Unitarity violation in sequential neutrino mixing in a model of extra dimensions

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Abstract

We investigate the possibility of unitarity violation in the sequential neutrino mixing matrix in a scenario with extra compact spacelike dimensions. Gauge singlet neutrinos are assumed to propagate in one extra dimension, giving rise to an infinite tower of states in the effective four-dimensional theory. It is shown that this leads to small lepton-number violating entries in the neutrino mass matrix, which can violate unitarity on the order of one per cent.

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1 Introduction

The ever-consolidating evidence in favour of neutrino masses and mixing has spawned a large volume of speculations on new physics possibilities that could be at their origin. Considering the three light sequential neutrinos, many proposed scenarios, including seesaw models of type I [1], II [2] or III [3], ensure unitarity to a high degree of precision in the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix describing mixing in the lepton sector [4]. A measured departure from such unitarity, evinced from precision data in the neutrino sector, may thus point towards some novel mechanism for the generation of neutrino masses. One such possibility arises when at least one small gauge singlet Majorana mass term enters into an extended neutrino mass matrix. Under certain conditions, this situation passes off as ‘inverse seesaw mechanism’ [5, 6, 7]. It has been demonstrated in a number of recent works that this can lead to a violation of unitarity at the level of about 1% or more in the \(3 \times 3\) light sequential neutrino mass matrix, due to mixing with additional sterile states [5, 6, 7, 8].

A pertinent question to ask is: does such a situation fit into some of the popular scenarios of new physics at the TeV scale?

The experimental constraints on the loss of unitarity as well as its testability in neutrino oscillation experiments has been investigated recently [8, 9]. As for theoretical models, a GUT-inspired scenario, based on \(SO(10)\) with a breaking chain involving an extra \(U(1)\) gauge symmetry surviving at low scale, has been considered recently for this purpose [6]. This scenario has been shown to lead to two-loop generation of some small Majorana masses and consequently lead to the inverse seesaw mechanism. It has also been suggested that a supersymmetric model including two types of gauge singlet neutrino superfields may produce effects of this kind [10]. In a number of other model-building ventures, too, the effect mentioned above emerges as a consequence [11].

Phenomenological implications of unitarity violation in the PMNS matrix, including its signatures in phenomena driven by neutrino oscillation, have been recently investigated [9]. In this paper, we point out that a loss of unitarity in the PMNS matrix can also arise if one has extra flat spacelike dimensions, with gauge singlet neutrinos propagating in one extra dimension, and lending small diagonal elements to an extended neutrino mass matrix.

Mechanisms of neutrino mass generation have been frequently suggested in models of compact extra spacelike dimensions, both flat [12, 13, 14, 15, 16, 17, 18] and warped [19]. Here we consider a minimal higher-dimensional framework where the standard model (SM) fields all lie on a 3-brane, while one or more gauge singlet neutrino propagate along one flat extra dimension [12, 13]. However, there can in principle be several extra spacelike
dimensions where gravity propagates, thereby evading the already established lower limits on the number of such dimensions [20]. An orbifold symmetry is further imposed along the compact direction containing the neutrino(s), so that one obtains only one (right-handed) chirality for the $n = 0$ Kaluza-Klein (KK) mode. It has been demonstrated earlier that this scenario can naturally suppress neutrino masses via a Type I [1] seesaw mechanism.

The gauge singlet neutrinos can have Majorana masses in five dimensions to start with. We are specially interested in the situation where a strong cancellation between this mass and the KK tower mass leads to a very small entry in the effective neutrino mass matrix in four dimensions. We show that the resulting mass matrix has additional ‘sterile’ states mixing appreciably with the sequential neutrinos. It is found that one can consequently expect the violation of unitarity in the $3 \times 3$ (PMNS) matrix in certain regions of the parameter space of such a model.

In Section 2, we outline some scenarios that lead to loss of unitarity of the PMNS matrix, including the inverse seesaw mechanism. The extra-dimensional model under investigation is briefly reviewed in Section 3. The viability of a substantial loss of PMNS unitarity is numerically demonstrated in Section 4. We summarise and conclude in Section 5.

