HARD BREMSSTRAHLUNG PHOTONS FROM GRAVITON EXCHANGE∗

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Abstract

We review photon Bremsstrahlung in extra-dimensional models with massive gravitons. The photon spectrum is harder than in the Standard Model. In the RS scenario, radiative return to gravitons below the c.m. energy can lead to a considerable increase of the cross section.

1 Introduction

In popular models involving extra dimensions [1, 2], the coupling of gravitons to ordinary matter can result in extraordinary phenomena. These effects lead to events with energy-momentum imbalance due to graviton emission, as well as effects due to virtual graviton exchange [3, 4, 5, 6].

The Randall–Sundrum (RS) model [2] predicts the existence of TeV-scale gravitons, whose couplings to Standard-Model (SM) fields have weak-interaction strength. They will therefore show up as narrow resonances [5]. We shall here report on a study of the effects of graviton exchange on the process

$$e^-(k_1) + e^+(k_2) \rightarrow \mu^-(p_1) + \mu^+(p_2) + \gamma(k), \quad (1)$$

focusing in particular on the emitted photon.

The coupling of gravitons to fermions and photons lead to additional Feynman diagrams, beyond those which have the topology of the SM diagrams, where the photon or Z is replaced by a graviton. The additional diagrams arise since the graviton also couples to two photons, and since there is a fermion-fermion-vector-graviton coupling [3, 8].

With the notation of (1) the graviton propagator will be characterized by either of two invariants: $$s = (k_1 + k_2)^2 = 4E^2$$, or $$s_3 = (p_1 + p_2)^2$$, and we classify the Feynman diagrams accordingly. The terminology “ISR” and “FSR”

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will thus refer to whether the photon is emitted “before” or “after” the graviton that is exchanged \[7\].

Since some of the “new” Feynman diagrams do not have the collinear and IR singularities of the SM contributions, one expects the photons to be harder and less aligned with the initial or final-state fermions. This turns indeed out to be the case in the ADD scenario. Another striking feature in the RS scenario is that radiative return can lead to peaks in the cross section related to resonant production of gravitons whose masses are below the c.m. energy.

## 2 Cross sections

The differential cross section for (1) is given in \[7\] in terms of the fractional energies of the muons \((x_1, x_2)\) and the photon \((x_3)\) and the angle \((\theta)\) between the photon and the initial beam. We shall here review integrated cross sections, and some photon \(k_\perp\) distributions, where “perpendicular” refers to perpendicular w.r.t. the beam direction.

The total cross section is defined as:

\[
\sigma_{ee\rightarrow\mu\mu\gamma} = \int_{x_3}^{x_3} dx_3 \int_{x_3-y_{\text{cut}}}^{x_3+y_{\text{cut}}} d\eta \frac{d^3\sigma_{ee\rightarrow\mu\mu\gamma}}{dx_3 dy d\cos \theta},
\]

where \(\eta = x_1 - x_2\) and and we impose the cuts \(|\cos \theta| < 1 - c_{\text{cut}}\), with \(c_{\text{cut}} = 0.005\), \(x_3^{\text{min}} < x_3 < \frac{1}{2}(1 - y_{\text{cut}})\) \(\equiv x_3^{\text{max}}\), \(x_3^{\text{min}} = \max (\xi_{\text{cut}}/\sqrt{1 - \cos^2 \theta}, y_{\text{cut}})\), with \(k_\perp^{\text{min}} = \xi_{\text{cut}} \sqrt{s}\), and \(y_{\text{cut}} = \xi_{\text{cut}} = 0.005\). In order to enhance the effects of graviton exchange, we consider a cut on radiative return to the \(Z\): \(s_3 > (m_Z + 3\Gamma_Z)^2 \equiv y_{\text{cut}}^2 s\).

## 3 The RS scenario

The discrete mass values of the RS scenario are given by \(m_n = (x_n/x_1)m_1\), \(J_1(x_n) = 0\), with \(J_1(x)\) a Bessel function. The Kaluza–Klein graviton coupling is given by \(\kappa = \sqrt{2}(x_1/m_1)(k/M_{\text{Pl}})\). We parametrize the coupling strength by \(k/M_{\text{Pl}}\), for which we consider three values: 0.01, 0.05, 0.1. For a range of mass values \(m_1\) assumed to be of interest for a linear collider \[9\], we show in Fig. 1 the masses of the next excited states.

If some RS graviton has a mass below the total c.m. energy, then an ISR photon can carry away the right amount of energy to yield a resonance in the \(s_3\) channel. This effect of radiative return can give a significant enhancement of the cross section as shown in Fig. 2 for values of \(m_1 < \sqrt{s}\), in particular when the coupling is large.

The radiative return to lower-lying gravitons is even more clear in the photon spectrum. In Fig. 2b we show the photon perpendicular-momentum distribution for two values of \(m_1 < \sqrt{s}\). Sharp peaks are seen for \(k_\perp \lesssim (s - m_i^2)/2\sqrt{s}\). (Note, however, that radiative return to the \(Z\) has been excluded.) For a range of mass values \(m_1\) assumed to be of interest for a linear collider \[9\], we show in Fig. 4 the masses of the next excited states.

In Fig. 4a we show the photon perpendicular-momentum distribution for two values of \(m_1 < \sqrt{s}\). Sharp peaks are seen for \(k_\perp \lesssim (s - m_i^2)/2\sqrt{s}\). (Note, however, that radiative return to the \(Z\) has been excluded.) The cross section at high \(k_\perp\) is larger for \(m_1 = 0.6\) TeV than for \(m_1 = 0.8\) TeV, since (for \(m_1 = 0.6\) TeV) \(m_2 = 1.1\) TeV is close to \(\sqrt{s}\).
**4 Summary**

The Bremsstrahlung process $e^+e^- \rightarrow \mu^+\mu^-\gamma$ may provide confirmation of graviton exchange in two-body processes. The photon spectrum is harder, because of additional Feynman diagrams. In the RS scenario, there is an enhancement of the cross section due to radiative return. Furthermore, the angular distribution is enhanced at $\theta \sim \pi/2$ [7].

A related study on $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ [10] also finds peaks in the photon spectrum that will help identify the RS scenario. This is a somewhat different signature, with a single photon plus missing energy in the final state, but the underlying physics is analogous to what is considered here.

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**Figure 1:** Left: Masses of RS gravitons vs. $m_1$. Right: Total cross sections for $e^+e^- \rightarrow \mu^+\mu^-\gamma$ vs. $m_1$, for $\sqrt{s} = 1$ TeV. Three values of $k/M_{Pl}$ are considered: from top and down: 0.1, 0.05 and 0.01.

**Figure 2:** Left: Photon perpendicular momentum distribution: solid, for $m_1 = 0.6$ and 0.8 TeV. Graviton-related contributions: dash-dotted, SM contribution: dotted. Right: bin-integrated $k_{\perp}$ distribution for $m_1 = 0.8$ TeV. Error bars (SM distribution) for $L_{\text{int}} = 300$ fb$^{-1}$. 

[10]
At the LHC, the corresponding mechanism also leads to similar signatures. In the ADD scenario, there are hard photons and in the RS scenario there are “steps” in the $k_{\perp}$ distribution caused by radiative return [11].

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