HIGH-FREQUENCY GRAVITATIONAL WAVES FROM SUPERMASSIVE BLACK HOLES: PROSPECTS FOR LIGO–VIRGO DETECTIONS

Bence Kocsis
Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138, USA; bkocsis@cfa.harvard.edu

Received 2012 September 20; accepted 2012 December 8; published 2013 January 16

ABSTRACT

It is commonly assumed that ground-based gravitational wave (GW) instruments will not be sensitive to supermassive black holes (SMBHs) because the characteristic GW frequencies are far below the ~10–1000 Hz sensitivity bands of terrestrial detectors. Here, however, we explore the possibility of SMBH GWs to leak to higher frequencies. In particular, if the high-frequency spectral tail asymptotes to $\tilde{h}(f) \propto f^{-\alpha}$, where $\alpha \lesssim 2$, then the spectral amplitude is a constant or increasing function of the mass $M$ at a fixed frequency $f \gg c^3/GM$. This will happen if the time-domain waveform or its derivative exhibits a discontinuity. Ground-based instruments could search for these universal spectral tails to detect or rule out such features irrespective of their origin. We identify the following processes which may generate high-frequency signals: (1) gravitational bremsstrahlung of ultrarelativistic objects in the vicinity of an SMBH, (2) ringdown modes excited by an external process that has a high-frequency component or terminates abruptly, and (3) gravitational lensing echoes and diffraction. We estimate the order of magnitude of the detection signal-to-noise ratio for each mechanism (1, 2, and 3) as a function of the waveform parameters. In particular for (3), SMBHs produce GW echoes of inspiraling stellar mass binaries in galactic nuclei with a delay of a few minutes to hours. The lensed primary signal and GW echo are both amplified if the binary is within a $\sim 10$ deg ($r/100M$)$^{-1/2}$ cone behind the SMBH relative to the line of sight at a distance $r$ from the SMBH. For the rest of the binaries near SMBHs, the amplitude of the GW echo is $\sim 0.1(r/100M)^{-1}$ of the primary signal on average.

Key words: black hole physics – gravitational waves – relativistic processes

Online-only material: color figures

1. INTRODUCTION

Gravitational wave (GW) observations are expected to open a new window on the universe within the decade. First-generation GW detectors (LIGO, Virgo, TAMA, GEO) have been successfully commissioned, and the development of the next advanced-sensitivity ground-based detectors (Advanced LIGO, Advanced Virgo, KAGRA) is well underway. However, the construction of the planned space-based detector LISA/NGO may be delayed until after 2020.¹

It is well known that the GW frequency, generated by a purely gravitational process involving mass $M$, typically does not exceed $M^{-1}$. In particular, for quasi-circular black hole (BH) inspirals, the spectrum peaks near twice the orbital frequency. As the separation shrinks, the GW frequency increases gradually until after $2020.1$

where $f_{\text{iso}} = (6^{3/2} \pi M)^{-1} = 4.4 M_6^{-1}$ mHz (where $M_6 = M/(10^6 M_\odot)$) corresponding to the innermost stable circular orbit. Then the BHs fall in quickly and the GW frequency reaches the fundamental quasinormal ringdown frequency $f_{\text{qnr}} = (5.4 \pi M)^{-1} = 12 M_6^{-1}$ mHz. Similarly, hyperbolic, parabolic, eccentric, and zoom-whirl encounters have a spectrum that peaks near $f_p \sim (2\pi r_p/v_p)^{-1} < 10 M_6^{-1}$ mHz, where $r_p \gg 3 M$ and $v_p < 1$ are the distance and relative speed at closest approach. These numbers correspond to nonspinning BHs, and vary by a factor of ~10 for spinning binaries (Berti et al. 2009). This suggests that GWs are in the sensitive frequency band of terrestrial instruments (10–1000 Hz) only for low-mass objects below $M_{\text{max}} \sim 10^4 M_\odot$ (Flanagan & Hughes 1998; Amaro-Seoane & Santamaria 2010).

While it is clear that LISA would be very sensitive to GWs emitted by supermassive black holes (SMBHs), we raise here the possibility of whether ground-based GW instruments could also detect massive sources. Since the instantaneous GW power output during a merger of compact objects is a constant multiple of $c^3/G \sim 10^{42}$W, independent of $M$, which persists for a long time of order $\Delta t \gtrsim 10 M = 50 M_6$ s, it is not obvious a priori whether a small portion of this enormous GW power leaking to high frequencies may be non-negligible compared to the detector sensitivity level.

We emphasize that the common assumption that BH space times cannot radiate at high frequencies $f \gg M^{-1}$ is not true in general. In this paper, we review previous results that show SMBHs are, at least in principle, capable of emitting or perturbing high-frequency GW signals that LIGO/VIRGO may detect. We present general arguments that can be used to search for high-frequency SMBH waves in the actual LIGO/VIRGO data, and motivate further theoretical research to identify astrophysical mechanisms driving such modes.

