Fuzzy node flow analysis of water distribution networks using Jaya algorithm

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Abstract. Fuzzy analysis helps in understanding how the uncertainty in various independent parameters of water distribution network such as, nodal demands, pipe roughness values, reservoir heads, pipe diameters and so forth will affect the dependent parameters such as pipe velocities, discharges and nodal pressures. The membership functions of dependent parameters are obtained by considering membership functions of uncertain independent parameters. The Impact Table method from literature suggests a repetitive analysis by considering the monotonous relationship between dependent and independent parameters. The Impact Table method was also employed for carrying out fuzzy analysis under pressure deficient condition to obtain fuzzy membership function of nodal outflows. Optimization based methods of fuzzy analysis are more useful when relationship between dependent and independent parameters are non-monotonous. A novel algorithm, Jaya is used in this study for performing fuzzy analysis on a benchmark network by pressure dependent approach. The analysis is performed by setting up hydraulic model in the software EPANET and linking it with the MATLAB for performing optimization through an EPANET-MATLAB toolkit. The propagation of the uncertainties of the input parameters for some non-linear hydraulic responses were identified and the algorithm was found to be a powerful tool for optimization.

Keywords: Fuzzy analysis, Jaya algorithm, MATLAB, Optimization, Water distribution network.

1. Introduction
A water distribution network (WDN) has various input parameters in intricate relationship with the hydraulic responses. The reliability of a system depends on how accurately the parameters are measured for the analysis and design. Aging of pipes, fluctuations in demand patterns, variations in the water levels in reservoirs etc. bring uncertainties to the independent input parameters which navigate to the hydraulic responses, affecting the functioning of the network. The traditional way of quantifying the uncertainties in a network is through probabilistic approach by methods such as Monte-Carlo simulation method. For attaining a probabilistic distribution pattern for the uncertainties, sufficient and precise knowledge is required. The input parameters of a WDN but undergo randomness in such a way
that it could not be precisely fitted into a probabilistic model. Fuzzy analysis therein provides a better solution by considering how the uncertainties could be distributed possibly as considered earlier by Revelli and Ridolfi (2002) [1], Bhave and Gupta (2006) [2], Branisavljevic and Ivetic (2006) [3], Gupta and Bhave (2007) [4], and Shibu and Reddy (2011) [5]. This paper aims in performing fuzzy analysis on pressure deficient networks to find out the variations in hydraulic functions using an optimization algorithm, Jaya.

2. Literature review

2.1 Fuzzy Analysis

Fuzzy logic is a mathematical means to represent the uncertainties or the vagueness in the data in the form of fuzzy numbers using membership functions. These membership functions are either triangular or trapezoidal and they represent the possibility of distribution of the values of data rather than their probability of occurrence. The parameters under consideration in the fuzzy set theory, will have a fuzzy number which is given by the membership function which defines the degree to which that particular value belong in the fuzzy set (Zadeh, 1965) [6]. The membership functions for a parameter $x$ are as represented in Figure 1.

![Figure 1. Triangular Fuzzy membership function and Trapezoidal Fuzzy Membership Function.](image)

This study uses triangular membership functions of the uncertainties for analysis. The membership functions are continuous functions which can be discretized using $\alpha$-cuts. For each of the $\alpha$-cut, a fuzzy optimization model is required to estimate the extreme responses by the hydraulic system (maximum and minimum). Bhave and Gupta (2006) [2], Gupta and Bhave (2007) [4] suggested the use of Impact Table method by considering a monotonous relationship between dependent and independent parameters. For parameters which vary non-monotonously, optimization algorithms are necessary. The evolutionary algorithms generally used in optimization of WDN such as genetic algorithm (Dongre and Gupta, 2017) [7], particle swarm optimization (Sabzkouhi and Haghighi, 2016) [8] requires algorithm specific parameters for the attainment of optimum results. Rao (2016) [9] introduced a new technique independent of algorithm specific parameters, Jaya algorithm, functioning similar to the teaching learning based optimization (TLBO) technique. The algorithm is employed for fuzzy optimization in the study.

2.2 Jaya Algorithm

Rao (2016) [9] proposed Jaya Algorithm, which depends only on some common controlling parameters such as population size, number of design variables and termination criteria. The algorithms which are dependent on the algorithm specific parameters tend to give results depending on the parameters considered (for example, Genetic algorithm uses mutation probability, crossover probability, selection operator; PSO uses inertia weight, social and cognitive parameters). But algorithms such as TLBO and Jaya gives the global optimum values as it does not involve any algorithm controlling parameter. The functioning of Jaya is as demonstrated: Let the function to be optimized be $f(x)$, ‘$m$’ be the number of design variables (that is, $j = 1,2,\ldots,m$), ‘$n$’ be the number of candidate solutions (i.e. $k$, population size $= 1,2,\ldots,n$) and $i$ be indicating the iterations.

