Electromagnetic radiation by electrons in corrugated graphene

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Electrons motion through the rippled graphene sheet induces an electromagnetic radiation. The mechanism of formation of the bremsstrahlung radiation in graphene is similar to one in the undulator or wiggler. While the electron trajectory in an undulator and wiggler deviates from the direct line due to the periodic system of the dipole magnets, electron trajectory in graphene is getting curved due to the ripples. A nature of undulations in the monolayer graphene are discussed. We analyzed radiation using a few distinct models. At first model we consider direct geometrical effect of ripples, at the second – the gauge field effect. Both regular and random ripple structures are considered. Our results can be useful in experimental observation of graphene inhomogeneities and in making electromagnetic radiation generators of infrared and terahertz bands on graphene.

I. INTRODUCTION

The recent experimental study has shown that a flat geometry of graphene is unstable that leads to forming of corrugations: topological defects and ripples [1]. A presence of these inhomogeneities results in modification of the electronic spectrum [2] and to rearrangement of the vibrational states. Nonequilibrium effects of the electron-lattice interaction in the presence of ripples were analyzed in [3]. Our aim is to study the electromagnetic radiation of the corrugated graphene in the presence of the transport electric current [4] in the ballistic regime. A spatial period of ripples in graphene could reach several hundreds of nanometers [1]. This makes the semiclassical approach feasible. Two of neglected quantum effects are most important: Zitterbewegung and the Wannier-Stark ladder quantization. The former becomes apparent, when the Fermi level is situated just near the Dirac point, while the latter appears, when the electron has enough time to cross the Brillouin zone ballistically at least once. We are planning to consider these effects elsewhere. Also, despite of the "ultrarelativistic" character of spectrum, the ratio of Fermi velocity to the speed of light is much smaller, than unity and we can neglect a retardation of the electromagnetic radiation. All these points give us a reason to consider a motion of electrons in graphene within the classical approach.

An electron emits the electromagnetic waves, when it moves with nonvanishing acceleration. The most important mechanisms are bremsstrahlung, cyclotron, and undulator radiation [5]. We investigate here the electromagnetic waves emission by the electrons moving in the corrugated graphene sheet. The emission mechanism under the consideration resembles one in the undulator but practically without retardation.

II. RIPPLES

Graphene can be viewed as a crystalline membrane. An ideal flat 2D-crystal could not exist at a finite temperature [6], and the long range order in the graphene takes place because of ripples and topological defects. Both of these factors facilitate to achieve thermodynamical stability in graphene [7] and they could be induced to relief the strain of the membrane. Equilibrium state of the membrane is determined by the effective potential in the Monge representation [8]:

$$F_{eff} = \int d^2 x \left[ \frac{\kappa_0}{2} (\Delta_2 h)^2 + \frac{K_0}{8} \left( P_{ij}^T \partial_i h \partial_j h \right)^2 \right],$$

where $\kappa_0$ is the bare membrane bending rigidity, $\Delta_2$ is the two-dimensional Laplacian, $K_0$ is the Gaussian elastic constant (Young’s module):

$$K_0 = \frac{4\mu (\mu + \lambda)}{2\mu + \lambda},$$

$\mu, \lambda$ are Lamè coefficients, $P_{ij}^T = \delta_{ij} - \partial_i \partial_j / \Delta_2$ is the transverse projection operator. Averaging over the in-plane elastic modes within the one-loop approximation, we obtain the renormalized bending rigidity [8]:

$$\kappa(L) = \kappa_0 - \frac{3k_BT}{4\pi} \log \frac{L}{a},$$

where $a$ is the lattice spacing, $L$ is the sample size. Strong two-dimensional fluctuations evidently lead to the flat state instability, but it is common opinion that ripples are induced by non-linear effects. However, only crossover at
$q_0 \sim \sqrt{K_0 k_B T} \cong 0.08 \text{Å}^{-1}$ is obtained up to now. This magnitude correlates with the observed ripples size, but no ripple-like solution is presented [9]. As a base for discussion we can suggest a hypothesis that fluctuations make the modulus $\kappa$ to be negative: the flat state is getting unstable, stability of the created inhomogeneous state is provided by nonlinear terms similarly to the case of incommensurate ferroelectrics [10]. The ripple state characteristic length is determined by the play of the constants $|\kappa|$ and $K_0$.

