Li-rich Giants in LAMOST Survey. III. The Statistical Analysis of Li-rich Giants

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Abstract

The puzzle of the Li-rich giant is still unsolved, contradicting the prediction of the standard stellar models. Although the exact evolutionary stages play a key role in the knowledge of Li-rich giants, a limited number of Li-rich giants have been observed with high-quality astroseismic parameters to clearly distinguish the stellar evolutionary stages. Based on the LAMOST Data Release 7 (DR7), we applied a data-driven neural network method to derive the parameters for giant stars, which contain the largest number of Li-rich giants. The red giant stars are classified into three stages of Red Giant Branch (RGB), Primary Red Clump (PRC), and Secondary Red Clump (SRC) relying on the estimated astroseismic parameters. In the statistical analysis of the properties (i.e., stellar mass, carbon, nitrogen, Li-rich distribution, and frequency) of Li-rich giants, we found that (1) most of the Li-rich RGB stars are suggested to be the descendants of Li-rich pre-RGB stars and/or the result of engulfment of planet or substellar companions; (2) the massive Li-rich SRC stars could be the natural consequence of Li depletion from the high-mass Li-rich RGB stars; and (3) internal mixing processes near the helium flash can account for the phenomenon of Li richness on PRC that dominated the Li-rich giants. Based on the comparison of [C/N] distributions between Li-rich and normal PRC stars, the Li-enriched processes probably depend on the stellar mass.

Unified Astronomy Thesaurus concepts: Chemically peculiar giant stars (1201); Giant stars (655); Stellar abundances (1577); Stellar evolution (1599); Classification (1907)

1. Introduction

Lithium is one of the most important elements produced at the early universe (Randich & Magrini 2021). It is easily burned at the temperature of several million Kelvin. As a low-mass star evolves out of the main sequence (MS), the Li material will be transported from the stellar surface to the interior by the first dredge-up (FDU) process, which leads to the depletion of surface Li (Iben 1967a, 1967b). The stellar evolution theory suggests that Li will be further depleted when the star climbs up to the red giant branch (RGB) bump, where the extra mixing conveys more Li to hot regions (Charbonnel 1994, 1995). Against the expectation of these models, a few percent of giants, namely Li-rich giants, present Li abundances A(Li) greater than 1.5 dex (Wallerstein & Sneden 1982; Brown et al. 1989; Kumar et al. 2011; Casey et al. 2016; Zhou et al. 2018; Yan et al. 2018; Gao et al. 2019; Deepak & Lambert 2021; Martell et al. 2021).

After the first Li-rich giant discovered by Wallerstein & Sneden (1982), researchers have paid much attention to such object (Charbonnel & Balachandran 2000; Drake et al. 2002; Kumar et al. 2011; Martell & Shetrone 2013; Casey et al. 2016; Smiljanic et al. 2018; Zhou et al. 2019). However, the sample size is limited. Currently, large-scale surveys have been available for studying these rare stars, such as the GALactic Archaeology with HERMES (GALAH, Deepak & Reddy 2019; Martell et al. 2021), the Large Sky Area Multi-Object Fiber Spectroscopic Telescope (LAMOST, Casey et al. 2019; Gao et al. 2019), and the number of Li-rich giants has reached about 10,000. A consensus one the origin of Li production has not been reached. Other than the extra mixing to destroy Li, the processes of “enhanced” extra mixing with a high mixing rate are proposed to be responsible for the Li enrichment, which can produce more Li because of high mixing efficiency. But the exact physical mechanism to trigger the mixing is still uncertain. The theories are mainly the thermohaline mixing (Eggleton et al. 2006; Charbonnel & Zahn 2007), rotational mixing (Denissenkov & Herwig 2004; Casey et al. 2019) and magneto-induced mixing (Busso et al. 2007).

Many explanations of Li enhancement are connected to the special evolutionary stages and the exact evolutionary stage of a star is hard to tell due to the overlap of different evolutionary stages, especially between the RGB bump and the red clump (RC). In the past few decades, the Li-rich giants are supposed to be the RGB bump within a narrow region of the Hertzsprung –Russell (HR) diagram (Charbonnel & Balachandran 2000). When the hydrogen-burning shell burns out the barrier of the mean molecular weight left after the FDU, it enables the extra
mixing such as the thermohaline instability (Charbonnel & Lagarde 2010). It is reasonable to consider the Li-rich giants at the RGB bump because of the long timescale in this stage associated with the extra mixing. The first Li-rich giant with helium-core burning is reported based on the asteroseismic analysis by Silva Aguirre et al. (2014). As Bedding et al. (2011) suggested, the RC and RGB stars can be well separated by the asteroseismic parameters, especially the large frequency separation ($\Delta \nu$) and the period spacing ($\Delta P$). Recently, Yan et al. (2021) provided a uniquely large sample with a clean determination of evolutionary stages based on the asteroseismology, they found that most ($\sim$80%) of the Li-rich giants belong to the phase of RC rather than RGB. A similar result has also presented by Singh et al. (2021).

