Abstract: We propose a new mechanism of spontaneous supersymmetry breaking in non-commutative gauge theories. We find that in $\mathcal{N} = 1$ noncommutative gauge theories both supersymmetry and gauge invariance are dynamically broken. Supersymmetry is broken spontaneously by a Fayet-Iliopoulos D-term which naturally arises in a noncommutative $U(n)$ theory. For a non-chiral matter content the Fayet-Iliopoulos term is not renormalized and its tree-level value can be chosen to be much smaller than the relevant string/noncommutativity scale. In the low energy theory, the noncommutative $U(n)$ gauge symmetry is broken down to a commutative $U(1) \times SU(n)$. This breaking is triggered by the IR/UV mixing and manifests itself at and below the noncommutativity mass scale $M_{NC} \sim \theta^{-1/2}$. In particular, the $U(1)$ degrees of freedom decouple from the $SU(n)$ in the infrared and become arbitrarily weakly coupled, thus playing the role of the hidden sector for supersymmetry breaking.

Keywords: Non-Commutative Geometry, Supersymmetry Breaking, Gauge Symmetry.
1. Introduction

There has been a lot of interest in gauge theories on noncommutative spaces. One of the reasons for this interest is the natural appearance of noncommutativity \([x^\mu, x^\nu] = i\theta^{\mu\nu}\) in the framework of string theory and D-branes [1–5]. Noncommutative gauge theories are also fascinating on their own right mostly due to a mixing between the infrared (IR) and the ultraviolet (UV) degrees of freedom discovered in [6]. This IR/UV mixing does not occur in \(\mathcal{N} = 4\) supersymmetric noncommutative gauge theories [7]. The \(\mathcal{N} = 4\) gauge/supergravity correspondence was analysed in [8, 9]. The low-energy dynamics of noncommutative \(\mathcal{N} = 2\) supersymmetric \(U(N)\) Yang-Mills theories in the Coulomb phase was recently examined in [10, 11], where exact results were derived for the leading terms in the derivative expansion of the Wilsonian effective action. In this case the IR/UV mixing is present in the \(U(1)\) sector [12,13], but does not affect the \(SU(N)\) degrees of freedom. This leads to a dynamical breaking of noncommutative \(U(N)\) gauge symmetry, \(U(N) \to U(1) \times SU(N)\), at momentum scales \(k \leq M_{NC} \sim \theta^{-1/2}\), with the \(U(1)\) degrees of freedom becoming arbitrarily weakly coupled and approaching a free theory, \(g_{U(1)} \to 0\), as \(k \to 0\). The remaining \(SU(N)\) degrees of freedom are strongly coupled in the IR and are described by the ordinary commutative Seiberg-Witten solution. At the same time, in the UV region, \(k \gg M_{NC} \sim \theta^{-1/2}\), the full noncommutative \(U(N)\) gauge invariance is restored.

In this paper we analyse noncommutative \(U(N)\) gauge theories with \(\mathcal{N} = 1\) supersymmetry. Our principal result is the observation that these theories generically exhibit dynamical supersymmetry breaking (DSB), which does not occur in theories with \(\mathcal{N} > 1\).

To illustrate this general point we will concentrate here on \(\mathcal{N} = 1\) noncommutative \(U(N)\) theories with fundamental non-chiral matter content such as SQCD. The standard commutative relative of this theory, \(SU(N)\) SQCD, has non-vanishing Witten index \(\mathcal{I}_{SU(N)} = N\), which is the main topological obstacle for breaking supersymmetry. However, in the noncommutative set-up the gauge group must be \(U(N)\) and the Witten index is zero, \(\mathcal{I}_{U(N)} = \mathcal{I}_{U(1)} \cdot \mathcal{I}_{SU(N)} = 0 \cdot 1 = 0\). Thus, one concludes that supersymmetry can be at least in principle broken spontaneously even in the non-chiral matter context. The simplest scenario of supersymmetry breaking can be immediately realized by introducing the Fayet-Iliopoulos D-term (FI)

\[
L_{FI} = \xi_{FI} \int d^2\theta d^2\bar{\theta} \, \text{tr}_N V ,
\]

where \(V\) is the real \(U(N)\) vector superfield\(^1\), and the trace over the \(N\) by \(N\) matrices selects the \(U(1)\)-component of \(V\). The FI action, \(\int d^4x L_{FI}\), is \(U(N)\) gauge invariant and can be naturally introduced at tree-level in our \(U(N)\) theory. It is well-known that \(\xi_{FI}\) is not renormalized.

