A REALISTIC SUPERUNIFICATION

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Abstract. It is argued that every quark flavor can be described as a lepton that has absorbed a superunified field, which is defined. In this context, a minimal irreducible representation of SUSY SU(5) is constructed exclusively of the chiral modes that correspond to three generations of leptons (or quarks) and their scalar superpartners. The proposed model predicts a new quark—a left-handed (non-strange) version of the strange quark.

1. Quarks as Superunified Excitations of Leptons

If one assigns isotopic spins

\[ I_3(e_L^-) = -1/2 \] (1.1)

\[ I_3(e_R^-) = 0 \] (1.2)

and

\[ I_3(\nu e^- L) = +1/2, \] (1.3)

to the light generation of fermions, then the hypercharges of the light quark generation and the strange quark are given by \( Y = 2Q - 2I_3 \):

\[ Y\text{(up)} = 2(Q(e^{-L}) + (2/3)) - 2I_3(e^{-L}) = 2[0 + (2/3)] - 2(1/2) = 1/3 \] (1.4)

\[ Y\text{(down)} = 2(Q(e^{-L}) + (2/3)) - 2I_3(e^{-L}) = 2(-1 + (2/3)) - 2(-1/2) = 1/3 \] (1.5)

and

\[ Y\text{(strange)} = 2(Q(e^{-R}) + (2/3)) - 2I_3(e^{-R}) = 2(-1 + (2/3)) - 2(0) = -2/3, \] (1.6)

suggesting that a quark can be interpreted as a lepton that has absorbed an electrical charge of 2/3.

It will now be demonstrated that transition from a single lepton to a single quark occurs if the lepton absorbs a gluon and a spin-2 field that carries a charge of 2/3, a color and a null isotopic spin. It will also be demonstrated that this interaction is a consequence of the heterotic superstring, and specifically of the graviton vertex operator, which emerges from this version of the superstring. Because the proposed spin-2 field interfaces the graviton with electrical charge, color and isotopic spin, this field is defined as a superunified field.

Let us now consider the heterotic superstring; e.g., let us consider the gravitino state of momentum \( k \) that is described by the vector-spinor \( u^\mu \). Graviton emission

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from this gravitino ground state is produced by interaction of the gravitino with a bosonic right-moving (Neveu-Schwarz) prescription, which is tensored with the fermionic left-moving (Ramond) prescription:

\[
\epsilon_{\mu\nu}(\partial_\tau X^\mu_R(0) + \frac{1}{2}(\psi^\mu_R k\psi_R(0))\psi^\nu_L(0)e^{-ikX})
\]

Expression 1.7 describes what is known as the graviton vertex operator [J. Bailen, 1994]. The supersymmetric vertex that results from the above-described interaction is depicted by Figure 1, or alternatively by Figure 2.

The supersymmetry of the Figure 2 vertex is clearly preserved if the outgoing spin-(3/2) field is replaced by a fermion-boson pair of like helicity, as depicted by Figure 3. Moreover, if the ingoing fermion in Figure 2 is an electron’s neutrino and if the ingoing boson is a photon, then clearly, the outgoing spin-(2) and spin-(3/2) fields are just the usual graviton and gravitino; i.e. then the interaction is a supergravitational interaction. However, if the ingoing boson in Figure 3 is a gluon, and if the ingoing fermion is any lepton, then the outgoing fermion is necessarily a quark if the outgoing spin-2 field is a superunified field (as defined above). This is specifically established by considering the three possibilities, which are depicted by Figures 4, 5 and 6. The ingoing fermions in Figures 4, 5 and 6 are respectively the electron’s neutrino, \(\nu^-_L\), the LH electron, \(e^-_L\) and the RH electron, \(e^-_R\); while the outgoing fermions are respectively (based upon the emerging quantum numbers) the up, down and strange quarks. Thus (given the assignments of isotopic spin that are indicated by expressions 1.1 through 1.3) the tree-level, superunified interactions depicted by Figures 4, 5 and 6 clearly require the interpretation of quarks that is indicated by expressions 1.4 through 1.6. Moreover, because fermionic generations are indistinguishable at ultra high energies (early in the cosmological process); i.e. because fermions are massless at ultra high energies, the proposed supersymmetric theory permits that the same isotopic spins be assigned to \(\mu^-_L\) and \(\tau^-_L\) as to \(e^-_L\); to \(\nu^\mu_R\) and \(\nu^-_L\) as to \(\nu^-_L\), and to \(\mu^-_R\) and \(\tau^-_R\) as to \(e^-_R\). In this context, one can assign hypercharges to those quarks which, as fermions assume massive status, become the top, bottom and charmed quarks:

