A Round-Robin Protocol for Distributed Estimation with $H_\infty$ Consensus

V. Ugrinovskii† and E. Fridman§

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Abstract

The paper considers a distributed robust estimation problem over a network with directed topology involving continuous time observers. While measurements are available to the observers continuously, the nodes interact according to a Round-Robin rule, at discrete time instances. The results of the paper are sufficient conditions which guarantee a suboptimal $H_\infty$ level of consensus between observers with sampled interconnections.

1 Introduction

The problem of distributed estimation is one of very active topics in the modern control theory and signal processing literature. Interest in this problem is motivated by a growing number of applications where a decision about the observed process must be made simultaneously by spatially distributed sensors, each taking partial measurements of the process.

When the process and measurements are subject to noise and disturbance, robustness aspects of the problem come into prominence. In the past several years, a number of results have been presented in the literature which develop the $H_\infty$ control and estimation theory for distributed systems subject to uncertain perturbations; e.g., see [2, 7, 8, 11, 18, 20, 19]. In particular, methodologies of distributed sampled-data $H_\infty$ filtering have been considered [16]. That reference emphasized several aspects of realistic sensor networks, among them coupling between sensor nodes through the information communicated between neighbouring sensor nodes and the sampled nature of that coupling, which is dictated by the digital communication technology.

In this paper we address some of the above challenges by developing a Round-Robin protocol for a network of distributed estimators. The Round-Robin protocol is a commonly used protocol for information transmission in networked control systems. From a hybrid systems perspective this protocol has been studied in details in [12, 5]. More recently, it has been considered in the context of time-delay systems in [9], where an analysis of exponential stability and $L_2$ properties of networked control systems with Round-Robin scheduling was presented using a delay switching system modeling. In this paper, we further develop this technique in the context of robust distributed estimation with intermittent communication between sensing nodes.

The objective of this paper is to develop a sufficient condition to enable synthesis of filter and interconnection gains for a network of distributed observers, whose aim is to track dynamics of a linear uncertain plant.

The first contribution of this paper is a version of the Round-Robin protocol of [8] to be used with the distributed estimation schemes proposed [18, 20, 19]. We show that instead of continuously exchanging...
information (the type of networks considered in those references), the node observers can achieve the relative $H_\infty$ consensus objective by exchanging information at certain sampling times, by polling one neighbour at a time. Our second contribution demonstrates that the Round-Robin design of [9] can be applied to derive a network of non-switching observers.

Our main result is a sufficient condition, expressed in the form of Linear Matrix Inequalities (LMIs), from which filter and interconnection gains for each node estimator can be computed, to ensure the network of sampled data observers under consideration converges to the trajectory of the observed plant. As in [18, 20, 19], our methodology relies on certain vector dissipativity properties of the large-scale system comprised of the observers’ error dynamics [4]. However, different form these references, to establish these vector dissipativity properties, we employ a novel class of generalized supply rates which reflect the sampled-data nature of interconnections between observers. The general idea behind introducing such generalized supply rates can be traced to [6] (also, see [20]), but our proposal here makes use of special properties of sampled signals. In the limit, when the maximum sampling period approaches zero, these generalized supply rates vanish, and one recovers the vector dissipativity properties of error dynamics established in [18].

The paper is organized as follows. The problem formulation, along with the graph theory preliminaries is presented in Section 2. The main results of the paper are given in Section 3. Section 4 concludes the paper.

Notation Throughout the paper, $\mathbb{R}^n$ denotes a real Euclidean $n$-dimensional vector space, with the norm $\|x\| \equiv (x'x)^{1/2}$; here the symbol $'$ denotes the transpose of a matrix or a vector. $L_2[0, \infty)$ will denote the Lebesgue space of $\mathbb{R}^n$-valued vector-functions $x(\cdot)$, defined on the time interval $[0, \infty)$, with the norm $\|x\|_2 \equiv \left(\int_0^\infty \|x(t)\|^2 dt\right)^{1/2}$ and the inner product $\int_0^\infty z_1(t)'z_2(t)dt$. $\otimes$ is the Kronecker product of matrices, $1_n \in \mathbb{R}^n$ is the column-vector of ones.

