Production and reconstruction of binary Fourier transform computer-generated holograms based on MATLAB

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Abstract. Based on MATLAB, the production and reconstruction of binary Fourier transform computer-generated holograms (CGHs) were realized using Roman III roundabout phase encoding method. The principle, method and step of CGHs were discussed. The digital reconstruction of holograms was completed using CGHs technology, and the simulation of whole holographic recording and reconstruction process was realized by a computer. Additionally, effects of the width of a sampling unit s and the threshold coefficient ef on the production and reconstruction of CGHs were studied, which can provide a theoretical reference for the improvement of the quality of the reconstruction image. Compared with experiments, the result of the CGHs technology is more intuitive and obvious. It is inferred that CGHs have the potential to be used to image encryptions and communications.

1. Introduction

Computer-generated holography (CGH) is a new technique for producing holograms based on digital computing and modern optics, which has been widely used in various fields such as optical components [1], three-dimensional displays [2-4], holographic interferences [5], optical information storages [6], laser scannings [7], and anti-counterfeiting encryptions [8]. The traditional holographic technique is to produce holograms using optical and interference-recording methods. But CGH is to produce holograms by computer coding, a hologram can be generated directly after the mathematical description of the material wave is entered into a digital computer. Instead of on-the-spot recording with the optical device, CGH can completely save the light source and doesn’t require the accuracy of the optical device highly, which can also produce holograms of various objects that do not actually exist. And CGH has a low noise and a high repeatability, which has more obvious simplicity and flexibility than optical holograms [9]. In the application of CGH, the most widely used and important one of computer-generated holograms (CGHs) is the Fourier transform CGHs. In addition, MATLAB is a combination of numerical calculations and graphics visualization functions [10], which is simpler and faster in the production of CGHs through the comparison with the traditional language (Fortran, C etc.).

CGH was firstly proposed by Kozma and Kelly in 1965, who used an artificial method to create a matching filter for detecting signals buried by noises [11]. Subsequently, Lohmann applied the sampling theorem in communication theory to the production of space filters, laid the theoretical foundation for the production of CGHs, and proposed circuitous phase CGHs [12]. In 1967, Burch added off-axis reference lights and offset components to the material-light-wave signal to obtain modified off-axis reference light CGHs [13]. In 1968, Lesem et al. proposed a new method of CGHs,
which is phase diagrams [14]. In 1974, Lee et al. proposed a method for producing CGHs, in which actually a computer was used to simulate the optical interference process, therefore, the method is more suitable for producing pure phase holograms and more convenient for applying to interference brightness technology [15]. In 1988, Leseberg and Frere considered that an object consists of multiple two-dimensional planes, then after calculating the Fourier transform of each plane, a hologram of a three-dimensional object could be obtained through a coordinate transformation [16]. In 1992, Lucente proposed a LUT method for the first time, which effectively improved the shortcoming of the slow-calculation speed of the point-source method, whose speed was raised 43 times on the basis of the original [17]. In 2008, Kim et al. proposed a N-LUT algorithm, which reduced the memory space occupied by the LUT algorithm [18]. In 2011, Jackin and Yatagai obtained distributions of material-light fields of three-dimensional planes on holographic planes by rotating spatial coordinates of plane Fourier spectrums, and made reference lights and material lights interfere to generate CGHs of three-dimensional objects [19]. In 2012, Ichikawa and Sakamoto proposed an algorithm that accelerated the point-source set using GPU [20]. In 2013, Jia et al. proposed a table-data-compression algorithm and a recursive-table-search method, successfully reduced the requirement of CGHs for storage spaces [21]. At the same time, by studying an occlusion relationship among three-dimensional objects, a fast algorithm for calculating an occlusion effect of holographic three-dimensional images was proposed [22]. Generation speeds of CGHs were improved by non-uniform sampling method, and 70% of redundant information was reduced by the N-LUT method, which can be used to 3D dynamic hologram displays [23,24]. Zhang et al. carried out a comprehensive and systematic research to CGH, its display algorithm, and related technologies, then proposed a CGH algorithm based on a principle of a conformal geometry [25]. In 2014, Yang et al. presented a new method for constructing CGHs using a precalculated triangular mesh [26]. In 2015, Im et al. presented an accelerated algorithm using the intrinsic sparsity in the polygon CGH pattern for the enhancement of computational efficiency effectively [27]. In 2016, Wang et al. proposed a new approach for designing the binary phase-only hologram with arbitrary-shaped polygonal apertures [28]. In 2017, Nishitsuji et al. reviewed fast calculation techniques for the CGH calculation of a point-light-source-based model [29]. In 2018, Wang et al. proposed a simple and fast algorithm for CGH based on pinhole-type II using a look-up table, results showed that the proposed method could easily achieve the real-time hologram generation with several CPU threads running simultaneously [30].

