Alteration of tunneling mechanism due to ferromagnetic insulator on Andreev spectroscopy for ferromagnet/superconductor junctions

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Abstract

We have theoretically elucidated the mechanism of the peak splitting of the tunneling conductance arisen from the effect of ferromagnetic insulator in ballistic ferromagnetic metal/superconductor junctions. On this peak splitting of the tunneling conductance, we have found that the exchange potential in the insulator gives crucial changes to the tunneling conductance through the coherence factor in superconductor. The peak splitting exists even though ferromagnetic metal is replaced by normal-metal. Moreover, in such junctions, the magnetization in ferromagnetic metal gives asymmetric tunneling conductance spectra between electron and hole injections. We have also clarified the difference of resultant influences of ferromagnetic insulator on tunneling conductance between spin-band asymmetry ferromagnet and standard Stoner ferromagnet junctions.

1. Introduction

Tunneling spectroscopy gives us many insights of the electronic properties of metals and superconductors. For instance, the symmetry of superconducting pair potential has been reflected sensitively on the differential conductance spectra in normal metal/unconventional superconductor junctions [1–11]. The junctions including magnetically active interface has also been an important subject from the viewpoint of fundamental physics as well as from an application perspective [12–16] on both the magnetism and the superconductivity in materials. Especially, ferromagnetic metal/superconductor (FM/S) junction has attracted much attention on investigations of the proximity effect, the transport phenomena, and so on [12, 17–20]. For example, in the FM/S junctions, the Andreev reflection (AR) is suppressed by the exchange potential in FM [12, 17, 18, 26–29]. Indeed, this modulation of AR has been utilized to measure the polarization of FM in the FM/S point contacts [21–24]. Furthermore, as one of the interesting magnetic effects, there is a spin active interface by the ferromagnetic insulator (FI). It has been shown theoretically that the double conductance peak around a bias voltage corresponding to the superconducting energy gap in semiconductor/S junction [25] appears due to the spin-flip scattering. In the FM/FI/d-wave (FM/FI/d) junction [26–29], it has been revealed that splitting of zero-bias conductance peak (ZBCP) occurs by the spin filtering effect of FI. Additionally, the spin filtering effect of FI can give electron-refrigeration in the N/FI/s-wave superconductor (N/FI/s) junctions [30]. In these theoretical investigations of FM/S junctions, ferromagnetic metals have been described by the Stoner model referred to as STF below. However, STF has not totally covered itinerant electron ferromagnetism in real materials. On the other hand, there is the ferromagnetism kinetically driven by a spin-dependent bandwidth asymmetry, or, equivalently, by an effective mass splitting between ↑- and ↓-spin particles [31–36]. We also consider this spin-dependent bandwidth asymmetry ferromagnet (SBAF) as well as STF to obtain more general results than the case of STF only. For SBAF/S junction, some results similar to STF case have been obtained [37–40]. However, the effect of FI in SBAF/FI/S junctions have not been studied yet. Accordingly, the study of the effect of FI on the tunneling conductance in SBAF/FI/S junctions becomes important to clarify the difference with those in STF/FI/S junctions.
In this paper, we apply our previous formula [40] based on the Blonder–Thinkham–Klapwijk theory [41] to the FM/Fl/s-wave and d-wave superconductor junctions, in which the polarization of FM can be hybridized between STF and SBAF. We show a significant role of the exchange potential in FI on the tunneling conductance. It is found that the FI gives specific difference in tunneling conductance between SBAF/Fl/S and STF/Fl/S junctions. Furthermore, we study in detail how the exchange potential in FI generates the spin filtering effect. As a result, a term leading the spin filtering effect that have not been discussed so far will be presented. And, we also investigate the effect of FI on tunneling conductance when the superconducting pair potential is a breaking time-reversal symmetry [42].

2. Model and formulation

In the present paper, we employ a two-dimensional ballistic FM/Fl/S junction (see Figure 1) with semi-infinite electrodes as in our previous paper [40]. The interface located at \( x = 0 \) is described by \( V_F(x) = (V_0 - \rho V_{ex})\delta(x) \), where \( \delta(x) \), \( V_0 \) and \( V_{ex} \) are the \( \delta \) function, a non-magnetic and a ferromagnetic barrier potentials, respectively. Here, \( \sigma = \uparrow \) or \( \downarrow \) is spin index and \( \rho = +1(-1) \) for \( \uparrow (\downarrow) \)-spin.

