Finite-time dissipative filter design for discrete-time stochastic interval systems with time-varying delays

Guici Chen¹,²*, Xin Zhou¹,² and Qing Zhang¹,²

Abstract
The finite-time dissipative filtering problem for a kind of discrete-time stochastic interval system with time-varying delays whose parameters are taken in a convex hull is investigated in this paper. Taking a representative subsystem from a stochastic convex hull system, based on convex analysis and matrix theory, a new interval matrix method is proposed to study the finite-time dissipative filter problem, which can deduct the conservativeness. Then, the finite-time dissipative filter is designed by employing a complex Lyapunov-Krasovskii functional together with the improved Wirtinger inequality technique. Correspondingly, some novel sufficient conditions are obtained to ensure the filtering error system with time-varying delays robustly stochastically finite-time bounded and the dissipative index is satisfied. Next, the desired filter gains are achieved in terms of linear matrix inequalities. Finally, the effectiveness of the designed filter is demonstrated by a numerical example with simulations.

Keywords
Dissipative filter, time-varying delays, stochastic interval system, finite-time

Introduction
Strictly speaking, time delays are ubiquitous in control systems.¹² As an effective method to characterize systems, time delays have caught many researchers’ attention. The research on system with time delays originated in the 1950s, there have existed lots of research results up to now.³–⁵ Furthermore, discrete-time systems, especially discrete-time systems with time-varying delays, are expanded rapidly.⁶–⁸ In recent years, some novel skills for the stability analysis of discrete-time systems with time-varying delays have been raised. For example, Zhang et al.⁹ have proposed the free-weighting-matrix method. A novel stability condition for discrete-time systems with interval time-varying delays was investigated by employing improved Wirtinger inequality method.¹⁰

The concept of dissipative was first proposed by Willems.¹¹ When the supply is greater than the storage for a system, the system is called dissipative system. Dissipative systems are thought to be open systems far from thermodynamic equilibrium state, which explain many previously unexplained phenomena in nature. For linear and nonlinear systems, dissipative theory plays a unique role in the aspect of designing feedback controllers. It is not only a powerful tool for researching economic systems and complex systems but also extensively used in solving the robust control problems of various systems.¹²–¹⁴

Finite-time stability was first proposed,¹⁵ since then lots of researchers began to study the finite-time control. In 2012, a novel global finite-time stability feedback controller was proposed.¹⁶ By combining the adaptive fuzzy control method with backstepping technique, an adaptive fuzzy finite-time backstepping control method was given.¹⁷ As a matter of fact, the system reaches a dissipative state in a finite-time interval is more practical. The finite-time dissipative problem has faster convergence speed and better robustness. Thus, the finite-time dissipative problem has become increasingly valuable.¹⁸–²¹

In practical engineering problems, we have to optimize the estimation of stochastic variables based on the
observation process, which is called as the filtering problem. In the past few decades, Wiener filter, Kalman filter, and other filtering methods have been proposed. It is worth noting that Kalman filter is a crucial research object. On one hand, it is always one of the most commonly used ways for state estimation because of its small computation and recursive real-time processing. On the other hand, it can be used in radio, computers, almost all videos or communication equipment and even in many engineering applications like radars and computer vision. Considering this, many results have been investigated. For example, the time-varying reliable $H_\infty$ filter design problem for semi-Markov jump systems over a lossy network was studied. The problem of event-triggered filtering for a class of nonlinear cyber-physical systems with deception attacks was considered.

As we known, there always exist uncertainties due to errors in systems modeling and changes in operating conditions. One way of describing uncertainty of the system is parameter uncertainty, so the system with interval parameter and stochastic interval parameter has attracted great attention of researchers. For example, the finite-time stabilization problem of memristor-based inertial neural networks with time-varying delays via interval matrix method has been studied.

Inspired by the above analysis, this paper is mainly devoted to research the finite-time dissipative filtering problem for a kind of discrete-time stochastic interval system, which is different from the existing literature. The main contributions of this paper are highlighted as follows. First, interval parameters are embedded into the discrete-time stochastic interval system, and the time-varying delays are taken into account in exploring its finite-time dissipative filtering problem. Second, improved Wirtinger inequality is further proposed as a novel summation inequality to reduce the conservatism of finite-time dissipative filtering. Third, there is few results on finite-time dissipative filter design for discrete-time systems by the interval matrix method.

The remainder of this paper is organized as follows. In section 2, the discrete-time stochastic convex hull system with time-varying delays is proposed and transformed to the discrete-time stochastic interval system with time-varying delays. Then, a linear filter is constructed to obtain the filter error stochastic system. Next, the relevant assumptions, definitions, and lemmas are given. Section 3 is the main section, which mainly analyzes the problem of finite-time dissipative filtering, and gives some novel sufficient conditions. Finally, the validity of the above method and the accuracy of the conclusion are illustrated by numerical simulation. A summary is provided in the last section.

**Notations:** $R^s$ stands for $s$-dimensional Euclidean space; $I$ and 0 denotes the identity matrix and zero matrix with appropriate dimensions; * denotes the symmetric elements of symmetric matrix; $P > 0$ means that $P$ is a positive definite and symmetric matrix; $Q^{-1}$ denotes the inverse of matrix $Q$; $E$ stands for mathematical expectation.

### Problem formulation and preliminaries

Consider the following discrete-time stochastic convex hull system:

$$
\begin{align*}
    x(k + 1) & \in \left[ co \left[ A_0, A_B \right] x(k) + co \left[ A_p, A_B \right] x(k - d(k)) \right] + G_{01} v(k) + G_{11} w(k), \\
    y(k) & \in \left[ co \left[ A_0, A_B \right] x(k) + co \left[ A_p, A_B \right] x(k - d(k)) \right] + G_{01} v(k) + G_{11} w(k), \\
    z(k) & = C_0 x(k) + C_d x(k - d(k)),
\end{align*}
$$

where $x(k) \in R^n$, $y(k) \in R^n$, and $z(k) \in R^l$ are the state vector, measured output, and control output respectively; $v(k) \in R^n$ is the disturbance input, which belongs to $L_2[0, \infty]$, where $L_2[0, \infty]$ denotes the space of nonanticipatory square-summable stochastic process with respect to $(F_t)_t$. $\omega(k)$ is a scalar Brownian motion on a complete probability space $(\Omega, F, P)$ with $E(\omega(t)) = 0$, $E(\omega^2(t)) = 1$. $d(k)$ is the time-varying delay satisfying $0 \leq d_1 \leq d(k) \leq d_2 \leq \infty$, where $d_1$ and $d_2$ are given nonzero constants. And $G_{11}$, $G_{11}(i, j) = 0, 1$, $C_0$ and $C_d$ are given matrix with appropriate dimensions.