In this paper, the production and reconstruction of binary Fourier transform CGHs were realized with MATLAB based on Roman III roundabout phase encoding method. Effects of the width of a sampling unit $s$ and the threshold coefficient $ef$ on the production and reconstruction of CGHs were studied. Good reconstruction images were obtained, and the whole process of holography was showed intuitively and vividly.

2. Principles and Steps for CGHs

Under a condition of linear recording, the optical hologram is sinusoidal and it is very difficult to draw sinusoidal stripes, but it can be done using computer control to draw black and white (or transparent and opaque) stripes. Black and white stripes become transparent-opaque interval stripe films by shrinkage, which are called computer-generated binary holograms. The main steps for producing binary Fourier transform CGHs include: Sampling the material surface according to the sampling theorem to obtain its discrete value at each discrete point; Calculate a distribution of light intensity on a holographic plane; According to various coding methods (this paper adopted Roman III circuitous phase encoding method), an optical field distribution on a holographic plane is expressed by calculating the permeability of holograms; Holograms are drawn using a plotter, a cathode-ray tube, or a computer-controlled microdensitometer. Holograms can also be recorded on a film with a special output directly.

2.1. Material surface and holographic sampling

Computers can only handle mathematical signals. For using a computer to produce holograms, material surfaces and holographic surfaces must be sampled firstly. Set a material-wave function as
\[ f(x, y) = a(x, y) \exp[\jmath \phi(x, y)] \]
\[ f(x, y) = 0, |x| \leq \frac{\Delta x}{2}, |y| \leq \frac{\Delta y}{2} \] (1)

The space size of a plane object is \( \Delta x \times \Delta y \). In eqn. (1), \( a(x, y) \) is the amplitude, \( \phi(x, y) \) is the phase, \( \Delta x \) and \( \Delta y \) are widths of the plane object in the direction of \( x \) and \( y \), respectively. The Fourier transform of material-wave function is
\[ F(u, v) = A(u, v) \exp[\jmath \phi(u, v)] \]
\[ F(u, v) = 0, |u| \leq \frac{\Delta U}{2}, |v| \leq \frac{\Delta V}{2} \] (2)

In eqn. (2), \( A(u, v) \) is the space spectra amplitude, \( \phi(u, v) \) is the space spectra phase, \( \Delta U \) and \( \Delta V \) are widths of the space spectra in the direction of \( U \) and \( V \), respectively.

The material-wave function \( f(x, y) \) is sampled according to the sampling theorem. The number of sampling units on the axis \( x \) and \( y \) is \( M \) and \( N \), and the sampling spacing is \( x_0 \) and \( y_0 \), then the material wave can be written as the following discrete form:
\[ f_{mn} = f(mx_0, ny_0) \] (3)

In eqn. (3), \( m \), \( n \) is the ordinal number of the sampling unit.
\[ -\frac{M}{2} \leq m \leq \frac{M}{2} - 1, -\frac{N}{2} \leq n \leq \frac{N}{2} - 1 \] (4)

The total number of samples is \( MN \).

The Fourier transform hologram \( F(u, v) \) is sampled in the same way. According to the principle that the spatial bandwidth product is invariant in the transformation, the number of samples in the hologram must at least be equal to (or greater than) the number of samples in the object field, and the total number of samples on the hologram surface is also taken as \( MN \).
\[ F_{mn} = F(mu_0, nv_0), -\frac{M}{2} \leq m \leq \frac{M}{2} - 1, -\frac{N}{2} \leq n \leq \frac{N}{2} - 1 \] (5)

In eqn. (5), \( u_0 \) and \( v_0 \) are the sample spacings in the direction of \( u \) and \( v \), respectively.

