Real and virtual $N\bar{N}$ pair production near the threshold

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Abstract

Nucleon-antinucleon optical potential, which explains the experimental data for the processes $e^+e^- \rightarrow p\bar{p}$ and $e^+e^- \rightarrow$ pions near the threshold of $p\bar{p}$ pair production, is suggested. To obtain this potential we have used the available experimental data for $p\bar{p}$ scattering, $p\bar{p}$ pair production in $e^+e^-$ annihilation, and the ratio of electromagnetic form factors of a proton in the timelike region. It turns out that final-state interaction via the optical potential allows one to reproduce the available experimental data with good accuracy. Our results for the cross sections of $e^+e^- \rightarrow 6\pi$ process near the threshold of $p\bar{p}$ pair production are in agreement with the recent experiments.

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I. INTRODUCTION

At present the study of nucleon-antinucleon interaction in the low-energy region is an actual topic. Several optical nucleon-antinucleon potentials [1–3] are usually used to describe the interaction in this region. All these nucleon-antinucleon potentials have been proposed to fit the nucleon-antinucleon scattering data. These data include elastic, charge-exchange, and annihilation cross sections of $p\bar{p}$ scattering, as well as some single-spin observables. These observables can be described very well by any of the models [1–3]. To discriminate different models one can use other observables. For example, calculations were made for double-spin observables in $p\bar{p}$ scattering [4–7], and the predictions of different optical potentials were indeed different. Unfortunately, the experimental data for these observables are still absent.

There is another set of data that one can hope to describe with the help of potential models, namely, the cross sections of nucleon-antinucleon production in $e^+e^-$ annihilation. It was shown in our previous papers [8, 9] that the cross sections of these processes in the energy region close to the threshold can be written in terms of the radial wave functions of nucleon-antinucleon pair at origin. These cross sections were measured at BABAR [10], CMD-3 [11] and SND [12]. The ratio of electromagnetic form factors of a proton in the timelike region, that was also measured [10, 11], can also be expressed via the wave functions. This ratio has quite strong energy dependence near the threshold, and one needs a nontrivial model to describe this. It was shown that some of these observables can be described by slightly modified Paris optical potential [8] or by the Julich model [13].

In this paper we go further and try to describe, with the help of an optical theorem, the contribution of virtual nucleon-antinucleon pair to the cross sections of meson production in the energy region close to the $p\bar{p}$ threshold. We refer the cross section of this process as inelastic cross section of nucleon-antinucleon pair production, while the cross section of real $N\bar{N}$ pair production is called the elastic cross section. The inelastic cross section can be expressed in terms of the Green’s function of the Schrödinger equation in the presence of an optical potential. First of all, we are interested in the processes $e^+e^- \rightarrow 3(\pi^+\pi^-)$
and $e^+e^- ightarrow 2(\pi^+\pi^-\pi^0)$ because the cross sections of $6\pi$ production have a sharp dip near the $p\bar{p}$ threshold [14–16], and this phenomenon is not well understood yet. This feature is expected to be a consequence of the interaction of virtual nucleons, because other contributions should be smooth functions in the energy region under consideration. There was an attempt to explain the behavior of meson production cross sections with the help of the Julich model [17]. Though the cross sections of some channels are reproduced, there are a few cross sections that are not described within this model, for instance, the cross section of the process $e^+e^- \rightarrow 3(\pi^+\pi^-)$. The Paris optical potential completely fails to describe the process $e^+e^- \rightarrow \text{mesons}$ via annihilation of virtual nucleon-antinucleon pair. This problem appears due to very large imaginary part of the central potential in this model. Such huge imaginary part results in the significant overestimation of the inelastic cross section compared with the elastic one. A strong potential appears as a result of an attempt to describe the data in wide energy and angle regions, i.e., in large region of momentum transfers. The Nijmegen optical potential has another shortcoming. This model implies that a complicated matching condition should be applied to the wave functions at radius about 1 fm, and it is not evident how to apply this condition to calculate the Green’s function at origin.