The Bogoliubov–de Gennes (BdG) equation describing the system is given by

\[
\begin{pmatrix}
H_0^\sigma(r) + \Delta(r) & \Delta^*(r) \\
\Delta^*(r) & -H_0^\sigma(r)
\end{pmatrix}
\begin{pmatrix}
u_r(r) \\
v_s(r)
\end{pmatrix}
= E
\begin{pmatrix}
u_r(r) \\
v_s(r)
\end{pmatrix},
\]

where \( \sigma = -\sigma \). Here, \( \nu_r(r) \) and \( \nu_s(r) \) are the wave functions for electronlike and holelike quasiparticles (ELQ and HLQ) with the eigenvalue \( E \), respectively. And \( H_0^\sigma(r) \) is the single particle Hamiltonian.

In the ferromagnet side \( (x < 0) \), the single particle Hamiltonian given is

\[
H_0^\sigma(r) = -\hbar^2 \nabla^2 / 2m_\sigma - \rho U_{ex} - E_{FM}
\]

where \( m_\sigma \) is the effective mass for \( \sigma \)-spin band particle, \( U_{ex} \) is the exchange potential and \( E_{FM} \) is the Fermi energy. Here, we assume a hybrid ferromagnet between SBAF and STF for FM. The pure SBAF and STF can be described by \( \gamma = m_\uparrow / m_\downarrow \geq 1 \) for \( U_{ex} = 0 \) and \( \gamma = U_{ex} / E_F(0 \leq \chi \leq 1) \) for \( m_\uparrow = m_\downarrow \), respectively. The magnetization \( M \) of the ferromagnet is given by \( M = P_{\uparrow} - P_{\downarrow} \) where the polarization \( P_{\sigma} \) for \( \sigma \)-spin is expressed as \( P_{\sigma} = (\gamma(1 + \rho)/2(1 + \rho\gamma))/(\gamma(1 + \rho)1 + 1 - \chi) \).

The \( H_0^\sigma(r) \) in the superconductor side \( (x > 0) \) is given by \( H_0^\sigma(r) = H_0^\sigma(r) = -\hbar^2 \nabla^2 / 2m_S - E_{ES} \) where \( m_S \) and \( E_{ES} \) are the effective mass of the quasiparticle and the Fermi energy, respectively. In this paper, we treat singlet superconductors. And the spatial dependence of the pair potential is simply taken as \( \Delta(r) = \Delta \Theta(x) \) where \( \Theta(x) \) is the Heaviside step function. In the following, we apply the quasiclassical approximation where \( E_E \gg (E, \Delta) \). In this approximation, the pair potential \( \Delta \) can be described by \( \Delta(\theta_0) = \Delta_0 \cos(2(\theta_0 - \alpha)) \) for d-wave superconductors, where \( \theta_0 \) is the angle between the direction of trajectory of quasiparticles in the superconductor and the interface normal, \( \alpha \) is the angle between the interface normal and the crystalline axis of superconductor. Moreover, for the wave vectors \( k_{\uparrow(\downarrow)} \) in the ferromagnets and \( k_{ELQ(HLQ)} = k_S \) in the superconductors, we consider a situation where \( k_1 < k_S < k_1 \) and \( m_\uparrow / m_S = m_\downarrow / m_S \). To describe the Fermi surface difference between \( \uparrow \)- and \( \downarrow \)-spin bands, we assume \( E_{FM} = E_{ES} = E_F \). And, from the momentum conservation on the direction parallel to the interface, we have \( k_S \sin \theta_0 = k_\sigma \sin \theta_0 = k_\sigma \sin \theta_0 \) where \( \theta_0 \) is incident angle of \( \sigma \)-spin particles in FM (see Figure 1). In addition, the BdG equation can be reduced to the effective one-dimensional equation due to the translational invariance along the interface of the Hamiltonian.
For ↑-spin electron injection with angle $\theta_i$ to the interface normal, there are four scattering trajectories; AR with angle $\theta_{IR}$, normal reflection (NR), transmission to superconductor as ELQ, and transmission as HLQ (see figure 1).

