Mutual Information-based State-Control for Intrinsically Motivated Reinforcement Learning

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Abstract
In reinforcement learning, an agent learns to reach a set of goals by means of an external reward signal. In the natural world, intelligent organisms learn from internal drives, bypassing the need for external signals, which is beneficial for a wide range of tasks. Motivated by this observation, we propose to formulate an intrinsic objective as the mutual information between the goal states and the controllable states. This objective encourages the agent to take control of its environment. Subsequently, we derive a surrogate objective of the proposed reward function, which can be optimized efficiently. Lastly, we evaluate the developed framework in different robotic manipulation and navigation tasks and demonstrate the efficacy of our approach. A video showing experimental results is available at https://youtu.be/CT4CKMWB3z0.

1. Introduction
In psychology (Sansone & Harackiewicz, 2000), behavior is considered intrinsically motivated when it originates from an internal drive. An intrinsic motivation is essential to develop behaviors required for accomplishing a broad range of tasks rather than solving a specific problem guided by an external reward.

Intrinsically motivated reinforcement learning (Chentanez et al., 2005) equips an agent with various internal drives via intrinsic rewards, such as curiosity (Schmidhuber, 1991; Pathak et al., 2017; Burda et al., 2018), diversity (Gregor et al., 2016; Haarnoja et al., 2018; Eysenbach et al., 2019), and empowerment (Klyubin et al., 2005; Salge et al., 2014; Mohamed & Rezende, 2015), which allow the agent to develop meaningful behaviors for solving a wide range of tasks. Mutual information is a core statistical quantity that has many applications in intrinsically motivated reinforcement learning. Still & Precup (2012) calculate the curiosity bonus based on the mutual information between the past and the future states within a time series. Mohamed & Rezende (2015) developed a scalable approach to calculate a common internal drive known as empowerment, which is defined as the channel capacity between the states and the actions. Eysenbach et al. (2019) use the mutual information between skills and states as an intrinsic reward to help the agent to discover a diverse set of skills. In multi-goal reinforcement learning (Schaul et al., 2015; Andrychowicz et al., 2017; Plappert et al., 2018), Warde-Farley et al. (2019) propose to utilize the mutual information between the high-dimensional observation and the goals as the reward signal to help the agent to learn goal-conditioned policies with visual inputs. To discover skills and learn the dynamics of these skills for model-based reinforcement learning, Sharma et al. (2020) recently designed an approach based on maximizing the mutual information between the next state and the agent’s controllable states, conditioned on the current state.

In this paper, we investigate the idea that agent’s “preparedness” to control the states to reach any potential goal would be an effective intrinsic motivation for Reinforcement Learning (RL) agents. We formulate the “preparedness” of control as the mutual information between the goal states and agent’s controllable states. This internal drive extends agent’s controllability from controllable states to goal states and subsequently prepares the agent to reach any goal. It makes learning possible in the absence of hand-engineered reward functions or manually-specified goals. Furthermore, learning to “master” the environment potentially helps the agent to learn in sparse reward settings. We propose a new unsupervised reinforcement learning method called Mutual Information-based State-Control (MISC). During the learning process of the agent, a Mutual Information (MI) estimator is trained to evaluate the mutual information between the goals and agent’s controllable states. Concurrently, the agent is rewarded for maximizing the MI estimation.

This paper contains the following five contributions. First, we introduce Mutual Information-based State-Control for in-
trinsically motivated RL. Secondly, we derive a scalable MI surrogate objective for optimization. Thirdly, we evaluate the developed framework for the robotic tasks of manipulation and navigation and demonstrate the control behavior that agents learned purely via the intrinsic reward. Fourthly, incorporating the intrinsic reward with the task reward, we compare our approach with state-of-the-art methods. Last but not least, we observe that the learned MI estimator from one task can be transferred to a different task and still accelerate learning.

2. Preliminaries

2.1. Environments

We consider multi-goal reinforcement learning tasks, like the robotic simulation scenarios provided by OpenAI Gym (Plappert et al., 2018), where four tasks are used for evaluation, including push, slide, pick & place with the robot arm, and a newly designed navigation task with a mobile robot in Gazebo (Koenig & Howard, 2004), as shown in Figure 1. Accordingly, we define the following terminologies for these scenarios.

