Dynamical systems theory for music dynamics

Jean Pierre Boon and Olivier Decroly

Physique Nonlinéaire et Mécanique Statistique
Université libre de Bruxelles, Campus Plaine CP 231
B-1050 Bruxelles, Belgium

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We show that, when music pieces are cast in the form of time series of pitch variations, the concepts and tools of dynamical systems theory can be applied to the analysis of temporal dynamics in music. (i) Phase space portraits are constructed from the time series wherefrom the dimensionality is evaluated as a measure of the global dynamics of each piece. (ii) Spectral analysis of the time series yields power spectra ($\sim f^{-\nu}$) close to red noise ($\nu \sim 2$) in the low frequency range. (iii) We define an information entropy which provides a measure of the local dynamics in the musical piece; the entropy can be interpreted as an evaluation of the degree of complexity in the music, but there is no evidence of an analytical relation between local and global dynamics. These findings are based on computations performed on eighty sequences sampled in the music literature from the 18th to the 20th century.

I. QUANTITATIVE APPROACH TO MUSIC ANALYSIS

Any attempt to investigate the forms of artistic expression through quantitative analysis calls necessarily, in one way or another, for the use of methods developed with scientific techniques, and thereby encounters unavoidably the difficulties inherent to the quantification

*email address: jpboon@ulb.ac.be
of art. Certainly music is the art form which by construction should be best amenable to scientific approach: (i) western music proceeds by vertical (harmony) and horizontal (counterpoint) structures; (ii) its language and syntax are based on mathematical properties where symmetries and symmetry breaking play an essential role; (iii) music, as performed, steers a one-dimensional course, the dimension of time. So time irreversibility is intrinsic to musical expression and a musical sequence can be considered as the time evolution of an acoustic signal. The signal detection (via human acoustical perception or via technical audio-equipment) triggers the sequence: measurement - coding - analysis, followed eventually by interpretation. As complicated as they may be (physiologically or technically), measurement and coding are essentially operational steps. Analysis is truly the key-point: it is a conceptual process whose results are indispensable to meaningful interpretation.

Starting from the idea that we receive musical messages as the time evolution of acoustic signals, we can associate to the development of the musical sequence the concept of time series and use the analytical and computational tools of physics and mathematics for the analysis of music dynamics \[1\]. Previous work along these lines \[2,3,4\] gave indication that time correlations and spectral characteristics could be identified in pieces of music chosen as typical examples. So the feasibility of the scientific approach to music analysis has been established, but clearly the results are limited in number and far from conclusive in scope. In particular they call for a more thorough investigation of a large number of pieces covering the whole history of music. In the next section we review earlier work by way of introduction to, and justification of the present work, whose methods are developed in section \[III\]. These methods were applied to the analysis of about eighty musical sequences and the results are discussed in section \[IV\]. We conclude with some comments.

II. 1/f MUSIC AND DIMENSIONALITY

Obviously there is more than one single time signature to a musical work: whether one considers the first successive few notes of a piece (say Bach’s third Brandenburg Concerto),
an entire movement, the full piece, or the complete set of such pieces, one finds different
time characteristics. Schematically however two time scales emerge: a short time scale and
a long time scale. The short time scale characterizes the dynamics of a musical cell, i.e. a
group of ten to twenty successive notes or sounds which are highly correlated in time. Such
short time correlations are found in almost any music (unless constructed on the basis of a
white noise algorithm) and are therefore not very useful for a quantitative analysis designed
to characterize works, composers or styles. Long time correlations have been investigated by
Voss and Clarke [2] by spectral analysis of the audio signal of recorded music. The analysis
proceeds with complete pieces of music (e.g. Bach’s first Brandenburg Concerto) as well
as with up to twelve hour stretches of data accumulated from radio stations. Low pass
filtering was performed to obtain spectra of these data in the low frequency domain. Voss
and Clarke found that loudness variations and pitch fluctuations exhibit $1/f$ power spectra
in the low frequency range ($f \leq 10Hz$) independently of the music considered. This finding
raised interest and stimulated further work amongst musicians and scientists (see [4] for
references). The claim that frequency fluctuations in music have a $1/f$ spectral density has
recently been critically discussed by Nettheim [4] in a constructive analysis. In particular
Nettheim questions the validity of the procedure when long duration stretches include various
pieces, different composers and styles as well as spoken announcements and comments. He
emphasizes that ”a single piece is normally the largest unit of artistic significance” ( [4],
p.136), a statement commonly endorsed by musicians. Nettheim provides new data for single
movements of 18th-19th century classical music yielding spectral analysis results which are
at variance with $1/f$ behavior, while he recognizes that it would be desirable to extend the
analysis beyond the five examples treated in his study.

