Abstract

The persistent current through a quantum dot inserted in a mesoscopic ring of length \( L \) is studied. A cluster representing the dot and its vicinity is exactly diagonalized and embedded into the rest of the ring. The Kondo resonance provides a new channel for the current to flow. It is shown that due to scaling properties, the persistent current at the Kondo regime is enhanced relative to the current flowing either when the dot is at resonance or along a perfect ring of same length. In the Kondo regime the current scales as \( L^{-1/2} \), unlike the \( L^{-1} \) scaling of a perfect ring. We discuss the possibility of detection of the Kondo effect by means of a persistent current measurement.
Electron transport through a quantum dot (QD) has been a subject of many experimental and theoretical studies in the last years. These small devices contain several millions of real atoms, but behave as if they were single artificial atoms.

Like real atoms they have a discrete spectrum of energy, which has been measured and theoretically understood on the basis of a confinement potential and a full many-body electron-electron interaction treatment. Unlike real atoms, electronic transport can be realized through a single QD. Experimental results show periodic oscillations of the conductance as a function of the electron density in the QD. These oscillations can be explained on the basis of a transport mechanism governed by Coulomb blockade and single-electron tunneling. Thanks to the quantization of charge the effect of the Coulomb interaction can be understood in terms of a charging energy $\sim e^2/C$ (where $e$ is the electronic charge and $C$ the capacitance of the dot) necessary to add an extra electron to an already charged dot. This energy is of the same order of the Coulomb repulsion $U$ between two electrons inside the dot.

Another manifestation of electron-electron interaction which was theoretically predicted some years ago is the Kondo effect in a QD coupled to external leads. In this case however the effect is due to correlations between the electrons inside the QD and the conduction electrons in the leads. When the system operates in the so-called Kondo regime a resonance in the vicinity of the Fermi level, localized at the QD, provides a new channel for the mesoscopic current to tunnel through, creating new phenomena which can be detected in a transport experiment. We have recently proposed an experiment based on the Aharonov-Bohm quantum interference phenomenon where the signature of the Kondo effect would be clearly reflected on the current. In this work the system studied consists of a ring connected to two leads having a QD inserted in one of its arms. Measurements of the current in a similar device have already demonstrated the coherent character of the electronic transport through the QD. However, in this last experiment the size of the QD and the experimental temperature were not appropriate to observe the Kondo effect. Its observation is a delicate task since it depends on several different energy scales and their relative sizes, such as the coupling constant between the QD and leads $t'$, the Kondo temperature $T_K$ and the energy spacing between the dot levels $\Delta \epsilon$. For example, if $t'^2/W > \Delta \epsilon$, where $W$ is the ring bandwidth, the charge and energy quantization in the QD is lost and the Kondo effect disappears. On the other hand, diminishing $t'$ leads to an exponential reduction of $T_K$.

So, in order to get simultaneously accessible temperatures and charge-energy quantization $\Delta \epsilon$ must be large enough, which implies small size QD. Very recently, such a very small QD was obtained in a shallow two-dimensional electron gas heterostructure fabricated by electron-beam lithography. The Kondo effect was, for the first time, observed in this system, although as a small effect.

Motivated by this experimental realization, we report in this letter the study of persistent currents going through a QD embedded in a mesoscopic ring of length $L$ threaded by a magnetic flux. As we show below the persistent currents in these systems are enhanced by the Kondo effect, making its detection possible. At very low temperature the Kondo resonance ($KR$) is a very sharp peak with width of the order of $T_K$, commonly much sharper than the dot resonance of width $\Delta_d \sim t'^2/W$. When the Kondo peak width is less than the energy spacing of the levels in the ring, that is $\Delta E > T_K$, it can be shown that the persistent current scales as $L^{-1/2}$ unlike the $L^{-1}$ scaling which occurs in a perfect ring. This gives rise
to an enhancement of the current intensity at the Kondo regime. In this case the current peak could be larger than \( I_0 (I_0 \sim e v_f/L) \), where \( v_f \) is the Fermi velocity, the intensity of the persistent current in a perfect ring [11]. This particular scaling of the current going through a very sharp resonance permits, in principle, a clear detection of the Kondo effect.

