Electronic transport in torsional strained Weyl semimetals

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Abstract
In a recent paper (Muñoz and Soto-Garrido 2017 J. Phys.: Condens. Matter 29 445302) we have studied the effects of mechanical strain and magnetic field on the electronic transport properties in graphene. In this article we extended our work to Weyl semimetals (WSM). We show that although the WSM are 3D materials, most of the analysis done for graphene (2D material) can be carried out. In particular, we studied the electronic transport through a cylindrical region submitted to torsional strain and external magnetic field. We provide exact analytical expressions for the scattering cross section and the transmitted electronic current. In addition, we show the node-polarization effect on the current and propose a recipe to measure the torsion angle from transmission experiments.

Keywords: Weyl semimetals, strain, magnetic field, node-polarization, electronic transport, scattering

(Some figures may appear in colour only in the online journal)
references therein), this is not the case for WSM. As in the case of graphene, different types of elastic strains give rise to different types of induced gauge fields in WSM [13–15]. These induced gauge fields manifest themselves in the chiral anomaly effect [16], in the energy spectrum [17], in quantum oscillations in the absence of a magnetic field [18] and in the collapse of the Landau levels [19]. However, most of the analysis are done using tight-binding and numerical methods. In contrast, in this work we focus on the combined effects of torsional mechanical strain and an external magnetic field on the electronic transport properties in WSM, using an analytic procedure developed in [1] by ourselves. Despite its relative simplicity, our model is able to demonstrate the node-filtering effect that can be achieved in WSMs under combined torsional strain and an external magnetic field. Moreover, we illustrate how this effect could be used in practice for strain sensing in WSMs.

2. Model

In the massless case, the relativistic Dirac equation in three spatial dimensions (3D) can be written as two separate equations for two 2-component spinors of opposite helicity. The Weyl fermions are described by the $2 \times 2$ Hamiltonian:

$$\hat{H}_b = \pm c \sigma \cdot \hat{p},$$

(1)

where $\sigma = (\sigma_1, \sigma_2, \sigma_3)$ are the three Pauli matrices, $c$ is the speed of light and $\hat{p}$ is the momentum operator in the three-dimensional momentum space [15]:

$$\hat{H}_b = \begin{pmatrix} H_0^+ & 0 \\ 0 & H_0^- \end{pmatrix} = v_F \begin{pmatrix} \sigma \cdot \hat{p} & 0 \\ 0 & -\sigma \cdot \hat{p} \end{pmatrix}$$

(2)

where $v_F$ is the Fermi velocity. For instance, the physical values of $b$ and $v_F$ in TaAs [5] are $b \sim 0.08 \AA^{-1}$ and $v_F \sim 1.3 \times 10^{-5}$ m s$^{-1}$.

2.1. Strain

In a recent paper Arjona and Vozmediano [15] show the appearance of pseudo-gauge fields associated to the presence of torsional strain. Closely following [15] in this section we write the effective model for such a strain. In 3D the antisymmetric part of the deformation tensor is given by:

$$\omega_y = \frac{1}{2} (\partial_y u_t - \partial_t u_y) = \epsilon_{ijk} \Omega_{jk},$$

(3)

where $\Omega \equiv \frac{1}{2} (\nabla \times \mathbf{u})$. Let us focus now in the case of torsional strain applied to a cylindrical region of a WSM. Assuming that the two Weyl nodes are separated by a vector in the $z$-direction $\mathbf{b} = b \hat{z}$ and that the cylinder has a length $L$ in the $z$-direction, the displacement vector is given by [15]:

$$\mathbf{u} = \theta \frac{z}{L} (r \times \hat{z}) = \theta \frac{z}{L} (y \hat{x} - x \hat{y}).$$

(4)

This vector gives rise to:

$$\Omega = \frac{1}{2} (\nabla \times \mathbf{u}) = \frac{\theta}{2L} (\hat{x} \hat{y} - 2 \hat{z} \hat{z}),$$

(5)

where the vector potential associated to the deformation is given by:

$$\mathbf{A} = \mathbf{b} \times \Omega = \frac{\theta b}{2L} (-y \hat{x} + x \hat{y})$$

(6)

and the associated induced pseudo-magnetic field is therefore:

$$\mathbf{B}_s = (\nabla \times \mathbf{A}) = \frac{\theta b}{L} \hat{z}.$$  

(7)

Let us notice that the pseudo-magnetic field will have different signs at each of the two nodes [13, 20], while a real magnetic field has the same sign at both nodes.

