Disorder-enhanced nonlinear delocalization in segmented waveguide arrays

Matthias Heinrich$^{1,4}$, Robert Keil$^1$, Yoav Lahini$^2$, Uta Naether$^3$, Felix Dreisow$^1$, Andreas Tünnermann$^1$, Stefan Nolte$^1$ and Alexander Szameit$^1$

$^1$ Institute of Applied Physics, Abbe Center of Photonics, Friedrich-Schiller-Universität, Max-Wien-Platz 1, 07743 Jena, Germany
$^2$ Department of Physics of Complex Systems, The Weizmann Institute of Science, Rehovot 76100, Israel
$^3$ Departamento de Física, Center for Optics and Photonics (CEFOP), and MSI-Nucleus on Advanced Optics, Faculdad de Ciencias, Universidad de Chile, Santiago, Chile

E-mail: p2hema@googlemail.com

New Journal of Physics 14 (2012) 073026 (10pp)
Received 30 April 2012
Published 12 July 2012
Online at http://www.njp.org/
doi:10.1088/1367-2630/14/7/073026

Abstract. Nonlinearity and disorder in discrete systems give rise to fascinating dynamics in various fields of physics. Photonic lattices allow investigation of them in an optical context. The very nature of discrete propagation allows perfect reconstruction of arbitrary initial wave packets by introducing phase shifts to specific lattice sites. We investigate, both numerically and experimentally, the interplay of nonlinearity with this so-called segmentation imaging in the presence of disorder. We find that whereas in the linear regime perfect imaging is achieved for arbitrary amounts of coupling disorder, the onset of nonlinear self-focusing generally destroys imaging. Interestingly, the influence of Anderson localization in strongly disordered lattices renders the imaging significantly more susceptible to nonlinear perturbations.
1. Introduction

One of the key tasks in optics is imaging: the faithful reconstruction of the input light distribution at the output of the transmission system. Arrays of evanescently coupled waveguides, so-called photonic lattices [1], offer several possibilities to tackle this challenge in the field of discrete optics. Exploiting the analogy to the ordered structure of crystals, it is possible to mimic self-imaging solid state phenomena such as discrete harmonic oscillation [2], Bloch oscillations [3], dynamic localization [4] and coherent destruction of tunneling [5]. However, this approach relies on imprinting specific external potentials [6] on the otherwise homogeneous system and consequently requires a high accuracy in fabrication. An alternative method makes use of the very nature of discrete propagation. By staggering the phases of adjacent waveguides at half the desired propagation distance, a wave packet’s evolution can be reversed, giving rise to a perfect reconstruction of the initial field distribution. This phase shift can be readily introduced by segmentation [7]. Recent investigations have shown that segmentation-based imaging persists for arbitrary amounts of disorder within the lattice, provided that the individual waveguides are identical (‘off-diagonal’ disorder) [8, 9]. Consequently, segmentation imaging is inherently robust to fabrication tolerances.

A fascinating aspect of disordered systems is Anderson localization [10]. Whereas each eigenmode of a homogeneous lattice spans the entirety of its transverse dimensions, disorder decreases the mean width of the supermodes. Hence, a narrow excitation yields significant occupation in fewer supermodes than it would in the corresponding homogeneous system. Consequently, the diffractive broadening of a launched wave packet is impeded compared to the uniform lattice. Since its discovery over 50 years ago, Anderson localization has been explored in a variety of physical systems [11] and under both on-diagonal [10] and off-diagonal manifestations of disorder [12]. In contrast to higher-dimensional systems, which undergo a transition from ballistic to diffusive transport before finally exhibiting Anderson localization above a certain level of disorder, one-dimensional disordered configurations show localization for any non-zero disorder [12]. Generally, the width of the localized states decreases with the degree of disorder.

