String Unification and Leptophobic $Z'$ in Flipped SU(5)

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We summarize recent developments in the prediction for $\alpha_s(M_Z)$, self-consistent string unification and the dynamical determination of mass scales, and leptophobic $Z'$ gauge bosons in the context of stringy flipped SU(5).

1. An oldie but goodie

Flipped SU(5) enthusiasts keep discovering hidden treasures, even after 10 years from its birth [1]. As is well known, the model attains its highest relevance in strings: efforts by several groups using different approaches have not (yet?) yielded appealing “string GUTs” [SO(10)]. Among level-one Kac-Moody models, only flipped SU(5) unifies SU(3) and SU(2), providing an explanation for the “LEP scale” [10^16 GeV]. The discrepancy between “observed” and predicted unification scales – $M_{\text{LEP}} \sim 10^{16}$ GeV versus $M_{\text{string}} \sim 10^{18}$ GeV – seems to have only way out: extra intermediate-scale states [2]. This solution was realized early on in stringy flipped SU(5) [3]. Here we summarize how this scenario may be achieved in practice [4], including the prediction for $\alpha_s(M_Z)$ [5], and also discuss the latest “flipped” goodie: a leptophobic $Z'$ [6].

2. Some basics first

Matter fields:

$F(10) = \{Q, d^c, \nu^c\}; \bar{f}(5) = \{L, u^c\}; \bar{l}^c(1) = e^c (\times 3)$

Higgs fields:

$H(10) = \{Q_H, d^c_H, \nu^c_H\}; \bar{H}^c(\bar{10}) = \{\bar{Q}_H, \bar{d}^c_H, \bar{\nu}^c_H\}$

$h(5) = \{H_2, H_3\}; \bar{h}(\bar{5}) = \{\bar{H}_2, \bar{H}_3\}$

GUT superpotential:

$W_G = H \cdot H \cdot h + \bar{H} \cdot \bar{H} \cdot \bar{h} + F \cdot \bar{F} \cdot \phi + \mu \bar{h} h$

The vevs $\langle \nu^c_H \rangle = \langle \bar{\nu}^c_H \rangle = M_U$ break $SU(5) \times U(1)$ down to $SU(3) \times SU(2) \times U(1)$. 

Double-triplet splitting:

$h = \left( \begin{array}{c} h_2 \\ h_3 \end{array} \right)$

mediates proton decay

$H \cdot H \cdot h \rightarrow d^c_H \langle \nu^c_H \rangle H_3$

$\bar{H} \cdot \bar{H} \cdot \bar{h} \rightarrow \bar{d}^c_H \langle \bar{\nu}^c_H \rangle \bar{H}_3$

The triplets get heavy, while the doublets remain light (“missing partner mechanism”).

Yukawa superpotential:

$\lambda_{d} F \cdot F \cdot h + \lambda_{u} F \cdot \bar{f} \cdot \bar{h} + \lambda_{e} \bar{f} \cdot \bar{l}^c \cdot h$

Neutrino masses: The GUT couplings $F \cdot \bar{f} \cdot h \rightarrow m_{\nu} \nu^c, F \cdot \bar{H} \cdot \phi \rightarrow \langle \nu^c \rangle \nu^c \phi$ entail

$M_{\nu} = \begin{pmatrix} \nu & \nu^c & \phi \\ m_u & 0 & m_U \\ 0 & M_U & M \end{pmatrix}$

See-saw mechanism: $m_{\nu_{e,\mu,\tau}} \sim m_{\nu_{e,\mu,\tau}}^2 / (M^2_H / M)$

Good for MSW mechanism, $\nu_\tau$ hot dark matter, and ($\nu^c$) baryogenesis.

Dimension-six proton decay: mediated by $X, Y$

GUT gauge bosons, the mode $p \rightarrow e^+ \pi^0$ may be observable at SuperKamiokande.

Dimension-five proton decay: very suppressed since no mixing exists, even though $H_3, \bar{H}_3$ are heavy via doublet-triplet splitting.

