Hadronization of $b \to c\bar{c} s$

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Abstract

The $b \to c\bar{c} s$ transition is usually believed to hadronize predominantly in $\overline{B} \to X_c D_s^{(*)-}$ with the $D_s^{(*)-}$ originating from the virtual $W$. We demonstrate in a variety of independent ways that other hadronization processes cannot be neglected. The invariant mass of $\bar{c}s$ has sizable phase-space beyond $m_{D}+m_{K}$. The rate for $\overline{B} \to D\overline{D} \ K X$ could be significant and should not be ignored as was done in previous experimental analyses. We estimate the number of charmed hadrons per $B$-decay, $n_c$, to be $\approx 1.3$ to higher accuracy than obtained in previous investigations. Even though $n_c$ is currently measured to be about 1.1, observing a significant $\overline{B} \to D\overline{D} \ K X$ would support $n_c \approx 1.3$. Many testable consequences result, some of which we discuss.
At present, there appears to be a conflict between experiment and theory for fitting both the inclusive semileptonic branching ratio and the number of charmed hadrons per $B$ decay

\[ n_c = 1 - B(b \rightarrow \text{no charm}) + B(b \rightarrow c\bar{c}s') \approx 1 + B(b \rightarrow c\bar{c}s') . \]  

(1)

The prime indicates that the corresponding Cabibbo-suppressed mode is included. Experimentally the inclusive semileptonic $BR$ has been measured accurately to be

\[ B(\overline{B} \rightarrow X\ell\nu) = (10.4 \pm 0.4)\% , \]  

(2)

and $n_c$ is measured as

\[ n_c = 1.10 \pm 0.06 . \]  

(3)

A value of $B(b \rightarrow c\bar{c}s') \approx 0.1$, suggested by Eqs. (1) and (3), would lead to a theoretical prediction of $B(\overline{B} \rightarrow X\ell\nu)$ that is too large—i.e., inconsistent with its measured value (2). On the other hand, theory predicts $n_c \approx 1.3$ when the observed semileptonic $BR$ is used as input, which is demonstrated below. Thus a conflict arises between (2) and (3) [2,3].

Recently, Bagan et al. and Voloshin made progress on the theoretical side [4–7]. Bagan et al. [4–6] performed a complete next-to-leading order analysis of inclusive $B$ decays, which included important final state mass effects in the QCD corrections. The predicted $B(\overline{B} \rightarrow X\ell\nu)$ agrees with (2), within uncertainties that are dominated by renormalization scale-dependences in the perturbative calculation [4–6]. Simultaneously, an enhancement of $B(b \rightarrow c\bar{c}s')$ was found [4–7], albeit with considerable uncertainties. Table I summarizes these recent theoretical findings [6]. The main sources of the large errors in those studies are dependence on the renormalization-scale ($m_b/2 < \mu < 2m_b$), dependence on the renormalization-scheme ($\overline{MS}$ versus pole mass), and uncertainties in quark masses. Although this theoretical analysis hints that $n_c$ may be larger than currently measured [5–7,2,3], it is difficult to draw firm conclusions from this direct calculation of $B(b \rightarrow c\bar{c}s')$ in view of the large uncertainties.
It should also be stressed at this point, that the experimental determination of $B(\bar{B} \to X\ell\nu)$ is reliable and accurate. In contrast, the measurement of $n_c$ is a sum over the inclusive yields of many charmed hadron species in $B$ decays. It is thus prone to large uncertainties, perhaps larger than currently realized.

Figure 1 displays the discrepancy graphically. We discuss now in some detail how the theoretical curve has been generated. Our objective is to draw the most accurate curve of $n_c$ versus semi-electronic $BR$ with presently available theoretical calculations. We do not use the prediction for $B(b \to c\bar{s}s')$ because it involves large errors, but rather proceed as follows. We start with

$$B(b \to c) = 1 - B(b \to \text{no charm}) ,$$

where $B(b \to \text{no charm})$ is small, typically at the percent level. We take

$$r_\ell \equiv \Gamma(b \to \text{no charm})/\Gamma(b \to ce\nu) = 0.25 \pm 0.10,$$

