Using two-dimensional Projections for Stronger Separation and Propagation of Bilinear Terms
Using two-dimensional Projections for Stronger Separation and Propagation of Bilinear Terms

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Abstract

One of the most fundamental ingredients in mixed-integer nonlinear programming solvers is the well-known McCormick relaxation for a product of two variables $x$ and $y$ over a box-constrained domain. The starting point of this paper is the fact that the convex hull of the graph of $xy$ can be much tighter when computed over a strict, non-rectangular subset of the box. In order to exploit this in practice, we propose to compute valid linear inequalities for the projection of the feasible region onto the $x$-$y$-space by solving a sequence of linear programs akin to optimization-based bound tightening. These valid inequalities allow us to employ results from the literature to strengthen the classical McCormick relaxation. As a consequence, we obtain a stronger convexification procedure that exploits problem structure and can benefit from supplementary information obtained during the branch-and-bound algorithm such as an objective cutoff. We complement this by a new bound tightening procedure that efficiently computes the best possible bounds for $x$, $y$, and $xy$ over the available projections. Our computational evaluation using the academic solver SCIP exhibit that the proposed methods are applicable to a large portion of the public test library MINLPLib and help to improve performance significantly.

1 Introduction

This paper is concerned with solving nonconvex mixed-integer quadratically constrained programs (MIQCPs) of the form

$$\begin{align*}
\min & \quad c^T x \\
\text{s.t.} & \quad x^T Q_k x + q_k^T x \leq b_k & \text{for all } k \in \mathcal{M}, \\
& \ell_i \leq x_i \leq u_i & \text{for all } i \in \mathcal{N}, \\
x_i \in \mathbb{Z} & \text{for all } i \in \mathcal{I},
\end{align*}$$

(1)

where $\mathcal{N} := \{1, \ldots, n\}$ is the index set of variables, $\mathcal{M} := \{1, \ldots, m\}$ the index set of constraints, $c \in \mathbb{R}^n$ is the objective function vector, $\ell \in \mathbb{R}^n$ and $u \in \mathbb{R}^n$ are the vectors of lower and upper bounds of the variables, $\mathcal{I} \subseteq \mathcal{N}$ is the index set of integer variables, and $Q_k \in \mathbb{R}^{n \times n}$ is a symmetric matrix for each $k \in \mathcal{M}$. Many real-world applications are inherently nonlinear and need to be tackled as MIQCPs or general mixed-integer nonlinear programs (MINLPs) that include
quadratic constraint functions. For a selection see, e.g., [23]. In this article, we develop new convexification and bound tightening techniques that are directly relevant to achieve improvements within the algorithmic framework of spatial branch-and-bound, which forms the basis of many modern solvers in global optimization, e.g., ANTIGONE [7], BARON [48], Couenne [17], and SCIP [49].

For clarity of presentation we assume that the MIQCP is equivalently reformulated as

\[
\begin{align*}
\min & \quad c^T x \\
\text{s.t.} & \quad \langle X, Q_k \rangle + q_k^T x \leq b_k \quad \text{for all } k \in \mathcal{M}, \\
& \quad \ell_i \leq x_i \leq u_i \quad \text{for all } i \in \mathcal{N}, \\
& \quad x_i \in \mathbb{Z} \quad \text{for all } i \in \mathcal{I}, \\
& \quad X = xx^T.
\end{align*}
\]

(2)

This reformulation is obtained by linearizing the original quadratic constraints via auxiliary variables and new constraints of the form \(X_{ij} = x_i x_j\) for \(i, j \in \mathcal{N}\). Usually, these constraints are only added for those \(i, j \in \mathcal{N}\) for which \(x_i x_j\) appears in at least one of the quadratic constraints of (1), i.e., if \((Q_k)_{ij} \neq 0\) for some \(k \in \mathcal{M}\). Formulation (2) is of major importance when using convex relaxations for solving MIQCPs to global optimality and allows us to focus on tight relaxations for the elementary nonconvex constraints of the form \(X_{ij} = x_i x_j\) with \(i \neq j\). The techniques presented in this paper extend fully to such bilinear constraints present in general reformulations that are applied when solving factorable MINLPs to global optimality [47, 53, 13]. For example, when a nonlinear constraint of the form \(f(x) g(x) \leq d\) is reformulated as

\[
\begin{align*}
\begin{array}{l}
w_1 = f(x), \\
w_2 = g(x), \\
w_1 w_2 \leq d,
\end{array}
\end{align*}
\]

(3)

with auxiliary variables \(w_1, w_2 \in \mathbb{R}\), our results can be directly applied to improve the convexification and propagation of the product \(w_1 w_2\).

Our initial motivation is as follows. Classically, a linear relaxation for the nonconvex constraint \(X_{ij} = x_i x_j, i \neq j\), is constructed by adding the four inequalities

\[
\begin{align*}
X_{ij} \geq & \ u_j x_i + u_i x_j - u_i u_j, \\
X_{ij} \geq & \ \ell_j x_i + \ell_i x_j - \ell_i \ell_j, \\
X_{ij} \leq & \ u_j x_i + \ell_i x_j - \ell_i u_j, \\
X_{ij} \leq & \ \ell_j x_i + u_i x_j - u_i \ell_j,
\end{align*}
\]

(4)

often called McCormick inequalities [39]. These inequalities are best possible on the domain \([\ell_i, u_i] \times [\ell_j, u_j]\) in the sense that they describe the convex and concave envelope of \(x_i x_j\) [3]. However, they do not take into account the presence of other linear and nonlinear inequalities of (2).

Suppose that for all feasible points \((x, X)\) of (2) the points \((x_i, x_j)\) are contained in a polytope \(P\) that is a strict subset of \([\ell_i, u_i] \times [\ell_j, u_j]\). As can be seen in Figure 1n, the convex hull of the graph of \(x_i x_j\) over \(P\) is not given by (4) and is not polyhedral. Tangent inequalities for the convex and concave envelope of \(x_i x_j\) over \(P\) lead to a stronger linear relaxation of \(X_{ij} = x_i x_j\) than (4).

In addition to tighter underestimators, knowledge about \(P\) can be exploited to construct tighter variable bounds. For example, consider the polytope

\[
P = \{(x_i, x_j) \in [0, 1]^2 \mid x_i + x_j \leq 3/2\}.
\]

(5)
Improving separation: Two functions that are valid underestimators for $x_i x_j$ (orange) on a polyhedral domain (cyan). The figure shows that a linearization of the convex envelope (blue) at a given point (yellow) is locally tighter than the McCormick relaxation (gray).

Improving propagation: The colored plot shows $x_i x_j$ over the polytope $P = [0, 1]^2 \cap \{(x_i, x_j) \mid x_i + x_j \leq \frac{3}{2}\}$ (cyan). The yellow point corresponds to the best possible bound of $x_i x_j$ on $P$, which is better than the best possible bound implied by the McCormick relaxation (4) on $P$ (green point).

Figure 1: Both separation and propagation can be improved by exploiting the presence of a non-rectangular, polyhedral domain.

The best possible upper bound for $X_{ij} = x_i x_j$ over $P$ is given as

$$\max\{X_{ij} \mid (x_i, x_j) \in P, X_{ij} = x_i x_j\} = \frac{9}{16}. \quad (6)$$

This improves upon the upper bound implied by the McCormick relaxation over $P$,

$$\max\{X_{ij} \mid (x_i, x_j) \in P, (4)\} = \frac{3}{4}. \quad (7)$$

An illustration is given in Figure 1.

These two examples show that a two-dimensional polytope $P \subseteq [l_i, u_i] \times [l_j, u_j]$ for $(x_i, x_j)$ can be exploited in order to improve the convexification and propagation of $X_{ij} = x_i x_j$. In order to leverage this potential in practice, one needs to determine how to efficiently compute

1. a suitable polytope $P$,
2. tangent inequalities for the convex and concave envelope of $x_i x_j$ over $P$, and
3. tighter variable bounds for $x_i$, $x_j$, and $X_{ij}$ over $P$.

For the second step, an algorithm to compute tangent inequalities for the envelopes of $x_i x_j$ over $P$ is presented in the recent paper by Locatelli [36]. One requirement of this algorithm is that $P$ needs to be explicitly given, as output of step one.

Ideally, the original formulation (2) already contains inequalities that only depend on the two variables of a bilinear term. A good example are symmetry-breaking inequalities in circle packing problems. For example, the instance pointpack08 from the MINLPLib [42] test library contains constraints of the form

$$(x_1 - x_2)^2 + (y_1 - y_2)^2 \geq 1,$$

$$x_1 - x_2 \leq 0,$$

$$(x_1, x_2, y_1, y_2) \in [0, 1]^4. \quad (8)$$
Here \((x_1, y_1)\) and \((x_2, y_2)\) are the centers of two circles. The quadratic constraint ensures a minimum distance between these centers and the linear constraint orders them along the \(x\)-axis. In this case the inequality \(x_1 \leq x_2\) can be directly used for convexifying \(x_1 x_2\) with Locatelli’s algorithm.

However, for many instances inequalities only depending on variables of a single bilinear term may not appear in the initial formulation of the MIQCP. Nevertheless, it might be possible that a substructure of (2) implies such inequalities. For example, consider the instance crudeoil_lee1_05 from MINLPLib. Aggregating the linear constraints
\[
\begin{align*}
  x_{260} + x_{292} + x_{324} + x_{356} + x_{451} &\leq 50, \\
  -x_{394} + x_{525} + x_{526} + x_{527} &= 0, \\
  x_{260} + x_{292} + x_{324} + x_{451} + x_{527} &= 50, \\
  x_{525} &\geq 0, \\
  x_{526} &\geq 0,
\end{align*}
\]
with the multiplier vector \((-\frac{1}{7}, -\frac{1}{7}, \frac{1}{7}, \frac{1}{7}, \frac{1}{7})\) shows that \(x_{356} \leq x_{394}\) is valid and thus it can be used for strengthening the relaxation of \(x_{356}, x_{394}\).

In this spirit, the first contribution of this paper is a fully general scheme for computing projected relaxations \(P\) in step one above. It solves a sequence of linear programs (LPs) to compute a polyhedral relaxation of the projection of the feasible region onto the space of two variables that appear bilinearly. The computed two-dimensional relaxation is described by at most eight inequalities. Second, we introduce a bound tightening procedure for forward and backward propagation that solves a reduced nonconvex optimization problem. This results in the best possible bounds for a bilinear term and its variables using the linear inequalities of the two-dimensional projection. Due to the construction of the projections, these optimization problems can be solved by inspecting only a constant number of points. Last, we propose an effective way of incorporating these techniques into an LP-based spatial branch-and-bound solver and provide a detailed computational analysis of their impact.

The remainder of the paper is organized as follows. Section 2 discusses relevant literature and provides an overview of convex relaxations for (2). In Section 3, we present a procedure for computing valid inequalities for the projections of the feasible region onto the space of two variables. Section 4 is dedicated to a bound tightening algorithm that exploits the computed projections. Section 5 provides a thorough computational study using the MINLP solver SCIP on publicly available benchmark instances based on three experiments. First, we measure the basic potential of the methods by analyzing how many instances of MINLPLib actually admit non-trivial two-dimensional projections. Second, we study the dual bound improvement in the root node of the branch-and-bound tree. Third, we evaluate the overall performance impact of the new methods on the full spatial branch-and-bound search. Section 6 gives concluding remarks.

2 Background

In this section, we give a brief overview of the relevant literature. First, we review important convex relaxations for MIQCPs and existing convexification methods for special nonconvex functions over non-rectangular domains. Second, we discuss basic bound tightening algorithms and their relation to convexification methods. Finally, we give a short summary of Locatelli’s algorithm and its complexity.
Convex relaxations for MIQCPs Two important convex relaxations for MIQCPs that have been exhaustively studied in the literature are semidefinite programming (SDP) \[55\] and the reformulation-linearization technique (RLT) \[51\]. Both relaxations utilize the $X_{ij}$ variables of (2) in order to linearize $x_i x_j$. For an SDP relaxation the nonconvex constraint $X = xx^T$ is relaxed to the convex constraint $X \succeq xx^T$, which is equivalent to

$$\begin{bmatrix}
1 & x \\
x & X
\end{bmatrix} \succeq 0,$$

via the Schur complement \[15\]. Even though the resulting SDP relaxation is efficiently solvable in theory, optimizing SDPs in practice is a numerically challenging task. We refer to \[14, 20, 24, 33, 38, 11\] for applications which utilize SDP relaxations to solve quadratic optimization problems.

While the construction of an SDP relaxation is independent of any linear or linearized constraints, an RLT-based relaxation uses them directly. After introducing auxiliary variables $X \in \mathbb{R}^{n \times n}$ and the nonconvex constraints $X = xx^T$, the idea is to linearize the product of all selections of two linear inequalities with the help of $X$. For example, consider the inequalities $x_i \geq 0$ and $\alpha^T x - \alpha_0 \geq 0$. Multiplying the second inequality by $x_i$ gives

$$\sum_{j=1}^{n} \alpha_j x_i x_j - \alpha_0 x_i \geq 0,$$

which is linearized with $X$ to

$$\sum_{j=1}^{n} \alpha_j X_{ij} - \alpha_0 x_i \geq 0.$$

These RLT inequalities can significantly improve a relaxation of (2), see \[52, 40, 5\]. Note that the McCormick relaxation \[4\] is a special form of RLT that uses variable bound constraints only.

To obtain a convex relaxation for (1), it is not mandatory to reformulate the MIQCP into (2). Following the ideas of McCormick \[39\], Vigerske \[56\] uses linear underestimators $\tilde{f}_k : [\ell, u] \to \mathbb{R}$ for each nonlinear function $f_k : [\ell, u] \to \mathbb{R}$ of a constraint $\sum_k f_k(x) \leq 0$ and obtains the valid cut $\sum_k \tilde{f}_k(x) \leq 0$ by summing the underestimators. The advantage of this approach is that it does not require the additional variables $X$ but Anstreicher \[6\] shows that even when replacing each quadratic function with its convex envelope, this is in general weaker than exploiting the extended formulation.

Convexification of bilinear terms Although RLT-based relaxations utilize the LP relaxation, they do not necessarily describe the convex hull of the constraint $X_{ij} = x_i x_j$ over this relaxation. For example, consider the set

$$\{(x_i, x_j, X_{ij}) \in [0,1]^3 \mid X_{ij} = x_i x_j, x_i \leq x_j\}.$$  \hspace{1cm} (10)

The RLT relaxation of (10) is equal to

$$\{(x_i, x_j, X_{ij}) \in [0,1]^3 \mid \{4\}, x_i^2 \leq X_{ij}\},$$

when keeping the convex constraint $x_i^2 \leq X_{ij}$. However, the convex hull of (10) is given by

$$\{(x_i, x_j, X_{ij}) \in [0,1]^3 \mid \{4\}, x_i^2 \leq (1 + x_i - x_j)X_{ij}\},$$

which is strictly tighter. This shows that RLT does not fully exploit the presence of linear inequalities.
In the literature, different cases for convexifying a bilinear term over special sets have been studied: Linderoth \cite{35} proposed a branch-and-bound algorithm for solving nonconvex quadratically-constrained quadratic programs. Variables of a bilinear term are partitioned into two-dimensional triangles and rectangles. He characterized the convex and concave envelope of $x_i x_j$ over a triangular domain and used it to improve upon (4). Based on perspective functions, Hijazi \cite{26} derived a closed formula for the convex and concave envelope on a polytope of the form $P := \{ (x_i, x_j) \in [\ell_i, u_i] \times [\ell_j, u_j] \mid x_i \leq x_j \}$. As mentioned above, an algorithm for computing tangent inequalities for the convex and concave envelope of $x_i x_j$ on a general two-dimensional polytope $P$ has been presented by Locatelli \cite{36}. Instead of using information on $x_i$ and $x_j$, Miller et al. \cite{41} showed a lifting procedure for cutting planes for $X_{ij} = x_i x_j$ that exploits bounds on $X_{ij}$ that are not implied by $x_i x_j$.

**Bound tightening methods**  As it is shown in \cite{4}, there is an interdependency between the variable bounds and the strength of the (convex) relaxation. Tighter variable bounds result in tighter relaxations for nonconvex constraints and vice versa. The two most practically relevant methods to tighten variable bounds are feasibility-based bound tightening (FBBT) and optimization-based bound tightening (OBBT). FBBT is based on interval arithmetic, see, e.g., \cite{12}, and computes activities of nonlinear expressions over the domain of the variables (forward propagation) and conversely propagating the bounds on the constraint activities back to the bounds of the variables (reverse propagation). Implementations usually rely on the representation of nonlinear term as nodes of a directed acyclic expression graph, see \cite{13} or \cite{56} for details. OBBT computes tighter lower and upper bounds for a variable $x_i$ by minimizing and maximizing $x_i$ over a linear relaxation of (2). These two linear programs are called OBBT LPs. Computing the best possible bounds for all variables over a fixed linear relaxation requires solving $2^n$ many OBBT LPs and thus OBBT is often too expensive to be applied in every node of a branch-and-bound tree. Gleixner et al. \cite{21} show how dual solutions of OBBT LPs can be used during the tree search as a fast approximation of OBBT.

**Locatelli’s algorithm**  Let $P \subset \mathbb{R}^2$ be a polytope and let $(x^*_i, x^*_j) \in P$. Locatelli showed that computing a tangent inequality of the convex and concave envelope of $x_i x_j$ at $(x^*_i, x^*_j)$ reduces to selecting at most three points in the boundary of $P$ such that $(x^*_i, x^*_j)$ is contained in the convex hull of these points. Figure 2 shows all possible cases that can occur. The resulting inequality is determined by either

1. three vertices of $P$,
2. a vertex and a point $p$ on a facet of $P$ such that the inequality is tangent at $p$, or
3. two points $p$ and $q$ on different facets of $P$ such that the inequality is tangent at $p$ and $q$.

Locatelli derived closed formulas for computing the inequalities in each of the three cases. When $P = [\ell_i, u_i] \times [\ell_j, u_j]$, they collapse to the first case and yield the McCormick inequalities \cite{4}. The third case only occurs if $P$ is described by at least two non-axis parallel facets that have both a positive or both a negative slope.

To find a valid inequality that is also tangent to the convex (concave) envelope, one needs to iterate through all possible selections of the points as discussed above, and select the inequality that has the smallest (largest) value at $(x^*_i, x^*_j)$. The computational cost for iterating through all possible choices and computing the tangent inequality is

$$O \left( \left( \binom{|V|}{3} + |V| \cdot |F| + \binom{|F|}{2} \right) \right),$$

6
Figure 2: All possibilities for computing a tangent linear inequality (blue) for the convex and concave envelope of $x_i x_j$ on a two-dimensional polytope $P \subset \mathbb{R}^2$ at a given point (black). The inequality is obtained by selecting at most three points (yellow) that are on the boundary of $P$ such that the given point is in the convex hull of the selected points (dashed lines).

where $|V|$ is the number of vertices and $|F|$ be the number of facets of $P$ that are not axis-parallel.

3 Two-dimensional projected relaxations

Consider a single nonconvex quadratic constraint $X_{ij} = x_i x_j$ of (2) with $i \neq j$, $x_i \in [\ell_i, u_i]$, $x_j \in [\ell_j, u_j]$, and $X_{ij} \in \mathbb{R}$. Let $\mathcal{F}$ be the set of feasible points of the original MINLP (2) and let

$$\mathcal{F}_{ij} := \{(x_i, x_j) \mid x \in \mathcal{F} \subseteq [\ell_i, u_i] \times [\ell_j, u_j]\} \subseteq \mathbb{R}^2$$

be the projection of $\mathcal{F}$ onto the $(x_i, x_j)$-space. The best possible convex relaxation for the nonconvex constraint $X_{ij} = x_i x_j$ is given by the convex hull of $\{(x_i, x_j, X_{ij}) \mid (x_i, x_j, X_{ij}) \in \mathcal{F}_{ij} \}$.

However, it is unclear how to enforce this relaxation in practice. First, the set $\mathcal{F}$ is unknown and in general even finding a single point in $\mathcal{F}$ is \textsc{Np}-hard. Second, $\mathcal{F}_{ij}$ can be a non-polyhedral, nonconvex, disconnected set and thus cannot be used by Locatelli’s algorithm.

Hence, instead of targeting $\mathcal{F}_{ij}$ directly, we propose to compute a polyhedral relaxation $P_{ij} \subset \mathbb{R}^2$ of $\mathcal{F}_{ij}$, i.e., $\mathcal{F}_{ij} \subseteq P_{ij}$. This relaxation is based on a polyhedral relaxation of $\mathcal{F}$, which we denote by

$$\mathcal{X} := \{(x, X) \mid A_1 x + A_2 X \leq d\} \subseteq \mathbb{R}^2 \times \mathbb{R}$$

where $X$ is assumed to be a vector. These relaxations are readily available in LP-based spatial branch-and-bound algorithms. They are constructed from linear constraints present in the original problem formulation, from cutting planes based on integrality information, and from other valid linearizations of quadratic constraints such as gradient cuts.

Similar to (11), let

$$\mathcal{X}_{ij} := \{(x_i, x_j) \mid x \in \mathcal{X} \subseteq [\ell_i, u_i] \times [\ell_j, u_j]\} \subseteq \mathbb{R}^2 \times \mathbb{R}$$

be the projection of $\mathcal{X}$ onto the $(x_i, x_j)$-space. The best polyhedral relaxation from $\mathcal{X}$ is $\mathcal{X}_{ij}$. Unfortunately, exponentially many inequalities may be necessary to describe $\mathcal{X}_{ij}$ \cite{45}. For this reason, exact projection methods such as standard homotopy procedures \cite{43} may be overly expensive in practice. This motivates the computation of a relaxation $P_{ij}$ of $\mathcal{X}_{ij}$. In view of the complexity of Locatelli’s algorithm, we would like for $P_{ij}$ to have few vertices and facets. Specifically, we propose an algorithm that yields a $P_{ij}$ described by at most four axis-parallel and at most four general inequalities. Later, we show that the quotient of the volume of $\mathcal{X}_{ij}$ and the volume of the constructed $P_{ij}$ is bounded by $1/2$ from below.
Figure 3: An example for computing heuristically one facet of $X_{ij}$. The idea is to find a facet with minimum distance with respect to the line connecting the center with a vertex of $[\ell_i, u_i] \times [\ell_j, u_j]$. The red colored facet in the left picture is closest to the center among the three facets that separate the top-right vertex. The right picture shows that using this facet-defining inequality together with the bound constraints of $x_i$ and $x_j$ defines a polytope $P_{ij}$ which is a relaxation of $X_{ij}$.

Remark 1. An even tighter relaxation can be achieved by also discarding feasible points from the set $F$ by using an objective cutoff $c^T x \leq U$. Typically, solutions with an objective value $U$ are found by heuristics during spatial branch-and-bound. Such a solution reduces the set of relevant feasible points to $F \cap \{(x, X) \mid c^T x \leq U\}$, which might later result in even tighter $P_{ij}$.

3.1 Computing polyhedral projections with linear programming

For $X_{ij} \subseteq [\ell_i, u_i] \times [\ell_j, u_j]$ to hold, there must be at least one valid (facet-defining) inequality that separates a vertex of $[\ell_i, u_i] \times [\ell_j, u_j]$ from $X_{ij}$. To find some of those facets, if they exist, we follow a procedure akin to the shooting experiment [27]. The idea is to shoot a ray from a point $(C_i, C_j) \in X_{ij}$ towards a vertex $(\bar{x}_i, \bar{x}_j)$ of $[\ell_i, u_i] \times [\ell_j, u_j]$. This ray is going to intersect the boundary of $X_{ij}$. If the intersection is at the vertex, then the vertex is feasible for $X_{ij}$. Otherwise, any active constraint at the intersection point separates the vertex from $X_{ij}$. If the intersection point is in the interior of a facet, then that facet is the only active constraint. See Figure 3 for an illustration of the idea. In our setting, the intersection point is $(x^*_i, x^*_j)$ where $(x^*, X^*, \theta^*)$ is the solution of the following LP:

$$\max \theta,$$

s.t. $A_1 x + A_2 X \leq d$

$$C_i + \theta(\bar{x}_i - C_i) = x_i,$$

$$C_j + \theta(\bar{x}_j - C_j) = x_j,$$

$$\theta \in \mathbb{R}. \quad (14)$$

Projecting out $\theta$ yields

$$\max \text{sign}(\bar{x}_i - C_i) x_i,$$

s.t. $A_1 x + A_2 X \leq d$

$$(x_j - C_j)(\bar{x}_i - C_i) = (x_i - C_i)(\bar{x}_j - C_j), \quad (15)$$

which is in the following denoted by $\text{LP}(\bar{x}_i, \bar{x}_j)$. As is shown next, the dual solution of this LP can be utilized to construct the inequality we are looking for.
Let \((x^*, X^*, \lambda^*, \mu^*)\) be an optimal primal-dual solution of \(\text{LP}(\bar{x}_i, \bar{x}_j)\), where \(\lambda^* \geq 0\) are the dual multipliers of the inequality constraints and \(\mu^* \in \mathbb{R}\) the dual multiplier for the equality constraint of (15). Note that the aggregation

\[ \lambda^*\mathbf{T}(A_1x + A_2X) \leq \lambda^*d \]

is valid for \(X\). Multiplying the stationarity condition

\[ \text{sign}(\bar{x}_i - C_i)e_i^T = \lambda^*\mathbf{T} \left( A_1 \middle| A_2 \right) - \mu^* \left( \frac{e_i^T}{\bar{x}_i - C_i} - \frac{e_j^T}{\bar{x}_j - C_j} \right) \]

of the Karush–Kuhn–Tucker \([29, 31]\) conditions by \((x^T, X^T)^T\) shows that

\[ \text{sign}(\bar{x}_i - C_i)x_i = \lambda^*\mathbf{T}(A_1x + A_2X) + \mu^* \left( \frac{x_i}{\bar{x}_i - C_i} - \frac{x_j}{\bar{x}_j - C_j} \right) \]

holds. Using \(A_1x + A_2X \leq d\) and reordering terms results in

\[ \left( \text{sign}(\bar{x}_i - C_i) - \frac{\mu^*}{\bar{x}_i - C_i} \right) x_i + \frac{\mu^*}{\bar{x}_i - C_i} x_j \leq \lambda^*d \tag{16} \]

which is valid for \(X\) and only depends on \(x_i\) and \(x_j\) and is tight at the intersection point.

For having a complete method we need to specify \((C_i, C_j) \in X_j\). The center of \([\ell_i, u_i] \times [\ell_j, u_j]\) is guaranteed to be in \(X_j\) after we applied OBBT on \(x_i\) and \(x_j\) for the relaxation \(X\), as the next Lemma shows. Recall that OBBT ensures \(\ell_i = \min_{(x,y)\in X} x_i\) and \(u_i = \max_{(x,y)\in X} x_i\).

**Lemma 1.** Let \(X \neq \emptyset\), \(\ell_i = \min_{(x,y)\in X} x_i\), \(u_i = \max_{(x,y)\in X} x_i\), \(\ell_j = \min_{(x,y)\in X} x_j\), and \(u_j = \max_{(x,y)\in X} x_j\). Denote by

\[ C := \left( \frac{\ell_i + u_i}{2}, \frac{\ell_j + u_j}{2} \right) \]

the center of \([\ell_i, u_i] \times [\ell_j, u_j]\). It holds that \(C \in X_j\).

**Proof.** Assume that \(C \not\in X_j\). It follows that there is an inequality \(\alpha_i x_i + \alpha_j x_j \leq \alpha_0\) that is valid for \(X_j\) and separates \(C\). The center \(C\) can only be separated if the inequality separates at least two adjacent vertices of the rectangular domain \([\ell_i, u_i] \times [\ell_j, u_j]\). Assume that it separates \((\ell_i, \ell_j)\) and \((u_i, u_j)\) (all other three cases work analogously), i.e., \(\alpha_i \ell_i + \alpha_j \ell_j > \alpha_0\) and \(\alpha_i u_i + \alpha_j u_j > \alpha_0\). This immediately shows that there is no feasible point in \(X_j\) with \(x_j = \ell_j\), which is a contradiction to \(\ell_j = \min_{(x,y)\in X} x_j\) and \(X \neq \emptyset\). Figure 4 illustrates the idea of the proof.

**Lemma 1** implies that each inequality that is valid for \(X_j\) can separate at most one vertex of \([\ell_i, u_i] \times [\ell_j, u_j]\) directly after OBBT has been applied to \(x_i\) and \(x_j\). However, if tighter bounds from OBBT are used to strengthen the linear relaxation \(X\) further by, e.g., computing tighter convexifications for nonconvex constraints or propagating variables bounds via FBBT, then the conditions in Lemma 1 may not be met anymore. For this reason, we solve (15) immediately after OBBT.

Finally, we are able to define the polytope \(P_{ij}\) by using the variable bounds \([\ell_i, u_i] \times [\ell_j, u_j]\) and the derived inequalities (16) for four choices of \(\bar{x}_i\) and \(\bar{x}_j\), namely the four vertices of \([\ell_i, u_i] \times [\ell_j, u_j]\). Defining \(P_{ij}\) like this has the advantage that it is described by at most eight inequalities and covers at least half of the volume of \(X_{ij}\) as it is shown in the following section.
Remark 2. The problem of computing a facet-defining inequality for a projection of a polyhedron has been extensively studied in the literature. It corresponds to the “project” step in lift-and-project cuts [9, 8]. The dual of (14) is
\[
\begin{align*}
\max \beta - \alpha_i C_i - \alpha_j C_j, \\
\text{s.t.} \quad \alpha_i e^T_i + \alpha_j e^T_j = \lambda^T A_1, \\
0 = \lambda^T A_2, \\
\beta = \lambda^T d, \\
\alpha_i (\bar{x}_i - C_i) + \alpha_i (\bar{x}_i - C_i) = 1, \\
\lambda \geq 0,
\end{align*}
\]
which can be interpreted as a cut generating linear program (CGLP) with the objective function of the so-called reverse polar CGLP [50, Chap. 2] and the normalization constraint of Balas and Perregaard [10]. We refer to the thesis of Serra [50, Chap. 2] for more details.

3.2 Volume bound
We are interested in how much we lose by not computing the exact projection of the polyhedral relaxation \( \mathcal{X} \). In the literature, the volume has been used as a measure for the strength of relaxations, see [32, 54]. Following this line of thought, we provide a lower bound on the quotient of the volume of \( \mathcal{X}_{ij} \) and \( P_{ij} \).

**Theorem 1.** Let \( \mathcal{X} \) be a relaxation of [2] with \( \ell_i = \min \{ x_i \mid (x, X) \in \mathcal{X} \} \), \( u_i = \max \{ x_i \mid (x, X) \in \mathcal{X} \} \), \( \ell_j = \min \{ x_j \mid (x, X) \in \mathcal{X} \} \), \( u_j = \max \{ x_j \mid (x, X) \in \mathcal{X} \} \) for two variable indices \( i, j \in \mathcal{N} \). Let \( \mathcal{X}_{ij} = \{(x_i, x_j) \mid (x, X) \in \mathcal{X} \} \) be the two-dimensional projection of \( \mathcal{X} \) onto the \((x_i, x_j)\)-space. Let \( P_{ij} \) be a polytope that is given by the intersection of \([\ell_i, u_i] \times [\ell_j, u_j]\) and (16) for the four choices \((\bar{x}_i, \bar{x}_j) \in \{\ell_i, u_i\} \times \{\ell_j, u_j\}\). Then, the inequality
\[
\frac{\text{Vol}(\mathcal{X}_{ij})}{\text{Vol}(P_{ij})} \geq \frac{1}{2}
\]
holds and the constant is best possible.

**Proof.** Since the volume quotient is invariant with respect to scaling and translating, we assume that all variable bounds are \([0, 1]\). By construction, \( P_{ij} \) is a relaxation of \( \mathcal{X}_{ij} \). Because the
conditions of Lemma 1 are met, it follows that the center point \( C = (1/2, 1/2) \) belongs to \( X_{ij} \) and thus also to \( P_{ij} \). Let \( p^k \in \mathbb{R}^2 \) for \( k \in \{1, 2, 3, 4\} \) be the four intersection points between \( P_{ij} \) and the line connecting the center \( C \). By construction, these four points belong to the set \( X_{ij} \).

First, we construct an example that shows that the constant is best possible. Let \((0,0), (0,1-a), (a,1), (1,1), (1,a), \text{ and } (1-a,0)\) be the vertices of \( P_{ij} \) and \( p^1 = (0,0), p^2 = (a/2, 1-a/2), p^3 = (1,1), p^4 = (1-a/2, a/2)\) the vertices of \( X_{ij} \) depending on a parameter \( a \in [0,1] \). See Figure 5 for an illustration of the construction. It follows that \( \text{Vol}(P_{ij}) = 1 - a^2 \) and \( \text{Vol}(X_{ij}) = 1 - a \) holds. As a consequence,

\[
\frac{\text{Vol}(X_{ij})}{\text{Vol}(P_{ij})} = \frac{1-a}{1-a^2} = \frac{1-a}{(1-a)(1+a)} = \frac{1}{1+a}
\]

converges to \( \frac{1}{2} \) for \( a \to 1 \). Note that for \( a = 1 \) the volume quotient exists but the polytopes \( P_{ij} \) and \( X_{ij} \) reduce to a single line.

