Earth Shielding and Daily Modulation from Electrophilic Boosted Dark Matter

Yifan Chen, Bartosz Fornal, Pearl Sandick, Jing Shu, Xiao Xue, Yue Zhao, and Junchao Zong

Abstract: Boosted dark matter represents an attractive class of models containing a fast-moving dark matter particle, which can lead to nonstandard nuclear or electron recoil signals in direct detection experiments. It has been shown that this interpretation successfully explains the excess of keV electron recoil events recently observed by the XENON1T experiment, and that a daily modulation of the signal in the detector is expected. In this paper we investigate the modulation feature in much greater detail and in a more general framework. We perform simulations of the dark matter interactions with electrons in atoms building up the Earth on its path to the detector, and we provide detailed predictions for the expected daily changes in the boosted dark matter signal for various direct detection experiments, including XENON1T, PandaX, and LUX-ZEPLIN.

Keywords: Boosted dark matter, daily modulation, direct detection, ionization
1 Introduction

Dark matter (DM) is certainly one of the greatest outstanding puzzles in modern particle physics. An enormous scientific effort has been undertaken, both on the theoretical and experimental sides, to shed more light on its nature, with great progress achieved in probing the parameter space of various particle physics models. Direct detection experiments offer a particularly promising way to search for DM, since in many models the DM particle is expected to undergo measurable recoils of nuclei and/or electrons in the detector.

Recently, a signal of this type has been hinted by the XENON1T experiment, where an excess of low-energy electron recoil events in the range \( \sim 2 - 3 \text{ keV} \) has been observed [1]. Although this effect might be the result of beta decays of tritium, a new physics explanation cannot be ruled out at this point due to large uncertainties in the determination of the
tritium concentration. One of the possible beyond Standard Model (SM) interpretations of the excess was put forward in [2] (see also [3]), where it is explained via boosted dark matter (BDM) scattering on electrons. If the BDM particle \( \chi \) is much heavier than the electron, the observed electron energy deposition signal implies \( \chi \) velocities of \( \mathcal{O}(10^{-1}) \, c \). Such fast-moving DM particles cannot come from the Milky Way halo and, instead, must be of astrophysical origin, produced, e.g., via semi-annihilation \( \bar{\chi} + \chi \rightarrow \chi + X \) (where \( X \) is a SM particle or a new particle eventually decaying to SM particles) [4], or via annihilation of a heavier dark sector particle, \( \psi \), \( \psi + \bar{\psi} \rightarrow \chi + \bar{\chi} \) [5]. Either the Galactic Center (GC) or the Sun can be the dominant source of the BDM flux.

Searches for such BDM particles have been proposed for large volume neutrino experiments, e.g., Super-Kamiokande [6–9], ProtoDUNE [10, 11], IceCube [12, 13], and DUNE [14–17] (see also [18–21] for related work). Here we focus on electrophilic BDM, however, our results are applicable not only to the XENON1T detector, but also to other experiments like PandaX [22] or LUX-ZEPLIN [23]. For electrophilic BDM, when the BDM-electron scattering cross section is sizable, the electron ionization signal in direct detection experiments is expected to exhibit daily modulation due to the Earth shielding effect [2]. This can be used to distinguish the BDM signal from various backgrounds. The information on the phase of the modulation can reveal the direction of the BDM flux, which would be of high importance for experimental analyses.

In this paper we extend the analysis of the daily modulation of the BDM signal of astrophysical origin, and explicitly account for the distribution of various elements in the Earth, calculating their contributions to the BDM-electron ionization cross section. Our results apply to any direct detection experiment measuring electron recoil energies. The software used in this research is publicly available. The ionization form factor is calculated using AtomIonCalc\(^1\), which is refined from the software DarkARC [24, 25]. The software realEarthScatterDM\(^2\) is used to simulate the DM propagation inside the Earth, and was independently developed for this research.

2 Boosted Dark Matter Model

The model we consider here is a simple extension of the SM that includes only two new particles: the BDM \( \chi \) and a dark mediator \( V \). The BDM interacts with electrons through the dark mediator, as described by the Lagrangian terms

\[
\mathcal{L} \supset g_\chi V_\mu \tilde{\chi} \gamma^\mu \chi + g_e V_\mu \bar{\epsilon} \gamma^\mu \epsilon .
\]

If the mediator mass \( m_V \) is much larger than the \( \sim \) keV momentum transfer, for the parameter space we are interested in, the cross section for the scattering of BDM on free electrons simplifies to

\[
\sigma_e = \frac{g_\chi^2 g_e^2 m_e^2}{\pi m_V^2} .
\]

\(^1\)[https://github.com/XueXiao-Physics/AtomIonCalc]
\(^2\)[https://github.com/XueXiao-Physics/realEarthScatterDM]
The benchmark points we consider here correspond to BDM-electron scattering cross sections of $\sigma_e = 10^{-28}$ cm$^2$, $10^{-31}$ cm$^2$, and $10^{-33}$ cm$^2$. For those scenarios, there is a wide range of values for the parameters $g_\chi$, $g_e$ and $m_V$ consistent with existing experimental bounds (see [2] for the relevant discussion).

### 3 Model of the Earth

The Earth consists essentially of two parts: the core and the mantle. The eight most abundant atomic elements in the core and mantle [26, 27] are shown in Table 1. The remaining elements contribute a mass fraction below 1%. Due to the lack of precise information regarding the density of each element in terms of the distance from the Earth’s center, we assume that a given element’s mass fraction is constant in the core and mantle, and we take the value in each region to be the average value in Table 1. The total density profile as a function of radius is taken from [28] and is shown in Fig. 1. The Earth is assumed to be isotropic, despite the complexity of its composition.

In the next section, we calculate the ionization form factor for all the elements in Table 1. Combining the result with the absolute abundance of elements at arbitrary radius $r$ as demonstrated in Fig. 1, we can fully determine the scattering behaviour of the BDM propagation in the Earth.