For these scattering processes, the probability coefficients $a_s$ and $b_s$ for AR and NR are obtained under appropriate boundary conditions for the BdG-wave functions $(u_s(r), v_s(r))^T$ at $x = 0$ as

$$a_s = \frac{4\gamma^{-1/2}\lambda_\nu \Gamma_\nu^+}{\Delta_+^{\sigma} - \Gamma_+^{\sigma} \Delta_d^{\sigma}}$$

$$b_s = \frac{\Delta_+^{\sigma} \lambda_\nu + \Gamma_+^{\sigma} \Delta_d^{\sigma}}{\Delta_+^{\sigma} - \Gamma_+^{\sigma} \Delta_d^{\sigma}}$$

with

$$\Lambda_+ = 1 + \gamma^{\rho/2} \lambda_\nu + 2iZ_\nu,$$

$$\Lambda_\nu = 1 - \gamma^{\rho/2} \lambda_\nu + 2iZ_\nu,$$

$$\Gamma_\nu = e^{i\phi_\nu} \Gamma_\nu, \quad \Gamma_\nu = \sqrt{E - \Omega_\nu}, \quad \Omega_\nu = \sqrt{E^2 - |\Delta_\nu|^2},$$

$$\phi_\nu = \frac{\Delta_\nu}{|\Delta_\nu|}, \quad \lambda_\nu = \sqrt{\gamma^{\rho/2} + \gamma^{\rho/2} \cos^2 \theta_\nu (1 - \gamma^{\rho/2} + \rho \chi)},$$

where $\Delta_+ = \Delta(\theta_\nu), \Delta_\nu = \Delta(\pi - \theta_\nu)$ and the barrier parameters $Z_\nu$ is defined as

$$Z_\nu = \frac{Z_{0,\nu}}{\cos \theta_\nu}, \quad Z_{0,\nu} = Z_0 - \rho Z_{ns},$$

$$Z_0 = m_s V_0 / \hbar^2 k_s, \quad Z_{ns} = m_s V_{ns} / \hbar^2 k_s.$$

The angle resolved conductance $G_{S,\sigma}$ for σ-spin at zero temperature can be calculated by using AR and NR coefficients as

$$G_{S,\sigma} = 1 + \frac{\psi_{\sigma}}{\psi_{\sigma}} |a_s|^2 - |b_s|^2,$$

(2.4)

where $\psi_{\sigma} = \hbar k_s / m_\sigma$ is the group velocity of the σ-spin particle in FM. Setting $\Delta_\nu = 0$ in $G_{S,\sigma}$, we can obtain the conductance $G_{N,\sigma} = 1 - |b_s|^2$ when the superconductor is in the normal state. Following our previous paper, the normalized conductance for σ-spin is defined as

$$G_T(eV) = \frac{\int_{-\pi/2}^{\pi/2} d\theta_s \cos \theta_s P_s}{\int_{-\pi/2}^{\pi/2} d\theta_s \cos \theta_s \sum_{\sigma} P_s G_{S,\sigma}},$$

(2.5)

where $V$ is the bias voltage. The normalized conductance is given by $G_T(eV) = \sum_{\sigma} G_{T,\sigma}$. In the calculation, we should take into account the critical angle $\theta_c$ for injected particles as $\theta_c = \sin^{-1} \sqrt{\gamma^{-1/2}(1 - \chi)} = \cos^{-1} \sqrt{\gamma^{-1/2}(\chi - 1 + \gamma^{1/2})}$.

### 3. Results

#### 3.1. Differential conductance

We begin with the $(\chi, \gamma)$-dependence of the normalized conductance $G_T(eV)$ in FM/FI/S junctions for $Z_0 = 5.0, Z_{ns} = 2.5$ under the fixed value of magnetization $M$.

