Backscatter Communications for the Internet of Things: A Stochastic Geometry Approach

Mudasar Bacha, Bruno Clerckx and Kaibin Huang

Abstract

Powering the massive number of small computing devices in the Internet of Things (IoT) is a major challenge because replacing their batteries or powering them with wires is very expensive and impractical. A viable option to enable a perpetual network operation is through the use of far-field Wireless Power Transfer (WPT). We study a large network architecture that uses a combination of WPT and backscatter communication. The network consists of power beacons (PBs) and passive backscatter nodes (BNs), and their locations are modeled as points of independent Poisson point processes (PPPs). The PBs transmit a sinusoidal continuous wave (CW) and the BNs reflect back a portion of this signal while harvesting the remaining part. A BN harvests energy from multiple nearby PBs and modulates its information bits on the composite CW through backscatter modulation. The analysis poses real challenges due to the double fading channel, and its dependence on the PPPs of both BNs and PBs. With the help of stochastic geometry, we derive the coverage probability and the capacity of the network in tractable and easily computable expressions. These expressions depend on the density of both PB and BN, both forward and backward path loss exponents, transmit power of the PB, backscattering efficiency, and number of PBs in a harvesting region. We observe that the coverage probability decreases with an increase in the density of the BNs, while the capacity of the network improves. We compare the performance of this network with a regular powered network in which the BNs have a reliable power source and show that for certain parameters the coverage of the former network approaches that of the regular powered network.

Index Terms

Stochastic geometry, backscatter communications, IoT, wireless power transfer, energy harvesting, Poisson point processes.

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I. INTRODUCTION

The emerging Internet of Things (IoT) is expected to connect billions of small computing devices to the Internet and to enable their device-to-device (D2D) communication [1]. These tiny devices have processing, sensing and wireless communications capabilities. They are supposed to be deployed everywhere and can be accessed from anywhere at any time. However, powering these small devices is one of the main challenge (others are interoperability, management, security and privacy) [2]- [3]. It is very expensive and impractical to replace the batteries of such a massive number of devices or power them with wires. Therefore, harvesting energy from external sources for perpetual operation is a viable option [3].

The advancement in Wireless Power Transfer (WPT) has made it possible to power IoT devices [4]. We consider a random network consisting of passive backscatter nodes, which are powered through WPT by power beacons (PBs) [5]. In contrast to conventional radio architecture relying on power hungry RF chains, the backscatter nodes do not have any active RF components. As a result the BN has miniature hardware and very low power consumption. The PBs are deployed for WPT and they transmit sinusoidal continuous wave (CW)\(^1\). The mobile nodes or backscatter nodes (BN), which are in the range of PBs will backscatter a portion of this CW to a nearby receiver by mismatching its antenna impedance, while harvesting the remaining power to operate its integrated circuit [8]. A BN modulates the data onto the backscattered signal by controlling the antenna impedance. A schematic diagram of backscatter communication system is shown in Fig. 1. For more details on the backscatter operation the readers are referred to [8] and [9], and the references therein. The aim of this work is to study the performance of this unique network, where the BNs are powered through wireless power transfer from the PBs. The analysis is performed by using tool from the stochastic geometry.

A. Related Work

Backscatter communication is traditionally used in the Radio Frequency IDentification (RFID) systems, where a RFID-reader reads data from nearby tag via backscatter modulation [8]- [10]. Recently, there have been some studies, which use backscatter communication by harvesting energy from ambient sources such as TV and Wi-Fi signals [11] [12] [13]. However, this

\(^1\)Significant enhancements can nevertheless be obtained by proper multisine waveform designs [6] [7].
Fig. 1: Backscattering communication system

| Notation | Description |
|----------|-------------|
| $\Phi_p, \Phi_b, \lambda_p, \lambda_b$ | PPP of the PBs, PPP of the BNs, density of the PBs, density of the BNs |
| $P_{\text{PC}, \beta, N_p}$ | transmit power of each PB, backscattering efficiency, average number of the PBs in the harvesting region |
| $\alpha^f, \alpha^b$ | path loss exponent of the forward channel, path loss exponent of the backward channel |
| $P_{\text{BN}}, \bar{P}_{\text{BN}}$ | received power at BN, average received power at BN |
| $d_{i,j}, x_{j,k}$ | distance between $i$th receiver and $j$th BN, distance between $j$th BN and $k$th PB |
| $\text{Ei}(z)$ | $\text{Ei}(z)$ is the exponential integral function $\text{Ei}(z) = \int_0^\infty \frac{e^{-t}}{t} \, dt$ |
| $\mathbb{P}_x, \mathbb{P}[\cdot], E_x$ | coverage probability, probability of an event, expectation with respect to $x$ |
| $\Theta_m, N_0, \gamma'_{R_0}, \gamma_{R_0}$ | SINR threshold, variance of the noise, SINR at typical receiver $R_0$, modified SINR at $R_0$ |
| $h^f_{i,j}$ | complex channel gain of the forward channel between the $i$th BN and $j$th PB while $h^f_{i,j} = |h^f_{i,j}|^2$ |
| $h^b_{j,k}$ | complex channel gain of the backward channel between the $j$th receiver and $k$th BN while $h^b_{j,k} = |h^b_{j,k}|^2$ |

ambient energy harvesting technique can not be used in the large scale IoT due to scalability issue [9], [14]. The authors in [15] consider a single cell network in which there is one reader in the center of the cell and the sensors node are distributed uniformly within this cell. They studied the decoding probability under different collisions resolution schemes such as using directional antennas, successive interference cancellation, and ultra-narrow band transmission. They observed that a combination of these techniques give significant gains. A full-duplex backscatter communication setup is considered in [16] where a multiple access scheme
based on time-hopping spread-spectrum is proposed to reduce the interference. A comparison between multistatic and monostatic radio architecture has been done in [17], where it has been demonstrated that multistatic architecture outperforms the monostatic architecture in term of diversity order, bit error rate, energy outage and coverage.

Most of the above works considered a single-reader single-tag or single-reader multiple-tags except [14], which consider a wireless powered backscatter communication network in which dense PBs are deployed to do WPT to the BNs. They model the network as Poisson cluster process (PCP), where PBs constitute the parent point process and the BNs form daughter point processes. Using tools from stochastic geometry they studied the coverage and network capacity of the network. However, they considered that a BN only harvests energy from a single PB and they also did not consider fading in the forward channel. Moreover, they derived lower bounds on the coverage and capacity of the network.

In our work, we study a more generic and close to practical network setup. In our setup both the PBs and the BNs are distributed according to PPPs and we consider fading in both the forward and backward channel. We also consider that the BNs can harvest and backscatter energy from multiple nearby PBs. Then we utilize tools from stochastic geometry to study the coverage and capacity of such network. Stochastic geometry has emerged as a powerful tool for the analysis of wireless networks due to its amazing tractability and accuracy [18]. It has been used in the analysis of the uplink cellular networks [19], the downlink cellular networks [20], ad-hoc networks [21], and millimeter wave networks [22]. Recently, it has been used to study the wireless powered network [14], [15], and [23].

