Displaced Vertices in Extended Supersymmetric Models

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Abstract

In extended supersymmetric models with additional singlet Higgs fields displaced vertices could be observed if the decay width of the next-to-lightest supersymmetric particle becomes very small due to a singlino dominated LSP. We study the supersymmetric parameter space where displaced vertices of the second lightest neutralino exist in the NMSSM and an E\(_6\) inspired model. For a mass difference between LSP and NLSP of more than 10 GeV the singlet vacuum expectation value has to be at least of the order of 100 TeV in order to obtain a lightest neutralino with a singlino component large enough for displaced vertices.

1 Introduction

Displaced vertices are often assumed to be a characteristic feature of the neutralino sector in extended supersymmetric models with gauge singlets. Especially in the Next-to-Minimal Supersymmetric Standard Model (NMSSM) with a singlino-like lightest supersymmetric particle (LSP) displaced vertices are expected in a large part of the parameter space [1]. They may appear if the mixing character of the lightest neutralino \(\tilde{\chi}_1^0\) – which is also the LSP – is dominated by the singlino component so that its couplings to the other supersymmetric particles are strongly suppressed. Then in models with conserved \(R\)-parity all supersymmetric particles first decay into the next-to-lightest SUSY particle (NLSP) which is assumed to be the second lightest neutralino \(\tilde{\chi}_2^0\). The NLSP finally decays into the LSP with a decay vertex displaced from the production vertex if the decay width is small enough.

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The direct production of singlino dominated neutralinos in the NMSSM and the $E_6$ model at an $e^+e^-$ linear collider with polarized beams has been discussed in [2]. Slow decays in models containing additional singlet superfields with intermediate-scale vacuum expectation values have been considered in [3]. The aim of this study is a detailed analysis of the low energy parameter regions where displaced vertices of $\tilde{\chi}_2^0$ exist in two representative extended supersymmetric models, the NMSSM and an $E_6$ model with an additional neutral gauge boson. These models are shortly described in Sec. 2.

For this purpose we first discuss in Sec. 3 scenarios where the lightest neutralino has a large singlino component. The vacuum expectation value $x$ of the singlet field turns out to be the crucial parameter responsible for a singlino dominated $\tilde{\chi}_1^0$ leading to a displaced $\tilde{\chi}_2^0$ decay vertex. Therefore we present in Sec. 4 the $\tilde{\chi}_2^0$ decay width in representative scenarios as a function of $x$ and show the parameter region where displaced vertices exist. Our main conclusion is that, depending on the neutralino masses and the mass difference between the lightest neutralinos, the singlet vacuum expectation value has to be in the range between several TeV and $10^4$ TeV in order to observe displaced vertices in these extended models.

2 NMSSM and $E_6$ model

The NMSSM is the minimal extension of the Minimal Supersymmetric Standard Model (MSSM) by a singlet Higgs field $S$ with vacuum expectation value $x$ [4]. The masses and couplings of the five neutralinos depend on the the gaugino mass parameters $M_2$ and $M_1$, the ratio of the vacuum expectation values of the doublet Higgs fields ($H_1, H_2$) $\tan \beta = v_2/v_1$ as in the MSSM, and on the singlet vacuum expectation value $x$ and the trilinear couplings $\lambda$ and $\kappa$ in the superpotential [5]

$$W \supset \lambda H_1 H_2 S - \frac{1}{3} \kappa S^3. \quad (1)$$

In this paper we always assume the GUT relation $M_1/M_2 = 5/3 \tan^2 \theta_W$. For large $x \gg |M_2|$ a singlino dominated neutralino decouples in the neutralino mixing matrix and could become very light for small values of the parameter $\kappa$. Then the heavier neutralinos have MSSM character with $\mu = \lambda x$. Such light or even massless singlino dominated neutralinos are not excluded by LEP [6].

Since also light neutral singlet-like Higgs bosons can generally exist in the NMSSM [4] the NMSSM Higgs sector may play an important role for the decay characteristics of the neutralinos. It contains five physical neutral Higgs bosons, three Higgs scalars and two pseudoscalars whose masses and mixings depend on the soft scalar masses $A_\lambda$ and $A_\kappa$ in addition to $\tan \beta$, $x$, $\lambda$ and $\kappa$.

