Mediation of Supersymmetry Breaking in Gauge Messenger Models

Radovan Dermišek*, Hyung Do Kim† and Ian-Woo Kim†

*School of Natural Sciences, Institute for Advanced Study, Princeton, NJ 08540, U.S.A.
†School of Physics and Astronomy and Center for Theoretical Physics, Seoul National University, Seoul, 151-747, Korea
E-mail: dermisek@ias.edu, hdkim@phya.snu.ac.kr, iwkim@phya.snu.ac.kr

ABSTRACT: We study gauge mediation of supersymmetry breaking in SU(5) supersymmetric grand unified theory with gauge fields as messengers. The generated soft supersymmetry breaking parameters lead to close to maximal mixing scenario for the Higgs mass and highly reduce the fine tuning of electroweak symmetry breaking. All gaugino, squark and slepton masses are determined by one parameter – the supersymmetry breaking scale. The characteristic features are: negative and non-universal squark and slepton masses squared at the unification scale, non-universal gaugino masses, and sizable soft-trilinear couplings. In this scenario, all soft supersymmetry breaking parameters at the unification scale can be smaller than 400 GeV and all the superpartners can be lighter than 400 GeV and still satisfy all the limits from direct searches for superpartners and also the limit on the Higgs mass. The lightest supersymmetric particle is gravitino or a sizable mixture of bino, wino and higgsino. We also consider a possible contributions from additional messengers in vector-like representations, and a contribution from gravity mediation, which is estimated to be comparable.
1. Introduction

Minimal Supersymmetric Standard Model (MSSM) is one of the most promising candidates for physics beyond the standard model. Gauge coupling unification, radiative electroweak symmetry breaking (EWSB) and the lightest supersymmetric particle (LSP) as a candidate for dark matter in the presence of R-parity indicate that MSSM might be the correct description of physics above the EW scale.

A natural explanation of EWSB being triggered by SUSY breaking requires the SUSY breaking scale to be near the EW scale. However, we have not observed any superparticles yet. Moreover, the Higgs quartic coupling in the MSSM is solely determined by gauge couplings, which gives a definite prediction for the physical Higgs mass. At tree level it is lower than the Z boson mass ($M_Z \simeq 91$ GeV),

$$m_h \leq M_Z |\cos 2\beta|,$$

(1.1)
where \( \tan \beta = v_u/v_d \) is the ratio of the vacuum expectation values (VEVs) of \( H_u \) and \( H_d \).

The dominant one loop correction \[1\] \[2\] \[3\], in case the stop mixing parameter is small, depends only logarithmically on stop masses and it has to be large in order to push the Higgs mass above the LEP limit, 114.4 GeV. A two loop calculation (we use \textit{FeynHiggs} 2.4.1 \[4\] \[5\] \[6\] with \( m_t = 172.5 \) GeV) reveals the stop masses have to be \( \gtrsim 900 \) GeV.

This constraint has a direct drawback in the electroweak symmetry breaking. The mass of the Z boson (or the EW scale), determined by minimizing the Higgs potential, is related to the supersymmetric Higgs mass parameter \( \mu \) and the soft SUSY breaking mass squared parameter for \( H_u \) as (for \( \tan \beta \geq 5 \))

\[
\frac{M_Z^2}{2} \approx -\mu^2(M_Z) - m_{H_u}^2(M_Z). \tag{1.2}
\]

The large stop mass affects the running of \( m_{H_u}^2 \),

\[
\delta m_{H_u}^2 \approx -\frac{3}{4\pi^2} m_t^2 \log \frac{\Lambda}{m_t}. \tag{1.3}
\]

and, since for \( \Lambda \sim M_{\text{GUT}} \sim 10^{16} \) GeV the loop suppression times large log is of order one, we find

\[
\delta m_{H_u}^2 \sim -m_t^2.
\]

Comparing it with Eq. \( (1.2) \) we immediately see that we need a miraculous cancelation between \( m_{H_u}^2 \) and \( \mu^2 \) to obtain the right \( M_Z \) for \( m_t \gtrsim 900 \) GeV. One possibility to keep \( \mu \) of order \( M_Z \) is to start with large enough \( m_{H_u}^2 \) at the GUT scale to cancel the large log correction \( -m_t^2 \) in which case the fine tuning is hidden in the boundary condition for \( m_{H_u}^2 \).

This is the so called “little hierarchy problem”.

The situation highly improves when considering large mixing in the stop sector. The mixing is controlled by the ratio of \( A_t - \mu \cot \beta \) and \( m_{\tilde{t}} \), where \( A_t \) is the soft SUSY breaking top trilinear coupling. Since we consider parameter space where \( \mu \) is small to avoid fine tuning and \( \tan \beta \gtrsim 5 \) in order to maximize the tree level Higgs mass \( (1.1) \), the mixing is simply given by \( A_t/m_{\tilde{t}} \). The Higgs mass is maximized for \( A_t(M_Z)/m_{\tilde{t}}(M_Z) \approx \pm \sqrt{6} \) and with such a mixing the limit on the Higgs mass can be satisfied with much lower stop masses, \( m_{\tilde{t}}(M_Z) \lesssim 300 \) GeV. Therefore in this “maximal mixing scenario” (scenario where mixing in the stop sector is such that the Higgs mass is maximized) the fine tuning in EWSB is highly alleviated. However it is very non-trivial to realize this scenario in models, since it usually requires very large \( A_t \) at the GUT scale, several times larger than other soft SUSY breaking parameters. The maximal mixing scenario and its possible realization in models will be discussed in more detail in Sec. 2.

A simple way of achieving close to maximal mixing was recently suggested in \[7\]. If we allow negative stop masses squared at the GUT scale several interesting things happen simultaneously. First of all, unless \( m_{\tilde{t}} \) is too large compared to \( M_3 \) it will run to positive values at the EW scale. At the same time the contribution to \( m_{H_u}^2 \) from the energy interval where \( m_{\tilde{t}}^2 < 0 \) partially or even exactly cancels the contribution from the energy interval
where $m_1^2 > 0$, see Eq. (1.3), and so the EW scale value of $m_{H_u}^2$ can be arbitrarily close to the starting value at $M_{GUT}$. No cancelation between initial value of $m_{H_u}^2$ (or $\mu$) and the contribution from the running is required. And finally, the stop mixing is typically much larger than in the case with positive stop masses squared. It turns out that in the region where $m_{H_u}^2$ gets negligible contribution from running, the radiatively generated stop mixing is close to maximal even when starting with negligible mixing at the GUT scale. Since in principle this scenario can eliminate fine tuning of EWSB completely, it is desirable to see how close to the radiatively generated maximal mixing scenario one can get in specific models.

In this paper we study gauge mediation of SUSY breaking in SU(5) supersymmetric grand unified theory (SUSY GUT) with an adjoint chiral multiplet and massive components of vector (gauge) multiplet playing the role of messengers. The soft susy breaking parameters in this “gauge messenger model” are similar to those discussed in [7] which were shown to lead to maximal mixing scenario for the Higgs mass. The characteristic features are: negative and non-universal squark and slepton masses squared at the GUT scale, non-universal gaugino masses, $|M_1| > |M_2| > |M_3|$, and sizable soft-trilinear couplings. Besides gauge messengers, we also consider a possible contributions from additional messengers in vector-like representations, e.g. 5 and $\bar{5}$ of SU(5). Finally, since the messenger scale is the GUT scale, and the gauge mediation is a one loop effect, the naively estimated size of gravity mediation induced by non-renormalizable operators (suppressed by $M_{Pl}$) is comparable to the contribution from gauge mediation. A combination of gauge mediation (with gauge and vector-like messengers) with gravity mediation opens completely new possibilities for model building. We show that already some of the simplest models lead to close to maximal mixing scenario for the Higgs mass and highly reduce the fine tuning of electroweak symmetry breaking. The SUSY spectrum is very different from other scenarios typically used for collider studies. All superpartners can be within 400 GeV with relatively light stop, $m_{\tilde{t}_1} \gtrsim 150$, while satisfying all experimental limits, including the limit on the Higgs mass. The lightest supersymmetric particle (LSP) is gravitino and the next to the lightest supersymmetric particle (NLSP) is neutralino, sneutrino, stau or stop in most of the parameter space.

We note that gauge messenger model has been considered in very early stages of MSSM history. After the work on inverted mass hierarchy [8], “geometric hierarchy model” has been constructed in [9] and soft scalar masses have been calculated in [10].\footnote{See also more recent attempts to break GUT symmetry and SUSY by the same field in [11, 12].} In this model the SUSY breaking scale is an intermediate scale and the messenger scale is the GUT scale. The explicit SUSY breaking model they considered has light (TeV scale) adjoint chiral superfields under the standard model gauge group and the gauge couplings unify at a scale beyond the Planck scale, $10^{20}$ GeV. We do not consider a specific model of SUSY breaking (although we assume it happens at the GUT scale). We only address the mediation of SUSY breaking. Therefore, we treat the number of fields in a model as discrete parameters and focus on minimal models with smaller number of fields.

This paper is organized as follows. In Sec. 2 we discuss the maximal mixing scenario
as a possible solution to the little hierarchy problem, and a possibility of it being generated radiatively without introducing large soft-trilinear couplings at the GUT scale. In Sec. 3 we present a gauge messenger model and briefly discuss possible contribution from gravity mediation of SUSY breaking. The results are given in Sec. 4 together with discussion of phenomenology. We conclude in Sec. 5. For convenience we summarize formulae necessary to derive soft SUSY breaking parameters from gauge messenger models in the Appendix A, and we discuss different possibilities for gravity mediated contributions in more detail in the Appendix B.

2. Maximal mixing scenario – a solution to the fine tuning problem

As mentioned in the Introduction, the physical Higgs boson mass receives an additional contribution from stop mixing [13],

\[ m^2_H \approx M_Z^2 \cos^2 2 \beta + \frac{3G_F m^4_t}{\sqrt{2} \pi^2} \left\{ \log \frac{m^2_t}{m^2_\tilde{t}} + \frac{A^2_t}{m^2_t} (1 - \frac{A^2_t}{12 m^2_\tilde{t}}) \right\}, \tag{2.1} \]

The last term has a maximum at \( |A_t/m_\tilde{t}| = \sqrt{6} \) which corresponds to the maximal mixing scenario. In this case the stop can be lighter, \( m_\tilde{t} \text{(maximal mixing)} = e^{-3/2} m_\tilde{t} \text{(no mixing)} \), and it can be as light as \( 250 \sim 300 \) GeV while fulfilling the physical Higgs mass bound from the LEP.

