Hadronic Decays Involving Heavy Pentaquarks

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Abstract

Recently several experiments have reported evidences for pentaquark $\Theta^+$. H1 experiment at HERA-B has also reported evidence for $\Theta_c$. $\Theta^+$ is interpreted as a bound state of an $\bar{s}$ with other four light quarks $udud$ which is a member of the anti-decuplet under flavor $SU(3)_f$. While $\Theta_c$ is a state by replacing the $\bar{s}$ in $\Theta^+$ by a $\bar{c}$. One can also form $\Theta_b$ by replacing the $\bar{s}$ by a $\bar{b}$. The charmed and bottomed heavy pentaquarks form triplets and anti-sixtets under $SU(3)_f$. We study decay processes involving at least one heavy pentaquark using $SU(3)_f$ and estimate the decay widths for some decay modes. We find several relations for heavy pentaquarks decay into another heavy pentaquark and a $B(B^*)$ or a $D(D^*)$ which can be tested in the future. $B$ can decay through weak interaction to charmed heavy pentaquarks. We also study some $B$ decay modes with a heavy pentaquark in the final states. Experiments at the current $B$ factories can provide important information about the heavy pentaquark properties.

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I. INTRODUCTION

Recently several experiments have reported evidences for pentaquarks $\Theta^+$ and other states\[1\], although there are also experiments reported null results\[2\]. The $\Theta^+(1540)$ pentaquark has strangeness $S = +1$ and has quark content $udud\bar{s}$. This particle is an isosinglet which is a member of the anti-decuplet multiplet\[3\] in flavor $SU(3)_f$ symmetry. At present there is very limited experimental information on the detailed properties such as the decay width, spin and parity. Several models have been proposed to accommodate these states\[3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\].

Replacing the $\bar{s}$ in $\Theta^+$ by a heavy quark such as a $\bar{c}$ or a $\bar{b}$, it is also possible to form bound heavy pentaquark states\[5, 6, 8, 10, 11, 12, 14, 15, 16, 17, 18\], $\Theta^c$ or $\Theta^b$. When implementing them into $SU(3)_f$, in the model of Jaffe and Wilczek\[5\] where pentaquark $\Theta^+$ is composed of two $(ud)$ diquarks with spin-0 and an $\bar{s}$ quark, heavy pentaquarks form a fundamental representation of $SU(3)_f$ triplet $R_{c,b}$ (the sub-indices $c$ and $b$ indicate whether the pentaquark is formed with a $\bar{c}$ or a $\bar{b}$), and an anti-sixtet $S_{c,b}$\[10\]. Discovery of these states and study of their properties can provide important information about the inner structure of matter. H1 collaboration has recently reported observation of a narrow resonance in $D^*^- p$ and $D^{*+}\bar{p}$ in inelastic $eP$ collisions at center-of-mass energies of 300 GeV and 320 GeV at HERA. This resonance has a mass of $3099 \pm 3(stat) \pm 5(syst)$ with a Gaussian width of $12 \pm 3$ MeV and can be interpreted as evidence for $\Theta_c$. If this observation is confirmed, there should be other related states exist. In this paper we study some properties of decay processes involving at least one heavy pentaquark using $SU(3)_f$ flavor symmetry.

The triplet $R_{c,b}$ transforms under $SU(3)_f$ in a similar way as the light quark $(u,d,s)$ triplet. To indicate this fact and also to distinguish the pentaquark triplet from the quark one, we use capital $U, D, S$ to indicate the elements in $R_{c,b}$. For the anti-sixtet $S_{c,b}$, we use $T_{c,b}, N_{c,b}$ and $\Theta_{c,b}$ to indicate the isospin triplet, doublet and singlet in $S_{c,b}$, respectively. We have

\[
R_c = (R_{c,i}) = (U_c^0, D_c^-, S_c^-),
\]
\[
S_c = (S_c^{ij}) = \begin{pmatrix}
T_c^{--} & T_c^- / \sqrt{2} & N_c^- / \sqrt{2} \\
T_c^- / \sqrt{2} & T_c^0 & N_c^0 / \sqrt{2} \\
N_c^- / \sqrt{2} & N_c^0 / \sqrt{2} & \Theta_c^0
\end{pmatrix},
\]
\[ R_b = (R_{b,i}) = (U_b^+, D_b^0, S_b^0), \]

\[ S_b = (S_{b}^{ij}) = \left( \begin{array}{ccc} T_b^- & T_b^0 / \sqrt{2} & N_b^0 / \sqrt{2} \\ T_b^0 / \sqrt{2} & T_b^+ & N_b^+ / \sqrt{2} \\ N_b^0 / \sqrt{2} & N_b^+ / \sqrt{2} & \Theta_b^+ \end{array} \right). \]  