A different approach towards a quantitative analysis of music has been proposed by Boon
et al. [3] who showed that the theory of dynamical systems could provide interesting tools
for the identification of complex dynamics in music. The basic material used by Boon et
al. is the printed score played on a synthesizer interfaced to a computer; pitch values are
converted into digital data which are stored in the computer memory so that the score is
converted into a time series, say \( X(t) \) for a single part score (see Fig.1a). Pieces with several parts are treated part by part to produce a set of time series \( X(t), Y(t), Z(t), \ldots \). Data processing is performed to construct the phase portrait: \( X(t), Y(t), Z(t), \ldots \) or \( X(t), X(t + n\Delta t), X(t + m\Delta t) \), using the time-delay method for single part pieces. The phase portraits (see Fig.1b) are used to compute the dimensionality, and the initial time series to calculate the correlation function \( C(t) \) (see Fig.1c) and the corresponding power spectrum \( S(f) \) (see Fig.1d) \[3\]. Here we also consider the information entropy which is discussed in section \[II\].

The phase portrait constitutes a spatial representation (in the abstract phase space) of the temporal dynamics of the music piece reconstructed from the time series obtained from the pitch variations as a function of time. An example is given in Fig.1b which shows the phase portrait of the three part Ricercar of Bach’s Musical Offering. The Hausdorff dimension is computed with the box-counting method. As references we consider two opposite cases. (i) A piece of random music constructed with a computer generated white noise algorithm exhibits a phase space trajectory which fills homogeneously the entire phase space and consequently has dimensionality three in a three-dimensional phase space. (ii) An elementary score constructed as a canon of three repeatedly ascending and descending chromatic scales yields a limit cycle with dimensionality one. For the six pieces of music analyzed by Boon et al. \[3\], the Hausdorff dimension takes values around two, a result which calls for further analysis in view of the limited number of data.

The power spectrum of the time series obtained from the scores was also considered by Boon et al. \[3\]; for instance the log – log plot of \( S(f) \) of the first part of Bach’s Ricercar shown in Fig.1d exhibits, in the range 0.03 – 3.0Hz, an average slope corresponding to \( 1/f^{\nu} \) with \( 1 < \nu < 2 \). However no systematic spectral analysis of the pieces studied was presented.

III. METHOD AND ANALYSIS

The pieces to be analyzed are played by a musician who uses a synthesizer interfaced to a computer where the pieces are stored. At this stage the computer is used as a multitrack
tape recorder: musical sequences can be played at will independently or simultaneously in any combination and quantitative corrections can be made to the recorded tracks to obtain perfect match with the original score. The pieces are stored in files which can be called for treatment and analysis. Digitizing the musical pieces requires double discretization, i.e. for pitch and for duration \[\frac{2}{100}\] sec. for a tempo set as quarter note 120. For the pitch scale we use the well-tempered clavier, i.e. the half-tone is the basic unit.

The choice of the music pieces was guided by several criteria. Since we are using the musical sequence as a time series, the pieces should ideally have parts with successions of exclusively single notes, like e.g. in woodwind chamber music. String trios, quartets,... fulfill this criterion if one accepts to perform a reduction of occasional double strings. Counterpoint pieces (e.g. fugues) are amenable to analysis after proper decryption of the individual parts. Jazz music as written on transcribed scores can be cast, for the purpose of the present investigation, into three parts: the melody, the bass line and the middle part (although such a drastic reduction certainly overlooks the essentials of jazz).

