Semiclassical Strings in Electric and Magnetic Fields
Deformed AdS$_5 \times S^5$ Spacetimes

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Abstract

We first apply the transformation of mixing azimuthal and internal coordinate or mixing time and internal coordinate to the 11D M-theory with a stack N M2-branes to find the spacetime of a stack of N D2-branes with magnetic or electric flux in 10 D IIA string theory, after the Kaluza-Klein reduction. We then perform the T duality to the spacetime to find the background of a stack of N D3-branes with magnetic or electric flux. In the near-horizon limit the background becomes the magnetic or electric field deformed AdS$_5 \times S^5$. We adopt an ansatz to find the classical string solution which is rotating in the deformed $S^5$ with three angular momenta in the three rotation planes. The relations between the classical string energy and its angular momenta are found and results show that the external magnetic and electric fluxes will increase the string energy. Therefore, from the AdS/CFT point of view, the corrections of the anomalous dimensions of operators in the dual SYM theory will be positive. We also investigate the small fluctuations in these solutions and discuss the effects of magnetic and electric fields on the stability of these classical rotating string solutions. Finally, we find the possible solutions of string pulsating on the deformed spacetimes and show that the corrections to the anomalous dimensions of operators in the dual SYM theory are non-negative.

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1 Introduction

It is known that the AdS/CFT correspondence plays important role in studying the gauge theories at strong coupling [1,2]. Extending it to less supersymmetric cases [3-16] may allow us to find simple string-theoretic descriptions of various dynamical aspects of gauge theories, from high-energy scattering to confinement in phenomena. The generalization of AdS/CFT duality to the non-BPS string mode sector can be guided by semiclassical considerations as suggested in [3-6]. In particular, the investigations of [5,6] had found that using the novel multi-spin string states one can carry out the precise test of the AdS/CFT duality in a non-BPS sector by comparing the $\lambda^J \ll 1$ expansion of the classical string energy with the corresponding quantum anomalous dimensions in perturbative SYM theory [7-9].

In recent Lunin and Maldacena [10] had demonstrated that certain deformation of the $AdS_5 \times S^5$ background corresponds to a $\beta$-deformation of $N = 4$ SYM gauge theory in which the supersymmetry being broken was studied by Leigh and Strassler [11]. Since then there are many studies of AdS/CFT correspondence in the $\beta$-deformation Lunin-Maldacena background [12-15].

In a previous publication [16] we considered the magnetic-flux deformed $AdS_5 \times S^4$ and $AdS_4 \times S^5$ background which are obtained by performing the dimensional reduction of the 10D spacetime of the N D3-branes through the transformation of mixing azimuthal and internal coordinate, the “twist” identification of a circle [17,18]. The new backgrounds may be regarded as those with Melvin magnetic flux [19]. As the supersymmetry in the deformed $AdS_m \times S^n$ backgrounds has been broken by the presence of magnetic field they offer the spacetimes to study the AdS/CFT correspondence with less supersymmetry.

In this paper we will apply the transformation of mixing azimuthal and internal coordinate [17,18] or mixing time and internal coordinate [20,21] to the 11D M-theory with a stack $N$ M2 branes [22] and then use T duality [23,24] to find the spacetime of a stack of N D3-branes with external magnetic or electric field. In the near-horizon limit the background becomes the magnetic or electric field deformed $AdS_5 \times S^5$. As the isometry groups of $AdS_5 \times S^5$ are $SO(2, 4) \times SO(6)$ which are exactly the conformal group and R-symmetry of $N = 4$ supper Yang-Mills theory the background studied in this paper will be more phenomenally interesting.

We will first use the method in [5,6] to find the classical rotating string solutions which are rotating in deformed $S^5$ with three equal angular momenta $J$ in the three rotation planes. The relations between the classical string energy and its angular momenta are found. The results show that the external magnetic and electric fluxes will increase the classical string energy. We also investigate the small fluctuations in these solution and discuss the effects of the magnetic and electric fields on the stability of these classical rotating string solutions.
As the corrections to the classical string energy, from the AdS/CFT point of view, give the anomalous dimensions of operators in the dual SYM theory our results therefore are of primary interest. We next use the method in [25,26] to find the classical pulsating string solutions and show that the corrections to the anomalous dimensions of operators in the dual SYM theory are non-negative.

In section II we study the case of classical rotating string in the electric field deformed $AdS_5 \times S^5$ and in section III the case in the magnetic field deformed $AdS_5 \times S^5$. In section IV we study the string pulsating on the deformed spacetimes. In section V we discuss our results.

2 Rotating String in Electric Field Deformed $AdS_5 \times S^5$

2.1 Electric Field Deformed $AdS_5 \times S^5$

We now apply the transformation of mixing time and internal coordinate [20,21] to the 11D M-theory with a stack $N$ M2-branes [22] and then use the T duality [23,24] to find the spacetime of a stack of N D3-branes with electric flux.

