DECOHERENCE FROM VACUUM FLUCTUATIONS

MARKUS BÜTTIKER
Département de Physique Théorique, Université de Genève, CH-1211 Genève 4, Switzerland

Vacuum fluctuations are a source of irreversibility and decoherence. We investigate the persistent current and its fluctuations in a ring with an in-line quantum dot with an Aharonov-Bohm flux through the hole of the ring. The Coulomb blockade leads to persistent current peaks at values of the gate voltage at which two charge states of the dot have the same free energy. We couple the structure to an external circuit and investigate the effect of the zero-temperature (vacuum fluctuations) on the ground state of the ring. We find that the ground state of the ring undergoes a crossover from a state with an average persistent current much larger than the (time-dependent) mean squared fluctuations to a state with a small average persistent current and large mean squared fluctuations. We discuss the spectral density of charge fluctuations and discuss diffusion rates for angle variables characterizing the ground state in Bloch representation.

1 Introduction

In this work we are interested in the coherence properties of the ground state of a mesoscopic system coupled to an environment. In the zero-temperature limit, the only source of decoherence are then provided by vacuum fluctuations. The work is motivated by a recent discussion in the mesoscopic physics community which largely insists that dephasing rates tend to zero (typically with some power law) as a function of temperature and that there is therefore no dephasing in the zero-temperature limit. For references to this discussion we direct the reader to Ref. 1.

A key argument is that in the zero-temperature limit a system can not excite a bath by giving away an energy quantum nor can a bath in the zero-temperature limit give an energy quantum to the system. This view holds that dephasing is necessarily associated with an energy transfer (a real transition) and since this is impossible there is in the zero-temperature limit no dephasing. However, this argument rests on the assumption that the system and the bath are in their ground state which they assume in the absence of any coupling. In the ground state of the system + bath, even at absolute zero, the energy of the small system fluctuates and the ground state of the small system is not a pure state.

To illustrate this we investigate a simple mesoscopic system, a quantum dot with its leads formed into a ring, as shown in Fig. 1. Such a ring, when the coupling between the dot and the arms of the ring is sufficiently weak, exhibits Coulomb blockade peaks in the persistent current at gate voltages which equalize the free energy of the N-th and N + 1-th charge state of the dot. We couple this system capacitively to an external circuit with ohmic resistance $R$ and investigate the persistent current near a resonance. At absolute zero temperature the gate voltage fluctuates due to vacuum fluctuations of the resistor. We show that the ground state of this system is not a pure coherent state. In the two state limit, the system shown in Fig. 1 maps onto the spin boson problem: for this system there exits a large literature and we emphasize that the work presented here is not directly related to weak localization but considers the persistent current (a ground state property).

2 Coulomb Blockade Peaks of the Persistent Current

Consider the system shown in Fig. 1. The arm of the ring contains electrons in levels with energy $E_{am}$ and the dot contains electrons in levels with energy $E_{dn}$. First let us for a moment neglect tunneling. Let $F_N$ be the free energy for the case that there are $N$ electrons in the dot and $M$ electrons in the arm. The transfer of an electron from the arm to the dot gives a free
Figure 1: Ring with an in-line dot subject to a flux $\Phi$ and capacitively coupled to an external impedance $Z$.

Energy $F_{N+1}$. The difference of these two free energies is

$$\hbar \epsilon_0 = F_{N+1} - F_N = E_{d(N+1)} - E_{aM} + \frac{e^2(N + 1/2 - C_0 V_e)}{C}. \quad (1)$$

