Noncommutative information is revealed from Hawking radiation as tunneling

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Abstract – We revisit the tunneling process from a Schwarzschild black hole in the noncommutative spacetime and obtain the nonthermal tunneling probability. In such nonthermal spectrum, the correlations are discovered, which can carry the information about the noncommutativity. Thus this enlightens a way to find the noncommutative information in the Hawking radiation. The entropy is also shown to be conserved in the whole radiation process, which implies that the unitarity is held even for the Hawking radiation from noncommutative black holes.

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Introduction. – Hawking’s semiclassical analysis\cite{1,2} of the black-hole radiation suggests that the information collapsed into the black hole will lose for ever since the thermal radiation cannot carry any information. This means that the unitarity as required by quantum mechanics is violated. However, it is found that the background geometry is considered fixed and the energy conservation is not enforced during the radiation process in the Hawking method. Recently, Parikh and Wilczek suggested\cite{3} a method based on energy conservation by calculating the particle flux in Painlevé coordinates from the tunneling picture. Their result recovered Hawking’s original result in leading order and gave the consistent temperature expression and the entropy relation. The method had also been discussed generally in different situations\cite{4–8} and showed that the formula was self-consistent even when checked by using the thermodynamic relation\cite{9–13}. Another important aspect is to give the nonthermal spectrum, as shown in ref.\cite{14} that there exist information-carrying correlation in the radiation spectrum and the entropy is conserved in the sequential tunneling process. Along this line the extension has been made in refs.\cite{15–18}.

It is noted that the noncommutativity had been introduced into the investigation of the Hawking radiation as tunneling\cite{11}. The noncommutativity\cite{19–22} can provide the minimal length scale upon the generalized uncertainty principle since the Heisenberg uncertainty principle may not be satisfied when quantum gravitational effects become important. Moreover, the noncommutativity also provides a totally different black hole and the noncommutative black-hole thermodynamics is also investigated in the Parikh-Wilczek tunneling picture\cite{11}. Particularly, the noncommutativity leads to the only remnant result of black-hole radiation since it can remove the so-called Hawking paradox where the temperature diverges as the radius of a standard black hole shrinks to zero. This is advantageous over the standard Schwarzschild black hole whose final radiation will lead to divergence of the temperature and to the quantum-corrected black hole whose final radiation is dependent on the quantum-corrected parameter which may be different in the string theory and loop quantum gravity theory\cite{5}. In this paper, we will show that there exist correlations among the radiated particles in the situation of noncommutative black holes and these correlations could carry the information about noncommutativity hidden in spacetime. We also check the entropy conservation in the radiation process, which is consistent with the unitarity of quantum mechanics.

The organization of the paper is as follows. In the second section we revisit the tunneling through the noncommutative black hole and discuss its thermodynamics.

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The third section is devoted to the investigation of correlation and entropy conservation for the radiation of the noncommutative black hole. Finally, we summarize our results in the fourth section.

In this paper we take the unit convention \( k = \hbar = c = G = 1 \).

**Tunneling in noncommutative space and thermodynamics.** In this section we will recalculate the particles’ tunneling probability from the Schwarzschild black hole in noncommutative space, along the line presented in ref. [11]. In order to include the noncommutative effect in gravity, we can change the mass of the gravitating object. The usual definition of mass density in commutative space is expressed in terms of the Dirac delta function, but in noncommutative space the form breaks down due to the position-position uncertainty relation. It is shown that noncommutativity eliminates point-like structures in favor of smeared objects in flat spacetime. The effect of smearing is implemented by redefining the mass density by a Gaussian distribution of minimal width \( \sqrt{\theta} \) instead of the Dirac delta function. Here \( \theta \) is the noncommutative parameter which is considered to be a small (Planck length) positive number and comes from the noncommutator of \([x^\mu, x^\nu] = i\theta \eta^{\mu\nu}\) with \( \theta^{\mu\nu} = \theta \text{diag}(\epsilon_1, \ldots, \epsilon_{D/2}) \). It is noted that the constancy of \( \theta \) is related to a consistent treatment of Lorentz invariance and unitarity. For the purpose of noncommutativity, the mass density is chosen as

\[
\rho_0 (r) = \frac{M}{(4\pi \theta)^2} \exp \left( \frac{-r^2}{4\theta} \right),
\]