### 4 Dark Matter Induced Ionization

In this section we briefly summarize how BDM particles ionize electrons bound inside atoms; for a more detailed discussion, see Appendices A, B, and C. The differential cross section for the ionization caused by an incoming BDM particle $\chi$ (with velocity $v_\chi$) is given by

$$\frac{d\sigma_{\text{ion}}}{dE_R}(v_\chi, E_R) = \frac{\sigma_e m_e q_0^2}{2\mu^2 v_\chi^2} \int_{q_-}^{q_+} q \left| F(q) \right|^2 K(E_R, q) \, dq,$$

where $q_-$ and $q_+$ are the lower and upper limits of integration, and $K(E_R, q)$ is a kernel function depending on the energy transfer $E_R$ and the momentum transfer $q$.
where $\mu$ is the reduced mass of the BDM-electron system, $a_0 = 1/(\alpha m_e)$ is the Bohr radius, $q_\pm = m_\chi v_\chi \pm \sqrt{m_\chi^2 v_\chi^2 - 2m_\chi E_R}$ is the range for momentum transfer $q$, and $F(q)$ is the BDM form factor, which for the model described by Eq. (2.1) is $F(q) = 1$.

The atomic form factor $K(E_R, q)$ for ionization describes the probability of obtaining a particular recoil energy of an ionized electron for a given momentum transfer $q$. We follow the calculation presented in [24, 25]. The wave functions of the electron initial states with quantum numbers $(n, \ell)$ are taken to be the Roothan-Hartree-Fock (RHF) ground state wave functions whose radial part is described by a linear combination of Slater-type orbitals,

$$R_{n\ell}(r) = a_0^{-3/2} \sum_j C_{j\ell n} \frac{(2Z_{j\ell})^{n'_{j\ell}+1/2}}{\sqrt{(2n'_{j\ell})!}} \left( \frac{r}{a_0} \right)^{n'_{j\ell}-1} \exp \left( -Z_{j\ell} \frac{r}{a_0} \right).$$  \hspace{1cm} (4.2)

The values of the parameters $C_{j\ell n}, Z_{j\ell}, n'_{j\ell}$, as well as the binding energies for each element are provided in [29]. The final state wave functions, which are asymptotically free spherical waves in a central potential, are given in [30]. The atomic form factor $K(E_R, q)$ defined in [31, 32] is related to the ionization response function $f_{\ell n}^{n\ell} (k', q)$ through

$$K(E_R, q) = \sum_{n\ell} \frac{|f_{\ell n}^{n\ell} (k', q)|^2}{2k'^2a_0^2} \Theta(E_R + E_{n\ell}^{n\ell}) ,$$  \hspace{1cm} (4.3)

where $\Theta$ is the Heaviside function. We have $E_R = -E_{n\ell}^{n\ell} + k'^2/2m_e$, where $E_{n\ell}^{n\ell}$ is the binding energy of the initial state electron, and $k'$ is the momentum of the final state ionized electron. We take into account contributions from all accessible states. A detailed calculation of $K(E_R, q)$ is presented in Appendices A and B.

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Figure 1. Number density of various atoms in the Earth’s core and mantle. The mantle-core border is indicated by a vertical black dotted line. The density profile was taken from [28], while the Earth’s composition was adopted from [27].
Figure 2. Here we show the atomic ionization form factor \( K(E_R, q) \) for different atoms listed in Table 1 in the tight binding limit. The radial wave functions are determined using the RHF ground state wave functions in Eq. (4.2) with the coefficients \( C_{j\ell n}, Z_{j\ell}, n'_{j\ell} \) and binding energies provided in [29]. The \( q \) and \( E_R \) distribution converges to \( E_R = q^2 / (2m_e) \) in the large recoil energy limit, which is labeled by the white solid line.
For the energy regime of the BDM scenario considered here, the energy losses are dominated by the ionization process. Scattering with the valence and conducting electrons, due to their small binding energy, should recover the elastic scattering limit at $E_R \sim O(1)$ keV. We thus treat the electron ionization in the tight binding approximation, where the electrons are assumed to have limited interactions with the neighboring atoms, so that the uncertainty of the molecular composition can be ignored. We also neglect the dissipation induced by the transitions of an electron among bound states (see [33, 34]), since these are subdominant compared to the ionization when the typical recoil energy $E_R$ is much larger than the binding energy of valence electrons. Under those assumptions, we calculate the ionization form factor $K(E_R, q)$ for each of the elements listed in Table 1 and show the results in Fig. 2. For all the cases, as expected, the ionization form factors approach the kinetic region of elastic scattering, i.e., $E_R = q^2/(2m_e)$, when $E_R$ is much larger than the binding energy. On the other hand, when $E_R$ is just enough to ionize an electron, $q$ has a broader distribution.

5 Propagation of Boosted Dark Matter Inside the Earth

5.1 Overview of the Monte Carlo Simulation

We assume that the BDM particles are produced monochromatically and arrive at Earth from a fixed direction [2]. Thus, the incoming BDM flux can be written as,

$$\frac{d\Phi_{\text{init}}}{d^3\vec{v}} = \Phi_0 \, \delta^3(\vec{v} - \vec{v}_0), \quad (5.1)$$

where $\Phi_0$ is the total initial flux directed towards the Earth. A schematic diagram of the model is shown in Fig. 3.