2 Loss of unitarity in the PMNS matrix

In general, the well-known Type I seesaw mechanism involving a light and a heavy neutrino also involves a departure from unitarity in the PMNS matrix. However, this departure is immeasurably tiny, since the seesaw mass scale is invariably much higher than the light neutrino masses [1]. An exception to this may occur if a very small Majorana mass is introduced. However, this choice is inhibited by (a) the need of justifying such a small $\Delta L = 2$ mass in terms of new physics, and (b) the need of introducing excessively suppressed Dirac masses for generating light neutrinos, which essentially destroys the motivation of the seesaw mechanism.

The situation can be different when one has more than one two-component sterile neutrinos. It has been shown in a number of recent works [5, 6, 7, 8] that this allows one to insert small lepton-number violating mass terms in diagonal positions of the neutrino mass matrix, and the off-diagonal entries need not all be much smaller. Its most noticeable consequence is a loss of unitarity at the level of 1% or more in the $3 \times 3$ (PMNS) part of the neutrino mass matrix.

Several kinds of scenarios that meet this description are found in the literature [5, 7, 8].
Here we mention two classes only among these, considering for illustration one sequential and two sterile species in each case. The first [5] is one of the form

\[ \mathcal{M} = \begin{pmatrix}
0 & m_D & 0 \\
m_D & m_R & m_N \\
0 & m_N & m_L
\end{pmatrix}, \]

(1)

in the basis \((\bar{\nu}_L, N_R, \bar{N}_L)\), where the last two are gauge singlets. The masses \(m_{L,R}\) arise from \(\Delta L = 2\) terms. For \(m_{L,R} << m_D, m_N\), this not only yields an active neutrino mass eigenstate in the right order, but also leads to active-sterile mixing at the level of 1% for appropriate choice of the mass parameters (say, for example, \(m_D \sim 10\) GeV, \(m_N \sim 1\) TeV, and \(m_L \sim 10\) keV). A corresponding situation with three sequential neutrinos will show unitarity violation at the same level in the PMNS matrix, but with an additional light sterile neutrino. Since the light (sequential) neutrino mass vanishes in the limit \(m_L \to 0\), it is often called an ‘inverse seesaw’ scenario.

Another situation that one can consider has the same choice of neutrino basis states, but a mass matrix of the form [8]

\[ \mathcal{M} = \begin{pmatrix}
0 & m_D & m_N \\
m_D & m_R & 0 \\
m_N & 0 & m_L
\end{pmatrix}, \]

(2)

with \(m_D << m_R\) and \(m_N^2/m_R << m_L << m_N << m_R\). It has been found that this situation, too, leads to light sequential neutrino(s) and unitarity violation at the same level (\(\sim 1\) %), for appropriate choice of parameters (say, for example, \(m_D, m_N \sim 1\) MeV, \(m_R \sim 1\) TeV and \(m_L \sim 100\) eV). The difference with the previous situation is that (a) one obtains a light sterile neutrino even with one sequential family, and (b) the sequential neutrino mass does not vanish in the limit \(m_L \to 0\). This makes it deviate from an inverse seesaw scenario in the strict sense, although it is equally interesting from the viewpoint of unitarity loss of the PMNS matrix. Since such unitarity loss is a very interesting consequence that is experimentally testable, it is worth exploring if it occurs in some otherwise well-motivated theories beyond the SM. In the next two sections we outline one such scenario, and go on to examine its potential for generating unitarity loss.
3 A model with extra dimensions

