Optimal Inverse Design Based on Memetic Algorithms—Part II: Examples and Properties

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Abstract—Optimal inverse design, including topology optimization and evaluation of fundamental bounds on performance, which was introduced in Part 1, is applied to various antenna design problems. A memetic scheme for topology optimization combines local and global techniques to accelerate convergence and maintain robustness. Method-of-moments (MoMs) matrices are used to evaluate objective functions and allow determination of fundamental bounds on performance. By applying the Shermann–Morrison–Woodbury identity, the repetitively performed structural update is inversion-free yet full-wave. The technique can easily be combined with additional features often required in practice, e.g., only a part of the structure is controllable, or evaluation of an objective function is required in a subdomain only. The memetic framework supports multifrequency and multiport optimization and offers many other advantages, such as an actual shape being known at every moment of the optimization. The performance of the method is assessed, including its convergence and computational cost.

Index Terms—Antennas, inverse design, numerical methods, optimization methods, shape sensitivity analysis, structural topology design.

I. INTRODUCTION

INVERSE design is a time-consuming process with no certainty regarding global minimum feasibility [1]. This is generally the case, no matter how sophisticated the method employed [2], [3], [4], [5], [6], [7], [8], [9], and there is no proof of convergence toward the global minimum [2], [3]. However, the global minimum is typically not needed in practice. A sufficiently good solution has to be found in a reasonable time. As such, a good balance between detailed local search and large-scale exploration of the solution space has to be achieved [10]. These properties are provided by the approach introduced in Part I [11] which lays down the groundwork for this second part.

The inverse design procedure proposed in this article relies on two equally important steps. In the first step, the fundamental bound on an optimized metric is evaluated. This provides a stopping criterion for the memetic scheme and delimits the performance of any realized device. The memetic scheme is initiated in the second step and attempts to minimize the distance from the fundamental bound. It may happen in some cases that the bound is not reachable, e.g., there are not enough feeders, or they are not properly placed. In such a case, the optimization can be restarted with improved initial conditions.

The memetic optimization method combines two distinct approaches—local and global steps. The local step is based on investigating the smallest topology perturbations, i.e., changes in the value of an objective function if one of the degrees-of-freedom (DOF) is removed or added to the optimized structure. The local step was proposed in [12] and [13], where only DOF removals were used to detect the local minima. The approach was extended by the possibility of adding DOF to the system in [14]. Satisfactory performance of the local step was confirmed on Q-factor minimization [14], as well as on the minimization of reflectance of a pixel antenna [15] or, recently, the design of surface unit cell [16]. Part 1 [11] merged both smallest perturbations (addition and removal of DOF) into a unified framework and combined it with the global step.

Since a fixed discretization grid is used, the differences calculated from the change of the objective function value under the smallest topology perturbations (topology sensitivities) represent a discrete analog to gradient over structural variables. As with gradient-based optimization schemes, these differences are used to search for a local minimum via an iterative greedy search [17].

The global step is designed to restore and maintain diversity when the local step finds a local minima. Heuristic approaches are known for their robustness [18], [19], applied, e.g., in the form of genetic algorithms operating over locally optimal shapes. The binary nature of genetic algorithms suits the combinatorial-type optimization solved in this work. While heuristics do not, in general, perform well [20], only good properties, including versatility, robustness, and easy implementation, are used here while the disadvantages (mainly computational requirements and slow convergence [21]) are mitigated by the underlying local step. Moreover, it is shown in this article that, in many cases, only the local step is needed to identify shapes good enough for practical purposes.
The above-mentioned properties and claims are confirmed in this article using four examples involving electrically small and medium-sized problems. Both scattering and antenna scenarios are treated. The minimizing functions are single and multiobjective.

The article is structured as follows. Implementation and benchmarking details are provided in Section II. Section III deals with the minimization of the Q-factor. Results for a rectangular plate and a spherical shell are consistently compared to fundamental bounds. The influence of discretization on precision and computational time is provided. Section IV generalizes the objective function to a multiobjective case studying the tradeoff between Q-factor and input impedance (matching). A code implementing the local step for minimizing Q-factor is published in freely accessible repository [22]. An array is synthesized in Section V to maximize realized gain. The different number of elements and different spacings are considered. Section VI describes how to optimize a region adjacent to a lossy chip in which the power absorbed from the incoming plane wave has to be maximized. Finally, the various aspects of the method are thoroughly discussed in Section VII and the article is concluded in Section VIII.

II. EXAMPLES—METHODOLOGY

The properties and overall performance of the proposed optimization procedure are discussed using various examples focusing on different aspects of the method. All parts were implemented in MATLAB [23], the matrix operators were evaluated in Antenna Toolbox for MATLAB (AToM) [24], and the genetic algorithm from Fast Optimization ProcedureS (FOPS) [25] was utilized. The surface method-of-moments (MoMs) for good conductors was utilized with Rao–Wilton–Glisson (RWG) basis functions [26] defined over a Delaunay triangulation [27]. The fifth-order quadrature rule [28] was applied to evaluate MoMs-based integrals.

The examples with recorded computational times were evaluated on a computer with an AMD Ryzen Threadripper 1950X CPU (16 physical cores, 3.4 GHz) with 128 GB RAM. The extensive studies, sweeping one or more parameters, were evaluated on an RCI cluster [29]. All parts of the code were implemented as described in [11] with the exception of the GPU evaluation of an objective function, i.e., the local step was vectorized. The global step is calculated with the help of parallel computing.