Although the evolutionary stages can be well distinguished by asteroseismology, the number of Li-rich giants with asteroseismic information is limited. More Li-rich RC stars are identified by the data-driven method though they are not directly determined from the asteroseismic data (Casey et al. 2019; Zhou et al. 2019; Kumar et al. 2020). Based on the location on the HR diagram associated with data-driven method, Kumar et al. (2020) found that the observed Li in the RC stars ($A(\text{Li}) \sim 0.7$) are enhanced by about 1.6 dex relative to the prediction of models. They claimed that all the low-mass RC stars have gone through a process of Li production. This process is suggested to operate during the helium flash or the RGB tip (Schwab 2020; Mori et al. 2021; Zhang et al. 2021), which is against by Chanamé et al. (2021). Chanamé et al. (2021) argued that this finding is misled by stellar masses because the masses estimated by Kumar et al. (2020) are higher than the reliable asteroseismic masses, which cause the Li abundances of RC stars to be larger than the calculation of models. In any case, the Li-rich giants with $A(\text{Li}) \geq 1.5$ remain a puzzle. A few trigger mechanisms are proposed to explain the Li enhancement in the RC stars, such as the internal gravity waves during helium flash (Schwab 2020), an additional energy loss related to the neutrino magnetic moment (Mori et al. 2021), tidal interaction in binary (Casey et al. 2019), and the merger of a helium-core white dwarf with a RGB star (Zhang et al. 2020c).

The spectroscopy reflects the information of stellar atmosphere, which could determine the evolutionary stages as well. Based on the APOGEE Data Release 12 (DR12) data, Masseron & Gilmore (2015) showed that a difference of [C/ N] between RGB stars and helium-core burning stars can reach 0.2 dex. It is thought to be caused by the extra mixing along with the RGB and the helium flash. Hawkins et al. (2018) suggested that the difference ensures the spectra alone to distinguish the RGB stars from the RC stars by Cannon, which is a data-driven method to map the relation of spectral flux and the asteroseismic parameters. Ting et al. (2018) expanded the idea to the LAMOST DR3 low-resolution spectra with neural network method, and achieved a very low contamination rate ($\sim$3%) for the RC stars.

A largest sample of Li-rich giants was constructed by (Gao et al. 2019, Paper I) based on the low-resolution spectra from the LAMOST DR7. In this work, we statistically investigated their properties in detail, such as the Li-rich distribution and frequency, stellar mass, and the abundances of C and N. Our data and method are described in Section 2. In Section 3, we compared the individual properties between Li-rich and normal giants with the same evolutionary stages. Possible origins of Li-rich were discussed in Section 4 and a brief summary is presented in Section 5.

2. Data and Method

LAMOST equipped 16 spectrographs with a spectral resolution ($R$) of about 1800. Its wavelength range is from 3800 to 9000 Å (Cui et al. 2012; Zhao et al. 2012; Yan et al. 2022). From 2011 October to 2019 June, LAMOST DR7 has publicly released about 10 million low-resolution spectra with SNR $\geq 10$. The red giant stars have been selected with log $g \leq 3.5$ dex and $T_{\text{eff}} < 5600$ K, which includes the Li-rich giants from Paper I. The typical errors for log $g$, $T_{\text{eff}}$, and [Fe/H] are about 0.05 dex, 30 K, and 0.03 dex, respectively, for the spectra with SNR $\geq 50$.

2.1. Method

Data-driven methods have been applied on the LAMOST low-resolution spectra by many researchers for deriving multiple information including the asteroseismic parameters, mass/age, and elemental abundances, etc., for example, The Cannon (Ho et al. 2017; Hawkins et al. 2018), KPCA (Wu et al. 2019; Huang et al. 2020), SLAM (Zhang et al. 2020a, 2020b), and neural network (Ting et al. 2018; Xiang et al. 2019). Suitable asteroseismic training data are required to establish the data-driven model, and the sample of Vrard et al. (2016) meets this requirement for this study, which contains 6100 giants with reliable $\Delta P$ and $\Delta \nu$. There are 2662 common stars with high-quality LAMOST spectra (SNR $\geq 50$) between the LAMOST DR7 giant stars and Vrard et al. (2016) catalog after cross-matching. We randomly pick 1800 stars to be the training set, and the rest are the test set for verifying the generalization of the neural network model.

In this work, a neural network with three layers is constructed to describe the mapping from the spectra to the asteroseismic parameters. We derive a highly nonlinear relation between the LAMOST normalized spectra and the asteroseismic parameters (i.e., $\Delta P$ and $\Delta \nu$). The labels of $\Delta P$ and $\Delta \nu$ can be represented as a function of the normalized flux, and they can be written as:

$$
(\Delta P, \Delta \nu) = \omega^j \sigma(\sigma^l \sigma^{\text{unf}} f_m + b_l) + b_j + b_i
$$

where $\sigma(x) = \text{max}(x, 0)$ is the activation function of Relu. $j$, $k$, and $l$ are the number of neurons of 512, 256, and 64 from the first to the last layers. The flux of normalized spectra is indicated as $f_m$, and $\omega$ and $b$ are the coefficients to be optimized during the training process. The training process stops when the $L1$ losses of validation set achieve constant tendency after tens of thousands of steps. The constant $L1$ losses indicate that the sum of difference between predicted and authentic labels ($\Delta P$, $\Delta \nu$) reaches the minimum. The training process to get a predictive model is implemented with the python Tensorflow package. Following Ting et al. (2018), the results were constrained within a convex hull of the LAMOST DR7 stellar atmospheric parameters (i.e., $T_{\text{eff}}$, log $g$, and [Fe/H]) to exclude
the extrapolation outside the range of stellar parameters of training sample.

