COMPUTING DEGREE-BASED TOPOLOGICAL INDICES OF JAHANGIR GRAPH

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Abstract. Topological indices are numerical numbers associated with a graph that helps to predict many properties of underlined graph. In this paper we aim to compute multiplicative degree based topological indices of Jahangir graph.

Index Terms: Zagreb index; Randić index; polynomial; degree; graph.

1. Introduction

The study of topological indices, based on distance in a graph, was effectively employed in 1947, in chemistry by Weiner [1]. He introduced a distance-based topological index called the Wiener index to correlate properties of alkenes and the structures of their molecular graphs. Topological indices play a vital role in computational and theoretical aspects of chemistry in predicting material properties [2, 3, 4, 5, 6, 7, 8]. Several algebraic polynomials have useful applications in chemistry [9, 10].

A graph $G$ is an ordered pair $(V, E)$, where $V$ is the set of vertices and $E$ is the set of edges. A path from a vertex $v$ to a vertex $w$ is a sequence of vertices and edges that starts from $v$ and stops at $w$. The number of edges in a path is called the length of that path. A graph is said to be connected if there is a path between any two of its vertices. The distance $d(u, v)$ between two vertices $u, v$ of a connected graph $G$ is the length of a shortest path between them. Graph theory is contributing a lion’s share in many areas such as chemistry, physics, pharmacy, as well as in industry.
We will start with some preliminary facts. The first and second multiplicative Zagreb indices are defined as

\[ MZ_1(G) = \prod_{u \in V(G)} (d_u)^2, \]
\[ MZ_2(G) = \prod_{uv \in E(G)} d_u d_v, \]

and the Narumi-Katayama index is defined as

\[ NK(G) = \prod_{u \in V(G)} d_u. \]

Like the Wiener index, these types of indices are the focus of considerable research in computational chemistry. For example, in 2011, Gutman characterized the multiplicative Zagreb indices for trees and determined the unique trees that obtained maximum and minimum values for \( M_1(G) \) and \( M_2(G) \). Wang et al. defined the following index for k-trees,

\[ W_1^s(G) = \prod_{u \in V(G)} (d_u)^s. \]

Notice that \( s = 1, 2 \) is the Narumi-Katayama and Zagreb index, respectively. Based on the successful consideration of multiplicative Zagreb indices, Eliasi et al. continued to define a new multiplicative version of the first Zagreb index as

\[ MZ_1^*(G) = \prod_{uv \in E(G)} (d_u + d_v), \]

Furthering the concept of indexing with the edge set, the first author introduced the first and second hyper-Zagreb indices of a graph. They are defined as

\[ HII_1(G) = \prod_{uv \in E(G)} (d_u + d_v)^2, \]
\[ HII_2(G) = \prod_{uv \in E(G)} (d_u d_v)^2, \]

In Kulli et al. defined the first and second generalized Zagreb indices

\[ MZ_1^a(G) = \prod_{uv \in E(G)} (d_u + d_v)^a, \]
\[ MZ_2^a(G) = \prod_{uv \in E(G)} (d_u d_v)^a, \]

Multiplicative sum connectivity and multiplicative product connectivity indices are defined as:

\[ SCII(G) = \prod_{uv \in E(G)} \frac{1}{(d_u + d_v)}, \]
\[
PCII(G)(G) = \prod_{uv \in E(G)} \frac{1}{(d_u d_v)}.
\]

Multiplicative atomic bond connectivity index and multiplicative Geometric arithmetic index are defined as

\[
ABCI(G) = \prod_{uv \in E(G)} \sqrt{d_u + d_v - 2} / d_u d_v,
\]

\[
GAI(G) = \prod_{uv \in E(G)} \frac{2 \sqrt{d_u d_v}}{d_u + d_v},
\]

\[
GA^aII(G) = \prod_{uv \in E(G)} \left( \frac{2 \sqrt{d_u d_v}}{d_u + d_v} \right)^a.
\]

(1)

In this paper we compute multiplicative indices of Jahangir graphs. The Jahangir graph \( J_{m,n} \) is a graph on \( 8n+2 \) vertices and \( m(n+1) \) edges for all \( n \geq 2 \) and \( m \geq 3 \). \( J_{m,n} \) consists of a cycle \( C_{mn} \) with one additional vertex which is adjacent to \( m \) vertices of \( C_{mn} \) at distance to each other. Figure 1 shows some particular cases of \( J_{m,n} \).