Instead of using Eq. (1.3) as a rough estimate of the contribution of stop mass to the running of \( m^2_{H_u} \) it is instructive to be more precise. For given \( \tan \beta \) we can solve RG equations exactly and express EW values of \( m^2_{H_u}, \mu^2 \), and consequently \( M_Z^2 \) given by Eq. (1.2), as functions of all GUT scale parameters. For \( \tan \beta = 10 \), we have:

\[ M_Z^2 \approx -1.9 \mu^2 + 5.9 M^2_3 - 1.2 m^2_{H_u} + 1.5 m^2_\tilde{t} - 0.8 A_t M_3 + 0.2 A^2_t + \cdots, \tag{2.2} \]

where parameters appearing on the right-hand side are the GUT scale parameters, we do not write the scale explicitly. Other scalar masses and \( M_1 \) and \( M_2 \) appear with negligible coefficients and we neglect them in our discussion. The coefficients in this expression depend only on \( \tan \beta \) (they do not change dramatically when varying \( \tan \beta \) between 5 and 50) and \( \log(M_{GUT}/M_Z) \).

Let us also express the EW scale values of stop mass squared, gluino mass and top trilinear coupling for \( \tan \beta = 10 \) in a similar way:

\[ m^2_\tilde{t}(M_Z) \approx 5.0 M^2_3 + 0.6 m^2_\tilde{t} + 0.2 A_t M_3 \tag{2.3} \]
\[ M_3(M_Z) \approx 3 M_3 \tag{2.4} \]
\[ A_t(M_Z) \approx -2.3 M_3 + 0.2 A_t. \tag{2.5} \]

In the case of \( m_\tilde{t} \) the coefficients represent averages of exact coefficients that would appear in separate expressions for \( m^2_{\tilde{t}_L} \) and \( m^2_{\tilde{t}_R} \).

From Eqs. (2.2), (2.3) and (2.4), we see the usual expectation from SUSY, \( M_Z \approx m_\tilde{t}_{1,2} \approx m_\tilde{g} \), when all the soft SUSY breaking parameters are comparable. Furthermore,
neglecting terms proportional to $A_t$ in Eqs. (2.3) and (2.3) we find that a typical stop mixing is

$$|\frac{A_t}{m_t}|(M_Z) \simeq \frac{2.3M_3}{\sqrt{5.0M_3^2 + 0.6m_t^2}} \lesssim 1.0,$$

(2.6)

and comparing it with Eqs. (2.1) we see that such a mixing only negligibly affects the mass of the Higgs boson. Due to the washout effect, see Eq. (2.5), a large mixing can be achieved only for $|A_t| \gg |M_3|$, $m_t$ at the GUT scale for opposite sign of $A_t$ compared to $M_3$, or even larger $A_t$ for the same sign.\(^2\) This is the reason why it is very difficult to build a model leading to the maximal mixing scenario.

Although the boundary condition for $m_t$ in the above discussion does not seem to be very important (it is mostly the gluino that drives the evolution of stop and thus $m_{H_u}^2$, and sets the mixing) it turns out that when considering negative stop masses squared it starts playing a major role as discussed recently in Ref. [1]. In spite of negative stop masses squared being somewhat suspicious, from Eq. (2.3) we see that unless $m_t$ is too large compared to $M_3$ it will run to positive values at the EW scale. At the same time, however, the contribution to $m_{H_u}^2$ from the energy interval where $m_t^2 < 0$ partially or even exactly cancels the contribution from the energy interval where $m_t^2 > 0$ and so the EW scale value of $m_{H_u}^2$ can be arbitrarily close to the starting value at $M_{GUT}$. From Eq. (2.2) we see that this happens for $m_t^2 \simeq -4M_3^2$ (neglecting $A_t$). No cancelation between initial value of $m_{H_u}^2$ (or $\mu$) and the contribution from the running is required, the electroweak scale is not sensitive to masses of colored particles in this case, and the situation when $M_Z \ll m_{t_{1,2}} \simeq m_3$ can be achieved without any fine tuning (provided there exists a model which generates negative stop masses squared and sets the ratio of gluino mass and the stop mass approximately to the required value). And finally, from Eqs. (2.3) and (2.5), or from Eq. (2.6), we see that the stop mixing is typically quite large. Unlike in the case with positive stop masses squared where mixing is typically less than one, in the case with negative stop masses squared it is typically greater than one, and it can be easily even maximal. The maximal mixing scenario can be entirely generated radiatively starting with no mixing at the GUT scale.

Very large $A_t$ term may cause dangerous color and/or charge breaking minimum to appear at around the EW vacuum. Considering cosmology, in order not to tunnel within the age of universe, the empirical bound is \([14]\ [15]\)

$$|A_t|^2(M_Z) + 3\mu^2(M_Z) \lesssim 7.5(m_{t_L}^2(M_Z) + m_{t_R}^2(M_Z)),$$

(2.7)

which is much weaker than the condition for the EW vacuum to be the global minimum, $|A_t|^2(M_Z) + 3\mu^2(M_Z) \lesssim 3(m_{t_L}^2(M_Z) + m_{t_R}^2(M_Z))$ \([14]\). Certainly the maximal mixing value is within the empirical bound and it is safe from the constraints of the CCB minima.

\(^2\)Extremely large $A_t$ in the case of the same sign as $M_3$ contributes significantly to the running of $m_t$ and consequently to the running of $m_{H_u}$. Therefore, $A_t \simeq m_t \simeq m_{H_u} \gg M_Z$ is required and the EW scale is a result of large cancelations.
2.1 Large (maximal) mixing in models

Since the radiatively generated maximal mixing scenario can in principle eliminate fine tuning of EWSB completely, it is desirable to see whether it is possible to get even close to it in specific models.

It is easy to see that this solution does not exist in mSUGRA. As a consequence of universalities in gaugino and scalar masses, when stop mass squared is negative enough to generate maximal stop mixing at the EW scale radiatively, sleptons remain tachyonic even at the EW scale because the bino contribution to the running of slepton masses is small. The EW scale slepton mass is \( m_{\tilde{e}_R}^2 \approx m_0^2 + 0.15 M_{1/2}^2 \). Imposing the slepton mass bound 100 GeV gives the following inequality

\[
m_0^2 \geq \left\{ -(0.4)^2 + \left(\frac{100 \text{ GeV}}{M_{1/2}}\right)^2 \right\} M_{1/2}^2.
\]  

The largest (negative) ratio of \( m_0^2 \) and \( M_{1/2}^2 \) is achieved in the limit \( M_{1/2} \to \infty \) (taking aside all the naturalness criteria) and even in this case it is only \( m_0^2 \approx -(0.4)^2 M_{1/2}^2 \) which makes negligible difference in the generated mixing at the EW scale, see Eq. (2.6). The maximal mixing solution can be achieved only when either gaugino masses are not universal at the GUT scale (bino should be heavier than gluino at the GUT scale) or scalar masses are not universal (sleptons are less negative than stops).

Usual gauge mediation\[18\][19][20] shares a common problem with mSUGRA due to its hierarchical spectrum at the weak scale. Gluino is almost 6 ~ 7 times heavier than bino and squarks are much heavier than sleptons. Anomaly mediation\[21\][22] also has a huge hierarchy in the EW scale spectrum and gluino is 10 times heavier than wino.

Recently proposed “mirage mediation” or “modulus-anomaly mixed mediation”\[23\][24][25][26][27][28][29][30] partially fulfills the criteria listed above. In the most interesting \( \alpha = 2 \) scenario of mirage mediation\[25\][28][29], the mirage scale is at TeV and the spectrum is more or less degenerate. In this case, squarks and sleptons are tachyonic except stop and \( H_u \) at the GUT scale and gaugino masses are non-universal at the GUT scale with the aid of anomaly mediation. The fine tuning in this model is highly reduced due to cancelation of RG running effects with anomaly mediation contribution. The stop mixing is predicted to be large but not close to the maximal, \( |A_t/m_{\tilde{t}}| \sim 1.4 \). The \( \alpha = 2 \) mirage mediation might be an alternative solution to the little hierarchy problem although the supersymmetry spectrum (except Higgs) can be at around TeV which is \( 4\pi \) times heavier than \( M_Z \). There are several common features between mirage mediation and gauge messenger model considered in this paper though the origin of supersymmetry breaking is very different.

In the next section we present a model of mediation of SUSY breaking which leads to close to maximal mixing scenario while all the SUSY breaking parameters at the GUT scale and also physical masses of all superpartners can be \( \lesssim 400 \) GeV.

3. Gauge Messenger Model

Let us consider \( N = 1 \) SU(5) supersymmetric grand unified theory (SUSY GUT). The
$N = 1$ vector multiplet $V$ transforms as an adjoint of SU(5), the three generations of matter fields are in chiral multiplets, $3 \times (10 + \bar{5})$, and the Higgs fields are in $5 + \bar{5}$. Besides these, we also introduce an adjoint chiral multiplet $\Sigma$, and we assume that both its scalar component, which we also denote $\Sigma$, and the auxiliary component, $F_{\Sigma}$, get vacuum expectation values. The VEV of $F_{\Sigma}$ breaks SUSY and the SUSY breaking is communicated to gauginos, squarks and sleptons through gauge interactions. The massive components of the gauge multiplet $V$ and $\Sigma$ play the role of messengers. This is the minimal field content we consider. In this case, the beta function coefficient of the unified gauge coupling is $b_G = 3$ and all soft SUSY breaking parameters at the GUT scale are calculable in terms of $b_G$ and the unified gauge coupling.

It is also possible to extend the messenger sector and introduce, for example, a pair of usual messenger fields $\Phi$ and $\Phi^c$ in 5 and $\bar{5}$ representations of SU(5). Additional messengers also change the beta function coefficient, $b_G = 3$ and the spectrum is in general given in terms of the number of messengers, $N_{\text{mess}}$, and $b_G$.

