Direct Location for Multiple Passive Radars without and with Reference

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Abstract. The problem of stationary target location by multiple passive radar sensors that using unknown and non-cooperative opportunity illuminator is considered. Traditional two-step approach which estimating the time difference of arrival (TDOA) and angle of arrival (AOA) firstly and locating using those parameters secondly. We explore the direct location with multiple passive radar sensors without estimating the intermediate parameters. As the reference path from the transmitter to receivers may be blocked in practice, we discuss the both cases that location without and with reference path. Two maximum likelihood algorithms of direct location are proposed for multiple passive radar sensors without and with reference. Monte Carlo simulations indicate that the direct location algorithm is superior to two-step approach with TDOA and AOA at low SNR for multiple passive radar sensors.

1. Introduction

Target location by passive radar sensors are researched in this paper. A passive radar sensor system employs the existing non-cooperative illuminators of opportunity, such as cellular, radio, television [1]. For passive radar sensors, both the direct-path (i.e. transmitter-to-receiver) and target-path (i.e. transmitter-to-target-to-receiver) signals are received by each sensor [2]. In the application of target location, the direct-path signal is always used for reference signal. Yet, the direct-path signal may be not received by sensors with directional antenna, or when the path from transmitter-to-receiver is blocked by some obstacles, such as mountains, houses and so on. So, the target location for multiple passive sensors without reference also needs to be considered.

Most location methods are proposed based on the direction of arrival (DOA), the time of arrival (TOA), the differential time of arrival (TDOA), the differential Doppler of arrival (FDOA), and the combination of them [3-5]. All of them are named as two-step method [6], as the intermediate parameters should be estimated firstly, and then, the target location is calculated by the intermediate parameters. Recently, the direct position determination method, also called direct location or one-step method, was proposed by A. Weiss, etcetera [7, 8], which obtains the target position from received signal directly. Following that, they proved the one-step method is prior to two-step method at low SNR, because of the information loss of two-step [8, 9]. From the view of system, the two-step method is a distributed method that the parameters are calculated at each sensor first, and then, the parameters are transmitted to the centre. However, the one-step method is a centralized method that the signals after pre-processing are transmitted to the centre directly. Besides, the direct location method has the advantage of multiple targets location [10], as the two-step needing for data-target association, but one-step not.
Though, the direct location method has been researched and applied in much field [10-12], and there is much literature, which is mainly concentrated on the transmitter location [7-11], and some on active radars [12]. However, the target location by passive radar sensors that using non-opportunity illuminator is little researched. Different from the transmitter location, there are several new problems which should be researched for the location of multiple passive sensors. Thus, from above analysis, the problem of direct location using multiple passive radars with and without reference is researched, and two direct location algorithms are proposed, which is demonstrated to be more precise than two-step approach with TDOA and AOA through simulations.

The manuscript structure is organized as follows. Section 2 describes the problem and multiple passive radars signal model. Section 3 derives the two direct location algorithms of multiple passive radars without reference and with reference. The numerical experiment results are shown in Section 4. Finally, conclusions are discussed in Section 5.

2. Problem Formulation
Consider a multiple passive sensors system as shown in figure 1, which involves a non-cooperative illuminator and $M$ widely distributed passive radar sensors, and each sensor has $K$ identical array elements. The dotted line denotes the target-path (i.e. transmitter-to-target-to-sensors), the solid line denotes the direct-path (i.e. transmitter-to-sensors). The stationary target which located at $\mathbf{p}$ is illuminated by a non-cooperative illuminator, which located at $\mathbf{t}$, and the echo from target is received by the sensor located at $\mathbf{r}_m$, for $m=1,...,M$.

The signal vector received by $m$ th sensor at $i$ th array element, denoted by $x_i^m$ is expressed as

$$
x_i^m = \eta_d^m e^{j\phi_d^m} \mathbf{D}_d^m \mathbf{u} + \eta_p^m e^{j\phi_p^m} \mathbf{D}_p^m \mathbf{u} + \mathbf{w}_i^m
$$

where $\mathbf{u}, \mathbf{w}_i^m \in \mathbb{C}^{N \times 1}$ denote unknown narrow-band signal vector transmitted by the non-cooperative illuminator and the complex Gaussian noise vector of $m$ th sensor at $i$ th element, respectively. $\eta_d^m$ and $\eta_p^m$ are $m$ th the complex attenuation coefficient at direct-path and target-path, respectively.

