Abstract—The performance and efficiency of internal combustion (IC) engines can be greatly improved by using a high-performance cooling system. This can be achieved by implementing robust control strategies and, also by building the cooling system with high-performance elements. The mechanical execution elements can be replaced with electrically controllable elements such as the pump and the thermostat valve. This will have a positive influence on the degree of controllability of the system. In order to develop high-performance control algorithms, it is necessary to have a model that best reflects the behaviors of the physical system. Thus, this paper presents a mathematical modeling approach for the cooling system using the principles of heat exchangers and the physical phenomena present in them.

Index Terms—Control system, engine cooling system, Heat exchangers.

I. INTRODUCTION

Reducing pollution causes the industry to build more and more efficient systems. In the case of vehicles, the cooling system is an important part of the optimal operation of the entire system. The cooling system has the role of dissipating the excessive heat created by the engine in the combustion process. The engine components used in the combustion process cannot withstand extraordinarily high temperatures due to their constructive properties. Running at high temperatures leads to the burning of the oil film, which ensures the lubrication of moving components. This can cause excessive wear or even engine seizure [1]. Even if some of the total energy of the fuel is lost to the cooling system, this is necessary for each component to perform well over time, increasing the reliability and efficiency of the system.

The heat transferred to the cooling system represents one third of the total heat produced by combustion, one third being transformed into mechanical work, and the remaining heat will be evacuated through exhaust [2]. The fact that the cooling system is responsible for handling a large part of the fuel’s energy shows that there is a great potential to increase the efficiency of the system [3].

The replacement of the classic components of the system (e.g. thermostat valve and water pump) and the addition of new functionalities leads to the increase of the degree of controllability, which opens the opportunity to implement high-performance control algorithms. Reducing the warm-up time of the engine at engine cold-start can be an important objective for the control algorithm. This will increase the operational efficiency of internal combustion engines [4]. Running the engine at the optimal temperature has the potential to reduce friction between moving parts due to good lubrication, improve carbon monoxide emissions and reduce fuel consumption by up to 10% for gasoline engines [3].

An accurate and complete mathematical model is key for implementing and testing the control algorithms.

The modeling of the cooling system has attracted many researchers both from academic and industrial areas. The validated model of a 4-cylinder gasoline engine is presented in [5]. Another researcher [6] proposes a model for the cooling system that is calibrated based on data from a Citroen C3 1.4L TDI.

Reference [3] presents a study on how the operation of the cooling system can improve the fuel consumption. To control the cooling system, the three-way valve is operated with a PD controller that uses an optimized parameter table. An improvement in fuel consumption of 1.5% for the urban mode, 1.42% for the extra-urban mode and 0.04% for the highway mode is obtained. In [7], the author chooses a complex control strategy using a robust model-based predictive control to reduce the warm-up time of the engine.

This paper focuses on the implementation of a mathematical model for a hypothetical cooling system in the form of a Simulink model. Chapter two describes the modeling of each component of the system. In chapter three is presented a simple PI control design and chapter four present the results.

II. MATHEMATICAL MODELING OF THE SYSTEM

A. System Overview

Traditional thermal management systems consist of radiator, radiator fan, water pump and thermostat valve. In newer systems the classic engine pump has been replaced with an electric pump. The belt-driven pump connected to the engine crankshaft produces a flow proportional to the engine speed. This method provides good thermal management in
engine loading conditions, but in certain scenarios the pump does not need to run, and the mechanical work produced by the engine is wasted [8]. An electric pump can provide energy improvement and enhances the controllability of the system through variable flow rates. The circuit structure used in this model is shown in the following figure.

![Heat Exchanger Diagram](image)

Fig. 2. The operating principle of heat exchangers.

The modeling of components that involve heat transfer in the heating system is implemented using the principle of heat exchangers. The primary agent with a higher temperature, passes through the heat exchanger and is cooled due to the heat transfer to its walls. The secondary agent, with a lower temperature, captures the temperature of the walls as it passes through the heat exchanger.

