AA-stacked bilayer graphene in an applied electric field: Tunable antiferromagnetism and coexisting exciton order parameter

R.S. Akzyanov,1,2,3 A.O. Sboychakov,2,4 A.V. Rozhkov,1,2,4 A.L. Rakhmanov,1,2,3,4 and Franco Nori4,5

1Moscow Institute of Physics and Technology, Dolgoprudny, Moscow Region, 141700 Russia
2Institute for Theoretical and Applied Electrodyamics, Russian Academy of Sciences, Moscow, 125412 Russia
3All-Russia Research Institute of Automatics, Moscow, 127055 Russia
4CEMS, RIKEN, Saitama, 351-0198, Japan
5Department of Physics, University of Michigan, Ann Arbor, MI 48109-1040, USA

PACS numbers: 73.22.Pr, 73.22.Gk, 73.21.Ac

I. INTRODUCTION

The electronic properties of graphene are a subject of active theoretical and experimental studies.1–3 In addition to single-layer graphene, bilayer graphene attracts significant research attention. This interest is partly driven by the desire to extend the family of graphene-like materials, and to create materials with a controllable gap in the electronic spectrum.

The most studied form of bilayer is the AB (or Bernal) stacked bilayer graphene (AB-BLG).4–8 The biased BLG has a tunable gap.9,10 Excitons can exist in AB-BLG under certain conditions.11–13

The AA-stacked bilayer graphene (AA-BLG) has received less attention.14–22 However, samples of AA-BLG have recently been produced17–19 and a detailed study of this system becomes necessary. A significant feature of the AA-BLG is the perfect nesting of the hole and electron Fermi surfaces. These degenerate Fermi surfaces are unstable with respect of an arbitrarily weak electron interaction, and the AA-BLG becomes an antiferromagnetic (AFM) insulator with a finite electron gap.20 This electronic instability is strongest at zero doping, when the bands cross at the Fermi level.

An interesting phenomenon, which occurs in bilayer systems, is exciton condensation.23–25 In graphene bilayers, exciton condensation attracted attention for both fundamental reasons23–25 and possible applications in devices, including ultra-fast switches and dispersionless field-effect transistors.26

The purpose of this paper is to investigate the influence of a transverse electric field on the properties of the AA-BLG. We show that such a field can partially suppress the AFM order parameter. However, the degree of suppression heavily depends on the effective value of the on-site Coulomb repulsion. Moreover, the transverse bias stabilizes the exciton order parameter. Namely, we found that the exciton order parameter coexists with the AFM order if a transverse electric field is applied. The exciton order is tuned by the voltage and tied to the AFM order. Since the magnitude of the gap is sensitive to the transverse field, it appears possible to control the transport properties of the bilayer with the help of a transverse bias, which can be created by, e.g., a gate electrode.

The paper is organized as follows. In section II we analyze the single-electron part of our model. Within the tight-binding approach we derive the degenerate electronic spectrum of the model. In section III we consider the on-site and inter-site inter-plane Coulomb repulsion using a mean-field theory. The electronic interaction removes the degeneracy of the single electron spectrum creating a gap. We found that the phase with coexisting AFM and exciton orders is the most stable one. We obtain the equations for the order parameters and solve them using both analytic and numerical methods.

FIG. 1: (Color online) Crystal structure of the AA-stacked bilayer graphene. The circles denote carbon atoms in the A (red) and B (blue) sublattices in the bottom (1), in grey, and top (2), in yellow, layers. The unit cell of the AA-BLG consists of four atoms A1, A2, B1, and B2. The hopping integrals t and t0 correspond to the in-plane and inter-plane nearest-neighbor hopping. A transverse electrical voltage V0 is applied to the planes.
II. TIGHT-BINDING HAMILTONIAN

The crystal structure of the AA-BLG is shown in Fig. 1. The AA-BLG consists of two graphene layers, 1 and 2. Each carbon atom of the upper layer is located above the corresponding atom of the lower layer. Each layer consists of two triangular sublattices A and B. The elementary unit cell of the AA-BLG contains four carbon atoms A1, A2, B1, and B2.

