The influence of nondipolar magnetic field and neutron star precession on braking indices of radiopulsars

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ABSTRACT

Some radiopulsars have anomalous braking index values \( n = \Omega \dot{\Omega}/\Omega^2 \sim \pm(10^3-10^4) \). It is shown that such anomalous values may be related to nondipolar magnetic field. Precession of a neutron star leads to rotation (in the reference frame of the star) of its angular velocity \( \Omega \) around the direction of neutron star magnetic dipole \( m \) with an angular velocity \( \Omega_p \). This process may cause altering of electric current flowing through the inner gap and consequently the current losses on the time-scale of precession period \( T_p = 2\pi/\Omega_p \). It occurs because the electric current in the inner gap is determined by Goldreich-Julian charge density \( \rho_{GJ} = -\Omega \cdot B/(2\pi c) \), that depends on the angle between direction of small scale magnetic field and the angular velocity \( \Omega \).

Key words: stars: pulsar: general – stars: neutron.

1 INTRODUCTION

Radio pulsars have been discovered more than 40 years ago (Hewish et al. 1968) and at present many thousand papers are devoted to these objects. Despite profound progress in the understanding of physical processes in pulsar magnetospheres many important questions are still unclear. One of them is the values of pulsar braking indices. If a pulsar were just a simple magnet with dipolar magnetic momentum \( m \) rotating with angular velocity \( \Omega \), then the braking index \( n = \Omega \dot{\Omega}/\Omega^2 \) would be equal to 3. Taking into account evolution of angle \( \chi \) between vectors \( m \) and \( \Omega \) yields \( n = 3 + 2\cot \chi \) (Davis & Goldstein 1970; Beskin 2006). Observations of some pulsars are well within theoretical predictions. For example, the braking index of pulsar Crab is equal to 2.5 and pulsar Vela has \( n \approx 1.4 \) (Beskin 2006). So it seems that more sophisticated magnetospheric models will be able to remove this discrepancy, see for example (Melatos 1997; Timokhin 2005, 2006; Contopoulos & Spitkovsky 2006; Contopoulos 2007; Timokhin 2007a,b). However, many isolated pulsars have very large positive or negative braking indices up to \( |n| \sim 10^4 \) (Manchester et al. 2005). Sometimes such large values may be due to unobserved glitches or timing noise Alpar & Baykal (2006). For example, Alpar & Baykal (2006) show that all the observed negative braking indices may be associated with unresolved glitches which occurred between intervals of observations of the pulsars. But in some cases refined measurements of braking indices show that at least some pulsars have large and sometimes negative braking indices \( n \sim \pm(10 \sim 10^3) \) (e.g. Johnston & Galloway 1999). For example, the pulsars B0656+14 and B1915+13 have braking indices \( n \approx 14.1 \pm 1.4 \) and \( n \approx 36.08 \pm 0.48 \), correspondingly, while pulsar B2000+32 and B1719-37 have very large negative braking indices \( n \approx -226 \pm 4.5 \) and \( n \approx -183 \pm 10 \), correspondingly (Johnston & Galloway 1999). There are, however, some explanations of such braking index values. For example, the values like \( n \sim 1 \pm 10 \) may be related to nonstandard mechanisms of pulsar braking, like neutron star slowing down due to neutrino emission (Peng, Huang & Huang 1982) or due to interaction with circumpulsar disk (Menou, Perna & Hernquist 2001; Malov 2004; Chen & Li 2006), see Malov (2001) for a review of possible mechanisms and their comparison with observations. The large braking indices may be explained by rapid changes of magnetic field. Such changes may be caused by Hall-drift instabilities (Rheinhardt & Geppert 2002; Geppert & Rheinhardt 2002; Pons & Geppert 2007). Also the positive braking indices of old pulsars may be explained by relaxation of angular velocities of neutron star crust and superfluid core between two glitches (Alpar & Baykal 2006) or by frictional instabilities caused by core-crust coupling (Shibazaki & Hirano 1995). The large braking indices of some pulsars may be related to Tkachenko waves (Popov 2008).

In this paper we present a model based on the works of Beskin, Biryukov & Karpov (2006) and Contopoulos (2007), see also Biryukov et al. (2007), Urama, Link & Weisberg (2006), Pons & Geppert (2007), where it has been shown that large braking indices may be explained by the existence of some internal cyclic process in a pulsar.

2 THE TORQUE

There are many mechanisms, that may be responsible for pulsar braking (Beskin 2006; Malov 2001). Let us shortly describe two of them. The first was proposed even before pulsar discovery by Pacini (1967). According to this mechanism, the rotation energy \( E \)
and the rotation momentum $M$ of a neutron star are carried away by magnetic dipole radiation, that, of course, leads to a torque acting on the star. Because of the presence of currents and charges in pulsar magnetosphere, calculation of the strength of this torque is a highly complicated and, at present, uncompleted task, (see for example Timokhin 2006, 2007a; Spitkovsky 2006; Beskin 2006). However, the torque can be estimated within a pure vacuum model of pulsar magnetosphere, where magnetospheric currents and charges are absent (Deutsch 1955). In this case, the rotation torque $K_{\text{dp}}$ acting on a neutron star, can be written as (e.g. Davis & Goldstein 1970; Melatos 2000):

$$K_{\text{dp}} = K_0 \cdot \left[ e_m \cos \chi - e_{\Omega} + R_{\text{dp}} (e_{a} \times e_{m}) \right].$$  \hfill (1)

where $\Omega = \Omega_{\text{dp}}$ is the angular velocity of the star, $m = me_m$ is the magnetic dipole moment of the star, $e_\Omega$ and $e_m$ are unit vectors, directed, correspondingly, along $\Omega$ and $m$, $\chi$ is the angle between $e_\Omega$ and $e_m$.

