The Structure of the Broad-line Region in Active Galactic Nuclei. II. Dynamical Modeling of Data From the AGN10 Reverberation Mapping Campaign

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Abstract

We present inferences on the geometry and kinematics of the broad-H\textbeta line-emitting region in four active galactic nuclei monitored as part of the fall 2010 reverberation mapping campaign at MDM Observatory led by the Ohio State University. From modeling the continuum variability and response in emission-line profile changes as a function of time, we infer the geometry of the H\textbeta-emitting broad-line regions (BLRs) to be thick disks that are close to face-on to the observer with kinematics that are well-described by either elliptical orbits or inflowing gas. We measure the black hole mass to be $M_{\text{BH}} = \log_{10}(M_{\text{BH}}) = 7.25_{-0.10}^{+0.10}$ for Mrk 335, $7.86_{-0.17}^{+0.20}$ for Mrk 1501, $7.84_{-0.15}^{+0.15}$ for 3C 120, and $6.92_{-0.24}^{+0.24}$ for PG 2130+099. These black hole mass measurements are not based on a particular assumed value of the virial scale factor $f$, allowing us to compute individual $f$ factors for each target. Our results nearly double the number of targets that have been modeled in this manner, and we investigate the properties of a more diverse sample by including previously modeled objects. We measure an average scale factor $\bar{f}$ in the entire sample of $\log_{10}(\bar{f}) = 0.54 \pm 0.17$ when the line dispersion is used to characterize the line width, which is consistent with the values derived using the normalization of the $M_{\text{BH}} - \sigma$ relation. We find that the scale factor $f$ for individual targets is likely correlated with the black hole mass, inclination angle, and opening angle of the BLR, but we do not find any correlation with the luminosity.

Key words: galaxies: active – galaxies: individual (Mrk 335, Mrk 1501, 3C 120, and PG 2130+099) – galaxies: Seyfert

1. Introduction

Over the past couple of decades, enormous improvements have been made in our understanding of the physics behind active galactic nuclei (AGNs) and their central engines. It is now widely accepted that AGNs contain a supermassive black hole (BH) with some form of accretion disk. The accretion disk photoionizes gas farther out in the broad-line region (BLR), from which we see emission lines that are Doppler-broadened due to the motion of the gas around the BH. In nearby quiescent galaxies, $M_{\text{BH}}$ is measured using stellar and gas dynamics (e.g., McConnell & Ma 2013), but the central regions of AGNs and galaxies farther away are too small in angular extent to allow such measurements. However, under the assumption that the motion of the gas in the BLR of AGNs is dominated by the gravity of the BH, we can directly measure the mass of the BH ($M_{\text{BH}}$) by employing reverberation mapping (RM) methods (e.g., Blandford & McKee 1982; Peterson 1993). RM makes use of the variability of AGNs to determine the time delay between signals in the continuum emission, which is thought to come from very close to the BH itself, and the response of the gas in the BLR. As the delay is due to the light-travel time between the central source and the BLR, a measurement of the time delay between these signals yields a distance of the BLR gas from the central source. Combined with a measurement of the velocity field of the BLR gas, one can measure $M_{\text{BH}}$ using the so-called virial relation:

$$M_{\text{BH}} = \frac{R \Delta V^2}{G},$$

where $R$ is the characteristic radius of the BLR, $\Delta V$ is the line-of-sight velocity of the gas, and $f$ is a dimensionless scale factor that accounts for the geometry, kinematics, and orientation of the BLR itself. All $M_{\text{BH}}$ estimates outside the local universe are made using the BLR in AGNs, making them powerful tools for exploring the BH population across the observable universe.

In Equation (1), $R$ is measured via RM or via single-epoch methods and $\Delta V$ is measured from the width of the emission line. Because the BLR is currently unresolved, the true value of $f$ in each target is unknown, so an average scale factor $\bar{f}$ has commonly been used to calculate $M_{\text{BH}}$ in AGNs. Typically, it is assumed that AGNs follow the same $M_{\text{BH}} - \sigma$ relation as quiescent galaxies, and the average scale factor $\bar{f}$ required to move the entire population of reverberation-mapped AGNs onto the quiescent $M_{\text{BH}} - \sigma$ relation (e.g., Onken et al. 2004; Graham et al. 2011; Park et al. 2012; Grier et al. 2013a; Woo et al. 2015; Batiste et al. 2016) is calculated. The unknown scale factor $f$ is the largest source of uncertainty in AGN $M_{\text{BH}}$ measurements today.
the BH population across the observable universe, it is in our best interest to refine these measurements to be as accurate as possible, and to do so involves the recovery of additional information on the environment within the BLR to determine individual scale factors in AGNs.

Until recently, RM efforts were typically only able to obtain measurements of the average time delay between signals in the BLR and the continuum—because the BLR gas is not confined to one specific radius, measuring the flux across an entire broad emission line yields some characteristic radius of the line-emitting region. Cross-correlation methods are the most common way of obtaining this average time delay (e.g., Peterson et al. 2004), although other methods have been examined in the past (such as linear inversion; e.g., Krolik & Done 1995). Recently, alternative methods that model the light curves and/or transfer functions have begun to be used (Zu et al. 2011; Grier et al. 2012b; Skielboe et al. 2015; Fausnaugh et al. 2016). The average time delay is difficult to obtain an $M_{BH}$ measurement, assuming $f$, but it does not yield information about the structure or kinematics within the BLR itself. However, with sufficiently high-quality RM data, we can actually resolve the time delays in velocity space and thus recover information about the possible phase-space structure within the BLR. This velocity-resolved analysis has been successfully done for several sets of data in recent years (Bentz et al. 2009; Denney et al. 2010; Barth et al. 2011a, 2011b; Doroshenko et al. 2012; Grier et al. 2013b; De Rosa et al. 2015; Du et al. 2016; Pei et al. 2017). In most cases, we see signatures of either gas in bound elliptical orbits or inflowing gas, although signatures of outflowing gas have also been seen (Denney et al. 2009; Du et al. 2016).

To obtain more detailed information about the BLR, a few recent studies have set out to recover the transfer function, or velocity-delay map, that shows exactly how the variations in the continuum emission are mapped onto variations in the broad-line emission as a function of the line-of-sight velocity of the gas (Bentz et al. 2010; Grier et al. 2013b). These studies used maximum entropy methods (MEM) implemented in a code called MEMECHO (Horne et al. 1991; Horne 1994) to recover the transfer functions. In Grier et al. (2013b), hereafter referred to as Paper I, we applied the MEMECHO code to five targets from a 2010 RM campaign (described in Grier et al. 2012b, hereafter G12) and successfully recovered velocity-delay maps for four of the targets. The velocity-delay maps confirmed the initial velocity-resolved time lag results, which also show signatures of both gas in elliptical orbits and inflowing gas, with possible disk-like or spherical geometries. Although the MEMECHO velocity-delay maps help us to determine qualitatively what kind of possible structures and kinematics we are seeing in the BLR, they yield no concrete parameters on either the geometry or kinematics of the BLR and ideally should be compared with models to make more detailed and precise inferences.

Recently, other approaches have been developed to model RM data sets directly to obtain quantitative constraints on both the geometry (Li et al. 2013) and kinematics of the BLR (Brewer et al. 2011; Pancoast et al. 2011, 2014a). Modeling both the geometry and kinematics yields a measurement of $M_{BH}$ independent of the scale factor $f$ that can be compared with values returned by the other RM methods. Pancoast et al. (2014a) improved the flexibility of the BLR model used in their approach and successfully applied their methodology to model five AGNs from the LAMP 2008 RM campaign (Walsh et al. 2009; Bentz et al. 2010) and demonstrated its power to fit the data and provide constraints on the BLR environment (Pancoast et al. 2014b, hereafter P14).

In this study, we continue our investigation of the structure and kinematics of the BLR that was begun in Paper I with the MEMECHO velocity-delay maps. Here we aim to obtain constraints on the BLR geometry and kinematics in our targets by directly modeling the data using the methods of Pancoast et al. (2014a). The aim of this study is to further add to the sample of targets with dynamical modeling information to learn about the RM population as a whole. We also aim to compare the transfer functions recovered with MEMECHO in Paper I to the information recovered from dynamical modeling. In Section 2, we describe the data we used to model the BLR and the spectral decomposition used to isolate the broad emission lines. In Section 3, we briefly discuss dynamical modeling methods, and in Section 4, we describe the modeling results for each individual AGN. In Section 5, we combine our results with those from the LAMP 2008 data set (P14) to discuss the dynamical modeling sample as a whole and any trends found, and to calculate the mean scale factor $f$ for the sample. We conclude in Section 6 with a summary of our results and their implications.

