skeletracks: automatic separation of overlapping fission tracks in apatite and muscovite using image processing

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ABSTRACT

One of the major difficulties of automatic track counting using photomicrographs is separating overlapped tracks. We address this issue combining image processing algorithms such as skeletonization, and we test our algorithm with several binarization techniques. The counting algorithm was successfully applied to determine the efficiency factor GQR, necessary for standardless fission-track dating, involving counting induced tracks in apatite and muscovite with superficial densities of about $6 \times 10^5$ tracks/cm$^2$.

INTRODUCTION

The fission track dating (FTD) is based on the spontaneous fission of $^{238}$U, an impurity in natural minerals such as apatite and zircon (Wagner and Van den Haute, 1992). The fission process releases two fragments. They trigger the displacement of atoms, leading to structural net alterations called latent tracks. After convenient etching, channels are formed along the latent track trajectory and become visible under an optical microscope. These channels are referred to as tracks, and their number is used to calculate the fission-track age. Tracks are counted at the microscope or in photomicrographs captured using a camera coupled to a microscope, in a time expensive process. Besides, track counting efficiency is observer dependent; to keep it constant, a trained observer must keep full attention during the several hours taken to analyze a sample for fission-track dating. The observer also must perform routine recalibration to maintain the same counting efficiency over time.

Algorithms for automatic track processing and counting in images from natural minerals have been proposed (e.g. Wadatsumi and Masumoto (1990); Petford and Miller (1992); Gleadow et al. (2009); de Siqueira et al. (2014); Donelick et al. (1999)). When associated with automatic systems for capturing photomicrographs, such algorithms have the potential to increase dating speed. However, these solutions still demand the counting results to be reviewed, and often adjusted, by the observer, being more time consuming than manual counting (Yasuda et al. 2005; Enkelmann et al. 2012). The major challenges to automatic track counting are detecting overlapping tracks, distinguishing tracks and material defects (e.g. surface scratches due to polishing), and identifying small tracks and defects of comparable size in the background of photomicrographs (Gleadow et al. 2009), which are straightforward tasks for experienced observers.

In this study we address the issue of automatically separating and counting overlapping tracks in apatite and muscovite photomicrographs, combining image processing algorithms. These techniques include the skeletonization algorithm, which was already proposed for separating overlapping tracks in a preliminary study (Lippold et al. 2007). The solution presented here does not exclude previous developments. Instead, it could be combined to other algorithms, and improve the speed and reliability of track counting. As a proof of concept, we apply the resulting algorithm to the determination of the efficiency factor GQR, fundamental for the standardless fission-track dating (Danhara and Iwano 2013).
MATERIAL AND METHODS

In this section we present the photomicrograph test set used in this study, along with the proposed methodology to separate tracks. It consists in:

1. Reading and filtering the input image.
2. Binarizing the filtered image.
3. Separating and skeletonizing each region containing candidate tracks in the binary image.
4. Characterizing each pixel in the skeletons according to their neighbors.
5. Classifying tracks in regions, based on the route and the Euclidean distance.

Apatite and muscovite photomicrographs

To illustrate and test the methodology presented here, we used photomicrographs of co-irradiated natural apatite samples from Durango, Mexico, and muscovite mica (Figure 1). To calculate GQR, tracks in both cases are generated by neutron-induced fission of $^{235}\text{U}$. This process will be further detailed in section . Durango is a yellowish fluorapatite, found as well-formed crystals in the Cerro de Mercado iron mine (Durango, Mexico). Its age, $31.44 \pm 0.18$ Ma (McDowell et al., 2005), and chemical compositions (Donelick et al., 1999) are well constrained. Durango apatite is widely used as age standard for (U-Th)/He and fission-track thermochronology. This sample is largely used also for methodological studies.

![Figure 1. Photomicrographs from the test dataset, presenting fission tracks in (a) muscovite mica and (b) apatite samples.](image)

Image filtering

We used median filters for smoothing the input images before further processing. After Tukey (Tukey, 1971) and others suggested the use of median filters for signal smoothing, Pratt and Frieden applied these filters for image processing (Pratt, 1975; Frieden, 1976). The median filtering of an input image when using an $k \times l$ window ($k$, $l$ being odd integers) is an image equal to the median of the gray levels of the pixels in a $k \times l$ window centered at each pixel in the input image (Huang et al., 1979).

To apply the median filters in the input images, we used the implementation available in scipy’s ndimage (Jones et al., 2001). The filtering window used has size $7 \times 7$. This smoothing could make small tracks to fade; however, small defects in the surface of the material would also fade, thus not being counted as tracks in further processing.

