Supporting information for “The energy budget and figure of Earth during recovery from the Moon-forming giant impact”

Simon J. Lock\textsuperscript{a,b,}\*, Sarah T. Stewart\textsuperscript{c}, Matija Ćuk\textsuperscript{d}

\textsuperscript{a}Division of Geological and Planetary Sciences, California Institute of Technology, 1200 E. California Blvd., Pasadena, CA 91125, U.S.A.
\textsuperscript{b}Department of Earth and Planetary Sciences, Harvard University, 20 Oxford Street, Cambridge, MA 02138, U.S.A.
\textsuperscript{c}Department of Earth and Planetary Sciences, University of California Davis, One Shields Avenue, Davis, CA 95616, U.S.A.
\textsuperscript{d}SETI Institute, 189 Bernardo Avenue, Mountain View, CA 94043, USA

Contents

1. Supporting Text S1-S2.
2. Figure S1-S7.
3. Table S1.

S1. Supplementary methods

\textit{S1.1. Smoothed particle hydrodynamics}

SPH self-consistently calculates the velocity, gravitational potential, and internal energy using a given equation of state (EOS). It is thus straightforward to calculate the energy budget of post-impact bodies. In SPH, Equation 1 can be simplified to

$$E_K = \frac{1}{2} \sum_i r_{xy,i}^2 \omega_i^2 m_i,$$

(S1)

where $r_{xy,i}$, $\omega_i$ and $m_i$ are the cylindrical radius, angular velocity and mass of particle $i$. Potential and internal energies are similarly calculated as a sum over all SPH particles:

$$E_{\text{pot}} = \frac{1}{2} \sum_i \Phi_i m_i,$$

(S2)

\*Corresponding author: slock@caltech.edu
\[ E_{\text{int}} = \sum_i \epsilon_i m_i, \quad (S3) \]

where \( \Phi_i \) and \( \epsilon_i \) are the gravitational potential and specific internal energy of each particle as determined from the distribution of mass and the EOS, respectively.

The modified specific impact energy (Lock and Stewart, 2017), \( Q_S \), takes into account the efficiency with which energy is coupled into the impacting bodies, and is defined by,

\[ Q_S = Q'_R \left( 1 + \frac{M_p}{M_t} \right) (1 - b), \quad (S4) \]

where \( Q'_R \) is a center-of-mass specific impact energy modified to include only the interacting mass of the projectile (Leinhardt and Stewart, 2012), \( M_p \) and \( M_t \) are the mass of the projectile and target respectively, and \( b \) is the impact parameter. \( Q'_R \) is given by

\[ Q'_R = \frac{\mu \alpha \mu}{\mu} Q_R. \quad (S5) \]

Note that there is a typographical error in the definition of \( Q'_R \) in equation 13 of Leinhardt and Stewart (2012). \( Q_R \) is the unmodified center of mass specific impact energy,

\[ Q_R = \frac{\mu V_i^2}{2 M_{\text{tot}}}. \quad (S6) \]

The reduced mass is defined as

\[ \mu = \frac{M_p M_t}{M_{\text{tot}}}, \quad (S7) \]

and, to consider only the interacting fraction of the projectile, a modified reduced mass is used, given by

\[ \mu_\alpha = \frac{\alpha M_p M_t}{\alpha M_p + M_t}. \quad (S8) \]

\( M_{\text{tot}} = M_p + M_t \), \( V_i \) is the impact velocity and \( \alpha \) is the mass fraction of the projectile that is involved in the collision. \( \alpha \) is defined as

\[ \alpha = \frac{m_{\text{interact}}}{M_p} = \frac{3R_p l^2 - l^3}{4R_p^3}, \quad (S9) \]

where \( m_{\text{interact}} \) is the interacting projectile mass, \( R_t \) and \( R_p \) are the radii of the target and projectile and \( B = (R_t + R_p) b \). \( l \) is the projected length of the projectile overlapping the target,

\[ l = \begin{cases} R_t + R_p - B & \text{when } B + R_p > R_t \\ 2R_p & \text{when } B + R_p \leq R_t \end{cases}. \quad (S10) \]

