MASSIVE NEUTRINOS AND LEPTON-FLAVOUR-VIOLATING PROCESSES

J.A. CASAS
Instituto de Estructura de la Materia, CSIC
Serrano 123, 28006 Madrid

A. IBARRA
Department of Physics, Theoretical Physics, University of Oxford
1 Keble Road, Oxford OX1 3NP, United Kingdom

If neutrino masses and mixings are suitable to explain the atmospheric and solar neutrino fluxes, this amounts to contributions to FCNC processes, in particular $\mu \rightarrow e, \gamma$. If the theory is supersymmetric and the origin of the masses is a see-saw mechanism, we show that the prediction for BR($\mu \rightarrow e, \gamma$) is in general larger than the experimental upper bound, especially if the solar data are explained by a large angle MSW effect, which recent analyses suggest as the preferred scenario.

1 See-saw, RG-induced LFV soft terms and $l_i \rightarrow l_j, \gamma$

In the pure Standard Model, flavour is exactly conserved in the leptonic sector since one can always choose a basis in which the (charged) lepton Yukawa matrix, $Y_e$, and gauge interactions are flavour-diagonal. If neutrinos are massive and mixed, as suggested by the observation of atmospheric and solar fluxes, this is no longer true and there exists a source of lepton flavour violation (LFV), in analogy with the Kobayashi–Maskawa mechanism in the quark sector. Unfortunately, due to the smallness of the neutrinos masses, the predicted branching ratios for these processes are so tiny that they are completely unobservable, namely BR($\mu \rightarrow e, \gamma$) $< 10^{-50}$. \cite{footnote}

In a supersymmetric (SUSY) framework the situation is completely different. Besides the previous mechanism, supersymmetry provides new direct sources of flavour violation in the leptonic sector, namely the possible presence of off-diagonal soft terms \cite{footnote}. In a self-explanatory notation, they have the form

$$- \mathcal{L}_{\text{soft}} = (m^2_L)_{ij} \bar{L}_i L_j + (m^2_{eR})_{ij} \bar{e}_{Ri} e_{Rj} + (A_{eij} \bar{e}_{Ri} H_1 L_j + \text{h.c.}) + \text{etc.}, \quad (1)$$

where we have written explicitly just the soft breaking terms in the leptonic sector, namely scalar masses and trilinear scalar terms. All the fields in the previous equation denote just the corresponding scalar components. Concerning flavour violation the most conservative starting point for $\mathcal{L}_{\text{soft}}$ is the assumption of universality, which corresponds to take

$$(m^2_L)_{ij} = m^2_0 \mathbb{1}, \quad (m^2_{eR})_{ij} = m^2_0 \mathbb{1}, \quad A_{eij} = A_0 Y_e_{ij}, \quad (2)$$

so that working in the $L_i$ and $e_{Ri}$ basis where $Y_e$ is diagonal, the soft terms do not contain off-diagonal (lepton flavour violating) entries.
It turns out, however, that even under this extremely conservative assumption, if neutrinos are massive, radiative corrections may generate off-diagonal soft terms.

The most interesting example of this occurs when neutrino masses are produced by a (supersymmetric) see-saw mechanism. This is based upon a superpotential

$$W = W_0 - \frac{1}{2} \bar{\nu}_R^c \mathcal{M} \nu_R^c + \bar{\nu}_R^c Y \nu L \cdot H_2,$$

(3)

where $W_0$ is the observable superpotential, except for neutrino masses, of the preferred version of the supersymmetric SM, e.g. the MSSM. The extra terms involve three additional neutrino chiral fields (one per generation; indices are suppressed) not charged under the SM group: $\nu_R^i (i = e, \mu, \tau)$. $Y$ is the matrix of neutrino Yukawa couplings, $L_i (i = e, \mu, \tau)$ are the left-handed lepton doublets and $H_2$ is the hypercharge +1/2 Higgs doublet. The Dirac mass matrix is given by

$$m_D = Y \nu \langle H_0^2 \rangle.$$ 

(4)

where $\langle H_0^2 \rangle^2 = v^2 = v^2 \sin^2 \beta$ and $v = 174$ GeV. The experimental data about neutrino masses and mixings are referred to the $\mathcal{M}_\nu$ matrix, or equivalently $\kappa$, evaluated at low energy (electroweak scale).

