Using Position-Based Dynamics for Simulating Mitral Valve Closure and Repair Procedures

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Abstract

To achieve the best treatment of mitral valve disease in a patient, surgeons aim to optimally combine complementary surgical techniques. Image-based in silico simulation as well as visualization of the mitral valve dynamics can support the visual analysis of the patient-specific valvular dynamics and enable an exploration of different therapy options. The usage in a time-constrained clinical environment requires a mitral valve model that is cost-effective, easy to set up, parameterize and evaluate. Working towards this goal, we develop a simplified model of the mitral valve and analyse its applicability for the sketched use-case. We propose a novel approach to simulate the mitral valve with position-based dynamics. The resulting mitral valve model can be deformed to simulate the closing and opening, and incorporate changes caused by virtual interventions in the simulation. Ten mitral valves were reconstructed from transesophageal echocardiogram sequences of patients with normal and abnormal physiology for evaluation. Simulation results showed good agreements with expert annotations of the original image data and reproduced valve closure in all cases. In four of five pathological cases, abnormal closing behaviour was correctly reproduced. In future research, we aim to improve the parameterization of the model in terms of biomechanical correctness and perform a more extensive validation.

Keywords: numerical analysis, methods and applications, biological modelling, modelling, medical imaging, visualization

CCS Concepts: • Applied computing → Life and medical sciences; Health informatics

1. Introduction

The mitral valve (MV) is part of the left heart (LH) and controls the blood flow between the left atrium (LA) and the left ventricle (LV). Anterior and posterior leaflets are attached to the mitral annulus (MA). The chordae tendineae (CT, tendinous cords) connect the leaflets to the papillary muscles (PMs), which extend from the heart muscle into the LV. The MV is open during the filling phase. Oxygenated blood coming from the lungs flows through the MV from LA to LV. When the oxygenated blood is ejected into the aorta in the contraction phase, fluid forces close the MV. In closed state, blood flow back to the LA is normally prevented by leaflet coaptation. This is a combined parachute-like action of leaflets, CT and PMs (cf. Figures 1(a) and (b)).

In Germany, mitral valve regurgitation (MR) is the second most common heart valve disease with an increasing incidence [KSH18]. Patients with MR exhibit defects in the leaflets, annulus or CT. This results in an incomplete closure and prolapsing of the leaflets into the atrium (cf. Figures 1(c)–(d). In turn, this leads to blood leaking back into the atrium during heart contraction (cf. Figure 1(a)). Heart function is impaired.

More than 12,300 MV procedures were performed in Germany in 2017 [BML*18]. Several therapy options and combinations of techniques exist to treat MR. The range spans from reconstruction using different catheter-based or surgical techniques to complete valve replacement. Surgical reconstruction is the gold standard therapy for MR with often times very good long-term results.
Forty four years ago, echocardiography was introduced as a diagnostic tool in clinical practice. It has since become an indispensable part of the diagnostic armamentarium for evaluating patients with heart disease. The introduction of transesophageal echocardiography (TEE) in the late 1980s and early 1990s has further enhanced the diagnostic capabilities of this non-invasive technique. TEE provides high-resolution images of the heart and great vessels, allowing for detailed visualization of the cardiac structures.

Figure 1: (a) Illustration of heart anatomy with mitral valve insufficiency (MR) (image adapted from elsewhere [Bla14]). The broad arrow depicts the normal blood flow direction through the aorta in the contraction phase. The smaller arrow depicts blood flowing into the atrium in MR. (b)–(d) Transesophageal echocardiography (TEE) images showing the heart in long axis view: (b) Normal valve, (c), (d) abnormal (prolapsing) leaflets. A disadvantage of TEE acquisitions is shadowing and signal dropout artifacts, which can result in imprecise mitral valve reconstructions and invisible small-scale anatomical structures such as the chordae tendineae. In the open state, the leaflets can be positioned very close to the myocardium, while in the closed state, the coaptation (zone where the leaflets meet) can be difficult to assess. Anatomical structures, landmarks and acquisition artifacts (shadowing, signal dropout at myocardium) are highlighted.

Building and simulating a patient-specific model that includes all details mentioned above has not yet been realized [GQF*17]. Even if all the necessary parameters and a reliable approximation of the in vivo geometry were available as a byproduct of the data gathered in clinical routine, using today’s methods, a simulation exploring all possible options of a single data set would require many days or even weeks of computation time on current workstation hardware [CMM*18]. While complex modeling approaches are valuable for biomedical research and education because of the detailed information that can be gained on all model levels (cf. Section 2 for more details), the system would be of no direct use in a clinical scenario where time is one of the constraints. Access to high-performance computing (HPC) and the personnel needed for setting up complex, problem-specific simulations on such clusters are not necessarily available for clinical purposes and, in particular, are not available in every clinic performing such interventions. Workstations for image-based diagnosis and therapy planning, however, are often times equipped with capable hardware for 3D image processing and are widely available. These workstations might as well be used for decision support using simplified models for simulation. Practical aspects to consider concern unknown model parameters, which cannot be derived from in vivo image data. This affects the quality of reconstructed MV shapes (which may be subject to imaging artifacts) or limits the available information for the characterization of material properties (e.g. patient-specific information about calcification, nonlinearities or fibre directions may be unknown). Therefore, in a DSS scenario, evaluating the model should be sufficiently precise under the given uncertainties in parameterization as well as fast enough in simulation setup and execution to be used on normal workstations found in a clinical setting.

We propose a simplified mitral valve and blood flow model using position-based modelling for the simulation of the MV. Position-based dynamics (PBD) is a simulation method that is commonly used in real-time computer graphics and games [MHHR07, BMO*14, MMC16, BMM17]. Especially when compared to mass–spring systems, which would be used traditionally for modelling simple elastic objects, PBD offers some advantages such as its
stability and a more elegant modelling of properties such as area or volume conservation, which would require multiple additional springs for each element in mass–spring systems. Instead of numerically integrating spring forces, PBD uses a different modelling approach in which positional constraints are combined with an iterative optimization technique. In contrast to other approaches like finite elements, PBD execution times are suitable for real-time visualizations at interactive frame rates and user interactions. This can be used for an interactive parameterization and to model missing information, e.g. approximate elasticity parameters or model CT quickly. A disadvantage compared to finite element models is that basic PBD models cannot be parameterized based on existing biomechanical models for biological soft tissue. In addition, PBD does not allow to directly extract physical quantities for analyses based on its parameterization. Besides fast computation and given all uncertainties, accuracy is of central importance. We analyse the applicability of the method based on subjective evaluations of the simulated valve shapes by experts in cardiac surgery as well as measured differences between generated valve shapes and expert annotations of the image data. We are satisfied with the results, if the generated shapes match the expectations of the experts and the simulated closed valve states match the closed state seen in the image data within bounds of previously published data. If present, this includes pathologies such as prolapsing leaflets. The simulation model should also be able to represent virtual interventions such as annuloplasty, PM re-location, cord manipulation and clipping. Here, we provide first examples for annuloplasty and clipping based on a single data set. The visual representation of these interventions is the main concern and can be used to gain insights into what ring size may be suitable in an upcoming intervention. The model enables the simulation of immediate effects of anatomical alterations. The estimation of long-term outcomes requires more comprehensive models, to which our approach might contribute in the future. The novel approach presented here is an extension of Walczak et al. [WGT*19, WTN*20] and represents a proof of concept study for a practical approach to MV simulation for DSS.

2. Related Work

In this section, we briefly review the state of the art concerned with the simulation of the MV behaviour. For a more in-depth review, the reader is referred to Gao et al. [GQF*17] or Peirlingk et al. [PCY*21]. We limit the review to material models as well as simulation methods and skip imaging, segmentation and reconstruction techniques that are essential for defining the geometry of the valve. Information on these topics can be found, e.g. in Tautz et al. [TNH*18].

2.1. Material models and parameterization

Most groups dealing with modelling the MV use solid mechanics models to represent the material of the valve. This mainly includes the well-known family of (hyper-)elastic material models [MV06]. These models sometimes include the description of fibre directions, which leads to transversely isotropic or anisotropic material models [GOH06, VCV*07, PSH07, VCV*08, PSSH10, DKS18]. The parameterization of these models is challenging and is usually done using autopsy material from human subjects or material obtained from animal experiments. Rausch et al. [RBK*11] measured in vivo strains of ovine anterior MV leaflets using marker tracking and reported significant temporal, regional and directional variations of leaflet strains. An understanding of these patterns could optimize surgical techniques for improved long-term results in MV repair or device implantation. Prot et al. [PSSH10] performed a series of elastic measurements of the human MV to develop a transversely isotropic hyper-elastic material model for the analysis of the MV. The complexity of the applied fluid mechanics models for blood can vary, e.g. non-Newtonian fluid models such as the Carreau–Yasuda model are used for modelling blood in the article by Vegguth et al. [VBG*18].

