ππ SCATTERING AT LOW ENERGY: STATUS REPORT

JÜRGGASSER
Institute of Theoretical Physics,
University of Bern, Sidlerstrasse 5, 3012 Bern, Switzerland

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ABSTRACT

I discuss the status of ongoing work to determine numerically the elastic ππ scattering amplitude at order $p^6$ in the framework of chiral perturbation theory.

1. Introduction

The interplay between theoretical and experimental aspects of elastic ππ scattering is illustrated in figure 1. On the theoretical side, Weinberg’s calculation of the scattering amplitude at leading order in the low–energy expansion gives for the isospin zero $S$–wave scattering length the value $a_{l=0}^{I=0} = 0.16$ in units of the charged pion mass. This differs from the experimentally determined value $a_{0}^0 = 0.26 \pm 0.05$ by two standard deviations. The one–loop calculation enhances the leading order term to $a_{0}^0 = 0.20 \pm 0.01$ – the correction goes in the right direction, but the result is still on the low side as far as the present experimental value is concerned. To decide about agreement/disagreement between theory and experiment, one should i) evaluate the scattering lengths in the theoretical framework at order $p^6$, and ii) determine them more precisely experimentally. My talk was concerned with the the former issue, indicated by the double arrow in the left column of the figure.

On the experimental side, several attempts are underway to improve our knowledge of the threshold parameters. The most promising ones among them are semileptonic $K_{14}$ decays with improved statistics (E865 and KLOE), and the measurement of the pionium lifetime (DIRAC) that will allow one to directly determine the combination $|a_{0}^0 - a_{2}^0|$ of $S$-wave scattering lengths. It was one of the aims of the workshop to discuss the precise relation between the lifetime of the pionium atom and the $ππ$ scattering lengths – I refer the interested reader to the numerous contributions to this workshop for details. In addition, Počanic has presented a nice review of the determination of scattering lengths from hadronic processes.

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As for theory, I will describe in the following sections our current effort to determine the scattering amplitude numerically at order $p^6$ in the framework of chiral perturbation theory (CHPT). As the title of my talk indicates, this work is still in progress—therefore, I will give an overview of what is going, omitting any details. For related work, see Refs. 11, 12, 13, 14.

| Theory | Experiment |
|--------|------------|
| $A = \frac{s-M_\pi^2}{F_\pi^2} + O(p^4)$ | $K \rightarrow \pi\pi\nu$ (30 000 decays) |
| $a_0^0 = 0.16$ | $a_0^0 = 0.26 \pm 0.05$ |
| + $O(p^4)$ | DIRAC $\parallel$ E865; KLOE |
| $a_0^0 = 0.20 \pm 0.01$ | |
| + $O(p^6)$ | |

Figure 1: Progress in the determination of the elastic $\pi\pi$ scattering amplitude. References are provided in the text.

2. Chiral perturbation theory

To set notation, I briefly describe the framework used to evaluate the $\pi\pi$ scattering amplitude. I consider the isospin symmetry limit where the two lightest quark masses are equal, $m_u = m_d \neq 0$, and where the electromagnetic coupling is set to zero, $\alpha_{QED} = 0$. I refer to Müller's contribution for a discussion of isospin violating effects in hadronic amplitudes. The effective lagrangian that describes the interaction of pions is given by a string of terms, $L_{\text{eff}} = L_2 + hL_4 + h^2L_6 + \cdots$, where $L_n$ contains $m_1$ derivatives of the pion fields and $m_2$ quark mass matrices, with $m_1 + 2m_2 = n$ (here, I consider the standard counting rules—see section 7 for a generalization thereof). The low–energy expansion corresponds to an expansion of the scattering matrix elements in powers of $h$.