2 The problem formulation

2.1 Graph theory

Consider a filter network with $N$ nodes and a directed graph topology $\mathcal{G} = (\mathcal{V}, \mathcal{E})$; $\mathcal{V} = \{1, 2, \ldots, N\}$, $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ are the set of vertices and the set of edges, respectively. The notation $(j, i)$ will denote the edge of the graph originating at node $j$ and ending at node $i$. In accordance with a common convention [13], we consider graphs without self-loops, i.e., $(i, i) \notin \mathcal{E}$. However, each node is assumed to have complete information about its filter and measurements.

For each $i \in \mathcal{V}$, we denote $\mathcal{V}_i = \{j : (j, i) \in \mathcal{E}\}$ to be the ordered set of nodes supplying information to node $i$, i.e., the neighbourhood of $i$. Without loss of generality, suppose the elements of $\mathcal{V}_i$ are ordered in the ascending order. The cardinality of $\mathcal{V}_i$, known as the in-degree of node $i$, is denoted $p_i$; i.e., $p_i$ is equal to the number of incoming edges for node $i$. Also, the out-degree of node $i$ (i.e., the number of outgoing edges) is denoted $q_i$.

Without loss of generality the graph $\mathcal{G}$ will be assumed to be weakly connected, [18] Proposition 1.

In the sequel, a shift permutation operator defined on elements of the set $\mathcal{V}_i$ will be used:

$$\Pi\{j_1, \ldots, j_{p_i-1}, j_{p_i}\} = \{j_{p_i}, j_1, \ldots, j_{p_i-1}\}. \tag{1}$$

Furthermore, $\Pi_k(\mathcal{V}_i)$ will denote the set obtained from $\mathcal{V}_i$ using $k$ consecutive shift permutations [1]. In regard to this set, the following notation will be used throughout the paper unless stated otherwise: for $\nu \in \{1, \ldots, p_i\}$, $j_\nu$ is the $\nu$-th element in the ordered set $\Pi_k(\mathcal{V}_i)$. Conversely, for $j \in \Pi_k(\mathcal{V}_i)$, $\nu_{j,k} \in \{1, \ldots, p_i\}$ is the index of element $j$ in the permutation $\Pi_k(\mathcal{V}_i)$. We will omit the superscript $k,i$ if this does not lead to ambiguity.

2.2 Distributed estimation with $H_\infty$ consensus

Consider a plant described by the equation

$$\dot{x} = Ax + B_2 w(t), \quad x(t) = x_0 \forall t \leq 0. \tag{2}$$


For simplicity, we assume that this schedule of updates is known to all participants in the network, and the distributed filtering problem under consideration is to estimate the state of the system using a network of filters connected according to the graph $\mathcal{G}$. Each node takes measurements

$$y_i(t) = C_i x(t) + D_{2i} w(t) + \bar{D}_{2i} v_i(t);$$

where $v_i \in \mathbb{R}^{m_w}$ is a measurement disturbance.

The measurements are processed by a network of observers connected over the graph $\mathcal{G}$. The key assumption in this paper is to allow the observers make use of their local measurements continuously, however, they can only interact with each other at discrete time instances $t_k$, $k = 0, 1, \ldots$, with $t_0 = 0$. For simplicity, we assume that this schedule of updates is known to all participants in the network, and therefore all nodes exchange information at the same time instance $t_k$. However, at every time instance $t_k$ only one neighbour in the set $\mathcal{N}_i$ is polled by each node $i$, according to the ‘Round-Robin’ rule. Formally, this leads us to define the following observer protocol: For $t \in [t_k, t_{k+1}]$, $k = 0, 1, \ldots$

$$\hat{x}_i(t) = A \hat{x}_i(t) + L_i (y_i(t) - C_i \hat{x}_i(t)) + K_i \sum_{j \in \Pi_i(\mathcal{N}_i)} H_i (\hat{x}_j(t_{k-\nu_j+1}) - \hat{x}_i(t_{k-\nu_j+1})), \quad (4)$$

where $\hat{x}_i(t)$ is the estimate of the plant state $x(t)$ calculated at node $i$, the matrices $L_i$, $K_i$ are parameters of the filters to be determined, and $H_i$ is a given matrix. The observers are initiated with zero initial condition, $\hat{x}_i(t) = 0$ for all $t \leq 0$.

