Dark Matter from Freeze-In via the Neutrino Portal

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Abstract

We investigate a minimal neutrino portal dark matter model (DM) where a right-handed neutrino both generates the observed neutrino masses and mediates between the SM and the dark sector, which consists of a fermion and a boson. In contrast to earlier work, we explore regions of the parameter space where DM is produced via freeze-in instead of freeze-out motivated by the small neutrino Yukawa couplings in case of $O(\text{TeV})$ heavy neutrinos. For a non-resonant production of DM we can predict the heavy neutrino mass to be $M_N \approx 10 \text{ TeV}$ and achieve a lower bound on the DM mass of $m_\chi \gtrsim 5 \text{ TeV}$. For the resonant production of DM, we find that it can be produced via freeze-in or freeze-out even with couplings of $O(10^{-5})$. In the latter case to produce the observed for DM density we find $m_\chi \approx 100 \text{ eV}$. 

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1 Introduction

Both Dark Matter (DM) and neutrino masses provide a strong hints for beyond standard model physics (BSM). A simple way to accommodate for neutrino masses is to introduce right-handed neutrinos which are SM singlets, thereby allowing for a Majorana mass term. This enables mass generation via the type I seesaw mechanism. Furthermore, the resulting heavy neutrino state $N$ is massive, electrically neutral and within a certain mass range even (cosmological) stable. If the heavy neutrino is considered to be a DM candidate it has to be stable. To ensure that its mass must satisfy $M_N < 2m_e$. Therefore, the Yukawa coupling has to be very small, namely $y_\nu \lesssim 10^{-6}$. As a consequence, the production rate is small, thereby allowing for DM production via the freeze-in mechanism. In freeze-in scenarios, DM production never becomes efficient, i.e. the interaction rate $\Gamma$ is always small compared to the Hubble parameter $H$, $\Gamma \ll H$. Thus, DM is never in thermal equilibrium with the SM. On the contrary, freeze-out scenarios require $\Gamma > H$ for a certain period of time (see figure 1).

![Figure 1: Freeze-in and freeze-out scenarios in comparison](image)

To account for the observed DM relic abundance via freeze-in of the decay $h \to N\nu$, the heavy neutrino mass should be of $O(10 \text{ keV})$. However, the possibility of keV sterile neutrino DM might be excluded by experiments soon, more precisely by the non observation of the decay $N \to \nu\gamma$ in gamma rays, see e.g. [5]. Moreover, in case of $M_N > 2m_e$, the heavy neutrino $N$ is not stable and therefore not a DM candidate. But even in this case, the right-handed neutrino can act as a mediator to DM

\footnote{A small DM production rate could also be generated by a large mediator mass as was pointed out in [1].}
since it is a SM singlet. This possibility is referred to as neutrino portal DM (NPDM) which has recently drawn attention in the literature [6–9]. However, these works only consider DM production via the freeze-out mechanism. This paper explores a minimal NPDM model where DM is produced via the freeze-in mechanism. The paper is organized as follows: In section 2 we present the model and its general properties. In section 3 we briefly review the Boltzmann equations which are solved for the considered model analytically within some special cases in section 4. Section 5 summarizes the results of a numerical solution of the Boltzmann equations. In section 6 we conclude.

2 Setup

In this section, we introduce the particle content and the general properties of the model. A model with the same particle content was investigated in [7] where DM production within freeze-out scenarios was explored. In addition to the SM particle content, the model includes a right-handed neutrino $\nu_R$ to accommodate for the observed neutrino masses via the type I seesaw mechanism. Moreover, the model contains a dark sector consisting of a fermion $\chi$ and a scalar $\phi$. While they are uncharged under the SM gauge groups, they are charged under a dark symmetry, e.g. a dark U(1). Assuming the SM particles to be uncharged under the dark symmetry renders the lighter particle of $\chi$ and $\phi$ to be a stable DM candidate since the dark symmetry forbids couplings between SM and dark sector particles. In this scenario, the resulting heavy neutrino $N$ acts as a mediator between the DM and the SM particles since the singlet $\nu_R$ can couple to $\chi$ and $\phi$ via a Yukawa coupling as long as the expression $\chi\phi$ is a singlet under all gauge groups. The parts of the Lagrangian relevant for the neutrino mass generation and the coupling to DM are given by

$$\mathcal{L} \supset -y_\nu \bar{\nu}_L h \nu_R - \frac{1}{2} M_N \bar{\nu}_R^c \nu_R - y_\chi \bar{\chi} \nu_R + h.c.$$  \hspace{1cm} (2.1)

Since the focus of this work is the DM production mechanism we restrict our analysis to only one generation of SM and DM particles. We do not take into account any contribution to the DM relic abundance from a possible Higgs portal interaction arising from the term $(\phi\phi^*) (hh^*)$ in the scalar potential and furthermore assume that $\phi$ does not acquire a VEV. Moreover, effects resulting from kinetic mixing of the vector mediators of the dark symmetry with the SM gauge bosons are neglected, thus our analysis focuses on the neutrino portal to DM only.

