CENTRALITY DEPENDENCE OF MULTIPLICITIES IN ULTRARELATIVISTIC NUCLEAR COLLISIONS

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Abstract

We compute the centrality dependence of multiplicities of particles produced in ultra-relativistic nuclear collisions at various energies and atomic numbers. The computation is carried out in perturbative QCD with saturated densities of produced gluons and by including effects of nuclear geometry. Numbers are given for Au+Au collisions at RHIC energies.

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1 Introduction

The initial transverse energies and multiplicities in central (zero impact parameter) ultrarelativistic heavy ion collisions have been computed in [1] from perturbative QCD supplemented by the crucial assumption of saturation of produced semihard gluons: a simple saturation criterion defines a saturation scale $p_{\text{sat}} \sim 1 (2) \text{ GeV}$ at RHIC (LHC). Doing the computation at transverse momenta $p_T \geq p_{\text{sat}}$ gives an estimate of the effect from all transverse momentum scales, both above and below $p_{\text{sat}}$. “Initial” here then means proper times of the order of $1 / p_{\text{sat}} \approx 0.2 \text{ fm}$ (RHIC) and $\approx 0.1 \text{ fm}$ (LHC). Assuming thermalisation at formation and further entropy conserving expansion, these initial gluon numbers can be converted to final hadron multiplicities. The predicted multiplicities agree well with the first results from RHIC [2].

The results of [1] are formulated in the form of scaling rules, quantity $\sim CA^a (\sqrt{s})^b$, in which the constants $C, a, b$ are determined for central $AA$ collisions. A further variable one has experimental control over is the centrality or impact parameter dependence of multiplicities and transverse energies. In fact, in [3] the centrality dependence of the multiplicity has been proposed as a means of distinguishing between various models for particle production in ultrarelativistic heavy ion collisions. In particular, in [3] one observed that the ratio of charged particle multiplicity to the number of participants, $N_{\text{ch}} / N_{\text{part}}$, grows when $b$ decreases while it decreases in the HIJING Monte Carlo model [4]. The purpose of this note is a systematic study of the behaviour of charged particle multiplicities with impact parameter in the pQCD + saturation model.

2 Impact parameter dependent saturation criterion

Since the present work is a simple extension of [1] to non-central $b \neq 0$ collisions, we refer there for a detailed exposition of dynamical ideas.

The tools of describing non-central $AA$ collisions are [3] the nuclear density $n_A(r)$, normalised to $A$, the standard nuclear thickness function $T_A(b) = \int dzn_A(r)$, normalised as $\int d^2b T_A(b) = A$, and the nuclear overlap function $T_{AA}(b) = \int d^2b T_{AA}(b) = A^2$, which is a 2-dimensional convolution in $b$-space of two $T_A(b)$’s. We shall systematically use the physically relevant Woods-Saxon nuclear density distribution with $R_A = 1.12 A^{1/3} - 0.86 A^{-1/3} \text{ fm}$, for which these integrals have to be computed numerically.

Experimentally, however, one does not have a direct control over the impact parameter. Instead, using forward calorimeters one can measure the number of participants $N_{\text{part}}$ in the collision; for central collisions $N_{\text{part}} \lesssim 2A$. This can be approximately converted to impact parameter by the formula

$$N_{\text{part}}(b) = \int d^2s T_A(b - s) [1 - \exp(-\sigma_{\text{in}} T_B(s))] + \int d^2s T_B(s) [1 - \exp(-\sigma_{\text{in}} T_A(b - s))]$$

$$= 2 \int d^2s T_A(b - s) [1 - \exp(-\sigma_{\text{in}} T_A(s))] \quad (A = B),$$

(1)
where $\sigma_{\text{in}} \approx 40$ mb is the inelastic pp cross section in the RHIC energy range. An example is shown in Fig. 3. The Poissonian $\exp(-\sigma_{\text{in}} T_A)$ in (1) can as well be replaced by a binomial-like $(1 - \sigma T A)^4$; the numerical effect is negligible. We have taken $\sigma_{\text{in}} = 35, 39$ and $42$ mb for $\sqrt{s} = 56, 130$ and $200$ GeV, respectively, based on [6].

