Remark on the Consistent Gauge Anomaly in Supersymmetric Theories

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ABSTRACT

We present a direct field theoretical calculation of the consistent gauge anomaly in the superfield formalism, on the basis of a definition of the effective action through the covariant gauge current. The scheme is conceptually and technically simple and the gauge covariance in intermediate steps reduces calculational labors considerably. The resultant superfield anomaly, being proportional to the anomaly $d^{abc} = \text{tr} T^a \{T^b, T^c\}$, is minimal without supplementing any counterterms. Our anomaly coincides with the anomaly obtained by Marinković as the solution of the Wess-Zumino consistency condition.

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The consistent gauge anomaly [1] might be conceptually more natural than the covariant gauge anomaly [2], as it is defined as gauge non-invariance of the effective action of the chiral fermion [3,4]. The consistent anomaly is important because it provides information on the Wess-Zumino Lagrangian [3]. To find an explicit form of the consistent anomaly, one may appeal to the algebraic-geometrical technique [5-8] or directly perform a field theoretical calculation with, say, the Pauli-Villars regularization. As is well-known, however, both approaches can be cumbersome for a theory in higher dimensions, or for a theory with a complicated gauge transformation. In particular, in the field theoretical calculation, gauge non-invariant normal terms (fake anomalies) generally appear. Then, to extract the intrinsic anomaly, one has to find suitable local counterterms to eliminate these normal terms.

The covariant gauge anomaly, on the other hand, has the quite restricted possible form due to the gauge covariance. The necessary calculational labors are consequently considerably less. Therefore, a practically useful calculational scheme might be formulated by relating the consistent anomaly with the covariant anomaly (or with a certain gauge covariant expression). In Ref. [9], Banerjee, Banerjee and Mitra gave a field theoretical prescription which provides this kind of calculational scheme. This prescription leads to basically equivalent consequences as the result due to Bardeen and Zumino [4], and that due to Leutwyler [10]. However, the prescription of Ref. [9] is more straightforward and flexible.

In this letter, we give a direct field theoretical calculation of the consistent gauge anomaly in supersymmetric theories, on the basis of the prescription of Ref. [9]. Generally, the treatment of the consistent anomaly with the superfield formalism [12–24] is quite complicated, because the gauge transformation is highly non-linear and because the gauge superfield has no mass dimension (i.e., an arbitrary function of the gauge superfield is a candidate of the counterterm). The advantage of our treatment in this letter is that the minimal superfield anomaly, being propor-

\footnote{It has the application even in chiral gauge theories on the lattice [11].}
tional to the anomaly $d^{abc} = \text{tr} T^a \{ T^b, T^c \}$, is directly obtained. This minimal-ness is guaranteed by the basic property of the prescription of Ref. [9]. Naturally, the resultant anomaly coincides with that due to Marinković [24], who applied the technique of Ref. [4] to this problem. Also our expressions below have some similarities with that of the work by McArthur and Osborn [23], in which the formulation of Ref. [10] was generalized to supersymmetric theories. Nevertheless, it seems worthwhile to report on our field theoretical calculation, because of simplicity of the basic idea and the treatment.

We consider the massless chiral superfield $\Phi$ coupled to the external gauge superfield $V = V^a T^a$ ($T^a$ is the representation of the gauge group to which $\Phi$ belongs). The classical action is given by

$$ S = \int d^8 z \, \Phi^\dagger e^V \Phi. \quad (1) $$

Following the prescription of Ref. [9], we define the effective action $\Gamma[V]$ as follows. We first introduce an auxiliary gauge coupling parameter $g$ by $V \to gV$. Then we may differentiate the effective action with respect to the parameter $g$ and integrate it over this parameter. Noting that the $g$-dependences arise only through the combination $gV$ and the original effective action is given by the value at $g = 1$, we have the following formal expression of the effective action

$$ \Gamma[V] = \int_0^1 dg \int d^8 z \, V^a(z) \frac{\delta \Gamma[gV]}{\delta g V^a(z)} $$

$$ = \int_0^1 dg \int d^8 z \, V^a(z) \left\langle \frac{\delta S}{\delta V^a(z)} \right\rangle_{V \to gV}. \quad (2) $$

Here the indication $V \to gV$ implies that all $V$-dependences involved are replaced by $gV$. The representation (2) is yet formal, because the regularization of the gauge

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* We basically follow the notational conventions of Ref. [25]. Our particular conventions and useful identities are summarized in the Appendix A.