\[
Y(\text{top}) = 2[Q(\nu^-_R) + (2/3)] - 2I_3(\nu^\tau^-_R) = 2[0 + (2/3)] - 2(1/2) = 1/3
\]

\[
Y(\text{bottom}) = 2[Q(\tau^-_L) + (2/3)] - 2I_3(\tau^-_L) = 2(-1 + (2/3)) - 2(-1/2) = 1/3
\]

and

\[
Y(\text{charmed}) = 2[Q(\nu^-_L) + (2/3)] - 2I_3(\nu^\mu^-_L) = 2[0 + (2/3)] - 2(1/2) = 1/3
\]

Although this result departs from traditional quark theory [D. Nordstrom, 1992], symmetry appears to permit these assignments of isotopic spin and hypercharge.

In the above context, one is also motivated to consider the hypercharge that is generated when \(\mu^-_L\) absorbs a superunified field:

\[
Y(? = 2[Q(\mu^-_L) + (2/3)] - 2I_3(\mu^-_L) = 2(-1 + (2/3)) - 2(-1/2) = 1/3
\]

Expression 1.11 clearly indicates a quark that is not currently recognized. The strange quark is usually regarded as the generational partner of charmed; but the strange quark cannot be inserted here (the hypercharge of strange is -2/3). Thus, expression 1.11 predicts a new quark—a quark that is characterized by the same quantum numbers that associate with the strange quark, except strangeness, which
is zero for the predicted quark (the new quark, having an isotopic spin of -1/2, is left-handed). Note that this prediction cures an anomaly that is characteristic of the traditional theory of quark generations. Traditionally the up and top quarks are complemented by left-handed generational partners: the down and bottom quarks; while the charmed quark is, according to traditional theory, complemented only by a right-handed generational partner: the strange quark. The proposed theory cures this anomaly by providing a left-handed (non-strange) version of the strange quark. In the proposed theory, the strange quark is an analogue of the right-handed electron.

A second consequence of the proposed interpretation of quarks as superunified excitations of leptons is a SUSY SU(5) model that precisely accommodates three generations of fermions. This model will now be considered.

2. A SUSY SU(5) Model of Three Leptonic Generations

In the context proposed by Section 1, a minimal irreducible representation \( 5 \oplus 10 \) of SUSY SU(5) can be precisely constituted by the chiral modes that associate with the three leptonic generations and their scaler superpartners. The strictly leptonic realization of the anti-symmetric \( 10 = [5,2] \) of SUSY SU(5) can be given by

\[
10_{\text{LEP}} = \begin{bmatrix}
0 & e_L^c & \nu_L^c & \tau_L^c & \nu_{\text{L}}^{-}
- e_L^c & 0 & e_R & \tau_R & S_{e\text{L}}^{-}
- \nu_L^c & - e_R & 0 & S_{\tau^{-}} & S_{\nu(e)}
\end{bmatrix}
\]

where \( S_{e\text{L}}^{-} \) and \( S_{\tau^{-}} \) represent the scaler superpartners of \( e^- \) and \( \tau^- \) and where \( S_{\nu(e)} \) and \( S_{\nu(\tau)} \) represent the superpartners of the neutrinos. The strictly leptonic realization of the symmetric \( 5 = [5,1] \), complementing \( 10_{\text{LEP}} \), can be given by

