Indirect Search for Supersymmetry

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Effects of supersymmetric particles may appear in various low energy experiments through loop diagrams. We discuss various flavor changing neutral current processes, the muon anomalous magnetic moment and lepton flavor violation in the context of the supergravity model. In particular the $B^0 - \overline{B}^0$ mixing and the muon anomalous magnetic moment are revisited taking into account the recent Higgs boson search result. We also consider $\mu^+ \to e^+\gamma$ and $\mu^+ \to e^+e^+e^-$ processes with polarized muons. We calculate the P-odd and T-odd asymmetries in these processes for SU(5) and SO(10) supersymmetric grand unified theories and show that these asymmetries are useful to distinguish different models.

1 Introduction

Unified theories based on supersymmetry (SUSY) have been studied as one of promising candidates beyond the Standard Model (SM) since early 1980’s because this symmetry could be a solution of hierarchy problem in the SM. Motivation for SUSY unification was strengthened when precisely determined values of three gauge coupling constants turned out to be consistent with the SU(5) supersymmetric grand unified theory (SUSY GUT). Experimental verification of SUSY is, therefore, one of the most important issues of current high energy physics.

In order to explore SUSY indirect search experiments are important in addition to direct search for SUSY particles at collider experiments. There are variety of possibilities that SUSY effects can appear in low energy experiments. SUSY can affect flavor changing neutral current (FCNC) processes and CP violation in $B$ and $K$ meson decays. Processes like proton decay, lepton flavor violation (LFV) such as $\mu \to e\gamma$ and neutron and electron electric dipole moments (EDM) are important because these are either forbidden or strongly suppressed within the SM.

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In the context of SUSY models flavor physics has important implications. Because the squark and the slepton mass matrices become new sources of flavor mixings generic mass matrices would induce too large FCNC and LFV effects if the superpartners’ masses are in a few-hundred-GeV region. For example, if we assume that the SUSY contribution to the $K^0 - \bar{K}^0$ mixing is suppressed because of the cancellation among the squark contributions of different generations, the squarks with the same $SU(3) \times SU(2) \times U(1)$ quantum numbers should be highly degenerate in masses. When the squark mixing angle is in a similar magnitude to the Cabibbo angle the requirement on degeneracy becomes as

$$\frac{\Delta m^2_{\tilde{q}}}{m^2_{\tilde{q}}} \lesssim 10^{-2} \left( \frac{m_{\tilde{q}}}{100 \text{GeV}} \right)$$

for at least the first and second generation squarks. Similarly, the $\mu^+ \rightarrow e^+ \gamma$ process puts a strong constraint on the flavor off-diagonal terms for the slepton mass matrices which is roughly given by

$$\frac{\Delta m^2_{\tilde{\mu} \tilde{\bar{e}}}}{m^2_{\tilde{\mu}}} \lesssim 10^{-3} \left( \frac{m_{\tilde{l}}}{100 \text{GeV}} \right)^2.$$

In the minimal supergravity model these flavor problems are avoided by taking SUSY soft-breaking terms as flavor-blind structure. The scalar mass terms are assumed to be common for all scalar fields at the Planck scale and therefore there are no FCNC effects nor LFV from the squark and slepton sectors at this scale. The physical squark and slepton masses are determined taking account of the renormalization effects from the Planck to the weak scale. This induces sizable SUSY contributions to various FCNC processes in the B and K decays. Detail calculations have been done for processes such as $B^0 - \bar{B}^0$ mixing, $\epsilon_K$, $b \rightarrow s l^+ l^-$ and $K \rightarrow \pi \nu \bar{\nu}$ in the minimal supersymmetric standard model (MSSM) and as well as in the minimal supergravity model. Also if there is interaction which breaks lepton flavor conservation between the Planck and the weak scales, LFV effects can be induced through the slepton mass matrices. An important example of this kind is SUSY GUT models where interaction at the GUT scale induces flavor mixing in the lepton/slepton sector as well as in the quark/squark sector.

In this talk we discuss two recent works on flavor physics in the supergravity model. The first topics is an update of FCNC processes in B and K decays. Detail calculations have been done for processes such as $B^0 - \bar{B}^0$ mixing, $\epsilon_K$, $b \rightarrow s l^+ l^-$ and $K \rightarrow \pi \nu \bar{\nu}$ in the minimal supersymmetric standard model (MSSM) and as well as in the minimal supergravity model. Also if there is interaction which breaks lepton flavor conservation between the Planck and the weak scales, LFV effects can be induced through the slepton mass matrices. An important example of this kind is SUSY GUT models where interaction at the GUT scale induces flavor mixing in the lepton/slepton sector as well as in the quark/squark sector.

In this work we took into account recent improvement of Higgs boson search at LEP II which turns out to be very important to constrain SUSY parameter space, especially for small $\tan\beta$ region. We also calculate SUSY contributions to the anomalous magnetic moment of muon in
this model. We show that the on-going experiment at BNL will put a strong constraint for large $\tan\beta$ region.