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\(^1\)Superfield formulation for noncommutative supersymmetric field theories was introduced in [14–16].
perturbatively or non-perturbatively [17–19] beyond the 1-loop level and the 1-loop correction trivially vanishes for theories with non-chiral matter. Hence, the tree-level value of $\xi_{FI}$ is a modulus of the theory and can be taken to be, for example, $0 < \xi_{FI}/g^2 \ll M_{NC}^2$. In this case we will see below that in our simple SQCD example the supersymmetry breaking scale will be of order $\sqrt{\xi_{FI}}/g \ll M_{NC}$. DSB in other theories, including 3-2-1 models, will be analysed elsewhere [20].

Is noncommutativity really necessary for this type of supersymmetry breaking? One might consider ordinary commutative theories with gauge group $U(1) \times SU(N)$ and introduce a $U(1)$ FI term as in [21–23]. Note however that the $U(1) \times SU(N)$ models can exist only as low-energy effective theories since in the commutative set-up $U(1)$ will necessarily have a Landau pole in the UV. It is well-known that any attempt to grand-unify a $U(1) \times SU(N)$ theory will render the FI term gauge non-invariant. On the other hand any unitary noncommutative gauge theory automatically contains the overall $U(1)$ factor which will be asymptotically free in the UV, and will not contain a Landau pole.

We now summarize the results of this paper. In section 2 we study the supersymmetry breaking pattern in the $\mathcal{N} = 1$ noncommutative SQCD with gauge group $U(N)$ and $N_f < N$ fundamental flavours. We find that in the presence of a FI term supersymmetry is always spontaneously broken. We find that the vacuum of the theory lifts the D-flatness condition, and breaks gauge symmetry down to $U(N - N_f)$. In section 3 we show that in the low-energy theory this gauge group manifests itself as a commutative $U(1) \times SU(N - N_f)$. We find that the leading order terms in the derivative expansion of the Wilsonian effective action read:

$$L_{\text{eff}} = -\frac{1}{4g_1^2(k)} \ F_{\mu\nu}^{U(1)} F_{\mu\nu}^{U(1)} - \frac{1}{4g_{N-N_f}^2(k)} \ F_{\mu\nu}^{SU(N-N_f)} F_{\mu\nu}^{SU(N-N_f)} + \cdots ,$$

where the dots stand for the superpartners of the gauge kinetic terms, terms involving matter fields and higher-derivative corrections. The multiplicative coefficients in front of the gauge kinetic terms in (1.2) define the Wilsonian coupling constants of the corresponding gauge factors. Their dependence on the Wilsonian scale $k$ is displayed in Figure 1. In particular the running of the $U(1)$ has the following asymptotic behaviour:

$$\frac{1}{g_1^2(k)} \rightarrow \frac{3N - N_f}{(4\pi)^2} \log k^2 , \quad \text{as } k^2 \rightarrow \infty , \quad (1.3)$$

$$\frac{1}{g_1^2(k)} \rightarrow \frac{3(N - N_f)}{(4\pi)^2} \log \frac{1}{k^2} , \quad \text{as } k^2 \rightarrow 0 , \quad (1.4)$$

while that for the $SU(N - N_f)$ gauge factor is

$$\frac{1}{g_{N-N_f}^2(k)} \rightarrow \frac{3N - N_f}{(4\pi)^2} \log k^2 , \quad \text{as } k^2 \rightarrow \infty , \quad (1.5)$$

$$\frac{1}{g_{N-N_f}^2(k)} \rightarrow \frac{3(N - N_f)}{(4\pi)^2} \log k^2 , \quad \text{as } k^2 \rightarrow 0 . \quad (1.6)$$
Notice that the two effective coupling constants are identical in the UV and run in opposite
directions in the IR (below $M_{NC}$). We interpret this as having full noncommutative $U(N)$ gauge
symmetry in the UV, which is dynamically broken to $U(1) \times SU(N - N_f)$ in the low-energy
theory. This intriguing breaking of the gauge symmetry is due to the IR/UV mixing which
affects only the $U(1)$ factor [11, 12].

![Running of the couplings as a function of the Wilsonian scale $k$. Here $m$ denotes a typical mass of gauge bosons and matter fields.](image)

**Figure 1:** Running of the couplings as a function of the Wilsonian scale $k$. Here $m$ denotes a typical mass of gauge bosons and matter fields.

