Radiative fluid flow of a nanofluid over an inclined plate with non-uniform surface temperature

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Abstract: In this paper, the convective heat transfer of nanofluid over an inclined plate including the effect of radiation along with variable surface temperature is investigated. Different kinds of nanofluid containing nano particles like Silver, Aluminum oxide and Copper were considered with base fluid water. Solutions are obtained using a crank Nicolson method; an efficient, tri-diagonal, iterative implicit finite difference method. The results are analyzed and presented through Graphs and tables for various parameters. The results are in excellent agreement with existing results.

1. Introduction

Nanofluid, a term introduced by Choi [1], a smart nanotechnology based fluid contains a suspended submicronic nano-scale particles has a main advantage that it enhances the base fluid thermal conductivity experienced by Masuda et al. [2]. Since for, many researchers like Xuan and Li, Xue [3, 4] are interested to do research in nanofluids in the last few years, because of long-established fluids like water, ethylene, mineral oils have a low thermal conductivity compared with nanofluids, many publications are in search of understanding the behavior of nanofluids in industrial and engineering systems, Nuclear reactors, electronics, advanced nuclear systems and also in medicine too. The applications carried out currently and also to be used in the future, which involves nanofluids were given by Wong and Leon [5]. A famous Rayleigh’s problem, dealt with the impulsively started infinite horizontal fluid through which the viscous incompressible fluid flow through got solved by Stokes [6] using mixed explicit implicit finite difference method. Followed by stokes, the main contribution for the fluid dynamics was explored by Prandtl [7] and Seigal [8]. Soundalgekar [9] concerned with the problem of stoke’s for the vertical plate under the effect of MHD. Later Soundalgekar and Ganesan [10] employed implicit finite difference method to explore the natural convection on an isothermal flat plate. Chen et al. [11] Gave an important result of studying the heat transfer in horizontal, vertical and inclined plates with varying wall temperature and heat flux.

In the situation of a transverse uniform magnetic field, the effects of radiation on an unsteady magneto hydrodynamic free convection flow past a semi-infinite vertical porous plate was worked by Abed El-Naby et al. [12]. Then the free convective viscous dissipative Radiative fluid flow over a vertical moving plate along with the internal heat generation and the boundary condition along the convective surface was read by Mohammed Ibrahim and Bhaskar Reddy [13] with chemical reaction. Free convective flow along finite sections of an inclined plate under magnetohydromagnetic effect with non uniform wall temperature was explored by Takhar et al. [14]. Kuznetsov and Nield [15] explored the results for the nanofluid past a vertical plate. Recently Selvarani and Govindarajan [16] studied
about the MHD effects on the nanofluid past a inclined plate.

Upto the knowledge of the author, there is no research undertaken for the free convective nanofluid past an inclined plate with Radiation and non-uniform surface temperature effects. Hence, the present work dealt with that particular study.

2. Mathematical Analysis

The present study considers free convective nanofluid flow along an inclined plate with MHD and heat generation/absorption effects. The schematic diagram and Cartesian coordinate system of the problem are shown in Figure 1 in which the x-axis is taken along the plate and the y-axis is taken normal to the plate. As shown in Figure 1, the inclined angle along the plate is assumed to be \( \phi \). \( T_{\infty} \) is the temperature of both the fluid and the plate at the initial stage, i.e. at time \( t \leq 0 \). Then at time \( t > 0 \), the temperature of the plate is raised to \( T_{x_0} \). The fluid taken into account here is a nanofluid containing nanoparticles like silver, aluminium oxide and copper. Also, the nanofluid considered to be an incompressible fluid.

All of the fluid physical properties are assumed to be constant except the density variations, which includes the buoyancy forces in the momentum equation.