In the following, the low–energy constants (LEC’s) hidden in the effective lagrangians $L_n$ play an important role. In $L_2$, there are two of them, the pion decay constant in the chiral limit ($F$) and the parameter $B$, which are related to the condensate by $F^2B = -\langle 0|\bar{u}u|0 \rangle$. In the loop expansion, these two parameters can be expressed in terms of the physical pion decay constant $F_\pi \simeq 92.4$ MeV and of the pion mass, $M_\pi = 139.57$ MeV. The $\pi\pi$ scattering amplitude contains, in the two–loop approximation, in addition
several LEC’s occurring in $\mathcal{L}_4$ and in $\mathcal{L}_6$,

$$
\begin{align*}
\mathcal{L}_2 & : F_\pi, M_\pi \\
\mathcal{L}_4 & : \bar{l}_1, \bar{l}_2, \bar{l}_3, \bar{l}_4 \\
\mathcal{L}_6 & : \bar{r}_1, \ldots, \bar{r}_6
\end{align*}
$$

occur in $\pi\pi \to \pi\pi$ (two–loop approximation). \hspace{1cm} (1)

These LEC’s are not determined by chiral symmetry – they are, however, in principle calculable in QCD\textsuperscript{19}.

3. $\pi\pi \to \pi\pi$ in CHPT

Elastic $\pi\pi$ scattering is described by a single Lorentz invariant amplitude $A(s, t, u)$, that depends on the standard Mandelstam variables $s, t, u$ (I use the same notation as Sainio\textsuperscript{20} – I refer the reader to his contribution for details). Here, we are concerned with the loop expansion of the scattering amplitude,

$$
A(s, t, u) = \begin{cases} 
A_{2} & + A_{4} & + A_{6} & + O(p^8) \,, \end{cases}
$$

where $A_{n}$ is of order $p^n$. In the following, I denote with the symbol $A^\chi$ the first three terms in (2),

$$
A^\chi = A_{2} + A_{4} + A_{6} \,.
$$

A dispersive evaluation of $A^\chi$ has been performed in Ref.\textsuperscript{12}. That calculation is not sufficient for the present purpose, because several tadpole diagrams that occur in the loop expansion have not been evaluated in that work. What is needed for the analysis outlined below is the complete two–loop expression of $A^\chi$. The relevant calculation has been performed in Ref.\textsuperscript{21}. The explicit form of the amplitude is not needed here, and I refer the interested reader to Sainio’s contribution\textsuperscript{20} for explicit formulae.

For the following discussion, it is useful to bring the amplitude into a canonical form. First, one makes use of the fact that $A^\chi$ can be expressed\textsuperscript{22} in terms of three functions $W_{0,1,2}$, that depend on a single kinematical variable ($s, t$ or $u$) and have only a right hand cut. Their imaginary part is given by the imaginary parts of the three lowest partial waves $t_{1}^{I}$, e.g., $\text{Im}W_{0} = \text{Im}t_{0}^{0}$, etc. We now write a four times subtracted dispersion relation for $W_{0}$,

$$
W_{0} = a_{0} + a_{1}s + a_{2}s^{2} + a_{3}s^{3} + \frac{s^{4}}{\pi} \int_{4M_{\pi}^{2}}^{\infty} \frac{dx}{x^{4}} \frac{\text{Im}t_{0}^{0}(x)}{x - s} \,.
$$

and similarly for $W_{1,2}$. Inserting these representations into the expression for $A^\chi$ gives

$$
\begin{align*}
A^\chi & = P^\chi + A^\chi_{\text{int}} \\
P^\chi & = \beta_{1}^{\chi} + \beta_{2}^{\chi}s + \beta_{3}^{\chi}s^{2} + \beta_{4}^{\chi}(t - u)^{2} + \beta_{5}^{\chi}s^{3} + \beta_{6}^{\chi}s(t - u)^{2} \,.
\end{align*}
$$

The coefficients $\beta_{i}^{\chi}$ contain the low–energy constants (4),

$$
\beta_{n}^{\chi} = \beta_{n}^{\chi}(\bar{l}_{1}, \ldots, \bar{l}_{4}; \bar{r}_{1}, \ldots, \bar{r}_{6}) \,.
$$

(6)
Here and in the following, I drop all dependence on $F_\pi$ and $M_\pi$, whose numerical value is taken from experiment. It is the aim of the enterprise to work out the consequences of the requirement that $A^\chi$ is a good approximation of the true amplitude at low energy. This requirement allows one in particular to work out the two $S$–wave scattering lengths $a_0^0$ and $a_0^2$, provided that we can determine the LEC’s occurring in the chiral amplitude.