Let $co \left[ A_0, A_B \right] = \{ a_{ij} \} \in R^{n \times n}$ and $A_0 = \{ a_{ij} \} \in R^{n \times n}$ are assumed to be known. By applying the theories of literature, there exist $A_i(k) \in co \left[ A_0, A_B \right]$, $H_i(k) \in co \left[ H_1, H_2 \right]$, $i, j = 0, d, p, q$ in stochastic convex hull system (1), such that

$$
\begin{align*}
    x(k + 1) & = \left[ A_0(k) x(k) + A_p(k) x(k - d(k)) \right] + G_{01} v(k) + G_{11} w(k), \\
    y(k) & = \left[ A_0(k) x(k) + A_p(k) x(k - d(k)) \right] + G_{01} v(k) + G_{11} w(k), \\
    z(k) & = C_0 x(k) + C_d x(k - d(k)).
\end{align*}
$$

Obviously, we have $A_i(k) \in co \left[ A_0, A_B \right] \subseteq \left[ A_0, A_B \right]$, $H_i(k) \in co \left[ H_1, H_2 \right] \subseteq \left[ H_1, H_2 \right]$, $i, j = 0, d, p, q$. Similar to the existing literature, by employing interval matrix method, one follows that

$$
\begin{align*}
    A_0(k) & = A_0 + \Delta A_0 = A_0 + D_1 F(k) G_1, \\
    A_p(k) & = A_p + \Delta A_p = A_p + D_2 F(k) G_2, \\
    A_d(k) & = A_d + \Delta A_d = A_d + D_3 F(k) G_3, \\
    A_q(k) & = A_q + \Delta A_q = A_q + D_8 F(k) G_8, \\
    H_0(k) & = H_0 + \Delta H_0 = H_0 + D_5 F(k) G_5, \\
    H_p(k) & = H_p + \Delta H_p = H_p + D_6 F(k) G_6, \\
    H_d(k) & = H_d + \Delta H_d = H_d + D_7 F(k) G_7, \\
    H_q(k) & = H_q + \Delta H_q = H_q + D_8 F(k) G_8,
\end{align*}
$$

where $F(k)$ is an unknown time-varying matrix with $F^T(k) F(k) \leq I$. 

$D_1, D_2, D_3, D_5, D_6, D_7, D_8$ are the positive definite matrices.
Remark 1. Noted that the interval matrix method not only overcomes the influence of countless subsystems switching casually in the system (1), but also greatly reduces computational complexity and conservativeness, which makes the results different from the existing ones.\textsuperscript{35-39}

Denote
\[
M_1 = \begin{bmatrix} D_1 & 0 & D_3 & 0 & D_5 & 0 & D_7 & 0 \end{bmatrix},
M_2 = \begin{bmatrix} 0 & D_2 & 0 & D_4 & 0 & D_6 & 0 & D_8 \end{bmatrix},
N_1^T = \begin{bmatrix} G_1^T & G_2^T \end{bmatrix} 0 0 0 0 0 0, 
N_2^T = \begin{bmatrix} 0 & 0 & G_3^T \end{bmatrix} G_4^T 0 0 0 0, 
N_3^T = \begin{bmatrix} 0 & 0 & 0 & G_5^T \end{bmatrix} G_6^T 0 0, 
N_4^T = \begin{bmatrix} 0 & 0 & 0 & 0 & G_7^T \end{bmatrix} G_8^T 0 0 0 0 0 0.
\]

Obviously, the uncertain parameters (3) can be rewritten as
\[
\begin{bmatrix} \Delta A_0 & \Delta A_d & \Delta H_0 & \Delta H_d \end{bmatrix} = \begin{bmatrix} M_1 & M_2 \end{bmatrix} F(k),
\times \begin{bmatrix} N_1 & N_2 & N_3 & N_4 \end{bmatrix}.
\]

Furthermore, we design the linear filter for (2) as follow:
\[
\begin{align*}
\hat{x}(k+1) &= A_K \hat{x}(k) + B_K v(k), \\
\hat{z}(k) &= C_K \hat{x}(k),
\end{align*}
\]

where $\hat{x}(k) \in R^a, \hat{z}(k) \in R^q$ are the estimates of $x(k)$ and $z(k)$, respectively: $A_K, B_K,$ and $C_K$ are the filter gain matrix to be calculated.

Denote $\xi(k) = [x^T(k), \hat{x}^T(k)]^T$ and the output error $e(k) = z(k) - \hat{z}(k)$. The filter error system can be described as:
\[
\begin{align*}
\xi(k+1) &= \begin{bmatrix} \Delta \hat{A}_{c0} & \Delta \hat{A}_{ci} \end{bmatrix} \xi(k) + \begin{bmatrix} H_{c0} & H_{ci} \end{bmatrix} \delta(k) + \begin{bmatrix} G_{c0} & G_{ci} \end{bmatrix} w(k), \\
\end{align*}
\]

where
\[
\Delta \hat{A}_{c0} = A_{c0} + \Delta A_{c0}, \quad \Delta \hat{A}_{ci} = A_{ci} + \Delta A_{ci}, \quad \Delta \hat{H}_{c0} = H_{c0} + \Delta H_{c0}(i,j = 1, 0),
\]
\[
\begin{bmatrix} A_{c0} & 0 \\ B_K A_{c0} & A_K \end{bmatrix}, \quad \begin{bmatrix} 0 & A_d \\ B_K A_q & 0 \end{bmatrix}, \quad \begin{bmatrix} H_0 & 0 \\ B_K H_p & 0 \end{bmatrix},
\quad \begin{bmatrix} G_{c0} & 0 \\ B_K G_{c0} & 0 \end{bmatrix}, \quad \begin{bmatrix} G_{c1} & 0 \\ B_K G_{c1} & 0 \end{bmatrix},
\quad \begin{bmatrix} C_{c0} & -C_K \\ C_d & 0 \end{bmatrix}.
\]

Define Assumption 1. The disturbance input vector $v(k)$ is assumed to be time-varying and satisfies the following constraint
\[
\mathbb{E}\left\{\sum_{k=0}^{K} v^T(k)v(k)\right\} \leq \delta, \delta > 0,
\]

where $K > 0$ is a given constant.

