Three-Dimensional Billiards with Time Machine*

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Self-collision of a non-relativistic classical point-like body, or particle, in the spacetime containing closed time-like curves (time-machine spacetime) is considered. A point-like body (particle) is an idealization of a small ideal elastic billiard ball. The known model of a time machine is used containing a wormhole leading to the past. If the body enters one of the mouths of the wormhole, it emerges from another mouth in an earlier time so that both the particle and its “incarnation” coexist during some time and may collide. Such self-collisions are considered in the case when the size of the body is much less than the radius of the mouth, and the latter is much less than the distance between the mouths. Three-dimensional configurations of trajectories with a self-collision are presented. Their dynamics is investigated in detail. Configurations corresponding to multiple wormhole traversals are discussed. It is shown that, for each world line describing self-collision of a particle, dynamically equivalent configurations exist in which the particle collides not with itself but with an identical particle having a closed trajectory (Jinnee of Time Machine).

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1 Introduction

Spacetimes containing closed time-like curves (time-machine spacetimes) were considered in the literature in the last decade (see [1]-[4] and references therein). We shall investigate dynamics of a point-like body (small billiard ball) in a time-machine spacetime, taking into account a self-collision of a particle. We use a wide-known model of the time-machine spacetime containing a wormhole. In such a model a body entering one of the mouths of the wormhole, goes out of another mouth at an earlier time.

In a series of recent papers (see for example [5]-[10]) the action of the standard laws of physics is investigated in different time-machine spacetimes. Specifically, it was argued in [11] that the principle of self-consistency for motion in time-machine spacetimes
(introduced earlier in [12]-[13]) may be derived from the principle of minimal action. Therefore, if the initial and final positions of all bodies are given, then the motion in the time-machine spacetime is completely determined by the principle of minimal action. Of course, it could happen that there are more than one local minimum of the action or even infinite number of them. Then different ways of motion are possible corresponding to different local minima.

In the present paper we shall apply this approach to investigate the motion of a non-relativistic classical point-like body (which will be called a particle) in the time-machine spacetime containing a wormhole. The point-like body may be thought of as an idealization of a small ideal elastic billiard ball. We restrict ourselves by the case when $\rho \ll r \ll l$ where $\rho$, $r$ and $l$ are correspondingly the radius of the ball, the radius of the mouths of the time machine and the distance between the mouths.

Complanar (and close to complanar) motions in a time-machine spacetime with a self-collision were analyzed in [7] for billiard balls of a finite size and in [11], from a different point of view, for a particle, i.e. infinitesimal billiard ball. In the present paper we shall investigate three-dimensional motion of a particle with a self-collision. Concrete examples will be given of the motion with a self-collision and repeated passages through the time machine.

2 The method and a simple solution

To be definite, we shall consider the time-machine spacetime containing the wormhole with two mouths, $A$ and $B$. If something (for example a particle) enters the mouth $B$ at time $t_B$, it immediately (counting in the proper time) goes out of the mouth $A$, but its time in the external spacetime turns out to be $t_A = t_B - \tau$. The parameter $\tau$ characterizes “power” of the time machine.

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1Possibility of three-dimensional motions was mentioned also in [11] though not analyzed there in detail.
During some time before entering the mouth $B$ the particle coexists with its “incarnation” escaped from the mouth $A$, and an interaction between them is possible during this period. We shall consider only the case of collision, i.e. short-distance interaction. Three-dimensional motion of the particle including such a self-collision and wormhole traversal will be considered in Sect. 3. Later, in Sect. 4, we shall discuss the case of multiple traversals.

Following the paper [11], we shall derive configuration of the process starting with the principle of minimal action, that guarantees self-consistency of the motion.

The trajectory of the particle begins (see Fig. 3a) at the space-time point (event) $i = (r_i, t_i)$ and ends at the event $f = (r_f, t_f)$. The collision occurs at the event $O = (r_0, t_0)$. Let us denote the entities characterizing the motion during the first period, between the events $i$ and $O$, by letters with the index 1. For example, position during this period will be denoted by $r_1(t)$. The entities corresponding to the last period, from the event $O$ to the event $f$, will be denoted also by the letters having the index 1 and prime. For example, position during this period will be denoted $r'_1(t)$.