In this section we describe the model adopted for illustrating our point. It assumes extra flat compact spacelike dimensions where gravity can propagate. The SM fields are confined to a 3-brane which constitutes a ‘slice’ in the higher-dimensional space. So far it is very similar to the Arkani Hamed-Dimopoulos-Dvali (ADD) scenario [21], excepting that it includes an effort to account for neutrino masses, through the introduction of one gauge singlet neutrino propagating in one extra dimension only [12, 13, 14, 15, 16, 17]. Thus, while all the phenomenology involving gravitons remains similar to that in the ADD framework with several extra dimensions, one can consider just the five-dimensional subspace for studying neutrino physics. We make our analysis simple by adhering to one generation of SM neutrinos. The fifth flat dimension \( y \), along which propagates the right handed neutrino \( N(x, y) \), is compactified over an \( S^1/Z_2 \) orbifold where \( R \) is the radius of compactification. The preservation of the \( Z_2 \) invariance necessitates the existence of at least two symmetrically placed branes, and the SM fields lie on either of them. Thus the complete leptonic field content of the model is

\[
L(x) = \begin{pmatrix} \nu(x) \\ \ell_L(x) \end{pmatrix}, \quad \ell_R(x), \quad N(x, y) = \begin{pmatrix} \xi(x, y) \\ \bar{\eta}(x, y) \end{pmatrix},
\]

Under \( S^1/Z_2 \), the latter may be associated with opposite parities:

\[
\xi(x, y) = \xi(x, -y), \quad \eta(x, y) = -\eta(x, -y).
\]

The brane where the SM is localised, can be assumed to be at \( y = a \) just for generality, instead of at the orbifold fixed point \( y = 0 \). We shall see later that this adds to the freedom of the model. The generic effective four-dimensional Lagrangian of this model is given by,

\[
L_{\text{eff}} = \int_0^{2\pi R} dy \left\{ \bar{N} \left( i\gamma^\mu \partial_\mu + \gamma_5 \partial_y \right) N - \frac{1}{2} \left( M N^T C^{(5)} \right)^{-1} \gamma_5 N + \text{ h.c.} \right\} + \delta(y - a) \left[ \frac{h_1}{(M_F)^{1/2}} L\tilde{\Phi}^* \xi + \frac{h_2}{(M_F)^{1/2}} L\tilde{\Phi}^* \eta + \text{ h.c.} \right] + \delta(y - a) L_{\text{SM}},
\]

where \( \tilde{\Phi} = i\sigma_2 \Phi^* \), \( L_{\text{SM}} \) is the SM Lagrangian, \( M \) is the Majorana mass for \( N \) (we do not specify its scale for the moment), \( C^{(5)} \) is the 5-dimensional charge conjugation operator and \( M_F \) is the fundamental gravity scale. The Yukawa couplings in five dimensions, \( h_{1,2} \), are assumed to be \( \mathcal{O}(1) \). For gravity propagating in a \( d \)-dimensional bulk,

\[
M_P = (2\pi M_F R)^{d/2} M_F,
\]

\[
(3)
\]

\[
(4)
\]

\[
(5)
\]

\[
(6)
\]
for the simple case where all the compactification radii are of equal size \( R \), \( M_P \) being the four-dimensional Planck scale. A Dirac mass term \( m_D \bar{N} N \) is not allowed in equation (5) because of the \( Z_2 \) symmetry.

Following equation (4), the two-component spinors \( \xi \) and \( \eta \) can be expanded as,

\[
\xi(x, y) = \frac{1}{\sqrt{2\pi R}} \xi_0(x) + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \xi_n(x) \cos \left( \frac{ny}{R} \right),
\]

(7)

\[
\eta(x, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \eta_n(x) \sin \left( \frac{ny}{R} \right),
\]

