Accretion of radiation and rotating Primordial black holes in anisotropic universe

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ABSTRACT

We consider Primordial black holes (PBHs) in a LRS (locally rotationally symmetric) Bianchi-I anisotropic space time and study the effect of accretion of radiation in the radiation dominated era. We show that the life time of PBHs in anisotropic universe becomes longer by accretion of radiation. We generalize the result by including nonzero angular momentum parameter and study the PBH evolution equation. We find that the evaporation time gets prolonged and it depends on the accretion efficiency as well as angular momentum parameter. This supports the conjecture that Primordial black holes can be considered as a viable candidate for dark matter.
1 Introduction

Primordial black holes (PBHs) are supposed to be formed during the early expansion of the universe. These black holes may have been produced due to density fluctuation in the early universe with extremely high temperature and pressure. The mass of the PBHs can cover a wide range. There are different theories regarding the formation of PBHs such as, initial inhomogeneities [1, 2], inflation [3, 4], phase transitions [5] bubble collisions [6, 7], decay of cosmic loops [8] etc. The formation of PBH can also play a very important role in understanding the cosmological inflation. According to the work of Stephen Hawking, black holes emit thermal radiation due to quantum effects near the event horizon [9]. As a result of Hawking radiation, the black holes can lose mass and evaporate. Smaller mass black holes are expected to evaporate quickly. The PBHs with a longer life time can act as seeds for structure formation [10]. PBHs with mass greater than $10^{15}g$ do not evaporate completely through Hawking radiation and the abundance of such black holes can be considered as suitable dark matter candidate [11].

In the context of standard cosmology, early work on the study of the effect of accretion of radiation on PBHs has led to several speculations regarding the possibility of increasing the mass of a PBH [1, 12]. Cosmological consequences of evaporation of PBHs in different eras have been studied quite well [13, 14] (see [15] for new cosmological constraints on PBHs). It has been realized during last couple of years that the effect of accretion in the radiation dominated era can result in longlived PBHs in braneworld scenario [16], in Brans-Dicke theory [17, 18] as well as in standard cosmology [19].

In this paper, we study the evolution of PBHs in an homogeneous and anisotropic universe by including the effect of accretion of radiation. In particular, we consider the LRS (locally rotationally symmetric) Bianchi-I space-time and study the PBH evolution equation in radiation dominated era and show that the evaporation time does depend on the accretion efficiency like in spatially flat FRW universe. We also consider the effect of accretion on the evaporation of PBHs with rotation and obtain the dependence of the evaporation time on the accretion efficiency and the angular momentum parameter.

2 Anisotropic universe and Primordial black holes

We consider PBHs in spatially homogeneous and anisotropic LRS (locally rotationally symmetric) Bianchi type-I universe. The corresponding metric in four space-time dimensions is given by,
\[ ds^2 = -dt^2 + a(t)^2 \, dx^2 + b(t)^2 \, (dy^2 + dz^2) \]  \hspace{1cm} (2.1)

where, \( a(t) \) and \( b(t) \) are the cosmic scale factors. We assume that the universe is filled with a perfect fluid described with the equation of state \( p = \gamma \rho \) (where \( \gamma = \frac{1}{3} \) for the radiation dominated era and \( \gamma = 0 \) for the matter dominated era), the energy-momentum tensor is given by,

\[ T_{\mu\nu} = (p + \rho) \, u_\mu \, u_\nu + p \, g_{\mu\nu} \]  \hspace{1cm} (2.2)

where, \( p \) is the pressure, \( \rho \) is the energy density and \( u_\mu \) is the 4-velocity of the fluid. The Einstein’s equations are then given by,

\[ \frac{2\dot{a}}{ab} + \left( \frac{\dot{b}}{b} \right)^2 = 8\pi G \rho \]  \hspace{1cm} (2.3)

\[ \frac{2\ddot{b}}{b} + \left( \frac{\ddot{b}}{b} \right)^2 = -8\pi G p \]  \hspace{1cm} (2.4)

\[ \frac{\dot{a}}{ab} + \frac{\ddot{b}}{b} + \frac{\dddot{a}}{a} = -8\pi G p \]  \hspace{1cm} (2.5)

The energy-momentum conservation equation is given by,

\[ \dot{\rho} + \left( \frac{\dot{a}}{a} + \frac{2\dot{b}}{b} \right) (p + \rho) = 0 \]  \hspace{1cm} (2.6)

From the above equations, we find that the scale factors \( a(t) \) and \( b(t) \) behave as follows:

for \( t < t_1 \),

\[ a(t) = a_0 \, t^{\frac{1}{2}} \]  \hspace{1cm} (2.7)

\[ b(t) = b_0 \, t^{\frac{1}{2}} \]  \hspace{1cm} (2.8)

For \( t > t_1 \), we find,

\[ a(t) = a_0 \, t^{\frac{2}{3}} \]  \hspace{1cm} (2.9)

\[ b(t) = b_0 \, t^{\frac{2}{3}} \]  \hspace{1cm} (2.10)

where \( a_0 \) and \( b_0 \) are constants and \( a_0 \neq b_0 \). Here, \( t < t_1 \) corresponds to radiation dominated era and \( t > t_1 \) corresponds to matter dominated era.


