Charming penguins in $B \to K^*\pi, K(\rho, \omega, \phi)$ decays

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We evaluate the decays $B \to K^*\pi, K(\rho, \omega, \phi)$ adding the long distance charming penguin contributions to the short distance: Tree+Penguin amplitudes. We estimate the imaginary part of the charming penguin by an effective field theory inspired by the Heavy Quark Effective Theory and parameterize its real part. The final results for branching ratios depend on only two real parameters and show a significant role of the charming penguins. The overall agreement with the available experimental data is satisfactory.

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I. INTRODUCTION

The CLEO II \cite{1,2}, the BaBar \cite{3,4} and the Belle \cite{5} collaborations have recently reported data on the $B$ meson non leptonic decay channels into a light vector and a pseudoscalar meson:

$$B \rightarrow PV.$$ (1)

These data are of the foremost importance for the determination of the angles of the unitarity triangle. In particular, if one of the two mesons in the final state is a strange particle the decays (1) offer several channels for the extraction of the $\gamma$ angle, thus providing alternatives to the $K\pi$ decay mode. These non leptonic decays have been proposed long ago \cite{6,7,8,9,10,11,12,13,14,15,16,17,18} as a method for the extraction of the $\gamma$ angle. By this strategy one relates the data to a theoretical amplitude given as a sum of Tree and Penguin short distance contributions, whose relative weak phase is indeed $\gamma$. The usual computational scheme is the factorization hypothesis, either in the naive or in the QCD based version \cite{19,20,21,22,23,24}. Here one neglects non factorizable contributions as one proves that they are of the order of $\Lambda/m_b$ and therefore negligible in the $m_b \rightarrow \infty$ limit.

There is a class of contributions however that, although suppressed in the infinite heavy quark mass limit, cannot be neglected a priori. These are long distance contributions known in the literature as charming penguin diagrams \cite{25,26,27,28,29,30}. Induced by the non leptonic hamiltonian $\propto V_{cb} V_{cs}^* \bar{b} \gamma^\mu (1-\gamma_5) c + \text{Fierz} + \text{h.c.}$, they cannot contribute in the vacuum saturation approximation. However, the insertion of at least two charmed particles (charm+anticharm) between the currents can produce sizeable contributions. As a matter of fact the $O(\Lambda \times m_b^{-1})$ suppression is compensated by the Cabibbo-Kobayashi-Maskawa enhancement and the actual role of these contributions can be established only as a result of some dynamical calculations. In \cite{28} and \cite{29} we estimated these contributions for the $B \rightarrow PP$ decay channels and found that for the $K\pi$ case they are indeed relevant. Their imaginary part can be evaluated with some confidence using the heavy meson chiral effective theory corrected for the light meson hard momenta. On the other hand, the real part is less predictable; for example in \cite{28} and \cite{29} the results strongly depend on the cutoff in the loop integrals.

In the present paper we give an estimate of the charming penguins in the charmless decays \cite{11} for the cases $K^*\pi$ and $K(\rho, \omega, \phi)$. We do not consider the particles $\eta, \eta'$ in the final state because the pseudoscalar $SU(3)$ singlet is most probably contaminated by the glue and the evaluation of these contributions cannot be reliably done within our theoretical scheme. Due to the above mentioned difficulties we compute by the effective theory only the imaginary part of the amplitude, $I_{LD}$. For the real part of the amplitude, $R_{LD}$, we assume a simple parametrization $R_{LD} \propto I_{LD}$, fixing the two proportionality constants (one for each of the $SU(3)_f$ multiplets comprising the light vector mesons) by a fit. A similar calculation has been recently attempted by \cite{30}; these authors assume a more phenomenological approach and parametrize the amplitude in the more general way (two complex numbers), using $SU(3)$ flavor symmetry to relate the different amplitudes. Therefore our calculation can provide a dynamical test of the model in \cite{30}.

The outline of the paper is as follows. In Section II we estimate the tree diagram contribution (or Short-Distance contribution), computed in the factorization approximation. In Section III we estimate the absorptive part of the charming penguin contribution. It is evaluated by use of the effective lagrangian for light and heavy mesons based on the Heavy Quark Effective Theory (for a review see \cite{31}). The main uncertainty of this approach is the extrapolation to hard light meson momenta and in this context we use an estimate of form factor given by us in \cite{28}. In Section IV we present numerical results for the branching ratios and the asymmetry. In our previous papers we estimated the real part of the charming penguin contributions by using a cut-off Cottingham formula; the results where strongly cut-off dependent and for this reason we give here a simple parametrization of the real part and we estimate the relevant two real parameters by
comparing with the data. The overall result for both of the branching ratios and the CP-asymmetries is rather satisfying and points to a significant role of the charming penguins in the $B \rightarrow K^{*}\pi$, $K(\rho, \omega, \phi)$ decay modes.

II. SHORT-DISTANCE NONLEPTONIC WEAK MATRIX ELEMENTS

The effective Hamiltonian for non-leptonic $B$ decays is given by

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ V_{ub}^* V_{us} (c_1 O_1^u + c_2 O_2^u) + V_{cb}^* V_{cs} (c_1 O_1^c + c_2 O_2^c) - V_{tb}^* V_{ts} \left( \sum_{i=3}^{10} c_i O_i + c_g O_g \right) \right]$$

(2)

where $c_i$ are the Wilson coefficients evaluated at the normalization scale $\mu = m_b$ and next-to-leading order QCD radiative corrections are included. $O_i$ $(i = 3, \ldots, 10)$ are the penguin operators and $O_g$ is the chromomagnetic gluon operator. The $c_i$ in eq. (2) are as follows [36]: $c_2 = 1.105$, $c_3 = -0.228$, $c_5 = 0.013$, $c_4 = -0.029$, $c_5 = 0.009$, $c_6 = -0.033$, $c_7/\alpha = 0.005$, $c_8/\alpha = 0.060$, $c_9/\alpha = -1.283$, $c_{10}/\alpha = 0.266$. Moreover, for the current quark masses we use the values

$$m_b = 4.6 \text{GeV} \quad m_u = 4 \text{MeV} \quad m_d = 8 \text{MeV} \quad m_s = 0.150 \text{GeV}.$$

