The complete study on the inclusive production of $\Upsilon + \gamma$ at the LHC

Rong Li$^{1,3}$ and Jian-Xiong Wang$^{2,3}$

$^1$School of Science, Xi’an Jiaotong University, Xi’an 710049, China.
$^2$Institute of High Energy Physics, Chinese Academy of Sciences, P.O. Box 918(4), Beijing, 100049, China.
$^3$Theoretical Physics Center for Science Facilities, CAS, Beijing, 100049, China.

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In this paper we investigate the inclusive associated production of $\Upsilon + \gamma$ at the hadron collider. We calculate the color-singlet sub-processes and all three color-octet sub-processes at the next-to-leading order. Seven sets of long distance matrix elements (LDMEs), which are extracted from the studies on the prompt production of $\Upsilon$ at hadron colliders, are used to give the numerical results and we find that there are three sets of LDMEs give the unphysical results on the yield and the polarization for this process. The yield is enhanced by several or two orders times comparing to that of color-singlet process and the polarization changes from longitudinal to slightly transverse or even mainly transverse in the large $p_t$ region. The estimation results indicate that to study the process at the large hadron collider (LHC) is difficult and it may be well investigated at the future Super Proton-Proton Collider (SPPC).

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The production and decay of heavy quarkonium have become an active area in physics since its discovery in 1974. Although there are some defects in the theoretical description, the color singlet model (CSM) had been the main tool to study the heavy quark system for about 20 years. In the early 1990s, in order to solve the $J/\psi$ surplus problem at the Tevatron [1] the non-relativistic quantum chromodynamics (NRQCD) had been proposed [2]. This new scheme also factorizes the physical process into the short distance part, which can be calculated perturbatively, and the long distance matrix elements (LDMEs), which can be extracted from matching the theoretical prediction and the experimental data or calculated using the lattice method. The extensive studies on the NRQCD and the heavy quarkonium can be found in the review references [3].

In reference [4] the authors had noticed that the LDMEs of $J/\psi$ extracted from the different process can not be consistent with each other. This may raise the doubt on the universality of the LDMEs in the NRQD factorization scheme. As for the $\Upsilon$, the heavy bottom quark mass makes it a better one to be described by the NRQCD and the dilepton decay channel of it also provides great convenience for experimental investigation. In 2007 in the color-singlet framework the yield of inclusive production of $\Upsilon$ at next-to-leading order (NLO) was presented in Ref. [5] and the polarization distribution was obtained in the next year [6]. The authors also estimate the results on the yield and the polarization at the partial next-to-next-to-leading order [7]. Although these works are impressive progress which enhanced the $p_t$ distribution largely in the large $p_t$ region and changed the polarization from transverse to longitudinal one in the CSM the theoretical predictions without the NLO results of color octet channel hardly can be proper results to compare with the experimental data. Neither the leading order (LO) results of CSM or NRQCD nor the NLO results of CSM can reconcile the conflict between theoretical results and the data on the $p_t$ distribution and the polarization simultaneously. In order to investigate the heavy quarkonium production at NLO in NRQCD we need not only to calculate the short distance parts at NLO but also to determine the value of corresponding LDMEs from matching the theoretical result with the experimental data. The NLO results of S wave color octet channel are calculated in Ref. [8]. The first theoretical calculation including the full NLO results of the color octet channel is presented in Ref. [9]. There are some other works on the full NLO theoretical investigation of inclusive hadroproduction of $\Upsilon$ [10–12]. By using different schemes on the chosen of NRQCD scale or the fitting procedure several sets of LDMEs were obtained. Although the relative satisfactory results on the yield and the polarization prediction were given in the above references there are large numerical difference among different sets of LDMEs.

Recent years, except the inclusive production of heavy quarkonium the associated production of quarkonium had also been an attractive topic. From 1992 some authors had studied the haroproduction of quarkonium associated with a gauge boson, photon, $W^\pm$ or $Z^0$ to investigate gluon content of the proton [13] or the color octet mechanism in the NRQCD at tree level [14]. The QCD NLO correction to some of these process had been studied in Ref. [15]. The color singlet channel of $pp \to \Upsilon + \gamma + X$ had been studied at QCD NLO [16] and the partial NNLO result also been given [17]. The hadroproduction of $\Upsilon + \gamma$ even proposed in reference [18] as an ideal tool to investigate the transverse dynamics and polarization of gluon in proton. In our previous paper [19] we give a full result including the NLO calculation on the color singlet and the color octet channel of the $J/\psi$ associated production with a photon. We had shown that several sets of LDMEs about $J/\psi$ production given different prediction on $J/\psi + \gamma$. Some of them even result in the unphysical distribution. How about the LDMEs.
on Υ which had given well description of yield and polarization on Υ inclusive production at hadron colliders. In this paper we extend our study to calculate the full results on Υ + γ at the NLO.