$\lambda_{d} F \cdot F \cdot h \supset QQH_3 \quad \lambda_{u} F \cdot \bar{f} \cdot \bar{h} \supset QL \bar{H}_3$
3. Prediction for $\alpha_s(M_Z)$

Starting from the low-energy Standard Model gauge couplings, and evolving them from low to high energies, first $\alpha_2$ and $\alpha_3$ unify at $M_{32}$.

\begin{align*}
\frac{1}{\alpha_2} - \frac{1}{\alpha_5} &= \frac{b_2}{2\pi} \ln \frac{M_{32}}{M_Z} \\
\frac{1}{\alpha_3} - \frac{1}{\alpha_5} &= \frac{b_3}{2\pi} \ln \frac{M_{32}}{M_Z}
\end{align*}

The hypercharge does not unify at $M_{32}$:

\begin{align*}
\frac{1}{\alpha_Y} - \frac{1}{\alpha_1'} &= \frac{b_Y}{2\pi} \ln \frac{M_{32}^{\max}}{M_Z}
\end{align*}

Above $M_{32}$ the gauge group is SU(5)$\times$U(1). Stringy unification occurs at $M_{31} \geq M_{32}$ – there is an $M_{32}^{\max}$

\begin{align*}
\frac{1}{\alpha_Y} - \frac{1}{\alpha_1'} &= \frac{b_Y}{2\pi} \ln \frac{M_{32}^{\max}}{M_Z}
\end{align*}

Solving for $\alpha_3$, to lowest order:

\[\alpha_s(M_Z) = \frac{\pi \alpha}{5 \sin^2 \theta_W - 1} \left( \frac{1}{\alpha} \ln \frac{M_{32}^{\max}}{M_{32}} \right)\]

compare with SU(5) where $M_{32} = M_{32}^{\max}$

$\alpha_s(M_Z)^{\text{Flipped SU(5)}} < \alpha_s(M_Z)^{\text{SU(5)}}$

What happens at next-to-leading order?

$\sin^2 \theta_W \to \sin^2 \theta_W - \delta_{\text{2loop}} - \delta_{\text{light}} - \delta_{\text{heavy}}$

Decreasing $\sin^2 \theta_W$ increases $\alpha_s(M_Z)$ [avoid!]:

$\delta_{\text{2loop}} \approx 0.0030$; $\delta_{\text{light}} \gtrsim 0$ (light SUSY thresholds); $\delta_{\text{heavy}}$ (GUT thresholds)

\[\delta_{\text{heavy}} = \frac{\alpha}{20\pi} \left[ -6 \ln \frac{M_{32}}{M_{H_3}} - 6 \ln \frac{M_{32}}{M_{H_3}} + 4 \ln \frac{M_{32}}{M_V} \right]\]

Since there is no problem with proton decay, $\delta_{\text{heavy}}$ can be negative. We obtain $\alpha_s(M_Z)$ as low as 0.108 (see Fig. 1). However, decreasing $M_{32}$ decreases the proton lifetime

\[\tau(p \to e^+\pi^0) \approx 1.5 \times 10^{33} \left( \frac{M_{32}}{10^{15}\text{GeV}} \right)^4 \left( \frac{0.042}{\alpha_5} \right)^2 \text{y}\]

The present lower bound $\tau(p \to e^+\pi^0)^{\exp} > 5.5 \times 10^{32}$ y implies $\alpha_s(M_Z) > 0.108$ (see Fig. 2). If $\alpha_s(M_Z) < 0.114$ then $p \to e^+\pi^0$ may be observable at SuperKamiokande (which should have a sensitivity of $\sim 10^{32}$ y). This is in contrast with minimal SU(5), where the preferred mode is $p \to \bar{\nu}K^+$. 

4. Stringy Flipped SU(5)

String construction in fermionic formulation

Gauge group: $G = G_{\text{observable}} \times G_{\text{hidden}} \times G_{U(1)}$

$G_{\text{observable}} = \text{SU(5)} \times \text{U(1)}$;

$G_{\text{hidden}} = \text{SO(10)} \times \text{SU(4)}$; $G_{U(1)} = \text{U(1)}^5$

Particle spectrum

Observable Sector:

$F^{10,1,2,3,4} [10]$; $f^{2,3,5} [5]$; $f^{10} [4,5]$; $F^{(4,5) [10]}$

$\tilde{h}^{(1,2,3,45) [5]}$; $\tilde{h}^{(1,2,3,45) [5]}$

Singlets: 20 charged under $U(1)$’s, 4 neutral
Hidden Sector:
\[ T^{(1,2,3)} \] of SO(10)
\[ D^{(1,2,3,4,5,6,7)} \] of SU(4)
\[ F^{(1,2,3,4,5,6)} \] of SU(4)
\[ \overline{F}^{(1,2,3,4,5,6)} \] of SU(4)

The \( \overline{F}_i, \overline{F}_j \) fields carry \( \pm 1/2 \) electric charges and exist only confined in hadron-like cryptons.

