Frequency dependence of induced spin polarization and spin current in quantum wells

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Dynamic response of two-dimensional electron systems with spin-orbit interaction is studied theoretically, on the basis of quantum kinetic equation taking into account elastic scattering of electrons. The spin polarization and spin current induced by the applied electric field are calculated for the whole class of electron systems described by p-linear spin-orbit Hamiltonians. The absence of non-equilibrium intrinsic static spin currents is confirmed for these systems with arbitrary (non-parabolic) electron energy spectrum. Relations between the spin polarization, spin current, and electric current are established. The general results are applied to the quantum wells grown in [001] and [110] crystallographic directions, with both Rashba and Dresselhaus types of spin-orbit coupling. It is shown that the existence of the fixed (momentum-independent) precession axes in [001]-grown wells with equal Rashba and Dresselhaus spin velocities or in symmetric [110]-grown wells leads to vanishing spin polarizability at arbitrary frequency ω of the applied electric field. This property is explained by the absence of Dyakonov-Perel-Kachorovskii spin relaxation for the spins polarized along these precession axes. As a result, a considerable frequency dispersion of spin polarization at very low ω in the vicinity of the fixed precession axes is predicted. Possible effects of extrinsic spin-orbit coupling on the obtained results are discussed.

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I. INTRODUCTION

The presence of spin-orbit interaction in solids provides a natural way of manipulating spin states of electrons by purely electrical means, without application of a magnetic field. This property is a subject of interest for the novel and rapidly developing field of spintronics.1 Some important manifestations of spin manipulation are the generation of non-equilibrium spin polarization of electrons and excitation of spin currents by a driving electric field which also leads to the usual charge current. Though the problem of field-induced spin polarization is an old one,2,3,4,5 it has been recently set at the focus of attention. The main reason for this is the appearance of experimental works6,7,8,9,10,11 which demonstrate the spin polarization generated under the current flow both in bulk and two-dimensional (2D) semiconductor layers with spin-orbit interaction. Another reason is the rapidly growing interest to the related phenomenon, the intrinsic spin-Hall effect, when the electric current in the presence of spin-orbit coupling leads to a non-equilibrium spin current in perpendicular direction (note that weak spin currents can exist even in equilibrium12). After the theoretical proposal13 of the intrinsic spin-Hall effect based on the Rashba model representing spin-orbit interaction for 2D electrons in quantum wells,14 it has been realized15,16,17,18,19,20,21 that this effect is absent in the infinite 2D system in the static (zero-frequency) limit and exists at non-zero frequency or in finite-size samples. The absence of the static intrinsic spin-Hall effect is not a general property, since it is related to the specific form of the spin-orbit Hamiltonian. It has been shown20,22,23,24,25,26 that the static spin-Hall effect exists for more complicated models involving higher-order (cubic) terms in the momentum dependence of the spin-orbit Hamiltonian. The experimental observation of the spin-Hall effect for the 2D hole system27 described by the spin-orbit Hamiltonian of this kind has confirmed this conclusion. The edge spin accumulation due to the spin-Hall effect has been also observed8,11,28 for 2D electron systems. Recently, it has been suggested29,30 that deviation of electron band dispersion from the parabolic one can lead to non-zero intrinsic static spin-Hall effect. This point, however, remains controversial, since the analysis given in Ref. 17 predicts zero static spin-Hall effect even in this case. The calculations presented in this paper also show the absence of static spin currents for non-parabolic electron band dispersion.

In contrast to the static regime, the frequency-dependent induced spin polarization and spin current have not been extensively studied. Theoretical calculations of the frequency-dependent spin-Hall current based on the Rashba model have been done in Refs. 16, 18 and 31 by using the methods of non-equilibrium Green’s functions, Kubo-Greenwood linear response theory, and quantum kinetic equation, respectively, with the same result. The authors of Ref. 31 also calculated the frequency dependence of the induced spin polarization. A more complicated case of frequency-dependent response, when both Rashba and Dresselhaus (linear in momentum) terms are included in the spin-orbit Hamiltonian, has been studied in Refs. 32 and 33 in the dynamical (collisionless) regime. A comparative numerical study of the frequency dependence of the spin-Hall effect has been done in Ref. 23 for linear, cubic, and modified Rashba models. The resonances in frequency dependence of spin-Hall conductivity in magnetic field have been described in
Ref. 34 for the Rashba model. Nevertheless, a systematic investigation of the frequency-dependent problem of the induced spin polarization and spin current is still missing.

A step towards systematic description of the frequency-dependent spin response is undertaken in this paper by considering the important class of spin-orbit Hamiltonians:

\[ \hat{h}_p = \hbar \Omega_p \cdot \hat{\sigma}, \quad \hbar \Omega_p^\alpha = \xi_p \mu_{\alpha\beta} \rho_\beta, \]

where \( \hat{\sigma} \) is the vector of Pauli matrices, \( \mathbf{p} = (p_x, p_y) \) is the 2D momentum of electrons, and \( \mu_{\alpha\beta} \) is the matrix of spin velocities. Next, \( \xi_p \) is an arbitrary function of the absolute value of electron momentum. This function describes possible isotropic corrections to spin-orbit interaction, which may have the same origin as the non-parabolicity of the band spectrum. The case \( \xi_p = 1 \) corresponds to the \( \mathbf{p} \)-linear spin-orbit Hamiltonian of the general form. The \( \mathbf{p} \)-linear spin-orbit coupling terms appear in quantum wells due to both the structural inversion asymmetry (Rashba term) and bulk inversion asymmetry (Dresselhaus term), the latter contribution is sensitive to orientation of the quantum well with respect to crystallographic axes. By solving the quantum kinetic equation for the matrix distribution function of electrons, with taking into account elastic scattering, the spin polarization and spin currents are found on an equal footing and a relation between them is established. The calculations not only provide analytical expressions for these quantities, but also demonstrate a need to reconsider the known results for static spin polarization based on the Hamiltonian (1) in special cases, when the symmetry of \( \mu_{\alpha\beta} \) allows zero spin precession for some chosen directions of the spin vector. The examples of this kind are the quantum wells grown in [001] crystallographic direction in the case of equal Rashba and Dresselhaus spin velocities and symmetric quantum wells (only the Dresselhaus term is present) grown in [110] crystallographic direction. In particular, it is shown that the induced spin polarization in these systems remains zero even when the frequency of the applied field goes to zero. The calculations also show that the static spin current in the electron systems described by the Hamiltonian (1) is zero even if a non-parabolicity of electron band spectrum is taken into account. The presented theory neglects the spin-orbit corrections to the scattering potential, which means that the effects of extrinsic spin-orbit coupling, such as the extrinsic spin currents and Elliot-Yafet spin relaxation, are not included in the calculations.

The paper is organized as follows. In Sec. II we consider the quantum kinetic equation and present its analytical solutions. The expressions for the induced spin density vector and spin current tensor, obtained on the basis of these solutions, are given in Sec. III. In that section we also establish a general relation between these quantities and present a detailed analysis of two important cases, [001]-grown and [110]-grown quantum wells with both Rashba and Dresselhaus spin-orbit coupling. A relation between the induced spin density and electric current is derived and analyzed in Sec. IV. The obtained results and the limits of their applicability are discussed in Sec. V.

II. GENERAL CONSIDERATION

The Hamiltonian of the problem is written in the form \( \hat{H}_p + \hat{h}_p + V_e + \hat{H}_t^{\text{ext}} \), where \( \hat{H}_p \) is the Hamiltonian of free electrons in the absence of spin-orbit interaction, \( \hat{h}_p \) is the spin-orbit Hamiltonian, \( V_e \) is the potential of impurities or other static inhomogeneities (the spin-orbit corrections to this potential are neglected, so only the intrinsic spin-orbit coupling is considered), and \( \hat{H}_t^{\text{ext}} = -e \mathbf{E}_i \cdot \mathbf{r} \) is the Hamiltonian of the external perturbation due to the applied time-dependent electric field \( \mathbf{E}_i \) (here \( e = -|e| \) is the electron charge). The calculations are based on the quantum kinetic equation for the matrix distribution function \( \hat{\rho}_{pt} \), which is a \( 2 \times 2 \) matrix over the spin indices (see, for example, Refs. 36 and 37). For the spatially-homogeneous problem considered below, this equation is written in the form

\[ \frac{\partial \hat{\rho}_{pt}}{\partial t} + \frac{i}{\hbar} \left[ \hat{h}_p, \hat{\rho}_{pt} \right] + e \mathbf{E}_i \cdot \frac{\partial \hat{\rho}_{pt}}{\partial \mathbf{p}} = \tilde{J}(\hat{\rho}|pt), \]

where \( \tilde{J} \) is the collision integral describing the elastic scattering. This integral is written below in the Markovian approximation and under the assumptions \( \hbar \omega \ll \tau \) and \( \hbar \tau \ll \tau_s \), where \( \omega \) is the frequency of the applied perturbation, \( \tau \) is the mean kinetic energy of electrons and \( \tau_s \) is the characteristic scattering time. One has (see Ref. 37, problem 13.10)