Another constraint concerns the length of the pieces to be analyzed: the number of notes must be sufficiently large (in practice not smaller than 500) to yield acceptable data for time series analysis. In addition to these technical criteria, we wanted a sample of pieces covering significantly the history of western classical music. Twenty-three pieces were chosen, which, accounting for each part in each piece, amounted to the coding of eighty-three sequences, from J.S. Bach to E. Carter, plus four jazz music scores. The complete list is given in Table 1.

The method was tested against two “idealized” sequences whose analysis provided reference values for the music pieces data. The first sequence constructed from repeated ascending and descending scales serves a typical example of deterministic (periodic) dynamics. On the other hand, we generated a sequence of random music (based on a white noise algorithm) as a prototype of unpredictable music. As the data to be obtained from the analysis of
these sequences can be predicted theoretically, these reference pieces also serve the purpose to evaluate quantitatively the influence of the number of notes in a sequence on the value of the measured quantities.

We developed and used three methods for the analysis of temporal dynamics in music: (i) the phase space portrait and its quantification through dimensionality; (ii) the autocorrelation function of the time series and its power spectrum; and (iii) the entropy.

(i) The phase portrait method which maps the time evolution of a dynamical process onto a spatial representation is straightforwardly borrowed from dynamical systems theory and its application to music dynamics was introduced by Boon et al. in [3]. We call attention to the fact that, since pitch and time variables are discretized (see above), the phase space is discrete and finite as it is spanned by the dynamical variables $X(t), Y(t), Z(t), \ldots$ corresponding to the time variations of the pitch in each part of the music score. The size of the phase space is set by the largest pitch extent in the piece. An example is shown in Fig.1b.

Since the phase space is discrete for musical trajectories, the Hausdorff dimension

$$D_H = \lim_{\lambda \to 0} \frac{\log \mathcal{N}(\lambda)}{\log(1/\lambda)}$$

must be redefined appropriately because the limit $\lambda \to 0$ is immaterial here. We denote by $\mu$ the size of the smallest box that can be constructed, i.e. $\mu$ is the half-tone unit. Then

$$\mathcal{N}(\lambda) \times (\lambda/\mu)^{D_f} = V/\mu^{D_f}$$

where the rhs is a constant. It follows that

$$D_f = \left[-\log \mathcal{N}(\lambda) + \text{const.}\right]/\log(\lambda/\mu)$$

and the dimensionality is obtained as the slope of $\log \mathcal{N}(\lambda)$ versus $\log \lambda$:

$$\log \mathcal{N}(\lambda) = -D_f \log \lambda + \text{const.'} ; \quad (\lambda \neq 0)$$

where $\log \mu$ has been absorbed in the constant.
The dimension $D_f$ is computed by the box-counting method and, as it characterizes the structure of the complete phase trajectory, its value yields a quantitative evaluation of the global dynamics of the musical piece.

(ii) Power spectrum methods are well known and need not be described here. $Log - log$ plots of the spectra obtained from the time series of the musical sequences are of interest in that the trend of their slopes yields a value which has been claimed to be "universally" typical of $1/f$ noise, a claim which has been critically discussed subsequently (see section [II]). In the next section we present the results obtained for the pieces analyzed in the present work.

(iii) While the Hausdorff dimension provides an evaluation of the global dynamics of a piece of music, further information on the local dynamics can be obtained from the application of information theory. The analysis proceeds on the basis of the data files. A sequence of notes can be viewed as a string of characters and as such can be analyzed from the point of view of its information content. The simplest string follows from straightforward coding of the pitch by assigning a character to each note. Duration of notes is included by repeating the character as many times as there are unit time steps until the next note (rests can be accounted for by incorporating a specific symbol for rest unit). Sequences of pitch intervals can be coded similarly with a signed number corresponding to the number of half-notes up or down from one note to the next. (Note that the interval coding is independent of the key).