The information of stellar mass and abundances of C and N are crucial to understanding the Li production. A similar data-driven method has been applied to derive these properties for the red giant stars. The precisions of [C/Fe] and [N/Fe] are 0.052 dex and 0.082 dex for stars with spectral SNR > 50 (Wang et al. 2022), which would make the uncertainty of [C/N] to be about 0.1 dex. The stellar masses with uncertainty of 0.21 \( M_\odot \) are estimated by mapping the relations from the high-fidelity asteroseismic mass of Yu et al. (2018) to the LAMOST spectra. The “ground truth” labels of C and N are from APOGEE DR16 with the corresponding flags = 0 (Jönsson et al. 2020).

2.2. Data and Sample

The comparison of predicted \( \Delta P \) and \( \Delta \nu \) and those from Vrard et al. (2016) imply an appropriate data-driven mapping from the LAMOST spectra to asteroseismic labels. Both \( \Delta P \) and \( \Delta \nu \) show a good consistency with one-by-one comparison, though a slight scatter is present at high \( \Delta P \) (Figure 1). A clear gap can be seen between the RC and RGB stars in \( \Delta P \). The RC stars occupy the high \( \Delta P \) region, while the RGB stars cluster at a lower range of \( \Delta P \), which are clearly separated in the diagram of \( \Delta P \) versus \( \Delta \nu \) (Figure 2). The criterion of \( \Delta P = 150 \) s is adopted to discriminate the RC from RGB stars suggested by Bedding et al. (2011).

Moreover, the primary RC (PRC) stars (the low-mass stars that experience helium flash with a degenerate helium core) could be properly identified from the secondary RC (SRC) stars (the massive stars that ignite helium with partly or non-degenerate core) with \( \Delta \nu > 5 \) \( \mu \)Hz (Mosser et al. 2014). We noted that part of SRC stars with low mass conflict with the fact that the SRC stars should be with a large mass. The discrepancy might be attributed to the limited number of SRC stars of the training set. For a reliable discussion, the SRC stars with mass \( \gtrsim 1.9 \) \( M_\odot \) have been adopted following the definition of Mosser et al. (2014) for solar metallicity PRC stars (see Section 3.3).

The test set is used to determine the completeness and purity, which is independent of the training processes of the data-driven model. Among the test set, 282, 501, and 79 stars are classified as RGB, PRC, and SRC by Vrard et al. (2016). The model can recall 280 RGB, 485 PRC, and 49 SRC stars, so the corresponding completenesses are 99.2% (280/282), 96.8% (485/501), and 62% (49/79), respectively. The model predicted 283 RGB, 515 PRC, and 62 SRC stars for the test set. Given the same number of stars at the authentic evolutionary stages, its accuracy of prediction for the RGB, PRC, SRC is 98.9% (280/283), 94.2% (485/515), and 79% (49/62), respectively. The purity of SRC stars will reach 97.8% (44/45) but with a reduction of completeness down to 59.5% (44/74) if we excluded the SRC stars with masses \( < 1.9 \) \( M_\odot \).

The completeness and purity are comparable to the results of Huang et al. (2015, 2020) and Ting et al. (2018), who applied a similar approach for LAMOST DR3.

Finally, we get a sample of 260,396 red giant stars (SNR \( \gtrsim 50 \)) with the determined evolutionary stages, including 50.9% (132,581) RGB, 43.8% (113,941) PRC, and 5.3% (12,874) SRC stars. With the method and selection criteria, 3,750 Li-rich giants have been derived with the properties of stellar mass, [C/Fe] and [N/Fe], which will be shown in Paper II (Yan et al. 2022, in preparation). The Li-rich sample consisting of 29.1% (1,093) RGB, 55.4% (2,079) PRC, and 15.4% (578) SRC stars is listed in Table 1.

3. Results

3.1. Distribution of Li Abundances

The Li-rich stars are rare in the red giant stage. Previous studies have found that about 1%–2% giants show Li excess (0.052 dex and 0.082 dex for stars with spectral SNR > 50 (Wang et al. 2022), which would make the uncertainty of [C/N] to be about 0.1 dex. The stellar masses with uncertainty of 0.21 \( M_\odot \) are estimated by mapping the relations from the high-fidelity asteroseismic mass of Yu et al. (2018) to the LAMOST spectra. The “ground truth” labels of C and N are from APOGEE DR16 with the corresponding flags = 0 (Jönsson et al. 2020).
i.e., A(Li) ≥ 1.5, Casey et al. (2019; Gao et al. 2019; Martell et al. 2021). A similar ratio (1.4%) of Li-rich giants is found for our giants’ sample.

A large fraction of the Li-rich giants have been confirmed as the RC with slightly different quantities, which are 68% (Deepak & Reddy 2019), 80% (Casey et al. 2019), 86% (Yan et al. 2021), and 64% (Martell et al. 2021), respectively. There are 70.9% of our Li-rich giants that belong to the RC. The different ratios might be attributed to the selection function, classified method, spectral resolution, and the determination of Li abundance.

