Preon stars: a new class of cosmic compact objects

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In the context of the standard model of particle physics, there is a definite upper limit to the density of stable compact stars. However, if a more fundamental level of elementary particles exists, in the form of preons, stability may be re-established beyond this limiting density. We show that a degenerate gas of interacting fermionic preons does allow for stable compact stars, with densities far beyond that in neutron stars and quark stars. In keeping with tradition, we call these objects “preon stars”, even though they are small and light compared to white dwarfs and neutron stars. We briefly note the potential importance of preon stars in astrophysics, e.g., as a candidate for cold dark matter and sources of ultra-high energy cosmic rays, and a means for observing them.

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I. INTRODUCTION

The three different types of compact objects traditionally considered in astrophysics are white dwarfs, neutron stars (including quark and hybrid stars), and black holes. The first two classes are supported by Fermi pressure from their constituent particles. For white dwarfs, electrons provide the pressure counterbalancing gravity. In neutron stars, the neutrons play this role. For black holes, the degeneracy pressure is overcome by gravity and the object collapses indefinitely, or at least to the Planck density.

The distinct classes of degenerate compact stars originate directly from the properties of gravity, as was made clear by a theorem of Wheeler and collaborators in the mid 1960s [1]. The theorem states that for the solutions to the stellar structure equations, whether Newtonian or relativistic, there is a change in stability of one radial mode of normal vibration whenever the mass reaches a local maximum or minimum as a function of central density. The theorem assures that distinct classes of stars, such as white dwarfs and neutron stars, are separated in central density by a region in which there are no stable configurations.

In the standard model of particle physics (SM), the theory of the strong interaction between

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quarks and gluons predicts that with increasing energy and density, the coupling between quarks asymptotically fades away [2, 3]. As a consequence of this “asymptotic freedom”, matter is expected to behave as a gas of free fermions at sufficiently high densities. This puts a definite upper limit to the density of stable compact stars, since the solutions to the stellar equations end up in a never-ending sequence of unstable configurations, with increasing central density. Thus, in the light of the standard model, the densest stars likely to exist are neutron stars, quark stars or the potentially more dense hybrid stars [4, 5, 6]. However, if there is a deeper layer of constituents, below that of quarks and leptons, asymptotic freedom will break down at sufficiently high densities, as the quark matter phase dissolves into the preon sub-constituent phase.

There is a general consensus among the particle physics community, that something new should appear at an energy-scale of around one TeV. The possibilities are, e.g., supersymmetric particles, new dimensions and compositeness. In this letter we consider “preon models” [7, 8], i.e., models in which quarks and leptons, and sometimes some of the gauge bosons, are composite particles built out of more elementary preons. If fermionic preons exist, it seems reasonable that a new type of astrophysical compact object, a preon star, could exist. The density in preon stars should far exceed that inside neutron stars, since the density of preon matter must be much higher than the density of nuclear and deconfined quark matter. The sequence of compact objects, in order of increasing compactness, would thus be: white dwarfs, neutron stars, preon stars and black holes.

II. MASS-RADIUS RELATIONS

Assuming that a compact star is composed of non-interacting fermions with mass $m_f$, the non-general relativistic (Chandrasekhar) expression for the maximum mass is [9, 10]:

$$M \simeq \frac{1}{m_f^2} \left( \frac{hc}{G} \right)^{3/2}.$$  

(1)

This expression gives a correct order of magnitude estimate for the mass of a white dwarf and a neutron star. For quark stars, this estimate cannot be used literally, since the mass of quarks cannot be defined in a similar way as for electrons and neutrons. However, making the simplifying assumption that quarks are massless and subject to a ‘bag constant’, a maximum mass relation can be derived [11]. The bag constant is a phenomenological parameter. It represents the strong interactions that, in addition to the quark momenta, contribute mass-energy to deconfined quark matter, i.e., in the same way as the bag constant for ordinary hadrons [12]. The result in [11] is somewhat similar to the Chandrasekhar expression, but the role of the fermion mass is replaced
by the bag constant $B$:

$$M = \frac{16\pi BR^3}{3c^2},$$

(2)

$$R = \frac{3c^2}{16\sqrt{\pi GB}}.$$  

(3)

