On RG-flow and the Cosmological Constant.

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Abstract. From the AdS/CFT correspondence we learn that the effective action of a strongly coupled large $N$ gauge theory satisfies the Hamilton-Jacobi equation of 5d gravity. Using an analogy with the relativistic point particle, I construct a low energy effective action that includes the Einstein action and obeys a Callan-Symanzik-type RG-flow equation. The flow equation implies that under quite general conditions the Einstein equation admits a flat space-time solution, but other solutions with non-zero cosmological constant are also allowed. I discuss the geometric interpretation of this result in the context of warped compactifications. This contribution is an expansion of the talk presented at the conference and is based on work reported in [1] and [2].

1. Introduction

The problem of the cosmological constant involves high energy as well as low energy physics. It is not just sufficient to have a zero cosmological constant at energies near the Planck scale, one also needs to explain the absence of vacuum contributions at much lower energy scales. This low energy aspect of the cosmological constant problem is the most puzzling, and seems to require a fundamental new insight in the basic principles of effective field theory, the renormalization group and gravity. String theory/M-theory, in whatever form it will eventually be formulated, has to provide a solution to this problem if it wants to earn a status as a true fundamental theory of our Universe. This resolution may depend on specific details of the underlying fundamental theory, but, whatever the mechanism is at high energies, it must manifest itself in some way in low energy terms as well.

So let us ask ourselves: what new insights did we get from string theory in recent years that could shed new light on this question? Clearly the most striking progress that has been made is the growing evidence of an intimate connection between quantum phenomena in gauge theory and classical aspects of gravity. Famous recent examples are D-branes, black hole entropy counting, matrix theory, and the AdS/CFT correspondence. At a deeper level these fundamental breakthroughs in string theory — and even less recent ones such as the Green-Schwarz anomaly cancellation — all seem to go back to the old observation that open string loops are dual to closed string propagators. It is a logical possibility that a quantum gauge theory/ classical gravity connection of this kind also exists in our low energy world, and plays a role in a cancellation mechanism for the cosmological constant.
2. The Holographic Correspondence and the Hamilton-Jacobi equation.

The clearest statement about the duality between gauge theory and gravity is made in the framework of the AdS/CFT correspondence \[3\]. The statement is: in the strong coupling and large \(N\) limit certain 4-d gauge theories have a dual descriptions in terms of 5-d supergravity theory defined on a space with a 4-d boundary. One of the main ingredients in this correspondence is the identification of the classical supergravity action, with specified boundary values for the metric \(g_{\mu\nu}(x)\) and the scalars \(\phi^i(x)\), with the generating function \(S_0\) of gauge theory correlators of gauge invariant observables \(O_i\).

\[
\langle O_{i_1}(x_1) \ldots O_{i_n}(x_n) \rangle = \frac{1}{\sqrt{g}} \frac{\delta}{\delta \phi^{i_1}(x_1)} \ldots \frac{1}{\sqrt{g}} \frac{\delta}{\delta \phi^{i_n}(x_n)} S_0[\phi, g].
\]  

It is a standard fact, well known in classical mechanics, that the classical action as a function of the boundary values satisfies the Hamilton-Jacobi equation. In a theory with gravity this is a bit more subtle, since in that case the equations of motion imply that the Hamiltonian vanishes. So instead of a H-J equation there is a H-J constraint. Generically, the H-J equation is obtained by replacing the canonical momenta by the derivatives of the action with respect to the conjugate variables. To write the H-J constraint we introduce the bracket notation

\[
\{S_0, S_0\} \equiv \frac{1}{\sqrt{g}} \left( \frac{1}{3} \left( g^{\mu\nu} \frac{\delta S_0}{\delta g_{\mu\nu}} \right)^2 - \frac{\delta S_0}{\delta g^{\mu\nu}} \frac{\delta S_0}{\delta g_{\mu\nu}} - \frac{1}{2} \frac{\delta S_0}{\delta \phi^i} \frac{\delta S_0}{\delta \phi^i} \right) \tag{2}
\]

