Zeno-anti-Zeno crossover via external fields in a one-dimensional coupled-cavity waveguide

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We have studied a hybrid system of a one-dimensional coupled-cavity waveguide with a two-level system inside, which subject to a external periodical field. Using the extended Hilbert space formalism, the time-dependent Hamiltonian is reduced into an equivalent time-independent one. Via computing the Floquet-Green's function, the Zeno-anti-Zeno crossover is controlled by the driven intensity and frequency, and the detuning between the cavity and the two-level system.

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I. INTRODUCTION

Inspired by modern microfabrication technology in photonic crystals\(^1\)\(^2\)\(^3\)\(^4\), optical microcavities\(^5\), superconducting devices\(^6\)\(^7\)\(^8\), and the realization of the quantum regime in the interaction of atomic-like structures and quantized electromagnetic modes in those systems\(^9\)\(^10\), considerable attention has been attracted to a family of models for coupled arrays of atom-cavity systems, since they offer a fascinating combination of condensed matter physics and quantum optics. Typically, these models are formed by an array of cavities with each cavity containing one or more atoms\(^11\), where photons hop between the cavities. Recently, due to the potential use for building a quantum switch for routing single-photons in quantum network, systems with one or two atoms inside the array of cavities have been extensively studied to revealed the intriguing features of photon transport in low dimensional environments\(^12\)\(^13\). It is found that the switch is formed by the interference between the spontaneous emission from atoms and the propagating modes in the one-dimensional (1D) continuum. Therefore, the search for a controllable switch is to finding a way to change the spontaneous emission of atoms.

The spontaneous emission results from the inevitable interaction of the atomic system with external influences. Decoherence of a quantum system, with a variety of couplings to a reservoir, has been investigated extensively in theory\(^14\)\(^15\). Basically, there are three ways to suppress or modify the rate of quantum transitions in a system. One is to engineer the state of the reservoir, as well as the form of the system-reservoir coupling\(^16\)\(^17\). Obviously, the coupled-cavity waveguide (CRW) meets this condition for its advantage of addressability of individual sites, extremely high controllability, and the great degree of flexibility in their geometric design. Two is to involve the quantum inference between multiple transition pathways of internal states, for example, the electromagnetically induced transparency technique\(^18\)\(^19\). The third way is to applies a succession of short and strong pulses, or measurement to the quantum system\(^20\)\(^21\).

Atoms have been particularly interested for acting as a quantum node in the extended communication networks and scalable computational devices, specially artificial atoms. Atoms in a time-varying field has been investigated long time before. However, the multiphoton resonance and quantum interference have been experimentally demonstrated in a strongly driven artificial atom\(^22\) until recently, which is important to superconducting approach of quantum computation, for instance, decreasing the time required for each gate operation\(^23\)\(^24\). Floquet theory is developed by Floquet centuries ago, it is a theory about the solutions of linear differential equations with periodic coefficients\(^25\). Later, it is applied to the two-level system (TLS) with the time-dependent problem, which is discussed by Autler and Townes\(^26\). And then such kind of the periodic time-dependent problem is reformulated as an equivalent time-independent infinite-dimensional Floquet matrix\(^27\), which is done by introducing the composite Hilbert space of square integrable and time-periodic wave functions. Now Floquet formalism is used as a theoretical tool to investigate time dependent phenomena\(^28\). In this paper, we study a two-level system (e.g., a flux qubit) interacting with a cavity which together with other cavities constructs a one-dimensional (1D) coupled-cavity waveguide (CRW). The CRW is modeled as a linear chain of sites with the nearest-neighbor interaction. Obviously, the dynamic of the TLS is irreversible, i.e., once the TLS is initially in its excited state, it never returns to its initial state spontaneously. Therefore, the CRW with a TLS inside is a typical system with a discrete state coupled to continua of states, which means the TLS is subject to decay. In order to control the decay rate of the TLS, an external periodical field is applied to the TLS through diagonal coupling. It is well known that periodic coherent pulses can either inhibit or accelerate the decay into its reservoir, however, such modification is based on off-diagonal time-dependent couplings between the TLS and the external field. In a atomic experiment, the off-diagonal couplings is caused by a transverse field perpendicular to the polarizing field, which is used to polarize the atomic spins. Then the spins of atoms and photon are aligned. However, diagonal couplings between the driving field and the TLS means that the driving field is parallel to the polar-
The Hamiltonian describing the TLS is can been experimentally realized using superconducting circuits\cite{36,45}. The Hamiltonian describing the TLS is

\[ H_I = g\left(\sigma_- a_0^\dagger + h.c.\right). \]

The operators \( \sigma_+ \) and \( \sigma_- \) are the usual raising and lowering operators of the TLS. The Hamiltonian governing the system is

\[ H = H_A + H_W + H_I. \]

By employing the Fourier transformation

\[ a_j = \frac{1}{\sqrt{N}} \sum_k e^{ikj} a_k, \]

the Hamiltonian \( H \) in the Hilbert space of configuration is now given by in the momentum space

\[ H = \sum_k \varepsilon_k a_k^\dagger a_k + \frac{1}{2} \Omega + A \cos(\nu t)\sigma_z \]

\[ + \frac{g}{\sqrt{N}} \sum_k \left( a_k^\dagger \sigma_+ + h.c.\right), \]

where the dispersion relation

\[ \varepsilon_k = \omega_c - 2\xi \cos k \]

describes an energy band of width \( 4\xi \) (the lattice constant is assumed to be unity). The second-quantization operators \( a_k^\dagger / a_k \) annihilate/create one photon in the \( k \)th mode of the 1D CRW. The periodical boundary condition is used to obtain the Hamiltonian \( H \) in Eq.\( \text{(6)} \).

It can be found that the number of quanta, which is defined by operator \( \mathcal{N} = \sum_k a_k^\dagger a_k + \sigma_z \), is conserved in this system, i.e. \( \mathcal{N} \) commutes with Hamiltonian \( H \). Therefore, if we have one quantum in the initial state, the state vector evolves restrictedly in the one-quantum space. We introduce the states \( |\bar{k}\rangle = a_k^\dagger |0\rangle \) which describes that there are one excitation in the \( k \)th mode of the CRW while the TLS stay in its ground state, and \( |\bar{e}\rangle = |0\rangle \) which denotes that the TLS has been flipped to its excited state while the CRW is in the vacuum state. The orthonormal basis set \( \{ |\bar{k}\rangle, |\bar{e}\rangle \} \) spans the one-quantum space. Therefore Hamiltonian \( H \) is rewritten as

\[ H = \frac{1}{2} \left[ \Omega + A \cos(\nu t)\right] \left( |\bar{e}\rangle \langle \bar{e}| - \sum_k |\bar{k}\rangle \langle \bar{k}| \right) \]

\[ + \sum_k \varepsilon_k |\bar{k}\rangle \langle \bar{k}| + \frac{g}{\sqrt{N}} \sum_k \left( |\bar{k}\rangle \langle \bar{e}| + h.c.\right) \]

in the one-quantum subspace. Hamiltonian \( H \) in Eq.\( \text{(8)} \) describes a single discrete state is coupled to a continuum when the periodical-driven field is absent, which means the discrete state is subject to decay. Consequently, the excited state of the TLS is an unstable state.
III. FLOQUET FORMULATION FOR DR
ATOM INSIDE A 1D WAVEGUIDE.

Since the external driving field is strictly per
time, Hamiltonian in Eq. (3) is a periodic fun-
tion, i.e., \( H(t) = H(t + T) \) with \( T = 2\pi/\nu \) be period. To study our problem, it is necessary to con-
sider solutions of Schrödinger equation with a t-
periodic Hamiltonian. In this section, the theoretic-
that we used is the Floquet representation of qu-
mechanical systems. A brief outline of this met-
been given in appendix.

We now employ the Floquet-state nomencl-
\(|\alpha_n\rangle = |\alpha\rangle \otimes |n\rangle\), where \( n \) is the Fourier in-
from \(-\infty\) to \( \infty \), \( \alpha = \tilde{\epsilon}, \tilde{k} \) is the system index. To deal with Hamiltonian \( H_A \) in Eq. (1). In the quan-
periodical function \( \cos (\nu t) \) is the system index. We first

\[
\cos (\nu t) = \frac{1}{2} \sum_n (|n+1\rangle \langle n| + \text{h.c.})
\]

In the one quantum subspace, Hamiltonian \( H_A \) be separated into two segments \( H_e + H_g \) with

\[
H_e = \sum_n \left( \frac{\Omega}{2} + n\nu \right) |\tilde{e}n\rangle \langle \tilde{e}n| + \sum_n \frac{A}{4} (|\tilde{e}n+1\rangle \langle n| + \text{h.c.})
\]

\[
H_g = \sum_{kn} \left( -\frac{\Omega}{2} + n\nu \right) |\tilde{k}n\rangle \langle \tilde{k}n| - \sum_{kn} \frac{A}{4} (|\tilde{k}n+1\rangle \langle \tilde{k}n| + \text{h.c.})
\]

It can be diagonalized by the following transform

\[
|\tilde{e}n\rangle = \sum_m J_{n-m} (-\chi/2) |\tilde{e}m\rangle \quad (11a)
\]

\[
|\tilde{k}n\rangle = \sum_m J_{n-m} (\chi/2) |\tilde{k}m\rangle \quad (11b)
\]

where \( J_n(x) \) is the Bessel function of the first kind and \( \chi = A/\nu \). In terms of states in Eq. (11) we reduce the solution of a periodic time-dependent Hamiltonian \( H \) to the problem of diagonalizing the time-independent Flo-
quen Hamiltonian \( H_F = H_0 + H_1 \) with

\[
H_0 = \sum_m E^\tilde{e}_m |\tilde{e}m\rangle \langle \tilde{e}m| + \sum_{km} E^\tilde{k}_m |\tilde{k}m\rangle \langle \tilde{k}m|
\]

\[
H_1 = \sum_{kmm'} \frac{gJ_{m-m'}(\chi)}{\sqrt{N}} (|\tilde{k}m\rangle \langle \tilde{e}m'| + \text{h.c.}) . \quad (12b)
\]

Here, \( E^\tilde{e}_m \) (\( E^\tilde{k}_m \)) is the eigenvalue of \( H_e (H_g) \) with

\[
E^\tilde{e}_m = \frac{\Omega}{2} + m\nu \quad (13a)
\]

\[
E^\tilde{k}_m = \varepsilon_k - \frac{\Omega}{2} + m\nu . \quad (13b)
\]

The eigenvectors of Hamiltonian \( H_e \) and \( H_g \) are coupled via the nonzero coupling strength \( g \). Figure ?? shows the energy diagram of Hamiltonian \( H_e \) and \( H_g \) in Eq. (10) when \( 2\xi < \nu \). When the eigenvalues \( E^\tilde{e}_m \) and \( E^\tilde{k}_m \) are close to each other, i.e. \( E^\tilde{e}_m \approx E^\tilde{k}_m \), Floquet Hamiltonian \( H_F \) is reduced to the following form

\[
H_R = H_{R0} + H_{R1} \quad (14)
\]

where

\[
H_{R0} = E^\tilde{e}_m |\tilde{e}\phi_m\rangle \langle \tilde{e}\phi_m| + \sum_k E^\tilde{k}_n |\tilde{k}\phi_n\rangle \langle \tilde{k}\phi_n| \quad (15a)
\]

\[
H_{R1} = \sum_k \frac{gJ_{n-m}(\chi)}{\sqrt{N}} (|\tilde{k}\phi_n\rangle \langle \tilde{e}\phi_m| + \text{h.c.}) \quad (15b)
\]

This approximation seems equivalent to the rotating wave approximation (RWA) which is traditionally used in the field of atomic field to neglect the counter-rotating term of the harmonic driving, however this approximation is different from the RWA, which is valid only for amplitudes of the driving field small compared to the energy difference between the atomic states and breaks down in the strong field.

The above discussion shows that if the TLS is initially in its excited state, it will emit a photon as a result of the interaction with the radiation field of the CRW, photons will gain or loss energy quantum \( h\omega \) due to the periodical modulation. Hence the state of the emitted photon is characterized by the set of energy \( E_q = \Omega - q\omega \) with \( q = n - m \). Once \( E_q \) equals to the energy \( \varepsilon_k \) of the CRW, photons go to the CRW. However, when ratio of the intensity to the frequency of the modulation is equal to the roots of the Bessel function \( J_q(\chi) \), this process is prevented due to the decoupling of the TLS and CRW as one can see in Eq. (15b). When the modulation is absent, the index \( q \) vanishes (i.e. \( q = 0 \)). The behavior of the

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**FIG. 2.** (Color online) Schematic Quasienergy diagram of the Hamiltonian \( H_e \) (a) and \( H_g \) (b) in Eq. (10) under the condition \( 2\xi < \nu \).
emitted photon is determined by whether $\Omega$ is equal to $\varepsilon_k$ or not. Here, we only give a intuitively discussion, more details will present in the next section.

IV. ZENO-ANTI-ZENO CROSSOVER

Time evolution of standard quantum theory assumes two principles: the continuous unitary evolution without measurement, and the projective measurement. Quantum Zeno effect (QZE) or anti-Zeno effect (AZE) is a phenomenon related to projective measurements, which says that repeated observations prolong or shorten of a lifetime of an unstable state. However, slowdown or speedup of the decay of an unstable state is also possible without measurements or observations.

We now investigate the lifetime of the periodically driven TLS in interaction with the CRW when the hopping energy and the driven frequency are chosen such that $2\xi < \nu$. An excited state of the TLS in one quantum subspace of this system will evolves into a superposition of itself and the states in which the atom is unexcited and has released a photon into the CRW. With the half width of the band smaller than the driven frequency, the dynamic of the TLS is major governed by Hamiltonian with $m = 0$. In terms of the Green function, the probability for finding an initial excited TLS still in the excited state reads

$$P_e = |\langle \phi_0 (E - H_R)^{-1} \phi_0 \rangle|^2$$

which is the Fourier-Laplace transform of the amplitude $C_e (t)$ for the TLS in its excited state at arbitrary time. Since the number of the cavities in the CRW is larger, the coupling between the TLS and the CRW is small. Therefore, Hamiltonian $H_{R0}$ can be regarded as the unperturbed part, while Hamiltonian $H_{R1}$ is treated as a perturbation part. Via Dyson’s equation, we compute the Floquet-Green’s functions up to the second nonvanishing order in the coupling strength $g/\sqrt{N}$. Then the amplitude reads

$$C_e (E) = \frac{1}{E - E_0} \left[ 1 + \frac{\tilde{g}_n (E)}{E - E_0} \right]$$

where $g_n (E)$ is the Fourier–Laplace transform of $g_n (t) e^{-i(\omega k - \Omega t + \nu t)}$

$$\tilde{g}_n (E) = \int_0^{\infty} d\tau g_n (\tau) e^{-i(\omega k - \Omega t + \nu t)} e^{\frac{i}{\hbar} E\tau}$$

$$= \sum_k \frac{g^2}{N} \frac{J_n^2 (\chi)}{E - E_n^k}.$$ 

The inverse Fourier–Laplace transform of $g (E)$ yields the memory function

$$g_n (t) = \frac{g^2}{N} \sum_k J_n^2 (\chi) e^{it2\xi \cos k}$$

or reservoir response function, which depends on the quasiexcitation in the $N$ modes of the CRW and characterize the spectrum of the reservoir. Comparing with the memory function $\Phi (t)$ in the absence of modulation, an extra factor $J_n^2 (\chi)$ has been involved. Obviously, by setting $n = 0$ and $A = 0$, $g_n (t)$ reduces to $\Phi (t)$.

The inverse Fourier-Laplace transform of Eq. (17) yields the time evolution of the amplitude $C_e (t)$

$$C_e = e^{iE_0 t} \left[ 1 - \frac{1}{1 - \frac{\tau}{\Omega}} e^{-i(\Delta + n\nu)\tau} \right].$$

where detuning $\Delta = \omega_e - \Omega$. The probability for finding the TLS in its excited state reads

$$P_e \approx \exp (-Rt).$$

Here the decay rate is the overlap of the modulation spectrum $f_n (\omega)$ and the reservoir coupling spectrum $g_n (\omega)$

$$R = 2\pi \int_{-\infty}^{+\infty} d\omega f_n (\omega) g_n (\omega),$$

where $f_n (\omega)$ and $g_n (\omega)$ is the Fourier transform of functions $f_n (\tau)$ and $g_n (\tau)$. The function $f_n (\tau)$ is defined as

$$f_n (\tau) = \frac{1}{1 - \frac{\tau}{\Omega}} e^{-i(\Delta + n\nu)\tau} \Theta (\tau - \theta),$$

where $\Theta (x)$ is the Heaviside unit step function, i.e., $\Theta (x) = 1$ for $x \geq 0$, and $\Theta (x) = 0$ for $x < 0$. Comparing with the form factor induced by the frequently measurement, an extra factor $e^{-i\nu t}$ has been introduced by the modulation. However one can find that when the modulation is absent ($n = 0$), the modulation spectrum reduces to the measurement-induced level-broadening function in Ref. [24].

The expression of functions $f_n (\tau)$ in Eq. (25) and $g_n (\tau)$ in Eq. (19) allows us to calculate the decay rate as

$$R = \frac{tg^2}{N} J_n^2 (\chi) \sum_k \sin c^2 \frac{(\Delta - 2\xi \cos k + n\nu) t}{2}$$

where $\sin c = \sin x/x$. It shows that the decay rate $R$ is determined by: 1) the parameters of the driving field, i.e. the driven intensity $A$ and frequency $\nu$; 2) the detuning $\Delta$ between the cavity and the TLS; 3) the modulation time $t$; 4) the number $N$ of cavities in the 1D waveguide. But only frequency $\nu$, intensity $A$, and the detuning $\Delta$ can be adjusted experimentally.

We first consider the long-time dynamics of the periodically driven TLS. As time $t \to \infty$, the decay rate in Eq. (24) is a sum of Dirac delta functions

$$R = \frac{tg^2}{N} J_n^2 (\chi) \delta (\Delta - 2\xi \cos k + n\nu).$$

Eq. (26) shows that depending on whether the matching condition

$$\Delta - 2\xi \cos k + n\nu = 0$$
is satisfied, the TLS is either (i) frozen to its initial excited state or (ii) the TLS moves to the lower state and stays in it for ever. Case (ii) appears at the matching condition, but case (i) occurs when the transition energy of the TLS is out of resonance with the nth energy band of the CRW. When the number of cavities in the CRW is infinity, the states in the reservoir are continuum, one can replace the sums over $k$ by integrals. Therefore, the properties of CRW are described by the reservoir spectral density

$$\rho(\omega) = N^{-1} \sum_k \delta(\omega + 2\xi \cos k)$$

(27)

$$= \begin{cases} 
0 & 2\xi < |\omega| \\
\infty & 2\xi = |\omega| \\
\frac{2/\pi}{\sqrt{4\xi^2 - \omega^2}} & 2\xi > |\omega|
\end{cases}.$$

And the Fourier transformation of the reservoir response function reads

$$g_n(\omega) = g^2 J_n^2(A/\nu) \rho(\omega).$$

(28)

The behavior of the TLS is determined by whether the transition energy of the TLS is inside or outside the energy band of the CRW. When $\varepsilon_{k=0} \leq \Omega - n\nu \leq \varepsilon_{k=\pi}$, the TLS is in its ground state and the single quantum stays in the modes of the CRW. The decay rate reads $R = 2\pi g_n(\Delta + n\nu)$. It is the extension of the golden rule to frequency. In Fig.3, we plot the decay rate as a function of time $t$ with the coupling strength $g=0.25$ and the number of cavities $N=41$. The width and center of the reservoir coupling spectrum $g_n(\omega)$ read $\Delta_f = t^{-1}$ and $\omega_f = \Delta + n\nu$. The width and center of the reservoir coupling spectrum $g_n(\omega)$ read $\Delta_f = \sqrt{2}\xi g_n(A/\nu)$ and $\omega_f = 0$, respectively. When $\Delta_f \gg \Delta, \omega_f$, the effective decay rate $R \sim 2\pi f_n(\Delta + n\nu)$, which grows with $t$, consequently, the quantum Zeno effect generally occurs. When $\Delta_f \ll |\omega_f - \omega_f|$, the effective decay rate $R \sim 2\pi g_n(\Delta + n\nu) \ll 2\pi g_n(\omega_f)$. As a result, the effective decay rate $R$ is a increasing function of the width $\Delta_f$, which leads to the acceleration of decay, i.e. the anti-Zeno effect.

V. CONCLUSION

In summary, we have considered a CRW with a periodically driven source, the Zeno-Anti-Zeno crossover can be adjusted by the detuning between the TLS and the CRW. For a given detuning $\Delta$, one can switch the occurrence of the QZE and AZE by controlling the driven intensity and frequency. We denote as the width and center of the reservoir coupling spectrum $g_n(\omega)$ read $\Delta_f = t^{-1}$ and $\omega_f = \Delta + n\nu$. The width and center of the reservoir coupling spectrum $g_n(\omega)$ read $\Delta_f = \sqrt{2}\xi g_n(A/\nu)$ and $\omega_f = 0$, respectively. When $\Delta_f \gg \Delta, \omega_f$, the effective decay rate $R \sim 2\pi f_n(\Delta + n\nu)$, which grows with $t$, consequently, the quantum Zeno effect generally occurs. When $\Delta_f \ll |\omega_f - \omega_f|$, the effective decay rate $R \sim 2\pi g_n(\Delta + n\nu) \ll 2\pi g_n(\omega_f)$. As a result, the effective decay rate $R$ is a increasing function of the width $\Delta_f$, which leads to the acceleration of decay, i.e. the anti-Zeno effect.
emission of the TLS is investigated in the one-quantum subspace via the Floquet representation of this system. Via computing the Floquet-Green’s function up to the second nonvanishing order in the coupling strength, the amplitude for the TLS still in its excited state is obtained. It is found that the band structure of the CRW in the temporal space $\mathcal{T}$ of time-periodic functions. The temporal part can be spanned by the orthonormal set of functions $\langle t \mid m \rangle = \exp(i mt)$, where $m = 0, \pm 1, \pm 2, \cdots$ is the Fourier index, and

$$
\langle n \mid m \rangle = \frac{1}{T} \int_0^T \exp[i (m - n) \nu t] \, dt = \delta_{mn}. \quad (A4)
$$

The composite Hilbert space $\mathcal{R}$ is called Floquet or Sambe space$^{[43]}$. In Floquet space, the Floquet Hamiltonian is linear and Hermitian, and the Floquet states provide a complete basis with the scalar product defined as

$$
\langle \Phi_\alpha(t) \mid \Phi_\beta(t) \rangle = \frac{1}{T} \int_0^T \langle \Phi_\alpha(t) \mid \Phi_\beta(t) \rangle \, dt. \quad (A5)
$$

The matrix elements of the time-evolution operator $U_{\alpha\beta}(t, t_0)$ propagates the state $|\alpha\rangle$ at time $t_0$ to the state $|\beta\rangle$ at time $t > t_0$ according to the time-dependent Hamiltonian $H$. In the Floquet representation, $U_{\alpha\beta}(t, t_0)$ is related to the Floquet Hamiltonian

$$
U_{\alpha\beta}(t, t_0) = \sum_n \langle \beta n \mid \exp[-i H_F(t - t_0)] \mid \alpha 0 \rangle e^{i \nu t} \quad (A6)
$$

and is interpreted in Ref.$^{[43]}$ as “the amplitude that a system initially in the Floquet state $|\alpha 0\rangle$ at time $t_0$ evolves to the Floquet state $|\beta n\rangle$ at time $t$ according to the time-independent Floquet Hamiltonian $H_F$, summed over $n$ with weighting factors $\exp[i \nu t]$”.$^{[43]}$. In experiment, the probability to go from the initial state $|\alpha\rangle$ to the final state $|\beta\rangle$ is the time-averaged transition probability between $|\alpha\rangle$ and $|\beta\rangle$

$$
P_{\alpha\beta} = \sum_n \langle \beta n \mid \exp[-i H_F(t - t_0)] \mid \alpha 0 \rangle^2. \quad (A7)
$$

The matrix elements $U_{\alpha\beta}(t, t_0)$ is related to the Floquet-Green’s functions via the Cauchy integral formula $U_{\alpha\beta}(t, t_0) = \oint e^{-iEt} G_{\alpha\beta}dE$ with the Floquet-Green’s functions$^{[46]}$

$$
G_{\alpha\beta} = \sum_n e^{i \nu t} G^{[n]}_{\alpha\beta}. \quad (A8)
$$

The right hand of equation (A8) is the Fourier expansion of $G_{\alpha\beta}$ with the Fourier coefficients

$$
G^{[n]}_{\alpha\beta} = \langle \beta n \mid (E - H_F)^{-1} \mid \alpha 0 \rangle. \quad (A9)
$$

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