Neutrino masses and mixing in $\mu\nu$SSM

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Abstract. $\mu\nu$SSM is an $R$-parity violating non-minimal supersymmetric model which uses right chiral neutrino superfields to solve the $\mu$-problem. The $R$-parity violation together with a TeV scale seesaw mechanism using right handed neutrinos are instrumental for the light neutrino mass generation in $\mu\nu$SSM. We show that it is possible to accommodate three flavour global neutrino data in $\mu\nu$SSM with three massive neutrinos at the tree level. Ingression of the one-loop corrections to neutrino masses and mixing shows certain variations over the tree level analysis depending on the specific hierarchy of neutrino masses involved. In $\mu\nu$SSM some of the $R$-parity violating decay branching ratios of the lightest neutralino show nice correlation with certain neutrino mixing angle. These correlations along with the presence of displaced vertices in the decay of the lightest neutralino can be further investigated as a test of $\mu\nu$SSM in collider experiments.

1. Introduction

Recent findings of neutrino oscillation experiments has put forward strong evidences for the massive neutrinos. A satisfactory explanation for the non-zero neutrino masses is beyond the Standard Model framework. Weak scale supersymmetry (SUSY) is a well motivated candidate for beyond the Standard Model (SM) physics and is of immense interest with the initiation of the Large Hadron Collider (LHC) era. However, SUSY has its own theoretical problems out of which a well-known name is the $\mu$-problem [1]. Solutions to the $\mu$-problem are addressed in a non-minimal version of SUSY model using a SM gauge singlet superfield ($\tilde{S}$). On the other hand, both the minimal supersymmetric standard model (MSSM) and next-to-minimal supersymmetric standard model (NMSSM) predicts massless neutrinos similar to the SM. Massive neutrinos in a SUSY framework can be achieved in two ways, either using the $R$-parity violation ($R_P$) or through canonical seesaw mechanism (initially proposed to prevent fast proton decay through sparticle mediated processes). $R_P$ is defined as $R_P = (-1)^{L+3B+2s}$ with $L(B)$ as lepton(baryon) number and $s$ is the spin. MSSM can accommodate neutrino masses through the bilinear or trilinear $R_P$ without solving the $\mu$-problem. However, these new bilinear $R_P$ violating terms will again cause naturalness problem similar to the $\mu$-problem [2].

The “$\mu$ from $\nu$” supersymmetric standard model ($\mu\nu$SSM) [3,4] invokes right-handed neutrino superfields to solve the $\mu$-problem and at the same time uses the same set of right-handed neutrino superfields to generate light neutrino masses. In a nutshell $\mu\nu$SSM is a minimalistic extension of MSSM (includes only right-handed neutrino superfields apart from the MSSM
superfields) for accommodating neutrino masses and simultaneously solving the $\mu$-problem.

The $\mu\nu$SSM can fit the three flavour global neutrino data even with flavour diagonal structure of the neutrino Yukawa couplings for various hierarchies of light neutrino masses [5] at the tree level. All three light neutrinos can acquire non-zero masses at the tree level. $R_P$ in $\mu\nu$SSM will lead to an unstable Lightest Supersymmetric Particle (LSP). Some ratios of the decay branching ratios of the LSP show nice correlations with certain neutrino mixing angles [5] depending on the LSP nature and the hierarchy in the light neutrino masses. These correlations can be investigated as the experimental signatures of $\mu\nu$SSM with possible discriminating features from other SUSY models. Another important feature of $\mu\nu$SSM is the possibility of having displaced vertices in the decay of the lightest neutralino, which can vary from a few $\text{mm}$ to $\sim 1\text{ meter}$ depending on the nature of the lightest neutralino [5]. Tree level results of neutrino masses and mixing show variations with the addition of the one-loop radiative corrections. The amount of variations were observed to be dependent on the choice of the hierarchy in light neutrino masses [6].

