Thermally driven ratchet motion of a skyrmion microcrystal and topological magnon Hall effect

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Spontaneously emergent chirality is an issue of fundamental importance across the natural sciences. It has been argued that a unidirectional (chiral) rotation of a mechanical ratchet is forbidden in thermal equilibrium, but becomes possible in systems out of equilibrium. Here we report our finding that a topologically nontrivial spin texture known as a skyrmion—a particle-like object in which spins point in all directions to wrap a sphere—constitutes such a ratchet. By means of Lorentz transmission electron microscopy we show that micrometre-sized crystals of skyrmions in thin films of Cu$_2$OSeO$_3$ and MnSi exhibit a unidirectional rotation motion. Our numerical simulations based on a stochastic Landau-Lifshitz-Gilbert equation suggest that this rotation is driven solely by thermal fluctuations in the presence of a temperature gradient, whereas in thermal equilibrium it is forbidden by the Bohr–van Leeuwen theorem. We show that the rotational flow of magnons driven by the effective magnetic field of skyrmions gives rise to the skyrmion rotation, therefore suggesting that magnons can be used to control the motion of these spin textures.

The formation of triangular arrays of skyrmions in chiral-lattice magnets under an applied magnetic field was theoretically predicted and experimentally observed in MnSi (refs 9,10), Fe$_{1−x}$Co$_x$Si (refs 11−13), FeGe (ref. 14) and Cu$_2$OSeO$_3$ (refs 15−17). The magnetization in such crystals is antiparallel to the magnetic field $B$ at the centre of each skyrmion and is parallel to $B$ at its periphery (Fig. 1a–c). Recently, real-space images of nanosized skyrmions have been successfully obtained by the Lorentz transmission electron microscope (TEM). While scanning temperatures and magnetic fields in the experiments, we have encountered a peculiar dynamical phenomenon: namely, in a wide temperature interval, excluding only the lowest temperatures, microscale regions of the skyrmion crystal (SkX) show, in addition to Brownian motion, a clearly discernible unidirectional rotation.

In Supplementary Movies 1 and 2, we show examples of this phenomenon observed in thin-plate (≈50-nm-thick) specimens of MnSi and Cu$_2$OSeO$_3$, respectively. These compounds are chiral-lattice magnets with a common space group $P2_13$. In zero field they undergo a phase transition from a paramagnetic to a helimagnetic phase at 29.5 K and 60 K, respectively. The helix period $\lambda_h$ is, respectively, 18 nm and 50 nm. Thin-plate (<100-nm-thick) specimens of these compounds host stable SkX phases with the skyrmion lattice spacing $\sim 2/\sqrt{3}\lambda_h$. The field strength for the

Supplementary Movies 1 and 2 is 175 mT and 65 mT, respectively, and $B$ is applied in the negative $z$-direction perpendicular to the plane plane: $B \parallel −z$. The observed (static) Lorentz-TEM image of SkX in a MnSi specimen is shown in Fig. 1c,d. In Fig. 1e–l, we show the time evolution (snapshots) of Fourier components of the magnetic configurations where a hexagon composed of six Bragg peaks rotates clockwise, indicating clockwise rotation of the SkX domains.

The rotation rate depends on the irradiation density of the electron beam of the microscope (Supplementary Information). The skyrmion rotation is observed only above a critical irradiation density and the rotation rate grows as the density increases. This indicates that this rotation is a non-equilibrium phenomenon induced by the electron beam. The similarity between the chiral rotations of SkX microdomains and their hexagon-shaped Fourier components observed in MnSi (Supplementary Movie 1) and Cu$_2$OSeO$_3$ (Supplementary Movie 2) is remarkable in view of the different origins of magnetism in these two compounds (MnSi is a metal, whereas Cu$_2$OSeO$_3$ is an insulator) and the difference in skyrmion parameters—for example, the transition temperature and the SkX lattice spacing. Hence, such a ratchet motion should be viewed as a generic feature of skyrmion systems. Then the question arises as to why and how it occurs.

A circular magnetic field induced by the electron beam of the Lorentz TEM is estimated to be five orders of magnitude smaller than the geomagnetic field, and thus cannot cause the rotation. Recently it was demonstrated that a spin-polarized electric current parallel to the sample plane can drive similar rotations of SkX domains in the presence of a thermal gradient. In our case, however, a possible effect of electric currents can be excluded as the electron beam of Lorentz TEM is three orders of magnitude smaller than the threshold current of $10^7−10^8$ A m$^{-2}$ for the current-driven skyrmion motions and, in addition, its direction is perpendicular rather than parallel to the film, which further reduces spin–torque effects. Furthermore, the energy of the electrons is very high and the interaction with the spins in the specimen is very weak, except through the magnetic field induced by the magnetization. This conclusion is corroborated by the fact that the skyrmion rotation is also observed in the insulating Cu$_2$OSeO$_3$. Thus, the unidirectional rotation is apparently induced by thermal effects.