Here the first two terms arise from the difference in kinetic energies. The third term results from the charging energy of the dot. $C_0 = C_L + C_R$ is the series capacitance of the internal capacitance $C_i = C_L + C_R$ and the external capacitance $C_e^{-1} = C_1^{-1} + C_2^{-1}$. The total capacitance is $C = C_i + C_e$. Tunneling through the barriers connecting the dot and the arm is described by amplitudes $t_L$ and $t_R$ and depends on the Aharonov-Bohm flux $\Phi$ in the following way,

$$\frac{\hbar \Delta_0}{2} = \left( t_L^2 + t_R^2 \pm 2t_L t_R \cos \frac{2\pi \Phi}{\Phi_0} \right)^{1/2}, \quad (2)$$

where $\Phi_0 = h c / e$ is the single electron flux quantum. The sign depends on the number of electrons in the dot and arm: it is positive if the total number is odd, and it is negative if the total number is even. The voltage across the system is $V = V_0 - V_\infty$. The Hamiltonian consists of the system part $H_0$, the coupling $H_c$ and the bath $H_B$ with

$$H_0 = \frac{\hbar \epsilon_0}{2} \sigma_z - \frac{\hbar \Delta_0}{2} \sigma_x, \quad H_c = \frac{C_0 eV}{C_i} \sigma_z. \quad (3)$$

$H_0$ has eigenstates with energies

$$E_{\pm} = \mp \frac{\hbar}{2} \sqrt{\epsilon_0^2 + \Delta_0^2} \equiv \mp \frac{\hbar \Omega_0}{2}. \quad (4)$$

In the presence of a flux $\Phi$ the ground state of the ring-dot system (see Fig. 1) permits a persistent current which is

$$< I > = -e \frac{dE_+}{d\Phi} = \mp e \frac{4\pi t_L t_R}{\Omega_0} \sin(2\pi \Phi/\Phi_0). \quad (5)$$

The equilibrium current is a pure quantum effect: only electrons whose wave functions are sufficiently coherent to reach around the loop contribute to the persistent current. Thus the persistent current is a measure of the coherence of the ground state. At resonance $\epsilon = 0$ the
current is of the order of $et$ with $t$ a transmission amplitude and it decreases and becomes of the order of $et^2/\epsilon$ as we move away from resonance.

In the two-level limit of interest here the transmission amplitudes $t_L$ and $t_R$ are taken to be very small compared to the level spacing in the dot and in the arm. For larger transmission amplitudes the system without a bath will already exhibit a Kondo effect \cite{7,8,9}. A completely equivalent model consists of a superconducting electron box \cite{10} which can be opened to admit an Aharonov-Bohm flux \cite{11,12}.

We are interested in the effect of the bath on the persistent current. With the bath included our two-level system becomes a spin-boson problem \cite{13}. Cedraschi, Ponomarenko and the author \cite{2} used known solutions based on a Bethe ansatz and perturbation theory (for the anisotropic Kondo model) to provide an answer. For the symmetric case $t_L = t_R, C_L = C_R$, the average current is shown in Fig. 2 as a function of the coupling parameter $\alpha$.

The persistent current is in units of the current for $\alpha = 0$. In addition Ref. \cite{2} also investigated the (instantaneous) fluctuations of the equilibrium current away from its average, $(<I(t)−<I(t)>^2>)^{1/2}$ shown in Fig. 2 in units of the average current $<I(t,\alpha)>$. With increasing resistance we have thus a cross over from a state with a well defined persistent current (small mean-squared fluctuations) to a fluctuation dominated state in which the mean-squared fluctuations of the persistent current are much larger than the average persistent current. For the derivation we refer the reader to Refs. \cite{2} and \cite{13}. Here we pursue a discussion based on Langevin equations \cite{14}. This approach is valid only for weak coupling constants $\alpha \ll 1$ but has the benefit of being simple.

We want to find the time evolution of a state $\psi(t)$ of the two-level system in the presence of the bath. We write the state of the two-level system

$$\psi(t) = e^{i\chi/2} \left( \cos \frac{\theta}{2} e^{i\varphi/2} \atop \sin \frac{\theta}{2} e^{-i\varphi/2} \right),$$

with $\theta, \varphi$ and $\chi$ real. This is the most general form of a normalized complex vector in two dimensions. In terms of $\theta, \varphi$ and the global phase $\chi$, the time dependent Schrödinger equation reads

$$\dot{\varphi} = -\epsilon_0 - \delta \epsilon(t) - \Delta_0 \cot \theta \cos \varphi,$$
\[
\dot{\theta} = -\Delta_0 \sin \varphi, \tag{9}
\]
\[
\dot{\chi} = \frac{\Delta_0 \cos \varphi}{\sin \theta}. \tag{10}
\]

