Controlled cavity-QED using a photonic crystal waveguide-cavity system

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We introduce a photonic crystal waveguide-cavity system for controlling single photon cavity-QED processes. Exploiting Bloch mode analysis, and medium-dependent Green function techniques, we demonstrate that the propagation of single photons can be accurately described analytically, for integrated periodic waveguides with little more than four unit cells, including an output coupler. We verify our analytical approach by comparing to rigorous numerical calculations for a range of photonic crystal waveguide lengths. This allows one to nano-engineer various regimes of cavity-QED with unprecedented control. We demonstrate Purcell factors of greater than 1000 and on-chip single photon beta factors of about 80% efficiency. Both weak and strong coupling regimes are investigated, and the important role of waveguide length on the output emission spectra is shown, for vertically emitted emission and output waveguide emission.

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I. INTRODUCTION

Single semiconductor quantum dots (QDs) are promising candidates for single photon emission applications because of their unique attributes, e.g., large exciton dipole moments, integrability with compact semiconductor cavity systems [1, 2, 3, 4, 5], and compatibility with telecommunication components. They also facilitate the study of light-matter interaction at a very fundamental level. However, semiconductor QDs suffer from environment-induced decoherence [6], that can have a detrimental influence on the desired “indistinguishable” and coherent nature of the emitted photons. In the last few years, there have been a number of experiments that show that these shortcomings can be largely overcome by increasing the spontaneous emission rate due to the Purcell effect [7], which is achieved by coupling the QD exciton to a target cavity mode. For example, planar photonic crystal (PC) cavities, such as those pioneered by Akane et al. [8], allow a pronounced modification of the single photon decay, by careful spatial and spectral positioning of an embedded QD exciton [9].

While new regimes of semiconductor cavity-QED (quantum electrodynamics) are being experimentally realized using photonic nanocavities, one major drawback of the monolithic cavity is that the photons are typically emitted out of the cavity and thus cannot be efficiently collected and manipulated. Moreover, it is against the general vision of planar integration, as one ultimately wants to emit the photons on-chip, into a target propagating mode; compared to regular microcavity systems, PC waveguides have the inherent advantage that they can collect and control the photons on-chip [10, 11, 12, 13, 14, 15, 16]. Moreover, enhanced spontaneous emission does not even need a quasi-closed cavity, and open system cavity-QED can be exploited to achieve photon emission enhancements by appropriate bandgap engineering of the propagation modes [17, 18, 19]. Related experiments on PC waveguides have been performed by Viasnoff-Schwoob et al. [20] and by Lund-Hansen et al. [21]; though only modest Purcell factors were achieved, the waveguide results of Ref. [21] demonstrated that large beta factors can be achieved for emission into an on-chip waveguide mode. However, several problems remain with long waveguide samples: since slow waveguide modes are required to increase the local density of states (LDOS), then large disorder-induced propagation losses occur [22, 23, 24] and the LDOS peak largely broadens [25]; in addition, for on-chip applications, one needs efficient output coupling, which requires a coupler and an output (non-PC) waveguide. Improvements for single photon gun applications have been proposed [26] using a small section of a PC waveguide that mimics a slow-light mode; although improved single photon applications were demonstrated, drawbacks of the finite-size PC waveguide include: i) longer waveguides are required to obtain large Purcell factors (> 100), and observing the strong coupling regime would be difficult; ii) lack of tunability and separation of the QD coupling region with the output coupling region; iii) complex Fabry-Perot ripples appear on the LDOS profile which can be challenging to overcome and engineer; iv) lack of theoretical insight using the known modes of the system, thus requiring a complex 3D numerical solution where parameter design sweeps are not practical; v) last, the waveguide looses many of the benefits of a PC nanocavity, e.g., local tuning and pronounced QD coupling using best-of-breed Q/Veff ratios, where Q is the quality factor and Veff is the effective mode volume.

