Torque Ripple Reduction in Sectored Multi Three-Phase Machines Based on PWM Carrier Phase Shift

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Abstract—The interest in multiphase machines for high power and reliable drives has been growing, and many control algorithms have been proposed to improve their torque performance. This work presents a new approach to the modeling of a multi three-phase drive, aiming at the minimization of the torque ripple introduced by Pulse Width Modulation (PWM) voltage excitation, by the shift of carrier phase angles among different three-phase inverters. The underlying idea is to use standard three-phase converters feeding the individual segment and to apply a phase shift between the PWM carriers. For the torque ripple analyzed in this paper, only the interaction between the armature field, resulting from the PWM voltage excitation, and the fundamental component of the permanent magnet field is considered. The proposed carrier phase shift angles are obtained for a case study of sectored triple three-phase synchronous permanent magnet machine. Analytical, numerical and finite element analysis (FEA) results are presented to explain how the carrier shift angles affect the current and torque ripple. Finally, experimental results are presented to validate the model and the control algorithm.

Index Terms—Analytical models, multiphase drives, machine vector control, permanent magnet machines, pulse width modulation, torque control.

NOMENCLATURE

$N$ Number of multi three-phase systems.

$\alpha_p$ Space phase shift angle between the $p^{th}$ three-phase system and the stator reference frame [rad].

$\omega_m$ Mechanical speed of the machine [rad/s].

$\omega_c$ Frequency of the carrier signal [Hz].

$\omega_a$ Frequency of the carrier signal [rad/s].

$\omega_{nm}$ Frequency of the modulating signal [rad/s].

$\omega_{mm}$ Frequency of the harmonic component with $m$ carrier signal index, and $n$ modulating signal index.

$f_c$ Frequency of the carrier signal [Hz].

$\theta_{c,p}$ Phase angle of the carrier signal of the $p^{th}$ three-phase system [rad].

$\theta_a$ Phase angle of the fundamental phase voltage (modulating signal) of the first three-phase system [rad].

$\theta'$ Phase angle of the fundamental phase current of the first three-phase system [rad].

$\theta''$ Time-varying angle of the fundamental phase voltage (modulating signal) of the first three-phase system [rad].

$y(t)$ Time-varying angle of the fundamental phase current of the first three-phase system [rad].

$R$ Stator phase resistance [$\Omega$].

$L_{X}, M_{XY}$ Self and mutual phase inductances of the machine matrix $L$ of a general machine winding, $X, Y \in \{A_1, B_1, C_1, A_2, B_2, C_2, A_3, B_3, C_3\}$ and $X \neq Y$. 
I. INTRODUCTION

Multiphase drives are well known for being a suitable solution for high power systems such as ship propulsion, electric vehicles and More Electric Aircraft applications [1]–[6]. The main advantage of a multiphase drive is the significant improvement in terms of flexibility in the design and control of the converters, and the reduced power rating requirement of the power electronic components [7]–[11]. Among the multiphase drives, the multi-three-phase layout offers the possibility to obtain a multiphase system by means of commercial three-phase inverters. Furthermore, the multi-three-phase layout with parallel or independent dc links offers a higher fault tolerance [12]–[15]. A scheme of a multi-three-phase drive is presented in Fig. 1.

To achieve high power and high power density drives, one of the main solution is to significantly increase the speed of the machine [16]–[19]. However, in high power systems the power electronics must bear high currents (or voltages) and the switching frequency of the power semiconductor is usually limited (below 30 kHz) [20]–[23]. This results in significant high frequency current ripple caused by the PWM of the DC/AC converter [24], [25]. The ripple affects the performance of the machine in terms of machine copper loss and torque [24]–[26]. In particular, the introduction of high frequency torque ripple is source of vibrations and noise that are undesired, especially for transport applications with high reliability requirements [27]–[28]. The possibility of reducing the torque ripple in PWM drive systems has been validated by previous works [24], [25], [27]. For example, in [27] the authors define a technique to eliminate the vibration in a phase shifted dual-three-phase machine. This paper proposes an analytical model of the torque ripple generated in a sectored multi-three-phase machine fed by a PWM control of multi-three-phase modular converters. Starting from the model, a method for the torque ripple reduction by applying CPS-PWM technique to the multi-three-phase inverters is defined. Results of analytical, numerical and FEA simulations are presented and validated by experimental tests.

