D4-D8-branes wrapped on a manifold with non-constant curvature

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Abstract

Employing the method applied to M5-branes recently by Bah, Bonetti, Minasian and Nar- doni, we study D4-D8-branes wrapped on a disk with a non-trivial holonomy at the boundary. In $F(4)$ gauged supergravity in six dimensions, we find supersymmetric $AdS_4$ solutions and uplift the solutions to massive type IIA supergravity. We calculate the holographic free energy of dual three-dimensional superconformal field theories. However, we could not express the holographic free energy in terms of the integers from flux quantization.

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1 Introduction

To our understanding of the AdS/CFT correspondence, [1], topological twisting has been essential in field theory, [2, 3, 4], and also in supergravity, [5]. Recently, there has been development from new examples of AdS/CFT beyond topological twisting, [6, 7, 8, 9, 10, 11] and [12, 13, 14].

In particular, a new way of preserving supersymmetry for solutions of gauged supergravity was developed in [12, 13] and applied to \( AdS_5 \) solutions from M5-branes. The dual field theory was proposed to be the Argyres-Douglas theory, [15], from 6d \( \mathcal{N} = (2,0) \) theories on a sphere with irregular punctures. This method was soon applied to \( AdS_3 \) solutions from D3-branes in [14]. See also [16].

In this paper, we apply the new method of obtaining supersymmetric solutions of gauged supergravity to the D4-D8 brane system. Five-dimensional superconformal field theories were first discovered in [17, 18] and their gravity dual was proposed, [19], and found to be supersymmetric \( AdS_6 \times S^4 \) solution, [20], of massive type IIA supergravity, [21]. This solution is also realized as the supersymmetric fixed point, [22], of \( F(4) \) gauged supergravity in six dimensions, [23].

From topological twisting in \( F(4) \) gauged supergravity, D4-D8-brane system wrapped on supersymmetric two- and three-cycles was studied in [24, 25] and also from massive type IIA supergravity in [26]. The D4-D8-brane system wrapped on a supersymmetric four-cycle provides the horizon geometry of supersymmetric \( AdS_6 \) black holes, [27], and the Bekenstein-Hawking entropy was shown to match the field theory calculation of topologically twisted index, [28, 29].
See [30] also for non-supersymmetric solutions. So far, solutions were obtained from D4-D8-branes wrapped on supersymmetric cycles with constant curvature.

In this paper, we study D4-D8-branes wrapped on a disc with non-trivial holonomy at the boundary. In particular, we construct supersymmetric $AdS_4$ solutions of $F(4)$ gauged supergravity and uplift the solutions to massive type IIA supergravity. We calculate the holographic free energy of dual three-dimensional superconformal field theories. However, due to the complexity of the expressions along the calculation, we were not able to express the holographic free energy in terms of the integers from flux quantization.

In section 2, we review $F(4)$ gauged supergravity in six dimensions. In section 3, we construct supersymmetric $AdS_4$ solutions and uplift the solutions to massive type IIA supergravity. In section 4, we conclude and discuss some open questions. The equations of motion are relegated in appendix A.

2 $F(4)$ gauged supergravity in six dimensions

We review $SU(2) \times U(1)$-gauged $\mathcal{N} = 4$ supergravity in six dimensions [23]. The bosonic field content consists of the metric, $g_{\mu\nu}$, a real scalar, $\phi$, an $SU(2)$ gauge field, $A^I_\mu$, $I = 1, 2, 3$, a $U(1)$ gauge field, $A_\mu$, and a two-form gauge potential, $B_{\mu\nu}$. The fermionic field content is gravitinos, $\psi_{\mu i}$, and dilatinos, $\chi_i$, $i = 1, 2$. The field strengths are defined by

\[
\begin{align*}
\mathcal{F}_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu, \\
F_{\mu\nu}^I &= \partial_\mu A^I_\nu - \partial_\nu A^I_\mu + ge^{IJK}A^J_\mu A^K_\nu, \\
G_{\mu\nu\rho} &= 3\partial_{[\mu}B_{\nu\rho]}, \\
\mathcal{H}_{\mu\nu} &= \mathcal{F}_{\mu\nu} + mB_{\mu\nu}.
\end{align*}
\]

(2.1)

The bosonic Lagrangian is given by

\[
\begin{align*}
e^{-1} \mathcal{L} &= -\frac{1}{4}R + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi + \frac{1}{8} \left( g^2 e^{2\phi} + 4gem^{-2\phi} - m^2 e^{-3\phi} \right) \\
&\quad -\frac{1}{4}e^{-\sqrt{2}\phi} \left( \mathcal{H}_{\mu\nu} \mathcal{H}^{\mu\nu} + F^I_{\mu\nu} F^{I\mu\nu} \right) + \frac{1}{12} e^{2\sqrt{2}\phi} G_{\mu\nu\rho} G^{\mu\nu\rho} \\
&\quad -\frac{1}{8} \epsilon^{\mu\nu\rho\sigma\tau\kappa} B_{\mu\nu} \left( \mathcal{F}_{\rho\sigma} \mathcal{F}_{\tau\kappa} + mB_{\rho\sigma} \mathcal{F}_{\tau\kappa} + \frac{1}{3} m^2 B_{\rho\sigma} B_{\tau\kappa} + F^I_{\rho\sigma} F^{I}_{\tau\kappa} \right),
\end{align*}
\]

