Abstract

In cryptography, the security of any algorithm relies on the strength of the key used and nonlinear mapping of the original information or data. It is desirable to have resistance against differential cryptanalysis, which assists in providing clues about the composition of keys, and linear secret system, where a simple approximation is created to copy the original cipher characteristics. The objective of the proposed work is to make the existing cipher techniques more prone to cryptanalysis by incorporating the proposed S-box in the design. In this paper, the use of nonlinear functional chaos-based substitution process is proposed which employs a set of differential equations called Lorenz equations with given initial parameters. The performance of the new substitution box is evaluated through simulation and data analytics tool. During testing, it has been found that the proposed technique produces high Standard Deviation (112.84) and negative correlation factor (-0.161) which makes it applicable where security against cryptanalysis is a major concern.

Keywords: Chaotic Function, Cipher, Cryptanalysis, Lorenz System, Substitution Box

1. Introduction

The purpose of providing security to the data is creating confusion to the eavesdropper via introduction of randomness in the data. The random or non-linear behavior of chaotic systems exhibit demanding properties suitable for nonlinear dynamic systems such as the substitution process in a cipher without independent round keys. All such chaotic systems are highly sensitive to initial conditions and exhibit random behavior, which is deterministic if the initial information is available. If this initial information is absent, the system appears to be random to any observer or intruder. These properties are desirable and suitable in the design of cryptographic systems. The application of chaotic sequences to the construction of substitution boxes, used in various cipher schemes such as Advanced Encryption Standard (AES) etc., is capable of creating confusion and applying diffusion to the original data. The substitution process in the AES encryption technique is the only nonlinear part, which creates confusion and secures the information. The substitution process is accomplished by the use of the Substitution-box (S-box) that is an array of size $n \times n$.

Chaotic ciphers are prone to attacks like denial of service and inability of resisting the privileged insider attack. These problems were formed as objective in their paper and provided a security mechanism to fight against those attacks. But to fight against attacks, a large order of keys with novel approach to construct those keys is required. It’s not only the key who plays the vital role against cryptanalysis but strong substitution box is also recommended. Different criteria’s can be used for the evaluation of the Substitution boxes like Strict Avalanche Criterion, Bit Independent Criterion, nonlinear criterion etc. Further the constituted S-box from this principle can be used in AES and with other techniques to find its optimized allocation. With these criteria’s, one can calculate the algebraic and statistical encryption strength cum weakness of the S-box. Variety of Substitution boxes are made over the time for different
applications. For image applications S-box substitution boxes are NCA chaotic sequences are suggested which can provide security against classic attacks\textsuperscript{11}. To analyze the strength of S-box in image encryption, root mean square error method\textsuperscript{13} and majority logic criterion can be used\textsuperscript{14}.

Several methodologies for the construction of cryptographically strong S-boxes have been implemented in the past. One such method is devised as linear fractional transform method. After testing, it is found that it has more confusion creating capabilities as compared with the counterparts i.e. AES, Skipjack etc\textsuperscript{17}. In\textsuperscript{24}, a method is proposed which relies on an in-depth search to construct a new S-box. Here the construction of new S-boxes with large values of \( n \) is computationally complex. Looking at the methods used by cryptanalysis\textsuperscript{20}, an S-box of size \( 5 \times 5 \) was suggested in\textsuperscript{1} with strong resistance to differential cryptanalysis. In today’s era, the theory of chaos is also used for the construction of S-boxes. In another construction method based on this technique\textsuperscript{6}, a 3D chaotic Baker map is used to generate an \( 8 \times 8 \) S-box. This method exhibited some attractive properties concerning to robustness and resistance to cryptanalysis. This method is again improved by the use of a continuous-time chaotic Lorenz system\textsuperscript{19}.

In order to obtain discrete data from the chaotic system, the system trajectory values are converted to digital numbers for selected time steps and a linear functional algorithm\textsuperscript{12} is applied to these coded discrete outputs. This method exhibits cryptographically strong properties as compared to other algorithms, which synthesize S-boxes based on chaotic methods. In this paper, we mainly relate our chaotic system with linear functional transformation in order to generate a strong S-box. Strength of an S-box lies in the unique combinations of the values present in the S-Box. AES uses a technique referred as affine-power-affine structure which creates 40320 unique instances\textsuperscript{18}.