To clarify the nature of this phenomenon, we performed numerical simulations. The SkX phase in chiral-lattice magnets is
described by a classical Heisenberg model on the two-dimensional square lattice, which contains ferromagnetic-exchange and Dzyaloshinskii–Moriya interactions as well as the Zeeman coupling to \( \mathbf{B} = (0, 0, B) \) normal to the plane. The Hamiltonian is given by

\[
\mathcal{H} = -J \sum_{\langle i,j \rangle} \mathbf{m}_i \cdot \mathbf{m}_j - D \sum_i \left( \mathbf{m}_i \times \mathbf{m}_{i+\hat{x}} \cdot \hat{x} + \mathbf{m}_i \times \mathbf{m}_{i+\hat{y}} \cdot \hat{y} \right) - B \cdot \sum_i \mathbf{m}_i
\]

where the magnetization vector \( \mathbf{m}_i \) is defined using the spin vector \( \mathbf{S}_i \) as \( \mathbf{m}_i = -\mathbf{S}_i / \hbar \), with the norm \( m = |\mathbf{m}| \). The spin turn angle \( \theta \) in the helical phase is determined by the ratio \( D/J \); for \( D/J = 0.27 \) used here, \( \theta = 11^\circ \), corresponding to the period of ~33 sites. With a typical lattice constant of 5 Å, this gives \( \lambda_m \sim 17 \) nm, which is comparable to the helical period ~18 nm in MnSi (our conclusions, however, are not affected by the choice of the value of \( D/J \)). Our Monte Carlo analysis shows that the SkX phase emerges in the range of \( 1.688 \times 10^{-2} < |B_z|/m < 5.67 \times 10^{-2} \) between the helical and ferromagnetic phases.

In this study, we treat SkX confined in a microscale circular disk with the diameter \( 2R = 137 \) sites, as shown in Fig. 2a. We impose the open boundary condition and simulate thermally-induced dynamics of this skyrmion microcrystal by numerically solving a stochastic Landau–Lifshitz–Gilbert equation using the Heun scheme. The equation is given by

\[
\frac{d\mathbf{m}_i}{dt} = -\frac{1}{1 + \alpha_G m} \left[ \mathbf{m}_i \times (\mathbf{B}^{\text{eff}}_i + \xi^\alpha_i(t)) + \frac{\alpha_G m}{2} \mathbf{m}_i \times [(\mathbf{B}^{\text{eff}}_i + \xi^\alpha_i(t))] \right]
\]

where \( \alpha_G \) is the Gilbert damping coefficient and \( \mathbf{B}^{\text{eff}}_i = -(1/\hbar) (\partial H/\partial \mathbf{m}_i) \) is the deterministic field derived from equation (1). The Gaussian stochastic field \( \xi^\alpha_i(t) \) describes the effects of a thermally fluctuating environment interacting with \( \mathbf{m}_i \), which satisfies \( \langle \xi^\alpha_i(t) \rangle = 0 \) and \( \langle \xi^\alpha_i(t) \xi^\beta_j(s) \rangle = 2 \kappa \delta_{ij} \delta_{\alpha \beta} \delta(t-s) \), where \( \beta \) and \( \lambda \) are Cartesian indices. The fluctuation-dissipation theorem gives a relation between \( \kappa \) and temperature \( T \): \( \kappa = \alpha_G k_B T / m \) (ref. 26). The initial spin configuration (Fig. 2a) is prepared by the Monte Carlo thermalization at low temperature and by further relaxing it in the Landau–Lifshitz–Gilbert simulation at \( T = 0 \). Starting from this initial configuration, we generate random numbers corresponding to the stochastic force \( \xi^\alpha_i(t) \) and solve equation (2). In what follows we use units in which the lattice constant \( a = 1 \), the exchange energy \( J = 1 \), the Boltzmann constant \( k_B = 1 \) and \( \hbar = 1 \).