Now, we prove the inequality. Since \( X_{ij} \) is a subset of \( P_{ij} \), it immediately follows that

\[
\text{Vol}(P_{ij}) = \text{Vol}(X_{ij}) + \text{Vol}(P_{ij} \setminus X_{ij})
\]

The inequality \( \text{Vol}(P_{ij}) \setminus X_{ij} \leq \text{Vol}(X_{ij}) \) is enough to show

\[
\text{Vol}(P_{ij}) = \text{Vol}(X_{ij}) + \text{Vol}(P_{ij} \setminus X_{ij}) \leq 2 \cdot \text{Vol}(X_{ij}),
\]

which proves the theorem. We still need to prove the following claim.

**Claim 1.** \( \text{Vol}(P_{ij} \setminus X_{ij}) \leq \text{Vol}(X_{ij}) \)

**Proof.** Let \( q^i \in \mathbb{R}^2 \) for \( i \in \{1, 2, 3, 4\} \) be four points in \( X_{ij} \) such that each point touches a different side of the \([\ell_i, u_i] \times [\ell_j, u_j]\) box. The left picture in Figure 6 shows how the points are labeled. The set

\[
X'_{ij} := \text{conv}\{q^1, p^1, q^2, p^2, q^3, p^3, q^4, p^4\}
\]

is by construction a subset of \( X_{ij} \). As \( \text{Vol}(P_{ij} \setminus X'_{ij}) \leq \text{Vol}(P_{ij} \setminus X_{ij}) \) and \( \text{Vol}(X'_{ij}) \leq \text{Vol}(X_{ij}) \), showing the claim for \( X'_{ij} \) implies the result for \( X_{ij} \).

The set \( X'_{ij} \) decomposes into eight triangles that are adjacent to the regions, see the right picture of Figure 6. In the following, we show that the area of each \( R_k \) is at least as big as the area of the two corresponding triangles, which proves the claim.

Consider the region \( R_1 \) in the left-bottom corner. If \((a_1, 0)^T \) and \((0, b_1)^T\) are the endpoints of the facet in \( P_{ij} \) that contains \( p^1 = (c, c)^T \), then \( c = a_1 b_1/(a_1 + b_1) \). Note that the claim is true if \( a_1 = 0 \) or \( b_1 = 0 \) because in this case the two adjacent triangles are empty. Let \( q^1 = (a_2, 0)^T \) and \( q^2 = (0, b_2)^T \). The area of the triangle \( \Delta_1 := \text{conv}\{(a_1, 0)^T, p^1, q^1\} \subseteq P_{ij} \setminus X_{ij} \) is

\[
\text{Vol}(\Delta_1) = \frac{c(a_2 - a_1)}{2}
\]

and the area of the second triangle \( \Delta_2 := \text{conv}\{(0, b_1)^T, p^1, q^2\} \subseteq P_{ij} \setminus X_{ij} \) is

\[
\text{Vol}(\Delta_2) = \frac{c(b_2 - b_1)}{2}.
\]
The area of the quadrilateral is given by the area of two triangles $\Delta_3 = \operatorname{conv}\{C, p_1, q_1\}$ and $\Delta_4 = \operatorname{conv}\{C, q_2, p_1\}$. Their areas are

$$\operatorname{Vol}(\Delta_3) = \frac{a_2}{4} - \frac{a_2c}{2} = \frac{a_2(1 - 2c)}{4}$$

and

$$\operatorname{Vol}(\Delta_4) = \frac{b_2}{4} - \frac{b_2c}{2} = \frac{b_2(1 - 2c)}{4}.$$  

Finally, we show that the area of $\Delta_1$ and $\Delta_2$ is less or equal than the area of $\Delta_3$ and $\Delta_4$, which proves the claim. After algebraic manipulation, we get

$$\operatorname{Vol}(\Delta_1) + \operatorname{Vol}(\Delta_2) - \operatorname{Vol}(\Delta_3) - \operatorname{Vol}(\Delta_4) = c(a_2 - a_1) + \frac{c(b_2 - b_1)}{2} - \frac{a_2 (1 - 2c)}{4} - \frac{b_2 (1 - 2c)}{4}$$

where the second step used the definition of $c$, i.e., $c = a_1b_1/(a_1 + b_1)$. Since the denominator of (18) is positive, showing that the nominator of (18) is non-positive implies

$$\operatorname{Vol}(\Delta_1) + \operatorname{Vol}(\Delta_2) - \operatorname{Vol}(\Delta_3) - \operatorname{Vol}(\Delta_4) \leq 0.$$

We consider two cases.

**Case 1:** $4a_1b_1 - a_1 - b_1 \leq 0$

The nominator of (18) consists of three non-positive terms.

**Case 2:** $4a_1b_1 - a_1 - b_1 > 0$

Since $a_2, b_2 \in [0, 1]$, it follows that

$$-2a_1^2b_1 - 2a_1b_1^2 + (a_2 + b_2)(4a_1b_1 - a_1 - b_1) \leq -2a_1^2b_1 - 2a_1b_1^2 + 2(4a_1b_1 - a_1 - b_1)
= 2a_1(2b_1 - b_1^2 - 1) + 2b_1(2a_1 - a_1^2 - 1)
= -2a_1(b_1 - 1)^2 - 2b_1(a_1 - 1)^2 \leq 0,$$

which proves that the nominator of (18) is non-positive.

**Remark 3.** The construction of the parametric example in the proof of Theorem 1 requires that $P_{ij}$ contains two facets that are not axis-parallel. If only one facet of $P_{ij}$ is not axis-parallel, the volume quotient is bounded by $2 + \sqrt{2} \approx 0.85$.

Theorem 1 and Remark 3 provide some theoretical justification why it suffices to compute a relaxation of the projection. From a practical point of view, spending more time in computing $X_{ij}$ exactly might not pay off because we are only projecting a relaxation of the feasible region.
Figure 5: Construction of a parametric example that shows that $\frac{\text{Vol}(X_{ij})}{\text{Vol}(P_{ij})}$ approaches $\frac{1}{2}$ when $a$ approaches 1. The gray region is $X_{ij}$ and the red region is $P_{ij}$.

Figure 6: Construction of $X'_{ij}$ in the proof of Claim 1. The inner polytope is $X'_{ij}$ (gray) is inscribed in $P_{ij}$. The main idea of the proof is that $P_{ij} \setminus X_{ij}$ (red) has smaller area than $X_{ij}$. The right picture illustrates that the region $R_1 = \Delta_3 \cup \Delta_4$ is larger than the two adjacent triangles $\Delta_1$ and $\Delta_2$.

3.3 Computational aspects

So far, we have only considered a single term $x_ix_j$, but in general contains up to $O(n^2)$ many bilinear terms. With growing number of variables, it may become computationally too expensive to solve for all bilinear terms. In order to save unnecessary solves of LP($x_i, x_j$), we observe the following: The existence of a feasible solution $(x^*, X^*) \in \mathcal{X}$ in which $(x^*_i, x^*_j)$ is a vertex of $[\ell_i, u_i] \times [\ell_j, u_j]$ proves that no useful inequality for $P_{ij}$ can be found that cuts off $(x^*_i, x^*_j)$. This observation is similar to the bound filtering in the branch-and-contract algorithm [59] and can be exploited as an aggressive filtering strategy, as it has been done in OBBT [21]. The idea of bound filtering is to use a solution $(x^*, X^*)$ of an LP relaxation $\mathcal{X}$ and to check for which variables $x_i$ the solution value $x^*_i$ is equal to $\ell_i$ or $u_i$. If $x^*_i = \ell_i$ ($x^*_i = u_i$) holds then OBBT cannot find a tighter lower (upper) bound for $x_i$. In addition to considering solutions from previous OBBT LPs, aggressive bound filtering solves auxiliary LPs with an objective function $v^T x$ for a vector $v \in \{-1, 0, 1\}^n$ to push as many unfiltered variables as possible to their lower or upper bounds. We refer to [21] for more details.

In the following, we present the generic Algorithm 1 that first applies OBBT to ensure that the center point $(C_i, C_j)$ belongs to $X_{ij}$ and afterwards computes a relaxation of $X_{ij}$ as discussed
Algorithm 1 Two-dimensional projections

Input: linear relaxation $\mathcal{X} = \{(x, X) \mid A_1x + A_2X \leq d\}$ of $\mathcal{P}$
Output: a list $\mathcal{P}$ of two-dimensional polytopes for each bilinear term $x_i x_j$

1: $K \leftarrow \{(i,j) \in \mathcal{N} \times \mathcal{N} \mid i < j \wedge \exists k \in \mathcal{M} : (Q_{kj})_{ij} \neq 0\}$ /* collect bilinear terms */
2: $\mathcal{P} \leftarrow \emptyset$, $F \leftarrow \emptyset$
3: for $i \in \mathcal{N}$ : $\exists j \in \mathcal{N}$ such that $(i,j) \in K$ do /* call OBBT */
4: $[\ell_i, u_i] \leftarrow$ apply OBBT on $x_i$; let $x^*$ be the OBBT LP solution
5: $F \leftarrow F \cup \{(i',j',x_{i'}^*,x_{j'}^*) \mid (i',j') \in K \wedge x_{i'}^* \in [\ell_{i'},u_{i'}] \wedge x_{j'}^* \in [\ell_{j'},u_{j'}]\}$
6: end for
7: for $(i,j) \in K$ do
8: $P_{ij} \leftarrow [\ell_i, u_i] \times [\ell_j, u_j]$
9: for $(\bar{x}_i, \bar{x}_j) \in [\ell_i, u_i] \times [\ell_j, u_j]$ do /* iterate through all vertices */
10: if $(i,j, \bar{x}_i, \bar{x}_j) \not\in F$ then /* consider unfiltered candidates */
11: $(x^*,X^*,\lambda^*,\mu^*) \leftarrow$ solve LP($\bar{x}_i, \bar{x}_j$)
12: $F \leftarrow F \cup \{(i',j',x_{i'}^*,x_{j'}^*) \mid (i',j') \in K \wedge x_{i'}^* \in [\ell_{i'},u_{i'}] \wedge x_{j'}^* \in [\ell_{j'},u_{j'}]\}$
13: if $x_i^* \neq \bar{x}_i \wedge x_j^* \neq \bar{x}_j$ then /* create linear inequality */
14: extract valid inequality (16) from dual solution ($\lambda^*,\mu^*$)
15: $P_{ij} \leftarrow P_{ij} \cap \{(x_i,x_j) \mid (16) \text{ holds}\}$
16: end if
17: end if
18: end for
19: add $P_{ij}$ to $\mathcal{P}$
20: end for
21: return $\mathcal{P}$

above.

In Line 1 Algorithm 1 computes an index set for all occurring bilinear terms. OBBT is called in Line 4 for each variable that appears bilinearly to ensure that the requirements of Lemma 1 are met. Afterward, in Line 11 to Line 12 for each term $x_i x_j$ the algorithm considers all vertices $(\bar{x}_i, \bar{x}_j)$ of $[\ell_i, u_i] \times [\ell_j, u_j]$ and solves LP($\bar{x}_i, \bar{x}_j$). The result is a primal-dual optimal solution ($x^*,X^*,\lambda^*,\mu^*$) that is used in Line 14 for generating a valid inequality for $\mathcal{X}_{ij}$. The LP solutions from Line 4 and Line 11 are used to update the set of filtered candidates $F$ in Line 5 and Line 12. In Line 10 a candidate $(x_i, x_j)$ for the direction $(\bar{x}_i, \bar{x}_j) \in [\ell_i, u_i] \times [\ell_j, u_j]$ is only considered if $(i,j, \bar{x}_i, \bar{x}_j)$ has not been filtered out.

In our implementation, all bilinear terms are ordered by how often they appear in different constraints of the original MIQCP. As a tie-break, we use the term $x_i x_j$ for which the volume of $[\ell_i, u_i] \times [\ell_j, u_j]$, i.e., $(u_i - \ell_i)(u_j - \ell_j)$ is maximized.

Algorithm 1 could either solve LP($\bar{x}_i, \bar{x}_j$) or its dual formulation (17) for deriving the two-dimensional projections. However, solving LP($\bar{x}_i, \bar{x}_j$) has two technical advantages:

1. The linear relaxation $\mathcal{X}$ is available in LP-based spatial branch-and-bound solvers and only needs to be extended by a single linear equality constraint for solving LP($\bar{x}_i, \bar{x}_j$). This is beneficial compared to constructing (17) for a relaxation that contains many variables and constraints.

2. Due to the close connection to OBBT, it is possible to warm start from a previously computed basis of an OBBT-LP. This would require to restructure Algorithm 1 in a way that it solves LP($\bar{x}_i, \bar{x}_j$) after the bounds of $x_i$ and $x_j$ have been tightened by OBBT. However, restoring a previous LP basis causes a significant overhead that cannot be compensated.
by the warm start capabilities of the LP solver. For this reason, our implementation of Algorithm 1 does not utilize a previously computed LP basis.

After computing inequalities of the form (10), we apply Locatelli’s algorithm to strengthen the linear relaxation of \( X_{ij} = x_ix_j \) through separation during the tree search. Moreover, the computed \( P_{ij} \) can not only be used to improve separation but also to strengthen variable bounds of \( X_{ij}, x_i, \) and \( x_j \), as shown in the next section.

4 Using 2D projections for propagation

Tight variable bounds are crucial when computing linear (or convex) relaxations for MIQCPs during spatial branch-and-bound. Stronger bounds on \( x_i, x_j, \) and \( X_{ij} \) not only affect the relaxation of \( X_{ij} = x_ix_j \) but also other constraints that involve these variables, including linear constraints. Propagating these constraints in turn might lead to further bound reductions of variables that appear in other nonconvex constraints [12, 45] and subsequently result in tighter relaxations.

In the following, we show how to use a two-dimensional projection \( P_{ij} \) to derive tighter bounds on \( x_i, x_j, \) and \( X_{ij} \) by solving nonconvex optimization problems that can be efficiently solved.

4.1 Forward propagation

Given a polytope \( P_{ij} \), the best possible lower/upper bound for \( X_{ij} \) on \( P_{ij} \) is given by

\[
\min / \max \{ X_{ij} \mid X_{ij} = x_ix_j, (x_i, x_j) \in P_{ij} \},
\]

which is a nonconvex optimization problem. Denote by \( F(P_{ij}) \) the facets of \( P_{ij} \), and let

\[
C(P_{ij}) := \left\{ \arg\max_{(x_i,x_j) \in F} x_ix_j \mid F \in F(P_{ij}) \right\} \cup \left\{ \arg\min_{(x_i,x_j) \in F} x_ix_j \mid F \in F(P_{ij}) \right\}
\]

be the set of optimal points for maximizing and minimizing \( x_ix_j \) over each facet of \( P_{ij} \). For example, if \( F = \{ (x_i, x_j) \in [\ell_i, u_i] \times [\ell_j, u_j] \mid a_ix_i + a_jx_j = a_0 \} \) is a facet of \( P_{ij} \) with \( a_i \neq 0 \) and \( a_j \neq 0 \), then \( x_ix_j \) restricted to \( F \) is \( -\frac{a_0}{a_j} x_i + \frac{a_0}{a_i} x_j \). The critical point of this function is \( \frac{a_0}{2a_i} \). Thus \( (\frac{a_0}{2a_i}, \frac{a_0}{2a_j}) \in C(P_{ij}) \) if and only if \( \frac{a_0}{2a_i} \in [\ell_i, u_i] \) and \( \frac{a_0}{2a_j} \in [\ell_j, u_j] \). Otherwise, both vertices of \( F \) belong to \( C(P_{ij}) \).

See Figure 7 for an illustration of the points \( C(P_{ij}) \). The following theorem shows that (19) can be solved by computing the minimum / maximum on the discrete set \( C(P_{ij}) \).

**Theorem 2.** Let \( P_{ij} \subset \mathbb{R}^2 \) be a polytope and let \( C(P_{ij}) \) be the optimal points of \( P_{ij} \). Then the equality

\[
\min \{ \alpha x_i x_j \mid (x_i, x_j) \in P_{ij} \} = \min \{ \alpha x_i x_j \mid (x_i, x_j) \in C(P_{ij}) \}
\]

holds for \( \alpha \in \{-1, 1\} \).

**Proof.** First, due to the fact that \( x_i x_j \) is bilinear, the minimum and maximum must be attained at the boundary of \( P_{ij} \). Restricted to a facet, \( x_i x_j \) achieves its maximum and minimum at a point in \( C(P_{ij}) \).

By construction, \( P_{ij} \) has at most four facets that are not axis-parallel. This bounds the number of points in \( C(P_{ij}) \) by 12. Computing these points requires only simple algebraic computations as illustrated in the example above.
Figure 7: An example of how to compute the minimum and maximum of $x_i x_j$ on $P_{ij} \subset [-1, 1] \times [-1, 2]$. The red points are the points in $C(P_{ij})$.

4.2 Reverse propagation

There are two ways to obtain tighter variable bounds for $x_i$ and $x_j$ by utilizing $P_{ij}$. First, after branching on $x_i$ or $x_j$, it is possible that a facet of $P_{ij}$ cuts off two vertices of the rectangular domain for some locally valid bounds. This implies that at least one of the variable bounds of $x_i$ or $x_j$ can be tightened.

Second, the bounds of $X_{ij}$ define a level set for the bilinear term $x_i x_j$. Intersecting the level set with $P_{ij}$ might imply tighter lower and upper bounds on $x_i$ and $x_j$. Even though the intersection is in general a nonconvex region, we show that the best possible variable bounds that are implied by the intersection can be computed by considering a finite set of points.

In the following, we give more details on the two possible types of bound reductions for $x_i$ and $x_j$.

Branching reductions  Even though the facets of $P_{ij}$ are valid and redundant inequalities for the relaxation $X$ that has been used for computing $P_{ij}$, they are still useful for deriving bound reductions on $x_i$ and $x_j$ during the tree search. Figure 8 shows that $P_{ij}$ implies tighter bounds on $x_j$ after branching on $x_i$. Note that optimizing $\pm x_j$ over $X$ leads to bounds that are at least as tight as the bounds implied by $P_{ij}$. However, finding these bounds either requires solving an expensive OBBT-LP or propagating several linear constraints with FBBT. The strength of using $P_{ij}$ together with variable bound changes due to branching is that the facets of $P_{ij}$ contain information of multiple inequalities of $X$ and are computationally cheap to propagate. Using the facets of $P_{ij}$ in this fashion is very similar to the so-called Lagrangian Variable Bounds of Gleixner et al. [22], which are aggregations of linear constraints that are learned during OBBT and used as a computationally cheap approximation for OBBT during the tree search.

Level set reductions  Let $[\ell_{ij}, u_{ij}]$ be bounds on $X_{ij}$ such that

$$
\ell_{ij} > \min \{ x_i x_j \mid (x_i, x_j) \in P_{ij} \}
$$

or

$$
u_{ij} < \max \{ x_i x_j \mid (x_i, x_j) \in P_{ij} \}
$$

holds. This means that the bounds on $X_{ij}$ are not implied by $x_i x_j$ on $P_{ij}$. The best possible lower/upper bound on $x_i$ (and analogous for $x_j$) using $P_{ij}$ and the bounds $[\ell_{ij}, u_{ij}]$ is given by

$$
\min / \max \{ x_i \mid \ell_{ij} \leq x_i x_j \leq u_{ij}, (x_i, x_j) \in P_{ij} \},
$$

(20)
Figure 8: The example shows that the upper bound of $x_j$ can be improved after using $x_i^*$ as branching point for $x_i$. The vertical red line is the branching point of $x_i$. The horizontal green lines correspond to a tighter lower bound $\ell'_j$ and a tighter upper bound $u'_j$ in both subproblems, respectively.

which is a nonconvex optimization problem. Figure 9 illustrates that intersecting the level set of $x_ix_j$ and $P_{ij}$ can imply stronger bounds on $x_i$ and $x_j$. Similar to Theorem 2, we show that (20) can be efficiently solved by scanning a finite set of points. Let $IP \subseteq P_{ij}$ consist of the vertices of $P_{ij}$ that satisfy $x_ix_j \in [\ell_{ij}, u_{ij}]$, and the intersection points of each facet of $P_{ij}$ with $\{(x_i, x_j) : x_ix_j = \ell_{ij}\}$ or $\{(x_i, x_j) : x_ix_j = u_{ij}\}$. In other words, $IP$ is the set of feasible points for which at least two constraints of (20) are active. Since the vertices and facets of $P_{ij}$ are explicitly given, computing points in $IP$ reduces to solving a univariate quadratic equation.

The following theorem shows that it suffices to consider the points in $IP$ to solve (20).

**Theorem 3.** Let $x_ix_j$ a bilinear term, $[\ell_{ij}, u_{ij}]$ bounds on $x_ix_j$, and $P_{ij} \subseteq \mathbb{R}^2$ a polytope. Then the equality
\[
\min \{ \alpha x_i : \ell_{ij} \leq x_ix_j \leq u_{ij}, (x_i, x_j) \in P_{ij} \} = \min \{ \alpha x_i : (x_i, x_j) \in IP \}
\]
holds for $\alpha \in \{-1, 1\}$.

**Proof.** We only prove the theorem for the objective function $x_i$ since $-x_i$ is analogous. Let $x_i^*$ be the optimal value. As the objective function is linear, every optimum is at the boundary. Therefore, at least one constraint is active. We will show that there is at least one optimum for which at least two constraints are active, i.e., is in $IP$. Let $(x_i^*, x_j^*)$ be any optimal point.

If the only active constraint is linear, then it must be $x_i \geq x_i^*$. Since the feasible region is bounded, there is an $M > 0$ such that $(x_i^*, x_j^* + M)$ is infeasible. Therefore, for some $x_j \in [x_j^*, x_j^* + M]$, $(x_i^*, x_j)$ is active for at least two constraints.

If the only active constraint is nonlinear, say, $x_ix_j = \phi$ with $\phi \in [\ell_{ij}, u_{ij}]$, then the region $\{(x_i, x_j) : x_ix_j = \phi\}$, in a neighborhood of $(x_i^*, x_j^*)$, must be contained in $x_i \geq x_i^*$. This can only happen when $x_i^* = 0$ and the same argument as above shows that there is an $x_j$ such that $(x_i^*, x_j)$ is active for another constraint.

5 Computational experiments

In this section, we present a computational study of the presented propagation and separation ideas for bilinear terms for publicly available instances of the MINLPLib [42]. We conduct three experiments to answer the following questions:
Figure 9: An example that shows bound reductions on $x_i$ and $x_j$ by utilizing $P_{ij}$ and bounds $[\ell_{ij}, u_{ij}]$ on $X_{ij}$. The left plot shows $P_{ij}$, the middle plot shows the points $(x_i, x_j)$ that satisfy $x_i x_j \leq u_{ij}$, and the right plot the intersection of both sets. Optimizing in the unit directions of the intersection, i.e., minimizing and maximizing $x_i$ and $x_j$, is equivalent to optimizing over the red points.

1. **AFFECTED**: Since it is unclear whether and to what extent MINLPs in practice allow for a nontrivial projection $X_{ij}$, we first investigate empirically how many instances have a linear relaxation that provides inequalities of the form (16) that are not axis-parallel.

2. **ROOTGAP**: How much gap can be closed when using the stronger separation and propagation of bilinear terms only in the root node of a branch-and-bound tree with aggressive root separation settings?

3. **TREE**: How much do the presented techniques affect the solvability and performance of MINLPs in spatial branch-and-bound? For this experiment, we discuss suitable working limits on the number of LP iterations to solve the projections and investigate the performance impact of the stronger separation and propagation individually.

Our ideas are embedded in the MINLP solver SCIP [49]. We refer to [1, 56, 57] for an overview of the general solving algorithm and MINLP features of SCIP.

5.1 Experimental setup

For the AFFECTED and ROOTGAP experiments, we disable the LP iteration limit of the OBBT propagator, enable the aggressive separation emphasis setting, and disable restarts. The choices for the parameters ensure that the root node has been completely processed and there are no further reductions possible by applying OBBT again. Afterward, we use the current linear relaxation to compute the two-dimensional projections $P_{ij}$ as described in Algorithm 1. The projections are then used to strengthen the separation and propagation of constraints of the form $X_{ij} = x_i x_j$.

In contrast to the first two experiments, the TREE experiment is based on default settings. The projections are utilized at every node of the branch-and-bound tree. Note that the convex hull of the graph of $x_i x_j$ on $P_{ij}$ is in general not polyhedral. To prevent a potential slowdown caused by too many separation rounds, at local nodes of the branch-and-bound tree, i.e., not at the root node, we use the inequalities only twice for separation. Additionally, we use a limit on the total number of LP iterations in order to bound to computational cost of solving (15).

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1 In a restart, SCIP aborts the current search process and preprocesses the problem again. Per default, this only happens in the root node when enough variable bound reductions could be found. We refer to [1 Section 10.9] for more details about restarts.

2 SCIP settings propagating/obbt/itlimfactor = -1, limits/restart = 0, limits/totalnodes = 1, and separation/emphasis/aggressive = TRUE
Similarly to Gleixner et al. [21], a limit of three times the LP iterations that are spent so far at the root node is imposed.

For the AFFECTED and ROOTGAP experiments, we use a time limit of 7200s and a memory limit of 30 GB to ensure that for each instance the root node could be completely processed. For our TREE experiment, all instances run with a time limit of 1800s, a memory limit of 30 GB, and an optimality gap limit of $10^{-4}$ to reduce the impact of tailing-off effects.

**Implementation** We extended two existing plug-ins of SCIP: the OBBT propagator, which can now additionally compute the two-dimensional projections for variables that appear in a bilinear term $x_i x_j$; and a so-called nonlinear handler that calls Locatelli’s algorithm and the propagation techniques described in Section 4 for each $x_i x_j$ individually. Bilinear terms that only appear in convex constraints or contain binary variables are ignored in both steps. To reduce side effects, we use a separate working limit for solving the LPs (15) after applying standard OBBT. This is similar to the structure of Algorithm 1.

Using OBBT in a local node of the tree search results in a significant slowdown of SCIP. For this reason, by default, SCIP applies OBBT only in the root node of the branch-and-bound tree.

**Test set** We used the publicly available instances of the MINLPLib [42], which at time of the experiments contained 1682 instances. This includes among others instances from the first MINLPLib, the nonlinear programming library GLOBALLib, and the CMU-IBM initiative minlp.org [16]. We selected the instances that were available in OSiL format and consisted of nonlinear expressions that could be handled by SCIP: 1671 instances.

**Hardware and software** The experiments were performed on a cluster of 64bit Intel Xeon X5672 CPUs at 3.2 GHz with 12 MB cache and 48 GB main memory. In order to safeguard against a potential mutual slowdown of parallel processes, we ran only one job per node at a time. We used a development version of SCIP that is based on version 6.0 with CPLEX 12.8.0.0 as LP solver [28], CppAD 20180000.0 [19], and Ipopt 3.12.11 as NLP solver [58, 18] with Mumps 4.10.0 [4].

**Averages and statistical tests** In order to evaluate algorithmic performance over a large set of benchmark instances, we compare geometric means, which provide a measure for relative differences. This avoids results being dominated by outliers with large absolute values as is the case for the arithmetic mean. In order to also avoid an over-representation of differences among very small values, we use the shifted geometric mean. The shifted geometric mean of values $v_1, \ldots, v_N \geq 0$ with shift $s \geq 0$ is defined as

$$\left(\prod_{i=1}^{N} (v_i + s)\right)^{1/N} - s.$$

See also the discussion in [1, 2, 25]. We use a shift value of 100 for LP iterations and a value of one second for the solving time.

5.2 Computational results

In the following, we present results for the three above described experiments.
In order to quantify how many instances are potentially affected by our ideas, we use the number of bilinear terms for which a useful two-dimensional projection could be found after processing the root node. We prioritize bilinear terms that appear in multiple quadratic constraints. In our analysis this is captured by taking the occurrence of a bilinear term in the original MIQCP \((1)\) into account. Denote by

\[
K_{ij} := |\{k \in M \mid (Q_k)_{ij} \neq 0\}|
\]

the number of constraints in \((1)\) that contain \(x_i x_j\). The value \(\phi_{ij} \in \{0, 1\}\) indicates whether a useful projection could be found for \(x_i x_j\) or not. Then

\[
\Psi := \frac{\sum_{i,j} K_{ij} \phi_{ij}}{\sum_{i,j} K_{ij}} \in [0, 1]
\]

defines a measure for the effectiveness of an MIQCP. The interpretation of \(K_{ij}\) in the definition of \(\Psi\) is that each bilinear term \(x_i x_j\) is counted as a separate term of \((1)\).

Figure 10 shows the effectiveness on the instances of the MINLPLib, where instances with \(\Psi = 0\) are filtered out. Detailed results for all instances that contain at least one bilinear term are given in Table 3 of the electronic supplement. Out of the 1682, 464 do not contain a bilinear term or are solved before computing the two-dimensional projections. In total, 564 instances provide a relevant projection for at least one bilinear term, i.e., \(\Psi > 0\). There are 97 instances with an effectiveness between 0 – 5% and 82 instances with an effectiveness of 95 – 100%. The average effectiveness among all instances is 0.19 and 0.40 for the subset of instances that have a strictly positive effectiveness.

Note that although we do not use an exact algorithm for computing the projection, we obtain the same number of relevant instances because if no nontrivial facet was found then the box is the exact projection, i.e., \([\ell_i, u_i] \times [\ell_j, u_j] = X_{ij}\). To analyze the computational cost of computing all projections, we use the total number of LP iterations and the time spent for solving all LPs \((15)\). Computing all projections takes
on average 2.6 seconds and 4454.6 LP iterations. On instances that do not provide any useful projection, we observe on average 875.3 LP iterations and spend 1.0 seconds in computing the projections. This time can be considered to be a constant slow-down because we could not learn anything for these instances which could pay off in the remaining solution process. For instances with a strict positive effectiveness, we use on average 9374.3 LP iterations and 3.6 seconds.

We briefly report on the success of filtering candidates by exploiting previously computed LP optima in Line 5 and 12 of Algorithm 1. Out of all 1682 instances, we could filter candidates on 797 instances. On these instances, the filtering rate is on average 48.1% and 51.0% on the 564 selected instances.

Last, we report on the impact of finding nontrivial inequalities when applying Algorithm 1 multiple times in the root node. As discussed in Section 3.1, tighter projections could be found when refining $X$ after calling OBBT. Indeed, we observed a slight improvement in the success when recomputing the projections. The first bar of Figure 10 decreases from 97 to 87, which means that for 10 more instances a relevant projection could be found that could not be found before. The average effectiveness improves from 18.9% to 19.2% on all instances, and improves from 40.3% to 41.0% on the affected instances.

Even though there is a slight improvement in the success when recomputing the projections in the root node, we observed that the tighter projections have almost no impact on the dual bounds of the ROOTGAP experiment. Due to the fact that recomputing the projections can be expensive, we only use Algorithm 1 once in the root node.

ROOTGAP experiment Aggregated results for the ROOTGAP experiment are shown in Table 1 and visualized in Figure 11. We refer to Table 4 in the electronic supplement for detailed, instance-wise results.

From the potentially 564 affected instances of the previous experiment, we filtered out all instances that have been detected to be infeasible, no primal solution is known, or we could not prove any finite dual bound with the above described settings. This leaves 547 instances. Let $I := \{1,...,547\}$ be the index set of these instances.

Definition 1. Let $p \in \mathbb{R}$ be a valid primal bound and $d_1 \in \mathbb{R}$ and $d_2 \in \mathbb{R}$ be two dual bounds for (1), i.e., $d_1 \leq p$ and $d_2 \leq p$. The function $GC : \mathbb{R}^3 \rightarrow [-1,1]$ with

$$GC(p, d_1, d_2) := \begin{cases} 0, & \text{if } d_1 = d_2 \\ +1 - \frac{p - d_1}{p - d_2}, & \text{if } d_1 > d_2 \\ -1 + \frac{p - d_1}{p - d_2}, & \text{if } d_1 < d_2 \end{cases}$$

measures the gap closed improvement when comparing the distance of $d_1$ and $d_2$ to $p$.

Denote by $d_1^i$ and $d_2^i$ the dual bounds of instance $i \in I$ obtained with and without using the two-dimensional projections for separation and propagation. A reference primal bound $p^i$ is given by the best known bound for $i \in I$ in the MINLPLib. We use the gap-closed values for comparing the bounds $d_1^i$ and $d_2^i$ with respect to $p^i$. Note that $GC(d_1^i, d_2^i, p^i) = 1$ implies $d_1^i = p^i$ and $d_2^i < d_1^i$, which means that the instance could be solved in the root node to optimality when using the two-dimensional projections, but could not be solved to optimality in the root node without them.

