An undergraduate laboratory experiment to accurately measure the speed of light

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Received 28 March 2020, revised 23 April 2020
Accepted for publication 12 May 2020
Published 9 June 2020

Abstract
In 1983 the speed of light was set as an exact quantity. It is now one of the fundamental constants in physics, with the meter being directly related to this speed and the definition of the second. As such, experiments that calculate the speed of light with high precision are important in university undergraduate laboratories. In the experiment discussed here, a method is described that allows the speed of light to be calculated using an apparatus that fits onto a 1 m bench-top. An advantage of this method is that a simple digital voltmeter can be used. This measures the amplitude-modulated output from a laser beam emerging from a multi-pass cell that is mixed with that from a reference beam. The relative phase shift is determined as the modulation frequency changes, allowing the speed of light to be ascertained. Full details of the apparatus and the electronics designed for the experiment are presented.

Keywords: light speed, undergraduate laboratory, speed of light measurement

(Some figures may appear in colour only in the online journal)

1. Introduction

The speed of light is now one of the fundamental constants in nature through which many other measurements are made. It is defined as precisely \( c = 299,792,458 \) meters per second in vacuum, whereas the second is defined precisely from the \(^{133}\)Cs ground state hyperfine transitions between the \( |F = 3, m_F = 0| \) and \( |F = 4, m_F = 0| \) states. In 1967 the definition of the second was set by defining the transition frequency between these states as exactly \( 9,192,631,770 \) oscillations each second [1]. The meter is hence defined as the distance travelled by light in 30.663 32 oscillations of the resonant radiation in Cs (to 7 significant figures). This definition

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Figure 1. Block diagram of the apparatus. A laser diode is amplitude modulated by an oscillator operating from $\sim1$ MHz to 50 MHz. The laser diode output is set to produce a well-defined sinusoidal modulation using the bias circuit. The beam is split by a 30:70 beamsplitter that directs 30% of the light onto a reference detector. The transmitted radiation passes through a beam expander before entering a multipass cell that creates a time delay. The delayed beam is then detected. A summing amplifier mixes the AC-signals to produce an output that changes with frequency, from which the speed of light is determined. The detector outputs can be monitored using a fast oscilloscope, and the peak of the mixer output is measured with a digital voltmeter.

replaced the conventional standard of the time, which was set in different regions of the earth by platinum–iridium bars measured to a precision of $\sim0.2\ \mu$m with respect to the ‘standard bar’ held in Paris.

These modern definitions mean that the associated measurements can be derived from a fundamental atomic property. The hyperfine transition frequency in $^{133}$Cs is insensitive to changes in the local environment that affects the standard meter bar such as pressure and temperature. The second can now be determined to a precision of $\sim1$ part in $10^{15}$ using atomic clocks (such as the Cs fountain clock [2]), from which the unit of distance can be derived.

The exact speed of light is defined in vacuum, and it is clearly important to measure this accurately. Experiments that determine this speed are hence fundamental for metrology, and so it is instructive to develop undergraduate experiments that can determine this to high accuracy, without the experiments being prohibitively expensive.

In practical terms, it is not easy to carry out undergraduate experiments in vacuum due to technical problems of maintaining a good vacuum. The apparatus detailed here hence measures the speed in air. A correction factor can then be applied to allow for the refractive index of air, by using one of the equations introduced by Ciddor or Edlén [3] that allow for the local pressure, temperature and humidity of the air in the laboratory.

The most straightforward method to measure the speed of light is to use a time-of-flight method, where a pulse of radiation is sent from a source to a detector through a well-known distance, and the delay time is measured [4–6]. This can be carried out in undergraduate laboratories with an accuracy of a few percent, by pulsing a laser beam and by using a fast photo-detector. Alternatively a continuous wave laser beam directed onto a mirror rotating at high speed can be used so the returning beam translates a small distance due to the flight time (Foucault’s method, see [7, 8] for modern versions of this experiment which can now produce accuracies of $\sim1\%$). In both cases the speed can be measured, however the experiments tend to require a large footprint in the laboratory.