\[ \tilde{J}(\hat{\rho}|pt) = \frac{1}{\hbar^2} \int \frac{dp'}{(2\pi \hbar)^2} W(| \mathbf{p} - \mathbf{p}' |) \int_0^\infty dt' e^{\lambda t'} \]

\[ \times \left\{ i(\varepsilon_p + \hbar \omega) t'/\hbar (\hat{\rho}_{pt} - \hat{\rho}_{pt}) e^{-i(\varepsilon_p + \hbar \omega) t'/\hbar} - (\mathbf{p}' \leftrightarrow \mathbf{p}) \right\}, \]

where \( W(| \mathbf{p} |) = \int d\Delta \varepsilon e^{-i\mathbf{p} \cdot \Delta \varepsilon /\hbar} \langle (V_e + \Delta V_e) \rangle \) is the spatial Fourier transform of the correlation function of the scattering potential, \( \varepsilon_p \) is the kinetic energy of electron in the absence of spin-orbit interaction (the energy spectrum is isotropic but not necessarily parabolic), \( \lambda \rightarrow +0 \), and \( (\mathbf{p}' \leftrightarrow \mathbf{p}) \) denotes the term obtained from the preceding one in the square brackets by permutation of momenta. The integration over \( t' \) in Eq. (3) is carried out elementary, but the resulting expression is rather lengthy and, for this reason, is not presented here.

Searching for the linear response to the Fourier component \( \mathbf{E} e^{-i\omega t} \) of the applied electric field, we represent the matrix distribution function in the form \( \hat{\rho}_{pt} = \hat{\rho}_p^{(eq)} + \hat{\rho}_p^{(an)} e^{-i\omega t} \), where \( \hat{\rho}_p \) is the Fourier component of the non-equilibrium part of the distribution function and \( \hat{\rho}_p^{(eq)} \) is the equilibrium distribution function,

\[ \hat{\rho}_p^{(eq)} = \frac{1}{\frac{1}{2}[f_{\varepsilon_p + \hbar \Omega_p} + f_{\varepsilon_p - \hbar \Omega_p}] + \frac{\Omega_p \cdot \hat{\sigma}}{2\hbar \Omega_p} [f_{\varepsilon_p + \hbar \Omega_p} - f_{\varepsilon_p - \hbar \Omega_p}]}, \]
which is expressed through the Fermi distribution $f_\epsilon$. Here and below, $\Omega_\mathbf{p} \equiv |\Omega_\mathbf{p}|$. The function (4) does not lead to spin polarization of electron system because its matrix part is antisymmetric in momentum. Since a substitution of the function (4) into Eq. (2) makes both the commutator and the collision integral (3) equal to zero, one has a closed integral equation for $f_\mathbf{p}$:

$$-i\omega f_\mathbf{p} + \frac{\hbar}{2} \left[ \mathbf{\hat{h}}_\mathbf{p}, \mathbf{\hat{f}}_\mathbf{p} \right] + e\mathbf{E} \cdot \frac{\partial f_\mathbf{p}(\epsilon)}{\partial \mathbf{p}} = \mathcal{J}(\mathbf{f}|\mathbf{p}),$$

(5) which determines the linear response of the electron system.

The induced spin density is defined as

$$s = \frac{1}{2} \int \frac{d\mathbf{p}}{(2\pi\hbar)^2} \text{Tr}(\sigma_\alpha \tilde{\mathbf{u}}(\mathbf{p}) \mathbf{f}_\mathbf{p}),$$

(6) and the induced (non-equilibrium) spin current density is given by the tensor

$$q^\gamma_\alpha = \frac{1}{2} \int \frac{d\mathbf{p}}{(2\pi\hbar)^2} \text{Tr} \left( \left\{ \sigma_\alpha, \mathbf{\tilde{u}}(\mathbf{p}) \right\} \mathbf{f}_\mathbf{p} \right),$$

(7) where $\mathbf{\tilde{u}}(\mathbf{p}) = \partial(\epsilon_\mathbf{p} + \tilde{h}_\mathbf{p})/\partial \mathbf{p}$ is the group-velocity matrix, Tr denotes the matrix trace, and $(\tilde{a}, \tilde{b}) = (\tilde{a} + \tilde{b})/2$ denotes the symmetrized matrix product. The expression (7) describes the flow of the spin polarized along $\alpha$ in the direction $\gamma$. One may also introduce the average spin $\mathbf{S} = s/n_{2D}$, where $n_{2D} = (2\pi\hbar)^{-2} \int d\mathbf{p} \text{Tr}(\mathbf{p})$ is the electron density. The tensors of spin polarizability, $\chi_{\alpha\beta}$, and spin conductivity, $\Sigma_{\gamma\beta}$, are introduced according to

$$s_\alpha = \chi_{\alpha\beta}(\omega)E_\beta, \quad q^\gamma_\alpha = \Sigma_{\gamma\beta}(\omega)E_\beta.$$  

(8)

It is assumed in the following that the spin-splitting energy $2\hbar\Omega_\mathbf{p}$ is small in comparison to the mean energy of electrons. Then it is convenient to apply an efficient method of solution of Eq. (5) based on the expansion of the collision integral in series with respect to the small parameter $\hbar\Omega_\mathbf{p}/\gamma$; see Refs. 5, 25, 31, and problem 13.11 in the book 37. Using the spin-vector representation $\mathbf{\hat{f}}_\mathbf{p} = f_0^\mathbf{p} + \sigma \cdot \mathbf{f}_\mathbf{p}$ and retaining only the terms of the first order in $\Omega_\mathbf{p}$ under the collision integral, we obtain coupled equations for scalar and vector parts of the distribution function:

$$-i\omega f_0^\mathbf{p} + \frac{1}{2} e\mathbf{E} \cdot \frac{\partial [f_{\mathbf{p} + \hbar\Omega_\mathbf{p}} + f_{\mathbf{p} - \hbar\Omega_\mathbf{p}}]}{\partial \mathbf{p}} = \frac{2\pi}{\hbar} \int \frac{d\mathbf{p}'}{(2\pi\hbar)^2} W(|\mathbf{p} - \mathbf{p}'|) \left[ (f_{\mathbf{p}'} - f_0^\mathbf{p}) \delta(\epsilon_{\mathbf{p}'} - \epsilon_{\mathbf{p}}) \right.$

$$\left. - \hbar(\Omega_\mathbf{p} - \Omega_{\mathbf{p}'}) \cdot (f_{\mathbf{p}'} - f_0^\mathbf{p}) \frac{\partial \delta(\epsilon_{\mathbf{p}'} - \epsilon_{\mathbf{p}})}{\partial \epsilon_{\mathbf{p}'}} \right],$$

(9) and

$$-i\omega f_\mathbf{p} + e\mathbf{E} \cdot \frac{\partial \Omega_\mathbf{p} [f_{\mathbf{p} + \hbar\Omega_\mathbf{p}} - f_{\mathbf{p} - \hbar\Omega_\mathbf{p}}]}{2\Omega_\mathbf{p}} = \frac{2\pi}{\hbar} \int \frac{d\mathbf{p}'}{(2\pi\hbar)^2} W(|\mathbf{p} - \mathbf{p}'|) \left[ (f_{\mathbf{p}'} - f_0^\mathbf{p}) \right.$

$$\times \delta(\epsilon_{\mathbf{p}'} - \epsilon_{\mathbf{p}}) - \hbar(\Omega_\mathbf{p} - \Omega_{\mathbf{p}'}) \left( f_0^\mathbf{p} - f_0^{\mathbf{p}'} \right) \frac{\partial \delta(\epsilon_{\mathbf{p}'} - \epsilon_{\mathbf{p}})}{\partial \epsilon_{\mathbf{p}'}} \right].$$

(10)

The expressions containing formal derivatives of the $\delta$-functions under the integrals should be evaluated using integration by parts. The Fermi distribution functions standing in the field terms also can be expanded in series of $\Omega_\mathbf{p}$. Then one can see that the iterative expansions of $f_0^\mathbf{p}$ and $f_\mathbf{p}$ start with the terms of zero and first order in $\Omega_\mathbf{p}$, respectively. For this reason, the last term under the collision integral in Eq. (9) can be neglected, and the solution of this equation is

$$f_0^\mathbf{p} \approx -\frac{e\mathbf{E} \cdot \mathbf{v}_\mathbf{p}}{\nu_\mathbf{p}^{(1)}} \frac{\partial f_{\mathbf{p}}}{\partial \epsilon_{\mathbf{p}}},$$

(11) where $\mathbf{v}_\mathbf{p} = \partial \epsilon_{\mathbf{p}}/\partial \mathbf{p}$ is the group velocity of electron in the absence of spin-orbit interaction. Here and below, the relaxation rates appearing in the problem are defined as

$$\nu_\mathbf{p}^{(n)} = \frac{2\pi}{\hbar} \int \frac{d\mathbf{p}'}{(2\pi\hbar)^2} W(|\mathbf{p} - \mathbf{p}'|) [1 - \cos(n\theta)] \delta(\epsilon_{\mathbf{p}'} - \epsilon_{\mathbf{p}}),$$

(12) where $\theta$ denotes the scattering angle $\hat{\mathbf{p}}'$. The rate $\nu_\mathbf{p}^{(n)}$ describes relaxation of the $n$-th angular harmonic of the distribution function.