In case of small rotation speed $\Omega a/c \ll 1$, where $a \approx 10$ km is the radius of the star, $K_0$ and $R_{\text{dp}}$ can be written as (Davis & Goldstein 1970):

$$K_0 = \frac{2}{3} m^2 \Omega^3 / c^3,$$ \hfill (2)

$$R_{\text{dp}} = \frac{3}{2} \frac{\xi}{\Omega a/c} \cos \chi.$$ \hfill (3)

In Davis & Goldstein (1970), Goldreich (1970) the coefficient $\xi$ is taken to be equal to 1. We suppose that the value $\xi = 3/5$ adopted in Melatos (2000) may be closer to reality, because it corresponds to absence of electric current sheet on the neutron star surface. Thus, we will further consider $\xi = 3/5$. Expression (1) is valid at any values of $\Omega a/c$, although coefficients $K_0$ and $R_{\text{dp}}$ slightly differ from their nonrelativistic values, defined by (2) and (3) (Melatos 2000).

The second mechanism (current losses) is due to the electric current $f$, that flows along open field lines (see for example Beskin, Gurevich & Istimon 1984; Beskin 2006; Beskin, Gurevich & Istimon 1993; Beskin & Nokhrina 2007). This current flows from the light cylinder, crosses the polar cap, intersects the star surface, and goes into deep crustal layers. Afterwards it begins to move back to the surface and then transforms into the backward current flowing along the separatrix between the open and closed field lines. It is worth to note that when the electric current travels inside the crust it sometimes flows across the magnetic field, that leads to a torque $K_{\text{cur}}$ acting on the star polar cap. The strength of this torque is calculated by Jones (1976) as:

$$K_{\text{cur}} = - K_0 \cdot \alpha \cdot e_m \cos \chi,$$ \hfill (4)

where parameter $\alpha(\Omega, \chi, \phi)$ characterizes the electric current value (Beskin & Nokhrina 2007; Beskin 2006). In the case of a circular pulsar tube cross-section the parameter can be estimated as (Beskin 2006; Beskin & Nokhrina 2007); Barsukov, Polyakova & Tsygan (2009):

$$\alpha = \frac{3}{4} \left[ \frac{j_s}{j_{\Omega}} \left( \frac{S_y(\eta)}{S_\Omega(\eta)} \right)^2 + \frac{j_s}{j_{\Omega}} \left( \frac{S_y(\eta)}{S_\Omega(\eta)} \right) \right],$$ \hfill (5)

where $j_s$ is the density of the electric current, that flows inside the northern pulsar tube, and $j_{\Omega}$ is the density of the electric current that flows inside the southern pulsar tube, $j_{\Omega} = \Omega B_0 \cos \chi/(2 \pi r)$ is the Goldreich-Julian current, $B_0 = 2 m/a^3$ is the magnetic field strength at the magnetic pole, $S_\Omega(\eta)$ and $S_y(\eta)$ are the areas of the northern and southern pulsar tubes, $S_y(\eta) = \pi a^2 (\Omega a/c) \eta^3$, $\eta = r/a$, $r$ is the distance from the center of the star.

Following Jones (1976), Xu & Qiao (2001), Wu, Xu & Gil (2003), we assume that the star can slow down by both mechanisms simultaneously and that the resulting torque $K$ can be described as a sum of partial torques $K_{\text{dp}}$ and $K_{\text{cur}}$:

$$K = K_{\text{dp}} + K_{\text{cur}}.$$ \hfill (6)

Thus, the equation of rotation momentum loss can be written as

$$\frac{dM}{dt} = K_{\text{dp}} + K_{\text{cur}}.$$ \hfill (7)

3 SPHERICAL SYMMETRY

At first we assume that a neutron star is an absolutely rigid sphere. Particularly, we neglect any star deformations and any viscosity and dissipation inside the star. Consequently, we can suppose that the star rotation momentum $M = I \Omega$, where $I$ is the momentum of inertia of the star. Under this assumption equation (7) can be rewritten as

$$\frac{d\Omega}{dt} = K = K_0 \cdot \left[ e_m \left( 1 - \alpha(\Omega, \chi, \phi) \right) \cos \chi - e_{\Omega} + R_{\text{dp}} (e_{a} \times e_{m}) \right].$$ \hfill (8)

Now let us introduce three orthogonal unit vectors $e_x$, $e_y$ and $e_z$ which rotate together with the star. Let $e_\Omega = e_m$, directions of vectors $e_x$ and $e_z$ may be arbitrary. Consequently, we have

$$\frac{de_x}{dt} = (\Omega \times e_x), \quad \frac{de_z}{dt} = (\Omega \times e_z), \quad \frac{de_{\Omega}}{dt} = (\Omega \times e_{\Omega}).$$ \hfill (9)

At any time $t$ the following relations are valid:

$$(e_x, e_z) = (e_x, e_z) = (e_x, e_z) = 0 \quad \text{and} \quad e_x^2 = e_z^2 = 1.$$ \hfill (10)

Hence, at any time $t$ we can treat these vectors as an orthogonal space basis. So it is possible to write

$$\Omega = \Omega \left( \sin \chi \cos \phi e_x + \sin \chi \sin \phi e_z + \cos \chi e_m \right).$$ \hfill (11)

Using the last expression, the equation (8) may be rewritten in the form

$$\frac{d\Omega}{dt} = - \frac{\Omega}{\tau} \cdot (\sin^2 \chi + \alpha \Omega, \chi, \phi) \cdot \cos^2 \chi \right) \hfill (12)

$$\frac{d\chi}{dt} = - \frac{1}{\tau} \cdot (1 - \alpha(\Omega, \chi, \phi)) \cdot \sin \chi \cos \chi \hfill (13)

$$\frac{d\phi}{dt} = - \frac{1}{\tau} R_{\text{dp}} = - \frac{2\pi}{\tau},$$ \hfill (14)

where

$$\tau = \frac{3}{2} \frac{I c^3}{m^2 \Omega^2} \approx 1.5 \cdot 10^4 \text{ years} \left( \frac{P}{1s} \right)^2 \frac{1}{B_{12}^2}.$$ \hfill (15)

$$T_p = \frac{2\pi}{R_{\text{dp}}} \approx 4\pi \frac{1}{\xi \cos \chi} \left( \frac{\Omega a}{c} \right) \cdot \frac{1}{\tau} \hfill (16)

\approx 1.2 \cdot 10^7 \text{ years} \left( \frac{1}{\xi \cos \chi} \right) \left( \frac{P}{1s} \right) \frac{1}{B_{12}^2},$$ \hfill (15)

where $B_{12} = B_0/10^{12}$ G. It is worth to note that equations (12–14) describe neutron star rotation in an inertial reference frame of ‘rigid stars’. But all vectors are represented by the basis $(e_x, e_y, e_z)$ altering in time.

