Non-Perturbative Regge Exchange in Meson-Meson Scattering:
An Analysis Based on the Stochastic Vacuum Model

A. I. Karanikas and C. N. Ktorides

University of Athens, Physics Department
Nuclear & Particle Physics Section
Panepistimiopolis, Ilisia GR 15771, Athens, Greece

Abstract

Employing the Worldline casting of the Dosch-Simonov Stochastic Vacuum Model (SVM) for QCD, a simulated meson-meson scattering problem is studied in the Regge kinematical regime. The process is modelled in terms of the ‘helicoidal’ Wilson contour first introduced by Janic and Peschanski in a related study based on an AdS/CFT-type approach. Using lattice supported estimations for the behavior of a two-point, field strength correlation function, as defined in the framework of the SVM, the reggeon slope and intercept are calculated in a semiclassical approximation. The resulting values are in good agreement with accepted phenomenological ones. Going beyond this approximation, the contribution resulting from boundary fluctuations of the Wilson loop contour is also estimated.
1. Introductory Remarks

In addition to confinement, which constitutes a profoundly non-perturbative, problem and whose solution is of quintessential importance, for fully establishing QCD as a fundamental theory for the strong interaction, there do exist specific *dynamical* processes, whose theoretical confrontation also calls for non-perturbative methods of analysis. One such situation arises in connection with the theoretical description of high energy scattering amplitudes for which the soft sector of the theory is involved. From the experimental point of view, one such case arises in connection with Regge kinematics, entering directly the theoretical description of, among others, diffractive and low-x physics processes. In this paper we shall apply, in this specific context, the Field Strength Correlator Method [1], in the framework of the SVM, as has been formulated in the preceding paper (I), *i.e.* in terms of its Worldline casting. In particular, we shall study a simulated case of a meson-meson scattering process whose quark-based description is of the general form

\[(1\bar{1}) + (2\bar{2}) \rightarrow (3\bar{3}) + (4\bar{4})\]

adopting a standard picture, already employed in the QCD literature -see, for example [2] and [3], according to which quark 1 from the first meson and antiquark 2 from the second meson are very heavy, in comparison to the incoming, total energy \(s\) -hence their worldlines are considered to remain intact from the gluon field action. In turn this means that they can be described in the framework of the eikonal approximation. The light pairs \(\bar{1}, 2\) and \(3, 4\), on the other hand, are annihilated and produced in the \(t\)-channel, where the eikonal approximation is not valid and a full treatment is called for their description. In the Worldline framework the process is schematically pictured in space-time by the straight eikonal lines \((1 \rightarrow 3)\) and \((\bar{2} \rightarrow \bar{4})\), describing an intact quark and anti-quark and by the curves \((\bar{1} \rightarrow 2)\) and \((3 \rightarrow 4)\) which correspond, respectively, to the annihilated and produced quark antiquark pairs. The structure of the field theoretical amplitude can be written as follows, see Fig.

\[G(x_4, x_3, x_2, x_1) = \langle \mathcal{A}iS_F(x_4, x_3 \mid A) iS_F(x_3, x_1 \mid A) iS_F(x_1, x_2 \mid A) iS_F(x_2, x_4 \mid A) \rangle . \quad (1)\]

In the above expression \(iS_F\) is the full fermionic propagator which, in the framework of the
Worldline formalism, assumes the form [4]

\[ iS_F(y, x | A) = \int_0^\infty dL e^{-Lm^2} \int_{x(0)=y}^{x(L)=y} Dx(\tau) e^{\frac{-i}{\hbar} \int_0^L d\tau \dot{x}^2} \left[ m - \frac{\gamma \cdot \dot{x}(L)}{2} \right] \Phi^{(1/2)}(L, 0) P \exp \left( i \int_0^L d\tau \dot{x} \cdot A \right), \]  

(2)

where \( \Phi^{(j)} \) is the so-called spin factor (see paper I) for the matter particles entering the system. For us, it means that \( j = \frac{1}{2} \).

Inserting the above formula into Eq. (1) we find

\[ G(x_4, x_3, x_2, x_1) = \prod_{i=1}^{4} \int_0^\infty d\tau_0 \delta[\tau_i - \tau_{i-1}] \int \int Dx(\tau) \delta[x(\tau_3) - x_3] \delta[x(\tau_2) - x_1] \times \]

\[ \times \delta[x(\tau_1) - x_2] \exp \left[ -\frac{1}{4} \int_0^\tau d\tau \dot{x}^2(\tau) \right] (\text{spin structure}) \left<P \exp \left( i \oint_C dx \cdot A \right) \right>_A, \]  

(3)

where the term \text{spin structure} corresponds to the following expression

\[ (\text{spin structure}) = \prod_{i=4}^1 \left[ m_i - \frac{1}{2} \gamma \cdot x(\tau_i) \right] \Phi^{(1/2)}(\tau_i, \tau_{i-1}), \ (\tau_0 \equiv 0). \]  

(4)

