H(z) diagnostics to discriminate dark energy models

Sergio del Campo, Ramón Herrera and Diego Pavón

\(^1\) Instituto de Física, Pontificia Universidad Católica de Valparaíso, Chile
\(^2\) Departamento de Física, Universidad Autónoma de Barcelona, Spain

E-mail: diego.pavon@uab.es

Abstract. To extract the invaluable information on the nature of dark energy hidden in the history of the Hubble factor we must have at our disposal simple and practical criteria to be used on the data. Here, we propose some diagnostics based on \(H(z)\) that may help discern whether the dark energy interacts with dark matter or not and whether it is phantom or quintessence in nature.

1. Introduction

The nature of the dark energy driving the present accelerated expansion of the Universe constitutes one of the biggest mysteries in cosmology. Within Einstein gravity dark energy may be simply a highly fine-tuned cosmological constant or something more complex, as an evolving scalar or a tachyon field with constant or variable equation of state, \(w\). In either case, dark energy may interact, or not, with dark matter and if it does, the interaction may take a variety of expressions. Interaction with baryonic matter is rather unlikely in view of the severe constraints arising from local gravity experiments [1, 2].

Given the ample manifold of possibilities, diagnostics to tell apart between them should be welcome especially if they are simple and based on the less possible number of assumptions.

In a recent work Sahni et al. [3] introduced the redshift dependent function \(\Omega_m(x) := (E^2(x) - 1)/(x^3 - 1)\), with \(E(x) := H(x)/H_0\) and \(x := 1 + z\), which for spatially flat Friedmann-Robertson-Walker cosmologies (\(\Omega_m + \Omega_x = 1\)) reduces to \(\Omega_m(x) = \Omega_{m0} + (1 - \Omega_{m0})/(x^{3(1+w)} - 1)(x^3 - 1)^{-1}\) with the interesting feature that for the \(\Lambda\)CDM model it yields \(\Omega_m(x) = \Omega_{m0}\), whereas for quintessence fields \((-1 < w < -1/3)\) and phantom fields \((w < -1)\) it gives \(\Omega_m(x) > \Omega_{m0}\) and \(\Omega_m(x) < \Omega_{m0}\), respectively. Because of this diagnostic does not rely on the density parameters \(\Omega_{m0}\) and \(\Omega_x0\) it avoids the drawbacks inherent to our comparatively poor knowledge of them. In fact, it is expected to be useful when accurate data of the Hubble factor, \(H(z)\), at different redshifts become available. This seems likely to come sooner than a substantial improvement in our knowledge of \(\Omega_{m0}\) and \(\Omega_x0\). However, this criterion is to fail if dark energy happens to interact non-gravitationally with dark matter. This is interesting because while the concordance \(\Lambda\)CDM model fits most cosmological data, within statistical errors, rather well recent data from 185 supernovae Ia, at redshifts as low as \(z < 0.08\) [4, 5], appear to cast serious doubts on the validity of the said model (even though \(w\), when considered constant, results very close to \(-1\)) and may suggest some non-gravitational interaction between both dark components [6]. Further, the dynamics of nearly one hundred relaxed galaxy clusters lends substantial support to a continuous transfer from dark energy to dark matter [7], something at variance with the concordance model.
Here we propose several criteria to discriminate interacting from non-interacting dark energy models based solely on the history of the Hubble factor. The underlying assumption of these diagnostics is the validity Friedmann-Robertson-Walker metric, they do not presuppose any specific cosmological model. Obviously, accurate data about $H(z)$ are needed for the criteria to be useful. Regrettably, present data are too scarce and exhibit large bar errors. Nevertheless the situation is expected to improve in a significant manner comparatively soon (though likely not up to the desired level) thanks to the Baryon Oscillation Spectroscopy Survey (BOSS) 5-years project [8] which aspires to measure the absolute cosmic distance scale and expansion rate with percent-level precision at redshifts $z < 0.7$ and $z \approx 2.5$. Likewise, $H(z)$ will be obtained with unprecedented precision, in a considerable redshift range, (i) by measuring the time drift of the redshift of cosmological objects. In particular, Liske et al [9] suggested that thanks to Mc Vittie’s formula, $\dot{\theta} \simeq H_0 (1 + z - E(z))$ [10], the next generation of Extremely Large Telescopes will be able to determine the redshift drift of high redshifts quasars in the interval $2 < z < 5$ using the absorption lines of the Lyman-\(\alpha\) forest of these far away objects. (ii) By observationally determining the the time shift of the angle, \(\theta\), subtended by clusters of galaxies and using the formula $H(z) \simeq (1 + z)(|θ \ln θ|)$ [11]. This may be achieved in the near future by means of micro-arcsecond astrometry [12].

