SMART Grid Evaluation Using Fuzzy Numbers and TOPSIS

Mohamed EL ALAOUI 1,2
1 ENSAM Meknes, Moulay Ismail University, Morocco
2 Laboratoire Systèmes et Environments Durables (LSED), Faculté des Sciences de l’Ingénieur (FSI), Université Privée de Fès (UPF), Fez, Morocco
mohamedelalaoui208@gmail.com

Abstract. In recent advent of smart grids, the end-users aims to satisfy simultaneously low electricity bills, with a reasonable level of comfort. While cost evaluation appears to be an easy task, capturing human preferences seems to be more challenging. Here we propose the use of fuzzy logic and a modified version of the TOPSIS method, to quantify end-users’ preferences in a smart grid. While classical smart grid focus only on the technological side, it is proven that smart grid effectiveness is hugely linked to end-users’ behaviours. The main objective here, is to involve smart grid users in order to get maximum satisfaction, preserving classical smart grid objectives.

1. Introduction
It is evident that actual power grid are under effective, whether in economical or environmental terms. The transition to smart grids seems to be a necessity more than a simple choice. This transition can be facilitated, given the number of intelligent measurement and control tools in the market. However, the main challenges are caused by the human activities. Hence, one of the main objectives, is to make end-users collaborate, especially in the period of peak demand, to consume electricity more efficiently [1]. Similarly to a classical supply chain, smart grids must take into consideration the end-users’ preferences. Several authors insisted in the role of users in the smart grid [2]–[5]. We can program the dishwasher, the cloth drier and other machines in order to function in off-peak periods. However, power consumption still depend mainly on end-users’ preferences and life style.

The main question is: “how to model users’ preferences?” Fuzzy set theory was first proposed by L. Zadeh [6] in 1965 in order to meet with human reasoning. Then, fuzzy theory was adopted in the decision making framework [7]. Several authors adopted the fuzzy decision making methodology for smart grids [8]–[11]. And employed Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) to rank preferences.

Section 2 gives a brief overview about smart grids .Section 3 presents the needed background of fuzzy logic. Section 4 presents TOPSIS. Section 5 gives an illustrative example. Section 6 concludes this work.

2. Smart grids
It is well known that the main problematic in smart grids is due to the peak load. Some countries, in the hope to influence their citizens’ electricity consumption, imposes a varying price, where the maximum price corresponds to the on-peak. So, to reduce the electricity bills, the users are invited to
shift their consumption from on-peak, to off-peak periods. Unfortunately, this move can result on end user discomfort. Authors in [12] represented the dissatisfaction function depending on some variables such as the difference between the desired temperature and the obtained temperature. However, the dissatisfaction, cannot be reduced, to linearly variables such as temperature or any other measurable variables. Hence, all crisp models will fail to capture human judgments.

3. Fuzzy logic
To deal quantitatively with the ambiguity of human judgement, L. Zadeh introduced fuzzy sets [6]. Let \( X \) be a universal set and \( F \) a fuzzy subset in \( X \), \( F \) is defined as follows [13]:

\[ F = \{ x, \mu_F(x) \mid x \in X \} \]

where \( \mu_F(x) \) is the degree of membership of \( x \) in \( F \) in the unity interval.

\[ \mu_F : X \to [0,1]. \]

A fuzzy set is convex if and only if:

\[ \mu_F(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_F(x_1),\mu_F(x_2)) \]

for all \( x_1, x_2 \in X \) and \( \lambda \in [0,1] \).

A fuzzy set \( F \) is normalized if \( \sup(\mu_F) = 1 \)

A fuzzy number is a convex normalized fuzzy set of the real line \( R^1 \) whose membership function is piecewise continuous.

A fuzzy number \( N \) is called positive, denoted by \( N > 0 \), if its membership function \( \mu_N(x) = 0 \) for all \( x < 0 \)

In order to achieve the compromise between simple representation and complexity [14], [15], the most used notation is based on positive trapezoidal fuzzy numbers, where a positive trapezoidal fuzzy number \( \tilde{A} \) is denoted by 4-tuple \((a_1, a_2, a_3, a_4)\), (fig. 1) as follows:

\[ \mu_{\tilde{A}}(x) = \begin{cases} 
\frac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\
1, & a_2 \leq x \leq a_3 \\
\frac{x - a_3}{a_4 - a_3}, & a_3 \leq x \leq a_4 \\
0, & \text{otherwise} 
\end{cases} \]

![Figure 1. Trapezoidal fuzzy number.](image)

If \( a_2 = a_3 \) then the trapezoidal fuzzy number is called a triangular fuzzy number. If \( a_1 = a_2 = a_3 = a_4 \) then we have a classical crisp number.