In principle, the Wilson loop appearing in Eq. (3) incorporates the dynamics (perturbative, as well as non-perturbative) of the process. In the framework of the Stochastic Vacuum Model (SVM) it assumes the form (see I)

\[ \left<P \exp \left( i \oint_C dx \cdot A \right) \right>_A = \exp \left[ \frac{-1}{2} \int_{S(C)} dS_{\mu\nu}(z) \int_{S(C)} dS_{\lambda\rho}(z') \Delta_{\mu\nu,\lambda\rho}^{(2)}(z - z') \right] \equiv e^{-A[C]}, \]  

(5)

The task to be undertaken in the present paper is to calculate the amplitude (3), using the above expression which, it is reminded, gives the structure of the Wilson loop in framework of the SVM. The particular strategy to be adopted in our relevant effort can be outlined as follows. In Section 2 we perform a semiclassical calculation based on a combined minimization of the action \( A[C] \), see Eq. (17) of I, with respect to the surface \( S[C] \) and of the surface \( S[C] \) with respect to the boundary \( C \). This procedure will allow us to determine the dominant contribution to the Worldline integral (3) in the stochastic limit \( T_g^2 \sqrt{\Delta} \ll 1 \), as determined in I.
As also noted in I, the first order approximation of the action $A[C]$, in terms of the correlation length $T_g$, is essentially the Nambu-Goto string action. The next order corrections give rise to terms which reveal different geometric characteristics of the surface $S[C]$, given its embedding in a 4-dimensional background, such as, e.g., its extrinsic curvature. The presence of these terms, the origin of which is completely different from the known quantum corrections of the Nambu-Goto string, points out the powerful structure of the SVM.

In Section 2 we proceed further to take into account the rigidity of the surface $S[C]$, which, as it will turn out, plays an important role for determining the reggeon intercept. Of lesser importance, but in any case computable, are the corrections related to the fluctuations of the (Wilson) ‘curve’ which forms the boundary of the surface and will be discussed in Section 3. In the same section we shall consider the contribution of the spin-factor. As a point of note it should be mentioned that, in order to compare our results with standard phenomenology, we shall adopt some lattice-based parametrizations of the two-point correlator. The technicalities of this matter will be discussed in an Appendix.

2. Semiclassical Calculation

According to Eq (3), in order to obtain the full amplitude it does not suffice to determine the minimal surface bounded by a, given, specific contour—a problem which, in general, is very hard to solve¹. One needs to proceed even further and sum over all possible boundaries with a weight of the form

$$S[x] = \frac{1}{4} \int_0^{\tau_4} d\tau \dot{x}^2 + A[C].$$  

(6)

The particular method we shall follow for getting an estimate of the scattering amplitude is the minimization of the correlator contribution to the action, i.e. $A[C]$, related to the contour $C$, while at the same time find the minimal surface corresponding to this specific boundary. In this way one obtains, in principle, a result which enables one to determine the dominant contribution to the path integral (3).

¹Indeed, known cases for which solutions have been found are limited and rather simple.
As was proved in I the variation of $A[C]$ under changes of the boundary reads:

$$g_\mu(\tau) = \frac{\delta A[C]}{\delta x_\mu(\tau)} = \dot{x}_\alpha(\tau) \int_{S(C)} dS_{\gamma\delta}(z') \Delta^{(2)}_{\alpha\mu,\gamma\delta}[x(\tau) - z(\tau', s')]$$

(7)

Accordingly, the correlator contributions become stationary for the “classical” trajectory

$$g_\mu[x_{cl}] = 0.$$  

(8)

Using the expansion for the correlator according to Eq(27) of the previous paper, it is easy to see that

$$g_\mu(\tau) = 2\dot{x}_\alpha(\tau)R_{\alpha\mu}[x] - \frac{1}{2}\dot{x}_\alpha(\tau)Q_{\alpha\mu}[x],$$

(9)

with

$$R_{\alpha\mu}[x] = \int_{S(C)} dS_{\alpha\mu}(z')[x(\tau) - z(s', \tau')]$$

(10)

and

$$Q_{\alpha\mu}[x] = \int d\tau' [\dot{x}_\mu(\tau')(x_\alpha(\tau') - x_\alpha(\tau)) - (\mu \leftrightarrow \alpha)] D_1[x(\tau) - x(\tau')]$$

(11)

where the quantities $D$ and $D_1$ are introduced in the framework of the SVM [1]. Their significance is of practical importance, as far as the credibility of the SVM is concerned. Our eventual numerical estimates in this work will use them as basic input. It should be further noted that the above expressions are reparametrization invariant. Also, in the last relation the integration covers the whole range of the $\tau$ variable. Our next step is to specify the minimal surface relevant to the problem under study.

Following Ref [3] the minimal surface bounded by two infinite rods at a relative angle $\theta$, has (in four-dimensional Euclidean space) the shape of a (three-dimensional) helicoid, which is the only surface that can be spanned by straight lines. In the considered process the eikonal lines $1 \rightarrow 3, \bar{2} \rightarrow \bar{4}$, play the role of the ‘rods’, while the angle $\theta$ is connected, via analytic continuation, to the logarithm of the incoming energy.