The new experiments described here were designed to replace older experiments in Manchester that used Foucault’s method. These needed a bench 6 m in length, as well as an expensive motorized system that rotated the mirror at high speed (and which was prone to regular failure).
Students redirected the laser beam back to the source and would then use a microscope eyepiece and graticule to estimate the displacement of the returning beam from the source beam. This required students to look directly at the beam through the eyepiece with its associated safety implications, and would also cause eyestrain. The experiment was not accurate, with measured uncertainties often greater than 30%–40%. It was hence decided to develop new experiments that needed a much smaller footprint (∼1 m of bench space), that could determine the speed of light with higher accuracy and which could be constructed for relatively low cost.

A phase-shift method was hence designed where a beam from a laser diode is amplitude modulated by a sinusoidal signal operating from 1 MHz to 50 MHz, and the phase shift between a time-delayed beam and reference beam measured over a range of frequencies, as shown in figure 1. The time delay is set by passing the beam through a multi-pass cell consisting of two parallel mirrors, before returning the beam to a high-speed detector. The reference beam is obtained from a 30:70 beam-splitter immediately after the laser diode and is directed to a second high-speed detector. The signals are then amplified and mixed together before passing to a peak detector and low-pass filter that removes high frequencies from the signal. As the modulation frequency changes, the two signals experience different phase shifts, and so vary from being in phase (producing a maximum signal from the low pass filter) to being out of phase (producing a minimum from the filter). This DC output is measured using a digital voltmeter, and is plotted as a function of the modulation frequency to determine the speed of light. By systematically measuring the signal over a large range of frequencies, a precise measurement can be made.

To detail this apparatus this paper is divided into six sections. Following this introduction, the theory of the modulation technique is presented and the appropriate equations derived for the signal from the peak detector and low-pass filter. The optical components are then described, after which the electronic control systems are detailed. These include schematics of the laser diode driver circuitry and biasing system, the high-speed detectors, the summing amplifier and the peaks detector and low-pass filter. Results from experiments are then presented which demonstrate how the speed of light is determined, with data taken from experiments in the undergraduate laboratories in Manchester. A summary is finally presented with conclusions drawn from the analysis and results.

2. Theory

The output power of a diode laser has a predominantly linear relationship to the driving current once threshold has been reached, as shown in figure 2. An input sinusoidal signal added to this bias current will hence produce an output intensity that is sinusoidally modulated. The example shown in figure 2 is for a red diode laser operating at 635 nm, with the DC bias set to 30 mA and the driving AC signal having a peak amplitude of 3 mA. This produces a corresponding sinusoidal output from the laser ranging from ∼1 mW to ∼4.2 mW over a wide range of frequencies. The modulation depth shown here is hence around 63%, with an average power of ∼2.5 mW.

The laser diode output can be described by equation (1):

\[ I_{\text{las}}(t) = (I_{\text{av}} + I_1 \cos(2\pi f_M t + \varphi_0)) \sin(\omega_{\text{Las}} t - k_{\text{Las}} z) \]  

where \( f_M \) is the amplitude modulation frequency in Hz, \( \varphi_0 \) is an arbitrary phase factor, \( \omega_{\text{Las}} \) is the angular frequency of the laser beam and \( k_{\text{Las}} \) is the amplitude of the laser wave-vector. The intensity \( I_{\text{av}} \) defines the average intensity around which modulation occurs, with \( I_1 \) the peak intensity of the modulation with respect to this average. In the experiments the offset...
Figure 2. Measured output power from a typical 5 mW visible laser diode as a function of the driving current. Once threshold is reached the output has a high degree of linearity with current, as shown by the fitted linear function. By setting a bias current appropriately, an injected sinusoidal signal added to this bias will produce a laser beam whose output is modulated sinusoidally.

signal is blocked by capacitors following the photodiode circuitry (see figure 1), and so only the AC-sinusoidal signal is amplified. The phase factor $\varphi_0$ is here set to zero without loss of generality.