In thermal equilibrium we only find Brownian motion of skyrmions and no unidirectional rotation. We then include a radial temperature gradient to examine whether it can give rise to chiral rotation. In the Lorentz-TEM experiment, the electron beam is irradiated onto a thin-plate specimen, which inevitably raises the temperature of the beam spot with respect to the outer region, resulting in the temperature gradient shown in Fig. 2b. We consider a constant temperature gradient with \( T = T_0 + \Delta T \) at the centre in the circular-disk system—that is, \(-dT/dr = \Delta T/R \).

We show snapshots of the calculated real-space magnetization dynamics at selected times in Fig. 3a–d, and the trajectory of a selected skyrmion indicated by solid rectangles in Fig. 3d. We find persistent rotation of SkX (Supplementary Movies 3 and 4). Figure 3e–l show the time evolution of the Fourier transform of the spin structure—the rotating hexagon composed of six Bragg peaks (Supplementary Movie 5). In the simulation, we apply \( B_z \parallel \zeta \) in accord with a set-up of the Lorentz-TEM experiment, and find clockwise rotations in agreement with the experimental observations. Remarkably, this unidirectional drive is driven purely by the thermal gradient because no other motive forces are considered in our simulation.

This nonreciprocal dynamics of SkX can be traced back to the algebra of spin operators, which determines the direction of spin precession in an applied field \( \mathbf{B} \). Equations of motion for the centre-of-mass coordinates of a skyrmion have the form (Supplementary Information)

\[
\begin{align*}
\mathbf{M} \ddot{\mathbf{Y}} + \alpha_G \Gamma \dot{\mathbf{Y}} - 4 \pi \mathbf{Q} \dot{\mathbf{S}} &= -\frac{\partial U}{\partial Y} + 4 \pi \mathbf{J}^{\text{magnon}}, \\
\mathbf{M} \ddot{\mathbf{X}} + \alpha_G \Gamma \dot{\mathbf{X}} + 4 \pi \mathbf{Q} \dot{\mathbf{Y}} &= -\frac{\partial U}{\partial X} - 4 \pi \mathbf{J}^{\text{magnon}}
\end{align*}
\]

where \( \mathbf{M} \) is the skyrmion mass, \( \mathbf{Q} \) is the topological charge \( Q = -1 \) for \( B_z < 0 \), \( \Gamma \approx 5.577 \pi S_0 \) is the external potential and \( \mathbf{J}^{\text{magnon}} = \left( J_{\text{magnon}}^x, J_{\text{magnon}}^y \right) \) is the magnon current density, defined in Supplementary Information, which drives skyrmion motion.
Figure 2 | Set-up of the numerical simulation. a, Magnetic configuration of the skyrmion microcrystal confined in a circular-shaped disk at $T=0$, where the in-plane magnetization components at sites $(i_x, i_y)$ are indicated by arrows when $\text{mod}(i_x, 2) = \text{mod}(i_y, 2) = 0$, whereas the distribution of the magnetization $z$-axis components, $m_{zi}$, is shown by a colour map. b, Schematics of the thermal gradient induced by the electron-beam irradiation in the Lorentz TEM experiment.

Figure 3 | Simulated thermally driven rotation of the skyrmion microcrystal. a–d, Snapshots of the simulated temporally changing distribution of the magnetization $z$-axis components $m_{zi}$ at $t = 1.9 \times 10^4$ (a), $t = 2.6 \times 10^4$ (b), $t = 3.3 \times 10^4$ (c), and $t = 4.0 \times 10^4$ (d), which show a clockwise rotation. Here the time unit is $\hbar/J$. Also shown in d is the trajectory of a selected skyrmion, indicated by rectangles. e–l, Time evolution of Fourier transforms of the magnetic structure in the reciprocal space $-0.2<\pi<k_x<0.2\pi$ and $-0.2<\pi<k_y<0.2\pi$ for $m=1, \beta_J/J = -0.03$, $\alpha_G = 0.01$, $k_B T_0/J = 0.1$ and $k_B \Delta T/J = 0.006$ at selected times between $t = 1.90 \times 10^4$ and $t = 4.35 \times 10^4$ with constant time intervals of $\Delta t = 0.35 \times 10^4$, which also show rotation.

through the spin transfer torque. The extension to a many-skyrmion problem is straightforward.

First, we note that, if one replaces the magnon current by the stochastic Langevin force, the dynamics of skyrmions becomes equivalent to that of classical particles in an external magnetic field. In this case, the famous Bohr–van Leeuwen theorem, which forbids orbital magnetism of classical particles in thermal equilibrium, precludes the spontaneous rotation of skyrmions, in agreement with our numerical results.