1These photomicrographs are contained in the folder orig\_figures, available in the Supplementary Material.
Image binarization

Photomicrographs captured from Durango apatite mounts were binarized using several algorithms:

- **Otsu (Otsu, 1979):** calculates the optimal threshold based on the minimal weighted sum of within-class variances chosen from pixels of regions of interest (ROI) and the background.

- **Yen (Yen et al., 1995):** calculates the threshold based on a maximum correlation criterion, which uses a cost function. It is a computationally efficient alternative to entropy measures.

- **Li (Li and Tam, 1998):** selects a threshold that minimizes the cross entropy between the original and thresholded images.

- **ISODATA (Ridler and Calvard, 1978):** chooses the threshold using iterations. At iteration \( n \), a new threshold \( T_n \) is calculated using the mean average of the classes “regions of interest” and “background”. The process is repeated until \( T_n - T_{n-1} \) becomes sufficiently small. ISODATA always converges when applied for two classes (Dias Velasco, 1980).

- **MLSS (de Siqueira et al., 2014):** the user chooses a threshold based on a list of results.

After binarizing the photomicrographs, we used three tools for enhancing the result:

- **Excluding small regions:** erasing small regions from the binary image avoid small scratches on the surface to be identified as tracks.

- **Filling regions:** tracks may have different gray levels within its extent. Filling closes the holes inside these tracks, when they are not separated completely by binarization; this avoid finding more tracks than desired when using skeletonization.

- **Clearing lower and right borders:** tracks crossing the image borders are shared by two counting areas. We use the “lower right corner” method to avoid counting these tracks twice, which means that objects touching the bottom and right edges are not counted. Counting tracks over many images reduce this kind of bias (Russ, 2011).

Separating and skeletonizing regions

After binarizing the input photomicrograph, we separate each distinct region in the binary image. Then, we reduce each region to a single pixel line using skeletonization (Lee et al., 1994) (Figure 2). This process makes it easier to find descriptors for overlapping tracks, as shown next.

Characterizing pixels

A major challenge of automatic track counting is distinguishing individual tracks in a cluster of overlapping tracks. Identifying the points where tracks intercept each other and where tracks end (their extremities) is a key step to distinguish cluster geometry. Using the skeletonized regions, intersections and extremities of tracks can be defined as two pixel sets:

- **Extremity pixels** have only one neighbor in the 8-pixel neighborhood (Figure 3(a)), thus indicating possible extremities of a track.

- **Intersection pixels** have more than two neighbors in the 8-pixel neighborhood (Figure 3(b)), indicating a possible overlapping between two or more tracks.

There is also the common case, where a pixel has two neighbors belonging to the same region (Figure 3(c)).

The simplest kind of track cluster contains only two tracks, which present three extremity and one intersection pixel (Figure 4). Considering these pixels, when the region does not have intersection pixels it represents only one track. When the region contains intersection pixels, it represents more than one track.
Figure 2. Binarizing and skeletonizing a region highlighted in an input photomicrograph. (a) Input photomicrograph containing the example region (red arrow). (b) Example region separated from the input photomicrograph. (c) Binarizing the region in (b) using the ISODATA algorithm (threshold: 133). (d) Skeletonizing the binary region in (c). Colormap: gray.

Classifying track candidates in regions
After obtaining the extremity and intersection pixels from a region, we choose the track candidates:

1. Extremity pixels are grouped in pairs. If there are only two extremity pixels, the region represents only one track.

2. We calculate the Dijkstra’s minimum cost path (Dijkstra,[1959]) passing by an intersection pixel, referred here as route, and the Euclidean distance between the two chosen pixels. The union of these curves form a region in the binary image.

3. The inner area from the region generated by the route and the Euclidean distance is given by the pixels within this region. The first track candidate is defined as the Euclidean distance between the pair of pixels yielding the smallest inner area (Figure 5).

After identifying the track candidate, its extremity pixels are excluded and the process starts again. The process continues until every extremity pixel in the skeleton has a pair (Figure 6). If the number of extremity pixels in the skeleton is odd, the last extremity pixel left will unite with the closest intersection pixel (Figure 6(b)).
Although it has been shown that these factors can have their values calculated separately (Jonckheere, which once were controversial (e.g. Hadler et al. (2003)) have been long resolved. Fission-track with neutrons in a nuclear reactor. The mica sheet is also etched after irradiation. The densities of tracks (a)neighbors; (4

