Improved handling of 4x4 vehicles with differential transmission in curved driving on support surfaces with low coupling properties

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Abstract. Studying the trends in the development of modern automotive industry, you can see that manufacturers are constantly increasing the level of control over the parameters of the movement of wheeled vehicles, achieving the maximum level of stability and controllability of cars. The purpose of this work is to improve the handling of vehicles with a 4x4 wheel formula with a differential transmission in curved motion on support surfaces with low coupling properties. An algorithm is proposed to improve the handling of vehicles with a 4x4 wheel formula with a differential transmission in curved motion on support surfaces with low coupling properties due to braking of individual wheels. The efficiency and efficiency of the proposed algorithm is proved by the methods of simulation of the movement of two-axle cars with a 4x4 wheel formula with a differential transmission.

Introduction
Studying the trends in the development of modern automotive industry, we can see that manufacturers are constantly increasing the level of control over the parameters of the movement of wheeled vehicles, achieving the maximum level of stability and controllability of cars.

Currently, the creation of active safety systems for cars that provide increased stability and manageability is actively engaged abroad. Research is conducted in two directions. First, dynamic stabilization systems (DSS) are created, the principle of operation of which is based on changing the torques supplied to the driving wheels, including by braking individual wheels. The use of various all-wheel drive systems is becoming more relevant and appropriate. The distribution of traction on all wheels allows you to use the entire weight of the car as a hitch, which has a positive effect on the dynamic qualities of the car, cross-country, and allows you to implement control algorithms that improve the handling and course stability of the car. The development of these methods is devoted, for example, to the work [1–6].

Secondly, increasing stability and controllability is provided by the introduction of automatic corrective changes in the angle of rotation of the controlled wheels (steering) [7–10]. Combined management based on these approaches is possible [11].

The purpose of this work is to improve the handling of 4x4 vehicles with differential transmission in curved motion on support surfaces with low coupling properties. To achieve this goal, you need to solve the following tasks:
• development of a mathematical model of curvilinear motion of 4x4 vehicles with differential transmission on support surfaces with low coupling properties;
• development of an algorithm for the operation of a 4x4 vehicle motion stabilization system with a differential transmission;
• checking the performance of the 4x4 vehicle traffic stabilization system with differential transmission using simulation methods.

Mathematical model of the differential transmission of a 4x4 car with a differential transmission

Consider the diagram (fig. 1) differential transmission of a four-wheel drive two-axle car.

Fig. 1. Differential transmission scheme for a four-wheel drive two-axle car: 1, 2 — front and rear axles; 3, 4 (D) — symmetrical inter-wheel differential; 5 — clutch; \( \omega_{13}, \omega_{24} \) — angular rotation speeds of the front and rear transmission shafts, respectively; \( \omega_{tr} \) — angular rotation speed of the primary transmission shaft; \( M_c \) — torque by the clutch; \( M_i, i=1,...,4 \) — resistance moments on the \( i \)-th wheel; \( M_e \) — torque by the engine; \( i_tr \) — the gear ratio of the transmission; \( i_m \) — the gear ratio of the main transmission; \( \omega_{tr}, i=1,...,4 \) — angular speed of rotation of the \( i \)-th wheel; \( \omega_e \) — angular rotation speed of the engine crankshaft; ENG — internal combustion engine; TR — transmission.
The operation of the transmission can be described by the following system of equations:

\[
\begin{align*}
J_1 \ddot{\omega}_1 &= \frac{M_c}{4} i_pr_m - M_1 \\
J_2 \ddot{\omega}_2 &= \frac{M_c}{4} i_pr_m - M_2 \\
J_3 \ddot{\omega}_3 &= \frac{M_c}{4} i_pr_m - M_3 \\
J_4 \ddot{\omega}_4 &= \frac{M_c}{4} i_pr_m - M_4 \\
\dot{\omega}_1 &= \frac{\dot{\omega}_1 + \dot{\omega}_2}{2} \\
\dot{\omega}_1 &= \frac{\dot{\omega}_2 + \dot{\omega}_3}{2} \\
\dot{\omega}_2 &= \frac{\dot{\omega}_3 + \dot{\omega}_4}{2} \\
J_e \dot{\omega}_e &= h_{pr} M_c - M_c
\end{align*}
\]

where \(J_c\) — moment of inertia of the wheel; \(J_e\) — the moment of inertia of moving engine parts brought to the crankshaft; \(h_{pr} = [0…1]\) — the degree of pressing a pedal of submission of fuel.

**Mathematical model of a single-disc friction clutch**

In mathematical models of friction clutches, the dependence of the dry friction force \(F_{mp}\) on the relative movement speed \(V_n\) of the contacting surfaces is given in the form shown in fig. 2.