III. MINIMUM Q-FACTOR

Minimization of Q-factor has a long history [30], [31] and is still a topical problem. The importance of this quantity for electrically small antennas is given by its inverse proportionality to fractional bandwidth [32], a parameter suffering greatly from small electrical size [33]. While the determination of the lower bound to quality factor has matured [34], the corresponding optimal shapes are known thanks only to empirical designs [31], [35], [36]. This inevitably limits their exclusive applicability to cases when they are known to be approximately optimal (e.g., a folded spherical helix for a spherical region [37] or a meander line for a rectangular region [31]).

Here, we demonstrate how to utilize the technique presented in Part I of this article [11] to get solutions close to the fundamental bounds in an automated manner. The optimization setup is chosen so that the results can be compared with empirically synthesized structures [31], i.e., with the region \( \Omega \) in the form of a rectangle, as, e.g., described in [11, Fig. 1 of Part I].

A. Inverse Design

The rectangular region of varying mesh grid density and varying electrical size expressed in \( ka \), where \( k \) is the wavenumber and \( a \) is the radius of the smallest circumscribing sphere, is considered first. The optimization was performed with \( N_A = 48 \) agents (three times the number of physical cores of the Threadripper 1950X processor) utilized during the global step [11]. The excitation is realized via a discrete delta gap feeder placed in the middle of the longer side, close to the boundary of the design region. The amplitude and phase of the excitation does not affect the result thanks to the quadratic form used to define the Q-factor as

\[
Q(g) = Q_U(g) + Q_E(g) = \frac{1}{2} \left( |I^{\text{H}}(g)W(g)|^2 + |I^{\text{H}}(g)R_\text{g}(g)|^2 \right),
\]

here decomposed into its untuned \( Q_U \) and tuning \( Q_E \) parts [11]. This corresponds to the classic Q-factor definition [32]

\[
Q = \frac{2\omega \max(W_m, W_c)}{P_{\text{rad}}} = \frac{\omega W}{P_{\text{rad}}} + \frac{|P_{\text{react}}|}{2P_{\text{rad}}}
\]

where \( W, W_m \), and \( W_c \) are cycle mean values of the total, magnetic, and electric stored energy, respectively, and where \( P_{\text{react}} \) and \( P_{\text{rad}} \) are the reactive and radiated powers, see [11, Appendixes B and C of Part I].

The algorithm was set as follows: the local step can potentially have infinitely many iterations \( I \), however, it is terminated when the relative difference \( \epsilon_{\text{loc}} = 10^{-7} \) between two consecutive iterations is reached. Similarly, the relative difference for the global step was set to \( \epsilon_{\text{glob}} = 10^{-7} \) with a maximum of \( J = 250 \) global iterations.

B. Comparison With the Fundamental Bounds

The results are compared with bounds \( Q_{\text{lb}}^{\text{TM}} \) and \( Q_{\text{lb}}^{\text{TM}} \), shown in Fig. 1, and confirm the good performance of the method. The placement of the feed and reflection symmetry of the optimized region makes it possible to excite purely TM modes [38], see Appendix A, therefore, \( Q_{\text{lb}}^{\text{TM}} \) is a tighter bound as compared to the \( Q_{\text{lb}} \) bound. The \( Q_{\text{lb}}^{\text{TM}} \) bound is almost reached in the \( ka \in [0.6, 1] \) region. The top of Fig. 1 shows that the structures meeting the bound are close to self-resonance, i.e., \( Q_{\text{E}}/Q_{\text{U}} \rightarrow 0 \), following the expectation that Q-factor reaches its minimum for a self-resonant current [39]. However, there are two regions where the bound is not met, namely, \( ka < 0.6 \) and \( ka > 1 \).

The region \( ka < 0.6 \) requires finer granularity of the optimized domain than what was used in this example. This can be achieved only at the cost of increasing...
Fig. 1. Topology optimization (inverse design) of an antenna minimizing Q-factor. The rectangular plate is used as the region \( \Omega \), see examples for grids \( 8 \times 5 \) and \( 16 \times 10 \) as the insets in the top-right corner. Due to cubic dependence on electrical size, Q-factor values are normalized to \((ka)^3\), [31]. The optimized performance for four mesh grids (different line styles) is compared with fundamental bounds \( Q_{b} \) and \( Q_{TM} \), [34], [38]. The self-resonance test \( Q_{b}/Q_{U} \) is depicted at the top. The number of meanders that are required to construct a self-resonant meander line antenna reaching the bound \( Q_{TM} \) and fitting the design region is indicated by diamond marks at the top. The number of meanders is inferred from the parameterization used in [31].

The performance of the optimization technique has also been evaluated as the average of all \( N_{A} = 48 \) agents at a given \( j \)th global iteration. No postprocessing of the shapes was performed to show the raw results of the optimization algorithm. However, additional regularity constraints might be imposed to improve the manufacturability of the designs [41], e.g., the topology sensitivity map of the final design can be used to manually remove parts of the structure that obstruct manufacturing but have only miniscule impact on the optimized metric. It is seen that, while all agents at all iterations are always local minima (due to the performance of the local step), there is still high uncertainty about the final structure at the beginning, while the global minimum becomes uniquely determined close to the end of the optimization. Notice also, that high uncertainty at the beginning (each agent sits in different local minima) means high diversity, which is required for nonconvex problems. On the other hand, the technique converges quickly, as shown in Section III-C.

