Vortex structure of thin mesoscopic disks in the presence of an inhomogeneous magnetic field

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(March 22, 2022)

The vortex states in a thin mesoscopic disk are investigated within the phenomenological Ginzburg-Landau theory in the presence of different “model” magnetic field profiles with zero average field which may result from a ferromagnetic disk or circulating currents in a loop near the superconductor. We calculated the dependences of both the ground and metastable states on the magnitude and shape of the magnetic field profile for different values of the order parameter angular moment, i.e. the vorticity. The regions of existence of the multi-vortex state and the giant vortex state are found. We analysed the phase transitions between these states and studied the contribution from ring-shaped vortices. A new transition between different multi-vortex configurations as the ground state is found. Furthermore, we found a vortex state consisting of a central giant vortex surrounded by a collection of anti-vortices which are located in a ring around this giant vortex. The limit to a disk with an infinite radius, i.e. a film, will also be discussed. We also extended our results to “real” magnetic field profiles and to the case in which an external homogeneous magnetic field is present.

PACS numbers: 74.60.Ec, 74.60.Ge, 74.80.-g, 74.25.Dw, 74.25.Ha

I. INTRODUCTION

Recent progress in microfabrication and measurement techniques makes it possible to study the properties of small superconducting structures, so called mesoscopic samples, with sizes comparable to the penetration depth $\lambda$ and the coherence length $\xi$. Mesoscopic disks have been one of the most popular and exciting study objects in this respect. The behavior of such mesoscopic samples in an external magnetic field is strongly influenced by the boundary conditions, sample size and geometry and may lead to various superconducting states and phase transitions between them.

Motivated by recent experiments we study the properties of a superconducting disk in the presence of a step-like external magnetic field. The step-like field profile is a model magnetic field profile for a ferromagnetic dot or current loop placed on top of the superconductor. These profiles have the important property that the average magnetic field is zero. We investigate the influence of step height, step profile, and ratio between step width and radius of the disk, on the superconducting phase diagram.

Previous investigations of structures with magnetic dots were limited to experiments with superconducting films deposited on regular arrays of magnetic dots and theoretical studies of single magnetic dots embedded in a thin superconducting film. The common problem was that, for magnetic dots, the strong field present inside the ferromagnet suppresses the superconducting order parameter, and in such situations it is appropriate to adopt a boundary condition in which the order parameter itself vanishes. This spoils the effect that leads to surface superconductivity, and it is not at first obvious why magnetic dots should support the relatively large supercurrents associated with multiple vortices. Therefore a possible oxide layer between the magnetic dot and the superconductor may restore the boundary condition to the one of a superconductor/insulator interface. In the present paper we put a single magnetic dot on top of the superconductor and study the behavior of our sample in such non-uniform magnetic field of the dot which enhances the possibility of obtaining various combinations of superconducting states. To better understand the problem we start from a simple theoretical model for the inhomogeneous magnetic field profile that, we believe, captures all aspects of the physics involved. Models used before vary from a representation of the magnetic dot by a perfect dipole to a magnetic field profile calculated numerically for an infinitely thin magnetic disk. To obtain a better insight we start first with a simple step-like field model and subsequently investigate the more complicated real magnetic field profiles, which we obtained numerically.

Theoretical studies have predicted that in mesoscopic disks surrounded by an insulating media three kinds of superconducting states can exist - giant vortex (a circular symmetric state with a fixed value of angular momentum), multi-vortex state (a collection of single vortices which can be obtained as a linear combination of giant vortices with different angular momentum), and ring-shaped vortex states with larger energy than giant and multi-vortex states. The ring-shaped two-dimensional vortex states have a cylindrically symmetric magnetic field profile and they
are different from the ring-vortices which are e.g. found in three-dimensional superfluid liquid helium. In the present paper we observe giant vortex states and first-order transitions between them and for sufficiently large disks, multi-vortex structures, which are the analogue of the Abrikosov flux-line lattice in a bulk superconductor. The latter results not only from a mixture of giant vortex states but also from giant-ring vortex combinations as well. The latter one may even lead to an off-center location of a single vortex or multi-vortices. Moreover, with changing the strength of the field there is a transition between giant-giant and giant-ring multi-vortex states. Increasing the step height of the magnetic field profile we found re-entrant behavior, i.e., transition from giant to multi-vortex state and back to the giant vortex state before superconductivity is destroyed. We found that for sufficiently large magnetic disks vortex/anti-vortex structures can be formed. In order to investigate these different vortex structures we use the method proposed by Schweigert et al.\( ^8\) and Palacios\( ^9\) with its semi-analytical extensions of Ref.\( ^{20}\) to determine the stability of the different multi-vortex configurations. In particular, the analysis of Ref.\( ^{20}\) showed that in a superconducting disk the ring-shaped vortices are unstable in the presence of a homogeneous magnetic field.

Our analysis is within the framework of the phenomenological Ginzburg-Landau (GL) theory. Although this theory has only a firm mathematical derivation in a narrow range of magnetic field close to the superconducting-normal state boundary, it has been found that it gives also very good results deep inside the superconducting phase diagram.\( ^{11,12}\)

The paper is organized as follows. In Sec. II we present our theoretical model. In Sec. III we discuss the giant vortex states and study the influence of a step magnetic field profile, with zero average which is centered at the disk or has a ring symmetry, on the superconducting state. These step profiles are limiting cases of the actual experimentally realizable profiles. The stability of multi-vortex states and transitions between them are investigated in Sec. IV. In the Appendix we give the analytical approach/solution to this subject. The excitement in this study is shown through different \(H_{in} - R\) phase diagrams in Sec. V, where \(H_{in}\) is the magnitude of the magnetic field profile and \(R\) represents the radius (of superconducting disk, magnetic dot, current loop, etc.). In Sec. VI we present the results of our approach applied to a superconducting disk in the presence of an experimentally realizable real magnetic dot and current loop field profile. Sec. VII is an extension of previous sections where the influence of an additional homogeneous background magnetic field is investigated. Our conclusions are given in Sec. VIII.

II. THEORETICAL MODEL

We consider a mesoscopic superconducting disk with radius \(R\) and thickness \(d\) surrounded by vacuum. The external magnetic field \(\vec{H} = (0, 0, H)\) is directed normal to the disk plane. In this paper we investigate two different magnetic field profiles: 1) step-like magnetic field in the center of the disk, and 2) a ring step-like magnetic field profile with inner radius of the ring \(R_d\). The magnetic field strenghts of the profile (Fig. 1) are chosen such that the total magnetic flux equals zero. These models should correspond to the magnetic field of a perpendicular magnetized magnetic disks vortex/anti-vortex structures can be formed.

\[ \frac{1}{2m} \left( -i\hbar \nabla - \frac{2e}{c} \vec{A} \right)^2 \Psi = -\alpha \Psi - \beta |\Psi|^2, \]  
\(\text{(1)}\)

\[ \nabla \times \nabla \times \vec{A} = \frac{4\pi}{c} \vec{j}, \]  
\(\text{(2)}\)

where the density of the superconducting current \(\vec{j}\) is given by

\[ \vec{j} = \frac{e\hbar}{im} \left( \Psi^* \nabla \Psi - \Psi \nabla \Psi^* \right) - \frac{4e^2}{mc} |\Psi|^2 \vec{A}. \]  
\(\text{(3)}\)

Here \(\vec{r} = (\vec{r}, z)\) is the three-dimensional position in space. Due to the circular symmetry of the disk we use cylindrical coordinates: \(\rho\) is the radial distance from the disk center, \(\varphi\) is the azimuthal angle and the \(z\)-axis is taken perpendicular to the disk plane, where the disk lies between \(z = -d/2\) and \(z = d/2\).

Eqs. (1\(\text{a}\)) has to be supplemented by boundary conditions (BC) for \(\Psi(\vec{r})\) and \(\vec{A}(\vec{r})\):

\[ \left( -i\hbar \nabla - \frac{2e}{c} \vec{A} \right) \Psi |_n = 0, \]  
\(\text{(4)}\)
where the subscript $n$ denotes the component normal to the disk surface. The boundary condition for the vector potential has to be taken far away from the disk where the magnetic field becomes equal to the external field $H$

$$\mathbf{A}\bigg|_{r\to\infty} = 0.$$  

(5)

Using dimensionless variables and the London gauge $\nabla \cdot \mathbf{A} = 0$ we can rewrite the system of equations (1) and BC (4) in the following form

$$\left(-i\nabla - \mathbf{A}\right)\psi = \psi - |\psi|^2\psi,$n

(6)

$$-\kappa^2\Delta \mathbf{A} = \frac{1}{2i}\left(\psi^* \nabla \psi - \psi \nabla^* \psi^*\right) - |\psi|^2 \mathbf{A},$$

(7)

$$\left(-i\nabla - \mathbf{A}\right)\psi|_{in} = 0,$n

(8)

Here all distances are measured in units of the coherence length $\xi = \hbar/\sqrt{2m|\alpha|}$, the order parameter in $\Psi_0 = \sqrt{|\alpha|}/\beta$, the vector potential in $c\hbar/2e\xi$, $\kappa = \lambda/\xi$ is the GL parameter, and $\lambda = c\sqrt{m/\pi\hbar|\alpha|}$ is the London penetration depth. We measure the magnetic field in $H_c^2 = c\hbar/2e\xi^2 = \kappa\sqrt{2}H_c$, where $H_c = \sqrt{4\pi\alpha^2/\beta}$ is the thermodynamical critical field.