Next, we discuss how a nonzero temperature gradient can induce a persistent rotation. One possible scenario is that the gradient $dT/dr$ modifies the radial distribution of skyrmions, resulting in a non-vanishing radial force $-\langle \partial U/\partial r \rangle$, which according to equation (3) gives rise to a nonzero angular velocity. This, however, is forbidden, as the Bohr–van Leeuwen theorem can be generalized to the case of local equilibrium with a spatially inhomogeneous temperature $T(r)$. What is required, is a nonequilibrium state with a heat flow forming a heat engine, in which an amount of heat
Q_{1} transferred from the high-temperature side (T = T_{1}) is partly transformed into work W, while the remaining heat Q_{2} = Q_{1} − W is absorbed on the low-temperature side (T = T_{2} < T_{1}). The ratchet rotation requires W > 0 and the engine efficiency η = W/Q_{1} is less than 1 − T_{2}/T_{1}.

The underlying microscopic mechanism involves the flow of the spin energy. There are two possible heat carriers: skyrmions and magnons. The dimensional analysis of the ratchet rotation frequency ν due to the thermal motion of skyrmions gives ν ∝ 1/R(−dT/dr) (in the J = h = k_{B} = α = 1 units), which is too small to explain the results of numerical simulations. Magnons, on the other hand, are much more efficient in driving the rotation of skyrmions through the spin transfer torque.\(^{28}\) Importantly, a skyrmion induces an effective magnetic field \(h_{z} = -\mathbf{m} \cdot \nabla \times \mathbf{m}\) with the total flux \(\int \mathbf{r} \cdot dh_{z} = 4\pi Q\), which exerts the Lorentz force on magnons\(^{28}\) (Supplementary Information). The skew scattering of magnons off skyrmions gives rise to the topological magnon Hall effect: in addition to the thermally-driven magnon current in the radial direction, \(J_{\text{magnon}} = \kappa_{xy}^{\text{magnon}}(−dT/dr)\), there is a current \(J_{\text{magnon}} = \kappa_{y}^{\text{magnon}}(−dT/dr)\) in the direction transverse to the temperature gradient, corresponding to the anticlockwise rotation of the magnon gas.

Figure 4 shows the result of simulations for the magnon current density. We show a real-space map of the magnon current density at a selected time in Fig. 4a, where the arrows point in the current directions and their lengths represent the current amplitudes. In this map, the currents may seem to flow in random directions, but we find a net positive value of the time-averaged quantity \(\langle J_{\text{magnon}} \rangle = \sum_{i} (\mathbf{r} \times J_{\text{magnon}})/|\mathbf{r}|\), which shows a finite and positive magnitude of the transverse magnon current in our numerical simulations, \(J_{\text{magnon}} \sim 10^{-2}\), which for \(−dT/dr \sim 10^{-4}\) corresponds to a rather large magnon Hall conductivity, \(\kappa_{xy}^{\text{magnon}} \sim 10^{2}\) (ref. 30).
The skew magnon scattering off skyrmions exerts a reaction force on skyrmions in the negative $\theta$ direction. The reaction force appears on the right-hand side of equations (3), for example, the force $F_\theta$ equals a product of the flux of the effective magnetic field $4\pi Q$ and the magnon current $j_{\text{magnon}}$. From the equations of motion (equation (3)), we obtain the estimate for the rotation rate of SkX (Supplementary Information)

$$v \sim \frac{j_{\text{magnon}}}{\pi R} \sim -5 \times 10^{-5}$$

(4)

for $j_{\text{magnon}} \sim 10^{-2}$. The minus sign corresponds to clockwise rotation of skyrmions. Equation (4) shows that the experimentally observed clockwise rotation of skyrmions is a consequence of the anticlockwise rotation of magnons. The estimate (equation (4)) is in good quantitative agreement with the numerical result for the SkX rotation rate, $3 \times 10^{-5}$.

We thus showed that the magnon current induced by the temperature gradient is deflected by the emergent magnetic field of skyrmions, which in turn gives rise to the rotation of SkX through the spin-transfer torque. As the sign of $j_{\text{magnon}}$ is governed by the sign of $dT/dr$, the rotation of SkX should be reversed on sign reversal of the temperature gradient, which is indeed what we find in our simulation (Fig. 4c and Supplementary Movie 6). Also the rotation direction becomes reversed on sign reversal of the magnetic field $B$, but not on sign reversal of the Dzyaloshinskii–Moriya parameter $D$, as seen in Fig. 4d, because the former changes the sign of $j_{\text{magnon}}$ but the latter does not. This shows that skyrmion–magnon interactions and thermal spin fluctuations provide a key to understanding the observed chiral rotation of skyrmions.

