Comprehensive Evidences of Octupole Vibration in $^{158}$Gd

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Abstract

Comprehensive evidences of the SU(3) limit in the spdf interacting boson model, a dynamical symmetry describing octupole vibration in rotational nucleus, are found in the spectrum, E2 and E1 transition rates, and relative intensities in $^{158}$Gd. This gives a good example of rotational nucleus with octupole vibration in rare-earth region.

21.60.Fw, 23.20Lv, 27.70+b

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There have been continued interests in the studies of octupole degree of freedom in nuclear structure recently [1–23]. In the boson model, negative parity states are described by the spdf interacting boson model (IBM) [9–17,19] or the sdf IBM [20,21]. Gamma soft octupole deformation has been found in Ba isotopes [16]. As for rotation with octupole deformation, corresponding to the SU(3) limit in the spdf IBM, the experimental evidence has not yet been found. It has been pointed that $^{232}$U and other actinide nuclei may be candidates for the SU(3) limit [12–14,19]. However, while the spectrum agrees with theoretical calculation very well, there are few electromagnetic transition data, in particular E1 transitions connecting positive and negative parity states. Experimental evidences of nuclei with such a dynamical symmetry is still lacking. Whereas these dynamic symmetries are very important because they can be used to classify states and characterize collective features of a nucleus. To this purpose we have studied the structures of over 30 deformed nuclei in the rare earth and actinide region, and found comprehensive evidences of octupole vibration in $^{158}$Gd. This result is somewhat out of the general expectation that a good example nucleus of the SU(3) octupole deformation dynamical symmetry should be in the actinide region [12–14,19].

The octupole vibration in rotational nucleus is characterized by the SU(3) group chain,

$$U(16) \supset U(6) \otimes U(10) \supset SU_{sd}(3) \otimes SU_{pf}(3) \supset SU_{spdf}(3) \supset O(3)$$

and the energy eigenvalue is

$$E = \epsilon_N N_{pf} + a_1 C_{SU_+} + a_2 C_{SU_-} + a_3 C_{SU(3)} + a_4 L(L+1).$$

The g.s.-band, $\beta$-band and $\gamma$-band are generated from (2N,0), (2N-4,2)K=0 and (2N-4,2)K=2 respectively, with N sd-bosons, where N is the valence nucleon pair number. In $^{158}$Gd, N is equal to 13. The low-lying negative parity are generalized by the SU(3) irreducible representation (IR) from the decomposition of (2N-2,0) $\otimes$ (3,0); that is $K^p = 0^-$ from (2N+1,0) and $K^p = 1^-$ from (2N-1,1) respectively. The value of $a_2$ is taken zero, since it is irrelevant to the spectrum of the low-lying states with only one pf-boson. The parameters are then determined by experimental data. They are: $a_1 = -7.793$ keV, $\epsilon_\gamma = 3.160$ MeV, $a_3 = -4.686$ keV. And $a_4$ is 11.917 keV for the positive parity states and 8.592 keV for the negative parity states, respectively. The smaller value of $a_4$ for the negative parity states reflects an increase of moment of inertia for the negative parity states due to octupole deformation, an effect which has also been observed in Uranium isotopes [24]. The spectrum of the spdf SU(3) is compared with data [21,25,26] in Fig.1. The general agreement between experiment and calculation is good. The five low-lying bands, 3 with positive parity and 2 with negative parity, are all well reproduced. But from the spectrum alone, it is not sufficient to determine the nature of the dynamical symmetry. The more tough criteria in determining the nature of the collective motion lie in the electromagnetic transition part.

E2 transitions among the positive parity states are calculated using the transition operator: $T(E2)^2 = e_2((s^4d + d^4s)^2 - \frac{47}{2}(d^4d)^2)$, the SU(3) generator. The calculated B(E2) values are compared with experimental data in table II. The agreement with experimental data is very good. The inband transitions in the ground state band agree with the data well. Since the SU(3) generator does not have matrix elements between different SU(3) IR's, all the inter-band E2 transitions from $\gamma$ or $\beta$ bands to the ground state band are zero. This is in
agreement with the experimental data. All the interband transitions are very weak. A small breaking in the SU(3) dynamical symmetry will produce nonzero interband transitions. This is a second order effect, and in this work, we are not going to pursue the details. At high spins ($L = 10$), the theoretical B(E2) is less than the data. This is the well-known reduction of collectivity problem in boson models and can only be solved by considering the g-boson \[27\].

There are also ample experimental data on the electric dipole transitions. We have calculated the E1 transitions using standard group theoretic method as in Ref. \[28\]. The E1 transition operator is taken the following form: $T(E1) = e((s^\dagger \tilde{p} + p^\dagger)^1 + \chi_{dp}(d^\dagger \tilde{d} + f^\dagger \tilde{d})^1)$. These parameters are determined by experimental data. The values are $\chi_{dp} = -3.825$, $\chi_{df} = 3.676$. The result of this parametrization is labeled as Cal1 in table II. The agreement between calculation and data is very well. The transitions from $0^-$ band to the ground state band transitions are perfectly reproduced. In particular, the transition from $2^- \to 2\beta$ is much less than the transition from $1^- \to$ ground state in experiment, and this is also reproduced by the calculation well. Considering the large range of variations in the data, the agreement is remarkable. It is worth pointing out that the transition operator is quite close the generator of O(10) \[14\], where $\chi_{dp} = -1.2649$ and $\chi_{df} = 1.1832$. This also shows the importance of the p boson in describing octupole collective motions, which has been pointed by many authors \[11–13,16–18\]. In order to see the goodness of the generator form, we made a calculation for the E1 transitions using just generator, the results are also listed in table II labeled as Cal2. It is apparent that the generator has already given a satisfactory agreement with the data. We have also calculated the relative intensities. The results using the E1 transition operator determined from experiment are listed in table III. We see that the agreement between calculation and data is very well. We have also calculated the relative intensities using the O(10) generator, whose results are not shown here. The agreement between calculation and the data is quite good.