The free energy of the superconducting state, measured in $F_0 = H_c^2V/8\pi$ units, is determined by the expression

$$F = \frac{2}{V}\left\{ \int dV \left[ -|\psi|^2 + \frac{1}{2}|\psi|^4 + \left(-i\nabla \psi - \mathbf{A}\mathbf{\psi}\right)^2 + \kappa^2 \left(\bar{H}(\mathbf{r}) - \bar{H}_0\right)^2 \right] \right\},$$

with the local magnetic field

$$\bar{H}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r}).$$

We restrict ourselves to sufficiently thin disks such that $d \ll \xi, \lambda$. In this case, to a first approximation, the magnetic field due to the circulating superconducting currents may be neglected and the total magnetic field equals the external one $\mathbf{H}_0$. Within this approximation we have to solve only the first GL equation (6) with $\mathbf{A} = \mathbf{A}_0$, where $\mathbf{H}_0 = \text{rot} \mathbf{A}_0$.

One should notice that in this approach, for different cases, we change only the vector potential profile $\mathbf{A}_0$. For our step profile

$$H_0(\rho) = \begin{cases} 0, & 0 \leq \rho \leq R_d, \\ \rho, & R_d \leq \rho \leq R_1, \\ H_{in}, & R_1 \leq \rho \leq R_2, \\ -\rho, & R_2 \leq \rho \leq R. \end{cases}$$

(10)

the vector potential is given by

$$A_0(\rho) = \begin{cases} 0, & 0 \leq \rho \leq R_d, \\ \frac{H_{in}}{2} \left(\rho - \frac{R_d^2}{\rho}\right), & R_d \leq \rho \leq R_1, \\ H_{in} \left(R_1^2 - \rho^2\right)/2\pi \left(\rho - \frac{R_d^2}{\rho}\right), & R_1 \leq \rho \leq R_2, \\ 0, & R_2 \leq \rho \leq R. \end{cases}$$

(11)

where $H_{in}$ describes the positive step and $H_{out} = -H_{in}R_2^2/(R_2^2 - R_1^2)$ determines the value of the negative step (see Fig. 4, Fig. 5, and the Appendix).

First, we determine the $z$-dependence of $\psi(\mathbf{r})$. Representing the order parameter as a series over cosines $\psi(\mathbf{r}) = \sum_{k=0}^{\infty} \psi_k(\mathbf{r}) \cos(k\pi z/d)$ and using the same BC (8) at the disk sides ($z = \pm d/2$) and using the first GL equation (6), one can verify that the uniform part of the order parameter, i.e. the $k=0$ term, gives the main contribution for $(\pi \xi/d)^2 >> 1$. Therefore, we may assume that the order parameter is uniform along the $z$ direction of the disk and average the first GL equation over the disk thickness. After this averaging and for fixed value of the angular
momentum it leads to \( \psi(\vec{p}) = f(\rho) \exp(iL\varphi) \), and the problem for \( f(\rho) \) is reduced to a one dimensional problem, like in Ref\(^2\):

\[
- \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial f}{\partial \rho} + \left( \frac{L}{\rho} - A \right)^2 f = f(1 - f^2),
\]

(12)

and for the vector potential

\[
- \kappa^2 \left( \frac{\partial}{\partial \rho} \frac{1}{\rho} \frac{\partial A}{\partial \rho} + \frac{\partial^2 A}{\partial z^2} \right) = \left( \frac{L}{\rho} - A \right) f^2 \theta(\rho/R) \theta(2|z|/d),
\]

(13)

where the function \( \theta(x) = 1 \) \((x < 1)\), 0 \((x > 1)\), and \( R, d \) are the dimensionless disk radius and thickness, respectively. The brackets \( \langle ... \rangle \) refer to averaging over the disk thickness.

### III. GIANT VORTEX STATES

The giant vortex state has cylindrical symmetry and consequently the order parameter can be written as \( \psi(\vec{p}) = f(\rho) \exp(iL\varphi) \). The stable states are obtained in the following way. From the linearized GL equation we find \( f(\rho) \) up to a multiplying constant. This function is then inserted into the free energy expression (9) which after minimization determines: (i) the constant in \( f(\rho) \), and (ii) the energy value corresponding to the stable state. It can be shown that, for the case of giant vortex states, the present approach and the one of Ref\(^2\) which was based on a solution of the non-linear GL equation, result into the same functional \( F(H_m) \) dependence.

The linearized GL equation for \( f(\rho) \) takes the form

\[
\dot{L}f = 0, \quad \dot{L} = - \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial f}{\partial \rho} + \left( \frac{L}{\rho} - A_0 \right)^2 - 1.
\]

(14)

The superconducting state starts to develop when the minimal eigenvalue of the operator \( \dot{L} \) becomes negative. For the zero angular momentum state, the normal state transforms to the superconducting one with decreasing magnetic field below the nucleation field \( H_{nuc} \). For nonzero angular momentum, the superconducting state appears when we cross either the lower \( H_{nuc,l} \) or the upper \( H_{nuc,u} \), critical magnetic field which depends on the disk radius. The eigenvalues and eigenfunctions of the \( \dot{L} \) operator are found from

\[
\dot{L}f_{n,L}(\rho) = \Lambda_{n,L} f_{n,L}(\rho),
\]

(15)

where \( f_{n,L}(\rho) \) satisfies \( \rho(\partial f/\partial \rho)|_{\rho=0} = 0 \) at the disk center. The index \( n = 1, 2, ... \) enumerates the different states for the same \( L \)-value.

In general, the eigenfunctions of Eq. (13) can be obtained analytically in the case of our step magnetic field profile. We present the complete calculation in the Appendix for both considered cases. Alternatively, we solve Eq. (13) numerically through the finite difference technique. We put the order parameter on a space grid and find numerically the eigenfunctions and eigenvalues of the operator \( \dot{L} \) using the Householder technique.

We start our analysis with the magnetic field profile shown in Fig. 2. First, we consider a magnetic dot on top of the center of the disk, i.e. with \( R_d = 0 \), and \( R_2 = R = 6.0\xi \) and study the influence of \( R_1 \), i.e. the width of the positive field region, on the superconducting state.

The magnetic field dependence of \( \Lambda \) for different angular momenta \( L \) are shown in Figs. 3(a-c) for the lowest radial state, i.e. \( n = 0 \), and in Figs. 3(d-f) for the first radial state, i.e. \( n = 1 \) for three different values of \( R_1 \). The top axis shows the flux corresponding to the positive magnetic field region \( \phi_{in} = H_{in}\pi R_1^2 \), which in dimensionless units becomes \( \phi_{in}/\phi_0 = (H_{in}/2H_{c2})(R_1/\xi)^2 \) where \( \phi_0 = ch/2e \) is the quantum of flux. All numerical calculations were done for a disk thickness \( d/\xi = 0.1 \) which is within the thin disk approximation. The negative \( L \) values (dashed curves in Fig. 3) correspond to “anti-vortices” in conventional superconductors. The thin horizontal line gives the \( \Lambda = 0 \) level. From Fig. 3 one notices that with increasing \( R_1 \): 1) the eigenvalues \( \Lambda \) of the states with the same \( L \) become more negative; 2) the magnetic field range over which solutions of Eq. (13) can be found decreases; 3) the number of possible solutions decreases also; and 4) the \( \Lambda(H_{in}) \) dependences become more parabolic. The latter can be explained by the fact that increasing \( R_1 \) corresponds to a more homogeneous magnetic field profile inside the disk, i.e. we reach the case considered in Ref\(^2\).

For small \( R_1 \) the curves \( L \) and \( L + 1 \) anti-cross for sufficiently large \( L \) values and consequently the low vorticity states have lower energy even with increasing strength of the magnetic field profile. E.g. for \( R_1/\xi = 1.5 \) this occurs...
when $L > 4 = L^*$ and for $R_1/\xi = 3.0$ when $L > 5 = L^*$. Notice that the slope of $\Lambda$ for the $L > L^*$ curves is substantially smaller than for the $L \leq L^*$ curves in the high field region. Notice also that the $n = 1$ states have a higher $\Lambda$ value than the $n = 0$ states and their energy is also larger than those of the anti-vortex states for the same vorticity.

In Figs. 3(a-c) the radial dependence of the superconducting density $|\psi|^2$ is shown for $|L| < 3$ at $H_{in} = 0.75H_{c2}$ for the corresponding profiles of Fig. 3. Notice that, with increasing $R_1$, the Cooper-pair density near the edge of the sample becomes more non-homogeneous. The $|L| \neq 0$ states have a vortex sitting in the center of the disk, i.e. $\psi(\rho = 0) = 0$, which becomes larger with increasing $|L|$. For the situation of Fig. 3(c) there is a narrow, very negative magnetic field region of $H_{out}/H_{c2} = 0.9643$ in the region $4.5 < \rho/\xi < 6.0$ which leads to a considerable suppression of the Cooper pair density. For example, the $L = 0$ vortex state, i.e. the Meissner state, has a strongly reduced Cooper pair density for $\rho/\xi > 1$. For the excited state $(n, L) = (1, 0)$ (dashed curve in Fig. 3) the order parameter vanishes inside the disk and a ring-shaped node in the wave function $\psi$ is formed which leads to a ring-like vortex.