Localization of wave packets may also occur due to nonlinear self-focusing. Giving rise to the so-called discrete solitons, the propagation of light in nonlinear photonic lattices has been the focus of intensive research [13–15]. The interplay of nonlinearity and disorder in such systems has been the subject of theoretical [16, 17] as well as experimental [18–20] efforts. More recently, the influence of dimensionality crossover [21] as well as nonlinear interfaces [22] on Anderson localization was investigated.

New Journal of Physics 14 (2012) 073026 (http://www.njp.org/)
A particularly fruitful platform for experimental investigations of discrete solitons is provided by femtosecond laser-written [23] arrays of waveguides [24]. Interestingly, linear and nonlinear localization mechanisms may mutually impede each other, as was demonstrated for dynamic localization [25] and solitons in disordered lattices [16].

The question arises as to how the self-imaging property of segmented waveguide arrays is impacted by nonlinear propagation effects and whether disorder may serve to stabilize it by virtue of Anderson localization. In this paper, we investigate this notion for the case of positive Kerr nonlinearity numerically as well as experimentally in laser-written waveguide arrays. We find that self-focusing distorts the imaging in both homogeneous and disordered lattices before tightly localized solitons emerge at high powers. Surprisingly, the presence of localized Anderson modes renders the self-imaging significantly more susceptible to nonlinear perturbations.

2. Numerical investigation

The propagation of light in photonic lattices is governed by a set of coupled differential equations [1] for the modal amplitudes $\varphi_n$ in the individual waveguides:

$$-i\partial_z \varphi_n = C_{n,n+1} \varphi_{n+1} + C_{n,n-1} \varphi_{n-1} + (\beta_n + \gamma|\varphi_n|^2)\varphi_n.$$  \hspace{1cm} (1)

In this so-called discrete nonlinear Schrödinger equation, $C_{k,\ell} = C_{\ell,k}$ are the symmetric coupling coefficients between adjacent waveguides. Finite lattices are described by limiting the waveguide index to a certain range, e.g. $n \in [1, N]$, and consequently $C_{1,0} = C_{N, N+1} = 0$. The propagation constants in the individual waveguides are described by $\beta_n$, which in the case of identical guides can be removed by transformation into a co-propagating reference frame. The strength of the focusing Kerr nonlinearity is given by the coefficient $\gamma$. The total power $P = \sum_n |\varphi_n|^2$ is a conserved quantity of equation (1). To model the evolution of a wave packet, we numerically solve the initial value problem given by the input field distribution $\varphi_n(0)$. In the case of segmented arrays we introduced the phase flip at half the propagation distance: $\varphi_n(L/2) \rightarrow e^{in\pi} \varphi_n(L/2)$. Disorder was realized by assigning uniformly distributed values to the coupling coefficients $C \in [\bar{C} - \sigma, \bar{C} + \sigma]$, where $\bar{C}$ is the mean coupling constant and the width $\sigma$ of the distribution is a measure of the degree of disorder. Values of $\sigma \in [0, 0.8\bar{C}]$ were considered to avoid zero (or near-zero) couplings while maintaining the mean value for all distributions.

Figure 1(a) illustrates the propagation of light in a homogeneous lattice comprised of $N = 31$ waveguides over a distance of six coupling lengths $L_C = \pi/2\bar{C}$. In the linear regime ($P \rightarrow 0$), a single-site excitation of the central channel yields the V-shaped pattern characteristic of discrete diffraction (first panel), whereas the segmented array facilitates self-imaging (second panel). In contrast, an input power of $P = 7\bar{C}/\gamma$ is sufficient for exciting a narrowly confined soliton, which emerges after a certain fraction of the light is radiated away as diffractive background (third panel). Interestingly, the soliton essentially shatters as it encounters a phase flip (fourth panel). This effect can be understood by considering that for focusing Kerr nonlinearity ($\gamma > 0$), the soliton resides in the semi-infinite gap above the propagation band and exhibits a smooth phase front [15]. Thus, the imposed staggered phase results in substantial amounts of light being radiated into the lattice before the wave packet can reshape itself to
Figure 1. (a) Left to right: simulated linear propagation, segmentation imaging, nonlinear propagation at $P = \bar{C}/\gamma$ and nonlinear delocalization at $P = 7\bar{C}/\gamma$ in a homogeneous array of $N = 31$ waveguides. (b) The corresponding panels for an exemplary realization of a disordered lattice with couplings uniformly distributed across the interval $[\bar{C} - \sigma, \bar{C} + \sigma]$ with $\sigma = 0.8\bar{C}$. In all cases the total propagation length is $6 L_C$, and segmentations are indicated by dashed lines.

