Vehicle emission control on road with temporal traffic information using deep reinforcement learning

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Abstract: The increased vehicle usage significantly aggravate the urban air pollution, which have great impact on the public health. Therefore, it is necessary to make proper traffic control policies and reduce traffic emissions. However, it is difficult to establish control strategies based on modeling methods, and carry out online control based on historical traffic information for the complex time-varying characteristics of emissions. In this paper, we present a deep reinforcement learning emission control strategy, which automatically learns the optimal traffic flow and speed limits to reduce traffic emission on the target road segment based on the temporal traffic information. The proposed approach is evaluated on real world vehicle emission data in Hefei. And the results demonstrate the effectiveness of the proposed approach against baseline methods.

Keywords: Urban air pollution, traffic emission control, deep reinforcement learning.

1. INTRODUCTION

Traffic emissions released by vehicles contain harmful substances, such as carbon monoxide (CO), carbon dioxide (CO$_2$), nitrogen oxides (NO$_x$), hydrocarbons (HC) and particulate matters (PM2.5), which have great impact on the public health. The increased vehicle usage significantly aggravate the urban air pollution. According to 'China Vehicle Environmental Management Annual Report (2018)', total CO emission is up to 33.273 million tons, NO$_x$ is 5.743 million tons in 2017. In some megalopolis of China, such as Beijing, Shanghai, Shenzhen, the vehicle emission share rate of PM2.5 is approximately 13.5%-41%. Therefore, it is necessary to study traffic emission control to help relevant government departments make proper traffic control policies and reasonable traffic planning Xu et al. (2000) and feedback control based approach and traffic management based approach.

The feedback control based method needs to establish macroscopic traffic flow model and microscopic traffic emission model for the controller to predict traffic emissions of traffic networks. Zhang and Zhu (2015) presented a discrete traffic flow model and designed a delay-feedback controller to suppress traffic jam and decrease traffic emission. Shen et al. (2013) proposed an integrated macroscopic traffic model to predict the traffic flow states and the emissions released by every vehicle at different operational conditions and designed the model predictive controller to reduce both travel delays and traffic emissions. Shinai et al. (2017) used multi-class macroscopic traffic flow and emission models for MPC for traffic networks to achieve a balanced trade-off between total time spent and total emissions. Uzunova et al. (2012) proposed non-integer robust control approach to analyse the speed and density variations due to perturbations on the road and to assure robust performances for the traffic velocity and evaluated pollution factor of CO2 emissions of the controlled and uncontrolled model. However, the macroscopic traffic flow model and microscopic traffic emission model are the simplification of the real world traffic condition, and the induced control strategy is definitely inaccurate.
The traffic management based approach mainly focus on the traffic speed and flow regulation to reduce the traffic emissions of the entire traffic networks. Panis et al. (2006) considers the effect of active speed management on traffic-induced emissions, suggesting vehicle acceleration and deceleration are important factors in determining traffic induced emission. Li et al. (2018) proposed an optimal dynamic credit charge scheme to redistribute the traffic flows to attain mobility and emission goals. Duell et al. (2014) presented a novel framework to estimate city-wide vehicle environmental effects that integrates a dynamic traffic assignment model with a novel application of a vehicle energy consumption model. Miles et al. (2018) proposed a decision support system (DSS) that uses an underlying traffic model to inform an atmospheric dispersion model. Bel et al. (2015) used quantile regression to evaluate whether speed management policies have been successful in promoting cleaner air, not only in terms of average pollutant levels but also during high and low pollution episodes. De Blasis et al. (2014) presented an integrated simulation tool to analyse the effects of traffic flow conditions on the pollutants emissions. Panis et al. (2011) analysed different traffic types (urban versus highway traffic) with different modelling approaches (microscopic versus macroscopic) to examines the impact of speed management policies on emissions. Carslaw et al. (2010) used Generalized Additive Models to describe how emissions from individual vehicles vary depending on their driving conditions, taking account of variable interactions and time-lag effects, and quantified the impact that vehicle speed control has on vehicle emissions of CO2 by road type, fuel type and driver behaviour. Dijkema et al. (2008) studied the lowering of the maximum speed limit effects on reducing traffic related air pollution. The traffic management based approach designed the traffic control strategies based on the history traffic flow information, which is an offline strategy with time-lag effects.