Given the above specifications, consider the following, helpful, parametrization of the boundary $C$: For $0 < \tau < \tau_1$ we have a straight line segment, $x^{(1)}$, going from the point $x_4$ to the point $x_2$. Introducing, moreover, for convenience the length $2T = |x_4 - x_2|$ and reparametrizing according to $\tau \rightarrow \frac{2T}{\tau_1} \tau - T$, we write

$$x^{(1)}_\mu = (\tau, 0, 0, 0), -T < \tau < T,$$

(12)
with $x^{(1)}_\mu(-T) = x_4$, $x^{(1)}_\mu(T) = x_2$.

The second eikonal line $x^{(3)}(\tau)$, $\tau_2 < \tau < \tau_3$, goes from the point $x_1$ to the point $x_3$ at a relative angle $\theta$ with respect to $x^{(1)}$, while a distance $b$ (impact parameter) separates the two linear contours in a transverse direction. Introducing the distance $2T_1 = |x_3 - x_1|$ and reparametrizing according to

$$\tau \to T_1 \left( \frac{2}{\tau_3 - \tau_2} \tau - \frac{\tau_3 + \tau_2}{\tau_3 - \tau_2} \right)$$

we write

$$x^{(3)}_\mu(\tau) = (-\tau \cos \theta, -\tau \sin \theta, b, 0), \quad -T_1 < \tau < T_1,$$

with $x^{(3)}_\mu(-T_1) = x_1$, $x^{(3)}_\mu(T_1) = x_3$.

In the following we shall assume, just for convenience, that

$$2T = |x_4 - x_2| \sim |x_3 - x_1| = 2T_1.$$

For $\tau_1 < \tau < \tau_2$, we have a helical curve $x^{(2)}_\mu(\tau)$, which joins the points $x_2 = x^{(2)}_\mu(\tau_1)$ and $x_1 = x^{(2)}_\mu(\tau_2)$, representing the exchanged light quarks. Performing, now, the change $s = \frac{b}{\tau_2 - \tau_1}(\tau - \tau_1)$, we write

$$x^{(2)}_\mu(s) = \left( \phi(s) \cos \frac{\theta s}{b}, \phi(s) \sin \frac{\theta s}{b}, s, 0 \right), \quad 0 < s < b.$$  

(14)

The continuity of the boundary requires

$$x^{(1)}_\mu(T) = x^{(2)}_\mu(0) = x_2 \quad \text{and} \quad x^{(2)}_\mu(b) = x^{(3)}_\mu(-T) = x_1,$$

or

$$\phi(0) = \phi(b) = T.$$  

(15)

The final helical curve is $x^{(4)}(\tau)$, which, for $\tau_3 < \tau < \tau_4$, joins the points $x_3 = x^{(4)}(\tau_3)$ and $x_4 = x^{(4)}(\tau_4)$. Making one more, final, parametrization, namely $s = \frac{b}{\tau_4 - \tau_3}(\tau - \tau_3)$ we write

$$x^{(4)}_\mu(s) = \left( -\phi(s) \cos \frac{\theta s}{b}, -\phi(s) \sin \frac{\theta s}{b}, s, 0 \right), \quad 0 < s < b.$$  

(16)

Once again, Eq. (15) takes care of the continuity of the boundary. Now, the minimal surface is bounded by the (four) curves specified by Eqs, (12)-(16) and can be spanned by straight lines parametrized as follows

$$z_\mu(\xi) = \frac{T - \tau}{2T} x^{(4)}_\mu(s) + \frac{T + \tau}{2T} x^{(2)}_\mu(s) = \left( \frac{\tau}{T} \phi(s) \cos \frac{\theta s}{b}, \frac{\tau}{T} \phi(s) \sin \frac{\theta s}{b}, s, 0 \right).$$  

(17)
It can be easily proved that the surface defined by the above equation is minimal, irrespectively of the function \( \phi \):

\[
\partial_\tau \left[ \frac{\hat{z} \cdot z'}{\sqrt{g}} \right] + \partial_s \left[ \frac{\hat{z} \cdot z' z'' - \hat{z}^2 z'}{\sqrt{g}} \right] = 0. \tag{18}
\]

One observes that the minimization of the surface is not enough for the complete specification of the parametrization of the helicoid. Accordingly, we go back to Eq. (8), which determines the boundary that dominates the path integration (3). A first observation is that, due to the antisymmetric nature of \( R_{\alpha\mu} \) and \( Q_{\alpha\mu} \), the function \( g_\mu \) vanishes when \( x_\mu(\tau) \) represents a straight line. Thus Eq. (8) is trivially satisfied for the eikonal sector of the boundary. Non-trivial contributions are coming only from the helices \( x^{(2)}_\mu \) and \( x^{(4)}_\mu \). One can simplify Eq. (9) by computing the leading behavior of the functions \( R_{\alpha\mu} \) and \( Q_{\alpha\mu} \) using the fact that the functions \( D \) and \( D_1 \), as defined in the SVM scheme -and measured in lattice calculations [5]- decay exponentially fast for distances which are large in comparison with the correlation length \( T_g \) [1]. In this connection and upon writing

\[ x(s') = x(s) + (s' - s)\dot{x}(s) + \frac{1}{2} (s' - s)^2 \ddot{x}(s) + \cdots, \]

we find, for the second term in Eq. (9),

\[
\dot{x}_\alpha Q_{\alpha\mu} = \frac{1}{2} \left[ (\dddot{x}^2) \ddot{x}_\mu - (\dddot{x} \cdot \ddot{x}) \dot{x}_\mu \right] \int_0^b ds'(s' - s)^2 D_1 \left[ \frac{x^2 (s' - s)^2}{T_g^2} \right] + \cdots =
\]

\[
= \frac{1}{|\dot{x}|} \left( \dddot{x} - \frac{\dddot{x} \cdot \ddot{x}}{x^2} \right) \frac{1}{T_g \alpha_1} + \cdots, \tag{19}
\]

where\(^2\)

\[
\frac{1}{\alpha_1} \equiv T_g^4 \int_0^\infty dw w^2 D_1(w^2).
\]

