Probabilistic finite element analysis of the deflection on a beam

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Abstract. The reliability of complex systems can be quantified using probabilistic finite element analysis. A widely accepted approach is interlacing of probabilistic methods with commercial finite element solvers for evaluating the fatigue life. In this paper, the deflection of a beam is calculated using finite element method (ABAQUS® commercial software), then the results of the simulated model are employed in MATLAB®, which is used to determine the probability, reliability and the reliability index. It was found that the reliability index is around 1.2428, while the reliability value is calculated to be around 0.8930. From the calculations the values of the probability of failure (POF) was found to be around 0.1070. Finally, the results of the calculated maximum deflection have been compared and a good agreement is found.

1. Introduction

Investigation of the probability of failure (POF) which is define as the probability of the failure at a given time in an equipment is significant, because it affects the risk analyses. POF analysis is safely executed using numerical simulation, which gives more accurate predictions in complex systems [1-3]. Globally, computational numerical simulations are now employed in the design and performance testing of engineering structures, because it reduces testing times and saves costs. However, when the research is related to the complex structures, it is important to develop the numerical model to suit the intended POF analysis, because of several uncertainties that may be present in the structural parameters or loadings definitions.

The Monte Carlo method is a probabilistic method and it can be used to process large works with high to low probabilities, however this method is not suitable for fitting of complex and extensive computations [4]. Other methods like the approximate fast probability integration (FPI) and the advanced mean value (AMV) have been proven to be more suitable for complex structures with fewer computational time and better accuracy [5]. The AMV also identifies the input parameters that affect the probabilistic sensitivity and reliability.

However, due to the above-mentioned problems there is still a lack in knowledge for finding a proper method for simulating uncertainties in geometry, loads, material behavior, random variables and other user-defined parameters that are used to predict the probabilistic response, sensitivity measures of the system and reliability. In this paper ABAQUS® commercial software is used to find the deflection of a beam. Then, MATLAB® software is employed in order to calculate the governing equations of the
probabilistic behavior. Finally, the POF, the reliability as well as, the reliability index for the model is found.

2. Methodology

2.1. Mathematical model (First order reliability method)

The performance function of form is written as shown in equation (1).

$$g(X) = X_1 - X_2$$

(1)

Where, the transformation of the normally distributed random variables $X_1$ and $X_2$, becomes

$$X = \mu + \sigma U$$

(2)

Therefore, the performance function becomes

$$g(X) = \mu_1 + \sigma_1 U_1 - (\mu_2 + \sigma_2 U_2)$$

(3)

The search process starts from the initial point $U_0 = (0, 0)$. At this starting point

$$\nabla g(U_0) = [\frac{\partial g}{\partial U_1}, \frac{\partial g}{\partial U_2}]$$

(4)

A cantilever beam is illustrated in figure 1. For a cantilevered beam the model fails when the tip displacement exceeds the allowable displacement ($D_0$). The performance function at the tip of the displacement is denoted as,

$$g = D_0 - \frac{4L^3}{Ewt} \left( \frac{P_y}{L^2} \right)^2 + \left( \frac{P_x}{w^2} \right)^2$$

(5)

Where $D_0=3''$ is the maximum deflection, $E=30 \times 10^6$ psi is the modulus of elasticity, $L=100''$ is the length, $P=1000$ (±100) is the external force, $w=2''$ (±0.2) is the width, and $t=4''$ (±1) is the height of the cross section.

![Figure 1. The cantilever beam.](image)

The POF at the tip of the displacement becomes

$$P_y = P \left\{ g = D_0 - \frac{4L^3}{Ewt} \left( \frac{P_y}{L^2} \right)^2 + \left( \frac{P_x}{w^2} \right)^2 \leq 0 \right\}$$

(6)

The normally distributed variables are transformed into standard normal variables using

$$U = (U_x, U_y) = \left[ \frac{P_x - \mu_{P_x}}{\sigma_{P_x}}, \frac{P_y - \mu_{P_y}}{\sigma_{P_y}} \right]$$

(7)

or

$$X = (P_x, P_y) = (\mu_{P_x} + U_x \mu_{P_x}, \mu_{P_y} + U_y \mu_{P_y})$$

(8)
The transformed performance function in U-space becomes

\[
g = D_0 - \frac{4L^3}{Ewt} \sqrt{\left(\frac{\mu_{ix} + U_{yx}\mu_{ix}}{t^2}\right)^2 + \left(\frac{\mu_{ix} + U_{yx}\mu_{ix}}{w^2}\right)^2}
\]  (9)

Where, the gradient of \( g(U) \) can be obtained from equation (10).

\[
\nabla g(U) = \frac{4L^3}{Ewt} \left( \frac{(\mu_x + U_x\sigma_x)\sigma_x}{w^4} \sqrt{\left(\frac{\mu_y + U_y\sigma_y}{t^2}\right)^2 + \left(\frac{\mu_x + U_x\sigma_x}{w^2}\right)^2} \right) \left( \frac{(\mu_y + U_y\sigma_y)\sigma_y}{t^4} \sqrt{\left(\frac{\mu_y + U_y\sigma_y}{t^2}\right)^2 + \left(\frac{\mu_x + U_x\sigma_x}{w^2}\right)^2} \right)
\]  (10)

2.2. Finite element descriptions

ABAQUS® has been chosen for modelling and analyzing the deflection of the beam due to its flexibility in creating geometry and material modelling [6-10]. The model of beam in 3D is shown in Figure 2. The three-level design is used to find the number of runs for the simulation. The material property is assumed the same as the mathematical model. In addition, B21 (2-node linear beam in a plane) is assumed to be as the element type.

