Manipulating polarization states of terahertz radiation using metamaterials

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*New Journal of Physics* **14** (2012) 115013 (11pp)
Received 11 June 2012
Published 21 November 2012
Online at [http://www.njp.org/](http://www.njp.org/)
doi:10.1088/1367-2630/14/11/115013

**Abstract.** We present a double-ring-chain metamaterial that enables efficient polarization conversion of terahertz waves. The experimental results and numerical simulations reveal that the linear-to-linear polarization rotation and linear-to-elliptic polarization transformation are simply accomplished by altering the dimensional parameters of the metamaterial unit cells. The polarization state conversion is found to be critically related to the resonant properties of the long bars and the rings in the unit geometries and is well described by the Jones matrix. This approach promises both passive and active polarization conversion of terahertz radiation using planar metamaterials.
1. Introduction

Manipulation of wave polarization states is one of the essential and challenging tasks in the terahertz regime [1]. Over the last few decades, various designs of polarizers and wave plates have been proposed, including sheet polarizers using anisotropic absorption media, prism polarizers, Brewster-angle polarizers, wire-grid polarizers [2], as well as birefringence utilizing paper [3], liquid crystals [4], multi-layer meander liners [5] and chiral effect [6–10]. However, more convenient and flexible approaches are always desirable to completely control the polarization states [11–15]. This has become possible with the advent of metamaterials that promise a variety of fascinating physical phenomena, such as negative refraction [16–19], invisibility cloaking [20, 21], superfocusing [22] and miniaturized antennas [23]. Here, we show that a specially designed planar metamaterial can be employed to manipulate the polarization state of terahertz waves. By altering the geometric parameters of the metamaterial unit cells, we experimentally and numerically demonstrate that the polarization of the incident linearly polarized terahertz waves can be efficiently converted.

2. Results and discussions

The basic building block of the proposed metamaterial is shown in figure 1(a), where each single unit cell of the structure consists of a pair of double-ring chains (DRCs). The DRC array was fabricated on a 200 nm thick aluminum film by the use of conventional photolithography and metallization processes on a 640 µm thick p-type silicon substrate. The array sample has a total 15 mm × 15 mm square area with a microscopy image shown in figure 1(b). Figure 1(c) illustrates the normal resonant transmission behavior of the metamaterial structure upon excitation with a linearly polarized terahertz wave.

The polarization-dependent terahertz responses of various DRC samples were measured by the use of terahertz time-domain spectroscopy (THz-TDS) with the help of four inserted polarizers [24, 25], as shown in figure 1(d). The first polarizer is placed in front of the sample to enable a linearly polarized terahertz wave that propagates through the DRC array. The other three analyzers are placed in front of the photoconductive antenna detector with designated directions of polarization. By rotating the first analyzer ($A_1$) with horizontal ($\theta = 90^\circ$) or vertical polarization ($\theta = 0^\circ$), we then are able to obtain the transmitted terahertz wave of orthogonal components through the sample.
Figure 1. (a) Schematic diagram of a DRC unit cell with line width \( d = 6 \, \mu m \) and other structural dimensions \( a = 38 \, \mu m, b = 40 \, \mu m, r = 8 \, \mu m, l = 100 \, \mu m, \) \( l_1 = l_2, \alpha = 45^\circ, \) and a lattice periodicity \( P = 120 \, \mu m. \) (b) Optical image of a DRC array with 45\(^\circ\) azimuth. (c) Schematic diagram of normal resonant transmission configuration upon excitation with a linearly polarized terahertz wave. (d) Experiment diagram of terahertz transmission measurement of a periodic array of planar metamaterials based on THz-TDS, using analyzer azimuth \( \theta = +90^\circ / 0^\circ \) to obtain two orthogonal components.

In the experiment, the linearly incident terahertz wave is oriented horizontally along the \( x \)-axis, and the transmitted electric fields along the \( x \) (\( \vec{E}_{xx}(\omega) \)) and \( y \) (\( \vec{E}_{xy}(\omega) \)) axes are then measured with respect to \( \theta = 90^\circ \) or \( 0^\circ \), respectively. The amplitude transmissions are further achieved as \( |\vec{I}_{xx}(\omega)| = |\vec{E}_{xx}^S(\omega) / \vec{E}_{xx}^R(\omega)| \) and \( |\vec{I}_{xy}(\omega)| = |\vec{E}_{xy}^S(\omega) / \vec{E}_{xy}^R(\omega)| \), where \( \vec{E}_{xx}^S(\omega), \vec{E}_{xy}^S(\omega), \vec{E}_{xx}^R(\omega) \) and \( \vec{E}_{xy}^R(\omega) \) denote the electric fields through the sample (superscript \( S \)) and the reference (blank Si substrate, superscript \( R \)). The phase difference \( \phi_{\text{diff}} = \phi_{xy} - \phi_{xx} = \arg(\vec{I}_{xy}(\omega)) - \arg(\vec{I}_{xx}(\omega)) \) between two orthogonal polarizations is further extracted from the measured data.

To determine the polarization state of the terahertz wave through measurements, four \textit{Stokes parameters} are introduced as [1]

\[
\begin{align*}
S_0 &= |\vec{I}_{xx}|^2 + |\vec{I}_{xy}|^2, \\
S_1 &= |\vec{I}_{xx}|^2 - |\vec{I}_{xy}|^2, \\
S_2 &= 2|\vec{I}_{xx}||\vec{I}_{xy}| \cos \phi_{\text{diff}}, \\
S_3 &= 2|\vec{I}_{xx}||\vec{I}_{xy}| \sin \phi_{\text{diff}}.
\end{align*}
\]

\text{New Journal of Physics 14 (2012) 115013 (http://www.njp.org/)}
Figure 2. Measured (a), simulated amplitude transmission spectra (b), PCR (c) and simulated emergent polarized light azimuth (d) with $\beta = 10^\circ$, 30$^\circ$ and 45$^\circ$.

where $S_0$ is the relative intensity of the wave in $x$-linear polarization and $y$-linear polarization, $S_1$ depicts the preponderance of $x$-linear polarization over $y$-linear polarization and $S_2$ and $S_3$ represent the phase information. According to equation (1), we have the polarization azimuth $\alpha$ and the ellipticity angle $\chi$ as

$$\tan 2\alpha = \frac{S_2}{S_1},$$

$$\sin 2\chi = \frac{S_3}{S_0}.$$  

2.1. Rotation of the polarization azimuth

The first row of figure 2(a) shows the measured amplitude transmissions $|\tilde{r}_{xx}|$ and $|\tilde{r}_{xy}|$ of the sample with structure azimuth $\beta = 10^\circ$, where a strong resonance is located at 0.66 THz of $|\tilde{r}_{xx}| = 0.89$, $|\tilde{r}_{xy}| = 0.16$ and a phase difference $\varphi_{\text{diff}} = 1^\circ$. The measured characteristic spectral responses of the chosen structures are further supported by a full wave numerical simulation using CST Microwave Studio, as shown in figure 2(b). The unit cell shown in figure 1(a) is used as a model in the simulations with periodic boundary conditions. The silicon substrate is modeled as a lossless dielectric $\varepsilon = 11.78$ and Al has a conductivity $\sigma = 3.72 \times 10^7$ S m$^{-1}$. The simulations reveal good agreement with the experimental results.

According to equations (2) and (3), we obtain the polarization azimuth $\alpha = 10^\circ$ and the ellipticity angle $\chi \approx 0^\circ$ for this sample, which indicates that the linearly polarized incident wave
along the $x$-axis is completely rotated $10^\circ$ transmitting through the metamaterial structure, but remains linearly polarized. The extracted $\alpha$ and $\chi$ from the measured data are also consistent with those in the simulations. In addition, the performance of frequency-dependent polarization conversion can be further described by polarization conversion rate (PCR), which is defined as \cite{14} \[
\text{PCR} = \frac{|\tilde{t}_{xy}(\omega)|}{|\tilde{t}_{xx}(\omega)| + |\tilde{t}_{xy}(\omega)|}.
\] Figure 2(c) shows that for the DRC structure with azimuth $\beta = 10^\circ$, PCR displays an average value of $\sim 0.15$ in the measured frequency region. This linear-to-linear polarization conversion effect can be visualized clearly in figure 2(d).

To gain insight into the linear polarization conversion of the proposed structure, we have further characterized different samples with various $\beta$. When the structure azimuth $\beta$ is increased from $10^\circ$ to $30^\circ$, it is interesting to observe that the polarization azimuth $\alpha$ is also increased, as shown in the second row in figures 2(a)–(d). For the sample of $\beta = 30^\circ$, the amplitude transmissions of two orthogonal components are $|\tilde{t}_{xx}| = 0.69$ and $|\tilde{t}_{xy}| = 0.40$ with $\psi_{\text{diff}} = 1^\circ$ at the resonance frequency, while PCR reaches 0.37. In this case, the measured $\alpha$ and $\chi$ are equal to $30^\circ$ and $0^\circ$, respectively. Further increasing $\beta$ to $45^\circ$, we obtain $|\tilde{t}_{xx}| = 0.47$, $|\tilde{t}_{xy}| = 0.49$ and a small phase difference $\psi_{\text{diff}} = 5^\circ$ with an increased PCR value to 0.51 at resonance frequency. In this case, we have $\alpha = 46^\circ$ and $\chi \approx 0^\circ$. The overall qualitative agreement between the experimental and simulation results is quite good. Figure 2(d) visualizes the polarization conversion processes.

So far, we found that a linear-to-linear polarization conversion of polarization azimuth $\alpha$ could be achieved based on the proposed metamaterial structure with an equal structure azimuth $\beta$, namely $\alpha = \beta$. Hence, an arbitrary linear-to-linear polarization conversion could be achieved through appropriate structure designs. To elucidate the underlying mechanism of these polarization conversion characteristics, the field distributions at the resonance frequency for the sample of structure azimuth $\beta = 30^\circ$, as an example, are given in figure 3. For either the horizontal incidence along the $x$-axis (figure 3(a)) or the vertical incidence along the $y$-axis (figure 3(b)), the resonant fields are both found to be mostly concentrated in the middle of the long bar. Therefore, the long bar in the chosen structure plays a key role in the polarization conversion process. In particular, we notice that the component of the electric field perpendicular to the bar makes a major contribution to the resonance excitation \cite{26}. As a result, if a plane wave of linear polarization is incident on the structure, there is a tendency for the transmitted...
wave to be predominantly polarized perpendicular to the bar. When the structure is further rotated, the polarization of the transmitted wave would be changed with the polarization vector perpendicular to the long bar.

An alternative approach based on the Jones matrix gives an apparent and straightforward description of the polarization conversion of the proposed DRC structure. We consider the incident electric field vector as

\[ \vec{E}_{\text{in}}(\omega) = E_0 e^{i(\kappa z - \omega t)} \hat{e}_x, \] (4)

and after normalization, it is written as \( \vec{E}_{\text{in}} = (1, 0) \), and

\[ \begin{pmatrix} A_{\text{out}} \\ B_{\text{out}} \end{pmatrix} = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \] (5)

where \( \vec{E}_{\text{out}} = (A_{\text{out}}, B_{\text{out}}) \) is the transmitted field, and \( G = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \) is the equivalent Jones matrix of the chosen metamaterial structure. Here, \( G \) can be estimated as

\[ G = \zeta \cdot \begin{pmatrix} \cos^2 \beta & \frac{1}{2} \sin 2\beta \\ \frac{1}{2} \sin 2\beta & \sin^2 \beta \end{pmatrix}, \] (6)

where the coefficient \( \zeta \) is 0.92 for the proposed structures.

2.2. Transformation of the polarization state

In addition to the evidence that the proposed structure could realize the linear-to-linear polarization rotation at resonance frequencies, we found that such a design can also be utilized to achieve transformation of the polarization state. While the other dimensional parameters are fixed as those in the above structure of \( \beta = 45^\circ \), we increase the radius of the rings in the DRCs to \( r = 12 \mu m \) and the distance of two chains to \( a = 34 \mu m \). If the distance of two rings \( b \) is varied, we observe that the transmitted wave is elliptically polarized from linear polarization with different phases.

Figure 4(a) shows the measured amplitude transmissions \( |\tilde{t}_{xx}| \) and \( |\tilde{t}_{xy}| \) of various samples. If we set \( b = 40 \mu m \), then \( |\tilde{t}_{xx}| \) and \( |\tilde{t}_{xy}| \) are 0.815 and 0.324 at the resonance frequency of 0.66 THz, but differ in phase by about 68°. The corresponding polarization azimuth and ellipticity angle are \( \alpha = 10^\circ \) and \( \chi = 20^\circ \), respectively. This indicates that the linearly polarized incident wave is to be left elliptically polarized with the principal axis azimuth of 10° and ellipticity angle of 20°. The measured results are in accordance with the simulations, as shown in figure 4(b). According to PCR, it reaches the peak value of 0.25 at 0.66 THz, as shown in figure 4(c). The emergent states are shown visually in figure 4(d).

The distance between two rings, \( b \), is further increased to 48 and 56 \( \mu m \), and the measured and corresponding simulated transmission spectra are represented in figures 4(a) and (b), respectively. With \( b = 48 \mu m \), \( |\tilde{t}_{xx}| = 0.62 \) and \( |\tilde{t}_{xy}| = 0.44 \) with \( \varphi_{\text{diff}} \approx 89^\circ \) at the resonance frequency, PCR reaches 0.42. The measured polarization azimuth and ellipticity angles are \( \alpha = 1^\circ \) and \( \chi = 35^\circ \), indicating that an elliptically polarized transmitted wave is realized. Further increasing \( b \) to 56 \( \mu m \), we have \( |\tilde{t}_{xx}| = 0.43, |\tilde{t}_{xy}| = 0.46, \varphi_{\text{diff}} \approx 84^\circ \) and \( \text{PCR} = 0.42 \) at resonance frequency, with measured values \( \alpha = -29^\circ \) and \( \chi = 42^\circ \). Since \( \sin 2\chi = 0.995 \sim 1.0 \), the corresponding linearly incident wave is now almost circularly polarized [21].