In order to understand the propagation of the BDM inside the Earth, we first need to consider the interaction between the BDM particle and Earth’s elements. According to Eq. (4.1), the probability distribution of the BDM final state after scattering is fully determined by the ionization form factor $K(E_R, q)$, where $E_R$ is the recoil energy and $q$ is the momentum transfer. From Eq. (4.1), the mean free path of the BDM particle inside the Earth can be calculated as,

$$l_{\text{fp}}(r, v_\chi, m_\chi) = \left[ \sum_a n^a(r) \sigma^a_{\text{ion}}(v_\chi, m_\chi) \right]^{-1}, \quad (5.2)$$

where the index $a$ denotes the type of the Earth’s element. $n^a(r)$ is the number density of element $a$ at radius $r$, which can be calculated from Fig. 1. $\sigma^a_{\text{ion}}(v_\chi, m_\chi)$ is the ionization cross section between element $a$ and a BDM particle with velocity $v_\chi$ and mass $m_\chi$, obtained by integrating out the recoil energy $E_R$ and momentum transfer $q$ in Eq. (4.1).

We have developed a Monte-Carlo simulation to study BDM propagation inside the Earth. The flow chart of the simulation is shown in Fig. 4, and a more detailed description can be found in Appendix D. We start with BDM particles of mass $m_\chi$ and velocity $\vec{v}_\chi^0$, evenly distributed on the plane perpendicular to $\vec{v}_\chi^0$. The main structure of the
Figure 3. The BDM flux comes from a given direction with velocity \( \vec{v}_0 \). The polar angle \( \theta \) is between the direction of the initial flux and the direction pointing from the Earth’s center to the detector. \( \delta \chi \) is the declination of the BDM flux direction in the equatorial coordinate system, ranging from \(-\pi/2\) in the south to \(\pi/2\) in the north. The blue and black solid lines denote the Earth’s rotation axis and the incident direction of the flux, respectively. The light and dark orange regions correspond to the Earth’s mantle and core. The cyan cylinder denotes the detector.

Simulation is the iteration of scattering (\(i\) and \(f\) denote the initial and final state for each step, respectively). In each iteration, we first calculate the mean free path \( l_{fp}^{\text{ion}}(x_i, v_i, m_\chi) \) using Eq. (5.2). Next, we use the exponential distribution \( e^{-l/l_{fp}^{\text{ion}}} \) to sample \( l \), which denotes the propagation distance for the BDM particle in this step of the iteration. The final position of the BDM particle, \( \vec{x_f} \), can thus be easily calculated from \( \vec{x_i}, \vec{v}_i \) and \( l \). Then, we sample the recoil energy \( E_R \) and the momentum transfer \( q \) whose probability distribution is proportional to \( q \times K(E_R, q) \) according to the differential cross section in Eq. (4.1). Meanwhile the azimuthal angle \( \beta \) on the transverse plane with respect to the initial velocity is drawn from a flat distribution between 0 and \( 2\pi \). The values of \( E_R, q, \) and \( \beta \) fully determine the momentum transfer vector \( \vec{q} \), which is used to calculate the final velocity \( \vec{v}_f \). Lastly, the pair \((\vec{x}_f, \vec{v}_f)\) is used as the input for the next iteration as \((\vec{x}_i, \vec{v}_i)\).

Additionally, in each iteration we check whether the trajectory crosses the mantle-core border. If it does, we recalculate the mean free path \( l_{fp}^{\text{ion}} \) and reset the starting point for this iteration at the spot where the crossing happens. Furthermore, the initial velocity \( \vec{v}_i \) remains unchanged. The location and velocity at each iteration are recorded. The simulation stops once the BDM particle exits the Earth or when its velocity is smaller than the threshold velocity which is either the DM virial velocity or the minimum velocity to ionize an electron in xenon. For more details, please see Appendix D.2. Finally, we perform the simulation with different BDM initial velocity directions to account for the effect of Earth’s rotation, as demonstrated in Fig. 3.
**5.2 Distortion of the Velocity Distribution**

Due to propagation inside the Earth, the BDM velocity distribution is distorted when reaching the detector. The amount of distortion depends on the polar angle $\theta$ between the incoming BDM flux and the direction pointing from the Earth’s center to the detector, as shown in Fig. 3. Before showing the results of the Monte Carlo simulation, we first present a qualitative estimate of the distortion of the BDM velocity distribution.

The distance traveled $l$ inside the Earth depends on the depth of the detector $d$ and the direction of the incoming BDM flux. In terms of $\theta$, it can be written as

$$l = \sqrt{R_E^2 - R_D^2 \sin^2 \theta} + R_D \cos \theta$$

(5.3)

where $R_D \equiv R_E - d$. In the limit $d \ll R_E$, $l$ ranges from $d$ to $\sqrt{2R_Ed}$ on the near side ($\frac{\pi}{2} < \theta \leq \pi$) and from $\sqrt{2R_Ed}$ to $2R_E$ on the far side ($0 \leq \theta < \frac{\pi}{2}$). The BDM kinetic energy, $E_{\text{kin}} \equiv m_\chi v_\chi^2/2$, is smeared due to dissipation from ionization. For each scattering, the typical energy loss in the elastic scattering limit is $m_e v_\chi^2$ when the BDM is much heavier than electrons [2] (see Appendix C for a more detailed discussion). Thus the
energy dissipation can be approximated in terms of the mean free path $l_{fp}$ as

$$\frac{dE_{\text{kin}}}{dx} \sim -\frac{m_e v^2}{l_{fp}} \mu \chi,$$  

(5.4)

from which one can derive the dissipation of velocity as

$$v(1) \approx v(0) \exp\left(-\int l_{fp} \frac{m_e}{m_{\chi}} (x) \, dx\right).$$  

(5.5)

In the elastic scattering approximation, the mean free path can be written as $l_{fp}(r) = \left[ n_e(r) \sigma_e \right]^{-1}$, where $n_e(r) = \sum_a n_a(r) Z^a$ is the electron density including the contributions of all elements inside the Earth and $\sigma_e$ is the scattering cross section between the BDM and a free electron. In Fig. 5, we compare the mean free path for ionization, $l_{fp}$, with the one from elastic scattering, $l_{fp}^{\text{free}}$. At low BDM velocities, the finite binding energy suppresses the ionization. On the other hand, when $v_{\chi} \gg 10^{-2} c$, $l_{fp}^{\text{free}}(r)$ serves as a good approximation for $l_{fp}(r)$. In this approximation, taking the electron number density as $1 \times 10^{24}/\text{cm}^3$ near the Earth’s surface, $1.3 \times 10^{24}/\text{cm}^3$ at the mantle, and $3 \times 10^{24}/\text{cm}^3$ at the core, the mean free path of the BDM in each region is $l_{fp}^{\text{free}} \sim 100 \times (10^{-28} \text{ cm}^2/\sigma_e)$, $l_{fp}^M \sim 75 \text{ m} \times (10^{-28} \text{ cm}^2/\sigma_e)$, and $l_{fp}^C \sim 33 \text{ m} \times (10^{-28} \text{ cm}^2/\sigma_e)$, respectively.

