MAGNETIC FLUX DENSITY FROM THE RELATIVE CIRCULAR MOTION OF STARS AND PARTIALLY IONIZED GAS IN THE GALAXY MID-PLANE VICINITY

JOANNA JALOCHA1, ŁUKAZS BRATEK2, JAN PEKALA2, SYZMON SIKORA3, AND MAREK KUTSCHERA4

1 Institute of Physics, Cracow University of Technology, PL-30084 Kraków, Poland
2 Institute of Nuclear Physics Polish Academy of Sciences, PL-31342 Kraków, Poland; Lukasz.Bratek@ifj.edu.pl
3 Astronomical Observatory, Jagiellonian University, PL-30244 Kraków, Poland
4 Institute of Physics, Jagiellonian University, PL-30059 Kraków, Poland

Received 2016 September 5; revised 2016 October 20; accepted 2016 October 21; published 2016 December 16

ABSTRACT

Observations suggest a slower stellar rotation relative to gas rotation in the outer part of the Milky Way Galaxy. This difference could be attributed to an interaction with the interstellar magnetic field. In a simple model, fields of order 10 μG are then required, consistently with the observed values. This coincidence suggests a tool for estimating magnetic fields in spiral galaxies. A north–south asymmetry in the rotation of gas in the Galaxy could be of magnetic origin too.

Key words: Galaxy: kinematics and dynamics – galaxies: magnetic fields – magnetohydrodynamics (MHD)

1. INTRODUCTION

The mechanism of the influence of turbulent and large-scale magnetic fields on the motion of the gaseous fraction was described in the context of galactic dynamics by Battaner et al. (1992). The interstellar gas is ionized to a degree sufficient for the magnetic field to freeze-in the gas, so that the resulting magnetic tension can be assumed to hold the gas and so influence its motion, modifying the predictions of purely gravitational models. Because the gas density decreases with the Galactocentric distance, the magnetic effect becomes important for larger radii.

There have been several works devoted to this problem so far, showing that magnetic fields of a few μG (typical for the interstellar medium in spiral galaxies) are capable of considerably influencing the kinematics of (at least) partially ionized gas (Battaner et al. 1992, 2008; Kutschera & Jalocha 2004; Battaner & Florido 2007; Ruiz-Granados et al. 2010b, 2012; Jalocha et al. 2012a, 2012b). The fields have the potential to modify the outer parts of gaseous rotation curves, and in turn, the predictions about the distribution of mass.

Magnetic fields cannot be ignored when one attempts to fully understand the problem of the rotation of spiral galaxies. Battaner et al. (1992) brought to attention the possibility that the observed flatness of outer disc rotation curves, or even their rise, could be simply the result of interaction with interstellar magnetic fields (not merely due to unseen dark matter halo). In particular, it might turn out, that taking magnetic fields into account would reduce the amount of non-baryonic dark matter required by models which ignore magnetic fields.

Interestingly, using position–velocity data for a sample of remote classical cepheids and for H II regions in the Galaxy, Pont et al. (1997) found that the rotation curve indicated by the stars in the outer disc is markedly lower than the rotation curve of gas in the same region. They attributed this difference to either nonaxisymmetric components in the gas motion, or to high uncertainties in the distances to H II regions. From (Pont et al. 1997) it is evident also that there is a south–north asymmetry of the gaseous rotation curve, not observed for the stellar rotation curve. Remarkably, the difference in the rotation curves concerns the outer part of the Galactic disc and increases with the Galactocentric distance. The difference seems too high to be solely due to the neglected velocity components. Rather, the differences may be a manifestation of the influence of magnetic fields acting upon the gaseous substructure, increasing the rotation of gas with respect to that of stars. By neglecting magnetic fields, this increase would be customarily attributed to higher amounts of dark matter distributed in outer regions.

So far, the studies on the influence of magnetic fields on rotation curves have been inconclusive. Some authors, e.g., Battaner & Florido (2007), Ruiz-Granados et al. (2012), state that magnetic fields are responsible for the flattening or rise of the outer parts of rotation curves, other authors provide sound arguments that the influence is unlikely, or even may impede the gravitationally supported rotation, leading to an even higher halo mass (Sánchez-Salcedo & Santillán 2013; Elstner et al. 2014). However, the effects are subject to modeling assumptions.