(2.2)
where $g$ is the $SU(2)$ gauge coupling constant and $m$ is the mass of the two-form gauge potential. The supersymmetry transformations of the fermionic fields are

\[
\delta \psi^i = \nabla_\mu \epsilon^i + g A^I_\mu (T^I)_i^j \epsilon_j - \frac{1}{8\sqrt{2}} \left( ge^{-\frac{\phi}{\sqrt{2}}} + me^{-\frac{3\phi}{\sqrt{2}}} \right) \gamma_\mu \gamma_7 \epsilon^i \\
- \frac{1}{8\sqrt{2}} e^{-\frac{\phi}{\sqrt{2}}} (F_{\nu\lambda} + m B_{\nu\lambda}) (\gamma^{\mu}_{\nu} \gamma^{\lambda}_{\mu} - 6 \delta^{\mu}_{\nu} \gamma^\lambda) \epsilon^i \\
- \frac{1}{4\sqrt{2}} e^{-\frac{\phi}{\sqrt{2}}} F^I_{\nu\lambda} (\gamma^{\mu}_{\nu} \gamma^{\lambda}_{\mu} - 6 \delta^{\mu}_{\nu} \gamma^\lambda) \gamma_7 (T^I)_i^j \epsilon_j \\
- \frac{1}{24} e^{\sqrt{2}\phi} G_{\nu\lambda\rho} \gamma_\mu \gamma^\rho \gamma_\mu \epsilon^i, (2.3)
\]

\[
\delta \chi^i = \frac{1}{\sqrt{2}} \gamma^\mu \partial_\mu \phi \epsilon^i + \frac{1}{4\sqrt{2}} \left( ge^{-\frac{\phi}{\sqrt{2}}} - 3me^{-\frac{3\phi}{\sqrt{2}}} \right) \gamma_7 \epsilon^i \\
+ \frac{1}{4\sqrt{2}} e^{-\frac{\phi}{\sqrt{2}}} (F_{\mu\nu} + m B_{\mu\nu}) \gamma^{\mu\nu} \epsilon^i \\
+ \frac{1}{2\sqrt{2}} e^{-\frac{\phi}{\sqrt{2}}} F^I_{\mu\nu} \gamma^{\mu\nu} \gamma_7 (T^I)_i^j \epsilon_j \\
- \frac{1}{12} e^{\sqrt{2}\phi} G_{\mu\nu\lambda} \gamma_7 \gamma^{\mu\nu} \epsilon^i, (2.4)
\]

where $T^I, I = 1, 2, 3,$ are the $SU(2)$ left-invariant one-forms,

\[
T^I = -\frac{i}{2} \sigma^I. (2.5)
\]

Described by the above Lagrangian, there are five inequivalent theories: $\mathcal{N} = 4^+ (g > 0, m > 0), \mathcal{N} = 4^- (g < 0, m > 0), \mathcal{N} = 4^g (g > 0, m = 0), \mathcal{N} = 4^m (g = 0, m > 0), \mathcal{N} = 4^0 (g = 0, m = 0).$ The $\mathcal{N} = 4^+$ theory admits a supersymmetric $AdS_6$ fixed point when $g = 3m$. At the supersymmetric $AdS_6$ fixed point, all the fields are vanishing except the $AdS_6$ metric.

### 3 Supersymmetric $AdS_4$ solutions

#### 3.1 Supersymmetry equations

We consider the background,

\[
ds^2 = f(r) ds^2_{AdS_4} + g_1(r) dr^2 + g_2(r) d\theta^2, (3.1)
\]

with the gauge fields,

\[
A^1 = A^2 = 0, \quad A^3 = A_\theta(r) d\theta, (3.2)
\]

and the scalar field, $\phi = \phi(r)$. The gamma matrices are given by

\[
\gamma^\alpha = \rho^\alpha \otimes \sigma^3, \quad \gamma^\theta = i\mathbb{I}_4 \otimes \sigma^1, \quad \gamma^\tilde{\theta} = i\mathbb{I}_4 \otimes \sigma^2, (3.3)
\]

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where $\alpha = 0, 1, 2, 3$ are four-dimensional flat indices and the hatted indices are flat indiced for the corresponding coordinates. $\rho^\alpha$ are four-dimensional gamma matrices with $\{\rho^\alpha, \rho^\beta\} = 2\eta^{\alpha\beta}$ and $\sigma^{1,2,3}$ are the Pauli matrices. The four- and six-dimensional chirality matrices are defined to be, respectively,

$$\rho_* = i\rho^0\rho^1\rho^2\rho^3, \quad \gamma_7 = \pm \rho_* \otimes \sigma^3,$$

(3.4)

The spinor is given by

$$\epsilon_i = n_i \vartheta \otimes \eta,$$

(3.5)

where $\vartheta$ is a Killing spinor on $AdS_4$ and $\eta = \eta(r, \theta)$. The Killing spinors satisfy

$$\nabla_{AdS_4}^\alpha \vartheta = \frac{1}{2} s \rho_\alpha \rho_* \vartheta,$$

(3.6)

where $s = \pm 1$.

The supersymmetry equations are obtained by setting the supersymmetry variations of the fermionic fields to zero. From the supersymmetry variations, we obtain

\begin{align*}
0 &= -s \frac{i}{2} \gamma^i \gamma_7 \epsilon_i + \frac{1}{4} f' f^{1/2} g_1^{1/2} \gamma^i \epsilon_i - \frac{1}{8\sqrt{2}} \left( g e^{\frac{\phi}{\sqrt{2}}} + m e^{-3\phi \sqrt{2}} \right) f^{1/2} \gamma_7 \epsilon_i \\
&\quad - \frac{1}{4\sqrt{2}} e^{-\frac{\phi}{\sqrt{2}}} A'_0 f^{1/2} g_1^{1/2} g_2^{1/2} \gamma^i \gamma_7 g_2 T^3 \epsilon_i,
\end{align*}

