Representations of Genetic Tables, Bimagic Squares, Hamming Distances and Shannon Entropy

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Abstract
In this paper we have established relations of the genetic tables with magic and bimagic squares. Connections with Hamming distances, binomial coefficients are established. The idea of Gray code is applied. Shannon entropy of magic squares of order $4 \times 4$, $8 \times 8$, and $16 \times 16$ are also calculated. Some comparison is also made. Symmetry among restriction enzymes having four letters is also studied.

Key words: Genetic Code, Codon, Magic Squares, Hamming distances, Probability distributions, Shannon entropy.

1 Introduction

Genetic code is the set of rules by which information encoded in RNA/DNA is translated into amino acid sequences in living cells. The bases for the encoded information are nucleotides. There are four nucleotide bases for RNA: Adenine, Uracil, Guanine, and Cytosine, which are labeled by $A$, $U$, $G$ and $C$ respectively, (in DNA Uracil is replaced by Thymine ($T$)). In canonical genetic code, codons are tri-nucleotide sequences such that each triplet relates to an amino acid. For example, the codon CAG encodes the amino acid Glutamine.

Amino acids are the basic building blocks of proteins. It stimulated interest of other researchers to study how genetic code was translated into amino acids. There are 20 different amino acids (plus start and stop codons), and since there are four nucleotide bases, $A$, $U$, $T$ and $C$, there are $4^n$ different combinations of bases, for a string of length $n$. Therefore, $n = 3$ is the smallest number of bases that could be used to represent the 20 different amino acids. There is degeneracy between the codons, i.e., more than one codon can represent the same amino acid; however, two different amino acids cannot be represented by the same codon. The following CODON table is well-known in the literature.

|     | T     | C     | A     | G     |
|-----|-------|-------|-------|-------|
| T   | TTT (Phe) | TTC (Phe) | TAT (Try) | TGT (Cys) |
| C   | TTA (Leu) | TCA (Ser) | TAA (Stop) | TGA (Stop) |
| A   | TTG (Leu) | TCG (Ser) | TAG (Stop) | TGG (Trp) |
| G   | CTT (Leu) | CTC (Leu) | CAT (His) | CGT (Arg) |
|     | CTA (Leu) | CTA (Leu) | CAA (Glu) | CGA (Arg) |
|     | CTT (Leu) | CTC (Leu) | CAC (His) | CGC (Arg) |
|     | CTA (Leu) | CTA (Leu) | CAG (Glu) | CGG (Arg) |

The DNA (Deoxyribonucleic acid) molecule residing in the cell nucleus encodes information conventionally represented as a symbolic string over the alphabet. The combination between single strands of DNA takes
place according to “Watson-Crick [12] complementarity” that says that the only permissible combinations between bases are $A - T$ or $T - A$ and $C - G$ or $G - C$ hence one strand can easily be used to predict the other in a double stranded chain. Let us consider the following configurations of $4^n$ for each value of $n$.

(i) For $n = 1$: In this case we have $4^1 = 4$. This gives

$$M_1 := \begin{bmatrix} C & A \\ T & G \end{bmatrix}.$$  

(ii) For $n = 2$: In this case we have $4^2 = 16$. This gives

$$M_2 := \begin{bmatrix} CC & AC & TC & GC \\ CA & AA & TA & GA \\ CT & AT & TT & GT \\ CG & AG & TG & GG \end{bmatrix}.$$  

(iii) For $n = 3$: In this case we have $4^3 = 64$. This gives

$$M_3 := \begin{bmatrix} CCC & ACC & TCC & GCC & CTC & ATC & TTC & GTC \\ CCA & ACA & TCA & GCA & CTA & ATA & TTA & GTA \\ CCT & ACT & TCT & GCT & CTT & ATT & TTT & GTT \\ CCG & ACG & TCG & GCG & CTG & ATG & TTG & GTG \\ CAC & AAC & TAC & GAC & CGC & AGC & TGC & GGC \\ CAA & AAA & TAA & GAA & CGA & AGA & TGA & GGA \\ CAT & AAT & TAT & GAT & CGT & AGT & TGT & GTG \\ CAG & AAG & TAG & GAG & CGG & AGG & TGG & GGG \end{bmatrix}.$$  

(iv) For $n = 4$, we have $M_4$ with $4^4 = 256$ combinations of blocks of four letters, for $n = 5$, we have $M_5$ with $4^5 = 1024$, etc.

2 Gray Codes: Binary representations

2.1 First Approach

Let us represent the letters $C, A, T$ and $G$ in two different ways:

(i) $C = 00, \ A = 01, \ T = 10$ and $G = 11$.

(ii) $C = 1, \ A = 2, \ T = 3$ and $G = 4$.

(iii) The CODON table given above is formed of three letters out of four, i.e., $A, T, G$ and $C$. According to (i), we can write, for example, $TTA \sim 101001, AGC \sim 011100$, etc. Thus we have six digit binary representations of 64 members available in CODON table. Let us apply the change of base 2 to base 10 (decimal) using the formula $(abcdef)_2 : = a \cdot 2^5 + b \cdot 2^4 + c \cdot 2^3 + d \cdot 2^2 + e \cdot 2^1 + f \cdot 2^0$ and then writing $(abcdef)_2 + 1$, we have, $TTA \sim 101001 \sim 41$ and $AGC \sim 011100 \sim 28$, etc. Similarly, we can write the four digits binary representation in decimal forms, such as $(abcd)_2 : = a \cdot 2^3 + b \cdot 2^2 + c \cdot 2^1 + d \cdot 2^0$, and then writing $(abcd)_2 + 1$. Just for simplicity, we have added 1.

