Proposal for testing Einstein’s moon using three-time correlations

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Quantum mechanics has predicted many counterintuitive phenomena in daily life, and has changed our view of the world. Among such predictions, the existence of a macroscopic object in superposition is especially unbelievable. As Einstein asked, “Do you really believe that the moon exists only when you look at it?” However, recent experimental results on a mesoscopic scale will ultimately require us to dismiss commonsense so-called macroscopic reality. Leggett and Garg applied the Bell scheme for testing local realism to the time evolution of a macroscopic two-state system, and proposed a temporal version of the Bell inequality (the Leggett-Garg (LG) inequality) for testing macroscopic realism. However, as with the Bell inequality, the statistical approach behind this scheme may be less effective in showing clear incompatibility. Here we propose a temporal version of the Greenberger-Horne-Zeilinger (GHZ) scheme without statistical treatment for testing Einstein’s moon using three-time correlations.

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The quantum mechanical formalism has been extraordinarily successful in accounting for a wide range of microscopic phenomena, while its applicability to macroscopic scales remains unclear. One striking example is Schrödinger’s dead-and-alive cat [1], which serves to demonstrate the apparent conflict between classical and quantum theory on a macroscopic scale. It has been examined in superconducting quantum interferometer devices (SQUIDs), i.e., a superconducting loop interrupted by a thin insulating layer with a small area. The supercurrent of about one microampere circulating in the loop, which is equivalent to the net magnetic flux threading the loop, is a relevant macroscopic variable characterizing the system. Under appropriate conditions, the potential energy of the system is a “double potential well” with minima sufficiently far apart, and so regarded as macroscopically distinct. We specify these two states using the dichotomic variable \( q = \pm 1 \). The ground states in the right and left well correspond to the current circulating in the clockwise (\( q = +1 \)) and anticlockwise (\( q = -1 \)) directions, respectively. According to standard quantum mechanics, a coherent oscillation of these counterflowing current states (\(|R\rangle \) and \(|L\rangle \)) occurs due to the superposition of the states if the system is suitably decoupled from its environment (see Fig. (1)(a)). At the end of the twentieth century, the tail of the cat fabricated by modern technology has been caught in superconducting nanocircuits by observing the energy splitting caused by the superpositions of macroscopically distinct states [2, 3].

However, this is insufficient to confirm the validity of quantum mechanics on a macroscopic scale. Closely related is Einstein’s moon paradox [4], which concerns the notion of objective reality. Einstein’s argument suggests that the moon (matter) must have a separate reality with definite values at all times independent of measurements. Leggett and Garg (LG) have challenged this philosophical question by moving the discussion to the level of mathematical demonstration and experiments [5]. They first define the notion of reality on a macroscopic scale as macrorealism per se, which is implicit in much of our thinking about the macroscopic world. They then derived Bell-type inequalities in the time domain under supplementary assumptions induction and noninvasive measurability, yielding constraints on the behavior of macroscopic systems that are incompatible with some predictions of quantum mechanics. These inequalities are similar to Bell’s inequalities for the Einstein-Podolsky-Rosen (EPR) experiment [6]. In contrast to the Bell test, LG involves successive measurements at different times on a single system. The measurement times, \( t_i \), play the role of the polarizer settings in the ordinary Bell inequalities.

Despite of considerable experimental and theoretical efforts [7, 8, 9, 10, 11, 12, 13], no violation has yet been reported basically due to the problem of ensemble preparation. Greenberger, Horne and Zeilinger (GHZ) developed a proof of the Bell theorem without inequalities [14]. The entanglement of more than two particles leads to a strong conflict between local realism and non statistical predictions of quantum mechanics. In this Letter, we propose a new scheme for testing Einstein’s moon without statistical treatments by combining LG and GHZ ideas, i.e., temporal GHZ scheme.