In Section II, the model of the sectored multi-three-phase machine is derived as well as the torque equation. Section III describes the torque ripple due to a PWM excitation. Section IV analyzes a case study of a sectored triple-three-phase PMSM. Section V and VI show the simulation and experimental results. Section VII draws the conclusion.

II. SECTORED MULTI THREE-PHASE MACHINE

A sectored multi-three-phase PMSM layout presents a set of three-phase windings symmetrically placed around the stator, covering one pole pair each. The advantage of this layout can be found in terms of fault tolerance and manufacturing. Indeed, the phases under one pole pair have not physical contact (overlapping) with the other phases. Therefore, the magnetic mutual coupling among phases of different windings are significantly lower than the one of other distributed winding solution [29]. The previous advantages allow for reducing the spread of faults. As example, Fig. 2 shows the triple-three-phase sectored machine analyzed in this work. The model of the PWM voltage waveform applied to this machine is based on a double Fourier transformation already used in some research works on the PWM topic [30], [31]. This work aims at extending the theory to an arbitrary number of phases.

As it is shown in Fig. 1, in the equivalent 2-pole space winding structure of the N three-phase systems, the space shift angles between the pth three-phase system (p ∈ {1, ..., N}) and the stator reference frame are represented by \( \alpha_1, \ldots, \alpha_N \) respectively. In addition, the back-EMFs generated on each phase are represented by \( e_{A_1}(t), e_{B_1}(t), e_{C_1}(t), \ldots, e_{A_N}(t), e_{B_N}(t), e_{C_N}(t) \) respectively. The total back-EMF space vector related to all the N three-phase systems \( \vec{e}_{total}(t) \) can be represented by (1):

\[
\vec{e}_{total}(t) = \frac{2}{\sqrt{3}} \sum_{p=1}^{N} [e_{A_p}(t)e^{j\alpha_p} + e_{B_p}(t)e^{j(\alpha_p + \frac{2\pi}{3})} + e_{C_p}(t)e^{j(\alpha_p - \frac{2\pi}{3})}] .
\]

The phase currents flowing through phase \( A_1, B_1, C_1, \ldots, A_N, B_N, C_N \), are represented by \( i_{A_1}, i_{B_1}, i_{C_1}, \ldots, i_{A_N}, i_{B_N}, i_{C_N} \) respectively. The phase current space vector representative of all the N three-phase systems \( \vec{i}_{total}(t) \) can be expressed by (2):

\[
\vec{i}_{total}(t) = \frac{2}{\sqrt{3}} \sum_{p=1}^{N} [i_{A_p}(t)e^{j\alpha_p} + i_{B_p}(t)e^{j(\alpha_p + \frac{2\pi}{3})} + i_{C_p}(t)e^{j(\alpha_p - \frac{2\pi}{3})}] .
\]
\[ T_{\text{total}}(t) = \frac{3N}{2\omega_m}[\tilde{r}_{\text{total}}(t) \cdot \tilde{\alpha}_{\text{total}}(t)]. \]  

Owing to the symmetrical distribution of the different three-phase windings under the different pole pairs, typical of a sectored machine, the space phase shift angles \(\alpha_p\) is 0.

### III. PWM RELATED TORQUE RIPPLE

For double-edge naturally sampled pulse width modulation, the harmonic components of the PWM waveform of each converter leg, and the resulting phase voltage, can be evaluated by using the double Fourier integration [31]. The voltage difference between the terminals \(a_1, b_1, c_1, \ldots, a_N, b_N, c_N\) and the middle point of the DC link (z in Fig. 1) is referred to as \(u_{kpz}\). Its time-varying expression \(u_{kpz}(t)\) can be represented by (4):

\[ u_{kpz}(t) = \frac{V_{dc}}{2} \cos[y(t) - \alpha_p] + \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} A_{mn} \cos[(m \cdot n) z] \cos[y(t) + \alpha_p], \]  

with:

\[ A_{mn} = \frac{2V_{dc}}{m \cdot n} 
\left( \left| \frac{m}{n} \right| \right) \sin [(m \cdot n) \pi/2], \]

\[ x_p(t) = \omega_z t + \theta_{cp}, \]

\[ y(t) = \omega_z t + \theta_o, \]

where \(k \in \{a, b, c\}\) and \(p \in \{1, \ldots, N\}\).