For a decision support scenario, there are inherent and practical limitations to model building. As mentioned in the introduction, imaging artifacts possibly limit the quality of the reconstruction of the MV shape. In addition, the mechanical properties of the valve cannot be easily and reliably measured in vivo, e.g. with regard to calcification, nonlinearities or exact fibre directions. Parameterization of complex material models is therefore limited to general data derived from literature (providing limited human or animal data), or assumptions based on individual geometric shapes, e.g. when defining fibre directions. Using an inverse approach for parameter optimization is limited by the complexity of the material model and the computational cost of the simulation method. Since our focus is on creating a plausible visual representation of the MV, we limit ourselves to a simple material model at the expense of biomechanical correctness, e.g. regarding stress–strain relationships. Similar arguments hold for the definition of boundary conditions modelling the surrounding blood as well as the interaction between fluid and valve. While blood flow velocities can be measured to a certain extent at a specific point in time, changing valvular and hemodynamics in different personal conditions caused, e.g. by drugs, stress or physical exercise cannot be predicted from one measurement alone. The range of possible parameter values is simply too large. Therefore, we limit ourselves to a set of forces for driving the valvular dynamics. In summary, while general material properties are known from literature, patient-specific peculiarities are not. Under all uncertainties mentioned in this paragraph, a simplified model with a limited set of parameters may therefore be useful starting point for an image-based approach reproducing the valvular motion depicted in an image series.

Modelling the CT is an integral part and is discussed in literature as well. Almost all publications cited above use some sort of cord model. Prot et al. [PSSH10] have conducted a series of mechanical measurements for parameterizing individual cords from different areas of the valve. Drach et al. [DKS18] have performed a high-resolution micro-computed tomography (μ-CT) scan of a MV specimen. They reconstructed the geometry of the valve as well as the placement of the tendinous cords with regard to the PM. Afterwards, they evaluated the original as well as the simplified cord distributions and simulated the influence of the configurations on material stresses with finite element methods (FEM). Similar work has been conducted by Feng et al. [FQG*18]. Their results indicate that an exact reconstruction of the cord distribution may not be necessary from a point of view where the focus is on the geometric approximation of the closed state. When analysing stresses, however, an exact cord model including all branches is needed.
This has practical implications for patient-specific modelling in DSS. While the tips of the PMs can often be identified in image data recorded in clinical routine, the tendinous cords that connect the PMs to the leaflets are so minuscule that they can only hardly be seen in most image data. When they form larger structures and thus can be seen, they can easily be mistaken for leaflet ends. Based on the aforementioned work, a simplification of the cord tree may be used in a simulation of MV closure. However, useful quantitative data such as material stresses cannot be obtained with simplified models.

2.2. Simulation approaches

Existing approaches for MV simulation can be grouped in three categories. The first category uses a structural mechanics approach and simulates deformation as well as material stresses under load with the FEM [VCV*07, PSH07, VCV*08, PSSH10, BMS12]. The simulation usually begins with the open-state valve and aims at reproducing the closure. Closure is achieved, e.g. by applying a pressure force [PSH07]. Image-focused approaches, on the other hand, use registration and morphing [DKS18] or tracking approaches in connection with a material model to achieve closure. Gribač et al. proposed an image-based tracking approach evaluated with ovine hearts based on the semi-automatic definition of an open-state valve and a finite element model including chordae distribution [GEM*17]. Burlina et al. suggested to start from an open valve surface and track the leaflets to the closed state. Starting from manual initialization, image-based active contours and an FE model with specific material properties mimic the valve dynamics [BSD*10]. The FE model incorporating the effect of the chordae proposed by Burlina et al. requires mechanical and anatomical information not easily extracted from the image data. They also report only limited validation data [BSD*10]. Another approach by Tautz et al. [TWG*19] combines image-based forces determined via gradient vector flow (GVF) field and PBD to deform and track the valve from open to closed state. The method is efficient, does not rely on modelling the chordae and uses only basic material modelling.

The second category has a different focus and uses a single fixed MV state of a specific heart phase for simulating blood flow through the valve [VBG*18, FQG*18]. The simulation methods applied are mostly finite volume- or finite element-based. The resulting pressure distribution, flow rates, velocity vector fields or flow jets are then further investigated.

The third category combines both aspects in a fluid–structure interaction (FSI) simulation [EKR*05, KEC07, CMM*18]. The simulations in the third category are by far the most challenging using today’s methods. In general, classical FSI using an arbitrary Lagrangian–Eulerian (ALE) approach has special demands with regard to mesh adaptation or remeshing, especially when dealing with large deformations. Other approaches use immersed boundary techniques [Pes02, GFQ*17] or approximate the fluid behaviour with methods such as smoothed particle hydrodynamics (SPH) [CMM*18]. Nevertheless, despite model simplifications, running times for FSI simulations are in the order of days or weeks on current workstation hardware [CMM*18]. Computation times like these are not practical in a DSS. Even when transferred to HPC clusters, the effort for simulation setups, meshing and computational cost for a larger number of different computer experiments limit the practical applicability.

Numerous publications deal with modelling virtual interventions such as annuloplasty [SMC*11, AEK*12, CRMK14, KPM*18, NTH*19] or clipping [MVG*12]. However, no publication combines all aspects and no publication claims to reproduce prolapse leading to regurgitation.

3. Material and Methods

For the use in a decision support software, we propose a simplified model for the MV in combination with a simulation method that can be combined with real-time visualization. The goal of this work is to build a simplified and practical model of the MV and provide an efficient method to evolve the reconstructed MV from the open state to the closed state and back, optionally modelling pathologies and virtual interventions. We focus on the visual appearance and not on strict biomechanical correctness. To be applicable in a time-constrained clinical scenario, simulations have to be set up and execute quickly. Certain simplifications have to be made regarding the complexity of the MV apparatus. The approach is relying on basic material assumptions for the valve and is suitable to virtually represent a set of interventions typically performed to treat MVD. This strategy avoids explicit modelling of complex mechanical properties associated with either healthy or pathological valves, e.g. fibre directions, nonlinearities or calcification. As stated above, the mechanical properties of the valve cannot be easily and reliably measured in vivo. The same is true for exact boundary conditions of the surrounding blood. In our approach, the dynamics is driven by a minimal set of external fluid forces representing the surrounding blood. Under the inherent uncertainties explained above, the method’s outcome should evaluate good enough to be valuable as an interactive tool in a DSS scenario. The evaluation will be based on subjective expectations of experts in cardiac surgery as well objective measurements of deviations between image data and simulated valve shapes in the following. The results should be within bounds described in literature. In addition, the approach should also include ways for future extensions that trade simulation time and precision to address certain shortcomings of the current model.

3.1. Initialization of the MV mesh

The geometry of the MV is reconstructed by the segmentation of the open-state valve in one volume of an image series. Open states can most likely be identified and reconstructed correctly. Here, the patient-specific MVs are constructed using the approach published by Tautz et al. [TNH*18]. The open-state valve geometry at time $t_0 = 0$ and the per-phase annulus positions at time frames $t_i$, $i \geq 0$ are semi-automatically defined on an image series. Landmarks for annulus and leaflet tips are placed in multi-planar reconstruction (MPR) planes rotating around the valve opening. An initial mesh triangulated from these landmarks is fitted to an image-derived cost image of leaflet positions to obtain a locally adapted leaflet mesh.
Assuming that the MV is in its open state at time \( t_0 \), the initial triangle mesh \( \mathcal{M}(t_0) \) consists of a set of vertices \( \mathcal{V}_i \), edges \( \mathcal{E}_{ij} \), and triangles \( \mathcal{T}_{ijk} \)

\[
\mathcal{V}_i, \quad i = 0, \ldots, N - 1, \tag{1}
\]

\[
\mathcal{E}_{ij} = (\mathcal{V}_i, \mathcal{V}_j), \quad i, j \in \{0, \ldots, N - 1\}, \quad i \neq j, \tag{2}
\]

\[
\mathcal{T}_{ijk} = (\mathcal{V}_i, \mathcal{V}_j, \mathcal{V}_k), \quad i, j, k \in \{0, \ldots, N - 1\}, \quad i \neq j \neq k. \tag{3}
\]

\( \mathcal{V}^{MA} \subset \mathcal{V} \) represent the annulus vertices. In addition, we define vertices for the PMs \( \mathcal{V}^{PM} \) and edges \( \mathcal{E}_{ij}^{CT} = (\mathcal{V}_i^{CT}, \mathcal{V}_j^{CT}) \) for the CT. The resulting model serves as a basis for the dynamics simulation.