### III. GEOMETRIC MECHANISM

Let us estimate the ratio of the radiation intensity to the thermal fluctuations. At first it is necessary to write the radiation intensity induced by electron using the Larmore formula [11]:

$$P_e = \frac{2}{3} \frac{e^2 \langle (v)^2 \rangle}{c^3} \sim \frac{2}{3} \frac{e^2 v_F^2 \omega^2}{c^3},$$

(4)

where $v_F$ is the Fermi velocity, and $\omega$ is the radiation frequency, which belongs to the terahertz band. The total radiated power density from graphene can be calculated as a product of single electrons power $P_e$ by charge carries density in the sample.

The power of thermal fluctuations reads

$$P_f = k_B T \Delta \omega = k_B T \omega \frac{\Delta \omega}{\omega},$$

(5)

where $\Delta \omega$ is the half-width of the radiation spectrum. The ratio of these powers is

$$\frac{P}{P_f} = \frac{2}{3} \frac{n}{c^3 h} \frac{\omega}{k_B T} \frac{\omega}{\Delta \omega},$$

(6)

where $k_B$ is the Boltzmann constant and $n$ is the electron gas density. Taking characteristic magnitudes for the ripples parameters and reasonable sizes of graphene sample, we estimate the ratio to be of order unity for the spectrum width $\Delta \omega \sim 10^{-13} \text{s}^{-1}$.

The Fermi velocity vector in the vicinity of the Dirac points in graphene has the constant absolute value of $10^8 \text{cm/s}$. However, its orientation changes with time leading to a time dependence of the vector components that is responsible for emission of the electromagnetic waves. In the next sections we will consider various models of electrons motion in corrugated graphene and will derive formula for the radiation intensity.

The ripple average amplitude is about 1 nm [1], whereas period could reach submicron values. Taking this into account, we could assume that velocity preserves a constant value in the direction of applied field. In other words, electrons in graphene could be considered as an oscillator with the Fermi velocity.

The real graphene sample has random corrugations period that stand out a little from mean value. The chaotic surface could be considered as a superposition of sinusoids with their own period and height. This approach widely used in the wave analysis in radiophysics. In order to describe the random process of electromagnetic radiation with limited spectrum we introduce the random function $h(x)$ that plays a role of the ripples height

$$v_z = v \frac{d}{dx} h(x).$$

(7)

It is clear that there is just one component, which is responsible for electromagnetic waves emission. The in-plane velocity components have constant values that makes $A_z$ to be the only component of the radiation vector potential. The Fourier transform of the out-of-plane vector potential component reads [11]

$$A_z = \frac{e v_F \exp (ikr)}{c r} \int_0^\infty dt \frac{dh(x)}{dx} \exp [i\omega t].$$

(8)
To find the radiation power in a selected point we have to calculate the square of the magnetic field absolute value in this point. Distribution of random function $h(x)$ could be considered as realizations, that could differ even in the same point. Therefore, while calculating the field intensity, two different realizations of $h(x)$ appear. The relation between them could be represented in terms of the correlation function, which can be obtained averaging the equations

$$
\langle |H_y|^2 \rangle = \left( \frac{e k_x}{c} \right)^2 \int_0^\infty dx \int_0^\infty dx' \frac{\partial h(x)}{\partial x} \frac{\partial h(x')}{\partial x'} \exp \left[ \frac{i \omega (x - x')}{v_F} \right],
$$

(9)

$$
\langle |H_z|^2 \rangle = \left( \frac{e k_y}{c} \right)^2 \int_0^\infty dx \int_0^\infty dx' \frac{\partial h(x)}{\partial x} \frac{\partial h(x')}{\partial x'} \exp \left[ \frac{i \omega (x - x')}{v_F} \right].
$$

(10)

With knowledge of the correlation function, it is possible to find other parameters of the random process. In our case, the statistical characteristics are invariant under a shift along the $x$-axis, and the random process is a stationary one. It means that the correlator depends on relative coordinate of $x - x'$. We assume that for normal stationary process the correlation function reads

$$
K(x) = \exp (-\alpha |x|) \cos(\gamma x),
$$

(11)

where $\alpha$ is the correlation radius, $\gamma = \frac{2 \pi}{L}$ is the inverse mean period and $L$ is a mean period of the ripples. After Fourier transforming our correlation function takes the form:

$$
K(k_x) = \frac{\alpha}{\alpha^2 + (\gamma + k_x)^2} + \frac{\alpha}{\alpha^2 + (\gamma - k_x)^2}.
$$

(12)

Substituting this result, we obtain the formula for radiation power

$$
P(\omega) \sim \frac{e^2 \omega^2}{4 \pi^2 c^3} \left( \frac{\alpha}{\alpha^2 + (\gamma + \frac{\omega}{c})^2} + \frac{\alpha}{\alpha^2 + (\gamma + \frac{\omega}{c})^2} \right).
$$

(13)

As we mentioned above, the Fermi velocity does not reach ultrarelativistic values and it means that the radiation of this model does not have synchrotron character and have out of plane distribution.

We see that the most part of electromagnetic radiation comes with frequency corresponding to the mean period of the ripples structure with some broadening due to its random character.

### IV. GAUGE FIELDS

In this part of our work we consider the synthetic gauge field effect on the electromagnetic waves emission. The elasticity theory gives the following relations for between the out-of-plane displacement $h(y, z)$ and the in-plane strain tensor components:

$$
A_x = -\frac{2 \beta}{a} u_{xy}, \quad A_y = \frac{\beta}{a} (u_{xx} - u_{yy}), \quad H_z = \frac{\partial A_x}{\partial y} - \frac{\partial A_y}{\partial x},
$$

(14)

$$
u_{xx} = \frac{1}{2} \left( \frac{\partial h}{\partial x} \right)^2, \quad u_{yy} = \frac{1}{2} \left( \frac{\partial h}{\partial y} \right)^2, \quad u_{xy} = \frac{1}{2} \frac{\partial h}{\partial x} \frac{\partial h}{\partial y},
$$

(15)

where $A_x, A_y, H_z$ are synthetic gauge fields and $u_{ij}$ is the strain tensor.

It is well known that electrons trajectory in undulator is forced to oscillate by the dipole system of magnets and this leads to radiation of electromagnetic energy. The motion through the undulated graphene could be interpreted
as a motion along the mentioned above device. We can set $OX$ axis as a direction of the electrons beam and assume
the mean ripples period $L_x$ as a period of bending magnets. The undulator-like structure of the synthetic magnetic
field can be mimicked by the following distribution of the height $h(x, y)$:

$$h(x, y) = h_0 \sin \left( \frac{2\pi x}{L_x} \right) \sin \left( \frac{2\pi y}{L_y} \right),$$

(16)

where $L_y$ is the graphene ribbon width.

In these terms, the equation of motion reads [14]:

$$\frac{\mathcal{E}}{v^2} \frac{d^2 x}{dt^2} = -e \frac{d}{dt} \mathcal{H} \cos \gamma x,$$

$$\frac{\mathcal{E}}{v^2} \frac{d^2 h}{dt^2} = e \frac{d}{dt} \mathcal{H} \cos \gamma x,$$

(17)

where $\mathcal{E}$ is the electrons energy.

The solution of the equation of motion could be written:

$$\cos \gamma x = \cosh \gamma h - \frac{1}{k} \sinh \gamma h,$$

(18)

and $k$ is denoted by

$$k = \frac{evF\mathcal{H}}{\gamma \mathcal{E}}.$$

The trajectory slightly differs from sinusoid.

Thus, the problem of the electronic trajectory in undulated graphene is similar to one in the undulator.

In the case of random function $h(x, y)$, nonlinear relation of $h$ and $\mathcal{H}$ [14], [15] leads to the presence of the
central peak in the radiation frequency distribution [13].

V. CONCLUSION

We considered here an impact of the ripples in the monolayer graphene on its electromagnetic properties. Possible
origin of the characteristic size of the ripples is discussed. Two mechanisms of the undulator-like radiation are
considered: geometric mechanism, directly connected with the presence of undulations and the pseudo gauge field
effect making trajectory to get curved in the base plane. The electromagnetic radiation was actually calculated with
a use of the standard retarded potential. For both of mechanisms two cases, of regular and random structures are
analyzed. Nonlinear relation between the random height function $h(x, y)$ and the gauge field $\mathcal{A}$ is shown to create
the radiation frequency distribution central peak.

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