Triple and Quartic Interactions of Higgs Bosons in the Two-Higgs-Doublet Model with CP violation

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Abstract

We consider the two-Higgs-doublet model with explicit CP violation, where the effective Higgs potential is not CP invariant at the tree-level. Three neutral Higgs bosons of the model are the mixtures of CP-even and CP-odd bosons which exist in the CP conserving limit of the theory. The mass spectrum and tree-level couplings of the neutral Higgs bosons to gauge bosons and fermions are significantly dependent on the parameters of the Higgs boson mixing matrix. We calculate the Higgs-gauge boson, Higgs-fermion, triple and quartic Higgs self-interactions in the MSSM with explicit CP violation in the Higgs sector and CP violating Yukawa interactions of the third generation scalar quarks. In some regions of the MSSM parameter space substantial changes of the self-interaction vertices take place, leading to significant suppression or enhancement of the multiple Higgs boson production cross sections.
1 Introduction

General interest to the models with two (and more) Higgs doublets is maintained by the absence of a convincing argument in favor of only one generation of Higgs bosons when there are three known generations of fundamental fermions. Models with extended Higgs sector provide richer physical possibilities than the standard scheme with one doublet. One of them is the possibility to introduce CP violation beyond the Cabibbo-Kobayashi-Maskawa (CKM) mechanism, by means of the Higgs boson exchange amplitudes with complex Higgs boson-fermion vertices. Complex couplings can be generated either spontaneously [1], when the vacuum expectation values of the Higgs fields are complex and couplings of the CP invariant tree-level Higgs potential are real, or explicitly inserted [2] on the level of $SU(2) \times U(1)$-invariant potential terms, when the complex vacuum expectation values of scalar fields correspond to the minimum of hermitian potential with complex couplings, which is not CP-invariant (CP-invariance softly broken by the mass terms).

Various representations of the $SU(2) \times U(1)$-invariant two-doublet Higgs potentials have been considered in the literature. The two-doublet models with spontaneous CP violation [1, 3] make use of the potential of general structure $-\mu^2 \phi^2 + \lambda \phi^4$ without the dimension two $\mu^2_{12}$-terms. Models with explicit CP violation use either the potential with trivial minimization ([2], see also [4]) or the potential with complex coupling $\mu^2_{12}$ of the dimension two terms and complex couplings $\lambda_5$, $\lambda_6$ and $\lambda_7$ in front of the dimension four potential terms [5, 6], similar to effective potential of the minimal supersymmetry (MSSM). Standard transformation (diagonalization) procedure from the level of primary fields which are the components of scalar doublets in $SU(2) \times U(1)$-invariant potential terms ($SU(2) \times U(1)$ eigenstates), to the physical fields (mass eigenstates) of Higgs bosons should be consistently performed to respect the $SU(2) \times U(1)$ invariance and the minimization of the potential. We consider the diagonalization for the two different two-Higgs-doublet potential forms in much details. A special case of the general two-Higgs-doublet model is represented by the Higgs sector of MSSM. Substantial radiative corrections to the Higgs boson masses and couplings are induced at the $m_Z$ scale mainly by the third generation quarks $t$, $b$ and the third generation scalar quarks [7]. In the special case of MSSM the multi-parameter space of general two-Higgs-doublet model is significantly reduced, providing possibilities of much less ambiguous phenomenological predictions.

Phenomenological consequences of the CP violating Higgs-third generation squark Yukawa interactions in the Higgs-fermion and the Higgs-gauge boson sectors have been considered in [5]. We focus mainly on the self-interactions of Higgs bosons. Experimental observation of the scalar boson
signals should be followed by the verification of Higgs mechanism as the essence of the gauge boson and fermion mass generation. Self-interactions of the Higgs fields lead to untrivial structure of the vacuum state with nonzero (and possibly complex) field tensions, initializing the spontaneous breakdown of $SU(2) \times U(1)$ symmetry. Reconstruction of the Higgs selfinteraction potential from the data on multiple (mainly double and triple) Higgs boson production cross sections [8] requires the experimental measurements of triple and quartic Higgs boson self-interaction vertices, which is nontrivial but valuable task for a future high luminosity colliders, such as LHC and TESLA.

In section 2 we discuss the diagonalization of CP invariant Higgs potential, represented in two different forms, in the general two-Higgs-doublet model (THDM) and consider the special case of the Higgs sector in the minimal supersymmetry (MSSM). In section 3 we introduce complex parameters of the $SU(2) \times U(1)$ invariant potential terms and discuss the diagonalization of the THDM potential which acquires explicit CP violation. In sections 4 and 5 we calculate the Higgs-gauge boson, Higgs-fermion and Higgs self-couplings in the MSSM with CP violation.

2 Diagonalization of the mass matrix in the general two-Higgs-doublet model

Two representations have been used for the two-doublet Higgs potential. The first representation [2, 4]

$$V(\varphi_1, \varphi_2) = \lambda_1(\varphi_1^+ \varphi_1 - \frac{v_1^2}{2})^2 + \lambda_2(\varphi_2^+ \varphi_2 - \frac{v_2^2}{2})^2$$

$$+ \lambda_3[(\varphi_1^+ \varphi_1 - \frac{v_1^2}{2}) + (\varphi_2^+ \varphi_2 - \frac{v_2^2}{2})]^2$$

$$+ \lambda_4[(\varphi_1^+ \varphi_1)(\varphi_2^+ \varphi_2) - (\varphi_1^+ \varphi_2)(\varphi_2^+ \varphi_1)]$$

$$+ \lambda_5[\text{Re}(\varphi_1^+ \varphi_2) - \frac{v_1 v_2}{2} \text{Re}(e^{i\xi})]^2 + \lambda_6[\text{Im}(\varphi_1^+ \varphi_2) - \frac{v_1 v_2}{2} \text{Im}(e^{i\xi})]^2$$

where $\lambda_i$ are real constants and the $SU(2)$ doublets $\varphi_{1,2}$ have the components

$$\varphi_1 = \{-iw_1^+, \frac{1}{\sqrt{2}}(v_1 + h_1 + iz_1)\}, \quad \varphi_2 = \{-iw_2^+, \frac{1}{\sqrt{2}}(v_2 + h_2 + iz_2)\}.$$

$w_{1,2}$ are complex fields and $z_{1,2}, h_{1,2}$ are real scalar fields. At positive $\lambda_1, \ldots \lambda_6$ each term of the potential $V(\varphi_1, \varphi_2)$ is obviously positive and its zero minimum is achieved if the vacuum expectation values of $<\varphi_1>$, $<\varphi_2>$ are taken in the form

$$<\varphi_1> = \frac{1}{\sqrt{2}}\{0, v_1\}, \quad <\varphi_2> = \frac{1}{\sqrt{2}}\{0, v_2 e^{i\xi}\}.$$
In the case of $\lambda_5 = \lambda_6$ (corresponding to the CP conserving MSSM-like potential, see below) the last two terms in (1) form the modulo squared, and the phase $\xi$ can be removed from the potential by the $U(1)$ rotation of $\varphi_2$, that does not affect minimization. In this section we will consider the case $\lambda_5 \neq \lambda_6$, $\xi = 0$. Substitution of (2) to (1) gives a bilinear form of the mass term with mixed components $w_i, h_i, z_i$, which can be diagonalized by an orthogonal transformation of the fields in order to define the tree level masses of Higgs bosons. In the CP conserving case the potential terms involving $z_1, z_2$ fields from the imaginary parts of $\varphi_1, \varphi_2$ doublets and $h_1, h_2$ fields from their real parts do not mix, so the mass terms are diagonalized by a separate two-dimensional rotations of the $z_1, z_2$ and the $h_1, h_2$ fields. The resulting spectrum of scalars consists of two charged $H^\pm$, three neutral $h, H, A^0$ scalar fields, and three Goldstone bosons $G$. This procedure is described in many papers (for instance, [4, 9]). The $w_{1,2}$ sector is diagonalized by the rotation of $w_1, w_2 \to H, G$

$$w_1^\pm = -H^\pm s_\beta + G^\pm c_\beta, \quad w_2^\pm = H^\pm c_\beta + G^\pm s_\beta$$

(4)
defined by the angle

$$\tan \beta = \frac{v_2}{v_1}$$

(5)
and leading to the massless $G$ field and the field of massive charged Higgs boson $H^\pm$, $m_{H^\pm}^2 = \lambda_4 (v_1^2 + v_2^2)/2$. The $z_{1,2}$ sector is diagonalized by the rotation $z_1, z_2 \to A^0, G'$ defined by the angle $\beta$ and giving again one massless field $G'$ and the field of CP-odd Higgs boson $A^0$ with the mass $m_A^2 = \lambda_6 (v_1^2 + v_2^2)/2$. Finally, the $h_1, h_2$ sector is diagonalized by the rotation $h_1, h_2 \to h, H$ defined by the angle $\alpha$

$$\sin 2\alpha = \frac{2m_{12}}{\sqrt{(m_{11} - m_{22})^2 + 4m_{12}^2}}, \quad \cos 2\alpha = \frac{m_{11} - m_{22}}{\sqrt{(m_{11} - m_{22})^2 + 4m_{12}^2}}$$

(6)
where

$$m_{11} = \frac{1}{4} [4v_1^2 (\lambda_1 + \lambda_3) + v_1^2 \lambda_5]$$

$$m_{22} = \frac{1}{4} [4v_2^2 (\lambda_2 + \lambda_3) + v_2^2 \lambda_5]$$

$$m_{12} = \frac{1}{4} (4\lambda_3 + \lambda_5) v_1 v_2$$
giving two massive fields of CP-even Higgs bosons $H, h$ with masses

$$m_{H, h}^2 = m_{11} + m_{22} \pm \sqrt{(m_{11} - m_{22})^2 + 4m_{12}^2}$$

(7)
The diagonal mass term of scalar fields and their triple and quartic self-interaction vertices can be explicitly obtained by the substitution of the following expressions for $\lambda_i$ to the potential $V(\phi_1, \phi_2)$ (1):

$$
\begin{align*}
\lambda_1 &= \frac{1}{2\pi^2 c_{\phi \phi}} \frac{1}{s_{\beta}} \frac{c_{\alpha-\beta} m_H^2}{s_{\beta}} - \frac{c_{\alpha} s_{\alpha-\beta} m_H^2}{c_{\beta}} + \frac{c_{\beta}}{4c_{\phi \phi}} \lambda_5, \\
\lambda_2 &= \frac{1}{2\pi^2 c_{\phi \phi}} \frac{1}{s_{\beta}} \frac{c_{\alpha-\beta} m_H^2}{s_{\beta}} + \frac{c_{\alpha} s_{\alpha-\beta} m_H^2}{c_{\beta}} - \frac{c_{\beta}}{4c_{\phi \phi}} \lambda_5, \\
\lambda_3 &= \frac{1}{2\pi^2} \left[-\frac{s_{\alpha} s_{\beta}}{s_{\beta} s_{\phi}} m_H^2 + \frac{s_{\alpha} s_{\beta}}{s_{\beta}} m_H^2 \right] - \frac{1}{2} \lambda_5, \\
\lambda_4 &= \frac{2}{v} m_H^2, \\
\lambda_6 &= \frac{2}{v} m_{A_0}^2
\end{align*}
$$

where we used the notation $v^2 = v_1^2 + v_2^2$, $s_{\alpha} = \sin \alpha$, $c_{\alpha} = \cos \alpha$. Diagonalization of the mass term takes place for arbitrary $\lambda_5$, which is a free parameter of the model.

