Automated higher-order mesh generation in MATLAB on biomaterials

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Abstract. We propose a method for automated higher-order triangle and tetrahedral mesh generation in MATLAB on biomaterials. The proposed mesh generator is based on the MATLAB function generate Mesh from the Partial Differential Equation Toolbox. Here, the inputs are STL file and an approximate upper bound on mesh edge length for the geometry under consideration using the higher-order triangle or tetrahedral elements. As output we get a higher-order triangle or tetrahedral meshes, connectivity matrix, coordinates of the nodes, and boundary nodes. These outputs can be efficiently used for solving any partial differential equations by finite element method which arises in many applications like biomaterials, computational materials science, crystal plasticity, materials engineering, mechanics of materials etc.

Keywords: Mesh generation, Biomaterials, Higher-order triangle element, Higher-order tetrahedral element

1. Introduction
Biomaterials are artificial or natural materials like ceramics, metals, composites, polymers etc. used to supplant ailing or cracked biological structures to restore its shape and functionality. Mesh generation (MG) and finite element (FE) analysis are widely used in Biomechanics for modelling and simulation of biomaterials using mechanical laws [1-4]. Three dimensional (3D) CAD geometries and STL files are extensively used in Dental Biomaterials [2]. It is also utilized in modelling of soft biomaterials such as brain, liver, tendon, fat, etc. MG is broadly applied to extract their uniaxial mechanical behaviours for further implementation in human body FE simulations under injurious mechanical loads [3]. Femur bone is the biggest and strongest part of human body. As of late, breakage of femur bone turns out to be the most common problem faced by ladies and older people groups. Therefore, automatic MG is in high demand to mesh a human femur. In [4], a brief review work is presented on the study of finite element analysis carried by researchers on different biomaterials.

It is well known that the higher-order (HO) techniques offer more noteworthy precision and productivity for scale-resolving simulations, in various material science and engineering applications including biomaterials. Therefore, these techniques can be used to reach designing error tolerance levels with less computational effort [5]. Nevertheless, for higher-order techniques to be powerful, they should be

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combined with efficient higher-order mesh generators [6-10]. Software to create higher-order meshes isn’t promptly accessible on the grounds that the algorithms to deliver higher-order meshes are still being worked on. For MATLAB users higher-order tetrahedral (from cubic-order onwards) automated mesh generating program is not available for 3D geometries.

Therefore, here we propose a simple and efficient method for automated higher-order mesh generator especially for MATLAB users in biomaterial applications. The same method can be efficiently applied to solve any other engineering and science problems. The methodology proposed in this paper is the extension of the work developed for automated higher-order finite element mesh generation exhibited in [8-9].

In this paper, we briefly explain the mathematical formulations and explanation for automated higher-order triangle and tetrahedral mesh generation method in MATLAB for biomaterial applications in section 2. In section 3, we discuss the outcomes and results of the proposed methodology. We conclude the work with directions for future research in section 4.

2. Mathematical formulations and explanation for higher-order triangle and tetrahedral mesh generation in MATLAB for geometries on biomaterials

In numerous applications including FE analysis, MG is a fundamental pre-step. Here, we give the technique on how a basic and proficient two-dimensional (2D) and 3D HO triangle and tetrahedral mesh can be created in MATLAB utilizing mesh generator in PdeModeler as the initial mesh which is accessible from the Partial Differential Equation Toolbox [10]. The outputs of the proposed MG code can be utilized for FE applications effectively as it delivers excellent HO meshes and ensures the absence of duplicate nodes, boundary nodes lay on the boundary with the uniform orientation of the node numbering of the finite elements as shown in figures 1-9.

In the case of 2D geometry, we place the nodes on the HO triangular FE as shown in figures 1-5 for linear to quintic order triangular elements using a point transformation formula. We use the following point transformations for any HO triangular element to define the nodes on the boundary and the interior [8]:

\[
x = \sum_{i=1}^{(n+1)(n+2)} N_i^{(n)}(\xi, \eta) x_i, \quad y = \sum_{i=1}^{(n+1)(n+2)} N_i^{(n)}(\xi, \eta) y_i.
\]