We write the single-particle tight-binding Hamiltonian of the AA-BLG in the form

\[
H_0 = -t \sum_{\langle \mathbf{nm} \rangle \sigma} \left( d_{n \mathbf{A} \sigma}^\dagger d_{m \mathbf{B} \sigma} + H.c. \right) + t_0 \sum_{\mathbf{n} \sigma} \left( d_{n \mathbf{A} \sigma}^\dagger d_{n \mathbf{B} \sigma} + H.c. \right) + \frac{V_0}{2} \sum_{\mathbf{n} \sigma} \left( d_{n \mathbf{A} \sigma}^\dagger d_{n \mathbf{B} \sigma} - d_{n \mathbf{B} \sigma}^\dagger d_{n \mathbf{A} \sigma} \right). \tag{1}
\]

Here \(d_{n \mathbf{A} \sigma}\) and \(d_{n \mathbf{B} \sigma}\) are the creation and annihilation operators of an electron with spin projection \(\sigma\) in the layer \(i = 1, 2\) on the sublattice \(\mathbf{A}, \mathbf{B}\) at the position \(\mathbf{n}\), and (...) denotes a nearest-neighbor pair inside a layer. The amplitude \(t(\mathbf{g})\) in Eq. (1) describes the in-plane (inter-plane) nearest-neighbor hopping. \(V_0\) is the voltage applied perpendicular to the layers. We assume that \(V_0 \ll t\), which corresponds to typical experimental conditions\(^{9,10}\).

For calculations, we use the values of the hopping integrals \(t = 2.57\, \text{eV}, t_0 = 0.36\, \text{eV}\), computed by the DFT method for multilayer carbon systems in Ref. 31.

After diagonalizing the Hamiltonian (1) we obtain four bands \(\varepsilon_{0k}^{(s)} (s = 1, \ldots, 4)\), which can be written as

\[
\varepsilon_{0k}^{(1)} = -\sqrt{t_0^2 + \frac{V_0^2}{4}} - t \zeta_k, \quad \varepsilon_{0k}^{(2)} = -\sqrt{t_0^2 + \frac{V_0^2}{4}} + t \zeta_k,
\]

\[
\varepsilon_{0k}^{(3)} = \sqrt{t_0^2 + \frac{V_0^2}{4}} - t \zeta_k, \quad \varepsilon_{0k}^{(4)} = \sqrt{t_0^2 + \frac{V_0^2}{4}} + t \zeta_k, \tag{2}
\]

where

\[
\zeta_k = |f_k|, \quad f_k = 1 + 2 \exp \left( \frac{3ik_x a_0}{2} \right) \cos \left( \frac{\sqrt{3}k_y a_0}{2} \right), \tag{3}
\]

and \(a_0 = 1.42\, \text{Å}\) is the in-plane carbon-carbon distance. The bands \(s = 2\) and \(s = 3\) cross the Fermi level near the Dirac points \(\mathbf{K} = 2\pi(\sqrt{3}, 1)/(3\sqrt{3}a_0)\) and \(\mathbf{K}' = 2\pi(\sqrt{3}, -1)/(3\sqrt{3}a_0)\). As it follows from Eqs. (2), the band \(s = 2\) is electron-like, while the band \(s = 3\) is hole-like. The band \(s = 1\) lies below and the band \(s = 4\) lies above the Fermi energy and, consequently, they do not form a Fermi surface.

In contrast to the Bernal stacking (where the bias voltage opens a gap at the Fermi level\(^{9,10}\)), the application of a transverse bias voltage does not qualitatively change the spectrum of the AA-BLG. The Fermi surface is given by the equation \(\zeta_k = t^{-1} \left( \sqrt{t_0^2 + V_0^2} / 4 \right) = \zeta_0\). Since \(\zeta_0 \ll 1\), we can expand the function \(\zeta_k\) near the Dirac points and find that the Fermi surface consists of two circles with radius \(k_r = 2\zeta_0/(3a_0)\).

One of the most important features of this tight-binding band structure is that the Fermi surfaces of both bands coincide. That is, the electron and hole components of the Fermi surface are perfectly nested. This property of the Fermi surface is quite robust against changes in the tight-binding Hamiltonian. It survives even if longer-range hoppings are taken into account, or a system with two non-equivalent layers is considered (e.g., the single-side hydrogenated graphene\(^{32}\)). However, the electron interactions can remove the degeneracy in the spectrum, creating a finite gap\(^{20}\).