The photodiode detectors are reverse biased, and so have a linear response to the intensity of light that illuminates them. Since photodiodes cannot respond at the carrier wave frequency $\omega_{Las}$, only the amplitude modulation of the laser beam is detected. The voltage signal from the detectors hence is also a sinusoidal signal with a frequency $f_M$.

The beam incident on the reference photodiode travels a distance $d_1$ from the beam splitter to the detector, and so the light arrives at a time $T_1 = n_{air}d_1/c$ later compared to when it was at the beam splitter, where $n_{air}$ is the refractive index of the air in the laboratory. The signal generated at the reference photodiode is hence given by equation (2):

$$\text{sig}_1(t) \propto k_{BS1} (I_{av} + I_1 \cos (2\pi f_M (t - n_{air}d_1/c))) = B_0 + B_1 \cos (\omega_M t - \varphi_1) \quad (2)$$

where $k_{BS1}$ is the reflection coefficient of the beam-splitter together with any efficiency losses of the detector, $B_0 = k_{BS1}I_{av}$, $B_1 = k_{BS1}I_1$, $\omega_M = 2\pi f_M$ and $\varphi_1 = \omega_M T_1 = 2\pi f_M n_{air}d_1/c$ is the associated phase shift due to this pathway.

The beam that passes through the beam splitter travels an optical distance $d_{21}$ from the beam splitter to the entrance of the multi-pass cell, travels a distance $d_{MP}$ inside the cell and a further distance $d_{22}$ from the exit of the cell to the delay photodiode. The total path length from beam splitter to detector is hence $L_{tot} = (d_{21} + d_{MP} + d_{22})$. The delay time along this route is given by $T_{delay} = \frac{L_{tot}}{c}$, yielding a signal generated at this detector given by equation (3):

$$\text{sig}_2(t) \propto k_{BS2} (I_{av} + I_1 \cos (\omega_M (t - n_{air}L_{tot}/c))) = B'_0 + B_2 \cos (\omega_M t - \varphi_2) \quad (3)$$

Here $k_{BS2}$ allows for the transmission coefficient of the beam-splitter, the detection efficiency of the delay photodiode and losses due to the optics (including attenuation in the multi-pass cell), $B'_0 = k_{BS2}I_{av}$, $B_2 = k_{BS2}I_1$ and $\varphi_2 = \omega_M T_{delay} = \frac{2\pi f_M n_{air}L_{tot}}{c}$.  

Eur. J. Phys. 41 (2020) 045704
The DC components of the signals \((B_0, B'_0)\) are blocked from the summing amplifier as shown in figure 1, and so only the AC-signals from the photodiodes are added. This then yields
\[
\text{sig}_{1+2}(t) = \text{sig}_{1}^{AC}(t) + \text{sig}_{2}^{AC}(t) = B_1 \cos(\omega_M t - \varphi_1) + B_2 \cos(\omega_M t - \varphi_2). \quad (4)
\]

This can be written as a single wave given by
\[
\text{sig}_{1+2}(t) = B_1 + 2 \cos(\omega_M t - \Delta) \quad (5)
\]
where
\[
B_1^2 = B_1^2 + B_2^2 + 2B_1B_2 \cos(\varphi_1 - \varphi_2) \quad & \Delta = \tan^{-1}\left(\frac{B_1 \sin(\varphi_1) + B_2 \sin(\varphi_2)}{B_1 \cos(\varphi_1) + B_2 \cos(\varphi_2)}\right). \quad (6)
\]

