From inflation to dark energy in the non-minimal modified gravity

Shin’ichi NOJIRI*1, Sergei D. ODINTSOV*)*2, and Petr V. TRETYAKOV*3

*1 Department of Physics, Nagoya University, Nagoya 464-8602, Japan
*2 Institució Catalana de Recerca i Estudis Avançats (ICREA) and Institut de Ciencies de l’Espai (IEEC-CSIC), Campus UAB, Facultat de Ciencies, Torre C5-Par-2a pl, E-08193 Bellaterra (Barcelona), Spain
*3 Sternberg Astronomical Institute, Moscow 119992, Russia

Abstract

We consider the modified gravity non-minimally coupled with matter Lagrangian for the description of early-time and late-time universe. Such \( F(R) \) \( (F(G)) \) gravity in the absence of non-minimal coupling is viable theory which passes the local tests and reproduces the \( \Lambda \)CDM era. For qualitatively similar choice of non-minimal gravitational coupling function it is shown that the unified description of early-time inflation and late-time cosmic acceleration is possible. It is interesting that matter (scalar) which supports the inflationary era is gravitationally screened at late times. Hence, it may be effectively invisible at current universe.
§1. Introduction

It has been realized recently that modified gravity (for a review, see 1)) may describe the universe expansion history quite realistically. In such a picture, the Einstein gravity is just the approximation to more complete (classical) gravity where some gravitational terms which are leading at early-time universe support the inflation while other terms which are dominant at late-time universe cause the cosmic acceleration. The simple gravitational model of such unification of inflation with late-time acceleration is suggested in ref.2). It also has been discovered the viable modified gravity\(^3\) (for related models, see 4)–6)) which describes the \(\Lambda\)CDM epoch similar to usual Einstein gravity with cosmological constant. Moreover, such theory passes the local tests\(^3,4\) (for review of confronting the observational data with modified gravity predictions, see 7)). Finally, such class of modified gravities may successfully describe the universe expansion history from the early-time inflation till late-time acceleration\(^4,5\) with correct intermediate epoch.

Another gravitational source of the inflation and dark energy may be the non-minimal coupling of some geometrical invariants function with matter Lagrangian. Such non-minimal modified gravity has been introduced in refs. 8), 9) for the study of gravity assisted dark energy occurrence. It may be also applied for realization of dynamical cancellation of cosmological constant.\(^10\) The viability criteria for such theory was recently discussed in refs. 11)–13). In the present work we consider non-minimal modified gravity for the class of functions introduced in ref. 3), 5). For simplicity, as the matter we consider just usual scalar kinetic term. It is shown that for such model one can easily achieve the unification of the inflation with cosmic acceleration. Moreover, it may suggest the explanation why inflaton is not seen at the late universe: because it is screened by gravitational function which quickly goes to zero with universe expansion.

§2. Unification of the inflation with late-time acceleration in the non-minimal modified gravity

In this section we obtain the equations of motion for general modified gravity non-minimally coupled with matter. These equations will be used for the investigation of the unified inflation-late-time acceleration epoch emergence for several realistic models.

The starting theory is:

\[
S = \int d^4x \sqrt{-g} \left[ \frac{1}{\kappa^2} R - f_1(A)L_d - f_2(A) \right],
\]  

(2.1)

where \(A\) is some function of geometrical invariants and \(L_d\) is the matter Lagrangian. In
this work we choose $A$ to be equal to $R$ or Gauss-Bonnet invariant $G$, for simplicity. We also assume $L_d$ is scalar function: $L_d = \frac{1}{2} n^{-2} \dot{\phi}^2$, where $\phi = \phi(t)$ and metric corresponds to spatially-flat FRW universe:

\[ g_{\mu\nu} = \text{diag}(-n(t)^2, a(t)^2, a(t)^2, a(t)^2) . \]  

\[ (2.2) \]

Varying (2.1) on $\phi$ one arrives to the scalar field equation:

\[ f_1(A)a^3 \dot{\phi} n^{-1} = q = \text{const.}, \]  

\[ (2.3) \]

