Deconfined SU(2) vector fields at zero temperature

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Markov chain Monte Carlo simulations of pure SU(2)⊗U(1) lattice gauge theory show a (zero temperature) deconfining phase transition in the SU(2) gluon sector when a term is added to the SU(2) and U(1) Wilson actions, which requires joint U(2) gauge transformations of the SU(2) and U(1) vector fields. Investigations of this deconfined phase are of interest as it could provide an alternative to the Higgs mechanism.

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The explicit breaking of gauge symmetry through massive vector bosons is avoided by the Higgs mechanism [1], which completes the standard electroweak theory [2] and ensures desired features such as renormalizability [3]. Nevertheless, the introduction of the Higgs particle remains quite adhoc and the discussion of alternatives is of interest as long as a Higgs boson has not been observed.

In Euclidean field theory notation the action of the electroweak gauge part of the standard model reads

\[ S = \int d^4x \, \mathcal{L}, \quad \mathcal{L} = \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{2} \text{Tr} F_{\mu\nu} F^{\mu\nu}, \]  

\[ F'_{\mu\nu} = \partial_\mu A'_{\nu} - \partial_\nu A'_{\mu}, \quad (2) \]

\[ F_{\mu\nu} = \partial_\mu A_{\nu} - \partial_\nu A_{\mu} + ig [A_{\mu}, A_{\nu}], \quad (3) \]

where \( A'_{\mu} \) are U(1) and \( A_{\mu} \) are SU(2) vector fields.

Typical textbook introductions of the standard model, e.g. [4], emphasize at this point that the theory contains four massless gauge bosons and introduce the Higgs mechanism as a vehicle to modify the theory so that only one gauge boson, the photon, stays massless. Such a presentation reflects that the introduction of the Higgs particle to electroweak interactions [1] preceded our non-perturbative understanding of non-Abelian gauge theories. In fact, there is already only the photon massless in the spectrum of [1]. The self interaction [3] of the SU(2) gauge part has dynamically generated a non-perturbative mass gap, and the SU(2) spectrum consists of massive glueballs. One of them can be used as mass scale at our disposal.

However, SU(2) gauge theory confines fermions, while the leptons are found as free particles. Coupling a Higgs field in the usual way causes a deconfining phase transition, so that fermions are liberated, a photon stays massless and glueballs break up into elementary massive vector bosons. Such a transition between the confinement and the Higgs phase has indeed been observed in pioneering lattice gauge theory (LGT) investigations [5].

The present study is in part motivated by the behavior of the Polyakov loop in pure U(1) LGT. It is well known that U(1) LGT confines fermions in the strong coupling limit of its lattice regularization [6]. Towards weaker coupling it undergoes a transition into the Coulomb phase in which the effective potential for the Polyakov loop assumes the Mexican hat shape that is also characteristic for a scalar Higgs doublet in the broken phase. From simulations on a 12^4 lattice with periodic boundary conditions Fig. 1 and 2 show for the U(1) Wilson action scatter plots of the Polyakov loop lattice averages from Monte Carlo (MC) simulations in the symmetric and in the broken phase, where the usual U(1) coupling is parametrized by \( \beta' = 1/g_0^2 \). As this transition happens for U(1) LGT at zero temperature, the pictures hold for Polyakov loops winding in any one of the four directions through the torus (ordered starts are used to avoid metastabilities). The similarity with the behavior of a Higgs field is evident. At \( \beta' = 2 \) we are deep in the broken phase, while in our simulations below we shall use \( \beta' = 1.1 \) where one has less problems with U(1) metastabilities of the MC algorithm.

One may like then to couple SU(2) gauge fields to the U(1) Polyakov loop, but a local operator is more attrac-
down remnants of a larger simple group [8]. It is tempting to pursue a related interpretation of the gauge transformation encountered here.

In the gauge for which the SU(2) factor of the U(1) matrices is transformed to the unit matrix and the phase factor of the SU(2) matrices to 1, the classical continuum limit [5] becomes

$$\lambda \text{Tr} F_{\mu\nu}^{\text{gad}}, \quad F_{\mu\nu}^{\text{gad}} = g_0 \partial_\mu A_\nu - g'_0 \partial_\nu A'_\mu. \quad (7)$$

This allows to couple matter fields with local U(2) invariance. Details are given in Ref. [9].

FIG. 2: Scatter plot for U(1) Polyakov loops on a $12^4$ lattice $\beta' = 0.9$ in the symmetric phase (center) and at $\beta = 2$ in the broken phase (ring).

FIG. 3: SU(2) string tension from Creutz ratios on a $12^4$ lattice as function of $\lambda$ with $\beta = 1.1$ and $\beta = 2.3$.