We study a two-level quantum dot embedded in a ring which is threaded by a magnetic flux and maintained at a fixed Fermi level. An external gate potential \( V_0 \) is applied to the QD in order to be able to change its one particle levels. Part of this system, consisting of a cluster of 8 atoms including the QD, is exactly solved by using a Lanczos algorithm. The cluster is then connected to the rest of the ring. We calculate the persistent current in the ring, the charge inside the QD and the density of states projected on the QD.

The system is represented by an Anderson-impurity first-neighbor tight-binding Hamiltonian. Although the electron-electron interaction exists in the entire system we assume it to be restricted to the dot where, due to quantum confinement, the electrons interact more strongly. The total Hamiltonian can be written as

\[
H = H_c + t \sum_{i,j,\sigma} c_{i\sigma}^\dagger c_{j\sigma} + \hat{T}
\]

where \( H_c \) is the cluster Hamiltonian, \( i \) and \( j \) represent nearest-neighbors atomic sites on the ring outside the cluster which interact through the hopping \( t \), and \( \hat{T} \) couples the cluster to the rest of the ring. Denoting the two QD states by \( \alpha \) and \( \beta \) and numbering the other six sites of the cluster from 1 to 3 and from 1 to 3, \( H_c \) can be written as,

\[
H_c = \sum_{r=\alpha,\beta} (V_0 + \varepsilon_r)n_{r\sigma} + U \sum_{r=\alpha,\beta} n_{r\sigma}n_{r\bar{\sigma}} + U \sum_{\sigma\sigma'} n_{\alpha\sigma}n_{\beta\sigma'}
\]

\[
+ \left[ t' \sum_\sigma (c_{\alpha\sigma}^\dagger + c_{\beta\sigma}^\dagger)(c_{1\sigma} + c_{1\bar{\sigma}}) + c.c. \right] + t \sum_{m,n} c_{m\sigma}^\dagger c_{n\sigma}
\]

and

\[
\hat{T} = t_\phi \left[ c_{3\sigma}^\dagger c_{4\sigma} + c_{3\bar{\sigma}}^\dagger c_{4\bar{\sigma}} \right] + c.c
\]

with

\[
t_\phi = t e^{-i\pi(\phi/\phi_0)}
\]

where \( U \) and \( V_0 \) correspond, respectively, to the electronic repulsion and the gate potential on the two states, \( \alpha \) and \( \beta \) with energies \( \varepsilon_\alpha \) and \( \varepsilon_\beta \), respectively. The hopping matrix element between these two states and their nearest-neighbor sites 1 and 1 is \( t' \) and \( m \) and \( n \) stand for the other atomic sites of the cluster which interact through the nearest-neighbor hopping \( t \). The external magnetic field producing the flux is incorporated in the matrix elements between the cluster and the rest of the ring, \( t_\phi \) where \( \phi \) is the magnetic flux crossing the ring, \( \phi = \oint \vec{A} \cdot d\vec{l} \), \( \phi_0 \) is the quantum of flux and \( \vec{A} \) is the vector potential of the external magnetic field.

The ring is supposed to be in contact with an external reservoir of electrons which fixes the Fermi level of the system. Therefore, a variation of the applied magnetic flux or the gate potential leads to a change of the total number of particles in the ring. To obtain the
persistent current we calculate the Green function $\hat{G}$ for the complete system, within the chain approximation of a cumulant expansion \[12\] for the dressed propagators, by solving the Dyson equation

$$\hat{G} = \hat{g} + \hat{g}\hat{T}\hat{G}. \quad (5)$$

where $\hat{g}$ is the cluster Green function matrix obtained by the Lanczos method. To take the charge fluctuation inside the cluster into account, we write $\hat{g}$ as a combination of the Green function of $n$ and $n+1$ particles with weights $(1-f)$ and $f$, respectively. The Green function and the charge of the cluster can be written as,

$$\hat{g} = \hat{g}_n(1-f) + \hat{g}_{n+1}f \quad (6)$$

$$Q_c = (1-f)n + f(n+1) \quad (7)$$

The charge can also be expressed as,

$$Q_c = \int_{-\infty}^{\epsilon_F} \sum_i ImG_{ii}(w)dw \quad (8)$$

where $i$ runs over the cluster sites. These equations are solved self-consistently in order to obtain $f$, $n$ and $\hat{G}$. The persistent current $J$ and the charge in the dot $Q_d$ are obtained from,

$$J = \frac{1}{2\pi} \int_{-\infty}^{\epsilon_F} Im [G_{i,i+1}(w) - G_{i+1,i}(w)] dw \quad (9)$$

and

$$Q_d = \int_{-\infty}^{\epsilon_F} Im [G_{\alpha,\alpha}(w) + G_{\beta,\beta}(w)] dw \quad (10)$$