The External Detector Method (EDM) is the dating approach chosen in most fission-track studies. Apatite grains containing fossil fission tracks are mounted in epoxy resin, grounded, polished (to reveal an internal plan surface) and etched. A muscovite mica sheet is coupled to the apatite mount and the set is irradiated with neutrons in a nuclear reactor. The mica sheet is also etched after irradiation. The densities of tracks in the apatite grains, \( \rho_f \), and in the co-irradiated mica, \( \rho_m \), are determined. The standardless fission-track age equation can be written as (Jonckheere, 2003):

\[
I = \frac{1}{\lambda} \ln \left( 1 + GQR \frac{\lambda R_U}{\rho_f \lambda_f C_{238}} \right)
\]

In Eq. 1 \( \lambda \) is the total decay constant, \( \lambda_f \) is the spontaneous fission decay constant, and \( C_{238} \) is the isotopic concentration of \( ^{238}U \). \( R_U \), the number of induced fission reactions per uranium atom, characterizes the neutron irradiation and is determined experimentally (for instance in (Iunes et al., 2002; Soares et al., 2014)). The decay constant \( \lambda_f \) and the direct quantification of neutron fluence, which once were controversial (e.g. Hadler et al. (2003)) have been long resolved. Fission-track determinations in the decade of 2000 (Guedes et al., 2000, 2003; Suzuki, 2005; Yoshioka et al., 2005) confirmed the value recommended by the International Union of Pure and Applied Chemistry (IUPAC), \( \lambda_f = (8.5 \pm 0.1) \times 10^{-17} a^{-1} \) (Holden and Hoffman, 2000). Extensive calibration of neutron quantification has also been carried out (De Corte et al., 1991; Van den Haute et al., 1988; Curvo et al., 2013; Soares et al., 2014).

The geometry factor \( G \) accounts for the different tracks source volumes generating tracks in apatite and mica. Apatite tracks are generated by fragments coming from below and above the exposed surface (4\( \pi \)-geometry) while mica tracks are generated only by fragments coming from the apatite (2\( \pi \)-geometry). Ideally, \( G = 1/2(2\pi/4\pi) \). \( Q \) is a procedural factor (Jonckheere, 2003) which considers the different etching and observation efficiencies between apatite and mica. \( R \) is the range deficit factor accounting for differences in the ability of mica and apatite to record etchable tracks (Iwano and Danhara, 1998). Although it has been shown that these factors can have their values calculated separately (Jonckheere, 2003), it is possible to determine the product GQR experimentally, usually using a standard sample.

To determine the value of GQR, we applied the technique of comparing the density of induced tracks in an internal apatite surface, \( \langle \rho_f \rangle_{IS} \), with the density of induced tracks in the external detector, \( \langle \rho_f \rangle_{ED} \),

![Figure 3. The 8-neighborhood characterized by a central pixel X\([0,0]\) and its neighbors. In this method, (a) X\([0,0]\) represents a extremity pixel (only one neighbor, X\([1,1]\]); (b) X\([0,0]\) represents a intersection pixel (three neighbors; X\([0,-1]\), X\([1,0]\) and X\([1,1]\)); (c) X\([0,0]\) represents a pixel in the common case (two neighbors; X\([0,-1]\) and X\([1,1]\)).](image)
coupled with the mineral during neutron irradiation. It has been shown (e.g., Jonckheere (2003); Soares et al. (2013)) that:

$$GQR = \frac{\rho_I}{ED} \left( \frac{\rho_I}{IS} \right)$$

(2)

Three crystals of Durango apatite were mounted in epoxy resin, grounded and polished. A plan sheet of muscovite mica was coupled with the apatite mount and this set was irradiated with neutrons in the IEA R1 reactor at the IPEN/CNEN, São Paulo, Brazil. Neutron fluence was monitored with a CN1 glass. The value of $R_U = (3.2 \pm 0.1) \times 10^{-8}$ fissions per uranium atom (equivalent to a neutron fluence of approximately $7.4 \times 10^{15}$ neutrons/cm$^2$) was adopted using the calibration proposed by (Soares et al., 2014). After irradiation, the apatite mount was grounded and polished again to reveal an internal surface, where tracks were etched in 1.1 molar nitric acid for 50s at 20°C. Muscovite mica was etched in 48% HF for 120 minutes at 15°C. Thirty apatite and 49 muscovite photomicrographs were captured with a nominal magnification of $500\times$ at a Zeiss Axioplan II optical microscope. The track densities were determined by manual counting, and using the counting algorithm proposed in this study.

License and reusability

The algorithms and functions implemented in this study were built using the Python packages Numpy (Oliphant, 2006; van der Walt et al., 2011), Scipy (Jones et al., 2001), Matplotlib (Hunter, 2007), scikit-image
Figure 5. Choosing track candidates in the region presented in Figure 2(b), obtaining extremity points two by two. Green pixels: Euclidean distance. Blue dots: route between the two extremity points. Yellow pixels: inner area of the region formed by Euclidean distance and route.