(5)

to account for the small fraction of $b \to s + \text{no charm}$ and charmless $b \to u$ transitions. Furthermore we use

$$r_\tau \equiv \frac{\Gamma(b \to c\tau\nu)}{\Gamma(b \to ce\nu)} = 0.25 ,$$

(6)

which is in accordance with the result of Ref. [11] and also agrees with a recent ALEPH measurement [12].

$$B(b \to X\tau\nu) = (2.75 \pm 0.30 \pm 0.37)\% .$$

(7)

The last required ratio is $\Gamma(b \to c\bar{u}d')/\Gamma(b \to ce\nu)$ where the dominant uncertainties in $|V_{cd}|^2$ and in fermion masses cancel. Bagan et al. [4] have presented a complete computation of this quantity in next-to-leading logarithmic approximation taking all final-state charm quark mass effects into account. Based on this perturbative calculation and also including nonperturbative corrections up to $O(1/m_b^2)$, the analysis of [4] yields,

$$r_{ud} \equiv \frac{\Gamma(b \to c\bar{u}d')}{\Gamma(b \to ce\nu)} = 4.0 \pm 0.4 .$$

(8)
Here the error comes almost entirely from the renormalization-scale uncertainty and represents a conservative estimate when working to order $O(1/m_b^2)$. Because nonperturbative effects at $O(1/m_b^3)$ could introduce rate-differences at the 10% level between $B^-$ and $\overline{B}_d$ decays governed by $b \to c \bar{u}d$ [13], there is considerable room for additional studies.

Combining Eqs. (4), (5), (6), (8), the $b \to c \bar{u}cs'$ branching fraction can be written as

$$B(b \to c\bar{c}ss') = 1 - (2 + r_\tau + r_{ud} + r_\ell) B(\overline{B} \to X_c e\nu)$$

$$= 1 - (6.50 \pm 0.40) B(\overline{B} \to X_c e\nu). \quad (9)$$

In this relation the very small contribution from $b \to u \bar{c}cs'$ transitions has been neglected. Eqs. (4) and (9) yield the number of charms per $B$ decay as

$$n_c = 2 - (2 + r_\tau + r_{ud} + 2r_\ell) B(\overline{B} \to X_c e\nu)$$

$$= 2 - (6.75 \pm 0.40) B(\overline{B} \to X_c e\nu), \quad (10)$$

where we note that $B(b \to c\bar{c}ss')$ drops out in the linear relation between $n_c$ and $\overline{B} \to X_c e\nu$, and that the relation is largely free from uncertainties in masses of $b$ and $c$ quarks since the error is dominated by the uncertainty in $r_{ud}$. Figure 1 shows the discrepancy between theory given by Eq. (10) and experiment.

The precisely measured semileptonic $BR$ together with Eqs. (4)- (10) gives

$$B(b \to c\bar{c}ss') = 0.32 \pm 0.05, \quad (11)$$

$$n_c = 1.30 \pm 0.05. \quad (12)$$

This is our central result. Our predictions for $B(b \to c\bar{c}ss')$ and for $n_c$ agree with the central values obtained in previous theoretical investigations [3,4,5,6,7] but have smaller errors. As discussed in more detail below, such a large value of $B(b \to c\bar{c}ss')$ requires a significant rate for $\overline{B} \to D\bar{D} K X$. We predict the observation of (a) $\overline{B} \to D^{(*)}\bar{D}^{(*)} K$ modes with significant $BR$’s, (b) enhanced $\ell^+ D$ and $\ell^- D$ correlations where the primary lepton originates from one $B$ and the charmed hadron from the other $B$ in the event, and (c) enhanced $DD$ and $\overline{D} \overline{D}$ correlations at the $\Upsilon(4S) \to B\overline{B}$.
If the predicted effects will be observed, then the $B(b \to c\bar{c}s')$ is larger than currently determined by experiment. The measured number of charm per $B$ will not change by those observations, but the larger $B(b \to c\bar{c}s')$ would indicate that the current experimental value of $n_c$ is underestimated. In that case, a careful re-evaluation of all errors involved in measuring $n_c$ would be in order, including re-assessments of absolute $BR$’s of the charmed hadrons some of which are poorly known. On the other hand, non-observation of our predictions would indicate an enhancement of the $b \to c\bar{u}d$ transition over the parton estimate \cite{14} and/or a larger rate than anticipated for charmless $b \to s$ transitions \cite{15,16}.