It is found that the percentage of Li-rich at the RC stage is 2.08%, which is 2.54 times of that (0.82%) of RGB stars (Table 1). Similar probabilities of RC and RGB stars to be Li-rich have been found by Martell et al. (2021) (1.9%; 0.76%) based on the GALAH and K2-HERMES surveys.

Yan et al. (2021) found that the distribution of Li-rich RC stars significantly differ from those of the Li-rich RGB stars. Figure 3 presents the distributions of Li abundances at the three evolutionary stages for our sample. The number of Li-rich stars decreases as the Li abundance increases. It is generally believed that less stars have a higher A(Li) because it is hard to maintain the Li due to further depletion.

For the Li-rich RGB stars, Li distribution more rapidly decreases compared to the Li-rich RC stars, which is in line with Yan et al. (2021). They showed a decreasing number toward the low log g, and more Li-rich RGB stars are found at higher log g as the bottom panel of Figure 3 shown. This trend was also found by Zhang et al. (2021) in the view of A(Li) versus stellar radius, which involves the RGB stars with A(Li) ≳ 0–2.5 based on the data of LAMOST medium-resolution and Kepler. A few exceptions (3.56%) of Li-rich RGB stars were shown to be with super high Li, which might be contaminated by the RC stars. It requires a firm verification by the direct asteroseismic data.

For the two subgroups of RC stars, the Li-rich ratio on the SRC (4.17%) is higher than that of PRC stars (1.82%). Conversely, the probability (22.66%) of Li-rich PRC stars to be super Li-rich is much higher than that of the Li-rich SRC stars (1.9%). As found by Martell et al. (2021), although the Li-rich ratio of PRC stars is lower than that of SRC stars, the PRC stars are more likely to be super Li-rich. As the bottom panel of Figure 3 shown, it is obvious that a few SRC stars exhibit very high A(Li) similar to the RGB stars.

In contrast to the Li-rich SRC stars, the Li-rich PRC stars spread at a large Li abundance range within a narrow log g, though the number decreases toward the high A(Li) as well. Since the ranges of metallicity are wider for the PRC stars relative to the SRC ones (Girardi 1999), Martell et al. (2021) suggested that it would lead to more complexity of Li related processes within the PRC stars.

3.2. Li-rich Frequency

The frequency of Li-rich stars among the field (Adamów et al. 2014) and the open clusters (Smiljanic et al. 2016) have been identified to be ~1%–2%. The Li-rich frequency likely increase toward high metallicity (Casey et al. 2019; Deepak et al. 2020; Martell et al. 2021). The different mechanisms of Li enhancement are usually tied to the stellar evolution with different metallicity, it is worth exploring the Li-rich fraction not only in a given evolutionary stage but also in the metallicity bins.

Figure 4 shows that the Li-rich RC stars occur more frequently with increasing [Fe/H], especially for the Li-rich SRC stars. Interestingly, the frequency of Li-rich SRC stars agrees with the prediction (~3.4%–9.6%) of the high-mass
RGB stars by Aguilera-Gómez et al. (2016), and they originally attempt to explain the Li-rich RGB stars by engulfment of substellar companions. The Li-rich frequency among the massive RGB stars ($\geq 1.9 \, M_\odot$) was estimated to be a high percentage of about 8.5% similar to the SRC stars.

The frequency of RC stars to be Li rich gradually grows toward higher $[\text{Fe}/\text{H}]$, although it is not so sharp as in Martell et al. (2021). It can be seen that the pattern is more consistent with that of Martell et al. (2021) if we combine the groups of PRC and SRC stars. The minor difference above the solar metallicity is negligible within the error bars. Moreover, rather than dramatically increasing as in Martell et al. (2021), the frequency of Li-rich RGB stars presents a turning point near the solar metallicity. At the low-metallicity end, the Li-rich frequency increases for both RC and RGB stars, which need further confirmation due to the limited number of metal-poor stars.

In the super-solar metallicity, our Li-rich frequencies are lower than that of Martell et al. (2021) for the RGB stage. The disagreement might be due to the selection function and the classified method. On one hand, it is noted that two samples have different fractions of Li-rich RGB. We got a lower fraction (29%) of Li-rich RGB stars than 42% of Martell et al. (2021). Figure 4 shows that our error bars are relatively small at the super-solar metallicity, which implies a more reliable statistical analysis. The error bar is adopted to be the Poisson

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### Table 1: Statistical Distributions in the Different Evolutionary Stages

| Stages | Total   | Li-rich | Super Li-rich | Completeness | Purity |
|--------|---------|---------|---------------|--------------|--------|
| Whole sample | 260,396 (100%) | 3,750 (1.4%) | 521 (13.9%) | ... | ... |
| RGB    | 132,581 (50.9%) | 1,093 (0.82%) | 39 (3.56%) | 99.2% | 98.9% |
| RC     | 127,815 (49.1%) | 2,657 (2.08%) | 482 (18.14%) | 92.1% | 92.5% |
| PRC    | 113,941 (43.8%) | 2,079 (1.82%) | 471 (22.66%) | 96.8% | 94.2% |
| SRC    | 12,874 (5.3%) | 578 (4.17%) | 11 (1.9%) | 59.5% | 97.8% |

*Note.* The stars are divided into the evolutionary stages of RGB, RC, PRC, and SRC. For the Total column, the percentage means the fraction of the stars at each stage. For the Li-rich and Super Li-rich columns, the percentages represent the probabilities of the normal stars to be Li-rich and the probabilities of the Li-rich stars to be super Li-rich (i.e., Li-rich/Total, Super Li-rich/Li-rich). Here, we adopted the stars with $A(Li) \geq 1.5$ as Li-rich and stars with $A(Li) \geq 3.3$ as super Li-rich.