According to the NRQCD factorization scheme, the differential cross section for this process can be expressed as,

\[
\sigma(p+p \to \Upsilon + \gamma + X) = \sum_{i,j} \int dx_1 dx_2 \sigma(i \to (Q\bar{Q})_n + \gamma + X)(O^n_1) \times G^u_p(x_1)G^d_p(x_2)\hat{\sigma}(ij \to (Q\bar{Q})_n + \gamma + X)(O^n_1).
\]

The above equation indicate that the cross section of this process is the convolution of the parton distribution function \(G^u_p(x_1)\) and the parton level short distance coefficients \(\hat{\sigma}\). The \(\langle O^n_1/\psi \rangle\) are the LDMEs of corresponding subprocesses. The parton level sub-process are,

\[
g + g \to Q\bar{Q}[s_{11}\,S_{0\,0}^8\,S_{11}^8\,P_{8}^3] + \gamma, \quad (2) \\
g + q \to Q\bar{Q}[s_{0\,1}\,S_{1\,0}^8\,S_{1\,1}^8\,P_{8}^3] + \gamma, \quad (3) \\
g(q, \bar{q}) + g(q, \bar{q}) \to Q\bar{Q}\left[\left(1\,s_{1\,1}^8\,S_{0\,0}^8\right)\right] + \gamma + g. \quad (4)
\]

For the description of \(J/\psi\) polarization we adopt the usual definition in helicity frame as

\[
\alpha(p_t) = \frac{d\sigma_{11}/dp_t - d\sigma_{00}/dp_t}{d\sigma_{11}/dp_t + d\sigma_{00}/dp_t}.
\]

As in our previous paper on the hadroproduction of \(J/\psi + \gamma\) we also use the same isolated condition in reference [20] to isolate a photon from the quark jet in some of the sub-processes. The isolated condition is

\[
p_t^i \leq p_t^i \frac{1 - \cos R_{\gamma_i}}{1 - \cos \delta_0} \quad \text{for} \quad R_{\gamma_i} < \delta_0. \quad (6)
\]

The definitions of the \(p_t^i, \ p_t^i, \ cos \ R_{\gamma_i}, \) and the \(\delta_0\) can be found in Ref. [20]. Here we set \(\delta_0 = 0.7\). The newly infrared divergence appearing in the calculation of p-wave subprocesses at the NLO is handled with the same method in Ref. [21]. We use the Feynman Diagram Calculation (FDC) package [22] to generate the analytic results and output the Fortran code for numerical evaluation.

Let us talk about the choice of parameters used in the numerical calculation. In the calculation, the bottom quark mass is set as 4.73GeV and will vary from 4.63 to 4.83 to estimate the related uncertainties. The factorization and the renormalization scales are set as \(\mu_F = \mu_R = \mu_0 = \sqrt{(2m_b)^2 + p_t^2}\) and will vary from \(\mu_0/2\) to \(2\mu_0\) to estimate the uncertainties. The CTEQ6L and CTEQ6M PDFs are used in the calculation of the LO and NLO convolutions and the \(\alpha_s\) running in these PDFs are used to calculate the cross section of the sub-processes. The center of mass energy and cut condition for the \(T\) or the final photon are set as \(\sqrt{s} = 7,8, 14\)TeV, \(|y_{T,\gamma}| \leq 3, \ |\eta_\gamma| \leq 1.45\) and \(p_t^i \geq 1.5,3,5,15\)GeV. The fine structure constant for the electromagnetic coupling is chosen as \(\frac{1}{12\pi}\). The different choice of the parameters are marked in the figures.

The extraction of the LDMEs in NRQCD is the key point to give the rational theoretical predictions. As for the LDMEs for the production of \(\Upsilon\) there are many sets of them extracted by two groups form Peking University (PKU) [9, 11] and the Institute of High Energy Physics [10, 12]. In reference [9] the authors had given the three LDMEs at the NLO for the first time. But they included the feed-down contribution from \(3S^1_0\) channel of P-wave bottomonium in the corresponding CO LDMEs of \(\Upsilon\). Therefore, we do not use the LDMEs in reference [9] in our numerical calculation. The authors in reference [10] extracted the LDMEs related to the production of \(\Upsilon\) individually by matching their theoretical prediction at the NLO with the yield and the polarization at the Tevatron and LHC. Thereafter, they updated their analysis with three different schemes by including the newly measured parameters, the mass of \(\Upsilon(3S)\) and the fraction for \(\chi_{bJ}(3P)\). The PKU group also extracted the individual LDMEs in the first version of reference [11] by matching the yield and the polarization data. In the second version of reference [11] they decomposed the contribution of P-wave color-octet subprocesses into the linear combination of the two S-wave subprocesses. Therefore, just as in the \(J/\psi\) case [23] they extracted two linear combination of the three LDMEs as