### 2. Data Preparation

#### 2.1. Spectroscopic Data

The spectra used in this analysis were taken during an RM campaign carried out primarily at MDM Observatory in late 2010, hereafter referred to as AGN10. Details on the data processing are discussed by G12. The spectra were obtained with the Boller and Chivens CCD spectrograph on the 1.3 m McGraw-Hill telescope over the course of 120 nights from 2010 August 31 to December 28. The continuum light curves consist of fluxes measured from both spectroscopic and photometric observations; they were constructed from data taken at multiple observatories, as discussed by G12. General information on the five targets we examine here is given in Table 1. Prior to modeling, the spectra were calibrated to the absolute flux of the narrow [O III] $\lambda5007$ emission line using the procedure of van Groningen & Wanders (1992).

Table 1 presents some basic information on the data used in the modeling, including the number of spectral epochs and the number of epochs in each continuum light curve. In Table 1, we reproduce previous RM results for each target from G12 to compare with the results from this study. These RM results from G12 were all obtained using JAVELIN to model the continuum and light curves (see Zu et al. 2011, 2013; Paper I; or G12 for details).

Paper I presents velocity-delay maps that include three emission lines: H$\beta$, H$\gamma$, and He II. However, the quality of the measurements for He II and H$\gamma$ is lower than that for H$\beta$. The He II emission line is broad, relatively weak, and very difficult to isolate from other components of the spectrum using spectral decomposition. The H$\gamma$ line, while stronger, lies near the blue end of the wavelength coverage of our spectra, where there are no strong narrow emission lines. As such, the relative flux calibration for H$\gamma$ is of lower quality than that for H$\beta$. We thus restrict our analysis here to only the H$\beta$ emission line and defer the discussion of additional emission lines to future RM studies with more favorable data quality for these line species.
2.2. Spectral Decomposition

In both Paper I and G12, and indeed in most prior RM studies, the broad emission-line fluxes were initially measured by subtracting off a linear continuum underneath the emission line, measured using local continuum windows on either side of the emission line. However, several groups (e.g., Barth et al. 2011b; Park et al. 2012; Barth et al. 2013; Hu et al. 2015) have developed methods to isolate various components of the AGN spectrum, allowing for the disentanglement of various broad emission-line features from the rest of the AGN, such as the host galaxy starlight, Fe II features, and various other species that often blend with the emission lines we are investigating. The recent success with isolating these different components of the AGN spectrum allows one to measure light curves for various AGN components despite strong starlight and blending that often blend with the emission lines we are investigating. These measurements were published by G12. Hβ measurements were produced using JAVELIN. σ\text{int} was measured from the root mean square residual spectrum, and the FWHM was measured from the mean spectrum.

Most of our targets show complex features, significant Fe II, and/or strong host galaxy starlight features in their spectra—these issues affect the H\beta line profiles and thus have the potential to affect the resulting BLR models inferred. Thus, we opted to perform spectral decomposition on our spectra to isolate the H\beta emission to allow us to subtract off all spectral components other than H\beta; this residual spectrum would then be used in our modeling. The spectral decomposition method used in our study is described in detail in Section 4.3 of Barth et al. (2015), although one modification was made: instead of using fourth-order Gauss–Hermite functions to model the broad H\beta component and [O III], we use a sixth-order Gauss–Hermite function because the line profiles in these objects were more complex than those examined by Barth et al. (2015). Because of the relatively small wavelength range of our MDM spectra, the fits were carried out over the entire range of the spectra, ranging from about 4200 Å to 5400 Å in the rest frame (with small variations depending on the object redshifts). The model components include host starlight, a power-law AGN continuum, [O III] λλ4959, 5007, H\beta, He II λ4686, H e I λ5876, and Fe II. The H\beta model includes both broad and narrow H\beta components as separate Gauss–Hermite models. We tried three different Fe II templates from Boroson & Green (1992), Véron-Cetty et al. (2004), and Kovačević et al. (2010), performing full decompositions with each template so we could compare the resulting H\beta profiles using each.

We were unable to adequately fit the extremely complex line profile of Mrk 6, which has a very broad, asymmetric H\beta profile with significant stellar, He II, and Fe II emission all overlapping with H\beta as well. Because of the difficulty in isolating the H\beta emission in this target, we were unable to successfully produce a model from it, and it is thus excluded from further analysis. However, the three different Fe II templates yielded remarkably consistent H\beta profiles in all four of our remaining sources, with the exception of some minor differences in the red wing of the H\beta emission. This region contains contributions from Fe II and He I, but the line profiles over the majority of the range spanning the H\beta line were nearly identical for each source. The one exception is Mrk 1501; this target has a broader H\beta profile than the others, and the exact shape of the red wing depends more heavily on the Fe II template used. We used the fits from the best Fe II template of the three, chosen based on both visual inspection and the χ² of the fit, for our modeling. In all four cases, the Boroson & Green (1992) template yielded the best fit to our data, though we again note that the differences were very minor and restricted to only the red wing of the H\beta profile.

We show the various components of the spectral fits using the Boroson & Green (1992) Fe II templates in Figure 1 for the mean spectrum of each of our AGNs. Because the He I components often appeared degenerate with portions of the red wing of the H\beta emission and the Fe II emission in that region, we opted to leave the possible He I emission in our spectra to model the case where all of the flux in this region is due to H\beta. Our final spectra for the modeling analysis are thus the original data with all other components subtracted except for He I. We note, however, that for our four objects, the contribution from He I is small. To minimize possible systematics caused by differences between the Fe II template fits, we include the red wing of the H\beta emission line only as far as the three templates were in general agreement on the H\beta line profile (see Figure 1 and Table 1 to see the exact wavelength ranges used in the modeling for each source).

### Table 1

| Object   | R.A. (J2000) | Decl. (J2000) | z  | A_V^a | N^b | N^c | N^d | Wavelength Range (Å) | τ_H^e | σ_{int}^e (days) | FWHM^e (km s⁻¹) |
|----------|--------------|---------------|----|-------|-----|-----|-----|-----------------------|-------|-------------------|-----------------|
| Mrk 335  | 00 06 19.5   | +20 12 10     | 0.0258 | 0.153 | 129 | 78 | 78 | 4800–4895 | 14.1±0.4 | 1293 ± 64 | 1273 ± 64 |
| Mrk 1501 | 00 10 31.0   | +10 58 30     | 0.0893 | 0.422 | 210 | 65 | 109 | 4800–4925 | 15.5±2.2 | 3321 ± 107 | 3494 ± 35 |
| 3C 120  | 04 33 11.1   | +05 21 16     | 0.0330 | 1.283 | 192 | 69 | 83 | 4800–4900 | 27.2±1.1 | 1514 ± 65 | 1430 ± 16 |
| Mrk 6   | 06 52 12.2   | +74 25 37     | 0.0188 | 0.585 | 204 | 72 | 265 | 4725–5050 | 9.2±0.5 | 3714 ± 68 | 2619 ± 24 |
| PG 2130+099 | 21 32 27.8 | +10 08 19     | 0.0630 | 0.192 | 235 | 68 | 81 | 4800–4895 | 12.8±1.0 | 1825 ± 65 | 1781 ± 5  |

Notes.

^a^ Galactic extinction values are from Schlegel et al. (1998).

^b^ Number of epochs in the continuum light curve.

^c^ Number of spectral epochs.

^d^ Number of pixels used in modeling analysis.

^e^ These measurements were published by G12.

3. Dynamical Modeling Method

We model individual RM data sets using a simply parameterized phenomenological modeling code for the BLR that is fully described by Pancost et al. (2014a). In addition,
we added the AGN redshift as a free parameter with a narrow Gaussian prior of width 1 Å to account for the imperfect determination of the redshift from nearby narrow lines. We also describe some systematic uncertainties in the model in Section 5.2.

The distribution of broad-line emission in position and velocity space is sampled using a number of massless point test particles that instantaneously and linearly reprocess the AGN continuum flux into broad-line flux. The position of the point particles determines the time lag with which the continuum flux is reprocessed, and the velocity of the point particles determines the Doppler-shifted wavelength at which the line flux is emitted. Given an input continuum light curve, we can use the positions and velocities of the point particles to generate model emission-line profiles that can be directly compared with the data.

