Time-dependent interpretation of the neutrino emission from Tidal Disruption Events

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Three tidal disruption event (TDE) candidates (AT2019\textsubscript{ds}g, AT2019\textsubscript{fdr}, AT2019\textsubscript{aalc}) have been associated with high energy astrophysical neutrinos in multi-messenger follow-ups. In all cases, the neutrino observation occurred $\mathcal{O}(100)$ days after the maximum of the optical-ultraviolet (OUV) luminosity. We discuss unified fully time-dependent interpretations of these events, where the neutrino delays are not a statistical effect, but rather the consequence of a physical scale of the post-disruption system. Noting that X-rays and infrared (IR) dust echoes have been observed in all cases, we consider three models in which quasi-isotropic neutrino emission is due to the interactions of accelerated protons of moderate, medium, and high energy energies with X-rays, OUV, and IR photons, respectively. We find that the neutrino time delays can be well described in the X-ray model assuming magnetic confinement of protons in a calorimetric approach, and in the IR model, where the delay is directly correlated with the time evolution of the echo luminosity (for which a model is developed here). The OUV model exhibits the highest neutrino production efficiency. In all three models, the highest neutrino fluence is predicted for AT2019\textsubscript{aalc}, due to its high estimated SMBH mass and low redshift. All models result in diffuse neutrino fluxes that are consistent with observations.

I. INTRODUCTION

Nearly a decade after their discovery, the high-energy extragalactic neutrinos seen by IceCube [1] – possibly indicating the production sites of the Ultra-High Energy Cosmic Rays (UHECRs) – are still largely a mystery, as their origin is still unresolved. Neutrino alert-triggered follow-up searches in electromagnetic data have proven successful to identify individual Active Galactic Nuclei (AGN) blazars as sources; the most prominent case is TXS 0506+056, which was found to be in a gamma-ray flaring state during the neutrino emission [2]. In time-integrated point source searches, individual neutrino sources are also emerging [3]: three AGN blazars (including TXS 0506+056), and a starburst galaxy (NGC 1068). The most sensitive limits for transient sources exist for the stacking of Gamma-Ray Bursts [4, 5], which indicate that they can only contribute to the diffuse neutrino flux at the percent level. Arguments from actual neutrino event detections and population statistics [6], as well as from spectral shape and directional information [7] point towards the contribution of multiple source populations contributing to the astrophysical diffuse neutrino flux; contributing candidate populations could be (apart from mis-identified atmospheric neutrino events) AGN blazars, AGN cores, starburst galaxies, neutrinos of Galactic origin, and Tidal Disruption Events (TDEs).

TDEs are phenomena in which a massive star passes close enough to a supermassive black hole (SMBH) to be ripped apart by its tidal forces. Following this process of tidal disruption, about half of the star’s matter remains bound to the SMBH and is ultimately accreted onto it. Observationally, this mass accretion results in a month- or year-long flare, with the emission of photons over a wide range of wavelengths, including a black body (BB) spectrum in the optical-ultraviolet (OUV) range, as well as sometimes X-ray, infrared (IR) and radio emission, see e.g. [8]. From observations and numerical modeling, the basic picture of the post-accretion phase of a TDE has emerged, including: an accretion disk, a semi-relativistic outflow, and possibly a jet, see e.g. [9]. Neutrinos have been associated with TDEs through follow-up searches; the Zwicky Transient Facility (ZTF) has been especially successful, leading to the identification of AT2019\textsubscript{ds}g [8] and AT2019\textsubscript{fdr} [10] as optical counterparts of two neutrinos (IceCube events IC191001A and IC200530A respectively). Afterwards, it was noticed that these TDEs were accompanied by an echo due to reprocessing of BB and X-ray radiation into the IR by surrounding dust, and this neutrino-dust link then led to the identification of a third TDE, AT2019\textsubscript{aalc}, as counterpart of the IceCube event IC191119A [11]. With three neutrino-TDE associations in less than one year, the case for TDEs as neutrino sources has become stronger\textsuperscript{1}, and

\textsuperscript{1} Note that AT2019\textsubscript{fdr} and AT2019\textsubscript{aalc} have not uniquely been identified as TDEs. Alternative interpretations are, e.g. AGN accretion flares or even luminous supernovae [12], although all events share a TDE-characteristic evolution of the BB light curve including the large and rapid optical flux increase [10] and the large dust echoes [11]. Also note these two events happened in AGN (i.e., black holes that were accreting prior to the optical flare). However the extreme properties of the flares compared to normal AGN variability suggest that the optical outbursts are likely induced by the disruption of a star. Here we adopt the hypothesis that these three objects are
it is therefore timely to revisit the neutrino production mechanism in TDEs, and the contribution of TDEs to the observed neutrino diffuse flux at IceCube, which was constrained to be $\lesssim 30\%$ in a stacking search \cite{13}.

Neutrino production in TDEs was proposed earlier in jetted models \cite{14,18}, mostly motivated by observations of the jetted TDE Swift J1644+57. Furthermore neutrino production in different disk states \cite{19} and ejecta-external medium interactions \cite{20} were considered: TDEs may also be candidates to accelerate and even power the UHECRs \cite{21,23}. For AT2019dsg, jets \cite{26,27}, outflow-cloud interactions \cite{28}, disk, corona, hidden winds or jets \cite{29} have been proposed, see \cite{30} for an overview. While a collimated outflow, such as a jet, has the advantage that it can provide the necessary power for the neutrino emission (see discussion in \cite{31}), no convincing direct jet signatures for AT2019dsg have been observed \cite{32}, and the observed radio signal might only be interpreted as jet signature in scenarios with purely leptonic radiative signatures for an unnaturally narrow jet \cite{33} or a steep density profile \cite{34}. For AT2019fdr, corona, hidden wind and jet models have been considered in \cite{10}, and in \cite{11} a disk model for all three TDEs is proposed. The neutrino production site is therefore uncertain, and comparative quantitative studies of all three TDEs do not yet exist.

In this work, we provide a unified quantitative description of the three observed neutrino-emitting TDEs, AT2019dsg, AT2019fdr, and AT2019aalc. We build on the fact that these TDEs have a few common characteristics beyond the detected IR dust echoes: (i) the most likely neutrino energies are in the 100 TeV range, and (ii) the neutrinos arrived $\mathcal{O}(100)$ days after the BB peak – where the BB luminosities already have decreased significantly, but the dust echoes have been close to their maxima. Moreover, (iii) X-rays from all the neutrino-associated sources have been detected, although X-ray detection is generally rare in TDEs, see \textit{e.g.} \cite{35}. Specifically, for AT2019dsg an exponentially decaying (with a timescale of tens of days) early signal was found \cite{8,35}, in AT2019fdr early upper limits have been imposed, with a detection about 600 days after the BB peak \cite{10}, and in AT2019aalc a roughly constant X-ray flux was found more than one year after the BB peak \cite{11}. In all cases, (iv) the estimated SMBH masses range (with large uncertainties) between about $10^{6.5}$ and $10^{7.5} M_\odot$ \cite{11}, close to the maximum mass for which tidal disruption is possible (often called Hills mass, see \cite{36}). Consequently, all events should have relatively high BB luminosities as a consequence of high Eddington luminosities – for which the measured values in the OUV range are only lower limits, due to obscuration effects.

These common observations immediately raise important questions: is the neutrino production associated with the X-ray, OUV, or IR signals, i.e., what is the smoking gun signature for the neutrino production? What causes the neutrino time delay with respect to the BB peak, and is that always expected? What can we learn from the predicted neutrino spectra in comparison with the observed neutrino energies? Are the neutrino spectra evolving with time? And: why do the neutrino-emitting TDEs seem to accumulate at the upper end of the allowed SMBH range? Here we address these questions by developing a fully quantitative, time-dependent model of neutrino production. We take the point of view that the neutrino time delays have a physical origin – in a characteristic time or length scale of the post-disruption system – instead of being statistical effects. We aim at keeping the model as minimal as possible, by only introducing the strictly necessary ingredients.

Our study is organized as follows: in Sec. II we introduce the model and describe its details. Results are presented for three realizations (named M-X, M-OUV and M-IR, after the three different photon targets used), in sections Sec. III, Sec. IV, and Sec. V. We present our results for the diffuses fluxes in Sec. VI and we compare the different TDEs and models in the comparison and discussion section Sec. VII. We finally summarize in Sec. VIII.

II. MODEL DESCRIPTION

In this section, we describe the spirit and the ingredients of our model. For a reader wanting a quick overview, Sec. II A, Fig. I and Table I summarize the qualitative features and numbers. Our method to describe the dust echo is described in Sec. II E.

A. Overview

To explain the emission of high energy neutrinos, protons must be accelerated to energies beyond $\sim$PeV, and interact with background photons and/or matter. The neutrinos are then natural results of these interactions. In the

Indeed TDEs, and therefore will be called as such; the wording “candidates” will be dropped from here on.

Note, however, that the neutrino observation itself points to a relatively large proton loading, perhaps similar to lepto-hadronic AGN models, which means that modifications of the radiative signatures (\textit{e.g.} the synchrotron self-absorption break) might be expected.
Elaborating on the latter point, let us examine the large scale structure of a TDE after the disruption has occurred, as shown in Fig. [1]. In the figure, the components where proton acceleration could take place, and their characteristic radial scales, are illustrated for a reference SMBH mass $M = 10^7 M_\odot$ (gravitational radius $R_S \approx 3 \cdot 10^{12}$ cm). The figure illustrates how the location of the proton acceleration (radial distance $R = R_{\text{acc}}$) can vary widely, from $R_{\text{acc}} \sim (3 - 30) R_S$ (the X-ray photosphere and the hot corona), to $R_{\text{acc}} \sim 10^3 R_S$ (the OUV photosphere, or the collision region inside a jet) or even larger values like $R_{\text{acc}} \sim 10^{16} - 10^{17}$ cm, for acceleration inside the outflow or in stream-stream collisions, or near the dust torus.\(^3\) In some of these cases, the neutrino delay might be reproduced by a geometric distance; for example higher-energy protons could interacting with the IR photons from the dust echo, which carries a delay from the size of the dust region. In other cases, the delay might arise from the dynamics of proton propagation: \textit{e.g.} lower-energy protons (such as leaking out from an off-axis jet; see App. [A] for a discussion) could be confined in magnetic fields over the diffusion time, and transfer energy into secondaries in the calorimetric limit.