Noting that

\[
z_\mu(s, \tau = T) = x^{(2)}_\mu(s), \quad z_\mu(s, \tau = -T) = x^{(4)}_\mu(s)
\]

\[
\partial_\tau z_\mu(s, \tau) = \dot{z}_\mu(s, \tau) = \frac{1}{2T} [x^{(2)}_\mu(s) - x^{(4)}_\mu(s)], \tag{20}
\]

\(^2\)We have omitted terms suppressed by powers of \( T_g^2 \).
the leading behavior of the first term of the rhs of (9) can be easily determined. One finds

\[ \dot{x}_\alpha R_{\alpha\mu} = \frac{1}{2} \ddot{x}^2 \left( \dddot{z}_\mu - \frac{\dot{x} \cdot \dddot{z}}{\dot{x}^2} \dddot{x}_\mu \right) T \langle \int_{-T}^{T} d\tau' \int_{0}^{b} ds' D \left[ \ddot{x}^2 \left( s' - s \right)^2 \right] + \cdots \right) \]

\[ = 2T |\dot{x}| \left( \dddot{z}_\mu - \frac{\dot{x} \cdot \dddot{z}}{\dot{x}^2} \dddot{x}_\mu \right) \frac{\mu^2}{T_g} + \cdots, \quad (21) \]

where we have introduced the parameter

\[ \mu^2 \equiv T_g \int_{0}^{\infty} dw D(w^2). \quad (22) \]

Thus, the function \( g \) takes, to leading order, the form

\[ g_\mu = \frac{1}{|\dot{x}|} T_g \left[ 4T \mu^2 \ddot{x}^2 \left( \dddot{z}_\mu - \frac{\dot{x} \cdot \dddot{z}}{\dot{x}^2} \dddot{x}_\mu \right) - \frac{1}{2\alpha_1} \left( \dddot{x}_\mu - \frac{\dot{x} \cdot \dddot{x}}{\dot{x}^2} \dddot{x}_\mu \right) \right]. \quad (23) \]

Now, we recall from its definition, cf Eq. (7) that the \( g \)-function provides a measure of the change of \( A[C] \) when the Wilson contour is altered as a result of some interaction which reshapes its geometrical profile. In this sense, it contains important information concerning the dynamics of the problem under study. The structure of the \( g \)-function, as it appears in the above equation, is quite general and exhibits its dependence, not only on the boundary but on the minimal surface as well. It is worth noting that this fact is strictly associated with the non-Abelian nature of the theory since the function \( D \) - and consequently \( \mu^2 \) - disappears [1] in an Abelian gauge theory.

Taking into account that for the helicoids parametrization the velocity \( \dot{x} \) has three non-zero components, while \( \dot{x} \) and \( \dot{z} \) have only two, we conclude that Eq. (8) can be satisfied only if

\[ 4T \mu^2 \ddot{x}^2 \dddot{z}_\mu - \frac{1}{2\alpha_1} \dddot{x}_\mu = 0. \quad (24) \]

Inserting in Eq. (24) the helical parametrization one easily finds that the function \( \phi \) must be a constant. Taking, now, into account Eq. (14) we determine this constant to be the length \( T \). It is then very easy to see that this result leads to the conclusions

\[ \dot{x} \cdot \dddot{z} = 0, \quad \dot{x} \cdot \dddot{x} = 0 \quad (25) \]

and

\[ \dddot{x}^2 = -\frac{1}{8\mu^2\alpha_1 b^2}, \quad (26) \]
where
\[ \dot{x}^2 = 1 + \frac{T^2 \theta^2}{b^2}. \]

This equation cannot be satisfied in Euclidean space. In Minkowski space the angle \( \theta \) becomes imaginary \( \theta \rightarrow -i \chi \simeq \ln \left( \frac{s}{2m^2} \right) \) and Eq. (26) has a positive definite solution:
\[ \frac{T^2 \chi^2}{b^2} = 1 - \frac{1}{8 \alpha_1} \mu^2 \frac{b^2}{\mu^2}. \] (27)

The above formula relates the impact parameter \( b \), the logarithm of the incoming energy \( \ln \left( \frac{s}{2m^2} \right) \) and the distance \( T \). These parameters must not be considered as independent from each other in a calculation of the leading behavior of the scattering amplitude. In fact, Eq. (27) indicates that the effective impact parameter must grow with the incoming energy: \( b \sim \ln s \), a conclusion which is in agreement with the landmark result of Cheng and Wu [6].