3.1. Geometric Model

We use a flexible model for the BLR geometry that parameterizes the positions of the point particles using radial and angular distributions. For the radial distribution of point particles, we use a Gamma distribution,

\[ p(x|\alpha, \theta) \propto x^{\alpha-1}\exp\left(-\frac{x}{\theta}\right), \]

that generates profiles ranging from Gaussian to exponential or steeper. The Gamma distribution is offset from the origin by the Schwarzschild radius, \( R_s = 2GM\text{in}/c^2 \), plus a minimum BLR radius, \( r_{\text{min}} \). The quantity \( r_{\text{min}} \) is measured relative to the radius at which the continuum emission is emitted; we here assume the continuum to be emitted at \( r = 0 \), but note that this assumption may not be correct (see Section 5.2). We assume that the outer edge of the BLR is small enough that the length of the RM campaign is more than sufficient to measure all time delays within it; we thus restrict the offset Gamma distribution to an outer radius \( r_{\text{out}} = c\Delta t_{\text{data}}/2 \), where \( \Delta t_{\text{data}} \) is the total time between the beginning of the continuum model light curve and the first epoch of the broad emission-line light curve. We perform a change of variables between \( r_{\text{min}} \) and \( F \), such that

\[ \mu = r_{\text{min}} + \alpha \theta, \]

\[ \beta = \frac{1}{\sqrt{\alpha}}, \]

\[ F = \frac{r_{\text{min}}}{r_{\text{min}} + \alpha \theta}, \]

where \( \mu \) is the mean radius, \( \beta \) determines the shape of the Gamma distribution, and \( F \) is a fractional radius corresponding to \( r_{\text{min}}/\mu \). The radial distribution has a standard deviation given by \( \sigma_r = \mu\beta(1 - F) \). As part of the modeling process, we also calculate the mean radius \( r_{\text{mean}} \), median radius \( r_{\text{median}} \), mean time lag \( \tau_{\text{mean}} \), and median time lag \( \tau_{\text{median}} \) for specific realizations of the point particle positions. We allow the system to deviate from spherical by including an opening angle (\( \theta_o \), defined as the half-opening angle of the BLR disk) that allows the geometry to range from a razor-thin disk to a sphere, and also allows the system to be inclined toward the observer.
by an inclination angle \( \theta_i \). Values of \( \theta_i = 0^\circ \) and \( \theta_i = 90^\circ \) correspond to thin-disk and spherical geometries, respectively; values of \( \theta_i = 0^\circ \) and \( \theta_i = 90^\circ \) correspond to face-on and edge-on geometries, respectively.

For additional flexibility, the BLR model also allows three different types of asymmetry. First, we allow for asymmetric line emission from each point particle. We weight the emission seen by the observer from each point particle as follows:

\[
W(\phi) = \frac{1}{2} + \kappa \cos \phi,
\]

where \( W \) is the weight given to each point particle (between 0 and 1), \( \phi \) is the angle between the observer’s and point particle’s line of sight to the central source, and \( \kappa \) is a parameter that allows for anisotropic emission from the point particles. The quantity \( \kappa \) ranges between \(-0.5 \) and \( 0.5 \): a value of \(-0.5 \) corresponds to the observer seeing more emission from the far side of the BLR due to the point particles emitting preferentially back toward the continuum source, and a value of \( 0.5 \) corresponds to the observer seeing more line emission from the near side of the BLR, with the point particles preferentially emitting away from the central ionizing source. Second, we allow for preferential emission from the outer faces of the disk by changing the angle for a point particle’s displacement from a flat to a thick disk defined by

\[
\theta = \cos^{-1}[\cos \theta_o + (1 - \cos \theta_o) \times U \gamma],
\]

where \( U \) is a random number drawn uniformly between the values of 0 and 1. The \( \gamma \) asymmetry parameter controls the extent to which BLR emission is concentrated in the inner regions or the outer faces of the disk. Values of \( \gamma \) range from 1 to 5, where \( \gamma = 1 \) corresponds to uniform concentrations of point particles in the disk and \( \gamma = 5 \) corresponds to more point particles along the faces of the disk. The third asymmetry parameter is \( \xi \), defined as two times the fraction of point particles below the disk mid-plane. \( \xi \) allows for the mid-plane of the BLR to range from transparent to opaque: for \( \xi = 0 \), the mid-plane is opaque, and as \( \xi \to 1 \), it becomes transparent.

### 3.2. Dynamical Model

The kinematics of the BLR are parameterized in the plane of the radial and tangential velocities of the point particles in the Keplerian potential of the BH (radiation pressure is presumed to be negligible). We allow for a fraction of particles \( f_{\text{ellip}} \) with elliptical orbits drawn from a distribution centered around the circular orbital velocity (near-circular elliptical orbits); \( f_{\text{ellip}} = 0 \) and \( f_{\text{ellip}} = 1 \) represent none and all of the particles having near-circular elliptical orbits, respectively. The remaining \( 1 - f_{\text{ellip}} \) fraction of particles are in inflowing \((0 < f_{\text{flow}} < 0.5)\) or outflowing \((0.5 < f_{\text{flow}} < 1)\) orbits drawn from a distribution centered around the radial escape velocity. The angle \( \theta_e \) adds flexibility to the dynamics by allowing the distributions for inflow and outflow velocities in the plane of the radial and tangential velocities of the point particles to be rotated toward the circular orbit velocity (for a more thorough discussion of this, see Section 2.5 in Pancoast et al. 2014a). As \( \theta_e \to 90^\circ \), the inflow and outflow velocity distributions approach the distribution for near-circular elliptical orbits, so models with low \( f_{\text{ellip}} \) at \( \theta_e = 90^\circ \) are the same as models with high values of \( f_{\text{ellip}} \).

We also allow for a small addition to the point particle velocity vector from macroturbulence, given by

\[
\nu_{\text{turb}} = \mathcal{N}(0, \sigma_{\text{turb}})|v_{\text{circ}}|, \tag{8}
\]

where \( \sigma_{\text{turb}} \) is the standard deviation of the Gaussian distribution from which a randomly oriented macroturbulent velocity component is drawn with a prior between 0.001 and 0.1, and \( v_{\text{circ}} \) is the circular orbit velocity.

### 3.3. Continuum Models and Implementation

In addition to a model for the BLR geometry and kinematics, we must also model the AGN continuum light curve in order to evaluate the continuum flux at arbitrary times for the calculation of the broad-line flux. We use Gaussian processes to model the stochastic AGN continuum variability and interpolate between the continuum flux data points, since it has been found to be a good model for larger samples of AGNs (e.g., Kelly et al. 2009; MacLeod et al. 2010; Zu et al. 2011; Fausnaugh et al. 2016; Kozlowski 2016). Using this model, we can incorporate the uncertainty in the interpolation into our constraints on the BLR geometry and kinematics as well as extrapolate beyond the ends of the data to evaluate the line flux from point particles with long time lags. We show examples of continuum Gaussian process models for each of our targets in the middle panels of Figure 2.

We pose this problem of fitting a model of the BLR to an RM data set in terms of Bayesian inference and use diffusive nested sampling (Brewer et al. 2010) of the BLR and AGN continuum model parameters. Diffusive nested sampling also allows for model comparison by calculating the “evidence” value that normalizes the posterior PDF. We compare the time series of the H/\( \beta \) emission-line profiles from the data with the time series of model line profiles using a Gaussian likelihood function. In general, the model cannot match the data completely to within the small quoted uncertainties, and the likelihood function must be softened by dividing the logarithm of the likelihood by a temperature \( T \), where \( T \gtrsim 1 \). This is equivalent to multiplying the spectral uncertainties in the Gaussian likelihood function by \( \sqrt{T} \). Using larger values of the temperature incorporates additional uncertainty into the likelihood function, which can be thought of as due to underestimated spectral flux errors or the use of a BLR model that does not include sufficient flexibility to match all features in the data.

Given the high-dimensional parameter space and high quality of the data, it is important to check the convergence of the BLR model inference. We can improve convergence by reducing the numerical noise of the model emission-line profiles, using 2000 point particles, and drawing 10 velocities for each. This results in numerical noise from changes in the model line profile for fixed BLR model parameters that is on the order of the spectral uncertainties in the data, \( \sigma_{\text{spectra}} \). However, given that we use temperature values greater than one \((T = 12–20\) for Mrk 335, \( T = 5\) for Mrk 1501, \( T = 5–7\) for 3C 120, and \( T = 35–55\) for PG 2130+099), the numerical noise is smaller than the effective spectral errors, \( \sigma_{\text{effective}} = \sigma_{\text{spectra}}\sqrt{T} \), for all but one epoch out of 275 for the four AGNs. As described in Section 4.4, the higher temperature for PG 2130+099 is due to a poor model fit at the end of the light curve. We test for convergence by running each
AGN multiple times using different starting parameter values and comparing the results. The final posterior PDFs for each AGN are then created by adding together an equal number of samples from each of the runs.

4. Results for Individual Objects

We here present the detailed dynamical modeling results for our sample of four AGNs. For each source, we discuss the quality of the model fit to the data, the constraints on the geometry and kinematics of the BLR, and the shape of the transfer function. When possible, we compare these constraints from dynamical modeling to the results from the analysis in G12 and the MEMECHO analysis presented by Grier et al. (2013b). The posterior median and 68% credible intervals for the BLR model parameters of each target are summarized in Table 2, while the individual $f$ values inferred for each target are listed in Table 3. We also provide histograms of the posterior distributions for a few of the most significant model parameters for each object in the Appendix.