2.2. Goal States and Controllable States

The goals $g$ in the manipulation tasks are the desired positions of the object. For the navigation task, the goal for the robot is to navigate to the ball. These goals are sampled from the environment. Note that in this paper we consider that the goals can be represented by states (Andrychowicz et al., 2017), which leads us to the concept of goal states $s^g$. The goal state $s^g$ has the same dimension as the real goal from the environment but represents the achieved states of the object being manipulated or the target ball position in the navigation task. The controllable state $s^c$ is the state that can be directly influenced by the agent (Borsa et al., 2019), such as the state of the robot and its end-effector. The goal states and the controllable states are mutually exclusive. The state split is under the designer’s control. Using states is common in reinforcement learning (Sutton & Barto, 2018).

2.3. Reinforcement Learning Settings

We consider an agent interacting with an environment. We assume the environment is fully observable, including a set of state $S$, a set of action $A$, a distribution of initial states $p(s_0)$, transition probabilities $p(s_{t+1} \mid s_t, a_t)$, a reward function $r: S \times A \rightarrow \mathbb{R}$, and a discount factor $\gamma \in [0, 1]$.

3. Method

We focus on agents learning to control goal states purely by using its observations and actions without supervision. Motivated by the idea that an agent capable of controlling the goal state $s^g$ to obtain high mutual information with its controllable state $s^c$ has been “prepared” to reach any future goal, we formulate the problem of learning without external supervision as one of learning a policy $\pi_\theta(a_t \mid s_t)$ with parameters $\theta$ to maximize intrinsic mutual information rewards, $r = I(S^g; S^c)$. In this section, we formally describe our method, mutual information-based state control.

3.1. Mutual Information Reward Function

Our framework simultaneously learns a policy and an intrinsic reward function by maximizing the mutual information between the goal states and the controllable states. Mathematically, the mutual information between the goal state random variable $S^g$ and the controllable state random variable $S^c$ is represented as Equation (1, 2):

\[
I(S^g; S^c) = H(S^g) - H(S^g \mid S^c) \quad (1)
\]

\[
= D_{KL}(P_{S^g} || P_{S^g} \otimes P_{S^c}) \quad (2)
\]

\[
= \sup_{T:pr_{S^g} \rightarrow P} \mathbb{E}_{pr_{S^g} \otimes P}[T] - \log([\mathbb{E}_{P_{S^g} \otimes P} e^T]) \quad (3)
\]

\[
\geq \sup_{\phi \in \Phi} \mathbb{E}_{P_{S^g} \otimes P}[T_\phi] - \log([\mathbb{E}_{P_{S^g} \otimes P} e^{T_\phi}]) \quad (4)
\]

\[
= I_\phi(S^g; S^c), \quad (5)
\]

where $P_{S^g} \otimes P_{S^c}$ is the joint probability distribution; $P_{S^g} \otimes P_{S^c}$ is the product of the marginal distributions $P_{S^g}$ and $P_{S^c}$; $D_{KL}$ denotes the Kullback-Leibler (KL) divergence. Equation (1) tells us that the agent should maximize the entropy of goal states $H(S^g)$, and concurrently, should minimize the conditional entropy of goal states given the controllable states $H(S^g \mid S^c)$. When the conditional entropy

Figure 1. Fetch robot arm manipulation tasks provided by OpenAI Gym and a navigation task based on the Gazebo simulator: FetchPush, FetchPickAndPlace, FetchSlide, SocialBot-PlayGround.
Figure 2. MISC Algorithm: We update the estimator to better predict the mutual information (MI), and update the agent to control goal states to have higher MI with the controllable states.

$H(S^g \mid S^c)$ is small, it becomes easy to predict the goal states based on the controllable states. For instance, we can see that the robot is controlling an object when it becomes easy to tell what the object state is, based on the robot state. Equation (2) gives us the mutual information in the KL divergence form.

