Scale-Free Overlay Topologies with Hard Cutoffs for Unstructured Peer-to-Peer Networks

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Abstract

In unstructured peer-to-peer (P2P) networks, the overlay topology (or connectivity graph) among peers is a crucial component in addition to the peer/data organization and search. Topological characteristics have profound impact on the efficiency of search on such unstructured P2P networks as well as other networks. It has been well-known that search on small-world topologies of \( N \) nodes can be as efficient as \( O(\ln N) \), while scale-free (power-law) topologies offer even better search efficiencies like as good as \( O(\ln \ln N) \) for a range of degree distribution exponents. However, generation and maintenance of such scale-free topologies are hard to realize in a distributed and potentially uncooperative environments as in the P2P networks.

A key limitation of scale-free topologies is the high load (i.e. high degree) on very few number of hub nodes. In a typical unstructured P2P network, peers are not willing to maintain high degrees/loads as they may not want to store large number of entries for construction of the overlay topology. So, to achieve fairness and practicality among all peers, hard cutoffs on the number of entries are imposed by the individual peers, which limits scale-freeness of the overall topology. Thus, efficiency of the flooding search reduces as the size of the hard cutoff does. We investigate construction of scale-free topologies with hard cutoffs (i.e. there are not any major hubs) and effect of these hard cutoffs on the search efficiency. Interestingly, we observe that the efficiency of normalized flooding and random walk search algorithms increases as the hard cutoff decreases.

I. INTRODUCTION

In decentralized P2P networks, the overlay topology (or connectivity graph) among peers is a crucial component in addition to the peer/data organization and search. Topological characteristics have profound impact on the efficiency of search on P2P networks as well as other networks. It has been well-known that search on small-world topologies can be as efficient as \( O(\ln N) \) [1], and this phenomenon has recently been studied on P2P networks [2], [3], [4], [5].

The best search efficiency in realistic networks can be achieved when the topology is scale-free (power-law), which offers search efficiencies like \( O(\ln \ln N) \). However, generation and maintenance of such scale-free topologies are hard to realize in a distributed and potentially uncooperative environments as in the P2P networks. Another key limitation of scale-free topologies is the high load (i.e. high degree) on very few number of hub nodes. In a typical unstructured P2P network, peers are not willing to maintain high degrees/loads as they may not want to store large number of entries for construction of the overlay topology. So, to achieve fairness and practicality among all peers, hard cutoffs on the number of entries are imposed by the individual peers. These hard cutoffs might limit scale-freeness of the overall topology, by which we mean having a network with a power-law degree distribution from which an exponent can be obtained properly. Thus, it is expected that the search efficiency reduces as the size of the hard cutoff does.

The primary focus of this paper is to (i) investigate construction of scale-free topologies with hard cutoffs (i.e. there are not any major hubs) in a distributed manner without requiring global topology information at the time when nodes join and (ii) to investigate the effect of these hard cutoffs on the search efficiency.

The rest of the paper is organized as follows: First, in the rest of this section, we provide motivation for this work, outline key dimensions to be considered, and briefly indicate major contributions and findings of the work. Then, we survey previous work on P2P networks in Section II. In Section III, we survey the previous work on scale-free topology generation and cover two specific models that we use: Preferential Attachment (PA), Configuration Model (CM). We introduce our practical topology generation methodologies, Hop-and-Attempt Preferential Attachment (HAPA) and Discover-and-Attempt Preferential Attachment (DAPA), in Section IV. In Section V, we present our simulations of three different search algorithms (i.e., Flooding (FL), Normalized Flooding (NF), and Random Walk (RW)) on topologies generated by the models PA, CM, HAPA, and DAPA. We conclude by summarizing the work and outlining future directions in Section VI.

A. Motivation and Key Considerations

The search efficiency on small-world and scale-free topologies is well-known. Although scale-free topologies are superior in search efficiency, their super-hub-based structure makes them vulnerable to threats and impractical due to unfair assignment
of network load on a very small subset of all nodes. As peers in a P2P network are typically not fully cooperative, protocols cannot rely on methods working with full cooperation of peers. For example, peers may not want to store large number of entries for construction of the overlay topology, i.e. connectivity graph. Even though characteristics of the overlay topology is crucial in determining the efficiency of the network, peers typically do not want to take the burden of storing excessive amount of control information for others in the network. Effect of this on the overlay topology maintenance is that peers impose hard cutoffs on the amount of control information to be stored. Since P2P overlay topology generation and maintenance are very important for realizing a scalable unstructured P2P network, the main focus of this paper is to investigate the effect of the hard cutoffs on the overall search efficiency.

A key issue is the construction of scale-free overlay topologies without global information. There are several techniques to generate a scale-free topology [6], [7], which rely on global information about the current network when a new node joins. Such global methods are not practical in P2P networks, and local heuristics in generating such scale-free overlay topologies with hard cutoffs is the key issue, which we investigate in this paper. In other words, each peer has to figure out the optimal way of joining the P2P overlay by only using the locally available (i.e. immediate/close neighbors) information, and also causing a minimal inefficiency to the search mechanisms to be run on the network.

B. Contributions and Major Results

This paper touches an uncovered set of research problems relating to tradeoffs between maximum number of links a peer can (or is willing to) store and the efficiency of search on an overlay topology composed of such peers. We defined the maximum number of links to be stored by peers as the hard cutoff for the degree of a peer in the network as compared to natural cutoff which occurs due to finite-size effects. Our contributions include:

- **Scale-free topology generation methods**: We studied two well-known scale-free topology generation mechanisms (i.e. Preferential Attachment (PA) and Configuration Model (CM)) which use global information about the overlay topology within the context of unstructured P2P networks. We introduced two novel mechanisms (i.e. HAPA and DAPA) that use local topology information solely or partially.
- **Search efficiency on scale-free topologies with hard cutoffs**: Through extensive simulations, we studied efficiency of FL, NF, and RW on the topologies generated by the four mechanisms PA, CM, HAPA, and DAPA.
- **Guidelines for designing peer join algorithms for unstructured P2P networks**: Our study yielded several guidelines for peers to join to a Gnutella-like unstructured P2P network, so that the search performance of the overall overlay topology remains high.

Our study of hard cutoffs enlightened several interesting issues. We found that hard cutoffs may not always affect the search performance adversely, unlike the expected wisdom that the value of the power-law exponent in the degree distribution of the topology is directly related to search performance. We showed that search performance depends on the particular search algorithm being used as well as on the topological characteristics including the exponent of the degree distribution, connectedness (minimum degree is a measure for it in scale-free networks), hard cutoff, and locality.

