Study on involute rotor profile based on pressure angle reduction

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Abstract. By adopting the method of reducing the pressure angle, this paper carried out an improvement on the profile of traditional involute rotor. It's whole tooth profile is described exactly by combining the meshing principle of gears with parameter equation. The study made clear that there were drops to some extent on the area utilization coefficient of the improved profile of involute rotor when compared with the traditional profile of the same structure, yet its coincident degree was increased along with the decreased of pressure angle. It raised the stability of rotor transmission and in the mean time raised the bearing capacity of rotor transmission.

1. Introduction
Roots vacuum pump is a kind of rotary displacement pump, the rotor is the core problem of the roots vacuum pump, the rationality of the design of the rotor type line will directly related to the merits of the pump suction performance[1,2]. At present, there are three types of rotor profile: involute profile, arc profile and cycloid profile. Cycloid profiles rarely used by area of low utilization coefficient, and arc profiles are not widely used because of the poor airtightness of the addendum and low volume efficiency of rotor profile[3,4].The design and process of the traditional involute profile are convenient, and the airtightness is well, however, which has profile interference and the decrease of area utilization coefficient[5].

2. Involute profile with reduced pressure angle
In roots vacuum pump, the whole rotor molded lines must be involved in the conjugate motion, so the arc replaced the addendum and dedendum of the original involute. The arc segment of the traditional involute profile is larger, and the instantaneous point meshing is implemented, which has a negative effect on the stability of gas flow and the area utilization coefficient of the tooth profile[6]. In order to improve its performance, this paper tries to reduce pressure angle in order to improve the area of utilization coefficient.

2.1. Establishment of the involute equation
As shown in figure 1, drawing two circles with O₁ and O₂ as the center and r₀ as the radius, and intersecting the line L-L at S₁ and S₂. S₁ and S₂ are the actual length of involute meshing line. As shown in figure the involute equation with O₁ K₁ as the starting axis of the involute is as follows:

\[
\begin{align*}
x &= r_0 \cos \phi + \phi \sin \phi \\
y &= r_0 \sin \phi - \phi \cos \phi
\end{align*}
\]  
(1)
The coordinate system with $O_1 K_1$ as the horizontal axis rotates clockwise by $\theta$ angle, and the coordinate rotation equation is:

$$
\begin{align*}
  x &= x\cos\theta - y\sin\theta \\
  y &= x\sin\theta + y\cos\theta
\end{align*}
$$

(2)

Then, the origin of coordinates is $(500,400)$, and the final equation of $K_1$ and $K_2$ involutes is:

\begin{align*}
  x &= 500 + r_{01} \left\{ (\cos\phi + \frac{\phi\sin\phi}{180})\cos 32.72^\circ - (\sin\phi - \frac{\phi\cos\phi}{180})\sin 32.72^\circ \right\} \\
  y &= 400 - r_{01} \left\{ (\cos\phi + \frac{\phi\sin\phi}{180})\sin 32.72^\circ + (\sin\phi - \frac{\phi\cos\phi}{180})\cos 32.72^\circ \right\}
\end{align*}

(3)

2.2. The establishment of the arc equation

2.2.1. The establishment of the arc equation of dedendum

With A as the center of the circle and $A K_1$ as the radius, the arc intersects the X-axis at point B, namely arc $B K_1$. Then the equation of the tooth valley of the rotor is:

\begin{align*}
  x &= r_{01} - r_1\cos\phi \\
  y &= r_1\sin\phi
\end{align*}

(4)

To keep the maximum radius of the rotor blade unchanged, that is, $R_2 = D/2$, take the center line of arc $B K_1$ intersecting X-axis at point $O_3$, as shown in figure 2. $O_3$ is the center of the dedendum arc, $O_3 B$ is the radius, then the $B K_1$ equation of the dedendum arc is:

\begin{align*}
  x &= 500 + (r + r_1) - r_1\cos\phi \\
  y &= 400 + r_1\sin\phi
\end{align*}

(5)

2.3. Establishment of the arc equation of addendum

The center line of arc $C K_3$ intersects Y-axis at point $O_4$, as shown in figure 2. The point $O_4$ is the center of the addendum arc, $O_4 C$ is the radius, then, the $K_2 C$ equation of the addendum arc is:
\[
\begin{aligned}
\begin{cases}
  x = 500 + r_1 \sin \phi \\
  y = 265 + (r_2 - r_1) \cos \phi
\end{cases}
\end{aligned}
\]  

Equation (3), equation (5) and equation (3) are the equation of the profile in which the pressure angle decreases by 1/4 of the involute type of the rotor.