Equation (11) describes the Drude response of the electron system. The next correction to $f_0^\mathbf{p}$ is of the order of $(\hbar\Omega_\mathbf{p}/\gamma)^2$. This correction is essential for calculation of the frequency-dependent conductivity and dielectric function of 2D electrons with spin-orbit splitting38,39 (see Sec. IV), but it is not important for calculation of the induced spin polarization and spin current. After substituting the expression (11) into the last term of the collision integral in Eq. (10), this term is unified with the field term on the left-hand side of Eq. (10). As a result, one gets a closed equation for the vector-function $f_\mathbf{p}$:

$$-i\omega f_\mathbf{p} - 2[\Omega_\mathbf{p} \times f_\mathbf{p}] + \mathbf{F}_\mathbf{p}(\omega)$$

$$= \frac{2\pi}{\hbar} \int \frac{d\mathbf{p}'}{(2\pi\hbar)^2} W(|\mathbf{p} - \mathbf{p}'|)(f_{\mathbf{p}'} - f_\mathbf{p}) \delta(\epsilon_{\mathbf{p}'} - \epsilon_{\mathbf{p})}.)$$

(13)

The vector $\mathbf{F}$ contains both isotropic and anisotropic contributions:

$$\mathbf{F}_\mathbf{p}(\omega) = e\mathbf{E}_\mathbf{E} \mathcal{F}^{(i)}(\omega) - e(\mathbf{E} \cdot \mathbf{p}) \Omega_\mathbf{p} \mathcal{F}^{(o)}(\omega),$$

(14)

$$\mathcal{F}^{(i)}(\omega) = R_\mathbf{p}(\omega)(\tilde{\nu}_\mathbf{p} - i\omega) - p^2 \frac{\partial R_\mathbf{p}(\omega)}{\partial p^2} \nu_\mathbf{p}^{(2)},$$

(15)

$$\mathcal{F}^{(o)}(\omega) = \frac{2\hbar}{p^2} \xi_\mathbf{p} \left[ R_\mathbf{p}(\omega)\tilde{\nu}_\mathbf{p} - p^2 \frac{\partial R_\mathbf{p}(\omega)}{\partial p^2} (\nu_\mathbf{p}^{(2)} - i\omega) \right],$$

(16)

where $\mathbf{E}_\mathbf{E}$ is the constant vector with components $\Omega_\mathbf{E} = \mu_{\alpha\beta} E_\beta$,

$$R_\mathbf{p}(\omega) = \frac{\xi_\mathbf{p}}{\nu_\mathbf{p}^{(1)}} - \frac{p^2}{2} \frac{\partial \xi_\mathbf{p}}{\partial \epsilon_{\mathbf{p}}} \left( \nu_\mathbf{p}^{(2)} - i\omega \right),$$

(17)

and

$$\tilde{\nu}_\mathbf{p} = \nu_\mathbf{p}^{(1)} - \nu_\mathbf{p}^{(2)} - \frac{p^2}{2} \frac{\partial \nu_\mathbf{p}^{(2)}}{\partial \epsilon_{\mathbf{p}}} \left( \nu_\mathbf{p}^{(2)} - i\omega \right).$$

(18)
is a relaxation rate. Note that $\bar{\nu}_p \to 0$ in the limit of short-range scattering potential.

It is convenient to search for the solution of Eq. (13) in the form

$$ f_p = \frac{e(\mathbf{E} \cdot \mathbf{p}) \Omega_p \bar{\nu}_p^{(s)}(\omega)}{\nu_p^{(2)} - i\omega} + \mathbf{g}_p. \tag{19} $$

Since the first term of this expression is proportional to $\Omega_p$, it does not contribute to the vector product in Eq. (13), thereby representing a non-precessing part of the solution. Note that the angular average of this term is directed along $\Omega_{BE}$. The substitution (19) leads to the following equation for $\mathbf{g}_p$:

$$ -i\omega \mathbf{g}_p - 2[\Omega_p \times \mathbf{g}_p] + i\omega \mathbf{G}_p(\omega) = \frac{m_p}{\hbar^2} \int_0^{2\pi} d\varphi' \frac{d\varphi'}{2\pi} |W(|\mathbf{p} - \mathbf{p}'|)(\mathbf{g}_p - \mathbf{g}_p)|_{\mathbf{p}' = |\mathbf{p}|}, \tag{20} $$

where $m_p = \frac{1}{2} (\partial p^2 / \partial \varepsilon_p)$ is the $p$-dependent effective mass and

$$ \mathbf{G}_p(\omega) = -\Omega_{BE} \frac{e\hat{e}_p(\nu_p^{(2)} + \hat{\nu}_p - i\omega)}{\nu_p^{(1)} - i\omega}(\nu_p^{(2)} - i\omega) \frac{\partial f_{\bar{\nu}_p}}{\partial \varepsilon_p}. \tag{21} $$

The collision integral in Eq. (20) is already reduced, by means of integration over the absolute value of $\mathbf{p}'$, to the integral over the angle $\varphi'$ of the vector $\mathbf{p}'$. Owing to the substitution (19), the inhomogeneous (field-dependent) term of Eq. (20), $i\omega \mathbf{G}_p(\omega)$, is proportional to the frequency $\omega$ and isotropic in the momentum space. With the aid of the definition (21), it is convenient to write the angular-averaged distribution function $\bar{f}_p \equiv (2\pi)^{-1} \int_0^{2\pi} d\varphi f_p$ as

$$ \bar{f}_p = -\mathbf{G}_p(\omega) + \mathbf{Q}_p(\omega) + \mathbf{g}_p, \tag{22} $$

$$ \mathbf{Q}_p(\omega) = -e\Omega_{BE} \frac{|p|^2 R_p(\omega)|}{\partial p^2}. $$

We also point out the exact relation

$$ \bar{f}_p = -\frac{2}{\hbar^2} \frac{i\omega}{\nu_p} [\Omega_p \times \mathbf{g}_p] + \mathbf{Q}_p(\omega), \tag{23} $$

which is obtained by applying the procedure of angular averaging to Eq. (20) and by using Eq. (22).

In spite of the isotropy of the term (21), Eq. (20) requires a numerical solution. The physical reason for this is the effect of precession in the presence of angular-dependent scattering. The angular dependence of the vector product $[\Omega_p \times \mathbf{g}_p]$, in the general case, is different from that of $\mathbf{g}_p$ standing there, and the standard method of solution, based on expansion of the distribution function in series of angular harmonics, leads to an infinite set of coupled equations. There are, however, a number of important situations when Eq. (20) can be solved analytically. These situations are described in the subsections below.

### A. Short-range scattering potential

Let us consider the limit of short-range scattering potential, when $W(|\mathbf{p} - \mathbf{p}'|)$ is replaced by a constant $W$ and $\nu_p^{(s)} = \nu_p = m_p W / h^3$ for any number $n$. The momentum dependence of the scattering rate is associated with possible non-parabolicity of the band spectrum and has to be ignored in the parabolic approximation. Since the right-hand side of Eq. (20) is reduced in this case to $\bar{\nu}_p (\mathbf{g}_p - \mathbf{g}_p)$, a regular way of solving exists. The solution is

$$ \mathbf{g}_p = -\frac{i\omega}{\nu_p(\nu_p - i\omega)} \frac{\partial f_{\bar{\nu}_p}}{\partial \varepsilon_p} \left\{ (\nu_p - i\omega)^2 \mathbf{A}_p + 2(\nu_p - i\omega)\Omega_p \times \mathbf{A}_p + 4\Omega_p(\Omega_p \cdot \mathbf{A}_p) \right\}, \tag{24} $$

where $\Delta_p^2 = (\nu_p - i\omega)^2 + 4\Omega_p^2$ and $\mathbf{A}_p$ is the vector with components

$$ A_p^α = e\xi_p \hat{T}_p^{-1} μ_{αβ} \Omega_p^β. \tag{25} $$

Here $\hat{T}_p^{-1}$ denotes the matrix inverse of the symmetric $3 \times 3$ matrix

$$ T_p^{αβ} = 4 \left\{ \Omega_p^α \Omega_p^β - δ_{αβ} \Omega_p^2 / \Delta_p^2 \right\} + δ_{αβ} \frac{i\omega}{\nu_p}, \tag{26} $$

obtained as a result of angular averaging. The whole solution, according to Eqs. (16)-(19), is

$$ f_p = -e(\mathbf{E} \cdot \mathbf{p}) \Omega_p \frac{2h}{\xi_p} \frac{\partial}{\partial p^2} \left[ \xi_p(\xi_p - i\omega) \frac{\partial f_{\bar{\nu}_p}}{\partial \varepsilon_p} \right] + \mathbf{g}_p, \tag{27} $$

and its angular average is written as

$$ \bar{f}_p = \frac{1}{\nu_p} \frac{\partial f_{\bar{\nu}_p}}{\partial \varepsilon_p} \left( e\xi_p \Omega_{BE} - \frac{i\omega}{\nu_p} \mathbf{A}_p \right) + \mathbf{Q}_p(\omega). \tag{28} $$

This vector determines the magnitude and the direction of the induced spin polarization.