The preceding analysis, obviously repeats itself for the two helical curves \( x^{(2)} \) and \( x^{(4)} \) and has led us to a specific parametrization for the Wilson loop, which plays the dominant role in the path integration in Eq. (3). We are now in position to determine the leading contribution to the action (6):
\[ S_{cl} = \frac{1}{4} \int_0^{\tau_4} d\tau \dot{x}_{cl}^2(\tau) + A[C]_{cl}. \] (28)

Our first step is to expand the second term of the integrand in powers of \( T^2 g \sqrt{\Delta} \). The first term of such an expansion is the familiar Nambu-Goto string. The next term, which reveals the rich structure of the SVM, is the so-called ‘rigidity term’, representing the extrinsic curvature of a surface embedded in a four-dimensional [7] background:
\[ A[C] = \sigma \int d^2 \xi \sqrt{g} + \frac{1}{\alpha_0} \int d^2 z \xi \sqrt{g g^{ab} \partial_a t_{\mu \nu} \partial_b t_{\mu \nu} + \cdots}, \] (29)
where, in the above expression,
\[ \sigma \equiv \frac{1}{2} T^2 g \int d^2 z D(z^2) \] (30)
enters as the string tension.

The coefficient of the rigidity term reads
\[ \frac{1}{\alpha_0} \equiv \frac{1}{32} T^4 g \int d^2 z z^2 (2 D_1(z^2) - D(z^2)). \] (31)
Terms proportional to $T_6^g$ entering the expansion in Eq. (28) will be considered negligible in our analysis. We have also omitted the term $\int d^2\xi \sqrt{g}R$, since in two dimensions the curvature is a total derivative. Using the helicoids parametrization (17), with $\phi = T$, the Nambu-Goto term in Eq. (29) takes the form

$$\int d^2\xi \sqrt{g} = \int_0^b d\tau \int_0^b ds \sqrt{1 + \frac{\tau^2q^2}{b^2}} = bT \left[ \sqrt{1 + p^2} + \frac{1}{p} \ln \left( \sqrt{1 + p^2} + p \right) \right], \quad (32)$$

where $p = \frac{Tb}{b}$.

To proceed further we analytically continue to Minkowski space where we can use Eq. (27) to determine

$$bT \sqrt{1 + p^2} \to bT \sqrt{1 - \frac{T^2\chi^2}{b^2}} \simeq b \left( 1 - \frac{1}{8\alpha_1\mu^2 b^2} \right)^{1/2} \frac{1}{\sqrt{8\alpha_1\mu^2}} \simeq \frac{b}{\sqrt{8\alpha_1\mu^2}} + O(T_6^3) \quad (33)$$

and

$$\frac{bT}{p} \ln \left( \sqrt{1 + p^2} + p \right) \to \frac{bT}{-iT\chi/b} \ln \left[ \sqrt{1 - \frac{T^2\chi^2}{b^2}} - i \frac{T\chi}{b} \right] \simeq \frac{\pi b^2}{2\chi} - \frac{b}{\sqrt{8\alpha_1\mu^2}} + O(T_6^3). \quad (34)$$

Thus

$$\sigma \int d^2\xi \sqrt{g} \to \frac{\sigma \pi b^2}{2\chi}. \quad (35)$$

In the same framework, the contribution of the rigidity term takes the form

$$\int d^2\xi \sqrt{g} g^{ab} \partial_a t_{\mu\nu} \partial_b t_{\mu\nu} = \int_0^b ds \left[ \frac{1}{\sqrt{1 + \frac{\theta^2}{b^2}}} \left( \frac{\theta^2}{b^2} + \frac{\theta^4}{2b^4} \right) \right] = \frac{3}{2} \ln \left( \sqrt{1 + p^2} + p \right) + \frac{1}{2}p \sqrt{1 + p^2}. \quad (36)$$

It follows that in Minkowski space we have

$$\frac{1}{\alpha_0} \int d^2\xi \sqrt{g} \sigma \simeq \frac{3\pi}{4\alpha_0} \chi. \quad (37)$$

For the full estimation of the classical action, cf Eq. (28), one should also take into account the presence of the classical kinetic term. Non trivial contributions come from the helical curves $x^{(2)}(1 \to 2)$ and $x^{(4)}(3 \to 4)$:

$$\frac{b}{4(\tau_2 - \tau_1)} \int_0^b ds (\dot{x}^{(2)})^2 + \frac{b}{4(\tau_4 - \tau_3)} \int_0^b ds (\dot{x}^{(4)})^2 = \frac{b^2 \dot{x}^2}{4(\tau_2 - \tau_1)} + \frac{b^2 \dot{x}^2}{4(\tau_4 - \tau_3)}. \quad (38)$$
Now we have to take into account that both $\tau_2 - \tau_1$ and $\tau_4 - \tau_3$ must be integrated with weights $e^{-(\tau_2 - \tau_1)m^2}$ and $e^{-(\tau_4 - \tau_3)m^2}$, respectively. These integrals, as it turns out, are dominated by the values $\tau_2 - \tau_1 = \tau_4 - \tau_3 = \frac{b|\dot{x}|}{2m}$, leading to a final kinetic contribution of the form

$$2m b |\dot{x}| = \frac{m}{\sqrt{8\alpha_1 \mu^2}} \chi.$$ \hspace{1cm} (39)$$

Here, $m$ is the mass of the light quarks, thus the result expressed by (34) can be considered negligible.