The notations given in (i) and (ii) can be seen in [4, 6, 2, 8, 10]. Decimal representation of the numbers is given by $C = (00)_2 \sim 0, A = (01)_2 \sim 1, T = (10)_2 \sim 2$ and $G = (11)_2 \sim 3$. For simplicity, we have added 1 and considered in (ii) as 1, 2, 3, and 4 instead of 0, 1, 2 and 3 respectively. We shall use frequently these three
representations and shall bring magic squares of different orders According to above notations we have

\[
M_1 := \begin{bmatrix}
C & A \\
T & G \\
\end{bmatrix} \sim \begin{bmatrix}
00 & 01 \\
10 & 11 \\
\end{bmatrix} \sim \begin{bmatrix}
1 & 2 \\
3 & 4 \\
\end{bmatrix},
\]

(1)

\[
M_2 := \begin{pmatrix}
0000 & 0100 & 1000 & 1100 \\
0001 & 0101 & 1001 & 1101 \\
0010 & 0110 & 1010 & 1110 \\
0011 & 0111 & 1011 & 1111 \\
\end{pmatrix},
\]

(2)

\[
M_2 := \begin{pmatrix}
11 & 21 & 31 & 41 \\
12 & 22 & 32 & 42 \\
13 & 23 & 33 & 43 \\
14 & 24 & 34 & 44 \\
\end{pmatrix},
\]

(3)

and

\[
M_2 := \begin{pmatrix}
1 & 5 & 9 & 13 \\
2 & 6 & 10 & 14 \\
3 & 7 & 11 & 15 \\
4 & 8 & 12 & 16 \\
\end{pmatrix}.
\]

(4)

The expressions appearing in (1) are due to (i), (ii) and (iii). The expression (2) is due to (i), (3) is due to (ii) and (3) is due to (iii). Similar tables can also be written for the matrix \(M_3\) Some of them can be seen in [4, 6, 7, 8, 10].

2.2 Second Approach

Following [2, 3], we use the following correspondence for the nucleotides and two-bit Gray codes:

\[
\begin{align*}
C & \sim \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \\
A & \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \\
T & \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } G \sim \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \\
\end{align*}
\]

The genetic code-based matrix, which will contain all nucleotide strings of length \(n\) is defined as \(M_n\). The Gray code sequences represented by \(M_n\) will be denoted by a \(2^n \times 2^n\) matrix. Here are corresponding Gray code representations

\[
M_1 := \begin{bmatrix}
C & A \\
T & G \\
\end{bmatrix} \sim \begin{bmatrix}
\begin{pmatrix} 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\
\begin{pmatrix} 1 \\ 0 \end{pmatrix} & \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\
\end{bmatrix} \sim \begin{bmatrix}
0 & 1 \\
0 & 1 \\
\end{bmatrix}.
\]

In information theory, the Hamming distance between two strings of equal length is the number of positions for which the corresponding symbols are different. Put another way, it measures the minimum number of substitutions required to change one into the other, or the number of errors that transformed one string into the other. Thus we observe that the Hamming distances of letters \(C\) and \(G\) is 0 and of letters \(A\) and \(T\) is 1. Replacing the same in the other cases we have

\[
M_2 := \begin{bmatrix}
CC & AC & TC & GC \\
CA & AA & TA & GA \\
CT & AT & TT & GT \\
CG & AG & TG & GG \\
\end{bmatrix} \sim \begin{bmatrix}
0 & 1 & 1 & 0 \\
1 & 2 & 2 & 1 \\
1 & 2 & 2 & 1 \\
0 & 1 & 1 & 0 \\
\end{bmatrix},
\]

and

\[
M_3 := \begin{pmatrix}
0 & 1 & 1 & 1 & 1 & 2 & 2 & 1 \\
1 & 2 & 2 & 1 & 2 & 3 & 3 & 2 \\
1 & 2 & 2 & 1 & 2 & 3 & 3 & 2 \\
1 & 1 & 1 & 0 & 1 & 2 & 2 & 1 \\
1 & 2 & 2 & 1 & 0 & 1 & 1 & 0 \\
2 & 3 & 3 & 2 & 1 & 2 & 2 & 1 \\
2 & 3 & 3 & 2 & 1 & 2 & 2 & 1 \\
1 & 2 & 2 & 1 & 0 & 1 & 1 & 0 \\
\end{pmatrix}.
\]
In the theory of discrete signal processing as a fundamental operation for binary variables, modulo-2 addition is utilized broadly. By definition, the modulo-2 addition of two numbers written in binary notation is made in a bitwise manner in accordance with the following rules:

\[
0 + 0 = 0, \quad 1 + 0 = 1, \quad 0 + 1 = 1, \quad 1 + 1 = 0
\]

For example, modulo-2 addition of two binary numbers 110 and 101, gives the result 110 ⊕ 101 = 011(3), where 3 is the decimal representation of 011. In case of 10 and 01, we have 10 ⊕ 01 = 11(3), where 3 is the decimal representation of 11(⊕ is the symbol for modulo-2 addition. The distance in this symmetry group is known as the Hamming distance. The modulo-2 addition of any two binary numbers always results in a new number from the same series. If any system of elements demonstrates its connection with diadic shifts, it indicates that the structural organization of its system is related to the logic of modulo-2 addition. In particular we have

\[
M_1 := \begin{bmatrix} C & A \\ T & G \end{bmatrix} \sim \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 \end{bmatrix}
\]

\[
M_2 := \begin{bmatrix} CC & AC & TC & GC \\ CA & AA & TA & GA \\ CT & AT & TT & GT \\ CG & AG & TG & GG \end{bmatrix} \sim \begin{bmatrix} 00 & 10 & 10 & 00 \\ 01 & 11 & 11 & 01 \\ 01 & 11 & 11 & 01 \\ 00 & 10 & 10 & 00 \end{bmatrix}
\]

and

\[
M_3 := \begin{bmatrix} 000 & 100 & 100 & 000 & 010 & 110 & 110 & 010 \\ 001 & 101 & 101 & 001 & 011 & 111 & 111 & 011 \\ 001 & 101 & 101 & 001 & 011 & 111 & 111 & 011 \\ 000 & 100 & 100 & 000 & 010 & 110 & 110 & 010 \\ 010 & 110 & 110 & 010 & 000 & 100 & 100 & 000 \\ 011 & 111 & 111 & 011 & 001 & 101 & 101 & 001 \\ 011 & 111 & 111 & 011 & 001 & 101 & 101 & 001 \\ 010 & 110 & 110 & 010 & 000 & 100 & 100 & 000 \end{bmatrix}
\]

The results are obtained by using the binary operations given above for example, \( GT \sim \begin{bmatrix} 11 \\ 10 \end{bmatrix} \sim 01 \), i.e., \( 11 \oplus 10 = 01 \), \( ACT \sim \begin{bmatrix} 001 \\ 100 \end{bmatrix} \sim 101 \), i.e., \( 001 \oplus 100 = 101 \), etc. The decimal transformations are

\[
(00)_2 \sim 0, \quad (01)_2 \sim 1, \quad (10)_2 \sim 2, \quad (11)_2 \sim 3
\]

and

\[
(000)_2 \sim 0, \quad (001)_2 \sim 1, \quad (010)_2 \sim 2, \quad (011)_2 \sim 3
\]

\[
(100)_2 \sim 4, \quad (101)_2 \sim 5, \quad (110)_2 \sim 6, \quad (111)_2 \sim 7
\]