III. ELECTRON-ELECTRON INTERACTION

The electronic spectrum changes drastically when considering the Coulomb interaction. To study the effects of this interaction on the electronic properties of our system, we use the following Hubbard-like Hamiltonian

\[
H_{\text{int}} = H_U + H_V. \tag{4}
\]

The first term, \(H_U\), is the on-site Coulomb repulsion between the electrons,

\[
H_U = U_0 \sum_{\mathbf{n} \mathbf{A} \sigma} \left( n_{\mathbf{n} \mathbf{A} \sigma} - \frac{1}{2} \right) \left( n_{\mathbf{n} \mathbf{B} \sigma} - \frac{1}{2} \right), \tag{5}
\]

where \(n_{\mathbf{n} \mathbf{A} \sigma} = d_{\mathbf{n} \mathbf{A} \sigma}^\dagger d_{\mathbf{n} \mathbf{A} \sigma}\) is the operator of the occupation number. The second term, \(H_V\), describes the nearest-neighbor Coulomb repulsion. It has a form

\[
H_V = U_{12} \sum_{\mathbf{n} \mathbf{A} \sigma} \left( n_{\mathbf{n} \mathbf{A} \sigma} - \frac{1}{2} \right) \left( n_{\mathbf{n} \mathbf{B} \sigma'} - \frac{1}{2} \right) + U_{ab} \sum_{\langle \mathbf{nm} \rangle \sigma \sigma'} \left( n_{\mathbf{n} \mathbf{A} \sigma} - \frac{1}{2} \right) \left( n_{\mathbf{m} \mathbf{B} \sigma'} - \frac{1}{2} \right), \tag{6}
\]

where the first term is the nearest-neighbor interaction between the electrons in different layers, while the second term describes the in-plane nearest-neighbor interaction. The terms \(1/2\) in the brackets in Eqs. (5) and (6) are added to keep the chemical potential corresponding to the half-filling (zero doping) equal to zero.

The value of the electron-electron interaction in graphene is relatively strong. According to DFT calculations\(^{30}\), the on-site repulsion energy, \(U_0\), is about 9–10eV, while the in-plane inter-site repulsion, \(U_{ab}\), is about 5–6eV. The nearest-neighbor inter-plane interaction in the bilayer graphene is unknown. We can estimate it as \(U_{12} \approx U_{ab} a_0 / c \approx 2.5\, \text{eV}\), where \(c = 3.35\, \text{Å}\) is the distance between the layers. It is commonly accepted that the mean-field calculations overestimate the
The mean-field expression for the inter-site part of the Hamiltonian has the following form

\[ H_{\text{MF}} = \mathcal{N} \left[ -\frac{(3U_{ab} - U_{12})\Delta n^2}{2} + \frac{4\Delta_{\text{exc}}^2}{U_{12}} \right] + \frac{(3U_{ab} - U_{12})\Delta n}{2} \sum_{n\sigma}(n_{11\sigma} - n_{22\sigma}) - \sum_{n\sigma}(\Delta_{\text{exc}}^d (d_{n1\alpha\uparrow}d_{n2\alpha\uparrow} - d_{n1\alpha\downarrow}d_{n2\alpha\downarrow}) + H.c.) \].

We introduce the four-component spinor

\[ \psi_{k\sigma} = (d_{k1\sigma\uparrow}, d_{k2\sigma\uparrow}, d_{k1\sigma\downarrow}, d_{k2\sigma\downarrow}). \]

In terms of this spinor, the mean field Hamiltonian

\[ H_{\text{MF}} = H_0 + H_{U}^{\text{MF}} + H_{V}^{\text{MF}} \]

can be written as

\[ H_{U}^{\text{MF}} = -\mathcal{N}U_0\Delta n^2 + \frac{U_0\Delta n}{4} \sum_{n\sigma}(n_{11\sigma} - n_{22\sigma}) + \frac{4\mathcal{N}\Delta_{\text{AFM}}^2}{U_0} - \sum_{n\sigma}\Delta_{\text{AFM}}^d n_{\sigma\uparrow} - n_{\sigma\downarrow}, \]

where \(\Delta n = \sum_{\sigma}(n_{11\sigma} - n_{22\sigma})\) is the difference in the electron densities in two graphene layers induced by the applied voltage \(V_0\) and \(\mathcal{N}\) is the number of unit cells in the sample.