According to Eq. (5.5), one can define the effective distance at which the velocity distortion is significant,

$$l_{\text{eff}} \equiv l_{fp} \frac{m_{\chi}}{m_e},$$  

(5.6)

This can be used to classify the distortion of the velocity distribution into several cases:

- $l_{\text{eff}} \ll d \approx 1.6 \text{ km} \left( \sigma_e \gg 1 \times 10^{-27} \text{ cm}^2 \right)$ for $m_{\chi} = 100 \text{ MeV}$: extremely strong interaction. No events are expected in the detector;
• $d \ll l_{\text{eff}} \ll \sqrt{2 R_E d} \simeq 143 \text{ km}$ ($2 \times 10^{-29} \ll \sigma_e \ll 1 \times 10^{-27} \text{ cm}^2$ for $m_\chi = 100 \text{ MeV}$): strong interaction. No BDM enters the detector if it is on the far side. The BDM velocity distribution may have a significant distortion when BDM enters the detector on the near side;

• $\sqrt{2 R_E d} \ll l_{\text{eff}} \ll 2 R_E \simeq 12740 \text{ km}$ ($1 \times 10^{-31} \ll \sigma_e \ll 2 \times 10^{-29} \text{ cm}^2$ for $m_\chi = 100 \text{ MeV}$): weak interaction. A significant distortion of the BDM velocity distribution may happen when the BDM enters the detector on the far side;

• $l_{\text{eff}} \gg 2 R_E$ ($\sigma_e \ll 1 \times 10^{-31} \text{ cm}^2$ for $m_\chi = 100 \text{ MeV}$): extremely weak interaction. The BDM flux experiences almost no distortion of its velocity distribution.

In Fig. 6, we show the results of our simulation for the BDM velocity distribution when it reaches the detector as a function of $\cos \theta$. Nine cases are presented, illustrating how the differential velocity distribution depends on various model parameters. The first row corresponds to different values of the BDM mass $m_\chi$. Equation (5.5) implies that the larger the mass, the less distorted the velocity distribution is after scattering inside the Earth. The second row corresponds to a variation of the initial velocity $v_0$. The third row compares three cases with various BDM-electron scattering cross sections $\sigma_e$, corresponding to scenarios with strong interaction, weak interaction, and extremely weak interaction, respectively.

6 Daily Modulation of Ionization Signals

Due to Earth’s rotation, the angle $\theta$ between the direction of the incoming BDM flux and the detector varies with a period of one day,

$$\cos \theta(t) = - \cos (\delta_\chi) \cos (\delta_D) \cos \left[2\pi \left(\frac{t - t_0}{24 \text{ h}}\right)\right] - \sin (\delta_\chi) \sin (\delta_D) ,$$  \hspace{1cm} (6.1)

where $\delta_\chi$ is the declination of the source of the BDM flux and $\delta_D$ is the detector’s declination projected onto the celestial sphere. The time $t_0$ is the time at which the BDM flux is on the upper culmination of the detector.

Assuming a BDM flux from the GC ($\delta_{\chi, \text{GC}} = -29.00^\circ$) as an example, $t_0$ is set to 18.65 h, 0.53 h and 10.87 h Greenwich Mean Sidereal Time (GMST) for XENON1T ($\delta_D = 42.25^\circ$), PandaX ($\delta_D = 28.20^\circ$) and LUX-ZEPLIN ($\delta_D = 44.35^\circ$), respectively. The relation between the GMST time and $\cos \theta$ is shown in the left panel of Fig. 7. Since the GC is on the southern hemisphere and the three detectors we consider in this study are on the northern hemisphere, the detectors are on the far side of the Earth with respect to the BDM flux for the majority of the time. Apart from BDM from the GC, one can also consider BDM from the Sun. In this case, the daily modulation is more conveniently described by the Coordinated Universal Time (UTC), shown in the right panel of Fig. 7. The value of $\delta_{\chi, \text{Sun}}$ varies from $-23.5^\circ$ on December 21st to $23.5^\circ$ on June 20th. We take $t_0 = 11.1$ h according to the longitude of Gran Sasso.
The BDM velocity distribution when it reaches the detector, after its propagation through the Earth. The three rows correspond to different choices of the BDM mass, the initial BDM velocity and the cross section, respectively. The value of the initial BDM velocity is indicated by the cyan dotted line. We use the color bar to characterize the normalized differential flux distribution as a function of the BDM velocity. \( \Delta \Phi \) is the flux within each bin of \( \log_{10}[v/c] \).

The signal rate for each experiment can be written as

\[
\frac{dR}{dE_R} = N_d \int \frac{d\sigma_{\text{ion}}}{dE_R}(v_\chi, E_R) \frac{d\Phi(v_\chi, \theta)}{dv_\chi} \, dv_\chi,
\]

where the differential cross section is provided in Eq. (4.1). \( N_d \) is the number of xenon atoms in the detector, which is \( N_d \approx 4.2 \times 10^{27} \) ton\(^{-1} \) for XENON1T. The electron recoil energy spectrum varies with time.