This gives

\[
M_2 := \begin{bmatrix} 0 & 2 & 0 \\ 2 & 2 & 0 \\ 1 & 3 & 1 \\ 3 & 3 & 1 \\ 0 & 2 & 2 \end{bmatrix}
\]

and

\[
M_3 := \begin{bmatrix} 0 & 4 & 4 & 0 & 2 & 6 & 6 & 2 \\ 1 & 5 & 5 & 1 & 3 & 7 & 7 & 3 \\ 1 & 5 & 5 & 1 & 3 & 7 & 7 & 3 \\ 0 & 4 & 4 & 0 & 2 & 6 & 6 & 2 \\ 2 & 6 & 6 & 2 & 0 & 4 & 4 & 0 \\ 3 & 7 & 7 & 3 & 1 & 5 & 5 & 1 \\ 3 & 7 & 7 & 3 & 1 & 5 & 5 & 1 \\ 2 & 6 & 6 & 2 & 0 & 4 & 4 & 0 \end{bmatrix}
\]
3 Reconfiguration Tables and Magic Squares

This section deals with the reconfigurations of matrices given above. These reconfigurations are made in such a way that using the notations given in section 2.1, lead use to magic squares or bimagic squares. Here below are the definitions of magic and bimagic squares

- A **magic square** is a collection of numbers put as a square matrix, where the sum of element of each row, each column and two principal diagonals are the same sum. For simplicity, let us write it as $S_1$

- **Bimagic square** is a magic square where the sum of squares of each element of rows, columns and two principal diagonals are the same. For simplicity, let us write it as $S_2$.

3.1 Reconfiguration Tables of order 4x4

Let us reconsider the matrix $M_2$ as

$$M_2^{4 \times 4} := \begin{bmatrix} AT & TG & CC & GA \\ CA & GC & AG & TT \\ GG & CT & TA & AC \\ TC & AA & GT & CG \end{bmatrix}.$$  \hspace{1cm} (5)

In the above configuration, we have permutations of letters $C, A, T$ and $G$ are the first and second places, in various situations, for example, in each row, in each column, main diagonals, each group of order $2 \times 2$, middle group of order $2 \times 2$, four corner elements, symmetrical diagonals, etc. These configurations are of the following type

$$\begin{bmatrix} AT & TG \\ CA & GC \end{bmatrix}, \begin{bmatrix} GC & AG \\ CT & TA \end{bmatrix}, \begin{bmatrix} CA & GC & AG & TT \end{bmatrix}, \begin{bmatrix} CC & AG \\ TA & GT \end{bmatrix}, \text{etc.}$$

Here below are 20 combinations where the first and second members are the permutations of the letters $C, A, T$ and $G$:

| 1 | 2 | 3 | 4 | 5 | 5 | 5 | 5 | 9 | 9 | 10 | 10 | 14 | 15 | 15 | 14 | 17 | 19 | 20 | 18 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 6 | 6 | 6 | 6 | 9 | 9 | 10 | 10 | 16 | 13 | 13 | 16 | 19 | 17 | 18 | 20 |
| 1 | 2 | 3 | 4 | 7 | 7 | 7 | 7 | 11 | 11 | 12 | 12 | 16 | 13 | 13 | 16 | 20 | 18 | 17 | 19 |
| 1 | 2 | 3 | 4 | 8 | 8 | 8 | 8 | 11 | 11 | 12 | 12 | 14 | 15 | 15 | 14 | 18 | 20 | 19 | 17 |

According to notations (i), (ii) and (iii) given in section 2.1, we have

| 0110 | 1011 | 0000 | 1101 |
|------|------|------|------|
| 23   | 34   | 11   | 42   |
| 7    | 12   | 1    | 14   |
| 0001 | 1100 | 0111 | 1010 |
| 12   | 41   | 24   | 33   |
| 2    | 13   | 8    | 11   |
| 1111 | 0010 | 1001 | 0100 |
| 44   | 13   | 32   | 21   |
| 16   | 3    | 10   | 5    |
| 1000 | 0101 | 1110 | 0011 |
| 31   | 22   | 43   | 14   |
| 9    | 6    | 15   | 4    |
In all the three situations we have magic squares of order $4 \times 4$ with $S_1^{4\times4} := 2222$, $S_1^{4\times4} := 110$ and $S_1^{4\times4} := 34$ respectively. The last one is well-known Khajurao magic square of order $4 \times 4$:

$$
\begin{array}{cccc}
7 & 12 & 1 & 14 \\
2 & 13 & 8 & 11 \\
16 & 3 & 10 & 5 \\
9 & 6 & 15 & 4
\end{array}
$$

The above magic square of order $4 \times 4$ is one of the most perfect magic square known in the literature. Its connections with genetic code can be seen in [11]. This is one of the very little work available on magic squares connecting DNA.

According to binary operations given in section 2.2, we have

$$
M_B^{4\times4} := \begin{bmatrix}
11 & 10 & 00 & 01 \\
01 & 00 & 10 & 11 \\
00 & 01 & 11 & 10 \\
10 & 11 & 01 & 00
\end{bmatrix} \sim \begin{bmatrix}
3 & 2 & 0 & 1 \\
1 & 0 & 2 & 3 \\
0 & 1 & 3 & 2 \\
2 & 3 & 1 & 0
\end{bmatrix}.
$$

We observe that the matrix $M_B^{4\times4}$ is a composition of two mutually orthogonal diagonalize Latin squares, while the matrix $M_B^{4\times4}$ is not a diagonalize Latin square.

### 3.2 Reconfiguration Tables of order 8x8

Here we shall reorganize the CODON table or the matrix $M_3$ given in section 1 in such way that it becomes as magic square of order $8 \times 8$. We shall present two different ways:

1. Four magic squares of order $4 \times 4$ of the sum $S_1^{4\times4}$ having all the properties of the configuration matrix $M_3^{4\times4}$ given by (5).
2. Binagic square of order $8 \times 8$.