We have now the tools to generalise the $b = 0$ saturation condition for initially produced partons,

$$N_{AA}(p_0, b = 0) = 2 T_A(0) \sigma_{pQCD}(p_0) = \sum_{k=g,q,\bar{q}} N_k(p_0) = \frac{p_0^2 R_A^2}{\pi},$$  \hspace{1cm} (2)

to arbitrary $b$. Perturbative QCD enters via an LO computation of the inclusive production cross section $\sigma_{pQCD}(p_0)$ of (mini)jets with $p_T \geq p_0$ and $|y| \leq 0.5$, with NLO contributions taken into account via an overall $K$-factor $K = 2$ according to [7]; all the formulas are spelled out in detail in [8]. Eq. (2) expresses the fact that at saturation $N_{AA}(p_0 = p_{\text{sat}})$ quanta each with transverse area $\pi/p_{\text{sat}}^2$ fill the whole nuclear transverse area $\pi R_A^2$. Numerical, group theory factors or powers of $g$ could be included [9], but these are anyway $O(1)$ unless one discusses a parametric weak coupling limit $g \to 0$.

All parton flavours are included, though at $p_0 = p_{\text{sat}}$ gluons clearly dominate even at lowest energies.

In terms of an average transverse area density in central collisions, $dN_{AA}/d^2s \approx N_{AA}(p_0, 0)/\pi R_A^2$, Eq. (2) becomes $dN_{AA}/d^2s \cdot \pi/p_{0}^2 = 1$. For arbitrary $b$ and $s$, the average saturation criterion thus generalizes to a local one,

$$\frac{dN_{AA}(p_0, b, s)}{d^2s} = 2 T_A(b - s) T_A(s) \sigma_{pQCD}(p_0) = \frac{p_0^2}{\pi}. \hspace{1cm} (3)$$

We shall choose the coordinate system at $b \neq 0$ so that $b = (b, 0)$ and so that the centers of the nuclei are at $(b/2, 0)$ and $(-b/2, 0)$. A solution of Eq. (3) gives us the local saturation momentum $p_{\text{sat}}(b, s)$ at the position $s$ of an A+A collision at impact parameter $b$. Fig. 1 shows $p_{\text{sat}}(b = 0, s)$, which only depends on $s$; Fig. 2 shows $p_{\text{sat}}(b = R_A, s = (s_x, 0))$ and $p_{\text{sat}}(b = R_A, s = (0, s_y))$.

The predicted initial multiplicity at fixed $b$ then simply is $dN_{AA}/d^2s$ at the solution of (3) integrated over $s$. In terms of the right-hand side:

$$N_{AA}(b) = \int d^2s \frac{p_{\text{sat}}^2(b, s)}{\pi}. \hspace{1cm} (4)$$

Here an important issue enters: how does one deal with large values of $s$ or small values of $p_{\text{sat}}(b, s)$ in (4)? Physically, how does one treat very peripheral collisions for which $p_{\text{sat}}$ becomes small, nonperturbative? One may note that even at SPS energies, where $p_{\text{sat}} \approx 0.7$ GeV, the data was reproduced by this model up to a 20% error. Clearly the pQCD+saturation model can be reliable only if the large $s$, small $p_{\text{sat}}$ region makes a negligible contribution to the integral (3). This, in fact, is the case: in central collisions, for example, the contribution of the range from $p_{\text{sat}} \geq 0.7$ GeV to $p_{\text{sat}} \geq 0.5$ GeV to
the integration (4) is only an ≈ 4% increase in $N_{AA}$. In Figs. 1 and 2 the decrease in $p_{sat}^2$ for $p_{sat} < 0.5$ GeV is very rapid and the surface integral (4) is negligibly affected. Values of $N_{AA}(b)$ as computed from (4) including only $p_{sat} \geq 0.5$ GeV are plotted in Fig. 3. The total initial particle production becomes thus described by the saturation model in an effective way in the sense that by pushing the local saturation criterion to its limits (to $p_{sat} \to 0$) no additional, non-saturated, components need to be included. For very peripheral collisions, or for pp collisions, the model cannot be applied.

