Phenomenological analysis of a dimension-two operator in QCD and its impact on $\alpha_s(M_\tau)$

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Abstract

Fits to the ARGUS data on hadronic decays of the tau-lepton, which determine the vector and axial-vector spectral functions, are used in order to determine the size of a dimension $d = 2$ term in the Operator Product Expansion. Constraints from the first Weinberg sum rule (in the chiral limit) are enforced in order to reduce the uncertainty of this determination. Results for the $d = 2$ operator are consistent with a quadratic dependence on $\Lambda_{QCD}$. The impact of this term on the extraction of $\alpha_s(M_\tau)$ is assessed.

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A technique based on analyticity and the Operator Product Expansion (OPE) of current correlators was proposed some time ago [1] to extract the strong coupling constant from data on $e^+e^-$ annihilation, and on the total hadronic width of the tau-lepton. Recent results [2] suggest this method to be unrivaled in precision. However, this high precision has been questioned recently [3] on the grounds that it relies on the assumption that no operators of dimension $d = 2$ (other than well known quark-mass insertion terms) enter the OPE of the two-point functions involving the vector and the axial-vector currents. While it is not possible to construct vacuum condensates of $d = 2$ with the quark and gluon fields of the QCD Lagrangian, one cannot e.g. rule out a-priori a term of the form $C_2 \propto \Lambda_{QCD}^2$. In fact, such terms have been suggested as resulting from large order perturbation theory [4]. Given the importance of this issue it becomes imperative to analyze the existing experimental data in order to decide if the presence of such a $d = 2$ term can be established. An important step in this direction has been taken in [5] by determining the size of this term from experimental data on $e^+e^-$ annihilation into hadrons, together with a zeroth-moment Finite Energy Sum Rule (FESR), and a Laplace transform QCD sum rule.

In this note I analyze this problem using the ARGUS data [6] on hadronic decays of the tau-lepton, which determine the hadronic spectral functions in the (non-strange) vector and axial-vector channels. These data have been used in the past [7] to extract the values of the vacuum condensates of dimension $d = 4, 6$ and 8 (assuming no term of $d = 2$ other than the one from the quark-mass insertion), as well as to check the saturation of the Weinberg sum rules in the chiral limit [8]. I shall perform the analysis by means of the zeroth-moment FESR, which has the advantage of decoupling the vacuum condensates of $d > 2$. In contrast, Laplace transform sum rules involve condensates of all dimensions whose values are affected by very large uncertainties. In addition, I shall consider the constraint imposed by the first Weinberg sum rule on the integrals of the vector and axial-vector spectral functions. This procedure effectively reduces the uncertainties induced by the experimental errors. I shall proceed without prejudice on the uncertainty in $\alpha_s(M_\tau)$, and determine $C_2$ separately for two different values of $\Lambda_{QCD}$, viz. $\Lambda_{QCD} = 100$ MeV, and $\Lambda_{QCD} = 300$ MeV. This will allow to establish a possible functional dependence of $C_2$ on $\Lambda_{QCD}$. Use of the tau-lepton decay data offers a comparative advantage over $e^+e^-$ annihilation in that $C_2$ determined from the vector channel can be confronted with the (independent) result from the axial-vector channel. Since chirality considerations require both values to be the same, one is offered a consistency check and a means of reducing...
the uncertainty on $C_2$.

The relevant two-point functions needed for this analysis are

$$\Pi^V_{\mu\nu}(q) = i \int d^4x e^{iqx} <0| T(V_\mu(x) V^\dagger_\nu(0))|0>$$

$$= -(g_{\mu\nu}q^2 - q_\mu q_\nu)\Pi_V(q^2),$$

$$\Pi^A_{\mu\nu}(q) = i \int d^4x e^{iqx} <0| T(A_\mu(x) A^\dagger_\nu(0))|0>$$

$$= -g_{\mu\nu} \tilde{\Pi}_A(q^2) + q_\mu q_\nu \Pi_A(q^2),$$

where $V_\mu = (\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)/2$, and $A_\mu = (\bar{u}\gamma_\mu \gamma_5 u - \bar{d}\gamma_\mu \gamma_5 d)/2$. Considering these (charge neutral) currents implies the normalization: $Im\Pi_V = Im\Pi_A = 1/(8\pi)$, at one loop order in perturbative QCD. This choice of normalization facilitates comparison with the $e^+e^-$ channel. The lowest moment FESR, which projects out a dimension $d = 2$ term in the OPE, is of the form

$$I_0(s_0)_{V,A} \equiv \frac{8\pi^2}{s_0} \int_0^{s_0} \frac{1}{\pi} \text{Im} \, \Pi_{V,A}(s) \, ds = F_2(s_0) + \frac{C_{2V,A}}{s_0},$$

