A Novel Fault Detection Model Based on Atanassov’s Interval-Valued Intuitionistic Fuzzy Sets, Belief Rule Base and Evidential Reasoning

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ABSTRACT

In engineering practice, fault detection, with the fundamental characteristics of randomness, uncertainty, and great damage, has attracted much attention from academia. The chief aim of this paper is to research the problem of flush air data sensing (FADS) system, an advanced airborne sensor, with Atanassov’s interval-valued intuitionistic fuzzy sets (AIVIFSs), belief rule base (BRB) and evidential reasoning (ER). First of all, this paper proposes some relevant concepts and similarity calculation operators between AIVIFSs, meanwhile, extracts a new similarity calculation approach based on previous researches. Then, a new score function that is applied to quantify the information contained in AIVIFSs is presented with the idea of p-norm, which is defined on the basis of the information measure from the reference level, regarding the positive and negative information depicted in terms of the AIVIFSs. Next, AIVIFSs and BRB that are characteristic of describing the randomness and uncertainty are combined to set up the fault detection model with ER. Finally, the algorithm process is presented and the proposed model is applied to the fault detection of FADS for the first time to confirm the validity and feasibility.

INDEX TERMS
Fault detection, flush air data sensing (FADS), Atanassov’s interval-valued intuitionistic fuzzy sets (AIVIFSs), belief rule base (BRB), evidential reasoning (ER).

I. INTRODUCTION

Safety and stability are important premises for a system, which are deeply related to human life and health. Once a failure occurs on a piece of certain equipment, it can cause property damage at least, and even more serious, huge casualties, including bridge collapse, nuclear power plant leakage, air crash, etc. The most tragic accident in the history of nuclear submarines, the sinking of a Russian nuclear submarine can be avoided if the torpedo fault was detected in time. Hence, it is necessary and significant to conduct the research of fault detection.

In the past several decades, considerable research efforts have been devoted to the field of fault detection, which can be classified into three categories.

The methods based on the analytical model:
Filter method [1], [2], least squares method [3], [4] and equivalent space method [5] are main solutions, the key idea of these methods is: Firstly, establish the model according to the connections between the components of the system. Then, the expected performance of the system under normal conditions is deduced by logical reasoning. Finally, compare the actual performance of the observed system with the expected performance. If there are differences, it indicates that there is a failure. Studies have shown that these methods are able to offer scientific detection results. The defect is that these methods require high-precision models, but it is difficult to guarantee accuracy when establishing the mathematical models of systems, which will lead to loss of detection performance to some extent.

The methods based on signal processing:
Unlike the above methods, the methods based on signal processing, including Fourier analysis, wavelet analysis, spectrum analysis, have no strict requirements on the accuracy of the established model [6]–[8]. All these methods tend to apply the parameters of signals, such as frequency, amplitude, phase, correlation, to detect abnormal signals. Although these methods can solve the problem to some extent, the study gives rise to some difficulties. For example, in terms of Fourier analysis, it is premised on the signal smoothness, but
most fault signals are contained in transient signals, which will lead to poor performance for non-stationary dynamic signals.

The methods based on knowledge:

A number of researches on the basis of knowledge were performed to solve the fault detection problem, including expert system [9], pattern recognition [10], [11], neural network [12]–[14] and other methods. Although these methods possess the ability to deal with the problem regardless of the accuracy of the established model, there is still much room for improvement in optimizing the structure and reducing the amount of computation.

It must be mentioned that a variety of uncertainties may coexist in real fault detection problems, e.g., random information may coexist with ignorance, thereby leading to the induction of knowledge without certainty. As a result, it is desirable to develop a new fault detection model to deal with different kinds of randomness and uncertainty in fault detection. An efficient solution is to make use of evidential reasoning (ER) to combine the approaches that can solve randomness and uncertainty.

Over the past two decades, evidential reasoning theory has been widely researched and developed as well as being successfully applied in various fields, including multi-criteria decision-making, fault detection, and state evaluation. In general, developments of the theory fall into three groups.

Conversions from different types of information to certitude structures:

In order to deal with the quantitative and qualitative information effectively, scholars put forward a series of approaches to achieve the transformation. In [15], an approach based on rules and utility information was suggested, which can convert the quantitative information described by semantic level, interval-valued form, and qualitative information, into certitude structures. On the basis of previous research, Sonmez M [16] proposed three new transformation approaches for different situations.

A theoretical extension of evidential reasoning:

Yang J B [17] proposed four axioms that evidential reasoning should satisfy, and further proposed a new evidential reasoning method, namely recursive evidence reasoning algorithm. Considering that the limitations of accurate reliability, Wang Y M [18] proposed the interval-valued certitude structure on the basis of rules and information transformation methods. In [19], an analytic evidential reasoning algorithm based on the recursive evidence reasoning algorithm was proposed, which was characterized by solving optimization problems more simple and suitable. In [20], weight was added to the reliability distribution of D-S theory, an evidential reasoning algorithm with weight and reliability was given.

Combination of evidential reasoning and belief rule base (BRB):

In order to make full use of the knowledge and experience of decision-makers and experts, a number of researches were performed to combine evidential reasoning with belief rule base. Yang J B [21] proposed a belief rule base inference methodology using the evidential reasoning approach. Later, Liu J [22], [23] proposed the extended certitude rule base that premise attributes and conclusions were both uncertain.

After sorting out a large amount of literature, it can be found that there is thereby an urgent need but it is feasible to solve the fault detection problem with fuzzy set theory, BRB and ER. For this objective, a new fault detection model on the basis of Atanassov’s interval-valued intuitionistic fuzzy sets (AIVIFSs), BRB, and ER is proposed in a more interpretable and transparent way. Although several scholars have investigated it [24]–[26], there are basically two shortcomings: 1) Until now, almost all relevant studies adopted the Atanassov’s intuitionistic fuzzy sets (AIFSs); however, compared with AIVIFSs, AIFSs still have many limitations in representing uncertain information. 2) In the calculation process, score functions and the approach of calculating similarity adopted in these studies are flawed and no improved solutions were proposed. Therefore, the results are not generally accepted and convincing.

The purpose of this paper is to design a new fault detection model to overcome the above shortcomings by combining AIVIFSs, BRB, and ER. Meanwhile, in order to demonstrate the effectiveness of the proposed model, we apply the model to analyze the flush air data sensing (FADS) system, which is important and advanced for measuring the angle of attack $\alpha$. It should be pointed out that, whether the failures of sensors measuring the angle of attack are detected in time is of vital importance to the safety of aircraft flight. It is because the failures of sensors measuring angle of attack were not detected, two tragic air crash incidents occurred in PT Lion Mentari Airlines and Ethiopian Airlines in 2018 and 2019, resulting in 346 deaths.

The key contributions of this paper are summarized down:

1) Atanassov’s interval-valued intuitionistic fuzzy sets that are characterized by the ability to reveal uncertainty with interval-valued membership and interval-valued non-membership, belief rule base, and evidential reasoning are employed to build a fault detection model.

2) A critical step for evidential reasoning in the proposed model is to obtain the similarity between AIVIFSs. After analyzing the previous studies, we propose a new solution based on implication operators, which can effectively solve the previous shortcomings.

3) Considering that score function has a crucial impact on the final result, a new score function for AIVIFSs based on p-norm is proposed and are strictly derived by formulas in this paper.

4) The proposed model is first applied to deal with the fault detection problem of the FADS system. Different from the traditional detection methods adopted in this system, such as the method based on the parity equation and the method based on chi-square $\chi^2$ distribution, the new model can better express the randomness and uncertainty.
The paper’s structure is as follows: In Section II, the FADS model is established. In Section III, a formula for calculating the similarity between AIVIFSs, as well as a new score function, are proposed. In Section IV, evidential reasoning of certitude degree is deduced. The proposed belief rule base inference framework and method are demonstrated in Section V. In Section VI, a fault detection model based on AIVIFSs, BRB and ER is proposed. In Section VII, an illustrative example and comparative studies are presented. Finally, the conclusion of this paper is given in Section VIII.

II. FLUSH AIR DATA SENSING MODEL

For FADS, the method of measuring α is called the “three-point method” and mainly relies on four measurement taps numbered 1, 3, 5, and 6. The basic layout of the measurement taps is shown in Fig. 1, where φ and λ represent the circumferential angle and cone angle. When it works, pressure values of the four taps are obtained by pressure sensors; then, three pressure values are selected for algorithm calculation to obtain four values α1, α2, α3, and α4, which are calculated by the combination of (1, 3, 5), (1, 3, 6), (1, 5, 6) and (3, 5, 6), respectively; finally, the average value of these results are considered as the expected angle of attack, see [27], [28] for details.

![Figure 1. Circumferential angle and cone angle of measurement taps in FADS.](image)

However, the sensor is usually applied for some high-performance airplanes such as X-33, X-43A, and X-31, the high surface temperature makes taps prone to fault and produce inaccurate readings. In view of this situation, a new fault detection model based on Atanassov’s interval-valued intuitionistic fuzzy sets (AIVIFSs) and belief rule base (BRB) is proposed, the main idea of which is to make use of AIVIFSs to describe the distribution laws of α1, α2, α3, and α4, and then set up the BRB model to determine whether the FADS system fails and which measurement taps are faulty. The fault detection model can be represented as follows:

$$ R_k : \text{IF } \alpha_1 \in A^k_1 \wedge \alpha_2 \in A^k_2 \wedge \alpha_3 \in A^k_3 \wedge \alpha_4 \in A^k_4, \text{THEN} $$

the fault detection result is \( (C^k, \beta^k) \)

with a rule weight \( w^k \) and attribute weight vector \( w_1, w_2, w_3, \) and \( w_4. \)

III. ATANASSOV’S INTERVAL-VALUED INTUITIONISTIC FUZZY SETS AND RELEVANT CONCEPTS

There are such cases that AIFSs cannot completely express the evaluation information of an object. For example, most people have their own opinions about whether an 18-year-old person belongs to the concept of “Youth”, but they would approve the range of age from 18 to 25. Therefore, AIVIFSs, whose membership and non-membership are interval values, are more flexible and practical. In the following part, some relevant concepts will be introduced.