Similar behavior can be observed in a different strain pattern as the one suggested by [19], where $u_x = u_0 (2xy + C)$, $u_y = u_0 [-(x^2 - Dz^2 + C)]$ and $u_z = 0$. Nevertheless, our analysis will focus on the azimuthally symmetric torsional strain. In addition to the torsional strain, we will also consider an external magnetic field that gives rise to the node-polarization effect on the electric current, similar to the valley-polarization effect observed in graphene [1].

3. Scattering through a cylindrical region with magnetic field and mechanical strain

We start by considering the problem of three-dimensional elastic scattering of an incident free spinor with momentum $\mathbf{k} = (k_x, 0, k_z)$ and energy $E_{\lambda k} = \lambda h v_F |\mathbf{k}|$, where $\lambda = \pm 1$ is the ‘band’ index and $\xi = \pm 1$ labels each of the Weyl nodes located at $K_x = \xi b / 2$. Since $k_z$ is a good quantum number, it is possible to decouple the $z-$direction from the plane and, therefore, most of the analysis done in [1] can be applied almost straightforwardly.

We consider a free fermion incident towards the cylindrical scattering center, described by the eigenvector of the equation

$$\hat{H}_0 \tilde{\psi}_{m,k}^{(\lambda, \xi)}(r, \phi, z) = \lambda h v_F |\mathbf{k}| \tilde{\psi}_{m,k}^{(\lambda, \xi)}(r, \phi, z),$$

(8)

which is given by the free spinor

$$\tilde{\psi}_{m,k}^{(\lambda, \xi)}(r, \phi, z) = \frac{1}{\sqrt{1 + (\lambda \xi |\mathbf{k}| - k_z)^2 / k_z}} \left( \begin{array}{c} 1 \\ \frac{1}{\xi \lambda |\mathbf{k}| - k_z} \end{array} \right) e^{i k_z z} e^{i \xi \lambda \phi - i k_z z}.$$  

(9)

We now proceed with the standard partial wave analysis for scattering [1, 21]. Using the symmetry around the $z-$axis, we choose as a quantum number the eigenvalue $\beta_m$ of the total angular momentum operator in the $z-$direction ($J_z = L_z + \sigma_3 / 2$) to label the solutions:

$$\tilde{\psi}_{m, k}^{(\lambda, \xi)}(r, \phi, z) = r^{-1/2} \left( \begin{array}{c} f_m(r) e^{i (m - 1/2) \phi} \\ -i g_m(r) e^{i (m + 1/2) \phi} \end{array} \right) e^{i k_z z}.$$  

(10)
Closely following [1] we found that:

\[ f_m(r) = c_1 \sqrt{k_x r} Y_{m+1/2}(k_x r) + c_2 \sqrt{k_x r} Y_{m-1/2}(k_x r), \]
\[ g_m(r) = c_3 \sqrt{k_x r} J_{m+1/2}(k_x r) + c_4 \sqrt{k_x r} Y_{m+1/2}(k_x r), \]

(11)

where

\[ c_3 = -\frac{k_x}{k_z + \lambda \xi |k|} c_1, \quad c_4 = -\frac{k_x}{k_z + \lambda \xi |k|} c_2. \]

(12)

In addition, as in standard elastic scattering theory [21], the phase shift captures the effect of a scattering region over the transmitted particle waves.