For any two positive trapezoidal fuzzy numbers \( \tilde{A}(a_1, a_2, a_3, a_4) \) and \( \tilde{B}(b_1, b_2, b_3, b_4) \) the fuzzy addition, subtraction, multiplication, inverse and division are respectively defined as follows:

\[ \tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4) \]

\[ \tilde{A} \ominus \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1) \]

\[ \tilde{A} \odot \tilde{B} = (a_1 \cdot b_1, a_2 \cdot b_2, a_3 \cdot b_3, a_4 \cdot b_4) \]

\[ 1 \odot \tilde{B} = (1/b_4, 1/b_3, 1/b_2, 1/b_1) \]

\[ \tilde{A} \oslash \tilde{B} = (a_1/b_4, a_2/b_3, a_3/b_2, a_4/b_1) \]

The distance between two trapezoidal fuzzy numbers \( \tilde{A}(a_1, a_2, a_3, a_4) \) and \( \tilde{B}(b_1, b_2, b_3, b_4) \) can be computed as follows:
\[ D(\tilde{A}, \tilde{B}) = \frac{1}{\sqrt{4}} \left[ (a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2 + (a_4 - b_4)^2 \right] \]

The distance between two triangular fuzzy numbers \( \tilde{A}(a_1, a_2, a_3) \) and \( \tilde{B}(b_1, b_2, b_3) \) can be computed as follows:

\[ D(\tilde{A}, \tilde{B}) = \frac{1}{\sqrt{3}} \left[ (a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2 \right] \]

For real numbers, the distance between fuzzy numbers becomes the Euclidean distance. Hence, two fuzzy numbers are identical if and only if \( D(\tilde{A}, \tilde{B}) = 0 \). Let \( \tilde{A}, \tilde{B} \) and \( \tilde{C} \) three fuzzy numbers, then \( \tilde{A} \) is closer to \( \tilde{B} \) than to \( \tilde{C} \) if \( D(\tilde{A}, \tilde{B}) \leq D(\tilde{A}, \tilde{C}) \).

4. TOPSIS

TOPSIS was proposed in [6] and adopted to the fuzzy context in [7]. The main idea, is that the furthest alternative from the worst solution should be ranked first. This can be calculated by the closeness coefficient for each alternative \( CC_i \), as follows:

\[ CC_i = \frac{D_i^-}{D_i^- + D_i^+} \quad (1) \]

where \( D_i^- \) is the distance between the alternative assessment and the worst solution (assessment) and \( D_i^+ \) the distance between the alternative assessment and the ideal solution (assessment).

The problem can be formulated as follows:

A set of \( K \) Decision Makers (End Users) \( E = (E_1, E_2, ..., E_K) \), are to evaluate a set of \( m \) alternatives (smart grids or parts of smart grids) \( A = (A_1, A_2, ..., A_m) \), according to \( n \) criteria \( C = (C_1, C_2, ..., C_n) \). The set of performance of \( A_i \) with accordance to \( C_j \), \( X = \{x_{ij}, i = 1, ..., m; j = 1, ..., n\} \). And criteria impotencies \( W = \{w_{ij}, j = 1, ..., n\} \)

Computing with words introduced by [16] helps meeting more conveniently with human reasoning. Hence, each end-user will make his decision, about criteria importance \( \tilde{w}_j \) (Table 1) and alternatives \( \tilde{x}_{ij} \) (Table 1), by using linguistic variables.