Figure 2 shows $G_T(eV)$ for some combinations $(\chi, \gamma)$ giving $M = 0.2$ when $S$ is the s-wave superconductor (FM/FI/s junction). All $G_T(eV)$ show the double peak around $eV = \pm \Delta_0$, which is asymmetric with respect to $eV = 0$. The height of the double peak decreases for the change from the pure SBAF (1.5, 0.0) to the pure STF (1.0, 0.2). It is noted that the double peak is different from those indicated by Zutic and Sarma [25] since any spin–flip scattering processes are not considered in our model. Thus, the double peak will be owing to the spin filtering effects of FI. In figure 3, we show $G_T(eV)$ in FM/FI/αd junction for $M = 0.75$. For pure STF, the results by Kashiwaya et al [26] have been reproduced. For $\alpha = 0$ (figure 3(a)), $G_T(eV)$ shows the double peak around $eV = \pm \Delta_0$, similar to those in s-wave superconductor case. For $\alpha = \pi/8$ and $\pi/4$ (figures 3(b) and (c)), the asymmetric splitting of ZBCP has been obtained. As in the case of s-wave superconductor, the height of peaks decreases as FM approaches pure STF. From these results, we can notice for both s- and d-wave cases that SBAF gives relatively larger double peaks than those in STF. As shown in our previous paper [40], the height of the peak can be attributed to the difference in the critical angle $\theta_c$ between SBAF and STF.
To see the constitution of the splitting peak in the tunneling conductance, each spin components of the conductance, $G_T^{\uparrow}$ and $G_T^{\downarrow}$ are shown in figure 4 with $G_{eVT}$ in the case of SBAF-STF mixed FM.

For the junction including $s$-wave superconductor (figure 4(a)), since the conductance peak at $eV_0 = \pm \Delta_0$ is suppressed. As a result, it can be seen for $eV_0 < \Delta_0$ that $G_T^{\uparrow}$ behaves like shifting to the negative $eV$ in contrast to $G_T^{\downarrow}$ shifting to the positive $eV$. Additionally, the double peak in $G_{eVT}$ around $eV_0 = - \Delta_0$ is comprised by both $G_T^{\uparrow}$ and $G_T^{\downarrow}$ while that around $eV_0 = \Delta_0$ is comprised $G_T^{\uparrow}$ and $G_T^{\downarrow}$ for the $d$-wave superconductor (figure 4(b)). $G_T^{\uparrow}$ and $G_T^{\downarrow}$ are shifted to the negative $eV$ and to the positive $eV$, respectively. Then, the resulting ZBCP is asymmetrically suppressed. As will be described below, these behaviors of the tunneling conductance can be interpreted by the impact of FI on the Andreev and NR coefficients.

3.2. Effect of FI

In this subsection, to see and to emphasize more clearly the effect of FI, we treat mainly N/FI/S junctions by setting parameters as $\chi = 0.0$, $\gamma = 1.0$, and $m_T = m_S$.

We show the $Z_{ex}$ dependence of $G_T(eV)$ for $Z_0 = 5.0$ in figure 5. For N/FI/$s$ junction (figure 5(a)), the conductance peak at $|eV| = \Delta_0$ is decreasing with increasing $Z_{ex}$. The double peak clearly appearing at $Z_{ex} = 2.0$ is also suppressed following the increment of $Z_{ex}$ and vanishes when $Z_{ex} = Z_0$. For N/FI/$d$ junction, the former results of [26] are reproduced. The ZBCP splits for all $Z_{ex}$ except $Z_{ex} = 0$ and vanishes at $Z_{ex} = Z_0$ (figure 5(b)). As it can be found directly in figure 5, $G_T(eV)$ is symmetric with respect to $eV = 0$ for both $s$-wave and $d$-wave cases. Thus, asymmetric behavior of $G_T(eV)$ as shown in figures 2-4 can be attributed to the
magnetization \( M \) in FM. The \( Z_{ex} \) dependence of \( G_T(eV) \) for \( Z_0 = 5.0 \) in FM/FI/S junctions are plotted for \( s \)-wave (figure 6) and \( d \)-wave (figure 7) cases. For \( s \)-wave case, \( G_T(eV) \) shows asymmetric behavior (figures 6(a)–(c)). However, as \( Z_{ex} \) approaches \( Z_0 \), the asymmetric behavior of \( G_T(eV) \) is weakened with suppression of peak around \( |eV| = \Delta_0 \). As a result, the difference between SBAF and STF will be difficult to identify when \( Z_{ex} \) is close to \( Z_0 \). For \( d \)-wave cases (figure 7), it is also clear that the asymmetric splitting of ZBCP is due to the influence of \( M \). In contrast to \( s \)-wave cases, asymmetric splitting of ZBCP is kept except \( Z_{ex} = Z_0 \). Although the height of peaks is suppressed as \( Z_{ex} \) increases, the magnitude of peaks in SBAF case is larger than that in STF case.

In order to clarify the origin of the double peak as a result of direct influence of the \( Z_{ex} \), we show the Andreev and NR coefficients because the conductance consists of these coefficients within the BTK model.