The second topics is LFV process in the SUSY GUT models. Here we consider $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$ processes with polarized muons. We show that parity-odd (P-odd) and time-reversal-odd (T-odd) asymmetries are useful to distinguish various sources of LFV interactions. In particular SU(5) and SO(10) SUSY GUT have different features of these asymmetries.

2 Update of FCNC Processes and Muon Anomalous Magnetic Moment in the Supergravity Model

2.1 $B^0 - \bar{B}^0$ mixing in the Supergravity Model

In the minimal SM various FCNC processes and CP violation in B and K decays are determined by the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The constraints on the parameters of the CKM matrix can be conveniently expressed in terms of the unitarity triangle. With CP violation at B factory as well as rare K decay experiments we will be able to check consistency of the unitarity triangle and at the same time search for effects of physics beyond the SM. In order to distinguish possible new physics effects it is important to identify how various models can modify the SM predictions.

Although general SUSY models can change the lengths and the angles of the unitarity triangle in variety ways, the supergravity model predicts a specific pattern of the deviation from the SM. Namely, we can show that the SUSY loop contributions to FCNC amplitudes approximately have the same dependence on the CKM elements as the SM contributions. In particular, if we assume that there are no CP violating phases from SUSY breaking sectors, the complex phase of the $B^0 - \bar{B}^0$ mixing amplitude does not change even if we take into account the SUSY and the charged Higgs loop contributions. In terms of the unitarity triangle this means that the angle measurements through CP asymmetry in B decays determine the CKM matrix elements as in the SM case. On the other hand the length of the unitarity triangle determined from $\Delta M_B$ and $\epsilon_K$ can be modified. The case with supersymmetric CP phases was also studied within the minimal supergravity model and it was shown that effects of new CP phases on the $B^0 - \bar{B}^0$ mixing amplitude and the CP asymmetry in the $b \rightarrow s \gamma$ process are small once constraints from neutron and electron EDMs are included.

We present the result of numerical calculation of $\Delta M_B$ in the supergravity model. We also calculate the branching ratios of $b \rightarrow s \gamma$ in this model and this is used as a constraint on SUSY parameter space. In the calculation we have used updated results of various SUSY search experiments at LEP II and...
Figure 1: $\Delta M_{B_d}$ normalized by the SM value for $\tan \beta = 2$. The square(dot) points correspond to the minimal (enlarged) parameter space of the supergravity model.

Tevatron as well as the next-to-leading QCD corrections in the calculation of the $b \to s\gamma$ branching ratio. Most important change in the experimental constraints for the last one year comes from improvement of SUSY Higgs boson search at LEP II.

It is well known that there is a strict upper bound on the lightest CP-even Higgs boson mass in the MSSM. Because the Higgs search in the LEP II experiment has reached to the sensitivity of 100 GeV for the SM Higgs boson and about 90 GeV for the case of the lightest CP-even Higgs boson in the MSSM, a meaningful amount of the SUSY parameter space is already excluded.

We first present the magnitude of $\Delta M_{B_d}$ in the supergravity model. In Fig. 1 we show $\Delta M_{B_d}/\Delta M_{B_d}^{SM}$ as a function of the lightest Higgs boson mass for $\tan \beta = 2$, where $\Delta M_{B_d}^{SM}$ denotes the SM value. As input parameters we take $m_t^{\text{pole}} = 175$ GeV, $m_b^{\text{pole}} = 4.8$ GeV, $\alpha_s(m_Z) = 0.119$. For the CKM matrix elements, we take $V_{us} = 0.2196$, $V_{cb} = 0.0395$, $|V_{ub}/V_{cb}| = 0.08$ and the CP violating phase in the CKM matrix as $\delta_{13} = \pi/2$. In the supergravity model $\Delta M_{B_d}/\Delta M_{B_d}^{SM}$ is the almost independent of the CKM parameters, so that the following results do not change very much if we take different values of the CKM matrix elements. We have calculated the SUSY particle spectrum based on two different assumptions on the initial conditions of RGE. The minimal case corresponds to the minimal supergravity where all scalar fields
Figure 2: Allowed ranges of $\Delta M_{B_d}/\Delta M_{B_d}^{SM}$ for several values of $\tan \beta$ with the present constraint of the Higgs boson mass. Two lines are shown according to the sign of the $b \to s \gamma$ amplitude for $\tan \beta = 10, 20, 40$. The left (right) lines correspond the case where the sign of the $b \to s \gamma$ amplitude is same (opposite) as that in the SM.

have a common SUSY breaking mass at the GUT scale. In the second case shown as “nonminimal” in the figures we enlarge the SUSY parameter space by relaxing the initial conditions for the SUSY breaking parameters, namely all squarks and sleptons have a common SUSY breaking mass whereas an independent SUSY breaking parameter is assigned for Higgs fields. From this figure we see that the deviation from the SM of $\Delta M_{B_d}$ is reduced to $+15\%$ for the nonminimal case if we require that the lightest CP-even Higgs mass should be larger than 95 GeV for $\tan \beta = 2$. If the mass bound is raised to 105 GeV no allowed parameter space remains for $\tan \beta = 2$.