Here, \( N_i^{(n)}(\xi, \eta) \) represents the Lagrange interpolation function for \( n^{th} \) order triangle element at node \( i \) in the natural coordinates \((\xi, \eta)\). For any HO triangular element, the transformation formula given by equation (1) reduces to the following after applying the section formula along each edge of the FE:

\[
x = x_3 + (x_1 - x_3)\xi + (x_2 - x_3)\eta, \quad y = y_3 + (y_1 - y_3)\xi + (y_2 - y_3)\eta.
\]
Similarly, in the case of 3D geometry, we place the nodes on the HO tetrahedral FE as shown in figures 6-9 for linear to quartic order tetrahedral elements using a point transformation formula. We use the following point transformations for any HO tetrahedral element to define the nodes on the boundary and the interior [9]:
Here, $N_{i}^{(n)}(\xi, \eta, \zeta)$ represents the Lagrange interpolation function for $n^{th}$ order tetrahedral element at node $i$ in the natural coordinates $(\xi, \eta, \zeta)$. After applying the section formula along each edge of the FE for any HO tetrahedral element, the transformation formula given by equation (2) reduces to the following:

\[
x = x_4 + (x_i - x_4)\xi + (x_2 - x_4)\eta + (x_3 - x_4)\zeta,
\]
\[
y = y_4 + (y_i - y_4)\xi + (y_2 - y_4)\eta + (y_3 - y_4)\zeta,
\]
\[
z = z_4 + (z_i - z_4)\xi + (z_2 - z_4)\eta + (z_3 - z_4)\zeta,
\]

Figure 6. Linear tetrahedron element.

Figure 7. Quadratic tetrahedron element.
We have set up an automated HO MG code in MATLAB with triangle and tetrahedral elements for 2D and 3D geometries. The techniques used for the MG are as follows:

Step 1. Input the STL files and an approximate upper bound on mesh edge length for the geometry of the biomaterial under consideration.

Step 2. Import the STL file in MATLAB utilizing the function importgeometry from the Partial Differential Equation Toolbox [10].

Step 3. Generate the linear mesh using the MATLAB function generateMesh from the Partial Differential Equation Toolbox [10].

Step 4. Generate the higher-order triangle meshes with the desired HO triangle elements as shown in figures 1-5, utilizing the MATLAB code HOMesh2d.m developed for automated higher-order triangle mesh generation presented in [8]. In a similar way, higher-order tetrahedral meshes are generated by using HO tetrahedral elements as shown in figures 8-9, ensuring the absence of duplication of nodes as explained in [9].

In the following section, the outcome of the proposed automated HO meshes is shown for few biomaterials. These MG methods produce high-quality meshes which can be efficiently utilized for FE analysis on biomaterials in MATLAB.

3. Results and discussions

In this section, the results obtained from the methodology discussed in section 2 applied to 2D and some complex biomaterial structures in 3D are provided. Here, the proposed code is successfully applied for 2D and some 3D biomaterial structures like arm cast, premolar crown in dentistry etc. The outputs from the code can be productively used in FE analysis of diverse biomaterial applications because here, the post-treatment requirement for FE operations are taken care like the absence of duplicate nodes, triangle node orientation are same for all mesh elements, boundary nodes are from the boundary etc. Therefore, automated mesh generation methodology can be efficiently applied for these structures to get more accurate numerical results with FE method.

3.1 Example. A planar 2D geometry

The following figures 10-13 show the MG outcome for 2D geometry from the developed MATLAB program as discussed in section 2:
Figure 10. Initial mesh of a planar 2D geometry.

Figure 11. Cubic mesh of a planar 2D geometry.

Figure 12. Quartic mesh of a planar 2D geometry.
In the following examples, we propose a 3D automated HO mesh generator in MATLAB as discussed in section 2, to mesh complex biomechanical structures like Premolar Crown and an adjustment pad in Arm Cast. The figures 15-20 show the MG outcome for these geometries:

3.2 Example. Premolar Crown geometry
FEM is becoming a viable research tool for biomechanical analyses in dentistry. It is a pervasive method for demonstrating complex structures and examining their mechanical properties. FEA in Implantology has been utilized to study the pressure designs in different implant parts and furthermore in the peri-implant bone [11]. It is additionally valuable for studying the biomechanical properties of implants and also to predict the performance of inserts in clinical condition [12]. So, the proposed technique can be a useful tool for the FE analysts in Implantology. The following figures 14-20 show the MG outcome for 3D premolar crown geometry:

Figure 13. Quintic mesh of a planar 2D geometry.

Figure 14. 3D premolar crown geometry.

Figure 15. Linear mesh of 3D premolar crown geometry.
Figure 16. Cubic mesh of 3D premolar crown geometry.

Figure 17. Cubic mesh of 3D premolar crown geometry from a different angle.

Figure 18. Quartic mesh of 3D premolar crown geometry.

Figure 19. Quartic mesh of 3D premolar crown geometry from a different angle.

