Modeling of processing technologies in food industry

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Abstract. Currently, the society is facing an urgent need to solve the problems of nutrition (products with increased nutrition value) and to develop energy-saving technologies for food products. A mathematical modeling of heat and mass transfer of polymer materials in the extruder is rather successful these days. Mathematical description of movement and heat exchange during extrusion of gluten-protein-starch-containing material similar to pasta dough in its structure, were taken as a framework for the mathematical model presented in this paper.

1. Introduction

The products with increased biological value include the so-called unconventional pasta products (products with wheat bran, dry gluten, fruit and vegetable pulp, starch, etc.). A considerable contribution to the production of unconventional pasta products was made by G.M. Medvedev, R.D. Polandova, V.D. Malkina, V.Ya. Chernykh, et al. In previous years, researchers designed various unconventional pasta production technologies. However, all of them are energy consuming and the resulting products have poor quality compared to traditional ones. At present, there is no uniform technology and equipment satisfying the requirements of both traditional and unconventional pasta production. In view of the aforesaid, there is a need to study the possibility of designing energy consuming and resource-saving equipment and technologies for traditional and unconventional pasta production.

Now there are various methods of pasta production [1, 2] that are characterized by different temperature and humidity of the dough itself [1-3]. According to G.M. Medvedev, the leading foreign manufacturers apply partial starch gelatinization in the production of pasta from unconventional raw materials (gluten-reduced or gluten-free materials) [2, 3]. This technology leads to a significant increase in energy intensity of production [3].

At the same time, the use of manufacturing processes ensuring direct contact of electric current with a product is considered as promising in the food industry. Simple equipment, high efficiency, short duration, relatively high temperature field uniformity, control and monitoring of energy parameters are very typical for the electrical contact method [4].

Yet the application of unconventional raw materials, partial starch gelatinization and electrical contact at the mixing stage leads to drastic changes of pasta rheological properties [5], and hence, causes the need to develop an operational tool that ensures their management during extrusion.
Rheological properties of pasta dough were studied by Kalinin Yu.V., Maksimov A.S., Nazarov N.I., Burov L.A., Machikhin Yu.A., Nikolaev B.A. [5-7].

Pasta is mainly formed via extrusion. The movement of pasta dough within the screw core is most accurately described by the Bingham plastic model [5, 8].

As a rule, extrusion is followed by drying pasta products. G.M. Medvedev, V.V. Mank, Jajima Mizio, Asama Chemical Co, Tecedor Silverio Luis, Schyr Richard F. etc. studied various drying methods applied to traditional and unconventional pasta production. They confirmed that the rheological properties obtained through extrusion of semi-finished products significantly influence the intensity and energy consumption of their drying [9, 10].

It should be noted that rheological properties of pasta dough during its extrusion were not previously studied, neither did rheological properties of pasta dough from unconventional raw materials and their measuring elements [11].

Mathematical modeling of heat and mass transfer of polymer materials in the extruder is rather successful these days. Tadmor Z., Rauvendaal K., Torner R.V., Vinogradov G.V., Volodin V.P., Shvartsman P., etc. made a great contribution to the study of heat and mass transfer in the extruder [12]. The models presented in their works imply processing of homogeneous easily plasticized materials, which, when heated, represent pseudoplastic bodies. These models are not suitable to describe the movement and heat exchange during extrusion of pasta dough since it is seen as an inhomogeneous elastoviscoplastic body containing starch kernels and protein threads, which cannot be considered in the above mentioned models.

Thus, it is relevant to develop a mathematical model for pasta dough extrusion.

Works by Kalinin Yu.V., Machikhin S.A., Nikolaev B.A., Timofeeva D.V., Korotkov V.G., Polishchuk V.Yu., Zubkova T.M., etc., devoted to the study of rheological properties of pasta dough and mathematical description of movement and heat exchange during extrusion of gluten-protein-starch-containing material similar to pasta dough in its structure, were taken as a framework for the mathematical model presented in this paper [5, 7].

The movement of pasta dough is described by the Bingham model for the general case:

\[
\dot{\gamma} = \frac{\dot{\tau}}{G} + \frac{(\tau - \tau_0)}{\eta_{sv}},
\]

where \(\dot{\gamma}\) – shear rate, s\(^{-1}\); 
\(\dot{\tau}\) – shear stress variation rate, Pa/s; 
\(\tau\) – shear stress, Pa; 
\(\tau_0\) – shear yield point, Pa; 
\(G\) – modulus of elasticity under shearing strain, Pa; 
\(\eta_{sv}\) – shear viscosity, Pa·s.

The authors suggest using an extruder output torque measured via special nozzles to assess the intensity of transformations within the extruder [12].