\[
M^T_{0,r_0} = \langle O^T(1S^0_0) \rangle + \frac{r_0}{m_b^2}\langle O^T(3P^0_0) \rangle, \quad (7)
\]

\[
M^T_{1,r_1} = \langle O^T(3S^0_0) \rangle + \frac{r_1}{m_b^2}\langle O^T(3P^0_0) \rangle, \quad (8)
\]

where \(r_0=3.8, \ r_1=-0.52\), \(M^T_{0,r_0} = 13.70 \times 10^{-2}\)GeV\(^3\) and \(M^T_{1,r_1} = 1.17 \times 10^{-2}\)GeV\(^3\). Here we use the same method as in our previous paper on \(J/\psi + \gamma\). By requiring the LDMEs to be positive we obtain two sets of LDMEs referring to as "Han Extension" (Han1 and Han2) in the following parts. The seven sets of LDMEs and related parameters, the \(\mu_A\) and the cuts on the transverse momentum of \(T(P_t^*),\) are listed in Table 1.

In figure 2, we show some features of the sub-processes. The ratios between the three sub-processes are plotted in the upper parts of the figure. We can see that with the increase of transverse momentum the cross sections for the \(3S^0_0\) and \(3P^0_0\) sub-processes at the parton level become much larger than that of the \(1S^0_0\) sub-process and there is no linear correlation among them, just like the associated production of \(J/\psi + \gamma\). Because the \(1S^0_0\) sub-process is unpolarized we just show the polarization of the other two sub-processes in the lower part of figure 2. Both of the previous two sub-processes show the transverse polarization in almost the whole \(P_t^*\) region. This two figures together with the choice of the LDMEs will give qualitative hints on the polarization of the final \(\Upsilon\).

Because the contribution from \(3S^0_0\) and \(3P^0_0\) sub-processes is dominant the numerical results with the Han set is almost the same as that of the Han1. We just show the results of two extension cases. There is similar situation between the Gong and the Feng1 sets. The sets Feng1 and Feng3 give the similar trend on the yield and
the polarization, which become unphysical in the large \( p_t \) region. Therefore, we just plot the numerical results with the LDMEs sets Han extension1, Han extension2, Feng1 and Feng2 in the following parts in our paper.

We plot three sets of results with different cut conditions and observables in three columns in figure 2. In the first column we investigate the dependence of cross section and the polarization on the \( p_t \) cuts of the associated photon with three sets of LDMEs which can give the physical results on the \( p_t \) distribution and the polarization. We choose three \( p_t^2 \) cuts and plot the corresponding curves. Comparing to the CSM results at the NLO even at the large \( p_t \) region (about 100 GeV) the cross section and the polarization do not have a manifestly convergence behavior in the \( p_t \) region being studied. This is due to the infrared divergence related to the photon which we handled by using the isolated condition. They almost have a parallel shift with different \( p_t^2 \) cuts.

The second column in figure 2 shows our results at \( \sqrt{s} = 14 \text{ GeV} \). The shaded band presents the dependence of physical observables on three parameters, bottom quark mass \( m_b \), renormalization scale \( \mu_r \) and the factorization scale \( \mu_f \). The numerical results show the similar feature as that in the \( J/\psi \) case \[19\]. At first by including the contribution of color-octet processes the cross section for the inclusive hadroproduction of \( \Upsilon + \gamma \) is enhanced about 2 or 70 times larger than that of the CSM result at NLO with the three sets of LDMEs which can give physical results. The polarization of \( \Upsilon \) changes from mainly longitudinal in CSM to slightly longitudinal, slightly transverse and even mainly transverse at the NLO with the three sets of LDMEs. The uncertainty of production rate becomes larger with the increase of \( p_t \) and at the same time the polarization parameter exhibit the more sophisticated feature. The polarization parameter obtained by using the Han extension sets of LDMEs converges with the increasing of \( p_t \) and the one with the Feng2 set of LDMEs does not. From the values of the LDMEs we can infer that the uncertainty mainly comes from the \( ^3S_1^0 \) channel. The results with the Feng1 set of LDMEs are also plotted in the figures, which give negative cross section in the \( p_t \) distribution at about \( p_t = 48 \text{ GeV} \) and result a polarization parameter lower than -1 when \( p_t > 32 \text{ GeV} \).

