Interdisciplinarity, Mathematical Modelling, and Poincare’s work: Comparing conceptions about knowledge construction

J Huincahue and K Vilches
1Departamento de Matemática, Física y Estadística, Universidad Católica del Maule, Talca, Chile
E-mail: jhuincahue@ucm.cl, karinavilch@gmail.com

Abstract. In this work we have applied the thematic analysis methodology to extract the most relevant conceptions about mathematical modelling practices using the collected opinions from a interdisciplinary team of researchers. The main feature of the team is the use of mathematical modelling in its practice to solve applied problems of the real world. So, five conceptions are obtained, and then all of them were recognized in two of the works published by Poincare’s. Consequently, these conceptions represent some key components of the interdisciplinary work supported by mathematical modelling practice.

1. Introduction

1.1. Interdisciplinarity and academic work

Disciplines are defined as intellectual structures, created to facilitate knowledge transfer from one generation to another through the educational system [1]. Interdisciplinarity concept emerges from the juxtaposition of two or more disciplines that work out problems mutually recognized, with the objective of obtaining novel and relevant solutions [2]. In this regard, the interdisciplinarity becomes a fundamental method to better understand complex problems of the real world. For such reason, some university institutions are focusing their efforts in training academic professionals and researchers with new skills and knowledge to face this complex reality, including the promotion of interdisciplinary practices [3]. From an epistemological perspective, the interdisciplinarity vision was first recognized in the middle of the 20Th century, in which the new reality demand the construction of new knowledge in were a single niche perspective was limited [4, 5]. An extensive literature is available about the interdisciplinarity foundations, including collaboration models [6], research strategies [7], spaces to learn, thinking, research or teach [8], recommendations for funding agencies and universities [3], boundaries of knowledge and practices [1,9], and risk of interdisciplinarity work [10].

1.2. Mathematical modelling and interdisciplinarity

Mathematics as an analytic discipline can be focused in the solutions of real problems. In this respect, the discipline acquires a remarkable development thought the use of mathematical modelling in an interdisciplinary context. Examples of such developments could be find in many field of the scientific and technological research, in which the conjunction of different scientific approaches with an integrative perspective is a key point of development [5]. From this perspective, is possible to define mathematical modelling as an interdisciplinary activity in which two or more disciplines converge together to solve a problem [5,11,12]. Considering such
points, mathematical modeller expert (MME) need to structure complex problem obtained from a wide range of discipline, in a collaborative way [13]. Thus, a MME should be prepared to work in interdisciplinary teams without establishing hierarchical disciplinary preferences [4], on the quest to solve real world problems.

Regarding the mathematical principles, Differential Equations (DE), constitute an epistemological example of new problems solved by mathematics techniques in the field of physics, promoting the development of new techniques and/or theories. In this regard, physical phenomenal have been supported by an analytic background [14]. According to [14, 15], the DE become a bridge between applied mathematics and the Physical Science, proposed by some classic works of Euler, D’Alambert, Lagrange, Hamilton and Laplace. From which foundations of the DE analysis are based. As a result, mathematics play an important role in the development of other complementary disciplines, to worse of definition of degree of duality. Such duality was already confirmed by some physicist-mathematicians from the 19th century, for instance H. Poincaré making reference to Laplace’s equation, said that: “...Les deux problemes, absolument differents au point de vue physique, sont identiques au point de vue analytique” [16, p. 212].

1.3. Research scope
In [17] the mathematical modelling professional activity was defined as: the construction of models based on quantitative data, in established theory or in varieties of already established models. Additionally, in [18], four components that characterize mathematical modelling professional activity was found: description, understanding, abstraction and negotiation. Inspired in such two works, the opinion of an interdisciplinary team (two mathematicians and two engineers) has been analyzed. The main objective was identified the conceptions about the role that plays mathematical modelling in the interdisciplinarity work. The conceptions obtained from informants opinion are contrasted with the Poincaré’s results in the study of three bodies problem [19, 20]. Similar studies can be find in [21, 22]. However, the interest of this study is focused on two original papers of Poincaré [19, 20]. We consider that the three bodies problem correspond to an interesting example that highlight the contribution of mathematical modelling in other disciplines, recognizing an effort for promote the construction of knowledge, either by mathematics, or else, another areas in where mathematical modelling give a theoretical support. In this regard, our purpose is give a first empirical answer to the following question: How do the conceptions coming from an interdisciplinary relationship among the MME’s can be evidenced in Poincaré’s work?

