Abstract

The rare decays $B^0 \rightarrow D_s^- K^+$ and $B^+ \rightarrow D_s^+ \bar{K}^0$ can occur only via annihilation type diagrams in the standard model. We calculate these decays in perturbative QCD approach. We found that the calculated branching ratio of $B^0 \rightarrow D_s^- K^+$ agreed with the data which had been observed in the KEK and SLAC $B$ factories. While the decay $B^+ \rightarrow D_s^+ \bar{K}^0$ has a very small branching ratio at $\mathcal{O}(10^{-8})$, due to the suppression from CKM matrix elements $|V_{ub}V_{cd}|$. 
1 Introduction

The rare $B$ decays are being measured at $B$ factories in KEK and SLAC. The generalized factorization approach has been applied to the theoretical treatment of non-leptonic $B$ decays \cite{1}. It is a great success in explaining many decay branching ratios \cite{2,3}. The factorization approach (F.A.) is a rather simple method. Some efforts have been made to improve their theoretical application \cite{4} and to understand the reason why the F.A. has gone well \cite{5,6}. One of these methods is the perturbative QCD approach (PQCD), where we can calculate the annihilation diagrams as well as the factorizable and nonfactorizable diagrams.

The rare decays $B \to D_s K$ are pure annihilation type decays. In the usual F.A., this decay picture is described as $B$ meson annihilating into vacuum and the $D_s$ and $K$ mesons produced from vacuum then afterwards. To calculate this decay in the F.A., one needs the $D_s \to K$ form factor at very large time like momentum transfer $O(M_B)$. However the form factor at such a large momentum transfer is not known in F.A. This makes the F.A. calculation of these decays unreliable. The annihilation amplitude is a phenomenological parameter in QCD factorization approach (QCDF) \cite{4}, and the QCDF calculation of these decays is also unreliable. In this paper, we will try to use the PQCD approach, where the annihilation amplitude is calculable, to evaluate the $B \to D_s K$ decays. By comparing the predictions with the experimental data, we can test the PQCD evaluation of the annihilation amplitude.

A $W$ boson exchange causes $\bar{b}d \to \bar{c}u$ or $\bar{b}u \to \bar{d}c$, and the $\bar{s}s$ quarks included in $D_s K$ are produced from a gluon. This gluon attaches to any one of the quarks participating in the $W$ boson exchange. This is shown in Figure \ref{fig1}. In the rest frame of $B$ meson, both $s$ and $\bar{s}$ quarks included in $D_s K$ have $O(M_B/2)$ momenta, and the gluon producing them also has $q^2 \sim O(M_B^2/4)$. This is a hard gluon. One can perturbatively treat the process where the four quark operator exchanges a hard gluon with $s\bar{s}$ quark pair. It is just the picture of PQCD approach.

In the next section, we explain the framework of PQCD briefly. In section \ref{sec2}, we give the analytic formulas for the decay amplitude of $B \to D_s K$ decays. In section \ref{sec3}, we show the predicted branching ratio from the analytic formulas and discuss the theoretical errors. It
is found that the prediction are is in good agreement with the data, and PQCD approach correctly gives the annihilation amplitude in consequence. Finally, we conclude this study in section 3.

2 Framework

PQCD approach has been developed and applied in the non-leptonic $B$ meson decays for some time. In this approach, the decay amplitude is separated into soft($\Phi$), hard($H$), and harder($C$) dynamics characterized by different scales. It is conceptually written as the convolution,

$$\text{Amplitude} \sim \int d^4k_1 d^4k_2 d^4k_3 \text{Tr}[C(t)\Phi_B(k_1)\Phi_{D_s}(k_2)\Phi_K(k_3)H(k_1, k_2, k_3, t)],$$

where $k_i$'s are momenta of light quarks included in each mesons, and Tr denotes the trace over Dirac and color indices. $C(t)$ is Wilson coefficient which results from the radiative corrections at short distance. In the above convolution, $C(t)$ includes the harder dynamics at larger scale than $M_B$ scale and describes the evolution of local 4-Fermi operators from $m_W$, $W$ boson mass, down to $t \sim \mathcal{O}(\sqrt{\Lambda M_B})$ scale, where $\Lambda \equiv M_B - m_b$. $H$ describes the four quark operator and the spectator quark connected by a hard gluon whose $q^2$ is on the order of $\Lambda M_B$, and includes the $\mathcal{O}(\sqrt{\Lambda M_B})$ hard dynamics. Therefore, this hard part $H$ can be perturbatively calculated. $\Phi_M$ is the wave function which describes hadronization of the quark and anti-quark to the meson $M$. While the $H$ depends on the processes considered, $\Phi_M$ is independent of the specific processes. Determining $\Phi_M$ in some other decays, we can make quantitative predictions here.