Substituting it to (2.1) one gets

\[ S = \int d^4 x \left[ \frac{1}{\kappa^2} R n a^3 - \frac{2 q n^2}{2 f_1(A)a^3} - f_2(A) n a^3 \right] = S_R + S_1 + S_2 . \]  

\[ (2.4) \]

Varying (2.4) on $n$ the FRW equation follows:

\[ \delta S_R = \frac{6}{\kappa^2} \int d^4 x a \dot{a}^2 \delta n . \]  

\[ (2.5) \]

(after variation we may put $n = 1$)

\[ \delta S_1 = \int d^4 x \left[ - \frac{q^2}{2 f_1(A)a^3} \dot{n} + \frac{q^2}{2a^3 f_1(A)^2} \frac{\partial f_1}{\partial A} \frac{\partial A}{\partial \delta n} + \frac{q^2}{2a^3 f_1(A)^2} \frac{1}{\partial A} \frac{\partial A}{\partial \delta n} \right] \]  

\[ = \int d^4 x \left[ - \frac{q^2}{2 f_1(A)^2 a^3} + \frac{q^2}{2a^3 f_1(A)^2} A_n + 3 \frac{q^2}{2a^3} H \frac{f_1'(A)}{f_1} A_n + \frac{q^2}{a^3 f_1^2} A_n \dot{A} \right. \]  

\[ \left. - \frac{q^2}{2a^3 f_1^2} A_n \dot{A} - \frac{q^2}{2a^3 f_1^2} \frac{d A_n}{d t} \right] \delta n , \]  

\[ \delta S_2 = - \int d^4 x \left[ a^3 f_2 + f_2' a^3 A_n - 3 a^3 H f_2' A_n - a^3 f_2' \frac{d A_n}{d t} - a^3 f_2'' A_n \dot{A} \right] \delta n . \]  

\[ (2.6) \]

Now FRW equation may be written in the following form:

\[ \frac{6}{\kappa^2} H^2 = \rho_1 + \rho_2 , \]  

\[ (2.7) \]

where

\[ \rho_1 = \frac{q^2}{f_1 a^6} \left[ \frac{1}{2} - \frac{1}{2} \frac{f_1'}{f_1} A_n - \frac{3}{2} \frac{H f_1'}{f_1} A_n - \frac{(f_1')^2}{f_1^2} A_n \dot{A} + \frac{1}{2} \frac{f_1''}{f_1} A_n \dot{A} + \frac{1}{2} \frac{f_1'}{f_1} \frac{d A_n}{d t} \right] , \]  

\[ (2.8) \]

\[ \rho_2 = f_2 + f_2' A_n - 3 H f_2' A_n - f_2' \frac{d A_n}{d t} - f_2'' A_n \dot{A} . \]  

\[ (2.9) \]

The important remark is in order. Our FRW-like equation contains higher derivative term which is multiplied by some function. If this function approaches to zero during evolution we discover so-called finite time singularity (for their classification, see 14)). To avoid the
singularity it is necessary that this function (multiplying by higher derivatives) is not equal to zero in any moment. It is clear from Eqs. (2.8)-(2.9) that higher derivatives appear only in \( \dot{A} \) term. Hence, the higher derivatives function is given by:

\[
HD = \dot{A} a \left[ \frac{q^2}{f_1 a^6} \left( \frac{1}{2} f_1'' - \frac{(f_1')^2}{f_1} \right) - f_2'' \right].
\]  

(2.10)

When it is not zero, the finite-time singularity does not occur.

Let us now consider the following choice of function \( A \): \( A = R \). Then \( R = \frac{\dot{a}}{a^2} \left[ \frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a} \right] \), \( R_n = 2R = -12(\dot{H} + 2H^2) \), \( R_n = -6H \).