Table 1 shows that using the projections for separation and propagation has a significant impact on the quality of the achieved dual bounds in the root node. On all 547 instances, the average gap closed improvement is 7.5%. The average improvement is 20.8% on 178 instances for which the gap closed values differ by at least 1%. Considering the affected instances with a minimum improvement or deterioration of 1% reveals that the dual bounds improve on 165 and
Figure 11: A bar diagram that visualizes the gap closed improvements for the 547 selected instances. Each bar maps to an interval with width 0.05 that corresponds to the gap closed improvement. The height of the bar displays the total number of instances that achieved a value in the corresponding interval.

```
| # instances | gap closed |
|-------------|------------|
| ALL         | 547        | 7.5%       |
| >1% change  | 178        | 20.8%      |
| >1% better  | 165        | 24.9%      |
| >1% worse   | 13         | -31.1%     |
```

Table 1: Aggregated results for the ROOTGAP experiments. The table shows the average gap closed values for different subsets of instances. ALL contains all instances, >1% better instances that improved by at least one percent, >1% worse instances that deteriorate by at least one percent, and >1% change is the union of instances in >1% better and >1% worse.

only get worse on 13 instances. The average gap improvement is 24.9% on the 165 instances and -31.1% on the 13 instances.

Next, we briefly report on the three instances in Figure 11 that have a gap closed value less than -80%. Those instances are crudeoil_lee4_05, crudeoil_lee4_06, and nuclear25b. The dual bounds obtained for both crudeoil instances are \( d_1 = 132.585 \) and \( d_2 = 132.548 \), and the dual bounds for nuclear25b are \( d_1 = -1.74673 \) and \( d_2 = -1.2208 \). The primal bounds are 132.548 for both crudeoil instances and -1.1136 for nuclear25b None of the three instances run into the time limit, which means that the differences in the dual bounds are caused by side effects or internal working limits in SCIP. Interestingly, it can be observed that SCIP applies 1.5 to 2 times more cutting planes when deactivating our developed methods for those three instances. However, we could not observe that the performance degradation is causally related to the new methods.

**TREE experiment** For the TREE experiment, we use five permutations for each of the 564 instances per setting in order to robustify the results against performance variability [30] [37].
Table 2: Aggregated results of SCIP on the 564 potentially affected instances of the MINLPLib. For each instance, five permutations including the default permutation have been solved. An instance is considered to be solved when all permutations of this could be solved by a setting. Three different settings for SCIP are used: default settings SCIP, SCIP+s for activating separation, and SCIP+s+p for activating separation and propagation for bilinear terms that provide a useful two-dimensional projection. The column “time” reports the change of solving time relative to SCIP+s+p.

|       | SCIP+s+p | SCIP+s | SCIP   |
|-------|----------|--------|--------|
| n     | # solved | # solved | time | # solved | time |
| ALL   | 564      | 249    | 247   | +0%     | 244  | +3%  |
| [1,tlim] | 166      | 159    | 158   | +1%     | 155  | +1%  |
| [10,tlim] | 109      | 102    | 101   | +2%     | 98   | +18% |
| [100,tlim] | 44       | 38     | 40    | −3%     | 33   | +36% |

permutation of an instance randomly changes the order of the variables and the constraints. This can have a large impact on the behavior and the performance of a MINLP solver. An instance is considered to be solved by a setting if all permutations could be solved by this setting. Hence, if a setting solves more instances it means that it could consistently solve more instances over all permutations. For comparing solving times between different settings, we use the shifted geometric mean with a shift value of one second for the five permutations of an instance and then consider the shifted geometric mean of all these values.

Aggregated results for the tree experiments are shown in Table 2 and more detailed results for each instance are contained in Table 5 of the electronic supplement. SCIP with its default settings solves 244 of the 564 instances. When activating the use of projections for separation 3 more instances are solved than with default SCIP; when activating it for both separation and propagation 5 more instances are solved. Considering the total time, we see that on average SCIP+s+p is 3% faster than SCIP. The groups [1,tlim], [10,tlim], and [100,tlim] are the subsets of instances for which at least one setting solved the instance in more than one, ten, or 100 seconds, respectively. These subsets form a hierarchy of increasingly difficult instance sets in an unbiased manner. Compared to SCIP, SCIP+s+p solves 4 more instances on [1,tlim], 4 more on [10,tlim], and 5 more on [100,tlim]. With respect to time, SCIP+s+p is 11% faster on [1,tlim], 18% on [10,tlim], and even 36% on [100,tlim] than SCIP.

A comparison of the second and the third column of Table 2 shows that both the separation and the propagation contribute to the larger number of solved instances. While activating separation alone does not improve the solving time, it can be seen that, more importantly, it does help to solve more instances in total.

6 Conclusion

In this article, we presented techniques to improve the separation and propagation of bilinear terms when solving MINLPs with spatial branch-and-bound and gave an extensive computational study on a large heterogeneous test set. Our ideas are based on projecting a linear relaxation onto two variables that appear bilinearly by solving a sequence of LPs that are similar to the ones in OBBT. Instead of computing the full projection, we compute a relaxation of the projection that is described by few inequalities. By applying known polyhedral results, we are able to strengthen
the separation of quadratic constraints by computing the convex and concave envelope of \( x_i x_j \) on the two-dimensional projections. Additionally, we presented that the projections also enables us to tighten variable bounds. Computing the best possible bounds of \( x_i, x_j, \) and \( x_i x_j \) on the projection is in general a nonconvex optimization problem. We proved that these problems can be efficiently solved by computing a discrete set of points. This allows us to efficiently solve these optimization problems at every node of the branch-and-bound tree.

Our experiments on the publicly available instances of the MINLPLib based on an implementation in the MINLP solver SCIP show that 564 of the 1682 instances provide nontrivial projections for at least one bilinear term. On these instances, it was possible to compute useful projections for 40.3\% of all bilinear terms. When using the projection exhaustively during the separation of the root node, we observed an improvement of the achieved dual bounds on 165 and a deterioration on only 13 instances. The average gap closed improvement on all instances for which a change of at least one percent could be observed is 20.8\%. Finally, our tree experiments showed that the new techniques improve performance by 36\% on difficult instances and enable us to consistently solve more instances.

There are two interesting extensions of the presented methods. First, our propagation techniques do not only apply to polyhedral projections but also for general two-dimensional convex sets. How to compute these convex sets efficiently by using a convex relaxation of a MINLP remains an open question. Second, for models that contain symmetric structures the tightness of the two-dimensional projections and the performance improvements gained might profit particularly from symmetry-breaking constraints of the form \( x_i \leq x_j \). These inequalities are in general not implied by a linear relaxation, but can be derived by considering formulation symmetry [34].

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# Notes

Table 3: Detailed results for the effectiveness of instances of the MINLPLib2 that contain bilinear terms. All instances for which Algorithm 1 has not been called are filtered out.

| Instance                  | $\sum_{i,j} K_{ij} \phi_{ij}$ | $\sum_{i,j} K_{ij}$ | # LPs | # iters | # filtered | time (in seconds) |
|---------------------------|--------------------------------|----------------------|-------|---------|------------|-------------------|
| spar30-100-1              | 0                              | 427                  | 376   | 29724   | 878        | 0.69              |
| spar30-100-2              | 0                              | 430                  | 493   | 43747   | 692        | 1.62              |
| spar30-100-3              | 0                              | 428                  | 375   | 30584   | 912        | 0.71              |
| spar30-60-1               | 0                              | 250                  | 178   | 10125   | 608        | 0.19              |
| spar30-60-2               | 11                             | 240                  | 313   | 16239   | 260        | 0.60              |
| spar30-60-3               | 0                              | 288                  | 410   | 22576   | 298        | 0.47              |
| spar30-70-1               | 0                              | 290                  | 246   | 16683   | 600        | 0.51              |
| spar30-70-2               | 0                              | 288                  | 376   | 22727   | 351        | 0.69              |
| spar30-70-3               | 0                              | 312                  | 390   | 26509   | 445        | 0.89              |
| spar30-80-1               | 0                              | 343                  | 248   | 18845   | 819        | 0.64              |
| spar30-80-2               | 0                              | 330                  | 334   | 21244   | 584        | 0.79              |
| spar30-80-3               | 0                              | 349                  | 599   | 42979   | 197        | 1.01              |
| spar30-90-1               | 0                              | 373                  | 259   | 20843   | 927        | 0.68              |
| spar30-90-2               | 0                              | 391                  | 455   | 36242   | 612        | 0.82              |
| spar30-90-3               | 0                              | 377                  | 443   | 33980   | 557        | 0.77              |
| genpool04                 | 2                              | 96                   | 68    | 17445   | 124        | 0.51              |
| genpool04i                | 2                              | 48                   | 72    | 16228   | 120        | 0.37              |
| genpool04paper            | 2                              | 96                   | 78    | 22765   | 114        | 0.66              |
| genpool10                 | 0                              | 600                  | 358   | 131149  | 842        | 8.69              |
| genpool10i                | 0                              | 300                  | 373   | 210402  | 827        | 17.02             |
| genpool10paper            | 0                              | 600                  | 365   | 130411  | 835        | 8.71              |
| genpool15                 | 6                              | 1350                 | 706   | 151715  | 1994       | 5.28              |
| genpool15i                | 1                              | 675                  | 858   | 1283245 | 1842       | 259.84            |
| genpool15paper            | 6                              | 1350                 | 703   | 83492   | 1997       | 2.12              |
| genpool20                 | 2                              | 2520                 | 1275  | 342223  | 3765       | 18.79             |
| genpool20i                | 2                              | 1260                 | 1375  | 2906822 | 3665       | 734.85            |
| genpool20paper            | 2                              | 2520                 | 1272  | 369178  | 3768       | 18.44             |
| mpss-basic-marvin-85-85    | 17                             | 32                   | 82    | 215187  | 46         | 84.02             |
| mpss-basic-ob25-125-125    | 25                             | 48                   | 122   | 543425  | 70         | 421.12            |
| mpss-basic-red-marvin-85-85| 17                             | 32                   | 82    | 200911  | 46         | 61.60             |
| mpss-basic-red-ob25-125-125| 25                             | 48                   | 123   | 417319  | 69         | 257.45            |
| mpss-extwarehouse-marvin-85-85| 108                         | 2414                 | 1918  | 1170408 | 7738       | 735.45            |
| mpss-extwarehouse-ob25-125-125| 50                          | 5428                 | 1011  | 1308812 | 16928      | 2315.00           |
| mpss-extwarehouse-red-ob25-125| 34                          | 5388                 | 711   | 1003194 | 16628      | 1294.87           |
| 4stufen                   | 13                             | 35                   | 86    | 2586    | 26         | 0.04              |
| alkyl                     | 4                              | 9                    | 24    | 167     | 10         | 0.00              |
| alkylation                | 2                              | 6                    | 13    | 90      | 6          | 0.01              |
| arki0003                  | 0                              | 360                  | 57    | 232     | 25         | 0.19              |
| Instance          | $\sum_{i,j} K_{ij}$ | $\sum_{i,j} \phi_{ij}$ | # LPs | # iters | # filtered | time  |
|-------------------|---------------------|-------------------------|-------|---------|------------|-------|
| arki0004          | 0                   | 5200                    | 2133  | 8072    | 6303       | 3.55  |
| arki0005          | 672                 | 3360                    | 48    | 52935   | 28         | 25.00 |
| arki0008          | 0                   | 2299                    | 583   | 422386  | 1305       | 226.68|
| arki0009          | 0                   | 90                      | 270   | 630     | 90         | 7.54  |
| arki0010          | 0                   | 45                      | 135   | 315     | 45         | 1.53  |
| arki0011          | 0                   | 135                     | 405   | 945     | 135        | 134.37|
| arki0012          | 0                   | 135                     | 405   | 945     | 135        | 173.27|
| arki0013          | 0                   | 135                     | 405   | 945     | 135        | 203.41|
| arki0014          | 0                   | 135                     | 405   | 945     | 135        | 161.04|
| arki0015          | 242                 | 704                     | 1634  | 835612  | 1182       | 831.92|
| arki0016          | 910                 | 4634                    | 12107 | 849097  | 3874       | 428.02|
| arki0017          | 562                 | 4027                    | 5383  | 343160  | 8636       | 127.38|
| arki0018          | 39                  | 9804                    | 10136 | 24930   | 19314      | 91.81 |
| arki0019          | 494                 | 1018                    | 839   | 939037  | 1886       | 182.31|
| arki0020          | 5                   | 2522                    | 2518  | 6849376 | 4081       | 2412.48|
| arki0022          | 35                  | 8302                    | 83    | 1408754 | 6805       | 5769.45|
| arki0024          | 424                 | 3452                    | 2486  | 189804  | 1742       | 39.97 |
| autocorr          | 0                   | 595                     | 176   | 16027   | 1758       | 2.84  |
| batch0812 nc      | 16                  | 37                      | 45    | 1648    | 69         | 0.03  |
| batch nc          | 17                  | 38                      | 39    | 1757    | 79         | 0.06  |
| bayes2.10         | 167                 | 382                     | 734   | 9412    | 794        | 0.21  |
| bayes2.20         | 199                 | 385                     | 739   | 18121   | 801        | 0.36  |
| bayes2.30         | 209                 | 385                     | 684   | 17985   | 856        | 0.38  |
| bayes2.50         | 192                 | 385                     | 846   | 24357   | 694        | 0.35  |
| bcocco05          | 9                   | 15                      | 31    | 472     | 16         | 0.02  |
| bcocco06          | 10                  | 21                      | 40    | 692     | 26         | 0.02  |
| bcocco07          | 19                  | 30                      | 58    | 3137    | 32         | 0.14  |
| bcocco08          | 10                  | 39                      | 78    | 1966    | 39         | 0.11  |
| beuster           | 7                   | 62                      | 74    | 4408    | 50         | 0.09  |
| blend029          | 22                  | 40                      | 60    | 2318    | 52         | 0.06  |
| blend146          | 76                  | 168                     | 163   | 14490   | 253        | 1.24  |
| blend480          | 66                  | 248                     | 251   | 68194   | 357        | 3.81  |
| blend531          | 84                  | 204                     | 209   | 14008   | 263        | 0.74  |
| blend718          | 92                  | 160                     | 166   | 21204   | 234        | 1.04  |
| blend721          | 64                  | 168                     | 162   | 20502   | 254        | 1.01  |
| blend852          | 80                  | 248                     | 234   | 29613   | 374        | 1.96  |
| btest14           | 0                   | 114                     | 19    | 220     | 277        | 0.02  |
| camcns            | 59                  | 282                     | 517   | 233911  | 39         | 11.03 |
| camshape100       | 101                 | 296                     | 268   | 6701    | 246        | 0.14  |
| camshape200       | 201                 | 596                     | 528   | 21711   | 527        | 0.64  |
| camshape400       | 398                 | 1196                    | 1044  | 77967   | 1094       | 1.82  |
| camshape800       | 787                 | 2396                    | 2075  | 309133  | 2232       | 10.31 |
| carton7           | 57                  | 168                     | 45    | 6177    | 31         | 0.19  |
| carton9           | 75                  | 207                     | 61    | 3976    | 39         | 0.25  |
| casctanks         | 52                  | 267                     | 337   | 16091   | 289        | 0.84  |
| case lscv2        | 147                 | 1792                    | 1614  | 1174876 | 1161       | 149.96|
| Instance       | $\sum_{i,j} K_{ij}\phi_{ij}$ | $\sum_{i,j} K_{ij}$ | # LPs | # iters | # filtered | time  |
|---------------|-------------------------------|---------------------|-------|---------|------------|-------|
| cesam2log     | 161                           | 185                 | 310   | 8280    | 357        | 0.64  |
| chain100      | 99                            | 99                  | 101   | 102570  | 97         | 6.91  |
| chain200      | 199                           | 199                 | 203   | 414072  | 196        | 26.95 |
| chain400      | 399                           | 399                 | 406   | 2640373 | 392        | 425.25|
| chain50       | 49                            | 49                  | 51    | 12615   | 47         | 0.56  |
| chem          | 10                            | 20                  | 39    | 565     | 1          | 0.01  |
| chenery        | 8                             | 22                  | 40    | 815     | 32         | 0.02  |
| chp_partload   | 572                           | 1041                | 2515  | 1860625 | 595        | 575.41|
| chp_shorttermplan2b | 0                              | 960                | 310   | 72032   | 319        | 17.40 |
| chp_shorttermplan2d | 261                          | 1632                | 351   | 83074   | 423        | 56.41 |
| clay0203h     | 36                            | 120                 | 56    | 3141    | 21         | 0.10  |
| clay0204h     | 52                            | 160                 | 79    | 7528    | 20         | 0.38  |
| clay0205h     | 36                            | 200                 | 97    | 16328   | 28         | 0.96  |
| clay0303h     | 0                             | 180                 | 52    | 1530    | 47         | 0.08  |
| clay0304h     | 0                             | 240                 | 63    | 4337    | 59         | 0.26  |
| clay0305h     | 0                             | 300                 | 87    | 7431    | 78         | 0.41  |
| contvar       | 38                            | 104                 | 168   | 33084   | 81         | 2.31  |
| crudeoill ee05 | 45                           | 48                  | 107   | 6223    | 85         | 0.21  |
| crudeoill ee06 | 60                           | 60                  | 114   | 35898   | 126        | 2.22  |
| crudeoill ee07 | 72                           | 72                  | 133   | 28405   | 155        | 2.18  |
| crudeoill ee08 | 83                           | 84                  | 149   | 80563   | 187        | 3.84  |
| crudeoill ee09 | 95                           | 96                  | 169   | 50603   | 215        | 4.63  |
| crudeoill ee10 | 108                          | 108                 | 191   | 123973  | 241        | 13.94 |
| crudeoill ee05 | 94                           | 106                 | 232   | 93047   | 192        | 4.09  |
| crudeoill ee06 | 128                          | 134                 | 278   | 156948  | 258        | 15.09 |
| crudeoill ee07 | 157                          | 162                 | 353   | 313322  | 295        | 33.52 |
| crudeoill ee08 | 184                          | 190                 | 404   | 197679  | 356        | 18.43 |
| crudeoill ee09 | 212                          | 218                 | 444   | 210673  | 428        | 22.26 |
| crudeoill ee10 | 240                          | 246                 | 505   | 382964  | 479        | 45.80 |
| crudeoill ee05 | 165                          | 212                 | 446   | 177718  | 402        | 16.99 |
| crudeoill ee06 | 220                          | 282                 | 615   | 476446  | 513        | 44.64 |
| crudeoill ee07 | 272                          | 352                 | 804   | 424006  | 604        | 39.58 |
| crudeoill ee08 | 327                          | 422                 | 986   | 1145102 | 702        | 105.02|
| crudeoill ee09 | 378                          | 492                 | 1149  | 1423496 | 819        | 244.22|
| crudeoill ee10 | 431                          | 562                 | 1322  | 1369558 | 926        | 279.00|
| crudeoill ee05 | 120                          | 146                 | 336   | 106502  | 248        | 7.57  |
| crudeoill ee06 | 145                          | 184                 | 405   | 242809  | 299        | 27.42 |
| crudeoill ee07 | 189                          | 222                 | 504   | 315668  | 384        | 52.81 |
| crudeoill ee08 | 231                          | 260                 | 606   | 555462  | 434        | 64.00 |
| crudeoill ee09 | 265                          | 298                 | 651   | 616486  | 541        | 134.83|
| crudeoill ee10 | 296                          | 336                 | 739   | 890562  | 605        | 210.96|
| crudeoil li01 | 39                            | 56                  | 146   | 18677   | 78         | 0.73  |
| crudeoil li02 | 0                             | 15                  | 54    | 50677   | 6          | 4.42  |
| crudeoil li03 | 59                            | 192                 | 405   | 227910  | 363        | 22.54 |
| crudeoil li05 | 58                            | 192                 | 413   | 259797  | 355        | 18.41 |
| crudeoil li06 | 39                            | 192                 | 391   | 160716  | 377        | 18.05 |
## Table 3 continued