Under Boussinesq approximation, Basic equations are

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
\rho_p \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) =
\begin{align*}
&= g \rho_p \beta_f \alpha_f \left[ T - T_{\infty} \right] \frac{\partial \phi}{\partial y} \\
&+ g \rho_p \beta_f \alpha_f \sin \phi \left[ T - T_{\infty} \right] + \mu \frac{\partial^2 u}{\partial y^2} \\
&= k_f \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_y}{\partial y}
\end{align*}
\]

(1)
Initial and boundary conditions are

\[ \begin{align*}
    t' \leq 0, u &= 0, v = 0, T' = T'_\infty \text{ for all } x \text{ and } y \\
    t' > 0, u &= 0, v = 0, T' = T'_\infty \text{ at } x = 0 \\
    u &= 0, v = 0, T' = T'_\infty + ax^n \text{ at } y = 0 \\
    u &\to 0, v \to 0, T' \to T'_\infty \text{ as } y \to \infty
\end{align*} \]

(4)

Radiation values are simplified as

\[ q_r = \frac{-4\sigma^* \partial T^d}{3k^* \partial y}, \quad T' = 4T'_\infty T - 3T'_\infty \frac{\partial q_r}{\partial y} - \frac{16\sigma^*T'_\infty^3}{3k^*} \frac{\partial^2 T}{\partial y^2} \]

Non - Dimensional quantities used to dimensionalize the basic equations are

\[ \begin{align*}
    X &= \frac{x}{L}, \quad Y = \frac{y}{L} Gr^{\frac{1}{2}}, \quad U = \frac{uL}{v Gr^{\frac{1}{2}}}, \quad V = \frac{vL}{v Gr^{\frac{1}{2}}}, \quad t = \frac{vt}{L Gr^{\frac{1}{2}}}, \quad T' = \frac{T' - T'_\infty}{T'_\infty - T'_\infty}, \\
    Gr &= \frac{g\beta L^3 (T'_w - T'_\infty)}{v^2}, \quad Pr = \frac{v}{\alpha (or) Pr = \frac{v (\rho C_p)}{k_f}}, \quad R = \frac{K * K}{4\sigma^*T'_\infty^3}
\end{align*} \]

(5)

Where \( u, v \) denotes the velocity. \( U, V \) denotes dimensionless velocity. \( Gr \) as Grashof number, \( t' \) denotes time and \( t \) denotes dimensionless time. \( G \) denotes acceleration due to gravity. \( Pr \) denotes Prandtl number. \( R \) denotes radiation parameter. \( q_r \) denotes Radiative heat flux. \( \alpha, \beta, \phi, \mu, v, \rho \) denotes thermal diffusivity, volumetric thermal expansion, angle of inclination, density, dynamoc viscosity, kinematic viscosity, nanoparticle volume fraction.

For nanofluids, the expressions of density \( \rho_{nf} \), thermal expansion coefficient \( (\rho \beta)_{nf} \) and heat capacitance \( (\rho c_p)_{nf} \) are given by

\[ \begin{align*}
    \rho_{nf} &= (1-\phi) \rho_f + \phi \rho_s \\
    (\rho \beta)_{nf} &= (1-\phi)(\rho \beta)_f + \phi(\rho \beta)_s \\
    (\rho c_p)_{nf} &= (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s
\end{align*} \]

(6)
\[ \mu_{nf} = \frac{\mu_f}{(1 - \varphi)^{2.5}} \]

\[ k_{nf} = k_f \left[ \frac{k_i + 2k_f - 2\varphi(k_f - k_i)}{k_i + 2k_f + \varphi(k_f - k_i)} \right] \]

Equations in Non-dimensional form after the simplification will be

\[ \frac{\partial U}{\partial X} + \frac{\partial U}{\partial Y} = 0 \] (7)

\[ \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{1 - \varphi + \varphi \left( \frac{\rho_f}{\rho_i} \right)}{1 - \varphi + \varphi \left( \frac{\rho_f}{\rho_i} \right)} \left( \frac{\rho_f}{\rho_i} \right) f Gr - \frac{1}{2} \cos \phi \frac{\partial}{\partial X} \int T dY + \frac{1 - \varphi + \varphi \left( \frac{\rho_f}{\rho_i} \right)}{1 - \varphi + \varphi \left( \frac{\rho_f}{\rho_i} \right)} T \sin \phi \]

\[ + \frac{1}{(1 - \varphi)^{2.5}} \frac{1}{1 - \varphi + \varphi \left( \frac{\rho_f}{\rho_i} \right)} \frac{\partial^2 U}{\partial Y^2} \] (8)

\[ \frac{\partial T}{\partial t} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \frac{1}{Pr} \frac{1}{k_{nf}} \frac{k_{nf}}{k_f} \frac{\partial^2 T}{\partial Y^2} \]

\[ + \frac{1}{Pr} \frac{1}{1 - \varphi + \varphi \left( \frac{\rho_f}{\rho_i} \right)} \frac{4}{3R} \frac{\partial^2 T}{\partial Y^2} \] (9)