Turning back to the structure of the SUSY soft-breaking terms, the universality condition (3) can only be imposed at a certain scale, typically at the scale at which the soft breaking terms are generated, e.g. $M_X$ in GUT models. Below that scale, the RGEs of the soft terms, which contain non-diagonal contributions proportional to $Y^T \mathcal{M}^{-1} Y$, induce off-diagonal soft terms. These contributions are decoupled at the characteristic scale of the right-handed neutrinos, $M$. More precisely, in the leading-log approximation, the off-diagonal soft terms at low-energy are given by

$$\begin{align*}
(m^2_L)_{ij} &\sim -\frac{1}{8\pi^2} (3m_0^2 + A_0^2) (Y^T \mathcal{M}^{-1} Y)_{ij} \log \frac{M_X}{M}, \\
(m^2_R)_{ij} &\sim 0, \\
(A_{ij}) &\sim -\frac{3}{8\pi^2} A_0 Y_{li} (Y^T \mathcal{M}^{-1} Y)_{ij} \log \frac{M_X}{M},
\end{align*}$$

(5)

aIt should be noted that eq.(4) is defined at the “Majorana scale”, $M$. Therefore, in order to compare to the experiment one has still to run $\kappa$ down to low energy through the corresponding RGE.

bWe use the leading-log approximation through the text in order to make the results easily understandable. Nevertheless, the numerical results, to be exposed below, have been obtained by integrating the full set of RGEs.
where \( i \neq j \) and \( Y_{li} \) is the Yukawa coupling of the charged lepton \( l_i \).

The previous off-diagonal soft terms induce LFV processes, like \( l_i \to l_j, \gamma \). The precise form of \( \text{BR}(l_i \to l_j, \gamma) \) that we have used in our computations is a rather cumbersome expression \( \text{7} \). However, for the sake of the physical discussion it is interesting to think in the mass-insertion approximation to identify the dominant contributions. As discussed in ref. \( \text{8} \), these correspond to the mass-insertion diagrams enhanced by \( \tan \beta \) factors. All of them are proportional to \( m_{L_{ij}}^2 \), and have the generic form shown in Fig. 1. Thus the size of the braching ratios is given by

\[
\text{BR}(l_i \to l_j, \gamma) \sim \frac{\alpha^3}{G_F^2 m_S^8} |m_{L_{ij}}^2| \tan^2 \beta
\]

where we have used eqs. (\text{5}). The \( Y_\nu^+ Y_\nu \) matrix is therefore the crucial quantity for the computation of \( \text{BR}(l_i \to l_j, \gamma) \). Hence, in order to make predictions on \( \text{BR}(l_i \to l_j, \gamma) \) we need to determine the most general form of \( Y_\nu \) and \( Y_\nu^+ Y_\nu \), compatible with all the phenomenological requirements. Recall that the latter are referred to the \( M_\nu \) matrix, evaluated at low energy, rather than to \( Y_\nu \) itself. So, this is a non-trivial task that we discuss in the next section. Notice also the strong dependence of \( \text{BR}(l_i \to l_j, \gamma) \) on \( \tan \beta \) and the fact that the larger (smaller) the initial scale at which universality is imposed, the larger \( \text{BR}(l_i \to l_j, \gamma) \).

2 General textures reproducing experimental data

Working in the flavour basis in which the charged-lepton Yukawa matrix, \( Y_e \), and gauge interactions are flavour-diagonal, the neutrino mass matrix, \( M_\nu \),

\[\text{cairo2: submitted to Rinton on March 25, 2022} \]
or equivalently the \( \kappa \) matrix defined in eq. (9), is diagonalized by the MNS matrix \( U \) according to
\[
U^T \kappa U = \text{diag}(\kappa_1, \kappa_2, \kappa_3) \equiv D_\kappa,
\]
where \( U \) is a unitary matrix that relates flavour to mass eigenstates. It is possible, and sometimes convenient, to choose \( \kappa_i \geq 0 \). Then, \( U \) can be written as \( U = V \cdot \text{diag}(e^{-i\phi/2}, e^{-i\phi'/2}, 1) \), where \( \phi \) and \( \phi' \) are CP violating phases (if different from 0 or \( \pi \)) and \( V \) has the ordinary form of a CKM matrix
\[
V = \begin{pmatrix}
c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\
-c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{13} \\
s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{13}
\end{pmatrix}.
\]

The experimental information about neutrinos consists of information about the low-energy spectrum of neutrinos, contained in \( D_\kappa \), and about the neutrino mixing angles (and CP phases), contained in \( U \). Let us discuss them in order.

