] A. M. Elaiw, X. Xia, and A. M. Shehata, “Hybrid DE-SQP method for solving combined heat and power dynamic economic dispatch problem,” Mathematical Problems in Engineering, vol. 2013, pp. 1-7, 2013.

[22] J. X. V. Neto, G. Reynoso-Meza, T. H. Ruppel et al., “Solving nonsmooth economic dispatch by a new combination of continuous grasp algorithm and differential evolution,” International Journal of Electrical Power & Energy Systems, vol. 84, pp. 13-24, Jan. 2017.

[23] C. L. Chiang, “An optimal economic dispatch algorithm for large scale power systems with cogeneration units,” European Journal of Engineering Research and Science, vol. 1, no. 5, pp. 10-16, Nov. 2016.

[24] M. Basu, “Group search optimization for combined heat and power economic dispatch,” International Journal of Electrical Power & Energy Systems, vol. 78, pp. 138-147, Jun. 2016.

[25] E. Davoodi, K. Zare, and E. Babaei, “A GSO-based algorithm for closed and heat and power dispatch problem with modified scrounger and ranger operators,” Applied Thermal Engineering, vol. 120, pp. 36-48, Jun. 2017.

[26] S. D. Beigvand, H. Abdi, and M. La Scala, “Combined heat and power economic dispatch problem using gravitational search algorithm,” Electric Power System Research, vol. 133, pp. 160-172, Apr. 2016.

[27] T. T. Nguyen, D. N. Vo, and B. H. Dinh, “Cuckoo search algorithm for combined heat and power economic dispatch,” International Journal of Electrical Power & Energy Systems, vol. 81, pp. 204-214, Oct. 2016.

[28] M. Mehidinejad, B. Mohammadi-Ivatloo, and R. Dadashzadeh-Bonab, “Energy production cost minimization in a combined heat and power generation systems using cuckoo optimization algorithm,” Energy Efficiency, vol. 9, no. 1, pp. 81-96, Feb. 2017.

[29] M. A. Mellal and E. J. Williams, “Cuckoo optimization algorithm with penalty function for combined heat and power economic dispatch problem,” Energy, vol. 93, no. 2, pp. 1711-1718, Dec. 2015.

[30] N. Jayakumar, S. Subramanian, S. Ganesan et al., “Grey wolf optimization for combined heat and power dispatch with cogeneration systems,” International Journal of Electrical Power & Energy Systems, vol. 74, pp. 252-264, Jan. 2016.

[31] E. Khorram and M. Jaberipour, “Harmony search algorithm for solving
ing combined heat and power economic dispatch problems,” *Energy Conversion and Management*, vol. 52, no. 2, pp. 1550-1554, Feb. 2011.

[32] A. Vasebi, M. Fesanghary, and S. Bathaei, “Combined heat and power economic dispatch by harmony search algorithm,” *International Journal of Electrical Power & Energy Systems*, vol. 29, no. 10, pp. 713-719, Dec. 2007.

[33] S. H. Huang and P. C. Lin, “A harmony-genetic based heuristic approach toward economic dispatching combined heat and power,” *International Journal of Electrical Power & Energy Systems*, vol. 33, pp. 482-487, Dec. 2013.

[34] T. Jayabarathi, A. Yazdani, V. Ramesh et al., “Combined heat and power economic dispatch problem using the invasive weed optimization algorithm,” *Front Energy*, vol. 8, no. 1, pp. 25-30, Mar. 2014.

[35] M. Basu, “Bee colony optimization for combined heat and power economic dispatch,” *Expert Systems with Applications*, vol. 38, no. 11, pp. 13527-13531, Oct. 2011.

[36] P. K. Roy, C. Paul, and S. Sultana, “Oppositional teaching learning based optimization approach for combined heat and power dispatch,” *International Journal of Electrical Power & Energy Systems*, vol. 57, pp. 392-403, May 2014.

[37] M. Basu, “Artificial immune system for combined heat and power economic dispatch,” *International Journal of Electrical Power & Energy Systems*, vol. 43, no. 1, pp. 1-5, Dec. 2012.

[38] T. Niknam, R. Azizipanah-Abarghoosae, A. Roosta et al., “A new multi-objective reserve constrained combined heat and power dynamic economic emission dispatch,” *Energy*, vol. 42, no. 1, pp. 530-545, Jun. 2012.

[39] A. Yazdani, T. Jayabarathi, V. Ramesh et al., “Combined heat and power economic dispatch problem using firefly algorithm,” *Front Energy*, vol. 7, no. 2, pp. 133-139, Jun. 2013.

[40] P. K. Adhvaryuu, P. K. Chattopadhyay, and A. Bhattacharjiya, “Application of bio-inspired krill herd algorithm to combined heat and power economic dispatch,” in *Proceedings of the 2014 IEEE Innovative Smart Grid Technologies-Asia (ISGTASIA)*, Kuala Lumpur, Malaysia, May 2014, pp. 338-343.

[41] A. Meng, P. Mei, H. Yin et al., “Crisscross optimization algorithm for solving combined heat and power economic dispatch problem,” *Energy Conversion and Management*, vol. 105, pp. 1303-1317, Nov. 2015.

[42] H. Salimi, “Stochastic Fractal Search: A powerful metaheuristic algorithm,” *Knowledge-based Systems*, vol. 75, pp. 1-18, Feb. 2015.

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