After collision at the event $(r_0, t_0)$ the particle enters the wormhole mouth $B$ at the event $(r_B, t_B)$. On escaping from the mouth $A$ at the event $(r_A, t_A)$ the particle travels to the event of collision, $(r_0, t_0)$. The entities between the events $(r_A, t_A)$ and $(r_0, t_0)$ will be denoted by the index 2, and those between $(r_0, t_0)$ and $(r_B, t_B)$ by the index 2 and prime (correspondingly $r_2(t)$ and $r'_2(t)$ for the positions).

The action of the particle may be calculated as a sum of the actions corresponding to the mentioned parts of the trajectory:

\[
S = \frac{m}{2} \left( \int \dot{r}_1^2 dt + \int \dot{r}_1'^2 dt + \int \dot{r}_2^2 dt + \int \dot{r}_2'^2 dt \right). \tag{1}
\]

2 These notations are not generally accepted, but they proved to be convenient for our aims.

3 A short-range interaction (collision) does not contribute the action. The proof of this for a specific form of the interaction potential may be found in [11], but the same is valid for much more wide class of potentials.
Figure 1: The trajectory with a self-collision and one passage through the wormhole:
(a) the action principle is not accounted; (b) the action principle is accounted between the key events.

Small variation of the part 1 of the trajectory (between the events \((r_i,t_i)\) and \((r_0,t_0)\))
does not change the action (1) only if the motion is linear in this part. Quite analogously,
the requirement that the action must be stationary separately for the part 2, or the part 1',
or the part 2' of the trajectory, leads to the conclusion that the motion is linear for
each of these parts.

Therefore, the trajectory has the form shown in Fig. 1b, and the action is given by
the following rather simple formula:

\[
S = \frac{m}{2} \left[ v_1^2 (t_0 - t_i) + v_1'^2 (t_f - t_0) + v_2^2 (t_0 - t_A) + v_2'^2 (t_B - t_0) \right] \tag{2}
\]

where the following notations are introduced for the constant velocities in the parts of the trajectory between the key events:

\[
v_1 = \frac{r_0 - r_i}{t_0 - t_i}, \quad v_1' = \frac{r_f - r_0}{t_f - t_0}, \quad v_2 = \frac{r_0 - r_A}{t_0 - t_A}, \quad v_2' = \frac{r_B - r_0}{t_B - t_0}. \tag{3}
\]

The principle of minimal action is not yet accounted completely. The boundary conditions \(r_i, t_i, r_f, t_f\)
as well as the locations of the mouths \(r_A\) and \(r_B\) and the time shift \(t_B - t_A = \tau\) are fixed, but the event of collision, \((r_0,t_0)\),
as well as the sum \(t_A + t_B\), may
be chosen arbitrarily. The principle of minimal action requires that a small variation of these parameters do not alter the action. The problem of minimum of the functional is reduced thus to the problem of minimum of the function of finite number of parameters.

There is one evident solution of this problem. To find it, one should consider the parts 1 and $1'$ of the trajectory jointly, as a single motion, and the parts $2$ and $2'$ as one more motion. The action has a minimum when each of these motions is linear, i.e.

$$v_1 = v_1', \quad v_2 = v_2'$$

(see Fig. 2). Indeed, any small change of the parameters $r_0$, $t_0$ and $t_A + t_B$ converts the straight line connecting the events $i$, $f$ and/or the straight line connecting $(t_A, r_A)$, $(t_B, r_B)$ into broken lines, increasing the action (2).

