Planck Scale Effects
and Axions in Supersymmetry

by

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ABSTRACT

The effects of possible explicit violation of the Peccei-Quinn symmetry responsible for the solution of the strong CP problem are studied in supersymmetric models. It is shown that automatic models with an abelian $U(1)$ gauge symmetry are easy to construct both in the context of fundamental and composite models of axions. It is argued that it is preferable to use abelian rather that nonabelian gauge groups in order to obtain automatic symmetries. A composite model with no exact R symmetry is studied and it is found that, unlike common belief, supersymmetry is broken.
One of the open problems of the standard model arises from nonperturbative effects in the QCD sector. Essentially, QCD instantons induce a term
\[ \frac{\theta g^2}{32\pi^2} F_{a\mu\nu} F^*_{a\mu\nu} \]
in the effective lagrangian which violates both P and CP symmetries [1]. As a consequence a neutron electric-dipole moment of order \( d_n \simeq 5 \times 10^{-16} \theta \) ecm will be induced which compared to the experimental measurements constrains \( \bar{\theta} \) to be less than \( 10^{-9} \). Here \( \bar{\theta} = \theta + \arg \det M_q \), whose \( M_q \) is the fermion mass matrix coming from the Higgs-fermion Yukawa interaction. The so-called strong CP problem is that there is no reason that \( \bar{\theta} \) fine-tune to zero to the required accuracy.

The candidate solutions are generally of two types. The first use Peccei-Quinn (PQ) type models [2] with an extra global \( U(1)_{PQ} \) symmetry that is spontaneously broken at a scale \( \Lambda_{PQ} \) giving rise to a Nambu-Goldstone boson a known as the axion [3]. This symmetry is explicitly broken by instanton effects which nonperturbatively generate an axion potential minimized by \( \bar{\theta} = 0 \). An axion mass of order \( m_a \sim \frac{A_{\text{QCD}}^2}{\Lambda_{PQ}} \) is generated which, combined with astrophysical and cosmological considerations gives us \( 10^9 \leq \Lambda_{PQ} \leq 10^{12} \text{GeV} \) [4]. Models along this line have been constructed with the invisible axion as a fundamental scalar particle [5] or a composite fermion-antifermion bound state [6].

A second candidate solution for the strong CP problem is given by the natural models [7]. There, CP is either explicitly (hard if complex Yukawa couplings are introduced and soft if complex scalar masses are allowed) or spontaneously broken.

Recently, the Peccei-Quinn mechanism was questioned on the basis
metry not protected by some gauge group. Even if these higher-dimensional operators are suppressed by inverse powers of the Planck scale, the fact that the Peccei-Quinn scale is not very far away generates contributions for the axion effective potential. Adding these terms to the usual one coming from the QCD color anomaly results in a vacuum with $\bar{\theta} \neq \theta$.

Evading models where constructed in ref. [9] and [11] using the notion of automatic symmetries [12]. In this case the gauge structure protects the appearance of low-dimension operators breaking the Peccei-Quinn symmetry which appears as an accidental consequence of the gauge symmetry.

The purpose of this note is to investigate the Planck scale effects in connection with the PQ solution in supersymmetric models. Solutions along the lines of ref. [10] and [11] will be analysed in order to find an example of a reasonable gauge group and chiral matter content leading to the required automatic symmetry. A simple solution is found to be a protecting U(1) gauge group with abelian charges which automatically forbid low-dimensional PQ breaking operator in the superpotential. Finally some consequences of supersymmetry breaking in a composite model are given.

A simple remark would be that we cannot identify the PQ symmetry with an $R$ symmetry because of terms of type $\frac{1}{M_p^{3n}} \int d^2\theta tr(W^\alpha W_\alpha)^n$ which cannot be avoided. So axial transformations commuting with supersymmetry will be the only possible candidates as automatic symmetries. A supersymmetric model using an axial PQ symmetry and no exact $R$ symmetry will be constructed, with fermion condensation breaking supersymmetry. This contradicts a general result [15] which states that a necessary condition for supersymmetry breaking is the existence of a nonanomalous continuous $R$ symmetry which is spontaneously broken.

There are two points which make the analysis different with respect to
- The first and the most important is the structure of the scalar potential in a global supersymmetric theory [13] which can be written as

\[ V(z_i) = \sum_i \left| \frac{\partial W}{\partial z_i} \right|^2 + \frac{1}{2} \sum_a (D_a)^2 + V_{\text{soft}}. \]  

(1)

In (1) \( W(\phi_i) \) is the superpotential constructed out of the chiral superfields \( \phi_i \) which contain the scalar complex fields \( z_i \). \( D_a \) are the auxiliary components of the real gauge superfields and \( V_{\text{soft}} \) contains terms breaking softly supersymmetry.