The entropy which measures the information content of a string of characters on the basis of their occurrence probability, will be defined such that its value has an upper bound ( = 1) for a fully random sequence. $H$ denotes the entropy when strings of notes are considered independently of their duration; when the latter is accounted for, we use $H^r$; $H^i$ is used for strings of intervals. Furthermore, a subscript indicates the order: $H_0$ is the zeroth order entropy which is a measure of the straight occurrence of each note, and the $\alpha$-th order entropies, $H_\alpha$ ($\alpha \neq 0$), follow from the successive conditional probabilities at increasing orders. We now formalize these quantities.
Consider a string of characters $S = \{s_i; i = 1, \ldots, N\}$ where the $s_i$’s can be chosen from an alphabet with $R$ characters. Subsequently the $N$ characters will be identified with the notes of a musical sequence - as its symbolic dynamics - and the alphabet will be identified with the degrees in the pitch range over which the piece of music extends. The set $\{S^\alpha_j; j = 1, \ldots, n; n = N - \alpha + 1\}$ of strings containing $\alpha$ characters ($\alpha \leq N$) is obtained by partitioning the initial set as follows

$$\{S^\alpha_1 = (s_1, \ldots, s_\alpha), \ldots, S^\alpha_j = (s_j, \ldots, s_{j+\alpha-1}), \ldots, S^\alpha_n = (s_{N-\alpha+1}, \ldots, s_N)\}$$

(5)

The occurrence probability of a given string $S^\alpha$ is defined by

$$P(S^\alpha) = \nu^\alpha / n_\alpha,$$

(6)

where $\nu^\alpha (= \nu(S^\alpha))$ is the number of occurrences of string $S^\alpha$ and $n_\alpha$ is the total number of substrings of $S$ with $\alpha$ characters. We now consider substrings with $(\alpha + 1)$ characters where the first $\alpha$ characters belong to a given string $S^\alpha$ and we define the probability

$$P(s \mid S^\alpha) = \nu_{s\mid\alpha} / \sum_{s'\in R} \nu_{s'\mid\alpha}$$

(7)

as the relative measure of the number of occurrences $\nu_{s\mid\alpha}$ of character $s$ given that the $\alpha$ previous characters belong to the string $S^\alpha$. The corresponding normalized entropy is given by

$$H(S^\alpha) = -\sum_{s\in R} P(s \mid S^\alpha) \log P(s \mid S^\alpha) / \log \nu^\alpha.$$  

(8)

Then performing the sum of the entropies corresponding to all possible strings with $\alpha$ characters, weighted by the occurrence probability of each $S^\alpha$ yields the $\alpha$-th order entropy

$$H_\alpha = \sum_{S^\alpha} H(S^\alpha) P(S^\alpha)$$

(9)

From (4) it is clear that $0 \leq H(S^\alpha) \leq 1$, and since by definition $\sum_{S^\alpha} P(S^\alpha) = 1$ (see(6)), it follows that $0 \leq H_\alpha \leq 1$. We now comment on the meaning of these quantities in the context of musical sequences. The meaning of $H_0$ is straightforward (see comment above), but not
very useful: $H_0 = 0$ when the score uses a single note and $H_0 = 1$ when all accessible degrees are equally visited (a chromatic scale or a random score). $H_1$ is related to the probability to find the note $s_{i+1}$ given that the previous note was $s_i'$: $H_1$ has the value one for a random score as well as for a descending and ascending scale. $H_2$ is a more interesting quantity in that it discriminates between deterministic sequences and random sequences: obviously $H_2 = 1$ for a random score and $H_2 = 0$ for a scale. Higher order entropies can be considered, but we found that they take fast decreasing values as the length of the string increases.

When these concepts were applied to musical sequences, it appeared in the course of the analysis, that the numerical results did not reflect consistent significance. The entropies are measures of the information content of the sequences and thereby provide an evaluation of the "diversity" of the notes in the score. However as most pieces analyzed are tonal music, there is one important feature which must be accounted for: tonality. We now introduce a new entropy which incorporates the property that the notes in a sequence belong or not to a reference scale (e.g. given by the clef, but not necessarily).

