Generation of entangled photons by trapped ions in microcavities under a magnetic field gradient

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We propose a potential scheme to generate entangled photons by manipulating trapped ions embedded in two-mode microcavities, respectively, assisted by a magnetic field gradient. By means of the spin-spin coupling due to the magnetic field gradient and the Coulomb repulsion between the ions, we show how to efficiently generate entangled photons by detecting the internal states of the trapped ions. We emphasize that our scheme is advantageous to create complete sets of entangled multi-photon states. The requirement and the experimental feasibility of our proposal are discussed in detail.

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I. INTRODUCTION

Entangled photons are important sources in quantum communication and quantum cryptography. To generate entangled photons, we can make use of atomic cascade decay, parametric down conversion in nonlinear crystals, and exciton emission in a semiconductor quantum dots. Besides, entangled photons can also be produced by using polarizing beam splitters. On the other hand, there have been a lot of proposals to entangle atoms by detecting the emitted photons, in which the entanglement of the photons produced by beam splitters is projected to internal degrees of freedom of the atoms. Since photons are flying qubits, this is a way towards future quantum network based on local atomic qubits.

In this paper, we study a scheme to project the entanglement in atomic states to the photons emitted from these atoms, a reverse step with respect to the proposals in Ref. 6. We noticed a recent work for the same purpose Ref. 7, in which the entangled photon pairs can be created from two distant dipole sources by means of the postselection and interference effects. In contrast, we create entangled photons by using modified ion traps, with which the trapped ions interact via spin-spin coupling produced by a magnetic field gradient and the Coulomb repulsion. The favorable features of our scheme include: (1) The entangled photons can be generated efficiently, and complete sets of entangled states of more than two photons are achievable. Because we put the trapped ions in microcavities with one ion in a microcavity, by adjusting the cavity decay rate and the detunings (mentioned specifically below), we can have a high success rate of photon generation. With more microtrap-microcavity setups added, the entangled states of more photons can be obtained straightforwardly. (2) The entanglement of the trapped ions is made by the spin-spin couplings, i.e., Ising terms, in which the vibrational modes of the ions remain unchanged throughout the scheme. So we don’t require the ions to be strictly cooled to the vibrational ground state and our quantum gating for the entanglement generation is in principle robust to heating of the ions. (3) The collection rate of the emitted photons from the ions can in principle approach to unity because the trapped ions are embedded in microcavities, and we collect the leaking photons from each microcavities. (4) Since the Ising term is widely used for quantum computing in other systems, e.g., the semiconductor quantum dots Ref. 11, our proposal can in principle be generalized to those systems.

II. GENERATION OF ENTANGLED PHOTON PAIRS

We first consider the simplest case, i.e., two trapped ions, for example, Yb$^+$, in a magnetic field gradient embedded respectively in two two-mode microcavities. These two ions are also confined in two microtraps, respectively, as discussed in Ref. 10 and shown in Fig. 1. Although in principle they are not required to be in the zero point of the trapping potential, the ions are required to be strictly within Lamb-Dicke limit regarding laser radiation. If we encode qubits in the levels $|g\rangle$ and $|e\rangle$, respectively, according to Ref. 6 Ref. 10, the two qubit states will interact via an Ising
coupling, i.e., $H_I = -(J/2)\sigma_1^x\sigma_2^x$, where $\sigma_z^m = |e\rangle_m\langle e| - |g\rangle_m\langle g| \ (m = 1, 2 \text{ here and hereafter})$ and

$$J = \sum_{n=1}^{2} \frac{1}{\nu_n} S_{n1} S_{n2} \frac{\partial \omega_1}{\partial z} \frac{\partial \omega_2}{\partial z} (\Delta z_n)^2,$$  

(1)

with $S_{nm}$ a unitary transformation matrix related to the Hessian of the potential, and $\nu_n$ and $\Delta z_n$ are respectively the frequency and the spatial spread of the ground state wavefunction of the mode $n$ of the ions’ collective vibration. $\omega_n = g \mu_B B_m$ with $B_m$ the magnetic field at the position the ion $m$ staying. $J$ can be changed by adjusting the trap frequencies, the distance between the two microtraps and the magnetic field gradient [10].