Mutual information is notoriously difficult to compute in real-world settings (Hjelm et al., 2019). Motivated by Belghazi et al. (2018), we use a lower bound to approximate the mutual information quantity $I(S^g; S^c)$. First, we rewrite Equation (2), the KL formulation of the mutual information objective, using the Donsker-Varadhan representation (Donsker & Varadhan, 1975), to Equation (3). The input space $\Omega$ is a compact domain of $\mathbb{R}^d$, i.e., $\Omega \subset \mathbb{R}^d$, and the supremum is taken over all functions $f$ such that the two expectations are finite. Secondly, we lower bound the mutual information in the Donsker-Varadhan representation with the compression lemma in the PAC-Bayes literature (Banerjee, 2006) and then derive Equation (4). The expectations in Equation (4) are estimated by using empirical samples from $P_{S^g,S^c}$ and $P_{S^g} \otimes P_{S^c}$. We can also sample the marginal distributions by shuffling the samples from the joint distribution along the axis (Belghazi et al., 2018). The derived mutual information reward function, $r = I_\phi (S^g; S^c)$, can be trained by gradient ascent. The statistics model $T_\phi$ is parameterized by a deep neural network with parameters $\phi \in \Phi$, which is capable of estimating the mutual information with arbitrary accuracy.

3.2. Efficient Learning State-Control

At the beginning of each episode, the agent takes actions $a_t$ following a partially random policy, such as $\epsilon$-greedy, to explore the environment and collects trajectories into a replay buffer. The trajectory $\tau$ contains a series of states, $\tau = \{s_1, s_2, \ldots, s_T\}$, where $t^*$ is the time horizon of the trajectory. Its random variable is denoted as $\mathcal{T}$. Each state $s_t$ consists of goal states $s_t^g$ and controllable states $s_t^c$. For training the mutual information estimator network, we first randomly sample the trajectory $\tau$ from the replay buffer. Then, the states $s_t^c$ used for calculating the product of marginal distributions are sampled by shuffling the states $s_t^c$ from the joint distribution along the temporal axis $t$ within the trajectory, see Equation (6,7). Note that we calculate the mutual information by using the samples from the same trajectory. If the agent does not alter the goal states during the episode, then the mutual information between the goal states and the controllable states remains zero.

We use back-propagation to optimize the parameter $\phi$ to maximize the MI lower bound, see Equation (7). However, for evaluating the mutual information, this lower bound, Equation (7), is time-consuming to calculate because it needs to process on all the samples from the whole trajectory. To improve its scalability and efficiency, we derive a surrogate objective, Equation (11), which is computed much more efficiently. Each time, to calculate the MI reward for the transition $r = I_\phi (S^g; S^c \mid \mathcal{T}^t)$, the new objective only needs to calculate over a small fraction of the complete trajectory, $\mathcal{T}'$. The trajectory fraction, $\mathcal{T}'$, is defined as adjacent state pairs, $\mathcal{T}' = \{s_t, s_{t+1}\}$, and $\mathcal{T}'$ represents its corresponding random variable. The derivation of the new MI surrogate objective Equation (11) is shown as follows:

\begin{align}
I_\phi (S^g; S^c \mid \mathcal{T}) &= \mathbb{E}_{P_{S^g,S^c} \mid \mathcal{T}}[T_\phi] - \log(\mathbb{E}_{P_{S^g} \mid \mathcal{T} \otimes P_{S^c} \mid \mathcal{T}}[e^{T_\phi}]) \\
&= \mathbb{E}_{P_{S^g,S^c} \mid \mathcal{T}}[T_\phi] - \mathbb{E}_{P_{S^g} \mid \mathcal{T} \otimes P_{S^c} \mid \mathcal{T}}[e^{T_\phi}] \\
&= \mathbb{E}_{P_{\mathcal{T}'}}[\mathbb{E}_{P_{S^g,S^c} \mid \mathcal{T}'}[T_\phi]] - \mathbb{E}_{P_{S^g} \mid \mathcal{T}' \otimes P_{S^c} \mid \mathcal{T}'}[e^{T_\phi}] \\
&= \mathbb{E}_{P_{\mathcal{T}'}}[I_\phi (S^g; S^c \mid \mathcal{T}')] \\
&= \mathbb{E}_{P_{\mathcal{T}'}}[\log(\mathbb{E}_{P_{S^g} \mid \mathcal{T}'} \otimes P_{S^c} \mid \mathcal{T}')[e^{T_\phi}]] \\
&= \mathbb{E}_{P_{\mathcal{T}'}}[I_\phi (S^g; S^c \mid \mathcal{T}')] \\
&= \mathbb{E}_{P_{\mathcal{T}'}}[\log(\mathbb{E}_{P_{S^g} \mid \mathcal{T}'} \otimes P_{S^c} \mid \mathcal{T}')[e^{T_\phi}]],
\end{align}
we use the symbol $\ll$ to denote a monotonically increasing relationship between two variables, for example, $\log(x) \ll x$ means that as the value of $x$ increases, the value of $\log(x)$ also increases and vice versa.