The full $N$ M2-branes solution is given by [22]

$$\begin{align*}
    ds^2_{11} &= H^{1/3} \left(-dt^2 + dx_1^2 + dx_2^2\right) + H^{1/3} \left(dx_3^2 + d\rho^2 + \rho^2 \Omega_5^2 + dx_{11}^2\right), \\
    d\Omega_5^2 &= d\gamma^2 + \cos^2\gamma d\varphi_1^2 + \sin^2\gamma (d\psi^2 + \cos^2\psi d\varphi_2^2 + \sin^2\psi d\varphi_3^2), \\
    A^{(3)} &= H^{-1} dt \wedge dx_1 \wedge dx_2.
\end{align*}$$

$H$ is the harmonic function defined by

$$H = 1 + \frac{R}{r^{D-p-3}}, \quad r^2 \equiv x_3^2 + \rho^2 + x_{11}^2, \quad R \equiv \frac{16\pi G_D T_p N}{D - p - 3},$$

in which $G_D$ is the D-dimensional Newton’s constant and $T_p$ the p-brane tension. In the case of (2.1), $D = 11$ and $p = 2$.

Now we transform the time $t$ by mixing it with the compactified coordinate $x_{11}$ in the following substituting [20,21]

$$t \rightarrow t - Ex_{11}.$$  

Using above substitution the line element (2.1) becomes

$$\begin{align*}
    ds^2_{11} &= \frac{-H^{2/3} dt^2}{1 - E^2 H^{-1}} + H^{1/3} \left(dx_1^2 + dx_2^2\right) + H^{1/3} \left(dx_3^2 + d\rho^2 + \rho^2 d\Omega_5^2\right) \\
    &\quad + \left(H^{1/3} - E^2 H^{2/3}\right) \left(dx_{11} + \frac{EH^{-1} dt}{1 - E^2 H^{-1}}\right)^2.
\end{align*}$$
Using the relation between the 11D M-theory metric and string frame metric, dilaton field and 1-form potential

\[ ds^2_{11} = e^{-2\phi/3} ds^2_{10} + e^{4\phi/3} (dx_{11} + 2A_\mu dx^\mu)^2, \]  

(2.6)

the 10D IIA background is described by

\[ ds^2_{10} = \frac{-H^{-1}}{\sqrt{1 - E^2 H^{-1}}} \left[ -dt^2 + \sqrt{1 - E^2 H^{-1}} \left( dx_1^2 + dx_2^2 \right) + H^{1/2} \left( dx_3^2 + d\rho^2 + \rho^2 d\Omega^2_5 \right) \right], \]

(2.7)

\[ e^{4\Phi} = H^{1/2} \left( 1 - E^2 H^{-1} \right), \quad A_t = \frac{EH^{-1}}{1 - E^2 H^{-1}}. \]

(2.8)

In this decomposition into ten-dimensional fields which do not depend on the \( x_{11} \), the ten-dimensional Lagrangian density becomes

\[ L = R - 2(\nabla \phi)^2 - e^{2\sqrt{3} \phi} \ F_{\mu\nu} F^{\mu\nu}, \]

(2.9)

and from (2.8) we see that the parameter \( E \) represents the magnitude of the external electric field. In the case of \( E = 0 \) the above spacetime becomes the well-known geometry of a stack of N D2-branes. Thus, the background describes the spacetime of a stack of N D2-branes with electric flux.

To find the spacetime of a stack of N D3-branes we now perform the T-duality transformation [23,24] on the coordinate \( x_3 \). After substituting the metric and dilaton field by

\[ g_{x_3 x_3} \rightarrow \frac{1}{g_{x_3 x_3}}, \quad e^\Phi \rightarrow \frac{e^\Phi}{\sqrt{g_{x_3 x_3}}}, \]

(2.10)

the metric describing a stack of N D3-branes with electric flux becomes

\[ ds^2_{10} = \frac{-H^{-1}}{\sqrt{1 - E^2 H^{-1}}} \left( dt^2 - dx_3^2 \right) + \sqrt{1 - E^2 H^{-1}} \left[ H^{1/2} \left( dx_1^2 + dx_2^2 \right) + H^{1/2} \left( d\rho^2 + \rho^2 d\Omega^2_5 \right) \right]. \]

(2.11)

Far from the sources, namely for \( \rho \gg R \), above metric approaches Minkowski space, since \( H \sim 1 \). In the “near-horizon” limit, namely the region \( \rho \ll R \), we can approximate \( H \sim \frac{R}{\rho} \), and the line element (2.11) becomes

\[ ds^2_{10} = -\frac{\rho^2}{R \sqrt{1 - E^2 \rho^2}} \left( dt^2 - dx_3^2 \right) + \sqrt{1 - \frac{E^2 \rho^4}{R}} \left[ \frac{R^2}{\rho^2} \left( dx_1^2 + dx_2^2 \right) + \frac{R}{\rho^2} \left( d\rho^2 + \rho^2 d\Omega^2_5 \right) \right]. \]

(2.12)

In the case of \( E = 0 \) the above spacetime becomes the well-known geometry of \( AdS_5 \times S^5 \). Thus, the background describes the electric field deformed \( AdS_5 \times S^5 \).
2.2 Rotating String Solution in Electric Field Deformed $S^5$

We will search the string solutions which is fixed on the spatial coordinates in deformed $AdS_5$ locating at $x_1 = x_2 = x_3 = 0$ in the electric field deformed spacetime (2.12) with a fixed value of $\rho$. The line element becomes

$$ds_6^2 = -\frac{1}{\sqrt{1 - E^2}} dt^2 + \sqrt{1 - E^2} \left[ d\gamma^2 + \cos^2\gamma d\varphi_1^2 + \sin^2\gamma (d\psi^2 + \cos^2\psi d\varphi_2^2 + \sin^2\psi d\varphi_3^2) \right].$$

(2.13)

In the above equation, as we merely want to see the effect of electric field on the string solution we let $R = \rho = 1$ for convenience, which means that we have used the scale $E^2 R \to E^2$.