(3)

We define the $T$-matrix element

$$(2\pi)^4 \delta^4(p_B - p_\pi - p_{K^*}) \times \mathcal{M}_{K^*\pi} = \langle K^*\pi| T | B \rangle,$$

(4)

with a similar definition for the $K(\rho, \omega, \phi)$ final state. We separate short-distance and long-distance contributions to the weak $B \rightarrow K^*\pi$ decay:

$$\mathcal{M} = \mathcal{M}_\text{SD} + \mathcal{M}_\text{LD},$$

(5)

and evaluate the short-distance part of the amplitude $\mathcal{M}_\text{SD}$ using the operators in (2) in factorization approximation (with $\mathcal{M}_\text{SD}(B \rightarrow f) = -\langle f | \mathcal{H}_{\text{eff}} | B \rangle$). The results are in Tab. I. Here $a_i = c_i + \frac{c_i+1}{3}$ (i=odd) and $a_i = c_i + \frac{c_i-1}{3}$ (i=even). Moreover, if $V$ is a vector meson, $P^{(i)}$ is a pseudoscalar meson we use the following definition for the matrix elements of weak currents:

$$\langle P(p)|A_\mu|0\rangle = -i f_P p_\mu,$$

(6)

and

$$\langle P(p)|V_\mu|0\rangle = f_V m_V \epsilon_\mu^*, \quad \langle V(\epsilon,p)|V_\mu|0\rangle = f_V m_V \epsilon_\mu^*,$$

(7)

and

$$\langle P(p)|V_\mu|0\rangle = F_1(q^2) \left[ (p_\mu + p'_\mu) - \frac{m_P^2 - m_{P'}^2}{q^2} q_\mu \right] + F_0(q^2) \frac{m_P^2 - m_{P'}^2}{q^2} q_\mu,$$

(8)

where

$$A_3(q^2) = \frac{M_V - M_P}{2M_V} A_2(q^2) + \frac{M_V + M_P}{2M_V} A_1(q^2).$$

(9)

and $A_3(0) = A_0(0)$.,
| Process | $\mathcal{M}_\text{SD}$ |
|---------|----------------|
| $B^+ \to K^+\pi^+$ | $+G_F \sqrt{2} F_1^{B \to \pi} (m_K^2) f_{K^*} m_{K^*} V_{tb}^* V_{ts} \left[ a_4 - \frac{a_{10} - 1}{2} \right] (\epsilon^* \cdot p_B)$ |
| $B^+ \to K^+\pi^0$ | $-G_F m_{K^*} \left\{ F_1^{B \to \pi} (m_K^2) f_{K^*} \left[ V_{ub}^* V_{us} a_2 - V_{tb}^* V_{ts} (a_4 + a_{10}) \right] + A_0^{B \to K^*} (m_K^2) f_{K^*} V_{ub}^* V_{us} a_1 + V_{tb}^* V_{ts} \left[ \frac{3}{2} (a_7 - a_9) \right] \right\} (\epsilon^* \cdot p_B)$ |
| $B^0 \to K^+\pi^0$ | $-G_F m_{K^*} \left\{ A_0^{B \to K^*} (m_K^2) f_{K^*} \left[ V_{ub}^* V_{us} a_1 - V_{tb}^* V_{ts} \left[ \frac{3}{2} (a_7 - a_9) \right] \right] + F_1^{B \to \pi} (m_K^2) f_{K^*} V_{tb}^* V_{ts} \left[ a_4 - a_{10} \right] \right\} (\epsilon^* \cdot p_B)$ |
| $B^0 \to K^+\pi^-$ | $-G_F \sqrt{2} F_1^{B \to \pi} (m_K^2) f_{K^*} m_{K^*} \left[ V_{ub}^* V_{us} a_2 - V_{tb}^* V_{ts} \left[ a_4 + a_{10} \right] \right] (\epsilon^* \cdot p_B)$ |
| $B^+ \to K^0\rho^+$ | $+G_F \sqrt{2} A_0^{B \to \rho} (m_K^2) f_K m_\rho V_{ts}^* \left[ a_4 - \frac{1}{2} a_{10} - \frac{2 (a_6 - a_8) m_K^2}{(m_b + m_u) (m_d + m_s)} \right] (\epsilon^* \cdot p_B)$ |
| $B^+ \to K^+\rho^+$ | $-G_F m_\rho \left\{ A_0^{B \to \rho} (m_K^2) f_K \left[ V_{ub}^* V_{us} a_2 - V_{tb}^* V_{ts} \left[ a_4 + a_{10} - \frac{2 (a_6 + a_8) m_K^2}{(m_b + m_u) (m_d + m_s)} \right] \right] + F_1^{B \to \rho} (m_K^2) f_\rho \left[ V_{ub}^* V_{us} a_1 - \frac{3}{2} V_{tb}^* V_{ts} \left[ a_7 + a_9 \right] \right] \right\} (\epsilon^* \cdot p_B)$ |
| $B^0 \to K^0\rho^+$ | $-G_F m_\rho \left\{ F_1^{B \to K} (m_K^2) f_\rho \left[ V_{ub}^* V_{us} a_1 - V_{tb}^* V_{ts} \left[ \frac{3}{2} (a_7 + a_9) \right] \right] + A_0^{B \to \rho} (m_K^2) f_K V_{tb}^* V_{ts} \left[ a_4 - a_{10} - \frac{2 (a_6 + a_8) m_K^2}{(m_b + m_u) (m_d + m_s)} \right] \right\} (\epsilon^* \cdot p_B)$ |
| $B^0 \to K^+\rho^-$ | $-G_F \sqrt{2} A_0^{B \to \rho} (m_K^2) f_K m_\rho \left[ V_{ub}^* V_{us} a_2 - V_{tb}^* V_{ts} \left[ a_4 + a_{10} - \frac{2 (a_6 + a_8) m_K^2}{(m_b + m_u) (m_d + m_s)} \right] \right] (\epsilon^* \cdot p_B)$ |
| $B^+ \to K^0\omega$ | $-G_F m_\omega \left\{ F_1^{B \to K} (m_K^2) f_\omega \left[ V_{ub}^* V_{us} a_1 - V_{tb}^* V_{ts} \left[ 2 (a_3 + a_5) + \frac{1}{2} (a_7 + a_9) \right] \right] + A_0^{B \to \omega} (m_K^2) f_K V_{tb}^* V_{ts} \left[ a_4 + a_{10} - \frac{2 (a_6 + a_8) m_K^2}{(m_b + m_u) (m_d + m_s)} \right] \right\} (\epsilon^* \cdot p_B)$ |
| $B^0 \to K^0\omega$ | $-G_F m_\omega \left\{ F_1^{B \to K} (m_K^2) f_\omega \left[ V_{ub}^* V_{us} a_1 - V_{tb}^* V_{ts} \left[ 2 (a_3 + a_5) + \frac{1}{2} (a_7 + a_9) \right] \right] - A_0^{B \to \omega} (m_K^2) f_K V_{tb}^* V_{ts} \left[ a_4 - \frac{1}{2} a_{10} - \frac{2 (a_6 - a_8) m_K^2}{(m_b + m_d) (m_d + m_s)} \right] \right\} (\epsilon^* \cdot p_B)$ |
| $B^{+, 0} \to K^{+, 0}\phi$ | $+G_F \sqrt{2} F_1^{B \to K} (m_K^2) f_\phi m_\phi V_{ts}^* \left[ a_3 + a_4 + a_5 - \frac{a_7 + a_9 + a_{10}}{2} \right] (\epsilon^* \cdot p_B)$ |
III. ABSORPTIVE PART