If \( B + R_p \leq R_t \) then the whole projectile is interacting with the target and \( \alpha = 1 \).
S1.2. HERCULES

In HERCULES, a body is described as a series of nested, concentric, constant-density spheroids. All the material between the surfaces of any two consecutive spheroids is called a layer. The energy components are calculated as a sum over these layers. The kinetic energy of a corotating body is simply

$$E_{K,co} = \sum_{i=0}^{N_{lay}-1} \frac{1}{2} I_i \omega^2_{rot},$$

(S11)

where $N_{lay}$ is the number of spheroids, $I_i$ is the moment of inertia of spheroid $i$, and $\omega_{rot}$ is the angular velocity of the body. Spheroids are numbered from the outside inwards starting at $i = 0$.

To calculate the potential energy, Equation 4 can be approximated as

$$E_{pot} = \sum_{i=0}^{N_{lay}-2} \frac{1}{2} \left[ \frac{\Phi_i + \Phi_{i+1}}{2} \right] \rho_i [V_i - V_{i+1}] + \frac{1}{2} \left[ \frac{\Phi_{N_{lay}-1} + \Phi_{\text{core}}}{2} \right] \rho_{N_{lay}-1} V_{N_{lay}-1},$$

(S12)

where $\rho_i$ is the real density of layer $i$, and $V_i$ is the volume of spheroid $i$. $\Phi_i$ is the potential at the surface of spheroid $i$ and $\Phi_{\text{core}}$ is the potential at the center of the body. The internal energy is given by the sum over each layer,

$$E_{int} = \sum_{i=0}^{N_{lay}-2} \rho_i [V_i - V_{i+1}] \epsilon(\rho_i, S_i) + \rho_{N_{lay}-1} V_{N_{lay}-1} \epsilon(\rho_{N_{lay}-1}, S_{N_{lay}-1}),$$

(S13)

where $\epsilon(\rho_i, S_i)$ is the specific internal energy of the material at the density and specific entropy of layer $i$, as determined by the EOS.

For this paper, we used the same HERCULES parameters as in Lock and Stewart (2017). The energy components calculated using HERCULES are only weakly dependent on the number of concentric potential layers (Figure S1), the number of points used to describe potential surfaces (Figure S2), and the maximum spherical harmonic degree included in the calculation (Figure S3). For the wide range of parameters we considered, each of the energy components of the body varied by less than 0.1%.

We considered bodies with two different thermal profiles. To calculate the properties of bodies just below the CoRoL (Stage II), we imposed thermally stratified mantles that emulate the thermal
structure of bodies after an impact and during condensation (Lock and Stewart, 2017; Lock et al., 2018). The core was assumed to be isentropic with a specific entropy of 1.5 kJ K\(^{-1}\) kg\(^{-1}\). This core isentrope has a temperature of 3800 K at the present-day core mantle boundary (CMB) pressure, similar to the present thermal state of Earth’s core (e.g., Anzellini et al., 2013). The lower mantle (75% by mass) was assumed to be isentropic with a specific entropy of 4 kJ K\(^{-1}\) kg\(^{-1}\), which is typical for the lower-mantle of calculated post-impact bodies (Lock and Stewart, 2017) (Figure S4 shows pressure-temperature profiles for forsterite isentropes). The upper 25 wt% of the mantle was isentropic with a specific entropy of \(S_{\text{outer}}\) at pressures above the liquid-vapor phase boundary. At pressures below the intersection of the isentrope with the phase boundary, the body was assumed to be pure vapor on the phase boundary. This thermal profile was called a stratified structure in Lock and Stewart (2017).

For condensed bodies (in Stages III to V), we used isentropic cores and mantles with specific entropies of 1.5 and 4 kJ K\(^{-1}\) kg\(^{-1}\) respectively. This mantle isentrope intersects the liquid-vapor phase boundary at low pressure (10 bar) and about 4000 K. The pressure at the surface was set at 10 bar so as not to resolve the silicate atmosphere. Our chosen thermal state approximates that of a well-mixed, liquid, magma-ocean planet with a volatile-dominated atmosphere.