conform with the profile of a lower-power soliton. Consequently, the imaging is distorted as soon as the Kerr effect leads to appreciable deviations from the phase distributions in the linear regime.

The corresponding behavior of light in an exemplary realization of a disordered lattice with $\sigma = 0.8\bar{C}$ is shown in figure 1(b). The linear propagation patterns clearly illustrate how the transverse transport is inhibited in the regime of Anderson localization (first panel) and that segmentation imaging persists [9] (second panel). Similar to the homogeneous lattice, a tightly confined soliton is excited at $P = 7\bar{C}/\gamma$ (third panel). Note that the diffractive background also remains Anderson localized in the vicinity of the excited guide. Interestingly, the introduction of the phase flip results in a weaker broadening of the emerging wave packet compared to the linear case (fourth panel).
Figure 2. Top: calculated power-dependent mean (100 realizations) output intensity distributions from a conventional lattice after a propagation distance of $6L_C$ for zero, intermediate and high disorder (left group). The corresponding behavior in the segmented lattice (right group). For reasons of visibility, the intensity distributions have been normalized with respect to their maxima of each row. Bottom: dependence of the mean (100 realizations) output intensity distribution’s width $w$ after a propagation distance of $6L_C$ on input power $P$ and degree of disorder $\sigma$. Left: conventional lattice; right: segmented lattice.

We systematically explore the power dependence of the output mean width

$$w = \sum_n |\varphi_n|^2 |n - n_0| / P,$$

arising from single-site excitations at waveguide $n_0$ in lattices with varying disorder. To ensure statistically meaningful results, we generated sets of $R = 100$ realizations for each $\sigma$ and evaluated the averaged intensity patterns after a propagation distance of $6L_C$. The upper part of figure 2 illustrates how the ensemble-averaged output patterns for the homogeneous lattice as well as intermediate ($\sigma = 0.4\bar{C}$) and strong ($\sigma = 0.8\bar{C}$) disorder change with increasing power. The initially broad light distribution in the conventional homogeneous lattice contracts abruptly around a normalized input power of $P \approx 3.5\bar{C}/\gamma$. In contrast, more power is required in the disordered systems to form a tightly localized soliton regardless of the intrinsically narrower
spread in the linear regime (left group of panels). Note that despite a more gradual localization behavior, the output pattern in the disordered lattices starts to contract at lower powers. The corresponding behavior in segmented lattices is shown in the right group of panels. Surprisingly, for the disordered systems nonlinear delocalization occurs at lower powers. Furthermore, in the disordered systems substantially higher powers are required to excite solitons, which are sufficiently localized to become impervious to delocalization.

The dependence of the mean output width on power and disorder is shown in the lower part of figure 2 in more detail. Here, $w$ is visualized as a false color plot with blue resembling broad distributions and red a narrow output. The plot for the conventional system (left panel) clearly demonstrates the impact of Anderson localization (i.e. decreasing $w$ for growing disorder at a fixed power) and soliton formation (i.e. decreasing $w$ for growing power at a fixed disorder). Likewise, the nonlinear delocalization in the segmented system (right panel) is apparent. Remarkably, despite the earlier onset of delocalization for higher disorder, Anderson localization allows the ejected light to spread less widely across the lattice for strong disorder, resulting in a lower maximum width.