Initial and boundary conditions in non-dimensional form is

\[ \forall \tau \leq 0, U = 0, V = 0, T = 0 \] for all \(X\) and \(Y\)

\[ \forall \tau > 0, U = 0, V = 0, T = X^0 \] at \(X = 0\)

\[ U = 0, V = 0, T = X^0 \] at \(Y = 0\)

\[ U \to 0, Y \to 0, T \to 0 \] as \(Y \to \infty \) (10)

By substituting
\[ E_1 = \frac{1}{(1-\varphi)^{2.5}} \frac{1}{1-\varphi + \varphi \frac{\rho_s}{\rho_f}}, E_2 = \frac{1-\varphi + \varphi \left( \frac{\rho_f}{\rho_f} \right)}{1-\varphi + \varphi \left( \frac{\rho_f}{\rho_f} \right)}, \]
\[ E_3 = \frac{1}{\text{Pr}} \frac{1}{1-\varphi + \varphi \left( \frac{\rho c_p}{\rho_f} \right)} k_{sf}, E_4 = \frac{1}{\text{Pr}} \frac{1}{1-\varphi + \varphi \left( \frac{\rho c_p}{\rho_f} \right)} \frac{4}{3R} \]  
\( (11) \)

In non-Dimensional equations, the basic equations become

\[ \frac{\partial U}{\partial X} + \frac{\partial U}{\partial Y} = 0 \]  
\( (12) \)

\[ \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = E_1 \frac{\partial^2 U}{\partial Y^2} + E_2 \left( Gr^{-2} \cos \phi \frac{\partial}{\partial X} \left[ T dY + T \sin \phi \right] \right) \]  
\( (13) \)

\[ \frac{\partial T}{\partial t} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = (E_3 + E_4) \frac{\partial^2 T}{\partial Y^2} \]  
\( (14) \)

Local Skin Friction and Local Nusselt Numbers are

\[ \tau_x = Gr^4 \frac{1}{(1-\varphi)^{2.5}} \left( \frac{\partial U}{\partial Y} \right)_{Y=0} \]  
\( (15) \)

\[ \tau = Gr^4 \frac{1}{(1-\varphi)^{2.5}} \int_0^1 \left( \frac{\partial U}{\partial Y} \right)_{Y=0} dX \]  
\( (16) \)

\[ Nu_x = Gr^4 \left( - \frac{k_{sf}}{k_f} \right) X \left( \frac{\partial T}{\partial Y} \right)_{Y=0} \]  
\( (17) \)

\[ Nu = Gr^4 \left( - \frac{k_{sf}}{k_f} \right) \int_0^1 \left( \frac{\partial T}{\partial Y} \right)_{Y=0} dX \]  
\( (18) \)

3. Numerical solution and discussion

The set of non-linear coupled equations along with the initial and boundary conditions are solved using a most reliable, unconditionally stable implicit finite difference method of the Crank-Nicolson type. The resulting problem attained from the partial difference equations are solved using a tri-diagonal system which is solved by using the Thomas Algorithm as described by Carnahan et al. [18] and the integrals in the equation are solved using the Newton-Cotes formula.
The results are illustrated graphically to discuss interesting features of the problem.

Figures 2 and 3 dealt with the velocity and temperature profile for the different values of angle $\phi$ and Gr. Increase in the value of angle increases the velocity. Time taken to reach the steady state for lower angle is maximum. Also for Gr, the velocity increases for the value of Gr reduced. Also Aluminum oxide as a nanoparticle shows more effect than copper and silver.