Chaotic ciphers are prone to attacks like denial of service and inability of resisting the privileged insider attack. These problems were formed as objective in their paper\textsuperscript{5} and provided a security mechanism to fight against those attacks. But to fight against attacks, a large order of keys with novel approach to construct those keys is required\textsuperscript{6}. It’s not only the key who plays the vital role against cryptanalysis but strong substitution box is also recommended\textsuperscript{7}. Different criteria’s can be used for the evaluation of the Substitution boxes like Strict Avalanche Criterion, Bit Independent Criterion, nonlinear criterion etc. Further the constituted S-box from this principle can be used in AES and with other techniques to find its optimized allocation\textsuperscript{6}. With these criteria’s, one can calculate the algebraic and statistical encryption strength cum weakness of the S-box.

2. Lorenz System of Equations

The Lorenz system was initially used to design atmospheric model\textsuperscript{19} in 1950 and is the first numerical study of chaos. It is a set of three differential equations with three variables. The system dynamics are represented by the equations shown below:

\[
\begin{align*}
\frac{dx}{dt} &= a(y - x), \\
\frac{dy}{dt} &= (\beta x - y - xz), \\
\frac{dz}{dt} &= (xy - yz)
\end{align*}
\]

**Equation 1**: Lorenz system of equations.

Where the values of the parameters are \( a = 10, \beta = 28 \) and \( \gamma = 8/3 \). The intervals used in the states of the system are \(-40 \leq x \leq 40, -40 \leq y \leq 40 \) and \(-40 \leq z \leq 40 \). The system exhibits chaotic behavior for the selected parameters and intervals\textsuperscript{5}.

Initial work depicts a new way to synthesize S-boxes based on chaotic system. The synthesis process is divided into a few different stages. The numerical methods are used to generate and solve the given chaotic system. The process continues by converting the results from the first step into integer values. The range of these values is from 0 and 255. In the last step, the proposed algorithm extracts only distinct values generated by this chaotic system. The refined results are then utilized in the construction of S-boxes used in the block ciphers, which exhibit strong encryption properties.

3. Proposed Algorithm

- First of all obtain \( x, y, z \) trajectories by solving the above set of equations with selected set of inputs and chaotic parameters in Matlab software. The plots of the \( x, y \) and \( z \) trajectories are shown in Figure 1, 2 and 3.
- Take any one trajectory from above mentioned trajectories of \( x, y \) and \( z \) and then code it at every step starting from 0 to 255.
- After removing all repeated values, 256 distinct values are obtained which is distributed randomly.
Now make 16 different 4 x 4 matrices say M1, M2…M16 from the array such that first value goes to M1, second value goes to M2 and so on.

Now reshape these 16 (4 x 4) matrices to form one 16 x 16 matrix which is the resultant S-box. Table 1 shows the resultant matrix prepared using the proposed algorithm.

4. Analysis of the Proposed S-box

For the purpose of the evaluating the cryptographic strength of S-box, SPSS tool is used where Paired Sample T Test is applied to compare the strength of existing S-box with the proposed S-box. Comparison is done on the following parameters:

4.1 Standard Deviation

It is a method used to measure the amount of variation of a set of data values. A standard deviation value which is close to 0 means that the data points tend to be very close to the mean (or the expected value) of the set, while a high value of standard deviation states that the data points are spread out over a much wider range of values.

4.2 Correlation

The more positive the correlation between two data sets more they are similar and more negative the value of correlation less is the similarity between data sets or more is dissimilarity in data sets.

So to get better results one's S-box must possess higher randomness or standard deviation and more negative value of the correlation.

5. Results

The comparison of the strong encryption capabilities shows that the performance of the new proposed S-box is better than some S-boxes used in past in the era of cryptography. The nonlinearity analysis done by comparing standard deviations depicts that the properties are comparable to the S-boxes use as a touchstone in this work. Table 2 presents a list of results of nonlinearity analysis based on standard deviation.

It can be understood graphically as well through Figure 4. As per discussion, that more negative the value of correlation less is the similarity between data sets or more is dissimilarity in data sets. The result of correlation is - 0.161 (refer Table 3) and it means that new S-box is stronger and so it assures the acceptability of this S-box to encryption application. Figure 5 clearly demonstrates the essence of correlation in the proposed S-box is more negative as compared to conventional one, hence providing more security and reliability.
6. Conclusions

In this paper, a new method is presented to construct an S-box with the application of a set of specific differential equations called Lorenz system of equations. This set of equations shows chaotic behavior in given intervals with specific initial parameters. In order to evaluate the performance of the new proposed S-box, a comparison
is presented by the application of standard deviation and correlation. The results yield that the new S-box have desirable properties which give more strength to cryptography techniques and is suitable for encryption applications used for secure communications.

7. References

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