In the framework of Brans-Dicke cosmology, we have studied the interacting power-law and logarithmic entropy-corrected holographic dark energy models with different cut-offs. The Brans-Dicke parameter is investigated versus the conditions for the acceleration and phantom phases to show that which entropy corrected model can exhibit acceleration with or without the phantom phase at early and present time universes. Moreover, the classical stability or instability of the interacting power-law and logarithmic entropy-corrected holographic dark energy models with different cut-offs is determined at early and present time.

Keywords: Brans-Dicke parameter, entropy correction, holographic dark energy

PACS numbers: 98.80.-k; 95.36.+x; 04.50.Kd.

I. INTRODUCTION

Recent cosmological and astrophysical data gathered from the observations type Ia supernovae (SNeIa), cosmic microwave background radiation (CMB) and large scale structure (LSS) convincingly suggest that the observable universe experiences an accelerated expansion phase [1] that this expansion may be driven by a mysterious energy component with negative pressure so called dark energy. The cosmological constant is the simplest candidate for dark energy, called ΛCDM model. Nowadays cosmologists know that the cosmological constant suffers from two difficulties, the fine-tuning and the cosmic coincidence problems [2, 3]. The cosmic coincidence problem needs that universe behaves in such a form that the ratio of dark matter to dark energy densities must be constant of order unity [4]. There are various models for dark energy including the cosmological constant, Quintessence, K-essence, Phantom, Quintom, Chaplygin gas, Thachyon and modified gravity [2, 6]. Recently, the holographic dark energy (HDE) as a new dark energy model based on the holographic principle was suggested [6].

\[ \rho_\Lambda = 3c^2 M_P^2 L^{-2} , \]

where \( c \) is a numerical constant, \( L \) is the cut-off radius and \( M_P \) is the reduced Planck mass. In the holographic dark energy model the Bekenstein-Hawking entropy relation \( S_{BH} = A/4G \) plays an essential role and is satisfied on the horizon [7] where \( A \sim L^2 \) is the area of horizon. Since this model is closely connected to the area law of entropy, hence any correction to this law will affect the energy density of the holographic dark energy model. These corrections may arise due to the quantum field theory [8, 9], thermal and quantum fluctuations in LQG [10, 11], and string theory [12].

One correction to the entropy is the power-law correction [8]:

\[ S = A/4G(1 - K_\alpha A^{-1/\alpha}) , \]

where \( \alpha \) is a dimensionless constant and

\[ K_\alpha = \frac{\alpha(4\pi)^{\frac{\alpha}{2}} - 1}{(4 - \alpha)r_c^{2-\alpha}} , \]
where \( r_c \) is the cross over scale. This correction results in a modification of holographic dark energy called power-law entropy corrected holographic dark energy (PLECHDE) as

\[
\rho_A = 3c^2 M_P^2 L^{-2} - \beta M_P^2 L^{-\alpha},
\]

(4)

where \( \alpha \) and \( \beta \) are two constants of the order of unity. In Ref. [8], it was demonstrated that the generalized second law of thermodynamics for the universe with the power-law corrected entropy is satisfied for \( \alpha > 2 \). For \( \alpha > 2 \) the second term in Eq. (7) is comparable to the first term when \( L \) takes a very small value, thus the correction has a physical meaning only at early universe and is ignorable when the universe becomes large [16].

Another corrected entropy takes on the following form [17]

\[
S_{BH} = \frac{A}{4G} + \tilde{\alpha} \ln(\frac{A}{4G}) + \tilde{\beta},
\]

(5)

where \( \tilde{\alpha} \) and \( \tilde{\beta} \) are two dimensionless constants. The logarithmic term also appears in a model of entropic cosmology, capable of unifying the early time inflation and late-time acceleration of the universe [18]. The entropy-area relation can be expanded in a series of infinite terms, however the contribution of extra terms are negligible due to the smallness of reduced Planck constant \( \hbar \). Hence, the most important leading term in the expansion is left as the logarithmic one, which has been considered in this paper. Considering the corrected entropy-area relation [5] and following the derivation of HDE, the corresponding energy density will be modified. Wei, has recently proposed the energy density \( \rho_A \) of the logarithmic entropy-corrected holographic dark energy (LECHDE) in the following form [19]

\[
\rho_A = 3c^2 \phi^2 L^2 + \frac{\alpha}{L^2} \ln(\frac{L^2 \phi^2}{4\omega}) + \beta L^4,
\]

(6)

where \( \alpha \) and \( \beta \) are dimensionless constants. The second and third terms in Eq. (6) are comparable to the first term only when \( L \) takes a very small value, hence the corrections given by these terms have a physical meaning only at early universe. When the universe becomes large, these corrections are ignorable and LECHDE reduces to the ordinary HDE. Therefore, in the following first we will investigate the state of the universe at present time and then study the effect of different cut-offs, considering the corrections at early time, on the state of the universe [20]. Since HDE density corresponds to a dynamical cosmological constant, we need a dynamical frame as alternative theory of general relativity to accommodate HDE density.

The scalar-tensor theory was first established as an alternative to general relativity [21, 22]. Then, it played the essential role in solving the main problems of standard cosmology in the context of inflationary scenario. The main motivation to use the scalar field models in quest for solving the recent DE problem in cosmology lies in the particle physics as well as string theory. In order to solve the DE problem, several models of dynamical dark energy with time evolving equation of state have been proposed in the context of scalar field models such as quintessence [23], k-essence [24], phantom [25], tachyon [26], and quintom [28]. Reconstruction of the holographic dark energy model with scalar fields is also one of the most attractive subjects studied in this direction [31–34]. The investigation on the holographic dark energy model in the framework of Brans-Dicke (BD) theory is one of these attempts with interesting results which, for example, implies that one can not generate phantom-like equation of state from a holographic dark energy model in non-flat universe in the framework of Brans-Dicke cosmology [29].

