Conference Title (Transport Research Arena– Europe 2012)

Assessment of Fatigue Cracks in Rails

George Kotsikos \(^{a}\) Marzio Grasso \(^{b}\)

\(^{a}\) School of Mechanical & Systems Engineering, Stephenson Building, Newcastle University, Newcastle upon Tyne NE17RU, UK
\(^{b}\) University Federico II, P.le V. Tecchio. 80, 80125 Naples, Italy

Abstract

The fatigue behaviour of cracks at the foot region of a rail subjected to bending load has been investigated. Depending on the position of initiation of a small semi-elliptical surface crack in a rail, the crack front changes shape during propagation to failure due to the large variation of the stress intensity factor from point to point round the crack front due to differences in the local stress field as each point round the crack front lies at different distances from the neutral axis of the rail. This condition implies a variable crack growth rate that transforms the crack front shape during fatigue crack propagation. SIF values have been estimated by means of both the finite element method and analytical solutions derived for a semi-elliptical crack in a finite rectangular cross-section beam. The SIF value predictions obtained with the two methods show good agreement suggesting that the analytical solutions can be used for a rapid assessment of the severity of a flaw in a rail. A predictive model for crack growth has been derived for an initial small crack at an initiation point at the foot/web corner of a rail tested under four point bending fatigue in the laboratory, showing a reasonably good prediction of both the shape and size of the crack at failure when compared with experiment.

© 2011 Published by Elsevier Ltd. Selection and peer-review under responsibility of TRA 2012

Keywords: fatigue, rail crack, fracture mechanics, FEA

1. Introduction

The fracture of a rail as a result of the development of fatigue cracking is considered as a serious event in the rail industry as it is likely to result in vehicle derailment with a high probability of loss of life. Cracks in rails have been appearing on the head of the rail as a result of mechanical damage due to wheel/rail contact stresses, or the foot of the rail emanating from corrosion pits. More recently, since the method of joining rails by fishplates was abandoned in favour of continuous welded rail, another source of
cracking was introduced arising from various weld defects (e.g. porosity, lack of fusion, shrinkage stresses etc.). Infrastructure managers are devoting a large amount of effort to ensure that the integrity of the rail track network is preserved by conducting regular inspections involving up to date NDE techniques. Nevertheless, the development of cracks in rails is unavoidable. The rail damage prediction tools (such as VAMPIRE) currently employed by the industry are based on empirical relations based on the cumulative damage sustained by the axle weight and volume of traffic passing over a rail (J. Evans, 2003). These models do not take into account the development or presence of flaws. It can be stated that decisions on replacement of a rail are not made on the basis of a fracture mechanics based flaw severity assessment (a damage tolerance approach).

This work has undertaken the study of fatigue cracks in rails using FEA modeling and analytical solutions to provide a tool to rail NDE inspectors for the quick assessment of the severity of surface breaking and embedded cracks in rails. The work does not investigate crack initiation and propagation phenomena on the heads of rails, but concentrates mainly on cracks initiating and propagating from bending loads in the foot of the rail as a result of passing trains.

1. Background

The propagation of a crack is driven by the stress field that develops ahead of the crack tip. In fracture mechanics, the stress and strain fields can be characterized by parameters such as the stress intensity factor, \( K \), under elastic conditions, or the \( J \)-integral or crack tip opening displacement (CTOD) under conditions of extended plasticity. Such parameters describe the mechanics of the crack in terms that include the applied load and the length of the crack. The resistance of a material to fast fracture is given by the fracture toughness. Under small scale yielding conditions, to predict crack propagation life and fracture strength, accurate stress-intensity factor solutions are needed both for initial, intermediate and final crack configurations. But, because of the complexities of such problems, exact solutions are not available. Instead, investigators have had to use approximate analytical methods, experimental methods, or engineering estimates to obtain the stress-intensity factors. Closed form solutions for stress intensity factors of elliptical cracks in infinite bodies have been derived by several authors (Green-1950, Irwin-1962, Kassir-1966, Kobayashi-1976, Newman-1979). For finite bodies, all solutions have required approximate analytical methods with the use of finite element analysis (FEA) or boundary element analysis (BEA) methods (Nishioka-1983, Raju-1979, Shah-1971, Smith-1967, Tracey-1973, Vijakumar-1981, Pan-1999).

