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LARGE MIXING ANGLES IN A $SU(2)_L$ GAUGE THEORY OF WEAK INTERACTIONS
AS A RESONANT EFFECT
OF 1-LOOP TRANSITIONS BETWEEN QUASI-DEGENERATE FERMIONS

B. Machet

Abstract: We show that 1-loop transitions between two quasi-degenerate fermions can induce a potentially large renormalization of their mixing angle, and a large renormalized Cabibbo (or PMNS) angle when the second fermion pair in the same two generations is far from degeneracy. At the resonance, the “Cabibbo angle” gets maximal and simply connected to masses without invoking any new physics beyond the standard model. This solution appears as the only one “perturbatively stable” (mixing angles are then renormalized with respect to their classical values by small amounts).

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1 Introduction

The origin of large mixing angles observed in leptonic charged currents is still unknown [1]. A common idea is that it is linked to a quasi-degeneracy of neutrinos, but this connection was never firmly established. And it cannot be on simple grounds, since homographic transformations on a (mass) matrix, while changing its eigenvalues, do not change its eigenvectors, hence mixing angles; accordingly, infinitely different spectra can be associated to a given mixing angle.

We show below, in the case of binary coupled systems, that large mixing can be associated with quasi-degeneracy. Indeed, small (perturbative) changes of parameters (for examples elements of mass matrix) can then trigger large variations of eigenstates. In the case under scrutiny, 1-loop transitions between two fermions generate perturbative $O(g^2)$ modifications of their kinetic terms. A (slightly non-unitary) transformation, which differs from a rotation only at $O(g^2)$, is needed to cast them back into their canonical form $\overline{\Psi} \not \Psi (\not I$ is the unit matrix). When the two fermions are quasi-degenerate, the induced transformation of their mass matrix is enough to trigger in turn large variations of its eigenvectors, such that its re-diagonalization requires a rotation by a large angle. The latter ultimately becomes the renormalized Cabibo angle that occurs in charged current.

In the following, we shall work with two generations of fermions, and take the example of two pairs of quarks $(d, s)$ and $(u, c)$. This can be easily translated to the (more appropriate) lepton case, when the two pairs are instead, for example, $(\nu_e, \nu_\mu)$ and $(e^-, \mu^-)$. Then, “Cabibbo angle” $\theta_{12}$ translates into “first PMNS angle” $\theta_{12}$, “quasi-degenerate $(d, s)$ system” into “quasi-degenerate neutrino pair”, $(u, c)$ far from degeneracy into $(electron, muon)$ far from degeneracy etc. Also, the the sake of simplicity, we shall work in a pure $SU(2)_L$ theory of weak interactions instead of the standard $SU(2)_L \times U(1)$ electroweak model.

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2 1-loop transitions between non-degenerate fermions. Re-diagonalizing the quadratic Lagrangian

2.1 1-loop transitions

Like in the Standard Model of electroweak interactions \[4\], the diagonalization of the classical mass matrix by a bi-unitary transformation leads to the classical mass eigenstates, for example \(s_0^m\) and \(d_0^m\), with classical masses \(m_s\) and \(m_d\). They are orthogonal with respect to the classical Lagrangian (no transition between them occurs at the classical level). However, at 1-loop, gauge interactions induce diagonal and non-diagonal transitions between them. For example, Fig. 1 describes non-diagonal \(s_0^m \to d_0^m\) transitions \[5\] mediated by the \(W^\pm\) gauge bosons. Diagonal transitions are mediated either by \(W^\pm\) or by the \(W^3\) gauge bosons.