In Fig. 2 the allowed ranges of $\Delta M_{B_d}/\Delta M_{B_d}^{SM}$ are shown for several values of $\tan \beta$ with the present constraint of the SUSY Higgs boson search and Fig. 3 corresponds to the case where $m_h > 105$ GeV. We take into account both $e^+e^- \to Zh$ and $e^+e^- \to Ah$ processes to constrain the SUSY parameter space from the SUSY Higgs boson search. For $\tan \beta \gtrsim 10$ there appears a separate region where the sign of the $b \to s \gamma$ amplitude is opposite to that of the SM amplitude. In this region the SUSY contribution to the $b \to s \gamma$ amplitude is large and opposite in sign to the SM amplitude. Two cases are distinguished according to the sign of the $b \to s \gamma$ amplitude for $\tan \beta = 10, 20, 40$. We
Figure 3: The same figure as Fig.2 for the case that $m_h > 105$ GeV.

can see that the present constraint allows the deviation of the SM for the $B^0 - \bar{B}^0$ mixing up to 12% for the minimal case and 30% for the nonminimal case. If the Higgs boson bound is raised to 105 GeV, the allowed deviation from the SM becomes less than 15% except for the small parameter space where the sign of the $b \to s \gamma$ amplitude becomes opposite to the SM case. Note that in such parameter space the $b \to s l^+ l^-$ branching ratio is expected to be enhanced.

Deviations from the SM of $\epsilon_K$, $K_L \to \pi^0 \nu \overline{\nu}$, and $K^+ \to \pi^+ \nu \overline{\nu}$ are also studied. If $\epsilon_K$ is normalized by the SM value the deviation from the SM is essentially the same as that of $\Delta M_{B_d}$. On the other hand, the both $K \to \pi \nu \overline{\nu}$ branching ratios are reduced up to 5% for the minimal case and 15% for the nonminimal case with the present Higgs bounds and further reduced to 7% for $m_h > 105$ GeV in the nonminimal case.

These deviations may be evident in future when B factory experiments provide additional information on the CKM parameters. It is expected that the one of the three angles of the unitarity triangle is determined well through the $B \to J/\psi K_S$ mode. Then assuming the SM, one more physical observable can determine the CKM parameters or $(\rho, \eta)$ in the Wolfenstein parameterization. New physics effects may appear as inconsistency in the determination of these parameters from different inputs. For example, the $\rho$ and $\eta$ parameters determined from CP asymmetry of B decay in other modes, $\frac{\Delta M_{B_d}}{\Delta M_{B_s}}$ and $|V_{ub}|$
can be considerably different from those determined through \( \Delta M_{B_d} \epsilon_K \) and \( B(K \to \pi \nu \bar{\nu}) \) because \( \Delta M_{B_d} \epsilon_K \) are enhanced and \( B(K \to \pi \nu \bar{\nu}) \) is reduced but \( \frac{\Delta M_{B_s}}{\Delta M_{B_d}} \) and \( |V_{ub}| \) are essentially independent of SUSY contributions in this model. The pattern of these deviations from the SM will be a key to distinguish various new physics effects.

### 2.2 Muon anomalous Magnetic Moment in the Supergravity Model

The muon anomalous magnetic moment is a very precisely measured quantity. The present experimental value of \( a_\mu = (g - 2)_\mu / 2 \) is \( a_\mu^{\text{exp}} = 1165925.0(73.0) \times 10^{-10} \) \( [12] \). This is also one of most accurately calculated quantities within the SM. The SM prediction is \( a_\mu^{\text{SM}} = 11659109.6(6.7) \times 10^{-10} \) where the error in the SM value is dominated by the hadronic contribution of the vacuum polarization diagram \( [13, 14] \). Because the QED corrections are very precisely known the experimental accuracy is now approaching to the level of detecting possible new physics contributions at the electroweak scale. Combining the above two values we can derive a possible new physics contribution to \( a_\mu \) as

\[
a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (75.5 \pm 73.3) \times 10^{-10}.
\]  

The current BNL experiment is aiming to improve \( a_\mu \) by a factor of 20 and the first result is reported as \( a_\mu = 1165925(15) \times 10^{-9} \) \( [12] \). In the context of SUSY models it is known that there is sizable contributions to \( a_\mu \) (\( a_\mu^{\text{SUSY}} \)) from the loop diagrams with sneutrino and chargino and with charged slepton and neutralino. The SUSY effects are particularly important for large \( \tan \beta \) region. The \( a_\mu^{\text{SUSY}} \) was calculated in the MSSM \( [15] \) and as well as in the supergravity model \( [16, 17] \).