which plays the role of a matter source and the mass is smeared around the region \( \sqrt{\theta} \) instead of locating at a point. To find a solution of the Einstein equation \( G_{\mu\nu} = 8\pi T_{\mu\nu} \) with the noncommutative mass density of type (1), the energy-momentum tensor is identified as \( T_{\mu\nu} = \text{diag}[-\rho_0, q_0, q_0, q_0] \) which provides a self-gravitating droplet of anisotropic fluid with the radial pressure \( q_r = -\rho_0 \) and the tangential pressure \( q_t = -\rho_0 - \frac{1}{2} \theta \partial_r \rho_0 = \left( \frac{3}{4\theta^2} - 1 \right) \frac{M}{(4\pi \theta)^2} \exp \left( -\frac{r^2}{4\theta} \right) \). It is seen easily that the pressure is anisotropic, but at the large values of \( r \) all the components of the energy-momentum tensor tend to zero very quickly and so the mass is again isotropic and the Schwarzschild vacuum solution is well applicable.

Solving the Einstein equation in the noncommutative space leads to the solution

\[
ds^2 = - \left( 1 - \frac{4M}{r\sqrt{\pi}} \right)^{\frac{3}{2}} \left( \frac{r^2}{4\theta} \right) \mathrm{d}t^2 + \left( 1 - \frac{4M}{r\sqrt{\pi}} \right)^{-1} \mathrm{d}r^2 + r^2 \mathrm{d}\Omega^2,
\]

where the lower incomplete gamma function is defined by

\[
\gamma \left( \frac{3}{2}, \frac{r^2}{4\theta} \right) = \int_0^{r^2/4\theta} t^{1/2} e^{-t} \mathrm{d}t.
\]

Note that when \( r \) goes to infinity, \( \gamma \) approaches \( \sqrt{\pi}/2 \). Comparing the the noncommutative coordinates (2) with the commutative one [3], we note that their difference lies in the mass term, that is to say, substituting the mass term of the Schwarzschild spacetime by \( m_\theta = \int_0^r 4\pi r^2 \rho_0 (r') \mathrm{d}r' = \frac{2M}{\sqrt{\pi}} \gamma \left( \frac{3}{2}, \frac{r^2}{4\theta} \right) \). It is noted that in the \( \theta \to 0 \) limit, the incomplete \( \gamma \) function becomes the usual gamma function and \( m_\theta (r) \to M \) that is the commutative limit of the noncommutative mass \( m_\theta (r) \). From the condition of \( g_{tt} (r_\theta) = 0 \), the event horizon can be found as

\[
r_h = \frac{4M}{\sqrt{\pi}} \gamma \left( \frac{3}{2}, \frac{r_h^2}{4\theta} \right) \equiv \frac{4M}{\sqrt{\pi}} \gamma_h.
\]

Keeping up to the leading order \( \frac{1}{\sqrt{\pi}} e^{-M^2/\theta} \), we find \( r_h \sim 2M \left( 1 - \frac{2M}{\sqrt{\pi}} e^{-M^2/\theta} \right) \).

In what follows, in order to describe cross-horizon phenomena of tunneling particles, we have to change the coordinates (2) to quasi-Painlevé coordinates, which are regular and not singular at the horizon. Doing the time Painlevé coordinate transformation, we obtain the new coordinates as

\[
ds^2 = - \left( 1 - \frac{4M}{r\sqrt{\pi}} \right) \mathrm{d}t^2 + 2 \left( 1 - \frac{4M}{r\sqrt{\pi}} \right) \frac{4M}{r\sqrt{\pi}} \gamma \left( \frac{3}{2}, \frac{r_h^2}{4\theta} \right) \mathrm{d}t \mathrm{d}r + r^2 \mathrm{d}\Omega^2,
\]

where \( \gamma \equiv \gamma \left( \frac{3}{2}, \frac{r_h^2}{4\theta} \right) \) and the spacetime described by (4) is still stationary. The radial null geodesics are obtained by setting \( \mathrm{d}s^2 = \mathrm{d}t^2 = 0 \) in (4),

\[
\dot{t} = \pm 1 - \frac{4M}{r\sqrt{\pi}} \gamma,
\]

where the upper (lower) sign can be identified with the outgoing (incoming) radial motion, under the implicit assumption that time \( t \) increases towards the future.