The second representation of the Higgs potential

$$
U(\phi_1, \phi_2) = -\mu_1^2 (\phi_1^+ \phi_1) - \mu_2^2 (\phi_2^+ \phi_2) - \mu_{12}^2 (\phi_1^+ \phi_2 + \phi_2^+ \phi_1)
$$

(9)

+ $\bar{\lambda}_1 (\phi_1^+ \phi_1)^2 + \bar{\lambda}_2 (\phi_2^+ \phi_2)^2 + \bar{\lambda}_3 (\phi_1^+ \phi_1)(\phi_2^+ \phi_2)$

+ $\bar{\lambda}_4 (\phi_1^+ \phi_2)(\phi_2^+ \phi_1) + \frac{\bar{\lambda}_5}{2} [(\phi_1^+ \phi_2)(\phi_1^+ \phi_2) + (\phi_2^+ \phi_1)(\phi_2^+ \phi_1)]$

originates from the general SUSY action after the integration over Grassman variables and introduction of the soft SUSY-breaking terms (see [4]). It is easy to check that the potentials (1) and (9) are equivalent if parameters $\bar{\lambda}_i$, $\mu_1^2$, $\mu_2^2$, and $\lambda_i$ are related by the formulae

$$
\begin{align*}
\bar{\lambda}_1 &= \lambda_1 + \lambda_3, & \bar{\lambda}_2 &= \lambda_2 + \lambda_3, & \bar{\lambda}_3 &= 2\lambda_3 + \lambda_4, \\
\bar{\lambda}_4 &= -\lambda_4 + \frac{\lambda_5}{2} + \frac{\lambda_6}{2}, & \bar{\lambda}_5 &= \frac{\lambda_5}{2} - \frac{\lambda_6}{2}, & \mu_{12}^2 &= \lambda_5 v_1 v_2 / 2
\end{align*}
$$

and

$$
\begin{align*}
\mu_1^2 &= \lambda_1 v_1^2 + \lambda_3 v_1^2 + \lambda_3 v_2^2, & \mu_2^2 &= \lambda_2 v_2^2 + \lambda_3 v_1^2 + \lambda_3 v_2^2
\end{align*}
$$

(11)

Unlike the potential (1) where the minization is obvious, the symbolic structure of (9) does not demonstrate evidently its minimum. The substitution of (2) to (9) gives linear terms in the component fields $z_{1,2}$, $h_{1,2}$ (or physical fields $h$, $H$, $A$) and unless some additional conditions to remove the linear terms are imposed, we are not in the minimum of the potential. So the equations (11) which set to zero the terms which are linear in component fields are the minimization conditions. The diagonalization of $U(\phi_1, \phi_2)$ takes place for arbitrary $\mu_{12}^2$ parameter.

Inverse transformation (10) has the form

$$
\begin{align*}
\lambda_1 &= \bar{\lambda}_1 - \frac{\bar{\lambda}_3}{2} - \frac{\bar{\lambda}_4}{2} - \frac{\bar{\lambda}_5}{2}, & \lambda_2 &= \bar{\lambda}_2 - \frac{\bar{\lambda}_3}{2} + \frac{\bar{\lambda}_4}{2} + \frac{\bar{\lambda}_5}{2}
\end{align*}
$$

(12)
\[
\lambda_3 = \frac{\lambda_3}{2} + \frac{\lambda_4}{2} + \frac{\lambda_5}{2} - \frac{\lambda_5}{2}, \quad \lambda_4 = -\bar{\lambda}_4 - \bar{\lambda}_5 + \lambda_5, \quad \lambda_6 = 2\bar{\lambda}_5 + \lambda_5
\]

so masses of the CP-even scalars and their mixing angle \(\alpha\) (6),(7) in the case of potential \(U(\varphi_1, \varphi_2)\) can be easily obtained using

\[
m_{11} + m_{22} = v_1^2\bar{\lambda}_1 + v_2^2\bar{\lambda}_2 + \frac{\mu_{12}^2}{s_{2\beta}}, \quad m_{11} - m_{22} = v_1^2\bar{\lambda}_1 - v_2^2\bar{\lambda}_2 - \text{ctg}2\beta\mu_{12}^2, \quad (13)
\]

\[
2m_{12} = v_1v_2(\bar{\lambda}_3 + \bar{\lambda}_4 + \bar{\lambda}_5) - \mu_{12}^2.
\]

The diagonal form of \(U(\varphi_1, \varphi_2)\) and the physical scalar boson self-interaction vertices are obtained by substitution of the following expressions for \(\bar{\lambda}_i\) and \(\mu_i\) to (9):

\[
\bar{\lambda}_1 = \frac{1}{2v^2}[(s_{\alpha\beta})^2m_H^2 + (s_{\alpha\beta})^2m_{H^\pm}^2 - \frac{\omega_{\alpha\beta}}{s_{2\beta}}\mu_{12}^2] \quad (14)
\]

\[
\bar{\lambda}_2 = \frac{1}{2v^2}[(s_{\alpha\beta})^2m_H^2 + (s_{\alpha\beta})^2m_{H^\pm}^2 - \frac{\omega_{\alpha\beta}}{s_{2\beta}}\mu_{12}^2] \quad (15)
\]

\[
\bar{\lambda}_3 = \frac{1}{2v^2}[2m_{H^\pm}^2 - \frac{\omega_{\alpha\beta}}{s_{2\beta}} + \frac{\omega_{\alpha\beta}}{s_{2\beta}}(m_H^2 - m_{H^\pm}^2)] \quad (16)
\]

\[
\bar{\lambda}_4 = \frac{1}{v^2}(\frac{\omega_{\alpha\beta}}{s_{2\beta}} + m_{H^\pm}^2 - 2m_{H^\pm}^2) \quad (17)
\]

\[
\bar{\lambda}_5 = \frac{1}{v^2}(\frac{\mu_{12}^2}{s_{2\beta}} - m_{H^\pm}^2) \quad (18)
\]

\[
\mu_1^2 = \bar{\lambda}_1v_1^2 + (\bar{\lambda}_3 + \bar{\lambda}_4 + \bar{\lambda}_5)\frac{v_1^2}{2} - \mu_{12}^2\text{ctg}\beta \quad (19)
\]

\[
\mu_2^2 = \bar{\lambda}_2v_2^2 + (\bar{\lambda}_3 + \bar{\lambda}_4 + \bar{\lambda}_5)\frac{v_2^2}{2} - \mu_{12}^2\text{ctg}\beta \quad (20)
\]

Conditions (14)-(18) ensure the diagonal form of the mass term expressed in physical fields \(h, H, A, H^\pm\) and (19)-(20) are the minimization conditions.

Two parametrizations for the Higgs boson self-interaction vertices can be used in THDM. In the first parametrisation [10] \(\mu_{12}^2\) is a free parameter and \(\bar{\lambda}_5\) is defined by (18). In the second one \(\bar{\lambda}_5\) is a free parameter and \(\mu_{12}^2\) is equal to \(s_{2\beta}c_\beta(v^2\bar{\lambda}_5 + m_A^2)\). Complete sets of Feynman rules (unitary gauge) for the triple \(\mu_{12}^2\) and \(\lambda_5\) parametrisations and quartic \(\mu_{12}^2\) parametrisation) Higgs boson interactions in the general two-Higgs-doublet model with CP-conservation are shown in Tables 1-2. In the case of MSSM potential at the scale \(M_{\text{SUSY}}\) (see (29)) \(\bar{\lambda}_5 = 0\) and it follows from (8),(10),(11) that \(\mu_{12}^2\) is fixed and equal to \(m_A^2s_{2\beta}c_\beta\).

Two additional terms of the dimension 4 can be constructed using the complete set of \(SU(2) \times U(1)\) invariants \(\varphi_1^+\varphi_1, \varphi_2^+\varphi_2, \text{Re}\varphi_1^+\varphi_2\) and \(\text{Im}\varphi_1^+\varphi_2\).

\(^1\)These sets were obtained by means of the LanHEP package [11], see http://theory.sinp.msu.ru/~semenov/lanhep.html Six misprints in sign of the second term in the factor \((c_{\alpha\beta}^2s_{2\beta} - s_{\alpha\beta}^2s_{2\beta})\) that have taken place in [10], are corrected in Table 3 in the expressions for vertices \(hhhh, AAAA, hhAA, H^+H^-H^+H^-, H^+H^-AA, H^+H^-hh\).
(a detailed discussion of all possible potential forms can be found in [12]). These terms are usually added to the $U(\varphi_1, \varphi_2)$ with the couplings $\lambda_6$ and $\lambda_7$.

$$\tilde{U}(\varphi_1, \varphi_2) = U(\varphi_1, \varphi_2)$$

\[ + \lambda_6(\varphi_1^1 \varphi_1) + (\varphi_2^1 \varphi_2) + \lambda_7(\varphi_1^2 \varphi_2 + (\varphi_2^1 \varphi_1)) \]  

The diagonal form of $\tilde{U}(\varphi_1, \varphi_2)$ at the local minimum takes place at arbitrary $\mu_{12}^2, \lambda_6, \lambda_7$ and can be achieved by means of the substitution (14)-(20) with additional $\lambda_6, \lambda_7$ terms in the right-hand side:

$$\lambda_1 = \frac{1}{2} v^2 \left[ \frac{8s^2}{c^2} \right] - \frac{4}{3} \left( \lambda_7 \tan^2 \beta - 3 \lambda_6 \tan \beta \right)$$

$$\lambda_2 = \frac{1}{2} v^2 \left[ \frac{c^2}{s^2} \right] - \frac{4}{3} \left( \lambda_6 \tan \beta - \lambda_7 \tan \beta \right)$$

$$\lambda_3 = \frac{1}{v^2} \left[ \frac{\mu_{12}^2}{s_{c2} c_{s2}} \right] - \frac{4}{3} \left( \lambda_6 \tan \beta - \lambda_7 \tan \beta \right)$$

$$\lambda_4 = \frac{1}{v^2} \left[ \frac{\mu_{12}^2}{s_{c2} c_{s2}} \right] - \frac{4}{3} \left( \lambda_6 \tan \beta - \lambda_7 \tan \beta \right)$$

$$\lambda_5 = \frac{1}{v^2} \left[ \frac{\mu_{12}^2}{s_{c2} c_{s2}} \right] - \frac{4}{3} \left( \lambda_6 \tan \beta - \lambda_7 \tan \beta \right)$$

$$\mu_1^2 = \lambda_1 v_1^2 + \lambda_3 + \lambda_4 + \lambda_5 \frac{v_2^2}{2} - \mu_{12}^2 \tan \beta + \frac{4}{3} \lambda_6 \tan \beta + \lambda_7 \tan \beta$$

$$\mu_2^2 = \lambda_2 v_2^2 + \lambda_3 + \lambda_4 + \lambda_5 \frac{v_1^2}{2} - \mu_{12}^2 \tan \beta + \frac{4}{3} \lambda_6 \tan \beta + \lambda_7 \tan \beta$$

Our expressions for the redefined $\lambda_4$ and $\lambda_5$ are the same as given in [13].