C. Computation Time

The performance of the optimization technique has also to be judged in terms of computation time and convergence.

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1It should be noted that (for given material support) to every optimal current vector \( \mathbf{I}_{opt} \) there exists an optimal vector of excitations \( \mathbf{V}_{opt} = \mathbf{ZI}_{opt} \). A study provided in [40] further shows that even for a lower number of controllable excitations, an optimally excited current density on realized structure attempts to mimic that of the fundamental bound.
rate. The optimization from Fig. 1 is repeated for \( ka = 0.5 \) for varying mesh grids, see Table I. Depending on the number of DOF \( N \), the number of required global iterations \( J \) is shown, together with the total required computational time \( t \), distance from the bound \( Q/\|\mathbf{Q}\|_b \), and number of investigated shapes for removals (\( \sum_i |\mathcal{R}(\mathbf{g}_i)| \)) and additions (\( \sum_i |\mathcal{A}(\mathbf{g}_i)| \)). The last two columns represent the number of antennas evaluated with the full-wave exact reanalysis method during the optimization. The number of optimized variables is \( B = N - 1 \) since one DOF is used for (fixed) delta gap feeding. It is seen that it reaches up to hundreds of millions of antenna shapes evaluated with (full-wave) MoMs.

The algorithmic complexity heavily depends on the implementation, computer architecture, and type of fitness function. In particular, the minimization of the Q-factor requires the evaluation of quadratic forms representing stored energy and reactive power, which is computationally expensive. From this perspective, the minimization of the Q-factor represents a computationally challenging example.

A graphical representation of the performance of the memetic algorithm (global and local steps together) is shown in Fig. 3 for a \( 12 \times 7 \) mesh grid and electrical size \( ka = 0.5 \), with computational time used as the \( x \)-axis. Only \( N_A = 16 \) agents are required for the memetic algorithm to reach very low values of the optimized metric. The remarkable convergence rate of the memetics stems from the local step. The local step, when it is applied for the first time in the second global iteration \( j = 2 \), already finds a solution (after 30 s) close to the final result. This occurs because of properties of the Q-factor’s solution space, which makes it possible to find a reasonable solution from an arbitrary starting point, see Monte Carlo analysis in [14].

The power of the local step within the memetic scheme is emphasized in Fig. 4 where the cost function for all \( N_A = 16 \) agents of memetics from Fig. 3 are shown for global iterations \( j \in \{2, 3, 5\} \). Iteration \( j = 2 \) (the top) is the first one when the local step is used (the first iteration only evaluates random initial seeds, see [11, Fig. 8 of Part I]). Although it takes the majority of the computational time, it is also capable of decreasing the value of the objective function by three orders in magnitude, reaching, in one case, \( Q/\|\mathbf{Q}\|_b \approx 1.15 \). Then, the next iteration (the middle) starts, on average, at a higher value of the objective function since the global operators were applied over the resulting words, consequently perturbing them and increasing the chance that a better local optimum will be found. Subsequently, the objective function is improved by one to two orders at the expense of a few seconds. Later, the fifth iteration slightly improves the local minima, almost reaching the final values of the objective function found for this grid, see Table I.

### TABLE I

| \( \) | \( N \) | \( J \) | \( t \) (s) | \( Q/\|\mathbf{Q}\|_b \) | \( \sum_i |\mathcal{R}(\mathbf{g}_i)| \) | \( \sum_i |\mathcal{A}(\mathbf{g}_i)| \) |
|---|---|---|---|---|---|---|
| \( 8 \times 5 \) | 227 | 27 | 24 | 1.16 | 8.75 \times 10^5 | 8.78 \times 10^5 |
| \( 12 \times 7 \) | 485 | 49 | 213 | 1.13 | 3.86 \times 10^6 | 4.02 \times 10^6 |
| \( 16 \times 10 \) | 934 | 85 | 2190 | 1.07 | 1.64 \times 10^7 | 1.84 \times 10^7 |
| \( 20 \times 12 \) | 1408 | 146 | 10400 | 1.06 | 3.85 \times 10^7 | 4.36 \times 10^7 |
| \( 24 \times 14 \) | 1978 | 182 | 43236 | 1.05 | 8.83 \times 10^7 | 1.05 \times 10^8 |

D. Comparison With Other Optimization Schemes

Apart from the performance of the memetic algorithm, the results obtained by sole heuristics (pure genetic algorithm, only the global step) and by density-based topology optimization [2] are also presented in Fig. 3. An amount of \( N_A = 80 \) agents was required for the genetic algorithm to reach values comparable to those offered by the memetic algorithm. The density-based topology optimization used the method of moving asymptotes [42] with the interpolation function from [7]. The density filter [43] had a fixed radius of a linearly decaying cone. A continuation scheme [44] was utilized to ensure convergence to the structure consisting of either PEC or vacuum only.

The performance offered by the sheer genetic algorithm (blue lines) is inferior to the memetic scheme.