Let us consider now the inter-site part of the interaction. The Hamiltonian \(H_{U}\) can produce several order parameters in the system. However, for zero-bias voltage all of them compete with the antiferromagnetism and only the antiferromagnetic order parameter survives, because \(U_0\) is the strongest interaction constant. A nonzero bias voltage breaks the symmetry between two graphene layers. In this case, there exists an order parameter driven by the electron-electron interaction. In the field approximation, the on-site interaction Hamiltonian, \(\Delta_{0}^d\), takes the form

\[ \Delta_{0}^d = \frac{U_{02}}{2} \left( (d_{n1\alpha\uparrow}d_{n2\alpha\downarrow} - d_{n1\alpha\downarrow}d_{n2\alpha\uparrow}) \right), \]

and the \(\Delta_{\text{exc}}^d\) is real. This order parameter corresponds to the bound state of the electron and the hole in different layers. We call it the exciton order parameter.

The mean-field expression for the inter-site part of the Hamiltonian has the following form

\[ H_{V}^{\text{MF}} = \mathcal{N} \left[ -\frac{(3U_{ab} - U_{12})\Delta n^2}{2} + \frac{4\Delta_{\text{exc}}^2}{U_{12}} \right] + \frac{(3U_{ab} - U_{12})\Delta n}{2} \sum_{n\sigma}(n_{11\sigma} - n_{22\sigma}) - \sum_{n\sigma}(\Delta_{\text{exc}}^d (d_{n1\alpha\uparrow}d_{n2\alpha\uparrow} - d_{n1\alpha\downarrow}d_{n2\alpha\downarrow}) + H.c.) \].

In Eq. (17) the quantity \(V\) is the effective bias voltage given by the relation

\[ V = V_0 + \alpha\Delta n, \quad \alpha = \frac{U_0 + 6U_{ab} - 2U_{12}}{2} > 0. \]

This equation describes the screening of the applied voltage due to the electron-electron interaction. Indeed, since \(\Delta n < 0\) for \(V_0 > 0\), we have \(V < V_0\). For the parameters \(U_0, U_{ab}\), and \(U_{12}\) under study the constant \(\alpha\) can be estimated as \(3.5t - 7t\).

The mean-field spectrum is obtained by the diagonalization of two \(4 \times 4\) matrices in Eq. (15). It consists of four bands doubly-degenerate with respect to spin

\[ \varepsilon_k^{(1)} = -\sqrt{A_k + 2B_k}, \quad \varepsilon_k^{(2)} = -\sqrt{A_k - 2B_k}, \]

\[ \varepsilon_k^{(3)} = \sqrt{A_k - 2B_k}, \quad \varepsilon_k^{(4)} = \sqrt{A_k + 2B_k}. \]
where
\[
A_k = \Delta_{AFM}^2 + \Delta_{exc}^2 + t^2 \xi_k^2 + t_0^2 + \frac{V^2}{4},
\]
\[
B_k = \sqrt{\left[-\Delta_{exc}t_0 + \Delta_{AFM} V \right]^2 + t^2 \xi_k^2 \left[ t_0^2 + \frac{V^2}{4} \right]}.
\]
The full gap in the spectrum \( \Delta \) is defined as \( \Delta = \min_k (\xi_k^{(3)} - \xi_k^{(2)})/2 \). It relates to the AFM and exciton order parameters as
\[
\Delta = \frac{2\Delta_{AFM} t_0 + \Delta_{exc} V}{\sqrt{4t_0^2 + V^2}}.
\]
To determine the values of the order parameters \( \Delta_{AFM} \) and \( \Delta_{exc} \) we should minimize the grand potential \( \Omega \). The grand potential per unit cell is
\[
\Omega = E_0 - 2T \sum_{\alpha=1}^{4} \int \frac{d\mathbf{k}}{\Omega_{BZ}} \ln \left[ 1 + e^{-\xi_{\mathbf{k}}/T} \right],
\]
where \( \Omega_{BZ} \) is the volume of the first Brillouin zone.

To calculate the integrals over the Brillouin zone, it is convenient to introduce the density of states
\[
\rho_0(\zeta) = \int \frac{d\mathbf{k}}{\Omega_{BZ}} \delta(\zeta - \xi_{\mathbf{k}}).
\]
This function is non-zero only if \( 0 < \zeta < 3 \). It is related to the single layer graphene density of states \( \rho_{gr}(\zeta) \) as \( \rho_{gr}(\zeta) = \rho_0(\zeta)/t \) (see Ref. 1).