The relative timings of the freezing of the mantle and tidal recession of the Moon are uncertain and so here we have used a single thermal profile for all condensed planets. However, the thermal state during tidal recession of a condensed body has little effect on the change in energy. Figure S5 shows the change in each energy term during tidal recession for bodies with mantle isentropes of 3, 3.2 and 4 kJ K\(^{-1}\) kg\(^{-1}\). Mantle specific entropies of 3 and 3.2 kJ K\(^{-1}\) kg\(^{-1}\) correspond to mantle potential temperatures of \(\sim 1600\) K and \(\sim 1900\) K, similar to the present-day and early terrestrial mantle respectively. The change in energy during tidal recession for different thermal states differs by only about a percent.

When comparing with SPH simulations to calculate the energy change upon condensation to a magma-ocean planet (transition from stage I to stage III), we did not directly calculate the structure for all the planets. Instead, we calculated a grid of planets with a range of masses, core-mass fractions, and AM. We used total mass increments of 0.1 \(M_{\text{Earth}}\), core-mass fraction increments of 0.05, and a base AM increment of 0.1 \(L_{\text{EM}}\). When HERCULES failed to converge at the next AM step, higher AM runs were performed using a smaller step. The AM step was sequentially halved five times to provide finer AM resolution just below the CoRoL. The energy components and total
energy were calculated for each body and linearly interpolated to determine the properties for a body of a given composition, mass and AM. The variation in energy for the range of parameters we considered is close to linear and this technique gives a very good approximation to the energy of directly calculated planets.

For comparison of SPH post-impact bodies (stage I) to bodies just below the CoRoL (stage II), we interpolated a grid of hot, stratified planets with varying mass, core-mass fraction, and \( S_{\text{outer}} \). We ran the same mass and core-mass fraction increments as for the magma-ocean planets and \( S_{\text{outer}} = 4, 4.25, 4.5, 4.75, 5.0, 5.5, 6.0, 6.5, 7.0, 7.5, \) and \( 8.0 \) kJ K\(^{-1}\) kg\(^{-1}\), using the same AM step procedure. We found the AM of the CoRoL for each set of parameters as described in Lock and Stewart (2017). The energies of a body at the CoRoL were found by linearly extrapolating from the calculated corotating planets in the same manner as in Lock and Stewart (2017) (see their Figure S8). For a post-impact body with a given mass, core-mass fraction and AM, the properties of the body when it has cooled to just below the CoRoL were found by linearly interpolating the grid of CoRoL properties in mass-core fraction-AM space.

S1.3. Comparison of methods

Lock and Stewart (2017) and Lock and Stewart (2019) have shown that, for corotating structures with a range of thermal states, the shape and pressure structure calculated by SPH and HERCULES are in good agreement. Here we examine the energy components determined using our two methods.

Figure S6 shows a comparison of the energies calculated using SPH and HERCULES for Earth-mass, corotating bodies with isentropic mantles of varying specific entropy. The shape and pressure contours for these bodies are shown in Figure 4 of Lock and Stewart (2017). The energies calculated using the two methods are the same to within a few percent. Given the agreement between the two different methods, we are confident in the conclusions we have made in this work.

S2. The effect of forming the Moon

In Figure 2, we neglected the effect of the formation of the Moon on the energy budget, and assumed that the magma-ocean planet had the same mass, AM and composition as the post-impact body. It is likely that little mass and AM is lost from the system during lunar accretion (Ida et al., 1997; Kokubo et al., 2000; Salmon and Canup, 2012, 2014), but the effect of the fully-formed Moon on the energy budget must be considered. Figure S7 shows the energy difference between a system
Figure S1: Caption on next page
Figure S1: Energies calculated using HERCULES are only weakly dependent on the number of concentric layers used ($N_{\text{lay}}$). Shown are the fractional differences in energies of Earth-mass, Earth-composition magma-ocean planets of varying angular momenta calculated using HERCULES with a given number of concentric layers (colors) and the same body calculated with $N_{\text{lay}} = 100$, as used elsewhere in this paper. Panels show the difference in the total (A), kinetic (B), gravitational potential (C), and internal (D) energies.