2. Methodology
Initially, it is required to establish what are the conceptions in the discourse of MME interdisciplinary team. For that, we applied the Thematic Analysis method characterized as more objective and flexible method [23]. We search the conceptions without considering a preliminary coding to analyze the discursive data. The informants are four researchers that work together in the Ph.D. program of Applied Mathematical Modelling in the University Catholic of Maule in Chile, and they work in three subjects of research: Epidemiological Analysis, Pattern Recognition, and Farming Systems. To know their conceptions about mathematical modelling, a semi-structured interview was conducted for the four researchers. Two of them, are mathematical modelers (MdrM), which are called Oliver and Jack, characterized by having a mathematical background with experiences in mathematical modelling projects in disciplines such as epidemiology, entomology, fisheries resources and ecology. The other two researchers have an engineers training (electronic and agronomist) that we are called as Daniel and Max, who are the context discipline experts (ExpCD). The four researchers have an extensive experience in interdisciplinary works, and they have supported and signed an informed consent to participate in the present investigation. After having found the conceptions, they will be recognized in [19, 20] by identification of common conceptions in both scientific development.
3. Analysis data: Interdisciplinary team

The coding and analysis are directed by the research question, which is part of an inductive analysis of the data [23]. The analytic practice in the coding process, the grounded categories and themes was developed using the Atlas.ti software. We start with an interview transcription for to begin the codify, resulting 82 codes. After generating a second analysis and adjustment of codes based on the research question, then five main conceptions are recognized. In the start of the Interdisciplinary Work (IW) is evidenced a process of approximation to define a common language, as a support the interlection. This allows the team to do some technical definition and purpose some ways in which the problem to be addressed. Subsequently, the mathematical modelling process is carried out providing a solution to the interdisciplinary problem treated. In this sense Daniel said: It contributes, because in reality [the mathematical model] is the solution to the problem... first of all, to understand the problem, to represent it, and then to extrapolate about it, [...] is the tool that is constructed to understand and to be able to predict the behaviour of a phenomenon in itself. On the other hand, the facing of this kind of problems, allows them to develop scientific and technological routes, both in IW and also in the disciplinary work (DW), as in the study of the three bodies problem, whose first results led the researchers to develop the relativity and the bifurcations theory [15]. In this way, the IW enriches cyclically knowledge. A systematization of this process is given in the Figure 1, recognizing five conceptions evidenced of the IW. The first two conceptions are associated with the scopes and methods of an IW, the second two are related with the relation between disciplines in IW, and the last one is referred to the vision of mathematics in IW. Such conceptions are deduced from the participants answers obtaining the thematic map Figure 2.

3.1. Scopes of IW

Far from a Platonic stance, Jack sees the results of the IW as a seedbed of knowledge, considering them as exploratory niches for the theoretical and experimental scientist: ... what one builds when doing mathematics is [...] always going to be inspired by reality, and in the end, it is looking for patterns of reality and systematizing... Aquis that!, that is why we give a lot of information so that there are people who are worried about general patterns that end [in] a theorem system, in a theorem of some theory, let’s say, therefore, in that sense, it is a mathematical activity... Daniel emphasizes that contributions are initially towards the Disciplinary Context (DC), although he recognizes current developments towards mathematics: there is one of the theses that is possible to do development of the mathematical area. However, the four researches affirm that the contribution is primarily in the DC.

3.2. Methods in the IW

When proposing the meaning of a mathematical model, Jack characterizes it according to the functionality of the model in the IW:... observed a phenomenon of objective reality, raise
a question and a level of precision of the answer, is to extract with others -with other views- the basic elements that allow to carry and recreate that reality. In a model that admits the rational, that admit the ethical deductive hypothetical process, and from there, transpose the questions to later inversely transpose the answer... Jack clarifies that the role of mathematics is developed by a deductive logic. However, certain hypotheses from other disciplines may or may not emerge from inductive logic. For instance, the mathematical models in the area of pattern recognition are validated applying the algorithms constructed and the models introduced in a number of finite samples (e.g., [24]), for compare the performance of the new algorithm introduced over other one.