The computation of the discontinuity of the charming penguins diagrams contributing to $B \to K^*\pi$ gives (cfr. analogous diagrams in figure 2 of Ref. [28])

$$\text{Disc}\,\mathcal{M}_{LD} = 2i \text{Im}\,\mathcal{M}_{LD} = i (2\pi)^2 \int \frac{d^4q}{(2\pi)^4} \delta_+ (q^2 - m_{D_s}^2) \delta_+ (p_{D_s}^2 - m_{D_s}^2) \times$$

$$\times \mathcal{M}(B \to D_s^{(*)} D^{(*)}) \mathcal{M}(D_s^{(*)} D^{(*)} \to K^*\pi) =$$

$$= i \frac{m_D}{16\pi^2 m_B} \sqrt{\omega^2 - 1} \int d^n \mathcal{M}(B \to D_s^{(*)} D^{(*)}) \mathcal{M}(D_s^{(*)} D^{(*)} \to K^*\pi) ,$$

where the integration is over the solid angle and a sum over polarizations is understood. A similar equation holds for the $K(\rho, \omega, \phi)$ final state. The amplitudes for the decays $B \to D_s^{(*)} D^{(*)}$ are computed by factorization and using information on the Isgur-Wise function (see below). Diagrams for the calculation of the amplitudes $D_s^{(*)} D^{(*)} \to K^*\pi$ are in Fig.1. A similar diagram can be drawn for the $D_s^{(*)} D^{(*)} \to K(\rho, \omega, \phi)$ amplitudes.

![Diagrams](image)

**FIG. 1:** Diagrams for the calculation of the amplitudes $D_s^{(*)} D^{(*)} \to K^*\pi$ amplitudes.

The effective lagrangian to compute these diagrams can be written as follows (see e.g. [31]):

$$\mathcal{L} = i < H_b v^\mu D_{\mu ba} \bar{H}_a > + ig < H_b \gamma_\mu \gamma_5 A_{\mu ba} \bar{H}_a > + i\beta < H_b v^\mu (V_\mu - \rho_\mu)_{ba} \bar{H}_a > + i\lambda < H_b \sigma^{\mu\nu} F_{\mu\nu} (\rho)_{ba} \bar{H}_a > .$$

(11)

Here $< ... >$ means the trace,

$$H = \frac{1 + \not{v}}{2} \left( -P_5 \gamma_5 - i\not{P} \right) ,$$

(12)

and $P_5$, $P^\mu$ are annihilation operators of the pseudoscalar $P$ and vector $V$ heavy mesons normalized as follows:

$$\langle 0 | P_5 | P \rangle = \sqrt{M_H} , \quad \langle 0 | P_\mu | V(\epsilon) \rangle = \epsilon_\mu \sqrt{M_H}$$

(13)
The first term in the lagrangian\(^{[14]}\) contains the kinetic term for the heavy mesons giving the \(P\) and \(P^*\) propagators, \(i/(2v\cdot k)\) and \(-i(g^\mu
u - v^\mu v^\nu)/(2v\cdot k)\) respectively. However, since the residual momentum is not small we will use the complete form of the propagators instead of their approximate expressions\(^{[28,29]}\). The interactions among heavy and light mesons are obtained by the other three terms. In the second term there are interactions among the heavy mesons and an odd number of pions coming from the expansion \(\xi\). Here \(\hat{A}_\mu = \frac{1}{2}(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger)\). Expanding the field \(\xi = \exp(i\pi/f)\) and taking the traces it provides the coupling in the lower vertices of Fig.\(^{[1]}\). The last two terms give the upper vertices of Fig.\(^{[1]}\) The octet of vector resonances (\(\rho, K^*, \text{etc.}\)) is introduced as the gauge multiplet according to the hidden gauge symmetry approach of Ref.\(^{[37]}\). We put

\[
\rho_\mu = i\frac{g_V}{\sqrt{2}}\hat{\rho}_\mu
\]

where \(\hat{\rho}\) is the usual hermitian \(3 \times 3\) matrix of flavor \(SU(3)\) comprising the nonet of light vector mesons and \(g_V\) is determined by vector meson dominance as follows: \(g_V \simeq 5.8\)\(^{[37]}\).

