Numerical Manifold Method and Its Application in the Study of Crustal Movements in the Sichuan-Yunnan Area

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Abstract  The numerical manifold method (NMM) can calculate the movements and deformations of structures or materials. Both the finite element method (FEM) for continua and the discontinuous deformation analysis (DDA) for block systems are special cases of NMM. NMM has separate mathematical covers and physical meshes: the mathematical covers define only fine or rough approximations; as the real material boundary, the physical mesh defines the integration fields. The mathematical covers are triangle units; the physical mesh includes the fault boundaries, joints, blocks and interfaces of different crust zones on the basis of a geological tectonic background. Aiming at the complex problem of continuous and discontinuous deformation across the Chinese continent, the numerical manifold method (NMM) is brought in to study crustal movement of the Sichuan-Yunnan area. Based on the GPS velocity field in the Sichuan-Yunnan area, a crustal strain and stress field is simulated and analyzed. Moreover, results show that the NMM is a more suitable method than DDA in simulating the movement of the Sichuan-Yunnan area. Finally, a kind of mechanism of crustal motion in the Sichuan-Yunnan area is discussed in the paper.

Keywords  numerical manifold method; continuous deformation; discontinuous deformation analysis; Sichuan-Yunnan area; strain-stress field

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Introduction

In the past decades, some experts have tried to study crustal surface movements on a large scale by the finite element method (FEM) and by discontinuous deformation analysis (DDA); however, there are inherent defects in both methods. The former is usually used to calculate continuous crustal deformation, and discontinuous deformation is not taken into account. Inversely, the latter can only solve for discontinuous deformation in the faults zone and neglects the continuous deformation of the crustal block interior. For global analysis, the numerical manifold method (NMM) was developed to deal with the complex deformation, and FEM and DDA are special cases of NMM.

During the recent 15 years, several repetitive GPS
networks have been conducted in the Chinese continent. Employing local deformation velocity fields derived from high precision GPS measurements (1991~2003) in the Sichuan-Yunnan area (China), we use the NMM model to simulate the crustal movement of the Sichuan-Yunnan area (SCYNA), which has one of the most frequent seismic activities and strongest earthquakes in the Chinese mainland, or even globally.

1 Geological mesh model

Before modeling the crustal deformation of the SCYNA, which is located at the geographical coordinates 20~34ºN and 96~106ºE in southwestern China, a proper geological mesh model is required. According to the newest distribution map of faults and active blocks in this area[3], our best mesh model considers several main faults, and other inactive faults are neglected. For simplicity, the faults are approximately divided into several segments, the frictions of which are uniform and are equal to 0.8.

Around the SCYNA, there are six subsidiary blocks which are locked on their outer boundary, and they do not represent the real blocks but substitute the back holding effect of the South China plate and the Burma plate. The boundary between the SCYNA and the subsidiary blocks has a relatively low friction of 0.4. Generally, the elastic parameters of the SCYNA are slightly different. For simplicity, this model uses the uniform elastic modulus $E$ equal to 80 Gpa, which is derived from the primary wave velocity $v_p$ of this area and the Poisson ratio $\nu$ equal to 0.25.

Especially, we model with $E_1=0.9E$, $E_2=0.1E$, $E_3=E$, $E_4=E$, $E_5=E$, $E_6=0.1E$ (as marked in Fig.1) in the six subsidiary blocks respectively, and with Poisson solids (with Poisson ratio $\nu=0.25$) so that the subsidiary blocks can hold back the movement of the SCYNA appropriately.

NMM incorporates the sum of the faults and the boundary physical mesh, which represents real geological structures. For calculating the discontinuous deformation in the block interior, a mathematical mesh is added artificially which covers the entire physical mesh. In our mesh model, the triangle mesh, with its side proportional spacing, is adopted as the mathematical mesh. Considering any node of a triangle, all triangular units having this node form a mathematical cover. If the faults or boundary divide a mathematical cover into two or more completely dis

Fig.1  Faults and blocks of SCYNA
connected domains, these domains are defined as physical covers. In algebraic topology, the manifold element is composed of the common region of three physical covers (i.e. three nodes of the manifold element). The sides of the manifold element can contain part of the mathematical mesh or physical mesh.

However, in the DDA case, each block (i.e. basic element of DDA) is both a mathematical cover and physical cover, and there are no overlaps between any two covers. In the two-dimensional FEM case, there is a common part between any abutting mathematical covers or physical covers. There are also no overlaps between any two covers. In the two-dimensional FEM case, there is a common part between any abutting mathematical cover or physical cover.

2 NMM model and method

2.1 NMM model

In the FEM, any side is shared by two abutting elements. However in the two-dimensional NMM model, the physical mesh represents two sides of different abutting manifold elements. These sides can split, slide or hustle so that the vertex may pass the side of another manifold element after deformation. Since this is likely untrue, NMM uses the spring against the vertex back to the side (i.e., entrance line) along the shortest distance. The displacement \((u(x, y) \ v(x, y))\) of any point \((x, y)\) in element \(e\) can be expressed with the displacement of three physical covers and the weighted function as follows:

\[
\begin{bmatrix}
  u(x, y) \\
  v(x, y)
\end{bmatrix} =
\begin{bmatrix}
  w_1(x, y) & 0 & w_3(x, y) & 0 & w_1(x, y) & 0 \\
  0 & w_1(x, y) & 0 & w_2(x, y) & 0 & w_1(x, y)
\end{bmatrix}
\begin{bmatrix}
  u_1(x, y) \\
  v_1(x, y) \\
  u_2(x, y) \\
  v_2(x, y) \\
  u_3(x, y) \\
  v_3(x, y)
\end{bmatrix}
\]

