Statistical Mechanics and Quantum Cosmology *

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(UMDPP#91-096, Nov. 1990)

Abstract

Statistical mechanical concepts and processes such as decoherence, correlation, and dissipation can prove to be of basic importance to understanding some fundamental issues of quantum cosmology and theoretical physics such as the choice of initial states, quantum to classical transition and the emergence of time. Here we summarize our effort in 1) constructing a unified theoretical framework using techniques in interacting quantum field theory such as influence functional and coarse-grained effective action to discuss the interplay of noise, fluctuation, dissipation and decoherence; and 2) illustrating how these concepts when applied to quantum cosmology can alter the conventional views on some basic issues. Two questions we address are 1) the validity of minisuperspace truncation, which is usually assumed without proof in most discussions, and 2) the relevance of specific initial conditions, which is the prevailing view of the past decade. We also mention how some current ideas in chaotic dynamics, dissipative collective dynamics and complexity can alter our view of the quantum nature of the universe.

*Invited talk given at the Second International Workshop on Thermal Field Theories and Their Applications, Tuskuba, Japan, July 1990. Published in Thermal Field Theories, edited by H. Ezawa, T. Aritmitsu and Y. Hashimoto (North-Holland, Amsterdam, 1991) p. 233-252.
†Work supported in part by the National Science Foundation under Grant No. PHY87-17155.
1 Introduction

In this talk I would like to present some general personal views on how the concepts and methodologies in statistical mechanics can be of use to facilitate a better understanding and a clearer formulation of some basic issues of quantum cosmology.

In keeping with the interdisciplinary nature of this workshop I will only discuss ideas here and avoid technicalities, knowing that one can always find the details in the original papers. Because of the general nature of discussions the viewpoints presented here could also appear to be tentative, sketchy and partial. I will, however, try to mention the important papers, as well as recent reviews and conference proceedings in this field for those who would like to get a better overall perspective. In this connection I may suggest Proceedings of the 1988 Osgood Hill Conference\(^1\), the 1989 Jerusalem Winter School\(^2\), and selected papers in the 1989 Santa Fe Institute Studies\(^3\). Quantum Cosmology began in the sixties with the work of DeWitt\(^4\), Wheeler\(^5\) and Misner\(^6\). The revival of interest in the eighties was brought about primarily by the works of Hartle, Hawking\(^7\) and Vilenkin\(^8\). A very complete bibliography of recent papers on quantum cosmology was compiled by Halliwell\(^9\), who himself contributed extensively to its recent development.

Information theory and quantum measurement ideas useful in quantum cosmology were introduced by Wheeler, Zurek and Unruh\(^10\), Joos, Zeh and Kiefer\(^11\), Griffith and Omnes\(^12\), Gell-Mann and Hartle\(^13\). Since this particular aspect of statistical mechanics in relevance to quantum cosmology is quite well noticed (see Ref. 3), I will not pursue it here. Neither will I belabor on the Hamiltonian\(^4\)–\(^6\) nor the path integral\(^7\)\(^,\)\(^8\) formulations, which make up most of the formal work in this field. Because I have difficulty understanding the physical meaning of results derived from Euclidean techniques I prefer to start from a firmer ground (quantum field theory in curved spacetime, or quantum mechanics) and analyze smaller problems (Brownian motion and the ubiquitous harmonic oscillator!) which I can understand and trust the results. In this sense I’ll actually not be doing quantum cosmology (as we shall see these two words given their present connotation can be intrinsically contradictory), but discuss quantum statistical processes like dissipation, fluctuation, correlation and decoherence in a much simpler context. These problems, which are basic to many physical phenomena and are believed to play an important role in quantum cosmology, have simply not been understood well enough in the more complicated conditions characteristic of quantum cosmology to warrant liberal generalization and excessive claims. In essence, then, I will take the layman’s approach, asking some intuitive and rudimentary questions, and trying to see if the basic tenets of quantum cosmology are sound and are compatible with what we understood in statistical and quantum physics. One of these sample problems, specifically on the stochastic properties of interacting quantum fields, has recently been treated in detail from first principles by Juan Pablo Paz, Yuhong Zhang and myself\(^14\)–\(^17\). It was reported in Dr. Paz’s talk. We are now in the process of trying to understand its implications in the context of stochastic inflationary universe\(^18\)–\(^22\) semiclassical gravity\(^23\),\(^24\) and quantum cosmology. The study of dissipation in quantum fields, semiclassical gravity and in quantum cosmology has been pursued by Esteban Calzetta and myself over the years\(^25\)–\(^33\) and furthered by Paz\(^34\),\(^35\).
recently. An earlier account of some views on the basic issues of minisuperspace cosmology can be found in my 1989 Erice lectures\textsuperscript{31}.