High-frequency spectral tails, at frequencies well beyond the characteristic scale, $M^{-1}$, may be expected to follow certain universal trends. A purely gravitational process involving BHs has only one-dimensional scale: the mass, $M$. The mass scaling of the GW spectrum follows from dimensional analysis. The Fourier transform of the dimensionless GW strain for a source at distance $D$ is most generally

\[ \tilde{h}(f) = \frac{M^2}{D} \psi(Mf), \]
where $\psi$ is an arbitrary dimensionless function which may depend on dimensionless source parameters.\(^3\) We are interested in whether there is any circumstance where the spectrum is an increasing function of $M$ at fixed frequency $f \gg M^{-1}$ if fixing all dimensionless parameters. Based on Equation (1), this is possible if $\tilde{h}(f) \propto f^{-\alpha}$ at high frequencies for $\alpha \leq 2$.

Similar effects, producing extended power-law spectral tails, are common in other fields of science and are often related to discontinuities in the signal or its derivatives. Examples are the “leakage” in data analysis when signals are abruptly terminated at the end of the sampling interval (Press et al. 1992), or “termination waves” and “wind contamination” in geophysical gravity waves (Pulido & Caranti 2000). A similar $1/f^\alpha$ spectrum is also produced by stochastic processes in physics, technology, biology, economics, psychology, language, and even music (for reviews, see Press 1978; Milotti 2002). Here, we demonstrate that spectral GW tails associated with (approximate) discontinuities have a universal high-frequency tail, $f^{-\alpha}$, where $\alpha$ is a positive integer, independent of the details of the waveform. Among these spectral profiles, Equation (1) shows that an increasing function of mass requires $\alpha = 1$ or 2.

The distinct universal shape of these high-frequency spectral tails allows to conduct searches for these GW burst sources irrespective of their origin. The relative arrival times and displacements of these and other possibilities generating high-frequency features which have nothing to do with the late-time temporal GW tail may constrain the model of gravity.

In this paper, we provide arguments which may guide future investigations to identify astrophysical mechanisms involving SMBHs that may produce high-frequency GW spectra for LIGO/Virgo detections. First, we highlight the general connection between spectral tails and discontinuities using Watson’s lemma. Then, we discuss three examples where high-frequency leakage may occur: ultrarelativistic encounters, quasinormal ringdown modes, and GW echoes. We highlight the need for further research in this direction to make more detailed assessments of these and other possibilities generating high-frequency spectral tails.

2. THEOREM ON SPECTRAL TAILS

Watson’s lemma states that any piecewise infinitely differentiable, asymptotically exponentially decaying function, $h(t)$, with a discontinuity in the $n$th time derivative at an instant $t_0$, has an asymptotic spectral tail\(^4\) $|\tilde{h}(f)| \propto f^{-(\alpha+1)}$ for $f \to \infty$ (Watson 1918).

\(^3\) To ensure a finite GW energy requires $\psi(Mf) < (Mf)^{-1/2}$ as $f \to \infty$.

\(^4\) To avoid confusion, note that the high-frequency GW tail discussed here is a spectral feature which has nothing to do with the late-time temporal GW tail of quasinormal modes due to the scattering of GWs off the curved background (Price 1972; Gleiser et al. 2008).

Watson’s lemma can be generalized to smooth waveforms, which do not have an exact discontinuity in any derivative, but for which the $n$th derivative undergoes a rapid transition within (comoving) time $\Delta t_0 \ll M$. Combining with the dimensional analysis argument, Equation (1),

$$\tilde{h}(f) \propto \frac{M^{1-n}}{d_0 f^{1+n}}, \quad \text{for } (\Delta t_0)^{-1} \gg f \gg M_\odot^{-1},$$

where for a source at cosmological redshift $z$, luminosity

$$d_L = (1+z)M, \quad \Delta t_0 = (1+z)\Delta t_0, \quad \text{and} \quad f = f_{\text{em}}/(1+z)$$

are the observed (cosmological-redshifted) quantities where $f_{\text{em}}$ is the local emission frequency near the source. The dimensionless constant of proportionality is independent of $M$, $d_L$, and $f$.

We conclude that the spectral amplitude may be an increasing function of the source mass $M$ at high frequencies for fixed dimensionless parameters if $n \leq 1$, i.e., when the signal amplitude or its derivative undergoes a rapid transition within a time $\Delta t_0 \ll 1/f$. In the following, we present specific examples where this may be expected.

3. GRAVITATIONAL BREMSSTRAHLUNG

Gravitational bremsstrahlung during ultrarelativistic flybys of compact objects with a high Lorentz factor $\Gamma$ is an example where the time-domain GW signal exhibits an approximate discontinuity in the first derivative (Kovacs & Thorne 1978). The previous dimensional arguments suggest that ground-based instruments may be sensitive to GWs from such high-speed flybys of objects near SMBHs. How large does the relative velocity have to be for detection?