Formally, this can be described by first applying the shift permutation operator $\Pi$ to the neighbourhood set at every time instance $t_k$, and then selecting the first element from the resulting permutation $\Pi^k(\mathcal{N}_i)$ for feedback.

Let $e_i = x - \hat{x}_i$ be the local estimation error at node $i$. This error satisfies the equation:

$$\dot{e}_i = (A - L_i C_i) e_i + (B - L_i D_i) \xi_i + K_i H_i \sum_{j \in \Pi_i(\mathcal{N}_i)} (e_{j_i}(t_{k-\nu_j+1}) - e_i(t_{k-\nu_j+1})), \quad (5)$$

Here we used the notation $\xi_i$ to represent the perturbation vector $[w' \nu']/\nu'$, and the matrices $B, D_i$ are defined as follows $B = [B_2 0]$, $D_i = [D_{2i} \bar{D}_{2i}]$. The initial conditions for (5) are $e_i(t) = x_0 \forall t \leq 0$. In particular in (5), $e_{j_i}(t_{k-\nu_j+1}) - e_i(t_{k-\nu_j+1}) = 0$ for $t_k - \nu_j + 1 < 0$.

Since the error dynamics (5) are governed by $L_2$ integrable disturbance signals $\xi_i$, we can only expect the node observers to converge in $L_2$ sense. To quantify transient consensus performance of the observer network (4) under disturbances, consider the cost of disagreement between the observers caused by a particular vector of disturbance signals $\xi(\cdot) = [\xi_1(\cdot) \ldots \xi_N(\cdot)]'$. 

$$J(\xi) = \frac{1}{N} \int_0^\infty \sum_{i=1}^N \sum_{j \in \Pi_i(\mathcal{N}_i)} \|\hat{x}_j(t) - \hat{x}_i(t)\|^2 dt = \frac{1}{N} \int_0^\infty \sum_{i=1}^N \sum_{j \in \Pi_i(\mathcal{N}_i)} \|e_j(t) - e_i(t)\|^2 dt, \quad (6)$$

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where $k$ is a time-dependent index, $k = 0, 1, \ldots$, defined so that for every $t \in [0, \infty)$, $t_k \leq t < t_{k+1}$. The functional (6) was originally introduced in [18] as a measure of consensus performance of a corresponding continuous-time observer network. It is worth noting that for each $t$, $\sum_{j \in \Pi_k^{(Y_i)}} \|\hat{x}_j(t) - \hat{x}_i(t)\|^2$ is independent of the order in which node $i$ polls its neighbours, so that

$$\sum_{j \in \Pi_k^{(Y_i)}} \|\hat{x}_j(t) - \hat{x}_i(t)\|^2 = \sum_{j \in Y_i} \|\hat{x}_j(t) - \hat{x}_i(t)\|^2.$$ 

Therefore, the inner summation in (6) can be replaced with summation over the neighbourhood set $Y_i$. This observation leads to the same expression for $J(\xi)$ as in the case of continuous-time networks [18],

$$J(\xi) = \frac{1}{N} \int_0^\infty \sum_{i=1}^N \left[ (p_i + q_i) \|e_i(s)\|^2 - 2e'_i \sum_{j \in Y_i} e_j(s) \right] ds. \quad (7)$$

The following distributed estimation problem is a version of the distributed $H_{\infty}$ consensus-based estimation problem originally introduced in [18][20], modified to include the Round-Robin protocol [4].

**Definition 1** The distributed estimation problem under consideration is to determine a collection of observer gains $L_i$ and interconnection coupling gains $K_i$, $i = 1, \ldots, N$, for the filters [4] which ensure that the following conditions are satisfied:

(i) In the absence of uncertainty, the interconnection of unperturbed systems [5] must be exponentially stable.