![Figure 1. Geometry of a multiple passive sensor system for target location](image)

Besides, $\phi_d^m$ (d) and $\phi_p^m$ (p) are the differential phase at $i$ th array element respect to the reference element at direct-path and target-path, respectively, written as

$$
\phi_d^m = 2\pi/\lambda(t-1)l \sin \theta_d
$$

$$
\phi_p^m = 2\pi/\lambda(t-1)l \sin \theta_p
$$
where $i=1,\ldots,K$, $\theta_d$ and $\theta_p$ are the incidence angle at direct-path and target-path, respectively. $N = [T f]$, where $T$ is the observation time, $f$ is the sample frequency, $\lceil \cdot \rceil$ denotes the function of floor. $D_d$ and $D_p$ are expressed as, respectively,

$$
D_d^T = F^H D \left( \frac{n_{r_d}}{N} \right) F
$$

$$
D_p^T = F^H D \left( \frac{n_{r_p}}{N} \right) F
$$

where $n_{r_d}$ and $n_{r_p}$ denote the time delay for direct-path and target-path, respectively, which are written as

$$
n_{r_d} = (\|t - r_m\|) f_s / c
$$

$$
n_{r_p} = (\|r_m - p\| + \|p - t\|) f_s / c
$$

where $c$ is the electromagnetic wave propagation speed. Besides, $D \in \mathbb{C}^{N \times N}$ is the time delay matrix, denoted by

$$
D(y) = \text{diag} \left( e^{-j2\pi y}, e^{-j2\pi[y]}, \ldots, e^{-j2\pi[N-1]y} \right)
$$

where $\text{diag}(\cdot)$ denotes the diagonal matrix. And, $F \in \mathbb{C}^{N \times N}$ is the unitary discrete Fourier transform (DFT) matrix, the $(l,k)^{th}$ element is

$$
F_{lk} = \frac{1}{N} \exp \left( -j2\pi \frac{lk}{N} \right)
$$

Then, let $x^m \in \mathbb{C}^{KN \times 1}$ denotes the concatenation of all elements at $m$th sensor, we have

$$
x^m = \eta_{a} \left( a_d \otimes D_d^m \right) u + \eta_{p} \left( a_p \otimes D_p^m \right) u + w^m
$$

where $x^m = [(x_1^m)^H, \ldots, (x_K^m)^H]^H$, $a_d = [e^{j\phi_1(1)}, \ldots, e^{j\phi_2(d)}]^H$, Similarly, we have $a_p$ and $w^m$, and

$x = [(x_1)^H, \ldots, (x_M)^H]^H \in \mathbb{C}^{KN \times M}$.  

3. Direct Location Algorithms

In the application for passive multiple sensors, the signal from direct-path is always used for reference signal, but sometimes, the direct-path maybe blocked by some buildings such as mountains, houses, and so on. Thus, the problem is divided into two parts. One is the location without references, the other is the location with references.  