For preon stars, one can naively insert a preon mass of $m_f \simeq 1$ TeV/c$^2$ in eq. (1) to obtain a preon star mass of approximately one Earth mass ($M_{\oplus} \simeq 6 \times 10^{24}$ kg). However, the energy scale of one TeV should rather be interpreted as a length scale, since it originates from the fact that in particle physics experiments, no substructure has been found down to a scale of a few hundred GeV ($\hbar c/\text{GeV} \simeq 10^{-18}$ m). Since preons must be able to give light particles, e.g., neutrinos and electrons, the “bare” preon mass presumably is fairly small and a large fraction of the mass-energy should be due to interactions. This is the case for deconfined quark matter, where the bag constant contributes more than 10% of the energy density. Guided by this observation, and lacking a quantitative theory for preon interactions, we assume that the mass-energy contribution from preon interactions can be accounted for by a bag constant. We estimate the order of magnitude for the preon bag constant by fitting it to the minimum density of a composite electron, with mass $m_e = 511$ keV/c$^2$ and “radius” $R_e \lesssim \hbar c/\text{TeV} \simeq 10^{-19}$ m. The bag-energy is roughly $4B\langle V \rangle$, so the bag constant is:

$$B \simeq \frac{E}{4\langle V \rangle} \gtrsim \frac{3 \times 511 \text{ keV}}{16\pi (10^{-19} \text{ m})^3} \simeq 10^4 \text{ TeV}/\text{fm}^3 \implies B^{1/4} \gtrsim 10 \text{ GeV}.$$  

(4)

Inserting this value of $B$ in eqs. (2), (3), we obtain an estimate for the maximum mass, $M_{\text{max}} \simeq 10^2 \text{ M}_{\oplus}$, and radius, $R_{\text{max}} \simeq 1$ m, of a preon star.

Since $B^{1/4} \simeq 10$ GeV only is an order of magnitude estimate for the minimum value of $B$, in the following, we consider the bag constant as a free parameter of the model, with a lower limit of $B^{1/4} = 10$ GeV and an upper limit chosen as $B^{1/4} = 1$ TeV. The latter value corresponds to an electron “radius” of $\hbar c/10^3 \text{ TeV} \simeq 10^{-22}$ m. In figs. 1 and 2 the (Chandrasekhar) maximum mass and radius of a preon star are plotted as a function of the bag constant.

Due to the extreme density of preon stars, a general relativistic treatment is necessary. This is especially important for the analysis of stability when a preon star is subject to small radial vibrations. In this introductory article we will neglect the effects of rotation on the composition. Thus, we can use the Oppenheimer-Volkoff (OV) equations [13] for hydrostatic, spherically symmetric equilibrium:

$$\frac{dp}{dr} = -\frac{G \left( p + \rho c^2 \right) \left( mc^2 + 4\pi r^3 p \right)}{r \left( rc^4 - 2Gmc^2 \right)},$$

(5)
FIG. 1: The maximum mass and corresponding central density \( \rho_c \) of a preon star vs. the bag constant \( B \). The solid lines represent the general relativistic OV solutions, while the dashed line represents the Newtonian (Chandrasekhar) estimate. Despite the high central density, the mass of these objects is below the Schwarzschild limit, as is always the case for static solutions to the stellar equations. \( M_\oplus \simeq 6 \times 10^{24} \text{ kg} \) is the Earth mass.

\[
\frac{dm}{dr} = 4\pi r^2 \rho.
\]  

Here \( p \) is the pressure, \( \rho \) the total density and \( m = m(r) \) the mass within the radial coordinate \( r \). The total mass of a preon star is \( M = m(R) \), where \( R \) is the coordinate radius of the star. Combined with an equation of state (EoS), \( p = p(\rho) \), obtained from some microscopic theory, the OV solutions give the possible equilibrium states of preon stars.