Figure 6. Track candidates chosen by the algorithm for the region in Figure 2(b). Green dots: extremity pixels. Green line: Euclidean distance between extremity pixels. Blue dots: route between extremity pixels.

Results

We counted all images in the test dataset using Otsu, Yen, Li, ISODATA, and MLSS binarizations. Counting times are smaller than $10^{-1}$ s when using most binarization algorithms (Figure 8), except for MLSS: it employs wavelet decompositions for segmenting the input image (de Siqueira et al., 2014), being more time demanding than the other algorithms. Also, further track counting could be impaired when using MLSS, since it adds artificial regions to the binary image as discussed in section.

Different binarization algorithms may yield different number of track candidates, and therefore different track counting, depending on the complexity of the input region (Figure 9). In general, the proposed algorithm can recognize the tracks in the binary regions.

Using the proposed algorithm, we counted the fission tracks contained in the test photomicrographs
Figure 7. Visual representation of each track labeled by the segmentation algorithm, when using the ISODATA binarization (threshold: 0.475). The numbers show how many tracks were counted in each region. Magenta lines: regions representing only one track. Green dots: extremity pixels. Green lines: Euclidean distance between extremity pixels. Blue paths: route between extremity pixels.

obtained from the Durango apatite and co-irradiated mica. Additionally, one of the authors (W. N.) manually performed the same measurements (Table 1). Apatite and mica counts have been pooled to calculate $\rho_{IS}$ and $\rho_{ED}$. The uncertainty presented is one Poisson standard deviation for the densities and the usual propagation of uncertainties for GQR:

$$\frac{\sigma_{GQR}}{GQR} = \sqrt{\frac{1}{N_{ED}} + \frac{1}{N_{IS}}}$$

To verify whether the counting distributions were drawn from the Poisson distribution, we used the one-sample, two-sided, Kolmogorov-Smirnov (KS) test, implemented in the R package “stats” (R Core Team 2018) (Table 1). The KS test compares cumulative distribution functions and its statistics, $D$, is defined as the maximum horizontal distance between them. We adopted the confidence level of $\alpha = 0.05$. In the KS test, higher p-values mean that there is no statistical evidence for differences between the tested distributions.

DISCUSSION

We proposed an algorithm for counting tracks in binary images obtained from mineral photomicrographs. Using this algorithm, we calculated GQR for a test dataset created to this purpose. Since the binarization
Another fact would be that MLSS adds artificial regions to the binary image; then, further track counting is impaired when using this binarization.

![Figure 8](image.png)  
Figure 8. Track counting time for images in the test dataset, according to each binarization algorithm. Counting times are usually small (around $10^{-2}$ s), except for MLSS, which employs wavelet decompositions in it (de Siqueira et al., 2014), being more time demanding than the other binarizations.

Table 1. Data used for calculating GQR. Tracks were counted in 49 mica and 30 apatite images. Image area: 3.58 cm$^2$. Counting detail: binarization algorithm combined with automatic counting, or manual counting; $N_{ED/IS}$: number of tracks counted in mica (apatite); $\rho_{ED/IS}$: induced fission track density in mica (apatite).

| Binarization algorithm | $N_{ED/IS}$ | $N_{ED/IS}/\text{image} (\pm 1\sigma)$ | $p$-value | $N_{IS}$ | $N_{IS}/\text{image} (\pm 1\sigma)$ | $p$-value | $p_{ED/IS}$ $\pm 10^6$ tracks/cm$^2$ | GQR |
|------------------------|-------------|----------------------------------------|-----------|-----------|--------------------------------------|-----------|-------------------------------------|-----|
| Otsu                   | 2482        | 48.3 $\pm$ 10.9                       | 0.10      | 2478      | 48.1 $\pm$ 10.9                      | 0.11      | 7.30 $\pm$ 0.12                     | 0.59 |
| Yen                    | 2482        | 48.1 $\pm$ 10.9                       | 0.12      | 2484      | 48.2 $\pm$ 10.9                      | 0.14      | 1.75 $\pm$ 0.00                     | 0.59 |
| Li                     | 2079        | 42.6 $\pm$ 9.9                        | 0.15      | 2130      | 71.7 $\pm$ 1.6                       | 0.14      | 3.60 $\pm$ 0.12                     | 0.59 |
| ISODATA                | 2136        | 45.6 $\pm$ 11.9                       | 0.11      | 2282      | 76.1 $\pm$ 1.6                       | 0.11      | 5.70 $\pm$ 0.06                     | 0.57 |
| MLSS                   | 5583        | 113.9 $\pm$ 1.5                      | 0.02      | 5346      | 111.5 $\pm$ 1.9                      | 0.01      | 0.86 $\pm$ 0.13                     | 1.02 |
| Manual                 | 2838        | 57.9 $\pm$ 1.1                       | 0.00      | 3154      | 103.8 $\pm$ 1.9                      | 0.00      | 4.91 $\pm$ 0.09                     | 0.56 |