The proposed physics behind the observed rotation is distinct from the skyrmion Hall effect discussed recently\(^\text{2},\text{3}\), in which it was theoretically proposed that a longitudinal skyrmion motion due to the magnon current along the thermal gradient is accompanied by a small transverse motion due to the Gilbert damping. This skyrmion Hall effect necessarily requires the longitudinal motion. In our case, however, the skyrmion motion in the radial direction (parallel to the temperature gradient) is forbidden owing to the geometrical confinement. Hence, no skyrmion Hall effect is possible and the topological magnon Hall effect is the only source of the observed skyrmion rotation. The reaction force from the magnon current deflected by the effective magnetic field of the topological skyrmion texture drives the peculiar chiral motion.

Note that the timescale of the rotation is microseconds in the simulation for $j \sim 1$ MeV, whereas it is a few seconds in the experiment. This discrepancy can be related to the strong sensitivity of the rotation rate to the shape of the boundary of the SkX, the magnitude of the temperature gradient and sample inhomogeneities, such as impurities and defects. Thus we observe no rotation in a rectangle-shaped system within a realistic simulation time (limited by a few milliseconds), apparently because the large friction between the SkX and the system edges makes the rotation rate extremely slow. In the circular-disk system, the rotation rate decreases as $\Delta T$ decreases, and vanishes when $\Delta T = 0$. In addition, the rotation is less pronounced and its rate is lower in a larger-sized disk, indicating the absence of rotation in the thermodynamic limit.

To summarize, we have found experimentally and explained theoretically that micrometre-sized skyrmion crystals behave as a Feynman’s ratchet. In the presence of the radial magnon flow, skyrmions exhibit persistent rotation in the direction determined by the sign of the topological charge of the skyrmions. The physical origin of this unusual phenomenon can be traced back to the chiral nature of spin dynamics. Our finding shows how thermal spin fluctuations can be harnessed to control topological spin textures by irradiating them with light and electrons. The manipulation of skyrmions with magnons can be used to build all-spin memory and logic devices with low dissipation losses by replacing the charge current by a magnon current, especially in insulating magnets\(^\text{11}\).

Methods

The single-crystal samples of MnSi were grown by the floating zone technique, whereas those of Cu$_2$OSeO$_4$ were grown by the chemical vapour transport method. For the real-space imaging of spin textures, their thin specimens with thickness of ~50 nm were prepared by mechanical polishing and subsequent argon-ion thinning with an acceleration voltage of 4 kV. All experiments were performed with a transmission electron microscope (JEM2100F, JEOL) at an acceleration voltage of 200 kV. Images of the SkX were obtained in the over-focused Lorentz-TEM mode. The movies of Lorentz-TEM image were taken with an exposure time of 50 ms and a frame rate of 18 fps (frames per second). A liquid helium cooling holder was used to investigate the $T$ dependence, by which $T$ at the specimen can be controlled from 6 to 300 K. The electron beam strength is $2.7 \times 10^4 \text{ Am}^{-2}$ ($3.6 \times 10^4 \text{ Am}^{-2}$) for the MnSi (Cu$_2$OSeO$_4$) specimen. A magnetic field of $B = 175 \text{ mT}$ ($65 \text{ mT}$) parallel to the electron beam was applied, and the measurement was performed at $8 \text{ K}$ ($40 \text{ K}$) for MnSi (Cu$_2$OSeO$_4$).

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Author contributions
M. Mochizuki carried out the numerical simulations and analysed the simulation data. X.Z.Y. carried out the Lorentz TEM measurement and analysed the experimental data. S.S. carried out the crystal growth of Cu$_2$OSeO$_3$. N.K. carried out the crystal growth of MnSi. The whole work has been led by N.N. and Y.T. The results were discussed and interpreted by M. Mochizuki, X.Z.Y., W.K., J.Z., M. Mostovoy, Y.T. and N.N. The draft was written by M. Mochizuki, M. Mostovoy, Y.T. and N.N.

Additional information
Supplementary information is available in the online version of the paper. Reprints and permissions information is available online at www.nature.com/reprints. Correspondence and requests for materials should be addressed to M. Mochizuki.

Competing financial interests
The authors declare no competing financial interests.
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In the version of this Letter originally published online, in Fig. 4c, the solid blue curve was missing. This error has now been corrected in all versions of the Letter.