3.1. Direct Location without Reference

From (8), the signal received at all sensors without references denotes as

$$
x^m = \eta_p^m \left( a_p^m \otimes D_p^m \right) u + w^m
$$

Assume the noise received at each sensor is independent, and $w^m \sim CN \left( 0, \sigma^2 I \right)$. The complex attenuation coefficient $\eta_p^m$ at target-path is unknown constant. Then, the probability density function of $x$ is expressed by
\[
\begin{align*}
    p_x &= (\pi \sigma)^{-MNK} \sum_{m=1}^{M} \exp \left\{ -\frac{1}{\sigma^2} \left\| x^m - \eta_p \left( D_p^m \right)^H \right\| \right\} \\
    \text{From above, the negative logarithm likelihood function is denoted as} & \quad L_1(p) = \sum_{m=1}^{M} \left\| x^m - \eta_p D_p^m u \right\|^2 \\
    \text{From (12), the maximum likelihood estimation (MLE) of } \eta_p & \quad \text{is given as} \\
    \hat{\eta}_p &= \frac{u^H \left( a_p^m \otimes D_p^m \right)^H x^m}{K \left\| u \right\|^2} \\
    \text{where the property } (D_p^m)^H D_p^m &= I \text{ has been used above.} \\
    \text{Substituting the estimation } \hat{\eta}_p & \quad \text{back into (12), we have} \\
    L_1(p) &= \sum_{m=1}^{M} \left\| \left( P_p^m \right)^H s_m \right\|^2 \\
    \text{where,} & \quad P_p^m = \frac{1}{K \left\| u \right\|^2} \left( a_p^m \otimes D_p^m \right) uu^H \left( a_p^m \otimes D_p^m \right)^H \\
    \text{is the projection matrix, and } P^\perp = I - P^\perp. & \quad \text{Then, the estimation of target location can be written as} \\
    \hat{p} &= \arg \min_p \left\{ \sum_{m=1}^{M} \left\| \left( P_p^m \right)^H s_m \right\|^2 \right\} \triangleq \arg \max_p \left( \sum_{m=1}^{M} s_m^H P_p^m s_m \right) \\
    \text{From, the formulation can be derived as, further,} & \quad \hat{p} = \arg \max_p \left\{ u^H U U^H u \right\} \\
    \text{and} & \quad U = \left[ (a_p^1 \otimes D_p^1)^H, \ldots, (a_p^M \otimes D_p^M)^H \right] \\
    \text{i.e. the } m \text{ th column of } U & \quad \text{is } (a_p^m \otimes D_p^m)^H x^m. \quad \text{As } D_p^m \text{ is delay time Matrix, the vector } U^H x^m \text{ is essentially the result after time compensation of received signal } x^m. \\
    \text{As shown at [7], the formulation (17) is Rayleigh quotient, so the cost function is maximized when} & \quad \text{the transmitted signal } u \text{ is the eigenvector corresponding to the largest eigenvalue. Further, as} \\
    \lambda_{\text{max}}(U U^H) = \lambda_{\text{max}}(U^H U), & \quad \text{we let } G_1 \in \mathbb{C}^{M \times M} \text{ and } G_1 = U^H U, \text{ the formula (14) can be expressed as} \\
    \hat{p} &= \arg \max_p \left( \lambda_{\text{max}}(G_1) \right), \text{ for } p \in \Omega \\
    \text{where } \Omega & \quad \text{is the interesting area where target may be exist in. From (18) and (19), we can see that, the} \\
    \text{estimation expression of target location by multiple passive radars without reference is the similar to} & \quad \text{the passive location of transmitter.} \\
    \text{For clearer, the procedures of direct location of multiple passive radars without reference is} & \quad \text{summarized in Algorithm 1.}
\end{align*}
\]
Algorithm 1. Direct Location of Multiple Passive Radars without Reference

Input: observation \( \mathbf{x} \), discretization \( \Omega \) which has \( L_x \times L_y \) points, steering vector \( \mathbf{a}_p^m \), location of transmitter and receiver, \( \mathbf{t} \) and \( \mathbf{r} \), \( m=1,\ldots,M \), respectively.

Output: target location estimation \( \mathbf{\hat{p}} \)

for \( l = 1,2,\ldots,L_x \times L_y \) do

1) compute \( \mathbf{a}_p^n \) by at each point of grid;

2) Compute the \( \mathbf{D}_p^m \) at each point of grid by (4) and (5);

3) Compute \( \mathbf{U} \) and \( \mathbf{G}_1 \) by (18) and (19);

4) Compute \( \lambda_{\text{max}}(\mathbf{G}_1) \) by eigenvalue decomposition of \( \mathbf{G}_1 \).

end for

Estimate \( \mathbf{\hat{p}} \) using for all \( \mathbf{p}_l \) by (19), \( l = 1,\ldots,L_x \times L_y \).

return the estimation of \( \mathbf{\hat{p}} \).

3.2. Direct Location with Reference

Similarly, the negative logarithm likelihood function for the direct location for multiple passive sensors with reference is written as

\[
\begin{align*}
\hat{L}_2(\mathbf{p}) & = \sum_{m=1}^{M} \left\| \mathbf{x}^m - \mathbf{\eta}_d^m (\mathbf{a}_d^m \otimes \mathbf{D}_d^m) \mathbf{u} - \mathbf{\eta}_p^m (\mathbf{a}_p^m \otimes \mathbf{D}_p^m) \mathbf{u} \right\|^2 \\
& \approx \sum_{m=1}^{M} \left\| \mathbf{x}^m - \mathbf{\mu}_d^m \mathbf{u} \right\|^2 + \left\| \mathbf{\mu}_p^m \mathbf{u} \right\|^2 + E_{(dp)}^m
\end{align*}
\]

(20)

where the \( E_{(dp)}^m \) denotes the overall energy within the orthonormal projection of \( \mathbf{x}^m \) into \( \langle \mathbf{a}_d^m, \mathbf{a}_p^m \rangle \) \(^\perp\) [2].