2.2. Discrete Fourier Transform (DFT)

After material-light waves are discretized, the discrete Fourier transform (DFT) is calculated for each sampling point, and the complex amplitude of each sampling point on the hologram surface is obtained. The using formula as follows
\[ F_{mn} = \sum_{m=\frac{M}{2}}^{\frac{M}{2}-1} \sum_{n=\frac{N}{2}}^{\frac{N}{2}-1} f_{st} \exp[-j2\pi(m \frac{s}{M} + n \frac{t}{N})] \] (6)

\( F_{mn} \) is a complex number after DFT, \( F_{mn} = C_{mn} + jD_{mn} \), \( C_{mn} \) and \( D_{mn} \) are real and imaginary parts, respectively. Then the amplitude and phase of each sampling point on the hologram are
\[ A_{mn} = \sqrt{C_{mn}^2 + D_{mn}^2}, \Phi_{mn} = \arctan \frac{D_{mn}}{C_{mn}} \] (7)

2.3. Coding

Coding is for a realization of a complex amplitude of each sampling point on a hologram by artificial method according to a given illumination-light wave. According to the Roman III encoding method, the recording of the amplitude and phase information can be used as follows: in a sampling unit of a hologram, a light aperture is placed, and the width \( w \) of a rectangular hole is taken as a constant value, the amplitude in the wave surface is modulated by the height of the rectangular hole; the phase of the wave surface is modulated by changing the position of the rectangular-hole center.
away from the sampling-unit center. In a MATLAB program implementation, selected encoding parameters are as follows:

The height of a rectangular hole: \( l_{mn} = A_{mn} \)

The distance of the rectangular-hole center away from the sampling-unit center: \( p_{mn} = \frac{\Phi_{mn}}{2\pi} \)

The width of rectangular hole: \( w = \frac{1}{2} \times (\text{the width } s \text{ of a sampling unit}) \)

For binary Fourier transform computer-generated holograms, a sampling unit of Roman III is shown in Fig. 1.

![Figure 1 Sampling unit of Roman III.](image)

2.4. Drawing or Display

After completing the amplitude and phase coding with a computer, CGHs can be produced and reduced to a suitable size.

2.5. Matlab program for making and reproducing computational holograms

Using the Roman III roundabout phase encoding method to compile a program to achieve the production and reconstruction of CGHs, the MATLAB program is as follows:

```matlab
close all;
clear;
A=zeros(64);
%---------------------------------------------------------
%E
%A(15:20,20:40)=1;A(15:50,20:25)=1; A(45:50,20:40)=1;A(30:34,20:35)=1;
%A(32:33,20:44)=1;A(14:52,32:33)=1;
%H
%A(15:50,20:25)=1;A(30:34,20:40)=1;A(15:50,40:45)=1;
%III
%A(15:50,15:20)=1;A(15:20,15:50)=1;A(15:50,45:50)=1;A(30:34,20:50)=1;A(15:50,30:35)=1;A(45:50,20:50)=1;
%---------------------------------------------------------
figure,imshow(A,[]);title('Object image', 'fontWeight', 'B');
Fa=fft2(fftshift(A));
Fs=fftshift(Fa);
Am=abs(Fs); % amplitude
Ph=angle(Fs); % phase
s=11;
cgh=zeros(64*s);
th=max(max(abs(Fs)));
```

qq=th/1.2;  
Am(Am>qq)=qq;  
q=1:s;w=(s+1)/2;  
for m=1:64;  
for n=1:64;  
h=round(Am(m,n)/qq*(w-1)-0.5);  
md=zeros(s);  
if h>0;  
    td=ones(h*2+1,3);  
    Pm=round(Ph(m,n)/pi*3);  
    kz=Pm+w;  
    md(w-h:w+h,kz-1:kz+1)=td;  
end  
cgh((m-1)*s+q,(n-1)*s+q)=md;  
end  
end  
figure;imshow(cgh,[]);  % Converse Phase Coding Results  
title('Converse Phase Coding Results', 'fontWeight', 'B');  
Re=ifft2(cgh);  Re=fftshift(Re);  
figure;imshow(abs(Re),[]);  % Reproducing image  
title('Reproducing image', 'fontWeight', 'B');  

The sentences between two double-dotted lines in the program are letters and characters generation sentences, which can be selected during program operations. In the program, \( s \) with a value of 11 is the width of a sampling unit, the width of optical propagation caliber \( w \) is half of \( s \). The threshold \( qq \) is inversely proportional to the threshold coefficient with a value of 1.2.