The classical topology optimization, represented by black lines in Fig. 3, excels in speed. Consequently, it will supersede the memetic scheme when the number of DOFs is scaled up. This can be mitigated by using techniques such as surrogate modeling [45, Fig. 2]. Density-based topology optimization is, however, not able to reach as low values of the optimized metric as compared to the memetic scheme. This is mostly given by the necessity of thresholding that converts the result-
The computational time spent with the local updates in global iteration $j$ simultaneously excite both TM and TE modes [37], [46], thereby needed. The spherical shell as a design region

E. Spherical Shell as a Design Region

Spherical helices fed by a delta gap source can simultaneously excite both TM and TE modes [37], [46], thereby reaching the lower bound $Q_{ib}$ [39], [47] and breaking the $Q_{TM}^{lb}$ bound. To construct them ad hoc without a long study of the topic and deep empirical knowledge is, nevertheless, difficult [48]. In this section, the Q-factor without the self-resonance constraint is minimized for a spherical shell of electrical size $ka = 0.2$, and both $Q_{ib}$ and $Q_{TM}^{lb}$ bounds are utilized to judge the performance of the optimized structures.

The spherical shell was discretized into $N = 2304$ DOF and optimized with $N_A = 64$ agents and $J = 250$ global iterations. The relative differences, both for global and local steps, were set once again to $\epsilon = 10^{-7}$. One discrete delta gap source was used (its position and orientation are irrelevant thanks to the spherical symmetry); the materials are either PEC or vacuum.

The optimization ran for 46 h and the resulting structure is shown in Fig. 5. While the computational time might seem enormous, in total $6.24 \cdot 10^9$ antennas were evaluated via the exact reanalysis local step (within a solution space containing $2^{2303} \approx 1.87 \cdot 10^{693}$ antenna variants). The crucial role of the local step is, once again, visible in Fig. 6 where the first iteration with a greedy search decreases the objective function from $(ka)^3 Q \approx 10^4$ to $(ka)^3 Q \approx 1.4$. Good performance in the Q-factor, similar to what was reported in [37], where the antenna was designed empirically, is observed, i.e., $Q/Q_{ib} \approx 1.27$ and $Q/Q_{lb} \approx 0.91$. The helix has a slope optimized for an optimal combination of TM and TE modes [39] and several turns to reach self-resonance. The input impedance is, however, only $Z_{in} \approx 2.14 \Omega$, which seriously limits the practical usage of such an antenna. It can be increased by prioritizing low reflectance instead of Q-factor in a multiobjective optimization as shown in Section IV.

IV. Tradeoff between Q-Factor and Input Impedance

It was demonstrated in the previous example that more than one metric is of concern in antenna design. For example, to minimize the Q-factor and to reasonably match the antenna
to a given input impedance $Z_{in} = R_{in} + jX_{in}$ simultaneously requires composite objective functions. It can, for example, be of the following form:

$$f(g) = \frac{Q(g)}{Q_{lb}} (1 + \zeta |\Gamma(g, Z_{in}^0)|^2)$$

(3)

where $\Gamma(g, Z_{in}^0)$ is the reflection coefficient

$$\Gamma(g, Z_{in}^0) = \frac{Z_{in}(g) - Z_{in}^0}{Z_{in}(g) + Z_{in}^0}$$

(4)

of a structure represented by a word $g$, evaluated for characteristic impedance $Z_{in}^0$ [50], and $\zeta$ is a weighting coefficient. For $\zeta = 0$ the optimization is the same as in Section III. Any value $\zeta > 0$ considers the matching as well. This results in a Pareto-type optimization with a scalarized objective function [51], where the parameter $\zeta$ sweeps over the feasible set.

The PEC rectangular plate of electrical size $ka = 0.7$ from Section III-B is considered first. Self-resonance is reachable for this electrical size, and only the real part of the input impedance has to be optimized. The desired characteristic impedance is set to $Z_{in}^0 = 50 \Omega$ and weight $\zeta$ is swept from $\zeta = 0$ to $\zeta = 5$ in 41 equidistant samples. The results are shown in Fig. 7. The Pareto-optimal solutions [52] are highlighted by red circles and interconnected to form an approximate Pareto frontier. It is seen that the Q-factor, in terms of $Q/Q_{lb}^{TM}$, can be minimized to $Q/Q_{lb}^{TM} \approx 1.02$ but at that point, the antenna is not matched. Conversely, a slight increase in $Q/Q_{lb}^{TM}$ leads to excellent matching. The two most distinct solutions, marked by (a) and (b) in Fig. 7 are shown in Fig. 8 in terms of optimal structure (top) and current density (bottom). The left structure in Fig. 8, denoted as (a), performs best in terms of the Q-factor. The right structure performs well in the Q-factor and is matched to 50 $\Omega$. The visual comparison reveals that the parallel stub was created in structure (b) to match the input impedance. This technique [36] is also used in practice [37, Fig. 9].