The motivation behind the present work is our conjecture, that the observed difference between the stellar and gaseous rotation curves of the Milky Way Galaxy can be explained by the influence of magnetic fields. With this assumption, we estimate the magnitude and profiles of the radial and azimuthal magnetic field components required to account for the difference in rotation.

2. GASEOUS AND STELLAR ROTATION CURVES

The Milky Way rotation curve is determined based on the measurements of various kinematical tracers moving in the Galactic plane vicinity. To obtain the stellar and gaseous rotation curves, we unified position–velocity data for 357 cepheids (Caldwell & Coulson 1987; Pont et al. 1994, 1997; Berdnikov et al. 2000; Mel’nik et al. 2015), 110 carbon stars (Demers & Battinelli 2007; Battinelli et al. 2013) and 255 H II regions (Blitz et al. 1982; Chini & Wink 1984; Fich et al. 1989; Brand & Blitz 1993). The data were originally used as tracers of rotation in the Galactic plane vicinity. Therefore, we do not perform any further selection. We transformed the radial distances, radial motions, and transverse motions (and the respective errors) from the heliocentric coordinate frame to the
Galactocentric coordinate frame, assuming the IAU Galactic constants $R_\odot = 8.5$ kpc and $V_\odot = 220$ km s$^{-1}$. For each data point, we found the projected distance and the azimuthal velocity component in the Galactic plane (that is, $R$ and $R \cdot d\Phi/dt$ in cylindrical coordinates). We denote them as $(r_i, v_i)$ pairs.

To obtain a rotation curve from a set $\mathcal{S} = \{(r_i, v_i)\}_{i=1}^n$, we start with a distribution function $\tilde{\rho}(r, v) = N^{-1}_S \sum_{i=1}^n \exp\left(-\frac{(r-r_i)^2}{2\Delta r^2} - \frac{(v-v_i)^2}{2\Delta v^2}\right)$. $\tilde{\rho}$ is a $S$-dependent normalization constant, and $\Delta r_i$ and $\Delta v_i$ the measurement uncertainties. To simplify calculations without noticeably changing the results, the integration regions were formally extended to $\pm \infty$ in both $R$ and $V$ variables, because the summands in $\tilde{\rho}$ are rapidly decreasing outside a convex region encompassing entire $\mathcal{S}$.

Next, we form a related smoothed-out distribution function $p$ by integrating $\tilde{\rho}$ within intervals $(R-w/2, R+w/2)$:

$$p(R, V) = N^{-1}_S \sum_{i=1}^n \exp\left(-\frac{(V-v_i)^2}{2\Delta v^2}\right) f_{w,i}(R) = \frac{1}{2w} \left( \text{erf} \left( \frac{R-r_i+w/2}{\sqrt{2}\Delta r} \right) - \text{erf} \left( \frac{R-r_i-w/2}{\sqrt{2}\Delta r} \right) \right)$$

are integrable to 1 on the interval $R \in (-\infty, +\infty)$. Then, corresponding to $p(R, V)$, the conditional expectation value of $V$ denoted by $E_V(R)$ and its dispersion measure $S_V(R)$ read (the summation is taken over all tracers):

$$E_V(R) = \frac{\sum_{i=1}^n v_i f_{w,i}(R)}{\sum_{i=1}^n f_{w,i}(R)},$$

$$S_V(R) = \sqrt{\frac{\sum_{i=1}^n ((E_V(R) - v_i)^2 + (\Delta v_i)^2) f_{w,i}(R)}{\sum_{i=1}^n f_{w,i}(R)}}.$$

Note that the $S_V(R)$ involves both the uncertainties $\Delta v_i$ and $\Delta r_i$.

We chose the window width $w \approx 1.5$ kpc for so-defined moving averages $E_V$ and $S_V$. The moving average lines were then smoothed-out using a smoothing spline interpolation method. The obtained gaseous and stellar rotation curves are plotted in Figure 1, together with their respective conditional density distributions.