We note in passing that various other aspects of $\mu\nu$SSM like LHC phenomenology, spontaneous CP-violation, gravitino dark matter, baryogenesis are discussed in refs. [7, 8, 9, 10] respectively. For a review of $\mu\nu$SSM and seesaw mechanism in $\mu\nu$SSM see refs. [11] and [12].

2. The model
The model superpotential is

$$\begin{align*}
W &= \epsilon_{ab}(Y_d^{ij}\hat{H}_2^a\hat{Q}_i^e\tilde{u}_j^c + Y_u^{ij}\hat{H}_1^a\hat{Q}_i^e\tilde{d}_j^c + Y_e^{ij}\hat{H}_1^a\hat{L}_i^b\tilde{e}_j^c + Y_{\nu}^{ij}\hat{H}_2^a\hat{L}_i^b\tilde{\nu}_j^c) \\
&\quad - \epsilon_{ab}\lambda_i^c\tilde{\nu}_i^c\hat{H}_1^a\hat{H}_2^b + \frac{1}{3}\kappa^{ijk}\tilde{\nu}_i^c\tilde{\nu}_j^c\tilde{\nu}_k^c,
\end{align*}$$

(1)

where $\tilde{\nu}_i^c$ are the right chiral neutrino superfields ingressed in $\mu\nu$SSM apart from the MSSM $SU(2)$ doublet ($\hat{H}_1, \hat{H}_2, \hat{Q}_i, \hat{L}_i$) and singlet ($\tilde{u}_i^c, \tilde{d}_i^c, \tilde{e}_i^c$) superfields. In eq.(1), $i, j, k$ represent generational indices. Appearance of any bilinear term is prohibited by imposing a $Z_3$ symmetry in the $\mu\nu$SSM superpotential. This superpotential explicitly breaks $R_P$ through lepton number violation by odd units ($5^{\text{th}}$ and $6^{\text{th}}$ terms of eq.(1)). There are corresponding entries in the soft SUSY breaking sector too [3].

After the electroweak symmetry breaking (EWSB) when the scalar component of Higgses and left and right sneutrino fields développe the respective Vacuum Expectation Values (VEVs) $v_1, v_2, v_1', v_2'$, we have an effective $\mu$-term and bilinear $R_P$ terms ($\epsilon_i$) as $\mu = \sum \lambda_i\epsilon_i^c$ and $\epsilon^c = \sum Y_{\nu}^{ij}v_{\nu}^c$, respectively. The $6^{\text{th}}$ term of eq.(1), with the coefficient $\kappa^{ijk}v_{\nu}^c$, is included in order to avoid an unacceptable axion associated to the breaking of a global $U(1)$ symmetry [13]. After EWSB this term generates effective Majorana masses ($m_{\nu_\alpha}$) for the right-handed neutrinos at the electroweak (EW) scale and are given by $2\kappa^{ijk}v_{\nu}^c$. These EW scale right neutrinos are further responsible for light neutrino mass generation through seesaw, thereby lowering the seesaw-scale within the reach of a $\text{TeV}$ scale collider.

Here, as a digression, let us mention that the spontaneous breakdown of the $Z_3$ symmetry through right-sneutrino VEV can in general lead to the formation of domain walls [14, 15, 16]. The associated problems can, however, be ameliorated through well-known methods [17, 18].

3. Tree level neutrino masses and mixing
The lepton number violating interactions in the superpotential and in the soft SUSY breaking part of the scalar potential allows mixing between states having zero lepton number with states having non-zero lepton number. As a consequence mass matrices in the scalar and fermion
sectors get enhanced over their MSSM structures \([4, 5]\). In the neutral fermion sector now the four MSSM neutralinos mix with three generations of left and right-handed neutrinos and hence the neutralino mass matrix enlarges to \(10 \times 10\). In the basis \(\Psi^0_T = (\tilde{B}_0^0, \tilde{W}_3^0, \tilde{H}_1^0, \tilde{H}_2^0, \nu_e, \nu_i)\), the neutralino mass matrix is given by

\[
M_n = \begin{pmatrix} M_{7 \times 7} & m_{3 \times 7}^T \\ m_{3 \times 7} & 0_{3 \times 3} \end{pmatrix}.
\] (2)

