Spintronic memristors for computing

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Abstract

The ever-increasing amount of data from ubiquitous smart devices fosters data-centric and cognitive algorithms. Traditional digital computer systems have separate logic and memory units, resulting in a huge delay and energy cost for implementing these algorithms. Memristors are programmable resistors with a memory, providing a paradigm-shifting approach towards creating intelligent hardware systems to handle data-centric tasks. To fulfill the promise, the memristors need to be high-speed, low-power, highly scalable, and capable of constructing dynamic complex systems. In this Review, we survey spintronic devices from a memristor point of view. We introduce spintronic memristors based on magnetic tunnel junctions, nanomagnet ensemble, domain walls, topological spin textures, and spin waves, which represent dramatically different state spaces. They can exhibit steady, oscillatory, chaotic, and stochastic trajectories in their state spaces, which have been exploited for neuromorphic computing, in-memory logic, stochastic and chaos computing. Finally, we discuss challenges and trends in realizing large-scale spintronic memristive systems for practical applications.
1 Introduction

The unprecedented development of artificial intelligence (AI), big data, and internet of things (IoTs) has redefined the concept of computing. To meet the ever-growing demands of computing performance, the hardware is expected to have more stringent requirements for computing throughput, power consumption, and form factor. This poses a great challenge to conventional complementary metal-oxide-semiconductor (CMOS) digital computing systems. Their physically separate memory and processing units lead to frequent data shuttling, which incurs large time latency and energy consumption, the so-called von Neumann bottleneck. In addition, the scaling of transistors is becoming increasingly cost-ineffective as the size of a transistor approaches its physical limit, which makes performance improvement of digital computing systems even more challenging. Thus, fundamental changes to the building blocks of our computers are imperative.

Spintronic devices provide a transformative solution for computing. Recent flourish of research on spintronic physics, materials, devices and applications render spintronics as one of the most topical fields in physics. Besides spin-transfer torque (STT) ¹, newly discovered switching mechanisms in the past decade include spin-orbit torque (SOT) ² and voltage control of magnetic anisotropy (VCMA) ³. Beyond conventional ferromagnetic materials, ferrimagnet⁴⁻⁶, antiferromagnet ⁷, and two-dimensional (2D) materials ⁹,¹⁰ have been employed in spintronic devices. In addition to traditional magnetic tunnel junctions (MTJs) for storage applications,¹¹,¹² domain wall devices ¹³,¹⁴, skyrmion devices ¹⁵, spin wave devices ¹⁶, and stochastic devices ¹⁷ are under heavy investigations for computing applications, such as brain-inspired computing,¹⁸⁻²⁰ digital logics¹²,²¹ and stochastic computing ¹⁷,²². Quite a few important results of spintronics for computing have been demonstrated. For example, spintronic devices are capable of storing and processing information in a bio-inspired manner based on underlying physical laws, which naturally overcomes the von Neumann bottleneck and achieve better efficiency for brain-inspired computing²³⁻²⁷. Its nonvolatile nature can also be leveraged to perform Boolean logics in-memory, which may mitigate the scaling bottleneck of transistors²⁸⁻³⁰. In addition, spintronic devices may work as probabilistic bits (P-bits), a concept bridging the gap of classical bits and quantum bits (Q-bits), for energy-efficient stochastic computing ¹⁷,²². This rapid development of spintronic computing is further augmented by the fast commercialization of STT-magneto-resistive random-access memory (STT-MRAM) by major foundries such as Samsung, Intel, GlobalFoundries and Taiwan Semiconductor Manufacturing Company (TSMC). It demands a unified and seamless integration of theoretical frameworks of spintronics, electronics, and computer science, which is yet to be developed.

To address this demand, we employ the memristor framework that has been extensively applied in describing generic nonlinear dynamic systems and unconventional computing circuits. The memristor framework has been successfully applied to redox resistive switches back in 2008 ³¹, one of the leading hardware contenders to revolutionize AI. Memristor-based computing has been extensively reported ³²⁻⁴⁰ and actively pursued by information technology giants.

We first show that the fundamental principles behind spintronics meet the criteria of memristors, so as that of the biological neural systems, forging the basis of spintronics-based brain-inspired computing. We then employ the circuit theory to examine the spintronic devices in terms of state space (Boolean domain, real vector, 1D complex field, 2D/3D scalar field, and 2D/3D complex field) and stability of their dynamics or trajectories in state space (convergence, oscillation, chaos, and stochasticity), a manifestation of the underlying physics and materials. Afterwards, we discuss
how these properties synergistically lead to various computing applications including neuromorphic applications, AI computing, digital logics, stochastic and chaos computing. At last, we discuss the challenges and point out potential research directions.

2  What is a memristor and why spintronic devices are memristors

Memristor is conceptualized by Leon Chua to describe the missing relation between flux and charge. Chua and Kang then redefined them as a form of nonlinear dynamic systems, with no connection to magnetic flux. Memristors differ from other commonly seen circuit building blocks such as resistors, capacitors, diodes, and transistors in the sense that the output signals of the latter are functions of their instantaneous input signals, or they do not possess internal state variables. However, memristors, as a generic nonlinear dynamic system, have their outputs depending on internal state variables, making their outputs reflecting the history of input signals. This can be translated to the evolution, usually a first-order differential equation over the state vector \( s(t) \), and transport equations in the state space constituted by dynamic variables:

\[
\frac{ds}{dt} = f(s, u, t) \quad \text{and} \quad y = g(s, u, t)u,
\]

where \( u(t) \) and \( y(t) \) are input and output vectors of the system, respectively. These equations equip memristors with three unique features that are (i) identical zero-crossing (ii) pinched hysteretic loop in the space formed by \( u(t) \) and \( y(t) \) as well as (iii) frequency dependence of the pinched loops. So far such dynamic systems have been observed at nanoscales empowered by different physics, for example, redox reactions, phase-transition in chalcogenide glasses, ferroelectric tunnel junctions, and spintronics.

