Interaction of vector solitons with a nonlinear interface

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We develop the analytical method of field momenta for analyzing the dynamics of optical vector solitons in photorefractive nonlinear media. First, we derive the effective evolution equations for the parameters of multi-component solitons composed of incoherently coupled beams and investigate the soliton internal oscillations associated with the relative motion of the soliton components. Then, we apply this method for analyzing the vector soliton scattering by a nonlinear interface.

In particular, we show that a vector soliton can be reflected, transmitted, captured, or split into separate components, depending on the initial energy of its internal degree of freedom. The results are verified by direct numerical simulations of spatial optical solitons in photorefractive nonlinear media.

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I. INTRODUCTION

Spatial solitons are self-trapped optical beams for which diffraction divergence is exactly compensated by nonlinear self-focusing. Such solitons possess many important properties which are believed to find a wide range of potential applications in optical scanning, switching, and processing devices. Spatial solitons can be generated in photorefractive crystals at low powers (of order of µW) and, therefore, such materials look promising from a viewpoint of their use for the soliton generation in all-optical information circuits.

One of the important properties of spatial solitons in photorefractive media is their stability against collapse and break-up even in a bulk medium due to the saturating nature of nonlinearity. Spatial solitons in (1+1)-dimensional systems can be realized for the light propagation in weakly guiding planar structures, where self-focusing properties of the core material lead to the light localization along the plane of a linear guiding structure.

Usually, a spatial soliton is viewed as a lowest-order (fundamental) mode of an effective waveguide it excites in a nonlinear medium. Moreover, higher-order modes of the induced waveguide, that are associated with an effective soliton-induced potential possessing more than one energy level, can be excited and such modes can be treated as the soliton internal modes. Such internal modes are responsible for oscillations of the soliton amplitude and width, and they have already been studied in a number of theoretical papers. A much more interesting situation takes place in the case of the so-called vector spatial solitons, i.e. multi-component optical solitons which consist of several mutually incoherent (or orthogonally polarized) co-propagating beams. This problem got a little of attention in the past, but it attracts much attention in view of recent experimental studies of multicomponent optical solitons.

The aim of the present paper is twofold. First, we study the internal dynamics of (1+1)-dimensional vector solitons, associated with the transverse relative oscillation of the centers of mass of the partial soliton components. Second, for the first time to our knowledge, we study the scattering of a vector soliton by a nonlinear interface and demonstrate that the scattering process can be modified dramatically by initial excitation of the soliton internal modes.

As an example, we consider the dynamics of solitons in photorefractive nonlinear media taking into account saturation of the medium nonlinearity. A standard way for studying the excited states of solitons is the analysis of the corresponding linear eigenvalue problem for small perturbations. However, in the case of nonintegrable systems, such excited eigenstates can be found only by rather complicated computer simulations (see, for example, Ref. and references therein). In contrast, here we suggest the method of momenta and applied it to analyze the scattering problem. As a matter of fact, such a method corresponds to the so-called quasi-classical approximation in quantum mechanics. Regardless of the fact that the method of momenta leads to the continuous spectrum of the eigenstates, it allows us to consider large perturbations (including the vector soliton splitting into individual components), since this approach is dealing with average characteristics of a vector soliton.

II. VECTOR SPATIAL SOLITONS.

THE METHOD OF MOMENTA

We consider spatial vector solitons consisting of N incoherently coupled components, which are described by
the following set of Schrödinger-like coupled nonlinear equations (see, e.g. Ref. [7] and references therein):

\[ i \frac{\partial \Psi_j}{\partial x} + \frac{\partial^2 \Psi_j}{\partial z^2} - \lambda_j \Psi_j + \epsilon_{NL}(I) \Psi_j = 0, \]

where \( x \) and \( z \) are the propagation and transverse coordinates, respectively, \( \Psi_j \) (\( j = 1, \ldots, N \)) are the dimensionless slowly varying amplitudes of the beam components, \( \lambda_j \) are the partial propagation constants, \( \epsilon_{NL}(I) \) is the nonlinear correction to the dielectric permittivity, \( I = \sum_{j=1}^{N} I_j \) is the total intensity of the beam, and \( I_j = |\Psi_j|^2 \) are the partial intensities of the soliton components. We assume that the nonlinear correction to the dielectric permittivity depends only on the total intensity of the light, \( I \), which can be written approximately as follows:

\[ \epsilon_{NL}(I) = I/(1 + sI), \]

where \( s \) is the saturation parameter. A ground state of a vector spatial soliton corresponds to the stationary solution of eq. (1) with \( \partial / \partial x = 0 \) in which \( \Psi_j = \Psi_j^{(0)}(z - Z_j) \), where \( \Psi_j^{(0)}(z) \) are real localized functions and \( Z_j \) is the center of mass of the \( j \)-th component. As a rule, the values \( Z_j \) for the vector solitons in the ground state are equal to each other and coincide with its center of mass. The dynamical behaviour of an excited vector soliton, such as oscillation of its amplitude and width, oscillation of the centers of mass of its components \( Z_j \), or superposition of these oscillations, depends on initially excited intrinsic degrees of freedom. Here, it is important to note an analogy between a vector spatial soliton and a bound state of quantum particles. Indeed, the difference lies only in self-consistent nonlinear nature of the potential that provides the confinement for soliton components.

To study excited states of vector solitons we use the method of momenta that has been initially developed for scalar solitons of a Kerr-type nonlinear medium [8]. Here we generalize this approach and technique to study multicomponent spatial solitons in photorefractive media.

The method of momenta describes the electromagnetic beams in terms of integral field characteristics, or field momenta [8]. The first momentum is defined as

\[ P_j = \int_{-\infty}^{\infty} I_j \, dz. \]

It has the obvious physical meaning of the energy flux in the \( j \)-th soliton component. The second momentum is defined as

\[ Z_j = \int_{-\infty}^{\infty} z I_j \, dz. \]

Here the value \( Z_j = Z_j / P_j \) is the center of mass of the \( j \)-th beam component.

The purpose of this section is to derive a close set of effective ordinary evolution equations for the momenta [8]. Similar procedure is used in semiclassical description of quantum systems (see, e.g. Ref. [8]). Let us multiply Eq. (1) by \( \Psi_j^* \) to obtain an equation for \( P_j \), and by \( z \Psi_j^* \) to obtain an equation for \( Z_j \), and then subtract the corresponding complex conjugated equations. The integration of the resulting equations over \( z \) yields

\[ \frac{dP_j}{dx} = 0, \]

\[ i \frac{dP_j}{dx} + \frac{d^2 Z_j}{dx^2} = \int_{-\infty}^{\infty} \left( \Psi_j^* \frac{\partial \Psi_j}{\partial z} - \Psi_j \frac{\partial \Psi_j^*}{\partial z} \right) \, dz. \]

Equation (3) means that the energy flux in each individual soliton component is a conserved quantity. Thus, there is no energy exchange between the components. After differentiation of Eq. (1) by \( x \) and subsequent integration over \( z \), we obtain

\[ P_j \frac{d^2 Z_j}{dx^2} = \int_{-\infty}^{\infty} I_j \frac{\partial \epsilon_{NL}}{\partial z} \, dz. \]

The physical meaning of the Eqs. (3) comes from an analogy with the geometrical optics: the Eqs. (3) determine optical rays that correspond to the partial beams forming a vector soliton. From the set of equations (3) one can see that the center of mass of the whole beam moves along a straight line:

\[ \frac{d^2 Z_j}{dx^2} = \frac{d^2}{dx^2} \left( \frac{Z_1 P_1 + Z_2 P_2 + \ldots + Z_N P_N}{P_1 + P_2 + \ldots + P_N} \right) = 0. \]