Definition 1.\textsuperscript{40} Given positive constants $c_1, c_2$ with $c_2 > c_1, K$ and a symmetric matrix $L$, the filter error system (5) is defined to be robustly stochastically finite-time bounded with respect to $(c_1, c_2, K, L, \delta)$, if $v(k)$ satisfies Assumption 1, it holds that $\mathbb{E}\left\{\xi^T(0)L\xi(0)\right\} < c_1 \Rightarrow \mathbb{E}\left\{\xi^T(k)L\xi(k)\right\} < c_2, k = \{1, 2, \cdots, K\}$.

Definition 2.\textsuperscript{35} The filter error system (5) is robustly stochastically finite-time dissipative with respect to $(c_1, c_2, K, L, \delta)$, for given a constant $\beta > 0$, matrix $S > 0$, $R > 0$ and $Q$, such that
\[(i) \quad \text{the filter error system (5) is robustly stochastically finite-time bounded with respect to } (c_1, c_2, K, L, \delta);
(ii) \quad \text{under the zero initial state, the following condition holds}
\]
\[
\sum_{k=0}^{K} v^T(k)Qe(k) + 2e^T(k)SV(k) + v^T(k)Rv(k) \geq \beta \sum_{k=0}^{K} v^T(k)v(k).
\]

Lemma 1.\textsuperscript{10} For a symmetric matrix $Z > 0$ and a variable $\xi \in [a, b]$, where $b - a \geq 1$, the following inequality holds
\[
\frac{b-1}{b-a} \sum_{i=a}^{b-1} \chi^T(i)Z\chi(i) \leq \frac{1}{b-a} \begin{bmatrix} \Theta_0 & 0 \\ 0 & Z \end{bmatrix} \begin{bmatrix} 0 & \Theta_0 \\ \Theta_1 & 0 \end{bmatrix},
\]

where $\chi(i) = \xi(i+1) - \xi(i), \quad \Theta_0 = \xi(b) - \xi(a), \quad \Theta_1 = \{b - a\} \sum_{i=a}^{b-1} \xi(i)$.

Remark 2. In this paper, the inequality in Lemma 1 is called the improved Wirtinger inequality. We know that the Wirtinger inequality is applied to the stability analysis of continuous-time systems. However, the improved Wirtinger inequality, which combines with the efficient representation of the improved reciprocal convex combination inequality, devotes to the stability analysis of discrete-time systems.\textsuperscript{10}

Lemma 2. (Schur complement). Given three matrix $S_1, S_2, S_3$, where $S_1 = S_1^T < 0$ and $S_3 = S_3^T < 0$, then $S_1 - S_2^{-1}S_3S_1 < 0$ if and only if $\begin{bmatrix} S_1 & S_2 \\ S_2^T & S_3 \end{bmatrix} < 0$ or $\begin{bmatrix} S_1 & S_2 \\ S_2^T & S_3 \end{bmatrix} < 0$.
Lemma 3. Define that $\Sigma$, $D$, $E$ are real matrix and $F(k)$ is a matrix function satisfying $F^T(k)F(k) \leq I$. There exists $\varepsilon > 0$, it holds that:

$$\Sigma + DF(k)E + E^T F^T(k)D^T \leq \Sigma + \varepsilon^{-1}DD^T + \varepsilon E^T E.$$

Main results

In this section, based on the complex Lyapunov-Krasovskii functional, improved Wirtinger inequality, some complicated mathematical skills, and linear matrix inequality technique, some novel sufficient conditions are presented to guarantee the robustly stochastically finite-time dissipative for the filter error system (5).

Theorem 1. For some given positive constants $d_1, d_2, \delta, \sigma, \beta, \rho, c_2 > c_1, \gamma > 1, K$, and a symmetric matrix $L$, the filter error system (5) is robustly stochastically finite-time dissipative with respect to $(c_1, c_2, K, L, \delta)$, if there exist symmetric definite matrix $P > 0, Q_i > 0, Z_i > 0(i, j = 1, 2)$ and constants $\lambda_i (i = 1, 2, \cdots, 5) > 0$ such that

$$\Theta = \begin{bmatrix}
\Theta_{11} & \Theta_{12} & \Theta_{13} & \Theta_{14} & \Theta_{15} & \Theta_{16} \\
* & \Pi \\
* & * & \Theta_{22} & \Theta_{23} & \Theta_{24} & \Theta_{25} & \Theta_{26} \\
* & * & * & \Theta_{33} & 0 & 0 \\
* & * & * & * & \Theta_{44} & 0 & 0 \\
* & * & * & * & * & \Theta_{55} & 0 \\
* & * & * & * & * & * & \Theta_{66}
\end{bmatrix},$$

$$L \prec P \leq 0,$$

\begin{align}
\alpha_1 c_1 + \alpha_2 c_2 + \rho \delta &< \frac{c_2}{\gamma^k}, \\
P \leq \lambda_1 I, Q_i \leq \lambda_2 I, Z_i \leq \lambda_3 I, Z_i \leq \lambda_4 I, Z_i \leq \lambda_5 I.
\end{align}

Proof. Construct a Lyapunov functional as follows

$$V(k) = \sum_{i=1}^{4} V_i(k),$$

where

$$V_1(k) = \xi^T(k) P \xi(k),$$

$$V_2(k) = \sum_{i=k-d_i}^{k-d_i-1} \gamma^{i-1} \xi^T(i) Q_i \xi(i),$$

$$V_3(k) = \sum_{j=-d_i}^{k-k-d_i-1} \sum_{i=k-j}^{k-d_i} \gamma^{j-1} \eta^T(i) Z_i \eta(i).$$
\[
V_4(k) = \hat{d} \sum_{j=-d_1}^{-d_1-1} \sum_{j=-d_1}^{k-1} \gamma^{k-j-1} \eta^T(i) Z_2 \eta(i) \\
+ a \sum_{j=d_2+1}^{-d_1} \sum_{j=d_2+1}^{k-1} \gamma^{k-j-1} \eta^T(i) Z_2 \eta(i).
\]

where \(\gamma > 1\), \(\hat{d} = d_2 - d_1\), and \(\eta(i) = \xi(i+1) - \xi(i)\).