### 3.2. Simulation of the dynamics

To model the evolution of the MV over time from open state \( \mathcal{M}(t_0) \) to closed state \( \mathcal{M}(t_{\text{closed}}) \) and back to open state \( \mathcal{M}(t_{\text{open}}) \), we only use simple material assumptions and a basic fluid approximation. The model is expressed using the extended position-based dynamics (XPBD) variant described in [MMC16, BMM17]. We briefly summarize the method in the following and describe the main simulation loop without repeating the derivation of the numerical method. For this, we refer the reader to relevant PBD literature [MHHR07, BMO*14, MMC16, BMM17].

In PBD, the vertices \( \mathcal{V}_i \) of \( \mathcal{M}(t) \) are associated with a mass \( m_i \), a position \( \mathcal{x}_i(t) \), a temporary position \( \mathcal{p}_i(t) \), and a velocity \( \mathcal{v}_i(t) \). At \( t_0 \), we initialize \( \mathcal{p}_i(t_0) = \mathcal{x}_i(t_0) \) and \( \mathcal{v}_i(t_0) = 0 \). The inward-facing normal \( \mathcal{n}_i(t) \) at \( \mathcal{V}_i \) is defined using the area-weighted average of the normals \( \mathcal{n}_{ijk} \) of the incident triangles \( \mathcal{T}_{ijk} \). The normalized direction vector for edges \( \mathcal{E}_{ij} \) is termed \( \mathcal{n}_{ij}(t) \). Depending on the context in which they are used, different coordinate sets, i.e. \( \mathcal{x}_i(t) \) or \( \mathcal{p}_i(t) \), are used.

To drive the dynamics of the system, a set of external forces

\[
f_i(\mathcal{x}_i, t), \quad i = 0, \ldots, N - 1, \quad l = 0, \ldots, L - 1 \tag{4}
\]

and a set of constraints

\[
C_m(\mathcal{p}_0(t), \ldots, \mathcal{p}_{N-1}(t)), \quad m = 0, \ldots, M - 1 \tag{5}
\]

are used. The forces act on the vertex positions \( \mathcal{x}_i(t) \) and are integrated using Newton's second law of motion whereas the constraint fulfilment is calculated based on the temporary positions \( \mathcal{p}_i(t) \). Each constraint is a function of all or a subset of the vertex positions. Constraints relate vertex positions to each other, can be equalities or inequalities, and allow to describe a simplified model of the valve's material behaviour. This is explained in the next section.

A two-step Verlet-like approach is used for time integration. In a first step, the velocities

\[
\mathcal{v}_i \left( t + \frac{1}{2} \Delta t \right) = \mathcal{v}_i(t) + \frac{\Delta t}{m_i} \sum_{l=0}^{L-1} f_i(\mathcal{x}_i, t) \tag{6}
\]

are integrated \( \forall i \) based on the external forces \( f_i(\mathcal{x}_i, t) \) with time step \( \Delta t \). Subsequently, the unconstrained positions \( \mathcal{z}_i(t) \) are integrated based on the velocities \( \mathcal{v}_i(t + \frac{1}{2} \Delta t) \) defining the temporary positions

\[
\mathcal{p}_i \left( t + \frac{1}{2} \Delta t \right) = \mathcal{z}_i(t) + \Delta t \mathcal{v}_i \left( t + \frac{1}{2} \Delta t \right) \tag{7}
\]

after force integration.

Instead of using a global constraint optimization method, a nonlinear Gauss–Seidel (GS)–type solver is used in PBD to iteratively satisfy each individual constraint \( C_m(\mathcal{p}_0(t), \ldots, \mathcal{p}_{N-1}(t)) \) while adjusting the respective current temporary positions \( \mathcal{p}_i(t + \frac{1}{2} \Delta t) \) only. For the iterative adjustment used in PBD constraint fulfilment, a correction vector \( \Delta \mathcal{p}_i \) is calculated for each individual constraint \( m \) and in each GS iteration \( q = 0, \ldots, Q - 1 \) based on the temporary positions \( \mathcal{p}_{i,m-1} \). The subscript \( m - 1 \) indicates the previous application of constraints \( 0, \ldots, m - 1 \) to temporary position \( \mathcal{p}_i \). In the following, we omit the subscript \( m - 1 \) for readability. The correction vector \( \Delta \mathcal{p}_i \) is chosen such that the equality or inequality constraint \( C_m(\mathcal{p}_0(t), \ldots, \mathcal{p}_{N-1}(t)) \) is fulfilled. Its component \( i \) corresponding to vertex \( \mathcal{V}_i \) reads

\[
\Delta \mathcal{p}_i = \frac{\lambda_{m,q}}{m_i} \nabla C_m(\mathcal{p}_0(t), \ldots, \mathcal{p}_{N-1}(t)). \tag{8}
\]

The gradient \( \nabla C_m \) arises in the linearization of the constraint function (first-order Taylor approximation) used in a Newton–Raphson step. In XPBD, the parameter \( \lambda_{m,q} \) (a Lagrange multiplier for each constraint \( m \), \( q \) indicates the GS iteration) can be interpreted as a measure of the forces acting upon the vertices based on constraint \( m \)

\[
\lambda_{m,q} = \lambda_{m,q-1} + \Delta \lambda_{m,q-1} \tag{9}
\]

\[
\Delta \lambda_{m,q} = \frac{-C_m(\mathcal{p}_0(t), \ldots, \mathcal{p}_{N-1}(t)) - \frac{\Delta t}{m_i} \lambda_{m,q-1}}{\nabla \lambda_{m,q} \cdot \nabla \lambda_{m,q}} \tag{10}
\]

The largest correction vectors occur during the first few iterations. The number of iterations \( Q \) can also be limited in case a certain tolerance is met, i.e. the value is chosen comparing the results for different values of \( Q \). Typically, the largest correction vectors occur during the first few iterations. The complete derivation of the XPBD method can be found elsewhere.
After the iterative evaluation of all constraints, the velocities
\[ \tilde{\mathbf{v}}(t + \Delta t) = \frac{\mathbf{p}(t + \Delta t) - \overline{\mathbf{v}}(t)}{\Delta t} \]  
and the positions
\[ \overline{\mathbf{x}}(t + \Delta t) = \mathbf{p}(t + \frac{1}{2} \Delta t) \]  
are finally updated in the second Verlet-like step. A single iteration of the simulation consists of one complete time-integration step and the inner GS loop with \( Q \) iterations. The method is applied until a steady state is reached. We define convergence when the sum of all positions changes between two time steps is lower than a constant \( \epsilon \) (chosen by the user)
\[ \sum_{i=0}^{N-1} |\overline{\mathbf{x}}(t + \Delta t) - \overline{\mathbf{x}}(t)| < \epsilon. \]  

3.3. Modelling the material of the MV

Since XPBD does not provide a way to define a material model in a classical continuum mechanics sense, the mechanical properties of the valve material need to be approximated using the constraint mechanism. Complex behaviour is created with simple building blocks such as the elements of a triangular mesh. The elements are combined by different constraints. The goal is a convincing visual deformation of the MV from open to closed state rather than strict biomechanical correctness. We neglect some properties of real MV mentioned in Section 2. The main ideas in no particular order are: the area of the leaflets should be conserved. Deformation of the leaflets should result in a smooth surface. Stretching of triangles or bending between incident triangles should be minimized. Contact has to be handled as well. To achieve this behaviour, we define the material model using four different types of constraints. First, we define stretching constraints
\[ C_0(\mathbf{p}(t), \mathbf{p}(t)) = |\mathbf{p}(t) - \mathbf{p}(t)| - l_0 = 0, \quad i \neq j \]
for each edge \( E_{ij} \) that limit movements in the direction \( \mathbf{n}_{ij}(t) \) based on the initial length \( l_0 = |\mathbf{p}(t_0) - \mathbf{p}(t_0)| \) of the edge \( E_{ij} \) at time \( t_0 = 0 \). This can be interpreted as a spring connecting to two vertices. Second, we use bending constraints
\[ C_1(\mathbf{p}(t), \mathbf{p}(t), \mathbf{p}(t)) \]
\[ = \arccos(\mathbf{n}_{ij}(t) \cdot \mathbf{n}_{ij}(t)) = \phi_0 = 0, \quad k \neq l \]
that preserves the dihedral angle \( \phi = \arccos(\mathbf{n}_{ij}(t) \cdot \mathbf{n}_{ij}(t)) \) between the normals \( \mathbf{n}_{ij}(t) \) and \( \mathbf{n}_{ij}(t) \) of the two neighbouring triangles \( T_{ik} \) and \( T_{il} \) either to the initial dihedral angle \( \phi_0 = \arccos(\mathbf{n}_{ij}(t_0) \cdot \mathbf{n}_{ij}(t_0)) \) or to \( \phi_0 = 0 \). This can be interpreted as a spring connecting two triangles maintaining the angle between them, i.e. to stay flat if \( \phi_0 = 0 \). Third, we apply area conservation constraints
\[ C_2(\mathbf{p}(t), \mathbf{p}(t), \mathbf{p}(t)) \]
\[ = |(\mathbf{p}(t) - \mathbf{p}(t)) \times (\mathbf{p}(t) - \mathbf{p}(t))|^2 - a_0 = 0 \]
for each triangle \( T_{ijl} \) that preserve the triangle area of the initial state \( a_0 = [(\mathbf{p}(t_0) - \mathbf{p}(t_0)) \times (\mathbf{p}(t_0) - \mathbf{p}(t_0))]^2 \). The associated parameters for the three constraint types are \( k_0, \phi_0 \) and \( k_2 \) with \( k_0 \in [0, 1] \). In addition, we have to set the compliance parameters \( a_{(0,1,2)} \in [0, 1] \) used in XPBD.