2 Viewpoint and Issues in Quantum Cosmology

Classical cosmology as a discipline, from Newtonian to relativistic, is based on the continuum notion of spacetime described by Riemannian geometry. There is no valid picture of quantum geometry as yet. When gravity is conceived as the force mediated by a spin-two particle, the graviton, one can talk about quantum gravity, even though grave problems exist in relation to its renormalizability; and for its higher derivative extensions, the $R^2$ theories, also unitarity and causality problems. Einstein’s relativity theory based on the Hilbert action $\int R \sqrt{-g} d^4x$ from which cosmology is derived, is in this view believed to be the long wavelength or low energy limit of a more complete and intricate theory, superstring theory being one serious candidate. Above the Planck scale quantum properties of spacetime become important. Here geometries with non-trivial topology can spontaneously be created and annihilated, rendering the smooth continuum picture of spacetime totally inadequate. At this time there are many attempts at constructing a picture of quantum spacetime but no success is yet in sight. The goal is however shared by workers on many fronts, including superstring theory\textsuperscript{36}, conformal field theory\textsuperscript{37}, topological field theory\textsuperscript{38}, wormholes and baby universes\textsuperscript{2}, and random geometry\textsuperscript{39}.

Therefore, according to this view, any talk on, say, superstring cosmology, which starts with a Robertson-Walker or de Sitter metric does not make much sense, because the spacetime constructed from and governing the superstrings cannot be the simple manifolds. One can at best be looking at the low energy limit (about, but not above the Planck scale) of such a theory, which can equally well be treated by the well-known semi-classical theories. If one views gravity as an effective theory resulting from elementary particle interactions, as Sakharov\textsuperscript{40} had proposed, the notion of continuum spacetime only makes sense in the long-range limit, much as elasticity is to electrodynamics. Then quantum cosmology is as meaningful as quantum elasticity. It is almost just as crazy (though doable in principle) to try to derive elasticity from QED, as is it hopeless (though we all seem to be trying) to deduce QED from elasticity. The incongruence ingrained in the words ”quantum cosmology” is a reflection of the deeper dilemma one faces today in probing the quantum structure of spacetime.

So just what exactly does one mean when one talks about quantum cosmology? and what does one gain in asking such almost impossible questions? Different people have different ideas about quantum cosmology. If one believes that the metric is a basic observable depicting a fundamental physical field quite at the opposite spectrum from the effective theory, then one would quantize the three-geometry $^3g_{ij}$ by imposing commutation relations between $^3g_{ij}$ and $\pi_{ij}$, its conjugate momentum, like what one usually does with any classical physical observable. The difficulties one encounters are that of quantum gravity proper. Quantum cosmology then refers to the quantization of a restricted set of the degrees of freedom allowed in quantum gravity. The geometries commonly encountered in (classical)
cosmology like Robertson-Walker, de Sitter, and mixmaster universes are only the homogeneous anisotropic solutions to Einstein’s equations. In a perturbative sense (e.g. the Lifshitz operator) one can view these geometries as the lowest modes of spacetime excitations\textsuperscript{31}, the higher modes corresponding to the inhomogeneous universes are being ignored. In the superspace picture\textsuperscript{4,5} (the space of all 3-geometries) homogeneous cosmology constitutes a highly truncated class, the so-called minisuperspace \textsuperscript{6}. The problem is now simplified from quantizing an infinite to only a few degrees of freedom. Most discussions of quantum cosmology make such a simplification. Whether this is acceptable has been an open question since the Hamiltonian cosmology of the sixties. We shall address this issue concerning the validity of minisuperspace approximation \textsuperscript{41} from the statistical mechanical viewpoint later.