B. Contributions and Outcomes

We develop a comprehensive model for the wireless powered backscatter IoT network in which the locations of PBs and BNs are distributed as independent PPPs. The BNs modulate the information on the unmodulated CW signal from the nearest PBs and reflect a portion of the signal to the receiver. The remaining energy is harvested for the operation of its integrated circuit. We study the coverage and capacity of this network for a typical receiver where the capacity is defined as the number of successful transmissions between the BNs and their receivers in per unit area of the network. In classical downlink cellular network, the coverage (and capacity) depends on a single channel (between a base station and a mobile terminal) and the density of the base stations (BSs), while in the uplink it depends on the channel and the density of
the mobile terminals (MTs). Furthermore, the BS has a constant transmit power in downlink while the mobile terminals use power control in the uplink. Whereas, in our setup, the coverage probability (and capacity) depends on the double fading channel, the PPP of BNs and the joint probability distribution function (PDF) of the distances of the nearest PBs. In addition, the reflected power of a BN is a random quantity and depends on the forward channel and the PPP of the PBs. Thus all these unique features make the analysis very challenging.

To tackle those challenges, we use dimensionality reduction technique in which we apply the expectation with respect to the forward channel to both the signal and interference power before the analysis is carried out. This makes the analysis tractable and we find insightful and accurate enough expressions for both the coverage and capacity of the network. Our analysis captures the effect of instantaneous energy harvesting from multiple PBs, backscatter modulations of the composite signal and double fading channel in a single tractable expression. This significantly contrasts with [14] that uses a constant transmit power for the BNs and derive only lower bound on the coverage probability and capacity of the network.

To get further insight, our coverage probability expression can be simplified into a single integral form when the BN harvests energy from the a single nearest PBs and the path-loss exponent of both the forward and backward channel is the same. It can be even simplified to a closed form solution when we consider that the BNs reflect the signal with the mean harvested power. We further consider a network scenario in which the BN harvests from all the PBs in the network and backscatter their signal. This case is even more challenging because the coverage probability depends on the PPPs of both PBs and BNs, and the double fading channel, therefore, we only find the approximation for this case. The accuracy of the analysis is verified with extensive Monte Carlo simulations.

Leveraging the above analysis, we are in a position to ask and answer some important and new questions/trade-offs that emerge in the wireless powered backscattered IoT network. Firstly, increasing the density of the PBs improves the harvested and reflected power, but at the same time it increases the amount of interference. So, how does the coverage of the network vary with the density of the PBs? Secondly, what is the impact of the density of the BNs on the coverage and the capacity of the network? Finally, can the wireless powered backscattered network give the same coverage as of the classical cellular network where BNs are powered with reliable power source rather than through backscattering? The answers to these questions are outlined below:
• We observe that increasing the density of the PBs increases the coverage probability of the BNs. However, when the SINR threshold is high then further increasing the density can decrease the coverage probability. This decrease in the coverage probability at higher SINR threshold is due to the increase in the interference power. The high SINR is achieved only by a very small fraction of the BNs, which are closer to the PBs and an increase in the density of the PBs increases both the signal and interfering power of the BNs. However, the increase in the interference power is more prominent than the increase in the signal power.

• The coverage probability can be considered as the individual link reliability while the capacity is the number of successful transmissions. Increasing the density of the BNs decreases the coverage probability, whereas it increases the capacity of the network. For higher SINR threshold it decreases the capacity of the network. Therefore, the network provider should take into account the tradeoff between the individual link-reliability and the number of successful transmissions.

• We compare the wireless powered backscatter IoT network with the regular powered network in which the BNs are supplied with reliable power source having the same transmit power as of the PBs. We observe that the coverage of the wireless powered backscatter network approaches to the coverage of the regular powered network for a very high density of the PBs. We further observe that up to a certain extent increasing the density of PBs gives rapid increase in the coverage probability. However, after that any further increase in the density of the PBs shows a very small improvement in the coverage probability, which suggests that it may not be cost-effective to deploy such a density of PBs beyond a certain level.

The rest of the paper is organized as follows. In Section II, we present our system model and discuss our performance metrics. In Section III, we conduct the analysis and find the coverage and capacity of the network, while in Section IV we present the numerical and simulations results. Finally, we conclude and provide future research directions in Section V.

The key notations used in this paper are given in Table 1.

II. System Model

A. Network Model

We consider an ad-hoc network consisting of PBs and BNs. The positions of both the PBs and BNs are modeled as independent homogeneous Poisson point processes (PPPs). Let $\Phi_p = \{X\}$ and $\Phi_b = \{Y\}$ be the PPPs with densities $\lambda_p$ and $\lambda_b$, and $X, Y \in \mathbb{R}^2$ denote the coordinates
of the PBs and BNs, respectively. We assume that PBs and BNs have single antenna. The PBs transmit a CW isotropically and all the PBs use the same frequency. A BN harvests some energy from this carrier and modulates its data on the carrier through backscatter modulation [8]. A BN receives a CW signal from multiple PBs and backscatters the signal. This reflected signal from the BN is received by a receiver, which is at a fixed distance from the BN. The network topology is shown in Fig. 2. The analysis is performed at a typical receiver $R_0$ located at the origin and the BN serving this receiver is called typical BN [24]. In order to make the analysis tractable, we do not consider the circuit-power constraint and assume that the BNs modulate and backscatter the signal all the time.

We assume that around each BN there is a region $z(Y, r_h) \subset \mathbb{R}^2$, which represents a disk of radius $r_h$ centered at $Y \in \Phi_b$. All the PBs inside this region contribute to the received power $P_{BN}$ at the BN. We further assume that the power received from the PBs which are outside the harvesting region is negligible. The number of PBs $\bar{N}_p$ inside $z(Y, r_h)$ is a Poisson random variable with mean $N_p = \pi r_h^2 \lambda_p$. We adjust $r_h$ such that the $N_p$ is an integer number. The $P_{BN}$ at a BN can be written as

$$P_{BN} = \left| \sum_{i=1}^{N_p} \sqrt{P_C h_i^f} x^{-\alpha_i/2}_i \right|^2,$$

where $P_C$ is the transmit power of a PB, $h_i^f$ represents the forward channel gain between the
PB and the BN and $h_f^i \sim \text{CN} (0, 1)$, $\alpha_f$ is the path loss exponent of the forward channel i.e., between the PBs and the BN, and $x_i$ is the distance between a BN and the $i$th nearest PB. A BN reflects $\beta P_{BN}$ amount of power while the remaining power $(1 - \beta) P_{BN}$ is harvested (as shown in Fig. 1), where $\beta$ is the backscattering efficiency and $\beta \in [0, 1]$ \cite{3}. The harvested power is used by the BN to run its integrated circuit (IC). It is important to mention that we use the average number of PBs $N_p$ in (1) and the rest of the paper instead of the random $\bar{N}_p$ for the sake of simplicity.