Since no theory for the interaction between preons yet exists, we make a simple assumption for the EoS. The EoS for a gas of massless fermions is \( \rho c^2 = 3p \) (see, e.g., [14]), independently of the degeneracy factor of the fermions. By adding a bag constant \( B \), one obtains \( \rho c^2 = 3p + 4B \). This is the EoS that we have used when solving the OV equations. The obtained (OV) maximum mass and radius configurations are also plotted in figs. 1 and 2.

### III. STABILITY ANALYSIS

A necessary, but not sufficient, condition for stability of a compact star is that the total mass is an increasing function of the central density \( dM/d\rho_c > 0 \) [14]. This condition implies that a slight compression or expansion of a star will result in a less favourable state, with higher total energy. Obviously, this is a necessary condition for a stable equilibrium configuration. Equally important,
FIG. 2: The maximum radius and the corresponding first three eigenmode oscillation frequencies \((f_0, f_1, f_2)\) vs. the bag constant. The solid line in the left-hand picture is the “apparent” radius, \(R^\infty = R/\sqrt{1 - 2GM/Rc^2}\), as seen by a distant observer. The dashed line represents the general relativistic coordinate radius obtained from the OV solution, while the dotted line represents the Newtonian (Chandrasekhar) estimate. Since the fundamental mode \(f_0\) is real \((\omega^2_0 > 0)\), preon stars with mass below the maximum are stable, for each value of \(B\).

A star must be stable when subject to radial oscillations. Otherwise, any small perturbation would bring about a collapse of the star.

The equations for the analysis of such radial modes of oscillation are due to Chandrasekhar\footnote{Chandrasekhar \cite{15}.}. An overview of the theory, and some applications, can be found in \footnote{An overview of the theory, and some applications, can be found in \cite{16}.}. For clarity, we reproduce some of the important points. Starting with the metric of a spherically symmetric equilibrium stellar model,

\[
ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2(\theta)d\phi^2),
\]

and the energy-momentum tensor of a perfect fluid, \(T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}\), the equation governing radial adiabatic oscillations can be derived from Einstein’s equation. By making an ansatz for the time dependence of the radial displacement of fluid elements of the form:

\[
\delta r(r, t) = r^{-2}e^{\nu(r)}\zeta(r)e^{i\omega t},
\]

the equation simplifies to a Sturm-Liouville eigenvalue equation for the eigenmodes\footnote{An overview of the theory, and some applications, can be found in \cite{15,16}}:

\[
\frac{d}{dr}\left(P\frac{d\zeta}{dr}\right) + (Q + \omega^2W)\zeta = 0.
\]
The coefficients $P$, $Q$ and $W$ are (in geometric units where $G = c = 1$):

\[ P = \Gamma r^{-2} p e^{\lambda(r)+3\nu(r)}, \]
\[ Q = e^{\lambda(r)+3\nu(r)} \left[ (p + \rho)^{-1} r^{-2} \left( \frac{dp}{dr} \right)^2 - 4r^{-3} \frac{dp}{dr} - 8\pi r^{-2} p(p + \rho) e^{2\lambda(r)} \right], \]
\[ W = (p + \rho) r^{-2} e^{3\lambda(r)+\nu(r)}, \]

where the adiabatic index $\Gamma$ is:

\[ \Gamma = \frac{p + \rho}{p} \left( \frac{\partial p}{\partial \rho} \right)_S. \]

The boundary conditions for $\zeta(r)$ are that $\zeta(r)/r^3$ is finite or zero as $r \to 0$, and that the Lagrangian variation of the pressure,

\[ \Delta p = - \frac{\Gamma p e^{\nu} d\zeta}{r^2 \frac{dr}{dr}}, \]

vanishes at the surface of the star.