We consider the $B$ meson at rest for simplicity. It is convenient to use light-cone coordinate $(p^+, p^-, p_T)$ to describe the meson’s momenta, where $p^\pm = \frac{1}{\sqrt{2}}(p^0 \pm p^3)$ and $p_T = (p^1, p^2)$. Using these coordinates we can take the $B$, $D_s$, and $K$ mesons momenta as $P_1 = \frac{M_B}{\sqrt{2}}(1, 1, 0_T)$, $P_2 = \frac{M_{D_s}}{\sqrt{2}}(1, r^2, 0_T)$, and $P_3 = \frac{M_K}{\sqrt{2}}(0, 1 - r^2, 0_T)$, respectively, where $r = M_{D_s}/M_B$ and we neglect the $K$ meson’s mass $M_K$. Putting the light (anti-)quark momenta in $B$, $D_s$ and $K$ mesons as $k_1$, $k_2$, and $k_3$, respectively, we can choose $k_1 = (x_1 P_1^+, 0, k_1 T)$, $k_2 = (x_2 P_2^+, 0, k_2 T)$,
and \( k_3 = (0, x_3 P^-_3, k_3 T) \). Then, integration over \( k_1^-, k_2^-, \) and \( k_3^+ \) in eq. (1) leads to

\[
\text{Amplitude} \sim \int dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 db_3 
\text{Tr} \left[ C(t) \Phi_B(x_1, b_1) \Phi_{D_s}(x_2, b_2) \Phi_K(x_3, b_3) H(x_i, b_i, t) S_t(x_i) e^{-S(t)} \right],
\]

(2)

where \( b_i \) is the conjugate space coordinate of \( k_i T \), and \( t \) is the largest energy scale in \( H \), as the function in terms of \( x_i \) and \( b_i \). The large logarithms (ln \( m_W/t \)) coming from QCD radiative corrections to four quark operators are included in the Wilson coefficients \( C(t) \). The large double logarithms (ln \( x^2 i \)) on the longitudinal direction are summed by the threshold resummation \[9\], and they lead to \( S_t(x_i) \) which smears the end-point singularities on \( x_i \). The last term, \( e^{-S(t)} \), contains two kinds of logarithms. One of the large logarithms is due to the renormalization of ultra-violet divergence ln \( t b \), the other is double logarithm ln \( x^2 b \) from the overlap of collinear and soft gluon corrections. This Sudakov form factor suppresses the soft dynamics effectively \[10\]. Thus it makes perturbative calculation of the hard part \( H \) applicable at intermediate scale, i.e., \( M_B \) scale. We calculate the \( H \) for \( B \to D_s K \) decays in the first order in \( \alpha_s \) expansion and give the convoluted amplitudes in next section.

3 Analytic formula

3.1 The wave functions

In order to calculate analytic formulas of the decay amplitude, we use the wave functions \( \Phi_{M,\alpha\beta} \) decomposed in terms of spin structure. In general, \( \Phi_{M,\alpha\beta} \) having Dirac indices \( \alpha, \beta \) are decomposed into 16 independent components, \( l_{\alpha\beta}, \gamma^\mu_{\alpha\beta}, \sigma^\mu_{\alpha\beta}, (\gamma^\mu \gamma^5)_{\alpha\beta}, \gamma_5\alpha\beta \). If the considered meson \( M \) is \( B \) or \( D_s \) meson, to be pseudo-scalar and heavy meson, the structure \( (\gamma^\mu \gamma^5)_{\alpha\beta} \) and \( \gamma_5\alpha\beta \) components remain as leading contributions. Then, \( \Phi_{M,\alpha\beta} \) is written by

\[
\Phi_{M,\alpha\beta} = \frac{i}{\sqrt{2N_c}} \left\{ (P_M \gamma_5)_{\alpha\beta} \phi^A_M + \gamma_5\alpha\beta \phi^P_M \right\},
\]

(3)

where \( N_c = 3 \) is color’s degree of freedom, \( P_M \) is the corresponding meson’s momentum, and \( \phi^A_P \) are Lorentz scalar wave functions. As heavy quark effective theory leads to \( \phi^P_B \simeq M_B \phi^A_B \), then \( B \) meson’s wave function can be expressed by

\[
\Phi_{B,\alpha\beta}(x, b) = \frac{i}{\sqrt{2N_c}} [(P_1 \gamma_5)_{\alpha\beta} + M_B \gamma_5\alpha\beta] \phi_B(x, b).
\]

(4)
According to ref. [11], a pseudo-scalar meson moving fast is parameterized by Lorentz scalar wave functions, $\phi$, $\phi_\mu$, and $\phi_\sigma$ as

$$\langle D_s^-(P)|\bar{s}(z)\gamma_\mu\gamma_5 c(0)|0\rangle \simeq -i f_{D_s} P_\mu \int_0^1 dx \ e^{ixPz}\phi(x), \tag{5}$$

$$\langle D_s^-(P)|\bar{s}(z)\gamma_5 c(0)|0\rangle = -i f_{D_s} m_{0D_s} \int_0^1 dx \ e^{ixPz}\phi_\mu(x), \tag{6}$$

$$\langle D_s^-(P)|\bar{s}(z)\gamma_5\sigma_{\mu\nu} c(0)|0\rangle = \frac{i}{6} f_{D_s} m_{0D_s} \left(1 - \frac{M_{D_s}^2}{m_{0D_s}^2}\right) (P_\mu z_\nu - P_\nu z_\mu) \int_0^1 dx \ e^{ixPz}\phi_\sigma(x), \tag{7}$$