![Diagram](image)

**Fig. 1:** \(s_0^m \to d_0^m\) transition at 1-loop

We investigate in this work how the Cabibbo procedure implements in the presence of these transitions \[5\]. The one depicted in Fig. 1 contributes as a left-handed, kinetic-like, \(p^2\)-dependent interaction of the type

\[
\sin \theta_c \cos \theta_c \left( h(p^2, m_u, m_W) - h(p^2, m_c, m_W) \right) \bar{d}_m^0 \not{\gamma} \not{s}_m^0,
\]

that we abbreviate, with transparent notations, into

\[
s_c c_c (h_u - h_c) \bar{d}_m^0 \not{\gamma} \not{s}_m^0.
\]

It depends on the classical Cabibbo angle \(\theta_c = \theta_d - \theta_u\). The function \(h\) is dimensionless. It is simple matter to realize that all (diagonal and non-diagonal) 1-loop transitions mediated between \(s\) and \(d\) mediated by \(W^\pm\) gauge bosons transform their kinetic terms into

\[
\left( \frac{\bar{d}_m^0}{d_m^0}, \frac{\bar{s}_m^0}{s_m^0} \right) \left[ \not{\gamma} + \left( \begin{array}{cc}
  c^2_h u + s^2_h c & s_c c_c (h_u - h_c) \\
  s_c c_c (h_u - h_c) & s^2_c h_u + c^2_c h_c
\end{array} \right) \not{\gamma} (1 - \gamma_5) \right] \left( \begin{array}{c}
  \bar{d}_m^0 \\
  \bar{s}_m^0
\end{array} \right),
\]

where we noted

\[
\mathcal{T}_x(\theta_c) = \frac{1}{2} \begin{pmatrix}
  \cos \theta & \sin \theta \\
  \sin \theta & -\cos \theta
\end{pmatrix}.
\]

To the contributions \[3\] we must add the diagonal transitions mediated by the \(W^3\) gauge boson. The kinetic terms for left-handed \(d_m^0\) and \(s_m^0\) quarks become (omitting the fermionic fields)

\[1\] Similar transitions occur between \(c\) and \(u\) quarks, and between their leptonic equivalent.
\[ K_d = \mathbb{I} + H_d; \]
\[ H_d = \frac{h_u + h_c}{2} + (h_u - h_c) T_x(2\theta_c) + \begin{pmatrix} h_d \\ h_s \end{pmatrix}, \quad (5) \]

where \( h_d = h(p^2, m_d, m_W) \) and \( h_s = h(p^2, m_s, m_W) \). Likewise, in the \((u, c)\) sector, one has

\[ K_u = \mathbb{I} + H_u; \]
\[ H_u = \frac{h_d + h_s}{2} + (h_d - h_s) T_x(2\theta_c) + \begin{pmatrix} h_u \\ h_c \end{pmatrix}. \quad (6) \]

We shall now diagonalize the quadratic part of the effective 1-loop Lagrangian, which means putting the pure kinetic terms back to the unit matrix and, at the same time, re-diagonalizing the mass matrix. This is accordingly a two-steps procedure.

Note that the kinetic terms of right-handed fermions are not modified, such that we shall only be concerned below with the left-handed ones.

2.2 First step: re-diagonalizing kinetic terms back to the unit matrix

The pure kinetic terms \( K_d \) for \((d, s)\) written in (5) can be cast back to their canonical form by a \( p^2 \)-dependent non-unitary transformations \( V_d \) according to

\[ V_d^\dagger K_d V_d = \mathbb{I}. \quad (7) \]

The procedure to find \( V_d \) is the following. Let \((1 + t_+)\) and \((1 + t_-)\), \( t_+, t_- = O(g^2) \), be the eigenvalues of the symmetric matrix \( K_d \)

\[ \text{It can be diagonalized by a rotation } \mathcal{R}(\omega_d) \equiv \begin{pmatrix} \cos \omega_d & \sin \omega_d \\ -\sin \omega_d & \cos \omega_d \end{pmatrix} \]

according to

\[ \mathcal{R}(\omega_d)^\dagger K_d \mathcal{R}(\omega_d) = \begin{pmatrix} 1 + t_+ \\ 1 + t_- \end{pmatrix}, \quad (9) \]

with

\[ \tan 2\omega_d = \frac{-(h_u - h_c) \sin 2\theta_c}{(h_u - h_c) \cos 2\theta_c + h_d - h_s} = \frac{-1}{1 + h_u - h_c} \tan 2\theta_c. \quad (10) \]

(10) defines \( \omega_d \) in particular as a function of the classical \( \theta_c \): \( \omega_d = \omega_d(\theta_c, \ldots) \).