For the purpose of this paper, the stellar and gaseous rotation curves were made to coincide at the point $(R_\odot, V_\odot)$, even though fractions of Galactic material in the neighborhood of the Local Standard of Rest perform a small relative motion. But, if we are to compute the magnetic field necessary to obtain a given difference between two rotation curves, first we have to exclude any contribution to the actual difference, which is known to be of nonmagnetic origin. The latter will certainly be dominating, because the density of gas in the vicinity of the circular orbit $R = R_\odot$ is too high for the magnetic field to be dynamically important, compared with the gravitational interaction.

Now, we come back to the observational fact reported by Pont et al. (1997), that the stellar rotation curve is markedly lower in the outer disc region than the rotation curve of gas. This is also true for the rotation curves in Figure 1, which shows that the separation effect is not changed by considering an extended sample of stars and by applying an independent averaging method. The south–north asymmetry in the rotation of gas has appeared too. In addition, a south–north asymmetry in the motion of stars is visible, but it is lower than that for gas and has variable sign—higher velocities closer to $R_\odot$ are observed for stars on the southern side, then, for greater $R$, on the northern side.

The dispersion measure $S_V(R)$ of observed azimuthal velocities of kinematical tracers about the mean value $E_V(R)$, defined earlier, and shown in Figure 1 separately for stars and for gas, is comparable with the separation of the stellar and gaseous rotation curves. It is true that the separation may be

---

5 Some of the data had to be recalculated from other Galactic constants; e.g., $R_\odot = 7.6$ kpc used in (Battinelli et al. 2013).
entirely due to measurement inaccuracies, and Pont et al. (1997) discuss a possible explanation, but the separation is also markedly large. The dilemma can be solved only by increasing the accuracy of measurements. Our hypothesis is that the whole effect is entirely due to measurement inaccuracies, and Pont et al. (1997) discuss a possible explanation, but the separation is also markedly large. The dilemma can be solved only by increasing the accuracy of measurements. Our hypothesis is that the whole effect is due to the presence of large-scale magnetic fields. Then, important is the real amount of the separation, although it might slightly differ from that seen in Figure 1.

The lack of complete information about the transversal motions for a number of heliocentric position-velocity data, might be consequential for the shape of the obtained stellar rotation curve and in a way sufficient to account for the observed difference in the rotation of stars and gas. Before proceeding further, this possibility should be test. A fraction of cepheids in the sample used to prepare the stellar rotation curve have all their three velocity components measured. We obtained for them two test rotation curves: one based on all three heliocentric velocity components, and the other based only on the radial heliocentric component (as if the transverse heliocentric components were unknown). The result is shown in Figure 2. There is a difference seen for the test curves, but it is very small in comparison to the separation between the stellar and gaseous rotation curves.

3. GAS DENSITY

Magnetic field cannot directly influence the motion of neutral gas. The ionized gas fraction of the interstellar medium is more diluted than the non-ionized fraction. But, as explained in the introduction, it is dense enough for the magnetic field to be frozen-in and to hold together the mixture of gases (including molecular gas). The gas then moves as a whole driven by magnetic tension. Its column mass density $\sigma(R)$ can be thus reliably approximated by that of the neutral hydrogen. The latter we adopt from (Nakanishi & Sofue 2016) and, reproject to a volume density of the form:

$$\varrho(R, Z) = \frac{\varrho_o(R)}{\cos h^2\left(\frac{1}{2}(Z/h)\right)}, \quad \varrho_o(R) = \frac{\sigma(R)}{4h}. \quad (1)$$

For $|Z| \gg h$, the profile function $\cos h^2\left(\frac{1}{2}(Z/h)\right)$ behaves as $\exp(-|Z|/h)$, however $\varrho(R, Z)$ is smoother and half as high in

the Galactic plane vicinity as the cuspy exponential profile $\frac{\sigma(R)}{2h} \exp(-|Z|/h)$ with the same integrated mass. The cos $h^2$ profile is a solution of the Jeans equation, under the assumption of negligible horizontal variations in the density, and almost uniform gravitational acceleration. The disk thickness is related to a width-scale parameter $h$, which is regarded as a single free parameter, assumed to be independent of $R$ (a region $-h < z < h$ comprises $\sim 46\%$ of total mass).