The experimental (solar and atmospheric) data strongly suggest a hierarchy of neutrino mass-splittings, \( \Delta \kappa^2_{\text{sol}} \ll \Delta \kappa^2_{\text{atm}} \). Numerically, \( \Delta \kappa^2_{\text{atm}} \sim 3 \times 10^{-3} \text{eV}^2/\nu^2 \) \([6]\), while the value of \( \Delta \kappa^2_{\text{sol}} \) depends on the solution considered to explain the solar neutrino problem \([2,4]\), i.e. large-angle, small angle or LOW MSW solutions (LAMSW, SAMSW and LOW respectively), or vacuum oscillations solution (VO). They require, in eV\(^2/\nu^2\) units, \( \Delta \kappa^2_{\text{sol}} \sim 3 \times 10^{-5}, 10^{-7}, 5 \times 10^{-8} \) and \( 8 \times 10^{-10} \) respectively. The most favoured one from the recent analyses of data \([2]\) is the LAMSW. In any case, there are basically three types of neutrino spectra consistent with the hierarchy of mass-splittings: hierarchical \( (\kappa_1^2 \ll \kappa_2^2 \ll \kappa_3^2) \), "intermediate" \( (\kappa_1^2 \sim \kappa_2^2 \gg \kappa_3^2) \) and "degenerate" \( (\kappa_1^2 \sim \kappa_2^2 \sim \kappa_3^2) \). In the usual notation, \( \Delta \kappa^2_{\text{atm}} \equiv \Delta \kappa^2_{\text{atm}} \), \( \Delta \kappa^2_{\text{sol}} \equiv \Delta \kappa^2_{\text{sol}} \).

Concerning the mixing angles, \( \theta_{23} \) and \( \theta_{13} \) are constrained by the atmospheric and CHOOZ data to be near maximal and minimal, respectively. The \( \theta_{12} \) angle depends on the solution considered for the solar neutrino problem: it should be either near maximal (LAMSW, LOW and VO) or near minimal (SAMSW). Hence, the two basic forms that \( U \) can present are either a single-maximal or (more plausibly) a bimaximal mixing matrix. Schematically,
\[
U \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad \text{or} \quad U \sim \begin{pmatrix} 1 & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix},
\]

Let us turn to our question of what is the most general form of \( Y_\nu \) and \( Y_\nu^+Y_\nu \), compatible with all the previous phenomenological requirements in a see-saw scenario. Notice, in the first place, that one can always choose to work in a basis of right neutrinos where \( \mathcal{M} \) is diagonal
\[
\mathcal{M} = \text{diag}(M_1, M_2, M_3) \equiv D_\mathcal{M},
\]
with $M_\nu \geq 0$. Then, from eqs. (11, 12), $D_\nu = U^T \nu U \sqrt{M_\nu} U^T D_{\sqrt{M_\nu}} \nu Y_\nu$, where, in an obvious notation, $D_{\sqrt{M_\nu}} \equiv + \sqrt{D_M}$. Consequently, the most general form of $Y_\nu$ and $Y_\nu^T Y_\nu$ is

$$Y_\nu = D_{\sqrt{M_\nu}} R \sqrt{U^+}$$

(11)

$$Y_\nu^T Y_\nu = U D_{R \sqrt{M_\nu}} D_{\sqrt{M_\nu}} R \sqrt{U^+}$$

(12)

where $R$ is any orthogonal matrix ($R$ can be complex provided $R^T R = 1$).