Gr increases implied the increase in temperature. Angle increases result in the decrease in temperature. The value of n increases results in the decrease of velocity and temperature. Velocity and temperature increases as a result of increment in nanoparticle volume fraction. Both the velocity and temperature decrease for the increase in radiation.
4. Conclusions:
In this paper, the free convective flow of a nanofluid past an inclined plate with radiation under the variable surface temperature is investigated. Here when the angel value increases, the velocity increases and it shows the reverse effect in the temperature. Also when both n and R increases, the velocity and temperature decreases. When the nanoparticle volume fraction increases gives result the increase in both the velocity and temperature. On taking into account the Grashof number’s increment resulted in the decrease of velocity and increases in temperature.

References

[1] Choi SU, Eastman JA. 1995 Oct. 1Enhancing thermal conductivity of fluids with nanoparticles. Argonne National Lab., IL (United States)

[2] Masuda H, Ebata A, Teramae K. 1993. Alteration of thermal conductivity and viscosity of liquid by dispersing ultra-fine particles. Dispersion of Al2O3, SiO2 and TiO2 ultra-fine particles. Netsu Bussei (Japan): 7: pp. 227-233

[3] Xuan Y, Li Q. 2000Heat transfer enhancement of nanofluids. International Journal of heat and fluid flow. Feb 29; 21: pp 58-64

[4] Xue QZ. 2003 Feb 10. Model for effective thermal conductivity of nanofluids. Physics letters A.; 307: pp 313-7
[5] Wong KV, De Leon O. 2010. Applications of nanofluids: current and future. 
Advances in Mechanical Engineering. 2:519659

[6] Stokes GG. 1851 Jan. On the effect of the internal friction of fluids on the motion of 
pendulums. Cambridge: Pitt Press

[7] Prandtl L. 1963. The essentials of fluid dynamics. Blackie & Son Limited

[8] Siegel R. 1958 Feb. Transient free convection from a vertical flat plate. Trans. Asme. 
80: 347

[9] Soundalgekar VM. 1977. Free convection effects on the Stokes problem for an 
infinite vertical plate. Journal of Heat Transfer. 99:499-501

[10] Soundalgekar VM, Ganesan P. 1981. Finite-difference analysis of transient free 
convection with mass transfer on an isothermal vertical flat plate. International 
Journal of Engineering Science. 19:757-70

[11] Chen TS, Tien HC, Armaly BF. 1986. Natural convection on horizontal, inclined, and 
vertical plates with variable surface temperature or heat flux. International journal 
of heat and mass transfer. 29:1465-78

[12] Abd El-Naby MA, Elbarbary EM, Abdelazem NY. 2003. Finite difference solution of 
radiation effects on MHD unsteady free-convection flow over vertical plate with 
variable surface temperature. Journal of Applied Mathematics. 2003:65-86.

[13] Ibrahim SM, Bhashar Reddy N. 2013. Similarity solution of heat and mass transfer 
for natural convection over a moving vertical plate with internal heat generation 
and a convective boundary condition in the presence of thermal radiation, viscous 
dissipation, and chemical reaction. ISRN Thermodynamics. 2013

[14] Takhar HS, Chamkha AJ, Nath G. 2003. Effects of non-uniform wall temperature or 
mass transfer in finite sections of an inclined plate on the MHD natural 
convection flow in a temperature stratified high-porosity medium. International 
journal of thermal sciences. 42:829-36

[15] Kuznetsov AV, Nield DA. 2010. Natural convective boundary-layer flow of a 
nanofluid past a vertical plate. International Journal of Thermal Sciences. 
49(2):243-7

[16] Selvarani M and Govindarajan A. 2017 MHD Effects On Natural Convective Flow 
Of A Nanofluid Past An Inclined Plate With Heat Generation/Absorption Proc., Fourth int.,l conf., on Nanosciences and Nanofluids (ICONN - 2017), 9-11 August 
2017, SRM University, Kattankulathur, India

[17] Schlichting H. Boundary-layer theory. 1968

[18] Carnahan B, Luther HA. Applied numerical methods. 1969.