### B. Engine Heat Transfer

The engine is modeled on the thermodynamic principles of a heat exchanger. The outlet coolant temperature is computed based on the amount of fuel burned in the combustion process and the inlet coolant temperature.

The primary agent is represented by the heat generated in the combustion process (1). The secondary agent is the coolant that absorbs the heat from the engine jacket. Knowing the heat that is lost from the engine walls to the coolant (2), the heat flux that passes through the engine block walls is calculated using the heat balance equation (3). The evolution of the engine block temperature over time can also be determined (4).

The heat produced in the combustion process:

\[ Q_{\text{combustion}} = k \cdot m_{\text{fuel}} \cdot Q_{\text{HV}} \]  
(1)

\( m_{\text{fuel}} \) - Fuel flow rate (kg/s); \( Q_{\text{HV}} \) - Fuel heat value (kJ/kg); \( k \) - Coefficient of the heat transfer combustion chamber to engine block.

The heat transferred from the engine to the cooling fluid:

\[ Q_{\text{eng.to.water}} = k_{\text{water}} \cdot S \cdot \Delta T \]  
(2)

\( k_{\text{water}} \) - Convective heat transfer coefficient (W/m²·grad); \( S \) - Contact surface with the coolant (m²); \( \Delta T \) - The average temperature difference across the exchange surface (°C).

Thermal balance equation:

\[ Q_{\text{engine}} = Q_{\text{combustion}} - Q_{\text{eng.to.water}} \]  
(3)

The evolution of the engine temperature:

\[ \frac{dT_{\text{engine}}}{dt} = \frac{1}{m_{\text{eng}} \cdot c_{\text{eng}}} \cdot Q_{\text{engine}} \]  
(4)

### C. Heat Dissipated by the Radiator

The radiator is a heat exchanger in which the secondary agent is not a liquid but a gas. The model was designed by dividing the functionality of the radiator into three parts: water part, metal part and air part. The water part calculates the evolution of the water temperature that cools by passing through the radiator (9). The metal part determines the evolution of the radiator wall temperature (11), and the air part calculates the evolution of the air temperature that cools the radiator (13).

The coolant heat evolution in the radiator is given by the heat of the water entering and leaving the radiator due to the circulation of the coolant and the heat dissipated in the radiator’s walls (8).

\[ Q_{\text{water}} = Q_{\text{water.in}} - Q_{\text{water.out}} - Q_{\text{water.to.rad}} \]  
(8)

The evolution of the water temperature, where \( V_{\text{water}} \) represents the volume of water from the radiator:

\[ \frac{dT_{\text{water}}}{dt} = \frac{q_{\text{water}} (T_{\text{water.in}} - T_{\text{water.out}}) - k_{\text{water}} S_{\text{water}} \Delta T_{\text{water}}}{\rho V_{\text{water}} c_{\text{water}}} \]  
(9)

\( m_{\text{eng}} \) - Engine mass (kg); \( c_{\text{eng}} \) - Iron specific heat (J/Kg·deg)

The thermal equilibrium in (5) gives the heat flowing from the engine jacket to coolant. Thus, the water outlet temperature (6) depends on the inlet and outlet temperature from the previous time step to which is added the influence of the temperature received from the engine walls:

\[ Q_{\text{water}} = Q_{\text{water.in}} - Q_{\text{water.out}} + Q_{\text{eng.to.water}} \]  
(5)

\[ \frac{dT_{\text{water.out}}}{dt} = \frac{q_{\text{water}} (T_{\text{water.in}} - T_{\text{water.out}})}{\rho V_{\text{water}} c_{\text{water}}} + \frac{k_{\text{water}} S_{\text{water}} \Delta T_{\text{water}}}{\rho V_{\text{water}} c_{\text{water}}} \]  
(6)

where the heat of the water brought or removed from the system by the coolant given by:

\[ Q_{\text{water.in/out}} = \rho \cdot c_{\text{water}} \cdot \frac{q_{\text{water}}}{ \frac{V_{\text{water}}}{\text{water}} } \cdot T_{\text{water.in/out}} \]  
(7)

\( \rho \) - Water density (Kg/m³); \( q_{\text{water}} \) - Water flow rate (m³/s); \( T_{\text{water.in/out}} \) - Water temperature (°C); \( c_{\text{water}} \) - Water specific heat (J/Kg deg.)