The string action could be written in the conformal gauge in terms of the independent global coordinates $x^m$

$$I = -\frac{1}{4\pi} \int d^2\xi \ G_{mn}(x) \partial_a x^m \partial^a x^n,$$

(2.14)

in which $\xi^a = (\tau, \sigma)$ and we let $\alpha' = 1$ for convenience. In the conformal gauge $\sqrt{-g} g^{ab} = \eta^{ab} = \text{diag}(-1, 1)$, the equations of motion following from the action should be supplemented by the conformal gauge constraints

$$G_{mn}(x) (\dot{x}^m \dot{x}^n + x^m x^m) = 0,$$

(2.15a)

$$G_{mn}(x) \dot{x}^m x^n = 0.$$  

(2.15b)

Following the method of Frolov and Tseytlin [5,6] we now adopt the ansatz

$$t = \kappa \tau, \quad \gamma = \gamma(\sigma), \quad \psi = \psi(\sigma), \quad \varphi_1 = \nu \tau, \quad \varphi_2 = \varphi_3 = \omega \tau.$$

(2.16)

to find the rotating string solution.

Substituting the ansatz (2.16) into metric form (2.13) the associated Lagrangian is

$$L = -\frac{1}{4\pi} \left[ \frac{1}{\sqrt{1 - E^2}} \kappa^2 - \sqrt{1 - E^2} \left( \nu^2 \cos^2\gamma(\sigma) + \omega^2 \sin^2\gamma(\sigma) - \gamma(\sigma)^2 - \psi(\sigma)^2 \sin^2\gamma(\sigma) \right) \right].$$

(2.17)

As the deformation we used does not change the properties of the translational isometries of coordinates $t$, $\varphi_1$, $\varphi_2$ and $\varphi_3$, there are the corresponding four integrals of motion:

$$E = P_t = \int_0^{2\pi} d\sigma \frac{1}{2\pi} \frac{1}{\sqrt{1 - E^2}} \partial_0 t,$$

(2.18)

which is the energy of the solution, and

$$J_1 = P_{\varphi_1} = \int_0^{2\pi} d\sigma \frac{1}{2\pi} \sqrt{1 - E^2} \cos^2\gamma(\sigma) \partial_0 \varphi_1,$$

(2.19)
\[ J_2 = P_{\varphi_2} = \int_{0}^{2\pi} \frac{d\sigma}{2\pi} \sqrt{1 - E^2} \sin^2 \gamma(\sigma) \cos^2 \psi(\sigma) \partial_0 \varphi_2, \quad (2.20) \]

\[ J_3 = P_{\varphi_3} = \int_{0}^{2\pi} \frac{d\sigma}{2\pi} \sqrt{1 - E^2} \sin^2 \gamma(\sigma) \sin^2 \psi(\sigma) \partial_0 \varphi_3, \quad (2.21) \]

which are the angular momenta of the rotating string in the electric field deformed $S^5$ space.

To find the values of energy and angular momenta we must know the function of $\gamma(\sigma)$ and $\psi(\sigma)$, and have relations between $\kappa$, $\nu$ and $\omega$. This can be obtained by solving the equations of $\gamma(\sigma)$ and $\psi(\sigma)$ associated to the Lagrangian (2.17), and imposing the conformal gauge constraints of (2.15). The field equation of $\psi(\sigma)$ is

\[ 0 = \left( \psi'(\sigma) \sin^2 \gamma(\sigma) \right)', \quad (2.22) \]

which could be easily solved by setting

\[ \psi(\sigma) = n \sigma, \quad \gamma(\sigma) = \gamma_0, \quad (2.23) \]

which are the same as those in the undeformed space [5]. In this case the field equation of $\gamma(\sigma)$ reduces to a simple relation

\[ \nu^2 = \omega^2 - n^2. \quad (2.24) \]

which is also the same as that in the undeformed space [5]. While the conformal gauge constraints (2.15b) is automatically satisfied the another conformal gauge constraints of (2.15a) implies

\[ \kappa^2 = \left( 1 - E^2 \right) \left( \nu^2 + 2n^2 \sin^2 \gamma_0 \right). \quad (2.25) \]

Using the above relations the energy and angular momenta of the string solution have the simple forms

\[ \mathcal{E} = \sqrt{\nu^2 + 2n^2 \sin^2 \gamma_0}, \quad J_1 = \sqrt{1 - E^2} \nu \cos^2 \gamma_0, \quad J \equiv J_2 = J_3 = \frac{1}{2} \sqrt{1 - E^2} \sqrt{\nu^2 + n^2 \sin^2 \gamma_0}, \quad (2.26) \]

respectively. We now use the above results to analyze two cases.