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Figure 4. Measured (a), simulated amplitude transmission spectra (b), PCR (c) and the simulated emergent polarized states (d) with \( b = 40 \), \( b = 48 \) and \( b = 56 \) \( \mu m \).

Figure 4(d) visualizes the emergent states, where the slight divergence between experiments and simulations is due to systematic errors in the phase measurement.

It can be seen from figure 4 that the ellipticity angle \( \chi \) experiences a significant increase with increasing \( b \). When \( b \) is increased from 40 to 56 \( \mu m \), \( \sin 2\chi \) is enhanced from 0.637 to 0.995, which means that the linearly polarized incident wave is converted into elliptically polarized or even circularly polarized wave after transmitting through the metamaterial structures.

The underlying mechanism in this situation can also be elucidated in the electric field distributions. Taking the sample of \( b = 48 \) \( \mu m \) as an example, fascinating features are observed, as shown in figure 5. Most of the energy is concentrated not only in the middle of the long bar (figure 5(a)), but also in the cross section of DRCs (figure 5(b)), although they have different phases. Therefore, both the long bar and circular rings contribute to the resonance of the structure, and the interactions of the linearly polarized incident wave with both of them thus lead to elliptical polarization conversion of the transmitted wave.

The Jones matrix can also be used to depict the polarization transformation effects. Considering equations (4)–(6), the equivalent Jones matrix \( G \) is given as

\[
G = \begin{pmatrix}
\zeta_1 e^{i\varphi_{\text{diff}}} \cos^2 \beta & \frac{1}{2} \zeta_2 \sin 2\beta \\
\frac{1}{2} \zeta_2 \sin 2\beta & \zeta_1 e^{i\varphi_{\text{diff}}} \sin^2 \beta
\end{pmatrix}.
\]

(7)

The coefficients are \( \zeta_1 = 1.63 \) and \( \zeta_2 = 0.65 \) for the sample of \( b = 40 \) \( \mu m \), \( \zeta_1 = 1.24 \) and \( \zeta_2 = 0.88 \) for the sample of \( b = 48 \) \( \mu m \), and \( \zeta_1 = 0.86 \) and \( \zeta_2 = 0.92 \) for the sample of \( b = 56 \) \( \mu m \), respectively.
2.3. Further systematic experiments

Furthermore, we investigate additional structures of different dimensional parameters, which can realize the linear-to-linear rotation, as well as the linear-to-elliptic transformation at various resonance frequencies. Figures 6(a) and (b) show the measured amplitude transmissions although a series of DRCs with dimensional parameters is listed in Table 1. It is seen that these structures could realize the polarization conversion at different resonance frequencies.

3. Conclusions

We show that a DRC structure enables efficient manipulation of terahertz polarization. The linear-to-linear polarization rotation and the linear-to-elliptic polarization transformation have been experimentally and numerically demonstrated. The polarization of the incident wave after transmitting through the proposed metamaterial structures can be well modulated at resonance frequencies. The resonances of the long bar and the circular rings in the unit cell play a key role in the polarization conversion process. The interactions of the linearly polarized incident wave

**Figure 5.** (a) and (b) Surface electric field distributions of two different phase positions.

**Table 1.** Various DRC structures of different dimensional parameters (µm) with the corresponding resonance frequencies.

| Parameters          | Samples | f (THz) | P  | a  | b  | r  | l  | d  |
|---------------------|---------|---------|----|----|----|----|----|----|
| Linear to linear    | 1       | 0.49    | 150| 46 | 53 | 12 | 130| 6  |
|                     | 2       | 0.63    | 120| 38 | 40 | 8  | 100| 6  |
|                     | 3       | 1.00    | 80 | 30 | 26 | 4  | 60 | 6  |
|                     | 4       | 1.27    | 65 | 26 | 19 | 2  | 45 | 6  |
| Linear to elliptic  | 5       | 0.51    | 150| 40 | 56 | 16 | 130| 6  |
|                     | 6       | 0.66    | 120| 34 | 48 | 12 | 100| 6  |
|                     | 7       | 1.00    | 80 | 22 | 28 | 6  | 60 | 6  |
|                     | 8       | 1.19    | 65 | 19 | 28 | 4  | 55 | 6  |
with both unit elements thus lead to rotation or transformation of the polarization state. Hence, one could manipulate the polarization state simply by altering the resonance behaviors through appropriate geometric designs. This approach offers a new way of implementing polarization conversion of terahertz waves using metamaterials, and such a design may also be extended to higher frequencies.

Acknowledgment

This work was partially supported by the US National Science Foundation, the National Science Foundation of China (grant numbers 61107053, 61007034, 61028011 and 61138001) and the Tianjin Sci-Tech Program (grant number 11JCYBJC25900).
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