Spin of the proton and orbital motion of quarks

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Effect of the quark intrinsic motion on the proton spin structure functions is demonstrated. It is shown, that the covariant version of the quark-parton model taking into account the orbital motion gives the consistent picture of the proton spin structure, which is based on the valence quarks. This picture is supported by the recent data, which indicate, that the spin contributions from the sea quarks and gluons are compatible with zero.

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INTRODUCTION

The spin of proton equals 1/2, which was proved first by Dennison [1]. Indeed, the spin of proton is equally well defined as the spin of electron despite the fact, that the proton is not elementary, pointlike Dirac particle. Generally, the spin is defined only by the corresponding representation of the Poincaré group according to which given state transforms. Allowed values of the spin quantum number are \( j = 0, 1/2, 1, 3/2, ... \) and corresponding angular momentum is given by the term \( \hbar \sqrt{j(j+1)} \). At the same time, component of the angular momentum in defined direction is \( \hbar j_z \), where \( j_z = -j, -j+1, ..., j-1, j \). This is the exact rule of quantum mechanics, one can observe e.g. anomalous magnetic moment, but never an anomalous spin.

Actually, the proton spin \( j = 1/2 \) well corresponds to the additive quark model: the spins of three quarks \((u,u,d)\) each has spin \(1/2\) are combined to give the observed proton spin. This simplified picture implied anticipation of the result on the proton spin structure function \( g_1 \), which is measured in the polarized deep inelastic scattering (DIS). The more formal aspects of the polarized DIS and related notions are explained in [2]. The first expectation was that just the three valence quarks contribute to the function \( g_1 \). Then this function, which measures quark spin contributions to the longitudinally polarized proton, is expressed in terms of the naive quark-parton model (QPM) as

\[
g_1(x) = \frac{1}{2} \sum_{q=u,d} e_q^2 \Delta q_{val}(x),
\]

where \( x \) is the Bjorken scaling variable and \( e_q \) are quark charges. This relation follows from a more general equality

\[
g_1(x) = \frac{1}{2} \sum_q e_q^2 (q^+(x) - q^-(x)) = \frac{1}{2} \sum_q e_q^2 \Delta q(x) = \frac{1}{2} \sum_{q=u,d} e_q^2 \Delta q_{val}(x) + \frac{1}{2} \sum_q e_q^2 \Delta q_{sea}(x),
\]

in which contribution from the sea quarks is set to zero. The functions \( q^{\pm}(x) \) represent distributions of quarks with polarization \( \pm \) related to the longitudinal orientation of the proton polarization. At the same time the assumption, that just the valence terms contribute, means:

\[
\sum_{q=u,d} \int_0^1 \Delta q_{val}(x) dx \simeq 1.
\]

So the assumption, that only valence quarks generate the proton spin can be checked by integrating \( g_1 \):

\[
\Gamma_1 = \int_0^1 g_1(x) dx.
\]

In fact one applies some model estimation for a proportion between contributions from \( u \) and \( d \) quarks and verifies compatibility of the relation [1] constrained by the condition [3], with the experimental quantity [1]. The first results

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distribution functions with spheric symmetry. However then the EMC experiment covering also a region of lower $x$ showed, that the moment $\Gamma_1$ is surprisingly low, $\Gamma_1 \approx 0.126$. This value was hardly compatible with the concept, that the proton spin is just sum of the valence spins. The next experiments confirmed the low $\Gamma_1$, which can correspond only to one third or even less of the value expected from spins of the valence quarks. So also another possible sources, like orbital momenta, sea quarks and gluons started to be considered. Unexpectedly low $\Gamma_1$ represents the essence of the known proton spin problem, which can be formulated: How is the spin of proton generated from the angular momenta (spins+orbital momenta) of the valence quarks, sea quarks and gluons? Shortly, what is the solution of the general balance equation

$$\frac{1}{2} = \langle j_{\text{val}} \rangle + \langle j_{\text{sea}} \rangle + \langle j_g \rangle ?$$

In the following I shall analyze in more detail the statements, which the proton spin problem resulted from and confront the last equation with the recent experimental data.