(8)

where the chiral spinors \( \xi_n(x) \) and \( \eta_n(x) \) form an infinite tower of KK fields. Using these expansions and integrating out the \( y \)-coordinate, the effective Lagrangian reduces to

\[
\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \bar{\xi}_0 (i\tilde{\sigma}^\mu \partial_\mu) \xi_0 + \left( \bar{h}_1^{(0)} L\tilde{\Phi}^* \xi_0 - \frac{1}{2} M \xi_0 \bar{\xi}_0 + \text{h.c.} \right) + \sum_{n=1}^{\infty} \left[ \bar{\xi}_n (i\tilde{\sigma}^\mu \partial_\mu) \xi_n \\
+ \bar{\eta}_n (i\tilde{\sigma}^\mu \partial_\mu) \eta_n + \frac{1}{R} \left( \xi_n \eta_n + \bar{\xi}_n \bar{\eta}_n \right) - \frac{1}{2} M \left( \xi_n \xi_n + \bar{\eta}_n \bar{\eta}_n + \text{h.c.} \right) \\
+ \sqrt{2} \left( \bar{h}_1^{(n)} L\tilde{\Phi}^* \xi_n + \bar{h}_2^{(n)} L\tilde{\Phi}^* \eta_n + \text{h.c.} \right) \right]
\]

(9)

in a basis in which \( M \) is positive, and with:

\[
\bar{h}_1^{(n)} = \frac{h_1}{(2\pi M_F R)^{1/2}} \cos \left( \frac{na}{R} \right) = \left( \frac{M_F}{M_P} \right)^{1/d} h_1 \cos \left( \frac{na}{R} \right) = \bar{h}_1 \cos \left( \frac{na}{R} \right).
\]

(10)

\[
\bar{h}_2^{(n)} = \frac{h_2}{(2\pi M_F R)^{1/2}} \sin \left( \frac{na}{R} \right) = \left( \frac{M_F}{M_P} \right)^{1/d} h_2 \sin \left( \frac{na}{R} \right) = \bar{h}_2 \sin \left( \frac{na}{R} \right).
\]

(11)

For deriving the last two equalities on the right hand sides of equations (10)- (11), we have made use of equation (6).

Equations (10) and equation (11) imply that the induced four-dimensional Yukawa couplings \( \bar{h}_{1,2}^{(n)} \) can get suppressed by many orders depending on the hierarchy between \( M_P \) and \( M_F \); for example, if gravity and the bulk neutrino feel the same number of extra dimensions, say \( d = 1 \), then these couplings are suppressed by a factor \( M_F/M_P \sim 10^{-15} \), for \( M_F \approx 10 \text{ TeV} \) (see also [12, 13]).

It is clear from equation (3) that \( \xi \) and \( \bar{\eta} \) have the same lepton number. Thus, the simultaneous presence of the two operators \( L\tilde{\Phi}^* \xi \) and \( L\tilde{\Phi}^* \eta \) in equation (9) leads to lepton number violation. Such coexistence of the two operators is possible only if we allow the brane to be shifted by an amount \( a(\neq 0) \) from the orbifold fixed points \( (y = 0, \pi R) \). Such a shifting of the brane, respecting the \( Z_2 \) invariance of the original higher dimensional Lagrangian, has been shown to be possible under certain restrictions in Type-I string theories [22].
indicated in [12, 18], the $Z_2$ invariance can be taken care of by allowing the replacements

$$\xi \delta(y-a) \rightarrow \frac{1}{2} \xi \left[ \delta(y-a) + \delta(y+a-2\pi R) \right],$$
$$\eta \delta(y-a) \rightarrow \frac{1}{2} \eta \left[ \delta(y-a) - \delta(y+a-2\pi R) \right],$$

(12)

with $0 \leq a < \pi R$ and $0 \leq y \leq 2\pi R$. Here we re-iterate that a $Z_2$-invariant implementation of brane-shifted couplings requires the existence of at least two branes placed at $y = a$ and $y = 2\pi R - a$.

A remarkable feature of the brane-shifted framework was pointed out in [18], where it has been shown that in such a framework it is possible to completely decouple the effective Majorana-neutrino mass $\langle m \rangle$ and the scale of light neutrino masses, so as to have $\langle m \rangle$ within an observable range. Therefore, the Lagrangian (9) contains two types of Majorana neutrino mass terms (involving respectively the parameters $M$ and $\tilde{h}_2^{(n)}$) both of which lead to a breaking of $L$. Such $L$-breaking is a necessary ingredient of leptogenesis.