The same formulation (3) is applied to an optimization of the spherical shell presented in Section III-E with an attempt to modify the structures from Fig. 5 to be matched to $Z_{in}^0 = 50 \Omega$. For this purpose, weight $\zeta$ in (3) is set to $\zeta = 1$. The optimization found a minimum with $Q/Q_{lb}^{TM} \approx 1.27$ and $Q/Q_{lb}^{TM} \approx 0.91$, i.e., the same as before, and $Z_{in} \approx (51.5 - j2.39) \Omega$, i.e., $|\Gamma|^2 \approx 7.72 \cdot 10^{-4}$, which is sufficient for a majority of applications [53]. The optimal structure is shown in Fig. 9. When the optimal shapes and currents for an unmatched antenna in Fig. 5 and a matched antenna in Fig. 9 are compared, a similar difference, as in Fig. 8, is observed, i.e., the optimal structure is only slightly perturbed in the vicinity of the feeder, using a stub-matching technique to divide the current flowing through the feeding port. Another improvement is a wider metallic strip used to
The width of the strips is \(\ell/5\) and is intentionally longer than \(\lambda/60\). The optimization was repeated for various separation distances denoted as \(d/\lambda\) to maximize realized gain. The reference impedance was set to \(Z_0 = 50\ \Omega\), polarization pointed along the dipoles, and gain was measured in the endfire direction. Excitation was performed by a delta gap feeder placed in the middle of the second dipole from the left, see Fig. 11. The vertical dotted lines point to the maximal realized gain found for various numbers of dipoles and denote the corresponding electrical length \(kd\). For convenience, the results are compared with fundamental bounds with prescribed input impedance (dashed lines). For one optimized sample, see Fig. 11.

with \(U\) being the radiation intensity, and \(\Gamma(g, Z_0)\) is reflection coefficient (4). From an implementation point of view, both radiation intensity and total power are one order cheaper to evaluate than the Q-factor. As the strip (1-D) structures are optimized, we can, therefore, expect relatively fast convergence.

The minimization of (6) was repeated for \(N_{\text{dip}} = \{3, 4, 5\}\) dipoles with separation distance selected from \(d/\lambda = 0.02\) to \(d/\lambda = 0.5\), see the solid lines in Fig. 10. To be as thorough as possible, the fundamental bound on the realized gain with \(Z_0 = 50\ \Omega\) was also evaluated (see the dashed lines). The procedure is detailed in Appendix B.

There are many conclusions that can be drawn from Fig. 10. First, there is a “sweet spot” in separation distance \(d/\lambda\) (or in \(kd\) which denotes the electrical length of an array) and this optimal separation distance is different for optimized structures and for fundamental bounds (fundamental bounds reach maxima for lower distance \(kd\)). Second, a higher number of array elements leads to significantly higher realized gain, see Table II for numerical comparison. This is consistent with array theory [55]. Finally, the maxima given by the fundamental bounds are closely followed by realized designs. As the desired input impedance was prescribed for the bounds, see Appendix B, this might indicate that all the realized arrays were sufficiently well matched and that the antenna gain for both bounds and designs are comparable. This has been verified by an inspection of the optimization data.

As an example, the optimal design for \(N_{\text{dip}} = 4\) dipoles and a separation distance chosen for the highest realized gain
Table II
Comparison of Realized Gain Maxima Found Via Memetic Topology Optimization and the Fundamental Bounds Depending on the Number of Dipoles Used. Both Separation Distance \(d/\lambda\) and Electrical Length of the Array \(kd\) Are Shown

| \(N_{\text{dip}}\) | \(d/\lambda\) | \(kd\) | \(G_1\) | \(d/\lambda\) | \(kd\) | \(G_1\) |
|---|---|---|---|---|---|---|
| 3 | 0.260 | 3.27 | 8.13 | 0.200 | 2.51 | 10.4 |
| 4 | 0.290 | 5.47 | 12.0 | 0.245 | 4.62 | 15.6 |
| 5 | 0.320 | 8.04 | 15.7 | 0.275 | 6.91 | 21.1 |

(d/\lambda = 0.29) is shown in Fig. 11. It is seen that the structure was significantly modified. A driven element was split by the removal of one DOF. This is indicated by the gray dashed line and a tag “cut” in Fig. 11. The same result occurred for the first and fourth elements (counted from the left). The third element was modified by the complete removal of material from its lower part. It is obvious that these modifications cannot be achieved by a simple parametric sweep to optimize the overall length of the dipoles. Such an approach would remove all material below the “cut” label, reducing the performance from \(G_1 \approx 12.0\) to \(G_1 \approx 10.9\). The resulting structure is not symmetrical, even though the initial problem was. This is partly because the underlying discretization grid is not symmetrical and partly because the optimization problem is nonconvex.

VI. Maximum Absorption in a Given Region

The last example deals with the maximization of power absorbed in a given region, e.g., an optimal receiving structure for an RFID chip. The optimization domain is, in this case, only a part of the entire structure, see Fig. 12 for a schematic layout. The controllable part, which is a subject of the optimization (highlighted by the yellow) is made of copper, \(\sigma = 5.96 \cdot 10^7\) Sm\(^{-1}\). The chip (pink) is made of carbon, \(\sigma = 1 \cdot 10^4\) Sm\(^{-1}\). The thin-sheet resistivity model (5) is applied. The operational frequency is 3 GHz, the shorter side has length \(\lambda/4\) and the longer side has length \(11\lambda/32\), and the incident plane wave is impinging from the endfire direction, i.e., along the \(-\hat{x}\)-direction with \(\hat{e} = \hat{y}\) polarization, see Fig. 12.