(3.7)

\begin{align*}
0 &= \partial_r \epsilon_i + \frac{1}{8\sqrt{2}} \left( g e^{\frac{\phi}{\sqrt{2}}} + m e^{-3\phi \sqrt{2}} \right) g_1^{1/2} \gamma^i \gamma_7 \epsilon_i + \frac{3}{4\sqrt{2}} e^{-\frac{\phi}{\sqrt{2}}} A'_0 \frac{1}{g_2} g_1^{1/2} \gamma^i \gamma_7 g_2 T^3 \epsilon_i,
\end{align*}

\begin{align*}
0 &= \partial_\theta \epsilon_i + \frac{1}{2} g A_0^2 T^3 \epsilon_i + \frac{3}{4\sqrt{2}} e^{-\frac{\phi}{\sqrt{2}}} A'_0 \frac{1}{g_1^{1/2} g_2} \gamma^i \gamma_7 \epsilon_i + \frac{1}{8\sqrt{2}} \left( g e^{\frac{\phi}{\sqrt{2}}} + m e^{-3\phi \sqrt{2}} \right) g_2^{1/2} \gamma^i \gamma_7 \epsilon_i + \frac{3}{4\sqrt{2}} e^{-\frac{\phi}{\sqrt{2}}} A'_0 \frac{1}{g_1^{1/2} g_2} g_2 T^3 \epsilon_i,
\end{align*}

where the first three and the last equations are from the spin-3/2 and spin-1/2 field variations, respectively. By multiplying suitable functions and gamma matrices and adding the last equation
to the first three equations, we obtain

\[ 0 = -s \left( \frac{i}{2} \gamma \gamma \epsilon_i + \frac{1}{2} g_1^{-1/2} f^{1/2} \left[ \frac{1}{2} f' + \frac{1}{\sqrt{2}} \phi' \right] \right) \gamma \gamma \epsilon_i - \frac{m}{2 \sqrt{2}} e^{-\frac{3 \phi}{\sqrt{2}}} f^{1/2} \gamma \gamma \epsilon_i, \]

\[ 0 = \partial \gamma \epsilon_i + \frac{1}{2 \sqrt{2}} \phi' \epsilon_i + \frac{m}{2 \sqrt{2}} e^{-\frac{3 \phi}{\sqrt{2}}} g_1^{1/2} \gamma \gamma \epsilon_i + \frac{1}{\sqrt{2}} g_2^{-1/2} e^{-\frac{\phi}{\sqrt{2}}} A_\theta \gamma \theta \gamma_2 (T_3)^i j \epsilon_j, \]

\[ 0 = \partial \gamma \epsilon_i + \frac{1}{2} g A_\theta (T_3)^i j \epsilon_j - \frac{1}{\sqrt{2}} g_1^{-1/2} e^{-\frac{\phi}{\sqrt{2}}} A_\theta \gamma \theta \gamma_2 (T_3)^i j \epsilon_j + \frac{1}{2} g_1^{-1/2} g_2^{1/2} \left[ \frac{1}{2} g_2 + \frac{1}{\sqrt{2}} \phi' \right] \gamma \theta \epsilon_i \]

\[ + \frac{m}{2 \sqrt{2}} e^{-\frac{3 \phi}{\sqrt{2}}} g_2^{1/2} \gamma \gamma \epsilon_i, \]

\[ 0 = \frac{1}{4 \sqrt{2}} \left( g e^{\frac{\phi}{\sqrt{2}}} - 3 m e^{-\frac{3 \phi}{\sqrt{2}}} \right) \gamma \gamma \epsilon_i + \frac{1}{2} \phi' g_1^{1/2} \gamma \epsilon_i + \frac{1}{\sqrt{2}} e^{-\frac{\phi}{\sqrt{2}}} A_\theta g_1^{-1/2} g_2^{-1/2} \gamma \gamma \epsilon_i (T_3)^i j \epsilon_j, \]

(3.8)

The spinor is supposed to have a charge under the \( U(1) \) isometry,

\[ \eta(r, \theta) = e^{im \tilde{g} - \eta(r)}, \]

(3.9)

where \( m \) is a constant. It shows up in the supersymmetry equations in the form of \( ( -i \partial \theta + \frac{1}{2} A_\theta ) \eta = ( n + \frac{1}{2} A_\theta ) \eta \) which is invariant under

\[ A^3 \mapsto A^3 - 2 \alpha_0 d \theta, \quad \eta \mapsto e^{i \alpha_0 \eta}, \]

(3.10)

where \( \alpha_0 \) is a constant. We also define

\[ \frac{1}{2} \tilde{A}_\theta = n + \frac{1}{2} A_\theta. \]

(3.11)

We solve the equation of motion for the gauge fields and obtain

\[ A_\theta = b e^{\sqrt{2} \theta} g_1^{1/2} g_2^{1/2} f^{-2}, \]

(3.12)

where \( b \) is a constant. Employing the expressions we discussed in (3.8) beside the second equation, we finally obtain the supersymmetry equations,

\[ 0 = -i s f^{1/2} \eta + g_1^{1/2} \left[ \frac{1}{2} f' + \frac{1}{\sqrt{2}} \phi' \right] (\sigma^1 \eta) \pm i \frac{m}{\sqrt{2}} e^{-\frac{3 \phi}{\sqrt{2}}} (\sigma^3 \eta), \]

\[ 0 = g g_2^{-1/2} \tilde{A}_\theta (\sigma^1 \eta) \mp i \sqrt{2} f^{-2} e^{\phi} b (\sigma^3 \eta) + g_1^{-1/2} \left[ \frac{1}{2} g_2 + \frac{1}{\sqrt{2}} \phi' \right] (i \sigma^2 \eta) \pm i \frac{m}{\sqrt{2}} e^{-\frac{3 \phi}{\sqrt{2}}} \eta, \]

\[ 0 = \mp i \frac{1}{2 \sqrt{2}} \left( g e^{\frac{\phi}{\sqrt{2}}} - 3 m e^{-\frac{3 \phi}{\sqrt{2}}} \right) \eta + \sqrt{2} g_1^{-1/2} \phi' (i \sigma^2 \eta) \mp i \frac{1}{\sqrt{2}} f^{-2} e^{\phi} b (\sigma^3 \eta). \]

(3.13)

The supersymmetry equations are in the form of \( M^{(i)} \eta = 0, i = 1, 2, 3 \), where \( M^{(i)} \) are three \( 2 \times 2 \) matrices, as we follow [12, 13],

\[ M^{(i)} = X_0^{(i)} \mathbb{1}_2 + X_1^{(i)} \sigma^1 + X_2^{(i)} (i \sigma^2) + X_3^{(i)} \sigma^3. \]