4.1. Mrk 335

Mrk 335 is a narrow-line Seyfert 1 galaxy that has been observed in several RM campaigns (Kassebaum et al. 1997; Peterson et al. 1998, 2004; G12). We use a number of different comparisons between the model and the data to illustrate the quality of the model fit in this target. First, we show the changing shape of the H$\beta$ emission-line profile as a function of time for both the model and the data, as seen in the top two panels of Figure 2 for a model drawn randomly from the posterior PDF. We also show two examples of the H$\beta$ emission-line profile in the middle panel of the left column in Figure 2 to see how well the model fits the detailed line shape. Finally, we compare the integrated emission-line flux for the same model along with the data (shown in the lowest panel in Figure 2) to illustrate how well the model matches the overall variability in the data. The BLR model fits the overall variability of the H$\beta$ emission and the detailed line shape in Mrk 335 very well.

To illustrate the possible geometries of the BLR, we show the geometry of a model randomly drawn from the posterior in Figure 3. By examining the inferred parameters in Table 2 and the posterior distributions of each parameter, we find that Mrk 335 is best described by a thick disk with preference for more emission at the faces of the disk, more emission at the far side of the BLR, and a multimodal distribution with solutions allowing for a mostly transparent or mostly opaque disk midplane. The radial Gamma distribution shape parameter corresponds to a distribution with a tail that is between exponential and Gaussian. The measured mean and median time lags are slightly higher than the time lag measured by Grier et al. (2012a) using JAVELIN; see Table 1. However, we
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Table 2

Dynamical Model Parameters

| Parameter       | Mrk 335 | Mrk 1501 | 3C 120 | PG 2130+099 |
|-----------------|---------|----------|--------|-------------|
| $r_{\text{out}}$ (lt-day) | 39.1796 | 39.6777  | 54.2524 | 50.1817     |
| $r_{\text{mean}}$ (lt-day) | 17.981–2.16 | 17.860–2.10 | 23.310–1.99 | 13.515–2.26 |
| $r_{\text{median}}$ (lt-day) | 16.911–2.00 | 16.441–2.00 | 21.591–1.39 | 10.631–2.44 |
| $r_{\text{min}}$ (lt-day) | 1.280–0.97 | 5.333–3.95 | 0.901–0.70 | 1.324–0.61 |
| $\sigma_r$ (lt-day) | 22.330–8.42 | 16.171–6.85 | 42.841–1.17 | 30.120–2.15 |
| $\tau_{\text{mean}}$ | 18.864–1.84 | 17.080–1.03 | 23.841–1.03 | 13.220–2.87 |
| $\tau_{\text{median}}$ | 16.382–2.19 | 14.961–1.41 | 20.621–1.08 | 7.792–2.57 |
| $\beta$ | 0.853–0.15 | 0.873–0.23 | 0.941–0.06 | 1.344–0.22 |
| $\theta_1$ (degrees) | 38.13–5.72 | 21.72–3.64 | 21.13–5.52 | 33.00–12.32 |
| $\theta_3$ (degrees) | 35.33–4.38 | 20.53–3.73 | 17.63–3.33 | 30.22–10.32 |

Note. $r_{\text{out}}$ is the maximum allowed distance of the point particles from the origin (see Section 3.1); it is the only parameter listed here that is set ahead of time and not calculated by the model.

Table 3

Inferred $f$ from Dynamical Modeling

| Object        | $\log_{10}(f_r)$ | $\log_{10}(f_{\text{FWMH}})$ |
|---------------|------------------|-------------------------------|
| Mrk 335       | 0.593±0.10       | 0.605±0.10                    |
| Mrk 1501      | 0.347±0.09       | 0.303±0.14                    |
| 3C 120        | 0.760±0.14       | 0.815±0.14                    |
| PG 2130+099   | 0.001±0.000      | 0.025±0.025                   |

Note. Virial products used to calculate $f_r$ were determined using the $\sigma$ measured from the root mean square residual spectrum and the FWHM from the mean spectrum, as presented in Table 1.

note that the model parameters $\tau_{\text{mean}}$ and $\tau_{\text{median}}$ are difficult to compare directly to previously published time delays, as JAVELIN assumes a simple top-hat transfer function when modeling the light curves, which is very different from the transfer functions recovered here and in Paper I.

There are two types of dynamical solutions dominating the posterior PDF for Mrk 335, as can be seen by the multiple peaks visible in the posterior distribution of the kinematic parameters shown in the Appendix. The first solution puts most of the point particles in near-circular elliptical orbits, with the remaining particles being in radial inflowing orbits. The second solution has almost no near-circular elliptical orbits, with the inflowing orbits having a larger component of tangential velocity compared to radial, making them more similar to elliptical orbits. Although the kinematics model also allows for the possibility of macroturbulent velocities, we find that macroturbulent velocities do not contribute significantly to the dynamics in Mrk 335.

A velocity-delay map was recovered for the H\beta emission line in Mrk 335 and presented in Paper I. This velocity-delay map is not well-resolved, but shows a hint of asymmetric structure that is consistent with inflowing gas (higher lags toward the blue and shorter lags toward the red). The majority of the signal in the velocity-delay map arises at low velocities at a range of radii from about 15–40 lt-day. We show a sample velocity-delay map from our model drawn from the posterior PDF in Figure 4; however, this transfer function is difficult to compare to a MEMECHO velocity-delay map due to differences in resolution. We do see much of the signal at similar radii (between 10 and 40 lt-day) in both sets of maps. We also compare the velocity-resolved RM lag measurements from Paper I to the mean values of the inferred transfer functions from dynamical modeling for the same wavelength bins, as shown in the right panel showing Mrk 335 in Figure 4.

The velocity-resolved lag measurements based on the models agree to within the uncertainties with those measured from the
not include the uncertainty in the scale factor \( f \) as asymmetric shape of \( H_\beta \) the spectral variations quite well, even estimated to be \( f \) from G12 only take into account the measurement errors in previous measurement of \( \log_{10} M_\text{BH} \) are using decomposed spectra. that there are some visible differences due to the fact that we \( \sigma \) the continuum light curves with the light curves generated from the decomposed spectra \( \text{decomposed spectra} \) (red crosses). We also cross-correlated the model spectra from all of the samples with the continuum light curve and measured the median time lag in each bin \( \text{Figure 2} \).

\[
\text{Figure 4. Transfer functions and velocity-resolved time delays for all four AGNs. For each source, the left panel shows a representative transfer function drawn from the posterior PDF. The right panel for each source shows the velocity-resolved time delays for a number of wavelength bins. The solid black line shows the mean time delays computed by the model corresponding to the transfer function shown, and the blue crosses show the median lag values recovered from all of the model fits. To compare the model lags (blue and black) with the lags measured via cross-correlation techniques, we also show the mean time delays measured via cross-correlation of the continuum light curves with the light curves generated from the decomposed spectra (red crosses). We also cross-correlated the model spectra from all of the samples with the continuum light curve and measured the median time lag in each bin (cyan crosses).}
\]

decomposed spectra (red). We note that the original velocity bins used in Paper I are different from those used in this work; we updated the bins due to the changes in wavelength ranges used in the modeling, and thus the velocity-binned results differ somewhat from Paper I. We show the original bins and measurements from Paper I in gray for comparison, but note that there are some visible differences due to the fact that we are using decomposed spectra.

The black hole mass in Mrk 335 is in agreement with the previous measurement of \( \log_{10}(M_\text{BH}) = 7.29 \pm 0.05 \) from the G12 analysis (all \( M_\text{BH} \) measurements from the G12 data have been updated using \( \log f = 0.63 \), corresponding to \( f = 4.31 \), following Grier et al. 2013a). Note that the uncertainties in \( M_\text{BH} \) from G12 only take into account the measurement errors in \( \tau \) and \( \sigma \) used to calculate the virial product—the uncertainties do not include the uncertainty in the scale factor \( f \), which is estimated to be \( \sim 0.4 \) dex.