The set of Eqs. (3) is not closed. It cannot be used directly since, in order to trace the dynamics of the soliton components, we should know the electric field at each point \((x, z)\) in space, i.e. effectively complete solutions of the Eqs. (3) are required. In this sense Eqs. (3) are equivalent to Eqs. (1), and so far we have not gained anything in addition. There are two ways to derive a closed set of equations from Eqs. (3). The first one is to obtain equations for higher order momenta which, nonetheless, does not guarantee that the system of equations will become closed. A simpler way, which allows us to obtain the desired result, is to employ a trial function to be substituted into Eqs. (3) instead of the unknown functions \( \Psi_j \). It is clear that, for small deviations of the soliton constituents from the lowest-order (ground) equilibrium state, we can choose the trial functions in the form of unperturbed stationary soliton components. Moreover, since within Eqs. (3) we are operating with the average (integral) characteristics of the soliton, one can guess that the result of the integration will not be sensitive to variations of the transverse soliton structure, even for large deviations from the ground state. This is a rather strong statement, and it will be verified below by means of direct numerical simulations of the Eqs. (3).
Fig. 1: (a) Effective potential of the internal motion of the soliton components, (b) transverse structure of the soliton with identical components, (c) solid – trajectories of the centers of mass of the beams calculated by the method of momenta; dashed – the same values found by numerical simulations of the Eqs. (9).

III. TWO-COMPONENT SOLITONS

To demonstrate the internal dynamics of the soliton excited states, we consider a particular case of two-component vector solitons. As discussed above, it is natural to choose the trial function for the Eqs. (7) in the form of a stationary solution \( \Psi_j(z) \). According to Eqs. (7), the relative shift between the soliton components, \( \Delta = Z_2 - Z_1 \), is governed by the following equation:

\[
\bar{P} \frac{d^2 \Delta}{dx^2} = F(\Delta),
\]

where \( \bar{P} = P_1 P_2 / (P_1 + P_2) \) is the reduced mass, and

\[
F(\Delta) = 2 \int_{-\infty}^{\infty} I_2(z + \Delta) \frac{\partial \epsilon_{NL}}{\partial z} [I_1(z) + I_2(z + \Delta)] dz.
\]

Let us now introduce the effective potential of the beams interaction as follows

\[
F(\Delta) = -\frac{dV(\Delta)}{d\Delta}.
\]

We have now come to the simple mechanical model for the internal dynamics of the two-component soliton, which describes two interacting quasi-particles in a bound state. This state can be thought of as a "photon molecule". Fig. 3 shows the effective potential \( V(\Delta) \) for numerically calculated one-hump soliton with two identical components. The minimum of the effective potential corresponds to the stationary soliton in a ground state. The internal oscillations of excited soliton components deduced from the solution of Eq. (9) (shown in Fig. 1 (c) by solid lines) are compared to the numerical solution of eqs.(1) (dashed lines in Fig. 1 (c)). The damping of the oscillations occurs due to radiation losses of the energy from the soliton originating from bends of the soliton component trajectories, which are not taken into account in the method of momenta.

The next example deals with the two-hump solitons with the transverse structures of the components shown in Fig. 2 (a,c) [11]. One can see that, when the amplitude of the two-hump component is significantly smaller than the amplitude of the one-hump one, the stationary state with coaxial constituents is unstable because of the maximum of the potential at \( \Delta = 0 \) (see Fig. 2 (b)). It is interesting to note that, in this case, the ground equilibrium state of the soliton corresponds to the soliton components with displaced centers. Such (1+1)D asymmetric vector solitons are well known from the theory of the two-component Kerr solitons described by Manakov model \( s \to 0 \) in eqn. (2) [12] and for the partially coherent multi-component solitons in photorefractive medium [13]. However, when the amplitudes of the components are comparable, the coaxial state becomes stable again (see Fig. 2 (d)). The dynamics of the unstable coaxial soliton state calculated by solving eqs.(6), as well as eqs.(9), is shown in Fig. 3. One can see that, in this
case, the agreement between the method of momenta and numerical simulations is much worse than for the one-hump solitons because of the strong reshaping of the two-hump beams. This reshaping, along with the radiation processes, violates the assumptions of the analytical method.

