Testing Chiral Symmetry Breaking at DAΦNE

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The spontaneous breakdown of the chiral symmetry of the QCD Lagrangian ensures that \( \pi\pi \) interactions are weak at low energies. How weak depends on the nature of explicit symmetry breaking. Measurements of \( K_{e4} \) decays at DAΦNE will provide a unique insight into this mechanism and test whether the \( q\bar{q} \)–condensate is large or small.

1. PIONS SCATTERING

Ever since Yukawa and certainly since Powell, pions have played a special role in the study of strong interactions. This is because pions are by far the lightest of all hadrons and so, to state the obvious, determine the range of the nuclear force. Even before we knew why these pseudoscalar mesons should be the lightest hadrons, the scattering of pions provided the prime process for studying the structure of scattering amplitudes. For instance, the high energy behaviour of a total cross-section is bounded with a scale set by the lightest exchange in the crossed channel. Since \( \pi\pi \) scattering is crossing symmetric, the nearest \( t \)–channel singularity is generated by two pion exchange and so the pion mass ( or rather \( 1/m_\pi^2 \) ) sets the scale for the Froissart bound [1] :

\[
\sigma_{\text{tot}}(\pi\pi) \leq \frac{\pi}{m_\pi^2} \ln^2(s/s_o) .
\] (1)

Now we can use the same fundamental properties of analyticity, crossing and unitarity, to bound not just the high energy behaviour but the low energy too.

The natural place to start is the interior of the Mandelstam triangle. There though the process is unphysical, the amplitude is real (as opposed to complex) and intimately connected to the three nearby physical regions, Fig. 1. In the case of the process \( \pi^0\pi^0 \rightarrow \pi^0\pi^0 \), which has manifest crossing symmetry, the mid-point of the Mandelstam triangle at \( s = t = u = \frac{4}{3}m_\pi^2 \) can be shown to be the minimum of the amplitude. The amplitude grows in all directions [2] away from this point because of the positivity of total cross-sections Fig. 2. If one is clever enough, and André Martin [2] is clever enough, one can deduce absolute upper and lower bounds on the magnitude of the amplitude at this symmetry point, Figs. 1, 2. Indeed Martin [2] found

\[
16 > F \left( s = t = u = \frac{4}{3}m_\pi^2 \right) > -100 ,
\] (2)

with the conventional normalization of the amplitude in S-matrix theory. For those more familiar with applications in Chiral Perturbation Theory (\( \chi \)PT) [3,4], then these values
should be multiplied by \(32\pi\), i.e. \(\sim 100\). These bounds set the size of what we would expect for a maximally strong interaction. The virtue of the symmetry point is that there all (kinematically) non-zero \(\pi\pi\) amplitudes are proportional:

\[
\frac{1}{3} F(\pi^0\pi^0 \to \pi^0\pi^0) = F(\pi^+\pi^- \to \pi^0\pi^0) = \frac{1}{5} F^{l=0} = \frac{1}{2} F^{l=2}.
\]

But why are pions so light? As discussed by Heiri Leutwyler here [3] and many times previously [4], the up and down quarks have current masses that are very small compared to \(\Lambda_{QCD}\). Indeed, if they were exactly massless, then the QCD Lagrangian would have a chiral invariance. While this is a symmetry of the underlying quark world, it does not occur at the hadron level — scalars and pseudoscalars are not degenerate for instance. Consequently, this chiral symmetry must be spontaneously broken.

A model that illustrates this simply is the \(\sigma\)–model of Gell-Mann and Levy [5] and especially that of Nambu and Jona-Lasinio [6]. One considers a world of just isosinglet scalars and isotriplet pseudoscalars. Then the potential produced by the interactions of these fields is found to be a minimum not when the scalar and pseudoscalar fields are zero, but rather when they have a non-zero vacuum expectation value, Fig. 3. Since there are degenerate minima, round the Mexican hat, nature chooses (for reasons that will become clear) a ground state, or vacuum, near where the pion field is zero but the expectation
value of the scalar field is definitely non-zero. The particle content is determined by the quantum fluctuations about this vacuum. Those fluctuations in the pseudoscalar direction are around the rim of the Mexican hat for which there is no resistance. Consequently, pions are massless: the Goldstone bosons of chiral symmetry breaking [7]. In contrast, the fluctuations in the scalar direction go up the sides of the Mexican hat and so the scalar field has a mass [8]. In this model, this comes about because the quark and antiquark fields are so strongly bound by non-perturbative gluon interactions that they form a condensate. The magnitude of this condensate determines the scalar mass. It is this that gives the (light) quarks a non-zero mass function, so that at low momenta ($\leq \Lambda_{QCD}$), they are effectively massive constituent quarks. In this picture, chiral symmetry breaking not only results in massless pseudoscalars, but gives a non-zero mass to all the other light hadrons — the $\rho$, the nucleon, etc. Their masses are determined by the scalar that acts as the Higgs boson of the strong interaction sector. This illustration of spontaneous chiral symmetry breaking has close analogies with ferromagnetism, as Nambu stressed long ago [9]. However, this is not the only way chiral symmetry need be broken. Massless Goldstone bosons always result, but a large $q\bar{q}$-condensate is not essential. Indeed, in analogy with an antiferromagnet, the condensate could be small or even zero, as Jan Stern and collaborators [10] have suggested. It is the difference between these two pictures that DAΦNE can test.