3 Accretion of radiation by PBHs

Here we consider the effect of accretion on the life time of the primordial black holes. Due to accretion in the radiation dominated era, the mass of the PBH increases and the accretion rate (which is taken to be proportional to the product of the surface area of the PBH and the energy density of the radiation [20]), is given by,

\[ \dot{M}_{\text{acc}} = 4 \pi f r_{BH}^2 \rho_r \] (3.1)

where \( \rho_r \) is the radiation energy density of the surrounding of the black hole; \( r_{BH} = 2 M G \) is the radius of the uncharged, nonrotating black hole with mass \( M \) and \( G \) is the gravitational constant. \( f \) is the accretion efficiency. The precise value of \( f \) is not known.

The accretion efficiency could in principle depend on complex physical processes such as the mean free paths of the particles comprising the radiation surrounding the PBHs.

From the Einstein’s equation, we can calculate \( \rho_r \) as,

\[ \rho_r = \frac{1}{8 \pi G} \left[ \left( \frac{b}{b'} \right)^2 + \frac{2 \dot{a} \dot{b}}{a b} \right] \] (3.2)

Substituting for \( \rho_r \) and \( r_{BH} \) and using the temporal behaviour of the scale factors \( a(t) \) and \( b(t) \) in the equation for the rate of change of mass due to accretion, one gets,

\[ \dot{M}_{\text{acc}} = \frac{3}{2} f G \frac{M_i^2}{t^2} \] (3.3)

Upon integration, the above equation gives,

\[ M(t) = \left[ M_i^{-1} + \frac{3}{2} f G \left( \frac{1}{t} - \frac{1}{t_i} \right) \right]^{-1} \] (3.4)

where, \( M_i \) is the mass of the black hole at its formation time. Using horizon mass which varies with time as \( M_H(t) = G^{-1} t \), as initial mass of PBH, we get

\[ M(t) = M_i \left[ 1 + \frac{3}{2} f \left( \frac{t_i}{t} - 1 \right) \right]^{-1} \] (3.5)

This behaviour is found to be the same as in standard cosmology with spatially flat FRW universe [19]. The above expression for \( M(t) \) indicates that the mass of the PBH increases with accretion efficiency (which is shown in Figure-1). For large \( t \), the maximum value of the black hole mass \( M_{BH} \) is given as,

\[ M_{\text{max}} = \frac{M_i}{1 - \frac{3}{2} f} \] (3.6)
which leads to an upper bound for the accretion efficiency \( f \) as,

\[
f < \frac{2}{3}
\]

The above result shows that the accretion can be effective in increasing the mass of the black hole and thereby increasing the life time of the primordial black hole.

It is worth mentioning here that in the hydrodynamic picture of the formation of the PBH during expansion of the early Universe \([1, 21, 22]\), it has been shown through numerical calculations that the pressure gradient plays an important role in impeding the formation of PBHs. The rate of accretion of PBHs can reduce drastically by the pressure gradient. In case of the relativistic equation of state, initial perturbations have to be large enough in order to allow for the formation of PBHs. In the present context, we have not considered the effect of pressure gradient on the accretion efficiency. Such consideration will need a full numerical computation which is beyond the scope of our paper.

### 4 Evaporation of PBH

Since the mass of the black hole decreases due to Hawking radiation, the rate of decrease of the mass of the PBH is given by,

\[
\dot{M}_{\text{evap}} = -4\pi r_{BH}^2 \sigma_H T_{BH}^4
\]
Table 1: The formation time and initial mass of the PBHs which are likely to evaporate now are displayed for different accretion efficiencies.

| $f$ | $t_i$          | $M_i$          |
|-----|----------------|----------------|
| 0   | $2.3669 \times 10^{-23}s$ | $2.3669 \times 10^{15}g$ |
| 0.2 | $1.6568 \times 10^{-23}s$ | $1.6568 \times 10^{15}g$ |
| 0.4 | $0.9467 \times 10^{-23}s$ | $0.9467 \times 10^{15}g$ |
| 0.6 | $0.23669 \times 10^{-23}s$ | $0.23669 \times 10^{15}g$ |

where $r_{BH}$ is the black hole radius given by $r_{BH} = 2 G M$, $\sigma_H$ is Stefan’s constant multiplied by number of degrees of freedom of radiation and $T_{BH} = \frac{1}{8 \pi G M}$ is the Hawking temperature. Substituting in the equation for the rate of evaporation, one gets,