The GWs have been calculated for ultrarelativistic encounters where $b$, the impact parameter, is sufficiently large to ensure that the two objects move on nearly linear trajectories (Kovacs & Thorne 1978). The time-domain GW signal is a “v-shaped” burst with a sharp discontinuity in the first derivative and is beamed in the direction of motion. In the rest frame of mass $m_1$,

$$\tilde{h}(f) \approx \frac{4m_1m_2(1+v^2)^3}{d_L^2 v^2} \left[ (1-v)K_2(w) - (1+v)K_2(u) \right]$$

$$\approx \frac{8}{\pi^2 b^2 d_L^2} \frac{\Gamma^3}{f^2}, \quad \text{if } \Gamma \geq \frac{f}{2\pi b} \geq \frac{\Gamma^2}{2\pi b} \quad \text{and} \quad \Gamma \gg 1,$$

in the forward direction at angles $\theta \ll \Gamma^{-1/2}$, where $\Gamma = (1-v^2)^{-1/2}$ is the Lorentz factor, $v$ is the relative velocity. Here, $K_2(x)$ is a modified Bessel function, $w = 2\pi f b/(v\Gamma)$, $w = 2\pi f b/(v(1+v)\Gamma^2)$, $q = m_2/m_1 < 1$ is the mass ratio, $b$ is the impact parameter, and $b = b/m_1$. Assuming that $m_1 \approx M \gg m_2$, self-consistency requires $b \gg 1$ (see Death 1978; D’Eath & Payne 1992; Segalis & Ori 2001, and discussion below for the limit $b \sim 0$). We find that the signal extends into the Advanced LIGO/Virgo frequency band ($f \gtrsim 10$ Hz) if $\Gamma \gtrsim 18 M_\odot^{1/2} b^{1/2}$. Equation (3) shows that the spectral amplitude is independent of $M$ for fixed dimensionless parameters and fixed $f$, as it should, based on the dimensional arguments of Equation (2). Figure 1 (bottom panel) shows that the amplitude may well exceed the Advanced LIGO/Virgo sensitivity level for SMBHs at 1 Gpc.

Whether or not such ultrarelativistic objects exist near SMBHs is unknown at present, but we argue that this possibility cannot be ruled out on theoretical grounds and may be tested observationally with Advanced LIGO and Virgo. Ultrarelativistic emission with Lorentz factors between $\Gamma \sim 5$
to 40 is observed in AGNs with jets (Urry & Padovani 1995; Jorstad et al. 2001; Sikora et al. 2005; Jorstad et al. 2005). These jets are believed to be launched by electromagnetic processes near a Kerr BH (Blandford & Znajek 1977; McKinney & Blandford 2009). The total mass outflow rate during flares, inferred from observations, reaches values comparable to the mass accretion rate of the BH, up to $M_c \, yr^{-1}$ (Celotti & Ghisellini 2008). These observations suggest that at least energetically, the magnetic field and angular momentum of a Kerr SMBH is sufficiently large to accelerate a neutron star or a white dwarf to ultrarelativistic velocities. Segalis & Ori (2001) considered the high-frequency GW signal during the acceleration of a stellar mass localized blob of matter. In this case, $h(t) \sim 4\Gamma^3 m/d_t \hat{L}$ in the forward direction and $2\Gamma m(1 + \cos \theta)/d_t \hat{L}$ at large angles $\theta \gg \Gamma^{-1}$. Here $m$ is the mass of the accelerated object, and the frequency extends to $f_{\text{cut}} \sim \Gamma^2/\Delta t$, where $\Delta t$ is the acceleration timescale in the SMBH rest frame.

Second, a merging SMBH binary may gravitationally accelerate stellar mass compact objects to ultrarelativistic velocities, if they happen to be in the vicinity of the merging binary (van Meter et al. 2010). The latter possibility is supported by several arguments. First, a widely separated SMBH binary gravitationally traps stellar objects in mean-motion resonances from the stellar cluster surrounding the binary. These objects are analogs of Trojan satellites in the solar system coorbiting with Jupiter. An accretion disk around the binary may also supply objects to mean-motion resonances by capturing stars crossing the disk or forming stars in the outskirts of the disk and transporting them radially inward close to the binary (Šubr & Karas 1999; Karas & Šubr 2001; Ayliffe et al. 2012). As the SMBH binary inspirals, objects locked in a mean-motion resonance are dragged inward to small radii down to 10 Schwarzschild radii (Schnittman 2010; Seto & Muto 2010, 2011). Numerical relativity simulations by van Meter et al. (2010) have shown that test particles surrounding an SMBH binary may be accelerated to high Lorentz factors during the final merger of the binary. If so, the GW bremsstrahlung of these objects due to the SMBH may be candidates for LIGO/VIRGO detections. Future studies should investigate this process in more detail for a variety of initial conditions, SMBH binary mass ratios and spins, and stellar mass gravitating particles to determine the likelihood of such detections.

### 4. BLACK HOLE RINGDOWN

A perturbed BH behaves as a damped harmonic oscillator, and emits GWs (for a review, see Berti et al. 2009):

$$h_{\pm}(t) = \sum_k A_k S_k \frac{e^{-\tau_k t}}{d_t} \left( \frac{\cos(2\pi f_k t)}{\sin(2\pi f_k t)} \right),$$

where $\pm$ and $\times$ denote the polarization, $t > 0$ is time from the end of the external perturbation, $f_k$ is the quasinormal ringdown frequency, $1/\tau_k$ is the decay rate, and $S_k$ are spin-weighted spheroidal harmonics which set the angular distribution of the emission with root mean square $(4\pi)^{-1/2}$. Here, $k$ is a discrete index which runs over all of the ringdown modes ($n, \ell, m$) analogous to the principal, angular momentum, and magnetic quantum numbers of the hydrogen atom. While $f_k$ and $\tau_k$ are universal properties of BHs which depend only on their mass and spin, the $A_k$ amplitude coefficients, called quasinormal excitation factors, are sensitive to the details of the perturbation. The excitations factors can be computed as a convolution of an external source with the Green’s function of the SMBH (Leaver 1986; Andersson 1995; Dorband et al. 2006; Berti & Cardoso 2006; Baker et al. 2007; Berti et al. 2010; Dolan & Ottewill 2011; Hadar et al. 2011).