We shall restrict ourselves to homogeneous and isotropic universes with flat space sections dominated by baryons, cold dark matter and dark energy (subscripts $b$, $c$, and $x$, respectively) satisfying the conservation equations

\[
\begin{align*}
\dot{\rho}_b & + 3H\rho_b = 0, \\
\dot{\rho}_c & + 3H\rho_c = Q, \\
\dot{\rho}_x & + 3H(1 + w)\rho_x = -Q,
\end{align*}
\]

where $Q$ stands for the interaction (coupling) term. In what follows, the relationship $\rho_b + \rho_c = \rho_m$ is to be understood. It is worthy of note that when $Q > 0$ -as we shall consider throughout- the coincidence problem is alleviated since $\rho_c$ decreases more slowly with expansion and $\rho_x$ more quickly, and it may even be solved in full [13, 14]. This is true no matter whether the dark energy corresponds to a scalar or tachyonic field or a Chaplygin gas [15]. Besides, if $Q$ were negative, the second law of thermodynamics could be violated [16] and one of the energy densities could become negative either at high or low redshifts.

Lacking a fundamental theory the quantity $Q$ cannot be derived from first principles. Nevertheless, we may surmise likely expressions for it by noting that $Q$ must be small\(^1\) (at least lower than $3H\rho_m$) and depend on the energy densities multiplied by a quantity with units of inverse of time -the natural choice being the Hubble factor. Therefore, we will consider that $Q = Q(H\rho_c, H\rho_x)$. By power law expanding this function and keeping the first term only, one follows $Q \simeq \epsilon_c H\rho_c + \epsilon_x H\rho_x$. Given the absence of information about the coupling, it appears advisable to work with just one parameter rather than two. Thence, three different options arise: $\epsilon_c = 0, \epsilon_x = 0$, and $\epsilon_c = \epsilon_x$. It should be noted that these expressions are obtainable from the scalar-tensor theory of gravity of Kaloper et al [17] -see e.g. [18, 19]. Below we shall consider successively three expressions for $Q$ with $0 < \epsilon < 1$, while keeping $w = \text{constant}$, and look for diagnostics -based in the history of $H(z)$- that may discriminate interacting from non-interacting models.

Interactions terms of the type $Q = \Gamma_c \rho_c + \Gamma_x \rho_x$, where the $\Gamma_i$ ($i = c, x$) quantities stand for constant rates, have also been proposed in the literature [20]. Unfortunately, either $\rho_c$ or $\rho_x$ becomes negative at early or late times and, in general, they cannot solve the coincidence

\(^1\) If it were large and positive, dark matter would dominate the expansion today. On the other hand, if $Q$ were large and negative, the Universe would have been dominated by dark energy practically from the outset.
problem. This is why we do not study them here (however, diagnostics for these cases can be found in [21]).

2. Diagnostics
In this section we set up criteria to tell apart interacting models with \( Q = Q(H \rho_\text{c}, H \rho_x) \) from non-interacting models and from decaying cosmological constant (DCC) models [22]. In all the cases \( w \) is assumed constant.

2.1. Interaction term proportional to \( H \rho_\text{c} \)
We begin by considering that the interaction term takes the form

\[
Q = 3 \epsilon H \rho_\text{c},
\]

where the factor 3 was introduced for mathematical convenience.

By integrating (1)-(3) and using Friedmann’s equation, \( 3H^2 = \kappa^2 (\rho_b + \rho_\text{c} + \rho_x) \), one follows

\[
E^2(x) = \Omega_{b0} x^3 + \Omega_{\text{c}0} x^{3(1-\epsilon)} + \Omega_{x0} x^{3(1+w)} + \left( \frac{\epsilon}{\epsilon + w} \right) \Omega_{c0} \left[ x^{3(1+w)} - x^{3(1-\epsilon)} \right].
\]