In Fig. 8, we present results for a BDM flux from the GC for three benchmark cross sections, \( \sigma_e = 10^{-28} \) cm\(^2 \) (upper panels), \( 10^{-31} \) cm\(^2 \) (middle panels), and \( 10^{-33} \) cm\(^2 \) (lower panels). From left to right, we show the velocity distribution for three different times, \((t - t_0) = 0 \) h, 6 h, and 12 h; the best fit electron recoil energy spectrum; and the time
Figure 7. Left: The value of $\cos \theta$ as a function of the sidereal time for XENON1T, PandaX and LUX-ZEPLIN, respectively, assuming the BDM flux originates in the GC. Right: The value of $\cos \theta$ as a function of UTC for the XENON1T experiment on four different days of the year, assuming the BDM flux arrives from the Sun.

The evolution of the event rate in three bins, [1, 2] keV, [2, 3] keV, [3, 4] keV, as well as the sum of the three bins. Here we see that the fine features in the experimental data can be used to extract the properties of the BDM in a comprehensive manner.

One way to examine the daily modulation signal is to perform a Fourier transform on the data \cite{35}. The signal can be parametrized as

$$\frac{dR}{dE_R} = A_0 + \sum_{n=1}^{\infty} \left[ A_n \cos \left( \frac{2\pi n t - t_n}{T} \right) \right], \quad (6.3)$$

where $T$ is the modulation period. For example, $T$ is either one sidereal day or one solar day (when the BDM source is the GC or the Sun, respectively). $A_n$ is the amplitude in the Fourier series and $t_n$ is the relative phase. For the signal, $t_n$ converges to $t_0$ for a given recoil energy. A fit to $t_n$ provides information on the direction of the BDM flux. If we correlate the time series of the signals for three different detectors, one expects the differences in $t_0$ for each detector to be related to the differences in the detector locations.

7 Discussion

In this paper, we carried out a detailed analysis of the daily modulation of the signal expected from boosted dark matter interacting with electrons. Such an effect can be searched for in terrestrial dark matter direct detection experiments like XENON1T, PandaX and LUX-ZEPLIN.

We developed a Monte Carlo code to simulate the dark matter’s trajectory through the Earth to the detector. In the case of the XENON1T detector, we focused on the recent hint of an electron recoil excess. Considering a benchmark scenario in which the boosted dark matter source is located at the Galactic Center, we calculated the expected
Figure 8. Velocity distribution, electron recoil energy spectrum, and time evolution of the event rate. The BDM flux is assumed to originate in the GC. The BDM mass $m_\chi$ and the initial velocity $v_\chi^0$ are taken to be 0.1 GeV and 0.06$c$ respectively. We consider three benchmark values of the cross section, $\sigma_e = 10^{-28}$ cm$^2$ (upper panels), $10^{-31}$ cm$^2$ (middle panels), and $10^{-33}$ cm$^2$ (lower panels). In the left column, the velocity distributions in terms of $\cos \theta$ are shown. To show the time dependence caused by the Earth shielding effect, the results at different sidereal times $(t - t_0) = 0$ h, 6 h, and 12 h are highlighted by the cyan, green, and blue dashed lines respectively, where $t_0$ corresponds to the time that the GC is at the upper culmination of the detector. In the middle column, with fixed $\sigma_e$, $m_\chi$, and $v_\chi^0$, we show the best-fit electron recoil energy spectrum for the XENON1T excess. We also present the spectra of the BDM signals at three choices of time. The averaged signals with and without the background are denoted by the orange and red solid lines, respectively. In the right column, the time evolution of the signals for different recoil energy bins is presented.
time variation in terms of the electron recoil event rate. Our predictions can be directly compared to the current and future data, to investigate the plausibility of the boosted dark matter interpretation of the XENON1T excess.

Another study regarding the daily modulation of the signal has been carried out in [36], where the BDM is produced by cosmic ray scattering. Instead of the BDM-electron scattering, the hadronic interaction of the dark matter particle is the focus. The hadronic form factor suppresses the interaction probability at the large momentum transfer, in which case the distortion of the flux becomes most pronounced in the intermediate energy regime. Combined with the difference in the initial velocity distribution of the BDM flux, this leads to substantially different predictions for the BDM energy spectrum in the detector after the signal propagated through Earth.

A possible future extension of this work includes calculating atmospheric or local geographic effects on the boosted dark matter signal. It may also be interesting to include the energy loss through the excitation among various atomic bound states, which requires a more in-depth knowledge of the chemical composition of the Earth.

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\section{From Elastic Scattering to Ionization}

The differential cross section for the $2 \rightarrow 2$ elastic BDM-electron scattering is,

$$\frac{d\sigma_{\text{free}}}{dE_e^\prime} = \frac{|M_{\text{free}}|^2}{4E_e E_e^\prime v_e} \frac{d^3p^\prime}{(2\pi)^3} \frac{d^3k^\prime}{(2\pi)^3} \delta^4 \left( k + p - k^\prime - p^\prime \right) ,$$

where $p$ and $k$ are the four-momenta of the BDM and electron initial state, while the prime denotes the final state. The $v_e$ is the relative velocity between the BDM and electron initial states. The $M_{\text{free}}$ is the matrix element for the elastic scattering which depends on the momentum transfer $\vec{q} \equiv \vec{p} - \vec{p}^\prime$ in the non-relativistic limit of electrons.
In the elastic scattering, the initial and final state electron wave functions are taken to be plane waves $|e_k\rangle$. However, for a process like ionization, both the initial bound state and the final unbound state are represented by a wavepacket in the momentum space, 

$$|e_k\rangle \rightarrow \int \frac{\sqrt{V}d^3k}{(2\pi)^3} \psi_i(\vec{k})|e_k\rangle,$$  