Minimization of \( \Omega \) with respect to \( \Delta_{AFM} \) and \( \Delta_{exc} \) gives the equations
\[
\frac{4\Delta_{AFM}}{U_0} = \int \frac{d\zeta}{3} \rho_0(\zeta) \left[ \Delta_{AFM} - t \theta(\zeta) \right] F(\xi^{(1)}(\zeta)) + \int \frac{d\zeta}{3} \rho_0(\zeta) \left[ \Delta_{AFM} - t \theta(\zeta) \right] F(\xi^{(2)}(\zeta)),
\]
\[
\frac{4\Delta_{exc}}{U_{12}} = \int \frac{d\zeta}{3} \rho_0(\zeta) \left[ \Delta_{exc} - t_0 \theta(\zeta) \right] F(\xi^{(1)}(\zeta)) + \int \frac{d\zeta}{3} \rho_0(\zeta) \left[ \Delta_{exc} - t_0 \theta(\zeta) \right] F(\xi^{(2)}(\zeta)),
\]
where
\[
F(\xi) = \frac{f(-\xi) - f(\xi)}{\xi}, \quad f(\xi) = \frac{1}{e^{\xi/T} + 1},
\]
\[
\theta(\zeta) = \sqrt{\frac{2\Delta_{exc} t_0 - \Delta_{AFM} V}{\sqrt{2\Delta_{exc} t_0 - \Delta_{AFM} V}^2 + t^2 (4t_0^2 + V^2) \zeta^2}},
\]
and \( \xi^{(1)}(\zeta) \) are given by Eqs. (20) and (21), in which \( \xi_{\mathbf{k}} \) is replaced by \( \xi \).

Equations (25) and (26) define the AFM and exciton order parameters as functions of the effective bias voltage \( V \). In order to find the dependencies of \( \Delta_{AFM} \) and \( \Delta_{exc} \) on the applied voltage \( V_0 \) we should use Eq. (19). To find the charge imbalance between two graphene layers \( \Delta n \), we apply the Hellman-Feynman theorem
\[
\Delta n = 2 \left( \frac{\partial H_{MF}}{\partial V} \right) = 2 \frac{\partial E}{\partial V},
\]
where \( E \) is the energy of the system per unit cell. It can be written as
\[
E = E_0 + \sum_{s=1}^{4} \int_0^3 d\zeta \rho_0(\zeta) \frac{\varepsilon^{(s)}(\zeta)}{f(\varepsilon^{(s)}(\zeta))}.
\]

As a result, the expression for the renormalized bias \( V \) takes the following form
\[
V = V_0 + \frac{2\alpha}{\delta(\zeta)} \frac{\partial E(V)}{\partial V}.
\]
This equation, together with Eqs. (25) and (20), define the AFM and exciton order parameters as functions of the applied voltage.

### B. Analytical results

In this subsection we obtain the solution of Eqs. (25), (26), and (30) in the limits \( \Delta_{exc} \ll \Delta_{AFM} \ll t_0 \) and \( T = 0 \). When these conditions hold, the functions \( \varepsilon^{(1,2)}(\zeta) \) and \( \theta(\zeta) \) become
\[
\varepsilon^{(1)}(\zeta) \approx -\sqrt{\Delta_{AFM}^2 + t^2 (\zeta - \zeta_0)^2},
\]
\[
\varepsilon^{(2)}(\zeta) \approx -t (\zeta + \zeta_0),
\]
\[
\theta(\zeta) \approx \frac{2\Delta_{exc} t_0 - \Delta_{AFM} V}{2t^2 \zeta_0},
\]
where
\[
\zeta_0 = \frac{\sqrt{t_0^2 + V^2/4}}{t}.
\]
Substituting \( \varepsilon^{(1,2)}(\zeta) \) with \( \Delta_{AFM} = 0 \) in Eq. (25), we obtain the following relation between \( V \) and \( V_0 \)
\[
V = \frac{V_0 t}{t + C \alpha \zeta_0}, \quad C = \left. \frac{\partial \rho_0(\zeta)}{\partial \zeta} \right|_{\zeta \rightarrow 0} \approx 0.37.
\]
For realistic parameter values, the renormalized bias voltage \( V \) depends almost linearly on \( V_0 \). Taking \( \alpha = 10 \text{ eV} \), we observe from Eq. (33) that \( V = 0.83 V_0 \), if \( V_0 \ll t_0 \). The numerical analysis shows that the estimation \( V \approx V_0 \) becomes even better for larger values of \( \Delta_{AFM}/t \) and \( \Delta_{exc}/t \).
the typical values of the system parameters insulator transition. This makes it possible to observe a voltage-driven metal-