(3.14)
We rearrange the matrices to introduce $2 \times 2$ matrices,

$$
A^{ij} = \det(v^{(i)}|w^{(j)}) , \quad B^{ij} = \det(v^{(i)}|v^{(j)}) , \quad C^{ij} = \det(w^{(i)}|w^{(j)}) ,
$$

from the column vectors of

$$
v^{(i)} = \begin{pmatrix} X^{(i)}_1 + X^{(i)}_2 \\ -X^{(i)}_1 - X^{(i)}_3 \end{pmatrix} , \quad w^{(i)} = \begin{pmatrix} X^{(i)}_0 - X^{(i)}_3 \\ -X^{(i)}_1 + X^{(i)}_2 \end{pmatrix} .
$$

From the vanishing of $A^{ij}$, $B^{ij}$ and $C^{ij}$, necessary conditions for non-trivial solutions are obtained. From $A^{ii} = 0$, we find

$$
0 = -\frac{1}{f} - \frac{1}{4g_1} \left( f' + \sqrt{2}\phi' \right)^2 + \frac{m^2 e^{-3\sqrt{2}\phi}}{2} ,
$$

$$
0 = 2b^2 e^{\sqrt{2}\phi} \frac{f'}{f^3} + \frac{1}{4g_1} \left( \frac{g'}{g_2} + \sqrt{2}\phi' \right)^2 - \frac{m^2 e^{-3\sqrt{2}\phi}}{2} - \frac{g^2 A^2}{g_2} ,
$$

$$
0 = \frac{2}{g_1} \frac{(\phi')^2}{2f^4} + \frac{b^2 e^{\sqrt{2}\phi}}{2f^4} - \frac{1}{8} \left( ge^{-\sqrt{2}\phi} - 3m e^{-\frac{3\phi}{\sqrt{2}}} \right)^2 .
$$

From $A^{ij} + A^{ii} = 0$, we find

$$
0 = -\frac{2b m e^{-\frac{2\phi}{\sqrt{2}}}}{f^2} - \frac{\sqrt{2} s m e^{-\frac{3\phi}{\sqrt{2}}}}{f^2} - \frac{g \hat{A}_G}{\sqrt{g_1} \sqrt{g_2}} \left( f' + \sqrt{2}\phi' \right) ,
$$

$$
0 = \frac{b m e^{-\frac{2\phi}{\sqrt{2}}}}{f^2} + \frac{s}{\sqrt{2} f^2} \left( ge^{-\sqrt{2}\phi} - 3m e^{-\frac{3\phi}{\sqrt{2}}} \right) ,
$$

$$
0 = \frac{2b^2 e^{\sqrt{2}\phi}}{g_1} \frac{g'}{g_2} + \frac{\sqrt{2}\phi'}{g_1} \left( \frac{g'}{g_2} + \sqrt{2}\phi' \right) + \frac{m e^{-\frac{3\phi}{\sqrt{2}}}}{g_1} \left( ge^{-\sqrt{2}\phi} - 3m e^{-\frac{3\phi}{\sqrt{2}}} \right) .
$$

From $A^{ij} - A^{ii} = 0$, we find

$$
0 = \frac{1}{2g_1} \left( f' \frac{f}{f} + \sqrt{2}\phi' \right) \left( \frac{g'}{g_2} + \sqrt{2}\phi' \right) - \frac{2\sqrt{2} s b e^{\frac{\phi}{\sqrt{2}}}}{f^{5/2}} - \frac{m^2 e^{-3\sqrt{2}\phi}}{2} ,
$$

$$
0 = \frac{\sqrt{2} s b e^{\frac{\phi}{\sqrt{2}}}}{f^{5/2}} - \frac{\sqrt{2} \phi'}{g_1} \left( f' \frac{f}{f} + \sqrt{2}\phi' \right) - \frac{m e^{-\frac{3\phi}{\sqrt{2}}}}{2} \left( ge^{\frac{\phi}{\sqrt{2}}} - 3m e^{-\frac{3\phi}{\sqrt{2}}} \right) ,
$$

$$
0 = -\frac{m b e^{-\sqrt{2}\phi}}{f^2} - \frac{b e^{-\frac{\phi}{\sqrt{2}}}}{f^2} \left( ge^{-\sqrt{2}\phi} - 3m e^{-\frac{3\phi}{\sqrt{2}}} \right) + \frac{2\sqrt{2} \phi' g \hat{A}_G}{\sqrt{g_1} \sqrt{g_2}} .
$$

(3.15)
From $B^{ij} + C^{ij} = 0$, we find

$$0 = -\frac{\sqrt{2} be^{\frac{2}{\sqrt{2}}} \left( \frac{f'}{f} + \sqrt{2} \phi' \right)}{f' \sqrt{g_1}} - \frac{s}{\sqrt{f' \sqrt{g_1}}} \left( \frac{g'_2}{g_2} + \sqrt{2} \phi' \right) - \frac{\sqrt{2} b e^{\frac{2}{\sqrt{2}}} m \hat{A}_\theta}{\sqrt{g_2}},$$

$$0 = \frac{b e^{\frac{2}{\sqrt{2}}} \left( \frac{f'}{f} + \sqrt{2} \phi' \right)}{\sqrt{2} f' \sqrt{g_1}} + \frac{4 s \phi'}{\sqrt{2} \sqrt{f' \sqrt{g_1}}},$$

$$0 = -\frac{1}{2 \sqrt{2} \sqrt{g_1}} \left( \frac{g'_2}{g_2} + \sqrt{2} \phi' \right) \left( g e^{\frac{2}{\sqrt{2}}} - 3 m e^{-\frac{3}{2}\phi} \right) - \frac{2 m e^{-\frac{3}{2}\phi'}}{\sqrt{g_1}} - \frac{\sqrt{2} b g e^{\frac{2}{\sqrt{2}}} \hat{A}_\theta}{f' \sqrt{g_2}}. \quad (3.20)$$

From $B^{ij} - C^{ij} = 0$, we find

$$0 = \frac{m e^{-\frac{3}{2}\phi}}{\sqrt{2} \sqrt{g_1}} \left( \frac{f'}{f} + \sqrt{2} \phi' \right) - \frac{m e^{-\frac{3}{2}\phi}}{\sqrt{2} \sqrt{g_1}} \left( \frac{g'_2}{g_2} + \sqrt{2} \phi' \right) - \frac{2 s g \hat{A}_\theta}{\sqrt{f' \sqrt{g_2}}},$$

$$0 = \frac{1}{2 \sqrt{2} \sqrt{g_1}} \left( \frac{f'}{f} + \sqrt{2} \phi' \right) \left( g e^{\frac{2}{\sqrt{2}}} - 3 m e^{-\frac{3}{2}\phi} \right) + \frac{2 m e^{-\frac{3}{2}\phi'}}{\sqrt{g_1}},$$

$$0 = -\frac{b e^{\frac{2}{\sqrt{2}}} \left( \frac{g'_2}{g_2} + \sqrt{2} \phi' \right)}{\sqrt{2} f' \sqrt{g_1}} \frac{4 b e^{\frac{2}{\sqrt{2}}} \phi'}{f' \sqrt{g_1}} \frac{g \hat{A}_\theta}{\sqrt{2} \sqrt{g_1}} \left( g e^{\frac{2}{\sqrt{2}}} - 3 m e^{-\frac{3}{2}\phi} \right). \quad (3.21)$$

### 3.2 Supersymmetric solutions

From the second equation of (3.18), we obtain

$$f = \frac{2^{1/3} b^{2/3} m^{2/3}}{e^{\frac{2}{\sqrt{2}}} \left( s \left( g e^{\frac{2}{\sqrt{2}}} - 3 m e^{-\frac{3}{2}\phi} \right) \right)^{2/3}}, \quad (3.22)$$

Then, from the third equation of (3.17) with (3.22), we obtain

$$g_1 = \frac{16^{2/3} b^{2/3} m^{2/3}}{\left( s \left( g e^{\frac{2}{\sqrt{2}}} - 3 m e^{-\frac{3}{2}\phi} \right) \right)^{2/3}} \left( 2^{1/3} b^{2/3} m^{8/3} - 2 e^{\frac{2}{\sqrt{2}}} \left( s \left( g e^{\frac{2}{\sqrt{2}}} - 3 m e^{-\frac{3}{2}\phi} \right) \right)^{2/3} \right) \frac{g \hat{A}_\theta}{\sqrt{2} \sqrt{g_1}} \left( g e^{\frac{2}{\sqrt{2}}} - 3 m e^{-\frac{3}{2}\phi} \right), \quad (3.23)$$

From the third equation of (3.18), we find an expression for $\sqrt{g_1} \sqrt{g_2}$,

$$\sqrt{g_1} \sqrt{g_2} = \frac{2^{1/3} g f^2 \hat{A}_\theta \phi'}{b e^{\frac{2}{\sqrt{2}}} \left( g e^{\frac{2}{\sqrt{2}}} - 2 m e^{-\frac{3}{2}\phi} \right)}. \quad (3.24)$$

Also from (3.12), we find another expression for $\sqrt{g_1} \sqrt{g_2}$,

$$\sqrt{g_1} \sqrt{g_2} = \frac{e^{-\frac{3}{2}\phi} f^2 \hat{A}_\theta'}{b}. \quad (3.25)$$

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Equating (3.24) and (3.25), we find an ordinary differential equation for $\hat{A}_\theta$ and it gives

$$\hat{A}_\theta = C e^{\frac{3\phi}{\sqrt{2}}} \left( g e^\frac{\phi}{\sqrt{2}} - 2m e^{-\frac{3\phi}{\sqrt{2}}} \right),$$

(3.26)

where $C$ is a constant. From (3.11), we find

$$A_\theta = C e^{\frac{3\phi}{\sqrt{2}}} \left( g e^\frac{\phi}{\sqrt{2}} - 2m e^{-\frac{3\phi}{\sqrt{2}}} \right) + n.$$  

(3.27)

Then, from (3.24) or (3.25), we obtain

$$g_2 = \frac{C^2 g^2 \left( 2^{1/3} b^{2/3} m^{8/3} - 2 e^{\frac{22\phi}{\sqrt{2}}} \left( s g e^\frac{\phi}{\sqrt{2}} - 3m e^{-\frac{3\phi}{\sqrt{2}}} \right)^2 \right)}{e^{\frac{4\phi}{\sqrt{2}}} \left( s g e^\frac{\phi}{\sqrt{2}} - 3m e^{-\frac{3\phi}{\sqrt{2}}} \right)^{2/3}}.$$  

(3.28)

Therefore, we have determined all functions in terms of the scalar field, $\phi(r)$, and its derivative. The solution satisfies all the supersymmetry equations in (3.17) to (3.21) and the equations motion which we present in appendix A. We can determine the scalar field by fixing the ambiguity in reparametrization of $r$ due to the covariance of the supersymmetry equations,

$$\phi(r) = \sqrt{2} \log r,$$

(3.29)

where $r > 0$.