Breaking supersymmetry at the Planck scale is not a welcomed possibility because a successful phenomenology and the hierarchy problem suggest the superpartners masses to be around 1 TeV [14]. Then more favoured possibilities seem to be the breaking at an intermediate scale or at low-energies. Hence we will allow only supersymmetric PQ violating terms in the potential (1) which come from the superpotential \( W \) (in global supersymmetry the Kahler function do not contribute to the scalar potential). Moreover, if \( W \) consists of two terms with different PQ charges

\[ W = W_1 + W_2 \]

then it is clear that its contribution

\[ \sum_i \left| \frac{\partial W_1}{\partial z_i} + \frac{\partial W_2}{\partial z_i} \right|^2 \]

(2)

to \( V \) may violate the PQ symmetry only through the interference terms in (2). By definition the automatic models have a tree level renormalisable superpotential necessary in order to accidentally generate the PQ symmetry. Then in order to avoid symmetry breaking operators with dimension less than, say 12, we must forbid in the superpotential terms with dimensions
renormalisable and gauge invariant term which can be written in the superpotential is of the form $27_1 27_{-1} 351_0$. This automatically gives rise to a PQ symmetry with PQ charges + 1 for the 27’s and - 2 for the 351. The lowest dimension PQ symmetry breaking operators in the superpotential consistent with gauge invariance are the terms $27_1^3 27_{-1}^3$, $351^6$ and $27_1 27_{-1} (351_0)^4$. The interference with the tree level renormalisable term gives dimension 7 operators breaking PQ symmetry which spoil the PQ solution. A simple modification of this model which do not have this problem is adding a supplementary chiral superfield multiplet $rX_4$ and impose the following system of equations on the $U(1)_X$ charges:

$$\text{Tr } X = 27(X_1 + X_2) + 351X_3 + rX_4 = 0$$

$$\text{Tr } X^3 = 27(X_1^3 + X_2^3) + 351X_3^3 + rX_4^3 = 0$$

$$X_1 + X_2 + X_3 = 0 \quad .$$ (3)

The first two are the conditions of anomaly cancellation for the $U(1)_X$ gauge group and the third one allows the construction of the same previous superpotential $27_1 27_{-1} 351_0$. Taking $X_4 = 0$ the unique solution is $X_1 + X_2 = 0$. That is why we must consider at least one supplementary chiral superfield multiplet.

As long as $\text{dim } r > 351$ we have real solutions for the system with $X_1 + X_2 \neq 0$ and no dangerous PQ breaking operators can be constructed. In fact imposing a supplementary term in the superpotential requires the supplementary equation

$$a \, X_1 + b \, X_2 + c \, X_3 + d \, X_4 = 0$$ (4)
we assume soft breaking terms such that all the corresponding particles will be superheavy of order the unification scale \( \Lambda_{\text{GUT}} \) and will not contribute to the running of \( \alpha_3 \) between \( \Lambda_{\text{GUT}} \) and \( \Lambda_{\text{QCD}} \).

- The second point is to check that the infrared confinement of QCD is not destroyed, a constraint especially for the composite axion models. We will take as example the model proposed in ref. [11] and try to supersymmetrize it. The gauge group is \( \text{SU}(N) \times \text{SU}(m) \times G \), with \( G \) the standard model gauge group. \( \text{SU}(N) \) with \( N > 3 \) has a coupling constant which becomes strong at the intermediate scale \( \Lambda_{\text{PQ}} \) and \( \text{SU}(m) \) is introduced in order to protect the low-dimension symmetry breaking operators. The supplementary fermions are left-handed transforming under \( \text{SU}(N) \times \text{SU}(m) \times \text{SU}(3)_c \) as

\[
(N,m,3) + 3(N,\bar{m},1) + m(\bar{N},1,\bar{3}) + 3m(\bar{N},1,1)
\]

(5)

Under the PQ symmetry the coloured fermions have charge +1 and the color neutral fermions -1. The lowest dimensional operator consistent with the gauge and Lorentz invariance which violates the PQ symmetry is the operator with 2m color neutral fermions, half of them N’s and half \( \bar{N} \)’s. For \( m \geq 4 \) the symmetry breaking effects are sufficiently suppressed and the PQ mechanism still works.