The eigenvalues $\Lambda$ determine the free energy $F$ of the giant vortex state. For the giant vortex state we consider only states which lie below the $F = 0$ level. In this approximation the order parameter is

$$\psi(\vec{r}) = \left( -\frac{I_2}{I_1} \right)^{1/2} f_{n,L}(\rho) \exp(iL\varphi), \quad (16)$$

and the minimal energy value is

$$F = -\Lambda^2 \frac{2\pi d I_2^2}{V I_1}, \quad (17)$$

where

$$I_1 = \int_0^R \rho d\rho f_{n,L}^2(\rho), \quad I_2 = \int_0^R \rho d\rho f_{n,L}^2(\rho).$$

The dependence of the free energy on the magnetic field strength of the inner core of the magnetic field profile, $H_{in}$, are shown in Figs. 3(a-c) for the angular momenta $|L| < 11$ for different values of $R_1$. The free energy is expressed in units of $F_0 = H_{c2}^2V/8\pi$. The highest value of vorticity in this disk is $L = 15$ (see Figs. 3(a-c)). From a comparison of the magnetic field dependence of $F(H_{in})$ for $R_1/\xi = 1.5$ (Fig. 3(a)) and the one with $R_1/\xi = 4.5$ (Fig. 3(c)) we clearly observe the reduction of superconductivity and the reduction of the maximal possible vorticity with increasing $R_1$. The envelope of the lowest parts of the curves in Fig. 3 represents the field dependence of the ground state energy. Notice that the increase of the width of the positive magnetic field region leads to an increase of the energy of the ground state. With increasing applied field the $L \rightarrow L + 1$ vortex transitions take place at the field where the corresponding curves cross (for example, the $0 \rightarrow 1$ transition occurs at $H_{in} = 0.8705H_{c2}$ for $R_1/\xi = 1.5$). The crossing points are shifted towards lower field values when increasing $R_1$. The $L \rightarrow L + 1$ transitions are of first order and lead to jumps in the magnetization of the sample. Notice that the positive flux captured in the superconducting disk for different $L$-states is not quantized. This makes it very clear in the $\phi_{in} - R_1$ diagram presented in Fig. 3(d) which shows the ground state vortex configurations. Notice that the flux in the positive magnetic field region has to increase with more than one flux quantum $\phi_0$ before the vorticity of the superconducting state increases with one unit. A similar phenomena was observed earlier for mesoscopic disks and rings in a homogeneous magnetic field. Notice also that for higher $L$ values quantization is slowly restored, and $\Delta \phi$ decreases with enlarging $R_1$, i.e. the radius of the positive field region.

Following our assumption that field inside the superconductor equals the external one, we obtain the expression for the superconducting current density $j_{n,L}$:

$$j_{n,L} = \frac{\Lambda_{n,L} A_{n,L}}{A_{n,L}} \left( \frac{L}{\rho} - A_0 \right) f_{n,L}^2(\rho), \quad (18)$$

where $\Lambda_{n,L}$ is determined by Eq. (13), $A_0$ represents the vector potential of the applied field and

$$A_{n,L} = \frac{2\pi d}{V} \int_0^R \rho d\rho f_{n,L}^2(\rho), \quad B_{n,L} = \frac{2\pi d}{V} \int_0^R \rho d\rho f_{n,L}^2(\rho).$$

The magnetic field due to the supercurrents, neglected in our first order approximation, is calculated from
\[ \text{rot} \overline{H_{sc}} = \frac{1}{\kappa^2} \overline{j}. \]  

Since the supercurrent has only an azimuthal component, and, is situated only in the superconductor plane, the \( \rho \) component of \( \overline{H_{sc}} \) can be neglected. Consequently, we obtain

\[ H_{sc}(\rho) = -\frac{1}{\kappa^2} \int j_\varphi(\rho) d\rho, \]  

and the magnetization of the superconductor is then defined as the magnetic field expelled from the superconductor.

\[ M = \int \frac{H_{total} - H_0}{8\pi} dV = \frac{d}{4} \int_0^R H_{sc}(\rho) \rho d\rho, \]  

where \( H_0 \) denotes the applied magnetic field, and \( d \) is the disk thickness.

The corresponding \( M(H_m) \) curves are given as insets in Figs. 3(a-c). The phase transition from the superconducting to the normal state is of second order (all curves \( F(H_{in}) \) reach the \( F = 0 \) line with zero derivative). The curves \( F(H_{in}) \) in Fig. 3 which are situated above the ground state energy correspond to metastable giant vortex states. With increasing applied field the transition from the Meissner state \( (L = 0) \) to the normal state goes through a set of consecutive first order transitions between the \( L \) and \( L+1 \) giant vortices which is finished by a second order transition to the normal state.

Our next step was to fix the magnetic field profile (we took \( R_d/\xi = 0.0, R_1/\xi = 4.5, R_2/\xi = 6.0 \)) and enlarge the disk radius \( R \). In Figs. 3(a-b) the radial dependence of the superconducting density \( |\psi|^2 \) is shown for \( |L| < 3 \) at \( H_{in} = 0.75H_{c2} \) and for a superconducting disk of radius \( R/\xi = 9.0 \), and 12.0, respectively. One can see that, even for \( L = 0 \), superconductivity in the center of the disk is destroyed, which is opposite to the homogeneous magnetic field case where \( |\Psi|^2 \) is maximal at \( \rho = 0 \). The shape of the \( L = 0 \) curve changes drastically and this occurs already for small enlargement of the disk. This transition is shown in Fig. 3(a). By increasing \( R/\xi \) from 6.1 to 6.5 the Cooper pair density in the center of the superconducting disk decreases from 0.73 to 0.03. Furthermore, in Fig. 3(b) we see also qualitative changes in the Cooper pair density of the excited state \( (n, L) = (1, 0) \) (dashed curve). Now the order parameter vanishes twice inside the disk and a double ring-like vortex is formed. This transition is shown in Fig. 3(b) with increasing disk size from \( R/\xi = 10.0 \) to 11.0.

The dependences of the free energy on the magnetic field \( H_{in} \) for different sizes of the superconducting disk are shown in Figs. 3(a-d). Notice that there are large differences in comparison with the previous cases. We still have the \( L \rightarrow L + 1 \) transitions, which are of first order and which lead to jumps in the magnetization of the sample (see Figs. 3(a-c)), but with increasing applied field the transition from the Meissner state \( (L = 0) \) to the normal state goes through a set of consecutive first order transitions and is finished by a first order transition back to the \( L = 0 \) state, sometimes with an intermediate \( L = 1 \) state. For \( R/\xi = 9.0 \) we have \( 0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 1 \rightarrow 0 \) transitions, for \( R/\xi = 12.0 \) we have \( 0 \rightarrow 1 \rightarrow 2 \rightarrow 1 \rightarrow 0 \), and for \( R/\xi = 18.0 \) only \( 0 \rightarrow 1 \rightarrow 0 \). Moreover, for a sufficiently large disk (see Fig. 3(d) which corresponds practically to the \( R/\xi \rightarrow \infty \) situation) we have no first order transitions but only the Meissner state as the ground state. The reason is that for \( R/\xi \rightarrow \infty \) the non-zero magnetic field region is limited (relatively) to a small area and we can always define a circle with radius \( \rho \) sufficiently large where the superconducting current is zero and, consequently one must have \( L = 0 \). But, this Meissner state is qualitatively different from the "usual" one. The radial distribution of the Cooper pair density is extremely inhomogeneous and superconductivity is strongly suppressed in the interior of the disk (see Figs. 3(b) and 3(a)).

IV. MULTI-VORTEX STATES

It is well known that for sufficiently large disks the giant vortex state can break up into multi-vortices. In our analysis, we considered different field profiles and disk geometries to investigate the properties of this transition. For smaller disks, the confinement effect dominates and we found that only the giant vortex states are stable and possible multi-vortex states, if they exist, have always larger energies. In order to investigate such structures in our case we use the method proposed by Schweigert et al. and extended it to determine the stability of the different multi-vortex configurations as proposed in Ref. 8. Following Refs. 8 and 14, the order parameter of the multi-vortex state is written as a linear combination of eigenfunctions of the linearized GL equation (14)

\[ \psi(\overline{\rho}) = \sum_{L_j=0}^{L} \sum_{n=0}^{\infty} C_{n,L_j} f_{n,L_j}(\rho) \exp(iL_j\varphi), \]  

where \( C_{n,L_j} \) are amplitudes to be determined.
where $L$ is, in the homogeneous magnetic field case, the value of the effective total angular momentum which is equal to the number of vortices in the disk, and $n$ enumerates the different radial states for the same $L_i$. Later on, we will see that for our inhomogeneous magnetic field case the assignment of the total vorticity can be more tricky.

Substituting (22) in the free energy expression (3) we obtain $F$ as a function of the complex parameters \{C_{n,L}\}. Minimization of $F$ with respect to these parameters allows us, to find the equilibrium vortex configurations, and to determine their stability. We use the procedure described in Refs. 23, 24 with full consideration of combinations of vortex configurations with different radial states.