This is a sinusoidally modulated signal also at frequency \(f_M\) whose amplitude now depends upon the frequency, since \(B_{1+2}^2 = B_1^2 + B_2^2 + 2B_1B_2 \cos\left(\frac{2\pi n_{\text{air}}(L_{\text{tot}} - d_1)}{c}\right)\). The peak detector and low-pass filter are set to detect only the positive component of this modulation, and so we may write
\[
y_B = \cos^{-1}\left(\frac{B_1^2 - B_2^2}{2B_1B_2}\right) = \frac{2\pi n_{\text{air}}(L_{\text{tot}} - d_1)}{c} f_M = m f_M \quad (7)
\]
which is a linear function with slope \(m\). By applying a linear least squares fit to \(y_B\), the speed of light is then given by
\[
c = \frac{2\pi n_{\text{air}}(L_{\text{tot}} - d_1)}{m}. \quad (8)
\]

The values \(B_1, B_2\) and \(B_{1+2}\) are obtained in three individual measurements at each frequency. \(B_1\) is obtained by switching off the delayed signal detector (see figure 1), \(B_2\) is measured with the reference detector switched off and \(B_{1+2}\) is measured with both detectors on. A maximum contrast is obtained for \(\text{sig}_{1+2}(t)\) by adjusting the light on the photodiodes so that \(B_1 \simeq B_2\). This is achieved by adjusting the optical setup (typically the focus into the detectors), and by monitoring the signals from each photodiode directly.

It should be noted that the above analysis assumes that the signal amplitudes from the photodetectors are independent of frequency, which is not possible in a real system. To compensate for this, the detector circuits use exactly the same components (photodiodes, amplifiers and passive components) and are built using the same printed circuit board (PCB) layout. The PCB’s are directly fixed to the photodiodes, and thin 50\(\Omega\) coaxial cables of identical length transmit the signals to the summing amplifier whose input impedance is also 50\(\Omega\) to prevent reflections and eliminate phase differences due to these feeds. Even with this provision there can be small differences in the electronic phase shifts associated with each detector circuit, and so it is sensible to measure these in a real system. This is done by placing the detectors at equal distance from the beam-splitter, so that the optical path difference is zero. A measurement can then be made of any frequency-related relative phase shift due solely to the electronics, which can be removed from the signal during the experiment. For the detectors described here the electronic phase difference was found to be less than 10° over the full range of frequencies from 1 MHz to 50 MHz.

A further assumption is that the output from the laser diode remains constant with frequency, which again is not possible. These variations can be eliminated by adjusting the drive voltage from the oscillator while ensuring the modulation remains sinusoidal (as monitored by an oscilloscope connected to the output from each detector).
3. The experimental apparatus

Figure 1 is a block diagram of the apparatus, which consists of a laser diode and driver circuit, two photodiode detectors and associated amplification, a summing amplifier, peak detector and low-pass filter, and associated power supplies. The oscillator used here is a commercial system, which can be obtained from many different suppliers. This needs to output signals that can drive a 50 Ω load with an output ~1 V peak to peak from 1 MHz to 50 MHz. The feed from the oscillator must adopt 50 Ω coaxial cable to minimise reflections along this line.

The laser diode, electronics, photo-detectors, beam steering optics and beam expander are all enclosed in a die-cast box as shown in figure 3. A 5 mm thick steel plate is screwed to the die-cast box directly below the optics, which are then secured to this plate. This was found to be necessary to minimise fluctuations in the long beam path of the external laser beam. A beam expander is used to expand the laser beam at the output to a diameter of ~10 mm. This has two advantages. First as is well known, the angular divergence of a Gaussian laser beam is inversely proportional to the beam diameter, and so an expanded beam is better for input to the multi-pass cell. Secondly, since the intensity of the larger diameter beam is reduced by a factor of ~100 compared to the source beam from the laser diode, this is much safer for use in undergraduate student laboratories.

The output from the beam expander is directed into the multi-pass cell as shown in figure 4. Mirror 1 launches radiation onto delay mirror 1, which redirects the beam onto delay mirror 2. These mirrors are carefully adjusted to be parallel and are separated by a distance of ~1 m. The beam that enters the multi-pass cell undergoes multiple reflections before exiting. Mirror 2 redirects the exit beam onto mirror 3, which redirects the beam through the 750 mm focal length lens onto the delay photodiode.