Calculating the forward difference by taking \(\Delta V(k) = V(k+1) - V(k)\) along the solution of the filter error system (5) and taking the mathematical expectation, it yields that

\[
E\{V_1(k) - (\gamma - 1) V_1(k)\} \\
= E\{V_1(k+1) - \gamma V_1(k)\} \\
= E\{\xi^T(k + 1) + P \xi(k+1) - \gamma \xi^T(k) P \xi(k)\} \\
= E\{-\gamma \xi^T(k) P \xi(k)\} \\
+ \lim_{N \rightarrow \infty} \int_0^N \int_C \sum_{i=k+1}^{k+N} \sum_{i=k+1}^{k+N} \gamma^{N-i} \eta^T(i) Z_2 \eta(i) \, d\eta(i) \, d\eta(i).
\]

and

\[
\xi^T(k) = \frac{1}{d+1} \sum_{i=-d_1}^{-1} \xi^T(i), \\
\xi^T(k) = \frac{1}{d+1} \sum_{i=-d_1}^{-1} \xi^T(i).
\]

From (14) and (15), it follows that

\[
E\{\Delta V_3(k) - (\gamma - 1) V_3(k)\} \\
\leq E\{\eta^T(k) v_1 Z_1 \eta(k) - 2 \Xi^T(k) W_1^T \gamma Z_1 W_1 \Xi(k)\} \\
- d_1 \sum_{i=-d_1}^{-1} \sum_{i=k+1}^{k+N} \gamma^{N-i} \eta^T(i) Z_2 \eta(i) \}
\]

Similarly, we get

\[
E\{\Delta V_4(k) - (\gamma - 1) V_4(k)\} \\
E\{\eta^T(k) v_2 Z_2 \eta(k) - 2 \Xi^T(k) W_2^T \gamma^{d+1} Z_2^T W_2 \Xi(k)\} \\
- \hat{d} \sum_{i=-d_1}^{-d_1} \sum_{i=k+1}^{k+N} \gamma^{N-i} \eta^T(i) Z_2 \eta(i) \}
\]

where \(v_2 = \frac{d_2(d_2 + d_3)}{2}\) and \(Z_2 = diag\{Z_2, 3Z_2\}\).

Substituting (12), (13), (16), and (17) into (11) yields

\[
E\{\Delta V(k) - (\gamma - 1) V(k) - \rho v^T(k) v(k)\} \\
\leq E\{\Xi^T(k) \left[ \Theta_1^T \Theta_2 + \Theta_2^T \Theta_3 + \Theta_3^T v_1 Z_1 \Theta_4 \right. \}
\]

\[
+ \Theta_4^T v_2 Z_2 \Theta_5 + \Theta_5^T v_1 Z_1 \Theta_6 + \Theta_6^T v_2 Z_2 \Theta_7 \Xi(k) \}
\]

\[
- d_1 \sum_{i=-d_1}^{-1} \sum_{i=k+1}^{k+N} \gamma^{N-i} \eta^T(i) Z_1 \eta(i) \\
- d_1 \sum_{i=-d_1}^{-d_1} \sum_{i=k+1}^{k+N} \gamma^{N-i} \eta^T(i) Z_2 \eta(i) \}
\]

where \(\rho > 0\) is the known coefficient to prove that the system is finite-time bounded and
Then, we can obtain the inequality from Assumption 1 transformed into a linear matrix inequality, which is implied in mathematical expectation of \(E^{21} \leq \sum_{k=0}^{\infty} \gamma^k \mathbf{v}^T(k) \mathbf{v}(k)\). (19)

Set \(\gamma > 1\), we have \(V(k) < \gamma V(k-1) < \cdots < \gamma^k V(0)\). Then, we can obtain the inequality from Assumption 1 as follows

\[
\mathbb{E}\{V(k)\} \leq \mathbb{E}\{V(0)\} + \rho \mathbb{E}\left\{\sum_{i=0}^{k-1} \gamma^{i-1} \mathbf{v}^T(i) \mathbf{v}(i)\right\}
\]

\[
\leq \mathbb{E}\{V(0)\} + \gamma^k \rho \delta. \quad \text{(20)}
\]

Assume that \(\eta^T(k) \eta(k) \leq \alpha (\sigma > 0)\) and if (10) holds, the mathematical expectation of \(V(0)\) is obtained as

\[
\mathbb{E}\{V_1(0)\} = \mathbb{E}\{\xi^T(0) P \xi(0)\} \leq \alpha_1 c_1
\]

\[
\mathbb{E}\{V_2(0)\} = \mathbb{E}\left\{\sum_{i=0}^{d_1-1} \gamma^{i-1} \xi^T(i) Q_i \xi(i)\right\}
\]

\[
\leq \lambda_1 \sigma_1 c_1.
\]

\[
\mathbb{E}\{V_3(0)\} = \mathbb{E}\left\{\delta d_1 \sum_{i=0}^{d_1-1} \gamma^{i-1} \eta^T(i) Z_1 \eta(i)\right\}
\]

\[
\leq \lambda_2 (\phi_1 + \phi_2) \delta \sigma_1.
\]

\[
\mathbb{E}\{V_4(0)\} = \mathbb{E}\left\{\delta \sum_{i=0}^{d_1-1} \sum_{j=0}^{d_2-1} \gamma^{i-1} \eta^T(i) Z_2 \eta(i)\right\}
\]

\[
\leq \lambda_3 (\phi_1 + \phi_2) \delta \sigma_1.
\]

By using the steps of (18), we derive

\[
\mathbb{E}\{V(k)\} \leq \sum_{k=0}^{K} \mathbb{E}\{e^T(k) Q \xi(k) + 2e^T(k) S \xi(k) + v^T(k) R \xi(k)\}
\]

\[
= \beta \sum_{k=0}^{K} \mathbb{E}\{v^T(k) v(k)\},
\]

where

\[
\Theta = \begin{bmatrix}
\Theta_{11} & \Theta_{12} & \Theta_{13} \\
* & \Theta_{22} & \Theta_{23} \\
* & * & \Theta_{33}
\end{bmatrix},
\]

\[
\Theta_{11} = \Theta_{111}, \Theta_{12} = \Theta_{112}, \Theta_{13} = \Theta_{113}, \Theta_{23} = \Theta_{123}, \Theta_{33} = \Theta_{133},
\]

\[
\Theta_{22} = \begin{bmatrix}
\Theta_{222} & 0 & 0 \\
0 & \Theta_{223} & \Theta_{224} \\
0 & * & \Theta_{225}
\end{bmatrix},
\]

\[
\Theta_{2} = \begin{bmatrix}
\Theta_{21} & \Theta_{22} & \Theta_{23} & \Theta_{24} & \Theta_{25}
\end{bmatrix},
\]

under conditions (8) and (9), combining (20) and (21) together, it can be concluded that

\[
\mathbb{E}\{\xi^T(k) L \xi(k)\} < c_2. \quad (22)
\]

Therefore, from Definition 1, we conclude that the filter error system (5) is robustly stochastically finite-time bounded. Next, sufficient conditions of the robustly stochastically finite-time dissipative for the filter error system (5) are given.