We calculated the SUSY contribution to \( a_\mu \) (\( a_\mu^{\text{SUSY}} \)) in the supergravity model discussed above. We require the radiative electroweak symmetry breaking condition and the various phenomenological constraints as before.

We first present \( a_\mu^{\text{SUSY}} \) in the minimal supergravity for \( \tan \beta = 10 \) as a function of \( B(b \to s \gamma) \) in Fig. 4. In this figure the present bound from the Higgs boson search is applied. The magnitude of \( a_\mu^{\text{SUSY}} \) becomes large for large \( \tan \beta \). The enhancement of \( a_\mu^{\text{SUSY}} \) for large \( \tan \beta \) comes from the fact that \( a_\mu^{\text{SUSY}} \) is dominated with the sneutrino-chargino loop diagram, which contains a contribution proportional to \( \mu \tan \beta \). As a result, SUSY contributions to both \( b \to s \gamma \) amplitude and \( a_\mu^{\text{SUSY}} \) are correlated with \( \text{sign}(\mu) \). We can see that \( a_\mu^{\text{SUSY}} \) becomes positive (negative) according to the suppression (enhancement ) of \( B(b \to s \gamma) \). This correlation was pointed out in the literature \( [17] \).

The predicted range of \( a_\mu^{\text{SUSY}} \) are shown for several values of \( \tan \beta \) with the present constraint on the Higgs search and with \( m_h > 105 \) GeV for the minimal
Figure 4: $\alpha^\text{SUSY}_\mu$ in the supergravity model for $\tan \beta = 10$. The square(dot) points correspond to the minimal (enlarged) parameter space of the supergravity model.

and the nonminimal cases in Fig. 5 and Fig. 6. Two cases according to the sign of the $b \to s \gamma$ amplitude are shown separately for $\tan \beta = 10, 20, 40$. This figures show that the muon anomalous magnetic moment is indeed expected to become very powerful to constrain the SUSY parameter space in near future. We see that even for $\tan \beta = 5$ the deviation is quite sizable considering future improvements on the $\alpha_\mu$ measurement.

3 $\mu^+ \to e^+\gamma$ and $\mu^+ \to e^+e^-e^-$ with Polarized Muons in SUSY GUT

3.1 P-odd and T-odd asymmetries in $\mu^+ \to e^+\gamma$ and $\mu^+ \to e^+e^-e^-$ processes

LFV processes such as $\mu^+ \to e^+\gamma$, $\mu^+ \to e^+e^-e^-$ and $\mu^- \to e^-\gamma$ conversion in atoms are another interesting possibility to search for SUSY effects through flavor physics. The current experimental upper bounds on these processes are $B(\mu^+ \to e^+\gamma) \leq 1.2 \times 10^{-11}$, $B(\mu^+ \to e^+e^-e^-) \leq 1.0 \times 10^{-12}$, and $\sigma(\mu^-Ti \to e^-Ti)/\sigma(\mu^-Ti \to \text{capture}) \leq 6.1 \times 10^{-13}$. Recently there are considerable interests on these processes because predicted branching ratios turn out to be close to the upper bounds in the SUSY GUT.

As discussed before no LFV is generated at the Planck scale in the context of the minimal supergravity model. In the SUSY GUT scenario, however, the LFV can be induced through renormalization effects on the slepton mass.
Figure 5: Allowed ranges of $a_{\mu}^{\text{SUSY}}$ for several values of $\tan \beta$ with the present constraint of the Higgs boson mass. Two lines are shown according to the sign of the $b \to s \gamma$ amplitude for $\tan \beta = 10, 20, 40$. The left (right) lines correspond the case where the sign of the $b \to s \gamma$ amplitude is same (opposite) as that in the SM.

Figure 6: The same figure as Fig.5 for the case that $m_h > 105$ GeV.
matrix because GUT interaction breaks lepton flavor conservation. In the minimal SUSY SU(5) GUT, the effect of the large top Yukawa coupling constant results in the LFV in the right-handed slepton sector. On the other hand in the SO(10) model both left- and right-handed sleptons induce LFV and the predicted branching ratio can be larger by two order of magnitudes. A similar enhancement is seen for SU(5) model with large tan $\beta$ when we take into account higher dimensional operators for the Yukawa couplings at the GUT scale to explain realistic fermion mass relations.

In this section we discuss $\mu^+ \rightarrow e^+ \gamma$ and $\mu^+ \rightarrow e^+ e^- e^-$ with polarized muons. Experimentally highly polarized $\mu^+$ beam is available because muons from decay of pions stopped at the target surface is initially 100% polarized opposite to the muon momentum direction. This muon beam is called surface muon. It was pointed out that polarized muon is useful to suppress background processes for $\mu^+ \rightarrow e^+ \gamma$ search because main background processes have particular angular distributions.