In this work, we aim to generalize the study in [29] and consider the interacting power-law and logarithmic entropy-corrected holographic dark energy model in Brans-Dicke cosmology and obtain the equations of state parameter for different viable cut-offs. We also consider the correspondence between the interacting power-law and logarithmic entropy-corrected holographic dark energy models with different cut-offs in Brans-Dicke cosmology in one hand, and quintessence and tachyon scalar field models in non-flat universe, on the other hand. Moreover, we perform the stability analysis of the models by determining the squared sound speed, \( v_s^2 = \frac{dp}{d\rho} \). If \( v_s^2 < 0 \), we have the classical instability against given perturbation. In contrast \( v_s^2 > 0 \) may lead to a stable universe against perturbations.
II. INTERACTING ENTROPY-CORRECTED HOLOGRAPHIC DARK ENERGY MODEL IN BRANS-DICKE COSMOLOGY

The action of Brans-Dicke theory in the canonical form may be written as 

\[ S = \int d^4x \sqrt{|g|} \left[ -\frac{\omega}{8\phi} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + L_M \right], \]

(7)

where \( g \) is the determinant of the tensor metric \( g^{\mu\nu} \), \( \omega \) is the Brans-Dicke parameter, \( R \) is the Ricci scalar curvature and \( L_M \) is the lagrangian of the matter. Variation of the action with respect to the metric \( g^{\mu\nu} \) and the Brans-Dicke scalar field \( \phi \) obtains

\[ \phi G_{\mu\nu} = -8\pi T^M_{\mu\nu} - \frac{\omega}{\phi} (\partial_{\mu} \phi \partial_{\nu} \phi - \frac{1}{2} g_{\mu\nu} \partial_{\lambda} \phi \partial_{\lambda} \phi + \Box \phi g_{\mu\nu}), \]

(8)

\[ \Box \phi = \frac{8\pi}{2\omega + 3} T^M_{\lambda\lambda}. \]

(9)

Here, \( T^M_{\mu\nu} \) is the energy-momentum tensor of the matter fields. The Friedman-Robertson-Walker (FRW) universe is defined by

\[ ds^2 = dt^2 - a(t)^2 \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \]

(10)

where \( a(t) \) and \( k \) are scale factor and curvature parameter, respectively. Using Eq. (10), the field equations (8) and (9) are simplified to

\[ \frac{3}{4\omega} \dot{\phi}^2 (H^2 + \frac{k}{a^2}) - \frac{1}{2} \dot{\phi}^2 - \frac{3}{2} \frac{H}{\omega} \dot{\phi} = \rho_m + \rho_\Lambda, \]

(11)

\[ -\frac{1}{4\omega} \dot{\phi}^2 \left( \frac{\ddot{a}}{a} + H^2 + \frac{k}{a^2} \right) - \frac{1}{\omega} H \dot{\phi} \phi \]

\[ -\frac{1}{2\omega} \ddot{\phi} \phi - \frac{1}{2} \left( 1 + \frac{1}{\omega} \right) \dot{\phi}^2 = p_\Lambda, \]

(12)

\[ \ddot{\phi} + 3H \dot{\phi} - \frac{3}{2\omega} \left( \frac{\ddot{a}}{a} + H^2 + \frac{k}{a^2} \right) \phi = 0, \]

(13)

where \( H = \dot{a}/a \), \( \rho_m \), \( \rho_\Lambda \) and \( p_\Lambda \) are the Hubble parameter, the pressureless matter density, the energy density and pressure of dark energy, respectively. We suppose that there is an interaction between entropy-corrected holographic model of dark energy and dark matter as follows

\[ \dot{\rho}_\Lambda + 3H (1 + \omega_\Lambda) \rho_\Lambda = -Q, \]

(14)

\[ \dot{\rho}_M + 3H \rho_M = Q, \]

(15)

where \( Q = 3b^2 H (\rho_\Lambda + \rho_m) \) is the interaction term and \( b^2 \) is a coupling constant. Moreover, following [36] we suppose that the BD field behaves as \( \phi = a^n \) so that

\[ \dot{\phi} = nH \phi , \quad \ddot{\phi} = (n^2 H^2 + nH) \phi. \]

(16)

A case of specific interest is when \( n \) is small while \( \omega \) is large, so that the product \( n\omega \) results in a value of order unity [30]. The fractional energy densities are given by

\[ \Omega_M = \frac{4\omega \rho_m}{3\dot{\phi}^2 H^2}, \]

(17)
\[ \Omega_k = \frac{k}{a^2 H^2}, \quad (18) \]

\[ \Omega_\Lambda = \frac{4\omega \rho_\Lambda}{3\phi^2 H^2}. \quad (19) \]

Taking time derivative of (11) gives

\[ H(z) = H_0 \left[ \frac{\Omega_{M_0} (1 + z)^{2n + 3}}{1 + \Omega_k + \frac{2}{3}n(3 - n \omega) - \Omega_\Lambda} \right]^{\frac{1}{2}}, \quad (20) \]

where we have converted time derivative to a derivative with respect to \( z \), and zero-indices quantities indicates the present time values. Also, we obtain the deceleration parameter as

\[ q = -\frac{\ddot{a}}{\dot{a}^2} = -1 - \frac{\dot{H}}{H^2}. \quad (21) \]

Taking time derivative of (18) yields

\[ \frac{d\Omega_k}{dz} = 2\Omega_k \left( (1 + z)^{-1} - \frac{dH}{H} \right), \quad (22) \]

where we have used using \( \frac{d\Omega_k}{dz} = -H(1 + z)\frac{d\Omega_k}{dt} \) and \( \frac{dH}{dt} = -H(1 + z)\frac{dH}{dz} \). On the other hand, taking time derivative of (19), using (14) and \( \frac{dH}{dt} = -H(1 + z)\frac{dH}{dz} \), yields

\[ \frac{d\Omega_\Lambda}{dz} = (1 + z)^{-1} \left[ 3b^2(\Omega_\Lambda + \Omega_m) + 3(1 + \omega_\Lambda)\Omega_\Lambda + 2n\Omega_\Lambda - 2\Omega_\Lambda(1 + z)\frac{dH}{dz} \right]. \quad (23) \]

Taking time derivative of (20), using Eqs. (22), (23) and \( \frac{dH}{dt} = -H(1 + z)\frac{dH}{dz} \), yields

\[ \frac{dH}{dt} = -\left( 2n + 3 + 3b^2 \right) (1 + \Omega_k - \frac{2}{3}n^2 \omega + 2n) \frac{-2\Omega_k + 3\Omega_\Lambda \omega_\Lambda}{2(1 + z)(-1 + \frac{2}{3}n^2 \omega - 2n)} - \frac{-2\Omega_k + 3\Omega_\Lambda \omega_\Lambda}{2(1 + z)(-1 + \frac{2}{3}n^2 \omega - 2n)}. \quad (24) \]

