Reserved Discrete Cosine Transform Coefficients Effects on Image Compression

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Abstract. Discrete Cosine Transform (DCT) is a special case of Discrete Fourier Transform (DFT) in which the sine component has been eliminated leaving only the cosine terms. DCT packing the energy into few numbers of transformed coefficients associated with low frequencies, the high frequency coefficients are discarded and replaced by zero coefficients. These coefficients may be discarded with little loss in energy and that will not effect that much on the image quality since the human eye doesn’t sense the high frequency components. The discarding process provides a trade-off between compression ratio and peak signal to noise ratio. Simulation programs were written using MATLAB with three images each of them of size (256×256) with 256 grayscale levels. Each line of these images is divided into blocks and each block is considered as a row vector of 128 pixels. 1D DCT is applied to convert each vector of pixels into a vector of 128 transformed coefficients. When the number of retained useful low frequency coefficient is 30% and less, the reconstructed images shows a noticeable degradation at all, in spite of increasing compression ratio. Two performance measurements have been employed, namely the objective and subjective.

1. Introduction

The Discrete Cosine Transform DCT is a special case of Discrete Fourier Transform DFT in which the sine component has been eliminated leaving only the cosine terms. The Discrete Cosine Transform is a method in which it’s used often in image compression [1]. DCT converts a block of image pixels into a block of transform coefficients of the same dimension. These DCT coefficients represent the original pixels values in the frequency domain. Any gray-scale 256 × 256 pixel block can be fully represented by a weighted sum of 65536 DCT basis functions where the weights are just the corresponding DCT coefficients. The forward N-point one-dimensional DCT and inverse DCT can be defined as follows [2]:

Forward N-point DCT:

\[
X(k) = \frac{2}{N} c_k \sum_{n=0}^{N-1} x(n) \cos \left( \frac{(2n + 1)k\pi}{2N} \right), \quad k = 0, 1, \ldots (1)
\]

Inverse N-point DCT:
\[ x(n) = \frac{2}{N} \sum_{k=0}^{N-1} c_k X(k) \cos \left[ \frac{(2n+1)k\pi}{2N} \right], \quad n=0,1,\ldots,N-1 \quad (2) \]

where \[ c_k = \begin{cases} 1/\sqrt{2}, & k = 0 \\ 1, & k \neq 0 \end{cases} \quad (3) \]

Both DCT and IDCT are orthogonal, separable and real transforms [3]. Being separable means that the multidimensional transform can be decomposed into successive application of one dimensional transforms in the appropriate directions. Similarly, orthogonal means if the matrices of DCT and IDCT are non-singular and real then their inverse is obtained merely by applying transpose operation. Like Fourier transform, DCT also considers the input sampled data to be a time invariant or stationary signal. A DCT encoder transforms the picture blocks and converts the signals into frequency components. The DCT has a strong “energy compaction” property and most of the signal information tends to be concentrated in a few low-frequency components [4]. The less important frequency (high) components are discarded from the image hence the use of lossy term. since the human eye will not notice or senses the high frequency components in the image so this gives the DCT an advantage of discarding the redundant data without affecting that much on the image. The low frequency components remain so that can be used in the reconstruction process decompression [5]. The proposed DCT system used in this paper uses the discarding theory instead of the quantization as shown in figure (1).

![DCT system block diagram](image)

**Figure 1.** DCT system block diagram

From the block diagram, the image first transformed into coefficients where the low frequency components kept, so these coefficients can be used in the reconstructing process and the high components are discarded, then these coefficients are stored or transmitted in DCT coding form. At the receiving point these coefficients are inverse transformed using the inverse DCT to reconstruct the image.
2. Material and Experimental Procedures

Referring to DCT system flow chart that illustrated in figure (1), here is the main stages of "DCT algorithm "that proposed and manipulated in this work:

![DCT system flow chart](image)

**Figure 2.** DCT system flow chart

DCT provides good energy compaction and several fast algorithms exist for calculating the DCT of a block of pixels [6], [7]. The advantage of DCT is that it can be expressed without complex numbers. Discrete Cosine Transform (DCT) is a transform method, which transforms a set of pixels to the frequency domain [8]. In the frequency domain the higher frequencies can be quantized or discarded more effectively than the lower frequencies without major loss in picture quality [9], [10]. The transformed DCT coefficients are then discarded to reduce the data size. When DCT based block-coding schemes are used for compressing images, undesirable blocking artifacts affect the reconstructed images. This problem becomes severe under high compression ratios or very low bit rates. This method gives moderate compression in a picture. (retained coefficients) is finer for lower DCT frequencies and coarser for higher DCT frequencies of the coefficients. Discarding regulates the amount of compression. Figure (3) shows “Building image” in different stages of the compression process.