We also showed that there is an interplay between the connectedness and the degree distribution exponent for a fixed cutoff; more specifically, if connectedness is too low in the topology, then one can improve search performance by applying smaller hard cutoffs. Our simulation experiments clearly showed that practical search algorithms like NF or repeated RWs can perform better on scale-free topologies with smaller hard cutoffs as long as peers join carefully, e.g. as in HAPA and DAPA mechanisms. As a particular guideline for optimizing joining techniques of peers, we showed that, as long as every peer is required to maintain a minimum of 2-3 links to the rest of the network rather than just one link, it is possible to diminish negative effects of hard cutoffs on search performance.

II. RELATED WORK

Our work is related to peer-to-peer (P2P) network protocol designs and topological analysis of complex networks. Previous work on P2P network protocols can be classified into centralized and decentralized ones. As centralized P2P protocols (e.g. Napster [8]) proved to be unscalable, the majority of the P2P research has focused on decentralized schemes. The decentralized P2P schemes can be further classified into sub-categories: structured, unstructured, and hybrid.

In the structured P2P networks, data/file content of peers is organized based on a keying mechanism that can work in a distributed manner, e.g. Distributed Hash Tables (DHTs) [9], CAN [10], Chord [11], Kademlia [12], Oceanstore [13]. The keying mechanism typically maps the peers (or their content) to a logical search space, which is then leveraged for performing efficient searches. Another positive side of the structured schemes is the guarantees of finding rare items in a timely manner. However, the cost comes from complexity of maintaining the consistency of mapping the peers to the logical search space, which typically causes considerable amount of control traffic (e.g. join/leave messaging) for highly dynamic P2P environments. Due to their capability of locating rare items, structured approaches have been very well suited to a wide-range of various applications, e.g. [14], [15], [16], [17], [18].
In contrast to the structured schemes, unstructured P2P networks do not include a strict organization of peers or their content. Since there is no particular keying or organization of the content, the search techniques are typically based on flooding. Thus, the searches may take very long time for rare items, though popular items can be found very fast due to possible leveraging of locality of reference [19], [20], [21] and caching/replication [22], [23].

To balance the tradeoffs between structured and unstructured schemes, hybrid approaches (e.g. [24], [25], [26]) have attempted attaining a middle-ground between the costly maintenance of global peer/data keying of structured schemes and the high cost searches of unstructured schemes. Typically, hybrid schemes include a localized (or reduced in size) data/peer keying (e.g. DHT among neighbors instead of among all nodes) to achieve faster discovery of rare items, and a probabilistic search among the partially organized peers.

Since our work is more applicable to unstructured P2P networks, we focus our survey in this section to that category of the previous work. Also, since we propose to use scale-free topologies in constructing the overlay P2P topology, we survey the literature on scale-free network topologies in the next section.

A. Unstructured P2P Networks

The main focus of the research on unstructured P2P networks has been the tradeoff between state complexity of peers (i.e. number of records needed to be stored at each peer) and flooding-based search efficiency. The minimal state each peer has to maintain is the list of neighbor peers, which construct the overlay topology. Optionally, peers can maintain forwarding tables (also referred as routing tables in the literature) for data items in addition to the list of neighbor peers. Thus, we can classify unstructured P2P networks into two based on the type(s) of state peers maintain: (i) per-data unstructured P2P networks (i.e. peers maintain both the list of neighbor peers and the per-data forwarding table), and (ii) non-per-data unstructured P2P networks (i.e. peers maintain only the list of neighbor peers).

Non-per-data schemes are mainly Gnutella-like schemes [27], where search is performed by means of flooding query packets. Search performance over such P2P networks has been studied in various contexts, which includes pure random walks [28], probabilistic flooding techniques [29], [30], and systematic filtering techniques [31].

Per-data schemes (e.g. Freenet [32]) can achieve better search performances than non-per-data schemes, though they impose additional storage requirements to peers. By making the peers maintain a number of <key,pointer> entries peers direct the search queries to more appropriate neighbors, where “key” is an identifier for the data item being searched and the “pointer” is the next-best neighbor to reach that data item. This capability allows peers to leverage associativity characteristics of search queries [33]. Studies ranged from grouping peers of similar interests (i.e. peer associativity) [5], [33] to exploiting locality in search queries (i.e. query associativity) [34], [20].

Our work is applicable to both per-data and non-per-data unstructured P2P networks, since we focus on the interactions between search efficiency and topological characteristics.

III. Scale-Free (Power-Law) Network Topologies

Recent research shows that many natural and artificial systems such as the Internet [35], World Wide Web [36], scientific collaboration network [37], and e-mail network [38] have power-law degree (connectivity) distributions. These systems are commonly known as power-law or scale-free networks since their degree distributions are free of scale (i.e. not a function of the number of network nodes N) and follow power-law distributions over many orders of magnitude. This phenomenon has been represented by the probability of having nodes with k degrees as $P(k) \sim k^{-\gamma}$ where $\gamma$ is usually between 2 and 3 [6]. Scale-free networks have many interesting properties such as high tolerance to random errors and attacks (yet low tolerance to attacks targeted to hubs) [39], high synchronizability [40], [41], [42], and resistance to congestion [43], [44].

The origin of the scale-free behavior can be traced back to two mechanisms that are present in many systems, and have a strong impact on the final topology [6]. First, networks are developed by the addition of new nodes that are connected to those already present in the system. This mechanism signifies continuous expansion in real networks. Second, there is a higher probability that a new node is linked to a node that already has a large number of connections. These two features led to the formulation of a growing network model first proposed by Barábasi and Albert that generates a scale-free network for which $P(k)$ follows a power law with $\gamma=3$. This model is known as preferential attachment (PA or rich get richer mechanism) and the resulting network is called Barábasi-Albert network [6], [7].

In this study, we use a simple version of the PA model [6]. The model evolves by one node at a time and this new node is connected to m (number of stubs) different existing nodes with probability proportional to their degrees, i.e., $P_i = \frac{k_i}{\sum_j k_j}$ where k_i is the degree of the node i. The average degree per node in the resulting network is 2m. Fig. 1(a) shows the degree distributions of scale-free networks generated by the PA model with different m values. The links are regarded as bidirectional links; however, the results can easily be generalized to directed networks as well [7]. The special case of the PA procedure is
When the number of stubs is one (i.e. \( m=1 \)) in which a scale-free tree without clustering (loops) is generated. The algorithm for the PA model is presented in Appendix A.