Finally, let us summarize the solution. The metric is given by

$$ds^2 = \frac{B r^{2/3}}{m^2 (s (g r^4 - 3m))^2/3} \left[ ds^2_{AdS_4} + \frac{32 m^2 r^{10/3}}{h(r) (s (g r^4 - 3m))^{4/3}} dr^2 + \frac{C^2 g^2 m^2 h(r)}{B} d\theta^2 \right],$$

(3.30)

where we define

$$h(r) = B - 2r^{16/3} (s (g r^4 - 3m))^{2/3},$$

(3.31)

with

$$B = 2^{1/3} b^{2/3} m^{8/3}.$$  

(3.32)

The gauge field is given by

$$\hat{A}_\theta = C (g r^4 - 2m).$$

(3.33)

The metric can also be written as

$$ds^2 = \frac{B r^{2/3}}{m^2 (s (g r^4 - 3m))^2/3} ds^2_{AdS_4} + \frac{32B r^4}{h(r) (s (g r^4 - 3m))^{2/3}} dr^2 + \frac{C^2 g^2 r^{2/3} h(r)}{(s (g r^4 - 3m))^{2/3}} d\theta^2.$$  

(3.34)

Now we consider the range of $r$ for regular solutions, i.e., the metric functions are positive definite and the scalar fields are real. We find regular solutions when we have

$$s = 1, \quad 0 < r < r_1,$$

(3.35)
where
\[
r_1 = \frac{1}{\sqrt{2g^{1/4}}} \left( 4m + 2^{1/3} \left( -1 + i\sqrt{3} \right) \left( bg^2m^4 + 4m^3 + \sqrt{bg^2m^7(bg^2m + 8)} \right)^{1/3} + \frac{2^{5/3} \left( -1 + i\sqrt{3} \right) m^2}{\left( bg^2m^4 + 4m^3 + \sqrt{bg^2m^7(bg^2m + 8)} \right)^{1/3}} \right)^{1/4},
\]
(3.36)
and \( r_1 \) is determined from \( h(r_1) = 0 \). We plot a representative solution with \( b = -0.1, C = 1 \) and \( g = 3m = 3 \) in Figure 1.

Approaching \( r = r_1 \), the metric becomes to be
\[
ds^2 = \frac{Br_r^{2/3}}{m^2 (s (gr_r^4 - 3m))^{2/3}} \left[ ds_{AdS_4}^2 + \frac{128m^2r^{10/3} [dr^2 + C^2\mathcal{E}^2(b; g, m)r^2d\theta^2]}{-h'(r_1) (s (gr_r^4 - 3m))^{4/3}} \right],
\]
where we introduced a new parametrization of coordinate, \( \rho^2 = r_1 - r \) and \( \mathcal{E}(b; g, m) \) is an unwieldy function of \( b \) which we do not reproduce here. Then, the \( \rho-\theta \) surface is locally an \( \mathbb{R}^2/\mathbb{Z}_l \) orbifold if we set
\[
C = \frac{1}{l\mathcal{E}(b; g, m)},
\]
(3.38)
where \( l = 1, 2, 3, \ldots \).

Employing the Gauss-Bonnet theorem, we calculate the Euler characteristic of \( \Sigma \), the \( r-\theta \) surface, from (3.30). The boundary at \( r = 1 \) is a geodesic and thus has vanishing geodesic curvature. The only contribution to the Euler characteristic is
\[
\chi(\Sigma) = \frac{1}{4\pi} \int_\Sigma R_\Sigma vol_\Sigma = \frac{2\pi}{4\pi b^{1/3}m^{1/3}} \left( gr_1^4 - 2m \right) \left( s (gr_r^4 - 3m) \right)^{1/3} = C\mathcal{E}(b; g, m) = \frac{1}{l},
\]
(3.39)
where \( 0 < \theta < 2\pi \). This result is natural for a disc in an \( \mathbb{R}^2/\mathbb{Z}_l \) orbifold at \( r = 1 \) with \( g = 3m \).
3.3 Uplift to massive type IIA supergravity

We introduce the bosonic field content of $F(4)$ gauged supergravity, \cite{23}, in the conventions of \cite{22}. There are the metric, the real scalar field, $\phi$, an $SU(2)$ gauge field, $A^I$, $I = 1$, 2, 3, a $U(1)$ gauge field, $A$, and a two-form gauge potential, $B$. Their field strengths are defined by

$$
\tilde{F}^I_2 = dA^I + \frac{1}{2}g e^{IJK} A_J \wedge A_K , \\
\tilde{F}_2 = dA + \frac{2}{3} u B , \\
\tilde{F}_3 = dB .
$$

(3.41)

The uplift formula for $F(4)$ gauged supergravity to massive type IIA supergravity was obtained in \cite{22}. For our solutions, the only non-trivial fields are the metric, the dilaton, and the four-form flux, respectively,

$$
ds^2_{10} = X^{1/8} \sin^{1/12} \xi \left( \Delta^{3/8} ds_6^2 + \frac{2}{g^2} \Delta^{3/8} X^2 d\xi^2 + \frac{1}{2g^2} \cos^2 \xi \Delta^{5/8} X ds_3^2 \right) ,
$$

(3.42)

$$
e^\Phi = \frac{\Delta^{1/4}}{X^{5/4} \sin^{5/6} \xi} ,
$$

(3.43)

$$
F_4 = -\frac{\sqrt{2} U}{6} \frac{\sin^{1/3} \xi \cos^3 \xi}{g^3 \Delta^2} d\xi \wedge \text{vol}_3 - \frac{\sin^{4/3} \xi \cos^4 \xi}{g \Delta^4 X^3} dX \wedge \text{vol}_3
$$

$$
+ \frac{1}{\sqrt{2}} \frac{\sin^{1/3} \xi \cos \xi}{g^2} \tilde{F}^I_2 \wedge h^I \wedge d\xi
- \frac{1}{4\sqrt{2}} \frac{\sin^{4/3} \xi \cos^2 \xi}{g^2 \Delta^2 X^3} \tilde{F}^I_2 \wedge h^I \wedge h^K \epsilon_{IK} .
$$

(3.44)

We employ the metric and the volume form on the gauged three-sphere by

$$
ds^2_{3/3} = \sum_{i=1}^{3} (\sigma^I - gA^I)^2 ,
$$

$$
\text{vol}_{3/3} = h_1 \wedge h_2 \wedge h_3 ,
$$

(3.45)

where

$$
h^I = \sigma^I - gA^I ,
$$

(3.46)