From the above analysis we conclude that

$$S_{cl} \approx \frac{\sigma \pi b^2}{2 \ln \left(\frac{s}{2m^2}\right)} - \frac{3\pi}{4\alpha_0} \ln \left(\frac{s}{2m^2}\right).$$ \hspace{1cm} (40)$$

Putting aside, for now, the possible corrections to $A[C]$ which arise from fluctuations of the boundary as well as the spin factor contribution, let us consider the result (39) as a whole, except for terms $\sim m$. To obtain the final expression for the scattering amplitude one must integrate over the impact parameter:

$$\int d^2b \exp \left(i\vec{q} \cdot \vec{b} - \frac{\sigma \pi b^2}{2\chi}\right) \propto \exp \left(-\frac{1}{2\pi \sigma} q^2 \chi\right).$$ \hspace{1cm} (41)$$

Combining (40) and (41) we find, for the scattering amplitude, a Regge behavior of the form $s^{\alpha'_R(0)t + \alpha_R(0)}$ with

$$\alpha'_R(0) = \frac{1}{2\pi \sigma} \quad \text{and} \quad \alpha_R(0) = \frac{3\pi}{4\alpha_0}.$$ \hspace{1cm} (42)$$

In the Appendix we present a certain parametrization [8] for the functions $D$ and $D_1$ entering the SVM scheme which give for the string tension the value $\sigma \approx 0.175$ GeV$^2$ and for the coefficient of the rigidity term the value $\frac{1}{\alpha_0} \approx 0.276$. With these numbers we obtain for the Regge slope the value $\alpha'_R(0) \approx 0.91$ GeV$^{-2}$ and for the Reggeon intercept the value $\alpha_R(0) \approx 0.65$, in good agreement with the phenomenological values $\alpha'_R(0) = 0.93$ GeV and $\alpha_R = 0.55$. [3]

3. Boundary Fluctuations and the Role of the Spin Factor

As repeatedly mentioned in our narration, corrections to the amplitude (3), beyond semiclassical ones, are expected to arise from fluctuations of the boundary of the surface on which the two-point correlator ‘lives’. Fluctuations of the surface itself can be taken into
account by higher order correlators. This, in fact, is the big difference which distinguishes the SVM approach, in comparison with Nambu-Goto type approaches.

We begin our related considerations by expanding the action (6) around the helicoid classical solution:

\[
S = S_{cl} - \frac{1}{2} \int_0^{\tau_4} d\tau \, y(\tau) \dddot{x}^{cl}(\tau) + \frac{1}{2} \int_0^{\tau_4} d\tau \int d\tilde{\tau} \, y_\alpha \tau \times \left[ -\frac{1}{2} \delta_{\alpha\beta} \frac{\partial^2}{\partial \tau^2} \delta(\tau - \tilde{\tau}) + \frac{\delta^2 A[C]}{\delta x^{cl}_\alpha(\tau) \delta x^{cl}_\beta(\tilde{\tau})} \right] y_\beta(\tilde{\tau}) + \cdots,
\]

where \( y = x - x^{cl} \).

Using the results of I one can easily determine that

\[
\frac{\delta^2 A[C]}{\delta x^{cl}_\alpha(\tau) \delta x^{cl}_\beta(\tilde{\tau})} = \dot{x}_\mu(\tau) \dot{x}_\nu(\tilde{\tau}) \Delta^{(2)}_{\mu_\alpha,\nu_\beta}[x(\tau) - x(\tilde{\tau})] - \frac{\partial}{\partial \tau} \delta(\tau - \tilde{\tau}) \int dS_{\lambda\rho}(\xi') \Delta^{(2)}_{\alpha_\beta,\lambda\rho}[z(\xi' - x(\tau)) + \dot{x}_\alpha(\tau) \int ds \, \alpha(\tilde{\tau}, s) \dot{z}_\lambda(\tilde{\tau}, s) z'_{\rho}(\tilde{\tau}, s) \epsilon^{\kappa\lambda\rho\mu} \Delta_{\kappa\alpha\mu}[z(\tilde{\tau}, s) - x(\tau)],
\]

where we have written

\[
\delta z_\mu(\tau, s) = \delta_{\mu\nu} \delta(\tau - \tilde{\tau}) a(\tilde{\tau}, s).
\]

The second term on the rhs of Eq.(44) is simply the area derivative which, as we have seen in I, has the general form \( \frac{\delta A[C]}{\delta x^{cl}_\alpha} \sim g_\alpha \dot{x}_\beta - g_\beta \dot{x}_\alpha \). Thus, for the classical solution \( g[x^{cl}] \) it gives zero contribution. It is, furthermore, easy to verify that the third term in (44) also disappears for \( x = x^{cl} \). We, therefore, conclude that

\[
\frac{\delta^2 A[C]}{\delta x^{cl}_\alpha(\tau) \delta x^{cl}_\beta(\tilde{\tau})} = \dot{x}^{cl}_\mu(\tau) \dot{x}^{cl}_\nu(\tilde{\tau}) \Delta^{(2)}_{\mu_\alpha,\nu_\beta}[x^{cl}(\tau) - x(\tilde{\tau})].
\]