The voltage difference between \(\alpha_p\) and \(z\) is referred to as the common mode voltage \(u_{opz}\). Since each three-phase system is star connected, according to Kirchhoff’s law, the common mode voltage will not generate any zero sequence current. Therefore, the common mode voltage will not lead to torque ripple, and its effects are not considered in this paper. The phase voltage is the voltage drop between the terminals \(a_1, b_1, c_1, \ldots, a_N, b_N, c_N\) and the terminals \(o_1, \ldots, o_N\), and are named as \(u_{A1}, u_{B1}, u_{C1}, \ldots, u_{AN}, u_{BN}, u_{CN}\) respectively. Removing all the common mode voltage components from the phase leg voltages \(u_{kpz}(t)\), the total phase voltage space vector of all the \(N\) three-phase systems \(\tilde{u}_{\text{total}}(t)\) can be defined as:

\[ \tilde{u}_{\text{total}}(t) = \frac{V_{dc}}{2} M_{E} \cos[y(t) - \alpha_p] + \sum_{p=1}^{N} \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} A_{mn} e^{j[(m \cdot n) y(t) + \alpha_p]}, \]

where \(l \) is an integer number. The voltage space vector \(\tilde{u}_{\text{total}}(t)\) contains the fundamental component \(\frac{V_{dc}}{2} M_{E} \cos[y(t)]\) and the harmonic components which are caused by the PWM. The harmonic components include both positive sequence components:

\[ \frac{1}{N} \sum_{p=1}^{N} \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} e^{j[m \cdot n y(t) + \alpha_p]}, \]

and negative sequence components:

\[ \frac{1}{N} \sum_{p=1}^{N} \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} e^{-(m \cdot n y(t) + \alpha_p)}]. \]

The main torque ripple caused by the PWM is due to the interaction of the high order winding field harmonics with the fundamental component of the permanent magnet field. The torque ripple caused by the interaction of the high order harmonics \(5\pi/3, 7\pi/3, 11\pi/3, 13\pi/3, \ldots\) is neglected in this analysis. Thus, only the fundamental component of the total back-EMF space vector \(\tilde{\alpha}_{\text{total}}(t)\) is considered while modelling the total phase current space vector \(i_{\text{total}}(t)\), which can be represented by (9):

\[ i_{\text{total}}(t) = I_{E} e^{jy(t)} + \frac{1}{N} \sum_{p=1}^{N} \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ e^{j[m \cdot n y(t) + \alpha_p]} - e^{-(m \cdot n y(t) + \alpha_p)} \right\}, \]

where:

\[ I_{E} e^{jy(t)} = \frac{1}{\pi(\omega_o)} \frac{V_{dc}}{2} M_{E} e^{jy(t)} e^{\tilde{\alpha}_{\text{total}}(t)}, \]

\[ y(t) = \omega_o t + \theta_o, \]

\[ \omega_{mn} = \omega_o + \omega_c, \]

Under the assumption of considering for only the fundamental component of the total back-EMF space vector \(\tilde{\alpha}_{\text{total}}(t)\), the instantaneous torque can be rewritten as:

\[ T_{\text{total}}(t) = \frac{3N}{2\omega_m} \left[ \tilde{r}_{\text{total}}(t) \cdot \tilde{\alpha}_{\text{total}}(t) \right], \]

It results from (13) that the average torque produced by the fundamental component \(I_{E}\) of the total current space vector \(i_{\text{total}}(t)\), whereas the main torque ripple caused by the PWM is generated by the harmonic components in the total current space vector \(i_{\text{total}}(t)\). Therefore, the minimization of the harmonic components of the total current space vector \(i_{\text{total}}(t)\) corresponds to the minimization of the related torque ripple. As it is expressed in (9), there are \(N\) degrees of freedom to change the carrier phase angle \(\theta_{cp}\) of each sub-three-phase system. Thus, different optimized carrier phase angles \(\theta_{cp}\) can be found for multi-three-phase drives with different values of three-phase systems \(N\) and related phase shift angles \(\alpha_p\).