Supersymmetry is broken due to the FI term in the $U(1)$ sector, which eventually becomes
arbitrarily weakly coupled in the IR. We thus provide a natural scenario for a gauge-mediated
supersymmetry breaking in which the $U(1)$ factor plays the role of the hidden sector. Both the
hidden sector and the messenger sector are naturally part of the noncommutative $U(N)$ gauge
theory.

## 2. Spontaneous Supersymmetry Breaking in Noncommutative SQCD

Here we will concentrate on $\mathcal{N} = 1$ supersymmetric noncommutative $U(N)$ QCD with $N_f$
flavours $Q$ and $\tilde{Q}$. For concreteness we will consider the case of $N_f \leq N - 1$. The matter
content of the theory is described by $N_f$ chiral fields $Q^I$ in the representation $N$, and $N_f$ chiral fields $\tilde{Q}_{iI}$ in the anti-fundamental representation $\bar{N}$. Here $i = 1, \ldots, N; I = 1, \ldots, N_f$. The physical component fields contained in $Q^I$ (resp. $\tilde{Q}_{iI}$) are the scalars $q^I$ and quarks $\psi^I$ (resp. $\tilde{q}_{iI}$ and $\tilde{\psi}_{iI}$). The noncommutative $U(N)$ gauge symmetry acts as

$$Q^I \rightarrow U^i_j * Q^I, \quad \tilde{Q}_{iI} \rightarrow \tilde{Q}_{jI} * (U^{-1})^j_i ,$$

where $*$ denotes the star-product,

$$(\phi * \chi)(x) \equiv \phi(x)e^{\frac{i}{2}g_{\mu\nu}\partial_{\mu}\partial_{\nu}\chi(x)} .$$

The $U(N)$ gauge multiplet is described by the real vector superfield $V = V^A T^A$, whose physical components are the vector fields $A^A_\mu$ and the gluinos $\lambda^A$, $A = 0, \ldots, N^2 - 1$. The field strength superfield is then given by

$$W_\alpha = -\frac{1}{4} D\dot{e}^{-2V} * D_\alpha e^{2V} .$$

Note that since the star-product only acts in the $x$-space, and does not affect Grassmann superspace coordinates $\theta$ and $\bar{\theta}$, the supercovariant derivatives $D$ and $\bar{D}$ behave as constants with respect to the star-product.

In the limit of massless flavours, the microscopic Lagrangian of SQCD is given by

$$L_{\text{micro}} = \frac{1}{4g^2}(\int d^2 \theta \ W^A * W^A + \text{h.c.}) + \int d^4 \theta \ (Q^I * e^{2V} * Q + \tilde{Q} * e^{-2V} * \tilde{Q}^I) .$$

The anomaly-free global symmetry of the massless theory is

$$G = SU(N_f)_{\text{left}} \times SU(N_f)_{\text{right}} \times U(1)_R .$$

Here $U(1)_R$ denotes the anomaly free combination of the axial symmetry $U(1)_A$ and the R-symmetry $U(1)_X$. Also note that the vector symmetry $U(1)_V$ is not included into (2.5) as it is a subgroup of the $U(N)$ gauge symmetry.

A tree-level superpotential can be added to the theory

$$W_0 = \int d^2 \theta \sum_{I=1}^{N_f} m_I J M^I_{J}$$

which introduces bare masses for flavours. Here we defined the meson superfield

$$M^I_{J} = \tilde{Q}_{iJ} * Q^{iI} ,$$

which is gauge invariant under the noncommutative $U(N)$ (2.1). $M$ transforms in the representation $(N_f, N_f)$ of the chiral group $U(N_f)_{\text{left}} \times U(N_f)_{\text{right}}$. Using the global symmetry (2.5)

\footnote{It appears to be challenging to construct a gauge invariant baryon operator.}
of the massless Lagrangian, one can diagonalize the mass matrix: \( m_I^J = m_J \delta_I^J \). The nonrenormalization theorems for F-terms were shown to hold in the noncommutative case as usual [14]. Hence the tree-level superpotential is not renormalized at any order in perturbation theory.