Here \(\delta \varepsilon(t) = (e/h)(C_0/C_i)V(t)\) arises from the gate voltage fluctuations. As shown by Eq. (10) the dynamics of the phase \(\chi\) is completely determined by the dynamics of the phases \(\theta\) and \(\varphi\) and has no back-effect on the evolution of \(\theta\) and \(\varphi\). While \(\chi\) is irrelevant for expectation values, like the persistent current or the charge on the dot, it plays an important role, in the discussion of phase diffusion times.

To close the system of equations, we have to find the voltage which drops across the system. The charge transfer between the dot and its arms permits a displacement current through the system which we have to include to find the voltage fluctuations. The charge operator on the dot for our problem is \(\hat{Q}_d = (1/2)(\sigma_z + 1)\). Its quantum mechanical expectation value is \(\langle \psi(t) | \hat{Q}_d | \psi(t) \rangle = e \cos^2(\theta/2)\). The displacement current is proportional to the time-derivative of this charge, \(\dot{Q}_d = -\frac{e}{2} \sin \theta \dot{\theta}\) multiplied by a ratio of capacitances which has to be found from circuit analysis. We find that the total current through the system is now given by \(C_0 \dot{V} - (C_0/C_i)(e/2) \sin \theta \dot{\theta}\). Using this result we find from conservation all currents that the fluctuating voltage across the system is determined by

\[
V = -C_0 R \dot{V} - R \frac{C_0}{C_i} \frac{e}{2} \sin \theta \dot{\theta} + RI_N(t). \tag{11}
\]

Eqs. (8, 9) and (11) form a closed system of equations in which the external circuit is incorporated in terms of an ohmic resistor \(R\) in parallel with a fluctuating current \(I_N(t)\) with spectral density \(S_{II}(\omega) = (\hbar \omega/R)c\coth(\hbar \omega/2kT)\). In the next section, we investigate Eqs. (8, 9) and (11) to find the effect of zero-point fluctuations on the persistent current of the ring.

3 Fluctuations around the ground state

In the absence of the noise term \(I_N(t)\), the stationary states of the system of differential equations, Eqs. (8, 9) and (11) are given by \(\varphi \equiv \varphi_0\), with \(\varphi_0 = 0\) or \(\varphi_0 = \pi\) and \(\theta \equiv \theta_0\), with \(\cot \theta_0 = \pm \frac{e}{\hbar \Omega_0}\). The lower sign applies for \(\varphi_0 = 0\). This is the ground state for the ring-dot system at fixed \(\varepsilon(t) \equiv \varepsilon_0\), and the upper sign holds for \(\varphi_0 = \pi\). The energy of the ground state is \(-\hbar \Omega_0/2\), thus the global phase is \(\chi_0(t) = \Omega_0 t\). We also introduce the “classical” relaxation time \(\tau_{RC} \equiv RC_0\).

Now, we switch on the noise \(I_N(t)\). We seek \(\varphi(t), \theta(t), \chi(t)\) and \(V(t)\) in linear order in the noise current \(I_N(t)\). We expand \(\varphi(t)\) and \(\theta(t)\) to first order around the ground state, \(\varphi = 0\) and \(\theta = \theta_0\). For \(\delta \varphi(t) = \varphi(t) - \varphi_0\), \(\delta \theta(t) = \theta(t) - \theta_0\), etc., we find in Fourier space,

\[
- i \omega \delta \varphi = - \Delta \varepsilon + \frac{\Omega_0^2}{\Delta_0} \delta \theta, \tag{12}
\]
\[
- i \omega \delta \theta = - \Delta_0 \delta \varphi, \tag{13}
\]
\[
- i \omega \delta \varepsilon = \frac{1}{\tau_{RC}} \left[ - \delta \varepsilon - \Gamma \delta \varphi + \frac{e}{\hbar} R \frac{C_0}{C_i} I_N \right], \tag{14}
\]
\[
- i \omega \delta \chi = \frac{\Omega_0 \varepsilon_0}{\Delta_0} \delta \theta. \tag{15}
\]