To model contact of the leaflets, e.g. in the coaptation zone, we define collision constraints \( C_i(\mathbf{p}(t), \mathbf{p}(t), \mathbf{p}(t)) \geq 0, \quad i \neq j \neq k \neq l \) with an AABB-type collision detection, ray-triangle intersection tests [MT97] and a collision response modifying the temporary positions \( \mathbf{p}_{(i,j,k,l)} \) in their respective opposite directions to resolve the collision, i.e. \( \mathbf{n}_{ij}(t) \) for \( V_i \) and \( \mathbf{n}_{ij}(t) \) for \( T_{ijl} \). The distance \( h \) is used to determine if a collision with vertex \( V_i \) has occurred on either side of the triangle \( T_{ijl} \) or not. This implies a valve thickness of \( 2h \). The constraint parameters are \( k_3 = 0 \) and \( \alpha_3 = 0 \). The existence of collision constraints is checked before the GS loop. If collisions are detected, the collision constraints for the respective elements are added to the solver. More details on the self collision constraint can be found elsewhere [MHHR07, MMC16].

3.4. Dirichlet boundary conditions

Annulus and PM positions for all phases of the series are included in our model. To reflect the fixed position of the annulus, the annulus vertex positions are added as Dirichlet boundary conditions on the evolution, i.e. for all indices \( i \) that belong to an annulus vertex \( V_i^{\text{ann}} \), we set \( \mathbf{x}(t) = I(t, \mathbf{x}(t), \mathbf{x}(t+1)) \). \( I \) represents an interpolation method that interpolates vertex positions between two annuli at \( t_0 \) and \( t_{n+1} \) at time \( t_0 \leq t \leq t_{n+1} \). The PMs are simply approximated by their tip positions and are represented by additional vertices \( V_i^{\text{PM}} \). Similar to the annulus, the positions serve as Dirichlet boundary conditions and are interpolated between states \( t_0 \) and \( t_{n+1} \).

3.5. Tendinous cords

To complete the MV model, a definition of the tendinous cords for the open state is needed. Since we are interested in modelling valve closure and not concerned with exact stress distributions in the leaflets, we follow previous works [DKS18, FQ+18] and simplify the whole chordae tree by a set of additional edges \( e_{ij}^{ct} \), which are defined between either manually or heuristically defined PM vertices \( V_i^{\text{PM}} \) and manually defined leaflet vertices \( V_j \) (cf. Figure 2(a)). The cord edges can be differentialed in primary and secondary depending on the position of the leaflet vertex. Edges ending close to the orifice (the loose ends of the leaflets) are labelled primary, edges ending on the belly of the leaflet are labelled secondary. For each such edge, we add an additional distance constraint \( C_3(\mathbf{p}(t), \mathbf{p}(t)) \) with length-dependent values for the parameter
\[ k_0^{ct} = \begin{cases} 0, & |\mathbf{p}(t) - \mathbf{p}(t)| - l_0 < 0 \quad (compression), \\ 1, & |\mathbf{p}(t) - \mathbf{p}(t)| - l_0 > 0 \quad (stretching). \end{cases} \]

This results in different behaviours for the two cases where the edge is compressed or stretched. When not under load, they can move freely. The compliance parameter for the modelled uniaxial deformation of the cords in direction \( n_{ij}^{ct}(t) \) corresponds to the
inverse Young’s modulus \( \alpha_{CT}^{E} = 1/E \) [MMC16]. Therefore, XPBD in principle allows to model linearly elastic behaviour for the cords based on physical measurements of \( E \). In this case, we choose to set \( \alpha_{CT} = 0 \). Using this parameterization, the tendinous cords cannot be stretched longer than their initial lengths \( b_0 \).

In case the PMs cannot be reliably determined in the image data, we propose a heuristic to initially define the simplified tendinous cords (cf. Figure 2(a)). As a first step, the user defines vertices \( V_i \) on the valve that correspond to the primary and secondary cords. As a second step, we fit a plane to the annulus, e.g. using principle component analysis (PCA). The normal \( \mathbf{n}^{AP} \) of the annulus plane (AP) doubles as a principal flow direction vector (cf. Equation (22)). Using the position information of the leaflet vertices and the AP, we define the model cords. We assume just two PM tips \( V_{PM}^{0,1} \) (postero-medial and anterolateral) in the following. The set of user-defined leaflet vertices is divided into two subsets connecting half of the vertices of the anterior and posterior leaflet (lighter coloured spheres in Figure 2(a)) to one PM tip and the other half (darker coloured spheres in Figure 2(a)) to the other tip. Light- and dark-coloured spheres are used for indicating the connectivity to one of the PM tips in Figure 2(a), while red- and blue-coloured spheres are used for indicating the connectivity to the other PM tip. Light- and dark-coloured vertices are connected to the other PM tip. The annulus plane (AP) is shown in blue. The AP is used for initial length determination of all \( E_{ij}^{CT} \) (see text). (b) Overview of the complete simulation setup (cf. Section 3). BC refers to boundary condition.

3.6. Modelling the fluid

In our model, the surrounding blood is not explicitly modelled, because FSI typically involves huge computational effort. To approximate the behaviour of the surrounding fluid and, coupled to that, the behaviour of the MV, we use a set of different and easy to evaluate external force fields, which depend on a consistent normal orientation. The main flow features are the direction vectors \( w(\vec{x}, t) \) as well as the pressure differences \( p(\vec{x}, t) \) between ventricle and atrium. A pressure force

\[
\vec{f}_0(\vec{x}, t) = p(\vec{x}, t) \cdot \mathbf{n}(t)
\]  

(19)

with pressure difference \( p(\vec{x}, t) = \pm p \) is applied in normal direction \( \mathbf{n}(t) \). \( p \) is set by the user (chosen by experimentation), and the sign changes depending on the current heart phase. This results in a movement of the leaflets towards each other and, because of the annulus boundary, towards the atrium in systole. In diastole, the leaflets move away from each other and towards ventricle wall. Initial tests using a pressure force show that bending constraints (maintaining the angle between triangles) and collision constraints (introduced by colliding leaflet ends far below the AP) interfere with each other. As a result, unrealistic coaptation shapes are formed. Therefore, we introduce additional forces for compensation. We define

\[
\vec{f}_1(\vec{x}, t) = \vec{w}(\vec{x}, t),
\]  

(20)

\[
\vec{f}_2(\vec{x}, t) = (\vec{w}(\vec{x}, t) \cdot \mathbf{n}(t)) \cdot \mathbf{n}(t).
\]  

(21)

The first force field \( \vec{f}_1 \) pushes the vertices \( V_i \) in the direction \( \vec{w}(\vec{x}, t) \), while the second force field \( \vec{f}_2 \) moves the vertices \( V_i \) depending on pressure changes.