#### 3.2.1 First Representations

Let us consider the following reorganization of matrix $M_3$ or the above CODON table:

| CCC | TAT | GTG | AGA | CAA | TCG | GGT | ATC |
|-----|-----|-----|-----|-----|-----|-----|-----|
| GTA | AGG | CCT | TAC | GGC | ATT | CAG | TCA |
| AGT | GTC | TAA | CCG | ATG | GGA | TCC | CAT |
| TAG | CCA | AGC | GTC | TCT | CAC | AIA | GGG |
| CTG | TGA | GCC | AAT | CTG | TTT | AAG | GAG |
| GCT | AAC | CTA | GGT | GAG | ACA | CGC | TTT |
| AAA | GCG | TGT | CTC | ACC | GAT | Ttg | CGA |
| TGC | CTT | AAG | GCA | TTA | CGG | ACT | GAC |

In the above configuration we have the same properties of the magic square of order $4 \times 4$ given by (5), i.e., there are many permutation of the letters $C$, $A$, $T$ and $G$ in the first, second and third places. Each block of order $4 \times 4$, half-row, half-column, half-principle diagonals etc. are also follow the same property. There are much more combinations of this type in the above configuration. In another way we can say there is a uniform distributions of letters $C$, $A$, $T$ and $G$. Using the notations (i), (ii) and (iii) given in section 2.1, we have
The above table brings three different magic squares of order $8 \times 8$, i.e., in each case we have $S_{1}^{8 \times 8} := 444444$, $S_{1}^{8 \times 8} := 2220$ and $S_{1}^{8 \times 8} := 260$ respectively. Moreover, the above magic square is also bimagic in columns, i.e., for each column we have $S_{2}^{8 \times 8} := 44893328844$, $S_{2}^{8 \times 8} := 717060$ and $S_{2}^{8 \times 8} := 11180$ respectively. Also, each block of order $4 \times 4$ is a magic square with $S_{1}^{4 \times 4} := 222222$. Sum of each block of order $2 \times 2$ also has the same sum as of $S_{1}^{4 \times 4}$.

### 3.2.2 Second Representation

We observe that the above magic square is bimagic only in columns. Here below we shall present a little different representation of CODON table resulting in bimagic square of order $8 \times 8$. Let us consider the following configuration:

| CGG | TTC | TCG | CAC | ATT | GGA | GAT | ACA |
|-----|-----|-----|-----|-----|-----|-----|-----|
| AFA | GGT | GAA | ACT | CGC | TGG | TCC | CAG |
| CCC | TAG | TGC | CTG | AAA | GCT | GTA | ACT |
| AAT | GCA | GTG | AGA | CCG | TAC | TGG | CTC |
| TAA | CCT | CIA | TGT | GCC | AAG | AGC | GTG |
| GCG | AAC | AGG | GTC | TAT | CCA | CTT | TGA |
| TTT | CGA | CAT | TCA | GAG | ATC | ACG | GAC |
| GCC | ATG | ACC | GAG | TTA | CGT | CAA | TCT |

In the above configuration we have permutations of four letters $C$, $A$, $T$ and $G$ only in the first and third place in each block of order 2x2, half-row, half-column, half-principal diagonals etc. Again, using the notations (i), (ii) and (iii) given in section 2.1, we have
The idea of Watson and Crick [12], they considered it as coefficients and:

3.4 Hamming Distances and Binomial Coefficients

In the first representation, we have bimagic many other combinations in the above table giving connection with (a), (b) and (c) each block of order 2.

Many authors [5, 6, 9] made connections with this prime number. Both the representations we have three cases. In the first, we can write half-sum of rows, columns and two principal diagonal as multiple of 37, i.e., in the first case we have \( S_1^{8\times8} = 444444 = 12012 \times 37 \);

(b) \( S_2^{8\times8} = 4489328844 = 1213333212 \times 37 \); and

(c) \( S_3^{8\times8} = 260; \quad S_2^{8\times8} = 11180 \).

We observe that (a) and (b) both \( S_1^{8\times8} \) and \( S_2^{8\times8} \) are multiple of 37. This is not true in case of (c). Still, in (a), (b) and (c) each block of order 2 \( \times \) 4 is also bimagic

3.3 Connections with Prime Number 37

Many authors [5, 6, 9] made connections with prime number 37. Here also we shall bring some interesting connections with this prime number. Both the representations we have three cases. In the first, we can write the sum \( S_1^{8\times8} = 444444 = 12012 \times 37 \) In the second case we have \( S_2^{8\times8} = 2220 = 37 \times 60 \). Also, in both cases these we have half-sum of rows, columns and two principal diagonal as multiple of 37, i.e., in the first case we have \( \frac{1}{2} S_1^{8\times8} = 222222 = 6 \times 1001 \times 37 \), and in the second case we have \( \frac{1}{2} S_2^{8\times8} = 1110 = 30 \times 37 \). There are many other combinations in the above table giving connection with 37. For example each block of 2 \( \times \) 2 is of sum \( \frac{1}{2} S_1^{8\times8} \). In case of \( S_2^{8\times8} \) we have \( S_2^{8\times8} = 4489328844 = 1213333212 \times 37 \) and \( S_2^{8\times8} = 717060 = 37 \times 19380 \). In the first representation, we have bimagic sum only in each column.

3.4 Hamming Distances and Binomial Coefficients

The idea of Hamming distances is given in section 2.2. Here we consider more representations to bring binomial coefficients and and bimagic squares. Kappraff and Adamson [4] considered \( C = G \) and \( A = U/T \). Following the idea of Watson and Crick [12], they [4] considered it as \( A = T = 2 \) and \( C = G = 3 \). For simplicity,
let us consider here $A = T = a$ and $C = G = b$, where it is understood that $TTG = a \times a \times b = a^2b$, $AGC = a \times b \times b = ab^2$, etc. Accordingly, we have

(i) For $n = 1$:

$$
\begin{array}{c|cc}
0 & 1 \\
\hline
b & a \\
1 & 0 \\
a & b \\
\end{array}
$$

(ii) For $n = 2$:

$$
\begin{array}{c|cc|cc}
1 & 2 & 0 & 1 & 1 \\
\hline
ab & b^2 & a^2 & ab & 1 \\
0 & 1 & 2 & a^2 & 1 \\
1 & 0 & 1 & ab & b^2 \\
a & ab & 2 & b^2 & 1 \\
\end{array}
$$