3.3. Discipline status
Here we evidenced a subordination from of mathematics to DC. Because in the first time a problem is established, which must be addressed using mathematics. However, the applied mathematical tools can come from any area of mathematics escaping from of mathematician expertise. In this regard, mathematician’s position must be “flexible”, in the sense explained by Oliver: [the mathematician] has certain specialities, but neither is it a matter of forcing what one knows based on the problem that arises, but the problem must be said with the mathematical tool to be used. This proposes that the problem delimitation and the solution scopes become of the DC, although “could occur in the development of the problem [mathematical results], but not as an end itself”. The focus is not placed on demonstrating theorems rigorously, on the contrary it is desired to solve the problem and the mathematical results will be developed in order to solve the problem.

3.4. Discipline uses
From an opinion as a mathematician and in agreement with the inductive-deductive component, the mathematical modelling for Jack offers a structure to the phenomenon of study providing a formalism to the IW: “the main task that mathematics does is to take a landscape with its objects and relations, to a language in which the deductive logical apparatus can function, and that is the formality sense...” While from the engineering, its multiple opportunities of disciplinary development, require studies of description, interpretation and optimization of processes, which are carried out via mathematical models: Imagine the environmental engineers who model the movement of suspension particles from one continent to another, to see what happens with the pollution of the air, how can you measure that? With probe balloons that are in the stratosphere [...] which are very expensive ... then, you model it! (Max). This type of use of mathematical models also allows engineering studies to establish quantitative results based on the experimentation of their results, which also impacts the technological development of society.

3.5. Vision of mathematics
By cultivating the competences to model mathematically is necessary to have a systemic perspective about knowledge construction. In this regard, Jack moves away from the world of ideas visualizing an indissoluble relationship between reality and mathematics: Well, if I am a platonice or neoplatonic, one could say, that is, if I place mathematics as a priory, as something that is, therefore I have the image of the discoverer that I am and I go for it, in that sense I can to be inspiring for people who do that: go to the ideal world to look for general patterns, it can be inspiring. For those of us who have the perspective that in reality that world does not exist, and therefore, what one builds when doing mathematics is simply ... that will always be inspired by reality ...

4. Analysis data: The three bodies problem
For this study, we analyzed the Poincaré original works corresponding to [19, 20]. In these originals, the analytic and interpretative aspects about his study of the three bodies problem are published, as well as the open problems in physical and mathematical terms that were proposed as consequence of such study. In general terms, the problem consists of determining
the trajectory of $n$ celestial bodies that attract each other in direct ratio with respect to their mass and in inverse ratio with respect to their distance, based on the analytic principles of mechanical physics published by Newton. In this sense H. Poincaré wrote in [20, p. 1]: Quel sera le movement de $n$ points matériels, s’attirant mutuellement en raison directe de leurs masses et en raison inverse du carré des distances. To treat such problem, a deterministic differential equations system is formulated, which represents the body interaction. First, a simplify scenario was studied, in which only are considered two bodies with non-zero mass ($n = 2$), so that they attract each other, obtaining a good approximation of the trajectories agree with the elliptical orbits determined by Kepler. Poincaré expected to extend the techniques from the two to three bodies case. However, it does not succeed, because news scenarios were found, which can not be explained with the techniques applied to analyze the reduced problem [19, p. 266] [20, p. 1]. We have that Poincaré affirmed: Je croyais en commençant ce travail, que la solution du problème, une fois trouvée pour le cas particulier que j’ai traité, se gérerait immédiatement sans qu’on ait à vaincre aucune difficulté nouvelle en dehors de celles qui sont dues aux nombre plus grand des variables et à l’impossibilité d’une représentation géométrique. Je me trompais. [19, p. 266]. From the description of these strange trajectories is that Poincaré questions the validity, scope and limitations of the Newtonian principles. Poincaré states that the deductions obtained from the model based on Newton’s laws and trajectory approximations are not enough to conjecture about what happens in reality. In addition, he suggested that the advances in instrument technology will allow the collection of data that may help verify or refute the relevance of these laws. At the same time, he questions the utility of seeking to prove a theorem rigorously, even though it does not have a concrete application in astronomy. Specifically, in [20, p. 1-2] we found: L’astronomie ne nous offre aucun exemple d’un système de trois ou de plusieurs corps dont les conditions initiales du mouvement soient telles qu’ils décrivent exactement des orbites périodiques ou asymptotiques. D’ailleurs a priori la probabilité pour que cette circumstances se présentant était manifestement nulle. On ne peut pas en conclure que les considérations précédentes ne sont intéressantes pour le géomètre et inutiles à l’astronome. Il peut arriver quelquefois que les condicion initiales du movement diffèrent peu de celles qui correspondent à une solution périodique. L’étude de cette solution présente alors un double intérêt [20, pp. 3]. The arguments that permit us identified the five components obtained from the thematic analysis in Poincaré’s works are presented below (see Table 1).