The effective energy-density becomes (below \( n = 1 \)):

\[
\rho_1 = \frac{q^2}{f_1 a^6} \left[ \frac{1}{2} + 3 \frac{f_1'}{f_1} (\dot{H} + 7H^2) + 6H \frac{(f_1')^2}{f_1^2} \dot{R} - 3H \frac{f_1''}{f_1} \dot{R} \right],
\]  

(2.11)

\[
\rho_2 = f_2 - 6f_2' (\dot{H} + H^2) + 6HF_2'' \dot{R}.
\]  

(2.12)

This FRW equation is used in the explicit analysis below.

Einstein gravity non-minimally coupled with matter. Let us consider the Einstein gravity non-minimally coupled with matter Lagrangian:

\[
S = \int d^4x \sqrt{-g} \left[ \frac{1}{\kappa^2} R - f_1(A) L_d \right],
\]  

(2.13)

where it is chosen \( f_2 \equiv 0 \). Note that recently observational bounds for such non-minimal coupling were discussed in 13). Let us take the following explicit choice for function \( f_1 \) which was discussed recently for modified \( F(R) \) gravity in 3), 4):

\[
f_1 = \frac{c_1 R^k}{c_2 R^k + 1},
\]  

(2.14)

where \( c_1 \) and \( c_2 \) are the arbitrary constants. The FRW equation looks as: \( \frac{6}{\kappa^2} H^2 = \rho_1 \).

Let us discuss the cosmologically important limit of this theory \( R \to 0 \) which corresponds to current universe. In this limit, the approximated FRW equation is obtained as follows:

\[
\frac{6}{\kappa^2} H^2 = \frac{18q^2}{a^6 c_1(6H + 12H^2)^{k+2}} [\dot{H}^2(k+1) + H^4(k+14k) + \dot{H} H^2(4 + 13k + 4k^2) + k(k+1)H \ddot{H}].
\]  

(2.15)

One can study the power-like solutions of (2.15): \( a = a_0 t^x, \ H = \frac{\dot{x}}{x}, \ \dot{H} = -\frac{\dot{x}}{x^2}, \ \ddot{H} = 2\frac{\ddot{x}}{x^3} \). Substituting these relations into (2.15) we find: \( x = \frac{k+1}{3} \). From another side \( a \propto t^{\frac{2}{3(k+1)}} \), where \( w \) is defined as \( p = w \rho \), and therefore we find \( w = \frac{1-k}{1+k} \). Hence, late-time accelerated universe \(( -1 < w < -\frac{1}{3} ) \) occurs when \( k > 2 \). Effective phantom regime emerges when \( k < -1 \). When \( w \) is negative, the universe shrinks when \( t \) is positive since \( x < 0 \). If
we replace \( t \) with \( t_s - t \) and assume \( t < t_s \), the universe is expanding. The universe has a singularity at \( t = t_s \), which corresponds to so-called “Big Rip” singularity. We should also note that exact de Sitter solution is impossible in this theory because there is scale factor \( a \) in equation (2.16). It always depends on time while all other parameters are constant. Nevertheless, de Sitter space may be realized asymptotically when \( k = -1 \).

At the early universe it is known that \( R \to \infty \). In this case \( f_1 \approx \frac{c_3}{c_4} - \frac{B}{R^m} \), where \( B \) is some constant. Then \( \rho_1 \) is about \( \rho_1 \sim f_1^{-1}a^{-6} \) where \( f_1^{-1} \approx \frac{c_3}{c_4} + R^{-k}/c_1 \) at this period. Hence, one sees that in this theory inflation (quasi-de Sitter stage) is possible in principal, because \( \rho_1 \) is sufficiently large at early times. The inflation stage is not stable as it should be. Indeed, \( a \) increases exponentially while \( f_1 \) decreases only as some power of time. Thus, the principal possibility to unify the inflation with late-time acceleration in above theory is proved. The important property of such unification is quick decay of gravitationally-coupled inflaton. That may suggest the scenario why the early time inflaton is not seen in current universe: it may be screened by gravitational coupling function which tends to zero with curvature decrease.