In order to calculate the phase shift \( \delta_n \) associated to each angular momentum channel \( m \equiv m_j - 1/2 \), it is necessary to match each spinor component of the free solution equation (10), and its first derivative, to the solution inside the region submitted to the effective magnetic field \( B_\xi = B_0 + \xi B_5 \) (where \( B_0 \) is the external magnetic field and \( B_\xi = \theta b_0/L \) is the pseudo-magnetic field induced by the torsional strain, equation (7)) at the boundary of the cylinder \( r = a \). The spectrum inside the cylindrical region corresponds to relativistic Landau levels, as seen in [1], with an effective magnetic field that is node-dependent,

\[ E_\xi(n) = \hbar v_F \sqrt{2n|B_\xi|/\phi_0 + k_z^2}, \]

(13)

with \( \phi_0 = h v_F / e \) the flux quantum defined in terms of the Fermi velocity in the WSM. As it is explained in detail in [1], one can find an exact analytical expression for the phase shift \( \delta_m \),

\[ \tan \delta_m = \frac{c_2}{c_1} \frac{J_{m+1}(k_x a) + \lambda \xi k_z |k| Y_{m+1}(k_x a)}{Y_{m+1}(k_x a) + \lambda \xi k_z |k| J_{m+1}(k_x a)} \frac{\sqrt{|m - m_j|/|2m_j|} \sqrt{|m - m_j|/|2m_j|}}{\sqrt{|m - m_j|/|2m_j|} \sqrt{|m - m_j|/|2m_j|}}. \]

(14)

where we have defined \( m \equiv m_j - 1/2 \).

### 3.1. Scattering cross section

Far from the cylindrical region where the effective magnetic field is present \( (r \gg a) \), the state is a linear combination of the incident and scattered spinor:

\[ \tilde{\Psi}_{out}(r, \phi, z) \sim \sqrt{\frac{1}{1 + \left( \frac{\lambda \xi |k| - k_z}{k_z} \right)^2}} \left( \frac{1}{|\lambda \xi |k| - k_z} \right)^{1/2} e^{ik_z r \cos \phi + ik_r z} + \left( f_1(\phi) \right) e^{ik_r r + ik_z z} \frac{1}{\sqrt{r}}, \]

(15)

with amplitudes \( f_1(\phi) \) and \( f_2(\phi) \) for each component of the scattered spinor. In the same region, we have that this expression must be equal to the asymptotic form of the solution, represented in terms of phase shifts as explained in [1]. We find the following scattering amplitudes:

\[ f_1(\phi) = \frac{e^{-i \phi/2}}{\sqrt{2 \pi k_F}} \sum_m \left( \frac{m \omega_{\phi} - \lambda \xi k_z}{\xi k_z} \right) \left( e^{2i \phi} - 1 \right). \]

(16)

For a very long cylinder, \( L \gg 1/k_F \), the differential scattering cross-section per unit length is given by the modulus of the vector above,

\[ \frac{d\sigma}{d\phi} = |f_1(\phi)|^2 + |f_2(\phi)|^2 = \frac{2}{\pi k_F} \left| k + \lambda \xi k_z \right| \left| k - \lambda \xi k_z \right| \sum_{m=-\infty}^{\infty} \sin^2 \delta_m e^{im(\phi - \phi_0)}, \]

(17)

and the total scattering cross section is then given by integrating over the scattering angle \( \phi \) (\( 0 \leq \phi \leq 2\pi \)) and over the length of the cylinder

\[ \sigma = \int_0^L dz \int_0^{2\pi} d\phi \frac{d\sigma}{d\phi} = L \int_0^{2\pi} \left( |f_1(\phi)|^2 + |f_2(\phi)|^2 \right) d\phi = 4 L \left| k + \lambda \xi k_z \right| \left| k - \lambda \xi k_z \right| \sum_{m=-\infty}^{\infty} \sin^2 \delta_m. \]

(18)