For \( N_f \leq N - 1 \), in addition to the tree-level superpotential \( W_0 \) one has to include a nonperturbative Affleck-Dine-Seiberg (ADS) superpotential [24] which is generated dynamically exactly in the same manner as in the ordinary commutative case [24–26],

\[
W_1 = (N - N_f) \Lambda^{\frac{3N-N_f}{N-N_f}} (\det \, *M)^{-\frac{1}{N-N_f}},
\]

where the determinant for a \( N_f \times N_f \) matrix \( M \) is defined by

\[
\det \, *M = \epsilon_{i_1 \cdots i_N} M_{i_1}^{j_1} * \cdots * M_{i_N}^{j_N} \epsilon_{i_1 \cdots i_N}.
\]

The origin of this potential is similar to that in the commutative case. The functional form of \( W_1 \) is determined by gauge invariance under the noncommutative \( U(N) \), the flavour symmetry \( SU(N_f)_{\text{left}} \times SU(N_f)_{\text{right}} \) and the fact that it must have R-charge equal to two.

Finally, and most importantly, we add a Fayet-Iliopoulos D-term (1.1). As already mentioned in the Introduction, the FI action is \( U(N) \) gauge invariant and can be naturally introduced at tree-level in our \( U(N) \) theory. The well-known result [17–19] that \( \xi_{FI} \) is not renormalized perturbatively or non-perturbatively beyond the 1-loop level also holds in the noncommutative case. The 1-loop correction is proportional to the sum of the \( U(1) \) matter charges and trivially vanishes in SQCD, and in general for theories with non-chiral matter. This means that a tree level FI term \( \xi_{FI} \) is protected from any quantum corrections, perturbative or nonperturbative, and is a modulus of the theory.

We can now determine the vacua of the theory. The scalar potential is given by

\[
V = V_1 + V_2, \quad V_1 = |f|^2 + |\tilde{f}|^2, \quad V_2 = \frac{1}{2q^2} D^2,
\]

where

\[
f_{iI} = \frac{\partial}{\partial q^{iI}} (W_0(q, \tilde{q}) + W_1(q, \tilde{q})) , \quad \tilde{f}^{iI} = \frac{\partial}{\partial \tilde{q}^{iI}} (W_0(q, \tilde{q}) + W_1(q, \tilde{q})) ,
\]

and

\[
D^A = \sum_{I=1}^{N_f} \left( q^{iI} T^{Ai} q^{iI} - \tilde{q}^{iI} T^{Ai} \tilde{q}^{iI} \right) - \xi_{FI} \delta^A_0.
\]

Including the mass term (2.6), the ADS superpotential (2.8) and the FI D-term (1.1), one obtains

\[
f_{iI} = -\Lambda^{\frac{3N-N_f}{N-N_f}} (\det \, *M)^{-\frac{1}{N-N_f}} \frac{1}{q^{iI}} + m_I \tilde{q}_{iI},
\]
and a similar expression for $\tilde{f}^i I$. In order to find the minimum of $V$, we first apply the global rotations in the colour and flavour spaces and bring $q, \tilde{q}$ to the form,

$$
q^I = v_I \delta^I, \quad \tilde{q}^I = \bar{v}_I \delta^I, \quad 1 \leq i \leq N_f,
q^I = \tilde{q}^I = 0, \quad \text{otherwise.}
$$

(2.14)

It is useful to note that

$$
\sum_{I=1}^{N_f} q^I q^\dagger_j = \begin{cases} 
\delta^{ij} |v_i|^2, & 1 \leq i, j \leq N_f, \\
0, & \text{otherwise},
\end{cases}
$$

(2.15)

and $D^A$ simplifies to

$$
D^A = \sum_{i=1}^{N_f} T^{A_i} (|v_i|^2 - |\tilde{v}_i|^2) - \xi_{fi} \delta^{A0}.
$$

(2.16)

Using the completeness relation for the $U(N)$ generators,

$$
\sum_{A=0}^{N^2-1} T^{A_i} T^{A_k} = \frac{1}{2} \delta_{jk} \delta_{il},
$$

(2.17)

one finds that $V_2$ is minimized for $(v_i, \tilde{v}_i)$ satisfying

$$
|v_i|^2 - |\tilde{v}_i|^2 = \frac{2}{N} \xi_{fi}, \quad i = 1, \ldots, N_f,
$$

(2.18)

or $v_i = \tilde{v}_i = 0$. We reject the latter solution since for vanishing VEVs the ADS superpotential diverges. We also note that the right hand side of (2.18) does not depend on $i$. Obviously, $V$ will attain its global minimum if there are $(v_i, \tilde{v}_i)$ among (2.18) that also satisfy

$$
f^I = \tilde{f}^I = 0.
$$

(2.19)

