Thermal analysis of a transmission line for Traveling Wave Tube TWT

Mounir CHBIKI1,2,*, Najib LARAQI1, Jean-François JARNO2, Jacques HERREWYN2, Tony DA SILVA BOTELHO3

1 Université Paris Ouest, Laboratoire Thermique Interfaces Environnement, EA 4415, PST Ville d’Avray, GTE, 50 R de Sèvres, F92410 Ville d’Avray
2 Thales Electron Devices, TED, 2 rue Marcel Dassault F78220 Vélizy-Villacoublay
3 SUPMECA, Laboratoire des Systèmes Mécaniques et des Matériaux, EA 2336, 3 rue Fernand Hainaut, F93400 Saint-Ouen

E-mail : mounir.chbiki@gmail.com or nlaraqi@gmail.com

Abstract. A new analytical method has been developed to study the delay line of Traveling Waves Tubes (TWT). Our study is focused on the analysis of the hot lines shrinking phenomenon. In the studied case, unlike brazed configuration, the contact areas are not perfect, resulting in a diminution of the heat transfer process. In this work, we highlight the influence of the macro-constriction on the heat transfer rate in the various parts of a TWT the geometry of which is also relatively complex. We propose in this work an analytical study of the thermal behavior of a transmission line in established regime. First, we determine the individual thermal resistance of each component. Secondly, we estimate the global resistance of the device according to the geometrical parameters and the respective conductivities of the various elements of this line. In this analytical model, we proceed to parametric studies in order to determine the geometrical configurations that will provide the lowest global thermal resistance. We will emphasize the potential gain according to the used materials and the increase of contact areas.

1. Introduction
Since their invention by Rudolf Kompfner in 1942 [1], performances of TWT have been continuously improving. The first vacuum tube was made of glass and achieved only a few milliwatts at a 2 GHz frequency. Nowadays TWT are more complex, and technical improvements enable to produce several hundreds watts at more than 20 GHz.

TWT (Figure 1) are basically vacuum tubes used in hyperfrequency to amplify the wave. An electron gun produces an electron beam. It is followed by a slow wave transmission line which is fed by a high frequency wave signal interacting with the beam, a part of whose kinetic energy is transferred to the high frequency wave. Finally, the electrons which have lost a fraction of their energy get to the collector [2].

In the objective of increasing the power and the bandwidth of traveling wave tubes (TWT), several innovations have to be investigated. The major constraint in the development of tubes is the power loss. Indeed, increasing on one hand the output power and decreasing on the other hand the size of the device results in some heating issues.
Few studies have been carried out on this subject. We can quote the works made by Calame and Abe [3] on the study of the materials properties used in the TWT. They studied the thermal effects in transmission lines using a simplified model and determined the temperature difference between the helix and the barrel. Their study aimed to assess the influence of the assemblies and its various configurations. But the distortion of flux lines in the various parts where not studied in their paper. Our model will offer a better study on the thermal point of view with more handiness. We can also mention the work of Crivello and Grow [4], with which our approach is similar. In our study, we will take into account the impact of the distortion of flux lines.

The present work gives particular attention to the shrinking of transmission line. The transmission line (Figure 2) is constituted by a helix of fireproof material which is suspended by three ceramic rods spaced circularly at 120°. The set is introduced into a barrel, which consists of a succession of pole pieces and spacers. The assembly methods can vary (hot shrinking, brazing and cold shrinking) resulting in varied contact areas and modifying the heat transfer. With this model we estimate the influence of macro-constriction. The helix is the center of a thermal loss as it intercepts electrons and because of RF losses. These thermal losses induce a local heating on the last spires of the helix, which are only cooled down through the real contacts with three rods. The rise of temperature depends on the nature of the materials used and on the quality of the contacts. An excessive rise of temperature can damage the smooth running of the tube.

Real rough surfaces under normal load results in the contribution of many local micro-contacts, thus the real contact area between nominally ideal surfaces is only a small fraction of the apparent contact area [6-7]. Many models where developed to assess the real contact area under a given normal load [8-10] and some experimental data are consistent with these models. Other models have been developed to calculate the thermal contact resistance according the real contact areas and boundary conditions [11-13].

This problem can be studied using a finite elements model but the system may be very complex, so the numerical simulations turn very cumbersome and time consuming. This is why an analytical model, which can produce very quick results, was developed. This model allows us to perform a thermal sizing of different parts of a transmission line. We look for the optimum of a configuration by varying the contact areas n°1 and n° 2 (Figure. 3). At first, we study the helix. We look at the influence of contact n°1 with the rod and the influence of the helix thickness. Then we perform a similar study on the rod by studying the influence of contact n°1 and n°2. And finally, we made a study on the barrel by taking into account all the elements together. We will notice the possible gain with the use of various materials and/or the optimization of the contact areas.