It has been observed in previous works that many spintronic devices have exhibited memristor-like behaviors. Here, we re-interpret the Landau–Lifshitz–Gilbert (LLG) equation from a memristor point of view. In the model structure – a MTJ, we describe this nonlinear dynamic system by evolution equations incorporating STT, SOT, VCMA and thermal fluctuation (Fig. 1):

\[
\frac{dm}{dt} = -\gamma m \times (H_{\text{eff}} + H_{\text{VCMA}}(I) + H_{\text{th}}) + \alpha m \times \frac{dm}{dt} + \gamma H_{\text{STT}}^{\text{DL}}(I)m \times m_p \times m + \gamma H_{\text{STT}}^{\text{FL}}(I)m \times m_p + \gamma H_{\text{SOT}}^{\text{DL}}(I)m \times \sigma + m + \gamma H_{\text{SOT}}^{\text{FL}}(I)m \times \sigma,
\]

where \( m \) is the unit magnetization vector of the magnetic free layer, \( I \) is the current, \( \gamma \) is the gyromagnetic ratio, \( H_{\text{eff}} \) is the effective field including contributions from the external field, exchange bias field, exchange field, and anisotropy field in the absence of an external voltage or current, \( H_{\text{VCMA}}(I) \) is the VCMA field, \( H_{\text{th}} \) is the stochastic thermal field, and \( \alpha \) is the Gilbert damping constant. Note that VCMA can be written as a function of current since the applied current is directly related to the applied voltage through the Ohm’s law. \( H_{\text{STT}}^{\text{DL}}(I) \) and \( H_{\text{STT}}^{\text{FL}}(I) \) are the effective fields arising from current-induced damping-like and field-like STTs, respectively. \( m_p \) is the magnetization vector of the magnetic pinned/reference layer. \( H_{\text{SOT}}^{\text{DL}}(I) \) and \( H_{\text{SOT}}^{\text{FL}}(I) \) are current-induced damping-like and field-like SOT effective fields, respectively. \( \sigma \) is the spin polarization vector induced by the current. Since \( \frac{dm}{dt} \) on the right side of the above formula can be moved to the left side by replacing it using the entire right side and algebraic reduction, \( \frac{dm}{dt} = f(m, I, t) \) can be explicitly written down.

3
The generalized transport equation builds on magnetoresistance effects (such as giant magnetoresistance and tunnel magnetoresistance), Hall effects (such as anomalous Hall effect), magneto-optical effects, and spin-to-charge conversion effects. In a MTJ, the state $m$ of the magnetic free layer can be electrically read out using the tunnel magnetoresistance (TMR) effect, where TMR ratio is defined as $(R_{AP} - R_P)/R_P$, where $R_{AP}$ and $R_P$ are resistance states when $m$ and $m_p$ are anti-parallel and parallel, respectively. The low-bias voltage $v$ as a function of the current can be written as $V(t) = [R_P + (R_{AP} - R_P)(1 - \cos \theta(m, m_p))]/2 \cdot I(t)$, where $\theta(m, m_p)$ is the angle between $m$ and $m_p$.

On the other hand, the brain is a well-known nonlinear dynamic system made of smaller scale memristor-like dynamic systems such as neurons and synapses. These dynamic systems operate on complicated electrochemical signal cascades, which yields the remarkable energy efficiency and intelligence of the brain.

The brain builds on ion channels, valves controlling ion fluxes across plasma membranes. Such valves can be electrically or chemically gated. The kinetics of ion channels are governed by finite state Markov chains, which can also be formulated by Langevin equations according to molecular dynamics simulations.\(^{51,52}\)

Synapses are junctions interfacing neurons. The presynaptic signal commands voltage-gated ion channels to release neural transmitters, which signify the ligand-gated ion channels of the postsynaptic cleft (Fig. 1)\(^{52}\). As a result, synapses transmit signals across neurons according to their internal states, or $g_{syn}(t) = g_{max}r(t)$ where $g_{max}$ and $r$ are the maximum transmission efficacy and fraction of open ion channels of the postsynaptic cleft. In addition, synapses update their states, or $r$, in parallel as formulated by the differential state evolution equation $\frac{dr}{dt} = \eta N(1 - r) - \beta r$ where $\eta$ and $\beta$ are the binding and unbinding constants, respectively. $N$ quantifies the total neurotransmitters released, $N(t) = \int_0^\infty n(\tilde{t})S(t - \tilde{t})d\tilde{t}$, where $S(t)$ is the presynaptic spike train (usually a sequence of $\delta$-functions) and $n(t)$ represents the neurotransmitter density as measured at the postsynaptic receptor. As a result, synapses naturally meet the definition of a memristor. Such kinetics also enable synapses to practice various local learning rules, like the short/long-term pulse facilitation and depression, as well as spike timing-dependent plasticity, which forges the basis of memory and learning.

Neurons are the sources of signals in the brain. The behavior of a neuron depends on its internal variable, which is frequently approximated by the membrane potential $u$, and can naturally be implemented using memristor-based circuits (Fig. 1)\(^{52}\). The rise and fall of membrane potential depend on the dendritic input $I$ to the neuron according to $\tau_m \frac{du}{dt} = -(u - u_{rest}) - RI$, where $\tau_m$, $u_{rest}$, and $R$ are time constant, rest membrane potential, and input resistance, respectively. In addition, the membrane potential dictates the output of the neuron according to $I = \frac{1}{R}(u - u_{rest}) + C \frac{du}{dt}$, where $C$ is the membrane capacitance, producing neural spikes once the membrane exceeds a threshold in the integrate-and-fire model, a behavior naturally reproduced by the switching process of a volatile memristor. In addition, the more advanced Hodgkin-Huxley model has also been proven a system built on memristors.\(^{53}\) As a matter of fact, the various spiking dynamics, including the three classes of excitability, of the neuron have also been experimentally realized on nanoscale memristors, illustrating their tight correlation\(^{54}\).
3 State space of spintronic memristors

The state-space representation of spintronic memristors includes Boolean domains for nanomagnets/macrospins, real vectors for nanomagnet ensemble/multi-domain magnets, 1D complex fields for domain walls, 2D scalar fields for skyrmions and other solitons, and 2D complex fields for spin waves/magnons (see Table 1). We will mathematically correlate the memristor state-space, a differentiable manifold in most cases, to the underlying physics and materials, and discuss their unique topological features.

Nano-scale MTJs, where the magnetization is in the single-domain state, are the representative model of a nanomagnet or macrospin, because the exchange interaction is strong enough to align all spins in the same direction. Therefore, the macrospin model can be used to approximate the statics and dynamics of collective atomic spins to high accuracy. The state is described by a single magnetization vector $\mathbf{m}$ of the free layer (Fig. 2a). The properties of these MTJs are well explained by the LLG equation shown above. The state of a MTJ can be controlled by many knobs, such as magnetic field, electric current, microwave magnetic field or current, heat current, etc., using many physical effects, such as Zeeman torque, STT, SOT, VCMA, stochastic thermal field, spin-Seebeck effect, etc. Typical binary switching of a MTJ is the foundation of today’s MRAM technologies, where the readout is achieved through the TMR effect (Fig. 2b)\textsuperscript{55}.

When the size of a MTJ gets larger, the entire magnetization of a magnetic free layer breaks down into multiple domains. The characteristic size of this transition from macrospin to a multi-domain state mainly depends on the competition of exchange energy and anisotropy energy, which, in turn, are determined by the geometry, material and structure parameters of the MTJ. In this sense, the number of available states is tunable. Besides, in special cases like ferromagnet/antiferromagnet heterostructures, the fine grains of the antiferromagnetic material can cause a distribution of exchange bias, resulting in a multi-domain state\textsuperscript{56}. The multi-domain magnet or nanomagnet ensemble state can be described by a few coupled and discrete macrospin models (Fig. 2c). If one only considers the analog resistance of the MTJ, a single averaged magnetization vector can be used to phenomenologically describe the state (Fig. 2d)\textsuperscript{56}, which can be controlled by many knobs like the macrospin case. However, since the coupling of magnetic domains and parameters of individual domains are hard to control in this naturally formed multi-domain state, one can assemble multiple nano-scale single-domain MTJs in an array to create a nanomagnet ensemble for particular applications. In this case, the state of individual MTJs and the coupling between them in a nanomagnet ensemble can be in principle precisely controlled. By doing this, one can truly utilize the strength of coupling in addition to the multiple states in the nanomagnet ensemble system.