It is easy to see that equation (12) describes the losses of neutron star rotation energy and equation (13) describes the evolution of the
inclusion angle $\chi$. In the case of $\alpha = 1$ the equation (12) may be rewritten as
\[
\frac{dP}{dt} = \frac{P}{\tau}.
\]
So in this case parameter $\tau$ is equal to two characteristic pulsar ages $\tau = P/(2P)$. For any other values of parameter $\alpha$ there is no so simple interpretation of the time $\tau$. But it is possible to treat the parameter $\tau$ just as some characteristic time over which large changes of the angular velocity $\Omega$ and the inclination angle $\chi$ occur. The equation (14) describes the precession of the neutron star and $T_p$ may be interpreted as precession period. It is worth to note that the period $T_p$ is 3–4 orders of magnitude smaller than the characteristic time $\tau$. So it seems that the neglecting of the altering of values $\Omega$ and $\chi$ times comparable with the precession period $T_p$ may be a good approximation. Consequently, if we assume $\alpha = \alpha(\Omega, \chi, \phi)$ it will be possible to write
\[
\frac{d\alpha}{dt} \approx \frac{\partial \alpha}{\partial \phi} \frac{d\phi}{dt}.
\]
In this case, in equation (12) only parameter $\alpha$ is able to change significantly over precession period and hence
\[
\frac{d^2\Omega}{dt^2} \approx -\frac{\Omega}{\tau} \cdot \frac{d\alpha}{dt} \cdot \frac{\cos^2 \chi}{(\sin^2 \chi + \alpha \cos^2 \chi)^2}.
\]
Thus, the braking index $n = \Omega \Omega / \dot{\Omega}^2$ can be estimated as (Good & Ng 1985)
\[
n \approx \frac{\Omega}{\Omega^2} \approx -\frac{\tau}{\tau} \cdot \frac{d\alpha}{dt} \cdot \frac{\cos^2 \chi}{(\sin^2 \chi + \alpha \cos^2 \chi)^2}.
\]
If we take into account the equation (14), expression (20) may be rewritten as
\[
n \approx R_{dp} \cdot \frac{\partial \alpha}{\partial \phi} \cdot \frac{\cos^2 \chi}{(\sin^2 \chi + \alpha \cos^2 \chi)^2}.
\]
If we assume that $\alpha \sim 1, \cos \chi \sim \sin \chi \sim 1$, then the braking index $n$ may be estimated as
\[
n \sim R_{dp} \cdot \frac{\partial \alpha}{\partial \phi}.
\]
If we take into account the equation (3) then the braking index estimation (21) takes the form
\[
n \approx \frac{3}{2 \cdot 5} \left( \frac{c}{\Omega a} \right) \cdot \frac{\partial \alpha}{\partial \phi} \cdot \frac{\cos^2 \chi}{(\sin^2 \chi + \alpha \cos^2 \chi)^2} \sim \left( \frac{c}{\Omega a} \right) \cdot \frac{\partial \alpha}{\partial \phi} \cdot 10^4 \left( \frac{1}{P} \right) \cdot \frac{\partial \alpha}{\partial \phi}.
\]
Hence, the braking index $n$ is positive when the electric current $j$ decreases and when the current $j$ increases it becomes negative. It is necessary to note that within this paper we assume that the magnetic dipole $m$ does not change significantly over the precession period $T_p$. Particularly, the expression (23) is valid only if the time corresponding to large changes of the magnetic dipole $m$ is large enough compared with the period $T_p$.

Now following Goldreich (1970), Barsukov et al. (2009) we define the value of function $f(\Omega, \chi, \phi)$ averaged over precession time as
\[
\langle f(\Omega, \chi, \phi) \rangle = \frac{1}{2\pi} \int_0^{2\pi} f(\Omega, \chi, \phi) \phi d\phi.
\]
Then, the averaged equations (12) and (13) may be written as
\[
\langle \frac{d\Omega}{dt} \rangle = -\frac{\Omega}{\tau} \left( \sin^2 \chi + (\alpha \cos^2 \chi) \right)
\]
It is easy to see that case $\langle \alpha \rangle = 1$ corresponds to equilibrium value of the inclination angle, when $\langle d\chi / dt \rangle = 0$ (Goldreich 1970; Gurevich & Istomin 2007; Istomin & Shabanova 2007). In this case the average rate of angular velocity decrease also does not depend on the inclination angle:
\[
\langle \frac{d\Omega}{dt} \rangle = -\frac{\Omega}{\tau}.
\]
The last equation may correspond to the results of Beskin et al. (2006), Biryukov et al. (2007) where it is shown that an average braking index of many pulsars is close to $n \approx 5$. It is worth to note that if the parameter $\langle \alpha \rangle$ decreases with increasing angle $\chi$, this equilibrium state is stable, (see Barsukov et al. 2009, for details).

4 NonDipolar Magnetic Field

It is widely accepted that besides large-scale dipolar magnetic field, small scale nondipolar magnetic field may exist near neutron star surface. This field has a spatial scale about $l \sim (0.3–3)$ km and rapidly falls to zero when the distance from the neutron star surface increases to become fully negligible at the light cylinder. This allows to suppose that nondipolar component does not exert any direct influence on magnetic dipole braking, that appears to occur near the light cylinder. It seems that the same reasons are applicable to the direct influence of the small scale component on current losses (Barsukov et al. 2009). However, the existence of the small scale component may lead to drastic changes of the electric current that flows through the inner ‘gaps’ of the pulsar (Shibata 1991). Thus, the indirect influence of the nondipolar field on current losses and torque $K_{\text{nu}}$ is supposed to play a decisive role in pulsar braking. Here, following Shibata (1991), by inner ‘gap’ we mean not only a true vacuum gap but also an arbitrary pulsar diode situated nearby the star surface. We only assume that a such diode must have an upper boundary created by electron-positron plasma that is necessary to generate the radio emission.