The cubic and non-renormalizable terms in the superpotential have been calculated \[4\], and more recently also the Kähler potential \[7\]. The properties of the Kähler potential illuminate the vacuum energy (which vanishes at tree level and possibly also at one loop) and determine the pattern of soft-supersymmetry-breaking masses, which has distinct experimental consequences \[8\].

5. String unification

Assume that

\[ SU(5) \times U(1) \to SU(3) \times SU(2) \times U(1) \]

breaks as in Standard Flipped SU(5) case. Cancellation of \( U_A(1) \) is consistent with

\[ M_{\text{lep}} \sim \langle \nu^c_H \rangle \sim 10^{15-16} \text{GeV} \]

Correct \( \sin^2 \theta_W \) and \( \alpha_3 \) obtained because of extra \( 10, \overline{10} \) present in string massless spectrum. String unification occurs at \( M_{\text{string}} \sim 10^{18} \text{GeV} \). This requires \( M_{10} \sim 10^{8-9} \text{GeV} \), which can be generated via VEVs of hidden matter fields.

Dynamical Determination of Scales \[4\]

\( 4, \overline{4} \) affect running of \( U(1) \) down to \( \Lambda_4 = M_{\text{string}} e^{8\pi^2/g^2\beta_4} \), where \( \beta_4 = -12 + \frac{1}{3} N_4 + N_6 \); \( \Lambda_4 \) depends on string spectrum of \( 4, \overline{4}, 6 \), and on actual decoupling of particles between \( M_{\text{string}} \) and \( \Lambda_4 \) [tricky]. Extra \( 10, \overline{10} \) affect running of \( SU(5) \times U(1) \) down to \( M_{10} \). Naively, if \( M_{4, \overline{4}} \sim \Lambda_4 \), a non-renormalizable term \( (10)(\overline{10})(4)(\overline{4}) \) implies \( M_{10} \sim \langle 4 \rangle / M \sim \Lambda_4^2 / M \). But in strings \( M_{4, \overline{4}} \sim \Lambda_4 \ll M \) is very unlikely; \( M_{4, \overline{4}} = 0 \) is more natural. In the actual string model we have a quintic term [and \( M_{4, \overline{4}} = 0 \]: \( (10)(\overline{10})(4)(\overline{4}) \phi \), where the cancellation of \( U_A(1) \) implies \( \langle \phi \rangle / M \sim 1/10 \).

With massless flavors \( (M_{4, \overline{4}} = 0) \) one expects \( \langle 4 \rangle \sim \infty \). Aharony, et. al. studied SU(\( N_c \)) with \( N_f \) “massless” flavors with supersymmetry-breaking scalar masses \[4\]. Supersymmetry-breaking masses \( (\tilde{m}) \) yield finite condensates

\[ \langle HH \rangle \sim \left[ \frac{N_c}{N_c - N_f} \right]^{(N_c - N_f)/2(N_c - N_f)} \]

In our case \( (N_c = 4, N_f = 2) \) we obtain

\[ \langle 4 \rangle \sim \Lambda_4^2 \left( \frac{\tilde{m}}{\Lambda_4} \right)^{-1/3} \approx \Lambda_4^2 \]

and we can calculate \( M_{10} \) from first principles

\[ M_{10} \sim \left( \frac{\Lambda_4}{M} \right)^2 \left( \frac{\tilde{m}}{\Lambda_4} \right)^{-1/3} \left( \frac{\phi}{M} \right) M \sim 10^{8-10} \text{GeV} \]

This result allows self-consistent string unification. The results for the various scales as a function of \( \alpha_s(M_Z) \) are shown in Fig. 3. The full evolution of the gauge couplings from the weak scale to the string scale is shown in Fig. 4 for the preferred choices of \( \alpha_s(M_Z) = 0.116 \) and \( N_4 = 2 \).
6. Leptophobic Z’