**3. Production and reconstruction of binary Fourier transform CGHs**

Material-light waves, which are recorded in the Fourier transform holograms, are Fourier spectrums of original objects. Objects, which are recorded in our study, are computer-generated English letters and Chinese characters “E, 十, H, 田”, which are binary images with a BMP format and used to calculate the Fourier transform of the distribution of original objects. Since operating numerically, the calculation process should first be discretized, then the calculation and encoding with MATLAB language are conducted, and CGHs are produced and reconstructed after running the program in section 2.5 of the paper. Fig. 2 shows original images of English alphabets “E, H” and Chinese characters “十, 田” used in our study whose sizes are all \( 64 \times 64 \) pixels, Fig. 3 is the corresponding CGHs, and Fig. 4 is the corresponding reconstruction images, which are clear. Therefore, it can be inferred that the production and reconstruction of CGHs with Matlab on the basis of the Roman III roundabout phase encoding method can be achieved.

![Figure 2](image_url)  
**Figure 2** Original images of English alphabets “E (a), H (c)” and Chinese characters “十 (b), 田 (d)”.  

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In addition, we studied effects of the width of a sampling unit $s$ and the threshold coefficient $ef$ on the production and reconstruction of CGHs. Take the character “E” as an example, the effect result on the production of CGHs is shown in Figs. 5 and 6, the effect result on the reconstruction of CGHs is shown in Figs. 7 and 8. Based on Figs. 5, 6, 7 and 8, it can be seen that when the size of the sampling unit $s$ is fixed, the production results show an increasing trend of rectangular holes with the increase of the threshold coefficient $ef$, reconstruction images gradually become clear from blur, but the intensity of the light was weakened. The threshold $th$ is inversely related to the threshold coefficient $ef$. The smaller the threshold coefficient, the greater the threshold. Therefore, more high-frequency components can be retained during the production process of CGHs, so that the image will gradually become clear. When the threshold coefficient $ef$ or the threshold $th$ is fixed, the number of reconstruction images presented in the field of view increases with the increase of the size of the sampling unit $s$, which is equivalent to the scaling of the image. For other characters “十, H, 田”, we also produced and reconstructed their CGHs, and it was indicated that effects of $s$ and $ef$ on the production and reconstruction of their CGHs is the same as the result of the character “E”.

**Figure 3** CGHs of English alphabets “E (a), H(c)” and Chinese characters “十 (b), 田 (d)”.

**Figure 4** Reconstruction images of English alphabets “E (a), H(c)” and Chinese characters “十 (b), 田 (d)”.
Figure 5 Effect of the threshold coefficient $ef$ on the production of CGHs. The width of a sampling unit $s$ is fixed as 9. (a) $ef = 0.8$; (b) $ef = 1.2$; (c) $ef = 2$; (d) $ef = 5$.

Figure 6 Effect of the width of a sampling unit $s$ on the production of CGHs. The threshold coefficient $ef$ is fixed as 0.8. (a) $s = 9$; (b) $s = 11$; (c) $s = 15$.

Figure 7 Effect of the threshold coefficient $ef$ on the reconstruction of CGHs. The width of a sampling unit $s$ is fixed as 9. (a) $ef = 0.8$; (b) $ef = 1.2$; (c) $ef = 2$; (d) $ef = 5$. 
Figure 8 Effect of the width of a sampling unit $s$ on the reconstruction of CGHs. The threshold coefficient $ef$ is fixed as 0.8. (a) $s = 9$; (b) $s = 11$; (c) $s = 15$.

4. Conclusion
We used MATLAB to design a program for producing and reconstructing binary Fourier transform holograms, and obtained good reconstruction images of different characters. The whole process of holography was realized completely through running the program. In addition, effects of $s$ and $ef$ on the production and reconstruction of CGHs were studied, it was seen that reconstruction images gradually became clear with the increase of the threshold coefficient $ef$, but the intensity of the light was weakened. The size of the sampling unit $s$ revealed the scaling of the image, which can provide a theoretical reference for the improvement of the quality of the reconstruction image. Compared with the experiment, the result of the CGHs technology is more intuitive and obvious, CGHs have the potential to be used to image encryptions and communications.

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