Manipulating Andreev and Majorana Bound States with microwaves

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We study the interplay between Andreev (Majorana) bound states that form at the boundary of a (topological) superconductor and a train of microwave pulses. We find that the extra dynamical phase coming from the pulses can shift the phase of the Andreev reflection, resulting in the appearance of dynamical Andreev states. As an application we study the presence of the zero bias peak in the differential conductance of a normal-topological superconductor junction - the simplest, yet somehow ambiguous, experimental signature for Majorana states. Adding microwave radiation to the measuring electrodes provides an unambiguous probe of the Andreev nature of the zero bias peak.

Andreev bound states and their topological counterparts, Majorana states, have seen considerable renewed interest springing from the possibility of using the latter for topologically protected quantum computation [1]. These Majorana states were first proposed to appear at the boundaries of so-called topological superconductors (TSs), rather exotic materials [2–4]. Although the search for a “natural” TS is very active, another route consists of engineering a TS by putting a regular (s-wave) superconductor into contact with another material that has strong spin-orbit interaction. A quantum spin Hall (QSH) topological insulator in contact with a superconductor is a popular possibility [5]. Another promising proposal is to use a semiconductor nanowire in contact with a superconductor [6, 7]. Indeed, a possible signature of the existence of Majorana states in a semi-conducting nanowire coupled to a superconducting electrode was recently reported [8–11].

In this letter we report on a dynamical generalization of Andreev/Majorana states in the presence of train of microwave pulses, i.e. the possibility to detect and manipulate these states by bringing the system to an out of equilibrium situation. Our proposal shares some features with other recent proposals that went under the name of “Floquet Majorana” states [12–15]: zero energy bound states that can be stabilized by a finite frequency perturbation. Our mechanism is, however, rather different and uses a simpler perturbation (oscillating voltage) applied to an electrode (as opposed to the whole system) together with much smaller frequencies (GHz to THz).

The dynamical Andreev states could be used to discriminate true Majoranas from other phenomena. Indeed, experimental evidence reported so far for Majorana states is based entirely on the presence (or lack) of a zero bias peak in the differential conductance. The peak could, however, originate from different sources [16–18]; the interpretation of the experiments so far is still debated. Here we propose to measure the (DC) differential conductance in the presence of a periodic train of voltage pulses. Our findings indicate that such a probe provides detailed insights into the structure of the Andreev or Majorana bound states and would lead to a clear signature of the presence of the latter.

Scattering approach to Andreev and Majorana states. While most of the considerations that follow are general, we focus on the particular setup shown in Fig. 1a): a
narrow connected to a superconducting electrode (S, yellow) on one side and to a normal electrode on the other side (N, blue). A short insulating barrier (I, red) (created from either a gate or the Schottky barrier at the semiconductor-metal (contact) interface) serves to confine any bound states at the superconducting-normal interface. Without the barrier, these states would hybridize strongly with the continuum and would be hardly visible. A weak hybridization transforms the bound states into resonances and allows one to probe them by measuring the DC differential conductance \( dI/dV \) through the wire. The superconducting electrode is grounded, while a bias voltage \( V_b + V(t) \) is applied to the normal electrode. \( V(t) \) is periodic with period \( T \).

Scattering theory provides a very intuitive way for understanding Andreev states in terms of an effective Fabry-Perot cavity. We start by following the approach developed in Ref. 5: an electron injected from the normal contact being reflected into the contact and the creation of a Cooper pair inside the superconductor – and the total phase accumulated over one period \( T \).