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Figure 3. The distributions of Li abundances for different evolutionary stages. In the top panel, the Li-rich giants in the RGB, PRC, and SRC are denoted by blue, red, and green, respectively, which each bin of Li abundances is about 0.20 dex in the histogram. Note that they have been normalized by the number of Li-rich stars at the corresponding evolutionary stage. The bottom panel is the Li abundance vs. $\log g$ with colors in number density.

Figure 4. The frequency of Li-rich giants vs. metallicity among the different evolutionary stages. The Li-rich frequency at each evolutionary stage is estimated for a metallicity bin of $\sim 0.1$ dex. The frequencies of Li-rich stars from Martell et al. (2021) are also presented as green points for comparison. Top panel: the frequency of Li-rich RC (red), Li-rich PRC (blue), and SRC stars (orange). Bottom panel: the frequency of Li-rich for RGB stars (red).
statistics error, as in Martell et al. (2021). On the other hand, our classifications of evolutionary stages achieve both higher purity and completeness. For instance, our RGB stars are determined with 98.9% purity and 99.2% completeness, while Martell et al. (2021) derived the RGB sample with 81% purity and 74% completeness, which would be more contaminated by the RC stars.

3.3. Mass Distribution

The extra mixing mostly depends on the stellar mass, metallicity, and evolutionary phase, which may impact the Li enhancement and dilution (Charbonnel & Lagarde 2010; Shetrone et al. 2019; Chanamé et al. 2021). Therefore, it is important to analyze the mass distribution of our Li-rich giants.

Deepak et al. (2020) found that the mass distribution of Li-rich stars at or below the RGB Bump is similar to the normal stars. With a more extensive Li-rich sample, our data show that the Li-rich RGB stars concentrate at a higher mass of 1.51 M⊙ than the Li-normal RGB stars of 1.33 M⊙, assuming a Gaussian profile, which is in agreement with the asteroseismic mass distribution of Yan et al. (2021) for RGB stars.

Rather than a straightforward mass definition of the SRC stars, the PRC sample could not be simply confined by a single mass definition considering their large metallicity range. The mass criteria to separate the PRC from SRC would vary from 1.7 to 2.5 M⊙ (Castellani et al. 2000; Girardi 2016), depending on the stellar models with different input physical parameters (e.g., convective mixing of the MS, initial composition, and metallicity).

With respect to the normal PRC stars, the Li-rich PRC stars present a bimodal distribution, which can be fitted with the Gaussian mixture models (GMMs). The normal stars assemble at the mass of 1.21 M⊙, while the Li-rich PRC stars seem to cluster at the mass of 1.0 M⊙ and 1.77 M⊙ (Figure 5). Even though the PRC stars might be slightly contaminated by the SRC stars, they still cover the intermediate-mass range of ~1.5–2.0 M⊙. It is in accord with Yan et al. (2021) and Deepak & Lambert (2021) for the RC stars, but noted that both of their RC samples include some of SRC stars, which have not been picked out from their RC sample.

For the pre-RGB stars in the open clusters, a bimodality is presented in the diagram of stellar masses versus Li abundances. In other words, there is a gap in the mass range of about 1.2–1.4 M⊙, which is known as the “Li dip” (Boesgaard & Tripicco 1986; Deliyannis et al. 2019; Twarog et al. 2020). It implies a relatively high Li abundances in the pre-RGB stars outside the region of Li dip. It is highly possible for these stars to become Li rich when they climb toward the giant branch. It is found that a bimodal mass distribution is present for the more evolved Li-rich PRC stars, but not for the Li-rich RGB ones as shown in Figure 5. A further discussion will be presented in Section 4.

For the SRC stars, if we only consider the SRC stars with moderately high mass above 1.9 M⊙, suggested as Mosser et al. (2014), the Li-rich and normal SRC stars do not show obviously different mass distributions, but with a slightly higher mass for the Li-rich stars. The distribution of SRC stars can not be well fitted with a Gaussian function due to the narrow mass range. As we can see from Figure 5, part (26%) of the Li-rich SRC stars are located in the low-mass region, which is possible due to the contamination (21%) from others. However, a relatively large ratio of the normal SRC stars with low mass is probably caused by a limited number of RC with high mass to train the data-driven model. Other than the PRC stars, the SRC stars only cover a narrow range of metallicity. To ensure a reliable discussion for the SRC stars, the SRC sample with mass greater than 1.9 M⊙ is adopted for the statistical analysis.

3.4. Carbon and Nitrogen

The abundances of carbon and nitrogen are valuable to trace the extra-mixing processes. The FDU will transport the CN-cycle processed materials with deficient C and enhanced N from the hot interior to the surface, causing the decline of [C/N] at the outer layer (Iben 1967b). After the FDU, a further decrease of [C/N] would be taken place caused by the extra mixing at the RGB bump (Charbonnel & Lagarde 2010; Masseron et al. 2017; Lagarde et al. 2019; Shetrone et al. 2019). The ratio of [C/N] is then sensitive to the internal mixing that might be responsible for the Li-rich.