**Definition 1:** Let \( X = \{x_1, x_2, \ldots, x_n\} \) be fixed and \( h_{0i}(x) \in H = \{h_{ij} = 0, 1, \ldots, 2^r, r \in N^*\} \) be a linguistic term set. An AIVIFS \( A \) in \( X \) is an object:

$$ A = \{(x, h_{0i}(x), \mu_A(x), v_A(x))| x \in X\} \quad (1) $$

where \( (\mu_A(x), v_A(x)) = ([\mu^L_A(x), \mu^U_A(x)], [v^L_A(x), v^U_A(x)]) \) denote the interval-valued membership and non-membership of the element \( x \) to \( A, [\mu^L_A(x), \mu^U_A(x)] \subseteq [0, 1] \) and \([v^L_A(x), v^U_A(x)] \subseteq [0, 1] \). The interval-valued hesitancy degree of \( x \) to \( A \) can be expressed as \( \pi_A(x) = [\pi^L_A(x), \pi^U_A(x)] = [1 - \mu^L_A(x) - v^L_A(x), 1 - \mu^U_A(x) - v^U_A(x)] \).

**Definition 2:** Given any two AIVIFSs \( A = (x, \mu_A(x), v_A(x)| x \in X \) \) and \( B = (x, \mu_B(x), v_B(x)| x \in X\), \( (\mu_A(x), v_A(x)) = ([\mu^L_A(x), \mu^U_A(x)], [v^L_A(x), v^U_A(x)]) \) and \( (\mu_B(x), v_B(x)) = ([\mu^L_B(x), \mu^U_B(x)], [v^L_B(x), v^U_B(x)]) \). The mathematical operators of \( A \) and \( B \) can be summarized as follows.

1) \( \overline{A} = ([v^L_A(x), v^U_A(x)], [\mu^L_A(x), \mu^U_A(x)]) \);  
2) \( A \cap B = ([\min(\mu^L_A(x), \mu^L_B(x)), \min(\mu^L_A(x), \mu^U_B(x))], \max(\nu^L_A(x), \nu^L_B(x)), \max(\nu^U_A(x), \nu^U_B(x))]) \);  
3) \( A \cup B = ([\max(\mu^L_A(x), \mu^L_B(x)), \max(\mu^U_A(x), \mu^U_B(x))], \min(\nu^L_A(x), \nu^L_B(x)), \min(\nu^U_A(x), \nu^U_B(x))]) \);  
4) \( A + B = ([\mu^L_A(x) + \mu^L_B(x) - \mu^L_A(x)\mu^L_B(x), \mu^L_A(x) + \mu^U_A(x) - \mu^U_A(x)\mu^U_B(x)], [v^L_A(x) + v^L_B(x) - \mu^L_A(x)v^L_B(x), v^L_A(x) + v^U_B(x) - \mu^L_A(x)v^U_B(x)]) \);  
5) \( AB = ([\mu^L_A(x)\mu^L_B(x), \mu^L_A(x)\mu^U_B(x)], [\mu^L_A(x)v^L_B(x) + v^L_A(x)\mu^L_B(x) - v^L_A(x)v^L_B(x) - v^L_A(x)v^U_B(x)], [\mu^L_A(x)v^U_B(x) + v^L_A(x)v^U_B(x) - v^L_A(x)v^U_B(x)] \);  
6) \( \lambda A = [(1 - \mu^L_A(x))^\lambda, \mu^L_A(x)^\lambda], [1 - (1 - \mu^L_A(x))^\lambda, (1 - \mu^L_A(x))^\lambda] \), \( \lambda > 0 \);  
7) \( \overline{A}^k = ([\mu^L_A(x)^k, \mu^L_A(x)^k], [1 - (1 - \mu^L_A(x))^k, 1 - (1 - \mu^L_A(x))^k]) \), \( \lambda > 0 \);

**Example 1:** Given AIVIFSs \( A = ([0.5, 0.6], [0.1, 0.2]) \) and \( B = ([0.65, 0.7], [0.05, 0.18]) \), then

1) \( \overline{A} = ([0.1, 0.2], [0.5, 0.6]) \);  
2) \( A \cap B = ([0.5, 0.6], [0.1, 0.2]) \);  
3) \( A \cup B = ([0.65, 0.7], [0.05, 0.18]) \);  
4) \( A + B = ([0.825, 0.88], [0.005, 0.036]) \);  
5) \( AB = ([0.325, 0.42], [0.145, 0.344]) \);  
6) when \( \lambda = 2, \lambda A = ([0.75, 0.84], [0.01, 0.04]) \);  
7) when \( \lambda = 2, A^k = ([0.25, 0.36], [0.19, 0.36]) \).

In addition, what needs to be emphasized is that the similarity between AIVIFSs is an important part of the model proposed in this paper; therefore, we conduct some related researches. First, offer the four well-known constraints of the similarity.
**Definition 3**: Suppose that $A$, $B$, and $C$ are three AIVIFSs, if $S(A, B)$ meets the following conditions, then it is called the similarity between $A$ and $B$ [29]:

1. $0 \leq S(A, B) \leq 1$;
2. If $A = B$, then $S(A, B) = 1$;
3. $S(A, B) = S(B, A)$;
4. If $A \subseteq B \subseteq C$, then $S(A, C) \leq \min(S(A, B), S(B, C))$;

Next, recall some typical similarity measurements.

**Definition 4**: Given any two AIVIFSs $A = \{(x, \mu_A(x), v_A(x))| x \in X \}$ and $B = \{(x, \mu_B(x), v_B(x))| x \in X \}$, $(\mu_A(x), v_A(x)) = ((\mu_A^L(x), \mu_A^U(x)), [v_A^L(x), v_A^U(x)])$ and $(\mu_B(x), v_B(x)) = ((\mu_B^L(x), \mu_B^U(x)), [v_B^L(x), v_B^U(x)])$.

a) Similarity measurement based on the normalized Hamming distance from Xu and Chen [18]:

$$S_1(A, B) = 1 - \frac{1}{4} \left[ (|\mu_A^L(x) - \mu_B^L(x)| + |\mu_A^U(x) - \mu_B^U(x)|) \right]$$

$$- \mu_B^L(x) + |v_A^L(x) - v_B^L(x)| + |v_A^U(x) - v_B^U(x)|;$$

(2)

b) Similarity measurement based on the normalized Hamming distance from Xu and Yager [30]:

$$S_2(A, B) = 1 - \frac{1}{4} \left[ (|\mu_A^L(x) - \mu_B^L(x)| + |\mu_A^U(x) - \mu_B^U(x)|) \right]$$

$$+ |v_A^L(x) - v_B^L(x)| + |v_A^U(x) - v_B^U(x)|) \right]$$

$$+ |v_A^L(x) - v_B^L(x)| + |v_A^U(x) - v_B^U(x)|;$$

(3)

c) Similarity measurement based on the normalized Euclidean distance from [13]:

$$S_3(A, B) = 1 - \frac{1}{4} \left[ (|\mu_A^L(x) - \mu_B^L(x)| + |\mu_A^U(x) - \mu_B^U(x)|) \right]$$

$$+ |v_A^L(x) - v_B^L(x)| + |v_A^U(x) - v_B^U(x)|;$$

(4)

d) Similarity measurement based on the Euclidean distance from [31]:

$$S_4(A, B) = 1 - \frac{1}{4} \left[ \sqrt{\frac{|\mu_A^L(x) - \mu_B^L(x)|^2 + |v_A^L(x) - v_B^L(x)|^2}{2}} \right]$$

$$+ \sqrt{\frac{|\mu_A^U(x) - \mu_B^U(x)|^2 + |v_A^U(x) - v_B^U(x)|^2}{2}} \right]$$

$$+ \sqrt{\frac{|\mu_A^L(x) - \mu_B^U(x)|^2 + |v_A^L(x) - v_B^U(x)|^2}{2}} \right]$$

$$+ \sqrt{\frac{|\mu_A^U(x) - \mu_B^L(x)|^2 + |v_A^U(x) - v_B^L(x)|^2}{2}} \right]$$

(5)

However, the common weakness of the above methods is that they only use a specific number to express similarity without considering membership and non-membership separately. For example, given $A = \{[0.1], [0.1], 1\}$, $B_1 = \{[1, 1], [0, 1]\}$, and $B_2 = \{[0, 1], [1, 1]\}$, we can obtain $S_1(A, B_1) = S_1(A, B_2)$ by using (2). However, it is obvious that $S_1(A, B_1)$ is acquired because of the difference of membership; for $S_1(A, B_2)$, the difference of non-membership is the main cause. In addition, since it is the similarity of the interval-valued fuzzy sets, the more reasonable way is to make use of interval to describe it. Therefore, a new approach to measure the similarity between AIVIFSs is proposed and **Definition 3** is applied to prove the rationality.

**Definition 5**: Given any two AIVIFSs $A = \{(x, \mu_A(x), v_A(x))| x \in X \}$ and $B = \{(x, \mu_B(x), v_B(x))| x \in X \}$, $(\mu_A(x), v_A(x)) = ((\mu_A^L(x), \mu_A^U(x)), [v_A^L(x), v_A^U(x)])$ and $(\mu_B(x), v_B(x)) = ((\mu_B^L(x), \mu_B^U(x)), [v_B^L(x), v_B^U(x)])$.