The quark contents of these particles are,

\[ U_{c,b}^{0,+} = (udus)(\bar{c}, \bar{b}), \quad D_{c,b}^{0,-} = (udds)(\bar{c}, \bar{b}), \quad S_{c,b}^{0} = (dus)(\bar{c}, \bar{b}), \]

\[ T_{c,b}^{-,-} = (dsds)(\bar{c}, \bar{b}), \quad T_{c,b}^{-,0} = (dus)(\bar{c}, \bar{b}), \quad T_{c,b}^{0,+} = (usus)(\bar{c}, \bar{b}), \]

\[ N_{c,b}^{-,0} = (udus)(\bar{c}, \bar{b}), \quad N_{c,b}^{0,+} = (udus)(\bar{c}, \bar{b}), \quad \Theta_{c,b}^{0,+} = (udud)(\bar{c}, \bar{b}). \]

The \( U_{c,b}, D_{c,b} \) and \( N_{c,b} \) particles have \( S = -1 \), \( S_{c,b} \) and \( T_{c,b} \) particles have \( S = -2 \), and \( \Theta_{c,b} \) particles have \( S = 0 \).

In the diquark model of Ref. [5], \( S_{c,b} \) have positive parity, whereas \( R_{c,b} \) have negative parity since there is no P-wave excitation between the diquarks. In our study we emphasis on the flavor \( SU(3)_f \) properties, the conclusions can be applied to both parity situations “+” or “-” for both \( R_{c,b} \) and \( S_{c,b} \).

**II. HEAVY PENTAQUARK STRONG DECAY COUPLINGS**

Whether heavy pentaquarks can have strong decay modes depends on their masses. With \( SU(3)_f \) symmetry, particles in each multiplet are supposed to have the same mass. Quark model estimates for the heavy pentaquark have been carried out by several groups. In the diquark model, the \( \Theta_{c,b} \) masses are estimated to be \( 2710 \text{ MeV} \) and \( 6050 \text{ MeV} \), respectively, which is below the strong \( pD \) and \( nB \) decay threshold. A lattice calculation in Ref. [8], gives \( m_{\Theta_c} \) about \( 3.5 \text{ GeV} \).

Removing the P-wave excitation energy, which is estimated using the mass difference of \( \Lambda_c \) and its excitation \( \Lambda_c' \), \( U_{P-wave} \approx m_{\Lambda_c'} - m_{\Lambda_c} = 310 \text{ MeV} \), from \( \Theta_{c,b} \), and adding a constituent strange quark contribution \( \Delta_s = m_{\Xi_c} - m_{\Lambda_c} \approx 184 \text{ MeV} \), Ref. [15] obtained \( 2580 \text{ MeV} \) for \( U_c, D_c \) masses Assuming the same \( U_{P-wave} \) and \( \Delta_s \) for beauty heavy pentaquarks, \( U_b, D_b \) masses are estimated to be \( 5920 \text{ MeV} \) [15].

The degeneracy of mass for the particles in a multiplet is lifted by quark mass differences, \( m_u, m_d \) and \( m_s \). The mass terms, up to linear corrections in light quark masses, are given...
by

\[
L = m_0^{R_{c,b}}Tr(\bar{R}_{c,b}R_{c,b}) + \alpha m_0^{R_{c,b}}Tr[\bar{R}_{c,b}(M + M^\dagger)R_{c,b}]
+ m_0^{S_{c,b}}Tr(\bar{S}_{c,b}S_{c,b}) + \alpha m_0^{S_{c,b}}Tr[\bar{S}_{c,b}(M + M^\dagger)S_{c,b}]
\]

(3)

We have neglected terms of the form \(Tr[\bar{R}(\bar{S})R(S)]Tr(M)\) which only re-scales \(m_0\). \(M\) is the quark mass matrix and is given by

\[
M = \begin{pmatrix}
    m_u & 0 & 0 \\
    0 & m_d & 0 \\
    0 & 0 & m_s
\end{pmatrix}.
\]

(4)

Neglecting small \(m_{u,d}\) masses, we obtain

\[
m_{U_{c,b}} = m_{D_{c,b}} = m_0^{R_{c,b}}, \quad m_{S_{c,b}} = m_0^{R_{c,b}} + 2\alpha m_0^{R_{c,b}} m_s,
\]

\[
m_{T_{c,b}} = m_0^{S_{c,b}}, \quad m_{N_{c,b}} = m_0^{S_{c,b}} + \alpha m_0^{S_{c,b}} m_s, \quad m_{\Theta_{c,b}} = m_0^{S_{c,b}} + 2\alpha m_0^{S_{c,b}} m_s.
\]

(5)

Taking into account of the \(SU(3)_f\) breaking effects by differences in light quark masses from \(\Delta_s = m_{\Xi_c} - m_{\Lambda_c}\) for constituent strange quark, the masses of \(S_c^-\) and \(S_b^0\) were estimated to be 2770 MeV and 6100 MeV, respectively in Ref. [15]. Making the same assumption we obtain the masses of \(N_c\), \(N_b\), \(T_c\) and \(T_b\) to be 2894 MeV, 6236 MeV, 3078 MeV and 6420 MeV, respectively. These values are similar to the estimates obtained in Ref. [16].

There are other model estimates for heavy pentaquark masses which give larger masses. For example in the model of Karliner and Lipkin, where the pentaquarks are formed from a triquark and a diquark bound states [6], the masses are estimated to be 2985 MeV and 6398 MeV, respectively, which are above strong \(pD\) and \(nB\) decay threshold. And the masses of \(N_c\), \(N_b\), \(T_c\) and \(T_b\) are estimated to be 3165 MeV, 6570 MeV, 3340 MeV and 6740 MeV, respectively [16]. Removing the P-wave excitation energy [6] \(\delta E^{P-wave} \approx 207\) between the diquark and triquark system from \(\Theta_{c,b}\) and adding the mass difference due to the replacement of a light \(u\) or \(d\) quark by an \(s\) quark, one obtains the masses of \(U_{c}^0\), \(D_{c}^-\) and \(U_{b}^+\), \(D_{b}^0\) to be 2858 MeV and 6533 MeV, respectively. \(S_c^-\) and \(S_b^0\) are approximately
3028 MeV and 6708 MeV. Clearly the above estimates for the masses are rather rough and should not be expected to hold to more than to within 50 MeV or even 100 MeV.

If the H1 narrow resonance of mass 3099 MeV is indeed the Θ_c particle, both the diquark and triquark-diquark model predictions for the mass are slightly lower than data. There is also the possibility that the narrow resonant state observed at H1 is a chiral partner of the Θ_c. At present the uncertainties involved in the estimates are large, it is too early to make a decisive conclusion. With a mass 3099 MeV for Θ_c, it is possible for it to decay into \( D^*^- p \) and \( D^+ \bar{p} \). Similar situation may happen for beauty pentaquarks. We therefore will consider processes involving both \( D, B \) and \( D^*, B^* \).

We now write down the strong decay amplitudes to the leading order using \( SU(3)_f \) symmetry for heavy pentaquark decays with a \( B \) or a \( D \) in the final states. We have

\[
L_{R_b N B} = c_{R_b N B} \bar{N}^i_j N^i_j \bar{B}_i + H.C.
\]

\[
L_{S_b N B} = c_{S_b N B} \bar{S}_{b, jk} N^i_j \bar{B}_i \epsilon^{jkl} + H.C.
\]

\[
L_{R_c N D} = c_{R_c N D} \bar{R}^i_c N^i_j \bar{D}_i + H.C.
\]

\[
L_{S_c N D} = c_{S_c N D} \bar{S}_{c, jk} N^i_j \bar{D}_i \epsilon^{jkl} + H.C.
\]  \( (6) \)

In the above \( N \) is the ordinary baryon octet, \( D^i \) and \( B^i \) are the charm and beauty mesons. They are given by

\[
N = (N^i_j) = \begin{pmatrix}
\frac{\Sigma^0}{\sqrt{6}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\
\Sigma^- & -\frac{\Sigma^0}{\sqrt{6}} + \frac{\Lambda}{\sqrt{6}} & n \\
\Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} 
\end{pmatrix}.
\]

\[
D = (D^i) = (D_u, D_d, D_s) = (c\bar{u}, c\bar{d}, c\bar{s}),
\]

\[
B = (B^i) = (B_u, B_d, B_s) = (b\bar{u}, b\bar{d}, b\bar{s}).
\]  \( (7) \)

In Tables II and III we list the couplings for \( \bar{N} R_b(S_b) B \). One can obtain the couplings for \( \bar{N} R_c(S_c) D \) by replacing \( B_i \) by \( D_i \) and the sub-index \( b \) to \( c \). Through out the paper, in equations for pentaquark couplings only group indices are properly labelled and all fields in the Lagrangian are going outwards. The Lorentz structures are suppressed in the equations, and the proper ones are given in the Tables. Also we will assume that heavy pentaquarks
are spin-1/2 particles. To obtain results for spin-3/2 heavy pentaquarks, one just uses at an appropriate place the Rarita-Schwinger vector-spinor for the relevant fields.