The values of $b$ in Figs. 1 to 3 can be converted to the experimentally more directly accessible number $N_{part}$ using Eq. (1) and Fig. 3. Concretely, Figs. 1 to 3 extend to $b = 1.8R_A$, which corresponds to $N_{part} \approx 30$; similarly $b = 1.6R_A$ would correspond to $N_{part} \approx 50$. For still larger $b$ one expects saturation effects to disappear and this region is thus beyond the scope of the current study.

We shall here not discuss the transverse energy $E_T$ production in detail, since for it the relation between the initially produced and finally observed $E_T$ per unit rapidity is likely to be more complicated than that for multiplicity, due to $pdV$ work and transverse flow effects in the course of expansion. However, a reasonable estimate for the initial
Figure 2: The saturation momentum $p_{\text{sat}}(b = R_A, s)$ along the symmetry axes of the overlap area. Solid curves are for $s = (s_x, 0)$ and the dotted ones for $s = (0, s_y)$, computed from Eq. (3) for a Woods-Saxon nuclear density distribution for $A = 197$ and $\sqrt{s} = 56, 130, 200$ GeV.

production is the same $E_T$ per quantum as at $b = 0$: $E_T(b)/N(b) \approx 1.35 \langle p_{\text{sat}}(b) \rangle$, where $\langle p_{\text{sat}}(b) \rangle$ is computed with a weight $T_A(s)T_A(b - s)$ for $p_{\text{sat}}(b, s)$. Note also that it is here that these pQCD+saturation results deviate significantly from the classical field computations [10, 11]: their $E_T/N$ is larger by a factor 3.

3 Comparison with experiment

The above were predictions for the multiplicity integrated along a curve $\tau_i \approx 1/p_{\text{sat}}(b, s)$ of initial time. Comparing the multiplicities with experiment requires assumptions on the expansion and decoupling of the system. These questions are discussed extensively in the literature and one is looking forward to experimental tests.

We shall here use the simplest assumption of early thermalisation and entropy conserving longitudinally boost invariant expansion. At the central slice $z = 0$ the local particle density is initially then $n(\tau_i(b, s), s) \approx dN/d^2s/(\tau_i\Delta \eta) = S(\tau_i, s)/3.6$, where $\eta$ is the space-time rapidity and $S$ is the entropy density. The initial total entropy per rapidity unit, $S_i$, can thus be computed from $S_i(b) = 3.6N_{AA}(b)$. This predicts that the relation of the number of initially produced particles (gluons) to the total number of particles in the final state (pions per unit rapidity) is $N_f/N_{AA} = (N_f/S_f)/(N_{AA}/S_i) \times (S_f/S_i) \approx 3.6/4$, inserting the number/entropy ra-
Figure 3: The initial ($\tau \approx 1/p_{\text{sat}}$) number of quanta $N_{AA}(b)$ produced in $^{197}$Au+$^{197}$Au collisions at $\sqrt{s} = 56$, 130 and 200 GeV as a function of $b/R_A$ (the upper set of three curves), as computed from Eq.(4) with $p_{\text{sat}} \geq 0.5$ GeV. The three lower curves show $N_{\text{part}}(b)$ computed from Eq.(1) for $\sigma_{\text{in}} = 35, 39, 42$ mb (bottom to top). The average values of $N_{\text{part}}(b)$ given by PHOBOS [2] are indicated by the two (almost overlapping) arrows on the vertical axis, the values of $N_{AA}(b)$ at the corresponding values of $b/R_A$ are marked by the asterisks.

tios for massless gluons and massive pions. To obtain the experimentally measured $dN_{\text{ch}}/d\eta$, $\eta=$pseudorapidity, we use the estimates $N_{\text{ch}} \approx \frac{2}{3}N_f$ and $dN/d\eta \approx 0.9dN/dy$. Fig. 4 then shows the quantity $(dN_{\text{ch}}(b)/d\eta)/(N_{\text{part}}/2)$, i.e., the number of charged particles produced in unit pseudorapidity per number of participant pairs, as a function of the number of participants. The parameters ($A$ and $\sqrt{s}$) are those for RHIC experiments.