Firstly, take into account the performance index as

\[
J(k) = \sum_{k=0}^{K} \mathbb{E}\{e^T(k) Q e(k) + 2e^T(k) S v(k) + v^T(k) R v(k)\}
\]

\[
- \beta \sum_{k=0}^{K} \mathbb{E}\{v^T(k) v(k)\},
\]

From Schur complement technique, the linear matrix inequality (7) are obtained from (23). Therefore, if (7) holds, it further yields \(\mathbb{E}\{\Delta V(k) - (\gamma - 1) V(k) - J(k)\} \leq 0\). It is clear that \(\mathbb{E}\{V(k+1)\} \leq \mathbb{E}\{\gamma V(k) + e^T(k) Q e(k) + 2e^T(k) S v(k) + v^T(k) (R - \beta I) v(k)\} \}

Consider the constant \(\gamma > 1\), it follows that

\[
\mathbb{E}\{V(k)\} \leq \mathbb{E}\left\{\gamma^k V(0) + \sum_{j=0}^{k-1} \gamma^{j-i} e^T(k) Q e(k) + 2 \sum_{j=0}^{k-1} \gamma^{j-i} e^T(k) S v(k) + \sum_{i=0}^{k-1} \gamma^{k-i} v^T(k) (R - \beta I) v(k)\right\}.
\]

\[
\sum_{j=0}^{k-1} \gamma^{j-i} e^T(k) Q e(k) + 2 \sum_{j=0}^{k-1} \gamma^{j-i} e^T(k) S v(k) + \sum_{i=0}^{k-1} \gamma^{k-i} v^T(k) (R - \beta I) v(k),
\]

\[
\sum_{i=0}^{k-1} \gamma^{k-i} v^T(k) (R - \beta I) v(k),
\]

\[
\sum_{i=0}^{k-1} \gamma^{k-i} v^T(k) (R - \beta I) v(k),
\]

\[
\sum_{i=0}^{k-1} \gamma^{k-i} v^T(k) (R - \beta I) v(k),
\]

\[
\sum_{i=0}^{k-1} \gamma^{k-i} v^T(k) (R - \beta I) v(k),
\]
What’s more, $V(k) > 0 (k = 1, 2, \cdots, K)$, we have

$$
E \left\{ \beta \sum_{i=0}^{K-1} y^{k-i-1} v^T(k) v(k) \right\}
\leq E \left\{ \sum_{i=0}^{K} y^{k-i-1} e^T(k) Q e(k) + 2 \sum_{i=0}^{K} y^{k-i-1} e^T(k) S v(k) + \sum_{i=0}^{K} y^{k-i-1} v^T(k) R v(k) \right\}
$$

which implies

$$
E \left\{ \beta \sum_{i=0}^{K} y^i v^T(k) v(k) \right\}
\leq E \left\{ \sum_{i=0}^{K} y^i e^T(k) Q e(k) + 2 \sum_{i=0}^{K} y^i e^T(k) S v(k) + \sum_{i=0}^{K} y^i v^T(k) R v(k) \right\}
$$

Therefore, by Definition 2, it shows that the filter error system (5) is robustly stochastically finite-time dissipative. The proof is completed.

**Remark 3.** As we all know, the stability of the system can be easily determined by choosing a reasonable Lyapunov functional in the stability analysis of the system. In this paper, the Lyapunov functional (11) containing triple summation terms is considered. It is evident that the considered Lyapunov functional increases the computational complexity. Meanwhile, the improved Wirtinger inequality in Lemma 1 is applied in calculating the forward difference and inequality scaling in Theorem 1. In fact, the considered Lyapunov functional in this paper can take fully the advantage of the improved Wirtinger inequality and reduces the conservatism caused by the optimal setting of linear matrix inequality with increasing the upper bound of time delays in control theory together with the improved Wirtinger inequality in the filter design, as discussed in Zhang et al.\(^{43}\) Chen and Sun.\(^{43}\)

Now, the design of the finite-time dissipative filter gain is presented in the following theorem.

**Theorem 2.** For some given positive constants $d_1, d_2, \delta, \sigma, \beta, \rho, c_2 > c_1$, $\gamma > 1$, $K$, and a symmetric matrix $L$, the filter error system (5) is robustly stochastically finite-time dissipative with respect to $(c_1, c_2, K, L, \delta)$, if there exist $P > 0, Q_i > 0, Z_i > 0 (i,j = 1, 2)$, any appropriate dimensioned matrix $Y (i = 1, 2, \cdots, 6)$, positive constants $\lambda_i (i = 1, 2, \cdots, 5)$ and $\varepsilon_k (i = 1, 2)$ such that the following linear matrix inequalities and (8)–(10) in Theorem 1 hold:

$$
\begin{bmatrix}
\Theta & B_1 N_1 & M_1 & B_2 N_2 & M_2 \\
* & -\varepsilon_1 I & 0 & 0 & 0 \\
* & * & -\varepsilon_2 I & 0 & 0 \\
* & * & * & -\varepsilon_2 I & 0 \\
\end{bmatrix}
< 0,
$$

where

$$
\Theta =
\begin{bmatrix}
\Theta_{11} & \Theta_{12} & \Theta_{13} & \Theta_{14} & \Theta_{15} & \Theta_{16} \\
* & \Theta_{22} & \Theta_{23} & \Theta_{24} & \Theta_{25} & \Theta_{26} \\
* & * & \Theta_{33} & 0 & 0 & 0 \\
* & * & * & \Theta_{44} & 0 & 0 \\
* & * & * & * & \Theta_{55} & 0 \\
* & * & * & * & * & \Theta_{66} \\
\end{bmatrix},
$$

$$
\Theta_{11} =
\begin{bmatrix}
\Pi_1 & 0 & -4\gamma Z_{11} & 0 \\
* & \Pi_2 & 0 & -4\gamma Z_{12} \\
* & * & \Pi_3 & 0 \\
* & * & * & \Pi_4 \\
\end{bmatrix},
$$

$$
\Theta_{12} =
\begin{bmatrix}
A_0^T P_1 & A_p^T Y_2 & H_1^T P_1 & H_p^T Y_2 & C_0^T S \\
0 & Y_1 & 0 & 0 & -C_k^T S \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix},
$$

$$
\Theta_{13} =
\begin{bmatrix}
(A_0^T - \lambda_1) Z_{11} & A_p^T Y_4 & H_1^T Z_{11} & H_p^T Y_4 \\
0 & Y_3 - Z_{12} & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix},
$$