In this context, the comparisons of [C/N] between the Li-rich and normal giants for different evolutionary stages are shown in Figure 6. To confirm the comparison, a subsample of
Li-rich giants with C and N from APOGEE DR16 is shown as green, which contains 138 RGB, 287 PRC, and 78 SRC stars. GMMs are applied to model the distributions of $[\text{C}/\text{Fe}]$, $[\text{N}/\text{Fe}]$, and $[\text{C}/\text{N}]$ for our sample stars.

For the Li-rich RGB stars, the distribution of $[\text{C}/\text{Fe}]$ looks like those of normal stars only with minor offset. Their $[\text{N}/\text{Fe}]$ share almost the same distribution with normal RGB stars in Figure 6. The $[\text{C}/\text{N}]$ ratios cluster at similar values of $-0.18$ and $-0.11$ for the Li-rich and normal RGB stars, respectively.

Similarly, in the bottom panels of Figure 6, the Li-rich SRC stars are indistinguishable from the normal giants in the $[\text{C}/\text{N}]$ distribution, although $[\text{C}/\text{N}]$ of the Li-rich SRC stars from the APOGEE data are not perfectly consistent with our sample due to the small number. It indicates that most of the Li-rich RGB and SRC stars might not experience any internal Li-enriched process, since $[\text{C}/\text{N}]$ would exhibit different distribution between Li-rich and normal RGB/SRC stars if an unusual mixing process operates to produce Li.

Compared to the other evolutionary phases, the Li-rich PRC stars display a bimodal $[\text{C}/\text{N}]$ distribution as their mass distribution, differing from the normal PRC stars with only one peak. Regardless of the whole sample or the subset from APOGEE, two Gaussian functions are good enough to interpret the distributions especially for $[\text{N}/\text{Fe}]$ and $[\text{C}/\text{N}]$. The Li-rich PRC stars mainly center on the $[\text{C}/\text{N}]$ with $-0.44$ and $0.07$ dex, while $[\text{C}/\text{N}]$ of the normal PRC stars locate on a medium value of $-0.12$. The $[\text{C}/\text{Fe}]$ seems distribute with two peaks but are not prominent as $[\text{N}/\text{Fe}]$. The feature of $[\text{N}/\text{Fe}]$ is consistent with that of Yan et al. (2021).

The bimodality occurs in both of mass and $[\text{C}/\text{N}]$ for the Li-rich PRC stars, which is suspected to be the reflection of Li dip shown in the MS stars of open clusters. Analogous to the study of Li in open clusters, we divide the PRC sample into three groups by the stellar masses below, near, and above the Li dip (i.e., $<1.2M_\odot$, $1.2M_\odot$ $\sim$ $1.4M_\odot$, and $>1.4M_\odot$). As Figure 7 shows, the Li-rich PRC stars below, near and above the Li dip cluster at the high, medium, and low $[\text{C}/\text{N}]$, respectively. It is clear to see that the Li-rich stars below and above the Li dip correspond to the two peaks in Figure 6. However, these two major portions exhibit contrary $[\text{C}/\text{N}]$ behaviors compared to their Li-normal counterparts. The Li-rich PRC stars with mass $<1.2M_\odot$ have larger $[\text{C}/\text{N}]$ than those of normal ones, but $[\text{C}/\text{N}]$ of the Li-rich above Li dip shift toward a lower value relative to the normal stars. For the mass region of Li dip, $[\text{C}/\text{N}]$...
with the stellar mass and result in the different distribution of Li-rich PRC stars might be affected by Li dip, the RGB stars with the help of asteroseismology − center of about $N$. The color indications are same as Figure 6. For each panel, the centers of [C/N] distributions of Li-rich and normal PRC stars are denoted as $\mu_1$ and $\mu_2$.

4. Discussion

4.1. Origins of the Li-rich on RGB

Recently, only minor fraction of Li-rich stars are identified as the RGB stars with the help of asteroseismology (Singh et al. 2021; Yan et al. 2021). The information of pre-RGB stars can provide the most important constraints on the understanding of Li-rich RGB stars.

First, Yan et al. (2021) found that the distribution of Li-rich RGB stars dramatically declines with an increase of Li abundance. With extensive sample size, we found that the distribution of the Li-rich RGB stars is anti-correlated with Li abundance but positively related to log $g$. It indicates that, as the stars evolve more toward low log $g$, fewer RGB stars are found to be Li-rich due to depletion. We do not find any uniform Li-enriched process at the specific evolutionary stage of RGB (e.g., RGB bump) similar to Zhang et al. (2021).

Second, the Li-rich frequency of RGB stars against metallicity might provide another clue. It appears to connect to the Li abundances of MS field stars. For the MS stars, the maximum Li abundance, $A(Li)_{max}$, climbs up to the solar metallicity and drops at [Fe/H] > 0 (Bensby & Lind 2018; Fu et al. 2018), which agrees with the pattern of Li-rich RGB frequency. Different dominant Li origins (e.g., novae, AGB stars) for the Galactic Li would result in such a pattern (Cescutti & Molaro 2019; Guiglion et al. 2019). As the “initial” $A(Li)$ varies with metallicity, the solar metallicity MS stars with higher $A(Li)_{max}$ would be more likely to be Li rich. Thus, the probability of Li-rich RGB stars is expected to be associated with the metallicity.