In this work, we introduce a hybrid solution for controlled cavity-QED, that combines the benefits of finite-size waveguides, on-chip couplers, and PC nanocavities, integrated together on a PC planar chip. Although a rather complicated structure to model and
quantum optics theory is built and measured, and we adopt, and optimize, the where $a$ is shown in Fig. 2. Similar integrated devices have been supported by numerically exact solutions of the 3D Maxwell equations. A schematic of the proposed device below, is found to be $L_{\text{eff}} \approx L + 0.38a$ [32]. The Green function is defined from

$$
\left[ \nabla \times \nabla \times - \frac{\omega^2}{c^2} \epsilon(r) \right] G(r, r'; \omega) = \frac{\omega^2}{c^2} \mathbf{1} \delta(r - r'), \tag{2}
$$

where $\mathbf{1}$ is the unit dyadic and $\epsilon(r)$ is the dielectric constant for the material, and $\mathbf{G} = \mathbf{G}^T + \mathbf{G}^L$ includes both transverse and longitudinal contributions. The one-ended waveguide Green function can be expressed as

$$
G^T_w(r, r'; \omega) = \sum \frac{\omega^2}{\omega_k^2 - \omega^2} f_k(r)f_k^*(r'), \tag{3}
$$

where $G^T_w$ is the transverse Green function. Without loss of generality, we assume $k > 0$. Replacing the $k-$summation ($k \equiv k_x$) by an integral, i.e. $\sum_k \rightarrow \int_{k=0}^{\infty} \frac{L_n}{2\pi} dk$, then

$$
G^T_w(r, r'; \omega) = \frac{L_n}{4\pi} \int_{\infty}^{\Omega} \frac{dk}{\omega_k - \omega - i\delta} f_k(r)f_k^*(r'),
$$

where the group velocity $v_g(\omega)$ is treated as positive and $\delta$ is a positive infinitesimal variable. Substituting $f_k$ from Eq. (1), and carrying out the complex integration,

$$
G^T_w(r, r'; \omega) \big|_{0 < x, x' < x_0} = \frac{iat\omega}{2v_g(1 - 2r\cos(2k_xL_{\text{eff}}) + r^2)} \big[ \Theta(x - x')e_{k_x}(r)e^*_{k_x}(r')e^{-ik_{\omega}(x - x')} + \Theta(x' - x)e^*_{k_x}(r)e_{k_x}(r')e^{-ik_{\omega}(x - x')} + e^*_{k_x}(r)e^*_{k_x}(r')e^{-ik_{\omega}(x' - 2x_0)} \big], \tag{5}
$$

and

$$
G^T_w(r, r'; \omega) \big|_{0 < x, x' < x_0} = \frac{iat\omega}{2v_g(1 - 2r\cos(2k_xL_{\text{eff}}) + r^2)} \big[ e_{k_x}(r)e^*_{k_x}(r')e^{-ik_{\omega}(x - x')} + e^*_{k_x}(r)e^*_{k_x}(r')e^{-ik_{\omega}(2x_0 - x')} \big]. \tag{6}
$$
where $\Theta(x - x')$ is the Heaviside function, and $t$ is the transmission amplitude of the PC waveguide mode into the output waveguide, which has a propagating mode $f_{k_0}^e(x, z) e^{ik_0 z}$, normalized through $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dydz \epsilon(y, z) |f_{k_0}^e(x, z)|^2 = 1$. In deriving the above equations, we are assuming that the PC waveguide has enough unit cells that a Bloch mode description is valid, and that the waveguide mode is below the light line with a frequency within the photonic band gap; later, we will quantify these assumptions with rigorous numerical calculations.