however, it can be fully suppressed by the bias voltage. for smaller

it follows from this figure, the gap suppression is stronger

increases. The dependence of the full gap $\Delta$ on $V$ zero. The dependence of the gap on the ratio $U_0/V_0$ for zero bias is shown in Fig. 2. The analytical expression Eq. (34) works well for $U_0 \leq 2.3t$.

If $V_0 \neq 0$, the exciton order parameter becomes non-

The full gap $\Delta$, however, decreases when $V_0$ increases. The dependence of the full gap $\Delta$ on $V_0$ calculated for three different values of $U_0$ is shown in Fig. 3. As it follows from this figure, the gap suppression is stronger for smaller $U_0$.

We consider here only the case of zero temperature. In this case the full gap never reaches zero for realistic values of the applied voltage. At finite temperatures, however, it can be fully suppressed by the bias voltage. This makes it possible to observe a voltage-driven metal-insulator transition.

Let us analyze now the dependencies of the AFM and exciton order parameters on the applied voltage. For the typical values of the system parameters $U_0 \simeq 2.2t$, $U_1/2U_0 \simeq 1/4$, and $V_0/t_0 \simeq 1$, the values of the order parameters are $\Delta_{AFM} \simeq 0.17$ eV and $\Delta_{exc} \simeq 8$ meV. We can rewrite the expressions for the order parameters in terms of magnetizations

$$S_z = \frac{2\Delta_{AFM}}{U_0} = \langle n_{1A\uparrow} \rangle - \langle n_{1A\downarrow} \rangle,$$

$$\phi = \frac{2\Delta_{exc}}{U_{12}} = \langle d_{1A\uparrow}^\dagger d_{2A\uparrow} \rangle - \langle d_{1A\downarrow}^\dagger d_{2A\downarrow} \rangle.$$

In these equations, the AFM magnetization $S_z$ is equal to the magnetization per site of the sublattice $A$ in layer 1. For the G-type AFM order, the magnetizations of electrons located at neighboring sites have opposite signs.

The exciton magnetization $\phi$ can be viewed as the spin located on the link connecting the sites $A1$ and $A2$ (for definitions of $A1$ and $A2$, see Fig. 1). The spin on the link connecting carbon atoms $B1$ and $B2$ has opposite sign. The dependence of $S_z$ on the applied bias voltage calculated for three different values of the on-site interaction constant $U_0$ is shown in Fig. 4 by the solid lines. This magnetization is suppressed by the bias voltage. The suppression is stronger for smaller $U_0$. When $U_0 > 2.4t$, the magnetization $S_z$ only slightly depends on $V_0$. The exciton magnetization $\phi$ is shown in Fig. 4 by dashed lines. It increases almost linearly with $V_0$. Nevertheless, $\phi$ is much smaller than $S_z$, even for relatively large $V_0$.

**D. Exciton order parameter**

In the limit of small interactions $U_{12}, U_0 \ll t$, the second equation in (33) simplifies and reduces to

$$\Delta_{exc} = \frac{\Delta_{AFM} V_{12}}{2t_0 U_0}.$$
Can we detect this order parameter? In principle, the exciton condensation can be observed experimentally by measuring the Coulomb drag. The experimental observation of Coulomb drag in bilayer graphene systems with a dielectric barrier between the layers has been reported. The execution and interpretation of similar experiment on bilayer graphene without the insulating layer might be a much more complicated issue.

All the above results were obtained at zero temperature. The detailed study of the temperature dependence of the AFM order parameter at zero bias voltage was performed in Ref. Since the graphene bilayer is a two dimensional system, it does not have a distinct magnetic phase transition. However, we can define a crossover temperature between the short-range antiferromagnetic and paramagnetic states. The calculations done in Ref. show that . For realistic values of the applied voltage the exciton order parameter is much smaller than the AFM order parameter. Consequently, . However the exciton order parameter is tied with the AFM order parameter, and we expect that they both have the same crossover temperature about, at . Since the AFM order parameter can be high enough, the exciton order parameter can survive at relatively high temperatures.

