I. INTRODUCTION

A major goal of the study of heavy-ion collisions at relativistic and ultra-relativistic collision energies is to explore the properties of strongly interacting matter at finite temperatures and densities [1, 2]. When two colliding nuclei hit each other, the nuclear matter is compressed, and a large amount of energy is deposited in a small spatial volume. This results in the creation of a deconfined state of quarks and gluons which is deposited in the nuclear collision; more precisely, they can leave the zone of hot and dense matter than the size and the lifetime of the fireball. Consequently, they can leave the zone of hot and dense matter undisturbed. Since electromagnetic probes are emitted in a large variety of processes over the whole lifetime of the fireball, the measured spectra reflect the time-integrated evolution of the thermodynamic properties of the system. While this allows to obtain convoluted information about
the properties of matter it also poses a serious challenge for the theoretical description. On the one hand, one needs to identify the relevant microscopic processes that contribute to dilepton and photon emission and to determine the corresponding production rates. On the other hand it is important to give a realistic description of the complete reaction dynamics.

The intense experimental study of photon and dilepton production in the high-energy regime (at SPS [7–9], RHIC [10–13], and LHC [14] energies), but also for very low collision energies as measured at SIS18 and BEVALAC [15–18] in comparison to theoretical model calculations has significantly enhanced our knowledge of the reaction dynamics and the properties of matter in the hot and dense medium created in a heavy-ion reaction. The importance of partonic emission for the correct theoretical description of the high-mass region of dilepton invariant mass spectra and the high-\(p_t\) photon spectra has been pointed out [19–21] and the various different hadronic contributions (especially for the photon production channels) could be identified [22–24]. Nevertheless, the most important finding was the large influence of the baryonic matter on the vector mesons’ spectral shape. Especially in the case of the \(\rho\) meson this causes a strong broadening of the spectral function with small mass shifts [25–28]. This effect has been observed as an enhancement in the low-mass region of the dilepton invariant mass spectra and also shows up as a stronger low-momentum thermal photon yield. Note that the \(\rho\) broadening is most dominant at low collision energies, where one obtains the largest baryochemical potentials, but even at RHIC energies baryonic effects are by far not negligible.

However, there still remains an up to now unexplored energy window between the \(E_{\text{lab}} = 1 – 2\) A GeV dilepton measurements by the DLS and HADES Collaborations and the CERES results for \(E_{\text{lab}} = 40\) A GeV. The future Compressed Baryonic Matter (CBM) experiment at the Facility for Anti-Proton and Ion Research (FAIR) with the SIS 100/300 accelerator provides the unique possibility to study heavy-ion collisions with beam energies from 2 up to 35A GeV and will therefore enable us to get an insight into exactly that regime of the phase diagram of highest baryon densities where no dilepton or photon measurements have been performed till now [29, 30]. In addition, also phase II of the Beam Energy Scan (RES) program at RHIC will allow to perform measurements in fixed-target mode at lab-frame energies of 7.7, 9.1, 11.5, 14.5 and 19.6A GeV, i.e., in the same collision energy range as FAIR [31]. Further complementary investigations are also planned for the NICA in Dubna [32].

From a theoretical point of view, the handling of this energy range is quite challenging, as the transition from a purely hadronic fireball at low collision energies to the creation of a partonic phase is expected here. Furthermore, at the high baryochemical potentials which still dominate the fireball at these energies, a first order phase transition from a hadron gas to the QGP is assumed, in contrast to the situation at RHIC or the LHC where a cross-over is predicted by Lattice QCD calculations [33].

Although transport models were applied successfully to describe electromagnetic observables in heavy-ion collisions [34–36], they generally have some shortcomings when describing very hot and dense systems. In detail, problems include the following aspects: Firstly, while the Boltzmann approach works quite well for quasi-particles of infinite lifetime, for broad resonances as the \(\rho\) meson a correct description is challenging. Furthermore, in dense matter the intervals between scatterings become extremely short and will consequently modify the spectral characteristics of the single particles (collisional broadening). To describe the off-shell dynamics correctly a transport description with dynamical spectral functions following the description of Kadanoff and Baym [37] is required. However, a practical implementation of this is currently not possible. Secondly, in a dense medium not only binary scatterings will occur but also multi-particle interactions play a role, which is beyond the capabilities of the common transport models. And finally, the microscopic models usually concentrate on either the transport of hadrons or partons. However, modelling a transition from an initially up-heating hadron gas to a deconfined phase and the later polarization when the system cools down is extremely difficult to realize within a transport approach.

There have been several investigations over the last years on these aspects (see, e.g., Refs. [38–44]), but a full treatment of all these issues is still beyond the scope of present investigations.

On the other side, the short mean free paths of particles in a medium might suggest to treat the reactions from a macroscopic point-of-view. However, approaches as simple fireball expansion models [45] or hydrodynamics [46], which have been successfully applied for SPS, RHIC and LHC energies, also have their shortcomings in the FAIR energy regime for three main reasons: Firstly, the separation of the fireball expansion from dynamics of the initial projectile-target dynamics is not applicable; secondly, the often applied simplification to assume a 2+1-dimensional boost invariant geometry is not possible; and finally, the time scale necessary for an approximate thermal equilibration of the fireball will be longer due to the slower overall evolution of the reaction and the lower temperatures reached.

To avoid the disadvantages of both pictures the coarse-graining method has been developed, based on previous studies [47], and was successfully applied to describe dilepton production at SPS and SIS18 energies [27, 28, 48]. The approach represents a combination of the microscopic picture from the underlying transport simulations with the resulting description of the dynamics in terms of the macroscopic quantities temperature and chemical potential. By averaging over many events one can extract the local energy and baryon densities at each space-time point from the transport simulations and use an equation of state to determine the corresponding
temperature and baryochemical potential. With this the calculation method of thermal dilepton and photon emission by application of full in-medium spectral functions is straightforward, employing the rates available from equilibrium quantum field theory.

In the present work the coarse-graining approach is used to calculate photon and dilepton spectra with focus on the FAIR energy regime, but naturally the results also serve as a theoretical prediction for the fixed-target measurements of the RHIC-BES since the prospected collision energies of both experimental programs overlap. Although the details of the future experimental set-ups are not yet determined, the results shall provide a general baseline calculation for the interpretation of the measurements to be conducted. Furthermore, it shall be investigated if and how one can obtain valuable information on the properties of matter from the measured spectra and discriminate between several effects that might influence the dilepton and photon results. In detail, we will concentrate on the following three aspects: The modification of the thermal emission pattern by high baryochemical potentials, signals for a phase transition or the creation of a deconfined phase and possible non-equilibrium effects on the thermal rates.

This paper is structured as follows. In Section II the coarse-graining approach will be presented. Thereafter, in Section III we will introduce the various microscopic sources for thermal emission of photons and dileptons and in short discuss the non-thermal cocktail contributions. In Section IV the results for the fireball evolution and the photon and dilepton spectra at FAIR energies are shown. The results are used to systematically analyse in which way it might be possible to discriminate between different scenarios for the fireball evolution in Section V. We conclude the present work with a summary and an outlook to subsequent investigations.

II. THE MODEL

While microscopic transport models describe the reaction dynamics of a heavy-ion collision in terms of many different degrees of freedom, the general idea of the coarse-graining approach is that in principle only a very reduced amount of the provided information is necessary to account for the thermal production of electromagnetic probes. The microscopic information about all individual particles and their specific properties – such as mass, charge and momentum – are ignored and the whole dynamics is reduced to macroscopic quantities which are assumed to fully determine the local thermodynamic properties: The energy and particle densities.

The coarse-graining method combines two advantages: On the one hand the collision dynamics is still based on the microscopic transport evolution and thereby gives a very nuanced picture of the entire collision evolution, on the other hand the reduction to macroscopic state variables enables an easy application of in-medium spectral functions from equilibrium quantum field theory calculations.

In the following the ingredients of the approach are presented in detail.

A. Ultrarelativistic Quantum Molecular Dynamics

The underlying microscopic input for the present calculations stems from the Ultra-relativistic Quantum Molecular Dynamics (UrQMD) approach [49–51]. It is a non-equilibrium microscopic transport model based on the principles of molecular dynamics [52–53]. It constitutes an effective Monte Carlo solution to the relativistic Boltzmann equation and connects the propagation of hadrons on covariant trajectories with a probabilistic description of the hadron-hadron scattering processes. To account for the quantum nature of the particles, each hadron is represented by a Gaussian density distribution, and quantum statistical effects such as Pauli blocking are considered [54].

The model includes all relevant mesonic and baryonic resonances up to a mass of 2.2 GeV/c². Production of particles occurs via resonant scattering of particles (e.g., $NN \rightarrow N\Delta$ or $\pi\pi \rightarrow \rho$) or the decay of higher resonances, e.g., the process $\Delta \rightarrow N\pi$. The individual interaction and decay processes are described in terms of measured and extrapolated hadronic cross-sections and branching ratios. For collision energies above $\sqrt{s_{NN}} = 3$ GeV also the excitation of strings is possible.

B. Coarse-graining of microscopic dynamics

Within the UrQMD model the particle distribution function $f(\vec{x},\vec{p},t)$ is determined by the space and momentum coordinates of all the different particles in the system at a certain time. However, due to the finite number $h$ of hadronic particles involved and produced in a heavy-ion collision, one needs to take the average over a large ensemble of events to obtain a smooth phase-space distribution of the form

$$f(\vec{x},\vec{p},t) = \left\langle \sum_h \delta^{(3)}(\vec{x} - \vec{x}_h(t))\delta^{(3)}(\vec{p} - \vec{p}_h(t)) \right\rangle. \quad (1)$$

Note that this distribution is Lorentz invariant if all particles are on the mass-shell, as provided in our case. Due to the non-equilibrium nature of the model, one will of course have to extract the particle distribution function locally. In the present approach, this is done by the use of a grid of small space-time cells where for each of these cells we determine the (net-)baryon four-flow and the
energy-momentum tensor according to the relations

\[
\begin{align*}
J_B^\mu &= \int d^3 p \frac{p^\mu}{p^0} f^B(\vec{x}, \vec{p}, t), \\
T^{\mu \nu} &= \int d^3 p \frac{p^\mu p^\nu}{p^0} f(\vec{x}, \vec{p}, t).
\end{align*}
\]

In practice, the integration is done by summing over the \( \delta \) functions. As we use cells of finite size, we have

\[
\delta^{(3)}(\vec{x} - \vec{x}_h(t)) = \begin{cases} 
\frac{1}{\Delta V} & \text{if } \vec{x}_h(t) \in \Delta V, \\
0 & \text{otherwise}
\end{cases}
\]

and in the limit for small volumes the density of some observable \( \hat{O} \) then becomes

\[
\int d^3 p \ \hat{O}(\vec{x}, \vec{p}, t) f(\vec{x}, \vec{p}, t) = \frac{1}{\Delta V} \left( \sum_{h} \hat{O}(\vec{x}_h, \vec{p}_h, t) \right).
\]

Consequently, Eqns. (2) and (3) take the form

\[
\begin{align*}
T^{\mu \nu} &= \frac{1}{\Delta V} \left( \sum_{i=1}^{N_\delta} \frac{p_i^\mu p_i^\nu}{p_i^0} \right), \\
J_B^\mu &= \frac{1}{\Delta V} \left( \sum_{i=1}^{N_\delta} \frac{p_i^\mu \pm p_i^\nu}{p_i^0} \right).
\end{align*}
\]

Having obtained the baryon flow, we can boost each cell into the rest frame as defined by Eckart, where \( J_B^\mu \) is \((p_B, \vec{0})\). The according transformation of the energy-momentum tensor provides the rest frame energy density.

C. Non-equilibrium dynamics

While macroscopic models usually introduce thermal and chemical equilibrium as an ad-hoc assumption, microscopic simulations – in the present case the UrQMD simulations – are based on the description of single particle-particle interactions and non-equilibrium will be the normal case. Consequently, we have to account for these deviations from equilibrium in such a manner that we can reliably apply equilibrium spectral functions to calculate the emission of photons and dileptons.

In general it is difficult to really determine to which degree a system has reached equilibrium. Basically there are two dominant effects, which may serve as indicators for thermal and chemical equilibration: The momentum-space anisotropies and the appearance of meson-chemical potentials.

1. Thermal non-equilibrium

Regarding thermal equilibration, it was found in microscopic simulations that independent of the collision energy the system needs a time of roughly 10 fm/c after the beginning of the heavy-ion collision until the transverse and longitudinal pressures are approximately equal. The pressure anisotropy stems from the initial strong compression along the beam axis when the two nuclei first hit and traverse each other. As thermal equilibrium requires isotropy, one will obtain too high values for the energy density in highly anisotropic cells. To obtain effective quantities that account for the thermal properties in the system we apply a description that explicitly includes the momentum-space anisotropies and in which the energy momentum-tensor is assumed to take the form

\[
T^{\mu \nu} = (\epsilon + P_\perp) u^\mu u^\nu - P_\perp g^{\mu \nu} - (P_\perp - P_\parallel) u^\mu v^\nu .
\]

where \( P_\perp \) and \( P_\parallel \) denote transverse or parallel pressure components, respectively; \( u^\mu \) and \( v^\mu \) are the cell’s four-velocity and the four-vector of the beam direction. The effective energy density \( \varepsilon_{\text{eff}} \) is obtained via the generalized equation of state for a Boltzmann-like system of the form

\[
\varepsilon_{\text{eff}} = \frac{\varepsilon}{r(x)},
\]

where the relaxation function \( r(x) \) and its derivative \( r'(x) \) are defined by

\[
r(x) = \begin{cases} 
\frac{x^{-1/3}}{2} & \text{for } x \leq 1 \\
1 + \frac{\arctan \sqrt{1-x}}{\sqrt{1-x}} & \text{for } x \geq 1
\end{cases}
\]

and \( x = (P_\parallel / P_\perp)^{3/4} \) denotes the pressure anisotropy.

As we have shown in our previous investigation at SPS energies, with this description the effective energy density deviates from the nominal one only for the very initial stage of the reaction, where the pressure components differ by orders of magnitude. Nevertheless, the effective energy density \( \varepsilon_{\text{eff}} \) allows to calculate meaningful \( T \) and \( \mu_B \) values for these cells. After the very initial collision phase the differences still exist, but have hardly any influence on the energy density, so that we can assume that these cells are in approximate local equilibrium.

2. Chemical non-equilibrium

Chemical equilibration is a more difficult problem, but one obvious deviation in microscopic models is the appearance of meson chemical potentials, especially for the case of pions as these are the most abundantly produced particles. As the meson number is not a conserved quantity in strong interactions (in contrast to, e.g., the baryon number) meson chemical potentials can only show up if the system is out of chemical equilibrium. While pion chemical potentials are introduced in fireball models for the stage after the chemical freeze-out to obtain the correct final pion yields, in non-equilibrium transport models they intrinsically appear in the early stages of the reaction when a large number of pions is produced in
many initial scattering processes [59]. At higher collision energies this mainly happens via string excitation. The pion (and kaon) chemical potentials have a large influence on the photon and dilepton production rates as an overpopulation of pions increases the reactions in many important channels, for example $\pi\pi \rightarrow p \rightarrow \gamma/\gamma'$ [60].

To implement the non-equilibrium effects in the calculations, we extract the pion and kaon chemical potentials in Boltzmann approximation as [61]

$$\mu_{\pi/K} = T \ln \left( \frac{2\pi^2 n}{g T^2 m^2 K_2 \left( \frac{m}{T} \right)} \right),$$  \tag{10}

where $n$ denotes the cell’s pion or kaon density and $K_2$ the Bessel function of the second kind. The degeneracy factor $g$ is 3 in the case of pions and 2 for kaons. Note that the Boltzmann approximation is in order here, as the mesons in the transport model also account for Boltzmann statistics and no Bose effects are implemented. However, while for a Bose gas the chemical potential is limited to the meson’s mass, in principle one can get higher values for $\mu_\pi$ or $\mu_K$ here in rare cases. As such values are non-physical, we assume that the maximum values to be reached are 140 MeV for $\mu_\pi$ and 450 MeV for $\mu_K$.

D. Equation of state

Once the rest-frame properties of each cell are determined, an equation of state (EoS) is necessary to describe the thermodynamic system of the hot and dense matter in the cell under the given set of state variables, i.e., the (effective) local energy density and the local net densities of conserved charges (for the strong interactions considered here the baryon number is the relevant quantity). For the present calculations we apply a hadron gas equation of state (HG-EoS) that includes the same hadronic degrees of freedom as the underlying transport model [62]. The EoS allows us to extract the temperature and baryochemical potential for an equilibrated hadron gas at a given energy and baryon density. It is similar to the result obtained for UrQMD calculations in a box in the infinite time limit, when the system has settled to an equilibrated state.

However, in the FAIR energy regime a purely hadronic description of the evolving hot and dense fireball will not be sufficient. As the temperatures will exceed the critical temperature $T_c$, a transition from hadronic to partonic matter has to be implemented, and the dynamic evolution of the created Quark-Gluon Plasma has to be considered. On the other hand, it is necessary to keep the EoS consistent with the underlying dynamics which is purely hadronic. In our previous study at SPS energies [27], we supplemented the HG-EoS with a Lattice equation of state [63] for temperatures above $T_c \approx 170$ MeV, in line with the lattice results. In the range around the critical temperature the results of the HG-EoS and the Lattice EoS match very well for $\mu_B \approx 0$, while significantly higher temperatures are obtained with the latter if one reaches temperatures significantly above $T_c$.

However, this procedure is problematic for the present study, as the transition from a hadronic to a partonic phase and back is assumed to take place at finite values of $\mu_B$ at FAIR energies, whereas the Lattice EoS is restricted to vanishing chemical potential. To avoid discontinuities in the evolution, we confine ourselves to the application of the HG-EoS, but with the assumption that the thermal emission from cells with a temperature above 170 MeV stems from the QGP (i.e., we employ partonic emission rates). This should be in order, as the temperatures will not lie too much above the critical temperature at the energies considered in the present work, where the deviations from a full QCD-EoS explicitly including a phase transition are expected to be rather moderate.

Nevertheless, we once again remind the reader that the underlying microscopic description is purely hadronic and it remains to be studied which consequences a phase transition has at the microscopic level of the reaction dynamics.

III. PHOTON & DILEPTON RATES

The mechanisms which contribute to the thermal emission of photons and dileptons are the same. Any process that can produce a real photon $\gamma$ can also produce a virtual (massive) photon $\gamma^*$, decaying into a lepton pair. However, due to the different kinematic regimes probed by photons and dileptons, the importance of the single processes varies. In the following the various sources of thermal radiation considered in this work are presented.

Determining quantity for the thermal emission of real photons as well as virtual photons (i.e., dileptons) is the imaginary part of the retarded electromagnetic current-current correlation function $\Pi^{\text{ret}}_{\text{em}}$, to which the rates are directly proportional. It represents a coherent summation of the cuts of those Feynman diagrams which are describing the different processes contributing to thermal $\gamma$ and $\gamma^*$ emission, and therefore accounts for the photon or dilepton self-energy. In the rest frame, the thermal emission can be calculated according to [29]

$$\frac{dN_B}{d^4x d\vec{q}} = -\frac{\alpha_{\text{em}}^2 L(M)}{\pi^2 M^2} f_B(q;T) \times \text{Im} \Pi_{\text{em}}^{\text{ret}}(M, \vec{q}; \mu_B, T),$$  \tag{11}

$$q_0 \frac{dN_\gamma}{d^4x d\vec{q}} = -\frac{\alpha_{\text{em}}^2}{\pi^2} f_B(q;T) \times \text{Im} \Pi_{\text{em}}^{T,\text{ret}}(q_0 = |\vec{q}|; \mu_B, T).$$  \tag{12}

Here $L(M^2)$ is the lepton phase-space factor (which plays a significant role only for masses close to the threshold $2m_l$ and is approximately one otherwise), $f_B$ the Bose distribution function, and $M$ the invariant mass of a lepton pair. Note that only the transverse polarization of the
current-current correlator enters for the photon rate, as the longitudinal projection vanishes at the photon point, i.e., for $M = 0$.

### A. Thermal rates from hadronic matter

#### 1. Vector meson spectral functions

In hadronic matter, all the spectral information of a hadron with certain quantum numbers is specified in $\text{Im} \Pi_{\text{had}}^{(\text{ret})}$. Assuming that Vector Meson Dominance (VMD) is valid, the correlator can be directly related to the spectral functions of vector mesons. The important challenge for theoretical models is to consider the modifications of the particle’s self-energy inside a hot and dense medium. Different calculations of in-medium spectral functions exist, however not many of them fully consider the effects of temperature and finite chemical potential. We here apply a hadronic many-body calculation from thermal field theory for the spectral functions of the $\rho$ and $\omega$ mesons, which has proven to successfully describe photon and dilepton spectra from SIS 18 to LHC energies. The different calculation of the different contributions to the $\rho$ spectral function takes three different effects into account: The modification due to the pion cloud, the direct scattering of the $\rho$ with baryons (nucleons as well as excited $N^*$ and $\Delta^*$ resonances) and with mesons ($\pi, K, \rho, \ldots$). While the pion cloud effects also contribute in the vacuum, the scattering processes only show up in the medium. For the $\omega$, the situation is slightly more complex since this meson basically constitutes a three-pion state. The vacuum self-energy is represented by a combination of the decays into $\rho + \pi$ or three pions, respectively. Further the inelastic absorption $\omega \pi \rightarrow \pi \pi$ and the scattering processes with baryons as well as the pion (i.e., $\omega \pi \rightarrow b_1$) are implemented. In the same manner as in our previous dilepton study at SIS 18 energy, we relinquish a treatment of thermal emission from the $\phi$ vector meson here, for reason of the minor in-medium broadening effects observed for this hadron and its still low multiplicities at least for lower FAIR energies. In the case of vanishing invariant mass, i.e., for real photons, only the $\rho$ vector meson will give a significant contribution. For the present calculation, we used the parametrization of the photon rates from the $\rho$ as given in Eqs. (2)-(7) in Ref. [78], while a more advanced parametrization is necessary for the dilepton rates from the $\rho$ and $\omega$, due to their dependence on invariant mass and momenta. Note that presently the photon parametrization of the $\rho$ contribution is limited to baryon chemical potentials lower than 400 MeV and momenta larger than 0.2 GeV/c. While we can easily neglect the lowest momentum region in the present study, the restriction to low $\mu_B$ is a problem for the lowest collision energies considered here, where one expects values of $\mu_B$ which significantly exceed this range. The difference will be dominant at lower momenta, where the influence of baryonic effects is known to be largest, while the effect of a finite chemical potential is rather small at higher momenta [78]. However, for the present work we can assume the photon contribution from the $\rho$ meson as a lower limit with regard to the baryonic effects.

#### 2. Meson gas contributions

The contribution from vector mesons is not the only hadronic source of thermal emission. The mass region above the $\phi$ meson, i.e., for $M > 1 \text{ GeV}/c^2$, is no longer dominated by well defined particles, but here one finds a large number of overlapping broad resonances constituting multi-meson (mainly four-pion) states, which have a significant impact on the dilepton yield. We here apply a description relying on model-independent predictions using a low-temperature expansion in the chiral-reduction approach.

While the multi-meson effects only show up for high invariant masses, i.e., in the time-like kinematic region probed by dileptons, when going to real photons with $M \rightarrow 0$ several scattering and bremsstrahlung processes become important, which can be mostly neglected at finite $M$. While baryonic bremsstrahlung processes such as $NN \rightarrow NN\gamma$ and $pN \rightarrow pN\gamma$ are included in the vector meson spectral functions, meson-meson bremsstrahlung has to be added in the case of photon emission. The most dominant part will here come from the meson-meson scatterings $\pi\pi \rightarrow \pi\pi\gamma$ and $\pi K \rightarrow \pi K\gamma$, for which we use the rates calculated within an effective hadronic model in form of the parametrization given by Eqs. (8) and (9) in Ref. [78]. Note that these bremsstrahlung processes are mainly contributing at low momenta, whereas they are rather subleading for photons of higher energy.

Besides the $\pi\pi$ bremsstrahlung, also other mesonic reactions contribute to the thermal photon production, such as strangeness bearing reactions and meson-exchange processes. In detail, these are $\pi p \rightarrow \pi\gamma$, $\pi K^* \rightarrow K\gamma$, $\pi K \rightarrow K^*\gamma$, $pK \rightarrow K\gamma$ and $K^* K \rightarrow \gamma\gamma$. The corresponding thermal rates were calculated for a hot meson gas in Ref. [22], which are applied here together with the respective form factors.

Since the $\omega$-$t$-channel exchange was found to give a significant contribution to thermal photon spectra via the $\pi p \rightarrow \pi\gamma$ process, it has been recently argued that also other processes including a $\pi\rho\omega$ vertex should be considered in the calculations, namely the $\pi p \rightarrow \gamma\omega$, $\pi\omega \rightarrow \gamma\rho$ and $\rho\omega \rightarrow \gamma\pi$ reactions for which the rates (including the form factors) are parametrized in Appendix B of Ref. [24]. Consequently, we also add these processes when calculating thermal photon spectra.
3. Influence of meson chemical potentials

As was already mentioned, we do not restrict ourselves to the consideration of emission from thermally and chemically equilibrated matter, but also include nonequilibrium effects in form of finite pion and kaon chemical potentials $\mu_\pi$ and $\mu_K$. It has been shown that the correct fugacity factors here.

\[
z^\pi_{M=\pi,K} = \exp \left( \frac{n_\pi M}{T} \right) \tag{13}\]

in the thermal dilepton and photon rates in Eqs. (11) and (12). The exponent $n$ depends on the difference in pion or kaon number $N_{\pi,K}$ between initial and final state of the process, i.e. $n = N_{\pi,K}^i - N_{\pi,K}^f$. Note that while the pion fugacity enters in most processes, the effects of a finite kaon chemical potential only play a role for the $\pi K$ bremsstrahlung and $\pi + K^+ \rightarrow \pi + \gamma$, $\pi + K \rightarrow K^+ + \gamma$ and $K^+ + K \rightarrow \pi + \gamma$ photon production channels. For the dilepton channels considered here, $\mu_K$ can be neglected.

While for the single mesonic channels the initial and final state are always well defined, several different types of processes are included in the $\rho$ and $\omega$ spectral functions, especially processes with baryons. For the $\rho$ not only processes with an initial two-pion state of the type $\pi \pi \rightarrow \rho \rightarrow \gamma/\gamma^*$ are accounted for, but also reactions including only one or no pion as ingoing particle (e.g., $\pi N \rightarrow A \rightarrow \gamma N$ or $NN \rightarrow \gamma NN$). However, as the correct fugacity depends on the initial pion number, one would obtain different enhancements for each channel. But as the different processes interfere with each other it is difficult to determine the exact strength of each channel and consequently one might hardly be able to account for some average enhancement factor. Instead we here apply a fugacity factor $z^2_\pi$ which would be correct for pure $\pi \pi$ annihilation processes. This can be interpreted as an upper estimate of the influence which the meson chemical potential might have on the thermal $\rho$ emission rates. The same procedure is applied for the $\omega$ meson, where we assume a fugacity of $z^2_\omega$. Note that while the multi-pion contribution also accounts for different initial states, they are each treated separately so that one can apply the correct fugacity factors here.

A full list of all hadronic contributions considered for the present calculation of thermal dilepton and photon emission, including the corresponding fugacity factors which account for the enhancement of the specific channel due to the meson chemical potentials, is given in Table I.

### Table I. Summary of the different dilepton contributions considered in the present calculations.

| Type         | Rates                  | Fugacity            | Ref.  |
|--------------|------------------------|---------------------|-------|
| Dilepton     | $\rho$ (incl. baryon effects) | $z^2_\rho$         | [73, 79] |
|              | $\omega$ (incl. baryon effects) | $z^2_\omega$       | [73, 79] |
|              | Multi-Pion             | $z^2_\pi / z^2_\omega$ | [15]   |
| Photon       | $\rho$ (incl. baryon effects) | $z^2_\rho$         | [73, 79] |
|              | $\pi \pi$ and $\pi K$ Bremsstr. | $z^2_\pi + 0.2z_\pi z_K$ | [22, 79] |
|              | $\pi \rho \rightarrow \gamma \pi$ | $z^2_\pi$          | [22]   |
|              | $\pi K^+ \rightarrow K \pi$ | $z^2_\pi z_K$      | [22]   |
|              | $\pi K \rightarrow K^+ \pi$ | $z^2_\pi z_K$      | [22]   |
|              | $\rho K \rightarrow K \gamma$ | $z^2_\pi$          | [21]   |
|              | $K^+ K \rightarrow \pi \gamma$ | $z_K$             | [21]   |
|              | $\pi \omega \rightarrow \gamma \rho$ | $z^2_\pi$        | [21]   |
|              | $\rho \omega \rightarrow \gamma \pi$ | $z^2_\pi$        | [21]   |
|              | $\pi \rho \rightarrow \gamma \omega$ | $z^2_\pi$      | [21]   |

B. Quark-Gluon Plasma

For the thermal emission of electromagnetic probes from the Quark-Gluon Plasma one is again confronted with the problem that different processes govern the dilepton production on the one side and photon emission on the other. Consequently, one has to apply two different descriptions for thermal rates, which are presented in the following.

In case of photon emission from a partonic phase of quarks and gluons the two main contributions stem from quark-antiquark annihilation ($q\bar{q}$) and Compton scattering processes ($qg \rightarrow q\gamma$ or $qg \rightarrow q\gamma^*$) [80]. However, it was shown that these processes are not sufficient to describe the production mechanism correctly and that it is necessary to (a) include Feynman diagrams accounting for bremsstrahlung and inelastic annihilation processes which are enhanced due to near-collinear singularities and (b) to implement the Landau-Pomeranchuk-Migdal effect [81, 82]. The results of a full calculation of the photon emission, to leading order in $\alpha_{em}$ and the QCD coupling $g(T)$, was evaluated by Arnold, Moore and Yaffe and takes the following form [83]:

\[
q_0 \frac{dR_\gamma}{dq} = \frac{\alpha_{em} \alpha_s}{\pi^2} \frac{5}{9} \left( \frac{q_T}{T} \right)^2 \ln \left( \frac{3}{4\pi \alpha_s} \right) \frac{1}{2} \ln \left( \frac{q_T}{T} \right) + C_{tot} \tag{14}\]

with

\[
C_{tot} = C_{2+2} \left( \frac{q_T}{T} \right) + C_{\text{annih}} \left( \frac{q_T}{T} \right) + C_{\text{brem}} \left( \frac{q_T}{T} \right), \tag{15}\]

\[
\alpha_s \approx \frac{6\pi}{27 \ln(T/0.022)}. \tag{16}\]

The functions $C_{2+2}$, $C_{\text{annih}}$ and $C_{\text{brem}}$ are approximated by the phenomenological fits given in Eqs. (1.9) and (1.10) in Ref. [83]. Note that this calculation assumes the chemical potential to be vanishing. However, the overall effect of finite values of a quark chemical potential (i.e., non-equal numbers of quarks and anti-quarks in the QGP phase) is known to be rather small.

\[
\ln \left( \frac{3}{4\pi \alpha_s} \right) \frac{1}{2} \ln \left( \frac{q_T}{T} \right) + C_{tot} \tag{14}\]

\[
C_{tot} = C_{2+2} \left( \frac{q_T}{T} \right) + C_{\text{annih}} \left( \frac{q_T}{T} \right) + C_{\text{brem}} \left( \frac{q_T}{T} \right), \tag{15}\]

\[
\alpha_s \approx \frac{6\pi}{27 \ln(T/0.022)}. \tag{16}\]
In case of the thermal dilepton emission from the QGP, the leading order contribution is the electromagnetic annihilation of a quark and an anti-quark into a virtual photon, $q\bar{q} \rightarrow \gamma^*$. This process is irrelevant in the light-cone limit for $M \rightarrow 0$, as the annihilation of two massive quarks into a massless photon is kinematically forbidden. The pure perturbative quark-gluon plasma rate was calculated for the mentioned leading order process [34] as

$$\frac{dR_{ll}}{d^3p} = \frac{\alpha_s^2}{4\pi^3} T^4 \rho_{ll}^{\text{EM}}(p_0; T) \sum_q \frac{e_q^2}{p} \times \ln \left( \frac{x_+ + y}{x_+ + \exp[-\mu_q/T]} \right) \left( \frac{x_+ + y}{x_+ + \exp[-\mu_q/T]} \right)$$

with $x_\pm = \exp[-(p_0 \pm p)/2T]$ and $y = \exp[-(p_0 + \mu_q)/T]$. The quark chemical potential $\mu_q$ which shows up here is equal to $\mu_q/3$. This calculation approximates the full QCD results quite well at high energies, but for soft processes of the order $g_\chi(T)$, i.e., for dileptons with low masses and momenta, the one-loop calculation is not sufficient and hard-thermal-loop (HTL) corrections to the result as given in Eq. (17) have to be considered [35]. It was found that the rate for soft dileptons is then by orders of magnitude larger than the simple leading order calculation [26].

Recent calculations from thermal lattice-QCD suggest an even stronger enhancement of the rates for low-mass dileptons [36]. These results, which are applied in the present work, have been extrapolated to finite three-momenta by a fit to the leading-order pQCD rates such that the correlation function takes the form [26]

$$- \text{Im} \Pi_{\text{EM}} = \frac{C_{\text{EM}}}{4\pi} M^2 \left( \hat{f}_2 \left( q_0, q; T \right) + Q_{\text{LAT}}^{\text{tot}} \left( M, q \right) \right),$$

where

$$Q_{\text{LAT}}^{\text{tot}} \left( M, q \right) = \frac{2\pi \alpha_s}{3} \frac{T^2}{M^2} \left( 2 + \frac{M^2}{4\theta} \right) \times K \left( M^2 \right) \ln \left( 1 + \frac{2.912 \theta}{4\pi \alpha_s \pi^2} \right)$$

with a form factor $F \left( M^2 \right) = 4T^2 \left( 4T^2 + M^2 \right)$ and a factor $K = 2$ to better fit the full IQCD rates. Note that in contrast to the pQCD result the rate in Eqs. (18) and (19) is calculated for $\mu_q = 0$ only, as a calculation for finite chemical potential is still beyond the current lattice calculations.

C. Hadronic decay contributions

While we restrict the calculation of photon yields to the thermal contribution, since all decay photons from long-lived hadronic resonances are usually subtracted from the experimental results, a full description of the dilepton spectra requires to take also the non-thermal contributions from the decay of pseudo-scalar and vector mesons into account. We here follow the same procedures as in our previous work for SIS 18 energies. In detail, we determine the following non-thermal dilepton contributions:

1. The Dalitz decays of the pseudo-scalar $\pi^0$ and $\eta$ mesons. To determine their contribution, we assume that each final state particle contributes with a weight of $\Gamma_{\rho M - e^+ e^-}/\Gamma_{\text{tot}}$.

2. The direct decay of the $\phi$ meson into a lepton pair. As the lifetime of the $\phi$ is relatively short, we apply a shinning procedure which takes absorption and re-scattering processes inside the medium into account.

3. Finally, we restricted the calculation of thermal dileptons to those cells where the temperature is larger than 50 MeV, as otherwise a thermal description becomes questionable. However, in principle one will of course also find $\rho$ and $\omega$ mesons at lower temperatures. To account for this, in the mentioned cases we calculate a “freeze-out” contribution from the $\rho$ and $\omega$ decays using the UrQMD results for these mesons.

For a more detailed description of the non-thermal hadronic contributions the reader is referred to Ref. [28]. We refrain from an extensive reproduction of the procedure here, as the cocktail contributions will not play a significant role in the present investigations.

IV. RESULTS

In the following we present the results of calculations with the coarse-graining approach for $Au+Au$ collisions in the energy range of $E_{lab} = 2 – 35$ AGeV. We restrict the analysis to the 10% most central reactions, as the medium effects will be largest here. In terms of the microscopic UrQMD results, this roughly corresponds to an impact parameter range of $b = 0 – 4.5$ fm. For the coarse-graining we use ensembles of 1000 microscopic events each. The length of the time-steps is chosen as $\Delta t = 0.6 \text{fm}/c$, and the size of the spatial grid is $\Delta x = \Delta y = \Delta z = 0.8$ fm. These grid parameters are similar to the ones used for the previous studies at SIS18 and SPS energies and constitute a good compromise between resolution and a sufficiently large hadron number per cell. To obtain enough statistics, especially for the non-thermal contributions, several runs with different ensembles are necessary.

A. Reaction dynamics

As the dilepton and photon production is directly related to the space-time evolution of the thermodynamic properties of the system, it seems natural to start with a study of the reaction dynamics obtained with the coarse-graining of UrQMD input.
In Fig. 1 the time evolution of temperature and baryochemical potential in the central cell of the grid is depicted for different beam energies. The evolution shows a significant increase of the temperature maxima from slightly above 100 MeV for 2 AGeV up to roughly 225 MeV for top SIS 300 energy. While the temperature is clearly below the critical temperature of 170 MeV for the lower energies, the highest energies covered by FAIR can probe also this deconfinement region of the phase diagram. The thermal lifetime of the central cell, i.e., the time for which it rests at temperatures above 50 MeV, increases slightly with increasing collision energy. This is mainly due to an earlier onset of thermalization after the first hadron-hadron collisions, which define the origin of the time axis. However, it is interesting that in the later phase of the collision the temperature curves for all energies show the same monotonous decrease and even lie on top of each other. A somewhat different behavior is observed for the evolution of the baryochemical potential $\mu_B$. For all energies it shows a clear peak with values between 700 and 900 MeV at the beginning of the collision, which is due to the high baryon densities reached in the central cell when the two nuclei first come into contact. Afterwards $\mu_B$ decreases and then remains on a plateau level for a significant fraction of the reaction time for the lower energies, while one observes a slight increase for the higher beam energies towards later times. If we neglect the peak in the early reaction stage, the chemical potential shows a clear decrease with increasing collision energy. While for 2 AGeV the baryochemical potential remains around 900 MeV for the whole thermal lifetime, $\mu_B$ is only 350 MeV at $t = 5$ fm for 35 AGeV and slowly increases up to 600 MeV after $t = 20$ fm.

Note that the results shown in Fig. 1 are only for one single cell at the center of the collision. The evolution in other cells of the grid may differ largely in dependence on their location (e.g., one finds in general lower temperature and chemical potential in more dilute peripheral cells). But yet it clearly depicts the influence of collision energy on $T$ and $\mu_B$. One finds two effects when going from the lowest to the highest FAIR energies: An increasing temperature combined with a decreasing baryochemical potential. This behavior is not specific for the central cell but reflected by the whole space-time evolution, as can be seen from Fig. 2. The two plots show the thermal four-volume in dependence of temperature $T$ (X-axis) and baryochemical potential $\mu_B$ (Y-axis). Results are shown for $E_{lab} = 4$ AGeV (a) and 35 AGeV (b). We see that for the lower energy the largest part of the four volume is concentrated at values of the baryochemical potential between 500 and 800 MeV, while the temperature remains below 160 MeV for all cells. In contrast, for $E_{lab} = 35$ AGeV the four-volume distribution extends to higher temperatures up to $T = 240$ MeV while at the same time the distribution is shifted to lower baryochemical potentials especially for higher temperatures while the lower-temperature cells are mainly dominated by high values of $\mu_B$. Interestingly, especially for the higher collision energy of 35 AGeV one finds some cells in a separate region with moderate to high temperature and very low baryochemical potential $\mu_B \approx 0$. These cells are mainly found in the more peripheral regions of the collision, where the baryon density (and particle density in general) is rather low and where nevertheless in some cases hadrons with large momenta are found, resulting in high energy density for these cells. However, compared to the large overall total thermal volume the relevance of these low-$\mu_B$ cells is negligible.

It is important to bear in mind that the dilepton and photon spectra will directly reflect the four-volume evolution in the $T-\mu_B$ plane, as presented here. The results show that at FAIR energies the region of the QCD phase diagram with temperatures above the critical temperature and large $\mu_B$ can be probed, in contrast to the situ-
tion at LHC or RHIC, where the transition from hadronic matter to a deconfined phase is assumed to happen at \( \mu_B \approx 0 \). However, note that the results presented here are obtained with a purely hadronic equation of state which does not include any effects of the phase transition itself. For an improvement the description one might need to implement the transition properly, to account, e.g., for the latent heat which would cause the cells to remain for a longer time at temperatures around \( T_c \). Nevertheless the present results can serve as a lower limit baseline calculation, assuming that we have a smooth crossing from hadronic to QGP emission. Significant deviations from this assumption might then show up in the photon and dilepton spectra. We will discuss this later.

As was pointed out before, the effects of chemical non-equilibrium show up in the form of finite meson chemical potentials for the \( \pi \) and \( K \); and \( \mu_\pi \) and \( \mu_K \) can have a significant effect on the population of several photon and dilepton production channels. The mean values of the pion chemical potential \( \mu_\pi \) and the kaon chemical potential \( \mu_K \) in dependence on the cell’s temperature for different collision energies are shown in Figure 3. Note that the results for the chemical potentials here are obtained by averaging the values of \( \mu_\pi \) and \( \mu_K \) over all space-time cells with a specific temperature. The study indeed indicates that regarding the pion density the system will be clearly out of equilibrium during the collision evolution. The value of \( \mu_\pi \) increases with temperature, which is not surprising since a large part of the pion production in the microscopic simulation takes place in initial scatterings and via string formation at the beginning of the reaction, when the system still heats up. At all temperatures one finds that the \( \mu_\pi \) decreases with increasing collision energy, which may indicate a faster and stronger equilibration of the system if more energy is deposited in the system. In addition, for top SIS300 energies the initial emission is dominated by QGP radiation at temperatures above \( T_c \) and consequently a larger fraction of cells with \( T < 170 \text{ MeV} \) is found later in the course of the fireball evolution, when the system is in a more equilibrated condition compared to the very beginning of the collision. For the higher collision energies we get average values up to \( \mu_\pi = 100 - 120 \text{ MeV} \) around the critical temperature of 170 MeV. Note that for \( E_{\text{lab}} = 4 \text{ AGeV} \) the maximum temperature found in the evolution is around 155 MeV, which explains the drop in the corresponding curve around this temperature.

In contrast to the large pion chemical potential, no
such dominant off-equilibrium effect is observed for the kaons, where $\mu_K \approx 0$ at all energies and temperatures. This is not surprising, as in the underlying microscopic simulations any inelastic reaction results in the creation of a $\pi$ whereas the cross-section for kaon production is rather low in the cases considered here (and especially for the lower FAIR energies). Consequently, the kaon production is a slow process which seems to happen synchronously with the equilibration of the system while a large amount of pions is produced in the initial hard nucleon-nucleon scatterings before any equilibration could take place.

The present results for the pion chemical potential are quite different from other model descriptions. For example, in fireball parametrizations the particle numbers are fixed at the chemical freeze-out of the system and consequently meson chemical potentials develop when the system cools down. However, in the fireball model this is just an ad-hoc assumption, as such macroscopic models are based on a presumed equilibrium within the system. In contrast, the overpopulation of pions is an intrinsic result stemming from the microscopic simulation in the case of the coarse-graining approach. Nevertheless, the very high pion chemical potentials in the temperature region close to the phase transition might be questionable, as one would assume that the transition from the Quark-Gluon Plasma to a hadronic phase should produce a system where the mesons are in an equilibrium state. A fully satisfying description of the chemical off-equilibrium evolution is not feasible within the present approach and would require a microscopic and dynamical description of the phase transition and its underlying dynamics.

B. Dilepton spectra

The dilepton invariant mass spectra in the low-mass range up to $M_{e^+e^-} = 1.2 \text{GeV}/c^2$ for four different beam...
energies ($E_{\text{lab}} = 2, 8, 15$ and $35 \, \text{AGeV}$) are presented in Fig. 4. The comparison shows some interesting similarities and differences: While the very low masses up to $0.15 \, \text{GeV/c}^2$ are generally dominated by the Dalitz decays of neutral pions, the region beyond the Dalitz peak is dominated by a strong thermal $\rho$ contribution. The thermal yield shows an absolute increase with $E_{\text{lab}}$, but its importance decreases relative to the non-thermal $\eta$ yield. This means that the thermal low-mass enhancement of the dilepton yield above a hadronic vacuum cocktail decreases with increasing collision energy. This observation is explained by the decrease of the baryon chemical potential at higher collision energies, as has been mentioned in the previous section. In contrast, the increasing temperature leads to a significantly flatter shape of especially the $\rho$ distribution in the invariant mass spectrum. While for low energies the thermal yield decreases strongly when going to higher invariant masses, at the top FAIR energies this effect is less prominent and the population of high masses is enhanced. This is can be clearly seen by the fact that the multi-pion yield shows a strong rise.

While at the lowest of the four energies the whole system is well below the critical temperature $T_c$, we know from the temperature evolution in Fig. 1(a) that the region around $T \approx 170 \, \text{MeV}$ from is reached $E_{\text{lab}} = 6-8 \, \text{AGeV}$ on. Consequently, in the dilepton invariant mass spectra of Fig. 4 the resulting QGP contribution is very small at $8 \, \text{AGeV}$, but even at $35 \, \text{AGeV}$ the partonic yield is suppressed by roughly an order of magnitude compared to the leading contributions in the mass range up to $1 \, \text{GeV/c}^2$.

The hadronic thermal yields in Fig. 4 are shown for the case of vanishing pion chemical potential. However, we also compare the result for $\mu_\pi = 0$ with the total yield assuming finite values of $\mu_\pi$. One can see that chemical non-equilibrium can increase the overall dilepton yield in the low-mass range up to a factor of two. For the region above $1 \, \text{GeV/c}^2$ the effect can be even larger as the fugacity factor enters the thermal rate with a power of four for the multi-pion contribution. It is important to bear in mind that this result should rather be seen as an upper estimate, as the approximation $\mu_\pi = 2 \mu_\pi$ is only correct for the rate $\pi \pi \to \rho$, which represents only one of the many processes included in the $\rho$ spectral function.

Furthermore, as UrQMD has no intrinsic description of the phase transition, the pion chemical potential might be overestimated in vicinity of the critical temperature. Nevertheless, the results show that a deviation from pion equilibrium has a huge impact on the thermal dilepton rates.

Considering possible signatures for a phase transition and the creation of a deconfined phase, the low-mass region is rather unsuited due to the dominance of the hadronic cocktail contributions and hadronic thermal emission from the vector mesons. Consequently, it might be more instructive to explore the mass range above the pole mass of the $\phi$, where one has a continuum dominated by thermal radiation. In this region the hadronic cocktail contributions can be neglected and thermal sources will dominate the spectrum. In previous works [27, 45] it has been shown for SPS energies that the dilepton invariant mass spectrum at very high masses $M_{\ell^+\ell^-} > 1.5 \, \text{GeV/c}^2$ could only be explained by including thermal radiation from the Quark-Gluon Plasma. In Fig. 5 the higher invariant mass region for $M_{\ell^+\ell^-} > 1.1 \, \text{GeV/c}^2$ is shown for the two collision energies $E_{\text{lab}} = 8$ and $35 \, \text{AGeV}$. Here we compare four different scenarios to study whether the high-mass invariant mass spectrum might help to iden-
tify the creation of a Quark-Gluon Plasma. Besides the two standard scenarios (hadron gas + partonic emission above \( T_c = 170 \text{ MeV} \)) for (i) finite and (ii) vanishing \( \mu_T \), we include a scenario with (iii) a 5-times enhanced emission from the partonic phase around the transition temperature to simulate the effect of a critical slowdown of the system due to a first order phase transition and, finally, (iv) a pure hadron gas scenario, where we assume all thermal radiation (also for \( T > 170 \text{ MeV} \)) to stem from hadronic sources. For (iii) and (iv) \( \mu_T = 0 \) is assumed, too. The comparison shows that the spectral shape of the total yield is very similar for all scenarios, at both energies considered here. While the results for a purely hadronic scenario and including QGP emission from temperatures above \( T_c \) give quite the same results within 10% deviation, also the artificially enhanced QGP emission does not significantly increase the overall yield. In contrast, one observes a very strong enhancement due to a finite pion chemical potential, which shows up in our calculation by an overall increase by a factor of 5 at 8 AGeV and still a factor of 2 at 35 AGeV. The results indicate that it will be difficult to draw unambiguous conclusions from single measurements of the higher mass region at a specific energy, as according to our calculations a stronger QGP yield and less hadronic contribution can finally result in the same overall dilepton spectrum. Furthermore, the non-equilibrium effects may lead to much larger modifications of the spectrum than caused by the dynamics of the phase transition.

The transverse-momentum spectra, plotted for two different energies in different invariant mass bins in Fig. 6, underline the previous finding. Again the slopes of the curves for hadronic and partonic emission are very similar for high masses, especially for \( M_{e^+e^-} > 1.1 \text{ GeV/c}^2 \) they are virtually identical. On the contrary we find the partonic contribution to be harder (i.e., having a stronger relative yield at high transverse momenta) than the hadronic contribution at lower masses. Together with the very similar invariant-mass spectra at high masses obtained with and without a partonic phase in the reaction evolution, the result confirms previous studies which showed a duality of emission rates in the transition-temperature region [20]. However, note that this is no longer true if one goes to a temperature regime significantly above \( T_c \). For this case clear differences between the hadronic and the partonic emission are observed. Unfortunately, even at the top...
SIS 300 energy only few cells reach temperature maxima above 200 MeV and one will not see clear differences between the partonic and hadronic emission patterns even at high \( p_t \) and high \( M_{e^+e^-} \).

The reason for the duality showing up only at high invariant masses (and momenta, respectively) is twofold: On the one hand, the low mass region is governed by the vector mesons with their specific spectral shapes and the baryonic effects on them. This effect has been called the “duality mismatch” [25] since the hadronic rates show an increase for finite baryochemical potentials, while the partonic emission rates are quite insensitive with regard to \( \mu_B \). On the other hand, while the spectra at low masses and momenta are populated by thermal emission at all temperatures, the production of dileptons for masses above 1 GeV/\( c^2 \) and for higher values of \( p_t \) is strongly suppressed at low temperatures. This is visible from Fig. 2(a), where the temperature dependent dilepton yield from thermal sources for Au+Au at \( E_{\text{lab}} = 35 \) AGeV is shown for different invariant-mass bins. (Note that the non-thermal lepton pairs directly extracted from the hadronic cocktail as calculated with UrQMD are not included.) The yields shown in this plot represent the sum of the contributions from all cells at a certain temperature. While for the lowest mass bin \( M_{e^+e^-} < 0.2 \) GeV/\( c^2 \) the total thermal dilepton yield is built up by roughly equal fractions stemming from the whole temperature range, with slight suppression of emission from temperatures above \( T_c \), one can see that the mass region above \( M_{e^+e^-} = 1.1 \) GeV/\( c^2 \) is dominated by emission from temperatures between 140 and 220 MeV, which is exactly the assumed transition region between hadronic and partonic emission. Dilepton emission at lower temperatures is strongly suppressed in this mass range. Furthermore, one finds a smooth behavior of the thermal emission in the highest mass bin, but at lower masses one observes a slight kink in the rates at \( T_c = 170 \) MeV. The finding indicates that for lower masses the partonic and hadronic rates do not perfectly match, as was discussed above. Another observation is the dominance of emission from the temperature range \( T = 100 - 140 \) MeV for the mass region from 0.6 to 0.9 GeV/\( c^2 \). This result is in contrast to the general trend of a shift of the emission to higher temperatures when going to higher masses. However, the very mass region covers the pole masses of the \( \rho \) and \( \omega \) meson. As the peak structures show a melting especially for finite baryon densities, one will get the largest yields in this mass range from cells for which \( \mu_B \approx 0 \). A comparison with the \( T-\mu_B \) distribution of the cells at this energy in Fig. 2(b) shows that the largest fraction of cells for which the baryochemical potential is below 200 MeV lies exactly in the temperature range from 100 to 140 MeV.

Finally, it is instructing not only to look at the temperature but also at the time dependence of thermal dilepton emission as presented in Figure 3(b). The total dilepton emission per timestep \( dN/dt \) as sum from all cells is shown for \( E_{\text{lab}} = 2 \) and 35 AGeV. In principle, the results reflect the findings from Fig. 2 and are similar to the temperature evolution depicted there. The system shows a faster heating for the top SIS 300 energy with higher temperatures, resulting in a larger number of emitted dileptons; partonic emission from cells with \( T > T_c \) is found for the first 5 - 10 fm/c and only very sporadically thereafter. At lower energies the evolution is retarded and \( T \) remains below the critical temperature. However, in contrast to the slow heating of the system (which is simply due to the fact that the nuclei are moving slower) the thermal emission drops much earlier for 2 AGeV (compared to 35 AGeV) and only few cells with thermal emission are found for \( t > 30 \) fm/c. The higher energy deposited in the system with increasing \( E_{\text{lab}} \) ob-
viously results not only in higher initial temperatures, but also in an enhanced emission at later stages, as it takes the system longer to cool down. Interestingly, for both energies one finds some sparse cells with thermal emission even after 60-70 fm/c; however, their contribution to the overall result is suppressed by 3-4 orders of magnitude compared to the early reaction stages.

C. Photon spectra

While we have two kinematic variables (momentum and invariant mass) which can be probed for virtual photons, real (i.e., massless) photons only carry a specific energy. In this sense, dileptons are the more versatile probes of the hot and dense medium and carry additional information, especially regarding the spectral modifications of the vector mesons. Nevertheless, the correct description of the experimental photon spectra has been a major challenge for theory at SPS and RHIC energies. In the kinematic limit $M \to 0$ several processes become dominant which are negligible in the time-like region probed by dileptons, as was discussed in Sec. [I]. Consequently, the study of photon production can provide complementary information for the theoretical description of thermal emission rates and the reaction dynamics.

As the parametrized photon emission rates have some restrictions with regard to the $\mu_B$ range of their applicability, it will be instructive to find out at the beginning under which thermodynamic conditions the photons are emitted at FAIR. Fig. 8 shows the dependence of thermal photon emission (at mid-rapidity) on temperature in the left plot (a) and on baryochemical potential in the right plot (b). As in Fig. 7, the yields are the sum of the thermal contributions from all cells with a certain temperature or baryochemical potential, respectively. For both results we consider the case of vanishing meson chemical potentials. Note that for the temperature dependence we consider only the thermal emission at higher transverse momentum values $p_t > 1$ GeV/c, as here the duality between hadronic and partonic rates should be approximately fulfilled, which is indeed visible from the continuous trend of the thermal photon emission around $T_c$. One can see that especially for lower collision energies the thermal emission is dominated by the cells which reach the maximum temperature, whereas the curves become flatter at higher energies. Even at the top energy of 35 AGeV with maximum temperatures above 220 MeV still a significant amount of emission also stems from the cells with temperatures around 100 MeV.

Regarding the photon emission related to baryochemical potential as presented in Fig. 8(b), a clear energy dependent trend is visible: While at low energies the largest fraction of emission stems from cells with very high values of $\mu_B$ around 900 MeV, the emission weighted average chemical potential drops continuously to 300–400 MeV at 35 AGeV. However, the emission from cells with higher values of the baryochemical potential is by far not negligible. In consequence, the findings once again underline that the strongest baryonic modifications of the spectral functions will be present at low energies. The results also show that to fully account for the baryonic effects on the photon emission the spectral functions should be able to reliably cover the whole $\mu_B$ region from 0 to the nucleon mass (i.e., $\approx$ 900 MeV). The presently used parametrization will provide only a lower limit for the photon yield, especially for the lower collision energies.

In Fig. 9(a)-(d) the transverse momentum spectra of thermal photons for four different collision energies are presented. As for the dilepton invariant mass spectra, we show the results with (full black line) and without meson chemical potentials (dashed black); once again the two calculations provide a lower and upper boundary for the off-equilibrium influence on the thermal yields, respectively. Two observations can be made when com-
comparing the results for the different energies: An overall increase of the photon $p_t$-yield with increasing energy and, secondly, a simultaneous hardening of the spectra, i.e., one gets a stronger relative contribution for higher momenta. This is similar to the dilepton invariant-mass spectra, where the yield in the higher mass region is suppressed for lower collision energies due to the lower overall temperatures in the fireball. (A more explicit comparison of the energy dependence of the results will be undertaken in Sec. [V.D].) Furthermore, one can see that at all energies the contribution from the $\rho$ meson dominates above the other hadronic contributions especially for low $p_t$, while the relative dominance of the $\rho$ decreases for higher momenta. The contribution from the Quark-Gluon Plasma is visible for $E_{\text{lab}} = 8 \text{ AGeV}$ and higher energies, giving an increasing fraction of the overall yield. Note the similarity between the low-mass dilepton and photon $p_t$ spectra for $35 \text{ AGeV}$: In both cases the slope of the (virtual or real) photons emitted from the QGP stage is significantly harder than the contribution from hadronic sources. Furthermore, looking only at the $\rho$ and the partonic contribution, one finds that the first is stronger for $p_t < 1 \text{ GeV/c}$, and the latter dominates for higher momenta - for both dileptons and photons. This behavior is expected as the real photon represents the $M_{\pi^+\pi^-} \rightarrow 0$ limit of virtual photon production. This is another requirement of consistency for the thermal rates.

Although finite values of $\mu_\pi$ and $\mu_K$ have an even more pronounced effect on the photon rates than on the dilepton rates, as several processes to be considered are very sensitive to an overpopulation of pions, it is remarkable that the overall effect leaves the shape of the photon $p_t$ spectra mostly unchanged: The yields are enhanced by the same factor at all transverse momenta. This is interesting as the effect of $\mu_\pi/\mu_K \neq 0$ on the different contributions is varying in strength. For example, the yield from the $\pi - \rho - \omega$ system shows a much stronger enhancement.
FIG. 10. (Color online) Comparison of the transverse-momentum spectra at mid-rapidity ($|y| < 0.5$) of the thermal photon yield for central Au+Au reactions resulting from different emission scenarios. In (a) the effect of a finite baryon chemical potential $\mu_B$ on the transverse momentum spectrum for the $\rho$ contribution and the total yield is shown for 2 AGeV (lower results) and 35 AGeV (upper results), comparing the standard scenario with $\mu_B \neq 0$ (full lines) with the results for $\mu_B = 0$ (dashed lines). Plot (b) shows the results for a purely hadronic scenario, i.e., for emission from the hadron gas also for $T > 170$ MeV and no partonic contribution. The single contributions are plotted as in Fig. 9 for comparison the total yield for the standard result including hadronic + partonic emission is shown (black dashed).

But not only meson chemical potentials influence the photon spectra, similar to the case of dileptons one expects also an enhancement of the $\rho$ contribution in the presence of baryonic matter. In Fig. 10(a) the effect of a finite baryon chemical potential $\mu_B$ on the transverse momentum spectrum for the $\rho$ contribution and the total yield is shown for 2 AGeV and 35 AGeV, comparing the standard scenario with $\mu_B \neq 0$ (full lines) with the results for $\mu_B = 0$ (dashed lines). The comparison shows that especially at lower momenta the $\rho$ contribution is significantly increased for finite baryochemical potential, while this effect is less dominant at larger $p_t$. Furthermore the effect is stronger for lower collision energies, where one obtains larger average values of $\mu_B$. However, one should bear in mind that the parametrization for the photon emission rates is limited to chemical potentials below 400 MeV, so that one can not fully account for the very large chemical potentials in this case. In consequence, one can expect an even larger enhancement in the experimental measurements than in the present calculation.

To conclude the study of the different influences on the photon spectra, we also consider whether the possible creation of a deconfined phase has any effect on the thermal emission pattern. Plot (b) of Fig. 10 shows the results for a purely hadronic scenario, i.e., for emission from the hadron gas also for $T > 170$ MeV and no partonic contribution. For comparison also the total yield for the standard scenario including hadronic + partonic emission is shown. (For both cases the meson chemical potentials are assumed to be zero.) Again, as for the high-mass dileptons (compare Fig. 5), the differences between the two scenarios are negligible, especially compared to the effect of the meson and baryochemical potentials. Only a very slight enhancement of the yield at low momenta is obtained for the pure hadron gas scenario, reflecting the different sensitivity of partonic and hadronic rates to finite $\mu_B$.

D. Excitation function of photon and dilepton yields

In the previous sections several differences and similarities between dilepton and photon spectra have already been discussed. However, it is instructing to do this in more detail and to compare the energy dependence of the emission patterns for photons and dileptons. Considering experimental measurements, an advantage of studying the excitation function of thermal yields might be that the trends and results obtained hereby at different energies are more robust and less sensitive to errors of measurement. It reflects the results of several different measurements in contrast to single spectra at a specific energy. For reason of comparison and because the baryonic effects are strongest in this case, all results in the following will be considered for mid-rapidity $|y| < 0.5$.

In Fig. 11 the total thermal photon $p_t$ and dilepton $M_{e^+e^-}$ spectra for eight different collision energies in the range $E_{lab} = 2–35$ AGeV are shown. It was already mentioned that—besides the hadronic structures due to the
The direct connection of the dilepton spectrum with the spectral function of the light vector mesons—the two spectra are strikingly similar. And also the change of the spectra with increasing collision energy is alike. At high masses/momenta the yield shows a stronger increase with $E_{\text{lab}}$ than in the low-mass/-momentum region. More quantitatively, this can be seen in Fig. 12 (a), where the relative increase of the thermal photon and dilepton yield for different transverse momentum or invariant mass regions, respectively, is shown. The results are normalized for different transverse momentum or invariant masses, the yield shows a stronger increase with increasing collision energy is alike. At high masses or momenta, respectively, the hadronic contribution becomes more dominant. This is not surprising, as one could already conclude that we probe lower temperatures at lower masses and momenta (compare Fig. 7). Further the baryonic influence increases here. In the dilepton spectra the direct connection to the vector meson spectral functions makes a comparison between the photon and dilepton results for low momenta and masses difficult.

Finally, we consider also the thermal photon yield in relation to the number of (neutral) pions which are produced in the heavy-ion collision, as presented in Fig. 13. This ratio is of theoretical and experimental interest: For the experimental study of photons, decays of neutral pions are the major background in the analysis. On the theoretical side, the number of pions gives an estimate of the freeze-out volume and is not sensitive to the details of the reaction evolution. While the electromagnetic emission takes place over the whole lifetime of the fireball and therefore reflects the evolution of the system in the phase diagram, the pion yield allows to scale cut trivial dependences. Previously, the thermal dilepton yield was found to scale with $N^{1/3}_\pi$ if one compares different system sizes at SIS 18 energies [28]. However, the situation is more complex here as we consider a large range of energies, which will be covered by FAIR. Several effects play a role, e.g., the lifetime of the fireball, the temperatures and baryochemical potentials which are reached and the different processes which contribute at different temperatures. While in our study at SIS 18 energies only the system size was modified (and all the other parameters could be assumed to remain quite constant, as one single energy was considered), in the present study only the size of the colliding nuclei is constant.
The investigation of the $N_γ/N_π^0$ ratio is here combined with a comparison of the different scenarios for the conditions of thermal emission, which were already studied in case of the photon and dilepton spectra (see Figs. 5 and 9). Varying $E_{\text{lab}}$ (and, in consequence, $T$ and $\mu_B$) might result in distinct excitation functions of the ratio $N_γ/N_π^0$ for the various scenarios, in contrast to the spectra for one specific energy where no unambiguous distinction was possible. In Fig. 13 one can see a strong increase of the photon to pion ratio for the lowest energies for both the lower $p_t$ range from 0.5 to 1 GeV/c and the high transverse momentum region above 1 GeV/c in case of the baseline scenario with QGP emission for $T > 170 \text{ MeV}$ and $\mu_\pi = 0$. However, for higher collision energies above 10 GeV we still observe a further increasing ratio for the higher momentum range, whereas at low $p_t$ the ratio remains relatively constant and even decreases for the highest collision energy. One can understand the decreasing ratio for lower $p_t$ reviewing again the energy dependence of $T$ and $\mu_B$ as shown in Fig. 1. The rise of temperature becomes less intensive for higher collision energies, while the baryochemical potential decreases for higher collision energies, causing a less pronounced increase of the thermal yield (compare also Fig. 12). Besides, the effects due to finite $\mu_B$ are more pronounced at low momenta, explaining the different trends for the two $p_t$ regions. Including the finite meson chemical potentials, we observe a strong increase of the ratio by factors 2-5 at all collision energies. The strongest effect in the present calculation is seen around 8 GeV, so that a slight peak structure builds up. However, as mentioned already several times, this scenario can only be seen as an upper limit for the non-equilibrium effects, most probably the increase will be smaller.

When comparing the two scenarios including enhanced QGP emission around $T_c$ on the one side and a pure hadronic scenario on the other side it is interesting that both cases lead to a similar result, namely an increase of the $N_γ/N_π^0$ ratio at higher collision energies. The effects show up more dominantly at high momenta, as this region is more sensitive with regard to emission from high temperatures. Note, however, that there are also significant differences between the two cases. The scenario with enhanced QGP emission around $T_c$ shows the most prominent increase at $E_{\text{lab}} = 8 - 10$ GeV whereas this enhancement becomes smaller again for higher energies. This can be explained by the fact that the relative fraction of emission from temperatures around 170-175 MeV is largest at those collision energies where the transition to a partonic phase is just reached. At higher $E_{\text{lab}}$ the corresponding higher temperatures may outshine any effects from the transition region. On the contrary, the enhancement over the baseline scenario increases with energy for the case of a pure hadron gas. However, for highest collision energies the difference seems to remain stable or even to drop again.

We remind again that for the experimental measurement the ratio of thermal photons from Fig. 13 is of importance, as almost all of the $\pi^0$ mesons decay into a photon. Therefore the vast majority will be decay photons, not stemming from direct (thermal or prompt) emission processes. Their spectra have to be subtracted in experiment to draw conclusions about the direct photons from thermal sources. This might be relatively difficult for the lowest energies available at FAIR, as here the ratio is suppressed by up to an order of magnitude compared to the higher collision energies.
V. DISCRIMINATING DIFFERENT SCENARIOS

It has so far become clear that one can extract only limited information regarding the properties of the hot and dense fireball from single photon and dilepton spectra, as there are usually several different effects that might interfere and finally lead to the same invariant-mass or transverse-momentum yields. However, the picture might be quite different if—in addition—the results in distinct mass or momentum regions, respectively, are systematically compared for several collision energies. In this case one might be able to discriminate the hadronic and partonic effects from each other. The FAIR energy regime will be ideally suited for such a study, as the transition from pure hadronic fireballs to the creation of a deconfined phase will take place somewhere around \( E_{\text{lab}} = 6 - 8 \text{ AGeV} \), as our results suggest. Nevertheless, there is no single observable that seems to allow for unambiguous conclusions on the details of the reaction evolution. On the contrary, it will still be necessary to carefully compare theoretical calculations and experimental results.

Based on the findings of the present work, one may consider the following scheme which might help to determine the strength of the different effects on the thermal rates and discriminate between the contributions:

1. The influence of baryonic matter leads to an enhancement which is most dominant for low transverse momenta and low masses. In general, it steepens the \( p_t \) slope of the overall yield. A large advantage is that today the spectral function of the \( \rho \) meson is quite well known from previous experimental and theoretical studies. Detailed and precise photon (dilepton) measurements for low momenta (low masses) in the FAIR and RHIC-BES energy regime might give further constraints for the spectral function in the region of extremely high baryon densities and can, vice versa, help to see whether the models correctly describe the fireball evolution in terms of \( \mu_B \).

2. In contrast to the baryonic effects on the emission rates, non-equilibrium effects caused by finite pion (and kaon) chemical potentials will show up as enhancement in the dilepton and photon spectra at all masses and momenta, and will be visible at all collision energies. The effect should be slightly more dominant in the high invariant mass or high \( p_t \) region, as here the multi-meson contributions become more pronounced. Ideally, one can discriminate between the \( \mu_B \) - and \( \mu_N \)-driven effects by comparing the modification of the slope and the overall enhancement in relation to baseline calculations.

3. If the baryon and non-equilibrium effects are under control, one might be able to find signals from the partonic phase in the dilepton and photon spectra for high \( p_t \) and \( M_{e^+e^-} \). In general, the dilepton and photon rates do not differ much from each other around \( T_c \), but effects such as a critical slowdown of the evolution might lead to an increased yield from the Quark-Gluon Plasma. On the other hand the “duality mismatch” might lead to a relative decrease of the yield, as hadronic rates are sensitive to finite baryon and meson chemical potentials while the QGP rates show hardly any modification. However, any effects connected to a phase transition can only show up if the obtained temperatures are large enough. Consequently, we would observe subsequent modifications of the spectra only for energies larger than \( E_{\text{lab}} = 6 - 8 \text{ AGeV} \), in contrast to non-equilibrium and baryonic effects which also appear at lower temperatures. Significant differences from calculations which only show up for the higher energies might then indicate the creation of a deconfined phase.

4. Furthermore, these effects should be dominant in the regions which are most sensitive to QGP forma-

![Figure 13](https://example.com/fig13.png)

**FIG. 13.** (Color online) Energy dependence for the ratio of thermal photon yield \( N_{\gamma}^{\text{thermal}} \) at mid-rapidity (\(|y| < 0.5\)) to the overall number of neutral pions \( N_{\pi^0} \). The results are shown for two different regions of the photon transverse momenta, \( 0.5 < p_t < 1.0 \text{ GeV/c} \) (green) and \( p_t > 1.0 \text{ GeV/c} \) (red). The baseline calculation assumes hadronic emission up to 170 MeV with vanishing \( \mu_{\pi/K} \) and partonic emission for higher temperatures (full line). In addition, the results with a five times enhanced emission around the critical temperature (dashed-dotted line), for a pure hadron gas scenario with emission at all temperatures from hadronic sources (short dashed line) and including meson chemical potentials (dashed line) are shown.
tion: The region for $M_{e^+e^-} > 1$ GeV in the dilepton spectra, and for high-$p_t$ in the photon respectively low-mass dilepton spectra (provided, it is possible to get control over the hadronic decay background). Another advantage in these regions is that they are relatively insensitive to the finite baryon chemical potential.

The different issues are not easy to disentangle and several interdependencies exist. Another aspect, which is not explicitly considered in our work but might further complicate the situation, is the influence of different EoS on the thermal yields. In the present work we use a Hadron-Gas EoS to provide consistency with the underlying microscopic model. However, previous investigations in a transport + hydrodynamics hybrid model showed that an MIT Bag Model EoS or a chiral EoS lead to different evolutions in the hydrodynamic phase compared to the HG-EoS, resulting in higher temperatures and, consequently, an increase of the emission rates.

VI. CONCLUSIONS & OUTLOOK

We have presented photon and dilepton spectra for the collision energy range $E_{\text{lab}} = 2 – 35$ AGeV, which will be covered by the future FAIR facility (and, in parts, by phase II of the BES at RHIC). The calculations were performed using a coarse-graining approach with transport simulations from the UrQMD model as input. In this approach local particle and energy densities are extracted from an ensemble average of the microscopic transport calculations, and an equation of state is used to calculate the corresponding values for temperature and chemical potential. The local thermodynamic properties are then used to determine the thermal emission rates.

The resulting spectra show a strong influence of finite baryochemical potentials and an enhancement due to non-equilibrium effects caused by finite meson chemical potentials. Regarding the search for signals of the deconfinement phase transition, there is no clear signal from which the creation of a partonic phase can be unambiguously inferred. Similarly, the results suggest that it is also hard to identify the type of the transition, whether it is a cross-over or a first-order phase transition, as effects due to a critical slowing down might be small compared to other influences on the spectra. The main difficulty is the dual connection between hadronic and partonic emission rates in the transition region around the critical temperature $T_c$, resulting in very similar slopes in the invariant-mass and transverse-momentum spectra.

For a clarification of the open issues, experimental input is needed. Our results suggest that one needs very precise and detailed measurements, as different evolution scenarios for the nuclear collisions are modifying the dilepton and photon spectra in a quite subtle manner. Systematic studies of several collision energies in the future FAIR energy range from $E_{\text{lab}} = 2$ AGeV to 35 AGeV are required to get more insights into the structure of the phase diagram of QCD matter and especially to find clues for the creation of a deconfined phase. Besides the experimental efforts, it will be similarly important to intensify the theoretical studies.

ACKNOWLEDGMENTS

The authors especially thank Ralf Rapp for providing the parametrizations of the spectral functions and many fruitful discussions. This work was supported by the Hessian Initiative for Excellence (LOEWE) through the Helmholtz International Center for FAIR (HIC for FAIR), the Bundesministerium für Bildung und Forschung, Germany (BMBF) and the Helmholtz-Gemeinschaft through the Research School for Quark-Matter Studies (H-QM).

[1] H. Stöcker and W. Greiner, Phys. Rept. 137, 277 (1986).
[2] P. Danielewicz, R. Lacey, and W. G. Lynch, Science 298, 1592 (2002).
[3] J. D. Bjorken, Phys. Rev. D 27, 140 (1983).
[4] R. Stock, in H. Schopper (ed.), Physics and Methods - Theory and Experiments, vol. 21A of Landolt-Börnstein - Group I Elementary Particles, Nuclei and Atoms (Springer, Berlin, 2010).
[5] E. L. Feinberg, Nuovo Cim. A 34, 391 (1976).
[6] E. V. Shuryak, Phys. Lett. B 78, 150 (1978).
[7] D. Adamova et al. (CERES/NA45), Phys. Rev. Lett. 91, 042301 (2003).
[8] M. M. Aggarwal et al. (WA98 Collaboration), Phys. Rev. Lett. 85, 3305 (2000).
[9] R. Arnaldi et al. (NA60 Collaboration), Phys. Rev. Lett. 96, 162302 (2006).
[10] L. Adamczyk et al. (STAR Collaboration), Phys. Rev. Lett. 113, 022301 (2014), [Addendum: Phys. Rev. Lett. 113, 049903 (2014)].
[11] S. S. Adler et al. (PHENIX Collaboration), Phys. Rev. Lett. 94, 232301 (2005).
[12] A. Adare et al. (PHENIX Collaboration), Phys. Rev. Lett. 104, 132301 (2010).
[13] A. Adare et al. (PHENIX), Phys. Rev. C 93, 014904 (2016).
[14] M. K. Koehler (ALICE), Nucl. Phys. A 931, 665 (2014).
[15] R. Porter et al. (DLS Collaboration), Phys. Rev. Lett. 79, 1229 (1997).
[16] G. Agakishiev et al. (HADES Collaboration), Phys. Rev. Lett. 98, 052302 (2007).
[17] G. Agakishiev et al. (HADES Collaboration), Phys. Lett. B 663, 43 (2008).
[18] G. Agakishiev et al. (HADES Collaboration), Phys. Rev. C 84, 014902 (2011).