Let us consider a positive-energy shell to cross the horizon in the outward direction from \( r_1 \) to \( r_f \). Along the method given by Parikh and Wilczek, the imaginary part of the action for that shell is given by

\[
\text{Im} I = \text{Im} \int_{r_1}^{r_f} p_r \mathrm{d}r = \text{Im} \int_{r_1}^{r_f} \int_0^{r_f} p_r' \mathrm{d}r' \mathrm{d}H' = \int_0^{H} \frac{r_f}{r_f - \frac{r_f}{\dot{r}}} \mathrm{d}H' = \text{Im} \int_{r_1}^{r_f} \int_0^{r_f} \frac{\mathrm{d}(M - E)}{\dot{r}} = \text{Im} \int_{r_1}^{r_f} \int_0^{r_f} \frac{\mathrm{d}E}{\dot{r}}.
\]

Moreover, if the self-gravitation is included, we have to make the replacement \( M \to M - E \) in eqs. (4) and (5), where \( E \) is the outgoing particle’s energy. Thus, the expression (6) is modified as

\[
\text{Im} I = \text{Im} \int_{r_1}^{M - E} \int_{r_1}^{r_f} \frac{\mathrm{d}M - E}{\dot{r}} = \text{Im} \int_{r_1}^{r_f} \int_0^{r_f} \frac{\mathrm{d}E}{\dot{r}}.
\]
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where we have changed the integration variable from \( H' \) to \( E' \). Then, inserting eq. (5) into the expression of the imaginary part of the action, we have

\[
\text{Im } I = -\text{Im} \int_0^E \int_{r_i}^{r_f} \frac{dr}{1 - \sqrt{\frac{M}{r \gamma}} E'}
\]

(8)
The \( r \)-integration is done by deforming the contour. A detailed calculation gives the tunneling rate

\[
\Gamma_N(E) \sim e^{-2\text{Im } I} = \exp \left( -8\pi E \left( M - \frac{E}{2} \right) \right) + 16 \sqrt{\frac{\pi}{\theta}} M^3 e^{\frac{M^2}{2}} - 16 \sqrt{\frac{\pi}{\theta}} (M - E)^3 e^{\frac{(M - E)^2}{2}}
\]

(9)
where the result is obtained up to the leading order \( \frac{1}{\sqrt{\theta}} e^{-M^2/\theta} \).

In what follows we will check the thermodynamics for tunneling from the noncommutative black hole. Generally, we can obtain the temperature from the tunneling probability by comparing it with the thermal Boltzmann distribution. Here it is difficult to expand the expression (9) to obtain the leading term. However, we could also consider the first law of thermodynamics to check whether the radiation temperature given by the thermodynamic relation \( \frac{1}{T} = \frac{dS}{dM} \) is equal to \( \frac{\kappa}{\theta} \), where \( \kappa \) is the surface gravity of the black hole. Since the metric (2) is static, the surface gravity can be calculated as \( \kappa = \frac{1}{2} \frac{d_{\theta}}{dr} \mid _{r=r_h} = \frac{1}{2} \left[ \frac{1}{r_h} - \frac{r_h^2}{4\theta^2 M^2} e^{-\frac{r_h^2}{2\theta M}} \right] \). Thus one can get the temperature

\[
T_N = \frac{\kappa}{2\pi} = \frac{1}{4\pi r_h \theta} \left[ 1 - \frac{r_h^3}{4\theta^2 M^2} e^{-\frac{r_h^2}{2\theta M}} \right].
\]

(10)
When the noncommutative parameter \( \theta \to 0 \), the temperature decays into the standard form, \( T_H = \frac{\kappa}{2\pi r_h M} \). Note that for the standard form of the black-hole temperature, in the limit \( M \to 0 \), the temperature will be infinite. But the consideration of noncommutative black-hole radiation will avoid the divergence problem. From the expression (10), we obtain that the temperature will fall down to zero at some definite value, i.e., \( r_h = r_0 \). As a result, the noncommutativity restricts the evaporation process to a remnant. In the region of \( r_h < r_0 \), there is no black hole since the temperature cannot be defined [20,22].

On the other hand, we can obtain the entropy of the noncommutative black hole by deforming the Bekenstein-Hawking relation,

\[
S_N = \pi r_h^2 = \frac{16M^2}{\pi} \frac{\gamma}{\theta} \approx 4\pi M^2 - 16 \sqrt{\frac{\pi}{\theta}} M^3 e^{-\frac{M^2}{2\theta}}.
\]

One can check easily that the temperature given by \( \frac{1}{T} = \frac{dS}{dM} \) is equivalent to that given by (10). More importantly, we can compare the tunneling probability (9) with the Boltzmann factor \( e^{-E/T} \) and find that the tunneling probability (9) gives a nonthermal spectrum. Moreover, a detailed calculation shows that \( \Delta S_N = S_{Nf} - S_{Ni} = -2\text{Im } I \) with \( r_i = 2M(1 - \frac{2M}{\sqrt{\theta}} e^{-M^2/\theta}) \) and \( r_f = 2(M - \omega) \left( 1 - \frac{2(M - \omega)}{\sqrt{\theta}} e^{-(M - \omega)^2/\theta} \right) \) up to the leading order \( \frac{1}{\sqrt{\theta}} e^{-M^2/\theta} \). So we have

\[
\Gamma = e^{\Delta S_N},
\]

(11)
which shows that the tunneling probability is related to the change of the noncommutative black-hole entropy.