The above volume density model with constant thickness is only a coarse-grained approximation used to map a given column density $\sigma(R)$ to a central volume density $\varrho_o(R)$. Real disk thickness grows quickly in an exponential fashion with the Galactocentric distance (Kalberla et al. 2007). Moreover, the gas disk warps (May et al. 1997) with asymmetric amplitude of a few kpc (Levine et al. 2006; Nakanishi & Sofue 2006). Thus, it is not a priori obvious how the single parameter $h$ of the model relates to the real variable thickness of the warped disk. Moreover, our simplified magnetohydrodynamical model (which we discuss in more detail later) neglects the vertical structure. Thus, the decrease in the density with $|Z|$ occurring when $h$ is very low must be accounted for. The H II regions, which are used to determine the Galactic rotation curve and whose motion is influenced by magnetic fields, are located within a strip $|Z| < 450$ pc in which we assume that the fields and volume density are independent of $Z$. Now, for $h$ large enough, say $h = 1.5$ kpc or more, the volume density would change insignificantly within the strip and could be assumed constant (equal to the central volume density). However, for small $h$, say $h = 450$ pc or less, the decrease of the density could not be neglected (the easiest way would be to take in place of the central density some reduced, say, average density within the strip).

In Figure 3, the central volume density (at $Z = 0$) corresponding to Equation (1), is shown for various $h$. A central volume density from (Kalberla & Kerp 2009) is also shown in the same figure. As can be seen, it is much higher than considered by us. We stress that surface density in (Nakanishi & Sofue 2016) (based on which we obtain our volume densities) and surface density in (Kalberla & Kerp 2009) are very similar. Therefore, the difference in the volume densities results from differences in the assumptions made about the shape and thickness of the vertical profile of gas. Kalberla & Kerp (2009) predict much lower half width at half maximum scale height than ours: in a region between $R_o$ and $2R_o$, it grows.
from $\approx 150$ to $\approx 360$ pc, see Figure 6 in Kalberla & Kerp (2009).

4. MAGNETIC FIELDS

A magnetic field, necessary to account for the measured difference in the circular velocities, can be obtained by finding a solution of the stationary Navier–Stokes equation for an inviscid and pressureless medium:

$$ (\nu \circ \nabla) \nu = - \nabla U + \frac{1}{4\pi \rho} (\nabla \times B) \times B. $$

The field $B$ can be decomposed into a series of modes, lower modes for larger scale structures and higher modes for lower scale structures; each mode with its own amplitude, possibly some of them dominating all the others. The dominant part of the large-scale magnetic field of the Galaxy is likely to be axisymmetric, according to the turbulent dynamo theory and in agreement with observations (Vallee 1991). Under axial symmetry and in the Galactic plane vicinity (where the vertical component $B_z$ and its first derivative can be assumed to vanish by reflection symmetry with respect to that plane), the system of Equation (2) reduces to

$$ V_R \frac{\partial V_R}{\partial R} = \frac{\delta v_\phi^2}{R} - \frac{1}{4\pi \rho} \frac{B_\phi^2}{R} - \frac{1}{4\pi \rho} B_\phi \frac{\partial B_\phi}{\partial R}, $$

$$ V_R \frac{\partial (R V_\phi)}{\partial R} = \frac{1}{4\pi \rho} B_\phi - \frac{\partial (R B_\phi)}{\partial R}. $$

Here, $\delta v_\phi^2 = V_{\phi G}^2 - V_{\phi IS}^2$ stands for a given difference of squares of circular velocities $V_{\phi G}$ and $V_{\phi IS}$, that of gas and stars, respectively, which is assumed to be of nongravitational origin and which, for the time being, we attribute to the interaction with magnetic field. We will see to what magnetic field magnitudes this hypothesis would lead to.