Therefore, in this scenario the mediation of supersymmetry breaking is a combination of two effects:

- **Gauge messenger contribution:**

  X and Y gauge bosons and gauginos contribute to the soft supersymmetry breaking terms. They become massive by the VEV of $\Sigma$ and gaugino masses get split due to $F_{\Sigma}$. Therefore, the messenger scale is the GUT scale. The ratio $|F_{\Sigma}|/|\Sigma|$ governs the common overall scale of soft SUSY breaking parameters given by gauge messengers. For convenience, we introduce $M_{\text{SUSY}}$ defined as:

  \[ M_{\text{SUSY}} = \frac{\alpha_{\text{GUT}}}{4\pi} \left| \frac{F_{\Sigma}}{\Sigma} \right|, \]

  which we use in expressions for all soft SUSY breaking parameters.

- **Matter messenger contribution:**

  If the additional vector-like messengers $\Phi$ and $\Phi^c$ couple to $\Sigma$,

  \[ W = \Phi \Sigma \Phi^c, \]

  they also contribute to the soft SUSY breaking terms.\(^3\) The matter messengers also become massive by $\Sigma$ VEV and mass splitting is given by $F_{\Sigma}$. The same $M_{\text{SUSY}}$ governs the common overall scale of soft SUSY breaking parameters given by the matter messengers.

The soft SUSY breaking parameters at the GUT scale (messenger scale) can be calculated by the powerful and convenient technique, so called “analytic continuation into superspace” \[^3\]. The results are derived in the Appendix A, here we only summarize them.

\(^3\)In principle it is possible to introduce an additional singlet superfield, whose $F$ component is non-zero, and which couples to the vector-like matter messengers. However, we consider only the minimal version in which $\Phi$ and $\Phi^c$ couple to the adjoint $\Sigma$ by which gauge messengers got their mass splitting.
Gaugino masses at the GUT scale ($\alpha_i = \alpha_{\text{GUT}}$) are (Eq. (A.12)):

$$M_i = [-2(5 - N_{C_i}) + N_{\text{mess}}]M_{\text{SUSY}},$$

where $N_{C_i}$ is the number of colors of the gauge group $SU(N_{C_i})$. More explicitly,

$$M_3 = (-4 + N_{\text{mess}})M_{\text{SUSY}}, \quad (3.3)$$
$$M_2 = (-6 + N_{\text{mess}})M_{\text{SUSY}}, \quad (3.4)$$
$$M_1 = (-10 + N_{\text{mess}})M_{\text{SUSY}}. \quad (3.5)$$

In the minimal messenger model ($N_{\text{mess}} = 0$), the gaugino masses at the messenger scale (the GUT scale) are

$$M_3 = -4M_{\text{SUSY}}, \quad (3.6)$$
$$M_2 = -6M_{\text{SUSY}}, \quad (3.7)$$
$$M_1 = -10M_{\text{SUSY}}. \quad (3.8)$$

Note that $|M_1| > |M_2| > |M_3|$ at the GUT scale ($|M_1| : |M_2| : |M_3| = 2.5 : 1.5 : 1$). As a result of RG evolution, at the weak scale we find $|M_1(M_Z)| : |M_2(M_Z)| : |M_3(M_Z)| \approx 1 : 2 : 1$. This is quite different from scenarios with the universal gaugino masses at the GUT scale which lead to gluino about 7 times heavier than bino at the EW scale.

Soft mass squared parameters for squarks and sleptons at the GUT scale with $N_{\text{mess}} = 0$ are given as (see Eq. (A.14)):

$$m_{\phi}^2 = \left( -2 \sum_i c_i b_{X_i} + 2\Delta c b_G \right) M_{\text{SUSY}}^2,$$

where $\Delta c = c_5 - \sum_{i=1}^3 c_i$ and $c_5, c_i$ are the quadratic casimirs of $\phi$ field under $SU(5)$ and standard model gauge groups, and $b_{X_i}$ are the contributions of messenger fields to the beta function coefficient. Detailed expression is given in the Appendix A. When there are additional chiral messengers, we would obtain (well known) additional gauge mediation contribution [31]. Explicit expressions for squark and slepton masses at the GUT scale are given as:

$$m_Q^2 = (-20 + 3b_G + \frac{21}{10}N_{\text{mess}})M_{\text{SUSY}}^2, \quad (3.10)$$
$$m_U^2 = (-16 + 4b_G + \frac{8}{5}N_{\text{mess}})M_{\text{SUSY}}^2, \quad (3.11)$$
$$m_D^2 = (-12 + 2b_G + \frac{7}{5}N_{\text{mess}})M_{\text{SUSY}}^2, \quad (3.12)$$
$$m_L^2 = (-12 + 3b_G + \frac{9}{10}N_{\text{mess}})M_{\text{SUSY}}^2, \quad (3.13)$$
$$m_{e}^2 = (-12 + 6b_G + \frac{3}{5}N_{\text{mess}})M_{\text{SUSY}}^2, \quad (3.14)$$
$$m_{H_u,H_d}^2 = (-12 + 3b_G + \frac{9}{10}N_{\text{mess}})M_{\text{SUSY}}^2. \quad (3.15)$$
In the minimal case ($N_{\text{mess}} = 0$), expressions are simplified:

\begin{align*}
    m_Q^2 &= (-20 + 3b_G)M_{\text{SUSY}}^2, \\
    m_{\ell^c}^2 &= (-16 + 4b_G)M_{\text{SUSY}}^2, \\
    m_{\tilde{e}^c}^2 &= (-12 + 2b_G)M_{\text{SUSY}}^2, \\
    m_L^2 &= (-12 + 3b_G)M_{\text{SUSY}}^2, \\
    m_{\tilde{e}^c}^2 &= (-12 + 6b_G)M_{\text{SUSY}}^2, \\
    m_{H_u, H_d}^2 &= (-12 + 3b_G)M_{\text{SUSY}}^2.
\end{align*}

(3.16) – (3.21)

Soft tri-linear terms are also calculated by adding individual contributions from three fields involved (Eq. (A.16)),

\begin{align*}
    A_{ijk} &= A_i + A_j + A_k, \\
    A_{\phi_i} &= 2\Delta c_{\phi_i} M_{\text{SUSY}}.
\end{align*}

(3.22) – (3.23)

More explicitly,

\begin{align*}
    A_t &= 10M_{\text{SUSY}}, \\
    A_b &= 8M_{\text{SUSY}}, \\
    A_\tau &= 12M_{\text{SUSY}}.
\end{align*}

(3.24) – (3.26)

The same result is given to the first and the second generation soft tri-linear terms as it just depends on gauge charges. Matter messengers ($N_{\text{mess}} \neq 0$) do not affect the boundary condition of soft tri-linear terms as in usual gauge mediation.

Negative sign in gaugino masses is absorbed by $U(1)_R$ symmetry rotation. $A$ and $\mu$ terms change sign accordingly. Thus, we choose the convention of $M_3 > 0$ in which $A < 0$ for $N_{\text{mess}} \leq 4$.

### 3.1 Characteristic Features

Gauge messenger models are very predictive, since the soft SUSY breaking parameters are calculable in terms of $M_{\text{SUSY}}$ and gauge quantum numbers of fields involved. The pattern of soft SUSY breaking terms is unique and distinctively different from other models. The most striking features are:

- Non-universal gaugino masses at the GUT scale:

\[ M_1 > M_2 > M_3 \]

The gaugino masses are non-universal even at the GUT scale though we started from the GUT models. It is the most interesting feature of the gauge messenger model. Furthermore, bino (and wino) is heavier than gluino at the GUT scale and the three gauginos have a tendency of gathering at the EW scale due to the usual running behavior of gauge couplings.
• (Non-universal) Negative squarks and sleptons masses squared at the GUT scale:

Gauge messenger contribution alone typically leads to the squarks and sleptons tachyonic at the GUT scale. However, this does not rule out the theory and just imply that we might live in a meta-stable vacuum rather than the true vacuum. From the discussion of fine tuning we learned that it actually might be more natural to live in a meta-stable vacuum. For \( 0 \leq b_G \leq 3 \), which is the case in realistic models due to a non-minimal content, squarks are even more negative, \( |m_{\tilde{q}}^2| > |m_{\tilde{l}}^2| \), \( m_{\tilde{q}}^2 < 0 \), \( m_{\tilde{l}}^2 < 0 \).

• Sizable \( A \) – terms:

Large \( A \) – terms is one of the unique feature of gauge messenger models which is absent in the usual gauge mediation. In usual gauge mediation, the soft tri-linear terms at the messenger scale are zero and are generated only by RG running. Here \( A_t \) is sizable and it will help to achieve close to maximal mixing scenario.

3.2 Contribution from Gravity Mediation

Since the messenger scale is the GUT scale, and the gauge mediation is a one loop effect, the naively estimated size of gravity mediation induced by non-renormalizable operators (suppressed by \( M_{\text{Pl}} \)) is comparable to the contribution from gauge mediation. The typical scale of gauge mediation is \( M_{\text{SUSY}} \), given in Eq. (3.1), and the typical size of gravity mediation is \( m_{3/2} = \left| \frac{F}{\sqrt{3}M_{\text{Pl}}} \right| \). Gravity to gauge mediation ratio is then

\[
\frac{m_{3/2}}{M_{\text{SUSY}}} = \frac{4\pi M_{\text{GUT}}}{\sqrt{3}g_{\text{GUT}}M_{\text{Pl}}} \simeq 1.5.
\]

Taking into account group theoretical factors appearing in the formulas for gauge mediation we see that the contribution of gravity mediation is of order 20% or 30% of gauge mediation for order one coupling of non-renormalizable operators.

There are several ways to deal with the contribution from gravity. It is possible to suppress this contribution entirely, e.g. by raising the cutoff scale of a theory beyond the Planck scale in superconformal frame or by lowering the GUT scale. Alternatively, one can actually use the contribution from gravity to generate the \( \mu \) term through the Giudice-Masiero mechanism \[\text{[32]}\]. The contribution from gravity can be also made universal, or sector dependent. Different possibilities for gravity contribution are discussed in detail in Appendix \[\text{[3]}\].