3. Experimental results

Based on our theoretical results, we expect segmentation imaging in disordered lattices to be more susceptible to nonlinear perturbations. In order to verify our findings, we fabricated conventional as well as segmented photonic lattices with vanishing ($\sigma = 0$), intermediate ($\sigma = 0.4\tilde{C}$) and strong ($\sigma = 0.8\tilde{C}$) disorder. For each of the two latter cases, ten independent realizations of coupling distributions were examined. The individual coupling coefficients were implemented by adjusting the waveguide separation [26]. In all cases, the sample length of 100 mm corresponded to $1.28L_C$ mean coupling lengths for $\tilde{C} = 0.02$ mm$^{-1}$. A Ti:sapphire laser system (Spectra Physics Tsunami/Spitfire), delivering 300 fs pulses with a repetition rate of 1 kHz at a carrier wavelength of $\lambda = 800$ nm, was used for single-site excitation. The waveguides were segmented over a distance of 5.2 mm to achieve optimum imaging [7]. We recorded the output intensity distributions arising from injected average powers between $0 < \bar{P} \leq 1.5$ mW. The upper panel of figure 3 shows the ensemble-averaged output patterns within the conventional (left group) and segmented (right group) lattices. Note that dynamic excitation effects generally prevent complete localization and give rise to a diffractive background even at powers sufficient to suppress coupling at the pulse center [27]. Furthermore, imperfections in the segmented waveguides lead to a non-unity fidelity of imaging even in the linear regime. This effect becomes more visible for higher degrees of disorder as the non-imaged parts of the wave function remain in closer proximity of the initial excitation due to Anderson localization. Despite these limitations, the principal behavior of the system agrees well with the theoretical predictions for continuous wave conditions. While nonlinear localization occurs at lower powers in disordered conventional lattices, nonlinear delocalization is substantially accelerated in the corresponding segmented systems. The widths according to equation (2) of the ensemble-averaged intensity distributions are shown in the lower part of figure 3. The onset of nonlinear delocalization, highlighted by arrows, occurs at $\bar{P} \approx 0.17$ mW in the uniform case. The presence of disorder decreases this threshold to $P \approx 0.14$ mW for $\sigma = 0.4\tilde{C}$ and $\bar{P} \approx 0.11$ mW for $\sigma = 0.8\tilde{C}$. 

New Journal of Physics 14 (2012) 073026 (http://www.njp.org/)
Figure 3. Top: measured power-dependent mean (ten experimental realizations) output intensity distributions from a conventional lattice after a propagation distance of $1.28L_C$ for zero, intermediate and high disorder (left group). The corresponding behavior in the segmented lattice (right group). For reasons of visibility, the intensity distributions have been normalized with respect to their overall maxima. Bottom: power-dependent width of the mean output intensity. The arrows indicate the onset of delocalization in the segmented lattices.

4. Interpretation

Nonlinear phase contributions perturb segmentation imaging as light propagation can no longer be described by a superposition of the system’s linear eigenmodes. In turn, segmentation introduces a phase distortion which shatters nonlinearly localized wave packets. The general interplay of these two competing mechanisms is valid for uniform as well as disordered lattices. However, a growing degree of disorder increasingly confines the eigenmodes (see exemplary amplitude profiles in figure 4(a)) and hence lowers the number of participating lattice sites. Conversely, the excitation of a single lattice site effectively results in the population of fewer states and thus a narrower maximum spread of the evolving wave packet. Consequently, a given total power induces larger nonlinear phase contributions. In conventional lattices this effect manifests itself as an accelerated onset of wave packet contraction, whereas light in segmented lattices is subject to delocalization at lower powers. From the perspective of
Figure 4. (a) Linear eigenvalue spectrum of a homogeneous lattice (left) and an exemplary disordered lattice with $\sigma = 0.8\bar{C}$ (right) of 15 waveguides each; amplitude profiles corresponding to the highest (flat phased states) and lowest eigenvalues (staggered states), respectively. (b) Nonlinear deformation of the uppermost eigenstates (homogeneous lattice: blue/dashed; disordered: red/solid) at powers $P = 2$, 3 and $5\bar{C}/\gamma$. The corresponding nonlinear eigenvalues are shown in panel (a) as haloed dots above the range of linear eigenvalues.

