Strong Coupling Hadron Masses in $1/d$ Expansion for Wilson fermions.

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Abstract

Motivated by the weak-strong coupling expansion [1], we calculate the spectrum of hadrons using a systematic $1/d$ ($d$ - dimensionality of spacetime) in addition to a strong coupling expansion in $\beta$. The $1/d$ expansion is pushed to the next to leading order in $(1/d)$ for mesons and next to next to leading order for baryons. We do the calculation using Wilson fermions with arbitrary $r$ and show that doublers decouple from the spectrum only when $r$ is close to the Wilson’s value $r = 1$. For these $r$ the spectrum is much closer to the lattice results and the phenomenological values than those obtained by using either the (nonsystematic) ”randomwalk” approximation or the hopping parameter expansion. In particular, the value of the nucleon to $\rho$ - meson mass ratio is lowered to $\frac{3\log d}{2\text{arccosh}^2} + O(1/d) \approx 1.48$. The result holds even for $\beta$ as large as 5, where the weak-strong coupling expansion is applicable and therefore these results are expected to be reasonable.

It is commonly believed that for low energy physical quantities in QCD, such as hadron masses, there is no small expansion parameter, and the theory is ”strongly coupled”. The theory only has an asymptotic weak coupling expansion successfully describes high energy quantities but breaks down at low energies. Recently one of us proposed a scheme combining weak and strong coupling expansion in a ”double expansion scheme” [1]. The high
energy modes are integrated out using expansion in weak coupling $\alpha_s$, the resultant effective Lagrangian is then expanded in derivatives and solved using strong coupling expansion in $\beta$.

*A priori* it would seem that the domain of applicability for the expansion in $\beta$ will have very little chance to intersect with that of the expansion in $\alpha$. Indeed, if the gauge coupling constant $g$ is small, $1/g$ is large and *vice versa*. Fortunately, however, a small $\alpha(g)$, does not necessarily imply that $\beta(g)$ will be large and *vice versa*. In fact, this scheme of a simultaneous weak and strong expansion has been tested on certain solvable low dimensional asymptotically free models like Ising chain and $d = 2$ Gross - Neveu models \[1\] and the results agree very well with the exact values.

As for QCD, looking at strong coupling expansion results \[2,3\] one notices that although the asymptotic scaling region is out of reach, the effective weak coupling $\alpha$ near the strong coupling radius of convergence is quite small. In the $SU(3)$ YM theory the radius of convergence (roughening phase transition), is larger than $\beta_{\text{max}} \equiv 6/g_{\text{min}}^2 \sim 5$ \[4\] which corresponds (using naive perturbative RG) to a relatively small effective weak coupling expansion parameter $\alpha_{\text{lat}} \equiv g_{\text{lat}}^2/4\pi \sim 0.1$. Even taking into account the fact that $\alpha_{\text{lat}}$ is a lattice one (not the $\overline{\text{MS}}, \alpha_{\overline{\text{MS}}} \sim 0.3$), this corresponds to the values at which perturbation theory is supposed to work for energies above $1.5 – 2 \text{ GeV}$. Recently, this fact has been fully understood for lattice weak coupling perturbation theory \[5\]. Therefore, there exists a (albeit smaller then the one for the Gross - Neveu model) window in the coupling in which both $\alpha$ and $\beta$ are small enough to produce a reasonable series. The well known “loop factors” $1/(4\pi)^2$ in the weak coupling expansion parameter $\alpha$ are partly responsible for this, although this generally is not sufficient since symmetry group factors tend to reduce the window. Note also that the leading order term in $\alpha$ coincides with the conventional “phenomenological” strong coupling model, in which the inverse lattice spacing $M$ is limited to values inside the weak-strong applicability window. This would seem to be a strong indication that the simultaneous weak-strong expansion will be applicable to QCD.