Throughout this letter the energies are taken in units of $t$. To study the effect of the scaling on the persistent current we consider two rings with lengths $L = 200$ and 2000, in units of the lattice parameter. According to our previous discussion we take a set of parameters compatible with an adequate Kondo temperature and a sharp charge quantization. The two dot level energies are chosen to be $\epsilon_\alpha = 0$ and $\epsilon_\beta = 2$ and $t' = 0.05$. The Fermi level is fixed at $\epsilon_F = 0$. The electron-electron interaction and the magnetic flux responsible for the persistent current are chosen to be $U = 2$ and $\phi = \phi_0/4$. The physics of the problem is not sensitive to the actual values of $U$ and $\phi$.

For $V_0 > 0$ the levels $\epsilon_\alpha$ and $\epsilon_\beta$ are above the Fermi energy and the dot has no electrons. The density of states of the system at the QD has two peaks at these one-particle dot states. As $V_0$ is lowered the level $\epsilon_\alpha$ goes through the Fermi level, permitting the entrance of electrons into the QD, increasing the charge in the whole ring and driving the system into the Kondo regime. This is shown in Fig.1 for $V_0 = -0.3$ and $L = 2000$ where the $KR$ appears as a sharp peak at the Fermi level and the dot resonance as a wider peak below it. Notice also the presence of a splitted Coulomb peak separated from the dot resonance by an energy of the order of $U$. In this situation a persistent current is installed in the ring.

The current and the charge inside the dot are shown in Fig.2 as a function of the gate potential. The current has a peak at the dot resonance ($V_0 \sim 0$) and, as $V_0$ decreases it changes its sign presenting a sharper and larger negative peak, as can be seen in Fig. 2(a).
This last behavior reflects the entrance of the system into the Kondo regime. The causes for the current sign change and the value of its peak intensity will be discussed later. The current in this regime is due to the KR which provides an extra channel for the electrons to go through the dot. Although almost pinned to the Fermi level, the KR moves slightly as $V_0$ decreases. This behavior, shown in the inset of Fig.1 for $L = 2000$, is to be expected since the system does not possess particle-hole symmetry [9]. Due to the sharpness of the KR its movement through the Fermi level results in a very sharp peak in the $I - V_0$ characteristic curve. Its intensity has a maximum at $V_0 \sim -0.5$ when the KR is at about $\varepsilon_F$. Decreasing further the gate potential the current promoted by the Kondo effect persists up to the point where the Coulomb peak aligns with the Fermi level and a second charge enters into the dot, as shown in Fig. 2(b). This is reflected in the current by the existence of another peak at $V_0 = -2.0$. The system has a completely different behavior when $V_0$ is still further reduced. The absence of a net spin when there are two electrons inside the dot inhibits the Kondo effect and the current goes to almost zero, as expected.

The current is an odd function with respect to $V_0 = -0.4$, as displayed in the Fig.2(a), which reflects the electron-hole symmetry of the system for this value of gate potential. The behavior of the dot charge as a function of the gate potential explains the change of the current sign. When the Kondo resonance is partially above the Fermi level the charge inside the dot is less than one, ($Q_d < 1$). The transit of the KR through the Fermi level produces a small cusp in the curve of the dot charge at $V_0 \sim -0.5$ and the current varies abruptly and changes its sign. This is a consequence of the extra charge at the dot that, according to Friedel sum rule [13], produces a phase shift of $\pi$ in the wave function at the Fermi level. This implies that the wave vector $k$ which contributes to the current changes sign during the charging process and so does the current.

