Numerical computations of the safe boundaries of complex technical systems and practical stability

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Abstract. In this article we discuss methods of computing the guaranteed values of the states of a technical system in order to estimate the safety of a technical system. The danger of a system functioning is a threat, possibility, probability of damage, system catastrophe, that is, potential damage to a technical system in certain conditions and situations. An analysis of the boundaries of the safety areas of technical systems helps to obtain quantitative estimates of the possibility of dangerous situations. The presence of such estimates may allow a more reasonable search and development of a set of measures to eliminate or mitigate the consequences of such situations. The article discusses the new results of computing the guaranteed boundaries of solution sets and the results of their application for assessing the boundaries of security areas and studying practical stability. Methods are used based on the approximation of the shift operator along the trajectory, and taking into account the influence of constantly acting perturbations on the solutions.

1. Introduction

In most practical problems, the safe operation of the system is controlled under conditions of a priori uncertainty either regarding external influences, or the current state of the object, or both at the same time. The analysis of safety areas is associated with the need to obtain quantitative assessments of the possibility of dangerous situations. This is an integral part of the security problem in the broad sense of the word [1], [2], [3], [4]. The presence of such assessments may allow a more reasonable search and development of a set of measures to eliminate or mitigate the consequences of such situations. Let the technical system be determined by a vector \( V(\tau) \), which in the general case can be considered as a quality vector. An area \( S(V) \) is specified for the quality vector. If at least one component of the vector \( V(\tau) \) goes beyond the boundaries of this region, then an emergency situation (dangerous state) occurs. In this case, the state of the object and external influences are not measured accurately, but with some errors. With a probabilistic approach to safety analysis, it is assumed that the uncertainty factors involved in the problem have the property of statistical stability and represent random variables or random functions with known probabilistic characteristics. For these uncertainty factors, probabilistic characteristics of the investigated controlled processes can be found, which in some cases can serve as indicators of the quality of safety management (for example, a widely used indicator of accuracy - the value of the standard error). However, a probabilistic approach to the problems of...
reaching security areas sometimes leads to conclusions that do not correspond to the actual conditions of the tasks being solved. This primarily relates to assessing the accuracy of the results obtained and choosing a strategy for solving the problem that provides maximum accuracy. In addition, statistical characteristics are the results of averaging over a large number of experiments.

Therefore, they fundamentally cannot guarantee a certain outcome of one particular experience. If it is necessary to guarantee a certain quality of the process in each individual case, it is necessary to focus on the most unfavorable (extreme) combination of uncertainty parameters. In this case, a deterministic approach is used to describe all phase states of dynamical systems, which estimates all phase states under inaccurate measurements. For example, it is possible to diagnose the functional state of technical systems. It is based on differential equations describing the functioning of serviceable and faulty systems. When conducting differential diagnostics of the state of technical systems, two independently solvable tasks can be distinguished: monitoring the criterion of malfunctions in the system, and diagnosing malfunctions. The criterion for the presence of malfunctions in the system is the exit of the object's trajectory to some preselected surface. A malfunction can occur at any previously unknown time moment of the object's movement at any point inside a given controlled surface. Faults arising in the control process lead to the fact that the control signal is formed incorrectly and in many cases the functioning of the system no longer ensures the proximity of the trajectory $V(\tau)$ to a certain program trajectory $V_{\text{damage}}(\tau)$. For example, studies of stress-strain, especially close to limiting, states of complex technical objects in some cases require taking into account contact interaction zones, high strain rates, including elastic-plastic, changes in the configuration of the object during damage and partial destruction [5], [6].