4. Effective couplings

The chiral amplitude $A^\chi$ contains two sets of LEC’s:

set 1 In this set we put the four LEC’s

$$\bar{l}_1, \bar{l}_2, \bar{r}_5, \bar{r}_6$$

that are related to the momentum and scattering angle dependence of the amplitude: the polynomial $P^\chi$ contains terms of the form $\bar{l}_1 s(t-u)^2, \bar{l}_2 (t-u)^2, \bar{r}_5 s^3, \bar{r}_6 s(t-u)^2$. As a result of this, these couplings can be determined from data above $\approx 800$ MeV, see below.

set 2 This set assembles the LEC’s whose effect disappears in the chiral limit, because they are multiplied with the square of the pion mass. Examples are the terms $M_\pi^6 \bar{r}_1, M_\pi^4 s \bar{r}_2$ in $P^\chi$. As nature does not allow us to vary the pion mass in the laboratory, we need input from outside $\pi\pi$ scattering to determine these couplings. I denote these with the symbol $\Theta$,

$$\Theta : \bar{l}_{3,4}; \bar{r}_{1,2,3,4} .$$

Using large–$N_c$ arguments, the constant $\bar{l}_3$ can be obtained from the chiral expansion of the ratio $M_K^2/M_\pi^2$ [17], whereas $\bar{l}_4$ is related to the scalar radius of the pion $\bar{l}_4$ [17, 23]. Finally, the effect of $\bar{r}_{1,2,3,4}$ is suppressed by powers of the pion mass – a rough estimate like the one provided in Ref. [21] (based on resonance saturation) thus suffices.

5. Matching

From the above discussion, we find that the coefficients $\beta_n^\chi$ in the polynomial $P^\chi$ of the chiral amplitude (5) are of the functional form

$$\beta_n^\chi = \beta_n^\chi(\bar{l}_1, \bar{l}_2, \bar{r}_5, \bar{r}_6; \Theta) .$$

In the following, I consider the couplings $\Theta$ as given.

Turning now to experiment, I write the corresponding amplitude $A^{exp}$ as

$$A^{exp} = P^{exp} + A^{exp}_{int}$$

where $P^{exp}$ has the same functional form as $P^\chi$, with $\beta_n^\chi \rightarrow \beta_n^{exp}$. As we will see in the following section, the Roy equations [24] fix these coefficients in terms of data above $\approx 800$ MeV and of the scattering lengths $a_0^0, a_0^2$,

$$P^{exp} \leftrightarrow \beta_n^{exp}(a_0^0, a_0^2) .$$
Furthermore, the amplitude $A_{int}^{\chi}$ coincides with $A_{int}^{\chi}$ up to terms of order $p^8$. We now require that the chiral amplitude agrees with the experimental one near threshold, as a result of which one obtains the relation

$$P^\chi(s, t, u) = P^{exp}(s, t, u) ,$$

matching condition (12).

In terms of the coefficients $\beta_n$, this matching condition amounts to the six relations

$$\beta_n^\chi(\bar{l}_1, \bar{l}_2, \bar{r}_5, \bar{r}_6; \Theta) = \beta_n^{exp}(a_0^0, a_0^2) ; \ n = 1, \ldots, 6$$

for six unknowns. Solving for these, we can determine the quantities

$$a_0^0, a_0^2; \bar{l}_1, \bar{l}_2, \bar{r}_5, \bar{r}_6 .$$

The remaining threshold parameters may be obtained from the Wanders sum rules).

6. Roy equations

It remains to show that indeed the polynomial coefficients $\beta_n^{exp}$ are determined in terms of the two $S$–wave scattering lengths $a_0^0, a_0^2$ and of data above 800 MeV.