![Building image](image)

**Figure 3.** a) Original image, b) DCT image, c) Reconstructed image, d) Error image.
3. Results and Discussion

The DCT system block diagram tested using three pictures with different details: Langair image, Bridge image and Flowers image by applying the MATLAB program on these pictures showing the peak signal to noise ratio (PSNR), mean square error (MSE), compression ratio (C.R) and bit rate (B.R). Table (2, 3, 4) shows the values of (the objective measurement) of each image used in the system with different number of retained coefficients.

**Table 1. objective measurement of image “Langair”**

| Reserved coefficients (%) | PSNR(dB) | MSE          | CR      | BR (bpp) |
|----------------------------|----------|--------------|---------|----------|
| 75%                        | 34.4725  | 23.2184      | 3.5556  | 2.2500   |
| 50%                        | 29.8920  | 61.0801      | 5.3333  | 1.5000   |
| 40%                        | 28.8375  | 84.9820      | 6.6667  | 1.2000   |
| 30%                        | 27.0353  | 128.6923     | 8.8889  | 0.9000   |
| 25%                        | 26.2075  | 155.7158     | 10.6667 | 0.7500   |
| 12.5%                      | 23.7243  | 275.8369     | 21.3333 | 0.3750   |

**Table 2. objective measurement of image “Bridge”**

| Reserved coefficients (%) | PSNR(dB) | MSE          | CR      | BR (bpp) |
|----------------------------|----------|--------------|---------|----------|
| 75%                        | 34.1970  | 24.7388      | 3.5556  | 2.2500   |
| 50%                        | 30.2718  | 66.6616      | 5.3333  | 1.5000   |
| 40%                        | 28.3415  | 95.2640      | 6.6667  | 1.2000   |
| 30%                        | 26.6197  | 141.6149     | 8.8889  | 0.9000   |
| 25%                        | 25.7562  | 172.7656     | 10.6667 | 0.7500   |
| 12.5%                      | 23.0133  | 324.9001     | 21.3333 | 0.3750   |

**Table 3. objective measurement of image “Flower”**

| Reserved coefficients (%) | PSNR(dB) | MSE          | CR      | BR (bpp) |
|----------------------------|----------|--------------|---------|----------|
| 75%                        | 39.0840  | 24.7388      | 3.5556  | 2.2500   |
| 50%                        | 33.0994  | 66.6616      | 5.3333  | 1.5000   |
| 40%                        | 30.6604  | 95.2640      | 6.6667  | 1.2000   |
| 30%                        | 28.0438  | 141.6149     | 8.8889  | 0.9000   |
| 25%                        | 26.8677  | 172.7656     | 10.6667 | 0.7500   |
| 12.5%                      | 23.0631  | 324.9001     | 21.3333 | 0.3750   |

As these coefficients decreases; the compression ratio increases but with trade off the image quality of the reconstructed one by monitoring the peak signal to noise ratio decreases with noticeable degradation in image quality especially at the edges of the reconstructed image, see Figures (4,5 and 6).
**Figure 4.** a- Langair image, b- (75%) Reconstructed image, c- (50%) Reconstructed image, d- (40%) Reconstructed image, e- (30%) Reconstructed image, f- (25%) Reconstructed image, g- (12.5%) Reconstructed image

**Figure 5.** a- Bridge image, b- (75%) Reconstructed image, c- (50%) Reconstructed image, d- (40%) Reconstructed image, e- (30%) Reconstructed image, f- (25%) Reconstructed image, g- (12.5%) Reconstructed image
4. Conclusions
This paper has summarized a modest DCT coding and effective energy depletion JPEG system, i.e. discarding theory. This approach reduces supply battery’s energy consumption necessities meaningfully while retaining all the advantages achieved through reserved adaptations to the JPEG software. Furthermore, the discarding process of high frequency DCT coefficients provides a tradeoff between CR and image quality using DCT by segmenting each image line into vectors may result in blocking artifacts. These artifacts are perceptually annoying and become prominent in the reconstructed images at very low bit-rates. A reenactment will be conveyed to give more complete and genuine outcomes in real time in future works.

5. References
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