Modified \( F(R) \) gravity non-minimally coupled with matter. Let us start from the theory (2.1) where \( f_1 \) is given by (2.14) and \( f_2 \) is taken in the similar form:

\[
f_2 = \frac{c_3R^m}{c_4R^m + 1}.
\]

The unification of early-time inflation with late-time acceleration in such theory without non-minimal coupling with matter has been recently studied in ref. 5). Let us estimate now the role of non-minimal coupling with matter Lagrangian.

At late-time universe when \( R \to 0 \) the cosmic acceleration was studied in ref. 3) in the theory without non-minimal coupling \((f_1 = 0)\). Moreover, it was demonstrated there that the late-time universe corresponds to usual \( \Lambda \)CDM epoch subject the corresponding choice of parameters of the theory. The theory also passes local tests.\(^{(3),(4)}\) Hence, when the parameters of the theory are chosen in such a way that \( \rho_1 \) may be neglected if compare with \( \rho_2 \) then late-time cosmic acceleration corresponds to \( \Lambda \)CDM epoch. The dark energy universe is realistic and complies with observational data. In the opposite situation, when \( \rho_1 \) gives the leading contribution to the effective energy-density one comes back to the case described above for pure Einstein gravity non-minimally coupled with matter where late-time acceleration is again possible.

At the early-time universe we have \( f_2 = \frac{c_3}{c_4} - \frac{B}{R^m} \), where \( B \) is some constant and respectively \( \rho_2 = C - \frac{B}{R^m} \), where \( C \) and \( B_1 \) are constants. When only \( \rho_2 \) presents in FRW equation, we have quasi-inflationary stage as the solution. From another side, one may estimate \( \rho_1 \) as \( \rho_1 \sim f_1^{-1}a^{-6} \) at this period. Since inflation starts from some \( a_s \neq 0 \), it is clear that \( \rho_1 \)
makes some contribution to the inflation. However, this contribution very rapidly decreases because $a$ increases exponentially while $f_1$ decreases only as some power of time. Thus, we again have the theory with unified inflation and late-time acceleration. It is important to note here that the presence of non-minimal coupling with matter (after some fine-tuning) may improve the theoretical estimations which are compared with the observational data like cosmological parameters (effective equation of state parameter, etc), and cosmological perturbations.

Another choice for functions $f_1, f_2$ is motivated by the realistic and viable model which was considered in ref. 5):

\begin{align}
  f_2(R) &= \frac{(R - R_0)^{2k+1} + R_0^{2k+1}}{c_3 + c_4 [(R - R_0)^{2k+1} + R_0^{2k+1}]}, \\
  f_1(R) &= \frac{(R - R_0)^{2k+2m+1} + R_0^{2k+2m+1}}{c_1 + c_2 [(R - R_0)^{2k+2m+1} + R_0^{2k+2m+1}]}. 
\end{align}

(2.17) (2.18)

It is known\(^5\) that the theory with $f_1 = 0$ leads to the unification of the early-time inflation and late-time acceleration with realistic universe expansion history(radiation/matter dominance, transition from deceleration to acceleration) between these two eras. This could be realized since $f_2(R)$ satisfies the following conditions:

\begin{align}
  \lim_{R \to \infty} f_2(R) &= A_i \equiv \frac{1}{c_4}, \\
  f_2(R_0) &= 2R_0 = \frac{R_0^{2k+1}}{c_3 + c_3 R_0^{2k+1}}, \\
  f_2'(R_0) &= 0. 
\end{align}

(2.19)

Here $R_0$ is current curvature $R_0 \sim (10^{-33}\text{eV})^2$. Then in the above model, the universe starts from the inflation driven by the effective cosmological constant $\Lambda_i$ at the early stage, where curvature is very large. As curvature becomes smaller, the effective cosmological constant also becomes smaller. After that the radiation/matter dominates. When the density of the radiation and the matter becomes small and the curvature goes to the value $R_0$, there appears the small effective cosmological constant $2R_0$. Hence, the current cosmic expansion could start. We also note that $f_2(R)$ satisfies the condition

\begin{align}
  \lim_{R \to 0} f(R) = 0, 
\end{align}

(2.20) which shows the existence of the flat spacetime solution.