As something of an aside, let us get some idea about the role of Alfvén speed $V_A = \sqrt{B \times B / 4\pi \rho}$ in our context. To simplify things, it is worth considering purely circular orbits when $\partial_R B_\phi = 0$, in which case Equation 3(a) implies

$$ \delta v_\phi^2 = \frac{B_\phi^2}{4\pi \rho}. $$

Then, the correction to $V_{\phi G}^2$ would be numerically equal to the Alfvén speed squared (if $B_\phi = 0$) or to a fraction of it (if $B(R) \neq 0$). In a more interesting case with circular orbits, when $\partial_R B_\phi = 0$, there is an additional term proportional to the gradient $\partial_R B_\phi$ in Equation 3(a), which can increase (if $\partial_R B_\phi > 0$) or reduce (if $\partial_R B_\phi < 0$) the contribution from the Alfvén term to $\delta v_\phi^2$, or even change the sign of $\delta v_\phi^2$, reducing the circular velocity to values lower than the $V_{\phi G}$ value valid in the absence of magnetic field. For noncircular orbits, when $V_R \neq 0$, the situation becomes more complicated.

The idealizations above, do not strictly reflect the reality and have some drawbacks. By the assumed symmetries, $B_\phi(0) = 0$ and $\partial_\rho B_\phi(0) = 0$, and so $B_\phi$ could be neglected in the Galactic plane vicinity. Then, the law $\nabla \circ B = 0$ would imply, for an axisymmetric field, a superposition of: a purely azimuthal field being an arbitrary function of $R$, and a radial field of the form $B_R = \text{const} \cdot R^{-1}$—a limitation, which seems not very realistic. However, we may allow for a small violation of the law $\nabla \circ B = 0$, because $B$ in Equation (3) does not strictly describe the total magnetic field. Furthermore, axial symmetry, although very convenient, holds only approximately; the large-scale field will neither be purely axisymmetric, nor reflection-symmetric, and the turbulent part will be devoid of all the symmetries, etc. But there will always be some correction to the simplified field, which restores the divergenceless of the exact total field. The purpose of our model is only to approximately determine the leading structure of the total horizontal magnetic field, and estimate its order of magnitude, consistently with the observed difference of rotation, without looking into the more sophisticated problem of modeling this structure in detail. To sum up, the assumption of axial symmetry and of vanishing $B_z$, is a simplification, which allows us to solve equations. We neglect the limitation $B_\phi \sim R^{-1}$ implied by the constraint $\nabla \circ B = 0$—although consistent with the earlier simplification, the condition would be too stiff. We consider it more realistic an approximate field, which is not quite divergence free, remembering that the violation is a result of necessary simplifying assumptions (the contribution to $\nabla \circ B$ introduced by $B_\phi$ could be balanced by a higher-order correction or, even more simply, by an antisymmetric $B_\phi$ which is zero at the symmetry plane and, therefore, not contributing to the radial Lorentz force).

For the reasons described above, we may assume a simple structure of $B$ in the disc plane vicinity admissible by axial and reflection symmetry:

$$ \{ B_R, B_\theta, B_Z \} = B(R) \{ \sin \varepsilon, \eta \cos \varepsilon, 0 \}, $$

$$ \eta = \pm 1, |\varepsilon| < \frac{\pi}{2}. $$

In this ansatz, $B(R)$ is assumed to be a positive function, then various directions of $B$ are realized by means of parameters $\varepsilon$ and $\eta$. Here, $\varepsilon$ may be considered as a small perturbation parameter. The law $\nabla \circ B = 0$ is violated by a term $\sin \varepsilon R^{-1} \partial_R (R B(R))$, which can be neglected when $\varepsilon$ is small enough or/and when $B(R)$ is close to a function const $\cdot R^{-1}$.

The form of Equation (2) involves products of components of $B$. Hence, the residual rotation $\delta V_\phi^2$ is insensitive to any change in the direction of $B$; similarly, the reduced (3) shows that $\delta V_\phi^2$ will be preserved when the sign of $B_\phi$ is altered. For $\varepsilon = 0$ there is no radial component: $B_R = 0$ and $V_R = 0$, and the solutions are purely azimuthal. This mimics the leading large-scale azimuthal magnetic field accounting for the $\delta V_\phi^2$. For greater $\varepsilon$, but still small, we obtain a perturbation of the previous solution—except for the leading azimuthal field $B_\phi$, a small radial field $B_R$ appears, which gives rise to a perturbation of the radial velocity component. This mimics a radial velocity dispersion one may expect to appear in the presence of turbulent $B_R$ when the sign of $\varepsilon$ is altered (this is equivalent to a composition of two reflections: $B \rightarrow -B$ followed by $B_\phi \rightarrow -B_\phi$), then $V_R$ also changes sign, while the leading component $B_\phi$ is still the same. Note also that the magnitudes of turbulent fields in the Galaxy may be comparable with or even exceeding the magnitudes of large-scale fields. The ansatz Equation (4) may be thus regarded as taking into account both the large-scale purely azimuthal field with variable sign, as well as a turbulent field—indeed perturbations to both components $B_\phi$ and $B_R$.