Notice that the assumption of a very long cylinder, \( L \gg 1/k_F \), is well founded. For instance, in TaAs where \( b \sim 0.08 \text{Å}^{-1} \) and \( v_F \sim 1.3 \times 10^5 \text{m s}^{-1}, 1/k_F \sim 9 \text{Å} \), so even a slab of a few microns is already in the range of validity of our expressions. Moreover, for Cd_{0.5}As_{0.5}, \( b \sim 0.2 \text{Å}^{-1} \) and \( v_F \sim 1.5 \times 10^6 \text{m s}^{-1}, 1/k_F \sim 0.8 \text{Å} \) [22] and hence the applicability of our expressions is even more striking in this second example.

### 4. Transmission and Landauer ballistic current

Let us consider a slab of a WSM of high \( L \) and width \( W \) (y-direction), which is connected to two semi-infinite WSM contacts held at chemical potentials \( \mu_L \) and \( \mu_R \), respectively. In addition, we consider that inside a cylindrical region of radius \( a \) a uniform magnetic field in the \( z \)-direction (along the cylinder axis) is applied (\( B_\xi \)). Moreover, we assume that in the same cylindrical region a torsional mechanical strain is applied, giving rise to an induced pseudo-magnetic field in the \( z \)-direction as it was mentioned in section 2.1. Using the Landauer ballistic approach, the net current along the slab (x-direction) is given by the net counterflow of the particle currents emitted from the left and right semi-infinite WSM contacts, respectively. Each contact is assumed to be in thermal equilibrium, with the Fermi–Dirac distributions \( f(E - \mu_j, T) \equiv f_L(E) \) and \( f_R(E) \equiv f(E - \mu_R, T), \)
corresponds to contacts, respectively, is defined as $T_0 = \theta = 0$ (for fixed $B_0 a^2 = 22 \delta_0$, and different values of the torsion angle $\theta$, and effective cross-section over charge transport can be expressed as an

\[ \text{Current (in units of } eV/a) = \text{Current (in units of } \hbar v_F/a), \text{ for fixed } B_0 a^2 = 25 \delta_0 \text{ and different values of the torsion angle } \theta. \]  

The solid (blue) line corresponds to $\theta = 0^\circ$ ($B_0 a^2 = 6.8 \delta_0$), the dotted (red) line corresponds to $\theta = 5^\circ$ ($B_0 a^2 = 13.6 \delta_0$) and the dashed (orange) line corresponds to $\theta = 15^\circ$ ($B_0 a^2 = 20.4 \delta_0$), with $\delta_0 \equiv \hbar v_F/e$. The subfigures (a)–(c) correspond to the different values of the temperature, $T = 0$, $T = 0.2 \hbar v_F/(k_B a)$ and $T = 0.4 \hbar v_F/(k_B a)$ respectively.

respectively. A pictorial description of the system is shown in figure 1.

The particle flux emitted by the left (L) and right (R) contacts, respectively, is defined as

\[ dJ_{L/R} = v_F D_{L/R}(E) f_{L/R}(E) dE, \]  

where $D_{L/R}(E)$ is the density of states at each contact.

The effect of the external magnetic field and the pseudomagnetic field over charge transport can be expressed as an effective cross-section $WLT_\xi(E, \phi)$, with $T_\xi(E, \phi)$ the transmission coefficient in the direction specified by the polar angle $\phi$, for an incident spinor arising from the node $K_\xi$. We thus define the effective cross-section in the $\phi$-direction by:

\[ WLT_\xi(E_\phi, \phi) = \sum_{n, \lambda} \frac{L}{\sigma(k)} \frac{d\sigma(k)}{d\phi} \delta \left( \lambda |k| - \frac{E_\phi(n)}{\hbar v_F} \right), \]  

with $E_\phi(n)$ the energy spectrum inside the cylindrical region, as defined in equation (13). We will take from now on that the incident spinor has a wave vector $k = (k_x, 0, 0)$, i.e. a plane wave propagating in the $x-$direction.