The similarity $S(A, B) = \{S_1^L(A, B), S_1^U(A, B), S_2^L(A, B), S_2^U(A, B), S_3^L(A, B), S_3^U(A, B), S_4^L(A, B), S_4^U(A, B)\}$ between $A$ and $B$ is defined below. Detailed proof processes are presented in the appendix.

$$S_1^L(A, B) = \min \left\{ \min \left\{ 1 - \mu_B^L(x) + \mu_B^U(x), \right\}, \min \left\{ 1 - \mu_A^L(x) + \mu_A^U(x), \right\} \right\};$$

$$S_1^U(A, B) = \max \left\{ \min \left\{ 1 - \mu_B^L(x) + \mu_B^U(x), \right\}, \min \left\{ 1 - \mu_A^L(x) + \mu_A^U(x), \right\} \right\};$$

$$S_2^L(A, B) = \min \left\{ \min \left\{ 1 - v_B^L(x) + v_B^U(x), \right\}, \min \left\{ 1 - v_A^L(x) + v_A^U(x), \right\} \right\};$$

$$S_2^U(A, B) = \max \left\{ \min \left\{ 1 - v_B^L(x) + v_B^U(x), \right\}, \min \left\{ 1 - v_A^L(x) + v_A^U(x), \right\} \right\};$$

$$S_3^L(A, B) = \min \left\{ \min \left\{ 1 - v_B^L(x) + v_B^U(x), \right\}, \min \left\{ 1 - v_A^L(x) + v_A^U(x), \right\} \right\};$$

$$S_3^U(A, B) = \max \left\{ \min \left\{ 1 - v_B^L(x) + v_B^U(x), \right\}, \min \left\{ 1 - v_A^L(x) + v_A^U(x), \right\} \right\};$$

(6)

**Example 2**: Given AIVIFSs $A = \{([0.5, 0.6], [0.1, 0.2])$ and $B = \{([0.65, 0.7], [0.05, 0.18])$, then

1. $S_1(A, B) = 0.92$;
2. $S_2(A, B) = 0.875$;
3. $S_3(A, B) = 0.906$;
4. $S_4(A, B) = 0.91$;
5. $S_1(A, B) = \{([0.85, 0.9], [0.95, 0.98])$.

In addition to the above concepts, for quantifying the information contained in AIVIFSs, experts proposed the concepts of score function and accuracy function. Here, we list some previous research results about them.

**Definition 6**: Given one AIVIFS $A = \{(x, \mu_A(x), v_A(x))| x \in X \}$, $(\mu_A(x), v_A(x)) = ((\mu_A^L(x), \mu_A^U(x)), [v_A^L(x), v_A^U(x)])$.

a) Xu’s score function $S$ and accuracy function $H$ [32]:

$$S(A) = \frac{\mu_A^L(x) + \mu_A^U(x) - v_A^L(x) - v_A^U(x)}{2};$$

$$H(A) = \frac{\mu_B^L(x) + \mu_B^U(x) + v_B^L(x) + v_B^U(x)}{2}. $$

(7)

(8)

**Remark 1**: There is a serious problem with both the score function and the accuracy function, i.e., information omission. The main reason is the insufficient use of the upper and lower bounds of the membership and non-membership.
b) Wang’s score function $W_S$ and accuracy function $W_H$ [33]:

$$W_S(A) = \left( \frac{\mu^k_A(x) + \lambda^k_A(x)}{2} - \frac{\nu^k_A(x) + \lambda^k_A(x)}{2} \right) \times \left( 1 + \frac{1 - \mu^k_A(x) - \nu^k_A(x) + 1 - \mu^k_A(x) - \nu^k_A(x)}{2} \right);$$

$$W_H(A) = \left( \frac{\mu^k_A(x) + \lambda^k_A(x)}{2} + \frac{\nu^k_A(x) + \lambda^k_A(x)}{2} \right) \times \left( 1 + \frac{1 - \mu^k_A(x) - \nu^k_A(x) + 1 - \mu^k_A(x) - \nu^k_A(x)}{2} \right) \tag{9}$$

Remark 2: It is worth noting that as long as the midpoints of the membership and the non-membership coincide, the score function is always 0, which is obviously unscientific.

c) Garg’s score function $G$ [34]:

$$G(A) = \frac{\mu^k_A(x) + \lambda^k_A(x)}{2} + k_1 \mu^k_A(x) \left( 1 - \mu^k_A(x) - \nu^k_A(x) \right) + k_2 \lambda^k_A(x) \left( 1 - \mu^k_A(x) - \nu^k_A(x) \right) \tag{10}$$

where $k_1 + k_2 = 1$, and $k_1, k_2 \geq 0$.

Remark 3: It is easily seen, when $\mu^k_A(x) = \lambda^k_A(x) = 0$, whatever $\nu^k_A(x), \nu^k_A(x), k_1$, and $k_2$ are, $G(A)$ is always equal to 0, which is flawed.

In order to overcome the shortcomings of the above methods, a score function based on the p-norm is proposed.

Definition 7: Given one intuitionistic fuzzy set $\tilde{A} = \left\{ (x, \mu_A(x), \nu_A(x)) | x \in X \right\}$ and the reference intuitionistic fuzzy set $\tilde{R} = \left\{ (x, 0, 0) | x \in X \right\}$, the p-norm distance of $\tilde{A}$ from $\tilde{R}$ taking into account the hesitancy degree is $\left( \mu^k_A(x) - 0 \right)^p + \left( \nu^k_A(x) - 0 \right)^p + \left( 1 - \mu^k_A(x) \right)^p$ and the p-norm variation between membership and non-membership is $\left( \mu^k_A(x) - \nu^k_A(x) \right)^p$. The generalized knowledge measure of $\tilde{A}$ is defined as:

$$K(\tilde{A}) = \frac{1}{2^{1/p} + 1} \left( \left[ (\mu^k_A(x))^p + (\nu^k_A(x))^p + (\mu^k_A(x))^p \right] + \left[ (\mu^k_A(x))^p - (\nu^k_A(x))^p \right] \right) + \frac{1}{2} \left( \mu^k_A(x) \right)^p \tag{11}$$

The p-norm knowledge-based score function of $\tilde{A}$ is:

$$J(\tilde{A}) = \left[ \frac{e^{\mu^k_A(x) - \nu^k_A(x)} - 1}{e^{\mu^k_A(x) - \nu^k_A(x)} + 1} \right] K(\tilde{A}) \tag{12}$$

Extend the score function from AIFSs to AIVIFSs.

Definition 8: Given one interval-valued intuitionistic fuzzy set $\tilde{A} = \left\{ (x, (\mu_A(x), \nu_A(x))) | x \in X \right\}$, $\left( \mu^k_A(x), \nu^k_A(x) \right) = \left[ \left[ \mu^k_A(x), \lambda^k_A(x) \right], \left[ \nu^k_A(x), \lambda^k_A(x) \right] \right]$, the score function $J(\tilde{A})$ is presented as follows:

$$J(\tilde{A}) = \frac{1}{2^{1/p} + 1} \left( \left[ \left( \frac{\mu^k_A(x)}{2} \right)^p + \left( \lambda^k_A(x) \right)^p \right] + \left( \mu^k_A(x) \right)^p + \left( \left( \frac{\lambda^k_A(x)}{2} \right)^p \right) \right) + \frac{1}{2} \left( \mu^k_A(x) \right)^p \tag{13}$$

IV. EVIDENTIAL REASONING OF CERTITUDE STRUCTURE

In the process of setting up the model, the main algorithm is based on evidential reasoning [35]. Different from previous researches, this paper mainly studies the evidential reasoning in the form of AIVIFSs, i.e., the certitude degree of case $\Psi$ is recorded as $Icd(\Psi) = \left[ Icd_{L^0}(\Psi), Icd_{U^0}(\Psi) \right], \left[ Icd_{L^0}(\Psi), Icd_{U^0}(\Psi) \right]$, where $0 \leq Icd_{L^0}(\Psi) \leq Icd_{U^0}(\Psi) \leq 1$ and $0 \leq Icd_{L^0}(\Psi) \leq Icd_{U^0}(\Psi) \leq 1$.

Here, some special values of $Icd(\Psi)$ will be introduced.

1) If the certitude degree of case $\Psi$ is 100%, then $Icd(\Psi) = \left[ (1, 1), (0, 0) \right]$.

2) If the certitude degree of case $\Psi$ is 0, then $Icd(\Psi) = \left[ (0, 0), (1, 1) \right]$.

3) If the certitude degree of case $\Psi$ is greater, then the membership of $Icd(\Psi)$ is closer to $[1, 1]$ and the non-membership is closer to $[0, 0]$; on the contrary, if the certitude degree of case $\Psi$ is smaller, then the membership of $Icd(\Psi)$ is closer to $[0, 0]$ and the non-membership is closer to $[1, 1]$.

Definition 9: Assume that the evidence sets of certitude structure is:

$$E = \left\{ (e_i, c_i) | c_i = \left( [c_i^{L_0}, c_i^{U_0}], [c_i^{L_0}, c_i^{U_0}] \right), i = 1, \ldots, T \right\}$$

with $w_l = \{ w_1, w_2, \ldots, w_T \}$.