Above, we have sketched our motivation behind the ansatz Equation (4) and its possible interpretation. We also draw to attention the fact that the ansatz is a special case of a field
configuration

\[ B(R) = \begin{cases} \sin(p) \cos(\chi(Z)), & \cos(p) \cos(\chi(Z)), \sin(\chi(Z)) \end{cases} \]

considered in the context of describing Galactic magnetic fields (Ruiz-Granados et al. 2010a) (with \( \sin p \neq 0 \) the latter would be divergence free for \( B(R) = B_r = \frac{\dot{R}}{\sin p} \exp\left(-\frac{kR}{\sin p}\right) \) and \( \chi(Z) = kZ + \chi_0 \)). In particular, the result in (Ruiz-Granados et al. 2012) shows that Equation (4) describes well an axisymmetric spiral pattern of 3 \( \mu G \), observed in the regular disk field of the Galaxy between 3 and 20 kpc (then \( p \approx 15^\circ \)).

Solutions. In the region where \( \partial_R (RV_\|) = 0 \), the second of the equations in Equation (3) can be solved for \( V_R \) and then substituted in the first equation. This gives a nonlinear second-order ordinary differential equation for \( B(R) \) with coefficients being functions of known functions \( \varphi(R), V_{\|0} \), and their derivatives. We solved this equation using numerical integration. After Ruiz-Granados et al. (2012), we put \( \varepsilon \approx \frac{\pi}{12} \).

The resulting magnitude of magnetic field, \( B(R) \), accounting for the observed difference between the rotation of gas and stars, is shown versus the Galactocentric distance in the Galactic plane in Figure 4, for three values of parameter \( h \). It follows that \( B(R) \) depends on the disk thickness, and hence is related to the density of gas in the disk plane. For the considered range of \( h \) values between 1.5 and 6 kpc, the required \( B(R) \) fall within a range between 8.5 and 13 \( \mu G \). For a volume density, as published in (Kalberla & Kerp 2009), the corresponding \( B(R) \) is higher, between \( \approx 11.8 \) and \( \approx 15.8 \) \( \mu G \). For comparison, in Figure 5 similar results are shown, accounting for the change between the rotation of stars and gas, obtained when the rotation curve of gas is split into two parts: one for the gas above the Galactic plane, and the other for the gas below that plane.

In Figure 6, we compare (in units of speed) the contributions of magnetic terms present in the azimuthal part of Equation (3). The square root of Alfvén term \( B_\theta^2/4\pi \rho \) (comparable with the Alfvén speed \( \sqrt{B \cdot B}/4\pi \rho \) for the particular solution), grows with radius, reaching values comparable with the circular velocity of gas. The square root of the absolute value of the gradient term \( B_\theta \partial_h B_\theta/4\pi \rho \), reaches values roughly twice lower, and depending on the sign, it increases or decreases the effect from the Alfvén term.

The radial velocity component \( V_R \) can be obtained for the found solutions from Equation 3(b). In Figure 7, the velocity is shown for a particular solution with \( h = 2 \) kpc. However, the velocity is almost the same for other \( h \), and the difference is noticeable only in the boundary region close to \( R = 9 \) kpc. As we have seen earlier, the \( V_R \) component can be interpreted in our model as a measure of the radial velocity dispersion due to the interaction with magnetic field that flops its radial direction, in which case the sign \( V_R \) changes following these flops. The \( V_R \) is of similar order of magnitude as the radial velocity dispersion observed for spiral galaxies, see Tamburro et al. (2009), which suggests that magnetic fields may be important in modeling the dispersion of gas. One can also use the \( V_R \) component to estimate the accretion rate connected with the...
radial flow of gas. The accretion rate is defined in our case as the amount of matter flowing through a cylindrical surface of radius $R$. Accordingly, using the density profile Equation (1), it can be estimated from $2\pi R \Sigma(R) V_R(R)$ or $8\pi R h \rho'(R,0) V_R(R)$. The resulting accretion rate for the same solution as above is shown in Figure 8. It will similarly be almost independent of $h$.