The potentials (1) and (9) can be reduced to the MSSM potential in some regions of the parameter space which we are going to discuss. The potential $V(\varphi_1, \varphi_2)$ (1) has eight parameters: two vev’s $v_1$, $v_2$ and six couplings $\lambda_i$ ($i=1,\ldots,6$). Eight parameters of the potential $U(\varphi_1, \varphi_2)$ (9) $\mu_1$, $\mu_2$, $\mu_{12}$ and $\lambda_i$ ($i=1,\ldots,5$) can be found using (10),(11). From the other side, in the Higgs sector we have eight physical parameters: the mixing angle $\beta$ and $W$-boson mass $m_W$, mixing angle $\alpha$, the parameter $\mu_{12}^2$ and four masses of scalars $m_h$, $m_H$, $m_A$, $m_{H\pm}$. The $m_W$ is fixed experimentally maintaining the constraint on the $v_1$, $v_2$, $v_2 = v_1^2 + v_2^2 = 4m_W^2/e^2 \cdot \sin^2 \theta_W$ which follows from the Higgs kinetic term $D_{\mu} \varphi D^{\mu} \varphi$ ($g = e/\sin \theta_W$, $\theta_W$ is the Weinberg angle). So the Higgs sector of THDM with the potentials (1) or (9) is described by a seven-dimensional parameter space. In the case of superpotential five additional constraints are imposed, relating all Higgs boson self-couplings $\lambda_i$, ($i=1,\ldots,5$)
to the gauge coupling constants at the energy scale $M_{\text{SUSY}}$ [14]:

$$\lambda_1^\text{SUSY} = \lambda_2^\text{SUSY} = \frac{g_2^2 + g_1^2}{8}, \quad \lambda_3^\text{SUSY} = \frac{g_2^2 - g_1^2}{4}, \quad \lambda_4^\text{SUSY} = \frac{g_2^2}{2}, \quad \lambda_5^\text{SUSY} = 0.$$  

(29)

It follows that the four Higgs boson masses and the two mixing angles are defined by two independent parameters. One can choose, for example, the $r_1, r_2$ parametrization [15] ($r_{1,2} = m_{h,H}/m_2^2$) or the well-known $m_A$, $\tan\beta$ parametrization. In order to reduce the general two-Higgs-doublet model vertices to MSSM at the scale $M_{\text{SUSY}}$ it is convenient to use the $\alpha, \beta$ parametrization:

$$m_h^2 = m_Z^2 c_{2\beta} \frac{s_{\alpha+\beta}}{s_{\alpha-\beta}}, \quad m_H^2 = m_Z^2 c_{2\beta} \frac{c_{\alpha+\beta}}{c_{\alpha-\beta}}, \quad m_A^2 = m_Z^2 \frac{s_{2(\alpha+\beta)}}{s_{2(\alpha-\beta)}}, \quad \mu_{12}^2 = m_A^2 s_{\beta} c_{\beta}.$$  

(30)

Substitution of these expressions to the vertex factors in Tables 1,2 after trivial trigonometric transformations reduces them [10] to a simpler MSSM factors (see [4]). However, (30) are no longer valid at the energy scale $m_W$ where $\lambda_i^{\text{SUSY}}$ couplings and masses of Higgs bosons are significantly changed by radiative corrections and the effective two-Higgs-doublet potential should be described in the complete seven dimensional parameter space. Practical calculations of the radiatively corrected masses and/or couplings can be conveniently carried out using results of the two approaches, renormalization group (the HMSUSY package [16] or the analytical representation [17]) and diagrammatic (the FeynHiggsFast package, see [18]). Two different parametrizations can be used for these approaches.

In the RG approach it seems convenient to use the two-Higgs-doublet model parameter space $m_A$, $\tan\beta$, $\lambda_1, ..., \lambda_5$. In the following we shall take into account the $\lambda_6, \lambda_7$ terms defined by (21), so the parameter space will be nine-dimensional. RG evolution of the coupling constants $\lambda_i$ from the energy scale $M_{\text{SUSY}}$ to the electroweak scale $m_W$ defines the $\lambda_1, ..., \lambda_5$ in (22)-(26) and the parameters $\lambda_6, \lambda_7$. At a given $m_A$, $\tan\beta$, $\lambda_6, \lambda_7$ the parameter $\mu_{12}^2$ and $m_{H^\pm}$ are fixed by the conditions (25) and (26), the parameters $\mu_1^2$ and $\mu_2^2$ are fixed by (27) and (28), $\alpha$, $m_h$ and $m_H$ can be found using the equations (22)-(24). If we denote the deviation from the coupling $\lambda_i^{\text{SUSY}}$ at the MSSM scale by $\Delta \lambda_i$

$$2(\lambda_{1,2}^{\text{SUSY}} - \lambda_{1,2}) = \Delta \lambda_{1,2}, \quad \lambda_{3,4}^{\text{SUSY}} - \lambda_{3,4} = \Delta \lambda_{3,4}, \quad -\lambda_{5,6,7} = \Delta \lambda_{5,6,7}$$

we find the mixing angle (introducing the notation $g_1^2 + g_2^2 = g^2 m_Z^2/m_W^2$,

$$g^2 - g_1^2 = g^2(2 - m_Z^2/m_W^2)$$

while using (22)-(26))

$$\tan 2\alpha = \frac{s_{2\beta}(m_A^2 + m_Z^2) + v^2((\Delta \lambda_3 + \Delta \lambda_4)s_{2\beta} + 2c_{\beta}\Delta \lambda_6 + 2s_{\beta}\Delta \lambda_7)}{c_{2\beta}(m_A^2 - m_Z^2) + v^2(\Delta \lambda_1 c_{\beta}^2 - \Delta \lambda_2 s_{\beta}^2 - \Delta \lambda_5 c_{2\beta} + (\Delta \lambda_6 - \Delta \lambda_7)s_{2\beta})}.$$  

(31)
CP-even Higgs boson masses and the $\mu_{12}^2$ parameter

$$m_H^2 = c_{\alpha+\beta}^2 m_Z^2 + s_{\alpha-\beta}^2 m_A^2$$

$$-v^2(\Delta\lambda_1c_{\alpha}^2 c_{\beta}^2 + \Delta\lambda_2 s_{\alpha}^2 s_{\beta}^2 + 2(\Delta\lambda_3 + \Delta\lambda_4)c_{\alpha}c_{\beta}s_{\alpha}s_{\beta} + \Delta\lambda_5(c_{\alpha}^2 s_{\beta}^2 + s_{\alpha}^2 c_{\beta}^2)) + 2s_{\alpha+\beta}(\Delta\lambda_6 c_{\alpha}c_{\beta} + \Delta\lambda_7 s_{\alpha}s_{\beta})$$

$$m_h^2 = s_{\alpha+\beta}^2 m_Z^2 + c_{\alpha-\beta}^2 m_A^2$$

$$-v^2(\Delta\lambda_1 s_{\alpha}^2 c_{\beta}^2 + \Delta\lambda_2 c_{\alpha}^2 s_{\beta}^2 - 2(\Delta\lambda_3 + \Delta\lambda_4)c_{\alpha}c_{\beta}s_{\alpha}s_{\beta} + \Delta\lambda_5(s_{\alpha}^2 s_{\beta}^2 + c_{\alpha}^2 c_{\beta}^2)) - 2c_{\alpha+\beta}(\Delta\lambda_6 s_{\alpha}c_{\beta} - \Delta\lambda_7 c_{\alpha}s_{\beta})$$

$$m_{H^\pm}^2 = m_W^2 + m_A^2 - \frac{v^2}{2}(\Delta\lambda_5 - \Delta\lambda_4)$$

$$\mu_{12}^2 = s_{\beta}c_{\beta}[m_A^2 - \frac{v^2}{2}(2\Delta\lambda_5 + \Delta\lambda_6 c_{\beta}g + \Delta\lambda_7 t_{\beta}g)]$$

with the minimization conditions

$$\mu_1^2 = \frac{1}{2} m_Z^2 c_{2\beta} - \mu_{12}^2 t_{\beta}g \beta$$

$$-\frac{v^2}{2}[(\Delta\lambda_1 c_{\beta}^2 + (\Delta\lambda_3 + \Delta\lambda_4 + \Delta\lambda_5)s_{\beta}^2 + 3\Delta\lambda_6 s_{\beta}c_{\beta} + \Delta\lambda_7 s_{\beta}^3_{\beta}]$$

$$\mu_2^2 = -\frac{1}{2} m_Z^2 c_{2\beta} - \mu_{12}^2 c_{\beta}g \beta$$

$$-\frac{v^2}{2}[(\Delta\lambda_2 s_{\beta}^2 + (\Delta\lambda_3 + \Delta\lambda_4 + \Delta\lambda_5)c_{\beta}^2 + \Delta\lambda_6 c_{\beta}^3 + 3\Delta\lambda_7 s_{\beta}c_{\beta}]$$

These expressions can be straightforwardly used to calculate the radiatively corrected masses of Higgs bosons and the mixing angle $\alpha$ in the MSSM with the help of a solution of the RG equations for $\lambda_1, \ldots \lambda_7$. Apparently, in the RG approach Feynman rules in terms of $\lambda_i$ couplings are more convenient than rules in terms of Higgs particle masses.

