Low complexity digital back propagation method using phase linear approximation for nonlinear distortion compensation for long haul transmission systems

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Abstract:
We propose a novel nonlinear distortion compensation technique using linear phase transition approximation for long haul optical communication system. Self-phase modulation (SPM) for phase change, and amplified spontaneous emission (ASE) from erbium-doped fiber amplifier (EDFA) are taken into consideration in simulation. It is confirmed that phase of 28Gbaud return to zero - quadrature phase shift keying (RZ-QPSK) signals compensated with split-step Fourier methods (SSFM) change almost linearly. They can be estimated using linear approximation with only two points of any span for low computational complexity. The performance of this technique is evaluated by bit error rate (BER) with numerical analysis. The achieved reduction of computational amount is 70.0% when transmission distance is 2500km and input power is 0dBm at a FEC limit of BER < 10^{-3}.

Keywords: digital signal processing, nonlinear distortion, SPM, chromatic dispersion

Classification: Fiber-optic transmission for communications

References

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1 Introduction

The reach of optical communication is greatly limited due to impact of nonlinear effect such as intra-channel (SPM: Self Phase Modulation, IXPM: Intra-channel Cross-phase Modulation, XPolM: Cross Polarization Modulation) and inter-channel interference (XPM: Cross Phase Modulation, FWM: Four Wave Mixing) to increase the total transmission capacity with advanced modulation format and several kinds of multiplexing. In order to use multilevel modulation format and to transmit the signal over a long distance, it is necessary to increase the input power and improve the SNR. However, it is difficult to increase the optical power and improve SNR for high capacity communication because of nonlinear Shannon limit [1]. To mitigate the impact of nonlinear distortion, several advanced digital back propagation (DBP) technique have been mainly proposed based on split-step Fourier methods (SSFM) [2],[3] and Volterra series nonlinear equalizers (VSNE) [4],[5]. Although nonlinear compensation with DBP has been regarded as a highly iterative technique, however, complex computation performance is required in real time, and that has been one of the significant technical issues to be overcome. Therefore, development of a novel DBP algorithm that relaxes the computation complexity is an important mission.

In this paper, we propose a novel efficient technique for nonlinear distortion compensation based on SSFM calculation. We confirmed that the phase transition of symbols per span with SSFM compensation changed almost linearly under the simulation with SPM, IXPM, and ASE taken into consideration. Hence, we estimate the phase of symbols on input point from the simulation results between two points from phase information per span obtained from linear approximation. We numerically assess the performance of linear approximation, and compared against the standard SSFM for 28Gbaud single polarized RZ-QPSK signals. We also evaluate the bit error rate (BER) performance, and report the number of steps per span and target spans for effective calculation, as well as the effect of reduction of computational amount.

The structure of this paper is as follow. First, we introduce the principle of the linear approximation technique based on SSFM. Next, the numerical results of the
proposed linear approximation performance evaluated by BER are presented. Finally, the conclusions of this study are stated.

2 Principle of linear approximation based on SSFM

The SSFM is based on the concept that original signals are evaluated from the received signals propagating in the opposite direction against the transmission one. This technique divides the fiber link into small steps and the linear and nonlinear distortion are taken into consideration in frequency and time domain. SSFM is useful for solving the nonlinear Schrödinger equation,

\[
\frac{\partial E}{\partial z} = -\frac{\alpha}{2} E + i \frac{\beta_2}{2} \frac{\partial^2 E}{\partial t^2} - i\gamma |E|^2 E, \tag{1}
\]

where \( E \) represents the electric field intensity. \( \alpha, \beta_2, \) and \( \gamma \) are the propagation loss, group velocity dispersion and nonlinear coefficient of the fiber, respectively. \( z \) and \( t \) are the variables of propagation direction and time.

Considering the attenuation term in electric field intensity of Eq. (1),

\[
A = e^{-\alpha z} E, \tag{2}
\]

\[
\frac{\partial A}{\partial z} = i \beta_2 \frac{\partial^2 A}{\partial t^2} - i\gamma e^{-\alpha z} |A|^2 A, \tag{3}
\]

and consider only the phase change amount \( \varphi_L \) when transmitting the distance \( L \) is

\[
\varphi_L = \int_0^L \Delta \varphi \, dz = \int_0^L \left( \frac{\omega^2 \beta_2}{2} - \gamma e^{-\alpha z} |A_0|^2 \right) \, dz \\
= \frac{\omega^2 \beta_2}{2} \cdot L - \gamma \frac{1 - e^{-\alpha L} |A_0|^2}{\alpha}, \tag{4}
\]

and in our proposed scheme, in order to make the analysis of the phase change easier, we compensate the linear distortion in advance, then the linear distortion term from Eq. (4) can be neglected and the phase change amount between EDFAs is

\[
\varphi_{\text{EDFA}} = -\gamma \frac{1 - e^{-\alpha L_{\text{EDFA}}}}{\alpha} |A_0|^2. \tag{5}
\]

Hence, the phase change amount between EDFAs are same as \( \varphi_{\text{EDFA}} \) in the whole transmit of optical signals, and the phase change can be estimated with linear approximation.

Fig.1 (a) shows the simulation results of the phase transition of a symbol. Setup parameters for the simulation are as follows; propagation loss of SMF is \( \alpha = 0.16 \text{dB/km} \), dispersion parameter of \( D = 16 \text{ps/nm/km} \), nonlinear coefficient of \( \gamma = 1.5 \text{W}^{-1} \text{km}^{-1} \), and core radius of 5.0\( \mu \text{m} \). An EDFA (erbium-doped fiber amplifier) that gain of 16dB is used for compensating fiber transmission loss. The noise figure of 3dB is used, and it is installed every 100km. A 28Gbaud single-polarized RZ-QPSK signal at a wavelength of 1550nm consists of pseudo-random bit sequences with word length of \( 2^{16} - 1 \). The presented symbol whose phase information is assigned to \( \pi / 4 \) at the launched point of the RZ-QPSK signal. It is transmitted through 5000km, then compensated with SSFM using linear approximation. The number of steps per span is 10. Here, a step is one section when fiber is separated by a small distance, a span is distance between EDFAs.
We can see that the simulated points are changing almost linearly. Hence, the phase in each span of symbols with SSFM can be approximated linearly and estimate the symbol phase on input point. Hence, the symbol phase on launched point can be estimated from the extension of the straight line. Linear approximation on Fig.1 (a) represents the linear line utilizing the results in span No.1 and 2, or in span No.1 and No.10. No.1 is defined as the first span from the received side. We can see that the point and the approximation line are almost agreed with the input point in the graph of No.10. On the other hand, as in the graph using No.1 and No.2, the phase is shifted far from the exact phase at the transmission point, thus it is necessary to find the suitable conditions.

In the calculation of standard SSFM in Fig.1 (a), we set the number of steps per span: \( N_{\text{step}} = 10 \), the number of target spans: \( N_{\text{span}} = 10 \), and the number of steps (required SSFM computational amount) to be calculated is \( N_{\text{step}} \times N_{\text{span}} = 100 \) steps. From this calculation, it is found that the phase of each step changes nonlinearly as superimpose, and the SPM phase change increases at the points of launched side where the light intensity is high.

Fig.1 (b) shows the analysis flow of SSFM for linear and nonlinear distortion compensation including the linear approximation. The first three blocks are conventional SSFM processing. CDC and NLC stands for chromatic dispersion and nonlinear dispersion compensator, respectively. As mentioned above, SSFM compensate linear and nonlinear distortion per divided steps (distance \( \Delta z \)). In the middle two blocks, the obtained phases of the symbols of each span are stored in the memories in the system. This process is performed until counting the number of spans \( N' \) reaches the target span \( N_{\text{span}} \) that is the number of spans required for linear approximation. In the last block, the phases of the memorized symbols are linearly approximated and phase estimation is performed. Phase information for each span is stored in the previous blocks. Linear approximation can be performed using two of these points, and the phase of the symbols at the input point can be estimated. In this report, one of two points is fixed to the first span.
3 Numerical simulation results

Fig. 1 (b) circuit is likely to get the great performance of compensating phase noise, however, intensity noise is concerned to limit the performance. We investigated the limit of the proposed scheme performance. Fig.2 (a) shows the performance of linear approximation evaluated by BER under the conditions of transmission length \( L = 5000 \) km, and input power \( P = 0 \) dBm. We considered the number of target spans ranging from 5 spans up to 45 spans when \( N_{\text{step}} = 2, 3, \) and 4. The horizontal dotted line is FEC limit of \( \text{BER} = 10^{-3} \). As can be seen in Fig.2 (a), there is a trade-off between the number of steps, target spans and the compensation performance.

Figs.2 (b) to (d) indicates the number of steps per span as a function of number of spans to reach \( \text{BER} = 10^{-3} \). These results are obtained from transmission distances \( L=2500, 5000, \) and \( 10000 \) km, input power \( P=0, 3, \) and \( 6 \) dBm, respectively. For \( N_{\text{span}} = 2\sim5, 8, 10, 20, \cdots, 60, N_{\text{step}} \) satisfying \( \text{BER} = 10^{-3} \) are plotted. The circles in the graph is marked when the calculation load \( N_{\text{step}} \times N_{\text{span}} \) becomes the minimum of this technique in each transmission distance and power. For example, \( N_{\text{step}} = 5, N_{\text{span}} = 3 \) are obtained in Fig.2 (b) when \( L=2500 \) km, and \( 0 \) dBm.
(c) Required number of steps and spans when $\text{BER} < 10^{-3}; L=5000\text{km}$

(d) Required number of steps and spans when $\text{BER} < 10^{-3}; L=10000\text{km}$

**Fig. 2.** Performance of linear approximation versus BER

Table I shows the maximum reduction of the calculation load by the proposed method compared with standard SSFM. The minus sign indicates that computational amount increase. For this result, At $P=0\text{dBm}$, this technique has great effect of the computational amount reduction and in the case that the input power is up to around $3\text{dBm}$, this technique can be expected to reduce the computational amount of SSFM.

**Table I.** The maximum reduction of the calculation load by linear approximation compared with standard SSFM

| Input power[dBm] | Distance[km] | 0     | 3     | 6     |
|------------------|--------------|-------|-------|-------|
|                  | 2500         | 70.00%| 30.00%| -6.70%|
|                  | 5000         | 55.00%| 40.00%| 4.00% |
|                  | 10000        | 36.30%| 13.30%| -145.50%|

**4 Conclusion**

We numerically assessed the proposed low computational complexity SSFM technique in 28Gbaud RZ-QPSK transmission system. The novel technique is performed by using linear approximation that uses two phase information of symbols in each span for estimating the phase at the transmission point, and reduce the computational amount by not performing the full span calculation with SSFM. The resultant reduction of the computational complexity of SSFM. Particularly, when input power is $0\text{dBm}$, this technique achieves reduction of the computational amount considerably. The reduction of computational amount is 70.0%, 55.0%, and 36.3% when transmission distance are 2500, 5000, and 10000km, respectively.