We begin our analysis with states built up by only two components in Eq. (22). This brings quantitative bounds in our calculation but, nevertheless, will give the correct qualitative behavior and facilitates the physical insight into the problem. The free energy of a two component state built out of \{(n_1, L_1)\} and \{(n_2, L_2)\} is

$$F = C_{n_1,L_1}^4 A_{n_1,L_1} + C_{n_2,L_2}^4 A_{n_2,L_2} + 4C_{n_1,L_1}^2 C_{n_2,L_2}^2 A_{n_1,n_2,L_1,L_2} + 2\Lambda_{n_1,L_1} C_{n_1,L_1}^2 B_{n_1,L_1} + 2\Lambda_{n_2,L_2} C_{n_2,L_2}^2 B_{n_2,L_2}, \tag{23}$$

where

$$A_{n_i,L_i} = \frac{2\pi d}{V} \int_0^R \rho d\rho f_{n_i,L_i}^4 (\rho) ,$$

$$A_{n_1,n_2,L_1,L_2} = \frac{2\pi d}{V} \int_0^R \rho d\rho f_{n_1,L_1}^2 (\rho) f_{n_2,L_2}^2 (\rho) ,$$

$$B_{n_i,L_i} = \frac{2\pi d}{V} \int_0^R \rho d\rho f_{n_i,L_i}^2 (\rho) .$$

One should notice that we leave the possibility of combination of states with different radial states, i.e. $n_1$ and $n_2$. Although, in general, $C_{n_1,L_1}$ is a complex number, for our two component state $C_{n_i,L_i}$ is a real number. Minimization of Eq. (23) with respect to $C_{n_1,L_1}$ and $C_{n_2,L_2}$ gives for the multi-vortex states:

$$C_{n_1,L_1}^{(0)} = \pm \left( \frac{-\Lambda_{n_1,L_1} A_{n_2,L_2} B_{n_1,L_1} + 2\Lambda_{n_2,L_2} A_{n_1,n_2,L_1,L_2} B_{n_2,L_2}}{A_{n_1,L_1} A_{n_2,L_2} - 4A_{n_1,n_2,L_1,L_2}^2} \right)^{1/2} , \tag{24}$$

$$C_{n_2,L_2}^{(0)} = \pm \left( \frac{-\Lambda_{n_2,L_2} A_{n_1,L_1} B_{n_2,L_2} + 2\Lambda_{n_1,L_1} A_{n_1,n_2,L_1,L_2} B_{n_1,n_1}}{A_{n_1,L_1} A_{n_2,L_2} - 4A_{n_1,n_2,L_1,L_2}^2} \right)^{1/2} ,$$

and inserting these expressions into Eq. (23) leads to the energy of the multi-vortex state

$$F_{n_1,n_2,L_1,L_2} = \frac{-\Lambda_{n_1,L_1}^2 A_{n_2,L_2} B_{n_1,L_1}^2 - \Lambda_{n_2,L_2}^2 A_{n_1,L_1} B_{n_2,L_2}^2 + 4\Lambda_{n_1,L_1} A_{n_1,L_1} A_{n_1,n_2,L_1,L_2} B_{n_1,L_1} B_{n_2,L_2}}{A_{n_1,L_1} A_{n_2,L_2} - 4A_{n_1,n_2,L_1,L_2}^2} \tag{25}$$

The corresponding conditions for the stability of the vortex state are

$$\frac{\partial^2 F}{\partial C_{n_1,L_1}^2} = \frac{8A_{n_1,L_1} (-\Lambda_{n_1,L_1} A_{n_2,L_2} B_{n_1,L_1} + 2\Lambda_{n_2,L_2} A_{n_1,n_2,L_1,L_2} B_{n_2,L_2})}{A_{n_1,L_1} A_{n_2,L_2} - 4A_{n_1,n_2,L_1,L_2}^2} > 0, \tag{26}$$

$$\frac{\partial^2 F}{\partial C_{n_2,L_2}^2} = \frac{8A_{n_2,L_2} (-\Lambda_{n_2,L_2} A_{n_1,L_1} B_{n_2,L_2} + 2\Lambda_{n_1,L_1} A_{n_1,n_2,L_1,L_2} B_{n_1,L_1})}{A_{n_1,L_1} A_{n_2,L_2} - 4A_{n_1,n_2,L_1,L_2}^2} > 0,$$

$$\frac{\partial^2 F}{\partial C_{n_1,L_1}^2} \frac{\partial^2 F}{\partial C_{n_2,L_2}^2} - \left( \frac{\partial^2 F}{\partial C_{n_1,L_1} \partial C_{n_2,L_2}} \right)^2 = \frac{64 (-\Lambda_{n_1,L_1} A_{n_2,L_2} B_{n_1,L_1} + 2\Lambda_{n_2,L_2} A_{n_1,n_2,L_1,L_2} B_{n_2,L_2})}{A_{n_1,L_1} A_{n_2,L_2} - 4A_{n_1,n_2,L_1,L_2}^2} (-\Lambda_{n_2,L_2} A_{n_1,L_1} B_{n_2,L_2} + 2\Lambda_{n_1,L_1} A_{n_1,n_2,L_1,L_2} B_{n_1,L_1}) > 0.$$

In our analysis we investigated the influence of the width of the positive field region and radius of the disk on the phase diagram, and especially on the stability of the multi-vortex states. For small values of $R_1/\xi$ these multi-vortex
states are always metastable. However, with enlarging $R_1/\xi$ these states can obtain lower energy. The energies of the equilibrium vortex states are plotted in Fig. 1(a) for $R_1/\xi = 5.25$ and $R_2/\xi = R/\xi = 6.0$ and in Fig. 1(b) for $R_1/\xi = 4.5$ and $R_2/\xi = 6.0$ with a larger disk radius $R/\xi = 9.0$. The giant vortex states are given by solid curves, anti-vortex states by dashed and the multi-vortex states (dotted curves) by $(L_1 : L_2)$, i.e. the angular momentum values they are composed of. As expected, when enlarging the disk, multi-vortex states become more stable and, moreover, in Fig. 1(b), we observe the existence of multi-vortices as a combination of a giant and a ring-like vortex. These states have surprisingly low energy and become the ground state for specific ranges of magnetic field. Furthermore, when increasing the field, we observe a phase transition between this giant-ring and giant-giant multi-vortex states, when another type of multi-vortices becomes the ground state. The giant-ring multi-vortex states with lowest energy are given in Fig. 1(b) by the quantum numbers $((n_1, L_1) : (n_2, L_2))$. It should be noted that there are many other metastable combinations possible, which are not shown in the figures.

As shown in Fig. 1, with increasing $R_1/\xi$ and the size of the superconducting disk the multi-vortices become more stable and they can even become the minimum of the $F(C_{n_1, L_1}, C_{n_2, L_2})$ function. We focus our attention to Fig. 2(a). The solid and dashed curves represent the giant vortex and anti-vortex states, respectively. The dotted curves are the energies of the multi-vortices. For example, lets follow the energies of the multi-vortices. For a fixed magnetic field profile and with increasing size of the superconducting disk, we obtain a variety of different superconducting states. Let us discuss the giant-ring configurations first. We observe transitions between the giant and multi-vortex states with the same vorticity. The contour plots of the $|\psi|^2$ distribution for different multi-vortex states as ground states $((0, 1) : (1, 0), (0, 2) : (1, 0), (0, 3) : (1, 0))$ are shown in Figs. 2(a-c) for a magnetic field profile with $R_1/\xi = 4.5$, $R_2/\xi = 6.0$ and $R/\xi = 9.0$, for different values of $H_{in}$ (dark regions correspond to high density and white regions to low density). The most interesting result we obtain for the $(0, 1) : (1, 0)$ combination. This configuration becomes the ground state configuration for $H_{in}/H_{c2} = 0.28$ and with further increase of the field becomes another giant vortex state, but now with $L = 4$, at $H_{in}/H_{c2} = 1.0$. The concomitant change of the Cooper pair density is illustrated in Fig. 10. Vortices enter the disk from the boundary and move to the middle with increasing field, and join into a giant vortex again.

For a fixed magnetic field profile and with increasing size of the superconducting disk, we obtain a variety of different superconducting states. Let us discuss the giant-ring configurations first. We observe transitions between the giant and multi-vortex states with the same vorticity. For example, lets follow the $|\psi|^2$ distribution for different multi-vortex states as ground states $((0, 1) : (1, 0), (0, 2) : (1, 0), (0, 3) : (1, 0))$ are shown in Figs. 2(a-c) for a magnetic field profile with $R_1/\xi = 4.5$, $R_2/\xi = 6.0$ and $R/\xi = 9.0$, for different values of $H_{in}$ (dark regions correspond to high density and white regions to low density). The most interesting result we obtain for the $(0, 1) : (1, 0)$ combination. This configuration becomes the ground state configuration for $H_{in}/H_{c2} = 0.75$. Surprisingly, the contour plot of the $|\psi|^2$ distribution is not circular symmetric - there is one vortex positioned off-center. In the lower part of the same figures we give the contour plots of the phase of the superconducting wave functions. The contour plot of the phase also nicely illustrates the off-center location of the vortex. For the other two cases more vortices are present which are located on a ring centered around the center of the superconducting disk. The giant-ring multi-vortex state can be seen as a transition between $(1, 0)$ and $(0, 5)$ state (see Fig. 1(b)). As function of the magnetic field we start from a ring vortex, for $H_{in}/H_{c2} = 0.03$, this state splits into a giant-ring vortex state, and finally, for $H_{in}/H_{c2} = 0.98$ we obtain a giant vortex state. This remarkable phenomenon is illustrated in Fig. 12.

However, giant-giant vortex combinations show completely different behavior. As shown in Fig. 1(b) with increasing magnetic field $H_{in}$, combinations $(0 : 7), (0 : 8), (0 : 9)$ etc. become the ground state. In the contour plot of the $|\psi|^2$ distribution we observe single vortices which are arranged on a ring with a low density area situated in the center of the disk (see Figs. 13(a-c)). This central area is associated with a giant vortex, and encuriling it leads to a phase change showing the vorticity 7. Fig. 13(d) shows the contour plot of the corresponding phase with seven anti-vortices arranged in a circle around the giant vortex and the total vorticity is $L = 0$. Figs. 13(e) and Fig. 13(f) correspond, respectively, to a giant vortex state with vorticity 8 and 9 where, respectively, 8 and 9 anti-vortices are located between the giant vortex and the boundary. Notice that the Cooper pair density for the $L = 0$ state for this disk geometry ($R_1/\xi = 4.5$, $R_2/\xi = 6.0$, $R/\xi = 9.0$) shows a similar behavior with a low density area in the center of the disk (see Fig. 1(b)). It is obvious that enlarging the superconducting disk enhances the influence of the negative part of the step-like magnetic field. In the case when multi anti-vortices are involved, the total vorticity of the giant-giant multi-vortex state is equal to the lowest vorticity of the two giant vortex states which compose the vortex state.