3. Analysis of rotor profile parameters with reduced pressure Angle

3.1. Area utilization coefficient

The area utilization coefficient is the ratio of the area contained between the outer surface of a rotor and the inner surface of a cylinder to the inner circle area of a cylinder, represented by \( C \), which represents the effective utilization of the cylinder space.

\[
C = \frac{2B}{\frac{\pi D^2}{4}} = 1 - \frac{4S}{\frac{\pi D^2}{4}}
\]

(7)

\( B \) - the projection of the end face of the maximum volume formed by the rotor and the cylinder. The key to find the area utilization coefficient is to find the rotor sectional area, as shown in figure 3:

\[
S = S_1 + S_2 + S_3
\]

(8)

![Figure 3. Sectional area of blade end face](image)

According to the polar coordinate parameter equation and area integral formula of the involute:

\[
S_2 = \frac{1}{2} \int_{0}^{\alpha_c} \left[ \frac{r_0^2}{\cos^2 \alpha} - 1 \right] \cos \alpha d\alpha
\]

(9)

\[
S_1 = \frac{1}{2} \int_{0}^{\gamma} (rr_1 \cos t + r^2) dt
\]

(10)

\[
S_3 = \frac{1}{2} \int_{0}^{\gamma} (rr_1 \cos t - r^2) dt
\]

(11)

Substituting the parameters into the formula, \( \alpha_c = 50.46^\circ \), \( C = 0.52 \); \( \alpha_c = 45^\circ \), \( C = 0.51 \).

3.2. Coincidence degree

The coincidence degree (\( \epsilon_c \)) is an important index to measure the transmission continuity and transmission load uniformity of the gear. In gear transmission design, it is usually necessary to check the coincidence degree, and it is inevitable to discuss the coincidence degree when studying the new tooth profile curve or the new gear joint curve [7]. According to the formula of reference [8], the traditional linearity \( \alpha_c = 50.46^\circ \), \( \epsilon_c = 0.500116 \); the modified profile \( \alpha_c = 45^\circ \), \( \epsilon_c = 0.500146 \). Although the area utilization coefficient of the rotor profile decreases with the decrease of the pressure angle, the coincidence degree increases with the decrease of the pressure angle, which not only improves the stability of the rotor drive, but also improves the bearing capacity of the rotor drive.
3.3. Synthesis curvature radius
The characteristic index of gear meshing is the radius of curvature. Because the two involute tooth profiles are engaged between convex surface and convex surface, the smaller the contact area is, the easier it is to wear when the contact stress is constant. According to the reference [9], the synthesis curvature radius of the traditional involute is 99.94 mm, and the synthesis curvature radius of the rotor profile after the reduced pressure angle is 77.75 mm. Reducing the pressure angle can reduce the overall curvature radius of the rotor profile, which indicates that the rotor profile with reduced pressure angle has better wear resistance than the traditional involute rotor profile.

4. Conclusion
(1) Although the area utilization coefficient of the improved involute rotor profile decreases, its coincidence degree increases with the decrease of the pressure angle, which improves the stability of the rotor drive, the wear resistance of the rotor and the bearing capacity of the rotor drive. The synthetic curvature radius decreases with the decrease of the pressure angle, which improves the wear resistance of the rotor and prolongs its service life.

(2) The parametric equation of gear tooth profile of the improved involute rotor profile can accurately and truly describe the complete tooth profile of the rotor at any corner position. It provides a new theoretical basis for meshing analysis, meshing process simulation, kinematics and dynamics analysis of involute gear.

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