The certitude structure consisting of case $e_i$ and cerititude degree $c_i$ represents the $i$th evidence; $w_l$ is the weight vector of $(e_i, c_i)$; the recognition frame is $\Theta = \{ \theta \}$; power set of $\Theta$ is $2^\Theta = \{ \emptyset, \{ \theta \} \}$. For any $l \in \{ 1, 2, \ldots, T \}$, the basic probability masses satisfy the following formulas:

$$m_{\Theta^l}(e_i, c_i) = 0;$$

$$m_{\Theta^l}(e_i, c_i) = \left[ m_{\Theta^l}(e_i, c_i), m_{\Theta^l}(e_i, c_i) \right];$$

$$m_{\Theta^l}(e_i, c_i) = \left[ c_i^{L_0}, c_i^{U_0} \right], \left[ c_i^{L_0}, c_i^{U_0} \right] = \left[ w_i c_i, c_i \right];$$

$$m_{\Theta^l}(e_i, c_i) = w_i c_i, \left[ c_i^{L_0}, c_i^{U_0} \right] = w_i c_i, \left[ c_i^{L_0}, c_i^{U_0} \right];$$

$$m_{\Theta^l}(e_i, c_i) = w_i c_i, \left[ c_i^{L_0}, c_i^{U_0} \right] = w_i c_i, \left[ c_i^{L_0}, c_i^{U_0} \right]. \tag{15}$$
$$m_{2\theta}^{\mu}(e_i, c_1) = \left( [m_{2\theta}^{W}(e_i, c_1), m_{2\theta}^{U}(e_i, c_1)], \right. \left. [m_{2\theta}^{L}(e_i, c_1), m_{2\theta}^{\mu}(e_i, c_1)] \right) d$$
$$= 1 - m_{\theta}(e_i, c_1) = 1 - w_i c_1.$$ (16)

where $m_{\theta}(e_i, c_1) = 0$ indicates that $\emptyset$ has no impact on case $\theta$; $m_{\theta}(e_i, c_1)$ indicates the assigned certainty degree caused by the evidence $(e_i, c_1)$; $m_{2\theta}(e_i, c_1)$ indicates the unassigned certainty degree caused by the evidence $(e_i, c_1)$; let the unassigned incertainty degree caused by the weight of evidence $(e_i, c_1)$ be $m_{2\theta}^{\mu}(e_i, c_1)$, then

$$\bar{m}_{2\theta}^{\mu}(e_i, c_1) = \left( [\bar{m}_{2\theta}^{W}(e_i, c_1), \bar{m}_{2\theta}^{U}(e_i, c_1)], \right. \left. [\bar{m}_{2\theta}^{L}(e_i, c_1), \bar{m}_{2\theta}^{\mu}(e_i, c_1)] \right).$$

$\bar{m}_{2\theta}^{\mu}(e_i, c_1) = 1 - w_i^{\prime}, \bar{m}_{2\theta}^{\mu}(e_i, c_1) = 1 - w_i^{\prime}. (17)$

Suppose that $O(t)$, $(t = 1, 2, \ldots, T)$ defines the first $t$ evidences; $m_{\theta}(O(t))$ is the basic probability mass obtained from the first $t$ evidences; $m_{2\theta}(O(t))$ indicates the unassigned synthetic certainty degree caused by the first $t$ evidences; $\bar{m}_{2\theta}(O(t))$ suggests the unassigned synthetic certainty degree caused by the weights of the first $t$ evidences.

When $t = 1$

$$m_{\theta}(O(1)) = \left( [m_{\theta}^{W}(O(1)), m_{\theta}^{U}(O(1))], \right. \left. [m_{\theta}^{L}(O(1)), m_{\theta}^{\mu}(O(1))] \right)$$
$$= m_{\mu}(e_1, c_1).$$ (18)

$$m_{\theta}^{L}(O(1)) = w_{1}^{L} c_1, \quad m_{\theta}^{U}(O(1)) = w_{1}^{U} c_1,$$
$$m_{\theta}^{L}(O(1)) = w_{1}^{U} c_1, \quad m_{\theta}^{L}(O(1)) = w_{1}^{L} c_1,$$
$$m_{2\theta}(O(1)) = \left( [m_{2\theta}^{W}(O(1)), m_{2\theta}^{U}(O(1))], \right. \left. [m_{2\theta}^{L}(O(1)), m_{2\theta}^{\mu}(O(1))] \right)$$
$$= m_{2\theta}(e_1, c_1),$$

$$m_{2\theta}^{L}(O(1)) = 1 - w_{1}^{U} c_1, \quad m_{2\theta}^{U}(O(1)) = 1 - w_{1}^{U} c_1,$$
$$m_{2\theta}^{L}(O(1)) = 1 - w_{1}^{L} c_1, \quad m_{2\theta}^{U}(O(1)) = 1 - w_{1}^{L} c_1.$$ (19)

When $t = 2, \ldots, T$, synthesize the first $t$ evidences and obtain the following formulas:

$$\text{(1) } m_{\theta}(O(t))$$

$$\text{max} \text{ / min : } m_{\theta}^{\mu}(O(t))$$

$$= m_{\theta}^{\mu}(O(t-1)) m_{\theta}^{\mu}(e_i, c_1) + m_{\theta}^{\mu}(O(t-1)) m_{\theta}^{\mu}(e_i, c_1) + m_{\theta}^{\mu}(O(t-1)) m_{\theta}^{\mu}(e_i, c_1);$$

$$\text{max} \text{ / min : } m_{\theta}(O(t))$$

$$= m_{\theta}^{\mu}(O(t-1)) m_{\theta}^{\mu}(e_i, c_1) + m_{\theta}^{\mu}(O(t-1)) m_{\theta}^{\mu}(e_i, c_1) + m_{\theta}^{\mu}(O(t-1)) m_{\theta}^{\mu}(e_i, c_1).$$ (20)

$$\text{s.t } m_{\theta}^{\mu}(O(t-1)) \leq m_{\theta}^{\mu}(O(t-1)) \leq m_{\theta}(O(t-1));$$
$$m_{\theta}(O(t-1)) \leq m_{\theta}^{\mu}(O(t-1)) \leq m_{\theta}(O(t-1));$$
$$m_{\theta}(e_i, c_1) \leq m_{\theta}^{\mu}(e_i, c_1) \leq m_{\theta}^{\mu}(e_i, c_1);$$
$$m_{\theta}(e_i, c_1) \leq m_{\theta}^{\mu}(e_i, c_1) \leq m_{\theta}(e_i, c_1);$$
$$m_{\theta}(e_i, c_1) \leq m_{\theta}(e_i, c_1) \leq m_{\theta}(e_i, c_1);$$
$$m_{\theta}(e_i, c_1) \leq m_{\theta}(e_i, c_1) \leq m_{\theta}(e_i, c_1).$$ (21)

Obviously, it is a typical non-linear programming problem. In the purpose of solving it, the simplified form of the problem is put forward.

Because

$$m_{\theta}^{\mu}(O(t-1)) + m_{\theta}^{\mu}(O(t-1)) = 1,$$
$$m_{\theta}^{\mu}(O(t-1)) + m_{\theta}^{\mu}(O(t-1)) = 1,$$
$$m_{\theta}^{\mu}(e_i, c_1) + m_{\theta}^{\mu}(e_i, c_1) = 1,$$
$$m_{\theta}^{\mu}(e_i, c_1) + m_{\theta}^{\mu}(e_i, c_1) = 1.$$ (22)

So

$$m_{\theta}^{\mu}(O(t-1)) = 1 - m_{\theta}^{\mu}(O(t-1));$$
$$m_{\theta}^{\mu}(O(t-1)) = 1 - m_{\theta}^{\mu}(O(t-1));$$
$$m_{\theta}^{\mu}(e_i, c_1) = 1 - m_{\theta}^{\mu}(e_i, c_1);$$
$$m_{\theta}^{\mu}(e_i, c_1) = 1 - m_{\theta}^{\mu}(e_i, c_1).$$ (23)

Therefore

$$m_{\theta}^{\mu}(O(t-1)) m_{\theta}^{\mu}(e_i, c_1) + m_{\theta}^{\mu}(O(t-1)) m_{\theta}^{\mu}(e_i, c_1) + m_{\theta}^{\mu}(O(t-1)) m_{\theta}^{\mu}(e_i, c_1) + m_{\theta}^{\mu}(O(t-1)) m_{\theta}^{\mu}(e_i, c_1)$$

$$= m_{\theta}^{\mu}(O(t-1)) m_{\theta}^{\mu}(e_i, c_1) + m_{\theta}^{\mu}(O(t-1)) m_{\theta}^{\mu}(e_i, c_1).$$
\[ m^\mu_{\Theta}(O(t)) = \min \left\{ m^\mu_{\Theta}(O(t)) \right\} \]
\[ m^\nu_{\Theta}(O(t)) = \min \left\{ m^\nu_{\Theta}(O(t)) \right\} \]

In addition
\[ m^L_{\Theta}(O(t)) = \min \left\{ m^\mu_{\Theta}(O(t)) \right\} \]
\[ = m^L_{\Theta}(O(t-1))m^\mu_{\Theta}(O(t)) \]
\[ + m^L_{\Theta}(O(t-1))m^\nu_{\Theta}(O(t)) \]
\[ m^U_{\Theta}(O(t)) = \max \left\{ m^\mu_{\Theta}(O(t)) \right\} \]
\[ = m^U_{\Theta}(O(t-1))m^\mu_{\Theta}(O(t)) \]
\[ + m^U_{\Theta}(O(t-1))m^\nu_{\Theta}(O(t)) \]

(2) \( m^\mu_{\Theta}(O(t)) \)

Similar to \( m^\mu_{\Theta}(O(t)) \), the following formulas about \( m^\nu_{\Theta}(O(t)) \) are obtained:
\[ \max / \min : m^\nu_{2\Theta}(O(t)) = m^\nu_{2\Theta}(O(t-1))m^\nu_{\Theta}(O(t)) \]
\[ \max / \min : m^\nu_{2\Theta}(O(t)) = m^\nu_{2\Theta}(O(t-1))m^\nu_{\Theta}(O(t)) \]

where
\[ m^\mu_{2\Theta}(O(t)) = \left[ m^L_{2\Theta}(O(t)), m^U_{2\Theta}(O(t)) \right] \]
\[ m^\nu_{2\Theta}(O(t)) = \left[ m^L_{2\Theta}(O(t)), m^U_{2\Theta}(O(t)) \right] \]