The influence of small scale magnetic field upon electric currents in the pulsar magnetosphere was considered in many papers (Arons & Scharlemann 1979; Gil & Mitra 2001; Gil, Melikidze & Geppert 2003; Gil, Melikidze & Zhang 2006a,b). In order to estimate its influence on current losses we will use a simple model, proposed by Palshin & Tsygan (1998) and used also by Kantor & Tsygan (2003), Barsukov et al. (2009). According to this model the inner ‘gaps’ fully occupy the pulsar tubes cross-sections and are situated close to the neutron star surface. It is assumed that magnetic field strength on the star surface (and, hence, work function) is not large enough to prevent free emission of electrons, so when the pulsar diode is placed on to the star surface it will operate in electron charge limited steady flow regime. In the neighbourhood of the inner ‘gap’ small scale magnetic field will be modelled by the field of a small magnetic dipole $m_1$, which is embedded into the neutron star crust under the magnetic (dipolar) pole at the depth $\Delta \cdot a$ (see Fig. 1 and Fig. 2). We assume that $\Delta \approx 1/10$ and that $m_1$ is perpendicular to the main dipole $m$. So the vector $m_1$ may be written in the form
\[
m_1 = -2m v \Delta^3 (e_1 \cos \gamma + e_2 \sin \gamma),
\]
where $v = B_1 / B_0$ is the ratio of the small scale magnetic field strength $B_1 = m_1 / (\Delta \cdot a^3)$ to the strength $B_0 = 2m / a^3$ of the large scale dipolar magnetic field at the magnetic (dipolar) pole. We also assume that the changes of values $v, \Delta$ and $\gamma$ as well as the value...
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In the case of a thin pulsar tube $R_t \ll \Delta \cdot a$, where $R_t$ is its radius, and at small altitudes $\eta = r/a \ll \kappa \sqrt{\Delta \Omega_1} \sim 10^2$, the Goldreich-Julian density (inside the pulsar tube) may be written as

$$\rho_{GJ} = -\frac{\Omega B}{2\pi c} f(\eta).$$

At $v \lesssim 1/3$ the function $f(\eta)$ may be approximated as (Palshin & Tsygan 1998):

$$f(\eta) = \frac{1}{\sqrt{1+\kappa^2}} \left[ \left(1 - \frac{\kappa}{\eta}\right) \cos \chi \right. \right.$$  

$$+ \left. \lambda \left(1 + \frac{\kappa}{2 \eta^2}\right) \sin \chi \cos(\phi - \gamma) \right].$$

where the coefficient $\kappa \approx 0.15$ describes general relativistic frame dragging (Muslimov & Tsygan 1992), $B$ – magnetic field strength, $\lambda = v (\Delta \eta) / (\eta - 1 + \Delta^3)$. In the case of $k = 0$ the function $f(\eta)$ is equal to the cosine of the angle between the direction of magnetic field $\mathbf{B}$ and angular velocity $\Omega$.

In the case of a thin long inner ‘gap’, when the radius $R_t$ of the pulsar tube is small compared with the gap height $z_c a$ (see Fig. 1), the density $j$ of electric current flowing through the inner ‘gap’ may be written as (Barsukov et al. 2009):

$$j \approx \frac{j_\Omega^G}{\eta_0} f(\eta_0).$$

where $\eta_0$ is the altitude where the inner ‘gap’ begins and $j_\Omega^G = \Omega B(\eta_0)/2\pi$. Following Barsukov et al. (2009) we assume that the electric potential $\Phi$ monotonically increases inside the pulsar diode and the diode is placed as close to the surface as possible.

Hence, using expression (30) one may write at $\cos(\phi - \gamma) < 0$:

$$j(\nu, \gamma) \approx j_\Omega^G \cdot \max(0, f(1)).$$

When $f(1) > 0$ the inner ‘gap’ is placed on the star surface $\eta_0 = 1$ and operates in the Arons-Scharlemann regime like normal polar cap pulsar diode (Scharlemann, Arons & Fawley 1978). When $f(1) < 0$ the inner ‘gap’ begins at an altitude $\eta_0 > 1$ where $f(\eta_0) \approx 0$ and operates in the Ruderman-Sutherland regime (Ruderman & Sutherland 1975), see also Barsukov et al. (2009) for details. In the last case we assume that the inner ‘gap’ is a true vacuum gap with zero electric current. In the case of $\cos(\phi - \gamma) > 0$ we assume that

$$j(\nu, \gamma) \approx j_\Omega^G \cdot \min(1, f(\eta), 1 \leq \eta \leq +\infty).$$

It means that the inner ‘gap’ begins at an altitude $\eta_0 \ll (c/\Omega a)$ where $\eta_0$ is a point where the function $f(\eta)$ achieves its minimum value. In this case the diode operates in Arons-Scharlemann regime (see Barsukov et al. 2009, for details). A similar configuration of pulsar diode is described by Shibata (1997). There are only two differences: in the present model the Goldreich-Julian density $\rho_{GJ}$ does not change sign and the potential $\Phi$ of the ‘last’ oscillation (pulsar diode) is supported only by the $\mathbf{V} \cdot \mathbf{E}_1 \sim \Phi/R_1^2$ term in the Poisson equation.