Therefore, using Eq. (24), the deceleration parameter is obtained

\[ q = -1 - \frac{(2n + 3 + 3b^2)(1 + \Omega_k - \frac{2}{3}n^2 \omega + 2n) - 2\Omega_k + 3\Omega_\Lambda \omega_\Lambda}{2(-1 + \frac{2}{3}n^2 \omega - 2n)}. \quad (25) \]

Moreover, using Eqs. (11), (19) and (14), we can obtain the following relations

\[ \beta = \frac{3c^2}{L^2 - \alpha} \left( 1 - \frac{L^2 H^2 \Omega_\Lambda}{c^2} \right), \quad (26) \]

\[ \alpha \ln \left( \frac{\phi L^2}{4\omega} \right) + \beta = L^4 \left( H^2 \Omega_\Lambda - \frac{c^2}{L^2} \right) \frac{3\phi^2}{4\omega}. \quad (27) \]

which will be used in the derivation of \( \omega_\Lambda \) in the following. For power-law and logarithmic entropy-corrected holographic dark energy models with any cut-off in Brans-Dicke cosmology, taking time derivative of (11) and (19), using (14), and using (26), (27) we obtain, respectively

\[ \omega_\Lambda = -1 - \frac{b^2}{\Omega_\Lambda} \left( 1 + \Omega_k - \frac{2}{3}n^2 \omega + 2n \right) - \frac{2c^2}{L^2 H \Omega_\Lambda} \left[ 1 + z \right] \left( 1 - \frac{\alpha}{2} \left( 1 - \frac{L^2 H^2 \Omega_\Lambda}{c^2} \right) + \frac{L^2 H^2 \Omega_\Lambda}{c^2} \right), \quad (28) \]

\[ \omega_\Lambda = -1 - \frac{b^2}{\Omega_\Lambda} \left( 1 + \Omega_k - \frac{2}{3}n^2 \omega + 2n \right) - (n - \frac{dL}{L}(1 + z)) \left[ \frac{2c^2}{3L^2 H^2 \Omega_\Lambda} + \frac{8\alpha n(1 + z)^2 n}{9L^4 H^2 \Omega_\Lambda} \right] - \frac{4}{3}(1 + z)\frac{dL}{L}. \quad (29) \]
One of the most important quantities for cosmological evolution of the universe is the squared sound speed, \(v_s^2 = \frac{dp}{d\rho} \). The squared sound speed of dark energy fluid is given by

\[
v_s^2 = \frac{dp}{d\rho} = -\frac{\dot{\rho}_\Lambda}{\dot{\rho}_\Lambda}
\]

Also, we have

\[
v_s^2 = \omega_\Lambda - \frac{\dot{\omega}_\Lambda}{3H(1 + \omega^{eff}_\Lambda)}.
\]

Here \(\omega^{eff}_\Lambda = \omega_\Lambda + \frac{\dot{\phi}}{\pi\rho_\Lambda}\) corresponds to the effective equation of state parameter for dark energy. For \(v_s^2 > 0\), there is a classical stability and for \(v_s^2 < 0\), there is a classical instability [38].

III. INTERACTING ENTROPY-CORRECTED HOLOGRAPHIC DARK ENERGY MODEL WITH HUBBLE CUT-OFF

The Hubble cut-off is considered as

\[L = H^{-1}.\]

A. PLECHDE model

Using Eqs. (20), (24), and (28), we obtain

\[
\omega_\Lambda = \left\{-1 - \frac{b^2}{\Omega_\Lambda} (1 + \Omega_k - \frac{2}{3} n^2 \omega + 2n) - \frac{2 c^2 H_0}{\Omega_\Lambda} \left[ \frac{\Omega_{M_0}(1 + z)^{2n+3}}{1 + \Omega_k + \frac{2}{3} n(3 - n\omega) - \Omega_\Lambda} \right] \right\} \times \left\{ \left[ \frac{(2n + 3 + 3b^2)(1 + \Omega_k - \frac{2}{3} n^2 \omega + 2n) - 2\Omega_k}{2(-1 - 2n + \frac{2}{3} n^2 \omega)} \right] \times \left( \frac{\Omega_{M_0}(1 + z)^{2n+3}}{1 + \Omega_k + \frac{2}{3} n(3 - n\omega) - \Omega_\Lambda} \right) \right\} \times \left\{ \left(1 - 2n + \frac{2}{3} n^2 \omega \right) \times \left[ 1 - \frac{\alpha}{2} (1 - \frac{\Omega_\Lambda}{c^2}) \right] \times \left[ 1 + \Omega_k + \frac{2}{3} n(3 - n\omega) - \Omega_\Lambda \right] \right\}^{-1}.
\]

For present time, we consider \(z \simeq 0, \alpha \simeq 0, c^2 \simeq 1.1\) [39], \(n \simeq 0.005\) [41], \(H_0 \simeq 67.8, \Omega_\Lambda \simeq 0.7, \Omega_{M_0} \simeq 0.27, \Omega_k \simeq 0\) and \(b^2 = 0.02\) [39]. The condition for acceleration as \(q < 0\), results in \(-10000 \lesssim \omega \lesssim 10000\) which is almost the same range for having the phantom phase \(\omega_\Lambda < -1\).

At early universe, using Eqs. (21), (28) and \(H \simeq const\) [40], we obtain \(q \simeq -1\) and

\[\omega_\Lambda = -1 - 2nH.\]

For early time, the condition \(\omega_\Lambda < -1\) requires \(n > 0\), so \(n \simeq 0.005\) [41] is consistent with the phantom phase. Therefore, the PLECHDE model with Hubble cut-off and \(-10000 \lesssim \omega \lesssim 10000\) supports the inflationary and accelerating universe with phantom phase at early and present universe, respectively.