For this purpose, one consider the partial waves $t_l^I$. As shown by Roy, their real and imaginary parts are related through

$$\text{Re} t_l^I = c_l^I(s) + \sum_{I^\prime=0}^{2} \sum_{l^\prime=0}^{\infty} \int_{4M^2_\pi}^{\infty} dx K_{II^\prime}^{lI^\prime}(s, x) \text{Im} t_{l^\prime}^I(x) ,$$

in the region $-28M^2_\pi < s < 60M^2_\pi$. Here, the quantities $K_{II^\prime}^{lI^\prime}$ are known kernels, whereas the subtraction polynomial $c_l^I$ contains the two scattering lengths $a_0^0, a_0^2$ as the only free parameters. Note that the relation (15) is linear in the partial wave amplitudes – it is therefore also true in ordinary perturbation theory, order by order in the coupling constant (modulo subtractions). Next, we observe that, at low energy, the most important contribution to the imaginary parts stems from the $S$– and $P$–waves, because the higher waves are suppressed, $\text{Im} t_l^I = O(p^8) ; \ l = 2, 3, \ldots$. For this reason, it is useful to perform the splitting

$$A^{exp} = A_{SP} + A_R ,$$

where the crossing symmetric amplitude $A_{SP}$ contains the exact $S$– and $P$–wave absorptive parts, whereas $A_R$ has only absorptive parts from $l = 2, 3, \ldots$ waves. Therefore, at low energies, the latter contribution can be approximated by a polynomial in $s, t$ and $u$. This allows one finally to write $A^{exp}$ in the form (14), where $P^{exp}$ is determined from $A_R$ and through the imaginary parts of the three lowest partial waves. I now show how one may determine these imaginary parts from experimental information above 800 MeV.

Using (14) and expanding $A_R$ as described, the partial wave relations (15) become for the $S$– and $P$– waves

$$\text{Re} t_m = c_m + \sum_{n=0}^{2} \int_{4M^2_\pi}^{\infty} dx K_{mn}(s, x) \text{Im} t_n(x) + d_m(s) ; \ m = 0, 1, 2$$

in the region $-28M^2_\pi < s < 60M^2_\pi$. Here, the quantities $K_{mn}$ are known kernels, whereas the subtraction polynomial $c_m$ contains the two scattering lengths $a_0^0, a_0^2$ as the only free parameters. Note that the relation (15) is linear in the partial wave amplitudes – it is therefore also true in ordinary perturbation theory, order by order in the coupling constant (modulo subtractions). Next, we observe that, at low energy, the most important contribution to the imaginary parts stems from the $S$– and $P$–waves, because the higher waves are suppressed, $\text{Im} t_l^I = O(p^8) ; \ l = 2, 3, \ldots$. For this reason, it is useful to perform the splitting

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where
\[(t_0^0, t_1^0, t_2^0) \to (t_0, t_1, t_2) ; (c_0^0, c_1^0, c_2^0) \to (c_0, c_1, c_2) . \] (18)

The kernels are $K_{00} = K_{00}^0$, etc. Finally, the so-called driving terms $d_m$ are obtained from the amplitude $A_R$. The crucial step\(^\text{[24]}\) is to invoke unitarity at this stage,
\[ t_m = \frac{1}{2i} \frac{1}{(1 - 4M_\pi^2/s)^{1/2}} (\eta_m \exp 2i\delta_m - 1) . \] (19)

We now assume that
- the elasticities $\eta_m$ are known for all $s$
- the $\delta_m$ are given above a cutoff energy $E_{\text{cut}} \simeq 800$ MeV
- the driving terms are known below $E_{\text{cut}}$.

At given $a_0^0, a_0^2$, the equations (17) then become a system of nonlinear integral equations for the unknown $\delta_m$ in the interval $2M_\pi < \sqrt{s} < E_{\text{cut}}$. This system is known as 'Roy equations' in the literature. Solving these\(^\text{[1]}\), one obtains the $S, P$ partial waves at low energies and thus the polynomial $P^{\text{exp}}$, as promised.

7. Generalized chiral perturbation theory

The above framework assumes the standard scenario of chiral counting, where the condensate is the leading order parameter, considered a quantity of order one. In recent years, an alternative picture\(^\text{[28]}\) has been implemented\(^\text{[18]}\) in an effective lagrangian framework. In that picture, the condensate may be small or even vanishing. Many predictions are lost in this case, a prominent one being the Gell–Mann–Okubo mass relation, that does not have a natural explanation in this framework. Furthermore, the scattering length $a_0^0$ cannot be predicted either – it is a free parameter, related to the size of the condensate. The philosophy of this approach is to let experiment tell the size of $a_0^0, a_0^2$, which allows one then to determine the size of the condensate. I refer the interested reader to Stern’s talk in Mainz\(^\text{[29]}\) for further details and for references.