In the diagrammatic approaches to calculation of the radiatively corrected masses [18] the corrections to $m_h$, $m_H$, $m_A$ and $m_{H^\pm}$ are extracted from the renormalized Higgs boson self-energies (usually radiative corrections to only the CP-even Higgs boson masses are calculated). The set of 7+2 independent parameters inherent for the diagrammatic approaches could be $m_A$, $t_{\beta}g$, $\alpha$, $\mu_{12}^2$, $m_h$, $m_H$, $m_{H^\pm}$, and $\lambda_6$, $\lambda_7$. At a given $m_A$, $t_{\beta}g$, $\lambda_6$, $\lambda_7$ the $\mu_{12}^2$ parameter can be fixed at the value $m_A^2 s_{\beta}c_{\beta}$, and $\alpha$ can be calculated using the renormalized self-energies correction [18] to the relation valid at the $M_{SUSY}$ scale $m_A^2 + m_Z^2 = -2s_{2\beta}/s_{2\beta}(m_H^2 - m_h^2)$. Then $\lambda_4$ is defined by (25) and $\lambda_1, \ldots, \lambda_3$ can be found using (22)-(24). In the diagrammatic calculations Feynman rules in terms of the radiatively corrected Higgs boson masses look more natural. Substitution of the radiatively corrected Higgs masses to the tree-level Higgs
vertex factors is expected to give results very close to those obtained from the loop corrections to Higgs vertex at the SUSY scale (see the discussion in the last ref. [8]). It has been shown in [19] on the example of $hhh$ and $hhhh$ vertices (and for the case of diagonal third generation squark mass matrix) that large radiative corrections to the vertex factors calculated diagrammatically can be absorbed in the radiatively corrected Higgs boson masses.

Other parametrizations in the two-Higgs-doublet model are of course possible, but they should be carefully introduced to respect the minimization and diagonalization conditions (22)-(28). The introduction of scalar particle masses and mixing angles inconsistent with them violates either diagonalization of the potential or its $SU(2)$ invariance, even if the minimization conditions remain valid.

3 CP violation in the two-Higgs-doublet model

CP transformation of the scalar doublet $CP \varphi^+P^+ = \zeta_{CP} \varphi^+$ (the phase factor $|\zeta_{CP}| = 1$) changes the sign of imaginary part $\text{Im}(\varphi^+_1\varphi_2)$ in the $\lambda_6$ term of potential (1), so if $\lambda_5 \neq \lambda_6$, $\lambda_6 \neq 0$ and $\xi \neq 0$, CP symmetry is broken there explicitly. In other words, the dimension two terms of (1) appear with the complex parameter $\mu_{12}^2$

$$\frac{\lambda_5}{4}[\varphi_1^+\varphi_2 + \varphi_2^+\varphi_1 - v_1 v_2 \cos \xi]^2 + \frac{\lambda_6}{4}[-i(\varphi_1^+\varphi_2 - \varphi_2^+\varphi_1) - v_1 v_2 \sin \xi]^2$$

$$\Rightarrow (\frac{\lambda_5}{4} - \frac{\lambda_6}{4})[(\varphi_1^+\varphi_2)^2 + (\varphi_2^+\varphi_1)^2] + (\frac{\lambda_5}{2} + \frac{\lambda_6}{2}) \varphi_1^+\varphi_2 \varphi_2^+\varphi_1 - \frac{v_1 v_2}{2}(\lambda_5 \cos \xi - i \lambda_6 \sin \xi) \varphi_1^+\varphi_2 - \frac{v_1 v_2}{2}(\lambda_5 \cos \xi + i \lambda_6 \sin \xi) \varphi_2^+\varphi_1$$

so we find

$$\mu_{12}^2 = \frac{v_1 v_2}{2}(\lambda_5 \cos \xi - i \lambda_6 \sin \xi).$$

CP is (softly) broken by the $\mu_{12}^2 \varphi_1^+\varphi_2 + \mu_{12}^2 \varphi_2^+\varphi_1$ terms. In this special case when the same phase $\xi$ is involved both in the potential and the vacuum expectation value of $\varphi_2$, diagonalization and minimization of CP transformed potential become less transparent. It is more convenient to analyse the potential form (21) for the general case of complex parameters with arbitrary phases. In the following we shall consider the hermitian potential which is the generalization of (21)

$$\bar{U}(\varphi_1, \varphi_2) = \frac{1}{2}[(-\mu_1^2(\varphi_1^+\varphi_1) - \mu_2^2(\varphi_1^+\varphi_1) - \mu_1^2(\varphi_2^+\varphi_2) - \mu_2^2(\varphi_2^+\varphi_2)]$$

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\[ -\mu^2_{12}(\varphi_1^+\varphi_2) - \mu^*_{12}(\varphi_2^+\varphi_1) \]
\[ + \frac{1}{2}[\lambda_1(\varphi_1^+\varphi_1)^2 + \lambda_2(\varphi_2^+\varphi_2)^2 + \lambda_3(\varphi_1^+\varphi_1)(\varphi_2^+\varphi_2) + \lambda_4(\varphi_1^+\varphi_2)(\varphi_1^+\varphi_1) + \lambda_5(\varphi_1^+\varphi_2)(\varphi_2^+\varphi_1)] \]
\[ + \lambda_6(\varphi_1^+\varphi_1)(\varphi_2^+\varphi_2) + \lambda_7(\varphi_1^+\varphi_2)(\varphi_2^+\varphi_2) \]

\( \ast \) denotes a complex conjugated. The potential terms with complex parameters \( \mu^2_{12} \) and \( \lambda_5, \lambda_6, \lambda_7 \) explicitly violate CP invariance. With the help of \( U(1)_Y \) hypercharge symmetry of the model the phases \( \theta_\mu \) and \( \theta_{5,6,7} \) of complex parameters \( \mu^2_{12} \) and \( \lambda_{5,6,7} \) can be removed by the phase rotation of scalar doublets \( \varphi_{1,2} \). In the case when \( v_1 \) in (3) is taken real and positive, and \( \eta \) is the overall phase of \( \varphi_2 \) doublet, the conditions to remove explicit phases from the potential are [20]

\[ \theta_\mu - \eta = n_\mu \pi, \quad \theta_5 - 2\eta = n_5 \pi, \quad \theta_{6,7} - \eta = n_{6,7} \pi \]

where \( n_\mu, n_i \) \( (i = 5,6,7) \) are arbitrary integer numbers. Or equivalently, in terms of complex parameters the conditions for the absence of explicit CP violation in the effective potential (37) are [5]

\[ \text{Im}(\mu^2_{12} \lambda_5) = 0, \quad \text{Im}(\mu^2_{12} \lambda_6) = 0, \quad \text{Im}(\mu^2_{12} \lambda_7) = 0. \]

The phase of complex \( \mu^2_{12} \) can be always rotated away, so \( \mu^2_{12} \) can be taken real. In the scenario of fine-tuning for the phases, when the conditions (38) or (39) are satisfied, \( \lambda_i \) are also real. Otherwise \( \lambda_i \) will be redefined after a phase rotation, keeping explicitly CP violating terms.

If the phases of \( \mu^2_{12}, \lambda_i \) are rotated away so there are no explicitly CP noninvariant potential terms, CP invariance can be nevertheless broken spontaneously. Using the convention when \( v_1 \) and \( v_2 \) in formula (3) are real and positive and selecting the \( \xi \) phase-dependent terms of (37), which are of the form \( acos^2\xi + bcos\xi \), where \( a = \lambda_5 v_1^2 v_2^2/2 \) and \( b = (\lambda_6 v_1^2/2 + \lambda_7 v_2^2/2 - \mu^2_{12})v_1 v_2 \) \( (0 \leq \xi \leq \pi) \) one can find their minimum (see the appendix of [20]) at \( \lambda_5 \geq 0 \) and

\[ \cos\xi = \frac{\mu^2_{12} - \frac{v_1^2}{2}\lambda_6 c^2_{\beta} - \frac{v_2^2}{2}\lambda_7 s^2_{\beta}}{\lambda_5 v_1^2 s_{\beta} c_{\beta}} \]

(40)
(the special case \( \mu_{12}^2 = 0 \) was found in [1, 3]). If \(| \mu_{12}^2 - \frac{v^2}{2} \bar{\lambda}_6 c_\beta^2 - \frac{v^2}{2} \bar{\lambda}_7 s_\beta^2 | \geq \bar{\lambda}_5 v^2 s_\beta c_\beta \) then the spontaneously CP violating extremum does not exist and the minimum of the potential takes place at the endpoints \( \cos \xi = \pm 1 \). For \( b > 0 \) the minimum is at \( \xi = \pi \) and for \( b < 0 \) the minimum is at \( \xi = 0 \). If \( \bar{\lambda}_5 < 0 \), the extremum inside the interval \( |\cos \xi| < 1 \) is the maximum, so the local minima are at the endpoints \( \xi = 0, \pi \). The case \( \xi = 0 \) is reduced to the case \( \xi = \pi \) by the change of sign for \( \bar{\lambda}_6, \bar{\lambda}_7, \mu_{12}^2 \) (change of sign for the \( b \)). The special case of CP conservation takes place at \( \xi = \pi/2 \) when \( \mu_{12}^2 = \frac{v^2}{2} \bar{\lambda}_6 c_\beta^2 + \frac{v^2}{2} \bar{\lambda}_7 s_\beta^2 \). In this case of purely imaginary \( < \varphi_2 > [21] \) our diagonalization procedure must be reconsidered. For instance, CP-even Higgs mass eigenstates are formed in this case by different orthogonal linear combinations of real and imaginary parts of scalar doublets, \( \text{Re} \varphi_1 \) and \( \text{Im} \varphi_2 \).