where $m_{0D_s} = M_{D_s}^2/(m_c + m_s)$. We ignore the difference between $c$ quark’s mass and $D_s$ meson’s mass in the perturbative calculation. This means, putting $\bar{\Lambda}' \equiv M_{D_s} - m_c$, the terms proportional to $\bar{\Lambda}'/M_{D_s}$ are neglected. In this approximation, the contributions of eq.(7) are of higher power than those of eqs.(5, 6) by $O\left(\frac{\Lambda'}{M_{D_s}}\right)$ because of the factor $1 - \frac{M_{D_s}^2}{m_{0D_s}^2}$ in eq.(4), and we neglect the $\gamma_5\sigma_{\mu\nu}$ component in the $D_s$ meson’s wave function. In addition, the eq.(5), eq.(6), and the relations

$$\frac{\partial}{\partial z_\mu}\langle D_s^-(P)|\bar{s}(z)\gamma_\mu\gamma_5 c(0)|0\rangle = i m_s \langle D_s^-(P)|\bar{s}(z)\gamma_5 c(0)|0\rangle, \tag{8}$$

$$\frac{\partial}{\partial z_\mu}\langle D_s^-(P)|\bar{s}(0)\gamma_\mu\gamma_5 c(z)|0\rangle = i m_c \langle D_s^-(P)|\bar{s}(0)\gamma_5 c(z)|0\rangle, \tag{9}$$

with equations of motion lead to

$$\phi_\mu(x) = \phi(x) + O\left(\frac{\bar{\Lambda}'}{M_{D_s}}\right). \tag{10}$$

Therefore the $D_s$ meson’s wave function can be expressed by one Lorentz scalar wave function,

$$\Phi_{D_s,\alpha\beta}(x, b) = \frac{i}{\sqrt{2N_c}} \left[(\gamma_5 \ P_2)_{\alpha\beta} + M_{D_s}\gamma_5\alpha\beta\right] \phi_{D_s}(x, b), \tag{11}$$

where $\phi_{D_s}$ is defined by

$$\phi_{D_s}(x) = \frac{f_{D_s}}{2\sqrt{2N_c}} \phi(x) = \frac{f_{D_s}}{2\sqrt{2N_c}} \phi_p(x). \tag{12}$$

The wave function $\phi_M$ for $M = B, D_s$ meson is normalized by its decay constant $f_M$

$$\int_0^1 dx \ \phi_M(x, b = 0) = \frac{f_M}{2\sqrt{2N_c}}. \tag{13}$$

In contrast to the $B$ and $D$ mesons, for the $K$ meson, being light meson, the $\gamma_5\sigma^{\mu\nu}$ component remains because the factor corresponding to $1 - \frac{M_{D_s}^2}{m_{0D_s}^2}$ in eq.(4) is $O(1)$. Then,
K meson’s wave function is parameterized by Lorentz scalar wave functions \( \phi_{K}^{A,P,T} \) as

\[
\Phi_{K, \alpha \beta}(x_3, b_3) = P_3^{(3)} \int \frac{dz^+}{2\pi} e^{-ik_{3}z} \langle P_3 | \bar{u}_{\beta}(z)s_{\alpha j}(0)|0\rangle \\
= \frac{i\delta_{ij}}{\sqrt{2N_c}} \left[ \gamma_5 P_3 \phi_{K}^{A}(x_3, b_3) + m_{0K}\gamma_5\phi_{K}^{P}(x_3, b_3) \right. \\
\left. + m_{0K}\gamma_5(\not{p} - 1 - \not{q})\phi_{K}^{T}(x_3, b_3) \right]_{\alpha \beta} 
\]

where \( v = (0, 1, 0_B) \propto P_3 \), \( n = (1, 0, 0_B) \propto z \), and \( m_{0K} = M_{K}^2/(m_u + m_s) \). In the numerical analysis we will use \( \phi_{K}^{A,P,T} \) which were calculated from QCD sum rule \[12\]. They will be shown in section \[4\].

### 3.2 \( B^0 \rightarrow D_s^- K^+ \) decay

We first consider the neutral \( B^0 \) decay \( B^0 \rightarrow D_s^- K^+ \). The effective Hamiltonian at the scale lower than \( M_W \) related to this decay is given as \[12\]

\[
H_{eff} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} [C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)] , 
\]

\[
O_1 = (\bar{b}d)_{V-A}(\bar{u}c)_{V-A} , \quad O_2 = (\bar{b}c)_{V-A}(\bar{u}d)_{V-A} , 
\]

where \( C_{1,2}(\mu) \) are Wilson coefficients at renormalization scale \( \mu \), and summation in SU(3)_c color’s index \( \alpha \) and chiral projection, \( \sum_{\alpha} \bar{q}_{a} \gamma^{\mu}(1 - \gamma_5)q'_{\alpha} \), are abbreviated to \( (\bar{q}q')_{V-A} \). The lowest order diagrams contributing to \( B^0 \rightarrow D_s^- K^+ \) are drawn in Fig.2 according to this effective Hamiltonian. As stated above, \( B \rightarrow D_s K \) decays only have annihilation diagrams.