4.2. Mrk 1501

Our data for Mrk 1501 constitute the first RM data set for this target, and our model of the BLR was once again able to fit the spectral variations quite well, even fitting the broad, asymmetric shape of \( H_\beta \) (Figure 2). Like Mrk 335, the geometry of Mrk 1501 is found to be a thick inclined disk. Unlike Mrk 335, however, the distribution of point particles in the BLR of Mrk 1501 is inferred to be fairly close to uniform throughout the disk, although solutions with more point particles at the edges of the disk are not ruled out. We see a preference for the emission from the far side of the BLR rather than that from the near side and mid-plane of the disk to be partially transparent. We show an example of a possible geometry in Figure 3. The radial profile of the \( H_\beta \) emission has a Gamma distribution shape parameter that is inferred to be close to exponential, and the inferred mean and median time delays are consistent with values found by G12. For Mrk 1501, we find the kinematics to be a combination of elliptical and inflowing orbits, where the fraction of elliptical orbits is not well-constrained but the remaining orbits are strongly preferred to be inflowing with a range of radial to tangential orbits. Although the contribution of macroturbulent velocities is not great, it approaches the maximum value of 0.1 allowed by the prior, and thus in this case could be larger but may be limited by the prior.

Velocity-binned results from Paper I indicate inflowing gas, with higher time lags toward the blue and lower time lags toward the red. A somewhat blurred velocity-delay map recovered using MEMECHO shows the same signature. As expected from the inferred parameters, we see that the transfer functions recovered from the dynamical modeling (Figure 4) also show strong signatures of inflowing gas. The velocity-binned results shown in the right Mrk 1501 panel of Figure 4 show consistent features, although the mean time delays measured in the central velocity bins from the model (shown in blue) and in the light curves created from the decomposed spectra (red) are somewhat higher on average than the time delays found using the RM techniques in Paper I (shown in gray).

We infer a black hole mass that is slightly lower than the previous measurement from the G12 data of \( \log_{10}(M_\text{BH}) = 8.16 \pm 0.06 \), suggesting that the scale factor \( f \) for this source deviates from the value \( \log f = 0.63 \) that is
commonly assumed. We note that there is a degeneracy between the black hole mass, inclination angle, and opening angle of the system, which limits the precision to which $M_{\text{BH}}$ can be measured with this approach (see Figure 13 in the Appendix). This degeneracy arises because the model is trying to match the width of the H$\beta$ emission-line profile, and all three of these parameters affect the measured line width. For a thin disk, viewing the BLR closer to face-on will decrease the measured line width, while increasing the opening angle of the disk will increase the measured line width. Although this degeneracy is partially broken by the transfer function, an independent method for measuring the inclination angle or opening angle of the BLR would allow for an even more precise measurement of the black hole mass (see Section 4.3).

4.3. 3C 120

3C 120 is a well-studied radio-loud galaxy that has been observed in multiple RM campaigns (e.g., Peterson et al. 1998, 2004). Our model of the BLR was able to fit the spectral variations and line shape from the 2010 data set quite well (Figure 2). We again see a preferred thick disk geometry in 3C 120, and the inclination angle of the system is well-constrained at $\theta_i = 17.6^{+5.4}_{-3.3}$ relative to the observer. 3C 120 is of particular interest because there are external indicators of inclination angle in the system obtained from the radio jet orientation, which has been shown to be linked to the BLR rotation axis (e.g., Wills & Browne 1986; Marscher et al. 2002; Jorstad et al. 2005; Agudo et al. 2012). Marscher et al. (2002) first determined the upper limit on the jet viewing angle to be 20$^\circ$, and further work by Jorstad et al. (2005) measured a viewing angle of 20$^\circ$5 $\pm$ 1$^\circ$.8. Later work by Agudo et al. (2012) determined a jet viewing angle of $\theta = 16^\circ$, and we estimate the uncertainty in this measurement from their paper to be about $\pm$3$^\circ$. Our $\theta_i$ measurement from dynamical modeling is both well-constrained and consistent with these measurements, indicating that in this system, the BLR orientation and jet orientation are aligned.

We see a preference for emission from the far side of the disk and for a mostly transparent disk mid-plane, although whether the emission is distributed equally throughout the disk or concentrated at the faces of the disk is not well-constrained. Figure 3 shows an example of a possible geometry of the H$\beta$-emitting BLR in 3C 120. The radial distribution of the H$\beta$ emission in 3C 120 has a Gamma distribution shape parameter that is close to exponential. We obtain a mean and median time delay consistent to within the uncertainties with the values reported by G12 as well as the recent work by Kollatschny et al. (2014), who report $\tau_{\text{H}\beta} = 27.9^{+3.4}_{-2.9}$ days.

The kinematics of 3C 120 are inferred to be a combination of near-circular elliptical orbits and inflow on mostly radial orbits. These kinematics are consistent with those recovered from velocity-delay maps using MEMECHO, which are the cleanest of the entire sample and show signatures consistent with those expected from elliptical orbits in an inclined disk or a spherical shell, with the He II λ4686 emission line showing signs of inflowing gas. Kollatschny et al. (2014) also performed a velocity-resolved analysis of 3C 120 RM data from a separate campaign and found similar features. The velocity-binned mean time delays shown in Figure 4 are also consistent with those measured in Paper I, though the measurements made directly from the spectra (red) deviate somewhat from the model (blue) in the second-bluest wavelength bin.

We infer a value for the black hole mass that is consistent with measurements from G12, who report log$_{10}(M_{\text{BH}})$ = 7.72 $\pm$ 0.04. As found in Mrk 1501, we see a strong correlation between $M_{\text{BH}}$, inclination angle, and opening angle for this object. The inclination measurement for 3C 120 made using the radio jet orientation can also provide additional external constraints on $M_{\text{BH}}$ that, if $M_{\text{BH}}$ were less well-constrained by the model, could be used to narrow down the black hole mass further. However, the uncertainties in the jet inclination angle measurement are not well-constrained and are likely similar to the inclination angle uncertainties inferred by the model, so in this case, considering the jet inclination angle would not result in a substantial increase in precision in $M_{\text{BH}}$. We note, however, that in other objects for which $M_{\text{BH}}$ is less well-constrained, the additional information provided by external measurements such as radio jet inclinations could significantly improve precision in $M_{\text{BH}}$ measurements, assuming the radio jets and BLR axes are aligned in all cases.

4.4. PG 2130+099

PG 2130+099 is a narrow-line Seyfert 1 galaxy that has been the target of several RM campaigns (Kaspi et al. 2000; Grier et al. 2008, 2012b). Figure 2 shows our model fit to the PG 2130+099 spectral time series. Overall, the model was able to reproduce the detailed spectral shape well, but was unable to reproduce the integrated line variability during the last third of the campaign, possibly due to the low levels of variability throughout the campaign. It is also possible that the mismatch occurs because of a nonlinear response by the H$\beta$ emission line (such behavior has been reported, for example, in NGC 5548 by Goad et al. 2016), and thus this particular model will not provide an optimal fit. Similar to our other targets, PG 2130+099 is well-described as a somewhat-inclined thick disk. We find preferential emission from the far side of the disk and more emission from the faces of the disk, although the transparency of the disk mid-plane is not well-constrained. Figure 3 shows an example of a possible geometry for the H$\beta$-emitting region of this target. The radial distribution of the H$\beta$ emission has a well-constrained Gamma distribution shape parameter corresponding to profiles steeper than exponential. We obtain mean and median time delays consistent with the value measured by G12.

The kinematics of the BLR are dominated by inflowing orbits, with a combination of radial and tangential velocities, although solutions with all near-circular elliptical orbits are not entirely ruled out. We do find a small contribution to the dynamics from macroturbulence. The velocity-delay maps recovered by these models (Figure 4) show asymmetries reminiscent of those shown in Paper I; however, it appears that much of the emission is concentrated at smaller time delays, and there is a stronger symmetric structure in the model velocity-delay maps that is more indicative of near-circular elliptical orbits. We also see that the two bluest velocity bins show much faster responses in the model than they do in the light curves created from the decomposed spectra (and also in Paper I).

The black hole mass in PG 2130+099 is significantly lower than the mass calculated in G12 of log$_{10}(M_{\text{BH}})$ = 7.56 $\pm$ 0.04. This suggests that the true scale factor $f$ in this system is different from the average value for the reverberation-mapped sample, again indicating the importance of individual scale factors when considering individual AGNs.
5. Discussion

5.1. Overview of Geometric and Kinematic Results

Overall, many of the BLR geometric and kinematic model parameters are well-constrained for the four sources in our sample. We find that the BLR geometries are best described by thick disks that are inclined such that they are close to face-on for the observer (θ < 45; this is unsurprising given that these are Type 1 AGNs). In all four sources, we find results that are consistent with preferential emission from the far side of the disk, although this is not always well-constrained by the model. The modeling results are generally consistent with previous analyses: we find that the BLR radii and time delays are mostly consistent with those measured by G12, with some minor deviations.