One more result should be noted. Following the ground state free energy diagram (Fig. 1(b)) we notice that the total vorticity doesn’t change uniformly ($0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 7 \rightarrow 8 \rightarrow 9$) and that the jumps in vorticity $|\Delta L|$ are not always equal to 1. This differs from Ref. 2 where it was claimed that the lowest barriers are those between the $L$ and $L + 1$ states. We found that the $L \rightarrow L \pm 1$ transitions take place in disks for a small maximum value of vorticity or at magnetic fields close to the “superconducting-normal state” transition point. Between these two limiting regimes $L \rightarrow L \pm N$ transitions are possible with $|\Delta L| = N > 1$. Our results are also in agreement with Ref. 4 where it was found numerically that several vortices can enter (or exit) at once for disks with sufficiently large radius. In Sec. III, where we investigated the influence of $R_1/\xi$ (with $R_2/\xi = R/\xi = 6.0$) we only found $L \rightarrow L + 1$ transitions.

In order to show the complexity of the system under study, we investigated one more field profile, the one shifted from the center of the disk i.e. a ring magnetic field. We keep all parameters from the previous case, and shift the field by $R_d = 2.0\xi (R_1 - R_d = 4.5\xi, R_2 - R_d = 6.0\xi, R/\xi = 9.0)$ towards the disk edge (a top view of this profile is given schematically in the inset of Fig. 14). As shown in Fig. 14 with $R_d \neq 0$ the multi-vortices become more stable.
and for $H_{in}/H_{c2} > 0.31$ are the ground state. The solid curves represent the giant vortex states, dashed curves denote the energy of the anti-vortex states while the dotted curves correspond to the energy of the multi-vortices. Since giant vortex states show similar, re-entrant behavior as for the $R_d = 0$ case, the multi-vortex configurations behave analogously. Giant-giant multi-vortex states dominate the free energy diagram while the giant-ring combinations are present only as metastable states and are not shown in Fig. [4]. However, one should notice the presence of giant–anti-vortex states, again strongly correlated with re-entrant behavior. The complete equilibrium phase diagram for $R_d \in [0.3, 3]$ is given in the next section.

We checked that for the disk parameters which we used, an increase of the number of components in Eq. (22) does not lead to different vortex configurations in the ground state. In order to investigate the region of stability of multi-vortex states in the above analysis we took the order parameter as a linear combination of three components in Eq. (22) and minimized the free energy with respect to the three variational parameters $C_{Li}$, the giant and multi-vortex states considered before correspond to the extremum points of the $F(C_{Li_1}, C_{Li_2}, C_{Li_3})$ function. This analysis has shown that accounting of third component in Eq. (22) gives all states obtained in two component consideration. Additionally, it results in 1) possible reducing of the region of existence of metastable multi-vortex states at the low magnetic field limit, 2) appearance of additional unstable states corresponding to the saddle points of the $F(C_{Li_1}, C_{Li_2}, C_{Li_3})$ function, and, 3) no new vortex configurations in the ground state. We did a similar investigation using a numerical approach of Schweigert and Peeters [2]. This was done for five component vortex configurations, and same results were obtained. Using two different approaches, for three and five components, no new ground state configurations are found. Moreover, energies of same states found in both analysis for different number of components differ less than 0.2%. Because all multi-vortex configurations considered in this paper are in the ground state, or near it, we conclude that two components in Eq. (22) are enough to describe the vortex structure in our system.

V. $H_{IN} – R$ PHASE DIAGRAMS

First, we investigate the influence of the width of the positive magnetic field region on the different vortex configurations. Having the free energies of the different giant vortex configurations for several values of $R_1/\xi$, we construct an equilibrium vortex phase diagram. Fig. [13] shows this phase diagram for a superconducting disk with radius $R = 6.0\xi$ and thickness $d = 0.1\xi$ where we took $R_0 = R$. The dashed curves indicate where the ground state of the free energy changes from one $L$ state to another and the solid curve gives the normal/superconducting transition. Notice that the superconducting/normal transition moves towards lower fields with increasing radius of the positive field region. Also notice that we don’t have any negative $L$ (anti-vortex) state as ground state even when the negative field area is much larger than the positive one.

As shown in previous section, an increase of $R_1/\xi$ is able to bring the energy of the multi-vortex states below those of the giant vortex states and they can become the ground state (see Fig. [3(a)]). In our phase diagram, the area bounded by the thick curves denotes the region of existence of the giant-giant multi-vortex states as ground state. One can see that, with enlargement of the positive field region, ground state multi-vortices appear for $R_1/\xi = 4.42$ and $H_{in}/H_{c2} = 0.91$. Further increase of $R_1/\xi$ broadens this multi-vortex area to almost the whole superconducting region. These multi-vortex states consist of vortices in a ring structure, and the total vorticity equals the highest vorticity of the giant vortex states involved (at least in the case when no anti-vortices are involved).

In order to show that the stabilization of the multi-vortex states due to an inhomogeneous magnetic field is not peculiar to the $R = 6.0\xi$ disks, we repeated the previous calculations for a larger superconducting disk. Furthermore, we investigated in detail the phenomena shown in previous section (Figs. [4] [12] [13]).

The effect of the size of the superconducting disk on the phase diagram is illustrated in Fig. [14]. The parameters considered are $R_d/\xi = 0.0$, $R_1/\xi = 4.5$, and $R_2/\xi = 6.0$. The solid lines indicate where the ground state of the free energy changes from one state to another (either giant or multi-vortex state) and dashed lines correspond to transitions between different giant vortex states as metastable states. One can clearly see the re-entrant behavior. For example, for $R/\xi = 9.0$ we observe the change of the total vorticity as $L = 0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 1 \rightarrow 0$. The $L = 0$ and $L = 1$ states as ground state cover the largest part of the phase diagram. With increasing disk size, all other giant vortex states are strongly suppressed in favor of the various multi-vortex states. Different giant vortex states are present as the ground state for small disk sizes but with increase of disk size, islands with different multi-vortex configurations (giant-ring and giant-giant multi-vortex states) dominate the ground state diagram. Precisely, for $R = 6.38\xi$, giant-ring multi-vortex state appear as the ground state, and for $R = 6.77\xi$ we obtain the giant-giant multi-vortex states as the ground state. In the previous diagram in Fig. [14] we have shown the existence of the giant-giant multi-vortex states as the ground state for $R = 6.0\xi$. With slight increase of the disk size, these states become metastable, and with further enlargement of the superconducting disk, they become the ground state again. However, these states are different from the previous ones and in the contour plots of the Cooper pair density of these “new” giant-giant
multi-vortex states (see Fig. 13) we found a giant vortex state in the middle of the superconducting disk surrounded by anti-vortices, and where the total vorticity is now equal to the lowest vorticity of the giant vortex states that the multi-vortex consists of. For $R > 12.33 \xi$, the multi-vortex states become metastable again and the $L = 0$ and $L = 1$ states become the only ground states. Moreover, for $R > 19.34 \xi$ we have the Meissner state for all values of the applied magnetic field.

To present the complexity and excitement in this study, we give one more diagram. As shown before, moving the step-like field profile along the radius of the superconducting disk, i.e. a ring magnetic field profile as in the case of a current loop, stabilizes the giant-giant multi-vortex states as the ground state. In Fig. 14, we present the phase diagram as function of $R_d$. The parameters of the magnetic field profile are $R_1 - R_d = 4.5 \xi$, $R_2 - R_d = 6.0 \xi$, and $R/\xi = 9.0$. Thick solid curves indicate the transitions between different vortex states and dashed lines denote the transitions between metastable giant vortex states. The re-entrant behavior, as in previous case, is clearly visible. The most important result is that the multi-vortex states dominate this diagram. Moreover, we have both types of multi-vortices, i.e. giant-giant and giant-ring, as stable and ground state, and with shifting the field profile towards the disk periphery, giant-giant multi-vortex configurations cover most of the superconducting region. The Giant-giant multi-vortex states in this case appear both as states with no anti-vortices present, and as giant-muti anti-vortex states. Following the transition lines, a correspondence between different giant-multi anti-vortex states ($L_1 : L_2$) can be seen: $L_2$ remains the same, while $L_1$ increases from 0 to 2. As one can see, the latter is strongly correlated with re-entrant behavior. In the rest of the diagram, the multi-vortex states with the “classical” geometry (both giant-giant and giant-ring) dominate. However, a difference between those two states exists. As shown in Fig. 12, in the case of a giant-ring state, vortices are preferentially distributed within a ring-shaped lower density area. Further, considering the high density areas at the disk periphery, a shift in phase of $\Delta \theta = \pi / L$ is observed, in comparison with corresponding giant-giant state (see, for example, Fig. 11), where $L$ is the total vorticity. Although these states have a different origin, sometimes they can exhibit a similar distribution of vortices, but the phase is always able to distinguish between them.

**VI. COMPARISON WITH REAL MAGNETIC FIELD PROFILE**

As emphasized before, our step-like field model is a simplification of the magnetic field profile of a ferromagnetic dot (see solid curve in Fig. 1) as shown schematically in Fig. 4. In this case, the radius of the dot corresponds to $R_1$, and $R_2$ is always equal to $R$. The magnetic field and vector potential were calculated through numerical integration of

$$H_z(\rho, z) = m_z \int_0^{R_1} \frac{4r' (z' - z)}{\sqrt{(r' - \rho)^2 + (z' - z)^2} ((r' + \rho)^2 + (z' - z)^2)} E\left[ -\frac{4r' \rho}{(r' + \rho)^2 + (z' - z)^2}\right] z'_0, \quad (27)$$

and

$$A_\varphi(\rho, z) = \frac{1}{\rho} \int_0^\rho r H_z(r, z) dr,$$

where $m_z$ denotes the magnetic moment of a dot directed along the $z$-axis, $R_1$ is the radius of the dot, $z'_0$ gives the distance from the plane of interest, and $d_d = z'_1 - z'_0$ is the thickness of the dot. $E(x)$ is the complete elliptic integral of the second kind. The calculations were done for $z \to 0$, which allowed us to neglect the radial component of the magnetic field.