As already mentioned, the observed neutrino masses are generated by a type I seesaw mechanism leading to mass eigenstates $\nu$ and $N$ with masses of approximately $m_\nu = \frac{y_\nu^2 v^2}{M_M}$ and $M_N = M_M$ in the limit of $y_\nu^2 v^2 \ll M_M$ where $v$ is the VEV of the Higgs. They are described
in terms of the interaction eigenstates by:

$$
\left( \begin{array}{c}
\nu \\
N
\end{array} \right) = U \left( \begin{array}{c}
\nu_L \\
\nu_R
\end{array} \right) \approx \left( \begin{array}{c}
\sqrt{1 - \frac{y^2 v^2}{M_N^2}} \\
\frac{y v}{M_N}
\end{array} \right) \left( \begin{array}{c}
\nu_L \\
\nu_R
\end{array} \right) .
$$

This mixing causes an interaction between the $\nu$, $N$ and the Higgs as well as a coupling of $N$ to the $SU(2)_L$ gauge bosons. As presented in [10], the resulting interactions between the heavy and the light neutrinos are given by:

$$
\mathcal{L}_W \supset -\frac{g_W}{2\sqrt{2}} l W^- \gamma^\mu (1 - \gamma_5) B_{1N} N + h.c. ,
$$

$$
\mathcal{L}_Z \supset -\frac{g_W}{4 \cos(\Theta_W)} \bar{Z}_\mu \{ \bar{\nu} \gamma^\mu [i \text{Im} (C_{\nu N}) - \gamma_5 \text{Re} (C_{\nu N})] \} N + h.c. ,
$$

$$
\mathcal{L}_H \supset -\frac{g_W}{4 M_W} h \{ 2 \bar{\nu} [(m_\nu + M_N) \gamma_5 \text{Re} (C_{\nu N})] + i (M_N - m_\nu) \text{Im} (C_{\nu N}) \} N
$$

The matrices $B$ and $C$ are defined as in [10] and in case of only one generation and real Yukawa couplings they yield:

$$
B_{1N} \approx \frac{y_\nu v}{M_N} , \quad C_{\nu N} \approx \frac{y_\nu v}{M_N} , \quad C_{NN} \approx \left( \frac{y_\nu v}{M_N} \right)^2 .
$$

Thus, the couplings relevant for heavy neutrino production are given by

$$
\mathcal{L}_W \supset -y_\nu \frac{M_W}{\sqrt{2 M_N}} l W^- \gamma^\mu (1 - \gamma_5) N + h.c. ,
$$

$$
\mathcal{L}_Z \supset \frac{y_\nu}{2 \cos(\Theta_W) M_N} \bar{Z}_\mu \{ \bar{\nu} \gamma^\mu \gamma_5 N - \frac{y_\nu v}{M_N} \bar{N} \gamma^\mu \gamma_5 N + h.c. \} ,
$$

$$
\mathcal{L}_H \supset -y_\nu h \bar{\nu} N - \frac{y_\nu^2 v}{M_N} h \bar{N} N ,
$$

whereas the coupling of the heavy neutrino to the dark sector is governed by:

$$
\mathcal{L}_\chi \supset -y_\chi \sqrt{1 - \frac{y^2 v^2}{M_N^2}} \phi \bar{\chi} N + h.c. \approx -y_\chi \phi \bar{\chi} N + h.c. .
$$

Note that the parameters $y_\nu$ and $M_N$ are not independent and related by the seesaw mechanism requiring $y_\nu = \sqrt{m_\nu M v^{-1}}$. Therefore, the couplings in the eq. (2.7)-(2.9) can be rewritten as:

$$
g_{h\nu N} = y_\nu = \sqrt{m_\nu M_N v} \quad g_{W1N, Z\nu N} = y_\nu \frac{M_W}{M_N} = \sqrt{\frac{m_\nu}{M_N} \frac{M_W}{v}}
$$

$$
g_{hNN} = \frac{y_\nu^2 v}{M_N^2} = \frac{m_\nu}{v} \quad g_{ZNN} = g_{Z\nu N} \frac{y_\nu v}{M_N} = \frac{m_\nu}{M_N}
$$

Thus, for $M_N \geq M_W$, the coupling $g_{h\nu N}$ is the dominant coupling and the $h\nu N$ vertex is the only relevant one for DM production whereas for $M_N \leq M_W$ the $W1N$ and $Z\nu N$ vertices are dominant as long as $M_N \gtrsim m_\nu$. 

3
3 Boltzmann Equations

Determining the relic abundance of the DM candidate requires solving the Boltzmann equations, which describe the time evolution of the particle number densities in the expanding universe. Adopting the formalism used in [11] the Boltzmann equations can be written as:

$$\dot{n}_N + 3Hn_N = - \sum_{a,i,j,\ldots} \left( \frac{n_N n_a \cdots}{n_{eq}^a n_{eq} \cdots} \gamma_{eq} (Na \cdots \rightarrow ij \ldots) - \frac{n_i n_j \cdots}{n_{eq}^i n_{eq}^j \cdots} \gamma_{eq} (ij \cdots \rightarrow Na \ldots) \right).$$

(3.1)

Here, $n_i$ is the number density of particle species $i$ and $H$ is the Hubble parameter. The $3Hn_N$ term takes the expansion of the universe into account while the right hand side governs the impact of scattering processes which occur with a certain thermal rate $\gamma_{eq}$. The equilibrium number densities $n_{eq}^i$ are given by the momentum integral over the distribution function $f$ of the respective particle species which is approximated with a Boltzmann distribution in our case:

$$n_{eq}^i = \int \frac{d^3 p}{(2\pi)^3} f_{eq}^i = \frac{g_i}{2\pi^2} \frac{m_i^2}{T} K_2 \left( \frac{m_i}{T} \right).$$