Provide an initial set of population, number of design variables and the termination criterion at the starting. From the given set of population of the design variables, the best ($f(x)_{best}$) and worst ($f(x)_{worse}$)
solutions are identified by evaluating the function for all candidate solutions available. Let $X_{j,k,i}$ represent the value of the $j^{th}$ variable for the $k^{th}$ population in the $i^{th}$ iteration. Then, $X_{j,best,i}$ will be representing the best value and $X_{j,worst,i}$ represents the worst value of the $j^{th}$ variable corresponding to the best and worst solutions of the function respectively during the $i^{th}$ iteration. The value for the decision variable $X_{j,k,i}$ is then modified for the next iteration using the Equation 1, which makes sure that the solution always moves towards the best one.

$$
X'_{j,k,i} = X_{j,k,i} + r_{1,j,i} \left( X_{j,best,i} - |X_{j,k,i}| \right) - r_{2,j,i} \left( X_{j,worst,i} - |X_{j,k,i}| \right)
$$

(1)

Where, $X'_{j,k,i}$ is the modified value of the variable and $r_{1,j,i}$ and $r_{2,j,i}$ are two random numbers generated in the range between 0 and 1 for the $j^{th}$ variable in the $i^{th}$ iteration. The modified variables are then evaluated. If found better than the previous solution, then the modified value is accepted and previous solution is replaced. Else, the previous solution is kept as such. It is then looked upon to whether the termination criteria (e.g., Number of iterations) is reached. The solution is reported then, otherwise the procedure is repeated till the best solution is obtained. The optimization model is set up in the platform of MATLAB and linked to the hydraulic model in EPANET.

### 3. Fuzzy node flow analysis (FNFA)

WDN analysis is traditionally done under the assumption that the demand requirements are met at all the nodes. In reality, a network could be met with various abnormal and uncertain conditions, resulting in variations at the demand nodes. Softwares such as EPANET 2, LOOP, KYPIPE, PIPE 2006, WaterCAD, etc analyze a network using node head analysis (NHA). Under pressure deficient conditions, NHA will not give satisfactory results. In such cases pressure-dependent or pressure driven analysis called as node flow analysis (NFA) [Bhave 1981] [10] is required, where outflows at the demand nodes are treated as functions of pressure using node head-flow relationships (NHFR). NFA can be performed on a network by direct as well as indirect approaches. Sayyed et al.(2014) [11] suggested a non-iterative method to use EPANET such that the NHFR is satisfied externally. This methodology involves attaching a Check Valve, Flow Control Valve and Emitter in series to the demand nodes in the network so that pressure deficient analysis could be carried on the network in a single run of the EPANET software. These modifications can be done in the network using the graphical user interface of EPANET or by using the toolkit, which would considerably reduce the time consumed and efforts required for a large network.

**The mathematical model for FNFA**:

For an $\alpha$-cut, $\alpha = \alpha^*$;

The head loss $h_x$ in a pipe $x$ is:

$$
(h_x)_a = \alpha^* = (R_xQ_x^n)_a = \alpha^*, \quad x=1,\ldots, X
$$

(2)

Continuity equation for all the nodes is:

$$
\sum_{x\in j}(Q_x)_a = \alpha^* + (q_j)_a = \alpha^* = 0
$$

(3)

Head loss relationship for all loops is:

$$
\sum_{x\in c}(h_x)_a = \alpha^* = \sum_{x\in c}(R_xQ_x^n)_a = \alpha^* = 0
$$

(4)

NHFR for flow through emitters:

$$
(q_j^{\text{em}})_a = \alpha^* = C_d \left( (H_j^{AVT})_a = \alpha^* - H_j^{\text{min}} \right)^\gamma ; \quad (H_j^{AVT})_a = \alpha^* \leq H_j^{\text{min}}
$$

(5)

$C_d$, the emitter coefficient and $\gamma$, the emitter exponent for each of the nodes are calculated using Eqs. (6) and(7)

$$
C_d = \frac{q_j^{\text{eq}}}{(H_j^{\text{des}} - H_j^{\text{min}})^{1/n_j}}
$$

(6)

$$
\gamma = \frac{1}{n_j}
$$

(7)

Where, $Q_x$ represents the discharge in the pipe number $x$; $H_j$ is the value of HGL at the node $j$; $R_x$ is the pipe resistance constant of the pipe $x$; $p$ is an exponent based on the head loss formula which is being applied; $q_j$ is the demand at node $j$. Equations (5),(6) and(7) were proposed by Rossman (2000) [12] for computing the flow through emitters and it establishes a NHFR. It takes care of the fact that,
during times when the available head is less than the desirable head, demand will not be fully supplied but will be according to the relationship with available head. For each of the α-cut, algorithm is run two times, one for maximization and another for minimization, with the hydraulic response as the objective function and input uncertainties as the constraints. The limits for input uncertainties are obtained from the fuzzy membership functions.