Suppose that the small scale magnetic field near the northern inner ‘gap’ may be described by a small dipole with parameters $\nu = \nu_N$ and $\gamma = \gamma_N$, and the small scale magnetic field in a neighbourhood of southern inner ‘gap’ (and, of course, inside it) may be described by a small dipole with $\nu = \nu_S$ and $\gamma = \gamma_S$. Then, the parameter $\alpha$ may be written as

$$\alpha = \frac{3}{4} \left[ j(\nu_N, \gamma_N) S_N(\eta) \right]^2 + \left[ j(\nu_S, \gamma_S) S_S(\eta) \right]^2,$$

where $S_N$ and $S_S$ are cross-section areas of the northern and southern pulsar tube.

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**Figure 1.** The position of the small dipole $\mathbf{m}_1$ is shown in the case of $\gamma = \Pi$ and $\phi = 0$. The gray square is the neutron star and the curved dark area is the pulsar diode. The pulsar diode is situated on star surface $\eta_0 = 1$, $R_t$ is the radius of the pulsar tube and $z_c a$ is the height of the diode.

**Figure 2.** The orientation of vectors $\Omega$, $\mathbf{m}$ and $\mathbf{m}_1$ in the case of $\gamma = \Pi$. Of magnetic dipole moment $\mathbf{m}$ are negligible, at least, over a period $T_{\nu}$ of neutron star precession. A similar model has been proposed by Gil, Melikidze & Mitra (2002a), where a small dipole $\mathbf{m}_1$ is placed not strictly under the magnetic pole and may have arbitrary direction. The case of pure axisymmetrical small scale magnetic field has been investigated by Gil et al. (2002a), Gil, Melikidze & Mitra (2002b), Asseo & Khechinashvili (2002), Tsygan (2000). It is worth to note that the field of dipole $\mathbf{m}_1$ is able to describe small scale magnetic field only in a small vicinity of the pulsar diode.

We do not have any intention to suppose that this dipole is able to describe small scale magnetic component over whole neutron star surface. We rather suppose that the small scale field is close to a sum of $10^2 - 10^6$ such small dipoles and that we use only one of them, which is the closest to the pulsar diode.
The value of $\alpha$ averaged over precession period $T_p$ may be written as

$$\langle \alpha \rangle = \frac{3}{2} \left( 1 - \kappa - \frac{v_N + v_S}{2\pi} \tan \chi \right). \quad (37)$$

Thus, the equilibrium angle $\chi$ is equal to

$$\chi_{eq} = \arctan \left[ \frac{2\pi}{v_N + v_S} \left( \frac{1}{3} - \kappa \right) \right]. \quad (38)$$

At this angle the expression (36) may be rewritten as

$$\alpha = \frac{3}{2} \left[ 1 - \kappa - \frac{\pi}{3} (1 - 3\chi) \times \right.$$

$$\times \left. \left( \frac{v_N}{v_N + v_S} \cos (\phi - \gamma_N) \Theta (\cos (\phi - \gamma_N)) \right) + \frac{v_S}{v_N + v_S} \cos (\phi - \gamma_S) \Theta (\cos (\phi - \gamma_S)) \right] \right] \quad (39)$$

and in the case of $v_N = v_S$ and $\gamma_N = \gamma_S = 0$ it may be estimated as

$$\alpha \approx \frac{3}{2} \left( 1 - \frac{1}{2} \cos \phi \cdot \Theta (\cos \phi) \right). \quad (40)$$

This expression shows that at the equilibrium value of angle $\chi$ parameter $\alpha$ is changing from $\approx \frac{1}{2}$ to $\approx \frac{1}{3}$ or, in other words, is changing in two times over the precession period. Consequently, the braking index $n$ may be crudely estimated as $n \sim 2\pi T_p \sim 10^3$.

Now let us discuss a more sophisticated case when the pulsar tube cross-section $S_t$ depends on angle $\chi$. We will use the dependence that was derived in Biggs (1990):

$$S_t(\eta) \approx S_0(\eta) g(\chi). \quad (41)$$

where

$$g(\chi) = \left[ \frac{(1 - \frac{\pi}{3})^{1/4}}{1 + \mu} \right] \quad (42)$$

and $\mu = 1 + \frac{1}{2} \cos^2 \chi - | \cos \chi | \sqrt{2 + \frac{1}{2} \cos^2 \chi}$. When $\chi$ increases from 0 to $\pi/2$ the value $\mu$ changes from 0 to 1. At $\chi = 0^\circ$ the coefficient $g(\chi)$ is equal to 1 and at $\chi = 90^\circ$

$$g \left( \frac{\pi}{2} \right) = \left( \frac{4}{27} \right)^{1/2} \approx 0.620, \quad (43)$$

so the area of pulsar tube cross-section decreases with increasing angle $\chi$. Some other dependencies of pulsar tube areas on the angle $\chi$ may be calculated, for example, by Dyks, Harding & Rudak (2004), Muslimov & Harding (2009), Beskin (2006). Such a dependence may be either decreasing or increasing, for example, in Beskin (2006) it is shown that the area of a pulsar tube increases with $\chi$ from 1.59 $S_0$ at $\chi = 0^\circ$ up to 1.96 $S_0$ at $\chi = 90^\circ$.

In the case of $v_N = v_S = v$ and $S_N = S_S = S_0$ the resulting braking indices $n$ for angle $\chi$ are shown in Figs 5 and 6. Braking indices at equilibrium values of the angle $\chi$ are shown in Figs 9 and 10.