(3.2)

For a two to two scattering involving only CP conserving interactions the quantity $\gamma_{eq}$ results in

$$\gamma_{eq} (Na \rightarrow ij) = \gamma_{eq} (ij \rightarrow Na) = \frac{T}{64\pi^4} \int_{s_{min}}^{\infty} ds \sqrt{s \hat{\sigma}} (s) K_1 \left( \frac{\sqrt{s}}{T} \right),$$

(3.3)

where $\hat{\sigma} (s) = 2s \sigma (s) \lambda \left[ 1, \frac{m_a^2}{s}, \frac{m_b^2}{s} \right]$ with $\lambda[a,b,c] = (a - b - c)^2 - 4bc$, $K_1 (x)$ is a Bessel function and $s_{min} = \max [(m_a + M_N)^2, (m_i + m_j)^2]$.

For a CP-conserving decay we obtain

$$\gamma_{eq} (N \leftrightarrow ij \ldots) = n_{eq}^N \frac{K_1 (z)}{K_2 (z)} \Gamma_N.$$  

(3.4)

Here $z = \frac{M_N}{T}$ holds and $\Gamma_N$ is the decay rate of the particle in its rest frame.

Next, to simplify the form of the Boltzmann equations they are written in terms of the quantity $Y = \frac{n}{s_E}$ instead of the number density where $s_E = \frac{2\pi^2 g_{*}^{s}}{45} T^3$ is the entropy density. This leads to

$$zHs_E \frac{dY_N}{dz} = - \sum_{a,i,j,\ldots} \gamma_{eq} (Na \cdots \leftrightarrow ij \ldots) \left[ \frac{n_N n_a \cdots}{n_{eq}^N n_{eq}^a \cdots} - \frac{n_i n_j \cdots}{n_{eq}^i n_{eq}^j \cdots} \right].$$

(3.5)

Finally, for a numerical solution, it is useful write eq. (3.5) in terms of $\log_{10} (Y_N)$ and $\log_{10} (z)$:

$$\frac{d \log_{10} (Y_N)}{d \log_{10} (z)} = \frac{z}{Y_N} \frac{dY_N}{dz} = - \sum_{a,i,j,\ldots} \frac{\gamma_{eq} (Na \cdots \leftrightarrow ij \ldots)}{H n_{eq}^N} \left[ \frac{n_N n_a \cdots}{n_{eq}^N n_{eq}^a \cdots} - \frac{n_i n_j \cdots}{n_{eq}^i n_{eq}^j \cdots} \right].$$

(3.6)
4 Relic Abundance: Analytic Estimates

The $2 \leftrightarrow 2$ scattering processes responsible for producing DM (the lighter particle of $\chi$ and $\phi$) can be classified into two categories: *SM Particle Scattering* and *Heavy Neutrino Scattering*. The SM particle scattering processes involve two SM particles in the initial state, have $\chi$ and $\phi$ in the final state and are mediated by the heavy neutrino. Consequently, we have $\sigma \sim y^2 \nu y^2 \chi$.

The heavy neutrino scattering processes have two heavy neutrinos in the initial state and produce a pair of $\chi$ or $\phi$. Here, we have $\sigma \sim y^4 \chi$.

In addition to the different dependence on the couplings $y_\nu$ and $y_\chi$, for SM particle scattering processes we can assume the SM particles to be in thermal equilibrium whereas the heavy neutrino $N$ is only in thermal equilibrium if the production via (inverse) decays as e.g. $(\nu h \rightarrow N) \rightarrow h \rightarrow \nu N$ is sufficiently efficient. Consequently, even in the case of $y_\chi \gg y_\nu$ the heavy neutrino scattering might not be the dominant production channel since $n_N n_{eq} N \ll 1$ could be the case. All contributing diagrams are displayed in figure 2.

4.1 SM Particle Scattering

For the rest of the discussion, we assume that the dark sector particles are roughly the same in mass and therefore replace $m_\phi = m_\chi$ and furthermore assume $M_N > M_W$. As discussed in the end of chapter 2, for $M_N \gg M_W$ the coupling of the heavy neutrino to the Higgs and a light neutrino is much larger than the coupling to the $SU(2)$ gauge bosons. Therefore, we only take the contribution of $vh \leftrightarrow \phi \chi$ into account. The relevant cross section is given in
appendix A (A.1) and for \( m_\phi = m_\chi \) results in

\[
\sigma_{vh\leftrightarrow\chi\phi}(s) \approx \frac{\sqrt{s \left( s - 4m_\chi^2 \right) \left( s + 4M_Nm_\chi + M_N^2 \right)}}{32\pi s \left[ (s - M_N^2)^2 + \Gamma_N^2M_N^2 \right]}.
\]  

(4.1)

Here, \( \Gamma_N \) is the total decay width of the propagating neutrino which can decay into \( vh \) for \( M_N > m_h \) and into \( \chi\phi \) for \( M_N > m_\chi + m_\phi \). The decay width is also given in appendix A (A.2). Next, we use eq. (3.3) to determine the thermal rate of the process. There are two cases to be distinguished:

- The resonant case with \( M_N \geq m_\chi + m_\phi \) where \( M_N^2 \geq s_{\text{min}} \) and thus it is integrated over the resonance of the cross section \( \sigma_{vh\leftrightarrow\chi\phi} \)

- The non-resonant case with \( M_N < m_\chi + m_\phi \) where \( M_N^2 < s_{\text{min}} \)

First, we discuss the non-resonant case where we neglect the contribution of the decay width \( \Gamma_N \). Moreover, for \( M_N \ll m_\chi \) the integral in eq. (3.3) is solvable analytically and results in:

\[
\gamma_{\text{eq}}(vh \leftrightarrow \chi\phi) = \frac{m_\chi^2T^2y_\nu^2y_\chi^2K_1\left(\frac{m_\chi}{T}\right)^2}{256\pi^5}.
\]  

(4.2)

Therewith, eq. (3.5) for the dark sector particles results in:

\[
zHsdY_{\chi(\phi)} = -\gamma_{\text{eq}}(vh \leftrightarrow \chi\phi) \left( \frac{n_\chi n_\phi}{n_\chi^{\text{eq}}n_\phi^{\text{eq}}} - \frac{n_h n_\nu}{n_h^{\text{eq}}n_\nu^{\text{eq}}} \right).
\]  

(4.3)

The number density of DM particles is given by \( n_\chi + n_\phi \) and we assume all SM particles to be in thermal equilibrium, i.e. \( n_{\text{SM}} = n_{\text{SM}}^{\text{eq}} \). Furthermore, since we are investigating a weakly interacting dark sector we assume \( n_{\chi(\phi)} \ll n_{\chi(\phi)}^{\text{eq}} \) during the time of production. Thus,

\[
zHsdY_{DM} = 2\gamma_{\text{eq}}(vh \leftrightarrow \chi\phi).
\]  

(4.4)

The factor of 2 arises due to the assumption that the slightly heavier particle of \( \chi \) and \( \phi \) is long lived but might eventually decay to the stable one, hence \( Y_{DM} = Y_\chi + Y_\phi \). Integrating from \( z = 0 \) to \( z = \infty \) yields:

\[
Y_{DM}(z \to \infty) = \frac{135M_{pl}}{2^{14}\pi^5g_{\text{eff}}^s\sqrt{g_{\text{eff}}}} \frac{y_\nu^2y_\chi^2}{m_\chi}.
\]  

(4.5)

Remarkably, the result is inverse proportional to the DM mass \( m_\chi \), i.e. the energy density is independent of \( m_\chi \). This allows for predicting the value of the product of the Yukawa couplings \( y_\nu y_\chi \) by setting \( Y_{DM}(z \to \infty) = Y_{DM,\text{exp}} \), with

\[
Y_{DM,\text{exp}} = \frac{\Omega_{DM}}{\Omega_B} \frac{m_B}{m_{DM}}Y_B \approx 10^{-10}m_Bm_\chi^{-1}.
\]  

(4.6)
The experimental values for $\Omega_{DM}$, $\Omega_B$, the density parameter for baryons, and $Y_B$, the baryon number density in a co-moving volume, are taken from [12] and $m_B$, the average baryon mass, is approximated with the proton mass.

Evaluating $Y_{DM} (z \to \infty) = Y_{DM, exp}$ results in:

$$\left(y_\nu y_\chi\right)^2 \approx 10^{-3} \frac{m_B}{M_{Pl}} \approx 10^{-21},$$

(4.7)

The implications of this result are discussed in chapter 4.3

Next, we discuss the resonant case, i.e. $M_N \geq m_\chi + m_\phi$. As it was pointed out in [13], in this case it is useful to approximate the Breit-Wigner peak in eq. (4.1) with:

$$\int_c^\infty dx \frac{f(x)}{(x-a)^2 + b^2} \approx \frac{f(a)}{b},$$

(4.8)

which is valid as long as $b \ll a$, i.e. $\Gamma_N \ll M_N$. With that we find

$$\gamma_{eq} (v_h \leftrightarrow \chi) = \frac{TK_1 \left(\frac{M_N}{T}\right)}{128\pi^3} \frac{y_\nu y_\chi}{y_\nu^2 + y_\chi^2} M_N^3.$$

(4.9)

The integration of the approximated Boltzmann equation (4.4) results in:

$$Y_{DM} (z \to \infty) = \frac{27M_{pl}}{4\pi^5 \sqrt{g_{eff}}} \frac{(y_\nu y_\chi)^2}{y_\nu^2 + y_\chi^2} \frac{1}{M_N^3}.$$  

(4.10)

Since the result is not proportional to $m_\chi^{-1}$ fitting the observed DM density depends on the DM mass $m_\chi$ itself. Again, we postpone the discussion of the result to chapter 4.3.