The substitution of (26) to the no-extremum condition \(| \mu_{12}^2 - \frac{v^2}{2} \bar{\lambda}_6 c_\beta^2 - \frac{v^2}{2} \bar{\lambda}_7 s_\beta^2 | \geq \bar{\lambda}_5 v^2 s_\beta c_\beta \) gives \( m_A^2 \geq 0 \). In the case \( \bar{\lambda}_5 < 0 \) the absolute minimum of the potential takes place at \( \xi = 0 \) (but not \( \xi = \pi \)) if \( \mu_{12}^2 - \frac{v^2}{2} \bar{\lambda}_6 c_\beta^2 - \frac{v^2}{2} \bar{\lambda}_7 s_\beta^2 \geq 0 \), which gives in combination with (26) \( m_A^2 \geq | \bar{\lambda}_5 | v^2 \). Even if \( \bar{\lambda}_5 \) is of the order of 1, the latter condition is valid when \( m_A \) is in the mass range of the order or greater than \( v \). In the case of real parameters \( \mu_{12}^2, \bar{\lambda}_i \) it is not straightforward to combine spontaneous CP violation with our procedure of diagonalization. In the case of complex parameters the situation may be changed. The extremum conditions can be found from the study of the fourth power equation with the coefficients depending on real and imaginary parts of \( \mu_{12}^2, \bar{\lambda}_i \). Nevertheless we are not going to consider spontaneous CP violation further on. With real parameters \( \mu_{12}^2, \bar{\lambda}_i \) and in the absence of spontaneous CP violation the minimum of the potential (37) can be taken at \( \xi = 0 \). At the same time insofar as the physical motivation of the fine-tuning conditions (38),(39) for CP conservation is not available, in the following we consider the general case of diagonalization and minimization of the two-Higgs-doublet potential with complex parameters.

For the diagonalization of (37) in the ground state we used the ansatz (22)-(28) to be taken for real parts of parameters. The real part of \( \mu_{12}^2 \) parameter is expressed through the real parts of \( \bar{\lambda}_{5,6,7} \) using (26)

\[
\text{Re} \mu_{12}^2 = m_A^2 s_\beta c_\beta + v^2 (s_\beta c_\beta \text{Re} \bar{\lambda}_5 + \frac{1}{2} c_\beta^2 \text{Re} \bar{\lambda}_6 + \frac{1}{2} s_\beta^2 \text{Re} \bar{\lambda}_7) \tag{41}
\]

defining also the real parts of \( \bar{\lambda}_{1,2,3,4} \) and \( \mu_1^2, \mu_2^2 \) by means of (22)-(25) and (27),(28). The substitution of complex \( \mu_i \) and \( \bar{\lambda}_i \) to the potential (37) leads to the linear term and the non-diagonal mass term which are dependent on the imaginary parts of \( \mu_{12}^2, \bar{\lambda}_i \)

\[
\bar{U}(\varphi_1, \varphi_2) = c_0 A + c_1 h A + c_2 H A \tag{42}
\]
We switch on the interaction of \( \phi \)

\[
+ \frac{m_h^2}{2} H^2 + \frac{m_H^2}{2} H^2 + \frac{m_A^2}{2} A^2 + m_{H^\pm}^2 H^+ H^- + \text{third and fourth order terms in } h, H, A, H^\pm
\]

where

\[
\begin{align*}
  c_0 &= -v \text{Im} \mu_{12}^2 + \frac{v^3}{2} s_\beta c_\beta \text{Im} \lambda_5 + \frac{v^3}{2} (c_\beta^2 \text{Im} \lambda_6 + s_\beta^2 \text{Im} \lambda_7) \\
  c_1 &= -s_{\alpha - \beta} \text{Im} \mu_{12}^2 + \frac{v^2}{4} (s_\beta s_{\alpha - \beta} - 2c_{\alpha + \beta}) \text{Im} \lambda_5 \\
  &\quad - \frac{v^2}{2} (s_\beta c_\beta c_{\alpha - \beta} - 3s_\alpha c_\beta) \text{Im} \lambda_6 + \frac{v^2}{2} (s_\beta c_\beta c_{\alpha - \beta} - 3s_\alpha c_\beta) \text{Im} \lambda_7 \\
  c_2 &= c_{\alpha - \beta} \text{Im} \mu_{12}^2 + \frac{v^2}{4} (c_\beta s_{\alpha - \beta} - 3s_{\alpha + \beta}) \text{Im} \lambda_5 \\
  &\quad - \frac{v^2}{2} (c_\beta^2 c_{\alpha - \beta} + 2c_\alpha c_\beta) \text{Im} \lambda_6 - \frac{v^2}{2} (c_\beta^2 c_{\alpha - \beta} + 2c_\alpha c_\beta) \text{Im} \lambda_7
\end{align*}
\]

In the case of CP conservation (38), (39) the linear and non-diagonal second order terms \( hA \) and \( HA \) do not appear because all imaginary parts of parameters can be removed. The linear term in \( A \) demonstrates that after the introduction of complex couplings we can be out of a local minimum of the potential \( \bar{U}(\varphi_1, \varphi_2) \). The minimization condition for the imaginary parts \( c_0 = 0 \) must be imposed. If \( \lambda_i, i = 5, 6, 7 \) are zero (this occurs in THDM if we additionally introduce a global \( U(1)_Q \) symmetry, softly broken by the dimension two \( \mu_{12}^2 \) term [22]), the imaginary part of \( \mu_{12}^2 \) can be removed by the phase rotation of \( \varphi_2 \), so the tree-level potential is CP invariant. In the general THDM with nonzero parameters the phase rotation of \( \varphi_2 \) which removes \( \text{Im} \mu_{12}^2 \) redefines \( \lambda_i, i = 5, 6, 7 \). These simple observations for the THDM potential isoflated from any other physical fields are no longer trivial if, keeping in mind the MSSM, we switch on the interaction of \( \varphi_1 \) and \( \varphi_2 \) with scalar quarks. CP invariance of the latter is (softly) broken by the dimension three terms with the Higgs mixing parameter \( \mu \) and the trilinear parameters \( A_{t,b} \) of the form \( f_{1,2} A_{b,t} \varphi_{1,2}^0 \bar{q}_L^* \bar{q}_R \), \( f_{2,1} \varphi_{1,2}^0 \bar{q}_L^* \bar{q}_R \) (\( q = t, \bar{b} \); \( \varphi_{1,2}^0 \) are the neutral components of the Higgs doublets, \( f_{1,2} = \sqrt{2m_{b,t}}/v_{1,2} \)). Then quartic scalar interaction parameters \( \lambda_i, i = 5, 6, 7 \) are affected by radiative corrections from one-loop diagrams with scalar quarks of the order of \( \mu^2 A^2/M_{\text{SUSY}}^4, \mu^2 A/M_{\text{SUSY}}^3 \) and \( \mu A^3/M_{\text{SUSY}}^3 \) [5], so the phases of \( \lambda_i, i = 5, 6, 7 \) are defined by the phases of complex \( \mu \) and \( A \), thus constraining the phase of \( \mu_{12}^2 \) in power of the conditions (41),(43). In the case of the Born level MSSM potential with a global \( U(1)_Q \) symmetry when \( \lambda_i = 0 \) (\( i = 5, 6, 7 \)) the complex \( \mu_{12}^2 \) parameter can still appear beyond the tree-level due to the same CP violating Yukawa interactions of scalar quarks with the Higgs fields. This possibility of the \( \mu_{12}^2 \) phase
induced in higher orders by radiative corrections calculated diagrammatically has been considered in [23]. The restoration of potential minimum can be achieved by means of the opposite sign quantum correction term, originating from the tadpole diagrams with the pseudoscalar $A$ connected to the squark loops $^2$.

In the classical minimum $c_0 = 0$ we find

$$
c_1 = \frac{v^2}{2} (s_\alpha s_\beta - c_\alpha c_\beta) \text{Im} \bar{\lambda}_5 + v^2 (s_\alpha c_\beta \text{Im} \bar{\lambda}_6 - c_\alpha s_\beta \text{Im} \bar{\lambda}_7) \tag{44}
$$

$$
c_2 = -\frac{v^2}{2} (s_\alpha c_\beta + c_\alpha s_\beta) \text{Im} \bar{\lambda}_5 - v^2 (c_\alpha c_\beta \text{Im} \bar{\lambda}_6 + s_\alpha s_\beta \text{Im} \bar{\lambda}_7)
$$

The second order terms $hA$ and $HA$ in (42) can be removed as usual by the orthogonal rotation $a_{ij}$ ($i, j = 1, 2, 3$) in $h, H, A$ sector

$$(h, H, A) M^2 \begin{pmatrix} h \\ H \\ A \end{pmatrix} = (h_1, h_2, h_3) a^T_{ik} M^2_{kl} a_{lj} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} \tag{45}$$

where the mass matrix has the form

$$M^2 = \frac{1}{2} \begin{pmatrix} m^2_h & 0 & c_1 \\ 0 & m^2_H & c_2 \\ c_1 & c_2 & m^2_A \end{pmatrix} \tag{46}$$

Squared masses of the physical states $h_1, h_2, h_3$, which are the Higgs bosons without definite CP-parity, are defined by the eigenvalues of mass matrix $M^2$ (roots of the cubic equation for eigenvalues are given by Cardano formulae)

$$m^2_{h1} = 2\sqrt{(-q)\cos(\theta + 2\pi/3) - a_2^2/3}$$

$$m^2_{h2} = 2\sqrt{(-q)\cos(\theta + 4\pi/3) - a_2^2/3}$$

where

$$\theta = \arccos \frac{r}{\sqrt{(-q)^3}}$$

$$r = \frac{1}{54} (9a_1 a_2 - 27a_0 - 2a_2^3), \quad q = \frac{1}{9} (3a_1 - a_2^2)$$

$$a_0 = c_1^2 m^2_H + c_2^2 m^2_h - m^2_H m^2_h m^2_A, \quad a_1 = m^2_h m^2_H + m^2_h m^2_A - c_1 - c_2$$

$$a_2 = -m^2_H - m^2_H - m^2_A$$

$^2$however, with nonzero $\lambda_5, \lambda_6$ and $\lambda_7$, the factor of the scalar-pseudoscalar Higgs counterterm is not explicitly proportional to the tadpole renormalization constant, or the tadpole parameter $c_0$. 

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One can see that in the limiting case of CP-conserving potential $c_{1,2} \to 0$ the following correspondence takes place: $m_{h_1} \to m_h, m_{h_2} \to m_H$ and $m_{h_3} \to m_A$. The normalized eigenvectors of the matrix $M^2$, which are at the same time the matrix elements of $a_{ij}$, $(h, H, A) = a_{ij} h_j$, have the form $a_{ij} = a'_{ij}/n_i$, where

$$a'_{11} = ((m_H^2 - m_{h_1}^2)(m_{h_1}^2 - m_{h_2}^2) - c_2^2), \quad a'_{21} = c_1 c_2, \quad a'_{31} = -c_1 (m_H^2 - m_{h_1}^2)$$

$$a'_{12} = c_1 c_2, \quad a'_{22} = ((m_H^2 - m_{h_2}^2)(m_{h_2}^2 - m_{h_3}^2) - c_1^2), \quad a'_{32} = -c_2 (m_H^2 - m_{h_2}^2),$$

$$a'_{13} = -c_1 (m_H^2 - m_{h_3}^2), \quad a'_{23} = -c_2 (m_H^2 - m_{h_3}^2), \quad a'_{33} = (m_H^2 - m_{h_3}^2)(m_H^2 - m_{h_3}^2)$$

and $n_i = \sqrt{(a'_{11}^2 + a'_{22}^2 + a'_{33}^2)}$. Representations for the triple and quartic Higgs boson self-interactions in the case of CP violating potential are given by the expansions of structures $a_{ij} h_j a_k h_k a_l h_l$, and $a_{ij} h_j a_k h_k a_l h_l a_m h_m$; they are bulky and not very telling, so we do not show them here. If the imaginary parts of $\bar{\lambda}_6$ and $\bar{\lambda}_7$ are not small, large off-diagonal elements of the mixing matrix $a_{ij}$ could appear leading to significant mass splittings of the Higgs states and modifications of the Higgs boson interactions.