A scale is defined by a tonality (A, B♭,...) and a mode (Major, minor,...); the degrees of the scale, starting with the tonic, yield a succession of well-defined degrees. $\theta$ is the set of notes containing all such degrees. In order to better quantify the "diversity" of the notes in a musical sequence and thereby attempting to take into account the "liberty" taken by the composer with tonality, we discriminate notes which belong to $\theta$ from those which do not. Mathematically, this is accomplished by separating the probability $P_s$ of occurrence of note $s$ into two contributions ($P'_s$) depending on whether $s$ belongs to $\theta$ or not. $P_s$ is the straight occurrence probability $P_s = n_s / \sum_{s'=1}^R n_{s'}$, where $n_s$ is the number of occurrences of note $s$, and the denominator is the total number of notes ($N$) in the sequence. Then we define

$$P'_s = \begin{cases} n_s / \sum_{s \in \theta} n_s & \text{if } s \in \theta; \\ n_s / \sum_{s \not\in \theta} n_s & \text{if } s \not\in \theta. \end{cases} \tag{10}$$

We now associate different weighting factors, $\gamma$ and $\delta$, to occurrences within and out of $\theta$ respectively, such that
\[ \gamma \sum_{s \in \theta} P'_s + \delta \sum_{s \notin \theta} P'_s = 1 \]  

(11)

with \( \gamma + \delta = 1 \). As \( \gamma \) and \( \delta \) are treated as parameters, we define the new normalized entropy

\[ H' = -[\sum_{s \in \theta} \gamma P'_s \log(\gamma P'_s) + \sum_{s \notin \theta} \delta P'_s \log(\delta P'_s)] / \log N \]  

(12)

as the parametric entropy. The idea being that deviations from tonality are considered as "unexpected events", thereby contributing more strongly to an increase of entropy, the values of \( \gamma \) and \( \delta \) will be set (empirically; see section [V]) such that \( \delta = m \gamma \) with \( m > 1 \): occurrences of notes off-tonality \((s \notin \theta)\) are given a more important weight than those of notes belonging to the reference scale. So \( H' \) should be low for pieces of music where most notes are within a well-set tonality, and should be high for nontonal music. Furthermore, for a strictly twelve-tone piece the value of \( H' \) should be independent of any reference key.

On the other hand we set \( \gamma = 1 \) and \( \delta = 0 \) for a sequence where all \( s \in \theta \). For random music, the parametric entropy is independent of \( \gamma \) and \( \delta \) and its value approaches one when the number of notes in the sequence becomes sufficiently large.

Parametric entropies can also be defined to successive orders

\[ H'_\alpha = \sum_{S^\alpha} H'(S^\alpha) P(S^\alpha) \]  

(13)

where

\[ H'(S^\alpha) = -[\sum_{s \in \theta} \gamma P(s \mid S^\alpha) \log(\gamma P(s \mid S^\alpha)) + \sum_{s \notin \theta} \delta P(s \mid S^\alpha) \log(\delta P(s \mid S^\alpha))] / (\log \nu^\alpha) \]  

(14)

and \( P(S^\alpha) \) is the weighted probability of occurrence of string \( S^\alpha \), i.e. if this string contains \( \kappa \) characters \( \in \theta \) (and \( (\alpha - \kappa) \notin \theta \))

\[ P(S^\alpha) = \gamma^\kappa \delta^{\alpha-\kappa} \nu^\alpha / n_\alpha. \]  

(15)

To first order the interpretation is as follows: \( H'_1 \) yields a measure of the information content of a musical sequence quantifying the transition probabilities from one note to the next
given that the transition can occur within the reference tonality \(((s_i, s_{i+1}) \in \theta)\), outside the 
tonality \(((s_i, s_{i+1}) \notin \theta)\), or from \(\theta\) to off \(\theta\) \((s_i \in \theta, s_{i+1} \notin \theta)\), and vice-versa. The operational result is that a large value of the parametric entropy is indicative of frequent excursions away from the tonality, with transitions over intervals distributed over a large number of notes. On the contrary, the entropy will assume a low value when a note determines almost unambiguously the next one, in particular when the next note remains in the range of tonality. Higher order parametric entropies are neither obviously interpreted nor very useful in practice: their values decrease uniformly very rapidly (because of the factors \(\gamma^\kappa \delta^{\alpha - \kappa}\) in (11)) and yield no significantly distinguishable data for the pieces analyzed.