We get the following analytic formulas by calculating the hard part \( H \) at first order in \( \alpha_s \). Together with the meson wave functions, the amplitude for the factorizable annihilation diagram in Fig.2(a) and (b) results in \( F_{a}^{(i=2)} \),

\[
F_{a}^{(i)} = -16\pi C_F M_{B}^2 \int_{0}^{1} dx_2 dx_3 \int_{0}^{\infty} b_2 db_2 b_3 db_3 \phi_{D_s}(x_2, b_2) \\
\times \left\{ (1 - r^2) \left( 1 - 2r^2 - (1 - r^2)x_3 \right) \phi_{K}^{A}(x_3, b_3) \right. \\
+ r \left( 3 - r^2 - 2(1 - r^2)x_3 \right) r_{K}\phi_{K}^{P}(x_3, b_3) \\
- r(1 - r^2)(1 - 2x_3) r_{K}\phi_{K}^{T}(x_3, b_3) \right\} E_{f}^{i}(t_{a}^{i}) h_a(x_2, x_3, b_2, b_3) \\
- \left\{ (1 - r^2)x_2 \phi_{K}^{A}(x_3, b_3) \\
+ 2r(1 - r^2 + x_2) r_{K}\phi_{K}^{P}(x_3, b_3) \right\} E_{f}^{i}(t_{a}^{2}) h_a(1 - x_3, 1 - x_2, b_3, b_2) , 
\]

(17)
where \( C_F = 4/3 \) is the group factor of SU(3)_c gauge group, and \( r_K = m_0K/M_B \), and the functions \( E_1, t_{1,2}^a, h_a \) are given in the appendix. The explicit form for the wave functions, \( \phi_M \), is given in the next section. The amplitude for the nonfactorizable annihilation diagram in Fig.2(c) and (d) results in

\[
M_a = \frac{1}{\sqrt{2}N_c} 64\pi C_F M_B^2 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \phi_{D_s}(x_2, b_2) \\
\times \left\{ \left( 1 - r^2 \right) \left( (1 - r^2)(1 - x_3) + r^2 x_2 \right) \phi_K^A(x_3, b_2) \right. \\
+ r \left( x_2 + (1 - r^2)(1 - x_3) \right) r_K \phi_K^P(x_3, b_2) \\
+ \left. \left( x_2 - (1 - r^2)(1 - x_3) \right) r_K \phi_K^T(x_3, b_2) \right\} E_m(t_1^a)h_a(x_1, x_2, x_3, b_1, b_2) \\
- \left\{ (1 - r^2) \left( (1 + r^2)x_2 - r^2 \right) \phi_K^A(x_3, b_2) \right. \\
+ r(2 + x_2 + (1 - r^2)(1 - x_3))r_K \phi_K^P(x_3, b_2) \\
\left. + r(-x_2 + (1 - r^2)(1 - x_3))r_K \phi_K^T(x_3, b_2) \right\} E_m(t_2^a)h_a(x_1, x_2, x_3, b_1, b_2), \tag{18}
\]

where \( x_1 \) dependence in the numerators of the hard part are neglected by the assumption \( x_1 \ll x_2, x_3 \). The total decay amplitude for \( B^0 \to D_s^- K^+ \) decay is given as

\[
A = f_B F_a^{(2)} + M_a, \tag{19}
\]

where the overall factor is included in the decay width with the kinematics factor. The decay width is expressed as

\[
\Gamma(B^0 \to D_s^- K^+) = \frac{G_F^2 M_B^3}{128\pi}(1 - r^2)|V_{cb}^* V_{ud}|^2. \tag{20}
\]

The decay width for CP conjugated mode, \( \overline{B}^0 \to D_s^+ K^- \), is the same value as \( B^0 \to D_s^- K^+ \), just replacing \( V_{cb}^* V_{ud} \) with \( V_{cb} V_{ud}^* \). Since there is only one kind of CKM phase involved in the decay, there is no CP violation in the standard model.

\footnote{We don’t apply this approximation to the denominators of the propagator which are sensitive to the variable \( x_1 \). Because such a \( x_1 \) behaves as the cut off, the resultant branching ratio is smaller than it given in ref. [13] where the whole \( x_1 \) in the hard part are neglected.}
3.3 $B^+ \rightarrow D_s^+ \bar{K}^0$ decay

The effective Hamiltonian related to $B^+ \rightarrow D_s^+ \bar{K}^0$ decay is given as

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{cd} \left[ C_1(\mu) O_1(\mu) + C_2(\mu) O_2(\mu) \right], \quad (21)$$

$$O_1 = (\bar{b}d)_{V-A} (\bar{c}u)_{V-A}, \quad O_2 = (\bar{b}u)_{V-A} (\bar{c}d)_{V-A}. \quad (22)$$

The amplitude for the factorizable annihilation diagram results in $-F_a^{(i=1)}$. The amplitude for the nonfactorizable annihilation diagram results in

$$M'_a = \frac{1}{\sqrt{2}N_c} 64\pi C_F M_B^2 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty db_1 db_2 db_2 \phi_B(x_1, b_1) \phi_{D_s}(x_2, b_2)$$

$$\times \left[ \left\{ (1-r^4)x_2 \phi^A_K(x_3, b_2) + r \left( x_2 + (1-r^2)(1-x_3) \right) r_K \phi^P_K(x_3, b_2) \right. \right.$$

$$+ r \left. \left( -x_2 + (1-r^2)(1-x_3) \right) r_K \phi^T_K(x_3, b_2) \right\} E_{m}(t_{m}^1) h_{a}^{(1)}(x_1, x_2, x_3, b_1, b_2)$$

$$- \left\{ (1-r^2) \left( (1-r^2)(1-x_3) - r^2 + r^2 x_2 \right) \phi^A_K(x_3, b_2) \right. \right.$$

$$+ r \left. \left( 2 + x_2 + (1-r^2)(1-x_3) \right) r_K \phi^P_K(x_3, b_2) \right\} E_{m}(t_{m}^2) h_{a}^{(2)}(x_1, x_2, x_3, b_1, b_2)$$

$$+ r \left. \left( x_2 - (1-r^2)(1-x_3) \right) r_K \phi^T_K(x_3, b_2) \right\} E_{m}(t_{m}^2) h_{a}^{(2)}(x_1, x_2, x_3, b_1, b_2). \quad (23)$$