IV. INTERACTION OF VECTOR SOLITONS WITH A NONLINEAR INTERFACE

The method of momenta looks promising for treating the problem of vector soliton dynamics in inhomogeneous media. In this section we consider interaction of a two-component vector soliton with an interface between two photorefractive media. Both linear and nonlinear properties of the materials are discontinuous on the interface. The problem of one-component photorefractive soliton interaction with nonlinear interface had recently been studied in [14], soliton interaction with interface in the Kerr-like medium can be found in [15, 16]. Let us represent the coordinate dependences of the partial propagation constants and nonlinear correction to dielectric permittivity by the following form:

\[
\begin{align*}
\lambda_{1,2}(z) &= \lambda_{1,2} + \delta \lambda_{1,2} \cdot 1(z), \\
\epsilon_{NL}(z) &= \epsilon_{NL}(I) \cdot [1 + \delta \epsilon_{NL} \cdot 1(z)]
\end{align*}
\]

where \(1(z) = \begin{cases} 1, & z > 0 \\ 0, & z < 0 \end{cases} \) is the unit step function. By the method of momenta one can obtain that the energy fluxes in both beams are conserved. For the centers of mass of the components we have:

\[
P_{1,2} \frac{d^2 Z_{1,2}}{dz^2} = \int_{-\infty}^{\infty} \left[ 2 + \delta \epsilon_{NL} \cdot 1(z) \right] I_{1,2} \frac{\partial \epsilon_{NL}(I)}{\partial z} dz - \\
- \delta \epsilon_{NL} I_{1,2} (Z_{1,2}) + \delta \lambda_{1,2} I_{1,2} (Z_{1,2}) \times \\
\times \epsilon_{NL} \left[ I_{1,2} (Z_{1,2}) + I_{2,1} (Z_{2,1}) \right]
\]

All the terms in the Eqs. (13) have clear physical meaning. The terms that do not contain the jumps of the medium parameters represent the internal forces acting between soliton components. They are responsible for an internal dynamics of the soliton and, in the homogeneous case, can be reduced to the equation (8). The terms proportional to \(\delta \lambda_{1,2}\) and \(\delta \epsilon_{NL}\) are the external forces perturbing the internal oscillations of the soliton components at the collisions of quasi-particles with an interface.

To derive the closed set of equations describing the interaction of a vector soliton with an interface we substitute into Eqs. (13) trial functions in the form of the stationary unperturbed soliton component (see Section II). One can expect that this ansatz will be sufficiently accurate whenever transverse structures of the beams are not strongly affected by large discontinuities of the medium parameters. We will verify this assumption by comparing direct computer simulation of the wave equations (1) (in the case of inhomogeneous media) with the results of quasi-particle theory obtained with the method of momenta. In what follows, we study the following effects of soliton interaction with an interface: (a) excitation of internal degrees of freedom; (b) the change of the type of soliton interaction with an interface (i.e. reflection, transmission and capture) as function of the initial internal energy of the vector soliton launched under different initial conditions; (c) the splitting of pre-excited soliton into individual components at the collision with an interface. It should be noted, with regard to the point (a) above, that a spatial soliton consisting of identical components, being initially in ground state, does not become excited at the collision with an interface. This happens because, due to identical character of interaction of every component with interface, the coaxial structure of the soliton state is preserved.

The results that we present in this section are obtained for the one-hump soliton shown in the inset in Fig. 1 (a) and interface parameters are as follows: \(\delta \lambda_{1,2} = -0.15; \delta \epsilon_{NL} = 0.2; ds = 0\). The solutions of the ordinary differential equations (13) are shown in Fig. 4. Excitation of internal oscillations after soliton reflection from an interface is demonstrated in Fig. 4 (a). Soliton dynamics near the interface depends dramatically on the initial energy level of the internal oscillations of the components. In Fig. 4 (b) soliton components were launched to the interface at the same angle \((dZ_{1,2}/dz = -0.5)\) and from the same position of the common center of mass. The only adjustable parameter, which determines the initial internal energy of oscillations, was the relative shift be-
FIG. 4: (a) Excitation of the internal oscillations of the soliton at the reflection from the interface, inset shows the transverse structure of the soliton, (b) soliton interaction with the interface at different values of $\Delta$: 1 – $\Delta = 0.0$, 2 – $\Delta = 0.4$, 3 – $\Delta = 0.8$, 4 – $\Delta = 1.2$, 5 – $\Delta = 1.4$, (c) $\Delta = 2.2$, beams make up a bound state without interface, (d) beams are split at the interface.

FIG. 5: Numerical simulations: (a) excitation of the internal oscillations of the soliton at the reflection from the interface, (b) soliton interaction with the interface at different values of initial value of $\Delta$, (c) separation of the beams at the interface, $\Delta = 2.2$.