The interaction of virtual nucleons in the process of $e^+e^-$ annihilation into mesons is very sensitive to the potential at small distances. Unfortunately, the short-range potential can’t be determined very well from $p\bar{p}$ scattering data alone. Therefore, one should also take into account other experimental data. This is what we perform in the present work. We consider the partial waves with the total angular momentum $J = 1$, that contribute to the processes of nucleon-antinucleon production in $e^+e^-$ annihilation, and assume that other partial waves can be described by any of the models mentioned above. For $^3S_1$ and $^3D_1$ partial waves, coupled by the tensor forces, we propose a new potential model based on the fit of experimental data for the cross sections of $p\bar{p}$ scattering, as well as the cross sections of nucleon-antinucleon pair production and the ratio of electromagnetic form factors of the proton. The account for the tensor potential is very important, because it is crucial for the description of the electromagnetic form factors ratio. Our model qualitatively reproduces the features of $6\pi$ production in $e^+e^-$ annihilation in the vicinity of the threshold of $e^+e^- \rightarrow p\bar{p}$ process.
II. AMPLITUDE OF THE PROCESS

It is shown in our recent paper [9] that in the non-relativistic approximation the amplitude $T_{I\mu}^I$ of $N\bar{N}$ pair production in $e^+e^-$ annihilation near the threshold can be presented for the certain isospin channel $I = 0, 1$ as follows (in units $4\pi\alpha/Q^2$, $\alpha$ is the fine structure constant, $\hbar = c = 1$):

$$T_{I\mu}^I = G_s^I \left\{ \sqrt{2} u_{1R}^I(0)(e_{\mu} \cdot e_{\lambda}^*) + u_{2R}^I(0) \left[ (e_{\mu} \cdot e_{\lambda}^*) - 3(\hat{k} \cdot e_{\mu})(\hat{k} \cdot e_{\lambda}^*) \right] \right\},$$

(1)

where $G_s^I$ is an energy-independent constant, $e_{\mu}$ is a virtual photon polarization vector, corresponding to the projection of spin $J_z = \mu = \pm 1$, $e_{\lambda}^*$ is the spin-1 function of $N\bar{N}$ pair, $\lambda = \pm 1, 0$ is the projection of spin on the nucleon momentum $k$, and $\hat{k} = k/k$. The radial wave functions $u_{nR}^I(r)$ and $w_{nR}^I(r)$, $n = 1, 2$, are the regular solutions of the equations

$$\frac{p_r^2}{M} \chi_n + V \chi_n = 2E \chi_n,$$

\[V = \begin{pmatrix} V_S^I & -2\sqrt{2} V_T^I \\ -2\sqrt{2} V_T^I & V_D - 2V_T^I + \frac{6}{Mr^2} \end{pmatrix}, \quad \chi_n = \begin{pmatrix} u_n^I \\ w_n^I \end{pmatrix}. \tag{2}\]

Here $M$ is the proton mass, $E = k^2/(2M)$, $V_S^I(r)$, $V_D^I(r)$, and $V_T^I(r)$ are the functions in the Hamiltonian $H^I$ of $N\bar{N}$ interaction for the isospin $I$,

$$H^I = \frac{p_r^2}{M} + V_S^I(r)\delta_{L0} + V_D^I(r)\delta_{L2} + V_T^I(r) S_{12},$$

$$S_{12} = 6 (\mathbf{S} \cdot \mathbf{n})^2 - 4,$$ \tag{3}

where $\mathbf{S}$ is the spin operator for the spin-one system of the produced pair, $(-p_r^2)$ is the radial part of the Laplace operator, $L$ denotes the orbital angular momentum, and $\mathbf{n} = r/r$. The asymptotic forms of the regular solutions (they have no singularities at $r = 0$) at large distances are [9]

$$u_{1R}^I(r) = \frac{1}{2ikr} \left[ S_{11}^I e^{ikr} - e^{-ikr} \right],$$

$$w_{1R}^I(r) = -\frac{1}{2ikr} S_{12}^I e^{ikr},$$

$$u_{2R}^I(r) = \frac{1}{2ikr} S_{21}^I e^{ikr},$$

$$w_{2R}^I(r) = \frac{1}{2ikr} \left[ -S_{22}^I e^{ikr} + e^{-ikr} \right].$$ \tag{4}
where $S_{ij}$ are some functions of energy, $S_{21}^{I} = S_{12}^{I}$, $|S_{11}^{I}|^2 + |S_{12}^{I}|^2 \leq 1$, and $|S_{22}^{I}|^2 + |S_{21}^{I}|^2 \leq 1$.