The matrix \(M_{7 \times 7}\) contains a \(4 \times 4\) block of MSSM neutralinos as well as a \(3 \times 3\) block of right-neutrinos and mixing terms between them \([4, 5]\). The null \(3 \times 3\) block in \(M_n\) signifies the absence of Majorana mass terms for the left handed neutrinos. The elements of \(m_{3 \times 7}\) contain either left handed sneutrino VEVs \((v'_i)\) or Higgs VEVs multiplied by neutrino Yukawa couplings \((Y^{ij}_\nu)\), and hence, are of much smaller magnitudes compared to the entries of \(M_{7 \times 7}\). This feature ensures a seesaw-like structure of \(M_n\) \([4, 5]\) and the light neutrino mass matrix using seesaw is given by

\[
M_{\text{seesaw}} = -m_{3 \times 7} M_{7 \times 7}^{-1} m_{3 \times 7}^T.
\] (3)

This seesaw mass matrix can be diagonalized (with \(m_{\nu_i}\) as mass eigenvalues) using a unitary matrix \(U_{PMNS}\) containing neutrino mixing angles (provided that the charged lepton mass matrix is already in the diagonal form) as follows

\[
U_{PMNS}^T M_{\text{seesaw}} U_{PMNS} = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}).
\] (4)

With a few simplifying assumptions \([5]\) one can get an approximate analytical form for the entries of seesaw mass matrix \(M_{\text{seesaw}}\) as

\[
M_{ij}^{\text{seesaw}} \approx \frac{2Av^e_{ij}}{3\Delta} b_i b_j + \frac{1}{6kv'^c_{ii}} a_i a_j (1 - 3\delta_{ij}),
\] (5)

where

\[
\begin{align*}
\Delta &= \lambda^2 (v'^1_1 + v'^2_2)^2 + 4\lambda v'^e v_1 v_2 - 4M\lambda A_i, \quad \mu = 3\lambda v^e, \\
A &= (\kappa v'^2 + \lambda v_1 v_2), \quad \frac{1}{M} = \frac{g_1^2}{M_1} + \frac{g_2^2}{M_2}, \\
a_i &= Y^{ei}_{\nu_i} v_2, \quad b_i = (Y^{iji}_{\nu_e} v_1 + 3\lambda v'_i),
\end{align*}
\] (6)

with \(i, j, k = e, \mu, \tau\). We have used the fact that \(v'_i \ll v_1, v_2\).

The seesaw structure in \(\mu\nu\)SSM can be well understood if we investigate eq.\((5)\) in the following limits\([5]\), (i) \(v^c \to \infty\) and \(v \to 0\) \((v^2 = (v'^1_1 + v'^2_2))\) and (ii) \(M \to \infty\).

In the limit (i) we end up with

\[
M_{ij}^{\text{seesaw}} \approx \frac{v'_i v'_j}{2M},
\] (7)

and in case (ii) we have

\[
M_{ij}^{\text{seesaw}} \approx \frac{v'^2_1}{6kv'^c_{ii}} Y^{iji}_{\nu} Y^{iji}_{\nu} (1 - 3\delta_{ij}).
\] (8)

The form of eq.\((7)\) is associated with the gaugino seesaw effect where the gauge coupling \(\times\) left sneutrino VEV \((v'_i)\) acts as the Dirac mass and the effective gaugino mass \((M)\) plays the role of the Majorana mass\([6]\). In the gaugino seesaw scenario because of the presence of left sneutrino VEV, an effective \(\Delta L = 2\) Majorana mass term for light neutrinos is generated from a pair of \(\Delta L = 1\) vertex involving left-handed neutrino and neutral wino/bino \((\tilde{B}/\tilde{W}^0_3)\). This is
analogous to Type-III seesaw because of the association of the hypercharge zero triplet fermion \([6]\). Another feature of gaugino seesaw is that it generates only one massive neutrino at the tree level. On the contrary eq.(8) is associated with the ordinary seesaw effect involving right-handed neutrinos. For this case one can get two massive neutrinos at the tree level. This is the well-known example of Type-I seesaw using singlet fermion (right-handed neutrino for \(\mu\nuSSM\)), where the \(\Delta L = 2\) effect is coming through right chiral neutrino Majorana masses \([5]\).