(ii) The filter must ensure a specified level of transient consensus performance, as follows

$$\sup_{x_0, \xi \neq 0} \frac{J(\xi)}{\|x_0\|^2_2 + \frac{1}{N} \|\xi\|^2_2} \leq \gamma^2. \quad (8)$$

Here, $\|x_0\|^2_2 = x'_0 P x_0$, $P = P' > 0$ is a matrix to be determined, and $\gamma > 0$ is a given constant.

## 3 The main results

Our approach to solving the problem in Definition 1 will follow the methodology for the analysis of stability and $L_2$-gain for networked control systems proposed in [9].

The proofs of the results are omitted due to space limitation. The key technical tools used in those proofs are the Wirtinger’s inequality [10] and the descriptor method [3][17].

As can be seen from (5), if the observer at node $i$ polls a channel at time $t_{k-p_i+1}$, the next time the same channel will be polled at time $t_{k+1}$. The longest time between polls of the same channel at node $i$ constitutes the maximum delay in communication between node $i$ and its neighbours, which will be denoted $\tau_i$:

$$\tau_i = \max_k (t_{k+1} - t_{k-p_i+1}).$$

The largest communication delay in the network is then $\tau = \max_i \tau_i$. It is easy to see from these definitions that $\tau = \max_k (t_{k+1} - t_{k-p+1})$, where $p = \max_i p_i$.

Consider the following Lyapunov-Krasovskii candidate for the system (5):

$$V_i(e_i) = e'_i Y_i^{-1} e_i + \int_{t_{i-1}}^t e^{-2\alpha_i(t-s)} e_i(s)' S_i e_i(s) ds + \tau_i \int_{t_{i-1}}^t e^{-2\alpha_i(t-s)} e_i(s)' (\tau_i + s - t) R_i e_i(s) ds, \quad (9)$$

where $Y_i = Y'_i > 0$, $R_i = R'_i \geq 0$, $S_i = S'_i \geq 0$ and $\alpha_i \geq 0$, $i = 1, \ldots, N$, are matrices and constants to be determined. $V_i(e_i)$ is a standard Lyapunov-Krasovskii functional used in the literature on exponential stability of systems with time-varying delays; e.g., see [9].
Given a matrix $W_i = W_i > 0$, define
\[
\mathcal{W}_i(u, z) = \frac{\pi^2}{4}(u - z)^TW_i(u - z).
\]

**Lemma 1** Suppose there exist gains $K_i$, $L_i$, matrices $W_i = W_i > 0$, and constants $\alpha_i > 0$, $0 < \pi_i < 2\alpha_i q_i^{-1}$, $i = 1, \ldots, N$, such that the following vector dissipation inequality holds for all $i = \ldots, N$: For $t \in [t_k, t_{k+1})$,
\[
\dot{V}_i(e_i) + 2\alpha_i V_i(e_i) - \sum_{j \in \mathcal{V}_i} \pi_j V_j(e_j)
+ \left( \sum_{j \in \mathcal{V}_i} \pi_j^2 \right)\dot{e}_i^TW_i\dot{e}_i - \sum_{j \in \mathcal{V}_i} \mathcal{W}_j(e_j, e_j(t_{k+1} - t_{k+1}^+))
+ \frac{1}{\gamma^2}(p_i + q_i)\|e_i\|^2 - \frac{2}{\gamma^2} \pi_i \sum_{j \in \mathcal{V}_i} e_j - \|\xi_i\|^2 \leq 0,
\]   \hspace{1cm} (10)

where and $\nu^{k,i}$ is the index of $j$ in the ordered permutation set $\Pi^k(\mathcal{V}_i)$. Then the system $[5]$ satisfies conditions of Definition [7].