(1)

where \((u_i(x, y) \ v_i(x, y))\) is the displacement of the physical cover \(i (i=1,2,3)\) in the latitudinal and longitudinal directions, respectively; \(w_i(x, y)\) is the weighted function, which is the ratio of the square of the angle composed of this point and any two nodes of \(e\) to the square of \(e\). Considering the elastic deformation of every manifold element and Eq.(1), the potential energies are integrated in the domain \(e\). Then the equilibrium equations are derived by minimizing the total potential energy with respect to the displacement variables of physical cover \(i\). After solving the equilibrium equations with successive overrelaxation (SOR), Eq.(1) will compute for the modeled GPS stations’ displacement based on their coordinates.

2.2 Driving mechanism of SCYNA and parameter estimation

One model has been proposed to describe the geodynamic mechanism. The major fault zones separating different active crustal blocks are viewed as lithospheric shear zones. Deformation in the upper part of the shear zone is characterized by brittle failure governed by friction laws\[^3\]. Stress is transmitted through the interaction of the blocks. Generally, the motion of SCYNA is mainly affected by movement of the Tibetan plateau, which is the grandest production of the collision and subsequent penetration of India into Eurasia\[^2\]. At the same time, crustal deformation is loaded by the total stress. Because stress on SCYNA is unknown, we assume that the deformation depends solely on stress change of the upper crustal shear zones. In taking this approach and the prior GPS velocities (Fig.2), the driving forces are imposed on the left top boundary of the geological model, and the blocks in the model are driven to interact with each other.

In this section, we will model the velocity vector from 1991 to 2003. It is necessary to adjust the boundary force and total time steps (i.e. the value of the accumulated displacement) so that the misfit between modeled and measured values is minimized by

\[
du = \frac{1}{n} \sum_{i=1}^{n} (u_i - \bar{u}_i)^2 = \min_{i=1,2,\cdots,n; \ j=1,2}
\]

(2)
where $u_{mi}^j$ is the GPS-measured velocity of station $I$; $u_{si}^j$ is the modeled velocity of the same station; $j$ represents the direction in longitude or latitude; $n$ is the number of GPS stations, and $du^j$ is the mean rootsquare (MRS) of the summed squared residual between modeled and measured velocities.

GPS data is adopted from the first bulletin of the Crustal Movement Observation Network of China (CMONOC), which published the velocities and coordinates of 176 stations in ITRF2000. Furthermore, we define a Eurasia-fixed reference frame by subtracting the Eurasia plate movement predicted by the NNR-NUVEL from GPS station motion in ITRF2000. The residual station velocities are deformations relative to a stable Eurasian continent.

After inverted calculations, we obtain a most fitted result (Fig.2(a)) when a clockwise torsion moment on the Qiangtang block is selected. Its RMS reaches 1.8 mm/a, 2.0 mm/a in the directions of longitude and latitude, respectively, which are equal to or less than the GPS stations’ measurement errors. However, the RMS is equal to 2.4 mm/yr and 2.8 mm/yr in the direction of the latitude and longitude in DDA, respectively(Fig.2(b)). For the Northern Sichuan block, Fig.2(a) shows that most misfits are near the XSHF; the modeled velocities are larger than the measured velocities. Maybe it is caused by the simplified XSHF which intersects GYF at Luhuo artificially because of mesh generation\(^4\) in the model. As a result, a possible small tension between the Northern Sichuan block and the Maerkang block is not considered. The figure also presents a relatively small modeled velocity in the Jinggu block, and it is probably affected by ignoring the viscosity between the Baoshan block and the Jinggu block, so that the compression to the Jinggu block is reduced. From the boundary force in the model, it appears that the motion of SCYNA may be produced together by right-rotating the Qiangtang block and the stable South China block. We also compute the stress on SCYNA by the NMM model (Fig.3(a)).

From Fig.3(a) we can see that it makes an extension in the S-N direction and a contraction in the E-W direction in the \(\Pi_1\) Northern Sichuan block, while it mainly takes an extension in the NE-SW direction and contraction in the NW-SE direction in the \(\Pi_2\) middle Yunnan block. We can also see that the direction of principal stress is different for different positions in the same block; however, there is only one direction of principal stress for different positions in the same block with the DDA method (Fig.3(b)).

### 3 Conclusions

In this paper, how to model the geological mesh and fit the GPS velocities by NMM is described. The velocity field model shows that the present-day movement of SCYNA can be explained well by block
boundary driving. The simulating results, such as the strain-stress field, show that motion of the SCYNA is mainly affected by movement of the Tibetan plateau, which is the grandest production of the collision and subsequent penetration of India into Eurasia. The results also show that the NMM is more suitable than DDA in simulating the movement of the Sichuan-Yunnan area.

The model, under assumptions of uniform faults and uniform media, is a simplified case of the SCYNA’s mechanical model in this paper. In the future, we should do much more detailed studies in this area based on NMM, associating it with geodetic, seismic and geological data, for example, considering a viscous medium.

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Notes to Contributors

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