A combination of gauge messengers with gravity mediation clearly opens an unexplored direction for model building. When we present results in the next section we take a pragmatic approach and consider only the simplest possibilities for the contribution from gravity.

4. Results: SUSY spectrum, the Higgs mass and the LSP

In this section, we discuss SUSY and Higgs spectra in gauge messenger models. SUSY spectrum is calculated with SoftSusy \[\text{[4]}\] and for the calculation of the lightest CP even
Higgs mass we use FeynHiggs 2.4.1 \cite{5,6} (with $m_t = 172.5$ GeV). We focus mainly on the minimal scenario of gauge messenger model, $N_{\text{mess}} = 0$, $b_G = 3$, and only briefly discuss other choices of $N_{\text{mess}}$ and $b_G$. Depending on the way gravity mediation contributes to the soft SUSY breaking parameters we distinguish the following cases:

- **Pure gauge mediation:**

  The model is the most predictive when we assume the gravity contribution is suppressed to a negligible level. The suppression does not have to be huge since gauge mediation already dominates over the gravity mediation. Given the particle content of a model ($N_{\text{mess}}$ and $b_G$), a single parameter $M_{\text{SUSY}}$ determines all the soft SUSY breaking parameters in terms of measured gauge couplings and group theoretical factors. We do not address the origin of $\mu$ and $B\mu$ terms in this case and we treat them as free parameters (as usual, we exchange $B\mu$ for $\tan \beta$).

  Independent parameters : $M_{\text{SUSY}}$, $\mu$ and $\tan \beta$.

- **Gauge mediation with gravity contribution in the Higgs sector:**

  In this case we consider that only the Higgs sector gets a sizable contribution from gravity mediation. This opens a possibility of generating the $\mu$ term through Giudice-Masiero mechanism. The soft masses squared of $H_u$, $H_d$, and the $\mu$ and $B\mu$ terms are determined by $m_{3/2}$ with order one couplings.

  Independent parameters : $M_{\text{SUSY}}$, $m_{H_u}^2$, $m_{H_d}^2$, $\mu$ and $\tan \beta$.

- **Additional universal gravity contribution to scalar masses:**

  We also consider a possibility of adding universal scalar masses to the two scenarios above. Adding universal scalar masses does not change the spectrum in a crucial way (unless this contribution is huge). However, small addition to scalar masses might change the LSP in some region of parameter space, and consequently be responsible for very different phenomenology.

  Additional independent parameters: $m_0$.

Finally, we also calculate fine tuning necessary for correct EWSB \cite{33,34}, defined as:

$$
\Delta_p \equiv \left| \frac{\partial \ln M_{\text{Z}}}{\partial \ln p} \right| .
$$

where $p$ spans over free parameters in a given model. It can be easily estimated from the formula for $M_{\text{Z}}^2$, Eq. (2.2), customized for a given case, e.g. in the case of pure gauge mediation we have

$$
M_{\text{Z}}^2 \simeq -1.9 \mu^2 + \alpha M_{\text{SUSY}}^2 ,
$$

where $\alpha$ depends on $N_{\text{mess}}$, $b_G$ and $\tan \beta$. The fine tuning, $\Delta_{\mu} \simeq \Delta_{M_{\text{SUSY}}}$ in this case, gives us the precision with which the two terms have to cancel each other.
4.1 Pure Gauge Mediation

Let us start with the case of pure gauge mediation, $N_{\text{mess}} = 0$ and $b_G = 3$. The absolute value of $\mu$ is fixed by requiring proper EWSB and so only the sign of $\mu$ can be chosen.\(^4\)

In Fig. 1 we plot renormalization group running of soft SUSY breaking parameters for a particular choice of $M_{\text{SUSY}}$ and $\tan \beta$ which leads to some of the lightest SUSY spectrum possible given the current experimental bound on SUSY and Higgs particles. The detailed information about this point is given in the first column of Table 1. Varying $\tan \beta$ does not qualitatively change results and increasing $M_{\text{SUSY}}$ scales the whole spectrum up.

The plot in Fig. 1 is unlike anything we are familiar with from other models of SUSY breaking. None of the soft SUSY breaking parameters at the GUT scale is larger than 400 GeV and none of the superpartner is heavier than 400 GeV, and yet all the limits from direct searches for SUSY particles and also the limit on the Higgs mass are satisfied. Squark and slepton masses squared start negative at the GUT scale (except right-handed sleptons, in this case) and are driven to positive values by gaugino masses. First two generations of squarks and sleptons are somewhat heavier as in scenarios starting with positive scalar masses at the GUT scale. Gluino is much lighter than in most models as a result of the hierarchical boundary condition at the GUT scale, $|M_1| > |M_2| > |M_3|$. The soft trilinear coupling, $A_t$, is sizable at the GUT scale, which helps to achieve close to maximal mixing scenario. On the other hand, sizable $A_t$ also contributes to the running of $m_{H_u}^2$ proportional to $-|A_t|^2$, see Eq. (2.2). The smallest possible $\mu$ in this case is about 270 GeV which require about 5% tuning between $\mu$ and $M_{\text{SUSY}}$ to recover the correct $M_Z$.\(^5\)

\(^4\)We chose the positive sign of $\mu$ in all results to be in principle consistent with $b \to s\gamma$.

\(^5\)The current limit on chargino mass requires $\mu \gtrsim 150$ GeV. Thus any model which does not relate the $\mu$ term in a calculable way to soft SUSY breaking parameters requires at least 20% tuning from $\mu$. 
Figure 2: Allowed region of parameter space and the degree of fine tuning in the $M_{\text{SUSY}} - \tan \beta$ plane for pure gauge mediation, $N_{\text{mess}} = 0$ and $b_G = 3$. The shaded regions are excluded by direct searches for SUSY and Higgs particles. We use the limits on the mass of the lightest Higgs boson, $m_{h_0} > 114.4$ GeV, the lightest stop, $m_{\tilde{t}} > 95.7$ GeV, the lightest stau, $m_{\tilde{\tau}} > 81.9$ GeV, and the lightest chargino, $m_{\chi^\pm} > 117$ GeV. The region denoted as “tachyon” is excluded due to tachyonic spectrum. The black dashed line separates regions where sneutrino or stau is NLSP.

Since there are only two parameters in this model, it is easy to explore the whole parameter space. In Fig. 2 we show allowed parameter space in $M_{\text{SUSY}} - \tan \beta$ plane, together with regions excluded by direct searches for SUSY and Higgs particles. Moderate to large $\tan \beta$ is allowed and, as usual, as small $M_{\text{SUSY}}$ which still satisfies the limit on the Higgs mass is preferred by naturalness. In most region of the parameter space sneutrino is NLSP for small $\tan \beta$ (gravitino is the LSP) and stau is NSLP for large $\tan \beta$ due to large mixing of the left and right-handed stau. A representative point from this region is given in the first (stau NLSP) and the third column (sneutrino NLSP) of Table 1. As we will discuss later, small contributions from gravity mediation can easily push sneutrino or stau above the lightest neutralino leading to a large region where neutralino is (N)LSP.

4.2 Gauge mediation with gravity contribution in the Higgs sector

Adding a contribution from gravity mediation opens a possibility of generating the $\mu$ term using Giudice-Masiero mechanism. Comparable in size soft masses squared for $H_u$ and $H_d$ are also generated. We parameterize additional contribution to the Higgs soft masses squared by: $c_{H_u} M_{\text{SUSY}}^2$ and $c_{H_d} M_{\text{SUSY}}^2$. An example of the renormalization group running of soft SUSY breaking parameters in this case is given in Fig. 3 and detailed information about this scenario can be found in the second column of Table 1. Adding gravity contribution to soft Higgs masses squared does not significantly affect running of other soft SUSY...
breaking parameters. The major advantage of adding a positive contribution to $m_{H_u}^2$ is that it allows smaller $\mu$ term. This further reduces fine tuning of EWSB, see the Table 1, because the original (somewhat large) contribution from gauge mediation can be canceled in an equal way by the additional contribution from gravity and by the $\mu$ term.

Exploring the whole parameter space in this case is more complicated. In Fig. 4 we present a typical cut through the parameter space in $M_{\text{SUSY}} - c_{H_u}$ plane with fixed $\tan \beta$ and $c_{H_d}$. We see that, depending on the size of $c_{H_u}$, fine tuning from any of the parameters can be reduced to the level of 10%. Besides excluded regions that already appeared in the case of pure gauge mediation, Fig. 2, there is also a region excluded by limits on the stop mass. This is due to a subtle effect of larger $m_{H_u}$ in the evolution of stop masses squared. Stop masses squared run to slightly smaller values which increases stop mixing and consequently leads to much smaller value of the lightest stop mass eigenstate. For the same reason, besides neutralino (N)LSP and stau NLSP, there is a region with stop NLSP. The NLSP situation can be easily changed when considering contribution from gravity mediation also to squarks and sleptons.

4.3 Other cases

Adding a universal contribution to all scalar masses from gravity mediation has a negligible effect on the EW scale value of $m_{H_u}^2$. This can be easily seen from Eq. (2.2) in which the terms containing $m_{H_u}^2$ and $m_l^2$ approximately cancel each other for $m_{H_u}^2 = m_l^2 = m_0^2$ at the GUT scale. Therefore, adding $m_0$ (unless it is very large) does not change fine tuning of EWSB. The contribution from $m_0$ also makes stops heavier and reduces the mixing. This reduces the Higgs mass and so only small values of $m_0$ are allowed for small $M_{\text{SUSY}}$ – the region we are interested in. Small $m_0$ is however sufficient to change the NLSP of a model. For smaller $\tan \beta$ it can highly enlarge the region where neutralino is (N)LSP instead of sneutrino or stop, and for larger $\tan \beta$ it can basically eliminate the region where stop is NLSP.
Figure 4: Allowed region of parameter space and the degree of fine tuning in the $M_{\text{SUSY}} - c_{H_u}$ plane for gauge mediation, $N_{\text{mess}} = 0$ and $b_G = 3$, with a contribution from gravity mediation in the Higgs sector. We fix $\tan \beta = 25$ and $c_{H_d} = 50$. The meaning of excluded regions is the same as in Fig. 2. The black dashed line separates regions where $\tilde{t}_1$ and $\tilde{\tau}_1$ are NLSP.