The path length \( L_{\text{tot}} \) is hence given by the distance the beam travels from the beam splitter through the multi-pass cell and back to the detector, whereas \( d_1 \) is from the beam-splitter to the reference detector. In the experiment students are given the values \( d_1 \) and \( d_2 \) (which includes the effect of passing through the beam expander lenses to the output port) since they cannot access this information without opening the die-cast box. They then have to carefully measure (or deduce) \( d_1 \) to \( d_n \) (\( n = 20 \) in the example in figure 4) to calculate \( L_{\text{tot}} - d_1 = \left( d_2 - d_1 \right) + \sum_{i=3}^{n} d_i \). The uncertainty in their path-length measurements can then be propagated through equation (8) to determine the uncertainty in their calculation of the speed of light.

The 60 mm × 270 mm first-surface delay mirrors used in the multi-pass cell were sourced from Edmunds optics (part number 32-279) [9] as these are inexpensive for their large size, and have a flatness of \( 4 - 6 \lambda \) which is adequate here. The mirrors were bonded to adjusters so that they could be made parallel. All beam folding mirrors are 25.4 mm diameter \( \lambda / 4 \) mirrors mounted on individual adjusters, as are widely available. The 30:70 beam-splitter was sourced from Thorlabs (part number EBPI) [10], whereas the 5.6 mm 5 mW laser diode was obtained from Roithner Lasertechnik (part no. S6305MG) [11]. The 10X Galilean beam expander is constructed from a short focal length biconcave input lens and plano-convex output lens, and is mounted in a custom built holder. The input lens position is adjusted to produce a 10 mm diameter beam in the far field at least 20 meters from the output, and is then locked in place. Short focal length aspheric lenses are positioned in front of the AEPX65 photodiodes [12] to produce a tight focus onto the active region of the detectors. The position of these lenses was adjusted in the initial setup to maximise the detected signals. The photodiode housings were placed in position using lens holders.

The die-cast box was secured to the external bench using four 35 mm long 25.4 mm diameter posts, as shown in figure 3. Once the optical components inside the box had been adjusted to
Figure 3. Placement of optics and electronics inside the $250 \text{ mm} \times 250 \text{ mm}$ die-cast box. All optics are mounted from a 5 mm thick steel plate secured externally to the bottom of the box. The laser diode, folding mirrors and beam-splitter are on mirror mounts to steer the beam into the reference photodiode and beam expander, which is adjusted to produce a 10 mm diameter output beam. The photodiodes are placed on lens mounts. Light from the external multi-pass cell (see figure 4) is redirected back into the box and is focussed using an external lens onto the delayed photo-detector board. The electronics use separate boards for the laser diode and detectors. The input DC supply is directed to the power supply board PCB1 that allows each detector to be switched on and off. This board also directs the oscillator input to the laser diode via an input BNC connector. The photodiode monitor board PCB2 consists of 2 BNC sockets for connection to an oscilloscope with $50 \Omega$ input impedance. The mixer board PCB3 includes the summing amplifier, peak detector and low-pass filter whose output is directly connected to a panel mounted BNC as shown.

produce a well-collimated output beam that was parallel to the workbench and which also gave a good signal on the reference detector, the box lid was fastened down. Too much force on any of the lid fixing screws would misalign the beam and so care was required when securing the lid. The multi-pass delay mirrors were then set in a fixed position on the aluminium bench top, as were the external folding mirror mounts. In the experiment the students are required to adjust all external components so as to direct the beam into the multi-pass cell, produce as
Figure 4. The external optics, including the multi-pass cell and mirrors that redirect the beam back to the detector. A 750 mm focal length lens focuses returning light onto the photodiode. All components are mounted on an aluminium bench plate for optical stability.