Under such circumstances, the torque equation is as follows:

\[
M_c = (\dot{\gamma} \cdot \eta_{sv} \cdot \tau_0) \left(1 - e^{-\frac{G}{\eta_{sv} \cdot \tau_0}}\right) \cdot 2\pi RL^2
\]

where

\(R\) – nozzle inner radius, m; 
\(t\) – passage time of processed material through a nozzle, s; 
\(L\) – length of a nozzle channel, m.
To assess the intensity of extrusion impact on the processed product in a matrix die, it is possible to use the average shear rate of pseudoplastic material, which can be defined according to the following expression:

$$\dot{\gamma}_{i.p.p} = R_{i} \left( \frac{\tau_{cm} - 2\tau_{0}}{4\eta_{lv}} \right) - \frac{2\tau_{cm}}{3G}$$  \hspace{1cm} (3)

where $\tau_{cm}$ – shear stress against a die wall;
$R_{i}$ – die radius, m.

Values $\tau_{0}, \eta_{sv}, G$ presented in expressions 2 and 3 shall be further identified.

To ensure identification of variables, the following regression equations were obtained:

$$\tau_{0} = 14.5127 - 1.6485 \cdot t_{i} + 0.9319 \cdot M_{C} \cdot t_{i} + 1.0241 \cdot t_{i} \cdot \omega_{y} +$$
$$+ 0.887 \cdot M_{C} \cdot \omega_{y} + 0.9825 \cdot M_{C} \cdot t_{i} \cdot \omega_{y} + 1.4076 \cdot M_{C}^{2} - 2.1291 \cdot \omega_{y}^{2}$$  \hspace{1cm} (4)

$$\eta_{sv} = 0.2113 + 0.0095 \cdot M_{C} - 0.019 \cdot t_{i} + 0.0117 \cdot t_{i} \cdot \omega_{y} +$$
$$+ 0.01424 \cdot M_{C} \cdot t_{i} \cdot \omega_{y} - 0.0229 \cdot M_{C}^{2} - 0.0249 \cdot \omega_{y}^{2}$$

$$G = 0.0659 - 0.0071 \cdot t_{i} + 0.0078 \cdot \omega_{y} + 0.0098 \cdot M_{C} \cdot t_{i} +$$
$$+ 0.0094 \cdot M_{C} \cdot \omega_{y} + 0.0072 \cdot M_{C} \cdot t_{i} \cdot \omega_{y} + 0.0207 \cdot M_{C}^{2} - 0.0212 \cdot \omega_{y}^{2}$$

where $M_{C}$ – reference mixing moment of raw materials (depends on chemical composition and properties of raw materials, defined using a special nozzle [12]);
$t_{i}$ – reference temperature of pasta dough;
$\omega_{y}$ – extruder screw, rpm.

Values $M_{C}, t_{i}, \omega_{y}$ are given in conventional units.

The following equations are used to transfer conventional units into physical ones:

$$M_{C}^{'} = 0.0013 \cdot M_{C} - 1.6667$$
$$t_{i}^{'} = 0.0667 \cdot t_{i} - 2.6667$$
$$\omega_{y}^{'} = 0.0333 \cdot \omega_{y} - 2$$  \hspace{1cm} (5)

where the UM values are as follows: $M_{C}^{'}$ - N·m, $t_{i}^{'}$ - c, $\omega_{y}^{'}$ - rad/s.

To define the boundaries of model application, it was recommended to establish the extrusion parameter value ranges.

To define the parameter value ranges of pasta extrusion, the three-factor experiments based on composite orthogonal design (complete factorial experiment (CFE) $2^3$) using the following parameters were made:

- mixing torque;
- angular rotation rate of a screw;
- resistance coefficient of an extruder matrix.
The following were accepted as the measures of efficiency:

- integrated quality index of pasta products (IQI);
- expert assessment of raw and finished pasta properties (EA);
- energy unit costs of the corresponding process (EUC).

The following regression equations, which precisely describe the experimental data at significance level $P = 0.95$, were obtained through experiments:

- for expert assessment of pasta properties:

$$
EA = 373.75 - 93 \cdot \omega_y - 12.12 \cdot M_c - 100.9 \cdot k_{cm} - 17.37 \cdot \omega_y \cdot M_c + \\
+ 20.2 \cdot M_c \cdot k_{cm} + 23.12 \cdot \omega_y \cdot M_c \cdot k_{cm} + 52.24 \cdot \omega_y^2 + 63.63 \cdot M_c^2 - 134.7 \cdot k_{cm}^2
$$

– integrated quality index of pasta products:

$$
IQI = 68.92 - 12.57 \cdot \omega_y - 2.3 \cdot M_c - 9.83 \cdot k_{cm} + 3 \cdot \omega_y \cdot k_{cm} + 19.5 \cdot \omega_y^2 + \\
+ 9.5 \cdot M_c^2 - 21.3 \cdot k_{cm}^2
$$

– for energy unit costs for mixing and pressing:

$$
EUC = 63.13 - 10.7 \cdot \omega_y + 18.18 \cdot M_c - 11.8 \cdot k_{cm} - 4.25 \cdot \omega_y \cdot M_c + \\
5.58 \cdot M_c \cdot k_{cm} - 4.16 \cdot \omega_y \cdot M_c \cdot k_{cm} - 4.19 \cdot \omega_y^2 - 1.8 \cdot M_c^2 + 1.3 \cdot k_{cm}^2
$$

where

- $\omega_y$ – angular rotation rate of a screw extrusion machine;
- $M_c$ – torque after mixing;
- $K_{cm}$ – matrix resistance coefficient.

