A New Approach for Low-resolution Image Registration Based on Weighted MI and Powell

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Abstract: Mutual information (MI) has been successfully applied to the registration of medical imagery and SR reconstruction because of its robustness. However, it also has obvious drawbacks that it is susceptible to local minima. For this purpose, an image registration scheme based on weighted mutual information is proposed in this paper. By introducing a reasonable weight coefficient, the gray-levels can exert greater impact on computation of entropy. And then, the weighted mutual information (WMI) which is designed as a new similarity measure is exploited by the Powell optimizer to refine the preregistration results. The experimental results suggest that the proposed algorithm has strong robustness and adaptive capacity for noise, and is convergent and efficient in resolving the given test problems.

Keywords: image registration; low-resolution; weighted MI; Powell optimizer

1. Introduction

Image registration is a vital process which determines the most precise match between two images of the same scene. Many feature-based[1-4] and intensity-based[5-6] image registration methods have been proposed. Mikolajczyk et al. [7] pointed that scale-invariant feature transform (SIFT) has good performances in scale change, rotation, and image blur. Three years later, Herbert Bay[8] presented SURF which has lower computation time than SIFT. Despite the attractive advantages of SURF, there exist some problems when it is directly applied to the LR images, e.g. feature vector is not unique, similarity measurement accuracy is not high enough, and the discrimination between the local features is small. The emergence of MI, which represents a measure of statistical dependence between two images, provides a new research direction for the study of the image registration. It has been demonstrated by Cole-Rhodes[9] that MI is robust to noise and nonlinear intensity relationship between images. It has been successfully applied to the registration of medical imagery [10-11], and SR reconstruction[12-13]. However, what should not be ignored is the mutual information alone is susceptible to local minima. It cannot be employed in SR problems as the Gaussian noise may lead to large mis-registrations.

Considering the high accuracy of MI and the influence of noise on it, a weighted MI-based registration scheme is proposed in this paper. By introducing a reasonable weight coefficient, the gray-levels, which play a significant role to improve the entropy, can exert greater impact on computation of entropy. So as to make the most promising registration area obtains more information. Finally, the Powell algorithm is exploited as an optimization strategy to achieve the most precise registration results, by searching rigid body transformation parameters when the mutual information calculated in the weighted MI reaches the maximum value.
2. Review of the registration model

Given a pair of 2-D gray-level image between which there exist some geometric and radiometric differences, let \( f_a(x, y) \) and \( f_s(x, y) \) represent the reference and sensed image, respectively, where coordinates \( (x, y) \in \Delta \subset \mathbb{R}^2 \) and \( \Delta \) is a region of interest. To register these two images is to find the optimal geometric transformation \( T_{\mu}(\cdot) \) by which \( f_s(T_{\mu}(x, y)) \) best matches \( f_a(x, y) \) for all \( (x, y) \), where \( \mu \) is a set of transformation parameters.

As being widely used in the image registration, the rigid body transformation model can be written as

\[
T_{\mu}(x, y) = \begin{bmatrix}
\cos \theta & -\sin \theta & \delta_x \\
\sin \theta & \cos \theta & \delta_y \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
1
\end{bmatrix}
\]  

(1)

Where the transformation origin is considered to be the upper left corner of the reference image; \( \theta \) represents the rotation angle and \( (\delta_x, \delta_y) \) are the shifts between the two images.

The main purpose of image registration is to find the optimal geometric transformation, which maximize (or minimize) some similarity metric between the two images. Therefore, the problem of image registration is mapped as a problem of searching the optimal transformation parameters \( \mu^* \), which can be expressed as:

\[
\mu^* = \text{arg} \, \text{opt}(M(f_a(x, y), T_{\mu}(f_s(x, y))))
\]  

(2)

Where \( T_{\mu} \) is the transformation model; \( \mu \) is the transformation parameter and \( M \) represents the similarity metric.

3. Image registration framework based on a weighted MI

3.1. The standard MI

As a basic concept of information theory, mutual information (MI) represents a measure of statistical dependence between two random variables, or represents the amount of information one variable containing another. For random variables A and B, the mutual information can be defined as equation (3). Where, \( P_a(a) \) and \( P_s(b) \) represent marginal probability distribution; \( P_{a,b}(a, b) \) represents joint probability distribution.

\[
MI(A, B) = \sum_{a,b} P_{a,b}(a, b) \log \frac{P_{a,b}(a, b)}{P_a(a)P_b(b)}
\]

(3)

In statistics, if the random variables A and B are independent of each other, \( P_{a,b}(a, b) = P_a(a) \cdot P_b(b) \) and \( MI(A, B) = 0 \); if they depend on each other completely, \( P_{a,b}(a, b) = P_a(a) = P_b(b) \) and \( MI(A, B) \) reaches the maximum value. Based on this theory, MI, as a similarity measure, is introduced into the field of image registration.