Third, for the MS stars of the open clusters, the distribution of stellar masses present two peaks with high Li abundances, i.e., the morphology of Li dip. These MS stars with high $A(Li)$ is most likely to be the Li-rich RGB stars, thereby it is supposed to present the bimodal morphology of mass for the Li-rich RGB. However, it is not found in our Li-rich sample, which only show higher masses than the Li-normal RGB stars.

The speculation is that some Li-enriched processes probably have happened, for example the engulfment of the planet or substellar companion. Recently, the calculation of Soares-Furtado et al. (2021) showed that planetary engulfment would be more efficient to enrich Li for the post-MS stars with $1.4 M_\odot$. Their models suggest that Li-enriched stars would pile up at $1.4-1.6 M_\odot$, which is consistent with mass distribution of our Li-rich RGB stars. Note that these scenarios are limited to accounting for the Li-rich stars with meteoritic $A(Li)$, but a small part of RGB stars show to be the super Li-rich (i.e., $A(Li) > 3.3$), which we could not exclude the possibility of binary interaction suggested by Casey et al. (2016).

Additionally, the similar [C/N] between Li-rich and normal RGB stars agrees with the suggestion that the Li-rich RGB stars unlikely go through the internal mixing process to produce Li. Considering stellar properties of Li-rich RGB stars associated the information from the pre-RGB, the possible origins for the Li-rich on RGB stars are the consequence of Li depletion from the Li-rich progenitors and/or caused by the engulfment of planet or substellar companions.

4.2. Li-rich or Li-normal SRC Stars?

The Li-rich RC stars are usually correlated with the helium flash since all the low-mass RC stars are supposed to go through this violent process. However, the Li-rich phenomenon on the SRC can not be connected with the scenario of helium flash because the massive SRC stars evolve too fast to develop a degenerate helium core (Girardi 1999). Therefore, another explanation is required to account for the Li-rich phenomenon on SRC.

Similar to the Li-rich RGB stars, few of the Li-rich SRC stars present super high Li abundances, and they have a

![Figure 7](image_url)
consistency of Li-rich frequency with the massive RGB stars. According to the PAdova and TRieste Stellar Evolution Code (PARSEC, Bressan et al. 2012), a star with $2 M_\odot$ only spends about 14 Myr in the RGB phase; it is too short to allow the star to experience any significant mixing at RGB. Thus, when those stars evolve to SRC, they are mostly assumed to inherit the Li abundances of high-mass RGB stars and the corresponding Li-rich frequency.

If this is true, the high-mass RGB stars should be with higher Li abundance than the SRC stars, because they would endure further Li depletion as they are more evolved. Given only Li abundances provided for the Li-rich giants, a rough comparison show that the mean A(Li) of massive Li-rich RGB stars ($\geq 1.9 M_\odot$) is higher with 0.22 dex than that of Li-rich SRC stars. The Li-rich phenomenon on SRC is probably the natural dilution from the Li-rich RGB stars with high mass.

There is only a slight difference on the mass distribution between Li-rich and normal SRC stars. Although the Li depletion has a mass dependency, it becomes less sensitive as the mass increases toward the higher range (Deepak & Lambert 2021). It fits in with the fact that the high-mass stars have a shallow convective envelope to make the Li depletion less effective. Moreover, the information of $[\text{C}/\text{N}]$ indicates that the extra-mixing process would not happen for producing fresh Li in the massive SRC stars. As Adamów et al. (2014) reported, the massive Li-rich stars show similar stellar properties with normal giants and no evidence points out the specific Li-rich process.

The Li-rich SRC stars might be the natural consequence of Li depletion from the Li-rich high-mass progenitors on the RGB. The probability to be Li-rich on SRC might depend on not only the Li depletion at SRC but also the initial Li at the massive RGB.

4.3. The Li Enhancement of PRC Stars

Since the majority of the Li-rich giants are identified as the RC stars rather than the RGB stars by the asteroseismology, the origins to enrich Li have attracted the community attention to how Li to be produced at this phase (Singh et al. 2021; Yan et al. 2021).

Based on the GALAH DR2, Kumar et al. (2020) even reported that all of PRC stars have undergone the Li-enriched process due to the helium flash, because they found an increase of 40 factors in Li abundance from the RGB tip to the RC for the low-mass stars, where the RC stars concentrate around $A(\text{Li}) = 0.7$. This contradicted the prediction of $A(\text{Li}) \lesssim -0.9$ by the stellar models. So they suggested a ubiquitous Li enhancement on the RGB tip or the helium-core ignition. However, Chamamé et al. (2021) claimed the increase of observed Li can be well reproduced by their models if considering different Li depletion rates with various stellar masses. They argued that the mass adopted by Kumar et al. (2020) is higher than the asteroseismic mass, thus it leads to a high Li at the RC due to the lower Li dilution for higher mass. Although it could explain the modest Li ($\sim 1.0 \text{ dex}$) during the stage of PRC, the PRC stars with $A(\text{Li}) \geq 1.5$ are indeed required another scenario.