1. $\nu = 0$: In this case, using (2.26) we see that $\mathcal{E} = \sqrt{2n \sin \gamma_0}$, $J_1 = 0$ and $J = \frac{1}{2} \sqrt{1 - E^2} n \sin \gamma_0$, thus

\[ \mathcal{E} = \frac{2}{\left( 1 - E^2 \right)^{\frac{1}{2}}} \sqrt{nJ} > 2\sqrt{nJ}. \quad (2.27) \]

Therefore the external electric fluxes will increase the string energy. To analyze the stability of the above solution we recall that, without the $E$ field the problems of the stability of the rotating string in the $S^5$ had been investigated in detail in [5] by Frolov and Tseytlin. The result of appendix A.2 in [5] had shown that the rotating string is stable only if

\[ 0 \leq \kappa^2 \leq \frac{3}{2}. \quad (2.28) \]
We claim that the above criterion could still be used in the electric field deformed $S^5$ background. This is because that the Lagrangian associated to the metric eq.(2.13) used to investigate the fluctuation field in the deformed case is equal to that used in the undeformed case, up to an overall constant value $\sqrt{1-E^2}$, after rescaling the time by $t \rightarrow \frac{t}{\sqrt{1-E^2}}$. Therefore, the criterion (2.28) would not be changed in the case with electric field deformation. Now, from (2.25), (2.26) and (2.27) we know that $\kappa^2 = (1-E^2)\mathcal{E}^2 = 4(1-E^2)^{\frac{1}{2}}nJ$. Thus the criterion (2.28) becomes

$$0 < J < J_c \equiv \frac{3}{8n(1-E^2)^{\frac{1}{2}}}.$$  

(2.29)

As $J_c > \frac{3}{8n}$ the electric fields therefore have inclinations to improve the stability of these classical rotating string solutions.

2. $\nu \gg n$: In this case, using (2.26) we see that $J_1 + 2J \approx \sqrt{1-E^2}\nu$ and $\sin^2\gamma_0 \approx \frac{2J}{\sqrt{1-E^2}\nu} = \frac{2J}{J_1+2J}$, thus

$$\mathcal{E} \approx \frac{J_1 + 2J}{\sqrt{1-E^2}}.$$  

(2.30)

Therefore the external electric fluxes will increase the string energy. To analyze the stability of the above solution we recall that, without the $E$ field the problems of the stability of the above rotating string in the $S^5$ had been investigated in detail in [6] by Frolov and Tseytlin. The result of eq.(2.36) in [6] had shown that the rotating string is stable only if

$$0 \leq \sin^2\gamma_0 \leq \frac{3}{4}.$$  

(2.31)

Following the discussion in the previous case we see that the above criterion could still be used in the electric field deformed $S^5$ background. In the case of $\nu \gg 1$ we can substitute the relation $\sin^2\gamma_0 \approx \frac{2J}{J_1+2J}$ into the criterion (2.23) and find that

$$0 < \frac{2J}{J_1+2J} \leq \frac{3}{4} \Rightarrow \frac{J}{J_1} \leq \frac{3}{2}.$$  

(2.32)

This means that the electric fields does not change stability of these classical rotating string solutions. Let us make two comments to conclude this section.

1. As the correction to the rotating classical string energy is positive then, from the AdS/CFT point of view, the correction of the anomalous dimensions of operators in the dual SYM theory will be positive.

2. In principle, there are the dilaton and anti-symmetric fields terms in the string Lagrangian. However, as the induced metric of the rotating string solution found in this section is flat and there is not NS-NS $B_2$ field in our background the action (2.14) is a proper one to be used. The property also reveals in the section III.
3 Rotating String in Magnetic Field Deformed $AdS_5 \times S^5$

3.1 Magnetic Field Deformed $AdS_5 \times S^5$

We now apply the transformation of mixing azimuthal and internal coordinate [17,18] to the 11D M-theory with a stack $N$ M2-branes [22] and then use the T duality [23,24] to find the spacetime of a stack of N D3-branes with magnetic flux.

Using the full N M2-branes metric described in (2.1) we can transform the angle $\varphi_1$ by mixing it with the compactified coordinate $x_{11}$ in the following substituting

$$\varphi_1 \to \varphi_1 + B x_{11}. \quad (3.1)$$

Using the above substitution the line element (2.1) becomes

$$ds_{11}^2 = H^{-\frac{2}{3}} \left( -dt^2 + dx_1^2 + dx_2^2 \right) + H^{-\frac{1}{3}} \left( dx_3^2 + d\rho^2 + \rho^2 \left( d\gamma^2 + \frac{\cos^2\gamma d\varphi_1^2}{1 + B^2 \rho^2 \cos^2\gamma} d\varphi_1^2 \right) \right. \right.$$

$$+ \left. \left. \sin^2\gamma d\Omega_3^2 \right) + H^{-\frac{1}{3}} \left( 1 + B^2 \rho^2 \cos^2\gamma \right) \left( dx_{11} + \frac{B \rho^2 \cos^2\gamma d\varphi_1^2}{1 + B^2 \rho^2 \cos^2\gamma} \right)^2, \right.$$

$$d\Omega_3^2 = d\psi^2 + \cos^2\psi d\varphi_2^2 + \sin^2\psi d\varphi_3^2. \quad (3.2)$$