To compare with the sensitivity levels of GW instruments, one needs to calculate $h(f)$, the Fourier transform of the waveform. For a damped harmonic oscillator, the high-frequency tail depends on the start of the waveform. Two conventions are used in the literature discussing the detectability of ringdown waveforms (Echeverria 1989; Finn 1992; Flanagan & Hughes 1998; Berti et al. 2007, 2006). In the Echeverria–Finn (EF) convention, the signal is assumed to vanish for $t < 0$ and follow Equation (4) for $t \geq 0$. In the Flanagan–Hughes (FH) convention, to avoid an unphysical discontinuity at $t = 0$, the signal is extrapolated symmetrically for $t < 0$ and the overall amplitude is renormalized by $2^{-1/2}$ to keep the total GW energy fixed. However, the signal used in the FH convention is also discontinuous in the first derivative, which has an impact at high frequencies. EF and FH introduced these conventions for convenience to calculate the detectability near the characteristic ringdown frequency of the signal, but here we explore the high-frequency asymptotics of these conventions and identify the corresponding physical conditions. Physically, an EF spectrum is appropriate in the case where the QNM is excited by an instantaneous pulse. The FH spectrum represents excitation processes that build up the mode continuously, but generate a sudden change in the derivative of the waveform as the excitation is terminated abruptly at $t = 0$, letting the BH ring freely thereafter. For waveforms excited smoothly on timescales $\tau_1$, we introduce a third convention which is continuous in all
derivatives, by replacing $e^{-t/\tau}$ by $\sqrt{2}\, \text{sech}(t/\tau_k) = \sqrt{8/(e^{t/\tau_k} + e^{-t/\tau_k})}$ in Equation (4).

The Fourier transform of the ringdown waveform can be obtained analytically in the three conventions for each mode $k$ comprising the waveform, independently. In the limit of asymptotically large frequencies $f \gg \max(f_k, \tau_k^{-1})$, we get

$$|\tilde{h}_{kx}^{\text{EF}}| \approx \frac{A_k}{2\pi d_L f} S_{2, k}, \quad |\tilde{h}_{kx}^{\text{FH}}| \approx \frac{A_k}{2\pi d_L f^2} S_{2, k},$$

(5)

$$|\tilde{h}_{kx}^{\text{FH}}(f)| \approx \frac{A_k}{\sqrt{\pi^2 \tau_k d_L}} f^2, \quad |\tilde{h}_{kx}^{\text{FH}}(f)| \approx \frac{A_k}{\sqrt{\pi^2 \tau_k d_L}} f^3,$$

(6)

$$\left(\frac{\tilde{h}_{kx}^{\text{secch}}(f)}{\tilde{h}_{kx}^{\text{secch}}(f)}\right) \approx \frac{\pi A_k}{\sqrt{2} d_L \cosh(\pi \tau_k f)} \left(\frac{\cosh(\pi Q_k)}{\sinh(\pi Q_k)}\right),$$

(7)

where $Q_k \equiv \pi \tau_k f_k$ is the quality factor of the damped oscillator. Equations (5)–(7) show that the asymptotic spectral tail may enter the detector band even if $f \gg M^{-1}$. For fixed $f$, the spectral GW amplitude scales very differently with $M$. Since $A_k$, $\tau_k$ and $f_k^{-1}$ are all proportional to $M$ for fixed $k$, we find that $|\tilde{h}_{kx}^{\text{EF}}(f)| \propto M$, $|\tilde{h}_{kx}^{\text{FH}}(f)|$ and $|\tilde{h}_{kx}^{\text{FH}}(f)|$ are independent of $M$, and $|\tilde{h}_{kx}^{\text{secch}}(f)|$ and $|\tilde{h}_{kx}^{\text{secch}}(f)|$ are asymptotically exponentially suppressed for $f \gg M^{-1}$. In the later case, the peak spectral amplitude is proportional to $A_k \tau_k \propto M^2$ for a fixed $k$ at frequencies $f \sim \tau_k^{-1}$. At $f \sim \tau_k^{-1}$ for some $k$, the orientation-averaged one-sided dimensionless GW spectral amplitude per logarithmic frequency bin for this signal is $(2 f \tilde{h}_{kx}^{\text{secch}}) \sim \left(4/5\right)^{(4/5)(2^{-1/2})} \tau_k^{1/2} d_L / d_L$. These mass scalings also follow from dimensional analysis once the general expectations from Watson’s lemma (Section 2) and are consistent with the general expectations from Watson’s lemma (Section 2). The EF and FH spectra become invalid beyond a frequency $f \gg \tau_k^{-1}$, where $\Delta \tau$ represents the characteristic timescale over which the excitation occurs generating the approximate discontinuity in the signal amplitude or derivative, respectively.