5. CONCLUSIONS

The difference between the stellar and gaseous rotation curves observed in the outer Galactic disk (Pont et al. 1997) is significant. Attributing it entirely to a magnetic interaction would require fields exceeding 10 $\mu$G, strongly depending on the gas density in the Galactic disk. The question arises, whether fields of this magnitude could be present in the Galaxy. According to the interpretation sketched before, the ansatz Equation (4) for the magnetic field we assumed for solutions of the Navier–Stokes equations, takes into account the influence of the total magnetic field, including both the regular and turbulent magnetic components. It is important to note that the magnitude of the turbulent component in the Galaxy is not negligible, and may exceed the magnitude of the regular component. The regular field observed in the Galaxy is estimated to have a value of 5 $\mu$G, while the turbulent field has 11 $\mu$G (Elstner et al. 2014). Furthermore, Elstner et al. (2014) estimate, based on their models, that each of the components $B_R$, $B_\theta$, and $B_Z$ of the total field may reach local values as high as 15 $\mu$G or more for lower radii, falling off with the radial distance, and exceeding 10 $\mu$G, even at the radius of the solar orbit. Our results seem consistent with these findings.

For lower densities (higher scale parameters $h$), the magnetic field, as can be seen in Figure 4, is a decreasing function of radius, while for higher densities (corresponding to $h = 2$ kpc, 1.5 kpc or lower), the field increases with radius or is approximately constant. Most often, the magnetic field is expected to decrease exponentially (Elstner et al. 2014), which may seem to disagree with our results at higher densities. However, it should be remembered that the real density of H II regions, used to determine the gas rotation curve, may be lower than the density obtained for H I distribution. This is because ionized gas, such as in H II regions, should have lower density than the neutral hydrogen H I. If this was the case, the densities corresponding to higher $h$ (above 2 kpc) would better approximate the density of the H II regions, which are influenced by magnetic fields. Then, the magnetic field found by us would have values and behavior consistent with the usual assumptions.

Interestingly, we observed that Alfvén velocity reaches values comparable with the circular velocity of gas. Alfvén velocities of more than 100 km s$^{-1}$ are about 10 times larger than the mid-plane turbulent velocity of 10 km s$^{-1}$. This means a 100 times higher magnetic energy compared to the kinetic energy of the turbulence.

The volume gas density, important for the magnetic field determination, is strongly dependent on the assumed vertical profile of the gas layer. Fortunately, this uncertainty seems not so essential to the modeled magnetic field magnitudes required by the observed separation of stellar and gaseous rotation curves. This is so, because a higher volume density in the Galactic plane means a lower postulated scale height $h$, which in turn requires taking into account the altitudes of the H II regions used to determine the rotation curve of gas. The ionized hydrogen of H II regions is much more diluted than the neutral hydrogen of H I regions. Both kinds of gas, H I and H II, may belong to the same gas cloud, which must move as a whole. It is, therefore, safer (in order not to reduce the magnetic field too much) to determine the (ionized) gas density such as if the gas consisted of H I hydrogen only. This approach overestimates the density of the ionized fraction.

As can be seen in Figure 5, the difference between circular velocities of gas in the northern and southern sides of the Galactic disk, if attributed to magnetic forces, requires a small difference in the magnetic field magnitudes, not greater than 1.3 $\mu$G. It is thus plausible that the difference in rotation may indeed be due to some asymmetry in the distribution of magnetic field.