The absorbed power \(P_{\text{lost}}\) is evaluated from

\[
P_{\text{lost}} = \frac{1}{2} \mathbf{I}^H \mathbf{D}_{\text{chip}}^H \mathbf{R}_e \mathbf{D}_{\text{chip}} \mathbf{I}
\]

where \(\mathbf{I}\) is the current on the optimized structure, \(\mathbf{R}_e\) is the lossy matrix, and matrix \(\mathbf{D}_{\text{chip}}\) is an indexation matrix having zeros everywhere except for diagonal positions corresponding to the DOF lying in the fixed region, see Fig. 12. To ease the computational burden, the matrix \(\mathbf{D}_{\text{chip}}\) is used to index out only the relevant entries required to evaluate (8).

The upper bound for this optimization problem is \(P_{\text{lost}}^{\text{ub}} \approx 11.2\) \(\mu\)W and was found using a procedure specified in Appendix C. The optimal current impressed in a vacuum is shown in Fig. 13. Even though this current does not fulfill \(\mathbf{Z} = \mathbf{V}\) with a plane wave excitation represented by a vector of expansion coefficients \(\mathbf{V}\), it is seen that it is a smooth function with maxima along the shorter sides and in the area of the chip. It must be noted that there is no guarantee that the bound is tight in this case.

The optimization was performed twice using the same settings and only varying the number of agents \(N_A\) used for the global step. As in all previous examples, the first application of the local step led to an immense improvement of the

Fig. 11. Optimized array of four thin-strip dipole elements made of copper for maximal realized gain at 1 GHz. The array is fed in the middle of the second dipole to the left.

Fig. 12. Schematic for maximization of power absorbed in the prescribed region made of carbon (\(\sigma = 1 \cdot 10^4\) Sm\(^{-1}\)). The operational frequency is \(f = 3\) GHz, and the shorter side has length \(\lambda/4\). The amount of power absorbed from the plane wave impinging from the \(d = -\hat{x}\)-direction with \(\hat{e} = \hat{y}\) polarization is evaluated only in this region denoted by the pink. The optimization domain is made of copper (\(\sigma = 5.96 \cdot 10^7\) Sm\(^{-1}\)) and denoted by the yellow.

Fig. 13. Current density representing the optimal performance for maximal power absorbed \(P_{\text{lost}}\) in the uncontrollable region, depicted by the pink in Fig. 12.
objective function value, here, of absorbed power \( P_{\text{lost}} \) from approximately \( 3.50 \cdot 10^{-3} \mu \text{W} \) to \( 5.10 \mu \text{W} \), i.e., by more than three orders in magnitude. For \( N_{A} = 80 \) agents (five times the number of cores of the Threadripper 1950X), the optimization ran for approximately 4.3 h with maximum \( P_{\text{lost}} \approx 6.96 \mu \text{W} \), see Fig. 14. Increasing the number of agents to \( N_{A} = 192 \) led to maximum \( P_{\text{lost}} \approx 7.75 \mu \text{W} \) found in 10.9 h. This value reaches 67.6\% of the upper bound realized by the current shown in Fig. 13. As compared to the original structure shown in Fig. 12, the power absorbed in the “chip” was increased by a factor of \( 4.52 \cdot 10^{4} \).

Comparing the convergence curves in Fig. 14, it is clear that increasing \( N_{A} \) increases the number of simultaneously used local minima during the evaluation of the global step, which leads to increased diversity and, consequently, increases the chance of finding a high-quality solution. This is, of course, at the expense of computational time, where the increase is approximately linear with \( N_{A} \). Notice, however, that with access to the computational cluster with \( C \) cores, there is almost no difference between computational times as far as \( N_{A} \leq C \) because all agents in the same global iteration \( j \) are evaluated simultaneously thanks to excellent scalability in parallel computing.

The optimal candidates for both runs are shown in the bottom-right corner of Fig. 14 which shows that the shape for \( N_{A} = 192 \) is more regular, consisting of three wire structures; see Fig. 15 for details and for surface current density excited by the impinging plane wave. The shape for \( N_{A} = 80 \) has not only worse performance in terms of absorbed power \( P_{\text{lost}} \) but also significantly more complex rendering its possible manufacturing risky.

VII. DISCUSSIONS AND FUTURE CHALLENGES

The method has some unique properties and offers unorthodox features which are discussed below together with a list of pros and cons.

A. Properties and Features

1) Full-Wave Evaluation: The entire approach is full-wave. The numerical errors occurring during the iterative updates are negligible, typically influencing the last digit in double precision.\(^4\)

2) Fixed Discretization Grid: The procedure is based on a fixed discretization grid. It may happen that some objective function tends to thin structures, i.e., only one DOF per width is chosen no matter what density of grid is used. A procedure with remeshing can be, however, applied in these cases. The objective function can also be penalized with ohmic losses, the incorporation of which always reflects physical reality better.

3) No Interpolation Function: As compared to the classic topology optimization [2], [8], there is no interpolation procedure which can change the actual value of an objective function when performed and which selection depends on a user.

4) Gradient-Based Procedure: As opposed to the pixeling [56], the proposed method involves the local step. Gradient-based optimization methods are typically preferred due to their rapid convergence [57]. The gradient-based methods may fail if a problem includes a significant amount of local minima with the solution space unable to be regularized [20]. To mitigate this, the global step is utilized. Being connected is, however, a question of whether the greedy search used in the local step is an appropriate choice as there are NP-hard problems known to be “greedy-resistant” [58]. Here we only rely on the experiment evidence presented in this article, which suggests that the greedy search provides appropriate designs.