**INTRINSIC MOTION OF THE QUARKS**

First, let me consider the relation or its more general version. The left side of the equation is the invariant function $g_1$ appearing in the antisymmetric part of the hadronic tensor related to the DIS, which is realized via one photon exchange. Experimentally, this function can be extracted from the corresponding differential cross section. The right side is the sum of polarized quark distributions with the charge factors. One should point out, that even the equality is not a generally valid identity between the structure function $g_1$ and a linear combination of of the quark distributions $\Delta q$, despite the fact, that both the terms are often used as synonyms. Distribution is a quantity defined in the framework of particular model, but structure function is the more general concept. The relation, similarly as e.g. equality

$$F_2(x) = x \sum_q e_q^2 q(x)$$

for the unpolarized structure function, is deduced within the naive QPM. Its standard formulation is related to the preferred reference system - the infinite momentum frame. The relations between the distribution and structure functions like are derived with the use of approximation

$$p_\alpha = x P_\alpha,$$

where $p$ and $P$ are the quark and proton momenta. This relation in the covariant formulation is equivalent to the assumption, that the quarks are static with respect to the proton, in particular that $p = 0$ in the proton rest frame.

In the papers the covariant QPM with non-static quarks was studied. Actually, in that approach the quark intrinsic motion is connected with the quark orbital momentum. The role of quark orbital motion in the context of nucleon spin was discussed in previous works. The aim of this letter is to show rigorously how the quark orbital momentum, which is manifested by intrinsic motion, directly modifies spin structure functions and to discuss, how is this effect compatible with the available experimental data. The quarks in the suggested model are represented by quasifree fermions, which are in the proton rest frame described by the set of distribution functions with spheric symmetry $G_i^\pm(p_0)$. These distributions measure the probability to find a quark in the state

$$u(p, \lambda n) = \frac{1}{\sqrt{N}} \left( \frac{\phi_{\lambda n}}{p_0 + m} \phi_{\lambda n} \right); \quad \frac{1}{2} m^2 \phi_{\lambda n} = \lambda \phi_{\lambda n},$$

where $m$ is the quark mass, $\lambda = \pm 1/2$ and $n$ coincides with the direction of target polarization $J$. The distributions allow to define the generic function $H$,

$$H(p_0) = \sum_q e_q^2 \Delta G_q(p_0), \quad \Delta G_q(p_0) \equiv G_q^+(p_0) - G_q^-(p_0),$$

from which the corresponding structure functions can be obtained. If one assumes $Q^2 \gg 4M^2 x^2$, where $-Q^2$ is the square of the photon momentum and $M$ is the proton mass, then:

$$g_1(x) = \frac{1}{2} \int H(p_0) \left( m + p_1 + \frac{p_1^2}{p_0 + m} \right) \delta \left( \frac{p_0 + p_1}{M} - x \right) \frac{d^3 p}{p_0}.$$
Although the procedure for obtaining these structure functions from the quark distributions \( g(x) \) can seem complicated, the task is well-defined and the result is unambiguous. The relation (10) is different from Eq. (2), but for the static quarks both the equalities become equivalent. Further, in the cited papers it is shown, that the structure functions (10), (11) satisfy the well known sum rules suggested by Burkhardt and Cottingham [21], Efremov, Leader and Teryaev [22], and by Wanzura and Wilczek [23]. For next discussion I assume, that the contribution of sea quarks in the generic distribution \( H \) is negligible, in other words, only the valence quarks contribute from the quark sector. Actually, this assumption corresponds to the result of recent analysis [8]. For unpolarized sea and assuming massless quarks, the relations between valence quark distributions and partial spin structure functions were obtained:

\[
\begin{align*}
g_{1q}(x) &= \frac{1}{2} \left[ q_V(x) - 2x^2 \int_x^1 \frac{q_V(y)}{y^2} dy \right], \\
g_{2q}(x) &= \frac{1}{2} \left[ -q_V(x) + 3x^2 \int_x^1 \frac{q_V(y)}{y^2} dy \right].
\end{align*}
\]

