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Robust Decentralized Supervisory Control in a Leader-Follower Configuration with Obstacle Avoidance

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Abstract: This paper presents a decentralized solution to control a leader-follower formation of unicycle wheeled mobile robots allowing collision and obstacle avoidance. It is assumed that only positions and orientations of the robots can be measured and that each robot is influenced by an additive input disturbance. To guarantee the problem solution a supervisory control algorithm is designed and finite-time differentiators are used to estimate the leader velocity. The supervisor orchestrates three control algorithms responsible for the following, rendezvous and collision avoidance manoeuvres. All controls ensure a finite-time regulation for the robots orientation and a practically finite-time fulfillment of the required performance constraints.

1. INTRODUCTION

The control of multi-agent systems has been thoroughly investigated in the last few years (Lewis (2014)). One of the first work in the field can be retrieved in Reynolds (1987) in which a model to simulate consensus is derived starting from the behavior of a herd of animals; a model derived from (Reynolds (1987)) is used in (Vicsek (1995)) to realize a efficient graphic simulation for elements called boids. In the literature there are the strategies commonly used to tackle the problem of multi-agent cooperation in robotics: leader-follower techniques where the leader can be physical (Defoort (2008); Gamage (2007); Ji-Wook Kwon (2012); Ghommam (2013)) or virtual (Yongcan (2010); Leonard (2014)), behavior based techniques (Antonelli (2009); Jad-babaie (2003); Sepulchre (2007); Olfati-Saber (2007)) and virtual structure techniques (Mehrjerdi (2011); Rezaee (2014)). In the leader-follower structure, one agent has the role of the leader and all other robots follow it according to a predefined rule. This structure is very sensible to the leader fail of course, in addition the leader has no feedback from the followers. In the virtual structure all the agents have to track the reference of a virtual shape/leader following some geometry constraints, then this strategy has the advantage to be modeled easily but it is not easy to handle it in the case of a need of reconfiguration. In the behavioral structure each agent has the instructions to react to different conditions, consequently a general group behavior is delineated. For the leader-follower approach, usually, the strategies like feedback linearization, backstepping and first order sliding mode control are used to guarantee the convergence to a stable formation, and they all rely on the knowledge of the leader velocity. There exist some works that achieved leader-follower formation without knowing it (Defoort (2008); Ghommam (2013)). The aim of this work is to present an original leader-follower approach for a group of wheeled mobile robots (WMRs), characterized by kinematic non integrable constraints and subject to additive input disturbances. The goal is to move the leader and the followers to a destination point doing that without sharing the leader velocity, and being able to avoid collision between the agents and external obstacles. Despite the classical $l-\lambda$ and $l-\omega$ schemes (Ghommam (2013)), where an angle and one or more distances were given to the agents to achieve the formation and avoid collisions, in the proposed solution just a desired distance to the leader is given to each agent, which means that the leader does not represent the most advanced robot of the formation but more a reference to follow. The goal is reached using the output stabilization and supervisory switching control frameworks: for each agent, except for the leader that is completely autonomous, three controllers are used to regulate two different outputs. The first controller is in charge of achieving the rendezvous part, that means to approach the agent to the leader. Once the rendezvous control achieved its task the second one, called the following control, assures the follower to maintain the heading and the velocity of the leader. These information, as specified previously, are not available and a finite-time observer is used to get the velocities of the leader. The third controller regulates a second output designed to avoid collisions between agents and obstacles. It is worth to remark that all three controls are robust with respect to additive input disturbances. A supervisor inspired by (Efimov (2006); Guerra (2013)) oversees the switches between three controls. The results are thus presented taking into account the notions of stability for switched systems (Liberzon (2003)) and output-to-state stability (Sontag...
In this work the communication topology issues are not taken into account, nevertheless the authors tried to achieve the results sharing the less information as possible, i.e. the robots positions and orientations.

2. PROBLEM STATEMENT

Let us consider a group of $N \in \mathbb{R}^+$ unicycle WMRs, in which the input is affected by additive disturbances:

$$\begin{align*}
\dot{x}_i &= \cos(\theta_i)(1 + d_{1,i})v_i, \\
\dot{y}_i &= \sin(\theta_i)(1 + d_{2,i})v_i, \\
\dot{\theta}_i &= (1 + d_{2,i})\omega_i,
\end{align*}$$