Since pions decay by the weak interaction, they couple to an off-shell $W$–propagator through an axial vector current. If pions were massless, then this current would be conserved. This leads to an important low energy theorem. In the massless world, when the Mandelstam triangle (Fig. 1) for $\pi\pi$ scattering shrinks to a point, the amplitude at this point vanishes, i.e. $F(s = t = u = 0) = 0$. This obviously satisfies the bounds of Eq. (2) (for massive pions). It highlights how the strong interaction of pions is far weaker
Figure 3: $\sigma$–model potential in the $\sigma$–$\pi$ plane. With no explicit chiral symmetry breaking, there are degenerate minima all around the rim of the Mexican hat. In practice, the hat is tilted so that there is a unique minimum close to the state labelled vacuum.

at low energies, than we would naively have expected. This allows pion amplitudes to be expanded round the symmetry point in powers of momenta, or of the Mandelstam invariants, for which the natural scale is the square of the mass of the scalar, or the $\rho$, or the nucleon, or $32\pi f_{\pi}^2$, so that

$$F(s, t, u, m_{\pi} = 0) = \frac{O(s, t, u)}{32\pi f_{\pi}^2}.$$  \hspace{1cm} (4)

$\chi$PT systematises this [11,4,12].

Of course pions are not massless. There is explicit breaking of chiral symmetry by the masses of the current quarks and hence by the pion mass. Indeed, it is this explicit breaking that lifts the vacuum degeneracy of the $\sigma$-model, so that there is a unique ground state close to the one previously discussed. Despite this explicit breaking of chiral symmetry, there is still an important low energy theorem. If we consider $\pi\pi \rightarrow \pi\pi$ with 3 massive pions and one massless, and let the 4-momentum of the massless one go to zero, then the amplitude again vanishes. This condition, deduced by Adler [13], means that at the symmetry point of the Mandelstam triangle, Fig. 1, in the world where $s + t + u = 3m_{\pi}^2$, the amplitude vanishes:

$$F\left( s = t = u = m_{\pi}^2 \right) = 0 \, .$$  \hspace{1cm} (5)
Now much closer to the real world, we can again make a Taylor expansion about the symmetry point in terms of not just the momenta squared, but explicit factors of $m_\pi^2$ and the value of the $q\bar{q}$-condensate (if this is small), all in $32\pi f_\pi^2$ units. This allows us to consider the world with $s + t + u = 4m_\pi^2$. Then at the symmetry point we have

$$F\left(s = t = u = \frac{4}{3}m_\pi^2\right) = \frac{\alpha m_\pi^2}{32\pi f_\pi^2}, \quad (6)$$

where in standard $\chi$PT with a large $q\bar{q}$-condensate, $\alpha$ is close to unity [4], while in generalized $\chi$PT with a smaller condensate [10] $\alpha$ can be as large as 4, when the condensate goes to zero. Thus either the amplitude is approximately

$$F\left(s = t = u = \frac{4}{3}m_\pi^2\right) \simeq 0.02 \quad (7)$$

for a large condensate or

$$0.04 \leq F\left(s = t = u = \frac{4}{3}m_\pi^2\right) \leq 0.09 \quad (8)$$

for a smaller condensate, while the general results for a strong interaction would mean the amplitude is between $-100$ and $+16$, Eq. (2). Having a small value for the $\pi\pi$ amplitude at the symmetry point speaks of the Goldstone nature of the pion. How small this value is tells us about the explicit breaking of chiral symmetry. It is at the symmetry point that the difference is maximal. This is because moving closer to the physical regions the parameters of the Chiral Lagrangians ($\bar{\ell}_i, \lambda_i$) [4,10] are necessarily fixed from experiment and the different expansions inevitably become more similar. Thus it is the amplitude at the symmetry point, Eqs. (7,8), that distinguishes the versions of explicit breaking.