Then the ordinary spin functions are calculated as

\[
g_i(x) = \sum_{q=u,d} s_q e_q^2 g_{iq}(x); \quad i = 1, 2,
\]

where \( s_q \) are weight factors controlling the spin contributions of different quark flavors. For the case of \( SU(6) \) symmetry one gets \( s_u = 2/3 \) and \( s_d = -1/3 \). So, using the known input on \( u_V \) and \( d_V \), one can directly calculate corresponding \( g_1, g_2 \). Fig. 1 shows, that the calculation gives quite reasonable agreement with the experimental data \( \text{[3]} \) for both the functions. At the same time, let me point out, that apart of the assumptions on \( SU(6) \) symmetry and massless quarks, no other parameters are fixed by hand in this approach. Nevertheless, one can observe, that the calculation of \( g_1 \) underestimates the experimental curve represented by the fit of world data \( \text{[4, 6, 7, 8, 9]} \). This result is qualitatively opposite to the situation representing the proton spin problem, where the theoretical expectation was substantially higher than the real data. I shall comment this difference in more detail in the next paragraph. Concerning the function \( g_2 \), it is obvious, that the agreement is very good. It is evident, that the \( g_2 \) has here a clear, well-defined meaning. On the other hand, it is known fact \( \text{[2]} \), that meaning of the \( g_2 \) in the standard QPM is inconsistent. Let me remark, that applied model was recently generalized to include also the transversity distribution \( \text{[24]} \). Now, let me analyze the magnitude of the \( g_1 \). After the integrating Eq. (10) one obtains

\[
\Gamma_1 = \int g_1(x) dx = \frac{1}{2} \int H(p_0) \left( \frac{1}{3} + \frac{2m}{3p_0} \right) d^3p,
\]
which, for the $SU(6)$ approach implies:

$$\frac{5}{18} \geq \Gamma_1 \geq \frac{5}{54}. \quad (16)$$

The upper limit corresponds to the static ($p_0 \simeq m$) and the lower, which is one third of the upper, to the non-static quarks in the limit $m \to 0$. Does it mean that more intrinsic motion results in less spin? What is the physical reason of this effect? First, let me remind the general rule concerning angular momentum in quantum mechanics: The angular momentum consists of the orbital and spin part $j = l + s$ and in the relativistic case the $l$ and $s$ are not conserved separately, but only the total angular momentum $j$ is conserved. This simple fact was in the context of quarks inside the nucleon pointed out in [25]. It means, that there are eigenstates of $j(j^2, j_z)$ only, which are for the fermions with the spin 1/2 represented by the spheric waves [26]

$$\psi_{kj,j_z} (p) = \frac{\delta(p-k)}{p \sqrt{2p_0}} \left( \begin{array}{c} i^{-l} \sqrt{p_0 + m} \Omega_{kj,j_z} (\omega) \\ i^{-l} \sqrt{p_0 - m} \Omega_{kj,j_z} (\omega) \end{array} \right), \quad (17)$$

where $\omega = p/p$, $l = j + \frac{1}{2}$, $\lambda = 2j - l$ ($l$ defines the parity) and

$$\Omega_{j,l,j_z} (\omega) = \left( \begin{array}{c} \sqrt{\frac{j+1}{2j}} Y_{l,j_z - 1/2} (\omega) \\ \sqrt{\frac{j+1}{2j}} Y_{l,j_z + 1/2} (\omega) \end{array} \right), \quad l = j - \frac{1}{2};$$

$$\Omega_{j,l,j_z} (\omega) = \left( \begin{array}{c} -\sqrt{\frac{j+1}{2j+2}} Y_{l,j_z - 1/2} (\omega) \\ \sqrt{\frac{j+1}{2j+2}} Y_{l,j_z + 1/2} (\omega) \end{array} \right), \quad l = j + \frac{1}{2}. \quad (18)$$

One can check, that the states are properly normalized:

$$\int \psi_{k,j',j_z} (p) \psi_{k,j,j_z} (p) d^3p = \delta(k-k')\delta_{jj'}\delta_{jj_z}\delta_{jj_z}. \quad (19)$$

The wavefunction [17] is simplified for $j = j_z = 1/2$ and $l = 0$. Taking into account that

$$Y_{00} = \frac{1}{\sqrt{4\pi}}, \quad Y_{10} = i \sqrt{\frac{3}{4\pi}} \cos \theta, \quad Y_{11} = -i \sqrt{\frac{3}{8\pi}} \sin \theta \exp(i \varphi),$$

one gets:

$$\psi_{k,j,j_z} (p) = \frac{\delta(p-k)}{p \sqrt{8\pi p_0}} \left( \begin{array}{c} \sqrt{p_0 + m} \left( \begin{array}{c} 1 \\ 0 \end{array} \right) \\ -\sqrt{p_0 - m} \left( \begin{array}{c} \cos \theta \\ \sin \theta \exp(i \varphi) \end{array} \right) \end{array} \right). \quad (20)$$