The particle flow (per unit time) along the $x$-direction emitted by the left $L$ (right $R$) contact and arising from the $K_\xi$ node can be written as

\[ dN_{\xi,L/R} = WLT_\xi(E, \phi) d\phi dJ_{L/R}(E), \]  

where $v_F = v_F \cos \phi$. The net electric current flowing across the region will be $I = I_+ + I_-$, with the node component given by

\[ I_\xi = \sum_{n, \lambda} \int_{k} \sigma(k) \frac{d\sigma(k)}{d\phi} \delta \left( \lambda |k| - \frac{E_\phi(n)}{\hbar v_F} \right) d\phi. \]  

Figure 2. Current (in units of $eV/a$) calculated from the analytical equation (23), as a function of applied bias $V$ (in units of $\hbar v_F/a$), for fixed $B_0 a^2 = 22 \delta_0$ and different values of the torsion angle $\theta$. The solid (blue) line corresponds to $\theta = 0$ ($B_0 a^2 = 0$), the dotted (red) line corresponds to $\theta = 5^\circ$ ($B_0 a^2 = 6.8 \delta_0$), the dot-dashed (green) line corresponds to $\theta = 10^\circ$ ($B_0 a^2 = 13.6 \delta_0$) and the dashed (orange) line corresponds to $\theta = 15^\circ$ ($B_0 a^2 = 20.4 \delta_0$), with $\delta_0 \equiv \hbar v_F/e$. The subfigures (a)–(c) correspond to the different values of the temperature, $T = 0$, $T = 0.2 \hbar v_F/(k_B a)$ and $T = 0.4 \hbar v_F/(k_B a)$ respectively.

Figure 3. Current (in units of $eV/a$) calculated from the analytical equation (23), as a function of applied bias $V$ (in units of $\hbar v_F/a$), for fixed $B_0 a^2 = 25 \delta_0$ and different values of the torsion angle $\theta$. The solid (blue) line corresponds to $\theta = 0$ ($B_0 a^2 = 0$), the dotted (red) line corresponds to $\theta = 5^\circ$ ($B_0 a^2 = 6.8 \delta_0$), the dot-dashed (green) line corresponds to $\theta = 10^\circ$ ($B_0 a^2 = 13.6 \delta_0$) and the dashed (orange) line corresponds to $\theta = 15^\circ$ ($B_0 a^2 = 20.4 \delta_0$), with $\delta_0 \equiv \hbar v_F/e$. The subfigures (a)–(c) correspond to the different values of the temperature, $T = 0$, $T = 0.2 \hbar v_F/(k_B a)$ and $T = 0.4 \hbar v_F/(k_B a)$ respectively.
\begin{equation}
I_\xi = e \int \left( dN_{\xi,L} - dN_{\xi,R} \right)
= e V F L \int_{-\infty}^{\infty} \frac{dE}{|E|} \left( D_L(E) f_L(E) - D_R(E) f_R(E) \right) T_\xi(E).
\end{equation}

Here, we have defined the net transmission coefficient for Dirac spinors at node \( \mathbf{K}_\xi \) as the angular average 
\[ T_\xi(E) = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} d\phi \cos \phi T_\xi(E, \phi). \]

Assuming that both contacts are identical semi-infinite regions, the density of states are equal, and given by
\begin{equation}
D_L(E) = D_R(E) = \frac{2}{\pi|E|} |E| \theta(|E|),
\end{equation}
where the factor of 4 arises from the spin and node degeneracy at each of the WSM semi-infinite contacts. With this consideration, the expression for the node component of the current \( I_\xi \) becomes
\begin{equation}
I_\xi = \frac{8e}{\pi^2} \sum_{\lambda,\sigma,m,p} \frac{(-1)^{p+1}}{E_\lambda^2(n)} \left[ f_\lambda(E_\lambda^\pm(n)) - f_\sigma(E_\sigma^\pm(n)) \right],
\end{equation}
with the total current given by \( I = I_+ + I_- \).