The muon polarization is also useful in identifying the nature of LFV interactions. Using angular distributions of final particles we can define asymmetries with respect to the initial muon polarization direction. For $\mu^+ \rightarrow e^+ \gamma$ we define a P-odd asymmetry which distinguishes between $\mu^+ \rightarrow e^+_L \gamma$ and $\mu^+ \rightarrow e^+_L \gamma$. For the $\mu^+ \rightarrow e^+ e^- e^-$ process we can define two P-odd asymmetries and one T-odd asymmetry. From these asymmetries we can obtain information on structure of LFV interactions. In particular we calculate branching ratios and asymmetries in the SU(5) and SO(10) SUSY GUT models and show that these asymmetries are useful to distinguish models.

Using the electromagnetic gauge invariance and the Fierz rearrangement the effective Lagrangian for $\mu^+ \rightarrow e^+ \gamma$ and $\mu^+ \rightarrow e^+ e^- e^-$ processes can be written without loss of generality:

$$\mathcal{L} = \frac{4G_F}{\sqrt{2}} \left( m_\mu A_R \sigma^\mu e_L F^\mu + m_\mu A_L \sigma^\mu e_R F^\mu + g_1 (\overline{\mu} e_L) (\overline{e_R} e_L) + g_2 (\overline{\mu} e_R) (\overline{e_L} e_R) + g_3 (\overline{\mu} \gamma^\mu e_R) (\overline{e_L} e_L) + g_4 (\overline{\mu} \gamma^\mu e_L) (\overline{e_R} e_L) + g_5 (\overline{\mu} \gamma^\mu e_R) (\overline{\mu} \gamma^\mu e_R) + h.c. \right),$$

where $G_F$ is the Fermi coupling constant and $m_\mu$ is the muon mass. The chirality projection is defined by the projection operators $P_R = \frac{1+\gamma_5}{2}$ and $P_L = \frac{1-\gamma_5}{2}$. $A_L(A_R)$ is the dimensionless photon-penguin coupling constant which contributes to $\mu^+ \rightarrow e^+_L \gamma$ ($\mu^+ \rightarrow e^+_R \gamma$). These couplings also induce the $\mu^+ \rightarrow e^+ e^- e^-$ process. $g_i$’s ($i = 1-6$) are dimensionless four-fermion coupling parameters.
constants which only contribute to $\mu^+ \to e^+ e^−$. $A_{L,R}$ and $g_i$ ($i = 1−6$) are generally complex numbers and calculated based on a particular model with LFV interactions.

The differential branching ratio for $\mu^+ \to e^+ \gamma$ is given by:

$$
\frac{d\mathcal{B}(\mu^+ \to e^+ \gamma)}{d \cos \theta} = 192\pi^2 \{ |A_L|^2 (1 + P \cos \theta) + |A_R|^2 (1 - P \cos \theta) \}
$$

(5)

$$
\frac{B(\mu^+ \to e^+ \gamma)}{2} (1 + A(\mu^+ \to e^+ \gamma) P \cos \theta),
$$

(6)

where the total branching ratio for $\mu^+ \to e^+ \gamma$ ($B(\mu^+ \to e^+ \gamma)$) and the P-odd asymmetry ($A(\mu^+ \to e^+ \gamma)$) are defined as

$$
B(\mu^+ \to e^+ \gamma) = \frac{384\pi^2}{11} (|A_L|^2 + |A_R|^2),
$$

(7)

$$
A(\mu^+ \to e^+ \gamma) = \frac{|A_L|^2 - |A_R|^2}{|A_L|^2 + |A_R|^2},
$$

(8)

and $P$ is the muon polarization.

Kinematics of the $\mu^+ \to e^+ e^−$ process with a polarized muon is determined by two energy variables of decay positrons and two angle variables which indicate the direction of the muon polarization with respect to the decay plane. In Fig.7 we take the $z$-axis as the direction of the decay electron momentum ($\vec{p}_e$) and the $z$-$x$ plane as the decay plane. Polar angles ($\theta, \varphi$) ($0 \leq \theta \leq \pi, 0 \leq \varphi < 2\pi$) indicate the direction of the muon polarization $\vec{P}$.

We take a convention that the decay positron having larger energy is named positron 1 and the other is positron 2 and $(p_1)_x \geq 0$.