$$\dot{M}_{\text{evap}} = - \frac{\sigma_H}{256 \pi^3 G^2 M^2}$$

Upon integration, one gets,

$$M = \left[ M_i^3 + 3 \alpha (t_i - t) \right]^{\frac{1}{3}}$$

where $\alpha = \frac{\sigma_H}{256 \pi^3 G^2}$ and $M_i$ is the black hole mass at its formation time $t_i$ and we assume $M_i$ to be the same as the horizon mass $[23, 24]$. Now, in the radiation dominated era, considering the effect of both accretion and evaporation simultaneously, the black hole evolution equation for the rate of change of mass of the PBH is given by,

$$\dot{M}_{PBH} = \frac{3}{2} f \frac{G}{t^2} \frac{M^2}{t^2} - \frac{\sigma_H}{256 \pi^3 G^2 M^2}$$

Since this equation can not be solved analytically, we solve it using numerical methods.

Let us assume that accretion is dominant up to a time $t = t_c$ where both the rates become equal and after that, evaporation effect becomes dominant. So for the radiation dominated era, this implies,

$$\frac{3}{2} f \frac{G}{t^2} \left( \frac{M_c^2}{t_c^2} \right) = \frac{\sigma_H}{256 \pi^3 G^2 M_c^2}$$

which gives,

$$M_c = \left( \frac{2 \alpha}{3 f G} \right)^{1/4} t_c^{1/2}$$
where, \( \alpha = \sigma_H^{2/3} \).

Using \( M_c = M_i \left[ 1 + \frac{3}{2} \left( \frac{t_i}{t_c} - 1 \right) \right]^{-1} \), we get,

\[
\frac{M_i}{1 + \frac{3}{2} \frac{t_i}{t_c} - 1} = \left( \frac{2 \alpha}{3 f G} \right)^{1/4} t_c^{1/2}
\]  

(4.7)

For \( t_c \gg t_i \), one gets,

\[
t_c^{1/2} = \left( \frac{3 f G}{2 \alpha} \right)^{1/4} \left( \frac{M_i}{1 - \frac{3}{2} f} \right)
\]  

(4.8)

Comparing the above two equations, we get,

\[
M_c = \frac{M_i}{1 - \frac{3}{2} f}
\]  

(4.9)

where \( M_c \) is the maximum mass \( M_{\text{max}} \) with an upper limit of \( f < 2/3 \). In the matter dominated era, accretion is zero and only the second term in r.h.s. of (4.4) contributes:

\[
\dot{M} = - \frac{\sigma_H}{256 \pi^3 G^2 M^2}
\]  

(4.10)

It is clear from Table-1 that Primordial black hole lives longer with increase in accretion efficiency.

5 Rotating black hole

In this section, we generalize the result to include rotation through a nonzero angular momentum and study the PBH evolution equation. Emission rates for massless particles from a rotating black hole and the subsequent evolution of the rotating black hole has been discussed in the nice work of Page [25] quite some time back. Here we consider the effect of accretion on the rotating hole. As before, the rate of change of mass due to the accretion of radiation is by,

\[
\dot{M}_{\text{acc}} = 4 \pi f R_{BH}^2 \rho_R
\]  

(5.1)

where, \( f \) is the accretion efficiency, \( R_{BH} \) is the radius of the outer horizon of the rotating black hole and is given by,

\[
R_{BH} = r_+ = M + \sqrt{M^2 - a^2}
\]  

(5.2)

with \( a (= J/M) \) being the rotation parameter and \( J \) is the angular momentum. The rotating black hole solution satisfies the inequality \( M^2 \geq a^2 \) in order to avoid a naked singularity. \( \rho_R \) can be calculated from the Einstein’s equation and is given by, \( \rho_R = \frac{3}{32 \pi r^2} \) (we have taken \( G = 1 \)). Using these values, we get,

\[
\dot{M}_{\text{acc}} = \frac{3 f}{8 t_c^2} \left( M + \sqrt{M^2 - a^2} \right)^2
\]  

(5.3)
This expression reduces to the nonrotating case in the limit $a = 0$. One can notice from here that when $a^2$ is comparable to that of $M^2$, the rate of change of mass reduces to one fourth of the corresponding value obtained in the nonrotating case. (5.3) can be integrated to obtain the value of $M_{acc}$.

In order to understand the effect of accretion in the radiation dominated era, we have numerically plotted the variation of mass with change in the angular momentum parameter $a$ for a particular PBH formed at $t = 10^{-23} \text{ sec}$ in Figure 2.