(A.2)

where $V \equiv (2\pi)^3\delta^3(\vec{0})$ is the volume of space. The momentum space wave functions satisfy the normalization condition $\int d^3k|\psi(\vec{k})|^2/(2\pi)^3 = 1$. In the non-relativistic limit, the scattering amplitude becomes

$$(2\pi)^3\delta^3\left(\vec{k} + \vec{q} - \vec{k}'\right) \mathcal{M}_{\text{free}}(\vec{q}) \rightarrow \int \frac{Vd^3k}{(2\pi)^3} \psi^*_j(\vec{k} + \vec{q})\psi_i(\vec{k}) \mathcal{M}_{\text{free}}(\vec{q})$$

(A.3)

where we define the transition form factor as

$$f_{i\rightarrow f}(\vec{q}) \equiv \int \frac{d^3k}{(2\pi)^3} \psi^*_j(\vec{k} + \vec{q})\psi_i(\vec{k}) = \int d^3x \psi^*_j(\vec{x})e^{i\vec{q} \cdot \vec{x}}\psi_i(\vec{x})$$

(A.4)

It describes the transition probability with a given momentum transfer $\vec{q}$. We can further rewrite the form factor in terms of the coordinate space wave functions, which becomes 

$$\psi(\vec{x}) = \exp(i\vec{k} \cdot \vec{x})/\sqrt{V}$$

in the plane wave limit.

For the ionization process, one needs to perform the summation on all the bound electrons in the initial state, as well as the final state phase space. Using the quantum numbers $(n, \ell, m)$ to label the initial bound state, one has

$$\sum_{\text{occupied}} = g_s \sum_{n,\ell, m},$$

(A.5)

where $g_s = 1$ or $2$ represents the occupancy due to the spin degeneracy. The final states are characterized by the asymptotically free spherical wavefunctions with phase space summation written as

$$\frac{d^3k'}{(2\pi)^3} \rightarrow \sum_{\ell' m'} \frac{k'^2dk'}{(2\pi)^3} = \sum_{\ell' m'} \frac{k'm_e dE_R}{(2\pi)^3} \Theta(E_R + E_B^{n\ell}),$$

(A.6)

where $\Theta$ is the heaviside function. $E_B^{n\ell} < 0$ is the binding energy for a given initial bound state $(n, \ell)$. The recoil energy $E_R$ is the sum of the final state electron kinetic energy and the amount of binding energy $E_R = k'^2/2m_e + |E_B^{n\ell}|$.

Finally, one has to perform the integral over the momentum of the BDM final state. With $d^3p' = d^3q$, the energy conservation leads to

$$\frac{d^3q}{(2\pi)^2} \delta\left(E_R + \frac{q^2}{2m_\chi} - qv_\chi \cos \theta_{qv_\chi}\right) = \frac{qdq}{2\pi v_\chi}$$

(A.7)

with integration limits

$$q_\pm = m_\chi v_\chi \pm \sqrt{m_\chi^2 v_\chi^2 - 2m_\chi E_R}.$$  

(A.8)
Putting everything together, in the non-relativistic limit, the ionization differential cross section for the BDM with velocity \( v \), can be written as

\[
\frac{d\sigma_{\text{ion}}}{dE_R} = \frac{\bar{\sigma}_e}{8\mu^2 v_\chi^2 (E_R + E_{B}^{\text{eff}})} \int_{q^-}^{q^+} q \left| F(q) \right|^2 \left| f_{\text{ion}}^{n\ell}(k', q) \right|^2 dq,
\]

where \( \bar{\sigma}_e \equiv \mu^2 |\mathcal{M}_{\text{free}} (\alpha m_e)|^2 / (16\pi^2 m_e^2 m_0^2) \) is the BDM-electron elastic scattering cross section evaluated at \( q = \alpha m_e \), and \( \mu \) is the reduced mass. The function \( F(q) \) is the BDM form factor, and we have \( F = 1 \) for the benchmark model studied here. The transition form factor defined in Eq. (A.4) for the ionization process can be written as

\[
\left| f_{\text{ion}}^{n\ell}(k', q) \right|^2 = \frac{4k^3V}{(2\pi)^3} \sum_{\ell' m'} \int d^3x \tilde{\psi}_{k'\ell'm'}(\vec{x}) \tilde{\psi}_{i\ell' m'}(\vec{x}) e^{i\vec{q} \cdot \vec{x}}. \tag{A.10}
\]

It has no dependence on the direction of \( \vec{q} \) due to the spherical symmetry. As mentioned in the main text, this is related to the factor \( K(E_R, q) \) defined in [31, 32] as

\[
K(E_R, q) = \sum_{n\ell} \left| f_{\text{ion}}^{n\ell}(k', q) \right|^2 \Theta(E_R + E_{B}^{\text{eff}}), \tag{A.11}
\]

where \( a_0 = 1/(\alpha m_e) \) is the Bohr radius.

### B Atomic Ionization Factor

The ionization form factor in Eq. (A.10) is obtained by calculating the spatial overlap between the initial and final electron wave functions, convoluted with the momentum transfer \( e^{i\vec{q} \cdot \vec{x}} \). Following [24, 37], we expand \( e^{i\vec{q} \cdot \vec{x}} \) as a linear combination of spherical harmonic functions, and the Eq. (A.10) can be written as

\[
\left| f_{\text{ion}}^{n\ell}(k', q) \right|^2 = \frac{4k^3V}{(2\pi)^3} \sum_{\ell' m'} \sum_{\ell=0}^{\infty} (2\ell + 1) (2\ell' + 1) (2L + 1) \left[ \ell \ell' L 0 0 0 \right]^2 \left| I_R(q) \right|^2. \tag{B.1}
\]

Here \( [ \cdots ] \) is the Wigner 3-\( j \) symbol, and \( I_R(q) \) is defined to be

\[
I_R(q) \equiv \int dr r^2 R_{k'\ell'}(r) R_{n\ell}(r) j_L(qr), \tag{B.2}
\]

in which \( j_L(qr) \) is the first order spherical Bessel function. The radial wavefunctions of the electron initial bound state can be written as a sum of Slater-type orbital wavefunctions

\[
R_{n\ell}(r) = a_0^{-3/2} \sum_j C_{j\ell n} \left( \frac{2Ze_j}{n_{j\ell}+1/2} \right)^{n_{j\ell}+1/2} \left( \frac{r}{a_0} \right)^{n_{j\ell}-1} \exp \left( -Z_j \frac{r}{a_0} \right), \tag{B.3}
\]

where the parameters \( C_{j\ell n}, Z_j \) and \( n_{j\ell} \), for each atom species, are given in [29].