Using \(\omega_\Lambda\) and its time derivative corresponding to early and present times, and using Eq. (31) with \(b^2 = 0, 0.02, 0.04, 0.06\), we get \(v_s^2 < 0\) and \(v_s^2 > 0\) respectively, indicating that PLECHDE model has classical instability at early time and classical stability at present time.
B. LECHDE model

For LECHDE model, using Eqs. (20), (24), and (29) we obtain

\[
\omega_{\Lambda} = \left\{ -1 - \frac{b^2}{3\Omega_{\Lambda}^2} + \frac{8\alpha \omega H_0^2 \Omega_{M_0}(1 + z)^{4n_+^3}}{9\Omega_{\Lambda}(1 + \Omega_k + \frac{2n}{3}n(3 - n\omega) - \Omega_{\Lambda})} \right. \\
\left. \times \left[ 1 - \frac{3\Omega_{\Lambda}}{2(-1 + \frac{2n}{3}n^2\omega - 2n)} \right] \right. \\
\left. \times \left[ \frac{2c^2}{3\Omega_{\Lambda}^2} + \frac{8\alpha \omega H_0^2 \Omega_{M_0}(1 + z)^{4n_+^3}}{9\Omega_{\Lambda}(1 + \Omega_k + \frac{2n}{3}n(3 - n\omega) - \Omega_{\Lambda})} \right] \right\}^{-1}. \tag{35}
\]

With the same numerical setup mentioned before, in order to have \( q < 0 \) at present time, we obtain \( \omega \lesssim 8383 \). However, \( \omega_{\Lambda} < -1 \) requires \( \omega \lesssim -2860 \). In other words, \( \omega \lesssim -2860 \) supports the inflationary and accelerating universe as well as an accelerating universe. For \( -2860 \lesssim \omega \lesssim 8380 \), we have an accelerating universe without the phantom phase.

At early universe, using Eqs. (21), (29), and \( H \simeq \text{const} \), we obtain \( q \simeq -1 \) and

\[
\omega_{\Lambda} = -1 - \frac{2c^2}{3\Omega_{\Lambda}^2} + \frac{8\alpha \omega H_0^2 (1 + z)^{2n_+^3}}{9\Omega_{\Lambda}}. \tag{36}
\]

Using the parameter values \( z \simeq 10^4 \), \( \alpha \simeq 1 \), \( c^2 \simeq 1.1 \), \( \Omega_{\Lambda} \simeq 1 \), the condition \( \omega_{\Lambda} < -1 \) or \( \omega_{\Lambda} > -1 \) requires \( \omega \gtrsim -H_{-2} \) or \( \omega \lesssim -H_{-2} \), respectively.

Therefore, LECHDE model with Hubble cut-off and \( \omega \lesssim -2860 \) supports the inflationary and accelerating universe with phantom phase at present universe; however, at early universe we may have inflationary universe with or without phantom phase corresponding to \( \omega \gtrsim -H_{-2} \) or \( \omega \lesssim -H_{-2} \), respectively.

Using the same procedure as that of PLECHDE model, for LECHDE model we obtain \( v_s^2 < 0 \) for both early and present times which accounts for the classical instability at both eras.

IV. INTERACTING ENTROPY-CORRECTED HOLOGRAPHIC DARK ENERGY MODEL WITH APPARENT HORIZON CUT-OFF

The apparent horizon cut-off is given as

\[
L = r_A = \frac{1}{\sqrt{H^2 + \frac{\dot{b}^2}{\omega}}}. \tag{37}
\]

A. PLECHDE model

For power-law entropy-corrected holographic dark energy model, using Eqs. (21), (24), (28) and (37), we can obtain EOS parameter at the present time

\[
\omega_{\Lambda} = \left\{ -1 - \frac{b^2}{3\Omega_{\Lambda}^2} (1 + \Omega_k + \frac{2n}{3}n^2\omega - 2n) \right. \\
\left. - \frac{2c^2 H_0^2 (1 + \Omega_k)}{\Omega_{\Lambda}^2} \left[ \frac{\Omega_{M_0}(1 + z)^{2n_+^3}}{1 + \Omega_k + \frac{2n}{3}n(3 - n\omega) - \Omega_{\Lambda}} \right] \right. \\
\left. \times \left[ 1 - \frac{n}{c^2 (1 + \Omega_k)} \right] \right. \\
\left. \times \left[ 1 - \frac{2}{c^2 (1 + \Omega_k)} \right] \right. \\
\left. \times \left[ \frac{2c^2}{3\Omega_{\Lambda}^2} + \frac{8\alpha \omega H_0^2 \Omega_{M_0}(1 + z)^{4n_+^3}}{9\Omega_{\Lambda}(1 + \Omega_k + \frac{2n}{3}n(3 - n\omega) - \Omega_{\Lambda})} \right] \right\}^{-1}. \tag{36}
\]
For the present time, we consider $z \simeq 0, \alpha \simeq 0, c^2 = 1.1$ \cite{39}, $n = 0.005$ \cite{41}, $H_0 = 67.8, \Omega_\Lambda = 0.7, \Omega_{M_0} = 0.27, \Omega_k \simeq 0$ and $b^2 = 0.02$ \cite{39}. By this set-up, the conditions for acceleration $q < 0$ and phantom phase $\omega_\Lambda < -1$ results in $\omega < 10000$ and $-10000 < \omega < 10000$, respectively. Therefore, for $-10000 < \omega < 10000$ we have both acceleration and phantom phase.

At early time, using Eqs. (28) and $H = constant$, we obtain $q \simeq -1$ and

$$\omega_\Lambda = -1 - \frac{2c^2H(1 + \Omega_k)}{\Omega_\Lambda} \left[ \frac{\Omega_k}{1 + \Omega_k} \left\{ 1 - \frac{\Omega_\Lambda}{2(1 - \frac{\Omega_\Lambda}{c^2(1 + \Omega_k)})} \right\} + \frac{n\Omega_\Lambda}{c^2(1 + \Omega_k)} \right].$$

(39)

For early time, the condition $\omega_\Lambda < -1$ requires $n > 0$, so $n \simeq 0.005 \cite{41}$ is consistent with the phantom phase. Therefore, the PLECHDE model with apparent cut-off and $-10000 \lesssim \omega \lesssim 10000$ supports the inflationary and accelerating universe with phantom phase at early and present universe, respectively.

Using $\omega_\Lambda$ and its time derivative corresponding to early and present times, and using Eq. (39) with $b^2 = 0, 0.02, 0.04, 0.06$, we get $v^2_c < 0$ and $v^2_c > 0$ respectively, indicating that PLECHDE model has classical instability at early time and classical stability at present time.