The variations are all with respect to the fields at the same point \(x\). Hence, the bracket — which can also be defined for two different actions \(S_1\) and \(S_2\) — is a local density on space-time. In this notation the H-J constraint reads

\[
\{S_0, S_0\} = \sqrt{g} \left[ V(\phi) + R + \frac{1}{2} (\nabla \phi)^2 \right] \tag{3}
\]

The right hand side represents the local lagrangian density of the 5-d gravity theory with potential \(V\) truncated to 4-d fields. The H-J equation \[3\] determines the behavior of \(S_0\) under rescalings of the metric, and thus may be interpreted as an RG-flow equation. Indeed, it reduces to the standard Callan-Symanzik equation in a certain limit \[3\], and it is in full accordance with earlier ideas on the holographic RG-flow \[3\]. The expression \[3\] has obvious similarities with the 4d Einstein action. It has, however, a different interpretation. The potential \(V(\phi)\) represents the vacuum energy contribution due to an infinitesimal RG-step. The other terms are interpreted in a similar way as modifications of Newtons constant and the scalar coupling constants.

The H-J equation is derived from the classical 5-d Einstein equations, and hence it is limited to certain special 4-d gauge theories for which this approximation is reliable. Now let us for the purpose of this note consider a Universe described by such a gauge theory, but then coupled to gravity. Is there something that the H-J constraint could teach us about the cosmological constant in that Universe? The action \(S_0\) is the effective action induced by integrating out the gauge theory, and has to be added to the usual Einstein action possibly with cosmological term. The problem of the cosmological constant
arises because there is no immediate reason why the cosmological term contained in the Einstein action would cancel the vacuum energy of the gauge theory. The two actions usually don’t know much about each other. In this note we want to argue that there actually is a natural relation between the Einstein action and the effective action of the gauge theory. To motivate the proposed relation I will use an analogy with the classical relativistic point particle. So let us, as an inter-mezzo, briefly discuss some facts about the H-J equation for point particles.

3. Inter-mezzo: Hamilton-Jacobi theory of the point particle.

The non-relativistic Hamilton-Jacobi equation may be regarded as the WKB approximation to the Schrödinger equation, in which one writes the wave-function as $\psi = e^{\frac{\pi}{h^2}S}$ and keeps only the leading order in $\hbar$. For a relativistic point particle one can do the same for the Klein-Gordon equation. Hence, the H-J equation for a free relativistic point particle takes the form

$$\frac{1}{c^2}(\partial_t S_0)^2 - (\partial_x S_0)^2 = m^2c^2$$

Once the classical action $S_0(t, x)$ is known as a function of $x$ at a given time $t$, one can use the H-J equation to determine it for later or earlier times. The solution is not unique, but depends on a choice of boundary conditions. For example, the classical action for a trajectory from $(x, t)$ to $(y, T)$ is

$$S_0(t, x) = -mc\sqrt{c^2(T - t)^2 - (y - x)^2}.$$ 

One easily verifies that it satisfies the H-J equation. It contains a rest-energy part that diverges in the non-relativistic limit $c \to \infty$; we have $S_0 \sim -mc^2(T - t) + m(y - x)^2/2(T - t)$. We will find that this rest-energy part plays an analogous role as the vacuum energy of the 4d gauge theory.

The coordinates $x$ and $t$ represent the starting point of a trajectory, and should not be regarded as variables to which the action has to be extremized: indeed, putting the variations of the action $S_0$ to zero would lead to nonsensical equations of motion. To obtain an action with a sensible equation of motion, we should add the action of another trajectory which has $(x, t)$ as its end-point. Let us choose as the additional term the classical action $S_E$ of a trajectory with a given velocity $v$ and energy $E$

$$S_E(t, x) = -E\left(t - \frac{v \cdot x}{c^2}\right).$$

To obtain consistent equations of motion, we should demand that this action $S_E$ satisfies the same H-J equation. One easily checks that this gives the usual relativistic formula for the energy $E$.