To solve the discrepancy between the standard stellar models and observation of Kumar et al. (2020), the novel physical processes, such as neutrino magnetic moment (Mori et al. 2021) and internal gravity waves (Schwab 2020), have been invoked to trigger the mixing to produce Li during the RGB tip or helium flash. But neither of the predictions of these models could achieve super Li-rich stars.

By comparing the predictions of stellar models and the observed asteroseismic parameters, a novel insight is proposed by Singh et al. (2021) to study the status of Li-rich stars during the RC stage. They reported that the super Li-rich PRC stars are exclusively located at the early stage of core-helium burning with low $\Delta P = 257$ s and the modest Li-rich stars are intermediate stage with $\Delta P = 290$ s between the early super Li-rich stars and the late Li-normal stars ($\Delta P = 306$ s). With their speculation, all super Li-rich stars should assemble at the early stage of RC. However, by inspecting the stellar evolution with the view of mass and radius, Zhang et al. (2021) found that the super Li-rich stars scatter at the RC phase rather than cluster at the early stage. Following Singh et al. (2021), we displayed the $\Delta P$ density distribution of PRC stars as the indicator of evolution (Figure 8). With a large number of super Li-rich PRC stars, we find that the super Li-rich truly concentrate at the early stage with $\Delta P = 260$ s, which is consistent with Singh et al. (2021). But they span a similar range of $\Delta P$ as Li normal and modest Li rich rather than a clear distinction as shown by Singh et al. (2021). It suggests that the Li-rich processes might happen during the early stage of core-helium burning.

Rather than the Li-rich process triggered by internal mechanisms, Denissenkov & Herwig (2004) suggested the rapid rotation to enhance the extra mixing in the binary is responsible for the Li-rich stars at the RGB bump. This scenario of binary interaction is generalized for both the RGB and RC by Casey et al. (2019). Based on the data of a nine-year radial-velocity monitoring, Jorissen et al. (2020) found a similar binary frequency between Li-rich and normal giants, therefore, the Li enhancement is not supported by the tidal interaction between binary systems.

We cross-match the Li-rich stars with the catalog of close-binary systems provided by Price-Whelan et al. (2020) based on multiple observations from APOGEE DR16. We can not find a higher binary fraction of the Li-rich giants (4.3%, 4/92)
than that of normal giants (6.4%, 350/5471). Additionally, Zhang et al. (2020c) proposed that the Li-rich RC stars are formed as the post-merger of a RGB stars and a helium white dwarf. Although it could reproduce the mass distribution of our PRC stars with this model of near solar metallicity, it does not account for the [C/N] and the change of Li-rich frequency with metallicity. The model needs to be confined with more information (e.g., [C/N]) due to the poor knowledge of the common envelope of a merger as Zhang et al. (2020c) stated.

As mentioned in Section 3.3, the MS stars outside mass range of Li dip have so high A(Li) that they are very likely to be the Li-rich. The bimodal distribution is not presented for the Li-rich RGB stars because of the planetary engulfment on the post MS. But it is shown at the more evolved Li-rich PRC stars, which is likely related to the Li dip (see Section 3.4). Even though the stars outside the Li dip have high “initial” Li abundance so that impact the distribution of Li-rich PRC stars, the processes of Li enhancement should be existed to be responsible for Li-rich PRC due to the further Li depletion during the RGB. In addition, when we divide the PRC stars into the below, near and above the mass regions of Li dip, [C/N] still exhibit different distributions between the Li-rich and normal PRC stars, which might be the reflection of the Li-enriched mixing processes.

5. Summary

A large sample of giants from LAMOST DR7 were classified into the evolutionary stages of RGB, PRC, and SRC by a data-driven approach. The purities can reach 97.9% for RGB stars, 93.6% for PRC stars, and 76.6% for SRC stars with the LAMOST spectral SNR ≳50. We established the sample of red giant stars with the informations of mass and elemental abundances of C and N, aiming to interpret the Li-rich problem at different evolutionary stages by the statistical analysis of the Li-rich distribution and properties between Li-rich and normal giants.

We find that most of the Li-rich RGB stars might be the descendant of Li-rich pre-RGB stars, and the scenario of engulfment of a planet or substellar companions are responsible for their Li richness. The Li-rich PRC stars, which dominate among the Li-rich giants, are probably interpreted by internal mixing processes near the helium flash. According to their [C/N], the Li-rich process seems depend on the stellar mass. Whereas, the Li-rich phenomena for the massive SRC stars are the consequence of natural Li depletion. Although several Li-rich models have been proposed to explain the Li-rich PRC stars, more information is required to be predicted to match observed features, such as Li-rich frequency, stellar mass, and [C/N]. From the view of observation, only a small fraction of giant stars are derived with the ratio of $^{12}\text{C}/^{13}\text{C}$, which is more sensitive to the mixing process. In this work, similar to the other data-driven approaches, our study is limited by the stellar parameters of the training set in order to exclude the extrapolation.

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Facility: LAMOST.

Software: Astropy (Astropy Collaboration et al. 2013, 2018), Matplotlib (Hunter 2007), TOPCAT (Taylor 2005).