Scale-free networks are very robust against random failures and attacks since the probability to hit the hub nodes (few nodes with very large degree) is very small and attacking the satellite nodes with just a few degree does not harm the network. On the other hand, deliberate attacks targeted to hubs through which most of the traffic goes can easily shatter the network and severely damage the overall communication in the network. For the same reason the Internet is called “robust yet fragile” [45] or “Achilles heel” [39], [46].

Scale-free networks also have small-world [47] properties. In small-world networks the diameter, or the mean hop distance between the nodes scales with the system size (or the number of network nodes) \( N \) logarithmically, i.e., \( d \sim \ln N \). The scale-free networks with \( 2 < \gamma < 3 \) have a much smaller diameter and can be named ultra-small networks [48], behaving as \( d \sim \ln \ln N \). When \( \gamma = 3 \) and \( m \geq 2 \), \( d \) behaves as in \( d \sim \ln N/\ln \ln N \). However, when \( m = 1 \) for \( \gamma = 3 \) the Barabási-Albert model turns into a tree and \( d \sim \ln N \) is obtained. Also when \( \gamma > 3 \), the diameter still behaves logarithmically \( d \sim \ln N \). These relationships are summarized in Table I. Since the speed/efficiency of search algorithms strongly depend on the average shortest path, scale-free networks have much better performance in search than other random networks.

### A. The Cutoff

One of the important characteristics of scale-free networks is the degree cutoff (or maximum degree) due to the finite-size effects. Aiello et al. [49] defined a natural cutoff \( k_{nc} = k_{max} \) value of the degree, above which the expected number of nodes is 1, that is

\[
NP(k = k_{nc}) \sim 1.
\]  

For a scale-free network, when one substitutes \( P(k) \propto k^{-\gamma} \)

\[
k_{nc}(N) \sim N^{1/\gamma}.
\]

This definition of natural cutoff degree used in many P2P network studies lacks some mathematical rigor since it considers the probability of a single point in a probability distribution, which is not completely well-defined in the continuous \( k \) limit for large \( N \) [50].

A more physical definition of cutoff was given by Dorogovtsev et al. [51], defining it as the value of the degree above which one expects to find at most one vertex. This meant the satisfaction of the condition:

\[
N \int_{k_{nc}}^{\infty} P(k)dk \sim 1.
\]
Choosing at random from the desired degree distribution

In CM, the vertices of the graph are assigned a fixed sequence of degrees expected degree distribution and no degree correlations; however, it allows self-loops and multiple-connections when it is used. Then, pairs of nodes are chosen randomly and connected by undirected edges. This model generates a network with the

B. Preferential Attachment with Hard Cutoffs

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is bounded with a finite hard cutoff value from above it is observed that the absolute value of degree distribution exponent decreases as in Fig. 1(c). A more detailed description of the PA model is provided in Appendix A.

Again by using the degree distribution for the scale-free network and the exact form of probability distribution (i.e., $P(k) = (\gamma - 1)m^{\gamma - 1}/k^\gamma$), one obtains

$k_{nc}(N) \sim mN^{1/(\gamma-1)}$, \hspace{1cm} (4)

which is known as the natural cutoff of the network. The scaling of the natural cutoff can also be calculated by using the extreme-value theory [50]. For the scale-free networks generated by PA model ($\gamma = 3$) the natural cutoff becomes

$k_{nc}(N) \sim m\sqrt{N}$. \hspace{1cm} (5)

B. Preferential Attachment with Hard Cutoffs

The natural cutoff may not be always attainable for most of the scale-free networks due to technical reasons. One main reason is that the network might have limitations on the number of links the nodes can have. This is especially important for P2P networks in which nodes cannot possibly connect many other nodes. This requires putting an artificial or hard cutoff $k_c$ to the number of links one node might have.

In order to see the effect of hard cutoff in PA, we simply did not allow nodes to have links more than a fixed hard cutoff value during the attachment process. This modified method generates a scale-free network in which there are many nodes with degree fixed to hard cutoff instead of a few very high degree hubs and the degree distribution still decays in a power law fashion. The degree distributions of scale-free networks generated by PA with different hard cutoff values are shown in Fig. 1(b). As can be seen in the figure they are slightly different than PA without a cutoff in terms of exponent except that they have an accumulation of nodes with degree equal to hard cutoff. PA model, in its original form, has a constant degree to the number of links one node can have.

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degree distribution exponent. Deleting this discrepancies also causes some very negligible number of nodes in the network to have degrees less than the fixed minimum degree \( m \) value, even zero, as seen in Fig. 2. One another characteristic of the CM is that the network is not a connected network when \( m=1 \), i.e., it has disconnected clusters (or components). For \( m>1 \), the network is almost surely connected having one giant component including all the nodes. The algorithm we use for CM can be seen in Appendix B.

The main disadvantage of PA and CM methods is that they require global knowledge about the network, i.e., the degrees of all peers or the maximum/total degree, which might be usually difficult to store and share by the nodes for unstructured P2P networks. This motivated us to modify the PA procedure so that it makes use of local information as much as possible. In the next section we explore the local heuristics in creating scale-free networks and introduce two new attachment models.

**IV. LOCAL HEURISTICS FOR SCALE-FREE OVERLAY TOPOLOGY CONSTRUCTION**

In the PA model as outlined in the previous section, the new node has to make random attempts to connect to the existing nodes with a certain probability depending on the degree of the existing node. To implement this in a P2P (or any distributed) networks, the new node has to have information about the global topology (e.g. the current number of degrees each node has for the PA model), which might be very hard to maintain in reality. Such global topology information is needed in the CM method as well.

Thus, in order for a topology construction mechanism to be practical in P2P networks, it must allow joining of new nodes by just using locally available information. Of course, the cost of using only local information is expected to be loss of scale-freeness (or any other desired characteristics) of the whole overlay topology, which will result in loss of search efficiency in return. In this section, we present two practical methods using local heuristics not necessarily using global information about the latest topology: HAPA and DAPA.

**A. Hop-and-Attempt Preferential Attachment (HAPA)**

In this method, the new node randomly selects an existing node and attempts to connect. Then it randomly selects a node which is a neighbor of the previously selected node and attempts to connect. Thus, the new node hops between the neighboring nodes and attempts to connect by using the existing links in the network until it fills all its stubs, i.e., the number of links it has reaches \( m \). The algorithm for HAPA model can be seen in Appendix C.