A domain wall forms between two domains with opposite directions (Fig. 2e). Electrical current can drive domain walls, which make them suitable for racetrack memory\textsuperscript{57}. Current-driven domain wall motion is also used to create nonvolatile magnetic logic circuits\textsuperscript{13}. In a thin-film racetrack, the state of a 180° domain wall can be described by the $\mathbf{m}(\mathbf{x}, \phi)$, where the $\mathbf{x}$ indicates the position and the $\phi$ is magnetization angle of the domain wall. $\phi = 0$ describes a Néel domain wall and $\phi = \frac{\pi}{2}$ describes a Bloch domain wall. The domain wall can be driven by heat current\textsuperscript{58} and spin waves\textsuperscript{59} in addition to the electric current. The information readout for domain wall-based devices is realized through the magneto-optical Kerr effect (Fig. 2f)\textsuperscript{30} or magnetoresistance effects.
Nanoscale skyrmions and other topological solitons have emerged to function as potential information carriers due to their small size, low drive current and topological stability. If one ignores the internal detail and only considers the function based on a mobile information bit, the state of a topological soliton is characterized by \( \mathbf{m}(\mathbf{x}, R) \), where \( \mathbf{x} \) and \( R \) indicate the position and the radius of the soliton, respectively. Skyrmions can be driven by electric current and their motion can exhibit the skyrmion Hall effect due to the Magnus force in the presence of nonzero topological charge. The solitons can also be driven by heat current. Due to the particle-like nature, their transport can be controlled by an applied bias voltage via VCMA effects. Current-driven dynamics of skyrmions can be detected by magneto-optical Kerr effect, transmission X-ray microscopy (Fig. 2h), Lorentz transmission electron microscopy, and neutron scattering.

Spin waves or magnons are the fundamental excitations of magnetization. Utilizing spin waves for information processing could have low power dissipation since it does not necessarily carry charge current and thus could be free of Joule heating. Spin wave solitons can be locally excited using electrical current or microwave magnetic fields and detected using an electrical voltage or microwave impedance (Fig. 2i). A spin wave soliton can be described by \( \mathbf{m}(\mathbf{x}, A, \phi) \), where \( \mathbf{x} \), \( A \) and \( \phi \) indicate the position, amplitude and phase of a spin wave soliton, respectively. Both the amplitude and phase can be used as information carriers. The wave-like interference can be naturally used for computing. Spin waves can be directly observed using micro-focused Brillouin light scattering (Fig. 2j).

4 State evolution of spintronic memristors

Spintronic memristors such as MTJs feature rich memristive dynamic behaviors under different drive conditions. A single MTJ’s governing equation is the LLG equation, which describes a nonlinear deterministic dynamical system. The unit magnetization vector is \( \mathbf{m} = (m_x, m_y, m_z) = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta) \) with two independent magnetization components (two out of \( x \), \( y \), \( z \) or \( \theta \) and \( \varphi \)). Coupled MTJs or higher dimensional spintronic systems can have more than two independent state variables. The trajectories of their solution space can be very rich as in other complex dynamic systems as pointed out by Henri Poincaré and later many others. These dynamics are enablers of various computing applications (see Table 1). In the following, we will explain more for MTJs and topological solitons while briefly mentioning the other states.

Steady dynamics

Here we use MTJs and skyrmions as examples to illustrate different types of trajectories in their state space. MTJs feature stable converging trajectories upon memristive switching (Fig. 3a), exhibiting stable binary states. As a result, they are utilized in information storage and in-memory logic devices. For long-term stability, the energy barrier between these two binary states is usually required to reach 40-80 \( k_B T \) depending on applications, where \( k_B \) is the Boltzmann constant and \( T \) is the working temperature. To write information into MTJs, we need to apply an electric current (via STT or SOT) or voltage (via VCMA effect) with a magnitude larger than a threshold value. Ultrafast measurements experimentally resolve the dynamics of magnetization upon the application of an electric current pulse, where the magnetization is electrically readout through TMR (Fig. 3b).
Stable converging trajectories observed in topological solitons such as skyrmions can be leveraged for memory applications. Skyrmions can be driven by electric current-induced STTs or SOTs. Experimentally, current-induced skyrmion motion has been demonstrated (Fig. 3c) where the information is encoded in the position of the skyrmion.

Nanomagnet ensemble or multi-domain magnets can exhibit stable states like MTJs. But different from the digital nature of individual MTJs, nanomagnet ensemble or multi-domain magnets can show analog behaviors, owing to multi-domain nature of varying magnetic properties across multiple MTJs. The trajectories of the multiple magnetizations can be controlled by electric current. Stable trajectories of domain walls can be achieved by applying charge current or heat current. A domain wall inside a MTJ can be utilized to create an analog resistance. Spin waves propagate in a magnetic media with a characteristic decay length of \( \lambda \), which is usually less than one micrometer for magnetic metals (due to presence of electron-magnon scattering) and can be up to centimeters for magnetic insulators. Interestingly, these (coherent) spin waves can be utilized to transmit information without Joule heating. Under certain conditions, spin waves can form extended or localized standing waves (or spin wave bullet modes), featuring a constant magnetization vector \( \mathbf{m}(x, A, \phi) \).

**Oscillatory dynamics**

In addition to stable trajectories, MTJs can also exhibit oscillatory behaviors under the combination of STT or SOT and an asymmetric energy barrier for parallel and antiparallel states (Fig. 3d). The STT or SOT is important to excite magnetization dynamics and the asymmetric energy barrier is important for destabilizing one state. The oscillation amplitude and frequency can be tuned by the magnitude of the current, which can be observed in both time domain using oscilloscope and frequency domain via spectrum analyzer (Fig. 3e).

Skyrmions and other topological solitons may exhibit oscillatory behaviors. In micromagnetic simulations, a locally injected spin current can create skyrmion oscillation in an extended circular magnetic thin film (Fig. 3f). Experimentally, vortex oscillations in a nanocontact structure have been observed. Multiple skyrmion oscillators can couple to form an oscillator neural network.

In a nanomagnet ensemble or multi-domain magnet, oscillations of individual domains and their coupling can be electrically or magnetically controlled. The controllable coupling in the nanomagnet ensemble system can be used to perform oscillator neural network-based computing. Direct current-induced steady oscillations of ferromagnetic domain walls are studied. Spin waves can be used in an oscillator system, where propagating spin waves with a fixed magnitude are created.