Let us now consider the equations of motion that follow from total action

$$S(t, x) = S_E(t, x) + S_0(t, x)$$
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The equations for $x$ are solved by the classical trajectory $x(t) = y + v(t - T)$. Somewhat unexpectedly, the equation for $t$ is automatically satisfied once we solve the equations for $x$. This can be understood from the fact that the total action $S$ satisfies a flow equation. When we insert $S_0 = S - S_E$ again in the H-J equation, and use the explicit form of $S_E$ we obtain

$$\gamma (\partial_t + v \cdot \partial_x) S = - \frac{1}{2m} (\partial_x S)^2 + \frac{1}{2mc^2} (\partial_t S)^2$$

(8)

This flow equation ensures that when $(x, t)$ is a point on a classical trajectory, that $(x + \epsilon, t + \epsilon)$ also is a point on that same classical trajectory.

Finally, from the explicit expressions for $S_0$ and $S_E$ it follows that when we take the limit $c \to \infty$ while keeping $v$ fixed, the term $mc^2 t$ in $S_0$ is canceled by the leading rest-energy part of $E$. This cancellation is automatically guaranteed by the fact that both actions satisfy the same H-J equation. It is this kind of cancellation that we also are

4. The Einstein action and the RG flow equations.

I will now repeat the same steps for the H-J equation of 5d-gravity. First let us identify the corresponding features of equation (4) and the more elaborate H-J equation (3) of the 5d gravity theory. The analogue of the time $t$ is obviously the conformal mode of the metric, while the coordinates $x$ represent the couplings $\phi$ and the off-diagonal modes of the metric. The role of the speed of light $c$ is played by the (square root of) the potential $V$. This potential represents the 5-d cosmological constant, and is also related to the 4d vacuum energy contained in $S_0$.

The magnitude of the vacuum energy term in $S_0$, which is the analogue of the energy term for the point particle, is restricted by the H-J equation. It is expected to be of the order $m_s^4 \sqrt{V}$, where $m_s$ denotes the cut-off scale. The (square of) $m_s$, which we identify with the string scale, plays the role of the mass $m$ in the point-particle analogy. So removing the cut-off is analogous to taking the infinite mass limit.

The large vacuum energy contained in the action $S$, is not yet a point of concern, since the action $S_0$ is not an action that has to be extremized, in particular not with respect to the conformal mode of the metric. To obtain sensible field equations we do like in the point particle case, and add an action $S_E$ of the form

$$S_E = \int \sqrt{\gamma} \left( U(\phi) + \frac{1}{\kappa^2(\phi)} \left[ R + (\nabla \phi)^2 \right] + \ldots \right)$$

(9)

We denote this action by $S_E$, since we will indeed identify it with the Einstein action. The geometric interpretation of $S_E$ in the AdS/CFT context will be explained below.

† The cut-off scale does not appear explicitly in our formulas, because it has been absorbed in the metric through a substitution $g_{\mu\nu} \to m_s^{-2} g_{\mu\nu}$.
For the moment we just follow what we did in the point-particle case, and require that \( S_E \) satisfies the same H-J equation as \( S_0 \). Thus we have

\[
\{ S_E, S_E \} = \sqrt{g} \left[ V(\phi) + R + \frac{1}{2} (\nabla \phi)^2 \right].
\] (10)

where we again used the bracket notation (2). This equation produces certain relations between the 4-d potential \( U \), Newton’s constant \( \kappa \), and the 5-d potential \( V \). Following the same argumentation as before we construct the total action

\[
S = S_0 + S_E
\] (11)

The fact that \( S_0 \) and \( S_E \) both satisfy the same H-J equation is now expected to lead to cancellations between the vacuum energy terms, just like the rest-energies canceled for the point-particle. We will return to this point below.

The action \( S \) also satisfies a flow equation. By inserting \( S_0 = S - S_E \) in the H-J equation we get

\[
2 \{ S_E, S \} = \{ S, S \}
\] (12)

where we used the bracket notation for the case of two different actions. Just like in the point particle case we want to identify the left-hand-side with the flow of the action \( S \).