### B. Isotropic spin splitting

The next exactly solvable situation is realized when the energy spectrum of electrons remains isotropic in the presence of spin-orbit coupling. In other words, $\Omega_p = \Omega_p$ depends only on the absolute value of $\mathbf{p}$. This imposes certain constraints on the matrix $μ_{αβ}$:

$$ μ_{xx}^2 + μ_{yy}^2 + μ_{zz}^2 = μ_{xy}^2 + μ_{yz}^2 + μ_{zx}^2 = μ_{yy} μ_{zz} + μ_{zz} μ_{xx} + μ_{xx} μ_{yy} = 0. \tag{29} $$

This situation is realized, for example, in [001]-grown quantum wells ($x \parallel [100]$, $y \parallel [010]$, $z \parallel [001]$) with only Rashba or only Dresselhaus type of spin-orbit coupling and in [111]-grown quantum wells ($x \parallel [11\bar{2}]$, $y \parallel [\bar{1}10]$, $z \parallel$
where \( \xi \) is directed along the collision integral equal to zero): \( U_{\mu}^{\beta} = 4(\Omega_{\mu}^{\beta} - \delta_{\alpha\beta} \Omega_{p}^{2})/\Delta_{p}^{2} + \delta_{\alpha\beta} \frac{i\omega}{\nu_{p}^{(2)}}, \)

we find \( g_{p} = -\frac{i\omega(\nu_{p}^{(2)} + \nu_{p} - i\omega)}{\nu_{p}^{(2)}(\nu_{p}^{(1)} - i\omega)(\nu_{p}^{(2)} - i\omega)} \frac{\partial f_{E}}{\partial E_{p}} \times \left\{ (\nu_{p}^{(1)} - i\omega)(\nu_{p}^{(2)} - i\omega)B_{p} + 2(\nu_{p}^{(2)} - i\omega)|\Omega_{p} \times B_{p}| + 4\Omega_{p}(\Omega_{p} \cdot B_{p}) \right\}. \)

It is easy to see that the results (24) and (32) become equivalent if one assumes isotropic spin splitting in Eq. (24) and short-range scattering in Eq. (32). The angular average of the whole solution is

\[
\bar{T}_{p} = \langle \nu_{p}^{(2)} + \nu_{p} - i\omega \rangle \frac{\partial f_{E}}{\partial E_{p}} \left( e_{E_{p}} \Omega_{p} - i\omega \nu_{p}^{(2)}B_{p} \right) + Q_{p}(\omega). \tag{33}
\]

### C. Fixed precession axis

There is also a special case, when Eq. (20) is solved in the most simple way. This happens when the vector product \( \Omega_{p} \times \Omega_{E} \) is zero for arbitrary \( p \) and \( E \). In other words, the symmetry of the matrix \( \mu_{\alpha\beta} \) should allow existence of a fixed (momentum-independent) precession axis. This imposes the following constraints:

\[
\mu_{xy}\mu_{yz} = \mu_{xz}\mu_{yz}, \quad \mu_{xy}\mu_{zx} = \mu_{xz}\mu_{zy} \tag{34}.
\]

Under these conditions, both \( \Omega_{p} \) and \( \Omega_{E} \) are directed along the fixed precession axis, without regard to directions of \( p \) and \( E \). The examples are \([001]\)-grown quantum wells \((x || [100], y || [010], z || [001])\) with equal absolute values of Rashba and Dresselhaus velocities, when the precession axis is in the quantum well plane at the angle of \( \pi/4 \) or \( -\pi/4 \) with respect to the main crystallographic axes, and \([110]\)-grown wells \((x || [\bar{1}10], y || [001], z || [1\bar{1}0])\) with only Dresselhaus type of coupling, when the precession axis is perpendicular to the quantum well plane. The solution of Eq. (20) in this case is non-precessing (makes the vector product equal to zero) and isotropic (makes the collision integral equal to zero):

\[
g_{p} = G_{p}(\omega). \tag{35}
\]

This vector is directed along \( \Omega_{E} \). The averaged whole solution \( \bar{T}_{p} \) is also directed along \( \Omega_{E} \):

\[
\bar{T}_{p} = Q_{p}(\omega). \tag{36}
\]

### D. Static limit

If the frequency \( \omega \) goes to zero, the solution of Eq. (20) is trivial, \( g_{p} = 0 \). Therefore, the function \( f_{p} \) from Eq. (19) with \( g_{p} = 0 \) and \( \omega = 0 \) describes the static spin-dependent response in the general case. The only exception is the special case considered in the previous subsection, when there exists a non-zero solution, \( g_{p} = G_{p}(0) \). The averaged distribution function for the static limit is

\[
\bar{T}_{p} = \Omega_{E} e_{E_{p}}(\nu_{p}^{(2)} + \nu_{p} - i\omega) \frac{\partial f_{E}}{\partial E_{p}} \left( \Omega_{p} \times B_{p} \right) + Q_{p}(0) \tag{37}
\]

for the general case. In the special case only the last term of this expression remains.

### III. SPIN RESPONSE

To describe the spin response, one should calculate the integrals over momentum \( p \) in Eqs. (6) and (7). It is convenient to separate the angular averaging from the integration over the squared absolute value of momentum, \( p^{2} \), according to \( dp = \frac{1}{2} dp^{2} d\varphi \). Then, after using the representation \( f_{p} = f_{p} + \sigma \cdot f_{p} \) and taking the matrix trace, the density of the induced spin polarization is given by

\[
s = \int_{0}^{\infty} \frac{dp^{2}}{4\pi h^{2}} \bar{T}_{p}, \tag{38}
\]

and the density of non-equilibrium spin current is

\[
q_{\gamma} = \int_{0}^{\infty} \frac{dp^{2}}{4\pi h^{2}} \left\{ \frac{p^{2}}{2} \frac{1}{m_{p}^{2} - \frac{1}{\hbar^{2}} \Omega_{p} \cdot \Omega_{p}} \right\} + \mu_{\alpha\beta} \frac{p_{\alpha}^{(1)\alpha} + \nu_{p}^{(2)}p_{\alpha}}{2(\Omega_{p} \cdot B_{p})} \right\}. \tag{39}
\]

The second term, which appears in the expression (39) owing to the spin-orbit correction to the group velocity, gives zero contribution because, according to Eq. (11), the scalar part of the distribution function is antisymmetric in momentum. If the frequency \( \omega \) is zero, the vector part of the distribution function is symmetric in momentum, so the first term of the expression (39) also gives zero contribution. Therefore, the non-equilibrium static spin currents do not exist for the model described by the spin-orbit Hamiltonian (1). At non-zero frequency, the spin currents are associated with \( g \)-contribution to the distribution function, because the first term of Eq. (19) is symmetric in \( p \):

\[
q_{\gamma} = \int_{0}^{\infty} \frac{dp^{2}}{4\pi h^{2}} \bar{B}_{p} p_{\gamma}; \tag{40}
\]

where \( q_{\gamma} = (q_{x\gamma}^{p}, q_{y\gamma}^{p}, q_{z\gamma}^{p}) \). Looking at the expressions (24) and (32), one can conclude that it is the second terms in the braces of these expressions that are responsible for the spin currents.
On the other hand, the induced spin density is determined by the angular-averaged symmetric part of $f_{\mathbf{p}}$, and exists in the static regime as well. Applying either the exact relations (22) and (23) or the expressions (28), (33), (36), and (37) describing different physical situations, one should always ignore the term $Q_{\mathbf{p}}(\omega)$ which represents a full derivative over $p^2$ and, for this reason, does not contribute to the integral (38). The static spin polarization obtained in this way is

$$s = \frac{e\Omega_E}{4\pi\hbar} \int_0^\infty dp^2 \epsilon_\mathbf{p} \frac{\partial f_{\mathbf{p}}}{\partial \epsilon_\mathbf{p}} \left[ \frac{1}{\nu_\mathbf{p}^{(2)}} \right] + \frac{p^2}{2\nu_\mathbf{p}^{(1)}\nu_\mathbf{p}^{(2)}} \frac{\partial}{\partial \epsilon_\mathbf{p}} \left( \frac{\nu_\mathbf{p}^{(2)}}{\nu_\mathbf{p}^{(2)}} \frac{\partial \epsilon_\mathbf{p}}{\partial \epsilon_\mathbf{p}} \right).$$

(41)