We define two P-odd asymmetries and one T-odd asymmetry as asymmetries of numbers of events as follows.

$$
A_{P1} = \frac{N(P_x > 0) - N(P_x < 0)}{N(P_x > 0) + N(P_x < 0)},
$$

(9)

$$
A_{P2} = \frac{N(P_y > 0) - N(P_y < 0)}{N(P_y > 0) + N(P_y < 0)},
$$

(10)

$$
A_T = \frac{N(P_y > 0) - N(P_y < 0)}{N(P_y > 0) + N(P_y < 0)}.
$$

(11)

Here, the $\vec{P}$ is the muon polarization vector and in the above definition muons are assumed to be 100% polarized. Using the coupling constants in the effective Lagrangian $A_T$ is expressed by

$$
A_T = \frac{64}{35} B(\mu^+ \to e^+ e^−) \left\{ 3Im(e A_R g_4^* + e A_L g_5^*) - 2Im(e A_R g_6^* + e A_L g_5^*) \right\}
$$

(12)
Figure 7: Kinematics of the $\mu^+ \to e^+ e^+ e^-$ decay in the center-of-mass system of muon. The plane I represents the decay plane on which the momentum vectors $\vec{p}_1$, $\vec{p}_2$, $\vec{p}_3$ lie, where $\vec{p}_1$ and $\vec{p}_2$ are momenta of two $e^+$'s and $\vec{p}_3$ is momentum of $e^-$ respectively. The plane II is the plane which the muon polarization vector $\vec{P}$ and $\vec{p}_3$ make.
This means that in order to have a T odd asymmetry there should be a relative phase between the photon penguin diagram term ($A_R$, $A_L$) and four fermion terms ($g_3, g_4, g_5, g_6$). As we show in the explicit numerical calculation, we need to introduce a CP violating phase other than the KM phase in order to have sizable T-odd asymmetry in the SU(5) SUSY GUT. This phase is provided by the complex phases in the SUSY breaking terms, for example, the phase in the triple scalar coupling constant (A term). Since this phase also induces electron and neutron EDMs, we have calculated the T odd asymmetry in the $\mu^+ \to e^+ e^+ e^-$ taking into account EDM constraints.

3.2 LFV branching ratios and asymmetries in the SU(5) and SO(10) SUSY GUT

We present results of our numerical calculation for SU(5) and SO(10) SUSY GUT. We present following quantities as contour plots in the SUSY parameter space.

$$
\frac{B(\mu^+ \to e^+ \gamma)}{|\lambda_\tau|^2}, \quad \frac{B(\mu^+ \to e^+ e^+ e^-)}{|\lambda_\tau|^2}, \quad \frac{B(\mu^+ \to e^+ e^-)}{B(\mu^+ \to e^+ \gamma)}, \quad A(\mu^+ \to e^+ \gamma), \quad A_{P_1}, A_{P_2}, A_T.
$$

Here $\lambda_\tau$ is defined as $\lambda_\tau = (V_R)_{23}(V_R)^{13}$ and $V_R$ is the right-handed lepton mixing matrix in the basis that the slepton mass matrix is diagonalized. In the minimal version of SUSY GUT this mixing matrix is related to the CKM matrix at the GUT scale, however in models which can explain realistic fermion mass spectrum the simple relationship is lost. Therefore we treat $\lambda_\tau$ as a free parameter and consider the quantities listed above which are independent of this overall factor of the LFV amplitudes.

In Figs. 8 and 9 we show the above quantities in the plane of $m_{\tilde{e}_R}$ and $|A_0|$ for $\tan \beta = 3$, $M_2 = 300$ GeV in the SU(5) SUSY GUT. There are two independent CP violating phases in the soft SUSY breaking parameters which we take the phases of A term ($\theta_{A_0}$) and the $\mu$ term ($\theta_\mu$) at the Planck scale. In this figure we take $\theta_{A_0} = \frac{\pi}{2}$ and $\theta_\mu = 0$. The experimental bounds from the electron, neutron and Hg EDMs are also shown in each figure. Because these bounds are sensitive to the small change of $\theta_\mu$, we also show the parameter region which is not allowed even if we vary $\theta_\mu$ around 0.