And with the rapid development of traffic data collection and artificial intelligence techniques, some researches are shifting to use deep reinforcement learning (RL) approaches on traffic management. The RL learns directly from the interactions between states and actions through episodes, the agents take optimal actions according to the long-term accumulation of rewards. In this paper, we present a deep reinforcement learning emission control strategy, which automatically learns the optimal traffic flow and speed limits to reduce traffic emission on the target road segment based on the temporal traffic information. The main contributions of this study are as followings:

1) We proposed a road vehicle emission control reinforcement learning model to establish the relationship of emissions and traffic information. And a compound emission environment state space is designed to leverage the emission temporal dependencies. Moreover, to deal with the large state and action space, a deep return valuation network (DQN) is applied to estimate the optimal long-term value function.

2) The proposed approach is evaluated on real world vehicle emission data in Hefei. The results demonstrate the effectiveness of the proposed approach against baseline methods.

2. RELATED WORK

There are some published work on traffic control in transportation field, i.e. traffic lights control, traffic flow optimization, and vehicle speed regulation. Li et al. (2016) applied the deep reinforcement learning method to design signal timing plan by implicitly modeling the control actions and the change of system states. Nishi et al. (2018) developed a RL-based traffic signal control method that employs a graph convolutional neural network to cope with broader traffic demand. Walraven et al. (2016) solved the traffic optimization problem with reinforcement learning, where the traffic congestion on the highway is reduced. Li et al. (2017) incorporated the reinforcement learning technique in variable speed limit control strategies to reduce system travel time at freeway bottlenecks. Chao et al. (2013) proposed an indirect reinforcement learning model based on Dyna-Q architecture to manage incident-induced congestion for ramp control. A more general application of reinforcement learning in this domain can be found in work Crucio et al. (2013), where different reward functions are investigated for decision-making in air traffic flow management with several stakeholders. Xu et al. (2019) proposed a deep spatiotemporal framework with multi-source urban data to predict the vehicle emissions in region scale. Yau et al. (2017) reviews various RL models and algorithms applied to traffic signal control in the aspects of the representations of the RL model (i.e., state, action, and reward), performance measures, and complexity to establish a foundation for further investigation in this research field.

3. OVERVIEW

3.1 Preliminary

The main notations used in this paper are shown in Table 1.

| Notation | Description |
|----------|-------------|
| $EF$     | An emission episode |
| $TA$     | Traffic agent to conduction emission |
| $tv_t$   | Traffic volume at time $t$ ($V/h$) |
| $ts_t$   | Traffic average speed at time $t$ (km/h) |
| $\pi$    | Traffic agent control policy |

Traffic Agent: The traffic agent $TA$ has the factors of traffic volume $tv$, and traffic average speed $ts$, which is the main body to interact with the emission environment.

Emission Episode: An emission episode $EF$ is a period emission sequence in a day, in which the total vehicle emission we want to minimize. The emission consists of Fuel, CO, HC, and NO at each time interval, which can be defined as $EF_t=\{Fuel_t,CO_{2t},HC_{2t},NO_{2t}\}$. And the sequences of emissions are all normalized respectively. In our problem, the length of emission episode is 24 hours, and the time interval is 1 hour.
3.2 Traffic data insight

Fig.1(a) and (b) show the daily speed and traffic volume distribution heatmap of the history traffic data. We find that in the morning rush hours, e.g. 10 a.m., the traffic pattern is with low speed and high traffic volume which easily contributes to high vehicle emissions, as can be seen in Fig.1(c).