Figure 20. Cubic and Quartic meshes with refinement of 3D premolar crown geometry.
3.3 Example. ARM CAST
FEM is extensively utilized in the design of lightweight structures module and the customizable, reusable, compatible (ARM) cast, which is a replacement for the cutting edge mortar or fiberglass cast [13-14]. The mortar or fiberglass cast is the present answer for setting and keeping bones stationary while mending after a medical procedure. The ARM cast was intended to enhance the present mortar or fiberglass cast and it comprehends a large number of the issues encompassing the mortar or fiberglass cast. Therefore, the proposed technique can be effectively utilized for the FEA in the design of mortar or fiberglass cast. The following figures 21-25 show the MG outcome for 3D Arm cast geometry:

![3D Arm cast geometry](image1)

**Figure 21.** 3D Arm cast geometry.

![Linear mesh of 3D Arm cast geometry](image2)

**Figure 22.** Linear mesh of 3D Arm cast geometry.

![Cubic mesh of 3D Arm cast geometry from different angles](image3)

**Figure 23.** Cubic mesh of 3D Arm cast geometry from different angles.
Figure 24. Quartic mesh of 3D Arm cast geometry from different angles.

Figure 25. Cubic and Quartic meshes with refinement of 3D Arm cast geometry from different angles.

4. Conclusions
We have provided in this paper, the details on the software development on automated mesh generation technique in MATLAB for biomaterial applications using higher-order triangle and tetrahedral elements. The proposed mesh generator is based on MATLAB function `generateMesh` and it works very well with refinement as well. It is an efficient tool for FE analysts in biomaterials and medical devices. This mesh can also be very effectively used in finite element method for solving any partial differential equation arising in various material sciences and engineering problems. This methodology of meshing can be adopted for curved geometries using curved finite elements.

5. References
[1] Shubhabrata Datta, J. Paulo Davim, Computational approaches to materials design: theoretical and practical aspects, *IGI Global*, Hershey, 2016.
[2] Ferracane, Jack, Luiz E. Bertassoni, and Carmem S. Pfeifer. Dental Biomaterials, An Issue of Dental Clinics of North America, E-Book. Vol. 61. No. 4. *Elsevier Health Sciences*, 2017.
[3] Prabhu, Rajkumar, Wilburn R. Whittington, Sourav S. Patnaik, Yuxiong Mao, Mark T. Begonia, Lakiesha N. Williams, Jun Liao, and M. F. Horstemeyer, A coupled experiment-finite element modeling methodology for assessing high strain rate mechanical response of soft
biomaterials, *Journal of visualized experiments: JoVE* 99, 2015.

[4] Kadam, Aparna G., Sanjay A. Pawar, and Smita A. Abhang, A Review on Finite Element Analysis of Different Biomaterials used in Orthopedic Implantation 4(4), 2017.

[5] Ims, Jeremy, and Z. J. Wang, Automated low-order to high-order mesh conversion, *Engineering with Computers* 1-13, 2018.

[6] M. Kardani, M. Nazem, J. P. Carter, A. J. Abbo, Efficiency of high-order elements in large-deformation problems of geomechanics, *Int. J. Geomech.* 15(6) 1-10, 2014.

[7] K. V. Nagaraja, V. Kesavulu Naidu, P. G. Siddheshwar, Optimal Subparametric Finite Elements for Elliptic Partial Differential Equations Using Higher-Order Curved Triangular Elements, *International Journal for Computational Methods in Engineering Science and Mechanics* 15(2) 83-100, 2014.

[8] T. V. Smitha, K. V. Nagaraja, J. Sarada, MATLAB 2D Higher-order triangle mesh generator with finite element applications using subparametric transformations, *Adv. Eng. Software* 115 327-356, 2018.

[9] T. V. Smitha, K. V. Nagaraja, MATLAB 3D higher-order tetrahedral mesh generator using the subparametric transformations, *Comput. Phys. Commun.*, (submitted).

[10] MathWorks, Partial Differential Equation Toolbox: User's Guide (R2015a), <https://in.mathworks.com/help/pdf_doc/pde/rn.pdf>, (2015).

[11] Trivedi, Shilpa, Finite element analysis: A boon to dentistry, Journal of oral biology and craniofacial research 4(3) 200-203, 2014.

[12] Roateş i, Iulia, and Simona Roateş i, Numerical FEM modeling in dental implantology, AIP Conference Proceedings, 1738 (1) AIP Publishing, 2016.

[13] Arm Cast, <https://grabcad.com/library/arm-cast-1>, March 7, 2017.

[14] Todd Pietila, Designing a Patient-Specific 3D-Printed Cast with the Lightweight Structures Module, <https://www.materialise.com/en/blog/3d-printed-cast>, May 27, 2016.

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