The top panel of Figure 1 shows the angular-averaged spectral amplitude for $M = 10^6 M_\odot$, $d_L = 100$ Mpc, and $A_k = 0.1 r$. Different line types correspond to different conventions at the start of the waveform as labeled. Two sets of curves peaked at $\sim 0.01$ and $30$ Hz represent $\tau = 10^8$ (weakly damped modes) and $10^{-3}$ (highly damped modes), respectively. The GW signal leaks into the LIGO/Virgo frequency band for FH-type excitation processes both for weakly and highly damped modes. If the excitation occurs smoothly over timescale $\tau$ (blue curves), then the signal may enter the LIGO/Virgo band if $\tau \sim 10^8 M_\odot$, but not if $\tau$ is much larger. Comparing to the minimum frequency and sensitivity level of Advanced LIGO/VIRGO, we conclude that SMBH ringdown waveforms may be detectable on average for $100$ Mpc if the quasinormal modes are excited to an amplitude exceeding $A_k \sqrt{0.1 \, \tau}$.\footnote{Note that the $\times$ polarization of the waveform is suppressed at high frequencies because the signal vanishes at $\tau = 0$ in this case; see Equation (4).}

Going beyond the statement that some QNMs may have a potentially detectable high-frequency tail, it would be desirable to identify conceivable astrophysical processes exciting these modes to detectable levels. The $A_k$ excitation factors of QNMs are explored only for a very limited number of configurations to date (Leaver 1986; Andersson 1995; Dorband et al. 2006; Berti & Cardoso 2006; Baker et al. 2007; Berti et al. 2010; Dolan & Ottewill 2011; Kocsis et al. 2011). For this discussion, we distinguish three classes of ringdown modes.

1. High-$\ell$ modes. For a Schwarzschild BH, $f_{\text{res}} \approx \ell \Omega_\text{c}/(2\pi)$ and $1/\tau_{\text{res}} \approx n \Omega_\text{c}$, where $\Omega_\text{c} = 27^{-1/2} M^{-1}$ is the orbital frequency at the light ring (Press 1971).\footnote{Similar expressions hold for Kerr BHs, see Ferrari & Mashhoon (1984), Yang et al. (2012) and Keshet & Ben-Meir (2012).} These modes may be identified with energy leakage from the photon sphere (Goebel 1972; Ferrari & Mashhoon 1984; Dolan & Ottewill 2011; Yang et al. 2012). They ring in the LIGO/VIRGO band if $10 H \lesssim f_{\text{res}} \lesssim 10^2$ Hz.

2. Weakly damped modes. Consider modes with $f_{\text{res}} \lesssim M^{-1}$, $1/\tau_{\text{res}} \sim M^{-1}$. Although, the ringdown frequency is well outside the LIGO/VIRGO frequency range in this case for large $M$, a rapidly changing external driving process on timescale $\Delta \tau \lesssim 0.1 s$ may lead to high-frequency tails for these modes (see, e.g., EF and FH above). This is analogous to driving a harmonic oscillator with a process that has a component well above the resonant frequency. Further studies are necessary to see if the high-frequency spectral amplitude of the QNM may be sufficiently large for any specific driving process in practice.

3. Highly damped modes. The decay rate of very high overtones may be arbitrarily large, $\lim \tau_{\text{res}} = \infty$ for all $\ell$ and $m$, which implies a high-frequency spectral tail. Although the quality factor of these modes approaches zero, their excitation may still be sufficient to reach levels required for detection as stated above ($A_k \approx 0.1 \tau$ at $100$ Mpc). For a damped oscillator driven by a stationary external process of amplitude less than unity, $A_k \approx \tau_k/(M f_k)$. These modes are detectable to $100$ Mpc with Advanced LIGO if $f_k \lesssim 100 M_\odot^{-1}$ and $\tau_k \sim 0.1 s$ (see the right blue curve on the top panel of Figure 1 and discussion above), which is satisfied for all modes with $\ell \lesssim 3000$.

We highlight two astrophysical processes which may generate high-frequency QNMs. First, a merging SMBH binary gravitationally accelerates particles and nearby stellar mass compact objects to ultrarelativistic velocities which may then escape or merge with the SMBH (van Meter et al. 2010, and see Section 3 above). Ultrarelativistic collisions with an SMBH (Davis et al. 1972; Berti et al. 2010) excite high-$\ell$ ringdown modes. The radiative energy scales as $E_\ell \propto 1/\ell^{-dS}$, where $d_S \approx 2$ for radial infall and 1 for grazing captures, implying that $h(f) \sim f^{-3/2}$ to $f^{-2}$ for high $f$. Thus, similar to ultrarelativistic gravitational bremsstrahlung, the high-frequency spectral GW amplitude of ultrarelativistic capture sources is a constant or increasing function of mass for a fixed mass ratio (see Equation (1)).