### IV. CASE STUDY ON A TRIPLE THREE-PHASE SECTORED PMSM

As a case study, an 18 slots and 6 poles triple three-phase PMSM is shown in Fig. 2. The machine has three sectors, sector 1, sector 2 and sector 3 respectively. Each sector has three phases (phase A, phase B, phase C) with an independent floating neutral point. Therefore, the three-phase back-EMF's generated.
in each sector are supposed to have no electrical degree phase shift with respect to other sectors.

For the mathematical model of the total current space vector \( \mathbf{r}_{\text{total}}(t) \) shown in (9), the number of three-phase systems is 3 (\( N = 3 \)) and the equivalent space phase shift angle is 0 for all of the three sectors (\( \alpha_p = 0, p \in \{1, 2, 3\} \)). By analyzing the effect of the carrier phase angle of the \( p^{\text{th}} \) three-phase system \( \theta_{c,p} \) \((\chi_p(t) = \alpha_p t + \theta_{c,p}) \) in the voltage and current total space vector equations (8)-(9), it can be found that when the carrier phase shift angles are \( \theta_{c,1} = 0, \theta_{c,2} = \frac{2\pi}{3} \) and \( \theta_{c,3} = \frac{4\pi}{3} \), for sector 1, 2 and sector 3 respectively, all of the harmonic components of the current space vector are cancelled out except the groups of harmonics around the frequency of \( 3m f_c \) \((m \in \{1, 2, 3, \ldots, \infty\}) \). As it is shown in (5) and (9), the amplitude of each PWM harmonic component changes under different modulation index \( (M) \). The modulation index is one of the main factors affecting the amplitudes of the harmonics. According to (13), the corresponding FFT spectrum of the normalized torque without and with applying the proposed carrier phase shift angles under different modulation index \( (M) \) is shown in Fig. 3.

The cross section of the considered machine is shown in Fig. 2. The self-inductance of each stator phase \( A_1, B_1, C_1, A_2, B_2, C_2, A_3, B_3, C_3 \) are represented by \( L_{A_1}, L_{B_1}, L_{C_1}, L_{A_2}, L_{B_2}, L_{C_2}, L_{A_3}, L_{B_3}, L_{C_3} \), respectively. The mutual inductance between the phases \( A_1, B_1, C_1, A_2, B_2, C_2, A_3, B_3, C_3 \) are represented by \( M_{A_1A_1}, M_{A_1A_2}, \ldots, M_{C_3A_3}, M_{C_3B_3} \) respectively. Due to the symmetrical design of the winding configuration in Fig. 2, the matrix inductance table \( L \) of the machine can be represented by:

\[
L =
\begin{bmatrix}
L & -M_1 & -M_2 & -M_3 & M_2 & M_3 & -M_2 & -M_3 & M_3 \\
-M_1 & L & M_2 & M_3 & -M_2 & -M_3 & M_2 & M_3 & -M_2 \\
-M_2 & -M_1 & L & M_3 & M_2 & -M_3 & M_2 & -M_3 & L \\
-M_3 & -M_2 & -M_1 & L & M_3 & M_2 & -M_3 & M_2 & -M_3 \\
M_1 & M_2 & M_3 & -M_2 & -M_3 & M_2 & M_3 & -M_2 & -M_3 \\
M_2 & M_1 & M_3 & -M_1 & M_2 & M_3 & -M_1 & M_2 & M_3 \\
M_3 & M_2 & M_1 & M_3 & -M_1 & M_2 & M_3 & -M_1 & M_2 \\
M_1 & M_3 & -M_1 & M_2 & M_3 & -M_2 & M_3 & -M_2 & M_1 \\
M_2 & M_3 & -M_2 & M_1 & M_3 & -M_1 & M_2 & M_3 & -M_1 \\
M_3 & M_2 & M_1 & M_3 & -M_1 & M_2 & M_3 & -M_2 & M_1 \\
-M_1 & M_2 & M_3 & -M_2 & -M_3 & M_2 & M_3 & -M_2 & -M_3 \\
-M_2 & M_1 & M_3 & -M_1 & M_2 & M_3 & -M_1 & M_2 & M_3 \\
-M_3 & M_2 & M_1 & M_3 & -M_1 & M_2 & M_3 & -M_2 & M_1 \\
M_1 & M_3 & -M_1 & M_2 & M_3 & -M_2 & M_3 & -M_2 & M_1 \\
M_2 & M_3 & -M_2 & M_1 & M_3 & -M_1 & M_2 & M_3 & -M_1 \\
M_3 & M_2 & M_1 & M_3 & -M_1 & M_2 & M_3 & -M_2 & M_1 \\
\end{bmatrix}
\]