Inserting Eq. (45) into Eq. (44) and taking into account that the dominant contribution to the two-point correlator comes from the region \( \tau \approx \tilde{\tau} \) we find

\[
S \approx S_{cl} + \int_0^b ds \, y_\alpha(s) \left[ -\frac{1}{2} \frac{m}{\dot{x}} \delta_{\alpha\beta} \frac{\partial^2}{\partial s^2} + \frac{\lambda^2}{T_g} \omega_{\alpha\beta}(s) \right] y_\beta(s).
\]

Let it be remarked that to arrive at the above relation we have adopted the expansion of the two-point correlator indicated in Eq. (21) of paper I. We have also used the helicoid
parametrization observing, at the same time, that the eikon al lines give null contribution. One further realizes that the contributions of the two helical curves to the linear term in (42) cancel each other, since $\ddot{x}_\mu^{(2)}(s) = -\ddot{x}_\mu^{(4)}(s)$ and $\tau_2 - \tau_1 \simeq \tau_4 - \tau_3 \sim \frac{\hbar |\dot{\gamma}|}{2m}$.

The non-trivial contribution of the helical curves is incorporated in the term
\[ \omega_{\alpha\beta} = \delta_{\alpha\beta} - \frac{1}{2x^2} \left( \ddot{x}_\alpha \ddot{x}_\beta + \dddot{x}_\alpha \dddot{x}_\beta \right), \] the origin of which is the second functional derivative, c.f. (48). The mass parameter $\lambda^2$ in (46) has the same source and is defined as
\[ \lambda^2 \equiv |\dot{x}| T^2_\alpha \int_0^\infty dw \left( D(w^2) + D_1(w^2) + \frac{d}{dw} D_1(w^2) \right). \] (48)

The differential operator entering Eq. (46) has no zero eigenvalues since the “classical” solution is, in fact, the one that annihilates the $g$-function. Accordingly, the calculation of the path integral over $y = x - x^{cl}$ does not require any particular regularization. A straightforward calculation shows that
\[ \text{det} \omega_{\alpha\beta} = \frac{1}{x^2} \left( 1 - \frac{1}{x^2} \right) = \frac{T^2 \theta^2 / b^2}{1 + \theta^2 / b^2}. \] (49)

Thus the matrix $\omega_{\alpha\beta}$ can be diagonalized and the $y$-integral can be easily performed. However, in the limit $m \to 0$ it can be immediately seen that the integration over the boundary fluctuations gives prefactors which are powers of the logarithm of the incoming energy and as far as Regge behavior is concerned, they cannot change the behavior that was determined in the previous section.

The next task is to take up the issue of the spin-field dynamics contribution to the scattering amplitude. As seen in I, a spin factor is associated with each segment of the worldline path. This factor receives contributions from two sources. The first one is
\[ \int d\tau \int dS \cdot \Delta^{(2)}(z - x) \cdot J = \int d\tau \frac{\dot{x}_\mu g_{\nu} - \dot{x}_\nu g_{\mu}}{x^2} \frac{i}{4} [\gamma_\mu, \gamma_\nu] \] (50)
and is obviously zero for the classical trajectory (8)

The other term has the form
\[ S = \frac{1}{8} \int d\tau \int d\tau' J_{\mu\nu} \Delta^{(2)}_{\lambda\mu,\lambda\nu}(x - x') J_{\lambda\rho} = \frac{3}{4} \int d\tau \int d\tau' (D + D_1) + \frac{3}{8} \int d\tau \int d\tau' (x - x')^2 D'_1. \] (51)
In the stochastic limit, within which we are working, the integrals in the above equation give appreciable contribution to (51) only for $|x(\tau) - x(\tau')| \approx |\dot{x}| |\tau - \tau'| \ll T_g$. More concretely, consider the contribution to (51) from the helical curve ($I \rightarrow 2$). A straightforward calculation shows that the analytically continued result is

$$S = -(t_2 - t_1)^2 \frac{M^4}{\chi},$$

(52)

where we have written $\tau = it$ for the time variable and denoted

$$M^4 = \left( \frac{\int \infty \delta D(w)\, dw}{8 \int \infty \delta w^2 D_1(w)\, dw} \right)^{1/2} \int \infty \delta \left( D(w^2) + D_1(w^2) + \frac{1}{2} \frac{d}{dw^2} D_1(w^2) \right).$$

(53)

As has been mentioned in $I$ and discussed in [9], contribution (52) has an interesting role as far as the form of the fermionic propagator is concerned, but it is obvious that it does not alter the basic Regge structure of the amplitude was calculated in the previous section.