To make the correspondence with an RG-flow equation more explicit we introduce the quantities

\[
\frac{1}{\sqrt{g}} \frac{\delta S_E}{\delta g_{\mu\nu}} = \gamma (3g_{\mu\nu} - \frac{1}{2} \beta_{\mu\nu})
\] (13)

with \( \beta_{\mu\nu} \) traceless and

\[
\frac{1}{\sqrt{g}} \frac{\delta S_E}{\delta \phi_i} = \gamma \beta_i
\] (14)

The quantities \( \gamma, \beta_{\mu\nu} \) and \( \beta_i \) represent the flow velocities of the metric and the scalar fields under the RG-transformations. Note that the off-diagonal mode of the metric also has been given a beta-function \( \beta_{\mu\nu} \). This means that the geometry in general may depend on the RG-scale in a way that differs from just a rescaling of the metric. In terms of these quantities the flow equation becomes

\[
\gamma \left( 2 g^{\mu\nu} \frac{\delta}{\delta g_{\mu\nu}} - \beta_{\mu\nu} \frac{\delta}{\delta g_{\mu\nu}} - \beta^i \frac{\delta}{\delta \phi^i} \right) S = \{ S, S \}
\] (15)

here the right hand side represent a kind of anomaly term, which however vanishes on shell. This equation is analogous to the point particle flow equation and guarantees that the solutions to the field equations describe RG-trajectories rather then particular field configurations. Namely, if \((g_{\mu\nu}, \phi^i)\) solves the field equations then \((g_{\mu\nu} + \epsilon g_{\mu\nu}, \phi^i + \epsilon \delta \phi^i)\) also represents a solutions with

\[
\frac{\delta}{\delta \phi^i} = \epsilon \beta^i(\phi)
\] (16)

where \( \epsilon \) is an arbitrary infinitesimal space-time dependent function. Note that in this framework it is natural to consider space-time dependent RG-flow equations, which reflects the fact that the formalism descents from 5d gravity.
The bracket of $S_E$ with itself can be expressed entirely in the $\gamma$-factor and the beta-functions $\beta_i$ and $\beta_{\mu\nu}$. We have
\[
\{S_E, S_E\} = 12\gamma^2 \left(1 - \frac{1}{48} \beta_{\mu\nu} \beta_{\mu\nu} - \frac{1}{24} \beta_i^2\right)
\] (17)
Thus the H-J constraint (10) gives a condition on the quantities on the right hand side. The similarity with the point-particle case should be obvious. In fact, the form of this relation suggests that there exist a Lorentz type of symmetry that acts on the metric, the scalar fields and on the beta-functions. The role of this symmetry is not yet clear to us.

5. The Einstein equations and the cosmological constant.

The field equations that follow from the total effective action are the Einstein equation and the scalar field equation
\[
\frac{1}{\kappa^2} \left( R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) + \frac{1}{2} U(\phi) g_{\mu\nu} = T^\phi_{\mu\nu} + \langle T_{\mu\nu} \rangle
\] (18)
\[
\nabla \left( \frac{2}{\kappa^2} \nabla \phi_i \right) + 2 \frac{\partial_i \kappa}{\kappa^3} \left[ R + (\nabla \phi)^2 \right] = \partial_i U + \langle O_i \rangle
\] (19)
where $T^\phi_{\mu\nu}$ denotes the stress tensor of the scalar fields and $\langle T_{\mu\nu} \rangle$ and $\langle O_i \rangle$ are the expectation values of the stress energy tensor and the operator $O_i$ in the gauge theory to which the scalar field $\phi^i$ couples. As we just mentioned, the flow equation implies that once we have a solution at a particular scale there is a solution along the whole RG-trajectory. Furthermore, the equations for the conformal mode of the metric are automatically implied once we have solved the other field equations.

These other equations do not depend on the cosmological constant, and naively one may think therefore, that one can simply shift the cosmological constant and still have a solution. This is not a valid procedure, however, since the solution of the other field equations determines the cosmological constant, and not visa versa – see also [1]. To explain this point in more detail, let us have a closer look at the trace of the Einstein equation
\[
\frac{1}{\kappa^2} R = 2 U(\phi) - \langle T^\mu_{\mu} \rangle
\] (20)
where we have taken the fields $\phi_i$ to be space-time independent. Here the two terms on right-hand-side together represents the cosmological constant. At first sight there appears to be no reason to suspect a relation between these two terms. However, in our approach there is additional information in the form of the H-J equation. It is important to realize that the H-J equation is not an equation of motion, but merely a condition on the variations of both the $S_0$ and $S_E$ part of the action. So we may consider this condition for any field configuration, including a preferred one, such as flat space with
constant scalar fields. In this case we find the following identities for $U$ and $\langle T_{\mu \nu} \rangle$ from the H-J equations

$$2U(\phi) = \sqrt{12V(\phi) + 6(\partial_i U)^2}$$

and

$$\langle T_{\mu \nu} \rangle = \sqrt{12V(\phi) + 6\langle O_i \rangle^2} \quad \text{for} \quad R = 0 \quad \text{and} \quad \nabla \phi^i = 0.$$  