where the radiative corrections to 4-loop order can be written as

$$F_2(s_0) = 1 + \frac{\alpha_s^{(1)}(s_0)}{\pi} + \left(\frac{\alpha_s^{(1)}(s_0)}{\pi}\right)^2 \left( F_3 - \frac{\beta_2}{\beta_1} \ln L - \frac{\beta_1}{2} \right)$$

$$+ \left(\frac{\alpha_s^{(1)}(s_0)}{\pi}\right)^3 \left[ \frac{\beta_2^2}{\beta_1^2} (\ln^2 L - \ln L - 1) + \frac{\beta_3}{\beta_1} \right],$$

\begin{align*}
-2 \left( F_3 - \frac{\beta_1}{2} \right) \frac{\beta_2}{\beta_1} \ln L + F_4 - F_3\beta_1 - \frac{\beta_2}{2} + \frac{\beta_1}{2} \right],
\end{align*}

with

$$\frac{\alpha_s^{(1)}(s_0)}{\pi} \equiv \frac{-2}{\beta_1 L},$$

where $L \equiv \ln(s_0/\Lambda^2_{QCD})$, and for two flavours: $\beta_1 = -29/6$, $\beta_2 = -115/12$, $\beta_3 = -48241/1728$, $F_3 = 1.756$, $F_4 = -9.057$. In writing Eq. (4) use has been made of the result [9]

$$\frac{\alpha_s^{(3)}}{\pi} = \frac{\alpha_s^{(1)}}{\pi} + \left(\frac{\alpha_s^{(1)}}{\pi}\right)^2 \left( -\frac{\beta_2}{\beta_1} \ln L \right) + \left(\frac{\alpha_s^{(1)}}{\pi}\right)^3 \left[ \frac{\beta_2^2}{\beta_1^2} (\ln^2 L - \ln L - 1) + \frac{\beta_3}{\beta_1} \right].$$
Notice that the convention for the sign of the $d = 2$ term in Eq.(3) is opposite to the one used in [3]. At dimension $d = 2$ there is a well known mass insertion contribution to $C_2$, e.g. in the vector channel it is given by

$$C_{2m} = -3 \frac{(\bar{m}_u^2 + \bar{m}_d^2)}{2 \ln s_0/\Lambda_{QCD}^2 - 4/\beta_i}$$

(7)

Using standard values for the current up and down-quark masses [10] this term is negligible. Hence, in the sequel the chiral limit will be used throughout.

The analysis of the ARGUS data [3] to extract the vector and axial-vector spectral functions entering Eq.(3) has been discussed in [5]. Using these fits to the data and performing the integrations leads to the results shown in Fig.1 for $I_0(s_0)_{V}$, and in Fig.2 for $I_0(s_0)_{A}$ (solid curves). The dashed lines correspond to $F_2(s_0)$, Eq.(4), for $\Lambda_{QCD} = 100$ MeV (lower line) and $\Lambda_{QCD} = 300$ MeV (upper line). The error bar is an estimate of the propagation of the experimental errors in the dispersive integrals. The agreement between the results of the fits and perturbative QCD is rather good in the interval $1.5\text{GeV}^2 < s_0 < 2.5\text{GeV}^2$, although there is some room left to accommodate a non-zero value of $C_2$. It is possible to reduce effectively the impact of the experimental uncertainties by considering the first Weinberg sum rule in the chiral limit

$$W_1(s_0) \equiv \int_0^{s_0} \left[ \frac{1}{\pi} \text{Im} \Pi_V(s) - \frac{1}{\pi} \text{Im} \Pi_A(s) \right] ds$$

(8)

where the pion-pole pole contribution, equal to $f_\pi^2$ ($f_\pi = 93.2$ MeV), is already included in the axial-vector spectral function. Using the fits to the data in Eq.(8) gives the result shown in Fig.3, which indicates a very good saturation of the sum rule. Notice that while strictly speaking only $W_1(\infty) \equiv 0$, for $s_0 > 2.5\text{GeV}^2$ both spectral functions are well approximated by their (identical) perturbative QCD expressions, as evidenced by the results of the fits (for more details see [7]-[8]). Hence, this sume rule becomes effectively a FESR. From $C_2^V = C_2^A$, this term does not enter Eq.(8). Hence, the first Weinberg sum rule provides an independent constraint on the areas under the vector and axial-vector spectral functions.