To decompose the lower bound Equation (7) into small parts, we make the following derivations, see Equation (8,9,10). Deriving from Equation (7) to Equation (8), we use the property that $\log(x) \ll x$. Here, the new form, Equation (8), allows us to decompose the MI estimation into the expectations over MI estimations of each trajectory fractions, Equation (9). To be more specific, we move the implicit expectation over trajectory fractions in Equation (8) to the front, and then have Equation (9). The quantity inside the expectation over trajectory fractions is the MI estimation using only each trajectory fraction, see Equation (9). We use the property, $\log(x) \ll x$, again to derive from Equation (9) to Equation (10).

The derived mutual information surrogate objective, Equation (11), brings us two important benefits. First, it enables us to estimate the MI reward for each transition with much less computational time because we only use the trajectory fraction, instead of the trajectory. This approximately reduces the complexity from $O(t^*)$ to $O(1)$ with respect to the trajectory length $t^*$. Secondly, this way of estimating MI also enables us to assign rewards more accurately at the transition level because now we use only the relevant state pair to calculate the transition reward.

Formally, we define the transition MI reward as the MI estimation of each trajectory fraction, $r^t = \{s_t, s_{t+1}\}$, mathematically,

$$r_\phi(a_t, s_t) = \text{clip} \left[ \alpha I_\phi(S^g; S^c \mid T^t), 0, 1 \right],$$

where $\alpha$ is a scale hyper-parameter, which is tuned in conjunction with the learning rate, in case that the estimated MI value, $I_\phi(S^g; S^c \mid T^t)$, is particularly small. The clipping function limits the range of the MI reward between 0 and 1.

### 3.3. Implementation

We combine MISC with both deep deterministic policy gradient (DDPG) (Lillicrap et al., 2016) and soft actor-critic (SAC) (Haarnoja et al., 2018) to learn a policy $\pi_\theta(a \mid s)$ that aims to control the goal states. In comparison to DDPG and SAC, the DDPG method improves the policy in a more “greedy” fashion, while the SAC approach is more conservative, in the sense that SAC incorporates an entropy regularizer $H(A \mid S)$ that maximizes the policy’s entropy over actions.

### 3.4. Complete Algorithm

Overall, the agent is rewarded for controlling the goal states to have higher mutual information with its controllable states, which is considered the “preparedness” to achieve any future goal. We summarize the complete training algorithm in Algorithm 1 and in Figure 2.

### 3.5. MISC Variants with Task Rewards

We propose three ways of using MISC to accelerate learning in addition to the task reward. The first method is using the MISC pretrained policy as the parameter initialization and fine-tuning the agent with rewards. We denote this variant as “MISC-F”, where “-F” stands for fine-tuning. The second variant is to use the MI intrinsic reward to help the agent to explore high mutual information states. We name this method as “MISC-r”, where “-r” stands for reward. The third approach is to use the mutual information quantity from MISC to prioritize trajectories for replay. We name this method as “MISC-p”, where “-p” stands for prioritization.

### 3.6. Skill Discovery with MISC and DIAYN

One of the most relevant works on unsupervised reinforcement learning, DIAYN (Eysenbach et al., 2019), introduces an information-theoretical objective $F_{\text{DIAYN}}$, which learns diverse discriminable skills indexed by the latent variable $Z$, mathematically,

$$F_{\text{DIAYN}} = I(S; Z) + H(A \mid S, Z),$$

$$\geq E_{p_Z} [log q_\phi(z \mid s) - log p(z)] + H(A \mid S, Z).$$

The first term, $I(S; Z)$, in the objective, $F_{\text{DIAYN}}$, is implemented via a skill discriminator, which serves as a variational lower bound of the original objective (Barber & Agakov, 2003; Eysenbach et al., 2019). The skill discriminator assigns high rewards to the agent, if it can predict the skill-options, $Z$, given the states, $S$. The second term, $H(A \mid S, Z)$, is implemented through SAC (Haarnoja et al., 2018) conditioned on skill-options (Szepesvari et al., 2014).

