Modeling of the maximum and minimum void ratios for binary-sized granular materials

Weimin Ye i), Zhangrong Liu ii), Yujun Cui iii), Zhao Zhang i) and Qiong Wang iv)

i) Professor, Key Laboratory of Geotechnical and Underground Engineering of Ministry of Education, Tongji University, 1239, Siping road, Shanghai 200092, China.
ii) Ph.D Student, Department of Geotechnical Engineering, Tongji University, 1239, Siping road, Shanghai 200092, China.
iii) Professor, Laboratoire Navier/CERMES, Ecole des Ponts ParisTech, 6-8 avenue Blaise-Pascal, Marne la Vallée 77455, France.
iv) Professor, Department of Geotechnical Engineering, Tongji University, 1239, Siping road, Shanghai 200092, China.

ABSTRACT

Maximum and minimum void ratios are two fundamental parameters for evaluating the packing efficiency of granular materials. For binary-sized granular materials composed of two size classes (coarse and fine), many packing models has been proposed for predicting their packing density (relating to void ratio). However, analytical packing models directly based on maximum and minimum void ratios are very limited in the field of geotechnical engineering. In this study, using a concept of dominant size class, a nonlinear packing model was developed for predicting the maximum and minimum void ratios with respect to fines content. Only two parameters (filling coefficient and embedment coefficient) were required in the proposed model and they were found to be closely related to the particle size ratios between the two size classes. The applicability and accuracy of the developed model were verified by experimental results of the crushed pellets of GMZ bentonite in this work and that of several other granular materials from literature. Furthermore, the relationship between the maximum and minimum void ratios was investigated. It appeared that the maximum void ratios were linearly related to the minimum void ratios with the same slope, regardless of the particle size ratio between the two size classes.

Keywords: maximum void ratio, minimum void ratio, granular materials, particle size ratio, particle packing model

1 INTRODUCTION

Granular materials composed of soil, sand or breakstone particles in different sizes are commonly used in geotechnical engineering. For instance, in the context of deep geological high-level radioactive waste disposal, bentonite pellets are considered as one of the candidate sealing materials thanks to their high swelling capacity, high radionuclide migration retardation properties and operational advantages: easy to manufacture and install (Molinero-Guerra et al., 2018). For many granular materials, it has been well revealed that their fundamental hydro-mechanical behaviors are particle size distribution dependent (Trani et al., 2010; Minh et al., 2013; Zhang et al., 2018). This is because the particle size distribution significantly affects the void ratio (the ratio of inter-particle void volume to the particle volume) or packing density (the ratio of the particle volume to the bulk volume of the sample), which is one of the most important parameters that governs the material properties (Chang et al., 2016). For bentonite pellets, it was found that the final swelling pressure increases exponentially while the corresponding saturated hydraulic conductivity decreases exponentially as the dry density increases (Imbert and Villar., 2006; Hoffmann et al., 2007; Karnland et al., 2008; Alonso et al., 2011). Obviously, to yield better performance, the void ratio (relating to dry density) of granular material need to be optimized. In this regard, traditionally a large number of laboratory tests were carried out for determining the optimal particle size distribution (Zhang et al., 2018). However, it is often time and labor consuming with limited data can be obtained. In view of this, many researchers devoted themselves to developing a particle packing model for modelling the variation of packing density or void ratio with particle size distribution, mostly on binary-sized granular materials which consist of two size classes of particles (Chan and Kwan, 2014).

Many particle packing models including empirical models and analytical models have been developed, mostly in terms of packing density. Among them, empirical models (Fedors and Landel, 1979; Yilmaz, 2009) were built by regression analysis on experimental data or derived for particles of uniform size, without much insight into the packing mechanisms. In comparison, analytical models were developed based on packing mechanisms, which were usually illustrated from the perspective of particle interactions, including the filling, loosening and wedging effect of fine particles.
particles, as well as the occupying and wall effect of coarse particles. Among them, the filling and occupying effects are fundamental and would improve packing density, while the others would decrease packing density. Early models developed by Westman and Hugill (1930), Furnas (1931) and Toufar et al. (1977) only considered the filling and occupying effect, and therefore often overestimated the packing density. Later proposed models (Stovall et al., 1986; Yu and Standish, 1987; Yu et al., 1996) incorporated the loosening effect and wall effect by introducing two parameters named loosening effect parameter and wall effect parameter, respectively. Most of these models turned out to be linear, i.e., the packing density related linearly to the volumetric fraction of fine particles. These linear 2-parameter models were proved to be appropriate for some granular materials, but not the case for some others, where unacceptable discrepancies in the vicinity of peak value degrade the performance of such models (de Larrard, 1999; Kwan et al., 2013). In comparison, the models developed in recent years describe a non-linear relationship between packing density and volumetric fraction of fine particles by incorporating a third parameter, called compaction index in the well-known CPM model (de Larrard, 1999) and wedging effect parameter in the 3-parameter models (Kwan et al., 2013; Wong and Kwan, 2014; Kwan et al., 2015). These models showed satisfactory accuracy but would turned out to be in intricate forms when it comes to multi-sized granular materials. Other models such as 4-parameter model (Roquier, 2017), geometrical model (Prior et al., 2013) and dual-skeleton model (Chang and Deng, 2017) also showed good performance but their expressions were too intricate to be calculated.