** As one would anticipate, this parameter $g$ becomes the integration variable appearing in the homotopy formula [5-8,4].
current $\langle \delta S/\delta V^a(z) \rangle$ has to be specified. The crucial point of the prescription of Ref. [9] is to adopt the covariant gauge current as the gauge current. Thus we introduce the proper time cutoff to regularize the gauge current in a gauge covariant manner

$$
\langle \frac{\delta S}{\delta V^a(z)} \rangle = \lim_{z' \to z} \text{tr} e^V \frac{\delta e^V}{\delta V^a(z)} \left\langle T^* \Phi(z) \Phi^\dagger(z') \right\rangle \\
\equiv -\frac{i}{16} \text{Tr} e^{-V} \frac{\delta e^V}{\delta V^a(z)} \int_{1/M^2}^\infty dt e^{\Box_+/M^2} \frac{1}{\Box_+} \nabla^2,
$$

(3)

where the trace Tr is taken with the full superspace measure $d^8z$ and $M$ denotes the cutoff mass parameter.\(^{***}\) In writing this expression, we have used the formal expression of the propagator of the chiral superfield in presence of the external gauge superfield $\langle T^* \Phi(z) \Phi^\dagger(z') \rangle = i\overline{D}^2 e^{-V} e^{-i\Lambda} \delta(z - z')/16$. (For the derivation of the propagator, see, for example, Ref. [26].) Note that all the derivatives $\nabla_\alpha$, $\overline{D}_\dot{\alpha}$ and $\nabla_m$ transform as $\nabla' = e^{-i\Lambda} \nabla e^{i\Lambda}$ under the gauge transformation $e^{V'} = e^{-i\Lambda^\dagger} e^{V} e^{i\Lambda}$ [25] and thus these are gauge covariant objects. Due to the gauge covariant definition (3), the gauge current transforms covariantly under the gauge transformation,

$$
\langle \frac{\delta S}{\delta V^a(z)} \rangle' = \frac{i}{16} \text{Tr} e^{-V} \frac{\delta e^V}{\delta V^a(z)} \overline{D}^2 e^{\Box_+/M^2} \frac{1}{\Box_+} \nabla^2
$$

(4)

as is formally expected.

\(^{***}\) One can generalize the regulator $e^{\Box_+/M^2}$ in the last line as $f(-\Box_+/M^2)$ where $f(x)$ is an arbitrary rapidly decreasing function with $f(0) = 1$. The result with $f(-\Box_+/M^2)$ is given by working with $e^{\Box_+/M^2}$ and then by multiplying $\int_0^\infty dp \ g(p)$; $g(p)$ is the inverse Laplace transformation of $f(x)$, $f(x) = \int_0^\infty dp \ g(p) e^{-px}$. Our results in $M \to \infty$ are independent of $M$ and thus of $p$. Therefore, all the results become independent of the choice of the regulator function $f(x)$ in the $M \to \infty$ limit because $\int_0^\infty dp \ g(p) = f(0) = 1$.\(^{***}\)
The definition of the effective action (2) through the covariant current (3) is perfectly legitimate. For UV convergent diagrams, it is equivalent to any conventional definition. For UV diverging diagrams, it may give a different value from the conventional definition but only by an amount expressed by local counterterms, because Eq. (3) reduces to the conventional gauge current in the $M \to \infty$ limit. As we will see below, the consistent anomaly derived from the effective action (2) with Eq. (3) is directly related to the covariant anomaly. Since the covariant anomaly [27,23] is proportional to the anomaly $d^{abc}$, the consistent anomaly thus obtained is also proportional to $d^{abc}$. This implies that, when the gauge representation of the chiral multiplet is anomaly-free, i.e., when $d^{abc} = 0$, the regularized effective action (2) with Eq. (3) automatically restores the gauge invariance without supplementing any counterterms. In this sense, a breaking of the gauge symmetry is kept to be minimal with the present prescription [28].