| Instance            | $\sum_{i,j} k_{ij} \phi_{ij}$ | $\sum_{i,j} k_{ij}$ | # LPs | # iters | # filtered | time  |
|---------------------|-------------------------------|---------------------|-------|---------|------------|-------|
| crudeoil_li11       | 47                            | 192                 | 390   | 225903  | 378        | 25.07 |
| crudeoil_li21       | 50                            | 192                 | 360   | 332654  | 408        | 44.15 |
| crudeoil_pooling_ct1| 9                             | 64                  | 135   | 17690   | 65         | 0.54  |
| crudeoil_pooling_ct2| 0                             | 70                  | 149   | 13541   | 131        | 0.62  |
| crudeoil_pooling_ct3| 12                            | 223                 | 324   | 159213  | 485        | 9.49  |
| crudeoil_pooling_ct4| 0                             | 95                  | 164   | 26917   | 216        | 1.80  |
| crudeoil_pooling_dt1| 10                            | 570                 | 1415  | 3316265 | 865        | 710.29|
| crudeoil_pooling_dt2| 215                           | 1106                | 2277  | 5868404 | 2147       | 2682.20|
| crudeoil_pooling_dt3| 12                            | 2707                | 3942  | 5403108 | 6886       | 3948.09|
| crudeoil_pooling_dt4| 147                           | 1121                | 2221  | 6070617 | 2263       | 1746.77|
| csched1             | 5                             | 7                   | 17    | 274     | 11         | 0.00  |
| csched1a            | 0                             | 7                   | 13    | 347     | 4          | 0.00  |
| csched2             | 16                            | 57                  | 102   | 24762   | 126        | 0.34  |
| csched2a            | 24                            | 57                  | 78    | 5105    | 67         | 0.25  |
| deb6                | 62                            | 88                  | 118   | 6152    | 58         | 1.35  |
| deb7                | 62                            | 176                 | 116   | 5958    | 60         | 1.18  |
| deb8                | 52                            | 176                 | 122   | 10403   | 54         | 1.62  |
| deb9                | 62                            | 176                 | 116   | 5958    | 60         | 1.18  |
| dispatch            | 0                             | 3                   | 12    | 62      | 0          | 0.00  |
| edgecross10-060     | 0                             | 982                 | 20    | 2679    | 69         | 0.38  |
| edgecross14-039     | 0                             | 625                 | 67    | 2645    | 454        | 0.56  |
| elec100             | 0                             | 14850               | 12235 | 98554   | 44955      | 575.46|
| elec25              | 0                             | 900                 | 824   | 21707   | 2586       | 2.86  |
| elec50              | 0                             | 3675                | 3529  | 182404  | 10608      | 45.22 |
| elf                 | 3                             | 3                   | 8     | 328     | 4          | 0.02  |
| eq6.1               | 0                             | 92                  | 74    | 1396    | 136        | 0.05  |
| etamac              | 0                             | 9                   | 24    | 392     | 12         | 0.03  |
| ethanolh            | 1                             | 4                   | 11    | 239     | 1          | 0.00  |
| ethanolm            | 1                             | 4                   | 7     | 183     | 5          | 0.01  |
| ex1226              | 1                             | 1                   | 1     | 7       | 1          | 0.00  |
| ex1233              | 0                             | 12                  | 7     | 190     | 21         | 0.01  |
| ex1243              | 0                             | 12                  | 16    | 618     | 13         | 0.03  |
| ex1244              | 0                             | 17                  | 24    | 1038    | 19         | 0.02  |
| ex1252              | 5                             | 15                  | 30    | 284     | 30         | 0.02  |
| ex1252a             | 7                             | 15                  | 27    | 330     | 33         | 0.01  |
| ex1263              | 0                             | 16                  | 20    | 2947    | 12         | 0.07  |
| ex1263a             | 3                             | 16                  | 24    | 457     | 8          | 0.00  |
| ex1264              | 1                             | 16                  | 16    | 1204    | 30         | 0.02  |
| ex1264a             | 0                             | 16                  | 22    | 633     | 16         | 0.01  |
| ex1265              | 2                             | 25                  | 33    | 2550    | 22         | 0.04  |
| ex1265a             | 3                             | 25                  | 31    | 1230    | 21         | 0.02  |
| ex1266              | 0                             | 36                  | 36    | 1859    | 70         | 0.03  |
| ex14_1_2            | 6                             | 23                  | 11    | 177     | 5          | 0.00  |
| ex14_1_3            | 2                             | 2                   | 3     | 14      | 1          | 0.00  |
| ex14_1_6            | 0                             | 10                  | 3     | 31      | 9          | 0.00  |
| ex14_1_8            | 4                             | 10                  | 14    | 149     | 6          | 0.00  |
| Instance | $\sum_{i,j} K_{ij}$ | $\sum_{i,j} \phi_{ij}$ | # LPs | # iters | # filtered | time |
|-----------|-----------------|-----------------|-------|---------|------------|------|
| ex2_1_9   | 22              | 22              | 9     | 65      | 3          | 0.01 |
| ex3_1_1   | 4               | 5               | 9     | 65      | 3          | 0.00 |
| ex3_1_2   | 9               | 9               | 15    | 120     | 1          | 0.00 |
| ex3_1_4   | 2               | 3               | 3     | 16      | 5          | 0.00 |
| ex5_2_2_case1 | 4       | 4               | 6     | 37      | 2          | 0.00 |
| ex5_2_2_case2 | 4       | 4               | 6     | 68      | 2          | 0.01 |
| ex5_2_2_case3 | 4       | 4               | 6     | 38      | 2          | 0.00 |
| ex6_2_4   | 4               | 14              | 8     | 99      | 9          | 0.00 |
| ex6_2_5   | 9               | 195             | 29    | 544     | 171        | 0.00 |
| ex5_3_2   | 12              | 12              | 28    | 255     | 20         | 0.01 |
| ex5_3_3   | 32              | 103             | 105   | 1352    | 175        | 0.04 |
| ex5_4_2   | 4               | 5               | 9     | 80      | 3          | 0.00 |
| ex5_4_3   | 7               | 10              | 22    | 250     | 6          | 0.01 |
| ex5_4_4   | 1               | 18              | 30    | 320     | 24         | 0.00 |
| ex6_1_1   | 2               | 18              | 30    | 266     | 25         | 0.00 |
| ex6_1_2   | 4               | 7               | 15    | 88      | 3          | 0.00 |
| ex6_1_3   | 3               | 39              | 63    | 475     | 51         | 0.03 |
| ex6_1_4   | 0               | 15              | 28    | 362     | 8          | 0.02 |
| ex6_2_10  | 15              | 51              | 95    | 1886    | 46         | 0.06 |
| ex6_2_11  | 8               | 24              | 45    | 686     | 10         | 0.01 |
| ex6_2_12  | 6               | 26              | 50    | 744     | 22         | 0.02 |
| ex6_2_13  | 8               | 37              | 69    | 1468    | 34         | 0.02 |
| ex6_2_14  | 11              | 34              | 63    | 703     | 31         | 0.03 |
| ex6_2_5   | 12              | 30              | 34    | 590     | 35         | 0.02 |
| ex6_2_6   | 12              | 12              | 24    | 261     | 0          | 0.01 |
| ex6_2_7   | 3               | 27              | 42    | 2409    | 20         | 0.03 |
| ex6_2_8   | 9               | 9               | 18    | 172     | 0          | 0.00 |
| ex6_2_9   | 10              | 34              | 62    | 1415    | 32         | 0.03 |
| ex7_2_1   | 2               | 11              | 13    | 255     | 7          | 0.01 |
| ex7_2_2   | 4               | 4               | 13    | 106     | 3          | 0.00 |
| ex7_2_3   | 0               | 4               | 4     | 44      | 7          | 0.00 |
| ex7_2_4   | 5               | 10              | 16    | 304     | 8          | 0.01 |
| ex7_3_1   | 0               | 11              | 22    | 95      | 7          | 0.01 |
| ex7_3_2   | 0               | 2               | 1     | 0       | 0          | 0.00 |
| ex7_3_3   | 0               | 3               | 8     | 24      | 0          | 0.00 |
| ex7_3_4   | 0               | 13              | 20    | 76      | 0          | 0.00 |
| ex7_3_5   | 0               | 20              | 7     | 21      | 2          | 0.00 |
| ex8_1_1   | 1               | 2               | 3     | 23      | 1          | 0.00 |
| ex8_1_7   | 0               | 1               | 2     | 26      | 2          | 0.01 |
| ex8_2_1b  | 0               | 50              | 100   | 1889    | 0          | 0.04 |
| ex8_2_2b  | 0               | 6156            | 8721  | 284173  | 5629       | 33.43 |
| ex8_2_3b  | 1               | 9065            | 10878 | 317649  | 7252       | 31.03 |
| ex8_2_4b  | 1               | 106             | 212   | 4847    | 18         | 0.16 |
| ex8_2_5b  | 0               | 12312           | 18468 | 966240  | 9681       | 83.38 |
| ex8_3_1   | 0               | 229             | 221   | 1721    | 535        | 0.09 |
| ex8_3_11  | 0               | 229             | 226   | 1916    | 530        | 0.07 |
| Instance | $\sum_{i,j} K_{ij} \phi_{ij}$ | $\sum_{i,j} K_{ij}$ | # LPs | # iters | # filtered | time   |
|----------|-------------------------------|-------------------|-------|---------|------------|--------|
| ex8_3.12 | 0                             | 244               | 204   | 3000    | 492        | 0.08   |
| ex8_3.13 | 0                             | 214               | 214   | 1783    | 482        | 0.08   |
| ex8_3.14 | 0                             | 214               | 208   | 6532    | 468        | 0.28   |
| ex8_3.2  | 0                             | 214               | 203   | 1175    | 513        | 0.05   |
| ex8_3.3  | 0                             | 214               | 202   | 1807    | 514        | 0.05   |
| ex8_3.4  | 0                             | 214               | 204   | 1748    | 512        | 0.08   |
| ex8_3.5  | 0                             | 214               | 212   | 2200    | 504        | 0.10   |
| ex8_3.7  | 0                             | 271               | 280   | 1435    | 622        | 0.08   |
| ex8_3.8  | 3                             | 269               | 257   | 2548    | 639        | 0.12   |
| ex8_3.9  | 0                             | 107               | 121   | 2092    | 247        | 0.07   |
| ex8_4.1  | 10                            | 10                | 40    | 1067    | 0          | 0.04   |
| ex8_4.2  | 8                             | 30                | 62    | 3235    | 58         | 0.09   |
| ex8_4.4  | 10                            | 12                | 41    | 1518    | 7          | 0.03   |
| ex8_4.5  | 14                            | 22                | 14    | 164     | 2          | 0.00   |
| ex8_4.6  | 0                             | 32                | 56    | 120     | 56         | 0.01   |
| ex8_4.7  | 0                             | 50                | 98    | 5689    | 22         | 0.11   |
| ex8_4.8.bnd | 10                       | 20               | 29    | 5894    | 15         | 0.12   |
| ex8_5.1  | 3                             | 12                | 8     | 12      | 26         | 0.01   |
| ex8_5.2  | 0                             | 8                 | 2     | 0       | 0          | 0.00   |
| ex8_5.5  | 0                             | 6                 | 1     | 0       | 1          | 0.00   |
| ex8_6.1  | 0                             | 85                | 85    | 107     | 255        | 0.02   |
| ex8_6.2  | 0                             | 85                | 93    | 2937    | 247        | 0.08   |
| ex9_1.2  | 0                             | 4                 | 5     | 0       | 3          | 0.00   |
| ex9_2.2  | 1                             | 2                 | 5     | 11      | 3          | 0.00   |
| ex9_2.3  | 6                             | 6                 | 16    | 89      | 8          | 0.01   |
| ex9_2.4  | 2                             | 2                 | 8     | 57      | 0          | 0.01   |
| ex9_2.5  | 0                             | 3                 | 3     | 0       | 3          | 0.00   |
| ex9_2.6  | 4                             | 6                 | 9     | 49      | 15         | 0.00   |
| ex9_2.7  | 0                             | 4                 | 12    | 67      | 4          | 0.00   |
| fdesign10 | 0                          | 1                 | 1     | 61      | 2          | 0.01   |
| fdesign25 | 0                          | 1                 | 1     | 137     | 2          | 0.00   |
| feedtray | 89                            | 259               | 617   | 36384   | 99         | 2.26   |
| filter   | 0                             | 3                 | 5     | 41      | 3          | 0.01   |
| fin2bb   | 21                            | 61                | 163   | 90885   | 1          | 5.72   |
| forest   | 27                            | 90                | 203   | 7570    | 157        | 0.17   |
| gabriel01 | 102                        | 336               | 333   | 49167   | 499        | 3.23   |
| gabriel02 | 220                        | 672               | 685   | 172756  | 979        | 23.59  |
| gabriel04 | 234                        | 992               | 909   | 366261  | 1523       | 46.82  |
| gabriel05 | 468                        | 1704              | 1746  | 342590  | 2382       | 79.47  |
| gabriel06 | 1301                       | 6112              | 6459  | 4493136 | 8261       | 3371.62|
| gabriel07 | 1638                       | 7640              | 7541  | 4339658 | 10859      | 3372.07|
| gabriel09 | 914                        | 5688              | 4392  | 1762991 | 7992       | 1425.87|
| gams02   | 174                         | 192               | 530   | 64798   | 238        | 2.19   |
| gams03   | 47                          | 53040             | 528   | 307254  | 150694     | 561.69 |
| gancns   | 5                           | 214               | 181   | 11647   | 129        | 0.72   |
| gasnet   | 14                          | 41                | 58    | 3054    | 66         | 0.11   |
| Instance               | $\sum_{i,j} K_{ij} \phi_{ij}$ | $\sum_{i,j} K_{ij}$ | # LPs | # iters | # filtered | time |
|-----------------------|-------------------------------|---------------------|-------|---------|------------|------|
| gasnet_al1            | 10                            | 145                 | 57    | 1348    | 31         | 0.29 |
| gasnet_al2            | 10                            | 145                 | 56    | 1333    | 32         | 0.15 |
| gasnet_al3            | 10                            | 145                 | 57    | 1210    | 31         | 0.16 |
| gasnet_al4            | 10                            | 145                 | 57    | 1406    | 31         | 0.20 |
| gasnet_al5            | 10                            | 145                 | 56    | 1217    | 32         | 0.21 |
| gasprod_sarawak01     | 3                             | 34                  | 88    | 7159    | 48         | 0.13 |
| gasprod_sarawak16     | 169                           | 544                 | 1514  | 470279  | 662        | 46.80 |
| gasprod_sarawak81     | 846                           | 2754                | 7094  | 5117307 | 3922       | 1523.17 |
| gastrans135           | 105                           | 228                 | 498   | 22679   | 169        | 3.98 |
| gastrans582_cold13    | 114                           | 221                 | 257   | 11320   | 71         | 1.76 |
| gastrans582_cold13_95| 114                           | 221                 | 255   | 11686   | 73         | 1.80 |
| gastrans582_cold17    | 120                           | 222                 | 289   | 11293   | 80         | 1.41 |
| gastrans582_cold17_95| 120                           | 222                 | 293   | 10928   | 76         | 1.25 |
| gastrans582_cool12    | 120                           | 221                 | 277   | 9241    | 86         | 2.15 |
| gastrans582_cool12_95| 111                           | 221                 | 278   | 10874   | 78         | 2.15 |
| gastrans582_cool14    | 111                           | 220                 | 283   | 8321    | 73         | 1.80 |
| gastrans582_cool14_95| 111                           | 220                 | 310   | 11716   | 65         | 2.69 |
| gastrans582_freezing27| 120                           | 221                 | 307   | 11652   | 68         | 1.57 |
| gastrans582_freezing27_95| 120               | 221                 | 317   | 13436   | 66         | 3.20 |
| gastrans582_freezing30| 120                           | 221                 | 322   | 10444   | 61         | 1.88 |
| gastrans582_mild10    | 111                           | 219                 | 269   | 8309    | 69         | 1.38 |
| gastrans582_mild10_95| 112                           | 219                 | 271   | 7451    | 68         | 1.55 |
| gastrans582_mild11    | 112                           | 220                 | 266   | 12155   | 78         | 1.40 |
| gastrans582_mild11_95| 111                           | 220                 | 269   | 12152   | 75         | 1.27 |
| gastrans582_warm15    | 115                           | 222                 | 264   | 10302   | 66         | 1.13 |
| gastrans582_warm15_95| 115                           | 222                 | 260   | 11044   | 70         | 2.02 |
| gastrans582_warm31    | 114                           | 224                 | 255   | 10034   | 81         | 1.38 |
| gastrans582_warm31_95| 114                           | 224                 | 266   | 9873    | 71         | 1.60 |
| genpooling_lee1       | 30                             | 72                  | 41    | 2765    | 55         | 0.06 |
| genpooling_lee2       | 45                             | 108                 | 56    | 5544    | 88         | 0.12 |
| genpooling_meyer04    | 0                              | 96                  | 69    | 17739   | 123        | 0.43 |
| genpooling_meyer10    | 4                              | 600                 | 376   | 211058  | 824        | 17.09 |
| genpooling_meyer15    | 0                              | 1350                | 740   | 942004  | 1960       | 123.95 |
| ghg_1veh              | 47                             | 63                  | 47    | 2601    | 24         | 0.06 |
| ghg_2veh              | 3                              | 148                 | 101   | 4668    | 63         | 0.12 |
| ghg_3veh              | 97                             | 255                 | 263   | 22022   | 41         | 0.95 |
| glider100             | 0                              | 1102                | 201   | 0       | 506        | 0.01 |
| glider200             | 0                              | 2202                | 401   | 0       | 1006       | 0.04 |
| glider400             | 0                              | 4402                | 801   | 0       | 2006       | 0.17 |
| glider50              | 0                              | 552                 | 101   | 0       | 256        | 0.00 |
| gsg_0001              | 18                             | 20                  | 80    | 2304    | 0          | 0.05 |
| haverly               | 0                              | 4                   | 3     | 0       | 1          | 0.00 |
| heatexch_gen1         | 4                              | 72                  | 77    | 2480    | 99         | 0.08 |
| heatexch_gen2         | 0                              | 67                  | 70    | 3032    | 98         | 0.17 |
| heatexch_gen3         | 6                              | 420                 | 373   | 120388  | 667        | 4.88 |
| Instance               | $\sum_{i,j} K_{ij} \phi_{ij}$ | $\sum_{i,j} K_{ij}$ | # LPs | # iters | # filtered | time |
|------------------------|-------------------------------|---------------------|-------|---------|------------|------|
| heatexch_spec1         | 0                             | 12                  | 8     | 232     | 17         | 0.02 |
| heatexch_spec2         | 0                             | 16                  | 11    | 865     | 25         | 0.07 |
| heatexch_spec3         | 0                             | 60                  | 25    | 5168    | 114        | 0.32 |
| heatexch_trigem        | 0                             | 18                  | 41    | 650     | 31         | 0.03 |
| hhfair                 | 0                             | 9                   | 6     | 9       | 0          | 0.00 |
| himmel11               | 9                             | 9                   | 24    | 212     | 2          | 0.00 |
| himmel16               | 0                             | 7                   | 20    | 345     | 8          | 0.03 |
| house                  | 0                             | 5                   | 7     | 0       | 0          | 0.00 |
| hs62                   | 0                             | 6                   | 4     | 19      | 11         | 0.00 |
| hvb11                  | 0                             | 32                  | 60    | 9550    | 4          | 0.32 |
| hybriddynamic_fixedcc  | 0                             | 20                  | 14    | 55      | 31         | 0.01 |
| hybriddynamic_var      | 7                             | 13                  | 22    | 413     | 6          | 0.01 |
| hybriddynamic_varcc    | 0                             | 68                  | 20    | 41      | 126        | 0.00 |
| hydroenergy1           | 40                            | 69                  | 179   | 15287   | 97         | 0.75 |
| hydroenergy2           | 42                            | 161                 | 364   | 100952  | 280        | 4.51 |
| hydroenergy3           | 66                            | 299                 | 713   | 244963  | 483        | 19.74|
| infeas1                | 1148                          | 1237                | 295   | 119888  | 121        | 9.50 |
| kall_circles_c6a       | 16                            | 43                  | 75    | 1647    | 57         | 0.03 |
| kall_circles_c6b       | 16                            | 43                  | 69    | 1158    | 76         | 0.02 |
| kall_circles_c6c       | 16                            | 57                  | 96    | 2667    | 80         | 0.07 |
| kall_circles_c7a       | 23                            | 57                  | 78    | 1742    | 102        | 0.04 |
| kall_circles_c8a       | 36                            | 73                  | 110   | 3036    | 110        | 0.05 |
| kall_circlespolygons_c1p12 | 3                         | 19                  | 36    | 958     | 40         | 0.01 |
| kall_circlespolygons_c1p13 | 3                       | 19                  | 47    | 855     | 29         | 0.01 |
| kall_circlespolygons_c1p5a | 4                         | 101                 | 217   | 4966    | 187        | 0.25 |
| kall_circlespolygons_c1p5b | 10                        | 611                 | 1242  | 40167   | 1202       | 4.18 |
| kall_circlespolygons_c1p6a | 11                        | 877                 | 1778  | 166686  | 1730       | 27.74|
| kall_circlesrectangles_c1r11 | 3                         | 21                  | 36    | 722     | 48         | 0.01 |
| kall_circlesrectangles_c1r12 | 3                         | 21                  | 38    | 666     | 46         | 0.01 |
| kall_circlesrectangles_c1r13 | 3                         | 21                  | 41    | 549     | 43         | 0.02 |
| kall_circlesrectangles_c6r1 | 9                          | 141                 | 264   | 7512    | 281        | 0.38 |
| kall_circlesrectangles_c6r29 | 23                        | 283                 | 596   | 35029   | 515        | 2.22 |
| kall_circlesrectangles_c6r39 | 30                        | 457                 | 967   | 49535   | 843        | 5.73 |
| kall_congruentcircles_c31 | 4                         | 7                   | 16    | 151     | 6          | 0.00 |
| kall_congruentcircles_c32 | 4                         | 7                   | 10    | 65      | 17         | 0.00 |
| kall_congruentcircles_c41 | 6                         | 6                   | 11    | 104     | 2          | 0.00 |
| kall_congruentcircles_c42 | 7                         | 13                  | 26    | 273     | 21         | 0.02 |
| kall_congruentcircles_c51 | 11                        | 21                  | 43    | 612     | 31         | 0.06 |
| kall_congruentcircles_c52 | 11                        | 21                  | 35    | 402     | 47         | 0.07 |
| kall_congruentcircles_c61 | 16                        | 31                  | 63    | 1342    | 46         | 0.08 |
| kall_congruentcircles_c62 | 16                        | 31                  | 41    | 515     | 80         | 0.04 |
| kall_congruentcircles_c63 | 16                        | 31                  | 43    | 1014    | 51         | 0.03 |
| kall_congruentcircles_c71 | 22                        | 43                  | 87    | 2185    | 64         | 0.03 |
| kall_congruentcircles_c72 | 22                        | 43                  | 80    | 876     | 71         | 0.02 |
| kall_diffcircles_10    | 2                             | 81                  | 88    | 1749    | 156        | 0.04 |
| kall_diffcircles_5a    | 5                             | 21                  | 33    | 490     | 31         | 0.01 |
| Instance               | $\sum_{i,j} K_{ij}\phi_{ij}$ | $\sum_{i,j} K_{ij}$ | # LPs | # iters | # filtered | time  |
|------------------------|-------------------------------|----------------------|-------|---------|------------|-------|
| kall_diffcircles_5b    | 1                             | 21                   | 32    | 749     | 32         | 0.02  |
| kall_diffcircles_6     | 1                             | 31                   | 45    | 553     | 47         | 0.01  |
| kall_diffcircles_7     | 2                             | 43                   | 45    | 1376    | 91         | 0.03  |
| kall_diffcircles_8     | 1                             | 49                   | 84    | 1145    | 65         | 0.03  |
| kall_diffcircles_9     | 2                             | 64                   | 88    | 1915    | 87         | 0.04  |
| kall_ellipsoids_tc02b  | 0                             | 48                   | 66    | 1608    | 108        | 0.07  |
| kall_ellipsoids_tc03c  | 0                             | 108                  | 134   | 3649    | 245        | 0.22  |
| kall_ellipsoids_tc05a  | 57                            | 960                  | 1569  | 240891  | 591        | 54.45 |
| kissing2               | 0                             | 73768                | 12356 | 309820  | 113138     | 154.96|
| knp3-12                | 0                             | 198                  | 120   | 3307    | 550        | 0.10  |
| knp4-24                | 0                             | 1104                 | 765   | 55857   | 2823       | 5.95  |
| knp5-40                | 0                             | 3900                 | 3609  | 415492  | 9024       | 59.93 |
| knp5-41                | 0                             | 4100                 | 2287  | 239255  | 10872      | 74.57 |
| knp5-42                | 0                             | 4305                 | 2718  | 366533  | 11148      | 36.76 |
| knp5-43                | 0                             | 4515                 | 3549  | 622735  | 10778      | 66.26 |
| knp5-44                | 0                             | 4730                 | 3007  | 499275  | 12199      | 89.59 |
| korcns                 | 23                            | 48                   | 93    | 7331    | 43         | 0.21  |
| launch                 | 2                             | 15                   | 41    | 650     | 12         | 0.01  |
| least                  | 0                             | 12                   | 4     | 6       | 0          | 0.00  |
| lnts100                | 0                             | 796                  | 313   | 566     | 515        | 0.27  |
| lnts200                | 0                             | 1596                 | 613   | 1166    | 1015       | 2.14  |
| lnts400                | 0                             | 3196                 | 1213  | 2366    | 2015       | 9.34  |
| lnts50                 | 0                             | 396                  | 163   | 266     | 265        | 0.10  |
| mathopt1               | 2                             | 2                    | 4     | 28      | 2          | 0.00  |
| mathopt4               | 1                             | 2                    | 4     | 35      | 0          | 0.00  |
| mathopt5_3             | 0                             | 2                    | 4     | 29      | 2          | 0.00  |
| mathopt5_6             | 0                             | 1                    | 2     | 4       | 0          | 0.00  |
| maxmin                 | 0                             | 132                  | 158   | 23362   | 222        | 0.55  |
| maxmineig2             | 0                             | 294                  | 354   | 716     | 822        | 0.12  |
| milinfract             | 0                             | 1                    | 2     | 28      | 0          | 0.07  |
| minlphi                | 0                             | 4                    | 4     | 15      | 4          | 0.00  |
| minlphix               | 0                             | 8                    | 4     | 24      | 7          | 0.01  |
| multiplants_mtg1a      | 49                            | 76                   | 117   | 20616   | 20         | 0.94  |
| multiplants_mtg1b      | 50                            | 71                   | 105   | 23828   | 28         | 0.97  |
| multiplants_mtg1c      | 46                            | 76                   | 105   | 45487   | 35         | 2.09  |
| multiplants_mtg2       | 58                            | 101                  | 158   | 28179   | 20         | 1.56  |
| multiplants_mtg5       | 93                            | 125                  | 156   | 10902   | 26         | 0.34  |
| multiplants_mtg6       | 121                           | 173                  | 222   | 35782   | 22         | 1.44  |
| multiplants_stg1       | 1                             | 67                   | 186   | 68380   | 82         | 2.35  |
| multiplants_stg1a      | 1                             | 49                   | 104   | 41238   | 92         | 1.48  |
| multiplants_stg1b      | 1                             | 55                   | 171   | 77258   | 49         | 2.87  |
| multiplants_stg1c      | 1                             | 43                   | 80    | 26555   | 92         | 0.94  |
| multiplants_stg5       | 1                             | 49                   | 162   | 63104   | 34         | 1.38  |
| multiplants_stg6       | 1                             | 65                   | 197   | 112837  | 63         | 4.47  |
| ndcc12                 | 528                           | 528                  | 1056  | 349712  | 492        | 15.61 |
| ndcc12persp            | 43                            | 124                  | 49    | 39797   | 80         | 2.23  |
| Instance | $\sum_{i,j} K_{ij}$ | $\sum_{i,j} \phi_{ij}$ | # LPs | # iters | # filtered | time  |
|----------|------------------|-----------------|-------|--------|-----------|-------|
| ndcc13   | 535              | 546             | 1092  | 326759 | 390       | 18.66 |
| ndcc13persp | 31               | 112             | 70    | 52843  | 43        | 4.13  |
| ndcc14   | 756              | 756             | 1512  | 505373 | 742       | 34.80 |
| ndcc14persp | 49              | 156             | 73    | 62300  | 85        | 4.04  |
| ndcc15   | 538              | 540             | 1080  | 296086 | 465       | 22.86 |
| ndcc15persp | 35              | 104             | 65    | 32859  | 36        | 2.52  |
| ndcc16   | 960              | 960             | 1920  | 928823 | 960       | 65.00 |
| ndcc16persp | 57             | 180             | 93    | 90509  | 85        | 10.33 |
| ngone    | 0                | 195             | 147   | 21301  | 485       | 6.99  |
| nous1    | 20               | 58              | 63    | 502    | 133       | 0.02  |
| nous2    | 20               | 58              | 80    | 717    | 116       | 0.03  |
| nuclear10a | 852              | 22962           | 226   | 4745248 | 1590   | 1362.17 |
| nuclear14a | 1905             | 2568            | 1108  | 2219487 | 428    | 227.05 |
| nuclear14b | 1449             | 1992            | 1082  | 1529710 | 454    | 288.17 |
| nuclear25a | 2058             | 2720            | 1131  | 1776885 | 469    | 201.09 |
| nuclear25b | 1624             | 2095            | 1328  | 2504727 | 272    | 424.74 |
| nuclear49a | 4788             | 7275            | 2820  | 22111293 | 708   | 5512.67 |
| nuclear49b | 3867             | 4874            | 2957  | 11491979 | 571   | 3982.06 |
| nuclearvb | 0                | 1036            | 14    | 0      | 154      | 0.00  |
| nuclearvc | 0                | 1036            | 14    | 0      | 154      | 0.00  |
| nuclearvd | 0                | 1474            | 14    | 0      | 154      | 0.00  |
| nuclearve | 0                | 1474            | 14    | 0      | 154      | 0.00  |
| nuclearvf | 0                | 1474            | 14    | 0      | 154      | 0.00  |
| nvs01    | 1                | 2               | 3     | 24     | 1        | 0.00  |
| nvs02    | 1                | 9               | 10    | 177    | 18       | 0.00  |
| nvs04    | 0                | 2               | 2     | 5      | 2        | 0.00  |
| nvs05    | 1                | 9               | 19    | 377    | 8        | 0.02  |
| nvs06    | 0                | 2               | 2     | 6      | 4        | 0.00  |
| nvs13    | 44               | 49              | 16    | 207    | 23       | 0.01  |
| nvs14    | 1                | 9               | 10    | 175    | 18       | 0.00  |
| nvs15    | 0                | 2               | 2     | 6      | 0        | 0.00  |
| nvs16    | 0                | 7               | 6     | 62     | 11       | 0.00  |
| nvs17    | 140              | 140             | 62    | 1486   | 20       | 0.01  |
| nvs18    | 64               | 87              | 20    | 316    | 39       | 0.01  |
| nvs19    | 206              | 206             | 78    | 3635   | 34       | 0.05  |
| nvs20    | 10               | 30              | 60    | 2095   | 9        | 0.08  |
| nvs21    | 0                | 2               | 3     | 5      | 3        | 0.00  |
| nvs22    | 0                | 10              | 20    | 275    | 12       | 0.02  |
| nvs23    | 290              | 290             | 109   | 4244   | 35       | 0.06  |
| nvs24    | 399              | 399             | 128   | 6503   | 52       | 0.10  |
| oil      | 77               | 331             | 688   | 133697 | 80       | 13.16 |
| oil2     | 80               | 270             | 596   | 6846   | 8        | 2.27  |
| ortez    | 5                | 24              | 58    | 861    | 18       | 0.03  |
| orth_d3m6 | 8               | 46              | 42    | 255    | 78       | 0.01  |
| orth_d3m6.pl | 15              | 225             | 240   | 939    | 460      | 0.22  |
| orth_d4m6.pl | 6              | 105             | 295   | 1352   | 125      | 0.09  |
| Instance     | $\sum_{i,j} K_{ij}$ | $\sum_{i,j} \phi_{ij}$ | # LPs | # iters | # filtered | time  |
|--------------|---------------------|------------------------|-------|---------|------------|-------|
| otop         | 11                  | 12                     | 48    | 2050    | 0          | 0.06  |
| parallel     | 65                  | 110                    | 120   | 12835   | 137        | 0.38  |
| pindyck      | 31                  | 32                     | 99    | 9237    | 29         | 0.37  |
| pointpack04  | 7                   | 12                     | 14    | 150     | 24         | 0.01  |
| pointpack06  | 16                  | 30                     | 34    | 538     | 72         | 0.02  |
| pointpack08  | 29                  | 56                     | 51    | 688     | 147        | 0.02  |
| pointpack10  | 46                  | 90                     | 97    | 1858    | 221        | 0.09  |
| pointpack12  | 67                  | 132                    | 159   | 3731    | 299        | 0.13  |
| pointpack14  | 92                  | 182                    | 239   | 6039    | 369        | 0.45  |
| pollut       | 3                   | 20                     | 29    | 281     | 16         | 0.01  |
| pooling_adhya1pq | 5             | 20                     | 16    | 286     | 64         | 0.01  |
| pooling_adhya1stp | 2           | 40                     | 40    | 704     | 120        | 0.01  |
| pooling_adhya2pq | 2           | 20                     | 35    | 520     | 45         | 0.01  |
| pooling_adhya2stp | 1            | 20                     | 22    | 81      | 58         | 0.00  |
| pooling_adhya3pq | 0             | 40                     | 51    | 415     | 109        | 0.01  |
| pooling_adhya3stp | 2           | 20                     | 28    | 252     | 52         | 0.01  |
| pooling_adhya3stp | 1           | 32                     | 36    | 184     | 92         | 0.01  |
| pooling_adhya3stp | 0            | 64                     | 71    | 473     | 185        | 0.03  |
| pooling_adhya3tp | 6            | 32                     | 59    | 808     | 69         | 0.02  |
| pooling_adhya4pq | 1             | 40                     | 42    | 428     | 118        | 0.01  |
| pooling_adhya4st | 0            | 80                     | 80    | 865     | 240        | 0.03  |
| pooling_adhya4tp | 6            | 40                     | 79    | 731     | 81         | 0.03  |
| pooling_bental14pq | 6           | 6                      | 17    | 109     | 7          | 0.01  |
| pooling_bental14st | 0            | 12                     | 21    | 158     | 27         | 0.01  |
| pooling_bental14tp | 6            | 6                      | 14    | 111     | 10         | 0.00  |
| pooling_bental15st | 0            | 120                    | 122   | 796     | 358        | 0.04  |
| pooling_digabel16 | 27           | 432                    | 197   | 6667    | 379        | 0.18  |
| pooling_digabel18 | 261          | 1080                   | 680   | 58487   | 760        | 2.57  |
| pooling_digabel19 | 159          | 636                    | 436   | 42436   | 412        | 1.03  |
| pooling_epa1  | 120             | 142                    | 153   | 17238   | 71         | 0.53  |
| pooling_epa2  | 292             | 351                    | 293   | 50899   | 151        | 2.99  |
| pooling_epa3  | 252             | 1480                   | 967   | 411230  | 673        | 96.18 |
| pooling_foulds2st | 0            | 32                     | 32    | 139     | 96         | 0.00  |
| pooling_foulds3st | 0            | 1024                   | 1006  | 9068    | 3090       | 0.93  |
| pooling_foulds4st | 0            | 1024                   | 1006  | 20704   | 3090       | 1.26  |
| pooling_foulds5st | 2            | 1024                   | 993   | 30287   | 3103       | 2.66  |
| pooling_haverly1pq | 4            | 4                      | 8     | 56      | 8          | 0.00  |
| pooling_haverly1stp | 4            | 8                      | 24    | 150     | 8          | 0.00  |
| pooling_haverly1tp | 4            | 4                      | 12    | 86      | 4          | 0.00  |
| pooling_haverly2pq | 4            | 4                      | 14    | 106     | 2          | 0.00  |
| pooling_haverly2st | 4            | 8                      | 20    | 138     | 12         | 0.01  |
| pooling_haverly2tp | 4            | 4                      | 10    | 72      | 6          | 0.01  |
| pooling_haverly3pq | 4            | 4                      | 12    | 127     | 4          | 0.00  |
| pooling_haverly3st | 6            | 8                      | 24    | 247     | 8          | 0.01  |
| pooling_haverly3tp | 4            | 4                      | 12    | 87      | 4          | 0.01  |
| pooling_haverly3tp | 1            | 18                     | 27    | 175     | 45         | 0.00  |
Table 3 continued