Quantum cosmology in the eighties works also mainly with minisuperspace, i.e., makes the same assumption of truncated degrees of freedom, but the attention was shifted to the issue of initial states or boundary conditions on the Euclidean path integrals. This opened up new avenues because a simple prescription\textsuperscript{7,8} on what 4-geometries to sum over in the Euclidean path integral leads to physically interesting results. For example, Hartle and Hawking’s\textsuperscript{7} no boundary condition of summing over all compact 4-geometries with boundary on \( ^3g \) and Vilenkin’s\textsuperscript{8} choice of outgoing modes on regions of the boundary where the 4-geometry is singular both have the following desirable features:

1. There are regions where the wave function of the universe \( \Psi \) is oscillatory, corresponding to classically allowed solutions.
2. There exist solutions with inflationary behavior (see, however, Ref. 2).
3. These boundary conditions on \( \Psi \) select a particular solution to the functional Schrödinger equation and define a particular vacuum state in the semiclassical limit of quantum field theory in curved spacetime.

Simply put, these boundary conditions admit classical solutions, some induce inflation, and give the correct vacuum state in the semiclassical limit. The prediction of classical spacetimes corresponding to the existence of oscillatory wave function is a very important result, because many features of our physical world\textsuperscript{18,19} are connected with the existence of a late universe (the flatness, age and entropy problems in cosmology are all related to this fact)\textsuperscript{20} described by classical spacetimes. Time is one of such features, as it is the only observable in quantum mechanics not represented by an operator, but enters as a parameter, thus lacking the interference effect intrinsic in all quantum phenomena. Because of its preferred status, time plays a special role in quantum mechanics and in general relativity, and brings about special problems in quantizing gravity. In this view, the issue of time is naturally linked to the issue of quantum to classical transition. In addressing these two issues statistical mechanical considerations enter in a fundamental way. Though occupying a central position, they are not, however, exclusive concerns of quantum cosmology. In fact the basic mechanisms which can bring about quantum to classical transitions like decoherence and correlation are common to all quantum phenomena and, in my view, should be better explored first outside of quantum cosmology without its particular problems. What makes these issues particularly relevant in quantum cosmology are 1) the appearance of classical spacetime and the associated special features of time in classical physics, and 2) the existence
of a late classical universe as "a consequence of the specific condition in a more general sum-over-history framework of quantum prediction". Thus according to this view quantum cosmology provides one with a pathway to connect these issues (of time and classicality) back to the issue of initial conditions. A parallel development also claims the usefulness of these initial conditions to predict the value of universal coupling constants in the context of summing over non-trivial topologies (wormholes and baby universes). The overwhelming emphasis in recent work on quantum cosmology is on the specificity of initial conditions. This is where our view differs: Put succinctly, we attach equal importance to these issues (time and classicality) which are amplified in the context of quantum cosmology, but we don’t see the choice of initial conditions as the most natural way to resolve these issues. Rather, we view the emergence of time and the quantum to classical transition more as consequences of dynamics and interactions of the system of interest with its environment, and attribute more importance to the working of statistical mechanical effects in the broadest senses, including the guiding principles of information-theory and chaotic dynamics. Let me explain what we mean by this.

3 How are Statistical Considerations Relevant?

In a broad sense, statistical mechanics deals with the issue of how to extract from a large, often infinite, degrees of freedom, a few variables which can capture the most essential physics of the whole system. Examples in physics are abound: Thermodynamics describes a system in equilibrium with its surrounding by a few macrovariables like temperature, pressure, entropy, etc; hydrodynamics with the transport functions, and critical phenomena with the critical exponents and universality classes. In all cases the skill rests upon 1) identifying (separation) the relevant variables describing the system of interest (often this involves not just taking a subset of the primitive microvariables but taking the average in some approximation and working in a different level of structure), 2) averaging away some information of the irrelevant variables (coarse-graining) which make up the environment, and 3) consideration of its overall effect on the system (backreaction). This schema of isolating a system of interest and the effective accounting for its interaction with the surroundings is not exclusive to statistical mechanics. In field theory renormalization and coupling hierarchy problems share the same spirit.