To learn about this, we must continue experimental information into the unphysical region, Fig. 1. The way to do this is by using dispersion relations. The $\pi\pi$ amplitudes satisfy fixed-$t$ dispersion relations that, thanks to the Froissart bound, need at most two subtractions. This would mean there would be 2 subtraction constants at each momentum transfer. However, by the clever use of crossing symmetry S.M. Roy [14] showed how these could be rewritten in terms of just two subtraction constants for a range of momentum transfers — a range that allows a rigorous type of partial wave dispersion relations, known as Roy equations. It is natural to take the two subtraction constants to be values of the $I = 0$ and $I = 2$ $S$-wave amplitudes at threshold, which give the scattering lengths $a_0^0$ and $a_0^2$. While, in general, these are independent, in the real world these two constants are closely correlated. This is because all experiments and models have an $I = 1$ cross-section below 1 GeV dominated by the $\rho$-resonance and have an $I = 2$ cross-section that is comparatively small [15]. This forces $a_0^0$ and $a_0^2$ to follow what is known as the universal curve of Morgan and Shaw [16], along which all phase-shift solutions and all models lie. Standard $\chi$PT [17] gives $a_0^0 = 0.21 \pm 0.01$, for example, at 2 loops with electromagnetic corrections, while in generalized $\chi$PT $a_0^0$ is bigger [18]. For instance if $\alpha = 3$ (Eq. (6)), then $a_0^0 \approx 0.31$. Though the one parameter $\pi\pi$ scattering depends on can be conveniently taken to be the $I = 0$ $S$-wave scattering length, $a_0^0$, it could equally well be the value of the $\pi\pi$ amplitudes at the symmetry point. As just noted changing this value by a factor 3, $a_0^0$ only changes by 50%. What do data on $\pi\pi$ scattering, tell us about this scattering length?
Figure 4: $S$-wave $\pi\pi$ phase-shifts $\delta_0^I$ with isospin $I = 0, 2$ as a function of $\pi\pi$ mass. The data are from different analyses of the CERN-Munich experiments [19,20]. For $\delta_0^0$, the open triangles below 620 MeV are from Estabrooks and Martin [21] (averaging their $s$ and $t$-channel treatments). Above 610 MeV are shown results of the energy-independent (open circles) and energy-dependent (solid triangles) analyses of Ochs [22]. The $I = 2$ phases, $\delta_0^2$, in 100 MeV bins are the results of the two analyses of Hoogland et al. [20]: method A (open circles), method B (solid squares). The solid line marks what are called the central phases and their extrapolation to threshold using the results of the study of the Roy equations with $a_0^0 = 0.2$, the dotted line has $a_0^0 = 0.1$ and the dashed line $a_0^0 = 0.3$. 
2. PION INTERACTIONS FROM EXPERIMENT

The classic CERN-Munich experiments [19,20] on dipion production at high energies and small momentum transfers for \( \pi^+p \to \pi^+\pi^+n \) give the \( \pi\pi \) phase-shifts [20-22] for the \( I = 0 \) and 2 \( S \)-waves, Fig. 4. While these data clearly indicate scattering lengths smaller than 1 (in pion mass units), as seen from the Roy equation solutions in Fig. 4 they allow a range of values for \( a_0^0 \) far larger than the tiny difference (corresponding to Eqs. (7,8)) we want to be able to detect if we are to test the nature of explicit chiral symmetry breaking. It is clear one needs precision data closer to threshold than 500 MeV. These will be provided by \( K_{e4} \) decays at DAΦNE [23]. With a branching ratio of 4.10\(^{-5}\), \( K^\pm \to e^\pm\nu \pi^+\pi^- \). Here, the pions interact in the femto-universe of the decay in a universal way and they remember how they interact. This means that each \( \pi\pi \) partial wave amplitude has the same phase as it does in all \( \pi\pi \) interactions, and so as in elastic scattering. It is these phases, or rather their differences, we want to measure. This is not an easy task, since the process of a decay into 4 particles, e.g. \( K^+ \to e^+\nu_e \pi^+\pi^- \), depends on 5 Lorentz invariants. These can be conveniently expressed in terms of 5 experimentally measurable kinematic variables. As proposed by Cabibbo and Maksymowicz [24], we imagine the \( K \)-decays into a dilepton and a dipion system back-to-back shown in Fig. 5 and the masses of these two systems \( M_{e\nu} \) and \( M_{\pi\pi} \) are the first two variables. Next we consider the dilepton rest frame, where the individual leptons go off back-to-back at an angle \( \theta_e \) relative to the initial dilepton direction. Similarly, the pions go off at an angle \( \theta_\pi \) relative to the initial dipion system. Then lastly, the plane of the two leptons is at an angle \( \phi \) to the two pion plane, Fig. 5. The decay distribution is then studied as a function of \( M_{e\nu}, M_{\pi\pi}, \theta_e, \theta_\pi \) and \( \phi \). Results from DAΦNE will not be limited by statistics alone, but as much or more by systematics. These can be ameliorated by having large samples of both \( K^+ \) and \( K^- \)'s. These decays are related by a \( CP \) transformation, so that their distributions should be the same, except that the dilepton and dipion planes are oppositely oriented, i.e. \( \phi \to -\phi \). This fact can help check how well the \( \phi \)-dependence of the KLOE detector is understood and so reduce systematic uncertainties [25].