A catalogue of various numerical methods for the solution of eq. (9) can be found in [17]. In principle, we first solve the OV equations, thereby obtaining the metric functions $\lambda(r)$ and $\nu(r)$, as well as $p(r)$, $\rho(r)$ and $m(r)$. Then, the metric functions $\lambda(r)$ and $\nu(r)$ must be corrected for, so that they match the Schwarzschild metric at the surface of the star (see, e.g., [14]). Once these quantities are known, eq. (9) can be solved for $\zeta(r)$ and $\omega^2$ by a method commonly known as the “shooting” method. One starts with an initial guess on $\omega^2$, and integrates eq. (9) from $r = 0$ to the surface of the star. At this point $\zeta(r)$ is known, and $\Delta p$ can be calculated. The number of nodes of $\zeta(r)$ is a non-decreasing function of $\omega^2$ (due to Sturm’s oscillation theorem). Thus, one can continue making educated guesses for $\omega^2$, until the correct boundary condition ($\Delta p = 0$) and number of nodes are obtained. This method is simple to use when only a few eigenmodes are needed.

Due to the time dependence in eq. (8), a necessary (and sufficient) condition for stability is that all $\omega_i^2$ are positive. Since $\omega_i^2$ are eigenvalues of a Sturm-Liouville equation, and governed by Sturm’s oscillation theorem, it is sufficient to prove that the fundamental mode, $\omega_0^2$, is greater than zero for a star to be stable. In fig. 3 the first three oscillation frequencies, $f_i = \omega_i/2\pi$, for various stellar configurations with $B^{1/4} = 100$ GeV are plotted. In agreement with the turning point theorem of Wheeler et al. [1], the onset of instability is the point of maximum mass, as $\omega_0^2$ becomes negative for higher central densities. Thus, for this value of the bag constant, preon stars are stable up to the maximum mass configuration. In order to see if the same is true for other
FIG. 3: The mass and the first three eigenmode oscillation frequencies ($f_0$, $f_1$, $f_2$) vs. the central density $\rho_c$ of preon stars. Here, a fixed value of $B^{1/4} = 100$ GeV has been used. For the maximum mass configuration, the fundamental mode $f_0$ has zero frequency, indicating the onset of instability. Preon stars with mass and density below the maximum mass configuration of this sequence are stable.

values of $B$, we plot the first three oscillation frequencies as a function of $B$, choosing the maximum radius configuration for each $B$. The result can be found in fig. 2. Indeed, the previous result is confirmed; all configurations up to the maximum mass are stable.

The eigenmode frequencies for radial oscillations of preon stars are about six orders of magnitude higher than for neutron stars. This result can also be obtained by making a simple estimate for the frequency of the fundamental mode. The radius of a preon star is a factor of $\sim 10^5$ smaller than neutron stars. Hence, if the speed of sound is similar in preon stars and neutron stars, the frequency would increase by a factor of $\sim 10^5$, giving GHz frequencies. If the speed of sound is higher in preon stars, say approaching the speed of light, the maximum frequency is $\sim 10^8$ m s$^{-1}$/0.1 m $\simeq$ 1 GHz. Thus, in either case, GHz oscillation frequencies are expected for preon stars.

IV. POTENTIAL ASTROPHYSICAL CONSEQUENCES AND DETECTION

If preon stars do exist, and are as small as $10^{-1} - 10^{-4}$ m, it is plausible that primordial preon stars (or “nuggets”) formed from density fluctuations in the early universe. As this material did not take part in the ensuing nucleosynthesis, the abundance of preon nuggets is not constrained by the hot big bang model bounds on baryonic matter. Also, preon nuggets are immune to Hawking radiation [18] that rapidly evaporates small primordial black holes, making it possible for preon
nuggets to survive to our epoch. They can therefore serve as the mysterious dark matter needed in many dynamical contexts in astrophysics and cosmology \[19, 20\].