Figure 2: (a) Setup for defining the chordae tendineae. The papillary muscle tips \( V_{PM}^{0,1} \) are connected to the leaflet vertices \( V_i \) via edge \( E_{ij}^{CT} \) (grey lines). In this case, light blue and light red coloured vertices connect to one papillary muscle (PM) tip, while the darker blue and darker red coloured vertices are connected to the other PM tip. The annulus plane (AP) is shown in blue. The AP is used for initial length determination of all \( E_{ij}^{CT} \) (see text). (b) Overview of the complete simulation setup (cf. Section 3). BC refers to boundary condition.
on the similarity of the normal direction \( \vec{n}(t) \) and the force vector \( \vec{w}(\vec{x}, t) \) at position \( \vec{x}(t) \). The vectors \( \vec{w}(\vec{x}, t) \) are defined as

\[
\vec{w}(\vec{x}, t) = \begin{cases} 
    w \cdot \vec{r}^{\text{AP}}, & \text{(systole),} \\
    -w \cdot \vec{r}^{\text{AP,}} & \text{(diastole)}
\end{cases}
\]

with \( w \) being a user-defined value that determines the magnitude of the force. All forces combined, this represents a basic form of a force field, which pushes the leaflet vertices in the respective flow directions. In addition to the pressure force, we use a principal direction vector that changes its sign depending on the heart phase. The result is a movement towards atrium during systole or a movement towards ventricle during diastole. Using this setup, we exert more force, if normal and force vector are already aligned. We exert less force, if normal and force vector are perpendicular, e.g. in the coaptation zone where the leaflets meet. Because of the local force differences, bending is encouraged in systole and a coaption zone is formed. The influence of the bending constraints, forcing the leaflets to stay flat, is countered.

The simulation parameters \( w \) or \( p \) themselves do not directly correspond to measured physical quantities and must be set up relative to each other and to the parameterization of the constraints. There are exceptions however. Distance is measured in [mm] and compliance can be expressed in [m\(^2\)/N] for distance constraints resembling springs. This has not been shown to hold for other compliance parameters. A connection of simulation time to real time can be established via a scaling factor \( t_{\text{scale}} \). This scaling factor is calculated based on the frame times of the image series as well as the number of frames between open and closed state of the anatomy.

A complete overview of the simulation setup including constraints, forces and Dirichlet boundary conditions can be found in Figure 2(b). In addition to the MV mesh and the definition of the cord end points, the user has to configure the following parameters:

- vertex mass \( m \),
- time step \( \Delta t \) and number of inner GS iterations \( Q \),
- constant \( \epsilon \) for definition when steady state is reached,
- compliance parameters \( k_{(0,1,2)} \) and weights \( k_{(0,1,2)} \),
- pressure \( p \) and flow magnitude \( w \) as well as
- collision distance \( h \).

Because of cross-influences, these parameters have to be set experimentally.

3.7. Left heart

In cases where a mesh representation of the LH is available, it can be included in the simulation. For consistency, it should at least follow the annulus movement (Dirichlet boundary condition), but it should not take part in the integration of the forces. The inclusion of the LH might be useful for visualization purposes or checks for collisions between leaflets and wall of the heart. Since blood flow is only approximated by a set of forces \( f_{\vec{r}} \), ventricular movement is not modelled in this work, but can be integrated in the future.

3.8. Virtual interventions

Using the methods described above, we are able to parameterize the model based on the preoperative images series and perform simulations that match the anatomy seen in the images. The parameterized model consisting of the material, forces and chordae is the basis for the virtual intervention steps. The set of modelled interventions presented here comprises annuloplasty [SMC*11, AEK*12, CRMK14, KPM*18, NTH*19] and the placement of clips [MVG*12], but other scenarios such as cord manipulation or PM relocation are possible as well [WTN*20]. The annuloplasty constrains the shape of the annulus to the chosen annuloplasty ring, which is represented by a new Dirichlet boundary condition for the annulus. PM tips are described by Dirichlet boundary conditions as well. In case of a cord manipulation, cords can be added or removed as well as elongated or shortened by changing the rest length of the edge [WTN*20].

Clips are placed by adding additional edges and additional distance constraints between the vertices of anterior and posterior leaflets. The distance constraints are parameterized like the ones for the tendinous cords but with zero initial length. Clip placement can be done either manually or automatically. The latter is especially useful for larger studies where automation is key. In this case, we suggest using a cylindrical coordinate mapping of the valve for defining clip positions at specific locations. The \((x, y, z)\) coordinates are transformed into \((r, \theta, z)\) coordinates with \( r \) being the radius, \( \theta \) the angle and \( z \) height. The \( z \) axis should be aligned with the AP normal \( \vec{r}^{\text{AP}} \). One of the commissure positions corresponds to \( \theta = 0 \). Using the arc length parameterization, clip end positions can be set consistently at specific radial locations \( \theta \) on both leaflets between the commissures. Clip vertices can be found by choosing a \( z \)-offset to the orifice contour. Since real clips act upon a part of the leaflet area, connecting single vertices of opposite leaflets may not be enough. The physical extent of a clip can be approximated by connecting multiple vertices using the aforementioned clip vertices as seeds, e.g. for a simple breath-first search (BFS) on the valve mesh. A subset of the vertices found by BFS at a distance smaller than a specific value can be used to define a set of distance constraints for a specific clip size.

4. Results

We applied the method described in Section 3 in two scenarios. At first, we applied the methods to ten reconstructed MVs. The ten data sets included five normal and five pathological valves with MR. We determined one material and fluid force parameter set for all cases, but, in contrast to Walczak et al. [WGT*19], we scaled the initial cord lengths for each model using the interactive editor described in Walczak et al. [WTN*20]. We simulated the closure and compared cross-sections to expert annotations of the respective TEE image sequences showing the closed state. Second, the parameterized models were used for the simulation of virtual interventions. Annuloplasty and clipping were performed for one data set. The cases were processed and the simulations were performed using a prototypical implementation based on the MeVisLab platform [RBH*11]. This implementation included all methods needed for setting up the simulation.
4.1. Parameterization and evaluation

The valves were defined based on TEE image sequences, five of which (cases A1–5) were acquired from patients with normal MVs, while the remaining five (cases B1–5) were acquired preoperatively from patients with MR. All images were acquired using a GE Vivid E9 system (GE Healthcare, Chicago, IL, USA). The spatial resolution in planes perpendicular to the acquisition direction ranged from 0.857 to 1.305 mm, and the resolution along the acquisition direction ranged from 0.591 to 0.9 mm. The frame time (duration between frames) was in the range from 24.2 to 49.4 ms (no frame delay), and the number of time frames per sequence ranged from 21 to 43. The number of frames from open to closed state ranged from 3 to 9. This equals 114.8 to 228.6 ms (A1–5 114.8–228.6 ms, B1–5 139.6–203.2 ms).

Two domain experts with multi-year experience in cardiac surgery used the approach from Tautz et al. [TNH*18] to define valve surfaces in one volume per image sequence that shows the valve in the open state. Annulus positions were marked in all phases between the open and the closed state. For the definition of the cords, the leaflet vertices were manually annotated, and the heuristic was used for determining the PM tips, setting up the edge distance constraints and initial lengths. Afterwards, the cord lengths were scaled (individually or in groups) using the interactive editor [WTN*20] to provide a good visual match to the image data.

The simulations were performed using the parameters given in Table 1. We tried to find a single common set of basis forces and material parameters for all cases. The reasoning behind this was that normal in vivo pressure differences are quite similar across healthy persons at rest. The same argumentation holds for MV material parameters of healthy subjects. The literature values for the Young’s modulus of the MV provided a good starting point for defining XPBD’s compliance parameters. Because our model includes a mixture of different constraints, we used the literature value of $E \approx 10^4$ N/m² [GR72] measured for MVs as an initial value. Material differences, e.g. regarding fibre directions or calcification, could not be derived from image data, and, therefore, were not modelled. The final parameterization given in Table 1 was experimentally found and suited most data sets in a way that MV closure was achieved, but no excessive leaflet billowing was observed. With an unbalanced parameterization, excessive billowing resulting in balloon-like protrusion of the leaflets into the atrium could be observed. This was the case with too large forces and compliance. The opposite was true for stiff material parameterization for which the valve would not move or deform properly. Using the final parameterization, the results matched physiological expectations of the experts as well as the range of compliance values measured for different load cases of MVs [GR72]. More inner loops were required to achieve a correct collision resolution at times where the external forces were too large. In the cases presented and with the forces used here, three iterations of the inner loop were sufficient to fulfill all constraints to a certain tolerance. The reader is referred to Table 2 where we present results for area conservation at $Q = 3$ GS iterations. The largest difference in the valve surface areas before and after deformation was $\approx 4$ mm². For most cases, the difference was less than $2$ mm² or 0.2%. We did not find any significant difference in area conservation for larger values such as $Q = 20$. In summary, the total leaflet area was conserved during deformation with just three GS iterations. The same holds for other constraints such as the distance constraints introduced in the cord model. Since $\alpha = 0$, the distance constraints basically cannot be stretched longer than the rest length of $l_0$.

| Parameter | $k_0$ | $k_1$ | $k_3$ | $m$ | $w$ | $\Delta$ | $\alpha_{[0, 1, 2]}$ | $h$ | $Q$ | $\epsilon$ |
|-----------|------|------|------|----|----|----------|----------------|----|----|-------|
| Value     | 0.1  | 0.1  | 1.0  | 1.0| ±50.0| ±100.0   | 0.005          | 1e⁻⁵| 0.5 mm | 0.5 mm |

Each simulation was run until a steady state was reached. The simulation time per case was in the order of minutes (longest $< 2$ min using 3 GS iterations/step; values for 20 GS iterations/step can be found in the article [WGT*19]). The time needed for the manual definition of the valve, cords, annuli and parameters was much larger [TNH*18]. Once set up, simulations could be executed quickly. Further details on the simulation input data and the surface meshes can be found in Table 2. The interactive frame rates that could be achieved with the prototypical implementation on a higher-end notebook equipped with current mobile workstation hardware ranged from 40 to 47 frames per second using three inner GS loops (values for 20 inner loops can be found in the article [WGT*19]). These numbers are based on a prototypical implementation running on the CPU.