Actually, these contacts are coated with a smooth conductive material, to enhance thermal transfer and to improve the real contact area. Due to the assembly process, these coating materials have elastoplastic behavior [14, 15] and quite temperature sensitive. These two effects are non linear but can be studied separately. In the present work, only the temperature is taken into account.
2. Formulation of the problem

The power dissipated in a transmission line evolves exponentially along the line. That means that the maximum of power loss is on the line exit. These losses give colossal levels of power densities. To avoid a progressive degradation of the transmission line a meticulous study on the thermal transfer must be carried out. The heat transfer occurs mainly by conduction considering that the tube is vacuumed. There is no convection and the radiation is negligible \[5\]. Thus, we resolve the equation of the heat in stationary state into cylindrical coordinates in the parts of a line (Eq.1):

\[
\frac{\partial^2 T(r, \theta)}{\partial r^2} + \frac{1}{r} \frac{\partial T(r, \theta)}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T(r, \theta)}{\partial \theta^2} + \frac{\partial^2 T(r, \theta)}{\partial z^2} = 0
\]  

(1)

2.1. Hypothesis

To simplify the problem the following hypothesis were taken: (i) the heat flux occurs axially, there is no heat transfer from a spire to the others, (ii) the helix is a ring, (iii) the barrel is monoblock. Thus, the study can be reduced to a twelfth of a step of helix and the model will be in two dimensions.

3. Solution for the different parts of a transmission line.

In all the following models, the contact areas n°1 and n°2 can vary. As a result the macro-constriction in the various parts may vary as well.

3.1. Helix

As we mentioned earlier, the helix is the center of thermal losses (see Figure 4) on its internal radius \(R_0\). So we have a flux \(q_0\) which enters by the internal radius and goes out by the contact area with the rod (Grey area, \(q_1\) flux). We have flux continuity through the helix, thus \(q_0=q_1\), \(\alpha_1\) is the contact angle with the rod, which will vary.

The heat equation to be resolved is:

\[
\frac{\partial^2 T(r, \theta)}{\partial r^2} + \frac{1}{r} \frac{\partial T(r, \theta)}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T(r, \theta)}{\partial \theta^2} = 0
\]  

(2)

The boundary conditions are:

- A flux is imposed on the internal radius: \(-\lambda \left( \frac{\partial T}{\partial r} \right)_{r=R_0} = q_0\)

- An exit flux on the zone of contact and isolated on the rest of the radius.

\[-\lambda \left( \frac{\partial T}{\partial r} \right)_{r=R_1} = \begin{cases} q_1 & (0 \leq \theta \leq \alpha_1) \\ 0 & (\alpha_1 \leq \theta \leq \alpha_0) \end{cases} \]
- The circumferential edges are insulated: \( \left( \frac{\partial T}{\partial \theta} \right)_{\theta=0} = 0 ; \left( \frac{\partial T}{\partial \theta} \right)_{\theta=\alpha_0} = 0 \)

The resolution gives the following solution:

\[
Z_{\text{helix}} = \frac{1}{\lambda \cdot a_1 \alpha_0} \left\{ \frac{1}{\ln \left( \frac{R_0}{R_i} \right)} - \frac{2}{\pi^2} \left( \frac{\alpha_0}{\alpha_i} \right)^2 \sum_{m=1}^{\infty} \frac{\sin^2 \left( m \pi \frac{\alpha_i}{\alpha_0} \right)}{m^3} \left( \frac{R_i}{R_0} \right)^{2m \pi^2 / \alpha_0} + 1 \right\} \tag{3}
\]

Where \( \lambda \) is the thermal conductivity and \( Z_{\text{helix}} \) the thermal impedance of the Helix.

3.1.1. Parametric study
During the development of a new project, a fast parametric study is always desirable. This model quickly gives an estimation of the thermal resistance according to the geometrical data.

In the TWT, the helix is made of Tungsten (W) or Molybdenum (Mo). Their respective thermal conductivities are 180 W/m.K and 150 W/m.K.

We study first a Tungsten helix and we investigate the influence of the contact area, which will depend on the chosen material. We can see on Figure 5 the influence of the contact area. (From perfect contact (100%) to no contact (0%))

![Figure 4: Helix](image)

![Figure 5: Influence of the contact area ratio on global thermal resistance](image)

The calculations are made for a surface ratio ranging from 1 % to 100 %. Problems can occur during the assembly process, which can be represented by 1% of contact, for the worse case. The perfect contact area (100% contact ratio) is the most optimistic case.