Most of the fore-mentioned particle packing models were developed in terms of packing density, which was not commonly used in the field of geotechnical engineering, where the maximum and minimum void ratios are of great interest because they represent the loosest and densest state of the soil. Unfortunately, packing models on void ratio are seldom reported in literature. Among others, based on an assumption that the void volume change was proportional to the solid volume of the added component, Chang et al. (2015, 2016, 2017) developed a serials of models for predicting the maximum and minimum void ratios of binary-sized and multi-sized sand-silt mixtures. Though in concise forms, these models present linear relationship between void ratio and fines content, resulting in underestimation of the void ratio that near to its peak value.

In this paper, firstly efforts were made on developing a nonlinear packing model for predicting the maximum and minimum void ratios of binary-size granular material with respect to fines content. Then the applicability and accuracy of the developed model were verified by experimental results of the crushed pellets of GMZ bentonite in this work and that of several other granular materials from literatures. Finally, the relationship between the maximum and minimum void ratios was investigated.

2 DEVELOPMENT OF A NEW PACKING MODEL

2.1 Model formulation

For binary-sized granular materials composed of two size classes, the size class with coarser particles is referred to as size class 1, and the other one size class 2. The mean particle sizes of the two size classes are denoted as \( d_1 \) and \( d_2 \), respectively. The volume of particles are denoted as \( V_{s1} \) and \( V_{s2} \), respectively. And their fractions in the mixture are denoted as \( y_1 \) and \( y_2 \), respectively, with \( y_1+y_2=1 \).

Firstly, size class 1 (coarse particles) is considered as the dominant size class. Before being mixed with size class 2 (fine particles), the void ratio can be calculated as:

\[
e_1 = \frac{V_{s1}}{V_{\text{Ve}}} \quad (1)
\]

where \( V_{s1} \) is the void volume in size class 1 (Fig. 1(a)). When adding size class 2 into size class 1, in a limiting case (Fig. 1(b)), all the fine particles will be filled into the voids among the coarse particles, without any disturbance to the original structure of size class 1. In such case, the bulk volume of the mixture remains unchanged, while the void volume was reduced by \( V_{s2} \). The void ratio of the mixture can be calculated as:

\[
e = \frac{(V_{s1} - V_{s2})}{(V_{s1} + V_{s2})} \quad (2)
\]

where the number ‘1’ marked at the superscript of \( e \) indicates that size class 1 is the assumed dominant size class.

![Fig. 1. Phase diagrams: (a) size class 1 (coarse); (b) mixture of size classes 1 and 2 (limiting case); (c) mixture of size classes 1 and 2 (general case).](image)

However, such limiting case is too ideal to take place. In a general case (Fig. 1(c)), with addition of size class 2, the original structure of size class 1 and the void volume of the mixture will also be changed. Therefore, the general form of void ratio for a coarse-dominant binary-sized system can be written as:

\[
e = \frac{V_{s1} + \Delta V}{V_{s1} + V_{s2}} \quad (3)
\]
where $\Delta V_v$ stands for void volume change due to the addition of size class 2.