The joint PDF of the $x_1, \cdots, x_{N_p}$ \cite{25} will be used throughout the paper, therefore, we present it here

$$f_x(x_1, \cdots, x_{N_p}) = (2\pi \lambda_p)^{N_p} x_1, \ldots, x_{N_p} e^{-\pi \lambda_p x_p^2}, (2)$$

where $x_1 < x_2 \cdots < x_{N_p}$. The SINR $\gamma_{R_0}$ at a receiver $R_0$, which is at the origin, when a BN $B N_0$ transmits under the above system model and in the presence of interference from other active BNs can be written as

$$\gamma'_{R_0} = \frac{\beta h_{0,0}^b d_{0,0}^{-\alpha_b} \left| \sum_{i=1}^{N_p} \sqrt{P_C} h_{0,i}^f x_{0,i}^{-\alpha_f/2} \right|^2}{\beta \sum_{j \in \Phi_b \setminus B N_0} h_{0,j}^b d_{0,j}^{-\alpha_b} \left| \sum_{k=1}^{N_p} \sqrt{P_C} h_{j,k}^f x_{j,k}^{-\alpha_f/2} \right|^2 + N_0}, (3)$$

where $h_{i,j}^b = |h_{i,j}^b|^2$ is the backward channel gain between $i$th receiver and $j$th BN and $h_{i,j}^b \sim \text{CN} (0, 1)$; $h_{m,n}^f$ is the forward channel between $m$th BN and $n$th PB and $h_{m,n}^f \sim \text{CN} (0, 1)$; $d_{i,j}$ is the distance between $i$th receiver and $j$th BN; $x_{m,n}$ is the distance between $m$th BN and $n$th PB; $\alpha_b$ is the path loss exponent of the backward channel i.e., between the BN and the receiver; $N_0$ is the variance of the additive white Gaussian noise (AWGN). We assume that the distance between the receiver and the typical BS $d_{0,0}$ is fixed. We consider Rayleigh fading and assume that all channel gains are independent and identically distributed (i.i.d).

### B. Performance Metrics

We consider two metrics to study the performance of this wireless powered backscatter communication network. The first metric is the coverage probability, $P_s$, which is the probability that the SINR at a typical receiver is greater than some threshold $\Theta_m$

$$P_s = P \left[ \gamma'_{R_0} \geq \Theta_m \right], (4)$$

which gives us the percentage of BN having successful transmission in the network. Our second metric is the transmission capacity, $C$, defined as

$$C = \lambda_b P_s, (5)$$
which is the number of successful transmissions between the BNs and their corresponding receivers per unit area of the network.

III. COVERAGE PROBABILITY ANALYSIS

In this section, we find the coverage probability, $P_s$, and the capacity, $C$, of the network. For the coverage probability, we consider two cases. In the first case, we consider that a BN harvests energy from the nearest $N_p$ PBs as mentioned in the previous section, whereas for the second case, we assume that the BNs harvest energy from all the PBs in the network. The analysis depends on both the PPPs of PBs $\Phi_p$ and of BNs $\Phi_b$, and both the forward channel $h_{i,j}^f$ and backward channel $h_{k,i}^b$. Whereas, in conventional wireless network, we do not have this extra tier of PBs and both the BS and the MT have reliable power source. The downlink analysis usually depends on the PPP of the BSs and the channel between the MT and the BS, while the uplink analysis depends on the PPP of MTs and the channel. Thus having the additional tier of PBs and the backscatter communications by the BNs make the analysis very challenging. Therefore, in order to make the analysis tractable we make some assumption as discussed in the sequel. The second case is more complicated than the first one, therefore, we find an approximation for the coverage probability. The details of the derivation are given in the following subsections.

A. Coverage Probability when a BN harvests energy from $N_p$ nearby PBs

The transmission of a typical BN is successful when the SINR $\gamma_{R_0}^f$ exceeds the threshold $\Theta_m$. It is very difficult to find the coverage probability of the network due to double fading (forward and backward channel) and its dependence on $\Phi_b$ and $N_p$ nearest PBs. Therefore, in order to make the analysis tractable, we use a slightly modified definition of the SINR. The harvested power at BN $\left| \sum_{i=1}^{N_p} \sqrt{P_h} h_{i}^f x_i^{\alpha_f/2} \right|^2$ is exponentially distributed with mean $P_C \sum_{i=1}^{N_p} x_i^{\alpha_f}$ [26]. We replace the harvested power with its mean value in (3). The accuracy of this assumption is verified through simulations in the next section. It is important to mention that by doing so, we take the expectation only with respect to the forward channel and not with the location of the $N_p$ PBs. Similar approaches have been used in the literature to reduce the dimensionality of the problem when the analysis is very complicated and leads to intractable results [27], [28], [23], [29], and [30]. More specifically in [27], and [28], the authors considered an averaging circle around the receiver of interest and model the interferers inside this region while for the interference emanating from outside the region they used the expected value of it. Similarly, in
The authors only considered the effect of two nearest interferers and for the rest of the interference, its average value is considered. We write the modified SINR $\gamma_{R_0}$ as

$$\gamma_{R_0} = \frac{h_{b,0}^{b_d} \sum_{i=1}^{N_p} x_{0,i}^{ -\alpha f}}{\sum_{j \in \Phi \setminus BN_0} h_{b,j}^{b_d} \sum_{k=1}^{N_p} x_{j,k}^{ -\alpha f} + \frac{N_0}{\beta P_C}}.$$ (6)

Before deriving our main result for the coverage probability, we first find the average received power by a random BN in the following lemma.

**Lemma 1.** The average received power at a BN when there are $N_p$ PBs in the harvesting zone is given by

$$\bar{P}_{BN} = P_C \left(2 \pi \lambda_pight)^N \int \cdots \int_{0<x_0,1<x_0,2<\cdots<x_0,N_p<\infty} x_1, \ldots, x_{N_p} \left(\sum_{i=1}^{N_p} x_i^{ -\alpha f}\right) e^{-\pi \lambda_p x_p^2} dx_1, \ldots, dx_{N_p}. \quad (7)$$

**Proof.** The average received power at a BN can be written as

$$\bar{P}_{BN} = \mathbb{E}[P_{BN}] = \mathbb{E}_{h_{i,1}, \ldots, h_{i,N_p}} \left[ \sum_{i=1}^{N_p} \sqrt{P_C h_{i}^{f} x_i^{\alpha f}/2} \right] \overset{(a)}{=} \mathbb{E}_{h_{i,1}, \ldots, h_{i,N_p}} \left[ P_C \sum_{i=1}^{N_p} x_i^{-\alpha f} \right], \quad (8)$$

where (a) follows because $h_{i}^f$ is i.i.d distributed $CN (0, 1)$ and after taking the expectation with respect to $x_1, \ldots, x_{N_p}$ we obtained the final result. \qed

It can be noticed from (7) that $\bar{P}_{BN}$ increases with an increase in $P_C$, $N_p$ and $\lambda_p$. It is important to remind that on average, a BN harvests $(1 - \beta) \bar{P}_{BN}$ while the $\beta \bar{P}_{BN}$ is reflected back through backscatter modulation.

We present the SINR coverage probability in the following theorem.