After confronting the left- and right-hand sides of Eq.(3) one obtains for $C_2$ the results shown in Fig.4. The upper set of curves corresponds to $\Lambda_{QCD} = 100$ MeV, and the lower set to $\Lambda_{QCD} = 300$ MeV. Curves (a) and (b) stand for $C_2$ obtained from the axial-vector and the vector channel, respectively. Using $C_2^V = C_2^A$, independent of $s_0$, determines an
overlap region (for each value of $\Lambda_{QCD}$) from which the following values are obtained

\begin{align}
C_2(\Lambda_{QCD} = 100\text{MeV}) &= -(0.028 \pm 0.012)\text{GeV}^2 \\
C_2(\Lambda_{QCD} = 300\text{MeV}) &= -(0.200 \pm 0.006)\text{GeV}^2.
\end{align}

Taking the ratio of the above results gives

\begin{equation}
\frac{C_2(\Lambda_{QCD} = 300\text{MeV})}{C_2(\Lambda_{QCD} = 100\text{MeV})} = 7 \pm 3,
\end{equation}

which is consistent with a ratio of 9, or $C_2 \propto \Lambda_{QCD}^2$, as if it would originate from renormalons [4].

With the sign convention used here, the result found in [5] using the $e^+e^-$ data in the FESR is $C_2 = +0.02 \pm 0.12\text{GeV}^2$, for an input value $\alpha_s(M_\tau) = 0.32 \pm 0.04$. If a more conservative error in $\alpha_s(M_\tau)$ had been used, e.g. some 50% bigger, then the value of $C_2$ from that analysis would have been consistent with the one obtained here, except for the sign. It is possible to understand the sign difference from the following observation. It is known from the $e^+e^-$ analysis of [11] that $I_0(s_0)V$ approaches the perturbative QCD result $F_2(s_0)$ from above, in contrast with the case of the tau-lepton analysis, where it approaches it from below. Inside the duality region, though, the areas under the vector spectral functions from the two analyses are consistent with each other. If one were to use $\text{Im}\Pi_V$ extracted from $e^+e^-$ data, and $\text{Im}\Pi_A$ from tau-lepton decays, then the first Weinberg sum rule would be saturated from above, rather than from below, as in Fig. 3. Hence, considering both analyses together makes the sign of $C_2$ indeterminate.

The current value of $\alpha_s(M_\tau)$ from the method of [1] is [12]

\[ \alpha_s(M_\tau) = 0.36 \pm 0.03. \]

This is obtained from the ratio of the hadronic to the leptonic widths of the $\tau$-lepton, $R_\tau$, which can be written as

\[ R_\tau = 3(1 + \delta^0 + \delta^2_m + \delta^6 + \delta^8 + ...), \]

where $\delta^0$ contains the perturbative corrections, $\delta^2_m$ is the quark-mass insertion at $d = 2$, $\delta^6$ stands for the $d = 6$ vacuum condensate contribution in the OPE, and so on. Corrections from the electro-weak sector can also be added to Eq.(12). The uncertainty in Eq.(11) includes, in addition to the experimental error on $R_\tau$, perturbative and non-perturbative uncertainties (except for a $d = 2$ term different from $C_{2m}$). At a scale $s_0 = M^2_\tau$, which is not too far away from the duality region found in this analysis, the results given in Eq.(9)
imply a correction of up to 6 % in the QCD evaluation of the τ hadronic width. This is not particularly negligible. While this correction is still a factor of 3 smaller than the perturbative QCD contribution (for δ⁰ ≃ 0.20), it may still have an impact on the final uncertainty in αₛ(Mₜ). For instance, for Λ_{QCD} = 300MeV the error in αₛ(Mₜ) could go up to ±0.06, which is a non-negligible 100 % effect. A better theoretical understanding of the origin of the d = 2 operator would clearly help in placing further constraints on its numerical contribution to Eq.(12).

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**Figure Captions**

Figure 1: The left-hand side of Eq. (3) in the vector channel (solid curve), and the perturbative QCD term $F_2(s_0)$ for $\Lambda_{QCD} = 300\text{MeV}$ (upper broken line), and $\Lambda_{QCD} = 100\text{MeV}$ (lower broken line).

Figure 2: The left-hand side of Eq. (3) in the axial-vector channel (solid curve), and the perturbative QCD term $F_2(s_0)$ for $\Lambda_{QCD} = 300\text{MeV}$ (upper broken line), and $\Lambda_{QCD} = 100\text{MeV}$ (lower broken line).

Figure 3: The Weinberg sum rule in the chiral limit, Eq. (8).

Figure 4: Curves (a) and (b) stand for $C_2$ obtained from the axial-vector and the vector channel, respectively. The upper set of curves corresponds to $\Lambda_{QCD} = 100\text{MeV}$, and the lower set to $\Lambda_{QCD} = 300\text{MeV}$.  

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