So far we have discussed only the case with $N_{\text{mess}} = 0$ and $b_G = 3$. In Fig. 3 we also present plots of renormalization group running of soft SUSY breaking parameters for $N_{\text{mess}} = 0$ and smaller values of $b_G$ which correspond to adding more content to the minimal GUT scenario. And for completeness, in the same figure we also include $N_{\text{mess}} = 1$, $b_G = 2$ case which corresponds to the minimal GUT content with one pair of additional vector-like messengers in 5 and $\bar{5}$ of SU(5). In all cases $M_{\text{SUSY}}$ and $\tan \beta$ are fixed to the same value which allows us to see trends in the spectrum from changing the content of a model. For exactly this reason we do not require that all the experimental limits are satisfied in all models. Detailed information about these five points is given in the last five columns in Table 1. The basic features of all presented cases are very similar. Lowering $b_G$ results in lighter squark and slepton spectrum but slightly larger stop mixing. As a result, the Higgs mass is decreasing very slowly. Adding additional pair of messenger leads to lighter spectrum because of the cancelation between contributions from gauge messengers and vector-like messengers and thus in order for this scenario to be viable, larger $M_{\text{SUSY}}$ is needed. We do not discuss possible addition of gravity mediation for these scenarios.

4.4 LSP and NLSP

When there is a sizable contribution to the Higgs soft parameters from gravity mediation, neutralino can be LSP or NLSP depending on the gravitino mass. In this region, neutralino
is a sizable mixture of bino, wino and Higgsino. Sizable mixture of bino with higgsino/wino can give the right amount of thermal relic density for dark matter when $\mu$, $M_1$ and $M_2$ are of order 100 GeV. In addition, the cross section for the direct detection is larger compared to bino LSP which gives a better chance to observe it.

In most region of allowed parameter space, sneutrino/stau or stop is NLSP and the LSP is the gravitino. Gravitino LSP scenario with the right-handed stau NLSP has been studied in the framework of supergravity [36] [37] [38] [39] [40] [41]. The life time of the stau NLSP is from $10^6$ sec to $10^{10}$ sec and we might be able to detect it using a stopper. As we provide a specific model in which the gravitino LSP is very plausible, we can get a more concrete prediction on NLSP lifetime and gravitino relic density. Similar analysis should be done for the stop NLSP.

The gauge messenger model considered here generally predicts a very light stop, $m_{\tilde{t}_1} \gtrsim 150$ GeV in the least fine tuned parameter space. The Fig. 4 shows that stop becomes NLSP if $c_{H_u} \gtrsim 30$. Stop NLSP has been studied in [35] in the framework of low scale gauge mediation. When gravitino is very light, the decay of stop NLSP can happen quickly, within a minute, and the search for a possible collider signal can be done. If gravitino mass is at around the weak scale, stop decays long after the big bang nucleosynthesis (BBN). Usually decays of particles having hadronic channels destroy the successful agreement of BBN and such scenarios are not considered.\footnote{We thank Michael Peskin for discussion on this point.} Nonetheless the detailed analysis of stop decay after BBN should be done to clear up this issue. If stop NLSP with weak scale gravitino mass is consistent with BBN, more natural parameter space is allowed.

5. Conclusions

In this paper we studied gauge mediation of supersymmetry breaking in SU(5) SUSY GUT with heavy gauge fields as messengers. We were led to consider this gauge messenger model by recently discussed possibility of generating the maximal mixing scenario for the Higgs mass radiatively [7]. In the optimal scenario colored particles do not contribute to the renormalization group running of the $m^2_{H_u}$ which in principle can eliminate fine tuning of EWSB. The gauge messenger model does not lead to the optimal scenario (only close to it), since stop masses are not negative enough at the GUT scale. However, it still highly reduces the fine tuning of EWSB and has many interesting features.

In this scenario negative scalar masses squared at the GUT scale, with squarks more negative than sleptons, together with non-universal gaugino masses, with $M_1 > M_2 > M_3$, lead to a viable spectrum at the EW scale in large portion of parameter space. None of the soft SUSY breaking parameters at the GUT scale has to be larger than 400 GeV and none of the superpartner has to be heavier than 400 GeV to satisfy all the limits from direct searches for SUSY particles and also the limit on the Higgs mass. There is no other existing scenario with similar features. And yet, just like anomaly mediation or the usual gauge mediation, also this scenario is governed by a single parameter - the SUSY breaking scale $M_{\text{SUSY}}$. The ratios of different soft SUSY breaking parameters are entirely
fixed by group theoretical factors. The main features of the spectrum do not change when considering more complicated GUT models than the minimal scenario we focused on in this paper. And finally, considering contributions from gravity mediation not only opens a possibility to generate the \( \mu \) term through the Giudice-Masiero mechanism, but also can lead to many variation of the minimal scenario with interesting consequences for ongoing and future SUSY and dark matter searches.

The LSP in this model is the lightest neutralino (in a limited region of parameter space) which is a sizable mixture of bino, wino and higgsino, or gravitino (in most region of parameter space) with sneutrino or stop NLSP. Stau NLSP might be detectable using stopper and similarly for stop NLSP, but a detailed study is needed. In the case of stop NLSP it is important to clarify whether such a scenario is consistent with BBN.

The model predicts light stop, \( m_{\tilde{t}_1} \gtrsim 150 \text{ GeV} \), and light gluino, \( m_{\tilde{g}} \gtrsim 400 \text{ GeV} \), in the least fine tuned region of parameter space. Light gluino should be easy to see at the LHC or even at the Tevatron. In spite of stop being considerably lighter than other squarks, it might be easier to search for the first two generations of squarks at the Tevatron. Indeed, recent results from D0 and CDF collaborations for jets + missing transverse energy search, put strong constraints on masses of the first two generations of squarks and the gluino mass, in the range \( \sim 300 - 400 \text{ GeV} \). These limits will further improve in near future. At this point we would like to note that the results of both collaborations are presented in \( m_{\tilde{q}} - m_{\tilde{g}} \) plane for mSUGRA scenario and exclusion limits cover only gluino masses little larger than squark masses because otherwise there is no mSUGRA solution. However, squarks quite lighter than gluino are well motivated by natural EWSB. In the model presented here masses of gluino and the first two generations of squarks lie very near the border with no mSUGRA solution, and, as we discussed, models that would further improve on naturalness (with more negative stop masses squared at the GUT scale) would lead to squarks even lighter compared to gluino. Therefore we strongly encourage both D0 and CDF collaborations to extend the search and explore full kinematically allowed region in the squark-gluino plane so that also these scenarios are covered in addition to the not so natural one.

Considerations of natural EWSB in MSSM together with the current experimental limits on SUSY and Higgs spectra lead us to conclusions that SUSY spectrum might be quite strange and perhaps complicated (not unifying at any scale) compared to the usual expectations based on models like mSUGRA, and that there is a good chance we live in a meta-stable vacuum. However, as we showed, these seemingly unattractive features might be a consequence of the same elegant idea that leads to an understanding of quantum assignments of standard model particles and gauge coupling unification.

**Acknowledgement**

We thank K. Agashe, K. Choi, J. Ellis, G. Giudice, A. Kusenko, K.-I. Izawa, K.-I. Okumura, M. Peskin, S. Raby and N. Weiner for discussions. HK thanks the Galileo Galilei Institute for Theoretical Physics and CERN for hospitality and the INFN for partial support during the work. IK thanks LBNL for hospitality during his visit. RD is supported in part by
U.S. Department of Energy, grant number DE-FG02-90ER40542. HK is supported by the ABRL Grant No. R14-2003-012-01001-0, the BK21 program of Ministry of Education, Korea and the Science Research Center Program of the Korea Science and Engineering Foundation through the Center for Quantum Spacetime (CQUeST) of Sogang University with grant number R11-2005-021.
Appendix

A. Calculation of supersymmetry breaking parameters at the GUT scale

We closely follow the approach and notation given in [42, 43]. The idea is to treat couplings (gauge, Yukawa, wavefunction renormalization) as superfields whose scalar components are the couplings and F components are the gaugino masses. The outcome is that we can extract renormalization group properties of supersymmetry breaking parameters from renormalization group equations of ordinary couplings. It simplifies the calculation of soft supersymmetry breaking parameters.

The running of gauge couplings at one loop is given by

\[ \frac{d\alpha_i^{-1}}{d\log \mu} = \frac{b_i}{2\pi} \]  

(A.1)

where \( b_i = (3, -1, -\frac{24}{5}) \) for the three gauge couplings of MSSM and \( \mu \) is the renormalization group scale. Wavefunction renormalization (\( Z_Q \)) of a chiral superfield \( Q \) is given by anomalous dimensions,

\[ \log Z_Q(\mu) = \int_{\Lambda_{UV}}^{\mu} \frac{d\mu'}{\mu'} \gamma_Q(\mu') = \sum_i \frac{c_i}{\pi} \int_{\Lambda_{UV}}^{\mu} \frac{d\mu'}{\mu'} \alpha_i, \]  

(A.2)

where

\[ \gamma_Q = \frac{d\log Z_Q}{d\log \mu} = \sum_i \frac{c_i}{\pi} \alpha_i, \]  

(A.3)

with \( c_i \), the quadratic casimir. It can be rewritten as

\[ \log Z_Q(\mu) = \log Z_Q(\Lambda_{UV}) + \sum_i \frac{2c_i}{b_i} \log \frac{\alpha_i(\Lambda_{UV})}{\alpha_i(\mu)}. \]  

(A.4)

\[ Z_Q(\mu) = Z_Q(\Lambda_{UV}) \prod_i \left( \frac{\alpha_i(\Lambda_{UV})}{\alpha_i(\mu)} \right)^{\frac{2c_i}{b_i}}. \]  

(A.5)

Suppose that there is an adjoint chiral superfield \( \Sigma \) which breaks SU(5) down to the standard model gauge group. At high energy, the beta function coefficient of the GUT group is given as \( b_G = 3 \times 5 - 5 - 3 \times 2 - 1 = 3 \) for SU(5). Each term represents the contribution from vector supermultiplet of SU(5), the adjoint chiral multiplet of SU(5), three generation of matter fields and Higgs fields respectively. At \( M_{GUT} \), \( X, Y \) gauge bosons become massive by eating wouldbe Goldstone bosons in \( \Sigma \). Let us define \( b_F \) as the beta function coefficient excluding \( X, Y \) gauge bosons and \( b_H \) as the one for the low energy theory. Gauge messengers give

\[ b_G - b_{Fi} = 3(N_C - N_{Ci}) - (N_C - N_{Ci}), \]  