**Theorem 1.** The SINR coverage probability $P_s$ when the BNs harvests from the nearest $N_p$ PBs is given by

$$P_s = \int \cdots \int_{0<x_0,1<x_0,2<\cdots<x_0,N_p<\infty} \exp(-\frac{sN_0}{\beta P_C}) L_I(s) f_x(x_{0,1}, \ldots, x_{0,N_p}) dx_{0,1}, \ldots, dx_{0,N_p} \quad (9)$$

where $s = \frac{\Theta_{m,0}^b}{\sum_{i=1}^{N_p} \bar{P}_{0,i}}$, $L_I(s)$ is the Laplace transform of the interference and is given by

$$L_I(s) = \exp \left( -\frac{2\pi^2 \lambda_b s^{2/\alpha^b}}{\alpha^b \sin \left( \frac{2\pi}{\alpha^b} \right)} \mathbb{E}_{x_1, \ldots, x_{N_p}} \left( \sum_{k=1}^{N_p} x_k^{-\alpha f} \right)^{2/\alpha^b} \right), \quad (10)$$
and \( f_x(x_{0,1}, \cdots, x_{0,N_p}) \) is the joint PDF of the distances to the \( N_p \) nearest PBs and given is in (2).

**Proof.** The proof is provided in Appendix A. \( \square \)

**Remark 1.** (Effect of \( \beta \) and \( P_C \) on the coverage probability) It can be observed from (9) that the coverage probability \( P_s \) increases by increasing the backscattering efficiency \( \beta \) or by increasing the transmit power \( P_C \) of the PBs.

We observe that as the number of PBs \( N_p \) in the harvesting region increases, the numerical computation of the coverage probability \( P_s \) becomes very tedious. This is due to the integration over the joint PDF of the \( x_1, \cdots, x_{N_p} \) in the \( \mathcal{L}_I(s) \) and in the final expression of (9). Nonetheless, the coverage probability \( P_s \) can be simplified for the following plausible special cases.

**Corollary 1.** When the BN harvests from the single nearest PB \((N_p = 1)\) and \( \alpha^f = \alpha^b = \alpha \) then the coverage probability simplifies to

\[
P_s = 2\pi \lambda_p \int_0^\infty x_1 \exp \left( - \left[ \frac{s N_0}{\beta P_C} + \pi \lambda_p x_1^2 + \frac{2\pi^2 \lambda_b s^{2/\alpha}}{\alpha \sin(\frac{2\pi}{\alpha})} \left[ 1 - e^{-\pi \lambda_p} - \pi \lambda_p \text{Ei}(-\pi \lambda_p) \right] \right] \right) \, dx_1,
\]

where \( s = \Theta_m d_0^0 x_1^\alpha \) and \( \text{Ei}(z) = \int_z^\infty e^{-t}t^{-1} \, dt \) is the exponential integral function [31]. The coverage probability can be computed by evaluating just a single integral.

**Proof.** The proof follows similar steps as in the proof of Theorem 1. However, instead of joint PDF of \( x_1, \cdots, x_{N_p} \), we use the marginal PDF \( x_1 \) of the nearest point in PPP network [25] and in the Laplace transform \( \mathcal{L}_I(s) \) we find the \( \mathbb{E}_{x_1}[x_1^{-2}] \). In order to find \( \mathbb{E}_{x_1} \) we use the path loss model \( \min \{ 1, x^{-\alpha} \} [24] \).

**Remark 2.** (Effect of \( \lambda_p \) on the coverage probability) The density of the PBs \( \lambda_p \) appears both inside the exponential and outside the exponential in (11). We expect that when \( \Theta_m \) is small then the \( \lambda_p \) outside the exponential dominates and as a result the coverage probability increases with an increase in \( \lambda_p \). However, when \( \Theta_m \) is large enough then the exponential-term dominates due to which the coverage probability decreases with an increase in \( \lambda_p \).

**Remark 3.** (Effect of \( \lambda_b \) on the coverage probability) The density of the BNs \( \lambda_b \) is only inside the exponential in (11), which suggests that an increase in \( \lambda_b \) always decreases the coverage probability.
Remark 4. (Effect of $\lambda_b$ on the capacity of the network) The capacity $C = \lambda_b P_s$, which shows that the density of BNs $\lambda_b$ is both inside the exponential term (in the expression for $P_s$) and outside the exponential term. This means that the $\lambda_b$ outside and inside exponential affects the capacity $C$ in a completely different way. We expect that for small $\Theta_m$ it will increase $C$, whereas for large $\Theta_m$ it will decrease $C$.

Corollary 2. When $N_p = 1, N_0 = 0$ and $\alpha_f = \alpha^b = \alpha$ then \((\text{11})\) can be further simplified to

\[
P_s = 2\pi \lambda_P \int_0^\infty x_1 \exp \left( -\left[ \pi \lambda_P x_1^2 + \frac{2\pi^2 \lambda_b s^{2/\alpha}}{\alpha \sin \left( \frac{2\pi}{\alpha} \right)} \left[ 1 - e^{-\pi \lambda_P} - \pi \lambda_P \text{Ei} (-\pi \lambda_P) \right] \right] \right) dx_1,
\]

where $s = \Theta_m d_{0,0}^\alpha$. The coverage probability becomes independent of the transmit power $P_C$ of the PB and the backscattering efficiency $\beta$.

Corollary 3. When $N_0 = 0$, but the BNs harvest energy from multiple nearby PBs then the coverage probability simplifies to

\[
P_s = \int \int \cdots \int_{0 < x_1 < x_2 < \cdots < x_{N_p} < \infty} \exp \left( -\frac{2\pi^2 \lambda_b s^{2/\alpha}}{\alpha \sin \left( \frac{2\pi}{\alpha} \right)} \mathbb{E}_{x_1, \cdots, x_{N_p}} \left( \sum_{k=1}^{N_p} x_k^{-\alpha_f} \right) \right) \times \frac{1}{f_x(x_1, \cdots, x_{N_p})} dx_1 \cdots dx_{N_p},
\]

where $s = \frac{\Theta_m d_{0,0}^\alpha}{\sum_{i=1}^{N_p} x_{0,i}^{-\alpha_f}}$ and $P_s$ is independent of the transmit power $P_C$ of PBs and the backscattering efficiency $\beta$.

Corollary 4. The coverage probability can be further simplified to

\[
P_s = \exp \left( -s \left[ \frac{N_0}{\beta d_{0,0}^{\alpha_f}} + \frac{2\pi^2 \lambda_b s^{2/\alpha}}{\alpha \sin \left( \frac{2\pi}{\alpha} \right)} \right] \right),
\]

if the BNs reflect the average power $\beta \bar{P}_{BN}$ instead of the instantaneous power $\beta P_{BN}$. The coverage probability is in closed form. The network behaves like a conventional ad-hoc network with BNs having the transmit power $\beta \bar{P}_{BN}$ and $s = \Theta_m d_{0,0}^\alpha$.