The same mechanism works also when one starts with the covariant current and then adds minimal corrections for ensuring the integrability of the whole current [10,23]. Our treatment is, however, more straightforward as it directly defines the effective action. Note that the prescription (2) and (3) is quite different from the direct proper time regularization of the effective action [14,15].

Now, from Eq. (2), we can read off a variation of the effective action under the infinitesimal gauge transformation [25]

$$
\delta_{\Lambda} e^V = e^V i\Lambda - i\Lambda^\dagger e^V,
\delta_{\Lambda} V = i\mathcal{L}_{V/2} \cdot \left[ (\Lambda + \Lambda^\dagger) + \coth(\mathcal{L}_{V/2}) \cdot (\Lambda - \Lambda^\dagger) \right],
$$

(5)

**** For anomaly-free cases, the prescription is equivalent [28] to the generalized Pauli-Villars regularization introduced in Refs. [29,30]. Since this is a Lagrangian level regularization, and the corresponding Hamiltonian is Hermitian, the S-matrix is manifestly unitary. (In the $M \to \infty$ limit, negative norm regulators cannot contribute to the out-state of the physical S-matrix.)
We then insert \( dg/dg = 1 \) into the first term and perform the integration by parts with respect to \( g \). By noting again that the \( g \)-dependences arise only through the combination \( gV \), we have the following representation of the consistent anomaly

\[
\delta\Lambda[\Gamma[\mathcal{V}]] = \int d^8 z \delta\Lambda V^a(z) \left\langle \frac{\delta S}{\delta V^a(z)} \right\rangle_{V \to gV} 
+ \int dg \int d^8 z \int d^8 z' V^a(z) \delta\Lambda V^b(z') \frac{\delta}{\delta V^b(z')} \left\langle \frac{\delta S}{\delta V^a(z)} \right\rangle_{V \to gV}.
\]

This anomaly must satisfy the Wess-Zumino consistency condition because it is a variation of the effective action (2). Eq. (7) shows that the consistent anomaly consists of two pieces: The first piece is the covariant gauge anomaly [27,23] that is obtained from Eqs. (3) and (5) as

\[
\int d^8 z \delta\Lambda V^a(z) \left\langle \frac{\delta S}{\delta V^a(z)} \right\rangle
= i \int d^8 z \lim_{z' \to z} \text{tr} \, i \Lambda e^{M^2/16} \frac{1}{16} \mathcal{D}^2 \nabla^2 \delta(z - z') + \text{h.c.}
= -\frac{i}{4} \int d^6 z \lim_{z' \to z} \text{tr} \, i \Lambda e^{M^2/16} \mathcal{D}^2 \delta(z - z') + \text{h.c.}
\]

where we have noted that \( \int d^8 z = \int d^6 z \left( -\mathcal{D}^2 / 4 \right) \) and the gauge parameter is chiral \( \mathcal{D}_\alpha \Lambda = 0 \). We have also used the identity (A.3). For the actual calculation of the
third line in plane wave basis, see Ref. [27], or the Appendix of Ref. [26]. The
calculation is quite simple due to the gauge covariance. Obviously the covariant
anomaly (8) is proportional to the anomaly $d^{abc}$, because $\Lambda, W_\alpha, \Lambda^\dagger,$ and $\overline{W}_\dot{\alpha}$ are
Lie algebra valued.

The second piece in Eq. (7), on the other hand, provides a difference between
the consistent anomaly and the covariant anomaly [4]. The difference is expressed
by the functional rotation of the covariant gauge current

$$\frac{\delta}{\delta V^b(z')} \left< \frac{\delta S}{\delta V^a(z)} \right> - \frac{\delta}{\delta V^a(z)} \left< \frac{\delta S}{\delta V^b(z')} \right>.$$  

(9)

The importance of this functional rotation has been noticed in various context [8–
10,31,23,32,11,33]. The gauge covariance (4) implies that the functional rota-
tion (9) possesses the following property:

$$\int d^8 z \delta_\Lambda V^a(z) \left[ \frac{\delta}{\delta V^b(z')} \left< \frac{\delta S}{\delta V^a(z)} \right> - \frac{\delta}{\delta V^a(z)} \left< \frac{\delta S}{\delta V^b(z')} \right> \right]$$

$$= \frac{\delta}{\delta V^b(z')} \int d^8 z \delta_\Lambda V^a(z) \left< \frac{\delta S}{\delta V^a(z)} \right>.$$  