3.3 Problem formulation

**Problem:** Given the history emission episodes $EF$, and traffic agent factors $TA$, design a control policy $\pi$ to minimize the total emission amount in an episode while ensuring a normal traffic volume, as shown in Fig.2.

4. METHODOLOGY

Obtaining the history emission episodes, a reinforcement learning based model (EFRL) is proposed to minimize the total emission in the episode by learning traffic agent control policy.

4.1 Framework

As shown in Fig.3, our model consists of two phases, offline learning and online controlling.

**Data Processing** Given the original vehicle emission measurements and traffic related information from the emission remote sensing systems (emission environment), the EFRL structures the batched data into states $s_t$ for generating the emission episodes.

**Simulation** In traffic emission variation process, there are interactions between vehicle emission and traffic information. Therefore, to train and evaluate a dynamic traffic emission model, a system simulator is required to simulate the system dynamics under reposition. During the simulation phase, the samples pool derived from real emission dataset to train our EFRL model. Specifically, the emission environment states, the traffic agent’s coordination actions and immediate reward at each time step are used to train our EFRL DQN model for estimating the long-term value function, from which the coordination optimal traffic controlling policy can be inferred.

**Online Controlling** After offline learning phase, we obtain the trained EFRL DQN model, which returns optimal traffic control policy with traffic flow limits and vehicle speed limits suggestions to reduce vehicle emissions.

4.2 EFRL Model

A reinforcement learning model Sutton et al. (1998) is usually defined as a six tuples, i.e. $(S,A,T,R,\pi,\gamma)$, where $S$ is the state space of environment, $A$ is the action space of agents, $T$ is the transition probability which an agent took action $a_t$ given state $s_t$, will transit to the next state $s_{t+1}$, $R$ represents the immediate reward after taking action under specific state, i.e. $r_t(s_t,a_t)$, $\pi$ is a policy $S \times A \rightarrow \pi$, describing the probability to take an action given specific state, $\gamma$ is a time discount parameter. The agent tries to select actions so that the sum of the discounted rewards it receives in the future is maximized, an action $a_t$ has a long-term rewards $G_t$ defined as Eq. 1

$$G_t = r_t + \gamma \cdot r_{t+1} + \gamma^2 \cdot r_{t+2} + \cdots + \gamma^k \cdot r_{t+k}$$ (1)

where $t+k$ is the final time step.

The objective of reinforcement learning is to learn an optimal policy $\pi$ such that given state $s_t$, and taken action $a_t$, the agent is able to receive the maximum expected long-term discounted reward. The optimal long-term value function is defined as Eq. 2

$$Q^*(s_t, a_t) = \max_\pi \mathbb{E}_r(G_t | s_t, a_t, \pi)$$ (2)

Bellman equation is usually adopted to calculate Eq. 2, which is defined as

$$Q^*(s_t, a_t) = \mathbb{E}_{s_{t+1}}(r_t + \gamma \cdot \max_{a_{t+1}} Q^*(s_{t+1}, a_{t+1}) | s_t, a_t)$$ (3)

Given the optimal long-term value of each action by each state, we can get the optimal policy with Eq. 4.

$$a^*_t = \arg \max_{a_t} Q^*(s_t, a_t)$$ (4)

Based on the traditional reinforcement learning theory, we can formulate the emission control model as following concepts.

**Observation** Each time the traffic agent requires new control, it has a real-time observation of the current emission environment. The observation is defined to include the emission environment states, and the traffic agent states. For the emission environment states, which includes current emission states $EF_t$, and the observed emission $\overline{EF}_t$ in the last period. For the traffic agent states, which includes the current observations of traffic volume $tv_{o,t}$ and traffic average speed $vs_{o,t}$. An observation can be denoted as $O_t = (EF_t, \overline{EF}_t, tv_{o,t}, vs_{o,t})$.