Proof. After plugging the $\bar{P}_{BN}$ in \((\text{6})\) the rest of the proof follows similar steps as of Theorem 1. \qed

Corollary 5. When $N_0 = 0$ and the BNs reflect the average power $\beta \bar{P}_{BN}$ then the coverage probability further simplifies to

\[
P_s = \exp \left( -\frac{2\pi^2 \lambda_b \Theta_m d_{0,0}^\alpha}{\alpha \sin \left( \frac{2\pi}{\alpha} \right)} \right),
\]
which depends only on the path loss exponent $\alpha^b$, density of the BN $\lambda_b$, and the distance $d_{0,0}$ between the BN and its receiver. The network acts like a conventional ad-hoc network and the coverage is independent of the transmit power of the BNs.

B. Coverage Probability when a BN harvests energy from all PBs

In contrast to the previous subsection, here we assume that a BN harvests energy from all the PBs in the network. We write the received power at a random BN as

$$P_{BN} = \left| \sum_{i \in \Phi_p} \sqrt{P_C h_i^f x_i^{-\alpha^f/2}} \right|^2,$$

where the summation is over the $\Phi_p$. Similarly, we write the SINR $\gamma'_{R_0}$ as

$$\gamma'_{R_0} = \frac{\beta h_{0,0}^b d_{0,0}^{-\alpha^b} \left| \sum_{i \in \Phi_p} \sqrt{P_C h_{0,i}^f x_{0,i}^{-\alpha^f/2}} \right|^2}{\beta \sum_{j \in \Phi_b \setminus B_{N_0}} h_{0,j}^b d_{0,j}^{-\alpha^b} \sum_{k \in \Phi_p} \sqrt{P_C h_{j,k}^f x_{j,k}^{-\alpha^f/2}}^2 + N_0}.$$

It is even more challenging than the previous section to find the coverage probability based on the above SINR expression because it depends on the PPPs of both PBs and BNs, and both the forward and backward channel. Therefore, in order to make the analysis tractable, we make some modifications to the above expression and consider SIR instead of SINR. We write the modified expression for SIR as

$$\gamma_{R_0} = \frac{h_{0,0}^b d_{0,0}^{-\alpha^b} \sum_{i \in \Phi_p} x_{0,i}^{-\alpha^f/2}}{\sum_{j \in \Phi_b \setminus B_{N_0}} h_{0,j}^b d_{0,j}^{-\alpha^b} \sum_{k \in \Phi_p} x_{j,k}^{-\alpha^f/2}},$$

where we use the average of $\left| \sum_{i \in \Phi_p} \sqrt{P_C h_i^f x_i^{-\alpha^f/2}} \right|^2$, which is $P_C \sum_{i \in \Phi_p} x_i^{-\alpha^f}$ [24]. It is important to mention that in (18), we only take the average with respect to the forward channel and not the $\Phi_p$. The coverage probability still depends on the $\Phi_p$, $\Phi_b$ and $h_{j,k}^b$. Similar approaches have been used in [27], [28], [23], [29], and [30] to reduce the complexity of the problem and to get tractable results.

Before presenting the main result of this section, we first find the average power at a BN in the following lemma.

**Lemma 2.** The average power at a BN when it harvests from all PBs in the network is given by

$$\bar{P}_{BN} = \frac{P_C \pi \lambda_p \alpha^f}{\alpha^f - 2}.$$
It is important to mention that \((1 - \beta) \bar{P}_{BN}\) power is harvested while the remaining \(\beta \bar{P}_{BN}\) is reflected through backscatter modulation. We do not use the dimensionality reduction in the proof of Lemma 1 and 2 and the expression in (7) and (19) are exact.

**Proof.** The average harvested power \(\bar{P}_{BN}\) in this setup can be written as

\[
\bar{P}_{BN} = \mathbb{E}[P_{BN}] = \mathbb{E}_{h,f,\Phi_p} \left[ \left( \sum_{i \in \Phi_p} \sqrt{P_C h_f^i x_i^{-\alpha/2}} \right)^2 \right] = \mathbb{E}_{\Phi_p} \left[ P_C \sum_{i \in \Phi_p} x_i^{-\alpha/2} \right],
\]

(20)

where \(a\) follows because \(h_f^i\) is i.i.d distributed \(\text{CN}(0,1)\) and the final result follows from Campbell’s theorem [24]. To find \(\mathbb{E}_{\Phi_p}\) we again use the bounded path loss model i.e., the path loss is \(\min\{1, x_i^{-\alpha/2}\}\) [24].

We know that SIR coverage probability depends on both the \(\Phi_p, \Phi_b\), and its exact expression can not be obtained. Therefore, we utilize Jensen’s inequality and find the approximated expression for the coverage probability in the following theorem.

**Theorem 2.** The SIR coverage probability when the BNs harvest energy from all the PBs is given by

\[
P_s \approx \exp\left( -\frac{2\pi^2 \lambda_b \Theta^2 / \alpha^b}{\alpha^b \sin\left(\frac{2\pi}{\alpha^b}\right)} \frac{\varrho^2}{d_{0,0}^2} \right).
\]

(21)

**Proof.** See Appendix B.

The above expression is the same as of Corollary 5, which makes sense because (15) is the SIR coverage probability when the BNs use the average transmit power and we know that SIR coverage is independent of the transmit power of the BNs.

**IV. Simulation Results and Discussion**

In this section, we present numerical results to validate the accuracy of the analysis and the tradeoffs that emerge in this wireless powered backscatter network. The system model of Section II are carefully reproduced in the Monte Carlo simulations. We fix \(P_C = 40\text{dBm}, \beta = 0.5, d_{0,0} = 1, N = -40\text{dB}, \) and \(\alpha_f = \alpha_b = \alpha.\) Both \(\lambda_p\) and \(\lambda_b\) are in per square meters (/m²) and we consider a unit area of 100m² when computing the capacity \(C\) of the network. If we do not mention \(\alpha\) and \(N_p\) then we keep \(\alpha = 4\) and \(N_p = 1,\) otherwise, we state it.
average received power at a random BN (W)

Lemma 1, $\alpha = 4$
Simulations, $\alpha = 4$
Lemma 1, $\alpha = 3$
Simulations, $\alpha = 3$

(a) Lemma 1 (harvesting energy from the nearest PB)

(b) Lemma 2 (harvesting energy from the whole network)

In Fig. 3 we plot the average received power at a BN $\bar{P}_{BN}$ against the density of the PBs $\lambda_p$. It can be seen that the numerical results obtained through Lemma 1 and 2 exactly match with the simulations results. It can be observed that $\bar{P}_{BN}$ is greater for small path loss exponent i.e., $\alpha = 3$. Furthermore, it can be noticed that $\bar{P}_{BN}$ increases with increase in $\lambda_p$ for both Fig. 3a and Fig. 3b. However, for Lemma 1 in Fig. 3a it saturates to 10 watts (40dBm) for $\lambda_p = 1$, which is the transmit power $P_C$ of a PB. This implies that when $\lambda_p = 1$, $N_p = 1$ and the path loss model is $\min \{1, x^{-\alpha}\}$, then the received power is on average equal to the transmit power of a PB. Whereas, in Lemma 2 the BNs harvest power from all the PBs, therefore, in Fig. 3b the $\bar{P}_{BN}$ does not saturate and keep increasing with $\lambda_p$.