Table S1: Summary of SPH impact simulations used in this paper and properties of their post-impact states. For each impact, the table includes: an index number; target mass $M_t$; number of SPH particles in target, $N_t$; target equatorial radius, $R_t$; target angular momentum, $L_t$; projectile mass, $M_p$; number of SPH particles in projectile, $N_p$; projectile equatorial radius, $R_p$; projectile angular momentum, $L_p$; impact velocity, $V_i$; impact parameter, $b$; modified specific energy, $Q_S$; final simulation time; bound mass of post-impact structure, $M_{\text{bnd}}$; core-mass fraction of post-impact structure, $f_{\text{core}}$; bound mass angular momentum, $L_{\text{bnd}}$; angular velocity of the dense ($\rho > 1000$ kg m$^{-3}$) region of post-impact structure, $\omega_\rho$; spin period of dense region, $T_\rho$; total energy of post-impact structure, $E_{\text{tot}}$; kinetic energy of post-impact structure, $E_{\text{K}}$; potential energy of post-impact structure, $E_{\text{pot}}$; internal energy of post-impact structure, $E_{\text{int}}$; total energy of a body of the same mass, core fraction, and angular momenta as the post-impact body but with a hot partially-vaporized, thermally-stratified thermal structure with an upper mantle entropy such that the body is at the CoRoL, $E_{\text{CoRoL}}$; kinetic energy at the CoRoL, $E_{\text{K}}^{\text{CoRoL}}$; potential energy at the CoRoL, $E_{\text{pot}}^{\text{CoRoL}}$; internal energy at the CoRoL, $E_{\text{int}}^{\text{CoRoL}}$; total energy of a magma-ocean planet with the same mass, angular momentum and core-mass fraction as the post-impact body, $E_{\text{MO}}$; kinetic energy of corresponding magma-ocean planet, $E_{\text{K}}^{\text{MO}}$; potential energy of corresponding magma-ocean planet, $E_{\text{pot}}^{\text{MO}}$; internal energy of corresponding magma-ocean planet, $E_{\text{int}}^{\text{MO}}$; and post-impact structure dynamical class.

with all the mass and AM in a single magma-ocean planet and a system of a magma-ocean Earth and Moon with the Moon orbiting at a range of semi-major axes. Forming the Moon affects the energy budget in two ways. First, the Moon carries with it orbital energy. Second, the extraction of mass and AM from Earth upon forming the Moon changes its kinetic, potential and internal energy (Section 4.2). For bodies with initially high AM, the energy budget of Earth decreases strongly as a function of AM, the reduction in Earth’s energy is dominant over the orbital energy, and the energy of the system is lower with a Moon than without. At lower AM, the orbital energy effect dominates and the energy of the total system is higher with a Moon than without. The semi-major axis of the Moon during condensation is uncertain, but, as the Moon is expected to stay close to Earth during the initial stages of cooling due to the limited tidal dissipation in a fluid body (e.g., Zahnle et al., 2015), the effect of the Moon on Earth’s energy budget is likely an order of magnitude smaller than that of condensation.
Figure S2: Caption on next page
Figure S2: Energies calculated using HERCULES are only weakly dependent on the number of points used to describe each equipotential surface ($N_{\mu}$). Shown are the fractional difference in the different energy components of Earth-mass, Earth-composition magma-ocean planets calculated using HERCULES with varying $N_{\mu}$ and bodies calculated using $N_{\mu} = 1000$. Panels show the difference in the total (A), kinetic (B), gravitational potential (C), and internal (D) energies.

References used in supplement

Anzellini, S., Dewaele, A., Mezouar, M., Loubeyre, P., Morard, G., 2013. Melting of iron at Earth’s inner core boundary based on fast X-ray diffraction. Science 340 (6131), 464–6.

Ida, S., Canup, R. M., Stewart, G. R., 1997. Lunar accretion from an impact-generated disk. Nature 389 (6649), 353–357.