![Characteristics of dry friction](image)

**Fig. 2.** Characteristics of dry friction

The torque \(M_c\) transmitted by the clutch is determined by the formula

\[
M_c = F_{mp} r_{ef}
\]

\[
r_{ef} = \frac{\pi \left(D_{akl}^3 - d_{akl}^3\right)}{12 S_{akl}}
\]

\[
S_{akl} = \frac{0.94 \pi \left(D_{akl}^2 - d_{akl}^2\right)}{4}
\]
where $r_{ef}$ — effective friction radius; $D_{a,kl}, d_{a,kl}$ — accordingly, the external and internal diameters of the clutch friction plates; $S_{a,kl}$ — area of clutch friction plates.

The strength of $F_{mp}$ is approximated using the following algorithm. If $|\omega_c - \omega_p| \geq \Delta \omega_n$ (where $\Delta \omega_n$ — threshold value), that

$$F_{mp} = F_c \left[ 1 + (k_b - 1) e^{-cw|\Delta\omega_n|} \right] \text{sign}(\omega_{dv} - \omega_{ep}) (1)$$

$$F_c = F_{pr} + f_{cfr} \frac{h_{ic} N_{max}}{\pi r_{ef}^2} (2)$$

$$F_{pr} = \mu h_{ic} N_{max} (3)$$

$$N_{max} = \frac{1.1 M_{emax}}{r_{ef} \mu} (4)$$

where $F_c$ — Coulomb friction; $k_b$ — the rate of change of force breakaway; $c_w$ — conversion factor; $F_{pr}$ — rest friction force; $f_{cfr}$ — the coefficient of Coulomb friction forces; $\mu$ — the coefficient of static friction; $N_{max}$ — maximum compression force of the clutch friction discs; $M_{emax}$ — maximum torque developed by the engine; $h_{ic} = [0...1]$ — the position of the pedal clutch control: 0 — clutch off; 1 — clutch on.

If $|\omega_c - \omega_p| < \Delta \omega_n$, that

$$F_{mp} = K_s (\omega_c - \omega_p)$$

where $K_s$— coefficient of proportionality.

$$K_s = \frac{F_c \left[ 1 + (k_b - 1) e^{-cw|\Delta\omega_n|} \right]}{\Delta \omega_n}$$

The numerical values of the parameters in formulas (1)–(4) used in the simulation are shown in the table 1.

**Table 1.** Numerical values of the parameters adopted in the simulation

| Parameter | Designation | Dimension | Value |
|------------|-------------|-----------|-------|
| The coefficient of Coulomb friction forces | $f_{cfr}$ | N/Pa | 0,00033 |
| The rate of change of force breakaway | $k_b$ | – | 1,1 |
| Conversion factor | $c_w$ | s | 1,24 |
| Threshold value | $\Delta \omega_n$ | 1/s | 0,5 |
| The coefficient of static friction | $\mu$ | – | 0,4 |

Features of the mathematical model of motion are considered in [12–17].

**Algorithm of operation of the 4x4 vehicle stabilization system with differential transmission due to individual braking of individual wheels**

The operation of the system is based on the recognition of the rear axle skidding (fig. 3a) or demolition of the front axle (fig. 3b) of the car.
Fig. 3. Braking of the outer front wheel in case of skidding of the rear axle (a) and the inner rear wheel in case of demolition of the front axle (b): C — center of mass of the vehicle; M_{DS} — dynamic stabilization moment

To prevent the development of the front axle demolition, it is necessary to first recognize the occurrence and development of this process. To do this, use the results of the work [18], where the parameter is used as a diagnostic sign of the onset of front or rear axle drift $\delta_V = \|V_{C1}\| - \|V_{C2}\|$, representing the difference in the estimation of linear speeds of the center of mass of the car, first using the linear speed of the center of the front axle (vector $V_{C1}$), and then — the linear speed of the center of the rear axle (vector $V_{C2}$).

In the event of a skid the rear axle is braked the outside front wheel (fig. 3A). If the front axle is demolished, the inner rear wheel is braked (fig. 3b) [19, 20].

The formula for determining the braking torque on each wheel is as

$$M_{sol} = h_{ESP}T_{max}, i = 1, \ldots, N,$$
where $T_{\text{max}}$ — maximum braking torque developed by the wheel brake mechanism; $N$ — number of car wheels; $h_{ESP_i} = [0...1]$ — the degree of reduction of the effective braking torque on the $i$-th wheel due to the operation of the algorithm to counteract the demolition of the front axle during braking.

Algorithm for determining the value $h_{ESP_i}, i = 1, \ldots, N$ taking into account the sign rule adopted in the simulation, it should be as follows.