In what follows we present a sufficient condition for the dissipation inequality $[10]$ to hold. We begin with a technical lemma which essentially restates the corresponding lemma of $[15]$ in the form convenient for the subsequent use in the paper. Consider a vector $\delta = [\delta_0', \ldots, \delta_n']$, $\delta \in \mathbb{R}^n$. Also, for given $n \times n$ matrices $R_i = R_i' \geq 0$ and $G_i$, define
\[
\Psi_i = \begin{bmatrix}
R_i & \frac{1}{2}(G_i + G_i') & \ldots & \frac{1}{2}(G_i + G_i') \\
\frac{1}{2}(G_i + G_i') & R_i & \ldots & \frac{1}{2}(G_i + G_i') \\
\vdots & \vdots & \ddots & \vdots \\
\frac{1}{2}(G_i + G_i') & \frac{1}{2}(G_i + G_i') & \ldots & R_i
\end{bmatrix}.
\]

**Lemma 2** Suppose the matrices $R_i = R_i' \geq 0$ and $G_i$ are such that
\[
\begin{bmatrix}
R_i & G_i \\
G_i^\top & R_i
\end{bmatrix} \geq 0.
\]   \hspace{1cm} (11)

Then
\[
\tau_i \left[ \frac{1}{t - t_k} \delta_0'R_i\delta_0 + \sum_{\nu = 1}^{p_i - 1} \frac{1}{t_{k-\nu + 1} - t_{k-\nu}} \delta_\nu'R_i\delta_\nu + \frac{1}{t_{k-p_i+1} - t + \tau_i} \delta_{p_i}'R_i\delta_{p_i} \right] \geq \delta^\top\Psi_i\delta.
\]

Next, we introduce a number of matrices. First, we introduce
\[
\bar{\Psi}_i = e^{-2\alpha_i\tau_i} T_i^\top \Psi_i T_i,
\]
it can be further partitioned in accordance with the partition of $\bar{e}_i$:
\[
\bar{\Psi}_i = \begin{bmatrix}
\bar{\Psi}_{i,11} & \bar{\Psi}_{i,12} & \bar{\Psi}_{i,13} \\
\bar{\Psi}_{i,12} & \bar{\Psi}_{i,22} & \bar{\Psi}_{i,23} \\
\bar{\Psi}_{i,13} & \bar{\Psi}_{i,23} & \bar{\Psi}_{i,33}
\end{bmatrix},
\]

Then we introduce the correspondingly partitioned matrix
\[
\bar{\Psi}_i = \begin{bmatrix}
\bar{\Psi}_{i,11} & \bar{\Psi}_{i,12} & \bar{\Psi}_{i,13} \\
\bar{\Psi}_{i,12} & \bar{\Psi}_{i,22} & \bar{\Psi}_{i,23} \\
\bar{\Psi}_{i,13} & \bar{\Psi}_{i,23} & \bar{\Psi}_{i,33}
\end{bmatrix},
\]   \hspace{1cm} (12)
where we let
\[
\tilde{\Psi}_{i,11} = \Psi_{i,11} - 2\alpha_i Y_i^{-1} - S_i, \\
\tilde{\Psi}_{i,33} = \tilde{\Psi}_{i,33} + e^{-2\alpha_i \gamma_i} S_i, \\
\tilde{\Psi}_{i,\mu\nu} = \Psi_{i,\mu\nu} 
\]
for all other elements of $\tilde{\Psi}_i$.

Also, the following matrices will be used in the sequel:
\[
\Phi_{i,11} = \begin{bmatrix}
\pi_j Y_{j,1}^{-1} + \frac{\pi_j}{W_{j,1}} & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \pi_j Y_{j,p_i}^{-1} + \frac{\pi_j}{W_{j,p_i}}
\end{bmatrix},
\]
\[
\Phi_{i,22} = \begin{bmatrix}
\frac{\pi_j}{W_{j,1}} & 0 & \cdots & 0 \\
0 & \frac{\pi_j}{W_{j,2}} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \frac{\pi_j}{W_{j,p_i}}
\end{bmatrix},
\]
\[
\Phi_{i,12} = \Phi_{i,21} = -\Phi_{i,22}.
\]