This configuration of the trajectory corresponds to the case when the velocities of the particle and its incarnation are exchanged in the collision. Of course, this solution exists only if the direct line (in the spacetime) connecting the events $(r_i, t_i)$ and $(r_f, t_f)$ intersects (see Fig. 2) the part of the plane between the world lines of the mouths, $r_A$ (arbitrary time) and $r_B$ (arbitrary time). The velocity $v_2 = v_2'$ in this case is directed from the

Figure 2: Simple trajectory minimizing the action: a collision with exchange of velocities.
mouth $A$ to the mouth $B$ and is equal to the distance between the mouths divided by $\tau$.

This simple configuration is complanar in the sense that all key points $r_i$, $r_f$, $r_A$, $r_B$ and $r_0$ lie in the same plane. Of course, this configuration is well known [7, 11] and is mentioned here only as an illustration of the method.

3 General solution for one traversal

Variation of the action (2) (with the notations (3)) in the parameters $r_0$, $t_0$ and $t_A + t_B$ gives the equations

$$v_1 + v_2 = v'_1 + v'_2, \quad v^2_1 + v^2_2 = v'^2_1 + v'^2_2, \quad v_2^2 = v_2'^2$$ (4)

The first and the second of these equations express correspondingly conservation of the momentum and energy in the collision. The third equation gives the rule that the particle exits from the time machine (wormhole) with the same velocity that it has had at the entrance.

It is easily seen that the simple motion considered in the preceding section (see Fig. 2) is a solution of the equations (4). The general solution has also very simple form [11]

$$v_1 = c_1 - c, \quad v'_1 = c_1 + c, \quad v_2 = c_2 + c, \quad v'_2 = c_2 - c,$$ (5)

where the vectors $c_1$, $c_2$ and $c$ satisfy the condition

$$c_1 c = 0, \quad c_2 c = 0$$ (6)

and are otherwise arbitrary.

If we know all the velocities, we can find all parameters of the motion (the spacetime points determining the trajectories of Fig. 4b). It is more difficult to solve the opposite problem and find $c_1$, $c_2$, $c$ given the characteristics of the time machine $r_A$, $r_B$, $\tau$ and boundary conditions $(r_i, t_i)$, $(r_f, t_f)$. This will be our task in the present section.

4They have been derived by this method in [11]
This problem may be in principle solved if one considers Eq. (4) (with the notations (3)) as equations not for velocities, but for the unknown space-time parameters $r_0, t_0$ and $t_A + t_B$. Indeed, substituting the expressions from (3) for the velocities in Eq. (4), we obtain one vector and two scalar equations for one vector and two scalar unknowns.

We shall slightly modify this scheme using the parametrization (5) for the velocities. Substituting the expressions (5) for the velocities in (3) and performing elementary operations with the resulting equalities, we have

\[ T c_1 + T_b c = L, \quad \tau c_2 - T_w c = l, \quad T_b c_1 - T_w c_2 + (T + \tau) c = d \]  \hspace{1cm} (7)

\[ 4r_0 = r_i + r_f + r_A + r_B - T_b c_1 - T_w c_2 - (T - \tau) c. \]  \hspace{1cm} (8)

where the following notations are introduced (“b” stands for “boundary” and “w” for “wormhole”):

\[
\begin{align*}
L &= r_f - r_i, \quad l = r_B - r_A, \quad d = (r_i + r_f) - (r_A + r_B), \\
T &= t_f - t_i, \quad T_b = (t_i + t_f - 2t_0), \quad T_w = (t_A + t_B - 2t_0).
\end{align*}
\]

Now we shall accept the following strategy. The linear equations (7) for the vectors $c_1, c_2, c$ may be easily solved giving the expressions of these vectors through the unknowns $T_b$ and $T_w$. Then the orthogonality conditions (6) give equations for $T_b$ and $T_w$. Finally $r_0$ may be found from Eq. (8).