Such a reduction procedure applied to the overall system and environment (closed system) results in the appearance of some unusual but commonly accepted behavior for the system alone (open system) which would otherwise be completely absent. The most salient feature of the dynamics of an open system is dissipation, and the associated appearance of an (thermodynamic) arrow of time for the system observer. Formally one sees this difference arises when one goes from the density matrix description of the system-environment obeying the unitary Louiville-von Neumann equation, to the reduced density matrix for the system alone obeying a non-unitary master equation. This is the basic reason for the apparent contradiction between the unitarity of microphysics and the apparent breakdown of macrophysics. This schema can be used to describe the so-called "collapse of the wave function".
quantum to classical transition in quantum measurement theory\textsuperscript{10,11}. The disappearance of interference between different branches of the wave functions associated with the vanishing of the off-diagonal components of the reduced density matrix is called decoherence. It is a consequence of information loss in the system through its coupling to the environment. In most practical cases, the environment can be simplified as a stochastic source. In fact, dissipative effects are oftentimes depicted by phenomenological equations such as the Langevin equation. Instead of an explicit description of the environment, one replaces it with a noise source which drives the dynamics of the system. The noise source can be from thermal or vacuum fluctuations which are related to dissipation through the fluctuation-dissipation relation.

This general scheme in statistical mechanics, often modeled by the motion of a Brownian particle (system) interacting with a collection of harmonic oscillators (bath) and depicted by a master equation for the reduced density matrix\textsuperscript{42–47} show clearly 1) the interconnection of fluctuation, noise, dissipation, decoherence and correlation; and as a consequence of these processes, 2) the nonspecificity of initial conditions in determining the long time behavior of the system.

We have applied this scheme to interpret quantum dissipative processes in semiclassical gravity\textsuperscript{29}. Let us now examine the issues of quantum cosmology in this light. The issues I raised earlier can be grouped into three distinct levels:

(1) Is gravity an effective theory? If yes, how do statistical concepts enter?
(2) Assuming that it is not, i.e., that the 3-geometries can be viewed as fundamental physical variables, then within this (conventional) framework of quantum cosmology,

a) How valid is the minisuperspace approximation in describing the full dynamics of quantum cosmology?

b) How specific are the initial conditions in yielding classical limits?

(3) What brings about decoherence and classicality?

Note that questions in level (3) are actually questions in quantum mechanics proper, without a proper understanding of which one cannot get a satisfactory answer to the corresponding questions in quantum cosmology. A thorough understanding of (2) alone may not help answering questions in level (1), but there is reason to believe that answers to questions in level (1) could help to resolve most questions on level (2). Cracking the mystery of (1) is, however, very difficult.

I will give here a general description of how statistical considerations enter into quantum cosmology and leave the more technical discussions on methodology and sample calculations to the next section.

As we mentioned early, cosmology is a study of spacetime dynamics and structure based on the observed facts in our universe. From the observed high degree of isotropy and homogeneity one constructs the standard model\textsuperscript{18} based on the Friedmann-Robertson-Walker universe, which is a fairly good depiction of the late history of the universe back to at least $10^{-43}$ sec. from the big bang. The flatness and entropy problems prompted one into speculating that at some very early time (the grand unification time $\sim 10^{-35}$ sec) the universe might have undergone a stage of inflation\textsuperscript{20}. This ushered in renewed interest of the de
Sitter universe. Mathematical analysis of Einstein’s equation indicates that near the singularity the universe could have been highly anisotropic and inhomogeneous, thus pointing to the relevance of the Bianchi models (spatially homogeneous) and the mixmaster universe (Type IX rotation group of motion) in particular. There is no a priori reason why the inhomogeneous cosmologies are usually ignored, except that they are very difficult to study, and for the belief that inhomogeneities and anisotropies in the very early universe could be dissipated away through various mechanisms, vacuum particle production near the Planck time being the most powerful.