![Figure 2. The model of beam in ABAQUS®.](image)

3. Results and discussion

A total of 13 runs was needed for the three-level three-factorial Box–Behnken experimental design, as shown in Table 1. The model takes the following form as shown by equation (11).

\[
y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \beta_{33} x_3^2 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{23} x_2 x_3
\]  (11)

Where the predicted response is \( y \); the model constant is \( \beta_0 \); the independent variables are \( x_1 \), \( x_2 \) and \( x_3 \); the linear coefficients are \( \beta_1 \), \( \beta_2 \) and \( \beta_3 \); the cross product coefficients are \( \beta_{12}, \beta_{13} \) and \( \beta_{23} \); and the quadratic coefficients are \( \beta_{11}, \beta_{22} \) and \( \beta_{33} \).
Table 1. The maximum deflection calculated by ABAQUS®.

| Run number | Calculated deflection in ABAQUS® |
|------------|---------------------------------|
| 1          | 1.042                           |
| 2          | 0.852                           |
| 3          | 1.273                           |
| 4          | 1.042                           |
| 5          | 2.222                           |
| 6          | 0.48                            |
| 7          | 2.716                           |
| 8          | 0.587                           |
| 9          | 2.743                           |
| 10         | 0.593                           |
| 11         | 2.245                           |
| 12         | 0.485                           |
| 13         | 1.042                           |

A comparison between the predicted values for the maximum deflection by MATLAB® and the numerical displacement measured by ABAQUS® is shown in figure 3. Achieving the goodness of fit (R-square) of 0.98 shows a good agreement between the results of both software in different numerical displacements. The process is run continues until convergence of the solution which occurred in iteration 6. Table 2 shows the maximum probability point (MPP) and the complete convergence history. The deformed shape of the model can be seen in figure 4.

Figure 3. A comparison between the maximum deflection in ABAQUS® (straight line) and MATLAB® (squares).
| Iteration Number | Value for Pressure | Value for (w) | Value for (t) | $\nabla g$ | $|\nabla g|$ | U (Space) | $\beta$ (Reliability index) |
|------------------|--------------------|---------------|---------------|---------|----------|-----------|-------------------------|
| 0                | 1050               | 2.1           | 4.5           | (-0.0713, 0.0739, 0.5114) | 0.5216 | (0.7287, -0.7552, -5.2268) | 5.3311 |
| 1                | 1.0729e+03         | 1.8490        | -1.22         | (-0.6430, 0.6570, 5.9455) | 6.0161 | (0.2731, -0.2791, -2.5252) | 2.5552 |
| 2                | 1.0273e+03         | 1.9442        | 1.4748        | (-0.3764, 0.3814, 3.3591) | 3.4015 | (0.1639, -0.1661, -1.4630) | 1.4815 |
| 3                | 1.0164e+03         | 1.9668        | 2.5370        | (-0.2718, 0.2754, 2.3562) | 2.3877 | (0.1427, -0.1446, -1.2370) | 1.2535 |
| 4                | 1.0143e+03         | 1.9711        | 2.7630        | (-0.2496, 0.2529, 2.1432) | 2.1724 | (0.1427, -0.1446, -1.2255) | 1.2422 |
| 5                | 1.0143e+03         | 1.9711        | 2.7745        | (-0.2484, 0.2517, 2.1326) | 2.1617 | (0.1428, -0.1447, -1.2254) | 1.2422 |
Figure 4. The deformed model created in ABAQUS® software.

The MPP is found at (0.1428, -0.1447, -1.2254) with the reliability index of $\beta=1.2422$. Then, the probability of failure ($P_f$) can be determined based on equation (12).

$$P_f = \Phi(\beta)$$  (12)

Therefore,

$$P_f = \Phi(1.2422) = 0.1070$$

It should be noted that, the reliability is calculated as,

$$R = 1 - P_f = 1 - 0.1070 = 0.8930$$

4. Conclusion

A computational tool was used to determine the rate of the deflection, reliability and POF of a beam. Three different variables including pressure width and the thickness were selected to change in different simulations in order to calculate the maximum values for the deflection of the beam in ABAQUS® commercial software. The original random variables of the probabilistic model were transformed from X-space to U-space. Also, the MPP in U-space, reliability index $\beta$ and reliability $R = F(\beta)$ were determined. The reliability index was found to be 1.2428, the reliability was equal to 0.8930 and the POF was measured to be 0.1070.

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