Before realizing this program, let us consider a simple special case

\[ T_b = 0, \quad T_w = 0. \]  \hspace{1cm} (9)

In this case we have (from (8))

\[ c_1 = \frac{L}{T}, \quad c_2 = \frac{l}{\tau}, \quad c = \frac{d}{T + \tau}. \]

The orthogonality conditions (8) should be imposed, and this means that this special case is realized if the vector $d$ is orthogonal to both $L$ and $l$. It is easy to verify that the opposite is also valid: if these vectors are orthogonal, then we necessarily have (8).
This simple example gives nevertheless a three-dimensional (not coplanar) configuration, if the vector $d$ is nonzero and the vectors $L$ and $l$ are not parallel. In this case the points $r_i$, $r_f$, $r_A$, $r_B$ and $r_0$ do not lie in the same plane.

If $d = 0$, we have a coplanar motion, in fact a symmetrical case of the configuration considered in Sect. 3, when the collision leads to exchange of the velocities between the particle and its incarnation. If $d \neq 0$, but the vectors $L$ and $l$ are parallel, a symmetrical case of another type of the coplanar motion is realized, so-called “mirror exchange” of the velocities [7]. The components of the velocities along the direction of $L$ and $l$ are exchanged in this case while the components along the direction $d$ are exchanged with simultaneous change of the signs.

Let us discuss now the general solution of the problem, i.e. attempt to find the parameters of the motion for arbitrary vectors $L$, $l$ and $d$.

According to what has been proposed above, we should find $c_1$, $c_2$ and $c$ from the equations (7) and require that they satisfy the orthogonality conditions (8). This will give the equations for the unknowns $T_b$, $T_w$. Solution of these equations will determine the configuration of the motion completely.

For convenience, we shall introduce the dimensionless unknowns instead of $T_b$, $T_w$:

$$\lambda_b = -\frac{T_b}{T}, \quad \lambda_w = \frac{T_w}{T}.$$  

Then Eq. (7) gives for the velocities expressed through these scalar unknowns

$$\theta c = \lambda_b L + \lambda_w l + d,$$

$$\theta c_1 = \left(\lambda_1 - \frac{T}{\lambda_w^2}\right) L + \lambda_b \lambda_w l + \lambda_b d,$$

$$\theta c_2 = \lambda_b \lambda_w L + \left(\lambda_2 - \frac{T}{\lambda_b^2}\right) L + \lambda_w d$$

(10)

where it is denoted

$$\lambda_1 = 1 + \frac{T}{\lambda_b}, \quad \lambda_2 = 1 + \frac{T}{\lambda_w}, \quad \theta = T + T \lambda_b^2 - \tau \lambda_w^2.$$
The orthogonality condition (6) imposed on the vectors (10) gives, after some algebra,

\[(d + \lambda_b L + \lambda_w l)(d + \frac{\lambda_1}{\lambda_b} L) = 0, \quad (d + \lambda_b L + \lambda_w l)(d + \frac{\lambda_2}{\lambda_w} L) = 0.\]  \hspace{1cm} (11)

This algebraic equations for \(\lambda_b, \lambda_w\) could be in principle solved (they may be reduced to a third-degree equation for one unknown). With the help of Eq. (8), this would result in the complete solution of the problem. Not trying to obtain this solution in an explicit form, we shall discuss the question of its existence.

It may be shown that, for arbitrary vectors \(L, l\) and \(d\), a unique pair of real numbers \(\lambda_b, \lambda_w\) exists satisfying the equations (11). However not all of these formal solutions give a solution of the physical problem. The reason is that, as a consequence of a natural inequalities

\[t_i < t_0 < t_f, \quad t_A < t_0 < t_B,\]

the following restrictions exist for the unknowns:

\[|\lambda_b| < 1, \quad |\lambda_w| < 1.\]  \hspace{1cm} (12)

The formal solution of Eq. (11) satisfies this restriction if the projection \(d_{\text{proj}}\) of the vector \(d\) onto the plane of the vectors \(L, l\) is small enough, but in general case this may be not valid.

Indeed, Eq. (11) means that among the three vectors

\[D = d + \lambda_b L + \lambda_w l, \quad D_b = d + \frac{\lambda_1}{\lambda_b} L, \quad D_w = d + \frac{\lambda_2}{\lambda_w} L\]

the first one should be orthogonal to the two others. These vectors are drawn in Fig. 3a. Geometrical analysis of this figure shows that the inequalities (12) are fulfilled (and therefore the physical solution of the problem exists) if the vector \(d_{\text{proj}}\) belongs to some region \(\Omega\) near zero.

The region \(\Omega\) is shown in Fig. 3b where a part of the plane \(L, l\) is drawn. This region coincides with the intersection of two parallelograms determined by the vectors \(L, l\) and having mutually perpendicular sides.
Figure 3: A self-collision with one passage through the wormhole. (a) The unknown parameters $\lambda_b$, $\lambda_w$ are determined by the condition that $D$ is orthogonal to $D_b$ and $D_w$. (b) The conditions $|\lambda_b| < 1$, $|\lambda_w| < 1$ (providing a correct physical interpretation) are satisfied if the vector $d_{proj}$ belongs to the intersection $\Omega$ of the two parallelograms constructed with the help of the vectors $L$ and $l$. 
The condition $d_{\text{proj}} \in \Omega$ is sufficient for existing a configuration with a self-collision and one passage through the wormhole. It is difficult to formulate a sufficient and necessary condition for this.

4 Multiple wormhole traversals

We considered in much detail the scheme of motion with a self-collision and one passage through the time machine (wormhole traversal). However this is not the only possibility. Multiple passages with a self-collision were discussed in [7]. Only complanar or very close to complanar motions were considered in this paper. This means that the space positions $(r_i, r_f, r_A, r_B, r_0)$ of all key events lied in the same plane. We shall consider the general situation (but with a point-like body instead of a finite billiard ball).

The scheme of consideration is quite analogous to that proposed in Sects. 2, 3. The kinematical scheme for a double passage is drawn in Fig. 4a. The action has in this case the form differing from Eq. (2) only by an additional term corresponding to an additional passage through the time machine:

$$S = \frac{m}{2} \left[ v_1^2(t_0 - t_i) + v_1^2(t_f - t_0) + v_2^2(t_0 - t_A) + v_2^2(t_B - t_0) + u^2(t'_B - t'_A) \right].$$

(13)

The following notations are used here:

$$v_1 = \frac{r_0 - r_i}{t_0 - t_i}, \quad v'_1 = \frac{r_f - r_0}{t_f - t_0}, \quad v_2 = \frac{r_0 - r_A}{t_0 - t_A}, \quad v'_2 = \frac{r_B - r_0}{t_B - t_0}, \quad u = \frac{r'_B - r'_A}{t'_B - t'_A}.$$ (14)

Variation of this action in the parameters $r_0, t_0, t_A + t'_B, t'_A + t_B$ gives the equations

$$v_1 + v_2 = v'_1 + v'_2, \quad v_1^2 + v_2^2 = v'_1^2 + v'_2^2, \quad v_2^2 = u^2 = v_2^2$$

(15)

(the momentum and energy conservation and the rule ‘the in-velocity is equal to the out-velocity’). The equalities (13) may be considered as equations for the unknown parameters of the motion: $r_0, t_0, t_A + t'_B, t'_A + t_B$. The concrete scheme of their solution, quite similar
Figure 4: Double (a) and multiple (b) wormhole traversals with a self-collision. Velocity $|u|$ is less than $|d|/\tau$, thus repeated passages through the wormhole bring the particle into the past.

to one proposed in Sect. 3, may be developed. It reduces the problem to the solution of a system of algebraic equations.

Now we can easily go over to the case of an arbitrary number $N$ of passages through the time machine. The scheme of such a motion is presented in Fig. 4b. It is evident how the action for this motion may be written and what is the result of its variation. In the notations evident from the figure, we have

$$v_1 + v_2 = v'_1 + v'_2, \quad v_1^2 + v_2^2 = v_1'^2 + v_2'^2, \quad v_2^2 = u_1^2 = u_2^2 = \ldots = u_N^2 = v_2^2. \quad (16)$$

To demonstrate existence of a solution to these equations, we shall consider a concrete example, suggesting that the collision occurs far from the wormhole (see Fig. 5). Suppose that the following inequalities are valid:

$$R \gg h \gg l$$

where $R$ is the distance, along the initial trajectory, from the collision point to the point
Figure 5: Many wormhole traversals with the collision far from the mouths.

closest to the mouth $B$, $h$ is the distance of the mouth $B$ from the initial trajectory, and $l = |l|$ the distance between the mouths.

As a consequence of the condition $R \gg l$, the velocity $v_2'$ with which the ball (particle) heads towards the wormhole and the velocity $v_2$ with which it returns are very nearly antiparallel. Using this fact and the rule $v_2 = v_2'$, it is easy to demonstrate that

$$v_2 = v_2' = v_1 \cos \frac{h}{R} = v_1 \left[ 1 - \frac{1}{2} \left( \frac{h}{R} \right)^2 \right],$$

where $v_1$ is the initial velocity of the particle.

After this, the simple argument based on the kinematical scheme Fig. 4b leads to the following relation for the number $n$ of the wormhole traversals:

$$n = \frac{2R/l}{v_1 \sqrt{1 - \frac{1}{2} \left( \frac{h}{R} \right)^2}} - 1.$$

Thus, if $v_1 > l/\tau$, there are infinite number of possible configurations of the motion for each initial trajectory. When $R$ tends to infinity, the number of traversals $n$ also tends to infinity. The ball may pass through the time machine many times provided the collision with the incarnation occurs far from the wormhole. There is a minimal distance

$$R_{\text{min}} = \frac{h}{\sqrt{2 \left( 1 - \frac{l}{v_1 \tau} \right)}}$$

for which there are permitted trajectories. While $R$ decreases from infinity to this $R_{\text{min}}$, the number $n$ decreases from infinity to some minimal value and after this increases to infinity again.
The inequality $v_1 < l/\tau$ leads to negative $n$ for any $R > h$ that is impossible. This means that self-collision in the considered configuration is impossible for such a small initial velocity of the particle.

Besides the configurations already discussed, one could in principle consider configurations with multiple self-collisions. Multiple wormhole traversals are also necessary to provide multiple self-collisions of the same particle. The equations for such configurations may be written down and analyzed in essentially the same way as it has been done in the case of one collision. However the analysis is in this case much more complicated. A preliminary analysis carried out for the case of two self-collisions (and two transversals) allows one to suggest that two-collision configurations, under the conditions $\rho \ll r \ll l$ (see Sect. 1) are impossible. However the question requires further investigation.

5 Jinnee of Time Machine

We have systematically considered different configurations of motion of a particle (point-like body) with traversals the time machine and a self-collision. It is a high time now to remark that each of these configurations may be interpreted in different ways.

Begin with the simplest configuration Fig. 1b. We supposed so far that the particle moved along the line 1, then, after collision, continued the motion along the line 2', then, on escaping from the wormhole, went along the line 2, and after the self-collision in the event $O$ finished the motion along the line 1'.

A different interpretation of the same dynamical scheme suggests that the particle moves only along the lines 1 and 1'. In the event $O$ it collides not with itself, but with another particle having the same mass and interaction (identical to the first one). The second particle emerges from the mouth $A$, travels along the line 2 to the event of collision $O$ and then, after the collision, returns to the wormhole, passing along 2' and entering the mouth $B$.

In this interpretation there are two particles. The particle 1 never enters the time
machine, and the particle 2 goes out of the time machine only to collide with the particle 1 and deflect it from the direct line of motion. After this the second particle returns to the wormhole. Such a particle has a closed time-like trajectory (world line). It is nothing else than Jinnee of Time Machine discussed in [14, 15] (see [16] for the discussion of a quantum version of Jinnee).

It is important that all characteristics of the particle 2 should be identical to those of the particle 1. All parameters of the motion are then quite the same for both interpretations of it. Of course, if finite dimension of the ball is taken into account, then the motions corresponding to the two interpretations turn out to be slightly different. For example, positions of the two colliding balls are different in the cases of the first and second interpretations (without or with the Jinnee).