The value of volume change, $\Delta V_v$, however, is related to particle interactions, which depend on both the original available void volume ($V_{v1}$) and the amount of newly added size class 2 ($V_{v2}$). Chang et al. (2015, 2016, 2017) assumed the void volume change is only related to $V_{v2}$, ignoring the influence of $V_{v1}$, and resulting in a linear relationship between $e^1$ and $y_2$. Instead, this study assumed that void volume after addition of size class 2, $V_{v1} + \Delta V_v$, is related to both $V_{v1}$ and $V_{v2}$, in the form of:

$$V_{v1} + \Delta V_v = \lambda y_1 V_{v1} + \alpha y_2 V_{v2}$$  \hspace{1cm} (4)

Where, $\lambda$ and $\alpha$ are parameters governed by particle interactions. Note that when $\lambda = 1/y_1$ and $\alpha = -1/y_2$, Eq. (3) turns out to be the same form of Eq. (2), which corresponds to the limiting case. For convenience, another symbol $a$ was used instead of $\alpha$, where $a = \alpha = -1/y_2$. Thus $\lambda = 1/y_1$ and $a = 0$ is corresponding to the limiting case. With these assumptions, Eq. (3) can be rewritten as:

$$e^1 = \lambda y_1^2 + \alpha y_2^2 - y_2$$  \hspace{1cm} (5)

For further simplification, it is postulated that $\lambda = 1$ constantly in the general case (Fig.1 (c)).

Then, oppositely, size class 2 (fine particles) is considered as the dominant size. Before being mixed with size class 1 (coarse particles), the void ratio can be calculated as:

$$e_2 = V_{v2}/V_{v1}$$  \hspace{1cm} (6)

where $V_{v2}$ is the void volume in size class 2 (Fig. 2(a)). When adding size class 1 into size class 2, in a limiting case (Fig. 2(b)), all the added coarse particles are discretely embedded in the matrix inducing no void volume change of size class 2. In such case, the void volume remains unchanged, while the bulk volume of the mixture was increased by $V_{v1}$. The void ratio of the mixture can be calculated as:

$$e^2 = \frac{V_{v2}}{V_{v1} + V_{v2}}$$  \hspace{1cm} (7)

where the number ‘2’ marked at the superscript of $e$ indicates that size class 2 is the assumed dominant size class.

However, such limiting case is too ideal to occur. In a general case (Fig. 2(c)), with addition of size class 1, the void volume of the mixture will also be changed. Therefore, the general form of void ratio for a fine-dominant binary-sized system can be written as:

$$e^2 = \frac{V_{v2} + \Delta V_v}{V_{v1} + V_{v2}}$$  \hspace{1cm} (8)

where $\Delta V_v$ stands for void volume change due to the addition of size class 1.

The value of volume change, $\Delta V_v$, however, is related to particle interactions, which depend on both the original available void volume ($V_{v2}$) and the amount of newly added size class 1 ($V_{v1}$). Chang et al. (2015, 2016, 2017) assumed the void volume change is only related to $V_{v1}$, ignoring the influence of $V_{v2}$, and resulting in a linear relationship between $e^2$ and $y_1$. Instead, this study assumed that void volume after addition of size class 1, $V_{v2} + \Delta V_v$, is related to both $V_{v2}$ and $V_{v1}$, in the form of:

$$V_{v2} + \Delta V_v = \kappa y_2 V_{v2} + b V_{v1}$$  \hspace{1cm} (9)

where $\kappa$ and $b$ are parameters governed by particle interactions. Note that when $\kappa = 1/y_2$ and $b = 0$, Eq. (8) turns out to be the same form of Eq. (7), which corresponds to the limiting case. Then Eq. (8) can be rewritten as:

$$e^2 = \kappa y_2^2 + b y_1^2$$  \hspace{1cm} (10)

For simplification, it is postulated that $\kappa = 1$ constantly in the general case (Fig.2 (c)).

Thus, for a binary-size mixture with a given component proportions ($y_1$ and $y_2$), two values of void ratio can be estimated, one from Eq. (5) and the other from Eq. (10). It is believed that the greater value is more closing to the ‘real’ one, due to a looser packing structure requires less energy to reach. Thus, the estimated void ratio of the binary-size mixture can be finally determined by,

$$e = \max(e^1, e^2)$$  \hspace{1cm} (11)

Where, $e^1$ and $e^2$ are calculated from Eq. (5) and Eq. (10), respectively.

