Azimuthal anisotropy: the higher harmonics

Arthur M. Poskanzer for the STAR Collaboration §

MS70R319, Lawrence Berkeley National Laboratory, Berkeley, CA 94720,
AMPoskanzer@LBL.gov

Abstract. We report the first observations of the fourth harmonic ($v_4$) in the azimuthal distribution of particles at RHIC. The measurement was done taking advantage of the large elliptic flow generated at RHIC. The integrated $v_4$ is about a factor of 10 smaller than $v_2$. For the sixth ($v_6$) and eighth ($v_8$) harmonics upper limits on the magnitudes are reported.

Anisotropic flow, an anisotropy of the particle azimuthal distribution in momentum space with respect to the reaction plane, is a sensitive tool in the quest for the quark-gluon plasma and the understanding of bulk properties of the system created in ultrarelativistic nuclear collisions. It is commonly studied by measuring the Fourier harmonics ($v_n$) of this distribution [1]. Elliptic flow, $v_2$, is well studied at RHIC and is thought to reflect conditions from the early time of the collision. Recently, Kolb [2] reported that the magnitude and even the sign of $v_4$ are more sensitive than $v_2$ to initial conditions in the hydrodynamic calculations. Besides one early measurement at the AGS [3], reports of higher harmonics have not previously been published. Some of the present work has already appeared [4].

Experiment — The data come from the reaction $Au + Au$ at $\sqrt{s_{NN}} = 200$ GeV. The STAR detector main time projection chamber (TPC) was used in the analysis of two million events. The main TPC covered pseudorapidity ($\eta$) from –1.2 to 1.2 and the low transverse momentum ($p_t$) cutoff was 0.15 GeV/c. In the present work all charged particles were analyzed, regardless of their particle type. The errors presented in the figures are statistical.

Analysis — The difficulty is that the signal is small and the non-flow contribution to the two-particle azimuthal correlations can be larger than the correlations due to flow. To suppress the non-flow effects the current analysis uses the knowledge about the reaction plane derived from the large elliptic flow. One method for eliminating the non-flow contribution in a case when the reaction plane is known was proposed in [1]. Results obtained with this method we designate by $v_4\{EP_2\}$. The analysis for $v_4$ was also done with three-particle cumulants [5] by measuring $\langle \cos(2\phi_a + 2\phi_b - 4\phi_c) \rangle$.

$p_t$-dependence — The results as a function of $p_t$ are shown in Fig. 1 (left) for minimum bias collisions (0–80% centrality). Shown for $v_4$ are both the analysis relative to the second harmonic event plane, $v_4\{EP_2\}$, and the three-particle cumulant, $v_4\{3\}$.

§ For the full author list and acknowledgments see Appendix "Collaborations" in this volume.
Both methods determine the sign of $v_4$ to be positive. As a function of $p_t$, $v_4$ rises more slowly from the origin than $v_2$, but does flatten out at high $p_t$ like $v_2$. The $v_n(p_t)$ values are consistent with zero. Ollitrault has proposed \cite{6} for the higher harmonics that $v_n$ might be proportional to $v_n^{n/2}$ if the $\phi$ distribution is a smooth, slowly varying function of $\cos(2\phi)$. In order to test the applicability of this $v_2$ scaling we have also plotted $v_2^2$ and $v_3^2$ in the figure as dashed lines. The proportionality constant has been taken to be $1.2$ in order to fit the $v_4$ data. The ratio, $v_4/v_2^2$, is shown in Fig. 1 (right) as a function of $p_t$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{(left) The minimum bias values of $v_2$, $v_4$, and $v_6$ with respect to the second harmonic event plane as a function of $p_t$ for $|\eta|<1.2$. The $v_2$ values have been divided by a factor of two to fit on scale. Also shown are the three particle cumulant values (triangles) for $v_4$ ($v_4\{3\}$). The dashed curves are $1.2 \cdot v_2^2$ and $1.2 \cdot v_3^2$. (right) The ratio $v_4/v_2^2$ is plotted against $p_t$. The dashed line is at the value of 1.2.}
\end{figure}

**Parton coalescence**—Assuming a simple parton coalescence model, for mesons one gets \cite{7}
\begin{equation}
\frac{v_4}{v_2^2} \approx 1/4 + 1/2(v_4^2/(v_2^2)^2). \tag{1}
\end{equation}

Since experimentally this ratio is 1.2, $v_4^2$ must be greater than zero. If one assumes that the hadronic $v_2$ scaling results from partonic $v_2$ scaling \cite{8}, then
\begin{equation}
v_4^2 = (v_2^2)^2 \tag{2}
\end{equation}

and
\begin{equation}
\frac{v_4}{v_2^2} = 1/4 + 1/2 = 3/4. \tag{3}
\end{equation}

But this is still less than 1.2. Therefore either $v_4^2$ is even greater than simple parton $v_2$ scaling would indicate, or the simple parton coalescence model is inadequate.

**Waist**—Kolb \cite{2} points out that for $v_2 > 10\%$, which occurs at high $p_t$, and no other harmonics, the azimuthal distribution is not elliptic, but becomes “peanut” shaped. He calculates the amount of $v_4$ (which looks like a four-leaf clover) needed to eliminate this waist. Our values of $v_4$ as a function of $p_t$ are about a factor of two larger than needed to just eliminate the waist.
Centrality-dependence— The values of $v_4(p_t)$ for eight centrality bins are shown in Fig. 2 (left). Integrating these values weighted with the yield gives Fig. 2 (right) which shows the centrality dependence of $v_2$, $v_4$, and $v_6$ with respect to the second harmonic event plane and also $v_4$ from three-particle cumulants ($v_4\{3\}$). The $v_6$ values are close to zero for all centralities. To again test the applicability of $v_n/2$ scaling we have also plotted $v_2^2$ and $v_3^2$ in the figure as dotted histograms. The proportionality constant has been taken to be 1.4 to approximately fit the $v_4$ data. The larger constant here compared to that used in Fig. 1 is understood as coming from the use of the square of the average instead of the average of the square, and because the integrated values weighted by yield emphasize low $p_t$, where the best factor is slightly larger.

![Figure 2](image_url)

Figure 2. (left) $v(p_t)$ for the centrality bins (bottom to top) 5 to 10 % and 10, 20, 30, 40, 50, 60, and 70 up to 80 %. (right) The $p_t$- and $\eta$- integrated values of $v_2$, $v_4$, and $v_6$ as a function of centrality. The $v_2$ values have been divided by a factor of four to fit on scale. Also shown are the three particle cumulant values for $v_4$ ($v_4\{3\}$). The dotted histograms are $1.4 \cdot v_2^2$ and $1.4 \cdot v_3^2$.

The $v_n\{EP_2\}$ values averaged over $p_t$ and $\eta$ ($|\eta|<1.2$), and also centrality (minimum bias, 0 – 80%), are (in percent) $v_2 = 5.18 \pm 0.01$, $v_4 = 0.44 \pm 0.01$, $v_6 = 0.043 \pm 0.037$, and $v_8 = -0.06 \pm 0.14$. Since $v_6$ is essentially zero, we place a two sigma upper limit on $v_6$ of 0.1%. Also, $v_8$ is zero, but the error is larger because the sensitivity decreases as the harmonic order increases.

**Blast Wave fits**— We have fitted the data with a modified Blast Wave model [9]:

$$
\rho(\phi) = \rho_0 (1 + 2f_2 \cos(2\phi) + 2f_4 \cos(4\phi))
$$

$$
v_n(p_t) = \frac{\int_{-\pi}^{\pi} d\phi \cos(n\phi) I_n(\alpha_t) K_1(\beta_t)(1 + 2s_2 \cos(2\phi) + 2s_4 \cos(4\phi))}{\int_{-\pi}^{\pi} d\phi I_0(\alpha_t) K_1(\beta_t)(1 + 2s_2 \cos(2\phi) + 2s_4 \cos(4\phi))},
$$

where $n = 0, 2, 4, 6$. Here $\rho_0$, $f_2$, $f_4$, $s_2$, and $s_4$ are fit parameters.
where $I_n$ and $K_1$ are modified Bessel functions, and $\alpha_t(\phi) = (p_t/T) \sinh(\rho(\phi))$ and $\beta_t(\phi) = (m_0/T) \cosh(\rho(\phi))$. In these equations, $\rho_0$ is the transverse expansion rapidity ($v_0 = \tanh(\rho_0)$) of the cylindrical shell. The parameters $f_2$ and $f_4$ are the harmonic amplitudes of the azimuthal variation of $\rho$, and $s_2$ and $s_4$ describe the spatial anisotropy of the source.

The Blast Wave fits to $v_2$ and $v_4$ are shown in Fig. 3 (left) and expanded in Fig. 3 (right), showing the approximate agreement with the ratio. A temperature of 0.1 GeV was assumed giving the fit parameters $\rho_0 = 0.49$, $f_2 = 1.4\%$, $s_2 = 9.1\%$, $f_4 = 0.0\%$, and $s_4 = 4.4\%$. It is interesting that in this large $p_t$ range the $s$ values are considerably larger than the $f$ values.

![Figure 3](image-url)  

**Figure 3.** (left) $v_2$ and $v_4$ as a function of $p_t$ with the lines showing the Blast Wave fits. (right) The ratio $v_4/v_2^2$ as a function of $p_t$ with the line showing the ratio of the Blast Wave fits.

**Conclusions**— We have measured $v_4$ as a function of $p_t$, and centrality. This is the first measurement of higher harmonics at RHIC. It is expected that these higher harmonics will be a sensitive test of the initial configuration of the system, since they provide a Fourier analysis of the shape in momentum space which can be related back to the initial shape in configuration space.

**References**

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