It should be noted that the safety assessment of mathematical models of technical systems is close to stability problems over a finite time interval [2], which were further developed as a theory of practical or technical stability over a finite time interval. Practical stability over a finite time interval means the uniform boundedness of solutions regarding the set of initial values and the totality of disturbing influences. These boundaries of the solutions are computed using methods based on the approximation of the shift operator along the trajectory [7] - [11] and take into account the influence of constantly acting perturbations on the solutions. It is possible to formulate mathematically rigorous results regarding practical sustainability for a fairly wide class of problems. The article describes numerical methods for assessing the safety and technical stability of mathematical models of technical systems. To find the guaranteed boundaries of the zones of dangerous states and the threshold values of the parameters of mathematical models that correspond to the boundaries of these zones, we apply the class of methods for guaranteed estimation of solutions of systems of differential equations.

2. System security and finding system parameters within acceptable limits

The dependence of safety and stability indicators (we will generally call these indicators a quality characteristic) of the functioning of the system is a multidimensional function of many variables. Each admissible region of the set of values of the quality indicator corresponds to an admissible region (subset) of parameter values. This subset has a complex configuration, or rather its construction is a difficult task, and at present it is only solvable for the simplest systems with a small number of parameters. We emphasize that all the parameters of the studied system are interconnected. Therefore, in the general case, the tolerance for each parameter, as a rule, is determined only approximately, replacing the actual subset (region) of permissible parameter values with a certain region with faces parallel to the coordinate planes.

Let consider the system of ordinary differential equations (ODE)

$$\frac{dy}{dt} = f(t,y(t),u(t)),$$

where initial data $y_0 \in Y_0$, $Y_0$ - set of initial data, the vector function $f(t,u)$ is defined for any values $t$ of the vector variable $y(t)$ and any values $u(t) \in U$ of external influences and controls. The
choice of possible implementations is constrained by restrictions reflecting the features of the problem under consideration. For many problems, the restrictions on the effects can be only geometric in nature. This means that at each moment of time, the value can be any of some convex compact set.

The problem of estimating security of an region is to determine the set of solutions (or its boundaries)

$$ Y(t) = \bigcup_{y_0 \in Y, u \in U} y(t, y_0, u). $$

Among the mathematical descriptions of such problems, we single out the problems of checking guaranteed security conditions and the problems of constructing "surviving" trajectories. To do this, it is necessary to verify the fulfillment of the “survival” conditions $y(t) \in N$ for $t_0 \leq t \leq \eta$ and $Y(\eta) \in M$ for any movement $y(\cdot)$ proceeding from points in the region of admissible initial positions $G^0$ and given sets $N, M$ when enumerating all control actions satisfying the constraint $u \in Q$. The algorithm for computing the guaranteed boundaries of solutions consists of several steps: realizing the construction of a symbolic solution formula, assessing the range of its possible values, and evaluating the global error.

The set of solutions at the time point of system (1) with the set of initial data $y_0 \in Y_0$ for $t = t_0$ is defined as the set of values $y(t_0, y_0, t)$ obtained for all possible permissible initial values

$$ Y(t_0, y_0, t) = \bigcup_{y_0 \in Y_0} y(t_0, y_0, t), $$

where the union is taken over all possible values of the initial data. In order for the algorithm for constructing this set to be carried out efficiently, it is necessary to determine in advance the properties of this set and the boundaries of the set of solutions. If problem (1) is linear, then the map from the set of initial data to the range of solutions of problem (1) will be an affine map that maps a convex set to a convex set, i.e., the boundary of the set of initial data goes to the boundary of the set of solutions at the moment.

Let the problem (1) is independent from external influence, then it is equivalent to the integral equation

$$ y(t) - y_0 - \int_{t_0}^{t} f(\tau, y(\tau)) d\tau = 0 $$

We write this equation as $F(y_0, y(\tau)) = 0$. Thus, $F$—this is an operator that maps the direct sum of space $E$ and the space $C^1_E[t_0, t_1]$ of continuously differentiable functions with values in to space $C^1_E[t_0, t_1]$. If the function $f(t, y)$ is continuous and has a continuous derivative, then the expression

$$ y(t) - \int_{t_0}^{t} f(\tau, y(\tau)) d\tau = 0 $$

defines a differentiable mapping of space into itself.