This hopping process gives a better chance to the new node to find the high-degree hubs in the network than the PA does since the hubs in scale-free networks are only a couple of hops away from the low-degree nodes and it is less likely to find hubs by random node selection. So, some nodes in the network (probably they are the initial nodes and their number is \( m+1 \) due to network generation algorithm) become dominant and attract almost all the nodes to themselves, thus deserve the name super hubs. The super hubs have degrees on the order of network size. It is easily seen that this procedure makes the topology of the system a star-like topology if the network is not limited by a cutoff. Naturally, without a hard cutoff the degree distribution is not a power-law and the average shortest path/diameter is very small with respect to scale-free networks generated by PA [see Fig. 3(a)]. As shown in Figs. 3(b) and 3(c), when a hard cutoff is introduced the degree distribution gets closer to a power law having an exponent \( \gamma = 3 \) but with possibly exponential factors making a degree exponent calculation very hard.
distribution on the unit interval. Then, any two randomly placed nodes are linked if they are closer to each other than some distance $R$. Two and three dimensional GRNs have been widely used in continuum percolation and real networks modeling (see references in [61]). GRNs have Poissonian degree distributions in the form $P(k) = e^{-\bar{k}k}/k!$, where $\bar{k}$ is the average degree.

In 2D, if $R > R_{critical}=0.012$ for $N=10^4$ corresponding to critical average degree $\bar{k}=4.52$, the GRN has a giant component [61]. Throughout the paper we use GRN as a substrate network with an average degree $\bar{k}=10$ and size $N = 2 \times 10^4$ because GRN is topologically closer to real life nodes in the Internet than a regular or highly random network.

In this model, initially, a few nodes are randomly selected from the substrate network and added to the previously empty network on this substrate network by using the PA method among the set of nodes visible/reachable to a specific node (the horizon of the node) in a number of steps, which we call local time-to-live (or local TTL) and represent with $\tau_{sub}$.

DAPA model starts with a substrate network $G_S$ with $N_S$ nodes. The topology for the substrate network may be a two-dimensional regular network (mesh with nodes connected to four neighbors in four different directions) or a geometric (Euclidean) random network (GRN) [61] with a giant component. GRN is a random network with a metric. It is constructed by assigning each node random coordinates in a $d$-dimensional box of unit volume, i.e., each coordinate is drawn from a uniform distribution on the unit interval. Then, any two randomly placed nodes are linked if they are closer to each other than some distance $R$. Two and three dimensional GRNs have been widely used in continuum percolation and real networks modeling (see references in [61]).

$B. \ Discover-and-Attempt \ Preferential \ Attachment \ (DAPA)$

DAPA model imitates the method for finding peers in Gnutella-like unstructured P2P networks. First, we assume that we have a network called substrate network with a predefined and preconstructed topology at hand. Then, we construct an overlay network called DAPA model imitates the method for finding peers in Gnutella-like unstructured P2P networks. First, we assume that we have a network called substrate network with a predefined and preconstructed topology at hand. Then, we construct an overlay network by using the PA method among the set of nodes visible/reachable to a specific node (the horizon of the node) in a number of steps, which we call local time-to-live (or local TTL) and represent with $\tau_{sub}$.

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overlay network, $G_O$, then these nodes are connected to each other in $G_O$. At each step one random node is chosen in the substrate network and let it send a query to its neighborhood reachable in $\tau_{sub}$ hops to get a list of peers in its horizon. Then by using the rules of preferential attachment the new node connects to $m$ peers with probability proportional to their degrees divided by the total degrees of the peers in its horizon. If the number of peers in the horizon is less than $m$, then the new node connects to peers it can find. The nodes which can find at least one peer in their horizon is added to the $G_O$ and becomes a peer. A peer belonging to $G_O$ can not be selected again to look for new peers. This process is continued until the number of peers in $G_O$ reaches the desired number $N_O$. The algorithm for DAPA model is presented in Appendix D.

The degree distribution of the network generated by DAPA model exhibits some interesting characteristics. For small values of $\tau_{sub}$, the nodes are shortsighted, i.e., they cannot see enough peers in their short horizon causing the degree distribution to be an exponential. For high enough $\tau_{sub}$ values, the degree distribution changes into a power-law. Thus, one can go from an exponential to a scale-free network by playing with the measure of locality ($\tau_{sub}$), as can be seen in Fig. 4. As the hard cutoff gets smaller the difference between the degree distributions becomes invisible. For higher values of $m$ (i.e., $m>1$), it is possible to find peers with degree less than $m$ as in Figs. 4(d-f), since some nodes cannot find enough peers in their horizon to fill all their stubs. The degree distribution exponent has a similar behavior to PA as we change the hard cutoff value, i.e., as the cutoff decreases the exponent increases [see Fig. 4(g)]. The data in Fig. 4(g) is very noisy and the data points contain quite large error bars because they are obtained from very scattered degree distribution tails.

A comparison of different network generation models in terms of locality can be seen in Table II. When a peer is to join the current overlay topology, the PA and CM do need global information about the current topology whilst HAPA and DAPA methods use local information partially or fully, respectively. Therefore, HAPA and DAPA methods are more practical in the context of unstructured P2P networks.

**C. Effect of Hard Cutoffs on Topological Characteristics**

Depending on the way one applies hard cutoffs to an initially scale-free topology results in different topological characteristics, such as the degree distribution, diameter (or expected search efficiency). Table I summarizes the inter-relationship between these three dimensions that PA models were studied in literature.

When we applied hard cutoffs to the regular PA topologies, their degree distribution looked like a power-law distribution except that a major jump happens in the frequency of nodes having degree equal to the hard cutoff [see Fig. 1(b)]. Unlike the original PA topologies without any cutoff [see Fig. 1(a)], these topologies exhibit different power-law exponents when the jump on the hard cutoffs is taken into account. We measured the estimated power-law exponent for these PA topologies with hard cutoffs and plotted Fig. 1(c), which shows the power-law exponent of the degree distribution with respect to the hard cutoff that was applied to the original PA topology. As expected, Fig. 1(c) shows that the degree distribution exponent $\gamma$ degrades to lower values when harder cutoffs are applied, suggesting that search efficiency (in connection with the diameter size) will also degrade for smaller cutoffs.