### 5 AN AXISYMMETRIC CASE

In this section we will suppose that a neutron star is an absolutely rigid axisymmetric body and that its symmetry axis coincides with magnetic dipole $\mathbf{m}$. The rotation momentum of the star may be written as

$$M = \sum_{\beta=n} I_{\beta} \cdot (e_{\beta} \cdot \Omega) = I_\perp \Omega - \Delta I e_m (e_m \cdot \Omega). \quad (44)$$
where $I_\perp = I_\perp$ and $I_\perp = I_\perp = I_\perp$ are momenta of inertia of the star. Hence, the equation (7) of momentum loss takes the form:

$$I_\perp \frac{d\omega}{dt} = K_{\text{eff}} = \frac{K_0}{I_\perp} \left[ (\sin^2 \chi + \alpha \cos^2 \chi) \right] \cos \chi,$$

where $K_{\text{eff}}$ is an effective rotation torque acting on the star

$$K_{\text{eff}} = K_0 \left[ (1 - \alpha(\Omega, \chi, \phi)) \cos \chi - e_\Omega + R_{\text{eff}}(e_\Omega \times e_m) \right].$$

Here we neglect the small term $\Delta I e_m(e_m \cdot e_m) = (\Delta I/I_\perp) e_m e_m - K_0$ and introduce the coefficient

$$R_{\text{eff}} = \frac{\Delta \omega}{\Delta t} + \frac{3\xi}{2} \left( \frac{c}{\Omega_0} \right) \cos \chi = 3 \cdot 10^3 \left( \frac{P}{1 \text{s}} \right) \left( \frac{\xi}{4} + \frac{\Delta I}{B_{12}} \right) \cos \chi,$$

where $\Delta I = \Delta I/10^{13}$ g cm$^2$.

It is easy to see that the only difference between equations (8) and (45) is the replacing of momentum of inertia $I$ by its component $I_\perp$ and the replacing of the coefficient $R_{\text{dp}}$ by the coefficient $R_{\text{eff}}$. Consequently, up to these two changes, all the formulas of previous section remain applicable to the axisymmetrical neutron star. Particularly, the braking index $n$ may be estimated as

$$n \approx R_{\text{eff}} \left[ \frac{\Delta \alpha}{\Delta \phi} \right] \cos^2 \chi \approx R_{\text{eff}} \frac{\Delta \alpha}{\Delta \phi}.$$

Again, supposing that $\alpha \sim 1$, $\cos \chi \sim \sin \chi \sim 1$, one can crudely estimate the braking index as

$$n \approx R_{\text{eff}} \sin \phi \sim 3 \cdot 10^4 \left( \frac{P}{1 \text{s}} \right) \left( \frac{\xi}{4} + \frac{\Delta I}{B_{12}} \right) \sin \phi \sim \frac{\tau}{250 \text{ years}} \left( \frac{\Delta I}{10^{-11} \text{I}} \right) I_{45} \left( \frac{18^2}{\frac{P}{1 \text{s}}} \right) \sin \phi.$$

where $\tau = P/(2\dot{P})$ is a characteristic pulsar age and $I \approx I_\perp$ is the moment of inertia of an undeformed (spherical) neutron star, $I_{45} = I/10^{45}$ g cm$^2$.\hfill

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**Figure 5.** The same as Fig. 3, but $S_N = S_S = S_0 \sin^2 \chi$.

**Figure 6.** The same as Fig. 5, but $\gamma_N = 0$ and $\gamma_S = \frac{3\pi}{4}$.

**Figure 7.** The dependence of braking index $n$ on precession phase $\phi$. The changing of phase $\phi$ from $-\pi$ to $\pi$ corresponds to one precession period and takes $T_p$ seconds. It is assumed that $S_N = S_S = S_0$, $\nu_N = \nu_S = \nu$, $\gamma_N = \gamma_S = 0$, $\chi = \chi_{eq}$, $P = 1 \text{s}$ and the neutron star is spherical. Equilibrium angles $\chi_{eq}$ are equal to $\chi_{eq} \approx 81^\circ$ at $\nu = 0.1$, $\chi_{eq} \approx 65^\circ$ at $\nu = 0.3$, $\chi_{eq} \approx 49^\circ$ at $\nu = 0.5$ and $\chi_{eq} \approx 33^\circ$ at $\nu = 0.7$.

**Figure 8.** The same as Fig. 7, but $\gamma_N = 0$ and $\gamma_S = \frac{3\pi}{4}$. Values of equilibrium angles $\chi_{eq}$ are the same.
The braking indices of radiopulsars

Some examples of dependence of braking index $n$ on the precession phase $\phi$ are shown in Fig. 11–13. This dependence may be found by calculating the braking index $n_{spher}$ for a spherically symmetrical neutron star and then multiplying it by $R_{\text{eff}}/R_{\text{dip}}$:

$$n = \frac{R_{\text{eff}}}{R_{\text{dip}}} n_{spher}. \quad (50)$$

The axisymmetrical deformations of the neutron stars may be caused by internal magnetic field that resides inside neutron star crust or inside its core (Goldreich 1970). The deformation caused by internal magnetic field may be estimated as

$$\Delta I \sim -\zeta B_{\text{fl}}^2 a^2 / GM^2, \quad (51)$$

where $B_{\text{fl}}$ is the strength of the internal magnetic field and $M$ is the mass of the star. The coefficient $\zeta$ depends on the magnetic field profile. In the case of dipolar magnetic field $\zeta$ may be estimated as $\zeta = 25/8$ (Ferraro 1954), the same value is used by Goldreich (1970). For other configurations it may be, for example, as small as $\zeta = 1/18$ (Haskell et al. 2008). If we assume that $B_{\text{fl}} = B_0 = 2m/a^3$, then the coefficient $R_{\text{eff}}$ can be estimated as

$$R_{\text{eff}} \approx \frac{3}{2} \left( \frac{c}{\Omega a} \right) \left( \frac{\xi - 8\zeta a}{r_g / Ma^2} \right) \cos \chi \approx 3 \cdot 10^4 \left( \frac{1s}{P} \right) \left( \frac{\xi - 12\zeta a}{3r_g I_{54}} \right) \cos \chi, \quad (52)$$

where $r_g = 2GM/c^2$ is the gravitational radius of the neutron star.

First, it is easy to see that in the case of $\zeta \sim 1/18$ the braking indices do not change significantly. Secondly, if $\zeta \gtrsim 1$ then the braking indices will be $\sim 20 \cdot \zeta$ times more than in the spherically-symmetrical case and, particularly, at the value $\zeta = 25/8$ used by Goldreich (1970) the braking index could become as large as $n \sim 10^5 - 10^6$.