(10)

The right hand side is nothing but the covariant anomaly (8). Quite interest-
ingly, the functional rotation (9) is a local functional of the gauge superfield, being
proportional to (a derivative of) the delta function $\delta(z - z')$. We will see this
shortly. Therefore, Eq. (10) implies that, when the covariant anomaly vanishes,
i.e., when $d^{abc} = 0$, the functional rotation vanishes and consequently our con-
sistent anomaly (7) entirely vanishes. Thus the minimal-ness of the anomaly is
guaranteed by construction; this is the advantage of the prescription (2) and (3).

In non-supersymmetric gauge theories, one can obtain the functional rotation (9)
by solving the relation corresponding to Eq. (10) [9]. Instead, one may directly
evaluate Eq. (9), as was performed in Ref. [10] for non-supersymmetric gauge the-
tories. Here we adopt the latter approach that seems much simpler for the present
supersymmetric case. This approach was also adopted in Ref. [23].
To evaluate the functional rotation (9), we consider

$$\delta_1 \langle \delta_2 S \rangle - \delta_2 \langle \delta_1 S \rangle,$$

where $\delta_1$ and $\delta_2$ are arbitrary variations of the gauge superfield $V$, being independent of $V$ itself. Here, in the same way as the gauge current (3), the composite operator $\langle \delta S \rangle$ is regularized in a gauge covariant manner

$$\langle \delta S \rangle \equiv -\frac{i}{16} \text{Tr} \Delta D^2 \int_{1/M^2}^{\infty} dt \ e^{t/16} \nabla^2$$

with the notation $\Delta \equiv e^{-V} \delta e^{V}$. By noting relations $\delta \nabla^2 = [\nabla^2, \Delta]$ and $\delta_1 \Delta_2 = -\Delta_1 \Delta_2 + (\text{symmetric on } 1 \leftrightarrow 2)$, we have

$$\delta_1 \langle \delta_2 S \rangle - \delta_2 \langle \delta_1 S \rangle$$

$$= \frac{i}{16} \int_{1/M^2}^{\infty} dt \ Tr \left\{ \int_0^t ds \left[ \Delta_2 D^2 e^{s/16} \Delta_1 \frac{\partial}{\partial t} e^{(t-s)/16} \nabla^2 \right] + \Delta_2 D^2 e^{t/16} \Delta_1 \nabla^2 \right\} - (1 \leftrightarrow 2)$$

In writing the last expression, we have used the identities (A.3) and (A.4). Note that, while originally the proper time in the gauge current (12) is belonging to the IR region $1/M^2 \leq t < \infty$, only the UV region $0 \leq \beta/M^2 \leq 1/M^2$ (or $0 \leq (1-\beta)/M^2 \leq 1/M^2$) is contained in the combination (13). Thus the functional
rotation (11) or (9) is a local quantity like the anomaly itself. Thanks to the gauge covariance, the evaluation of the last expression in plane wave basis is again simple as is shown in Appendix B. We have

$$
\delta_1 \langle \delta_2 S \rangle - \delta_2 \langle \delta_1 S \rangle
$$

$$
M \to \infty \frac{1}{64\pi^2} \int d^8z \text{tr} \Delta \left( \left[ D^\alpha \Delta_2, W_\alpha \right] + \left[ \overline{D}_\dot{\alpha} \Delta_2, W^{\dot{\alpha}} \right] + \{ \Delta_2, D^\alpha W_\alpha \} \right).
$$

(14)

In spite of the asymmetric appearances of 1 and 2 in this expression, one can confirm by using the reality constraint (A.2) that this is actually odd under the exchange 1 ↔ 2. From Eq. (14), we can read off the left hand side of Eq. (10):

$$
\int d^8z \delta_A V^a(z) \left[ \frac{\delta}{\delta V^b(z')} \left( \frac{\delta S}{\delta V^a(z)} \right) - \frac{\delta}{\delta V^a(z)} \left( \frac{\delta S}{\delta V^b(z')} \right) \right]
$$

$$
M \to \infty \frac{1}{64\pi^2} \int d^8z \int_0^1 d\beta \times \text{tr} e^{-\beta V^b} \delta(z - z') e^{\beta V} \left( D^\alpha \{ i\Lambda, W_\alpha \} - \overline{D}_{\dot{\alpha}} \{ e^{-V} i\Lambda \dagger e^V, W^{\dot{\alpha}} \} \right),
$$