IV. RESULTS AND COMMENTS

A list of the pieces of music which were analyzed is given in Table 1. The analysis was performed for each part of each piece; thirteen quantities were computed for eighty-three sequences yielding a large number of numerical data wherefrom a selection was made by significant criteria analysis. The selected results are shown in Table 2. The Hausdorff dimension \(D_f\) (whose value is given in the first column of Table 2) is the mean value of the dimensions computed for each part by the time delay method \(D_f^R\). The reason for using this quantity (rather than the "global" dimensionality obtained directly from the phase portrait in \(D\)-dimensional space) is that the other quantities (entropies, spectral characteristics) are computed for each part and so must be averaged to characterize the complete piece. Furthermore we obtain good agreement between the value of the "global" dimensionality and those of the reconstructed trajectories (a property which is consistent with Ruelle-Takens theorem, despite the fact that here the phase space is discrete).

The computation of the parametric entropy was performed not only with the tonality given by the clef as the reference tonality but also with respect to other tonalities. It would be expected that, as a general rule, the minimum value of the entropy should be obtained for the tonality corresponding to the clef. We found that this is not necessarily the case. The
values given in Table 2 are the minimum entropies obtained and they may as well correspond to a neighboring tonality or to the corresponding major tonality for a piece written in minor tonality. Optimization criteria were also used to set the values of the parameters $\gamma(=0.2)$ and $\delta(=0.8)$.

An interesting result is obtained by considering the Hausdorff dimension (of the reconstructed trajectories) as a function of the first order parametric entropy: figure 2 shows $\log D_f^R = f(H'_1)$. While there is no evidence of quantitative correlation between the global dynamical structure (as measured by $D_f^R$) and local dynamics (the information content evaluated by $H'_1$), it is interesting that about all data are clustered within a limited zone of the plane $(D_f^R, H'_1)$. In figs. 3 and 4 the values of $D_f^R$ and $H'_1$ respectively are presented in chronological order. Note that the value $D_f^R = 3$ for the randomly generated sequence is obtained when the number of data is large ($5 \times 10^3$ notes). Since none of the musical sequences contains as many notes, we show the values of $D_f^R$ for random sequences generated with 500 and 2000 notes for reference. The important observation here is that - with very few exceptions - there is no obvious clustering of pieces by composer or by period of composition; this holds for dimensionality as well as for parametric entropy.

A similar comment can be made when considering Fig. 5. Here - as in Fig. 2 - we plot the dimensionality versus parametric entropy, but we use the values obtained by averaging over all parts in order to characterize the piece globally (see Table 2 for data). No systematic grouping appears neither by composer nor by style. While physicists, who are keen to logical rules, may be disappointed by the lack of systematic ordering that could be drawn from these results, musicians may find it comforting that music resists simple formalization. Nevertheless some general comments are in order.

(i) We investigated systematically the possible relations between all measured quantities (entropies and dimensionality; see Table 2). All plots produced widely scattered data without any indication of obvious correlations, except $D_f = f(H'_1)$. While no analytical relation could be conjectured for $D_f = f(H'_1)$, Fig. 5 suggests that a trend might emerge from a statistical analysis performed on a sufficiently large sampling of music pieces. An exten-
sion of the analysis should also be considered where the present treatment is generalized to
harmonic dynamics, that is instead of considering the melodic lines from each part sepa-
ately, consider the sequence of the vertical structures of the score obtained by constructing
the vectors of the simultaneous notes at each time step, define the corresponding transition
probability matrix in terms of interval changes, and generalize accordingly the entropies of
section II, in particular by introducing chordal references to define the parametric entropy.