| Instance          | $\sum_{i,j}K_{ij}\phi_{ij}$ | $\sum_{i,j}K_{ij}$ | # LPs | # iters | # filtered time | time |
|-------------------|-------------------------------|--------------------|-------|---------|-----------------|------|
| pooling_rt2stp    | 10                            | 36                 | 56    | 1416    | 88              | 0.02 |
| pooling_rt2tp     | 10                            | 18                 | 34    | 833     | 38              | 0.01 |
| pooling_sppa0pq    | 6                             | 329                | 413   | 61146   | 903             | 3.23 |
| pooling_sppa0stp   | 2                             | 658                | 803   | 177185  | 1829            | 7.43 |
| pooling_sppa0tp    | 8                             | 329                | 442   | 147332  | 874             | 6.07 |
| pooling_sppa5pq    | 9                             | 968                | 968   | 204838  | 2904            | 14.61|
| pooling_sppa5stq   | 4                             | 1936               | 2030  | 397767  | 5714            | 28.79|
| pooling_sppa5tp    | 16                            | 968                | 1007  | 290852  | 2865            | 13.58|
| pooling_sppa9pq    | 8                             | 1828               | 1851  | 115410  | 5461            | 10.06|
| pooling_sppa9stq   | 9                             | 1992               | 2153  | 111430  | 5815            | 9.78 |
| pooling_sppb0pq    | 14                            | 1153               | 1176  | 209657  | 3436            | 31.45|
| pooling_sppb0stq   | 2                             | 2306               | 2745  | 496741  | 6479            | 94.31|
| pooling_sppb0tp    | 18                            | 1153               | 1429  | 482396  | 3183            | 39.62|
| pooling_sppb2pq    | 17                            | 3093               | 3161  | 1133060 | 9211            | 262.35|
| pooling_sppb2stq   | 2                             | 6186               | 6342  | 4395622 | 18402           | 1047.31|
| pooling_sppb2tp    | 16                            | 3093               | 3723  | 1372710 | 8649            | 145.49|
| pooling_sppb5pq    | 16                            | 7947               | 8215  | 1113027 | 25733           | 1572.60|
| pooling_sppb5stq   | 36                            | 15894              | 15290 | 25499298| 47718           | 6806.60|
| pooling_sppc0pq    | 35                            | 2826               | 2925  | 4783872 | 8379            | 1451.83|
| pooling_sppc0stq   | 15                            | 5652               | 5693  | 10640160| 16915           | 4767.84|
| pooling_sppc0tp    | 27                            | 2826               | 3958  | 5526943 | 7346            | 1774.29|
| pooling_sppc1pq    | 25                            | 4770               | 5557  | 13864945| 13523           | 4173.37|
| pooling_sppc1stq   | 50                            | 9540               | 5206  | 15821587| 28614           | 6955.07|
| pooling_sppc1tp    | 43                            | 4770               | 6021  | 14955663| 13059           | 4526.10|
| pooling_sppc3pq    | 26                            | 9116               | 1947  | 9173015 | 26745           | 6385.09|
| pooling_sppc3stq   | 5                             | 18232              | 858   | 10582833| 53501           | 4778.78|
| pooling_sppc3tp    | 13                            | 9116               | 714   | 9725129 | 25461           | 4958.52|
| powerflow0009r     | 22                            | 116                | 44    | 1314    | 92              | 0.04 |
| powerflow0014r     | 48                            | 264                | 162   | 11311   | 142             | 0.32 |
| powerflow0030r     | 192                           | 584                | 391   | 80692   | 249             | 4.58 |
| powerflow0039r     | 192                           | 700                | 290   | 100404  | 438             | 6.71 |
| powerflow0057r     | 166                           | 1088               | 508   | 43675   | 708             | 3.25 |
| powerflow0118r     | 276                           | 2748               | 1116  | 175707  | 1700            | 28.98|
| powerflow0300r     | 978                           | 6020               | 2778  | 2390185 | 3742            | 893.46|
| primary            | 2879                          | 2917               | 1731  | 65916   | 2097            | 2.68 |
| prob07             | 0                             | 30                 | 6     | 0       | 14              | 0.00 |
| prob09             | 1                             | 1                  | 3     | 56      | 1               | 0.00 |
| process            | 0                             | 3                  | 9     | 104     | 2               | 0.01 |
| procurement1large  | 68                            | 136                | 79    | 211     | 198             | 1.83 |
| procurement1mot    | 12                            | 24                 | 23    | 423     | 37              | 0.03 |
| prolog             | 0                             | 8                  | 4     | 0       | 4               | 0.00 |
| qp3                | 50                            | 50                 | 50    | 850     | 82              | 0.02 |
| rbrock             | 1                             | 1                  | 2     | 17      | 0               | 0.00 |
| ringpack_10_1      | 50                            | 660                | 104   | 4242    | 255             | 0.13 |
| ringpack_10_2      | 60                            | 840                | 106   | 2041    | 252             | 0.09 |
| Instance               | $\sum_{i,j} K_{ij} \phi_{ij}$ | $\sum_{i,j} K_{ij}$ | # LPs | # iters | # filtered | time  |
|-----------------------|-------------------------------|---------------------|-------|---------|------------|-------|
| ringpack_20_1         | 712                           | 4674                | 405   | 22259   | 1092       | 1.19  |
| ringpack_20_2         | 782                           | 5434                | 426   | 64877   | 1076       | 2.59  |
| ringpack_20_3         | 0                             | 6110                | 429   | 35598   | 1066       | 1.29  |
| ringpack_30_1         | 2672                          | 14866               | 1097  | 156015  | 2356       | 9.75  |
| ringpack_30_2         | 2856                          | 16606               | 898   | 96546   | 2548       | 6.66  |
| robot100              | 0                             | 1489                | 594   | 1485    | 594        | 0.89  |
| robot200              | 0                             | 2989                | 1194  | 2985    | 1194       | 4.23  |
| robot400              | 0                             | 5989                | 2394  | 5985    | 2394       | 24.30 |
| robot50               | 0                             | 739                 | 294   | 735     | 294        | 0.15  |
| rocket100             | 0                             | 995                 | 299   | 5150    | 897        | 0.13  |
| rocket200             | 0                             | 1995                | 599   | 20302   | 1797       | 0.97  |
| rocket400             | 0                             | 3995                | 1199  | 80604   | 3597       | 4.17  |
| rocket50              | 0                             | 495                 | 149   | 1325    | 447        | 0.02  |
| rsyn0805h             | 1                             | 9                   | 9     | 117     | 10         | 0.01  |
| rsyn0805m02h          | 1                             | 18                  | 21    | 370     | 17         | 0.04  |
| rsyn0805m03h          | 4                             | 27                  | 32    | 1049    | 24         | 0.07  |
| rsyn0805m04h          | 4                             | 36                  | 42    | 1746    | 28         | 0.21  |
| rsyn0810h             | 2                             | 18                  | 16    | 240     | 17         | 0.01  |
| rsyn0810m02h          | 0                             | 36                  | 35    | 880     | 36         | 0.06  |
| rsyn0810m03h          | 5                             | 54                  | 51    | 1502    | 56         | 0.23  |
| rsyn0810m04h          | 8                             | 72                  | 75    | 3184    | 61         | 0.33  |
| rsyn0815h             | 0                             | 33                  | 29    | 910     | 27         | 0.03  |
| rsyn0815m02h          | 1                             | 66                  | 59    | 2346    | 54         | 0.12  |
| rsyn0815m03h          | 5                             | 99                  | 91    | 6038    | 81         | 0.58  |
| rsyn0815m04h          | 7                             | 132                 | 123   | 10680   | 102        | 0.91  |
| rsyn0820h             | 0                             | 42                  | 41    | 810     | 33         | 0.05  |
| rsyn0820m02h          | 1                             | 84                  | 71    | 2896    | 79         | 0.26  |
| rsyn0820m03h          | 3                             | 126                 | 111   | 7518    | 117        | 1.04  |
| rsyn0820m04h          | 5                             | 168                 | 147   | 11900   | 159        | 1.81  |
| rsyn0830h             | 1                             | 60                  | 46    | 640     | 73         | 0.05  |
| rsyn0830m02h          | 0                             | 120                 | 108   | 3897    | 110        | 0.32  |
| rsyn0830m03h          | 0                             | 180                 | 159   | 8164    | 166        | 1.08  |
| rsyn0830m04h          | 1                             | 239                 | 204   | 11142   | 229        | 1.26  |
| rsyn0840h             | 1                             | 84                  | 64    | 1314    | 98         | 0.18  |
| rsyn0840m02h          | 4                             | 168                 | 153   | 4281    | 145        | 0.99  |
| rsyn0840m03h          | 1                             | 252                 | 219   | 13505   | 234        | 2.26  |
| rsyn0840m04h          | 8                             | 335                 | 307   | 28075   | 281        | 6.40  |
| saa_2                 | 5424                          | 8992                | 7386  | 958322  | 254        | 634.91|
| sepl                  | 2                             | 6                   | 24    | 216     | 0          | 0.01  |
| sepasequ_complex      | 70                            | 611                 | 643   | 51204   | 799        | 7.13  |
| sepasequ_convent      | 192                           | 1046                | 1144  | 110380  | 794        | 8.26  |
| sfacloc1_2_95         | 2                             | 14                  | 49    | 2259    | 7          | 0.06  |
| sfacloc1_3_95         | 3                             | 21                  | 62    | 2242    | 22         | 0.06  |
| sfacloc1_4_95         | 0                             | 28                  | 81    | 2321    | 31         | 0.14  |
| sjup2                 | 0                             | 44400               | 2836  | 0       | 80478      | 0.04  |
| smallinvSNPr1b010-011 | 541                           | 4950                | 6855  | 288946  | 7964       | 11.35 |
| Instance     | $\sum_{i,j} K_{ij} \phi_{ij}$ | $\sum_{i,j} K_{ij}$ | # LPs | # iters | # filtered time | time |
|--------------|-------------------------------|----------------------|-------|---------|----------------|------|
| smallinvSNPr1b020-022 | 772 | 4950 | 6678 | 461149 | 8138 | 17.51 |
| smallinvSNPr1b050-055 | 1183 | 4950 | 4827 | 304980 | 10007 | 6.60 |
| smallinvSNPr1b100-110 | 649 | 4950 | 6483 | 521834 | 8331 | 11.54 |
| smallinvSNPr1b150-165 | 647 | 4950 | 6493 | 456477 | 8320 | 10.18 |
| smallinvSNPr1b200-220 | 634 | 4950 | 6590 | 599430 | 8222 | 20.05 |
| smallinvSNPr2b010-011 | 358 | 4950 | 7255 | 396469 | 7560 | 15.99 |
| smallinvSNPr2b020-022 | 1249 | 4950 | 6577 | 393384 | 8238 | 9.65 |
| smallinvSNPr2b050-055 | 716 | 4950 | 4984 | 270082 | 7869 | 19.52 |
| smallinvSNPr2b100-110 | 562 | 4950 | 6765 | 396274 | 8054 | 15.25 |
| smallinvSNPr2b150-165 | 882 | 4950 | 6762 | 585985 | 8055 | 12.92 |
| smallinvSNPr2b200-220 | 1545 | 4950 | 6762 | 585985 | 8055 | 12.92 |
| smallinvSNPr3b010-011 | 360 | 4950 | 7737 | 386626 | 7077 | 14.93 |
| smallinvSNPr3b020-022 | 450 | 4950 | 7342 | 390364 | 7471 | 14.49 |
| smallinvSNPr3b050-055 | 406 | 4950 | 3577 | 233254 | 11266 | 12.21 |
| smallinvSNPr3b100-110 | 1136 | 4950 | 4886 | 286460 | 9952 | 6.59 |
| smallinvSNPr3b150-165 | 585 | 4950 | 7335 | 524362 | 7477 | 21.95 |
| smallinvSNPr3b200-220 | 875 | 4950 | 4575 | 244438 | 10259 | 10.09 |
| smallinvSNPr4b010-011 | 433 | 4950 | 8298 | 625317 | 6508 | 19.41 |
| smallinvSNPr4b020-022 | 1097 | 4950 | 5470 | 222705 | 9385 | 8.40 |
| smallinvSNPr4b050-055 | 1039 | 4950 | 3741 | 213594 | 1107 | 4.69 |
| smallinvSNPr4b100-110 | 851 | 4950 | 4119 | 288191 | 10720 | 6.06 |
| smallinvSNPr4b150-165 | 629 | 4950 | 4201 | 343772 | 10636 | 7.07 |
| smallinvSNPr4b200-220 | 860 | 4950 | 4324 | 193061 | 10511 | 7.40 |
| smallinvSNPr5b010-011 | 378 | 4950 | 8319 | 492164 | 6488 | 14.18 |
| smallinvSNPr5b020-022 | 595 | 4950 | 8319 | 477858 | 6486 | 13.18 |
| smallinvSNPr5b050-055 | 1004 | 4950 | 5163 | 297039 | 9676 | 12.19 |
| smallinvSNPr5b100-110 | 999 | 4950 | 4972 | 246117 | 9866 | 8.84 |
| smallinvSNPr5b150-165 | 967 | 4950 | 4783 | 225261 | 10057 | 5.27 |
| smallinvSNPr5b200-220 | 966 | 4950 | 4617 | 248387 | 10220 | 9.68 |
| space25 | 0 | 86 | 12 | 225 | 20 | 0.00 |
| space25a | 0 | 86 | 24 | 658 | 32 | 0.02 |
| space960 | 12 | 3740 | 2365 | 7577269 | 9330 | 6504.43 |
| spectra2 | 1040 | 1080 | 354 | 32394 | 186 | 1.60 |
| sporttournament14 | 0 | 168 | 80 | 2589 | 222 | 0.23 |
| spring | 3 | 3 | 10 | 142 | 0 | 0.00 |
| sssd08-04 | 0 | 12 | 4 | 261 | 4 | 0.00 |
| sssd08-04persp | 12 | 36 | 24 | 734 | 12 | 0.02 |
| sssd12-05 | 0 | 15 | 5 | 415 | 5 | 0.01 |
| sssd12-05persp | 15 | 45 | 30 | 1347 | 15 | 0.05 |
| sssd15-04 | 0 | 12 | 4 | 170 | 4 | 0.00 |
| sssd15-04persp | 12 | 36 | 24 | 1165 | 12 | 0.05 |
| sssd15-06 | 0 | 18 | 6 | 655 | 6 | 0.02 |
| sssd15-06persp | 18 | 54 | 36 | 1758 | 18 | 0.06 |
| sssd15-08 | 0 | 24 | 8 | 1045 | 8 | 0.02 |
| sssd15-08persp | 24 | 72 | 48 | 3258 | 24 | 0.16 |
| sssd16-07 | 0 | 21 | 7 | 617 | 7 | 0.02 |
| Instance       | $\sum_{i,j} K_{ij} \phi_{ij}$ | $\sum_{i,j} K_{ij}$ | # LPs | # iters | # filtered time |
|----------------|--------------------------------|---------------------|-------|---------|-----------------|
| sssd16-07persp| 21                             | 63                  | 42    | 2037    | 21              | 0.10            |
| sssd18-06     | 0                              | 18                  | 6     | 905     | 6               | 0.03            |
| sssd18-06persp| 16                             | 54                  | 34    | 3788    | 18              | 0.13            |
| sssd18-08     | 0                              | 24                  | 8     | 1077    | 8               | 0.04            |
| sssd18-08persp| 24                             | 72                  | 48    | 7449    | 24              | 0.16            |
| sssd20-04     | 0                              | 12                  | 4     | 309     | 4               | 0.01            |
| sssd20-04persp| 12                             | 36                  | 24    | 1056    | 12              | 0.02            |
| sssd20-08     | 0                              | 24                  | 8     | 1211    | 8               | 0.03            |
| sssd20-08persp| 24                             | 72                  | 48    | 3969    | 24              | 0.18            |
| sssd22-08     | 0                              | 24                  | 8     | 1186    | 8               | 0.05            |
| sssd22-08persp| 24                             | 72                  | 48    | 6662    | 24              | 0.31            |
| sssd25-04     | 0                              | 12                  | 4     | 214     | 4               | 0.01            |
| sssd25-04persp| 12                             | 36                  | 24    | 2117    | 12              | 0.05            |
| sssd25-08     | 0                              | 24                  | 8     | 647     | 8               | 0.03            |
| sssd25-08persp| 24                             | 72                  | 48    | 3496    | 24              | 0.10            |
| st_bpafla     | 3                              | 5                   | 9     | 57      | 3               | 0.00            |
| st_e03        | 5                              | 5                   | 11    | 170     | 1               | 0.01            |
| st_e04        | 1                              | 1                   | 1     | 11      | 1               | 0.00            |
| st_e05        | 2                              | 2                   | 7     | 38      | 1               | 0.00            |
| st_e07        | 4                              | 4                   | 4     | 35      | 4               | 0.00            |
| st_e08        | 1                              | 1                   | 2     | 8       | 0               | 0.00            |
| st_e09        | 2                              | 2                   | 4     | 18      | 0               | 0.00            |
| st_e11        | 1                              | 1                   | 4     | 18      | 0               | 0.00            |
| st_e16        | 10                             | 10                  | 26    | 491     | 2               | 0.01            |
| st_e17        | 0                              | 1                   | 2     | 8       | 0               | 0.00            |
| st_e23        | 1                              | 1                   | 2     | 10      | 0               | 0.00            |
| st_e25        | 4                              | 6                   | 12    | 77      | 1               | 0.00            |
| st_e28        | 9                              | 9                   | 24    | 212     | 2               | 0.01            |
| st_e30        | 2                              | 6                   | 17    | 211     | 7               | 0.01            |
| st_e31        | 2                              | 6                   | 9     | 108     | 15              | 0.00            |
| st_e32        | 308                            | 314                 | 413   | 25878   | 230             | 0.66            |
| st_e33        | 0                              | 4                   | 4     | 24      | 4               | 0.00            |
| st_e35        | 0                              | 19                  | 20    | 108     | 6               | 0.01            |
| st_e36        | 3                              | 3                   | 3     | 31      | 1               | 0.00            |
| st_e38        | 2                              | 4                   | 6     | 55      | 3               | 0.00            |
| st_e40        | 1                              | 3                   | 4     | 29      | 8               | 0.01            |
| st_e41        | 14                             | 14                  | 24    | 285     | 4               | 0.01            |
| st_e42        | 0                              | 1                   | 3     | 0       | 1               | 0.00            |
| st_glm邳fp1    | 0                              | 1                   | 2     | 8       | 0               | 0.00            |
| st_glm邳fp2    | 0                              | 1                   | 2     | 9       | 0               | 0.00            |
| st_glm邳fp3    | 1                              | 1                   | 2     | 8       | 0               | 0.00            |
| st_glm邳kk90   | 0                              | 1                   | 2     | 11      | 0               | 0.00            |
| st_glm邳kk92   | 1                              | 1                   | 2     | 11      | 0               | 0.00            |
| st_glm邳kkky   | 2                              | 2                   | 4     | 24      | 0               | 0.00            |
| st_glm邳ss1    | 1                              | 1                   | 2     | 11      | 0               | 0.00            |
| st_glm邳ss2    | 1                              | 1                   | 2     | 10      | 0               | 0.00            |
| Instance     | $\sum_{i,j} K_{ij} \phi_{ij}$ | $\sum_{i,j} K_{ij}$ | # LPs | # iters | # filtered | time |
|--------------|--------------------------------|----------------------|-------|---------|------------|------|
| st_iqpbk1    | 26                             | 28                   | 53    | 3135    | 9          | 0.06 |
| st_iqpbk2    | 28                             | 28                   | 52    | 2565    | 9          | 0.06 |
| st_jcbpaf2   | 3                              | 5                    | 9     | 137     | 3          | 0.00 |
| st_qpc-m1     | 10                             | 10                   | 13    | 112     | 7          | 0.00 |
| st_qpc-m3a    | 45                             | 45                   | 46    | 863     | 110        | 0.02 |
| st_qpc-m3b    | 45                             | 45                   | 45    | 1079    | 72         | 0.03 |
| st_qpk1      | 1                              | 1                    | 2     | 10      | 1          | 0.00 |
| st_robot     | 1                              | 5                    | 5     | 45      | 7          | 0.01 |
| steenbrf     | 0                              | 72                   | 36    | 68      | 0          | 0.00 |
| super3t      | 193                            | 483                  | 1241  | 364974  | 395        | 35.29|
| syn05h       | 0                              | 9                    | 10    | 73      | 9          | 0.00 |
| syn05m02h    | 0                              | 18                   | 20    | 238     | 17         | 0.01 |
| syn05m03h    | 0                              | 27                   | 28    | 345     | 30         | 0.02 |
| syn05m04h    | 0                              | 36                   | 35    | 524     | 44         | 0.02 |
| syn10h       | 0                              | 18                   | 15    | 125     | 18         | 0.00 |
| syn10m02h    | 0                              | 36                   | 28    | 441     | 40         | 0.02 |
| syn10m03h    | 0                              | 54                   | 42    | 880     | 64         | 0.03 |
| syn10m04h    | 0                              | 72                   | 56    | 1298    | 86         | 0.05 |
| syn15h       | 0                              | 33                   | 27    | 420     | 32         | 0.02 |
| syn15m02h    | 0                              | 66                   | 52    | 1135    | 66         | 0.04 |
| syn15m03h    | 0                              | 99                   | 75    | 2006    | 107        | 0.13 |
| syn15m04h    | 0                              | 132                  | 110   | 4722    | 124        | 0.35 |
| syn20h       | 0                              | 42                   | 35    | 499     | 43         | 0.02 |
| syn20m02h    | 0                              | 84                   | 73    | 2120    | 82         | 0.15 |
| syn20m03h    | 0                              | 126                  | 99    | 3920    | 137        | 0.25 |
| syn20m04h    | 0                              | 168                  | 131   | 5471    | 183        | 0.31 |
| syn30h       | 0                              | 60                   | 52    | 653     | 58         | 0.04 |
| syn30m02h    | 0                              | 120                  | 102   | 3874    | 118        | 0.39 |
| syn30m03h    | 0                              | 180                  | 157   | 9203    | 176        | 0.62 |
| syn30m04h    | 0                              | 239                  | 205   | 13332   | 223        | 1.58 |
| syn40h       | 0                              | 84                   | 70    | 1141    | 81         | 0.10 |
| syn40m02h    | 0                              | 168                  | 146   | 6402    | 157        | 0.72 |
| syn40m03h    | 0                              | 252                  | 211   | 13533   | 244        | 1.17 |
| syn40m04h    | 0                              | 335                  | 313   | 37671   | 281        | 2.74 |
| synheat      | 0                              | 12                   | 7     | 171     | 18         | 0.01 |
| tanksize     | 11                             | 36                   | 44    | 1066    | 0          | 0.03 |
| tln12        | 0                              | 144                  | 118   | 28891   | 337        | 1.13 |
| tln4         | 12                             | 16                   | 18    | 622     | 21         | 0.01 |
| tln5         | 25                             | 25                   | 25    | 1666    | 34         | 0.04 |
| tln6         | 0                              | 36                   | 31    | 2491    | 79         | 0.06 |
| tln7         | 0                              | 49                   | 43    | 3542    | 122        | 0.08 |
| tls12        | 0                              | 252                  | 28    | 8908    | 35         | 1.64 |
| tls2         | 1                              | 4                    | 3     | 72      | 2          | 0.00 |
| tls4         | 0                              | 25                   | 12    | 842     | 8          | 0.03 |
| tls5         | 0                              | 72                   | 9     | 1000    | 11         | 0.03 |
| tls6         | 0                              | 91                   | 14    | 1909    | 6          | 0.08 |
| Instance         | $\sum_{i,j} K_{ij} \phi_{ij}$ | $\sum_{i,j} K_{ij}$ | # LPs | # iters | # filtered | time |
|------------------|-------------------------------|---------------------|-------|---------|------------|------|
| tls7             | 0 49 64                       | 13676               | 45    | 1.09    |            |      |
| tltr             | 2 27 18                       | 1915                | 23    | 0.05    |            |      |
| torsion100       | 0 19700 512                   | 5967479             | 27014 | 3603.99 |            |      |
| torsion25        | 0 4850 2736                   | 2430701             | 6964  | 525.13  |            |      |
| torsion50        | 0 9800 5852                   | 12862528            | 13748 | 6125.10 |            |      |
| torsion75        | 0 14750 678                   | 6370943             | 19954 | 4492.02 |            |      |
| transswitch0009r | 14 140 83                     | 3931                | 53    | 0.13    |            |      |
| transswitch0014r | 0 344 71                      | 1905                | 233   | 0.13    |            |      |
| transswitch0030r | 12 736 268                    | 13058               | 372   | 1.43    |            |      |
| transswitch0039r | 2 872 257                     | 9100                | 467   | 1.76    |            |      |
| transswitch0057r | 12 1396 702                   | 86215               | 514   | 29.72   |            |      |
| transswitch0118r | 16 3452 1785                  | 406369              | 1027  | 254.48  |            |      |
| transswitch0300r | 324 7492 3752                 | 390991              | 2764  | 897.23  |            |      |
| tricp            | 0 462 272                     | 19337               | 870   | 0.57    |            |      |
| uselinear        | 0 5570 4526                   | 1047666             | 12239 | 70.06   |            |      |
| util             | 5 5 19                       | 330                 | 1     | 0.01    |            |      |
| var_con10        | 112 240 322                   | 10126               | 158   | 1.50    |            |      |
| var_con5         | 108 240 324                   | 6706                | 156   | 1.34    |            |      |
| wager            | 2 114 63                      | 2309                | 30    | 0.11    |            |      |
| waste            | 5 1230 77                     | 8385                | 187   | 0.21    |            |      |
| wastepaper3      | 0 54 10                       | 379                 | 14    | 0.02    |            |      |
| wastepaper4      | 0 88 9                       | 268                 | 23    | 0.01    |            |      |
| wastepaper5      | 0 130 9                      | 216                 | 31    | 0.01    |            |      |
| wastepaper6      | 0 180 10                     | 343                 | 38    | 0.01    |            |      |
| wastewater02m1   | 6 8 17                       | 87                  | 12    | 0.02    |            |      |
| wastewater02m2   | 9 12 48                      | 836                 | 0     | 0.04    |            |      |
| wastewater04m1   | 12 16 24                     | 186                 | 36    | 0.00    |            |      |
| wastewater04m2   | 6 18 68                      | 1154                | 4     | 0.03    |            |      |
| wastewater05m1   | 0 45 42                      | 537                 | 129   | 0.01    |            |      |
| wastewater05m2   | 0 48 88                      | 3589                | 104   | 0.22    |            |      |
| wastewater11m1   | 24 63 115                    | 5498                | 130   | 0.07    |            |      |
| wastewater11m2   | 0 112 202                    | 19060               | 246   | 0.20    |            |      |
| wastewater12m1   | 55 120 223                   | 8320                | 247   | 0.18    |            |      |
| wastewater12m2   | 3 220 520                    | 67647               | 360   | 1.35    |            |      |
| wastewater13m1   | 64 255 484                   | 13076               | 521   | 0.24    |            |      |
| wastewater13m2   | 0 480 746                    | 153382              | 1174  | 3.14    |            |      |
| wastewater14m1   | 15 70 121                    | 1717                | 149   | 0.03    |            |      |
| wastewater14m2   | 6 90 224                     | 12706               | 136   | 0.17    |            |      |
| wastewater15m1   | 18 45 72                     | 1793                | 99    | 0.03    |            |      |
| wastewater15m2   | 0 48 98                      | 4075                | 94    | 0.18    |            |      |
| water            | 0 4 2                       | 0                   | 2     | 0.00    |            |      |
| water3           | 0 2 2                       | 0                   | 2     | 0.00    |            |      |
| water4           | 0 2 2                       | 0                   | 2     | 0.00    |            |      |
| watercontamination0202 | 16 137 205     | 45947               | 192   | 3.43    |            |      |
| watercontamination0202r | 2080 4371 5008 | 457197              | 7879  | 184.96  |            |      |
| watercontamination0303 | 36 243 322   | 118630              | 360   | 13.13   |            |      |
| Instance          | $\sum_{i,j} K_{ij} \phi_{ij}$ | $\sum_{i,j} K_{ij}$ | # LPs | # iters | # filtered | time    |
|-------------------|-------------------------------|---------------------|-------|---------|------------|---------|
| watercontamination0303r | 649                           | 17020               | 16916 | 10107150 | 35068      | 4023.40 |
| waterful2         | 0                             | 58                  | 171   | 23830   | 57         | 1.26    |
| waternd1          | 8                             | 32                  | 40    | 2543    | 56         | 0.04    |
| waternd2          | 32                            | 174                 | 164   | 22967   | 472        | 1.13    |
| waterno2_01       | 11                            | 35                  | 39    | 579     | 45         | 0.01    |
| waterno2_02       | 30                            | 72                  | 86    | 3485    | 122        | 0.24    |
| waterno2_03       | 49                            | 108                 | 129   | 11353   | 195        | 1.00    |
| waterno2_04       | 62                            | 144                 | 170   | 14312   | 262        | 1.03    |
| waterno2_06       | 89                            | 216                 | 247   | 19875   | 401        | 2.74    |
| waterno2_09       | 137                           | 324                 | 388   | 34319   | 584        | 4.72    |
| waterno2_12       | 185                           | 432                 | 485   | 53046   | 811        | 8.99    |
| waterno2_18       | 296                           | 648                 | 713   | 89453   | 1231       | 27.22   |
| waterno2_24       | 382                           | 864                 | 990   | 91084   | 1602       | 32.75   |
| waters            | 0                             | 2                   | 3     | 0       | 1          | 0.00    |
| watersbp          | 0                             | 2                   | 2     | 0       | 2          | 0.01    |
| watersym1         | 0                             | 30                  | 89    | 36236   | 27         | 0.71    |
| watersym2         | 0                             | 28                  | 82    | 4189    | 26         | 0.21    |
| watertreatnd_conc | 0                             | 140                 | 143   | 2570    | 397        | 0.05    |
| watertreatnd_flow | 0                             | 150                 | 187   | 4163    | 413        | 0.11    |
| waterund01        | 17                            | 42                  | 41    | 1818    | 98         | 0.03    |
| waterund08        | 86                            | 112                 | 236   | 15908   | 148        | 0.37    |
| waterund11        | 41                            | 84                  | 157   | 10278   | 110        | 0.21    |
| waterund14        | 183                           | 240                 | 497   | 31332   | 235        | 1.35    |
| waterund17        | 70                            | 96                  | 173   | 15347   | 146        | 0.23    |
| waterund18        | 60                            | 81                  | 150   | 8187    | 112        | 0.13    |
| waterund22        | 118                           | 232                 | 480   | 91648   | 233        | 2.97    |
| waterund25        | 80                            | 150                 | 311   | 21370   | 179        | 0.56    |
| waterund27        | 173                           | 608                 | 1174  | 226273  | 647        | 13.99   |
| waterund28        | 591                           | 2760                | 3958  | 1218908 | 6602       | 192.67  |
| waterund32        | 519                           | 1840                | 4521  | 3305101 | 2519       | 591.14  |
| waterund36        | 183                           | 702                 | 1444  | 218473  | 936        | 17.61   |
| waterx            | 0                             | 16                  | 58    | 3051    | 2          | 0.09    |
| waterz            | 0                             | 2                   | 2     | 0       | 2          | 0.00    |
| weapons           | 0                             | 3                   | 6     | 117     | 0          | 0.01    |
| windfac           | 9                             | 9                   | 16    | 123     | 12         | 0.01    |
| ibell3a           | 1                             | 51                  | 7     | 76      | 7          | 0.01    |
| ivalues           | 20                            | 3620                | 4711  | 124353  | 6162       | 10.13   |
| 10bar1A           | 0                             | 10                  | 40    | 1923    | 0          | 0.04    |
| 10bar1B           | 0                             | 10                  | 40    | 1036    | 0          | 0.02    |
| 10bar1C           | 1                             | 10                  | 40    | 1888    | 0          | 0.08    |
| 10bar1D           | 0                             | 10                  | 40    | 2857    | 0          | 0.05    |
| 10bar2            | 0                             | 20                  | 80    | 3328    | 0          | 0.06    |
| 10bar3            | 3                             | 10                  | 40    | 1087    | 0          | 0.05    |
| 10bar4            | 9                             | 20                  | 80    | 2341    | 0          | 0.13    |
| 200bar            | 0                             | 595                 | 2380  | 231423  | 0          | 84.82   |
| 25bar             | 0                             | 50                  | 200   | 8284    | 0          | 0.40    |
Table 3 continued

| Instance | $\sum_{i,j} K_{ij} \phi_{ij}$ | $\sum_{i,j} K_{ij}$ | # LPs | # iters | # filtered | time |
|----------|-------------------------------|---------------------|-------|---------|------------|------|
| 72bar    | 0                             | 144                 | 576   | 86396   | 0          | 6.20 |
| 90bar    | 2                             | 180                 | 720   | 156190  | 0          | 11.32|

Table 4: Detailed results for the root gap experiments. Dual bounds are highlighted if there is an improvement or deterioration of at least one percent. Aggregated results are presented in Table 1.

$GC(d_1, d_2, p)$ — gap closed improvement

$d_1$ — dual bound obtained with additional separation and propagation for bilinear terms

$d_2$ — dual bound obtained without additional separation and propagation for bilinear terms