In order to show the quality of our previous model we repeated the analysis from previous sections. First, we enlarge $R_1$, i.e. the radius of the magnetic dot, investigating the influence of the magnetic field on the vortex structure and especially on the stability of the multi-vortex states, and secondly we studied the vortex configurations resulting from an increase of the superconducting disk size.

For small values of $R_1 / \xi$ the multi-vortex states are always metastable states. However, with enlarging this parameter these states lower in energy. The energies of the equilibrium vortex states as function of magnetic moment of the dot measured in units of $m_0 = H_{c2}$, are plotted in Fig. 14(a) for radius of the dot $R_1 / \xi = 4.0$ and $R / \xi = 6.0$ and in Fig. 14(b) for $R_1 / \xi = 4.0$ with a larger disk radius $R / \xi = 9.0$. Dashed curves correspond to anti-vortex states and dotted curves represent the energy of the multi-vortex states. The giant-giant multi-vortex states are given by $(L_1 : L_2)$, i.e. the angular momentum values they are composed of, and the giant-ring multi-vortex states with lowest energy are given in Fig. 14(b) by $((n_1, L_1) : (n_2, L_2))$. It should be noted that there are many other metastable combinations.
possible, which are not shown in the figures. Notice from Fig. 19(a) that the results are qualitative similar to the results obtained with our step magnetic field model (see Fig. 13). The difference is caused by the fact that part of the negative magnetic field does not penetrate the superconducting disk. Because the positive field region in the center dictates the behavior of the phase diagram even when the total flux is zero (see, for example, Fig. 13(a)), we obtain very similar results with the real magnetic field profile, but with an increased number of possible superconducting states. However, increasing the size of the superconductor brings some qualitative changes. First, it is clear from Fig. 19(b) that there is no re-entrant behavior, and, second, no giant-giant multi-vortex configurations are ground state. But, this was expected since in our model field profile the whole magnetic flux is trapped in the center of the disk, with total flux equal zero. For the real profile, the total flux would be zero if our disk is infinitely extended, and, more importantly, the flux is now spread over the whole disk area. Naturally, smaller magnetic dots in combination with large superconductors would make the correspondence better, since most of the flux would be captured inside the disk.

A similar discussion holds for the results for the real magnetic field profile of a current loop placed on top of a superconductor. This field profile (solid curve) is compared with our model in Fig. 21 (thick dashed curve). The magnetic field and vector potential were calculated numerically from

\[ H_z(\rho, z) = \frac{I k}{4\sqrt{R_0 \rho}} \left[ \frac{R_1^2 - \rho^2 - z^2}{(R_1 - \rho)^2 + z^2} E(k^2) + K(k^2) \right], \]  

and

\[ A_\varphi(\rho, z) = \frac{I}{k} \sqrt{\frac{R_1}{\rho}} \left[ (1 - \frac{k^2}{2}) K(k^2) - E(k^2) \right], \]

with

\[ k = 2 \sqrt{\frac{R_1 \rho}{(R_1 + \rho)^2 + z^2}}, \]  

where \( K(x) \) is the complete elliptic integral of the first kind and \( R_1 \) denotes the radius of the loop with current \( I \).

The free energy is shown in Fig. 21 for a loop with radius \( R_1 = 7.5 \xi \), with superconductor disk size \( R = 9.0 \xi \), as a function of current \( I \) measured in units of \( I_0 = \pi \xi H_{c2}/\mu_0 \). Typical values of \( I_0 \) are 3.29 mA for aluminum, to 0.823 A for high-temperature superconductors. One should compare these results with those presented in the \( H_{in} - R_d \) phase diagram (Fig. 17) and we find that our previous model contained all the essential physics of the system. The free energy diagrams of both Fig. 14 and Fig. 21 show the re-entrant behavior in total vorticity and the existence of the giant-giant multi-vortex configurations over a large region of the phase diagram.

### VII. SUPERCONDUCTING DISK WITH A MAGNETIC DOT OR CURRENT LOOP ON TOP OF IT IN THE PRESENCE OF A BACKGROUND HOMOGENEOUS EXTERNAL MAGNETIC FIELD

In addition, we investigated the vortex structure of a superconducting disk in the presence of an inhomogeneous magnetic field profile resulting from a ferromagnetic dot and a homogeneous external background magnetic field. In Figs. 22(a-c) we present the free energy as function of the external homogeneous field, for different magnetic dot thicknesses. The parameters were - radius of the dot \( R_1/\xi = 4.0 \), radius of the superconducting disk \( R/\xi = 6.0 \) with thickness \( d = 0.1 \xi \) and fixed ferromagnetic dot profile which is shown as inset of Figs. 22(a-c). As expected, when a negative external field overwhelms the average of the positive magnetic field of the dot in the disk center, the anti-vortex states become energetically more favorable, and, opposite, when the positive external field becomes larger than the average negative value of the ferromagnetic dot field, we see that the ground state goes through successive giant vortex states towards the normal state. For example, for a thickness of the magnetic dot \( d \eta = 0.1 \xi \) we obtain an almost symmetrical figure with respect to \( H_{ext} = -0.14 H_{c2} \). However, with increasing magnetic dot thickness, the magnetic field of the ferromagnetic dot (given as insets in Figs. 22(a-c)) becomes more pronounced and we obtain two sets of curves corresponding with the vortex and anti-vortex states, each of which having two minima. These local minima occur at an external field value which is approximately equal to the average field of the positive or negative region of the magnetic dot profile. The region between these two minima is characterized by a strong interplay of states.

For a current loop on top of the superconducting disk in a homogeneous external field we obtained qualitatively similar results. Also two sets of curves, with two local minima are visible in the free energy, for sufficiently large
found that with an increase of the disk size, re-entrant respectively, where \( R \) the two GL equations and thus we neglected the magnetic field created by the superconducting currents.

### ACKNOWLEDGMENTS

This work was supported by the Flemish Science Foundation (FWO-VI), the Belgian Inter-University Attraction Poles (IUAP-IV), the “Onderzoeksaad van de Universiteit Antwerpen” (GOA), and the ESF programme on “Vortex matter”.

APPENDIX: Eigenfunctions of the linearized first GL equation in the presence of an inhomogeneous magnetic field
A. Disk magnetic field profile

The vector potential distribution is determined by the piecewise function

\[
A(\rho) = \begin{cases} 
H_0\rho/2, & 0 \leq \rho \leq R_1, \\
-H_{\text{out}}\rho/2 + H_{\text{out}}R_2^2/2\rho, & R_1 \leq \rho \leq R_2, \\
0, & R_2 \leq \rho \leq R. 
\end{cases} 
\]  
(A.1)

The eigenfunctions of Eq. (13) are expressed in the following way:

\[
f_{L,n}(\rho) = \begin{cases} 
f^I(\rho), & (I) \\
b_1f^{II}_{1}(\rho) + b_2f^{II}_{2}(\rho), & (II) \\
d_1f^{III}_{1}(\rho) + d_2f^{III}_{2}(\rho), & (III) 
\end{cases} 
\]  
(A.2)

where

\[
\begin{align*} 
f^I(\rho) &= (H_0\rho^2/2)^{|L|/2} \exp(-H_0\rho^2/4) M(-\nu_{n,1}(\Lambda), |L| + 1, H_0\rho^2/2), \\
f^{II}_{1}(\rho) &= (H_{\text{out}}\rho^2/2)^{|L_{II}|/2} \exp(-H_{\text{out}}\rho^2/4) M(-\nu_{n,II}(\Lambda), |L_{II}| + 1, H_{\text{out}}\rho^2/2), \\
f^{II}_{2}(\rho) &= (H_{\text{out}}\rho^2/2)^{|L_{II}|/2} \exp(-H_{\text{out}}\rho^2/4) U(-\nu_{n,II}(\Lambda), |L_{II}| + 1, H_{\text{out}}\rho^2/2), \\
f^{III}_{1}(\rho) &= J_{|L|}(\sqrt{1 + \Lambda\rho}), \\
f^{III}_{2}(\rho) &= Y_{|L|}(\sqrt{1 + \Lambda\rho}), 
\end{align*}
\]

with

\[
L_{II} = L - H_{\text{out}}R_2^2/2, \\
\nu_{n,1}(\Lambda) = -\frac{1 + |L| - L}{2} + \frac{1 + \Lambda}{2H_0}, \\
\nu_{n,II}(\Lambda) = -\frac{1 + |L_{II}| + L_{II}^*}{2} + \frac{1 + \Lambda}{2H_{\text{out}}}. 
\]

\(J_m(x)\) and \(Y_m(x)\) are the Bessel functions of the first and second kind, \(M(a,c,y)\) and \(U(a,c,y)\) are the Kummer functions. To find the unknown constants \(b_{1(2)}\), \(d_{1(2)}\) and the eigenvalue \(\Lambda\) we have to join the different parts of \(f_{L,n}(\rho)\) and its derivatives at \(R_1\) and \(R_2\) as well as to use the boundary condition \((\partial f/\partial \rho)|_{\rho=R} = 0\).