For our purpose we also need to know the non-regular solutions of Eqs. (2) which have the asymptotic forms at large distances

$$u_{1N}^{I}(r) = \frac{1}{kr} e^{i\kappa r}, \quad \lim_{r \to \infty} ru_{1N}^{I}(r) = 0, \quad \lim_{r \to \infty} ru_{2N}^{I}(r) = 0,$$

$$w_{2N}^{I}(r) = -\frac{1}{kr} e^{i\kappa r}. \quad (5)$$

### III. CROSS SECTION AND THE SACHS FORM FACTORS

Performing summation over the polarization of nucleon pair and averaging over the polarization of virtual photon by means of the equations,

$$\sum_{\lambda=1,2,3} \epsilon^{*}_i \lambda \epsilon^j \lambda = \delta^{ij}, \quad \frac{1}{2} \sum_{\mu=1,2} \epsilon^{*}_i \mu \epsilon^j \mu = \frac{1}{2} \delta^{ij} = \frac{1}{2} (\delta^{ij} - P^i P^j / P^2), \quad (6)$$

where $P$ is the electron momentum, we obtain the cross section corresponding to the amplitude (1) in the center-of-mass frame (see, e.g., Ref. [18])

$$\frac{d\sigma}{d\Omega} = \frac{\beta \alpha^2}{4Q^2} \left[ |G_M^I(Q^2)|^2 (1 + \cos^2 \theta) + \frac{4M^2}{Q^2} |G_E^I(Q^2)|^2 \sin^2 \theta \right]. \quad (7)$$

Here $\beta = k/M$, $Q = 2(M + E)$, and $\theta$ is the angle between the electron (positron) momentum $P$ and the momentum of the final particle $k$. In terms of the form factor $G_s^I$, the electromagnetic Sachs form factors have the form

$$G_M^I = G_s^I \left[ u_{1R}^I(0) + \frac{1}{\sqrt{2}} u_{2R}^I(0) \right],$$

$$\frac{2M}{Q} G_E^I = G_s^I \left[ u_{1R}^I(0) - \sqrt{2} u_{2R}^I(0) \right]. \quad (8)$$

Thus, in the non-relativistic approximation the ratio $G_E^I / G_M^I$ is independent on the constant $G_s^I$,

$$\frac{G_E^I}{G_M^I} = \frac{u_{1R}^I(0) - \sqrt{2} u_{2R}^I(0)}{u_{1R}^I(0) + \frac{1}{\sqrt{2}} u_{2R}^I(0)}. \quad (9)$$

Note that the electromagnetic interaction is important only in the narrow region $\beta \sim \pi \alpha$ where the nucleon energy is $E = M\beta^2/2 \sim 0.3$ MeV. In this paper we do not consider this narrow region and neglect the electromagnetic interaction in the potential. The contribution
of the isospin \( I \) to the total cross section of the nucleon pair production (the elastic cross section) reads
\[
\sigma^I = \frac{2\pi \beta \alpha^2}{Q^2} |G_{sI}|^2 \left[ |u_{1R}(0)|^2 + |u_{2R}(0)|^2 \right].
\] (10)

Thus, to describe the energy dependence of the ratio \( G_{EI}/G_{MI} \) and the cross section \( \sigma^I \) in the non-relativistic approximation, it is necessary to know the functions \( u_{1I}^I(0) \) and \( u_{2I}^I(0) \).

In order to describe the total cross section, a sum of elastic and inelastic cross sections (the production of mesons via annihilation of virtual \( N\bar{N} \) pair), we use the method of the Green’s function. Let us introduce the Green’s function \( D(r, r'|E) \),
\[
\left( \frac{p^2}{M} + V - 2E \right) D(r, r'|E) = -\frac{1}{rr'} \delta(r-r').
\] (11)

Then the total cross section, \( \sigma^I_{\text{tot}} \) can be written as [19]
\[
\sigma^I_{\text{tot}} = -\frac{2\pi \alpha^2}{M^2 Q^2} |G_{sI}|^2 \text{ Sp} \left[ \text{Im} D(0, 0|E) \right].
\] (12)