From these comparisons of the spectrum, the E2 and E1 transition rates, and relative intensities, we can conclude that $^{158}$Gd is a good example of the SU(3) dynamical symmetry in the spdf IBM, a dynamical symmetry describing the rotations with octupole vibration. In particular, all existing electric dipole transition data varying over a large range agree with the SU(3) limit dynamical symmetry results very well. This has passed through the stringent test on the validity of the dynamical symmetry, that is, the check on the wave functions of a dynamical symmetry. This has established firmly the experimental evidence of rotation with octupole vibration in $^{158}$Gd. Because of the similarities of the Gd isotopes with other rare-earth nuclei in many properties, it is hoped that this finding will be helpful in the studies of the octupole collectivity in this region.

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TABLES

| $I_i$ | $I_f$ | Exp      | Cal      | $I_i$ | $I_f$ | Exp      | Cal      |
|-------|-------|----------|----------|-------|-------|----------|----------|
|       |       |          |          |       |       |          |          |
| Intra band transitions | | | | Interband transitions | | | |
| $2_g$ | $0_g$ | 198(6)   | 198      | $2\gamma$ | $4_g$ | 1.39(15) | 0.0      |
| $4_g$ | $2_g$ | 289(5)   | 279      | $2\beta$  | $0_g$ | 0.31(4)  | 0.0      |
| $6_g$ | $4_g$ | 300      |          | $2\gamma$ | $4_g$ | 0.27(4)  | 0.0      |
| $8_g$ | $6_g$ | 320(30)  | 302      | $4\gamma$ | $2_g$ | 5.9(7)   | 0.0      |
| $10_g$| $8_g$ | 330(30)  | 296      | $2\gamma$ | $0_g$ | 3.5(4)   | 0.0      |
| $12_g$| $10_g$| 310(30)  | 282      |         |       |          |          |

TABLE I. B(E2) values among the positive parity states.

| $I_i$ | $I_f$ | Exp.(W.u.) | Cal.1(W.u.) | Cal.2(W.u.) |
|-------|-------|------------|-------------|-------------|
| $1^-_2$| 0$_1^+$ | 0.0035(8) | 0.0028 | 0.0035 |
| $1^-_2$| 2$_1^+$ | 0.0063(16)| 0.0056 | 0.0068 |
| $3^-_1$| 2$_1^+$ | 0.00033(10)| 0.00029 | 0.00047 |
| $3^-_1$| 4$_1^+$ | 0.00029(8)| 0.00041 | 0.00088 |
| $2^+_2$| 1$_{-1}$ | 6.4(8) $\times 10^{-5}$ | 1.30 $\times 10^{-5}$ | 6.2 $\times 10^{-5}$ |
| $2^+_2$| 2$_{-1}$ | 1.21(5) $\times 10^{-5}$ | 1.19 $\times 10^{-5}$ | 3.16 $\times 10^{-5}$ |
| $2^+_2$| 3$_{-1}$ | 1.89(24) $\times 10^{-4}$ | 1.11 $\times 10^{-5}$ | 2.54 $\times 10^{-4}$ |

TABLE II. Comparison of B(E1) values in $^{158}$Gd.
| Nucleus  | $E_{level}(keV)$ | $K^\pi$ | $I_i$ | $I_f$ | Cal. | Exp. |
|----------|------------------|---------|-------|-------|------|------|
| $^{158}$Gd | 977 | $1^-$ | $I_1$ | $0_{gs}^+$ | 100 | 100(5) |
|          | 1042 | $3_{1^-}$ | $2_{gs}^+$ | 100 | 100(20) |
|          | 1176 | $5_{1^-}$ | $4_{gs}^+$ | 100 | 100(6) |
|          | 1260 | $2_{\beta}^+$ | $1_{1^-}$ | 5.1 | 5.1(46) |
|          | 1263 | $0^-$ | $1_{2^-}$ | $0_{gs}^+$ | 64 | 68(4) |
|          | 1403 | $3_{2^-}$ | $2_{gs}^+$ | 100 | 100(6) |
|          | 1407 | $1^-$ | $4_{\beta}^+$ | $3_{1^-}$ | 19.9 | 19.9(12) |
|          | 1639 | $0^-$ | $5_{2^-}$ | $4_{gs}^+$ | 100 | 100(8) |

TABLE III. Comparisons of relative intensities in $^{158}$Gd
FIG. 1. The spectra of $^{158}\text{Gd}$. The left part is the experimental spectrum, and the right part is the calculated spectrum.