Fig. 4 illustrates the impact of the number of PBs $N_p$ in the harvesting zone on the average received power $\bar{P}_{BN}$. It can be noticed that for Lemma 1 the average received power $\bar{P}_{BN}$ increases with an increase in $N_p$, whereas for Lemma 2 the $\bar{P}_{BN}$ is constant because in Lemma 2 the harvesting zone is the whole network. The important observation that can be noticed is that most of the power is harvested from the nearest PBs especially for $\alpha = 4$. In case of $\alpha = 4$ the contribution from the far away PBs is very small, therefore, $\bar{P}_{BN}$ quickly converges to the maximum value, whereas for $\alpha = 3$ the contribution from those far away nodes is still significant and therefore convergence is not achieved.

In Fig. 5 we compare the Monte Carlo simulations results with the numerical results for various network parameters. It can be noticed that the numerical and simulations curves are
matching well for both the Theorem 1 and Corollary 4, which shows the accuracy of the analysis. It is important to remind that in the proof of Theorem 1, we used a simplified expression for the SINR in order to make the analysis tractable. Similarly, in Corollary 4 we used the averaged received power $\bar{P}_{BN}$ as the harvested power at BN.

Similar to Fig. 5, we compare the numerical and simulation results in Fig. 6 for Theorem 2. It can be noticed that the numerical and simulations curves are close to each other, which shows the validity of our analysis. It should be reminded that in the proof of Theorem 2, we first used the modified SIR expression by taking average with respect to the forward channel and then use approximation for the expectation with respect to $\Phi_p$. However, both Fig. 5 and Fig. 6 show that our assumptions are quite reasonable.

Fig. 7 illustrates the impact of the density of the PBs on the coverage probability for different modulation thresholds. It can be noticed from Fig. 7a that the coverage probability increases with an increase in density of the PBs for different values of $\lambda_b$. This increase in coverage probability is due to the increase in received power at BNs due to which the power of the backscatter signal also increases. However, for large value of the SINR threshold the increase in $\lambda_p$ decreases the coverage probability as shown in Fig. 7b. This decrease in the coverage probability at higher SINR threshold is due to the increase in the interference power. The high SINR is achieved only by a very small fraction of the BNs, which are very close to the PBs. The distance between the
PBs and BNs decreases with an increase in $\lambda_p$, which improves the harvested and reflected power of the BNs. As a result the interference power increases, therefore, the coverage probability of the those BNs, which are very close to the PBs drops. This confirms our earlier observation in Remark 2 and suggests that in order to achieve the optimal coverage, the density of the PBs should be adjusted by taking $\Theta_m$ into account.

Fig. 8 depicts the impact of the density of the BNs on the coverage probability and capacity.
of the network. It can be observed from both Fig. 8a and Fig. 8c that an increase in the density of the BNs decreases the coverage probability, which is due to the increase in the number of interferers. However, a more appropriate metric in this scenario might be the capacity of network, which is the number of successful transmissions in some unit area. Therefore, we plot the capacity corresponding to Fig. 8a and Fig. 8c in Fig. 8b and Fig. 8d respectively. In contrast to the coverage probability, both Fig. 8b and Fig. 8d show that the capacity of the network increases with increase in the density of the BNs. However, at higher threshold i.e., above 0 dB the capacity decreases with an increase in the density of the BNs. The high SINR threshold is achieved by the BNs which are in close vicinity of the PBs. The number of interfering BNs and the interference increase with an increase in $\lambda_b$ due to which the BNs can not achieve such a high $\Theta_m$ and therefore, the capacity drops. The insights from Fig. 8 again validate our earlier observations in Remark 3 and 4. The coverage probability can be seen as an objective function of an individual user whereas the capacity is a network wide metric and the service provider would be more interested in it. The network provider would like to maximize the capacity of the network given that certain minimum quality of service (coverage probability here) should be satisfied. Therefore, in order to get an optimal performance the tradeoff between the coverage and capacity should be considered along with $\Theta_m$.

In Fig. 9 we compare the coverage of regularly powered network (RPN) with the wireless
powered backscatter communications network (WPBN). In RPN there is no PB tier and each BN have constant transmit power equal to the transmit power of a PB i.e., $P_C$. Since, the transmit power of each BN is $P_C$, its coverage probability is the same for different values of $\lambda_p$ as shown in the figure. Whereas for the WPBN, the coverage probability increases with an increase in the density of the PBs. It can be noticed that when $\lambda_p = 1$ then the coverage probability of the WPBN approaches to the coverage probability of the RPN. However, from the zoomed-in region,
Fig. 8: Impact of density of the BNs $\lambda_b$ on the coverage probability $P_s$ and capacity $C$ of the network.

It can be seen that there is still some gap between the coverage of both types of networks. Another key point that can be observed from both Fig. 9a and 9b is that the coverage of WPBN increases very rapidly with increase in $\lambda_p$ initially, while after a certain increase in $\lambda_p$ the gain in coverage is very small. For instance, in both Fig. 9a and 9b the increase in the coverage probability is very significant when $\lambda_p$ goes from 0.025 to 0.4, whereas from 0.4 to 1 the improvement in the coverage is almost negligible. This suggests that after certain point further densification of the WPBN with more PBs would not improve the coverage significantly.
In this paper, a general framework to study the performance of wireless powered backscattered IoT network is presented. To make the analysis tractable, a dimensionality reduction technique has been applied. The coverage and the capacity of the network in the form of tractable and numerically computable expressions are derived with the help of stochastic geometry. These expressions can be further simplified for some relevant special cases. It has been noticed that...

V. CONCLUSION

Fig. 9: Regularly powered vs. wireless powered backscatter network
the coverage of the network can either increase or decrease with the increase in the density of the PBs depending upon the SINR threshold. Furthermore, the capacity of the network increases with an increase in the density of the BNs, whereas the coverage probability decreases. Finally, the coverage of the wireless powered backscatter network has been compared with the coverage of the regularly powered network and it has been observed that for very high density of the PBs the coverage of the wireless powered backscattered network approaches to the coverage of the regular powered network.

A future study might consider a minimum circuit-power threshold on the harvest power of the BN and if the harvested power is not satisfied then the BN will not backscatter the signal. It would also be interesting to incorporate different collisions resolution schemes such as using directional antennas and/or successive interference cancellation and study its effect on the performance of this wireless powered backscatter network.

APPENDIX A

Proof of Theorem 1: By using the definition of γ₉₀ in (6), we write 𝑃ₜₚ as

\[ 𝑃ₜₚ = P[γ₉₀ ≥ Θₘ] = \int \cdots \int P[\gamma₉₀ ≥ Θₘ, x_{0,1}, \ldots, x_{0,Nₚ}] \]

\[ f(x_{0,1}, \ldots, x_{0,Nₚ})dx_{0,1}, \ldots, dx_{0,Nₚ}, (22) \]

where the integration is over the joint PDF of the distances \( x_{0,1}, \ldots, x_{0,Nₚ} \) to nearest \( Nₚ \) PBs.