(3) \( \overline{m}^\mu_{\Theta}(O(t)) \)

\[ \max / \min : \overline{m}^\mu_{\Theta}(O(t)) = \overline{m}^\mu_{\Theta}(O(t-1))\overline{m}^\mu_{\Theta}(O(t)) \]
\[ \max / \min : \overline{m}^\nu_{\Theta}(O(t)) = \overline{m}^\nu_{\Theta}(O(t-1))\overline{m}^\nu_{\Theta}(O(t)) \]

(4) \( Icd(\Theta) \)

Synthesize all \( T \) evidences and get the certitude degree of case \( \Theta \):
\[ Icd(\Theta) = \left[ Icd^\mu(\Theta), Icd^\nu(\Theta) \right] \]
where

\[
Icd^{I_w}(\theta) = \frac{m_{I_w}^L((O(T))}{1 - m_{I_w}^L((O(T))} \\
= \frac{\prod_{t=1}^T (1 - (w_t^L(c_t)^L))}{1 - \prod_{t=1}^T (1 - (w_t)^L)}
\]

\[
Icd^{I_U}(\theta) = \frac{m_{I_U}^L((O(T))}{1 - m_{I_U}^L((O(T))} \\
= \frac{\prod_{t=1}^T (1 - (w_t^U(c_t)^U))}{1 - \prod_{t=1}^T (1 - (w_t)^U)}
\]

\[
Icd^{U_v}(\theta) = \frac{m_{I_v}^U((O(T))}{1 - m_{I_v}^U((O(T))} \\
= \frac{\prod_{t=1}^T (1 - (w_t)(c_t)^{U_v}))}{1 - \prod_{t=1}^T (1 - (w_t))(c_t)^{V}}
\]

V. BELIEF RULE BASE INFERENCE METHOD

Regarding the FADS fault detection problem, signals incline to fluctuate due to the presence of measurement noise. It seldom occurs that a relevant method can express randomness and uncertainty. For solving the problem, BRB and AIVIFSs are combined to depict the uncertain information.

Definition 10: The BRB model in the form of AIVIFSs is described as:

\[
IR = \langle X, A \rangle, \langle Y, C \rangle, ICD, \Omega, W \rangle
\]

where \( X = \{X_i|i = 1, 2, \cdots, I\} \) is the premise attribute set; \( A \) is the premise attribute value set of \( X_i \), i.e., \( A = \{A(X_i)|i = 1, 2, \cdots, I\} \); \( Icd(X_i)|i = 1, 2, \cdots, I \rangle = \{Icd(X_i)|i = 1, 2, \cdots, I\} \); \( Y = \{Y_j|j = 1, 2, \cdots, J\} \); \( \Omega = \{w^1, w^2, \cdots, w^K\} \) is the weight vector of rules and \( 0 \leq w^k \leq 1; W = \{w_1, w_2, \cdots, w_I\} \) is the weight vector of premise attributes and \( 0 \leq w_j \leq 1; \sum_{j=1}^J w_j = 1 \).

Definition 11: The kth rule \( IR^k \) in the BRB model IR = \( \langle X, A \rangle, \langle Y, C \rangle, ICD, \Omega, W \rangle \) means:

If \( X_i = A_k \), \( Icd^k(X_i) = A_k \rangle \) then \( Y_j = C_j \), \( Icd^k(Y_j) = C_j \rangle \) with \( Icd^k(R^k), w^k, w_i \).
and

\[ w_{IR}^k = \left(\left[w_{IR}^k, \sum_{k=1}^{K} w_{IR}^k, (w_{IR}^k)^2\right]\right) \left(\left(\gamma k\right)^2, \sum_{k=1}^{K} (\gamma k)^2, (\gamma k)^2\right), \right. \]

\[ \left(\left(\gamma k\right)^2, \sum_{k=1}^{K} (\gamma k)^2, (\gamma k)^2\right) \left(\left(\hat{\alpha}^k\right)^2, \sum_{k=1}^{K} (\hat{\alpha}^k)^2, (\hat{\alpha}^k)^2\right) \right) \right] \right). \]

where \( w_{IR}^k \) is the interval-valued weights of rule \( IR^k \) and \( w_k^k \) is the interval-valued weights of premise attributes \( \wedge A^k \), \( \sum_{k=1}^{K} w_{IR}^k \leq 1, \sum_{k=1}^{K} (w_{IR}^k)^2 \leq 1, \sum_{k=1}^{K} w_k^k \geq 1 \) and \( \sum_{k=1}^{K} w_k^k \geq 1 \).

Exploiting the Definition 9 to derive the basic portability masses formula:

\[ m_{cgb}(\{IR^k, y^k\}) = \left(\left[m_{cgb}(\{IR^k, y^k\}), m_{cgb}(\{IR^k, y^k\}), \right] \left[\left[m_{cgb}(\{IR^k, y^k\}), m_{cgb}(\{IR^k, y^k\})\right] \right] \right. \]

\[ \left(\left[m_{cgb}(\{IR^k, y^k\}), m_{cgb}(\{IR^k, y^k\}), \right] \left[\left[m_{cgb}(\{IR^k, y^k\}), m_{cgb}(\{IR^k, y^k\})\right] \right] \right). \]

where \( m_{cgb}(\{IR^k, y^k\}) \) represents the assigned basic probability mass caused by \( IR^k \); \( m_{cgb}(\{IR^k, y^k\}) \) represents the unassigned basic probability mass caused by \( IR^k; m_{cgb}(\{IR^k, y^k\}) \) represents the unassigned basic probability mass caused by the activation interval-valued weights of \( IR^k \).

By Similarity,

\[ m_{cgb}(\{\wedge A^k, \hat{\alpha}^k\}) = \left(\left[m_{cgb}(\{\wedge A^k, \hat{\alpha}^k\}), m_{cgb}(\{\wedge A^k, \hat{\alpha}^k\}), \right] \left[\left[m_{cgb}(\{\wedge A^k, \hat{\alpha}^k\}), m_{cgb}(\{\wedge A^k, \hat{\alpha}^k\})\right] \right] \right. \]

\[ \left(\left[m_{cgb}(\{\wedge A^k, \hat{\alpha}^k\}), m_{cgb}(\{\wedge A^k, \hat{\alpha}^k\}), \right] \left[\left[m_{cgb}(\{\wedge A^k, \hat{\alpha}^k\}), m_{cgb}(\{\wedge A^k, \hat{\alpha}^k\})\right] \right] \right). \]

where \( m_{cgb}(\{\wedge A^k, \hat{\alpha}^k\}) \) represents the assigned basic probability mass caused by \( \wedge A^k \); \( m_{cgb}(\{\wedge A^k, \hat{\alpha}^k\}) \) indicates the unassigned basic probability mass caused by \( \wedge A^k \); \( m_{cgb}(\{\wedge A^k, \hat{\alpha}^k\}) \) represents the unassigned basic probability mass caused by the activation interval-valued weights of \( \wedge A^k \).

Combine uncertainty of evidence to obtain the composite probability mass:

\[ m_{cgb}(\{\{IR^k, y^k\}, \{\wedge A^k, \hat{\alpha}^k\}\}) = \left(\left[m_{cgb}(\{\{IR^k, y^k\}, \{\wedge A^k, \hat{\alpha}^k\}\}), m_{cgb}(\{\{IR^k, y^k\}, \{\wedge A^k, \hat{\alpha}^k\}\}), \right] \left[\left[m_{cgb}(\{\{IR^k, y^k\}, \{\wedge A^k, \hat{\alpha}^k\}\}), m_{cgb}(\{\{IR^k, y^k\}, \{\wedge A^k, \hat{\alpha}^k\}\})\right] \right] \right. \]

\[ \left(\left[m_{cgb}(\{\{IR^k, y^k\}, \{\wedge A^k, \hat{\alpha}^k\}\}), m_{cgb}(\{\{IR^k, y^k\}, \{\wedge A^k, \hat{\alpha}^k\}\})\right] \right). \]
where
\[ m_\text{L}^{k\mu} = m_\text{L}^{k\nu} (IR^k, \nu^k) m_\text{L}^{k\mu} (\land A^k, \alpha^k); \]
\[ m_\text{U}^{k\mu} = m_\text{U}^{k\nu} (IR^k, \nu^k) m_\text{L}^{k\mu} (\land A^k, \alpha^k); \]
\[ m_\text{L}^{k\nu} = m_\text{L}^{k\nu} (IR^k, \nu^k) m_\text{U}^{k\nu} (\land A^k, \alpha^k); \]
\[ m_\text{U}^{k\nu} = m_\text{U}^{k\nu} (IR^k, \nu^k) m_\text{U}^{k\nu} (\land A^k, \alpha^k). \]
(63)