The complexity of such a calculation depends on the quantity and the precision one would like to achieve. Usually strong coupling series are relatively easy to evaluate to very high orders. Consequently the scale $M$ can be chosen in such a way that $\beta_M$ is just
below the strong coupling expansion radius of convergence. Simultaneously, the scale $M$ should be sufficiently high or alternatively the relevant energy scale sufficiently low so that just a few orders in the derivative expansion are needed to achieve the desired accuracy. Consequently, this method is limited to low energy quantities only. Therefore, inside this weak-strong coupling window the usual lattice action with renormalized coupling can serve as a reasonable effective action [6]. The expected accuracy is rather low: up to corrections of order $\alpha_{\overline{MS}} \sim 0.3$ due to weak coupling expansion. On the top of this we moreover will then have to perform the strong coupling expansion. Fortunately it is well known that in the hadronic sector (unlike the glueball or pure glue sector in which the above estimates of the radius of convergence of the strong coupling were taken) the situation is much better. The strong coupling limit $\beta = 0$ already produces a reasonable spectrum of hadrons. In addition, it is known that the next to leading order terms in $\beta$ for the hadronic spectrum are numerically very small [10,9]. Even for the window value of $\beta = 5$ the corrections do not exceed 15%.

However even in this limit QCD is nontrivial and an additional expansion parameter should be utilized. This may be the hopping parameter, $1/N_c$ or $1/d$ expansions. The hopping expansion parameter $\kappa \approx 1/2$ is defined as a bare coupling in units of lattice spacing and therefore cannot be easily related to a physical quantity [2,7]. The $1/N_c = 1/3$ has been extensively used, but is notoriously difficult to perform beyond the leading order. Indeed, it is mostly the nonsystematic random walk approximation [8,10,8] that has been used to estimate the spectrum at strong coupling. In this approximation the hopping parameter expansion is partially summed up, so that quark-lines form “collapsed paths” [8,10]. Although it seems to be superior than the simple hopping parameter expansion, one does not find a controllable expansion parameter within this approximation. Moreover, the results of this approximation as well as those of the hopping parameter expansion were rather discouraging. Although the ordering of the lowest hadronic states is correct, some mass ratios are grotesque. An especially bad example is the nucleon to $\rho$-meson mass ratio which is about 2.2 instead of the phenomenological 1.2 or (at $\beta = 5$) lattice MC simulation value of $\sim 1.4$.

In this paper we use the $1/d = 1/4$ expansion to calculate the hadron mass spectrum, which allows manageable higher order calculations. This was first applied to Yang-Mills
theories for staggered fermions in [11]. The results for the nucleon to ρ mass ratio is \( \sim 1.7 \), which, although better the previously mentioned strong coupling result is still so different from the phenomenological value that it cannot be accounted for by next to leading order corrections. Moreover, the interpretation of particle spectrum for staggered fermions is by no means straightforward. We, therefore, shall consider Wilson fermions which allows us to discuss splitting due to three flavors and for which the interpretation is straightforward.

In view of these above results for the spectrum, one of the following should be valid: (a) something is nevertheless wrong with the argument for the existence of the weak-strong coupling window and the quenched lattice results are not precise enough. The spectrum at \( \beta = 5 \) is indeed very different from the observed experimental one because the continuum limit is still far from this point or (b) the random walk and the hopping expansion (and to a smaller degree the staggered fermion) results are inaccurate. We show in this paper that when a systematic \( 1/d \) expansion is applied to the Wilson action within the weak-strong window, a spectrum is obtained which is in agreement with the above lattice MC results within the expected accuracy of the expansion.