(A.6)

which is \( (4, 6, 10) \) for \( i = 3, 2, 1 \) gauge group respectively. There still remain (diagonal) adjoints of \( \Sigma \) under the low energy gauge group which we call \( \Sigma_3 \) and \( \Sigma_2 \) given by

\[ b_{Fi} - b_{Hi} = -N_{Ci}, \]  

(A.7)
which is \((-3, -2, 0)\) respectively. We call \(b_{X_i} = b_G - b_{F_i}\) as the beta function coefficient coming from fields that become massive by \(\Sigma\).

\[
\begin{align*}
  b_F &= b_{M_a} + b_H \\
  b_G &= b_X + b_{M_a} + b_H.
\end{align*}
\] (A.8)

At low energy, the degrees of freedom would be the usual gauge bosons (or vector multiplets) of 3,2,1 and matter and Higgs fields.

\[
b_{Hi} = 3N_{Ci} - 7,
\] (A.9)

which is \((2, -1, -7)\) respectively. We assume that the Higgs triplet mass is just below the GUT scale to simplify the discussion.\(^7\) The expression for the running of a gauge coupling is then written as follows:

\[
4\pi \alpha^{-1}(\mu) = 4\pi \alpha^{-1}(\Lambda) + b_X \log \frac{\Sigma^\dagger \Sigma}{\Lambda^2} + b_{M_a} \log \frac{M_a^2}{\Lambda^2} + b_H \log \frac{\mu^2}{\Lambda^2}.
\] (A.10)

Gaugino masses at the messenger scale are determined by analytic continuation of gauge couplings into superspace.

\[
M_i = -b_{X_i} \frac{\alpha_i}{4\pi} \left\lfloor \frac{F}{\Sigma} \right\rfloor = -b_X M_{\text{SUSY}},
\] (A.11)

where \(b_{X_i}\) is the contribution of fields which become massive by \(\Sigma\) and \(M_{\text{SUSY}}\) is defined in Eq. (3.1). If there are gauge messengers and matter messengers at the same time, \(b_{X_i} = 2(5 - N_{Ci})\) for massive \(X\) and \(Y\) superfields and \(b_{X_i} = -1\) for \(5\) and \(\bar{5}\) messengers. The explicit expression for the gauge messenger contribution with \(N_{\text{mess}}\) matter messengers is

\[
M_i = [-2(5 - N_{Ci}) + N_{\text{mess}}] M_{\text{SUSY}}.
\] (A.12)

For soft scalar masses, we consider the case in which \(M_a\) is slightly lower than the messenger scale, \(\Lambda \geq \Sigma \geq M_a \geq \mu\) so that we can write

\[
\log Z_Q(\Sigma, \Sigma^\dagger, \mu) = \log Z_Q(\Lambda) + \frac{2c_G}{b_G} \log \frac{\alpha_G(\Lambda)}{\alpha_G(\Sigma)} + \sum_i \frac{2c_i}{b_{M_{ai}} + b_{Hi}} \log \frac{\alpha_i(\Sigma)}{\alpha_i(M_a)} + \sum_i \frac{2c_i}{b_{Hi}} \log \frac{\alpha_i(M_a)}{\alpha_i(\mu)}.
\] (A.13)

We can assume that the scale difference between \(M_a\) and \(\langle \Sigma \rangle\) is negligible. With \(\xi_i = \frac{\alpha_i(\Sigma)}{\alpha_i(\mu)}\), the same calculation as in the previous subsection gives

\[
m_Q^2 = \left(2c_G b_G - \sum_i \frac{2c_i}{b_{M_{ai}} + b_{Hi}} b_G^2 + \sum_i \left(\frac{2c_i}{b_{M_{ai}} + b_{Hi}} - \frac{2c_i}{b_{Hi}} b_{X_i}^2 \right) \right) M_{\text{SUSY}}^2.
\]

\(^7\)In case when Higgs triplet is heavier than the GUT scale, the final expression becomes slightly complicated since it cannot be written in terms of single parameter \(b_G\). As Higgs triplet contribution does not make a significant change in the result, we take the simplest case (Higgs triplet slightly lighter than the GUT scale).
At $\mu = M_a \sim (\Sigma)$, we obtain soft scalar masses,
\[
m^2_Q = \left( 2c_G b_G + \sum_i \frac{2c_i}{b_{M_{a,i}} + b_{H_i}} (-b_{G_i} + b_{X_i}^2) \right) M^2_{\text{SUSY}}
\]
\[
= \left( 2\Delta c b_G - 2 \sum_i c_i b_{X_i} \right) M^2_{\text{SUSY}},
\]
(A.14)
where $\Delta c = c_5 - \sum_i c_i$. For the minimal content ($V, \Sigma, \text{Higgs}$ and matter fields), we have $b_G = 3$. By adding one extra $5 + \bar{5}$ messenger, $b_G$ is lowered by one.

The $A$ terms at the messenger scale are calculated by canonically normalizing the scalar fields,
\[
A_i(M) = \left. \frac{\partial \log Z_{Q_i}(\Sigma, \Sigma^\dagger, \mu)}{\partial \log \Sigma} \right|_{\Sigma = M} \frac{F}{M},
\]
(A.15)
In the gauge messenger model,
\[
A_Q(M) = 2\Delta c_Q M_{\text{SUSY}},
\]
(A.16)
and similarly for others. From the Table 2 we see $2\Delta c_Q = 3$, $2\Delta c_{H_u} = 3$ and $2\Delta c_{u^c} = 4$.

The $A$ term for top Yukawa coupling is then
\[
A_t(M) = A_Q + A_{H_u} + A_{u^c} = 10 M_{\text{SUSY}}.
\]
(A.17)

Quadratic casimirs and related parameters (e.g., $\Delta c, \sum_i c_i b_{X_i}$) used in the calculations are summarized in Table 2.

B. Suppression of Gravity Mediation

B.1 Large cutoff scenario

Gravity mediated contribution can not be neglected in gauge messenger model due to high messenger scale $M_{GUT} \left( \frac{m_{3/2}}{M_{\text{SUSY}}} \simeq 1.5 \right)$. The problem can be overcome either by raising up the cutoff scale of the theory beyond the Planck scale or lowering the messenger scale (GUT scale). There would be various ways of achieving it and here we illustrate some possibilities.

We consider superconformal frame and Einstein frame to discuss the problem. Fine tuning of electroweak symmetry breaking is not sensitive to the choice of frames but neutralino LSP (or NLSP) region can be enlarged in Einstein frame.

- Sequestering (Large cutoff in superconformal frame)

Conformal symmetry guarantees the stability of the sequestering once it happens at tree level.

\[
S_{\text{SUGRA}} = \int d^4 x \left[ \int d^4 \theta \bar{E} \left( -3M^2_{\text{Pl}} e^{-\frac{X}{3M^2_{\text{Pl}}}} \right) + \left\{ \int d^2 \theta \left( \frac{1}{4} f_{\alpha} W^{\alpha \beta} W_{\alpha \beta} + W \right) + \text{h.c.} \right\} \right],
\]
If Kähler potential is minimal in the superconformal frame,

\[-3M_{Pl}^2 e^{-\frac{K}{3M_{Pl}^2}} = \Phi^\dagger \Phi + \Sigma^\dagger \Sigma,\]

there would be no dangerous gravity mediation effect. We do not address how the conformal sequestering can be realized in our specific setup. Sequestered form of Kähler potential is understood by conformal symmetry. Conformal symmetry prevents higher dimensional operators. Sequestering means a large cutoff for possible non-renormalizable interactions with order one coefficients.

\[
-3M_{Pl}^2 e^{-\frac{K}{3M_{Pl}^2}} = \Phi^\dagger \Phi + \Sigma^\dagger \Sigma + \frac{1}{M_*^2} \Sigma^\dagger \Sigma \Phi^\dagger \Phi + \cdots,
\]

with \(M_* \gg M_{Pl}\). Note that \(M_* \sim 5M_{Pl}\) is enough to keep an accuracy of gauge messenger model within 1 or 2 percent. Required suppression is very small and slightly large cutoff might work without building a sequestering model.\(^8\)

- Large cutoff in Einstein frame

Universal soft scalar masses appear for the minimal Kähler potential in Einstein frame, \(K = \Phi^\dagger \Phi + \Sigma^\dagger \Sigma\).

\[
-3M_{Pl}^2 e^{-\frac{K}{3M_{Pl}^2}} = -3M_{Pl}^2 + K - \frac{1}{6M_{Pl}^2} K^2 + \cdots,
\]

\[
= \Phi^\dagger \Phi + \Sigma^\dagger \Sigma - \frac{1}{3M_{Pl}^2} \Sigma^\dagger \Sigma \Phi^\dagger \Phi + \cdots.
\]

The last term gives universal soft scalar masses to all \(\Phi\)s once \(F_\Sigma\) is nonzero\(^9\) and we have \(\delta V = m_{3/2}^2 \Phi^\dagger \Phi\). The problem associated with other unpredictable soft terms due to nonrenormalizable operators can be solved if a large cutoff of the theory is assumed [16, 17]. Let the cutoff of the theory be \(M_*\). We can imagine that matter sector couples weakly while gravity sector happens to couple strongly.

There are two ways to explain large cutoff. Firstly, we can start with the cutoff \(M_*\) and the observed Planck scale happens to be small due to the cancelation with loop corrections \(\delta M_*\).

\[
S = \int d^4x (M_*^2 + \delta M_*^2) R = \int d^4x M_{Pl}^2 R + \cdots.
\]

Numerically \(M_{Pl} \sim \frac{M_*}{4\pi} \sim \frac{M_*}{10}\). The observed Planck scale appears to be lower than the cutoff of the theory due to an accidental cancelation of the bare parameter and the quantum corrections. The other explanation comes with a strong coupling.

\[
S = \int d^4x \frac{1}{g^2} [M_*^2 R + \cdots].
\]

If the theory couples strongly, \(g \sim 4\pi\), we would get an effective Planck scale \(M_{Pl} = \frac{M_*}{g} \sim \frac{M_*}{4\pi}\). Now if \(M_* \sim 3.0 \times 10^{19}\) GeV, we would get the reduced Planck scale \(M_{Pl} = 2.4 \times 10^{18}\) GeV.