**Chaotic dynamics**

Since the LLG equation for a single MTJ only has two independent variables, chaos is precluded for a direct current. The existence of chaotic dynamics in MTJs in the presence of an alternating current can be judged by the Poincaré-Melnikov method (Fig. 3g). If a system is chaotic, at least one of its corresponding Lyapunov exponents is larger than zero. Indeed, chaotic dynamics of MTJ has been theoretically predicted and experimentally observed in MTJs. Skyrmions and other topological solitons may exhibit chaotic behaviors. Through theoretical calculations and magnetic simulations, an antiferromagnetic bimeron, which is an in-plane...
analogue of the magnetic skyrmion, can exhibit chaotic dynamics in the presence of an ac drive current (Fig. 3i)\textsuperscript{109}. Experimentally, chaos in magnetic vortex nanocontacts has been observed\textsuperscript{110,111}.

While a direct current cannot induce chaos in a single MTJ, it can induce rich dynamics including chaos for coupled multi-domain magnets or nanomagnet ensemble when more than two variables are present. The system should have more than one tunable magnetic layer\textsuperscript{112,113} or more than one resonance mode\textsuperscript{114}. Chaotic ferromagnetic and antiferromagnetic domain walls are theoretically studied\textsuperscript{115,116}. Chaotic spin wave soliton dynamics are experimentally observed in an yttrium iron garnet (YIG) delay line with feedback\textsuperscript{117}.

**Stochastic dynamics**

When the thermal noise dominates, the dynamics of spintronic memristors can be stochastic. There are two major types of stochasticity in MTJs\textsuperscript{118}. First, the MTJ switching is probabilistic due to the presence of thermal noise and the switching probability is highly tunable by adjusting the current amplitude and the pulse amplitude. The stochastic nature of switching can be used for true random number generation\textsuperscript{119,120} and stochastic computing\textsuperscript{121}. Second, low-energy barrier magnets have stochastic trajectories in the absence of external current, which can benefit low-power hardware stochastic and probabilistic computing (Fig. 3j)\textsuperscript{17}. The occurrence of this random fluctuations can be greatly tuned by the voltage or current, where the retention time can be from microseconds to seconds (Fig. 3k)\textsuperscript{17}. Recently, through engineering the energy landscape of the free layer magnetization, nanosecond random telegraph spectra have been demonstrated in in-plane MTJs\textsuperscript{122}.

Skyrmions may have stochastic trajectories driven by the thermal noise\textsuperscript{22}. Experiments also show that the stochastic processes are skyrmion topology-\textsuperscript{123} and symmetry-dependent\textsuperscript{124}. When the topological charge changes from +1 to -1, the stochastic trajectories of skyrmions are changed (Fig. 3l)\textsuperscript{123}.

Multiple stochastic MTJs with interactions can be used to construct a stochastic neural network to perform object recognition\textsuperscript{121} and integer factorization\textsuperscript{17}. Current-driven domain wall motion is naturally stochastic due to the thermal fluctuation induced by the Joule heating and random defects present in magnetic materials. On the one hand, this poses a challenge on using domain walls to construct a reliable racetrack memory. On the other hand, this intrinsic randomness can be utilized to build a secure hardware\textsuperscript{125}. Stochastic spin waves are thermally excited spin waves, of which the frequency, amplitude and phase are fluctuating. These thermal spin waves can also be used to transmit information\textsuperscript{70}.

5 **Spintronic memristive computing (3 page)**

Memristive properties of spintronic devices make them advantageous in implementing neuromorphic computing, in-memory logic, and stochastic and chaos computing, compared to conventional digital computing hardware. Readers may refer to the existing reviews for a general comparison between spintronic memristors and other types of memristors\textsuperscript{20,37,38}. 
5.1 Neuromorphic computing

As synapses and neurons can be formulated as memristors or using memristors, this common feature allows efficient emulation of synapses, neurons, and their networks with spintronic devices, which may revolutionize AI hardware.\textsuperscript{18,19,48}

Synapses

The resemblance to LLG equation allows representing the state of an artificial synapse via spin configurations, such as discrete spins or magnetic textures. For chemical synapses, the evolution of state variables and thus transmission efficacy is driven by the combined presynaptic and postsynaptic stimulus, leading to different local learning rules at different timescales, such as the widely observed long-term plasticity and spike timing-dependent plasticity (Fig. 4a).

Long-term plasticity. Long-term potentiation and depression, particularly those responses are linear to input signals, can be leveraged for in-memory acceleration of machine learning. The synapses are tunable weights, typically optimized using gradient-based approaches in minimizing a loss or energy function.

Macrospins, like MTJs, have long been demonstrated as binary synapses. One approach to encode analogue values with binary macrospins is probabilistic programing of macrospin to encode analogue values in its expectation. This is because, strictly speaking, the evolution of both synapses and LLG at nonzero temperatures are governed by stochastic differential equations. Whether this stochasticity can be manifested or not depends on the ratio between potential barrier and energy fluctuation. It is also reported that such stochasticity is critical to efficient learning in biological systems\textsuperscript{126}. This makes spintronic devices even more appealing over digital alternatives that rely on tedious pseudo random number generation.

Analogue long-term memory can be physically realized using magnetic textures, such as domain wall and skyrmion motions. For example, it has been shown that domain wall displacement in a spin valve is equivalent to a bipolar non-volatile memristor, where potentiation and depression are due to the motion of walls towards different directions\textsuperscript{50}. Such long-term magnetoresistance changes induced by external electrical stimuli mimicking presynaptic signals have been experimentally demonstrated on MTJs, featuring a large dynamic range and a low operating power\textsuperscript{80,81,127}. In addition, the long-term synaptic potentiation and depression may also be built on the current-induced creation, displacement, and annihilation of skyrmions\textsuperscript{128} that were experimentally observed\textsuperscript{27} (Figs. 4b and 4c).

Spike timing-dependent plasticity (STDP). Chemical synapses possess rich dynamic behaviors in addition to long-term plasticity of simple monotonic resistance changes upon electrical stimulus. Such dynamics can be harnessed to implement time-dependent local learning rules, such as STDP, a famous Hebbian rule that is widely used for learning in spiking neural networks\textsuperscript{129}. According to STDP rule, the synaptic weight changes according to the relative timing difference between a presynaptic and a postsynaptic spike. Such a STDP rule can be implemented on a stochastic binary switch, using STT\textsuperscript{121} or SOT\textsuperscript{130}. In addition, paired current pulses are used to switch antiferromagnet/ferromagnet heterostructures using SOT, where the Hall resistance shows a clear STDP like behavior (Figs. 4d and 4e). This timing effect can be modelled by incorporating Joule heating where the temperature rise due to electrical pulse impacts on the subsequent switching\textsuperscript{26}.

Neurons
Neurons exhibit rich dynamic behaviors including self-sustained oscillation, leaky integrate-and-fire, chaos, resting states, burst-number adaptation, spike latency, and refractory period, which can be reproduced using memristors. Among them, self-sustained oscillation and leaky integrate-and-fire have been popular for spintronic memristors in developing computing applications (Fig. 4f).

**Neural oscillation.** Such oscillations are the rhythmic or repetitive patterns of neural activity in the brain, which plays important roles in advanced cognitive functions. Injecting a charge current to MTJs can lead to sustained magnetization precession of the free layer, resulting in oscillating magnetoresistance or voltage that mimics the neural oscillations (Figs. 4g and 4h). In addition, the LLG equation endows this oscillator with a fading memory. As a result, the evolution of the oscillator not only depends on the input current but also its state, allowing a single oscillator to function as a delayed feedback system that mathematically parallels systems of coupled oscillators, which has wide applications including reservoir computing.

For high dimensional magnetic textures, such as domain walls and skyrmions, their oscillating trajectories in state spaces can emulate oscillating neurons if they are driven by external changing field or injecting current. Similarly, the memristive dynamics equip those oscillators with short-term memory that oscillators can modulate their outputs under the same excitation, mimicking the neuromodulation and self-adaptability.

**Leaky integrate-and-fire.** Another widely pursued dynamic behavior of neurons is leaky integrate-and-fire. The neuron spikes once the integrated input stimulus, reflected as the membrane potential, exceeds a threshold. This can be both implemented on macrospins or magnetic solitons such as domain walls or skyrmions.

For macrospins like MTJs, the magnetization switching driven by STT in combination with back-hopping can output spikes like that of neurons. In antiferromagnet/ferromagnet devices, the switching or firing probability strongly depends on the intensity or frequency of the incoming stimulus, reproducing the leaky integrate-and-fire functionality (Figs. 4i and 4j). In addition, the stochastic switching of an MTJ due to VCMA may follow a sigmoid probability density function, that naturally performs the nonlinear activation.

For high dimensional magnetic features, magnetic solitons such as domain walls and skyrmions can be manipulated and moved over large distances using STTs and SOTs. The spatial motion of domain walls and skyrmions can be mapped to the membrane potential of biological neurons, exhibiting leaky integrate-and-fire and lateral inhibition (the firing of one neuron prevents others from firing) on nanoscale ferromagnetic tracks.

**Neural networks**

The brain computes with coupled synapses and neurons, where synapses are edges and neurons are vertices in a graph, forming a composite nonlinear dynamic system. Following the same principle, spintronic synapses and neurons could be grouped towards building a spintronic brain. Such a brain may employ different methods to encode signals, leading to artificial neural networks, spiking neural networks, and oscillatory networks.

**Artificial neural networks (ANNs).** Spintronic ANNs encode signals (in each discrete time step if any) using time-invariant physical quantities such as amplitudes of voltages or currents. The key computations performed by spintronic ANNs are vector-matrix multiplications, where the matrices can be physically mapped to synapses such as the electrical conductance of MTJs grouped in...
crossbar arrays. When rows (or columns) or such arrays are biased to input voltage/current vectors, the output current/voltage vectors compute the products between the matrix and the input vectors, offering significantly improved parallelism. In addition, unlike digital computers, here the data are processed right at where they are stored, thus eliminating the von Neumann bottleneck and bringing predicted advantages in various computing architectural designs such as computing-in-memory and computational random-access memory (Figs. 4k and 4l) 79,143–146.

Spintronic ANN has been physically realized on a Hopfield recurrent network, also a dynamic system with multiple attractors, consisting of 36 weights (half-lower triangle of a 9 × 9 weight matrix due to symmetry). The output of the network serves as its input at the next discrete time step. The trajectory of the 9 neurons, representing pixels of a 3 × 3 pattern, falls into one of the attractors after evolution upon different initial conditions, thus a way to associate the input with one of the memorized patterns. The matrix-multiplications were physically carried out by 36 discrete SOT Hall devices where the Hall resistances were programmed to pre-computed values representing patterns followed by in-situ fine tuning using Hebbian rules 23.

In addition to encode the weights of ANNs using discrete macrospins, wave mechanics have been harvested for AI in the form of optical, acoustic, and spin-wave neural networks. The latter performs the cascaded linear and nonlinear transformation of input signals by propagating spin-wave across a customized magnetic-field pattern, which serves as the weights of neural networks. The network is trained by refining the field pattern to realize the desired input-output mapping 147.

**Spiking neural networks (SNNs).** SNNs encode signals using timing or rate (frequency) of spikes. A popular SNN for spintronic memristors are liquid state machines, a variant of reservoir computing. As revealed by its name, liquid state machine echoes the idea that dropping a stone (input signal) into a still body of water generates ripples (state of the reservoir). The latter is usually in a high-dimensional state space following a trajectory at the chaos boundary, making the corresponding state vector much more linearly separable than that of the input vector (Fig. 4m).

Macropins oscillatory neurons with fading memory or spin-wave active resonators could work as delayed-feedback systems, capable of implementing liquid state machines 24,131,148. The inputs, usually spatial temporal patterns, drive the evolution of the reservoir. Its internal states sampled at different time points, or virtual nodes, serve as the outputs. A simple fully connected readout map is trained to perform regression or classification (Fig. 4n).

The spatial evolution of magnetic textures can also be exploited to nonlinearly map the input to the state of a dynamic system. For example, a liquid state machine made of skyrmions can map the temporal spatial voltage input patterns to the spatial configuration of skyrmions thanks to the spin torques and pinning. This configuration, or state of the reservoir, can be probed by fixed position electrodes on ferromagnet tracks 149. An ANN variant of liquid state machine, echo state network, uses skyrmion fabric, where skyrmions are pinned by grain boundaries to nonlinearly map input voltage waveform to output current waveform without displacing skyrmions, which functions as a recurrent network of random and fixed weights 150.

**Oscillator neural network.** Oscillatory neural networks encode information using the phases and frequencies of oscillators. These individual oscillators exhibit phase and frequency synchronization when they couple with each other. For example, individual spintronic oscillators can couple through spin wave exchange interaction 94,96,97,100 or microwave current 98,101. Dynamics of these coupled systems can be very useful for oscillator-based computing 102.
The phase and frequency dynamics of coupled oscillators, such as those using spin-torque oscillators forming an oscillatory Hopfield network, under the influence of subharmonic injection locking, are governed by Lyapunov functions that are related to associative memory, which can retrieve a pre-stored memory upon a given input \(^{151,152}\).

In addition, synchronization of two coupled oscillators reveals a strong inter-connection, or equivalently a large synaptic weight in the coupling matrix. The coupling strength can be adjusted by tuning the natural frequency of each oscillator where a smaller frequency difference between two oscillators results in a larger tendency to couple. As a result, each input triggers a specific synchronization pattern of the neurons. Experimentally, a neural network of four coupled spin-torque oscillators can take two input frequencies that encode vowel information (Fig. 4o and 4p) and classify vowels by generating distinct synchronization frequency patterns \(^{25}\) (Fig. 4q).

5.2 In-memory logic

Spintronic memristors combine both logic and memory in a single entity. So far, they have been applied to both combinatorial logics and sequential logics. Thanks to the maturity of MTJ technology, hybrid MTJ-CMOS chips have been extensively investigated, where embedded MTJs offer non-volatility to CMOS logic gates for combinatorial logics and replace CMOS registers and caches for sequential logics \(^{143,153-155}\). This hybrid approach can not only bring intelligent power management in integrated circuits for ultralow-power IoT devices and edge computing \(^{156,157}\), but also provide significant improvement in memory accessing bandwidth \(^{158,159}\).

Pure spintronic memristors have been investigated to implement in-memory logic, which can result in even lower power consumption and better performance for data-centric cognitive tasks \(^{19,146}\). For combinatorial logics, various approaches are proposed based on a variety of spintronic states \(^{12}\). Here, we mainly introduce logics based on spin waves and domain walls. Amplitude and phase of spin waves can be utilized to encode information and their modulation in magnonic circuits enable logic applications \(^{16,71,160}\). NOT gate \(^{161}\), XOR and NAND gates \(^{162}\), majority gate \(^{163,164}\), and spin wave transistor \(^{165}\) were experimentally demonstrated. Furthermore, an all-spin logic with spin wave interconnects was proposed to eliminate the overhead of spin-charge conversion processes \(^{166}\). One concern is that the spin current is not conservative and decays in the interconnect, making cascaded gates difficult. Recently, magnetoelectric spin-orbit logic (MESO) with a charge interconnect is proposed as a potential logic/memory solution for beyond 3 nm technology nodes \(^{28}\). MESO logic could enable ultralow-power building blocks like inverters and majority gates (Figs. 5a-c) provided that a high spin-charge conversion efficiency can be achieved \(^{28}\).

In addition to spin waves, spin textures, such as domain walls driven by a magnetic field or current, have been used to implement logic functions. Early demonstrations of domain wall logic require external magnetic fields \(^{13,14}\). Recently, chiral interactions between domain walls were discovered and then utilized to construct purely electrically controlled NOT, NAND, NOR gates, and full adders (Figs. 5d-f) \(^{29,30}\). The purely electrical control promises better scalability.

Besides spin waves and domain walls, we briefly mention other approaches here, which are mostly at the conceptual level. Dipolar interaction between nanomagnets in a nanomagnet ensemble can be utilized to build a majority logic gate, which can be a fundamental building block for many other logic gates \(^{167}\). Spin field-effect transistor \(^{168}\) and spin accumulation-based semiconductor
logic have been theoretically proposed. Skyrmions as a potentially more compatible version of domain walls could enable more scalable and low-power logic circuits.

For sequential logic, domain walls on a racetrack have been exploited as shift registers. Electric pulses with desired duration and amplitude can be utilized to create and shift domain walls in in-plane magnetized nanowires (Figs. 5g-i). Careful design of the magnetic energy landscape could enable a ratchet-like motion in a perpendicularly magnetized nanowire, which can potentially enable more scalable shift registers due to the benefit of smaller domain sizes in it.

Besides domain walls, skyrmion shift memory was also experimentally demonstrated, where individual skyrmions can be created and shifted using well-defined train pulses.

A considerable challenge of in-memory logic is that spintronic devices are often prone to bit errors. For example, current industrial MRAM has to use relatively strong error corrections codes (ECC) to ensure perfectly reliable operation. Therefore, the ultimate success of spintronic-based in-memory logic will have to require extensive device optimization, the integration of ECC within in-memory circuits, or the use of approximate computing strategies that tolerate errors.

5.3 Stochastic and chaos computing

Utilizing the stochastic trajectories in state space, spintronic memristors can leverage the entropy from thermal fluctuation to perform useful computing. As we discussed earlier, the switching probability of a MTJ depends on the current pulse amplitude and duration. As a result, the trajectory of a MTJ state can have tunable stochasticity that can be utilized as a synapse with probabilistic plasticity, which can mimic plasticity in a stochastic manner. This is very different from the previous synapse with determined plasticity. Simulations have shown that stochastic switching of spintronic memristors leads to probabilistic synapses in a stochastic neural network, with applications to unsupervised learning (Figs. 6a-d).

An alternative way to utilize the stochasticity is to employ low-energy barrier magnets, which have highly tunable stochasticity even in the absence of the external current supply. This intrinsic randomness has been widely used for true random number generators, which are essential for security and encryption applications. Recently, the concept of probabilistic bit (P-bit) is revived with a concrete realization based on a manufacturable and compatible MTJ hardware solution. These P-bits can serve as a bridge between ordinary bits and quantum bits. Very much like quantum bits, the P-bits can solve some problems that are challenging to classical computers. Researchers have utilized a network of P-bits with carefully designed interconnections and bias inputs to solve integer factorization problem (Figs. 6e-h).

In addition to the above-mentioned neuromorphic applications, stochastic spintronic memristors can be utilized to achieve probabilistic computing. For two uncorrelated stochastic bitstreams with up and down states, multiplication of the probability for up state is equivalent to the result of AND operation for these two bitstreams. However, one major obstacle is that when the two bitstreams are correlated, this kind of calculation fails (Fig. 6k). The key is to preserve the probability of up state but reshuffle the appearance of up states in the bitstream. Skyrmion reservoirs (Fig. 6i) have been utilized to achieve this reshuffler due to two important features. First, the skyrmion number is a conserved number. Second, the skyrmion motion is highly stochastic under the low drive current (Fig. 6j). Experiments have shown prototype shufflers based on skyrmions (Fig. 6l).
Stochastic spintronic devices are also under investigation for security applications including but not limited to recycling sensors, physically unclonable functions, true random number generators, and encryption. The sources of entropy and randomness for a single MTJ mainly come from the thermal noise-induced stochastic spin-torque switching and random telegraph signals. For nanomagnet ensembles and MTJ arrays, the sources could include all kinds of process-induced variations in device properties such as magnetic anisotropy, MTJ area, tunnel barrier oxide thickness, intrinsic switching current and time.

Chaotic dynamics of spintronic memristors can also be utilized for security applications. Unlike stochastic dynamics, chaotic dynamics are intrinsically deterministic, and thus the recovery of encrypted information is easy to implement using the same system that generates the dynamics. We introduce one chaos-based image encryption here. As illustrated in Fig. 6m, the original image is converted to seed numbers using the Secure Hash Algorithm and these numbers together with private keys are used as inputs for a chaotic spintronic memristor system that will generate extremely dynamics and thus unpredicted outputs. These outputs can be used in different encoding schemes to encrypt the original image.

Another important application is to use chaotic dynamics to assist the global optimization. Since the chaos is deterministic, which is different from stochasticity, controlled reduction of fluctuation amplitude in chaos could help find the global minimum of a designed energy landscape in a more deterministic manner. Recent experiments have shown that one can use a direct current to tune a nanocontact vortex oscillator between commensurate phase-locked and incommensurate chaotic states. As a result, a nanocontact vortex oscillator can generate highly unpredictable bitstreams or symbolic dynamics in a controllable manner.