For generality, here we do not specify the proton accelerator in detail, but rather characterize it by the maximal proton energy achieved, $E_{p,\text{max}}$, and the proton luminosity evolution. We focus instead on the radiation zone, i.e., the region at $R \gtrsim R_{\text{acc}}$ where the neutrinos are produced. We start by observing that the preferred photon target

\(^3\) We do not consider core models, where the acceleration takes place at $R \ll 10^{14}$ cm and the corresponding travel times are too short.
depends on the maximal proton energy provided by the accelerator. Indeed, for $p\gamma$ interactions, the observed neutrino energies in the 100 TeV range indicate black body target photon temperatures:

$$T \simeq 80\,\text{eV} \left( \frac{E_\nu}{100\,\text{TeV}} \right)^{-1},$$

in the (soft) X-ray range. Since, however, depending on the spectral shape the actual neutrino energies may be significantly higher for observed muon tracks, lower target photon temperatures in the optical-UV (OUV) or infrared (IR) ranges paired with neutrinos peaking at higher energies may work as well, provided that the accelerator is efficient enough. The requirement

$$E_{p,\text{max}} \gtrsim 20\,E_\nu \simeq 160\,\text{PeV} \left( \frac{T}{\text{eV}} \right)^{-1}$$

translates then into $E_{p,\text{max}} \gtrsim 2\,\text{PeV}$, $E_{p,\text{max}} \gtrsim 100\,\text{PeV}$, and $E_{p,\text{max}} \gtrsim 1\,\text{EeV}$ for X-rays, OUV, and IR targets, respectively. Detailed parameters for the individual TDE electromagnetic spectra are listed in Table II. We will study these options systematically, increasing $E_{p,\text{max}}$ within three models called M-X, M-OUV, and M-IR, making additional target photon fields accessible for the interactions. In some cases, $pp$ interactions with the outflow will also contribute significantly, especially if no high target photon densities are accessible.

Note that our model assumptions will be as universal as possible, which means that common parameters are chosen for all TDEs if physically motivated; this assumption simplifies the comparison, whereas the very different redshifts of the TDE candidates with associated neutrinos jeopardize the direct comparison of neutrino fluences or event rates, see Table II in fact, we will see that the predicted neutrino emission is in fact not as different as one may expect if the neutrino luminosity or spectra (at the source) are compared.

Compared to other models in the literature, our model M-X shares some similarities with the hidden wind model in [29], and our model M-OUV is in fact a time-dependent numerical implementation of the idea proposed in [8]. Our model M-IR is motivated by the dust echo connection of the neutrino-observed TDEs [11], postulating a direct connection. Therefore, we develop our own dust model to obtain the time-dependent luminosity of the IR echo (see Sec. II E). Compared to jetted models, the main challenge for the presented models (and in fact most quasi-isotropic emission models) are very high required transfer efficiencies of material of the disrupted star into non-thermal protons; this problem can be avoided in models with a collimated outflow, see discussion in [31]. However, the difference between isotropic and collimated emission models is more fundamental: are TDEs inefficient neutrino emitters each with the contribution of a fraction of an event on average, invoking the Eddington bias argument [39], or are they more efficient neutrino emitters perhaps just not pointing into our direction or are they too far away in most cases? Since we focus on the isotropic case in this study, the predicted neutrino event number per TDE will be an interesting indicator if the Eddington bias argument has to be invoked for each individual TDE.

In the following subsections, we describe the elements that are common to all the models, whereas specifics will be discussed in the following respective results sections.

### B. Numerical time-dependent simulation of radiation zone

We solve the coupled differential equation system for the in-source densities $N_i$ (differential in energy)

$$\frac{\partial N_i(E,t)}{\partial t} = \frac{\partial}{\partial E} \left( \frac{E}{t_{\text{cool}}(E,t)} N_i(E,t) \right) - \frac{N_i(E,t)}{t_{\text{esc}}(E,t)} \text{Injection} + \text{Cooling} - \text{Escape}$$

for the protons ($i = p$) and neutrons ($i = n$) fully time-dependent using the NeuCosmA software [37, 43, 44]. Here the cooling rate is given by $t^{-1}_{\text{cool}} = E^{-1} |dE/dt|$, the escape rate by $t^{-1}_{\text{esc}}$, and the injection $\tilde{Q}_p(E) = Q_p(E) + Q_{j\rightarrow p}(E)$ (differential in energy and time) contains the injection from the acceleration zone $Q_p$ as well as the re-injection $Q_{j\rightarrow p}$ from interacting protons at higher energies and interacting neutrons ($j = p, n$) – which couples the differential equations; for neutrons corresponding terms are used, but there is no injection from an acceleration zone, i.e., $Q_n(E) \equiv 0$.

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4 In this estimate, both the pitch-angle averaged cross section (see Fig. 4 in [37]) and the peak of the photon number density located at $\simeq 2.8\,T$ [38] are taken into account.
Photohadronic interactions are treated as discrete energy losses as described in [45], which means that escape terms with the interaction rate $t_{p\gamma}^{-1}$ are added and the interaction products are re-injected in $Q_{j\to p}$ at lower energies; we use the efficient but accurate treatment of [37] based in the physics of SOPHIA [45]. The emitted neutrinos are integrated over time, while the in-source densities $N_i$ are evolved over the lifetime of the system (as shown in our figures). Since the proton injection rate varies with time, a steady state will be never reached. Protons also cool via Bethe Heitler pair production; note that all these injection, escape and cooling rates are explicitly time-dependent through the time-dependent evolution of the target photon fields.

Protons escape diffusively, see below, or by interactions, whereas neutrons escape over the free-streaming timescale $t_{fs} \simeq R/c$. We also assume that protons escape over the duration of the TDE, referred to as dynamical timescale $t_{dyn}$ here (see below for its definition), such as by advection, to take into account the transient nature of the event; however, the impact of that term on the neutrino production is small. We also include synchrotron losses for all charged species (including the secondary muons, pions, kaons).\(^5\)

Our approach can properly treat the optically thick case (see App. C of [45]), which we define as $\tau_{p\gamma}^{\text{cal}} \equiv t_{fs}/t_{p\gamma} > 1$ (protons interact efficiently while crossing the radiation zone), and the calorimetric case, which we define as $t_{p\gamma}^{\text{cal}} \equiv t_{dyn}/t_{p\gamma} > 1$ (magnetically confined protons interact efficiently over the duration of the system). We will find the calorimetric case for interactions in model M-OUV, where the proton energies are low, and the optically thick case for M-OUV, where the interaction rates are high (M-IR is in between). Note that the frequently used concept displaying effective photohadronic cooling rates $t_{p\gamma, \text{cool}}^{-1} \simeq 0.2 t_{p\gamma}^{-1}$ does not apply to the (optically thin) calorimetric case, since neutrons produced in the interactions can escape over the free-streaming timescale, and hence the effective cooling rate will be much higher; consequently, we only show interaction rates where applicable, while our numerical

\(^5\) Synchrotron losses play only a minor role in this study and because non-thermal electrons are not explicitly considered. Even for the neutrino flavor composition, the effects are negligible because of very high critical energies due to the relatively small values of $B$; see e.g. App. A.1 of [47].
discuss. The confinement condition \( R \) may support a stable production region. If, however, the non-relativistic outflow is indicative for the expansion of the system, which for instance has been detected in AT2019\( \text{disg} \) supported by radio data, then \( t_{\text{ad}}^{-1} \simeq 4/3 \cdot v/R_{\text{rad}} \sim 10^{-3} t_{\text{dyn}}^{-1} \sim t_{\text{peak}}^{-1} \) for a non-relativistic outflow with \( v \simeq 0.1 \) c which is small enough to support the calorimetric picture. Since the adiabatic expansion rate is uncertain and also depends on the nature of the accelerator, we do not include it in the simulation, but we take note of that requirement; especially for models M-X, our predicted neutrino event rates can be reduced if \( t_{\text{ad}} \ll t_{\text{pe}} \).

C. Energetics, proton injection, and proton confinement

The Eddington luminosity \( L_{\text{edd}} \simeq 1.26 \times 10^{45} \, (M/10^7 M_\odot) \) erg s\(^{-1}\) with the SMBH mass \( M \) is a measure for the energy re-processing rate through the SMBH, where super-Eddington mass accretion rates are expected for TDEs at peak; we use the inferred SMBH masses listed in Table \( \text{I} \). Following \([9]\), we assume that the mass accretion rate \( \dot{M} \) exceeds \( L_{\text{edd}} \) by a factor \( F_{\text{peak}} \simeq 100 \) at peak. While the mass fallback rate is expected to scale \( \propto t^{-5/3} \) generically, we more accurately implement that the mass accretion rate roughly follows the observed BB evolution for each individual TDE; for details and references, see Table \( \text{I} \). We furthermore roughly estimate the dynamical timescale \( t_{\text{dyn}} \) of the TDE from the time period for which \( \dot{M} > L_{\text{edd}} \) from the (extrapolated) lightcurve, since qualitative changes are expected for \( \dot{M} \ll L_{\text{edd}} \) (e.g. transition of the disk accretion state or jet cessation).

We parameterize the proton injection spectrum \( Q_p \) (density differential in energy and time) as

\[
Q_p = Q_0 \, E_p^{-2} \exp \left( -\frac{E_p}{E_{p,\text{max}}} \right),
\]

where \( E_{p,\text{max}} \) is a parameter depending on the model (M-X, M-OUV, or M-IR). The non-thermal proton injection luminosity \( L_p \) is dynamically following the mass accretion rate \( \dot{M} \) by \( L_p = \varepsilon_{\text{diss}} \dot{M} \), where we need to postulate relatively high dissipation efficiencies \( \varepsilon_{\text{diss}} \simeq 0.2 \) into non-thermal protons, see discussion sections. Note that on the other hand, we use 1 GeV as lower energy of the proton spectrum, as we anticipate that the protons are picked up from a thermal bath; this assumption is conservative, as a lot of energy will be transferred into non-thermal protons which are below the \( p_T \) threshold. The proton spectrum normalization \( Q_0 \) in the radiation zone is obtained from the proton luminosity \( L_p \)

\[
\int dE_p \, E_p \, Q_p(E_p) = \frac{L_p}{\frac{4}{3} R^3 \pi}.
\]

We note that the relation between the size of the radiation zone \( R \) and the acceleration location \( R_{\text{acc}} \lesssim R \) depends on the model, see next sections.