The diagonal matrix obtained in (9) is not yet the unit matrix, but one gets to it by a simple renormalization of the columns of \( \mathcal{R}(\omega_d) \) respectively by \( \frac{1}{\sqrt{1 + t_+}} \) and \( \frac{1}{\sqrt{1 + t_-}} \). The looked for non-unitary matrix \( V_d \) writes finally

\[ V_d(p^2, \ldots) = \begin{pmatrix} c_{\omega_d} & s_{\omega_d} \\ -s_{\omega_d} & c_{\omega_d} \end{pmatrix} \frac{1}{\sqrt{1 + t_+}} + \begin{pmatrix} s_{\omega_d} & c_{\omega_d} \\ -c_{\omega_d} & s_{\omega_d} \end{pmatrix} \frac{1}{\sqrt{1 + t_-}}. \quad (11) \]

\[ t_\pm = \frac{h_u + h_c + h_d + h_s}{2} \pm \frac{1}{2} \sqrt{(h_u - h_c)^2 + (h_d - h_s)^2 + 2 (h_u - h_c) (h_d - h_s) \cos 2\theta_c}. \quad (8) \]

One has explicitly

\[ t_\pm = \frac{h_u + h_c + h_d + h_s}{2} \pm \frac{1}{2} \sqrt{(h_u - h_c)^2 + (h_d - h_s)^2 + 2 (h_u - h_c) (h_d - h_s) \cos 2\theta_c}. \]
It differs from the rotation $\mathcal{R}(\omega_d)$ only at $\mathcal{O}(g^2)$ and satisfies

$$
\mathcal{V}_d \mathcal{V}^\dagger_d = \frac{1}{(1+t_+)(1+t_-)} \left( \mathbb{I} + \frac{t_+ + t_-}{2} - (t_+ - t_-) T_\epsilon (-2\omega_d) \right), \quad \mathcal{V}^\dagger_d \mathcal{V}_d = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}. \tag{12}
$$

For $|h_d - h_s| \ll |h_u - h_c|$, eq. (10) shows that $\omega_d(\theta_c) \approx -\theta_c$. So, when the pair $(d, s)$ is close to degeneracy and $(u, c)$ far from it (see also footnote 3), $\mathcal{V}_d$ becomes close to a rotation $\mathcal{R}(-\theta_c)$. This property plays, as shown in subsection 4.2, an important role in the determination of the renormalized Cabibbo angle.

### 2.3 Second step: re-diagonalization of the mass matrix

By the flavor transformation $\mathcal{V}_d$ acting on left-handed fermions in the bare mass basis, $M_d$ transforms into $\mathcal{V}^\dagger_d M_d \mathcal{V}_d$, which needs to be re-diagonalized. To this purpose, a new bi-unitary transformation is needed. The transformation acting on left-handed fermions is the rotation $\mathcal{R}(\xi_d)$ that diagonalizes the real symmetric matrix $\mathbb{R}$,

$$
\mathcal{V}^\dagger_d M_d \mathcal{V}_d = \mathcal{V}^\dagger_d \begin{pmatrix} m^2_d \\ m^2_s \end{pmatrix} \mathcal{V}_d = \begin{pmatrix} m^2_d \cos \theta_d m^2_s \sin \theta_d & \frac{m^2_d \sin \theta_d + m^2_s \cos \theta_d}{1 + t_+} \\ \frac{m^2_d \sin \theta_d + m^2_s \cos \theta_d}{1 + t_-} & \frac{m^2_d \sin \theta_d - m^2_s \cos \theta_d}{1 + t_-} \sqrt{(1 + t_+)(1 + t_-)} \end{pmatrix}, \quad \tag{13}
$$

according to \[[\text{[1]}][1]

$$
\mathcal{R}(\xi_d)^\dagger \begin{pmatrix} \mathcal{V}^\dagger_d M_d \mathcal{V}_d \end{pmatrix} \mathcal{R}(\xi_d) = \begin{pmatrix} \mu_d^2 \\ \mu_s^2 \end{pmatrix}. \tag{14}
$$