where $(x_i, y_i) \in \mathbb{R}^2$ define the Cartesian position of each robot, and $\theta_i \in [0, 2\pi)$ is the orientation of the robots with respect to the world reference frame, $v_i$ and $\omega_i$ are the control inputs (the linear velocity and the angular velocity respectively). The additive disturbances on the inputs are unknown, but supposed to be bounded as: $-1 < d_{min} \leq d_{k,i} \leq d_{max}$, $k = 1, 2, i = 1, \ldots, N$. The aim is to design control laws providing the rendezvous and leader-following (the robots must create a formation around the leader) with collision/obstacle avoidance capability for the specified group of unicycle WMRs. The proposed solution uses a supervisor which articulates the activation of three controllers (designed below) depending on the needs. To proceed the following sets have to be defined:

$$\begin{align*}
\Delta_i &= \{0, 1 + \Lambda_i, 1 + d_{ci}, 1\}, \\
\Lambda_i &= \max\{0, 1 + \Lambda_i, 1 + d_{ci}, 1\}.
\end{align*}$$

where $z_{1i}$ being the distance from the leader (i.e. $(x_L, y_L)$ is the leader Cartesian position), and $z_{2i}$ is an output function of the distance $d_{ci} = \sqrt{(x_i - x_L)^2 + (y_i - y_L)^2}$ from a point with the coordinates $(x_c, y_c)$ (defined in Section 3.4, and dependent on other robot positions). The first output $z_{1i}$ is used to manage the switch between the rendezvous and following controllers, while the second output $z_{2i}$ is the one designed to tackle the collision/obstacle avoidance part and design the dedicated controller. To proceed, several assumptions must be introduced. Firstly the maximum leader velocity must be smaller than the maximum followers velocities, i.e. $\omega_{L,max} \leq \omega_{i,max}$ and $v_{L,max} \leq v_{i,max}$, where the suffix max defines the maximum velocity. Then each follower enters the following mode when it reaches a distance $\delta_i$ from the leader, which can be different for different robots and bounded: $\delta_{min} < \delta_i < \delta_{max}$, where $\delta_{min}$ is tied to the collision avoidance minimum distance and $\delta_{max}$ is proportional to the number $N$ of robots. There is a safe distance around each robot $\lambda_i$, which ensures absence of collisions. We will also assume that the linear velocity of the leader $v_L$ is nonnegative (i.e. it is moving forward).

## 2.1 Theoretical Problem Formulation

The problem can be generalized as follows. Consider $N \in \mathbb{R}^+$ dynamical systems

$$\dot{q}_i = f(q_i, u_i, d_i), \quad z_{1i} = h_1(q_i), \quad z_{2i} = h_2(q_i),$$

where $q_i \in \mathbb{R}^n$ is the state, $q = [q_1^T, \ldots, q_N^T]^T$, $u_i \in \mathbb{R}^m$ is the control input and $d_i \in \mathbb{R}^{m_i}$ is a disturbance, with $d_i \in \Omega = \{d_i \in \mathbb{R}^m: ||d_i|| \leq D\}$ for some $D \in \mathbb{R}^+$ ($L^\infty$ denotes the set of essentially bounded functions $d_i: \mathbb{R}_+ \rightarrow \mathbb{R}^m$). We want to regulate the outputs $z_{1i}$ and $z_{2i}$ assuming that the functions $h_1$, $h_2$, and $\lambda_i$ are continuous and locally Lipschitz. It is needed to design the controls $u_i: \mathbb{R}^n \rightarrow \mathbb{R}^m$ guaranteeing that both outputs $z_{1i}$ and $z_{2i}$ will be kept under certain thresholds: i.e. for all $1 \leq i \leq N$ and all initial conditions $q_{0i} \in \mathbb{R}^n$, $q_0 = [q_{01}^T, \ldots, q_{0N}^T]^T$, all $d_i \in \Omega$ and $t \geq t_0 > 0$:

$$\begin{align*}
|z_{1i}(t, q_0, d_i)| &\leq \sigma_{1i}(|\Delta_i, \lambda_i(q_{0i})|), \\
|z_{2i}(t, q_0, d_i)| &\leq \sigma_{2i}(|\Delta_i, \lambda_i(q_{0i})|),
\end{align*}$$

where the values of $\Delta_i$ and $\lambda_i$ are given (they are related with $\delta_i$ and $\lambda_i$), whereas $\sigma_{1i}$, $j = 1, 2$, are functions from the class $\mathcal{K}$ (continuous strictly increasing functions, $\sigma(0) = 0$). The first output, (5), must be smaller than $\sigma_{1i}(\Delta_i)$, in the case $h_1(q_{0i}) > \lambda_i$ the trajectory should converge to a subset where $|h_1(q_i)| \leq \sigma_{1i}(\Delta_i)$. In the same way (6) must be smaller than $\sigma_{2i}(\lambda_i)$. In the case $|h_2(q_{0i}) > \lambda_i$, the trajectory should converge to a subset where $|h_2(q_i)| \leq \sigma_{2i}(\lambda_i)$. For the designed outputs, the restriction (5) implies that all robots should find their positions efficiently close to the leader (on the distance $\sigma_{1i}(\Delta_i)$), and a safe distance should be preserved between the robots and obstacles ($\sigma_{2i}(\lambda_i)$).