Consequently,

\[
\frac{dE^2(x)}{dx^3} = \Omega_{b0} + \Omega_{\text{c}0} (1-\epsilon) x^{-3\epsilon} + \Omega_{x0} (1+w) x^{3w}
\]

\[
+ \left( \frac{\epsilon}{\epsilon + w} \right) \Omega_{c0} \left[ (1+w) x^{3w} - (1-\epsilon) x^{-3\epsilon} \right],
\]

and

\[
\frac{d^2E^2(x)}{d(x^3)^2} = -(1-\epsilon) \Omega_{c0} x^{-3(1+\epsilon)} + w(1+w) \Omega_{x0} x^{3(w-1)}
\]

\[
+ \left( \frac{\epsilon}{\epsilon + w} \right) \Omega_{c0} \left[ w(1+w) x^{3(w-1)} + \epsilon (1-\epsilon) x^{-3(1+\epsilon)} \right].
\]

In view of the hypothesis \( 0 < \epsilon << 1 \) and the fact that observation reveals that \( |1+w| << 1 \), the right hand side of (6) is dominated by its two first terms. Likewise, the first term dominates the right hand side of (7). Therefore, for not large redshifts (i.e., \( z < 3 \)) the normalized Hubble function, \( E^2(x) \), is growing and concave. Thus, its shape will not tell a decaying energy field (with \( w \neq -1 \)) from non-interacting quintessence or phantom models with constant equation of state. However, discrimination will likely come at higher redshifts since the first derivative may tell interacting from non-interacting cosmologies depending on whether \( dE^2(x)/dx^3 \) tends to \( \Omega_{b0} \) or \( \Omega_{\text{c}0} \) at large redshifts. Since, observationally, \( \Omega_{b0} \) and \( \Omega_{m0} \) are separated by a non-small gap (the latter is about six or seven times larger than the former) this may serve to discriminate interacting from non-interacting models.

It should be noted that the \( \Omega m(x) \) diagnostic would mistake an interacting dark energy model for a noninteracting quintessence (phantom) model if applied at low (high) redshifts.

Recalling that \( \Omega_{b0} \) is reasonably well determined by nucleosynthesis measurements\(^2\), another criterion can be established by means of the ratio

\[
R_\Delta := \left[ \frac{\Delta(x_1,x_2) - \Omega_{b0} (x_2^4 - x_1^4)}{\Delta(x_1,x_2) - \Omega_{b0} (x_2^4 - x_1^4)} \right]_A,
\]

\[
R_{\Delta} := \left[ \frac{\Delta(x_1,x_2) - \Omega_{b0} (x_2^4 - x_1^4)}{\Delta(x_1,x_2) - \Omega_{b0} (x_2^4 - x_1^4)} \right]_B,
\]

\(^2\) 0.036 \leq \Omega_{b0} \leq 0.047, see, e.g., Ref. [23].
where $\Delta(x_1, x_2) := E^2(x_2) - E^2(x_1)$. In the present case (8) reduces to

$$R_\Delta \simeq \frac{x_2^{3(1-\epsilon_A)} - x_1^{3(1-\epsilon_A)}}{x_2^{3(1-\epsilon_B)} - x_1^{3(1-\epsilon_B)}}.$$  \hfill (9)

In DCC models an extra $(1 - \epsilon_B)/(1 - \epsilon_A)$ factor appears multiplying the right hand side [21]. Thanks to this, we might hope to discriminate DCC models from interacting dark energy models with $w$ close to the cosmological constant value, $-1$, and also discriminate models featuring the interaction (4).

2.2. Interaction term proportional to $H \rho_x$

We now assume that the interaction term takes the form

$$Q = 3\epsilon H \rho_x$$  \hfill (10)

which may explain the non-vanishing temperature of the gas of sterile neutrinos [24, 25].

Then,

$$E^2(x) = x^3 - \Omega_{x0} \left( \frac{w}{w + \epsilon} \right) \left[ x^3 - x^{3(1+w+\epsilon)} \right].$$  \hfill (11)

For $w = -1$ (i.e., DCC with interaction given by Eq. (10)), it reduces to

$$E^2(x) = x^3 - \frac{\Omega_{\Lambda 0}}{1 - \epsilon} \left[ x^3 - x^{3\epsilon} \right].$$  \hfill (12)

Again, $E^2(x)$ is growing and concave whence its graph will tell neither DCC models with interaction term given by (10) from DCC models with interaction term given by (4) nor these models from non-interacting quintessence models with $w = \text{constant}$.