![Figure 1. Jahangir graph.](image)

2. Computational Results

In this section, we present our computational results.

**Theorem 2.1.** Let \( J_{m,n} \) be the jahangir’s graph. Then

1. \( MZ^1_1(J_{m,n}) = (4)^{am(n-2)} \times (5)^{2am} \times (3 + m)^{am} \),
2. \( MZ^2_2(J_{m,n}) = (4)^{am(n-2)} \times (6)^{2am} \times (3m)^{am} \),
3. \( G^aII(J_{m,n}) = \left( \frac{2 \sqrt{6}}{5} \right)^{2am} \times \left( \frac{2 \sqrt{3 \times m}}{3+m} \right)^{am} \).

**Proof.** Let \( G \) be the graph of \( J_{m,n} \). It is clear that the total number of vertices in \( J_{m,n} \) are \( 8n+2 \) and total number of edges are \( 10n+1 \). The edge set of \( J_{m,n} \) has following three partitions,

\( E_1 = E_{2,2} = \{ e = uv \in E(J_{m,n}) : d_u = 2, d_v = 2 \} \),

\( E_1 = E_{2,3} = \{ e = uv \in E(J_{m,n}) : d_u = 2, d_v = 3 \} \).
and

\[ E_1 = E_{3,m} = \{ e = uv \in E(J_{m,n}) : d_u = 3, d_v = m \}. \]

Now,

\[ |E_1(J_{m,n})| = m(n - 2). \]
\[ |E_2(J_{m,n})| = 2m, \]
and

\[ |E_1(J_{m,n})| = m. \]

(1)

\[ MZ^a(J_{m,n}) = \prod_{uv \in E(G)} (d_u + d_v)^a \]
\[ = \prod_{uv \in E_1(J_{m,n})} (d_u + d_v)^a + \prod_{uv \in E_2(J_{m,n})} (d_u + d_v)^a + \prod_{uv \in E_3(J_{m,n})} (d_u + d_v)^a \]
\[ = (d_u + d_v)^a|E_1(J_{m,n})| + (d_u + d_v)^a|E_2(J_{m,n})| + (d_u + d_v)^a|E_3(J_{m,n})| \]
\[ = (2 + 2)a^{m(n-2)} + (2 + 3)a^{2m} + (3 + m)a^m \]
\[ = (4)a^{m(n-2)} \times (5)^2a^m \times (3 + m)a^m. \]

(2)

\[ ^{2}MZ^a(J_{m,n}) = \prod_{uv \in E(G)} (d_u d_v)^a \]
\[ = \prod_{uv \in E_1(J_{m,n})} (d_u d_v)^a + \prod_{uv \in E_2(J_{m,n})} (d_u d_v)^a + \prod_{uv \in E_3(J_{m,n})} (d_u d_v)^a \]
\[ = (d_u d_v)^a|E_1(J_{m,n})| + (d_u d_v)^a|E_2(J_{m,n})| + (d_u d_v)^a|E_3(J_{m,n})| \]
\[ = (2.2)^a^{m(n-2)} + (2.3)^a^{2m} + (3m)^a^m \]
\[ = (4)a^{m(n-2)} \times (6)^2a^m \times (3m)a^m. \]

(3)

\[ GAII(J_{m,n}) = \prod_{uv \in E(G)} \left( \frac{2\sqrt{d_u d_v}}{d_u + d_v} \right)^a \]
\[ = \prod_{uv \in E(G)} \left( \frac{2\sqrt{d_u d_v}}{d_u + d_v} \right)^a + \prod_{uv \in E(G)} \left( \frac{2\sqrt{d_u d_v}}{d_u + d_v} \right)^a + \prod_{uv \in E(G)} \left( \frac{2\sqrt{d_u d_v}}{d_u + d_v} \right)^a \]
\[ = \left( \frac{2\sqrt{d_u d_v}}{d_u + d_v} \right)^a|E_1(J_{m,n})| \times \left( \frac{2\sqrt{d_u d_v}}{d_u + d_v} \right)^a|E_2(J_{m,n})| \times \left( \frac{2\sqrt{d_u d_v}}{d_u + d_v} \right)^a|E_3(J_{m,n})|. \]
\[
= \left(\frac{2\sqrt{2\cdot2}}{2+2}\right)^{am(n-2)} \times \left(\frac{2\sqrt{2\cdot3}}{2+3}\right)^{a(2m)} \times \left(\frac{2\sqrt{3\cdot3}}{3+m}\right)^{am}
\]
\[
= \left(\frac{2\sqrt{6}}{5}\right)^{2am} \times \left(\frac{2\sqrt{3\cdot3}}{3+m}\right)^{am}.
\]

\[\Box\]