Now let us check at which conditions $f_1$ term (almost) does not change late-time acceleration. To satisfy this, it is necessary that $\rho_1$ tends to zero at $R \to R_0$ more rapidly than $\rho_2$ tends to constant. Simple check shows that acceptable choice of parameters corresponds to $c_1 = -c_2 R_0^{2k+2m+1}$. For $m > 0$ there is no any influence on late-time acceleration from scalar term, because near $R = R_0$ one finds $\rho_2 \sim \text{const} + (R - R_0)^{2k+1}$, while $\rho_1 \sim (R - R_0)^{2k+2m+1}$.
From another side, there is some contribution from scalar term to early-time inflation and, therefore, to cosmological perturbations. However, this contribution very rapidly decreases as $\frac{1}{a^6}$. Moreover, if $2k+2m+1 > 1$ there may be one more accelerating regime in the future. Thus, it is shown the principal possibility to unify the inflation with late-time acceleration in the modified gravity model\(^5\) even in the presence of non-minimal coupling with matter.

§3. Inflation and late-time acceleration in non-minimal $F(G)$-gravity

Let us study the theory with $A = G$, where $G = R^2 - 4R_{\mu\nu}^2 + R_{\mu\nu\alpha\beta}^2$ is Gauss-Bonnet invariant. Such $F(G)$ gravity has been introduced in ref. 15) as gravitational alternative for dark energy (for recent study of local tests in such theory with power-law function $F(G)$, see\(^16\)). The dark energy application of its non-minimal coupling with matter was investigated in ref. 17).

Let us consider the same qualitative choice for functions $f_1, f_2$ as in previous section:

$$f_1(G) = \frac{G^k}{c_1G^k + c_2}, \quad f_2(G) = \frac{G^m}{c_3G^m + c_4}. \quad (3.1)$$

It is known\(^5\) that in the absence of non-minimal coupling $f_1 = 0$ such a model naturally leads to unification of the inflation with late-time acceleration being viable theory. The presence of $f_1$ term does not qualitatively change the situation at the early universe. Indeed, when $t \to 0$, $\rho_{G2} \simeq const$ while $\rho_{G1} \propto a^{-6}$. Hence, $\rho_{G1}$ rapidly decreases at the expanding universe and can be neglected. The inflationary epoch emerges. At the late-time universe ($t \to \infty$) one can see that $\rho_{G2} \to 0$ supporting the acceleration, while $\rho_{G1} \propto a^{-6}G^{-k}$. Hence, for sufficiently big $k$, we may get the situation where $\rho_{G1} \to const$ or it is growing. In this case, the dominant contribution to the late-time acceleration is due to non-minimal coupling with matter. In this case, the matter perturbations of such theory may be significantly changed if compare with the case without non-minimal coupling.

Let us discuss the occurrence of cosmic acceleration in more detail. In this case one can put $\rho_{G2} = 0$ and $f_1 = G^k/c_2$. Searching for the power-like solutions: $a = a_0t^x$, $H = \frac{x}{t}$, $\dot{H} = -\frac{x}{t^2}$, $\ddot{H} = 2\frac{x}{t^3}$, $G \propto t^{-4}$, $f_1 \propto t^{-4k}$ in FRW Eq.(7), we find: $-2 = -6x + 4k$ or $x = \frac{2k+1}{3}$. Thus, the effective equation of state parameter is given by $w = \frac{1-2k}{1+2k}$. The late-time accelerating universe ($-1 < w < -\frac{1}{3}$) occurs if $k > 1$. Effective phantom era emerges if $k < -\frac{1}{2}$ while at the very large $k$ the universe is described by $\Lambda$CDM cosmology. The case of $k = -\frac{1}{2}$ leads to asymptotically de Sitter space. Thus, the principal possibility of the unification of the early-time inflation with late-time acceleration is again established. The realistic intermediate epoch follows in the same way as it is shown in ref. 5).
§4. Discussion