To transfer the values that appear in equations into physical values, it is possible to use the expressions stated below:

$$
\omega_y' = 5 \cdot \omega_y + 15
$$

$$
M_c' = 750 \cdot M_c + 1250
$$

$$
k_{cm}' = 18 \cdot k_{cm} + 22
$$

A semigraphical optimization of the process was carried out by building response surfaces and their overlapping. Fig. 1 shows an example of the obtained response surface.
**Figure 1.** Dependence of integrated quality index on extruder shaft frequency rate and torque after mixing at $K_{cm} = 4\%$ (-1 c.u.)

Horizontal projections of obtained response planes were used in semigraphical optimization.

**Figure 2.** Extrusion optimization, $K_{cm} = 4\%$ (= -1 c.u.)
Figure 3. Pressing optimization ($K_{cm} = -0.5$)

The diagram (Fig. 3) obtained as a result of surface overlapping (Z1, Z1, I1) shows that for $k_{cm} = -0.5$ the optimal area is limited by lines: EA = 550 points, IQI = 110 points, EUC = 60 W/kg. At the same time, the angular rate of the extruder screw shall make from 30 to 38 rad/s (-1÷-0.73 in dimensionless expression), the torque after mixing shall amount from 500 to 562 N*m (-1÷-0.917 in dimensionless expression).
Figure 4. Pressing optimization ($K_{cm} = 0$)

The diagram (Fig. 4) obtained as a result of surface overlapping (Zh2, Z2, I2) shows that for $k_{cm} = 0$, the optimal area is limited by lines: $EA = 550$ points, $IQI = 110$ points, $EUC = 50$ W/kg. At the same time, the angular rate of the extruder screw shall make from 30 to 33 rad/s ($-1 \div -0.9$ in dimensionless expression); the torque after mixing shall amount from 500 to 575 N*cm ($-1 \div -0.9$ in dimensionless expression).
Figure 5. Pressing optimization (Kcm = 0.5)

The diagram (Fig. 5) obtained as a result of surface overlapping (Zh3, Z3, I3) shows that for $k_{cm} = 0.5$, the optimal area is limited by lines: EA = 450 points, IQI = 90 points, EUC = 30 W/kg. At the same time, the angular rate of the extruder screw shall make from 30 to 37.4 rad/s ($-1÷0.753$ in dimensionless expression); the torque after mixing shall amount from 500 to 650 N*m ($-1÷0.8$ in dimensionless expression).
Figure 6. Pressing optimization (K\textsubscript{cm} = 1)

The diagram (Fig. 6) obtained as a result of surface overlapping (Zh4, Z4, I4) shows that for K\textsubscript{cm} = 1, the optimal area is limited by lines: EA = 300 points, IQI = 70 points, EUC = 40 W/kg. At the same time, the angular rate of the extruder screw shall make from 30 to 36.9 rad/s (-1÷-0.77 in dimensionless expression); the torque after mixing shall amount from 500 to 687 N*m (-1÷-0.75 in dimensionless expression).

Eventually, the best effect parameters were obtained for a matrix with K\textsubscript{cm} = 0 c.u. The reduction of the matrix resistance coefficient leads to a minor decline in the quality of semi-finished products and to the increase in energy unit costs. The increase of the matrix resistance coefficient leads to the decrease in energy unit costs alongside with considerable deterioration of semi-finished products.

The results of semigraphical optimization for mixing and pressing are shown in Tab. 1.

| K\textsubscript{cm}, % | \(\omega\) \text{rpm} | \(M\) \text{N*m} |
|---|---|---|
| 4 | 30…34 | 500…665 |
| 13 | 30…38 | 500…562 |
| 22 | 30…33 | 500…575 |
| 31 | 30…37.4 | 500…650 |
| 40 | 30…36.9 | 500…687 |
The developed mathematical model will make it possible to calculate rheological properties of pasta dough in any given point of an extruder by measuring the torque at extruder outlet, which, in turn, will provide for accurate calculation of extruder design.

The obtained equations of relationship between model equation coefficients and process parameters allow considering characteristics of processed material within the mathematical model.

The given efficiency ranges ensure the highest quality of pasta products.

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