**Correlation and entropy conservation in the radiation process.** – In the previous section it has been shown that the tunneling probability satisfied the relation \( \Gamma = e^{\Delta S} \) and gave a nonthermal spectrum for the noncommutative black hole. Such a spectrum is intriguing and could give some suggestions for the black-hole information loss paradox. In this section, along the line outlined by us earlier [14], we will show that there exists a correlation between the tunneling particles and the entropy which is conserved in the tunneling process.

Considering two emissions with energies \( E_1 \) and \( E_2 \) and using the expression (9), we have

\[
\Gamma_N(E_1) = \exp \left( -8\pi E_1 \left( M - \frac{E_1}{2} \right) \right) + 16 \sqrt{\frac{\pi}{\theta}} M^3 e^{\frac{M^2}{2}} - 16 \sqrt{\frac{\pi}{\theta}} (M - E_1)^3 e^{\frac{(M - E_1)^2}{2}},
\]

\[
\Gamma_N(E_2|E_1) = \exp \left( -8\pi E_2 \left( M - \frac{E_1 - E_2}{2} \right) \right) + 16 \sqrt{\frac{\pi}{\theta}} (M - E_1)^3 e^{\frac{(M - E_1)^2}{2}} - 16 \sqrt{\frac{\pi}{\theta}} (M - E_1 - E_2)^3 e^{\frac{(M - E_1 - E_2)^2}{2}},
\]

where the tunneling probability for the second radiation is a conditional probability given the occurrence of tunneling of the particle with energy \( E_1 \). According to the definition of joint probability in statistical theory [23], we get

\[
\Gamma_N(E_1, E_2) = \Gamma_N(E_1) \Gamma_N(E_2|E_1)
\]

\[
= \exp \left( -8\pi (E_1 + E_2) \left( M - \frac{E_1 + E_2}{2} \right) \right) + 16 \sqrt{\frac{\pi}{\theta}} M^3 e^{\frac{M^2}{2}} - 16 \sqrt{\frac{\pi}{\theta}} (M - E_1 - E_2)^3 e^{\frac{(M - E_1 - E_2)^2}{2}}.
\]

We can check that \( \Gamma_N(E_1, E_2) = \Gamma_N(E_1 + E_2) \) which is the emission of a particle with energy \( E_1 + E_2 \).

In order to evaluate the statistical correlation which says that two events are correlated if the probability of the two events arising simultaneously is not equal to the product
An amount of correlation is exactly equal to the mutual
probabilities of each event occurring independently, we
have to integrate the $E_1$ variable in $\Gamma_N(E_1, E_2)$ to attain the
independent probability $\Gamma_N(E_2)$,

$$\Gamma_N(E_2) = \Lambda \int_0^{M-E_2} \Gamma_N(E_1, E_2) \, dE_1$$

$$= \exp\left( -8\pi E_2 \left( M - \frac{E_2}{2} \right) + 16 \sqrt{\frac{\pi}{\theta}} M^3 e^{-\frac{M^2}{\theta}} \right) - 16 \sqrt{\frac{\pi}{\theta}} (M - E_2)^3 e^{-\frac{(M - E_2)^2}{\theta}},$$

where $\Lambda$ is the normalized factor which is the function of
the black-hole mass $M$, stemmed from the normalization
of the tunneling probability $\Lambda \int \Gamma_N(E) \, dE = 1$. Now we
can calculate the statistical correlation between the two radiations

$$C(E_1, E_2, \theta) = \ln \Gamma(E_1 + E_2) - \ln[(\Gamma(E_1) \Gamma(E_2))] \neq 0.$$  

Thus, the adoption of a noncommutative spacetime does
not change our statement that a nonthermal spectrum
affirms the existence of correlation, as is illustrated for
a Schwarzschild black hole. We find that the information
associated with noncommutativity is factored out in the
relation even in the early stage of Hawking radiation. This
even though noncommutativity only exists at the small scale, we can still test its effect through correlations
contained in the nonthermal spectrum of Hawking radiation.
 Especially, the LHC experiment could produce the
micro black-hole [24] and the analogous black-hole radiation
experiment has been realized through the laser [25]
or BEC [26], so one can observe such radiations to check
whether there is the information about noncommutativity in
them.