Just as in the case of \( J/\psi + \gamma \) the associated production of \( \Upsilon + \gamma \) provides an opportunity to distinguish or to assess the different sets of LDMEs obtained with different schemes on \( \Upsilon \) production. We calculate the theoretical prediction for the yield and the polarization of \( \Upsilon \) in this process with the \( \sqrt{s} = 8 \text{ GeV} \). The large hadron collider (LHC) has \( 23 \text{ fb}^{-1} \) integrated luminosity at \( \sqrt{s} = 8 \text{ GeV} \). We choose \( p_t^2 > 15 \text{ GeV} \) to suppress the background and \( \eta_\gamma < 1.45 \) for photon reconstruction efficiency consideration. The numerical results show that in the large \( p_t \) region the differential cross section which including the COM contribution is about 2 to 20 times larger than the CSM result. The ratio is smaller than that of the \( J/\psi + \gamma \). The Feng1 set of LDMEs

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**TABLE I:** The NRQCD LDMEs \( \langle O^T(n) \rangle \) extracted by two groups at the NLO with \( \langle O^T(3S_1^0) \rangle\) = 9.28 GeV³. The other three color-octet LDMEs are listed as following (LDMEs in unit of \( 10^{-2} \text{ GeV}^3 \), \( P_t^3 \) and \( \mu_A \) in unit of GeV.)

|     | \( O^{(1)S_0^0} \) | \( O^{(3)S_1^0} \) | \( O^{(3)P_0^0}/m_b^2 \) | \( P_t^3 \) | \( \mu_A \) |
|-----|------------------|------------------|------------------|--------|--------|
| Han | 0.017            | 2.97             | 3.83             | 15     | \( m_b \) |
| Han1| 0                | 3.04             | 3.61             | 15     | \( m_b \) |
| Han2| 13.7             | 1.17             | 0                | 15     | \( m_b \) |
| Gong| 11.15            | −0.41            | −0.67            | 8      | \( m_b \)u |
| Feng1| 13.6            | 0.61             | −0.93            | 8      | \( m_b \)u |
| Feng2| 10.1            | 0.73             | −0.23            | 8      | \( m_b \)u |
| Feng3| 11.6            | 0.47             | −0.49            | 8      | \( m_b \)u |

**FIG. 1:** The ratio of \( d\sigma(3P_0^0)/d\sigma(1S_0^0) \) and \( d\sigma(3S_1^0)/d\sigma(1S_0^0) \) as functions of \( P_t \).
also gives the unphysical predictions on the $p_t$ yield and the polarization. The photon reconstruction efficiency under the above pseudo-rapidity and $p_t$ cut condition is 0.7 \cite{24}. The branch ratio of $\Upsilon$ di-lepton decay channel is $Br(\Upsilon \rightarrow \mu^+\mu^-\text{ann} e^+e^-) = 0.05$. Therefore we can expect there are 0.5~5 events at $p_T^\Upsilon = 100\text{GeV}$ or 9~30 events at $p_T^\Upsilon = 60\text{GeV}$ with $\sqrt{s} = 8\text{TeV}$. If we raise the center of mass energy to 100\text{TeV} and keep the other conditions unchanged the event number can be enhanced to 32~161 or 322~782 respectively.

In summary, we investigate the inclusive production of $\Upsilon + \gamma$ at hadron collider. By including the three different color-octet sub-processes we give complete theoretical predictions on the yield and the polarization at the QCD NLO within NRQCD framework. Comparing to our previous work \cite{19} the cross section for this process is enhanced several times or even two orders in the large $p_t$ region. The inclusion of color-octet sub-processes at the NLO changes the polarization from longitudinal one to transverse one. Although the scale chosen and the variation of the bottom quark mass can bring the uncertainties to some extent the main uncertainty of the theoretical prediction comes from the difference among the LDMEs. We investigate seven sets of LDMEs and only four of them, Han, Han extension1, Han extension2, and Feng2 can give the physical predictions on the yield and the polarization in the $p_t$ region that we studied. From figure 1 it can be seen that the p-wave color-octet sub-process give the dominate contribution in the large $p_t$ region. Therefore we infer that any set of LDMEs with negative $O(3P^8_0)$ will results in the negative prediction on the yield at sufficiently large $p_t$ region and the Feng2 set may meet this problem with the increase of $p_t$. The estimate with the LHC integrated luminosity at $\sqrt{s} = 8\text{TeV}$ indicate that it is not optimistic to investigate this process at the LHC. The result with $\sqrt{s} = 100\text{TeV}$ shows us this process can be studied in the next generation hadron colliders, such as the Super Proton-Proton Collider(SPPC).

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