Following the notations of reference [12], we now introduce the weak basis for the KK Weyl-spinors, by defining

$$\chi_{\pm n} = \frac{1}{\sqrt{2}} (\xi_n \pm \eta_n),$$

(13)

followed by a rearrangement of the states $\xi_0$ and $\chi_{\pm n}$, such that, for a given value of $n$ (say, $n = k_0$), the smallest diagonal entry of the neutrino mass matrix is

$$\varepsilon = \min \left( \left| M - \frac{k_0}{R} \right| \right) \leq 1/(2R).$$

(14)

After re-ordering, we can define the multiplet $\Psi_\nu$ consisting of the Majorana spinors

$$\Psi_\nu^T = \begin{bmatrix}
(\chi_{\nu 0}), & (\chi_{k_0}), & (\chi_{k_0-1}), & (\chi_{k_0+1}), & \cdots, & (\chi_{k_0-n}), & (\chi_{k_0+n}), & \cdots
\end{bmatrix},$$

(15)

while the effective Lagrangian for right handed neutrinos reduces to

$$\mathcal{L}_{\text{kin}} = \frac{1}{2} \bar{\Psi}_\nu \left( i \not{\partial} - \mathcal{M}_\nu^{KK} \right) \Psi_\nu,$$

(16)
where $\mathcal{M}_{\nu}^{KK}$ is the corresponding neutrino mass matrix given by

$$
\mathcal{M}_{\nu}^{KK} = \begin{pmatrix}
0 & m^{(0)} & m^{(-1)} & m^{(1)} & m^{(-2)} & m^{(2)} & \cdots \\
m^{(0)} & \varepsilon & 0 & 0 & 0 & 0 & \cdots \\
m^{(-1)} & 0 & \varepsilon - \frac{1}{R} & 0 & 0 & 0 & \cdots \\
m^{(1)} & 0 & 0 & \varepsilon + \frac{1}{R} & 0 & 0 & \cdots \\
m^{(-2)} & 0 & 0 & 0 & \varepsilon - \frac{2}{R} & 0 & \cdots \\
m^{(2)} & 0 & 0 & 0 & 0 & \varepsilon + \frac{2}{R} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}.
$$

(17)

The most important consequence of such a rearrangement is that the mass scale $M$, which we did not specify earlier but which could be arbitrarily large, is now replaced by the light mass scale $\varepsilon$. The entries in the first row and the first column of $\mathcal{M}_{\nu}^{KK}$ are given by the relation,

$$
m^{(n)} = \frac{v}{\sqrt{2}} \left[ \tilde{h}_1 \cos \left( \frac{(n - k_0)a}{R} \right) + \tilde{h}_2 \sin \left( \frac{(n - k_0)a}{R} \right) \right] = m \cos \left( \frac{na}{R} - \phi_h \right),
$$

(18)

with

$$
m = \frac{v}{2} \sqrt{\frac{h_1^2 + h_2^2}{\pi M_FR}} = \frac{m_{\text{max}}}{\sqrt{M_FR}},
$$

(19)

$$
\phi_h = \tan^{-1} \left( \frac{h_2}{h_1} \right) + k_0 \frac{a}{R},
$$

(20)

where $v$ is the vacuum expectation value of the SM Higgs boson.

### 4 Unitarity loss with extra dimensions: some numerical illustrations

Here we show that a substantial loss of unitarity of the PMNS matrix can occur in different allowed regions of the parameter space of the model described in the previous section. The first issue is, of course, insuring at least one small entry in diagonal positions of the neutrino mass matrix $\mathcal{M}_{\nu}^{KK}$. Equation (14) tells us that $\varepsilon \leq 1/2R$. There is no other theoretical or phenomenological constraint on $\varepsilon$. Thus $\varepsilon$ qualifies to be the small diagonal element which can be potentially responsible for a departure from unitarity.