Corollary 2.2. Let \(J_{m,n}\) be the Jahangir's graph. Then

1. \(MZ_1(\cdot) = (4)^{m(n-2)} \times (5)^{2m} \times (3+m)^{m}\),
2. \(MZ_2(\cdot) = (4)^{m(n-2)} \times (6)^{2m} \times (3m)^{m}\),
3. \(GAI(\cdot) = \left(\frac{2\sqrt{2\cdot2}}{2+2}\right)^{am} \times \left(\frac{2\sqrt{3\cdot3}}{3+m}\right)^{am}\).

Proof. We get our result by putting \(\alpha = 1\) in Theorem 2.1. \(\Box\)

Corollary 2.3. Let \(J_{m,n}\) be the Jahangir's graph. Then

1. \(HI_1(\cdot) = (4)^{2m(n-2)} \times (5)^{4m} \times (3+m)^{2m}\),
2. \(HI_2(\cdot) = (4)^{2m(n-2)} \times (6)^{4m} \times (3m)^{am}\).

Proof. We get our desired results by putting \(\alpha = 2\) in Theorem 2.1. \(\Box\)

Corollary 2.4. Let \(J_{m,n}\) be the Jahangir's graph. Then

1. \(XII(\cdot) = (\frac{1}{2})^{m(n-2)} \times \left(\frac{\sqrt{3}}{\sqrt{2}}\right)^{2m} \times \left(\frac{\sqrt{3+m+1}}{\sqrt{m+1}}\right)^{mn}\),
2. \(\chi_II(\cdot) = (\frac{1}{2})^{m(n-2)} \times \left(\frac{\sqrt{3}}{\sqrt{2}}\right)^{2m} \times \left(\frac{\sqrt{3+m+1}}{\sqrt{m+1}}\right)^{mn}\).

Proof. We get our desired results by putting \(\alpha = \frac{-1}{2}\) in Theorem 2.1. \(\Box\)

Theorem 2.5. Let \(J_{m,n}\) be the Jahangir's graph. Then

\[ABCII(\cdot) = \left(\frac{1}{\sqrt{2}}\right)^{mn} \times \left(\frac{\sqrt{m+1}}{\sqrt{3m}}\right)^{mn}\].

Proof. By using the edge partition of the Jahangir graph given in Theorem 2.1

\[ABCII(\cdot) = \prod_{uv \in E(J_{m,n})} \sqrt{\frac{d_u + d_u - 2}{d_u d_u}}\]
\[= \prod_{uv \in E_1(J_{m,n})} \sqrt{\frac{d_u + d_u - 2}{d_u d_u}} \times \prod_{uv \in E_2(J_{m,n})} \sqrt{\frac{d_u + d_u - 2}{d_u d_u}}\]
\[\times \prod_{uv \in E_3(J_{m,n})} \sqrt{\frac{d_u + d_u - 2}{d_u d_u}}\]
\[= \left(\sqrt{\frac{d_u + d_u - 2}{d_u d_u}}\right)^{|E_1(J_{m,n})|} \times \left(\sqrt{\frac{d_u + d_u - 2}{d_u d_u}}\right)^{|E_2(J_{m,n})|}\]
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\[
\times \left( \sqrt{\frac{d_u + d_u - 2}{d_u d_u}} \right)^{m(n-2)} \times \left( \frac{d_u + d_u - 2}{d_u d_u} \right)^{2m} \\
= \left( \sqrt{\frac{1}{2}} \right)^{m(n-2)} \times \left( \sqrt{\frac{1}{2}} \right)^{2m} \times \left( \sqrt{\frac{m+1}{3m}} \right)^{mn} \\
= \left( \frac{1}{\sqrt{2}} \right)^{mn} \times \left( \sqrt{\frac{m+1}{3m}} \right)^{mn}
\]

\[\square\]

Competing Interests

The authors declare that they have no competing interests.

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