## 3. THE SUPERVISORY CONTROL

To proceed the following sets have to be defined:

$$\begin{align*}
X_{\delta_i} &= \{q_i \in \mathbb{R}^n: |h_1(q_i)| \leq \delta_i\}, \\
X_{\Delta_i} &= \{q_i \in \mathbb{R}^n: |h_1(q_i)| \leq \Delta_i\},
\end{align*}$$

where $X_{\delta_i} = \{j \in \{1, \ldots, N\} \setminus \{i\}: \sqrt{(x_{jo} - x_i)^2 + (y_{jo} - y_i)^2} \leq \lambda_i\} \cup \{j \in \{1, \ldots, N\}: \sqrt{(x_{jo} - x_i)^2 + (y_{jo} - y_i)^2} \leq \lambda_i\}$

$$\begin{align*}
X_{\lambda_i} &= \{j \in \{1, \ldots, N\} \setminus \{i\}: \sqrt{(x_{jo} - x_i)^2 + (y_{jo} - y_i)^2} \leq \lambda_i\} \cup \{j \in \{1, \ldots, N\}: \sqrt{(x_{jo} - x_i)^2 + (y_{jo} - y_i)^2} \leq \lambda_i\}.
\end{align*}$$

where $N_o$ is the number of (static) obstacles with the coordinates $(x_{jo}, y_{jo})$ (the number $N_o$ could be considered finite without any loose of generality). $\lambda_i, \Lambda_i, \delta_i, \Delta_i \in \mathbb{R}_+$ are given values of parameter ($\lambda_i < \Lambda_i$ and $\delta_i < \Delta_i$), whose meaning will be explained below. Once defined those sets we can describe the switching sequence: the controller $U_{1i}$ (following) is activated when $q_i \notin X_{\lambda_i}$ and the distance from the leader is less than the threshold $\delta_i$ and it is kept active while the output $h_1(q_i)$ remains less than $\Delta_i$ (this is a safety measure to avoid continuous switching between $U_{1i}$ and $U_{2i}$ controllers); the controller $U_{2i}$ (rendezvous) is active when $q_i \notin X_{\lambda_i}$ and the output $h_1(q_i)$ is greater than $\delta_i$. The third controller $U_{3i}$ (collision/obstacle avoiding) becomes active as soon as $q_i \in X_{\lambda_i}$ and it is kept active until $q_i \notin X_{\lambda_i}$; also in this case a hysteresis is added to avoid continuous switching,
or chattering, between the controllers (Liberzon (2003)). Therefore, the supervisory control law $u_i$ for all $i=1,\ldots,N$ can be summarized as follows

$u_i(t) = U_{p_i(t,)}(q(t)), \; p_i : \mathbb{R}_+ \to \{1, 2, 3\}$ (7)

with the initial conditions

$t_0 = 0, \; p_i(t_0) = \begin{cases} 1 & \text{if } q_i(t_0) \in X_0, \text{and } q(t_0) \notin X_\delta, \\ 2 & \text{if } q_i(t_0) \notin X_0, \text{and } q(t_0) \notin X_\delta, \\ 3 & \text{if } q_i(t_0) \in X_\delta, \end{cases}$

and $p_i(t) = p_i(t_0)$ for $t \in [t, t_{+1})$, where

$p_i(t_{+1}) = \begin{cases} 1 & \text{if } q_i(t_{+1}) + 1 \in X_0, \text{and } q(t_{+1}) \notin X_\delta, \\ 2 & \text{if } q_i(t_{+1}) - 1 \notin X_0, \text{and } q(t_{+1}) \notin X_\delta, \\ 3 & \text{if } q_i(t_{+1}) \notin X_\delta, \end{cases}$ (8)

with $t_0$ is the generic switching instant defined as follows:

$t_{+1} = \inf_{t \geq t_0} q_i(t) \notin X_\delta$.

Now let us define the estimators and all $U_{hi}, k = 1, 2, 3$.