The wavefunction of the ionized electron in the final state is the unbound solution of the Schrödinger equation with a hydrogen-like potential \(-Z_{\text{eff}}^{n\ell}/r\). The numerical results are provided in [24], for example. This recovers the free plane wave solution when the kinetic energy is much larger than the binding energy. The effective charge, \( Z_{\text{eff}}^{n\ell} \), is related to the binding energy as \( Z_{\text{eff}}^{n\ell} = n\sqrt{-E_{B}^{\text{eff}}/13.6 \text{ eV}} \) [29].
Figure 9. Here we show the effective electron number for various xenon shells in the large velocity and heavy BDM limit. Particularly, we have \( n_{\text{eff}}^{n} \equiv \sum \ell n_{\text{eff}}^{n \ell} \), defined in Eq. (C.2).

C Comparison Between Elastic Scattering and Ionization

Let us first consider the elastic scattering, assuming the electron is free and at rest. For a contact interaction with \( F(q) = 1 \), in the non-relativistic approximation of the final state electron, one can derive the differential cross section as a function of the recoil energy, \( E_R = k'^2/2m_e \), as

\[
\frac{d\sigma_{\text{free}}}{dE_R} = \frac{\sigma_e m_e}{2\mu^2 v^2} \Theta \left( \frac{2\mu^2 v^2}{m_e} - E_R \right),
\]

which is a flat distribution for \( E_R < 2\mu^2 v^2/m_e \).

We now include the effect of the binding energy and consider the ionization process. This requires the velocity of the incoming BDM particle and its mass to be large so that the momentum transfer is enough to trigger the ionization. In this limit, the integration range \((q_-, q_+)\) in Eq. (A.9) is \((E_R/v_\chi, 2m_\chi v_\chi)\) at leading order, which effectively becomes \((0, \infty)\) for the integral. Thus below the energy cutoff in Eq. (C.1), the ratio between the differential cross section for the ionization of an electron with \((n, \ell)\), i.e. Eq. (A.9), and that of a free electron scattering, i.e. Eq. (C.1), can be written as

\[
n_{\text{eff}}^{n \ell}(E_R) \equiv \int_0^{\infty} q \left| f_{\text{ion}}^{n \ell}(E_R, q) \right|^2 \frac{\Theta(E_R + E_B^{n \ell})}{4m_e(E_R + E_B^{n \ell})} dq.
\]

This is defined to be the effective electron number for a given atomic level. The numerical results for xenon are shown in Fig. 9. The results converge to the number of the electrons for that shell. Notice that it requires the sum of the final state angular momentum number \( \ell' \) to a large number to properly mimic the final state wave function when \( E_R \) is large.

This result is also consistent with the kinetic distribution of the ionization form factor in the large recoil energy limit. In Fig. 10, we show the result for xenon. In the limit of a large recoil energy, the form factor peaks at \( E_R \simeq q^2/(2m_e) \), which recovers the kinetic distribution of elastic scattering.
Figure 10. The atomic ionization form factor $K(E_R,q)$ for xenon is shown. The white solid line is for $E_R = q^2/(2m_e)$.

D Monte Carlo Simulation

In this study, we develop a Monte Carlo simulation in order to derive the velocity distribution of the BDM flux when it reaches the detector. Here we provide a comprehensive description of the simulation procedure.

D.1 Initial Setup

The simulation starts with four input parameters: $m_\chi$, $\sigma e$, $v_0^x\chi$, and $N$, which correspond to the BDM mass, the BDM-electron elastic scattering cross section evaluated at $q = \alpha m_e$, the initial incident BDM velocity, and the number of simulation events, respectively. We consider a BDM flux from the GC or the Sun. We set the direction of the $z$-axis to always coincide with the direction of the incoming BDM flux, see Fig. 3 for an illustration.

To generate the initial BDM flux evenly distributed on the plane orthogonal to the $z$-direction, we first draw a random value from the uniform distribution $[0,R_E^2]$ where $R_E$ is the Earth’s radius, then we define $\rho_{xy}$ as the square root of the previously generated number. Next, we draw a random azimuthal angle $\phi_E$ from a uniform distribution $[0,2\pi)$. With this choice, we can calculate the position of each BDM particle entering the Earth in the Cartesian coordinate $(x,y,z)$ as,

$$
\begin{align*}
  x_0 &= \rho_{xy} \cos \phi_E, \\
  y_0 &= \rho_{xy} \sin \phi_E, \\
  z_0 &= \sqrt{R_E^2 - x_0^2 - y_0^2}.
\end{align*}
$$

(D.1)
D.2 Propagation Inside the Earth

Next we consider the propagation of the BDM inside the Earth. The simulation procedure of the BDM propagation is shown on the flow chart in Fig. 4. For each iteration, we first calculate the mean free path, \( l_{fp} \), of the BDM particle traveling inside the Earth, using Eq. (5.2). Next, we determine the travel distance between two successive scatterings in the Earth’s mantle or core according to an exponential probability distribution,

\[
f(l; \frac{1}{l_{fp}}) = \frac{1}{l_{fp}} \exp \left( -\frac{l}{l_{fp}} \right),
\]

(D.2)

Combining \( l \) with the final velocity calculated from the previous step, we obtain the endpoint of the trajectory in each iteration where the scattering occurs. It becomes subtle when the trajectory hits the mantle-core border before it ends. If this happens, we set the location where the trajectory hits the mantle-core border as the new starting point \( \vec{x}_i \) of this iteration while the velocity is left unchanged.