$p$ — reference primal bound

$obj$ — objective sense

| Instance | GC($d_1, d_2, p$) | $d_1$ | $d_2$ | $p$ | $obj$ |
|----------|-------------------|-------|-------|-----|-------|
| 10bar1C  | 0.00              | 1485.26 | 1485.26 | 1623.09 | min   |
| 10bar3   | 0.00              | 1705.74 | 1705.74 | 5156.64 | min   |
| 10bar4   | 0.00              | 1934.78 | 1934.78 | 5647.06 | min   |
| 4stufen  | 0.00              | 100948 | 100948 | 116330 | min   |
| 90bar    | 0.00              | 90.3833 | 90.3619 | 97.5374 | min   |
| alkylation | 0.00            | 2374.08 | 2374.08 | 1768.81 | max   |
| alky1    | 0.01              | -1.99632 | -1.99912 | -1.765 | min   |
| arki0005 | 0.00              | 0      | 0      | 372.605 | min   |
| arki0015 | 0.78              | -287.002 | -337.945 | -272.3 | min   |
| arki0016 | 0.09              | -1291.09 | -1506.76 | 867.973 | min   |
| arki0017 | 0.19              | -1337.94 | -1623.99 | -121.833 | min   |
| arki0018 | 0.00              | -2.96278 | -2.96946 | 0.0104566 | min   |
| arki0019 | 0.05              | -33.8267 | -34.6682 | -17.543 | min   |
| arki0020 | 0.00              | -71.8188 | -71.8188 | -41.075 | min   |
| arki0022 | 0.00              | -138.599 | -138.599 | -87.0991 | min   |
| arki0024 | 0.00              | -8361.94 | -8361.94 | -7431.03 | min   |
| batch0812_nc | 0.09 | 2.67193e+06 | 2.67051e+06 | 2.68703e+06 | min   |
| batch_nc | 0.00              | 238021 | 238021 | 285507 | min   |
| bayes2_50| 0.00              | 3.53496e-11 | 3.53496e-11 | 0.520208 | min   |
| bchoo05  | 0.00              | 0.9999964 | 0.999973 | 0.951903 | max   |
| bchoo06  | 0.00              | 0.999984 | 0.999989 | 0.962776 | max   |
| bchoo07  | 0.00              | 0.999997 | 0.999991 | 0.962992 | max   |
| beuster  | 0.00              | 19360.2 | 19360.2 | 116330 | min   |
| blend029 | 0.09              | 15.2087 | 15.3846 | 13.3594 | max   |
| blend146 | -0.00             | 47.704 | 47.7034 | 45.2966 | max   |
| blend480 | 0.02              | 10.2616 | 10.2873 | 9.2266 | max   |
| blend531 | 0.00              | 20.9429 | 20.9429 | 20.039 | max   |
| blend718 | 0.00              | 20.5011 | 20.5011 | 7.3936 | max   |
| blend721 | 0.00              | 14.3443 | 14.3443 | 13.5268 | max   |
| blend852 | 0.03              | 54.5897 | 54.6086 | 53.9627 | max   |
| camshape100 | 0.00 | -5.02505 | -5.02799 | -4.28441 | min   |
| camshape200 | 0.00 | -5.1528 | -5.15304 | -4.27952 | min   |
| Instance          | $GC(d_1, d_2, p)$ | $d_1$     | $d_2$     | $p$     | obj  |
|-------------------|--------------------|-----------|-----------|---------|------|
| camshape400       | 0.00               | -5.24612  | -5.24612  | -4.27976| min  |
| camshape800       | 0.00               | -5.31217  | -5.31217  | -4.29034| min  |
| carton7           | 0.00               | 45        | 45        | 191.73  | min  |
| carton9           | -0.25              | 46.928    | 86.8414   | 205.137 | min  |
| casctanks         | 0.00               | 6.48234   | 6.48234   | 9.16347 | min  |
| case_1scv2        | 0.41               | 2.08988e+11| 3.56906e+11| 7888.57 | max  |
| cesam2log         | 0.33               | -436.742  | -502.327  | 5.06976 | min  |
| chain100          | 0.00               | -145.233  | -121.952  | 5.06862 | min  |
| chain200          | 0.03               | -115.157  | -120.213  | 5.07226 | min  |
| chain400          | 0.04               | -115.157  | -120.213  | 5.07226 | min  |
| chain50           | 0.03               | -61.9221  | -62.3813  | -47.7065| min  |
| chenery            | 0.15               | -1180.41  | -1202.43  | -1058.92| min  |
| chp_partload       | 0.01               | 20.2007   | 20.1587   | 23.2981 | min  |
| chp_shorttermplan2d| 0.00              | 462599    | 462599    | 489382  | min  |
| clay0203h         | -0.01              | 200       | 524.695   | 41573.3 | min  |
| clay0204h         | 0.00               | 1585.48   | 1585.48   | 6545    | min  |
| clay0205h         | 0.00               | 1241.72   | 1241.72   | 8092.5  | min  |
| contvar            | 0.01               | 391188    | 386632    | 809150  | min  |
| crudeoil_lee1.05  | 0.11               | 79.9167   | 79.9854   | 79.35   | max  |
| crudeoil_lee1.06  | -0.00              | 80        | 80        | 79.75   | max  |
| crudeoil_lee1.07  | 0.00               | 80        | 80        | 79.75   | max  |
| crudeoil_lee1.08  | 0.00               | 80        | 80        | 79.75   | max  |
| crudeoil_lee1.09  | 0.00               | 80        | 80        | 79.75   | max  |
| crudeoil_lee1.10  | 0.00               | 80        | 80        | 79.75   | max  |
| crudeoil_lee2.05  | -0.02              | 102.768   | 102.656   | 96.1699 | max  |
| crudeoil_lee2.06  | 0.11               | 102.794   | 102.999   | 101.175 | max  |
| crudeoil_lee2.07  | -0.01              | 102.811   | 102.8     | 101.175 | max  |
| crudeoil_lee2.08  | -0.11              | 102.996   | 102.8     | 101.175 | max  |
| crudeoil_lee2.09  | 0.11               | 102.8     | 103       | 101.175 | max  |
| crudeoil_lee2.10  | -0.00              | 103       | 103       | 101.175 | max  |
| crudeoil_lee3.05  | 0.23               | 97.746    | 97        | 85.4489 | max  |
| crudeoil_lee3.06  | 0.21               | 93.5643   | 95.744    | 85.4489 | max  |
| crudeoil_lee3.07  | 0.19               | 94.5867   | 96.7917   | 85.4489 | max  |
| crudeoil_lee3.08  | 0.12               | 95.4342   | 96.8583   | 85.4489 | max  |
| crudeoil_lee3.09  | 0.14               | 95.3685   | 97        | 85.4489 | max  |
| crudeoil_lee3.10  | 0.11               | 95.6961   | 97        | 85.4489 | max  |
| crudeoil_lee4.05  | -0.97              | 132.585   | 132.548   | 132.548 | max  |
| crudeoil_lee4.06  | -0.97              | 132.585   | 132.548   | 132.548 | max  |
| crudeoil_lee4.07  | 0.00               | 132.585   | 132.585   | 132.548 | max  |
| crudeoil_lee4.08  | 0.34               | 132.572   | 132.585   | 132.548 | max  |
| crudeoil_lee4.09  | 0.00               | 132.585   | 132.585   | 132.548 | max  |
| crudeoil_lee4.10  | 0.00               | 132.585   | 132.585   | 132.548 | max  |
| crudeoil_li01     | 0.00               | 5239.26   | 5239.65   | 5122.56 | max  |
| crudeoil_li03     | 0.00               | 3578.4    | 3578.45   | 3483.65 | max  |
| crudeoil_li05     | 0.00               | 3471.07   | 3471.07   | 3129.84 | max  |
| Instance          | $GC(d_1,d_2,p)$ | $d_1$  | $d_2$  | $p$  | obj  |
|-------------------|-----------------|--------|--------|------|------|
| crudeoil_li06     | -0.00           | 3375   | 3375   | 3355 | max  |
| crudeoil_li11     | 0.00            | 4779.68| 4779.73| 4868.79| max  |
| crudeoil_li21     | -0.01           | 4943.89| 4942.79| 4799.58| max  |
| crudeoil_pooling_ct1 | -0.00    | 50155.1| 50155.9 | 210538| min  |
| crudeoil_pooling_ct3 | 0.00    | 53685.3| 53685.3 | 287000| min  |
| crudeoil_pooling_dt1 | -0.00    | 140292 | 140625 | 209585| max  |
| crudeoil_pooling_dt2 | 0.00    | 12774.2| 12774.2 | 10239.9| max  |
| crudeoil_pooling_dt3 | 0.00    | 180879 | 180879 | 284781| min  |
| crudeoil_pooling_dt4 | 0.00    | 14333  | 14333.2 | 132547.6| max  |
| csched1           | 0.04           | -46735.1| -47322.6| -30639.3| min  |
| csched2           | 0.00           | -1.83098e+07 | -1.83098e+07 | -166102| min  |
| csched2a          | -0.00          | -9.8738e+06 | -9.87377e+06 | -165399| min  |
| deb6              | 0.00           | 0      | 0      | 201.739| min  |
| deb7              | 0.00           | 0      | 0      | 116.585| min  |
| deb8              | 0.00           | 0      | 0      | 116.585| min  |
| deb9              | 0.00           | 0      | 0      | 116.585| min  |
| elf               | 0.00           | 0      | 0      | 0.191667| min  |
| ethanolh           | 0.00           | -157.663| -157.663 | -157.587| min  |
| ethanolin          | 0.00           | -157.589| -157.589 | -157.587| min  |
| ex1252            | 0.00           | 6329.03| 6329.03| 128894| min  |
| ex1252a           | 0.00           | 0      | 0      | 128894| min  |
| ex1263a           | 0.00           | 19.3   | 19.3   | 19.6| min  |
| ex2_1_9           | 0.56           | -1.10713| -2.05246| -0.375001| min  |
| ex3_1_1           | 0.00           | 2835.87| 2825.06| 7049.25| min  |
| ex3_1_2           | 1.00           | -30665.5| -30670 | -30665.5| min  |
| ex3_1_4           | 0.00           | -6     | -6     | -4| min  |
| ex5_2_2_case1     | 0.00           | -590.563| -590.563| -400| min  |
| ex5_2_2_case2     | 0.00           | -1200  | -1200  | -600| min  |
| ex5_2_2_case3     | 0.00           | -868.384| -868.384| -750| min  |
| ex5_2_4           | 0.07           | -2765.38| -2933.33| -450| min  |
| ex5_2_5           | 0.00           | -9700  | -9700  | -3500| min  |
| ex5_3_2           | 0.00           | 0.9979 | 0.9979 | 1.86416| min  |
| ex5_3_3           | 0.00           | 1.63436| 1.63132| 3.23402| min  |
| ex5_4_2           | 0.00           | 3100.82| 3095.61| 7512.23| min  |
| ex5_4_3           | 0.01           | 4199.84| 4193.84| 4845.46| min  |
| ex5_4_4           | 0.00           | 4250.21| 4250.21| 10077.8| min  |
| ex6_1_1           | 0.44           | -4.91971| -8.75722| -0.0201983| min  |
| ex6_1_2           | 0.48           | -3.7855| -7.21873| -0.0324638| min  |
| ex6_1_3           | 0.44           | -5.16504| -8.91322| -0.352498| min  |
| ex6_2_10          | 0.13           | -458.252| -527.568| -3.05198| min  |
| ex6_2_11          | 0.11           | -551.081| -622.387| -2.6724e-06| min  |
| ex6_2_12          | 0.18           | -133.39| -163.217| 0.289195| min  |
| ex6_2_13          | 0.13           | -92.6301| -106.243| -0.216209| min  |
| ex6_2_14          | 0.12           | -81.8774| -93.3293| -0.695358| min  |
| ex6_2_5           | 0.30           | -19460.2| -27805.5| -70.7521| min  |
| ex6_2_6           | 0.23           | -281.148| -364.721| -2.6025e-06| min  |
| Instance | GC \((d_1, d_2, p)\) | \(d_1\) | \(d_2\) | \(p\) | obj |
|----------|----------------|--------|--------|------|-----|
| ex6_2_7  | 0.00 | -173.868 | -174.146 | -0.160848 | min |
| ex6_2_8  | 0.34 | -221.639 | -337.127 | -0.027064 | min |
| ex6_2_9  | 0.18 | -556.992 | -680.203 | -0.034066 | min |
| ex7_2_1  | 0.00 | 1088.8 | 1088.8 | 1227.22 | min |
| ex7_2_2  | 0.31 | -0.512116 | -0.568234 | -0.388811 | min |
| ex7_2_4  | 0.02 | 1.17141 | 1.11347 | 3.91801 | min |
| ex8_2_3b | 0.00 | -3731.59 | -3731.59 | -3731.08 | min |
| ex8_2_4b | 0.00 | -1197.49 | -1197.49 | -1197.14 | min |
| ex8_3_8  | 0.00 | -10 | -10 | -3.25612 | min |
| ex8_4_1  | 0.14 | 0.426895 | 0.396777 | 0.618573 | min |
| ex8_4_2  | 0.00 | -5.54815e-07 | -5.72607e-07 | 0.485152 | min |
| ex8_4_4  | 0.07 | 0.0633458 | 0.0515233 | 0.21246 | min |
| ex8_4_8_bnd | 0.00 | -194025 | -194025 | 3.32185 | min |
| ex9_2_2  | 1.00 | 99.9996 | 78.9871 | 99.9996 | min |
| ex9_2_3  | 0.00 | -30 | -30 | -0 | min |
| ex9_2_4  | 1.00 | 0.499999 | -6.6236e-07 | 0.499999 | min |
| ex9_2_6  | 0.53 | -1.23022 | -1.4901 | -1 | min |
| feedtray | 0.00 | -68.6842 | -68.6842 | -13.406 | min |
| forest   | 0.00 | 2.04172e+07 | 2.04172e+07 | 1.43962e+07 | max |
| gabriel01| 0.01 | 47.61 | 47.6436 | 45.2444 | max |
| gabriel02| 0.01 | 47.3304 | 47.4432 | 39.6097 | max |
| gabriel04| 0.09 | 10.2055 | 10.2972 | 9.2266 | max |
| gabriel09| 0.00 | 134.625 | 134.647 | 112.42 | max |
| gans02   | 0.00 | 2.42232e+06 | 2.4998e+06 | 8.94669e+06 | min |
| gans03   | 0.00 | 90473.2 | 90473.2 | 10182 | max |
| gasnet   | 0.00 | 2.33965e+06 | 2.3245e+06 | 6.99938e+06 | min |
| gasnet_al1| 0.00 | 6320.77 | 6320.77 | 7438.04 | min |
| gasnet_al2| 0.00 | 6122.87 | 6122.87 | 7114.13 | min |
| gasnet_al3| 0.00 | 6306.98 | 6306.98 | 7363.32 | min |
| gasnet_al4| 0.00 | 6312.36 | 6312.36 | 7429.71 | min |
| gasnet_al5| 0.00 | 6279.89 | 6279.89 | 7385.11 | min |
| gasprod_sarawak01| 0.00 | -33085.4 | -33085.4 | -32445.4 | min |
| gasprod_sarawak16| 0.03 | -32928.7 | -32947.5 | -32271.2 | min |
| gasprod_sarawak81| 0.00 | -33027.7 | -33028.2 | -32273 | min |
| gastrans582_warm31| 0.00 | -33027.7 | -33028.2 | -32273 | min |
| genpool15 | 0.00 | 463471 | 463471 | 4.3491e+06 | min |
| genpool15i | 0.00 | 649403 | 649403 | 2.0790e+06 | min |
| genpool15paper | 0.00 | 552573 | 552573 | 4.3491e+06 | min |
| genpool20 | 0.00 | 702565 | 702565 | 3.72126e+06 | min |
| genpool20i | 0.00 | 918071 | 918071 | 3.9463e+06 | min |
| genpool20paper | 0.00 | 702572 | 702572 | 3.72126e+06 | min |
| genpooling_lee1 | 0.00 | -5494.43 | -5494.43 | -4640.08 | min |
| genpooling_lee2 | 0.01 | -5086.9 | -5093.33 | -3849.27 | min |
| genpooling_meyer10 | 0.00 | 648966 | 648966 | 1.08619e+06 | min |
| ghg_1veh | 0.02 | 6.26042 | 6.22463 | 7.78163 | min |
| ghg_2veh | 0.00 | 0 | 0 | 7.7709 | min |
| Instance       | $GC(d_1, d_2, p)$ | $d_1$    | $d_2$    | $p$    | obj   |
|---------------|------------------|---------|---------|-------|------|
| ghh_3veh      | 0.00             | 0       | 0       | 7.75401 | min  |
| gsg_0001      | 0.00             | 2342.42 | 2342.42 | 2378.16 | min  |
| heatexch_gen1 | 0.00             | 100500  | 100500  | 154896 | min  |
| himmel11      | 1.00             | -30665.5 | -30670 | -30665.5 | min  |
| hybriddynamic_var | 0.08       | 1.48182 | 1.47681 | 1.53641 | min  |
| hydroenergy1  | 0.14             | 213642  | 214293  | 209721 | max  |
| hydroenergy2  | 0.08             | 379231  | 379866  | 371812 | max  |
| hydroenergy3  | 0.04             | 763522  | 764383  | 744964 | max  |
| ibell3a       | 0.00             | 874307  | 874307  | 878785 | min  |
| inffeas1      | 0.43             | 17300.9 | 20012.4 | 13685.6 | max  |
| ivalues       | 0.00             | -11.4099 | -11.4099 | -1.16568 | min  |
| kall_circles_c6a | 0.00       | 0       | 0       | 2.11171 | min  |
| kall_circles_c6b | 0.00       | 0       | 0       | 1.9736 | min  |
| kall_circles_c6c | 0.00       | 0       | 0       | 2.7977 | min  |
| kall_circles_c7a | 0.00       | 0       | 0       | 2.66281 | min  |
| kall_circles_c8a | 0.00       | 0       | 0       | 2.54092 | min  |
| kall_circlespolygons | 0.00   | 0       | 0       | 0.339602 | min  |
| kall_circlespolygons | 0.00   | 0       | 0       | 0.339602 | min  |
| kall_circlespolygons | 0.00   | 0       | 0       | 2.84872 | min  |
| kall_circlespolygons | 0.00   | 0       | 0       | 3.84872 | min  |
| kall_circlespolygons | 0.00   | 0       | 0       | 3.87051 | min  |
| kall_circlesrectangl | 0.00 | 0       | 0       | 0.214602 | min  |
| kall_circlesrectangl | 0.00 | 0       | 0       | 0.339602 | min  |
| kall_circlesrectangl | 0.00 | 0       | 0       | 6.29517 | min  |
| kall_circlesrectangl | 0.00 | 0       | 0       | 6.63339 | min  |
| kall_circlesrectangl | 0.00 | 0       | 0       | 7.1645 | min  |
| kall_congruentcircle | 0.00 | 0       | 0       | 0.643805 | min  |
| kall_congruentcircle | 0.00 | 0       | 0       | 0.858407 | min  |
| kall_congruentcircle | 0.00 | 0       | 0       | 0.858407 | min  |
| kall_congruentcircle | 0.00 | 0       | 0       | 1.07301 | min  |
| kall_congruentcircle | 0.00 | 0       | 0       | 1.28761 | min  |
| kall_congruentcircle | 0.00 | 0       | 0       | 1.28761 | min  |
| kall_congruentcircle | 0.00 | 0       | 0       | 1.37586 | min  |
| kall_congruentcircle | 0.00 | 0       | 0       | 1.50221 | min  |
| kall_congruentcircle | 0.00 | 0       | 0       | 1.53711 | min  |
| kall_congruentcircle | 0.00 | 0       | 0       | 1.96631 | min  |
| kall_diffcircles_10 | 0.00 | -1e-09 | -1e-09 | 11.9355 | min  |
| kall_diffcircles_5a | 0.00 | 1.80501 | 1.80501 | 5.11618 | min  |
| kall_diffcircles_5b | 0.00 | 0       | 0       | 5.11618 | min  |
| kall_diffcircles_6 | 0.00 | 0       | 0       | 7.78789 | min  |
| kall_diffcircles_7 | 0.00 | 0       | 0       | 7.15313 | min  |
| kall_diffcircles_8 | 0.00 | -1e-09 | -1e-09 | 14.4813 | min  |
| kall_diffcircles_9 | 0.00 | -1e-09 | -1e-09 | 13.3503 | min  |
| kall_ellipsoids_tc05 | 0.00 | 20.9921 | 20.9921 | 5.68507e+06 | min  |
| launch         | 0.00             | 1831.92 | 1831.92 | 2257.8 | min  |
| Instance                        | $\text{GC}(d_1, d_2, p)$ | $d_1$  | $d_2$  | $p$       | obj        |
|--------------------------------|---------------------------|--------|--------|-----------|------------|
| mathopt1                       | 0.52                      | -1368.41 | -2863.63 | -3.81757e-08 | min        |
| mathopt4                       | 0.00                      | -226.705 | -226.705 | 0         | min        |
| mpss-basic-marvin-85           | 0.09                      | 1462.84  | 1537.76  | 670.296   | max        |
| mpss-basic-ob25-125-           | 0.06                      | 103.975  | 107.778  | 44.5005   | max        |
| mpss-basic-red-marvi           | 0.08                      | 1464.18  | 1536.74  | 669.167   | max        |
| mpss-basic-red-ob25-           | 0.06                      | 104.203  | 107.931  | 44.454    | max        |
| mpss-extwarehouse-ma           | 0.00                      | 714.673  | 714.673  | 687.608   | max        |
| mpss-extwarehouse-ob           | 0.00                      | 50.0001  | 50.0001  | 0         | max        |
| mpss-extwarehouse-re           | 0.00                      | 50.0228  | 50.0228  | 3.23207   | max        |
| mpss-extwarehouse-re           | 0.00                      | 715.069  | 715.069  | 688.452   | max        |
| multiplants_mtg1a              | 0.08                      | 1872.9   | 1995.65  | 391.613   | max        |
| multiplants_mtg1b              | 0.02                      | 3205.84  | 3261.81  | 450.548   | max        |
| multiplants_mtg1c              | 0.03                      | 5228.07  | 5375.64  | 683.971   | max        |
| multiplants_mtg2               | 0.02                      | 10076.1  | 10137.8  | 7099.19   | max        |
| multiplants_mtg6               | 0.20                      | 7101.6   | 7387.06  | 5924.65   | max        |
| multiplants_mtg6               | 0.24                      | 6765.9   | 7221.84  | 5314.43   | max        |
| multiplants_stg1               | 0.01                      | 10984    | 11056.3  | 355.087   | max        |
| multiplants_stg1a              | 0.01                      | 9080.55  | 9178.87  | 390.966   | max        |
| multiplants_stg1b              | 0.00                      | 21546    | 21651.2  | 471.75    | max        |
| multiplants_stg1c              | 0.00                      | 19346.7  | 19394.6  | 708.44    | max        |
| multiplants_stg5               | 0.00                      | 30760.8  | 30760.8  | 5843.27   | max        |
| multiplants_stg6               | -0.00                     | 38984.4  | 38984.4  | 5166.12   | max        |
| ndcc12persp                    | 0.00                      | 47.8141  | 47.6512  | 106.354   | min        |
| ndcc12                         | 0.02                      | 48.3895  | 47.1944  | 106.354   | min        |
| ndcc13                         | 0.00                      | 65.0181  | 65.0181  | 84.625    | min        |
| ndcc13persp                    | 0.01                      | 65.4329  | 65.2812  | 85.8919   | min        |
| ndcc14                         | 0.00                      | 60.6594  | 60.6288  | 110.328   | min        |
| ndcc14persp                    | -0.00                     | 61.0086  | 61.0624  | 111.27    | min        |
| ndcc15                         | 0.00                      | 68.788   | 68.787   | 94.6112   | min        |
| ndcc15persp                    | -0.00                     | 69.4158  | 69.4164  | 94.6112   | min        |
| ndcc16                         | 0.00                      | 60.7739  | 60.7532  | 112.071   | min        |
| ndcc16persp                    | 0.00                      | 60.7714  | 60.7714  | 113.546   | min        |
| nous1                          | 0.05                      | -0.182865 | -0.272505 | 1.56707   | min        |
| nous2                          | 0.11                      | 0.256654  | 0.208944  | 0.625967   | min        |
| nuclear14a                     | -0.00                     | -12.258  | -12.258  | -1.12955   | min        |
| nuclear14b                     | 0.87                      | -1.19898  | -1.7092  | -1.12589   | min        |
| nuclear25a                     | 0.00                      | -12.3207  | -12.307  | -1.12051   | min        |
| nuclear25b                     | -0.83                     | -1.74673  | -1.2208  | -1.11362   | min        |
| nuclear49a                     | -0.00                     | -12.3598  | -12.3598 | -1.15144   | min        |
| nuclear49b                     | -0.13                     | -1.76511  | -1.68278 | -1.14      | min        |
| nvso1                          | 0.00                      | 6.42354   | 6.42354  | 12.4697   | min        |
| nvso5                          | 0.00                      | 2.02222   | 2.0222   | 5.47093   | min        |
| nvso13                         | 0.00                      | -588.801  | -588.801 | -585.2    | min        |
| nvso17                         | 0.01                      | -1104.55  | -1104.61 | -1100.4   | min        |
| nvso18                         | 0.00                      | -782.618  | -782.618 | -778.4    | min        |
| nvso19                         | -0.00                     | -1104.17  | -1104.16 | -1098.4   | min        |
| Instance         | GC\((d_1, d_2, p)\) | \(d_1\)   | \(d_2\)   | \(p\)   | obj    |
|------------------|---------------------|-----------|-----------|--------|-------|
| nvs20            | 0.00                | 197.377   | 197.377   | 230.922| min   |
| nvs23            | 0.00                | -1130.63  | -1130.63  | -1125.2| min   |
| nvs24            | 0.00                | -1035.77  | -1035.77  | -1033.2| min   |
| oil              | 0.00                | -1.05575  | -1.05579  | -0.853598| min   |
| orth_d3m6        | 0.00                | 0         | 0         | 0.707107| min   |
| orth_d3m6_pl     | 0.00                | 0         | 0         | 0.707107| min   |
| orth_d4m6_pl     | 0.00                | 0         | 0         | 0.649519| min   |
| parallel         | -0.00               | -67742.6  | -67740.6  | 924.296| min   |
| pindyck          | 0.62                | -2239.98  | -3972.57  | -1170.49| min   |
| pointpack04      | 0.18                | 1.16325   | 1.2       | 0.361111| max   |
| pointpack06      | 0.35                | 0.9375    | 1.25      | 0.267949| max   |
| pointpack08      | 0.32                | 0.931118  | 1.25      | 0.177476| max   |
| pointpack10      | 0.30                | 0.931117  | 1.25      | 0.151111| max   |
| pointpack12      | 0.29                | 0.931118  | 1.25      | 0.121742| max   |
| pointpack14      | 0.28                | 0.931118  | 1.25      | 0.121742| max   |
| pooling_adhya1pq  | 0.00                | -840.271  | -840.271  | -549.803| min   |
| pooling_adhya1stp| 0.00                | -840.271  | -840.271  | -549.803| min   |
| pooling_adhya1tp  | 0.00                | -856.251  | -856.251  | -549.803| min   |
| pooling_adhya2pq  | 0.00                | -574.783  | -574.783  | -549.803| min   |
| pooling_adhya2tp  | 0.00                | -573.624  | -573.624  | -549.803| min   |
| pooling_adhya3pq  | 0.00                | -574.783  | -574.783  | -549.803| min   |
| pooling_adhya3tp  | 0.00                | -574.783  | -574.783  | -549.803| min   |
| pooling_adhya4pq  | 0.00                | -961.932  | -961.932  | -877.646| min   |
| pooling_adhya4tp  | 0.00                | -976.439  | -976.439  | -877.646| min   |
| pooling_bental14pq| 0.04                | -464.888  | -465.574  | 450   | min   |
| pooling_bental14tp| 0.39                | -496.855  | -527.165  | -450   | min   |
| pooling_digabel16 | 0.00                | -2513.72  | -2513.72  | -2410.69| min   |
| pooling_digabel18 | 0.00                | -799.853  | -799.853  | -689.161| min   |
| pooling_digabel19 | 0.03                | -4552.22  | -4552.55  | -4539.91| min   |
| pooling_epa1      | 0.00                | -509.399  | -509.85   | -280.806| min   |
| pooling_epa2      | 0.00                | -4649.45  | -4649.45  | -4567.36| min   |
| pooling_epa3      | 0.00                | -14998.6  | -14998.6  | -14965.2| min   |
| pooling_haverly1tp| 0.59                | -247.273  | -466.667  | -400   | min   |
| pooling_haverly2pq| 0.00                | -1134.63  | -1134.63  | -4391.83| min   |
| pooling_haverly2stp| 0.00               | -658.906  | -658.906  | -600   | min   |
| pooling_haverly2tp | 0.00            | -1134.63  | -1134.63  | -4391.83| min   |
| pooling_haverly3tp| 0.33                | -833.951  | -875      | -750   | min   |
| Instance            | GC($d_1, d_2, p$) | $d_1$       | $d_2$       | $p$     | obj     |
|---------------------|--------------------|-------------|-------------|---------|---------|
| pooling_sppa9pq      | 0.00               | -21934      | -21934      | -21933.9 | min     |
| pooling_sppa9stp     | 0.00               | -21934      | -21934      | -21864.2 | min     |
| pooling_sppa9tp      | 0.00               | -21934      | -21934      | -21929.6 | min     |
| pooling_sppb0pq      | 0.00               | -45466.5    | -45466.5    | -43412.4 | min     |
| pooling_sppb0stp     | 0.00               | -45466.5    | -45466.5    | -42546.3 | min     |
| pooling_sppb0tp      | 0.00               | -45466.5    | -45466.5    | -43372.8 | min     |
| pooling_sppb2pq      | 0.00               | -56537.4    | -56537.4    | -53734.4 | min     |
| pooling_sppb2stp     | 0.00               | -56537.4    | -56537.4    | -54092.4 | min     |
| pooling_sppb2tp      | 0.00               | -60696.4    | -60696.4    | -60599.2 | min     |
| pooling_sppb5pq      | 0.00               | -45466.5    | -45466.5    | -42546.3 | min     |
| pooling_sppb5stp     | 0.00               | -60696.4    | -60696.4    | -53800.4 | min     |
| pooling_sppb5tp      | 0.00               | -60696.4    | -60696.4    | -60438   | min     |
| pooling_sppb0pq      | 0.00               | -99776.3    | -99776.3    | -84775.4 | min     |
| pooling_sppb0stp     | 0.00               | -93052.2    | -93052.2    | -86043.6 | min     |
| pooling_sppb0tp      | 0.00               | -9616.4     | -9616.4     | -86439.1 | min     |
| pooling_sppc1pq      | 0.00               | -120594     | -120594     | -99870.2 | min     |
| pooling_sppc1stp     | 0.00               | -120154     | -120154     | -9257.9  | min     |
| pooling_sppc1tp      | 0.00               | -120222     | -120222     | -96689.6 | min     |
| pooling_sppc3pq      | 0.00               | -130315     | -130315     | -114741  | min     |
| pooling_sppc3stp     | 0.00               | -130315     | -130315     | -87023.7 | min     |
| pooling_sppc3tp      | 0.00               | -130315     | -130315     | -118490  | min     |
| powerflow0009r       | 0.00               | 2244.81     | 2244.81     | 5296.69  | min     |
| powerflow0014r       | 0.00               | 0           | 0           | 8082.58  | min     |
| powerflow0030r       | 0.00               | 0           | 0           | 576.893  | min     |
| powerflow0039r       | 0.00               | 27035.8     | 27035.8     | 41869.1  | min     |
| powerflow0118r       | 0.00               | 0           | 0           | 129657   | min     |
| primary              | 0.00               | -100        | -100        | -1.28797 | min     |
| prob09               | 0.00               | -100        | -100        | 0        | min     |
| procurement1large    | 0.26               | 17806.5     | 22683.8     | 3796.23  | max     |
| procurement1mot      | 0.26               | 2632.87     | 3454.16     | 291.542  | max     |
| qp3                  | 0.86               | -0.384106   | -2.81695    | 0.0000809315 | min |
| rbrick               | 0.86               | -0.384106   | -2.81695    | 0.0000809315 | min |
| rsyn0805h            | -0.18              | 4718.12     | 4279.01     | 2238.4   | max     |
| rsyn0805m02h         | -0.01              | 9929.58     | 9888.71     | 7174.22  | max     |
| rsyn0805m03h         | -0.01              | 9929.58     | 9888.71     | 7174.22  | max     |
| rsyn0805m04h         | -0.01              | 9929.58     | 9888.71     | 7174.22  | max     |
| rsyn0810h            | 0.00               | 8304.2      | 2722.45     | 2238.4   | max     |
| rsyn0810m03h         | 0.00               | 8304.2      | 2722.45     | 2238.4   | max     |
| rsyn0810m04h         | 0.00               | 8304.2      | 2722.45     | 2238.4   | max     |
| rsyn0815m02h         | -0.02              | 4535.62     | 4475.06     | 1774.4   | max     |
| Instance           | GC\((d_1,d_2,p)\) | \(d_1\)  | \(d_2\)  | \(p\)  | obj  |
|--------------------|--------------------|--------|--------|--------|------|
| rsyn0815m03h       | 0.00               | 6609.81| 6609.81| 2827.93| max  |
| rsyn0815m04h       | 0.00               | 8382.33| 8382.33| 3410.86| max  |
| rsyn0820m02h       | 0.00               | 3750.17| 3750.17| 1092.09| max  |
| rsyn0820m03h       | 0.00               | 6384.02| 6384.02| 2028.81| max  |
| rsyn0820m04h       | 0.00               | 8241.34| 8241.34| 2450.77| max  |
| rsyn0830h          | 0.00               | 1227.18| 1227.18| 510.072| max  |
| rsyn0830m04h       | 0.00               | 5994.57| 5994.57| 2529.07| max  |
| rsyn0840h          | 0.00               | 1014.56| 1014.56| 325.555| max  |
| rsyn0840m02h       | 0.00               | 2861.44| 2861.44| 734.984| max  |
| rsyn0840m03h       | 0.00               | 5188.82| 5188.82| 2742.65| max  |
| rsyn0840m04h       | 0.00               | 6741.84| 6741.84| 2564.5 | max  |
| saa_2              | 0.00               | 2.65718| 2.65048| 12.1613| min  |
| sep1               | 0.00               | -523.873|-523.873| -510.081| min  |
| sepequ_complex     | 0.00               | 32.9115| 32.9115| 368.762| min  |
| sepequ_convent     | 0.00               | 206.958| 206.958| 482.5 | min  |
| sfacloc1_2_95      | 0.00               | 0.052573| 0.0449145| 18.8501| min  |
| sfacloc1_3_95      | 0.00               | 0        | 0 | 12.3025 | min  |
| smallinvSNPr1b010-01| 0.14            | -0.857926|-1.01251| 0.0665282| min  |
| smallinvSNPr1b020-02| 0.24           | -3.86333|-5.13799| 0.249427 | min  |
| smallinvSNPr1b050-05| 0.26           | -27.5666|-37.9197| 1.47655 | min  |
| smallinvSNPr1b100-11| 0.29           | -105.148|-150.539| 5.81308 | min  |
| smallinvSNPr1b150-16| 0.28           | -244.298|-342.634| 12.9912 | min  |
| smallinvSNPr1b200-22| 0.26           | -437.043|-596.091| 23.0993 | min  |
| smallinvSNPr2b010-01| 0.12           | -0.627074|-0.724322| 0.0665282| min  |
| smallinvSNPr2b020-02| 0.18           | -2.99537|-3.71319| 0.261127 | min  |
| smallinvSNPr2b050-05| 0.19           | -22.0866|-27.5066| 1.50222 | min  |
| smallinvSNPr2b100-11| 0.28           | -78.1266|-111.553| 5.97772 | min  |
| smallinvSNPr2b150-16| 0.19           | -200.438|-250.187| 13.3605 | min  |
| smallinvSNPr2b200-22| 0.18           | -364.768|-452.011| 23.7525 | min  |
| smallinvSNPr3b010-01| 0.10           | -0.555164|-0.627159| 0.0840684| min  |
| smallinvSNPr3b020-02| 0.24           | -2.05927|-2.77977| 0.263234 | min  |
| smallinvSNPr3b050-05| 0.13           | -17.6986|-20.5786| 1.58959 | min  |
| smallinvSNPr3b100-11| 0.07           | -77.8361|-83.9851| 6.19995 | min  |
| smallinvSNPr3b150-16| 0.10           | -143.638|-161.416| 13.7965 | min  |
| smallinvSNPr3b200-22| 0.15           | -289.597|-343.486| 24.4776 | min  |
| smallinvSNPr4b010-01| -0.16          | -0.23764|-0.181442| 0.111102| min  |
| smallinvSNPr4b020-02| 0.02           | -1.8303|-1.87246| 0.282352 | min  |
| smallinvSNPr4b050-05| 0.03           | -11.4576|-11.8816| 1.6412 | min  |
| smallinvSNPr4b100-11| 0.04           | -48.2906|-50.6166| 6.36716 | min  |
| smallinvSNPr4b150-16| 0.01           | -95.795|-96.3841| 14.2952 | min  |
| smallinvSNPr4b200-22| 0.01           | -200.894|-204.051| 25.2689 | min  |
| smallinvSNPr5b010-01| 0.09           | -0.313774|-0.356436| 0.111102| min  |
| smallinvSNPr5b020-02| 0.08           | -0.801615|-0.900147| 0.308004 | min  |
| smallinvSNPr5b050-05| 0.01           | -8.10856|-8.16781| 1.67257 | min  |
| smallinvSNPr5b100-11| 0.02           | -31.1772|-31.8599| 6.6369 | min  |
| smallinvSNPr5b150-16| 0.01           | -72.1961|-72.8221| 14.8804 | min  |
Table 4 continued

| Instance                  | $GC(d_1, d_2, p)$ | $d_1$      | $d_2$      | $p$       | obj     |
|---------------------------|-------------------|------------|------------|-----------|---------|
| smallinvSNPr5b200-22     | 0.01              | -130.563   | -132.309   | 26.2652   | min     |
| space960                  | 0.00              | 6.5265e+06 | 6.5265e+06 | 1.713e+07 | min     |
| spar30-60-2               | 0.04              | 1657.79    | 1668.25    | 1377.17   | max     |
| spectra2                  | 0.00              | 12.7052    | 12.7052    | 13.9783   | min     |
| spring                    | 0.00              | 0.109406   | 0.109406   | 0.846246  | min     |
| sssd08-04persp            | 0.21              | 120380     | 103803     | 539635    | min     |
| sssd12-05persp            | 0.05              | 172054     | 166435     | 281409    | min     |
| sssd15-04persp            | 0.18              | 94591.5    | 70169      | 205054    | min     |
| sssd15-06persp            | 0.21              | 330469     | 274824     | 347691    | min     |
| sssd15-08persp            | 0.06              | 321695     | 305983     | 562618    | min     |
| sssd16-07persp            | 0.00              | 222034     | 222034     | 417189    | min     |
| sssd18-06persp            | 0.22              | 191815     | 133659     | 397992    | min     |
| sssd18-08persp            | 0.06              | 414610     | 389225     | 832796    | min     |
| sssd20-04persp            | 0.11              | 175838     | 154942     | 347691    | min     |
| sssd20-08persp            | 0.16              | 278920     | 214257     | 469644    | min     |
| sssd22-08persp            | 0.08              | 300168     | 281075     | 508714    | min     |
| sssd25-04persp            | 0.13              | 175594     | 156354     | 300177    | min     |
| sssd25-08persp            | 0.06              | 284833     | 272002     | 472093    | min     |
| st_e03                    | 0.40              | -1540      | -1788.27   | -1161.34  | min     |
| st_e04                    | 0.40              | 4115.28    | 3393.64    | 5194.87   | min     |
| st_e05                    | 0.78              | 6694.62    | 5400.39    | 7049.25   | min     |
| st_e07                    | 0.85              | -404.388   | -428.571   | -400      | min     |
| st_e09                    | 0.67              | -0.5       | -0.502066  | -0.5      | min     |
| st_e16                    | 0.28              | 11947.3    | 11814.3    | 12292.5   | min     |
| st_e23                    | 0.99              | -1.08334   | -1.27683   | -1.08333  | min     |
| st_e25                    | 0.49              | 0.873575   | 0.856691   | 0.890193  | min     |
| st_e28                    | 1.00              | -30665.5   | -30670     | -30665.5  | min     |
| st_e30                    | 0.00              | -3         | -3         | -1.58114  | min     |
| st_e31                    | 0.00              | -3         | -3         | -2        | min     |
| st_e32                    | 0.00              | -9.30136   | -9.30952   | -1.43041  | min     |
| st_e36                    | 0.00              | -304.5     | -304.5     | -246      | min     |
| st_e38                    | -0.37             | 5914.6     | 6387.74    | 7197.73   | min     |
| st_e40                    | -0.02             | 16.2643    | 16.4806    | 30.4142   | min     |
| st_glm_fp3                | 1.00              | -12.0009   | -32        | -12       | min     |
| st_glm_kk92               | 1.00              | -12.0004   | -29.3529   | -12       | min     |
| st_glm_kky                | 0.28              | -14.3598   | -18.9731   | -2.5      | min     |
| st_glm_ssa1               | 0.59              | -26.5597   | -29.4627   | -24.5174  | min     |
| st_iqpbk1                 | 0.76              | -795.139   | -1356.4    | -621.488  | min     |
| st_iqpbk2                 | 0.75              | -1567.78   | -2695.49   | -1195.23  | min     |
| st_jcobaif                | 0.00              | -802.914   | -802.914   | -794.856  | min     |
| st_qpc-m1                 | 0.70              | -503.999   | -573.604   | -473.778  | min     |
| st_qpc-m3a                | 0.45              | -546.304   | -679.23    | -382.695  | min     |
| st_qpc-m3b                | 1.00              | -1.276e-09 | -8.59191   | -1.276e-09| min     |
| st_qpk1                   | 0.50              | -7         | -11        | -3        | min     |
| super3t                   | 0.00              | -1         | -1         | -0.684104 | min     |
| tanksize                  | 0.17              | 0.99787    | 0.942958   | 1.26864   | min     |
Table 4 continued

| Instance                  | GC($d_1, d_2, p$) | $d_1$  | $d_2$  | $p$    | obj    |
|---------------------------|-------------------|--------|--------|--------|--------|
| tln4                      | 0.20              | 4.2    | 3.2    | 8.3    | 5.3 min |
| tln5                      | 0.00              | 4.1    | 4.1    | 10.3   | 5.3 min |
| tln2                      | 0.00              | 1.72281| 1.72281| 5.3    | 5.3 min |
| transswitch0009r          | 0.00              | 1188.75| 1188.75| 5296.69| 5296.69 min |
| transswitch0030r          | 0.00              | 2      | 2      | 4186.61| 4186.61 min |
| transswitch0039r          | 0.00              | 0      | 0      | 12946.9| 12946.9 min |
| util                      | 0.00              | 999.554| 999.554| 999.579| 999.579 min |
| var_con10                 | 0.00              | 0      | 0      | 444.214| 444.214 min |
| var_con5                  | 0.00              | 0      | 0      | 278.145| 278.145 min |
| wager                     | 0.00              | 19584.2| 19584.2| 20339.4| 20339.4 min |
| waste                     | 0.00              | 297.326| 297.326| 598.919| 598.919 min |
| wastewater02m1            | 0.01              | 101.191| 101.034| 130.703| 130.703 min |
| wastewater02m2            | 0.00              | 127.924| 127.924| 130.703| 130.703 min |
| wastewater04m1            | 0.00              | 69.2386| 69.2386| 89.8361| 89.8361 min |
| wastewater04m2            | -0.00             | 75.4002| 75.4002| 89.8361| 89.8361 min |
| wastewater11m1            | 0.00              | 1024.8 | 1024.8 | 2127.12| 2127.12 min |
| wastewater12m1            | 0.00              | 648    | 648    | 1201.04| 1201.04 min |
| wastewater13m1            | 0.00              | 1017.2 | 1017.2 | 1564.96| 1564.96 min |
| wastewater14m1            | 0.00              | 213.266| 213.266| 513.001| 513.001 min |
| wastewater14m2            | 0.00              | 337.654| 337.654| 513.001| 513.001 min |
| wastewater15m1            | 0.00              | 975.484| 975.484| 2446.43| 2446.43 min |
| watercontamination02      | 0.00              | -18.869| -18.869| 125.196| 125.196 min |
| watercontamination02      | 0.00              | -302.016| -302.016| 97.9045| 97.9045 min |
| watercontamination03      | 0.00              | -1069.49| -1069.49| 207.985| 207.985 min |
| watercontamination03      | 0.00              | -1100.73| -1100.73| 424.544| 424.544 min |
| waternon2_01              | 0.10              | 13.0648| 13.0648| 19.4567| 19.4567 min |
| waternon2_02              | 0.05              | 4.38405| 4.38405| 39.5714| 39.5714 min |
| waternon2_03              | 0.00              | 2.10392| 2.10392| 115.005| 115.005 min |
| waternon2_04              | 0.00              | 0      | 0      | 145.44 | 145.44 min |
| waternon2_06              | 0.00              | 2.14943| 2.14943| 285.227| 285.227 min |
| waternon2_09              | 0.01              | 19.4313| 19.4313| 933.293| 933.293 min |
| waternon2_12              | 0.00              | 17.9703| 17.9703| 2302.51| 2302.51 min |
| waternon2_18              | 0.00              | 0      | 0      | 5269.64| 5269.64 min |
| waternon2_24              | 0.00              | 5.68434e-14| 0    | 7349.04| 7349.04 min |
| waterund01                | 0.26              | 81.2351| 81.2351| 86.8333| 86.8333 min |
| waterund08                | 0.07              | 149.475| 149.475| 164.49 | 164.49 min |
| waterund11                | 0.09              | 90.5111| 90.5111| 104.886| 104.886 min |
| waterund14                | 0.02              | 312.678| 312.678| 329.57 | 329.57 min |
| waterund17                | 0.15              | 148.434| 148.434| 157.094| 157.094 min |
| waterund18                | 0.13              | 223.241| 223.241| 238.733| 238.733 min |
| waterund22                | 0.05              | 258.205| 258.205| 323.505| 323.505 min |
| waterund25                | 0.00              | 290.968| 290.968| 410.635| 410.635 min |
| waterund27                | 0.00              | 456.211| 456.118| 556.675| 556.675 min |
Table 4 continued

| Instance    | GC($d_1, d_2, p$) | $d_1$ | $d_2$ | $p$ | obj |
|-------------|-------------------|------|------|-----|-----|
| waterund28  | 0.04              | 1681.79 | 1676.23 | 1812.17 | min |
| waterund32  | 0.01              | 256.41  | 252.716 | 638.735 | min |
| waterund36  | 0.03              | 551.77  | 548.818 | 662.807 | min |
| windfac     | 0.00              | 0      | 0      | 0.254487 | min |

Table 5: Detailed results for the tree experiments. The table reports shifted geometric means for the number of branch-and-bound nodes and the solving time for five different permutations for each of the 564 relevant instances of the affected instances experiment, see Table 3. Aggregated results are presented in Table 2.