(15)

which satisfies Eq. (10) in conjunction with Eq. (8); this fact provides the consistency check of Eq. (14). Finally, from Eqs. (7), (8) and (14), we obtain the consistent gauge anomaly

$$
\delta_A \Gamma[V]
$$

$$
M \to \infty - \frac{1}{64\pi^2} \int d^6z \text{tr} i\Lambda W^\alpha W_\alpha + \frac{1}{64\pi^2} \int d^6z \text{tr} e^{-V} i\Lambda \dagger e^V \overline{W}^{\dagger}_{\dot{\alpha}} \overline{W}^{\dot{\alpha}}
$$

$$
+ \frac{1}{64\pi^2} \int d^8z \int_0^1 dg \int_0^1 d\beta \text{tr} e^{-\beta g V} \delta_A V e^{\beta g V}
$$

$$
\times \left( \left[ D^\alpha V, W_\alpha \right] + \left[ \overline{D}_{\dot{\alpha}} V, W^{\dot{\alpha}} \right] + \{ V, D^\alpha W_\alpha \} \right)_{V \to gV}.
$$

(16)

Here, as indicated, the quantities inside the round bracket are defined by substituting the gauge superfield $V$ involved by $gV$. On the other hand, the gauge variation $\delta_A V$ is given by Eq. (5) as it stands without setting $V \to gV$. It is obvious that our consistent anomaly is proportional to the anomaly $d^{abc}$, as expected.
As the simple but non-trivial check of Eq. (16), we may consider the Abelian case for which the expression is considerably simplified. By noting \( \delta \Lambda V = i \Lambda - i \Lambda^\dagger \) in this case, we have

\[
\delta \Lambda \Gamma[V] = (-1 + 2/3) \int d^6z \, \text{tr} \, i \Lambda W^\alpha W_\alpha/(64\pi^2) + \text{h.c.}
\]

This is one-third the covariant anomaly (8) and reproduces the correct result of the consistent Abelian anomaly. We note that, in our treatment, nothing special (except simplicity of the expression) occurs in the Abelian case. In approaches based on the Wess-Zumino consistency condition, strictly speaking, it is necessary to start with the non-Abelian case and then to take the Abelian limit, because the consistency condition becomes trivial in the Abelian case.

One might ask whether the anomaly (16) actually satisfies the Wess-Zumino consistency condition. In fact, Eq. (16) is identical to the consistent anomaly due to Marinković [24] up to the overall normalization factor (ours is four times smaller). See Eq. (5) of Ref. [24], where \( g \) is denoted as \( t \) and \( \int_0^1 d\beta \, e^{-\beta g V} g \delta \Lambda V e^{\beta g V} \) is abbreviated as \( S_g \). Since the consistent anomaly in Ref. [24] was constructed as the solution of the consistency condition, we may claim that we already know Eq. (16) actually satisfies the consistency condition.

It is interesting to examine the form of the anomaly (16) in the Wess-Zumino (WZ) gauge \( V = -\theta \sigma^m \bar{\theta} + i \theta^2 \bar{\theta} \sigma^i - i \partial^i \theta \partial \lambda + \theta^2 \sigma^2 D/2 \) [25]. We first set \( \Lambda(z) = a(y) \) for reproducing the usual gauge transformation (\( a \) is real). Then we have

\[
\delta \Lambda \Gamma[V] \rightarrow \infty = \left[ \varepsilon^{mnkl} \partial_m \left( v_n \partial_k v_l + \frac{i}{4} v_n v_k v_l \right) - \frac{1}{2} \partial_m (\bar{\lambda} \sigma^m \lambda - \lambda \sigma^m \bar{\lambda}) \right].
\]