We now fix notation and outline the formalism, which is well-known, focusing on the differences with random walk approach. No details of higher order calculations will be given. The standard lattice Wilson action is

\[
S = -\frac{\beta}{2N_c} \sum_{\text{plaquettes}} (TrU^2 + TrU^\dagger) + \sum_{x,\mu} \left\{ J^{AB}_{\mu} U^{BA}_{\mu} + U^{BA}_{\mu} J^{AB}_{\mu} \right\} - \sum_{x} m \bar{c}^{A}_{a} \psi^{A}_{a} \tag{1}
\]

where \( J_{\mu}^{AB}(x) \equiv \bar{\psi}^{A}_{a}(x + \mu) P_{\mu}^{+} \psi^{B}_{a}(x) \) and \( P_{\mu}^{\pm} = (r \pm \gamma_{\mu})/2 \). \( U^{AB} \) is the usual compact gauge field on the lattice and the script letters run over \( N_c \) colors. The lower latin letters runs over \( N_f \) flavors. First, we shall limit ourselves to the \( \beta = 0 \) limit and then discuss the effects of finite \( \beta \).

Integration over the gauge fields \( U \) give in the leading order [9],

\[
S_0 = -\frac{1}{N_c} \sum_{x,\mu=1}^{d} J^{AB}_{\mu} \bar{J}^{BA}_{\mu}(x) - \frac{1}{N_c} \sum_{x,\mu} \left[ \det c J^{AB}_{\mu}(x) + \det c \bar{J}^{AB}_{\mu}(x) \right] + \ldots \tag{2}
\]

where the determinant is over the color indices. The \( \ldots \) indicates a finite number of terms which contribute to higher order terms in \( 1/d \). Considering first the mesonic sector, we note that although we have introduced different flavors, for the leading order in \( 1/d \), they
will not play a role and we shall suppress them for now. We then introduce the mesonic fields through $M^A(x) = \frac{1}{N_c}\sqrt{2} \bar{\psi}^A \Gamma^A \psi^A$. The “channel” index $A$ runs from 0 to 15 and normalization of matrices $\Gamma$ is chosen so that $\text{Tr} \Gamma^A \Gamma^B = \delta^{AB}$ [10]. In terms of these fields the mesonic part of the action becomes

$$
S^{\text{mes}}_0 = -\frac{N_c}{2} \sum_{xy\mu} M^A(x) D_{AB}^{-1}(x - y) M^B(y) \tag{3}
$$

where unless otherwise stated summation over repeated channel indices will be understood.

We now integrate over the fermion fields. This is done by first introducing auxiliary fields conjugate to $M^A$ [12]:

$$
e^{-S^{\text{mes}}_0} = \int D M^A \exp \left\{ N_c \sum_x M^A(x) M^A(x) - \frac{N_c}{2} \sum_{xy} M^A(x) D_{AB}^{-1}(x - y) M^B(y) \right\} \tag{4}
$$

Then the remaining gaussian integral over fermionic fields can be done:

$$
Z_0 = \int D M^A \exp \left\{ -\frac{N_c}{2} \sum_{xy} M^A(x) D_{AB}^{-1}(x - y) M^B(y) - N_c \sum_x \text{Tr}_D \log \left( \Gamma^A M^A(x) + 2\overline{m} \right) \right\} 
\equiv \int D M^A e^{-A[M(x)]} \tag{5}
$$

where $\overline{m} = m/\sqrt{2d}$. The factor $\sqrt{d}$ was introduced in the mass to facilitate the $1/d$ expansion [11]. The functional $A[M(x)]$ of hadronic fields is an effective hadronic action describing dynamics of the color invariant ”basic” fields only. In the meson sector, which is being considered now, these fields interpolate between the pseudoscalar and vector mesons. They correspond to the lowest energy states of the naive quark model. Later on baryonic fields interpolating between the octet and decouplet ($N$ and $\Delta$) fields will be introduced.

The quadratic part of the mesonic action to lowest order is then

$$
A^{\text{mes}}_0 = -\frac{N_c}{2} \sum_{xy} M^A(x) G^{-1}_{AB}(x - y) M^B(y) \tag{6}
$$

where

$$
G^{-1}_{AB}(x - y) = D_{AB}^{-1}(x - y) + \frac{\delta_{AB}}{\lambda^2_0} \tag{7}
$$

and $\lambda_0 = \overline{m} + \sqrt{m} + 1 - r^2$ comes from the solution of the gap equation [11]. To find the mass spectrum of the theory, we need to find the zeros of $G^{-1}$ in momentum space. This would seem to be difficult, but note that $G^{-1}$ and $k \equiv \lambda^2_0 G^{-1} D$ have the same zero
eigenvalues, as long as \( D \) does not vanish and finding the zeros of the latter quantity is fairly straightforward.