In this paper, a numerical approach that takes into account not only the finite geometry of the defected component but also the real elastoplastic behaviour of the component material has been applied. The implemented numerical model has been validated by means of full scale crack propagation test on defected rails.

2. Analytical solution

The stress intensity factor expression used in this work is given by: 
\[
K = S \cdot F_b \cdot \sqrt{\pi \cdot a/Q}
\]
where, \( S \) is the remote applied stress (tensile or bending), \( a \) is the maximum crack depth, \( Q \) is the shape factor for an ellipse given by the square of the complete elliptic integral of the second kind, and \( F_b \) is the boundary correction factor which accounts for the influence of the various boundaries. The definition of \( F_b \) is important and various authors have provided numerous expressions based on FEA parametric
studies. As a starting point the expressions derived by Newman & Raju have been used in this work. A detailed description of these expressions can be found in reference Raju & Newman (1979).

3. Experimental

Mechanical property tests were carried out according to BS EN 10002-1:2001 to obtain the rail material mechanical properties for the analysis. Specimens were obtained from the rail head, web and base along the longitudinal and transverse direction to the rail. There was little variation in the tensile properties between the different regions of the rail. The average yield strength was determined as 540MPa and the ultimate strength 940MPa with an elongation of approximately 12%.

Fracture mechanics tests were conducted according to BS7448 Part 2. DCB specimens were extracted from the head, web and base of the rail so that the notch (and crack extension path) would lie in the transverse direction to the rail, which is also the direction cracks grow in service.

![Figure 1. Position of extraction of tensile and fracture mechanics test samples from a UIC60 rail.](image1)

The fracture toughness $K_{IC}$ of the material could not be determined because the size of the specimens that could be extracted from the rail would not yield valid results. The $K_Q$ value was determined and was found to be approximately 27 MN·m$^{3/2}$ (±2 MN·m$^{3/2}$) at all specimen positions in the rail.

Constant load amplitude four point bending fatigue tests were carried out on a section of a rail to assess the accuracy of the numerical and analytical solutions. A UIC60 rail section, the most common rail profile in European railways, was used. The rail support span was 750mm and the loading point span 190mm. The maximum applied fatigue load was 582kN with a frequency of 1Hz and an $R$ ratio of 0.3 ($R=P_{\text{min}}/P_{\text{max}}$). The fatigue tests were allowed to continue until complete fracture of the rail took place. Once a crack had initiated, the test was stopped and the crack heat tinted in order to determine the shape of the crack front just after initiation and how this changes as the crack front propagates to failure.

![Figure 2. Flaws on fatigued samples, semicircular crack at rail base (left), elliptical crack at bottom web corner (right)](image2)
4. FEA

The finite element analysis has been carried out using the ANSYS package. Singular quarter-point elements for the small region around the crack front, where the stress field is K-dominated, have been used. The quarter-point elements allow the modelling of singular displacement fields and thus the evaluation of the stress intensity factor (SIF) with great accuracy and a relatively coarse mesh. Several methods are actually available to obtain the SIF values during the post processing of the finite element solution. Among them, the Displacement Correlation Method (DCM), applicable only when the material has linear elastic isotropic behaviour, makes use of the well-known relations between the SIF’s values and the displacement field around the crack front. Otherwise, knowing the stress and strain field, the J values along the crack front have to be numerically evaluated and from them the SIF values can be derived.

The scientific code FRANC3D version 5.0, developed by Cornell University, was used to generate the mesh around the crack front, which allows for both the preprocessing, i.e. it allows the creation of the starter crack in the model, and the postprocessing, i.e. the calculation of the SIF in all the nodes of the crack front created. Starting from these SIF values, a prediction of crack growth has been attempted using Paris’ law which relates the crack growth rate with the stress intensity factor in the form of the expression:

\[ \frac{da}{dN} = C \Delta K^m \]

In this work, the parameters \( C \) and \( m \) were obtained by means of crack propagation tests on DCB specimens, extracted from a rail. The values were: \( C = 3.3 \times 10^{-13} \) and \( m = 2.63 \)

![Figure 3. Outline of fatigue crack fronts at failure.](image)

Figure 3 shows a number of crack fronts of fatigue cracks at failure during fatigue testing. The crack geometries were carefully measured and used for comparison with the FEA model and the analytical solutions. Comparative results of the FEA and analytical solution for crack 1 and crack 3 are given in Figure 4. It can be seen that there is a good agreement of the SIF values between FEA and analytical solution for both crack fronts. The FEA shows an increase in the SIF values as the crack front ends approach the surface. This is a result of how the FRANC3D code treats the singularity near the free surface of the material. The singularity is described by \( 1/r^{0.5} \) and is approximated by the quarter point.
element. This is true along the crack front but not at the point where the crack front intersects the external surface and the quarter point element does not describe in a proper manner the singularity because the power of the singularity depends on the angle between the front and the external surface and the Poisson ratio.

![Figure 4](image1.png)

Figure 4. Comparison of SIF values from FEA and analytical solution for web corner Crack 1 (left) and crack 3 (right).

Also, in the present calculation, the M-integral method was used to evaluate stress intensity factors. Therefore, the stress intensity factor calculated for the surface point was in fact an average value over the element size. The stress intensity factor for the surface point of the surface crack should be considered a reasonable physical approximation of the state of affairs at the surface.

![Figure 5](image2.png)

Figure 5. Semi-circular crack on base of rail 30mm from the CL

The modelling then examined the development of cracks from the foot of the rail, (Figure 2-left). A comparison of the results is shown in Figure 5. In a similar manner to the web corner cracking, the analytical model also adopted the Newman & Raju solutions for bending stress incorporating a bending correction factor, $H_b$, for the component geometry. For the rail geometry the bending factor overestimated the stress level and was not appropriate for prediction of SIFs. A better agreement between the FEA models and the analytical solution was achieved by considering the loading on the rail as a variable tensile stress the magnitude of which is a function of the distance from the neutral axis according to the equation $s=My/I$ where, $M$ is the bending moment, $I$ the moment of inertia of the rail and $y$ the distance from the neutral axis.
5. Crack Propagation predictive model

It was observed that the crack initially has a semi-circular profile which changes to elliptical as the crack propagates. The crack front extends further towards the foot of the rail and less towards the web. This is reasonable as the stress increases the further away from the neutral axis towards the foot of the rail.

![Fatigue tested rail showing semicircular crack near initiation site (solid line) and semi-elliptical crack at failure (dotted line).](image)

Since the aforementioned practice does not take into account the material ductility, a special code has been implemented in MATLAB, to correct the crack front shape with the plastic radius and evaluate the effective SIF values along the front. By the theoretical SIF values, \( K_I \), and the following expression:

\[
r_p = \left(\frac{\pi}{8}\right)\left(\frac{K_I}{Y}\right)^2
\]

where, \( Y \) is the material yield stress. The plastic radius values, \( r_p \), along the front have been evaluated, and a virtual crack front equivalent to the real one, but embedded in an elastic medium, has been generated. The front so created was imported in FRANC3D to evaluate the effective \( \Delta K \) and the local increment \( \Delta a \) of the crack front, in order to obtain the new “theoretical” crack front corresponding to a given increment of number of cycles.

![Crack fronts obtained numerically starting from an initial semicircular flaw with radius equal to 2 mm.](image)
In Figure 7 a series of crack fronts during fatigue crack propagation to failure obtained from an initial flaw of semicircular shape and 2 mm radius are presented. The initial crack is located near the corner between the foot and web of the rail where the center of the semicircle is positioned on the external surface and 27 mm distance from the underside of the rail foot. This simulates the fatigue crack of figure 6. A comparison of the crack fronts at failure of Figures 6 and 7 shows not a perfect but a reasonably close prediction of the fatigue crack final front. This result also confirms the need of take account of the plastic region around the crack tip when the fatigue crack growth phenomenon in a ductile material has to be simulated.