Theory alone or experimental measurements alone have large inherent uncertainties for $B(b \to c\bar{c}s')$. We therefore adopted a hybrid approach which uses well measured quantities from experiment in conjunction with reliably calculated quantities from theory to determine $B(b \to c\bar{c}s')$ to higher accuracy \cite{8}.

One conventional way to determine $B(b \to c\bar{c}s)$ is to add the inclusive yield of $D_s$ \cite{9,14,15}

$$R_{D_s} \equiv B(\bar{B} \to D_{s}^{-}X) + B(\bar{B} \to D_{s}^{+}X)$$

(13)

to the other observed final states governed by $b \to c\bar{c}s$ \cite{18},

$$B(b \to c\bar{c}s) = R_{D_s} + B(\bar{B} \to \Xi_c\bar{\Lambda}_cX) + B(\bar{B} \to (c\bar{c})X) = 0.12 + 0.01 + 0.03 = 0.16 \pm 0.02 .$$

(14)

$(c\bar{c})$ denotes charmonia not seen in $D\bar{D}X$ such as $J/\psi, \psi', \eta_c, \eta'_c, \chi_c, h_c, 1^{3}D_2$. Within errors, this agrees with the experimental measurement of $n_c$

$$B(b \to c\bar{c}s') = n_c - 1 + B(b \to \text{no charm}) = 0.13 \pm 0.06 .$$

(15)

The agreement appears to support the low value of $n_c$.

Our determination of $B(b \to c\bar{c}s')$ suggests a different picture as to how $b \to c\bar{c}s$ hadronizes. A systematic classification shows that five classes of hadronization can occur, see Table II. Conventional wisdom \cite{9,14,15} assumes that most of the inclusive $D_s$ production in $B$ decays originates from the virtual “W”. Motivated by the observed inclusive
momentum spectrum of $D_s$ in $B$ decays and by a simple theoretical argument given below, we predict instead that only about 70% of the inclusive $D_s$ yield in $B$ decays contribute to $\bar{B} \to DD_s^-X$ processes. The remaining $D_s$ (about 30%) could occur in conjunction with $s\bar{s}$ fragmentation. We will return to this point below.

The branching ratio for class (a) is thus depleted and becomes about $0.7R_{D_s}$. [This branching ratio can be at most $R_{D_s}$, which would soften our conclusion by a small amount only]. The branching ratios of the observed classes (a)-(c), do not add up to 30%. Thus class (d) must have a sizable branching fraction of about 20%,

$$B(\bar{B} \to D\bar{D}KX) \sim 20\%.$$  \hfill (16)

There are several interesting experimental implications. Those modes can be studied at CLEO and at LEP. CLEO has higher statistics, whereas LEP has the ability to separate one $B$ from the other $b$ hadron. Thus far, however, they have not been seriously searched for. The low $Q$ value in this process suggests that a significant portion will be three body [23],

$$\bar{B} \to D^{(*)}\bar{D}^{(*)}K.$$  

Because the responsible Hamiltonian is isospin zero, many isospin relations can be used to facilitate the observation of those modes [24].

Finally, the class (e) processes involve $s\bar{s}$ fragmentation. Their branching ratio could be non-negligible, at the few percent level [22]. A few exclusive final states would then carry the lion’s share of the class (e) branching ratio, because of limited phase-space.

Before proposing a number of tests, we discuss briefly a few additional indications that support our hypothesis from

(a) a naive Dalitz plot analysis [25],
(b) measured inclusive kaon yields in $B$ decays, and
(c) measured inclusive $D$ momentum spectra in $B$ decays.

Figure 2 shows the $b \to c\bar{c}s$ Dalitz plot resulting from the $(V - A) \times (V - A)$ matrix element, where the initial and final spins were averaged and summed. In this simple model,
the $\bar{c}s$ system hadronizes as a $D_s^- X$ for invariant $\bar{c}s$ masses below $m_D + m_K$. In contrast, for

$$m_{\bar{c}s} > m_D + m_K$$

the $\bar{c}s$ is not seen as a $D_s^- X$ but rather as $D K X$. The Dalitz plot region contributing to $D_s$ production in $b \to c \bar{c}s$ decay is $m_{\bar{c}s} < m_D + m_K$, and one obtains