(iii) For $n = 3$: According to configuration given in section 3.2.1, we have

$$
\begin{array}{c|cc|cc|cc|cc|cc|cc|cc|cc}
0 & a^3 & 3 & 1 & 2 & 2 & 1 & 2 & 2 & a^2b \\
\hline
3 & b^3 & a^{b^2} & 1 & 2 & a^2b & a^2b & 1 & ab^2 & ab^2 \\
2 & a^{b^2} & ab & 1 & 2 & 1 & 3 & 0 & a^2b & a^2b \\
2 & a^2b & 0 & 1 & 2 & ab^2 & a^2b & ab^2 & ab^2 \\
2 & a^2b & ab^2 & 1 & 2 & ab^2 & a^2b & 0 & 1 & 3 & ab^2 \\
1 & ab^2 & 2 & 0 & 3 & ab^2 & a^3 & 1 & ab^2 & a^3 \\
1 & ab^2 & 3 & 0 & 0 & a^3 & a^3 & 1 & ab^2 & 0 & 3 & b^3 \\
3 & b^3 & ab^2 & 2 & 1 & ab^2 & a^2b & 2 & a^2b & 2 & a^2b & 1 & ab^2 \\
1 & ab^2 & 2 & 0 & 3 & ab^2 & a^3 & 0 & a^3 & 3 & ab^2 \\
\end{array}
$$

According to configuration given in section 3.2.2 we have
The interesting fact in the above tables is that in the first case, it is symmetric in rows, columns and principal diagonals, while it is not true in the second case. In the second case it holds only in rows. The tables studied above gives us the following frequency distributions:

| n | Hamming distances | Frequency distributions | Binomial coefficients | Sum |
|---|-------------------|-------------------------|----------------------|-----|
| 1 | 0 1 0 1 2 1 1 3 | 2^1 = 2 a b 1 2 a 2 | (a + b)^1 |
| 2 | 0 1 2 1 1 0 1 3 | 2^2 = 4 a^2 2ab b^2 2 | (a + b)^2 |
| 3 | 0 1 2 1 1 3 2 1 | 2^3 = 8 a^3 3a^2b 3ab^2 b^3 2 | (a + b)^3 |

For more properties of above table refer to [H].

### 3.5 Binary Operations

Considering the notations and binary operations given in section 2.2, i.e., $CGT := \begin{pmatrix} 011 \\ 010 \end{pmatrix} \sim 001$, $ATC := \begin{pmatrix} 010 \\ 100 \end{pmatrix} \sim 110$, etc. This operations gives us eight possibilities, i.e., 000, 001, 010, 011, 100, 101, 110 and 111. Let us represent 000 → a, 001 → b, 010 → c, 011 → d, 100 → e, 101 → f, 110 → g and 111 → h. Instead of decimal representations as 0, 1, 2, 3, 4, 5, 6 and 7 we have considered here the letters a, b, c, d, e, f, g and h respectively. According to section 3.2.1, we have the following table:

| a | b | c | d | e | f | g |
|---|---|---|---|---|---|---|
| d | e | b | g | a | h | c |
| f | c | h | a | g | b | e |
| g | b | e | d | f | c | h |
| e | f | a | h | b | g | d |
| b | g | d | e | c | f | a |
| h | a | f | c | e | d | g |
| c | d | g | b | h | a | f |
We observe that we have 16 matrices of order $2 \times 2$ divided in two groups formed by the elements $(a, d, e, h) \sim (000, 011, 100, 111)$ and $(b, c, f, g) \sim (001, 010, 101, 110)$. The above configuration is well-known diagonalize Latin square of order $8 \times 8$. In the second case, i.e., for the section 3.2.2, we don’t have symmetric configuration. See below:

![Matrix Configuration](attachment:image.png)

4 Restriction Enzymes

There are (ref. Reiner [11]) 402 known restriction enzymes. Out of these 402 enzymes, 108 cut at a tetrameric sequence containing the four different bases. Of these 108, 100% have either $AT$ or $GC$ dimers (or both) in the sequence. None of 108 enzymes have $G$ apart from $C$ and $A$ apart from $T$, as in $AGTC$. Thus, all 108 enzymes cut at the tetrameric sequence which is complementary to its reverse cyclic permutation. The specific antiparallel sequences and the enzymes at which they cut are listed below. Reiner [11] considered following two different combinations of four letters having together, either $AT − TA$ or $GC − CG$ specifying antiparallel enzymes. See the table below:

| Antiparallel $A − T, G − C$ in same orientation (88) | Antiparallel $A − T, G − C$ in opposite orientation (20) |
|-----------------------------------------------------|-----------------------------------------------------|
| $AGCT (9)$                                           | $TAGC (0)$                                           |
| $CGTA (0)$                                           | $ACGT (2)$                                           |
| $TACG (0)$                                           | $GTA C (4)$                                          |
| $CTAG (9)$                                           | $GCTA (0)$                                           |
| $GCAT (1)$                                           | $TGC A (11)$                                         |
| $TCGA (32)$                                          | $ATCG (0)$                                           |
| $ATGC (0)$                                           | $CATG (3)$                                           |
| $GATC (45)$                                          | $CGAT (0)$                                           |

Interestingly, the above pairs follows the same cyclic permutations, i.e., for example, $A − G − C − T − A − G − C$.

4.1 Distribution of four Letter Combinations

Very less work can be seen in literature having the combinations of four letters in four places. Most of the work is towards codon representation given above. Thus we observe that we $4^4 = 256$ possibilities of writing combinations of four letters in four places. Here below is a configuration $16 \times 16$ having all the 256 possibilities.
The construction of above table is based on the same techniques of the magic of $M_{4 \times 4}^4$. It has the same properties as of $M_{4 \times 4}^4$. Moreover, the antiparallel pairs appearing in the above table are in the same block in each case. They lies in the last eight blocks of order $4 \times 4$.

### 4.2 Bimagic Squares of Order 16x16

Let us consider now the representations (i) and (ii) of the letters $C$, $A$, $T$ and $G$ as given in section 2.1. These representations lead us to following two bimagic square of order $16 \times 16$.