According to the classical approach in TOPSIS, to aggregate \( p \) fuzzy numbers \( (\tilde{F}_1, \tilde{F}_2, ..., \tilde{F}_p) \), such that \( \tilde{F}_q = (F_{q1}, F_{q2}, F_{q3}, F_{q4}) \) where \( q = 1, ..., p \) is \( \tilde{F} = (f_1, f_2, f_3, f_4) \) where \( f_1 = \min(F_{q1}) \), \( f_2 = \frac{1}{p} \sum_{q=1}^{p} F_{q2} \), \( f_3 = \frac{1}{p} \sum_{q=1}^{p} F_{q3} \) and \( f_4 = \max(F_{q4}) \). The main problematic with the classical TOPSIS aggregation is its sensitivity to extremal values, suppose aggregating the three following fuzzy numbers: \( (1,2,3,4), (7,8,9,10) \) and \( (7,8,9,10) \), the result according to the classical TOPSIS method is \( (1,6,7,10) \), which is hugely influenced by the first tuple of the first fuzzy number. Subsequently, we adopt the average value as used for the second and third tuple in the classical TOPSIS method. In the previous example, the result would be \( (5,6,7,8) \), that represent conveniently the three aggregated fuzzy numbers.

The method can be summarized as follows:

Step 1: DMs assess criteria importance by linguistic variables (Table 1)  
Step 2: DMs assess alternatives with respect to each criterion (Table 1)  
Step 3: convert the linguistic variables to trapezoidal/triangular fuzzy numbers (Table 1).  
Step 4: construct the weighted fuzzy decisions: Having the individual assessments for each alternative \( \tilde{x}_{ij} \), the collective assessment for each alternative is: \( \tilde{x}_{ij} = \frac{1}{K} [\tilde{x}_{ij} \oplus \tilde{x}_{ij} \oplus ... \oplus \tilde{x}_{ij}] \). And having the individual assessments for weight \( \tilde{w}_j \), the collective assessment for weights is: \( \tilde{w}_j = \frac{1}{K} [\tilde{w}_j \oplus \tilde{w}_j \oplus ... \oplus \tilde{w}_j] \). Hence the weighted fuzzy decision is: \( \tilde{G}_{ij} = \tilde{x}_{ij} \otimes \tilde{w}_j \)
\[ G = \begin{bmatrix} \tilde{x}_{11} \tilde{\omega}_1 & \ldots & \tilde{x}_{m1} \tilde{\omega}_1 \\ \vdots & \ddots & \vdots \\ \tilde{x}_{1n} \tilde{\omega}_n & \ldots & \tilde{x}_{mn} \tilde{\omega}_n \end{bmatrix} = \begin{bmatrix} \tilde{g}_{11} & \ldots & \tilde{g}_{m1} \\ \vdots & \ddots & \vdots \\ \tilde{g}_{1n} & \ldots & \tilde{g}_{mn} \end{bmatrix} \]

Since all fuzzy variables tuples are in the unity interval, there is no need to the normalizing step required in classical TOPSIS.

Step 5: determine the fuzzy positive ideal solution (FPIS, \( \tilde{A}^+ \)) and the fuzzy negative ideal solution (FNIS, \( \tilde{A}^- \)). In the example (Section 5): \( \tilde{A}^+ = (0.7,0.9,0.9) \) and \( \tilde{A}^- = (0.1,0.1,0.3) \)

Step 6: compute the distance to each candidate

\[ D_i^- = \sum_{j=1}^{n} D(\tilde{g}_{ij}, \tilde{A}_j^-) \quad i = 1, \ldots, m \]

\[ D_i^+ = \sum_{j=1}^{n} D(\tilde{g}_{ij}, \tilde{A}_j^+) \quad i = 1, \ldots, m \]

Step 7: compute the closeness coefficients (Eq. 1)

Step 8: rank alternatives according to the closeness coefficients

**Table 1.** Criteria importance / Alternative evaluation.

| Linguistic variable | Fuzzy number |
|---------------------|--------------|
| Very low            | (0.1,0.1,0.3) |
| Low                 | (0.1,0.3,0.5) |
| Medium              | (0.2,0.5,0.7) |
| High                | (0.6,0.7,0.9) |
| Very high           | (0.7,0.9,0.9) |

5. **Illustrative example and comparison**

Authors in [9], proposed a scenario with two users. The first user, who is rich, do not care about cost and wants a maximum comfort using energy. While the second user, who is a green consumer, is concerned about environmental issues. Both users have all needed information that includes cost energy, carbon tax and greenhouse gas GHG emission in home areas; \( A_1 \): Kitchen, \( A_2 \): bedrooms, \( A_3 \): living rooms and \( A_4 \): laundry. The collected information, should give users an idea about how efficient they are compared to their neighbors. Table 2 resumes criteria of evaluation and their impotencies assessment according to both users.