In the case of nonlinear ODE systems, each internal point of the solution domain goes to the internal point of the solution domain, which has a neighborhood composed again of the solutions of the original system. For ODE systems, the conditions for the existence and uniqueness of solutions are satisfied, therefore, for boundary points lying on the boundary of this solution region, the only possibility remains to be reflected at the boundary points of this region along the trajectories given by equation (4). This means that it is enough to study the behavior of the points of the solution domain (integral curves) starting at the boundary of the initial data.
In the general case, the method proposes a model of computations (transformations and computations) of symbolic formulas based on a phased static storage of information and its transformation in the final stage of the method. Thus, the formula will represent a recursive structure, the size of which varies. To write such a formula in a computer, linear dynamic structures are used. Due to this, the model of computations (transformations and computations) of symbolic formulas is carried out without explicit writing out of the superposition of the components of the formula defined at each step. The relationship between these components is determined by setting the addressing mechanism. Links to addresses of various levels are stored in the stack memory in the form of a tree. The formula computation code is generated in the process of traversing this tree, starting from the vertices.

3. Applications
Safety bounds of oscillations of a technical system.
Safety bounds of oscillations of a technical system with control actions \( u_1, u_2, y_1, y_3 \) – generalized coordinates, \( y_2, y_4 \) – generalized impulses are computed when solving the ODE system

\[
\frac{dy_1}{dt} = 2y_2 + y_4, \quad \frac{dy_2}{dt} = -2y_1 + y_4 + u_1, \quad \frac{dy_3}{dt} = y_2 + 10y_4, \quad \frac{dy_4}{dt} = 2y_1 - 2y_3 + u_2.
\]  

(5)

Control actions are limited \( u_1^2 + u_2^2 \leq 1 \). The computations were carried out at \( t \in [0, 4] \). The boundaries of the regions of solutions (security regions) are marked in figure 1.

![Figure 1](image1.png)

**Figure 1.** The computed boundaries of the sets of solutions of system (5) - the projection onto the plane \( t - y_2 \).

![Figure 2](image2.png)

**Figure 2.** The computed boundaries of the sets of solutions of system (6) - the projection onto the planes \( t - \delta_1 \).

![Figure 3](image3.png)

**Figure 3.** The computed boundaries of the sets of solutions of system (6) - the projection onto the planes \( t - \delta_2 \).

4. Estimates of security areas bounds with emergency control
With large disturbances in electric power systems (EES), electromechanical transients occur, characterized by mutual movements of the rotors of synchronous machines in the system, significant
changes in voltages in the nodes and currents in the elements of the EES. If the dynamic stability is not violated, then the mutual movements of the rotors of the synchronous machines occur in a certain limited area. With the development of emergencies in complex EPS, there are long transient processes associated with frequency deviations in individual parts of the system. In the event of an unfavorable development of accidents, these deviations may be unacceptable under the operating conditions of the system equipment and consumers. The boundaries of the set of solutions in the simplest model of the power system were calculated: stations - buses of infinite power (figures 2, 3).

For the equation of the speed controller, the equations of the perturbed motion of a multi-machine electric power system are written out in the form [12]

\[
\frac{d\delta_i}{dt} = y_i, \quad \frac{dy_i}{dt} = a_i z_i - d_i y_i - \psi_i, \quad \frac{dz_i}{dt} = -l_i y_i - g_i z_i - a_i y_i. \tag{6}
\]

Here, \( m \) – the number of machines, \( \delta_i \) – the deviation of the angle of the rotor of the machine from the steady-state value \( \delta_i^0 \), \( y_i \) – the absolute angular velocity of rotation of the rotor of the \( i \) – machine, \( z_i \) – the coordinate of the regulator, \( a_i, d_i, l_i, g_i \neq 0 \) are constants. Nonlinear functions \( \psi_i \) are expressed in terms of known quantities. When calculating the boundaries of the sets of solutions (safety areas), the values from [12] were used.

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