Motivation: Original “smoking gun” of string: \( R_{b}, R_{c} \) ‘crisis’ has revived interest in \( Z' \) models, although this time the \( Z' \) must not couple to leptons. Leptophobia is natural in flipped SU(5) \( [6] \)

\[
10 = \{Q, d^c, u^c\}; \quad 5 = \{L, u^c\}; \quad 1 = e^c
\]

If the leptons are uncharged, most quarks may be charged under \( U' \). Compare with regular SU(5) \( 10 = \{Q, u^c, e^c\}; \quad 5 = \{L, d^c\} \), where uncharged leptons imply uncharged quarks. Dynamic leptophobia (via RGE U(1) mixing) is also possible, as in the \( \eta \)-model in Ref. \( [5] \).

Any \( Z-Z' \) mixing shifts the usual \( Z \) couplings (\( C_{V,A}^{0} \)): \( C_{V} = C_{V}^{0} + \theta(g_{Z'}/g_{Z})C_{V}', \quad C_{A} = C_{A}^{0} + \theta(g_{Z'}/g_{Z})C_{A}' \), where \( \theta \) is the \( Z-Z' \) mixing angle (small); \( g_{Z}, g_{Z'} \) are the \( Z, Z' \) gauge couplings; and \( C_{V,A}' \) the fermion couplings to the \( Z' \). In flipped SU(5) we have \[
\begin{pmatrix} C_{V}^{0} & C_{A}^{0} & Q_{L} & Q_{R} & C_{V}' & C_{A}' \\
\end{pmatrix}
\]

\[
\begin{array}{cccc}
u & \frac{1}{2} - \frac{1}{3} \xi_{w} & \frac{1}{2} & c & 0 & c & -c \\
d & -\frac{1}{2} + \frac{1}{3} \xi_{w} & -\frac{1}{2} & c & c & 2c & 0
\end{array}
\]

We can determine the first-order shifts in \( \Gamma_{cc}, \Gamma_{bb}, \) and \( \Gamma_{\text{had}} \), allowing for non-universal \( c_{1,2,3} \) charges picked from

\[
F_{0} = -\frac{1}{2}, \quad F_{4} = \frac{1}{2} \quad f_{2,3,5} = 0
\]

\[
F_{1} = -\frac{1}{2}, \quad F_{3} = 0, \quad f_{2,3,5} = 0
\]

\[
F_{2} = 0, \quad F_{3} = 1
\]

\[
F_{4} = -\frac{1}{2}
\]

This \( U' \) charge space satisfies specific requirements: The leptons (in \( f_{2,3,5}, f_{2,3,5}' \)) are uncharged. Uncharged \( (10, \overline{10}) \) pair \( (F_{2}, F_{3}) \) so that \( U' \) remains unbroken upon SU(5)×U(1) breaking; Tr\( U' = 0 \) enforced; extra \( (10, \overline{10}) \) to allow string unification. The actual string model underlies these choices.

There are 13 possible charge assignments that can be made. Phenomenology demands \( \Delta \Gamma_{\text{had}} \lesssim 3 \text{ MeV} \), as the SM prediction and LEP agree well. Since \( R_{b}^{SM} = 0.2157 \) and \( R_{b}^{exp} = 0.2202 \pm 0.0016 \) (\( R_{c} \) fixed to SM value), we demand \( \Delta R_{b} = 0.0030 - 0.0060 \). Fig. \( [5] \) shows \( \Delta R_{b} \) versus \( \Delta \Gamma_{\text{had}} \). An analogous plot for \( \Delta R_{c} \) versus \( \Delta R_{b} \), demanding \( \Delta R_{c}, \Delta R_{b} \) shifts in opposite directions can be found in Ref. \( [5] \). We should keep in mind that experimentally there appears to be a trend of \( R_{c} \) converging to the Standard Model prediction and \( R_{b} \) approaching it significantly.