For tunneling of two particles with energies $E_1$ and $E_2$,
we find the entropy

$$S_N(E_1) = -\ln \Gamma_N(E_1) = 8\pi E_1 \left( M - \frac{E_1}{2} \right) - 16 \sqrt{\frac{\pi}{\theta}} M^3 e^{-\frac{M^2}{\theta}} + 16 \sqrt{\frac{\pi}{\theta}} (M - E_1)^3 e^{-\frac{(M - E_1)^2}{\theta}},$$  

$$S_N(E_2|E_1) = -\ln \Gamma_N(E_2|E_1) = 8\pi E_2 \left( M - \frac{E_2}{2} \right) - 16 \sqrt{\frac{\pi}{\theta}} (M - E_2)^3 e^{-\frac{(M - E_2)^2}{\theta}} + 16 \sqrt{\frac{\pi}{\theta}} (M - E_2 - E_2)^3 e^{-\frac{(M - E_2 - E_2)^2}{\theta}}.$$  

It is seen easily that they satisfy the definition for conditional entropy $S(E_1, E_2) = -\ln \Gamma(E_1 + E_2) = S(E_1) + S(E_2|E_1)$. A detailed calculation confirms that the amount of correlation is exactly equal to the mutual
information described in ref. [14], and this shows that it
is the correlation that carries away the information. If
we count the total entropy carried away by the outgoing
particles, we find

$$S_N(E_1, E_2, \ldots, E_n) = \sum_{i=1}^{n} S_N(E_i|E_1, E_2, \ldots, E_{i-1}).$$  

Thus we show that entropy is conserved in Hawking radiation for a Schwarzschild black hole in a noncommu-
tative spacetime. Note that the temperature will be zero
before the black hole vanishes, that is, the black hole will
evolve into a remnant which is in a high-entropy state with
entropy $S_{NC} = 4\pi E_2^2$. So finally the black-hole entropy
can be found as

$$S_{NB} = S_N(E_1, E_2, \ldots, E_n) + S_{NC}.$$  

Especially, if the reaction is included, the tunneling
probability is

$$\Gamma_{NR}(E) \sim \left[ 1 - \frac{2E(M - \frac{E}{2})}{M^2 + \alpha} \right]^{-4\alpha} \exp\left[ -8\pi E \left( M - \frac{E}{2} \right) \right]$$

$$\times \exp\left[ 16 \sqrt{\frac{\pi}{\theta}} M^3 e^{-\frac{M^2}{\theta}} - 16 \sqrt{\frac{\pi}{\theta}} (M - E)^3 e^{-\frac{(M - E)^2}{\theta}} + \text{const (independent of } M) \right].$$  

Using the same method as above, we can also show
that there exists correlations among radiations and the
entropy is conserved in the tunneling radiation process.
Our method for this case, however, does not solve the
remaining problem of whether the black hole will evaporate
to exhaustion or will halt at some value of a critical
mass because of the reaction effect or the use of a
noncommutative spacetime. As we discussed before, when
the quantum reaction parameter $\alpha$ is negative, a black
hole will leave behind a remnant instead of radiating into
exhaustion [27]. If noncommutative spacetime is intro-
duced, however, the parameter $\alpha$ cannot be negative, in
order to avoid a divergent temperature at the end of radiation.
That is to say, when the noncommutative parameter
$\theta \neq 0$, a reasonable value for $\alpha$ would be positive, and thus
when the mass of the black hole is reduced to a certain
value, the temperature will drastically decrease to zero to
form an extreme black hole [11]. Despite these subtleties,
we have shown in this section that the entropy is conserved in
the tunneling process and so the information could not be
lost even for the noncommutative Schwarzschild black
hole by taking into account the information carried away
by correlations in emitted particles.

**Conclusion.** – Using the tunneling method and considering the noncommutative effect in the Schwarz-
schild spacetime, the modified tunneling probability is
derived. Based on this probability, we have shown that
the adoption of a noncommutative spacetime supports that there exist correlations in a nonthermal spectrum and that the correlation can carry all information, even including the information about the noncommutativity, which may be observed in the future LHC experiment or simulating the experiment in laboratory. The entropy conservation is also investigated, which implies that the radiation process of a black hole is unitary in the background of noncommutative spacetime.

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