5) Second Topology Derivative: Considering genetic algorithms as a particular choice for the global step, the mutation operator [59] serves as a second derivative. It is

\(^4\)This was verified by optimizing the shape with the proposed method first and solving the MoMs for the resulting shape directly after that. The difference in objective function was compared for these two current densities.
performed for random perturbations, but its application can be controlled by a proper choice of metaparameters.

6) MoMs Compatibility: The approach is compatible with any MoMs formulation based on piecewise basis functions and a solution found with a direct (noniterative) solver. Important differences may, however, exist depending on the type of basis functions used. For example, overlapping basis functions (e.g., RWG [26]), do not coincide one by one with discretization elements, therefore, removing one DOF does not correspond to removing one discretization element.

7) Compatibility With Fundamental Bounds: The formulation within the framework of MoMs allows evaluating fundamental bounds on many metrics of practical interest. These bounds then serve as stopping criteria for topology optimization, a nonconvex problem in which the proximity to the global extremum is unknown.

8) Multimodality: The algorithm is multimodal since, in principle, $N_A$ local minima are found during each global step thanks to the application of the local step [60]. The number of unique local minima decreases as the optimization converges to the global minimum.

9) Big Data: A huge amount of data is gathered during the optimization, see Table I which, together with excellent control over the entire optimization, makes this approach an ideal candidate for real-time data processing with machine learning.

10) Flexible Objective Function: Topology sensitivity is evaluated as the difference between the performance of the actual current $I(g_i)$ and the performance of the current flowing on the perturbed structure $I(g_{i+1})$. As such, the objective function can be easily extended toward:

1) an objective function evaluated only within a subregion (see Section VI);
2) an objective function considering multiple frequency samples; and
3) an objective function involving port quantities defined for multiple ports.

11) Multiobjective Optimization: Multiobjective optimization can easily be performed with the proposed memetics by scalarization, see Section IV. Only convex tradeoffs can, however, be found with a weighted sum, such as in (1) or (3) where more advanced techniques would have to be adopted for complex Pareto frontiers, e.g., a rotated weighted metric method [60].

B. Generalization of Variable Space

1) Feeding Synthesis: The optimal placement of the feeder(s), including optimal amplitudes and phases, can be incorporated into optimization. For one feeder, it is, technically, not needed.\(^5\) For more feeders, it is an extra combinatorial problem that can run over or run together with topology optimization.

2) Optimization Variables: It is possible to generalize the entire algorithm so that discretization elements (e.g., triangles) are used as the optimization variables instead of DOF (basis functions). One possibility is to define the smallest blocks manually and optimize them only. This slows down the evaluation since vectorization is not as efficient because block inversion has to be applied. On the other hand, the number of unknowns is reduced depending on the number and size of the optimized blocks. The initial tests indicate that computational time is comparable or lower than with DOF as the optimization unknowns.

3) Optimization Domain: Topology sensitivity can be evaluated only for removals or additions offering a substantial speed-up. Alternatively, as often required in practice, only a part of the structure can be optimized, see Fig. 12, reducing computational time proportionally to the size of the optimized region. Another possibility is to investigate only DOF, which separates two regions filled by different materials (the number of unknowns is reduced approximately by one order).

4) Multimaterial Representation: The two-state, binary, material representation (PEC × vacuum) can easily be generalized to a multistate optimization at the cost of the linear increase of variable space size.

C. Deficiencies and Possible Remedies

1) Slow Evolution: Shape modification is performed via the removal or addition of a DOF decreasing the value of the objective function the most. Following this greedy search, the DOF corresponding to the thickest green line in Fig. 16, i.e., the connections located to the left and on the top right section of the feeder are to be removed in order to eliminate the short-circuit. Such an approach is, however, relatively slow,\(^6\) requiring in many cases $O(N)$ local updates. The possible remedy might be to average the sensitivity in a small area and update all DOF inside at once.

\(^5\)It is assumed that the optimal shape is adjusted according to the initial placement of the feeder.

\(^6\)As compared to the adjoint formulation of topology optimization [2] where all DOF are updated at once.
2) Algorithm Complexity: Numerical complexity is unpleasant as it favors small to mid-size structures. Realistically, with the current implementation and state-of-the-art hardware, thousands of unknowns are possibly optimized in hours. Billions of DOF are, however, common in FEM [9], but this is because of different algebraic properties of the stiffness matrix as compared to the impedance matrix.

3) Shape Irregularity: The method might produce highly irregular optimized shapes. This is a common issue of topology optimization, no matter what technique is used, see Fig. 17, comparing representatives of this method, pixeling based on genetic algorithm [56], and gradient-based topology optimization [8]. Hence, the next step could be to find a way to regularize the shapes.

VIII. Conclusion

A novel memetic procedure for optimizing electromagnetic devices was presented, consisting of local and global steps to mitigate the disadvantages of both approaches when used alone. The fixed discretization grid is assumed, allowing the MoMs system matrix to be inverted only once while storing it and other required matrices in the computer’s memory. The iterative full-wave evaluation of all the smallest topology perturbations is done using an inversion-free (Sherman–Morrison–Woodbury) formula and an exact-reanalysis-based procedure. The local step provides gradient-type information about the topology, and local updates are performed in a greedy sense, i.e., with the part of the optimized shape enhancing the performance the most being updated in each local iteration. The global step used in this article is based on a genetic algorithm that maintains diversity and increases the chance of the algorithm’s convergence toward a global minimum. Genetic algorithms match well with the discrete form of the optimization problem and, as a result, the memetic procedure is robust, fast, and versatile.