It is not easy to distinguish phenomenologically a small condensate from the standard case with the present precision of low–energy experiments\(^\text{[29]}\). It is, however, expected that new precise measurements of the $\pi\pi$ amplitude at low energy will shed more light on the issue.

On the theoretical side, a recent interesting investigation by Knecht and de Rafael\(^\text{[33]}\) has shown that, in vector–like gauge theories like QCD, and at large values of $N_c$, the ordering pattern of narrow vector and axial–vector states is correlated with the size of possible local order parameters of chiral symmetry breaking. The authors find that, from a duality point of view, the option of a vanishing condensate seems unnatural. I refer the reader to their work for more details. Recent lattice calculations\(^\text{[30, 31]}\) do not support a small condensate either. Indeed, Ecker\(^\text{[32]}\) has compared the results of Ref.\(^\text{[30]}\) on the quark mass dependence of the meson masses with the predictions of standard and generalized CHPT and concludes that “...lattice QCD is incompatible with a small quark condensate”. The authors of Ref.\(^\text{[31]}\) have made a considerable effort to pin down systematic uncertainties in a direct evaluation of $\langle 0|\bar{q}q|0 \rangle$ on the lattice in the quenched approximation. They come up with the standard value, with remarkably small error bars.

\(^{1)}\)For $E_{\text{cut}} \simeq 800$ MeV, the solution is unique\(^\text{[27]}\).
8. Summary

Status

• The analytic form of the two-loop amplitude in standard CHPT is known.
• We can numerically construct the solution of the system for given scattering lengths, input data and driving terms.
• The determination of the uncertainties in the final values of the parameters, in particular in the scattering lengths, is in progress.
• For related work, see Refs.

Outlook

• GCHPT provides a framework where the condensate may be small or even vanishing.
• In this scenario, the isospin zero $S$-wave scattering length $a_0$ can be large, in contrast to the standard case presented here, and in contrast to lattice calculations in the quenched approximation.
• We all hope that DIRAC, E865, KLOE and CHAOS will soon provide additional information on the issue from the experimental side.

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References

1. S. Weinberg, Phys. Rev. Lett. 17 (1966) 616.
2. L. Rosselet et al., Phys. Rev. D15 (1977) 574.
3. J. Gasser and H. Leutwyler, Phys. Lett. B125 (1983) 325.
4. Experiment E865 at Brookhaven AGS; S. Pislak, talk given at the $K_{l4}$ meeting in Bern, Switzerland, June 29–30, 1998.
5. M. Baillargeon and P.J. Franzini, in Ref., p. 413; J. Lee–Franzini, in Ref., p. 761; P.J. Franzini, in Ref., p. 117.
6. L. Maiani, G. Pancheri and N. Paver (eds.), The Second DAΦNE Physics Handbook (INFN, Frascati, 1995).
7. A.M. Bernstein and B.R. Holstein (eds.), Chiral Dynamics: Theory and Experiment, Proceedings of the Workshop held at MIT, Cambridge, MA, USA, 25–29 July 1994 (Springer, Berlin and Heidelberg, 1995).
8. B. Adeva et al., Proposal to the SPSLC: Lifetime measurement of $\pi^+\pi^-$ atoms to test low–energy QCD predictions, CERN/SPSLC/P 284, December 15, 1994; L. Nemenov, J. Schacher, these proceedings.

9. D. Počanić, these proceedings.

10. B. Ananthanarayan et al., work in progress.

11. M.R. Pennington and S.D. Protopopescu, *Phys. Rev.* D7 (1973) 1429; 2591; J.L. Basdevant, C.D. Froggatt and J.L. Petersen, *Nucl. Phys.* B72 (1974) 413; C.D. Froggatt and J.L. Petersen, *Nucl. Phys.* B91 (1975) 454; ibid. B104 (1976) 186 (E); ibid. B129 (1977) 89; O.O. Patarakin, V.N. Tikhonov and K.N. Mukhin, *Nucl. Phys.* A598 (1996) 335; B. Ananthanarayan and P. Büttiker, *Phys. Rev.* D54 (1996) 1125; 5501.

