Is the island universe model consistent with observations?

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Abstract

We study the island universe model, in which initially the universe is in a cosmological constant sea, then the local quantum fluctuations violating the null energy condition create the islands of matter, some of which might corresponds to our observable universe. We examine the possibility that the island universe model is regarded as an alternative scenario of the origin of observable universe.

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Recent observations of the cosmic microwave background (CMB) have implied that the inflation [1, 2] is a very consistent cosmological scenario. The inflation stage took place at the earlier moments of the universe, which superluminally stretched a tiny patch to become our observable universe today, and ended with a period of reheating that bring the universe back to the usual FRW evolution. The quantum fluctuations in the inflation period are drew out the horizon and, when reentering into the horizon during radiation/matter domination after the end of inflation, are responsible for the formation of cosmological structure.

However, the inflation has still its own problem of fine tuning, in addition it is not also able to address the initial singularity problem, the cosmological constant problem, super Planck fluctuations problem, as was described in Ref. [3]. Thus it seems desirable to search for an alternative to the inflation model. Recently, Dutta and Vachaspatian have proposed a model [4], in which initially the universe is in a cosmological constant (Λ) sea, then the local quantum fluctuations violating the null energy condition (NEC) [5, 6] create the islands of matter, some of which might corresponds to our observable universe. The headstream of this model in some sense may be backward to the eternal inflation [7, 8], and especially the recycling universe proposed by Garriga and Vilenkin [9], in which the thermalised regions asymptotically approaching dS spacetime may be fluctuated and recycled back to the false vacuum and then the nucleated false vacuum region will serve as a seed for a new (eternally) inflating domain. The difference of island universe is that after the NEC violating fluctuation is over, instead of the inflation followed, the thermalisation occurs instantly and the radiation fills the volume rapidly. Though the island universe model brings us an interesting environment, people are still able to doubt whether the island universe model is consistent with our observations.

Whether we can live in “Islands”? We firstly attempt to phenomenally (semiclassically) understand it, following closely Ref. [10]. We set $8\pi/m_p^2 = 1$ and work with the parameter $\epsilon = -\dot{h}/h^2$, where $h \equiv \dot{a}/a$ is the local Hubble constant. The “local” means here that the quantities, such as $a$ and $h$, only character the value of the NEC violating region. In the island universe model, the scale of the NEC violating region is required to be larger than that of present horizon, which has been clearly stated by Dutta and Vachaspatian [4], and is consistent with Refs. [11, 12], see also Refs. [13, 14, 15, 16] for relevant discussions. This result provides the initial value of local evolution of $a$ and $h$. The $\epsilon$ can be rewritten as $\epsilon \simeq \frac{1}{h\Delta t} \frac{\Delta h}{h}$, thus in some sense $\epsilon$ actually describes the change of $h$ in unit of Hubble time.
During the NEC violating fluctuation, \( \dot{h} > 0 \), thus \( \epsilon < 0 \) can be deduced. The time of this fluctuation can be given by

\[
T = \int dt = - \int_i^e \frac{dh}{\epsilon h^2} \approx \frac{1}{|\epsilon| h_i},
\]

where the subscript i and e denote the initial and end value of the NEC violating fluctuation respectively. In this model \( h_i^2 \equiv h_0^2 \sim \Lambda \), where the subscript 0 denotes the present value.

The reasonable and simplest selection for the local evolution of scale factor \( a(t) \), due to \( \dot{h} > 0 \), is \[10, 17\] \( a(t) \sim (-t)^n \), where \( t \) is from \(-\infty \) to \( 0_- \), and \( n \) is a negative constant. We have \( h = n/t \), and thus \( \dot{\epsilon} = 1/n. \) To make \( T \to 0 \) in which the NEC violating fluctuation can be so strong as to be able to create the islands of our observable universe \[4\], from \[11\], \( |\epsilon| \to \infty \) is required, which results in \( n \to 0_- \). Thus though the change of \( h \) is large, the expanding proportion of the scale factor is very small. Further in some sense before and after the fluctuation \( a \) is nearly unchanged, which can be also seen from following discussions.