In the following results from MC simulations with the action [4] on $6^4$ and $12^4$ lattice with periodic boundary conditions are reported. Our MC procedure proposes the usual U(1) and SU(2) changes and relies on a variant of the biased Metropolis-heatbath algorithm [10]. In contrast to ordinary MC simulations of lattice gauge theories, updates stay within the fixed gauge introduced before Eq. [7]. The gauge freedom remaining within the MC procedure is a global transformation $U \rightarrow GUG^{-1}$ with the same $G$ for all $U$.

The U(1) coupling is kept at $\beta' = 1.1$ and for the SU(2) coupling the values $\beta = 2.2$, $2.3$ and $2.4$ are used. At $\lambda = 0$ without interaction, $\beta'$ is in the U(1) Coulomb phase and $\beta$ in the SU(2) scaling region. On the $12^4$ lattice Creutz ratios [11] for the SU(2) string tension $\kappa$ were calculated from Wilson loops up to size $5 \times 5$. Increasing $\lambda$, Fig. 3 shows their behavior at $\beta = 2.3$. Units with lattice spacing $a = 1$ are used. Data points are based on a statistics of at least $2 \times 10^5$ sweeps without measurements and subsequently $32 \times 2 \times 10^5$ sweeps with measurements. Error bars are calculated with respect to the 32 jackknife
bins. Corresponding plots for $\beta = 2.2$ and $\beta = 2.4$ look very similar, only that a considerably larger statistics is needed for low $\lambda$ values at $\beta = 2.2$ to overcome the statistical noise.

Figure 4 shows a strong first order transition, which is also directly visible in the average SU(2) plaquette action $\langle \text{Tr} U_p \rangle$ depicted in Fig. 4 and in the U(1) variables. See Fig. 5 for the U(1) string tension $\kappa'$ and Fig. 6 for the U(1) plaquette action. The $6^4$ lattice appears to be slightly more ordered than the $12^4$, which is presumably due to the periodic boundary conditions. While the jump in the plaquette actions is similar for SU(2) and U(1), this is not the case for the string tensions. The SU(2) string tension decreases at the transition by a factor 3.5, whereas the drop for U(1) is just 25% of the original value (the metastability which is for both string tensions encountered for the disordered start at $\lambda = 0.9$ is not supposed to be of relevance).

Figure 5: U(1) string tension from Creutz ratios on a $12^4$ lattice as function of $\lambda$ with $\beta' = 1.1$ and $\beta = 2.3$.

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FIG. 6: U(1) plaquette action on $6^4$ and $12^4$ lattice as function of $\lambda$ with $\beta' = 1.1$ and $\beta = 2.3$.

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FIG. 7: U(1) string tension from Creutz ratios on a $12^4$ lattice as function of $\beta'$ ($\beta = \lambda = 0$).
The interpretation of our figures is that the U(1) string tension signal of the deconfined phase on both sides of the transition, while the SU(2) string tension is characteristic for the confined phase at small $\lambda$ and for a zero-temperature deconfined phase at large $\lambda$. The latter point is supported by comparison with the behavior of the U(1) string tension for $\beta = \lambda = 0$ at the U(1) deconfining phase transition as shown in Fig. 7. The discontinuity in the string tension is as in Fig. 3 for SU(2), only that no strong metastabilities are observed for U(1) for which the opinion has remained that the transition is weakly first order as first reported in [12].

\begin{equation}
W_{i,\mu}(x) = -i \text{Tr} \left[ \tau_i \alpha^\dagger(x + \hat{\mu}) U_{\mu}(x) \alpha(x) \right]
\end{equation}

where $\tau_i$, $i = 1, 2, 3$ are the Pauli matrices and $\alpha(x)$ collects the angular variables of the scalar field. In contrast to Higgs model simulations on the lattice, these scalar fields should have positive squared masses and no self-interactions. They serve only as trial operators which allow one to test in a gauge invariant way for massive vector bosons. Assuming that the SU(2) gauge sector can be driven from a confined in a deconfined phase by the mixing with U(1) introduced in this paper, one would expect a degenerate $W$ triplet in the deconfined and scalar bound states in the confined phase, similarly as in Higgs model simulations. Such investigations will be the subject of future work.

To summarize, relying on LGT simulations, a SU(2) deconfining phase transition is obtained from interactions between the SU(2) and the U(1) vector fields. As this is achieved through spontaneous breaking of the SU(2) center symmetry one may expect that a renormalizable and asymptotically free continuum theory can be defined for suitable limits of the couplings $g^0_0, g_0$ and $\lambda$. Though our model introduces a number of unusual features, the associated difficulties do not appear to be unsurmountable. Whether the observed transition can provide an alternative to the Higgs mechanism by generating a $W$ boson mass dynamically remains to be seen.

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