---

\(^8\)Gauginos can get a correction 10 to 15 percent in this case but this contribution does not lead to flavor changing neutral currents.

\(^9\)When there are several sources of supersymmetry breaking, all of them contribute to \(m_{3/2}\).
GeV. $M_*$ is much larger than $M_{Pl}$. It is natural to have a reduced Planck scale if the gravity couples strongly.

Similarly we can imagine that each sector can couple with a different strength. Naive dimensional analysis \[48\] \[49\] \[50\] tells us that
\[ K = \frac{M_*^2}{g^2} \tilde{K}(\frac{g_\Sigma}{M_*}, \frac{g_\Sigma'}{M_*}) \] (B.3)
where $\Sigma$ couples strongly with $g \sim 4\pi$. When there is a weakly coupled sector, we can add them to the Kähler potential as follows.
\[ K = \frac{M_*^2}{g^2} \tilde{K}_1(\frac{g_\Sigma}{M_*}, \frac{g_\Sigma'}{M_*}, \frac{e_\Phi}{M_*}, \frac{e_\Phi'}{M_*}) + \frac{M_*^2}{e^2} \tilde{K}_2(\frac{e_\Phi}{M_*}, \frac{e_\Phi'}{M_*}), \] (B.4)
where $\Phi$ represents all fields that couple weakly by itself with $e \sim 1$ and $\tilde{K}$ has polynomials with order one coefficients. Expanding Kähler potential up to quartic terms, we get
\[ K = \Sigma^\dagger \Sigma + \Phi^\dagger \Phi + \frac{g^2}{M_*^2} (\Sigma^\dagger \Sigma)^2 + \frac{e^2}{M_*^2} (\Phi^\dagger \Phi)^2 + \frac{e^2}{M_*^2} \Sigma^\dagger \Sigma \Phi^\dagger \Phi. \] (B.5)
Note that $M_{Pl} \sim \frac{M_*}{g} \sim \frac{M_*}{4\pi}$. We can consider the case in which matter fields $\Phi$ (quarks and leptons) couple weakly ($e \sim 1$) while Higgs fields $H$ and $\bar{H}$ couple strongly ($g \sim 4\pi$). Relevant terms in the Kähler potential would be
\[ K = \Sigma^\dagger \Sigma + H^\dagger H + \bar{H}^\dagger \bar{H} + \Phi^\dagger \Phi \\
+ \frac{1}{M_{Pl}^2} \Sigma^\dagger \Sigma H^\dagger H + \frac{1}{M_{Pl}^2} \Sigma^\dagger \Sigma \bar{H}^\dagger \bar{H} + \frac{1}{M_*^2} \Sigma^\dagger \Sigma \Phi^\dagger \Phi. \] (B.6)
Giudice-Masiero term
\[ K = \frac{1}{M_{Pl}} H \Sigma^\dagger \bar{H} + \frac{1}{M_{Pl}^2} \Sigma^\dagger \Sigma \bar{H} H, \] (B.7)
is suppressed only by $M_{Pl}$. This setup explains $\mu \sim m_{3/2}$ and $B\mu \sim m_{3/2}^2$. Let us summarize gravity mediated contributions on various fields when only Higgs fields couple strongly at $M_{Pl}$ and the large cutoff is realized for other fields in superconformal frame.
\[ m_{H_u}^2, \quad m_{H_d}^2, \quad \mu^2, \quad B\mu \sim m_{3/2}^2, \]
\[ m^2(\text{squarks, sleptons}) \sim \frac{m_{3/2}^2}{16\pi^2}, \]
\[ M_{4\pi}, \quad A \sim \frac{m_{3/2}}{4\pi}. \]
Note that gravity mediation is suppressed in squark and slepton soft scalar masses and gaugino masses. In Einstein frame, a common universal $m_{3/2}^2$ is added to all squarks, sleptons and Higgs soft scalar masses.
B.2 Lowering the GUT scale

Another way of suppressing gravity mediation is to lower the GUT scale. Although we have an indirect evidence that three gauge couplings meet at the GUT scale, $M_{\text{GUT}} = 2 \times 10^{16}$ GeV, any hints of $X$ and $Y$ gauge bosons have not been observed yet. The lower bound on their mass due to proton decay from dimension six operators is about $10^{15}$ GeV. If we can suppress the proton decay from dimension five operators related to color triplet Higgses, we can lower the GUT scale (more precisely the messenger scale, the mass of $X$, $Y$ gauge bosons and $X$, $Y$ gauginos). GUT scale threshold correction would then explain the illusion of having $M_{\text{GUT}} = 2 \times 10^{16}$ GeV. Furthermore, by adding extra matter fields in full multiplets of $SU(5)$, we can also make $\alpha_{\text{GUT}}$ larger than $1/24$ while keeping unification. This would enhance the gauge mediation effects even for messenger scale being the GUT scale. Finally, a combination of both effects might suppress gravity contribution to a negligible level.

References

[1] Y. Okada, M. Yamaguchi and T. Yanagida, “Upper bound of the lightest Higgs boson mass in the minimal supersymmetric standard model,” Prog. Theor. Phys. 85, 1 (1991).

[2] H. E. Haber and R. Hempfling, “Can the mass of the lightest Higgs boson of the minimal supersymmetric model be larger than $m_Z$?,” Phys. Rev. Lett. 66, 1815 (1991).

[3] J. R. Ellis, G. Ridolfi and F. Zwirner, “Radiative corrections to the masses of supersymmetric Higgs bosons,” Phys. Lett. B 257, 83 (1991).

[4] B. C. Allanach, “SOFTSUSY: A C++ program for calculating supersymmetric spectra,” Comput. Phys. Commun. 143, 305 (2002) [arXiv:hep-ph/0104145].

[5] S. Heinemeyer, W. Hollik and G. Weiglein, “FeynHiggs: A program for the calculation of the masses of the neutral CP-even Higgs bosons in the MSSM,” Comput. Phys. Commun. 124, 76 (2000). [arXiv:hep-ph/9812320].

[6] S. Heinemeyer, W. Hollik and G. Weiglein, “The masses of the neutral CP-even Higgs bosons in the MSSM: Accurate analysis at the two-loop level,” Eur. Phys. J. C 9, 343 (1999). [arXiv:hep-ph/9812472].

[7] R. Dermisek and H. D. Kim, “Radiatively generated maximal mixing scenario for the Higgs mass and the least fine tuned minimal supersymmetric standard model,” Phys. Rev. Lett. 96, 211803 (2006) [arXiv:hep-ph/0601036].

[8] E. Witten, “Mass Hierarchies In Supersymmetric Theories,” Phys. Lett. B 105, 267 (1981).

[9] S. Dimopoulos and S. Raby, “Geometric Hierarchy,” Nucl. Phys. B 219, 479 (1983).

[10] V. Kaplunovsky, “Nosonomy Of An Upside Down Hierarchy Model. 2,” Nucl. Phys. B 233, 336 (1984).

[11] K. Agashe, “GUT and SUSY breaking by the same field,” Phys. Lett. B 444, 61 (1998) [arXiv:hep-ph/9809421].

[12] K. Agashe, “Improved GUT and SUSY breaking by the same field,” Nucl. Phys. B 588, 39 (2000) [arXiv:hep-ph/0003236].
[13] J. R. Ellis, G. Ridolfi and F. Zwirner, “On radiative corrections to supersymmetric Higgs boson masses and their implications for LEP searches,” Phys. Lett. B 262, 477 (1991).

[14] A. Kusenko, P. Langacker and G. Segre, “Phase Transitions and Vacuum Tunneling Into Charge and Color Breaking Minima in the MSSM,” Phys. Rev. D 54, 5824 (1996) [arXiv:hep-ph/9602414].

[15] A. Kusenko and P. Langacker, “Is the vacuum stable?,” Phys. Lett. B 391, 29 (1997) [arXiv:hep-ph/9608340].

[16] J. A. Casas, A. Lleyda and C. Munoz, “Strong constraints on the parameter space of the MSSM from charge and color breaking minima,” Nucl. Phys. B 471, 3 (1996) [arXiv:hep-ph/9507294].

[17] J. L. Feng, A. Rajaraman and B. T. Smith, “Minimal supergravity with $m_0^2 < 0$,” arXiv:hep-ph/0512172.

[18] M. Dine and A. E. Nelson, “Dynamical supersymmetry breaking at low-energies,” Phys. Rev. D 48, 1277 (1993) [arXiv:hep-ph/9303230].

[19] M. Dine, A. E. Nelson and Y. Shirman, “Low-energy dynamical supersymmetry breaking simplified,” Phys. Rev. D 51, 1362 (1995) [arXiv:hep-ph/9408384].

[20] M. Dine, A. E. Nelson, Y. Nir and Y. Shirman, “New tools for low-energy dynamical supersymmetry breaking,” Phys. Rev. D 53, 2658 (1996) [arXiv:hep-ph/9507378].

[21] L. Randall and R. Sundrum, “Out of this world supersymmetry breaking,” Nucl. Phys. B 557, 79 (1999) [arXiv:hep-th/9810155].

[22] G. F. Giudice, M. A. Luty, H. Murayama and R. Rattazzi, “Gaugino mass without singlets,” JHEP 9812, 027 (1998) [arXiv:hep-ph/9810442].

[23] K. Choi, A. Falkowski, H. P. Nilles, M. Olechowski and S. Pokorski, JHEP 0411, 076 (2004) [arXiv:hep-th/0411066].

[24] K. Choi, A. Falkowski, H. P. Nilles and M. Olechowski, Nucl. Phys. B 718, 113 (2005) [arXiv:hep-th/0503216].

[25] K. Choi, K. S. Jeong and K. i. Okumura, “Phenomenology of mixed modulus-anomaly mediation in fluxed string compactifications and brane models,” JHEP 0509, 039 (2005) [arXiv:hep-ph/0504037].