\[
5_{\text{LEP}} = \begin{bmatrix}
\mu_L & \nu_L^c & \mu_R & S_{\mu^{-}}
\nu_L & 0 & \mu_R & S_{\nu(\mu)}
\end{bmatrix}
\]

where \( S_{\mu^{-}} \) is the scaler superpartner of \( \mu^- \) and \( S_{\nu(\mu)} \) is the superpartner of the muon’s neutrino. (Note that \( \mu_L \) and \( \mu_R \) correspond to the same scaler superpartner.) The realization of the anti-symmetric \( 10 = [5,2] \) in terms of quarks can be given by

\[
10_{\text{QRK}} = \begin{bmatrix}
0 & d & u & b & t
-d & 0 & s, S_B & s, S_A & S_d
-u & -s, S_B & 0 & S_b & S_t
-b & -s, S_A & -S_b & 0 & S_u
-t & -S_d & -S_t & -S_u & 0
\end{bmatrix}
\]
and the realization in terms of quarks of the symmetric 5 of SU(5) can be given by

\[
5_{\text{QRK}} = \begin{bmatrix}
? \\
c \\
s \\
S_d \\
S_t
\end{bmatrix}
\]

where the particle designated ‘?’ represents the new, predicted quark, where \(S_d, S_t, \) and \(S_b\) respectively represent the scaler superpartners of the down (d), top (t) and bottom (b) quarks, and where components \((s, S_A)\) and \((s, S_B)\) respectively represent the simultaneous production of a strange quark and boson that has absorbed a Higgs scalar \(S_A\); and the simultaneous production of a strange quark and a boson that has absorbed a Higgs scalar \(S_B\). (Note that the charmed quark \((c)\) and the quark designated by ‘?’ correspond to the same scaler superpartner.)

The scalars \(S_A\) and \(S_B\) are postulated to solve a problem that is intrinsic to the model under consideration. Specifically, the interaction that is depicted by Figure 6, which involves an RH electron and an LH gluon produces a strange quark, whether the right-handed lepton is an \(e^- R\), an \(\mu^- R\) or a \(\tau^- R\). It may be however, that this problem can be solved if one considers the chiral degrees of freedom, described above, that constitute the quark and lepton realizations of the irreducible representation \(5 \oplus 10\) of the proposed SUSY SU(5). The lepton realization of \(5 \oplus 10\) precisely accommodates the chiral modes that are represented by the three generations of left and right handed electrons, neutrinos and their scaler superpartners; but the chiral modes that correspond to the quark realization of this representation do not exhaust the available degrees of freedom, unless they include the modes that are distinguished by the introduction of scaler particles \(S_A\) and \(S_B\)—Higgs scalers, in terms of which the above-stated problem may find a solution. Specifically, the interaction that produces a strange quark from an \(e^- R\) (Figure 6) is understandable in terms of a hypothesis that the outgoing RH boson has absorbed an anti-scaler \(\Sigma_A\) of approximate mass \(200 \text{ GeV}/c^2\) (mass(\(e^-\)) \(\approx 51 \text{ GeV}/c^2\), mass(\(s\)) \(\approx 200 \text{ GeV}/c^2\)); and the interaction that produces the strange quark from a \(\tau^- R\), as depicted by Figure 7, is understandable in terms of a hypothesis that the outgoing RH boson of Figure 7 has absorbed a scaler \(S_B\) of approximate mass \(10^8 \text{ GeV}/c^2\) (mass(\(\tau^-\)) \(\approx 10^5 \text{ GeV}/c^2\), mass(\(s\)) \(\approx 200 \text{ GeV}/c^2\)). Finally, the outgoing RH boson of the interaction that is depicted by Figure 8 can be interpreted as a boson that has not absorbed a scaler (the masses of the strange quark and the \(\mu^- R\) are of the same order of magnitude).