Using the relation between the 11D M-theory metric and string frame metric, dilaton field and 1-form potential described in (2.6) the 10D IIA background is then described by

$$ds_{10}^2 = H^{-\frac{1}{2}} \sqrt{1 + B^2 \rho^2 \cos^2\gamma} \left( -dt^2 + dx_1^2 + dx_2^2 \right) + H^{-\frac{1}{2}} \sqrt{1 + B^2 \rho^2 \cos^2\gamma} \left( dx_3^2 + d\rho^2 \right.$$

$$\left. + \rho^2 \left( d\gamma^2 + \frac{\cos^2\gamma d\varphi_1^2}{1 + B^2 \rho^2 \cos^2\gamma} d\varphi_1^2 + \sin^2\gamma d\Omega_3^2 \right) \right). \quad (3.3)$$

$$e^{\Phi} = \sqrt{1 + B^2 \rho^2 \cos^2\gamma} H^{-\frac{1}{4}}, \quad A_{\varphi_1} = \frac{B \rho^2 \cos^2\gamma}{2 (1 + B^2 \rho^2 \cos^2\gamma)}. \quad (3.4)$$

In this decomposition into ten-dimensional fields which do not depend on the $x_{11}$, the ten-dimensional Lagrangian density will be described by (2.9) and the parameter $B$ is the magnetic field defined by $B^2 = \frac{1}{2} F_{\mu\nu} F^{\mu\nu} |_{\rho=0}$. In the case of $B = 0$ the above spacetime becomes the well-known geometry of a stack of N D2-branes. Thus, the background describes the spacetime of a stack of N D2-branes with magnetic flux.

To find the spacetime of a stack of N D3-branes we now perform the T-duality transformation [23,24] on the coordinate $x_3$. Using the substitution (2.10) the background describing a stack of N D3-branes with magnetic flux therefore becomes

$$ds_{10}^2 = H^{-\frac{1}{2}} \left[ \sqrt{1 + B^2 \rho^2 \cos^2\gamma} \left( -dt^2 + dx_1^2 + dx_2^2 \right) + \frac{1}{\sqrt{1 + B^2 \rho^2 \cos^2\gamma}} dx_3^2 \right]$$
\[ H^{1/2}\sqrt{1 + B^2 \rho^2 \cos^2 \gamma}\left( d\rho^2 + \rho^2 \left( d\gamma^2 + \frac{\cos^2 \gamma}{1 + B^2 \rho^2 \cos^2 \gamma} d\varphi_1^2 + \sin^2 \gamma \ d\Omega_3^2 \right) \right) . \] (3.5)

\[ e^\Phi = \sqrt{1 + B^2 \rho^2 \cos^2 \gamma}, \quad A_{\varphi_1} = \frac{B \rho^2 \cos^2 \gamma}{2 (1 + B^2 \rho^2 \cos^2 \gamma)} . \] (3.6)

Far from the sources, namely for \( \rho \gg R \), above metric approaches Minkowski space, since \( H \sim 1 \). In the “near-horizon” limit, namely the region \( \rho \ll R \), we can approximate \( H \sim \frac{R}{r} \), and the line element (3.5) becomes

\[ ds^2_{10} = R \sqrt{1 + B^2 R^2 Z^{-2} \cos^2 \gamma} \left[ \frac{1}{Z^2} (-dt^2 + dx_1^2 + dx_2^2 + \frac{1}{1 + B^2 R^2 Z^{-2}} dx_3^2 + dZ^2) + \right. \]

\[ \left. \left( d\gamma^2 + \frac{\cos^2 \gamma d\varphi_1^2}{1 + B^2 R^2 Z^{-2} \cos^2 \gamma} + \sin^2 \gamma \ d\Omega_3^2 \right) \right] , \] (3.7)

in which we define \( Z \equiv R^2/\rho \). In the case of \( B = 0 \) the above spacetime becomes the well-known geometry of \( AdS_5 \times S^5 \). Thus, the background describes the magnetic-deformed \( AdS_5 \times S^5 \).

### 3.2 Rotating String Solution in Magnetic Field Deformed \( S^5 \)

We will search the string solutions which is fixed on the spatial coordinates in deformed \( AdS_5 \) locating at \( x_1 = x_2 = x_3 = 0 \) in the magnetic field deformed spacetime (3.7) with a fixed value of \( Z \). The line element becomes

\[ ds^2_5 = \sqrt{1 + B^2 \cos^2 \gamma} \left[ -dt^2 + d\gamma^2 + \frac{\cos^2 \gamma d\varphi_1^2}{1 + B^2 \cos^2 \gamma} + \sin^2 \gamma \ d\Omega_3^2 \right] , \] (3.8)

in which, as we merely want to see the effect of magnetic Melvin field on the string solution, we let \( R = \rho = 1 \) for convenience. This means we have used the scale \( B^2 R^2 Z^{-2} \to B^2 \).