The solution of (2.19) is easily determined. It is

$$
v_I \tilde{v}_I = r^2 e^{2\pi in/N}, \quad n = 1, \ldots, N,
$$

(2.20)

where

$$
r^2 = \Lambda^2 \prod_{I=1}^{N_f} m_I^{N_f} \Lambda^{N - N_f}.
$$

(2.21)

Writing $v_I = R_I e^{i\theta_I}, v_I = \tilde{R}_I e^{i\tilde{\theta}_I}$, the vacua of the theory are given by the solutions $(R, \tilde{R})$ to the equations

$$
R_I^2 - \tilde{R}_I^2 = \frac{2}{N} \xi_{fi}, \quad R_I \tilde{R}_I = \frac{r^2}{m_I},
$$

(2.22)
and $\theta_I - \tilde{\theta}_I = 2\pi n/N$. Note that at the minimum one can write

$$\frac{1}{\xi_{FI}} D^A = \sqrt{\frac{2}{N}} \text{tr}(T^A P_f) - \delta^{A0},$$

where

$$P_f = \begin{pmatrix} \mathbb{1}_{N_f \times N_f} & 0_{N_f \times (N-N_f)} \\ 0_{(N-N_f) \times N_f} & 0_{(N-N_f) \times (N-N_f)} \end{pmatrix}$$

is the unit matrix in the flavour space. From Eq. (2.23) it is easy to read off which components of $D^A$ are lifted. The minimum value of $V$ can be easily computed,

$$\langle V \rangle = \frac{1}{2} g^2 \xi_{FI}^2 (1 - \frac{N_f}{N}),$$

and it does not depend on $m_I$ and $\Lambda$. Since $\langle V \rangle > 0$ supersymmetry is spontaneously broken. The mass spectrum of the matter field components is affected by the FI term $\xi_{FI}$. In particular, the mass squares of the matter scalars are shifted by $\pm \xi_{FI}/g^2$ while the fermion masses are unaffected in the leading order. This boson-fermion mass non-degeneracy is the consequence of supersymmetry breaking.

Vacuum expectation values of the scalars also break the $U(N)$ gauge symmetry down to $U(N - N_f)$. Then $N_f^2$ of the broken gauge multiplet degrees of freedom acquires masses $g \sqrt{R^2 + \tilde{R}^2}$, and the remaining $2N_f(N - N_f)$ ones get masses $g \sqrt{(R^2 + \tilde{R}^2)/2}$. Here there are no splittings of the superpartner masses. Finally, since the F-flatness (but not the D-flatness) condition is satisfied in the vacuum, the Goldstino is simply given by $\sum_A (D^A) \lambda^A$.

### 3. Dynamical Breaking of Gauge Symmetry

In [11, 13, 27], the background field perturbation theory was applied to noncommutative $U(N)$ gauge theories. The gauge field $A_\mu$ is decomposed into a background field $B_\mu$ and a fluctuating quantum field $N_\mu$,

$$A_\mu = B_\mu + N_\mu. \quad (3.1)$$

The effective action $S_{\text{eff}}[B]$ is obtained by functionally integrating out all quantum fluctuations. The leading term in the derivative expansion of the effective action takes the form

$$S_{\text{eff}}[B] = 2 \int \frac{d^4 k}{(2\pi)^4} B^A_\mu(k) B^B_\nu(-k) \Pi_{\mu\nu}^{AB} + \cdots$$

$$(3.2)$$
Here $\Pi^{AB}_{\mu\nu}$ is the Wilsonian polarization tensor [13], and in supersymmetric theories it has the form

$$\Pi^{AB}_{\mu\nu}(k) = \Pi^{AB}(k^2, \tilde{k}^2)(k^2\delta_{\mu\nu} - k_{\mu}k_{\nu}) .$$  \hspace{1cm} (3.3)

Introducing the matrix of Wilsonian couplings through the relation

$$\begin{bmatrix} \frac{1}{g^2_{\text{eff}}(k)} \end{bmatrix}^{AB} = \frac{\delta^{AB}}{g^2_{\text{micro}}} + 4\Pi^{AB}(k^2, \tilde{k}^2) ,$$  \hspace{1cm} (3.4)

one can write the effective action as

$$S_{\text{eff}} = -\left[ \frac{1}{4g^2_{\text{eff}}} \right]^{AB} \int d^4x F_{\mu\nu}^A F_{\mu\nu}^B + \cdots$$  \hspace{1cm} (3.5)