To simplify the next calculation let us introduce the following notations: \(K_1 = f^I(R_1), I_1 = f^{II}_{1}(R_1), N_1 = f^{II}_{2}(R_1), I_3 = f^{II}_{1}(R_2), N_3 = f^{II}_{2}(R_2), P_3 = f^{III}_{1}(R_2), Q_3 = f^{III}_{2}(R_2), K_2 = \frac{\partial f^I(\rho)}{\partial \rho}|_{\rho=R_1}, I_2 = \frac{\partial f^{II}_{1}(\rho)}{\partial \rho}|_{\rho=R_1}, N_2 = \frac{\partial f^{II}_{2}(\rho)}{\partial \rho}|_{\rho=R_1}, I_4 = \frac{\partial f^{III}_{1}(\rho)}{\partial \rho}|_{\rho=R_2}, N_4 = \frac{\partial f^{III}_{2}(\rho)}{\partial \rho}|_{\rho=R_2}, P_4 = \frac{\partial f^{III}_{1}(\rho)}{\partial \rho}|_{\rho=R_2}, Q_4 = \frac{\partial f^{III}_{2}(\rho)}{\partial \rho}|_{\rho=R_2}, P_5 = \frac{\partial f^{III}_{1}(\rho)}{\partial \rho}|_{\rho=R}, Q_5 = \frac{\partial f^{III}_{2}(\rho)}{\partial \rho}|_{\rho=R}\). With these notations the system of equations which determine the coefficients \(b_{1(2)}, d_{1(2)}\) and \(\Lambda\) have the simple form

\[
\begin{align*} 
b_1I_1 + b_2N_1 &= K_1, \\
b_1I_2 + b_2N_2 &= K_2, \\
b_1I_3 + b_2N_3 - d_1P_3 - d_2Q_3 &= 0, \\
b_1I_4 + b_2N_4 - d_1P_4 - d_2Q_4 &= 0, \\
d_1P_5 + d_2Q_5 &= 0. 
\end{align*} 
\]  
(A.3)

From the first four equations of the system (A.3) we obtain:

\[
\begin{align*} 
b_1 &= \frac{K_1N_2 - K_2N_1}{I_1N_2 - I_2N_1}, \\
b_2 &= \frac{I_1K_2 - I_2K_1}{I_1N_2 - I_2N_1}.
\end{align*} 
\]  
(A.4)
\[ d_1 = \frac{(K_1 N_2 - K_2 N_1) (I_3 Q_4 - Q_3 I_4) + (I_1 K_2 - I_2 K_1) (Q_4 N_3 - Q_3 N_4)}{P_3 Q_4 - P_4 Q_3}, \]
\[ d_2 = \frac{(K_1 N_2 - K_2 N_1) (P_3 I_4 - I_3 P_4) + (I_1 K_2 - I_2 K_1) (N_4 P_3 - N_3 P_4)}{P_3 Q_4 - P_4 Q_3}. \]

Inserting these results into the last equation of system (A.3) results into the non-linear equation for \( \Lambda \):
\[ P_5 [(K_1 N_2 - K_2 N_1) (I_3 Q_4 - Q_3 I_4) + (I_1 K_2 - I_2 K_1) (Q_4 N_3 - Q_3 N_4)] \]
\[ + Q_5 [(K_1 N_2 - K_2 N_1) (P_3 I_4 - I_3 P_4) + (I_1 K_2 - I_2 K_1) (N_4 P_3 - N_3 P_4)] = 0. \]

To obtain the correct \( \Lambda \) values we have to exclude from the spectrum of solutions of Eq. (A.5) those which result in zeros of both denominators \( I_1 N_2 - I_2 N_1 \) and \( P_3 Q_4 - P_4 Q_3 \).

In the large radius limit \( R \to \infty \), i.e. which is equivalent to the thin film limit, the Bessel functions in the \( \text{(III)} \)-region have to be replaced by their asymptotics. Substituting them in the equation \( d_1 P_5 + d_2 Q_5 = 0 \) we obtain
\[ \Lambda = -1 + \frac{1}{R^2} \left[ \frac{\pi}{4} (3 + 2|L|) - \arctan \frac{d_1}{d_2} \right]^2. \]  

\( f_{L,n} (\rho) \) near the edge of the sample is equal to \( \sqrt{d_1^2 + d_2^2} \).

**B. Ring magnetic field profile**

The vector potential distribution in this case is
\[ A (\rho) = \begin{cases} 0, & 0 \leq \rho \leq R_d, \quad (I) \\ H_0 \rho/2 - H_0 R_d^2/2 \rho, & R_d \leq \rho \leq R_1, \quad (II) \\ -H_{out} \rho/2 + H_{out} R_d^2/2 \rho, & R_1 \leq \rho \leq R_2, \quad (III) \\ 0, & R_2 \leq \rho \leq R, \quad (IV) \end{cases} \]  

and, therefore, the eigenfunctions of Eq. (A.7) are expressed as follows:
\[ f_{L,n} (\rho) = \begin{cases} f^I (\rho), & (I) \\ b_1 f^I_1 (\rho) + b_2 f^I_2 (\rho), & (II) \\ d_1 f^I_1 (\rho) + d_2 f^I_2 (\rho), & (III) \\ e_1 f^IV_1 (\rho) + e_2 f^IV_2 (\rho), & (IV) \end{cases} \]  

where
\[ f^I (\rho) = f^IV (\rho) = J_{|L|} \left( \sqrt{1 + \Lambda \rho} \right), \]
\[ f^I_1 (\rho) = (H_0 \rho^2/2)^{|L^I_1|/2} \exp (-H_0 \rho^2/4) M \left( -\nu_{n,II} (\Lambda), |L^I_1| + 1, H_0 \rho^2/2 \right), \]
\[ f^I_2 (\rho) = (H_0 \rho^2/2)^{|L^I_1|/2} \exp (-H_0 \rho^2/4) U \left( -\nu_{n,II} (\Lambda), |L^I_1| + 1, H_0 \rho^2/2 \right), \]
\[ f^II_1 (\rho) = (H_{out} \rho^2/2)^{|L^II_1|/2} \exp (-H_{out} \rho^2/4) M \left( -\nu_{n,III} (\Lambda), |L^II_1| + 1, H_{out} \rho^2/2 \right), \]
\[ f^II_2 (\rho) = (H_{out} \rho^2/2)^{|L^II_1|/2} \exp (-H_{out} \rho^2/4) U \left( -\nu_{n,III} (\Lambda), |L^II_1| + 1, H_{out} \rho^2/2 \right), \]
\[ f^IV_2 (\rho) = Y_{|L|} \left( \sqrt{1 + \Lambda \rho} \right). \]

with
\[ L^I_1 = L + H_0 R_d^2/2, \]
\[ L^II_1 = L - H_{out} R_d^2/2, \]
\[ \nu_{n,II} (\Lambda) = -\frac{1 + |L^I_1| - L^I_1}{2} + \frac{1 + \Lambda}{2H_0}, \]
\[ \nu_{n,III} (\Lambda) = -\frac{1 + |L^II_1| + L^II_1}{2} + \frac{1 + \Lambda}{2H_{out}}. \]
The unknown constants \( b_{1(2)} \), \( d_{1(2)} \), \( e_{1(2)} \) and the eigenvalue \( \Lambda \) are found by joining the different parts of \( f_{L,n}(\rho) \) and its derivatives at \( R_d, R_1 \) and \( R_2 \) as well as from the boundary condition \( (\partial f/\partial \rho)|_{\rho=R} = 0 \). The corresponding system of equations has the form:

\[
\begin{align*}
\ b_1 I_1 + b_2 N_1 & = K_1, \\
\ b_1 I_2 + b_2 N_2 & = K_2, \\
\ b_1 I_3 + b_2 N_3 - d_1 P_3 - d_2 Q_3 & = 0, \\
\ b_1 I_4 + b_2 N_4 - d_1 P_4 - d_2 Q_4 & = 0, \\
\ d_1 P_5 + d_2 Q_5 - e_1 S_5 - e_2 T_5 & = 0, \\
\ d_1 P_6 + d_2 Q_6 - e_1 S_6 - e_2 T_6 & = 0, \\
\ e_1 S_7 + e_2 T_7 & = 0.
\end{align*}
\]

where the following notations were introduced:

\[
\begin{align*}
\ N_1 & = f^I(R_d), \ I_1 = f_1^{II}(R_d), \ N_2 = f_2^{II}(R_d), \ K_2 = \frac{\partial f^I(\rho)}{\partial \rho}|_{\rho=R_d}, \ I_2 = \frac{\partial f^{II}(\rho)}{\partial \rho}|_{\rho=R_d}, \\
\ N_3 & = f_3^{II}(R_1), \ I_3 = f_1^{IV}(R_1), \ N_4 = f_4^{II}(R_1), \ P_3 = f_3^{IV}(R_1), \ Q_3 = f_4^{IV}(R_1), \ I_4 = \frac{\partial f^{IV}(\rho)}{\partial \rho}|_{\rho=R_1}, \\
\ N_5 & = f_5^{IV}(R_2), \ P_4 = \frac{\partial f^{IV}(\rho)}{\partial \rho}|_{\rho=R_1}, \ Q_4 = \frac{\partial f^{IV}(\rho)}{\partial \rho}|_{\rho=R_1}, \ P_5 = f_1^{IV}(R_2), \ Q_5 = f_2^{IV}(R_2), \ S_5 = f_3^{IV}(R_2), \\
\ T_5 & = f_4^{IV}(R_2), \ T_6 = \frac{\partial f^{IV}(\rho)}{\partial \rho}|_{\rho=R_2}, \ S_6 = \frac{\partial f^{IV}(\rho)}{\partial \rho}|_{\rho=R_2}, \ T_7 = \frac{\partial f^{IV}(\rho)}{\partial \rho}|_{\rho=R_2}.
\end{align*}
\]

The constants \( b_{1(2)} \) and \( d_{1(2)} \) are determined by the expressions (A.4) and

\[
\begin{align*}
\epsilon_1 & = \frac{(P_5 T_6 - T_5 P_6) d_1 + (Q_5 T_6 - T_5 Q_6) d_2}{S_5 T_6 - S_6 T_5}, \\
\epsilon_2 & = \frac{(S_5 P_6 - P_5 S_6) d_1 + (Q_5 Q_6 - Q_5 S_6) d_2}{S_5 T_6 - S_6 T_5}.
\end{align*}
\]

From the last equation of system (A.9)

\[
\begin{align*}
\left[ (K_1 N_2 - K_2 N_1) (I_3 Q_4 - Q_3 I_4) + (I_1 K_2 - I_2 K_1) (Q_4 N_3 - Q_3 N_4) \right] \\
\times \left[ S_7 (P_5 T_6 - T_5 P_6) + T_7 (S_5 P_6 - P_5 S_6) \right] + \\
\left[ (K_1 N_2 - K_2 N_1) (P_3 I_4 - I_3 P_4) + (I_1 K_2 - I_2 K_1) (N_4 P_3 - N_3 P_4) \right] \\
\times \left[ S_7 (Q_5 T_6 - T_5 Q_6) + T_7 (S_5 Q_6 - Q_5 S_6) \right] = 0
\end{align*}
\]

we obtain the spectrum of \( \Lambda \) values (notice, that we have to exclude the zeros of the denominators \( I_1 N_2 - I_2 N_1, \ P_3 Q_4 - P_4 Q_3 \) and \( S_7 T_5 - S_5 T_7 \)).