To proceed, we use the classical rotating string solution ansatz (2.16) to find the associated Lagrangian

\[ L = -\frac{1}{4\pi} \sqrt{1 + B^2 \cos \gamma(\sigma)} \left[ -\kappa^2 + \frac{\nu^2 \cos^2 \gamma(\sigma)}{1 + B^2 \cos^2 \gamma(\sigma)} + \omega^2 \sin^2 \gamma(\sigma) - \gamma(\sigma)^2 - \psi(\sigma)^2 \sin^2 \gamma(\sigma) \right] , \] (3.9)

Using the Lagrangian we can find that the field equation of \( \psi(\sigma) \) is

\[ 0 = \left( \psi'(\sigma) \sin^2 \gamma(\sigma) \right)' , \] (3.10)

which could be easily solved by setting

\[ \psi(\sigma) = n \sigma, \quad \gamma(\sigma) = \gamma_0 . \] (3.11)
which are the same as those in the undeformed space [5]. Using this relation the field equation of $\gamma(\sigma)$

$$0 = \frac{\partial}{\partial \gamma_0} \left( \sqrt{1 + B^2 \cos^2 \gamma_0} \left[ -\kappa^2 + \frac{\nu^2 \cos^2 \gamma_0}{1 + B^2 \cos^2 \gamma_0} + \omega^2 \sin^2 \gamma_0 - n^2 \sin^2 \gamma_0 \right] \right), \quad (3.12)$$

implies a simple relation

$$\kappa^2 = \left[ \frac{2}{B^2} - \frac{\cos^2 \gamma_0}{1 + B^2 \cos^2 \gamma_0} \right] \nu^2 - (\omega^2 - n^2) \left[ \frac{2}{B^2} + 3 \cos^2 \gamma_0 - 1 \right]. \quad (3.13)$$

In the case of $B^2 \to 0$ above equation reduces to the relation $\omega^2 = \nu^2 + n^2$ as that in [5]. To proceed, we see that while the conformal gauge constraints (2.15b) is automatically satisfied the another conformal gauge constraints of (2.15a) implies that

$$\kappa^2 = \frac{\cos^2 \gamma_0}{1 + B^2 \cos^2 \gamma_0} \nu^2 + (n^2 + \omega^2) \sin^2 \gamma_0. \quad (3.14)$$

From (3.13) and (3.14) we can find that

$$\omega^2 = \frac{\nu^2}{(1 + B^2 \cos^2 \gamma_0)^2} + \frac{n^2 (1 + B^2 (2 \cos^2 \gamma_0 - 1))}{1 + B^2 \cos^2 \gamma_0}. \quad (3.15)$$

Now we can follow the definition in (2.18)-(2.21) to evaluate the energy and angular momenta of the rotating string. The formula are

$$E = \kappa \sqrt{1 + B^2 \cos^2 \gamma_0} = \sqrt{1 + B^2 \cos^2 \gamma_0}$$

$$\times \sqrt{\frac{\cos^2 \gamma_0}{1 + B^2 \cos^2 \gamma_0} \nu^2 + \left( \frac{\nu^2}{(1 + B^2 \cos^2 \gamma_0)^2} + \frac{n^2 (2 + B^2 (3 \cos^2 \gamma_0 - 1))}{1 + B^2 \cos^2 \gamma_0} \right) \sin^2 \gamma_0}, \quad (3.16)$$

$$J_1 = \frac{\nu \cos^2 \gamma_0}{\sqrt{1 + B^2 \cos^2 \gamma_0}}. \quad (3.17)$$

$$J \equiv J_2 = J_3 = \frac{\omega}{2} \sqrt{1 + B^2 \cos^2 \gamma_0} \sin^2 \gamma_0 = \frac{1}{2} \sqrt{1 + B^2 \cos^2 \gamma_0} \sin^2 \gamma_0$$

$$\times \sqrt{\frac{\nu^2}{(1 + B^2 \cos^2 \gamma_0)^2} + \frac{n^2 (1 + B^2 (2 \cos^2 \gamma_0 - 1))}{1 + B^2 \cos^2 \gamma_0}}. \quad (3.18)$$

From (3.17) and (3.18) we can, in principle, express $\nu$ and $\gamma_0$ as the functions of $J_1$ and $J$. Then, substituting the functions into (3.16) we can express the energy $E$ as the functions of $J_1$ and $J$. However, as the algebra in there is too complex we will first consider the case of $\nu = 0$ under a small magnetic flux.
1. $\nu = 0$: In this case, using (3.18) we can express $\cos^2 \gamma_0$ as a function of $J$, i.e.

$$
\cos^2 \gamma_0 = \left(1 - \frac{2J}{n}\right) + \frac{J}{n} \left(1 - \frac{4J}{n}\right) B^2 + O(B^4).
$$

(3.19)

Substituting this relation into (3.16) we obtain a simple form of the string energy

$$
E = 2\sqrt{n}J + \frac{1}{2} \sqrt{n}J \left(1 - \frac{2J}{n}\right) B^2 + O(B^4).
$$

(3.20)

The second term is the corrected energy raised from the Melvin magnetic flux which deforms the $S^5$. As a rotating string with $\frac{1}{2} \leq \frac{J}{n}$ will be an unstable solution, which is investigated in the following, the above result shows that the stable rotating string has a positive corrected energy.