$$
\Theta_{14} =
\begin{bmatrix}
(A_0^T - \lambda_2) Z_{21} & A_p^T Y_6 & H_1^T Z_{21} & H_p^T Y_6 \\
0 & Y_5 - Z_{22} & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix},
$$

$$
\Theta_{15} =
\begin{bmatrix}
-A_0^T Q_{11} & 0 & 0 & 0 & -C_0^T S \\
* & -A_p^T Q_{12} & 0 & 0 \\
* & * & \Pi_5 & 0 & 0 \\
* & * & * & \Pi_6 & 0 \\
\end{bmatrix},
$$

$$
\Theta_{16} =
\begin{bmatrix}
-A_0^T Q_{21} & 0 & 0 & 0 & -C_0^T S \\
* & -A_p^T Q_{22} & 0 & 0 \\
* & * & \Pi_5 & 0 & 0 \\
* & * & * & \Pi_6 & 0 \\
\end{bmatrix},
$$

$$
\Theta_{22} =
\begin{bmatrix}
\Pi_1 - \gamma d_1 Q_{11} - 8\gamma d_2 Z_{11}, & \Pi_2 - \gamma d_1 Q_{12} - 8\gamma d_2 Z_{12}, \\
\Pi_3 - \gamma d_1 Q_{21} - 8\gamma d_2 Z_{21} - 8\gamma d_2 + 1 Z_{22}, & \Pi_4 - \gamma d_1 Q_{22} - 8\gamma d_2 Z_{22} - 8\gamma d_2 + 1 Z_{22} \\
\end{bmatrix}. \]
Furthermore, the desired filter gain can be given by

\[ A_K^T = Y_1 Z_1 Z_2^{-1}, \quad B_K^T = Y_4 Z_1 Z_2^{-1}, \quad C_K = C_K. \]  \hfill (28)

**Proof.** From (7), it can derive that

\[ \Theta + \sum_{i=1}^{2} \Phi_i F^T(k) \Phi_i^T + \sum_{i=1}^{2} \Phi_i F(k) \Phi_i^T < 0, \]  \hfill (29)

where

\[
\begin{bmatrix}
\Theta_{11} & \Theta_{12} & \Theta_{13} & \Theta_{14} & \Theta_{15} & \Theta_{16} \\
* & \Theta_{22} & \Theta_{23} & \Theta_{24} & \Theta_{25} & \Theta_{26} \\
* & * & \Theta_{33} & 0 & 0 & 0 \\
* & * & * & \Theta_{44} & 0 & 0 \\
* & * & * & * & \Theta_{55} & 0 \\
* & * & * & * & * & \Theta_{66}
\end{bmatrix}
\]

\[ \Theta_{11} = \Theta_{11}, \]

\[ \Theta_{12} = \Theta_{12}, \Theta_{13} = \Theta_{13}, \Theta_{22} = \Theta_{22}, \Theta_{23} = \Theta_{23}, \]

\[ \Theta_{33} = \Theta_{33}, \Theta_{44} = \Theta_{44}, \Theta_{55} = \Theta_{55}, \Theta_{66} = \Theta_{66}, \]

\[ \Theta_{14} = \begin{bmatrix} A_{10} & H_{10} & C_{10} \end{bmatrix}, \Theta_{15} = \begin{bmatrix} A_{10} - I_n & H_{10} \end{bmatrix}, \]

\[ \Theta_{16} = \begin{bmatrix} A_{10} - I_n & H_{10} \end{bmatrix}, \Theta_{24} = \begin{bmatrix} A_{10} & H_{10} & C_{10} \end{bmatrix}, \]

\[ \Theta_{25} = \begin{bmatrix} A_{10} & H_{10} \\ G_{10}^T & G_{10}^T \end{bmatrix}, \Theta_{26} = \begin{bmatrix} A_{10} & H_{10} \\ G_{10}^T & G_{10}^T \end{bmatrix}, \]

\[ \Phi_{i1} = \begin{bmatrix} N_1 & N_2 & 0 \cdots 0 \\ 0 & 0 \cdots 0 \\ N_4 & 0 \cdots 0 \end{bmatrix}, \]

\[ \Phi_{i2} = \begin{bmatrix} 0 \cdots 0 \\ 0 \cdots 0 \\ 0 \cdots 0 \end{bmatrix}, \]

\[ \Phi_{21} = \begin{bmatrix} N_3 & N_4 & 0 \cdots 0 \\ 0 & 0 \cdots 0 \end{bmatrix}, \]

\[ \Phi_{22} = \begin{bmatrix} 0 \cdots 0 \\ 0 \cdots 0 \\ 0 \cdots 0 \end{bmatrix}, \]

By virtue of Lemma 3, we know that there exist \( \epsilon_i (i = 1, 2) > 0 \), such that (29) can be rewritten as

\[ \Theta + \sum_{i=1}^{2} \epsilon_i \Phi_i \Phi_i^T + \sum_{i=1}^{2} \epsilon_i^{-1} \Phi_i^T \Phi_i < 0. \]  \hfill (30)

By implementing Schur complement lemma, it follows that

\[ \begin{bmatrix} \Theta & \epsilon_1 \Phi_{i1} & \epsilon_2 \Phi_{i2} & \Phi_{i3} \\
* & -\epsilon_1 I & 0 & 0 \\
* & * & \epsilon_2 I & 0 \\
* & * & * & \epsilon_2 \epsilon_1 I
\end{bmatrix} < 0. \]  \hfill (31)

Denote \( \phi = \text{diag}\{I_7, I_4, P, P, I, Z_1, Z_1, Z_2, Z_2, I_4, I_4\} \), and

\[ P = \text{diag}\{P_1, P_2\}, Z_1 = \text{diag}\{Z_{11}, Z_{12}\}, Z_2 = \text{diag}\{Z_{21}, Z_{22}\}, \]
can be converted into (27). The proof is completed.

Then, left-multiply \( \phi^T \) and right-multiply \( \phi \) on (31), the inequality (31) can be converted into (27). The proof is completed.

**Numerical simulation**

A numerical example with simulation results is provided in this section to ensure the effectiveness of the designed filter. Consider the systems (2) with

\[
Q_1 = \text{diag}\{Q_{11}, Q_{12}\}, Q_2 = \text{diag}\{Q_{21}, Q_{22}\}.
\]

Figure 1. Trajectories of \( x_i(k) \) and its filter \( \hat{x}_i(k) \) with \( x^T(0) = (-3, 3) \).

Figure 2. Trajectories of \( x_2(k) \) and its filter \( \hat{x}_2(k) \) with \( x^T(0) = (5, -5) \).