So cosmology is the study of the dynamics of a very restricted set of spacetime (with high symmetries) to begin with. This is similar to the first task of statistical mechanics, i.e., selecting a few relevant parameters which can capture the physical essence of the whole system. In late cosmology, these parameters are given by observation (e.g. Hubble expansion, microwave background), but in early cosmology especially in the realm where quantum phenomena are dominant, which parameters are important is not as straightforward. (It may be that spacetime is an averaged, composite concept and even the simple scale factor in the metric function loses its obvious meaning. See ideas from Regge calculus, random geometry, cellular automata, causal sets, etc). Within the conventional framework, the minisuperspace approximation in quantum cosmology assumes that the infinite degrees of freedom ignored have little effect on the selected ones (see however calculation of Halliwell and Hawking). Even so, using the schema in statistical mechanics I described above, this is highly questionable. Viewing the scale factor a and the anisotropy parameters $\beta_{\pm}$ as our system, say, in a Bianchi Type IX universe and treating the remaining infinite degrees of freedom corresponding to other anisotropic ($\theta_{1,2,3}$) and inhomogeneous universes as the environment, one would get additional terms in the (non-unitary) master equation associated with the Wheeler-DeWitt equation (an energy constraint condition). With an eye towards matching the classical limit, one can use the Wigner functional $f(g, \pi)$ for the 3-geometries $g$ and the conjugate momenta $\pi$ (which describes the distribution of states in a superphase space) and obtain a Wheeler-DeWitt Vlasov equation, or a Fokker-Planck equation. These equations embody the dissipative and diffusive effects in the dynamics of the minisuperspace variables. An immediate consequence is that the late time behavior will no longer depend sensitively on the specific choice of initial conditions. One can show that for reasonable conditions on the bath and the coupling, memory loss (near Markovian behavior) is a rather general phenomena. This viewpoint supports the chaotic cosmology philosophy, which the mixmaster and the inflationary programs both share - i.e., that the present state of the universe depends only weakly on the stipulation of initial conditions, but results largely from its own dynamics and interactions.

For cosmology, splitting the system and the bath may pose some conceptual difficulty if one envisages the Universe as containing "everything" - spacetime and matter. Though this is by definition true, it is in reality often not so stringently applied. The observable "Universe" often refers to the causally connected parts which are much smaller, and relevant physics goes on largely within the particle horizons. One can take the part beyond as the environment. For spacetimes with event horizons (e.g. the de Sitter universe) physics
within and without can be quite different (e.g., Starobinsky’s stochastic inflation). One can also view topological fluctuations such as wormholes and baby universes as making up the environment, which can have interesting effects on the universe proper. For an earlier discussion on how nontrivial global structures of spacetime and quantum processes in the universe can lead to entropy generation see Ref. 52.

Another way statistical considerations enter into cosmology is from chaos and dynamical systems. As is well-known, the Einstein equations for some classical cosmological models (e.g. Bianchi Type IX) admit chaotic behavior. The criteria of chaos in this context (e.g. Liapunov exponent) are still under investigation, but the existence of even slight chaos will render the specificity of initial conditions highly unstable in its prediction of late time behavior. Whether quantum dynamics exhibits the same degree of chaos is unknown but it is unlikely that the initial conditions could remain highly regulative and predictive, as the prevailing view in quantum cosmology seeks to establish.

A related criticism of a statistical nature of the ”initial condition” school of thought comes from complexity and information theory considerations. C. H. Woo pointed out that in addition to a simple and elegant initial condition for the wave function of the universe, the macroscopic variables require arbitrary inputs for a more detailed description of the classical history of the universe. He estimated that ”the number of bits needed to encode the algorithm for the wave function and its simple initial condition is relatively small, of the order of $10^3$. Thus, ”such a wave function can predict at most the specific behavior of a few hundred macroscopic variables”, a far cry from the sweeping optimism implied by the initial conditions school of thought.

So far we have assumed that gravity is viewed as a fundamental field. If instead gravity is regarded as a composite force or an effective theory, then statistical considerations would enter in an even more basic and familiar way. Think about how we construct molecules from atoms, then gas, fluids and solids. Statistical mechanics is an almost indispensable tool when one wants to extract meaning and structure from a collection of more elementary constituents. In addition to the effective theory of Sakharov mentioned above, I have also toyed with the idea of viewing gravity as the result of a ”time-dependent” Hartree-Fock interaction among the unspecified basic constituents, similar to the nuclear collective model, where the ”normal modes” of nuclear rotation and vibration are regarded as the dynamical variables. Note that for the description of many gross features one does not need to know the details of the nucleons, which are the more elementary constituents, but only their collective motion. This is in contradistinction to the independent particle model, where the starting point is the individual nucleon. From this maybe we can learn something about the relationship of the two approaches to quantum gravity, vis, starting with a collective structure like geometry or starting with the more basic constituents like superstrings or pre-geometry. The other idea I have toyed with somewhat is to view Einstein’s theory as the hydrodynamic (long wavelength) limit of a micro-theory of gravity. There is of course no implied ”collision” of the basic constituents in the corresponding gravitational ”kinetic theory” other than their nonlinear interaction. But the reduction of the unitarity dynamics of a microsystem to a macrosystem exhibiting irreversibility like the Boltzmann equation is
an interesting analogous scheme. The incorporation of hydrodynamic fluctuations and phase
transition ideas into the consideration of transition from quantum to classical gravity may
bring forth some new insight. In the more mainstream recent developments, how statistical
mechanics enters into random geometry, conformal field theory and superstring as ways to
relate to quantum gravity should be quite apparent. It is not my intention to delve into these
areas here but I hope at least I have convinced you that a good knowledge of non-equilibrium
statistical mechanics (field theory) is essential for understanding the basic issues of quantum
cosmology.