This expression of the usual gauge anomaly in the WZ gauge is surprisingly simple compared to the result of existing field theoretical calculations. We emphasize again that we obtained Eq. (17) without supplementing any counterterms. The first term is celebrated Bardeen’s minimal anomaly [1] with the coefficient for a single chiral fermion. The second term, if one wishes, may be eliminated by adding a non-supersymmetric local counterterm \( C \) as \( \delta \Lambda \Gamma[V] + \delta_a C \), where \( \delta_a \) is the usual gauge transformation \( \delta_a v_m = 2D_m a \) and \( \delta_a \lambda = -i[a, \lambda] \). The counterterm is given
As another interesting application, we may consider the anomalous breaking of the supersymmetry in the WZ gauge, the so-called ε-SUSY anomaly [34,35]. The super-transformation in the WZ gauge is a combination of the supersymmetric transformation generated by $\varepsilon Q + \bar{\varepsilon}Q$ (which is not anomalous in the present formulation [26]) and the gauge transformation $\delta_\Lambda$ with the gauge parameter $\Lambda(z) = -i\theta\sigma^m v_m(y) - \theta^2\bar{\lambda}(y)$ [25]. Therefore we have the (apparent) breaking of supersymmetry as the consequence of the gauge anomaly. By setting the gauge parameter $\Lambda$ to this form in Eq. (16), we have after some calculation,

$$\delta_\Lambda \Gamma[V] \rightarrow \int d^4 x \ tr v^m (\bar{\lambda} \sigma_m \lambda - \lambda \sigma_m \bar{\lambda}) \left\{ 3 \bar{\lambda} \sigma_m \lambda - \varepsilon^m_{\ nkl} \left[ 2 v_n (\partial_k v_l) + 2 (\partial_n v_k) v_l + \frac{3i}{2} v_n v_k v_l \right] \right\} - \delta_\varepsilon C,$$

(19)

where $\delta_\varepsilon$ is the super-transformation in the WZ gauge $\delta_\varepsilon v^m = i\varepsilon \sigma^m \lambda + \text{h.c.}$, $\delta_\varepsilon \lambda = \sigma^m \varepsilon v_m + i\varepsilon D$ and $\delta_\varepsilon D = -D_m \lambda \sigma^m \varepsilon + \text{h.c.}$ Eq. (19) shows that Eq. (16) reproduces the ε-SUSY anomaly given in Ref. [35] again with the non-supersymmetric local counterterm $C$ (18). Note that the structure of the counterterm (18) is quite simple, compared to that of the counterterm required in Ref. [35] for obtaining the above form. Our anomaly is proportional to $d^{abc}$ from the beginning and thus the possible (non-supersymmetric) counterterm also must be proportional to $d^{abc}$. This fact severely restricts the possible form of counterterms.

In this letter, we have presented a (yet another) field theoretical calculation of the consistent gauge anomaly in the superfield formalism. As we have shown, it is possible to fully utilize the advantage of gauge covariance, by defining the effective action through the covariant gauge current (Eqs. (2) and (3)). Although our result (16) itself has been known in the literature [24] (see also Ref. [23]), this
is the first time to our knowledge that an explicit field theoretical calculation in
the superfield formalism directly led to the minimal consistent anomaly.

APPENDIX A

Notational conventions:

$$\eta_{mn} = \text{diag}(-1, +1, +1, +1),$$
$$z = (x^m, \theta^\alpha, \bar{\theta}_{\dot{\alpha}}), \quad y^m = x^m + i\theta^m \bar{\theta},$$
$$d^8 z = d^4 x d^2 \theta d^2 \bar{\theta}, \quad d^6 z = d^4 x d^2 \theta, \quad d^6 \bar{z} = d^4 x d^2 \bar{\theta},$$
$$\delta(z) = \delta(x) \delta(\theta) \delta(\bar{\theta}),$$
$$W_\alpha = -\frac{1}{4} \tilde{D}^2 (e^{-V} D_\alpha e^V), \quad \bar{W}_{\dot{\alpha}} = \frac{1}{4} e^{-V} \tilde{D}^2 (e^V \bar{D}_{\dot{\alpha}} e^{-V}) e^V,$$
$$\nabla_\alpha = e^{-V} D_\alpha e^V, \quad \{ \nabla_\alpha, \bar{D}_{\dot{\alpha}} \} = -2i\sigma^m_{\alpha\dot{\alpha}} \nabla_m,$$
$$D^\alpha A = \{ \nabla^\alpha, A \},$$
$$\Box_+ = \frac{1}{16} \tilde{D}^2 \nabla^2 + \frac{1}{16} \nabla^2 \tilde{D}^2 - \frac{1}{8} \delta_{\alpha} \nabla^2 \tilde{D}^\dot{\alpha} = \nabla^m \nabla_m - \frac{1}{2} W^\alpha \nabla_\alpha - \frac{1}{4} (D^\alpha W_\alpha),$$
$$\Box_- = \frac{1}{16} \nabla^2 \tilde{D}^2 + \frac{1}{16} \tilde{D}^2 \nabla^2 - \frac{1}{8} \delta^\alpha \nabla^2 \nabla_\alpha = \nabla^m \nabla_m + \frac{1}{2} \bar{W}_{\dot{\alpha}} \bar{D}^\dot{\alpha} + \frac{1}{4} (\bar{D}_{\dot{\alpha}} \bar{W}^\dot{\alpha}).$$