The engine physical parameters used in the model are displayed in Table I. The values were chosen for a hypothetical Internal Combustion engine.

| **Engine** | **$S$** | **$m^e$** | **$P_{\text{water}}$** | **$m_{\text{eng}}$** | **$k_{\text{water}}$** | **$w_{\text{water}}$** |
|-----------|---------|-----------|------------------------|---------------------|----------------------|---------------------|
| Engine    | 0.62    | m³        | 0.003                  | 65                  | 3018                 | W/m²·grad           |

\( m_{\text{eng}} \) - Engine mass (kg); \( c_{\text{eng}} \) - Iron specific heat (J/Kg·deg)
In the metal part, the heat flux through radiator walls (10) is given by the heat received from the coolant side and the heat transferred to the air side. Both heats are calculated according to the principle presented in (2), the difference being the surface and the convective heat transfer coefficient. To have a higher heat transfer efficiency the air contact surface is much larger than the water contact surface.

\[
Q_{\text{radiator}} = Q_{\text{water, to, rad}} - Q_{\text{rad, to, air}} \tag{10}
\]

The temperature of the radiator walls:

\[
m_{\text{rad}} c_{\text{rad}} \frac{dT_{\text{radiator}}}{dt} = k_{\text{water}} S_{\text{water}} \Delta T_1 - k_{\text{air}} S_{\text{air}} \Delta T_2 \tag{11}
\]

In the air part, the heat evolution of air from the radiator is given by the heat brought by the incoming air stream, the heat evacuated by the outgoing air and the heat received from the walls of the radiator.

\[
Q_{\text{air}} = Q_{\text{air, in}} - Q_{\text{air, out}} + Q_{\text{rad, to, air}} \tag{12}
\]

The temperature of the air coming out of the radiator:

\[
\frac{dT_{\text{air, out}}}{dt} = \frac{q_{\text{air}}}{V_{\text{air}}} (T_{\text{air, in}} - T_{\text{air, out}}) + \frac{k_{\text{air}} S_{\text{air}} \Delta T_2}{\rho_{\text{air}} V_{\text{air}} c_{\text{air}}} \tag{13}
\]

Both the inlet and outlet heat brought by the water and the inlet and outlet heat brought by the air used in the thermal balance (8), (10) are expressed according to the principle in (7). \(V_{\text{air}}\) represents the volume of air from the radiator. Radiator parameters are presented in the Table II and were acquired from literature and measurements from real radiator.

| TABLE II. RADIATOR PARAMETERS |
|-----------------------------|
| \(S_{\text{water}}\) | 1.4 | m² |
| \(V_{\text{water}}\) | 0.0013 | m³ |
| \(m_{\text{rad}}\) | 4.5 | kg |
| \(k_{\text{water}}\) | 4000 | W/m²²·grad |
| \(S_{\text{air}}\) | 8.32 | m² |
| \(V_{\text{air}}\) | 0.0065 | m³ |
| \(k_{\text{air}}\) | 60 | W/m²²·grad |

Based on the same principles as the cooling radiator (8-13), the cabin radiator was added to the system thus modeling additional disturbances to the cooling system. It can be disabled by a bypass valve. The cabin radiator model has different constructive parameters.