Another measure for the available energy is the mass of the disrupted star. Since \( L_p(t) = \varepsilon_{\text{diss}} \dot{M}(t) \) and taking about half of the stellar debris accreted towards the SMBH, we can estimate the mass of the disrupted star as \( M_* \simeq 2 \cdot \int \dot{M} \, dt \), which we list in terms of solar masses \( M_\odot \simeq 1.8 \times 10^{34} \, \text{erg} \) as a result of our computation in Table \( \text{I} \).

If the magnetic field in the radiation zone is given by \( B \), protons gyrate with the Larmor radius

\[
R_L \simeq 3.3 \times 10^{12} \, \text{cm} \left( \frac{E_p}{\text{PeV}} \right) \left( \frac{B}{\text{G}} \right)^{-1},
\]

which means that PeV protons are magnetically confined for Gauss-scale magnetic fields for the region sizes \( R \) we discuss. The confinement condition \( R_L \lesssim R_{\text{acc}} \) directly imposes constraints on the size of the accelerator; for example, \( R_{\text{acc}} \gtrsim 3 \times 10^{16} \, \text{cm} \) for \( E_{p,\text{max}} \simeq 1 \, \text{EeV} \) and \( B = 0.1 \, \text{G} \) (see model M-IR).

Assuming Bohm-like diffusion with a diffusion coefficient \( D \approx R_L \, c \), protons can be displaced by

\[
\Delta x \simeq 3 \times 10^{15} \, \text{cm} \left( \frac{E_p}{\text{PeV}} \right)^{1/2} \left( \frac{B}{\text{G}} \right)^{-1/2} \frac{t_{\text{dyn}}}{(1000 \, \text{days})}^{1/2}.
\]

\[ \text{Integration ranges in time will correspond to the ranges shown in our figures, e.g. Fig. 4 upper row. Note that for TDEs in AGN the mass of the disrupted star may be smaller, because material from the existing accretion disk may be attracted; in general TDEs in AGN seem to be more luminous; see e.g. [35]. In that cases, half of the reported } M_* \text{ correspond to the accreted material.} \]
over the dynamical timescale \( t_{\text{dyn}} \). This means that protons with PeV energies will be magnetically confined by Gauss-scale magnetic fields the region of size \( R \approx 3 \times 10^{15} \) cm over the lifetime of the TDE, and the system will be calorimetric if \( t_{p,\gamma} < t_{\text{dyn}} \), see e.g. model M-X. Since the Larmor radius in Eq. (6) is for our parameters considerably smaller than the region \( R \), only protons with very high energies can escape directly (ballistically), and protons with \( E_p \approx 1 - 10 \) PeV (relevant for the X-ray interactions) are confined. Here we follow \([9]\) and define a self-consistent escape rate for the protons given by

\[
I_{p,\text{esc}}^{-1} = \min(I_{p,\text{diff}}^{-1}, I_{p,\text{fs}}^{-1}) \quad \text{with} \quad I_{p,\text{diff}}^{-1} = \frac{D}{(c t_{p,\text{fs}})^2},
\]

the diffusion coefficient \( D \), and the free-streaming timescale \( t_{p,\text{fs}} \approx R/c - \), which implies that the diffusive escape rate is limited by the free-streaming rate.\(^7\) Note that magnetic confinement also leads to isotropization of the proton-photon pitch-angles (the angles between incoming protons and photons in the interaction frame), if not already isotropized.

**D. Photon and proton targets**

Protons encounter target photons and protons within the radiation zone of radius \( R \). All photon targets are described by quasi-thermal spectra with temperature \( T \), motivated by observations. As a short summary, for X-rays, which stem from the accretion disk, we hypothesize that the (highest) detected X-ray flux is indicative for the actually emitted X-ray flux, and obscuration, such as from a complicated geometry or outflow, leads to flux fluctuations – whereas the intrinsic flux within \( R \) is relatively stable (see e.g. \([51]\)). For OUV, we take the BB evolution from observations directly, but we correct for absorption by inferring the unabsorbed luminosity from the dust echo, see Sec. [II]. For IR, we model the dust echo both in terms of time-dependence and normalization, as described in the same subsection. More details will be also given in the respective model sections later. The in-source photon density \( n(\varepsilon) \) (typically units \([\text{GeV}^{-1} \text{ cm}^{-3}]\)) is then computed from the luminosity \( L \) as

\[
\int d\varepsilon \varepsilon n(\varepsilon) = \frac{L}{4 \pi R^2 c},
\]

which is a good estimate for X-rays, a lower limit estimate for the OUV black body if \( R < R_{\text{BB}} \) and a better estimate if \( R \gtrsim R_{\text{BB}} \), and a rough estimate for the IR target if \( R \) corresponds to the scale of the dust scattering. We discuss the optical thickness to the respective target photons in detail later.

We also consider \( pp \) interactions with a mildly relativistic outflow because the outflow is a plausible model ingredient, as it may be the reason for the X-ray obscuration, the outflow is expected in numerical simulations e.g. \([49]\), and it has been directly observed for AT2019\( \text{ds}g \) \([8]\). Note that there may be interactions of other components, such as debris, clumps or clouds as well, which we however do not describe in view of major geometric and density uncertainties.

From \([9]\), the outflow densities are high up to about 1000 gravitational radii, which is about \( 10^{15} \) cm for \( M \approx 10^7 M_{\odot} \) in consistency with Eq. (7); therefore even calorimetric effects may be expected. The relevant target density is computed by assuming that \( \varepsilon_{\text{Outflow}} \approx 0.2 \) of the mass accretion is re-processed into the outflow \( L_{\text{outflow}}(t) = \varepsilon_{\text{outflow}} M(t) \) (it is scaling with the mass accretion rate) \([9]\). Since \( M_{\text{outflow}} \approx \frac{L_{\text{outflow}}}{\varepsilon_{\text{outflow}} c} t_{p,\text{fs}} \) in the production volume, smaller velocities mean higher densities. The free-streaming optical thickness can be estimated as

\[
\tau_{p,\text{fs}}^{-1} = \frac{t_{p,\text{fs}}}{t_{p,\text{pp}}} \approx 0.01 \left( \frac{M}{10^7 M_{\odot}} \right) \left( \frac{R}{10^{15} \text{ cm}} \right)^{-1} \frac{v}{0.5 c}^{-1},
\]

where \( v \) is the velocity of the outflow. A comparison to \([9]\) reveals that \( v \approx 0.5 c \) towards the poles, whereas \( v \approx 0.1 c \) perpendicular to that – where the densities are higher. We conservatively use \( v \approx 0.5 c \) We will see that \( pp \) interactions can be important if the X-ray luminosity is low, or at low energies (below the \( p,\gamma \) threshold).

**E. Dust echo and inferred bolometric luminosities**

For each TDE, an IR lightcurve was measured at several times after the peak by neoWISE in the W1 and W2 frequency bands. It has been interpreted as thermal emission from a dust torus which is illuminated and heated by

\(^7\) This definition mimicks the transition between the diffusive and free-streaming escape regimes for magnetic field turbulence coherence lengths \( l_c \sim R \), see discussion in \([50]\).
the OUV and X-ray radiation emitted by the TDE accretion disk (see, e.g., [11]). The main features of this IR dust echo are: (i) the delayed emission with respect to the primary OUV and X-ray emission, due to the dust being at an angle with respect to line of sight (see Fig. 1); and (ii) a thermal IR spectrum with temperature at or below the dust sublimation temperature of $T_{IR} \simeq 0.16 \text{eV} \ (T_{IR} \simeq 1850 \text{K})$ [10, 11] (see also, e.g., $52$ for a general description of dust properties in TDEs). We use this value unless it has been measured, see Table I.

Assuming that the contribution of the X-rays to the dust echo is negligible in the present case, the energy emitted in IR in the neoWISE bands can be expressed as:

$$ E_{IR} = \epsilon_{bol} \epsilon_\Omega \epsilon_{dust} E_{BB}^{bol}. \quad (11) $$

Here $E_{BB}^{bol}$ is the total (bolometric) energy in the OUV spectrum, $\epsilon_{bol}$ is a correction factor describing the ratio between the neoWISE measured luminosity and the bolometric luminosity, $\epsilon_\Omega = \Omega/4\pi$ is the geometric covering factor of the dust, and $\epsilon_{dust} \leq 1$ is an efficiency expressing the fraction of the incident radiation that is re-emitted by the dust in the IR.

As shown in [10], the time evolution of the IR luminosity for AT2019fdr is well described by convolving the observed OUV luminosity with a (normalized) box function $B(t)$ centered at time $\Delta t$ and having width $2\sigma_t$ (i.e., $B(t) = 1/(2\sigma_t)$ if $\Delta t - \sigma_t \leq t \leq \Delta t + \sigma_t$ and $B(t) = 0$ elsewhere, with $\sigma_t \leq \Delta t$). Such a function models the fact that a wide spread in time delays is expected due to the extended shape of the dust torus, see Fig. 1; the quantity $\sigma_t$ describes such a spread, with $\Delta t$ being the central value. Following [10], we choose $\sigma_t = \Delta t$, which accounts for a portion of the IR flux to have zero delay due to some dust being along the line of sight. The time-differentiated version of Eq. (11) reads

$$ L_{IR}(t) = \epsilon_{bol} \epsilon_\Omega \epsilon_{dust} \int_{-\infty}^{+\infty} L_{BB}^{bol}(t') B(t - t') \, dt'. \quad (12) $$

We performed a least-squares fit of the IR luminosity measurements at different times (taken from [10, 11]), with the goal of obtaining: (i) a best-fit IR lightcurve; (ii) an estimate of the unabsorbed OUV luminosity and temperature and (iii) information on the size and geometry of the dust torus. Results are given in Table I; below a more detailed description of the methodology is given.