Our main idea is to map the entanglement from the qubit states to the photons emitted from the ions. We consider the configuration of each ion as in Fig. 1(a), where for convenience the hyperfine levels are defined to be: $(1, 1) = |e\rangle$, $(0, 0) = |g\rangle$, $(1, 0) = |e'\rangle$, $(1, -1) = |g'\rangle$ in $S_{1/2}$, and $(1, 0) = |r\rangle$ in $P_{1/2}$. $\Omega_{jm}$ and $h_{jm}$ (here and hereafter $j=g,e$) are coupling constants with respect to lasers and cavity modes to the $m$th ion, respectively. We assume that the laser radiation and the cavity modes are detuned from $|r\rangle$ by $\delta_{jm}$. Due to these large detunings, we have two Raman processes in each cavity with effective Rabi frequencies $\tilde{\Omega}_{jm} = \Omega_{jm} h_{jm}/\delta_{jm}$. For simplicity, we assume $\Omega_{gm} = \Omega_{cm} = \Omega_m$ by appropriately adjusting $\Omega_{jm}$, $h_{jm}$ and $\delta_{jm}$.

Considering the cavity decay, we have following non-Hermitian Hamiltonian for each cavity,

$$H_m = \tilde{\Omega}_m (c_{gm}|g'\rangle_m\langle g| + c_{gm}^\dagger |g\rangle_m\langle g'| + c_{em}|e'\rangle_m\langle e| + c_{em}^\dagger |e\rangle_m\langle e'|) - i\kappa_m (c_{gm}^\dagger c_{gm} + c_{em}^\dagger c_{em}),$$  

(2)

where we suppose the same decay rate $\kappa_m$ for the two cavity modes. Consider the ion to be initially prepared in the state

$$|\Psi_0\rangle_m = \frac{1}{\sqrt{2}}(|g\rangle_m + |e\rangle_m)|00\rangle_m,$$  

(3)

with the vacuum states of $\sigma_0$ and $\sigma_+$ cavity modes in $m$th cavity denoted by $|00\rangle_m$. Before any photon leaking out of each cavity, we can solve Eq. (2) following the idea in [12]. After an interaction time $\tau_m$ with $\tilde{\Omega}_m$ satisfying $\tan(\tilde{\Omega}_m\tau_m) = 2\tilde{\Omega}_m/\kappa_m$ and $\tilde{\Omega}_m' = \sqrt{\tilde{\Omega}_m^2 - \kappa_m^2}/4$, the system evolves to

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|g\rangle_m|10\rangle_m + |e\rangle_m|01\rangle_m),$$  

(4)

with the success probability

$$P_m = \exp(-\kappa_m\tau_m) \sin^2(\tilde{\Omega}_m\tau_m)/(\tilde{\Omega}_m/\tilde{\Omega}_m')^2.$$  

(5)

When $t > \max\{1/\kappa_1, 1/\kappa_2\}$, photons will leak out of the cavities, which yields the total wavefunction

$$|\Phi\rangle = \frac{1}{2} \prod_{m=1}^{2} (|g\rangle_m|\sigma_0\rangle_m + |e\rangle_m|\sigma_+\rangle_m).$$  

(6)

Before we discuss how to collect and store the emitted photons, we first focus on how to make Bell measurement on the two ions, which projects entanglement to the two leaking photons no matter where they are. Our steps of Bell measurement follow the ideas in [12, 14, 15], i.e., performing Hadamard and two-qubit gates before detection on the qubit states by standard fluorescence techniques [10]. Due to the Ising coupling between the ions, we can carry out the two-qubit gate by using the sophisticated method in NMR quantum computing [12].