It satisfies

$$
\tan 2\xi_d = \frac{-(m_d^2 - m_s^2)\sqrt{(1 + t_+)(1 + t_-)} \sin 2\omega_d}{(m_d^2 - m_s^2) \left( \frac{1 + t_+ + t_-}{2} \right) \cos 2\omega_d - (m_d^2 + m_s^2) \frac{t_+ - t_-}{2}} \tag{15}
$$

(15) defines in particular $\xi_d$ as a function of $\omega_d$, and thus as a function of the classical $\theta_c$; $\xi_d = \xi_d(\theta_c, \ldots)$. It also defines 1-loop mass eigenstates $d_{mL}(p^2)$ and $s_{mL}(p^2)$. Since it is in particular unitary, it preserves the canonical form of the kinetic terms that had been recovered in the first step of the procedure. By construction, at any given $p^2$, there is no 1-loop transition between $d_{mL}(p^2)$ and $s_{mL}(p^2)$.

The main property of (15) is the presence of a pole. It occurs for

$$
2 \frac{m_d^2 - m_s^2}{m_d^2 + m_s^2} \cos 2\omega_d(\theta_c) \approx t_+ - t_- \tag{[16]},
$$

which is, through (10), a relation between $\theta_c$, $m_d$, $m_s$, $m_u$, $m_c$, $m_W$ (and $p^2$).

We shall see in subsection 4.2 that, for quasi-degenerate $(d, s)$ and largely split $(u, c)$, $\xi_d(\theta_c)$ ultimately becomes the renormalized Cabibbo angle, which is accordingly implicitly expressed by (15) as a function of the masses of fermions and gauge fields, and of $p^2$.

---

1. From now onwards, to lighten the notations, we shall frequently omit the dependence on $p^2$ and on the masses.

2. The re-diagonalization of kinetic terms indirectly contributes to a renormalization of the masses: $m_d \rightarrow \mu_d, m_s \rightarrow \mu_s$. 

4. We shall see in subsection 4.2 that, for quasi-degenerate $(d, s)$ and largely split $(u, c)$, $\xi_d(\theta_c)$ ultimately becomes the renormalized Cabibbo angle, which is accordingly implicitly expressed by (15) as a function of the masses of fermions and gauge fields, and of $p^2$. 

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4
3 Individual mixing matrix and renormalized mixing angle

3.1 1-loop and classical mass eigenstates are non-unitarily related

The left-handed 1-loop mass eigenstates are related to the bare ones by

\[
\begin{pmatrix}
  d^0_{mL} \\
  s^0_{mL}
\end{pmatrix} = \mathcal{V}_d \mathcal{R}(\xi_d) \begin{pmatrix}
  d_{mL} \\
  s_{mL}
\end{pmatrix}.
\] (17)

They are thus deduced from the latter by the product of a \(p^2\)-dependent non-unitary transformation \(\mathcal{V}_d\) and a \(p^2\)-dependent unitary one \(\mathcal{R}(\xi_d)\). The two basis are accordingly non-unitarily related \([3]\). In particular, on mass-shell (respectively at \(p^2 = m_d^2\) and \(p^2 = m_s^2\)), one has for the physical mass eigenstates

\[
\begin{align*}
  d^{\text{phys}}_{mL} &= d_{mL}(p^2 = m_d^2) = [\mathcal{V}_d \mathcal{R}(\xi_d)]_{11}(p^2 = m_d^2) d^0_{mL} + [\mathcal{V}_d \mathcal{R}(\xi_d)]_{12}(p^2 = m_d^2) s^0_{mL}, \\
  s^{\text{phys}}_{mL} &= s_{mL}(p^2 = m_s^2) = [\mathcal{V}_d \mathcal{R}(\xi_d)]_{21}(p^2 = m_s^2) d^0_{mL} + [\mathcal{V}_d \mathcal{R}(\xi_d)]_{22}(p^2 = m_s^2) s^0_{mL}.
\end{align*}
\] (18)

Since bare mass states are unitarily related to bare flavor states, the physical mass eigenstates are also non-unitarily related to bare flavor states.