2.2 Characteristics of the new model

It is noted that both Eq. (5) and Eq. (10) show non-linear relationships between void ratio and component proportions ($y_1$ and $y_2$). When size class 1 is dominant, in the limiting case ($\lambda = 1/y_1$ and $a = 0$), the curve represented by Eq. (5) is shown in Fig. 3 as the line along ACD, which intercepts the vertical axis at $e = -1$ where $y_2 = 1$. It can be observed that the void ratio
of the mixture decreases linearly from $e_1$ to -1 with the increase of volume fraction of size class 2, $y_2$. The negative void ratio means that the total particle volumes of size class 2 is greater than the available void volume of size class 1. When size class 2 is dominant, in the limiting case ($x = 1/y_2$ and $b = 0$), the curve represented by Eq. (10) is shown in Fig. 3 as the line along BCE, which intercepts the vertical axis at $e = 0$, where $y_2 = 0$. It can be observed that the void ratio of the mixture decreases linearly from $e_2$ down to 0 with the decrease of volume fraction of size class 2, $y_2$. The zero void ratio means that the total volume of size class 2 is completely occupied by the solid particles from size class 1, remaining no any inter-particle voids. The solid lines AC and CB are determined by Eq. (11), which gives the lower limit of the estimated void ratio. The lower limit lines presented here are in accordance with those in Chang et al. (2015).

$$e^1 = e_1 y_1^2 - e_2 y_2^2 + 2e_2 y_2 - a_{12}(2e_2 + 1 - e_2 y_2) y_2 \quad (12)$$

and

$$e^2 = e_2 y_2^2 - e_1 y_1^2 + 2e_1 y_1 - b_{12}(2e_1 - e_1 y_1) y_1 \quad (13)$$

respectively. Where, $a_{12}$ and $b_{12}$ are termed as filling coefficient and embedment coefficient similar to that given by Chang et al. (2015), respectively.

### 3 PARAMETER CALIBRATION

In the newly developed model, there are two parameters including the filling coefficient, $a_{12}$, and embedment coefficient, $b_{12}$. Theoretically, the values of $a_{12}$ and $b_{12}$ are between 0 and 1. They value 0 in the special case of $d_2/d_1 = 1$ and 1 in the limiting case of $d_2/d_1 \approx 0$. While in a general case (i.e., $0 < d_2/d_1 < 1$), they are between 0 and 1. In order to reveal the relationships between $a_{12}$, $b_{12}$, and $d_2/d_1$, and further validate the proposed model, the results of packing tests on crushed pellets of GMZ bentonite were used.

The pellets used in this study were crushed from high-density (1.94~1.95 Mg/m$^3$ in dry) bentonite blocks, which were statically compacted from GMZ bentonite powder with initial water content of 10.6 %. The as crushed pellet mixtures were sieved into nine size classes with particle size ranges of 15~10 mm, 10~7 mm, 7~5 mm, 5~2 mm, 2~0.9 mm 0.9~0.5 mm, 0.5~0.25 mm, 0.25~0.075 mm and less than 0.075 mm. For convenience, hereafter they were denoted in turn by P15, P10, P7, P5, P2, P0.9, P0.5, P0.25 and P0.075, respectively. Their mean particle sizes, each simply taken as the arithmetic mean value of the upper and lower sieve sizes, were 12.5 mm, 8.5 mm, 6 mm, 3.5 mm, 1.45 mm, 0.7 mm, 0.375 mm, 0.1625 mm and 0.0375 mm, respectively. Then every two selected size classes (Table 1) were mixed and poured into a cylindrical cell for packing tests.

| Components | Particle size distribution |
|------------|---------------------------|
| (1) P15 and P10, P7, P5, P2,P0.9, P0.5, P0.25 or P0.075; | The volume fraction of fine size class varies from 10 % to 90 % in steps of 10 %. |
| (2) P10 and P7, P5, P2,P0.9, P0.5, P0.25 or P0.075; | |
| (3) P7 and P5, P2, P0.9, P0.5, P0.25 or P0.075; | |
| (4) P5 and P2, P0.9, P0.5, P0.25 or P0.075. | |

Following procedures similar to the methods specified in the ASTM standards (ASTM D 4253-16, 2016; ASTM D 4254-16, 2016), the maximum and minimum void ratios of each single size class and
binary-size mixtures (Table 1) were determined. For each binary-sized mixtures, the mass fraction of fine size class was varied from 10% to 90%. Due to the limited space, the measured maximum and minimum void ratios of P7 & P5, P7 & P2, P7 & P0.9, P7 & P0.5 were shown as symbols in Fig. 4. It appears that as the volume fraction of fine size class \( f\) increases, both the maximum and minimum void ratios firstly decrease to a lowest value when \( f\) equals around 25% to 40% and then increase again with further increase of \( f\).