3.2. A weighted MI

According to the equation (3), the mutual information of the reference image \( f_a(x, y) \) and the sensed image \( f_s(x, y) \), can be defined as equation (4):

\[
MI(f_a, f_s) = \sum_{i=0}^{L-1} \sum_{j=0}^{L-1} P_{a,b}(i, j) \log \frac{P_{a,b}(i, j)}{P_a(i)P_b(j)}
\]

(4)

Where, \( L \) \( (L = 256) \) is the gray-level contained by \( f_a(x, y) \) and \( f_s(x, y) \); \( P_{a,b}(i, j) \) is their joint
probability distribution; \( P_s(i) \) and \( P_s(j) \) are their marginal probability distribution. And they are defined as :

\[
P_{s,s}(i, j) = \frac{h(i, j)}{\sum_{i=0}^{L-1} \sum_{j=0}^{L-1} h(i, j)} \tag{5}
\]

\[
P_s(i) = \sum_{j=0}^{L-1} P_{s,s}(i, j) \tag{6}
\]

\[
P_s(j) = \sum_{i=0}^{L-1} P_{s,s}(i, j) \tag{7}
\]

\( h(i, j) \) is the joint histogram of \( f(x, y) \) and \( f_s(x, y) \), and it is a 2-D metric as follows:

\[
h = \begin{bmatrix}
h(0,0) & h(0,1) & \ldots & h(0,L-1) \\
\vdots & \vdots & \ddots & \vdots \\
h(L-1,0) & \ldots & \ldots & h(L-1,L-1)
\end{bmatrix} \tag{8}
\]

It’s obvious that the correlation between \( f(x, y) \) and \( f_s(x, y) \) can be well reflected by the MI-based registration. However, this method is also insufficient in some aspects: 1) it is easy to appear mismatching when the noise of the image is larger, the correlation between the images is weaker or large deformation in the image occurs; 2) the convergence precision is damaged due to MI. Therefore, a weighted MI-based registration scheme is designed in this section to overcome this problem. By introducing a reasonable weight coefficient, the gray-levels, which play a significant role to improve the entropy, can exert greater impact on computation of entropy. So as to make the most promising registration area obtains more information.

The entropy of an image \( I(x, y) \) after introducing the weight coefficient can be defined as:

\[
H(I) = -\sum_{a=0}^{a\in\omega(a)} \omega(a) \times P(a) \times \log(P(a)) \tag{9}
\]

Where, \( \omega(a) \) represents the extent of the impact on computation of entropy of the pixel with \( a \) gray-level. In this definition, \( \omega(a) \) also should satisfy the following constraints:

\[
\sum \omega(a) = 1 \quad (0 < \omega(a) < 1) \tag{10}
\]

In order to meet the needs of two aspects, i.e. calculate conveniently and can effectively highlight the difference between extreme values, the proportion of each gray-level in the total number of pixels is employed to define the weighted coefficient of the mutual information: \( \omega(a) = P(a) \). Under this condition, a normalized measure of mutual information can be rewritten as:

\[
NMI_w(f_k, f_s) = \frac{H_u(f_k) + H_u(f_s)}{H_u(f_k) + H_u(f_s)} \tag{11}
\]

\[
H_u(f_k) = -\sum_{a=0}^{a\in f_k} P^2(a) \times \log(P(a)) \tag{12}
\]

\[
H_u(f_s) = -\sum_{b=0}^{b\in f_s} P^2(b) \times \log(P(b)) \tag{13}
\]

\[
H_u(f_k, f_s) = -\sum_{a=0}^{a\in f_k} \sum_{b=0}^{b\in f_s} P(a, b) \times \log(P(a, b)) \tag{14}
\]