Adding in the cavity. Next, we add a finite single-mode cavity to the finite waveguide plus output waveguide system (see Fig. 2). As above, all frequencies are assumed to be deep inside the in-plane photonic band gap. The eigenmode of the cavity $f_c$ with a resonance frequency of $\omega_c$. Note that these values are for the cavity system shown, including perfectly matched layers at the PC waveguide interface (and thus no scattering back from the $x = x_0$ interface); the presence of this interface causes a resonance shift and broadening in comparison to the infinite PC bare cavity eigenmode $f_c$, e.g., a cavity surrounded by an infinite PC. For this waveguide-cavity system, we can obtain the photon Green function following a similar approach of Cowan and Young [28], and Hughes and Kamada [29]. Specifically, we expand the transverse Green function $G_{wc}^T$ of the PC waveguide-cavity system in terms of the cavity and waveguide eigenmodes, $G_{wc}^T = \sum_{\alpha, \beta} B_{\alpha \beta} f_\alpha \otimes f_\beta^*$, where $f_\alpha \otimes f_\beta^*$ are the transverse eigenmodes of the uncoupled (separate) waveguide and cavity. From the definition of $G_{wc}^T$, we then obtain a set of equations in matrix form: $MBT = T$. The matrix $M$ has the form

$$
M = \begin{pmatrix}
M_{cc} & M_{ck} & \cdots \\
M_{ck} & M_{kk} & \cdots \\
\vdots & \vdots & \ddots 
\end{pmatrix},
$$

with $M_{11} = (\omega_c^2 - \omega^2)/\omega^2$, $M_{ck} = -\langle f_c | V_c f_k \rangle$, and $M_{kk} = (\omega_c^2 - \omega^2)/\omega^2$. The shorthand notation $V_c$, represents the perturbation in the dielectric constant that results from adding in the cavity, else there is a perfect PC (mirror) for $x > x_0$. Without the cavity, then $V = V_c$.

After solving the equation set by matrix inversion, the Green function is obtained analytically for the complete waveguide-cavity system. One obtains

$$
G_{wc}^T \mid_{x > 0} = G_{w,c}^T + \frac{\omega^2 |f_c \rangle \langle f_c|}{\omega^2 - \omega^2 - \omega_i^2} (\int V_c G_{w,c}^V |f_c \rangle \\
+ \omega^2 G_{w,c}^T V_c |f_c \rangle \langle f_c| \\
+ \omega^2 \omega_i^2 G_{w,c}^V V_c |f_c \rangle \langle f_c| \\
+ \omega^2 G_{w,c}^T V_c |f_c \rangle \langle f_c| \frac{G_{w,c}^V |f_c \rangle}{\omega_i^2 - \omega^2 - \omega_i^2} |f_c \rangle \\
+ \omega^2 G_{w,c}^T V_c |f_c \rangle \langle f_c| \frac{G_{w,c}^V |f_c \rangle}{\omega_i^2 - \omega^2 - \omega_i^2} |f_c \rangle) \langle f_c| \cdot \frac{G_{w,c}^V |f_c \rangle}{\omega_i^2 - \omega^2 - \omega_i^2} |f_c \rangle),
$$

where $G_{wc}^T$ is in operator form, and by spatial projection: $G_{wc}^T(r, r') = \langle r | G_{wc}^T | r' \rangle$, $f_c(r) = (r | f_c \rangle$. Thus, the components of $G_{wc}^T(r, r')$ projected onto $f_c(r) \otimes f_c^*(r')$, and onto $f_k(r) \otimes f_k^*(r')$, are

$$
G_{wc}^T(r, r'; \omega) \mid_{x > 0} = \frac{\omega^2 f_c(r) \otimes f_c^*(r')}{\omega^2 - \omega^2 - i\omega(\Gamma^0_c + \Gamma_{wc})}, \quad (8)
$$

and

$$
G_{wc}^T(r, r'; \omega) \mid_{x < x_0} = \frac{ia A_{sk} \omega^3 V_{kc} f_k(y, x) e^{ikx} \otimes f_c^*(r')}{2v_g \omega_k^2 - \omega^2 - i\omega(\Gamma^0_c + \Gamma_{wc})}, \quad (9)
$$