The duration of the simulated MV closure ranged from 3.54 to 14.84Δt in simulated time. The scaling factor $l_{scal}$ ranged from 12.0 to 41.9Δt ms. For the healthy cases (A1–5), the scaling factor ranged from 21.8 to 41.9Δt ms, while for the MR cases (B1–5), the scaling factor ranged from 12.0 to 24.0Δt ms.

We compared the steady-state results with the annotations of the images series that depicted the closed valve. With the chosen parameterization, MV closure was achieved in all cases. For a detailed quantitative analysis of the deviations, we used manually drawn tracings of the closed valve. To reduce the amount of annotation work, we limited the annotations to every $n$th image or to certain images that the two experts picked. More details can be found in Walczak et al. [WTN*20]. Using this annotation, we compared the distances of the tracings to the surface of the simulated closed valve. We computed the minimal distance to all triangles for all points in the expert annotations. This yielded the deviations provided in Table 2. The median deviation ranged from 0.87 to 1.85 mm, the maximum deviation ranged from 4.47 to 7.94 mm. The maximum deviations were no larger than ±6 mm for most normal cases. Maximum deviations for pathological cases were no larger than ±8 mm. On average, the deviation was in the order of one to two times the voxel size with a standard deviation in the order of one voxel size. Detailed results for each case can be found in Table 2.

In addition to measuring the deviations, the experts classified the results for the pathological cases by ranking them in three quality levels using an adapted rating scheme of the article [WGT*19]: A simulation result was ranked as satisfactory if the closure as well as the pathologies (prolapse, regurgitation orifice) were reproduced, and the simulated outcome resembled the anatomy seen in the images with minimal measured deviations; passable if the closure was...
Table 2: Case-specific input mesh statistics (left columns), algorithm performance and iterations (It) until convergence (middle columns) with $Q = 3$ inner Gauss–Seidel (GS) loops, and deviations to the expert annotations. Cases A refer to normal mitral valves (MVs), cases B to pathological MVs. Time refers to the time for one complete simulation step. CD stands for the time used for collision detection and Conv for the number of iterations until convergence. Area is given at the time of convergence. The value provides a reference of how well the area constraints preserve the surface area of the valve (compare with values in Area column). MinD, MedD, MaxD and AvgD refer to minimum, median, maximum and average deviation from the manually determined close valve contours.

| Case | Vertices | Edges | Faces | Area [mm$^2$] | Time [ms/It] | CD [ms/It] | Conv It | Area [mm$^2$] | MinD [mm] | MedD [mm] | MaxD [mm] | AvgD [mm] |
|------|----------|-------|-------|--------------|-------------|-----------|---------|--------------|----------|----------|----------|----------|
| A1   | 1365     | 3934  | 2569  | 1707         | 19          | 2         | 1068    | 1706         | 0.00     | 1.00     | 4.47     | 1.23 ± 0.92 |
| A2   | 1643     | 4759  | 3116  | 2076         | 20          | 3         | 1052    | 2073         | 0.00     | 1.72     | 5.99     | 1.96 ± 1.37 |
| A3   | 2252     | 6547  | 4293  | 2780         | 21          | 4         | 1648    | 2784         | 0.01     | 1.85     | 7.33     | 2.21 ± 1.51 |
| A4   | 1491     | 4311  | 2820  | 1860         | 19          | 2         | 708     | 1858         | 0.01     | 0.87     | 4.54     | 1.12 ± 0.94 |
| A5   | 2066     | 5813  | 4293  | 3805         | 21          | 3         | 920     | 2515         | 0.01     | 1.31     | 4.93     | 1.50 ± 1.07 |
| B1   | 2699     | 7834  | 5133  | 3312         | 22          | 4         | 1692    | 3311         | 0.00     | 1.77     | 7.39     | 2.06 ± 1.60 |
| B2   | 2825     | 8188  | 5359  | 3531         | 22          | 5         | 1552    | 3531         | 0.00     | 1.66     | 7.94     | 2.11 ± 1.70 |
| B3   | 2344     | 6758  | 4408  | 2860         | 21          | 4         | 1996    | 2860         | 0.00     | 1.77     | 5.96     | 1.93 ± 1.31 |
| B4   | 1830     | 5305  | 3475  | 2312         | 20          | 3         | 800     | 2310         | 0.01     | 1.74     | 4.86     | 1.88 ± 1.13 |
| B5   | 3008     | 8778  | 5770  | 3855         | 23          | 5         | 2968    | 3855         | 0.00     | 1.55     | 5.54     | 1.77 ± 1.26 |

Figure 3: Example for a satisfactory simulation (case B3). From left to right: (a) open valve, tendinous cords and leaflet vertices shown from atrium, (b) open valve, view of posterior leaflet, (c) tracing (blue) and cross-section of closed valve (orange) overlaid on image data (posterior leaflet on the left, anterior leaflet on the right), (d) closed valve shown from atrium, (e) closed valve. Views (a) and (d) as well as (b) and (e) show the valve in the same orientation. The pathology (prolapse leading to MR) is reproduced in the right location. The regurgitation orifice cannot be seen in these views, it is shown in Figure 9.

Figure 4: Example for a satisfactory simulation (case B5). From left to right: (a) open valve, tendinous cords and leaflet vertices shown from atrium, (b) open valve, view of posterior leaflet, (c) tracing (blue) and cross-section of closed valve (orange) overlaid on image data (posterior leaflet on the left, anterior leaflet on the right), (d) closed valve shown from atrium, (e) closed valve. Views (a) and (d) show the valve in the same orientation, (b) and (e) show the valve rotated 180 degrees to be able to see the regurgitation orifice. The pathology (prolapse leading to MR) is reproduced in the right location, see view (e) (off-centre to the left next to the red star symbol). The regurgitation orifice is shown in more detail in Figure 9.

reproduced, but there were larger than average deviations from the anatomy seen in the images or the pathologies; or unsatisfactory if the valve closure was reproduced, but the deviations of the leaflets were comparatively large or the pathologies were not reproduced (no regurgitation orifice). Using this rating scheme, three of the processed pathological cases were ranked as satisfactory, one as passable, and one as unsatisfactory. The results for each pathological case are shown in Figures 3–7. Prolapse leading to MR was reproduced in cases B2, B3, B4 and B5 (cf. Figure 9), but not in B1 where the valve was fully closed and only leaflet billowing could be observed. An exemplary result for a normally closing valve can be seen in Figure 8. The experts stated that all the leaflet surfaces generally exhibited folding patterns comparable to the ones observed in in vivo valves and found them realistic (cf. Figure 9).

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Figure 5: Example for a satisfactory simulation (case B2). From left to right: (a) open valve, tendinous cords and leaflet vertices shown from atrium, (b) open valve, view of posterior leaflet, (c) tracing (blue) and cross-section of closed valve (orange) overlaid on image data (posterior leaflet on the left, anterior leaflet on the right), (d) closed valve shown from atrium, (e) closed valve. Views (a) and (d) as well as (b) and (e) show the valve in the same orientation. The regurgitation orifice is shown in the middle of view (e) next to the red star symbol. Although the prolapse is reproduced in the right spot, the approximation in other areas is slightly worse than in the other satisfactory cases.

Figure 6: Example for a passable simulation (case B4). From left to right: (a) open valve, tendinous cords and leaflet vertices shown from atrium, (b) open valve, view of posterior leaflet, (c) tracing (blue) and cross-section of closed valve (orange) overlaid on image data (posterior leaflet on the left, anterior leaflet on the right), (d) closed valve shown from atrium, (e) closed valve. Views (a) and (d) as well as (b) and (e) show the valve in the same orientation. The pathology (prolapse leading to MR) is reproduced in the right location. Because of the overlapping leaflets and the annulus shape, the regurgitation orifice cannot be seen in these views.