So, besides the physical and measurable low-energy parameters, contained in $D_\nu$ and $U$, $Y_\nu$ and $Y_\nu^T Y_\nu$ depend on the three (unknown) positive mass eigenvalues of the right-handed neutrinos, contained in $D_{M_\nu}$, and on the three (unknown) complex parameters defining $R$. We will see, however, that in practical cases the number of relevant free parameters becomes drastically reduced. Notice also that $Y_\nu^T Y_\nu$, and therefore eq. (12), does not depend on the $\nu_R$-basis, and thus on the fact that $M$ is diagonal or not.

Two (very) special cases of eqs. (11, 12) occur when $R = 1$ and when $Y_\nu$ has the form $Y_\nu = W D_\nu$, where $D_\nu$ is a diagonal matrix and $W$ is a unitary matrix. Then, there exists a basis of $L_i, \nu_R$, where all the leptonic flavour violation arises from the $Y_\nu$ (i.e. from the charged leptons) or from the $M$ (i.e. the right neutrinos) matrices respectively. In the second case $Y_\nu^T Y_\nu$ is diagonal, so the predictions for $BR(l_i \to l_j, \gamma)$ are negligible.

Next, we study the general predictions for $BR(l_i \to l_j, \gamma)$, focussing on $BR(\mu \to e\gamma)$, by considering, in a separate way, some interesting scenarios that often appear in the literature. A more detailed discussion can be found in Ref. [3].

3 Predictions for $BR(l_i \to l_j, \gamma)$

$\nu_L$’s and $\nu_R$’s completely hierarchical

In this case $D_\nu \simeq \text{diag}(0, \kappa_2, \kappa_3)$, $D_{M_\nu} \simeq \text{diag}(0, 0, M_3)$.

* If $R$ is a generic matrix, with $R_{32} \neq 0$ or $R_{33} \neq 0$, $(Y_\nu^T Y_\nu)_{ij}$ is given by

$$(Y_\nu^T Y_\nu)_{ij} \sim (Y_\nu)^{3i}_{3j} (Y_\nu)_{3j} \sim M_3 \left[ \sum_{i=2,3} R_{3i}^* \sqrt{\kappa_i} U_{id} \right] \left[ \sum_{j'=2,3} R_{3j'} \sqrt{\kappa_j' U_{j'd}} \right]$$

Parameterizing $R$ as

$$R = \begin{pmatrix}
\hat{c}_2 \hat{c}_3 & -\hat{c}_1 \hat{s}_3 - \hat{s}_1 \hat{s}_2 \hat{c}_3 & \hat{s}_1 \hat{s}_3 - \hat{c}_1 \hat{s}_2 \hat{c}_3 \\
\hat{c}_2 \hat{s}_3 & \hat{c}_1 \hat{c}_3 - \hat{s}_1 \hat{s}_2 \hat{s}_3 & -\hat{s}_1 \hat{c}_3 - \hat{c}_1 \hat{s}_2 \hat{s}_3 \\
\hat{s}_2 & \hat{s}_1 \hat{c}_2 & \hat{c}_1 \hat{c}_2
\end{pmatrix},$$

(13)
Figure 2. BR(µ → e, γ) vs. the unknown angle ˆθ for the case of hierarchical (left and right) neutrinos, a typical set of supersymmetric parameters, and different values of the largest neutrino Yukawa coupling, Y0, at the “unification” scale, Mx. Yl denotes the value of the top Yukawa coupling at that scale. ˆθ is taken real for simplicity, so the two limits ˆθ = 0, π of the horizontal axis represent the same physical point. The dashed lines correspond to the present and forthcoming experimental upper bounds 15.

where ˆθ1, ˆθ2, ˆθ3 are arbitrary complex angles [eq.(13) is sufficiently general for this case], one obtains in particular

\[
(Y^\nu Y)_{21} \sim \frac{|Y_0|^2}{|s_1|^2\kappa_2 + |c_1|^2\kappa_3} c_1^* s_1 \sqrt{\kappa_3 \kappa_2} U_{23}^* U_{12}^* \quad (14)
\]

Here |Y0|^2 is the largest eigenvalue of YνYν and ˆθ is an arbitrary complex angle. The branching ratio just depends on |Y0|^2 and ˆθ.