The procedure has many advantages and unique features, such as being able to find minima close to the fundamental bounds, implying that the local minima are close to the global one. The number of full-wave evaluated shapes spans from millions for small problems to billions for larger problems with thousands of unknowns. The optimization approach is multimodal and capable of finding many local minima at once during one global step. As a result, the global scheme optimizes shapes only in a significantly reduced solution space containing only local minima. Since the optimized shape is known at every step, multiport, multifrequency, or multimaterial optimization can be performed only at the expense of the linear increase of computational time.

Examples shown in this part proved the efficiency of the method. For the first time, the performance of optimized structures has been directly compared with the fundamental bounds of the optimized metrics. This practice provides an ultimate measure of optimization efficiency, but also naturally scales optimized metrics and establishes the straightforward terminal criterion for optimization. In many cases, the performance of the resulting shapes closely follows the bounds. The excellent performance of the algorithm demonstrates its wide applicability in various inverse design problems in electromagnetics.

There are still many future challenges that could broaden the usage of the proposed method to make it even more efficient, such as including lumped elements and their synthesis. It can be shown that this technique is similar to adjoint formulation over gray-scaled material. Another possibility is to introduce a set of geometrical operators, representing geometrical metrics, such as the area spanned by the material, or the curvature of the shape. Having complete freedom in formulating the objective function, these operators may improve the regularity of shapes and eliminate manufacturing difficulties. Technical, and still extremely relevant, improvements include GPU implementation or a detailed study of metaparameters used for optimization settings and their tuning at the beginning or throughout an optimization. Another way to accelerate the evaluation is to use an adaptive scheme with a successively refined discretization grid to impose the optimal structure from a coarse grid as the initial shape for the finer grid.

APPENDIX A

Fundamental Bound on Antenna Q-Factor

The lower bound on radiation Q-factor $Q_{lb}$ is found via a quadratically constrained quadratic program (QCQP)

$$\min_i I^H W I$$

s.t. $I^H R_0 I = 1$

$I^H X I = 0$  \hspace{1cm} (9)

where $R_0$ is a radiation and $X$ is the reactance part of impedance matrix $Z$, and $W$ is the stored energy matrix. The problem is recast into its dual form and solved via a generalized eigenvalue problem as described in [62].

Q-factor $Q_{lb}^{TM}$ is a tighter bound for all cases when only the TM modes are involved. This includes, e.g., planar antennas with a discrete feeder [31], [35], i.e., the case studied in Section III. The formula (9) still applies with a change $R_0 \rightarrow R_0^{TM}$, where matrix $R_0^{TM}$ is evaluated as $R_0^{TM} = (U_1^{TM})^H R_0 U_1^{TM}$ with $U_1^{TM}$ being a projection matrix from the basis of TM spherical vector waves into an MoMs basis [63].
**APPENDIX B**

**FUNDAMENTAL BOUND ON REALIZED GAIN WITH PRESCRIBED INPUT IMPEDANCE**

The upper bound on realized gain was evaluated using (6) with current vector \( \mathbf{I} \) being the solution to

\[
\min I^H F^H (\tilde{d}, \tilde{e}) F (\tilde{d}, \tilde{e}) \mathbf{I} \\
\text{s.t.} \quad I^H \mathbf{Z} \mathbf{I} = I^H \mathbf{V} \\
V^H \mathbf{I} = \frac{|V_{\text{in}}|^2}{Z_{\text{in}}^0} 
\tag{10}
\]

where the last affine constraint enforcing matching \( \Gamma = 0 \), see (6), is removed from the optimization using the affine transformation described in [62]. Voltage \( V_{\text{in}} \) imposed on the delta-gap source together with a real impedance \( Z_{\text{in}}^0 \) is related to input power and matrix \( F(\tilde{d}, \tilde{e}) \) gives the electric far-field.

**APPENDIX C**

**FUNDAMENTAL BOUND ON ABSORBED POWER IN AN UNCONTROLLABLE SUBREGION**

This fundamental bound assumes partitioning

\[
\begin{bmatrix}
Z_{cc} & Z_{cu} \\
Z_{uc} & Z_{uu}
\end{bmatrix}
\begin{bmatrix}
I_c \\
I_u
\end{bmatrix}
= \begin{bmatrix}
V_c \\
V_u
\end{bmatrix}
\tag{11}
\]

into “controllable” (index c) and “ uncontrollable” (index u) subregions. The QCP leading to the desired optimal current reads

\[
\min I^H \begin{bmatrix}
0_{cc} & 0_{cu} \\
0_{uc} & R_{uu}
\end{bmatrix} \mathbf{I} \\
\text{s.t.} \quad I^H \mathbf{Z} \mathbf{I} = I^H \mathbf{V} \\
\begin{bmatrix}
Z_{cc} & Z_{cu} \\
Z_{uc} & Z_{uu}
\end{bmatrix} \mathbf{I} = \mathbf{V}_u 
\tag{12}
\]

where the first constraint enforces the conservation of complex power [64] and where the last affine constraint enforcing the bottom row of the partitioned system (11) is, as in Appendix B, removed from the optimization using the affine transformation described in [62].

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