There are two coupled channels for the mesonic sector. First, the pseudoscalar couples with the time component of the axial vector giving the mass term for the pion. In this channel,

\[
G^{-1} D \lambda^2 = \begin{pmatrix}
\lambda^2_0 - (1 + r^2) + \frac{(1+r^2)}{d} (1 - \cosh m) & -i \frac{2\pi}{d} \sinh m \\
-i \frac{2\pi}{d} \sinh m & \lambda^2_0 + (1 - r^2) - \frac{1-r^2}{d} - \frac{(1+r^2)}{d} \cosh m
\end{pmatrix}
\]

(8)

where we included the correct powers of \( 1/d \) and have taken the \( d \)-momentum to be \((0, ..., 0, im)\). Within the \( 1/d \) expansion, we can generically write

\[
cosh[m] = xd + y + O(1).
\]

(9)

where, of course, \( y \) cannot be determined as yet since higher orders in \( 1/d \) terms have not been included. Using eq.(9) and expanding the eigenvalue equation of the matrix eq.(7) in orders of \( 1/d \), we find two solutions. The one which is finite for all relevant \( r \), is

\[
x_\pi = \frac{1}{(1 - r^2)^2} \left\{ (\lambda^2_0 - r^2)(1 + r^2) - \left[ (1 - r^2)^2 + 4r^2(\lambda^2_0 - r^2)^2 \right]^{1/2} \right\}
\]

(10)

and determines the mass of the mass of the pion. The second eigenvalue \( x_d \) is nonzero and describes a doubler (it is a bound state of two fermionic doublers at the opposite corners of the Brillouin zone):

\[
cosh m_d = \frac{2d(1 + r^2)}{(1 - r^2)^2}
\]

(11)

One then requires that the pion be massless for all \( r \), and sets \( x_\pi = 0 \) and \( y_\pi = 1 \). This, to lowest order, determines the trajectory in parameter space relating bare mass to \( r \): \( \lambda^2_0 = 1 + r^2 \).

In the \( \rho \) meson channel the corresponding matrix (on the trajectory) is:

\[
\begin{pmatrix}
\lambda^2_0 - (1 + r^2) + \frac{1}{d} (3 + r^2 - (1 + r^2) \cosh m) & -i \frac{2\pi}{d} \sinh m \\
i \frac{2\pi}{d} \sinh m & \lambda^2_0 (1 - r^2) + \frac{1}{d} [-3 + r^2 - (1 + r^2) \cosh m]
\end{pmatrix}
\]

(12)
where we have considered only one spatial component of the vector field interpolating $\rho$ mesons. On this trajectory we write $\cosh m_\rho = x_\rho d + y_\rho + O(1/d)$ and obtain

$$
\begin{pmatrix}
-(1 + r^2)x_\rho & -2irx_\rho \\
2irx_\rho & 2 - (1 + r^2)x_\rho
\end{pmatrix}
$$

(13)
to the leading order in $1/d$. There are two eigenvalues to this matrix. The first is $x_\rho = 0$. This means that the $\rho$ meson’s mass does not have a “natural” order of $\log d$, but is, in fact, smaller – just of order $1$. It is, however, inconsistent to to determine $y_\rho$ by solving equation for vanishing determinant of eq.(12). The $1/d$ corrections to this matrix, considered in the following, must be taken into account. Other channels, scalar, tensor, etc, contain doublers only.