For the LAMSW scenario the previous equation generically gives BR(µ → e, γ) above the present experimental limits, at least for Y0 = O(1), as it occurs in the unified scenarios. (This holds until m0 > 1.5 TeV, except for a narrow range at low m0.) Actually, even for Y0 = O(10^{-1}), most of the parameter space will be probed in the forthcoming generation of experiments experiments. This is illustrated in Fig. 2.

* The only exceptions to the previous result are

- If ˆθ is such that (Y31) or (Y32) ≃ 0. Then (Yν Y)_{21} ≃ 0 and BR(µ → e, γ) is small. These two special values of ˆθ are visible in Fig. 2 and correspond to tan ˆθ ≃ −√(ν3 ν21) and tan ˆθ ≃ −√(ν3 ν22). The associated textures in our basis are

\[
Y_\nu \propto \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & V_{31} & -V_{21} \end{pmatrix} \propto \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} , \quad (15)
\]
where the numerical values of the entries correspond to a bimaximal mixing $V$ matrix. For these cases $(Y_\nu^+Y_\nu)_{21} \propto M_2$ (instead $\propto M_3$), which can still be sizeable. In any case other processes, as BR$(\tau \to \mu, \gamma)$, are not suppressed, normally lying above the forthcoming experimental upper bound.

$\nu_L$’s hierarchical and $\nu_R$’s degenerate

If $R$ is such that $Y_\nu^+Y_\nu$ is diagonal. This requires a very special form of $R$, which in particular has $R_{32}, R_{33} \simeq 0$.

$\nu_L$’s quasi-degenerate

In this case $D_\kappa \simeq \text{diag}(0, \kappa_2, \kappa_3), \quad D_M \simeq \text{diag}(M, M, M)$.

* If $R$ is real, this scenario is very predictive. Then $(Y_\nu^+Y_\nu)_{ij}$ does not depend on $R$

$$ (Y_\nu^+Y_\nu)_{ij} = M\kappa_i U_{il} U_{lj}^* \simeq \mathcal{M} [\kappa_2 U_{i2} U_{j2}^* + \kappa_3 U_{i3} U_{j3}^*] , $$

In particular, taking into account $U_{13} \simeq 0$,

$$ (Y_\nu^+Y_\nu)_{21} \simeq \mathcal{M}\kappa_2 U_{22} U_{12}^* = |Y_0|^2 \frac{\kappa_2}{\kappa_3} U_{22} U_{12}^* , $$

where $|Y_0|^2 \simeq \mathcal{M}\kappa_3$ is the largest eigenvalue of $Y_\nu^+Y_\nu$.

The corresponding BR$(\mu \to e, \gamma)$ is illustrated in Fig. 3 for the LAMSW scenario.

The branching ratio turns out to be already above the present experimental limits except for a rather small region of $m_0$ values which should be probed by the next generation of experiments. Here are not special textures where the branching ratio becomes suppressed.

* If $R$ is complex, the analysis is more involved since it contains more arbitrary parameters. But in general the conclusion is the same: BR$(\mu \to e, \gamma)$ is at least of the same order as in the real case. Now there exists, however, the possibility of a (fine-tuned) cancellation.

$\nu_R$’s degenerate

In this case $D_\kappa \simeq \text{diag}(\kappa_1, \kappa_2, \kappa_3)$, with $\kappa_1 \sim \kappa_2 \sim \kappa_3 \equiv \kappa$. Then, it is logical to assume that $\mathcal{M}$ has degenerate eigenvalues, otherwise a big conspiracy would be needed between $Y_\nu$ and $\mathcal{M}$. Hence $D_M \simeq \text{diag}(\mathcal{M}, \mathcal{M}, \mathcal{M})$.