**Action** The action of traffic agent is denoted as a vector describing the traffic volume limits and the traffic average speed limits, which is can be represented as $a_t = (tv_{1,t}, vs_{1,t})$. For example, action $a_1 = (120, 40)$ denotes that the current traffic volume is limited to 120 V/h and the vehicle speed is limited to 40 km/h on the target road segment in the next hour.

**State** The state of the emission environment is defined as interleaved sequences of observations and actions combined with current time, which can be denoted as $s_t = (O_{t-k}, a_{t-k}, \cdots, O_{t-1}, a_{t-1}, O_t, t)$, $k$ is the observation time lag. And $k$ is set to 1 in this work.

**Immediate reward** To control the emission of an episode at low level while ensuring a quite traffic volume. The immediate reward is defined as a combination gain of emission and traffic volume $r_t = G_f(EF_t - \overline{EF}_{f+1}) + G_i(tv_{1,t} - tv_1)$ after taking action $a_t$ under $s_t$ and transit to $s_{t+1}$ in period $(t,t+1]$. And the definition of $G_i$ is as

$$G_f(x) = \begin{cases} 1 & x > 0 \\ -1 & \text{otherwise} \end{cases}$$ (5)

**Optimal value network** Defining the EFRL model, we can design DQN to estimate the optimal long-term value function (Eq. 3), i.e. $Q^*(S,A,\theta) : S \times A \rightarrow Q^*$, where $\theta$ are the learning parameters. Our EFRL model is optimized by minimizing the Bellman equation square error, the objective function is formulated as
Fig. 1. Daily traffic and emission distribution: (a) low speed in the morning rush hours; (b) high traffic in the morning rush hours; (c) high emissions in the morning rush hours.

Fig. 2. Emission control policy problem definition

And the pseudo algorithm for training the optimal long-term value network is detailed in Algorithm 1.

Algorithm 1 EFRL DQN training algorithm

Require:
1: Sample replay buffer $D$; emission episodes $EF = \{EF^1, EF^2, \ldots, EF^N\}$; episode length $T$; behavior network parameter $\theta$; target network parameter $\theta^−$.

Return: Learned EFRL DQN model $Q^∗(\theta)$

2: $D \leftarrow \emptyset$ //Initialize the replay buffer $D$
3: Random initialize $Q^∗(S, A, \theta)$, $\theta^− \leftarrow \theta$
4: for each episode $EF^i (1 \leq i \leq N)$ do
5: for each time step $t (1 \leq t \leq T)$ do
6: random select an action $a_t$ with probability $\epsilon \in [0, 1]$/\epsilon is a parameter for exploration
7: otherwise select $a_t = \arg\max_{a \in A} Q^∗(s_t, a, \theta)$
8: the traffic agent executes action $a_t$ and transits to next state $s_{t+1}$, receiving reward $r_t$. storing new sample $(s_t, a_t, s_{t+1}, r_t)$ to $D$
9: sample random minibatch $(s_t, a_t, s_{t+1}, r_t)$ to $D$
10: sample random minibatch $(s_t, a_t, s_{t+1}, r_t)$ from $D$
11: $Q^∗(s_t, a_t, \theta) = Q^∗_{\text{target}}(s_t, a_t, \theta^−)$
12: minimize $L$ (Eq. 6)
13: $\theta^− \leftarrow \theta$
14: end for
15: end for

5. EXPERIMENTS

5.1 Data and setup

The proposed method is implemented in a high performance server with GeForce GTX 1080Ti GPU.

And we use real-world road segment to evaluate the emission control reinforcement learning model. Specifically, we collect emission data from vehicle remote sensing systems deployed on a main road in Hefei, which is authorised by Hefei Environmental Protection Bureau. As a sensor station may not have records sometimes, the missing entries are filled by average value. And the data details are summarized in Table 2.

5.2 Baselines and Metric

To evaluate the performance of the proposed EFRL model, we compare it with the following baselines.