where $\Gamma^0_c$ is the vertical decay rate of the cavity, and $\Gamma_{wc} = A_{gs} \Gamma_{wc}$ is the coupling coefficient between the finite-size waveguide and the cavity, with $A_{gs} (\text{Eff}, \omega) = 1/[1 + \rho^2 - 2r \cos(2k_{wc} \text{Eff})]$. Without the cavity, then $|V_{ec}|^2 = |V_{kc}|^2 \approx \int dr f_k^*(r) V_{kc}(r) f_c(r) e^{ikx}$. In practical calculations, and in what follows below, we will compute this coupling exactly using a straightforward numerical simulation. Comparing with the side-coupling waveguide-cavity system [18], we highlight two important differences: the expression for $\Gamma_{wc}$ is doubled with unidirectional coupling (for a side-coupled cavity, $\Gamma_{wc}' = \frac{2\omega}{2\omega_g} |V_{kc}|^2$), and there is a finite-size dependent coupling factor $A_{fs}$.

The Green function that describes propagation from the dot to the output waveguide can again be calculated by mode coupling theory, yielding

$$
G_{wc}^T(r, r'; \omega) \mid_{x < x_0} = \frac{ia t A_{fs} \omega^3 V_{kc} f_k^e(y, x) e^{ikx} \otimes f_c^*(r')} {2v_g \omega_k^2 - \omega^2 - i\omega(\Gamma^0_c + \Gamma_{wc})}, \quad (10)
$$

B. Enhanced Spontaneous Emission Rate

We can invoke the electric-dipole approximation to derive the medium-dependent spontaneous emission rate, or (Einstein $A$ coefficient), defined through

$$
\Gamma(r_d, \omega_d) = \frac{2d \cdot \text{Im}[G^T(r_d, r_d; \omega_d)] \cdot d}{\hbar \epsilon_0}, \quad (11)
$$

where $d = n_d d$ is the optical dipole moment of the photon emitter’s electronic resonance, and $r_d$ is the spatial position of the QD. Therefore, the enhancement of spontaneous emission rate, i.e., the Purcell factor, can be expressed analytically via $F = \Gamma / \Gamma_h$, where $\Gamma_h$ is the spontaneous emission rate in a corresponding homogeneous medium. It is noted that the concept of spontaneous emission rate only makes sense for weak and intermediate coupling regime. In other words the application of Fermi’s Golden Rule assumes the weak coupling regime, which is an assumption that must be used with care for this system. However, our formalism is not restricted to this regime, and strong coupling effects will also be investigated later. Using the derived Green functions [39],
then the on-resonance Purcell factor,
\[
F(L_{\text{eff}}, \omega_d = \omega_c) = \frac{\Gamma_{\text{PC}}}{\Gamma_h} = \frac{6\pi^3|n_d \cdot \mathbf{f}_c(r_d)|^2}{w^2\sqrt{\varepsilon_h}(\Gamma_0 + A_{fs}\Gamma_0^o)}, \quad (12)
\]
and the on-resonance beta factor,
\[
\beta(L_{\text{eff}}, \omega_d = \omega_c) = \frac{\Gamma_{\text{target}}}{\Gamma_{\text{target}} + \Gamma_{\text{others}}},
\]
\[
= \int_{S_d} \text{Re}(\{G_{cc}^T(r, r_d; \omega_c) \cdot n_d\}) \{\nabla \times (G_{cc}^T(r, r_d; \omega_c) \cdot n_d)\}^* \cdot ds \times \int_{S_d} \text{Re}(\{G_{cc}^T(r, r_d; \omega_c) \cdot n_d\}) \{\nabla \times (G_{cc}^T(r, r_d; \omega_c) \cdot n_d)\}^* \cdot ds
\]
\[
= A_{fs}(L_{\text{eff}}, \omega_d)B_{\text{coup}}, \quad (13)
\]
where \(B_{\text{coup}}\) depends on the coupling out to the target waveguide, and is determined from the full numerical simulation of a dipole, including the coupler region; \(s_o\) and \(s_d\) refer to surface regions perpendicular to the output propagating waveguide (at \(x \ll 0\)) and to a surface surrounding the dipole, respectively. The target output mode represents the output waveguide, and we have neglected the influence of non-radiative decay since we are considering QD coupling regimes at low temperature in an enhanced emission regime. These analytical formulas are valid for well defined PC waveguides, and, as we will show below, can even be used to accurately describe emission for integrated systems with only four unit cells in the waveguide section.