* If $R$ is real, $Y_\nu^+Y_\nu = M U D_\kappa U^+$. In particular, since $U_{13} \simeq 0$, 

$$ (Y_\nu^+Y_\nu)_{21} = \mathcal{M} [U_{21} U_{11}^* (\kappa_1 - \kappa_2)] = |Y_0|^2 U_{21} U_{11}^* \frac{\Delta \kappa_{sol}^2}{\kappa^2} . $$

$$ (19) $$
Figure 3. BR(\(\mu \to e, \gamma\)) vs. the universal scalar mass, \(m_0\), for the case of hierarchical (degenerate) left (right) neutrinos, \(R\) real and typical sets of supersymmetric parameters. The dashed lines correspond to the present and forthcoming upper bounds. A top-neutrino “unification” condition has been used to fix the value of the largest neutrino Yukawa coupling at high energy. The curves do not fall below the present bound until \(m_0 > 1.6\) TeV.

where, again, \(|Y_0|^2 \simeq \mathcal{M}\kappa\) is the largest eigenvalue of \(Y_\nu^\dagger Y_\nu\). This equation is identical to eq.\((18)\), multiplied by \(\kappa^{-2}\sqrt{\Delta \kappa^2_{\text{sol}} \Delta \kappa^2_{\text{atm}}}\). This is a factor \(\sim 10^{-4}\) for the LAMSW. Therefore all the plots representing BR(\(\mu \to e, \gamma\)) in the previous scenario (Fig. 3) are valid here, but with the vertical axis re-scaled eight orders of magnitude smaller. Consequently, BR(\(\mu \to e, \gamma\)) is naturally suppressed below the present (and even forthcoming) limits.

\* If \(R\) is complex, \(Y_\nu^\dagger Y_\nu = \mathcal{M} U D \sqrt{\kappa} R^+ R D \sqrt{\kappa} U^+\), which may have sizeable off-diagonal entries. Hence, BR(\(\mu \to e, \gamma\)), could be very large in this case.

\* If the (quasi-) degeneracy is only partial: \(\kappa_3 \ll \kappa_1 \simeq \kappa_2 \equiv \kappa \sim \sqrt{\Delta \kappa^2_{\text{atm}}}\), \((Y_\nu^\dagger Y_\nu)_{21}\) is given (for \(R\) real) by eq.\((18)\), multiplied now by \(\sqrt{\Delta \kappa^2_{\text{atm}}} / \Delta \kappa^2_{\text{atm}}\). This represents a suppression factor \(\sim 10^{-1}\) for the LAMSW, which means that Fig. 3 should be re-scaled by a factor \(\sim 10^{-2}\). As a consequence, BR(\(\mu \to e, \gamma\)) for this partially degenerate scenario should be testable within the next generation of experiments. The conclusion is similar for BR(\(\tau \to \mu, \gamma\)).

For generic complex \(R\), the value of BR(\(\mu \to e, \gamma\)) does not get any suppression and falls naturally above the present experimental limits.
4 Conclusions

If the origin of the neutrino masses is a supersymmetric see-saw, which is probably the most attractive scenario to explain their smallness, then the leptonic soft breaking terms acquire off-diagonal contributions through the RG running, which drive non-vanishing $\text{BR}(l_i \rightarrow l_j, \gamma)$. These contributions are proportional to $(Y_\nu^+ Y_\nu)_{ij}$, where $Y_\nu$ is the neutrino Yukawa matrix.

Therefore, in order to make predictions for these branching ratios, one has first to determine the most general form of $Y_\nu$ and $Y_\nu^+ Y_\nu$, compatible with all the phenomenological requirements. This is summarized in eqs. (11).

Then, we have shown that the predictions for $\text{BR}(\mu \rightarrow e, \gamma)$ are normally above the present experimental limits if the three following conditions occur

1. The solution to the solar neutrino problem is the LAMSW, as favoured by the most recent analyses.

2. $Y_0(M_X) = \mathcal{O}(1)$, where $|Y_0|^2$ is the largest eigenvalue of $(Y_\nu^+ Y_\nu)$. This occurs e.g. in most grand-unified scenarios.

3. The soft-breaking terms are generated at a high-energy scale, e.g. $M_X$, above the Majorana mass of the right-handed neutrinos, $M$.

These conditions are very plausible. In our opinion, the most natural scenarios fulfill them, but certainly there exists other possibilities. E.g. it may happen that supersymmetry is broken at a scale below $M$. This is the case of gauge-mediated scenarios, where there would be no generation of off-diagonal leptonic soft terms through the RG running.