**Table 2.** Linguistic assessment for criteria.

| Criteria          | Users  | Aggregated fuzzy weights |
|-------------------|--------|--------------------------|
|                   | User 1 | User 2       |                     |
| \( C_1 \): Energy cost | H      | VH           | (0.6,0.8,0.9)      |
| \( C_2 \): Budget   | M      | H            | (0.4,0.6,0.8)      |
| \( C_3 \): Urgency  | H      | M            | (0.4,0.6,0.8)      |
| \( C_4 \): Comfort level | VH    | M           | (0.5,0.7,0.8)      |
| \( C_5 \): GHG emission | VL    | VH          | (0.4,0.5,0.6)      |
| \( C_6 \): Energy efficiency score | L     | VH           | (0.4,0.6,0.7)      |
| \( C_7 \): Carbon tax | M     | H            | (0.4,0.6,0.8)      |
| \( C_8 \): Occupancy level | H    | H            | (0.5,0.7,0.9)      |
The main objective is to know the energy distribution according to users’ preferences in smart grids. Fuzzy logic was used to model these preferences, and a modified version of TOPSIS to rank them. A successful comparison proving the consistency of the proposed approach compared to existing approaches in the literature was made.

Table 3. Linguistic assessment for alternatives.

| Criteria | A1 Aggregated decision (10^1) | A2 Aggregated decision (10^1) | A3 Aggregated decision (10^1) | A4 Aggregated decision (10^1) |
|----------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| Home areas | Use r 1 Use r 2 | Use r 1 Use r 2 | Use r 1 Use r 2 | Use r 1 Use r 2 |
| C1 | H | M | 4,6,8 | M | H | 4,6,8 | M | VH | 5,7,8 | L | H | 3,5,7 |
| C2 | L | VH | 4,6,7 | M | L | 2,4,6 | H | L | 3,5,7 | H | L | 3,5,7 |
| C3 | M | H | 4,6,8 | M | L | 2,4,6 | M | L | 2,4,6 | VH | M | 5,7,8 |
| C4 | VH | M | 5,7,8 | VH | M | 5,7,8 | VH | H | 6,8,9 | L | VL | 1,2,4 |
| C5 | L | H | 3,5,7 | M | H | 4,6,8 | L | VH | 4,6,7 | VL | VH | 4,5,6 |
| C6 | L | M | 2,4,6 | H | H | 5,7,9 | L | VH | 4,6,7 | VL | H | 3,4,6 |
| C7 | M | M | 3,5,7 | M | VH | 5,7,8 | L | VL | 1,2,4 | M | H | 4,6,8 |
| C8 | VH | M | 5,7,8 | L | VL | 1,2,4 | VL | M | 2,3,5 | L | M | 2,4,6 |

Hence, Table 4 resumes the ranking of alternative after computing all needed parameters.

Table 4. Ranking according to closeness coefficients.

| Alternatives | CCl1 | A1 | A2 | A3 | A4 |
|--------------|------|----|----|----|----|
| Proposed     | 0,331 | 0,297 | 0,277 | 0,201 |
| Ranking      | 1     | 2   | 3   | 4   |
| [9] and [11] | 0,3587 | 0,3602 | 0,3539 | 0,3742 |
| Ranking      | 3     | 2   | 4   | 1   |

The ranking proposed and the ranking in [9], [11] are completely different. While the proposed ranked the first alternative A1 as first, the ranking in [9], [11] ranked the 4th alternative A4 as first. Comparing these two alternatives according to their assessments (Table 3) and the criteria impotencies (Table 4), we observe that A1 is better than A4 according to criteria C1, C2, C4 and C8. A4 is better than A1 according to C3, C6 and C7. While there is no preference for neither alternatives according to criterion C5. Moreover, criteria C3, C6 and C7, that favor A4, are less or equivalently important compared to other criteria according to their assessments individually.

Discordances between results in our method and other authors [9], [11], are mainly due to the aggregation method proposed in [9], [11], especially the normalizing step, which is unnecessary in our case, since all variables are chosen in the unity interval.

6. Conclusion

The main objective is to know the energy distribution according to users’ preferences in smart grids. Fuzzy logic was used to model these preferences, and a modified version of TOPSIS to rank them. A successful comparison proving the consistency of the proposed approach compared to existing approaches in the literature was made.

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