We can see that for large parameter region the ratio of two branching fraction is enhanced. $A(\mu^+ \to e^+ \gamma)$ is close to 100% except for small region where the almost exact cancellation occurs. The two P-odd asymmetries can be large and depend on SUSY parameter space. $A_{P_1}$ changes from $-30\%$ to $40\%$ and $A_{P_2}$ changes from $-10\%$ to $15\%$. Within the allowed region of EDM constraints.
Figure 8: The observables in the SU(5) model with the SUSY CP violating phases in the $m_{\tilde{e}_R}-|A_0|$ plane. We fix the SUSY parameters as $\tan \beta = 3$, $M_2 = 300$ GeV, $\theta_{A_0} = \pi$ and $\theta_{\mu} = 0$ and the top quark mass as 175 GeV. (a) Branching ratio for $\mu^+ \rightarrow e^+\gamma$ normalized by $|\lambda_r|^2 \equiv |(V_R)_{23}(V_R)^*_{13}|^2$. (b) Branching ratio for $\mu^+ \rightarrow e^+e^+e^-$ normalized by $|\lambda_r|^2$. (c) The ratio of two branching fractions $\frac{B(\mu^+ \rightarrow 3e)}{B(\mu^+ \rightarrow \mu^+\gamma)}$. (d) The P-odd asymmetry for $\mu^+ \rightarrow e^+\gamma$. The experimental bounds from the electron, neutron and Hg EDMs are also shown in each figure. The left upper solid line corresponds to the electron EDM, the right upper solid line to the neutron EDM and the right lower solid line to the Hg EDM. The lower side of each bound is allowed by these experiments. The upper side of the bold line is excluded by the EDM bounds even if we allow $\theta_{\mu}$ taking slightly different value from 0.
Figure 9: Continued from the previous figure for the observables in the SU(5) model with the SUSY CP violating phases in the $m_{\tilde{e}_R}-|A_0|$ plane. (e) The $P$-odd asymmetries $A_{P1}$ for $\mu^+ \rightarrow e^+e^+e^-$. (f) The $P$-odd asymmetries $A_{P2}$ for $\mu^+ \rightarrow e^+e^+e^-$. (g) The $T$-odd asymmetry for $\mu^+ \rightarrow e^+e^+e^-$. 
constraints, the maximum value of $A_T$ is 15%. Note that in the SU(5) case only $g_3$, $g_5$ and $A_L$ can be sizable because the LFV is induced in the right-handed slepton sector as long as we take not too large $\tan \beta$ region. Using these asymmetries and branching ratios we can in principle determines these coupling constants up to an overall phase.

In Figs. 10 and 11 we show similar plots for SO(10) SUSY GUT case. The $\mu^+ \rightarrow e^+ \gamma$ asymmetry $A(\mu^+ \rightarrow e^+ \gamma)$ varies from $-20\%$ to $-90\%$. This is in contrast to the previous belief that $A_L$ and $A_R$ have a similar magnitude in this model. Although the diagram proportional to $m_\tau$ gives the same contribution to the $A_L$ and $A_R$, there is a chargino loop diagram which only contributes to $A_R$. In spite of no $m_\tau$ enhancement, the contribution from the latter diagram can be comparable to that from the former one, especially when the slepton mass is larger than the chargino mass. The P-odd asymmetries for $\mu^+ \rightarrow e^+e^+e^-$ are proportional to $A(\mu^+ \rightarrow e^+\gamma)$ for the SO(10) case. In fact we obtain approximate relations

$$A_{P_1} \sim -\frac{1}{10} A(\mu^+ \rightarrow e^+\gamma),$$

$$A_{P_2} \sim -\frac{1}{6} A(\mu^+ \rightarrow e^+\gamma).$$

For the branching ratio it is known that the following relation is satisfied since the photon-penguin diagram give dominant contributions.

$$\frac{B(\mu^+ \rightarrow e^+e^+e^-)}{B(\mu^+ \rightarrow e^+\gamma)} \sim 0.0062.$$ (16)

The above new relationship of the asymmetries also arises since the $\mu^+ \rightarrow e^+e^+e^-$ amplitude is dominated by two photon-penguin amplitudes ($A_R, A_L$). As we see in the figure the T-odd asymmetry is very small in the SO(10) model. This is also a consequence of the photon penguin dominance of the $\mu^+ \rightarrow e^+e^+e^-$ amplitude because the four-fermion amplitudes should be comparable in magnitude to the photon-penguin amplitude to get sizable T-odd asymmetry.

We have investigated the branching ratios and asymmetries of two processes with different SUSY parameters. Qualitative features are the same as in Figs. 8 - 11. Results can be summarized in Table 1 for SU(5) and SO(10) models.

4 Conclusions

In this talk we have discussed various processes sensitive to loop diagrams of SUSY particles. For the quark sector FCNC processes of $B$ and $K$ de-
Figure 10: The observables in the SO(10) model with the SUSY CP violating phase in $m_{\tilde{e}R}$-$|A_0|$ plane. The input parameters are same as in Figs. 8 and 9. The small $m_{\tilde{e}R}$ region bounded by the left bold line is not allowed in the minimal SUGRA model. The upper right bold line shows the bound from EDM constraints set in the same way as in Figs. 8 and 9.
Figure 11: Continued from the previous figure for the observables in the SO(10) model with the SUSY CP violating phase in $m_{\tilde{e}} - |A_0|$ plane.
SU(5) SUSY GUT | SO(10) SUSY GUT
---|---
$A(\mu^+ \to e^+\gamma)$ | +100% | -20% - -100%
$B(\mu^+ \to e^+e^+e^-)/B(\mu^+ \to e^+\gamma)$ | 0.007 - $O(1)$ | constant ($\sim 0.0062$)
$A_{P_1}$ | -30% - +40% | $\sim -\frac{1}{4} A(\mu^+ \to e^+\gamma)$
$A_{P_2}$ | -20% - +20% | $\sim -\frac{1}{4} A(\mu^+ \to e^+\gamma)$
$|A_T|$ | $\lesssim 15\%$ | $\lesssim 0.01\%$

Table 1: Summary of branching ratios and asymmetries for $\mu^+ \to e^+\gamma$ and $\mu^+ \to e^+e^+e^-$ processes in the SU(5) and SO(10) SUSY GUT.