The above hypothesis may also provide an explanation of the two mass scales \(M_X\) and \(M_Y\) that characterize SUSY SU(5) theories generally. Because the difference \([\Delta M]_{X-Y} \equiv M_X - M_Y \approx 10^{15} \text{ GeV}/c^2\) between the mass scales \(M_X = 10^{18} \text{ GeV}/c^2\) and \(M_Y = 10^9 \text{ GeV}/c^2\), is so large that the difference \(M_X - M_Y\) appears unrelated to \(M_A - M_B\). But \([\Delta M]_{X-Y}\) and \([\Delta M]_{A-B}\) may represent two states of a running hierarchy that depends upon the level of energy per particle. In this hypothesis, the sum of \([\Delta M]_{X-Y}\) and the energy level to which \([\Delta M]_{X-Y}\) corresponds would always equal \(10^{18} \text{ GeV}\); e.g. \([\Delta M]_{X-Y}\) at 1 TeV is about \(10^{15} \text{ GeV}/c^2\). In this context, a lower bound on the energy level where fermions can be massless can evidently be calculated; e.g. Since \([\Delta M]_{X-Y} \approx 10^5 \text{ GeV}/c^2\) at the energy level where the interactions depicted by Figure 6 and Figure 7 occur, the energies at
which fermions can be massless must be greater than or equal to $10^{13}$ GeV. Note that each component of $10_{LEPij}$: $i,j=1,2,3,4,5$ is transformed into its counterpart $10_{QRKij}$ by tree level interactions like those depicted by Figures 4, 5 and 6, and that each component $5_{LEP_i}$ is transformed into its counterpart $5_{QRK_i}$ by the same interactions; so that the postulated SUSY SU(5) symmetry is preserved by the proposed superunified interactions.

3. **Conclusion**

The graviton vertex operator was derived as usual from a bosonic, right-moving Neveu-Schwarz prescription that is tensored with a fermionic, left-moving Ramond recipe. It was observed that the supersymmetry of the graviton vertex is preserved if an outgoing gravitino is replaced with a fermion–boson pair of like helicity. Secondly, it was observed that if the ingoing fermion is a lepton of given $I_3$ (the third component of isospin), and if the ingoing boson is a gluon, then the outgoing fermion is a quark of the same isotopic spin if the outgoing spin-2 field is a superunified field, which was defined as a spin-2 field of color and of charge $2/3$, and null isotopic spin. In this context, every quark was characterized as a lepton that has absorbed a superunified field (it was noted that generations were not distinguished during the very early cosmological processes, and in this context it was argued that those quarks, which ultimately constituted the heavier generations of quarks can also be characterized as leptons that have absorbed superunified fields). A consequence of this result is that the chiral modes that correspond to the three leptonic generations and their superpartners precisely constitute a minimal irreducible representation $5 \oplus 10$ of SUSY SU(5). It was shown that this representation can also be constructed of quarks, and that this construction predicts a new quark--a left-handed (non-strange) version of the strange quark.

The construction of $5 \oplus 10$ in terms of quarks requires that two initially unused degrees of freedom be accounted for in terms of two additional scalers. These scalers are interpreted as two Higgs scalers, and are regarded as distinguishing the productions of a strange quark from the $e^+_R$, from the $\mu_R$ and from the $\tau_R$ at the energy level where fermions are massive. These scalers were also identified with the scalers that distinguish the X and Y particles of SUSY SU(5) theory. Specifically, it was postulated that the difference $[\Delta M]_{X-Y}$ is a function of energy level per particle; e.g. while $[\Delta M]_{X-Y}$ is approximately $10^{15}$ GeV/$c^2$ at a 1 TeV energy level, it was argued that the energy difference $[\Delta M]_{X-Y}$ is much smaller at a high energy level. Accordingly, the energy difference $[\Delta M]_{X-Y}$ was identified as equivalent to the difference $[\Delta M]_{X-Y}$ evaluated at the energy level below which fermions become massive. In the context of this hypothesis, it was concluded that $[\Delta M]_{X-Y} \equiv 10^{13}$ GeV/$c^2$ represents a lower bound for massless fermions. According to the proposed hypothesis, the difference $[\Delta M]_{X-Y}$ approaches zero as the energy level approaches $10^{18}$ GeV.

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