It is found in figure 8 that the behavior of the coefficients for \( \uparrow \)- and \( \downarrow \)-spin particles are inverted with respect to \( eV = 0 \) for \( -\Delta_0 \leq eV \leq \Delta_0 \). The mechanism of this inversion can be explained naturally from the viewpoint of symmetry between electron and hole bounded by AR. That is, the injection of electron with \( \uparrow \)-spin for \( 0 \leq eV \leq \Delta_0 \) corresponds to that of hole with \( \downarrow \)-spin for \( -\Delta_0 \leq eV < 0 \), and vice versa. For example, in the situation \( Z_1 < Z_0 \), the probability of AR, \( |a_{\uparrow}|^2 \) for \( \uparrow \)-spin electron injection is higher than that of \( \downarrow \)-spin electron injection, even though the AR is suppressed by \( Z_1 \) for \( 0 \leq eV \leq \Delta_0 \). In contrast to \( |a_{\downarrow}|^2 \), the probability of NR, \( |b_{\downarrow}|^2 \) for \( \downarrow \)-spin electron is higher than that of \( \uparrow \)-spin electron injection. On the other hand, as mentioned above, \( |a_{\uparrow}|^2 \) and \( |a_{\downarrow}|^2 \) for \( -\Delta_0 \leq eV < 0 \) correspond to \( \downarrow \)-spin and \( \uparrow \)-spin holes injection, respectively. Therefore, for \( -\Delta_0 \leq eV < 0 \), the probability of AR for \( \uparrow \)-spin electron is lower than that for \( \downarrow \)-spin electron.
Figure 5. The $Z_{ex}$ dependence of $G_T(eV)$ for (a) $s$-wave and (b) $d(\alpha = \pi/4)$-wave superconducting junctions. We have set $Z_{ex} = 0.0, 1.0, 2.0, 3.0, 4.0, 5.0$ for $Z_0 = 5.0$.

Figure 6. The $Z_{ex}$ dependence of $G_T(eV)$ in FM/FI/s junctions. The FM is (a) $\text{SBAF}(\chi = 0.0, \gamma = 1.5)$, (b) $\text{MIX}(\chi \sim 1.2, \gamma \sim 0.1)$, (c) $\text{STF}(\chi = 0.2, \gamma = 1.0)$ for $M = 0.2$. We have set $Z_{ex} = 0.0, 1.0, 2.0, 3.0, 4.0, 5.0$ for $Z_0 = 5.0$. 
This inverted behavior is also applicable to the case of NR coefficients because the density of probability flow is preserved \(|b_1|^2 = 1 - \frac{\gamma}{\gamma^*}|a_0|^2\). Consequently, \(G_T(G_T)\) for \(|eV| \leq \Delta_0\) has a peak at \(eV \approx \Delta_0(-\Delta_0)\).

In the case of \(d\)-wave superconductor, as shown in figure 9, since AR is depressed by \(Z_{ex}\), \(|d_{1(1)}|^2\) is seemed shifting to the negative (positive) \(eV\). On the other hand, for \(|eV| \leq \Delta_0\), \(|d_{1(1)}|^2\) shows symmetric behavior with respect to \(eV = 0\). There is, however, slight difference between \(|b_1|^2\) and \(|b_1|^2\) due to \(Z_{ex}\) (equation (A.6)). Nevertheless, this difference of the \(|b_0|^2\) for \(|eV| \leq \Delta_0\) does not appear in \(G_T(eV)\) [26], since the conservation of the probability density flow is satisfied. These obtained results indicate that the split of the conductance peak has not been a straightforward shift in bias voltage.

We can approach to the origin of the split of the conductance peak (figures 2–5) by analyzing the denominator of the conductance. Subtracting the denominators between \(G_S,\uparrow\) and \(G_S,\downarrow\), we have the term \(D\) (equation (A.5))

\[
D = 4i(Z_{ex}^2 - Z_0^2)Z_{ex}(\Gamma_{\uparrow,\uparrow} - \Gamma_{\uparrow,\downarrow}) \cos \theta_S.
\]