Figure 3. The tracking error \( e(k) \) with \( e(k) = z(k) - \hat{z}(k) \).

The related parameters are taken as \( d_1 = 1.5, d_2 = 5, \gamma = 1.65, \rho = 1, c_1 = 8, c_2 = 30, \delta = 4.3, K = 80, \sigma = 0.1, S = R = 2I, Q = -0.1I, L = I \) and the finite-time dissipative index \( \beta^* = 1.5 \). By exploiting the LMI toolbox of MATLAB, the linear matrix inequalities (8)-(10) in Theorem 1 and (27) in Theorem 2 are solved, the filter gain matrix are calculated as follows:

\[
A_0(k) = \begin{bmatrix}
-0.24 & 0 \\
-0.12 & -0.13
\end{bmatrix},
A_p(k) = \begin{bmatrix}
-0.25 & 0.12 \\
-0.50 & 0
\end{bmatrix},
A_d(k) = \begin{bmatrix}
-0.35 & 0.30 \\
0 & 0.25
\end{bmatrix},
A_d(k) = \begin{bmatrix}
-0.20 & 0.15 \\
0 & 0.001
\end{bmatrix},
H_0(k) = \begin{bmatrix}
0 & 0.35 \\
0 & -0.2
\end{bmatrix},
H_p(k) = \begin{bmatrix}
0.40 & 0.45 \\
0.1 & 0.01
\end{bmatrix},
H_d(k) = \begin{bmatrix}
0 & 0.25 \\
0.1 & -0.10
\end{bmatrix},
H_d(k) = \begin{bmatrix}
0.16 & 0.20 \\
0.20 & -0.50
\end{bmatrix},
C_0 = \begin{bmatrix}
0.20 & 0 \\
-0.10 & -0.40
\end{bmatrix},
C_d = \begin{bmatrix}
0.10 & 0 \\
0 & 0.10
\end{bmatrix},
G_{10} = \begin{bmatrix}
-0.80 & 0 \\
0 & -0.80
\end{bmatrix},
G_{11} = \begin{bmatrix}
-0.80 & 0 \\
0 & 0.00
\end{bmatrix},
\tilde{G}_{10} = \begin{bmatrix}
-0.10 & 0 \\
0 & -0.10
\end{bmatrix},
\tilde{G}_{11} = \begin{bmatrix}
-0.10 & 0 \\
0 & 0.00
\end{bmatrix}.
\]

To illustrate the effectiveness of the designed filter, the simulation is carried out. Furthermore, the simulation results are presented in Figures 1 to 6, in which Figures 1 and 2 are the state trajectories \( x_i(k) \) and their filters \( \hat{x}_i(k) \) \( (i = 1, 2) \) with different initial conditions. The tracking error state estimate of \( z(k) - \hat{z}(k) \) is shown in Figure 3. It is evident that the error state estimate trends to 0, that is to say, the designed filter is observed to be effective.
Figure 4 gives the trajectory of $E\{\xi^T(k)\xi(k)\}$ and $c_2$.

Figure 5. Trajectories of $\sum_{k=0}^{K} v^T(k)v(k)$.

Figure 6. Trajectory of $\beta(k)$.

$\beta(k) = \frac{\sum_{k=0}^{K} [v^T(k)Qv(k) + 2v^T(k)Sr(k) + v^T(k)Rv(k)]}{\sum_{k=0}^{K} v^T(k)v(k)}$.

From Figure 6, we can conclude that $\beta(k)$ is always higher than $\beta^* = 1.5$, which implies (6) holds. It can be seen from the simulation results that the designed filter can effectively satisfy desired tracking performance.

**Conclusion**

The finite-time dissipative filtering problem for a kind of discrete-time stochastic interval system with time-varying delays whose parameters are taken in a convex hull has been addressed in this paper. With the help of the complex Lyapunov functional and improved Wirtinger inequality technique, the finite-time dissipative filter has also been given. Some new sufficient conditions are obtained to ensure the filter error system (5) robustly stochastically finite-time bounded and dissipative. Finally, a numerical example with simulations has demonstrated the effectiveness of the desired filter.

In the future, it is well worth to further investigate the event-triggered finite-time dissipative filtering problem for a class of Markov switching systems via interval matrix methods. We are also interested in exploring the fault detection filter design problem and the finite-time filtering problem for networked control systems via interval matrix method. Furthermore, the integration of our research with practical applications could be considered.

**Declaration of conflicting interests**

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

**Funding**

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This work is supported by National Natural Science Foundation of China with Grant No: 61473213, 61671338, and Hubei Province Key Laboratory of Systems Science in Metallurgical Process (Wuhan University of Science and Technology) with Grant No: Z201901.

**ORCID iD**

Guici Chen https://orcid.org/0000-0002-3069-0829

**References**

1. Liu Q, Liu Y and Pan X. Global stability of a stochastic predator-prey system with infinite delays. *Appl Math Comput* 2014; 235: 1–7.

2. Meng L and Bai C. Global asymptotic stability of a stochastic delayed predator-prey model with beddington-deangelis functional response. *Appl Math Comput* 2014; 226: 581–588.
3. Park P, Ko JW and Jeong C. Reciprocally convex approach to stability of systems with time-varying delays. *Automatica* 2011; 47: 235–238.

4. Zhang CK, He Y, Jiang L, et al. Notes on stability of time-delay systems: bounding inequalities and augmented Lyapunov-Krasovskii functionals. *IEEE Trans Automat Contr* 2016; 62: 5331–5336.

5. Lee TH and Park JH. A novel Lyapunov functional for stability of time-varying delay systems via matrix-refined function. *Automatica* 2017; 80: 239–242.

6. Li X, Wang R, Du S, et al. An improved exponential stability analysis method for discrete-time systems with a time-varying delay. *Int J Robust Nonlinear Control* 2022; 32(2): 669–681.

7. Conte G, Perdon A, Zattoni E, et al. Disturbance decoupling and model matching problems for discrete-time systems with time-varying delays. *Nonlinear Anal Hybrid Syst* 2021; 41: 101043.

8. Dai Z, Xu L and Ge SS. Attracting sets of discrete-time Markovian jump delay systems with stochastic disturbances via impulsive control. *J Franklin Inst* 2020; 357(14): 9781–9810.

9. Zhang CK, He Y, Jiang L, et al. Delay-variation-dependent stability of delayed discrete-time systems. *IEEE Trans Automat Contr* 2016; 61(9): 2663–2669.