D. Mathematical Model of the Pump

The pump used in this cooling system is a centrifugal pump driven by a 24 V electric motor. Through the DC motor block presented in Fig. 3, we can calculate the rpm produced by the electric motor by applying different input voltages (14).

\[
\text{In the cooling system the pressure that the pump must overcome is variable due to the degree of valves openness that will vary the pressure drop on each branch. The calculation of the consumed current is made according to the efficiency of the electric motor-pump assembly, which is given by the ratio between the hydraulic power and the electric power (15).}
\]

\[
\text{Speed of the electric motor:}
\]

\[
n = Ke \cdot (U - IR - L \cdot \frac{dI}{dt}) \tag{14}
\]

\(Ke\) - Motor constant (rpm/V); \(R\) - Motor resistance (ohm); \(L\) - Motor impedance (H);

\[
P_h = q \cdot \rho \cdot g \cdot p_p; \quad P_{el} = U \cdot I \tag{15}
\]

\(\eta\) - Efficiency; \(q\) – Flow of the liquid through the pump (l/min); \(\rho\) – Density of water; \(g\) – Gravitational constant; \(p_p\) – The pressure produced by the pump

The behavior of the pump implemented in the Pump block is given by its characteristic curve [6]. The pressure produced depends on the pump flow and the blade speed (16). The pressure consumed is given by the pressure drops on each component of the cooling system (17).

\[
\Delta p_p = A(n) \cdot q^2 + B(n) \cdot q + C(n) \tag{16}
\]

\(A(n), B(n), C(n)\) – Functions that calculate the coefficients of the pump characteristic curve; \(n\) – The speed of the pump (rpm); \(q\) – Flow of the liquid (l/min).

\[
\Delta p_e = K_{eq} \cdot q^2 \tag{17}
\]

\(q\) - Flow of the liquid through the pump (l/min); \(K_{eq}\) - The sum of the pressure loss constants.

The equivalent constant depends on the arrangement of the components in the cooling system and may vary depending on the opening of the valves. The placement of the components is in series (18) or a parallel (19).

\[
K_{eq} = K_1 + K_2 \tag{18}
\]

\[
K_{eq} = (K_1 \cdot K_2)/(K_1 + K_2 + 2\sqrt{K_1 \cdot K_2}) \tag{19}
\]

The operating point at a given moment is determined by equaling the pressure produced by the pump with the consumed pressure.
\[ A(n) \cdot q^2 + B(n) \cdot q + C(n) = K \cdot q^2 \]  

(20)

\[ q = \frac{B(n) - \sqrt{B(n)^2 - 4 \cdot (A(n) - K) \cdot C(n)}}{2 \cdot (A(n) - K)} \]  

(21)

The parameters of the characteristic equation describing the behavior of the pump and the constants of the pressure drops are presented in Table III. These parameters were taken from [6] and adapted to the present model.

| TABLE III: CENTRIFUGAL PUMP PARAMETERS |
|-----------------------------------------|
| Pump | A(n) | B(n) | C(n) |
|------|------|------|------|
| n [rpm] | -0.022 | -0.041 | -0.081 | -0.105 | -0.166 | -0.098 |
| 1500 | 0.603 | 1.15 | 2.29 | 3.15 | 5.01 | 5.86 |
| 2000 | 183 | 357 | 471 | 658 | 852 | 1778 |

E. Three-Way Valve

Based on the pressure drop on the bypass circuit and radiator circuit the flow rate through each branch is computed. The three-way valve acts as a flow splitter. After the coolant passes through radiator it reaches a combiner where the mixed water temperature is computed. The weighted average is used to model the liquid mixture on the two paths, with the weight being given by the value of the flows.

\[ T_{out} = \frac{T_1 \cdot q_1 + T_2 \cdot q_2}{q_1 \cdot q_2} \]  

(22)

F. Delay

The propagation of the liquid temperature through the system is not instantly. The time in which the temperature reaches from one end to the other of an element in the circuit depends on the flow generated by the pump and the volume of the element. A variable delay function model the temperature propagation for each element of the system.

III. CONTROL

This chapter will analyze the nonlinearity of the system and will show a PI [9] control implementation capable of rejecting the disturbances appearing into the system.

A. The Nonlinearity of the Engine and Radiator

To nonlinear behavior of the engine model is emphasized through a staircase experiment. For this experiment the engine model is initialized with the initial values such that the engine temperature is 90 °C. The input coolant temperature was kept constant and the coolant flow rate was varied.

A series of consecutive steps (“staircase event”) on the coolant flowrate were applied, increasing the value of the coolant flow by a constant step. Thus, it is observed that the response of the system differs at each stage both in terms of amplitude and response time.

As with the engine, the radiator has a non-linear behavior, and it was tested by a similar staircase experiment. The initial conditions for the radiator subsystem were chosen in such a way that the output temperature is 45 °C and initial flowrate is 2.3 l/min. The coolant input temperature is kept constant at 90 °C. Then, a series of consecutive steps on the coolant flow rate with a constant value of 5 l/min was applied.

![Fig. 4. Nonlinear behavior of motor temperature at different flow values.](image)

![Fig. 5. The nonlinear behavior of the coolant temperature in the radiator at different flow values.](image)

B. Implementation of Control

The nonlinearity of the system makes its behavior differ from one operating point to another, thus being a difficult system to control. The control strategy adopted for this case consists of using a PI controller to maintain the engine temperature at the desired value by manipulating the supply voltage of the pump.

To compute the parameters of the control system it was necessary to identify the linear model of the system around an operating point where the engine temperature is 90 °C. This point is reached when the vehicle is running at a speed of 50 km/h at an engine speed of 1500 rpm, the valve is open at 70%, and the voltage applied to the pump is 4.1V.

![Fig. 6. Identify the transfer function used to calculate the controller.](image)

For this purpose, the step response of the system was used to identify a linear model of the system in the form of a first order transfer function. The system would have the supply voltage of the pump as an input and the engine temperature as output. A +10% step on the supply voltage was applied and the response of the system was analyzed. The resulted linear model is shown in (23). The tuning of the PI controller was
carried out by imposing the performance of the closed system in the form of a first order transfer function with the response time $T_0 = 20$ s. Using the expression (24) we obtain a PI controller whose parameters are $K_p = -4.1$ and $K_i = -0.012$.

$$H(s) = \frac{-40.2}{330s + 1}$$

$$R(s) = \frac{1}{H(s)1 - H_0(s)} = -0.41 - 0.0012 \frac{1}{s}$$

IV. RESULTS

The testing of the control strategy was performed using the following scenario: the vehicle is running at a constant speed of 50 km/h and a series of steps are applied on the engine speed. This will simulate the cases in which the vehicle runs at the same speed but in different gears. For these tests, the reference value of the control loop was set at 90 °C.

![Testing the control strategy](image1)

**Fig. 7.** Rejection of disturbances by the control system for different engine speeds.

![ECE-15 Driving Cycle](image2)

**Fig. 8.** Rejection of disturbances by the control system using ECE-15 driving cycle.

The ECE-15 test cycle was used to test the control strategy in a situation as close to reality as possible. This test simulates a scenario in which the vehicle is driven in the city and includes accelerations and decelerations in a short period of time. As can be seen in Fig.8, the controller manages to keep the temperature within $+ - 3$ °C around the reference.

V. CONCLUSIONS

This paper presents the mathematical modeling of the cooling system used in the internal combustion vehicles. The modeling approach is based on the principle of operation of heat exchangers and on the principles of hydraulics and electricity. The model manages to simulate the evolution of the temperatures present in the engine block and in the radiators and the way in which the flow in the cooling system is influenced by the pump and the three-way valve. Following the implementation of the model, a simple control strategy was calculated to maintain the engine temperature at the desired reference. The PI control manages to reject the disturbances that act on the system that works in different scenarios. Once the model is implemented, more advanced control schemes remain the subject of future developments.

CONFICT OF INTEREST

The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

C. Mureșan carried out the research, developed the model and simulation and wrote the paper. G. Harja was involved in the process of modelling and control of the cooling system.

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