The interval-valued intuitionistic certitude degree of \( C^k \) is:
\[
\beta^k = \left( (\tilde{\beta}^k)_{Y^\mu}, (\tilde{\beta}^k)_{U^\mu}, (\tilde{\beta}^k)_{Y^\nu}, (\tilde{\beta}^k)_{U^\nu} \right) = \left( \min \left\{ \frac{m_\text{L}^{k\mu}}{1 - m_\text{U}^{k\mu}}, 1 \right\}, \min \left\{ \frac{m_\text{L}^{k\nu}}{1 - m_\text{U}^{k\nu}}, 1 \right\} \right); \]
(64)

where
\[
(\tilde{\beta}^k)_{Y^\mu} = \min \left\{ \frac{m_\text{L}^{k\mu}}{1 - m_\text{U}^{k\mu}}, 1 \right\}; \\
(\tilde{\beta}^k)_{U^\mu} = \min \left\{ \frac{m_\text{L}^{k\mu}}{1 - m_\text{U}^{k\mu}}, 1 \right\}; \\
(\tilde{\beta}^k)_{Y^\nu} = \min \left\{ \frac{m_\text{L}^{k\nu}}{1 - m_\text{U}^{k\nu}}, 1 \right\}; \]
and
\[
(\tilde{\beta}^k)_{U^\nu} = \min \left\{ \frac{m_\text{L}^{k\nu}}{1 - m_\text{U}^{k\nu}}, 1 \right\}. \]
(65)

Obtain the interval-valued intuitionistic certainty degree of conclusion attributes on the basis of \( Input^k, \tilde{\beta}^k \) and the method shown in Definition 5:
\[
\tilde{\beta}_j^k = \left( (\tilde{\beta}_j^k)_{Y^\mu}, (\tilde{\beta}_j^k)_{U^\mu}, (\tilde{\beta}_j^k)_{Y^\nu}, (\tilde{\beta}_j^k)_{U^\nu} \right); \]
(66)

where
\[
(\tilde{\beta}_j^k)_{Y^\mu} = \min \left\{ \tilde{\beta}_j^k (\tilde{\beta}_j^k)^{Y^\mu}, 1 \right\}; \\
(\tilde{\beta}_j^k)_{U^\mu} = \min \left\{ \tilde{\beta}_j^k (\tilde{\beta}_j^k)^{U^\mu}, 1 \right\}; \\
(\tilde{\beta}_j^k)_{Y^\nu} = \min \left\{ \tilde{\beta}_j^k (\tilde{\beta}_j^k)^{Y^\nu}, 1 \right\}; \]
and
\[
(\tilde{\beta}_j^k)_{U^\nu} = \min \left\{ \tilde{\beta}_j^k (\tilde{\beta}_j^k)^{U^\nu}, 1 \right\}. \]
(67)

\[ VI. \ ALGORITHM \ PROCESS \]

In this section, an algorithm based on AIVIFSs, BRB, and ER is proposed to solve the fault detection problem.

**Step 1:** Obtain the attribute weight vector \( w_i \), \( i = 1, \ldots, I \).

Usually, the weight vector is given according to the importance of different attributes. The more important the premise attribute, the greater its weight.

**Step 2:** Determine if the input matches the rule \( IR^k \).
If \( A_i^k = a_i \) or \( A_i^k = \phi \), the Input matches the rule \( IR^k \) and \( IR^k \) is activated successfully.

**Step 3:** Compare the similarity between AIVIFSs.
Use Definition 5 to obtain the similarity between the AIVIFSs indicating the input and the AIVIFSs indicating premise attributes.

**Step 4:** Obtain the certitude degree of attributes \( \tilde{\alpha}^k \).
Utilize Definition 12 to obtain \( \tilde{\alpha}^k \).

**Step 5:** Calculate the activation weights \( w_A^k \) of premise attributes and the weights \( w_{IR}^k \).

Use the certitude degree of attributes and certainty degree of each rule to get the activation weights.
\[
w_A^k = \left( \left( w_A^k \right)^{Y^\mu}, \left( w_A^k \right)^{U^\mu}, \left( w_A^k \right)^{Y^\nu}, \left( w_A^k \right)^{U^\nu} \right) \]
\[
= \left( \left( \tilde{\alpha}^k \right)^{Y^\mu} + \left( \tilde{\alpha}^k \right)^{U^\mu}, \left( \tilde{\alpha}^k \right)^{Y^\nu} + \left( \tilde{\alpha}^k \right)^{U^\nu} \right). \]
(69)

and
\[
w_{IR}^k = \left( \left( w_{IR}^k \right)^{Y^\mu}, \left( w_{IR}^k \right)^{U^\mu}, \left( w_{IR}^k \right)^{Y^\nu}, \left( w_{IR}^k \right)^{U^\nu} \right) \]
\[
= \left( \left( \tilde{\alpha}^k \right)^{Y^\mu} + \left( \tilde{\alpha}^k \right)^{U^\mu}, \left( \tilde{\alpha}^k \right)^{Y^\nu} + \left( \tilde{\alpha}^k \right)^{U^\nu} \right). \]
(70)

**Step 6:** Obtain the certitude degree of \( \bigwedge C^k \).
Combine the uncertainty of evidence and acquire the composite probability masses \( m_{\text{L}}^{k\mu} \) and \( m_{\text{R}}^{k\mu} \). The certitude degree of conclusion can be obtained by (64) and (65).

**Step 7:** Obtain the certainty degree \( \tilde{\beta}_j^k \) of conclusion attributes under the input condition.

Utilize the approach of calculating similarity and the certainty degree of each conclusion attribute in the rule, obtain the certitude degree of the conclusion attribute under the input condition by (66), (67), and (68).

**Step 8:** Determine the final fault detection result.
Utilize the score function proposed in Definition 8 to rank the conclusion attributes, and then, regard the conclusion attribute with the highest score as the final result.

\[ VII. \ ILLUSTRATIVE \ EXAMPLE \]

The method introduced in Section VI should be put into practice; therefore, suppose a FADS fault situation to verify the feasibility of the method. Assume that the actual angle of attack \( \alpha_0 \) is 5°; after many tests, we collect and summarize...
all measurement data under normal condition, the results are illustrated in Fig. 2. We can see that $\alpha_1$, $\alpha_2$, $\alpha_3$, and $\alpha_4$ fluctuate within a reasonable range due to the presence of measurement noise. In addition, as can be seen from the figure, $\alpha_2$ and $\alpha_3$ are farther from the actual value $\alpha_4$ and the measurement accuracy is not as good as that of $\alpha_1$ and $\alpha_4$; therefore, we set attribute weights $W = \{w_1, w_2, w_3, w_4\} = \{0.3, 0.2, 0.2, 0.3\}$ to express the difference.

**FIGURE 2.** Measurement results under normal condition.

However, once the measurement tap is faulty, all measurement results will be abnormal. Given that there are many fault conditions, four typical fault conditions are analyzed to prove the feasibility of the proposed model: 1) tap 1 faults; 2) tap 3 faults; 3) tap 1 and tap 5 fault; 4) tap 3 and tap 6 fault. Fig. 3 highlights that when different taps fault, $\alpha_1$, $\alpha_2$, $\alpha_3$, and $\alpha_4$ will have different performance.

**FIGURE 3.** Measurement results under faulty conditions.

First, set domains $d_1 = [-2^\circ, 1^\circ]$, $d_2 = [1^\circ, 4^\circ]$, $d_3 = [4^\circ, 7^\circ]$, $d_4 = [7^\circ, 10^\circ]$ and $d_5 = [10^\circ, 13^\circ]$, which are given subjectively and will not affect the results.

Then, the premise attributes can be obtained:

- Tap 1 fails: $\alpha_1 = \langle d_3, [0.5, 0.6] \rangle$, $\alpha_2 = \langle d_2, [0.55, 0.6] \rangle$, $\alpha_3 = \langle d_5, [0.51, 0.54] \rangle$ and $\alpha_4 = \langle d_2, [0.55, 0.63] \rangle$.
- Tap 2 fails: $\alpha_1 = \langle d_3, [0.5, 0.6] \rangle$, $\alpha_2 = \langle d_2, [0.55, 0.6] \rangle$, $\alpha_3 = \langle d_5, [0.51, 0.54] \rangle$ and $\alpha_4 = \langle d_2, [0.55, 0.63] \rangle$.
- Tap 3 and tap 5 fail: $\alpha_1 = \langle d_3, [0.5, 0.6] \rangle$, $\alpha_2 = \langle d_2, [0.55, 0.6] \rangle$, $\alpha_3 = \langle d_5, [0.51, 0.54] \rangle$ and $\alpha_4 = \langle d_2, [0.55, 0.63] \rangle$.
- Tap 1 and tap 5 fail: $\alpha_1 = \langle d_3, [0.5, 0.6] \rangle$, $\alpha_2 = \langle d_2, [0.55, 0.6] \rangle$, $\alpha_3 = \langle d_5, [0.51, 0.54] \rangle$ and $\alpha_4 = \langle d_2, [0.55, 0.63] \rangle$.

Tap 3 fails: $\alpha_1 = \langle d_3, [1, 1] \rangle$, $\alpha_2 = \langle d_3, [1, 1] \rangle$, $\alpha_3 = \langle d_4, [0.6, 0.65] \rangle$, $\alpha_4 = \langle d_3, [1, 1] \rangle$.

Tap 1 and tap 5 fail: $\alpha_1 = \langle d_3, [0.9, 0.96] \rangle$, $\alpha_2 = \langle d_2, [0.64, 0.7] \rangle$, $\alpha_3 = \langle d_5, [0.59, 0.65] \rangle$, $\alpha_4 = \langle d_2, [0.8, 0.9] \rangle$.

Tap 3 and tap 6 fail: $\alpha_1 = \langle d_3, [0.6, 0.7] \rangle$, $\alpha_2 = \langle d_2, [0.78, 0.84] \rangle$, $\alpha_3 = \langle d_5, [0.66, 0.76] \rangle$, $\alpha_4 = \langle d_2, [0.78, 0.84] \rangle$.

Normal condition: $\alpha_1 = \langle d_3, [1, 1] \rangle$, $\alpha_2 = \langle d_3, [1, 1] \rangle$, $\alpha_3 = \langle d_3, [1, 1] \rangle$, $\alpha_4 = \langle d_3, [1, 1] \rangle$.

The approach of determining the lower and upper boundaries of these intervals goes through roughly two steps: analyze the proportion of all measurement data in domains $d_1, d_2, d_3, d_4, d_5$ and obtain a specific proportion value; then, extend the proportion value into interval form on the basis of many tests and general understandings of the situation to express the randomness and uncertainty. Similar to it, the lower and upper boundaries of conclusion attributes, and certainty degrees can be obtained. The BRB model in the forms of AIVIFSs can be constructed, as shown in Table 1.

### A. SIMULATION RESULTS UNDER A CERTAIN INPUT

In order to illustrate the flow and the rationality of the method, an example based on a certain input is going to be performed.

Assume that $\text{Input} = \{a_1 = (d_3, [0.7, 0.75], [0.1, 0.2]), a_2 = (d_2, [0.65, 0.7], [0.2, 0.25]), a_3 = (d_5, [0.55, 0.62], [0.34, 0.39]), a_4 = (d_2, [0.78, 0.8], [0.08, 0.09])\}$.

**Step 1:** Obtain the attribute weight vector $w_i (i = 1, \ldots, I)$.

The weight vector has been gotten according to the measurement accuracy, $w_1 = 0.3$, $w_2 = 0.2$, $w_3 = 0.2$, and $w_4 = 0.3$.

**Step 2:** Determine if the input matches the rule $IR^k$.

For the $\text{Input}$, $\{a_1, a_2, a_3, a_4\} = \{d_3, d_2, d_5, d_2\}$, which is the same as the premise attributes of rules $IR^1$, $IR^3$, and $IR^4$; therefore, the input is considered to match the three rules successfully.

**Step 3:** Compare the similarity between AIVIFSs.

By using **Definition 5** to obtain the similarity between the input and premise attributes of rules $IR^1$, $IR^3$ and $IR^4$, represented by $\tilde{\delta}_1^i$, $\tilde{\delta}_2^i$ and $\tilde{\delta}_4^i$.

\[
\begin{align*}
\tilde{\delta}_1^i &= ([0.8, 0.85], [0.8, 0.85]), \\
\tilde{\delta}_2^i &= ([0.9, 0.9], [0.85, 0.9]) \\
\tilde{\delta}_3^i &= ([0.92, 0.96], [0.94, 0.94]), \\
\tilde{\delta}_4^i &= ([0.77, 0.82], [0.69, 0.76]) \\
\tilde{\delta}_5^i &= ([0.79, 0.8], [0.87, 0.9]) \\
\tilde{\delta}_6^i &= ([0.99, 1], [0.9, 0.93]) \\
\tilde{\delta}_7^i &= ([0.96, 0.97], [0.89, 0.94]), \\
\tilde{\delta}_8^i &= ([0.91, 0.98], [0.94, 0.98]) \\
\tilde{\delta}_9^i &= ([0.9, 0.95], [0.98, 1]), \\
\tilde{\delta}_10^i &= ([0.86, 0.87], [0.92, 0.93])
\end{align*}
\]
TABLE 1. Belief rule base model of fads system when actual angle of attack is 5°.

| Rule | $\theta$ | $\tilde{X}$ | $\tilde{Y}$ | $\tilde{Y}_j$ | $\tilde{Y}_k$ | $\tilde{Y}_l$ | $\tilde{Y}_m$ | $\tilde{Y}_n$ | $\tilde{Y}_o$ | $\tilde{Y}_p$ |
|------|----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 1 $\theta^1$ | $\theta^2$ | $\theta^3$ | $\theta^4$ | $\theta^5$ | $\theta^6$ | $\theta^7$ | $\theta^8$ | $\theta^9$ | $\theta^{10}$ | $\theta^{11}$ |
| 2 $\theta^2$ | $\theta^1$ | $\theta^3$ | $\theta^4$ | $\theta^5$ | $\theta^6$ | $\theta^7$ | $\theta^8$ | $\theta^9$ | $\theta^{10}$ | $\theta^{11}$ |
| 3 $\theta^3$ | $\theta^1$ | $\theta^2$ | $\theta^4$ | $\theta^5$ | $\theta^6$ | $\theta^7$ | $\theta^8$ | $\theta^9$ | $\theta^{10}$ | $\theta^{11}$ |
| 4 $\theta^4$ | $\theta^1$ | $\theta^2$ | $\theta^3$ | $\theta^5$ | $\theta^6$ | $\theta^7$ | $\theta^8$ | $\theta^9$ | $\theta^{10}$ | $\theta^{11}$ |
| 5 $\theta^5$ | $\theta^1$ | $\theta^2$ | $\theta^3$ | $\theta^4$ | $\theta^6$ | $\theta^7$ | $\theta^8$ | $\theta^9$ | $\theta^{10}$ | $\theta^{11}$ |

Step 4: Obtain the certitude degree of attributes $\tilde{\alpha}^k$. Given that

$$\tilde{\alpha}_1 = 0.3, \quad \tilde{\alpha}_2 = 0.2, \quad \tilde{\alpha}_3 = 0.3$$

therefore

$$\tilde{\alpha}_1 = 1, \quad \tilde{\alpha}_2 = 1, \quad \tilde{\alpha}_3 = 1$$

and

$$\tilde{\alpha}_4 = 0.96, \quad \tilde{\alpha}_5 = 0.99$$

Step 5: Calculate the activation weights $w_A^k$ of premise attributes and the weights $w_{IR}^k$. Apply the ceritude degree of attributes and each rule to get the activation weights.

$$w_A^1 = 0.49, \quad w_A^2 = 0.53, \quad w_A^3 = 0.81, \quad w_A^4 = 0.90$$

Step 6: Obtain the certitude degree of $\bigwedge C^k$. The ceritude degree of the conclusions can be obtained by (46) and (65).

$$m_{\tilde{y}_1}^{\mu} = 0.64, \quad m_{\tilde{y}_2}^{\mu} = 0.71, \quad m_{\tilde{y}_3}^{\mu} = 0.67, \quad m_{\tilde{y}_4}^{\mu} = 0.79$$

Step 7: Obtain the ceritude degree $\tilde{p}_j^k$ of conclusion attributes under the input condition. Utilize the approach of calculating similarity and the ceritude degree of each conclusion attribute, obtain the ceritude degree of the conclusion attribute under the input fact condition by (66), (67), and (68).

$$\tilde{p}_1^1 = 0.86, \quad \tilde{p}_2^1 = 0.89, \quad \tilde{p}_3^1 = 0.89, \quad \tilde{p}_4^1 = 0.88, \quad \tilde{p}_5^1 = 0.90, \quad \tilde{p}_6^1 = 0.95$$

Step 8: Determine the final fault detection result. Using the proposed score function, we can get:

$$J(\tilde{p}_1^1) = 0.662, \quad J(\tilde{p}_2^1) = 0.654, \quad J(\tilde{p}_3^1) = 0.668.$$
Let us take an example to explain the Table. When the input is \textit{Input 1}, the premise attributes do not match the rules $IR_2$ and $IR_5$. As a result, scores are 0 for the two rules. Then, by utilizing the BRB model, scores of the three rules $IR_1$, $IR_3$, and $IR_4$ can be obtained respectively. The rest situations are similar to this. It can be found that the rankings are the same with the conclusion attributes in the BRB model for the five inputs, which shows that the established model is reasonable to some extent.

| Input | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ | ranking of results |
|-------|-------|-------|-------|-------|-------|-------------------|
| Input1 | 0.6908 | 0 | 0.6213 | 0.6590 | 0 | $c_1 > c_4 > c_3 > c_2 = c_5$ |
| Input2 | 0 | 1 | 0 | 0 | 0 | $c_3 > c_1 = c_3 = c_4 = c_5$ |
| Input3 | 0.6573 | 0 | 0.6710 | 0.6599 | 0 | $c_3 > c_4 > c_3 > c_2 = c_5$ |
| Input4 | 0.6625 | 0 | 0.6432 | 0.6870 | 0 | $c_4 > c_1 > c_3 > c_2 = c_5$ |
| Input5 | 0 | 0 | 0 | 0 | 1 | $c_5 > c_1 = c_3 = c_3 = c_4$ |

### C. NUMERICAL SIMULATION RESULTS UNDER AN ACTUAL SITUATION

In order to further confirm the feasibility of this model in solving the fault detection problem of the FADS system, we conduct a numerical simulation. In this case study, the actual angle of attack $\alpha_a = 5^\circ$, the sampling frequency is 50Hz, and the variance of noise for the four measurement taps is the same, which is 40. As we can see in Fig. 4, during the first 5 seconds, pressures of the four measurement taps fluctuate within a reasonable range. However, at the 5th second, tap 3 and tap 6 fault, their readings are reduced by 2000 [Pa], the remaining measurement taps are still normal. Resulting parameters $\alpha_1$, $\alpha_2$, $\alpha_3$ and $\alpha_4$ are shown in Fig. 5, from which we can see that results remain normal during the first 5 seconds; however, subsequent measurement results are abnormal because of the fault of tap 3 and tap 6.

### D. COMPARATIVE STUDIES

For illustrating the rationality of the proposed model, a comparison is going to be performed with the other two most widely used fault detection methods in FADS system [36], [37]. The comparison analysis is based on the same illustrative example.