### 4.2 Heavy Neutrino Scattering

The cross sections for the heavy neutrino scattering processes are given in Appendix A. For the case of $M_N \ll m_\chi \approx m_\phi$ they result in

$$\sigma_{NN \to \chi \chi} = y_\chi^4 \frac{\sqrt{1 - \frac{4m_\chi^2}{s}}}{16\pi s},$$

(4.11)

$$\sigma_{NN \to \phi \phi} = \frac{2 \text{arctanh} \left(\sqrt{1 - \frac{4m_\chi^2}{s}}\right) - \sqrt{1 - \frac{4m_\chi^2}{s}}}{16\pi^2 s}$$

(4.12)

Within this limit, we find an analytic expression for the thermal rate of the process $NN \to \chi \chi$

$$\gamma_{eq} (NN \to \chi \chi) = y_\chi^4 \frac{m_\chi^2 T^2 K_1 \left(\frac{m_\chi}{T}\right)^2}{128\pi^6}.$$  

(4.13)
Although it is not possible to find an analytic expression for the process $NN \rightarrow \phi\phi$ it turns out that $\sigma_{NN \rightarrow \chi\chi} \approx \sigma_{NN \rightarrow \phi\phi}$ for all $s$. Therefore, we approximate its contribution to the DM production in the derivation of an analytic result by the contribution of the process $NN \rightarrow \chi\chi$.

The Boltzmann equation for the DM density $Y_{DM} = Y_\chi + Y_\phi$ results in

$$zHSE \frac{dY_{DM}}{dz} = 4\gamma_{eq} (NN \rightarrow \chi\chi), \quad (4.14)$$

by again assuming $n_\chi \ll n_{eq}^\chi$ and the factor four arises since two $\chi/\phi$ particles are produced and due to $\gamma_{eq} (NN \rightarrow \phi\phi) \approx \gamma_{eq} (NN \rightarrow \chi\chi)$. Furthermore, we assumed $n_N = n_{eq}^N$. This assumption is only valid if the processes that keeps $N$ in thermal equilibrium, e.g. the inverse decay $\nu h \rightarrow N$, are sufficiently efficient.

The integration of eq. (4.14) yields:

$$Y_{DM} (z \rightarrow \infty) = \frac{27M_{Pl}}{212^{\pi}g_{s}^{eff}\sqrt{g_{eff}} m_\chi}. \quad (4.15)$$

As for the SM particle scattering in the limit of $M_N \ll m_\chi$, the DM density is inverse proportional to its mass and thus allows for a prediction of $y_\chi^4 \approx 10^{-5}$. However, for a realistic result we have to consider both - SM particle scattering and heavy neutrino scattering - processes. This is discussed in the next chapter.

For the case where the SM scattering processes are in the resonant regime, i.e. $M_N > m_\chi + m_\phi$, the cross section given by eq. (A.5) consists of two terms. The first can be integrated in the limit $M_N \gg m_\chi$ to obtain the thermal rate

$$\gamma_{eq} \approx y_\chi^4 \frac{T^3 M_N^3}{64\pi^2} G_{1,3}^3 \left( \frac{M^2}{T^2} \left| -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2} \right. \right). \quad (4.16)$$

Even within this limit it is not possible to obtain an analytic result for the second term in the cross section. However, the contribution of this term to the cross section is smaller or equal compared to the contribution of the first term for all center of mass energies $s$. Hence we approximate the thermal rate by two times (4.16). Here, the integration of eq. (4.14) results in:

$$Y_{DM} (z \rightarrow \infty) = \frac{3^3 y_\chi^4 M_{Pl}}{512\pi^6 \sqrt{g_{eff} g_{eff}} M_N}. \quad (4.17)$$

### 4.3 Discussion of the Analytic Results

In the limit of $M_N \ll m_\chi \approx m_\phi$ we found analytic solutions for the DM relic density for both types of processes. Combining both results yields:

$$Y_{DM} (z \rightarrow \infty) = \frac{27M_{Pl}}{212^{\pi}g_{eff}^{s} \sqrt{g_{eff}} m_\chi} \left( \frac{5}{2} \pi y_\chi^2 y_\chi^2 + y_\chi^4 \right). \quad (4.18)$$
By comparing this expression with the observed DM density (4.6) one obtains
\[ \left( \frac{5}{2} \pi y_\nu^2 y_\chi^2 + y_\chi^4 \right) \approx 10^{-21} . \] (4.19)

Therefore, the coupling \( y_\chi \) is required to be smaller than \( 10^{-5} \) in order to not overproduce DM. In principle, the couplings \( y_\chi \) and \( y_\nu \) are unrelated. However, both describe a coupling to the right-handed neutrino and - if the heavy neutrino is lighter than \( \mathcal{O}(10^{15} \text{ GeV}) \) - both couplings are required to be relatively small. This motivates the idea that they might be suppressed by the same mechanism, thus resulting in \( y_\nu \approx y_\chi^2 \). Considering a model which generates \( y_\chi \approx y_\nu \) allows for constraining the mass of the heavy neutrino since with that eq. (4.19) reads
\[ \frac{5\pi + 2}{2} y_\nu^4 = \frac{5\pi + 2}{2} \left( \frac{m_\nu M_N}{v^2} \right)^2 \approx 10^{-21} . \] (4.20)

Thus, to fit the observed DM density (4.6), \( M_N \approx 10 \text{ TeV} \) is required.

However, we achieved this result by assuming that the heavy neutrino is always in thermal equilibrium, \( m_\chi \gg M_N \) and by only taking into account the dominant processes of the SM particle and heavy neutrino scattering each. From eq. (4.18), we see that the contribution of the heavy neutrino scattering processes only accounts for roughly ten percent of the produced DM in case of \( y_\chi = y_\nu \). Thus, the result will not be altered significantly if the heavy neutrino is out of equilibrium. Also, taking into account the sub-dominant processes does not have a significant impact since they are suppressed by \( M_N^2 / M_W^2 \). The only significant change will occur in areas of the parameter space where \( m_\chi \approx M_N \), thereby violating the assumption of \( m_\chi \gg M_N \).