where: \( L = L_{kp}, k \in \{A, B, C\} \), \( p \in \{1, 2, 3\} \); \( M_{A_pB_p} = M_{kIP} = M_1 \), \( k \in \{B, C\} \); \( M_{B_pC_p} = M_{C_pB_p} = M_2 \); \( M_{k_pk_p'}, k, k' \in \{A, B, C\}, p, p' \in \{1, 2, 3\}, p \neq p' \).

The PWM related phase voltage harmonics and phase current harmonics of the \( p^{\text{th}} \) three-phase system can be represented by \( u_{k_p}, i_{k_p}, (p \in \{1, 2, 3\}) \) respectively. Since the three phases in each sector are star-connected, the sum of three-phase currents is zero in any sector, leading to the following constraint:

\[
i_{A_p,h} + i_{B_p,h} + i_{C_p,h} = 0 \quad (14)
\]

According to the electric principle, the corresponding \( p^{\text{th}} \) phase voltage harmonic \( u_{A_p,h}, u_{B_p,h}, u_{C_p,h} \) can be represented by (15) (16) and (17) respectively:

\[
u_{A_p,h} = Ri_{A_p,h} + (L + M_1) \frac{d}{dt} i_{A_p,h} - (2M_3) \frac{d}{dt}(i_{A_p,h} + i_{A_p,h'}), \quad (15)
\]

\[
u_{B_p,h} = Ri_{B_p,h} + (L + M_1) \frac{d}{dt} i_{B_p,h} - (2M_3) \frac{d}{dt}(i_{B_p,h} + i_{B_p,h'}), \quad (16)
\]

\[
u_{C_p,h} = Ri_{C_p,h} + (L + M_1) \frac{d}{dt} i_{C_p,h} + (M_1 + M_2) \frac{d}{dt} i_{C_p,h} - (2M_3) \frac{d}{dt}(i_{C_p,h} + i_{C_p,h'}), \quad (17)
\]
\[ u_{cp,h} = R_i + (L + M_1 - 4M_3) \frac{dt}{dt} i_{cp,h} \]
\[ + (M_1 + M_2 - 4M_3) \frac{dt}{dt} b_{cp,h}, \]
\[ (19) \]
\[ u_{Bp,h} = R_i + (L + M_1 + 2M_3) \frac{dt}{dt} i_{Bp,h} \]
\[ + (M_1 + M_2 + 2M_3) \frac{dt}{dt} c_{Bp,h}, \]
\[ (20) \]

Whereas, by applying the proposed CPS-PWM method \((\theta_{c,1} = 0, \theta_{c,2} = \frac{2\pi}{3}, \theta_{c,3} = \frac{4\pi}{3})\), there are two conditions of phase current harmonics according to (9). First, for the harmonic components of the total current space vector which cannot be cancelled out by the CPS-PWM method (around the frequency of \(3f_c, z \in \{1, 2, ..., \infty\}\)), the relevant phase current harmonics of each sector are the same \((i_{k_1,h} = i_{k_2,h} = i_{k_3,h}, k \in \{A, B, C\})\). Therefore, the corresponding \(p^{th}\) phase voltage harmonics \(u_{Ap,h}, u_{Bp,h}, u_{Cp,h}\) with applying CPS-PWM can be represented by (18), (19) and (20) respectively. Secondly, for the harmonic components of the total current space vector which can be cancelled out by the CPS-PWM method (around the frequency of \(2f_c\) and \(2f_c, z \in \{1, 2, ..., \infty\}\), the sum of phase current harmonics from different sectors is equal to 0, which can be represented by (21):
\[ i_{k,h} = \sum_{p=1}^{\infty} i_{k,p,h} = i_{k_1,h} + i_{k_2,h} + i_{k_3,h} = 0, \]
\[ (21) \]
where \(k \in \{A, B, C\}\). It results that the corresponding \(p^{th}\) phase voltage harmonic \(u_{Ap,h}, u_{Bp,h}, u_{Cp,h}\) with applying CPS-PWM can be represented by (22) and (23) and (24) respectively:
\[ u_{Ap,h} = R_i + (L + M_1 + 2M_3) \frac{dt}{dt} i_{Ap,h}, \]
\[ (22) \]
\[ u_{Bp,h} = R_i + (L + M_1 + 2M_3) \frac{dt}{dt} i_{Bp,h} \]
\[ + (M_1 + M_2 + 2M_3) \frac{dt}{dt} c_{Bp,h}, \]
\[ (23) \]
\[ u_{Cp,h} = R_i + (L + M_1 + 2M_3) \frac{dt}{dt} i_{Cp,h} \]
\[ + (M_1 + M_2 + 2M_3) \frac{dt}{dt} c_{Bp,h}. \]
\[ (24) \]