Afterwards, the measured relationship between maximum (minimum) void ratios and volume fraction of fine size class were fitted by the proposed model (Eqs. (11), (12) and (13)). The fitted results were shown as solid lines in Fig. 4. It appears that the measured results can be well fitted by the proposed model, indicating the good applicability of the proposed model to binary-sized mixtures. The calibrated parameters \( a_{12} \) and \( b_{12} \) versus particle size ratio \( d_2/d_1 \) for maximum and minimum void ratios were shown as symbols in Fig. 5 and Fig. 6, respectively. As can be seen, for both maximum and minimum void ratios, both the filling and embedment coefficients increase rapidly to a constant value as the particle size ratio \( d_2/d_1 \) decreases from 1 to 0. It seems that both the filling coefficient \( a_{12} \) and embedment coefficient \( b_{12} \) can be fitted by a power function of the particle size ratio \( d_2/d_1 \):}

\[
a_{12} = A(1-d_2/d_1)^p \quad (14)
\]

and

\[
b_{12} = B(1-d_2/d_1)^s \quad (15)
\]

respectively. Where, \( A \) and \( B \) are the limiting values of \( a_{12} \) and \( b_{12} \) as \( d_2/d_1 \) decreases down to 0, respectively. The \( p \) and \( s \) are filling and embedment indexes, respectively, that govern the shape of the curves.
Fig. 6. The calibrated embedment coefficients ($b_{12}$) versus particle size ratio ($d_2/d_1$) for maximum (red squares) and minimum (blue circles) void ratios.

The fitted curves of $a_{12}$ vs. $d_2/d_1$ and $b_{12}$ vs. $d_2/d_1$ were shown as solid (for maximum void ratio) and dash (for minimum void ratio) lines in Fig. 5 and Fig. 6, respectively. It can be seen that the fitted curve of filling coefficient ($a_{12}$) for maximum void ratio shares a comparable trend with that for minimum void ratios (Fig. 5), though somewhat differences exist. The fitted curves of embedment coefficient ($b_{12}$) shows almost the same trends for both maximum and minimum void ratios (Fig. 6).

4 MODEL VERIFICATION

After calibration, the filling coefficient ($a_{12}$) and embedment coefficient ($b_{12}$) at different particle size ratios can be determined by Eqs. (14) and (15), respectively. Then they were used to estimate the maximum and minimum void ratios of binary-sized mixtures with different particle size distributions (Table 1). The predicted results versus the measured ones are presented in Fig. 7. It can be seen that the predicted values, both maximum and minimum void ratio, are in good agreement with the measured ones, with an average discrepancy of about 2.3 % and 1.9 %, respectively.

Fig. 7. Comparison of the measured and predicted results: (a) maximum void ratios; (b) minimum void ratios.

In order to further examine the accuracy of the proposed model, the test results of binary-size mixtures of spherical glass beads (Kwan et al., 2013), Pasabahce silica sand (Yilmaz, 2009) and crushed granite rock aggregate (Kwan et al., 2015) were borrowed from literature. These test results were in terms of maximum packing density ($\rho_{\text{max}}$), which can be converted to minimum void ratio ($\epsilon_{\text{min}}$) by

$$\epsilon_{\text{min}} = 1/\rho_{\text{max}} - 1$$  \hspace{1cm} (16)

Following the procedures shown in section 3, firstly the parameters were calibrated. Then they were used to estimate the minimum void ratio at particle size distributions considered the literature. The predicted results versus the measured ones are presented in Fig. 8. Still a good agreement between the predicted and measured results is obtained, with an average discrepancy of 1.6 %.

Fig. 8. Comparison of the measured and predicted minimum void ratios for other granular materials from literature.

The above verifications imply that the deviations between predicted and measured results are rather slight, indicating an excellent accuracy of the new model for different binary-sized granular materials.
The comparable filling coefficient \((a_{12})\) and embedment coefficient \((b_{12})\) for maximum and minimum void ratios suggests that there is a certain relationship between the maximum and minimum void, regardless of the particle size ratio. This deduction can be confirmed by Fig. 9, in which the predicted maximum void ratios were plotted against the predicted minimum void ratios. It is identified that the data can be well fitted by a straight line, indicating a linear relationship between the predicted maximum and minimum void ratio. Accordingly, the maximum and minimum void ratio can be roughly estimated from each other when it is necessary.

5 CONCLUSIONS

Based on the concept of dominant size class, a nonlinear packing model was developed for predicting the maximum and minimum void ratios of binary-sized granular materials. The proposed model incorporates only two parameters (filling coefficient and embedment coefficient) which were closely related to the particle size ratios between the two size classes. The accuracy of the developed model was verified by experimental results of the crushed pellets of GMZ bentonite in this work and that of several other granular materials from literature. Furthermore, a linear relationship between the predicted maximum and minimum void ratio was identified.

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