We adapt DIAYN to goal-oriented tasks by replacing the full states, $S$, with goal states, $S^g$, as $I(S^g; Z)$. In comparison, our method MISC proposes to maximize the mutual information between the controllable states and the goal states, $I(S^c; S^g)$. These two methods can be combined as follows:

$$F_{\text{MISC-DIAYN}} = I(S^c; S^g) + I(S^g; Z) + H(A \mid S, Z).$$

The combination of MISC and DIAYN helps the agent to learn control primitives via skill-conditioned policy for hierarchical reinforcement learning (Eysenbach et al., 2019).

### 4. Experiments

To evaluate the proposed methods, we used the robotic manipulation tasks provided by OpenAI Gym and also a newly designed navigation task using Gazebo, see Figure 1 (Brockman et al., 2016; Plappert et al., 2018). First, we analyze the
control behaviors learned purely with the intrinsic reward (refer to the video starting from 0:04 and Figure 8 in Appendix B). Secondly, we show that the pretrained models can be used for improving performance in conjunction with the task rewards. Interestingly, we show that the pretrained MI estimator can be transferred among different tasks and still improve performance. We compared MISC with other methods, including DDPG (Lillicrap et al., 2016), SAC (Haarnoja et al., 2018), DIAYN (Eysenbach et al., 2019), PER (Schaul et al., 2016), VIME (Houthooft et al., 2016), ICM (Pathak et al., 2017), and Empowerment (Mohamed & Rezende, 2015). Thirdly, we show some insights about how the MISC rewards are distributed across a trajectory. The experimental details are shown in Appendix C. Our code is available at https://github.com/ruizhaogit/misc and https://github.com/HorizonRobotics/alf.

4.1. Analysis of the Learned Behaviors

**Question 1.** What behavior does MISC learn?

We tested MISC in the robotic manipulation tasks. The object is randomly placed on the table at the beginning of each episode. During training, the agent only receives the intrinsic MISC reward. In all three environments, the behavior of reaching objects emerges. In the push environments, the agent learns to push the object around on the table. In the slide environment, the agent learns to slide the object into different directions. Perhaps surprisingly, in the pick & place environment, the agent learns to pick up the object from the table without any task reward. All the observations are shown in the uploaded video starting from 0:04.

We implemented MISC with both DDPG and SAC and ran the experiments with 5 different random seeds. To compare DDPG+MISC and SAC+MISC, we ran 20 trials using the learned policy in the pick & place environment with each seed. We observed that, in all the 5 random seed settings, SAC+MISC learns the picking-up behavior, while DDPG+MISC learns to pick up an object in only 1 out of 5 random seed settings. Mostly, the agent learns to push, flip, or grip the object. These observations show that the entropy bonus, $H(A \mid S)$, of SAC can incorporate with MISC and helps the agent to better explore the behavior space.

**Question 2.** Can we use learned behaviors to directly maximize the task reward?

We tested our method in the navigation task, which is based on the Gazebo simulator. The task reward is 1 if the agent reaches the ball, otherwise, the task reward is 0. We combined our method with PPO (Schulman et al., 2017) and compared the performance with ICM (Pathak et al., 2017) and Empowerment (Mohamed & Rezende, 2015). During training, we only used one of the intrinsic rewards such as MISC, ICM, or Empowerment to train the agent. Then, we used the averaged task reward as the evaluation metric.

The experimental results are shown in Figure 3 (left). The y-axis represents the mean task reward and the x-axis denotes the training epochs. From Figure 3 (left), we can see that the proposed method, MISC, has the best performance. Empowerment has the second-best performance. Figure 3 (right) shows that the MISC reward signal $I(S^c, S^g)$ is relatively strong compared to the Empowerment reward signal $I(A, S^g)$. Subsequently, higher mutual information reward encourages the agent to explore more states with higher mutual information. A theoretical connection between Empowerment and MISC is shown in Appendix A. Furthermore, the ICM method does not enable the agent to navigate to the ball because it seeks only novel states and does not control these states. The uploaded video starting from 1:44 shows the learned navigation behaviors.