10. Seuret A, Gouaisbaut F and Fridman E. Stability of discrete-time systems with time-varying delays via a novel summation inequality. *IEEE Trans Automat Contr* 2015; 60(10): 2740–2745.

11. Willems JC. Dissipative dynamical systems part I: general theory. *Arch Ration Mech Anal* 1972; 45(5): 321–351.

12. Zhang X, He S, Stojanovic V, et al. Finite-time asynchronous dissipative filtering of conic-type nonlinear Markov jump systems. *Sci China Inf Sci* 2021; 64(5): 1–12.

13. Brogliato B, Lozano R, Maschke B, et al. Dissipative systems analysis and control: theory and applications. *Meas Sci Technol* 2013; 12(12): 2211.

14. Mathiyalagan K and Ragul R. Observer-based finite-time dissipativity for parabolic systems with time-varying delays. *Appl Math Comput* 2022; 413: 126605.

15. Dorato P. Short-time stability in linear time-varying systems. New York, NY: Polytechnic Institute of Brooklyn, 1961.

16. Shen Y and Huang Y. Global finite-time stabilisation for a class of nonlinear systems. *Int J Syst Sci* 2012; 43: 73–78.

17. Lv W, Wang F and Zhang L. Adaptive fuzzy finite-time control for uncertain nonlinear systems with dead-zone input. *Int J Control Autom Syst* 2018; 16: 2549–2558.

18. Lu S, Wang X and Li Y. Adaptive neural network finite-time control filtered tracking control of fractional-order permanent magnet synchronous motor with input saturation. *J Franklin Inst* 2020; 357(18): 13707–13733.

19. Chen M, Sun J and Karimi HR. Input-output finite-time generalized dissipative filter of discrete time-varying systems with quantization and adaptive event-triggered mechanism. *IEEE Trans Cybern* 2019; 50(12): 5061–5073.

20. Gao H, Shi K, Zhang H, et al. Finite-time event-triggered extended dissipative control for a class of switched linear systems. *Int J Control Auton Syst* 2021; 19(8): 2687–2696.

21. Liu Y and Ma Y. Finite-time non-fragile extended dissipative control for ts fuzzy system via augmented Lyapunov-Krasovskii functional. *ISA Trans* 2021; 117: 1–15.

22. Li L, Wang T, Xia Y, et al. Trajectory tracking control for wheeled mobile robots based on nonlinear disturbance observer with extended Kalman filter. *J Franklin Inst* 2020; 357(13): 8491–8507.

23. Yim HYA and Cheng K-KM. Novel dual-band planar resonator and admittance inverter for filter design and applications. In: *Proceedings of the 2005 IEEE MTTS International Microwave Symposium Digest*, Long Beach, CA, USA, June 2005, pp.2187–2190.

24. Mauldin FW, Lin D and Hossack JA. The singular value filter: a general filter design strategy for pca-based signal separation in medical ultrasound imaging. *IEEE Trans Med Imaging* 2011; 30(11): 1951–1964.

25. Gao X, Deng F, Zhang H, et al. Reliable $H_{\infty}$ filtering of semi-Markov jump systems over a lossy network. *J Franklin Inst* 2021; 358: 4528–4545.

26. Gu Z, Zhou X, Zhang T, et al. Event-triggered filter design for nonlinear cyber-physical systems subject to deception attacks. *ISA Trans* 2020; 104: 130–137.

27. Chang XY and Yang GH. Non-fragile $H_{\infty}$ filter design for discrete-time fuzzy systems with multiplicative gain variations. *Inf Sci* 2014; 266: 171–185.

28. Su X, Shi P, Wu L, et al. A novel approach to filter design for T-S fuzzy discrete-time systems with time-varying delay. *IEEE Trans Fuzzy Syst* 2012; 20(6): 1114–1129.

29. Wei F, Chen G and Wang W. Finite-time stabilization of memristor-based inertial neural networks with time-varying delays combined with interval matrix method. *Knowledge Based Syst* 2021; 230: 107395.

30. Mao X, Lam J, Xu S, et al. Razumikhin method and exponential stability of hybrid stochastic delay interval systems. *J Math Anal Appl* 2006; 314(1): 45–66.

31. Zhang H, Wang Z and Liu D. Robust stability analysis for interval Cohen–Grossberg neural networks with unknown time-varying delays. *IEEE Trans Neural Netw* 2008; 19(11): 1942–1955.

32. Liu J, Ran G, Huang Y, et al. Adaptive event-triggered finite-time dissipative filtering for interval type-2 fuzzy Markov jump systems with asynchronous modes. *IEEE Trans Cybern* 2021; PP(99): 1–13.

33. Wei F, Chen G and Wang W. Finite-time synchronization of memristor neural networks via interval matrix method. *Neural Networks* 2020; 127: 7–18.

34. Chen G, Gao Y and Zhu S. Finite-time dissipative control for stochastic interval systems with time-delay and Markovian switching. *Appl Math Comput* 2017; 310: 169–181.

35. Song J, Niu Y and Wang S. Robust finite-time dissipative control subject to randomly occurring uncertainties and stochastic fading measurements. *J Franklin Inst* 2016; 354(9); 3706–3723.

36. Chen H, Li Z and Xia W. Event-triggered dissipative filter design for semi-Markovian jump systems with time-varying delays. *Math Prob Eng* 2020; 2020(1C3): 1–13.

37. Saktivel R, Nithya V, Ma YK, et al. Finite-time nonfragile dissipative filter design for wireless networked systems with sensor failures. *Complexity* 2018; 2018: 1–13.

38. Kim SH. Asynchronous dissipative filter design of non-homogeneous Markovian jump fuzzy systems via relaxation of triple-parameterized matrix inequalities. *Inf Sci* 2019; 478: 564–579.

39. Aslam MS and Li Q. Quantized dissipative filter design for Markovian switch T–S fuzzy systems with time-varying delays. *Soft Comput* 2019; 23: 11313–11329.
40. Amato F, Ariola M and Dorato P. Finite-time control of linear systems subject to parametric uncertainties and disturbances. *Automatica* 2001; 37(9): 1459–1463.

41. Chen G and Yi S. Robust $H_{\infty}$ filter design for neutral stochastic uncertain systems with time-varying delay. *J Math Anal Appl* 2009; 353(1): 196–204.

42. Wang Y, Xie L and De Souza CE. Robust control of a class of uncertain nonlinear systems. *Syst Control Lett* 1992; 19(2): 139–149.

43. Chen M and Sun J. $H_{\infty}$ finite time control for discrete time-varying system with interval time-varying delay. *J Franklin Inst* 2018; 355(12): 5037–5057.