With a suitable choice of model parameters one can treat the second term of eq.(5) as a perturbation over the first term. We would like to emphasize that for most of the parameter choice \(a_i \sim b_i\), hence the relative weight difference is coming from the co-efficient in front \((\frac{2A\kappa}{10\kappa} and \frac{1}{3\kappa})\) \([4]\). Appearance of \(b_ib_j\) in the first term of eq.(5) gives only one non-zero neutrino mass \(\propto \sum b_i^2\). The other two masses emerge due to the effect of ordinary seesaw \([5]\). Approximate analytical expressions for the masses and mixing for normal hierarchical schemes of light neutrino masses were obtained in ref. \([5]\) which show good agreement with the complete numerical analysis without any approximation \([5]\). As an illustrative example, in case of normal hierarchy the atmospheric mixing angle is given by

\[
\sin^2 \theta_{23} = \frac{b_\mu^2}{b_\mu^2 + b_\tau^2},
\]

with \(b_i\)'s are given by eq.(6). It is clear from the analytical formula in eq.(9), that the maximal mixing in the atmospheric sector indicates \(b_\mu = b_\tau\).

Note that using \(b_i = (a_i \cot \beta + 3\lambda c_i)\) with \(c_i = \nu_i'\) and \(\tan \beta = \frac{\nu_2}{\nu_1}\), one ends up with more elucidate form of eq.(5) as \([6]\)

\[
(M_\nu^{\text{seesaw}})_{ij} = f_1 a_i a_j + f_2 c_i c_j + f_3 (a_i c_j + a_j c_i),
\]

where (using eq.(6))

\[
f_1 = \frac{1}{6 \kappa c} (1 - 3 \delta_{ij}) + \frac{2 A \nu \cot^2 \beta}{3 \Delta}, \quad f_2 = \frac{2 A \lambda \mu}{\Delta}, \quad f_3 = \frac{2 A \mu \cot \beta}{3 \Delta}.
\]

It is very clear from eq.(10), that the 1\(^{st}\) and the 2\(^{nd}\) term can contribute to only one mass eigenvalue and are \(\propto \sum a_i^2\) and \(\sum c_i^2\) with suitable co-efficient in front. It is the 3\(^{rd}\) term or the mixing term which is responsible for giving masses to all three light neutrinos \([6]\).

4. One-loop corrected neutrino masses and mixing

It is clear from the above discussion that in \(\mu\nuSSM\) all three light neutrinos get seesaw masses at the tree level even with the flavour diagonal choice of neutrino Yukawa couplings \((Y_\nu)\) \([5]\) consistent with the three flavour global neutrino data \([19]\). With the inclusion of one-loop radiative corrections tree level results of neutrino masses and mixing receive corrections over their tree level values depending on the concerned mass hierarchy \([6]\).

There are seven possible sources \([6]\) of one-loop corrections to the light neutrino masses in \(\mu\nuSSM\) and they are listed below

(i) neutralino - neutral scalar in the loop,
(ii) neutralino - neutral pseudoscalar in the loop,
(iii) neutralino - Z-boson in the loop,
(iv) chargino - charged scalar in the loop,
(v) chargino - W\(^{\pm}\)-boson in the loop,
(vi) up-type quark - up-type suqark in the loop,
(vii) down-type quark - down-type suqark in the loop.