SCI{P+s+p} — SCIP using stronger separation and propagation for bilinear terms
SCI{P+s} — SCIP using stronger separation for bilinear terms
SCI{P} — default SCIP

$t$ — shifted geometric mean of solving times (shift value 1.0)
nodes — shifted geometric mean of branch-and-bound nodes (shift value 100.0)

| Instance     | SCI{P+s+p} | SCI{P+s} | SCI{P} |
|--------------|------------|----------|--------|
|              | $t$ nodes  | $t$ nodes | $t$ nodes |
| 10bar1C      | 1.2        | 246.8    | 1.2    | 246.8 |
| 10bar3       | 142.5      | 18.9K    | 85.3   | 170.6K |
| 10bar4       | 296.0      | 172.4K   | 274.6  | 170.6K |
| 4stufen      | 1800.0     | 215.9K   | 1800.0 | 210.8K |
| 90bar        | 640.3      | 23.6K    | 723.3  | 26.4K |
| alkyl        | 0.8        | 99.0     | 0.8    | 95.0 |
| alkylation   | 1800.0     | 2486.9K  | 1800.0 | 2499.5K |
| arki0005     | 1800.0     | 1.0      | 1800.0 | 1.0 |
| arki0015     | 1800.0     | 78.8     | 1800.0 | 143.0 |
| arki0016     | 1800.0     | 634.1    | 1800.0 | 664.5 |
| arki0017     | 1800.0     | 1.2K     | 1800.0 | 1.2K |
| arki0018     | 1800.0     | 1.0      | 1800.0 | 1.0 |
| arki0019     | 177.2      | 1.0      | 176.6  | 1.0 |
| arki0020     | 538.3      | 1.0      | 538.6  | 1.0 |
| arki0022     | 1596.7     | 1.0      | 1596.1 | 1.0 |
| arki0024     | 1800.0     | 9.1K     | 1800.0 | 9.2K |
| batch0812 nc | 1.6        | 180.7    | 1.6    | 180.7 |
| batch nc     | 1.3        | 468.8    | 1.2    | 473.5 |
| bayes2_10    | 1800.0     | 70.8K    | 1800.0 | 64.9K |
| bayes2_20    | 1800.0     | 73.6K    | 1800.0 | 75.2K |
| bayes2_30    | 1800.0     | 78.8K    | 1800.0 | 74.1K |
| bayes2_50    | 1800.0     | 80.5K    | 1800.0 | 80.4K |
| bchoco05     | 1800.0     | 192.4    | 1800.0 | 194.7 |
| bchoco06     | 1800.0     | 173.4    | 1800.0 | 327.1 |
| bchoco07     | 1800.0     | 692.0    | 1800.0 | 355.6 |
| bchoco08     | 1800.0     | 1.1K     | 1800.0 | 533.7 |
| beuster      | 1800.0     | 168.3K   | 1800.0 | 162.1K |
| blend029     | 2.9        | 813.3    | 2.9    | 868.5 |

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| Instance       | t   | nodes | t   | nodes | t   | nodes |
|---------------|-----|-------|-----|-------|-----|-------|
| blend146      | 1800.0 | 568.8K | 1719.7 | 536.0K | **1630.8** | 553.3K |
| blend480      | 108.5 | 14.9K  | 96.5  | 12.0K  | 124.3 | 20.0K  |
| blend531      | 56.4  | 10.3K  | **34.9**  | 5.6K  | 41.7  | 7.6K  |
| blend718      | **522.1** | 189.3K | 659.0 | 244.3K | 612.5 | 227.2K |
| blend721      | 42.7  | 12.4K  | **28.6**  | 7.5K  | 48.3  | 14.3K  |
| blend852      | **59.9** | 9.4K  | 74.3  | 11.9K  | 166.1 | 37.8K  |
| camcns        | 1800.0 | 1.2K  | 1800.0 | 1.1K  | 1800.0 | 1.3K  |
| camshape100   | 1800.0 | 142.4K | 1800.0 | 145.5K | 1800.0 | 146.3K |
| camshape200   | 1800.0 | 42.5K  | 1800.0 | 42.3K  | 1800.0 | 42.3K  |
| camshape400   | 1800.0 | 13.5K  | 1800.0 | 13.6K  | 1800.0 | 13.6K  |
| camshape800   | 1800.0 | 6.3K   | 1800.0 | 6.3K   | 1800.0 | 6.3K   |
| carton7       | 1439.6 | 647.8K | 1411.9 | 638.9K | **1276.8** | 583.5K |
| carton9       | 1800.0 | 484.5K | 1800.0 | 486.0K | 1800.0 | 489.9K |
| casctanks     | 230.1 | 18.7K  | **208.2**  | 17.4K  | 352.9 | 33.0K  |
| case_lscv2    | 1800.0 | 1.2K   | 1800.0 | 1.2K   | 1800.0 | 1.2K   |
| cesam2log     | 1800.0 | 88.2K  | 1800.0 | 89.0K  | 1800.0 | 202.1K |
| chain100      | 1800.0 | 699.7  | 1800.0 | 699.2  | 1800.0 | 773.6  |
| chain200      | 1800.0 | 94.3   | 1800.0 | 95.1   | 1800.0 | 167.2  |
| chain400      | 1800.0 | 11.0   | 1800.0 | 35.5   | 1800.0 | 37.0   |
| chain500      | 1800.0 | 23.7K  | 1800.0 | 23.8K  | 1800.0 | 26.6K  |
| chem          | **1593.3** | 860.1K | 1800.0 | 1203.8K | 1800.0 | 1160.2K |
| chenery        | 8.8   | 2.5K   | 8.0   | 2.2K   | **5.3**  | 1.5K   |
| chp_partload   | 1800.0 | 2.5K   | 1800.0 | 2.5K   | 1800.0 | 2.5K   |
| chp_shorttermplan2d | 1800.0 | 1.1K | 1800.0 | 1.1K | 1800.0 | 1.1K |
| clay0203h      | 4.0   | 170.3  | 4.0   | 170.3  | 3.9   | 170.3  |
| clay0204h      | 13.6  | 1.2K   | 13.5  | 1.2K   | 13.6  | 1.2K   |
| clay0205h      | 112.9 | 10.4K  | 113.0 | 10.4K  | 112.4 | 10.4K  |
| contvar        | 1800.0 | 8.2K   | 1800.0 | 6.8K   | 1800.0 | 9.3K   |
| crudeoil_lee1_05 | 1.9   | 3.4    | 1.8   | 9.2    | 1.9   | 6.5    |
| crudeoil_lee1_06 | 3.1   | 39.9   | 3.6   | 52.2   | 3.2   | 44.4   |
| crudeoil_lee1_07 | 4.8   | 55.3   | 4.2   | 56.3   | 4.4   | 39.9   |
| crudeoil_lee1_08 | 7.4   | 73.4   | 7.8   | 71.3   | **6.7**  | 61.7   |
| crudeoil_lee1_09 | **11.1** | 58.9 | 12.6 | 75.3   | 11.3  | 75.2   |
| crudeoil_lee1_10 | **13.1** | 100.2 | 14.6 | 84.7   | 13.7  | 96.8   |
| crudeoil_lee2_05 | 8.5   | 8.9    | 8.8   | 11.7   | 9.0   | 8.9    |
| crudeoil_lee2_06 | **11.9** | 33.3 | 12.5 | 85.8   | 13.0  | 71.1   |
| crudeoil_lee2_07 | **21.5** | 208.4 | 23.3 | 268.0  | 21.7  | 142.2  |
| crudeoil_lee2_08 | 32.7  | 319.9  | 32.7  | 411.8  | **29.0**  | 319.7  |
| crudeoil_lee2_09 | **41.3** | 393.6 | 56.6 | 542.8  | 53.5  | 588.7  |
| crudeoil_lee2_10 | 71.6  | 598.5  | 80.1  | 742.4  | **66.5**  | 565.1  |
| crudeoil_lee3_05 | **37.1** | 1.5K  | 45.2  | 2.0K   | 37.5  | 2.2K   |
| crudeoil_lee3_06 | **107.1** | 7.6K  | 136.0 | 11.4K  | 126.2 | 10.8K  |
| crudeoil_lee3_07 | 182.1 | 13.0K  | **175.2** | 12.7K  | 176.1 | 12.7K  |
| crudeoil_lee3_08 | 242.4 | 13.4K  | **234.1** | 13.3K  | 311.4 | 19.0K  |
| crudeoil_lee3_09 | **336.6** | 14.4K | 345.4 | 16.1K  | 420.0 | 21.4K  |
| Instance         | SCIP+s+p | SCIP+s | SCIP |
|------------------|----------|--------|------|
| crudeoil_lee3_10 | 435.9    | 17.1K  | 429.8 |
| crudeoil_lee4_05 | 10.1     | 36.9   | 10.8 |
| crudeoil_lee4_06 | 15.5     | 43.4   | 15.4 |
| crudeoil_lee4_07 | 27.8     | 45.7   | 31.4 |
| crudeoil_lee4_08 | 38.0     | 75.2   | 34.1 |
| crudeoil_lee4_09 | 60.3     | 53.1   | 55.0 |
| crudeoil_lee4_10 | 57.7     | 36.8   | 66.7 |
| crudeoil_li01   | 1800.0   | 304.1K | 1800.0 |
| crudeoil_li03   | 1800.0   | 93.2K  | 1800.0 |
| crudeoil_li05   | 1800.0   | 102.4K | 1800.0 |
| crudeoil_li06   | 511.6    | 28.1K  | 504.0 |
| crudeoil_li11   | 1800.0   | 82.5K  | 1800.0 |
| crudeoil_li21   | 1800.0   | 76.9K  | 1800.0 |
| crudeoil_pooling_ct1 | 1800.0 | 393.3K | 1800.0 |
| crudeoil_pooling_ct3 | 1800.0 | 138.2K | 1800.0 |
| crudeoil_pooling_dt1 | 1800.0 | 13.2K  | 1800.0 |
| crudeoil_pooling_dt2 | 1800.0 | 6.5K   | 1800.0 |
| crudeoil_pooling_dt3 | 1800.0 | 400.6K | 1800.0 |
| crudeoil_pooling_dt4 | 1800.0 | 7.9K   | 1800.0 |
| sched1           | 1800.0   | 801.9K | 1800.0 |
| sched2           | 1800.0   | 174.1K | 1800.0 |
| sched2a          | 1800.0   | 451.1K | 1800.0 |
| deb6             | 1800.0   | 12.9K  | 1800.0 |
| deb7             | 1800.0   | 7.0K   | 1800.0 |
| deb8             | 1800.0   | 6.2K   | 1800.0 |
| deb9             | 1800.0   | 13.2K  | 1800.0 |
| elf              | 0.9      | 243.7  | 0.9  |
| ethanoll         | 2.7      | 69.9   | 3.0  |
| ethanolm         | 1.4      | 2.0    | 1.4  |
| ex1226           | 0.0      | 1.0    | 0.0  |
| ex1252           | 102.7    | 46.3K  | 27.5 |
| ex1252a          | 22.4     | 9.3K   | 27.3 |
| ex1263a          | 0.3      | 107.6  | 0.3  |
| ex1264           | 0.4      | 88.3   | 0.4  |
| ex1265           | 0.7      | 186.9  | 0.7  |
| ex1265a          | 0.2      | 88.5   | 0.2  |
| ex14_1_2         | 0.1      | 13.8   | 0.1  |
| ex14_1_3         | 0.0      | 1.0    | 0.0  |
| ex14_1_8         | 0.1      | 4.9    | 0.1  |
| ex2_1_9          | 3.8      | 3.6K   | 3.9  |
| ex3_1_1          | 1.0      | 1.0K   | 1.0  |
| ex3_1_2          | 0.0      | 1.0    | 0.0  |
| ex3_1_4          | 0.1      | 33.0   | 0.1  |
| ex5_2_2_case1    | 0.3      | 39.0   | 0.3  |
| ex5_2_2_case2    | 75.8     | 170.8K | 76.3 |
Table 5 continued