### IV. CONCLUSIONS

In this paper we have studied theoretically the electronic properties of biased AA stacked bilayer graphene. The model Hamiltonian was analyzed in the mean-field approximation. At zero bias, the ground state of the system is antiferromagnetic. We found that the applied transverse voltage stabilizes the exciton order parameter coexisting with the AFM order. This new order parameter couples the electrons and holes in different graphene layers. The AFM phase with the coexisting exciton order parameter is the most stable phase if the bias voltage is non-zero. The electronic gap is partially suppressed by the bias voltage leading to a tunable metal-insulator transition. The value of the exciton order parameter can be about several tens of meV. Despite this small value, the exciton order parameter can survive at relatively high temperatures due to its coexistence with the AFM phase.

**Acknowledgments**

This work was supported in part by RFBR (Grants Nos. 14-02-00276, 14-02-00058, 12-02-00339), the RIKEN iTHES Project, MURI Center for Dynamic Magneto-Optics, and a Grant-in-Aid for Scientific research (S).
Here we derive the analytical formulas for the AFM and exciton order parameters. We introduce dimensionless quantities $\delta_{\text{AFM}} = \Delta_{\text{AFM}}/t$ and $\delta_{\text{exc}} = \Delta_{\text{exc}}/t$. It is convenient to rewrite the exciton order parameter in the following form

$$\delta_{\text{exc}} = \delta_{\text{AFM}} \frac{V}{2t_0} b,$$

(A1)
where \( b \) is the new variable. Using this substitution we can rewrite Eqs. (25) and (26) in the form

\[
\frac{4t}{U_0} = \int_{0}^{\infty} d\zeta \rho_0(\zeta) \left[ \frac{1}{\sqrt{\delta_{AFM}^2 + (\zeta - \zeta_0)^2}} + \frac{1}{\zeta + \zeta_0} \right] - (A2)
\]

\[
\frac{4t}{U_{12}} = \int_{0}^{\infty} d\zeta \rho_0(\zeta) \left[ \frac{1}{\sqrt{\delta_{AFM}^2 + (\zeta - \zeta_0)^2}} + \frac{1}{\zeta + \zeta_0} \right] + (A3)
\]

\[
\frac{4t}{U_0} = \int_{0}^{\infty} d\zeta \rho_0(\zeta) \left[ \frac{1}{\sqrt{\delta_{AFM}^2 + (\zeta - \zeta_0)^2}} + \frac{1}{\zeta + \zeta_0} \right] + (A4)
\]

Performing the similar integration in Eq. (A4) and expressing the logarithmic term using Eq. (A3) we obtain in the limit of small \( b \ll 1 \) the following equation for \( b \)

\[
b = \frac{4t}{U_{12}} - \frac{\eta_1(\zeta_0) - \eta_2(\zeta_0)\zeta_0}{\sqrt{2} \frac{\rho_0(\zeta_0)}{\zeta_0} + \frac{1}{\sqrt{2} \frac{\rho_0(\zeta_0)}{\zeta_0}}} .
\]

For the range of parameters \( U_0 \) and \( U_{12} \) under study, we have \( b < 0.05 \), so the assumption \( b \ll 1 \) is well satisfied. The expression for \( \Delta_{\text{exc}} \) is written as follows

\[
\Delta_{\text{exc}} = \Delta_{\text{AFM}} V \left[ \frac{4t}{U_{12}} - \frac{\eta_1(\zeta_0) - \eta_2(\zeta_0)\zeta_0}{\sqrt{2} \frac{\rho_0(\zeta_0)}{\zeta_0} + \frac{1}{\sqrt{2} \frac{\rho_0(\zeta_0)}{\zeta_0}}} \right] .
\]

The antiferromagnetic gap is found from Eq. (A4), where we can neglect \( b \) in the first and third terms. As a result, we obtain

\[
\Delta_{\text{AFM}} = 2t/\sqrt{\zeta_0(\zeta_0 - \zeta_0)} \exp \left[ \frac{\eta_1(\zeta_0) + \eta_2(\zeta_0)\zeta_0 - 4t}{2\rho_0(\zeta_0)\zeta_0} \right] .
\]