Genericly, in any next to leading order calculation in $1/d$ there are two types of contributions. The first is the ”tree” contribution which arises from additional terms in the integral over gauge fields eq.(2), while the second is the one loop diagrams involving the propagator and vertices of the leading effective action eq.(5). Keeping the pion mass zero, the “next to leading” contribution to the $\rho$ meson’s mass $y_\rho$ (which is actually the leading since $x_\rho = 0$) is:

$$
y_\rho = \frac{3 + r^2}{1 + r^2}
$$

(14)

This value is consistent with the $r = 0$ result obtained in [11] for staggered fermions. Note that for $r = 1$ the $\rho$ mass is significantly larger then in the random walk approximation.

The purpose of introducing the chiral symmetry breaking mass and Wilson’s terms was to remove doublers. The value of $r$ should, in principle, be optimized in such a way that on the one hand doublers do not interfere with physical particles and, on the other hand, the chiral symmetry is minimally violated. As is well known, setting the pion mass to zero does not mean that the chiral symmetry is somehow restored on the trajectory. It just means that we are situated on the spontaneous parity breaking phase transition line. On Fig.1 we show the $r$ dependence of various doublers masses compared to the $\rho$ meson mass. We can see that the doublers are significantly heavier then $\rho$ mesons only near Wilson’s value of $r = 1$, where they all become infinitely heavy. Therefore we conclude that there is no great advantage to work with $r < 1$ contrary to some lattice [13] and random walk approximation [13] results in which doublers were not considered.
Turning our attention to the baryonic sector, we note that the general expression for the exponent of the baryon masses contains, half integer as well as integer powers of $1/d$:

$$e^m = xd^{3/2} + x'd + yd^{1/2} + y' + O(1/\sqrt{d})$$  \hspace{1cm} (15)

We calculated $x, x', y$ and $y'$. To the leading order the $\Delta$ mass on the $m_\pi = 0$ trajectory is

$$x_\Delta = \frac{2\sqrt{2}(1 + r^2)^{3/2}}{(1 + r)^3}$$  \hspace{1cm} (16)

Notice that for $r = 0$, $x_\Delta = 2\sqrt{2}$ which reproduces the result obtained in [11] for staggered fermions calculation. The nucleon is degenerate with $\Delta$ up to the order of $1/d$ we have considered when $r = 1$. The formulas for the higher order corrections to the baryon masses for other values of $r$ are cumbersome and will be given elsewhere. Instead, for $r = 1$

$$x'_\Delta = -\frac{1}{4}, \quad y_\Delta = -\frac{1}{12}, \quad y'_\Delta = \frac{29}{144}$$  \hspace{1cm} (17)

We also calculated the leading order $\beta$ corrections to the $\rho$ and baryons. These types of corrections has been studied for staggered fermions in [11] and within the random walk approximation scheme in [10]. They vanish for $r = 1$ and are small for other values of $r$ near the window range.

To summarize, we have systematically studied the spectrum of the Wilson action with arbitrary $r$ using strong coupling and the $1/d$ expansions. The Lagrangian is considered as a phenomenological low energy effective Lagrangian for the values of $\beta$ at which the expansion is still reasonable. It turns out that the doublers decouple from the physical spectrum only when $r$ is quite close to the Wilson’s value $r = 1$. The results of the systematic $1/d$ expansion for the nucleon to $\rho$ mass ratio is $\frac{3 \log d - 1/4}{2 \arccosh 2} + O(1/d) \approx 1.48$. This value is within the range of the next correction of the Monte Carlo results.

We now compare the $1/d$ results for mesons with those obtained within the random walk approximation in $d = 4$. To understand the difference with the random walk approach, we have extended the random walk calculation [10] to arbitrary $d$ and obtained the mass of $\rho$ meson for $r = 1$ as:

$$\cosh m_\rho = \frac{2d - 1}{d + 1} = 2 - \frac{3}{d} + \frac{3}{d^2} - \frac{3}{d^3} + \ldots$$  \hspace{1cm} (18)
The first term in this expansion coincides with the systematic $1/d$ expansion. There is however no reason to expect that the next order term in $1/d$ will resemble that in eq.(18). A priori this is not obvious since at the special value $r = 1$, many contributions to the next to leading order term vanish. This is due to the well-known result from the random walk approximation \[10\] that to this order all the propagators travel along a single direction. As such, we will necessarily encounter product of two projectors $P^+ \mu P^- \mu = 0$. However if we go to the next to next to leading order there will certainly be many contributions which will not vanish for the simple reason that certain diagrams will not “linear”, but rather “planar”.