In the supersymmetrized version of the model the representations given in eq. (5) describe chiral superfields. Denote by \( \Lambda_{\text{GUT}} \) the energy where supplementary superheavy fields come into play. Computing the running of \( \alpha_3 \) between \( \Lambda_{\text{GUT}} \) and an arbitrary scale \( \mu < \Lambda_{\text{PQ}} \) and taking for simplicity a step-type decoupling of the heavy fields we obtain

\[
\frac{1}{\alpha_3(\mu)} = \frac{1}{\alpha_3(\Lambda_{\text{GUT}})} - \frac{1}{2\pi} (3-mN)\ell n\frac{\Lambda_{\text{GUT}}}{\Lambda_{\text{PQ}}} - 3\frac{\ell n\Lambda_{\text{PQ}}}{2\pi \ell n \mu}.
\]

(6)
Take interesting values for $\Lambda_{\text{PQ}} \sim 10^7 - 10^{12} \text{ GeV}$ and $\Lambda_{\text{GUT}} \sim 10^{15} - 10^{18} \text{ GeV}$, for $N > m \geq 4$. Then using a perturbative value for $\alpha_3(\Lambda_{\text{GUT}})$ we are not allowed to get a strong coupled QCD at low energies, because the coloured exotic matter fields tends to decrease $\alpha_3$ above $\Lambda_{\text{PQ}}$ such that $\alpha_3(\Lambda_{\text{QCD}}) < \alpha_3(\Lambda_{\text{GUT}})$.

The problems with the unmodified version of the first model $E_6 \times U(1)_X$ was that the PQ was not sufficiently protected and with the second composite model that protecting it with a non abelian gauge group $SU(m)$ we lost the infrared confinement of QCD.

A simple way to protect the PQ symmetry without affecting QCD is to use a $SU(N) \times U(1)_X$ gauge group and matter multiplets with abelian charges such that an appropriate automatic symmetry naturally arises. Probably it is not the only way to construct models with suppressed Planck scale effects, but a very simple one. We will consider a composite model in the spirit of ref.[11] and ask for the simultaneous breaking of supersymmetry and PQ symmetry at the scale $\Lambda_{\text{PQ}}$.

The chiral superfields content of the model transforms under $SU(N) \times SU(3)_c \times U(1)_X$ as

$$\begin{align*}
\phi_1(N, 1, 1)_{X_1} &+ \phi_2(\bar{N}, 1, 1)_{X_2} + \phi_3(N, 3)_{X_3} + \phi_4(\bar{N}, 3)_{X_4} + S_1(1, 1)_{X_5} + S_6(1, 1)_{X_6}
\end{align*}$$

(7)

where $X_i$ are the abelian charges. At $\Lambda_{\text{QCD}}$ condensates of type $< \psi_1 \psi_2 >$ and $< \psi_3 \psi_4 >$ are formed breaking $U(1)_{\text{PQ}}$ and supersymmetry.

In the globally supersymmetric case in most cases the fermion condensation does not breaks supersymmetry [14]. A necessary (but not sufficient) condition is the existence of a non-anomalous continuous $R$ symmetry which
which do not have it and still supersymmetry is broken.

Imposing the $U(1)_X$ anomaly cancellation condition we will obtain two equations for $X_i$. Another one is obtained by imposing the existence of a term $S_1 \phi_1 \phi_2$ in the superpotential useful in defining the automatic PQ symmetry. A fourth equation, needed for the dynamical breaking of supersymmetry comes by imposing a term $S^+_2 \phi_3 \phi_4$ in the Kahler potential $K$.

Hence we arrive at the following system of equations

\[
\begin{align*}
N(X_1 + X_2) + 3N(X_3 + X_4) + X_5 + X_6 &= 0 \\
N(X_1^3 + X_2^3) + 3N(X_3^3 + X_4^3) + X_5^2 + X_6^2 &= 0 \\
X_1 + X_2 + X_5 &= 0 \\
X_3 + X_4 + X_6 &= 0
\end{align*}
\]

(8)

which always has nontrivial real solutions.

Write the lowest dimensional terms allowed by the gauge symmetry for $N \geq 4$ (but $N \neq 5$ where accidentally we can construct a dimension 6 operator breaking PQ symmetry in $W$)

\[
W = \lambda_1 S_1 \phi_1 \phi_2 + \frac{\lambda_2}{M_p^{5N-4}} S_2^{3N+1} (\phi_1 \phi_2)^{N-1} + \frac{\lambda_3}{M_p^{8N-3}} (\phi_1 \phi_2)^{N-1} (\phi_3 \phi_4)^{3N+1} + \cdots
\]

\[
K = \frac{1}{2} \frac{K_3}{M_p} (S_2^+ \phi_3 \phi_4 + S_2 \phi_3^+ \phi_4^+) + \frac{K_4}{M_p^2} (\phi_1^+ \phi_1 \phi_2^+ \phi_2 + \cdots) + \cdots
\]

(9)

where $\phi_i$ is the set of all chiral superfields and $M_p$ is the Planck mass scale.