The figure 5 shows that the perfect contact reduces the thermal resistance of a factor 2.8 compared to a 1% ratio contact (quasi no contact at all). We also notice that the improvement of the contact area between the helix and the rod from 30% to 100% results in a 30% drop the thermal resistance, with a quasi linear evolution in this range of contact ratio.
Then we study the influence of the helix thickness. This part should help the engineers to choose the optimal helix thickness with regard to thermal criteria. The first step consists in a numerical study in order to investigate the distortions of the thermal flux lines for different helix thicknesses (figure 6.1 to 6.3) thicknesses are investigated: “thin”, “wide” and “optimized” (according to Thales standards).

The distortion of the flux lines is a representation of the thermal resistance. We have two resistances that add up. The resistance due to the thickness $R_p$ (macroscopic Phenomenon bound to materials and to the geometry) and the resistance due to the constriction $R_{cs}$ (deformation of the flux lines due to the geometry).

![Figure 6.1: Thickness is thin.](image1.png)  
![Figure 6.2: Thickness is optimized.](image2.png)  
![Figure 6.3: Thickness is wide.](image3.png)

The flux lines have difficulty in passing by a narrow zone while the thickness is short.
- $R_{cs}$ is high
- $R_p$ is low

A good balance is established between both resistances.
- $R_{cs}$ is moderate
- $R_p$ is medium

The flux lines do not have difficulty to pass by a wide zone. While the thickness is long.
- $R_{cs}$ is low
- $R_p$ is high

Figure 6.4 presents the influence of the thickness on the thermal resistance for a tungsten helix at 20°C and 350°C and a molybdenum helix at 350°C.

The contact area is supposed to be perfect (100 %). Figure 6.4 shows that if the thickness is too thin, the thermal resistance is high. It is explained by the distortion of flux line (Figure 6.1). An optimal thickness is clearly seen on this parametric study, showing the importance of optimizing the helix thickness towards thermal resistance. Comparing tungsten results at 20°C and 350°C shows that for the same contact ratio (100%) and the same helix thickness, the thermal resistance improves when temperature decreases.

### 3.2. Rod

When the heat flux $q_1$ evacuated by the helix reaches the rod, it is evacuated by the contact area with the barrel (Figure 7). Since the actual contact between the helix and the internal surface of the rod is partial (i.e. only a small fraction of the inner surface of the rod is in contact with the helix) a 3D model is required in order to take into account the deformations of flux line without contact.

The resolution of the problem gives the following solution:

$$
Z_{rod} = \frac{1}{\lambda \cdot a_0} \left\{ \frac{1}{\alpha_0} \ln \left( \frac{R_0}{R_1} \right) + B_{m0} + B_{n0} + B_{mn} \right\}
$$

Where

$$
B_{0m} = \frac{2a_0}{\alpha_2 R_1 d_2} \sum_{m=1}^{\infty} \left[ \tilde{F}_{n0}(R_1) \sin \left( \frac{m \pi \alpha_2}{\alpha_0} \right) - \tilde{F}_{n0}(R_2) \sin \left( \frac{m \pi \alpha_3}{\alpha_0} \right) \right] \left( \frac{m \pi \alpha_2}{\alpha_0} \right)
$$

5
3.2.1. Parametric study
The rods are located between the helix and the barrel, resulting in two contacts: one between the helix and the rod and the other between the rods and the barrel. In the following, we will inspect the influence of the contacts areas on the global thermal resistance. In this analysis, we vary the contact area from 1% to 100% like in the helix study. The material of the rod may change. In order to highlight the influence of materials properties: Beryllia BeO (material 1), Anisotropic Boron Nitride APBN (material 2) and Diamond (material 3).

Figure 8 presents the evolution of the thermal resistance for these three materials as a function of the contact area ratio. Contact ratios less than 20% are not representative of a production device, thus we just focus on the values ranging from 20% to 100% of contact area ratio. The difference between the conductivity of material 1 and material 2 is 60% in this range of contact area ratio. We notice that an increase of the conductance involves a decrease of the thermal resistance in the same order.

3.3. Barrel
Power densities found at the interface between the rod and the barrel are less critical than the ones found between the helix and the rod. The heat flux penetrates the barrel by the contact area with the rod, and exits by all of the rod’s outside surface (Figure 9). This model is similar to that of the helix. As for the others models, we can vary the contact area between the rod and the barrel. These modifications increase or decrease the macro-constriction (i.e. the distortion of the flux lines).