In the parabolic approximation $\epsilon_\mathbf{p} = p^2/2m$ and at $\xi_\mathbf{p} = 1$ this expression is rewritten as

$$s = \frac{\Omega_E e\mu m}{2\pi\hbar} \int_0^\infty d\epsilon \frac{\partial f_\epsilon}{\partial \epsilon} \left[ \tau_\epsilon^{(2)} + \tau_\epsilon^{(1)} \frac{1}{2} \frac{\partial \ln \tau_\epsilon^{(2)}}{\partial \ln \epsilon} \right].$$

(42)

where $\tau_\epsilon^{(n)} = 1/\nu_\epsilon^{(n)}$ are the relaxation times. For degenerate electron gas, when $(\partial f_\epsilon/\partial \epsilon) \sim -\delta(\epsilon - \epsilon_F)$ and $\epsilon_F$ is the Fermi energy, the integral over energy is taken in a straightforward way. In the case of short-range scattering potential ($\tau_\epsilon^{(n)} = \tau = 1/\nu$) one arrives at the well-known result $s_\alpha = \chi_{\alpha\beta}E_\beta$ with $\chi_{\alpha\beta} = -\mu_{\alpha\beta}[\hbar\Omega/2\pi\hbar^2]$ valid for any linear spin-orbit Hamiltonian including the case of Rashba spin-orbit coupling studied in the early papers.\(^{3,4}\)

It is important that Eqs. (41) and (42) are not valid in the special situation when a fixed precession axis exists; see subsection C of Sec. II. In fact, a straightforward substitution of Eq. (36) into Eq. (38) shows that, at arbitrary frequency of the applied field, the spin polarization does not appear in this special situation. The physical explanation of this remarkable property is based on the facts that in the case of a fixed precession axis (a) there is no spin relaxation\(^{40}\) by the Dyakonov-Perel-Kachorovskii (DPK) mechanism\(^{41}\) for the spins directed along this axis and (b) the induced spin polarization can, in principle, appear only along this axis, without regard to the direction of the applied electric field. Imagine that the electric field is abruptly turned on. The anisotropic distribution of electrons over momenta, which determines the electric current, is established during the momentum relaxation time $1/\nu_\epsilon^{(1)}$. However, the distribution of electrons over spins, which determines the spin density, is established during the spin relaxation time, and the corresponding transient process becomes infinitely slow if the spin relaxation is absent. Therefore, if a periodic alternating field acts on electrons in a sample with a fixed precession axis, the spin density cannot react to the field at any frequency $\omega$, and the spin polarizability is zero. The absence of the static spin polarization in the case of a fixed precession axis cannot be revealed by consideration of the static spin response alone. From the formal point of view, this paradox is related to the fact that in the static limit the additional part of the distribution function, $g_\mathbf{p} = G_\mathbf{p}(0)$, which cancels the contribution of the first term in the expression (19), still exists, only in the case of a fixed precession axis. One can say that the dependence of the induced spin polarization on the parameters (components $\mu_{\alpha\beta}$) of the spin-orbit Hamiltonian is non-analytic in the region of parameters where the fixed precession axis appears. This means that the result for the spin polarization depends on the order of limiting transitions. If first $\omega$ is aimed to zero and then the precession axis is fixed, the polarization is finite. If first the precession axis is fixed and then $\omega$ is aimed to zero, the polarization is zero. More details on the issue of non-analyticity will be given below in this section, by considering concrete examples. It is important to state that the consideration given above does not provide spin relaxation mechanisms other than the DPK mechanism. Inclusion of the Elliot-Yafet mechanism (see Refs. 42 and 43 for the 2D case) can lead to a finite relaxation of the spins oriented along the fixed precession axis and, therefore, to a finite induced spin polarization for this special case; see Sec. V for more discussion.

Using Eq. (23) together with Eqs. (20), (38), and (40), one can obtain an exact relation connecting the induced spin polarization and spin current. From Eqs. (38) and (23) one has $s = -(2\pi\omega)^{-1} \int_0^\infty dp^2 \langle \mathbf{f}_\mathbf{p} \times \mathbf{g}_\mathbf{p} \rangle$. Since $\mathbf{f}_\mathbf{p} \times \mathbf{g}_\mathbf{p} = \xi_\mathbf{p} \mu_\beta \mathbf{g}_\mathbf{p} \times \mathbf{q}_\beta$, where $\mu_\beta$ is the vector with components $\mu_{\alpha\beta}$, we find, for the case of parabolic band and $\xi_\mathbf{p} = 1$,}

$$s = -\frac{2m}{i\hbar \omega} \mu_{\beta} \times \mathbf{q}_\beta.$$
spin density and spin current recently derived\textsuperscript{18,32} for this model. The corresponding relations\textsuperscript{37,44} for the Rashba model also follow from Eq. (43).

Since it is established that the non-parabolic corrections to the band spectrum and the corrections to the spin-orbit Hamiltonian do not lead to qualitative modifications of the induced spin polarization and spin current, we neglect these corrections in the following, by assuming $\varepsilon_p = p^2/2m$ and $\zeta_p = 1$. Let us consider first the frequency behavior of the induced spin polarization and spin current in the quantum wells described by the Rashba model. The non-zero components of the spin-velocity matrix are $\mu_{yx} = -\mu_{xy} = v_R$, where $v_R$ is the Rashba velocity. The spin splitting described by the Hamiltonian (1) is isotropic in this situation. Using Eq. (33) and taking into account that the matrix (31) is diagonal, we obtain

$$\mathbf{s} = -\mathbf{\Omega}_s \frac{em}{2\hbar} \mathbf{T}(\omega, v_R),$$

where the frequency-dependent function

$$\mathbf{T}(\omega, v) = \int_0^\infty d\varepsilon_p \left( \frac{-\partial f_{\varepsilon_p}}{\partial \varepsilon_p} \right) \left( 1 - \frac{\varepsilon_p \partial \nu_p^{(2)} / \partial \varepsilon_p}{2 \nu_p^{(1)} - i\omega} \right) \frac{2(\nu_p/h)^2}{2(\nu_p/h)^2 - i\omega((\nu_p^{(1)} - i\omega)(\nu_p^{(2)} - i\omega) + 4(\nu_p/h)^2)}$$

has dimensionality of time. Equations (44) and (45) generalize the result of Ref. 31 obtained in the limit of short-range scattering potential to the case of arbitrary scattering potential. They also describe \textsuperscript{[111]}-grown quantum wells with both Rashba and Dresselhaus spin-orbit coupling, where $\mu_{yx} = -\mu_{xy} = v_R + 2v_D/\sqrt{3}$ and $v_D$ is the Dresselhaus velocity. In this case, $v_R$ in Eq. (44) should be merely replaced by $v_R + 2v_D/\sqrt{3}$ and the direction of the spin polarization vector $\mathbf{s}$ with respect to $\mathbf{E}$ is the same as for the pure Rashba coupling. In particular, $\mathbf{s} = \{em\nu_T(\omega, v) / 2\pi \hbar^2(-E_y, E_x, 0)$ where $v$ is either $v_R$ or $v_D + 2v_D/\sqrt{3}$, respectively. Next, in symmetric [001]-grown quantum wells, where only the Dresselhaus spin-orbit coupling is present and the non-zero components are $\mu_{yy} = -\mu_{xx} = v_D$, Eq. (44) with $\mathbf{T}(\omega, v_D)$ is also valid, leading to $\mathbf{s} = \{em\nu_D(\omega, v_D) / 2\pi \hbar^2(E_y, -E_x, 0)$. In all these cases, the spin polarization is in the quantum well plane ($s_z = 0$), and the direction of this polarization is frequency-independent. The spin currents are expressed in the applied field:

$$\left( \begin{array}{c} q_x \\ q_y \\ q_z \end{array} \right) = \frac{ie\omega}{4\pi \hbar} \mathbf{\Omega}_s \frac{em}{\nu_T} \int_0^\infty d\varepsilon_p \left( \frac{-\partial \nu_p}{\partial \varepsilon_p} \right) \left( \begin{array}{c} -E_y \\ E_x \end{array} \right),$$

where $\mathbf{T}(\omega, v)$ is given by Eq. (45) with $v = v_R$ for the pure Rashba coupling and $v = v_R + 2v_D/\sqrt{3}$ for [111]-grown quantum wells. For symmetric [001]-grown quantum wells one should use $v = v_D$ and change the sign of the right-hand side. Therefore, the symmetry properties and frequency dependence of the spin currents remain the same for all important cases of isotropic spin splitting considered in this paragraph. In the limit of short-range scattering potential and in the case of degenerate electrons, Eqs. (46) and (45) with $v = v_R$ give the results obtained previously in Ref. 16. The universal behavior\textsuperscript{13} of the spin-Hall conductivity $\Sigma_{xy} = -\Sigma_{yx}$ exists in the limit $\nu_T/\hbar \gg \nu_p^{(2)}$, where $\nu_T$ is the Fermi momentum.

Now we turn to more complicated situations when the anisotropy of spin splitting is essential due to combined effect of both Rashba and Dresselhaus spin-orbit coupling. These cases are considered in the following subsections in the limit of short-range scattering potential, when the expressions obtained in subsection A of Sec. II are valid.