For these reasons in section 5, we solve the Boltzmann equations numerically for the case of \( y_\nu = y_\chi \).

In addition, we found an analytic solution for the DM relic density in the limit \( M_N \gg m_\chi \) where the SM particle scattering processes are in the resonant regime:
\[ Y_{DM} (z \to \infty) = \frac{3^3 M_{Pl}}{2^9 \pi^6 g_{\text{eff}} \sqrt{g_{\pi}} M_N} \left( \frac{2^7 \pi}{y_\nu^2 + y_\chi^2} \right) \approx 10^{-1} . \] (4.21)

Again, we obtain an upper bound on \( y_\chi \) from the measurement of the DM energy density by setting \( y_\nu = 0 \) which results in \( y_\chi \lesssim 10^{-5} (M_N m_\chi^{-1})^{1/2} \). Moreover, in case of \( y_\chi \ll y_\nu \) we find the observed DM energy density if \( y_\chi \approx 10^{-12} \sqrt{\frac{M_N}{m_\chi}} \).

However, if \( y_\chi \ll y_\nu \) does not hold the approximation of \( n_\chi \ll n_\chi^{eq} \) we used to derive (4.18) does not apply anymore. To illustrate that we look at the case \( y_\chi = y_\nu \), where (4.18) results in:
\[ Y_{DM} (z \to \infty) \approx \frac{3^3 M_{Pl}}{2^9 \pi^6 g_{\text{eff}} g_{\text{eff}} M_N} \left( \frac{m_\nu M_{Pl}}{v^2} \right) \approx 10^{-1} . \] (4.22)

\footnote{For example, such a mechanism could be an extra dimensional model where the right-handed neutrino in contrast to all other particles propagates in an extra dimension since it is uncharged under all considered gauge groups. Thereby, its coupling gets suppressed by the reduced wave function overlap \cite{14,15}. This concrete possibility is not explored within this paper but might be explored in a future work.}
Using eq. (3.2) we find that $Y_{\chi}^{\text{eq}} \lesssim 10^{-2}$. Therefore, $n_\chi \ll n_\chi^{\text{eq}}$ cannot be satisfied. Hence, the freeze-in scenario does not apply here. Nevertheless, it is still possible to account for the correct amount of DM. In this case, we recover a freeze-out like scenario since due to the resonance the interaction rate becomes as large as the Hubble parameter although the system is only weakly coupled. Thus, DM comes into equilibrium with the SM and freezes out as soon as the interaction rate becomes smaller than the Hubble parameter. This occurs approximately at $T = M_N$. Consequently, the number density can be estimated by the equilibrium density at freeze-out and since we assumed $M_N \gg m_\chi$ the number density (3.2) at freeze out yields:

$$Y_{\text{DM}} (z \to \infty) = Y_{\chi}^{\text{eq}} (T \approx M_N) = \frac{45g_\chi}{2\pi^4g_{\text{eff}}} \approx 10^{-3}.$$  \hspace{1cm} (4.23)

Equating this result with eq. (4.6) yields a DM mass of $m_\chi = \mathcal{O} (100 \text{ eV})$.

We summarized our results for the case $y_\chi = y_\nu$ in a schematic plot (see fig. 3).

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This is due to the fact that the main contribution to the interaction rate comes from the resonance at $s = M_N^2$, i.e. as soon as the temperature drops below $M_N$ the resonance cannot be reached efficiently anymore and therefore the interaction rate decreases significantly.
Figure 4: The numerically obtained DM density $Y_{th}$ is compared to the observed DM density $Y_{exp}$ for different values of the DM mass $m_\chi$ and the mediator mass $M_N$: The solid black line shows the points where the observed DM density is reproduced. For points within the purple area DM gets overproduced whereas the DM density is too small in the green area. The hatched area displays the resonant production regime and was not scanned.

5 Numerical Analysis

In this section we present the results of the numerical solution of the Boltzmann equation for the DM candidate $\chi$. We solved (3.5) for $\chi$ and $N$ in the non-resonant regime, including the processes $\nu h \leftrightarrow \chi \phi$, $NN \leftrightarrow \chi \chi$, $NN \leftrightarrow \phi \phi$ and $N \leftrightarrow \nu h$ or $h \leftrightarrow N \nu$. Furthermore, we set $y_\nu \approx y_\chi$ and $m_\phi = m_\chi$. As a consequence of our analytic approximations we expect the scenario to work for $M_N \approx 10 \text{ TeV}$ and $m_\chi \gtrsim 5 \text{ TeV}$. However, especially if $M_N \approx m_\chi$ our approximations from the analytic computation do not apply. Also $N$, as shown in appendix [3] will reach thermal equilibrium via the process $N \leftrightarrow \nu h$ around $T = M_N$. Thus, the contribution of $NN \leftrightarrow \chi \chi$ and $NN \leftrightarrow \phi \phi$ might be smaller than expected. We solved the problem numerically for $M_N < 2m_\chi$ while assuming that during the time of the DM production $n_\chi = n_\phi$ and that $\phi$ eventually decays into $\chi$ so that the DM relic density is given by $n_\chi + n_\phi$. The initial conditions are that the SM particles are in thermal equilibrium while $N$, $\chi$ and $\phi$ are not present. The interpolation of the thermal rates and the numerical solution of the Boltzmann equations are performed with Mathematica. The obtained results are summarized in figure [4] where we compared the obtained DM density $Y_{th}$ to the experimentally observed DM density $Y_{exp}$ for different values of $m_\chi$ and $M_N$. As expected, for $M_N \ll m_\chi$ the correct density is obtained for a constant value of $M_N$ of $M_N \approx 30 \text{ TeV}$. Also we see that, in fact, there is a lower bound on the DM mass which results
in $M_N \approx 5$ TeV.