The novel similarity measure \( NMI_w(f_k, f_s) \) is adopted as the objective function in the fine-tuning registration. It corresponds to registration of the images when maximization of their \( NMI_w(f_k, f_s) \) is achieved. Therefore, the process of registration is mapped as a problem of searching the optimal transformation parameters \( \mu' \), and the equation(2) is revised as follows:

\[
\mu' = \arg \max NMI_w(f_k(x, y), T'_s(f_s(x, y))) \tag{15}
\]
4. Optimal search strategy based on the Powell
Powell’s optimization method is a reasonably fast yet generally successful optimization method. Given an n-dimensional search space, Powell’s method performs one-dimensional optimizations for each of the n dimensions in turn. For each one-dimensional optimization, the position of the optimum found in one direction is the starting point for the optimization in the next direction. In one iteration, the registration function will be optimized once in all n dimensions. The method ceases iterating when the difference in the registration measure at the beginning and the end of each iteration falls below a user-defined tolerance. At the end of each iteration, the average direction of movement is determined by calculating the direction vector between the starting point and the end point of the iteration. This new direction will replace the direction in which the largest increase in function value was found. This direction is chosen because it is most similar to the new direction and this choice prevents convergence of the directions. In some cases the new direction is rejected, for example when an additional optimization in the new direction leads to a decrease in function value.

The one-dimensional optimizations are performed with Brent’s method. Beginning with a small interval around the starting position of optimization, this method changes the position and size of the interval until it incorporates a maximum. The position of the maximum is then determined within the interval found. Note that this method does not necessarily return the global optimum in a certain direction. The detailed pseudo-code of registration parameters optimization based on the Powell algorithm is presented as Table1 shown.

Table1 Registration parameters optimization based on the Powell algorithm

| 01 | Initialize the starting transform parameter vector \( \bar{\mu}_0 \), independent search direction vectors \( \bar{d}_i = \bar{e}_i \), \( i = 1, 2, 3 \), the tolerance for stop criteria \( \epsilon \), set \( \bar{\mu}_0 = \bar{\mu} \) and \( \bar{\mu} \) is the transform parameter vector obtained by the coarse registration, \( f_0 = f(\mu_0) \), \( k = 1 \n\)
| 02 | **While** (stopping criterion is not met, namely \( |\Delta f| > \epsilon \) ) **do**
| 03 | **for** \( i = 1: 3 \) **do**
| 04 | \( d_{\eta} = \arg \max (NMI_w) \), \( \mu_i = \mu_{i-1} + \eta d_i \), \( f_i = f(\mu_i) \), \( i = i + 1 \)
| 05 | **End for**
| 06 | \( d_\eta = \mu_\eta - \mu_0 \), \( \eta = \arg \max (NMI_w) \), \( \mu_\eta = \mu_{i-1} + \eta d_\eta \), \( f = f(\mu_\eta) \) \( \Delta \text{diff} = |f - f_1 - f_2 - f_3| \)
| 07 | temp = \( \text{sqrt}((f - f)/\max(\text{diff})) \) \( \Delta f = \| \mu_\eta - \mu_0 \| \) \( \mu_0 = \mu_\eta \) \( k = k + 1 \)
| 08 | **End While**

5. Experiments and analysis
In order to validate the performance of the proposed algorithm, a pair of outdoor car images with LR resolution 360×470 has been employed as the reference image and sensed image respectively (shown as in Fig.1(a) and Fig.1(b)). All the experiments were implemented in Matlab and run on a PC with Intel(R) Core(TM)2 2.33 Gz CPU, and 2 GB RAM.

5.1 Convergence and robustness of the weighted MI
The convergence speed and accuracy of a matching algorithm can be evaluated according to several properties, e.g. the sharpness of peak and smoothness, of the criterion function curve between the reference image and sensed image. The statistical curve of an outstanding criterion function for matching should be smooth. The sharper the curve waves, the easier the optimizing course is trapped the local extreme value and the poorer robustness the algorithm has.
The peak of curve which corresponds to the correct match in an algorithm should be sharp and stand out clearly. The extent of sharpness and standing out of the curve directly determines the convergence speed and accuracy of a matching algorithm. The clearer the peak of curve stands out from the other extreme values and the sharper the peak of curve is, the easier the optimizing course converges to the maximum of the criterion function accurately. Meanwhile, faster convergence speed and higher convergence accuracy are achieved simultaneously.

Therefore, two experiments are performed in this section to evaluate the weighted MI similarity measure (WMI) and the standard MI similarity measure (SMI).

**Experiment-A** the sensed image \( f(x, y) \) which contains no noise, as shown in Fig.1(b), is transformed by altering just one element of the transformation parameter vector while keeping other transformation parameters constant. For example, \( f(x, y) \) is moved in the horizontal direction on a scale of [-30 30] while the transformation parameters in the vertical and rotation direction retain the same. The criterion function curve of WMI and SMI in the horizontal direction has been shown as Fig 1(c) and Fig 1(d). In the same way that the criterion function curves of WMI and SMI in vertical direction on a scale of [-30 30] (Fig 1(e)-Fig 1(f)) and rotation direction on a scale of [-5 5] (Fig 1(g)-Fig 1(h)) have been performed.