6 Summary and concluding remarks (0.5 page)

In this review, we provide a holistic picture of spintronic devices as memristors. Our picture correlates the memristive dynamics (trajectories in state space), a manifestation of the underlying physics and materials, to various computing applications. Spintronic memristor provide considerable benefits with regards to other memristor technologies. As they do not rely on the motion of atoms, they feature much higher reliability than most memristor technologies. Also, as they rely on the well-controlled and well-understood physics of magnetism, spintronic memristors can exhibit the incredibility diversity of behaviors that we have described throughout this review. Spintronic memristors also come with some challenges. In the following, we discuss these challenges and trends for spintronic memristor-based neuromorphic computing, in-memory logic, stochastic and chaos computing.

Toward better spintronic memristive devices

Better spintronic memristive devices concern lower write energy, larger readout signal and lower area cost. Most demonstrations of spintronic memristors are based on STTs. To improve the energy efficiency of spintronic memristors, researchers have employed novel materials with large SOT efficiencies such as topological insulators to drive the magnetization dynamics to achieve memristive behaviors. Besides, utilizing voltage instead of current can enable even lower power consumption. Existing spintronic memristors have a relatively small read margin due to a small TMR ratio (around 200% for a typical MTJ), whereas other memristor technologies such as
valence-change resistive switching devices or phase change memories provide much higher read margins. The fundamental breakthrough lies in the improvement in the read mechanism. Some 2D material-based spin-filter TMR can be more than 10000% at low temperatures. Scaling down individual spintronic memristors is critical for future large-scale integration. Thermally stable MTJs down to a diameter of 2.3 nm have been demonstrated with the perpendicular shape anisotropy technique.

**Toward high-order and high-connectivity systems**

So far, experimental demonstrations of neuromorphic computing with spintronic memristors are based on first-order (resistance) and second-order (oscillation) dynamics. Emulating biological neurons that exhibit periodic bursting (third-order), chaotic oscillation (third-order), and hyperchaos (fourth-order) demands spintronic memristors based on higher-order dynamics. Besides, the number of coupled spintronic oscillators and stochastic MTJs in recent demonstrations is still very limited and the connection topology lies in a 2D plane. Further expansion into a larger array and higher dimension could significantly enhance the representation capability to solve more complex problems.

**Toward cross-layer device, circuit, and algorithm design**

Most studies on spintronic memristors are still at the individual theory, material, device, circuit, and algorithm levels. There is a large gap between the device concept and the final functional system that utilizes the state-of-the-art algorithm. In the past two decades, extensive efforts have been put into developing algorithms for solving various challenging tasks. Thanks to the fast-increasing computation capability, the algorithms have performed very well. However, the electricity in utilizing these algorithms has been enormously increasing since they do not fit with the existing computer architectures and the scaling of CMOS technology is slowing down. Understanding the characteristics of different algorithms is essential for choosing the spintronic memristors and designing proper circuits. Spintronic memristors are an emerging computing paradigm that is full of opportunities. The current demonstrations remain at the device and small system level. Cross-layer design by employing better devices with rich dynamics and investigations into larger circuits and systems is in an urgent need to unleash the potential of spintronic memristor-enabled computing.

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Q. S. and Z. W. drafted the manuscript with extensive contributions from Y. Z. All authors edited and modified the manuscript.

Competing interests
The authors declare no competing interests.
Table 1. Summary of experiments on computing applications categorized by state-space representations and state evolutions. Theoretical studies or experimental observations are cited if there is no experimental study on computing-related applications.

| State-space representation | Type of evolution | Steady | Oscillatory | Chaotic | Stochastic |
|----------------------------|-------------------|--------|-------------|---------|------------|
| Nanomagnet/ macrospin      | MTJ-enabled nonvolatile CMOS logic 153,154,156,157; spin logic 166; magneto-electric spin-orbit logic 28; | Spin-torque oscillator-based reservoir computing 24,133–135 | Experimental observation in MTJ 108 | Random number generators 119,120,122 |
| Nanomagnet ensemble/ multi-domain magnets | Synapse 26; artificial neural network (ANN) for associate memory 23; ANN for deep learning (experiment/theory) 79,145; neuron logic 26; majority logic gate 167 | Spin-torque oscillator neural network 25,94,152 | Secure hardware (theory) 125 | Probabilistic bit network for probabilistic computing 17 |
| Domain walls               | Synapse 80,127; domain wall logic 13,29,30; shift registers 172,173 | Domain wall oscillator (theory) 103,104 | Secure hardware (theory) 125 | Secure hardware (theory) 125 |
| Topological spin textures  | Synapse 27,128 | Skyrmion oscillator (theory) 89 | Unpredicted pattern generation using chaotic vortex dynamics 181 | Skyrmion reshuffler 22 |
| Spin waves / magnons       | Spin wave logic 161–165 | YIG spin wave reservoir computing 148; information carrier 69 | Experimental observation in YIG 117 | Information carrier 70 |
Figures and captions

