RELATION BETWEEN THE CHIRAL AND
DECONFINEMENT PHASE TRANSITIONS

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Lattice QCD simulations at finite temperature have shown that the chiral phase transition in the chiral limit and the deconfinement phase transition in the quenched limit are continuously connected. I emphasize the nontriviality of this result and propose an unconventional scenario which naturally explains the existing lattice data. The continuity of the two phase transitions is a manifestation of the familiar glueball-meson mixing, which can be traced back to the properties of QCD at zero temperature.

1. Introduction and motivation

In QCD at finite temperature, there are two kinds of phase transitions in two different limits of the quark mass parameter. The chiral phase transition in the chiral limit and the deconfinement phase transition in the quenched limit. Away from these limits, the meanings of these phase transitions become less transparent. Nevertheless, the notions of deconfinement and chiral symmetry restoration are indispensable for our understanding of the quark-gluon plasma phase. In this talk we discuss the possible deep relation between the two phase transitions in the intermediate quark mass region.

In Table 1, some key properties of the two phase transitions are listed.

| Property              | Chiral Phase Transition | Deconfinement Phase Transition |
|-----------------------|-------------------------|--------------------------------|
| Quark Mass            | 0                       | ∞                              |
| Symmetry              | Chiral Symmetry         | Center Symmetry                |
| Order Parameter       | Quark Condensate        | Polyakov Loop                  |
At first sight, it does not make sense to talk about the relationship between the two phase transitions. They are defined in completely different theories in the first place. Symmetries are different, and the order parameters are different. Just by looking at the table, little do we suspect that the two phase transitions have anything to do with each other. Therefore, it is quite natural that they have long been considered as distinct phase transitions and questions like “Which phase transition occurs first?” have been addressed many times in the literature.

However, putting these theoretical speculations aside, finite temperature lattice simulations have repeatedly shown that the two phase transitions occur at the same critical temperature. Fig 1. is the most commonly accepted phase diagram of QCD in the temperature-quark mass plane. This figure shows that, for all values of the quark mass, there is a single (crossover) phase transition which smoothly connects the chiral phase transition in the chiral limit and the deconfinement phase transition in the quenched theory.

Figure 1. The QCD phase diagram in the temperature-quark mass plane. $T$ is the temperature and $m$ is the quark mass. Solid (dotted) lines represent first order (crossover) phase transition. $C$ and $D$ are second order phase transition points.

We emphasize that this is a very nontrivial result. Theorists were able to predict the order of phase transitions at $m = 0$ and $m = \infty$. Theorists could also predict that the first order chiral (deconfinement) phase
transition would turn into a crossover if the quark mass was increased from 0 (decreased from $\infty$). However, no one could predict the *global* structure of the phase diagram shown in Fig. 1 because *physics at the intermediate quark mass region is non-universal*. One cannot invoke the usual universality argument of phase transitions to predict anything in this region. The continuity of the two phase transitions is a consequence of the non-perturbative dynamical effect of QCD, which is far from obvious. And, of course, such a *non-*universal phenomenon is one of the most interesting aspects of a given theory. In this report, we will propose a novel scenario which naturally explains the puzzling lattice data, Fig. 1.

2. The level repulsion scenario

The interplay between the two transitions is an old but interesting problem. First, Gocksch and Ogilvie observed that what is responsible for the breaking of center symmetry is (the inverse of) the *constituent* quark mass rather than the current quark mass. This suggests that the Polyakov loop and the chiral dynamics are closely coupled. See, also, Ref. for more recent works. So far, most of the works on this subject focused on the coupling of the Polyakov loop and the sigma field. (See, however, Ref.) Here we point out that, in the presence of dynamical quarks, the deconfinement phase transition can equivalently be characterized in terms of the glueballs. Specifically, we have predicted that the screening mass of the $0^+$ electric glueball goes to zero with the specified critical exponent at the second order deconfinement phase transition. In the case of color SU(2), this screening state is responsible for the weak divergence of the specific heat at the Z(2) phase transition but is distinct from the true order parameter field, namely, the Polyakov loop. However, in the case of color SU(3), at the point $D$, nonzero expectation value of the Polyakov loop induces mixing between the glueball and the Polaykov loop. Therefore, the glueball is an equivalent critical field at $D$. Datta and Gupta observed a significant decrease of the $0^+$ glueball screening mass near the SU(2) phase transition in the quenched simulation. We believe that the mass will go to zero in