$$CNOT_{12} = e^{-i\pi/4}e^{-i(\pi/4)\sigma_z^1}e^{i(\pi/4)\sigma_z^2}e^{i(\pi/4)\sigma_z^1}e^{-i(\pi/4)\sigma_z^2}e^{i(\pi/4)\sigma_z^1}e^{i(\pi/4)\sigma_z^2},$$  

(7)

where the ions 1 and 2 are control and target, respectively, and $\sigma_z^m$ is the Pauli matrix of the qubit states of the ion $m$ as defined in $H_I$. The Hadamard gate on ion $m$ is defined as $U_{Hm}: |g\rangle_m \rightarrow |g\rangle_m + (-)|e\rangle_m)/\sqrt{2}$. All these operations can be done with high fidelity by microwave pulses involving refocusing [12, 17].

After a two-qubit gate CNOT$_{12}$, followed by a Hadamard gate $U_{H1}$, we have from Eq. (6),

$$\frac{1}{2\sqrt{2}}|gg\rangle_{12}(|\sigma_+\rangle_{12} + |\sigma_0\rangle_{12}) +$$  

$$\frac{1}{2\sqrt{2}}|ee\rangle_{12}(|\sigma_0\rangle_{12} - |\sigma_+\rangle_{12}) +$$  

(7)
(1/2\sqrt{2})|eg\rangle_{12}(|\sigma_0\sigma_0\rangle_{12} - |\sigma_+\sigma_+\rangle_{12}) + \\
(1/2\sqrt{2})|ge\rangle_{12}(|\sigma_0\sigma_+\rangle_{12} + |\sigma_+\sigma_0\rangle_{12}). \tag{8}

So a certain detection on the ions would yield a certain entangled photon pair. In other words, if we have two photons produced, we can entangle them deterministically by our scheme.

To have a highly efficient generation of entangled photons, we require 0 < \kappa_m < \tilde{\Omega}_m. In the limit of \kappa_m = 0, the success rate \( P = 1 \). Current ion-trap-cavity setup could not meet our requirement because \( \kappa_m \gg \tilde{\Omega}_m \) (i.e., \( \kappa_m = 0.64 \) MHz, \( \Omega_{jm} \sim h_{jm} = 0.02 \) MHz, and \( \delta_{jm} = 0.1 \) MHz) \cite{18}. But in cavity QED, \( h_{jm} \) and \( \kappa_m \) are dependent on the size \( R \) and the quality of the cavity. We have following relations \cite{19}: \( h_{jm} \sim R^{-3/4} \) and \( \kappa_m \sim (T/R) \) with \( T \) the total loss of the cavity field. For the cavity in our case of the size 10 \( \mu \)m and with the same finesse as in \cite{12}, we have \( h_{jm} = 138.4 \) MHz and \( \kappa_m = 960 \) MHz. Suppose \( \Omega_{jm} = 10 \) MHz, and \( \delta_{jm} = 0.1 \) MHz, we have \( \tilde{\Omega}_m \gg \kappa_m \). Fig.2 presents different cases regarding different \( \delta_{jm} \) and \( \kappa_m \).

How to collect the emitted photons is important to the application of our scheme. The frequency difference between \( |g\rangle_m \) and \( |e\rangle_m \) is different from the ion to ion due to the applied magnetic field gradient. By choosing suitable detunings, however, we can have all the generated photons with the same frequency, but with different polarizations associated with the deexcited levels \( |g\rangle_m \) and \( |e\rangle_m \). This implies that we need two high-Q two-mode (i.e., \( \sigma_0 \) and \( \sigma_+ \)) cavities \cite{20} to collect the photons from the microcavities involving the ions, respectively, as shown in Fig.1 (b). Moreover, the minimum photon storage time is the time for the two leaking photons becoming entangled. Table I gives \( J \) of the order of KHz. So considering the time for Hadamard gate and refocusing pulses, generation of an entangled photon pair in our scheme takes time at least of the order of millisecond.