Second, the high-frequency gravitational effects of a stellar (or intermediate) mass ($M_b$) BH binary may excite high-frequency QNMs of the SMBH. Stellar mass compact binaries form during close encounters and merge due to GW emission (O’Leary et al. 2009; Kocsis & Levin 2012) or they may be driven to merger due to the Kozai effect (Wen 2003; Antonini & Perets 2012; Naoz et al. 2012). The QNM excitation function of these sources has a high-frequency component and the driving force terminates on a short timescale proportional to $M_b$. Based on the arguments outlined above, it is then plausible to expect a high-frequency ringing of the SMBH.
and the detectability of the signal to assess the likelihood of further studies are necessary to examine the excitation factors. The frequency GWs are scattered by the gravitational field of the black hole binary source in a galactic nucleus near an 
SMBH, generating GW echoes. The scattered waves represent (or intermediate) mass compact binaries are scattered by the 
Figure 2. Gravitational lensing magnification of a high-frequency GW source by an SMBH as a function of deflection angle \( \alpha \). Colored curves use the thin lens equation; black dotted lines show the plane wave limit. The three sets of curves correspond to different source distances \( r \) from the SMBH as labeled; different line styles represent relativistic images of different order. Solid curves show the primary signal; dashed and dotted-dashed curves show the amplitude of the first GW echo. The peak at small deflection angles corresponds to strong lensing at the Einstein radius \( \alpha_E = (4r/M)^{1/2} \); the second peak at 180° represents the glory phenomenon in geometrical optics. Additional peaks appearing at \( \alpha = n\pi \), where \( n \) is an integer, are not shown beyond \( n = 1 \). The GW frequency is assumed to greatly exceed \( M^{-1} \). Diffraction fringe patterns (not shown) appear with spacing \( h_0 \sim (34 M f)^{-1} \) and sharper features are smoothed. (A color version of this figure is available in the online journal.)

Furthermore, the high-frequency GWs emitted by such stellar (or intermediate) mass compact binaries are scattered by the SMBH, generating GW echoes. The scattered waves represent high-r ringdown modes, which we discuss in Section 5 below. Further studies are necessary to examine the excitation factors and the detectability of the signal to assess the likelihood of detecting these waveforms.

5. GW LENSING ECHOES AND DIFFRACTION

Consider the GWs emitted by an inspiraling stellar mass BH binary source in a galactic nucleus near an SMBH. High-frequency GWs are scattered by the gravitational field of the SMBH (e.g., Futterman et al. 1988) producing GW echoes (Dolan & Ottewill 2011; Zenginoğlu & Galley 2012). The GW frequency is not effected to leading order in the WKB limit if the frequency of the incoming wave exceeds \( M^{-1} \). The scattered GWs, or GW echoes, are completely analogous to the relativistic images of electromagnetic waves for rays going around the SMBH in opposite directions and/or multiple times. However, GW echoes may be easier to distinguish from astrophysical foregrounds than the relativistic images of EM sources since high-frequency GW sources are sufficiently rare, have a short duration, and a distinct frequency evolution (O’Leary et al. 2009; Kocsis & Levin 2012). Furthermore, GW detectors are sensitive to the signal phase and amplitude, i.e., the square-root of the intensity, which is suppressed much less than the EM intensity (see Figure 2 below).

The GW echo arrives at the detector after the primary signal with a delay corresponding to the projected light-travel time between the object and the source,

\[
\Delta T = (1 - \cos \alpha) r \sim 14 \text{ hr} \times (1 - \cos \alpha) M_6 (r/10^4 M) \tag{8}
\]

where \( r \) is the distance between the binary and the SMBH and \( \alpha \) is the deflection angle. Note that the probability distribution of inspiraling stellar mass BH binaries in galactic nuclei formed by GW emission during close encounters increases toward the SMBH (O’Leary et al. 2009). A significant fraction (\( \sim 30\% \)) of sources are within \( r \lesssim 10^4 M \) for which the GW echoes occur within hours of the primary signal.

We calculate the amplitude of the GW echo for a source at radius \( r \) from the SMBH in the geometrical optics limit using the Virbhadra & Ellis (2000) lens equation valid for arbitrary deflection angles. We direct the reader to that paper for details (see also Bozza 2008). For a sanity check, we also calculate the scattering cross-section of plane waves impinging on the SMBH, which approaches the lensing cross-section in the large deflection angle limit if \( r \gg M \). The amplitude of the GW echo may be larger or smaller than the primary GW signal of the source depending on the relative position of the source, the SMBH, and the line of sight as shown in Figure 2. The lensing effect may strongly increase the intensity of the wave if the lens and the source are nearly collinear along the line of sight (Einstein 1936). For \( r \ll D \), the deflection angle corresponding to the Einstein radius for strong magnification is \( \alpha_E \sim (4/r M)^{1/2} \), i.e., \((11°, 3.6°, 1.1°)\) for \( r = (10^3, 10^4, 10^5) M \), respectively. For an isotropic distribution of sources, this implies that \((2, 0.2, 0.02\%)\) of sources are within the Einstein radius for the three values of \( r \), respectively. For larger deflection angles, the GW amplitude decreases as \( h \propto h_0/(r\alpha^2) \), where \( h_0 \) is the GW amplitude without lensing. The peak at \( \alpha \sim \pi \) (parallel backscattering) is known as the glory effect of geometrical optics (Futterman et al. 1988). The figure shows that the amplitude of relativistic images may be significant even for large deflection angles if \( r \lesssim 10^3 M \) and the primary signal has a signal-to-noise ratio larger than \( \sim 50 \).