Thus, the total decay amplitude $A'$ and decay width $\Gamma$ for $B^+ \rightarrow D_s^+ \bar{K}^0$ decay is given as

$$A' = -f_B F_a^{(1)} + M'_a, \quad (24)$$

$$\Gamma(B^+ \rightarrow D_s^+ \bar{K}^0) = \frac{G_F^2 M_B^2}{128\pi} (1-r^2) |V_{ub}^* V_{cd} A'|^2. \quad (25)$$

The decay width for CP conjugated mode, $B^- \rightarrow D_s^- K^0$, is the same value as $B^+ \rightarrow D_s^+ \bar{K}^0$. Similar to the $B^0$ decay, there is also no CP violation in this decay within standard model.

4 Numerical evaluation

In this section we show numerical results obtained from the previous formulas. At the beginning, we give the branching ratios predicted from the same parameters and wave functions that are adopted in the other works. Secondly, we show the theoretical errors due to uncertainty of some parameters.
For the $B$ meson’s wave function, there is a sharp peak at the small $x$ region, we use

$$\phi_B(x, b) = N_B x^2 (1 - x)^2 \exp \left[ -\frac{M_B^2}{2\omega_b^2} - \frac{1}{2} (\omega_b b)^2 \right],$$

(26)

which is adopted in ref. [3, 4]. This choice of $B$ meson’s wave function is almost a best fit from the $B \to K\pi, \pi\pi$ decays. For the $D_s$ meson’s wave function, we assume the same form as $D$ meson’s one except for the normalization

$$\phi_{D_s}(x) = \frac{3}{\sqrt{2N_c}} f_{D_s} x (1 - x) \{1 + a_{D_s} (1 - 2x)\}.$$  

(27)

Since $c$ quark is much heavier than $s$ quark, this function is peaked at $c$ quark side, i.e. small $x$ region, too. The wave functions $\phi_A, P, T$ of the $K$ meson are expanded by Gegenbauer polynomials,

$$\phi_A^K(x) = \frac{f_K}{2\sqrt{2N_c}} 6x (1 - x) \left\{1 - a_1^K \cdot 3\xi + a_2^K \cdot \frac{3}{2} (-1 + 5\xi^2)\right\},$$

(28)

$$\phi_P^K(x) = \frac{f_K}{2\sqrt{2N_c}}  \left\{1 + a_{p1}^K \cdot \frac{1}{2} (-1 + 3\xi^2) + a_{p2}^K \cdot \frac{1}{8} (30\xi^2 + 35\xi^4)\right\},$$

(29)

$$\phi_T^K(x) = \frac{f_K}{2\sqrt{2N_c}} (1 - 2x) \left\{1 + a_T^K \cdot 3(-3 + 5\xi^2)\right\},$$

(30)

where $\xi = 2x - 1$. The coefficients $a_{1,2,p1,p2,T}^K$ calculated from QCD sum rule are given in ref.[11], and their values are

$$a_1^K = 0.17, \quad a_2^K = 0.2, \quad a_{p1}^K = 0.212, \quad a_{p2}^K = -0.148, \quad a_T^K = 0.0527,$$

(31)

for $m_{0K} = 1.6$ GeV. In addition, we use the following input parameters:

$$M_B = 5.28 \text{ GeV}, \quad M_{D_s} = 1.969 \text{ GeV},$$

(32)

$$f_B = 190 \text{ MeV}, \quad f_K = 160 \text{ MeV}, \quad f_{D_s} = 241 \text{ MeV},$$

(33)

$$m_{0K} = 1.6 \text{ GeV}, \quad \omega_b = 0.4 \text{ GeV}, \quad a_{D_s} = 0.3.$$  

(34)

With these values and eq.(13) we get the normalization factor $N_B = 91.745$ GeV. We show the decay amplitudes calculated with the above parameters at Table I. For the neutral decay $B^0 \to D_s^- K^+$, the dominant contribution is the nonfactorizable annihilation diagrams, where the contribution $M_a$ is proportional to the Wilson coefficient $C_2(t)$, which is of order one. The factorizable annihilation diagram contribution is proportional to $a_2 = C_1 + C_2/3$, which is
one order magnitude smaller. For the charged decay $B^+ \rightarrow D_s^+ \overline{K}^0$, it is the inverse situation. The Wilson coefficient in $M'_a$ is $C_1(t)$, which is smaller than the one in $F_a^{(1)}$, $a_1 = C_1/3 + C_2$. Both amplitudes $A$ and $A'$ are at the same order magnitude here.