3.2 Individual mixing matrix and renormalized mixing angle

Classical flavor eigenstates and 1-loop mass eigenstates are related to each other according to

\[
\begin{pmatrix}
  d^0_{fL} \\
  s^0_{fL}
\end{pmatrix} = C_{d0} \begin{pmatrix}
  d^0_{mL} \\
  s^0_{mL}
\end{pmatrix} \mathcal{R}(\xi_d) \mathcal{V}_d \begin{pmatrix}
  d_{mL} \\
  s_{mL}
\end{pmatrix},
\] (19)

where \(C_{d0} = \mathcal{R}(\theta_d)\) is the classical mixing matrix in the \((d, s)\) sector. The individual mixing matrix at 1-loop is thus given by

\[
C_d = C_{d0} \mathcal{V}_d \mathcal{R}(\xi_d) = \mathcal{R}(\theta_d) \mathcal{V}_d \mathcal{R}(\xi_d).
\] (20)

Since \(\mathcal{V}_d \approx \mathcal{R}(\omega_d) + \mathcal{O}(g^2)\) (see \([1]\)), one has

\[
C_d \approx \mathcal{R}(\theta_d + \omega_d + \xi_d).
\] (21)

The quantity \(\omega_d + \xi_d\) is seen on \(\mathcal{V}_d\) to renormalize the classical mixing angle \(\theta_d\). From \([5]\), one deduces that it satisfies the general relation

\[
\tan 2(\omega_d + \xi_d) \approx -\frac{\tan 2\omega_d \left[ \frac{t+\frac{1}{2}}{t\frac{1}{2}} \frac{m_u^2 - m_d^2}{m_d^2} \frac{1}{\cos 2\omega_d} \right]}{1 + \tan^2 2\omega_d \left[ \frac{t-\frac{1}{2}}{t\frac{1}{2}} \frac{m_u^2 + m_d^2}{m_d^2} \frac{1}{\cos 2\omega_d} \right]}.
\] (22)

Let us suppose now that \(d\) and \(s\) are quasi-degenerate and that \(u\) and \(c\) are, at the opposite far from degeneracy. Then (see subsection \(2.3\)), \(\omega_d(\xi_c) \approx -\xi_c\), and \((21)\) becomes \(C_d \approx \mathcal{R}(\theta_d - \theta_c + \xi_d(\theta_c)) = \)

\[\text{The subscript}_L\text{ refers to left-handed fermions.} \]
$R(\theta_u + \xi_d(\theta_c))$. Furthermore, at the pole $[10]$ of $[13]$, i.e. when $\xi_d(\theta_c)$ becomes maximal $\xi_d = \pm \pi/4$, it is easy to show that $\omega_d + \xi_d$ gets small $\overline{1}$; $\theta_d$ is then renormalized only by a small amount $[11]$

4 The renormalized Cabibbo angle

4.1 The effective gauge-invariant and hermitian Lagrangian at 1-loop

After 1-loop radiative corrections to $s_m^0 \leftrightarrow d_m^0$ and $c_m^0 \leftrightarrow u_m^0$ have been accounted for, the kinetic terms for the first two generations of left-handed fermions, once cast into their standard form $\bar{\Psi} \overline{D} \Psi \equiv \frac{1}{2}(\bar{\Psi} D \Psi - (D\bar{\Psi})\Psi)$, write, in the bare mass basis

$$\mathcal{L} \in \left( \begin{array}{ccc} \bar{u}_m^0 & \bar{c}_m^0 & \bar{s}_m^0 \\ \bar{d}_m^0 & \bar{d}_m^0 & \bar{s}_m^0 \end{array} \right) \left( \begin{array}{c} A \gamma^\mu - \frac{ig}{2}(A T^i + \overline{T}^j A) \gamma^\mu + \ldots \end{array} \right) \left( \begin{array}{c} u_m^0 \\ c_m^0 \\ s_m^0 \end{array} \right), \quad (24)$$

with

$$A = \left( \begin{array}{cc} K_u & K_d \\ K_d & -1 \end{array} \right) = I + \left( \begin{array}{cc} H_u & H_d \\ H_d & -1 \end{array} \right). \quad (25)$$

$SU(2)_L$ gauge invariance, by requesting the replacement of the partial derivative by the covariant one, is at the origin of the gauge couplings that appear in $[24]$. $\mathcal{L}$ is hermitian and involves the (Cabibbo rotated) $SU(2)_L$ generators $T^i$

$$T^3 = \frac{1}{2} \left( \begin{array}{c} 1 \\ -1 \end{array} \right), \quad T^+ = \left( \begin{array}{c} C_0 \\ -C_0 \end{array} \right), \quad T^- = \left( \begin{array}{c} C_0^\dagger \\ -C_0^\dagger \end{array} \right), \quad (26)$$

where $C_0$ is the classical Cabibbo matrix

$$C_0 = \mathcal{R}(\theta_c) = \left( \begin{array}{cc} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{array} \right) = C_{u0}^\dagger C_{d0} = \mathcal{R}(\theta_u)^\dagger \mathcal{R}(\theta_d). \quad (27)$$

6When the pair $(d, s)$ is quasi-degenerate and $(u, c)$ far from degeneracy, $\omega_d \overline{[16]} - \theta_c$ such that $[16]$ is approximately a second degree equation in $\cos 2\theta_c$. Furthermore, one has, then (in the unitary gauge), $|h_d - h_u| \approx \frac{m_u^2 - m_s^2 - m_d^2}{\pi \alpha^2}$, $\approx \frac{m_d^2 - m_s^2 - m_u^2}{\pi \alpha^2}$, which, added to $|h_u - h_c| \approx \frac{m_s^2 - m_d^2}{\pi \alpha^2}$, enables to write the approximate solution of $[16]$ as

$$\cos 2\theta_c \approx \frac{1}{2}(h_u - h_c) \approx \frac{m_s^2 - m_d^2}{\pi \alpha^2} \approx \frac{m_d^2 - m_u^2}{\pi \alpha^2} \approx \frac{m_s^2 - m_d^2}{\pi \alpha^2} \approx \frac{m_u^2 - m_s^2}{\pi \alpha^2}. \quad (23)$$

Since the r.h.s of $[23]$ is $\ll 1$, it corresponds to a classical $\theta_c$ itself close to maximal. Then, so does $\omega_d(\theta_c)$.

At the pole $[16]$, $1 + \frac{\alpha^2 - \cos \omega_d}{\sin \omega_d} = 1$ and the relation $[20]$ becomes $\tan(2(\omega_d + \xi_d)) = -1/\tan 2\omega_d$, which vanishes when $\omega_d$ becomes maximal. Then, $\omega_d + \xi_d \to 0$, q.e.d.

7Ones finds numerically from $[23]$ and $[16]$ that $(\omega_d + \xi_d)(\theta_c)$ only vanishes at the pole $[16]$, i.e. when $\theta_c \approx -\omega_d$ is close to maximal.
4.2 The renormalized Cabibbo angle

From (24), one deduces that, in the bare mass basis, the renormalized Cabibbo matrix is, at $O(g^2)$

$$C_{bm}(p^2, \ldots) = \frac{1}{2} \left[ (\mathbb{I} + H_u) C_0 + C_0 (\mathbb{I} + H_d) \right]$$  \hspace{1cm} (28)

which, in particular, is not unitary. Using (17), it becomes in the basis of 1-loop mass eigenstates

$$C(p^2, \ldots) = [V_u(\xi_u)]^T C_{bm}(p^2, \ldots) [V_d(\xi_d)],$$  \hspace{1cm} (29)

Since $H_u$ and $H_d$ in (28) are $O(g^2)$, the terms proportional to them in (29) can be calculated with the expressions of $R(\xi_d)$ and $V_u$ at $O(g^0)$, that is, for $t_+ = 0 = t_-$; one can accordingly take in there $R(\xi_d) \rightarrow R(-\omega_d)$ and $V_d \rightarrow R(\omega_d)$, such that $V_d R(\xi_d) \rightarrow \mathbb{I}$. The same approximation can be done in the $(u, c)$ sector. The resulting expression for $C$ is

$$C(p^2, \ldots) \approx [\mathcal{O}(g^2)] R(\xi_u)^T C_0 V_d R(\xi_d) + \frac{1}{2} (H_u C_0 + C_0 H_d)$$

$$= C_0^d C_d + \mathcal{O}(g^2),$$  \hspace{1cm} (30)

in the second line of which we have used (17), (20) and its equivalent for $C_u$.

Let us now get an approximate expression for $C_u^d C_d$ when $d$ and $s$ are close to degeneracy, while $u$ and $c$ are far from it. Then (see subsection 3.2), $\omega_d(\theta_c) \approx -\theta_c$ such that, by (11), one has $V_d \approx R(-\theta_c)$, which cancels the $C_0$ in $R(\theta_c)$ in (30). Likewise, from the equivalent $\tan 2\omega_u = \frac{-h_d - h_s \sin 2\theta_c}{h_u - h_s \cos 2\theta_c + h_s - h_u}$ of (14) in the $(u, c)$ sector, we deduce that, since $|h_u - h_c| \gg |h_d - h_s|$, $\omega_u \rightarrow 0$ such that, from the equivalent of (14), $V_u \approx \mathbb{I}$. Also, since $\tan 2\xi_u$ is proportional to $\sin 2\omega_u$ in the equivalent of (15), $\xi_u$ becomes small, such that $R(\xi_u) \rightarrow \mathbb{I}$, too. Finally, (15) becomes

$$C(p^2, \ldots) \approx [\mathcal{O}(g^2)] + \mathcal{O}(g^2)$$ when $(d, s) \approx$ degenerate and $(u, c)$ far from degeneracy.  \hspace{1cm} (31)

This is our main result: the renormalized value of the Cabibbo angle finally becomes $\xi_d(\theta_c)$ as given by (13); it can become large and eventually maximal at the resonance (14). If so, since $\theta_c$ is then close to maximal, too (see footnote 8), the Cabibbo angle gets renormalized by a small amount (like $\theta_d$ (see subsection 3.2) and $\theta_u$ (see footnote 8)).

5 Summary and prospects

We have shown that, in a $SU(2)_L$ gauge model of weak interactions, 1-loop transitions between two fermions can strongly modify their mass eigenstates and generate a large mixing angle when:

* this pair is close to degeneracy;
* the other pair in the same two generations is, at the opposite, far from degeneracy.

While the classical mixing angle $\theta_u$ of the largely split pair undergoes a small renormalization, the one $\theta_d$ of the quasi-degenerate pair gets renormalized by $\xi_d(\theta_c) - \theta_c$, which play the following roles: the rotation $R(\theta_c)$ casts the kinetic terms of the quasi-degenerate pair back to the unit matrix and $R(\xi_d(\theta_c))$ puts its mass matrix back to diagonal. The Cabibbo angle gets accordingly renormalized from $\theta_c \equiv \theta_d - \theta_u$ to, approximately, $(\theta_d + \xi_d(\theta_c) - \theta_c) - \theta_u$, that is, up to corrections $O(g^2)$, $\xi_d(\theta_c)$ itself. In the vicinity of the pole of $\tan 2\xi_d$, both $\theta_c$ and $\xi_d$ become close to maximal. 1-loop renormalizations of $\theta_u$, $\theta_c$ and $\theta_d$ are then small. A maximal value for the Cabibbo angle appears in these conditions as the only perturbatively stable solution (see footnote 7).

\footnote{$\nu_u R(\xi_u) \approx 1$, such that the renormalization of the mixing angle of the largely split pair is small.}
This non-trivial effect of 1-loop radiative corrections could explain the large mixing angles observed in charged leptonic currents if the classical PMNS angles are close to fulfilling the leptonic equivalent of conditions (16) and (23). To our knowledge, it is the first time that such relations connecting masses and angles could be established on simple perturbative grounds without invoking physics beyond the standard model.

A more quantitative analysis is currently under investigation.

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