Let me now start over from the beginning and discuss some techniques which we find
useful to treat problems of this nature.

4 Formalisms and Sample Problems

As I mentioned in the beginning, because there exist many subtleties and conceptual pitfalls
in quantum cosmology and our current understanding of the statistical meaning of quantum
mechanics is still vague, it is perhaps more fruitful to begin the investigation on a more
familiar and firmer ground before putting them to test in the volatile setting of quantum cos-

mology. These phenomena are: fluctuation, noise, dissipation, particle creation, decoherence
and correlation; the processes are: separation (definition of open system), coarse-graining,
backreaction semiclassical limit, and quantum to classical transition. Various authors have
tried to attack individual phenomenon without due consideration of the interrelation with
others - e.g. decoherence without taking into account dissipation, semiclassical limit with-
out taking into account the nonlinearity of fields and nonseparability of the background.
And because they want to do these in the complexity of quantum cosmology and to see the
result in one strike, they have to make many simplifying assumptions which may actually
wash away the many features special to quantum cosmology (e.g. the Born-Oppenheimer
separability for geometry and matter, the WKB approximation for the wave function, the
linearity of gravitational normal modes).

The platform we choose to work on in this first stage of investigation is quantum field
theory, first in flat space and then in curved space, the latter being the semiclassical limit
of quantum gravity. We have some previous knowledge of how dynamical excitation of the
vacuum in the form of particle creation can act as a dissipative force in changing the dynamics
of spacetimes. We know how the closed-time-path formalism can yield a real and causal
equation of motion for the geometry, and provide a correct statistical interpretation of
particle creation as a dissipative process. We recognize the effective action as a suitable
object for studying backreaction effects. We also gained some experience in the properties
of Wigner function as a classical distribution function, and in dealing with near-uniform
kinetic systems. These were the ground posts we have established to explore the statistical
properties of quantum fields with an eye on the cosmological applications.

The one big missing piece in this mosaic is noise and fluctuation. We believe there
must exist some fluctuation-dissipation relation even for nonequilibrium systems (not just
for close to equilibrium linear response systems), as suggested in the particle creation and
backreaction problems \textsuperscript{29}. Even though the effect of noise, fluctuation and dissipation are rarely mentioned in quantum cosmology\textsuperscript{60}, we believe they should play as essential a role as decoherence and correlation, as they are intrinsically related, although they address different aspects of the basic issues.

Thus the first task we set for ourselves in this program is to formulate a stochastic theory of quantum fields from first principle. The criterion is that it should contain the interconnection of all the above-mentioned processes and can address all statistical properties of quantum fields in curved spacetime. The two major useful ingredients we found are the influence functional and the coarse-grained effective action, which I will now briefly describe.

These ideas are of course not new. The former was established by Feynman and Vernon\textsuperscript{45}, the latter is a recasting of the Zwanzig-Mori\textsuperscript{43} projection operator formalism in effective action forms. The influence functional captures the overall averaged effect of the environment on the system, from which one can obtain the quantum master equation for the reduced density matrix. This formalism was lately popularized by Caldeira and Leggett\textsuperscript{41} in their work on dissipative tunneling. It has also seen application to the inflationary cosmology\textsuperscript{61}. We have worked with the closed-time-path integral formalism of Schwinger and Keldysh\textsuperscript{46} for particle creation and backreaction problems\textsuperscript{25} and we know that it is suitable for treating non-equilibrium quantum fields\textsuperscript{26}, but did not realize that it is just the influence functional formalism of Feynman-Vernon (in fact Schwinger’s paper was on Brownian motion)\textsuperscript{62}. One can interpret the $x$ and $x'$ paths as one propagating forward and the other backwards in time. After we discovered this connection all of our previous results on CTP formalism can be carried over easily. The other pivotal point is the incorporation of noise and stochastic sources in purely quantum field theory terms without the presence of a thermal bath. This lifts the restriction of near-equilibrium conditions customarily imposed in the treatment (e.g. linear response theory) of noise, fluctuation and dissipation. Indeed one major result we obtained from this program is to show that a general fluctuation-dissipation relation exists for non-equilibrium quantum systems. We have used this to improve on a recent result of Unruh and Zurek, as well as making new predictions.