To sum up, as the above magnetic field estimates show, it is plausible that most of the observed difference in the circular velocities of the stellar and gaseous fractions in the Galaxy may be caused by the presence of interstellar magnetic fields, frozen in the partially ionized gas, and holding the gaseous mass component together by the resulting magnetic tension. Within the possible range of higher densities corresponding to $h$ about 1–2 kpc or less, the magnetic field needs to increase with the distance (it is bound within 12 and 16 $\mu$G) or at least to be approximately constant between 10 and 20 kpc. Only for very
low gas density at the Galactic plane, lower than \(10^{-25} \text{ g cm}^{-3}\), does the magnetic field decrease with the Galactocentric distance, as observations suggest; the difference in rotation could then be explained by magnetic fields of order \(10 \mu \text{G}\). If the real fields are different, then only a fraction of the rotation difference could be explained by the presence of magnetic fields. In any case, the difference in the rotation could be used as a means to estimate the intensity of the total magnetic field in spiral galaxies. This is also a manifestation of the influence of magnetic fields on the dynamics of spiral galaxies. Apart from the influence on the rotation, the radial component of magnetic field will modify the radial velocity dispersion.

We would like to thank the referee for carefully reading the manuscript and for various constructive suggestions.

REFERENCES

Battaner, E., & Florido, E. 2007, AN, 328, 92
Battaner, E., Florido, E., Guijarro, A., Rubiño-Martín, J. A., & Ruiz-Granados, A. Z. 2008, LNEA, 3, 83
Battaner, E., Garrido, J. L., Membrado, M., & Florido, E. 1992, Natur, 360, 652
Battinelli, P., Demers, S., Rossi, C., & Gigoyan, K. S. 2013, Ap, 56, 68
Berdnikov, L. N., Dambis, A. K., & Vozyakova, O. V. 2000, A&AS, 143, 211
Blitz, L., Fich, M., & Stark, A. A. 1982, ApJS, 49, 183
Brand, J., & Blitz, L. 1993, A&A, 275, 67
Caldwell, J. A. R., & Coulson, I. M. 1987, AJ, 93, 1090
Chini, R., & Wink, J. E. 1984, A&A, 139, L5
Clemens, D. P. 1985, ApJ, 295, 422
Demers, S., & Battinelli, P. 2007, A&A, 473, 143
Elstner, D., Beck, R., & Gressel, O. 2014, A&A, 568, A104
Fich, M., Blitz, L., & Stark, A. A. 1989, ApJ, 342, 272
Jalocha, J., Bratek, L., Pękala, J., & Kutschera, M. 2012a, MNRAS, 427, 393
Jalocha, J., Bratek, L., Pękala, J., & Kutschera, M. 2012b, MNRAS, 421, 1555
Kalberla, P. M. W., Dedes, L., Kerp, J., & Haud, U. 2007, A&A, 469, 511
Kalberla, P. M. W., & Kerp, J. 2009, ARA&A, 47, 27
Kutschera, M., & Jalocha, J. 2004, AcPPB, 35, 2493
Levine, E. S., Blitz, L., & Heiles, C. 2006, ApJ, 643, 881
May, J., Alvarez, H., & Bronfman, L. 1997, A&A, 327, 325
Mešník, A. M., Rautiainen, P., Berdnikov, L. N., Dambis, A. K., & Rastorguev, A. S. 2015, AN, 336, 70
Nakanishi, H., & Sofue, Y. 2006, PASJ, 58, 847
Nakanishi, H., & Sofue, Y. 2016, PASJ, 68, 5
Pont, F., Mayor, M., & Burki, G. 1994, A&A, 285, 415
Pont, F., Queloz, D., Bratschi, P., & Mayor, M. 1997, A&A, 318, 416
Ruiz-Granados, B., Battaner, E., Calvo, J., Florido, E., & Rubiño-Martín, J. A. 2012, ApJL, 755, L23
Ruiz-Granados, B., Rubiño-Martín, J. A., & Battaner, E. 2010a, A&A, 522, A73
Ruiz-Granados, B., Rubiño-Martín, J. A., Florido, E., & Battaner, E. 2010b, ApJL, 723, L44
Sánchez-Sáezed, F. J., & Santilán, A. 2013, MNRAS, 433, 2172
Tamurru, D., Rix, H.-W., Leroy, A. K., et al. 2009, AJ, 137, 4424
Vallee, J. P. 1991, ApJ, 366, 450