$M_F$, the Planck mass in five dimensions, is expected to be $\gtrsim \text{TeV}$, since gravitational effects will otherwise become important in low-energy physics. At the same time, in order
to ensure that physics along the compact dimension(s) is not plagued with trans-Planckian effects, one should have $1/R \leq M_F$. Thus, in the expression for $m$ in equation (19), $M_FR$ is at least of order unity. Given the fact that the five-dimensional Yukawa couplings $h_{1,2}$, too, are *prima facie* of the order of unity, this implies that $m$ can at most be around $v$, the electroweak symmetry breaking scale.

We indicated two scenarios of unitarity loss in section 2: (i) where the off-diagonal elements in the extended neutrino mass matrix are all larger than the diagonal ones, and (ii) where diagonal elements excepting the smallest one are larger than the off-diagonal ones. Let us first examine whether the extra-dimensional model under scrutiny answers to both of these scenarios.

The first possibility demands

$$m > \frac{1}{R} \quad (21)$$

since, with $a \neq 0$, $m^{(n)}$ can approach $m$ for some value of $n$ along the tower. Using equation (19) in (21), one obtains

$$\frac{m_{\text{max}}^2}{M_F} > \frac{1}{R} \quad (22)$$

The inequality should hold for the maximum value of the right-hand side for a given $M_F$, which is $M_F$ itself. Thus we have

$$M_F < m_{\text{max}} \quad (23)$$

Therefore, demanding $m > 1/R$ implies that the five-dimensional Planck scale has to be brought down below $m_{\text{max}}$ which is just about the electroweak symmetry breaking scale, and hence is inadmissible.

The first scenario is thus disfavoured in this model. On the other hand, since the diagonal elements $M_{\nu KK}(i,i), i \geq 3$ are always greater in magnitude than $m^{(n)}$ for all values of $n$, one can say that for sufficiently small $\varepsilon$, this model provides an opportunity for unitarity loss in the PMNS matrix in the sense of the second scenario mentioned in Section 2.

The value of $1/R$, on the other hand, is not subject to any general constraint stronger than that arising from the validity of Newton’s law of gravitation down to about $10^{-2} \text{mm}$, which essentially allows $1/R$ to have any value $\gtrsim 10^{-8} \text{ MeV}$ [20]. Precision electroweak constraints do not tighten the constraint, since the tower resulting from the compactification of the extra dimension corresponds to gauge singlet neutrinos only. Thus, in order to have loss of unitarity in the PMNS matrix, we are faced with two possibilities, namely, (a) $\varepsilon << 1/R$, and (b) $\varepsilon \approx 1/R$. We show below that both of these situations are possible.

Our principal aim is to check if it is possible to have the sequential neutrino masses in the right order ($\sim 10^{-2} \text{ eV}$), and at the same time have substantial violation of unitarity.
The latter requires that the squares of elements of some particular column in the full mixing matrix beyond the PMNS block add up to $\mathcal{O}(10^{-4})$. This sum is defined as $\delta^2$ here. \footnote{In references \cite{8, 9}, the violation of unitarity has been defined in terms of a parameter $\eta$. It is easy to check that $2\eta = \delta^2$.}

We carry out this investigation in the simplified situation, with one sequential neutrino flavour and just one gauge singlet neutrino in the bulk. We shall comment later on the generalisations necessary to generate the actual pattern of masses and mixing. As far as the violation of PMNS unitarity is concerned, however, the conclusions we reach below remain valid even when such generalisations are made.

**Case (a):** This implies a fine cancellation between the bulk mass $M$ and some integral multiple of $1/R$. While a dynamical explanation of this is difficult, it is not entirely unlikely, as both $M$ and $1/R$ can rather naturally be around the TeV-scale, and there is a distinct possibility of the two of them having near-coincident values.