Decays receive contributions from flavor mixing in the squark sector. The muon anomalous magnetic moment is sensitive to the slepton loop diagram. LFV processes such as $\mu^+ \to e^+\gamma$ and $\mu^+ \to e^+e^+e^-$ are induced if there are flavor off-diagonal terms in the slepton mass matrices.

We first updated the numerical analysis for FCNC processes in $B$ and $K$ decays and the muon anomalous magnetic moment in the supergravity model. Taking account of the recent progress in the Higgs boson search, we showed that a small $\tan\beta$ region is almost excluded for $\tan\beta \lesssim 2$. The maximal deviation from the SM value in the $B^0-B^0$ mixing is 12% for the minimal supergravity case and 30% for the nonminimal case. If the Higgs mass bound is raised to 105 GeV the deviation is further reduced. We also calculate the SUSY contribution to the muon anomalous magnetic moment. $a^\mu_{\text{SUSY}}$ and $B(b \to s\gamma)$ show a strong correlation and $a^\mu_{\text{SUSY}}$ becomes very large for a large $\tan\beta$ region. We find that the SUSY contribution $a^\mu_{\text{SUSY}}$ can be $(-30 - +80) \times 10^{-10}$ for $\tan\beta = 10$ for the minimal supergravity case. Along with the $B(b \to s\gamma)$ constraint, $a^\mu_{\text{SUSY}}$ will soon become a very important constraint on the parameter space in the supergravity model.

LFV processes in muon decay are particularly important in the SUSY GUT because the flavor mixing in the quark sector naturally induces LFV in the lepton sector through the interaction at the GUT scale. This effect can be observed through the renormalization effects on slepton mass matrices. The structure of the LFV effective Lagrangian at the muon scale reflects nature of the LFV interaction at the high energy scale.

In order to see how we can distinguish various terms in the effective Lagrangian we developed the model-independent formalism for $\mu^+ \to e^+\gamma$ and $\mu^+ \to e^+e^+e^-$ with polarized muon and defined convenient observables such as the P-odd and T-odd asymmetries. Using explicit calculation based on the SU(5) and SO(10) SUSY GUT, we show that various combination of LFV cou-
pling constants can be determined from the measurement of branching ratio and asymmetries. In the SO(10) case the P-odd asymmetry in $\mu^+ \rightarrow e^+\gamma$ varies from $-20\%$ to $-100\%$ whereas it is $+100\%$ for the SU(5) case. We can define two kinds of P-odd asymmetries of $\mu^+ \rightarrow e^+e^+e^-$. These asymmetries can be large and vary over SUSY parameter space for the SU(5) case so that they can be used to determine effective coupling constants. For the SO(10) case these two asymmetries are proportional to the P-odd asymmetry in $\mu^+ \rightarrow e^+\gamma$. We also calculated the T-odd asymmetry in the $\mu^+ \rightarrow e^+e^+e^-$ process with the SUSY CP violating phases and compare it with the neutron, electron and Hg EDMs. The T-odd asymmetry can reach $15\%$ within the constraints of the EDMs for the SU(5) case whereas it is very small for the SO(10) case. Thus these quantities are useful to distinguish different models.

The experimental prospects for measuring these quantities depend on the branching ratio. For the SO(10) model we expect the $\mu^+ \rightarrow e^+\gamma$ branching ratio can be $10^{-12}$ when the $\lambda_\tau$ is given by the corresponding CKM matrix elements. In such a case the $\mu^+ \rightarrow e^+\gamma$ asymmetry can be measurable in an experiment with a sensitivity of $10^{-14}$ level. For the SU(5) model, to get the $\mu^+ \rightarrow e^+\gamma$ branching ratio of order $10^{-12}$ and the $\mu^+ \rightarrow e^+e^+e^-$ branching ratio of $10^{-14}$, we have to assume $\lambda_\tau$ is larger than a several times $10^{-3}$. If the branching ratio turns out to be as large, the $\mu^+ \rightarrow e^+e^+e^-$ experiments with a sensitivity of $10^{-16}$ level could reveal various asymmetries. Because various asymmetries are defined with respect to muon polarization, experimental searches for $\mu^+ \rightarrow e^+\gamma$ and $\mu^+ \rightarrow e^+e^+e^-$ with polarized muons are very important to uncover the nature of the LFV interactions.

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