\(G_S,\uparrow\) and \(G_S,\downarrow\) have half of this term with mutually opposite sign in their denominator, respectively (see appendix). Thus, we can conclude that \(D\) leads the spin filtering effect. Therefore, it is notable that the balance between reflections for \(\uparrow\)-spin and \(\downarrow\)-spin particle injections is broken when \(D = 0\). Conversely, the spin filtering effect does not occur for \(D = 0\). It is obvious from equation (3.1) that \(D\) becomes 0 for \(Z_{ex} = 0\) and \(Z_{ex} = Z_0\). It goes without saying about \(Z_{ex} = Z_0\). However, for \(Z_{ex} = Z_0\), an interesting reflection process takes...
place at the interface. In this case, NR occurs for ↑-spin electrons in spite of $Z_{0,\uparrow} = 0$ similarly to that for ↓-spin electrons owing to $Z_{0,\uparrow} = 2Z_0$. The reason for this NR is that the AR holes with ↓-spin caused by ↑-spin electrons completely transmitted interface are converted to the AR electrons with ↑-spin again after being reflected by $Z_{0,\downarrow}$. The probabilities of NR for both ↑-spin and ↓-spin electrons injections have the same weight, because these NR are arisen from $Z_{0,\downarrow}$. It is also noted that $\Gamma_\uparrow$ becomes real number in $|eV| > \Delta_0$. As a result, $D$ becomes 0 for $0 < Z_{ex} < Z_0$, since $\Gamma_\uparrow, \Gamma_\downarrow, -\Gamma_\downarrow^*\Gamma_\uparrow^* = 0$ when $\Gamma_\downarrow$ is real number. Reflection processes for $0 < Z_{ex} < Z_0$ is depicted when ↑-spin electrons with $|eV| \leq \Delta_0$ are injected from N to the interface in figure 10. In addition to the usual NR ((a) in figure 10) by $Z_\uparrow$, there is the NR ((b) in figure 10) through AR by $Z_\downarrow$ as explained above.

From the above discussion, we define $D_{res}$ and $\Gamma_{res}$ to quantitatively describe the effect of $Z_{ex}$ as

$$D_{res} = (Z_{ex}^2 - Z_0^2)Z_{ex} \Gamma_{res},$$

$$\Gamma_{res} = \int_{\pi/2}^{\pi/2} (\Gamma_\uparrow, \Gamma_\downarrow, -\Gamma_\downarrow^*\Gamma_\uparrow^*) \cos \theta_d d\theta_d. \tag{3.3}$$

In order to simply grab the energy dependence of $D_{res}$ and $\Gamma_{res}$, we integrated $(\Gamma_\uparrow, \Gamma_\downarrow, -\Gamma_\downarrow^*\Gamma_\uparrow^*) \cos \theta_d$ on the angle $\theta_d$ from $-\pi/2$ to $\pi/2$ in equation (3.3). The $eV$ dependence of $\Gamma_{res}$ and $D_{res}$ for $G_{\uparrow \uparrow}$ are shown in figure 11. For the $s$-wave case, since $D_{res}$ has a minimum value at a certain $eV$ nearly $\Delta_0$, the resulting $G_{\uparrow \uparrow}$ has the conductance peak at $eV \approx \Delta_0$ (figure 4(a)). On the other hand, for $G_{\uparrow \downarrow}$, since $D_{res}$ has opposite sign, $G_{\uparrow \downarrow}$ has the peak at $eV \approx -\Delta_0$. The $eV$ dependence of $|a|^2$ and $|b|^2$ shown in figure 6 can be understood with same reason, because the denominator is common among $|a|^2, |b|^2$, and $G_{\uparrow \downarrow}$. For $d$-wave case, we show the results for $\alpha = \pi/4$. The sign of $\Gamma_{res}$ is reversed with comparing to $s$-wave case as shown in figure 11. Thus, $D_{res}$ has minimum value at a certain $eV < 0$, the resulting conductance peak of $G_{\uparrow \downarrow}$ emerges at a certain negative $eV$ nearly 0. In contrast to this, the peak of $G_{\uparrow \downarrow}$ is at a certain positive $eV$ close to 0 (figure 4(b)) because $D_{res}$ has opposite sign. From these
obtained results, it can be interpreted that the spin-filtering effect is due to the imaginary part of the coherence factor $\Gamma_0$ induced by $Z_{ex}$. Additionally, for the emergence of the splitting of the conductance peak for both $s$- and $d$-wave superconductors, it seems that there is optimal combination of $Z_0$ and $Z_{ex}$. Indeed, it can be easily found that $(Z_{ex}^2 - Z_0^2)Z_{ex}$ has a minimum value at $Z_{ex} = Z_0/\sqrt{3}$. For example, as shown in figure 5, it is found that the splitting peak is sharper when $Z_{ex}$ close to $Z_0/\sqrt{3}$.