\[a\text{See, for example, Ref.}^{9}\text{ for the meaning of these quantum numbers. 'Electric' means that the glueball interpolating operator contains } A_0\text{'s or timelike links. One can also consider } 0^+ \text{ 'magnetic' glueballs (composed only from spacelike links) which, in principle, mix with the electric ones. However, near the critical temperature and above, it has been observed}^{10} \text{ that the mixing is very weak, could be absent.}\]

\[b\text{Such a mixing always takes place at generic end-points. See, Ref.}^{11}\text{ for an explicit example in the case of the end-point at finite density.}\]
the infinite lattice volume limit.

Next we observe that the glueball field $G$ must mix with the sigma field $\sigma$ so that the correct massless field at $D$ is a linear combination of the two;

$$\phi = G \cos \theta + \sigma \sin \theta, \quad \sin \theta \approx 0. \quad (1)$$

The orthogonal linear combination with large sigma field content,

$$\phi' = -G \sin \theta + \sigma \cos \theta, \quad (2)$$

is massive. Now the key question is the behavior of the mixing angle $\theta$ as the quark mass varies. If the mixing angle remains small at small values of the quark mass, the $\phi'$ field, which is massive at $D$, would become massless at $C$ because the critical field at $C$ is dominantly sigma-like.12 However, if this were the case, the coincidence of the two critical temperatures for all values of the quark mass would be a pure accident. Fig. 1 is most naturally explained by postulating that the critical field at $C$ is again the $\phi$ field. Namely, the two second order phase transitions at $C$ and $D$ are driven by the same field. This is possible only if the mixing angle changes from $\theta \approx 0$ to $\approx \pi/2$. Such a continuous variation of the mixing angle is typical of a level repulsion in quantum mechanics. Thus we have arrived at a novel scenario of the finite temperature QCD phase transition: Due to a level repulsion between the $\phi$ and the $\phi'$ fields, the $\phi$ field continues to be the lightest screening state for all values of the quark mass. Simultaneous divergences and peaks in various susceptibilities are simply caused by the dropping of the $\phi$ field screening mass.

Moreover, we conjecture that this scenario is realized at all temperatures, not only near the critical temperature. The level repulsion between the scalar glueball and the sigma meson takes place already at zero temperature as one can easily convince oneself by considering the scalar meson and glueball mass spectrum at zero temperature.4 Therefore, our scenario can be naturally embedded in the entire phase diagram, Fig. 1. Note that this argument is made possible only when one characterizes the deconfinement phase transition in terms of the glueball. (The Polyakov loop cannot be defined at zero temperature.)

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1The mixing angle will go to exactly $\pi/2$ at the chiral symmetry restoration point in the chiral limit.

4We assume that, in accordance with popular belief, the lightest glueball mass in the real world does not change appreciably from its value in the quenched theory.
3. Conclusion

We have pointed out the importance of the glueball screening states for the understanding of the QCD phase diagram for all values of the quark mass and the temperature. Compared to Polyakov loops, glueballs have been much less studied on a lattice at finite temperature with or without dynamical quarks.\textsuperscript{9,10,13,14} We expect that further glueball measurements will provide rich information on the nature of the thermal QCD phase transition. On the other hand, it turned out to be very difficult to reproduce the lattice result, Fig. 1, in model calculations. The main difficulty is to keep track of the important coupling between the two fields in the intermediate quark mass region where there is no guiding principle (no symmetry) to construct the effective potential. (For details and discussions, see Ref.\textsuperscript{15}.) This remains to be a theoretical challenge.

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