### B. LECHDE model

For interacting logarithmic entropy-corrected holographic dark energy model at present time, using Eqs. (20), (24), and (29) one can write

$$\omega_\Lambda = \left\{ -1 - \left( n + \frac{\Omega_k}{1 + \Omega_k} \right) \frac{2c^2(1 + \Omega_k)}{3\Omega_\Lambda} + \frac{8\alpha\omega H^2\Omega_{M_0}(1 + z)^{4n+3}(1 + \Omega_k)^2}{9\Omega_\Lambda(1 + \Omega_k + \frac{2n^2(3 - n\omega)}{3} - \Omega_\Lambda)} + \frac{4\Omega_k}{3(1 + \Omega_k)} + \frac{-2\Omega_k + (2n + 3 + 3b^2)(1 + \Omega_k - \frac{2n^2\omega}{3} + 2n)}{2(-1 + \frac{2}{3}n^2\omega - 2n)(1 + \Omega_k)} \times \left[ \frac{2c^2(1 + \Omega_k)}{3\Omega_\Lambda} + \frac{8\alpha\omega H^2\Omega_{M_0}(1 + z)^{4n+3}(1 + \Omega_k)^2}{9\Omega_\Lambda(1 + \Omega_k + \frac{2n^2(3 - n\omega)}{3} - \Omega_\Lambda)} - \frac{4}{3} \right] \right\}^{-1} \times \left[ 1 - \left( \frac{3\Omega_\Lambda}{2(-1 + \frac{2}{3}n^2\omega - 2n)(1 + \Omega_k)} \right) \times \left[ \frac{2c^2(1 + \Omega_k)}{3\Omega_\Lambda} + \frac{8\alpha\omega H^2\Omega_{M_0}(1 + z)^{4n+3}(1 + \Omega_k)^2}{9\Omega_\Lambda(1 + \Omega_k + \frac{2n^2(3 - n\omega)}{3} - \Omega_\Lambda)} - \frac{4}{3} \right] \right].$$

(40)

For the present time, we consider $z \simeq 0, c^2 = 1.1$ \cite{39}, $n = 0.005 \cite{41}, H_0 = 67.8, \Omega_{M_0} = 0.27, \Omega_k \simeq 0$ and $b^2 = 0.02 \cite{39}$. By this set-up, for $q < 0$ and $\omega_\Lambda < -1$ we obtain $-29032 \lesssim \omega \lesssim 34254$ and $16908 \lesssim \omega \lesssim 38858$, respectively. Therefore, we may have acceleration with the phantom phase for some regions of $\omega$.

At early time, using Eqs. (20) and (37) and $H = constant$, we obtain $q \simeq -1$ and

$$\omega_\Lambda = -1 - \left( n + \frac{\Omega_k}{1 + \Omega_k} \right) \frac{2c^2(1 + \Omega_k)}{3\Omega_\Lambda} + \frac{8\alpha\omega H^2(1 + z)^{2n}(1 + \Omega_k)^2}{9\Omega_\Lambda} + \frac{4\Omega_k}{3(1 + \Omega_k)}.$$  

(41)

where $\alpha$ is dimensionless constant of order unity.

Using the parameter values $z \simeq 10^4, \alpha \simeq 1, c^2 \simeq 1.1$ and $\Omega_\Lambda \simeq 1$, the condition $\omega_\Lambda < -1$ or $\omega_\Lambda > -1$ requires $\omega \gtrsim -H^{-2}$ or $\omega \lesssim -H^{-2}$, respectively.

Therefore, LECHDE model with apparent cut-off and $16908 \lesssim \omega \lesssim 38858$ supports the inflationary and accelerating universe with phantom phase at present universe; however, at early universe we may have inflationary universe with or without phantom phase corresponding to $\omega \gtrsim -H^{-2}$ or $\omega \lesssim -H^{-2}$, respectively.

Using the same procedure as that of PLECHDE model, for LECHDE model at present time, we obtain that the squared speed $v^2_c$ is negative for $0.5 < \Omega_\Lambda < 1$ and is positive for $0 < \Omega_\Lambda < 0.5$. That is, we have classical instability $v^2_c < 0$, for $0.5 < \Omega_\Lambda < 1$ and the classical stability $v^2_c > 0$, for $0 < \Omega_\Lambda < 0.5$.

At early time, we obtain that the squared speed is always negative and that there is a classical instability.
V. INTERACTING ENTROPY-CORRECTED HOLOGRAPHIC DARK ENERGY MODEL WITH EVENT HORIZON CUT-OFF

The event horizon cut-off is considered as

\[ L = a(t)r(t), \]  \hspace{1cm} (42)

and

\[ r(t) = \frac{\sin n(\sqrt{|k|}y)}{\sqrt{|k|}} = \begin{cases} 
\sin y & k = 1, \\
y & k = 0, \\
\sinh y & k = -1,
\end{cases} \]

where

\[ y = \frac{R_h}{a(t)} = a(t) \int_{a(t)}^{\infty} \frac{da(t)}{a(t)^2H}. \]  \hspace{1cm} (43)

Here, \( L \) and \( R_h \) are the radius of the event horizon measured on the sphere of the horizon and the radial size of the event horizon, respectively \[42\].

A. PLECHDE model

For power-law entropy-corrected holographic dark energy model, Using Eqs. \([20], [21], [28], [12]\) and \([13]\) at the present time we can write

\[ \omega_{\Lambda} = -1 - \frac{b^2}{\Omega_{\Lambda}}(1 + \Omega_k - \frac{2}{3}n^2 \omega + 2n) - \frac{2H_0}{\gamma_c} \left[ \frac{\Omega_{M_0}(1 + z)^{2n + 3}}{1 + \Omega_k + \frac{2}{3}n(3 - n\omega) - \Omega_{\Lambda}} \right]^{\frac{1}{2}} \left[ -1 + n\gamma_c + \sqrt{\Omega_{\Lambda} - \Omega_k} \right], \]  \hspace{1cm} (44)

where

\[ \gamma_c = 1 - \frac{\beta}{3c^2}L^{2-\alpha}. \]  \hspace{1cm} (45)

Moreover, for PLECHDE at early time, using Eqs. \([28], [12], [33]\) and \( H \approx constant \), we obtain \( q \approx -1 \) and

\[ \omega_{\Lambda} = -1 - 2Hc^2 \left[ \left(1 + \sqrt{\Omega_{\Lambda} - \Omega_k}\right) \times \left(1 - \frac{\alpha}{2} \left(1 - \frac{1}{c^2}\right)\right) \right] + \frac{n}{c^4}, \]  \hspace{1cm} (46)

For the present time, we consider \( z \approx 0, \alpha \approx 0, c^2 = 1.1 \) \[39\], \( n = 0.005 \) \[41\], \( H_0 = 67.8, \Omega_{\Lambda} = 0.7, \Omega_{M_0} = 0.27, \Omega_k \approx 0 \) and \( b^2 = 0.02 \) \[39\]. By this set up, The condition for acceleration \( q < 0 \) results in \(-5000 < \omega < 0 \) and no Brans-Dicke parameter is obtained for the phantom phase condition \( \omega_{\Lambda} < -1 \). Therefore, we have acceleration without the phantom phase.