The IR lightcurve was modeled as in Eq. (12), with the assumption that the time profile of $L_{BB}^{bol}$ is the same as that of the observed OUV luminosity, $L_{BB}$. The results of the fit are the time delay, $\Delta t$, and the normalization $E_{IR}$. For all three TDEs the obtained best-fit IR lightcurve is in good agreement with the data. For AT2019fdr, it is consistent with the one shown in [10]. For AT2019dsd, the lightcurve underestimates the earliest-measured IR flux, but fits the later data points very well.8 Setting the unknown coefficients in Eq. (11) to optimistic (large) values, where we took $\epsilon_\Omega \epsilon_{dust} = 0.5$, and estimating the correction factor $\epsilon_{bol}$ for the W1 and W2 bands9, a minimum value $E_{BB}^{bol}$ (min.) and, therefore $L_{BB}^{bol}$ (min.) = $E_{BB}^{bol}$ (min.) $/ L_{BB}/E_{BB}$ was obtained. From that estimate $L_{BB}^{bol}$ (min.), the temperature of the OUV spectrum can be inferred from the Stefan-Boltzmann law; the result was adopted as best estimate available for AT2019aalc (in the absence of an estimate from the observed spectrum), and served as a consistency check for the remaining two TDEs. See Table I for our results, the $L_{BB}^{bol}$ (min.) and $L_{IR}^{bol}$ = $L_{BB}^{bol}/\epsilon_{bol}$ light curves are included in our results figures (Fig. 4, Fig. 7 and Fig. 10, upper rows) as black dotted and dashed-dotted curves, respectively.

### III. MODERATE-ENERGY PROTONS INTERACTING WITH X-RAYS (MODEL M-X)

**Model-specific description.** Our model M-X uses a low maximal $E_{p,max} = 5 \times 10^6 \text{GeV}$, universal for all TDEs, to guarantee interactions with the X-ray targets in all TDEs, but low enough to suppress the interactions with the OUV; it therefore has the lowest requirement on the proton acceleration efficiency. The microphysics is illustrated in the cartoon in Fig. 2 left panel, and model-specific assumptions and results are listed in the table in Fig. 2 right panel. The radiation zone is determined by the location of the accelerator $R \simeq R_{acc} \gtrsim R_{BB}$. For the sake of simplicity, we choose $R = 5 \times 10^{15} \text{cm}$ together with $B = 1 \text{G}$ for all three TDEs to satisfy the confinement condition in Eq. (7) with

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8 The discrepancy at early times might be an indication that the simple model in Eq. (12) is inaccurate. A possible improvement on it could be to use a bi-modal distribution instead of a box function, to account for the presence of dust on both sides of the line of sight, with one side being closer to the observer (resulting in a smaller time delay) than the other.

9 Here we use thermal spectra with either the measured IR temperature (for AT2019fdr) or a theoretically motivated value close to the dust sublimation temperature (for AT2019dsd and AT2019aalc): we find $(\epsilon_{bol}^{IR})^{-1} \simeq 1.26, 1.38$ in the two cases (factors correcting the combined W1+W2 luminosity $\nu L_{\nu}$).
FIG. 2: Model M-X. Left panel: Microphysics cartoon. Right panel: Model assumptions and results. See caption of Fig. 4 for the definition of the GFU (Gamma-Ray Follow-Up) and PS (Point Source) effective areas.

FIG. 3: Relevant inverse timescales (rates) for protons and neutrons for M-X and AT2019dsg in the source frame as a function of energy (in the observer’s frame); the shaded areas is beyond \( E_{p,\text{max}} \) for M-X. Solid curves refer to the \( t_{\text{peak}} \), whereas dashed curves to the time of the neutrino emission \( t_{\nu} \) (unless the rate is constant in time). Note that for \( p\gamma \) and \( pp \) interactions the interaction rates (and not the cooling rates) are shown.

\( R \approx \Delta x \) for the maximal proton energies used here.\(^{10}\) This means that injected protons will gyrate in magnetic fields and interact with X-rays and protons of the outflow to produce neutrinos, as illustrated in the cartoon.

All three TDEs have in common that they have been observed in X-rays, which are the prime target for the neutrino production of 100 TeV neutrinos, see Eq. (1). The X-ray observations have, however, very different characteristics: an exponential early decay (AT2019dsg) \(^{8}\), a late-time observation with strongly varying limits at different times (AT2019fdr) \(^{10}\), and a late-term constant flux (“plateau”) significantly after the neutrino observation (AT2019aalc) \(^{11}\). Since X-rays are expected to originate from the accretion disk, the TDE unified model predicts obscuration effects depending on the viewing angle \(^9\), and large temperature fluctuations on short timescales may be unlikely, we hypothesize that the (highest) detected X-ray signal is indicative for the actually emitted X-ray flux, and that obscuration beyond \( R \), such as from a complicated geometry or outflow, leads to the observed fluctuations.\(^{11}\) For example, following the slim disk model \(^{51}\), the unobscured flux will be relatively stable, whereas it is expected to change or cease if the mass accretion rate drops below the Eddington luminosity – such as if there is a transition of the

\(^{10}\) One could also introduce a dependence on \( M \) here scaling the parameters such that \( R \approx \Delta x \) in Eq. (1) holds, but this would be more complicated.

\(^{11}\) For AT2019dsg the same effect has led to X-ray isotropization of external target photons in the jetted model \(^{26}\), where the assumed \( R_{\text{acc}} \approx R_{BB} \) was somewhat smaller.
accretion disk state. We therefore exponentially suppress it with a factor $\propto \exp(-L_{\text{edd}}/\dot{M})$, which implies that the X-ray flux available for interactions is stable over $t_{\text{dyn}}$. Consequently, we use the measured X-ray temperature $T_X$ for each TDE, and we normalize the spectrum at the time of the highest measured energy flux to the X-ray measurement in the respective energy range – we however quote bolometric X-ray luminosities in Table I. The in-source photon density $n_{\gamma}$ can be computed with Eq. (1) from $L_X$.

Interaction rates and calorimetric behavior. It is useful to look at the rates (inverse timescales) as a function of energy for one example to illustrate the calorimetric behavior; we therefore show in Fig. 3 the rates for AT2019dsg at peak time (solid curves) and at the time of the neutrino emission (dashed curves, where time-dependent). First of all, we note that $p\gamma$ interactions with X-rays are possible for $E < E_{p,\text{max}}$, whereas the interactions with the OUV blackbody are suppressed because the proton energies are too low (the gray-shaded area marks $E > E_{p,\text{max}}$). However, since the OUV luminosity is much higher than the X-ray luminosity, and Bethe-Heitler pair production has a lower threshold, the pair production rate $t_{p,\text{BH}}^{-1}$ off OUV photons (see Sec. IV for the implementation) can be substantial. Depending on the ratio between OUV and X-ray luminosities, X-ray $p\gamma$ interactions may be suppressed. Since the OUV scales with the mass accretion rate, the corresponding Bethe-Heitler rate will be lower at the neutrino emission time $t_{\nu}$ – whereas the X-ray part of $t_{p,\text{max}}^{-1}$ is quite stable (dashed curves). We observe that while this effect suppresses the overall neutrino production and leads to low neutrino event rates, see Fig. 2 (right panel), it prefers a late-term neutrino production. Proton-proton interactions are always relevant for the neutrino production below the $p\gamma$ threshold, but inefficient in the shown example compared to the X-ray interactions; a counter-example is AT2019aalc for which the observed X-ray flux is low.

We can also discuss the proton confinement and calorimetric behavior with Fig. 3. Here the free-streaming escape rate $t_{fs,\gamma}^{-1}$ is much larger than $t_{\text{dyn}}^{-1}$, and X-ray interactions are effective over the dynamical timescale, but not the free-streaming timescale. Since protons at the highest energies are confined ($t_{p,\text{esc}}^{-1} \ll t_{\text{dyn}}^{-1} \ll t_{p,\gamma}^{-1}$), the in-source proton density will accumulate and $p\gamma$ interactions will be stretched over a longer timescale – leading to a delay of the neutrino production scaling with $t_{p,\gamma}$. This can be also seen in analytical estimates: the optical thicknesses for the free-streaming and calorimetric cases can, for the X-rays, be analytically roughly estimated as

$$
\tau_{p,\gamma}^{\text{fs}} \equiv \frac{t_{fs}}{t_{p,\gamma}} \simeq 0.06 \left( \frac{L_X}{10^{44} \text{erg s}^{-1}} \right) \left( \frac{T_X}{100 \text{eV}} \right)^{-1} \left( \frac{R}{5 \times 10^{15} \text{cm}} \right)^{-1},
$$

$$
\tau_{p,\gamma}^{\text{cal}} \equiv \frac{t_{\text{dyn}}}{t_{p,\gamma}} \simeq 18 \left( \frac{L_X}{10^{44} \text{erg s}^{-1}} \right) \left( \frac{T_X}{100 \text{eV}} \right)^{-1} \left( \frac{R}{5 \times 10^{15} \text{cm}} \right)^{-2} \left( \frac{t_{\text{dyn}}}{600 \text{days}} \right),
$$

which means that $\tau_{p,\gamma}^{\text{fs}} < 1$, but $\tau_{p,\gamma}^{\text{cal}} > 1$ – the system is optically thin, but calorimetric. A small subtlety are neutrons produced in $p\gamma$ interactions, for which free-streaming escape dominates over interactions and decays (see black line); the effective proton cooling rate is therefore closer to the (shown) interaction rate than $0.2 t_{p,\gamma}^{-1}$. Our numerical code treats all these effects self-consistently, as pointed out earlier.