To derive this result we did not make use of the equations of motion. But now we observe that when we do insert the equation of motion for $\phi$, which for constant fields reads

$$\partial_i U + \langle O_i \rangle = 0$$

that both terms indeed cancel! However, this is no surprise, since we already knew that once the other field equations are solved that also the trace of the Einstein equation holds. Indeed, in a similar way one could argue that there always is a solution to the field equations for any value of the cosmological constant. This is natural from the point of view of the local flow equation, because metrics with different constant curvature are related by a conformal rescaling. In other words: there is whole family of solutions, all related by the local RG-flow, among which there is in general one that looks like flat space. So even though we have not resolved the cosmological constant problem completely, we did make important progress: we have now the possibility of having a flat space solution. This leaves the question of why Nature would choose that particular solution and not some other value for the cosmological constant. The answer to this question requires more physical input, such as the boundary conditions in the UV and in the IR. I will not say much about that here, and refer to [2] for more details on this point.

6. Geometric interpretation in terms of Warped Compactifications.

The equations that we described have a natural geometric interpretation as a compactification of the holographic AdS/CFT correspondence. In the usual non-compact AdS/CFT set-up gravity is decoupled from the dual boundary theory, because the graviton modes that extend all the way to the UV boundary are not normalizable. However, as pointed out in [5], this situation changes as soon as one truncates the UV region by introducing a brane-like structure (a “Planck brane”). As a result there will exist normalizable modes of the 5-d metric that couple as 4-d gravitons to the dual gauge theory with the correct gravitational strength [5, 6].

§ This assumes that a solution exists. There are arguments that in a theory with an invariance of the type [10] not all equations can be satisfied, unless one fine tunes the parameters in the action [11]. I believe that these arguments do not apply here, since it would mean that there does not exist a solution to the 5-d Einstein equations for generic boundary conditions.
The warped compactification manifold is split up in two parts $\Sigma_0$ and $\Sigma_E$ by cutting at a radial location $r$ close to the end of the tube, or at a location $r = r_1$ inside the tube.

Instead of cutting off the UV-region with a real Planck Brane one can alternatively consider the holographic region as a part of a bigger warped compactification manifold with geometry

$$ds^2 = a^2(r)\eta_{\mu\nu}dx^\mu dx^\nu + h_{mn}(r)dr^m dr^n. \quad (24)$$

where the fifth coordinate has become one of the coordinates of a compact 6-manifold $K_6$. This kind of warped geometry naturally arises in a class of type IIB string compactifications based on type IIB with a large number of $D3$ branes. These $D3$-branes wrap the 4-d uncompactified world, and are localized as point-like objects inside the $K_6$. When a relatively large number of $D3$-branes are located near or at the same point one typically gets a warped geometry of the kind \((24)\), in which warp factor in the neighbourhood of that point essentially depends only the radial distance $r$ from the $D3$-branes. By cutting the warped compactification manifold at some finite value of $r$ one splits the manifold in two parts: one part, which we call $\Sigma_0$ describes the near horizon region of the $D3$ branes, and the other part $\Sigma_E$ the rest of the compactification manifold. In this geometric picture the two actions $S_0$ and $S_E$ can be identified with the classical supergravity action of that part of the field configuration that extends in $\Sigma_0$ or in $\Sigma_E$.