Finally, to formulate our first result concerned with the analysis of consensus performance of the observer network (13), we introduce the matrix
\[
\Xi_i = \begin{bmatrix}
\Xi_{aa} & \Xi_{ab} & \Xi_{ac} & 0 & 0 & \Xi_{af} & \Xi_{ag} \\
\ast & \Xi_{bb} & \Xi_{bc} & -\tilde{\Psi}_{i,13} & \Xi_{be} & \Xi_{bf} & \Xi_{bg} \\
\ast & \ast & -\tilde{\Psi}_{i,22} & -\tilde{\Psi}_{i,23} & 0 & \Xi_{ef} & 0 \\
\ast & \ast & \ast & -\tilde{\Psi}_{i,33} & 0 & 0 & 0 \\
\ast & \ast & \ast & \ast & -\tilde{\Phi}_{i,11} & -\tilde{\Phi}_{i,12} & 0 \\
\ast & \ast & \ast & \ast & \ast & \Xi_{ff} & \Xi_{fg} \\
\ast & \ast & \ast & \ast & \ast & \ast & -I
\end{bmatrix}
\]  
(13)

where we have used the following notation
\[
\Xi_{aa} = \tau_i^2 R_i + \left( \sum_{j_i \in \mathcal{Y}_j} \tau_j^2 \right) W_i - Z_i - Z_i',
\]
\[
\Xi_{ab} = Y_i^{-1} - X_i + Z_i'(A - L_i C_i),
\]
\[
\Xi_{ac} = -Z_i'(1_{p_i} \otimes K_i H_i),
\]
\[
\Xi_{af} = 1_{p_i} \otimes (-Q_i + Z_i'K_i H_i),
\]
\[
\Xi_{ag} = Z_i'(B - L_i D_i),
\]
\[
\Xi_{bb} = \frac{(p_i + q_i)}{\gamma^2} I - \tilde{\Psi}_{i,11}
\]
\[
+ X_i'(A - L_i C_i) + (A - L_i C_i)'X_i,
\]
\[
\Xi_{bc} = -\tilde{\Psi}_{i,12} - (1_{p_i} \otimes X_i'K_i H_i),
\]
\[
\Xi_{be} = -\frac{1}{\gamma^2} (1_{p_i} \otimes I),
\]
\[
\Xi_{bf} = 1_{p_i} \otimes (X_i'K_i H_i + (A - L_i C_i)'Q_i),
\]
\[
\Xi_{bg} = X_i'(B - L_i D_i),
\]
\[
\Xi_{cf} = -1_{p_i} 1_{p_i} \otimes (H_i'K_i Q_i),
\]
\[
\Xi_{ff} = 1_{p_i} 1_{p_i} \otimes (Q_i'K_i H_i + H_i'K_i Q_i) - \Phi_{i,22},
\]
\[
\Xi_{fg} = 1_{p_i} \otimes Q_i'(B - L_i D_i).
\]
Theorem 2 Suppose there exists matrices $Y_i = Y_i' > 0$, $X_i$, $Z_i$, $Q_i$, $W_i = W_i' \geq 0$, $S_i = S_i' \geq 0$, $R_i = R_i' \geq 0$, $G_i$, constants $\alpha_i > 0$, $0 \leq \pi_i < 2\alpha_i q_i^{-1}$, and gain matrices $K_i$, $L_i$, $i = 1, \ldots, N$, which satisfy the LMI (17) and

$$\Xi_i < 0.$$  \hspace{2cm} (14)

Then the corresponding observer network [4] solves the problem posed in Definition 1. The matrix $P$ in condition (15) corresponding to this solution is $P = \frac{1}{\alpha_i} \sum_{i=1}^{N} (Y_i^{-1} + S_i \frac{1 - e^{-\pi_i}}{\pi_i})$.