(ii) The last column of Table 2 shows the value of the slope $\nu$ of the log–log plot of
the power spectrum $S(f) \sim f^{-\nu}$, obtained by least-squares fit computation in the range
$0.03\, \text{Hz} < f < 3.0\, \text{Hz}$. The values shown here are obtained by averaging over the value of
$\nu$ from each part of each piece (there are but very weak differences between the values of
each separate part). The results are remarkably consistent: $1.79 \leq \nu \leq 1.97$ and indicate
values which are close to the red noise value. The data of the spectral analysis given in
Table 2 confirm and quantify the qualitative result of Nettheim based on the analysis of five
melodic lines (Bach, Mozart, Beethoven, Schubert and Chopin) [4]. These findings are at
variance with Voss and Clarke $1/f$ claim [2]. It must be re-emphasized that in the present
work as well as in Nettheim's, the musical sequences considered are single pieces taken as
"the largest unit(s) of artistic significance" ([4], p.136) whereas Voss and Clarke considered
long stretches of recorded music (see Section II). That such long time sequences yield a
$1/f$ spectrum found a plausible explanation in a theoretical analysis by Klimontovich and
Boon [7]. However if the musical dynamics analysis is meant as a procedure to identify and
characterize elements of musical significance, the single piece is the commonly recognized
object to be studied. In this respect the meaning of long stretches of blended musical pieces
is unclear.

Note the value $\nu \approx 0$ for the random sequences (*500, *2000, and *5000 in Table 2).
These sequences were constructed by distributing, on equal time intervals, pitch values
according to white noise generated data; they may qualify as examples of "white noise
music" as they exhibit a flat power spectrum [8].

(iii) The unexpected elements in a piece of music can be found in the deviations from
established rules and the violation or even the mere rejection of such rules. In the context of classical forms, these deviations are mostly related to the liberty taken by the composer with respect to tonality. Thus when Leibowitz considers the complexity of musical language, he argues that Bach’s and Haendel’s complex polyphonic style is commonly opposed to what has been called the homophony of Haydn and Mozart (...). According to which criteria does one evaluate simplicity and complexity? Only one: the counterpoint (...). However, continues Leibowitz, the counterpoint is hardly the only constituting element in music, and, even more, it should be obvious that music can be simple or complex independently of any notion of counterpoint. Leibowitz then considers the problem of harmony and so observes that the composer’s audacity as well as harmonic complexity may and must be evaluated according to further criteria. Those then invoked concern the principles of tonality expansion and here - as argued on the basis of a few specific examples - Haydn’s and Mozart’s works appear more audacious than those of their precursors. Obviously the argument is of considerable importance as it leads Leibowitz to the concept of increasing complexity which should determine the overall evolution of musical tradition. Considering that entropy provides a quantitative measure of the degree of complexity the present results show that complexity - in contrast to Leibowitz’ hypothesis - appears to be characteristic of the composition rather than of the composer. Accordingly we find no indication of a systematic increase in complexity paralleling historically the evolution of classical music.

V. ACKNOWLEDGEMENTS

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[1] Dynamics is to be understood in the sense of the physicist as the (set of) process(es) governing the
time evolution of the system considered and should not be confused with the definition commonly
used by musicians for loudness variations.

[2] R.F.Voss and J.Clarke, ”1/f noise” in music: Music from 1/f noise,” J.Acoust.Soc.Am. 63, 258
(1978); M.Gardner, “White and brown music, fractal curves and 1/f fluctuations,” Sci.Am. 238,
4 (1978).

[3] J.P.Boon, A.Noullez, and C.Mommen, “Complex dynamics and musical structure,” Interface-
J.New.Mus.Res. 19, 3 (1990).

[4] N.Nettheim, “On the spectral analysis of melody,” Interface- J.New.Mus.Res. 21, 135 (1992).