Here we have also expanded the global phase \(\chi(t)\) around its evolution in the ground state \(\chi_0(t) = \Omega_0 t\), and define \(\delta \chi(t) = \chi(t) - \chi_0(t)\). We note that there is no effect of the global shift in energy, \(\hbar \nu(t)\), as it is quadratic in the voltage \(\delta V\), and we are only interested in effects up to linear order in \(\delta V\).
4 Mapping onto a harmonic oscillator

Let us assume that the charge relaxation time of the external circuit $\tau_{RC}$ is very short compared to the dynamics of the two-level system $\tau_{RC} \ll \Omega_0$. Eliminating $\delta \theta$ with the help of Eq. (13) and $\delta \varepsilon$ with the help of Eq. (14) we find

$$\langle \omega^2 - i \omega - \Omega_0^2 \rangle \delta \theta = \Delta_0 \frac{e}{\hbar} \frac{C_0}{C_i} R I_N. \quad (16)$$

Thus we have mapped the dynamics of the fluctuations away from the ground state of this two-level system on the quantum Langevin equation of a damped harmonic oscillator. $\delta \theta$ plays the role of the charge, $\delta \varphi$ the role of the current and $\Gamma$ (defined in Eq. (5)) takes the role of the friction constant. The spectral density $S_{\theta \theta}(\omega)$ is just that of the coordinate of the harmonic oscillator,

$$S_{\theta \theta}(\omega) = \frac{2 \pi \alpha \Delta_0^2 |\omega|}{(\omega^2 - \Omega_0^2)^2 + \Gamma^2 \omega^2}.$$  \quad (17)

In the literature it is often the correlation function of $\sigma_z$ which is of interest. We have $\langle \psi | \sigma_z | \psi \rangle = \cos(\theta)$ and thus for the fluctuations away from the average $\Delta \langle \psi | \sigma_z | \psi \rangle = -\sin(\theta) \delta \theta$. Since $\sin(\theta_0) = \Delta_0/\Omega_0$, we find in the zero-temperature limit $S_{\psi \sigma_z}(\omega) = (\Delta_0^2/\Omega_0^2) S_{\theta \theta}(\omega)$. This result agrees with an expression given by Weiss and Wollensak \cite{13} and Görlich et al. \cite{14} who have used an entirely different approach. For non-zero temperatures Weiss and Wollensak find in addition a Debye peak around zero-frequency. The essential point is that the peaks are broadened with a relaxation rate $\Gamma$. Using Eqs. (13) and (14) we obtain similarly the spectral densities $S_{\varphi \varphi}(\omega)$, $S_{\chi \chi}(\omega)$ and cross-correlations like $S_{\theta \varphi}(\omega)$.

5 Suppression of the Persistent Current

Let us next examine the reduction of the persistent current using the approach outlined above. We consider only the case of a symmetric ring $t_R = t_L \equiv t$ and $C_R = C_L$ (see Fig. 1). The persistent current is the quantum and statistical average of the operator $\hat{I}_c = \mathcal{J} \sigma_z$ where $\mathcal{J}$ is given by $\mathcal{J} = \hbar c \partial \Delta_0 / \partial \Omega_0$. In general, in the non-symmetric case, the operator for the persistent current depends also on the capacitances (see Appendix B of Ref. 13). The quantum mechanical expectation value of the persistent current for the state given in Eq. (7) reads $\langle I(t) \rangle = \langle \psi(t) | \hat{I}_c | \psi(t) \rangle = \frac{1}{2} \text{Re}(\mathcal{J} \sin \theta e^{-i \varphi})$. We are interested in the statistically averaged persistent current $\langle I(t) \rangle$. Therefore, we have to calculate the correlator $\langle \sin \theta e^{-i \varphi} \rangle$. Taking into account that for a harmonic oscillator the fluctuations are Gaussian, we find $\langle \sin \theta e^{-i \varphi} \rangle = (\sin \theta) \langle e^{-i \varphi} \rangle$. and $\langle e^{-i \varphi(t)} \rangle = e^{-i \varphi_0} \langle e^{-i \delta \varphi(t)} \rangle = \langle \exp(-\delta \varphi^2(t)/2) \rangle$, where we have used that $\varphi_0 = 0$. In the weak coupling limit, and in the extreme quantum limit, $T = 0$, we find for the time averaged mean-squared fluctuations to leading order in $\Gamma$,