1) PARITY EQUATION METHOD

For FADS, when it is normal, (71) is always true, which is derived from the mathematical formulas of the system.
The left side of the formula is regarded as the parity equation. However, because of the measurement noise, the parity equation cannot always be equal to 0; so, experts put forward an efficient solution to the problem, i.e., select a monitoring threshold $O_p$. The system can still be considered normal as long as (72) is achieved.

$$
\begin{bmatrix}
\cos^2 \theta_k - \cos^2 \theta_j \\
\cos^2 \theta_i - \cos^2 \theta_k \\
\cos^2 \theta_j - \cos^2 \theta_i
\end{bmatrix}^T
\begin{bmatrix}
P_i \\
P_j \\
P_k
\end{bmatrix}
= 0. \\
(71)
$$

$$
\begin{bmatrix}
\cos^2 \theta_k - \cos^2 \theta_j \\
\cos^2 \theta_i - \cos^2 \theta_k \\
\cos^2 \theta_j - \cos^2 \theta_i
\end{bmatrix}^T
\begin{bmatrix}
P_i \\
P_j \\
P_k
\end{bmatrix}
\leq |O_p|. \\
(72)
$$

where $\theta_i$, $\theta_j$, and $\theta_k$ stand for the airflow incidence angles of measurement taps, which are affected by the measurement angle of attack; $P_i$, $P_j$, and $P_k$ represent the pressures of the measurement taps; $O_p$ can be set according to the engineering experience.

For this example, the values of the parity equation are shown graphically in Fig. 7. Obviously, the values fluctuate around 0 during the first five seconds, and then when tap 3 and tap 6 fault, the values appear abnormal. It can be seen that $-100$ is an advisable choice to be the monitoring threshold, which is able to distinguish between normal conditions and faulty conditions.

2) CHI-SQUARE $\chi^2$ DISTRIBUTION METHOD

The working principle of this method is that the difference between the pressures obtained from the measurement taps and the pressures derived from the pressure model is statistically different under normal and faulty situations.

In the case of no-fault, the residual of a single tap is subject to the normal distribution after a large number of tests in [38]; therefore, the sum of squares of the residuals belonging to the four taps should roughly obey the distribution of $\chi^2(4)$.

The process based on the method goes through roughly two steps: First, determine whether the system has failed; followed by determine which tap is faulty. According to the characteristics of $\chi^2(4)$, if $\chi^2 = 7.78$, the probability of system failure is more than 90% and the FADS system is considered faulty; on the contrary, the FADS system is normal. After identifying the system failure, the measurement taps will be checked one by one and recalculate the chi-square values, if the chi-square values return to normal, then the excluded pressure tap is the faulty one.

As shown in Fig. 8, chi-square values are always less than 7.78, which indicates that the system is normal in the first 5 seconds. Then, the chi-square values are abnormal, and the system is detected to fault. According to the test procedure, we exclude all the measurement taps one by one, and Fig. 9 shows the chi-square values obtained by removing both tap 3 and tap 6. It can be seen that the values return to normal and the system is not faulty, which indicates that the two taps fault at the 5th second.

E. DISCUSSION

Based on the above researches, it is clear that the parity equation method, $\chi^2$ distribution method and the proposed method
are applicable. However, as known for FADS system, there are various factors that will generate measurement noise, including temperature, humidity, and even air density. The variance of the noise is randomly changing and the value may be large. Hence, we consider comparing the accuracy of these three methods in the face of noise with different variance.

For the parity equation method, when the variance increases, the values of the parity equation will fluctuate with greater amplitude under both normal and faulty conditions. For example, when the variance is equal to 90, the values of the parity equation are shown in Fig. 10, and we can see that it is impossible to distinguish the two cases by setting a suitable monitoring threshold. So, the method is not applicable when the variance of the noise is relatively large.

![FIGURE 10. Values of the parity equation when the variance is equal to 90.](image)

As is known to all, the stability of a series of data and the variance are negatively correlated, i.e., the stability of data decreases when the variance increases. When it comes to the $\chi^2$ distribution method, the probability that describes the abnormal values of pressure residuals will inevitably increase when the measurement noise increases, which will lead to the fact that the sum of squares after the normalization of the residuals don’t particularly fit $\chi^2(4)$ and affect the detect accuracy. All in all, these methods cannot adequately express the randomness and uncertainty caused by the measurement noise.

Unlike the above two methods, the proposed method tends to apply AIVIFSs and BRB to summarize the distribution law of data, rather than focusing on a single point. In other words, this method better reflects the randomness and uncertainty in the fault detection problem; therefore, even if the variance increases, this method will not be greatly affected.

VIII. CONCLUSION

In this paper, a fault detection model for the FADS system is introduced. Generally speaking, this paper makes four contributions with respect to the existing studies. First, a new approach to calculate the similarity between AIVIFSs is proposed. Next, for measuring the information contained in AIVIFSs, a new score function is put forward. Then, Atanassov’s interval-valued intuitionistic fuzzy sets, belief rule base and evidential reasoning is combined to express the randomness and uncertainty of fault detection problem. Finally, the proposed model is firstly applied to deal with the fault detection problem of FADS system, an important airborne sensor. To verify the effectiveness of the proposed method, a comprehensive comparative analysis is conducted with the other two methods that have been successfully applied for solving the same problem. In future research, we will consider the combination of the BRB model with other artificial intelligence algorithms.

APPENDIX

PROOF PROCESSED OF SIMILARITY BETWEEN AIVIFSs

Proof 1: Given three AIVIFSs $A = \{ (x, \mu_A(x), \nu_A(x)) | x \in X \}$, $B = \{ (x, \mu_B(x), \nu_B(x)) | x \in X \}$, and $C = \{ (x, \mu_C(x), \nu_C(x)) | x \in X \}$, where $(\mu_A(x), \nu_A(x)) = ([\mu^L_A(x), \mu^U_A(x)], [\nu^L_A(x), \nu^U_A(x)])$, $(\mu_B(x), \nu_B(x)) = ([\mu^L_B(x), \mu^U_B(x)], [\nu^L_B(x), \nu^U_B(x)])$, and $(\mu_C(x), \nu_C(x)) = ([\mu^L_C(x), \mu^U_C(x)], [\nu^L_C(x), \nu^U_C(x)])$, the proof process is as follows:

1) $0 \leq S(A, B) \leq 1$.

Because

$$0 \leq \mu^L_A(x) \leq 1, \quad 0 \leq \mu^U_A(x) \leq 1;$$
$$0 \leq \nu^L_A(x) \leq 1, \quad 0 \leq \nu^U_A(x) \leq 1;$$
$$0 \leq \mu^L_B(x) \leq 1, \quad 0 \leq \mu^U_B(x) \leq 1;$$
$$0 \leq \nu^L_B(x) \leq 1, \quad 0 \leq \nu^U_B(x) \leq 1.$$

So

$$-1 \leq \mu^L_A(x) - \mu^L_B(x) \leq 1, \quad -1 \leq \mu^U_A(x) - \mu^U_B(x) \leq 1;$$
$$-1 \leq \nu^L_A(x) - \nu^L_B(x) \leq 1, \quad -1 \leq \nu^U_A(x) - \nu^U_B(x) \leq 1;$$
$$0 \leq (\mu^L_A(x) - \mu^L_B(x))^2 \leq 1, \quad 0 \leq (\mu^U_A(x) - \mu^U_B(x))^2 \leq 1;$$
$$0 \leq (\nu^L_A(x) - \nu^L_B(x))^2 \leq 1, \quad 0 \leq (\nu^U_A(x) - \nu^U_B(x))^2 \leq 1.$$

Therefore, $S(A, B) \in [0, 1]$.  

2) If $A = B$, then $S(A, B) = 1$.

If

$$\mu^L_A(x) = \mu^L_B(x), \quad \mu^U_A(x) = \mu^U_B(x),$$
$$\nu^L_A(x) = \nu^L_B(x), \quad \nu^U_A(x) = \nu^U_B(x).$$

So

$$\min \left\{ \min \{ 1 - \mu^L_A(x) + \mu^L_B(x), 1 - \mu^U_A(x) + \mu^U_B(x) \} \right\} = 1;$$
$$\max \left\{ \min \{ 1 - \mu^L_A(x) + \mu^U_B(x), 1 - \mu^U_A(x) + \mu^L_B(x) \} \right\} = 1;$$
$$\min \left\{ \min \{ 1 - \mu^L_A(x) + \mu^L_B(x), 1 - \mu^U_A(x) + \mu^U_B(x) \} \right\} = 1;$$
$$\min \left\{ \min \{ 1 - \nu^L_A(x) + \nu^L_B(x), 1 - \nu^U_A(x) + \nu^U_B(x) \} \right\} = 1;$$
$$\min \left\{ \min \{ 1 - \nu^L_A(x) + \nu^U_B(x), 1 - \nu^U_A(x) + \nu^L_B(x) \} \right\} = 1;$$
$$\min \left\{ \min \{ 1 - \nu^L_A(x) + \nu^U_B(x), 1 - \nu^U_A(x) + \nu^L_B(x) \} \right\} = 1;$$

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\[ \min \{1 - v_A^U(x) + v_B^U(x), 1 - v_A^L(x) + v_B^L(x)\} = 1; \]
\[ \max \{1 - v_A^U(x) + v_B^U(x), 1 - v_A^L(x) + v_B^L(x)\} = 1. \]

3) \( S(A, B) = S(B, A) \).
\[
S(A, B) = \left( \left[ \begin{array}{c} S_A^L(A, B), S_B^L(A, B) \\ S_A^U(A, B), S_B^U(A, B) \end{array} \right] \right)
\]
\[= \left[ \begin{array}{c} \min \{1 - \mu_A^L(x) + \mu_B^L(x), 1 - \mu_A^L(x) + \mu_B^L(x)\} \\ \max \{1 - \mu_A^L(x) + \mu_B^L(x), 1 - \mu_A^L(x) + \mu_B^L(x)\} \end{array} \right]. \]

So
\[ S(A, C) \leq \min \left\{ S(A, B), S(B, C) \right\}. \]

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