5. Results and discussion

In analogy to the case of graphene discussed in [1], we calculate in this section the total current \( I = I_+ + I_- \) (in units of \( eV/F \)) equation (23) for the two nodes components \( I_\xi \), at zero and finite temperatures (figures 2(a)–(c), 3(a)–(c) and 4(a)–(c)), for the particular choice of contact chemical potentials \( \mu_L = eV \) and \( \mu_R = 0 \).

Pikulin and collaborators [16] estimated the value of the induced magnetic field \( B_S \approx 1.8 \times 10^{-3} \) T per angular degree of torsion for a cylinder of length \( L \approx 100 \) nm and radius \( a \approx 50 \) nm. In addition, we define the scaled flux quantum \( \tilde{\phi}_0 \equiv \frac{h}{e} = \frac{1}{2} \frac{\pi}{L} = \frac{4}{1.5604} \cdot 4.14 \times 10^{-5} \text{ T}^{-2} \approx 330 \text{ T}^{-2} \).

Using these values we can write:
\begin{equation}
B_5 a^2 = 1.36 \theta \tilde{\phi}_0,
\end{equation}
where \( \theta \) is the torsion (twist) angle in degrees. In the following, we plot the current and conductance for different values of the torsion angle \( \theta \) and different values of the external magnetic field.
and different values of the torsion angle and for a fixed and controlled value of the external magnetic field. The quasi-continuum distribution of energy values in the contacts allows for this condition to be always fulfilled, for an interval within the window imposed by one of the eigenstates in the cylindrical region submitted to the effective magnetic field. The quasi-continuum distribution of energy values in the contacts allows for elastic scattering (see equation (20)) restriction, which gives the condition that for the incident particle to be transmitted across the scattering region its energy must be resonant with one of the eigenstates in the cylindrical region submitted to the effective magnetic field. The quasi-continuum distribution of energy values in the contacts allows for this condition to be always fulfilled, for an interval within the window imposed by the external bias voltage. Although the plateaus can still be seen at low finite temperature (see figure 3(b)), they are smoother than at zero temperature and they eventually disappear at high enough temperatures, as can be appreciated in figure 3(c).

In figure 2 at \(B_0 a^2 = 22.5 \tilde{\phi}_0\), figure 3 at \(B_0 a^2 = 25 \tilde{\phi}_0\) and figure 4 at \(B_0 a^2 = 28 \tilde{\phi}_0\), we compare the effect of the external magnetic field \(B_0\) on the current–voltage curves. At zero and finite temperatures, one can observe that for the same values of torsion angle, i.e. \(\theta = (5^\circ, 10^\circ, 15^\circ)\), the total current decreases as the external magnetic field is increased from \(B_0 a^2 = 22 \tilde{\phi}_0\) (in figure 2) towards \(B_0 a^2 = 28 \tilde{\phi}_0\) (in figure 4). This effect can be understood looking at the density of the pseudo-Landau level spectrum at the nodes. For a fixed value of \(B_0\) the effective pseudomagnetic field \(|B_\perp| = |B_0 + \xi B_S|\) increases as \(B_0\) increases (we are taking \(B_0 > B_S\) in figures 2–4). This produces a lower spectral density at the nodes \(E_{n}^{(\perp)} \sim \sqrt{|B_\perp|} n\), giving less channels to the incident particles to be transmitted and decreasing the total current \(I\) across the contacts.

Another feature of the current–voltage curves that is important to mention is the relative enhancement of the node-polarized contribution arising from the \(K_-\) node, as can be clearly observed in figure 5, where the two nodes components \(I_+\) and \(I_-\) of the total current are represented at finite temperature. This effect is even stronger when \(B_0\) is closer to \(B_0\) and can be understood by noticing that the effective pseudomagnetic field \(|B_\perp| = |B_0 - B_S|\) decreases as \(B_S\) increases, thus leading to a higher spectral density associated to the \(K_-\)-node where \(E_{n}^{(\perp)} \sim \sqrt{|B_\perp|} n\). This gives more channels to the incident particles to be transmitted per finite interval of bias voltage imposed at the contacts, thus increasing the current \(I_+\) associated to the \(K_-\)-node. This effect can also be appreciated at finite temperatures, as seen in figure 5, and it could be used to design a node-sensitive filter.