C. Emitted Spectra and Strong Coupling Regime

Assuming an incoherently excited QD, the exact electric field operator can be written as
\[
\hat{E}(R, \omega) = \frac{1}{\varepsilon_0}G_T^T(R, r_d; \omega) \cdot d\hat{\sigma}^-(\omega) + \hat{\sigma}^+(\omega), \quad (14)
\]
where \(\hat{\sigma}^{\pm}\) are the Pauli operators of the electron-hole pair (exciton). The spectrum, detected at position \(R\), is
\[
S(R, \omega) = |G_T^T(R, r_d; \omega) \cdot d|^2 \times \left|\frac{\alpha_0(\omega)}{(1 - \alpha_0(\omega) n_d \cdot \mathbf{G}_{cc}(r_d, r_d; \omega) \cdot n_d)}\right|^2, \quad (15)
\]
where \(\alpha_0(\omega) = 2\omega_d d^2/\hbar\varepsilon_0(\omega_d^2 - \omega^2)\) is the bare polarizability, with \(\omega_d\) the exciton resonance frequency. It is noted that the contribution from continuous radiation modes have been neglected, which could be thought of not existing anyway in the definition of \(\omega_d\), it can be safely neglected. Using \(G_T^T(R, r_d; \omega)\) and \(G_{cc}^T(R, r_d; \omega)\) from Eqs. (13), we obtain the spectrum at any relevant spatial point, e.g., above the cavity, or along the output waveguide. For example, when the photon is emitted on-chip along the waveguide, then
\[
S_{\text{side}}(R, \omega) \approx |G_{cc}^T(R, r_d; \omega) \cdot d|^2 \times \left|\frac{\alpha_0(\omega)}{(1 - \alpha_0(\omega) n_d \cdot \mathbf{G}_{cc}(r_d, r_d; \omega) \cdot n_d)}\right|^2, \quad (16)
\]
and when the photon is emitted vertically, above the cavity:
\[
S_{\text{vert}}(R, \omega) \approx |\mathbf{G}_{cc}^T(R, r_d; \omega) \cdot \mathbf{d}|^2 \times \left|\frac{\alpha_0(\omega)}{1 - \alpha_0(\omega) n_d \cdot \mathbf{G}_{cc}(r_d, r_d; \omega) \cdot n_d}\right|^2. \quad (17)
\]
We now have all the relevant formulas to compute the Purcell factor, beta factor, and emission spectrum for the integrated waveguide-cavity system shown in Fig. (1).

III. CALCULATIONS

A. Weak Coupling Regime

In order to validate the above Green function theory, a direct 3D finite-difference time-domain (FDTD) calculation of the Green function terms is first performed [30, 31]. We use parameters representative of the popular L3 cavity [8] and a nominal W1 (removed row of holes) waveguide, with the following parameters: semiconductor slab dielectric constant \(\varepsilon = 12\); lattice constant \(a = 420\) nm (PC pitch); the two holes as indicated in Fig. 1 are shifted outwards by a distance of 0.15 \(a\); the thickness of the slab and radius \(R\) of the air holes are 0.5 \(a\) and 0.275 \(a\), respectively; the width of the output waveguide is 470 nm, which was optimized to give the largest beta factor. The TE-like band gap ranges from 0.760 eV to 0.935 eV (corresponding to 0.26–0.32 \(c/a\) in normalized frequency units, or 85 to 228 THz), and the band structure of the waveguide mode is shown in Fig. 2(a). In the frequency range of our interest, the waveguide is single mode and under the light line (gray shaded region). In Fig. 2(b), we show the enhancement of the spontaneous emission versus frequency for a maximally positioned and \(y\)-aligned QD exciton, with \(L = 6a\); the Purcell factor spectra exhibits a typical Lorentzian line shape, that agrees with the analytical expression of Eq. (1). The electric-field distribution at the resonant frequency (indicated by red circle in Fig. 2) is also shown in Fig. 3. The local field strength in the cavity is pronounced and the energy is mainly guided into the coupled PC waveguide, and subsequently into the target output waveguide; both a significant Purcell factor and an enhanced beta factor are obtained. Although the Purcell factor is reduced in comparison to a bare waveguide, the emphasis here is on achieving an enhanced Purcell factor while still obtaining a large on-chip \(\beta\)-factor. These Purcell factors give a quantitative measure of the enhancement in the projected LDOS, and are already large enough, with suitable QD coupling, to facilitate strong coupling. The effective mode volume of the cavity system is found to be \(V_{\text{eff}} \approx 0.063 \mu\text{m}^3\), which can be related to the cavity mode position at the peak field antinode, through \(|\mathbf{E}_c(r_{\text{antinode}})|^2 = 1/V_{\text{eff}}\varepsilon\).