Preon stars born out of the collapse of massive ordinary stars \[21\] cannot contribute much to cosmological dark matter, as that material originally is baryonic and thus constrained by big bang nucleosynthesis. However, they could contribute to the dark matter in galaxies. Roughly 4% of the total mass of the universe is in baryonic form \[22\], but only 0.5% is observed as visible baryons \[23\]. Assuming, for simplicity, that all dark matter \( \rho_{DM} = 10^{-25} \text{g/cm}^3 \) in spiral galaxies, \( e.g., \) our own Milky Way, is in the form of preon stars with mass \( 10^{24} \text{kg} \), the number density of preon stars is of the order of \( 10^4 \) per cubic parsec (1 parsec \( \simeq 3.1 \times 10^{16} \text{m} \)). This translates into one preon star per \( 10^6 \) solar system volumes. However, even though it is not ruled out a priori, the possibility to form a very small and light preon star in the collapse of a large massive star remains to be more carefully investigated. In any case, preon nuggets formed in the primordial density fluctuations could account for the dark matter in galaxies. The existence of such objects can in principle be tested by gravitational microlensing experiments.

Today there is no known mechanism for the acceleration of cosmic rays with energies above \( \sim 1\times 10^{17} \text{eV} \). These so-called ultra-high energy cosmic rays \(24\) (UHE CR) are rare, but have been observed with energies approaching \( 10^{21} \text{eV} \). The sources of UHE CR must, cosmologically speaking, be nearby (\( \lesssim 50 \text{Mpc} \simeq 150 \text{million light years} \)) due to the GZK-cutoff energy \( \sim 10^{19} \text{eV} \) \[25, 26\], since the cosmic microwave background is no longer transparent to cosmic rays at such high energies. This requirement is very puzzling, as there are no known sources capable of accelerating UHE CR within this distance. Preon stars open up a new possibility. It is known that neutron stars, in the form of pulsars, can be a dominant source of galactic cosmic rays \[27\], but cannot explain UHE CR. If for preon stars we assume, as in models of neutron stars, that the magnetic flux of the parent star is (more or less) frozen-in during collapse, induced electric fields more than sufficient for the acceleration of UHE CR become possible. As an example, assume that the collapse of a massive star is slightly too powerful for the core to stabilize as a 10 ms pulsar with radius 10 km, mass \( 1.4M_\odot \) (\( M_\odot \simeq 2 \times 10^{30} \text{kg} \) is the mass of our sun) and magnetic field \( 10^8 \text{T} \), and instead collapses to a preon star state with radius 1 m and mass \( 10^2 M_\odot \). An upper limit estimate of the induced electric field of the remaining “preon star pulsar” yields \( \sim 10^{34} \text{V/m} \), which is more than enough for the acceleration of UHE CR. Also, such strong electric fields are beyond the limit where the quantum electrodynamic vacuum is expected to break down, \( |\mathbf{E}| > 10^{18} \text{V/m} \), and spontaneously start pair-producing particles \[28\]. This could provide an intrinsic source of charged particles that are accelerated by the electric field, giving UHE CR. With cosmic ray detectors, like
the new Pierre Auger Observatory [29], this could provide means for locating and observing preon stars.

V. CONCLUSIONS

In this letter we argue that if there is a deeper layer of fermionic constituents, so-called preons, below that of quarks and leptons, a new class of stable compact stars could exist. Since no detailed theory yet exists for the interaction between preons, we assume that the mass-energy contribution from preon interactions can be accounted for by a ‘bag constant’. By fitting the bag constant to the energy density of a composite electron, the maximum mass for preon stars can be estimated to \( \sim 10^2 M_\odot \) (\( M_\odot \approx 6 \times 10^{24} \) kg being the Earth mass), and their maximum radius to \( \sim 1 \) m. The central density is at least of the order of \( 10^{23} \) g/cm\(^3\). Preon stars could have formed by primordial density fluctuations in the early universe, and in the collapse of massive stars. We have briefly noted their potential importance for dark matter and ultra-high energy cosmic rays, connections that also could be used to observe them. This might provide alternative means for constraining and testing different preon models, in addition to direct tests [8] performed at particle accelerators.

VI. ACKNOWLEDGEMENTS

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