For phase A in each sector, the effective inductance \((L + M_1 - 4M_3)\) in (18) is smaller than the effective inductance \((L + M_1 + 2M_3)\) in (22). For phase B and C in each sector, the effective self-inductance \((L + M_1 - 4M_3)\) in (19) and (20) is smaller than the effective self-inductance \((L + M_1 + 2M_3)\) in (23) and (24); the effective mutual inductance between phase B and C \((M_1 + M_2 - 4M_3)\) in (19) and (20) is smaller than the effective mutual inductance \((M_1 + M_2 + 2M_3)\) in (23) and (24). Overall, for the considered sectored machine, the effective inductance without applying CPS-PWM method is smaller than the effective inductance with applying CPS-PWM. The amplitudes of phase voltage harmonics with and without CPS-PWM are the same [31]. Therefore, the amplitudes of phase current harmonics with applying CPS-PWM method are smaller than the ones obtained without the CPS-PWM method. The numerical results obtained in PLECS of the normalized phase current FFT spectrum with and without CPS-PWM under different modulation index \((M = 0.3, M = 0.6\) and \(M = 0.9\) ) is shown in Fig. 4.

It results that applying the CPS-PWM method to the triple three-phase sectored PMSM machine brings two major advantages to the machine. First, the torque ripple of machine is reduced, hence the noise and vibration of the machine is effectively reduced. Secondly, the phase current harmonic is reduced as well, hence less copper loss of the machine is expected.

V. ANALYTICAL, NUMERICAL AND FEA RESULTS

Analytical, numerical and FEA simulations have been carried out in order to evaluate and validate the advantages of the proposed control of the carrier phase angles. Analytical results are obtained using the time-varying equations (13) and (22) in Matlab. The numerical results are obtained by using variable-step simulations in PLECS. The operating condition concerning the numerical results is no load condition. There is no external load on the machine and the machine operating power is to overcome the mechanical (friction) power loss and the electromagnetic power loss of the machine itself. FEA results are finally realized by Magnet with the triple three-phase machine model (Fig. 2) excited by the currents resulting from the PLECS simulation. The numerical and FEA results are used to validate the analytical model described by (13) and (22), and quantify the phase current and torque ripple with and
without CPS-PWM. The main converter and machine parameters are shown in Table I.

Fig. 5 shows the block diagram for the control of the nine-phase machine fed by its three independent PWM converters. The CPS-PWM method is applied to the three three-phase systems with carrier phase shift angles $\theta_{c,1}$, $\theta_{c,2}$, and $\theta_{c,3}$ respectively. The machine in the PLECS simulations (numerical results) is controlled in speed, by a simple proportional-integral (PI) controller which provides the same current reference ($i_q$) as input to the internal current PI regulator of each three-phase system.

Fig. 6 shows the comparison of the analytical and numerical models in terms of phase current (phase $A_1$ is considered). Fig. 6a and Fig. 6c show the analytical and numerical results without applying CPS-PWM method. Fig. 6b and Fig. 6d show the same while applying the CPS-PWM method. Fig. 6d shows that there is slightly difference between the analytical and numerical results at the groups of harmonics around 2 kHz, 4 kHz, 8 kHz, 10 kHz, 14 kHz and 16 kHz. The reason is that the analytical model is based on a simplification of the system considering the equations in electrical degrees for phase $A_p$, $B_p$ and $C_p$ ($p \in \{1, 2, 3\}$) independent from the sector where they are placed. In this machine, due to the sectored stator winding structure, applying CPS-PWM in numerical model will lead to small harmonic phase current difference among phase $A_p$, $B_p$ and $C_p$ in each sector, which is shown in Fig. 4.