The one-loop polarization tensor for the $U(N)$ theory was analysed and it was found in [11] that the planar and non-planar contributions are given by

$$[\Pi^{AB}]_{\text{planar}} = N \delta^{AB} \Pi_{\text{planar}} ,$$  \hspace{1cm} (3.6)

$$[\Pi^{AB}]_{np} = N \delta^{A0} \delta^{B0} \Pi_{np} ,$$  \hspace{1cm} (3.6)

where

$$\Pi_{\text{planar}}(k^2) = \frac{2}{(4\pi)^2} \left( \sum_j \alpha_j C(j) C(r) \right) \left[ \frac{2}{\epsilon} - \gamma_E - \int_0^1 dx \log \frac{x(1-x)k^2}{4\pi\mu^2} \right] + O(\epsilon) ,$$  \hspace{1cm} (3.7)

$$\Pi_{np}(k^2, \tilde{k}^2) = -\frac{4C(G)}{(4\pi)^2} \sum_j \alpha_j C(j) \int_0^1 dx K_0(\sqrt{x(1-x)||\tilde{k}||}) .$$  \hspace{1cm} (3.8)

Here $\tilde{k}^\mu = \theta^{\mu\nu}k_\nu$, $j$ is a spin index and $\alpha_j$ is equal to $+1$ ($-1$) for ghost (scalar) fields and to $+1/2$ ($-1/2$) for Weyl fermions (gauge fields). Moreover

$$C(j) \equiv \begin{cases} 0 & \text{for scalars} \\ \frac{1}{2} & \text{for Weyl fermions} \\ 2 & \text{for vectors} \end{cases}$$  \hspace{1cm} (3.9)

and $C(r)$ is the Dynkin index for the representation $r$. The sum in (3.7) is extended to all fields in the theory, including ghosts, whereas the sum in (3.8) excludes fields in the (anti)-fundamental representation, which do not contribute to the non-planar diagrams. For the sake of simplicity the expressions in (3.7) and (3.8) are written for the case when all the fields propagating in the loops are massless. For massive fields these expressions have to be modified accordingly [11, 13]. In particular, when the momentum scale $k$ falls below the mass of a particular field, the contribution of this field should not be included in the summations.

We now examine more closely the Wilsonian polarization function (3.6). Note that the planar piece is UV divergent and has the gauge invariant structure $\delta^{AB}$. These terms, as
well as the other UV divergent terms that arise in 3- and 4-point functions, were examined in [12, 28]. It was found in that [28] these UV divergences can be subtracted in the traditional way using counter terms, thus proving that the theory is 1-loop renormalizable. However, a closer examination of the nonplanar contributions shows that the quadratic part of the effective action is not gauge invariant due to the emergence of the gauge non-invariant structure $\delta A_0 \delta B_0$.

It was also argued in [29] that in the $\mathcal{N} = 4$ gauge theory, the complete (all-orders) effective action is gauge invariant. What we see here is that, as far as the low energy effective action (3.5) is concerned, the noncommutative gauge symmetry is broken and is replaced by the commutative $U(1) \times SU(N)$ gauge symmetry. We stress that this does not imply that the theory is inconsistent. By including all the higher derivative terms, one might hope to recover the noncommutative gauge symmetry. But our results show that in the low energy effective theory the noncommutative gauge symmetry is replaced by the commutative one. Temporarily neglecting the masses of the fields in the loops, the running of the Wilsonian coupling constant is given by (3.4)

$$\left[\frac{1}{g^2_{\text{eff}}(k^2)}\right]^{AB} = \frac{3N - N_f}{(4\pi)^2} \left(\log \frac{k^2}{\Lambda^2} - 2\right) \delta^{AB} + \frac{6N}{(4\pi)^2} \int_0^1 dx K_0(\sqrt{x(1-x)} |k||\tilde{k}|) \delta^{A0}\delta^{B0}.$$  

(3.10)

In the UV regime this directly leads to Eqs. (1.3), (1.5). In the IR regime one has to decouple the $N_f$ flavours of massive matter as well as all the massive gauge multiplets. This amounts to setting in (3.10) first $N_f = 0$ and then replacing $N \rightarrow N - N_f$. This leads to Eqs. (1.4), (1.6). The general running of the Wilsonian couplings is represented in Figure 1. The change of slope is due to a decoupling of massive fields.

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