\footnote{The couplings and fields in the uplift formula for pure $F(4)$ gauged supergravity to massive type IIA supergravity in \cite{22} are related to the ones of \cite{23} by}

$$
\tilde{g} = 2g , \quad X = e^{-\phi/\sqrt{2}} = e^{\tilde{\phi}/\sqrt{2}} , \quad \tilde{A}^I_\mu = \frac{1}{2} A^I_\mu , \quad \tilde{A}_\mu = \frac{1}{2} A_\mu , \quad \tilde{B}_{\mu\nu} = \frac{1}{2} B_{\mu\nu} ,
$$

(3.40)

where the tilded ones are of \cite{23}.
and $\sigma^I, I = 1, 2, 3,$ are the $SU(2)$ left-invariant one-forms which satisfy
\begin{equation}
\quad \quad \quad \quad \quad \quad \quad \quad d\sigma^I = -\frac{1}{2} \epsilon_{IJK} \sigma^J \wedge \sigma^K .
\end{equation}

A choice of the left-invariant one-forms is
\begin{align*}
\sigma^1 &= -\sin \alpha_2 \cos \alpha_3 d\alpha_1 + \sin \alpha_3 d\alpha_2 , \\
\sigma^2 &= \sin \alpha_2 \sin \alpha_3 d\alpha_1 + \cos \alpha_3 d\alpha_2 , \\
\sigma^3 &= \cos \alpha_2 d\alpha_1 + d\alpha_3 .
\end{align*}

We also defined quantities
\begin{align*}
X &= e^{\frac{\phi}{\sqrt{2}}} , \\
\Delta &= X \cos^2 \xi + X^{-3} \sin^2 \xi , \\
U &= X^{-6} \sin^2 \xi - 3 X^2 \cos^2 \xi + 4 X^{-2} \cos^2 \xi - 6 X^{-2} .
\end{align*}

For our solutions, we have $X = r$. In particular, for our solutions, the metric can also be written by
\begin{equation}
ds_{10}^2 = \frac{B^3}{m^2 (s (gr^4 - 3m))^2/3} \left[ ds_{AdS_4}^2 + \frac{32m^2 r^{10/3}}{h (s (gr^4 - 3m))^{1/3}} dr^2 + \frac{C^2 g^2 m^2 h}{B} d\theta^2 
\right. \\
+ \frac{8m^2 r^2 (s (gr^4 - 3m))^{2/3}}{B g^2 r^{2/3}} d\xi^2 + \frac{2m^2 \cos^2 \xi (s (gr^4 - 3m))^{2/3}}{B g^2 r^{5/3} \Delta} ds_{S^3}^2 \right] ,
\end{equation}

where we have
\begin{equation}
\Delta = r \cos^2 \xi + r^{-3} \sin^2 \xi .
\end{equation}

### 3.4 Flux quantization

We consider the flux quantization condition for the five-form flux. The integral of the four-form flux over any four-cycle in the internal space is an integer, see, e.g., [26],
\begin{equation}
\frac{1}{(2\pi l_s)^3} \int_{M_4} F_{(4)} \in \mathbb{Z} .
\end{equation}

First, we consider the $F_{(4)} \xi_{\alpha_1 \alpha_2 \alpha_3}$ component of the four-form flux and we obtain
\begin{equation}
\frac{1}{(2\pi l_s)^3} \int F_{(4)} \xi_{\alpha_1 \alpha_2 \alpha_3} = \frac{1}{(2\pi l_s)^3} \int \left( -\frac{\sqrt{2} U \sin 1/3 \xi \cos^3 \xi}{g^2 \Delta^2} \right) d\xi \wedge \text{vol}_{S^3} = \frac{3}{16\sqrt{2\pi l_s^3} g^3} \equiv N ,
\end{equation}

\footnote{For the scalar field, $\phi$, we follow the normalization of [23] than [22].}
where \( \text{vol}_{\mathbb{R}^3} = 2\pi^2 \) and \( N \in \mathbb{N} \) is the number of D4-D8-branes wrapping the two-dimensional manifold, \( \Sigma \).

Second, we consider the \( F_{(4) r\theta\alpha\beta\gamma} \) component of the four-form flux and we obtain

\[
\frac{1}{(2\pi l_s)^3} \int F_{(4) r\theta\alpha\beta\gamma} = \frac{1}{(2\pi l_s)^3} \int \frac{1}{\sqrt{2}} \frac{\sin^{1/3} \xi \cos \xi}{g^2} \tilde{F}_{(2)} \wedge d\alpha \wedge d\xi = \frac{3C r_1^4}{8\sqrt{2}2l_s^3 g} .
\]  

(3.54)

Plugging \( C \) from (3.38) and \( l_s \) from (3.53), we obtain

\[
\frac{1}{(2\pi l_s)^3} \int F_{(4) r\theta\alpha\beta\gamma} = \frac{N l_s^2}{l} \mathcal{E}(b; g, m) \equiv K
\]

(3.55)

where \( K \in \mathbb{Z} \) is another integer. However, due to the complexity of \( r_1(b; g, m) \) and \( \mathcal{E}(b; g, m) \), we were not able to solve for \( b \) and \( C \) in terms of two integers, \( N \) and \( K \).