4. The electronic systems

The main electronic components are the visible laser diode driver, the photodiode amplifiers, mixer and amplifier, peak detector and low-pass filter, and the power supply. Figure 5 is a schematic of the different circuits that are used. The power supply is located in a small die-cast box (box 2) remote from the apparatus as shown in the lower circuit. A 12 VA power pack supplies 15 VAC to box 2, and this delivers ±15 VDC and ±5 VDC through a panel-mounted connector on the main die-cast box. The required voltages are then distributed from the power supply board PCB1 to the different circuits. PCB1 also brings in signal from the oscillator and distributes this to the laser diode board using 0.3 mm OD 50Ω micro-coaxial cable. Switches on PCB1 allow the photodiode amplifiers to be switched on and off, so that the different signals (B1, B2, B1+2) can be measured.

The laser diode driver board is mounted from the laser diode, and uses all surface mount components. The LM7171 operational amplifier [13] mixes signal from the oscillator with the bias set by the 500Ω potentiometer and 220Ω resistor. The LM7171 acts as an attenuator with a gain of around 0.25, set by the 510Ω feedback resistor and 2.2 kΩ resistor. This is a very high-speed voltage feedback amplifier that can deliver up to 100 mA, and so is ideal for driving the laser diode that has a threshold around 25 mA (see figure 2). The laser diode anode is directly connected to 0 V and so this circuit sinks current from the diode through the 33Ω resistor. When the laser is operating normally the voltage drop across the diode remains essentially constant at around 2.2 VDC, and so to a good approximation the diode drive current through
the resistor is proportional to the voltage from the amplifier. The output intensity variation from the laser diode is hence proportional to the sinusoidal drive from the oscillator.

The photodiode boards use identical components so that their characteristics closely match. An AD8015 high-speed photodiode amplifier [14] is used to amplify signal from the AEPX65 photodiodes. This transimpedance amplifier has a set gain of 90, and provides separate differential outputs allowing signal delivery to the mixer board and for monitoring the signals via the BNC on monitor board PCB2. Both outputs have an impedance of 50\Ohm and so these feeds also use micro-coax between the different boards. The 470 nF capacitors decouple the DC component in the amplified signal, so that the output is symmetric around 0 V.

The mixer board uses an LM7171 unity gain differential amplifier to mix the signals from the two photodiode amplifiers, before feeding a second non-inverting LM7171 with a gain of \approx 1.25. The output from this stage feeds an LTC5507 peak detector [15] set to detect the positive modulation from the mixed signal. An OPA277 high precision amplifier [16] buffers and filters the peak detector signal before passing signal to the final OPA277 amplifier. This amplifier removes any residual DC components that arise from the peak detector, using the 2 k\Ohm trim potentiometer. The output from this stage is measured using an external digital voltmeter to produce \( B_1, B_2 \) and \( B_{1+2} \).

The high speed of the components used here necessitate careful decoupling of all power supplies, with the decoupling capacitors being placed as close as possible to each component. Surface mount capacitors are used throughout, with values as shown in figure 5. The layout of the printed circuit boards is also important to prevent parasitic oscillation at high frequencies, and so double-sided boards are used with carefully placed ground planes on both sides. This was particularly relevant for the photodiode amplifiers due to the very high 240 MHz bandwidth of the AD8015 amplifiers.
Figure 6. (a) Calculation of the speed of light from the data by non-linear least squares fitting a cosine function to the data using a Marquardt method. (b) Calculation of the speed of light by taking the inverse cosine of the data, allowing for the accumulated phase change as the frequency changes. This allows a linear fit to be carried out. The error bars are calculated by propagating the uncertainty in the measurements through the equations in the usual way. Variations in the individual signals were estimated as ±5 mV, which arises mostly due to vibrations that slightly move the laser beams as they illuminate the AEPX65 photodiodes.