Lastly we comment on the case $y_\nu \neq y_\chi$: Here, the coupling $y_\chi$ which is determined by the mediator mass via eq. (4.19) is larger than $y_\nu$ when $M_N \lesssim 10$ TeV. In this case, most of the DM gets produced via heavy neutrino scattering. Since we overestimated its contribution to the relic density by assuming $n_N = n_N^{eq}$ a larger coupling $y_\chi$ will be required and therefore relax the upper bound on $y_\chi \lesssim 10^{-5}$.

The numerical analysis of the resonant case is not performed within this paper. As in the non-resonant region, we expect the most significant deviations from the analytic result for cases where $M_N \approx m_\chi$.

6 Conclusion

We have investigated a minimal neutrino portal DM model where the dark sector consists of a scalar $\phi$ and a fermion $\chi$. In addition, the SM is extended by a right-handed neutrino which generates the neutrino masses via a type I seesaw mechanism and, furthermore, acts as a mediator between the SM and DM. Motivated by the small Yukawa couplings of the type I seesaw mechanism in case of small heavy neutrino masses of $M_N \lesssim \mathcal{O}(\text{PeV})$ we studied DM production via the freeze-in mechanism which, in contrast to the freeze-out mechanism, requires interaction rates of $\Gamma \ll H$.

We derived analytic solutions for the resonant ($M_N > m_\chi + m_\phi$) and non-resonant ($M_N < m_\chi + m_\phi$) DM production regime. Adding the requirement that the coupling of the right-handed neutrino to the SM is of the same order of magnitude as its coupling to the dark sector increases the predictivity of the model and allows for predictions of the mediator or the DM mass respectively. In the non-resonant regime, we find $M_N \approx 30$ TeV and a lower bound on the DM mass $m_\chi \gtrsim 5$ TeV. Within the resonant regime, however, for $y_\chi = y_\nu$ the resonant production of DM is strong enough to bring DM into equilibrium with the SM. Thus, we recover the usual freeze-out mechanism although the couplings between DM and the SM are very small. Moreover, in this scenario we can predict a DM mass of $m_\chi \approx 100$ eV. For $y_\chi \ll y_\nu$, nonetheless, DM production via freeze-in is still possible. To satisfy the observed DM energy density the coupling of the right-handed neutrino to DM has to be $y_\chi \approx 10^{-12} \sqrt{\frac{M_N}{m_\chi}}$.

Thus, we demonstrated that producing the observed DM energy density within this model of neutrino portal DM is viable even with small couplings between the SM and the dark sector. An interesting extension of this model would be the explanation of the small Yukawa couplings of the right-handed neutrino to the SM and DM by a suppression mechanism. One explanation could be an extra dimensional model where the heavy neutrino in contrast to the SM and DM particles propagates in an extra dimension since it is the only singlet under all considered gauge groups. Thus, all couplings to the right-handed neutrino are suppressed by a volume factor which can even generate $y_\chi \approx y_\nu$.

Due to the tiny couplings of the right-handed neutrino to SM and DM direct detection
seems not viable. Therefore, exploring the detectability of this model might be an interesting prospect for a future work. Moreover, we only demonstrated a few working cases of this model. For example we considered only the case of \( m_\chi = m_\phi \) and \( M_N > M_W \). Thus, an exploration of \( M_N < M_W \) which leads to different dominant interactions between the heavy neutrino \( N \) and the SM or a hierarchic dark sector might be interesting.

A  Cross Sections

\[
\sigma_{\nu h \to \chi \phi} (s) = y_\nu^2 y_\chi^2 \frac{\sqrt{m_\phi^4 + (s - m_\chi^2)^2 - 2m_\phi^2 (s + m_\chi^2)}}{16\pi^2 s^2} \left[(s - M_N^2)^2 + \Gamma_N^2 M_N^2\right] \]

(A.1)

Here, \( \Gamma_N \) is the total decay width of the propagating neutrino which can decay into \(\nu h\) for \( M_N > m_h \) and into \( \phi \) for \( M_N > m_\chi + m_\phi \). The decay width is given by:

\[
\Gamma_N = y_\nu^2 (M_N^2 - m_\phi^2)^2 \frac{8\pi}{8\pi M_N^2} + y_\chi^2 (M_N - m_\phi + m_\chi) (M_N + m_\phi + m_\chi) \times \frac{1}{\sqrt{(M_N - m_\phi - m_\chi) (M_N - m_\phi + m_\chi) (M_N + m_\phi - m_\chi) (M_N + m_\phi + m_\chi)}}
\]