[5] Intensity variations (loudness) and timbre are not considered here.

[6] For instance conjecturing a power law $D_f \propto (H_1)^\xi$, we found that a fit to the data produced a
correlation coefficient whose value rules out the quantitative validity of the power law as representa-
tive of a possible relation between dimensionality and entropy. From Fig.5 it can be guessed that
a third degree polynomial would yield an acceptable fit for which we found indeed a correlation
coefficient of .75; however the interpretation of a cubic function could hardly be justified.

[7] Y.L.Klimontovich and J.P.Boon, “Natural flicker noise ("1/f") in music,” Europhys.Lett. 3, 395
(1987).

[8] Note that white noise algorithm data redistributed independently over both pitch and time vari-
ables produce long time correlations and consequently the power spectrum of the corresponding
sequence is not flat. Sequences so generated may qualify as examples of random music but should
not be considered as white noise music if the criterion is the flatness of the spectrum. This point,
which has been overlooked by some authors [2], was discussed by Nettheim [4].

[9] R.Leibowitz, L’évolution de la musique de Bach à Schöenberg (Correa, Paris, 1951), Chap.2.
Tables.

TABLE 1 : Data for the pieces analyzed, presented in chronological order: Composer, Year of composition, Title, Movement analyzed (unless single movement piece), Key (except for Carter’s Quartet), Measure. The first column gives the acronym for identification (ID) in Table 2 and in figs.3 and 4.

TABLE 2 : Numerical results for the pieces analyzed. Chronological order as in Table 1 where the acronyms are defined (chromatic scale = SCA; random sequences are indicated by the number of notes: 500, 2000, 5000; * indicates a computer generated sequence). The second column shows the number of notes in each piece. The numerical values are averages taken over the values obtained for each part of the piece (unless single part piece, BPS). $H'_{1}$ and $\nu$ are not relevant quantities for the chromatic scale (SCA).
Figure Captions.

FIGURE 1: *Ricercar* of J.S.Bach’s *Musical Offering*.
(a) Time series: Pitch variations $X(t)$ as a function of time for the first part of the *Ricercar*. Pitch unit is the half-tone and the value 60 corresponds to the keyboard midrange C ($C3 = 60$, $C3^\# = D3^\flat = 61$, $D3 = 62$, ...); time unit (on the horizontal axis) is the beat as given by the measure.

(b) Phase portrait: three-dimensional phase space trajectory obtained from the time series of the three parts of the *Ricercar* (see e.g. $X(t)$ in Fig.1.a).

(c) Time correlations: Normalized autocorrelation function $C(t)$ of part one of the *Ricercar* ($X(t)$, Fig.1.a); $C(t) = (1/T) \sum_{\tau=0}^{T}(X(\tau) - \langle X \rangle)(X(\tau + t) - \langle X \rangle)/{(X(\tau) - \langle X \rangle)^2}$.

(d) Power spectrum: log–log representation of $S(f)$, the spectral density of $X(t)$ (Fig.1.a), obtained by FFT of $C(t)$. Frequency unit is reciprocal time unit. The straight lines correspond to $1/f$ and $1/f^2$ spectra respectively.

FIGURE 2: Log–log plot of the dimensionality versus parametric entropy for all sequences analyzed. $D^R_f$ is the value computed from the reconstructed trajectory in three-dimensional phase space (by the time delay method). Different symbols are used for classical music (●), jazz music (□), and computer generated music (◇)(random sequences).

FIGURE 3: Dimensionality presented in chronological order of the pieces. $D_f$ is the value obtained for each piece by averaging over the values $D^R_f$ of each part of each piece (unless single part piece, BPS). The acronyms are defined in Table 1.

FIGURE 4: First order parametric entropy $H'_1$; same as for $D_f$ (see caption of Fig.3).

FIGURE 5: Log–log representation of $D_f$ (see Fig.3) as a function of $H'_1$ (see Fig.4). Symbols are: ● for classical music, □ for jazz music, ◇ for random sequences.