To analyze the stability of the above rotating string solution we can also use the criterion (2.28), $0 \leq \kappa^2 \leq \frac{3}{2}$, in the case of weak magnetic flux. The reason is that, from (3.8) we see that the magnetic flux $B^2$ only appears in the combination form $"1 + B^2 \cos^2 \gamma(\sigma)"$. Thus, during considering the fluctuation of the field $\gamma$ we shall replace $\gamma \rightarrow \gamma_0 + \tilde{\gamma}(t, \sigma)$ in the original Lagrangian. Then the combination form could be approximated by $1 + B^2 \cos^2(\gamma_0 + \tilde{\gamma}(t, \sigma)) \approx 1 + B^2 \cos^2 \gamma_0$, in the case with a small value of $B^2$. Thus, the Lagrangian used to investigate the fluctuation field $\gamma$ in the deformed case is equal to that used in the undeformed case, up to an overall constant value $"\sqrt{1 + B^2 \cos^2 \gamma_0}"$, after rescaling the field by $\varphi_1 \rightarrow (1 + B^2 \cos^2 \gamma_0)^{-1/2} \varphi_1$. Therefore, the criterion (2.28) would not be changed under a small magnetic-flux deformation. Now, as found in (3.19) we have expressed $\cos^2 \gamma_0$ as a function of $J$, we can now substitute this relations into (3.14) and (3.15) and, therefore can express $\kappa$ as a function of $J$. Substituting the function into the criterion (2.28) we finally obtain a simple relation

$$
0 \leq J \leq \frac{3}{8n} \left(1 + \left(\frac{1}{2} - \frac{3}{8n^2}\right) B^2\right) + O(B^4).
$$

(3.21)

which is the stability criterion of the rotating string in the deformed spacetime. We thus see that magnetic fluxes have inclination to improve the stability of the string solutions.

Note that, although the metric (3.7), which represents a deformed $AdS_5 \times S^5$, is different form that in our previous paper [16], which represents a deformed $AdS_4 \times S^5$, as the string we considered is fixed on the spatial coordinates in $AdS_5$ and $AdS_4$ respectively, the background considered in the both cases indeed have the same deformed $S^5$ space. (Some calculation errors of the previous paper are corrected in this paper.)

2. $\nu \gg n$: In this case, using (3.17) and (3.18) we can find that

$$
\cos^2 \gamma_0 = \frac{J_1}{J_1 + 2J}.
$$

(3.22)
As (3.17) tells us that \( \nu \) can be expressed as a function of \( J_1 \) and \( \cos^2 \gamma_0 \), we can therefore substitute (3.22) into (3.16) to obtain a simple form of the string energy

\[
\mathcal{E} = (2J + J_1) \sqrt{1 + \left( \frac{J_1}{2J + J_1} \right)^2 B^2},
\]

(3.23)

which tells us that there is a positive corrected energy raised from the Melvin magnetic flux which deforms the \( S^5 \). To analyze the stability of the above rotating string solution we can follow the previous argument to see that the criterion (2.31), \( 0 \leq \sin^2 \gamma_0 \leq \frac{3}{4} \), could also be used here. Now, substituting the relation (3.22) into the criterion (2.31) we finally obtain the criterion of a stable rotating string

\[
\frac{J}{J_1} \leq \frac{3}{2},
\]

(3.24)

which shows that the magnetic fluxes does not change the stability property of the string solutions. Note that as the correction to the rotating classical string energy is positive then, from the AdS/CFT point of view, the correction of the anomalous dimensions of operators in the dual SYM theory will be positive.

4 Pulsating Strings in Electric and Magnetic Fields

Deformed \( AdS_5 \times S^5 \)

We will follow the method in [25] to find the possible pulsating string solutions in the electric and magnetic field deformed spacetimes.

4.1 Pulsating String Solution in Electric Field Deformed \( S^5 \)

To proceed, we first identify \( t \) with \( \tau \) and \( \varphi_1 \) with \( m\sigma \) to allow for multiwrapping and consider a circular pulsating string expanding and contracting on \( S^5 \). The Nambu-Goto action corresponding to the metric (2.13) becomes

\[
S = -m \int dt \left( 1 - E^2 \right)^{1/4} \cos \gamma \sqrt{(1 - E^2)^{-1/2} - \dot{\gamma}^2 - \sin^2 \gamma g_{ij} \dot{\phi}^i \dot{\phi}^j},
\]

(4.1)

where \( g_{ij} \) is the metric on \( S_3 \) described in (2.13) and \( \phi^i \) refers to the coordinates on \( S_3 \). The canonical momenta calculated from (4.1) are

\[
\Pi_\gamma = \frac{m (1 - E^2)^{1/4} \cos \gamma \dot{\gamma}}{\sqrt{(1 - E^2)^{-1/2} - \dot{\gamma}^2 - \sin^2 \gamma g_{ij} \dot{\phi}^i \dot{\phi}^j}},
\]

(4.2)
\[ \Pi_i = \frac{m (1 - E^2)^{1/4} \cos \gamma \sin^2 \gamma g_{ij} \dot{\phi}^j}{\sqrt{(1 - E^2)^{-1/2} - \dot{\gamma}^2 - \sin^2 \gamma g_{ij} \dot{\phi}^i \dot{\phi}^j}}. \]  
(4.3)

In terms of the canonical momenta the Hamiltonian becomes

\[ H^2 = \frac{1}{\sqrt{1 - E^2}} \left[ \Pi_i^2 + g^{ij} \Pi_i \Pi_j \right] + m^2 \cos^2 \gamma. \]  
(4.4)

