Optimisation of composite superconducting bulks made from (RE)BCO coated conductor stacks using pulsed field magnetization modelling

A Patel¹ and B A Glowacki¹,²,³
¹Applied Superconductivity and Cryoscience Group, Department of Materials Science and Metallurgy, University of Cambridge, Pembroke Street, Cambridge, CB2 3QZ, UK
²Institute of Power Engineering, ul. Augustówka 6, 02-981 Warsaw, Poland
³Department of Physics and Energy, University of Limerick, Plassey, Ireland
E-mail: ap604@cam.ac.uk

Abstract. Coated conductors, although designed to carry transport current, are ideally suited to carrying persistent current and can therefore be cut and stacked to form a type of composite bulk which has superior thermal properties compared to existing bulks despite having less than 2% superconductor by volume. The magneto-thermal modelling reported follows on from previous experimental work on pulsed magnetization of a 12 mm square tape stack. The magnitude of the applied field has a strong effect on the trapped field and flux. The optimum applied field depends on sample height and the maximum trapped field and flux saturates as the height reaches the diameter of the stack. The nature of a composite bulk made from a stack of tapes gives complete control over the height of the stack which needs to be optimised for pulsed magnetization.

1. Introduction
The ability of superconducting bulks to trap high fields once magnetized allows them to act as powerful permanent magnets thanks to persistent currents circulating in the bulks. However, for high trapped fields and temperatures below 77 K, bulks suffer from poor thermal stability [1] and lack mechanical strength. A stack of tapes created from commercially available 12 mm wide tape produced by SuperPower Inc has already been shown to trap up to 2 T when pulse magnetized [2] and over 7 T between two stacks when field cooled [3]. Given the stack size and $I_c$ properties, these results show great potential for a stack of tapes to be used as a trapped field magnet. Optimisation of the stack parameters is now needed including critical state modelling of applied field and sample height as reported here.

2. Modelling framework and thermal properties
The modelling of pulsed field magnetization was carried out using the H-formulation for magnetic fields, coupled with heat transfer in the finite element modelling package COMSOL Multiphysics 4.2a. The framework used the E-J power law and is the same as that used in [4]. The applied pulsed field was half a sinusoid. The full mathematical description of heat generation and the temperature and field dependent critical current density for the model can all be found in [4]. The total cooling power applied to the cold head (see figure 1) used in the simulation is described by the following function, equation (1), which gives realistic cooling saturating at a maximum of 30 W for a temperature rise greater than
15 K. These parameters approximately reflect the real cooling power in cryocoolers, such as the one used in [5].

\[ P_0 = 0 \quad T < T_0 \]
\[ P_0 = Q_0 (T - T_0) \quad T_0 \leq T \leq (T_0 + 15) \]
\[ P_0 = 15Q_0 \quad T > (T_0 + 15) \]

(1)

The simulation was split into two time domains, \(0 < t < 0.1\) s (which includes the 28 ms pulsed field) and \(0.1 \leq t < 30\) s, with the second time domain having a higher \(n\) value to give realistic flux creep rates as described in [4]. The equilibrium temp, \(T_0\), was 38K with a starting temp of 40 K. All trapped field and flux values were evaluated at 0.8 mm above sample to match previous experiments.

![Figure 1](image1.png)

**Figure 1.** Geometry and mesh used in model for a 12 mm diameter stack with variable sample height \(h_0\).

![Figure 2](image2.png)

**Figure 2.** (a) Effective thermal conductivity of the tape stack in the radial and azimuthal directions compared to constituent materials. (b) Effective volumetric heat capacity of tape stack compared to constituent materials.

Modelling of individual stack layers is a very challenging task given the number of mesh elements required due to thin layers. An approximation can be made for the thermal properties of the stack which are highly anisotropic. The effective thermal conductivity parallel and perpendicular to layers of a 3 layer structure can be shown analytically to be:

\[
\bar{\kappa}_r = f_1 \kappa_1 + f_2 \kappa_2 + f_3 \kappa_3, \quad \bar{\kappa}_z = \begin{pmatrix} \frac{\kappa_1}{f_1} \\ \frac{\kappa_2}{f_2} \\ \frac{\kappa_3}{f_3} \end{pmatrix} + \begin{pmatrix} \kappa_2 \\ \kappa_3 \\ \kappa_1 \end{pmatrix} + \begin{pmatrix} \frac{\kappa_1}{f_1} \\ \frac{\kappa_2}{f_2} \\ \frac{\kappa_3}{f_3} \end{pmatrix}
\]

(2)

where \(f_n\) is the volume fraction of each layer. Applying this equation to the stack of tapes magnetized in [2], gives an enhanced radial thermal conductivity as shown in figure 2(a), but the azimuthal conductivity is dominated by Hastelloy as well as the average volumetric heat capacity (figure 2(b)).