The CM does not allow changes in the degree distribution exponent because the degree sequence is drawn from a predefined distribution generated by using a specific degree exponent [see Fig. 2]. The only change in degree distribution exponent is due to the deletion of self loops and multiple connections and can be considered negligible. It is also observed that applying harder (smaller) cutoffs to the degrees decreases the probability to have self loops and multiple connections.

In HAPA model, it is not even possible to say that we still have power-law degree distributions. Without a hard cutoff the degree distribution decreases very fast as the degree increases and there are a few nodes with degree on the order of system size, i.e., star-like topology [see Fig. 3(a)]. Applying a cutoff destroys the star-like topology and changes the degree distribution into a one similar to power-law with exponential cutoff dependent correction as can be seen in Figs. 3(b,c).

DAPA model is qualitatively very similar to PA for high enough $\tau_{sub}$ values [see Fig. 4]. The small $\tau_{sub}$ makes the network an exponential one. By tuning this parameter, one can change the degree distribution from exponential to power-law. As in the PA model, applying harder cutoff increases the degree distribution exponent as can be seen from Fig. 4(g).

**V. Simulations**

In P2P networks that do not have a central server including Gnutella and Freenet files are found by forwarding queries to one’s neighbors until the target is found. In the previous sections, in addition to studying well-known techniques like Preferential

| Procedure | Usage of Global Information |
|-----------|-----------------------------|
| PA        | Yes                         |
| CM        | Yes                         |
| HAPA      | Partial                     |
| DAPA      | No                          |
Attachment (PA) and Configuration Model (CM) for scale-free topology construction, we introduced new algorithms (i.e. HAPA and DAPA) with the same purpose within the context of unstructured P2P networks. Here, we study a number of message-passing algorithms that can be efficiently used to search items in P2P networks utilizing the power-law (the presence of hubs) degree distribution in sample networks generated by our topology construction algorithms. These algorithms are completely decentralized and do not use any kind of global knowledge on the network. We consider three different search algorithms: flooding (FL), normalized flooding (NF), and random walk (RW). Goals of our simulation experiments include:

- **Effect of hard cutoffs on search efficiency:** Applying hard cutoffs on power-law topologies reduces the degree distribution exponent, which should affect the search efficiency (i.e. number of hits per unit time) on such topologies. We are interested in observing this effect for the three search algorithms on the topologies constructed by our topology construction algorithms.
- **Topology construction with global vs. local information:** Though we showed in the previous section that using local information when a peer is joining yields a less scale-free topology, the effect of this on search efficiency still needs to be shed light on. Our simulations aim to investigate this too.
- **Messaging complexity:** One side-effect of changing topology characteristics is that it will affect the messaging complexity (i.e. number of messages per search request) of the search algorithms. We would like to observe this effect as well.

### A. Search Algorithms

1) **Flooding (FL):** FL is the most common search algorithm in unstructured P2P networks. In search by FL, the source node \(s\), sends a message to all its nearest neighbors. If the neighbors do not have the requested item, they send on to their nearest neighbors excluding the source node [see Fig. 5(a)]. This process is repeated a certain number of times, which is usually called...
message, it forwards the message only to randomly chosen its neighbors excluding the node forwarded the message in the previous step. When a node with larger degree receives the performance in terms of number of messages per distinct number of discovered nodes. To overcome this problem, search definition in PA and HAPA, whereas in CM an DAPA it is not guaranteed that the minimum degree will be randomly chosen two neighbors and these neighbors forward the message to their randomly chosen two neighbors. In the third step, the message reaches its destination.

In search by FL, when large degree nodes (hubs) are reached, the number of neighbors traversed, and is equal to the shortest path length. Since the average shortest path for small-world networks, including scale-free ones generated by the PA model, is proportional to the logarithm of system size \( N \) or even slower, the average delivery time \( T_N \) is logarithmic as well, i.e.,

\[
T_N = \log(N).
\]

The main disadvantage with FL is that it requires a very large amount of messaging traffic because most of the nodes are visited and forced to exchange messages, which makes search by FL unscalable. Another disadvantage is that FL has poor granularity, i.e., each additional step in the search significantly increases the number of nodes visited [23]. Yet, search efficiency of FL (i.e., number of hits per search) provides a way of determining how other realistic and scalable search algorithms can perform in comparison to the best possible, i.e., the search efficiency of FL.

2) Normalized Flooding (NF): In search by FL, when large degree nodes (hubs) are reached, the number of neighbors for the next step in FL increases dramatically leading to a poor granularity. This also causes a lot of shared edges reducing the performance in terms of number of messages per distinct number of discovered nodes. To overcome this problem, search by NF algorithm was introduced in [30]. In NF, the minimum degree in the network \( k_{\text{min}} \) is an important factor. NF search algorithm proceeds as follows: When a node of degree \( k_{\text{min}} \) receives a message, the node forwards the message to all of its neighbors excluding the node forwarded the message in the previous step. When a node with larger degree receives the message, it forwards the message only to randomly chosen \( k_{\text{min}} \) of its neighbors except the one which forwarded the message. The NF procedure is illustrated in Fig. 5(b). In this simple network with \( k_{\text{min}}=2 \), the source node sends a message to its randomly chosen two neighbors and these neighbors forward the message to their randomly chosen two neighbors. In the third step, the message reaches its destination.

The NF search algorithm is based on the minimum degree in the network. The fixed minimum degree is equal to \( m \) by definition in PA and HAPA, whereas in CM an DAPA it is not guaranteed that the minimum degree will be \( m \).
of self-loops and multiple links reduce the minimum degree to values less than \( m \) down to 1. In DAPA, however, the minimum degree might be less than \( m \) because of the short range of horizon for some peers which are geographically far from other peers. But still since the ratio of nodes with degree less than \( m \) is small we ignored them and ran NF algorithm based on the predefined minimum degree value \( m \).

3) Random Walk (RW): RW or multiple RWs have been used as an alternative search algorithm to achieve even better granularity than NF. In RW, the message from the source node is sent to a randomly chosen neighbor. Then, this random neighbor takes the message and sends it to one of its random neighbors excluding the node from which it got the message. This continues until the destination node is reached or the total number of hops is equal to \( \tau \). A schematic of RW can be seen in Fig. 5(c). RW can also be seen as a special case of FL where only one neighbor is forwarded the search query, providing the other extreme situation of the tradeoff between delivery time and messaging complexity. RW search is inherently serial (sequential), which causes a large increase in the delivery time [23], [28]. In particular, computer simulations performed on a generalized scale-free network with degree exponent \( \gamma = 2.1 \), which is equal to the value observed in P2P networks, yield the result [62]:

\[
T_N = N^{0.79}. \tag{7}
\]

Although the RW search is worse than a FL search in scale-free networks in terms of the time needed to locate a given node, the average total traffic in the network is equal to \( T_N \), and therefore scales sublinearly with \( N \), better than the linear growth of FL search.

