Velocity Response Analysis of the SCR Rigid Body Swing under Wave and Z-direction Motion

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Abstract. Under the action of waves or current, steel catenary riser(SCR) has rigid body oscillation. On the basis of the slender beam model and the wave model and the rigid body swing, the influence of rigid body swing on the cross flow of structure velocity is researched. Calculation shows that under the action of wave and top motion, the change of speed response of in 10th-200th is the largest in 10th, and then decreases gradually to the minimum of 200th. Under the action of wave superimposed rigid body oscillation and top motion, this phenomenon of decreasing with the increase of water remain existing, indicating that rigid body oscillation has little effect and the maximum impact of rigid body swing on structure velocity is approximately 2%. The percentage change of speed response in 10th-200th is the largest in 140th, and is the least in 10th. For the middle interval of 80th-140th, working condition 3 has the largest influence. It is hoped that some reasonable and practical suggestions can be provided for the calculation of transverse flow velocity.

1. Introduction
Steel catenary riser is the connecting channel between sea surface and submarine wellhead, and it can be used in the application of floating offshore platform[1-3].

Tian Yao et al.[4] studied unstable stage differential pressure fluctuation and the liquid phase. Falin Liao et al.[5] established a slender rod model to analyze the steep-wave risers. Wanhai Xu et al.[6] predicted a flexible cylinder undergoing VIV by hydrodynamic coefficients. Dongyang Chen et al.[7] studied the flexible riser’s wet modal vibration. Jianxing Yu et al.[8] used response surface method for the fatigue reliability. Leijian Song et al.[9] researched the flexible riser’s structural responses and forces. Fang Guo et al.[10] studied a flexible riser’s vibration control.

In this paper, the influence of rigid body oscillation on the structure under the influence of the top z-direction motion and wave motion is studied.

2. SCR Rigid Body Rotation Mode
The vibration control equation[1] can be deduced.

\[ \dddot{\textit{M}} + (Br^*)'' - (\dot{\lambda}r')' = q \]  \hspace{1cm} (1)

\( \dddot{\textit{M}} \) : Quality; \( B \) : Beam stiffness; \( \dot{\lambda} \) : the Lagrange operator, \( q \) : External forces; \( r \) : displacement.
Fig. 1 Rigid rotation coordinate of SCR[4]

The riser rotation equation[1-3] is:

\[(m + m_a)s^2 \ddot{\alpha}_r + c_a s^2 \dot{\alpha}_r + mgc_1 s \alpha_r = \]

\[q_s \sqrt{s_1^2 + s_2^2} + q_s c_2 s_3 \]

\[\alpha_r, \dot{\alpha}_r, \ddot{\alpha}_r -\text{angular displacement, angular velocity and angle acceleration of the rigid body rotation.} \]

The Morison equation[1-3] is:

\[f_R = \frac{1}{2} C_D \rho A (u_x - \dot{x}) |u_x - \dot{x}| \]

\[+ C_M \rho \frac{\pi D^2}{4} \frac{\partial u_x}{\partial x} - C_m \rho \frac{\pi D^2}{4} \ddot{x} \]

\[u_x -\text{water velocity, } u_x -\text{water acceleration, } \dot{x} -\text{structure velocity, } \ddot{x} -\text{structure acceleration.} \]

The Rotation Morison Vibration(RMV) control equation is equation(4).

Add equation(3) and equation (2) as a coupling term to equation(1):

\[M \dddot{r} + (Br^*)'' - (\ddot{\lambda}r')' = q + mg \]

\[-(m + m_s) \dddot{r} - c_s \dddot{r} + f_R \]

SCR top motion boundary condition
The z-direction motion is added to the calculation file and the new program is for the rigid body rotation.

3. SCR Velocity Response Comparison.

3.1. Calculation Parameters.
X direction is the the wave incident direction. Table 1 shows the z direction motion parameters.

| Calculation parameter | Numerical value |
|-----------------------|-----------------|
| SCR outer diameter (m) | 0.355           |
| SCR inner diameter (m) | 0.305           |
| Modulus of elasticity (Gpa) | 207             |
Table 2. Motion parameters of working conditions[3]

| No | A/m | T/s | f(Hz) | w(Hz) |
|----|-----|-----|-------|-------|
| 1  | 3   | 10.8| 0.093 | 0.5815|
| 2  | 2   | 9.90| 0.101 | 0.6343|
| 3  | 1   | 9.00| 0.111 | 0.6978|

A-amplitude, T-period, f- frequency, w- angular frequency.

3.2. Response in z direction.

Fig.2 Velocity response of 10th in condition 1

Fig.3 Velocity response of 80th in condition 1

Fig.4 Velocity response of 140th in condition 1
Fig. 5 Velocity response of 200th in condition 1

Fig. 6 Velocity response of 10th in condition 2

Fig. 7 Velocity response of 80th in condition 2

Fig. 8 Velocity response of 140th in condition 2
Fig. 9 Velocity response of 200th in condition 2