Figure 1 Spintronic devices are memristors, so as components in the brain. (Central panel) Memristor is a nonlinear dynamic system. The unique transport equation results in pinched hysteresis loops in the I(t)-V(t) plane. (Upper and middle left panel) Spintronic systems are memristors. Their state evolutions are governed by the Landau–Lifshitz–Gilbert (LLG) equation taking into consideration of the spin-orbit torque (upper left panel), spin-transfer torque (upper middle panel) and voltage-controlled magnetic anisotropy (upper right panel). (Lower and middle right panel) Biological synapses and neurons are general memristors or can be formulated using memristors. Their dynamic evolution builds on the voltage or ligand gated ion channels (lower left panel), where their state variables are the portion of ion channels that are open in synapses (lower middle panel) and the membrane potential in neurons (lower right panel). This analogy enables spintronic memristor-based neuromorphic computing. The detailed description of the formulas here is explained in the main text.
Figure 2 State-space representations and transport equations of spintronic memristors. a. Schematic of a magnetic tunnel junction (MTJ) as a typical example for nanomagnet/macrospin systems and detection of magnetization using the tunnel magnetoresistance effect. b. Experimental result of current-induced binary switching in a perpendicular MTJ. c. Schematic of multi-domain magnet/nanomagnet ensemble systems with coupled magnetization states in a heavy metal/ferromagnet bilayer and detection of overall magnetization using the anomalous Hall effect. d. Experimental result of current-induced analog resistance switching in an antiferromagnet/ferromagnet heterostructure. e. Schematic of a Néel-type domain wall and detection of magnetization map using magneto-optical Kerr effect (MOKE). f. Experimental observation of a domain wall and the current-driven domain wall motion in a racetrack using MOKE. g. Schematic of a Néel-type skyrmion and its detection using transmission X-ray microscopy (TXM). h. Experimental observation of skyrmions and the current-driven skyrmion motion in a racetrack using scanning TXM. i. Schematic of spin waves and their excitation and detection using microwave antenna. The spin waves can be spatially and temporally resolved using Brillouin light scattering (BLS). j. Micro-focused BLS microscope image of a standing spin wave (upper panel), where amplitude and phase of spin waves are shown (lower panel). Part b reprinted with permission from REF. 55, Springer Nature Limited. Part d reprinted with permission from REF. 56, Springer Nature Limited. Part f reprinted with permission from REF. 30, Springer Nature Limited. Part h reprinted with permission from REF. 66, Springer Nature Limited. Part j adapted with permission from REF. 72, Copyright © 2015 Sebastian, Schultheiss, Obry, Hillebrands and Schultheiss.
Figure 3 Steady, oscillatory, chaotic and stochastic trajectories in state spaces for MTJs and skyrmions (bimerons). a. The Bloch sphere representation of a stable converging trajectory for magnetization from up to down. b. Real-time detection of magnetization switching in a MTJ by reading its resistance. c. Schematic of experimentally observed multiple frames of current-induced skyrmion motion in a racetrack. d. The Bloch sphere representation of an oscillatory trajectory. e. Frequency spectra of direct current-induced magnetization oscillations in a MTJ with different current amplitudes. f. Schematic of micromagnetic simulations of current-induced skyrmion oscillation. g. The Bloch sphere representation of a chaotic trajectory. h. Threshold ac drive voltage as a function of the ac drive frequency as an evidence of low-dimensional chaos-assisted magnetization reversal. i. Theoretical results of current-induced bifurcation and chaos in antiferromagnetic bimeron systems, where bimerons in in-plane magnetized magnets are analogues to skyrmions in out-of-plane magnetized magnets. j. The Bloch sphere representation of a stochastic trajectory. k. Experimentally observed random telegraph signals of a MTJ-based probabilistic system. l. Simulated Brownian motion trajectories of skyrmions with a positive (left panel) and negative (right panel) topological charge. Part b reprinted with permission from REF. 77, Springer Nature Limited. Part c adapted with permission from REF. 78, American Chemical Society. Part e reprinted with permission from REF. 88, Springer Nature Limited. Part f adapted with permission from REF. 89, IOP Publishing. Part h reprinted with permission from REF. 108, Springer Nature Limited. Part i reprinted with permission from REF. 109, American Physical Society. Part k reprinted with permission from REF. 17, Springer Nature Limited. Part l reprinted with permission from REF. 123, American Physical Society.
Figure 4 Brain-inspired computing. a. Schematic of a biological synapse operating on neural transmitters and ions channels. b. Spintronic memristor operating on the creation and annihilation of skyrmions. c. Long-term potentiation and depression of the synapse in b. d. Spintronic memristor consisting of multiple domains. e. Spike timing-dependent plasticity of the synapse in d. f. Schematic of a biological neuron operating on ion dynamics. g. Spintronic memristor neuron using a single MTJ. h. The neural oscillation of the neuron in g. i. Spintronic memristor neuron with temperature as a dynamic variable. j. Integrate-and-fire behaviors of the neuron in i. k. A simple layer of an artificial neural network and its matrix representation. l. A possible hardware artificial neural network using spintronic memristors. m. A special spiking neural network, the liquid state machine. n. A liquid state machine implemented on a single delayed-feedback system with a spintronic oscillatory neuron. o. A small oscillatory neural network with coupling between output neurons. p. Physical implementation of oscillatory neural networks with spintronic oscillatory neurons. q. Vowel recognition using the network in p. Each color corresponds to a different spoken vowel. Part c reprinted with permission from REF. 27, Springer Nature Limited. Parts d, e, i, and j reprinted with permission from REF. 26, © 2019 WILEY - VCH Verlag GmbH & Co. KGaA, Weinheim. Part n adapted with permission from REF. 24, Springer Nature Limited. Parts o and q reprinted with permission from REF. 25, Springer Nature Limited.
Figure 5 **In-memory Boolean logic.** Combinatorial logic using spintronic memristors. **a.** A majority logic gate constructed from NAND gates. **b.** Schematic of majority logic gate constructed from magneto-electric spin-orbit (MESO) logic. **c.** Input-output transfer curve in MESO logic. The magnetoelectric spin-orbit devices can implement majority gates. **d.** A full adder constructed from NOT and NAND gates. **e.** Schematic of current-driven domain wall inverter using chirally coupled domains through a chiral domain wall. **f.** Magnetic force microscopy image of the full adder logic operation, A (0) + B (1) = Sum (1) + C_{out} (0). The current-driven domain wall motion is used to construct full adders. Sequential logic using spintronic memristors. **g.** Schematic of a three-bit shift register (upper panel) and scanning electron microscopy image of a permalloy nanowire (bottom panel). The data is encoded in three domains in the nanowire. **h.** Schematic of shift-register operation. **i.** Electrical detection of the operations in h using the nanowire resistance change. Part **c** reprinted with permission from REF. 28, Springer Nature Limited. Parts **e** and **f** reprinted with permission from REF. 30, Springer Nature Limited. Parts **g**, **h**, and **i** reprinted from REF. 172, Copyright © 2008 The American Association for the Advancement of Science.
Figure 6 Stochastic and chaotic memristor systems and applications. a. Schematic of a MTJ, where the free layer magnetization switching probability is controlled by the pulse width and amplitude as shown in b. c. MTJ-based probabilistic synapses are used for hardware encoding synaptic plasticity, which is tuned according to the spike timing-dependent plasticity rule. d. Demonstration of clustering images using unsupervised learning in simulation. e. MTJ-based spintronic memristor system for binary stochastic neuron or probabilistic bit (P-bit) in the absence of a significant excitation. f. Random telegraph output signals under different input voltages, where more “0 V” and “5 V” are observed for lower and higher input voltages, respectively. g. Network of P-bits is configured according to the nature of a problem to solve the problem. h. Network of six P-bits is used to solve a simple integer factorization problem, 161 = 23 × 7. i. Skyrmion-based stochastic re-shuffler device. j. Experimentally observed stochastic trajectories of four skyrmions at room temperature. k. Stochastic computing using skyrmion gas-based re-shufflers that eliminate the correlation impact in ordinary stochastic multiplication. l. Demonstration of re-shuffling operation to a stochastic bitstream. The radius of the reshuffling chamber is 40 μm. m. Chaos-assisted image encryption using current-tunable chaotic dynamics in nanomagnet ensembles, where the private keys are encoded as input currents and voltages. n. Chaos-aided global
optimization using current-tunable dynamics in nanomagnet ensembles, where the chaos to order transition is controlled by time-dependent input currents and voltages. Parts b and d reprinted with permission from REF. 121, Copyright © 2015, IEEE. Parts f and h reprinted with permission from REF. 17, Springer Nature Limited. Parts j and l reprinted with permission from REF. 22, Springer Nature Limited. Part n adapted with permission from REF. 180, AAAS.
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