3.1 The Homogeneous Estimator

As specified in Section 2 each follower can access just the position and the orientation of the leader robot; to gather information about the leader velocities $v_L$ and $\omega_L$, linear and angular, an observer is necessary. We will assume that the leader has the following dynamics:

$\dot{x}_L = \cos(\theta_L)v_L,$

$\dot{y}_L = \sin(\theta_L)v_L,$

$\dot{\theta}_L = \omega_L,$

where the terms $v_L$ and $\omega_L$ may contain perturbations with respect to some reference controls, but for the cooperation objective we need to estimate not the reference controls applied to the leader, but its real inputs $v_L$ and $\omega_L$. If $\dot{x}_L, \dot{y}_L$ and $\dot{\theta}_L$ were available for all followers, then

$v_L = \sqrt{\dot{x}_L^2 + \dot{y}_L^2}, \; \omega_L = \dot{\theta}_L.$

Thus since $x_L, y_L$ and $\theta_L$ are only available, then the estimates of the derivatives of these variables have to be calculated. To estimate the derivatives the following homogeneous finite-time differentiator (Perruquet (2008)) has been adopted:

$\dot{\xi}_1 = -\alpha|e|^{0.75}\text{sign}(e) + \xi_2,$

$\dot{\xi}_2 = -\beta|e|^{0.5}\text{sign}(e),$ \; $e = \xi_1 - f,$

where $\xi_1, \xi_2$ are the states of the differentiator, $f$ is the measured signal to be differentiated (i.e. $x_L, y_L$ or $\theta_L$), $\dot{f} = \xi_2$ is the estimate of derivative we are looking for (i.e. $\dot{x}_L, \dot{y}_L$ and $\dot{\theta}_L$). The use of this kind of observer helps also to filter the disturbances and to have a better estimation of both velocities. It has been proven in Perruquet (2008) that $\max\{|\xi'_1|, |\xi'_2|\} \leq \bar{\eta}$ where $\eta_0 = v_L - \tilde{v}_L, \eta_0 = \omega_L - \tilde{\omega}_L$ are the estimation errors and

$\tilde{v}_L = \sqrt{\tilde{x}_L^2 + \tilde{y}_L^2}, \; \tilde{\omega}_L = \dot{\tilde{\theta}}_L.$

Therefore, in all calculations below the estimates $\tilde{v}_L, \tilde{\omega}_L$ can be used assuming presence of bounded errors $\eta_0$ and $\eta_0$.

3.2 Following

The Following control $U_1 = (v_i, \omega_i)$, which should be activated when the follower reaches the circle of radius $\delta_i$ around the leader, it forces the orientation of the ith robot to track the leader’s one. Defining a deviation angle $\epsilon_f = \theta_L - \theta_i$, the dynamics of this error can be easily derived from the WMR model (1) where $\theta_L$ is the leader heading:

$\dot{\epsilon}_f = \dot{\theta}_L - \dot{\theta}_i = \tilde{\omega}_L + \omega_0 - \omega_i(1 + d_{2i}).$ (9)

Select a Lyapunov function $V_f = \frac{1}{2}\epsilon_f^2$ with $\dot{V_f} = \epsilon_f=[\tilde{\omega}_L + \omega_0 - \omega_i(1 + d_{2i})]$, then the following control can be proposed:

$\omega_0 = \tilde{\omega}_L + [K_F\epsilon_f + \rho_f + \rho_o]\text{sign}(\epsilon_f), (10)$

where $K_F$ and $\rho_f$ are the design parameters, $\rho_o = |\omega_L|\frac{d_{\min} - d_{\min}}{d_{\min}} + \bar{\eta}$. An upper estimate for $\epsilon_f$ can be evaluated:

$|\epsilon_f(t)| \leq \begin{cases} \left|\epsilon_0 + \frac{\rho_f}{K_F} \right| e^{K_F(t - t_0)} & \text{if } t < t_0 + \hat{T}_{f}, \\ 0 & \text{if } t \geq t_0 + \hat{T}_{f}, \end{cases}$ (11)

where $t_0$ is the instant in which the control is switched on and $\epsilon_0 = \epsilon(t_0)$ is the value of the angle error at $t_0$ with $\epsilon_0 \in [-\pi, \pi]$. Thus the following control stabilizes the orientation of the robot in a finite time, and this time has the upper bound $\hat{T}_f = \sup_{\epsilon_0 \in [-\pi, \pi]} T_{f_e} = \frac{K_F^{-1}}{\rho_f} \ln \left[ 1 + \frac{K_F^2}{\rho_f^2} \right]$. The velocity part of the following control $v_i$ cannot be a simple estimation of the leader velocity $v_L$ because of the disturbances $d_{1i}$ acting on the follower. To explain the idea of the control used in this case, consider the Lyapunov function $W_f = \frac{1}{2}\tilde{v}_L^2$, with $\tilde{W}_f = -z_1(1 + d_{1i})v_i|\cos(\alpha_i - \theta_L) + z_1(\tilde{v}_L + \eta_0)|\cos(\alpha_i - \theta_L)$, where $\alpha_i = \alpha_L + \frac{\eta_0 - \eta_0}{d_{1i} + \bar{\eta}}$ is the angle between the leader and the follower robots. For the sake of simplicity hereafter, denote $C_{\alpha} = |\cos(\alpha_i - \theta_L)|$ then the chosen linear control can be proposed:

$v_i = \begin{cases} C(z_1)|\sin(\alpha_i) + d_{\max} - \tilde{v}_L + \tilde{v}_L | & \text{if } \epsilon_f = 0, \\ 0 & \text{otherwise,} \end{cases}$

where

$\zeta(z_1) = \begin{cases} 0 & \text{if } z_1 < \delta, \\ 1 & \text{if } z_1 > \delta, \\ \frac{z_1 - \delta}{\delta - \delta} & \text{otherwise,} \end{cases}$

with $\delta < \hat{\delta}$ are the design parameters. Substitution of this control gives:

$W_f < 0 \; \forall z_{1i} < \hat{\delta}$

if $\epsilon_f = 0$. While the error $\epsilon_f$ goes to zero in the finite time $T_f$, then the distance $z_{1i}$ may increase its value by $v_{L,max}T_f$. Therefore, the control assures the follower to stay in the zone $z_{1i}(t) \leq \Delta = \delta_i + v_{L,max}T_f$ (the control...
is activated at the instant when \( z_{i1} \leq \delta_i \). The following control can be summarized as follows:

\[
U_{1i} = \begin{cases} 
\zeta(z_{i1}) \text{sign}(C_a) \frac{\bar{v}_L}{1 - d_{\text{min}}} + \bar{v}_L + \bar{v}_L & \text{if } \epsilon_f = 0, \\
0 & \text{otherwise}, \\
\omega_i = \frac{\bar{\omega}_L + [K_f(\epsilon_f) + \rho p + \rho_0] \text{sign}(\epsilon_f)}{1 - d_{\text{min}}} & \text{if } |\epsilon_{rdv}| \leq \frac{\pi}{2},
\end{cases}
\]

We have proven the following result.

**Lemma 1.** The controller (12) for the system (1) provides an uniform finite-time stabilization for the variable \( \epsilon_{rdv} = \theta_L - \theta_i \) (with an upper estimate (10)) and practical output stability for the output \( z_{i1} \).

### 3.3 Rendezvous

The Rendezvous control, \( U_{2i} = (v_i, \omega_i) \), assures the robot to approach the leader. Define a desired orientation angle as \( \epsilon_{rdv} = \theta_i - \alpha, \) where \( \alpha = \text{atan} \left( \frac{2 \bar{v}_L - \bar{v}_L - x_i}{y_i} \right) \). Consider a Lyapunov function \( W_{rdv} = \frac{1}{2} z_{i1}^2 \), whose derivative admits the estimate:

\[
\dot{W}_{rdv} = z_{i1} (C_a (\bar{v}_L + \eta_i) - \cos(\epsilon_{rdv}) v_i (1 + d_{\text{min}})).
\]

To preserve the semi-definiteness of the function \( W_{rdv} \) the proposed control \( v_i \) has the form:

\[
v_i = \begin{cases} 
\frac{\cos(\epsilon_{rdv}) C_a (\bar{v}_L + \eta_i) + \rho_{rdv}}{1 - d_{\text{min}}} & \text{if } |\epsilon_{rdv}| \leq \frac{\pi}{2}, \\
0 & \text{otherwise},
\end{cases}
\]

where \( 0 < \kappa < 1, \rho_{rdv} > \frac{\bar{v}_L + \eta_i}{\cos(\epsilon_{rdv})} + \rho_1 \) and \( \rho_1 > 0 \) are design parameters. Then we define the Lyapunov function \( V_{rdv} = \frac{1}{2} \epsilon_{rdv}^2 \) and evaluate its derivative:

\[
\dot{V}_{rdv} = \epsilon_{rdv} \left[ \omega_i - \frac{(x_L - x_i)(\bar{v}_L - y_i) - (\bar{v}_L - y_i)(\bar{v}_L - \bar{v}_L)}{z_{i1}^2} \right],
\]

which brings to the following expression for \( \omega_i \):

\[
\omega_i = -\frac{S_a \bar{v}_L + |S_a| y_i + \sin(\epsilon_{rdv})(1 + d_{\text{max}}) v_i}{z_{i1}^2} - \rho_{rdv} \text{sign}(\epsilon_{rdv}) - \left( K_{rdv} + \frac{d_{\text{max}} - d_{\text{min}}}{z_{i1}} \right) \epsilon_{rdv},
\]

where \( K_{rdv} > 0 \) and \( \rho_{rdv} > 0 \) are design parameters. Applying this control, the Lyapunov function derivative \( \dot{V}_{rdv} \) can be rewrittten as follows:

\[
\dot{V}_{rdv} \leq -2K_{rdv} V_{rdv} - \rho_{rdv} \sqrt{2V_{rdv}}.
\]