#### 4.2.1 First representation

This representation is according (i) given in section 2.1, by choosing $C = 00$, $A = 01$, $T = 10$ and $G = 11$. This we have written in two parts:

**Part 1:**

| 00000000 | 10011001 | 11101110 | 01110111 | 00010110 | 10001111 | 11110000 | 01100001 |
| 11100111 | 01111110 | 00001001 | 10001000 | 11110001 | 01101000 | 00011111 | 10000110 |
| 01111001 | 11100000 | 10010111 | 00001110 | 01101111 | 11110110 | 10000001 | 00011000 |
| 10011110 | 00000111 | 01100000 | 11101001 | 10001000 | 00010001 | 01101110 | 11111111 |
| 00101101 | 10110100 | 11000011 | 01011000 | 00110011 | 10100110 | 11010101 | 01000110 |
| 11001010 | 01010011 | 00100100 | 10111011 | 11011100 | 01001010 | 00110100 | 10101101 |
| 01010100 | 11001101 | 01100100 | 00100011 | 10111011 | 11011100 | 11010101 | 00110101 |
| 10110000 | 00100110 | 01011101 | 11001000 | 01001101 | 10110010 | 10111100 | 01010100 |
| 00110011 | 10111110 | 11101100 | 01000000 | 01001000 | 10110101 | 10110011 | 01001110 |
| 10010000 | 10110100 | 11101101 | 11001100 | 01000100 | 10101011 | 10110111 | 01001100 |
Part 2:

|      |      |      |      |      |      |      |      |      |      |      |      |      |
|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 00101011 | 10110010 | 11000101 | 01011100 | 00111101 | 10100100 | 11010111 | 01011010 | 00110011 | 10101101 | 01010101 | 01100101 | 11100110 |
| 11011000 | 01010101 | 00100010 | 10111011 | 11010110 | 01000011 | 00110100 | 11110111 | 01001010 | 00110010 | 11010100 | 01100100 | 11111100 |
| 01010100 | 10110011 | 10111100 | 00100010 | 01000101 | 10100111 | 01001101 | 00110110 | 11011000 | 00010110 | 11101100 | 01101100 | 10001110 |
| 11001111 | 00000101 | 01100110 | 11110111 | 10001101 | 01010101 | 00101100 | 11101001 | 01000110 | 00111000 | 10011010 | 00100010 | 11111100 |

Let us combine the parts 1 and 2 as given below

\[
\begin{array}{|c|c|}
\hline
\text{Part 1} & \text{Part 2} \\
\hline
\end{array}
\]

This gives us a bimagic square of order 16 \(\times\) 16 with \(S_{16\times16}^1 := 88888888\) and \(S_{16\times16}^2 := 897867554657688\). Each block of order 4 \(\times\) 4 is also a magic square with \(S_{4\times4}^1 := 22222222\). Square of sum of each term in each block of order 4 \(\times\) 4 is also \(S_{16\times16}^{2,2} := 897867554657688\). Here only \(S_{16\times16}^{2,2}\) is divisible by 37 i.e., \(S_{16\times16}^{2,2} := 897867554657688 = 24266690666424 \times 37\).

4.2.2 Second representation

This representation is according to (ii) given in section 2.1 by choosing \(C = 1\), \(A = 1\), \(T = 3\) and \(G = 4\).
Here again we have bimagic square of order $16 \times 16$ with $S_{16}^{16} = 444440$ and $S_{2}^{16} = 143634120$. Each block of order $4 \times 4$ is also a magic square with $S_{16}^{16} = 111110$. Square of sum of each term in of each block of $4 \times 4$ is also $S_{2}^{16} = 143634120$.

4.2.3 Third representation

Applying the change of base 2 to base 10 (decimal) in first case, i.e.,

$$(abdefgh)_{2} := a \cdot 2^{7} + b \cdot 2^{6} + c \cdot 2^{5} + d \cdot 2^{4} + e \cdot 2^{3} + f \cdot 2^{2} + g \cdot 2^{1} + h \cdot 2^{0}$$

then writing $(abdefgh)_{2} + 1$, we get the bimagic square of order $16 \times 16$ with sum $S_{16}^{16} = 2056$ and $S_{2}^{16} = 351576$. Also each block of order $4 \times 4$ is a magic square with sum $S_{4}^{16} = 514$.

Here again we have bimagic square of order $16 \times 16$ with $S_{16}^{16} = 444440$ and $S_{2}^{16} = 143634120$. Each block of order $4 \times 4$ is also a magic square with $S_{16}^{16} = 111110$. Square of sum of each term in of each block of $4 \times 4$ is also $S_{2}^{16} = 143634120$.

| 1  | 154 | 239 | 120 | 23 | 144 | 249 | 98 | 44 | 179 | 198 | 93 | 62 | 165 | 212 | 75 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 232 | 127 | 10 | 145 | 242 | 105 | 32 | 135 | 205 | 86 | 35 | 188 | 219 | 68 | 53 | 174 |
| 122 | 225 | 152 | 15 | 112 | 247 | 130 | 25 | 83 | 204 | 189 | 38 | 69 | 222 | 171 | 52 |
| 159 | 8 | 113 | 234 | 137 | 18 | 103 | 256 | 182 | 45 | 92 | 195 | 164 | 59 | 78 | 213 |
| 46 | 181 | 196 | 91 | 60 | 163 | 214 | 77 | 7 | 160 | 233 | 114 | 17 | 138 | 255 | 104 |
| 203 | 84 | 37 | 190 | 221 | 70 | 51 | 172 | 226 | 121 | 16 | 151 | 248 | 111 | 26 | 129 |
| 85 | 206 | 187 | 36 | 67 | 220 | 173 | 54 | 128 | 231 | 146 | 9 | 106 | 241 | 136 | 31 |
| 180 | 43 | 94 | 197 | 166 | 61 | 76 | 211 | 153 | 2 | 119 | 240 | 143 | 24 | 97 | 250 |
| 55 | 176 | 217 | 66 | 33 | 186 | 207 | 88 | 30 | 133 | 244 | 107 | 12 | 147 | 230 | 125 |
| 210 | 73 | 64 | 167 | 200 | 95 | 42 | 177 | 251 | 100 | 21 | 142 | 237 | 118 | 3 | 156 |
| 80 | 215 | 162 | 57 | 90 | 193 | 184 | 47 | 101 | 254 | 139 | 20 | 115 | 236 | 157 | 6 |
| 169 | 50 | 71 | 224 | 191 | 40 | 81 | 202 | 132 | 27 | 110 | 245 | 150 | 13 | 124 | 227 |
| 28 | 131 | 246 | 109 | 14 | 149 | 228 | 123 | 49 | 170 | 223 | 72 | 39 | 192 | 201 | 82 |
| 253 | 102 | 19 | 140 | 235 | 116 | 5 | 158 | 216 | 79 | 58 | 161 | 194 | 89 | 48 | 183 |
| 99 | 252 | 141 | 22 | 117 | 238 | 155 | 4 | 74 | 209 | 168 | 63 | 96 | 199 | 178 | 41 |
| 134 | 29 | 108 | 243 | 148 | 11 | 126 | 229 | 175 | 56 | 65 | 218 | 185 | 34 | 87 | 208 |