The differential conductance \(G(V,T) = dI/dV\) (in units of \(e^2/h\)) at finite temperature \(T = 0.1 \hbar v_F/(k_B a)\) is displayed in figure 6(a) for different values of the torsion angle \(\theta\) and fixed \(B_0 = 22.5 \tilde{\phi}_0/a^2\). A characteristic trend of oscillations is observed, which are consistent with the staircase behavior of the current observed in figures 2–4.

### 5.1 Torsional strain sensing

As it was explained in [1] for the case of graphene, the dependence of the current–voltage characteristics on the magnitude of the torsional strain, could be in principle used to design a piezoelectric sensor in WSM. The recipe to determine the torsion angle starts by experimentally measuring the differential conductance \(G = dI/dV\) for a fixed and controlled value of...
the external magnetic field $B_0$, as displayed in figure 6(a). As aforementioned, the plateaus in the current, and hence the peaks in the conductance, arise from the bias voltage window imposed by two consecutive Landau levels (mainly from the $K_\parallel$-node), i.e., $\Delta V = eV_{n+1} - eV_n \sim E_{n+1}^{(-)} - E_n^{(-)} = \frac{\hbar v}{2} \sqrt{2|B_0 - a^2/\phi_0|} (\sqrt{n + 1} - \sqrt{n})$, where in the equation we measure energies in units of $h/e^2$ and magnetic field in units of $\phi_0/a^2$. Therefore, by reading the locus of two consecutive peaks $V_{n+1} > V_n$ in the conductance curve (see figure 6(b) for an example), it is possible to extract the value of the corresponding integer $n$ from the ratio:

$$V_{n+1} \sim E_n^{(-)} / E_{n+1}^{(-)} = \sqrt{1 + \frac{1}{n}} \Rightarrow n = \left\lfloor \left( \frac{V_{n+1}}{h/e^2} \right)^2 - 1 \right\rfloor^{-1}.$$ (25)

Here, the symbol $\lfloor x \rfloor$ represents the nearest integer to $x$. With the value of $n$, one can solve for the effective pseudo-magnetic field:

$$|B_n| = \left( \frac{E_n^{(-)} / (h/e^2) \phi_0/a^2}{2n} \right)^2 \phi_0/a^2 \sim \left( \frac{eV_n / (h/e^2) \phi_0/a^2}{2n} \right)^2 \phi_0/a^2 = |B_0 - B_n|.$$ (26)

As a concrete example, let us take the values in figure 6(b). We have that $n \approx \lfloor (7.3/5.9)^2 - 1 \rfloor = [1.88] = 2$ and from equation (26) we have that $B_0 - B_2 = 5.92 / (2 \cdot 2) \phi_0/a^2 = 8.7 \phi_0/a^2$. Using that $B_2 a^2 = (22.5 - 8.7) \phi_0 = 13.8 \phi_0$. Using equation (24), this pseudomagnetic field corresponds to $\theta \approx 10.1^\circ$, which is within 1% of the value ($\theta = 10^\circ$) used to generate the conductance curve in figure 6(b) in the first place. This simple procedure can be applied in general, and used to read off the effective torsional angle from the conductance curves.