Therefore, the proposed control stabilizes the variable \( \epsilon_{rdv} \) heading the robot toward the leader in a finite time, and as for the previous controller the time of orientation can be evaluated referring to the \( \epsilon_{rdv} \) variable dynamics. The estimation for \( \epsilon_{rdv} \) is

\[
|\epsilon_{rdv}(t)| \leq \left\{ \begin{array}{ll}
|\epsilon_0| + \frac{\rho_{rdv}}{K_{rdv}} e^{K_{rdv}(t_0 - t)} - \frac{\rho_{rdv}}{K_{rdv}} & \text{if } t < t_0 + T_{rdv}^\epsilon \\
|\epsilon_0| + \frac{\rho_{rdv}}{K_{rdv}} e^{K_{rdv}(t_0 - t)} + \frac{\rho_{rdv}}{K_{rdv}} & \text{if } t \geq t_0 + T_{rdv}^\epsilon
\end{array} \right.
\]

where \( t_0 \) is the instant in which the control is switched on and \( \epsilon_0 = \epsilon(t_0) \) is the value of the angle error at \( t_0 \) with \( \epsilon_0 \in [-\pi, \pi] \). \( T_{rdv}^\epsilon = -K_{rdv}^{-1} \ln \left( \frac{\rho_{rdv}}{K_{rdv} |\epsilon_0| + \rho_{rdv}} \right). \) Thus the the upper bound of the orientation time is \( T_{rdv}^\epsilon = -K_{rdv}^{-1} \ln \left( \frac{\rho_{rdv}}{K_{rdv} |\epsilon_0| + \rho_{rdv}} \right). \) Then for any \( \rho_{rdv} > 0 \) and \( \rho_{rdv} > 0 \) any initial orientation \( \epsilon_0 \), the time of reaching the zone where \( |\epsilon_{rdv}| \leq \frac{\pi}{2} \) is less than \( T_{rdv}^\epsilon \). The controller can be resumed as:

\[
U_{2i} = \begin{cases} 
\frac{\cos(\epsilon_{rdv}) C_a (\bar{v}_L + \eta_i) + \rho_{rdv}}{1 - d_{\text{min}}} & \text{if } |\epsilon_{rdv}| \leq \frac{\pi}{2}, \\
0 & \text{otherwise},
\end{cases}
\]

From the inequality \( \dot{V}_{rdv} \leq -\rho_1 \sqrt{2W_{rdv}} \), for the case \( |\epsilon_{rdv}| \leq \frac{\pi}{2} \), it follows that for \( t \geq t_0 + T_{rdv}^\epsilon \) the distance \( z_{i1} \) is uniformly decreasing to zero in a finite time. Then the distance \( z_{i1} \) may increase on the value \( v_{L,max} T_{rdv} \) during the orientation phase, and after \( t_0 \leq t \leq t_0 + T_{rdv}^\epsilon \) when \( |\epsilon_{rdv}(t_1)| \leq \frac{\pi}{2} \) for the first time \( (z_{i1}(t_1) \leq z_{i1}(t_0)) + v_{L,max} T_{rdv} \), then:

\[
z_{i1}(t) \leq \begin{cases} 
z_{i1}(t_0) + v_{L,max} T_{rdv} & \text{if } t_0 \leq t \leq t_1, \\
z_{i1}(t_1) - \rho_1 (t - t_1) & \text{if } t_1 \leq t \leq t_1 + T_1, \\
0 & \text{if } t \geq t_1 + T_1,
\end{cases}
\]

\[
T_1 = \frac{z_{i1}(t_1) - z_{i1}(t_0)}{\rho_1} \equiv \frac{T_{rdv} \epsilon}{\rho_1}.
\]

In this case, to achieve the task the robot has to reach a distance \( \delta_i \) from the leader. The necessary time to travel till this distance is \( t_{rdv} = \frac{z_{i1}(t_1) - \delta_i}{\rho_1} \). Considering the worst case scenario the time in which the control (16) will achieve his task would be \( T_{rdv} = t_{rdv} + T_{rdv}^\epsilon \). Thus the following claim has been proven.

**Lemma 2.** The control (16) provides for the system (1): 1. Uniform finite-time stability with respect to the variable \( \epsilon_{rdv} \) (see (15)); 2. Uniform boundedness and finite-time convergence with respect to the variable \( z_{i1} \) (see (17)); 3. \( \forall T_{rdv} \in \mathbb{R}_+ \) such that \( z_{i1}(T_{rdv}) \leq \delta_i \).