Also the parameter \(\beta\) can be fixed by vector meson dominance. This corresponds to assume that, for the heavy pseudoscalar mesons \(H\), the coupling of the electromagnetic current to the light quarks is dominated by the \(\rho, \omega, \phi\) vector mesons. This produces the numerical result

\[
\beta = \frac{\sqrt{2}m_V}{g_Vfv} \approx 0.9.
\]

For \(g\) we take the recent experimental result from the CLEO collaboration, obtained by the full width of \(D^{*+}\)\(^{[38]}\); this gives a value: \(g = 0.59 \pm 0.07 \pm 0.01\).

It can be noted that while these determinations consider soft pions we are interested in the coupling of a hard pion to \(D\) and \(D^*\). This introduces a correction that can be parametrized by a form factor, see below for a discussion.

Let us now consider the parameter \(\lambda\) in Eq.\(^{[11]}\). In\(^{[31]}\) one can found an estimate of this parameter based on the effective chiral lagrangian for heavy mesons. We present here an update of this analysis and new numerical results.

It is useful to begin by writing down the phenomenological heavy-to-heavy current in the leading \(1/m_Q\) approximation:

\[
J^{cb}_\mu = -\xi(v\cdot v')<H^a_\mu(1-\gamma_5)H^b_\mu>\quad ,
\]

which gives rise to

\[
< D(v')|\bar{c}\gamma_\mu b|B(v) > = \sqrt{M_B M_D} \xi(v\cdot v')(v + v'_\mu), \\
< D^*(v',\epsilon)|\bar{c}\gamma_\mu b|B(v) > = \sqrt{M_B M_D} \xi(v\cdot v')\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu\alpha\beta}v^\nu v'^\beta , \\
< D^*(v',\epsilon)|\bar{c}\gamma_\mu\gamma_5 b|B(v) > = \sqrt{M_B M_D} \xi(v\cdot v')i \left[(1 + v\cdot v')\epsilon^\mu_\mu - (\epsilon^*\cdot v)v'_\mu\right],
\]

where \(\xi(v\cdot v')\) is the Isgur-Wise form factor. Note that, by this definition, the choice of phases in Eq.\(^{[14]}\) agrees with\(^{[8]}\) and\(^{[9]}\). We also write the phenomenological heavy-to-light leading current for coupling to light pseudoscalar mesons of the heavy mesons in the multiplet \(H\)

\[
L^b_\mu = i\frac{\hat{F}}{2} <\gamma^\mu(1-\gamma_5)H_b\xi_{ba}^\dagger > .
\]

Here \(\hat{F}\) is related to leptonic decay constant of the pseudoscalar heavy meson \(B\) appearing in\(^{[6]}\) by

\[
f_B = \frac{\hat{F}}{\sqrt{M_B}} .
\]
Several numerical analyses based on QCD sum rules have been performed, see for a review e.g. 31. The result we shall use is

\[ \tilde{F} = 0.30 \pm 0.05 \text{ GeV}^{3/2} \] (20)

which is obtained neglecting radiative corrections since they are neglected also in the evaluation of the Isgur-Wise form factor the parameterization we use below is based on.

The effective theory approach gives predictions for the form factor \( V(q^2) \) at high \( q^2 \) and relates it to the parameter \( \lambda \). We can match this result with the theoretical calculations coming from Light Cone Sum Rules 39, 40 (LCSR) and lattice QCD 41, 42, 43. To do this we compute the form factor at \( q^2 \approx q_{\text{max}}^2 \) where it is dominated by the nearest low-lying vector meson pole. Computing the form factor at \( q^2 \approx q_{\text{max}}^2 \equiv (M_B - M_V)^2 \) and at leading order in \( 1/m_Q \), one gets 41:

\[ V(q_{\text{max}}^2) = \frac{g_V}{\sqrt{2}} \frac{\sqrt{M_P + M_V}}{\sqrt{M} + \Delta} \] (21)

where \( \Delta \) is the appropriate mass splitting, i.e., for the transition \( B \to K^* \), \( \Delta = m_{B^*} - m_B \approx 135 \text{ MeV} \). From the LCSR and lattice QCD analyses for the transition \( B \to K^* \) we infer the value \( V = 1.5 \) at \( q^2 = \tilde{q}^2 = 17 \text{ GeV}^2 \). In order to compare with Eq. (21) we assume that in the range \( q^2 \in (q^2, q_{\text{max}}^2) \) the form factors are dominated by the \( B_s^* \) simple pole. This gives \( V(q_{\text{max}}^2) = 1.8 \) and, as a result,

\[ \lambda = +0.56 \text{ GeV}^{-1} \] (22)

We note that to determine \( \lambda \) we used a positive sign of \( V(q_{\text{max}}^2) \). The same sign would be obtained using HQET and by assuming that the strange mass is large, see eq. 17. We also note that a different result for the phase and the magnitude of \( \lambda \) was obtained in 31. The difference in magnitude was due to the different methods used. Here we use LCSR and lattice QCD, while in 31 an extrapolation is used from the \( D \to K^* \) form factor. As to the phase, it is due to a different choice we use here for the phase factor of the heavy vector meson, see eq. 12.