3.1 Illustrative example 1
A looped distribution network considered by Ozger and Mays (2003) [13], as shown in Figure 2 is considered for the analysis. Network consists of 21 pipes and 13 junctions and pipe 3 is kept closed. Elevation of the junctions and their base demands are given in Table 1 and length, diameter and Hazen William’s coefficient of roughness of the pipes are given in Table 2. Emitter exponent is taken as 2/3 for the example.

![Figure 2. Water distribution network for the illustrative example.](image)

**Table 1. Pipe data for the illustrative example.**

| Node | Elevation(m) | Base Demand(m³/h) | Desired head (m) |
|------|--------------|-------------------|-----------------|
| 1    | 27.43        | 0                 | 42.43           |
| 2    | 33.53        | 212.4             | 48.53           |
| 3    | 28.96        | 212.4             | 43.96           |
| 4    | 32           | 640.8             | 47              |
| 5    | 30.48        | 212.4             | 45.58           |
| 6    | 31.39        | 684               | 46.39           |
| 7    | 29.56        | 640.8             | 44.56           |
| 8    | 31.39        | 327.6             | 46.39           |
| 9    | 32.61        | 0                 | 47.61           |
| 10   | 34.14        | 0                 | 49.14           |
| 11   | 35.05        | 108               | 50.05           |
| 12   | 36.58        | 108               | 51.58           |
| 13   | 33.53        | 0                 | 48.53           |
| Reservoir 1 | 60.96        |                   |
| Reservoir 2 | 60.96        |                   |
Table 2. Node data for the illustrative example

| Pipes | Length(m) | Diameter(mm) | Roughness CHW |
|-------|-----------|--------------|---------------|
| 1     | 609.6     | 762          | 130           |
| 2     | 243.8     | 762          | 128           |
| 3     | 1524      | 609          | 126           |
| 4     | 1127.76   | 609          | 124           |
| 5     | 1188.72   | 406          | 122           |
| 6     | 640.08    | 406          | 120           |
| 7     | 762       | 254          | 118           |
| 8     | 944.88    | 254          | 116           |
| 9     | 1676.4    | 381          | 114           |
| 10    | 883.92    | 305          | 112           |
| 11    | 883.92    | 305          | 110           |
| 12    | 1371.6    | 381          | 108           |
| 13    | 762       | 254          | 106           |
| 14    | 822.96    | 254          | 104           |
| 15    | 944.88    | 305          | 102           |
| 16    | 579       | 305          | 100           |
| 17    | 487.68    | 203          | 98            |
| 18    | 457.2     | 152          | 96            |
| 19    | 502.92    | 203          | 94            |
| 20    | 883.92    | 203          | 92            |
| 21    | 944.88    | 305          | 90            |

The uncertain parameter taken in this example for FNFA is nodal demand at all the nodes. ±15% of the crisp value of the demands is taken as the range of uncertainty. The objective functions which are evaluated are (1) nodal pressures and (2) the actual demand at the nodes under the uncertain conditions for α cuts = 0, 0.25, 0.5, 0.75 and 1. The membership function considered is triangular. The network is evaluated for FNFA analysis using Jaya algorithm and the results are provided below. Table 3 shows the hydraulic heads at all the junctions corresponding to various values of α cuts and Table 4 show the results for the actual flow available at the junctions corresponding to the propagating uncertainties.
4. Summary and conclusion

Estimation of the uncertainties that can occur in an engineering system is very important to provide a safe and reliable system. Fuzzy analysis is the approach adopted in this study to quantify the vagueness of a water distribution network. An optimization model is employed to find out the extremities in the responses of the system, as the relationship between independent and dependent variables.

