Development of PET projection data correction algorithm

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Development of PET projection data correction algorithm

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Abstract. Positron emission tomography is modern nuclear medicine method used in metabolism and internals functions examinations. This method allows to diagnosticate treatments on their early stages. Mathematical algorithms are widely used not only for images reconstruction but also for PET data correction. In this paper random coincidences and scatter correction algorithms implementation are considered, as well as algorithm of PET projection data acquisition modeling for corrections verification.

1. Introduction
PET method is based on 511 keV photon pairs registration: radionuclide injected to patient before the procedure generates positrons annihilating with electrons in internal tissue after a short free passage. The main forms of interaction between photons with energy value 511 keV and tissues involve absorption in the result of photoelectric effect and Compton scattering. Both effects affect on images obtained during the procedure. Random coincidences factor, radiation attenuation during decay process, geometric factors connected with detectors and patient motion also influence on images quality. Random coincidences arise when pair of photons from different annihilation acts are registered in the same time window.

Various mathematical algorithms are using in PET data processing [1]. Radiation attenuation during decay process and geometric factors are usually corrected under reconstruction, for example in iteration maximum likelihood estimation method [2]. For random coincidences correction delayed time window method is generally used [3]. Patient motion can be corrected by using an algorithm based on cross-correlation function method [4]; the approaches based on the determination of the velocity field can be helpful for the motion correction as well [5]. For scatter correction the following approaches are used: multiple energy-window (spectral-analytic) approaches and methods based on scatter coincidences distribution estimation from linear attenuation coefficients and activity distributions obtained during emission and transmission tomography data reconstruction [6].

2. PET projection data acquisition modeling algorithm
PET projection data acquisition modeling algorithm is based on Monte-Carlo methods and can be presented in the following diagram.
A free passage length $\varepsilon$ is taken in the following way [7]:

$$\varepsilon = -\frac{1}{\mu} \cdot \ln \gamma, \gamma \in [0,1].$$

An equally probable direction of photon flight $\bar{\omega}$ is chosen as

$$\bar{\omega} = i \cdot \omega_i + j \cdot \omega_j + k \cdot \omega_k,$$

$$\omega_i = \sqrt{1 - \cos^2 \theta \cdot \cos \varphi},$$

$$\omega_j = \sqrt{1 - \cos^2 \theta \cdot \sin \varphi},$$

$$\omega_k = \cos \theta,$$

$$\cos \theta = 2 \cdot \gamma_1 - 1, \gamma_1 \in [0,1],$$

$$\varphi = 2\pi \gamma_2, \gamma_2 \in [0,1].$$

The form of interaction between particle and tissue is defined as following: $\alpha = \max_{x \in V} (\mu(x))$, random value $\gamma \in [0,1]$ is chosen. Now if

- $\gamma \leq \frac{\mu_x}{\alpha}$, then photon is absorbed;
- $\frac{\mu_x}{\alpha} < \gamma \leq \frac{\mu_x + \mu_e}{\alpha}$, then photon is scattered;
- $\gamma > \frac{\mu_x + \mu_e}{\alpha}$, then photon keeps moving in the same direction.

The considered algorithm is implemented by C# program. The input data for the algorithm are activity and linear attenuation distributions given as three-dimensional arrays, data acquisition time, radionuclide decay half-life and detectors configuration. PET projection data acquisition modeling results for different phantoms are given below.

**Figure 1.** PET projection data acquisition modeling algorithm diagram.

**Figure 2.** Sinogram for acquisition without attenuation account, uniform cylinder as a phantom is used.

**Figure 3.** Sinogram for acquisition with attenuation account, uniform cylinder as a phantom is used.

**Figure 4.** Sinogram for acquisition without attenuation account, set of five uniform cylinders as a phantom is used.
3. Random coincidences correction algorithm

Random coincidences correction algorithm is based on random coincidences sinogram generation with the use of delayed time window method. The coincidence list is constructed from a single event registration list and a given time window which is maximal difference among single events from the same annihilation act. Coincidence time window value depends on detectors ring radius and usually takes about 10 ns. If time window is much more than coincidence time window then we obtain a coincidences list without true coincidences. That coincidences list is an approximation for random coincidences sinogram construction [3].

Random coincidences correction algorithm consists of three steps:

1. Coincidence list obtaining with the use of delayed time window;
2. Random coincidences sinogram construction;
3. Subtraction of obtained sinogram from uncorrected sinogram.

The considered algorithm is implemented by C# program, the results are given below.

![Figure 5](image1) Uncorrected sinogram.

![Figure 6](image2) Random coincidence sinogram.

![Figure 7](image3) Corrected sinogram.

4. Single scatter correction algorithm

Single scatter correction algorithm for sinogram consists of the following steps [8]:

- Scattering point distribution in reconstructed volume;
- Single scatter quantity estimation for each sinogram line of response;
- Scattered sinogram construction;
- Subtraction of obtained sinogram from initial sinogram.

Single scatter quantity estimation for a given line of response is obtained as follows: \( N = N_{ij} + N_{ji} \), where \( i \) and \( j \) are detectors indexes identifying line of response,

\[
N_{ij} = P_{D_iM} \cdot \int_{D_i}^{M} \lambda(\tau) d\tau \cdot P_s \cdot P_{D_jM},
\]

\[
P_{D_iM} = \exp \left( \int_{D_i}^{M} \mu_e(\tau) d\tau \right),
\]

\[
P_j = \frac{\mu_s(M)}{\sigma} \cdot \frac{d\sigma}{d\Omega} \cdot \Omega_j,
\]

where \( \lambda(r) \) is a uncorrected activity distribution, \( D_i, D_j \) are detectors centers, \( M \) is one of scattering points, \( \mu_e(\tau), \mu_s(\tau) \) are linear attenuation coefficients of attenuation and scattering, \( \sigma \cdot \frac{d\sigma}{d\Omega} \)
are total and differential scattering cross sections, $\Omega_j$ is a solid angle for detector $j$ and with vertex in point $M$.

The considered algorithm is implemented by C# program, the results are given below.

![Figure 8. Initial sinogram.](image1)

![Figure 9. Scattered sinogram.](image2)

![Figure 10. Corrected sinogram.](image3)

Hence the sinograms have less noise level with the use of this algorithm.

5. Conclusion
In the course of this work PET projection data acquisition modeling algorithm which considers radiation attenuation and scattering is implemented. The obtained results may be used for PET scanners construction and verification of correction algorithms. Also single scattering and random coincidences correction algorithms are implemented and verified by using model PET projection data.

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