Let us consider the measurement of the \( q \) value of the system at a certain time \( t \). One requirement for macrorealistic theories is that any measurements must be noninvasive. A negative-result measurement is one possible scheme that has been previously considered [2, 5]. Here we employ an orthodox noninvasive measurement [7], i.e., a microscopic probe. Here it is assumed that the back action of the measurement is too small to cause any great change in the consequent motion of the macroscopic sys-
the ancilla bit initially prepared as $|\pm\rangle$. In the first step of the indirect measurement, the state of the system is indirectly measured via

$$|\pm\rangle = (|R\rangle + |L\rangle)/\sqrt{2},$$

and the bonding state $|u\rangle = (|R\rangle - |L\rangle)/\sqrt{2},$ and the $z$ axis represents $|R\rangle$ and $|L\rangle$. The Pauli operators are defined as $\hat{\sigma}_x = |R\rangle\langle R| - |L\rangle\langle L|$, $\hat{\sigma}_y = i(|L\rangle\langle R| - |R\rangle\langle L|)$, and $\hat{\sigma}_z = -(|R\rangle\langle R| + |L\rangle\langle L|)$.

The Hamiltonian for the double well system in the Pauli representation is expressed as $H = \hbar \omega \hat{\sigma}_z/2$ where $\hbar \omega$ is the energy separation between the states $|\pm\rangle$.

FIG. 1: (a) The double-well system and (b) its spin representation (Bloch sphere). The $z$ axis of the Bloch sphere in (b) represents the bonding state $|\pm\rangle = (|R\rangle + |L\rangle)/\sqrt{2}$ and the antibonding state $|\mp\rangle = (|R\rangle - |L\rangle)/\sqrt{2}$, and the $x$ axis represents $|R\rangle$ and $|L\rangle$. The Pauli operators are defined as $\hat{\sigma}_x = |R\rangle\langle R| - |L\rangle\langle L|$, $\hat{\sigma}_y = i(|L\rangle\langle R| - |R\rangle\langle L|)$, and $\hat{\sigma}_z = -(|R\rangle\langle R| + |L\rangle\langle L|)$. The Hamiltonian for the double well system in the Pauli representation is expressed as $H = \hbar \omega \hat{\sigma}_z/2$ where $\hbar \omega$ is the energy separation between the states $|\pm\rangle$.

The operation in the first step of the indirect measurement, the ancilla bit probes the system by interaction with it, the system information is registered by flipping state of the ancilla bit initially prepared as $|0\rangle$ if the system has a value $q = +1$, otherwise keeps it unchanged. The ancilla then acts a memory by taking the state $|1\rangle$ or $|0\rangle$ corresponding to the system value $q = +1$ or $q = -1$, respectively. The result of this measurement step is stored tentatively in the ancilla bit. Then, the ancilla bit’s state is measured directly in the usual way as the second step.

The operation in the first step of the indirect measurement is considered to be a controlled NOT (CNOT) operation in terms of quantum information technology and is represented as a circuit diagram in Fig. 2 (a). The unitary operator for the CNOT operation can be expressed as

$$\hat{S} = |R\rangle\langle R| \otimes \hat{X} + |L\rangle\langle L| \otimes \hat{I}$$
$$= \frac{1}{2} (\hat{1} + \hat{\sigma}_z) \otimes \hat{X} + \frac{1}{2} (\hat{1} + \hat{\sigma}_x) \otimes \hat{I}$$

(1)

where $\hat{1} = |R\rangle\langle R| + |L\rangle\langle L|$ is an identical operator for the system. $\hat{X}$ and $\hat{I}$ are a flip operator and an identical operator for the ancilla bit. $\hat{\sigma}_x$ is a spin operator of the $x$ component defined as $\hat{\sigma}_x = |R\rangle\langle R| - |L\rangle\langle L|$. In the Heisenberg picture, the operator representing the CNOT operation at $t$ is expressed as

$$\hat{s}(t) = \hat{u}(t) \hat{s} \hat{u}(t) = \hat{R}_z(-\omega t) \hat{s} \hat{R}_z\dag(-\omega t)$$
$$= \frac{1}{2} \{ \hat{1} + \hat{\sigma}(-\omega t) \} \otimes \hat{X} + \frac{1}{2} \{ \hat{1} - \hat{\sigma}(-\omega t) \} \otimes \hat{I},$$

(2)

where $\hat{\sigma}(-\omega t) = \hat{\sigma}_x \cos \omega t - \hat{\sigma}_y \sin \omega t$ and the time-translational operator $\hat{u}(t)$ is equivalent to the rotational operator around the $z$ axis:

$$\hat{u}(t) = \exp \left[-i \frac{\omega t}{2} \hat{\sigma}_z \right] = \hat{R}_z(\omega t).$$

Thus the state evolution of two-level system is regarded as the spin precision under a static magnetic field directed towards the $z$ axis. This shows that time-translational operators in the Heisenberg picture play a role of the change of the polarizer angle in a spatial scheme.