Figure 5. Correlated shifts in \( R_{b} \) and \( \Gamma_{\text{had}} \) for the various \( U' \) charge assignment combinations. Dashed lines delimit the experimental limits on \( \Delta \Gamma_{\text{had}} \) and \( \Delta R_{b} \). Circled charge assignments \( (2,5,10,11,12) \) agree with experiment.
6.1. String scenario
Consider $G_{U(1)} = U_1 \times U_2 \times U_3 \times U_4 \times U_5$, with $\text{Tr} \ U_1 = 0$, $\text{Tr} \ U_{1,2,3,5} \neq 0$. The anomalous combination is $U_A = U_1 - 3U_2 + U_3 + 2U_5$, with three orthogonal traceless combinations: $U'_1 = U_3 + 2U_5$; $U'_2 = U_1 - 3U_2$; $U'_3 = 3U_1 + U_2 + 4U_3 - 2U_5$. The lepton charges under $U'_1$, $U'_2$, $U'_3$ are $\tilde{f}_{2,5}, \ell_{2,5}^c : (0, \frac{1}{2}, \frac{1}{2})$; $\tilde{f}_3, \ell_3^c : (\frac{3}{2}, 0, 1)$.

There is a unique $U'$ that is leptophobic

$U' \propto 2U'_1 - U'_2 - 3U'_3 \propto U_1 + U_3 - U_5$

and by construction $\text{Tr} \ U' = 0$. Higgs fields charged under $U'$ exist ($Z\cdot Z'$ mixing). The D- and F-flatness conditions may be satisfied, leaving $U'$ unbroken, but breaking the hidden group.

Model building: $F_1$ should contain 3rd generation (top Yukawa); $F_2, F_3$ neutral under $U'$; symmetry breaking: $F_4$: string unification; $R_b, R_c$ inputs: four charge assignments allowed

|   | $c_1$ | $c_2$ | $c_3$ |
|---|---|---|---|
| (2) | 0 | $-\frac{1}{2}$ | 1 |
| (5) | $-\frac{1}{3}$ | 0 | 1 |
| (11) | $-\frac{1}{2}$ | 1 | $-\frac{1}{2}$ |
| (12) | $-\frac{1}{2}$ | $-\frac{1}{2}$ | 1 |

Unlike any considered before. Unnatural? Obtained from string! Top-quark Yukawa coupling, and $R_b, R_c$ select scenario (11) uniquely

$\Delta R_b \approx 0.0042 \left( \frac{\Delta \Gamma_{\text{had}}}{-3 \text{MeV}} \right)$, \hspace{1em} $\Delta R_c \approx -0.76 \Delta R_b$.

Dynamics: Running of $U'$ from $M_Z$ up looks good: $\beta' = \frac{16}{7}$ (c.f. $b_Y = \frac{33}{8}$). Sufficiently small $Z\cdot Z'$ mixing appears to require radiative $U'$ symmetry breaking via singlet $\phi$.

6.2. Experimental prospects
$Z'$ width and branching ratios for preferred case:

$\frac{\Gamma_{Z'}}{M_{Z'}} \approx 0.033 \left( \frac{g_{Z'}}{g_Z} \right)^2$ [narrow]

Experimental limits:

$$\frac{\sigma(u\bar{u} \rightarrow Z')}{\sigma(u\bar{u} \rightarrow Z')_{SM}} \approx 0.58 \left( \frac{g_{Z'}}{g_Z} \right)^2$$

$$\frac{\sigma(d\bar{d} \rightarrow Z')}{\sigma(d\bar{d} \rightarrow Z')_{SM}} \approx 0.90 \left( \frac{g_{Z'}}{g_Z} \right)^2$$

Average up/down; multiply by \frac{B(Z' \rightarrow jj)}{B(Z' \rightarrow jj)_{SM}} \approx 1.4,

$$\sigma(p\bar{p} \rightarrow Z' \rightarrow jj) \approx \left( \frac{g_{Z'}}{g_Z} \right)^2 \sigma(p\bar{p} \rightarrow Z' \rightarrow jj)_{SM}$$

Only limit from UA2: $M_{Z'} > 260$ GeV, but only if $g_{Z'} = g_Z$.

$Z'$ contributes to top-quark cross section (see Fig. 3 in Ref. [1]) at a level that may be observable if $M_{Z'} \sim 500$ GeV. Parity-violating spin asymmetries at RHIC may also show deviations from Standard Model expectations because of the $t$-channel exchange of our parity-violating $Z'$.

In sum, flipped SU(5) continues to provide unsolicited solutions to unanticipated problems, as evidenced most recently by the self-consistent string unification and the possible existence of a leptophobic $Z'$ gauge boson.

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