(A.1)

Identities:

$$D^\alpha W_\alpha = \bar{D}_{\dot{\alpha}} \bar{W}^\dot{\alpha}. \quad (A.2)$$
$$\tilde{D}^2 \Box_+ = \Box_+ \tilde{D}^2 = \frac{1}{16} \tilde{D}^2 \nabla^2 \tilde{D}^2. \quad (A.3)$$
$$\nabla^2 \Box_- = \Box_- \nabla^2 = \frac{1}{16} \nabla^2 \tilde{D}^2 \nabla^2. \quad (A.4)$$
$$D_\alpha \text{tr}(AB) = \text{tr} D_\alpha AB \pm \text{tr} A D_\alpha B. \quad (A.5)$$

Note that the last identity (A.5) allows the integration by parts on $D_\alpha$. 

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APPENDIX B

In this appendix, we illustrate the calculation of Eq. (13) in plane wave basis. Basically the same calculation was performed in Ref. [23] by using the heat kernel expansion. First, in Eq. (13), we note

\[ \delta(x - x') = M^4 \int d^4k \, e^{iMk(x-x')/(2\pi)^4} \]

and

\[ e^{-iMkx} \nabla_m e^{iMkx} = \nabla_m + iMk_m, \]
\[ e^{-iMkx} \nabla_\alpha e^{iMkx} = \nabla_\alpha - \sigma^m_{\alpha\dot{\alpha}} \partial^{\dot{\alpha}} M_k m, \] (B.1)
\[ e^{-iMkx} \mathcal{D}_{\dot{\alpha}} e^{iMkx} = \mathcal{D}_{\dot{\alpha}} + \theta^\alpha \sigma^m_{\alpha\dot{\alpha}} M_k m. \]

Then we have

\[ \delta_1 \langle \delta_2 S \rangle - \delta_2 \langle \delta_1 S \rangle = -\frac{i}{16} \int d^8 z \int_0^1 d\beta \int \frac{d^4k}{(2\pi)^4} \]
\[ \times \text{tr} \Delta_2 \exp \left\{-\beta \left[k^m k_m - 2ik^m \nabla_m / M \right. \right. \]
\[ \left. \left. - \nabla^m \nabla_m / M^2 + W^\alpha \nabla_\alpha / (2M^2) + (\mathcal{D}^\alpha W_\alpha) / (4M^2) \right] \right\} \mathcal{D}^2 \]
\[ \Delta_1 \exp \left\{-(1 - \beta) \left[k^m k_m - 2ik^m \nabla_m / M \right. \right. \]
\[ \left. \left. - \nabla^m \nabla_m / M^2 - W^{\dot{\alpha}} \mathcal{D}_{\dot{\alpha}} / (2M^2) - (\mathcal{D}_{\dot{\alpha}} W^{\dot{\alpha}}) / (4M^2) \right] \right\} \nabla^2 \]
\[ \times \delta(\theta - \theta') \delta(\bar{\theta} - \bar{\theta}) \bigg|_{\theta = \theta' = \bar{\theta} = \bar{\theta}} - (1 \leftrightarrow 2). \]