Using \( \omega_{\Lambda} \) and its time derivative corresponding to early and present times, and using Eq.\([31]\) with \( b^2 = 0, 0.02, 0.04, 0.06 \), we get the classical stability for the range \( 0 < \Omega_{\Lambda} < 0.2 \) and the classical instability for the range \( 0.2 < \Omega_{\Lambda} < 1 \) at present time. At early time, we find that the squared speed \( v_s^2 \) is positive indicating the classical stability for early time.

B. LECHDE model

For logarithmic entropy-corrected holographic dark energy model, Using Eqs. \([20], [21], [29], [12]\) and \([13]\) at the present time we can write

\[ \omega_{\Lambda} = -1 - n \left[ \frac{2}{3\gamma_\alpha} + \frac{8\alpha \omega (1 + z)^{2n}}{9L^2 c^2 \gamma_\alpha} \right] + \left(1 - \sqrt{\frac{\Omega_{\Lambda}}{c^2 \gamma_\alpha} - \Omega_k} \right) \left[ -\frac{2}{3\gamma_\alpha} - \frac{8\alpha \omega (1 + z)^{2n}}{9L^2 c^2 \gamma_\alpha} + \frac{4}{3} \right] - \frac{b^2}{\Omega_{\Lambda}} (1 + \Omega_k - \frac{2}{3}n^2 \omega + 2n), \]  \hspace{1cm} (47)
where
\[
\gamma_\alpha = 1 + \frac{4\omega(1 + z)^{2n}}{3L^2c^2} \left[ \alpha \ln\left( \frac{L^2}{4\omega(1 + z)^{2n}} + \beta \right) \right].
\]  
Equation (48)

Also, for LECHDE at the early time, using Eqs. (53) \(\), (54) and \(H = constant\), we obtain \(q \approx -1\) and

\[
\omega_\Lambda = -1 - \frac{n}{9} \left[ \frac{2c^2}{3} + \frac{8\omega(1 + z)^{2n}H^2}{9} \right] + \left( 1 - \sqrt{\Omega_\Lambda - \Omega_k} \right) \left[ -\frac{2c^2}{3} - \frac{8\omega(1 + z)^{2n}H^2}{9} + \frac{4}{3} \right].
\]  
Equation (49)

VI. INTERACTING ENTROPY-CORRECTED HOLOGRAPHIC DARK ENERGY MODEL WITH SCALAR RICCI CUT-OFF

We consider IR cut-off as \(L = R^{-\frac{1}{2}}\) where \(R\) is Ricci scalar curvature. The scalar Ricci cut-off is given by

\[
R = 6(\dot{H} + 2H^2 + \frac{k}{a^2}).
\]  
Equation (50)

Here \(\dot{H}\) is the derivative of the hubble parameter with respect to the cosmic time \(t\). Using Eqs. (11) and (10), we get

\[
H^2 + \frac{k}{a^2} = \frac{4\omega}{3\phi^2} (\rho_\Lambda + \rho_M) + 2nH^2(-1 + \frac{n\omega}{3}).
\]  
Equation (51)

Now, using Eqs. (10) and (51), we can write

\[
R = 6\left( \dot{H} + H^2 + \frac{4\omega}{3\phi^2} (\rho_\Lambda + \rho_M) + 2nH^2(-1 + \frac{n\omega}{3}) \right). \]  
Equation (52)

Differentiating Eq. (51) with respect to the cosmic time \(t\) and using Eqs. (14), (15) and (51), we can derive

\[
\dot{H} + H^2 = \frac{4\omega}{\phi^2} \left[ \frac{\rho_\Lambda (\frac{2}{9}\omega + \frac{1}{9} + n) + (n + \frac{1}{n})\rho_M}{-1 + 2n(n\omega - 1)} \right].
\]  
Equation (53)

Inserting Eq. (53) in Eq. (52), we obtain

\[
\omega_\Lambda = \left( \frac{\phi^2(2n^2\omega - 6n - 3)}{\rho_\Lambda} \right) \left( \frac{R}{36\omega} - \frac{nH^2(n\omega - 3)}{9\omega} \right) - \frac{(1 + \Omega_k - \frac{4}{9}n^2\omega + 2n)(4n^2\omega - 6n - 3)}{9\Omega_\Lambda}.
\]  
Equation (54)

Moreover, using Eqs. (13), (16), (59), (21) and \(P_\Lambda = \rho_\Lambda \omega_\Lambda\), we find

\[
q = \frac{1}{2(n + 1)} \times \left[ 3\Omega_\Lambda \omega_\Lambda + (2n + 1)^2 + 2n(n\omega - 1) + \Omega_k \right].
\]  
Equation (55)
A. PLECHDE model

Using Eq. (4) and \( L = R^{-\frac{1}{2}} \), we find

\[
\rho_{\Lambda} = \frac{3c^2\varphi^2 R}{4\omega} \gamma_{\mu},
\]

where

\[
\gamma_{\mu} = 1 - \frac{\beta}{3c^2 R^{\frac{3}{2}} - 1}. 
\]

For early time, using Eqs. (28), (50) and \( H = \text{constant} \), we can obtain

\[
\omega_{\Lambda} = \omega_{A} = -1 + \frac{12c^2 H}{\Omega_{\Lambda}} \left(-1 + \frac{2}{3}n^2 - 2n + \frac{\Omega_{\Lambda}}{2\omega} \right) - 2nH. 
\]

Here \( \alpha \) is dimensionless constants of order unity. For early time, we consider \( c^2 = 1.1, \Omega_k \approx 0 \) and assume \( \alpha > 2 \). In order to have \( \omega_{\Lambda} < -1 \), it turns out that \( n > 0 \) for any value of \( \omega \).

For 0 < \( R < 1 \) at the early time, taking time derivative of \( \omega_{\Lambda} \), and using Eqs. (31) and (58), one finds that the squared speed is always negative for early time. This means that we have a classical instability.