B. Results

We simulated the three search algorithms FL, NF, and RW on the topologies generated by the four methods PA, CM, HAPA, and DAPA, and provide results all the combinations with various hard cutoffs. Through the PA, CM, HAPA, and DAPA methods, we generated topologies with 10000 nodes. We used cutoff values of 10 and 40 (or 50 in some cases), in addition to the natural cutoff, i.e., no hard cutoff. When generating DAPA topologies, we used \( \tau_{sub} \) values of 2, 4, 6, 8, 10, 20, and 50 with expectation that larger \( \tau_{sub} \) should yield better search efficiency. Minimum degree values (or \( m \)) in our topologies were 1, 2, or 3. We varied the \( \tau \) values of search queries in FL up to the point we reach the system size and for NF/RW up to
values of HAPA has better hits results due to the star-like topology. The FL in DAPA is less efficient than in PA, although for higher values of hard cutoff. For small values of cutoff, PA and HAPA give similar performances in FL, whereas for higher values of cutoff, PA and HAPA have better hits results. For small cutoffs, PA and HAPA give similar performances in FL, whereas for higher values of cutoff, PA and HAPA have better hits results due to the star-like topology. The FL in DAPA is less efficient than in PA, although for higher values of cutoff, PA and HAPA give similar performances in FL, whereas for higher values of cutoff, PA and HAPA have better hits results due to the star-like topology. The FL in DAPA is less efficient than in PA.

10. To compare search efficiencies of RW and NF fairly in our simulations, we equated $\tau$ of RW searches to the number of messages incurred by the NF searches in the same scenario. Thus, for the search efficiency graphs of RW [e.g., Fig. 1] when $\tau$ is equal to a particular value such as 4, this means that the number of hits data-point corresponding to that $\tau = 4$ value is obtained by simulating a RW search with $\tau$ equal to the number of messages that were caused by an NF search using a $\tau$ value of 4. A similar normalization was done in [30].

1) Search Efficiency: As the $\tau$ varies, Fig. 6(a) shows the number of hits achieved by FL on various topologies generated by the PA method. Similarly, Fig. 7 shows search efficiency of FL on the topologies generated by the CM. In both of these figures, as expected, when there is no hard cutoff in the topology, the FL algorithm can achieve higher search efficiency by capturing more of the peers in the network for a specific $\tau$ value. Also, the effect of imposing a hard cutoff reduces when minimum degree in the topology is higher. One interesting feature of CM is when the minimum number of links is one (i.e., $m = 1$) the number of hits cannot reach system size even for very large $\tau$ values because the network is not a connected one for $m = 1$. Fig. 6(b) shows a similar search efficiency behavior for FL on HAPA topologies, with even more apparent effect of hard cutoff. For small values of cutoff, PA and HAPA give similar performances in FL, whereas for higher values of cutoff, HAPA has better hits results due to the star-like topology. The FL in DAPA is less efficient than in PA, although for higher values of $\tau_{sub}$ it gets closer to PA and efficiency of FL increases as can be seen in Fig. 8.

A minimum of three links for all peers eliminates negative effects of hard cutoffs: An interesting observation is that negative
effect of hard cutoffs on the FL performance on the PA and HAPA topologies can be easily reduced to negligible values by increasing the number of stubs $m$ (or connectedness). The number of stubs as small as 3 leaves virtually no difference between the search performances of overlay topologies with or without hard cutoffs. This result provides the guideline that to achieve a better FL performance a requirement of having at least three links to the rest of the network will be adequate to assure that no one else in the network will need to maintain unbearably large number of links. However, the necessity of complete or partial global information about the overall when constructing a PA topology is a major discouragement of using the PA and HAPA methods for overlay topologies of unstructured P2P networks.

There exists an interplay between connectedness and the degree distribution exponent for a fixed cutoff: As the DAPA method is a purely local method, it is more interesting to observe search performance on the DAPA topologies. Figs. 8(a), 8(b), and 8(c) show the FL performance on DAPA topologies generated with minimum degrees (or the number of stubs) of 1, 2, and 3 respectively. In each of these figures, search performance is shown for different $\tau_{sub}$ values 2, 4, 6, 8, 10, 20, and 50. Interestingly, when there is weak connectedness (i.e. $m = 1$), Fig. 8(a) shows that imposing hard cutoffs improves the search performance. This is due to the fact that hard cutoffs increase the connectedness of the topology by moving the links that were normally go to a hub in a topology without a hard cutoff. However, when the number of stubs is larger, in Figs. 8(a-c), we observe an interplay between the degree distribution exponent and connectedness for a fixed cutoff. We observe that improvement caused by hard cutoffs depend on the value of the hard cutoff, suggesting that reducing hard cutoff value hurts the search performance after a while. That is, potential improvements by having smaller hard cutoffs diminishes as the performance starts to become dominated by the degree distribution exponent rather than the connectedness. Another observation to be made is that impact of local information plays a major role in the search performance, as can be seen from Figs. 8(a-c).

Hard cutoffs may improve search efficiency in NF and RW: More interestingly, for NF and RW, improvements due to having hard cutoffs are apparent in all three topology generation methods, including the PA topologies, regardless of the number of stubs $m$. The only exception to this behavior is the CM, as shown in Figs. 9(b) and 11(b) for NW and RW respectively. This means that practical search algorithms like NF and multiple RWs are affected better by having hard cutoffs on the overlay topology. For NF, this is evidenced by Figs. 9(a,d) and 9(c,f) for the PA and HAPA topologies respectively. As it can be seen, having a little more local connectivity to the network by having a minimum of 2-3 links in every peer, the search performance

![Fig. 11. RW results for PA, CM, and HAPA models.](image-url)
increases rapidly for the same $\tau$ values (i.e. by comparing Fig. 9(a) and Fig. 9(d)). For RW, a very similar behavior is exhibited in Figs. 11(a,d) and 11(c,f), with only difference that effect of hard cutoffs is more apparent due to the fact that NF does better averaging of search possibilities. The observed behavior of RW illustrates how bad the effect of hard cutoffs can be on the search performance.