| Table 3. Pressure at the junctions by FNFA approach |
|---------------------------------------------------|
| Pressure at nodes for various \( \alpha \text{cuts} \) (m) |
| node no | \( \alpha \text{cut} = 1 \) | \( \alpha \text{cut} = 0.75 \) | \( \alpha \text{cut} = 0.5 \) | \( \alpha \text{cut} = 0.25 \) | \( \alpha \text{cut} = 0 \) |
| normal | min | max | min | max | min | max | min | max | min | max |
| 1 | 60.51 | 60.5048 | 60.5156 | 60.4994 | 60.5226 | 60.4943 | 60.5336 | 60.4893 | 60.5485 |
| 2 | 60.33 | 60.3174 | 60.3327 | 60.3098 | 60.3425 | 60.3027 | 60.3589 | 60.2955 | 60.3797 |
| 3 | 41.98 | 41.9458 | 42.0121 | 41.9153 | 42.0482 | 41.9145 | 42.2551 | 41.9137 | 42.5221 |
| 4 | 42.02 | 41.9814 | 42.0485 | 41.9505 | 42.0849 | 41.9498 | 42.2872 | 41.949 | 42.5484 |
| 5 | 47.25 | 47.1848 | 47.3174 | 47.1239 | 47.3862 | 47.1234 | 47.574 | 47.1228 | 47.7844 |
| 6 | 42.02 | 41.9865 | 42.0526 | 41.9558 | 42.0887 | 41.9546 | 42.2893 | 41.9538 | 42.5488 |
| 7 | 42.74 | 42.709 | 42.7603 | 42.6852 | 42.791 | 42.6838 | 43.0655 | 42.6824 | 43.4087 |
| 8 | 42.71 | 42.682 | 42.7355 | 42.6573 | 42.7692 | 42.6559 | 43.0149 | 42.6546 | 43.3465 |
| 9 | 51.6 | 51.5862 | 51.607 | 51.5762 | 51.7231 | 51.5719 | 51.9398 | 51.5676 | 52.3239 |
| 10 | 53.47 | 53.4631 | 53.4816 | 53.4542 | 53.5906 | 53.4493 | 53.7784 | 53.4443 | 54.1018 |
| 11 | 49.18 | 49.1745 | 49.1892 | 49.1674 | 49.4394 | 49.1639 | 49.8047 | 49.1604 | 50.4021 |
| 12 | 48.95 | 48.9415 | 48.9568 | 48.9341 | 49.1305 | 48.9307 | 49.4317 | 48.9273 | 50.0353 |
| 13 | 44.76 | 44.7134 | 44.808 | 44.6699 | 44.8592 | 44.669 | 45.0671 | 44.6681 | 45.3345 |

| Table 4. Actual demands at the nodes by FNFA approach |
|------------------------------------------------------|
| Actual demand at nodes for various \( \alpha \text{cuts} \) (m³/h) |
| node no | \( \alpha \text{cut} = 1 \) | \( \alpha \text{cut} = 0.75 \) | \( \alpha \text{cut} = 0.5 \) | \( \alpha \text{cut} = 0.25 \) | \( \alpha \text{cut} = 0 \) |
| normal | min | max | min | max | min | max | min | max | min | max |
| 1 | 0 | 0 | 0 | 0 | 188.505 | 236.295 | 180.54 | 244.26 |
| 2 | 212.4 | 204.435 | 220.365 | 196.47 | 228.33 | 188.505 | 236.295 | 180.54 | 244.26 |
| 3 | 193.27 | 192.949 | 193.606 | 192.647 | 193.962 | 188.505 | 195.44 | 180.54 | 197.194 |
| 4 | 489.54 | 488.452 | 490.637 | 487.444 | 491.822 | 487.419 | 498.38 | 487.394 | 506.78 |
| 5 | 212.4 | 204.435 | 220.365 | 196.47 | 227.71 | 188.505 | 228.554 | 180.54 | 229.805 |
| 6 | 543.71 | 542.589 | 544.843 | 541.538 | 546.07 | 541.5 | 552.823 | 541.472 | 561.275 |
| 7 | 587.73 | 586.985 | 588.511 | 586.277 | 589.421 | 568.71 | 591.145 | 544.68 | 593.41 |
| 8 | 271.54 | 271.123 | 271.979 | 270.727 | 272.517 | 270.704 | 276.425 | 270.683 | 281.045 |
| 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11 | 103.8 | 103.765 | 103.837 | 99.9 | 103.872 | 95.85 | 104.062 | 91.8 | 105.089 |
| 12 | 94.98 | 94.9394 | 95.0177 | 94.9018 | 95.9049 | 94.8842 | 97.0045 | 91.8 | 98.1476 |
| 13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
variable are not necessarily monotonous. The hydraulic system is analyzed by node flow analysis to take care of the head-demand relationships under pressure deficient conditions. The results showed that, FNFA gives better insight into the identification of the vulnerable zones in a network under pressure deficient scenario. As most of the hydraulic simulation software perform analysis considering that demands are met at all the nodes, the results will vary when there is a pressure deficiency in the network. From Table 3 the nodes where pressure deficiency occurs can be identified in comparison with the desired head given in Table 1 and from Table 4 the demand at those pressure deficient nodes are accurately estimated. This assessment of network can be incorporated into the better designing of a WDN. This study has considered only the uncertainties in the nodal demands, but it can be further extended to any input parameter just by considering them as variables in the algorithm and extreme responses can be obtained.

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