Fig. 7 shows the numerical current results of phase $A_1$, $A_2$ and $A_3$ with and without applying CPS-PWM. Fig. 7b and Fig. 7d are the zoom of the waveform in between the cursor ranges of Fig. 7a and Fig. 7c respectively. Comparing Fig. 7b and Fig. 7d, the phase current harmonics in different sectors are effectively shifted with applying the CPS-PWM method. Comparing Fig. 7a and Fig. 7c, the amplitudes of phase current harmonics obtained by applying CPS-PWM are reduced compared with those obtained by not applying the CPS-PWM. The corresponding FFT spectrum of Fig. 7a and Fig. 7c are shown in Fig. 6c and Fig. 6d respectively. For all of the harmonic components except the harmonics around 6 kHz and 12 kHz in Fig. 6c and Fig. 6d (numerical result), their amplitudes in Fig. 6d are reduced by 45.18% compared with the ones in Fig. 6c.

Fig. 8 shows the comparison of analytical, numerical and FEA models in terms of the machine electromagnetic torque. Fig. 8(a-d) show that the analytical results match with the numerical results for both without and with applying CPS-PWM. Fig. 8(a-b) show that the FEA results match with the analytical and numerical results with a good approximation considering for the analyzed ripple. Fig. 8(c-d) show that there are low order harmonics ($6^{th}$ at 300Hz; $12^{th}$ at 600Hz) in FEA results, which are not shown in analytical and numerical results. One reason is that only fundamental component of back-EMF is considered in analytical and numerical models, which has been mentioned in chapter II. In the FEA machine model, the interaction between the fundamental component of winding field and the $5^{th}$, $7^{th}$, $11^{th}$, $13^{th}$…permanent magnet harmonics results in the $6^{th}$, $12^{th}$…harmonics of the torque ripple. The other reason is that the machine model in Magnet is a 6 poles, 18 slots PMSM, and

### Table I

| Parameter                          | Value                          |
|-----------------------------------|--------------------------------|
| DC voltage ($V_{dc}$)             | 60 [V]                         |
| Switching frequency ($f_s$)       | 2 [kHz]                        |
| Modulating frequency ($f_m$)      | 50 [Hz]                        |
| Pole pair number                  | 3                              |
| Power rating of the machine       | 1.5 [kw]                       |
| Rated torque of the machine       | 5 [Nm]                         |
| Rated current of the machine      | 11.5 [Amp]                     |
| Rated voltage of the machine      | 28.5 [Vpk]                     |
| Phase resistance ($R$)            | 0.08 [Ω]                       |
| Stator inductance matrix($L$)     | $L \approx 0.31$; $M_1 = 0.087$; $M_2 = 0.03$; $M_3 = 0.029$ [mH] |
| Mechanical speed ($\omega_m$)     | 104.72 [rad/s] (1000rpm)       |
| Back-EMF coefficient ($K_B$)      | 0.085 (phase peak back-EMF is 8.9V at 50Hz) |
The interaction between the permanent magnet rotor and stator slots generate $6^\text{th}$, $12^\text{th}$,... harmonics in the torque caused by the cogging effect. In addition, Fig. 8(c-d) show that the FEA result presents slightly higher amplitudes compared with analytical and numerical results, this is due to machine parameter uncertainties in the model, for example the changes of them with working operation due to saturations and non-linear effects.

Comparing the torque waveform with and without CPS-PWM, Fig. 8(a-b) show that the peak-to-peak torque are reduced by 79.5%, 78.5% and 63.8% with applying CPS-PWM in analytical, numerical and FEA results respectively. Fig. 8(c-d) show that the harmonic components of the torque FFT spectrum around 2 kHz, 4 kHz, 8 kHz, 10 kHz, 14 kHz and 16 kHz obtained by applying CPS-PWM are effectively cancelled out in analytical, numerical and FEA results.