5. Experimental results

Testing was carried out after alignment of the optics as described above, and the peak to peak signals \( B_1 \) and \( B_2 \) were set to be around 300–400 mV from each detector. This ensured that the peak detector was not driven into a non-linear regime. The path length difference \( (L_{\text{tot}} - d_1) \) was determined to be 18.248 m ± 0.020 m using a tape measure, the uncertainty arising due to accumulated errors in the addition of individual path lengths as in figure 4. Measurements of \( B_1, B_2 \) and \( B_{1+2} \) were then taken in 1 MHz steps from 1 MHz to 50 MHz. The value of \( \left( \frac{B_{1+2}^2 - B_1^2 - B_2^2}{2B_1B_2} \right) \) was then calculated (see figure 6(a)), and the inverse cosine determined as in figure 6(b). A non linear least squares Marquardt fit was used in figure 6(a) to calculate the speed of light directly, whereas a linear least squares fit to the slope was used for the data in figure 6(b). The small differences in the calculated light speed in each case arise from the different techniques used for fitting the functions.
In figure 6(a) the data are fitted to a function of the form

\[
\left( \frac{B_{2}^{3} - B_{1}^{3} + B_{2}^{3} - B_{2}^{3}}{2B_{1}B_{2}} \right) = \cos \left( \frac{2\pi n_{\text{air}} (L_{\text{tot}} - d_{1})}{c} \right) f_{M}
\]  

(9)

where the refractive index of air was set to \( n_{\text{air}} = 1.0002765 \) [3] and \((L_{\text{tot}} - d_{1}) = 18.248 \pm 0.020m\). The uncertainty in the speed was determined by fitting the function using different values of the path-length as set by the estimated uncertainty.

In figure 6(b) a linear least squares fit to the phase shift yielded a slope \( m = 0.3838 \pm 0.0003\), and so the speed was obtained using the expressions:

\[
c = \frac{2\pi n_{\text{air}} L_{\text{tot}}}{m}, \quad \sigma (c) = \pm \frac{2\pi n_{\text{air}} L_{\text{tot}}}{m} \left( \frac{\sigma^{2} (L_{\text{tot}})}{L_{\text{tot}}^{2}} + \frac{\sigma^{2} (m)}{m^{2}} \right)^{1/2}.
\]  

(10)

In both methods the speed of light is calculated to within 0.5% of the accepted value. The difference is probably due to a small measurement error in the \( z \) distance travelled by the delayed beam, since exact light speed would be obtained if \((L_{\text{tot}} - d_{1})\) was increased by 0.36% of the measured value. This discrepancy could be due to inaccuracies in measurement of the path lengths in the multi-pass cell, or due to inaccuracies in the tape measure that was used.

6. Conclusion

In this paper a method is described for determining the speed of light in university undergraduate laboratories that has high accuracy, and which uses an apparatus that occupies a small footprint. The method uses an amplitude-modulated laser beam, and determines the relative phase shift of the modulation as it propagates to a reference detector and a second detector after travelling a fixed distance between mirrors in a multi-pass cell. By sweeping the frequency of the modulation through a range of values, an accurate determination of the speed of light can be made.

Although the method appears similar to a Michelson interferometer, it does not use the same technique. In the present apparatus the beam travelling at the speed of light is the carrier of information encoded onto the beam using amplitude modulation. The laser hence does not require a long coherence length, as would be required for a Michelson interferometer that adopted similar path lengths as used here. As such, inexpensive laser diodes that can be easily modulated are used.

Four experiments are now operating in the first-year undergraduate physics laboratory in Manchester using this technique, allowing students to ascertain the speed of light with high accuracy. These replace older rotating mirror systems that required 6 m of lab bench for each experiment. These older experiments gave poor results and required students to look directly at the returning laser beam through an eyepiece, which is potentially hazardous. The new experiments detailed here expand the beam to reduce the intensity to class 2 levels, ensuring the experiments are safe within an undergraduate laboratory environment. The complete apparatus requires only a small footprint of \( \sim 1 \) m of bench space, can be constructed for relatively low cost and only requires a simple digital voltmeter to carry out the measurements.

Further details (CAD drawings, etc) are available from the author on request.

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