Figure 7: Example for an unsatisfactory simulation (case B1). From left to right: (a) open valve, tendinous cords and leaflet vertices shown from atrium, (b) open valve, view of posterior leaflet, (c) tracing (blue) and cross-section of closed valve (orange) valve overlaid on image data (posterior leaflet on the left, anterior leaflet on the right), (d) closed valve shown from atrium, (e) closed valve. Views (a) and (d) as well as (b) and (e) show the valve in the same orientation. Although the approximation is good, the pathology is not reproduced. The leaflets are too large and block each other because of the collision handling.

For comparison, we provide an image showing an endoscopic view of an in vivo valve during minimal-invasive heart surgery with arrested heart (cf. Figure 9). The valve is shown next to the simulation results at a similar viewing angle (viewed from atrium to ventricle).

4.2. Virtual interventions

This section briefly showcases the results of virtual interventions that were performed with the proposed methods on preoperative data. These give an impression of what analyses may be done in future DSS. In Figure 10, images of a simulation result of an open valve and a closed valve with ring annuloplasty are shown. The altered LH geometry was generated on a preoperative LH reconstruction using the purely geometric approach by Neugebauer et al. [NTH*19]. For deformation of LH, a CT-scanned ring shape with 38-mm diameter was used. The LH mesh was included in the MV simulation. The MV was deformed interpolating the annulus from its normal shape to the ring shape. Notice the bulging in the open valve introduced due to the adaption of the ring shape. In its preoperative state, the valve used in this showcase did not close properly. In the simulated postoperative state using the 38-mm ring, the former incompletely closing valve was completely closed. To judge MV stenosis, the geometric orifice area was assessed in the open state. One could compare different ring sizes and their geometric orifice area.
Figure 9: Visual comparison of a simulated valve closure (top row A1–5, middle row B1–5) and an endoscopic view on a mitral valve during minimal-invasive heart surgery with arrested heart (bottom row, left side, different patient). Note the folding and bulges of the leaflet surface in both the surface models and the in vivo valve. The shape of the simulated prolapse of cases B3 and B5 as well as the improper coaptation is depicted in the image located in the bottom row, in the middle (B3) and on the right side (B5). These results match the indication for surgery of the MR patients.

Figure 8: Exemplary result for a normally closing valve (case A5). From left to right: (a) open valve, tendinous cords and leaflet vertices shown from atrium, (b) open valve, view of posterior leaflet, (c) tracing (blue) and cross-section of closed valve (orange) overlaid on image data (posterior leaflet on the left, anterior leaflet on the right), (d) closed valve shown from atrium, (e) closed valve. Views (a) and (d) as well as (b) and (e) show the valve in the same orientation. There is a good approximation of the expert annotations.

5. Discussion

The simulation outcome heavily depends on a multitude of aspects, which makes an evaluation rather difficult. At first, the manual annotations performed by the experts are heavily influenced by image quality. Imaging artifacts like signal dropout or shadowing lead to uncertainties that need to be filled in with expert knowledge. Because of the subjective nature of this process, manual annotations and segmentations of the anatomy are susceptible to possible deviations. In particular, the differentiation between leaflet and the transition to the CT is a challenge in TEE images. The same holds for structures that are close to each other, such as leaflets and heart wall in the open state or coaptating free leaflets ends directly below

areas (not shown here). Using our system, an 38-mm ring might be identified as a possible treatment option.

Figure 11 shows images of simulations with a single clip in different positions at diastole. The clips were placed automatically connecting opposite vertices at specific positions along each leaflet. The positions were chosen using a cylindrical mapping of the open valve. Seed vertex positions for the clips were placed at $\left(\frac{1}{2}, \frac{1}{2}, \frac{2}{3}\right)$ are length of each leaflet at a distance of 3 mm to the orifice. Additional vertices were added using BFS with a distance smaller than three. Using this set of vertices, the distance constraint end points were defined. The clips were placed in preoperative MV geometry and the remaining geometric orifice area could be assessed simulating diastole. As an alternative to the automatic placement, an interactive placement was possible as well, see the article [WGT*19] for examples. By choosing a position and simulating valve closure, the user is able to visualize the outcome and judge the chosen position of the clip, e.g. with regard to prolapse removal in systole or remaining orifice area in diastole.
the atrium facing leaflet parts in closed state. In addition, since we are reconstructing a surface mesh and not a volumetric structure, annotations have to carefully trace the centre line of the structure in the rotated MPR view. Temporal and spatial resolution are crucial, especially when imaging rapidly moving leaflets and small-scale structures like CT. The simulation has a much higher temporal resolution ($t_{\text{scale}} \in [12.0\Delta t, 41.9\Delta t]$ ms) than the recorded images. This impedes comparisons between the simulated dynamics and single individual images. A comparison would involve temporal averaging.

Second, the boundary conditions at the time of image acquisition are usually unknown. Simulation results can only be compared to one example. Comparisons of the complete dynamics remain difficult. The superposition of forces in Section 3.6 results in time- and position-dependent force fields that change with heart phase and geometry but do not necessarily resemble the exact fluid flow at image acquisition. They are designed to close the valve, but cannot reflect real blood flow properties. The same holds for material parameter definitions (calcification, nonlinearities, fibre directions), cord distributions and cord length definition. Since we are dealing with diseased MVs, the underlying pathology plays a role as well. What causes the disease is typically not fully known and cannot be inferred from the image data alone. There are a multitude of effects and even combinations of them, e.g. a widened annulus might not necessarily be combined with ruptured or elongated chordae to result in prolapse and MR. In the end, an evaluation judges not just the quality of the model and simulation method used but depends on all of the aspects and uncertainties above. Because of this, we are limiting the discussion here mostly to two aspects: how well is the closure of the valve and how well is the pathology reproduced by our approach compared with the annotated image data. The latter is an even harder task than the former.

The results show that our approach is able deform all ten models from open to closed state. The deviations between simulated results and expert annotations are similar to other results found in literature for finite element models. Burlina et al. [BMS12] report an average deviation of $1.86 \pm 1.72$ mm, which is comparable, but with higher mean and maximum values compared to our results in Table 2. According to Mansi et al. [MVG*12], the average deviation is $1.49 \pm 0.62$ mm, which is comparable to our results. Although we are not using the most complex continuum mechanics models available, we are able to achieve a good accuracy. This indicates that Dirichlet boundary conditions as well as cord structures have a significant influence on the result. The role of the material model has to be analysed further. Please note that the values elsewhere [BMS12, MVG*12] were calculated for normal valve closure only.
and were achieved with a different methodology. In addition, overlays of mesh cross sections on the image data were only provided in one of the two sources.

In pathological cases, we found no other values to compare our results to. Looking at the satisfactory case B3 in Figures 3 and 9 (bottom row, middle image) for instance, the prolapse into the atrium and the associated regurgitation orifice is reproduced in the right location. However, there are deviations in the flatness of leaflet shape in Figure 3(c). When compared to the manual annotations, the deviations are between the other two satisfactory data sets B2 and B5 (cf. Table 2).

In the satisfactory case B5 (shown in Figures 4 and 9 (bottom row, right image)), changes in the parameterization of the cords compared to the results in the article [WGT*19] led to a very good approximation of the closed valve as well as the regurgitation orifice position and size. This was achieved using the interactive editor [WTN*20]. The general shape of the simulated closed valve B5 matches the annotations and the form seen in the image data well. Remaining deviations may be attributed to cord placement or material modelling.

This change in the cord length parameterization using the interactive editor is worthy of being discussed a bit more in depth. Since our simulation is paired with a real-time visualization, the cord lengths can be altered at run time and changes to the MV geometry can be seen immediately. In fact, changes to every parameter can be made while the simulation is running. Because of all uncertainties involved in modelling in vivo valves, this is a strong point for our approach. It can be set up compared to image information. Moreover, the valve can be opened and closed by using the different force fields to reproduce the results multiple times in a row. This way, changes, e.g. in the dynamics of closing and prolapse development can directly be studied on screen. This enables the user to come up with a (cord) parameterization that fits to the anatomy seen in the image overlay well. Doing so while the simulation is running was not possible before. Another aspect concerns the fact that cords are modelled. Since their exact configuration is unknown and cannot be seen in the image data, the user has to experiment and try to find a parameterization. In the end, the user could assign cords to every vertex and pretend to have come up with a perfect solution. This is not what we are trying to achieve. In combination with our material modelling and collision handling, the cords in the vicinity of the prolapse can be either left out or can be assigned a length that is at least as long as necessary. If it is longer, the material modelling will limit overall stretching or bending of the leaflet and may not even utilize the maximum length assigned. We think this is a valid approach to modelling prolapse and the idea is supported, e.g. by Mansi et al. [MVG*12] for optimizing the approximation of normal closure.