In summary, we considered $f(R)$ or $F(G)$ modified gravity non-minimally coupled with matter Lagrangian which is chosen to be the scalar kinetic energy, for simplicity. It is demonstrated that in such theory with the special choice of gravitational functions the realistic universe expansion history emerges quite naturally. In particular, the unification of the early-time inflation with late-time acceleration occurs within the same theory. In addition, as non-minimal gravitational coupling with matter goes to zero at late times, there appears the scenario explaining why scalars may be not seen in the present universe. Moreover, one can conjecture that it is really modified gravity which is responsible for early-time inflation. The success of some scalar models of inflation with specific potentials may be explained by occasional reason, i.e. by the fact that non-minimal gravitational coupling at very early universe is approximately constant.

It is not hard to extend our formulation for more complicated theories. For instance, one can include the scalar potential into consideration in matter Lagrangian. Other fields like spinors or vectors may be considered in the similar fashion of non-minimal gravitational coupling with matter Lagrangian. From another point, other types of realistic modified gravity may be investigated: the model unifying $R^m$ inflation with LambdaCDM epoch\(^{18}\) or non-local gravity.\(^{19,20}\) The non-minimal versions of such models will be investigated elsewhere.

Acknowledgements.

The research by S.N. has been supported in part by the Monbusho grant no.18549001 and 21st Century COE Program of Nagoya University provided by JSPS (15COEG01). The research by S.D.O. has been supported in part by the projects FIS2006-02842, FIS2005-01181 (MEC, Spain), by RFBR grant 06-01-00609 (Russia) and by YITP, Kyoto.

Appendix A

Classically equivalent forms of non-minimal modified gravity

One can rewrite the action (2.1) with $A = R$ by using the auxiliary field(s). First we introduce two scalar field $\zeta$ and $\eta$ and rewrite (2.1) as

\[
S = \int d^4x\sqrt{-g} \left\{ \frac{1}{\kappa^2} \zeta^2 - f_1(\zeta) L_d - f_2(\zeta) + \eta (R - \zeta) \right\} .
\] (A.1)
Using the equation $\zeta = R$ given by the variation over $\eta$, the action (A.1) is reduced into the original one (2.1). Varying over $\zeta$, we obtain

$$\eta = \frac{1}{\kappa^2} - f'_1(\zeta)L_d - f'_2(\zeta).$$  \hspace{1cm} (A.2)

By substituting (A.2) and deleting $\eta$ in (A.1), one gets

$$S = \int d^4x \sqrt{-g} \left\{ \left( \frac{1}{\kappa^2} - f'_2(\zeta) \right) R - (f'_1(\zeta)R + f_1(\zeta)) L_d - f_2(\zeta) - f'_2(\zeta) + \frac{1}{\kappa^2} \right\}. \hspace{1cm} (A.3)$$

By rescaling the metric as $g_{\mu\nu} \rightarrow e^\sigma g_{\mu\nu}$ with

$$e^{-\sigma} = 1 - \kappa^2 f'_2(\zeta), \hspace{1cm} (A.4)$$

the Einstein frame action follows:\(^{(8),\ 12)}\)

$$S = \int d^4x \sqrt{-\tilde{g}} \left\{ \frac{1}{\kappa^2} \left[ R - \frac{3}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma \right] - \left\{ f'_1(\zeta(\sigma)) e^\sigma \left( R - 3 \Box \sigma - \frac{3}{2} \partial_\mu \sigma \partial^\mu \sigma \right) + e^{2\sigma} f_1(\zeta(\sigma)) \right\} L_d \left( e^\sigma g_{\mu\nu}, \phi \right) - e^{2\sigma} f_2(\zeta(\sigma)) + e^\sigma \zeta(\sigma) \right\}. \hspace{1cm} (A.5)$$