4.1. Scopes of IW

We have collected two statements made by Poincaré evidencing that it is necessary to consider the possibility of other forces governing the movement of the planets: ...Il peut en effet être soumis à d’autres forces que celle de Newton, et les astres ne se reduisent pas à des points matériels [20, pp. 4]. Furthermore, it is possible to develop new mathematical techniques: Ce, fait, pour peu qu’on prenne la peine d’y réfléchir, semblera une preuve éclatante de la complexité du problème des trois corps et de l’impossibilité de le resoudre avec les instruments actuels de l’Analyse [20, pp. 2].

4.2. Methods in IW

We remark that Poincaré applied different methods to analyze the system associated to the three bodies problem, either by approximating solutions or working in particular cases: Incapables por le moment de résoudre le problème général, nous pouvons nous borner un cas particulier [20, pp. 4]. He affirmed that the technological advance will allow us to obtain real observations of the trajectories, which will allow to obtain a greater adjustment in the inequalities: Aussi l’approximation dont nous pouvons nous contenter aujourd’hui deviendra-t-elle un jour insuffisante. Et, en effet, en admettant même, ce qui est très improbable, que les instruments de mesure ne se perfectionnent plus, l’accumulation seule des observations pendant plusieurs siècles nous fera connaître avec plus de précision les coefficients des diverses inégalités [20, pp. 1-2]. Thus, we observe experimental, analytic and approximation techniques. Therefore, we can infer that Poincaré’s applied inductive and deductive methods.
4.3. Disciplines status
We can observe that Poincaré was interested in developing theories or mathematical techniques that lead to a coherent interpretation with the real problem, visualizing the problem as a direct component of scientific development: "Dans une autre solution plus étrange encore, la lune passe trois fois par le premier quartier entre la nouvelle lune et la pleine lune; dans cet intervalle, elle croît d’abord, décroît ensuite, pour se mettre de nouveau à croître. Ces solutions sont trop différentes des véritable trajectoires des astres, pour pouvoir jamais être réellement utiles à l’Astronomie. Elles n’ont qu’un intérêt de curiosité... [20, pp. 2]."

4.4. Discipline uses
Poincaré was interested in develop theorem proofs associated to concrete contributions in the real problem: "...On ne peut s’empêcher de le penser, et c’est ce qui donne quelque prix aux rares théorèmes susceptibles d’une démonstration rigoureuse, quand même ils ne semblent pas immédiatement applicables à l’astronomie [19, pp. 2]. On the other hand, he proposes that the stability problem of the solar system is mainly mathematical, and concludes how mathematics would contribute in this case, from the astronomical perspective.