Results. Our main results for model M-X are presented in Fig. 4 for all three TDEs (columns). The upper row shows the time evolution of the luminosities (in the SMBH frame), the lower row the muon neutrino fluences and expected event rates, as well as the contributions from different targets. Flavor mixings are taken into account using the mixing angles in [55]; however, as explained earlier, non-trivial effects are not expected here. Concerning the time evolution (upper row), the proton injection follows the OUV BB (black dotted curves) in all cases, scales with respect to the Eddington luminosity (green dashed curves) such that at $t_{\text{peak}}$ (by our assumptions) $20 \times L_{\text{edd}}$ goes into non-thermal proton injection. The in-source proton density $L_{p,\text{cal}}$, which determines the neutrino production, is illustrated by the green dashed-dotted curves (arbitrary units, integrated over two highest orders of magnitude in energy): it first increases with the proton injection, then it decays with a delay determined by $p\gamma$ interactions. The X-ray targets (unattenuated) are shown as blue curves, normalized to the highest observed luminosity (dots). The neutrino light curves, to leading order, follows the product $L_{p,\text{cal}}^{\text{eff}} \times L_X$, but OUV and pp interactions also contribute somewhat. The observed neutrino times are marked by arrows, where the best description of the neutrino delay is obtained for AT2019aalc and the worst for AT2019dsg (because of the quickly decaying BB). It is noteworthy that in all cases the neutrino luminosities (at source) are comparable, and the fluxes are strongly affected by redshift.
The GFU effective area includes the trigger probability of the gamma-ray follow-up pipeline, which implies that it has a higher threshold than the PS effective area.

15 The GFU effective area includes the trigger probability of the gamma-ray follow-up pipeline, which implies that it has a higher threshold than the PS effective area.
Discussion. Our model shares some characteristics with the hidden wind model \[29\], which may serve as an accelerator: particle acceleration may occur in high-velocity winds embedded in the TDE debris or shocks from stream crossings. Compared to \[29\], our production region is typically slightly smaller than \(10^{16}\) cm and our required cosmic-ray injection luminosity (which was normalized to the BB luminosity in that paper) is higher, as it can be seen in Fig. 4 (upper row), which means that it may be difficult to achieve high enough proton luminosities in the hidden wind models at least from the estimates in \[29\]. Magnetic confinement is also considered in the hidden wind model, where however adiabatic losses are assumed to limit the optical thickness, which corresponds to an intermediate assumption between our calorimetric and free-streaming cases for an expansion velocity \(v \simeq 0.1 \sim 0.5 c\) of the production region. In the hidden wind model, \(pp\) interactions with the debris can also play a major role, where however large uncertainties are connected to, such as from the geometry and the time evolution of the system. There are also some similarities with the TDE outflow model for AT2019\[\text{ds}g\] in \[28\]: outflow-cloud interactions may lead to particle acceleration, and \(pp\) interactions with the clouds may lead to neutrino production. While the production region in that model is a bit further out \(\simeq 10^{16}\) cm, the \(pp\) interactions are efficient in the clouds, which are assumed to have a size of about \(\simeq 10^{14}\) cm and act as calorimeters. Our \(pp\) interactions with the outflow itself are less efficient because our production radius is large (the target density is about a factor 50 smaller), and therefore the effect is sub-dominant for AT2019\[\text{ds}g\] see Eq. (10). We note however that the shocks generated by the outflow-cloud interactions may be an interesting acceleration sites. For a comparison/discussion of jetted models, see App. \[\text{A}\].

Another question is if we can have Gauss-scale magnetic fields over such a large region \(R\). Ref. \[27\] obtain a magnetic field \(\simeq 10^{32}\) G cm\(^2\) at about 80 \(R_g\), which translated to about 1 Gauss in a distance of \(10^{15}\) cm. Ref. \[58\] obtain a field of about \(B \simeq 0.07\) G from the radio equipartition analysis at \(R \simeq 7 \times 10^{16}\) cm for near-contemporaneous radio epoch to the neutrino. Assuming that the field \(B \propto 1/R\) for a toroidal configuration, Gauss-scale fields at \(10^{15}\) cm are plausible. In the hidden wind model, Gauss-scale magnetic fields are estimated from the kinetic wind luminosity as well \[10\] \[29\], in fact up to 30 G for AT2019\[\text{fdr}\]. Therefore, the assumption seems consistent.

Finally we note that in all cases an average neutrino delay \(t_{\nu} - t_{\text{peak}}\) in the right ball park is expected, originating in the calorimetric behavior of the system. However, the distribution is wide in time, and a slight preference of an early neutrino emission closer to the peak is implied in the cases with higher \(p_{\gamma}\) efficiencies – which have the effect that they deplete the available in-source protons. Thus a high neutrino production efficiency and a long neutrino delay are kind of anti-correlated in M-X. An exception is AT2019\[\text{aal}\]c where (slow) \(pp\) interactions dominate and the neutrino flux peaks at lower energies. Apparently, better results for the neutrino delay can be expected if the non-thermal proton injection does not follow the mass fallback rate. Note that in all cases the actual X-ray emission may be even higher than the observation, since the observed flux may be obscured as well. However, exactly for AT2019\[\text{aal}\]c the observed X-ray flux was stable over the duration of a few hundred days (called “X-ray plateau” in figures), which raises questions in this hypothesis in our view.

IV. MEDIUM-ENERGY PROTONS INTERACTING WITH OPTICAL-UV PHOTONS (MODEL M-OUV)

Model-specific description. In order to foster interactions with the OUV target photons, \(E_{p,\text{max}} = 1 \cdot 10^8\) GeV, higher than for M-X, is used. The microphysics is illustrated in the cartoon in Fig. 5 left panel, and model assumptions and results are listed in the table in Fig. 5 right panel. The radiation zone is determined by the black body radius \(R \simeq R_{\text{BB}}\) for M-OUV if \(R_{\text{acc}} \ll R_{\text{BB}}\), where we use measured or estimated values listed in the table. While possible accelerators could (in principle) be disk, corona or a jet, we assume for the sake of simplicity that \(R_{\text{acc}} \sim R \sim R_{\text{BB}}\).\[16\]

While the OUV luminosity has been measured for all three TDEs, a dust echo has been measured as well – and consequently, the actual luminosity at \(R_{\text{BB}}\) must be significantly higher than the observed luminosity: we therefore derived in Sec. \[\text{III}\] an estimate for the minimal bolometric luminosity, see Table 1. Furthermore, the target photon density in the BB photosphere is estimated by the free-streaming assumption Eq. (5), which is conservative if \(R_{\text{acc}} < R_{\text{BB}} \simeq R\), and more accurate if \(R_{\text{acc}} \simeq R \gtrsim R_{\text{BB}}\). We therefore anticipate that our neutrino prediction for M-OUV is on the conservative side, and the actual neutrino fluence could be somewhat higher.

Interaction rates and optically thick behavior. We show in Fig. 6 the rates (inverse timescales) as a function of energy for AT2019\[\text{dsg}\] as an example, for the parameters chosen for M-OUV; again solid curves refer to peak time \[\text{---}\], for X-rays, the optical thickness increases with decreasing injection radius \(R_{\text{acc}}\) as long as \(R_{\text{acc}} > R_X\) (beyond the X-ray photosphere). Generic core models therefore typically have higher production efficiencies with respect to X-rays. Here we adopt a more conservative point of view for the X-ray interactions, and focus on the OUV target.

\[16\] For X-rays, the optical thickness increases with decreasing injection radius \(R_{\text{acc}}\) as long as \(R_{\text{acc}} > R_X\) (beyond the X-ray photosphere).
and dashed curves to the time of the neutrino emission. Here $E_{p,\text{max}}$ is high enough such that interactions with the OUV target are possible – and, in fact, they dominate even at the time of the neutrino emission. The optical thickness to $p\gamma$ interaction can be analytically estimated as

$$\tau_{fs} \approx \frac{t_{fs}}{t_{p\gamma}} \simeq 300 \left( \frac{L_{BB}}{10^{45} \text{ erg s}^{-1}} \right) \left( \frac{T_{BB}}{\text{eV}} \right)^{-1} \left( \frac{R}{10^{15} \text{ cm}} \right)^{-1},$$

which means that typically $\tau_{fs} \gg 1$ at peak, and the system is optically thick. Neutrons produced in $p\gamma$ interactions off the OUV target will also interact beyond the threshold, which means that the effective proton cooling time is indeed $\simeq 0.2 t_{p\gamma}$ here; from Eq. (14), protons and neutrons cool efficiently by $p\gamma$ interactions.

X-ray and $pp$ interactions may still contribute at lower energies over the dynamical timescale, as the $p\gamma$ interaction rate off X-rays is higher than the (diffusive) escape rate. While the OUV interaction time is order hours, the X-ray and $pp$ signals tend to come later. Nevertheless, it is clear already from the interaction rates that the dominant contribution to the neutrino signal (from OUV interactions) will follow the BB evolution on the hour scale.

**Results.** Fig. 7 displays our main results in the same format as Fig. 4. Here the neutrino light curves (red curves in upper panels) follow the product of $L_{p,\text{cal}}^{\text{eff}}$ (green dashed-dotted curve) and $L_{BB}^{\text{bol}}$ (min.) (black dotted curve) to leading order. The calorimetric behavior, i.e., the delay between $L_{p,\text{cal}}^{\text{eff}}$ and the injection luminosity $L_p$ is less pronounced than for M-X since the high-energy protons interact efficiently; there is however some contribution from lower energies, and AT2019aalc also is a better calorimeter because of the larger $R$. As expected, it is more difficult to re-produce the neutrino time delay marked by the red arrows.

The neutrino fluences in the lower row (see also Fig. 5 right table) are higher than for model M-X, and are all within the expected ranges in Table I. The peaks are all dominated by the BB interactions (green dotted curves)
thanks to the high enough proton energies, whereas X-rays (blue dashed-dotted curves) and pp (orange dashed curves) interactions contribute at lower energies. The predicted neutrino energies are significantly higher than the most likely energies (black arrows), but plausible if the possible ranges (gray areas) are taken into account. It is noteworthy that AT2019aalc in fact, for the given assumptions and in spite of the relatively large assumed value for $R$, exhibits event rates $N_\mu > 1$ for both the GFU and PS effective areas. This means that, while the observation of AT2019dsg and AT2019fdr may have been a coincidence motivated by the Eddington bias, AT2019aalc is a neutrino source which must have been seen (in neutrinos).