The solution of Eq. (11) can be written in the form
\[
D(r, r'|E) = -Mk \sum_{n=1,2} \left[ \partial (r' - r) \chi_{nR}(r) \chi_{nN}^T(r') + \partial (r - r') \chi_{nN}(r) \chi_{nR}^T(r') \right],
\] (13)

where \( \chi^T \) denotes transposition of \( \chi \), if the following relations hold:
\[
\sum_{n=1,2} \left[ \chi_{nR}(r) \chi_{nN}^T(r) - \chi_{nN}(r) \chi_{nR}^T(r) \right] = 0,
\]
\[
\sum_{n=1,2} \left[ \chi_{nR}'(r) \chi_{nN}^T(r) - \chi_{nN}'(r) \chi_{nR}^T(r) \right] = \frac{1}{kr^2} 1.
\] (14)

Here \( \chi'(r) = \partial \chi(r)/\partial r \), 0 and 1 stand for zero and unit matrix, respectively. The validity of Eq. (14) is a consequence of the relations
\[
\chi_{1R}(r) \chi_{2N}'(r) - \chi_{2N}(r) \chi_{1R}'(r) = 0, \quad \chi_{2R}(r) \chi_{1N}'(r) - \chi_{1N}(r) \chi_{2R}'(r) = 0,
\]
\[
\chi_{1N}(r) \chi_{1R}'(r) - \chi_{1R}(r) \chi_{1N}'(r) = \frac{1}{kr^2}, \quad \chi_{2N}(r) \chi_{2R}'(r) - \chi_{2R}(r) \chi_{2N}'(r) = \frac{1}{kr^2},
\] (15)

following from Eq. (2), symmetry of the matrix \( V \) in that equation, and the asymptotic forms (4) and (5).
IV. RESULTS OF THE CALCULATIONS

We propose a simple potential model to describe the nucleon-antinucleon interaction in the state with the total angular momentum \( J = 1 \). This is the only state that contributes to the processes of nucleon-antinucleon production in \( e^+e^- \) annihilation. The interaction in other partial waves can be described very well by the models [1–3]. The optical potential of nucleon-antinucleon interaction in Eq. (16) can be written as

\[
V_n(r) = V_{n0}(r) + V_{n1}(r) (\tau_1 \cdot \tau_2) , \quad n = S, D, T ,
\]

(16)

where \( \tau_i \) are the Pauli matrices in the isospin space, so that the potentials corresponding to \( I = 0, 1 \) channels read

\[
V_n^0(r) = V_{n0}(r) - 3V_{n1}(r) , \quad V_n^1(r) = V_{n0}(r) + V_{n1}(r) .
\]

(17)

We use a potential which is the sum of a long-range pion-exchange potential and a short-range potential well

\[
V_{n0}(r) = (U_{n0} - i W_{n0}) \theta (a_{n0} - r) , \\
V_{n1}(r) = (U_{n1} - i W_{n1}) \theta (a_{n1} - r) + \tilde{V}_n(r) \theta (r - a_{n1}) ,
\]

(18)

where \( \theta(x) \) is the Heaviside function, \( \tilde{V}_n(r) \) is the pion-exchange potential, \( U_{nI}, W_{nI}, \) and \( a_{nI} \) are free parameters fixed by fitting the experimental data. The pion-exchange potential of nucleon-antinucleon interaction for the total spin \( S = 1 \) is given by the expression (see, f.i., [20])

\[
\tilde{V}_S(r) = \tilde{V}_D(r) = -f^2 \pi e^{-m_\pi r} , \\
\tilde{V}_T(r) = -f^2 \pi \left( \frac{1}{3} + \frac{1}{m_\pi r} + \frac{1}{(m_\pi r)^2} \right) e^{-m_\pi r} ,
\]

(19)

where \( f^2_\pi = 0.075, m_\pi \) is the pion mass. At small \( r \) the tensor potentials \( \tilde{V}_T \) are regularized by the factor

\[
F(r) = \frac{(cr)^2}{1 + (cr)^2}
\]

with \( c = 10 \text{ fm}^{-1} \). Our analysis shows that one can take the radii of real and imaginary parts of the potentials (18) to be the same.
The electromagnetic form factors of the proton and neutron are expressed via the isoscalar and isovector form factors (8) by the relations

\[ G_E^p = \frac{G_0^p + G_1^p}{\sqrt{2}}, \quad \frac{G_M^p}{\sqrt{2}} = \frac{G_0^p + G_1^p}{\sqrt{2}}, \]

\[ G_E^n = \frac{G_0^n - G_1^n}{\sqrt{2}}, \quad \frac{G_M^n}{\sqrt{2}} = \frac{G_0^n - G_1^n}{\sqrt{2}}. \] (20)

