Introduction

We propose to analyse quantum protocols by applying the formal verification techniques developed in classical computing for the analysis of communicating concurrent systems. Typically, the first step in formal verification is to define a model of the system to be analysed, in a well-founded mathematical notation. Experience has shown that this step in itself is a valuable way of eliminating ambiguities from an informal description of the system. Next, an automated analysis tool, based on the same underlying theory, is used to reason about the system. This might consist either of checking that the system is behaviourally equivalent to another system which is viewed as a specification, or of checking that the system satisfies properties expressed in a separate specification language.

One area of successful application of these techniques is that of classical security protocols [16], exemplified by Lowe’s [12] discovery and fix of a flaw in the well-known Needham-Schroeder authentication protocol which had been proposed several years previously. Secure quantum cryptographic protocols are also notoriously difficult to design: although protocols for quantum bit-commitment [6] were believed to be secure for several years, it has recently been shown not only that such protocols are insecure, but that secure quantum bit-commitment is impossible [10, 13]. Quantum cryptography is therefore an obvious and interesting target for formal verification, and provides our first example; we expect the approach to be transferable to more general quantum information processing scenarios.

Our example is the quantum key distribution protocol proposed by Bennett and Brassard [5], commonly referred to as BB84. We present a model of the protocol in the process calculus CCS [15] and the results of some initial analyses using the Concurrency Workbench of the New Century (CWB-NC) [1]. Similar work could be carried out with other combinations of modelling language and toolset, such as CSP [9] and FDR [2], or Promela and SPIN [4].

Proofs of unconditional security of the BB84 protocol exist [4, 1] and we have no reason to doubt their correctness. Nevertheless, we argue that the modelling/analysis approach has merit for the study of this and other quantum security protocols. Gottesman and Lo [8] point out that “the proof of security of QKD is a fine theoretical result, but it does not mean that a real QKD system would be secure. Some known and unknown security loopholes might prove to be fatal. Apparently minor quirks of a system can provide a lever for an eavesdropper to break the encryption”. The analysis techniques which we are proposing can be applied to models at a range of levels of abstraction, from an idealised description to a concrete implementation. Moreover, a real system for security in information processing has components other than key distribution—authentication or authorisation, for example. In the future, some of these components may be quantum, but others could still be classical. We should be able to apply our methods in a uniform fashion to various components and their interactions and thus provide certification of complex systems. Finally, the analysis tools are oriented towards debugging: if a desired property is not satisfied, then their output enables us to understand the reason.

Modelling BB84

We use a version of the BB84 protocol in which Alice reveals the polarisation basis she used for each photon as soon as the photon is received and measured by Bob. The CCS model is based on processes and actions, both of which may be parameterised. In this particular model, all parameters are binary valued.

The quantum communication channel is modelled by a pair of processes: Empty and Full. The action put(d, b) indicates that it is possible to send a bit d into the channel; the bit is encoded with respect to one of two polarisation bases, represented by another binary parameter b. A process which actually sends data into the channel will contain the
complementary action \texttt{put} with a particular choice of parameters. The dot stands for sequencing, so that the \textit{Empty} channel becomes \textit{Full} after receiving the data.

\begin{equation*}
\text{Empty} = \text{put}(d, b).\text{Full}(d, b)
\end{equation*}

A \textit{Full} channel allows an observer to measure its contents with respect to a particular basis; the channel then uses the action \texttt{get} to release a binary value, and becomes \textit{Empty}. If the basis used for the measurement is different from the basis which was originally used to encode the transmitted bit, then the value released by the channel may be either 0 or 1. The \texttt{+} operator indicates nondeterminism. Note that we are only modelling possibilities, not probabilities. Modelling languages and analysis tools for probabilistic systems are available, for example PRISM \cite{3}. Probabilistic modelling and reasoning about quantum protocols is an area for future work.

\begin{equation*}
\text{Full}(d, b) = \text{measure}(b').
\end{equation*}
\begin{equation*}
\text{if } b' = b \text{ then } \overline{\text{get}}(d).\text{Empty}
\end{equation*}
\begin{equation*}
\text{else } (\text{get}(0).\text{Empty} + \text{get}(1).\text{Empty})
\end{equation*}

Alice and Bob interact with the channel via the actions \texttt{put}, \texttt{measure} and \texttt{get}. The actions \texttt{go}, \texttt{\overline{go}} are used for additional synchronisation, so that Alice does not choose and \texttt{put} repeatedly before Bob has finished processing what he received; this facilitates later analysis.

\begin{equation*}
\text{Alice} = \text{choose}(x). (\text{put}(x, 0).\overline{\text{reveal}}(0).\text{go}.\text{Alice} + \text{put}(x, 1).\overline{\text{reveal}}(1).\text{go}.\text{Alice})
\end{equation*}
\begin{equation*}
\text{Bob} = \text{measure}(0).\overline{\text{get}}(x). (\text{reveal}(0). \text{if } b = 0 \text{ then } \overline{\text{keep}}(x).\text{\overline{go}}.\text{Bob} \text{ else } \overline{\text{go}}.\text{Bob})
\end{equation*}
\begin{equation*}
\text{+ } \text{measure}(1).\overline{\text{get}}(x). (\text{reveal}(1). \text{if } b = 1 \text{ then } \overline{\text{keep}}(x).\text{\overline{go}}.\text{Bob} \text{ else } \overline{\text{go}}.\text{Bob})
\end{equation*}