In the conformal time \( d\eta = dt/a \), i.e. \( -\eta \sim (-t)^{-n+1} \), we obtain

\[
a(\eta) \sim (-\eta)\frac{-n}{1-n} \equiv (-\eta)^{\frac{n-1}{n}}
\]

\[
h = \frac{a'}{a^2} = \frac{1}{(\epsilon - 1)an},
\]

where the prime denotes the derivative with respect to \( \eta \). The perturbations leaving the horizon during the NEC violating fluctuation might reenter the horizon during radiation/matter domination after the end of fluctuation, which are responsible for the structure formation of our observable universe. The efolding number which measures the quantity of perturbation leaving the horizon during the NEC violating fluctuation can be defined as

\[
\mathcal{N}_{ei} \equiv \ln \left( \frac{a_e h_e}{a_i h_i} \right).
\]

From \[2\] and \[3\], we obtain

\[
a \sim \left( \frac{1}{(1-\epsilon)a_h} \right)^{\frac{1}{1-n}}
\]

Thus for the constant \( \epsilon \), we have

\[
\frac{a_e}{a_i} = \left( \frac{a_i h_i}{a_e h_e} \right)^{\frac{1}{1-n}} = e^{\mathcal{N}_e}.
\]

We can see that for the negative enough \( \epsilon \), the change \( \Delta a/a = (a_e - a_i)/a_i \simeq \mathcal{N}/(1-\epsilon) \) of \( a \) will be very small.
FIG. 1: The sketch of evolution of local $\ln(1/ah)$ with respect to the scale factor $\ln a$, in which the yellow region denotes the NEC violating fluctuation and the red lines denote the evolution of primordial perturbation modes. The perturbation modes exit the Hubble horizon during the NEC violating fluctuation and then reenter the horizon during the radiation/matter domination at late time.

Taking the logarithm in both sides of (5), we obtain

$$\ln \left( \frac{1}{ah} \right) = (\epsilon - 1) \ln a \equiv \left( 1 - \frac{n}{n} \right) \ln a.$$  \hspace{1cm} (7)

We plot Fig.1 to further illustrate the island universe model, in which $\ln(1/ah)$ is regarded
as the function of \( \ln a \). In the NEC violating region, \( n \to 0_- \), thus the slope \( (1 - n)/n \to -\infty \), which makes the evolution of \( \ln(1/ah) \) during the fluctuation correspond to a nearly vertical line in Fig.1. The \( \ln n \to 0- \), thus the slope \( (1 - n)/n \to -\infty \), which makes the evolution of \( \ln (1/ah) \) during the fluctuation correspond to a nearly vertical line in Fig.1. The \( \ln (1/ah) \) can be also shown and actually applied for the expansions with arbitrary constant \( n \). Thus during the \( \Lambda \), radiation and matter domination, we have the slope \( -1, 1 \) and \( 1/2 \) respectively, which have been reflected in the Fig.1.

The NEC violating fluctuations in the \( \Lambda \) sea with the observable value of cosmological constant create some thermalised matter islands, which subsequently evolve as the usual FRW universe. The radiation and matter will be diluted with the expansion of island and eventually this part of volume will return to the \( \Lambda \) sea again. The total evolution may be depicted in the Penrose diagram of Fig.2. We can see that there are always some perturbations (b) never reentering and remaining outside the Hubble scale after their leaving from it during the NEC violating fluctuation, which means that a part of island is permanently inaccessible to any given observer in the island. This may be reanalysed as follows. The wavelength of perturbations grows with the scale factor \( \sim a \), thus for the perturbations with the present horizon scale, we have \( a_0h_0 = a_ch_c \), where the subscript \( c \) denotes the value at the time when this perturbation leaves the horizon during the NEC violating fluctuation. We define

\[
N_{ec} \equiv \ln\left(\frac{a_ch_c}{a_0h_0}\right) = \ln\left(\frac{a_ch_c}{a_0h_0}\right)
\]

as the efolding number when the perturbation with the present horizon scale leaves the horizon during the NEC violating fluctuation, and from \( \text{(7)} \), we have

\[
N_{ei} - N_{ec} = \ln\left(\frac{a_0h_0}{a_i h_i}\right).
\]

Thus for the island universe model, in which \( h_i = h_0 \) and \( a_0 \gg a_i \geq 1/h_i \), we obtain \( N_{ei} > N_{ec} \). This implies that as long as the islands created are suitable for our existence, the efolding number required to solve the horizon problem of FRW universe may be always enough, independent of the energy scale of thermalisation.

Then we discuss the the primordial perturbations from the island universe model. Dutta and Vachaspatian \[4\] have shown that the spectrum from the quantum fields other than the NEC violating field can be scale invariant, but the amplitude of perturbation is too small to seed the structure of observable universe. However, they in their calculations regarded the NEC violating region as an instant link between the \( \Lambda \) sea and FRW evolution, which
to some extent leads that the effect of NEC violating region on the perturbation spectrums is lose. Thus we here will focus on the NEC violating region.

To simplification, we adopt the work hypothesis in which the NEC violating fluctuation behaves like the phantom energy. The simplest implementation of phantom energy is a scalar field with the reverse sign in its dynamical term, in which the NEC $\rho + p = -\dot{\phi}^2 < 0$ is violated. To make $\rho > 0$, the potential $\mathcal{V}(\phi) > \phi^2/2$ of phantom field is also required. Thus in this case the local evolution of $h$ can be written as $h^2 \sim -\dot{\phi}^2/2 + \mathcal{V}(\phi)$. This in some sense is equal to introducing a creation field, see Ref. [18]. Thus we have

$$w + 1 = \frac{p + \rho}{\rho} = \frac{-\dot{\phi}^2}{-\dot{\phi}^2/2 + \mathcal{V}(\phi)} = \frac{2}{3} \epsilon \to -\infty.$$  \hspace{1cm} (10)

Due to $\mathcal{V}(\phi) > \dot{\phi}^2/2$, this inequality actually requires the denominator $\sim 0$, which gives $\mathcal{V}(\phi) \sim \dot{\phi}^2/2$. Further, combining $\dot{\phi}^2/2 = \dot{h} = |\epsilon|h^2$, we may approximately deduce

$$\dot{\phi}^2/2 \simeq \mathcal{V}(\phi) \simeq |\epsilon|h^2.$$  \hspace{1cm} (11)

This operation does not mean that the phantom field is actually required in the island universe model. The aim that we appeal to the phantom field here is only to phenomenally simulate the NEC violating fluctuation, which may be convenient to calculate the primordial perturbation spectra from the NEC violating region. But it maybe also a possibility that the phantom field only exists instantly for producing the NEC violating fluctuation and after the end of fluctuation, the phantom field thermalises into the radiation, see Ref. [19] for a different scenario.

In the momentum space, the equation of motion of gauge invariant variable $u_k$, which is related to the Bardeen potential $\Phi$ by $u_k \equiv a\Phi_k/\phi'$, is

$$u''_k + \left(k^2 - \frac{z''}{z}\right) u_k = 0.$$  \hspace{1cm} (14)

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1 In the momentum space, the equation of motion of gauge invariant variable $v_k$, which is related to the tensor perturbation by $v_k \equiv ah_k$, is

$$v''_k + \left(k^2 - \frac{a''}{a}\right) v_k = 0.$$  \hspace{1cm} (12)

We have, from (2), $a''/a = (2 - \epsilon)/(\epsilon - 1)^2 \eta^2$. In the regime $k\eta \to \infty$, the mode $v_k \sim e^{-ik\eta}/(2k)^{3/2}$ can be taken as the initial condition. In long wave limit, the expansion of the Bessel functions to the leading term of $k$ gives

$$k^{3/2}v_k/a \sim k^{-\nu+3/2}$$  \hspace{1cm} (13)

where $\nu = \sqrt{(2 - \epsilon)/(\epsilon - 1)^2} + 1/4$. 

6
We have, from (2),
\[
\frac{z''}{z} = \frac{(1/a)''}{(1/a)} = \frac{\epsilon}{(\epsilon - 1)^2 \eta^2}
\]
(15)
The general solutions of this equation are the Bessels functions. In the regime \(k\eta \to \infty\), the mode \(u_k\) are very deep in the horizon of \(\Lambda\), see Fig.1 and Fig.2. Thus \(u_k \sim e^{-ik\eta/(2k^{3/2})}\) can be taken as the initial condition. In the regime \(k\eta \to 0\), the mode \(u_k\) are far out the horizon, and become unstable and grows. In long wave limit, the expansion of the Bessel functions to the leading term of \(k\) gives
\[
k^{3/2}u_k \sim \frac{\sqrt{\pi}}{2^{3/2}} \frac{1}{\sin (\pi \nu) \Gamma(1 - \nu)} \left(\frac{-k\eta}{2}\right)^{-\nu + 1/2},
\]
(16)
where \(\nu = \sqrt{\epsilon/(\epsilon - 1)^2 + 1/4}\). We can see that if \(\nu \simeq 1/2\), the spectrum of \(\Phi_k\) will be nearly scale invariant. This requires
\[
|\frac{\epsilon}{(\epsilon - 1)^2}| \ll 1.
\]
(17)
Thus for the NEC violating fluctuation with \(\epsilon \to -\infty\), we will have the nearly scale invariant spectrum [10].

In \(k\eta \to 0\), the Bardeen potential can also be rewritten as \(\Phi = C + hD/a\), where \(C\) and \(D\) are constants dependent on the mode \(k\). In the inflation model in which \(\epsilon \simeq 0\), \(h\) is nearly unchanged while \(a\) increases exponentially, thus \(D\) is a decay mode, and \(\Phi\) is dominated by the constant mode \(C\), and thus is nearly constant in superhorizon scale. This is the reason why people can briefly obtain the amplitude of perturbation reentering into the horizon by calculating the amplitude of perturbation leaving the horizon during the inflation. But for \(\epsilon \to -\infty\), the case is just inverse. \(a\) is nearly unchanged while \(h\) increases rapidly, thus instead of being regarded as the decay mode, \(D\) is a growing mode, which can be seen from (18), and compared with the constant mode \(C\), it will dominate the Bardeen potential \(\Phi\), which means that the amplitude of perturbation in the superhorizon scale will be still increasing during the NEC violating fluctuation, while during the radiation/matter domination after the thermalisation, \(D\) will not increase any more but decrease, and thus the

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\[\text{We notice from (17) that for } \epsilon \simeq 0_-, \text{ in which the scale factor expands exponentially, the spectrum is also nearly scale invariant. This is the so called phantom inflation proposed and discussed in detail in Ref. [20], see also Refs. [21, 22, 23, 24]. This result is actually also a direct reflection of the dualities of primordial perturbation spectra, in which the spectrum index of } \Phi \text{ is invariant under the change } \epsilon \to 1/\epsilon, \text{ which has been pointed out in Refs. [25, 26] and further studied in Refs. [17, 27, 28], and see also Refs. [29, 30, 31] for other discussions on the dualities.}\]
constant mode $C$ becomes dominated. Thus the key of obtaining the scale invariant spectrum during the radiation/matter domination is making the growing mode of $\Phi$ spectrum have an opportunity to be inherited by the constant mode after the thermalisation. However unfortunately, it has been shown $[34]$ that in the simple scenario the growing mode of $\Phi$ can hardly be matched to the constant model after the transition. This result is disappointing. However, it may be conceivable that our simplified operation in the calculation of primordial spectrum may have missed what. We here only phenomenally simulate the NEC violating process by using the scale field violating the NEC, and actually do not know the details of this NEC violating quantum fluctuation and subsequent thermalisation. The latter may significantly affect the final matching result. The example that in the gauge, in which instead of occurring a constant energy hypersurface the thermalisation may be everywhere simultaneous, the growing mode of $\Phi$ spectrum can be inherit by the constant mode at late time has been pointed out $[33, 35]$. We will tentatively take this optimistic matching for the following calculations and leave behind some further comments in the final conclusion.

Because in superhorizon scale the Bardeen potential is increased during the NEC violating fluctuation and constant after the end of fluctuation, it may be reasonable to take the value at the end of fluctuation to calculate the amplitude of perturbation,

$$k^{3/2} \Phi_k \simeq \frac{1}{2^{3/2}} \left( \frac{\zeta'}{a} \right) \simeq V_e^{1/2} / 2. \tag{18}$$

where \[[11]\] has been used.

The observations of CMB constrain the amplitude of perturbation is $k^{3/2} \zeta_k \sim k^{3/2} \Phi_k \sim 10^{-5}$, which require $V_e \sim 10^{-10}$, which corresponds to $h_e^2 \sim 10^{-10} / |\epsilon|$. We assume the thermalisation is almost instantaneous, thus $h_e^2 \sim \rho_r$, where $\rho_r$ is the energy density of radiation after the thermalisation. For example, for $\rho_r \sim m_{ew}^4 \sim 10^{-60}$, where $m_{ew}$ is the electroweak scale, which may be regarded as a loose lower limit of reheating temperature though the lowest limit might be in nucleosynthesis scale, we obtain $|\epsilon| \sim 10^{50}$. The change of $a$ is far smaller than that of $h$, thus from \[[11]\], we have $N \simeq \ln \left( h_e / h_i \right) \sim \ln \left( V_e^{1/2} / |\epsilon|^{1/2} \Lambda^{1/2} \right) \sim 127 - \ln |\epsilon| / 2 \simeq 69$, which is enough to solve the horizon problem of our observable universe.

\[\text{The recent numerical studies} \[36, 37\] \text{shown that the comoving curvature perturbation $\zeta$ passes continuous through the transition and therefore the spectrum inherited by the Bardeen potential at late times is the one of $\zeta$ and not the spectrum of the growing mode of the Bardeen potential. However, the examples (or classes) discussed in these studies are still limited, thus whether the result is universal remains open.}\]
These results are also implicitly reflected in the depiction of causal structure of island universe of Fig. 2.

How do we distinguish the island universe from the inflation model by the observations? For \( \epsilon \to -\infty \), from (13), we have \( \nu \approx \frac{1}{2} \), and thus the index of tensor perturbation is 2, which means that the island universe model will produce a more blue gravitational wave spectrum than the inflation model. This character is also actually a reflection of rapid change of background during the NEC violating fluctuation, compared with the nearly unchanged background during the inflation. This result generally leads to the intense suppression of gravitational wave amplitude on large scale. Thus it seems for the island universe model to be hardly possible to search for the imprint of gravitational wave in the CMB, while a stochastic gravitational wave will be consistent with the inflation model.

In conclusion, it seems to be possible that we can live in “Islands”, but slightly not optimistic, since the loophole how the perturbations propagate through the thermalisation surface remains, which may be fatal to the island universe model. However, it should be mentioned that the NEC violating fluctuation is actually a quantum phenomenon, and thus the innocence about the thermalisation and matching hypersurface may to some extent reflect the lack of our understanding on the quantum characters of gravity. We think that (at least) at present it will be premature to either confirm or deny the island universe model. The deep insights into the quantum gravity in the future may show us the final answer. However, whatever the outcome, it appears that the viability of the island universe model hangs in the balance and the different outcomes will either hold out or end its competing as an alternative scenario of the origin of observable universe. We will back to the relevant issues in the future.

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