### III. GENERATION OF ENTANGLED MULTI-PHOTON STATES

Our scheme is suitable for not only repeatedly producing entangled photon pairs, but also generating entangled multi-photon states. For example, with three ions confined in three microtrap-microcavity setups, respectively, after laser pulse radiations on these ions, followed by the cavity-induced emission, we perform CNOT\(_{12}\), CNOT\(_{13}\), and a Hadamard gate \( U_{H1} \), which yields

\[
(1/4)|ggg\rangle_{123}(|\sigma_+\sigma_+\sigma_+\rangle_{123} + |\sigma_0\sigma_0\sigma_0\rangle_{123}) + \\
(1/4)|egg\rangle_{123}(|\sigma_0\sigma_0\sigma_0\rangle_{123} - |\sigma_+\sigma_+\sigma_+\rangle_{123}) + \\
(1/4)|gge\rangle_{123}(|\sigma_0\sigma_0\sigma_+\rangle_{123} + |\sigma_+\sigma_0\sigma_+\rangle_{123}) + \\
(1/4)|geg\rangle_{123}(|\sigma_0\sigma_+\sigma_+\rangle_{123} + |\sigma_+\sigma_0\sigma_0\rangle_{123}) + \\
(1/4)|ege\rangle_{123}(|\sigma_0\sigma_0\sigma_+\rangle_{123} - |\sigma_+\sigma_0\sigma_0\rangle_{123}) + \\
(1/4)|gee\rangle_{123}(|\sigma_0\sigma_0\sigma_0\rangle_{123} + |\sigma_+\sigma_+\sigma_0\rangle_{123}) + \\
(1/4)|gge\rangle_{123}(|\sigma_0\sigma_0\sigma_0\rangle_{123} - |\sigma_+\sigma_+\sigma_0\rangle_{123}). \tag{9}
\]

So once we have produced the photons from the ions embedded in the microcavities, we can generate any entangled three-photon state with a certain probability by measuring the internal states of the ions. In principle, for N ions confined in N microtrap-microcavity setups, respectively, we can generate \( 2^N \) different entangled states by implementing CNOT\(_{12}\), \ldots, CNOT\(_{1N}\), and \( U_{H1} \), followed by measurements on the internal states.

Entangling more photons, however, takes longer time because more operations should be taken. Besides, to make our scheme work well, we hope that the couplings between any two of the ions are on the same order of magnitude so that we can carry out CNOT\(_{ij}\) (\( j > i + 1 \)) efficiently. As shown in Table II with different cases for three ions
under consideration, the nearest-neighbor coupling is almost equivalent to the second nearest-neighbor one. To reduce infidelity due to the vibrations of the ions, we restrict $\epsilon \leq 0.071$ in our calculation, where $\epsilon$ is an additional Lamb-Dicke parameter regarding the magnetic field gradient \cite{21}. Therefore, the magnetic field gradient is also restricted to be smaller than a certain number and thereby the spin-spin coupling could not be very large. Due to the weak coupling, in addition to more steps to take, the implementation time in three-ion case is much longer than in the two-ion case.

More specifically, to carry out our scheme for $N$ (e.g., $N < 10$) ions, we need a minimum time $(N - 1) \times t_0 + t_1$ to generate the entangled photons, where we suppose the time for a single CNOT$_{ij}$ to be $t_0$, and other time for Hadamard gate and measurement to be $t_1$. On the other hand, for more photons entangled, the success rate in our scheme would be lower. This is due to the enhanced failure probabilities of emitting desired photons from individual ions $(x (1 - P_1 \cdots P_N))$ with $P_m$ the success rate for each ion), and of the detection for a certain entangled photon states.