The $E_6$ model with an additional $U(1)'$ factor in the low energy gauge group and therefore a new neutral gauge boson $Z'$ is a further extension of the MSSM beyond the NMSSM. It can be motivated by superstring theory [8] and implies an $E_6$ group as effective GUT group, which is broken to an effective low energy gauge group of rank 5. We consider an $E_6$ model with one additional singlet superfield in the Higgs sector [9]. To respect the experimental mass bounds for new gauge bosons the singlet vacuum expectation value $x$
must be larger than about 1.5 TeV [10]. The extended neutralino sector in this model contains six neutralinos being mixtures of photino, zino, \( Z' \) gaugino, doublet higgsinos and singlino [3, 11, 12]. The 6 × 6 neutralino mass matrix depends on six parameters: the SU(2)\(_L\), U(1)\(_Y\) and U(1)' gaugino mass parameters \( M_2, M_1 \) and \( M' \), \( \tan \beta \), \( x \) and the trilinear coupling \( \lambda \) in the superpotential

\[
W \supset \lambda H_1 H_2 S.
\] (2)

In the E\(_6\) model the 4 × 4 submatrix of the MSSM-like neutralinos and the 2 × 2 submatrix of the exotic ones approximately decouple because of the large values of \( x \). Then for very large values of \( |M'| \gg x \) light singlino-like neutralinos occur because of a see-saw-like [13] mechanism in the 2 × 2 submatrix [14].

3 Scenarios

Since displaced vertices are only expected for an LSP with a large singlino component we discuss in the following scenarios with a singlino-dominated \( \tilde{\chi}_1^0 \).

In the NMSSM the singlino content of \( \tilde{\chi}_1^0 \) is described by the squared matrix element \( |N_{15}|^2 \) of the unitary 5 × 5 matrix \( N \) which diagonalizes the neutralino mass matrix in the basis \((\tilde{\gamma}, \tilde{Z}, \tilde{H}_1, \tilde{H}_2, \tilde{S})\) of the photino, zino, the two doublet higgsinos and the singlino. The MSSM content of \( \tilde{\chi}_1^0 \), described by \( 1 - |N_{15}|^2 \), is crucial for the couplings of the heavier neutralinos to the LSP and especially for the decay width of the second lightest neutralino.

Similarly in the E\(_6\) model the 6 × 6 neutralino mass matrix is diagonalized by an unitary 6 × 6 matrix \( N \) with \( |N_{16}|^2 \) describing the singlino content of the LSP. Here for \( |M'| \gg x \) the lightest neutralino can be a nearly pure singlino. Since the very heavy \( Z' \) with mass \( \mathcal{O}(M') \) decouples nearly completely [15, 16], \( 1 - |N_{16}|^2 \) describes the MSSM content of the LSP in very good approximation.

Fig. 1 shows the contour lines of the singlino content of the LSP in the NMSSM and the E\(_6\) model \( |N_{15}|^2 \) and \( |N_{16}|^2 \), respectively, in the \( M_2-x \) parameter space for \( \tan \beta = 3 \) and two values \( \lambda x = 200 \) and 400 GeV. The mass of the LSP \( m_{\tilde{\chi}_1^0} = 50 \) GeV is fixed by the parameter \( \kappa \) in the NMSSM or \( M' \) in the E\(_6\) model. While the singlino content of the LSP significantly increases with \( x \) it depends only weakly on the other parameters \( M_2 \) and \( \lambda x \). In Fig. 1(b) the sign of \( M' \) is chosen positive. Then the contour lines for positive \( M_2 \) in the NMSSM correspond to those with negative \( M_2 \) in the E\(_6\) model and vice versa because of the relative sign between the submatrices of the minimal and exotic neutralinos in the neutralino mass matrices which is reflected in the signs of the mass eigenvalues of \( \tilde{\chi}_1^0 \) and \( \tilde{\chi}_2^0 \). They have the same sign for positive \( M_2 \) in the NMSSM, but for negative \( M_2 \) in the E\(_6\) model. For \( M' < 0 \) in the E\(_6\) model the mass eigenvalue of the \( \tilde{\chi}_1^0 \) flips sign and thus becomes the same as in the NMSSM, so that in this case the contour lines of the singlino content of the LSP are similar to those in the NMSSM.