Our program was executed in successive stages\textsuperscript{14–16}, starting with the quantum mechanical problem of a Brownian particle bilinearly (Cxq) coupled to a collection of harmonic oscillators as bath, generalizing the result of Caldeira-Leggett and Unruh-Zurek to non-local dissipation and colored noise. (We call the dissipation local and the noise white if the dissipation and noise kernels are both delta functions.) We obtain (for the first time, we believe) an exact quantum master equation for these more general cases. Contrary to what we originally anticipated, this equation describing a non-Markoffian process turns out to be not so complicated as an integro-differential equation, but only an ordinary differential equation with complicated time dependent coefficients, and is exact (see Eq. 7 of Ref. 63). When we apply the Wigner transform to this quantum master equation, we obtain for the Wigner distribution function the quantum Fokker-Planck equation with time-dependent diffusion coefficients. The second problem we looked at was a biquadratic ($\lambda x^2q^2$) coupling between the system and bath variables, still in the quantum mechanical context. Here we carried out a perturbation analysis (in preparation for the $\lambda\phi^4$ theory) up to quadratic order in $\lambda$ and de-
rived a nonstationary quantum master equation with nonlinearly generated dissipation and colored noise. The third problem was to do everything in field theory, first in flat space then in de Sitter space making use of its conformally flat property, thus deriving for the Brownian field with nonlinear nonlocal dissipation and colored noise a quantum functional master equation and the associated Fokker-Planck equation for the Wigner distribution functional.

This completes the first stage of our program. We have used this equation to study the loss of coherence in some interesting quantum mechanical problems and cosmological models. We are now in the process of applying this formalism to problems in semiclassical gravity (connecting dissipation due to particle creation to noise and fluctuation), stochastic inflation (noise generation) and quantum cosmology (may need a different framework). In this path-integral framework one recovers the nice results of Unruh and Zurek on decoherence, and can also see its relation with dissipation and other processes. One can also relate the degree of decoherence with correlation (between the coordinate and momenta variables) as one set of criteria for classicality. We have indicated before that particle creation in quantum fields can bring about both dissipation and decoherence. Calzetta and Mazitteli have recently shown in the context of quantum field theory in curved spacetime that particle creation is a necessary and sufficient condition for decoherence. For stochastic inflation we have some doubts on the validity of Starobinsky’s scheme in generating white noise for a linear field via dynamic truncation at the event horizon. Instead, with nonlinear mode coupling, one can generate noise without assuming a dynamic truncation. We applied the coarse-grained effective action method to calculate the effect of bath (high frequency modes) on the system (low-frequency modes) which are nonlinearly coupled. Indeed, colored noise is generated with nonlocal dissipation in this problem, which is what our general program predicts.

The problem of backreaction and of semiclassical approximation was brought up recently in the context of quantum cosmology. We feel that without incorporating dynamical fluctuations both in the fields and the geometry, and dealing with the nonlinearity and nonadiabaticity condition squarely one cannot bring forth too much new beyond what we already know from quantum field theory in curved spacetime. Quantum gravity is an intrinsically nonlinear theory. Backreaction is only an approximate concept - it is meaningful only if one can separate some background geometry apart from the remaining (field or gravitational) degrees of freedom which is only possible in linearized theory and at energies lower than the Planck energy. For quantum gravity at the Planck energy where full nonlinearity is at work, this separation is not easily attainable. There is of course still particle creation - not only of quantum fields, but also gravitons - but the way to treat these quantum processes is not by the background field method to which quantum field theory in curved spacetime belongs, but rather by dealing directly with the nonlinear interaction of fluctuations of both gravity and fields and their dynamical excitations (which give rise to particle creation). Our formulation can shed some light on the nonlinearity aspects if one models the gravity-field interaction in the form of field-field interaction, although factorizability assumption for the density matrix of a closed system in the influence functional method is still a limitation (see however, Grabert et al in Ref. 47).