Let me now explain how the results in the previous subsection fit in this geometric picture. The 4d metric and fields $\phi_i$ represent the values of the higher dimensional metric and field $\phi_i$ at a particular slice in the geometry. The flow equation is a reflection of the arbitrariness in the choice of that slice. The generic existence of a flat space-time solution is due to the fact that it is possible to choose slices for which the 5d cosmological term is entirely by the expansion (or contraction) of the warp factor $a(r)$ and the flow velocities of the scalar fields. The effective 4d cosmological constant is

$$\Lambda = a^2V(\phi) - 12a^2 + \frac{1}{2}a^2\dot{\phi}_i^2 \quad (25)$$

where the dot denote differentiation with respect to the radial coordinate. A cancellation between these three terms seems to require fine tuning. But one easily checks that, for space-time independent fields $\phi_i$, once these terms cancel at one value of $r$, they remain
equal at other values of \( r \) by the equations of motion. Mathematically this calculation is identical – except for the change from 4 to 5 dimensions and from a time to a radial variable— to the cosmological evolution of an inflationary Universe at critical density \((k = 0)\). Indeed, it follows trivially from the Friedmann equations that a Universe with a flat spatial geometry will stay flat, even if some phase-transition occurs. One can also see this fact in the Hamilton-Jacobi formalism. The flow velocities of \( a \) and \( \phi^i \) are given by
\[
\dot{a} = \frac{1}{6} U a \quad \text{and} \quad \dot{\phi}_i = \partial_i U,
\]
while through the H-J equation the potential \( V \) is expressed as
\[
V = \frac{1}{3} U^2 - \frac{1}{2} (\partial_i U)^2.
\]
Adding the three contributions in (25) indeed gives a zero total result. This is precisely the condition that was noted to be sufficient to have stable brane-like structures with vanishing curvatures \([8]\). But again, in a similar way one can also construct solutions that have a constant non-zero curvature. The equations again look very much like the Friedmann equations for an inflationary Universe with non-zero spatial curvature \( k \neq 0 \). Except now the curvature is that of the whole Universe itself.

7. Discussion and Conclusions

The main result that we derived, the flow equation for the effective action, is based on the assumption that the 5d gravity equations are applicable and describe the radial propagation of the effective action. This requires that the gauge theory must have a large coupling constant, and large enough gauge group, and furthermore our energy scale should be small enough so that we can ignore string and quantum gravity corrections. Outside this low energy, strongly coupled, large \( N \) regime one should therefore expect to get significant modifications of the flow equations. Before discussing what we have learned about the cosmological constant, let me describe what I expect some of these corrections will look like.

First of all, \( 1/N \) corrections would change the flow equation \([16]\) by adding a second term which is expressed as a second order variation of the action \( S \). This is analogous to the transition of the H-J equation to the Schroedinger equation. Indeed, the flow equation should in that case be derived from the Wheeler-DeWitt constraint rather than the H-J constraint. Although it is hard to give a precise definition of the WDW equation, it does at least formally exhibit the same flow invariance. Secondly, at some energy scale we expect that the \( \alpha' \) corrections in the 5d gravity would also become more important. One might try to add some higher derivative terms, but probably a better way to do it is to go to a non-local form of the equation like a loop equation \([10]\) or use string field theory. Finally, when quantum gravity effects in 4d gravity become important one has to go to the full string language, and think about the gauge theory
as open strings and the gravity part as closed strings. Also in this case one expects to have a flow equation that relates the generating function of open string amplitudes to a part of the closed string action \[9\].

Finally let us return to the issue of the cosmological constant. The ideas presented in this note shed new light on the issue of the cosmological constant, but clearly more work needs to be done to provide a solution to the problem. In geometric terms I have shown that by embedding our 4d world as a slice in a higher dimensional space that there is a natural way in which the vacuum energy that is generated by phase transition is canceled by an ever decreasing (towards the IR) warp factor. Via the H-J formalism this translates to the statement that the potential \(U\) is compensated by the trace of the stress energy tensor at all scales, once this is the case at one particular scale. Both these statements only hold if one restricts to a particular conformal frame of the metric, or in terms of the warped geometry, to a particular slicing.

Thus, in the presented scenario the remaining open question is to explain why these particular slicings or conformal frames are the right ones. In a specific warped compactification it is possible that one can identify such a preferred slicing due to the presence of domain wall like structures that represent a physical transition at a certain given scale, like a Planck Brane. It is also possible that a better understanding of 5d holography – and possibly 4d holography – will reveal an underlying principle that can be used to answer this questions. I will leave this question for future research.

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