(A.2)

\[
\sigma_{\nu l \to \chi \phi} = y_\nu^2 y_\chi^2 \frac{3M_W^2}{24\pi^2 s M_N (s - M_N^2)^2} \left[(M_W^2 - m_l^2) (M_W^2 + 2 (m_l^2 - M_W^2) - 4M_N m_\chi) + (M_N^2 + m_l^2 - M_W^2 + 4M_N m_\chi)\right] \sqrt{\frac{(s - 4m_\chi^2)}{m_l^4 + (s - M_W^2)^2 - 2m_l^2 (s + M_W^2)}}
\]

(A.3)

\[
\sigma_{\nu \to \chi \phi} = y_\nu^2 y_\chi^2 \frac{3M_W^2}{16\pi^2 M_N^2 (s - M_N^2)^2 (s - M_Z^2)} \left[(s + M_Z^2) M_N^2 + 4M_N m_\chi (s - M_Z^2) + s^2 - s M_Z^2 - 2M_Z^2\right]
\]

(A.4)

\[
\sigma_{NN \to \chi \chi} = \frac{y_\chi^4}{4\pi} \left(1 - \frac{4m_\chi^2}{s}\right) \sqrt{\frac{s}{s - 4M_N^2}} \frac{4M_N^4 + 8M_N^2 m_\chi + 2m_\chi^2 s}{M_N^4 - 4M_N^2 m_\chi^2 + m_\chi^4} - \frac{8M_N (M_N + 2m_\chi)}{\sqrt{(s - 4M_N^2) (s - 4m_\chi^2)}} \arctanh\left[\frac{\sqrt{(s - 4M_N^2) (s - 4m_\chi^2)}}{2M_N^2 - s}\right]
\]

(A.5)
\[ \sigma_{NN\rightarrow\phi\phi} = y_{\chi}^4 \left( 1 - \frac{4m_{h}^2}{s} \right) \left[ -\sqrt{(s - 4M_N^2)} \left(s - 4m_{h}^2\right) \left(M_N^2 s + 2M_N^4 + 4M_N^2 m_{\chi}\right) \right. \\
\left. + 2 \left[2M_N \left(2m_{\chi} M_N\right) + s\right] \left[M_N^2 \left(s - 4M_N^2\right) + M_N^4\right] \right] \times \text{arctanh} \left( \frac{\sqrt{(s - 4M_N^2)} \left(s - 4m_{h}^2\right)}{s - 2M_N^2} \right) \] (A.6)

B  The Decay \( N \rightarrow \nu h \)

The decay rate for the process \( N \rightarrow \nu h \) for \( M_N > m_h \) is given by:

\[ \Gamma_N = \frac{y_{\nu}^2 (M_N^2 - m_h^2)^2}{8\pi M_N^3}. \] (B.1)

With that we find the interaction rate:

\[ \Gamma = \gamma_{\text{eq}} n_{\text{eq}}^{-1} = \frac{K_1(z)}{K_2(z)} \gamma_{\text{eq}} = \frac{K_1(z)}{K_2(z)} \left( \frac{y_{\nu}^2 (M_N^2 - m_h^2)^2}{8\pi M_N^3} \right). \] (B.2)

Comparing this result to the Hubble Parameter with \( M_N \gg m_h \) yields:

\[ \frac{\Gamma}{H} = \frac{3m_{\nu} M_{\text{pl}} z^2 K_1(z)}{40\pi \sqrt{g_{\text{eff}} v^2} K_2(z)}. \] (B.3)

This quantity is greater than one for Temperatures of roughly \( T \leq M_N \). Consequently we can expect the heavy neutrino to be in thermal equilibrium with the SM for \( T \lesssim M_N \).

References

[1] S.-L. Chen and Z. Kang, JCAP 1805, 036 (2018), 1711.02556.

[2] L. J. Hall, K. Jedamzik, J. March-Russell, and S. M. West, JHEP 03, 080 (2010), 0911.1120.

[3] S. Dodelson and L. M. Widrow, Phys. Rev. Lett. 72, 17 (1994), hep-ph/9303287.

[4] M. Drewes et al., JCAP 1701, 025 (2017), 1602.04816.

[5] K. Perez et al., Phys. Rev. D95, 123002 (2017), 1609.00667.

[6] M. Escudero, N. Rius, and V. Sanz, JHEP 02, 045 (2017), 1606.01258.

[7] M. Escudero, N. Rius, and V. Sanz, Eur. Phys. J. C77, 397 (2017), 1607.02373.

[8] M. G. Folgado, G. A. Gomez-Vargas, N. Rius, and R. Ruiz De Austri, (2018), 1803.08934.
[9] B. Batell, T. Han, D. McKeen, and B. Shams Es Haghi, Phys. Rev. D97, 075016 (2018), 1709.07001.

[10] A. Pilaftsis, Z. Phys. C55, 275 (1992), hep-ph/9901206.

[11] G. F. Giudice, A. Notari, M. Raidal, A. Riotto, and A. Strumia, Nucl. Phys. B685, 89 (2004), hep-ph/0310123.

[12] Particle Data Group, C. Patrignani et al., Chin. Phys. C40, 100001 (2016).

[13] M. Blennow, E. Fernandez-Martinez, and B. Zaldivar, JCAP 1401, 003 (2014), 1309.7348.

[14] N. Arkani-Hamed, S. Dimopoulos, G. R. Dvali, and J. March-Russell, Phys. Rev. D65, 024032 (2001), hep-ph/9811448.

[15] M. Becker and H. Pas, Eur. Phys. J. C78, 273 (2018), 1707.02882.