Rib layout optimization of stiffened square plates based on a meshfree model and a hybrid genetic algorithm

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Abstract. In order to minimize the central deflection of stiffened square plates under different loads, stiffener/rib layout optimization of stiffened plate is studied in the present work based on meshfree method and a hybrid genetic algorithm. The stiffened plates are composed of flat plates and straight beams with constant cross section. In order to combine the flat plates and beams, the displacement compatibility conditions are introduced. The calculate method of the stiffened plate is the meshfree method, and the ribs can be placed beyond mesh lines. During the entire process of rib layout optimization, no remeshing of the plate is required, which is convenient in programming and cost-efficient in computation. The optimization is carried out through the hybrid genetic algorithm which combines the genetic algorithm (GA) and the controlled random search algorithm. The controlled random search algorithm can effectively shorten the computation time of the genetic algorithm and increase the probability to find the efficient points. The validity and adaptability of the proposed method are tested by calculating several numerical examples.

1. Introduction

Stiffened plates are tougher than flat plates while adding little weight. There are various circumstances that stiffened plates have been applied to, such as bridge decks, ship hulls, and aircrafts. Early models for stiffened plates were the grillage model [1] and the orthotropic model [2]. Due to too much simplification, these models could not give satisfying solution for generalized stiffened plate problems. Nowadays, researchers always regard a stiffened plate as a structure composed of flat plate and ribs/stiffeners, and combine them by applying the displacement compatibility conditions. Based on this model, analysis methods for stiffened plates were developed, including the original Rayleigh- Ritz approaches [3-6], finite element methods (FEM) which discretizes the structure into a finite number of small elements [7-9] and the constraint method based on finite elements [10]. Among the methods, the finite element methods are widely applied and has been formed commercial software.

In the past decades, meshless or meshfree methods [11-13] have been introduced. Unlike the finite element methods, the meshfree methods discretize a problem domain with orderly or randomly
distributed points, so it can be more flexible in some instances, including the moving discontinuity and phase transformation problems, where the FEMs may encounter difficulties. Early meshfree methods are the Smoothed Particle Hydrodynamics (SPH) \cite{14, 15}, the diffuse element method (DEM) was proposed by Nayroles et al. \cite{16} in 1992. Belytschko et al. \cite{17} modified and improved the DEM, therefore the element-free Galerkin method (EFG) was developed. Melenk and Babuska \cite{18} confirmed that the methods on the basis of moving least-square approximations were specific cases of partitions of unity. Then Liu et al. \cite{19} proved the convergence of this methods. Perrone and Kao \cite{20} developed the generalized finite difference method, but the more robust instance of the method was introduced by Liszka and Orkisz \cite{21} in the 1980s. In 2016 the smoothed finite element method (SFEM) that incorporates the cell-wise strain smoothing treatment into the finite elements was proposed by Liu G.R. \cite{22}. Comparing with the common FEM, the SFEM gives results of higher accuracy with little increase of computational cost. Zhang X. \cite{23} used least square method and collocation method to solve problems like elastoplastic problems, the spread of the flow field and nonlinear and singularity problem. Liew and Peng et al. \cite{24-27} studied the bending, vibration, buckling and nonlinear problems of various plates, such as folded plates, stiffened plates and so on. 

With the development of computation techniques, engineering design pursues not only stability but also economy. Therefore, optimal design has been paid more and more attention. At present, optimization of structures is divided into three branches, which includes size optimization, shape optimization and topology optimization. And the frequently-used optimization methods are the optimality criteria method, mathematical programming approach and bionics algorithm. Shape optimization determines the optimal shape of a structure component subjected to specified requirements. For the past several decades, much advance has been achieved in shape optimization. It can solve the nonlinear problems and multiple physical fields problems of the structure of the shape optimization. Zou et al. \cite{28} presented a meshfree method for shape design optimization of continua based on partition of unity and demonstrated the effectiveness and efficiency of the method. Roque et al. \cite{29} used meshless method to form a differential evolution optimization scheme and found that it can produce excellent solutions. Tan \cite{30} studied a new meshless optimization method employing the conjugate gradient method to carry out geometric optimization on heating devices. The results showed that the method could be a successful design tool for the radiate. Grindeanu et al. \cite{31} carried out the shape design optimization on hyper-elastic structures by a meshless method. Besides the aforementioned work, few literatures on the rib layout optimization of stiffened plates have been found, which motives this paper.

In this article, a meshfree model for stiffened plates proposed by the author \cite{24} is extended and combined with the hybrid genetic algorithm to form a meshfree method for rib layout optimization of stiffened plates under different loads. Because of the meshfree advantages of the model, remeshing of the flat plate is absolutely avoided for every change of the rib layout in the optimization process, which reduces the computation cost and simplifies the programming. The hybrid genetic algorithm is one of the bionics algorithms. It is associated with the controlled random search algorithm in this paper to assembly the advantages of the two in the optimization. Several numerical examples are calculated to test the efficiency and effectiveness of the proposed method.

2. Meshfree model

The meshfree model of a stiffened plate including two stiffeners is shown in figure 1. In this figure, the hollow dots represent the nodes on the beams and the flat plate. The degree of freedom (DOF) of each node on the flat plate is defined as \((u_p, v_p, w_p, \varphi_{px}, \varphi_{py})\), and for a node of the stiffener, the DOF is defined as \((u_s, w_s, \varphi_s)\). From the degrees of freedom of the stiffeners, we can get that the torsion and bending which in the flat plate of the stiffeners were not considered.
2.1. The shape functions
According to moving least-square approximation (MLS) [17], the function $u(x)$ can be approximated by $u^h(x)$, and

$$u^h(x) = \sum_{i=1}^{N} N_i(x)u_i$$  \hspace{1cm} (1)

where $N$ defines the number of the nodes on the structure, $N_i(x)$ are the shape functions and $u_i$ are the nodal parameters. For detailed derivation of $N_i(x)$, the readers may refer to [17] or [25].

2.2. Transformation equations
On the basis of MLS [17] and the first-order shear deformation theory (FSDT) [32], the approximate displacements of the plate are

$$\begin{align*}
    u_p(x,y,z) &= \sum_{i=1}^{n} N_i(x,y)u_{0pi} - z\sum_{i=1}^{n} N_i(x,y)\phi_{pi} \\
    v_p(x,y,z) &= \sum_{i=1}^{n} N_i(x,y)v_{0pi} - z\sum_{i=1}^{n} N_i(x,y)\phi_{pi} \\
    w_p(x,y,z) &= \sum_{i=1}^{n} N_i(x,y)w_{pi}
\end{align*}$$  \hspace{1cm} (2)

where $u_{0pi}$, $v_{0pi}$, $\phi_{pi}$, and $w_{pi}$ are plate nodal parameters. $n$ is the number of the nodes, and $N_i(x,y)$ are the shape function. The approximate displacements of the x-rib/stiffener are

$$\begin{align*}
    u_s(x,z) &= u_{0si}(x) - z\phi_{si} \\
    &= \sum_{i=1}^{N_s} \Phi_i(x)u_{0si} - z\sum_{i=1}^{N_s} \Phi_i(x)\phi_{si} \\
    w_s(x) &= \sum_{i=1}^{N_s} \Phi_i(x)w_{si}
\end{align*}$$  \hspace{1cm} (3)

where $u_{0si}$, $\phi_{si}$, and $w_{si}$ are the nodal parameters. $N_s$ is the number of the nodes of the x-stiffener, and $\Phi_i(x)$ are the shape functions.
As shown in figure 2, on a vertical cross section of the stiffened plate, the node “S” is on the x-stiffener, “C” and “P” are the points on the flat plate. Corresponding to the node on stiffener, “C” and “P” are the corresponding points on the surface of the plate and the middle surface of the plate. Their displacement compatibility includes

\[ \begin{bmatrix} w_p \end{bmatrix}_p = \begin{bmatrix} w_i \end{bmatrix}_i \]  
(4)

\[ \begin{bmatrix} \varphi_{pm} \end{bmatrix}_p = \begin{bmatrix} \varphi_i \end{bmatrix}_i \]  
(5)

\[ \begin{bmatrix} u_p \end{bmatrix}_p = \begin{bmatrix} u_i \end{bmatrix}_i \]  
(6)

According to equation (6), we have

\[ [u_p]_i = [u_i]_i \ (I = 1, \ldots, Ns) \]  
(7)

or

\[ u_p(x_j, y_j, -h_p / 2) = u_i(x_j, h_i / 2) \]  
(8)

The thickness of plate and stiffeners are \( h \) and \( h_p \), respectively. According to equation (2) and equation (3), equation (8) can be written as

\[ u_{op}(x_j, y_j) + \varphi_{pm}(x_j, y_j) \cdot h_p / 2 = u_{oi}(x_j) - \varphi_i(x_j) \cdot h / 2 \]  
(9)

From equation (5), we have

\[ \varphi_{pm}(x_j, y_j) = \varphi_i(x_j) \]  
(10)

Therefore,

\[ u_{op}(x_j, y_j) = u_{oi}(x_j, y_j) + \frac{h_p + h}{2} \varphi_i(x_j) \]  
(11)

or

\[ \sum_{i=1}^n N_i(x_j, y_j) u_{op} = \sum_{j=1}^{Ns} \Phi_j(x_j) u_{oi} - es \sum_{j=1}^{Ns} \Phi_j(x_j) \varphi_j, \quad (I = 1, \ldots, Ns) \]  
(12)

Where \( es = (h_p + h) / 2 \). For concentric stiffeners, \( es = 0 \).

equation (12) can be rewritten as

\[ T_s \delta_{pu} = T_s \delta_{su} - es T_s \delta_{sp} \]  
(13)

where

\[ T_s = \begin{bmatrix} N_1(x_1, y_1) & N_2(x_1, y_1) & \cdots & N_s(x_1, y_1) \\
\vdots & \vdots & \ddots & \vdots \\
N_1(x_N, y_N) & N_2(x_N, y_N) & \cdots & N_s(x_N, y_N) \end{bmatrix}, \]  
\[ T_p = \begin{bmatrix} \Phi_1(x_1) & \Phi_2(x_1) & \cdots & \Phi_N(x_1) \\
\vdots & \vdots & \ddots & \vdots \\
\Phi_1(x_N) & \Phi_2(x_N) & \cdots & \Phi_N(x_N) \end{bmatrix}, \]

\[ \delta_{pu} = \begin{bmatrix} u_{op1}, u_{op2}, \ldots, u_{opm} \end{bmatrix}^T, \quad \delta_{su} = \begin{bmatrix} u_{oi1}, u_{oi2}, \ldots, u_{oin} \end{bmatrix}^T \]

Equation (13) can be rewritten as

\[ \delta_{su} = es T_s^{-1} T_p \delta_{pxp} + T_s^{-1} T_p \delta_{pu} \]  
(14)

The other two formulas have similar derivation, and detailed derivation process can be seen in literature [24, 25]. The relation between plates and stiffeners can be given

\[ \delta_x = T_w \delta_p \]  
(15)

Where

\[ \delta_s = \begin{bmatrix} \delta_{s1} & \delta_{s2} & \cdots & \delta_{sn} \end{bmatrix}^T = \begin{bmatrix} u_{o1}, w_{o1}, \varphi_{o1}, \ldots, u_{on}, w_{on}, \varphi_{on} \end{bmatrix}^T, \]
\[ \delta_p = \begin{bmatrix} \delta_{p1} & \delta_{p2} & \cdots & \delta_{pm} \end{bmatrix}^T = \begin{bmatrix} u_{p1}, v_{p1}, w_{p1}, \varphi_{p1}, \varphi_{p1}, \ldots, u_{pm}, v_{pm}, w_{pm}, \varphi_{pm}, \varphi_{pm} \end{bmatrix}^T. \]

\( T_{sp} \) is a \( 3Ns \times 5n \) matrix and \( T_w = T_w^T T_p \).
It was noted that Point “P” is the corresponding point on the plate and is not necessary the nodes of the plate. When the location of x-stiffener varies, we do not need to recalculate the whole structure just have to compute T<sub>sp</sub>. Which is quite convenient and will save lot of computation efforts: During the optimization process of rib layout, repeatedly remeshing of the flat plate due to rib position change is totally avoided.

2.3. Governing equation

The flat plate under pressure has the potential energy

\[ U_p = \frac{1}{2} \int \int_{\Omega} \left[ \epsilon_{xx}^p + \epsilon_{yy}^p + \epsilon_{xy}^p + \gamma_{xx}^p + \gamma_{yy}^p + 4 \gamma_{xy}^p \right] dxdy + \int q w_p dxdy \]

where \( \epsilon_{xx}^p, \gamma_{xx}^p \) and \( \gamma_{yy}^p \) are the strains, and \( k = \frac{5}{6} \) is the shear correction factor [32].

\[
D = \frac{E h}{12(1-\mu^2)} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & (1-\mu) / 2 \end{bmatrix}, \text{ and } G \text{ is the shear modulus can be get from } D.
\]

The stiffener has the potential energy

\[ U_s = \frac{1}{2} \int \int_{\Omega_s} E \left( u_{w,x} - u_{w,x} \right)^2 dxdy + \frac{1}{2} \int \int G A \left( w_{s,x} - \phi_s \right)^2 dx \]

where \( A_s \) is the cross section area of the rib and \( W_s \) is the width of the rib. Then we obtain the total potential energy of the ribbed plate

\[ U = U_p + U_s \]

Similarly to [24, 25], we have the governing equation for the stiffened plate

\[ K \delta_p = f \]

The full transformation treatment by Chen et al. [33] is used to introduce the essential boundary conditions.

3. The hybrid genetic algorithm

The genetic algorithm [34] finds the target point by imposing a lot of iterations to simulate the process of biological evolutionary. It needs to pre-determine the number of populations, the way of encoding, genetic probability, crossover probability and mutation probability so as to make the initial population evolve according to the probabilities. The more precise results can be obtained with a lot of repetitively iterative computations, usually. The genetic algorithm is good at global optimization, but always has a low convergence speed.

Controlled random search algorithm [35] simplifies engineering problems into mathematical models, which contain design variables, objective function and constraints, and seeks the extremum of the objective function under the constraints. It determines the target direction by looking for the steepest descent direction from a large number of produced search directions. But this method is only applicable to small-scale problems and good at local optimization. Because the results are more susceptible to the initial points and directions, we usually need to determine the final result via several calculations.

Hybrid genetic algorithm combines the advantages of the aforementioned two methods by employing the controlled random search method to identify the best individual each time in a population generated from the genetic algorithm. This treatment can effectively accelerate the convergence speed of the genetic algorithm. By adding the controlled random search algorithm, the hybrid genetic algorithm becomes more efficient. Figure 3 displays the flow chart of the algorithm.
Set the number of population \( M \); the way to encode; selective probability; crossover probability \( P_c \); mutation probability \( P_m \)

Code and then generate \( M \) individuals

Decode and get \( M \) meaningful values

Calculate corresponding \( M \) individual fitness function values

Find the best individual

Generate a series of random directions

Find the steepest descent direction

Determine whether meet the terminating conditions of constrained random direction search method

get the optimized best individual

Calculate the corresponding fitness function value and code

select operation

crossover operation

mutation operation

Determine whether to suit the requirements of iterative times in GA

Fig.3. The process of the hybrid genetic algorithm.
When the relative difference between the two values of objective function in consecutive iteration is smaller than the specified threshold, the iteration in the controlled random search algorithm will stop. The number of population in the genetic algorithm often takes between 20 and 100. Although small population can improve the computing speed of the algorithm, the diversity of population will suffer. Crossover probability generally takes a value between 0.4 and 0.99. If the value is too small, the process to produce new individual may be slowed down. If the value is too large, it is easy to destroy the current model. Mutation probability typically takes a value between 0.0001 and 0.1, and the number of iteration usually takes between 100 and 1000. Hybrid genetic algorithm can significantly reduce the number of genetic iterations.

4. Results and Discussion
Several numerical examples are calculated for the optimum layout of ribs under the target of minimizing the plate central deflection. In all examples, the square flat plate and stiffeners have the same material properties and size: \( E = 1.7 \times 10^7 \text{ Pa} \), \( \mu = 0.3 \), \( h_p = 0.01 \text{ m} \), \( h_s = 0.1 \text{ m} \), \( W_s = 0.01 \text{ m} \). The width of the plate is 1m. The flat plate is discretized by \( 9 \times 9 \) nodes, and the stiffener by 9 nodes. The influence domain of (DOI) the plate nodes is four times of the distance between the adjacent nodes, and the DOI of the stiffener nodes twice of the distance. For the genetic algorithm, the population takes 20, crossover probability 0.4 and mutation probability 0.05. The specified threshold for iteration is \( 10^{-5} \) in the controlled random search algorithm. The plates are simply supported.

4.1. Verification study
A square plate stiffened by two ribs is under a uniform load of 1 Pa. Obviously, the optimum layout of ribs for the minimum plate central deflection is that both ribs are set along the centerlines of the plate (figure 4). The optimized rib positions calculated by the proposed method after iterating once, twice and three times respectively are list in tables 1 to 3. In the tables, “x-stiffener’s position” refers to the distance between x-stiffener (the horizontal one) and lower edge of the plate, and “y-stiffener’s position” represents the distance between y-stiffener and left edge of the plate.

![Figure 4. A plate with two ribs subjected to a uniformly distributed load.](image)

The relative error is defined by

\[
RE = \frac{w - w_0}{w_0} \times 100 \%
\]

where \( w \) is the central deflection under current ribs layout, \( w_0 \) is the central deflection when the ribs are on center of the plate. The value of \( w_0 \) is \( 7.8360 \times 10^{-5} \text{ m} \).

| Table 1. Optimized rib layout (one generation). | }
Calculation No. | x-stiffener’s position (m) | y-stiffener’s position (m) | plate central deflection ($10^{-5}$m) | relative error (%) |
--- | --- | --- | --- | --- |
1 | 0.492351 | 0.508675 | 7.8447 | 0.11 |
2 | 0.456594 | 0.471543 | 8.0037 | 2.14 |
3 | 0.508780 | 0.498524 | 7.8423 | 0.05 |
4 | 0.494979 | 0.505495 | 7.8415 | 0.07 |
5 | 0.505910 | 0.500177 | 7.8400 | 0.05 |
6 | 0.574803 | 0.519685 | 8.1934 | 4.56 |
7 | 0.330709 | 0.488189 | 9.4275 | 20.31 |
8 | 0.494859 | 0.484825 | 7.8384 | 0.03 |
9 | 0.500731 | 0.500827 | 7.8400 | 0.05 |
10 | 0.499389 | 0.502667 | 7.8392 | 0.04 |

Table 2. Optimized rib layout (two generations).

Calculation No. | x-stiffener’s position (m) | y-stiffener’s position (m) | plate center deflection ($10^{-5}$m) | relative error (%) |
--- | --- | --- | --- | --- |
1 | 0.498856 | 0.494220 | 7.8400 | 0.05 |
2 | 0.507330 | 0.505917 | 7.8368 | 0.07 |
3 | 0.497634 | 0.505210 | 7.8415 | 0.07 |
4 | 0.497372 | 0.507484 | 7.8407 | 0.06 |
5 | 0.497890 | 0.504321 | 7.8400 | 0.05 |
6 | 0.502074 | 0.494367 | 7.8400 | 0.05 |
7 | 0.510037 | 0.500426 | 7.8431 | 0.09 |
8 | 0.494829 | 0.488680 | 7.8462 | 0.13 |

Table 3. Optimized rib layout (three generations).

Calculation No. | x-stiffener’s position (m) | y-stiffener’s position (m) | plate center deflection ($10^{-5}$m) | relative error (%) |
--- | --- | --- | --- | --- |
1 | 0.501330 | 0.501675 | 7.8368 | 0.01 |
2 | 0.497630 | 0.492361 | 7.8415 | 0.07 |
3 | 0.499221 | 0.504984 | 7.8400 | 0.05 |
4 | 0.494744 | 0.495802 | 7.8407 | 0.06 |
5 | 0.504226 | 0.496857 | 7.8400 | 0.05 |
6 | 0.497027 | 0.496031 | 7.8400 | 0.05 |
7 | 0.506822 | 0.498247 | 7.8407 | 0.06 |
8 | 0.495694 | 0.505655 | 7.8407 | 0.06 |
9 | 0.504969 | 0.504737 | 7.8407 | 0.06 |
10 | 0.502937 | 0.503236 | 7.8392 | 0.04 |

From table 1 to table 3, it can be observed that the rib layout results calculated by the proposed method after the individuals iterating twice have already meet the requirements. Considering efficiency and accuracy, two generations of evolution are employed in the calculation of the following examples.

4.2. A square plate with one x-stiffener under a patch load

A one-stiffener square plate is subjected to a uniform patch load of 1 Pa, which occupies a quarter of the plate (figure 5). The optimized rib position is list in table 4.
Figure 5. A plate with single rib subjected to patch load.

Table 4. Optimized rib layout for the plate with single stiffener.

| x-stiffener’s position (m) | plate central deflection (10^{-5}m) |
|---------------------------|----------------------------------|
| 0.421846                  | 1.8408                           |

Therefore, the plate central deflection reaches the minimum while the distance between x-stiffener and the lower edge of the plate is 0.422m. The optimization process is shown in figures 6-9.

Figure 6. Position of x-rib in the first generation.

Figure 7. Position of x-rib in the second generation.

Figure 8. Plate central deflection in the first generation.

Figure 9. Plate central deflection in the second generation.

Comparing figure 6 with figure 7, we find that there are more solid points distributing near the dot line (which represents the optimum value) in figure 7. In the second generation, the variable of the
proposed method gradually converge, and the central deflection also has a corresponding centralized trend.

4.3. A square plate with one x-stiffener under a patch load

A square plate with two x-stiffeners is subjected to the same patch load as that in Section 4.2 (figure 10). The interval between the two ribs is fixed to 0.3 m. The optimized rib positions are listed in Table 5.

![Figure 10. A square plate with two x-stiffeners subjected to patch load.](image)

**Table 5. Optimized rib layout for the plate with two x-stiffeners.**

| the first x-rib’s position $y_1$ (m) | the second x-rib’s position $y_2$ (m) | plate central deflection $(10^{-5}$m$)$ |
|-------------------------------------|--------------------------------------|----------------------------------------|
| 0.412949                            | 0.712949                             | 1.8369                                 |

Therefore, the plate central deflection reaches the minimum while the distance between the first and second x-stiffener and the lower edge of the plate are 0.413m and 0.713m, respectively. The optimization process is shown in Figures 11-14.

![Figure 11. Position of the first x-rib in the first generation.](image)

![Figure 12. Position of the first x-rib in the second generation.](image)
Figure 13. Plate central deflection in the first generation.

It is found that, there are more solid points close to the dot line of 0.413 in figure 12, and the central deflection also has a corresponding converged trend in figure 14.

4.4. A square plate with x- and y- stiffeners subjected to a patch load

A square plate stiffened by x- and y- stiffeners is subjected to the same patch load as that in Section 4.2 (figure 15). The optimized rib positions are list in table 6.

Table 6. Optimized rib layout for the stiffened plate.

| x-rib’s position (m) | y-rib’s position (m) | central deflection (10^{-5}m) |
|----------------------|----------------------|-------------------------------|
| 0.73357              | 0.433192             | 0.9090                        |

Therefore, the plate central deflection reaches the minimum while the distance between the x-stiffener the lower edge of the plate is 0.734m, and the distance between the y-stiffener the left edge of the plate is 0.433m, respectively. The optimization process is shown in figures 16-19.
It can be observed that, there are more solid points in the fourth quadrant in figure 16, and the central deflection also has a corresponding converged trend in figure 18.

5. Conclusions
An optimization study on rib layout for the minimum central deflection of stiffened plates is studied through meshless method and hybrid genetic algorithm. Stiffened plate is considered as beams and flat plate model and combined them by employing displacement compatibility conditions. The calculate function of the model is given by supposing the energy of the all part of components and invoking the principle of minimum potential energy. Due to the meshfree characteristics of the model and the transformation equations derived in this paper, ribs are not limited to mesh lines. Therefore, in the entire process of rib layout optimization, no remeshing of the plate is required. A hybrid genetic algorithm which combines the controlled random search method and the genetic algorithm is employed to carry out the optimization. The controlled random search algorithm increases the efficiency of the genetic algorithm. The accuracy and efficiency of the proposed method are tested with different numerical examples.

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