In some cases the neutron star interiors may be superconductive (Baym, Pethick & Pines 1969). If the neutron star matter is a superconductor of the second type, the internal magnetic field is able to form magnetic flux tubes. In this case the coefficient $\zeta$ increases substantially as $\zeta \sim B_{\text{fl}}/B_{\text{in}} \sim 10^3$, where $B_{\text{fl}} \sim 10^{15}$ G is the strength of magnetic field inside magnetic flux tubes and $B_{\text{in}}$ is average strength of internal magnetic field (Akgun & Wasserman 1985).
2008). This leads to $B_{fl}/B_{in} \sim 10^3$ time larger deformation of the star and, consequently, to very short precession period

$$P_p \sim \frac{50}{\cos \chi} \left( \frac{P}{s} \right) \text{years.}$$

(53)

Hence, the coefficient $R_{eff}$ increases significantly and leads to the increasing of braking index, that can reach $|n| \sim 10^{-7} - 10^{-8}$. As the absolute majority of normal isolated radio pulsars do not have such large braking indices it may be possible to conclude that there is no superconductivity of the second type in neutron star interiors.

The dependence of

$$f = \frac{2}{3} n \left( \frac{\Omega_a}{c} \right)$$

on characteristic pulsar age $\tau$ is shown in Fig. 14. The pulsar data is taken from ATNF Pulsar Catalogue (Manchester et al. 2005). If the estimation (49) of the braking index and the estimation (52) of neutron star deformation were valid then value $f$ would depend only on parameter $\zeta$. The solid line shows the estimation of value $f$ at $\zeta = 25/8$ and dashed line corresponds to the case of $\zeta = 1/18$. As the estimation (49) yields only the upper limit of possible values of braking indices the observation data are in good agreement with the case of $\zeta = 25/8$. Although the part of pulsar data does not contradict the case of $\zeta = 1/18, 10$.

In Fig. 15 the dependence of value $f$ on dipolar magnetic field $B_0$ is shown. The pulsar data are taken from ATNF Pulsar Catalogue (Manchester et al. 2005). The horizontal lines correspond to constant values of parameter $\zeta$. The solid line corresponds to $\zeta = 25/8$ and the dashed line corresponds to $\zeta = 1/18$. The two dot-dashed lines correspond to the case of $\zeta = (25/8) \cdot (B_{fl}/B_0)$ (upper line) and $\zeta = (1/18) \cdot (B_{fl}/B_0)$ (lower line), $B_0 = 10^{15} \text{G}$. The last two lines correspond to the case of neutron star deformations caused by magnetic flux tubes and the average internal magnetic field $B_{in}$ is equal to $B_0$.

The formula (49) allows to estimate the lower limit of neutron star deformation $\Delta I/I$. The corresponding values are shown in Fig. 16. It shows that the braking indices of majority of pulsars may be explained by the existence of deformation $\Delta I/I \sim 10^{-13} - 10^{-11}$ that agrees with estimation of $\Delta I$ obtained by Goldreich (1970).
6 CONCLUSION

In this paper we present an explanation of large values of braking indices of pulsars. The proposed model is based on four main assumptions:

(i) The neutron stars have small scale magnetic fields. The strength of these fields must be large enough to curve pulsar tube but small enough to allow free emission of electrons from the star surface.

(ii) There are inner ‘gaps’ in pulsar tubes and the electric current flowing across the inner ‘gaps’ depends on the angle between the small scale magnetic field and the angular velocity $\Omega$ of the star.

(iii) The braking torque depends on the current that flows across the inner ‘gaps’.

(iv) The neutron stars precess with periods about $10^3 - 10^4$ years.

In this paper we assume that the spatial scale $\ell$ of the small scale magnetic field is about 1 km. Also it is assumed that it has enough strength $B_1$ to curve the pulsar tube. For example, in the case of $B_1 \lesssim 10^{-2} B_0$ the small scale field is not able to curve pulsar tube very much and, consequently, the pulsar tube is only slightly different from the pure dipolar case $B_1 = 0$. In the present model in the case of a spherical star the strength $B_1 \gtrsim 0.1 B_0$ is necessary to obtain braking indices $n \sim 10 - 10^2$ and in order to obtain braking indices $n \sim 10^2$ it is necessary to assume $B_1 \sim B_0$. However the magnetic field must be weak enough to allow free electron emission from polar cap surface. This limit, of course, depends on the type of surface and its temperature $T$ (see, for example, Medin & Lai 2007). In the case of ‘middle age’ $\tau \sim 10^5$ years old pulsars the temperature of neutron star surface may be as high as $T \sim (0.5 - 1) \times 10^6$ K (Yakovlev & Pethick 2004). In the case of $\tau \sim 10^6$ years old radiopulsars the polar caps may be heated by backflowing positrons up to $T \sim (1 - 3) \times 10^6$ K (see, for example, Gil et al. 2008). So the upper limit of magnetic field strength can be estimated as $B_1 + B_0 \lesssim (1 - 3) \times 10^{13}$ G for an iron surface and $B_1 + B_0 \lesssim 10^{14}$ G for a He atmosphere (Medin & Lai 2007). There are some observational evidences that small scale magnetic field may be as strong as $B_1 \sim 3 \times 10^{13} - 10^{15}$ G (see, for example, Sanwal et al. 2002). Although, in the case of $\tau \sim 10^6$ years old radiopulsars small scale magnetic field may be already as weak as $B_1 \sim (2 - 5) \times 10^{15}$ G due to ohmic decay in the crust (see, for example, Urpin & Gil 2004). So if we assume that $B_1 \sim (0.1 - 2) \times 10^{15}$ G the present model may be applicable to old radiopulsars with dipolar magnetic field $B_0 \lesssim 3 \times 10^{15}$ G and polar cap temperature $T \sim (1 - 3) \times 10^6$ K. In this paper we also neglect the contribution of the outer gaps. If a part of electric current flows through the outer gaps then this part is determined only by the outer gap electrodynamics and, consequently, does not depend on small scale magnetic field. It decreases the variation of current losses over the precession period and, consequently, leads to decreasing of the braking index. In the case of young pulsars like Crab and Vela, which have small braking indices, we suppose that current losses are almost fully determined by currents flowing through the outer gaps and the contribution of the inner ‘gaps’ current is negligible. In this paper it is also assumed that magnetic dipole braking exists and does not depend on the electric current flowing across the inner ‘gaps’. This assumption is widely used, cf. Eliseeva, Popov & Beskin (2006), Yue, Xu & Zhu (2007), Gurevich & Istimon (2007), Istimon & Shabanova (2007), but, as mentioned by Beskin (2006), it must be treated with caution. It is shown that in the case of force free magnetosphere and absence of electric current flowing along the pulsar tube a orthogonal pulsar ($\chi = \pi/2$) does not slow down at all (Beskin, Gurevich & Istimon 1983; Beskin et al. 1984; Mestel, Panagi & Shibata 1999). It gives the reason to suppose that magnetic dipole braking does not exist or, at least, must depend on the electric current (Beskin 2006). In such a case the presented model can provide the only a qualitative explanation of the existence of large braking indices. And the quantitative estimations, of course, will strongly depend on the relation between magnetic dipole braking and current losses torques.