Although the generation rate of entangled photon pairs with our scheme (i.e., 1 pair/ms) is lower than with other methods, e.g. in Refs. \cite{2} \cite{3}, we argue that our proposal is good at generation of entangled multi-photon states. The previous scheme \cite{3} by means of biexcitons in semiconductor quantum dots is hard to extend to creation of entangled state for more than two photons. Although with the ideas in Refs. \cite{1} \cite{4} entangled multi-photon states could be generated, our scheme is advantageous to create complete sets of entangled multi-photon states. Moreover, the proposals based on linear optical elements suffer from low success rate because half of outputs have to be discarded. For example, in a recent experiment for entangled five-photon states \cite{22}, the success rate after the photons going through several beam splitters is lower than 1/128, even if we neglect other imperfect factors. In contrast, with our scheme, any entangled state of five photons can be obtained in an ideal implementation with equal success rate of 1/32.

IV. DISCUSSION AND CONCLUSION

There are some points we have to emphasize for realizing the present scheme. First, the long-range Coulomb force regarding the spin-spin coupling between non-neighboring ions makes the scheme hard to scale up to a large number of ions. We have noticed that a scalable approach is proposed in \cite{10} by making the nearest-neighbor coupling prominent by properly choosing the trapping potentials of individual traps as well as the distance between the traps. However, in the discussion above, we prefer nearly constant coupling strength between any two ions. So our scheme is only valid for few-photon entanglement.

Second, although the vibrational modes of the ions remain unchanged throughout the scheme, the spin-spin coupling we employ here is from virtually exciting the vibrational mode \cite{10}, which is similar to the idea in hot-ion scheme \cite{23}. As an example, we estimate the two-qubit gating time to be 2 ms, and only 10% vibrational modes are actually excited during the gating. This implies that our scheme would work as long as the heating times of the vibrational states are longer than 0.2 ms. However, the heating rate changes with the trap size as $L^{-2}$, where $L$ is the trap size and $s$ can be 2, 4 and 5 depending on different traps \cite{23}. Since the heating time for a trap with $L = 100 \mu m$ is 4 ms \cite{14}, the microtrap smaller than 10 $\mu m$ would be of very heating time. Therefore, more advanced techniques to improve current microtraps are highly expected to achieve our scheme.

Alternatively, we can give up the microtraps, but put the ions with each confined in a microcavity into a linear trap. Current experiments in linear trap have achieved the ultracold ions confined with the spacing of the order of $\mu m$ and the heating time of the order of microsec \cite{13}. While in this case, the ions experience the same trap frequency, and thereby we will not have a nearly constant spin-spin coupling between any two ions. This might more or less prolong the implementation time, but will not be a serious problem for the few-ion case anyway.

Third, by choosing different detunings, we may generate photons with different energies (i.e., frequencies) but with polarization entangled, which are also useful \cite{3}. But in this case, we need microcavities with two modes of different frequencies and different polarizations to generate the photons and to collect the leaking photons, which makes the experimental requirements more challenging.

Fourth, to make our scheme work well, we should have the Lamb-Dicke parameter of each ion much smaller than 1 to avoid any vibrational mode excitation. As referred to above, however, besides the usually defined Lamb-Dicke parameter $\eta$ related to the laser radiation, there is an additional contribution $\epsilon$ in our case from the magnetic field gradient. Provided $\eta = 0.1$, the effective Lamb-Dicke parameter $\eta_1 = \sqrt{\eta^2 + \epsilon^2} \leq 0.12$, where we have used the numbers in Table 1. Anyway, a larger Lamb-Dicke parameter, even if a little bit larger, is not good for our implementation of the scheme.

Finally, although we have mentioned in \cite{21} that no detailed discussion regarding the techniques of photon collection and storage would be given in this paper, we have to point out that both the collection and storage of the entangled multi-photon would be more difficult than that of photon pairs. Since the leaking photons from the mirrors of each microcavities would be directed toward the corresponding cavities for photon collection, we neglected any imperfect
factors due to the devices in above discussion and assumed that the collection rate is unity. As known from [19], however, the bigger the cavity, the longer survival time the inside photons. Because the implementation time in the multi-photon case is much longer than the case of photon pairs, we have to have much bigger cavities for collecting leaking photons in the multi-photon cases. This will probably reduce the collection rate due to the size difference between the microcavities for photon generation and cavities for photon collection. For example, the current optical cavity (of the size of 0.8 \sim 10 \text{ mm}) is bigger than our required cavity for photon collection by at least 80 times [13,19]. To store the photons with high fidelity in the three-photon case, however, by using the numbers in Tables I and II, we require the size of the currently available optical cavity to be further enlarged by at least 5 times. This big difference in size is evidently not good for highly efficiently collecting the leaking photons, and the solution of this problem relies on the further advance in cavity QED: Reducing the cavity size by two orders of magnitude and increasing the finesses of the cavity by two to three orders of magnitude [12].