Now, the conditional probability \( P[γ₉₀ ≥ Θₘ, x_{0,1}, \ldots, x_{0,Nₚ}] \) can be written as

\[ P[γ₉₀ ≥ Θₘ, x_{0,1}, \ldots, x_{0,Nₚ}] = P\left[ \sum_{i=1}^{Nₚ} x_{i}^{-\alpha_f} \geq \Thetaₘ \right] \]

\[ = \mathbb{E}_I \left[ \exp(-s(I + \frac{N₀}{\beta Pₖ})) \right] = \exp(-s\frac{N₀}{\beta Pₖ})L_I(s), \quad (23) \]

where \( s = \frac{\Thetaₘd_{0,0}^{-\alpha_b}}{\sum_{i=1}^{Nₚ} x_{i}^{-\alpha_f}} \), \( I = \sum_{j\in\Phi_b\setminus BN₀} h_{0,j}^{-\alpha_b} \sum_{k=1}^{Nₚ} x_{j,k}^{-\alpha_f} \), (a) follows because \( h_{0,0} \sim \exp(1) \), and \( L_I(s) = \mathbb{E}_I[\exp(-sI)] \) is the Laplace transform of the interference \( I \) evaluated at \( s \). Next,
we find the Laplace transform as

\[
\mathcal{L}_I(s) = \mathbb{E}_I[\exp(-sI)] = \mathbb{E}_{b_{0,j},d_{0,j},x_{j,1},\ldots,x_{j,N_p}} \left[ \exp \left( -s \sum_{j \in \Phi_0 \setminus BN_0} h_{0,j} d_{0,j}^{-\alpha f} \sum_{k=1}^{N_p} x_{j,k}^{-\alpha f} \right) \right]
\]

\[
= \mathbb{E}_{d_{0,j},x_{j,1},\ldots,x_{j,N_p}} \left[ \prod_{j \in \Phi_0 \setminus BN_0} \mathbb{E}_{b_{0,j}} \left[ \exp \left( -s h_{0,j} d_{0,j}^{-\alpha f} \sum_{k=1}^{N_p} x_{j,k}^{-\alpha f} \right) \right] \right]
\]

\[
= \mathbb{E}_{d_{0,j}} \left[ \prod_{j \in \Phi_0 \setminus BN_0} \mathbb{E}_{x_{j,1},\ldots,x_{j,N_p}} \left[ \frac{1}{1 + s d_{0,j}^{-\alpha f} \sum_{k=1}^{N_p} x_{j,k}^{-\alpha f}} \right] \right]
\]

\[
= \exp \left( -2\pi \lambda_b \int_0^\infty \mathbb{E}_{x_{1,\ldots,x_{N_p}}} \left[ \frac{1}{1 + \frac{u}{\sum_{k=1}^{N_p} x_{k}^{-\alpha f}}} \right] du \right),
\]

\[
= \exp \left( -\frac{2\pi^2 \lambda_b s^{2/\alpha f}}{\alpha f \sin \left( \frac{2\pi}{\alpha f} \right)} \mathbb{E}_{x_{1,\ldots,x_{N_p}}} \left( \sum_{k=1}^{N_p} x_{k}^{-\alpha f} \right)^{2/\alpha f} \right)
\]

where (a) follows due to the independence of \( h_{0,j} \), (b) follows from the fact that \( h_{0,j} \sim \exp(1) \) and the independence of \( \mathbb{E}_{x_{j,1,\ldots,x_{j,N_p}}}, (c) \), and (d) follows by changing the order of integration and expectation and then computing the integral. The last expectation \( \mathbb{E}_{x_{1,\ldots,x_{N_p}}} \) can be computed by using the joint PDF of \( x_1, \ldots, x_{N_p} \) given in (2). By plugging (24) in (23), and then (23) in (22) we get the final expression for \( \mathbb{P}_s \).

**APPENDIX B**

**Proof of Theorem 2:** The SIR coverage probability can be written as

\[
\mathbb{P}_s = \mathbb{P}[\gamma_{R_0} \geq \Theta_m] = \mathbb{P} \left[ \frac{h_{0,0}^{-\alpha f} \sum_{i \in \Phi_0} x_{0,i}^{-\alpha f}}{\sum_{j \in \Phi_0 \setminus BN_0} h_{0,j} d_{0,j}^{-\alpha f} \sum_{k \in \Phi_0} x_{j,k}^{-\alpha f}} \geq \Theta_m \right],
\]

where we put (13) for \( \gamma_{R_0} \). We know that \( h_{0,0} \) is exponentially distributed with unit mean due to which the above expression becomes

\[
\mathbb{P}_s = \mathbb{E}_{\Phi_0,\Phi_p, h_{0,j}} \left[ \exp \left( -\Theta_m \sum_{j \in \Phi_0 \setminus BN_0} h_{0,j} d_{0,j}^{-\alpha f} \sum_{k \in \Phi_0} x_{j,k}^{-\alpha f} \right) \right],
\]
where the expectation should be with respect to the $\Phi_b$, $\Phi_p$, and $h_{0,j}^b$. Due to the independence of $h_{0,j}^b$ the previous expression can be written as

$$ P_s = \mathbb{E}_{\Phi_b, \Phi_p} \left[ \prod_{j \in \Phi_b \setminus B_{N_0}} \mathbb{E}_{h_{0,j}^b} \left[ \exp \left( -\frac{\Theta_m h_{0,j}^b d_{0,j}^{\alpha_b}}{d_{0,0}^{\alpha_b} \sum_{i \in \Phi_p} x_{0,i}^{-\alpha_f}} \sum_{k \in \Phi_p} x_{j,k}^{-\alpha_f} \right) \right] \right] \geq \mathbb{E}_{\Phi_b} \left[ \prod_{j \in \Phi_b \setminus B_{N_0}} \mathbb{E}_{\Phi_p} \left[ \frac{1}{1 + \frac{\Theta_m u^{\alpha_b}}{d_{0,0}^{\alpha_b} \sum_{i \in \Phi_p} x_{0,i}^{-\alpha_f}}} \sum_{k \in \Phi_p} x_{j,k}^{-\alpha_f} \right] \right] \left( 1 - \mathbb{E}_{\Phi_p} \left[ \frac{1}{1 + \frac{\Theta_m u^{\alpha_b}}{d_{0,0}^{\alpha_b} \sum_{i \in \Phi_p} x_{0,i}^{-\alpha_f}}} \sum_{k \in \Phi_p} x_{j,k}^{-\alpha_f} \right] \right) u^2 du = 1, \quad (27) $$

where (a) follows because $h_{0,j}^b \sim \exp(1)$ and both $\Phi_b$ and $\Phi_p$ are independent, (b) follows due the probability generating functional of PPP \cite{24}, while in (c) we utilize the Jensen’s inequality.

Now to find the last expectation $\mathbb{E}_{\Phi_p}$, we use $\mathbb{E}_{\Phi_p} \left[ \frac{1}{1 + \frac{\Theta_m u^{\alpha_b}}{d_{0,0}^{\alpha_b} \sum_{i \in \Phi_p} x_{0,i}^{-\alpha_f}}} \sum_{k \in \Phi_p} x_{j,k}^{-\alpha_f} \right] \geq \frac{\mathbb{E}_{\Phi_p} \left[ \sum_{k \in \Phi_p} x_{j,k}^{-\alpha_f} \right]}{\mathbb{E}_{\Phi_p} \left[ \sum_{i \in \Phi_p} x_{0,i}^{-\alpha_f} \right]} = 1$, where to find $\mathbb{E}_{\Phi_p}$ we utilize Lemma 2. The final expression in (21) is obtained by doing some algebra.