To compute the matrix elements we use the following kinematics:

\[ p^\mu = m_B v^\mu = (m_B, \vec{0}) , \quad \rho^{\mu}_{D(*)} = m_D v^\mu , \quad q = p - p_{D(*)} \] (23)

where \( \omega^* = \frac{m_B^2 + m_D^2 - m_{D^*}^2}{2m_D m_B} \) and the angular integration is over the directions of the vector \( \vec{v}' = \vec{v}/\sqrt{\omega'^2 - 1} \). Our results are as follows:

\[ \mathcal{M}(B(v) \to D_s(q) D^*(\epsilon, v')) = -K (m_B + m_D) \epsilon^* \cdot v , \] (24)

\[ \mathcal{M}(B(v) \to D^*_s(q, \eta) D(v')) = -K m_{D^*} \eta^* \cdot (v + v') , \] (25)

\[ \mathcal{M}(B(v) \to D^*_s(q, \eta^*) D^*(\epsilon, v')) = -iK m_D \eta^* \epsilon^{\alpha*} (i\epsilon_{\alpha\lambda\mu\sigma} \epsilon^{\lambda\mu\sigma} - g_{\mu\alpha}(1 + \omega^*) + v_{\alpha} v_{\mu}' ) , \] (26)

where \( K = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs} a_2 \sqrt{m_B m_D} f_D \xi_I W(\omega^* ) \). On the other hand, for the \( K^*\pi \) final state we have

\[ \mathcal{M}(D_s(q) D^*(\epsilon, v') \to K^*(p_K, \epsilon) \pi(p_\pi)) = -\frac{2g F^2(|p_\pi|)}{f_\pi} \frac{g_V}{\sqrt{2}} \frac{\sqrt{m_{D^*}}}{m_D} \epsilon_{\lambda} \tilde{\epsilon}_{\sigma}^* \]

\times \left[ \frac{2\beta m_D q^2_p^2}{(m_D^2 - p_\pi^2 - m_D^2)} + \frac{4 \lambda m_{D^*} G^2(p_\pi, p_K, v')}{(m_D^2 - v'^2 - m_D^2)} \right] , \] (27)
\[ M(D_\pi^*(\eta, q)D(\epsilon)) \rightarrow K^*(p_K, \hat{\epsilon})(p_\pi) = -\frac{2g m_{D_\pi} F^2(|q_\pi|) g_V}{f_\pi} \sqrt{2} \sqrt{\frac{m_D}{m_{D_\pi}}} \frac{\eta \hat{\epsilon}_\sigma \hat{e}_\rho}{(m_{D_\pi}^2 - m_{D_\pi}^2)^2} \times \left[ 2\beta q^\sigma \left( +p_\pi^\alpha - \frac{v^\alpha p_\pi}{m_{D_\pi}} p_K^\alpha \right) - 4\lambda m_{D_\pi} H^{\sigma \lambda}(p_\pi, p_K, \epsilon, \epsilon') \right], \quad (28) \]

\[ M(D_\pi^*(\eta, q)D^*(\epsilon, \epsilon') \rightarrow K^*(p_K, \hat{\epsilon})(p_\pi) = +\frac{2g m_{D_\pi} F^2(|q_\pi|) g_V}{f_\pi} \sqrt{2} \sqrt{\frac{m_D}{m_{D_\pi}}} \frac{\epsilon^\alpha \epsilon_\beta \epsilon_\gamma \epsilon_\rho}{(m_{D_\pi}^2 - m_{D_\pi}^2)^2} \times \left[ 4\lambda m_{D_\pi} q_\alpha(p_K) p_{\pi 2}^\beta \delta_\alpha^\sigma \delta_\beta^\lambda + \frac{v_\alpha(p_\pi) \mu \delta_\rho^\lambda}{(m_{D_\pi}^2 - m_{D_\pi}^2)^2} (2\beta q^\sigma \delta_\alpha^\beta + 4\lambda m_{D_\pi}(p_{K 2}^\alpha \delta_\lambda^\beta - (p_K)^{\lambda \sigma} \hat{\epsilon}_\tau)) \right], \quad (29) \]

where

\[ G^{\sigma \lambda}(p_\pi, p_K, \epsilon, \epsilon') = -\left( (q \cdot q') \left( g^{\sigma \lambda}(p_K \cdot p_\pi) - p_{\pi 2}^\alpha p_K^\alpha \right) - (q \cdot p_\pi) \left( v^\sigma p_K^\lambda - g^{\sigma \lambda}(v' \cdot p_K) \right) - q^\lambda \left( p_{\pi 2}^\alpha (p_K \cdot \epsilon') - v^\sigma (p_K \cdot p_\pi) \right) \right), \quad (30) \]

\[ H^{\sigma \lambda}(p_\pi, p_K, \epsilon, \epsilon') = g^{\sigma \lambda} \left( p_K \cdot p_\pi - \frac{v_\rho' p_\pi (m_{K 2}^2 - p_K \cdot q)}{m_{D_\pi}^2} \right) - p_{K 2}^\lambda \left( p_{\pi 2}^\rho + \frac{v_\sigma p_\pi}{m_{D_\pi}^2} q^\tau \right). \quad (31) \]

Eq. (28) corresponds to the sum of diagrams (a) and (b) of Fig. 1. Eq. (29) corresponds to diagram (c) and, finally, Eq. (30) corresponds to the sum of diagrams (d) and (e). Similar results hold for the \( K(\rho, \omega, \phi) \) final states and we do not reproduce them here for brevity sake. \( F(|q_\pi|) \) is a form factor taking into account that in the vertex \( DD^*\pi \) the pion is not soft and therefore the coupling constant should be corrected. Its determination by a quark potential model is discussed in [28] using a quark model. The central value we use is \( F(|q_\pi|) = 0.065 \). In absence of more detailed information, the same form factor is adopted for the upper vertices in Fig. 1 as well.