Leggett and Garg introduced a two-time correlation function on a single object to test the validity of the macrorealistic theory. This scheme is essentially a temporal version of the Bell scheme, which tests the local realistic theory by using the correlation between two particles, and the same experiment must be repeated many times to obtain the average correlation value precisely in statistical treatments. On the other hand, the GHZ scheme can tests the local realistic theory without statistical treatment by using the correlation between three particles. Therefore we consider the three-time correlation of a single object for testing the macrorealistic theory without statistical treatment.

From a macrorealistic viewpoint, the system has a definite $q$ value at any time. Three CNOT operations then flip the ancilla-bit’s state in an odd-number time for $q_k q_j q_i = +1$, resulting in $|1\rangle$ and in an even-number time for $q_k q_j q_i = -1$ with the output $|0\rangle$. As a result, the system’s state is stored in the ancilla bit after all the operations and the output of the ancilla bit represents the product of the system’s values at each CNOT time, i.e., $q_k q_j q_i = \pm 1$. This product is the key to our temporal scheme as with the spatial GHZ scheme.

In a spatial GHZ test, two of three spin components, say $x$ and $y$, are used for each particle of three entangled particles. The correlation measurement is performed for a total of six variables. Similarly, a temporal GHZ scheme requires six different choices of measurement times $(t_i, i = 1 - 6)$, from which three-time combinations such as $(t_1, t_2, t_3)$ are selected for CNOT operations. We can assume that the CNOT operations for combinations $(t_1, t_4, t_5), (t_2, t_5, t_6)$, and $(t_3, t_4, t_6)$ generate $|1\rangle$ as the
Here we use the identity for the dichotomic variable \( q \). These conditions should simultaneously hold because the q-values are predetermined at the start of system evolution whether or not measurements are performed. Therefore, three successive CNOT operations cannot be satisfied in macrorealistic theories, thus there is no clear correspondence between ancilla bit value and \( \sigma_1 \) from a macrorealistic viewpoint. Therefore, \( \sigma_1 \) becomes \( \sigma_x \), resulting in the ancilla state |0⟩ for the system initial state |L⟩. This is completely opposite to the prediction inferred from macrorealistic condition (iv). Therefore, we can rule out macrorealistic theories from the clear discrepancy as regards the ancilla bit in a single run of experiments simultaneously satisfied in (8) and (9). There are a number of solutions that satisfy the above four conditions of the six successive times. One example is \((t_1, t_2, t_3, t_4, t_5, t_6) = \left\{ \frac{\pi}{2}, 1, \frac{\pi}{2}, 3, \frac{\pi}{2}, \frac{\pi}{2} \right\} \) for \((k, l, m, n) = (0, 1, 2, 1)\) as shown in Fig. 3.

A possible experimental setup that can be performed with current technology is shown in Fig. 4. A SQUID (system) is inductively coupled to the center quantum dot with a giant g-factor in a quantum nanostructure with three quantum dots. An electron probes the SQUID state through a tunneling between left and right dots since the energy level configuration in the center dot depends on the SQUID state. Fig. 4 shows an example of the CNOT operation in this system. Three CNOT operations designed in three different definitely planned times provide a final result in question.

In summary, we have proposed a nonstatistical scheme for testing the validity of macrorealistic theories using three-time correlations on a single object based on the GHZ scheme. This proposal allows us to clarify directly the incompatibility between quantum mechanics and a

ancilla-bit state without loss of generality. In terms of macroscopic realism, this is equivalent to the following three GHZ-like relations:

\[
\begin{align*}
(\text{i}) & \quad q_3 q_4 q_1 = +1 \\
(\text{ii}) & \quad q_0 q_3 q_2 = +1 \\
(\text{iii}) & \quad q_6 q_4 q_3 = +1.
\end{align*}
\]

These conditions should simultaneously hold because the \( q_i \) values are predetermined at the start of system evolution whether or not measurements are performed. Therefore, the product of these conditions yields

\[
(\text{iv}) \quad q_1 q_2 q_3 = +1.
\]

Here we use the identity for the dichotomic variable \( q^2 = 1 \). This is a key relation of a temporal GHZ scheme. Any violation of Eq. (5) completely rules out macrorealistic theories.