individual realizations, this behavior can be understood more intuitively: for a given degree of disorder, light propagating in different realizations undergoes (de-)localization at slightly different powers than in the uniform system. The ensemble average thus leads to a smoothing over transition, which becomes more pronounced with increasing disorder. Moreover, even for an individual eigenstate in a specific disorder realization, the nonlinear contraction may progress more slowly than in the homogeneous case (see figure 4(b)). Consequently, the presence of disorder delays the contraction of the ensemble average at high powers in both conventional and segmented lattices alike.

Note that while Anderson localization relies on the system being invariant along the propagation direction, the segmentation technically breaks this symmetry. However, the phase shift is introduced ‘instantaneously’, i.e. over a distance much smaller than the coupling length $L_C$. Consequently, the system effectively behaves as $z$-invariant: the supermodes and their corresponding eigenvalues are the same on all points along the $z$-axis. The influence of the segmentation then is merely a one-time reshuffling of the occupation levels of the supermodes.
5. Conclusion

In conclusion, we found that Anderson localization renders segmentation imaging in disordered lattices significantly more susceptible to nonlinear perturbations. A systematic study of this phenomenon revealed that while in disordered lattices nonlinear delocalization sets in at lower powers, substantially higher powers are required for overcoming it. Nevertheless, even in strongly disordered photonic lattices, segmentation imaging can withstand a certain amount of nonlinearity. Although our investigations were carried out in the context of photonic lattices, our findings have much wider implications. Discreteness, disorder and nonlinearity can be found in manifold manifestations throughout nature. Likewise, the need to control the evolution of wave packets under these conditions arises in various fields of physics beyond optics, such as quantum mechanics and solid state physics.

Acknowledgments

The authors gratefully acknowledge funding from the German Federal Ministry of Education and Research (ZIK 03Z1HN31 ‘ultra optics 2015’) and the Thuringian Ministry for Education, Science and Culture (research group ‘Spacetime’, grant no. 11027-514). MH was financially supported by the German National Academy of Sciences Leopoldina (grant no. LPDS 2012-01). UN acknowledges partial support from Programa ICM P10-030-F, Programa de Financiamiento Basal de CONICYT (FB0824/2008) and a CONICYT doctoral fellowship. FD was financed by Graduate Research School OptiMi (B514-10061).