We have not yet embarked on a full scale investigation of the basic issues in quantum
cosmology using the framework we constructed, as the issue of time still poses a nontrivial problem for the path-integral formalism (see Ref. 13 and Kiefer in Ref. II). However we have earlier done a few smaller pilot problems to show 1) how dissipation in quantum cosmology can efface the memory of initial conditions and 2) how the higher gravitational modes usually discarded can introduce an effective dissipative term in the equation of motion for the lower (minisuperspace) modes. The first problem was illustrated by Calzetta with the example of a linear scalar field in a Robertson-Walker universe. The reduced density matrix obeys an equation of motion with a viscosity term containing a nonlocal kernel, signifying the existence of non-Markovian dissipation processes. In the second problem Sinha used a $\phi^4$ scalar field in a Robertson-Walker universe to mimic the nonlinear gravitational interaction. She viewed the scale factor $a$ and the lowest (conformal) scalar field mode $\chi_0$ as the background and studied the coupling of the higher scalar field modes $\chi_n$ with themselves and their backreaction on the background modes via the coarse-grained effective action method. Backreaction shows up as a dissipative term in the equation of motion for the lowest modes, in addition to the usual renormalized mass and a shifted natural frequency. These results exemplify our earlier claim that statistical effects can alter ones view on the importance of initial conditions and the validity of the minisuperspace truncation. We still need to understand better how the dissipative effect manifests itself in the dynamics described by different "times" chosen in quantum cosmology. We would also like to see how the "thermodynamic" arrow of time emerges from the dissipation and decoherence processes in a quantum to classical transition. These are part of our future work.

5 Summary

I have arranged the issues, processes and methodologies discussed in this talk into a Table below. Also noted therein are some related problems in gravity and cosmology where statistical mechanical considerations can be fruitful. To conclude we think more attention need be paid to the following aspects:
1) Relevance of statistical considerations in quantum cosmology.
2) Their role in addressing basic issues in theoretical physics.
3) Interconnectedness of statistical processes such as
   a) decoherence, dissipation and correlation
   b) noise, fluctuation and
   c) particle creation, backreaction and semiclassical approximation.
For example, quantum to classical transition involves all processes in a) and requires considerations of c), but b) also affects a) and engenders c).

Inquires on the statistical properties of the vacuum (noise, fluctuation, excitation by dynamics, constraint by event horizon) in curved spacetime and quantum gravity can also shed light on the interconnection of basic issues in quantum mechanics, general relativity and statistic mechanics (as manifested in the Hawking effect).
Issues:

1) How large is the class of initial conditions which can admit classical spacetimes as solutions? How regulative and predictive are the specific initial conditions?
2) How valid is the minisuperspace approximation?
3) How does time emerge? Is classical spacetime a necessary condition for rendering time as we perceive it?
4) Quantum to classical transition - criteria for classicality.
5) Semiclassical limit - relation of quantum field theory in curved spacetime with quantum cosmology.
6) Separability of background and field, validity of the adiabatic condition, backreaction and consistency.

Processes

1) Coarse-graining: how sensitive are the final results to the averaging measure.
2) Decoherence, correlation and dissipation
3) Noise, fluctuation and dissipation
4) Particle creation

Frameworks

1) Nonunitary evolution equations: master equation, Fokker-Planck equation and Langevin equation.
2) Closed-time-path integral formalism, influence-functional formalism
3) Superscattering (§) matrix formalism\textsuperscript{69}.

Techniques

1) Subdynamics and projection operator formalism; coarse-grained effective action (for coarse-graining).
2) Wigner distribution function, coherent state representation (for classical limits).
3) BBGKY hierarchy, nth order correlation function (for correlation).

Other Related Problems

1) Gravitational entropy\textsuperscript{70} and Hawking effect: a stochastic field theoretical interpretation.
2) Tunneling, decoherence and dissipation.
3) Dynamical critical phenomena and noise-induced transition.

Acknowledgement

The work I described in this talk was done jointly in stages with Esteban Calzetta, Juan Pablo Paz, Sukanya Sinha, and Yuhong Zhang with whom I have enjoyed many interesting discussions and correspondences. (They should however not be held responsible for any outlandish comment or crazy idea I advanced here.) I would like to thank the organizers of this workshop, Professor Arimatsu in particular, for the effort they put in, and the warm hospitality they extended to us. This work is supported in part by the National Science Foundation under grant No. PHY-8717155
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