The presence of a long period precession is the weakest point of the model. At present the precession is discovered only in a few isolated neutron stars. And these pulsars have the precession periods like $T_p \sim 1-10$ years (Link 2007; Haberl 2007). The presence of pinned superfluid in neutron star crust substantially increases precession speed (Shaham 1977). The precession periods $T_p$ larger than $(10^2-10^4) \times P$ may exist only when vortices of superfluid can not be pinned anywhere throughout the star, are able to move freely and do not coexist with magnetic flux tubes (Alpar & Ogelman 1987; Link 2006, 2007). The small number of isolated pulsars with observed precession force us to exclude the triaxial precession and to assume that neutron star is axisymmetrical and symmetry axis coincides with magnetic dipole $m$. In this case, in the reference frame related to ‘rigid stars’ vector $m$ rotates with a constant angular velocity $\omega$. And because $\omega \approx \Omega$ its trajectory coincides with the case of unprecessing star, see Appendix A. Also, in order to prevent the observation of such precession it is necessary to assume that pulsar tube structure does not precess too. It particularly means that pulsar tube crossection is circular or depends only on vectors $\Omega$ and $m$ and does not depend on small scale magnetic field. Also, it means that distributions of energy of primary electrons and pair multiplicity over pulsar tube crossection are close to axisymmetrical.

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\[ \chi = R \] and, consequently, 
\[ \cos \beta = \arctan (\alpha) \] 
and calculate 
\[ \times \beta e/\Omega_1 \sin \omega_\chi \]
its time derivative 
\[ \frac{d \omega}{d t} = - (\sin \beta \omega_\chi + \cos \beta \omega_m) \frac{d \beta}{d t} \]
\[ + \left( \frac{R_e}{\tau} \cos \beta - \sin \beta \right) (\omega_\chi \times \omega_m) = 0. \]
Thus, in order to make vector \( \omega \) constant in time, it is enough to choose the angle \( \beta \) as 
\[ \beta = - \arctan \left( \frac{R_e}{\Omega \tau} \right). \]
In this case the first of equations (9) may be rewritten as 
\[ \frac{d \omega}{d t} = \Omega \cos \beta (\omega \times \omega_m) \]
It means that \( \omega \equiv e_\chi \) and, consequently, \( m \) just rotate with the constant angular velocity \( \omega = \omega_\chi / \cos \beta \) around the constant vector \( \omega \) (see for example Link 2003). As \( \Omega = \omega + \omega_\chi \tan(\beta) \), \( \Omega \) would also rotate with the angular velocity \( \omega_\chi \) around the constant direction \( e_\chi \).
It is worth to note that the angle \( \beta \) is very small: 
\[ \beta = \arctan \left( \frac{m^2}{T \omega_\chi^2} + \Delta I / I \right) \cos \chi \]
\[ \sim 10^{-12} \left( \frac{\xi B_{12}^2}{4 L_5} + \frac{\Delta I}{4 I \chi} \right) \cos \chi. \]

APPENDIX A: PRECESSION OF AN AXISYMMETRICAL STAR

Consider the neutron star rotation in an extreme case when it is possible to neglect the first and second terms in expression (46) which contain vectors \( e_\omega \) and \( e_\chi \) compared with the last term which contains \( (e_\chi \times e_\omega) \). Particularly, it means that any rotation energy losses are neglected and it is assumed that the angular velocity \( \Omega \) and the inclination angle \( \chi \) do not change with time. Consequently, this approximation is valid only over time-scales that are small compared with the characteristic time \( \tau \) or pulsar age \( \tau \), although this time-scales may be comparable with or larger than the precession period \( T_\omega \). In this case the equation (45) may be written as 
\[ \frac{d \Omega}{d t} = \frac{1}{I} R_e (\chi \times \omega_m). \]
where
\[ \Omega_p = -e_m \Omega \tan(\beta) \]  \hspace{1cm} (A6)

Take, for example, any vector \( e_\alpha \) rotating together with the star as (see Fig. A1):
\[ \frac{d e_\alpha}{d t} = (\Omega \times e_\alpha) \]  \hspace{1cm} (A7)

Hence in the reference frame \( K_\omega \) it is possible to write
\[ \frac{d e_\alpha}{d t} = (\Omega \times e_\alpha) - (\omega \times e_\alpha) = - (\Omega_p \times e_\alpha). \]  \hspace{1cm} (A8)

As radio waves may be generated at some distance from the center of the pulsar tube, the precession may be manifested as subpulses drift or variation of pulse profile (Asseo & Khechinashvili 2002; Ruderman & Gil 2006). In order to exclude such manifestations it is necessary to assume that average distribution of radio sources in the pulsar tube is axisymmetrical. Also, one needs to assume that pulsar tube crosssection is circular or, at least, its profile depends on vectors \( \Omega \) and \( m \) and does not depend on small scale magnetic field. The model of pulsar tube presented by Biggs (1990) satisfies these criteria.