B. LECHDE model

Using Eq. (6) and \( L^2 = R^{-1} \), one can obtain

\[
\rho_{\Lambda} = \frac{3c^2\varphi^2 R}{4\omega} \gamma_{\phi},
\]

where

\[
\gamma_{\phi} = 1 + \frac{4c^2 R}{3c^2 \varphi^2} \left[ \alpha \ln(\frac{\varphi^2}{4c^2 R}) + \beta \right]. 
\]

For the early time, using Eqs. (50) and (29), \( H \approx \text{constant} \), we obtain

\[
\omega_{\Lambda} = -1 + \frac{128c^2 R^2}{\Omega_{\Lambda}} \left[ \frac{1}{\Omega_{\Lambda}} + \frac{128c^2 R^2}{\Omega_{\Lambda}} \left( \frac{1 + \frac{3}{2}n^2 - 3n}{\Omega_{\Lambda}} \right) \right].
\]

Where \( \alpha \) is dimensionless constants of order unity. For the early time, we consider \( c^2 = 1.1, \Omega_k \approx 0 \). In order to have \( \omega_{\Lambda} < -1 \), we obtain the range \( -10^{-98} < \omega < -10^{-42} \) and \( n > -0.1 \). For 0 < \( R < 1 \) at the early time, taking time derivative of Eq. (61), and using Eqs. (50) and (61), one finds that the squared speed is always negative for early time. This means that we have a classical instability.

For both PLECHDE and LECHDE models at the present time, we consider \( \alpha = \beta = 0, \gamma_{\mu} = 1, \gamma_{\phi} = 1 \). Using Eqs. (20), (54), (56) and (59) we obtain

\[
\omega_{\Lambda} = \left[ 4\frac{2n^2 - 6n - 3}{3c^2} \right] \times \left[ \frac{1}{36} - \frac{nH_0^2(nw - 3)\Omega_{M_0}(1 + z)^{2n+3}}{9R(1 + \Omega_k + \frac{3}{2}n(3 - nw) - \Omega_{\Lambda})} \right] - \frac{(1 + \Omega_{k} - \frac{4}{3}n^2 \omega + 2n)(4n^2 \omega - 6n - 3)}{9\Omega_{\Lambda}}.
\]

Now, for both PLECHDE and LECHDE models, we consider \( c^2 = 1.1 \), \( n = 0.005 \), \( H_0 = 67.8 \), \( \Omega_{M_0} = 0.27, \Omega_k \approx 0 \) and 0 < \( R < 1 \). By this set up, for \( q < 0 \) and \( \omega_{\Lambda} < -1 \), we obtain 20000 < \( \omega < 45000 \) and \( -100000 < \omega < -100000 \), respectively. Therefore, we have acceleration without the phantom phase.

Taking time derivative of Eq. (62), and using Eqs. (51) and (58) for \( \beta^2 = 0, 0.02, 0.04, 0.06 \) and 0 < \( R < 1 \) at the present time, one finds that the squared speed is always positive for present time. This means that we have a classical stability.
For PLECHDE and LECHDE models, using Eq. (4) and Eq. (6), we find respectively

\[ L = (\dot{\alpha} H^2 + \tilde{\beta} \dot{H})^{-\frac{1}{2}}, \]

where \( \dot{\alpha} \) and \( \tilde{\beta} \) are constant. By inserting Eq. (16) in Eq. (12) and using \( P_\Lambda = \rho_\Lambda \omega_\Lambda \), we obtain

\[ \rho_\Lambda = -\frac{\dot{\phi}^2 H^2}{4 \omega_\Lambda} \left\{ 3 + \Omega_k + 4n + 2n^2(2 + \omega) + \frac{\dot{H}}{H^2}(2n + 2) \right\}. \] (64)

moreover, By inserting Eq. (24) in Eq. (64), we find

\[ \omega_\Lambda = \left[ 3 + \Omega_k + 4n + 2n^2(2 + \omega) + (n + 1) \times \left( \frac{(2n + 3 + 3b^2)(1 + \Omega_k - \frac{2}{3} n^2 \omega + 2n) - 2\Omega_k}{-1 - 2n + \frac{2}{3} n^2 \omega} \right) \right] \times \left[ -\frac{4\omega_\Lambda}{\dot{\phi}^2 H^2} - \frac{3(n + 1)\Omega_\Lambda}{-1 - 2n + \frac{2}{3} n^2 \omega} \right]^{-1}. \] (65)

For PLECHDE and LECHDE models, using Eq. (11) and Eq. (10), we find respectively

\[ \rho_\Lambda = \frac{3c^2 \dot{\phi}^2}{4 \omega L^2 \gamma_c}, \] (66)

and

\[ \rho_\Lambda = \frac{3c^2 \dot{\phi}^2}{4 \omega L^2 \gamma_\alpha}, \] (67)

where \( L \) is the Granda-Oliveros cut off. For both PLECHDE and LECHDE models, considering \( \alpha = \beta = 0 \), and using \( \omega_\Lambda \), (24), (63) and (64) at present time we can find

\[ \omega_\Lambda = \left\{ 3 + \Omega_k + 4n + 2n^2(2 + \omega) + (n + 1) \times \left[ \frac{(2n + 3 + 3b^2)(1 + \Omega_k - \frac{2}{3} n^2 \omega + 2n) - 2\Omega_k}{-1 - 2n + \frac{2}{3} n^2 \omega} \right] \right\} \times \left\{ -\frac{3c^2 (1 + \Omega_k - \frac{2}{3} n(3 - n\omega - \Omega_\Lambda))}{L^2 H_0^2 \Omega_{M_0}(1 + z)^{2n+3}} - \frac{3(n + 1)\Omega_\Lambda}{-1 + \frac{2}{3} n^2 \omega - 2n} \right\}^{-1}, \] (68)

where \( q \) is given by Eq. (65). For the present time, we consider \( c^2 = 1.1 \), \( n = 0.005 \), \( H_0 = 67.8 \), \( \Omega_{M_0} = 0.27 \), \( \Omega_k \simeq 0 \) and \( b^2 = 0.02 \), \( z = 0 \) and \( 0 < L < 1 \). For both LECHDE and PLECHDE models, the demand for \( q < 0 \) and \( \omega_\Lambda < -1 \) results in \( 10000 < \omega < 12000 \) and \( 11100 < \omega < 12000 \), respectively. Hence, we may have acceleration with phantom phase for \( 11100 < \omega < 12000 \).