According to above three constructions the Reiner [11] table of antiparallelism is given by

| Antiparallel | AGCT | CGTA | TACG | CTAG | GCAT | TCAG | AGTC | GATC | SUM |
|-------------|------|------|------|------|------|------|------|------|-----|
| A-T, G-C    | 01110010 | 00110001 | 10010011 | 00010011 | 11001001 | 10001101 | 01010110 | 11010100 | 44444444 |
| in same     | 2413 | 1432 | 3214 | 1324 | 4123 | 3142 | 2341 | 4231 | 22220 |
| orientation | 115 | 58 | 40 | 48 | 19 | 14 | 10 | 7 | 1028 |

| Antiparallel | TAGC | ACCG | GTAC | GCTA | TGCA | ATCG | CATG | CGAT | SUM |
|-------------|------|------|------|------|------|------|------|------|-----|
| A-T, G-C    | 10011001 | 01001110 | 11100100 | 11001001 | 10110001 | 01100011 | 00100111 | 10101110 | 44444444 |
| in opposite | 3241 | 2143 | 4321 | 4132 | 3412 | 2314 | 1234 | 1423 | 22220 |
| orientation | 157 | 79 | 229 | 202 | 178 | 100 | 28 | 55 | 1028 |

We have the same sum in both the situations, i.e., in the same as well as in the opposite orientation of the genetic letters.

4.3 Hamming Distances and Binomial Coefficients

Here also we shall consider more representations to bring Hamming distances and binomial coefficients. Using the same notations of section 3.4, we have the following table with Hamming distances and binomial coefficients:

Here we observe the symmetry in elements in each row, each column and each block of order $4 \times 4$. The same symmetry we have in principal diagonals too. This don't happened in case of order 8x8. Thus there is a straight relationship with the binomial coefficients and Hamming distances, i.e., $0 \rightarrow b^{1}$, $1 \rightarrow ab^{3}$, $2 \rightarrow a^{2}b^{2}$, $3 \rightarrow a^{3}b$ and $4 \rightarrow a^{4}$. Accordingly, we have the following frequency distribution table:
[Table]

- Frequency distribution

| n   | Hamming distances | Frequency distributions | Binomial coefficients | Sum   |
|-----|-------------------|-------------------------|-----------------------|-------|
| 1   | 0 1               | 2^2 = 2                 | ab                    | (a + b)^2 |
| 2   | 0 1 2             | 2^2 = 4                 | a^2 2ab b^2           | (a + b)^2 |
| 3   | 0 1 2 3           | 2^3 = 8                 | a^3 3a^2b 3ab^2 b^3   | (a + b)^3 |
| 4   | 0 1 2 3 4         | 2^4 = 16                | a^4 4a^3b 6a^2b^2 4ab^2 a^4 | (a + b)^4 |
| 5   | 0 1 2 3 4 5       | 2^5 = 32                | a^5 5a^4b 10a^3b^2 10a^2b^3 5ab^4 b^5 | (a + b)^5 |
| 6   | 0 1 2 3 4 5 6     | 2^6 = 64                | a^6 6a^5b 15a^4b^2 20a^3b^3 15a^2b^4 6ab^5 b^6 | (a + b)^6 |

The above table allow us to extend the results for the next values of n. Some studies having combinations of four letters can be seen in [1].

5 Shannon’s Entropy and Genetic Tables

The idea of Shannon entropy is well-known in the literature on information theory. It is defined as

\[ H(P) = - \sum_{i=1}^{n} p_i \log p_i \]

where \( P = (p_1, p_2, ..., p_n) \), \( p_i > 0 \), \( \sum_{i=1}^{n} p_i = 1 \) is a set of probability distribution associated with a random variable \( X = \{x_1, x_2, ..., x_n\} \). Applications of Shannon entropy to genetic code can be seen in many works. In [13, 14], authors introduce the idea of genome order index given by

\[ S(P) = \sum_{i=1}^{n} p_i^2 \]

In Information theory the expression \( S(P) \) is famous as quadratic entropy. Thus based the magic squares given above we shall calculate Shannon entropy and genome order index. First, we shall transform values in probabilities dividing by sum of each row or column. Here we shall consider only the first case.
5.1 Shannon Entropy of Order 4x4

In section 3.1, we have three different kind of magic squares of order 4 × 4. The first one is with binary digits. In this let us divide the each value by the magic sum. This gives us the following probability distributions:

- **Probability distributions**

|        | 0.4500 | 0.0050 | 0.0455 | 0.4995 |
|--------|--------|--------|--------|--------|
| 0.0495 | 0.4955 | 0.4550 | 0.0000 |
| 0.5000 | 0.0450 | 0.0045 | 0.4505 |
| 0.0005 | 0.4545 | 0.4950 | 0.0500 |

- **Shannon entropy**

Based on above probability distributions, let us calculate the values of Shannon entropy. The table below give these values.

|        | 0.3683 |
|--------|--------|
| 0.0400 | 0.0114 |
| 0.0646 | 0.1511 |
| 0.1505 | 0.0606 |
| 0.0005 | 0.2567 |
| 0.0256 | 0.3783 |
| 0.0256 | 0.3783 |
| 0.0256 | 0.3716 |
| 0.0256 | 0.2667 |

We observe that the value of Shannon entropy varies from 0.2567 to 0.3777.

5.2 Shannon Entropy of Order 8x8

In sections 3.2.1 and 3.2.2, we have two different kinds of magic squares of order 8 × 8. The magic square appearing in section 3.2.2 is bimagic. In both the cases, we considered here below the first one with binary digits. In these cases let us divide the each value by their magic sum. This gives us the following probability distributions:

5.2.1 First case

Here below is a table for probability distributions based on the binary magic square of order 8 × 8 given in section 3.2.1.