12. M. Knecht, B. Moussallam, J. Stern and N.H. Fuchs, *Nucl. Phys.* B457 (1995) 513; ibid. B471 (1996) 445.

13. G. Wanders, *Phys. Rev.* D56 (1997) 4328; *Helv. Phys. Acta* 70 (1997) 287.

14. B. Ananthanarayan, D. Toublan and G. Wanders, *Phys. Rev.* D51 (1995) 1093; ibid. D53 (1996) 2362; B. Ananthanarayan and P. Büttiker, *Phys. Lett.* B415 (1997) 402.

15. G. Müller, these proceedings.

16. S. Weinberg, *Physica* 96A (1979) 327.

17. J. Gasser and H. Leutwyler, *Ann. Phys. (N.Y.)* 158 (1984) 142; *Nucl. Phys.* B250 (1985) 465.

18. N.H. Fuchs, H. Sazdjian and J. Stern, *Phys. Lett.* B269 (1991) 183; J. Stern, H. Sazdjian and N.H. Fuchs, *Phys. Rev.* D47 (1993) 3814; M. Knecht and J. Stern, in [Ref. 8], p. 169, and references cited therein.

19. S. Myint and C. Rebbi, *Nucl. Phys.* B421 (1994) 2416; A.R. Levi, V. Lubicz and C. Rebbi, *Phys. Rev.* D56 (1997) 1101; *Nucl. Phys. Proc. Suppl.* 53 (1997) 275.

20. M.E. Sainio, these proceedings.

21. J. Bijnens et al., *Phys. Lett.* B374 (1996) 210; *Nucl. Phys.* B508 (1997) 263; ibid. B517 (1998) 639 (E).

22. J. Stern, H. Sazdjian and N.H. Fuchs, in Ref. 8.

23. J. Bijnens, G. Colangelo and P. Talavera, *J. High Energy Phys.* 05 (1998) 014 (hep-ph/9805389).

24. S.M. Roy, *Phys. Lett.* 36B (1971) 353; *Helv. Phys. Acta* 63 (1990) 627.

25. G. Wanders, *Helv. Phys. Acta* 39 (1966) 228.

26. G. Mahoux, S.M. Roy and G. Wanders, *Nucl. Phys.* B70 (1974) 297.

27. G. Wanders, to appear.

28. M.D. Scadron and H.F. Jones, *Phys. Rev.* D10 (1974) 967; H. Sazdjian and J. Stern, *Nucl. Phys.* B94 (1975) 163; N.H. Fuchs, *Phys. Rev.* D14 (1976) 1709.

29. J. Stern, *Light quark masses and condensates in QCD*, hep-ph/9712438, to appear in: Proceedings of the Chiral Dynamics Workshop in Mainz, Germany, September 1-5, 1997.

30. S. Aoki et al. (CP-PACS), *Nucl. Phys. Proc. Suppl.* 60A (1998) 14.

31. L. Giusti et al., *The QCD Chiral Condensate from the Lattice*, Edinburgh preprint 98/10 (hep-lat/9807014).

32. G. Ecker, *Chiral symmetry*, hep-ph/9805500, to appear in: Proc. of 37. Internationale Universitätswochen für Kern- und Teilchenphysik, Schladming, Austria, Feb. 28 - March 7, 1998.
33. M. Knecht and E. de Rafael, *Phys. Lett.* B424 (1998) 335.

34. M.E. Sevior, *Determination of the $\pi^\pm p \rightarrow \pi^\pm \pi^\mp n$ Cross-Section Near Threshold*, in: Working group on $\pi\pi$ and $\pi N$ interactions, p.11 (hep-ph/9711361), to appear in: Proceedings of the Chiral Dynamics Workshop in Mainz, Germany, September 1-5, 1997.