Even under the previous 1–3 conditions, there are physical scenarios compatible with the present $\text{BR}(\mu \rightarrow e, \gamma)$ experimental limits. Namely

- Whenever all the leptonic flavour violation can be attributed to the sector of right-handed neutrinos. In this case there is no RG generation of non-diagonal soft terms.

- In the scenario of hierarchical (left and right) neutrino masses, if $Y_\nu$ has (in our basis) one of the two special textures shown in eqs. (15, 16).

- If the left-handed neutrinos are quasi-degenerate and the $R$ matrix in eq. (11) is real.

In our opinion, the scenario of quasi-degenerate neutrinos and the one with gauge mediated supersymmetry breaking represent the most plausible explanations to the absence of $\mu \rightarrow e, \gamma$ observations, specially if the absence persists after the next generation of experiments.
As a final conclusion, the discovery of neutrino oscillations makes much more plausible the possibility of observing lepton-flavour-violation processes, specially $\mu \rightarrow e, \gamma$, if the theory is supersymmetric and the neutrino masses are generated by a see-saw mechanism. Large regions of the parameter space are already excluded on these grounds, and there exists great chances to observe $\mu \rightarrow e, \gamma$ in the near future (PSI, 2003). This means that, hopefully, we will have signals of supersymmetry before LHC.

References

1. See e.g. talks by Y. Oyama and P. Harris in this conference.
2. S. M. Bilenkii, S. T. Petcov and B. Pontecorvo, Phys. Lett. B 67 (1977) 309; T. P. Cheng and L. Li, Phys. Rev. Lett. 45 (1980) 1908; W. J. Marciano and A. I. Sanda, Phys. Lett. B 67 (1977) 303; B. W. Lee, S. Pakvasa, R. E. Shrock and H. Sugawara, Phys. Rev. Lett. 38 (1977) 937 [Erratum-ibid. 38 (1977) 937].
3. See e.g. talks by A. Masiero and D. Carvalho in this conference.
4. M. Gell-Mann, P. Ramond and R. Slansky, proceedings of the Supergravity Stony Brook Workshop, New York, 1979, eds. P. Van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam); T. Yanagida, proceedings of the Workshop on Unified Theories and Baryon Number in the Universe, Tsukuba, Japan 1979 (edited by A. Sawada and A. Sugamoto, KEK Report No. 79-18, Tsukuba); R. Mohapatra and G. Senjanović, Phys. Rev. Lett. 44 (1980) 912, Phys. Rev. D23 (1981) 165.
5. F. Borzumati and A. Masiero, Phys. Rev. Lett. 57 (1986) 961.
6. See e.g. J. Ellis, M. E. Gomez, G. K. Leontaris, S. Lola and D. V. Nanopoulos, data,” Eur. Phys. J. C 14 (2000) 319; M. E. Gomez, G. K. Leontaris, S. Lola and J. D. Vergados, Phys. Rev. D 59 (1999) 116009; D. F. Carvalho, M. E. Gomez and S. Khalil, hep-ph/0101250; R. Barbieri, L. Hall and A. Strumia, Nucl. Phys. B 445 (1995) 219.
7. J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, Phys. Rev. D 53 (1996) 2442.
8. J. Hisano and D. Nomura, Phys. Rev. D 59 (1999) 116005.
9. Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. 28 (1962) 870.
10. See e.g. M. C. Gonzalez-Garcia, M. Maltoni, C. Pena-Garay and J. W. Valle, Phys. Rev. D 63 (2001) 033005.
11. B. Pontecorvo, Sov. Phys. JETP26 (1968) 984.
12. S. P. Mikheev and A. Y. Smirnov, Sov. J. Nucl. Phys. 42 (1985) 913; L. Wolfenstein, Phys. Rev. D 17 (1978) 2369.
13. G. Altarelli and F. Feruglio, Phys. Lett. B439 (1998) 112, JHEP 9811 (1998) 021, and Phys. Lett. B451 (1999) 388.
14. J. A. Casas and A. Ibarra, hep-ph/0103065.
15. M. L. Brooks et al. [MEGA Collaboration], Phys. Rev. Lett. 83 (1999) 1521; L. M. Barkov et al., Research Proposal for an experiment at PSI
(1999), see the webpage: http://meg.psi.ch/