For each scattering, we determine the electron recoil energy \( E_R \) and the momentum transfer \( q \) using the ionization form factor. According to Eq. (4.1), \( q \times K(E_R, q) \) describes the joint probability of \( E_R \) and \( q \) in an ionization process. In Fig. 2, \( K(E_R, q) \) for all elements listed in Table 1 are demonstrated. It is worth noting that when the binding energy of an electron is much smaller than the BDM kinetic energy, \( E_R \) and \( q \) become closely correlated, and the most probable values are those satisfying \( E_R \approx q^2/(2m_e) \). We use the generalized acceptance-rejection method [38] to acquire a \((E_R, q)\) pair corresponding to the probability distribution given by \( q \times K(E_R, q) \). A further dynamical constraint on the \((E_R, q)\) pair is imposed,

\[
E_R \leq q v_i - q^2/(2m_\chi),
\]

which is equivalent to the condition of \( q \in (q_-, q_+) \) used in Eq. (4.1). The magnitude of the BDM final velocity, \( v_f \), after the scattering is written as,

\[
v_f = \sqrt{v_i^2 - 2E_R/m_\chi}.
\]

(D.3)

The polar angle of the final velocity, \( \alpha \), respect to the direction of the initial velocity can be calculated as

\[
v_i^2 + v_f^2 - 2v_iv_f \cos \alpha = \frac{q^2}{m_\chi^2}.
\]

(D.4)

To fully determine the direction of the BDM final state after the scattering, we sample the azimuthal angle \( \beta \) following a uniform distribution \([0, 2\pi]\). The final state of the BDM in each iteration is thus determined, including its location \( \vec{x}_f \) and velocity \( \vec{v}_f \). These will be used as the inputs for the next iteration.

There are two conditions for the simulation to stop. First, there is a minimal recoil energy required to ionize an electron in xenon. It can be used to set a lower bound for the BDM velocity as \( v_{i,\text{min}}^{\text{ion}} = \sqrt{2E_{R,\text{min}}^{\text{ion}}/m_\chi} \), with \( E_{R,\text{min}}^{\text{ion}} \equiv 10 \text{ eV} \) is set in this study. Thus the threshold velocity in our simulation is chosen to be the maximum value between the DM virial velocity, i.e. \( 10^{-3} c \), and \( v_{i,\text{min}}^{\text{ion}} \). Second, the BDM may reach the surface \( r = R_D \) before its velocity becomes smaller than the threshold velocity, in which case it is no longer relevant. Under both conditions, the simulation of the event will be stopped automatically.
D.3 Reconstructing the BDM Flux Distribution

After the simulation, we define the “hit events” as the ones which reach the surface $r = R_D$. We collect the velocity and position of each event. If the BDM-electron interaction is strong, the BDM loses its energy rapidly in the Earth, and the BDM particles can only penetrate the $r = R_D$ sphere at most once, i.e. when they just enter the Earth. On the other hand, when the interaction is weak, most of the BDM particles travel across the $r = R_D$ sphere twice, this leads to a doubling of the number of events, to $2N$. In this subsection, we explain how to convert the distribution of “hit events” to the BDM velocity distribution that can be used to calculate the event rate in a detector.

In Fig. 11, we show an example of the “hit event” distribution projected to the $x − y$ plane in both the near side ($z < 0$) and the far side ($z ≥ 0$) of the Earth. On the near side, the events are almost equally distributed on the $x − y$ plane, which is consistent with the initial setup in Sec. (D.1). However on the far side, the events are more densely distributed near $\sqrt{x^2 + y^2} \approx R_E$ where $R_E$ is the radius of the Earth.

For the parameter space we are interested in, the transverse component of the BDM velocity is much smaller than the one along the $z$–axis after the propagation, thus the event rate in a detector can be approximately calculated using the modified BDM flux along the $z$–axis. With a proper normalization, the “hit event” rate per area on the $r = R_D$ sphere is simply related to the modified BDM flux by a factor of $1/\cos \theta$.

D.3.1 Reconstruction of the General Velocity Distribution

In more general cases, the BDM can reach the detector from all directions. In this section, we study the conversion from the “hit event” rate per area to the general velocity distribution.

The number density of BDM particles with velocities within $d^3\vec{v}_\chi$ is

$$n_\chi f(\vec{v}_\chi) d^3\vec{v}_\chi,$$

where

$$f(\vec{v}_\chi) = \frac{1}{(2\pi)^3} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}},$$

with $v^2$ the magnitude of the velocity $\vec{v}_\chi$. The factor $1/(2\pi)^3$ normalizes the distribution such that $\int f(\vec{v}_\chi) d^3\vec{v}_\chi = 1$.
where \( n_\chi \) is the BDM number density and \( f(\vec{v}_\chi) \) is the 3-velocity distribution. For an infinitesimal area element \( d\vec{s} \), the rate of particles hitting this area with velocities within \( d^3\vec{v}_\chi \) is

\[
\frac{dh(\vec{v}_\chi)}{d^3\vec{v}_\chi \, dt} = n_\chi \, f(\vec{v}_\chi) \, (\vec{v}_\chi \cdot d\vec{s}).
\]  
(D.6)

This can be used to calculate the differential BDM event rate at a detector,

\[
\frac{dR}{dE_R} = N_d \int d^3\vec{v}_\chi \, n_\chi \, f(\vec{v}_\chi) \, |\vec{v}_\chi| \, \frac{d\sigma_{\text{ion}}}{dE_R}
\]  
(D.7)

where \( N_d \) is the number of target atoms in the detector.

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