### 3.5 Holographic free energy

Now we calculate the holographic free energy of dual 3d SCFTs. Consider the metric of the form,

\[
ds_{10}^2 = e^{2A} (ds_{AdS_4}^2 + ds_{M_6}^2) ,
\]

(3.56)

in the Einstein frame. The formula for holographic free energy is given in e.g., [26],

\[
\mathcal{F} = \frac{16\pi^3}{(2\pi l_s)^8} \int_{M_6} e^{8A - 2\Phi} \text{vol}_{M_6} ,
\]

(3.57)

where the ten-dimensional metric is in the string frame, \( ds_{\text{string}}^2 = e^{\Phi/2} ds_{\text{Einstein}}^2 \) and \( l_s \) is the string length. Employing the formula, we find

\[
\mathcal{F} = \frac{16\pi^3}{(2\pi l_s)^8} \int 32\sqrt{2} B^{3/2} C r_3 \cos^3 \xi \sin^{1/3} \xi \frac{g^3 m^2 (s (gr^4 - 3m))^2}{g^3 m^2 (s (gr^4 - 3m))^2} \text{vol}_{\mathbb{R}^3} dr d\theta d\xi = \frac{16\pi^3}{(2\pi l_s)^8} \left[ \frac{36 bC m^2}{5g^4 (s (gr^4 - 3m))^2} \right]_{r_{\text{min}}}^{r_{\text{max}}} 2\pi 2\pi^2 ,
\]

(3.58)

where we have \( 0 < \theta < 2\pi, 0 < \xi < \frac{\pi}{2} \), and \( \text{vol}_{\mathbb{R}^3} = 2\pi^2 \). For the solutions in (3.35) with \( r_{\text{min}} = 0 \) and \( r_{\text{max}} = r_1 \), we obtain the holographic central charge,

\[
\mathcal{F} = \frac{3}{5\pi^2 l_s^8} \frac{bC m r_1^4}{g^3 (gr_1^4 - 3m)} ,
\]

(3.59)

where \( r_1 \) is given in (3.36). Even though the uplifted solutions have singularities, we obtain a well-defined finite result for central charge. However, as we were not able to solve for \( b \) and \( C \) in terms of two integers, \( N \) and \( K \), due to the complexity of \( r_1(b; g, m) \) and \( \mathcal{E}(b; g, m) \), we could not express the holographic free energy in terms of two integers, \( N \) and \( K \).
4 Conclusions

Employing the method applied to M5-branes recently by [12, 13], we constructed supersymmetric \( AdS_4 \) solutions from D4-D8-branes wrapped on a two-dimensional manifold with non-constant curvature. We uplifted the solutions to massive type IIA supergravity and calculated the holographic free energy of dual three-dimensional superconformal field theories. However, as a function of magnetic charge, \( b \), the expression of \( r_1(b; g, m) \) of the range of solution, \( 0 < r < r_1 \), turns out to be unwieldy and it prevents us from obtaining the holographic free energy in terms of two integers, \( N \) and \( K \), from flux quantization.

Therefore, the first natural question would be to obtain a simpler expression of holographic free energy in order to compare with the free energy from field theory calculations. For this purpose, the three-dimensional superconformal field theory which is dual to the solution should be identified as well.

In this work, we only constructed a class of \( AdS_4 \) fixed points from D4-D8-branes on a non-constant curvature manifold. The holographic RG flow from the \( AdS_6 \) fixed point dual to 5d superconformal field theories would enable us to understand more details of the solution.

From matter coupled \( F(4) \) gauged supergravity, [31], wrapped D4-D8-brane solutions on constant-curvature manifolds were previously studied in [32] [33] [34]. We would like to generalize our solutions in matter coupled \( F(4) \) gauged supergravity. See also [35].

Among twist compactifications of branes and their dual field theories, D4-D8-brane system is lesser understood and we look forward to seeing development to come in the future.

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A The equations of motion

In this appendix, we present the equations of motion of $F(4)$ gauged supergravity,

$$R_{\mu\nu} = 2\partial_\mu\phi\partial_\nu\phi + \frac{1}{8}g_{\mu\nu}(g^2e^{\sqrt{2}\phi} + 4ge^{-\sqrt{2}\phi} - m^2e^{-3\sqrt{2}\phi}) - 2e^{-\sqrt{2}\phi}\left(H_\mu^\rho H_\nu^\rho - \frac{1}{8}g_{\mu\nu}H_\rho H_\rho^\rho\right)$$

$$- 2e^{-\sqrt{2}\phi}\left(F^I_\mu F^I_\nu - \frac{1}{8}g_{\mu\nu}F^I_\rho F^I_\rho\right) + e^{\sqrt{2}\phi}\left(G_\mu^\rho G_\nu^\rho - \frac{1}{6}g_{\mu\nu}G_\rho G_\rho^\rho G^\rho\rho\right),$$

(A.1)

$$\frac{1}{\sqrt{-g}}\partial_\nu\left(\sqrt{-g}g^{\mu\nu}\partial_\mu\phi\right) = \frac{1}{4\sqrt{2}}\left(g^2e^{\sqrt{2}\phi} - 4ge^{-\sqrt{2}\phi} + 3m^2e^{-3\sqrt{2}\phi}\right) + \frac{1}{2\sqrt{2}}e^{-\sqrt{2}\phi}\left(H_{\mu\nu}H^{\mu\nu} + F^I_\mu F^{I\mu}\right) + \frac{1}{3\sqrt{2}}e^{2\sqrt{2}\phi}G_{\mu\nu\rho}G^{\mu\nu\rho},$$

(A.2)

$$\mathcal{D}_\nu\left(e^{-\sqrt{2}\phi}H^{\nu\mu}\right) = \frac{1}{6}e^{\mu\nu\rho\sigma\tau\kappa}H_{\nu\rho}G_{\sigma\tau\kappa},$$

(A.3)

$$\mathcal{D}_\nu\left(e^{-\sqrt{2}\phi}F^{I\nu\mu}\right) = \frac{1}{6}e^{\mu\nu\rho\sigma\tau\kappa}F^I_{\nu\rho}G_{\sigma\tau\kappa},$$

(A.4)

$$\mathcal{D}_\rho\left(e^{2\sqrt{2}\phi}G^{\rho\mu}\right) = -\frac{1}{4}e^{\mu\nu\rho\sigma\tau\kappa}\left(H_{\rho\sigma}H_{\tau\kappa} + F^{I}_{\rho\sigma}F^I_{\tau\kappa}\right) - me^{-\sqrt{2}\phi}H^{\mu\nu}.$$  

(A.5)

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