### 3. Optimisation for pulse magnetization of a 12mm diameter stack

#### 3.1. Optimisation of applied field

The applied pulsed field was varied from 0 to 10 T for three different sample heights. The trapped field and flux results are shown in figure 3. Trapped flux needs to be considered as much as trapped field when optimising the sample as flux is as important for some applications such as motors and generators. Significant relaxation has occurred 30 s after the pulse but the general dependence of trapped field and flux on applied field is similar to 0.1 s. There exists a height dependent optimum applied field as too
low an applied field failed to induce current deep enough into the sample, whereas too high an applied field generates excessive heat leading to a larger temperature rise. This matches experimental behaviour for trapped flux in a bulk [6]. Interestingly, the optimum applied field did not fully penetrate any of the heights modelled and less so for the largest height. This is a clear indication that trying to saturate the sample is not beneficial for pulse magnetization at lower temperatures (< 50 K). Experimental applied fields need to be optimised depending on the sample height.

Figure 3. Effect of applied field magnitude on trapped field and flux shortly after pulsed field (a) and (b), and after relaxation (c) and (d), for different sample heights.

3.2. Optimisation of sample height
The optimum applied field dependence on height was estimated by interpolating the optimum fields determined for the three heights modelled in figure 3. The trapped field data in figure 3(c) was used for the optimisation of applied field to account for relaxation effects. Once the dependence of applied field on height was determined, 11 different sample heights were modelled from 2 mm to 12 mm as shown in figure 4. The dotted lines show the expected dependence if the samples have constant uniform current density. Therefore the dotted lines show the form of behaviour resulting purely from geometric scaling of the sample. In order to make a comparison to the full thermal critical state model, the uniform current density for this approximation was chosen to be that of the \( h_0 = 6 \) mm critical state model. Looking at the dotted lines, it is clear that the trapped field and flux above a sample saturate for larger heights, because for the point above a sample, the contribution to field from far away layers is small. The trapped field and flux is dominated by the top few mm of the sample. The full thermal critical state results using optimised applied field have remarkably close behaviour to the uniform \( J_c \) scaling curves. This correlation is related to the average temperature rise (figure 4(c)) being the same for all the heights modelled. If the same applied field was used for all the heights, there would be a large difference in temperature rise between small and large heights resulting in greater differences in trapped field and flux. The surface area to volume changes significantly with height for small heights (inset of figure 4(a)), but surprisingly, for the optimum applied field used for each height, the enhanced surface cooling expected for the smallest sample heights is not so apparent in the trapped field and flux results. The
overall result from the data is that large sample heights close to the sample diameter (12 mm) can be used to maximise trapped field. There does not appear to be a maxima for the trapped field and flux for the sample size and cooling power modelled. In practice, to minimise cost whilst still having high performance, a sample height of approximately 8 mm would be a good compromise which corresponds to 145 layers of the tape tested in [2].

![Figure 4](image)

**Figure 4.** Effect of sample height on trapped field (a) and flux (b), where the applied field used for each height was optimised to give maximal trapped field. (c) The average sample temperature shortly after the pulse for each height simulated.

4. Conclusions

A thermal critical state model was applied to a 12 mm diameter stack of tapes assuming homogeneous material properties. The effective thermal conductivity was calculated based on Hastelloy, YBCO and silver contributions and found to be highly anisotropic compared to bulk YBCO. There exists a height dependent optimum applied field for pulsed magnetization of the samples. When using optimum applied fields for various sample heights, the maximum trapped field and flux possible varies almost as expected from geometric scaling of a sample with uniform constant persistent current density. The conclusion from this behaviour is that increasing the sample height always increases trapped field and flux for the sample modelled but there is saturation as the height approaches the sample diameter. Given materials cost, the results suggest that there is no benefit in creating a stack with height greater than or equal to its width or diameter. The model made a number of assumptions which do not completely match experimental reality. In future, to make the model more realistic, multiple pulses will be modelled as well as eddy current heating in the silver layer which was not taken into account. Future modelling will also consider magnetization of larger sized stack currently under experimental investigation.

References

[1] Fujishiro H, Naito T, and Oyama M, 2012 *Physics Procedia*. 36 687-692
[2] Patel A, Hopkins S C, and Glowacki B A, 2013 *Supercond. Sci. Technol.* 26 032001
[3] Patel A, Filar K, Nizhankovskii V I, Hopkins S C, and Glowacki B A, 2013 *Appl Phys Lett.* 102 102601-5
[4] Patel A and Glowacki B A, 2012 *Supercond. Sci. Technol.* 25 125015
[5] Patel A, Giunchi G, Albisetti A F, Shi Y, Hopkins S C, Palka R, Cardwell D A, and Glowacki B A, 2012 *Physics Procedia*. 36 937-942
[6] Mizutani U, Oka T, Itoh Y, Yanagi Y, Yoshikawa M, and Ikuta H, 1998 *Appl Supercond.* 6 235-246