Determination of measurement fidelity for a superconducting photon-number resolving detector with micron-wide strips

M Dryazgov\textsuperscript{1, 2}, N Simonov\textsuperscript{2}, Yu. Korneeva\textsuperscript{2} and A Korneev\textsuperscript{1, 2}

\textsuperscript{1}Higher School of Economics - National Research University, 101000, Russia
\textsuperscript{2}Department of Physics, Moscow State Pedagogical University, 119992, Russia

mdryazgov@hse.ru

Abstract. We report a study of multiphoton detection fidelity (or accuracy) for sequential photon-number resolving detectors based on micron-wide superconducting strips. It was found that an increase in the width of the superconducting strips by a factor of 5 leads to an improvement in the measurement accuracy by more than a factor of 10.

1. Introduction
Photon-number resolving detectors (PNR detectors) \cite{1, 2} based on superconducting strips have found applications in the areas where it is necessary not only to validate the fact of photon detecting, but to find out the number of photons: quantum computers \cite{3} and quantum cryptography \cite{4}.

The standard architecture of a PNR detector consists of several superconducting single-photon detectors (SSPD) \cite{5} connected either in series with parallel resistors or in parallel with series resistors. Distinguishing the number of photons occurs using spatial multiplexing, when several sections are triggered simultaneously by several photons, and the voltage pulses from the sections are summed up resulting in the amplitude proportional to the number of photons.

For a PNR detector with a series connection of sections (figure 1), the maximum pulse height is limited by the value of the bias current and does not depend on the total number of sections. At a low bias current, as in samples with a superconducting strip width of 100 nm, and a large number of sections, the pulses are difficult to distinguish due to thermal noise. As a consequence, when \( i \) photons are absorbed actual voltage may be larger or smaller due to thermal noise, and resulting amplitude will correspond to \( i+1 \) or \( i-1 \) photon pulse.

Here we address the issue of fidelity (or accuracy) of the photon number identification: due to electric noise the resulting amplitude may be smaller or larger, thus giving the wrong measurement result. We consider the detection fidelity as the probability to detect exactly \( i \) photons, when \( i \) sections are triggered, when the internal quantum efficiency of the detector is 100%.

The following is a description of the method for determining detection fidelity and its comparison for a PNR detector with a series number of sections with a strip width of 0.5 \( \mu \text{m} \) and 100 nm.

2. Measurement procedure
Voltage pulses are obtained for a serial PNR detector with a strip width of 0.5 \( \mu \text{m} \), 7 sections, the length of one section is 240 \( \mu \text{m} \), and the total area is 35x30 \( \mu \text{m}^2 \). The pulses pass through 2 Mini-
Circuits ZFL-1000LN amplifiers and are measured using a Rohde&Schwarz RTO1012 digital oscilloscope (figure 2).

Figure 1. (a) A simplified electrical scheme of the PNR with 7 series connection of sections. Here \( I_b \) is bias current, \( R_p \) is parallel resistors, \( Z_0 \) is resistance of the coaxial line. (b) SEM image of PNR detector, the width of the superconducting strips is 0.5 \( \mu m \). One section and the corresponding resistor are highlighted in color.

We obtain a histogram of the frequencies for the occurrence of a sample of a certain voltage in the noise waveform. This histogram represents the normal distribution (figure 2 (in the inset)). Using the normal distribution formula for relative frequencies, the probability function of detecting noise with amplitude \( U_{\text{noise}} \) is obtained:

\[
P(U_{\text{noise}}) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(\frac{-U_{\text{noise}}^2}{2\sigma^2}\right)
\]

The mean value for the normal distribution in this case \( \mu = 0 \), the deviation value \( \sigma \) is selected by the least squares method. The \( P(U_{\text{noise}}) \) values were also plotted with a solid line (figure 2 (in the inset)). The \( P(U_{\text{noise}}) \) values were also plotted with a solid line (figure 2 (in the inset)).

To detect the \( i-th \) pulse with the amplitude \( U_{\text{imp}}^i \), the trigger value is \( U_{tr}^i = (U_{imp}^i - U_{imp}^{i-1})/2 \). This means that if the noise value is \( U_{\text{noise}} \leq U_{tr}^i - U_{imp}^i \), the equipment will fix \( i-1 \) photon. Similarly, if \( U_{\text{noise}} \geq U_{tr}^{i+1} - U_{imp}^i \), the equipment will capture \( i + 1 \) photons.

Then we define fidelity as the probability of detecting exactly \( i \) pulses, taking into account thermal noise:

\[
F = P\left(U_{tr}^i - U_{imp}^i < U_{\text{noise}} < U_{tr}^{i+1} - U_{imp}^i\right) = \int_{u_{tr}^{i+1} - u_{imp}^i}^{u_{tr}^i - u_{imp}^i} \frac{1}{\sigma \sqrt{2\pi}} \exp\left(\frac{-U_{\text{noise}}^2}{2\sigma^2}\right) dU_{\text{noise}}
\]

As shown in figure 2, with a superconducting strip width of 0.5 \( \mu m \) and 7 sections, the desired fidelity will be 100% for each pulse. But with an increase in the number of sections, the difference in heights of neighboring pulses decreases, as well as with a decrease in the width of the superconducting strips.

To assess this effect, the classical electro-thermal model [7], rewritten for an SSPD with micron-wide superconducting strips [8], was modified for a PNR-detector with a series connection of sections. The simulation result is shown in figure 2 (dashed red lines). Good agreement with the experimental result is seen.

Using this model, it is possible to obtain voltage pulses for a PNR detector with any number of sections and any width of the superconducting strip. This makes it possible to obtain the fidelity value
for each voltage pulse and compare them with each other for a PNR detector based on superconducting strips 0.5 μm and 100 nm wide.

Figure 2. Experimentally obtained voltage pulses for a PNR detector with a 7 series connection of sections, the width of the superconducting strips is 0.5 μm. The noise track is highlighted in purple. Red lines are simulation voltage pulses. The inset shows a histogram of the voltage probability distribution of the noise track, the purple curve is the normal distribution of relative frequencies.

3. Results
In the course of the simulation, it was found that when using 0.5-μm-wide superconducting strips, a significant effect of noise occurs only if the total number of sections of the PNR detector exceeds 40.

Figure 3. Dependence of fidelity on the number of triggered sections for a PNR detector with a series connection of sections with a width of superconducting strips of 0.5 μm. Inset for 100 nm stripes.
Figure 3 shows the dependence of the calculated fidelity on the number of triggered sections for a PNR detector with a different number of sections. As can be seen from the graphs, with the total number of sections less than or equal to 30, the noise does not affect the probability of detecting N voltage pulses when N sections are triggered. With an increase in the total number of sections, the difference in amplitude of neighboring impulses decreases, which causes a decrease in fidelity.

The inset in figure 3 shows similar plots for a PNR detector with a series connection of sections with a superconducting strip width of 100 nm. It is clearly seen that even with 7 sections, the fidelity is significantly reduced, and with the number of sections more than 9, it becomes less than 50%.

4. Conclusion
We demonstrate a method for determining the fidelity of detection of i-photon pulses of the PNR-detector for different widths of superconducting strips and different total number of sections. The use of micron-wide superconducting strips makes it possible to improve the fidelity of detecting voltage pulses for a PNR-detector with a series connection of sections due to higher critical current. This fact and low kinetic inductance give an advantage to micron-wide superconducting strips.

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