$$\frac{\Gamma(b \to c + D_s^-)}{\Gamma(b \to c\bar{c}s)} \approx 0.35 .$$

This argument suggests that a large fraction of $b \to c\bar{c}s$ transitions has not been accounted for in previous investigations [9,17]. (See however the analyses of Refs. [23,26] which reach similar conclusions to ours.) Of course, the naive Dalitz plot argument is rather crude. It does not address issues of hadronization, resonance bands and their interferences, QCD-corrections, and interferences between penguin-amplitudes ($b \to s$) with the dominant spectator-amplitude ($b \to c\bar{c}s$). Nevertheless, the Dalitz plot conveys the important message that a significant fraction of $b \to c\bar{c}s$ processes could be seen in $D \bar{D} K X$.

The surplus of the inclusive kaon yield in $B$ decays beyond all the conventional sources again indicates a significant $B(\bar{B} \to D \bar{D} K X)$ [22]. The indication is further strengthened by the large observed $K$-flavor correlation with its parent $B$-flavor at time of decay [27,28]. The flavor of the kaon in $\bar{B} \to D \bar{D} K X$ is 100% correlated with its parent $b$-flavor. The momentum spectra of the inclusive $D$ yields in $B$ decays indicates an excess of low momentum $D$’s over conventional sources [29]. A natural explanation can be found in $\bar{B} \to D \bar{D} K X$.

We are now ready to suggest several tests. In addition to the “indirect” measurement using $B(b \to c\bar{c}s') \approx n_c - 1$ which involves large errors, we suggest to directly determine $B(b \to c\bar{c}s')$ by adding up the “wrong-sign” charm in tagged $B$ decays [3,8],

$$B(b \to c\bar{c}s') \approx B(b \to \bar{c'}) = B(\bar{B} \to D_s^- X) + B(\bar{B} \to \bar{D} X) + B(\bar{B} \to \Xi_c X) + B(\bar{B} \to \Xi_c X) + B(\bar{B} \to (c\bar{c})X) .$$

The traditional lepton and $K^{\pm}$ tags could be supplemented by other tags, such as $K^*$ and jet charge techniques. Further, the number of $DD$ and $DD_s$ events per $\Upsilon (4S) \to B\bar{B}$ decay can
be combined with the single, inclusive $D$ and $D_s$ yields in untagged $B$ decay to determine $B(\overline{B} \to \overline{D}X)$ and $B(\overline{B} \to D_s^- X)$ [8]. Of course, $B^0 - \overline{B}^0$ mixing effects must be corrected for [28]. No tagging is required to measure $B(\overline{B} \to (c\bar{c})X)$.

A sizable $B(\overline{B} \to D\overline{D} KX)$ would show up as a $D^{(*)}K$ (from $cs$) enhancement. The background at the $\Upsilon(4S)$ is much reduced because

$$\Upsilon(4S) \to \overline{B}B \to \overline{D} \to K \to D,$$  \hspace{1cm} (18)

which naturally yields a $DK$ correlation, while its $D\overline{K}$ correlation is suppressed. The Dalitz plot allows to enhance the $D^{(*)}\overline{K}$ signal correlation further by assuming

$$\frac{d\Gamma}{dm_{D^{(*)}\overline{K}}^2} \approx \frac{d\Gamma}{dm_{cs}^2}.$$  

The invariant mass spectrum of the $b \to c\bar{c}s$ transition indicates that $D^{(*)}\overline{K}$ (from $cs$) tends to have a large invariant mass, see Fig. 2.

The inclusive $D_s$ yield in $B$ decays, $R_{D_s}$, has two roughly equal contributions. Figure 3 shows the measured momentum spectrum [19]. Whereas the high peak is dominated by the exclusive two-body modes $\overline{B} \to D^{(*)}D_s^{(*)-}$, the underlying dynamics of the remainder had been unclear. The factorization assumption is successful in predicting ratios of rates for the two-body modes $\overline{B} \to D^{(*)}D_s^{(*)-}$ [19]. Thus we assume factorization and predict that $b \to c + D_s^{(*)-}$ is dominated by the exclusive two-body decays $\overline{B} \to D^{(*)}D_s^{(*)-}$ in analogy to semileptonic decay of $B$ mesons. We calculate that