FIG. 6: Numerical simulations: strong reshaping of the two-hump soliton at the interaction with an interface takes place, surface and contour plots. Initial conditions are taken in the form of the soliton shown in Fig. 2 (a). Parameters of the interface are the same as in Fig. 1.

One can see that, depending on the value of $\Delta$, the transmission (curve 5 – $\Delta = 0.12$), reflection (curves 2, 3, 4 – $\Delta = 0.4, 0.8, 1.2$, respectively) and capture into the unstable nonlinear surface mode (curve 5 – $\Delta = 0.0$) take place.

It is interesting to note that the character of soliton interaction depends upon the parameter $\Delta$ non-monotonically, which can be explained by considering the phase of the internal oscillations at the interface. This phase depends on the initial value of $\Delta$ by virtue of anharmonicity of internal oscillations in the effective potential well. Fig. 3 demonstrates the results of numerical calculations of equations (1), in which $\lambda_{1,2}$ and $\epsilon_{NL}$ are taken according to eq. (12). Excitation of the internal degree of freedom takes place at the reflection from the interface Fig. 3 (a). The calculations shown in Fig. 3 (b) were carried out for the same initial conditions as for curves 1, 2, and 3 in Fig. 3 (b). The higher values of the initial relative shift $\Delta$ between individual components do not result in the same behaviour of the trajectories as in Fig. 3 (b). It can be easily explained by the strong damping of oscillations due to the radiation effects, which takes place for large values $\Delta$. As a result, the amplitude of the internal oscillations is small at the point where soliton meets the interface and the outcome of interaction is very similar to the one for the smaller initial values of $\Delta$. To conclude, one can check that method of momenta leads to the satisfactory agreement with the direct numerical solution of nonlinear Schrödinger equations for the values of $\Delta$ up to the half width of the soliton.

The splitting of the vector soliton into individual components is shown in Fig. 4. We have revealed that the splitting occurs at the angles of incidence of the soliton beam close to the angle of total internal reflection and for large enough values of initial relative shift between components ($\Delta = 2.2$, $dZ_1/dx = dZ_2/dx = -0.5$). Without interface, the soliton components make up a bound state (Fig. 4 (c)). At the presence of interface the method of momenta predicts that splitting into components takes place (Fig. 4 (d)). This effect can be treated in terms of the quasi-particle theory. Indeed, when the depth of ef-
fective potential well is of the same order that the energy level of the bound state, than even a small perturbation (at the interface) of this state can lead to its destruction.

We have obtained the similar splitting effect [shown in Fig. 5 (c)] by solving the equations (1) numerically for the initial conditions used in the method of momenta. In numerical simulations, we can observe the separation of the beam components, but in the case of such large values of the initial transverse shift, the value of the first momentum (4) fails to determine the position of the center of mass of the localized beams. Significant part of the energy is transferred into radiation, and the integration in (4) gives us the coordinate of the common center of mass of both localized and nonlocalized waves. It is clear that the less the radiation losses are, the better the position of the localized beam is described by (4).

Finally, it is necessary to note that the dramatic reshaping of the multi-hump solitons at the interaction with the interface takes place. Numerical simulations show that these complex beams can be transformed into a set of single hump solitons and a considerable part of the electromagnetic energy goes into the radiation even for a very small change of the parameters of the medium at the interface (see Fig. 6). This reshaping makes it impossible to apply the method of momenta to the problems of multi-hump soliton dynamics in the inhomogeneous media.

V. CONCLUSIONS

We have developed the method of momenta to analyze the dynamics of vector solitons consisting of two (or more) incoherently coupled components, in both homogeneous and inhomogeneous photorefractive nonlinear media. First, we have demonstrated that this method provides an effective analytical tool for the study of internal oscillations of vector solitons. Second, we have studied the scattering of a vector soliton by a nonlinear interface, and we have demonstrated that this type of the soliton interaction can lead to the excitation of the soliton’s internal oscillations. In particular, we have demonstrated that the same vector soliton incident onto an interface can become reflected, transmitted, captured by the interface, or even split into its constituents, depending solely on the energy of the initially excited internal oscillations.

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