$$\langle \delta \varphi^2(t) \rangle = \int_0^{\omega_c} \frac{d\omega}{\pi} S_{\varphi \varphi}(\omega) = \frac{\Omega_0}{\Delta_0^2} [2 \Gamma \ln \frac{\omega_c}{\Omega_0} - \Gamma + \pi \Omega_0] \approx 2 \alpha \ln \frac{\omega_c}{\Omega_0}. \quad (18)$$

Here we have assumed that the cut-off frequency is so large that the logarithmic term dominates. In the limit $\omega_c \gg \Omega_0$, we can neglect $\langle \delta \theta^2(t) \rangle$ against $\langle \delta \varphi^2(t) \rangle$. We insert $\langle \delta \varphi^2(t) \rangle$ and $\sin \theta_0 = \Delta_0/\Omega_0$ into $\langle \sin \theta e^{-i \varphi} \rangle$, and find a noise averaged persistent current in the ring given by

$$\langle I(t) \rangle = -\frac{\hbar c}{2} \frac{\partial \Delta_0}{\partial \Omega_0} \frac{\Omega_0}{\omega_c} \left( \frac{\Omega_0}{\omega_c} \right)^\alpha. \quad (19)$$

The weak coupling limit corresponds to $\alpha \ll 1$. The power law for the persistent current obtained in Eq. (19), as well as the exponent $\alpha$, Eq. (8) coincide in this limit with the result obtained by Cedraschi et al. \cite{14} using a Bethe ansatz solution. We next characterize the fluctuations of the ring-dot subsystem in more detail.
6 Phase Diffusion Times

Due to the vacuum fluctuations the ground state of the two-level system is a dynamic state. To see this we project the actual state of the system on the ground state
\[ \psi_- = \cos \left( \frac{\theta_0}{2} \right), \sin \left( \frac{\theta_0}{2} \right), \]
with eigenvalue \(-\hbar \Omega_0/2\), and the excited state \[ \psi_+ = \left( -\sin \left( \frac{\theta_0}{2} \right), \cos \left( \frac{\theta_0}{2} \right) \right) \]
with eigenvalue \(\hbar \Omega_0/2\) of the isolated system. Instead of the wave function \[ \psi(t) \]
we consider
\[ \psi_R(t) = \exp \left( i \hat{H}_0 t/\hbar \right) \psi(t) \]
To first order in \(\delta \varepsilon\), we find for the wave function
\[ \psi_R(t) = (1 + ic_-) \psi_- + c_+ \psi_+ e^{i\Omega_0 t} \]
(20)
Expressing the fluctuations of the angle variables in terms of their fluctuation spectra, we find
\[ \left\langle (c_+(t) - c_+(0))^2 \right\rangle = t/\tau_\pm \]
diffusion times
\[ \tau_- = \frac{\hbar^2}{2 \pi \alpha kT} \frac{\Omega_0^2}{\varepsilon_0^2}, \quad \tau_+ = \frac{1}{\Gamma} \tanh \left( \frac{\hbar \Omega_0}{2 kT} \right). \]
(21)
Note that \(\tau_-\) depends on the detuning \(\varepsilon_0\). In particular, at resonance \(\varepsilon_0 = 0\), the phase diffusion time \(\tau_-\) diverges for any temperature. The long time behavior of \(\tau_+\), is determined by frequencies near \(\Omega_0\). Eq. (21) holds for finite temperatures as well as in the quantum limit. In the low-temperature or quantum limit, however, \(\tau_+\) saturates to a value \(1/\Gamma\). Thus at short and intermediate times the ground state is not a coherent state but exhibits diffusion. The dephasing rate is one-half of the sum of the two rates \(1/\tau_\pm\).

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