- **Probability distribution**

|        | 0.00000 | 0.22525 | 0.24977 | 0.02498 | 0.00023 | 0.22502 | 0.25000 | 0.02475 |
|--------|---------|---------|---------|---------|---------|---------|---------|---------|
| 0.24975 | 0.02500 | 0.00020 | 0.22523 | 0.24998 | 0.02477 | 0.00025 | 0.22500 |
| 0.02500 | 0.24975 | 0.22523 | 0.00020 | 0.02477 | 0.24998 | 0.22500 | 0.00025 |
| 0.22525 | 0.00000 | 0.24977 | 0.22502 | 0.00023 | 0.02475 | 0.25000 | 0.02475 |
| 0.00227 | 0.22748 | 0.24750 | 0.22725 | 0.00250 | 0.22725 | 0.24773 | 0.02252 |
| 0.24752 | 0.02273 | 0.22750 | 0.22750 | 0.00225 | 0.22750 | 0.24775 | 0.00248 |
| 0.02273 | 0.24752 | 0.22750 | 0.00225 | 0.02250 | 0.24775 | 0.22727 | 0.00248 |
| 0.22748 | 0.00227 | 0.02275 | 0.24750 | 0.22725 | 0.00250 | 0.02252 | 0.24773 |
• Shannon entropy

Based on above probability distributions, let us calculate the values of Shannon entropy. The table below give these values.

|        | 0.0000 | 0.1458 | 0.1505 | 0.0400 | 0.0008 | 0.1458 | 0.1505 | 0.0398 | 0.6766 |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.1505 | 0.0401 | 0.0001 | 0.1458 | 0.1505 | 0.0398 | 0.0009 | 0.1458 | 0.6734 |
| 0.0400 | 0.1505 | 0.1458 | 0.0001 | 0.0398 | 0.1505 | 0.1458 | 0.0009 | 0.6734 |
| 0.1458 | 0.0000 | 0.0400 | 0.1505 | 0.1458 | 0.0008 | 0.0398 | 0.1505 | 0.6732 |
| 0.0060 | 0.1463 | 0.1501 | 0.0374 | 0.0065 | 0.1462 | 0.1501 | 0.0371 | 0.6797 |
| 0.1501 | 0.0373 | 0.0060 | 0.1463 | 0.1501 | 0.0371 | 0.0065 | 0.1462 | 0.6796 |
| 0.0374 | 0.1501 | 0.1463 | 0.0060 | 0.0371 | 0.1501 | 0.1462 | 0.0065 | 0.6796 |
| 0.1463 | 0.0060 | 0.0374 | 0.1501 | 0.1462 | 0.0065 | 0.0371 | 0.1501 | 0.6797 |
| 0.6761 | 0.6761 | 0.6761 | 0.6768 | 0.6768 | 0.6769 | 0.6769 | 0.6763 |

We observe that the values of Shannon entropy varies from 0.6732 to 0.6797

5.2.2 Second case

In this case we shall deal with bimagic square given in section 3.2.2 with binary digits. The table below is the probability distributions table:

• Probability distribution

|        | 0.00250 | 0.22725 | 0.22502 | 0.00023 | 0.02477 | 0.24998 | 0.24775 | 0.02250 |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.02475 | 0.25000 | 0.24773 | 0.02252 | 0.00248 | 0.22727 | 0.22500 | 0.00025 |
| 0.00000 | 0.22525 | 0.22748 | 0.00227 | 0.02273 | 0.24752 | 0.24975 | 0.02500 |
| 0.02275 | 0.24750 | 0.24977 | 0.02498 | 0.00002 | 0.22523 | 0.22750 | 0.00225 |
| 0.22523 | 0.00002 | 0.00225 | 0.22750 | 0.24750 | 0.02275 | 0.02498 | 0.24977 |
| 0.24752 | 0.02273 | 0.02500 | 0.24975 | 0.22525 | 0.00000 | 0.00227 | 0.22748 |
| 0.22727 | 0.00248 | 0.00025 | 0.22500 | 0.25000 | 0.02475 | 0.02252 | 0.24773 |
| 0.24998 | 0.02477 | 0.02250 | 0.24775 | 0.22725 | 0.00250 | 0.00023 | 0.22502 |

• Shannon entropy

|        | 0.0065 | 0.1462 | 0.1458 | 0.0008 | 0.0398 | 0.1505 | 0.1501 | 0.0371 | 0.6763 |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0398 | 0.1505 | 0.1501 | 0.0371 | 0.0065 | 0.1462 | 0.1458 | 0.0009 | 0.6769 |
| 0.0000 | 0.1458 | 0.1463 | 0.0060 | 0.0374 | 0.1501 | 0.1505 | 0.0400 | 0.6761 |
| 0.0374 | 0.1501 | 0.1505 | 0.0400 | 0.0001 | 0.1458 | 0.1463 | 0.0060 | 0.6761 |
| 0.1458 | 0.0001 | 0.0060 | 0.1463 | 0.1501 | 0.0374 | 0.0400 | 0.1505 | 0.6761 |
| 0.1501 | 0.0373 | 0.0401 | 0.1505 | 0.1458 | 0.0000 | 0.0060 | 0.1463 | 0.6761 |
| 0.1462 | 0.0065 | 0.0009 | 0.1458 | 0.1505 | 0.0398 | 0.0371 | 0.1501 | 0.6769 |
| 0.1505 | 0.0398 | 0.0371 | 0.1501 | 0.1462 | 0.0065 | 0.0008 | 0.1458 | 0.6768 |
| 0.6763 | 0.6763 | 0.6766 | 0.6766 | 0.6764 | 0.6763 | 0.6766 | 0.6766 | 0.6763 |
We observe that the values of Shannon entropy varies from 0.6761 to 0.6769. Thus conclude that in the second case the variation is much less than in the first case. Moreover in the second case the values are very much close to each other. Since we know that magic square with probability distributions is bimagic square. In this case we have $S_{2^4 \times 4} := S(P) = 0.2273$

5.3 Shannon Entropy of Order $16 \times 16$

Here we shall give directly the Shannon entropy table based on the first representation of bimagic square of order $16 \times 16$ given in section 4.

| $P$ | $S$ |
|-----|-----|
|     |     |

In this case, the sum of the lines or columns are very much near to each other. Thus we observe that in case of bimagic squares of order $8 \times 8$ and $16 \times 16$, sums representing Shannon entropy are very much close to each other in each case. In both the cases the Shannon entropy is same upto three digit decimal. This gives that the bimagic squares give better results. The above magic square of order $16 \times 16$ of probability distributions is bimagic square. In this case we have $S_{2^4 \times 4} := S(P) = 0.11364$ This value is much more less than the value of Shannon’s entropy, i.e., approximately, $H(P) = 0.9775$.

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