We also compare the $M_{BH}$ measurements from dynamical modeling to those recovered from the RM analysis from G12, shown in Figure 5. The previous measurements of $M_{BH}$ (hereafter referred to as $M_{BH,RM}$) were again calculated assuming a value for $f$ of 0.63 ($f = 4.31$), following Grier et al. (2013a). As discussed above, the uncertainties quoted for these $M_{BH,RM}$ measurements include only the measurement uncertainties in $\tau$ and $\sigma$—they do not include the uncertainty in $f$ that is introduced by using the $f$ calculated from the $M_{BH,\theta,\gamma} - \sigma_*$ relation, which is estimated to be ~0.4 dex (shown by the yellow error bars in Figure 5). With the exception of PG 2130+099, the $M_{BH}$ measurements from G12 are consistent with those from dynamical modeling when this additional uncertainty in $f$ is also taken into account. The mass inferred for PG 2130+099 is marginally inconsistent with that reported by G12, suggesting that the scale factor $f$ in this object is lower than the average scale factor generally adopted for the entire AGN population, or that the model does not contain enough flexibility to fit the data sufficiently in this case. Given the relatively poor fit of the PG 2130+099 model light curves to the data at the end of the campaign (see Figure 2), we suspect that the latter is a likely scenario.

P14 show their $M_{BH}$ measurements from dynamical modeling on the $M_{BH} - \sigma_*$ relation of quiescent galaxies with the dynamical $M_{BH}$ measurements presented by McConnell & Ma (2013), showing that the $M_{BH}$ measured via dynamical modeling places their five sources in positions consistent with the quiescent $M_{BH} - \sigma_*$ relationship. Unfortunately, stellar velocity dispersion ($\sigma_*$) measurements only exist for two of our targets: 3C 120, with $\sigma_* = 162 \pm 20$ km s$^{-1}$ (Nelson & Whittle 1995), and PG 2130+099, with $\sigma_* = 163 \pm 19$ km s$^{-1}$ (Grier et al. 2013a). However, for completeness, we reproduce Figure 23 from P14 in our Figure 6 with our two additional measurements included. We see that the locations of both PG 2130+099 and 3C 120 are also consistent with the distribution of dynamical black hole mass measurements.

All four of our targets exhibit kinematics with either mostly near-circular elliptical orbits or near-circular elliptical orbits combined with inflowing gas. In addition, all of these kinematic results are consistent with the qualitative interpretation of the MEMECHO analysis we performed in Paper I; the velocity-delay maps we recovered all show signatures of elliptical motion and/or inflow. Our results are consistent, with all of the targets favoring dynamics that are dominated by the gravitational potential of the black hole, supporting the use of RM to measure $M_{BH}$.

RM data for three of our targets (3C 120, Mrk 335, and PG 2130+099) have also recently been analyzed by Li et al. (2013), who use a geometry-only model of the BLR within the framework that is based on the model proposed by Pancost et al. (2011). The main difference between this work and the analysis of Li et al. (2013) is that we model the H/β emission-line profile while Li et al. (2013) model the integrated H/β flux. We also use slightly different models for the BLR geometry, with Li et al. (2013) including nonlinear response and this analysis including additional asymmetry parameters for the geometry ($\kappa$, $\gamma$, and $\lambda$). Although formally consistent, we find smaller inclination angles (between 25° and 30° rather than closer to 50°) and smaller opening angles (also ranging from about 15°–30° rather than closer to 50°) for 3C 120 and Mrk 335. The measurements for PG 2130+099 differ.
significantly from those reported by Li et al. (2013), likely due to the differences in modeling approaches. The BLR radii reported by Li et al. (2013) for the three targets are formally consistent with those found in this work, though our models constrain these angles much more tightly; this highlights the importance of using full dynamical models when data of sufficient quality are available.

5.2. Systematic Uncertainties in the Model

The formal uncertainties from our model fitting are very small (particularly in the case of Mrk 335): with this particular data set, we are approaching the regime where the uncertainties in the inferred parameters are dominated by the systematic limitations of the model rather than the observational signal-to-noise ratio or cadence of the data. Because our formal (statistical) uncertainties are derived from the data, it is possible that the true uncertainties are somewhat larger than the statistical uncertainties alone. We now discuss possible sources of systematic uncertainty from both correlations between model parameters and from physical assumptions made by the model.

5.2.1. Systematic Uncertainties from Correlations between Model Parameters

There are two primary ways that correlations between the model parameters influence the inferred uncertainties in BLR properties. First, constraints provided by the data may introduce degeneracies in parameter space that appear unphysical when observed for a sample of AGNs as a whole. The best example of this for the current sample of nine AGNs with BLR modeling is the correlation between the inclination and opening angles. As shown in Figure 13, the inclination and opening angles show a positive tight correlation and approximately equal values. Inspecting emission-line profiles produced by the BLR model with similar parameters as inferred for these data suggests that the opening angle is forced to be at least as large as the inclination angle in order to generate a single-peaked emission-line profile. As the opening angle becomes larger than the inclination angle, the transfer function becomes more spread out in wavelength and time-lag space. If the data prefer a more compact transfer function, it will thus force the opening angle to be as small as possible while still producing a single-peaked line profile. This condition is met when the opening angle is very close to the inclination angle. Since there is no obvious upper bound to possible values of the opening angle, it is likely that the values we infer are closer to upper limits; in other words, to match a single-peaked emission-line profile, the effective prior on the opening angle is from $\theta_0 \rightarrow 90^\circ$, so an inference of $\theta_0 \sim \theta$ could also mean that $\theta_0 \lesssim \theta$. Although it is impossible to quantify the magnitude of this systematic uncertainty without comparing to models that include other methods for making single-peaked emission lines, we can test whether the inclination or opening angle has a greater effect on the model transfer function. We varied $\theta_i$ and $\theta_o$ by $5^\circ$ to $30^\circ$ around a fiducial value of $\theta_i = \theta_o = 30^\circ$ and compared the resulting transfer functions. We found that the inclination angle has a qualitatively greater effect by causing more extreme changes in the line profile shape. Comparing the relative dispersion in the differences of the transfer function to that derived from the fiducial parameters confirms this result quantitatively. This suggests that the inclination angle may be more robustly determined than the opening angle; indeed, the consistency between the radio jet inclination measurement for 3C 120 and our model (see Section 4.3) supports the idea that our model with a more robust modeling constraints the inclination angle.

Second, with such a flexibly parameterized model, there can be multiple distinct parameter combinations that end up producing the same distribution of point particles in position and velocity space. A clear example of this is for inferred models where the inclination and opening angles approach $90^\circ$ with $\gamma \sim 5$, as for Mrk 335, Mrk 1501, and PG 2130+099. This combination of parameter values corresponds to a spherical distribution of point particles, but where the particles are concentrated along the face-on axis that is perpendicular to the observer’s line of sight, such that they form a jet-like structure. Since rotations in the plane of the sky cannot be resolved by RM, which is only sensitive to time delay and line-of-sight velocity, this jet perpendicular to the line of sight is equivalent to a face-on thick disk. This may increase the statistical uncertainty in certain inferred parameters compared to the true uncertainty in, for example, disk thickness.

![Figure 7](image-url)
However, Figures 9–12 show that the number of posterior samples in the solutions with larger inclination and opening angles just described is a small fraction of the total and our use of the median instead of the mean value of the posterior PDF in Table 2 minimizes the contribution from posterior samples in the tails of the distribution.

5.2.2. Systematic Uncertainties from Assumptions of the Model

In order to develop a simply parameterized and flexible model for the BLR, many assumptions were made about BLR physics, including the following:

1. We assume spatially and temporally uniform responsivity across the BLR without optical depth effects. However, Korista & Goad (2004) show that spatially and temporally constant responsivity is not necessarily a good assumption for Balmer lines.

2. We assume that the only non-negligible force at play is gravity; thus, any force that has a functional form of $1/r^2$ (such as radiation pressure from electron scattering from a distant source) is subsumed under this, and we cannot differentiate between them.

3. We assume that the driving continuum light curve is emitted by a source much smaller than the BLR—in our case, a point-like source at $r = 0$ (i.e., we are neglecting to account for the size of the ionizing continuum-emitting region).

To fully address the first point above would require a self-consistent BLR model that includes photoionization models, constraining both the emission properties and gas distribution simultaneously. We note, however, that other BLR models (e.g., MEMECHO) and codes to measure time lags (e.g., JAVELIN) currently all include a linear response of the emission lines, working under the assumption that the changes in AGN luminosity within the time spanned by a single RM campaign are small enough that the response will not deviate far from linearity. To address the second assumption above would require an updated model for the inner accretion disk and the emission of ionizing photons, which is quite challenging to constrain. As such, for these two assumptions, we are unable to calculate or estimate the possible magnitude of systematic uncertainty introduced in our measurements.