Random policy (RP) This emission control policy means...
Table 2. Experimental Emission Datasets

| Dataset Measurement | Location | Time Span | Time interval | Evaluation stations | Pollutants measurements | Traffic volume | Vehicle speed | Available time interval |
|---------------------|----------|-----------|---------------|---------------------|-------------------------|---------------|--------------|------------------------|
|                     | Hefei    | 2017/5/1-2017/7/31 | 1 hour        | 4                   | Fuel, CO, HC, NO        | [0.3512](V · h⁻¹) | [0.77.9](km · h⁻¹) | 2091 (117 missing)    |

that the traffic agent selects a random action for traffic volume and traffic average speed limits policy each time.

Monte Carlo (MC) MC estimates optimal actions based on experience episodes without complete knowledge of the environment Sutton and Barto (2018). With this strategy, the traffic agent revises its policy and value estimates based on emission episodes experience.

Q-learning (QL) The QL is the most commonly used reinforcement learning algorithms Dayan and Watkins (1992)Mahadevan (1996). The QL agent chooses the optimal action based on the largest Q-value.

Metric evaluation To evaluate the EFRL control policy, we compare the total emission in average episode with the baseline policies. The average episode emission is defined as follows.

$$EF_{k, average} = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{T} \sum_{t=1}^{T} E_{F_k(i,t)}$$  \hspace{1cm} (7)$$

where k ∈ \{Fuel, CO, HC, NO\}, N is the total emission episodes, T is the episode length.

5.3 Results

In the emission policy evaluation experiment, we used the policy algorithms as described in previous section to run different traffic emission simulations with traffic and speed control. Fig.4 show the total emission of different emission controlling policies. And the emission reduction ratio respect to the total emission without controlling policy is shown in Table3, the emission reduction is little while taking the random policy. Moreover, the policies perform unstably on different pollution, i.e. MC and QL performs well on CO, HC reduction, MC and QL achieve similar results on NO reduction. And our EFRL achieves the best results on all pollution reduction.

Table 3. Emission reduction comparison on average episode

| Control Policy | Fuel | CO  | HC  | NO  |
|----------------|------|-----|-----|-----|
| RP            | ↑ 1.28% | ↑ 0.11% | ↓ 5.54% | ↓ 6.01% |
| MC            | ↓ 56.42% | ↓ 76.86% | ↓ 86.95% | ↓ 61.93% |
| QL            | ↓ 59.34% | ↓ 78.08% | ↓ 96.38% | ↓ 62.37% |
| EFRL          | ↓ 62.39% | ↓ 89.54% | ↓ 97.72% | ↓ 65.56% |

The traffic statistics of different policies are shown in Table 4 and Fig.5, comparing with the none control policy and random policy, the reinforcement learning based approaches are more stable on the traffic volume and speed control. We can find that the EFRL policy can significantly reduce the traffic emission in average episode while ensuring a quite normal traffic volume and speed.

And to evaluate the training performance of EFRL, Fig.6 shows the total reward of training iterations. We can find that as the training process, there is a increasing trend of total reward.

Fig. 4. Emission controlling comparison

Fig. 5. Policy quality comparison on Fuel:(a) traffic volume distribution;(b) traffic speed distribution.

Fig. 6. Total reward of training iterations
6. CONCLUSION

In this paper, we solved the traffic emission management based on deep reinforcement learning model, namely EFRL. Different from the existing works which leverage the macroscopic traffic flow model and microscopic traffic emission model to simulate the emission environment, the EFRL directly learned from the history emission sequence, and used a compound emission state to capture the system temporal dependencies. To deal with the large state and action space, a DQN is applied to estimate the optimal long-term value function. The proposed policy is evaluated on real world vehicle emission data in Hefei. And the comparing result demonstrated the effectiveness of the proposed method.

In the future, we will extend the single road segment control policy to other spatiotemporal emission controlling on a regional scale.

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