VI. EXPERIMENTAL RESULTS

In order to validate the analytical model and the simulation results, experimental tests have been carried out by means of the platform shown in Fig. 9. The parameters and the control algorithm used in the experimental platform is the one explained in chapter V. The operating condition concerning the experimental results (same as the numerical result) is no load condition. There is no external load and the machine operating power is to overcome the mechanical (friction) power loss and the electromagnetic power loss of the machine itself. The experimental setup consists of three three-phase inverters with standard IGBT modules, a sectored triple three-phase PMSM with its cross section shown in Fig. 2, and a centralized controller (uCube [32]). Optical fiber is used to communicate between the power module gate drives and the uCube.

Fig. 10 shows the experimental current results of phase $A_1$, $A_2$ and $A_3$ with and without applying CPS-PWM. Fig. 10b and Fig. 10d are the zoom of the waveform in between the cursor ranges of Fig. 10a and Fig. 10c respectively. Comparing Fig. 10b and Fig. 10d, the phase current harmonics in different sectors are effectively shifted with applying the CPS-PWM method. Comparing Fig. 10a and Fig. 10c, the amplitudes of the phase current harmonics obtained by applying CPS-PWM are
reduced compared with those obtained by not applying CPS-PWM. The corresponding FFT spectrum of Fig. 10a and Fig. 10c shows that the amplitudes of harmonic components around 2 kHz, 4 kHz, 8 kHz, 10 kHz, 14 kHz and 16 kHz obtained by applying CPS-PWM are reduced by about 36.1% 38.4%, 36.1%, 34.7%, 34.1%, 29.5% respectively compared with those obtained by not applying CPS-PWM. The improvement achieved with CPS-PWM in the experimental results is reduced compared with the improvement in the numerical results, due to the back-EMF distortion, the machine parameter uncertainties, the inverter non-linearity and the dead time effect. Another effect is the switching noise that causes current spikes during the commutations, as visible in Fig. 10b and Fig. 10d, that accounts for 4.6% of torque ripple increase (evaluated by numerically removing the switching noise).

The experimental phase currents are used to calculate the equivalent electromagnetic torque, and only the fundamental component of the back-EMF is considered. The equivalent electromagnetic torque is calculated based on (2) and (13), due to the bandwidth limitation of commercial torque meters. The equivalent electromagnetic torque waveforms and their corresponding FFT spectrum with and without CPS-PWM, are shown in Fig. 11. Comparing the torque waveform with and without CPS-PWM, Fig. 11a shows that the peak-to-peak torque is reduced by 58.3%, with applying CPS-PWM. The experimental torque ripple reduction of 58.3% is smaller than the analytical (numerical, FEA) results, but it still represents a major improvement compared to the control without CPS-PWM. Fig. 11b shows that the harmonic components of the torque FFT spectrum around 2 kHz, 4 kHz, 8 kHz, 10 kHz, 14 kHz are effectively cancelled out.

VII. CONCLUSION

This work proposes a new mathematical modeling approach to multi three-phase drive systems in order to improve the torque performance of multi three-phase machines by applying carrier phase shift among three-phase inverters (CPS-PWM method). Numerical, FEA simulations and experimental tests validate the analytical model shown in Chapter III and IV. The carrier phase shift angles obtained by the developed theory are applied on a case study of a sectored triple three-phase machine. The peak-to-peak values of the torque waveforms obtained by applying CPS-PWM are reduced by 79.5%, 78.5%, 63.8% and 58.3% compared with those obtained by not applying CPS-PWM in analytical, numerical, FEA and experimental results respectively. The PWM related harmonic components of the torque FFT spectrum obtained by applying CPS-PWM are effectively cancelled out. In addition, the phase current harmonics in different sectors are effectively shifted with applying the CPS-PWM method. For this case study on the sectored triple three-phase machine, while the CPS-PWM method is applied, the amplitudes of PWM related harmonic components of the phase current FFT spectrum (except the components around 6 kHz and 12 kHz) are reduced by 45.18% and about 35% in numerical and experimental results respectively.

Therefore, applying CPS-PWM method to multi three-phase drives can effectively improve the torque performance of the machine, guaranteeing major benefits in terms of current and torque ripple without additional computational burden.
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