The sharpness of the KR compared to the energy spacing of the ring, which restricts the participation of only one ring state in the current, permits the elaboration of an explanation for the enhancement of the Kondo peak current relative to $I_0$ (the current of a perfect ring of the same size). The argument goes as follows: the persistent current of a ring with a sharp Kondo localized peak weakly coupled to the ring states can be calculated, within the one-particle framework provided by the mean field approximation of the slave bosons formalism [14], using degenerate perturbation theory. If the localized level width is much smaller than the ring energy spacing, that is $\Delta_d \ll \Delta E$, this level is strongly hybridized only with an almost degenerate ring state giving rise to a bonding and anti-bonding states, which carry opposite currents. The only situation in which a net current could go along the ring is when the bonding and the anti-bonding states are respectively below and above the Fermi level. The bonding state is therefore the only one participating in the current. In this limit it is straightforward to show that the diagonalization of the $2 \times 2$ sub-matrix of the degenerated states at the Fermi level gives a contribution to the current which scales as the norm of the ring state, namely $L^{-1/2}$. In a perfect ring, the $L^{-1}$ dependence of the persistent current results from the partial cancellation among the currents carried by the different states participating in the transport. On the other hand, if the ring localized level does not satisfy the condition $\Delta_d \ll \Delta E$, perturbation theory is no longer valid and the current has an intermediate scaling between the $L^{-1}$ and $L^{-1/2}$ regimes. This behavior can be obtained by studying the persistent current of a ring with a dot at resonance as a function of the ring length $L$, for different values of the dot resonant width $\Delta_d \sim t'^2/W$. The results
are displayed in Fig.3 where the persistent current maximum intensity as a function of the ring length $L$ is plotted in a Log-Log scale for several values of $t'$. The figure shows the $L^{-1}$ scaling for a perfect ring ($t'=1$), the $L^{-1/2}$ scaling for a weakly connected dot ($t'=0.01$) and an intermediate behavior for other situations. Notice that for a ring of length $L = 100$ the current for $t' = 0.05$ is about 0.4 of the current of the perfect ring, while for $L = 2000$ the persistent currents circulating along both rings have almost the same value.

The width of the $KR$ is of the order of the Kondo temperature $T_K$ which decreases exponentially with the dot state energy relative to the Fermi energy [9]. This allows the coexistence of very narrow peaks with moderate values of the coupling between the ring and the dot. These conditions are consistent with having simultaneously a $L^{-1/2}$ scaling and relatively high current intensities proportional to $t'$. On the contrary, in one-body problems the very small $t'$ required to obtain a narrow peak reduces the current intensity. In a recent measurement of Kondo promoted mesoscopic currents [10], the values of $T_K$ and of the width of the $KR$ were found to be of the order of 0.01 meV. If we consider a typical ring of length $L = 2000$ the energy spacing of the ring states satisfies $\Delta E >> T_K$ and so, according to our above discussion, the persistent current in the Kondo regime is enhanced relative to its value for a perfect ring and for a ring with a dot at resonance. This enhancement can be clearly seen in Fig.2(a). The maximum intensity of the Kondo current ($V_0 \sim -0.5$) is greater than $I_0$ and also greater than the dot resonance current intensity ($V_0 \sim 0$).

The current intensity of a ring with $L = 200$ is also shown in Fig.2(a). As expected, it results to be larger than the one for $L = 2000$. Although the mesoscopic nature of the persistent current appears for all gate potentials, by increasing $L$ the Kondo current decreases much less than the current circulating when the dot is at resonance. This is a consequence of the already discussed different scaling behavior of these two regimes.

Summarizing, we have analyzed the persistent current going through a quantum dot embedded in a mesoscopic ring. Unlike the current of a perfect ring the persistent current in this system depends only upon the states at the vicinity of the Fermi level. Our study shows that the Kondo resonance provides a new channel for the electron to go along the system. As the Kondo resonance is very sharp the contribution to the current comes from the state at the Fermi level and the current scales as $L^{-1/2}$.

The Kondo current results to be greater than the current of a perfect ring of same length. This property makes the detection of the Kondo effect in this structure a feasible and very interesting possibility.

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FIGURE CAPTIONS:

FIG.1: Local density of states at the QD for $V_0 = -0.3$ and a ring length $L=2000$. The inset shows the detail of the Kondo peak for three values of $V_0$.

FIG2.: a) Persistent current as a function of $V_0$ through a QD inserted in a ring of size $L = 2000$ (continuous line) and $L = 200$ (dotted line) relative to the persistent current ($I_0$) of a perfect ring of size $L = 2000$, for $t' = 0.05$. (b) Charge at the QD as a function of $V_0$ for a ring with $L = 2000$.

FIG. 3: Enhancement of the persistent current through a QD resonance (without e-e correlation) relative to the persistent current of a perfect ring of size $L = 2000$ as a function of $L$, in a Log*Log scale, for four values of $t'$. 