6. Conclusions

Analytical solutions based on the Newman & Raju equations to obtain the stress intensity factors round the crack front of cracks in rails have been derived. These were compared with FEA data using ANSYS incorporating the FRANC3D code. The results show a good agreement if a modification is made to the Newman & Raju equations whereby the applied stress is treated as a variable tensile stress rather than bending. The results were validated via experimental four point fatigue tests in rails.

A finite element analysis of the fatigue crack growth near the foot of a rail was carried out by means of a commercial code integrated with an ad hoc macro written to iteratively evaluate the plastic radius, since the rail material is ductile, and the effective ΔK value that has to be used to evaluate the local growth of the crack front.

The good agreement between numerical result and experimental data obtained by fatigue testing of a UIC60 rail segment with the same geometry, made of the same material, having the same defect and subjected to same load used in the numerical simulation, confirms the validity of the adopted modeling procedure and the need of directly introduce the effect of the plastic region at the crack tip when the problem of the crack growth in a ductile material has to be faced.

7. References

J. Evans. “Whole life rail model application and development for RSSB- Dynamic modeling of rolling contact fatigue”, AEA Technology Rail report AEATR-VTI-2003-048 Issue 1, www.rssb.co.uk, (2003).

Green, A. E. and Sneddon, I. N.: The Distribution of Stress in the Neighbourhood of a Flat Elliptical Crack in an Elastic Solid, Proc. Cambridge Phil. Soc., Vol. 47, 1950, pp. 159-164.

Irwin, G. R.: The Crack Extension Force for a Part-Through Crack in a Plate, ASME, Journal of Applied Mechanics, Vol. 29, No. 4, 1962, pp. 651-654.

Irwin, G. R.: The Crack Extension Force for a Part-Through Crack in a Plate, ASME, Journal of Applied Mechanics, Vol. 29, No. 4, 1962, pp. 651-654.

Kassir, M. K. and Sih, G. C., Three-Dimensional Stress Distribution Around an Elliptical Crack Under Arbitrary Loadings, Journal of Applied Mechanics, Vol. 88, 1966, pp. 601-611.

Kobayashi, A. S.: Crack-Opening Displacement in a Surface Flawed Plate Subjected to Tension or Plate Bending, Proc. Second Intl. Conf. on Mechanical Behaviour of Materials, ASM, 1976, pp. 1073-1077.

Newman, J. C., Jr._ and Raju, I. S.: Analyses of Surface Cracks in Finite Plates under Tension or Bending Loads, NASA TP-1578, Dec. 1979.
Nishioka, T.; and Atiuri, S. N.: Analytical Solution for Embedded Elliptical Cracks, and Finite Element-Alternating Method for Elliptical Surface Cracks, Subjected to Arbitrary Loadings, *Engineering Fracture Mechanics*, Vol. 17, 1983, pp. 247-268.

Raju, I. S.; and Newman, J. C., Jr.: Stress-Intensity Factors for a Wide Range of Semi-Elliptical Surface Cracks in Finite-Thickness Plates, *Engineering Fracture Mechanics J.*, Vol. ii, No. 4, 1979, pp. 817-829.

Shah, R. C.; and Kobayashi, A. S.: Stress-Intensity Factor for an Elliptical Crack Under Arbitrary Normal Loading, *Engineering Fracture Mechanics*, Vol. 3, 1971, pp. 71-96.

Smith, F. W.; Emery, A. F.; and Kobayashi, A. S.: Stress Intensity Factors for Semi-Circular Cracks, Part 2 - Semi-Infinite Solid, *Journal of Applied Mechanics*, Vol. 34, No. 4, Trans. ASME, Vol. 89, Series E, Dec. 1967, pp. 953-959.

Tracey, D. M.;: 3D Elastic Singularity Element for Evaluation of K Along an Arbitrary Crack Front, *Int. J. of Fracture*, Vol. 9, 1973, pp. 340-343.

Vijayakumar, K.; and Atluri, S. N.: An Embedded Elliptical Flow in an Infinite Solid, Subject to Arbitrary Crack-Face Traction, *Journal of Applied Mechanics*, Vol. 48, 1981, pp. 88-96.

Pan, E.; Amadei, B.: Boundary Element analysis of fracture mechanics in anisotropic materials, *Engineering analysis with Boundary Elements*, Vol 23, (1999), pp. 683-691