Classification of the circumstellar disc evolution during the main accretion phase

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ABSTRACT
We performed hydrodynamical simulations to investigate the formation and evolution of protostars and circumstellar discs from the pre-stellar cloud. As the initial state, we adopted the molecular cloud core with two non-dimensional parameters representing the thermal and rotational energies. With these parameters, we derived 17 models and calculated the cloud evolution $\sim 10^4$ yr after the protostar formation. We found that early evolution of the star–disc system can be qualitatively classified into four modes: the massive-disc, early-fragmentation, late-fragmentation, and protostar-dominant modes. In the ‘massive-disc mode’, to which the majority of models belong, the disc mass is greater than the protostellar mass for over $10^4$ yr and no fragmentation occurs in the circumstellar disc. The collapsing cloud shows fragmentation before the protostar formation in the ‘early-fragmentation mode’. The circumstellar disc shows fragmentation after the protostar formation in the ‘late-fragmentation mode’, in which the secondary star gains most of its mass from the circumstellar disc after fragmentation and has a mass comparable to that of the primary star. The protostellar mass rapidly increases and exceeds the circumstellar disc mass in the ‘protostar-dominant mode’. This mode appears only when the initial molecular cloud core has a very small rotational energy. Comparison of our results with observations indicates that the majority of protostars have a fairly massive disc during the main accretion phase: the circumstellar disc mass is comparable to or more massive than the protostar mass. It is expected that such a massive disc promotes gas-giant formation by gravitational instability in a subsequent evolutionary stage.

Key words: hydrodynamics – planets and satellites: formation – circumstellar matter – stars: formation.

1 INTRODUCTION
There are two major mechanisms for gas-giant planet formation: one is the core accretion mechanism, in which a massive solid core forms first and the disc gas then accretes onto the core (Goldreich & Ward 1973; Pollack et al. 1996), and the other is the gravitational instability (GI) mechanism, in which the circumstellar disc directly fragments into gas-giant planets via GI (Cameron 1978). The recent discovery of extra-solar planets at a great distance from the central star, such as HR8799b, c, d and e (Marois et al. 2008, 2010) and GJ579b (Thalmann et al. 2009), creates a new problem for planetary formation in that it is difficult to form planets in regions far from central stars according to the core accretion mechanism, because massive solid-core formation takes too long compared to the dissipation of the gaseous disc (Dodson-Robinson et al. 2009).

The GI mechanism may be more plausible for the formation of these planets. Many studies of disc fragmentation have been performed using either an analytic approach (Rafikov 2005) or numerical simulations (e.g. Pickett et al. 2003; Mejía et al. 2005; Boley et al. 2006; Cai et al. 2008; Stamatellos & Whitworth 2008; Meru & Bate 2010). These efforts, however, seem to lead to the consensus that the formation of planets by GI within $\sim 50$ au is very difficult when the disc-to-stellar mass ratio $M_{\text{disc}}/M_{\text{star}} \lesssim 0.1$, a ratio suggested by observations (see e.g. Kitamura et al. 2002).

However, Inutsuka, Machida & Matsumoto (2010) showed that the circumstellar disc is comparable to or more massive than the protostar ($M_{\text{disc}}/M_{\text{star}} \gtrsim 1$) during the early (i.e. Class 0 or Class I stages) and is highly gravitationally unstable. Recently, such massive discs have been observed around very young protostars (e.g. Eisner & Carpenter 2006; Enoch et al. 2009). The fragmentation in such massive discs may potentially account for
the formation of gas-giant planets with a wide separation, such as HR 8799 or GJ579. Thus, we may have to focus on fragmentation and gas-giant planet formation in a massive disc during the main accretion phase. There are several unsolved problems, for example how long such a massive disc exists, and whether the massive disc can fragment to form sub-stellar objects.

So far, few authors have investigated the early evolution of the circumstellar disc and its fragmentation from the pre-stellar core. Using the thin-disc approximation, Vorobyov & Basu (2010a) calculated the formation and evolution of the circumstellar disc in the molecular cloud core with a barotropic equation of state (or including cooling and heating effects, Vorobyov & Basu 2010b). They found that a gravitationally unstable disc frequently appears in the star formation process and tends to show fragmentation during the main accretion phase. Using an isothermal equation of state, Kratter et al. (2010) investigated the formation of the circumstellar disc in their three-dimensional calculation. They also showed massive-disc formation during the main accretion phase and concluded that the disc fragments if the disc-to-stellar mass ratio \( M_{\text{disc}}/M_{\text{star}} \geq 1 \). Using an adiabatic equation of state and radiative cooling,Walch et al. (2009) studied circumstellar disc formation with a coarse spatial resolution. They showed that a massive disc becomes hot during the early stage of the main accretion phase and fragmentation is suppressed.

Machida, Inutsuka & Matsumoto (2010) also investigated disc formation but with a finer spatial resolution than Walch et al. (2009) and using a barotropic equation of state. In the collapsing cloud, the adiabatic heating dominates the radiative cooling at \( n \approx 10^{10} \, \text{cm}^{-3} \), and the first (adiabatic) core with a size of \( \sim 1 \, \text{au} \) forms before the formation of the protostar (Larson 1969; Masunaga & Inutsuka 2000). Machida et al. (2010) pointed out that the first core directly evolves into the circumstellar disc after the protostar formation. Thus, to investigate the (early) formation of the disc, it is necessary to resolve the first core, which has a size of \( \sim 1 \, \text{au} \). In their calculation, however, they implicitly assumed a point-symmetric structure because they fixed the protostellar location to the centre of the computational domain. Michael & Durisen (2010) pointed out that the stellar motion weakens GI activity to some degree when the disc-to-stellar ratio is \( M_{\text{disc}}/M_{\text{star}} \sim 0.1 \). Thus, a more adequate treatment may be necessary for the central protostar.

All previous studies of circumstellar disc formation have shown that the circumstellar disc can become more massive than the protostar during the mass accretion phase. However, in such studies, the disc evolution was investigated in a limited parameter range (Walch et al. 2009; Machida et al. 2010; Machida & Matsumoto 2011). Thus, we cannot conclude whether such a massive disc that is a favourable site for gas-giant planet formation generally appears in the star formation process. To determine this, we need to investigate the disc formation, resolving \( \sim 1 \, \text{au} \) structure over a wider parameter range.

The initial cloud for star formation is conventionally characterised by two parameters: the ratios of thermal energy to gravitational energy, \( \alpha \), and rotational energy to gravitational energy, \( \beta \) (for a detailed description, see Section 2). With these parameters, many studies have calculated and classified the cloud evolution in the isothermal (Miyama 1984; Boss 1993; Tsuribe 2002) and adiabatic (Cha & Whitworth 2003; Matsumoto & Hanawa 2003) gas contraction phases to investigate fragmentation before the protostar formation. However, the evolution and fragmentation of the circumstellar disc that is formed after the protostar formation has not been investigated with these parameters.

In this study, using parameters \( \alpha \) and \( \beta \), we simulate the formation and evolution of a circumstellar disc from the molecular cloud core until \( \sim 10^4 \, \text{yr} \) after the protostar formation using a smoothed particle hydrodynamics (SPH) code with sufficient spatial resolution. The parameters \( \alpha \) and \( \beta \) significantly influence the disc formation, because they determine the accretion rate onto the circumstellar disc and the size and mass of the circumstellar disc (see Section 4). We calculate the cloud evolution for 17 models using the barotropic equation of state.

In Section 2, we describe the numerical method and initial conditions. The evolution of the circumstellar disc is presented in Section 3. We discuss our results and compare them with previous works in Section 4. Finally, we summarize our results in Section 5.

2 NUMERICAL METHOD AND INITIAL CONDITIONS

Our simulations are carried out using an SPH code newly developed for this work. The code includes an individual time-steps technique and uses the Barnes–Hut tree algorithm to calculate self-gravity with an opening angle \( \theta = 0.5 \). We use an adaptive softening length according to Price & Monaghan (2007). Artificial viscosity is included according to the prescription of Monaghan (1997), with \( \alpha = 1 \) plus the Balsara switch (Balsara 1995). Our code is parallelized with message passing interface (MPI).

To mimic the thermal evolution of a molecular cloud, we use the barotropic equation of state as

\[
P = c_s^2 \rho \left[ 1 + \left( \frac{\rho}{\rho_s} \right)^{2/3} \right],
\]

where \( c_s = 190 \, \text{m} \, \text{s}^{-1} \) and \( \rho_s = 4 \times 10^{-14} \, \text{g} \, \text{cm}^{-3} \) is adopted. As the initial state, we take a uniform-density sphere with an axisymmetric density perturbation. The density perturbation \( \delta \rho \) is given as \( \delta \rho = 0.01 \times \cos 2\phi \). The initial cloud is rigidly rotating. We parametrize the ratios of the thermal and the rotational energy to the gravitational energy of the initial cloud. They are characterized by two non-dimensional parameters:

\[
\alpha = \frac{E_{\text{thermal}}}{E_{\text{grav}}} = \frac{5 R_0 c_s^2}{2GM},
\]

and

\[
\beta = \frac{E_{\text{rotation}}}{E_{\text{grav}}} = \frac{\Omega_0^2 R_0^3}{3GM},
\]

where \( R_0, \Omega_0 \) and \( M \) are the initial cloud radius, angular velocity and mass of the cloud core, respectively. We vary the parameters \( \alpha \) and \( \beta \) in the range \( 0.4 \leq \alpha \leq 0.8 \) and \( 3 \times 10^{-4} \leq \beta \leq 7 \times 10^{-2} \). In this study, the total cloud mass is fixed to \( 1 \, M_\odot \) in all models in order to enable comparison of the evolution of clouds with the same mass. In addition, we fix the initial cloud temperature to 10 K. Thus, the parameter \( \alpha \) and the initial cloud density are determined when the initial cloud radius is given. Note that such treatment changes the gravitational energy of the cloud. After the cloud radius is fixed with arbitrary \( \alpha \), the parameter \( \beta \) is determined when the initial angular velocity \( \Omega_0 \) is given. The model name, parameters \( \alpha \) and \( \beta \), initial cloud radius, initial angular velocity and initial cloud density are listed in Table 1. The initial cloud is modelled with about 500,000 SPH particles.

To calculate the disc formation several \( 10^5-10^4 \, \text{yr} \) after the protostar formation, we adopt a sink particle technique according to the prescription by Bate, Bonnell & Bromm (1995). Starting from...
the pre-stellar core stage, the cloud evolution is calculated without sink particles. Then, we assume the protostar formation and dynamically introduce a sink particle when the gas particle density exceeds the threshold density, $\rho_{\text{sink}} = 4 \times 10^{-9} \text{ g cm}^{-3}$. The threshold density roughly corresponds to the density at which the second collapse begins. As shown in Larson (1969) and Masunaga & Inutsuka (2000), the second collapse begins when the gas density reaches $\rho \sim 10^{-8} \text{ g cm}^{-3} \sim \rho_{\text{sink}}$, at which point the gas temperature exceeds $T \gtrsim 2 \times 10^4 \text{ K}$ and molecular hydrogen begins to dissociate. The protostar forms immediately after the second collapse. Thus, in this paper, we safely define the protostar formation epoch as that at which the gas density exceeds $\rho > \rho_{\text{sink}}$.

To treat the gas accretion onto the sink particle after the creation of the sink particle, we set the accretion radius $r_{\text{acc}} = 1 \text{ au}$. Then, within the accretion radius, we allow gas accretion onto the sink particle when the following conditions are fulfilled: (1) the gas particle density exceeds the accretion density $\rho_{\text{acc}} = 4 \times 10^{-11} \text{ g cm}^{-3}$, (2) the gas particle is gravitationally bound to the sink particle, and (3) the specific angular momentum of the gas particles is less than that required for it to form a circular orbit at $r_{\text{acc}}$. We did not implement a boundary condition for the sink particle.

### 3 RESULTS

As listed in Table 1, we calculated the cloud evolution for 17 models in total, and classified them into four modes as follows:

(i) Massive-disc mode: the disc mass dominates the protostellar mass for over $10^4 \text{ yr}$ after the protostar formation.

(ii) Early-fragmentation mode: fragmentation occurs in the collapsing cloud before the protostar formation.

(iii) Late-fragmentation mode: fragmentation occurs in the circumstellar disc after the protostar formation.

(iv) Protostar-dominant mode: the protostellar mass rapidly dominates the disc mass within $10^4 \text{ yr}$ of the protostar formation.

### Notes:

$a$ ‘M’, ‘EF’, ‘LF’, ‘P’ mean ‘Massive disc mode’, ‘Early fragmentation mode’, ‘Late fragmentation mode’ and ‘Protostar dominant mode’, respectively.

$b$ ‘Massive disc era’ is defined as the period during which the disc mass is greater than the protostar mass. Some of simulations are terminated during the massive disc era due to computational limits and we represent it with $\gtrsim$.

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**Table 1. Model parameters and Calculation Results.**

| Model | $\alpha$ | $\beta$ | $R$ (au) | $\Omega_0$ (s$^{-1}$) | $\rho_{\text{int}}$ (g cm$^{-3}$) | Mode | number of star | length of massive disc era (years) |
|-------|----------|---------|----------|-----------------------|-----------------------------------|------|----------------|----------------------------------|
| 1     | 0.8      | $1 \times 10^{-2}$ | 7866     | $4.94 \times 10^{-14}$ | $2.9 \times 10^{-19}$ | M    | 1              | $\gtrsim 1.3 \times 10^4$        |
| 2     | 0.8      | $7 \times 10^{-3}$ | 7866     | $4.14 \times 10^{-14}$ | $2.9 \times 10^{-19}$ | M    | 1              | $\gtrsim 3.9 \times 10^4$        |
| 3     | 0.8      | $3 \times 10^{-3}$ | 7866     | $2.71 \times 10^{-14}$ | $2.9 \times 10^{-19}$ | M    | 1              | $\gtrsim 1.6 \times 10^4$        |
| 4     | 0.8      | $1 \times 10^{-3}$ | 7866     | $1.56 \times 10^{-14}$ | $2.9 \times 10^{-19}$ | P    | 1              | $\gtrsim 4.5 \times 10^3$        |
| 5     | 0.8      | $3 \times 10^{-4}$ | 7866     | $8.56 \times 10^{-15}$ | $2.9 \times 10^{-19}$ | P    | 1              | $\gtrsim 1.2 \times 10^3$        |
| 6     | 0.6      | $7 \times 10^{-3}$ | 5900     | $2.01 \times 10^{-13}$ | $6.9 \times 10^{-19}$ | M    | 1              | $\gtrsim 3.4 \times 10^4$        |
| 7     | 0.6      | $1 \times 10^{-3}$ | 5900     | $7.6 \times 10^{-14}$  | $6.9 \times 10^{-19}$ | M    | 1              | $\gtrsim 2.0 \times 10^4$        |
| 8     | 0.6      | $7 \times 10^{-3}$ | 5900     | $6.37 \times 10^{-14}$ | $6.9 \times 10^{-19}$ | M    | 1              | $\gtrsim 3.4 \times 10^4$        |
| 9     | 0.6      | $3 \times 10^{-3}$ | 5900     | $4.17 \times 10^{-14}$ | $6.9 \times 10^{-19}$ | M    | 1              | $\gtrsim 2.1 \times 10^4$        |
| 10    | 0.6      | $1 \times 10^{-3}$ | 5900     | $2.41 \times 10^{-14}$ | $6.9 \times 10^{-19}$ | P    | 1              | $\gtrsim 4.5 \times 10^3$        |
| 11    | 0.4      | $7 \times 10^{-2}$ | 3933     | $3.70 \times 10^{-13}$ | $2.3 \times 10^{-18}$ | EF   | 4              |                                   |
| 12    | 0.4      | $1 \times 10^{-2}$ | 3933     | $1.40 \times 10^{-13}$ | $2.3 \times 10^{-18}$ | EF   | 4              |                                   |
| 13    | 0.4      | $9 \times 10^{-3}$ | 3933     | $1.32 \times 10^{-13}$ | $2.3 \times 10^{-18}$ | EF   | 4              |                                   |
| 14    | 0.4      | $7 \times 10^{-3}$ | 3933     | $1.17 \times 10^{-13}$ | $2.3 \times 10^{-18}$ | LF   | 2              |                                   |
| 15    | 0.4      | $3 \times 10^{-3}$ | 3933     | $7.66 \times 10^{-14}$ | $2.3 \times 10^{-18}$ | LF   | 2              |                                   |
| 16    | 0.4      | $1 \times 10^{-3}$ | 3933     | $4.42 \times 10^{-14}$ | $2.3 \times 10^{-18}$ | P    | 1              | $\gtrsim 2.2 \times 10^3$        |
| 17    | 0.4      | $3 \times 10^{-4}$ | 3933     | $2.42 \times 10^{-14}$ | $2.3 \times 10^{-18}$ | P    | 1              | $\gtrsim 9.0 \times 10^2$        |

Notes:

$a$ ‘M’, ‘EF’, ‘LF’, ‘P’ mean ‘Massive disc mode’, ‘Early fragmentation mode’, ‘Late fragmentation mode’ and ‘Protostar dominant mode’, respectively.

$b$ ‘Massive disc era’ is defined as the period during which the disc mass is greater than the protostar mass. Some of simulations are terminated during the massive disc era due to computational limits and we represent it with $\gtrsim$.

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**Figure 1.** The classification of simulation results in the $\alpha$–$\beta$ plane. The circles, open triangles, filled triangles and crosses indicate the massive-disc mode, early-fragmentation mode, late-fragmentation mode and protostar-dominant mode, respectively. $M$ and $r_{\text{cent}}$ are the mass accretion rate and centrifugal radius, respectively.

The calculation results and its classification are summarized in Fig. 1.

### 3.1 Massive-disc mode

The massive-disc mode is indicated by circles in Fig. 1. Figs 2 and 3 show the time evolution of the centre of the cloud for model 8 ($\alpha = 0.6$ and $\beta = 7 \times 10^{-3}$), which is a typical model for the massive-disc mode. The upper left and upper middle panels in these figures show snapshots before the protostar formation. A strong bar-like structure appears in the central high-density gas region in the top left panel in Fig. 2. Then, the bar structure effectively transfers
Figure 2. Time sequence of the logarithm of the face-on surface density before and after protostar formation for model 8 ($\alpha = 0.6$ and $\beta = 7 \times 10^{-3}$). The top left and top middle panels show snapshots about $1.1 \times 10^3$ and $6.9 \times 10^2$ yr before the protostar formation. The top right panel shows a snapshot just when the protostar forms. The bottom left, middle and right panels show snapshots $1.4 \times 10^2$, $1.0 \times 10^3$ and $1.0 \times 10^4$ yr after the protostar formation. The elapsed time from the beginning is noted in each panel.

Figure 3. As Fig. 2, but for an edge-on view.

the angular momentum outwards and the high-density gas region shrinks. As a result, a bimodal structure composed of the central high-density core and its surrounding disc appears as seen in the top middle panel. These figures clearly show the disc formation before the protostar formation. The top right panel shows a snapshot of just when the second collapse occurs (i.e. the protostar forms). The time when the protostar forms is $8.6 \times 10^5$ yr after the start. This roughly corresponds to the free-fall time-scale of the initial cloud. By this epoch, the spiral arm has developed around the central object. The bottom left, middle and right panels show snapshots $1.4 \times 10^2$, $1.0 \times 10^3$ and $1.0 \times 10^4$ yr after the protostar formation. These figures show that the disc gradually increases its size with time, retaining the global spiral structure.

Fig. 4 shows the mass evolution of the disc and protostar. For this model, the disc mass is greater than the protostellar mass for more than $10^4$ yr after the protostar formation. Such a disc is expected to be gravitationally unstable. To investigate the disc stability, the contours of Toomre’s $Q$ parameter at $1.4 \times 10^2$ and $1.0 \times 10^3$ yr after the protostar formation (the same epoch as in the bottom left and middle panels of Fig. 2) are plotted in Fig. 5. Toomre’s $Q$ parameter is described as

$$Q = \frac{c_s \kappa}{\pi G \Sigma},$$

where $c_s$, $\kappa$, and $\Sigma$ are the sound velocity, epicyclic frequency and surface density of the disc, respectively. For this model, the circumstellar disc did not show fragmentation, even though the circumstellar disc includes the region $Q < 1.4 \times 10^2$ yr after the protostar formation. This is because the GI cannot grow sufficiently fast in the circumstellar disc. The characteristic time-scale of Toomre’s analysis, $\tau = 2c_s/[G \Sigma (1 - Q^2)^{1/2}]$, is comparable to the orbital period of the disc (both are several thousand years). In this case, there are non-linear stabilizing mechanisms against GI. As seen in the right panel of Fig. 5 or bottom middle panel of Fig. 2, the spiral arms globally redistribute the mass and angular momentum within a short time, $\sim 10^3$ yr ($\lesssim T_{\text{orb}}$).

Furthermore, the disc–star configuration also dynamically changes in a short time-span. Fig. 6 shows the trajectory of the protostar for $8.0 \times 10^3$ yr after the protostar formation. The asterisk

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indicates the position where the protostar forms. The central protostar drifts towards the dense part of the spiral arm as a result of the gravitational interaction between the protostar and spiral arms. Because the strength of Keplerian shear is proportional to $\frac{d}{dr}(r\Omega^2) \propto r^{-3}$, its radial dependence is very strong and the drift motion increases the shear stress of the dense region and suppresses the fragmentation of the disc. As described in Gammie (2001), the thermal evolution of the disc also may play an important role in suppressing/promoting fragmentation. Gammie showed that no fragmentation occurs when the cooling time is much longer than the orbital period. In our simulation, because we adopted the barotropic equation of state (see Section 2.2), the disc evolves adiabatically (the disc mid-plane density is greater than $\rho_c = 4 \times 10^{-14}$ g cm$^{-3}$ practically all the time). With these effects, the disc is stabilized, as seen in the right panel of Fig. 5. For disc fragmentation, the mass in-fall must be sufficiently fast to overcome these non-linear back reactions.

3.2 Early-fragmentation mode

The open triangles in Fig. 1 indicate the early-fragmentation mode, in which the collapsing cloud undergoes fragmentation before the protostar forms. Fig. 7 shows the time evolution of the centre of the cloud for model 11 ($\alpha = 0.4$ and $\beta = 7 \times 10^{-2}$). The figure shows that a ring pattern arises (top left panel) and fragments to form four clumps (or protostars). These protostars have roughly the same mass when they form. After fragmentation, they interact with each other. Finally, a quadruple stellar system composed of two close binaries appears, as seen in the lower right panel of Fig. 7. The condition of the ring-like structure was investigated in Cha & Whitworth (2003). Because they fixed the parameter $\alpha$ (but changed the rotation law), we cannot quantitatively compare their results with ours. However, our result does not contradict theirs.

Figure 5. The contours of Toomre’s $Q$ parameter at $1.4 \times 10^2$ (left) and $1.0 \times 10^3$ yr (right) after the protostar formation.

Figure 6. The protostellar trajectory for $8.0 \times 10^3$ yr after the protostar formation. The asterisk indicates the position where the protostar forms.
In models showing fragmentation before the protostar formation (i.e. the early-fragmentation mode), the fragmentation pattern qualitatively changes as the rotational energy of the initial cloud decreases. Fig. 8 shows the time evolution of the centre of the cloud for model 12 ($\alpha = 0.4$ and $\beta = 1.0 \times 10^{-2}$). A clear bar-like structure arises before the protostar formation for model 12. The bar fragments into two clumps. Then, a binary system appears around the centre of the collapsing cloud (upper middle panel of Fig. 8). Furthermore, the remnant of the bar fragments form two extra protostars about $10^3$ yr after the first fragmentation, as seen in the upper right panel of Fig. 8. Then, one protostar is ejected from the central region by gravitational interaction with protostars, and a triple stellar system remains around the centre of the cloud at the end of the calculation.

The fragmentation and subsequent evolution of fragments is very complicated in models belonging to the early-fragmentation mode. However, because fragmentation before the protostar formation has been well investigated in previous studies (see Tsuribe 2002; Goodwin et al. 2007; Bodenheimer et al. 2000 for a review), we do not comment on it further in this paper.

### 3.3 Late-fragmentation mode

The filled triangles in Fig. 1 indicate the late-fragmentation mode, in which fragmentation occurs in the circumstellar disc after the protostar formation. Models 13 and 14 belong to this mode. Fig. 9 shows snapshots for model 14 ($\alpha = 0.4$ and $\beta = 7 \times 10^{-3}$). As seen in the massive-disc mode, after a strong bar develops (top left panel), the bar fragments into two clumps and then a binary system appears. The remnant of the bar fragments form an extra protostar about $10^3$ yr after the first fragmentation, as seen in the lower right panel of Fig. 9. Then, one protostar is ejected from the central region by gravitational interaction with protostars, and a triple stellar system remains around the centre of the cloud at the end of the calculation.
Classification of circumstellar disc evolution

Figure 9. Time sequence of the logarithm of the face-on surface density for model 14. The top left panel shows a snapshot about $1.2 \times 10^2$ yr before the protostar formation, while the top middle panel shows a snapshot of just when the protostar forms. The top right, bottom left, middle and right panels show snapshots $2.6 \times 10^2$, $3.5 \times 10^2$, $1.4 \times 10^3$ and $5.9 \times 10^3$ yr after the protostar formation. The elapsed time from the beginning is noted in each panel.

Figure 10. Mass evolution of the primary star (solid line) and the secondary star (dashed line).

3.4 Protostar-dominant mode

The crosses in Fig. 1 indicate the protostar-dominant mode, in which the protostar rapidly increases its mass and exceeds the circumstellar disc mass within $10^4$ yr. Figs 11 and 12 show the time evolution of the centre of the cloud for model 16 ($\alpha = 0.4$ and $\beta = 10^{-3}$), which is a typical model for the protostar-dominant mode. The top left and middle panels show snapshots before the protostar formation for model 16. Unlike the massive-disc mode, the first core keeps an almost axisymmetric structure without developing the non-axisymmetric perturbation. The second collapse occurs $4.4 \times 10^4$ yr after the start. Even after the protostar formation, the axisymmetric structure is maintained until the disc radius grows sufficiently ($r \gtrsim 10$ au).

To investigate the disc stability for model 11, Toomre’s $Q$ parameter at $1.1 \times 10^3$ yr after the protostar formation (the same epoch as the bottom middle panel of Fig. 11) is plotted in Fig. 13. At this epoch, the disc mass is still greater than the protostellar mass (see Fig. 14). Nevertheless, the $Q$ parameter is greater than unity in the whole region of the disc owing to the compactness of the disc.

The masses of the protostar and circumstellar disc are plotted against time after the protostar formation in Fig. 14. The figure indicates that the accretion rate onto the protostar is higher than that of the circumstellar disc, and the protostellar mass exceeds the circumstellar disc mass $\sim 2.0 \times 10^3$ yr after the protostar formation.

4 DISCUSSION

4.1 Effect of radiative cooling

In this study, we used the barotropic equation of state (equation 1), in which the gas in the disc behaves adiabatically when the disc
Figure 11. Time sequence of the logarithm of the face-on surface density for model 16 ($\alpha = 0.4$ and $\beta = 1 \times 10^{-3}$). The elapsed time from the beginning is noted in each panel.

Figure 12. As Fig. 11, but for edge-on snapshots.

mid-plane density is greater than $\rho_c = 4 \times 10^{-14} \text{ cm}^{-3}$. However, in reality, the disc cools radiatively with time. According to Rafikov (2005), we can roughly estimate the cooling time of the disc $t_{\text{cool}}$ as

$$t_{\text{cool}} \simeq \frac{\Sigma c_s^2 f(\tau)}{\gamma - 1 \frac{\sigma T^4}{2}} \simeq 1.2 \times 10^4 \left( \frac{\Sigma}{200 \text{ g cm}^{-2}} \right)^2 \left( \frac{T}{100 \text{ K}} \right)^{-3} \left( \frac{\tilde{\mu}}{2.3} \right) \times \left( \frac{\kappa}{10 \text{ cm}^2 \text{ g}^{-1}} \right) \text{ (yr)},$$

(5)

where $\Sigma, \sigma, T, \tau$ and $\tilde{\mu}$ are the surface density, Stephan–Boltzmann constant, mid-plane temperature, optical depth of the disc and mean molecular weight, respectively. In equation (5), we estimate the vertical optical depth $f(\tau)$ of the disc as $f(\tau) = (1 + \tau^2)/\tau$. In addition, we assume that the optical depth is $\tau \simeq \kappa \Sigma/2$, where $\kappa$ is the opacity. We assume that the disc is optically thick, and derive the right-hand side of equation (5) with $f(\tau) \simeq \tau$.

Equation (5) indicates that the cooling time-scale of the circumstellar disc is typically about $10^4 \text{ yr}$. Thus, radiative cooling may play an important role in investigations of the thermal evolution of the disc and its fragmentation over $10^5 \text{ yr}$. In this study, however, we showed that the evolution of the circumstellar disc can be qualitatively classified into four modes during the very early stages of star formation ($t \lesssim 10^4 \text{ yr}$). Thus, we expect that our classification will not change qualitatively even when radiative effects are included. However, the radiative cooling of the disc is important when investigating the disc evolution and fragmentation for $t \gtrsim 10^4 \text{ yr}$.

4.2 Treatment of protostar and sink

In this study, no model shows the formation of a star–planet system during the main accretion phase because the secondary object continues to increase its mass and finally exceeds the hydrogen-burning limit ($M \gtrsim 0.08 \text{ M}_\odot$). In contrast, Vorobyov & Basu (2010a) and Machida et al. (2010) showed the formation of a star–planet system during the main accretion phase. The difference is thought to be caused by the treatment of the protostar and sink.

In Vorobyov & Basu (2010a) and Machida et al. (2010), the protostar (or sink cell) is fixed at the centre of the computational domain. It is expected that such treatment promotes fragmentation. In reality, the density fluctuation arising around the protostar can be cancelled out by the movement of the protostar. Thus, fragmentation tends to occur when the protostar is fixed.
early stages of the main accretion phase, and claimed that fragmentation frequently occurs when the disc-to-stellar mass ratio is greater than unity. However, their assumption of isothermality seems not to be valid for the disc evolution of the early main accretion phase, because the gas becomes opaque and behaves adiabatically when the gas density exceeds the critical density of \( \rho_c \simeq 10^{-13} - 10^{-14} \text{ g cm}^{-3} \) (e.g. Masunaga & Inutsuka 2000). In addition, radiative cooling can affect the disc evolution \( \sim 10^4 \) yr after the protostar formation, as described in Section 4.1. Because the adiabatic equation of state stabilizes the circumstellar disc, fragmentation barely occurs in our calculation, even when the disc-to-stellar mass ratio exceeds unity. Thus, Kratter et al. (2010) may overestimate the fragmentation condition, while our calculation may underestimate it because of the lack of radiative cooling.

Walch et al. (2009) also studied circumstellar disc formation with the adiabatic equation of state and radiative cooling. They showed no fragmentation in the circumstellar disc, because the disc becomes very hot during the early stages of the main accretion phase. Their spatial resolution is, however, somewhat coarse: the minimum smoothing length of their study is \( h_{\text{min}} = 2 \) au, whereas we use \( h_{\text{min}} \sim 0.3 \) au in our simulations. In addition, they restricted their initial conditions to rapidly rotating cases \( (\beta > 10^{-2}) \) to investigate disc evolution before the central density becomes high. Observations suggest that molecular cloud cores have a rotational energy of \( 10^{-4} < \beta < 0.07 \) with a typical value of \( \beta \simeq 0.02 \) (Caselli et al. 2002). Thus, they studied cloud evolution in a limited parameter range.

5 SUMMARY

We have carried out hydrodynamical simulations to investigate the evolution of circumstellar discs with two non-dimensional parameters representing the thermal and rotational energy of the initial cloud. The thermal energy \( \alpha \) is related to the mass accretion rate onto the circumstellar disc as \( \dot{M}_{\text{disc}} = \alpha^{-3/2} c_s^3 G^{-1} \) (see Machida, Inutsuka & Matsumoto 2011). Thus, smaller values of \( \alpha \) provide a high accretion rate onto the circumstellar disc and vice versa. On the other hand, the initial rotational energy that is represented by the parameter \( \beta \) is related to the disc radius. The centrifugal radius of the initial cloud is related to \( \beta \) as \( r_{\text{cent}} = 3R_0\beta \), where \( R_0 \) is the initial cloud radius. Thus, with larger \( \beta \), the cloud forms a larger disc in the main accretion phase. In other words, with larger \( \beta \), a large fraction of the in-falling matter accretes onto the disc, rather than directly onto the primary protostar. As a result, smaller \( \alpha \) and larger \( \beta \) increase the disc surface density and make a gravitationally unstable disc.

However, the non-axisymmetric structure arising in such an unstable disc can stabilize the disc, because it can redistribute the angular momentum and promote mass accretion onto the protostar. Thus, no fragmentation occurs when a strong non-axisymmetric structure grows and transfers sufficient angular momentum outwards. By contrast, the disc becomes highly gravitationally unstable and shows fragmentation when non-axisymmetric structure does not grow sufficiently or when the growth time-scale of the non-axisymmetry is much longer than the disc-growth time-scale. Thus, the fragmentation condition depends also on the growth of the non-axisymmetricity, which is closely related to the parameters \( \alpha \) and \( \beta \) because they determine the evolution of the mass and angular momentum of the disc.

Using the parameters \( \alpha \) and \( \beta \), we found that the disc evolution is qualitatively classified into four modes: the protostar-dominant, massive-disc, early-fragmentation and late-fragmentation modes.
The schematic classification of the circumstellar disc is shown in
Fig. 1, which covers a wide parameter range for the rotational energy
supported by observations \(10^{-4} < \beta < 0.07\); Caselli et al. 2002).
We describe each mode below.

(i) \textit{Protoplast-massive mode.} For the protostar-massive mode, the
protostellar mass exceeds the circumstellar disc mass within \(10^4\)
yr after the protostar formation. This mode appears in the range of \(\beta \lesssim (1 - 3) \times 10^{-3}\). When the protostar forms, the circumstellar disc is
more massive than the protostar. However, for this mode, because the
mass increase rate of the protostar is larger than that of the
circumstellar disc, the protostellar mass exceeds the circumstellar disc mass within \(\sim 10^4\) yr of the protostar formation, as described in Section 3.4.

(ii) \textit{Massive-disc mode.} For the massive-disc mode, the circumstellar disc is more massive than the protostar \(10^4\) yr after protostar formation. As seen in Fig. 1, this mode appears when the initial cloud has larger rotational and thermal energies. In our calculation, a majority of models (8 out of 17 models) belong to this mode. This indicates that a circumstellar disc that is more massive than the protostar frequently appears in a star formation process. The circumstellar disc is self-regulated for this mode. When the circumstellar disc becomes massive, it decreases its mass because a non-axisymmetric structure develops and promotes the mass accretion onto the protostar. When the disc is relatively less massive, the disc mass increases because a relatively stable disc acquires its mass from the in-falling envelope without effective angular momentum transfer.

(iii) \textit{Early-fragmentation mode.} For the early-fragmentation mode, fragmentation occurs before the protostar formation. As seen in Fig. 1, this mode appears when the initial cloud has large rotational but small thermal energy. If the initial rotational energy is large enough, the gas cannot condense to the centre, and a ring-like structure appears as seen in Fig. 7. This structure is unstable against perturbation and it fragments into protostars. As the rotational energy decreases, the gas can condense to the centre and a bar-like structure develops. However, unlike in the massive-disc mode, because the bar cannot transfer angular momentum outwards fast enough, fragmentation occurs and a binary system appears, as seen in Fig. 8.

(iv) \textit{Late-fragmentation mode.} For the late-fragmentation mode, fragmentation occurs after the protostar formation. The rotational energy for this mode is smaller than that for the early-fragmentation mode. Thus, without fragmentation before the protostar formation, the central bar-like structure shrinks, transferring angular momentum outwards and forming a single protostar and a disc.

In this study, because we did not include a proper radiation treatment, we cannot discuss whether a gas-giant planet can form in such a massive disc. However, such a massive disc is a plausible site for gas-giant formation by GI. To determine whether a gas-giant planet forms in the circumstellar disc by GI, we need to perform longer-term calculations including radiative effects.

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