### A. [001]-grown quantum wells

Let us study the case of [001]-grown quantum wells. If the Cartesian coordinate axes are chosen along the principal crystallographic directions, there are four components of the spin-velocity tensor:

$$\mu_{xy} = -\mu_{yx} = v_R, \quad \mu_{yy} = -\mu_{xx} = v_D.$$  \hspace{1cm} (47)

The spin splitting depends on the angle $\varphi$ of the vector $p$ according to $\Omega_s^2 / m = (v_R^2 + v_D^2)/(p/h)^2 - 2v_D v_R/(p/h)^2 \sin(2\varphi)$ and $\Delta_p^2 = a - b \sin(2\varphi)$, where

$$a = (v - i\omega)^2 + 4(v_R^2 + v_D^2)/(p/h)^2, \quad b = 8v_R v_D/(p/h)^2.$$ \hspace{1cm} (48)

The angular averaging in Eq. (26) results in $T_{xx}^T = T_{yy}^T = (v - i\omega)^2/2r - 1/2 + i\omega/v$ and $T_{xx}^{xy} = T_{yy}^{xy} = b/2r - (a/r - 1)(v_R^2 + v_D^2)/(4r v_D v_R)$, where $r = \sqrt{a^2 - b^2}$. After some transformations, we obtain the relation defining the tensors of spin polarizability and spin conductivity:

$$\left( \begin{array}{c} s_x \\ s_y \end{array} \right) = \frac{em}{2\pi \hbar^2 \nu_T} \int_0^\infty d\varepsilon_p \left( \frac{-\partial \nu_p}{\partial \varepsilon_p} \right) \times \left( \begin{array}{c} \kappa_p(\omega) - \bar{\kappa}_p(\omega) \\ \bar{\kappa}_p(\omega) - \kappa_p(\omega) \end{array} \right) \left( \begin{array}{c} E_x \\ E_y \end{array} \right),$$

and

$$\left( \begin{array}{c} q_x \\ q_y \\ q_z \end{array} \right) = \frac{ie\omega}{4\pi \hbar} \frac{v_D^2 - v_R^2}{4v_D v_R} \int_0^\infty d\varepsilon_p \left( \frac{-\partial \nu_p}{\partial \varepsilon_p} \right) \times \left( \begin{array}{c} \theta_p(\omega) - \bar{\theta}_p(\omega) \\ \bar{\theta}_p(\omega) - \theta_p(\omega) \end{array} \right) \left( \begin{array}{c} E_x \\ E_y \end{array} \right),$$

where the denominator is given by

$$D_p(\omega) = \frac{(v_R^2 - v_D^2)^2}{8e^2 v_D} \left( \frac{a}{r} - 1 \right),$$

$$+ \frac{i\omega}{v} \left( \frac{v - i\omega}{r} \right)^2 - 1 + \left( \frac{i\omega}{v} \right)^2.$$ \hspace{1cm} (51)
The elements of the matrix in Eq. (49) are

\[ \kappa_{\mu}(\omega) = \frac{v_R^2 - v_D^2}{4v_D} \left\{ \left[ \frac{v_R^2 - v_D^2}{2v_R^2} \right] - \frac{i\omega}{\nu} \right\} \times \left( \frac{a}{r} - 1 \right) + \frac{i\omega b}{\nu v_R r} \right\}, \]

and \( \bar{\kappa}_{\mu}(\omega) \) is obtained from this expression by the permutations \( v_R \leftrightarrow v_D \). The elements of the matrix in Eq. (50) are given by

\[ \theta_{\mu}(\omega) = \left( 1 - \frac{i\omega}{\nu} \right) \left( \frac{a}{r} - 1 \right) \]

and

\[ \bar{\theta}_{\mu}(\omega) = \frac{v_R^2 + v_D^2}{2v_Dv_D} \left( \frac{a}{r} - 1 \right) - \frac{i\omega b}{\nu r}. \]

Equations (49) and (50) demonstrate the symmetry relations \( \chi_{xx} = -\chi_{yy} \), \( \chi_{xy} = -\chi_{yx} \), \( \Sigma_{xx} = -\Sigma_{yy} \), and \( \Sigma_{xy} = -\Sigma_{yx} \).

Though the matrix in Eq. (49) retains the symmetry of \( \mu, \beta \), the ratio \( \bar{\kappa}_{\mu}(\omega)/\kappa_{\mu}(\omega) \) is not equal to \( v_R/v_D \). For this reason, the direction of spin polarization in the plane is different from the direction of \( \Omega_E \) and depends on the frequency. The direction of the spin current is not perpendicular to the direction of the field and is also frequency-dependent. The components of the spin density and spin current are related according to Eq. (43), which can be written, for this particular case, in the form

\[ q^x = (i\hbar \omega/2m) (v_D s_x + v_D s_y)/(v_R^2 - v_D^2) \] and \[ q^y = (i\hbar \omega/2m) (v_D s_x + v_D s_y)/(v_R^2 - v_D^2). \]

In the collisionless limit, \( \nu \rightarrow 0 \), Eqs. (49) and (50) are reduced to the results obtained in Ref. 32. Note that the formal substitution \( \nu \rightarrow 0 \) makes the spin currents finite at \( \omega \rightarrow 0 \) because the denominator \( D_{\mu}(\omega) \) is reduced to \( -\omega^2/\nu^2 \), while the functions (53) and (54) become proportional to \( \omega/\nu \). The static spin currents for the systems described by the spin-velocity matrix (47) have been also studied in the collisionless limit in Refs. 45 and 46. All these studies show that, as the ratio \( v_R/v_D \) is varied, the spin current reverses its sign going through zero at \( v_R^2 = v_D^2 \). This general property, also reflected in Eq. (50), follows from the Berry phase analysis.\(^{45}\)

In the case \( v_R^2 = v_D^2 \), as already mentioned, there exists a fixed precession axis directed at the angle of \( \pi/4 \) (or \(-\pi/4 \)) in the quantum well plane. Therefore, not only the spin current, but also the induced spin density goes to zero in this case. This behavior is demonstrated in Figs. 1-3, where the calculated real parts of the components of spin polarizability and spin conductivity tensors, \( \text{Re} \chi_{\alpha \beta} \) and \( \text{Re} \Sigma_{\alpha \beta} \), are plotted as functions of the ratio of Rashba and Dresselhaus velocities. The case of degenerate electron gas is assumed. The spin polarizability is expressed in the units of static polarizability for symmetric [001]-grown quantum wells, \( \chi_0 = emv_D/2\pi \hbar^2 \nu \). If the frequency decreases, both \( \chi_{xx} \) and \( \chi_{yx} \), as expected, approach their static values \( \chi_0 \) and \( (v_R/v_D) \chi_0 \), respectively. However, this never happens at \( v_R = v_D \), when the polarizability remains exactly zero at arbitrary \( \omega \). The real part of the spin conductivity (Fig. 2) shows prominent peaks near the point \( v_R = v_D \) at small \( \omega \), though its behavior is analytic. The imaginary parts of \( \chi_{xx} \) and \( \chi_{yx} \) (not shown) also have sharp peaks in the vicinity of \( v_R = v_D \) at small \( \omega \). The depression of the spin polarizability in the region \( v_R \approx v_D \) is extended with the increase of the disorder, as shown in Fig. 3. This also means that, especially for the "dirty" case \( v_D \nu \ll \hbar \nu \), the frequency dispersion of the spin polarizability remains significant at very small frequencies if \( v_R \) is close enough to \( v_D \). The corresponding behavior is illustrated in Fig. 4 and is explained by the reduc-
tion and disappearance of the DPK spin relaxation as \( v_R \) approaches to \( v_D \).

![FIG. 3: Effect of disorder on the real part of spin polarizability. The "clean" and the "dirty" cases correspond to \( v_{DPF}/\hbar \nu = 10 \) and \( v_{DPF}/\hbar \nu = 0.1 \), respectively.](image)

![FIG. 4: Low-frequency dispersion of the real part of spin polarizability in [001]-grown quantum wells in the vicinity of the fixed precession axis (\( v_R \) is close to \( v_D \)).](image)

**B. [110]-grown quantum wells**

The next case we consider is [110]-grown quantum wells. If the Cartesian coordinate axes are chosen with \( OZ \) perpendicular to the quantum well plane and \( OY \) along the principal crystallographic direction (\( x \parallel [110], y \parallel [001], z \parallel [110] \)), there are three components of the spin-velocity tensor:

\[
\mu_{xy} = -\mu_{yx} = v_R, \quad \mu_{xz} = v_D/2.
\]

One has \( \Omega_p^2 = (v_R^2 + v_D^2)/8(p/\hbar)^2 + (v_D^2/8)(p/\hbar)^2 \cos(2\varphi) \) and \( \Lambda_p^2 = c + d \cos(2\varphi) \), where

\[
c = (\nu - i \omega)^2 + (4v_R^2 + v_D^2/2)(p/\hbar)^2, \quad d = v_D(p/\hbar)^2/2.
\]