More global information is more important when target connectedness is high: Fig. 10 shows the performance of NF on various DAPA topologies with different parameters. Figs. 10(a-c) shows the search performance on linear scale when $m = 1$, while Figs. 10(d-i) show it on semi-logarithmic scale when $m = 2$ and $m = 3$. We observe, again, that as the hard cutoff is getting smaller, the search efficiency improves regardless of the connectedness $m$. Also, having a little better connectedness (e.g. $m = 3$) improves the search performance greatly. An interesting observation is that, when constructing the overlay topology, having more information (i.e. larger $\tau_{sub}$) about the global topology (thus more scale-freeness in the overall topology) yields more important improvements on the search performance for topologies with more connectedness, i.e. larger $m$. This means that, for the purpose of constructing topologies with better search performance, when the target connectedness value is high one needs to be more patient and obtain as much information as possible before finalizing its links to the rest of the peers.

**DAPA and HAPA models perform almost as optimal as the CM:** An interesting characteristic to observe is how close the performances of DAPA and HAPA are to the best possible correspondent CM for the NF and RW search algorithms. Unlike the other topology construction mechanisms studied in this paper, CM achieves a perfect scale-freeness for a given target hard
cutoff value, with the cost of global information. Specifically, topologies generated by the CM do not have big jumps at the hard cutoff values [e.g., Fig. 1(b)] in their degree distributions, in such a way that the links are configured in the perfect manner to assure that no node has links more than the target hard cutoff and the degrees of nodes follow exactly a power-law. This can be seen by comparing Fig. 2 with its counterparts Figs. 1, 3, and 4. As can be seen from Figs. 9(e) and 9(f), with connectedness \( m = 2 \) or \( m = 3 \), HAPA performs slightly worse than CM when using NF. Similarly, DAPA performance for moderate \( \tau \) sub values (e.g. 6) is very close to the optimal possible by the CM. For small or no connectedness \( m = 1 \), the behavior is the same, in that DAPA and HAPA performance is close to the CM performance as shown in Figs. 10(a)–(c), 9(c), and 9(b) for DAPA, HAPA, and CM respectively.

2) Messaging Complexity: We also looked at the complexity of messaging overhead for the search algorithm and topology combinations\(^1\). We specifically looked at the average number of messages incurred by a search request. As the FL algorithm is an extreme and not scalable in terms of messaging complexity, we did not study its performance. In all cases, NF performs better than RW consistently, though the difference between the two algorithms diminishes as \( \tau \) increases for weak connectedness, i.e. \( m = 1 \). But, for stronger connectedness, i.e. \( m > 1 \), the difference between NF and RW is more apparent. More importantly, although the effect of hard cutoffs is negative in terms of messaging complexity, we observed that this negative effect is very minimal and negligible, given that improvements on the search performance were observed for smaller hard cutoffs.

VI. SUMMARY AND DISCUSSIONS

We studied effects of the hard cutoffs peers impose on the number of entries they store on the search efficiency. Specifically, we showed that the exponent of the degree distribution reduces as hard cutoffs imposed by peers become smaller. We introduced new scale-free topology generation mechanisms (e.g., HAPA and DAPA) that use completely or partially local information unlike traditional scale-free topology generation mechanisms (i.e., PA and CM) using global topology information. We showed that topologies generated by our mechanisms allow better search efficiency in practical search algorithms like normalized FL and RW. Our study also revealed that interplay between the degree distribution exponent with a fixed hard cutoff and connectedness is likely to occur when using our mechanisms. We also showed that this interplay can be exploited by enforcing simple join rules to peers such as requiring each peer to have a minimum of 2-3 links to the rest of the unstructured P2P network.

Future work will include study of join/leave scenarios for the overlay topologies while attempting to maintain the scale-freeness of the overall topology. The challenge is to achieve minimal messaging overhead for join and leave operations of peers while keeping the scale-freeness in a topology with a hard cutoff.

APPENDIX

The pseudo-codes of the algorithms we used for this work are presented here. In all these codes it is assumed that the programmer had routines to create and maintain a network data structure at hand such as \( \text{ADD}_\text{EDGE}(i,j) \) creating an undirected edge between nodes \( i \) and \( j \). The other routines commonly used by these algorithms are \( \text{RANDOM}(i,j) \) creating a random integer \( x \) such that \( i \leq x \leq j \), \( \text{RAN DOM}() \) creating a real-valued random number in \([0,1]\); and the variables are \( k_i \) (stores the degree of the node \( i \)), \( m \) (the number of stubs or initial links), \( \text{Adj}[i] \) (all the nodes connected to node \( i \)), \( k_{\text{total}} \) (the total number of degrees in the network) and \( k_c \) (the hard cutoff value).

A. PA

The preferential attachment is a simple model to generate a scale-free network as in Alg. 1. The algorithm assumes that the user has already created a network with \( m + 1 \) fully connected nodes and adds one node at a time starting with the \( m + 2^{nd} \) node and fills its stubs by attempting to connect it to the existing nodes until the total number of nodes in the network reaches \( N \). The repeat-until loop (lines 3-10) repeated \( m \) times is responsible for making sure that the new node \( i \) is connected to one of the old nodes (line 7). To link the new node to the old one some conditions have to be satisfied (line 6), i.e., they should not be already connected, a random real number generated during the execution should be less than the ratio of the degree of the old node and the total degree (preference in the attachment), and the old node should have less than \( k_c \) connections.

B. CM

The main characteristic of the CM is that it uses a predefined degree sequence. We assume that the user has already generated a degree sequence consisting of random integers drawn from a specific distribution such as a power-law distribution. The CM algorithm (Alg. 2) uses this user-supplied degree sequence \( \{k_i\}_{i=1}^{n} \) in a for loop and connects the node \( i \) to randomly selected another node (line 3) until all the connections are made. After that the algorithm deletes the self-loops and multiple connections (line 9-10). The implementations of these functions depend on the data structures used.