However, in light of recent developments, the assumption that the driving optical continuum originates at $r = 0$ warrants further discussion. As part of a multiwavelength RM campaign using the Hubble Space Telescope, Fausnaugh et al. (2016) measure a time delay between the optical continuum at various wavelengths and the ultraviolet (UV) continuum at 1367 Å. In particular, they found that the V-band continuum lags the UV continuum by about two days, indicating that the V-band emitting region is at least 2 lt-day farther out than the UV-emitting region. This is on par with the mean lag of 2.5 days measured in the optical and UV He II emission lines—the size of the optical continuum-emitting region is therefore far from negligible. It is possible that our assumption of continuum emission originating at $r = 0$ could cause us to underestimate $M_{BH}$, although it is also worth noting that NGC 5548 was in an exceptional state during the course of this RM campaign, and the emission-line time delays were all shorter than predicted from the radius–luminosity relation (Pei et al. 2017). Time delays between the EUV and optical of varying quality and significance have been reported by several other studies as well (Collier et al. 1998; Sergeev et al. 2005; McHardy et al. 2014; Shappee et al. 2014; Edelson et al. 2015), suggesting that the situation is not unique.

For NGC 5548, Pei et al. (2017) determine that the Hβ–UV lag is about two days longer than the Hβ–optical lag (consistent with the optical-to-UV lag reported by Fausnaugh et al. 2016), so the BLR radius is underestimated by about 50% in this particular case. The magnitude of the effect on $M_{BH}$ caused by the non-negligible accretion disk size will depend on the AGN,
as the accretion disk size depends on luminosity, $M_{\text{BH}}$, and the slope of the temperature profile. However, Pei et al. (2017) examine this in detail and find that the scaling with luminosity is expected to be slow and the scatter in the radius–luminosity relationship (Bentz et al. 2013) is small—thus, this effect is likely small for most AGNs. Additional studies to measure the relationship between the optical and UV lags would be useful to confirm this for sources of different $M_{\text{BH}}$ and luminosities.

It is worth noting that this particular systematic will not have an effect on BH masses measured via traditional RM (by measuring the average time lag within the BLR via cross-correlation or some other method), as the use of the $M_{\text{BH}}-\sigma_*$ relation to calculate the average scale factor $f$ automatically makes up for this by requiring that AGNs fall on the quiescent $M_{\text{BH}}-\sigma_*$ relation. For our sample of four AGNs, the scale factors measured from the dynamical modelling are consistent with the average scale factor measured from the $M_{\text{BH}}-\sigma_*$ relation (with the exception of PG 2130+099), which also suggests that any systematic effect caused by ignoring the size of the continuum-emitting region is likely to be small in these targets.

5.3. The Scale Factor $f$

The scale factor $f$ summarizes the relationship between the observables from an RM experiment ($\Delta V$ and $R$) and $M_{\text{BH}}$. The line width and BLR radius are related as $\Delta V \propto R^{1/2}$, indicating that the quantity $\Delta V^2/R$, or the virial product, is proportional to $M_{\text{BH}}$—however, there are other quantities that affect $M_{\text{BH}}$ measurements that are not measured in traditional RM. For example, if the BLR is a disk or disk-like, $f$ should depend strongly on the inclination of the disk relative to the line of sight. In addition, it should depend on the kinematics of the BLR, which could include infall, outflow, or circular-like motion. The scale factor $f$ should also depend on both the radial distribution of gas within the BLR and its responsivity. It will depend on how well the characteristic $R$ measured by RM actually reflects the typical size of the system. If the BLR environment is dependent on accretion rate, one might expect to see a correlation between $f$ and the AGN luminosity and/or the Eddington ratio. We do not expect $f$ to correlate with $M_{\text{BH}}$—such a correlation would indicate that the BLR structure and kinematics are mass dependent or likely that we have some sort of selection bias in our sample.

As discussed in Section 1, traditional RM studies do not yield measurements of $f$; the most common way to estimate $f$ to obtain $M_{\text{BH}}$ measurements using RM is by assuming that AGNs follow the same $M_{\text{BH}}-\sigma_*$ relationship as quiescent galaxies to calculate $f$ for the entire set of AGNs. There has been a range of many different $f$ measurements reported in the literature, ranging from $\log_{10} f = 0.58$ (Graham et al. 2011) to 0.74 (Onken et al. 2004), though most are consistent with one another to within the uncertainties. It is also possible that galaxies with different morphologies follow different $M_{\text{BH}}-\sigma_*$ relations, which would require the use of different values of $f$ depending on the galaxy type (e.g., Ho & Kim 2014). However, by measuring $f$ in individual AGNs using dynamical modeling, we can consider the values of $f$ for our sample as a whole, as described in Sections 5.3.1 and 5.3.2.

5.3.1. $f$ Measurements

We calculate the mean scale factor ($f$) in our sample to compare with the external measurements reported in previous studies. To obtain a measurement of $f$ from a sample of sources with dynamical modeling results, we model the distribution of posterior PDFs of individual $f$ values using a Gaussian with a mean $f$ and dispersion or scatter in individual values of $f$ (see P14 for further details). The posterior PDFs of individual $f$ values are calculated by dividing the posterior PDF of the black hole mass by the virial product for each target. This analysis yields uncertainties on both the mean scale factor $f$ and the dispersion or scatter. For the AGN10 sample, we calculate the mean value of $\log_{10} f$ to be $\log_{10} f = 0.45 \pm 0.32$ and the dispersion in $\log_{10} f$ to be $0.49 \pm 0.35$. We also calculate the mean value of $\log_{10} f_{\text{FWHM}}$ to be $\log_{10} f_{\text{FWHM}} = 0.45 \pm 0.33$ and the dispersion in $\log_{10} f_{\text{FWHM}}$ to be $0.52 \pm 0.36$. Our $\log_{10} f$ is consistent to within the uncertainties with the measurement made by P14 (who report a mean value of $\log_{10} f$ of $\log_{10} f = 0.68 \pm 0.40$ and a dispersion in $\log_{10} f$ of $0.75 \pm 0.40$), and our value for $\log_{10} f_{\text{FWHM}}$ is also consistent with their measurement of $\log_{10} f_{\text{FWHM}} = -0.07 \pm 0.40$ and dispersion of $0.77 \pm 0.38$.

When we combine our sample with the P14 sample for a total of nine AGNs (hereafter referred to as the “combined sample”), we measure a mean value of $\log_{10} f$ of $\log_{10} f = 0.54 \pm 0.17$ with a dispersion in $\log_{10} f$ of $0.39 \pm 0.23$ and $\log_{10} f_{\text{FWHM}} = 0.18 \pm 0.23$ with a dispersion in $\log_{10} f_{\text{FWHM}}$ of $0.59 \pm 0.22$. We show the posterior and predictive distributions for $\log_{10} f$ and $\log_{10} f_{\text{FWHM}}$ in Figure 7. The predictive distributions are the distributions from which new measurements of $f$ or $f_{\text{FWHM}}$ are drawn and are generated from linear combinations of Gaussians weighted by the posterior probability of the model parameters fit to the nine measured values of $f$. The $f$ value is consistent with the measurements of $f$ found using the $M_{\text{BH}}-\sigma_*$ relation listed above, suggesting that the $M_{\text{BH}}-\sigma_*$ relation yields a reasonable calibration for AGN $M_{\text{BH}}$ measurements.

5.3.2. Correlations between $f$ and AGN Properties

RM is a very time- and observation-intensive practice; the data used for modeling in this work and by P14 have very high signal-to-noise ratio and high cadence. RM $M_{\text{BH}}$ measurements, determinations of the scale factor $f$, and BLR information are impractical to obtain in large samples of AGNs due to the stringent data quality requirements. With our expansion of the sample of AGNs with dynamical modeling results, we not only learn about the BLR in individual sources, but also aim to uncover any potential correlations between $f$ and other more easily measured AGN parameters that may offer information without the heavy requirements of RM campaigns and enable improvements in single-epoch $M_{\text{BH}}$ measurements. To search for such correlations, we combine the AGN10 sample with the P14 sample, nearly doubling the number of sources with detailed constraints on the BLR. However, there is one other AGN, Mrk 50, which has dynamical modeling results (Pancoast et al. 2012). The results for this source were obtained using a less flexible BLR model that did not allow for unbound infalling and outflowing orbits. The narrow-line model has also been updated, and more asymmetry parameters ($\gamma$ and $\xi$) have been added to account for the complexities of the BLR geometry. Since it is difficult...
to directly compare the results for Mrk 50 to our new results and those of P14, we omit this source from our analysis.