**Question 3.** How does MISC compare to DIAYN?

We compared MISC, DIAYN and MISC+DIAYN in the pick & place environment. For implementing MISC+DIAYN, we first pre-train the agent with only MISC, and then fine-tune the policy with DIAYN. After pre-training, the MISC-trained agent learns manipulation behaviors such as, reaching, pushing, sliding, and picking up an object. Compared to MISC, the DIAYN-trained agent rarely learns to pick up the object. It mostly pushes or flicks the object with the gripper. However, the combined model, MISC+DIAYN, learns to pick up the object and moves it to different locations, depending on the skill-option. These observations are shown in the video starting from 0:48. In short, MISC helps the agent to learn the DIAYN objective. The agent first learns to control the object with MISC, and then discovers diverse manipulation skills with DIAYN.

4.2. Accelerating Learning with MISC

**Question 4.** How can we use the learned behaviors or the trained MI estimator to accelerate learning?

We investigated three ways of using MISC to accelerate learning in addition to the task reward, see Section 3.5. We combined these three variants with DDPG and SAC and tested them in the multi-goal robotic tasks. The environments, including push, pick & place, and slide, have a set of predefined goals, which are represented as the red dots,
Figure 4. **Mean success rate with standard deviation**: The percentage values after colon (:) represent the best mean success rate during training. The shaded area describes the standard deviation.

As shown in Figure 1, the task for the RL agent is to manipulate the object to the goal positions. We ran all the methods in each environment with 5 different random seeds and report the mean success rate and the standard deviation, as shown in Figure 4. The percentage values alongside the plots are the best mean success rates during training. Each experiment is carried out with 16 CPU-cores.

From Figure 4, we can see that all these three methods, including MISC-f, MISC-p, and MISC-r, accelerate learning in the presence of task rewards. Among these variants, the MISC-r has the best overall improvements. In the push and pick & place tasks, MISC enables the agent to learn in a short period of training time. In the slide tasks, MISC-r also improves the performances by a decent margin.

We also compare our methods with more advanced RL methods. To be more specific, we compare MISC-f against the parameter initialization using DIAYN (Eysenbach et al., 2019); MISC-p against Prioritized Experience Replay (PER), which uses TD-errors for prioritization (Schaul et al., 2016); and MISC-r versus Variational Information Maximizing Exploration (VIME) (Houthooft et al., 2016). The experimental results are shown in Figure 5. From Figure 5 (1\textsuperscript{st} row), we can see that MISC-f enables the agent to learn, while DIAYN does not. In the 2\textsuperscript{nd} row of Figure 5, MISC-r performs better than VIME. This result indicates that the mutual information between states is a crucial quantity for accelerating learning. The mutual information intrinsic rewards boost performance significantly compared to VIME. This observation is consistent with the experimental results of MISC-p and PER, as shown in Figure 5 (3\textsuperscript{rd} row), where the MI-based prioritization framework performs better than the TD-error-based approach, PER. On all tasks, MISC enables the agent to learn the benchmark task more quickly.

### 4.3. Transfer Learning with MISC

**Question 5.** Can the learned MI estimator be transferred to new tasks?

It would be beneficial if the pretrained MI estimator could be transferred to a new task and still improve the performance (Pan et al., 2010; Bengio, 2012). To verify this idea, we directly applied the pretrained MI estimator from the pick & place environment to the push and slide environments, respectively. We denote this transferred method as “MISC-t”, where ‘-t’ stands for transfer. The MISC reward function trained in its corresponding environments is denoted as “MISC-r”. We compared the performances of DDPG baseline, MISC-r, and MISC-t. The results are shown in Figure 6. Perhaps surprisingly, the transferred MISC still improved the performance significantly. Furthermore, as expected, MISC-r performed better than MISC-t.
Figure 5. **Performance comparison**: We compare the MISC variants, including MISC-f, MISC-r, and MISC-p, with DIAYN, VIME, and PER, respectively.

Figure 6. **Transferred MISC**

in both tasks. We can see that the MI estimator can be trained in a task-agnostic (Finn et al., 2017) fashion and later utilized in unseen tasks.