Theorem 1 serves as the basis for derivation of the main result of this paper, given below in Theorem 2 which is a sufficient condition for synthesis of distributed observer networks of the form (4). Consider the following matrix

$$\Xi_i = \begin{bmatrix}
\Xi_{aa} & \Xi_{ab} & \Xi_{ac} & 0 & 0 & \Xi_{af} & \Xi_{ag} \\
* & \Xi_{bb} & \Xi_{bc} & -\Psi_{i,13} & \Xi_{be} & \Xi_{bf} & \Xi_{bg} \\
* & * & -\Psi_{i,22} & -\Psi_{i,23} & 0 & \Xi_{ef} & 0 \\
* & * & * & -\Psi_{i,33} & 0 & 0 & 0 \\
* & * & * & * & -\bar{\Phi}_{i,11} & -\bar{\Phi}_{i,12} & 0 \\
* & * & * & * & * & \Xi_{ff} & \Xi_{fg} \\
* & * & * & * & * & * & -I
\end{bmatrix}$$  \hspace{2cm} (15)

where

$$\Xi_{aa} = \tau_i^2 R_i + \left( \sum_{j \in \mathcal{F}_i} \tau_j^2 \right) W_i - \epsilon_i X_i - \epsilon_i X_i',$$

$$\Xi_{ab} = Y_i^{-1} - X_i + \epsilon_i (X_i'A - U_i C_i),$$

$$\Xi_{ac} = -\epsilon_i (1_{p_i} \otimes F_i H_i),$$

$$\Xi_{af} = 1_{p_i} \otimes (-\epsilon_i X_i + \epsilon_i F_i H_i),$$

$$\Xi_{ag} = \epsilon_i (X_i'B - U_i D_i),$$

$$\Xi_{bb} = \left( \frac{p_i + q_i}{\gamma^2} \right) I - \Psi_{i,11},$$

$$\Xi_{bc} = -\Psi_{i,12} - 1_{p_i} \otimes (F_i H_i),$$

$$\Xi_{be} = -\frac{1}{\gamma^2} (1_{p_i} \otimes I),$$

$$\Xi_{bf} = 1_{p_i} \otimes (F_i H_i + \bar{\epsilon}_i X_i - \bar{\epsilon}_i C_i' U_i'),$$

$$\Xi_{bg} = X_i'B - U_i D_i,$$

$$\Xi_{cf} = -1_{p_i} 1_{p_i} \otimes (\bar{\epsilon}_i H_i' F_i'),$$

$$\Xi_{cf} = \bar{\epsilon}_i 1_{p_i} 1_{p_i} \otimes (F_i H_i + H_i' F_i') - \bar{\Phi}_{i,22},$$

$$\Xi_{fg} = \bar{\epsilon}_i 1_{p_i} \otimes (X_i'B - U_i D_i).$$

Theorem 2 Suppose there exists matrices $Y_i = Y_i' > 0$, $X_i$, det $X_i \neq 0$, $F_i$, $U_i$, $S_i = S_i' \geq 0$, $R_i = R_i' \geq 0$, $W_i = W_i' \geq 0$, $G_i$, and constants $\alpha_i > 0$, $0 \geq \pi_i < 2\alpha_i q_i^{-1}$, $\epsilon_i > 0$, $\bar{\epsilon}_i > 0$, $i = 1, \ldots, N$, which satisfy the LMI (17) and

$$\Xi_i < 0.$$  \hspace{2cm} (16)
Then the network of observers \(\mathcal{G}\) with

\[ K_i = (X'_i)^{-1} F_i, \quad L_i = (X'_i)^{-1} U_i, \quad (17) \]

solves the distributed estimation problem posed in Definition \(\mathcal{G}\). The matrix \(P\) in condition \(\mathcal{G}\) corresponding to this solution is

\[ P = \frac{1}{N} \sum_{i=1}^{N} (Y_i^{-1} + S_i \frac{1-e^{-\alpha_i \tau_i}}{2\alpha_i}). \]

4 Conclusions

The paper has presented a sufficient LMI condition for the design of a Round-Robin interconnection protocol for networks of distributed observers. We have shown that the proposed protocol allows one to use sampled-data communications between the observers in the network, and does not require a combinatorial gain scheduling. As a result, the node observers are shown to be capable of achieving the \(H_{\infty}\) consensus objective introduced in \([18, 20, 19]\).

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