The relevant Feynman rules are given in ref. [6]. In the absence of any fine cancellation, the dominant contribution to loop corrected neutrino masses arises from (i) and (ii) above, when right sneutrinos are in the loop. This contribution is proportional to squared mass difference between right sneutrino scalar and pseudoscalar mass eigenstates [7].

Using the dimensional reduction (DR) scheme in the 't-Hooft-Feynman gauge (\(\xi = 1\)) one can write down the expression for one-loop corrected neutrino mass matrix in the basis, where the tree level mass matrix is diagonalized. This one-loop corrected neutrino mass matrix can be further diagonalized to get one-loop corrected mass eigenvalues. In order to obtain the neutrino mixing matrix we rotate back to the flavour basis using neutralino mixing matrix [6]. In this way we obtain the one loop corrected neutrino mass matrix in the flavour basis and further apply the known methods of seesaw mechanism to get the one-loop corrected mass eigenvalues ([\(m^'_{\nu_i}\)]. The corresponding unitary matrix \(U^'_{PMNS}\), which diagonalizes this matrix contain the one-loop corrected mixing angles [6].

The one-loop corrected neutrino mass matrix as shown in ref. [6] looks like

\[
(M_{\text{seesaw loop corrected}})^{ij} = A_1 a_i a_j + A_2 c_i c_j + A_3 (a_i c_j + a_j c_i),
\]  

(12)

where \(A_i\)'s are functions of our model parameters and the Passarino-Veltman functions \((B_0, B_1)\) [23, 24]. The form of the loop corrected mass matrix thus obtained is identical to the tree level one (see, eq. (10)) with different coefficients \(A_i\)'s in front arising because of the one-loop corrections.

The effect of one-loop corrections to neutrino masses and mixing varies with the neutrino mass hierarchy. We found it relatively easier to accommodate the normal hierarchical spectrum of light neutrino masses compared to inverted or quasi-degenerate spectrum when the effect of one-loop correction was taken into account [6]. The allowed region of tree level parameter space were observed to shrink severely in the case of inverted hierarchy of light neutrino mass and thereby only a very little window of allowed parameter space remains open with the inclusion of loop-effect [6]. The effect of one-loop correction produces practically vanishing allowed region in parameter space for the quasi-degenerate scheme. However, we must emphasize here that these conclusion are parameter dependent and the huge parameter space of \(\mu\nu\)SSM always left us with enough room to observe some different phenomena in a entirely different corner of parameter space.

5. Decays of the LSP

\(R_P\) in \(\mu\nu\)SSM leaves no room for a stable LSP. The lightest SUSY particle in this model will eventually decay into the SM particles making this model testable in collider experiment. It is an well-known feature of the SUSY models with bilinear \(R_P\) to show nice correlations among neutrino oscillation parameters and LSP decay patterns [25, 26, 27]. In the \(\mu\nu\)SSM one observes similar correlations among certain neutrino mixing angles with ratio of decay branching ratios [5, 7]. These correlations vary with the concerned neutrino mass hierarchy and with the LSP nature [5]. In \(\mu\nu\)SSM, apart from a gaugino or higgsino LSP one can also have a right-handed neutrino or singlino like LSP. This third possibility is a special feature of \(\mu\nu\)SSM and is of great interest because of its direct relation with the seesaw scale. A proper detection may be followed by a faithful mass reconstruction tool to probe the right-handed neutrino mass scale or the seesaw scale which is hitherto unseen for its gauge-singlet nature [5]. Decays of fermionic LSP can produce multiple leptons and jets in the final state decay products, for which a comparative
analysis can be performed for obtaining signatures of $\mu\nu$SSM at colliders [7]. In $\mu\nu$SSM right-
sneutrino is also an eligible candidate to be the LSP [5].

One more crucial experimental signature of $\mu\nu$SSM can come from the study of displaced
vertices. Depending on the LSP composition the decay length can vary several orders of
magnitude, which is $\sim$ a few meters for a singlino like LSP. As a corollary a dedicated
investigation of the displaced vertices can provide characteristic signature of $\mu\nu$SSM.

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