The complete protocol, without an eavesdropper, consists of the parallel composition (operator \texttt{\mid}) of Alice, Bob, and an \textit{Empty} channel. The operator \texttt{\\{}\texttt{put}, \texttt{get}, \texttt{measure}, \texttt{go}, \texttt{\overline{reveal}}\texttt{\} indicates that these actions, and their complements, are hidden; they are used for internal interaction, but are not visible outside. Parallel composition means that individual processes run independently, only synchronising on actions and their complements. For example, Alice’s \texttt{put} must synchronise with the \texttt{put} in \textit{Empty}. This means that Alice cannot do \texttt{reveal}, synchronising with Bob, until after Bob has done \texttt{get}; Alice has to wait.

\begin{equation*}
\text{BB84} = (\text{Alice}\mid\text{Bob}\mid\text{Empty}) \texttt{\\{}\texttt{put}, \texttt{get}, \texttt{measure}, \texttt{go}, \texttt{\overline{reveal}}\texttt{\}
\end{equation*}

An eavesdropper Eve can be modelled similarly, allowing us to define the attacked protocol BB84′. This particular eavesdropper simply guesses a basis, then measures, extracts and returns the sent bit. More generally, following a standard approach to the analysis of classical security protocols, we could consider an eavesdropper who arbitrarily attempts to use any actions with any parameters derivable from information available to her.

\begin{equation*}
\text{Eve} = \overline{\text{measure}}(0).\text{get}(x).\overline{\text{put}}(x, 0).\text{Eve} + \overline{\text{measure}}(1).\text{get}(x).\overline{\text{put}}(x, 1).\text{Eve}
\end{equation*}
\begin{equation*}
\text{BB84′} = (\text{Alice}\mid\text{Bob}\mid\text{Eve}\mid\text{Empty}) \texttt{\\{}\texttt{put}, \texttt{get}, \texttt{measure}, \texttt{go}, \texttt{reveal}\texttt{\}
\end{equation*}

The process \texttt{Spec} can be viewed as a specification of the protocol in the absence of an eavesdropper. \texttt{Spec} is a description of how the protocol should behave: Alice chooses and sends a sequence of bits some of which Bob can discard (when polarisation bases do not match) but whenever he keeps a bit it is the same as what Alice sent. This results in the generation of a sequence of bits common to both parties—the key.

\begin{equation*}
\text{Spec} = \text{choose}(x). (\text{Spec} + \overline{\text{keep}}(x).\text{Spec})
\end{equation*}

\subsection*{Analysing BB84}

Using the tool CWB-NC, we have established that BB84 is equivalent to \texttt{Spec} and that BB84′ is not equivalent to \texttt{Spec}. “Equivalent” refers to trace equivalence, which means equality of the set of possible sequences of observable actions. The tool discovers that \texttt{choose}(0)\texttt{keep}(1) is a trace of BB84′ but not of \texttt{Spec}. This trace arises from an execution in which Bob measures with the correct basis but Eve has already corrupted the channel by measuring with the wrong basis. Alternatively, the property \langle\langle\text{choose}(0)\rangle\rangle\langle\langle\text{keep}(1)\rangle\rangle\texttt{true} (expressed in the modal \mu-calculus) specifies that a process may choose 0 and end up keeping 1. CWB-NC establishes that BB84 does not satisfy this property, whereas BB84′ does. This shows the possibility of interference by Eve. Note that once the processes and properties have been defined, CWB-NC carries out verification automatically and without human intervention.

\subsection*{Conclusions and Future Work}

We have introduced techniques for formally modelling and analysing quantum protocols. As far as we are aware, this is the first proposal to use formal modelling and analysis in the field of quantum information processing. As a specific demonstration,
we have modelled components of the BB84 protocol in CCS, and analysed the model with the CWB-NC tool.

Future work will include the development of a framework based on our initial investigation, for detailed analysis of quantum information systems. We are already considering tools that enable us to incorporate probabilities into our model and we will also include methods to reason about errors and error-correction. We aim to be able to generalise the model of the attacker, in order to analyse collective or coherent attacks, for example. The modelling of entanglement-based quantum key distribution [7], other quantum cryptographic protocols and quantum communication protocols is another goal.

An alternative approach, which we also plan to investigate, is to use machine-assisted theorem-proving technology to formalise conventional proofs about quantum systems, such as the unconditional security proofs.

Quantum cryptography is already viable and prototype implementations are being seriously considered. If verification efforts are begun early and proceed in tandem with implementations, the resulting systems are likely to be highly secure.

References

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