References

[1] Christodoulides D, Lederer F and Silberberg Y 2003 Discretizing light behaviour in linear and nonlinear optical waveguide lattices Nature 424 817
[2] Gordon R 2004 Harmonic oscillation in a spatially finite array waveguide Opt. Lett. 29 2752
[3] Chiodo N, Valle G D, Osellame R, Longhi S, Cerullo G, Ramponi R, Laporta P and Morgner U 2006 Imaging of Bloch oscillations in erbium-doped curved waveguide arrays Opt. Lett. 31 1651
[4] Szameit A, Garanovich I, Heinrich M, Sukhorukov A A, Dreisow F, Pertsch T, Nolte S, Tünnemann A and Kivshar Y S 2009 Polychromatic dynamic localization in curved photonic lattices Nature Phys. 5 271
[5] Zhang P, Efremidis N K, Miller A, Hu Y and Chen Z 2010 Observation of coherent destruction of tunneling and unusual beam dynamics due to negative coupling in three-dimensional photonic lattices Opt. Lett. 35 3252
[6] Longhi S 2009 Quantum-optical analogies using photonic structures Laser Photonics Rev. 3 243
[7] Szameit A, Dreisow F, Heinrich M, Pertsch T, Nolte S, Tünnemann A, Suriun E, Louradour F, Barthelemy A and Longhi S 2008 Image reconstruction in segmented femtosecond laser-written waveguide arrays Appl. Phys. Lett. 93 181109
[8] Lahini Y, Bromberg Y, Shechtman Y, Szameit A, Christodoulides D N, Morandotti R and Silberberg Y 2011 Hanbury Brown and Twiss correlations of Anderson localized waves Phys. Rev. A 84 041806
[9] Keil R, Lahini Y, Shechtman Y, Heinrich M, Pugatch R, Dreisow F, Nolte S and Szameit A 2012 Perfect imaging through a disordered waveguide lattice Opt. Lett. 37 809
[10] Anderson P W 1958 Absence of diffusion in certain random lattices Phys. Rev. 109 1492
[11] Lagendijk A, van Tiggelen B and Wiersma D S 2008 Fifty years of Anderson localization Phys. Today 62 24
[12] Soukoulis C M and Economou E N 1981 Off-diagonal disorder in one-dimensional systems Phys. Rev. B 24 5698
[13] Christodoulides D and Joseph R 1988 Discrete self-focusing in nonlinear arrays of coupled waveguides Opt. Lett. 13 794
[14] Eisenberg H, Silberberg Y, Morandotti R, Boyd A and Aitchison J 1998 Discrete spatial optical solitons in waveguide arrays Phys. Rev. Lett. 81 3383
[15] Lederer F, Stegeman G, Christodoulides D, Assanto G, Segev M and Silberberg Y 2008 Discrete solitons in optics Phys. Rep. 463 1
[16] Efremidis N K and Hizanidis K 2008 Disordered lattice solitons Phys. Rev. Lett. 101 143903
[17] Ghosh S, Pal B P and Varshney R K 2012 Role of optical nonlinearity on transverse localization of light in a disordered one-dimensional optical waveguide lattice Opt. Commun. 285 2785
[18] Pertsch T, Peschel U, Nolte S, Tuennermann A, Lederer F, Kobelke J, Schuster K and Bartelt H 2004 Nonlinearity and disorder in two-dimensional fiber arrays Phys. Rev. Lett. 93 053901
[19] Schwartz T, Bartal G, Fishman S and Segev M 2007 Transport and Anderson localization in disordered two-dimensional photonic lattices Nature 446 52
[20] Lahini Y, Avidan A, Pozzi F, Sorel M, Morandotti R, Christodoulides D N and Silberberg Y 2008 Anderson localization and nonlinearity in one-dimensional disordered photonic lattices Phys. Rev. Lett. 100 013906
[21] Jovic D M, Belic M R and Denz C 2011 Transverse localization of light in nonlinear photonic lattices with dimensionality crossover Phys. Rev. A 84 043811
[22] Jovic D M, Belic M R and Denz C 2012 Anderson localization of light at the interface between linear and nonlinear dielectric media with an optically induced photonic lattice Phys. Rev. A 85 031801
[23] Itoh K, Watanabe W, Nolte S and Schaffer C 2006 Ultrafast processes for bulk modification of transparent materials MRS Bull. 31 620
[24] Heinrich M, Keil R, Dreisow F, Tünnermann A, Szameit A and Nolte S 2011 Nonlinear discrete optics in femtosecond laser-written photonic lattices Appl. Phys. B 104 469
[25] Szameit A et al 2008 Observation of diffraction-managed discrete solitons in curved waveguide arrays Phys. Rev. A 78 031801
[26] Szameit A, Dreisow F, Pertsch T, Nolte S and Tuennermann A 2007 Control of directional evanescent coupling in fs laser written waveguides Opt. Express 15 1579
[27] Heinrich M, Szameit A, Dreisow F, Keil R, Minardi S, Pertsch T, Nolte S, Tünnermann A and Lederer F 2009 Observation of three-dimensional discrete–continuous X waves in photonic lattices Phys. Rev. Lett. 103 113903

New Journal of Physics 14 (2012) 073026 (http://www.njp.org/)