\(^1\)Due to space constraints we have not included the results for our messaging complexity study. The results are available upon request from authors.
Algorithm 1 PA algorithm

1: for $i = m + 2$ to $N$ do
2:     for $j = 1$ to $m$ do
3:         repeat
4:             try ← true
5:             node ← $\text{RANDOM}(1, i - 1)$; $\text{rnd} ← f\text{RANDOM}()$
6:             if node $\notin \text{Adj}[i]$ AND $\text{rnd} < k_{node}/k_{total}$ AND $k_{node} < k_c$ then
7:                 $\text{ADD\_EDGE}(i, node)$
8:                 try ← false
9:         end if
10:     until try = false
11: end for
12: end for

Algorithm 2 CM algorithm

1: for $i = 1$ to $N$ do
2:     while $k_{pre}^i > 0$ do
3:         node ← $\text{RANDOM}(1, N)$
4:         $\text{ADD\_EDGE}(i, node)$
5:         $k_{pre}^i ← k_{pre}^i - 1$
6:         $k_{node}^i ← k_{node}^i - 1$
7:     end while
8: end for
9: $\text{DELETE\_SELF\_LOOPS}()$
10: $\text{DELETE\_MULTIPLE\_LINKS}()$

C. HAPA

The HAPA algorithm (Alg. 3) also assumes, like PA algorithm (Alg. 1), that the user created a network with $m + 1$ fully connected nodes. Then the new node tries to attempt to a randomly selected node in this initial network by using the preferential attachment rule and hard cutoff condition (lines 4-7). Independently from the success of this attempt the new node hops by using the existing links (lines 10-14) until it fills all of its stubs (or the number of connections reaches $m$). The new node avoids connecting itself with an additional condition in line 11. This process is repeated for all new nodes until the number of nodes in the network reaches $N$. For this algorithm we define an additional function $\text{RANDOM\_LINK}(i)$ returning a randomly chosen neighbor of the node $i$.

D. DAPA

The DAPA algorithm (Alg. 4) is different than previously defined algorithms in the sense that it maintains two different networks; a substrate network $G_S$ and an overlay or P2P network $G_O$ on top of $G_S$. We assumed that the user has already created the substrate network $G_S$ with number of nodes $N_S$, and with a GRN topology or a two-dimensional regular mesh topology as explained in Section IV-B. Throughout our simulations we used a GRN with size $N_S = 2 \times 10^4$ and average degree $\bar{k} = 10$ as a substrate to generate an overlay network with size $N_O = 10^4$. We also assume that the user created an overlay network $G_O$ with some nodes (2 in our simulations) randomly chosen from the substrate network. A new function $\text{ADD}(G, i)$ which adds the node $i$ to the network or list $G$ is also defined.

The algorithm has a big while loop checking at every time step the size of the overlay network, or the number of nodes in it, $|G_O|$ (line 1). Then a random node is chosen from the substrate network and if that node is not already in the overlay network the algorithm enters a big if block. In this if block first a breadth-first search algorithm is run on the substrate network to find the distances of the nodes to node and the values are stored in $\{d_i\}$. After all the distances to node are determined the nodes which belong to $G_O$, having distances to node less than $\tau_{sub}$ and degree less than the hard cutoff are added to the list $L_{PH}$ (the list of peers in the horizon of node) as in lines 6-10. If the number of peers in the horizon ($|L_{PH}|$) in the list is less than $m$ then all the peers are connected to node (lines 11-15). Otherwise the preferential attachment procedure is used, i.e. a random node peer is selected from the list $L_{PH}$ and conditions are checked (lines 18-31). This process is continued until all the stubs are filled. Every selected node becomes a peer in $G_O$ once they are connected to any other peer in their horizon. This algorithm does not guarantee that the minimum degree will be $m$ since in some cases the peer candidates may not find
Algorithm 3 HAPA algorithm

1: for $i = m + 2$ to $N$ do
2: $j \leftarrow 0$
3: node $\leftarrow$ RANDOM(1, $i - 1$); $rnd \leftarrow f\text{RANDOM}()$
4: if $i \notin \text{Adj}[\text{node}]$ AND $rn < \frac{k_{\text{node}}}{k_{\text{total}}}$ AND $k_{\text{node}} < k_{c}$ then
5: $\text{ADD}_{\text{EDGE}}(i, \text{node})$
6: $j \leftarrow j + 1$
7: end if
8: node $\leftarrow i$
9: while $j < m$ do
10: node $\leftarrow$ RANDOM_LINK(node); $rnd \leftarrow f\text{RANDOM}()$
11: if $i \neq \text{node}$ AND $i \notin \text{Adj}[\text{node}]$ AND $rn < \frac{k_{\text{node}}}{k_{\text{total}}}$ AND $k_{\text{node}} < k_{c}$ then
12: $\text{ADD}_{\text{EDGE}}(i, \text{node})$
13: $j \leftarrow j + 1$
14: end if
15: end while
16: end for

| Acronym | Meaning |
|---------|---------|
| PA      | Preferential Attachment |
| CM      | Configuration Model |
| HAPA    | Hop-and-Attempt Preferential Attachment |
| DAPA    | Discover-and-Attempt Preferential Attachment |
| FL      | Flooding |
| NF      | Normalized Flooding |
| RW      | Random Walk |

| Symbol  | Meaning |
|---------|---------|
| $N$     | Number of Nodes |
| $k$     | Node Degree |
| $P(k)$  | Probability that node degree is $k$ |
| $\gamma$ | Power-law exponent, Exponent of the degree distribution |
| $k_c$   | Hard cutoff on the node degree |
| $m$     | Number of Stubs, Connectedness |
| $\tau$  | Time-to-live (TTL) |
| $\tau_{\text{sub}}$ | Local time-to-live in DAPA |

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Algorithm 4 DAPA algorithm

1: while |G_O| < N_O do
2:     node ← RANDOM(1, N_S)
3:     if node ∉ G_O then
4:         BFS(G_S, node)
5:     L_PH ← NULL
6:     for i = 1 to N_S do
7:         if i ∈ G_O AND d_i > 0 AND d_i ≤ τ sub AND k_i < k_c then
8:             ADD(L_PH, i)
9:     end if
10: end for
11: if |L_PH| ≤ m then
12:     for i = 1 to |L_PH| do
13:         ADD_EDGE(node, L_PH[i])
14:     end for
15:     ADD(G_O, node)
16: else
17:     j ← 0
18:     while j < m do
19:         repeat
20:             try ← true
21:             peer ← RANDOM(1, |L_PH|)
22:             rnd ← fRANDOM()
23:             if node ∉ Adj[peer] AND rnd < k_peer/k_total AND k_peer < k_c then
24:                 ADD_EDGE(node, peer)
25:             try ← false
26:         end if
27:     until try = false
28:     j ← j + 1
29: end while
30:     ADD(G_O, node)
31: end if
32: end if
33: end while

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