Applying the equations listed in subsection A of Sec. II, we find that the field \( E_x \) can induce both \( y \)- and \( z \)-polarized spins, while the field \( E_y \) induces only \( x \)-polarized spins:

\[
s_x = -\frac{emv_RE_y}{2\pi\hbar^2\nu} \int_0^\infty d\varepsilon_p \left( \frac{\partial f_{\varepsilon_p}}{\partial \varepsilon_p} \right) \left( 1 - i\frac{\omega}{\nu}K_p^x(\omega) \right), \tag{57}
\]

\[
s_y = \frac{emv_RE_x}{2\pi\hbar^2\nu} \int_0^\infty d\varepsilon_p \left( \frac{\partial f_{\varepsilon_p}}{\partial \varepsilon_p} \right) \left( 1 - i\frac{\omega}{\nu}K_p^y(\omega) \right), \tag{58}
\]

and \( s_z = -(v_D/2v_R)s_y \), where

\[
K_p^x(\omega) = \left( \frac{4v_R^2}{v_D^2} + 1 \right) \left( \sqrt{\frac{c-d}{2}} - 1 \right) + i\frac{\omega}{\nu}, \tag{59}
\]

and

\[
K_p^y(\omega) = -\frac{4v_R^2}{v_D^2} \left( \sqrt{\frac{c-d}{2}} - 1 \right) + i\frac{\omega}{\nu}. \tag{60}
\]

The component \( s_z \) is a symmetric function of \( v_R \), while \( s_x \) and \( s_y \) are antisymmetric functions of \( v_R \).

The spin currents appear for \( y \)- and \( z \)-polarized spins and flow in the direction perpendicular to the applied field:

\[
q^y_x = -\frac{i\hbar\omega}{mv_D(4v_R^2/v_D^2 + 1)}s_x, \quad q^y_y = \frac{i\hbar\omega v_D}{4mv_R^2} s_y, \tag{61}
\]

and

\[
q^z_x = (2v_R/v_D)q^y_x, \quad q^z_y = (2v_R/v_D)q^y_y. \tag{62}
\]

The relations (61) and (62) satisfy the general requirement (43). The components \( q^z_x \) and \( q^z_y \) are symmetric functions of \( v_R \), while \( q^y_x \) and \( q^y_y \) are antisymmetric functions of \( v_R \).

For symmetric quantum wells, where \( v_R = 0 \), both the polarization and the spin currents disappear for arbitrary \( \omega \). If both \( v_R \) and \( \omega \) go to zero, the \( z \)-component of the spin polarizability tensor, \( \chi_{zz}(\omega) \), is described by a simple formula applied to the case of degenerate electrons:

\[
\chi_{zz} \approx -\frac{\gamma_F}{2} \left( \frac{2v_R/v_D}{(v_D/v_R)^2 \gamma_F - i\omega} \right), \tag{63}
\]

where \( \gamma_F = \sqrt{\nu^2 + (v_{DPF}/\hbar)^2} - \nu \). Equation (63) illustrates the non-analytic behavior of the spin polarizability in the vicinity of the fixed precession axis. The calculated dependence of the real part of \( \chi_{xx} \) on the ratio \( v_R/v_D \), shown in Fig. 5 for the case of degenerate electron gas, is similar to the dependence of \( \chi_{xx} \) shown in Fig. 1 for [001]-grown wells. The region of depression near \( v_R = 0 \) is extended in the "dirty" limit \( v_{DPF} \ll \hbar \nu \), as it is seen directly from Eq. (63). The behavior of the component \( \chi_{xy} \) remains analytic at \( v_R \to 0 \) and \( \omega \to 0 \), though it is strongly affected in the vicinity of \( v_R = 0 \) at low frequencies. In contrast, the component \( \chi_{xy} \) (not shown in
Fig. 5) stays close to its static value \(-(v_R/v_D)\chi_0\) up to the frequency region \(\omega \sim \nu\). The real part of the spin conductivity \(\Sigma^z_{\mu z}\) shows peaks in the vicinity of \(v_R = 0\) at low frequencies, while the low-frequency behavior of \(\Sigma^z_{\nu z}\) is monotonic in this region, see Fig. 6. According to Eq. (63), the frequency dispersion of the spin polarizability remains significant at very low frequencies if \(v_R\) is small enough. It is remarkable that in the "dirty" limit the characteristic rate describing this dispersion, \((2v_R/v_D)^2\gamma_F\), is equal to \(2(v_Rp_F/h)^2/\nu\), which is the DPK spin relaxation rate for the Rashba model.

\[
\nu_{\text{DPK}} = 2(v_Rp_F/h)^2/\nu
\]

FIG. 6: Dependence of the real part of spin conductivity in [110]-grown quantum wells on the ratio \(v_R/v_D\) at \(v_Dp_F/h\nu = 1\) for several frequencies. The polarizability is expressed in the units of \(\chi_0 = emvD/2\pi h^2\nu\).

IV. CURRENT RESPONSE

It is important to relate the behavior of spin polarization and spin currents studied in the previous section to the frequency dispersion of the conductivity (or dielectric function) of 2D electron layers with spin-orbit interaction. Some relations of this kind have been established previously\(^{33,39}\) in the collisionless approximation. In this section the corresponding relations are obtained and analyzed for the general \(p\)-linear model of spin-orbit coupling described by the Hamiltonian (1), with taking into account the electron-impurity interaction. The calculation is based on the kinetic equation (2). The electric current density is defined as

\[
j = e \int \frac{dp}{(2\pi\hbar)^2} \text{Tr}(\hat{u}(p)\hat{f}_p).
\]

Below we neglect the non-parabolicity of the energy spectrum and the deviation of the Hamiltonian (1) from the linearity, by putting \(m_p = m\) and \(\xi_p = 1\). Let us multiply Eq. (2) by \(ep/m\), sum it over \(p\), and take the matrix trace. Using Eqs. (6) and (64), one has the exact relation

\[
-\nu j_{\beta} - \frac{2e\mu_\beta \delta_{\alpha \alpha}}{m} = \frac{e^2E_{\beta}n_{2D}}{m} = e \int \frac{dp}{(2\pi\hbar)^2} \frac{p_\beta}{m} \text{Tr}(\hat{f}(\hat{p})).
\]

The right-hand side of this equation should be set at zero in the collisionless approximation. To consider the collision-induced contribution in the general case, one should calculate the distribution function \(\hat{f}_p\) with the accuracy up to the terms \(\sim (\hbar\Omega_p/\nu)^2\). Instead of doing this, we consider the limit of short-range scattering potential, when the right-hand side of Eq. (65) is exactly transformed to \(-\nu j_{\beta}\). Therefore,

\[
j_{\beta} = \frac{e^2n_{2D}}{m(\nu - i\omega)} E_{\beta} = \frac{2i\nu}{\nu - i\omega} \mu_\alpha \mu_\beta.
\]

Equation (66) contains the usual Drude term and the term induced by the spin-orbit interaction.\(^{38,39}\) Together with Eq. (43), this equation establishes the relationship between the electric current, spin current, and induced spin density. The validity of both Eq. (43) and Eq. (66) is not restricted by the assumption of linear response.

Expressing the currents through the electric conductivity \(\sigma_{\beta\lambda}(\omega)\) and spin conductivity \(\Sigma^\alpha_{\beta\lambda}(\omega)\), one can write the equation that relates these tensors:

\[
\sigma_{\beta\lambda} = \delta_{\beta\lambda} \frac{e^2n_{2D}}{m(\nu - i\omega)} + \sigma_{\beta\lambda}^{SO},
\]

\[
\sigma_{\beta\lambda}^{SO} = \frac{4me}{\hbar(\nu - i\omega)} \left[\mu_\beta \times \mu_\alpha\right] \delta_{\alpha\beta} \Sigma^\delta_{\alpha\lambda}.
\]

In general, the spin-orbit term \(\sigma^{SO}_{\beta\lambda}\) brings non-diagonal contributions to \(\sigma_{\beta\lambda}\). This should lead to a weak Hall effect in the absence of magnetic field at finite frequencies. For the Rashba model, when only the components \(\mu_{xy} = -\mu_{yx} = v_R\) exist, the conductivity is diagonal and isotropic,\(^{47}\) \(\sigma^{SO}_{\beta\lambda} = \sigma^{SO}\delta_{\beta\lambda}\), where

\[
\sigma^{SO} = \frac{4mev_R^2}{\hbar(\nu - i\omega)} \Sigma^z_{xy}.
\]
In the collisionless limit, this equation gives a relation\textsuperscript{49} between the imaginary part of \( \sigma^{(SO)} \) and the real part of the spin-Hall conductivity \( \Sigma_{xy} \).