Expression (12) will help constrain $\epsilon$ without need of knowing $\Omega_{\Lambda 0}$ since the ratio

$$\left[ \frac{E^2(x) - x^3}{E^2(x) - x^{3\epsilon}} \right]_{x=x_1} = \frac{x_1^3 - x_1^{3\epsilon}}{x_2^3 - x_2^{3\epsilon}}$$  \hfill (13)

do not depend on that quantity.

Unlike the previous case, as inspection of the right hand side of Eq. (11) shows, the behavior at high redshifts of $dE^2(x)/dx^3$ will not discriminate interacting from non-interacting cosmological models. For this to be feasible $\epsilon$ should take unrealistic high values.

By deriving (11) twice we get $d^2E^2(x)/d(x^3)^2 = \Omega_{x0} w(1 + w + \epsilon) x^{3(w+\epsilon-1)}$. If eventually this quantity is accurately determined, it will enable us to set useful constraints on $w$ and $\epsilon$ since its ratio for two interacting dark energy models (say $A$ and $B$) does not depend on the $\Omega$ parameters,

$$\frac{d^2E^2/d(x^3)^2}{d^2E^2/d(x^3)^2} |_A = \frac{w_A(1 + w_A + \epsilon_A)}{w_B(1 + w_B + \epsilon_B)} \frac{x^{3(w_A+\epsilon_A)}}{x^{3(w_B+\epsilon_B)}}.$$  \hfill (14)

Once again $E^2(x)$, given by (11), is a growing function of redshift, concave for quintessence fields and convex for phantom fields. Accordingly, the shape of $E^2(x)$ will tell coupled quintessence fields from coupled phantom fields but not from non-interacting dark energy fields.
2.3. Interaction term proportional to $H (\rho_c + \rho_x)$

Assuming the interaction term

$$Q = 3\epsilon H (\rho_c + \rho_x),$$

widely considered in the literature -see, e.g. [7, 13, 14, 20, 26]-, we get

$$E^2(x) = \Omega_{b0} x^3 + \frac{1}{2\epsilon} \left\{ \Omega_C (2\epsilon - (A + B)) - \Omega_C (A - B) x^{-3B} \right\} x^{\gamma_1} + \Omega_C x^{\gamma_2},$$

where the constants appearing in this expression are all positive-definite and depend on $\epsilon$, $\omega$, $\rho_{c0}$, and $\rho_{x0}$ only [21].

Further, $E^2$ is a growing function of $x^3$, concave for quintessence and convex for phantom (Fig.1) which will help discriminate phantom from quintessence dark energy in interacting models with $Q$ given by Eq. (15). However, as is apparent, for $x^3 \geq 5$ the graphs show a nearly straight line behavior whereby one must focus on redshifts below, say, 1.7.

 Inspection of the right hand side of Eq. (16) shows that for interacting models ($\epsilon > 0$) the derivative $dE^2(x)/dx^3$ tends to $\Omega_{b0}$ at high redshifts. Thus if observation reveals that behavior, it would be suggestive of interaction.

![Figure 1](image_url). Left panel: Graphs of $E^2$ vs. $x^3$ as given by Eq. (16) for three values of $w$. Right panel: Graphs of the second derivative of $E^2$ with respect to $x^3$ for the same three values of $w$. Solid, dotted, and dashed lines are for $w = -0.8$, $-1.0$, and $-1.2$, respectively. Note that only phantom models present convex curves for reasonable values of the parameters. In drawing the graphs we took $\Omega_{b0} = 0.04$, $\Omega_{c0} = 0.26$, $\Omega_{x0} = 0.70$, and $\epsilon = 0.05$.

3. Discussion

To extract the invaluable information hidden in the history of the Hubble factor on the nature of dark energy one must first set up simple and practical criteria to be used on the data. We proposed several criteria, of notable mathematical simplicity, based on an accurate knowledge of the said history -something we might hope to have at our disposal comparatively soon.

Specifically if $d^2E^2/d(x^3)^2$ results positive (negative), then the dark energy is of phantom type (either quintessence or a decaying cosmological constant). Only if the dark energy is a conserved cosmological constant, the graph of $E^2$ vs. $x^3$ will result in a straight line.