Thus, the cross sections of nucleon-antinucleon production read

\[ \sigma_{\bar{p}p} = \frac{\pi \beta^2}{Q^2} \left[ |G_0^0 u_{1R}^0(0) + G_1^0 u_{1R}^1(0)|^2 + |G_0^0 u_{2R}^0(0) + G_1^0 u_{2R}^1(0)|^2 \right]^2, \]

\[ \sigma_{\bar{n}n} = \frac{\pi \beta^2}{Q^2} \left[ |G_0^0 u_{1R}^0(0) - G_1^0 u_{1R}^1(0)|^2 + |G_0^0 u_{2R}^0(0) - G_1^0 u_{2R}^1(0)|^2 \right]^2, \] (21)

and the ratio of electromagnetic form factors of the proton is given by

\[ \frac{G_E^p}{G_M^p} = \frac{G_0^0 u_{1R}^0(0) + G_1^0 u_{1R}^1(0) - \sqrt{2} [G_0^0 u_{2R}^0(0) + G_1^0 u_{2R}^1(0)]}{G_0^0 u_{1R}^0(0) + G_1^0 u_{1R}^1(0) + \frac{1}{\sqrt{2}} [G_0^0 u_{2R}^0(0) + G_1^0 u_{2R}^1(0)]}. \] (22)

The data used for fitting the parameters of the potential include the cross sections of \( \bar{p}p \) and \( \bar{n}n \) production [10–12], the ratio of electromagnetic form factors of the proton [10] and the partial contributions of \( J = 1 \) waves to the elastic, change-exchange and total cross sections of \( \bar{p}p \) scattering. The partial cross sections of \( \bar{p}p \) scattering were calculated from the Nijmegen partial wave \( S \)-matrix (Tables VI, VII of Ref. [2]). The results of the fit are shown in Table I. The accuracy of the fit can be seen from Figs. 1–3.

The number of free parameters in our model is \( N_{fp} = 20 \). The total number of experimental data points for the cross sections of \( \bar{p}p \) and \( \bar{n}n \) production and for the ratio \( |G_E^p/G_M^p| \) is \( N_{dat} = 35 \). Thus, we have \( N_{df} = N_{dat} - N_{fp} = 15 \) degrees of freedom. The minimum \( \chi^2 \) per degree of freedom is \( \chi_{\min}^2/N_{df} = 29/15 \), and is rather large. However, large \( \chi_{\min}^2 \) value is originated mainly from poor accuracy of some data points for \( \bar{n}n \) production cross section. Excluding two less accurate data points gives \( \chi_{\min}^2/N_{df} = 16/13 \), which is good enough. The errors in Table I correspond to the values of the parameters that give \( \chi^2 = \chi_{\min}^2 + 1 \).

As soon as the potential is determined, we can calculate the Green’s function and the total cross section of pion production through nucleon-antinucleon intermediate state (12). The elastic cross section of \( N\bar{N} \) production, the total cross section and the cross section of annihilation into mesons for different isospins are shown in Fig. 4. A dip in the total cross section of \( e^+e^- \) annihilation into mesons is predicted close to the \( N\bar{N} \) threshold. This
Fig. 1. The cross sections of $p\bar{p}$ (red/dark line) and $n\bar{n}$ (green/light line) production (left) and the ratio of electromagnetic form factors of the proton (right) as a function of total energy $2\varepsilon = 2M + 2E$. The experimental data are from Refs. [10–12].

Fig. 2. $^3S_1$ (first row) and $^3D_1$ (second row) contributions to the elastic and total cross sections of $p\bar{p}$ scattering compared with the Nijmegen data [2], $\varepsilon = M + E$. 
Table I. The results of the fit for the short-range potential (18) and the constants $G^I_S$.