$$\frac{\Gamma(\overline{B} \to D^{(*)}D_s^{(*)-})}{\Gamma(b \to c + D_s^{(*)-})} = 0.7 \pm 0.2,$$  \hspace{1cm} (19)

where the quoted error refers to a variation in the $b$-quark mass, $4.4 \leq m_b \leq 5.2$ GeV, and in the slope of the Isgur-Wise function [30], $\rho^2 = 0.84 \pm 0.15$. The numerator is the sum over the four exclusive two-body rates obtained [31,32] by using the heavy quark limit [33]. The denominator is the sum of two rates $b \to c + D_s$ and $b \to c + D_s^*$. It treats the $b \to c$ transition as if it were that of free quarks [13]. The decay constant $f_{D_s}$, the CKM elements
and the factorization parameter \( a_1 \) all cancel in the ratio. The prediction Eq. (13) can currently be tested since the ratio \( \Gamma(B \to D^{(*)} D_s^{(*)-})/\Gamma(b \to c + D_s^{(*)-}) \) is an observable in which the uncertainty due to \( B(D_s \to \phi \pi) \) cancels. The prediction Eq. (19) together with the measured ratio \( |R_{D_s}^b| = 0.46 \pm 0.04 \),

\[
\frac{B(\overline{B} \to D^{(*)} D_s^{(*)-})}{R_{D_s}} = 0.46 \pm 0.04 ,
\]

yields that \( B(\overline{B} \to D D_s^- X) \approx B(b \to c + D_s^{(*)-}) = (0.7 \pm 0.2) R_{D_s}. \)

The remainder of the inclusive \( D_s \) yield in \( B \) decays \( [R_{D_s} - B(\overline{B} \to D D_s^- X) = (0.3 \pm 0.2) R_{D_s}] \) could be a significant fraction of the lower momentum \( D_s \) mesons. One sizable source for it could be the \( b \to c\bar{c}s \) transition with \( \bar{s}s \) fragmentation \( \|22\| 

\[
B(b \to c\bar{c}s + \bar{s}s) \approx 0.01 - 0.03. \]

One generally expects one \( D_s^- \) per such a transition, as long as \( D_{s**}^- \) and higher \( D_s^- \) resonance production in \( b \to c\bar{c}s + \bar{s}s \) transitions is negligible. The total \( D_s^- \) production in flavor-tagged \( \overline{B} \) decays is thus expected to be

\[
B(\overline{B} \to D_s^- X) \approx B(b \to c\bar{c}s + \bar{s}s) + B(\overline{B} \to D D_s^- X) \approx 0.1. \]

The \( D_s^+ \) yield in flavor-tagged \( \overline{B} \) decays is governed by the \( b \to c \) transition with \( \bar{s}s \) fragmentation, and may be non-negligible

\[
B(\overline{B} \to D_s^+ X) = R_{D_s} - B(\overline{B} \to D_s^- X) \sim 10^{-2}. \]

For a model of the relative contributions to the \( D_s^+ \) yield from \( b \to c\bar{c}d, c\ell\nu, c\bar{c}s \) transitions with \( \bar{s}s \) fragmentation, we refer the reader to Ref. \( \|22\| \). The \( D_s^+ \) yield in flavor-tagged \( \overline{B} \) decays has been traditionally neglected \( \|15,17,4\| \).

In conclusion, by combining reliable theoretical calculations and precise experimental measurements \( \|8\| \), we obtain a more accurate estimate of \( B(b \to c\bar{c}s') \) and of \( n_c \) than previous investigations \( \|14,17,2,5,7\| \). We predict
\[ B(b \to c\bar{c}s') = 0.32 \pm 0.05 \text{ and } n_c = 1.30 \pm 0.05, \] (25)

which is significantly larger than the low experimental value \( n_c|_{\text{exp}} = 1.10 \pm 0.06 \). We believe that (25) is on firm ground, and expect an increase in the measured \( n_c \) in the future. Our prediction can be tested in a variety of ways. First we advocate to measure \( B(b \to c\bar{c}s') \) by counting up the number of anticharmed hadrons (the “wrong” charm flavor) per \( \bar{B} \)-decay.