Figure 2 assumes \( f \gg M^{-1} \) and neglects diffraction effects due to the interference of rays going around the SMBH in opposite directions and the QNM of the SMBH (Matzner et al. 1985; Nakamura 1998). This is a good approximation for typical large deflection angles of stellar mass GW inspirals at the typical distances \( r \gg 100 M \), since the timescale for the increase of the GW frequency is much less than \( \Delta T \) (Equation (8)). However, interference effects may become significant for sources that happen to be behind or in front of the SMBH close to the line of sight. This smears out the magnification peak and produces a GW fringe pattern in amplitude and phase on the angular scale \( \delta \alpha \sim 1/(34 f M) \sim 5.9 \times 10^{-4} x M_6^{-1}(f/100 \text{ Hz})^{-1} \) for a nonscattering SMBH (similar expressions with different prefactors hold for spinning SMBHs; see Dolan 2008).

The frequency and direction dependence of the GW diffraction patterns introduce interesting time variations in the signal. The diffraction pattern scale shrinks in time for inspiralling binary sources as \( f \) increases (Dolan 2008). Furthermore, the relative motion of the lens and the source lead to a varying GW amplitude analogous to the microlensing events of electromagnetic signals (Mao & Paczynski 1991) and phase shift. Assuming that the source moves along a circular orbit around the SMBH with a Keplerian angular velocity, \( \Omega_k \), it travels across the diffraction fringe within \( \delta t_f = \delta t/\Omega_k \sim 9.3 s \times (r/10^3 M)^{1/2}(f/100 \text{ Hz})^{-1} \). This is much less than the source lifetime for inspiraling stellar mass binaries. Based on the strong magnification rates estimated above, we conclude that a small fraction of sources (\( \lesssim 2\% \)) might exhibit these features.

If measured, the GW echo may be used to learn about the parameters of the SMBH beyond its distance from the merging stellar mass binary. Time-dependent effects of GW diffraction patterns of an inspiral signal and/or the microlensing effect
discussed above carry information about the mass and spin of the SMBH. The polarization of the scattered waves relative to that of the primary signal is also sensitive to the SMBH spin (Dolan 2008).

6. DISCUSSION

Ground-based instruments may, in principle, be sensitive to GWs emitted by SMBHs with arbitrarily large \( M \) at frequencies \( f \gg M^{-1} \). Dimensional analysis shows that the spectrum \( \tilde{h}(f) \) is a growing function of mass or is mass independent at large frequencies if \( \tilde{h}(f) \propto f^{-1} \) or \( f^{-2} \), respectively. Watson's lemma shows that such a spectrum corresponds to a waveform having an abrupt change in amplitude or, in its first time derivative, within a short time (\( \Delta t \ll 1/f \)). Discontinuous GW shock fronts with arbitrarily small \( \Delta t \) are self-consistent, exact solutions of general relativity (Vaidya 1951; Aichelburg & Sexl 1971; and see also cosmic strings for which \( \tilde{h}(f) \propto f^{-1/3} \), Damour & Vilenkin 2000, 2005), so one should not discard this possibility a priori.

Ground-based GW detectors could provide observational constraints on the event rates of such sources independent of their origin. Unlike other GW burst sources, these sources have a well-defined broadband spectral shape, \( \tilde{h}(f) \propto f^{-1} \) or \( f^{-2} \), allowing the development of optimal search algorithms for their efficient detection. These signals may be distinguished from instrumental glitches if they show up coincidently in all of the different instruments in a detector network (i.e., the two LIGO sites, Virgo, and Kagra) with a consistent spectral amplitude.

We have discussed three hypothetical examples involving SMBHs potentially capable of generating high-frequency GW signals: ultrarelativistic sources, QNM ringdown waveforms excited externally by high-frequency processes or those that terminate abruptly, and the scattering and diffraction of GWs. First, an ultrarelativistic stellar mass object moving in the gravitational field of an SMBH emits gravitational bremsstrahlung. The signal derivative undergoes a rapid transition on an observed timescale \( f_{\text{cut}}^{-1} \sim \Delta t/\Gamma^2 \), where \( \Gamma \) is the Lorentz factor and \( \Delta t \) is the acceleration timescale proportional to the impact parameter, \( b \). Then, for \( M^{-1} < f < f_{\text{cut}} \), the GW spectrum \( \tilde{h}(f) \propto f^{-2} \) is proportional to the mass of the object but independent of the mass of the SMBH, \( M \), extending well into the sensitive range of ground-based GW instruments if \( \Gamma \gtrsim 5.7 \, M_\odot b \). Here \( M_\odot = M/10^5 \, M_\odot \) and \( b = b/M \). Second, SMBHs, if perturbed, ring as damped harmonic oscillators, and emit ringdown GWs. We have shown that weakly damped ringdown waveforms leak into the LIGO frequency band if the excitation process has a high-frequency component or terminates within 0.1 s. These GWs may be detectable for arbitrary \( M \gg 100 \, M_\odot \), with Advanced LIGO/Virgo-type instruments to 100 Mpc if the GW amplitude satisfies \( A_\gamma \gtrsim 0.01 \tau \), where \( \tau \) is the decay time of the QNM. However, detailed calculations of excitation factors are necessary to determine whether or not this level may be reached for specific realistic configurations.