The resolution of the equation gives the following solution:
\[ Z_{\text{barrel}} = \frac{1}{\lambda \cdot \alpha_3} \left[ \frac{1}{\alpha_0} \ln \left( \frac{R_3}{R_2} \right) - \frac{2}{\pi} \left( \frac{\alpha_0}{\alpha_3} \right)^2 \sum_{m=1}^{\infty} \frac{\sin^2 \left( \frac{m \pi \alpha_3}{\alpha_0} \right)}{m^2} \left( \frac{R_2}{R_3} \right)^{-2m \pi \alpha_3/\alpha_0} + 1 \right] \] (5)

\[ Z_t = Z_{\text{helix}} + R_{ch} + Z_{\text{rod}} + R_{cb} + Z_{\text{barrel}} \] (6)

The study of barrel gives similar comments as for the helix: a reduction of the thermal resistance with the increase of contact area ratio and a quasi linear evolution in the range 30% to 100% of contact area ratio.

4. Global resistance

4.1. Results

The global resistance of the line is a succession of serial resistances. We calculate separately every resistance according to the zones of contacts then we add them. Three resistances are separated by two contact resistances (\( R_{ch} \), for the contact with the helix, and \( R_{cb} \), for the contact with the barrel), what gives:

The analytical model allows us to determine a global resistance according to materials and contact area ratios.

Then, we study the influence of several configurations corresponding to several industrial applications. Each configuration has its limitations in terms of performance. Table 1 shows, the varied configurations we studied.

The comparison of thermal resistances of these four configurations is presented Figure 10. We do not take into account resistances of contacts but only macro-constriction because we do not have enough data at the present time. The conductance goes up by about 30% when the contact areas increase from 0% to 100% of contact area ratio.

As we did previously, we observe the possible power gain on the optimization of contact surfaces and materials proprieties. For example, comparing configurations 1 and 2 exhibits a thermal resistance improvement up to 24%.
4.2. Helix temperature

During the life in service of the TWT tube, we have no data on the helix temperature. Considering the small size of the device, a direct measure is impossible without disturbing the RF wave and the vacuum. Nevertheless, this temperature should be between 300°C and 450°C. The final goal of this work is to give an approximation of helix temperature by computing it with previously obtained values of the thermal resistance and thermal loses.

The thermal losses may vary depending on the applications but we can evaluate it. For example, we take as thermal losses 5 W/mm. Figure 11 presents the helix temperature variation with thermal resistance with that assumption. A linear variation of the thermal resistance towards the helix temperature is exhibited.

| Configuration | Helix Material | Conductivity (W/m.K) | Rods Material | Conductivity (W/m.K) | Barrel Material | Conductivity (W/m.K) |
|---------------|----------------|----------------------|---------------|----------------------|----------------|----------------------|
| 1             | Molybdenum     | 150                  | Beryllia      | 200                  | Iron           | 65                   |
| 2             | Tungsten       | 180                  | Anisotropic Boron Nitride (APBN) | 66              | Iron           | 65                   |
| 3             | Molybdenum     | 150                  | Diamond       | 1200                 | Iron           | 65                   |
| 4             | Tungsten       | 180                  | Diamond       | 1200                 | Iron           | 65                   |

Table 1: Studied configurations

The temperature on the barrel is measured using a thermocouple. It is about 180°C.

\[ Z_r = \frac{T_{\text{helix}} - T_{\text{barrel}}}{\theta_{\text{estimated}}} \]  

(7)

We noticed a decrease of about 45°C when the contact areas are optimized from 30 % to 100 %.

4.3. Discussion

Throughout this work, we do not take into account the Thermal Resistance of Contact (TRC) because we have not data on this parameter yet. We showed the influence of contact areas on the thermal
resistance. The increase of contact areas results in a decrease of the line resistance, since the thermal fluxes travel better through the contact. But it also affects the TRC. Indeed, if the shrinking strength remains constant while the areas increase, then the TRC will increase. So increasing the contact area will have two effects. A mechanical study and TRC study will give us more information about the weight of each resistance. This study will allow us to see the potential gain on a configuration over the other one by taking into account resistances of contacts. We should keep in mind that the contact resistance depends on materials, pressure and the other parameters. As two opposite effects will be in competition, a possible optimal dimensioning may occur.

5. Conclusion
This easy-to-use implementation model allows a fast parametric study. It allows us to investigate the potential gain from a configuration to the other one and an optimization of the geometry is demonstrated. This model may also be used as a tool to check of the helix temperature with well defined resistances of contact and the well identified thermal losses. It allows us a design of the structures helix and rod corresponding to prescribed performances.

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