**References**

[1] “Internet of things (iot),” [Online]. Available: http://www.cisco.com/c/en/us/solutions/internet-of-things/overview.html

[2] M. Elkhodr, S. Shahrestani, and H. Cheung, “The internet of things: New interoperability, management and security challenges,” arXiv preprint arXiv:1604.04824, 2016.

[3] “Powering iot devices: Technologies and opportunities,” [Online]. Available: http://iot.ieee.org/newsletter/november-2015/powering-iot-devices-technologies-and-opportunities.html

[4] Y. Zeng, B. Clerckx, and R. Zhang, “Communications and signals design for wireless power transmission,” IEEE Transactions on Communications, vol. 65, no. 5, pp. 2264–2290, May 2017.

[5] K. Huang and V. K. Lau, “Enabling wireless power transfer in cellular networks: architecture, modeling and deployment,” Wireless Communications, IEEE Transactions on, vol. 13, no. 2, pp. 902–912, 2014.

[6] B. Clerckx and E. Bayguzina, “Waveform design for wireless power transfer,” IEEE Transactions on Signal Processing, vol. 64, no. 23, pp. 6313–6328, Dec 2016.

[7] B. Clerckx, Z. B. Zawawi, and K. Huang, “Wirelessly powered backscatter communications: Waveform design and snr-energy tradeoff,” IEEE Communications Letters, vol. PP, no. 99, pp. 1–1, 2017.
[8] C. Boyer and S. Roy, “Backscatter communication and rfid: Coding, energy, and mimo analysis,” Communications, IEEE Transactions on, vol. 62, no. 3, pp. 770–785, 2014.

[9] W. Liu, K. Huang, X. Zhou, and S. Durrani, “Backscatter communications for internet-of-things: Theory and applications,” arXiv preprint arXiv:1701.07588, 2017.

[10] R. Want, “An introduction to rfid technology,” IEEE Pervasive Computing, vol. 5, no. 1, pp. 25–33, Jan 2006.

[11] V. Liu, A. Parks, V. Talla, S. Gollakota, D. Wetherall, and J. R. Smith, “Ambient backscatter: wireless communication out of thin air,” ACM SIGCOMM Computer Communication Review, vol. 43, no. 4, pp. 39–50, 2013.

[12] B. Kellogg, A. Parks, S. Gollakota, J. R. Smith, and D. Wetherall, “Wi-fi backscatter: Internet connectivity for rf-powered devices,” in ACM SIGCOMM Computer Communication Review, vol. 44, no. 4. ACM, 2014, pp. 607–618.

[13] G. Yang, Y.-C. Liang, R. Zhang, and Y. Pei, “Modulation in the air: Backscatter communication over ambient ofdm carrier,” arXiv preprint arXiv:1704.02245, 2017.

[14] K. Han and K. Huang, “Wirelessly powered backscatter communication networks: Modeling, coverage, and capacity,” IEEE Transactions on Wireless Communications, vol. 16, no. 4, pp. 2548–2561, 2017.

[15] C. Psomas and I. Krikidis, “Backscatter communications for wireless powered sensor networks with collision resolution,” arXiv preprint arXiv:1707.02999, 2017.

[16] W. Liu, K. Huang, X. Zhou, and S. Durrani, “Full-duplex backscatter interference networks based on time-hopping spread spectrum,” IEEE Transactions on Wireless Communications, 2017.

[17] P. N. Alevizos, K. Tountas, and A. Bletsas, “Multistatic scatter radio sensor networks for extended coverage,” arXiv preprint arXiv:1706.03091, 2017.

[18] H. ElSawy, A. Sultan-Salem, M.-S. Alouini, and M. Z. Win, “Modeling and analysis of cellular networks using stochastic geometry: A tutorial,” IEEE Communications Surveys & Tutorials, vol. 19, no. 1, pp. 167–203, 2017.

[19] M. Bacha, Y. Wu, and B. Clerckx, “Downlink and uplink decoupling in two-tier heterogeneous networks with multi-antenna base stations,” IEEE Transactions on Wireless Communications, vol. 16, no. 5, pp. 2760–2775, 2017.

[20] M. Di Renzo and P. Guan, “A mathematical framework to the computation of the error probability of downlink mimo cellular networks by using stochastic geometry,” IEEE Transactions on Communications, vol. 62, no. 8, pp. 2860–2879, 2014.

[21] A. M. Hunter, J. G. Andrews, and S. Weber, “Transmission capacity of ad hoc networks with spatial diversity,” IEEE Transactions on Wireless Communications, vol. 7, no. 12, pp. 5058–5071, 2008.

[22] M. Di Renzo, “Stochastic geometry modeling and analysis of multi-tier millimeter wave cellular networks,” IEEE Transactions on Wireless Communications, vol. 14, no. 9, pp. 5038–5057, 2015.

[23] M. A. Kishk and H. S. Dhillon, “Joint uplink and downlink coverage analysis of cellular-based rf-powered iot network,” CoRR, vol. abs/1705.06799, 2017. [Online]. Available: http://arxiv.org/abs/1705.06799

[24] M. Haenggi and R. K. Ganti, Interference in large wireless networks. Now Publishers Inc, 2009.

[25] D. Moltchanov, “Distance distributions in random networks,” Ad Hoc Networks, vol. 10, no. 6, pp. 1146–1166, 2012.

[26] G. Nigam, P. Minero, and M. Haenggi, “Coordinated multipoint joint transmission in heterogeneous networks,” Communications, IEEE Transactions on, vol. 62, no. 11, pp. 4134–4146, 2014.

[27] R. K. Mungara, X. Zhang, A. Lozano, and R. W. Heath, “Analytical characterization of itlinq: Channel allocation for device-to-device communication networks,” IEEE Transactions on Wireless Communications, vol. 15, no. 5, pp. 3603–3615, 2016.

[28] G. George, R. K. Mungara, and A. Lozano, “An analytical framework for device-to-device communication in cellular networks,” IEEE Transactions on Wireless Communications, vol. 14, no. 11, pp. 6297–6310, 2015.
[29] S. Weber, J. G. Andrews, and N. Jindal, “The effect of fading, channel inversion, and threshold scheduling on ad hoc networks,” *IEEE Transactions on Information Theory*, vol. 53, no. 11, pp. 4127–4149, Nov 2007.

[30] P. Madhusudhanan, J. G. Restrepo, Y. Liu, T. X. Brown, and K. R. Baker, “Downlink performance analysis for a generalized shotgun cellular system,” *IEEE Transactions on Wireless Communications*, vol. 13, no. 12, pp. 6684–6696, Dec 2014.

[31] D. Zwillinger, *Table of integrals, series, and products*. Elsevier, 2014.