The satisfactory case B2 (cf. Figure 5) reproduces the prolapse shape in the right location, but the resulting maximum and average deviations sum up to values that are larger compared to the other two satisfactory data sets. The slightly larger deviations occur in other parts of the valve, which are not visible in Figure 5(c). The reason for this is that the leaflet sizes were probably slightly overestimated in segmentation. Large leaflets have to somehow form a coaptation, but this can lead to additional folds and bulging on other parts of the valve (apart from the section shown in Figure 5(c)).

The passable case B4 (cf. Figure 6) reproduces the regurgitation orifice with an overall good and second best approximation. However, the shape and deviations between the leaflet and the expert annotations indicate a slightly too large leaflet or missing annotations. Closely examined, there is noise above the annotations that may or may not be a part of the leaflet. The lowered contour in the middle of the right leaflet is the result of a cord pulling down this part of the leaflet. Because of this, it was classified passable.

Although a good approximation could be achieved in the unsatisfactory case B1 (cf. Figure 7) because of the cord length, the leaflet motion is blocked by the collision handling, whereas in the other data sets, the leaflets glide off each other. Here, the overestimated leaflets block each other before being able to form the prolapse. In the end, the leaflets exhibit billowing and a fully closed valve without regurgitation orifice. The prolapsing valve shown in Figure 7(c) could not be reproduced by the simulation. The bottom line is that the open-state valve model and the annulus positions have to match the image data well to achieve the necessary precision.

Case A5 (cf. Figure 8) is shown for reference of a normally closing valve. When comparing the cross-section of the valve to manually defined tracing of the closed valve in Figure 8(c), billowing of the anterior leaflet can be observed. This may have multiple reasons that are related to the segmentation, to the parameterization of the material, the definition of the forces or the manual placement of the secondary cords. Results using more sophisticated models do not show such billowing [MVG*12].

An exact cord placement in number and position seems not to be necessary for closure, which confirms other statements in literature. But as seen, lengths as well as length proportions for primary and secondary cords defined by the user are essential for the simulation outcome. Prolapsing valves can be simulated using the heuristic cord setup as a starting point for length parameterization. To reproduce the prolapse in cases B1–B5, individual or group length cord scaling can be applied to approximate the anatomy using the interactive editor [WTN*20]. This could be achieved in four of the five cases. To fully answer the question what cord configuration results in the depicted prolapse, an automated, inverse problem solver that can add or subtract and elongate or shorten cords may be beneficial for an easier parameterization. Doing this by hand is rather tiresome, even with a specialized editing tool [WTN*20]. This may be combined with image-based tracking [TWG*19], but all these approaches do not consider a possible overestimation of leaflet sizes.

The simplification of forces may in many cases not be critical for closure, but in case of B1, the current modelling may have been too simple. Closure can be reproduced very well, but in the prolapsing case B1, the external forces were not able to push the leaflets to the recorded end position because of a combination of leaflet size and the collision handling. The cord length could be scaled to achieve a good approximation in general, but the mentioned aspects prevented better results for case B1. In addition, the shape of normal and abnormal leaflets could be improved as well. We attribute this
shortcoming to be caused by material (no fibre direction modelling as well as nonlinearities) and force modelling. Full FSI might be necessary to provide a more realistic set of forces that fully address this aspect.

6. Conclusion

We presented first results of a practical simulation method for MVs that runs at interactive frame rates with user interaction. The method can even be sped up in the future using a GPU implementation. Because of the interactiveness, the parameters can be explored and set while comparing the simulation results to image data in real-time. The information needed to parameterize the cord model cannot be gained any other way in a clinical scenario. The approach combines a simple material model (expressed in three types of constraints) with a set of external forces (two scalars representing the fluid flow). This simplicity has drawbacks however. While being visually convincing and interactive, the approach cannot be used for usual biomechanical evaluations such as the analysis of stress distributions. Visually exploring the changes in valve behaviour caused by virtual interventions, e.g. when using different annuloplasty ring sizes or clips in different positions, are possible and might be useful in future DSS.

The evaluation with ten normal and abnormal cases showed that the deformation of our model forms closed valve shapes that resemble normal valves and is even able to reproduce pathologies such as prolapse leading to regurgitation in four of the five cases. The shapes of the simulated closed valves fit to the anatomy and pathology depicted in the image series and annotations provided by the experts well. The absolute deviations of our approach are similar to values found in literature for normal closure. For prolapse leading to regurgitation, there are no values for comparison. In total, the deviations between normal and abnormal cases are comparable.

Regarding the simulation times and the accuracy achieved, we think the chosen ansatz is suitable in a decision support scenario with some additions. There are still local deviations that probably stem from uncertainties in the segmentation and parameterization, e.g. with regard to unknown local material properties or fluid influences as well as subjective definitions for leaflets, annuli or tendinous cords. In particular, the segmentation process has to be improved in order to avoid an overestimation of the valve’s surface. Further analyses and validations are needed.

In the future, we plan to investigate the aspects that lead to the described shortcomings. A natural extension of the current approach consists of using more sophisticated position-based material models [BMM17] that also describe local variations, nonlinearities and fibre directions. For an easier setup, the parameterization of the valve material should be automated, e.g. by adopting an inverse approach such as Noe [Noe18]. To fully answer the question of what cord configuration results in a prolapse leading to regurgitation, an inverse problem solver that can automatically add or subtract and elongate or shorten cords may be beneficial. The results should be compared to other simulation results, e.g. generated using the FEM. Benefits and drawbacks of the specific methods should be analysed. To complete the model and achieve more realistic force fields, the movement of the LH as well as FSI should be added to the model. This can be achieved using, e.g. position-based fluids [MM13] in a unified framework. In addition, the generated geometry may serve as a basis for mesh generation and computational fluid dynamics (CFD) analysis. CFD has been used to quantify blood flow through the valve in virtual intervention scenarios with prescribed ventricle boundary conditions, but fixed MV [VGB*18, VBT*19]. In addition, the results of the virtual interventions should be explored further and have to be validated on postoperative image data.

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References

[AEK*12] Amini R., Eckert C. E., Koomalsingh K., McGarvey J., Minakawa M., Gorman J. H., Gorman R. C., Sacks M. S.: On the in vivo deformation of the mitral valve anterior leaflet: Effects of annular geometry and referential configuration. Annals of Biomedical Engineering 40, 7 (July 2012), 1455–1467. https://doi.org/10.1007/s10439-012-0524-5.

[Bla14] Blausen.com staff: Medical gallery of Blausen Medical 2014. WikiJournal of Medicine 2(2):10. https://doi.org/10.15347/wjm/2014.010.

[BLG*18] Bonis M. D., Lapenna E., Giambuzzi I., Meneghin R., Affronti G., Pappalardo F., Castiglia N., Trumello C., Buzzatti N., Giacomini A., Lucchetti M. R., Alfieri O.: Second cross-clamping after mitral valve repair for degenerative disease in contemporary practice. European Journal of Cardio-Thoracic Surgery 54 (2018), 91–97. https://doi.org/10.1093/ejcts/ezx507.

[BMO*14] Bender J., Müller M., Otaduy M. A., Teschner M., Macklin M.: A survey on position-based simulation methods in computer graphics. In EUROGRAPHICS 2017 Tutorials (2017), Eurographics Association. https://doi.org/10.2312/egt.20171034.

[BMS12] Burlina P., Mukherjee R., Sprouse C.: A personalized mitral valve closure simulator. In 2012 Annual Interna-
[WGT*19] Walczak L., Georgii J., Tautz L., Neugebauer M., Wamala I., Sündemann S., Falk V., Hennemuth A.: Using position-based dynamics for simulating the mitral valve in a decision support system. In Proceedings of the Eurographics Workshop on Visual Computing for Biology and Medicine (2019), B. Kozlíková, L. Linsen, P.-P. Vázquez, K. Lawonn, R. G. Raidou (Eds.), The Eurographics Association. https://doi.org/10.2312/vcbm.20191242.

[WTN*20] Walczak L., Tautz L., Neugebauer M., Georgii J., Wamala I., Sündemann S., Falk V., Hennemuth A.: Interactive editing of virtual chordae tendineae for the simulation of the mitral valve in a decision support system. International Journal of Computer Assisted Radiology and Surgery (2020). https://doi.org/10.1007/s11548-020-02230-y.