It will be an interesting problem to compare the splitting of ZBCP by the broken time-reversal symmetry (BTRS) state [26, 38, 42] with those by $Z_{ex}$. For the BTRS, we choose $\Delta_0 \cos(2(\theta_0 - \pi/4)) + i\Delta_{btrs}$ for the $d^\uparrow_{x-y} + i$-wave or $\Delta_0 \cos(2(\theta_0 - \pi/4)) + i\Delta_{btrs} \sin(2(\theta_0 - \pi/4))$ for the $d^\downarrow_{x-y} + i\Delta_{btrs}$-wave symmetry. Here, we ignore the effect of FI on BTRS pairing state for simplicity. In the $d^\uparrow_{x-y}$-wave case, the width on $eV$ of the splitting peak position depends on the magnitude of $\Delta_{btrs}$. Since the difference in the splitting of ZBCP between BTRS and $Z_{ex}$ cases is obviously on broad peak-to-peak width, we set here $\Delta_{btrs} = 0.1\Delta_0$ and also calculate for $Z_0 = 5.0$ to investigate changes on ZBCP. In these cases, since $d^\uparrow_{x-y}$-wave pair potential is dominant, the angle averaged $D_{ex}$ for both cases show almost similar behavior as in the case of N/FI/ $d^\uparrow_{x-y}$-wave junction (see figures 12(a) and 13(a)). This means that the $\uparrow$-spin and $\downarrow$-spin components of $G_T(eV)$ shift in opposite directions on $eV$ by the change of the minimum value of $D_{ex}$ owing to $Z_{ex}$. Therefore, as shown in figure 12(b), the double peak structure of $G_T(eV)$ originated from the imaginary $s$-wave component collapses with increasing $Z_{ex}$. On the other hand, as shown in figure 13(b), $G_T(eV)$ in N/FI/ $d^\downarrow_{x-y} + i\Delta_{btrs}$-wave junction behaves similarly to that in N/FI/ $d^\uparrow_{x-y}$-wave junction since the zero-energy Andreev bound state exists. The structures of $G_T(eV)$ around $eV = 0$ for both cases consists of superposition of shifted peaks of $G_T,_{\uparrow}$ and $G_T,_{\downarrow}$ (see inset at the upper right in figures 12(b) and 13(b)). $G_T(eV)$ around $eV = 0$ will be useful not only to investigate the difference between BTRS and $Z_{ex}$ in the origin of ZBCP splitting but also to distinguish between BTRS pair potentials. In figure 14, we show the $Z_{ex}$-dependence of $G_T(eV)$ in FM/FI/ $d^\uparrow_{x-y} + i\Delta_{btrs}$-wave junctions. We can see that the height of splitting peak due to the BTRS is suppressed with change from pure SBAF to pure STF. With the increase of $Z_{ex}$, it can be seen for all FM cases that the position of splitting peak is shifted to negative $eV$ direction and the structure of peak is broken with the suppression of height. For $d^\uparrow_{x-y} + i\Delta_{btrs}$-wave state (see figure 15), $G_T(eV)$ shows the almost similar behavior as that in $d^\downarrow_{x-y}$-wave case. From obtained results, in these cases, it becomes difficult to identify the BTRS for $Z_{ex} \geq 2.0$, because the behavior of $G_T(eV)$ is similar to those in FM/FI/ $d^\downarrow_{x-y}$-wave junctions shown in figure 3(c) and figure 7. However, for $Z_{ex} < 2.0$, the height of double peak may provide an useful information on the difference between SBAF and STF.