The energy \( E \) is measured from the Fermi energy, \( E_F \). When \( E \) is smaller than the superconducting gap \( \Delta \) – the only situation considered hereafter – \( r_A \) is a pure phase, due to the Andreev reflection, given by \( r_A^2 = (-1)^2 2 e^{-i 2 \arccos(E/\Delta)} \) where \( Z_2 = 0 \) for a conventional superconductor and \( Z_2 = 1 \) for a topological one [19]. The factor \( z \) accounts for the phase accumulated in the normal region, \( z = e^{i E_F T / \hbar} \), where \( T_F \) is the time of flight between the superconducting interface and the barrier (long junction limit). The geometric series \( \Phi(1) \) can be readily resummed into \( r_{he}(E) = d^2 r_A/(1 - r_A e^{i \Phi(E)}) \) with,

\[
\Phi(E) = 2\arccos(E/\Delta) + \frac{4E_F}{\hbar} + \pi Z_2
\]

When the insulating barrier is very high (perfect reflection \( r = 1 \) ) one observes true bound states in the spacer between I and S with energies \( E_p \) satisfying \( \Phi(E_p) = 2\pi p \) with \( p = 0, 1, 2, \ldots \). The zero energy state \( E_1 = 0 \) present for TS is the Majorana bound state [32 20]. Upon increasing the transparency of the insulating barrier, the Andreev states hybridize with the conducting electrode allowing one to probe them through the differential conductance \( dI/dV_b = (4e^2/\hbar)(1 + Z_2)|r_{ch}(eV_b)|^2 \) of the system (note that the TS is spinless, hence the factor \( 1/2 \)).

The simplest situation is when one sends a series of up-right localized pulses (of widths much shorter than \( \tau_F \) ). Fig. 2 sketches the phase accumulated along the different paths through the cavity; it corresponds to a flattened description of the paths shown in Fig. 1b). Whenever one crosses a pulse one picks up a phase \( 2\pi n \) while a phase \( \pi/2 \) is picked up upon Andreev reflection at the NS interface (for conventional superconductors; for topological ones, one alternately picks up \( \pi/2 \) and \( -\pi/2 \), [16]). When the period of the pulse train exactly matches the period, due to the time-dependent potential \( V(t) \). This phase is translated by \( 4\tau_F \) for different trajectories to account for the different times of propagation from the normal contact. It is this phase that will be used to manipulate the resonances. We will consider trains of pulses of different shapes as shown in Fig. 1b). They are \( T \)-periodic and we denote \( 2\pi n \) the phase accumulated over one period \( \phi(T) \) (upright pulses) or half a period \( \phi(T/2) \) (alternating and sine pulses). In analogy with the DC case, the time-dependent differential conductance is,

\[
\frac{dI(t)}{dV_b} = \frac{4e^2}{\hbar(1 + Z_2)} |r_{eh}(t, eV_b)|^2
\]

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As a result, \( \Phi_m(t, E) = m \Phi(E) + m(2\pi \bar{n}) \) and Eq. 5 takes the form of a geometric series, i.e. the (only) effect of the pulse is to shift the resonance by \( 2\pi \bar{n} \):

\[
\tau_{\text{he}}(E) = \frac{d^2 F}{1 - r e^{\Phi(E) + 2\pi \bar{n}}} \tag{6}
\]

An example of differential conductance for increasing \( \bar{n} \) is given in Fig. 3(b). In particular, for \( \bar{n} = 1/2 \) one transforms the spectrum of a conventional superconductor into the spectrum of a TS and vice versa. Also, for \( \bar{n} = 1/2 \), the train of pulses is equivalent to a train of alternating pulses; Fig. 3(b) corresponds to this alternating train. We see that upon increasing \( \bar{n} \) (using the amplitude of the pulses), a zero-bias peak develops while the Andreev peaks that were initially present shrink. Fig. 3(c) shows the effect of a similar alternating train on a system in the topological phase. We see that the initial zero-bias (Majorana) peak is effectively destroyed when \( \bar{n} = 0.5 \) while new Andreev peaks appear; the dual to the previous situation. Anticipating what follows, we find that a simple monochromatic sine pulse \( \propto \cos \omega t \) (which can be seen as a distortion of the alternating pulse train) will have a similar, although slightly less marked, effect.