Fig. 10 Velocity response of 10th in condition 3

Fig. 11 Velocity response of 80th in condition 3

Fig. 12 Velocity response of 140th in condition 3
Table 3 Calculated values of Velocity of each working condition node

| No | Node | Cab (m/s) | Csw (m/s) | growth rate |
|----|------|-----------|-----------|-------------|
| 1  | 10th | 1.4768    | 1.4766    | -0.01%      |
| 1  | 80th | 0.4813    | 0.4812    | -0.02%      |
| 1  | 140th| 0.2877    | 0.2921    | 1.53%       |
| 1  | 200th| 0.2127    | 0.2119    | -0.38%      |
| 2  | 10th | 1.1366    | 1.1362    | -0.04%      |
| 2  | 80th | 0.4253    | 0.4259    | 0.14%       |
| 2  | 140th| 0.2674    | 0.272     | 1.72%       |
| 2  | 200th| 0.1797    | 0.1799    | 0.11%       |
| 3  | 10th | 0.6953    | 0.6952    | -0.01%      |
| 3  | 80th | 0.3331    | 0.335     | 0.57%       |
| 3  | 140th| 0.228     | 0.233     | 2.19%       |
| 3  | 200th| 0.1729    | 0.1733    | 0.23%       |

$V_{cab}, V_{csw}$ The formula is shown below

\[
V_{cab} = V_{wave} + V_z \quad (5)
\]

\[
V_{csw} = V_{wave} + V_z + V_{rbs} \quad (6)
\]

\[
\eta = \left( V_{csw} - V_{cab} \right) / V_{cab} \quad (7)
\]

$V_{cab}$ - the wave and top z-direction motion of velocity,

$V_{csw}$ - the wave and top z-direction motion response superimposed rigid body swing velocity,

$V_{wave}$ - the wave motion of velocity,

$V_z$ - the z-direction motion of velocity.

3.3. Response Comparison.

Figure 2-5 shows the speed response graph of 10th-200th under structural condition 1. In terms of the overall response, the structure shows relatively stable amplitude oscillation during 10th-200th. Under
the action of wave and top motion, the nodes 10th, 80th, 140th and 200th calculated results are 1.4768 m/s, 0.4813 m/s, 0.2877 m/s and 0.2127 m/s. Under the action of wave superimposed rigid body oscillation and top motion, the nodes 10th, 80th, 140th and 200th calculated results are 1.4766 m/s, 0.4812 m/s, 0.2921 m/s and 0.2119 m/s. The wave and rigid body rotation effect and top effect are simply regarded as a linear superposition, and the nodes 10th, 80th, 140th and 200th calculated results are to increase by -0.01%, -0.02%, 1.53% and -0.38%. The change of speed response of in 10th-200th is the largest in 10th, and then decreases gradually to the minimum of 200th. The percentage change of speed response in 10th-200th is the largest in 140th, and is the least in 10th.

Table 4 Percentage change of speed response Maximum and Minimum in each condition

| Condition | Maximum | Value | Minimum | Value |
|-----------|---------|-------|---------|-------|
| 1         | 140th   | 1.53% | 10th    | -0.01%|
| 2         | 140th   | 1.72% | 10th    | -0.04%|
| 3         | 140th   | 2.19% | 10th    | -0.01%|

From table 4, the maximum influence of the superimposed rigid body oscillation of condition 1-3 is the largest in 140th, and is the least in 10th. The median vector diameter of the structure corresponding to the 140th node is the largest. The 10th node is the top suspension point, which has a small vector diameter.

As the amplitude of motion in conditions decreases, the nodes 10th, 80th, 140th and 200th calculated response velocity decreases. Under the action of wave and top motion, the nodes 10th calculated response velocity in conditions 1-3 are 1.4768 m/s, 1.1366 m/s, 0.6953 m/s. Under the action of wave superimposed rigid body oscillation and top motion, the nodes 10th calculated response velocity in conditions 1-3 are 1.4766 m/s, 1.1362 m/s, 0.6952 m/s. The change of speed response of in condition 1-3 is the largest in 1, and then decreases gradually to the minimum of condition 3. The nodes 10th calculated results are to increase by -0.01%, -0.04% and -0.01%. The difference in speed response is most obvious in condition 2.

Table 5 Maximum and Minimum condition of each node

| Growth Rate | Maximum Condition | Value | Minimum Condition | Value |
|-------------|-------------------|-------|-------------------|-------|
| 10th        | 1/3               | -0.01%| 2                 | -0.04%|
| 80th        | 3                 | 0.57% | 1                 | -0.02%|
| 140th       | 3                 | 2.19% | 1                 | 1.53% |
| 200th       | 2                 | -0.11%| 1                 | -0.38%|

For the 10th node, the biggest impact is condition 2. For the 80th node, the biggest impact is condition 3. For the 140th node, the biggest impact is condition 3. For the 200th node, the biggest impact is condition 1. For the middle interval of 80th-140th, working condition 3 has the largest influence, while working condition 3 has the smallest amplitude and the highest frequency. For 10th in the top area, condition 2 had the greatest influence. The amplitude and frequency of condition 2 were in the middle value. For the touchdown area of 200th, working condition 1 has the largest influence, while working condition 1 has the largest amplitude and the lowest frequency.

The rest of the conditions have a similar situation, as shown in Figures 6-9, 10-13 and Table3.
4. Conclusion
Under the action of wave and top motion, the change of speed response of in 10th-200th is the largest in 10th, and then decreases gradually to the minimum of 200th. The waves and the response of the top z direction motion decrease as the depth of water increases. So the structural response value is weakened.

Under the action of wave superimposed rigid body oscillation and top motion, this phenomenon of decreasing with the increase of water remain existing, indicating that rigid body oscillation has little effect.

In this paper, the structural response is linearly related to the vector diameter s. The percentage change of speed response in 10th-200th is the largest in 140th, and is the least in 10th. For the middle interval of 80th-140th, working condition 3 has the largest influence.

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