From (24), the MLE of \( \mu_d^m \) and \( \mu_p^m \) are computed as

\[
\hat{\mu}_d^m = \mathbf{u}^H (\mathbf{b}_d^m \otimes \mathbf{D}_d^m)^H \mathbf{x}^m / \| \mathbf{u} \|^2
\]

(25)
\[ \hat{\mu}_p^n = \mathbf{u}^H \left( \mathbf{b}_p^n \otimes \mathbf{D}_p^n \right)^H \mathbf{x}^n / \| \mathbf{u} \|^2 \]  

(26)

Moreover, the total signal energy can be written as

\[ E = \sum_{m=1}^{M} \left\| \left( \mathbf{b}_d^n \otimes \mathbf{D}_d^n \right)^H \mathbf{x}^* \right\|^2 + \left\| \left( \mathbf{b}_p^n \otimes \mathbf{D}_p^n \right)^H \mathbf{x}^n \right\|^2 + E_{(q_n)}^m. \]  

(27)

Then, substituting (25)-(27) into (24), we have

\[ \tilde{L}_z(p) = E - L_z(p) \]  

(28)

and

\[ L_z(p) = \sum_{m=1}^{M} \frac{1}{\| \mathbf{u} \|^2} \left\| \mathbf{u}^H \left( \mathbf{b}_p^m \otimes \mathbf{D}_p^m \right)^H \mathbf{x}^m \right\|^2 - \frac{1}{\| \mathbf{u} \|^2} \left\| \mathbf{u}^H \left( \mathbf{b}_d^m \otimes \mathbf{D}_d^m \right)^H \mathbf{x}^m \right\|^2 \]  

(29)

### Algorithm 2. Direct Location of Multiple Passive Radars with Reference

**Input:** observation \( \mathbf{x} \), discretization \( \Omega \) which has \( L_x \times L_y \) points, steering vector \( \mathbf{a}_d^n \) and \( \mathbf{a}_p^n \), location of transmitter and receiver, \( \mathbf{t} \) and \( \mathbf{r}_m \), \( m=1,...,M \), respectively

**Output:** target location estimation \( \hat{\mathbf{p}} \)

**for** \( l = 1,2,...,L_x \times L_y \) **do**

1) Compute \( \mathbf{b}_d^n \) and \( \mathbf{b}_p^n \) by (21) and (22);
2) Compute \( \mathbf{D}_d^n \) and \( \mathbf{D}_p^n \) by (4), (5) and (6);
3) Compute \( \mathbf{B} \) and \( \mathbf{G}_2 \) by (32);
4) Compute \( \lambda_{\text{max}} \left( \mathbf{G}_2 \right) \) by eigenvalue decomposition of \( \mathbf{G}_2 \).

**end for**

Estimate \( \hat{\mathbf{p}} \) using for all \( \mathbf{p}^l \) by (33), \( l = 1,...,L_x \times L_y \).

**return** the estimation of \( \hat{\mathbf{p}} \).

Thus, the estimation of target position is written as

\[ \hat{\mathbf{p}} = \arg \min_p \left( \tilde{L}_z(p) \right) = \arg \max_p \left( L_z(p) \right) \]  

(30)

Further, the formula (29) yields

\[ L_z(p) = \frac{\mathbf{u}^H \mathbf{B} \mathbf{B}^H \mathbf{u}}{\| \mathbf{u} \|^2} \]  

(31)

where,

\[ \mathbf{B} = \left[ \left( \mathbf{b}_d^1 \otimes \mathbf{D}_d^1 \right)^H \mathbf{x}^1, \ldots, \left( \mathbf{b}_p^M \otimes \mathbf{D}_p^M \right)^H \mathbf{x}^M, \left( \mathbf{b}_d^1 \otimes \mathbf{D}_d^1 \right)^H \mathbf{x}^1, \ldots, \left( \mathbf{b}_d^M \otimes \mathbf{D}_d^M \right)^H \mathbf{x}^M \right] \]  

(32)

Similarly, let \( \mathbf{G}_2 = \mathbf{B}^H \mathbf{B} \), from (30) and (31), the MLE of target position is expressed as

\[ \hat{\mathbf{p}} = \arg \max_p \left( \lambda_{\text{max}} \left( \mathbf{G}_2 \right) \right), \text{ for } \mathbf{p} \in \Omega \]  

(33)
Similarly, the proposed direct location algorithm of multiple passive radars with reference is summarized in Algorithm 2.

4. Numerical Experiment Results

In this section, numerical experiment results are provided to demonstrate the performance of proposed method. The LFM signal with bandwidth 50 KHz and carrier frequency 100 MHz is used. For the simplification, the two-dimension is considered. And, the transmitter is located at [20, 15] km, the target is located at [5, 12] km, and the four passive radar sensors are located at [-2, 6] km, [0.5, 2] km, [4.5, -0.3] km, [8.5, 0.5] km, respectively, as shown in figure 2. The interesting area where target may be presence is [-5, 2] km to [15, 22] km, and the search grid is 100 m. Besides, the array of each sensor has 8 elements, and the space between two adjacent elements is half of a wavelength.