IV. NUMERICAL RESULTS

Let us first consider the short distance contribution. Using, for the CKM matrix elements, \( \rho = 0.2296 \), \( \eta = 0.3249 \), \( A = 0.819 \), and for the involved form factors the value collected in Table IV of Ref. [47], one gets the results reported in Tab. [H] These values correspond to the following value of the angle \( \gamma \) of the unitarity triangle:

\[ \gamma = \arctan \left( \frac{\eta}{\rho} \right) \simeq 54.8^\circ. \quad (32) \]

Next we consider the long distance absorptive part. For its parameters we assume the values of previous Section. For the \( D_s \) decay constant we use \( f_{D_s} = \hat{F}/\sqrt{m_{D_s}} \), with \( \hat{F} \) given in (20); for the Isgur-Wise form factor we use the parameterization

\[ \xi(v \cdot v') = \left( \frac{2}{1 + v \cdot v'} \right)^{\hat{\beta}^2} \quad (33) \]

with \( \hat{\beta}^2 \simeq 1 \). This parametrization agrees with the results for the \( b \rightarrow c \) exclusive decays obtained by the CLEO and Belle Collaboration. It is a fit to the results obtained by the QCD sum rules method, see [31].
TABLE II: Theoretical values for $M_{SD}$, $M_{LD}$. Units are GeV.

| Process          | $M_{SD} \times 10^6$ | $\text{Im } M_{LD} \times 10^6$ |
|------------------|----------------------|---------------------------------|
| $B^+ \to K^{+} \pi^-$ | $+1.45$              | $-2.14$                        |
| $B^+ \to K^{+} \pi^0$ | $+1.02 - 0.79i$     | $-1.52$                        |
| $B^0 \to K^{*0} \pi^0$ | $-0.60 - 0.08i$     | $+1.52$                        |
| $B^0 \to K^{*0} \pi^-$ | $+0.85 - 1.01i$     | $-2.14$                        |
| $B^+ \to K^0 \rho^+$ | $+0.17$              | $-2.74$                        |
| $B^+ \to K^0 \rho^0$ | $+0.39 - 0.64i$     | $-1.94$                        |
| $B^0 \to K^0 \rho^0$ | $+0.48 - 0.11i$     | $+1.94$                        |
| $B^0 \to K^0 \rho^-$ | $-0.28 - 0.74i$     | $-2.74$                        |
| $B^+ \to K^+ \omega$ | $+0.27 - 0.63i$     | $-1.94$                        |
| $B^0 \to K^0 \omega$ | $+0.03 - 0.10i$     | $-1.94$                        |
| $B^+ \to K^+ \phi$  | $1.30$               | $-2.58$                        |
| $B^0 \to K^0 \phi$  | $1.30$               | $-2.58$                        |

The numerical results for the imaginary part of $M_{LD}$ are given in Tab. II. Typical sizes of the different contributions to $B^- \to \overline{K}^0 \pi^-$ are as follows. From the two terms in Eq. (27): $\text{Im } M_{LD} = -1.32 \times 10^{-8}$; from the terms in Eq. (28): $\text{Im } M_{LD} = -0.16 \times 10^{-8}$; from the terms in Eq. (29): $\text{Im } M_{LD} = -0.66 \times 10^{-8}$. For the $B^- \to K^0 \rho^-$ channel we find: $\text{Im } M_{LD} = -1.52 \times 10^{-8}$; $\text{Im } M_{LD} = -0.70 \times 10^{-8}$; $\text{Im } M_{LD} = -0.52 \times 10^{-8}$. It can be noted that the phase of $M_{SD}$ is purely weak while the phase in $M_{LD}$ is only due to strong interactions.

A. Branching ratios

From the results of Tab. II we can compute the Branching Ratios ($B$) and the CP asymmetries. As explained in the Introduction, we have not presented an evaluation of the real part of $A_{LD}$. Attempts for the $K\pi$ channel can be found in Ref. [28, 29]. Typical results are that the real part is of the same order of the imaginary part, but the uncertainties are large. To estimate the CP averaged Branching Ratios we add therefore to the imaginary part a real part as follows:

$$M_{LD} = R_{LD} + i I_{LD}$$

and consider the values $R_{LD} = 0, \pm I_{LD}$. The results are reported in Tab. III for the three values of $R_{LD}$, together with the results obtained considering only the short distance amplitude. They show the relevance of the long-distance contribution. The "best value" of $R_{LD}$ might be obtained by a fit to the available experimental data; these data are reported in Tab. IV and a comparison with the results of the previous table shows that the preferred values, except for the channels with $\phi$ in the final state, satisfy $R_{LD} \approx - I_{LD}$. Instead, the decay in $K^0 \phi$ prefer $R_{LD} \approx + I_{LD}$. To find a "best value" of $R_{LD}$ we defined a $\chi^2$ as:

$$\chi^2 = \sum_i \left( \frac{B_i^{th} - B_i^{exp}}{\sigma(B_i)} \right)^2$$

(35)

where the $B_i^{exp}$ are reported in Tab. IV and the $\sigma(B_i)$ are obtained summing quadratically the statistic and systematic errors in the same table. When the experiment provides an asymmetric error $\sigma_i$, a conservative error was assumed: $\sigma = \text{Max}(\sigma_1, \sigma_2)$. For the decay $B^0 \to K^*0 \pi^0$ we put $B_i^{exp} = 0$ and the corresponding
two multiplets (octet+singlet) of the light vector mesons. This gives as a result not expect the validity of the nonet hypothesis we have also tried a two parameter fit, corresponding to the same $\chi$-error was fixed to be equal to the experimental upper limit $\sigma = 3.6 \times 10^{-6}$. We have first attempted a fit with only one real parameter $r = R_{LD}/I_{LD}$, which corresponds to the use of the SU(3) nonet symmetry for the light vector mesons. The minimum value of $\chi^2(=12.1)$ is obtained for $r = 1.207$. Since however we do not expect the validity of the nonet hypothesis we have also tried a two parameter fit, corresponding to the two multiplets (octet+singlet) of the light vector mesons. This gives as a result $R_{LD} = -0.954 I_{LD}$ for the ($\rho$, $K^*$, $\omega$) set of particles and two solutions, $R_{LD} = -0.201 I_{LD}$ and $R_{LD} = +1.21 I_{LD}$, for the $\phi$ with the same $\chi^2 = 7.8$. The former solution would be preferred because it represents a less important deviation of the SU(3) nonet symmetry. Even if the two solutions produce the same values of the Branching Ratios they would produce different CP asymmetries. The results we find are in Tab. LV. Besides our data, we also present two model calculations presented in [30], where an analysis of all the PV (not only strange) final states is performed. The first model (called in that paper Scenario 1) contains short distance terms (QCD factorization) and unconstrained $\gamma$ angle. Other theoretical determinations based on QCD factorization are in Ref. [40], e.g. $B(B^0 \to K^+\pi^-) = 12.1$. The authors of [30] do not agree with the conclusions of Du et al., Ref. [40], because, differently from them, they include the $K^*\pi$ channels. The second model (Scenario 2) uses QCD factorization and charming penguins as in the present paper, but differently from this paper, where only two real free parameters are used, they employ two universal complex amplitudes multiplied by a computed Clebsch-Gordan coefficient. It is worthwhile to note that a significant agreement with the data is obtained with our simple hypothesis. We also note that the value $\gamma$ of Eq. [42] obtained by a global fit to the CKM matrix [47] is compatible with the $B \to K^*\pi, K(\rho,\omega,\phi)$ Branching Ratios only if the charming penguin diagrams are included.

| Process          | $B (SD$ only) | $B (SD+LD)$ |
|------------------|--------------|-------------|
| $B^+ \to K^{*0}\pi^+$ | 1.96         | (4.71, 6.22, 16.3) |
| $B^+ \to K^{*+}\pi^0$ | 1.56         | (5.20, 5.95, 11.0) |
| $B^0 \to K^{*0}\pi^0$ | 0.32         | (2.51, 2.10, 5.66) |
| $B^0 \to K^{*+}\pi^-$ | 1.50         | (9.94, 9.13, 16.2) |
| $B^+ \to K^0\rho^0$   | 0.03         | (13.1, 6.99, 14.8) |
| $B^0 \to K^0\rho^0$   | 0.21         | (7.88, 3.07, 4.71) |
| $B^0 \to K^+\rho^-$   | 0.54         | (18.1, 10.4, 15.6) |
| $B^+ \to K^+\omega$   | 0.44         | (10.7, 6.21, 8.73) |
| $B^0 \to K^0\omega$   | 0.01         | (6.70, 3.58, 6.91) |
| $B^+ \to K^+\phi$     | 1.55         | (7.58, 7.63, 19.8) |
| $B^0 \to K^0\phi$     | 1.42         | (6.98, 7.02, 18.3) |
The result we find agrees with the data:

We also get

We can similarly compute the asymmetries for the $K^*\pi$, $K\rho$ channels, defined by

with analogous definitions for the $K\rho$ channel, with $A_\pi \rightarrow A_\rho$ and the changes $K^* \rightarrow K$ and $\pi \rightarrow \rho$. 

TABLE IV: CP averaged Branching Ratios $\mathcal{B}$ (units $10^{-6}$). In the second column theoretical values computed using the present model. In third column and fourth column theoretical computations based on Ref. [30]: Scenario (Sc.) 1 refers to QCD with factorization and free $\gamma$; Scenario 2 refers to QCD+Charming penguins with constrained $\gamma$ (see text). Experimental data are from CLEO [1,2], BaBar [3,4], and Belle [5,6] or averages from these data.

| Process | $\mathcal{B}$ (this paper) | $\mathcal{B}$ ([30], Sc. 1) | $\mathcal{B}$ ([30], Sc. 2) | $\mathcal{B}$ (Exp.) |
|---------|-----------------|-----------------|-----------------|-----------------|
| $B^+ \rightarrow K^{*0}\pi^0$ | 15.6 | 7.889 | 11.080 | 12.1 ± 3.1 (av.) |
| $B^- \rightarrow K^{*-}\pi^0$ | 8.44 | 7.303 | 8.292 | 7.1^{+1.4}_{-1.9} ± 1.0 (< 31) (CLEO) |
| $B^0 \rightarrow \bar{K}^{*0}\pi^0$ | 5.61 | < 3.6 (CLEO) |
| $B^0 \rightarrow K^{*+}\pi^-$ | 12.0 | 9.760 | 10.787 | 19.3 ± 5.2 (av.) |
| $B^+ \rightarrow K^0\rho^+$ | 14.1 | 7.140 | 14.006 |
| $B^+ \rightarrow K^+\rho^0$ | 8.56 | 1.882 | 5.665 | 8.9 ± 3.6 (av.) |
| $B^0 \rightarrow K^0\rho^0$ | 4.86 | 5.865 | 8.893 |
| $B^0 \rightarrow K^0\rho^+$ | 11.6 | 6.531 | 14.304 | 15.9 ± 4.7 (av.) |
| $B^+ \rightarrow K^+\omega$ | 6.19 | 2.398 | 6.320 | 9.2^{+2.5}_{-2.4} ± 1.0 (CLEO); < 4 (BaBar); < 7.9 (Belle) |
| $B^0 \rightarrow K^0\omega$ | 6.27 | 2.318 | 5.606 | 6.3 ± 1.8 (av.) |
| $B^+ \rightarrow K^+\phi$ | 9.11 | 8.941 | 9.479 | 8.9 ± 1.2 (av.) |
| $B^0 \rightarrow K^0\phi$ | 8.39 | 8.360 | 8.898 | 8.7 ± 1.4 (av.) |
TABLE V: Theoretical values for the asymmetries; for the definitions see (39).

| Asymmetry | $A_{e}^{0}$ | $A_{e}^{0+}$ | $A_{e}^{0-}$ | $A_{u}^{0}$ | $A_{u}^{0+}$ | $A_{u}^{0-}$ | $A_{p}^{0}$ | $A_{p}^{0+}$ | $A_{p}^{0-}$ |
|------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| $A_{CP}$   | 0.27        | 0           | 0.31        | -0.04       | 0.27        | 0           | 0.30        | -0.08       |

The results are presented in Table V. They have a peculiar pattern and will therefore provide a crucial test of the present model when future experimental data for these asymmetries are available.

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[1] C. P. Jessop et al. (CLEO), Phys. Rev. Lett. **85**, 2881 (2000), hep-ex/0006008.
[2] E. Eckhart et al. (CLEO), Phys. Rev. Lett. **89**, 251801 (2002).
[3] B. Aubert et al. (BABAR) (2001), hep-ex/0109007.
[4] B. Aubert et al. (BABAR), Phys. Rev. Lett. **87**, 221802 (2001), hep-ex/0108017.
[5] R. S. Lu et al., Phys. Rev. Lett. **89**, 191801 (2002), hep-ex/0207019.
[6] J. R. Buras and R. Fleischer, Phys. Lett. **11**, 93 (1999), hep-ph/9810260.
[7] M. Gronau and D. Wyler, Phys. Lett. **265**, 172 (1991).
[8] R. Fleischer and T. Mann, Phys. Rev. **D57**, 2752 (1998), hep-ph/9704423.
[9] M. Gronau, J. L. Rosner, and D. London, Phys. Rev. Lett. **73**, 21 (1994), hep-ph/9404282.
[10] O. F. Hernandez, D. London, M. Gronau, and J. L. Rosner, Phys. Lett. **B333**, 500 (1994), hep-ph/9404281.
[11] M. Gronau, O. F. Hernandez, D. London, and J. L. Rosner, Phys. Rev. **D50**, 4529 (1994), hep-ph/9404283.
[12] M. Gronau, O. F. Hernandez, D. London, and J. L. Rosner, Phys. Rev. **D52**, 6356 (1995), hep-ph/9504326.
[13] R. Fleischer, Eur. Phys. J. **C6**, 451 (1999), hep-ph/9804233.
[14] M. Gronau and J. L. Rosner, Phys. Rev. **D57**, 6843 (1998), hep-ph/9711246.
[15] M. Neubert and J. L. Rosner, Phys. Lett. **B411**, 403 (1998), hep-ph/9808493.
[16] M. Neubert and J. L. Rosner, Phys. Rev. Lett. **81**, 5076 (1998), hep-ph/9809311.
[17] M. Neubert, JHEP **02**, 014 (1999), hep-ph/9812396.
[18] A. J. Buras and R. Fleischer, Eur. Phys. J. **C6**, 97 (2000), hep-ph/0003323.
[19] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999), hep-ph/9905312.
[20] Y.-Y. Keum, H.-n. Li, and A. I. Sanda, Phys. Lett. **B504**, 6 (2001), hep-ph/0004004.
[21] Y. Y. Keum, H.-N. Li, and A. I. Sanda, Phys. Rev. **D63**, 054008 (2001), hep-ph/0004173.
[22] Y.-Y. Keum and H.-n. Li, Phys. Rev. **D63**, 074006 (2001), hep-ph/0006001.
[23] C.-D. Lu, K. Ukai, and M.-Z. Yang, Phys. Rev. **D63**, 074009 (2001), hep-ph/0004213.
[24] E. Kou and A. I. Sanda, Phys. Lett. **B525**, 240 (2002), hep-ph/0106159.
[25] P. Colangelo, G. Nardulli, N. Paver, and Riazuddin, Z. Phys. **C45**, 575 (1990).
[26] M. Ciuchini, F. Franco, G. Martinelli, and L. Silvestrini, Nucl. Phys. **B501**, 271 (1997), hep-ph/9703353.
[27] M. Ciuchini, F. Franco, G. Martinelli, M. Pierini, and L. Silvestrini, Phys. Lett. **B515**, 33 (2001), hep-ph/0104126.
[28] C. Isola, M. Ladisa, G. Nardulli, T. N. Pham, and P. Santorelli, Phys. Rev. **D64**, 014029 (2001), hep-ph/01011118.
[29] C. Isola, M. Ladisa, G. Nardulli, T. N. Pham, and P. Santorelli, Phys. Rev. **D65**, 094005 (2002), hep-ph/0110411.
[30] R. Aleksan, P. F. Giraud, V. Morenas, O. Pene, and A. S. Safir (2003), hep-ph/0301165.
[31] R. Casalbuoni et al., Phys. Rept. **281**, 145 (1997), hep-ph/9605342.
[32] R. Fleischer, Z. Phys. **C58**, 483 (1993).
[33] R. Fleischer, Z. Phys. **C62**, 81 (1994).
[34] M. Ciuchini, E. Franco, G. Martinelli, L. Reina, and L. Silvestrini, Phys. Lett. B316, 127 (1993), hep-ph/9307364.
[35] M. Ciuchini, E. Franco, G. Martinelli, and L. Reina, Nucl. Phys. B415, 403 (1994), hep-ph/9304257.
[36] A. J. Buras, in Probing the Standard Model of Particle Interactions, F. and Gupta, R. eds. (Elsevier Science B.V., 1998), hep-ph/9806471.
[37] M. Bando, T. Kugo, and K. Yamawaki, Phys. Rept. 164, 217 (1988).
[38] S. e. a. Ahmed (CLEO), Phys. Rev. Lett. 87, 251501 (2001), hep-ex/0108013.
[39] P. Ball and V. M. Braun, Phys. Rev. D58, 094016 (1998), hep-ph/9805422.
[40] P. Colangelo and A. Khodjamirian (2000), hep-ph/0010175.
[41] J. M. Flynn et al. (UKQCD), Nucl. Phys. B461, 327 (1996), hep-ph/9506398.
[42] D. R. Burford et al. (UKQCD), Nucl. Phys. B447, 425 (1995), hep-lat/9503002.
[43] L. Del Debbio, J. M. Flynn, L. Lellouch, and J. Nieves (UKQCD), Phys. Lett. B416, 392 (1998), hep-lat/9708008.
[44] R. Casalbuoni et al., Phys. Lett. B299, 139 (1993), hep-ph/9211248.
[45] A. Ali and C. Greub, Phys. Rev. D57, 2996 (1998), hep-ph/9707251.
[46] D.-s. Du, J.-f. Sun, D.-s. Yang, and G.-h. Zhu, Phys. Rev. D67, 014023 (2003), hep-ph/0209233.
[47] M. Ciuchini et al., JHEP 07, 013 (2001), hep-ph/0012308.