![Figure 1](image1.jpg)

**Figure 1** The criterion function curve of WMI and SMI in three directions
(a) the reference image; (b) the sensed image; (c),(e), (g) are the criterion function curves of SMI in horizontal, vertical and rotation direction respectively; (d),(f) ,(h)are the criterion function curves of WMI in horizontal, vertical and rotation direction respectively

**Experiment-B** WMI and SMI as similarity measures to register the reference image and the sensed image \( f(x, y) \) which is polluted by Gaussian noise and shown in Fig.2 (b). With the same test method as that of the experiment-A, the criterion function curves in three directions have been drawn and shown in Fig.2(c)-(h).

![Figure 2](image2.jpg)
Figure 2 shows the criterion function curves of WMI and SMI in three directions. (c), (e), (g) are the criterion function curves of SMI in horizontal, vertical, and rotation direction, respectively; (d), (f), (h) are the criterion function curves of WMI in horizontal, vertical, and rotation direction, respectively.

Results in experiment A have shown that the criterion function curves of WMI have obtained higher matching degree as proved by the mutual information on vertical axis. Meanwhile, the WMI similarity measure is smoother and sharper than that of SMI, which indicates the WMI similarity measure is easier to accurately converge to excellent approximate results. This tendency shows more evidence in the situation that the sensed image is polluted by more noise, as the results of experiment B shown.

In Fig. 2 the criterion function curves of SMI fluctuate sharply, even having little or no access to convergence the maximum value in horizontal (Fig. 2(c)) and vertical direction (Fig. 2(e)). And yet this phenomenon is significantly improved in the proposed method. The criterion function curves of WMI are smoother and have more outstanding peak than that of SMI. It means it’s less likely trapped local extreme value in the search process for optimal transformation parameters. The results of comparing two groups of experimental data show that the method presented in this paper has stronger robustness.

5.2 Efficiency and accuracy of the Powell optimization strategy
The Powell optimization strategy is utilized to search the optimal transformation parameters with WMI as a similarity measure. In order to verify the efficiency and accuracy of this process, three simulation images \( S_i(x, y) \) \( (i = 1, 2, 3) \), as shown in Fig. 3(a-c), are obtained by transforming the reference image \( f(x, y) \) as known rigid transformation model \( P_i(i = 1, 2, 3) \) whose parameters have been shown in Table 2.

The accuracy of the presented method in this context was evaluated through visual inspection by generating the checkerboard mosaic images using the corresponding registration parameters between simulation images and the reference image, as shown in Fig. 3(d-f).

Table 2 Parameters of three known transformation models

| Model | Parameters |
|-------|------------|
| \( P_1 \) | \( \delta_x = 36 \) (pixel), \( \delta_y = -14 \) (pixel), theta = 5 (deg) |
| \( P_2 \) | \( \delta_x = 17 \) (pixel), \( \delta_y = -13 \) (pixel), theta = 3 (deg) |
| \( P_3 \) | \( \delta_x = -21 \) (pixel), \( \delta_y = 18 \) (pixel), theta = -3 (deg) |

The checkerboard mosaic image [Fig. 3 (a-c)] validates that the image features of the reference image and the simulated images such as the body part, other than several erroneously matched positions marked in red ovals, are precisely overlapped, which demonstrates the accuracy of our obtained registration parameters.
In this paper, an image registration algorithm based on WMI and the Powell’s optimization is proposed. Based on the standard MI, a weighted coefficient of the mutual information between the reference image and the sensed image is defined. A normalized novel similarity measure which is less sensitive to changes in overlap is adopted as the objective function. And then the Powell algorithm is exploited as an optimization strategy to refine the preregistration results to search the most precise registration results. In this process, the weighted mutual information reaches the maximum value, the optimal rigid body transformation parameters are obtained.

Results of experiments have demonstrated two aspects of the proposed method. First, it is convergent and effective because it is easier to accurately convergence to excellent approximate result and prevent optimizing course from trapping the local minimum. Moreover, it is robust and highly accurate, especially in the case of containing noise. However, the proposed method is not perfect. When the angles between LR images vary greatly, the rigid body transformation model is not suitable. Therefore, both the affine transformation model and thin-plate-spline are alternative models according to research objects in the future study.

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