In the case of \( E = 0 \) the square of \( H \) looks like the Hamiltonian for a particle on \( S_5 \) with an angular dependent potential. If we are interested in large quantum numbers, the potential may be considered as a perturbation and we can proceed by considering free wavefunctions on \( S_5 \) and then do first order perturbation theory to find the correction. Denoting the total \( S_5 \) angular momentum quantum number by \( L \) and the total angular momentum quantum number on \( S_3 \) by \( J \) the corresponding zero-order wavefunction had been found in [25]. To the first order correction it was found that

\[ H^2|_{E=0} = L(L + 4) + m^2 \frac{L^2 - J^2}{2L^2}, \]  
(4.5)

which was calculated in eq.(2.9) of [25]. Therefore the spectrum corresponding to the Hamiltonian (4.4) becomes

\[ H^2 = \frac{1}{\sqrt{1 - E^2}} L(L + 4) + m^2 \frac{L^2 - J^2}{2L^2}. \]  
(4.6)

As \( H^2 > H^2|_{E=0} \) the corrections to the anomalous dimensions of operators in the dual SYM theory are positive.

### 4.2 Pulsating String Solution in Magnetic Field Deformed \( S^5 \)

As before, we identify \( t \) with \( \tau \) and \( \varphi_1 \) with \( m \sigma \) to allow for multiwrapping and consider a circular pulsating string expanding and contracting on \( S^5 \). The Nambu-Goto action corresponding to the metric (3.8) becomes

\[ S = -m \int dt \cos \gamma \sqrt{1 - \dot{\gamma}^2 - \sin^2 \gamma g_{ij} \dot{\phi}^i \dot{\phi}^j}, \]  
(4.7)

where \( g_{ij} \) is the metric on \( S_3 \) described in (3.8) and \( \phi^i \) refers to the coordinates on \( S_3 \). As the action is independent of magnetic field \( B \) the pulsating string solution is exactly like that in [25].

It shall be remarked that this trivial behavior may be traced to the special pulsating string we assumed. Other pulsating string solutions like those described [26] are difficult to be investigated and are expected to have nontrivial behaviors. Note also that we have only considered the strings on the deformed \( S^5 \) spacetimes in this paper. The coordinates of the deformed \( AdS_5 \) spacetime in (2.12) or (3.7) are mathematically complex and it is difficult to study the strings therein. The problems remain to be investigated.
5 Conclusion

In this paper we apply the transformation of mixing azimuthal and internal coordinate or mixing time and internal coordinate to the 11D M-theory with a stack N M2-branes to find the spacetime of a stack of N D2-branes with magnetic or electric flux in 10 D IIA string theory, after the Kaluza-Klein reduction. We perform the T duality to the spacetimes to find the backgrounds of a stack of N D3-branes with magnetic or electric flux. In the near-horizon limit the background becomes the magnetic or electric field deformed $AdS_5 \times S^5$. In contrast to the previous study in which the background were magnetic-flux deformed $AdS_5 \times S^4$ and $AdS_4 \times S^5$, the isometry groups of $AdS_5 \times S^5$ is $SO(2,4) \times SO(6)$ which are exactly the conformal group and R-symmetry of $N = 4$ supper Yang-Mills theory, the backgrounds studied in this paper will be more phenomenally interesting.

We have found the classical spinning string solutions which are rotating in the deformed $S^5$ with three angular momenta $J$ in the three rotation planes. The relations between the classical string energy and its angular momenta are obtained and results show that the external magnetic and electric fluxes will increase the string energy. Therefore, from the AdS/CFT point of view, the corrections to the anomalous dimensions of operators in the dual SYM theory will be positive. We have investigated the small fluctuations in these solutions and discuss the effects of magnetic and electric fields on the stability of these classical rotating string solutions. We have also found the possible solutions of string pulsating on the deformed spacetime. From the corrections to the pulsating string energy we see that the corrections to the anomalous dimensions of operators in the dual SYM theory are non-negative.

As the supersymmetry is broken by the magnetic or electric field there will in general appear tachyon when the string is in the magnetic or electric deformed spacetime. However, as the classical solutions studied in this paper correspond to the states with large quantum number the tachyon will not be shown. It will be interesting to find the string spectrum by following the method in [27] to see more properties of the effects of the magnetic or electric field on the rotating and pulsating string solutions. Also, the correspondence between spin chain [28] and classical strings in the electric or magnetic filed deformed $AdS^5 \times S^5$, which can improve our understanding of the AdS/CFT correspondence, are deserved to be investigated.

Finally, it is known from $AdS_5 \times S^5$ case that in order to establish the correspondence relation between the energy and spins to the anomalous dimension it is important to prove that the non Cartan elements of the angular momentum are vanishing. Also, as the energy of a string state is conjectured to be equal to the scaling dimension of the dual gauge theory operator it is important to find the Yang-Mills operators corresponding to the classical rotating and pulsating strings in the magnetic or electric field deformed spacetimes. (The corresponding operators in the undeformed backgrounds had been studied in [4,5,6,25].)
To complete the analysis from AdS/CFT point of view we shall also develop Bethe ansatz analysis [29,30] and compare the above result to that in SYM side. These important problems are left to further research.

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