In summary, we have proposed a potentially practical scheme to generate entangled photons by trapped-ions embedded in microcavities in a magnetic field gradient. We use microtraps to fix the ions, and by adjusting the microtraps we can have different spin-spin couplings. The (micro)cavities play the role in photon generation and collection. Compared to other previous proposals [6], our scheme gives a reverse step, and could provide an efficient generation of entangled photons. Although the necessary microtraps and microcavities are not within reach of the present techniques, our proposal has advantages of creating complete sets of entangled multi-photon states, which would be a stable and efficient source of entangled photons useful for future large-scale quantum information processing.

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Note added: After finishing this paper, we become aware of a work [25], in which the Ising coupling between ions is obtained by changing the laser intensities and the polarizations. As it is mathematically identical to the models in [3,8,9,10], our scheme can in principle be applied to it.

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[20] Since the photons leaking out of the two microcavities are of the same energy (or say, frequency), but probably of different polarizations, the two collection cavities should also be of two modes and are designed to be resonant with the photons getting in. However, the current optical cavity decays on the order of milliseconds, which is of the same order as the minimum storage time calculated below for the entangled photon pairs. Alternatively, we may use two high-Q fibers to
collect the photons. While according to the minimum storage time calculated below, the fibers should be at least thousands of kilometers long. Therefore, both of these collection methods are experimentally challenging at present, and the detailed discussion in technique is beyond the scope of this paper. We here argue that, even if we neglect the collection step, our scheme is good for an efficient source of entangled photons.

[21] As shown in [8, 9, 10, 15], this additional Lamb-Dicke parameter is related to the different confined ions and the different vibrational modes. So it is denoted by $\epsilon_{nl}$ for the nth ion and the lth vibrational mode. Here we choose the biggest $\epsilon_{nl}$ to be $\epsilon$, and restrict it to be smaller than 0.071.

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Table I. Ising coupling $J$ in two-ion case, where $d$ is the center-to-center distance between the two microtraps, $\nu_m (m = 1, 2)$ is the frequency of the microtrap $m$, $\Delta_m (m = 1, 2)$ is the deviation of the equilibrium position of ion 1 or ion 2 from their respective trap centers, and $h = d + \Delta_1 + \Delta_2$ is the inter-ion distance. $\epsilon$ is the biggest one in $\epsilon_{nl}$ which is related to the nth ion and the lth vibrational mode. The listed $J$ is the largest spin-spin coupling in our calculation under $\epsilon < 0.071$ and a specific $d$.

| $d$ ($\mu$m) | $\nu_1$ (MHz) | $\nu_2$ (MHz) | $\partial B/\partial z (T/m)$ | $\Delta_1$ ($\mu$m) | $\Delta_2$ ($\mu$m) | $h$ ($\mu$m) | $\epsilon$ | $J$ (KHz) |
|--------------|---------------|---------------|-----------------------------|---------------------|---------------------|-------------|-----------|-----------|
| 6.0          | 5.55          | 5.55          | 550                         | 0.521               | 0.521               | 7.042       | 7.066e-002| 6.328     |
| 7.0          | 4.50          | 4.50          | 400                         | 0.588               | 0.588               | 8.176       | 7.038e-002| 4.980     |
| 8.0          | 3.75          | 3.75          | 300                         | 0.653               | 0.653               | 9.307       | 6.939e-002| 3.959     |
| 9.0          | 3.30          | 3.30          | 250                         | 0.681               | 0.681               | 10.362      | 7.005e-002| 3.370     |
| 10.0         | 2.35          | 2.35          | 150                         | 1.001               | 1.001               | 12.001      | 6.994e-002| 2.875     |

