Study on the Scattering Properties of Cylinders in Suspensions Containing Sediment Particles

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Abstract. A method is presented that combining the Mclements suspension model with the cylindrical model to solve the complexity of calculating the scattered acoustic pressure around the cylinder, when suspended particles are contained in the water. The scattered acoustic pressure of an infinitely long cylinder in water is derived based on the boundary conditions. Then, the wave number of the particle-containing suspension is calculated by the Mclements suspension model, which is combined with the scattered acoustic pressure of an infinitely long rigid cylinder in water. The scattered acoustic pressure of an infinitely long rigid cylinder in suspension is obtained. The results show that the content of sediment particles has a greater impact on the distribution of scattered acoustic pressure. The scattered acoustic pressure is more directional with the increase of the content of sand particles, laying a foundation for further research on other target objects in suspension.

1. Introduction
Acoustic scattering is a very interesting physical phenomenon and is widely used in practical applications, such as exploration engineering [1], underwater acoustics [2], medical equipment and material testing, etc. The application of early scattering theory to cylinders has been reported in radiation simulation of fiber composites [3]. At present, there are many studies on the characteristics of target objects by studying the scattering of acoustic waves from water objects, but in actual situations, sediment particles are often mixed in water, which makes the problem more complicated. More and more scholars are paying attention to the application of ultrasonic method in particle size measurement. [4] However, there are very few studies on the scattering characteristics of target objects in suspensions, especially the scattering of cylinders in suspensions containing sediment particles. Therefore, to study the scattering characteristics around cylinders in suspensions containing sediment particles is of great significance. Both sediment particles and cylinders have a scattering effect on acoustic waves. In the present research, the scattering characteristics of cylinders in water and suspensions containing sediment particles are mainly studied.

2. Theoretical analysis and numerical study of acoustic pressure scattered by a cylinder in water
When acoustic waves travel through water and encounters an infinitely long rigid cylinder, the cylinder will have a scattering effect on the acoustic wave. The scattering problem of a cylinder regards the incident wave and the scattered wave as a combination of various orders of cylindrical acoustic waves,
and then substitutes the boundary conditions of the cylindrical surface to obtain the scattered wave amplitude.

2.1. Theoretical analysis of scattered acoustic pressure in pure water

As shown in Figure (1), take the cross section of the cylinder to establish a coordinate system. The cylinder radius is \( a = 0.2 \text{m} \). The acoustic wave incident along the \( x \) direction perpendicular to the cylinder axis, and the cylinder axis coincides with the \( z \) axis.

![Acoustic scattering model of a rigid cylinder.](image)

\( \text{Figure 1. Acoustic scattering model of a rigid cylinder.} \)

The incident acoustic pressure is

\[
p_i = e^{j(\omega t - kr)} = e^{-jkr\cos\theta} e^{j\omega t}
\]

Where \( k \) is incident wave number, \( \omega \) is angular frequency, \( \theta \) is scattering angle.

The incident wave is decomposed into cylindrical waves of various orders and superimposed as:

\[
p_i = e^{j\omega t} [J_0(kr) + 2\sum_{n=1}^{\infty} (-j)^n J_n(kr) \cos n\theta]
\]

Where \( J_n(x) \) denotes the Bessel function of the first kind.

Scattered waves are represented by cylindrical waves of various orders as:

\[
p_s = e^{j\omega t} \sum_{n=0}^{\infty} b_n H_n^{(2)}(kr) \cos n\theta
\]

Where \( H_n(x) \) denotes the Hankel function of the second kind, \( b_n \) is the scattering coefficient to be solved.

According to the cylindrical boundary condition:

\[
\frac{\partial}{\partial r}(p_i + p_s) \bigg|_{r=a} = 0
\]

Therefore,

\[
b_n = (-j)^n \varepsilon_n \frac{d}{d(ka)} J_n(ka)
\]

When \( n = 0 \), \( \varepsilon_n = 1 \), otherwise \( \varepsilon_n = 2 \). Substituting the coefficient \( b_n \) can obtain the scattered acoustic pressure as:

\[
p_s = \sum_{n=0}^{\infty} (-j)^n \varepsilon_n \frac{d}{d(ka)} J_n(ka) H_n^{(1)}(kr) \cos(n\theta)
\]
2.2. Distribution of scattered acoustic pressure in water.
In order to study the distribution of scattered acoustic pressure, three different wave numbers \( ka = 5 \), \( ka = 10 \), and \( ka = 15 \) is selected for calculation. Figures 2, 3, and 4 show the scattered acoustic pressure distribution around a cylinder at different wave numbers.

It can be seen from Fig. 1 to Fig. 3 that the scattered acoustic pressure has obvious directivity and changes with the wave number, that is, when the frequency increases, the scattered acoustic pressure distribution is different. At low wave numbers, that is, at low frequencies, the backscatter acoustic pressure is weak, and as the frequency increases, the backscatter acoustic pressure gradually increases.

Figure 2. Scattered acoustic pressure distribution at \( ka = 5 \).

Figure 3. Scattered acoustic pressure distribution at \( ka = 10 \).

Figure 4. Scattered acoustic pressure distribution at \( ka = 15 \).
3. Theoretical Analysis and Numerical Study of Scattered Acoustic Pressure of a Cylinder in Suspension

3.1. Theoretical analysis of scattered acoustic pressure in suspension

When the water contains sediment particles, the Mclements suspension model is introduced in this paper to obtain the sonic wave number expression [5].

\[
\left( \frac{k}{k} \right)^2 = 1 + \frac{3\varphi}{ikR} \sum_{n=0}^{\infty} (2n+1)A_n
\]  

(7)

In formula (7), the particle volume content is represented by \( \varphi \), the complex wave number of the continuous phase of the fluid is represented by \( k \), \( A_n \) is the n-th order ultraacoustic scattering coefficient matrix, and the particle size is represented by \( R \). Based on the wave equation and the boundary conditions on the particle surface, \( A_n \) can be obtained by solving a 6th order linear equation. Substituting the wave number \( k \) into the scattered acoustic pressure can obtain the scattered acoustic pressure distribution of the cylinder in the suspension containing sediment particles.

3.2. Distribution of scattered acoustic pressure in suspension.

In order to study the influence of sediment particles on the scattered acoustic pressure around the cylinder, particle size of 10 \( \mu m \) is selected here to calculate the distribution of scattered acoustic pressure when the content of the sand particles is different.

**Figure 5.** Scattered acoustic pressure distribution with a sediment content of 0.8%.

**Figure 6.** Scattered acoustic pressure distribution with a sediment content of 1.0%
Fig. 4 and Fig. 5 show that the sediment particles make the scattering acoustic pressure more complicated, and forward scattering and back scattering exhibit a certain symmetry. Compared with the scattered acoustic pressure around a cylinder in pure water, as the sediment content increases, the scattered acoustic pressure around a cylinder in a particle-containing suspension decreases and is more directional.

4. Conclusion
In this paper, a cylinder scattering model in water and a Mcclements suspension model are combined to compare and analyze the effect of sediment particles on the scattering characteristics around the cylinder. Through the numerical results, several major concluding remarks can be drawn as follows:

1. In water, as the frequency increases, the backscattering of the cylinder gradually increases.
2. In suspension, the scattered acoustic pressure becomes more directional as the sediment particle content increases.

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