A few illustrative points in the parameter space for this case are shown in Table 1. We have confined ourselves to $1$ MeV $\leq 1/R \leq 10$ TeV. It is found that, to get a substantial unitarity violation ($\delta \geq 0.5$ %) and neutrino mass in the right order, the largest possible value of $\varepsilon$ that we can take is $\simeq 10^{-6}$. In this case, the $2 \times 2$ block in the upper left corner of the neutrino mass matrix effectively determines the masses of the sequential and the lightest sterile neutrino, given as $(m^{(0)})^2/\varepsilon$ and $\varepsilon$ respectively. One further has:

$$\delta \sim \left[ \frac{(m^{(0)})^2}{\varepsilon} \right] \frac{1}{m^{(0)}} \quad (24)$$

Therefore if $\varepsilon$ is increased, the concomitant enhancement in $m^{(0)}$, required to keep the sequential neutrino mass unaffected, ends up suppressing $\delta$. The values of $m^{(0)}$ required point towards $\phi_h \simeq \pi/2$. On the other hand, the fact that $m^{(1)}$ can vary over a wide range, implies that the brane-shift parameter $a$ can vary from zero to $0.1R$ approximately. It should be noted that Table 1 includes one sample corresponding to $m^{(0)} = m^{(1)}$, which means $a = 0$. Thus, in this case, large unitarity violation is consistent with both the cases where the brane is located at orbifold fixed point and where it is noticeably shifted.

**Case (b):** In this case $\varepsilon$ and $1/R$ can be relatively close to each other. Therefore, no drastic cancellation is required between them, and no allegation of fine-tuning can be levelled against such a scenario.

Some sample results for this case are presented in Table 2. It should be noted that values of $1/R$ of comparable smallness as that of $\varepsilon$ implies that the part of $\mathcal{M}_\nu^{KK}$ beyond the upper left $2 \times 2$ block no longer tends to decouple. An immediate consequence is that
| 1/R  | $\varepsilon$ | $m^{(0)}$ | $m^{(-1)} = m^{(+1)}$ | $\delta$ (%) |
|------|--------------|-----------|------------------|-----------|
| 10,000 | $10^{-7}$ | $10^{-9}$ | 100 | 1.7 |
|       | $10^{-9}$ | $10^{-10}$ | 0.001 | 0.99 |
|       | $10^{-9}$ | $10^{-7}$ | 0.001 | 1.0 |
|       | $10^{-8}$ | $3 \times 10^{-10}$ | 0.001 | 3.3 |
|       | $10^{-7}$ | $10^{-9}$ | 0.001 | 1.0 |
|       | $10^{-7}$ | $10^{-9}$ | 0.005 | 1.0 |
|       | $10^{-7}$ | $10^{-9}$ | 0.01 | 1.0 |
|       | $10^{-7}$ | $10^{-9}$ | 0.1 | 1.0 |
|       | $10^{-7}$ | $10^{-9}$ | 1.0 | 1.0 |
|       | $10^{-7}$ | $10^{-9}$ | 10.0 | 1.7 |
|       | $10^{-7}$ | $10^{-9}$ | 20.0 | 3.0 |
|       | $10^{-7}$ | $10^{-9}$ | 25.0 | 3.6 |
| 1000  | $3 \times 10^{-8}$ | $3 \times 10^{-10}$ | $3 \times 10^{-10}$ | 1.0 |
| 10    | $10^{-7}$ | $10^{-9}$ | 0.001 | 1.0 |
| 1     | $10^{-7}$ | $10^{-9}$ | 0.001 | 1.0 |
|       | $10^{-6}$ | $3 \times 10^{-9}$ | 0.005 | 0.8 |
|       | $10^{-6}$ | $6 \times 10^{-9}$ | 0.005 | 0.9 |
| 0.01  | $10^{-7}$ | $10^{-9}$ | $10^5$ | 1.2 |
| 0.001 | $10^{-7}$ | $10^{-9}$ | $10^5$ | 1.7 |

Table 1: Different sample points in the parameter space of the model where substantial unitarity violation takes place, for $\varepsilon << 1/R$. All mass parameters are in GeV.