Now the way the decays depend on the kinematic variables is specified by the weak matrix element for \( K^\pm \to \pi^+\pi^- \) \( W^\pm \to \pi^+\pi^-e^\pm\nu \). The \( W^\pm \) are described by vector and axial vector currents. Their matrix elements can be expressed in terms of the Lorentz vectors [24] that can be formed from the kaon and the two pion momenta. Each independent combination is multiplied by a formfactor \( F, G, R \) and \( H \), which will depend on \( M_{\pi\pi} \) and \( M_{e\nu} \). Taking the modulus squared of these matrix elements and using the Dirac equation, the formfactor \( R \) is multiplied by \( m_e^2/M_{e\nu}^2 \), so if we avoid the difficult measurements at very small dilepton masses, this term is negligible. Consequently, the 5-fold differential decay distribution depends on the unknown formfactors \( F, G \) and \( H \). These can be decomposed in terms of \( \pi\pi \) partial waves. Analysis shows that \( D \) and higher waves are negligible [26], so that only \( S \) and \( P \)-waves are needed. The decay distribution involves terms that depend on the \( S-P \) phase difference, \( \delta_S - \delta_P \). By Watson’s final state interaction theorem and the \( \Delta I = \frac{1}{2} \) rule, this difference is just equal to the difference between the \( I = 0 \) \( S \)-wave and \( I = 1 \) \( P \)-wave phases, \( \delta_0^0 - \delta_1^1 \), of \( \pi\pi \) elastic scattering. Since \( \delta_1^1 \) is accurately determined from the tail of the \( \rho \) by the use of dispersion relations, these measurements allow \( \delta_0^0 \) to be determined. As seen from Eq. (20) of [27] and Eq. (5.19) of [28], it is the \( \phi \)-dependence that mainly determines this phase differ-
ence. Pais and Treiman [27] proposed a method to determine this independently of the unknown formfactors. The method involves the double differential decay distribution for \( \cos \theta_e \) and \( \phi \). However the crucial \( \phi \)-dependence is experimentally very weak and the use of information on just two variables is insufficient for a precision determination. Rather a maximum likelihood analysis of the full five-fold differential decay distribution is needed. With 7000 events from the University of Pennsylvania group [29] and 30000 events from the Geneva-Saclay group [30] give the phase differences shown in Fig. 6. A free fit to the Geneva-Saclay results yields \( a_0^0 = 0.31 \pm 0.11 \), while incorporating the Roy equation constraints gives [15]

\[
a_0^0 = 0.28 \pm 0.05 \quad \text{or} \quad a_0^0 = 0.26 \pm 0.05
\]

(9)

depending on the solutions used. These can be translated into values of the amplitude at the symmetry point, Eq. (6), with [10,18]

\[
1.30 \leq \alpha \leq 3.02
\]

(10)

for the second values of \( a_0^0 \) in Eq. (9). Clearly, this range cannot distinguish between the different forms of explicit chiral symmetry breaking and tell whether the \( q\bar{q} \)-condensate is large or small. The aim is that with higher statistics and better control of systematics, DAΦNE will do better. Clearly greater precision on the phases would be achieved if the
Figure 6: The phase difference $\delta^0_0 - \delta^1_1$ determined from $K_{e4}$ decays in the Pennsylvania (open triangles) [29] and Geneva-Saclay (solid dots) [30] experiments as a function of $\pi\pi$ mass. The curves are the predictions of two loop $\chi$PT: the Standard result [17] for which $\alpha \simeq 1.2$ and for $\alpha = 2, 3$ of Generalised $\chi$PT [18].

formfactors $F$, $G$, $H$ were modelled as in $\chi$PT [28], but this would be prejudicing the result. Further study will be needed in the specific context of the KLOE detector [23, 25].