All counting was carried out in images with track densities of about $6 \times 10^5$ tracks/cm$^2$. The counting algorithm should perform well in samples with track densities within this magnitude or less. From the KS test, we did not find evidence that most distributions of number of tracks per image were not drawn from Poisson distribution ($p$ - value > 0.05), as it should be expected from measurements of processes such as the neutron induced fission of $^{235}$U. Only counting with the MLSS binarization for both apatite and mica tracks resulted in a KS $p$ - value < 0.05.

Except when using MLSS binarization, the GQR values found in this study with automatic counting, $0.57 - 0.59 \pm 0.02$, are in the range commonly found in the literature: $0.51 \pm 0.02$ (Gleadow and Lovering, 1977), $0.55 \pm 0.02$ (Iwano and Danhara, 1998), $0.56 \pm 0.03 - 0.61 \pm 0.02$ (Soares et al., 2013), $0.58 \pm 0.02$ (Iwano et al., 2018), $0.61 \pm 0.01$ (Enkelmann and Jonckheere, 2003). The same holds for the manual counting.

The number of tracks identified as such and counted by the automatic algorithm is lower than tracks counted manually by the observer, again with the exception of MLSS binarization. Due to previous processing given in filtering and binarization, smaller and/or lighter gray tracks are not counted. The
algorithm also does not count tracks in certain cluster configurations. Although the number of counted tracks is smaller in relation to manual counting, this feature prevents the count of small artifact objects like etching figures, avoiding false positives. Once the counting efficiency remains constant when counting tracks for dating, the age results should not be affected. On the other hand, MLSS binarization produces a greater number of counted tracks than manual counting (Figure [10]). In this case, the overidentification of tracks is due to the counting artifacts (false positives), which compromises dating.

Track counting times for the proposed algorithm are usually small (Figure [8]), except for MLSS. This binarization algorithm employs wavelet decompositions in it (de Siqueira et al., 2014), being more time demanding than the other ones. Since MLSS adds artificial regions to the binary image (Figure [10], Table [1]), the further track counting procedure becomes more time expensive.

Due to its need of external points to recognize track candidates, the presented algorithm has some limitations. For instance, when the skeletonization of a certain region results in a single pixel, the algorithm does not have two external points to process, and will consider the region as having zero tracks (regions labeled with the number 0 in Figure [7]).

The Euclidean distance of the track candidates may provide a measure of the track projection in the X axis (green lines in Figure [5]). However, this distance is obtained from the external pixels, defined according to the skeletonization algorithm (magenta lines in Figures [7] and [10]). Since the skeleton of a region is smaller than the binary region itself, the attributed distance is slightly smaller than the actual Euclidean distance of the track extremities. Methods such as Fewss dynamic perimeter positioning (Fews, 1992) could emphasize gray levels in the borders of fission tracks, highlighting them, thus improving the counting results for overlapped tracks.

**CONCLUSION**

In this study, we present an algorithm to separate and count overlapping fission tracks in photomicrographs. This method is based on the definition of extremity and intersection pixels over the skeleton of a binary region. Once these pixels are determined, we obtain the inner area of the region formed by the route between two pixels and their Euclidean distance. Smallest areas lead to possible track candidates.

Results found for GQR are encouraging since the values found are in the expected range and false positives are avoided when using most binarization algorithms. The only exception is the MLSS binarization, which performs poorly in avoiding false positives, and have higher processing times. The proposed method is not a definitive solution, but is certainly a step forward in the automatic counting of tracks. It also can be applied jointly with other algorithms. The counting algorithm should perform...
Figure 10. Counting tracks in Figure 10(a). MLSS binarization creates artifacts in the resulting binary image, thus misleading the track counting algorithm, which counts 115 tracks. (a) MLSS binary image obtained from Figure 10(a) presenting the generated artifacts. (b) Results of the automatic counting algorithm. Manual counting: 54 tracks. Automatic counting using ISODATA, Li, Otsu, and Yen binarizations, respectively: 41, 43, 41, and 44 tracks.

well in samples with track densities of about $6 \pm 10^5$ tracks/cm$^2$ or less.

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