Table II. Ising couplings in three-ion case, where $d$ is the center-to-center distance between two neighboring traps, $\nu_m (m = 1, 2)$ is the frequency of the microtrap $m$, $\nu_3 = \nu_1$ due to the symmetry. So we omit it. $\Delta$ is the deviation of the equilibrium position of ion 1 or ion 3 from their respective trap centers, and $h = d + \Delta$ is the distance between two neighboring ions. Due to symmetry, ion 2 (i.e., the middle one) oscillates at the center of the trap confining it, and $J_{12} = J_{23}$. Other parameters are defined in the text.

| $d$ ($\mu$m) | $\nu_1$ (MHz) | $\nu_2$ (MHz) | $\partial B/\partial z (T/m)$ | $\Delta$ ($\mu$m) | $h$ ($\mu$m) | $\epsilon$ | $J_{12}$ (KHz) | $J_{13}$ (KHz) |
|--------------|---------------|---------------|-----------------------------|-------------------|-------------|-----------|----------------|----------------|
| 6.0          | 2.75          | 7.75          | 240                         | 2.037             | 8.037       | 6.994e-002| 1.455         | 1.448          |
| 7.0          | 2.55          | 7.25          | 210                         | 1.922             | 8.922       | 7.048e-002| 1.141         | 1.149          |
| 8.0          | 2.05          | 5.80          | 150                         | 2.252             | 10.25       | 6.962e-002| 0.922         | 0.922          |
| 9.0          | 1.45          | 4.10          | 90                          | 3.186             | 12.19       | 6.810e-002| 0.747         | 0.747          |
| 10.0         | 1.20          | 3.40          | 70                          | 3.688             | 13.69       | 6.996e-002| 0.670         | 0.672          |

Captions of the figures

Fig. 1 (a) Level scheme for a single Yb ion, where the states regarding $F = 1$ for $S_{1/2}$ and $P_{1/2}$ are split into three levels respectively, and the lowest level (0,0) is for $F = 0, S_{1/2}$. We encode qubits in $|g\rangle$ and $|e\rangle$, respectively, and take $|r\rangle$, $|g'\rangle$, and $|e'\rangle$ to be auxiliary states. $\Omega_{jm}$ and $h_{jm}$ ($j = g,e$ and $m = 1,2$) are coupling constants regarding lasers and cavity modes, respectively. $\sigma_0$ and $\sigma_+$ are for polarization of the classical and quantum fields. $\delta_{jm}$ is the detuning. (b) Schematic plot for generation of two leaking photons from the two-mode microcavites (on the left) involving the ions by laser radiation and for collection by two two-mode cavities (on the right) with high-Q, where $B(z)$ is the magnetic field gradient, and the two ions are confined in two microtraps, respectively.

Fig. 2 Success rate of two-photon generation vs. cavity decay rate $\kappa$ based on Eq. (5) for $P = P_1 P_2$, where for simplicity we suppose the situation in the two microcavites to be the same, i.e., $\Omega_{j1} = \Omega_{j2} = \Omega$, $h_{j1} = h_{j2} = H$, and $\delta_{j1} = \delta_{j2} = \delta$. We set $H = 138$ MHz, $\Omega = 10$ MHz. The curves from top to bottom correspond to $\delta = 0.1$ MHz, 0.25 MHz, 0.5 MHz, and 1.0 MHz. If $\delta \leq 10^{-3}$ MHz, $P$ is almost 1 even when $\kappa = 10^9$ Hz. For generating two photons with the same energy but entangled in polarization, $\delta_{em}$ should be different from $\delta_{gm}$. While in that case the results are very similar to those shown here.