It is interesting to consider the Jinnee interpretation for a special case of the same configuration, namely for the motion presented in Fig. 2. In the usual interpretation (accepted in the preceding sections) the particle, because of the collision, is deflected from the direct motion and enters the wormhole. Escaping from the wormhole, it collides itself, providing deflection of its first incarnation, and proceeds further to the final event $f$. The motion is rather complicated.

The Jinnee interpretation of the same motion is quite trivial. The particle freely moves along the direct line from the event $i$ to the event $f$ without any collision. Simultaneously the second particle (Jinnee) exists. It goes out of the wormhole only to move along the direct line from $A$ to $B$ and return to the wormhole. It does not collide the first particle, but it passes very nearly to it (that is possible in the approximation of point-like bodies and short-range interaction between them).

Thus, in the configuration Fig. 2 we have alternatively non-trivial motion of a single particle or trivial motion of two particles, one of which is Jinnee. In the general case Fig. 1b both interpretations (without Jinnee or with Jinnee) lead to non-trivial motions.

The preceding argument evidently applies to a similar, but more complicated configu-
ration presented in Fig. 4ab. Again we may interpret the motion i) either as a motion of a single particle entering the wormhole and undergoing self-collision, ii) or as a motion of the particle travelling outside the wormhole, but colliding with Jinnee of Time Machine. Now we have Jinnee that passes through the wormhole two or several times. Usually Jinnee travels freely outside the wormhole. However once, being outside the wormhole, Jinnee goes far from it, collides the by-passing particle and returns to the wormhole.

Thus, when possible existence of bodies with closed time-like world lines (trajectories) is taken into account, different interpretations may be given for each concrete configuration of the motion with a self-collision and traversals of the time machine. From a certain point of view, interpretations with Jinnee are simpler than those with no Jinnee.

Since different interpretations are dynamically quite equivalent, one of them may be taken for the analysis arbitrarily. For example, when we need to specify all configurations characterized by a definite number of traversals, it is convenient to look for those configurations where the particle never enters the wormhole but collides with Jinnee. Afterwards each configuration found in such a way may be interpreted in terms of a single particle, with no Jinnee.

We should underline that different interpretations are dynamically equivalent only in the approximation of point-like bodies. For small balls instead of points, slight differences arise in mutual positions these balls have in different interpretations.

Moreover, the interpretations with Jinnee seem to be much more natural for simple objects such as elementary particles, than for complicated bodies such as billiard balls. One may suppose that Jinnee-interpretations may become physically significant only for elementary particles. In this case existence of completely identical objects is not at all astonishing. The objects identical to one under investigation but moving along closed

\[5\text{By the way, this is reflected in the fact that the notations for velocities chosen in Sect. 3 only for mathematical convenience, turned out to correspond to the Jinnee interpretation rather than to the no-Jinnee one.}\]

\[6\text{in the approximation of point-like bodies}\]
world lines, evidently resemble vacuum loops arising in quantum theory.

Thus, classical theory of bodies moving in presence of a time machine naturally leads to some concepts, otherwise arising only in quantum theory. This property seems very interesting and deserving further exploration.

6 Conclusion

We considered the motion (including a self-collision) of a point-like body in the time-machine spacetime. The set of configurations of such a motion turned out to be richer than it has been known earlier. The approximation of a point-like body and the method of minimal action proved to be efficient for investigating three-dimensional configurations of the motion.

We found main features of such motions (for a specific model of a time machine) and found that, given boundary conditions (initial and final positions and times), many configurations of motion are possible, differing by the number of traversals through the time machine. An interesting feature of these configurations is discovered: they may be interpreted in different ways if the bodies with closed time-like world lines (Jinnee of Time Machine) are introduced. There is a certain analogy between (classical) trajectories of Jinnee and vacuum loops typical for relativistic quantum theory.

With all this taken into account, it seems interesting to consider scattering of a particle on the time machine and the role played in this process by different configurations of motion. It may be supposed that correct formulation of this problem may be given only in the framework of quantum theory.

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