4.4. Insights and More

**Question 6.** How does MISC distribute rewards over a trajectory?

To understand why MISC works and how MISC distributes rewards, we visualize the learned MISC rewards in Figure 7 and in the uploaded video starting from 1:32. From Figure 7, we can observe that the mutual information reward peaks between the fourth and fifth frame, where the robot quickly picks up the cube from the table. Around the peak reward value, the middle range reward values are corresponding to the relatively slow movement of the object and the gripper (see the third, ninth, and tenth frame). When there is
no contact between the gripper and the cube (see the first two frames in Figure 7), or the gripper holds the object still (see the sixth to eighth frames) the intrinsic reward remains nearly zero. From this example, we see that MISC distributes positive intrinsic rewards when the goal state has correlated changes with the controllable state.

**Question 7.** Can MISC help the agent to learn control behaviors when there are no objects involved?

In the navigation task, we define the MISC objective to be the MI between the left wheel and the right wheel. We observe that the agent learns to balance itself and run in a straight line, as shown in the video starting from 2:14.

**Question 8.** What happens if there are multiple objects involved?

When there are multiple objects to control, we define the MISC objective as follows:

\[ F_{\text{MISC}} = \sum_i I(S^c_i; S^g_i) \]

In the case that there is a red and a blue ball on the ground, with MISC, the agent learns to reach both balls and sometimes also learns to use one ball to hit the other ball. The results are shown in the uploaded video starting from 2:29.

4.5. Summary

From these examples, we can see that, with different combinations of the goal states and the controllable states, the agent is able to learn different control behaviors. When there are no specific goal states involved, we can train a skill-conditioned policy corresponding to different combinations of the two sets of states and later use the pretrained policy for the tasks at hand.

5. Related Work

Deep RL led to great successes in various tasks (Ng et al., 2006; Peters & Schaal, 2008; Mnih et al., 2015; Levine et al., 2016; Zhao & Tresp, 2018a,c). However, RL via intrinsic motivation is still a challenging topic. Intrinsic rewards are often used to help the agent learn more efficiently to solve tasks. For example, Jung et al. (2011) and Mohamed & Rezende (2015) use empowerment, which is the channel capacity between states and actions, for intrinsically motivated RL agents. A theoretical connection between MISC and empowerment is shown in Appendix A. VIME (Houthoof et al., 2016) and ICM (Pathak et al., 2017) use curiosity as intrinsic rewards to encourage the agents to explore the environment more thoroughly.

Another line of work on intrinsic motivation for RL is to discover meaningful skills. Variational Intrinsic Control (VIC) (Gregor et al., 2016) proposes an information-theoretical objective (Barber & Agakov, 2003) to jointly maximize the entropy of a set of options while keeping the options distinguishable based on the final states of the trajectory. Recently, Eysenbach et al. (2019) introduced DIAYN, which maximizes the MI between a fixed number of skill-options and the entire states of the trajectory. Eysenbach et al. (2019) show that DIAYN can scale to more complex tasks compared to VIC and provides a handful of low-level primitive skills as the basis for hierarchical RL.

Intrinsic motivation also helps the agent to learn goal-conditioned policies. Warde-Farley et al. (2019) proposed DISCERN, a method to learn a MI objective between the states and goals, which enables the agent to learn to achieve goals in environments with continuous high-dimensional observation spaces. Based on DISCERN, Pong et al. (2019) introduced Skew-fit, which adapts a maximum entropy strategy to sample goals from the replay buffer (Zhao & Tresp, 2019; Zhao et al., 2019) in order to make the agent learn more efficiently in the absence of rewards. More recently, Hartikainen et al. (2019) proposed to automatically learn dynamical distances, which are defined as a measure of the expected number of time steps to reach a given goal that can be used as intrinsic rewards for accelerating learning to achieve goals.

Based on a similar motivation as previous works, we introduce MISC, a method that uses the MI between the goal states and the controllable states as intrinsic rewards. MISC enables the agent to learn control behaviors without supervision. Our method is complementary to the previous works, such as DIAYN, and can be combined with them. The idea of MISC is to encourage the agent to learn to be “prepared” to reach any goal, as one step forward towards mastery of the environment. Inspired by previous works (Schaul et al., 2016; Houthoof et al., 2016; Zhao & Tresp, 2018b; Eysenbach et al., 2019), we additionally demonstrate three variants, including MISC-based fine-tuning, rewarding, and prioritizing mechanisms, to accelerate learning in the case when the task rewards are available.