For values of \( x < 2 \) TeV the singlino content of the LSP is always smaller than about 0.99 in the whole parameter space in both models. For \( x = \mathcal{O}(10 \text{ TeV}) \) it reaches values of about 0.9997 in the NMSSM and 0.9995 in the E\(_6\) model. In this \( x \) range the decay
widths of the $\tilde{\chi}_2^0$ are typically suppressed by a factor $10^{-2}$ to $10^{-3}$ compared to the MSSM and so are still too large to yield displaced vertices.

Obviously, the singlet vacuum expectation value $x$ is the crucial parameter in order to increase the singlino component of $\tilde{\chi}_1^0$ and generate displaced vertices of the $\tilde{\chi}_2^0$. Therefore we discuss in the following the dependence of the $\tilde{\chi}_1^0$ singlino content and the $\tilde{\chi}_2^0$ decay width from the parameter $x$ in four representative supersymmetric scenarios presented in Table 1. In all scenarios the lightest neutralino is singlino dominated with a mass fixed by the parameters $\kappa$ (NMSSM) or $M'$ (E6 model). In order to study the impact of the neutralino mixing and masses we choose scenarios with gaugino and higgsino dominated $\tilde{\chi}_2^0$ and mass differences $m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0} = 10$ GeV and 50 GeV.

For these scenarios, the MSSM content of the LSP (1 − $|N_{15}|^2$ and 1 − $|N_{16}|^2$, respectively) is plotted in Fig. 2 as a function of $x$. In the E6 model the scenarios (c), (d) differ from (e), (f) by the sign of the parameter $M'$ that determines the sign of the mass eigenvalue of the lightest neutralino because of the see-saw-like mechanism in the $2 \times 2$ submatrix described in Sec. 2.

The MSSM content of the lightest neutralino decreases as $1/x^2$ in very good approximation. For the larger mass difference between $\tilde{\chi}_1^0$ and $\tilde{\chi}_2^0$ it does not significantly depend on the parameters $M_2$ and $\lambda x$ which determine the mixing character of the $\tilde{\chi}_2^0$ and reaches values of $10^{-8}$ for singlet vacuum expectation values of about $3 \times 10^3$ TeV.

Smaller mass differences lead to a larger neutralino mixing and consequently to a
Figure 2: MSSM content of the LSP with mass 50 GeV (solid) and 90 GeV (dashed) for the scenarios given in Table 1 in (a) the NMSSM with gaugino-like $\tilde{\chi}_2^0$, (b) the NMSSM with higgsino-like $\tilde{\chi}_2^0$, (c) the $E_6$ model with $M' < 0$ and gaugino-like $\tilde{\chi}_2^0$, (d) the $E_6$ model with $M' < 0$ and higgsino-like $\tilde{\chi}_2^0$, (e) the $E_6$ model with $M' > 0$ and gaugino-like $\tilde{\chi}_2^0$ and (f) the $E_6$ model with $M' > 0$ and higgsino-like $\tilde{\chi}_2^0$. 
Table 1: Parameters of the supersymmetric models in representative scenarios with a singlino dominated lightest neutralino. The mass of the LSP is fixed by the parameters \( \kappa \) (NMSSM) and \( M' \) (E\(_6\) model).

| Scenario | \( M_2\)/GeV | \( \lambda x\)/GeV | \( \tan \beta \) | \( m_{\tilde{\chi}_0^1}\)/GeV | \( m_{\tilde{\chi}_0^2}\)/GeV | \( \tilde{\chi}_2^0 \) mixing type |
|----------|-------------|-------------------|----------------|----------------------|----------------------|----------------------|
| G50      | 211         | 400               | 3              | 50                   | 100                   | gaugino              |
| G90      | 211         | 400               | 3              | 90                   | 100                   | gaugino              |
| H50      | -400        | 107               | 3              | 50                   | 100                   | higgsino             |
| H90      | -400        | 107               | 3              | 90                   | 100                   | higgsino             |

smaller singlino component of the LSP if the mass eigenvalues of the the light neutralinos have the same sign. Then the decoupling of the submatrices of the MSSM and exotic neutralinos becomes weaker and the MSSM content of the LSP is approximately one order of magnitude larger in Fig. 2 (a, c, f). If, however, the mass eigenvalues have opposite sign, the MSSM content is less affected by the smaller mass difference in Fig. 2 (b, d, e). The impact of the MSSM content of the LSP on the decay width of the NLSP will be discussed in the next section.