The propagators of inner quark and gluon in Figure 2 and 3 are usually proportional to $1/x_i$. One may suspect that these amplitudes are enhanced by the endpoint singularity around $x_i \sim 0$. This can be explicitly found in eqs. (58, 59), where the Bessel function $Y_0$ diverges at $x_i \sim 0$ or 1. However this is not the case in our calculation. First we introduce the transverse momentum of quark, such that the propagators become $1/(x_i x_j + k_T^2)$. Secondly, the Sudakov form factor $\exp[-S]$ suppresses the region of small $k_T^2$. Therefore there is no singularity in our calculation. The dominant contribution is not from the endpoint of the wave function. As a proof, in our numerical calculations, for example, an expectation value of $\alpha_s$ in the integration for $M_a$ results in $\langle \alpha_s/\pi \rangle = 0.10$, which gives the dominant contribution to $B^0 \rightarrow D_s^- K^+$ decay. Therefore, the perturbative calculations are self-consistent.

|                | $B^0 \rightarrow D_s^- K^+$ | $B^+ \rightarrow D_s^+ \overline{K}^0$ |
|----------------|-------------------------------|------------------------------------------|
| $f_B F_a^{(2)}$| $-0.84 + 1.57 i$              | $-f_B F_a^{(1)}$                         |
| $M_a$          | $1.99 - 18.36 i$              | $M'_a$                                   |
| $A$            | $1.15 - 16.79 i$              | $A'$                                    |
|                | $-7.80 + 14.93 i$             |                                         |

Table 1: Amplitudes($10^{-3}$ GeV) with parameters eqs. (32-34).

Now we can calculate the branching ratio according to eqs. (19, 20, 24, 25). Here we use CKM matrix elements and the life times[15],

$$|V_{ud}| = 0.9734 \pm 0.0008, \quad |V_{ub}| = (3.6 \pm 0.7) \times 10^{-3}, \quad (35)$$
$$|V_{cb}| = (41.2 \pm 2.0) \times 10^{-3}, \quad |V_{cd}| = 0.224 \pm 0.016, \quad (36)$$
$$\tau_{B^\pm} = 1.67 \times 10^{-12} \text{ s}, \quad \tau_{B^0} = 1.54 \times 10^{-12} \text{ s}. \quad (37)$$

The predicted branching ratios are

$$\text{Br}(B^0 \rightarrow D_s^- K^+) = 4.57 \times 10^{-5}, \quad (38)$$
$$\text{Br}(B^+ \rightarrow D_s^+ \overline{K}^0) = 2.01 \times 10^{-8}. \quad (39)$$
The $B^0 \rightarrow D_s^- K^+$ decay is observed at Belle\cite{16} and BaBar\cite{17},

$$\text{Br}(B^0 \rightarrow D_s^- K^+) = (4.6^{+1.2}_{-1.1} \pm 1.3) \times 10^{-5}, \quad \text{Belle}, \quad (40)$$
$$\text{Br}(B^0 \rightarrow D_s^- K^+) = (3.2 \pm 1.0 \pm 1.0) \times 10^{-5}, \quad \text{BaBar}. \quad (41)$$

For the $B^+ \rightarrow D_s^+ K^0$ decay, there is only upper limit given at 90\% confidence level\cite{15},

$$\text{Br}(B^+ \rightarrow D_s^+ K^0) < 1.1 \times 10^{-3}. \quad (42)$$

It is easy to see that our results are consistent with the data.

Despite the calculated perturbative annihilation contributions, there is also hadronic picture for the $B^0 \rightarrow D_s^- K^+$ decay: $B^0 \rightarrow D^- \pi^+ (\rho^+)$ then $D^- \pi^+ (\rho^+) \rightarrow D_s^- K^+$ through final state interaction. Our numerical results show that the PQCD contribution to this decay is already enough to account for the experimental measurement. It implies that the soft final state interaction is not important in the $B^0 \rightarrow D_s^- K^+$ decay. This is consistent with the argument in ref.\cite{18}.

The branching ratios obtained from the analytic formulas may be sensitive to various parameters, such as parameters in eqs.(31), eqs.(34). It is important to give the limits of the branching ratio when we choose the parameters to some extent. Table\ref{table2} shows the sensitivity of the branching ratio to 30\% change of parameters in eqs.(31), eqs.(34). It is found that uncertainty of the predictions on PQCD is mainly due to $m_0K$ and $\omega_b$. Below we show the limits of the branching ratio within the suitable ranges on $m_0K$ and $\omega_b$. The appropriate extent of $m_0K$ can be found in ref.\cite{11},

$$1.4 \text{ GeV} \leq m_0K \leq 1.8 \text{ GeV}. \quad (43)$$

$a_{P,T}^{K_{p1,p2,t}}$ in the wave functions $\phi_K^{P,T}$ are given as functions with respect to $m_0K$, $a_2^K$, and some input parameters, $\eta_3$, $\omega_3$ in ref.\cite{11}. Within eq. (43), the branching ratios normalized by the decay constants and the CKM matrix elements result in