We searched for correlations between $f$ and all of the model parameters reported in Table 2; however, only a few correlations were found. In Figure 8, we show the scale factors measured as a function of five different parameters: $\log_{10} L_{\mathrm{AGN}}$, Eddington ratio ($\log_{10} f = L_{\mathrm{edd}}/L_{\mathrm{edd}}$), $M_{\mathrm{BH}}$ recovered from dynamical modeling, $\theta_{i}$, and $\theta_{o}$. Spearman rank test results between $f$ and these parameters are given in Table 4. Luminosities ($L_{\mathrm{5100}}$) were measured by G12 and $L_{\mathrm{edd}}$ values were computed using these luminosities, a bolometric correction factor of 9, and the $M_{\mathrm{BH}}$ measurements reported by our dynamical modeling. The luminosities have all been corrected for host galaxy contamination by Bentz et al. (2013), with the exception of Mrk 1501, which does not yet have imaging data of sufficient quality to make these measurements. The host galaxy contribution to the luminosity of Mrk 1501 causes the AGN luminosity to be overestimated. Although our data do not span a sufficient wavelength range to make precise measurements of the host contamination, we use our spectral decomposition of the mean spectrum to estimate that the stellar component contributes roughly 20% of the total flux at 5100 Å. This is comparable to that seen in the other three objects that have more well-measured host components (see G12). Applying this correction would change the AGN luminosity of Mrk 1501 from $\log_{10} L_{\mathrm{AGN}} = 44.32$ to $\log_{10} L_{\mathrm{AGN}} = 44.22$, which has no significant effect on the position of Mrk 1501 in Figure 8. Since this estimate is so uncertain and has no effect on the correlation, we choose to use the uncorrected value in this work. We note that the AGN10 sample occupies a higher range of AGN luminosities than the P14 sample, as all five P14 targets have $\log_{10} L_{\mathrm{AGN}} < 43.0$ while all four AGN10 targets have $\log_{10} L_{\mathrm{AGN}} > 43.0$.

Figure 8 shows no significant correlation between $f$ and the 5100 Å luminosity of the AGNs in either the AGN10 sample or the combined sample. However, there is a possible correlation between $f_{\mathrm{FWHM}}$ and $L_{\mathrm{edd}}$, though it is not seen with $f_{\mathrm{FWHM}}$; such a correlation may be expected if radiation pressure forces provided an important contribution. However, we note that interpretation of the $L_{\mathrm{edd}}$ panel is not entirely straightforward, as both of the parameters being examined ($f$ and $L_{\mathrm{edd}}$) are computed using the $M_{\mathrm{BH}}$ measured by our model (as a reminder, $f$ is calculated by dividing the $M_{\mathrm{BH}}$ measured by the dynamical modeling by the virial product measured from the line width and mean BLR radius, and $L_{\mathrm{edd}}$ is also calculated using the $M_{\mathrm{BH}}$ from the dynamical modeling).

Figure 8 also suggests a possible correlation between $f$ and $M_{\mathrm{BH}}$, particularly for $f_{\mathrm{FWHM}}$. This correlation would be expected only if the BLR geometry or dynamics depends on the size of the BH itself. We also see a correlation between $f$ and the inclination angle and also between $f$ and the opening angle, and these correlations are strengthened when the two samples are combined. P14 also observed the correlation between $f$ and the inclination angle of the system in their sample, noting that this correlation was predicted by Goad et al. (2012) but could also be an effect of correlated errors. We see strong correlations between $M_{\mathrm{BH}}$ and the inclination angle (and between the inclination and opening angles; see Section 5.2.1 and Figure 13), so the correlations seen between $f$ and these parameters could be related to this.

We have a broad range in line profile shape among the combined sample, so we also investigated potential correlations between the line shape, parameterized by the ratio $\text{FWHM}/\sigma$ (with the FWHM measured from the mean spectrum and $\sigma$ measured from the root mean square residual spectrum), and various BLR quantities recovered by the models such as inclination angle, $f$, and $M_{\mathrm{BH}}$. The H$\beta$ profiles in the AGN10 sample have similar FWHM/$\sigma$ that are much lower than those of the P14 sample. We see no evidence for any correlation between the BLR parameters and $\log_{10}(\text{FWHM}/\sigma)$ among the AGN10 sample or the combined sample—the tentative correlation seen by P14 between $\log_{10}(\text{FWHM}/\sigma)$ and $M_{\mathrm{BH}}$ disappears when the combined sample is examined.

6. Summary

We used the dynamical modeling techniques of Pancoast et al. (2014a) to constrain the geometry and kinematics of the H$\beta$-emitting BLR in a sample of four AGNs from a 2010 RM campaign. The main results of this work are as follows.

1. In all cases, we find that the H$\beta$-emitting BLR is best described by a thick, inclined disk that is closer to face-on than edge-on relative to the observer.

2. In the case of 3C 120, our measurements of the inclination angle are consistent with measurements made independently using the radio jet inclination. This indicates that the BLR and jet axes are aligned in this system, and demonstrates the potential combined power of dynamical modeling and radio jet measurements in the future (see Section 4.3).

3. As with the P14 sample, our results are consistent with most of the broad-line emission originating from the far side of the BLR. This is consistent with photoionization modeling (e.g., Ferland et al. 1992; O’Brien et al. 1994; Korista & Goad 2004).

4. We see signatures of gas in near-circular elliptical orbits as well as inflowing movement of the gas within the BLR in all four cases. This is in good agreement with the signatures seen in the velocity-delay maps of these sources in Paper I.

5. From our recovered models, we obtain $M_{\mathrm{BH}}$ measurements for all four targets: $\log_{10}(M_{\mathrm{BH}}) = 7.25^{+0.10}_{-0.10}$ for Mrk 335, $\log_{10}(M_{\mathrm{BH}}) = 7.86^{+0.20}_{-0.17}$ for Mrk 1501, $\log_{10}(M_{\mathrm{BH}}) = 7.84^{+0.16}_{-0.15}$ for 3C 120, and $\log_{10}(M_{\mathrm{BH}}) = 6.92^{+0.24}_{-0.23}$ for PG 2130+099. These measurements are independent of $f$.
and largely consistent with previous RM $M_{\text{BH}}$ measurements, with the exception of PG 2130+099. This lack of agreement with previous measurements could be because PG 2130+099 has a different scale factor $f$, or because its behavior is not well-represented within the constraints of this particular model.

6. We compute the scale factors ($f$) for all four objects using the virial products measured by G12 and the $M_{\text{BH}}$ measurements from our models (tabulated in Table 3). Three of the four have values close to the average value typically used in RM $M_{\text{BH}}$ calculations; however, PG 2130+099 has a much lower scale factor $f$ than the others. We note that previous studies of PG 2130+099 (e.g., Grier et al. 2008) have had difficulties measuring a reliable lag for this source; the pattern of variability in this object has always led to ambiguous results.

7. We find that $f$ is possibly correlated with $M_{\text{BH}}$, $\ell_{\text{Edd}}$, the inclination angle, and the opening angle of the system, but not with any other parameters that we examined. We similarly see no correlations between the ratio of the FWHM to the line dispersion of the H$\beta$ emission line and any of the following: $M_{\text{BH}}$, inclination angle, or $f$.

8. We combine the posterior distributions of $f$ for each AGN in our sample and measure the mean scale factor $\bar{f}$ for our sample of four AGNs to be $\log_{10} f_\odot = 0.45 \pm 0.32$. We measure this for the combined sample from this study, which includes objects from P14, to be $\log_{10} f_\odot = 0.54 \pm 0.17$. Our $f$ measurements are consistent with nearly all measurements of $\bar{f}$ made using the $M_{\text{BH}}-\sigma_*$ relationship.

The dynamical modeling method from P14 was strikingly successful in providing constraints for these four AGNs, likely due to the high S/N and high cadence of the RM campaign, allowing us to nearly double the size of the sample of objects having been modeled in this manner. Though we have expanded the size of the sample with dynamical modeling measurements, our sample is still small and not entirely representative of the reverberation-mapped population of AGNs, much less the AGN population as a whole. Thus, while promising, it may still be premature to use our average measured scale factor $\bar{f}$ to calibrate the $M_{\text{BH}}$ scale. Future RM experiments with similar data quality would yield additional constraints on the larger population of AGNs by allowing us to improve the statistical significance of observed correlations between various parameters and eventually allow us to build a large enough sample to apply the average scale factor $f$ and its scatter to the broader AGN sample with confidence.

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Appendix

We here present the posterior distributions for some of the key model parameters for each target. Each parameter is defined in Section 3. We also show two-dimensional posterior distributions (Figure 13) to demonstrate the correlations between $M_{\text{BH}}$, $\theta_i$, and $\theta_o$.

Figure 9. Posterior distributions for key model parameters for Mrk 335.
**Mrk 1501**

Figure 10. Posterior distributions for key model parameters for Mrk 1501.

**3C 120**

Figure 11. Posterior distributions for key model parameters for 3C 120.
Figure 12. Posterior distributions for key model parameters for PG 2130+099.

Figure 13. Two-dimensional posterior distributions for all four AGNs, showing the correlations between $M_{BH}$, $\theta_1$, and $\theta_o$.

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