In the case of [001]-grown quantum wells with both Rashba and Dresselhaus types of coupling,

\[
\begin{pmatrix}
\sigma_{xy}^{(SO)} \\
\sigma_{yx}^{(SO)}
\end{pmatrix} = \frac{4me(v_R^2 - v_D^2)}{\hbar(\nu - i\omega)} \begin{pmatrix}
-\Sigma_{xy} \\
\Sigma_{yx}
\end{pmatrix}.
\]  

(69)

The non-diagonal part of \( \sigma_{xy}^{(SO)} \) is related to the diagonal components of the spin conductivity, while the diagonal part is expressed through the spin-Hall conductivity (such an expression has been recently established\textsuperscript{33} in the collisionless regime). The tensor \( \sigma_{\beta\lambda}^{(SO)} \) can be diagonalized by in-plane rotation of the Cartesian coordinate axes \( OX \) and \( OY \). In contrast to the spin polarization, the quantity \( \sigma_{\beta\lambda}^{(SO)} \) is analytic at \( v = v_D \).

In the case of [110]-grown quantum wells the conductivity is diagonal (in the chosen coordinate system) but anisotropic:

\[
\begin{pmatrix}
\sigma_{xy}^{(SO)} \\
\sigma_{yx}^{(SO)}
\end{pmatrix} = \frac{4me(v_R^2 + v_D^2/4)}{\hbar(\nu - i\omega)} \begin{pmatrix}
-\Sigma_{xy} \\
\Sigma_{yx}
\end{pmatrix}.
\]  

(70)

The expressions for the components \( \Sigma_{\alpha\beta} \) entering Eqs. (69) and (70) are obtained from the expressions for spin currents presented in subsections A and B of the previous section. Although the contribution \( \sigma_{\beta\lambda}^{(SO)} \) is small as \( (\hbar\Omega_p/\pi)^2 \) with respect to the Drude conductivity, its frequency dependence has qualitatively new features. If \( \Omega_p \gg \nu \), so that the spin-split states are well-defined, the term \( \sigma_{\beta\lambda}^{(SO)} \) describes resonance absorption of electromagnetic radiation, typically in the THz region, associated with transitions between these states. In the case of isotropic spin splitting\textsuperscript{48} (the Rashba model is considered below) and degenerate electron gas, the resonance takes place\textsuperscript{48} at \( \omega \approx \omega_p \equiv 2|v_R|p_F/\hbar \). Substituting in Eq. (68) the detailed expression for the spin conductivity, see Eq. (46) and Eq. (45) with \( \nu_p^{(1)} = \nu_p^{(2)} = \nu \), one has an equation

\[
\sigma^{(SO)} = \frac{e^2mv_R^2i\omega}{\pi\hbar^2(\nu - i\omega)} \frac{\omega^2}{\omega^2 + (\nu - i\omega)^2 + \omega_D^2}.
\]  

(71)

which describes the resonance under consideration at \( \omega \gg \nu \). Of course, this resonance also exists in the frequency dependence of the spin conductivity and spin polarizability.\textsuperscript{31} Another important feature following from Eq. (71) is the presence of low-frequency dispersion under the opposite condition, \( \omega \ll \nu \). This dispersion appears when \( \omega \) is comparable to the DPK spin relaxation rate. Since this rate, for in-plane spin polarization, is given by \( \nu_{DP} = 2(\nu_p^2p_F^2/\hbar^2) \nu_p \), Eq. (71) in the limits \( \omega \ll \nu \) and \( \omega \approx \nu \) gives

\[
\text{Re}\sigma^{(SO)} \approx -\frac{e^2mv_R^2\nu_D}{\pi\hbar^2} \frac{\omega^2}{\omega^2 + \nu_D^2}.
\]  

(72)

Therefore, \( \sigma^{(SO)} \) essentially depends on \( \omega \) in the region of frequencies \( \omega \ll \nu \), when the Drude conductivity still remains frequency-independent. Another contribution to the frequency dependence of the conductivity in this region exists owing to weak localization.\textsuperscript{49} Though the weak-localization correction is larger in magnitude than \( \text{Re}\sigma^{(SO)} \), its frequency dependence is slow (logarithmic), and can be distinguished from the dependence given by Eq. (72).

V. SUMMARY AND DISCUSSION

In this paper, the electric-field-induced spin density \( s \) and intrinsic spin current \( q_\beta \) in 2D electron layers described by the general \( p \)-linear spin-orbit interaction Hamiltonian \( \hat{\sigma}_\alpha \mu_\alpha p_\beta \equiv \hat{\sigma} \cdot \mu p_\beta \) are studied in the classical region of frequencies \( \omega \). The consideration is done for macroscopic systems and at zero magnetic field. The quantities \( s \) and \( q_\beta \) are closely related to each other through Eq. (43) following from the balance equation for spin density. To find them, a careful analysis of the quantum kinetic equation is carried out taking into account interaction of electrons with impurities or other static inhomogeneities. The presented results are valid for arbitrary correlation between the spin splitting energy \( 2\hbar\Omega_p \), disorder-induced broadening \( h\nu \), and energy \( h\omega \), under condition that all these energies are small in comparison with the characteristic kinetic energy of electrons. A complete analytical solution of the linear-response problem for the matrix distribution function is given in the limit of short-range scattering potential (see subsection A of Sec. II). This solution is applied to [001]- and [110]-grown quantum wells with both Rashba and Dresselhaus types of spin-orbit coupling (subsections A and B of Sec. III). All other situations when analytical solutions exist are described in subsections B, C, and D of Sec. II. The theory also takes into account the isotropic energy-dependent corrections to the effective mass \( m \) (non-parabolicity effect) and to the spin-velocity matrix \( \mu_\alpha \). This is reflected by the substitutions \( m \to m_p \) and \( \mu_\alpha \to \hat{\xi}_\alpha \mu_\alpha \) assumed from the beginning of the consideration. Since these weak corrections do not lead to qualitative effects (in particular, it is shown that the static spin currents remain equal to zero in the presence of these corrections), they are ignored in the most part of the applications, starting from Eq. (42).

The main approximation of the present consideration is the neglect of the spin-dependent contribution to the scattering potential. This contribution, also caused by the spin-orbit interaction, is often referred to as the extrinsic spin-orbit coupling. In the first order with respect to the extrinsic spin-orbit coupling, there appear spin currents which are not equal to zero in the static limit. This leads to the extrinsic spin-Hall effect,\textsuperscript{50,51,52} which is beyond the scope of the present paper. The extrinsic spin-orbit coupling also leads to an additional induced spin polarization, which is considered, in the limit...
with quite different positions. In \([001]\)-grown quantum wells the \([001]\)- and \([110]\)-grown quantum wells appear to be in vicinity of the fixed precession axes can be influenced by filtering and amplification in future spintronics devices. Phenomena are of interest for the purposes of frequencyimention. As concerns possible technological use, these of the quantum well via a bias applied to an external the Rashba velocity is variable by modifying the shape quantum wells this dispersion is given by Eq. (63). Since electrical bias to the 2D sample. For \([110]\)-grown quan-
tion is significant in the low-frequency region, when the quantum wells, the frequency dispersion of spin polariza-
tion is close to zero for \([110]\)-grown symmetric quantum wells, where the fixed precession axis is per-
pendicular to the plane of motion, the applied electric field cannot excite the in-plane polarized spins even in the presence of the extrinsic spin-orbit coupling. Excitation of the current of \(z\)-polarized spins (the extrinsic spin-Hall current) does not lead to spin polarization in \(z\) direction far from the boundaries of the sample. Finally, the Elliot-Yafet relaxation of \(z\)-polarized spin density is absent in \([110]\)-grown symmetric quantum wells. In conclusion, the extrinsic spin-orbit coupling does not lead to a finite spin polarization in \([110]\)-grown symmetric quantum wells and cannot modify Eq. (63). Modification of the low-frequency behavior described by this equation may occur owing to non-Markovian memory effects and effects of spatial inhomogeneity due to finite sample size.

The other important result is Eq. (66), which establishes a simple relation between the electric current and induced spin polarization. Though the validity of this equation is restricted by the approximation of short-range scattering potential, Eq. (66) is applicable to the whole class of the systems described by the \(p\)-linear spin-orbit Hamiltonians and does not require the linear response approximation. It can be useful in applications and in analysis of experiments where both spin polarization and electric conductivity are measured. Equation (66) also shows that the spin-orbit term in the electric conductivity is of purely dynamic origin, this term vanishes at \(\omega \to 0\). It is not clear whether this property is specific for the short-range scattering potential or remains valid for arbitrary scattering potential. The corresponding calculations are now in progress. In combination with Eq. (43), the result (66) relates the electric current with the spin current and leads to a unified description of spin and charge response to the applied electric field.

The results for frequency dispersion of the spin polarization and spin current obtained in this paper can be directly reformulated for description of transient spin response,\textsuperscript{18,29,31} which is investigated experimentally by the time-resolved spectroscopy.\textsuperscript{5,11,56} If the spin-split electron states are well-defined, the transient process
shows coherent oscillations. If the spin splitting is suppressed by collisional broadening, there should appear long-time transients associated with the DPK spin relaxation.\textsuperscript{18,31} Under conditions close to the appearance of the fixed precession axis, for example, when \(v_{Q}\) is close to zero in [110]-grown quantum wells, the duration of the transient process, according to Eq. (63), is expected to increase dramatically.

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