$$\text{Br}(B^0 \rightarrow D_s^- K^+) = (4.57^{+0.26}_{-0.10}) \times 10^{-5} \left( \frac{f_B f_{D_s}}{190 \text{ MeV} \cdot 241 \text{ MeV}} \right)^2 \left( \frac{|V_{cb} V_{ud}|}{0.0412 \cdot 0.9734} \right)^2, \quad (44)$$
$$\text{Br}(B^+ \rightarrow D_s^+ K^0) = (2.01^{+0.16}_{-0.18}) \times 10^{-8} \left( \frac{f_B f_{D_s}}{190 \text{ MeV} \cdot 241 \text{ MeV}} \right)^2 \left( \frac{|V_{ub} V_{cd}|}{0.0036 \cdot 0.224} \right)^2. \quad (45)$$
Table 2: The sensitivity of the branching ratio to 30% change of parameters in eqs.(31), eqs.(34). Here we don’t present the sensitivity to $a_{p1,p2,T}^K$ because the branching ratios are insensitive to them.
From the $B \to K$ transition form factor $f_+^K(0)$, the appropriate extent of $\omega_b$ can be obtained. $f_+^K(0)$ calculated from PQCD at $m_{0K} = 1.6$ GeV in the region

$$0.35 \text{ GeV} \leq \omega_b \leq 0.46 \text{ GeV},$$

is consistent with $f_+^K(0)$ by QCD sum rules given in ref. [11]. The branching ratios calculated at the region of eqs. (46) are found within

$$\text{Br}(B^0 \to D_s^- K^+) = (4.57^{+0.77}_{-0.59}) \times 10^{-5} \left( \frac{f_B f_{D_s}}{190 \text{ MeV} \cdot 241 \text{ MeV}} \right)^2 \left( \frac{|V_{cb}^* V_{ud}|}{0.0412 \cdot 0.9734} \right)^2,$$

$$\text{Br}(B^+ \to D_s^+ \bar{K}^0) = (2.01^{+0.20}_{-0.10}) \times 10^{-8} \left( \frac{f_B f_{D_s}}{190 \text{ MeV} \cdot 241 \text{ MeV}} \right)^2 \left( \frac{|V_{ub}^* V_{cd}|}{0.0036 \cdot 0.224} \right)^2.$$

In the factorizable contribution, the $B$ meson’s wave function is integrated and normalized by the decay constant $f_B$. Thus, in the factorizable dominant decay, $B^+ \to D_s^+ \bar{K}^0$, its branching ratio is insensitive to the change of $\omega_b$.

5 Conclusion

In two-body hadronic $B$ meson decays, the final state mesons are moving very fast, since each of them carry more than 2 GeV energy. There is not enough time for them to exchange soft gluons. The soft final state interaction is not important in the two-body $B$ decays. This is consistent with the argument based on color-transparency [19]. We thus neglect the soft final state interaction in the PQCD approach. The PQCD with Sudakov form factor is a self-consistent approach to describe the two-body $B$ meson decays. Although the annihilation diagrams are suppressed comparing to other spectator diagrams, but their contributions are not negligible in PQCD approach [3, 4].

In this paper, we calculate the $B^0 \to D_s^- K^+$ and $B^+ \to D_s^+ \bar{K}^0$ decays, which occur purely via annihilation type diagrams. The branching ratios are still sizable. The $B^0 \to D_s^- K^+$ decay has been observed in the $B$ factories [16, 17]. This is the first channel measured in $B$ decays via annihilation type diagram. The fact that the predicted branching ratio is in good agreement with the data means that PQCD approach gives the annihilation amplitude correctly, the soft final state interaction is probably small in $B$ decays, and it is one of the evidences to justify PQCD approach.
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A Some functions

The definitions of some functions used in the text are presented in this appendix. In the numerical analysis we use one loop expression for strong coupling constant,

\[ \alpha_s(\mu) = \frac{4\pi}{\beta_0 \log(\mu^2/\Lambda^2)}, \]  

where \( \beta_0 = (33 - 2n_f)/3 \) and \( n_f \) is number of active flavor at appropriate scale. \( \Lambda \) is QCD scale, which we use as 250 MeV at \( n_f = 4 \). We also use leading logarithms expressions for Wilson coefficients \( C_{1,2} \) presented in ref.\[12\]. Then, we put \( m_t = 170 \) GeV, \( m_W = 80.2 \) GeV, and \( m_b = 4.8 \) GeV.

The function \( E_{f_i} \), \( E_m \), and \( E'_m \) including Wilson coefficients are defined as

\[ E_{f_i}(t) = a_1(t)\alpha_s(t) e^{-S_B(t)-S_D(t)-S_K(t)}, \]  
\[ E_m(t) = C_2(t)\alpha_s(t) e^{-S_B(t)-S_D(t)-S_K(t)}, \]  
\[ E'_m(t) = C_1(t)\alpha_s(t) e^{-S_B(t)-S_D(t)-S_K(t)}, \]

where

\[ a_1(t) = \frac{C_1(t)}{N_c} + C_2(t), \quad a_2(t) = C_1(t) + \frac{C_2(t)}{N_c}, \]

and \( S_B, S_D, \) and \( S_K \) result from summing both double logarithms caused by soft gluon corrections and single ones due to the renormalization of ultra-violet divergence. The above
$S_{B,D,K}$ are defined as

$$S_B(t) = s(x_1 p^+_1, b_1) + 2 \int_{-1}^{t} \frac{d\mu'}{\mu'} \gamma_q(\mu'),$$  \hspace{1cm} (54)