We assume that in the Yukawa sector $\varphi_1$ couples only to down fermions

$$\sum_{a=d,s,b} V_{ua} \frac{e m_a}{\sqrt{2} m_W s_W c_\beta} [\bar{\psi}_1 (1 + \gamma_5) \psi_{2a} \varphi_1 + \bar{\psi}_{2a} (1 - \gamma_5) \psi_1 \varphi_1^+]$$

(48)

where $(h, H, A) = a_{ij} h_j$, for the first generation quarks $\bar{\psi}_1 = \{\bar{u}, V_{ud} \bar{d} + V_{us} \bar{s} + V_{ub} \bar{b}\}$, $\psi_{2a} = (d, s, b)$ and analogous terms for $c$ and $t$ quarks ($V_{ab}$ denotes the CKM matrix elements), and $\varphi_2$ couples only to up fermions (model of type II [24]):

$$\frac{e m_u}{\sqrt{2} m_W s_W s_\beta} [\bar{\psi}_1 (1 + \gamma_5) i \tau_2 \psi_2 \varphi_2^+ + \bar{\psi}_2 (1 - \gamma_5) i \tau_2 \psi_1 \varphi_2]$$

(49)

where again physical $h_1, h_2, h_3$ states are introduced by means of the $a_{ij}$ rotation, $\bar{\psi}_1 = \{\bar{u}, V_{ud} \bar{d} + V_{us} \bar{s} + V_{ub} \bar{b}\}$, $\psi_2 = u$ and analogous terms for $c$ and $t$ quarks.

---

3No ordering of masses $m_{h_1} < m_{h_2} < m_{h_3}$ is required. If we want to keep this ordering, then $a_{ij}$ written above, valid for the case $m_H > m_A$, must be changed. For the case $m_H < m_A$ one should replace $m_{h_2} \leftrightarrow m_{h_3}$ and change the sign of $a_{12}, a_{33}$ in the expressions for $a_{ij}$. 

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4 Higgs-gauge boson and Higgs-fermion couplings in the MSSM with explicit CP violation

In the following we shall focus on the MSSM scenario for the two-Higgs-doublet model, which allows to restrict strongly the THDM parameter space. It is not the only one possible, Standard Model-like scenarios in the general two-Higgs-doublet model have been discussed in [25]. Detailed consideration in the framework of MSSM has been performed in [5] (also [6]). In this section we would like only to compare qualitatively our results with the results of these approaches. Our calculation follows somewhat different scheme. In [5] the tree-level two-Higgs doublet potential is CP invariant. The phase \( \xi \) of \( \mu_1^2 \) is radiatively induced by the tadpole diagrams and can be absorbed in the definition of the \( \mu \) parameter which appears in the stop mixing matrix off-diagonal element \( A_t - \mu / \tan \beta \). The \( \bar{\lambda}_5, \bar{\lambda}_6 \) and \( \bar{\lambda}_7 \) terms are also radiatively induced by the threshold effects. At the same time the trilinear couplings \( A_t, A_b \) also carry a phase \(^4\), so both the radiatively induced and the trilinear phases contribute to the phase \( \text{arg}(\mu A) \) of the \( \bar{\lambda}_5, \bar{\lambda}_6 \) and \( \bar{\lambda}_7 \) terms. We do not account for the radiatively induced phase which is calculated diagrammatically. In the case under consideration the fine-tuning conditions (38),(39) are not fulfilled, so CP invariance of the two-doublet potential is explicitly broken by complex parameters at the tree level. The real and imaginary parts of \( \mu_1^2 \) parameter are defined by means of the condition (41) for the real parts of \( \bar{\lambda}_{5,6,7} \) parameters and the minimization condition (43) (where \( c_0 =0 \)) for their imaginary parts. In the following calculations complex parameters \( \bar{\lambda}_{5,6,7} \) are specified in the framework of the MSSM.

We used the two-loop symbolic results for \( \bar{\lambda}_i, \ i =1,...,7 \), which were obtained in the RG approach [17] and extended to the case of CP violation in [5]. The parameters \( \bar{\lambda}_5, \bar{\lambda}_6 \) and \( \bar{\lambda}_7 \) are nonzero in the next-to-leading order approximation (RG improved leading order approximation), so using (41) and (43)

\[
\text{Re}\mu_{12}^2 = m_A^2 s_\beta c_\beta + v^2 (s_\beta c_\beta \text{Re}\bar{\lambda}_5 + \frac{1}{2} c_\beta^2 \text{Re}\bar{\lambda}_6 + \frac{1}{2} s_\beta^2 \text{Re}\bar{\lambda}_7) \tag{50}
\]

\[
\text{Im}\mu_{12}^2 = \frac{v^2}{2} (s_\beta c_\beta \text{Im}\bar{\lambda}_5 + c_\beta^2 \text{Im}\bar{\lambda}_6 + s_\beta^2 \text{Im}\bar{\lambda}_7) \tag{51}
\]

where \( \bar{\lambda}_{5,6,7} \) depend on the finite term corrections to the leading logarithmic result which appear from the one-loop diagrams with trilinear couplings.

\(^4\)for a review see e.g. [26]
where $V$ masses of the third generation quarks, and $t$ Im invariance conditions (39) can be rewritten in the form

$$\lambda_5 = - \frac{3}{192\pi^2} h_t^4 \frac{\mu^2 A_h^2}{M_{SUSY}^4} \left[ 1 - \frac{1}{16\pi^2} (2h_b^2 - 6h_t^2 + 16g_s^2) t \right]$$

$$\lambda_6 = - \frac{3}{192\pi^2} h_t^4 \frac{\mu^2 A_h^2}{M_{SUSY}^4} \left[ 1 - \frac{1}{16\pi^2} (2h_b^2 - 6h_t^2 + 16g_s^2) t \right]$$

$$\lambda_7 = \frac{3}{96\pi^2} h_t^4 \frac{\mu^2 A_h^2}{M_{SUSY}^4} \left[ 1 - \frac{1}{16\pi^2} (2h_b^2 - 6h_t^2 + 16g_s^2) t \right]$$

$$\lambda_8 = \frac{3}{96\pi^2} h_t^4 \frac{\mu^2 A_h^2}{M_{SUSY}^4} \left[ 1 - \frac{1}{16\pi^2} (2h_b^2 - 6h_t^2 + 16g_s^2) t \right]$$

$A_t, A_b$ and $\mu$ are the factors in front of Higgs-squark(left)-squark(right) trilinear terms, $M_{SUSY}$ is the SUSY energy scale, $m_{top}$, $m_b$ are the on-shell running masses of the third generation quarks, and $t = \log \frac{M_{SUSY}^2}{m_{top}}$, $h_t = \frac{\sqrt{2m_t}}{\sin \beta}$, $h_b = \frac{\sqrt{2m_b}}{\cos \beta}$, $g_s = \sqrt{4\pi\alpha_s}$. The trilinear parameters $A_t$, $A_b$ and $\mu$ of the Higgs boson interaction with the third generation squarks can be generally speaking complex. In this case the $\lambda_i$, $i = 1, \ldots, 7$ parameters of the two-doublet Higgs potential are defined by $\tan \beta$, SUSY scale $M_{SUSY}$, and six relevant parameters in the sector of Higgs boson interaction with the third generation squarks: $\mu$, $\arg(\mu)$, $A_t$, $\arg(A_t)$, $A_b$, $\arg(A_b)$. In the following consideration for simplicity we assume $|A_t| = |A_b|$ and assign the universal phase $\theta$ to $\mu A_t$ and $\mu A_b$ so that $\theta = \arg(\mu A_t) = \arg(\mu A_b)$. Then using the explicit structure of (52) CP invariance conditions (39) can be rewritten in the form $\text{Im}(\mu_1^2 \mu A) = 0 [5]$. The couplings of $W$ and $Z$ bosons to the $h_1, h_2, h_3$ scalars have the form

$$V_\mu V_\nu h_1 = f_V g_{\mu\nu}(c_{\alpha-\beta}a_{21} - s_{\alpha-\beta}a_{11})$$

$$V_\mu V_\nu h_2 = f_V g_{\mu\nu}(c_{\alpha-\beta}a_{22} - s_{\alpha-\beta}a_{12})$$

$$V_\mu V_\nu h_3 = f_V g_{\mu\nu}(c_{\alpha-\beta}a_{23} - s_{\alpha-\beta}a_{13})$$

where $V = W, Z$, $f_W = \frac{e}{s_W} m_W$ and $f_Z = \frac{e}{s_W c_W} m_W$. The couplings of $h_1, h_2, h_3$ bosons to the $t$ and $b$ quarks have the form

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\[ f_{t,b} = -\frac{e}{2s_W m_W} \]

The Higgs boson mass spectrum of the CP conserving limit \( \theta = 0 \) (in this limit \( a_{ij} = \text{diag}\{1,1,1\} \)) is shown in Fig.1. For the case of explicit CP violation in the two-doublet Higgs potential we take the parameter set \( \mu = -2 \) TeV, \( A_t = A_b = -1.8 \) TeV, \( M_{SUSY} = 0.5 \) TeV, \( m_A = 220 \) GeV, \( \tan \beta = 4 \) which is typical for the region of MSSM parameter space where the imaginary parts of \( \lambda_5, \lambda_6 \) and \( \lambda_7 \) are large (of the order of 0.1-1) \(^5\). We demonstrate in Fig.2 the neutral Higgs boson masses given by (47) and the mixing matrix elements \( a_{ij} \) as a function of the universal phase \( \theta = \arg(\mu A_{t,b}) \). The Higgs boson masses of the CP conserving limit are substantially changed when the phase \( \theta \) is not small. The \( m_h \) in Fig.2 is always smaller than \( m_h \) and \( m_h \) has a downfall at the phase values around \( \pi/4 \). The Higgs-vector boson \( WW_h \), \( ZZ_h \) and the Higgs-fermion \( q\bar{q}h_i \) (\( q=t,b \)) interaction vertices as a function of the phase \( \theta \) are shown in Fig.3. One can observe that the \( h_1 \) couplings to gauge bosons \( W,Z \) decrease by about 15% if the phase of \( \lambda_5, \lambda_6, \lambda_7 \) is large enough. Nonzero couplings of \( h_3 \) to gauge bosons appear. The changes of the \( bbh_1 \) and the \( bbbh_2 \) coupling regime are also rather pronounced (see Fig.3). In the region of MSSM parameter space where the \( m_{h_3} \) is around 150-250 GeV and the \( \mu \) and \( A_{t,b} \) parameters are of the order of TeV the regime of strong mixing in the Higgs sector takes place. As a result the light Higgs boson \( h_1 \) could have not been observed at LEP2 \( (\sqrt{s} = 200 \) GeV) in the production channels \( e^+e^- \rightarrow h_1Z, e^+e^- \rightarrow \nu\bar{\nu}h_1 \) for the reason of suppressed couplings to gauge bosons, while the \( h_2, h_3 \) bosons are sufficiently heavy to be not produced on mass-shell at the LEP2 energy. Detailed analysis of this scenario can be found in [5, 6].

\(^{5}\)A detailed discussion of possible combined constraints on the MSSM parameter space from the cosmology, direct searches and indirect measurements (rare decays) can be found in [27].
5 Triple and quartic Higgs boson couplings in the MSSM with explicit CP violation

In the regions of the MSSM parameter space where the couplings of lightest Higgs boson $h_1$ to gauge bosons and top quark are suppressed, traditional channels of Higgs boson production by radiation from $W,Z$ or $t$ line and $WW, ZZ$ fusion can have too small rate to be experimentally observed. For this reason it is interesting to consider the possibility of double Higgs production (like $gg \rightarrow h_2 \rightarrow h_1 h_1$) defined by the self-coupling vertices. Such calculations are known in the CP conserving limit [8], when the cross sections of double and triple Higgs boson production turn out to be very small. Only some of them are accessible for observation at high luminosity colliders. In the case of CP violation some self-couplings can be substantially increased, providing better possibilities for the experimental reconstruction.

The $\lambda_5, \lambda_6$ and $\lambda_7$ potential terms can modify significantly the Higgs boson self-interaction vertices calculated in the leading one-loop approximation with $\lambda_i = 0$ ($i = 5, 6, 7$). At the next-to-leading order approximation the $\lambda_i$ couplings (52) include the terms of the order of $h_{t,b}^4 \mu^2 A_{t,b}^2 / M_{SUSY}^4$ and $h_{t,b}^4 \mu A_{t,b} / M_{SUSY}^2$, so they can reach the values of the order of 0.1-1 at moderate values of $M_{SUSY}$ and $\mu$ and $A_{t,b}$ taken at TeV energy scale. For example, in the CP conserving limit $\theta = 0$ the $hhh$ vertex in the mass parametrization has the form

$$g_{hhh} = \frac{3e}{m_W s_W s_{2\beta}} \left[ -(c_\beta c_\alpha - s_\beta s_\alpha)m_h^2 + c_\beta - a c_\beta + a m_A^2 \right]$$

The contributions of $\lambda_{5,6,7}$ terms and the $m_{h,A}^2$ terms in this expression are of the same order if $\lambda_{5,6,7} \sim O(1)$. The rotation of the $\theta = 0$ mass eigenstates by the matrix $a_{ij}$ defined by (45) gives the $g_{h_1 h_1 h_1}$ vertex of different form but also with substantial contributions of the $\lambda_{5,6,7}$ terms.

Using the parameter set described in the previous section, we show the values of various triple and quartic Higgs boson self-interaction vertices as a function of the universal phase $\theta = \arg(\mu A_t) = \arg(\mu A_b)$ in Fig.4. The values of Higgs boson self-interaction vertices in the CP-conserving limit $\theta = 0, \pi$ and in the leading order approximation $\lambda_5 = \lambda_6 = \lambda_7 = 0$ are marked in Fig.4 by horizontal arrows. The $\lambda_5$, $\lambda_6$ and $\lambda_7$ potential terms induced in the next-to-leading order approximation introduce very large corrections to the triple and quartic self-interactions of Higgs bosons. In the region of the MSSM parameter space under consideration the difference of the leading order and the next-to-leading order vertex factors can be several times in some ranges of the phase variation.
6 Summary

We demonstrated the tree-level equivalence of the two-Higgs-doublet model potentials (1) and (9), where CP-invariance can be explicitly broken by the \( \lambda_6 \) term in (1) or by the complex \( \mu_1^2, \mu_2^2, \mu_{12}, \lambda_i \) (i=1,...6) of (9). The parameters \( \lambda_i \) (i=1,...6) of (1) and \( \mu_1^2, \mu_2^2, \mu_{12}, \lambda_i \) (i=1,...5) of (9) are related by the equations (10). In the case of real parameters the diagonalization of potential (9) in the ground state can be performed by means of the substitutions (14)-(20) which express the \( \lambda_i \) and \( \mu_1^2, \mu_2^2 \) parameters through the Higgs boson masses \( m_h, m_H, m_A, m_{H^\pm} \), the mixing angles \( \alpha, \beta \) and the \( \mu^2_{12} \) parameter. In the general case the \( \lambda_5, \lambda_6 \) and \( \lambda_7 \) potential terms (21) should be also considered with the diagonalization and minimization conditions (22)-(28). If the complex parameters \( \mu_1^2, \mu_2^2, \lambda_i \) (i=1,...7) are introduced, CP invariance of the hermitian potential (37) is explicitly violated at the tree-level unless the fine-tuning conditions (38) or (39) for parameters are satisfied. So in the following we consider the problem of diagonalization in the local minimum for the two-Higgs-doublet potential which is not CP invariant. For the diagonalization of potential (37) again we use the substitution (22)-(28) to be taken for real parts of parameters. The minimization of potential (37) at the tree level takes place with the condition \( c_0 = 0 \) (43) for the imaginary parts of parameters. The imaginary parts of \( \lambda_5, \lambda_6 \) and \( \lambda_7 \) give rise to the CP-odd/CP-even Higgs boson off-diagonal terms, which are removed by the orthogonal rotation in \( (h, H, A) \) space, giving mass eigenstates \( h_1, h_2, h_3 \) without definite CP-parity and with the mass spectrum and couplings substantially different from the masses and couplings of CP-even and CP-odd states \( h, H, A \), if the imaginary parts of parameters \( \lambda_5, \lambda_6 \) and \( \lambda_7 \) are sufficiently large.

In the framework of MSSM the real parts of \( \lambda_i \) (i=1,...5) couplings are fixed at the SUSY energy scale by the conditions (29). Radiative corrections to the \( \lambda_i^{SUSY} \) (i=1,...7) couplings are generated at the \( m_W \) energy scale. The equations (31)-(33) express the mixing angle \( \alpha \) and masses of Higgs bosons in terms of the radiative corrections to \( \lambda_i^{SUSY} \) (i=1,...7) couplings (e.g. given by the RG evolution). They are valid independently on the particular scheme which is used for calculation of radiative corrections to the \( \lambda_i^{SUSY} \) (i=1,...7).

In the next-to-leading order approximation the complex \( \lambda_5, \lambda_6 \) and \( \lambda_7 \) parameters are generated by the soft CP violating Yukawa interactions of Higgs bosons with the scalar quarks. Using the results of [5] we calculated the Higgs-gauge boson, Higgs-fermion and the Higgs triple and quartic couplings for a representative MSSM parameter set, when the off-diagonal elements of the Higgs boson mixing matrix are large. The \( \lambda_5, \lambda_6 \) and \( \lambda_7 \) parameters introduce significant corrections to the Higgs self-interaction, even in the case when their effects on the Higgs-gauge boson and Higgs-fermion couplings
are rather small. These corrections could rather strongly (by one-two orders of magnitude in comparison with the case of CP conservation) enhance or suppress some channels of multiple Higgs boson production at next colliders, providing discriminative tests of CP violation in the Higgs sector and improved feasibility to reconstruct experimentally the Higgs potential.

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### Variational derivative of Lagrangian by fields

| Fields in the vertex | Variational derivative of Lagrangian by fields |
|----------------------|-----------------------------------------------|
| \( h \ h \ h \)     | \[ -s_2\beta(c^3_\alpha c_\beta - s^3_\alpha s_\beta)m^2_H + 2c^2_\alpha-\beta c_{\alpha+\beta}\mu^4_{12} \] |
| \( H \ H \ H \)     | \[ -s_2\beta(c^3_\alpha s_\beta + s^3_\alpha c_\beta)m^2_H + 2s^2_\alpha-\beta s_{\alpha+\beta}\mu^4_{12} \] |
| \( H \ H \ h \)     | \[ -(2m^2_H + m^2_h)s_{2\alpha} + 2s_\beta c_{\alpha+\beta}\mu^2_{12} \] |
| \( H \ h \ h \)     | \[ -(m^2_H + 2m^2_h)s_{2\alpha} + 4c_\beta c_{\alpha+\beta}\mu^2_{12} \] |
| \( H \ A \ A \)     | \[ s_2\beta(c_\alpha c^3_\beta - c_\alpha c^3_\beta)m^2_H + 2s^2_\beta s_{\alpha+\beta}\mu^2_{12} \] |
| \( h \ A \ A \)     | \[ s_2\beta(c_\alpha c^3_\beta - c_\alpha c^3_\beta)m^2_H + 2s^2_\beta s_{\alpha+\beta}\mu^2_{12} \] |
| \( h \ H^+ \ H^- \)  | \[ s_2\beta(c_\alpha c^3_\beta - c_\alpha c^3_\beta)m^2_H + 2s^2_\beta s_{\alpha+\beta}\mu^2_{12} \] |
| \( H \ H^+ \ H^- \)  | \[ -s_2\beta(c_\alpha c^3_\beta - c_\alpha c^3_\beta)m^2_H + 2s^2_\beta s_{\alpha+\beta}\mu^2_{12} \] |

**Table 1.** Triple Higgs boson interaction vertices in the general two-Higgs-doublet model, \( \mu_{12} \) parametrisation.

| Fields in the vertex | Variational derivative of Lagrangian by fields |
|----------------------|-----------------------------------------------|
| \( h \ h \ h \)     | \[ -(c^3_\alpha c_\beta - s^3_\alpha s_\beta)m^2_H + c^2_\alpha-\beta c_{\alpha+\beta}(m^2_A + v^2\lambda_5) \] |
| \( H \ H \ H \)     | \[ -(c^3_\alpha s_\beta + s^3_\alpha c_\beta)m^2_H + s^2_\alpha-\beta s_{\alpha+\beta}(m^2_A + v^2\lambda_5) \] |
| \( H \ H \ h \)     | \[ -(2m^2_H + m^2_h)s_{2\alpha} + 2(3s_\alpha c_\alpha + s_\beta c_\beta)(m^2_A + v^2\lambda_5) \] |
| \( H \ h \ h \)     | \[ -(m^2_H + 2m^2_h)s_{2\alpha} - 2(3s_\alpha c_\alpha - s_\beta c_\beta)(m^2_A + v^2\lambda_5) \] |
| \( H \ A \ A \)     | \[ (s_\alpha c^3_\beta + c_\alpha s^3_\beta)m^2_H + c_\beta s_{\alpha+\beta}(m^2_A + v^2\lambda_5) \] |
| \( h \ A \ A \)     | \[ (s_\alpha c^3_\beta - c_\alpha s^3_\beta)m^2_H + c_\beta s_{\alpha+\beta}(m^2_A + v^2\lambda_5) \] |
| \( h \ H^+ \ H^- \)  | \[ (s_\alpha s^3_\beta - c_\alpha c_\beta)m^2_H + s_\alpha s_{\alpha+\beta}(m^2_A + v^2\lambda_5) \] |
| \( H \ H^+ \ H^- \)  | \[ -(s_\alpha s^3_\beta + c_\alpha c_\beta)m^2_H + s_\alpha s_{\alpha+\beta}(m^2_A + v^2\lambda_5) \] |

**Table 2.** Triple Higgs boson interaction vertices in the general two-Higgs-doublet model, \( \lambda_5 \) parametrisation.
| Fields in the vertex | Variational derivative of Lagrangian by fields |
|----------------------|------------------------------------------------|
| \( h' h' h' h' \)   | \(-\frac{3}{4} \frac{e^2}{M_W^2 s^2 \beta^2} [4s_23(c^\alpha_\alpha s^\beta_\beta - s^\alpha_\beta s^\beta_\alpha)^2 m_H^2 + s_23 s_23 a^2_\alpha - \beta m_H^2 - 8c^2_\alpha - \beta c^2_\alpha + \beta \mu_2] \) |
| \( H H H H \)       | \( \frac{3}{4} \frac{e^2}{M_W^2 s^2 \beta^2} [s_23(c_\alpha^3 s_\beta^3 + c_\alpha s^3_\beta)^2 m_H^2 s_23(c_\alpha c_\beta - s_\alpha s^3_\beta)^2 m_H^2 - 2c^2_\beta \mu_2] \) |
| \( A^0 A^0 A^0 A^0 \) | \(-3 \frac{e^2}{M_W^2 s^2 \beta^2} [s_23(c_\alpha^3 s_\beta^3 + c_\alpha s^3_\beta)^2 m_H^2 s_23(c_\alpha c_\beta - s_\alpha s^3_\beta)^2 m_H^2 - 2c^2_\beta \mu_2] \) |
| \( H H H h \)       | \(-\frac{3}{4} \frac{e^2}{M_W^2 s^2 \beta^2} [s_23(c_\alpha^3 s_\beta^3 + c_\alpha s^3_\beta)^2 m_H^2 + s_23 s_2 a^2_\alpha - \beta m_H^2 + 4s_\alpha^2 - \beta c^2_\beta \mu_2] \) |
| \( H H H h \)       | \( -\frac{3}{4} \frac{e^2}{M_W^2 s^2 \beta^2} [s_23(c_\alpha^3 c_\beta - s_\alpha s^3_\beta)^2 m_H^2 + s_23 s_2 a^2_\alpha - \beta m_H^2 + 4s_\alpha^2 - \beta c^2_\beta \mu_2] \) |
| \( H H H h \)       | \(-\frac{1}{4} \frac{e^2}{M_W^2 s^2 \beta^2} [s_23(c_\alpha^3 c_\beta - s_\alpha s^3_\beta)^2 m_H^2 + s_23 s_2 a^2_\alpha - \beta m_H^2 + 4s_\alpha^2 - \beta c^2_\beta \mu_2] \) |
| \( H H A^0 A^0 \)   | \( \frac{1}{4} \frac{e^2}{M_W^2 s^2 \beta^2} [s_23(c_\alpha^3 c_\beta - s_\alpha s^3_\beta)^2 m_H^2 + s_23 s_2 a^2_\alpha - \beta m_H^2 - 2s_23 c^2_\alpha - \beta m_2^2] \) |
| \( h h A^0 A^0 \)   | \( \frac{1}{4} \frac{e^2}{M_W^2 s^2 \beta^2} [-s_23 s_2 a^2_\alpha - \beta(c_\alpha c^3_\beta - s_\alpha s^3_\beta)^2 m_H^2 + 2s_\alpha^2 - \beta c^2_\beta (c^3_\beta - s_\alpha s^3_\beta)^2 \mu_2] \) |
| \( H A^0 A^0 h \)   | \(-2 \frac{e^2}{M_W^2 s^2 \beta^2} [s_23(c_\alpha^3 c_\beta - s_\alpha s^3_\beta)^2 m_H^2 + s_23 s_2 a^2_\alpha - \beta m_H^2 - 2s_23 c^2_\alpha - \beta m_2^2] \) |
| \( H H H^+ H^- H^- H^- \) | \(-2 \frac{e^2}{M_W^2 s^2 \beta^2} [s_23(c_\alpha^3 c_\beta - s_\alpha s^3_\beta)^2 m_H^2 + 2s_\alpha^2 - \beta c^2_\beta (c^3_\beta - s_\alpha s^3_\beta)^2 \mu_2] \) |
| \( H H H^+ A^0 A^0 \) | \(-\frac{e^2}{M_W^2 s^2 \beta^2} [s_23(c_\alpha^3 c_\beta - s_\alpha s^3_\beta)^2 m_H^2 + s_23 s_2 a^2_\alpha - \beta m_H^2 - 2s_23 c^2_\alpha - \beta m_2^2] \) |
| \( H H H^- h \)     | \( \frac{e^2}{M_W^2 s^2 \beta^2} [s_23(c_\alpha^3 c_\beta - s_\alpha s^3_\beta)^2 m_H^2 + s_23 s_2 a^2_\alpha - \beta m_H^2 - 2s_23 c^2_\alpha - \beta m_2^2] \) |
| \( H H H^- h \)     | \(-2 \frac{e^2}{M_W^2 s^2 \beta^2} [s_23(c_\alpha^3 c_\beta - s_\alpha s^3_\beta)^2 m_H^2 + s_23 s_2 a^2_\alpha - \beta m_H^2 - 2s_23 c^2_\alpha - \beta m_2^2] \) |
| \( H H H^- h \)     | \( \frac{1}{2} \frac{e^2}{M_W^2 s^2 \beta^2} [-s_23 s_2 a^2_\alpha - \beta(c_\alpha c^3_\beta - s_\alpha s^3_\beta)^2 m_H^2 + 2s_\alpha^2 - \beta c^2_\beta (c^3_\beta - s_\alpha s^3_\beta)^2 \mu_2] \) |

Table 3. Quartic Higgs boson interaction vertices in the general two-Higgs-doublet model, \( \mu_2 \) parametrisation.
Figure 1: Masses of the neutral and charged Higgs bosons $h, H, H^\pm$ versus the pseudoscalar mass $m_A$ and the trilinear constants $A_t, A_b$ calculated by means of (31)-(33) with the analytical $\bar{\lambda}_i$ (i=1,...7) parametrization of [5]. The $\Delta \bar{\lambda}_5$ is chosen to be positive. The CP-conserving limit $\theta = 0$ is taken. (a) $\tan \beta = 4, M_{\text{SUSY}} = 0.5$ TeV, $A_t = A_b = \mu = 0$; (b) $\tan \beta = 4, M_{\text{SUSY}} = 0.5$ TeV, $A_t = A_b = 0.9$ TeV, $\mu = -1.5$ TeV; (c) $\tan \beta = 4, M_{\text{SUSY}} = 0.5$ TeV, $m_A = 220$ GeV, $\mu = 0$, $A_t = A_b$; (d) $\tan \beta = 4, M_{\text{SUSY}} = 0.5$ TeV, $m_A = 220$ GeV, $\mu = -2$ TeV, $A_t = A_b$. Very small variations of the charged Higgs boson mass $m_{H^\pm}$ in (d) are due to the cancellation of leading power terms $\sim \mu^2 A_{t,b}^2/M_{\text{SUSY}}^4$, see [5], in the difference of $\Delta \bar{\lambda}_4$ and $\Delta \bar{\lambda}_5$, see (33). If $\Delta \bar{\lambda}_5$ is chosen to be negative (purely imaginary $\mu A$ in (52)), $m_H$ increases in comparison with the case $A_t = A_b = \mu = 0.$
Figure 2: Masses of the neutral Higgs bosons and the mixing matrix elements as a function of the $\bar{\lambda}_6$ and $\bar{\lambda}_7$ phase. The $\bar{\lambda}_i$ parameters are taken from [5] at the MSSM parameter values $\tan \beta = 4$, $m_A = 220$ GeV, $M_{SUSY} = 0.5$ TeV, $A_t = A_b = -1.8$ TeV, $\mu = -2$ TeV.
Figure 3: Higgs-gauge boson and Higgs-fermion vertex factors as a function of the $\bar{\lambda}_6$ and $\bar{\lambda}_7$ phase. The $\bar{\lambda}_i$ parameters are taken from [5] at the MSSM parameter values $\tan \beta = 4$, $m_A = 220$ GeV, $M_{SUSY} = 0.5$ TeV, $A_t = A_b = -1.8$ TeV, $\mu = -2$ TeV. For the coupling with fermions we plot $\sqrt{\text{Im}^2 g_{ffh} + \text{Re}^2 g_{ffh}}$. 
Figure 4: Triple and quartic Higgs boson vertex factors as a function of the $\bar{\lambda}_6$ and $\bar{\lambda}_7$ phase. The $\bar{\lambda}_i$ parameters are taken from [5] at the MSSM parameter values $\tan\beta = 4$, $m_A = 220$ GeV, $M_{SUSY} = 0.5$ TeV, $A_t = A_b = -1.8$ TeV, $\mu = -2$ TeV. Horisontal arrows indicate the values of vertex factors in the CP-conserving limit $\theta = 0$ and the leading order approximation $\bar{\lambda}_5 = \bar{\lambda}_6 = \bar{\lambda}_7 = 0$. 