To proceed with the generic case where \( T \neq 4T_F \), we expand \( \phi(t) \) as a Fourier series, 

\[
e^{i \phi(t)} = e^{i \phi(T)/T} \sum_{p=\infty}^{+\infty} c_p e^{ip\omega t}.
\]

For a simple monochromatic pulse \( V(t) = V_0 \cos \omega t \), the Fourier coefficients are given by \( c_p = J_p(eV_0/\hbar \omega) \) where \( J_p(x) \) is the \( p^{th} \) Bessel function of the first kind. Keeping only the DC part of the current, we get

\[
dI 
dV_b = \frac{4e^2|d|^4}{h(1 + Z_2)} \sum_{p = \infty}^{+\infty} \frac{c_p}{1 - |r|^2 e^{i \Phi(E, V_b)} + 4\omega T_F (\phi(T)/2\pi + p)} \tag{7}
\]

The appearance of the resonances in Eq. 7 corresponds to minimizing the denominator, i.e. the phase in the denominator is a multiple of \( 2\pi \). We thus expect (for \( eV_b \ll \Delta \)) sharp peaks at positions given by

\[
e^{V_b} = \frac{\omega}{2\pi} (p + \phi(T)/2\pi) = \frac{1}{4T_F} \left[ q - \frac{1}{2} (1 + Z_2) \right] p, q \in \mathbb{Z}
\]

The very simple structure of Eq. 8 allows an intuitive visualization. Fig. 4 shows colorplots of the DC conductance for the system in the topological regime in the presence of a sinusoidal time-dependent voltage. The different lines visible on Fig. 4 correspond to different pairs of the integers \( p \) and \( q \) of Eq. (8); the slope corresponding to \( p \) and the offset corresponding to \( q \). It now becomes clear that the presence of the resonant peaks with conductance \( 2e^2/h \) occur when lines for all values of \( p \) coincide. Resonant peaks with smaller conductance values occur where subsets of the lines coincide. One can understand each of the terms in Eq. 7 as referring to a \( p \)-photon absorption process, where \( |c_p|^2 \) can be interpreted as the probability to absorb \( p \) photons. The presence of this characteristic structure in the transport measurements could be used to identify 0-bias peaks as being Andreev/Majorana bound states – as opposed to Kondo resonances or Shiba states, as proposed in Ref. 17 – as the frequency dependence is closely linked to the time of flight of the resonant cavity. One could, for instance, vary the length of the cavity (for an insulating barrier produced by a gate) by having several gates coupled to the nanowire at different distances from the superconductor. Fig. 4 shows that, although the upright and alternating pulses have the strongest signatures, a sinusoidal perturbation can still produce a clear signal. This is clearly advantageous from an experimental point of view. If we take a cavity length of 200 nm (similar to that used in experiments [9]) and a Fermi velocity of \( 10^4–10^5 \text{m.s}^{-1} \).
we obtain a time of flight $\tau_F \sim 2\text{-}20 \text{ps}$. We thus require signal frequencies of $10\text{-}100 \text{GHz}$ (microwaves) to perturb the system in the regime of interest. This is within reach of current experimental techniques [22].

*Microscopic (numerical) calculations.* We now turn to a microscopic treatment of the system following the proposal made in Refs. [7, 24]. We consider a one-dimensional semiconductor wire with (strong) spin-orbit interaction and an external magnetic field. The combination of these two ingredients makes the system effectively spinless close to the Fermi level, so that a nearby (s-wave) superconductor induces p-wave topological superconductivity, and the presence of Majorana fermions (s-wave) superconductor induces p-wave topological superconduction [3] (shown in Fig. 4 (numerics not shown)). The conductance of the simulations is presented in Fig. 3.

The results of the simulations are presented in Fig. 3. We see a perfect agreement between the direct numerical simulations (symbols) and the analytical result (lines). The transmission probability, $|d|^2$, and time of flight, $\tau_F$, needed for the analytical curves are obtained from DC transport simulations using the Kwant package [25].

**Conclusion** We have shown that the interplay between a train of pulses and Andreev/Majorana states can provide a differential conductance identical to the DC case except for a tunable dynamical phase (upright pulses) as well as a spectroscopy of the states (sine pulses). Beyond the present study, we anticipate that the same line of thought could be used for more elab-
orate manipulations such as those needed for quantum computation.

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