|                  | $V_{S0}$ | $V_{D0}$ | $V_{T0}$ | $V_{S1}$ | $V_{D1}$ | $V_{T1}$ |
|------------------|----------|----------|----------|----------|----------|----------|
| $U$ (MeV)        | $-433 \pm 3$ | $-140^{+40}_{-36}$ | $58 \pm 4$ | $2.4^{+0.7}_{-0.6}$ | $798^{+165}_{-140}$ | $7.1 \pm 0.1$ |
| $W$ (MeV)        | $224 \pm 10$ | $0^{+28}_{-27}$ | $19 \pm 1$ | $0^{+0.9}_{-1.3}$ | $456^{+215}_{-107}$ | $-0.3 \pm 0.3$ |
| $a$ (fm)         | $0.564 \pm 0.002$ | $1.02^{+0.06}_{-0.09}$ | $1.03 \pm 0.02$ | $1.86^{+0.08}_{-0.09}$ | $0.49^{+0.04}_{-0.02}$ | $2.4 \pm 0.02$ |
| $G^0_S$          | $0.179 \pm 0.006$ |                        |                    |                        |                        |             |
| $G^1_S$          |                      |                        |                    | $0.044 + 0.29i \pm 0.014$ |                        |             |

Fig. 3. $^3S_1$ and $^3D_1 \rightarrow ^3S_1$ contributions to the charge-exchange cross section compared with the Nijmegen data [2], $\varepsilon = M + E$.

behavior seems to be the consequence of some quasi-bound $NN$ state near the threshold.
To check this hypothesis we have searched for bound states in the potential considered. Our analysis shows that there are no near-threshold bound states in the $I = 1$ channel. However, we have found a state with energy $E_B = (10 - i \ 32)$ MeV in the $I = 0$ channel. This state is located above the $NN$ threshold, but it moves to $E_B = -21$ MeV if the imaginary part of the potential is turned off. This is an unstable bound state in the terminology of Ref. [21]. This result is quite similar to the result obtained in Ref. [3], where $4.8 \text{ MeV} < \text{Re} \ E_B < 21.3 \text{ MeV}$ and $-74.9 \text{ MeV} < \text{Im} \ E_B < -60.6 \text{ MeV}$ in the $I = 0$ channel.

The total contribution of nucleon-antinucleon intermediate states to the cross section of $e^+e^-$ annihilation is given by the sum of $I = 0$ and $I = 1$ terms. The states with $I = 0$ contribute to the production of odd number of pions, while the states with $I = 1$ contribute to the production of even number of pions. However, we don’t know accurately the pion multiplicity distribution in meson production. This distribution was analyzed for
Fig. 4. Elastic (blue/dark line), inelastic (green/light line) and total (red/medium line) cross sections of $e^+e^-$ annihilation through nucleon-antinucleon intermediate states with different isospins, $\varepsilon = M + E$.

Fig. 5. The prediction for the cross section of $6\pi$ production (red thick line). The thin line shows the contribution of non-$N\bar{N}$ channels. The data for total $6\pi$ production are calculated from BABAR [14] and CMD-3 [15, 16] data. $3(\pi^+\pi^-)$ and $2(\pi^+\pi^-\pi^0)$ channels are taken into account, $\varepsilon = M + E$.

$p\bar{p}$ annihilation at rest [22, 23], and the cross section of six pion production gives about 55% of the total cross section with $I = 1$. We fit the cross section of $6\pi$ production in the energy region between 1.7 GeV and 2.1 GeV with the sum of inelastic $I = 1$ contribution and a linear function describing the contribution of other intermediate states (see Fig. 5). We obtain the best coincidence when the contribution of $6\pi$ events to the inelastic cross section is 56%, which is very close to the expectation.
V. CONCLUSIONS

We have proposed an optical potential describing simultaneously the experimental data for \(N\bar{N}\) scattering and \(e^+e^-\) annihilation to \(N\bar{N}\) and \(6\pi\) close to the threshold of \(N\bar{N}\) production. Our model predicts the dip in the total cross section of \(e^+e^-\) annihilation to mesons, which is consistent with the observed behavior of the cross section of \(6\pi\) production (see Fig. 5). The calculation of the inelastic cross section of the process \(e^+e^- \rightarrow \text{mesons}\) is based on the use of the Green’s function method. The Green’s function of the Schrödinger equation in the optical potential is derived with the tensor forces taken into account.

It is worth noting that we found several sets of potential parameters that fit the experimental data for \(N\bar{N}\) scattering and for the cross sections \(e^+e^- \rightarrow N\bar{N}\) with good \(\chi^2\). However, the account for the data for the cross section of \(e^+e^- \rightarrow 6\pi\) close to the threshold of \(N\bar{N}\) production leads to the unique set of parameters presented in the Table I. Diminishing of uncertainties of experimental data for the cross sections of another channels of \(e^+e^-\) annihilation into mesons would be very important for better determination of the optical potential.

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