The coefficient in front of $1/d$ in eq.(18) is negative and large numerically for $d = 4$. This leads to significant underestimate for the $\rho$ mass and consequently for the overestimation of $m_N/m_\rho$. Let us emphasize that this term cannot be taken seriously at this point since corrections to the mass at this order in $1/d$ have not been done as yet. Indeed, it would be very interesting to calculate this next $(1/d$ for $\cosh m_\rho$) order term to obtain better estimate of the $\rho$ mass.

This feature of random walk approximation does not carry over to baryon sector, however. The corresponding expansion for the nucleon mass in powers of $1/d$ is:

$$e^{m_N} = d^{3/2} - \frac{1}{4} d - \frac{5}{48} d^{1/2} + \frac{119}{576} + \ldots$$

The first two terms of these now coincide with our systematic $1/d$ expansion results. When a similar expansion is done for the Delta mass, we find that for some unknown reason all four terms now coincide with eq.(17). Once again, however, any terms which is of higher order than $d$ in eq. (19) are unreliable, since they will be almost certainly be changed by higher order $1/d$ corrections. In particular, we see that small splitting between the nucleon and $\Delta$ found in random walk approach is due these unreliable higher order terms. Within the systematic $1/d$ expansion, the Delta and Nucleon are degenerate.

Of course there are numerous other corrections to the weak-strong approximation scheme. Here we discuss few of the many which have not been calculated. Our previous discussion revealed the peculiar fact most of the contributions to the next to leading order terms in $1/d$ vanish for $r = 1$. We can ask the following question: What are the corrections that do not vanish? In particular, we note that to the next to leading order there is no splitting between Goldstone bosons and flavour singlet - the $\eta$ particles. As is well known, this splitting is
due to an anomaly. The plaquette correction also does not lead to the splitting. This is easy to understand. The arguments of Frohlich and King [14] are applicable to the $1/d$ expansion. We therefore expect these anomaly effects to appear only at very high orders in $1/d$ or $\beta$, when a diagram which spreads in all four directions can be constructed. Another possibility is that the main mechanism for the splitting is not due to these corrections but rather to direct anomaly breaking terms which are proportional to $\alpha_s$. These appear due to the presence of instantons at energies higher than the scale $M$.

The actual splitting between $\Delta$ and the nucleon is also probably due to higher order terms in the effective Lagrangian. Note that in lattice simulations at relatively small $\beta$ ($\beta \sim 5$) the splitting is also smaller than the phenomenological values. The presence of higher dimension terms are also crucial for two other purposes: restoration of the Lorentz invariance and the chiral symmetry. Chiral symmetry is explicitly broken by the mass and Wilson terms. As we have mentioned, the vanishing of the pion mass is not sufficient to restore chiral symmetry. Instead it is simply a signal criticality with respect to a discrete symmetry. It would be therefore be interesting to investigate whether the chiral properties are gradually restored once higher dimensional operators are introduced. For example the pion scattering at small momenta is nonvanishing [9] without higher dimensional terms. It is reasonable to expect that with the inclusion of the term due to next order correction in $1/M^2$ of the derivative expansion of the effective action the correct zero momentum limit at least will be recovered. Existing results, which are quite scarce within the random walk approximation scheme, in which only part of the higher dimensional (improvement) operators are considered did not address this question.

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Fig.1 Masses of nucleon, $\rho$ meson, mesonic and baryonic doublers as function of Wilson parameter $r$. 