### 3.4 Collision/Obstacle Avoidance

The Collision/Obstacle Avoidance control becomes active when either the leader or other robots of the group (or an external obstacle, or all of them) enter the safety zone around a robot, which is specified by the circle of radius \( \lambda_1 \). This control is kept active until all the robots exit a bigger circle of radius \( \lambda_1 \); the annulus delimited the two radii can be considered as a hysteresis to avoid Zeno/chattering phenomena. To achieve the task, an effective strategy has been designed. Firstly, each robot who finds itself in a collision avoidance condition, evaluates a point \( (x_c, y_c) \) as follows:

\[
x_c = \frac{1}{M} \sum_{j=1}^{M} x_j, \quad y_c = \frac{1}{M} \sum_{j=1}^{M} y_j,
\]

where \((x_c, y_c) \in X_{\lambda_1}\) is the medium point among all robots/obstacles participating in the collision avoidance maneuver, with \( X_{\lambda_1} \) defined as in Section 3. \( 0 < M \leq N + N_o \) is the number of robots and obstacles. The point \( (x_c, y_c) \) represents the point from which the robot has to go away to exit the collision avoidance conditions. In order
to maximize the distance $d_{ci}$ from the point $(x_c, y_c)$ for all participating robots, the following Lyapunov function is introduced $W_{ca}(x) = z_2 = \max\left\{0, \frac{1 + d_{ci}}{1 + d_{ca}} - 1\right\}$. Let $\bar{\gamma}_i = \arctan\left(\frac{y_i - y_c}{x_i - x_c}\right)$ be the angle between the robot and the point $(x_c, y_c)$. The derivative $\dot{W}_{ca} = 0$ if $d_{ca} > \Lambda_i$ (the avoiding is performed), and for $d_{ca} \leq \Lambda_i$ it has the form:

$$W_{ca} = \frac{v_i \cos(\theta_i - \bar{\gamma}_i)(1 + d_{i1}) - \sum_{j \neq i} v_j \cos(\theta_j - \bar{\gamma}_i)(1 + d_{ij})}{(\Lambda + 1)^{-1}(1 + d_{ca})^2},$$

(18)

Let us introduce the desired orientation that the robot has to reach to go away from the point $(x_c, y_c)$. It is given by the angle $\gamma_i = \theta_i - (\bar{\gamma}_i + \pi)$, where $\pi$ is the natural choice to get away from that point. The proposed controller has the form:

$$U_{3i} = \begin{cases} v_i = \begin{cases} v_{\max} & \text{if } |\gamma_i| \leq k\pi, \\ 0 & \text{otherwise,} \end{cases} \\ \omega_i = \frac{-\rho_1 + \rho_2|\text{sign}(\gamma_i)|}{1 - d_{\min}} \end{cases},$$

(19)

where $\rho_1 \geq \frac{v_{\max}(1+d_{\max})}{d_{ca}}$ and $\rho_2 > 0$. Substituting the control (19) in the derivative of the Lyapunov function $\dot{V}_{ca} = \dot{W}_{ca}$ we obtain:

$$\dot{V}_{ca} \leq -\rho_2 \sqrt{2V_{ca}},$$

which gives us a finite time convergence on the variable $\gamma_i(t)$. This time can be evaluated from the estimation of $\gamma_i(t)$:

$$|\gamma_i(t)| \leq |\gamma_0| - \rho_2(t - t_0),$$

(20)

where $\gamma_0 \in [-\pi, \pi]$ is the initial value of $\gamma_i$ at the instant $t_0$ when the collision avoidance control has been switched on. Thus, when the condition $|\gamma_i| \leq k\pi$ can be verified, is $t_{ca} = \frac{\max(0, |\gamma_0|) - k\pi}{\rho_2}$, and for the worst case scenario $t_{ca} = \sup_{\gamma_0 \in [-\pi, \pi]} t_{ca} = \frac{(1-k)\pi}{\rho_2}$. Following this result and (19), the value of $\lambda_i$ has to satisfy $\lambda_i > t_{ca}v_{\max}$ (the maximal movement velocity for the point $(x_c, y_c)$ is $v_{\max}$). Denote $C_{\infty} = \cos(k\pi)$, then (18) with the control (19) satisfies the estimate:

$$W_{ca} \leq v_{\max} \frac{\Lambda + 1}{(1 + d_{ca})^2},$$

where the upper bound $M = 2$ on the number of terms in the sum appears since one term leaves for $j = i$ and at least one (in the worst case) has a negative value of $\cos(\theta_j - \bar{\gamma}_i)$. If we can assume that the quantity in the square brackets is negative, then we can assure decreasing $W_{ca}$. It can be shown that there exist sufficiently small values $d_{\min}, d_{\max}$ and $k$ close to 1 such that this term is negative (it is easy to see that it is true for $d_{\min} = d_{\max} = 0$ and $C_{\infty} > \frac{M - 2}{M}$, next it will be true by continuity for sufficiently small values of $d_{\min}, d_{\max}$ and some $k$). To conclude, $z_{2i}$ may increase during the orientation phase $t_{ca}$ (due to constraint $\lambda_i > t_{ca}v_{\max}$ a collision is not possible), but next is decreasing to zero, thus this distance is bounded and there is a finite time $T_{ca} > 0$ such that $z_{2i}(t)$ becomes sufficiently small for $t \geq t_0 + T_{ca}$ and $q(t_0 + T_{ca}) \notin \mathcal{X}_i$, thus the collision avoiding is finished.

Lemma 3. The system (1) with the control (19) admits the properties: 1. Uniform finite-time stability with respect to the variable $\gamma_i$ (see the estimate (20)); 2. $2d_{\min}, d_{\max}$ and $k$ such that the variable $z_{2i}$ is bounded and there exists $T_{ca} > 0$ such that $q(t_0 + T_{ca}) \notin \mathcal{X}_i$.

Remark 4. A static obstacle is considered as a robot, which has zero linear and angular velocities.

3.5 Supervisory control

Summarizing the results obtained so far and using the results of Efimov (2006); Guerra (2013), the following statement can be obtained.

Conjecture 5. The system (4) with the supervisor (8) and controls (7) is forward complete and for all $q_0 \in \mathbb{R}^n$, $d \in \Omega$

$$|z_1(t, q_0, d_i)| \leq \max(D_i, |l_i(q_0)|),$$

$$|z_2(t, q_0, d_i)| \leq \max(Y_i, |k_i(q_0)|)$$

for $t \geq 0, Y_i = \frac{1 + \Lambda_i}{\gamma_i - t_{ca}v_{\max}} - 1$.

4. SIMULATIONS

In the simulations the number of WMRs is $N = 4$, with sampling time $t_s = 0.01$ [sec]; the maximum velocity for the leader is set to $v_{L,max} = 0.5$ while the maximum velocity for the followers is $v_{i,max} = 2$. The disturbances have form $d_i = \chi \sin(t) + 0.1 * \text{rand}$ where rand is a pseudo-random values drawn from the standard uniform distribution on the open interval (0,1) with $\chi \leq 0.5$. The following controller has $\delta_1 \in \{x \in \mathbb{R} : 0.7 < x < 0.9 + 0.1\gamma\}$, $K_f = 5$ and $\rho_f = 0.01$; for the rendezvous control the values are: $\rho_1 = 2$, $\rho_{rdc} = 0.1$. For the obstacle avoidance $\rho_2 = 0.1$. Fig. 1 and Fig. 2 represent how the agents behave when the presented strategy is implemented. The leader follows a predefined path, while the followers are placed randomly with random orientation at $t = 0$. Indeed, each agent activates the following controller and the formation movement is accomplished. The distance of each robot from the leader is shown in Fig. 2 and the straight horizontal lines of the same colors represent the corresponding values of $\Delta_i$. While the black line represents the limit distance beyond which the collision/obstacle avoidance is activated. When the collision avoidance control is not active, the followers reach the following mode and remain in it if no external perturbation are applied (as an obstacle could be). If necessary, at the end of the collision avoidance maneuver, they switch back to the rendezvous control to reach again the minimum distance, which is necessary to switch back in the following mode. Thoroughly analyzing Fig. 2 though, it can be noticed that the activation of the collision avoidance controller not always forces the WMR to switch back to the rendez-vous one once the maneuver is accomplished switching back directly to the following one.

5. CONCLUSION

A switching-based solution has been presented to the leader-follower formation problem for a group of WMR in the presence of additive input disturbances with obstacle/collision avoidance. A supervisor, able to regulate two different outputs, orchestrates three different controls to regroup the robots (rendezvous controller), make them follow the leader (following controller) and avoiding collisions/obstacles when necessary during the motion. It is worth to remark that no assumption has been made about a priori knowledge of the positions of obstacles or leader velocities. It has been formally shown that each
control robustly achieves the task it is designed for and, in addition, the robots orientations are provided in a finite-time. Simulations are performed for a group of 4 WMRs to prove the effectiveness of the strategy. Future research will consider the case in which the leader takes into account the distances during the rendez-vous and the collision avoidance maneuvers.

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