Kokubo, E., Ida, S., Makino, J., 2000. Evolution of a circumterrestrial disk and formation of a single moon. Icarus 148 (2), 419–436.

Leinhardt, Z. M., Stewart, S. T., 2012. Collisions between gravity-dominated bodies. I. Outcome regimes and scaling laws. The Astrophysical Journal 745 (1), 79.

Lock, S. J., Stewart, S. T., 2017. The structure of terrestrial bodies: Impact heating, corotation limits, and synestias. Journal of Geophysical Research: Planets 122 (5), 950–982.

Lock, S. J., Stewart, S. T., 2019. Giant impacts stochastically change the internal pressures of terrestrial planets. Science Advances 5 (9), eaav3746.

Lock, S. J., Stewart, S. T., Petaev, M. I., Leinhardt, Z., Mace, M. T., Jacobsen, S. B., Cuk, M., 2018. The origin of the Moon within a terrestrial synestia. Journal of Geophysical Research: Planets 123 (4), 910–951.

Salmon, J., Canup, R. M., 2012. Lunar accretion from a Roche-interior fluid disk. The Astrophysical Journal 760 (1), 83.

Salmon, J., Canup, R. M., 2014. Accretion of the Moon from non-canonical discs. Philosophical transactions. Series A, Mathematical, physical, and engineering sciences 372 (2024), 20130256.

Zahnle, K. J., Lupu, R., Dobrovolskis, A., Sleep, N. H., 2015. The tethered Moon. Earth and Planetary Science Letters 427, 74–82.

S9
Figure S3: Caption on next page
Figure S3: Energies calculated using HERCULES are only weakly dependent on the maximum spherical harmonic degree included ($2k_{\text{max}}$). Shown are the fractional difference in the different energy components of Earth-mass, Earth-composition magma-ocean planets calculated using HERCULES with varying $k_{\text{max}}$ and bodies calculated using $2k_{\text{max}} = 12$, as used elsewhere in this paper. Panels show the difference in the total (A), kinetic (B), gravitational potential (C), and internal (D) energies.

Figure S4: Isentropes for the M-ANEOS derived forsterite EOS used in this work in pressure-temperature space. Each colored line is an isentrope for the specific entropy given by the number of the same color in kJ K$^{-1}$ kg$^{-1}$. The black line is the liquid-vapor phase boundary. The black dot is the critical point. Adapted from Lock and Stewart (2017).
Figure S5: Caption on next page
Figure S5: The change in Earth’s energy budget during tidal recession is only weakly dependent on its thermal state. Shown are the differences between Earth-mass, Earth-composition bodies of given angular momenta and equivalent non-rotating bodies. Color indicate bodies with different mantle specific entropies (see Sec S1.2).
Figure S6: Planetary structures calculated using SPH and HERCULES have similar energies. Rows show the absolute (top) and fractional (bottom) difference in different energy components between SPH and HERCULES calculations. Columns show the total (A), kinetic (B), gravitational potential (C), and internal (D) energies in Earth-mass bodies with different angular momenta (colors) and isentropic mantles of varying specific entropy (x-axis). Comparisons were made to HERCULES planets with a bounding pressure of both 10 bar (+) and a pressure equivalent to the lowest pressure in the midplane of the SPH structure (). The solid black and red lines in the bottom row show the mean fractional difference using HERCULES planets with bounding pressures of 10 bar and the maximum SPH pressure respectively. The shaded area shows one standard deviation in the fractional errors. The shape and pressure contours for the SPH bodies plotted here are shown in Figure 4 of Lock and Stewart (2017).
Figure S7: Forming the Moon changed the energy budget but the effect was likely small compared to the effect of condensation. Shown is the difference in the total energy between an Earth-like magma-ocean planet orbited by a tidally-locked Moon at a given semi-major axis (y-axis) and a system with the same total angular momentum, mass, and composition, but with all the mass combined into a single magma-ocean planet. The orbital energy was calculated treating Earth and the Moon as point masses and the Moon was assumed to have a mantle entropy of 4 kJ K$^{-1}$ kg$^{-1}$. The dashed line indicates the locus of points for which there is no difference in energy between the two systems.