Next, we carry out a systematic investigation of the Purcell factor as a function of length \(L\), where \(L\) is in-
Of course, the increased by integer multiple of $a$. The results are shown in Fig. 4(a), and the data is successfully fitted with the analytical form introduced earlier; the main parameters required for the analytical formulas are extracted from carrying out one numerical simulation, to obtain $r = 0.21$ and $R_0^0/T_0^0 = 0.31$. The center wave vector $k(\omega_c) = 0.75\pi/a$ is obtained from the band structure. From Fig. 5, we know when $L$ is larger than $3a$, then the Bloch mode theory becomes valid. Of course, the analytical theory fit is only effective for integer multiple of $a$ because of the coupler-termination dependence of $r$. If we want to show the case of a continuously varying $L$, we should first calculate $r$ (and $t$) for various unit cell truncations at the output coupler. However, since we have optimized this coupler region, the most practical case is for the integer number of unit cells. Importantly, our calculations include the output reflection coefficient and the length of waveguide. In addition, one can also tune the properties of the cavity, e.g., to the target exciton resonance, and still overlap with the broadband coupling region of the PC waveguide mode (20–40 meV bandwidth below the light line, cf. Fig. 2(a)).

We also investigate the single-photon $\beta$-factor; this parameter quantifies the efficiency of emitting a single photon into the desired output mode, namely the non-PC waveguide mode after the coupler (cf. Figs. 1 and 2). The beta factor is first calculated by using the numerically-exact FDTD technique, by computing the emitted fields at the left of the coupler; these fields are subsequently mode-overlapped with the desired waveguide mode and normalized with respect to the total power flowing out of the lossless device. We first obtain the total emitted power $P_t$ by having six field monitors completely surrounding the emitting dipole; we also record the propagation power after the field travels through the coupler $P_{out}$, and calculate the field distribution $\mathbf{E}_i(r)$ and $\mathbf{H}_i(r)$, including all bound and radiation mode contributions. We then adopt a mode overlap integral technique, and calculate the projection of the scattered field that overlaps with the target output waveguide mode. Labeling the electric and magnetic field of the target mode as $\mathbf{E}_{out}(r)$ and $\mathbf{H}_{out}(r)$, respectively, then the overlap integral can be expressed as

$$OI = \frac{\text{Re} \left[ \int \mathbf{E}_{out}(r) \times \mathbf{H}_{out}^*(r) \cdot dS \right] \int \mathbf{E}_{i}(r) \times \mathbf{H}_{i}(r) \cdot dS}{\text{Re} \left( \int \mathbf{E}_{i}(r) \times \mathbf{H}_{i}(r) \cdot dS \right)} \quad (18)$$