If the $q\bar{q}$-condensate is large, then in the NJL model [6], the scalar mesons form the Higgs sector of the strong interaction. Much of our present information about this comes from the 25 year old CERN-Munich experiments [19, 20], mentioned earlier. These were first analysed assuming the unnatural parity exchange component of the $\pi N \rightarrow \pi\pi N$ cross-section at 17.2 GeV/c to be controlled by $\pi$ exchange alone, Fig. 7. This gives the Ochs and Wagner [22], or Estabrooks and Martin [21] phases shown in Fig. 4. A test of these assumptions was provided by the later ACCMOR collaboration measurements [31] on a polarized target. These showed that the unnatural parity component was consistent with one-pion-exchange below 1 GeV. However, taking new data at 5.95 and 11.85 GeV/c [32] and using the ACCMOR results up to 900 MeV, Svec [33] claimed that an $up$ solution [15] for $\delta^0_0$ was also possible, indeed favoured, indicating a narrow “$\sigma$” with $\rho$-like mass and width. It was known that this could not possibly be correct [15], since this solution could not describe the sharp change in behaviour of the integrated cross-section and $S - P$ interference at $K\bar{K}$ threshold. These require the phase $\delta^0_0$ to rise steeply through 180° near $K\bar{K}$ threshold and the phase could not have already reached this value below 900 MeV as Svec proposed. Nevertheless, these claims prompted the Krakow group [34] to go
back and reanalyse their data from ACCMOR taken nearly twenty years ago. They find one solution (the so called \textit{down-flat} solution, Fig. 8a) which is very close to that of Ochs and Wagner [22], and of Estabrooks and Martin [21] of Fig. 4. However, they do find a second solution (\textit{up-flat}, Fig. 8b) that does give a steeper rise of the phase below 900 MeV and a not so steep rise through $K\overline{K}$ threshold. (There are incidentally 2 other solutions but they are not consistent with unitarity.) These two solutions must have different $\pi$ and other unnatural parity exchange, e.g. $a_1$, components, Fig. 7.

These can be tested by measuring $\pi^-p \to \pi^0\pi^0n$ on a polarized target and extracting the $\pi^-\pi^+ \to \pi^0\pi^0$ amplitude. The advantage of this $\pi^0\pi^0$ channel is that there are only even $\pi\pi$ partial waves, with no $\rho$-contribution, but no data on a polarized target are likely to be taken. Nonetheless, the Krakow group can predict for their solutions what the $S$-wave cross-sections for unnatural parity exchange in an unpolarized $\pi^-p \to \pi^0\pi^0n$ experiment at 17.2 GeV/c should be. Fortunately, the E852 collaboration at BNL [35] has measured this at 18 GeV/c already, as Alex Dzierba [36] has described here. So if the Krakow group make these predictions, E852, once their full statistics are analysed, should be able to distinguish between these solutions. This is an essential test of the $S$-wave $\pi\pi$ amplitudes, on which any view of the scalar mesons is necessarily based. Both DAΦNE [23] at Frascati and CEBAF [36] at TJNAF can add to this by their radiative $\phi$-decay experiments. With sufficient statistics the $\pi\pi$ mass spectrum can be mapped out and this too will aid our understanding of the universality of $\pi\pi$ interactions.

The explicit breaking of chiral symmetry can only be tested in very low energy $\pi\pi$ processes. Measurements of $K_{e4}$ decays at DAΦNE [23], or of the lifetime of $\pi^+\pi^-$ atoms [37] at CERN, alone can achieve this. These have the potential to provide a unique insight.
Figure 8: The down-flat and up-flat solutions for the $I = 0$ $S$–wave phase-shift, $\delta_0^0$, as functions of $\pi\pi$ mass, from the recent re-analysis of the CERN-Munich and ACCMOR data [19,31]. The down-flat solution is very similar to that displayed in Fig. 4 from Refs. [21,22].

into one of the most fundamental properties of QCD.

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