In the large radius limit \( R \to \infty \), similar considerations have to be made as for the previous case of the disk profile. \( f_{L,n}(\rho) \) near the sample edge is equal to \( \sqrt{\epsilon_1^2 + \epsilon_2^2} \) and

\[
\Lambda = -1 + \frac{1}{R^2} \left[ \frac{\pi}{4} (3 + 2|L|) - \arctan \frac{\epsilon_1}{\epsilon_2} \right]^2.
\]

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The insets depict the magnetic field dependence of the disk magnetization for the ground giant vortex state for different values $L$.

The corresponding curves for the first radial excited state, i.e. $n = 1$, are presented in figures (d-f). The top axis gives the flux through the positive magnetic field region in units of the flux quantum.

The Cooper pair density for the giant vortex states with angular momenta $L = 0, 1, 2$ (solid curves), the antivortex states $L = -1, -2$ (dotted curves) and the ring vortex state $n = 1, L = 0$ (dashed curve) at the magnetic field $H_m = 0.75H_{C2}$ for (a) $R_1/\xi = 1.5$, (b) $R_1/\xi = 3.0$, and (c) $R_1/\xi = 4.5$.

The free energy of the giant vortex states with different angular momenta $L$ as a function of the applied magnetic field in the positive region for (a) $R_1/\xi = 1.5$, (b) $R_1/\xi = 3.0$, and (c) $R_1/\xi = 4.5$. Only the states with $|L| \leq 10$ are shown. The insets depict the magnetic field dependence of the disk magnetization for the ground giant vortex state for different values of the $R_1/\xi$ parameter. Figure (d) gives the positive flux captured in the superconducting disk for the different $L$-ground states as the ground state as function of $R_1/\xi$.

The Cooper pair density for the giant vortex states with angular momenta $L = 0, 1, 2$ (solid curves), the anti-vortex states $L = -1, -2$ (dotted curves) and the ring-shaped vortex state $n = 1, L = 0$ (dashed curve) at the magnetic field $H_m = 0.75H_{C2}$ for (a) $R/\xi = 9.0$, and (b) $R/\xi = 12.0$.

The Cooper pair density for (a) the Meissner state ($L = 0$), and (b) the ring-shaped vortex state $n = 1, L = 0$ at the magnetic field $H_m = 0.75H_{C2}$ for different sizes of the superconducting disk.

The free energy of the giant vortex states with different angular momenta $L$ as a function of the applied magnetic field for (a) $R/\xi = 9.0$, (b) $R/\xi = 12.0$, (c) $R/\xi = 18.0$, and (d) $R/\xi = 20.0$, with $R_1/\xi = 4.5$, and $R_2/\xi = 6.0$. Dashed curves represent the energy of the anti-vortex states.
FIG. 8. The magnetic field dependence of the disk magnetization for the ground giant vortex state corresponding to the states in Fig. 7 for (a) $R/\xi = 9.0$, (b) $R/\xi = 12.0$, and (c) $R/\xi = 18.0$.

FIG. 9. The free energy of the giant vortex states with different angular momenta $L$ as a function of the external magnetic field for (a) $R_1/\xi = 5.25$, $R_2/\xi = R/\xi = 6.0$, and (b) $R_1/\xi = 4.5$, $R_2/\xi = 6.0$, $R/\xi = 9.0$. Dashed curves illustrate the energy of the anti-vortex states. In (a) dotted curves represent the giant-giant multivortex states while in (b) dotted curves depict the free energy of the giant-ring multi-vortex states, and the solid curves correspond to the giant-giant states. Thick solid curve illustrates the energy of the ring vortex state, i.e. $(n, L) = (1, 0)$.

FIG. 10. Transition between two giant vortex states shown through the contour plots of the superconducting wave function density for the $(0:4)$ giant-giant multi-vortex state for different values of the magnetic field in the positive field region (corresponding to the magnetization of the dot).

FIG. 11. Contour plots of the superconducting density for the ground state and the corresponding phase contour plots (see Fig. 10(b)) for different values of the magnetic field in the positive region.

FIG. 12. Contourplot of the superconducting density for the $(5,0):(0,1)$ giant-ring multi-vortex state (see Fig. 10(b)) for different values of the applied magnetic field.

FIG. 13. (a-c) Contour plots of the superconducting density for the $(0:7)$, $(0:8)$, and $(0:9)$ giant-multi anti-vortex states, respectively, for different values of the magnetic field $H_{in}$; (d-f) the corresponding contour plots of the phase of the superconducting wave function density. Notice that the phase near the boundary is near 0 or $2\pi$ but due to the finite numerical accuracy it oscillates between $2\pi - \varepsilon$ and $2\pi + \varepsilon$ where $\xi \sim 10^{-3}$.

FIG. 14. The free energy of the giant vortex states with different angular momenta $L$ as a function of the external magnetic field for $R_d = 2.1\xi$, $R_1 = 6.6\xi$, $R_2 = 8.1\xi$, $R/\xi = 9.0$. Dashed curves depict the anti-vortex states and dotted curves represent the free energy of the giant-giant multi-vortex states. A bird view of the magnetic field profile is given in the inset.

FIG. 15. The $H_{in} - R_1$ equilibrium vortex phase diagram for a thin superconducting disk with $R_d/\xi = 0.0$, $R_2/\xi = 6.0$, and $R/\xi = 6.0$. Dashed curves indicate transitions between different giant vortex states and the thick shaded area denotes the multi-vortex region. The normal/superconducting state transition is given by the thin solid curve.

FIG. 16. The $H_{in} - R$ phase diagram for the ground state of a thin superconducting disk with $R_d/\xi = 0.0$, $R_1/\xi = 4.5$, and $R_2/\xi = 6.0$. Solid curves indicate transitions between different vortex states including multi-vortex (giant-multi anti-vortex and ring-giant) regions. Dashed curves denote the transitions between different metastable giant vortex states.

FIG. 17. The $H_{in} - R_d$ phase diagram for the ground state of a thin superconducting disk with $R_1 - R_d = 4.5\xi$ and $R_2 - R_d = 6.0\xi$, i.e. a ring inhomogeneous magnetic field distribution. The same curve convention is used as in Fig. 14.

FIG. 18. The configuration: a superconducting disk with radius $R$ and thickness $d$ with a ferromagnetic dot with radius $R_1$ and thickness $d_d$, which is placed on top of it.

FIG. 19. The free energy of different vortex states as a function of the magnetic moment of the ferromagnetic dot with (a) $R_1/\xi = 4.5$, $R_2/\xi = R/\xi = 6.0$, and (b) $R_1/\xi = 4.5$, $R_2/\xi = R/\xi = 9.0$. In (a), dotted curves depict the free energy of the giant-giant multi-vortex states while in (b) dotted curves denote giant-ring states. The bold curve gives the energy of the ring vortex state, i.e. $(n, L) = (1, 0)$. The energy of the anti-vortex states is given by the dashed curves.
FIG. 20. The magnetic field profile as produced by a current loop (solid curve) and its corresponding model profile (dashed curve).

FIG. 21. The free energy of different vortex states as a function of the current in a loop with (a) \( R_1/\xi = 7.5 \), \( R_2/\xi = R/\xi = 9.0 \). Dashed curves depict the free energy of the anti-vortex states and dotted curves denote the giant-giant multi-vortex states.

FIG. 22. The free energy of giant vortex states as a function of the external magnetic field in a superconducting disk with magnetic dot on top of it with parameters \( R_1/\xi = 4.0 \), \( R/\xi = 6.0 \) and (a) \( d_d = 0.1\xi \), (b) \( d_d = 0.5\xi \), and (c) \( d_d = 1.0\xi \). Dashed curves depict the free energy of the anti-vortex states. Insets show the magnetic field profile inside the superconducting disk created by the magnetic dot.

FIG. 23. The free energy of giant vortex states as a function of the external magnetic field in a superconducting disk with current loop on top of it with parameters \( R_1/\xi = 7.5 \), \( R/\xi = 9.0 \) and (a) \( I/I_0 = 5.0 \), (b) \( I/I_0 = 10.0 \), (c) \( I/I_0 = 15.0 \), and (d) \( I/I_0 = 20.0 \). Dashed curves depict the free energy of the anti-vortex states. Insets show the magnetic field profile inside the superconducting disk created by the current loop.
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