where $S$ is on the $y-z$ plane perpendicular to the target waveguide at the left of the coupler region ($x \ll 0$). The beta factor is then simply $\beta = OI \times P_{out}/P_t$. The key advantages in the present proposal are as follows: i) the Purcell factors, in comparison to a bare finite-size waveguide, are substantially higher when over-coupled to the cavity, ii) the Purcell factor and beta factor can be controlled in a systematic way, and iii), the conceptional understanding and coupling can be described analytically; to show that this latter point is also true for the beta factor, we have fitted the analytical form with the previous parameters and found good agreement when $L \geq 4a$ (Fig. 4(b)).
Since the coupled QD-PC system results in significant LDOS enhancements, we can naturally probe finite-size coupling coefficient $A_{fs}$ (black dotted curve). (a-c) $d = 30$ Debye for $L = 9a$, 10$a$, 11$a$, respectively. (d-f) $d = 50$ Debye and $r = 0.21$, for $L = 109a$, 110$a$, 111$a$, respectively.

**B. Strong Coupling Regime**

Finally, we turn our attention to the strong coupling regime. Since the coupled QD-PC system results in significant LDOS enhancements, we can naturally probe strong coupling and non-perturbative cavity-QED, either above the cavity or on-chip.

We study emission spectra for detectors that are placed above the cavity and at the target waveguide, for various waveguide lengths. These represent the spectra emitted off-chip and on-chip. For these strong coupling calculations, one must include the dispersion of the propagating PC waveguide mode; using a linear dispersion model, $k_\omega = k_{\omega_d} + (\omega - \omega_d)/v_g$, where $v_g = 14$ is obtained from the slope of the waveguide band in Fig. 3(a), at the indicated red circle.

In Fig. 6(a-c), we display the emitted spectra, both vertically and for the output waveguide, for a $9 - 11a$ unit-cell PC waveguide, and a dipole moment of $d = 30$ Debye; to ensure maximum coupling, the exciton is resonant with the cavity mode ($\omega_d = \omega_e$). Clearly the emitted spectra are qualitatively different depending upon the PC waveguide length, which is due to the modal propagation characteristics of the cavity and the waveguide; in particular, one can see the broadening and thus the Purcell factor increases as we go from ($L = 9a$) to ($L = 11a$), showing that the QD coupling depends sensitively upon the length of the waveguide section. Ideally, for a side-coupled waveguide-cavity system, with no external reflection from the waveguide ends, the shape of spectra emitted vertically from cavity mode and on-chip via the waveguide mode are symmetric and identical. However, for any real system, the effect of finite size is always there, and in general will result in different spectral shapes for the vertically and horizontally emitted spectra, e.g., proportional to $A_{fs}(L_{eff}, \omega)$.

We next choose a slightly larger dipole moment of 50 Debye and a longer PC guide, with $L = 109 - 111a$ unit-cells. A PC waveguide length of $109 - 111a$ has the same peak PF as those with $9 - 11a$, since the length difference between them (100$a$) is an integer multiple of the period, $8a/3$ ($k_{\omega_d} = 0.75\pi/a$); at the resonance frequency, the Purcell factor is periodic, however, for off-resonance, it will be more complicated because the field is propagating back and forth between the coupler and the cavity, which appears differently in general for on-chip emission and out-of-plane (vertical) emission (cf. the presented spectrum equations (16) and (17)). As shown in Fig. 6(d-f), we recognize a much larger frequency-dependence on the coupling parameter $A_{fs}(L_{eff}, \omega)$, which can produce significant asymmetries in the emission spectra of a single QD exciton: indeed, there is now a substantial difference between the vertically emitted light and the emitted light on chip, and, in principle, these different spectra could be probed in experiments by placing detectors above the cavity and at the output of the exit waveguide.

**IV. CONCLUSIONS**

We have proposed and investigated the spontaneous emission properties of an embedded single QD in a photonic crystal waveguide-cavity system. To describe the quantum light-matter interactions in this system, an intuitive Green function formalism has been developed which is confirmed by detailed numerical calculations. The structure can achieve both large Purcell factors and high extraction rates, and allow the investigation of weak and strong coupling regimes, both on- and off-chip. These waveguide-cavity systems are timely with recent improvements in PC fabrication, and offer a very rich degree of fundamental control of the ensuing light-matter interactions.

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