For central collisions (approximately; as shown in Fig.3, the number of participants in [2] is somewhat less than $2A$) the agreement with experiment is good. The rest of the curve is a prediction.

The prediction in Fig.4 has the striking feature of being essentially constant in $N_{\text{part}}$. This is in agreement with the estimate in [3], shown by the dotted lines. The small difference is due to the fact that in [3] one did not actually perform the saturation computation at arbitrary $b$ but instead used $b = 0$ results at $A_{\text{eff}} = N_{\text{part}}(b)/2$. Due to the computed [4] $A$-scaling of $N_{AA}$ at $b = 0$, $N_{AA} \sim A^{0.922}$, this immediately results in $N_{AA}/N_{\text{part}} \sim 1/N_{\text{part}}^{0.08}$, a slow increase as $N_{\text{part}}$ decreases. At very small $N_{\text{part}}$ this would seem to lead to a striking effect, but then one enters the very peripheral region in which saturation does not work.
4 Conclusions

In this note we have extended the pQCD + saturation model for initial production of partons in $A + A$ collisions, studied in quantitative detail in [1], from central ($b = 0$) to non-central collisions, up to perhaps $b = 1.6R_A$. This extension is based on a simple geometric argument. Subject to the usual uncertainties in relating initial to final observed numbers, predictions for particle multiplicities and transverse energies can now be made and tested in terms of three variables, $A$, $\sqrt{s}$ and $b$ (or number of participants).

It is impossible to give a systematic estimate of the theoretical error in the predictions. Corrections to pQCD can, in principle, be computed, but the use of the saturation scale adds a nonperturbative, even phenomenological, element. What one can say is that the model is unambiguously determined, fits the data at SPS within 20% and predicted well the first measurements at RHIC, so maybe it is an acceptable way of at least extrapolating to so far unobserved parameter values. It thus offers a strongly constrained framework for an analysis of experimental data in a 3-parameter ($A, \sqrt{s}, b$) space and one should be able to make a distinction between different models along the lines suggested in [3].
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References

[1] K. J. Eskola, K. Kajantie, P. V. Ruuskanen and K. Tuominen, Nucl. Phys. B570 (2000) 379 [hep-ph/9909456].

[2] B. B. Back et al. [PHOBOS Collaboration], “Charged particle multiplicity near mid-rapidity in central Au + Au collisions at \( \sqrt{s} = 56 \) and 130 AGeV,” hep-ex/0007030.

[3] X.-N. Wang and M. Gyulassy, “Energy and centrality dependence of rapidity densities at RHIC,” nucl-th/0008014.

[4] X.-N. Wang and M. Gyulassy, Phys. Rev. D44 (1991) 3501.

[5] K. J. Eskola, K. Kajantie and J. Lindfors, Nucl. Phys. B323 (1989) 37.

[6] The Review of Particle Physics, The European Physical Journal C15 (2000) 1, Fig. 37.19.

[7] K. J. Eskola and K. Tuominen, “Production of transverse energy from minijets in next-to-leading order perturbative QCD,” hep-ph/0002008, Phys. Lett. B, in press.

[8] K. J. Eskola and K. Kajantie, Z. Phys. C75 (1997) 515 [nucl-th/9610015].

[9] A. H. Mueller, Nucl. Phys. B572 (2000) 227 [hep-ph/9906322].

[10] A. Krasnitz and R. Venugopalan, “The initial gluon multiplicity in heavy ion collisions,” hep-ph/0007108.

[11] A. Krasnitz and R. Venugopalan, Phys. Rev. Lett. 84 (2000) 4309 [hep-ph/9909203].