**Discussion.** While M-OUV exhibits higher event rates than M-X, it is more difficult to describe the neutrino time delay in the model, and the size of the system plays only a minor role in this. A possibility might be that the neutrino delay comes from a delayed proton injection itself, for which we could however not identify a physical motivation yet. The strength of model M-OUV is, however, that high neutrino fluences can be produced, given that the estimates for $\varepsilon_{\mathrm{dis}}$ (efficiency into non-thermal protons) can be mitigated.

It is interesting to note that our model M-OUV is a quantitative implementation of the original analytical estimate in AT2019dsg; we have therefore performed numerical cross-checks for similar parameters to isolate the differences. From Fig. 7, upper left panel, we can read off that the neutrino luminosity is about $L_p/40$ at $t_{\mathrm{peak}}$, whereas the original model predicts a factor $L_p/8$ in the optically thick case. The main difference comes from the bolometric correction: our $L_p$ is related to the full non-thermal proton spectrum, not only to the part beyond the pion production threshold; a smaller part comes from the pitch-angle averaging and width of the cross section taken into account in numerical computations. Furthermore, note that the optical thickness drops as a function of time, which overall leads to a significantly lower neutrino event rate prediction than the analytical estimate taken at the BB peak.
V. HIGH-ENERGY PROTONS INTERACTING WITH INFRARED PHOTONS (MODEL M-IR)

Model-specific description. The fact that all three neutrino-observed TDEs have been associated with a strong dust echo and, in fact, AT2019aalc has been found that way \[11\], makes a direct connection of the neutrino production with the IR photons from the dust echo interesting. Even more: the dust echo light curves shown as dashed-dotted black curves in our results plots (e.g. Fig. 4) seem to directly suggest their origin of the neutrino time delay (see arrows for neutrino arrivals), because the neutrinos arrive at or close to the peak of the dust echo.\(^{17}\)

The microphysics of model M-IR is illustrated in the cartoon in Fig. 8 left panel, and model assumptions and results are listed in the table in Fig. 8 right panel. The challenge, however, are very high required proton and corresponding neutrino energy; we use a TDE-universal \(E_{p,\text{max}} = 5 \times 10^9\) GeV here, see Eq. (1) and discussion thereafter. The scattering and re-processing of the OUV (perhaps even X-ray) photons in the dust leads to a delayed IR signal to the observer, as outlined in Sec. II E, the in-source density is computed with Eq. (9).

The dust model luminosities and light curves are taken from our own dust model in Fig. 4) seem to directly suggest their origin of the neutrino time delay (see arrows for neutrino arrivals), because the neutrinos arrive at or close to the peak of the dust echo.\(^{17}\)

The chosen value for \(B = 0.1\) G for all three TDEs leads to confinement of protons over such a large region, see Eq. (7), which somewhat helps the production off the IR target and contributes to the neutrino time delay. The dust model luminosities and light curves are taken from our own dust model in Sec. II E, the in-source density is computed with Eq. (9).

Interaction rates and optically thick/calorimetric behavior. We show in Fig. 9 the rates (inverse timescales) as a function of energy this time for AT2019fdr as an example, for the parameters chose for M-IR; again solid curves refer to peak time and dashed curves to the time of the neutrino emission. Here \(E_{p,\text{max}}\) is high enough such that interactions with all targets are possible. We that the free-streaming and dynamical timescales are much closer to each other because of the larger size of the region; interactions with X-rays and the outflow (pp) play a minor role here, so does Bethe Heitler pair production.

The analytical estimates for the optical thicknesses in the free-streaming and calorimetric cases and the IR target

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\(^{17}\) For AT2019dag, the dust echo peaks much later, but the OUV light curve much earlier, which may compensate the delay.
FIG. 9: Relevant inverse timescales (rates) for protons and neutrons for M-IR and AT2019fdr in the source frame as a function of observed energy. For details, see caption of Fig. 3.

\[ \tau_{p\gamma}^{fs} \equiv \frac{t_{fs}}{t_{p\gamma}} \approx 3 \left( \frac{L_{\text{IR}}}{10^{44} \text{ erg s}^{-1}} \right) \left( \frac{T_{\text{IR}}}{0.1 \text{ eV}} \right)^{-1} \left( \frac{R}{10^{17} \text{ cm}} \right)^{-1}, \]

\[ \tau_{p\gamma}^{\text{cal}} \equiv \frac{t_{\text{dyn}}}{t_{p\gamma}} \approx 60 \left( \frac{L_{\text{IR}}}{10^{44} \text{ erg s}^{-1}} \right) \left( \frac{T_{\text{IR}}}{0.1 \text{ eV}} \right)^{-1} \left( \frac{R}{10^{17} \text{ cm}} \right)^{-2} \left( \frac{t_{\text{dyn}}}{900 \text{ days}} \right), \] respectively. This means that the neutrino production off the IR target is guaranteed to be efficient over the dynamical timescale, and may be optically thick. Assuming \( t_{p\gamma,\text{cool}} \approx 5 \times t_{p\gamma} \) (such as in the optically thick case), it is also interesting that the proton interactions are slow

\[ t_{p\gamma,\text{cool}} \approx 70 \text{ days} \left( \frac{T_{\text{IR}}}{0.1 \text{ eV}} \right) \left( \frac{L_{\text{IR}}}{10^{44} \text{ erg s}^{-1}} \right)^{-1} \left( \frac{R}{10^{17} \text{ cm}} \right)^{2}, \]

which helps the neutrino time delay, as the injected in-source protons will be depleted over that time scale.

**Results.** We present our main results for model M-IR in Fig. 10. The neutrino light curves (red curves in upper panels) follow the product of \( L_{p,\text{cal}} \) (green dashed-dotted curves) and OUV (black dotted curves) or IR (black dashed-dotted curves). Therefore, a different time evolution for the neutrinos from OUV and IR is expected. Here we re-discover the calorimetric behavior similar to M-X for AT2019fdr and AT2019aalc in which cases the neutrino time delay (red arrows) can be easily reproduced.

The neutrino fluences in the lower row (see also Fig. 8, right table) are more comparable to M-X rather than to M-OUV, but lie within the expected ranges in Table 1. The peaks are all dominated by the IR interactions (red dashed-dotted curves) because of the high enough proton energies, but OUV interactions (blue dotted curves) contribute significantly in all cases. This is unavoidable, since IR and OUV are directly connected, which means that a part of the OUV luminosity is re-processed into the IR dust echo – and consequently the OUV interactions (which also have a lower threshold) cannot be avoided. The peaks are all dominated by the BB interactions (green dotted curves) thanks to the high enough proton energies, whereas X-rays (blue dashed-dotted curves) and \( pp \) (orange dashed curves) interactions contribute at lower energies. The predicted neutrino energies are significantly higher than the most likely energies (black arrows), which is the biggest limitation of model M-IR.

**Discussion.** Model M-IR provides the best description of the neutrino time delay due to the time evolution of the dust echo target. A slight exception is the AT2019dsg case: Here the early strong OUV peak and the large contribution from OUV interactions prohibit a very good description of the time delay. The neutrino delay description can be somewhat improved by a larger value of \( R \) (because the protons interact slower), at the expense of the overall neutrino production efficiency. Therefore there is no good tradeoff between time delay and neutrino fluence for AT2019dsg. Major disadvantages of model M-IR are the predicted neutrino energies peaking at very high values (which are significantly above the detected energies), and the relatively low predicted neutrino fluences. Note that
all AGN have large regions with hot dust which could be targets for neutrino production; however, there may be no corresponding proton acceleration site.

It is an interesting question if the model M-IR can be connected with origin of the UHECRs. Depending on nuclear disintegration and air shower models, a maximum rigidity $R_{\text{max}} \simeq 1.4 - 3.5 \times 10^9$ GV was found in [57] to describe data from the Pierre Auger Observatory [58] in the rigidity-dependent maximal energy model in multi-dimensional parameter space fit. This range is sufficiently close to the assumption made for protons here $E_{p,\text{max}} = 5 \times 10^9$ GeV, which means that our TDEs could also be the sources of the UHECR protons. For heavier nuclear compositions, needed to describe UHECR data, a similar result for the neutrino flux is expected (see [59] for the dependence of $\nu$ interactions on the mass number) as long as the source is optically thin to nuclear disintegration at the highest energies, see [24, 59] for corresponding examples. Of course, the nuclear composition must be powered by appropriate progenitor disruptions, such as ONe or CO white dwarfs. Concerning the energy output in UHECRs, we know that the energy in non-thermal protons per TDE is $E_p \simeq \varepsilon_{\text{diss}} \int M dt \simeq 0.1 M_\star$ in our model, where $M_\star$ is given in Table I. Taking into account the bolometric correction (only protons at the highest energies are relevant here) and that only a fraction of UHECR protons escape (see e.g. [60] for a corresponding discussion in GRBs), one may estimate that $E_{p,\text{esc}} \lesssim 0.01 M_\star$ can be re-processed into non-thermal protons at the highest energies per TDE; for a solar-mass star, that is about $2 \times 10^{52}$ erg. This would require a local white dwarf disruption rate of about $5 \, \text{Gpc}^{-3} \, \text{yr}^{-1}$ to match a local injection rate of $10^{14} \, \text{erg} \, \text{Mpc}^{-3} \, \text{yr}^{-1}$ – which has been perceived as too high in the literature, see e.g. discussion in [24]. While more recent observations find much higher local rates $500 \, \text{Gpc}^{-3} \, \text{yr}^{-1}$ [61], it is critical here how the population extends to large SMBH masses (as the UHECR output in our model scales with $M \propto L_{\text{edd}}$); a detailed population model is beyond the scope of this study.
VI. PREDICTED DIFFUSE NEUTRINO FLUXES

Using the results in the previous sections, we have computed the expected diffuse flux of neutrinos from TDEs, following the method outlined in [17]. The neutrino flux of a given flavor $\alpha$ – differential in energy, time, area and solid angle – is given by:

$$\Phi_{\alpha}(E) = \frac{\eta c}{4\pi H_0} \frac{\int_{M_{\min}}^{M_{\max}} dM \int_{z_{\min}}^{z_{\max}} dz \dot{\rho}(z, M) \epsilon_{\alpha}(E(1+z), M)}{\sqrt{\Omega_M (1+z)^3 + \Omega_\Lambda}}, \quad (17)$$

where $\dot{\rho}(z, M)$ is the cosmological rate of TDEs differential in redshift and SMBH mass (taken mainly from Shankar et al. [62], see also [63, 64]). The function $\epsilon_{\alpha}$ is the number of neutrinos emitted per unit energy in the SMBH frame (inclusive of neutrino flavor oscillations in vacuum), and $E' = E(1+z)$ is the neutrino energy in the same frame: $E$ is the energy observed at Earth. Here $\eta$ is the fraction of TDEs where the neutrino production mechanism is active and efficient. Eq. (17) also includes the speed of light, c, the Hubble constant, $H_0$, and the fractions of cosmic energy density in dark matter and dark energy, $\Omega_M \simeq 0.3$, and $\Omega_\Lambda \simeq 0.7$.