4.5. Vision of mathematics
Poincaré makes reference to find solutions, but without applying the geometric rigour to describe them. In reference to the affirmation "solid ground", we interpret it as the support and structure that mathematics provide to build new theoretical knowledge, and the limitations proper of mathematical knowledge developed until such period are identified. Poincaré wrote: "Tout ce que nous pouvons dire, c’est que le problème des trois corps ne peut être résolu avec les instruments dont nous disposons actuellement;... [20, pp. 3]."

| Table 1. Synthesized comparative results. |
|------------------------------------------|
| **Conceptions** | Poincaré works | Interdisciplinary team |
| Scopes | Contributions to mathematics, physics and astronomy have been evidenced. He was interested in proof properties in coherence with the real problem | Mainly, the team direct their effort to solve a real problem, focusing the work in the context discipline |
| Methods | Inductive-Deductive | Inductive-Deductive |
| Status | Subordination of mathematics to obtain results with astronomical interest | Subordination of mathematics to solve the real problem |
| Uses | Analysis, structure and approximation for physical phenomenon | Approximation formalism and theoretical support |
| Vision | Support and structure with limitations to solve some complex problems and it is presented the need of continue developing new mathematical knowledge | Mathematical knowledge is constructed from the observed reality from the need of solve real problems |

5. Conclusion
We note that methodological interaction and disciplinary subordination are fundamental for the development of an IW, where mathematical modelling is the support. The delimitation of the problem and the use of a common language to both disciplines involved is necessary to obtain approximate solutions. Regarding the relationships between the disciplines, we can mention that to solve a becoming problem from another disciplinary, mathematical knowledge must be subordinated to the techniques and needs of the context in which the solution makes sense. We also conclude that the present conceptions induced from the thematic analysis coincide with some of the opinions written by Poincaré.

Acknowledgement
For K Vilches this work was funding by CONICYT PAI/Academia 79150021 2016-2018.
References

[1] Stehr N and Weingart P 2000 Practising interdisciplinarity (Toronto: University of Toronto Press)
[2] Klein J T 1990 Interdisciplinarity: History, theory, and practice (Detroit: Wayne State University Press)
[3] Eisenberg L, Pellmar T C et al. 2000 Bridging disciplines in the brain, behavioral, and clinical sciences (Washington: National Academies Press)
[4] Uribe Mallarino C 2012 Universitas Humanística 73(73) 148–172
[5] Williams J, Roth W, Swanson D, Doig B, Groves S, Omuwwe M, Borromeo Ferri R and Mousoulides N 2016 Interdisciplinary mathematics education. A state of the art (Heidelberg: Springer International Publishing)
[6] Amey M J and Brown D F 2005 New Directions for Teaching and Learning 2005(102) 23–35
[7] Sá C M 2008 Higher Education 55(5) 537–552
[8] Lattuca L R 2002 The journal of Higher Education 73(6) 711–739
[9] Klein J T 1996 Crossing boundaries: Knowledge, disciplinarities, and interdisciplinarities (Charlottesville: University of Virginia Press)
[10] Rhoten D and Parker A 2004 Science 306(5704)
[11] Klein J T 2012 Humanities, culture, and interdisciplinarity: The changing American academy (New York: State University of New York Press)
[12] Samson G 2014 Canadian Journal of Science, Mathematics and Technology Education 14(4) 346–358
[13] Epstein S L 2005 Interdisciplinary collaboration: An emerging cognitive science (New Jersey: Lawrence Erlbaum Associates)
[14] Dyson F J 1972 Bulletin of the American Mathematical Society 78(5) 635–652
[15] Brezis H and Browder F 1998 Advances in Mathematics 135(1) 76–144
[16] Poincaré H 1890 American Journal of Mathematics 12(3) 211–294
[17] Frejd P and Bergsten C 2016 Educational Studies in Mathematics 91(1) 11–35
[18] Frejd P and Bergsten C 2018 ZDM 50(1-2) 117–127
[19] Poincaré H 1890 Acta Mathematica 13(1) A3–A270
[20] Poincaré H 1891 Bulletin Astronomique 8 12–24
[21] Ariza R P, del Pozo R M and Toscano J M 2002 Teaching and Teacher Education 18(3) 305–321
[22] Lederman N G 1992 Journal of Research in Science Teaching 29(4) 331–359
[23] Braun V and Clarke V 2006 Qualitative Research in Psychology 3(2) 77–101
[24] Aubin V, Mora M and Santos-Peñas M 2018 Pattern Recognition 79 414–426