For the early time, using Eqs. (54), (60), (67) and \( H = constant \), we can obtain \( q \simeq -1 \) and

\[ \omega_\Lambda = \frac{-3 + \Omega_k + 4n + 2n^2(2 + \omega)}{3\Omega_\Lambda}. \] (69)

Considering \( c^2 = 1.1 \) and \( \Omega_k \simeq 0 \), we find that the demand for \( \omega_\Lambda < -1 \), results in \( n > 0 \).

For present time, taking time derivative of \( \omega_\Lambda \) and using Eqs. (31) and \( \omega_\Lambda \) for \( b^2 = 0.02, 0.04, 0.06 \) and \( 0 < L < 1 \), we obtain the classical instability. For both PLECHDE model and LECHDE model, one finds that the squared speed is negative. This means that we have a classical instability.

For early time, taking the time derivative of \( \omega_\Lambda \), and using Eqs. (31) and \( \omega_\Lambda \), we find that for both PLECHDE and LECHDE models the squared speed is always negative. This means that we have a classical instability.
### Table 1. Brans-Dicke parameter versus the phantom phase $\omega_\Lambda < -1$ at present time.

| Cut-Off          | PLECHDE                   | LECHDE       |
|------------------|---------------------------|--------------|
| Hubble           | $-10000 < \omega < 10000$ | $\omega \lesssim -2860$ |
| Apparent         | $-10000 < \omega < 10000$ | $16908 \lesssim \omega \lesssim 38858$ |
| Event Horizon    | $-\infty < \omega < -\infty$ | $\omega \lesssim -225706$ |
| Scalar Ricci     | $-100000 < \omega < -10000$ | $-10000 < \omega < -10000$ |
| Granda-Oliveros  | $11100 < \omega < 12000$  | $11100 < \omega < 12000$ |

### Table 2. Brans-Dicke parameter versus the acceleration phase $q < 0$ at present time.

| Cut-Off          | PLECHDE                   | LECHDE       |
|------------------|---------------------------|--------------|
| Hubble           | $-10000 < \omega < 10000$ | $\omega \lesssim 8383$ |
| Apparent         | $-10000 < \omega < 10000$ | $-29032 \lesssim \omega \lesssim 34254$ |
| Event Horizon    | $-5000 < \omega < 0$     | $-51628 \lesssim \omega \lesssim 59086$ |
| Scalar Ricci     | $20000 < \omega < 45000$ | $20000 < \omega < 45000$ |
| Granda-Oliveros  | $10000 < \omega < 12000$  | $10000 < \omega < 12000$ |

### Table 3. Brans-Dicke parameter versus the phantom phase $\omega_\Lambda < -1$ at early time.

| Cut-Off          | PLECHDE                   | LECHDE       |
|------------------|---------------------------|--------------|
| Hubble           | $n > 0, -\infty < \omega < \infty$ | $\omega > -H^{-2}$ |
| Apparent         | $n > 0, -\infty < \omega < \infty$ | $\omega > -H^{-2}$ |
| Event Horizon    | $n > 0, -\infty < \omega < \infty$ | $\omega > -H^{-2}$ |
| Scalar Ricci     | $n > 0, -\infty < \omega < \infty$ | $-10^{-98} < \omega < -10^{-42}, n > -0.1$ |
| Granda-Oliveros  | $n > 0, \omega > 0$       | $n > 0, \omega > 0$ |

### Table 4. Brans-Dicke parameter versus the acceleration phase $q < 0$ at early time.

| Cut-Off          | PLECHDE                   | LECHDE       |
|------------------|---------------------------|--------------|
| Hubble           | $\omega = -\frac{3(nH - 2n - 1)}{n(2n + 3)}, n \neq 0, -\frac{3}{2}$ | $\omega = 6n + 8.7 \frac{2n^2 + 3n + 4H^2(10001)^2}{2n^2 + 3n + 4H^2(10001)^2}, -\infty < n < \infty$ |
| Apparent         | $\omega = -\frac{3(nH - 2n - 1)}{n(2n + 3)}, n \neq 0, -\frac{3}{2}$ | $\omega = 6n + 8.7 \frac{2n^2 + 3n + 4H^2(10001)^2}{2n^2 + 3n + 4H^2(10001)^2}, -\infty < n < \infty$ |
| Event Horizon    | $\omega = -\frac{3(nH - 2n - 1)}{n(2n + 3)}, n \neq 0, -\frac{3}{2}$ | $\omega = 6n + 8.7 \frac{2n^2 + 3n + 4H^2(10001)^2}{2n^2 + 3n + 4H^2(10001)^2}, -\infty < n < \infty$ |
| Scalar Ricci     | $\omega = \frac{375H^2 - 10.5H - 1.25 - 1.25n}{2n(125H + 0.625)}, n \neq 0$ | $\omega = 6n + 33.0 \frac{-3n + 6H^2(10001)^2}{-3n + 6H^2(10001)^2}, -\infty < n < \infty$ |
| Granda-Oliveros  | $n = 0$                   | $n = 0$      |
| Cut-Off | PLECHDE | LECHDE |
|---------|---------|--------|
| Hubble  | Stability | Instability |
| Apparent| Stability | Instability ($0 < \Omega_\Lambda < 0.5$) |
|         |          | Stability ($0.5 < \Omega_\Lambda < 1$) |
| Event Horizon | Stability ($0 < \Omega_\Lambda < 0.2$) | Instability |
|           | Instability ($0.2 < \Omega_\Lambda < 1$) |
| Scalar Ricci | Stability | Stability |
| Granda-Oliveros | Instability | Instability |

Table 6. Classical stability or instability of models at early time.

| Cut-Off | PLECHDE | LECHDE |
|---------|---------|--------|
| Hubble  | Instability | Instability |
| Apparent| Instability | Instability |
| Event Horizon | Stability | Stability |
| Scalar Ricci | Instability | Instability |
| Granda-Oliveros | Instability | Instability |

VIII. CONCLUDING REMARKS

We have studied the interacting power-law and logarithmic entropy-corrected holographic dark energy models with different cut-offs in the framework of Brans-Dicke cosmology. For comparison between the two entropy corrected models at early and present time universes, we have obtained the Brans-Dicke parameter versus the conditions for the acceleration and phantom phases. Moreover, using the squared sound speed, the classical stability or instability of the interacting power-law and logarithmic entropy-corrected holographic dark energy models with different cut-offs is determined. This study shows which entropy corrected model can exhibit acceleration with or without the phantom phase, and which entropy corrected model is stable or unstable, at early and present time.

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