| Instance      | SCIP+s+p t | SCIP+s t | SCIP t |
|---------------|------------|----------|--------|
|               | nodes      | nodes    | nodes  |
| ex5_2_case3   | 0.2 15.0   | 0.2 15.0 | 0.2 15.0 |
| ex5_2         | 0.8 311.0  | 0.8 311.0| 0.8 311.0 |
| ex5_2_5       | 1800.0 644.5K | 1800.0 647.5K | 1800.0 642.1K |
| ex5_2_3       | 1.4 1.1K   | 0.8 448.3 | 0.9 302.1 |
| ex5_2_3       | 1800.0 542.4K | 1800.0 545.6K | 1800.0 565.5K |
| ex5_4_2       | 0.6 326.8  | 0.7 322.6 | 0.7 297.2 |
| ex5_4_3       | 0.2 13.0   | 0.2 13.0 | 0.2 13.0 |
| ex5_4_4       | 7.5 6.3K   | 7.6 6.5K | 24.6 20.5K |
| ex6_1_1       | 98.2 40.2K | 95.7 39.8K | 92.9 39.5K |
| ex6_1_2       | 0.7 105.8  | 0.5 111.0 | 0.4 112.0 |
| ex6_1_3       | 1800.0 373.5K | 1800.0 375.2K | 1800.0 389.0K |
| ex6_2_10      | 1800.0 176.1K | 1800.0 175.9K | 1800.0 177.1K |
| ex6_2_11      | 1800.0 1102.8K | 1800.0 1044.2K | 1800.0 1043.9K |
| ex6_2_12      | 1800.0 701.4K | 1800.0 731.4K | 1800.0 717.7K |
| ex6_2_13      | 1800.0 268.0K | 1800.0 268.2K | 1800.0 266.8K |
| ex6_2_14      | 77.6 18.8K | 77.6 18.6K | 76.0 18.9K |
| ex6_2_5       | 1800.0 336.4K | 1800.0 336.4K | 1800.0 363.7K |
| ex6_2_6       | 19.5 11.8K | 19.0 11.8K | 18.5 11.5K |
| ex6_2_7       | 1800.0 328.8K | 1800.0 330.8K | 1800.0 329.4K |
| ex6_2_8       | 10.1 6.5K | 9.6 6.1K | 9.4 5.9K |
| ex6_2_9       | 1800.0 558.8K | 1800.0 560.8K | 1800.0 574.1K |
| ex7_2_1       | 1800.0 2624.6K | 1800.0 1910.5K | 1800.0 2002.2K |
| ex7_2_2       | 0.4 83.1 | 0.4 83.1 | 0.4 92.7 |
| ex7_2_4       | 8.7 3.2K | 8.8 3.3K | 9.5 3.5K |
| ex8_1_1       | 0.0 1.0 | 0.0 1.0 | 0.0 1.0 |
| ex8_2_3b      | 1800.0 11.5K | 1800.0 11.6K | 1800.0 11.7K |
| ex8_2_4b      | 0.6 3.0 | 0.6 3.0 | 0.6 3.0 |
| ex8_3_8       | 1800.0 104.5K | 1800.0 97.8K | 1800.0 108.5K |
| ex8_4_1       | 285.7 12.5K | 290.6 11.7K | 288.2 11.2K |
| ex8_4_2       | 1800.0 3.8K | 1800.0 3.8K | 1800.0 3.6K |
| ex8_4_4       | 2.5 187.5 | 2.5 187.5 | 2.6 205.5 |
| ex8_4_5       | 43.7 3.2K | 40.6 2.9K | 48.5 3.3K |
| ex8_4_8_bnd   | 1800.0 30.5K | 1800.0 22.5K | 1800.0 8.5K |
| ex8_5_1       | 1800.0 1.0 | 1800.0 1.0 | 1800.0 1.0 |
| ex9_2_2       | 0.0 1.0 | 1.0 0.2 | 28.1 |
| ex9_2_3       | 0.3 35.0 | 0.2 35.0 | 0.3 35.0 |
| ex9_2_4       | 0.0 1.0 | 0.2 23.0 | 0.1 23.0 |
| ex9_2_6       | 0.3 11.0 | 0.2 19.0 | 0.2 19.0 |
| feedtray      | 1800.0 16.4K | 1800.0 18.4K | 1800.0 19.2K |
| forest        | 1800.0 245.9K | 1800.0 237.9K | 1800.0 225.8K |
| gabriel01     | 616.8 95.2K | 1072.3 166.3K | 1393.7 235.2K |
| gabriel02     | 1687.1 162.0K | 1633.4 168.7K | 1636.9 168.6K |
| gabriel04     | 838.5 52.1K | 421.2 20.8K | 806.6 51.6K |
| gabriel05     | 1800.0 44.4K | 1800.0 44.0K | 1800.0 43.3K |
| gabriel06     | 1800.0 1.3K | 1800.0 1.3K | 1800.0 1.4K |
| Instance          | t  | nodes | t  | nodes | t  | nodes |
|-------------------|----|-------|----|-------|----|-------|
| gabriel07         | 1800.0 | 136.6 | 1800.0 | 126.3 | 1800.0 | 131.8 |
| gabriel09         | 1800.0 | 4.4K  | 1800.0 | 4.5K  | 1800.0 | 5.1K  |
| gams02            | 1800.0 | 23.1K | 1800.0 | 25.7K | 1800.0 | 40.6K |
| gams03            | 1800.0 | 1.0   | 1800.0 | 1.0   | 1800.0 | 1.0   |
| gancns            | 1800.0 | 200.8 | 1800.0 | 247.2 | 1800.0 | 396.1 |
| gasnet            | 1800.0 | 90.9K | 1800.0 | 85.8K | 1800.0 | 107.2K |
| gasnet_al1        | 1800.0 | 112.1K| 1800.0 | 108.8K| 1800.0 | 105.7K |
| gasnet_al2        | 1800.0 | 108.3K| 1800.0 | 101.1K| 1800.0 | 102.9K |
| gasnet_al3        | 1800.0 | 111.7K| 1800.0 | 107.3K| 1800.0 | 109.3K |
| gasnet_al4        | 1800.0 | 111.6K| 1800.0 | 110.1K| 1800.0 | 107.0K |
| gasnet_al5        | 1800.0 | 113.0K| 1800.0 | 116.8K| 1800.0 | 116.7K |
| gasprod_sarawak01 | 2.4 | 116.4 | 1.8 | 67.8 | 1.8 | 67.8 |
| gasprod_sarawak16 | 1800.0 | 58.7K | 1800.0 | 54.3K | 1800.0 | 54.9K |
| gasprod_sarawak81 | 1800.0 | 6.5K  | 1800.0 | 6.7K  | 1800.0 | 6.9K  |
| gastrans135      | **178.3** | 282.7 | 449.0 | 1.9K | 734.8 | 7.9K |
| gastrans582_cold13 | **22.0** | 41.4  | 58.3  | 298.4 | 40.3  | 219.6 |
| gastrans582_cold13_95 | 55.1 | 347.4 | **22.3** | 19.6 | 187.8 | 2.4K |
| gastrans582_cold17 | **23.9** | 45.6  | 38.5  | 176.6 | 463.8 | 9.6K |
| gastrans582_cold17_95 | 24.3 | 23.8  | 31.3  | 144.3 | 82.7  | 975.5 |
| gastrans582_cool12 | **24.0** | 49.4  | 26.0  | 47.5  | 52.0  | 394.3 |
| gastrans582_cool12_95 | 48.6 | 314.6 | **31.0** | 90.4  | 370.6 | 7.7K |
| gastrans582_cool14 | **22.0** | 19.8  | 44.4  | 300.3 | 184.9 | 2.7K |
| gastrans582_cool14_95 | 51.7 | 325.9 | 30.2  | 92.5  | **29.6** | 49.2 |
| gastrans582_freezing27 | **18.8** | 12.6  | 24.6  | 2.0   | 130.1 | 1.4K |
| gastrans582_freezing27_19 | **18.3** | 2.0   | 29.3  | 92.4  | 47.1  | 282.0 |
| gastrans582_freezing30 | **68.8** | 847.4 | 83.4  | 677.4 | 145.0 | 2.3K |
| gastrans582_freezing30_19 | 80.0 | 759.2 | **65.0** | 440.7 | 279.4 | 5.1K |
| gastrans582_mild10 | **39.3** | 251.4 | 62.6  | 459.3 | 55.5  | 501.7 |
| gastrans582_mild10_95 | 21.4 | 22.0  | 26.3  | 87.1  | 90.2  | 882.0 |
| gastrans582_mild11 | **26.5** | 75.5  | 79.9  | 765.1 | 39.1  | 270.9 |
| gastrans582_mild11_95 | 22.6 | 25.4  | 73.9  | 481.4 | 31.0  | 93.4 |
| gastrans582_warm15 | **22.4** | 51.4  | 49.1  | 347.7 | 27.8  | 126.7 |
| gastrans582_warm15_95 | 96.6 | 1.0K  | 31.3  | 159.7 | **22.0** | 59.2 |
| gastrans582_warm31 | 23.5 | 39.8  | 18.7  | 19.6  | **17.4** | 20.2 |
| gastrans582_warm31_95 | **14.8** | 17.0  | 16.2  | 15.3  | 31.0  | 164.4 |
| genpool04         | 1800.0 | 727.9K | 1800.0 | 725.9K | 1800.0 | 724.9K |
| genpool04i        | 1800.0 | 23.2K | 1800.0 | 3.7K  | 1800.0 | 3.7K  |
| genpool04paper    | 1800.0 | 708.4K | 1800.0 | 725.0K | 1800.0 | 718.8K |
| genpool15         | 1800.0 | 66.0K | 1800.0 | 65.9K | 1800.0 | 66.0K |
| genpool15i        | 1800.0 | 52.6K | 1800.0 | 54.0K | 1800.0 | 55.6K |
| genpool15paper    | 1800.0 | 66.0K | 1800.0 | 66.4K | 1800.0 | 66.4K |
| genpool20         | 1800.0 | 15.1K | 1800.0 | 15.2K | 1800.0 | 15.2K |
| genpool20i        | 1800.0 | 15.3K | 1800.0 | 16.4K | 1800.0 | 15.8K |
| genpool20paper    | 1800.0 | 25.2K | 1800.0 | 25.3K | 1800.0 | 25.3K |
| genpooling_lec1   | **4.1** | 1.3K  | 6.0   | 1.5K  | 6.2   | 1.4K  |
| Instance                  | SCIP+s+p | SCIP+s | SCIP |
|---------------------------|----------|--------|------|
| genpooling_loe2           | 16.1     | 3.3K   | 14.0 |
| genpooling_meyer10        | 1800.0   | 144.8K | 1800.0|
| ghg_1veh                  | 18.4     | 1.7K   | 1800.0| 14.9 |
| ghg_2veh                  | 1800.0   | 62.5K  | 1800.0| 14.8 |
| ghg_3veh                  | 1800.0   | 41.2K  | 1800.0| 12.8 |
| gsg_0001                  | 166.9    | 112.5K | 112.3 | 1800.0| 111.9|
| heatexch_gen1             | 1800.0   | 266.1K | 1800.0| 278.1K|
| heatexch_gen3             | 1800.0   | 80.2K  | 1800.0| 78.3K |
| ghm11                     | 1800.0   | 375.1K | 1800.0| 382.7K|
| ghm2                      | 1800.0   | 223.2K | 1800.0| 224.9K|
| ghm3                      | 1800.0   | 97.2K  | 1800.0| 96.7K |
| gsg                      | 1800.0   | 645.2K | 1800.0| 656.0K|
| hydroenergy1              | 1800.0   | 492.0K | 1800.0| 493.9K|
| hydroenergy2              | 1800.0   | 504.4K | 1800.0| 503.9K|
| hydroenergy3              | 1800.0   | 42.7K  | 1800.0| 105.1K|
| infeas1                   | 1800.0   | 718.3K | 1800.0| 714.3K|
| ivalue                    | 1800.0   | 26.2K  | 1800.0| 26.4K |
| kall_circles_c6a          | 1725.2   | 679.3K | 1800.0| 757.2K|
| kall_circles_c6b          | 776.0    | 301.9K | 721.4 | 444.4K|
| kall_circles_c6c          | 1800.0   | 453.9K | 1800.0| 493.0K|
| kall_circles_c7a          | 1800.0   | 492.0K | 1800.0| 491.7K|
| kall_circles_c8a          | 1800.0   | 504.4K | 1800.0| 503.9K|
| kall_circlespolygons_c1   | 1800.0   | 562.9K | 1800.0| 885.7K|
| kall_circlespolygons_c2   | 1800.0   | 652.9K | 1800.0| 1145.1K|
| kall_circlespolygons_c3   | 1800.0   | 227.8K | 1800.0| 227.1K|
| kall_circlespolygons_c4   | 1800.0   | 49.1K  | 1800.0| 49.6K |
| kall_circlespolygons_c5   | 1800.0   | 33.2K  | 1800.0| 35.6K |
| kall_circlesrectangles_c1 | 0.1      | 1.0    | 1.0  |
| kall_circlesrectangles_c2 | 1703.1   | 679.6K | 1471.6| 511.0K|
| kall_circlesrectangles_c3 | 579.6    | 405.5K | 498.6 | 524.7K|
| kall_circlesrectangles_c4 | 1800.0   | 201.4K | 1800.0| 202.5K|
| kall_circlesrectangles_c5 | 1800.0   | 98.6K  | 1800.0| 101.0K|
| kall_circlesrectangles_c6 | 1800.0   | 111.7K | 1800.0| 114.0K|
| kall_congrentcircles_c    | 0.7      | 226.1  | 0.7  | 268.0 |
| kall_congrentcircles_c2   | 0.5      | 163.4  | 0.5  | 163.4 |
| kall_congrentcircles_c3   | 0.3      | 11.0   | 0.2  | 19.0  |
| kall_congrentcircles_c4   | 1.1      | 272.2  | 1.1  | 304.6 |
| kall_congrentcircles_c5   | 16.3     | 8.9K   | 16.7 | 9.5K  |
| kall_congrentcircles_c6   | 6.7      | 2.5K   | 8.0  | 3.3K  |
| kall_congrentcircles_c7   | 261.8    | 111.7K | 314.1 | 138.1K|
| kall_congrentcircles_c8   | 48.8     | 17.9K  | 66.5 | 27.9K |
| kall_congrentcircles_c9   | 10.5     | 3.0K   | 6.2  | 1.9K  |
| kall_congrentcircles_c10  | 1800.0   | 718.3K | 1742.4| 714.6K|
| kall_congrentcircles_c11  | 472.6    | 144.9K | 511.8 | 172.2K|
| kall_congrentcircles_c12  | 1621.0   | 465.5K | 1800.0| 515.2K|
| kall_congrentcircles_c13  | 177.7    | 133.8K | 128.8| 96.1K | 128.8 | 96.1K |
| Instance             | t  | nodes | t  | nodes | t  | nodes |
|----------------------|-----|-------|-----|-------|-----|-------|
| kall_diffcircles_5b  | 55.2| 35.9K | 55.4| 35.9K | 55.0| 35.9K |
| kall_diffcircles_6   | 39.8| 18.5K | 44.7| 19.0K | 31.1| 12.9K |
| kall_diffcircles_7   | 1515.0| 682.8K | 1137.6| 442.5K | 1073.1| 432.7K |
| kall_diffcircles_8   | 200.7| 93.3K | 79.9K| 362.5| 155.6K |
| kall_diffcircles_9   | 401.7| 133.2K | 281.7| 100.8K | 256.6| 96.2K |
| kall_diffcircles     | 1800.0| 3.5K | 1800.0| 3.5K | 1800.0| 3.5K |
| kall_diffcircles_5b  | 323.8| 14.8K | 625.6| 25.3K | 444.7| 20.4K |
| launch               | 1.6 | 69.1 | 1.6 | 69.1 | 1.6 | 69.1 |
| mathopt1             | 0.2 | 47.0 | 0.2 | 46.0 | 0.2 | 46.0 |
| mathopt4             | 0.2 | 33.0 | 0.3 | 33.0 | 0.2 | 33.0 |
| mpss-basic-marvin-85-85 | 1800.0| 41.8K | 1800.0| 40.4K | 1800.0| 54.7K |
| mpss-basic-ob25-125-125 | 1800.0| 852.1 | 1800.0| 446.1 | 1800.0| 463.4 |
| mpss-basic-red-marvin-85-85 | 1800.0| 5.3K | 1800.0| 2.6K | 1800.0| 6.6K |
| mpss-basic-red-ob25-125-1 | 1800.0| 711.0 | 1800.0| 547.9 | 1800.0| 898.8 |
| mpss-extwarehouse-marvin | 1800.0| 1.0 | 1800.0| 1.0 | 1800.0| 1.0 |
| mpss-extwarehouse-red-mar | 1800.0| 28.4 | 1800.0| 174.8 | 1800.0| 605.5 |
| mpss-extwarehouse-red-ob2 | 1800.0| 1.0 | 1800.0| 1.0 | 1800.0| 1.0 |
| multiplants_mtg1a    | 1800.0| 136.4K | 1800.0| 136.6K | 1774.8| 146.2K |
| multiplants_mtg1b    | 1800.0| 45.7K | 1800.0| 107.6K | 1800.0| 88.0K |
| multiplants_mtg1c    | 1800.0| 111.5K | 1800.0| 21.0K | 1800.0| 124.2K |
| multiplants_mtg2     | 1800.0| 126.1K | 1800.0| 122.2K | 1800.0| 130.4K |
| multiplants_mtg5     | 1800.0| 79.8K | 1800.0| 81.4K | 1800.0| 86.3K |
| multiplants_stg1     | 1800.0| 16.3K | 1800.0| 16.3K | 1800.0| 16.3K |
| multiplants_stg1a    | 1800.0| 45.6K | 1800.0| 45.5K | 1800.0| 45.5K |
| multiplants_stg1b    | 1800.0| 11.1K | 1800.0| 6.1K | 1800.0| 6.1K |
| multiplants_stg1c    | 1800.0| 2.8K | 1800.0| 4.9K | 1800.0| 5.5K |
| multiplants_stg5     | 1800.0| 7.9K | 1800.0| 7.9K | 1800.0| 7.9K |
| multiplants_stg6     | 1800.0| 22.8K | 1800.0| 22.8K | 1800.0| 22.8K |
| ndce12               | 1800.0| 89.5K | 1800.0| 88.8K | 1800.0| 186.6K |
| ndce12persp          | 1800.0| 310.0K | 1800.0| 323.4K | 1800.0| 389.3K |
| ndce13               | 1800.0| 100.3K | 1800.0| 90.3K | 1800.0| 97.1K |
| ndce13persp          | 1800.0| 370.7K | 1800.0| 357.8K | 1800.0| 367.8K |
| ndce14               | 1800.0| 46.4K | 1800.0| 44.6K | 1800.0| 97.1K |
| ndce14persp          | 1800.0| 243.9K | 1800.0| 250.4K | 1800.0| 285.1K |
| ndce15               | 1800.0| 102.3K | 1800.0| 106.7K | 1800.0| 136.2K |
| ndce15persp          | 1800.0| 400.0K | 1800.0| 452.3K | 1800.0| 455.9K |
| ndce16               | 1800.0| 39.1K | 1800.0| 40.4K | 1800.0| 90.8K |
| ndce16persp          | 1800.0| 178.7K | 1800.0| 186.0K | 1800.0| 186.5K |
| nous1                | 1581.2| 515.4K | 1556.3| 504.8K | 1609.7| 534.7K |
| nous2                | 4.3 | 990.2 | 3.3 | 779.4 | 4.9 | 1.4K |
| nuclear10a           | 1800.0| 1.0 | 1800.0| 1.0 | 1800.0| 1.0 |
| nuclear14a           | 1800.0| 8.2K | 1800.0| 8.8K | 1800.0| 8.8K |
| nuclear14b           | 1800.0| 21.6K | 1800.0| 20.3K | 1800.0| 19.7K |
| Instance      | t | nodes | t | nodes | t | nodes |
|---------------|---|-------|---|-------|---|-------|
| nuclear25a    | 1800.0 | 7.9K | 1800.0 | 8.1K | 1800.0 | 8.1K |
| nuclear25b    | 1800.0 | 18.4K | 1800.0 | 19.6K | 1800.0 | 19.8K |
| nuclear49a    | 1800.0 | 605.2 | 1800.0 | 566.1 | 1800.0 | 568.3 |
| nuclear49b    | 1800.0 | 424.0 | 1800.0 | 313.8 | 1800.0 | 320.9 |
| nvs01         | 0.1 | 10.0 | 0.1 | 10.0 | 0.1 | 10.0 |
| nvs02         | 0.1 | 18.0 | 0.0 | 18.0 | 0.0 | 18.0 |
| nvs05         | 2.0 | 245.6 | 2.0 | 245.6 | 2.0 | 245.6 |
| nvs13         | 0.3 | 28.0 | 0.2 | 27.0 | 0.2 | 27.0 |
| nvs14         | 0.0 | 15.0 | 0.0 | 15.0 | 0.0 | 15.0 |
| nvs17         | 1.3 | 66.7 | 1.2 | 54.3 | 1.3 | 61.2 |
| nvs18         | 0.5 | 41.6 | 0.5 | 41.9 | 0.5 | 41.9 |
| nvs19         | 2.8 | 148.6 | 2.9 | 159.0 | 2.8 | 158.4 |
| nvs20         | 1.3 | 68.6 | 1.2 | 68.6 | 1.3 | 68.6 |
| nvs23         | 5.7 | 225.4 | 5.3 | 213.3 | 5.5 | 231.1 |
| nvs24         | 9.0 | 307.0 | 8.3 | 270.6 | 8.8 | 298.5 |
| oil           | 1800.0 | 16.4K | 1800.0 | 17.9K | 1800.0 | 16.5K |
| oil2          | 3.7 | 5.0 | 3.6 | 5.0 | 3.6 | 5.0 |
| ortez         | 0.1 | 2.0 | 0.1 | 2.0 | 0.1 | 2.0 |
| orth_d3m6     | 1800.0 | 65.3K | 1800.0 | 64.6K | 1800.0 | 65.7K |
| orth_d3m6_pl  | 1800.0 | 203.9K | 1800.0 | 229.0K | 1800.0 | 213.2K |
| orth_d4m6_pl  | 1800.0 | 142.5K | 1800.0 | 142.8K | 1800.0 | 145.2K |
| otop          | 9.7 | 18.4 | 9.7 | 18.4 | 9.7 | 18.4 |
| parallel      | 19.2 | 2.3K | 20.3 | 2.5K | 20.3 | 2.5K |
| pindyck       | 1800.0 | 233.6K | 1800.0 | 240.0K | 1800.0 | 265.6K |
| pointpack04   | 0.2 | 7.0 | 0.3 | 7.0 | 0.2 | 7.0 |
| pointpack06   | 8.3 | 4.4K | 8.0 | 4.6K | 10.0 | 6.1K |
| pointpack08   | 213.1 | 86.7K | 227.9 | 90.5K | 206.8 | 98.2K |
| pointpack10   | 1800.0 | 481.0K | 1800.0 | 496.6K | 1800.0 | 508.7K |
| pointpack12   | 1800.0 | 324.7K | 1800.0 | 343.0K | 1800.0 | 373.4K |
| pointpack14   | 1800.0 | 306.6K | 1800.0 | 325.1K | 1800.0 | 312.3K |
| pollut        | 0.0 | 1.0 | 0.0 | 1.0 | 0.0 | 1.0 |
| pooling_adhya1pq | 3.5 | 1.6K | 3.0 | 1.4K | 3.3 | 1.6K |
| pooling_adhya1stp | 10.7 | 3.0K | 10.1 | 2.5K | 10.5 | 2.7K |
| pooling_adhya1tp | 2.0 | 847.0 | 2.1 | 847.0 | 2.0 | 847.0 |
| pooling_adhya2pq | 2.5 | 1.1K | 2.3 | 987.4 | 2.3 | 1.0K |
| pooling_adhya2tp | 2.3 | 529.9 | 2.4 | 543.9 | 2.4 | 543.9 |
| pooling_adhya3pq | 4.3 | 625.0 | 4.3 | 638.1 | 4.5 | 649.2 |
| pooling_adhya3tp | 5.6 | 587.1 | 5.5 | 587.1 | 5.5 | 587.1 |
| pooling_adhya4pq | 1.6 | 200.1 | 1.5 | 200.1 | 1.5 | 200.1 |
| pooling_adhya4tp | 2.3 | 292.1 | 2.4 | 297.6 | 2.6 | 340.4 |
| pooling_bental4pq | 0.2 | 3.0 | 0.2 | 3.0 | 0.2 | 3.0 |
| pooling_bental4tp | 0.2 | 3.0 | 0.3 | 3.0 | 0.2 | 3.0 |
| pooling_divg16 | 16.6 | 1.8K | 18.6 | 1.9K | 18.7 | 1.9K |
| pooling_divg18 | 1454.8 | 74.5K | 1684.3 | 86.8K | 1783.2 | 82.9K |
| pooling_divg19 | 1800.0 | 167.8K | 1800.0 | 167.2K | 1800.0 | 171.1K |
| Instance            | SCIP+s+p t | SCIP+s t | SCIP t |
|---------------------|------------|----------|--------|
| pooling_cpa1        | 33.6       | 2.6K     | 31.2   |
| pooling_cpa2        | 1754.9     | 48.7K    | 1800.0 |
| pooling_cpa3        | 1800.0     | 14.3K    | 128.8K |
| pooling_f oud s5stp | 1064.9     | 25.5K    | 738.7K |
| pooling_ha ver ly1pq| 0.1        | 1.0      | 0.1    |
| pooling_ha ver ly1stp| 0.0       | 1.0      | 0.1    |
| pooling_ha ver ly2pq| 0.1        | 7.0      | 0.1    |
| pooling_ha ver ly2stp| 0.2       | 6.2      | 0.2    |
| pooling_ha ver ly3pq| 0.0        | 1.0      | 0.1    |
| pooling_ha ver ly3stp| 0.4       | 1.0      | 0.1    |
| pooling_ha ver ly3tp| 0.1        | 5.0      | 0.1    |
| pooling_r t2pq       | 1.7        | 677.6    | 692.4K |
| pooling_r t2stp      | 2.4        | 154.5    | 121.4K |
| pooling_r t2tp       | 0.6        | 61.6     | 62.6K  |
| pooling_spp a0pq     | 1800.0     | 125.4K   | 129.1K |
| pooling_spp a0stp    | 1800.0     | 86.7K    | 87.3K  |
| pooling_spp a0tp     | 1800.0     | 122.4K   | 123.3K |
| pooling_spp a5pq     | 1800.0     | 30.7K    | 31.1K  |
| pooling_spp a5stp    | 1800.0     | 23.3K    | 23.5K  |
| pooling_spp a5tp     | 1800.0     | 28.8K    | 26.8K  |
| pooling_spp a9pq     | 1800.0     | 22.8K    | 22.8K  |
| pooling_spp a9stp    | 1800.0     | 22.8K    | 22.8K  |
| pooling_spp a9tp     | 1800.0     | 22.8K    | 22.8K  |
| pooling_spp b0pq     | 1800.0     | 13.6K    | 13.7K  |
| pooling_spp b0stp    | 1800.0     | 14.8K    | 15.1K  |
| pooling_spp b0tp     | 1800.0     | 6.9K     | 6.9K   |
| pooling_spp b2pq     | 1800.0     | 3.2K     | 3.8K   |
| pooling_spp b2stp    | 1800.0     | 4.2K     | 4.3K   |
| pooling_spp b2tp     | 1800.0     | 2.8K     | 3.0K   |
| pooling_spp b5pq     | 1800.0     | 107.8K   | 187.3K |
| pooling_spp b5stp    | 1800.0     | 883.5    | 890.0K |
| pooling_spp b5tp     | 1800.0     | 203.6    | 249.7K |
| pooling_spp c0pq     | 1800.0     | 524.3    | 525.6K |
| pooling_spp c0stp    | 1800.0     | 2.6K     | 2.6K   |
| pooling_spp c0tp     | 1800.0     | 609.2    | 610.0K |
| pooling_spp c1pq     | 1800.0     | 313.7    | 313.7K |
| pooling_spp c1stp    | 1800.0     | 1.1K     | 1.1K   |
| pooling_spp c1tp     | 1800.0     | 872.3    | 753.7K |
| pooling_spp c3pq     | 1800.0     | 96.5     | 96.1K  |
| pooling_spp c3stp    | 1800.0     | 1.2      | 1.4    |
| pooling_spp c3tp     | 1800.0     | 77.2     | 70.9   |
| powerflow0009r       | 1800.0     | 109.4K   | 109.4K |
| powerflow0014r       | 1800.0     | 70.6K    | 81.2K  |
| Instance                   | t  | nodes | t  | nodes | t  | nodes |
|----------------------------|----|-------|----|-------|----|-------|
| powerflow0030r             | 1800.0 | 16.8K | 1800.0 | 21.0K | 1800.0 | 19.8K |
| powerflow0039r             | 1800.0 | 2.1K  | 1800.0 | 2.1K  | 1800.0 | 2.1K  |
| powerflow0057r             | 1800.0 | 9.0K  | 1800.0 | 8.7K  | 1800.0 | 423.7 | 1800.0 | 489.5 |
| powerflow0300r             | 1800.0 | 63.7  | 1800.0 | 65.1  | 1800.0 | 66.5  |
| primary                    | 1800.0 | 47.4K | 1800.0 | 45.6K | 1800.0 | 47.2K |
| prob09                     | 1800.0 | 9022.8K | 1800.0 | 8684.7K | 1800.0 | 8728.4K |
| procurement1large          | 1800.0 | 33.0K | 1800.0 | 33.7K | 1800.0 | 46.2K |
| procurement1mot            | 1800.0 | 448.2K | 1800.0 | 451.8K | 1800.0 | 730.3K |
| qp3                        | 1800.0 | 303.1K | 1800.0 | 282.5K | 1800.0 | 326.2K |
| r Brock                    | 1800.0 | 9191.8K | 1800.0 | 8885.9K | 1800.0 | 8949.0K |
| ringpack_10_1              | 1800.0 | 63.2K | 1800.0 | 63.3K | 1800.0 | 64.3K |
| ringpack_10_2              | 1800.0 | 57.2K | 1800.0 | 54.9K | 1800.0 | 54.6K |
| ringpack_20_1              | 1800.0 | 5.4K  | 1800.0 | 5.1K  | 1800.0 | 5.1K  |
| ringpack_20_2              | 1800.0 | 4.0K  | 1800.0 | 4.4K  | 1800.0 | 3.9K  |
| ringpack_30_1              | 1800.0 | 719.7 | 1800.0 | 700.0 | 1800.0 | 691.7 |
| ringpack_30_2              | 1800.0 | 441.1 | 1800.0 | 443.7 | 1800.0 | 509.0 |
| rsyn0805h                  | 0.8  | 65.2  | 0.7  | 21.2  | 0.6  | 21.2  |
| rsyn0805m02h               | 2.6  | 502.3 | 2.8  | 616.8 | 2.7  | 616.8 |
| rsyn0805m03h               | 10.1 | 1.5K  | 11.0 | 1.8K  | 11.2 | 1.8K  |
| rsyn0805m04h               | 8.1  | 681.2 | 4.0  | 208.4 | 4.1  | 208.4 |
| rsyn0810h                  | 1.7  | 438.8 | 1.7  | 436.7 | 1.7  | 436.7 |
| rsyn0810m03h               | 1800.0 | 216.2K | 1800.0 | 217.6K | 1800.0 | 217.1K |
| rsyn0810m04h               | 1740.6 | 174.6K | 1771.7 | 177.1K | 1774.8 | 177.0K |
| rsyn0815m02h               | 331.8 | 65.4K | 408.3 | 80.4K | 407.7 | 80.4K |
| rsyn0815m03h               | 1800.0 | 142.0K | 1800.0 | 147.7K | 1800.0 | 148.0K |
| rsyn0815m04h               | 1800.0 | 105.2K | 1800.0 | 106.3K | 1800.0 | 106.1K |
| rsyn0820m02h               | 1800.0 | 258.5K | 1800.0 | 53.7K | 1800.0 | 53.8K |
| rsyn0820m03h               | 1800.0 | 96.8K | 1800.0 | 89.1K | 1800.0 | 89.0K |
| rsyn0820m04h               | 1800.0 | 75.6K | 1800.0 | 75.9K | 1800.0 | 76.0K |
| rsyn0830h                  | 1180.0 | 329.6K | 1006.9 | 294.9K | 1005.1 | 294.9K |
| rsyn0830m04h               | 1800.0 | 64.4K | 1800.0 | 62.8K | 1800.0 | 62.5K |
| rsyn0840h                  | 1718.1 | 415.5K | 1755.6 | 432.1K | 1754.9 | 431.8K |
| rsyn0840m02h               | 1800.0 | 110.5K | 1800.0 | 109.4K | 1800.0 | 109.2K |
| rsyn0840m03h               | 1800.0 | 60.5K | 1800.0 | 58.4K | 1800.0 | 58.1K |
| rsyn0840m04h               | 1800.0 | 41.7K | 1800.0 | 42.3K | 1800.0 | 42.2K |
| saa_2                      | 1800.0 | 1.0   | 1800.0 | 1.0   | 1800.0 | 1.0   |
| sep1                       | 0.4  | 28.1  | 0.5  | 28.1  | 0.5  | 28.1  |
| sepa_sequ_complex          | 1800.0 | 24.6K | 1800.0 | 24.3K | 1800.0 | 24.3K |
| sepa_sequ_convert          | 13.7 | 10.9  | 13.4 | 13.3  | 13.3 | 8.2   |
| sfacloc1_2_95              | 1800.0 | 429.6K | 1800.0 | 432.2K | 1800.0 | 431.9K |
| sfacloc1_3_95              | 1800.0 | 319.7K | 1800.0 | 320.5K | 1800.0 | 319.9K |
| smallinvSNPr1b010-011      | 35.5 | 75.5  | 35.1 | 75.5  | 43.1 | 151.2 |
| smallinvSNPr1b020-022      | 143.5 | 919.2 | 148.4 | 899.2 | 111.3 | 896.0 |
| smallinvSNPr1b050-055      | 1780.8 | 13.4K | 1716.6 | 12.0K | 1800.0 | 19.1K |
| Instance                        | SCIP+s+p | SCIP+s | SCIP |
|--------------------------------|----------|--------|------|
|                                | $t$      | nodes  | $t$  | nodes | $t$  | nodes |
| smallinvSNPr1b100-110          | 1800.0   | 5.7K   | 1800.0 | 7.7K  | 1800.0 | 4.8K  |
| smallinvSNPr1b150-165          | 1800.0   | 3.9K   | 1800.0 | 4.2K  | 1800.0 | 5.7K  |
| smallinvSNPr1b200-220          | 1800.0   | 5.0K   | 1800.0 | 4.7K  | 1800.0 | 6.2K  |
| smallinvSNPr2b010-011          | 42.9     | 64.4   | 47.5  | 184.7 | 39.8  | 63.5  |
| smallinvSNPr2b020-022          | 78.7     | 376.6  | 78.8  | 376.6 | 58.9  | 172.1 |
| smallinvSNPr2b050-055          | 859.4    | 7.8K   | 1080.9 | 11.1K | 623.0 | 4.0K  |
| smallinvSNPr2b100-110          | 1800.0   | 8.4K   | 1800.0 | 5.9K  | 1800.0 | 3.9K  |
| smallinvSNPr2b150-165          | 1800.0   | 6.0K   | 1800.0 | 3.6K  | 1800.0 | 4.0K  |
| smallinvSNPr2b200-220          | 1800.0   | 4.5K   | 1800.0 | 4.8K  | 1800.0 | 3.9K  |
| smallinvSNPr3b010-011          | 49.8     | 145.3  | 45.4  | 96.4  | 40.3  | 101.3 |
| smallinvSNPr3b020-022          | 106.2    | 555.2  | 105.2 | 555.2 | 81.9  | 172.1 |
| smallinvSNPr3b050-055          | 1015.1   | 10.3K  | 1352.7 | 14.7K | 1356.0 | 14.7K |
| smallinvSNPr3b100-110          | 1800.0   | 8.7K   | 1613.9 | 9.4K  | 1614.0 | 9.4K  |
| smallinvSNPr3b150-165          | 1800.0   | 6.4K   | 1800.0 | 7.9K  | 1800.0 | 5.9K  |
| smallinvSNPr3b200-220          | 1800.0   | 6.9K   | 1800.0 | 6.3K  | 1800.0 | 6.3K  |
| smallinvSNPr4b010-011          | 65.7     | 401.3  | 61.3  | 265.9 | 56.7  | 265.9 |
| smallinvSNPr4b050-055          | 562.7    | 4.3K   | 501.1  | 2.9K  | 500.5  | 2.9K  |
| smallinvSNPr4b100-110          | 1800.0   | 9.5K   | 1742.9 | 8.4K  | 1800.0 | 10.3K |
| smallinvSNPr4b150-165          | 1800.0   | 6.4K   | 1800.0 | 7.9K  | 1800.0 | 5.9K  |
| smallinvSNPr4b200-220          | 1800.0   | 6.9K   | 1800.0 | 6.3K  | 1800.0 | 6.3K  |
| smallinvSNPr5b010-011          | 51.7     | 275.7  | 48.4  | 219.6 | 34.7  | 142.0 |
| smallinvSNPr5b020-022          | 55.3     | 201.1  | 58.7  | 260.1 | 50.9  | 216.2 |
| smallinvSNPr5b050-055          | 356.9    | 1.7K   | 350.5  | 1.6K  | 350.7  | 1.6K  |
| smallinvSNPr5b100-110          | 1300.6   | 7.9K   | 1613.9 | 9.4K  | 1614.0 | 9.4K  |
| smallinvSNPr5b150-165          | 1665.4   | 9.0K   | 1742.9 | 8.4K  | 1800.0 | 10.3K |
| smallinvSNPr5b200-220          | 1800.0   | 7.6K   | 1800.0 | 10.0K | 1714.8 | 8.4K  |
| space960                       | 1800.0   | 90.7   | 1800.0 | 93.2  | 1800.0 | 93.1  |
| spar30-60-2                    | 3.5      | 175.0  | 3.5   | 175.0 | 3.6   | 175.0 |
| spectra2                       | 11.1     | 16.2   | 11.0  | 16.2  | 11.0  | 16.2  |
| spring                         | 0.3      | 54.7   | 0.3   | 64.4  | 0.3   | 64.4  |
| sssd08-04persp                 | 19.0     | 13.9K  | 19.4  | 14.7K | 62.4  | 58.4K |
| sssd12-06persp                 | 1800.0   | 1057.8K| 1800.0 | 1137.2K| 1800.0 | 1309.0K|
| sssd15-04persp                 | 1800.0   | 742.2K | 1800.0 | 746.2K| 1800.0 | 989.1K|
| sssd15-06persp                 | 1800.0   | 686.5K | 1800.0 | 742.1K| 1800.0 | 933.3K|
| sssd15-08persp                 | 1800.0   | 682.0K | 1800.0 | 720.9K| 1800.0 | 887.1K|
| sssd16-07persp                 | 1800.0   | 726.6K | 1800.0 | 720.4K| 1800.0 | 995.7K|
| sssd18-06persp                 | 1800.0   | 571.1K | 1800.0 | 588.5K| 1800.0 | 935.6K|
| sssd18-08persp                 | 1800.0   | 615.5K | 1800.0 | 622.2K| 1800.0 | 872.7K|
| sssd20-04persp                 | 1800.0   | 488.0K | 1800.0 | 489.3K| 1800.0 | 733.9K|
| sssd20-08persp                 | 1800.0   | 541.0K | 1800.0 | 538.9K| 1800.0 | 842.0K|
| sssd22-08persp                 | 1800.0   | 517.7K | 1800.0 | 529.2K| 1800.0 | 838.3K|
| sssd25-04persp                 | 1800.0   | 450.7K | 1800.0 | 455.9K| 1800.0 | 765.2K|
| sssd25-08persp                 | 1800.0   | 431.6K | 1800.0 | 431.0K| 1800.0 | 758.4K|
| st_bpaf1a                      | 0.0      | 1.0    | 0.0   | 1.0   | 0.0   | 1.0   |
Table 5 continued

| Instance     | SCIP+s+p t nodes | SCIP+s t nodes | SCIP t nodes |
|--------------|------------------|----------------|-------------|
| ste03        | 1.2 544.7        | 1.2 562.9       | 1.2 486.7   |
| ste04        | 1.5 11.0         | 0.8 11.0        | 0.5 13.0    |
| ste05        | 0.4 164.0        | 0.4 164.0       | 0.5 164.0   |
| ste07        | 0.1 3.0          | 0.1 3.0         | 0.1 5.0     |
| ste08        | 0.0 1.0          | 0.0 1.0         | 0.0 1.0     |
| ste09        | 0.0 1.0          | 0.1 3.0         | 0.1 3.0     |
| ste11        | 0.4 1.0          | 0.4 1.0         | 0.4 1.0     |
| ste16        | 0.2 9.0          | 0.2 9.0         | 0.2 11.0    |
| ste23        | 0.0 1.0          | 0.0 1.0         | 0.1 33.4    |
| ste25        | 0.2 7.0          | 0.2 7.0         | 0.2 9.0     |
| ste28        | 0.0 1.0          | 0.0 1.0         | 0.1 3.0     |
| ste30        | 0.3 72.9         | 0.3 72.9        | 0.3 72.9    |
| ste31        | 1.5 491.3        | 1.6 555.9       | 1.6 556.6   |
| ste32        | 9.7 921.6        | 9.0 817.9       | 8.6 790.2   |
| ste36        | 0.3 209.8        | 0.3 209.8       | 0.3 209.8   |
| ste38        | 0.1 5.0          | 0.1 9.0         | 0.1 9.0     |
| ste40        | 0.1 15.0         | 0.1 14.0        | 0.1 16.0    |
| st_glmp_fp3  | 0.1 1.0          | 0.1 8.0         | 0.1 10.0    |
| st_glmp_kk92 | 0.1 1.0          | 0.1 7.4         | 0.1 11.0    |
| st_glmp_kky  | 0.1 7.0          | 0.1 9.0         | 0.1 7.0     |
| st_glmp_ss1  | 0.2 32.0         | 0.2 28.0        | 0.2 29.0    |
| st_glmp_ss2  | 0.0 1.0          | 0.0 1.0         | 0.0 1.0     |
| st_jqbk1     | 0.8 25.0         | 0.8 25.0        | 0.7 31.0    |
| st_jqbk2     | 0.6 23.0         | 0.6 27.0        | 0.7 31.0    |
| st_jcbpf2    | 0.1 5.0          | 0.1 5.0         | 0.1 5.0     |
| st_qpc-m1    | 0.1 5.0          | 0.1 5.0         | 0.1 7.0     |
| st_qpc-m3a   | 0.4 41.0         | 0.4 91.0        | 0.4 81.0    |
| st_qpc-m3b   | 0.4 3.0          | 0.5 3.0         | 0.5 5.0     |
| st_qpk1      | 0.1 5.0          | 0.1 5.0         | 0.1 7.0     |
| st_robot     | 0.0 1.0          | 0.0 1.0         | 0.0 1.0     |
| super3t      | 1800.0 7.3K      | 1800.0 7.0K     | 1800.0 7.0K |
| tanksize     | 4.0 1.4K         | 4.0 1.4K        | 4.6 1.7K    |
| tln4         | 2.4 1.8K         | 1.9 1.4K        | 1.6 1.1K    |
| tln5         | 58.3 36.2K       | 50.4 30.4K      | 43.4 26.5K  |
| tls2         | 0.3 14.4         | 0.3 14.4        | 0.3 14.4    |
| tltr         | 0.3 3.6          | 0.3 3.6         | 0.3 3.2     |
| transswitch009r | 1800.0 127.4K   | 1800.0 121.0K   | 1800.0 127.5K |
| transswitch0030r | 1800.0 40.8K    | 1800.0 36.1K    | 1800.0 33.3K |
| transswitch0039r | 1800.0 71.1K    | 1800.0 64.9K    | 1800.0 66.4K |
| transswitch0057r | 1800.0 19.8K    | 1800.0 20.4K    | 1800.0 20.3K |
| transswitch0118r | 1800.0 16.8K    | 1800.0 14.7K    | 1800.0 14.8K |
| transswitch0300r | 1800.0 1.7K     | 1800.0 2.5K     | 1800.0 2.5K  |
| transswitch2383wpr | 1800.0 1.0      | 1800.0 1.0      | 1800.0 1.0  |
| util         | 0.4 50.9         | 0.4 51.8        | 0.4 51.8    |
| var_con10    | 1800.0 36.2K     | 1800.0 37.6K    | 1800.0 33.1K |
| Instance                  | \( t \)  | \( \text{nodes} \) | \( t \)  | \( \text{nodes} \) | \( t \)  | \( \text{nodes} \) |
|--------------------------|---------|------------------|---------|------------------|---------|------------------|
| var_con5                 | 1800.0  | 32.6K            | 1800.0  | 40.4K            | 1800.0  | 34.3K            |
| wager                    | 2.3     | 35.2             | 2.3     | 35.2             | 2.3     | 35.2             |
| waste                    | 1800.0  | 120.0K           | 1800.0  | 119.2K           | 1800.0  | 118.4K           |
| wastewater02m1           | 0.4     | 43.7             | 0.3     | 36.8             | 0.4     | 43.7             |
| wastewater02m2           | 0.7     | 16.2             | 0.4     | 17.7             | 0.4     | 17.7             |
| wastewater04m1           | 0.7     | 62.8             | 1.1     | 81.8             | 0.9     | 77.7             |
| wastewater04m2           | 0.7     | 23.3             | 0.8     | 30.1             | 0.7     | 30.1             |
| wastewater11m1           | 1800.0  | 1142.6K          | 1800.0  | 1183.0K          | 1800.0  | 1178.7K          |
| wastewater12m1           | 1800.0  | 756.8K           | 1800.0  | 779.5K           | 1800.0  | 778.5K           |
| wastewater13m1           | 1800.0  | 478.3K           | 1800.0  | 501.3K           | 1800.0  | 492.2K           |
| wastewater14m1           | 1800.0  | 969.5K           | 1800.0  | 987.2K           | 1800.0  | 983.9K           |
| wastewater14m2           | 1800.0  | 365.3K           | 1800.0  | 369.1K           | 1800.0  | 423.5K           |
| wastewater15m1           | **357.6** | **203.4K**          | 379.2   | **218.2K**          | 397.7   | **228.5K**          |
| watercontamination0202   | 1800.0  | 233.5K           | 1800.0  | 224.6K           | 1800.0  | 223.9K           |
| watercontamination0202r  | 1800.0  | 20.5K            | 1800.0  | 20.6K            | 1800.0  | 20.7K            |
| watercontamination0303   | 1800.0  | 93.0K            | 1800.0  | 93.5K            | 1800.0  | 93.4K            |
| watercontamination0303r  | 1800.0  | 1.0              | 1800.0  | 1.0              | 1800.0  | 1.0              |
| waternd1                 | 15.1    | 3.0K             | 14.0    | 3.6K             | **12.1** | 2.4K             |
| waternd2                 | 1800.0  | 362.5K           | 1800.0  | 375.3K           | 1800.0  | 378.8K           |
| waterno2_01              | 0.4     | 6.7              | 0.4     | 6.7              | 0.5     | 8.2              |
| waterno2_02              | 3.5     | 154.7            | 3.3     | 150.3            | 3.6     | 124.9            |
| waterno2_03              | **529.8** | **53.2K**          | 591.7   | **65.3K**          | 736.9   | **80.0K**          |
| waterno2_04              | 1800.0  | 113.4K           | 1800.0  | 115.6K           | 1800.0  | 115.8K           |
| waterno2_06              | 1800.0  | 64.6K            | 1800.0  | 66.0K            | 1800.0  | 67.7K            |
| waterno2_09              | 1800.0  | 30.1K            | 1800.0  | 29.8K            | 1800.0  | 34.1K            |
| waterno2_12              | 1800.0  | 19.8K            | 1800.0  | 20.6K            | 1800.0  | 24.0K            |
| waterno2_18              | 1800.0  | 10.1K            | 1800.0  | 10.2K            | 1800.0  | 11.9K            |
| waterno2_24              | 1800.0  | 6.0K             | 1800.0  | 5.6K             | 1800.0  | 7.6K             |
| waterund01               | 1800.0  | 973.8K           | 1800.0  | 998.9K           | 1800.0  | 1010.8K          |
| waterund08               | **14.9** | 2.4K             | 18.4    | 3.1K             | 16.9    | 2.8K             |
| waterund11               | 306.3   | 97.8K            | **211.5** | 74.0K           | 316.8   | 95.0K            |
| waterund14               | 1800.0  | 232.8K           | 1800.0  | 228.0K           | 1800.0  | 266.5K           |
| waterund17               | 1800.0  | 480.8K           | 1800.0  | 486.2K           | 1800.0  | 575.3K           |
| waterund18               | 1800.0  | 564.7K           | 1800.0  | 576.8K           | 1800.0  | 626.4K           |
| waterund22               | 1800.0  | 246.7K           | 1800.0  | 229.2K           | 1800.0  | 228.1K           |
| waterund25               | 1800.0  | 304.8K           | 1800.0  | 308.9K           | 1800.0  | 315.7K           |
| waterund27               | 1800.0  | 69.4K            | 1800.0  | 68.3K            | 1800.0  | 69.8K            |
| waterund28               | 1800.0  | 6.9K             | 1800.0  | 7.9K             | 1800.0  | 8.8K             |
| waterund32               | 1800.0  | 9.3K             | 1800.0  | 9.6K             | 1800.0  | 9.3K             |
| waterund36               | 1800.0  | 57.7K            | 1800.0  | 58.9K            | 1800.0  | 60.2K            |
| windfac                  | 0.1     | 15.4             | 0.1     | 20.0             | 0.1     | 20.6             |