6. Conclusion

This paper introduces Mutual Information-based State-Control (MISC), an unsupervised RL framework for learning useful control behaviors. The derived efficient mutual information-based theoretical objective encourages the agent to control states without any task reward. MISC enables the agent to self-learn different control behaviors, which are non-trivial, intuitively meaningful, and useful for learning and planning. Additionally, the pretrained policy and the mutual information estimator significantly accelerate learning in the presence of task rewards. We evaluated three MISC-based variants in different environments and demonstrate a substantial improvement in learning efficiency compared to state-of-the-art methods.
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A. Connection to Empowerment

The state $S$ contains the goal state $S^g$ and the controllable state $S^c$. For example, in robotic tasks, the goal state and the controllable state represent the object state and the end-effector state, respectively. The action space is the change of the gripper position and the status of the gripper, such as open or closed. Note that, the agent’s action directly alters the controllable state.

Here, given the assumption that the transform, $S^c = F(A)$, from the action, $A$, to the controllable state, $S^c$, is a smooth and uniquely invertible mapping (Kraskov et al., 2004), then we can prove that the MISC objective, $I(S^c, S^g)$, is equivalent to the empowerment objective, $I(A, S^g)$.

The empowerment objective (Klyubin et al., 2005; Salge et al., 2014; Mohamed & Rezende, 2015) is defined as the channel capacity in information theory, which means the amount of information contained in the action $A$ about the state $S$, mathematically:

$$\mathcal{E} = I(S, A).$$

(12)

Here, we replace the state variable $S$ with goal state $S^g$, we have the empowerment objective as follows,

$$\mathcal{E} = I(S^g, A).$$

(13)

Theorem 1. The MISC objective, $I(S^c, S^g)$, is equivalent to the empowerment objective, $I(A, S^g)$, given the assumption that the transform, $S^c = F(A)$, is a smooth and uniquely invertible mapping:

$$I(S^c, S^g) = I(A, S^g)$$

(14)

where $S^g$, $S^c$, and $A$ denote the goal state, the controllable state, and the action, respectively.

Proof.

$$I(S^c, S^g) = \int \int ds^c ds^g p(s^c, s^g) \log \frac{p(s^c, s^g)}{p(s^c)p(s^g)}$$

(15)

$$= \int \int ds^c ds^g \left| \frac{\partial A}{\partial S^c} \right| p(a, s^g) \log \frac{\| \frac{\partial A}{\partial S^c} \| p(a, s^g)}{\| \frac{\partial A}{\partial S^c} \| p(a)p(s^g)}$$

(16)

$$= \int \int ds^c ds^g J_A(s^c)p(a, s^g) \log \frac{J_A(s^c)p(a, s^g)}{J_A(s^c)p(a)p(s^g)}$$

(17)

$$= \int \int da ds^g p(a, s^g) \log \frac{p(a, s^g)}{p(a)p(s^g)}$$

(18)

$$= I(A, S^g)$$

(19)

B. Learned Control Behaviors without Supervision

The learned control behaviors without supervision are shown in Figure 8

C. Experimental Details

The experiments of the robotic manipulation tasks in this paper use the following hyper-parameters:

- Actor and critic networks: 3 layers with 256 units each and ReLU non-linearities
- Adam optimizer (Kingma & Ba, 2014) with $1 \cdot 10^{-3}$ for training both actor and critic
- Buffer size: $10^6$ transitions
Figure 8. **Learned Control behaviors with MISC**: Without any reward, MISC enables the agent to learn control behaviors, such as reaching, pushing, sliding, and picking up an object. The learned behaviors are shown in the uploaded video starting from 0:04.

- Polyak-averaging coefficient: 0.95
- Action L2 norm coefficient: 1.0
- Observation clipping: $[-200, 200]$  
- Batch size: 256  
- Rollouts per MPI worker: 2  
- Number of MPI workers: 16  
- Cycles per epoch: 50  
- Batches per cycle: 40  
- Test rollouts per epoch: 10  
- Probability of random actions: 0.3  
- Scale of additive Gaussian noise: 0.2  
- Scale of the mutual information reward: 5000

All hyper-parameters are described in greater detail at [https://github.com/ruizhaogit/misc/tree/master/params](https://github.com/ruizhaogit/misc/tree/master/params).