The remaining spin structure is summarized in the chain

$$I = \prod_{i=4}^{1} m_i \left[ 1 - \frac{1}{2m_i} \gamma \cdot \dot{x}^{(i)}(\tau_i) \right],$$

(54)

which must be sandwiched between the external spinor wavefunctions representing the incoming and outgoing quarks (in the simple picture wherein the meson wavefunction is just the product of free spinors). The non-trivial dynamics of the process are now incorporated into the fact that the vectors $x^{(i)}_\mu, i = 1, 2, 3, 4$ forming the boundary of the helicoids, are 3-dimensional vectors with $|\dot{x}^{(i)}|^2 = \text{const.}$ For $i = 1, 3$ turns the factor in (54) to the operator $1 - \frac{\gamma \cdot p^{(i)}}{|p^{(i)}|}$.

For $i = 2, 4$ the matrices

$$I_2 = 1 - \frac{b}{2m_1} \frac{\gamma \cdot \dot{x}^{(2)}(b)}{|\dot{x}^{(2)}|} \rightarrow 1 - \frac{\gamma \cdot \dot{x}^{(2)}(b)}{|\dot{x}^{(2)}|}$$

(55)

and

$$I_4 = 1 - \frac{b}{2m_3} \frac{\gamma \cdot \dot{x}^{(4)}(b)}{|\dot{x}^{(4)}|} \rightarrow 1 - \frac{\gamma \cdot \dot{x}^{(4)}(b)}{|\dot{x}^{(4)}|}$$

(56)

are also representations of projection operators. As shown in [3] the matrices (55) and (56) are the direct product of two $2 \times 2$ matrices each of which are by themselves projection operators. Given these observations it becomes a matter of simple algebra to find that the standard kinematics are reproduced.
Appendix

In this Appendix we present a parametrization of the functions $D$ and $D_1$, already referred to in I and used extensively in the present paper. This parametrization is supported by lattice data and is extensively discussed in Ref. [8].

The exact relations defining the functions are

$$D = \frac{\pi^2(N_C^2 - 1)}{2N_C} \frac{G_2}{24} \kappa D_N, \quad D_1 = \frac{\pi^2(N_C^2 - 1)}{2N_C} \frac{G_2}{24} (1 - \kappa) D_{1,N},$$

where $D_N$ and $D_{1,N}$ are functions which determine the structure of the two-point correlators, as defined in [8]. The factor $G_2$ is defined as follows

$$G_2 \equiv \langle 0 | \frac{g^2}{4\pi} F^\alpha_{\mu\nu}(0) F^\alpha_{\mu\nu}(0) | 0 \rangle = \frac{2N_C}{4\pi^4} \Delta^{(2)}_{\mu\nu,\mu\nu}(0).$$

For the above correlator we shall adopt the value given in Ref [8], namely $G_2 = (0.496)^4 GeV^4$. The value of the numerical quantity $\kappa$ in (A.1) is estimated in the same reference to be 0.74.

The ansatz for the function $D_N$ is [8]

$$D_N(z) = \frac{27}{64} \frac{1}{a^2} \int d^4ke^{ik\cdot z} \frac{k^2}{k^2 + \left(\frac{3\pi}{8a}\right)^2}^4,$$

where

$$a \equiv \int_0^\infty dz D_N(z).$$

A simple calculation shows that

$$D_N(z) = wK_1(w) - \frac{1}{4} w^2 K_0(w), \quad w = \frac{3\pi}{8a} | z |,$$
with \( K_\nu \) denoting a Bessel function. The correlation length \( T_g \), introduced in I and frequently used in the text, can be deduced from Eq. (A.5):

\[
T_g = \frac{8a}{3\pi} \quad (A.6)
\]

The estimated value of \( a \) is

\[
a \approx 0.35 \text{ fm or } T_g \approx 0.297 \text{ fm}. \quad (A.7)
\]

With the help of ansatz (A.3) and using (A.7) one can determine the string tension:

\[
\sigma = \frac{1}{2} T_g^2 \int d^2 w D(w) = \frac{1}{2} T_g^2 \frac{\pi^2 (N_C^2 - 1)}{24} \kappa \int d^2 w \left[ w K_1(w) - \frac{1}{4} w^2 K_0(w) \right], \quad (A.8)
\]

or

\[
\sigma = \frac{1}{2} T_g^2 \frac{\pi^2 (2N_C^2 - 1)}{24} \kappa = a^2 G_2 \kappa \pi \frac{32}{81} \approx 0.175 \text{ GeV}^2 \quad (A.9)
\]

The ansatz for the function \( D_{1,N} \) is deduced from the equation [8]

\[
\left( 4 + z \mu \frac{\partial}{\partial z} \right) D_{1,N}(z) = 4 D_N(z) \quad (A.10)
\]

or

\[
D_{1,N}(z) = \frac{1}{z^4} \int_0^z dw [4w^4 K_1(w) - w^5 K_0(w)] \quad (A.11)
\]

The coefficient of the rigidity term entering Eq. (31) can now be calculated:

\[
\frac{1}{\alpha_0} = \frac{1}{32} T_g^4 \int d^2 w w^2 [2D_1(w) - D(w)] = \frac{1}{32} T_g^4 \frac{\pi^2 (N_C^2 - 1)}{24} \kappa \int d^2 w w^2 [2(1 - \kappa) D_{1,N}(w) - \kappa D_N(w)] = \frac{1}{32} T_g^4 \frac{\pi^2 (N_C^2 - 1)}{24} 2(1 - \kappa)32 \pi \approx 0.276. \quad (A.12)
\]

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