Let me remark, that $j = 1/2$ is the minimum angular momentum for particle with the spin 1/2. Generally, one can make the superposition

$$\Psi (p) = \int a_k \psi_{k,j,j_z} (p) dk; \quad \int a_k^* a_k dk = 1 \quad (21)$$

and calculate the average spin contribution to the total angular momentum as

$$\langle s \rangle = \int \Psi^\dagger (p) \Sigma_z \Psi (p) d^3p; \quad \Sigma_z = \frac{1}{2} \left( \begin{array}{cc} \sigma_z & 0 \\ 0 & -\sigma_z \end{array} \right).$$

After inserting from Eqs. (19), (20) into (21) one gets

$$\langle s \rangle = \int a_k^* a_p \left( p_0 + m \right) \left( p_0 - m \right) \left( \cos^2 \theta - \sin^2 \theta \right) d^3p = \frac{1}{2} \int a_k^* a_p \left( \frac{1}{3} + \frac{2m}{3p_0} \right) dp. \quad (22)$$

Since $j = 1/2$, the last relation implies for the orbital momentum:

$$\langle l \rangle = \frac{1}{3} \int a_k^* a_p \left( 1 - \frac{m}{p_0} \right) dp. \quad (23)$$
It means, that:

i) For the fermion at rest ($p_0 = m$) it follows $\langle s \rangle = j = 1/2$, which is obvious, since without kinetic energy no orbital momentum can be generated.

ii) Generally, for $p_0 \geq m$, one gets $1/3 \leq \langle s \rangle / j \leq 1$.

In other words, for the states with $p_0 > m$ part of the total angular momentum $j = 1/2$ is necessarily generated by the orbital momentum. At the same time, localized states must satisfy $\langle p_0 \rangle > m$. These two statements are general consequences of quantum mechanics, and not a consequence of the particular model. The integrals (15) and (22) involve the same kinetic term, so the interpretation of dependence on the ratio $m/p_0 \in \langle s \rangle$ is obviously valid also for $\Gamma_1$. In fact, this comparison only confirms the general fact, that $\Gamma_1$ measures contributions from the net quark spins.

**DISCUSSION AND CONCLUSION**

Now, one can reinterpret the "small" experimental value of the moment $\Gamma_1$. This value is small for the scenario of static quarks, for which the relations (2), (7) and the upper limit in (16) hold. On the other hand, the experimental value of $\Gamma_1$ does not contradict the inequalities (16). The $\Gamma_1$ is close to the lower limit, corresponding to the quarks undergoing intrinsic motion characterized by the small ratio $m/p_0$. In fact, this ratio is a free parameter of the model and some arbitrariness is connected also with the choice of $SU(6)$ symmetry. This symmetry originates in the non relativistic approach, where the flavor and spin variables are included in the symmetry scheme. However, in the applied relativistic model the quark total angular momentum $j = 1/2$ is used instead of the spin $s = 1/2$. The intrinsic orbital motion generates in the function $q_1$ kinetic term, which effectively reduces the integral $\Gamma_1$. On the other hand, this reduction of the spin content is compensated by orbital momentum. In this way it appears, that in Eq. 5 only the valence term is sufficient for generating the proton spin. Contribution of the sea quarks can be considered zero, as it is proved experimentally. So it follows, that in this approach virtually no room is left for the gluonic term in Eq. 5. But this does not contradict the recent data in gluon polarization $\Delta G/G$. The measured values are very small or compatible with zero, but have still rather big experimental errors. So, the suggested model predicts, that the gluonic term, or at least the integral $\langle \Delta G \rangle = \int \Delta G dx$, will be compatible with zero or will be rather small, even if more precise data are available. So it seems, that the original assumption that the proton spin is generated by spins of the valence quarks can be correct, provided that one replaces the word "spins" by the words "total angular momenta". Generally, the intrinsic motion, regardless of its source, is a delicate effect, which can essentially modify the spin structure functions, if the covariant approach is properly applied. At the same time it is an effect, which is invisible in the usual infinite momentum approach. As a matter of fact, roughly the same amount of orbital motion is predicted in models (16). Recent experimental analysis (11) also suggests significant presence of orbital momentum.

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