![Accretion for rotating PBH](image)

Figure 2: Variation of PBH mass for $a = M_i, \frac{M_i}{2}, \frac{M_i}{4}; f = 0.5$

Now let us consider the evaporation of the rotating hole due to Hawking radiation. The rate of change of mass due to evaporation is given by,

$$\dot{M}_{\text{evap}} = -4 \pi R_{BH}^2 \sigma_H T^4$$

The Hawking temperature for the rotating, uncharged black hole is obtained as,

$$T = \frac{\sqrt{M^2 - a^2}}{4 \pi M \left( M + \sqrt{M^2 - a^2} \right)}$$

Using the expression for $T$ and the radius of the outer horizon, one gets,

$$\dot{M}_{\text{evap}} = -\left( \frac{\sigma_H}{64 \pi^3} \right) \left( \frac{M^2 - a^2}{M^4 \left( M + \sqrt{M^2 - a^2} \right)} \right)^2$$

Here one can see that when $a^2$ becomes comparable to that of $M^2$, the rate of change of mass during evaporation becomes negligibly small. In principle, one should also
\[ t_i = 10^{-22} \text{ s}; M_i = 10^{14} \text{ g}; f = 0.5 \]

\[ a^2 \quad t_{\text{evap}} \]
\[ \begin{array}{|c|c|} \hline
10^{-9} M_i^2 & \sim 10^{13} \text{ s} \\
10^{-7} M_i^2 & \sim 10^{18} \text{ s} \\
10^{-5} M_i^2 & \sim 10^{22} \text{ s} \\
10^{-3} M_i^2 & \sim 10^{24} \text{ s} \\
10^{-1} M_i^2 & \sim 10^{28} \text{ s} \\
\hline
\end{array} \]

Table 2: A rough estimate of evaporation time with change in angular momentum parameter

consider the rate of change of angular momentum of the black hole due to the emission of the particles together with the rate of change of mass. However, for simplicity of the problem, we only look at the rate of change of mass due to evaporation. So the total rate of change of mass including both accretion and evaporation for the rotating PBH is given by,

\[ \dot{M} = \frac{3f}{8t^2} \left(M + \sqrt{M^2 - a^2}\right)^2 - \left(\frac{\sigma_H}{64\pi^3}\right) \frac{\left(M^2 - a^2\right)^2}{M^4 \left(M + \sqrt{M^2 - a^2}\right)^2} \tag{5.7} \]

In the matter dominated era, only the second term on the r.h.s. of (5.7) contributes. We may note from Table-2 that the life time of PBH increases with increase in the angular momentum parameter.

From the above equation, we obtain the expression for the time \( t = t_c \) in terms of maximum mass \( M_c \) and the accretion efficiency:

\[ t_c = \left(\frac{3f}{32}\right)^{1/2} \alpha^{-1/2} \left(\frac{M_c^2 \left(M_c + \sqrt{M_c^2 - a^2}\right)^2}{M_c^2 - a^2}\right) \tag{5.8} \]

where, \( M_c \) is the mass obtained from the accretion equation, i.e. \( M_c = M_{\text{max}} \). One can see that the life time for the rotating PBH becomes longer by including the effect of radiation. One can check that in the limit \( a = 0 \), the above expression for \( t_c \) reduces to that of the nonrotating PBH i.e. \( t_c = \sqrt{\frac{3f}{2\alpha^{1/2}}} M_c^2 \).

We would like to mention here that in earlier work [26, 27, 25], it has been shown that angular momentum itself decreases with time nearly in the same order as the mass of the PBH. This decrease in angular momentum also depends on the emission of particle species. So both accretion as well as evaporation of a PBH may be affected by this variation. Since we are interested in a general analysis of the evolution of the rotating PBH, we have not included the variation of angular momentum with time and the issues related to emission of massless or nearly massless particles.
6 Summary and discussion

In this work, we have considered the effect of accretion on Primordial black holes in homogeneous and anisotropic Bianchi type-I space-time. Like in the case of FRW universe, we find that due to the effect of accretion of radiation in the radiation dominated era, the life time of the PBH becomes longer and the result depends on the accretion efficiency $f$, with an upper bound $f < 2/3$. We have also generalised the analysis of the evolution equation by including nonzero angular momentum and we find that rotation increases the life time of PBHs. The mass of the PBH increases with accretion efficiency as well as the angular momentum parameter. Since Hawking radiation is supposed to carry away the angular momentum, it is worthwhile to have a detailed analysis of the evolution of the rotating PBHs taking into account the rate of change of angular momentum in the context of emission of massless or nearly massless particles with different spins. Here, we have not considered the effect of back reaction of the PBH evaporation [28] which is supposed to modify the radius of the horizon and the Hawking temperature of the black hole [29]. It is expected that such effects might affect the evolution of PBHs and also there has been arguments that evaporation may stop after some time leaving a stable remnant which can be a viable candidate for cold dark matter.

It is worth investigating these issues further in the context of Primordial black holes with and without rotation.

Acknowledgement

We would like to thank L.P.Singh for useful discussions.
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