To discern whether dark energy is interacting or not the behavior of $dE^2(x)/dx^3$ at high redshifts must be studied. If this derivative tends to the present value of the fractional baryon
density, \( \Omega_{m0} \), then there will be grounds to believe that is it interacting with the coupling term \( Q \) given either by Eqs. (4) or (15). If it tends to \( \Omega_{m0} \), then we may strongly suspect that it is not interacting; and, in particular, if it coincides with \( \Omega_{m0} \) independently of redshift, then it will most likely be a conserved cosmological constant -see [21] for details. Finally, if the redshift function \( dE^2(x)/dx^3 \) does not tend to any of these two quantities, we may conclude that either dark energy is interacting with \( Q \) given by Eq.(10) or some other law not considered here, or that in reality \( w \) is not a constant but a function of redshift. Obviously, this would complicate matters very much because it would call for the introduction of further parameters in the analysis.

We restricted ourselves to constant \( w \) and vanishing spatial curvature. While this automatically limits the scope of our work we feel it does not do it seriously, at least no more than otherwise since it would introduce additional unknown parameters that would compromise the said scope in other ways.

**Acknowledgments**

This work was supported from the “Comisión Nacional de Ciencias y Tecnología” (Chile) through the FONDECYT Grant No. 1070306 (SdC), No. 1090613 (RH and SdC) and the Spanish Ministry of Education and Science under Grant FIS2006-12296-C02-01. D P acknowledges “FONDECYT-Concurso incentivo a la cooperación internacional” No. 7090081.

**References**

[1] Peebles P J E and Ratra B 2003 *Rev. Mod. Phys.* **75** 559
[2] Hagiwara K et al 2002 *Phys. Rev.* D **66** 010001
[3] Sahni V, Shafieloo A and Starobinsky A A 2008 *Phys. Rev.* D **78** 103502
[4] Hicken M et al 2009 *Astrophys. J.* **700** 331
[5] Hicken M et al 2009 *Astrophys. J.* **700** 1097
[6] Shafieloo A, Shani V and Starobinsky A A 2009 *Phys. Rev.* D **80** 101301(R)
[7] Abdalla E, Abramo L R and de Souza J C C 2009 Signature of the interaction between dark energy and dark matter in observations Preprint arXiv:0901.5236 [gr-qc]
[8] Schlegel D, White M and Eisenstein D 2009 The Baryon Oscillation Spectroscopic Survey: Precision measurements of the absolute distance scale Preprint arXiv:0902.4680 [astro-ph.CO]
[9] Liske J et al 2008 *Mon. Not. R. Astron. Soc.* **386** 1192
[10] Mc Vittie G C 1962 *Astrophys. J.* **136** 334
[11] Zhang H and Zhu Z-H 2009 *J. High Energy Phys.* (in press)
[12] Brown AGA 2008 Getting ready for the micro-arcsecond era Preprint arXiv:0802.3568 [astro-ph]
[13] del Campo S, Herrera R and Pavón D 2009 *Phys. Rev.* D **78** 021302(R)
[14] del Campo S, Herrera R and Pavón D 2009 *J. Cosmol. Astropart. P.* JCAP01(2009)020
[15] Chimento L P, Jakubi A S and Pavón D 2003 *Phys. Rev.* D **67** 087302
[16] Pavón D and Wang B, 2009 *Gen. Rel. Grav.* **41** 1
[17] Kaloper N and Olive K A 1988 *Phys. Rev.* D **57** 811
[18] Curbelo R, González T and Quirós I 2006 *Class. Quantum Grav.* **23** 1585
[19] Zhang H and Zhu Z-H 2006 *Phys. Rev.* D **73** 043518
[20] Caldera-Cabral G, Maartens R and Ureña-López L A 2009 *Phys. Rev.* D **79** 063518
[21] del Campo S, Herrera R and Pavón D 2009 H(z) diagnostics on the nature of dark energy (work in progress)
[22] Overduin J M and Cooperstock F I 1998 *Phys. Rev.* D **58** 043506
[23] Olive K A 2003 TASI lectures on dark matter Preprint arXiv:astro-ph/0301505.
[24] Hansen S H, Lesgourgues J, Haehnelt M G, Matarrese S and Riotto A 2005 *Phys. Rev.* D **71** 063534
[25] Zhou J, Wang B, Pavón D and Abdalla E 2009 *Mod. Phys. Lett.* A **24** 1689
[26] Olivares G, Atrio-Barandela F and Pavón D 2005 *Phys. Rev.* D **71** 063523