Finally, lensing and diffraction effects of high-frequency GWs by SMBHs may also generate detectable features. In particular, we expect a \( 0.02\%\)–\( 2\% \) of LIGO/Virgo sources in galactic nuclei (e.g., inspiral and merger of stellar mass BH or neutron star binaries) could be strongly magnified by the central SMBH. Diffraction limits the maximum magnification of these sources. The rest of the high-frequency sources in the vicinity of SMBHs produce weak GW echoes due to large deflection angle scattering by the SMBH. The GW echo amplitude is typically between 0.1\% and 10\% of the primary signal for these sources. The time delay between the primary and scattered signals in this case is typically tens of minutes.

It remains to be seen whether these processes occur in reality. To date, numerical simulations of binary mergers did not find evidence for waveforms having such sharp features for quasi-circular mergers (Baker et al. 2007), except for spurious GWs related to the improper choice of initial conditions (Field et al. 2010). However, these simulations are limited by resolution to comparable mass binaries (see Lousto & Zlochower 2011 for the most unequal mass ratio) and cannot resolve high-order ringdown modes. Recently, Berti et al. (2010) carried out simulations for head-on, grazing, and off-axis collisions with nonzero initial velocity. They found that the cutoff frequency is shifted to larger values for higher Lorentz factors and multipole moments. Future studies should investigate other possible examples, involving SMBHs. These may include high-frequency ringdown resonances in extreme mass ratio inspirals or three-body encounters. High-frequency GWs of SMBHs may also arise in alternative theories of gravity or cosmology due to the self-gravity of GWs (Fodor & Rácz 2003; Kaloper 2005).

One should also be aware of these spectral effects when constructing time-domain templates matching the inspiral, merger, and ringdown waveforms (Babak et al. 2007; Santamaria et al. 2010). A high-frequency power-law spectral tail is sensitive to a discontinuity in the derivative of the time-domain waveform at the matching boundaries. We have shown that the high-frequency QNMs are very sensitive to the initial conditions for both the EF and the FH conventions. This theorem may also be useful for debugging numerical codes.

I thank Emanuele Berti, Ryan O’Leary, Dan Holz, Lorenzo Sironi, Zoltan Haiman, and Scott Tremaine for discussions, and Nico Yunes and Cole Miller for useful comments after carefully reading the manuscript. The author acknowledges support by NASA through Einstein Postdoctoral Fellowship grant number PF9-00063 awarded by the Chandra X-ray Center (operated by the SAO, contract NAS8-03060).

REFERENCES

Abadie, J., Abbott, B. P., Abbott, R., et al. 2012, A&A, 541, A155
Aichelburg, P. C., & Sexl, R. U. 1971, GReGr, 2, 303
Amaro-Seoane, P., & Santamaria, L. 2010, ApJ, 772, 1197
Andersson, N. 1995, PhRvD, 51, 353
Antonini, F., & Perets, H. B. 2012, ApJ, 757, 27
Ayliffe, B. A., Laihe, G., Price, D. J., & Bate, M. R. 2012, MNRAS, 423, 1450
Babak, S., Fang, H., Gair, J. R., Glampedakis, K., & Hughes, S. A. 2007, PhRvD, 75, 024005
Baker, J. G., McWilliams, S. T., van Meter, J. R., et al. 2007, PhRvD, 75, 124024
Berti, E., Cardoso, J., Cardoso, V., & Cavaglià, M. 2007, PhRvD, 76, 104044
Berti, E., & Cardoso, V. 2006, PhRvD, 74, 104020
Berti, E., Cardoso, V., & Starinets, A. O. 2009, CQGra, 26, 163001
Berti, E., Cardoso, V., & Will, C. M. 2006, PhRvD, 73, 064030
Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433
Bozza, V. 2008, PhRvD, 78, 103005
Celotti, A., & Ghisellini, G. 2008, MNRAS, 385, 283
Dalal, N., Holz, D. E., Hughes, S. A., & Jain, B. 2006, PhRvD, 74, 063006
Damour, T., & Vilenkin, A. 2000, PhRvL, 85, 3761
Damour, T., & Vilenkin, A. 2005, PhRvD, 71, 063510
Davis, M., Ruffini, R., Tjonno, J., & Zerilli, F. 1972, PhRvL, 28, 1352
Death, P. D. 1978, PhRvD, 18, 990
D’Eath, P. D., & Payne, P. N. 1992, PhRvD, 46, 694
Deffayet, C., & Menou, K. 2007, ApJL, 668, 143
Dolan, S. R. 2008, CQGra, 25, 235002
Dolan, S. R., & Ottewill, A. C. 2011, PhRvD, 84, 104002

The Astrophysical Journal, 763:122 (7pp), 2013 February 1

Kocsis