6. Conclusions and summary

We studied the electronic transport in WSM submitted to torsional strain and an external magnetic field. For this purpose we consider a cylindrical region embedded in a bulk WSM connected to two semi-infinite regions that act as reservoirs held at a different bias voltage. Using the partial wave method of scattering theory [21] and the analytical method developed in [1], we find analytical expressions for the transmission and Weyl node current components through the cylindrical region. Our analytical results predict a node-polarization effect on the current (analogous to the valley-polarization effect observed in graphene [23, 24]), due to the combined effect of the local torsional strain field and the externally imposed magnetic field. This is due to the asymmetry on the spectral density arising from the two different nodes $K_x$. We remark that the node filtering effect is a consequence of the symmetry breaking between the two nodes induced by the linear combination of the strain pseudomagnetic field and the $z$-component of the external magnetic field, that have different relative signs at each node: The physical magnetic field breaks time-reversal symmetry, whereas the strain field does not. Therefore, the node-filtering effect is robust in different geometries, and is not only a property of the cylindrical configuration chosen for the explicit analytical solution provided in our work.

As in the case of graphene [1], the results obtained in this paper could be used for the construction of a torsional strain sensor for WSMs. To the best of our knowledge, there are no reports in the literature about experimental measurements of torsional strain in WSM. We assume that this is due to the technical difficulties to assess small spatial displacements on the scale of a few lattice constants of the material. However, as it was explained in detail in the previous section, our theoretical results suggest that by performing electronic conductance measurements the magnitude of such strain could in principle be inferred, opening the possibility to develop a torsional strain-meter in WSMs.

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References

[1] Muñoz E and Soto-Garrido R 2017 J. Phys.: Condens. Matter 29 445302
[2] Weyl H 1929 Z. Phys. 56 330
[3] Hirata K S, Kajita T, Koshiha M, Nakahata M, Ohara S, Oyama Y, Sato N, Susuki A, Takita M and Totsuka Y 1988 Phys. Lett. B 205 416
[4] Armitage N P, Mele E J and Vishwanath A 2018 Rev. Mod. Phys. 90 015001
[5] Hasan M Z, Xu S Y, Belopolsky I and Huang S M 2017 Annu. Rev. Condens. Matter Phys. 8 289
[6] Yan B and Felser C 2017 Annu. Rev. Condens. Matter Phys. 8 337
[7] Burkov A 2018 Annu. Rev. Condens. Matter Phys. 9 359
[8] Wan X, Turner A M, Vishwanath A and Savrasov S Y 2011 Phys. Rev. B 83 205101
[9] Huang L et al 2016 Nat. Mater. 15 1155
[10] Morimoto T and Nagaoa N 2016 Phys. Rev. Lett. 117 146603
[11] Amorim B et al 2016 Phys. Rep. 617 1
[12] Naumis G G, Barraza-Lopez S, Oliva-Leyva M and Terrones H 2017 Rep. Prog. Physics 80 096501
[13] Cortijo A, Ferreiros Y, Landsteiner K and Vozmediano M A H 2015 Phys. Rev. Lett. 115 177202
[14] Cortijo A, Kharzeev D, Landsteiner K and Vozmediano M A 2016 Phys. Rev. B 94 241405
[15] Arjona V and Vozmediano M A 2017 arXiv:1709.02394
[16] Pikulin D, Chen A and Franz M 2016 Phys. Rev. X 6 041021
[17] Grushin A G, Venderbos J W, Vishwanath A and Ilan R 2016 Phys. Rev. X 6 041046
[18] Liu T, Pikulin D and Franz M 2017 Phys. Rev. B 95 041201
[19] Arjona V, Castro E V and Vozmediano M A 2017 Phys. Rev. B 96 081110
[20] Cortijo A, Ferreiros Y, Landsteiner K and Vozmediano M A 2016 2D Mater. 3 011002
[21] Sakurai J I and Napolitano J 2017 Modern Quantum Mechanics (New York: Cambridge University Press)
[22] Neupane M et al 2014 Nat. Commun. 5 3786
[23] Low T and Guinea F 2010 Nano Lett. 10 3551
[24] Settnes M, Power S R, Brandbyge M and Jauho A P 2016 Phys. Rev. Lett. 117 276801