4 Decay widths

In this section the decay widths of the second lightest neutralino \( \tilde{\chi}_2^0 \) will be discussed in the scenarios of Table 1. Since in the \( E_6 \) model the lightest Higgs particle is always a MSSM-like state and no light singlet dominated Higgs boson exists for \( x > 5 \) TeV, two-body decay channels, which could result in much larger decay widths with displaced vertices appearing at even higher values of \( x \), are kinematically forbidden. Therefore we only have to consider three-body decays into leptons, neutrinos or quarks (\( \tilde{\chi}_2^0 \to \tilde{\chi}_1^0 + \ell^+ \ell^- \), \( \nu \bar{\nu} \), \( q \bar{q} \)) and the radiative decay \( \tilde{\chi}_2^0 \to \tilde{\chi}_1^0 + \gamma \).

In the NMSSM, however, light singlet dominated Higgs bosons are not experimentally excluded. Consequently we discuss in this model also the case of a light singlet dominated Higgs scalar \( S_1 \) or pseudoscalar \( P_1 \) and thus include into our analysis the two-body decays \( \tilde{\chi}_2^0 \to \tilde{\chi}_1^0 + S_1(P_1) \).

The analytical formulae for the neutralino decay width are well known \([1, 12, 18]\). For the numerical calculations we used the masses of the scalar leptons and quarks given in Table 4. They are motivated by renormalization group equations with the scalar mass parameter \( m_0 = 135 \) GeV and \( M_2 = 300 \) GeV in the MSSM and NMSSM \([11]\). In the \( E_6 \) model the same mass values are chosen in order to compare the decay widths in NMSSM and \( E_6 \) model without sfermion mass effects. Larger or smaller sfermion masses may obviously reduce or enhance the decay width. All qualitative results, however, remain valid.

Displaced vertices are expected for decay lengths of the order \( 10^{-3} \) – 1 m, which correspond to decay widths of \( 10^{-13} \) – \( 10^{-16} \) GeV. At even smaller decay widths the particle escapes detection and cannot be identified.
Figure 3: Total decay widths of the $\tilde{\chi}_2^0$ with mass 100 GeV and a mass of the LSP of 50 GeV (solid) and 90 GeV (dashed) for the scenarios given in Table 1 in (a) the NMSSM with gaugino-like $\tilde{\chi}_2^0$, (b) the NMSSM with higgsino-like $\tilde{\chi}_2^0$, (c) the $E_6$ model with $M' < 0$ and gaugino-like $\tilde{\chi}_2^0$, (d) the $E_6$ model with $M' < 0$ and higgsino-like $\tilde{\chi}_2^0$, (e) the $E_6$ model with $M' > 0$ and gaugino-like $\tilde{\chi}_2^0$ and (f) the $E_6$ model with $M' > 0$ and higgsino-like $\tilde{\chi}_2^0$. The thin lines in (a) and (b) show the total decay width in the case of a light singlet-like scalar Higgs of mass 25 GeV in the NMSSM. The shaded area marks the region of decay widths where displaced vertices should be visible. Below this area the decaying particle escapes detection.
In Fig. 3 the dependence of the total decay width of the $\tilde{\chi}_2^0$ on the singlet vacuum expectation value $x$ is shown. In all scenarios without open two-body decay channels the decay widths decrease approximately as $1/x^2$ similar as the MSSM content of the LSP studied in Sec. 3. For the larger mass difference $m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0} = 50$ GeV between the decaying $\tilde{\chi}_2^0$ and the LSP the decay widths reach values small enough for displaced vertices for $x$ between $8 \times 10^2$ TeV and $4 \times 10^3$ TeV.