[26] M. Endo, M. Yamaguchi and K. Yoshioka, “A bottom-up approach to moduli dynamics in heavy gravitino scenario: Superpotential, soft terms and sparticle mass spectrum,” Phys. Rev. D 72, 015004 (2005) [arXiv:hep-ph/0504036].

[27] A. Falkowski, O. Lebedev and Y. Mambrini, “Susy Phenomenology Of Kklt Flux Compactifications,” JHEP 0511, 034 (2005) [arXiv:hep-ph/0507110].

[28] K. Choi, K. S. Jeong, T. Kobayashi and K. i. Okumura, “Little SUSY hierarchy in mixed modulus-anomaly mediation,” Phys. Lett. B 633, 355 (2006) [arXiv:hep-ph/0508029].

[29] R. Kitano and Y. Nomura, “A solution to the supersymmetric fine-tuning problem within the MSSM,” Phys. Lett. B 631, 58 (2005) [arXiv:hep-ph/0509039].

[30] O. Lebedev, H. P. Nilles and M. Ratz, “A note on fine-tuning in mirage mediation,” arXiv:hep-ph/0511320.
[31] G. F. Giudice and R. Rattazzi, “Theories with gauge-mediated supersymmetry breaking,” Phys. Rept. 322, 419 (1999) [arXiv:hep-ph/9801271].

[32] G. F. Giudice and A. Masiero, “A Natural Solution To The Mu Problem In Supergravity Theories,” Phys. Lett. B 206, 480 (1988).

[33] J. R. Ellis, K. Enqvist, D. V. Nanopoulos and F. Zwirner, “Observables In Low-Energy Superstring Models,” Mod. Phys. Lett. A 1, 57 (1986).

[34] R. Barbieri and G. F. Giudice, “Upper Bounds On Supersymmetric Particle Masses,” Nucl. Phys. B 306, 63 (1988).

[35] C. L. Chou and M. E. Peskin, “Scalar top quark as the next-to-lightest supersymmetric particle,” Phys. Rev. D 61, 055004 (2000) [arXiv:hep-ph/9909536].

[36] J. L. Feng, “SuperWIMPs in supergravity,” arXiv:hep-ph/0308201.

[37] J. R. Ellis, K. A. Olive, Y. Santoso and V. C. Spanos, “Gravitino dark matter in the CMSSM,” Phys. Lett. B 588, 7 (2004) [arXiv:hep-ph/0312262].

[38] J. L. Feng, S. f. Su and F. Takayama, “SuperWIMP gravitino dark matter from slepton and sneutrino decays,” Phys. Rev. D 70, 065019 (2004) [arXiv:hep-ph/0404198].

[39] J. L. Feng, S. Su and F. Takayama, “Supergravity with a gravitino LSP,” Phys. Rev. D 70, 075019 (2004) [arXiv:hep-ph/0404231].

[40] J. R. Ellis, K. A. Olive and E. Vangioni, “Effects Of Unstable Particles On Light-Element Abundances: Lithium Versus Deuterium And He-3,” Phys. Lett. B 619, 30 (2005) [arXiv:astro-ph/0503023].

[41] F. D. Steffen, “Gravitino dark matter and cosmological constraints,” arXiv:hep-ph/0605306.

[42] G. F. Giudice and R. Rattazzi, “Extracting supersymmetry-breaking effects from wave-function renormalization,” Nucl. Phys. B 511, 25 (1998) [arXiv:hep-ph/9706540].

[43] N. Arkani-Hamed, G. F. Giudice, M. A. Luty and R. Rattazzi, “Supersymmetry-breaking loops from analytic continuation into superspace,” Phys. Rev. D 58, 115005 (1998) [arXiv:hep-ph/9803290].

[44] V. M. Abazov et al. [D0 Collaboration], “Search for squarks and gluinos in events with jets and missing transverse energy in p anti-p collisions at √s = 1.96 TeV,” Phys. Lett. B 638, 119 (2006) [arXiv:hep-ex/0604029].

[45] CDF preliminary results for jets + missing transverse energy search can be found on the web site, http://www-cdf.fnal.gov/physics/exotic/r2a/20060420.squarkgluino_metjets/

[46] M. Ibe, K. I. Izawa and T. Yanagida, “Realization of minimal supergravity,” Phys. Rev. D 71, 035005 (2005) [arXiv:hep-ph/0409203].

[47] M. Ibe, K. I. Izawa, Y. Shinbara and T. T. Yanagida, “Minimal supergravity, inflation, and all that,” Phys. Lett. B 637, 21 (2006) [arXiv:hep-ph/0602192].

[48] S. Weinberg, “Phenomenological Lagrangians,” Physica A 96, 327 (1979).

[49] M. A. Luty, “Naive dimensional analysis and supersymmetry,” Phys. Rev. D 57, 1531 (1998) [arXiv:hep-ph/9706255].

[50] A. G. Cohen, D. B. Kaplan and A. E. Nelson, “Counting 4pi’s in strongly coupled supersymmetry,” Phys. Lett. B 412, 301 (1997) [arXiv:hep-ph/9706275].
Figure 5: Renormalization group running of relevant soft SUSY breaking parameters for pure gauge mediation, for different choices of \( N_{\text{mess}} \) and \( b_G \), with \( M_{\text{SUSY}} = 45 \text{ GeV} \) and \( \tan \beta = 8 \).
| $(N_{\text{mess}}, b_G)$ | (0, 3) | (0, 3) | (0, 3) | (0, 2) | (0, 1) | (0, 0) | (1, 2) |
|----------------|--------|--------|--------|--------|--------|--------|--------|
| GUT parameter |        |        |        |        |        |        |        |
| $M_{\text{SUSY}}$ | 37     | 40     | 45     | 45     | 45     | 45     | 45     |
| $\tan \beta$   | 23     | 29     | 8      | 8      | 8      | 8      | 8      |
| $c_{H_u}$       | 38     |        |        |        |        |        |        |
| $c_{H_d}$       | 37     |        |        |        |        |        |        |
| EW scale parameter |        |        |        |        |        |        |        |
| $m_{Q_3}$       | 274    | 277    | 338    | 334    | 329    | 324    | 254    |
| $m_{U_3}$       | 219    | 211    | 254    | 248    | 242    | 236    | 173    |
| $m_{D_3}$       | 290    | 299    | 373    | 369    | 365    | 360    | 292    |
| $m_{L_3}$       | 133    | 130    | 178    | 161    | 141    | 118    | 138    |
| $m_{E_3}$       | 135    | 115    | 196    | 163    | 120    | 49.3   | 154    |
| $M_1$           | 149    | 161    | 183    | 183    | 183    | 183    | 162    |
| $M_2$           | 171    | 185    | 208    | 208    | 208    | 208    | 173    |
| $M_3$           | 369    | 400    | 440    | 440    | 441    | 441    | 345    |
| $\mu$           | 270    | 210    | 336    | 335    | 334    | 333    | 283    |
| $m_{\tilde{t}}$ | 245    | 242    | 293    | 288    | 282    | 276    | 210    |
| $A_t/m_{\tilde{t}}$ | -1.78  | -1.93  | -1.81  | -1.85  | -1.89  | -1.93  | -2.15  |
| Physical spectrum |        |        |        |        |        |        |        |
| $m_{h_0}$       | 114.4  | 115.6  | 115.3  | 115.2  | 114.9  | 114.1  | 110.1  |
| $m_A$           | 248    | 265    | 374    | 365    | 355    | 345    | 306    |
| $\tilde{t}_1$   | 138    | 101    | 192    | 182    | 171    | 159    | 44.0   |
| $\tilde{b}_1$   | 263    | 266    | 350    | 345    | 339    | 334    | 258    |
| $\tilde{\tau}_1$ | 103    | 88.2   | 182    | 159    | 123    | 61.0   | 141    |
| $\tilde{\nu}_r$ | 118    | 114    | 168    | 149    | 128    | 102    | 123    |
| $\tilde{u}_1, \tilde{c}_1$ | 340   | 365    | 405    | 396    | 386    | 375    | 309    |
| $\tilde{d}_1, \tilde{s}_1$ | 328   | 352    | 390    | 385    | 380    | 374    | 298    |
| $\tilde{e}_1, \tilde{\mu}_1$ | 158   | 169    | 188    | 172    | 135    | 78.7   | 166    |
| $\tilde{g}$     | 379    | 406    | 451    | 450    | 449    | 447    | 348    |
| $\chi_{1/2}^\pm$ | 158    | 152    | 204    | 203    | 203    | 203    | 165    |
| $\chi_{1/2}^0$  | 141    | 137    | 179    | 178    | 178    | 178    | 156    |
| $\Psi_{3/2}$    | > 55.5 | > 60   | > 67.5 | > 67.5 | > 67.5 | > 67.5 | > 67.5 |
| Fine Tuning     |        |        |        |        |        |        |        |
| $\Delta_{M_{\text{SUSY}}}$ | 17.6  | 10.6   | 26.4   | 26.2   | 26.1   | 26.0   | 18.8   |
| $\Delta_{\mu}$ | 18.2   | 11.1   | 30.6   | 30.4   | 30.2   | 30.0   | 21.9   |
| $\Delta_{c_{H_u}}$ | 8.96   |        |        |        |        |        |        |
| $\Delta_{c_{H_d}}$ | 0.462  |        |        |        |        |        |        |

Table 1: GUT input parameters, EW scale parameters, physical spectrum and fine tuning for gauge messenger models specified by $N_{\text{mess}}$ and $b_G$. All the mass parameters are understood in GeV units. Here $m_{\tilde{t}} = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$. We present only masses of the lightest squark and slepton in each generation.
$$\Delta c = \frac{3}{2} \quad 2 \quad 1 \quad \frac{3}{2} \quad 3$$

$$-2 \sum_i c_i b_{X_i} = -20 \quad -16 \quad -12 \quad -12 \quad -12$$

**Table 2:** Quadratic casimirs and their combinations relevant for calculation of soft SUSY breaking masses. $c_5$ is the quadratic casimir under $SU(5)$ and $c_{10}$ is the one under $SO(10)$. In the final expression, the minimal messenger model, $b_X = (4, 6, 10)$, has been used.