In computing the expression in Eq. (17), $z_{\max} = 6$ was used, its value influences the result only weakly. We took $M_{\min} = 2 \times 10^6 \ M_\odot$ and $M_{\max} = 5 \times 10^6 \ M_\odot$, which is justified by the estimated masses in Table I when considering their uncertainty (at least a factor of two). The upper cutoff $M_{\max}$ is also expected because tidal disruption becomes increasingly inefficient for increasing $M$, see, e.g. [64].

The integration in $M$ was approximated by a discrete sum over three mass bins. Specifically, we worked under the assumption that the entire TDE population is represented, although roughly, by the three neutrino-detected TDEs, each of which corresponding to a different value of $M$. For each model, our benchmark scenario assumed that the three neutrino spectra found for AT2019dal, AT2019dar and AT2019aalc contribute equally to the diffuse flux, so they were assigned equal weights: $(w_1, w_2, w_3) = (1/3, 1/3, 1/3)$. This choice is the best inference that can be obtained from observations. It is also plausible theoretically: considering the three mass values in Table I with uncertainties, it is possible that the observed TDEs may fall in the mass bins $M/ M_\odot = [2 \times 10^6, 4 \times 10^6], [4 \times 10^6, 10^7], [10^7, 4 \times 10^7]$, which correspond to equal TDE rates for our chosen $\dot{\rho}(z, M)$. To describe the uncertainty on the neutrino spectrum and normalization, we also varied over all the possible weights $(w_1, w_2, w_3)$, thus obtaining an envelope of curves with the purpose to quantify the uncertainty of how representative each TDE is for the full population.

The resulting $\nu_\mu + \bar{\nu}_\mu$ flux is shown in Fig. 11 for the benchmark case (central curve) as well as the varying weights one (shaded area). Also shown are the corresponding flux measurements from cascade-like [65] and track-like [66] events at IceCube. In the figure, the fraction of neutrino-emitting TDEs, $\eta \leq 1$, has been adjusted to reproduce the data; chosen values are $\eta = 10^{-0.5}, 10^{-2.3}, 10^{-2.0}$ for M-X, M-OUV and M-IR respectively. From these values, as well as with the comparison measured spectrum, we conclude that, for M-OUV and M-IR, TDEs can not be the main contributors to the astrophysical flux observed by IceCube, but they may significantly contribute at the highest energies $E \gtrsim 1$ PeV. In contrast, for M-X the predicted spectrum reproduces the data over three orders of magnitude of energy. We note that this conclusion depends strongly on the spectral weights $w_i$; in the case where the spectrum for AT2019dal (which is more suppressed at low energy compared to the others, see Fig. 1) carries a large weight, the observed flux at $E \lesssim 0.1$ PeV can not be accounted for by TDEs. We also note a degeneracy between $\eta$ and other parameters. One of them is the dissipation efficiency, $\epsilon_{\text{diss}}$: we have that $\epsilon_{\alpha}(E(1+z), M) \propto \epsilon_{\text{diss}}$, and therefore $\phi \propto \eta \epsilon_{\text{diss}}$. This implies that the data can be reproduced with a more moderate requirements for the dissipation efficiency into non-thermal protons over the whole TDE population, at the expense of a larger fraction of neutrino emitting TDEs (which could be as large as one); see e.g. discussion in App. A for off-axis jets. There is also a degeneracy with $M_{\min}$: due to the negative evolution of the TDE rate with mass, $\Phi_{\alpha}$ increases when lowering $M_{\min}$. For $M_{\min} \sim 10^5 \ M_\odot$ an order of magnitude increase is expected with respect to our results (see, e.g., [17]), thus leading to lower requirements for $\eta \epsilon_{\text{diss}}$.

It is an interesting question if the chosen values of $\eta$ are roughly consistent with the number of neutrino-TDE associations. Generically one expects $\eta \approx 1 \text{yr}^{-1}/(N N_{\text{GFU}})$ for about one neutrino-TDE association per year (assuming that most neutrino events are followed up), where $N$ is the number of observable TDEs per year and $N_{\text{GFU}}$ is the predicted neutrino event rate from our models. In the quoted mass range, we obtain $N \approx 4 \times 10^{4}$ yr$^{-1}$ at $z < 0.3$, and $N \approx 400$ yr$^{-1}$ at $z < 0.1$, which serves as the range of uncertainty for this estimate. Taking the average values for $N_{\text{GFU}}$ over the three TDEs (see Fig. 2, Fig. 3, Fig. 4, right tables), one would expect ranges $\eta \in [10^{-2.6}, 10^{-1.2}], [10^{-3.7}, 10^{-2.3}]$, and $[10^{-2.4}, 10^{-1.0}]$ for M-X, M-OUV, and M-IR, respectively. Comparing these expectations to reproduce the number of TDE associations with the numbers to reproduce data in the respective energy ranges above, we can estimate the expected contribution to the diffuse flux from the ratio: $0.8\% - 20\%$, $\gtrsim 4\%$, and $\gtrsim 38\%$ for M-X, M-OUV, and M-IR, respectively. This means that the diffuse neutrino flux for M-X shown in Fig. 11 would probably lead to too many neutrino neutrino-TDE associations, while the other two are plausible. For comparison, a range between 5\% and 59\% of the diffuse flux is given in [2] at the 90\% CL: this range is roughly consistent with our
FIG. 11: $\nu_e + \bar{\nu}_e$ diffuse fluxes for the three models, for the case where the three single-TDE spectra contribute with equal weights (solid curves) or arbitrary weights are allowed (shaded areas). The fractions of contributing TDEs have been set to $\eta = 10^{-0.5}, 10^{-2.3}, 10^{-2.0}$ for M-X, M-OUV and M-IR respectively, to reproduce the observed diffuse flux. Data points represent the IceCube measurements from cascades 63 (blue, data with narrower energy bins) and tracks 65 (magenta, data with larger energy bins).

| Model criterion                        | M-X                          | M-OUV                        | M-IR                          |
|----------------------------------------|------------------------------|------------------------------|------------------------------|
| Probable acceleration region           | $10^{15}$ to $10^{16}$ cm    | Around BB photosphere $\sim$ $10^{17}$ cm | $10^{16}$ to $10^{17}$ cm    |
| Main targets                           | X-rays, protons              | Optical-UV blackbody         | IR photons from dust echo    |
| Observational evidence/correlation     | High $L_{\text{BB}}$         | Dust echoes                  |                              |
| Origin of neutrino time delay          | Diffusion (high $B$)         | Unrelated to size of system  | Dust echo travel times       |
| Description neutrino time delay        | Intermediate                 | Poor                         | Good                         |
| Neutrino event rate                    | Low                          | Intermediate-High            | Low                          |
| Required $E_{\nu_{\text{max}}}$       | Moderate                     | Intermediate                 | High                         |
| Neutrino energy                        | Matches                      | Somewhat high                | Very high                    |
| Neutrino spectral time evolution       | Matches                      | Right direction              | Wrong direction              |
| Diffuse flux spectral shape            | Matches                      | High $E$ only                | Highest $E$ only             |
| Diffuse flux contribution              | $0.8\% - 20\%$              | $\gtrsim 4\%$ (high $E$ only) | $\gtrsim 38\%$ (highest $E$ only) |

TABLE II: Qualitative comparison of different models regarding different aspects; best matches are boldface.
VII. COMPARISON AND DISCUSSION

Let us take a comparative look at the three models we have proposed; a first question that arises is which model, if any, is favored by observations. We give a qualitative comparison of the three models in Table II. From the table, it appears that at present there is no clear preference for a single model. In terms of neutrino signal or time delay, M-X describes the neutrino energy very well and the required $E_{\text{p, max}}$ is moderate; M-OUV can most easily describe the expected neutrino fluence, and therefore has the best requirement for the transfer efficiency of accretion power into non-thermal protons; M-IR describes the neutrino time delay best. Major drawbacks are the poor time delay description in model M-OUV, and the high proton and neutrino energies in model M-IR, so that, overall M-X seems to be the most attractive choice. However, M-X can only describe up to 20% of the diffuse neutrino flux as estimated from the neutrino-TDE associations.

As was noted before, all three neutrino-TDE associations require SMBH masses around $10^7 M_\odot$, close to the theoretical upper limit. Our models are consistent with this. Indeed, for M-OUV, both the proton injection rate and the BB luminosity are proportional to $L_{\text{edd}}$, and therefore to $M$, therefore one expects $L_\nu \propto M^2$, which means that the neutrino-TDE associations will be dominated by the upper end of the SMBH range. For M-IR, the same argument holds, since intensity of the dust echo is proportional to the one in OUV. Instead, predicting a scaling with $M$ for M-X is more difficult. For AT2019aalc, the neutrino production is dominated by $pp$ interactions, for which the same scaling of the other models applies. For the other two TDEs, one could apply the prediction by Mummery [67] (see also [68]) that $L_X$ should be nearly constant over a wide mass range, leading to $L_\nu \propto M$ and to the same qualitative agreement with the three measured SMBH masses (note however, that the X-ray spectrum would have a non-trivial dependence on $M$ [67], which would affect the neutrino spectrum).