In the NMSSM scenarios G50 and H50 also the impact of a light singlet dominated Higgs scalar on the decay width is shown in Fig. 3. As a typical example we set $A_\lambda = 0$ GeV and fix a mass of 25 GeV for the light Higgs scalar by the parameter $A_\kappa$. Then for small $x$ the two-body decay dominates but decreases approximately as $1/x^4$. So in G50 slightly larger $x$ values are required in order to observe displaced vertices whereas in H50 the two-body width plays no role for $x > 100$ TeV. Similarly, also the $\tilde{\chi}_2^0$ decay into a light Higgs pseudoscalar is suppressed at high $x$ values and does not affect the region of displaced vertices.

The decay widths in the scenarios with smaller mass difference $m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0} = 10$ GeV are two to three orders of magnitude smaller reaching the region of displaced vertices already for $x$ between 30 TeV and $2 \times 10^2$ TeV. They may even become so small that the $\tilde{\chi}_2^0$ escapes the detector for about $x > 10^3$ TeV.

Although the MSSM content of the LSP becomes larger (Fig. 2), the decay widths decrease with smaller mass differences between the NLSP and the LSP. Obviously phase space effects clearly outweigh the impact of the singlino character of the LSP on the decays of the $\tilde{\chi}_2^0$. Even smaller mass differences, which, however, make it difficult to detect the decay products of the $\tilde{\chi}_2^0$ and thus the displaced vertex, allow displaced vertices already for $x$-values of some TeV.

These results remain valid in all scenarios with a singlino-like LSP and mass differences between $\tilde{\chi}_1^0$ and $\tilde{\chi}_2^0$ small enough for two-body decays in Z and MSSM-like Higgs bosons to be forbidden. The $\tilde{\chi}_2^0$ decay width is suppressed by the singlino purity of the LSP which is governed mainly by the singlet vacuum expectation value $x$ and does only weakly depend on the MSSM parameters which fix the character of the MSSM-like $\tilde{\chi}_2^0$.

### 5 Conclusion

Displaced vertices of the NLSP $\tilde{\chi}_2^0$ decaying into a singlino dominated LSP $\tilde{\chi}_1^0$ may occur in certain parameter regions of the NMSSM and an E$_6$ model with an additional singlet Higgs field compared to the MSSM. They can only be expected for singlet vacuum expectation values $x$ in the TeV range significantly above the electroweak symmetry breaking scale. For singlet vacuum expectation values of the same order of magnitude as the vacuum

| $m_{\tilde{\chi}_L}/\text{GeV}$ | $m_{\tilde{\nu}_L}/\text{GeV}$ | $m_{\tilde{\nu}_R}/\text{GeV}$ | $m_{\tilde{\mu}_L}/\text{GeV}$ | $m_{\tilde{\mu}_R}/\text{GeV}$ | $m_{\tilde{d}_L}/\text{GeV}$ | $m_{\tilde{d}_R}/\text{GeV}$ |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 295             | 300             | 200             | 994             | 963             | 996             | 963             |

Table 2: Masses of the scalar leptons and quarks used for the numerical calculation of the neutralino decay width.
expectation values of the doublet Higgs fields displaced $\tilde{\chi}^0_2$ vertices cannot be observed.

In the studied supersymmetric scenarios, minimum singlet vacuum expectation values between $10^2$ and $10^4$ TeV are required for observable displaced $\tilde{\chi}^0_2$ vertices, depending on the mixing character of $\tilde{\chi}^0_2$ and the mass difference between $\tilde{\chi}^0_1$ and $\tilde{\chi}^0_2$. Then the lightest neutralino approaches a singlino state of high purity with a reduced MSSM content of $10^{-4} - 10^{-8}$. The $\tilde{\chi}^0_2$ decay into a light singlino-like Higgs boson, if kinematically allowed, does not significantly affect the $x$ region of displaced vertices. If two-body $\tilde{\chi}^0_2$ decay channels in $Z$ and MSSM-like Higgs bosons are open, however, the decay widths are much larger and displaced vertices appear at considerably higher values of $x$. For smaller neutralino mass differences $m_{\tilde{\chi}^0_2} - m_{\tilde{\chi}^0_1} = 10$ GeV, $\tilde{\chi}^0_2$ escapes the detector for $x$ values larger than about $10^3$ TeV.

Our results indicate that the appearance of displaced vertices in extended supersymmetric models with a singlino dominated LSP is mainly triggered by the singlet vacuum expectation value and does not significantly depend on the supersymmetric parameters of the MSSM.

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