$m^{(\pm 1)}$ have to be as small as $m^{(0)}$ when $1/R$ is small. This in turn drives the value of the brane-shift parameter $a$ to very small values. Therefore, unlike in the previous case, the brane is compelled to be close to the orbifold fixed points.

The figures included in Table 2 are self-explanatory; the provision for substantial unitarity violation is clearly there. However, there is a relative paucity of available points compared to the previous case. This is because the part of $M^{\nu KK}$ beyond the $2 \times 2$ block is not ineffectual in determining the sequential neutrino mass and its mixing with light sterile states. In order to comply with all constraints there, one therefore requires a correlation between $\varepsilon$ and $1/R$, in contrast to the situation with $\varepsilon << 1/R$, thereby restricting the allowed points in the
parameter space.

The numerical results presented by us are obtained through the diagonalisation of the $4 \times 4$ neutrino mass matrix, including a tower of states up to the first KK excitation only. We have, however, checked that the results do not change qualitatively upon the inclusion of additional towers and the resulting augmentation of the mass matrix.

$$\frac{1}{R} \quad \varepsilon \quad m^{(0)} \quad m^{(-1)} = m^{(+1)} \quad \delta \; (\%)$$

| $1/R$ | $\varepsilon$ | $m^{(0)}$ | $m^{(-1)}$ | $\delta$ |
|-------|---------------|------------|------------|---------|
| $10^{-5}$ | $10^{-6}$ | $10^{-8}$ | $3.5 \times 10^{-8}$ | 1.0 |
| $10^{-5}$ | $2 \times 10^{-6}$ | $10^{-8}$ | $3.5 \times 10^{-8}$ | 0.7 |
| $10^{-5}$ | $5 \times 10^{-6}$ | $10^{-8}$ | $3.0 \times 10^{-8}$ | 0.7 |
| $10^{-5}$ | $5 \times 10^{-6}$ | $3 \times 10^{-8}$ | $3 \times 10^{-8}$ | 0.9 |
| $10^{-6}$ | $5 \times 10^{-7}$ | $3 \times 10^{-9}$ | $10^{-8}$ | 2.2 |

Table 2: Different sample points in the parameter space of the model where substantial unitarity violation takes place, for $\varepsilon \simeq 1/R$. All mass parameters are in GeV.

## 5 Summary and conclusions

We have studied a popular model of flat extra compact spacelike dimensions. A gauge singlet neutrino is assumed to propagate in one extra dimension. The ‘bulk’ mass possessed by this neutrino can undergo cancellation with the KK tower mass for some member of the tower, giving rise to at least one small diagonal entry ($\varepsilon$) in the infinite-dimensional neutrino mass matrix in four dimensions. We show that this can cause substantial mixing between the sequential and sterile neutrinos without violating any existing constraint. The consequence is a departure from unitarity of the PMNS matrix, both for $\varepsilon << 1/R$ and $\varepsilon \approx 1/R$, $R$ being the radius of the compact dimension housing the gauge singlet neutrino.

For the sake of simplicity, we have presented our results for one sequential neutrino. It can be easily checked that the conclusions are valid with additional generations. In fact, the constraints on $\delta$ are easy to satisfy, since the strongest constraint on the PMNS matrix is on its $(1,2)^{th}$ element [8, 9]. The relatively unconstrained mixing of, for example, $\nu_\tau$ with a sterile neutrino can accommodate the values of $\delta$ obtained here. On the other hand, it may be difficult to accommodate the neutrino mixing data and mass hierarchies with one sterile bulk neutrino only [23]. At least two such neutrinos can, however, accommodate everything
rather easily, thanks to the additional Yukawa couplings available, which are essentially free parameters. Our general conclusions are unaffected by such extensions.

In conclusion, the phenomenon of unitarity violation in the PMNS matrix can be motivated rather well in a model of extra dimensions. This brings to the fore the likely connection between subtleties of the neutrino sector and theories which advocate strikingly new physics around the Tev scale.

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