Here we have solved (A.4) with respect to $\zeta$ as $\zeta = \zeta(\sigma)$. Such the non-linear action includes Brans-Dicke type scalar $\sigma$ and the scalar $\phi$ corresponding to the dark energy, that is, two scalars appear. Such classical equivalence of two formulations does not mean the physical equivalence as is explained in.\(^{(21),\ 22)}\)

References

1) S. Nojiri and S. D. Odintsov, [arXiv:hep-th/061213]; J. Phys. Conf. Ser. 66, 012005 (2007) [arXiv:hep-th/0611071].
2) S. Nojiri and S. D. Odintsov, Phys. Rev. D 68, 123512 (2003) [arXiv:hep-th/0307288].
3) W. Hu and I. Sawicki, [arXiv:0705.1158][astro-ph].
4) S. Nojiri and S. D. Odintsov, Phys. Lett. B 652, 343 (2007), [arXiv:0706.1378].
5) S. Nojiri and S. D. Odintsov, [arXiv:0707.1941][hep-th].
6) S. Appleby and R. A. Battye, [arXiv:0705.3199][astro-ph]; L. Pogosian and A. Silvestri, [arXiv:0709.0296][astro-ph]; S. Tsujikawa, [arXiv:0709.1391][astro-ph].
7) S. Capozziello and M. Francaviglia, [arXiv:0706.1146][astro-ph].
8) S. Nojiri and S. D. Odintsov, Phys. Lett. B 599, 137 (2004) [astro-ph/0403622]; PoS WC2004, 024 (2004) [arXiv:hep-th/0412030].
9) G. Allemandi, A. Borowiec, M. Francaviglia, and S. D. Odintsov, Phys. Rev. D 72, 063505 (2005) [arXiv:gr-qc/0504057].
10) S. Mukohyama and L. Randall, Phys. Rev. Lett. 92, 211302 (2004); T. Inagaki, S. Nojiri and S. D. Odintsov, JCAP 0506, 010 (2005) [arXiv:gr-qc/0504054]; A. D. Dolgov and M. Kawasaki, arXiv:astro-ph/0307442.
11) O. Bertolami, C. Boehmer, T. Harko, and F. Lobo, Phys. Rev. D 75, 104016 (2007) [arXiv:0704.1733]; T. Koivisto, Class. Quant. Grav. 23, 4289 (2006) [arXiv:gr-qc/0505128].
12) V. Faraoni, [arXiv:0710.1291[gr-qc]].
13) O. Bertolami and J. Paramos, arXiv:0709.3988[astro-ph].
14) S. Nojiri, S. D. Odintsov, and S. Tsujikawa, Phys. Rev. D 71, 063004 (2005) [arXiv:hep-th/0501025].
15) S. Nojiri and S. D. Odintsov, Phys. Lett. B 631, 1 (2005) [arXiv:hep-th/0508049].
16) G. Cognola, E. Elizalde, S. Nojiri, S. D. Odintsov, and S. Zerbini, Phys. Rev. D 73, 084007 (2006) [arXiv:hep-th/0601008]; B. Li, J. Barrow and D. Mota, Phys. Rev. D 76, 044027 (2007) [arXiv:0705.3795]; S. Davis, arXiv:0709.4453[hep-th].
17) S. Nojiri, S. D. Odintsov, and P. Tretyakov, Phys. Lett. B 651, 224 (2007) [arXiv:0704.2520].
18) S. Nojiri and S. D. Odintsov, arXiv:0710.1738[hep-th].
19) S. Deser and R. Woodard, Phys. Rev. Lett. 99, 111301 (2007).
20) S. Nojiri and S. D. Odintsov, arXiv:0708.0924[hep-th].
21) S. Capozziello, S. Nojiri, S. D. Odintsov, and A. Troisi, Phys. Lett. B 639, 135 (2006) [arXiv:astro-ph/0604431]; S. Nojiri and S. D. Odintsov, Phys. Rev. D 74, 086005 (2006) [arXiv:hep-th/0608008].
22) V. Faraoni and S. Nadeau, Phys. Rev. D 75, 023501 (2007).