If $\Theta > 0^\circ$ (turn left) & $\delta_v > 0$ (demolition of the front axle), that $h_{ESP1} = h_{ESP3} = h_{ESP4} = 0$; $h_{ESP2} = C_1 [\delta_v] + C_2 \delta_v$.

If $\Theta > 0^\circ$ (turn left) & $\delta_v < 0$ (the drift of a back axis), that $h_{ESP1} = h_{ESP2} = h_{ESP4} = 0$; $h_{ESP3} = C_1 [\delta_v] + C_2 \delta_v$.

If $\Theta < 0^\circ$ (turn right) & $\delta_v > 0$ (demolition of the front axle), that $h_{ESP1} = h_{ESP2} = h_{ESP3} = 0$; $h_{ESP4} = C_1 [\delta_v] + C_2 \delta_v$.

If $\Theta < 0^\circ$ (turn right) & $\delta_v < 0$ (the drift of a back axis), that $h_{ESP4} = h_{ESP2} = h_{ESP3} = 0$; $h_{ESP1} = C_1 [\delta_v] + C_2 \delta_v$.

In the above formulas $C_1$ and $C_2$ — regulator gain factors that can be adjusted individually for each vehicle.

**Performance criteria for the 4x4 vehicle stabilization system with differential transmission due to individual braking of individual wheels**

As criteria for the effectiveness of the stabilization system of a 4x4 vehicle with a differential transmission due to individual braking of individual wheels, you can take the following.

1) Type of vehicle trajectory (no rear axle skid or front axle drift).

2) The case when $\delta_v = 0$, corresponds to the vehicle's neutral turnability (ideal value). Therefore, the diagnostic parameter $\delta_v$ it can be used to formulate a criterion for the efficiency $KE$ operation of motion stabilization algorithms:

$$KE = \sqrt{\frac{1}{n_r-1} \sum_{i=1}^{n_r} \delta_{v_i}^2},$$

where $n_r$ — the number of points in the implementation process $\delta_{v_i}$.

**Study of the effectiveness of the proposed algorithm for the system of stabilization of a 4x4 car with a differential transmission due to individual braking of individual wheels**

The health check algorithm of the system stabilization system of a vehicle 4x4 with differential transmission due to the individual braking of each wheel were conducted on the example of vehicle 2400 kg.

Was carried out a theoretical study of the motion of the vehicle using simulation mathematical modeling.

We study the movement on the support base "ice with snow" (with the coefficient of interaction of the engine with the support base when fully skidding $\mu_{\text{max}} = 0.35$). Note that the term "support base" refers only to a solid non-deformable support surface. The front wheels of the car are controllable. The steering angle changes from zero to the set value within 3 seconds and then remains unchanged.

The mode of entering a turn and moving in a turn with a fixed radius is simulated, while the speed $V=25$ km/h, which the driver tries to keep constant during the movement.

**4x4 car with no controls braking of the wheels (the base for comparison)**

The trajectory of the 4x4 vehicle on the ice when making a turn is shown in fig.4.
Analysis of vehicle trajectories (fig. 4) shows that when performing a maneuver on the support surface "ice with snow" the car loses control (develops a skid of the rear axle).

The value of the $KE$ efficiency criterion for motion stabilization algorithms is 1,00 m/s.

**4x4 vehicle with wheel braking control**

The path of movement on the ice of a 4x4 vehicle with the control of braking the wheels when making a turn is shown in the fig. 5.

![Graph showing vehicle trajectory](image)

**Fig. 4.** The trajectory of the 4x4 vehicle in a turn on the ice in a turn

**Fig. 5.** The trajectory of the car 4x4 with control braking wheels in a turn on the ice

![Graph showing time variation of $h_{ESP}$](image)

**Fig. 6.** The time variation of the parameters $h_{ESP}$ for the front left wheel of a 4x4 vehicle with wheel braking control when cornering
Analysis of the movement paths of a 4x4 vehicle with wheel braking control (fig. 5) shows that when performing a maneuver on the "ice with snow" support surface, the car remains controllable.

On fig. 6 shows the change in time of the control parameter $h_{ESP}$ for the front left wheel of a 4x4 vehicle with wheel braking control when driving in a turn.

As seen in fig. 6, braking of the front left wheel occurs all the time of movement in the turn to prevent the development of skidding of the rear axle, the level of braking torque does not exceed 10...20% of its maximum value.

The value of the KE efficiency criterion for motion stabilization algorithms is 0.39 m/s.

Conclusions
1. An algorithm is proposed to improve the handling of 4x4 vehicles with differential transmission in curved motion on support surfaces with low coupling properties due to braking of individual wheels.
2. The efficiency and efficiency of the proposed algorithm is proved by the methods of simulation of the movement of two-axle 4x4 vehicles with differential transmission.

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