For simplicity we take the gauge kinetic function $f = 1$. The PQ symmetry is defined by the first term in $W$ and the second one in $K$ being described by the charges

\[
R_1 = R_2 = -3 \quad R_3 = R_4 = 1 \quad R_{S1} = 6 \quad R_{S2} = -2
\]

(10)

The lowest dimensional terms in the scalar potential breaking the PQ sym-
Consider now the $U(1)^2 \times U(1)_R$ axial symmetries of the model defined in eq.(7) and (9). Denoting the corresponding parameters by $\alpha$, $\beta$, and $\delta$, the fields transform as follows:

\[
\begin{align*}
\theta' &= e^{-\frac{3i\delta}{2}} \theta \\
\Phi'_{1,2}(\theta') &= e^{i\alpha} \Phi_{1,2}(\theta) \\
\Phi'_{3,4}(\theta') &= e^{i\beta} \Phi_{3,4}(\theta) \\
S'_1(\theta') &= e^{-2i\alpha} S_1(\theta) \\
S'_2(\theta') &= e^{2i\beta} S_2(\theta).
\end{align*}
\]

All these symmetries are anomalous and the variation of the Lagrangian gives

\[
\delta \mathcal{L} \propto \left[ \alpha + \delta + 3(\beta + \frac{\delta}{2}) - 3\frac{\delta}{2} N (FF^*)_N + [N(\beta + \frac{\delta}{2}) + l.e.](FF^*)_3 + [N(X_1^2 + X_2^2)(\alpha + \frac{\delta}{2}) \\
+ 3N(X_3^2 + X_4^2)(\beta + \frac{\delta}{2}) + X_5^2(-2\alpha + \frac{\delta}{2}) + X_6^2(2\beta + \frac{\delta}{2})](FF^*)_X \right].
\]

where l.e. is the contribution of $(FF^*)_3$ to the anomalies of the axial symmetries coming from the low-energy sector. But the low-energy color anomalous axial symmetries are forbidden due to the Weinberg-Wilczek axion [3] which is experimentally excluded, so this contribution vanishes. Then to get a nonanomalous symmetry we must separately put to zero the contribution to $\delta \mathcal{L}$ of the three gauge groups $SU(N) \times SU(3)_c \times U(1)_X$, but the only solution of the three equations is the trivial one. So we have no nonanomalous axial symmetry.

To check that at $\Lambda_{PQ}$ susy is dynamically broken we consider the v.e.v. of the auxiliary component $F_{S2}$ given by

\[
F_{S2} = - (K^{-1})_{S_2}^{S_2} K_{S_2}^{34} < \psi_3 \psi_4 > .
\]

In the local supersymmetry case $K$ will be replaced by $G$, where [16]

\[
G = K + \log |W|^2.
\]
generated through terms of style

\[ R_{ijkl} \left( \overline{\psi}_i \psi_j \right) \left( \overline{\psi}_k \psi_l \right). \]  

(15)

where \( R_{ijkl} = \partial_i \partial_j g_{kl} - g_{mn} g_{in,k} g_{mj,l} \) is the curvature in the Kahler space and \( g_{ij} = \frac{\partial^2 K}{\partial z_i \partial z_j^*} \). Actually the soft masses are of order \( \frac{\Lambda_{PQ}^3}{M_p^2} \) so larger values of \( \Lambda_{PQ} \) are preferred by this scenario. This is allowed by the PQ breaking operator of dimension 5N in the scalar potential. Gaugino masses will be generated at one-loop level due to the breaking of R symmetry but will have rather small values. The interesting property of this model is that it have no continuous nonanomalous R symmetry and nevertheless supersymmetry is broken.

To conclude, we can suppress the Planck scale effects and protect the PQ symmetry in supersymmetric theories with an abelian U(1)X gauge group. The superfield content must be such that imposing the anomaly cancellation and a single renormalisable term in the superpotential we can have nontrivial abelian charges that forbid low dimensional operators breaking the PQ symmetry. It is easy to construct models where no polynomial nonrenormalisable superpotential can be written at all and Planck scale effects vanish identically.

After completion of this paper I became aware of the ref.[17] where models with dynamical supersymmetry breaking and no R symmetry are constructed.

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