The performance of proposed algorithm is examined by the root mean squared error (RMSE), which is expressed as

\[
RMSE = \left( \frac{1}{Q} \sum_{q=1}^{Q} \left\| \hat{\mathbf{p}} - \mathbf{p}_{\text{real}} \right\|^2 \right)^{1/2}
\]

where, \(Q\) is the number of Monte Carlo experiment, \(\mathbf{p}_{\text{real}}\) is the real position of target. Afterward, the method proposed is compared with traditional two-step location method with AOA and TDOA at different signal to noise ratio (SNR). The AOA is estimated by multiple signal classification (MUSIC) algorithm [13]. And, the TDOA is estimated by searching the maximum value of cross ambiguity function of two received signals that coming from the reference sensor and the one of other sensors [14]. Then the position is located through weighted least squares (WLS) algorithm with TDOA and AOA parameters.
Figure 3 demonstrates the results of RMSE with two direct location algorithms and two-step algorithms at SNR levels of -20 dB to 5 dB. It’s clear that the proposed direct location algorithms for multiple passive radars is better than traditional two-step with or without reference, especially for low SNR. Besides, as shown at the Figure 3, the location precision with reference path is superior to the precision without reference path because the extra useful information from direct-path is employed.

5. Conclusions
Direct location has been widely researched recently, which has been proved to be better than two-step approach for the location of emitter at low SNR. Different from the location of emitter, there are more problems needing to be explored for the target location by multiple passive radar sensors using unknown and non-cooperative opportunity illuminator. We discuss the target location by multiple passive radar sensors without and with reference path and propose the two direct location algorithms through the MLE and grid search. Meanwhile, the two-step methods combined TDOA and AOA are applied to compare with the proposed algorithms. Simulations show that the performance of proposed algorithms is better than traditional two-step approaches.

References
[1] X. Zhang, H. Li, B. Himed 2017 Multistatic detection for passive radar with direct-path interference IEEE Trans. Aerospace and Electronic Systems 53 pp 915-925.
[2] D. E. Hack, L. K. Patton, B. Himed 2014 Detection in passive MIMO radar networks IEEE Trans. Signal Process. 62 pp 2999-3012.
[3] X. Qu, L. Xie, W. Tan 2017 Iterative constrained weighted least squares source localization using TDOA and FDOA measurements IEEE Trans. Signal Process. 65 pp 3990-4003.
[4] A. Noroozi, M. A. Sebt 2018 Algebraic solution for three-dimensional TDOA/AOA localisation in multiple-input–multiple-output passive radar IET Radar, Sonar & Navigation 12 pp 21-99.
[5] Y. H. Kim, D. G. Kim, J. E. Han 2017 Analysis of sensor-emitter geometry for emitter localisation using TDOA and FDOA measurements IET Radar, Sonar & Navigation 11 pp 341-349.
[6] X. X. Cui, K. Yu, S. S. Lu 2018 Approximate closed-form TDOA-based estimator for acoustic direction finding via constrained optimization IEEE Sensors Journal 18.
[7] A. J. Weiss 2004 Direct position determination of narrowband radio frequency transmitters IEEE Signal Processing Letters 11 pp 513-516.
[8] A. J. Weiss 2011 Direct geolocation of wideband emitters IEEE Trans. Signal Process. 59 pp 2513-2521.
[9] A. J. Weiss, J. Picard 2013 Maximum-likelihood direct position estimation in dense multipath IEEE Trans. Vehicular Technology 62 pp 2069-2079.
[10] T. Tire, A. J. Weiss 2017 Performance analysis of a high-resolution direct position determination method IEEE Trans. Signal Process. 65 pp 544-554.
[11] N. Vankayalapati, S. Kay, Q. Ding 2014 TDOA based direct positioning maximum likelihood estimator and the Cramer-Rao bound IEEE Trans. Aerospace and Electronic Systems 50.
[12] Ofer Bar Shalom and A. J. Weiss 2011 Direct positioning of stationary targets using MIMO radar Signal Processing 91 pp 2345-2358.
[13] M. Mohanna, M. L. Rabeh, E. M. Zieur and S. Hekala 2013 Optimization of MUSIC algorithm for angle of arrival estimation in wireless communications Signal Processing 2 pp 116-124.
[14] Say Song Goh and Tim N. T. Goodman 2013 Estimation maxima of generalized cross ambiguity functions and uncertainty principles Electronics Letters 52 pp 235-237.