A sizable \( BR \) for \( \bar{B} \to DDKX \) is our second prediction. It shows up as a large \( \ell^- D \) and \( \ell^+ \bar{D} \) correlation after removing \( B^0 - \bar{B}^0 \) mixing effects [28], where the primary lepton comes from one \( B \) hadron and the charmed meson from the other \( B \)-hadron in the event. It can also be seen by observing the exclusive modes \( \bar{B} \to D^{(*)}D^{(*)}K \), and/or by searching for \( D^{(*)}K \) (from \( cs \)) correlations. There are many additional implications, consequences and tests which we hope to discuss in a forthcoming report [22].

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TABLE I. The predicted semileptonic branching ratio, the $B(b \to c\bar{c}s')$ and $n_c$ taken from Bagan et al. [6].

| Scheme      | $B(\overline{B} \to X_c\ell\nu)$ | $B(b \to c\bar{c}s')$ | $n_c$      |
|-------------|----------------------------------|------------------------|------------|
| MS          | $0.112 \pm 0.017$                | $0.35 \pm 0.19$       | $1.35 \pm 0.19$ |
| Pole mass   | $0.120 \pm 0.014$                | $0.27 \pm 0.07$       | $1.27 \pm 0.07$ |

TABLE II. The five classes of hadronization of $b \to c\bar{c}s$. ($c\bar{c}$) denotes charmonia not seen in $D\bar{D}X$, and class (e) involves $\bar{s}s$ fragmentation.

| Class | Mode                | $BR$              | Reference |
|-------|---------------------|-------------------|-----------|
| (a)   | $\overline{B} \to Ds^-X$ | $(0.7 \pm 0.2)R_{Ds} \approx 0.08$ |           |
| (b)   | $\overline{B} \to \Xi_c\bar{\Lambda}_cX$ | $0.01$ |           |
| (c)   | $\overline{B} \to (c\bar{c})KX$ | $0.03$ |           |
| (d)   | $\overline{B} \to DD\bar{K}X$ | $\sim 0.2$ |           |
| (e)   | $b \to c\bar{c}s + \bar{s}s$ | $\sim few \times 10^{-2}$ |           |
| Total:| $b \to c\bar{c}s$ | $0.31 \pm 0.05$ |           |
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The inclusive yields of D_s[19] and Λ_c[20] in B decay can be expressed in terms of the B(D_s → φπ) and B(Λ_c → pK^−π^+) as,

R_{D_s} = (0.12 ± 0.01) \frac{0.035}{B(D_s → φπ)} ,

R_{Λ_c} = (0.041 ± 0.008) \frac{0.044}{B(Λ_c → pK^−π^+)} .

We choose the current central values B(D_s → φπ) = 0.035 and B(Λ_c → pK^−π^+) = 0.044. We alert the reader that smaller absolute BR’s for D_s and Λ_c decays increase the yield of charm per B, and would lessen the discrepancy between experiment and theory regarding n_c. B(\bar{B} → Ξ_cΛ_cX) is obtained by combining R_{Λ_c} and the relevant ℓ^±Λ_c measurement[21] where the primary lepton comes from one B and the Λ_c from the other B in the Υ(4S) event. The inclusive BR into (c\bar{c}) charmonia is obtained[22] to be 0.026 ± 0.004 by summing over their observed and estimated BR’s which is larger than previous estimates[9].

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A detailed analysis of inclusive K^* yields in B decays suggests that the dominant source
of $K^*$ in $B^-$ decays comes from intermediate charmed hadrons. Thus there is not much room for $K^*$ in the process $B \rightarrow D\bar{D}K^*X$.

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FIG. 1. Number of charm per $B$ decay ($n_c$) is plotted against the $B$ meson semileptonic branching ratio. The uncertainty in the theoretical prediction is indicated by dashed lines.
FIG. 2. Dalitz plot of the decay $b \to c\bar{c}s$ as a function of $u = m_{cs}^2/m_b^2$ and $s = m_{cs}^2/m_b^2$. The projection onto the $s$ axis is shown at the bottom where the $\overline{D}K$ threshold is indicated by an arrow.
FIG. 3. Momentum spectrum of inclusive $D_s$ mesons produced in untagged $B$ decays at the \( \Upsilon(4S) \) as measured by the CLEO collaboration. The parameter $x$ is defined by $x = p_{D_s}/p_{\text{max}}$ where $p_{\text{max}}^2 \equiv E_{\text{beam}}^2 - M_{D_s}^2$. The continuum background has been subtracted.