In writing this expression, we have omitted terms in which \( \theta^\alpha \) or \( \bar{\theta} \) explicitly appears; the reason for this is the following. The superfield, such as Eq. (13), cannot have a term which explicitly contains \( \theta^\alpha \) or \( \bar{\theta} \), because such a term has no first (\( \theta = \bar{\theta} = 0 \)) component and thus has no higher components (the higher components of the superfield are uniquely determined by the linearly realized supertransformation of the first component [25]). Therefore those terms in which \( \theta^\alpha \) or \( \bar{\theta} \) explicitly appears must eventually be canceled out, or, if these contribute, \( \theta^\alpha \) must
be eliminated by $D_\alpha$ (or $\bar{\theta}_\dot{\alpha}$ by $\bar{D}^i\dot{\alpha}$). However, in the original form of (B.2), one can confirm that when $D_\alpha$ eliminates $\theta^\alpha$ (or when $\bar{D}^i\dot{\alpha}$ eliminates $\bar{\theta}_\dot{\alpha}$), the corresponding term does not have enough powers of $M$ to survive in the $M \to \infty$ limit, or does not have a sufficient number of spinor derivatives to eliminate the delta function: In the equal point limit $\theta = \theta'$ and $\bar{\theta} = \bar{\theta}'$, only those terms with which just four spinor derivatives acting on the delta function survive [25]:

$$
\nabla_\alpha \nabla_\beta \delta(\theta - \theta')|_{\theta = \theta'} = -2\varepsilon_{\alpha\beta}, \quad \bar{D}_\dot{\alpha} \bar{D}_\dot{\beta} \delta(\bar{\theta} - \bar{\theta}')|_{\bar{\theta} = \bar{\theta}'} = 2\varepsilon_{\dot{\alpha}\dot{\beta}}.
$$

In short, the terms in which $\theta^\alpha$ or $\bar{\theta}_\dot{\alpha}$ explicitly appears must be canceled out. This cancellation may directly be verified as was done in Ref. [26] in a similar calculation.

The expansion of Eq. (B.2) in powers of $1/M$ is easy. After the integration over $k_m$ and $\beta$, one can readily verify (say, by substituting $\nabla_m = \partial_m + \Gamma_m$) that the terms contain the vector covariant derivative $\nabla_m$ are combined into a total divergence. Thus vector covariant derivatives do not contribute. In this way, we have

$$
\delta_1 \langle \delta_2 S \rangle - \delta_2 \langle \delta_1 S \rangle 
= \frac{1}{64\pi^2} \int d^8 z \, \text{tr} \left[ \Delta_2 W^\alpha \nabla_\alpha \bar{D}^2 \Delta_1 \nabla^2 + \frac{1}{2} \Delta_2 (W^\alpha W_\alpha) \bar{D}^2 \Delta_1 \nabla^2 
- \Delta_2 \bar{D}^2 \Delta_1 W^\alpha \bar{D}^\alpha \nabla^2 - \frac{1}{2} \Delta_2 \bar{D}^2 \Delta_1 (\bar{D}_\dot{\alpha} \bar{W}^\alpha) \nabla^2 \right] (B.4)
\times \frac{1}{16} \delta(\theta - \theta')\delta(\bar{\theta} - \bar{\theta}')|_{\theta = \theta', \bar{\theta} = \bar{\theta}'} - (1 \leftrightarrow 2).
$$

By noting again that four spinor derivatives have to act on the delta function as
in Eq. (B.3), we find
\[
\delta_1 \langle \delta_2 S \rangle - \delta_2 \langle \delta_1 S \rangle \\
\lim_{M \to \infty} - \frac{1}{64\pi^2} \int d^8 z \\
\times \text{tr} \left[ \Delta_2 W^\alpha D_\alpha \Delta_1 + \frac{1}{2} \Delta_2 (D^\alpha W_\alpha) \Delta_1 + \Delta_2 \bar{D}_{\dot{\alpha}} (\Delta_1 W^{\prime \dot{\alpha}}) - \frac{1}{2} \Delta_2 \Delta_1 \bar{D}_{\dot{\alpha}} W^{\prime \dot{\alpha}} \right] \\
- (1 \leftrightarrow 2).
\] (B.5)

Finally, we obtain Eq. (14) after some rearrangements with use of the reality constraint (A.2) and the identity (A.5).

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