$$S_D(t) = s(x_2 p^+_2, b_3) + 2 \int_{-1}^{t} \frac{d\mu'}{\mu'} \gamma_q(\mu'),$$  \hspace{1cm} (55)

$$S_K(t) = s(x_3 p^+_3, b_3) + s((1-x_3) p^+_3, b_3) + 2 \int_{-1}^{t} \frac{d\mu'}{\mu'} \gamma_q(\mu'),$$  \hspace{1cm} (56)

where $s(Q, b)$, so-called Sudakov factor, is given as \cite{20}

$$s(Q, b) = \int_0^Q \frac{d\mu'}{\mu'} \left[ \frac{2}{3} (2\gamma_E - 1 - \log 2) + C_F \log \frac{Q}{\mu'} \right] \frac{\alpha_s(\mu')}{\pi}$$

$$+ \left\{ \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27} n_f + \frac{2}{3} \beta_0 \log \gamma_E \right\} \left( \frac{\alpha_s(\mu')}{\pi} \right)^2 \log \frac{Q}{\mu'},$$  \hspace{1cm} (57)

$\gamma_E = 0.5772 \cdots$ is Euler constant, and $\gamma_q = \alpha_s/\pi$ is the quark anomalous dimension.

The functions $h_a$, $h_a^{(1)}$, and $h_a^{(2)}$ in the decay amplitudes consist of two parts: one is the jet function $S_i(x_i)$ derived by the threshold resummation\cite{9}, the other is the propagator of virtual quark and gluon. They are defined by

$$h_a(x_2, x_3, b_2, b_3) = S_i(1-x_3) \left( \frac{\pi i}{2} \right)^2 H_0^{(1)}(M_B \sqrt{1-r^2} x_2(1-x_3) b_2)$$

$$\times \left\{ H_0^{(1)}(M_B \sqrt{1-r^2}(1-x_3) b_2) J_0(M_B \sqrt{1-r^2}(1-x_3) b_3) \theta(b_2-b_3) + (b_2 \leftrightarrow b_3) \right\},$$  \hspace{1cm} (58)

$$h_a^{(j)}(x_1, x_2, x_3, b_1, b_2) =$$

$$\left\{ \frac{\pi i}{2} H_0^{(1)}(M_B \sqrt{1-r^2} x_2(1-x_3) b_1) J_0(M_B \sqrt{1-r^2} x_2(1-x_3) b_2) \theta(b_1-b_2)$$

$$\times \left\{ K_0(M_B F_{(j)} b_1), \text{ for } F_{(j)}^2 > 0 \right\} \right\} \times \left\{ \frac{\pi}{2} H_0^{(1)}(M_B \sqrt{|F_{(j)}^2|} b_1), \text{ for } F_{(j)}^2 < 0 \right\},$$  \hspace{1cm} (59)

where $H_0^{(1)}(z) = J_0(z) + i Y_0(z)$, and $F_{(j)}$ are defined by

$$F_{(1)}^2 = (1-r^2)(x_1-x_2)(1-x_3), \quad F_{(2)}^2 = x_1 + x_2 + (1-r^2)(1-x_1-x_2)(1-x_3).$$  \hspace{1cm} (60)

We adopt the parametrization for $S_i(x)$ of the factorizable contributions,

$$S_i(x) = \frac{2^{1+c} \Gamma(3/2+c)}{\sqrt{\pi} \Gamma(1+c)} [x(1-x)]^c, \quad c = 0.3,$$  \hspace{1cm} (61)
which is proposed in ref. [21]. In the nonfactorizable annihilation contributions, \( S_t(x) \) gives a very small numerical effect to the amplitude\([22]\). Therefore, we drop \( S_t(x) \) in \( h_a^{(1)} \) and \( h_a^{(2)} \).

The hard scale \( t \)'s in the amplitudes are taken as the largest energy scale in the \( H \) to kill the large logarithmic radiative corrections:

\[
t_1^a = \max(M_B \sqrt{(1-r^2)(1-x_3)}, 1/b_2, 1/b_3), \quad \text{(62)}
\]
\[
t_2^a = \max(M_B \sqrt{(1-r^2)x_2}, 1/b_2, 1/b_3), \quad \text{(63)}
\]
\[
t_m^j = \max(M_B \sqrt{|F_{ij}^2|}, M_B \sqrt{(1-r^2)x_2(1-x_3)}, 1/b_1, 1/b_2). \quad \text{(64)}
\]

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Figure Captions

Figure 1: $D_sK$ are produced only by annihilation between $\bar{b}$ and $d$ quark in the $B$ meson from point of view of quark model. While $W$ boson exchange causes $\bar{b}d \rightarrow \bar{c}u$, the $\bar{s}s$ quarks included in $D_sK$ are produced from a hard gluon.

Figure 2: Diagrams for $B^0 \rightarrow D_s^- K^+$ decay. The factorizable diagrams (a),(b) contribute to $F_a^{(2)}$, and nonfactorizable (c), (d) do to $M_a$.

Figure 3: Diagrams for $B^+ \rightarrow D_s^+ K^0$ decay. The factorizable diagrams (a),(b) contribute to $F_a^{(1)}$, and nonfactorizable (c), (d) do to $M_a'$. 
Figure 1:
Figure 2:
Figure 3: