Some generalized Pythagorean 2-tuple linguistic Bonferroni mean operators in multiple attribute decision making

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Abstract
Green supply chain management is a strategy which strengthens and integrates environmental consideration into whole supply chain. The green strategic supplier plays an important role in the implementation of green supply chain strategy. In the selection methodology of a strategic green supplier, some special requirements are needed which are different from the traditional supplier selection practices. In this paper, we combine the generalized weighted BM operator with Pythagorean 2-tuple linguistic numbers to propose the generalized Pythagorean 2-tuple linguistic-weighted Bonferroni mean operator, and then the multiple attribute decision-making methods are developed based on this operator. Finally, we use an example for green supplier selection to illustrate the multiple attribute decision-making process of the proposed methods.

Keywords
Multiple attribute decision making, Pythagorean 2-tuple linguistic numbers, generalized weighted BM operator, green supplier selection

Introduction
More recently, Pythagorean fuzzy set (PFS)\(^1\)\(^2\) has emerged as an effective tool for depicting the uncertainty of the multiple attribute decision making (MADM) problems. The PFS is also characterized by the membership degree and the non-membership degree, whose sum of squares is less than or equal to 1; the PFS is more general than the intuitionistic fuzzy set (IFS). In some cases, the PFS can solve the problems that the IFS cannot, for example, if a DM gives the membership degree and the non-membership degree as 0.8 and 0.6, respectively, then it is only valid for the PFS. In other words, all the intuitionistic fuzzy degrees are a part of the Pythagorean fuzzy degrees, which indicates that the PFS is more powerful to handle the uncertain problems. Zhang and Xu\(^3\) defined the detailed mathematical expression for PFS and developed a Pythagorean fuzzy TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) for handling the MADM problem within PFSs. Peng and Yang\(^4\) proposed the division and subtraction operations for PFSs and also developed a Pythagorean fuzzy superiority and inferiority ranking method to solve multiple attribute group decision making (MAGDM) with PFSs. Afterwards, Beliakov and James\(^5\) focused on how the notion of “averaging” should be treated in the case of PFSs. Reformat and Yager\(^6\) applied the PFSs in handling the collaborative-based recommender system. Gou et al.\(^7\) investigated the properties of continuous Pythagorean fuzzy information. Ren et al.\(^8\) proposed the Pythagorean fuzzy TODIM approach to MADM. Garg\(^9\) proposed the new generalized Pythagorean fuzzy information aggregation by using Einstein operations. Zeng et al.\(^10\) developed a hybrid method for the Pythagorean fuzzy multiple-criteria decision making. Garg\(^11\) studied a novel accuracy function under interval-valued Pythagorean fuzzy environment for solving the MADM problem. Liang et al.\(^12\) developed the projection model for fusing the information of Pythagorean fuzzy multicriteria group decision making based on geometric Bonferroni mean. Peng et al.\(^13\) defined some Pythagorean fuzzy information measures. Garg\(^14\) proposed the generalized Pythagorean fuzzy...
geometric aggregation operators using Einstein t-Norm and t-Conorm for multicriteria decision-making process. Wei and Lu\textsuperscript{15} proposed some Pythagorean fuzzy Maclaurin Symmetric Mean operators in MADM. Wei\textsuperscript{16} developed some Pythagorean fuzzy interaction aggregation operators for MADM. Wei and Lu\textsuperscript{17} proposed some Pythagorean fuzzy power aggregation operators, such as Pythagorean fuzzy power average operator, Pythagorean fuzzy power geometric operator, Pythagorean fuzzy power weighted average operator, Pythagorean fuzzy power weighted geometric operator, Pythagorean fuzzy power ordered weighted average operator, Pythagorean fuzzy power ordered weighted geometric operator, Pythagorean fuzzy power hybrid average operator and Pythagorean fuzzy power hybrid geometric operator in MADM. Wei and Lu\textsuperscript{18} proposed some dual hesitant Pythagorean fuzzy Hamacher aggregation operators in MADM. Lu et al.\textsuperscript{19} defined the concept of hesitant PFSs and utilized Hamacher operations to develop some hesitant Pythagorean fuzzy aggregation operators. Wei et al.\textsuperscript{20} defined the concept of Pythagorean 2-tuple linguistic sets (P2TLSs) and utilized arithmetic and geometric operations to develop some Pythagorean 2-tuple linguistic aggregation operators.

Obviously, these established Pythagorean 2-tuple linguistic aggregation operators cannot be used to fuse the arguments which are correlated.\textsuperscript{21} Meanwhile, the Bonferroni mean (BM)\textsuperscript{22-29} is a very practical tool to tackle the arguments which are correlated. How to effectively extend the mature BM mean to P2TLN environment is a significant research task which is the focus of this paper.

The organization of this manuscript is as follows. The next section reviews P2TLNs and some other basic definitions. The ‘GP2TLWBM operator’ section introduces the extended GWBM operator\textsuperscript{30} which can be used to fuse the P2TLNs, and describes some properties of these operators. Then, we study the MADM problem with P2TLNs based on the GP2TLWBM operator. The penultimate section illustrates the functions of the proposed operators with an example for green supplier selection in green supply chain management area. Finally, the conclusions of the study are given.

**Preliminaries**

**P2TLSs**

Wei et al.\textsuperscript{20} proposed the concepts and basic operations of the P2TLSs on the basis of the PFSs\textsuperscript{1,2} and 2-tuple linguistic model.\textsuperscript{30-38}

**Definition 1.**\textsuperscript{20} A P2TLSs $A$ in $X$ is given

$$P = \{(s_{0(x)}, \rho), (\mu_{P}(x), \nu_{P}(x)), x \in X\}$$  \hspace{1cm} (1)

where $s_{0(x)} \in S, \rho \in [-0.5, 0.5], u_{P}(x) \in [0, 1]$ and $v_{P}(x) \in [0, 1]$, with the condition $0 \leq (u_{P}(x))^{2} + (v_{P}(x))^{2} \leq 1, \forall x \in X$. The numbers $\mu_{P}(x), \nu_{P}(x)$ represent, respectively, the degree of membership and degree of non-membership of the element $x$ to linguistic variable $(s_{0(x)}, \rho)$. Then, for $x \in X$, $\pi_{P}(x) = \sqrt{1 - (u_{P}(x))^{2} + (v_{P}(x))^{2}}$ could be called the degree of refusal membership of the element $x$ to linguistic variable $(s_{0(x)}, \rho)$.

For convenience, Wei et al.\textsuperscript{20} call $p = \langle (s_{p}, \rho), (u_{p}, v_{p}) \rangle$ a Pythagorean 2-tuple linguistic number (P2T LN), where $\mu_{p} \in [0, 1], \nu_{p} \in [0, 1]$, $(\mu_{p})^{2} + (\nu_{p})^{2} \leq 1, s_{0(p)} \in S$ and $\rho \in [-0.5, 0.5]$.

**Definition 2.**\textsuperscript{20} Let $p_{1} = \langle (s_{p_{1}}, \rho_{1}), (u_{p_{1}}, v_{p_{1}}) \rangle$ and $p_{2} = \langle (s_{p_{2}}, \rho_{2}), (u_{p_{2}}, v_{p_{2}}) \rangle$ be two P2TLNs, $S(p_{1}) = \Delta(\Delta^{-1}(s_{0(p_{1})}, \rho_{1}) \cdot \frac{1 + (\mu_{p_{1}})_{2} - (\nu_{p_{1}})^{2}}{2})$ and $S(p_{2}) = \Delta(\Delta^{-1}(s_{0(p_{2})}, \rho_{2}) \cdot \frac{1 + (\mu_{p_{2}})_{2} - (\nu_{p_{2}})^{2}}{2})$ be the scores of $p_{1}$ and $p_{2}$, respectively, and let $H(p_{1}) = \Delta(\Delta^{-1}(s_{0(p_{1})}, \rho_{1}) \cdot \frac{(\mu_{p_{1}})^{2} - (\nu_{p_{1}})^{2}}{2})$ and $H(p_{2}) = \Delta(\Delta^{-1}(s_{0(p_{2})}, \rho_{2}) \cdot \frac{(\mu_{p_{2}})^{2} - (\nu_{p_{2}})^{2}}{2})$ be the accuracy degrees of $p_{1}$ and $p_{2}$, respectively, then if $S(p_{1}) < S(p_{2}), p_{1} < p_{2}$; if $S(p_{1}) = S(p_{2})$, then equation (1) if $H(p_{1}) = H(p_{2}), p_{1} = p_{2}$; (2) if $H(p_{1}) < H(p_{2}), p_{1}$ is smaller than $p_{2}$, denoted by $p_{1} < p_{2}$.

Wei et al.\textsuperscript{20} defined some operational laws of P2TLNs.

**Definition 3.**\textsuperscript{20} Let $p_{1} = \langle (s_{p_{1}}, \rho_{1}), (u_{p_{1}}, v_{p_{1}}) \rangle$ and $p_{2} = \langle (s_{p_{2}}, \rho_{2}), (u_{p_{2}}, v_{p_{2}}) \rangle$ be two P2TLNs, then

$$ \begin{align*}
  p_{1} \oplus p_{2} &= \Delta(\Delta^{-1}(s_{0(p_{1})}, \rho_{1}) + \Delta^{-1}(s_{0(p_{2})}, \rho_{2})) , \\
  &\sqrt{(\mu_{p_{1}})^{2} + (\nu_{p_{2}})^{2} - (\mu_{p_{1}})^{2}(\nu_{p_{2}})^{2}, \nu_{p_{1}}\nu_{p_{2}}}; \\
  p_{1} \otimes p_{2} &= \Delta(\Delta^{-1}(s_{0(p_{1})}, \rho_{1}) \cdot \Delta^{-1}(s_{0(p_{2})}, \rho_{2})) , \\
  &\mu_{p_{1}}\mu_{p_{2}}, \sqrt{(u_{p_{1}})^{2} + (v_{p_{2}})^{2} - (u_{p_{1}})^{2}(v_{p_{2}})^{2}, (\mu_{p_{1}})^{2}}; \\
  \lambda p_{1} &= \Delta(\Delta^{-1}(s_{0(p_{1})}, \rho_{1})) , \\
  &\sqrt{1 - (1 - (\mu_{p_{1}})^{2})^{2}, (\nu_{p_{1}})^{2}}; \\
  (p_{1})^{2} &= \Delta(\Delta^{-1}(s_{0(p_{1})}, \rho_{1})^{2}) , \\
  &((\mu_{p_{1}})^{2}, \sqrt{1 - (1 - (\nu_{p_{1}})^{2})^{2}}) .
\end{align*} $$
GWBM operator

Xia et al.\textsuperscript{24} defined the generalized weighted BM (GWBM) operator.

**Definition 4.**\textsuperscript{24} Let \( p, q, r > 0 \) and \( a_i (i = 1, 2, \ldots, n) \) be a collection of non-negative crisp numbers with the weights vector being \( \omega = (\omega_1, \omega_2, \ldots, \omega_n)^T \), thereby satisfying \( \omega_i \in [0, 1] \) and \( \sum_{i=1}^{n} \omega_i = 1 \). The GWBM operator is defined as follows

\[
\text{GWBM}^{\rho, q, r}(a_1, a_2, \ldots, a_n) = \left( \sum_{i,j,k=1}^{n} (\omega_i a_i^p) \odot (\omega_j a_j^q) \odot (\omega_k a_k^r) \right)^{1/(\rho+q+r)}
\]  

(2)

The GP2TLWBM operator

This section extends GWBM to fuse the Pythagorean 2-tuple linguistic operators and proposes several new Pythagorean 2-tuple linguistic operators.

\[
\text{GP2TLWBM}^{\alpha, \beta, \gamma}(p_1, p_2, \ldots, p_n) = \left( \bigoplus_{i,j,k=1}^{n} (\omega_i p_i^\alpha) \odot (\omega_j p_j^\beta) \odot (\omega_k p_k^\gamma) \right)^{1/(\alpha+\beta+\gamma)}
\]

(3)

**Definition 5.** Let \( \alpha, \beta, \gamma > 0 \) and \( p_i = ((s_{p_i}, \rho_{p_i}), (\mu_{p_i}, v_{p_i}))(i = 1, 2, \ldots, n) \) be a collection of P2TLNs with their weights vector being \( \omega = (\omega_1, \omega_2, \ldots, \omega_n)^T \), thereby satisfying \( \omega_i \in [0, 1] \) and \( \sum_{i=1}^{n} \omega_i = 1 \). If

\[
\text{GP2TLWBM}^{\alpha, \beta, \gamma}(p_1, p_2, \ldots, p_n)
\]

where GP2TLWBM\textsuperscript{\alpha, \beta, \gamma} is called the Generalized Pythagorean 2-tuple linguistic weighted Bonferroni mean (GP2TLWBM) operator.

We can obtain the following theorem according to Definition 3.

**Theorem 1.** Let \( \alpha, \beta, \gamma > 0 \) and \( p_i = ((s_{p_i}, \rho_{p_i}), (\mu_{p_i}, v_{p_i}))(i = 1, 2, \ldots, n) \) be a collection of P2TLNs. The aggregated value by GP2TLWBM is also a P2TLN and

\[
\omega_i p_i^\alpha = \left( \Delta \left( \omega_i \left( \Delta^{-1}(s_{p_i}, \rho_{p_i}) \right)^\beta \right), \sqrt{1 - \left(1 - (1 - v_{p_i}^\beta)^{\omega_i} \right)^2 \omega_i} \right)
\]

(5)

**Proof:** According to Definition 3, we can obtain

\[
\omega_i p_i^\alpha = \left( \Delta \left( \omega_i \left( \Delta^{-1}(s_{p_i}, \rho_{p_i}) \right)^\beta \right), \sqrt{1 - \left(1 - (1 - v_{p_i}^\beta)^{\omega_i} \right)^2 \omega_i} \right)
\]

(6)
\[ \omega_k p_k^\gamma = \left( \Delta \left( \omega_k \left( \Delta^{-1}(s_{p_k}, \rho_k) \right)^\gamma \right), \left( \sqrt{1 - (1 - \mu_k^2)}^\gamma, \sqrt{1 - (1 - v_k^2)}^\gamma \right) \right) \] (7)

Thus

\[ (\omega_k p_k^\gamma) \otimes (\omega_l p_l^\rho) \otimes (\omega_l p_l^\gamma) \]

\[ = \left( \Delta \left( \omega_i \left( \Delta^{-1}(s_{p_i}, \rho_i) \right)^\alpha \cdot \omega_j \left( \Delta^{-1}(s_{p_j}, \rho_j) \right)^\beta \cdot \omega_k \left( \Delta^{-1}(s_{p_k}, \rho_k) \right)^\gamma \right), \left( \sqrt{1 - (1 - \mu_i^2)^\alpha}, \sqrt{1 - (1 - \mu_j^2)^\beta}, \sqrt{1 - (1 - \mu_k^2)^\gamma} \right) \right) \] (8)

Thereafter

\[ \bigoplus_{i,j,k=1}^n (\omega_i p_i^\gamma) \otimes (\omega_j p_j^\rho) \otimes (\omega_k p_k^\gamma) \]

\[ = \left( \Delta \left( \sum_{i,j,k=1}^n \left( \omega_i \left( \Delta^{-1}(s_{p_i}, \rho_i) \right)^\alpha \cdot \omega_j \left( \Delta^{-1}(s_{p_j}, \rho_j) \right)^\beta \cdot \omega_k \left( \Delta^{-1}(s_{p_k}, \rho_k) \right)^\gamma \right) \right), \left( \prod_{i,j,k=1}^n \left( 1 - (1 - \mu_i^2)^\alpha \cdot (1 - \mu_j^2)^\beta \cdot (1 - \mu_k^2)^\gamma \right), \prod_{i,j,k=1}^n \left( 1 - (1 - v_i^2)^\rho, 1 - (1 - v_j^2)^\beta, 1 - (1 - v_k^2)^\gamma \right) \right) \right) \] (9)

Therefore

\[ \text{GP2TLWBM}^{2,\beta,\gamma}(p_1, p_2, \ldots, p_n) = \left( \bigoplus_{i,j,k=1}^n (\omega_i p_i^\gamma) \otimes (\omega_j p_j^\rho) \otimes (\omega_k p_k^\gamma) \right)^{1/(\alpha + \beta + \gamma)} \]

\[ = \left( \Delta \left( \left( \sum_{i,j,k=1}^n \left( \omega_i \left( \Delta^{-1}(s_{p_i}, \rho_i) \right)^\alpha \cdot \omega_j \left( \Delta^{-1}(s_{p_j}, \rho_j) \right)^\beta \cdot \omega_k \left( \Delta^{-1}(s_{p_k}, \rho_k) \right)^\gamma \right) \right)^{1/(\alpha + \beta + \gamma)}, \left( \prod_{i,j,k=1}^n \left( 1 - (1 - \mu_i^2)^\alpha \cdot (1 - \mu_j^2)^\beta \cdot (1 - \mu_k^2)^\gamma \right), \prod_{i,j,k=1}^n \left( 1 - (1 - v_i^2)^\rho, 1 - (1 - v_j^2)^\beta, 1 - (1 - v_k^2)^\gamma \right) \right) \right) \] (10)
Hence, equation (4) is maintained. Thereafter

\[
\omega_i \left( \Delta^{-1}(s_i, \rho_i) \right)^x \cdot \omega_j \left( \Delta^{-1}(s_j, \rho_j) \right)^y \cdot \omega_k \left( \Delta^{-1}(s_k, \rho_k) \right)^z \leq \omega_i \omega_j \omega_k \left( \Delta^{-1}(s_{\text{max}}, \rho_{\text{max}}) \right)^{x+y+z} \tag{11}
\]

\[
\sum_{i,j,k=1}^n \left( \omega_i \left( \Delta^{-1}(s_i, \rho_i) \right)^x \cdot \omega_j \left( \Delta^{-1}(s_j, \rho_j) \right)^y \cdot \omega_k \left( \Delta^{-1}(s_k, \rho_k) \right)^z \right) \leq \sum_{i,j,k=1}^n \omega_i \omega_j \omega_k \left( \Delta^{-1}(s_{\text{max}}, \rho_{\text{max}}) \right)^{x+y+z} \tag{12}
\]

\[
\Delta \left( \left( \sum_{i,j,k=1}^n \left( \omega_i \left( \Delta^{-1}(s_i, \rho_i) \right)^x \cdot \omega_j \left( \Delta^{-1}(s_j, \rho_j) \right)^y \cdot \omega_k \left( \Delta^{-1}(s_k, \rho_k) \right)^z \right) \right)^{1/(x+y+z)} \right) \leq (s_{\text{max}}, \rho_{\text{max}}) \tag{13}
\]

Similarly

\[
(s_{\text{min}}, \rho_{\text{min}}) \leq \Delta \left( \left( \sum_{i,j,k=1}^n \omega_i \left( \Delta^{-1}(s_i, \rho_i) \right)^x \cdot \omega_j \left( \Delta^{-1}(s_j, \rho_j) \right)^y \cdot \omega_k \left( \Delta^{-1}(s_k, \rho_k) \right)^z \right)^{1/(x+y+z)} \right) \tag{14}
\]

Thereafter

\[
0 \leq \left( 1 - \prod_{i,j,k=1}^n \left( 1 - \left( 1 - \left( 1 - \mu_i^2 \right)^{c_i} \right) \cdot \left( 1 - \left( 1 - \mu_j^2 \right)^{c_j} \right) \cdot \left( 1 - \left( 1 - \mu_k^2 \right)^{c_k} \right) \right)^{1/(x+y+z)} \right) \tag{15}
\]

\[
0 \leq 1 - \left( 1 - \prod_{i,j,k=1}^n \left( 1 - \left( 1 - \left( 1 - v_i^2 \right)^{a_i} \right) \cdot \left( 1 - \left( 1 - v_j^2 \right)^{a_j} \right) \cdot \left( 1 - \left( 1 - v_k^2 \right)^{a_k} \right) \right) \right)^{1/(x+y+z)} \tag{16}
\]

Because \( \mu_i^2 + v_i^2 \leq 1 \)

\[
0 \leq \left( 1 - \prod_{i,j,k=1}^n \left( 1 - \left( 1 - \left( 1 - v_i^2 \right)^{a_i} \right) \cdot \left( 1 - \left( 1 - v_j^2 \right)^{a_j} \right) \cdot \left( 1 - \left( 1 - v_k^2 \right)^{a_k} \right) \right) \right)^{1/(x+y+z)} \tag{17}
\]
Therefore

\[
\left(1 - \left(1 - \prod_{i,j,k=1}^{n} \left(1 - \left(1 - (1 - \mu_{i,j,k}^{2})^{\omega_{0}}\right) \cdot \left(1 - (1 - \mu_{i,j,k}^{2})^{\omega_{0}}\right)\right)\right)^{1/(\alpha + \beta + \gamma)\right)^{2} \right.
\]

\[
+ \left(1 - \prod_{i,j,k=1}^{n} \left(1 - \left(1 - (1 - \mu_{i,j,k}^{2})^{\omega_{0}}\right) \cdot \left(1 - (1 - \mu_{i,j,k}^{2})^{\omega_{0}}\right)\right)\right)^{1/(\alpha + \beta + \gamma)\right)^{2} \right.
\]

\[
\leq 1 \left(1 - \prod_{i,j,k=1}^{n} \left(1 - \left(1 - (1 - \mu_{i,j,k}^{2})^{\omega_{0}}\right) \cdot \left(1 - (1 - \mu_{i,j,k}^{2})^{\omega_{0}}\right)\right)\right)^{1/(\alpha + \beta + \gamma)\right)^{2} \right.
\]

\[
= 1
\]

Therefore, the proof of Theorem 1 is completed.

Moreover, GP2TLWBM has the following properties.

**Property 1** (Idempotency). If \(p_{i}(i = 1, 2, \ldots, n)\) are equal, that is \(p_{i} = p = ((s, \rho), (\mu, v))\), then

\[
\text{GP2TLWBM}^{\alpha, \beta, \gamma}(p_{1}, p_{2}, \ldots, p_{n}) = p
\]

**Proof:**

\[
\begin{align*}
\text{GP2TLWBM}^{\alpha, \beta, \gamma}(p_{1}, p_{2}, \ldots, p_{n}) & = \left(\bigoplus_{i,j,k=1}^{n} (\omega_{0}p_{i}) \otimes (\omega_{0}p_{j}) \otimes (\omega_{0}p_{k})\right)^{1/(\alpha + \beta + \gamma)} \\
& = \left(\bigoplus_{i,j,k=1}^{n} (\omega_{0}p_{i}) \otimes (\omega_{0}p_{j}) \otimes (\omega_{0}p_{k})\right)^{1/(\alpha + \beta + \gamma)} \\
& = p_{i,j,k=1}^{(\alpha, \beta, \gamma)} (\omega_{0}p_{i}) \otimes (\omega_{0}p_{j}) \otimes (\omega_{0}p_{k}) \\
& = p
\end{align*}
\]

**Property 2** (Monotonicity). Let \(p_{i} = ((s_{p_{i}}, \rho_{p_{i}}), (\mu_{p_{i}}, v_{p_{i}}))(i = 1, 2, \ldots, n)\) and \(q_{i} = ((s_{q_{i}}, \rho_{q_{i}}), (\mu_{q_{i}}, v_{q_{i}}))(i = 1, 2, \ldots, n)\) be two collections of GP2LNs.

\[
\begin{align*}
\sum_{i,j,k=1}^{n} \omega_{i} \left(\Delta^{-1}(s_{p_{i}}, \rho_{p_{i}})\right)^{x} \cdot \omega_{j} \left(\Delta^{-1}(s_{p_{j}}, \rho_{p_{j}})\right)^{y} \cdot \omega_{k} \left(\Delta^{-1}(s_{p_{k}}, \rho_{p_{k}})\right)^{z} & \leq \sum_{i,j,k=1}^{n} \omega_{i} \left(\Delta^{-1}(s_{q_{i}}, \rho_{q_{i}})\right)^{x} \cdot \omega_{j} \left(\Delta^{-1}(s_{q_{j}}, \rho_{q_{j}})\right)^{y} \cdot \omega_{k} \left(\Delta^{-1}(s_{q_{k}}, \rho_{q_{k}})\right)^{z}
\end{align*}
\]

If \((s_{p_{i}}, \rho_{p_{i}}) \leq (s_{q_{i}}, \rho_{q_{i}})\) and \(\mu_{p_{i}} \leq \mu_{q_{i}}\) and \(v_{p_{i}} \geq v_{q_{i}}\) hold for all \(i\), then

\[
\text{GP2TLWBM}^{\alpha, \beta, \gamma}(p_{1}, p_{2}, \ldots, p_{n}) \leq \text{GP2TLWBM}^{\alpha, \beta, \gamma}(q_{1}, q_{2}, \ldots, q_{n})
\]

**Proof:**

Let \(\text{GP2TLWBM}^{\alpha, \beta, \gamma}(p_{1}, p_{2}, \ldots, p_{n}) = ((s, \rho), (\mu, v))\) and \(\text{GP2TLWBM}^{\alpha, \beta, \gamma}(q_{1}, q_{2}, \ldots, q_{n}) = ((s_{q}, \rho_{q}), (\mu_{q}, v_{q}))\), given that \(\Delta^{-1}(s_{p}, \rho_{p}) \leq \Delta^{-1}(s_{q}, \rho_{q})\), we can obtain

\[
\omega_{i} \left(\Delta^{-1}(s_{p_{i}}, \rho_{p_{i}})\right)^{x} \cdot \omega_{j} \left(\Delta^{-1}(s_{p_{j}}, \rho_{p_{j}})\right)^{y} \cdot \omega_{k} \left(\Delta^{-1}(s_{p_{k}}, \rho_{p_{k}})\right)^{z} \leq \omega_{i} \left(\Delta^{-1}(s_{q_{i}}, \rho_{q_{i}})\right)^{x} \cdot \omega_{j} \left(\Delta^{-1}(s_{q_{j}}, \rho_{q_{j}})\right)^{y} \cdot \omega_{k} \left(\Delta^{-1}(s_{q_{k}}, \rho_{q_{k}})\right)^{z}
\]

Therefore
That means \((s_p, \rho_p) \leq (s_q, \rho_q)\), and we also can obtain

\[
\left(1 - \left(1 - \left(1 - \frac{2}{C_0}\right)^0\right) \cdot \left(1 - \left(1 - \frac{2}{C_0}\right)^0\right) \cdot \left(1 - \left(1 - \frac{2}{C_0}\right)^0\right)\right)
\geq \left(1 - \left(1 - \left(1 - \frac{2}{C_0}\right)^0\right) \cdot \left(1 - \left(1 - \frac{2}{C_0}\right)^0\right) \cdot \left(1 - \left(1 - \frac{2}{C_0}\right)^0\right)\right)
\]

\[
\prod_{i,j=1}^{n} \left(1 - \left(1 - \left(1 - \frac{2}{C_0}\right)^0\right) \cdot \left(1 - \left(1 - \frac{2}{C_0}\right)^0\right) \cdot \left(1 - \left(1 - \frac{2}{C_0}\right)^0\right)\right)
\]

\[
\geq \prod_{i,j=1}^{n} \left(1 - \left(1 - \left(1 - \frac{2}{C_0}\right)^0\right) \cdot \left(1 - \left(1 - \frac{2}{C_0}\right)^0\right) \cdot \left(1 - \left(1 - \frac{2}{C_0}\right)^0\right)\right)
\]

\[
1 - \prod_{i,j=1}^{n} \left(1 - \left(1 - \left(1 - \frac{2}{C_0}\right)^0\right) \cdot \left(1 - \left(1 - \frac{2}{C_0}\right)^0\right) \cdot \left(1 - \left(1 - \frac{2}{C_0}\right)^0\right)\right)
\]

Therefore

\[
\left(1 - \prod_{i,j=1}^{n} \left(1 - \left(1 - \left(1 - \frac{2}{C_0}\right)^0\right) \cdot \left(1 - \left(1 - \frac{2}{C_0}\right)^0\right) \cdot \left(1 - \left(1 - \frac{2}{C_0}\right)^0\right)\right)\right)^{1/(\alpha+\beta+\gamma)}
\]

\[
\leq \left(1 - \prod_{i,j=1}^{n} \left(1 - \left(1 - \left(1 - \frac{2}{C_0}\right)^0\right) \cdot \left(1 - \left(1 - \frac{2}{C_0}\right)^0\right) \cdot \left(1 - \left(1 - \frac{2}{C_0}\right)^0\right)\right)\right)^{1/(\alpha+\beta+\gamma)}
\]

Thus

\[
\left(\left(1 - \prod_{i,j=1}^{n} \left(1 - \left(1 - \left(1 - \frac{2}{C_0}\right)^0\right) \cdot \left(1 - \left(1 - \frac{2}{C_0}\right)^0\right) \cdot \left(1 - \left(1 - \frac{2}{C_0}\right)^0\right)\right)\right)^{1/(\alpha+\beta+\gamma)}\right)^2
\]

\[
\leq \left(\left(1 - \prod_{i,j=1}^{n} \left(1 - \left(1 - \left(1 - \frac{2}{C_0}\right)^0\right) \cdot \left(1 - \left(1 - \frac{2}{C_0}\right)^0\right) \cdot \left(1 - \left(1 - \frac{2}{C_0}\right)^0\right)\right)\right)^{1/(\alpha+\beta+\gamma)}\right)^2
\]

which means \(\mu_p^2 \leq \mu_q^2\); similarly, we can obtain \(v_p^2 \geq v_q^2\)

If \((s_p, \rho_p) < (s_q, \rho_q)\) and \(\mu_p^2 < \mu_q^2\) and \(v_p^2 > v_q^2\), then

\[
\text{GP2TLWBM}^{\alpha,\beta,\gamma}_{\omega}(p_1, p_2, \ldots, p_n) \leq \text{GP2TLWBM}^{\alpha,\beta,\gamma}_{\omega}(q_1, q_2, \ldots, q_n)
\]

If \((s_p, \rho_p) < (s_q, \rho_q)\) and \(\mu_p^2 < \mu_q^2\) and \(v_p^2 = v_q^2\), then

\[
\text{GP2TLWBM}^{\alpha,\beta,\gamma}_{\omega}(p_1, p_2, \ldots, p_n) \leq \text{GP2TLWBM}^{\alpha,\beta,\gamma}_{\omega}(q_1, q_2, \ldots, q_n)
\]
Table 1. P2LN decision matrix.

|       | G₁       | G₂       | G₃       | G₄       |
|-------|----------|----------|----------|----------|
| A₁    | <(s₂, 0), (0.60, 0.70)> | <(s₂, 0), (0.80, 0.50)> | <(s₂, 0), (0.40, 0.60)> | <(s₃, 0), (0.70, 0.70)> |
| A₂    | <(s₂, 0), (0.50, 0.70)> | <(s₄, 0), (0.70, 0.20)> | <(s₄, 0), (0.20, 0.60)> | <(s₁, 0), (0.40, 0.60)> |
| A₃    | <(s₄, 0), (0.70, 0.60)> | <(s₃, 0), (0.50, 0.70)> | <(s₅, 0), (0.50, 0.30)> | <(s₄, 0), (0.60, 0.70)> |
| A₄    | <(s₄, 0), (0.90, 0.20)> | <(s₄, 0), (0.60, 0.50)> | <(s₇, 0), (0.20, 0.50)> | <(s₂, 0), (0.50, 0.60)> |
| A₅    | <(s₄, 0), (0.60, 0.10)> | <(s₁, 0), (0.40, 0.70)> | <(s₂, 0), (0.70, 0.50)> | <(s₃, 0), (0.30, 0.80)> |

Table 2. The aggregating results of the green suppliers by the GP2TLWBM operator \((x = \beta = \gamma = 3)\).

|       | GP2TLWBM       |
|-------|----------------|
| A₁    | <(s₂, 0.4987), (0.6852, 0.6078)> |
| A₂    | <(s₄, -0.1947), (0.5658, 0.5038)> |
| A₃    | <(s₄, 0.2377), (0.5816, 0.5239)> |
| A₄    | <(s₅, 0.3924), (0.6777, 0.4788)> |
| A₅    | <(s₄, -0.3444), (0.6057, 0.5373)> |

Table 3. The score functions of the green suppliers.

|       | GP2TLWBM       |
|-------|----------------|
| A₁    | (s₁, 0.3744)   |
| A₂    | (s₂, 0.0288)   |
| A₃    | (s₂, 0.2540)   |
| A₄    | (s₃, 0.3164)   |
| A₅    | (s₂, -0.0293)  |

Table 4. Ordering of the emerging technology enterprises.

|       | Ordering       |
|-------|----------------|
| GP2TLWBM | A₄ > A₃ > A₂ > A₅ > A₁ |

If \((s₀, p₀) = (s₁, q₁)\) and \(\mu_{p₀}^2 > \mu_{q₁}^2\), then

\[
\text{GP2TLWBM}^{2,\beta,\gamma}(p₁, p₂, \ldots, pₙ) < \text{GP2TLWBM}^{2,\beta,\gamma}(q₁, q₂, \ldots, qₙ);
\]

If \((s₀, p₀) = (s₁, q₁)\) and \(\mu_{p₀}^2 = \mu_{q₁}^2\) and \(\nu_{p₀}^2 = \nu_{q₁}^2\), then

\[
\text{GP2TLWBM}^{2,\beta,\gamma}(p₁, p₂, \ldots, pₙ) = \text{GP2TLWBM}^{2,\beta,\gamma}(q₁, q₂, \ldots, qₙ);
\]

If \((s₀, p₀) = (s₁, q₁)\) and \(\mu_{p₀}^2 = \mu_{q₁}^2\) and \(\nu_{p₀}^2 > \nu_{q₁}^2\), then

\[
\text{GP2TLWBM}^{2,\beta,\gamma}(p₁, p₂, \ldots, pₙ) < \text{GP2TLWBM}^{2,\beta,\gamma}(q₁, q₂, \ldots, qₙ);
\]

Therefore, the proof of Property 2 is completed.

Property 3 (Boundedness). Let \(pᵢ = ((sᵢ, pᵢ), (μᵢ, vᵢ))\) \((i = 1, 2, \ldots, n)\) be a collection of P2TLNs. If \(p^+ = (\max(sᵢ, pᵢ), (\max(μᵢ), \min(vᵢ)))\) and \(p^- = (\min(sᵢ, pᵢ), (\min(μᵢ), \max(vᵢ)))\), then

\[
p^- \leq \text{GP2TLWBM}^{2,\beta,\gamma}(p₁, p₂, \ldots, pₙ) \leq p^+ \tag{29}
\]

Proof:

From Property 1, we can obtain

\[
\text{GP2TLWBM}^{2,\beta,\gamma}(p⁺, p⁺, \ldots, p⁺) = p⁺\tag{30}
\]

From Property 2, we can obtain

\[
p⁻ = \text{GP2TLWBM}^{2,\beta,\gamma}(p⁻, p⁻, \ldots, p⁻) \leq \text{GP2TLWBM}^{2,\beta,\gamma}(p₁, p₂, \ldots, pₙ) \leq \text{GP2TLWBM}^{2,\beta,\gamma}(p⁺, p⁺, \ldots, p⁺) = p⁺\tag{31}
\]
Table 5. Ranking results for different operational parameters of the GP2TLWBM operator.

| \((\alpha, \beta, \gamma)\) | \(S(A_1)\) | \(S(A_2)\) | \(S(A_3)\) | \(S(A_4)\) | \(S(A_5)\) | Ordering |
|--------------------------|------------|------------|------------|------------|------------|-----------|
| (1, 1, 1)                | (S_2, -0.3909) | (S_2, -0.0093) | (S_1, -0.2259) | (S_2, 0.2329) | (S_2, -0.0470) | A4 > A3 > A2 > A5 > A1 |
| (2, 2, 2)                | (S_1, 0.4865) | (S_2, 0.2463) | (S_2, -0.2861) | (S_2, 0.4689) | (S_2, 0.1864) | A4 > A3 > A2 > A5 > A1 |
| (3, 3, 3)                | (S_1, 0.4981) | (S_2, 0.4486) | (S_2, -0.2675) | (S_2, -0.2411) | (S_2, 0.4375) | A4 > A3 > A2 > A5 > A1 |
| (4, 4, 4)                | (S_2, -0.4631) | (S_2, -0.4005) | (S_2, -0.2304) | (S_2, 0.0306) | (S_2, -0.3133) | A4 > A3 > A2 > A5 > A1 |
| (5, 5, 5)                | (S_2, -0.4193) | (S_2, -0.2858) | (S_2, -0.1875) | (S_2, 0.2662) | (S_2, -0.1284) | A4 > A5 > A3 > A2 > A1 |
| (6, 6, 6)                | (S_2, -0.3768) | (S_2, -0.1958) | (S_2, -0.1437) | (S_2, 0.4649) | (S_2, 0.0441) | A4 > A5 > A3 > A2 > A1 |
| (7, 7, 7)                | (S_2, -0.3379) | (S_2, -0.1230) | (S_2, -0.1016) | (S_2, -0.3685) | (S_2, 0.1884) | A4 > A5 > A3 > A2 > A1 |
| (8, 8, 8)                | (S_2, -0.3029) | (S_2, -0.0627) | (S_2, -0.0623) | (S_2, -0.2286) | (S_2, 0.3086) | A4 > A5 > A3 > A2 > A1 |
| (9, 9, 9)                | (S_2, -0.2718) | (S_2, -0.0115) | (S_2, -0.0262) | (S_2, -0.1102) | (S_2, 0.4090) | A4 > A5 > A3 > A2 > A1 |
| (10, 10, 10)             | (S_2, -0.2444) | (S_2, 0.0326) | (S_2, 0.0068) | (S_2, -0.0091) | (S_2, 0.4936) | A4 > A5 > A2 > A3 > A1 |

GP2TLWBM: generalized Pythagorean 2-tuple linguistic-weighted Bonferroni mean.

Table 6. Ordering of the green suppliers.

| Ordering     | P2TLWA | P2TLWG |
|--------------|--------|--------|
|              | A_4 > A_3 > A_2 > A_5 > A_1 | A_4 > A_3 > A_1 > A_2 > A_5 |

Therefore

\[
p^+ \leq GP2TLWBM_{\alpha; \beta; \gamma}(p_1, p_2, \ldots, p_n) \leq p^- \tag{32}
\]

Model for MADM with P2TLN

Based on the GP2TLWBM operator, in this section, we shall propose the model for MADM with P2TLNs. Let \(A = \{A_1, A_2, \ldots, A_m\}\) be a discrete set of alternatives, and \(G = \{G_1, G_2, \ldots, G_n\}\) be the set of attributes, \(\omega = (\omega_1, \omega_2, \ldots, \omega_n)\) is the weighting vector of the attribute \(G_i(j = 1, 2, \ldots, n)\), where \(\omega_j \in [0, 1]\), \(\sum_{j=1}^{n} \omega_j = 1\).

Suppose that \(R = (r_{ij})_{m \times n} = ((s_{ij}, \rho_{ij}), (\mu_{ij}, v_{ij}))_{m \times n}\) is the P2TLN decision matrix, where \(r_{ij}\) takes the form of the P2TLNs, where \(\mu_{ij}\) indicates the degree that the alternative \(A_i\) satisfies the attribute \(G_j\) given by the decision maker, \(v_{ij}\) indicates the degree that the alternative \(A_i\) does not satisfy the attribute \(G_j\) given by the decision maker, \(\mu_{ij} \in [0, 1], v_{ij} \in [0, 1], (\mu_{ij})^2 + (v_{ij})^2 \leq 1, \pi_{ij} = \sqrt{1 - ((\mu_{ij})^2 + (v_{ij})^2)}, s_{ij} \in S, \rho_{ij} \in [-0.5, 0.5]\).

In the following, we apply the GP2TLWBM (GP2TLWGBM) operator to the MADM problems with P2TLNs.

Step 1. We utilize the decision information given in matrix \(R\), and the GP2TLWBM operator

\[
\hat{p}_i = GP2TLWBM_{\alpha; \beta; \gamma}(p_1, p_2, \ldots, p_m)
\]

\[
= \left( \sum_{i,j,k=1}^{n} (\omega_i p_{ij}^\alpha) \odot (\omega_j p_{ij}^\beta) \odot (\omega_k p_{ij}^\gamma) \right)^{1/(\alpha + \beta + \gamma)}
\]

\[
= \left( \sum_{i,j,k=1}^{n} \omega_i \left( \Delta^{-1}(s_{pi}, \rho_{pi}) \right)^\alpha \cdot \omega_j \left( \Delta^{-1}(s_{pj}, \rho_{pj}) \right)^\beta \cdot \omega_k \left( \Delta^{-1}(s_{pk}, \rho_{pk}) \right)^\gamma \right)^{1/(\alpha + \beta + \gamma)}
\]

\[
= \left( \sum_{i,j,k=1}^{n} \frac{1}{1 - \prod_{i,j,k=1}^{n} \left( 1 - (1 - (1 - \mu_{pi}^2 \omega_i)^{1/2}) (1 - (1 - \mu_{pj}^2 \omega_j)^{1/2}) (1 - (1 - \mu_{pk}^2 \omega_k)^{1/2}) \right)^{1/(\alpha + \beta + \gamma)} \cdot \left( 1 - (1 - (1 - v_{pi}^2 \omega_i)^{1/2}) (1 - (1 - v_{pj}^2 \omega_j)^{1/2}) (1 - (1 - v_{pk}^2 \omega_k)^{1/2}) \right)^{1/(\alpha + \beta + \gamma)} \right)^{1/(\alpha + \beta + \gamma)}
\]

\[
l = 1, 2, \ldots, m
\]
to derive the overall preference values $\tilde{p}_i(l = 1, 2, \ldots, m)$ of the alternative $A_i$.

**Step 2.** Calculate the scores $S(\tilde{p}_i) (l = 1, 2, \ldots, m)$ of the overall P2TLNs $\tilde{p}_i(l = 1, 2, \ldots, m)$ to rank all the alternatives $A_i(l = 1, 2, \ldots, m)$ and then to select the best one(s). If there is no difference between two scores $S(\tilde{p}_i)$ and $S(\tilde{p}_j)$, then we need to calculate the accuracy degrees $H(\tilde{p}_i)$ and $H(\tilde{p}_j)$ of the overall P2TLNs $p_i$ and $p_j$, respectively, and then rank the alternatives $A_i$ and $A_j$ in accordance with the accuracy degrees $H(\tilde{p}_i)$ and $H(\tilde{p}_j)$.

**Step 3.** Rank all the alternatives $A_i(l = 1, 2, \ldots, m)$ and select the best one(s) in accordance with $S(\tilde{p}_i) (l = 1, 2, \ldots, m)$.

**Step 4.** End.

### Numerical example and comparative analysis

#### Numerical example

In this section, we shall present a numerical example to select green suppliers in green supply chain management with P2TLNs in order to illustrate the method proposed in this paper. There is a panel with five possible green suppliers in the green supply chain management $A_i(i = 1, 2, 3, 4, 5)$ to select. The experts select four attributes to evaluate the five possible green suppliers: ① $G_1$ is the product quality factor; ② $G_2$ is the environmental factors; ③ $G_3$ is the delivery factor; ④ $G_4$ is the price factor. The five possible green suppliers $A_i(i = 1, 2, 3, 4, 5)$ are to be evaluated using the P2TLNs by the decision maker under the above four attributes (whose weighting vector $\omega = (0.15, 0.25, 0.35, 0.25)$), as listed in Table 1.

In the following, we utilize the approach developed to select green suppliers in green supply chain management.

**Step 1.** According to P2LN $r_j(i = 1, 2, 3, 4, 5, j = 1, 2, 3, 4)$, we can aggregate all P2LN $r_j$ by using the GP2TLWBM operator to obtain the overall P2LN $A_i(i = 1, 2, 3, 4, 5)$ of the green supplier $A_i$. The aggregating results are shown in Table 2.

**Step 2.** According to the aggregating results shown in Table 2, the score functions of the green suppliers are shown in Table 3.

**Step 3.** According to the score functions shown in Table 3 and the comparison formula of score functions, the ordering of the green suppliers is shown in Table 4. Note that “>” means “preferred to”. As we can see, depending on the aggregation operators used, the ordering of the green suppliers is slightly different, but the best green supplier is $A_4$.

### Influence of the parameters on the final result

In order to show the effects on the ranking results by changing parameters of $(\alpha, \beta, \gamma) \in [1, 10]$ in the GP2TLWBM operator, all the results are shown in Tables 5.

### Comparative analysis

Then, we compare our proposed method with other existing methods including P2TLWA operator and P2TLWG operator proposed by Wei et al. The comparative results are shown in Table 6.

From the above, we can obtain the same results to show the practicality and effectiveness of the proposed approaches. However, the existing aggregation operators, such as P2TLWA operator and P2TLWG operator, do not consider the information about the relationship between arguments being aggregated, and thus cannot eliminate the influence of unfair arguments on the decision result. Our proposed GP2TLWBM operator considers the information about the relationship between arguments being aggregated.

### Conclusion

In this paper, we focused on P2TLN information aggregation operators, as well as their applications in MADM. To aggregate the P2TLNs, the GP2TLWBM operator has been developed. Further research has been conducted to explore the desirable properties of this operator. In addition, we demonstrated the effectiveness of the GP2TLWBM operator in practical MADM problems. At the end of this study, we use an example about green supplier selection in the green supply chain management process to illustrate the applicability of this operator; meanwhile, the analysis of the comparison when the parameters take different values also has been studied. In the future, we shall extend the proposed models to other uncertain and fuzzy MADM problems.

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