If the system obeys quantum mechanics, each CNOT operation only results in an entanglement between the system and the ancilla bit, and then the system does not have a definite \( q \) value until the ancilla is detected. Therefore, three successive CNOT operations cannot be regarded as a measurement of the product \( q_4 q_3 q_1 \) from a quantum-mechanical standpoint. Thus, there is no clear correspondence between ancilla bit value and \( q_4 q_3 q_1 \). However, despite this lack of correspondence, the violation of Eq. (5) can still be tested if the ancilla state is measured, since the condition (iv) implies that the ancilla state is in |1⟩ from a macrorealistic viewpoint. Therefore, the observation of the |0⟩ state for the CNOT combination \((t_1, t_2, t_3)\) can be clear evidence for the violation of macrorealism. We now show that condition (5) is indeed violated when the system obeys quantum mechanics. The successive CNOT operations at \( t_1, t_2, \) and \( t_3 \) are described as

\[
\hat{s}(t_k)\hat{s}(t_j)\hat{s}(t_i) = \frac{1}{2} \left[ \hat{X} \otimes \{ \hat{I} + \hat{\sigma}(-\omega t_k)\hat{\sigma}(-\omega t_j)\hat{\sigma}(-\omega t_i) \} \right] + \hat{I} \otimes \{ \hat{I} - \hat{\sigma}(-\omega t_k)\hat{\sigma}(-\omega t_j)\hat{\sigma}(-\omega t_i) \}.
\]

Since the relation

\[
\hat{\sigma}(-\omega t_k)\hat{\sigma}(-\omega t_j)\hat{\sigma}(-\omega t_i) = \hat{\sigma}(-\omega(t_k - t_j + t_i)),
\]

the product of three operators is formed into a single spin operator. This describes a spin operator rotated clockwise with an angle \( \omega(t_k - t_j + t_i) \) from the \( x \) in the \( x-y \) plane of the Bloch sphere. The outputs of the ancilla bit become |1⟩ if \( t_i \) satisfies the following conditions:

\[
\begin{align*}
t_5 - t_4 + t_1 & = (2k + 1)\pi/\omega \\
t_6 - t_5 + t_2 & = (2l + 1)\pi/\omega \\
t_6 - t_4 + t_3 & = (2m + 1)\pi/\omega,
\end{align*}
\]

where \( k, l, \) and \( m \) are integers. In macrorealistic terms, this means situations (i), (ii), and (iii) are achieved.

Now let us consider the following additional relation for \( t_1, t_2 \) and \( t_3 \):

\[
t_3 - t_2 + t_1 = \frac{2n\pi}{\omega}, \quad n = 0, \pm 1, \ldots
\]

This relation does not change the condition (iv) as long as the relations (8) holds in macrorealistic theories, equivalently |1⟩ for the ancilla bit. In this case, the operator (7) becomes \( \sigma_x \), resulting in the ancilla state |0⟩ for the system initial state |L⟩. This is completely opposite to the prediction inferred from macrorealistic condition (iv). Therefore, we can rule out macrorealistic theories from the clear discrepancy as regards the ancilla bit in a single run of experiments simultaneously satisfied in (8) and (9). There are a number of solutions that satisfy the above four conditions of the six successive times. One example is \((t_1, t_2, t_3, t_4, t_5, t_6) = \left\{ \frac{\pi}{2}, 1, \frac{\pi}{2}, 3, \frac{\pi}{2}, \frac{\pi}{2} \right\} \) for \((k, l, m, n) = (0, 1, 2, 1)\) as shown in Fig. 3.
realistic framework, showing clear discrepancies from realistic predictions from a series of definite measurement outputs without statistical treatment.

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