To elaborate on the consistency between the models and detected neutrino events, let us examine the time-dependent evolution of the neutrino spectrum. Especially if different targets contribute to the neutrino production, we have seen that the interaction rates will be very different; this means that e.g. neutrinos from OUV interactions will be closely following the proton injection because the system is optically thick, whereas neutrinos from X-ray and IR interactions appear intrinsically later. Therefore, there is a connection between the expected neutrino energy and the time delay.

We illustrate this at the example of AT2019dsg for the three different models in Fig. 12, where the spectral evolution of the neutrino spectrum is shown as a function of time (from red–early to blue–late, black curves show the time of the neutrino emission). For M-X, the early spectrum from OUV interactions peaks at higher energies than the spectrum at the actual time of the neutrino emission; in all cases the spectrum matches well. For M-OUV, the overall neutrino emission will be dominated by earlier times and relatively high neutrino energies within the possible range, whereas at the time of neutrino emission the spectrum is relatively flat in the expected (gray-shaded) range; the spectral evolution thus goes into the right direction. For M-IR, it is actually the early emission (from the OUV target) which
peaks closer to the observed neutrino energy, whereas at the time of the neutrino emission, the spectrum peaks at too high energies; the temporal evolution of the spectrum actually increases the tension with observations. These qualitative observations are also listed in our Table II.

At a more detailed level, one could wonder which of the three observed TDEs is best described by our models. We find that for AT2019aaic the neutrino fluence and time delay are well reproduced because of its relatively large SMBH mass (resulting in higher luminosities), low redshift, and slower decline of the BB luminosity. AT2019dsg on the other hand, has a faster declining BB luminosity, which makes the description of the neutrino time delay more difficult because non-thermal proton injection will peak early. While AT2019fdr is well described in the source frame, the neutrino fluence at Earth is too suppressed due to the high redshift. Interestingly, for AT2019aaic very high point source event rates are found for models M-X and M-OUV, which leads to the conclusion that the source must have been seen in neutrinos. Instead, for the other two TDEs the Eddington bias argument has to be invoked.

VIII. SUMMARY AND CONCLUSIONS

We have studied fully time-dependent quasi-isotropic neutrino production models for the three TDEs associated with high-energy astrophysical neutrinos, postulating that the observed neutrino time delays with respect to the BB peak come from the physical size of the post-disruption system, such as confinement of protons in magnetic fields over a large enough region, or propagation time delays. We have pointed out that the dominant photon target for proton interactions depends on the available maximal proton energy provided by the acceleration region; we have not specified the accelerator, examples could be hidden winds, shocks from outflow-environment interactions, or (truly) off-axis jets.

We have focused on observations common to the three TDEs, which, apart from the neutrino time delays, are: a) X-ray signals, b) relatively high SMBH masses, and, correspondingly, also relatively high BB luminosities, and c) strong dust echoes. Our models consequently have adopted X-ray (model M-X), optical-UV BB (model M-OUV), and infrared dust echoes (model M-IR) as main targets, selected by the maximally available proton energy, targeting the question what the smoking gun signature for the neutrino production actually is. A qualitative comparison is given in Table II. Model M-X describes the observed neutrino energies and time delays well due to the confinement of moderate-energy protons; model M-OUV describes the highest neutrino event rates at the expense of small neutrino time delays; model M-IR provides a good description of the neutrino time delays because of the correlation with dust echoes with similar delays and it may actually power the diffuse neutrino flux at the highest energies, at the expense of very high proton and neutrino energies. Note that TDEs may also be the sources of UHECRs if model M-IR can be established, but a more quantitative approach requires further study.

Since our models predicts a neutrino luminosity roughly scaling $L_\nu \propto M^2$ (except for M-X, where it could be weaker), it is not surprising that neutrino-TDE associations are found at the upper end of the available SMBH mass range close to the Hills mass even if the differential TDE rate $\propto M^{-1.6}$, see discussion in [17]. From that perspective, the newly found TDE AT2019aaic, which was associated with the highest SMBH mass $M$ and the lowest redshift, is expected to produce the highest neutrino fluence, which we have seen in all models. In fact, in two models the event number using the point source effective area was larger than one, which means that this source could be seen in (transient) multiplet or point source searches, whereas the detection of the other two was predicted to be a matter of luck (from a larger sample of sources with low predicted event rates each, invoking the Eddington bias). Note, however, that the information on AT2019aaic is sparse, which means that there are larger uncertainties (that for the other two). Especially in this case, the search for/comparsion to additional electromagnetic signatures, e.g. from secondaries produced in the photo-pion production, might be interesting, which may also follow the time-dependence of the neutrino spectra.

We point out that the nature of the proton accelerator cannot be easily identified from our models, other than it sits relatively far away from the SMBH (about $10^{15}$ to $10^{17}$ cm, depending on the model). While for M-OUV and M-IR disk or corona could serve as accelerators as well, additional contributions from regions closer to the SMBH are expected, which we do not include. A major limitation of our approach are relatively large required dissipation efficiencies $\varepsilon_{\text{diss}}$ into non-thermal protons for the individual TDEs, which can be mitigated in the core models by more compact production regions and in jetted models by relativistic beaming. Our diffuse flux computation has however demonstrated that the average $\varepsilon_{\text{diss}}$ of the whole population could be lower, at the expense of a larger fraction of neutrino-emitting TDEs. As far as the origin of the neutrino time delays is concerned, its description has been especially challenging by our assumption that the non-thermal proton injection follows the mass accretion rate; we have proposed a calorimetric approach paired with a relatively low (free-streaming) optical thickness (models M-X and M-IR), or paired with target photons evolving in favor of the neutrino delay (dust echo model M-IR). There could have also other reasons, such as a time evolution of the proton injection different from the mass accretion rate, or a transition of the disk state in core models.
We conclude that a decision of the neutrino production model based on the available information cannot be made; future observations will show if X-ray signals are associated with neutrino from TDEs (points towards M-X), or if dust echoes are observed (points towards M-IR); in both these cases time delays of the neutrinos are expected as well. If, on the other hand, BB-luminous TDEs with neutrinos close to the BB peak are found, M-OUV will be preferred. Note that other signatures initially gauged interesting for AT2019dsg \cite{3}, such as the outflow and radio signals, may actually be of secondary importance for the neutrino production in the light of AT2019fdr and AT2019aalc. Important clues will also come from the observed neutrino energies, which will have to scrutinized with more realistic spectra; small correlations between neutrino arrival time and neutrino energy are also expected in the models. Depending on the scenario, protons, such as in an outflow or the debris stream, may also be a target for neutrino production; in our cases, these have not affected the qualitative conclusions, but may help to describe the soft diffuse neutrino flux at the low energies. A more challenging questions may be the origin of the accelerated protons: can these be associated with other non-thermal signature in the electromagnetic spectrum, which would allow for an identification of the acceleration region, or will the origin of the non-thermal protons remain a mystery?

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**Appendix A: Discussion of jetted models (on-axis, off-axis)**

The isotropic models presented in this study very different from jetted models (with jets pointed towards us, or slightly off-axis), see \cite{26, 27} (AT2019dsg) and \cite{10} (AT2019fdr). The major limitation of the quasi-isotropic emission models is the very large fraction of energy has to go into non-thermal protons compared to models with a collimated emission, see \cite{31} for a discussion. In jetted models about 20% of the total accretion power was assumed to go into the jet, and the transfer efficiency from kinetic energy to non-thermal radiation was 20% as well, which makes an overall \( \varepsilon_{\text{diss}} \approx 0.04 \) into non-thermal protons – which is about a factor of five lower than \( \varepsilon_{\text{diss}} \) needed here; high isotropic-equivalent proton luminosities are reached by relativistic beaming. Note, however, that we use the most conservative \( E_{p,\text{min}} \approx 1 \text{ GeV} \) in all cases, which means that a large fraction of non-thermal protons cannot interact by \( p\gamma \) interactions because they are below threshold; increasing \( E_{p,\text{min}} \) or harder acceleration spectra would reduce the efficiency challenge for the individually observed TDEs. A big question for jetted models has been the non-observation of non-thermal internal radiation, and the interpretation of the radio observations for AT2019dsg, which indicate that the jet may have been unusually narrow (\( \theta \leq 1^\circ \)) \cite{33}. This immediately raises the question why the neutrino-TDE associations can be so abundant, as most jetted TDEs would not be seen in our direction for such a narrow jet.

Strongly off-axis jets acting as proton accelerators would be a possible solution to the puzzle, as the particle acceleration itself is known to work efficiently in astrophysical jets. In this case escaping non-thermal protons need to escape on the scale within our radiation zone, and to be trapped within the calorimeter and isotropize – which is in principle possible in our calorimetric models M-X and M-IR (see Fig. 4 and Fig. 9) energy ranges where \( t_{p,\text{esc}}^{-1} < t_{\nu}^{-1} \). Similar ideas have been proposed on different scales, see e.g. discussions in \cite{69, 70}. Note that here the cosmic-ray escape mechanism from the jet is also critical: for example, neutron escape does not contribute as neutrons cannot be magnetically confined, and advective escape may not work at the relatively small scales \( R \) proposed for M-X, whereas direct or diffusive escape of high-energy protons could work \cite{49}, i.e., of protons for which the Larmor radius reaches the size of the accelerator. The advantage of an off-axis jet is that the jet can serve as proton accelerator, while other radiation signatures of the jet are not expected or too weak if the viewing angle is large enough; in addition, all TDEs with jets would serve as neutrino sources (not only the ones pointing towards us). Since the required non-thermal proton luminosity is comparable to the typically expected physical (kinetic) jet luminosity, the jets would have to be very efficient to transfer kinetic energy into non-thermal particles for the three TDEs discussed here. For the diffuse flux, note that not all TDEs are expected to produce relativistic jets; using a fraction \( \simeq 0.1 \) assumed in \cite{17} (corresponding to the fraction of neutrino-emitting TDEs) and \( \varepsilon_{\text{diss}} \approx 0.04 \), a corresponding \( \eta \lesssim 0.02 \) in Fig. 11 is obtained, which is e.g. compatible with the shown diffuse flux for M-IR, and the estimated diffuse flux ranges from...
the number of neutrino-TDE associations for M-X and M-IR within the uncertainties.

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