4. Conclusion

In this paper, we have studied the tunneling conductance in two-dimensional ballistic ferromagnet metal/FI/ superconductor (FM/FI/S) junctions. For both $s$-wave and $d$-wave superconductor cases, it has been unveiled...
that the ferromagnetic barrier potential $Z_{ex}$ has given rise to a split of the conductance peak reflecting AR bound states. It is also found that the height of the conductance peak becomes larger with increasing the magnitude of $Z_{ex}$ for SBAF/FI/S junction in contrary to those for STF/FI/S junction. From these obtained results, we can speculate that differences between STF and SBAF may be more clearly observed in the tunneling conductance for junctions including FI rather than non-magnetic insulator case. Furthermore, as a most important result, it has been clarified that the exchange potential in FI induces the imaginary part of superconducting coherence factor as a cause of conductance peak splitting. The resulting symmetrical splitting peak are made asymmetric by the magnetization of FM. It is a scenario of asymmetric splitting peak in FM/FI/S junction. In accordance with this scenario, we have investigated the double peak structure in a junction including the $d_{x^2-y^2} + i$s-wave or
Although it is an example of a specific magnitude of subdominant paring potential, the tunneling conductance in junctions including the BTRS is also effective for discriminating between SBAF and STF. In this study, we have never considered other magnetic effects such as spin mixing effects and proximity effects. Inclusion of these effects would be necessary for a realistic theory and be an important future problem.

**Appendix. Denominator of \( G_{S, \sigma} \)**

Here, we give the denominator of \( G_{S, \sigma} \) for N/FI/S junction. From equations (2.2)–(2.4) with \( \chi = 0, \gamma = 1 \), and \( m_1 = m_q = m_S \), we have

\[
\begin{align*}
    a_S &= \frac{\Gamma_+}{(1 + iZ_\sigma)(1 - iZ_\sigma) - Z_{e_z}Z_{\sigma} \Gamma_+ \Gamma_-}, \\
    b_{\sigma} &= \frac{-iZ_\sigma(1 - iZ_\sigma) + iZ_\sigma(1 - iZ_{e_z}) \Gamma_+ \Gamma_-}{(1 + iZ_\sigma)(1 - iZ_\sigma) - Z_{e_z}Z_{\sigma} \Gamma_+ \Gamma_-}.
\end{align*}
\]
\[ G_{\sigma, \sigma} = \frac{(1 + Z_0 Z_\sigma + Z_{\sigma}^2) (1 + Z_0^2) + (1 + Z_0 Z_\sigma + Z_{\sigma}^2) (1 + Z_0^2) (1 + Z_0 Z_\sigma + Z_{\sigma}^2)}{(1 + Z_0^2) (1 + Z_0 Z_\sigma + Z_{\sigma}^2) (1 + Z_0^2)^2 + D_\sigma}, \quad (A.3) \]

where

\[ D_\sigma = - Z_0 Z_\sigma [(1 - i Z_\sigma) (1 + i Z_\sigma) \Gamma_+ \Gamma_- + (1 + i Z_\sigma) (1 - i Z_\sigma) \Gamma_-^\dagger \Gamma_+^\dagger] \quad (A.4) \]

and also \( D_\sigma \) as an explicit function of \( Z_0 \) and \( Z_{\sigma} \) can be written by

\[ D_\sigma = \left( Z_0 - Z_{\sigma}^2 \right) \left( \cos^2 \theta_S + Z_{\sigma}^2 - Z_{\sigma}^2 \right) (\Gamma_+ \Gamma_- + \Gamma_-^\dagger \Gamma_+^\dagger) + 2i \mu_{Z_{\sigma}} (\Gamma_+ \Gamma_- + \Gamma_-^\dagger \Gamma_+^\dagger) \cos \theta_S \]

with \( \rho = 1 (-1) \) for \( \sigma = \uparrow (\downarrow) \).

Therefore, the subtraction between denominators of \( G_{\uparrow, \uparrow} \) and \( G_{\downarrow, \downarrow} \) gives us

\[ D \equiv D_{\uparrow} - D_{\downarrow} = 4i (Z_{\sigma}^2 - Z_0^2) Z_{\sigma} (\Gamma_+ \Gamma_- - \Gamma_-^\dagger \Gamma_+^\dagger) \cos \theta_S \quad (A.5) \]

In addition, the numerator of \( |b_\sigma|^2 \) has similar term with equation \((A.6)\) which consists of \( Z_{\sigma} \) and the coherence factor \( \Gamma_+ \Gamma_- - \Gamma_-^\dagger \Gamma_+^\dagger \). For example, \( |b_\sigma|^2 \) includes

\[ 2i Z_{\sigma} (\Gamma_+ \Gamma_- - \Gamma_-^\dagger \Gamma_+^\dagger) \cos \theta_S \quad (A.6) \]

in its numerator, hence, the probability of the NR by the ferromagnetic insulator can be modified by \( Z_{\sigma} \) (figure 8) besides equation \((A.4)\) in its denominator.

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