We would like to point out that the above equations can be equally applied to processes with $B$ and $D$ replaced by $D^*$ and $B^*$, respectively. We will not distinguish them in equations in later discussions unless specifically indicated.

**TABLE I**: Couplings for $B\bar{N}R_b(D\bar{N}R_c)$ in unit $c_{RNB}(c_{RND})$. The Lorentz structure for the bi-spinor product is of the form $\bar{N}\Gamma_{\mu}\gamma_5\Gamma_{\nu}R_b$. 

\[ \begin{array}{|c|c|} 
\hline
B_u & (\frac{1}{\sqrt{6}}\bar{\Lambda} + \frac{1}{2}\bar{\Sigma}^0)U^+_b + \Sigma^-D^0_b + \bar{\Xi}^-S^0_b \\
B_d & \Sigma^+U^+_b + (\frac{1}{\sqrt{6}}\bar{\Lambda} - \frac{1}{2}\bar{\Sigma}^0)D^0_b + \bar{\Xi}^0S^0_b \\
B_s & \bar{p}U^+_c + \bar{n}D^0_c - \sqrt{\frac{2}{3}}\bar{\Lambda}S^0_b \\
D_u & (\frac{1}{\sqrt{6}}\bar{\Lambda} + \frac{1}{2}\bar{\Sigma}^0)U^+_c + \Sigma^-D^-_c + \bar{\Xi}^-S^-_c \\
D_d & \Sigma^+U^+_c + (\frac{1}{\sqrt{6}}\bar{\Lambda} - \frac{1}{2}\bar{\Sigma}^0)D^-_c + \bar{\Xi}^0S^-_c \\
D_s & \bar{p}U^+_c + \bar{n}D^-_c - \sqrt{\frac{2}{3}}\bar{\Lambda}S^-_c \\
\hline
\end{array} \]

**TABLE II**: Couplings for $B\bar{N}S_b(D\bar{N}S_c)$ in unit $c_{SNB}(c_{SND})$. Lorentz structure is the same as in Table I.

\[ \begin{array}{|c|c|} 
\hline
B_u & \frac{1}{2}[\sqrt{2}\bar{\Xi}^- - \bar{T}^0_b + 2\bar{\Xi}^0T^+_b - \sqrt{2}\Sigma^-N^0_b - \sqrt{3}\bar{\Lambda}N^+_b + \Sigma^0N^+_b - 2\bar{n}\Theta^+_b] \\
B_d & \frac{1}{2}[\bar{T}^-_b - \sqrt{2}\bar{\Xi}^0T^-_b + \sqrt{2}\Sigma^+N^+_b + \sqrt{3}\Lambda N^+_b + \Sigma^0N^+_b + 2\bar{p}\Theta^+_b] \\
B_s & \frac{1}{2}[\bar{T}_b - 2\bar{\Xi}^0T^-_b + \Sigma^-N^+_b + \bar{T}^+_b - 2\bar{p}N^+_b + \sqrt{2}\bar{n}N^+_b] \\
D_u & \frac{1}{2}[\sqrt{2}\bar{\Xi}^- - \bar{T}^0_c + 2\bar{\Xi}^0T^+_c - \sqrt{2}\Sigma^-N^+_c - \sqrt{3}\bar{\Lambda}N^-_c + \Sigma^0N^-_c - 2\bar{n}\Theta^-_c] \\
D_d & \frac{1}{2}[\bar{T}^-_c - \sqrt{2}\bar{\Xi}^0T^-_c + \sqrt{2}\Sigma^+N^+_c + \sqrt{3}\Lambda N^-_c + \Sigma^0N^-_c + 2\bar{p}\Theta^-_c] \\
D_s & \frac{1}{2}[\bar{T}_c - 2\bar{\Xi}^0T^-_c \Sigma^-N^-_c - \sqrt{2}\bar{p}N^+_c + \sqrt{2}\bar{n}N^-_c] \\
\hline
\end{array} \]

If the diquark model for pentaquarks is the right one, we see that all the strong decay modes are forbidden due to restriction of phase space. However, if the masses are close to the triquark-diquark model predictions, strong decays are allowed. The H1 data indicate that the above strong decays are possible. One can use the allowed decay modes to determine the parameter $c_{abc}$ and therefore the widths of the heavy pentaquarks. Here we give the formula for the couplings in terms of decay widths assuming the decays are allowed. For decays with $B$ in the final states, we have,
\[
\begin{align*}
\mathcal{C}_{R_b}^2 & = \frac{16\pi m_{S_b}^0 \Gamma(S_b^0 \to \Xi^- \bar{B}_u)}{[(\hat{\pi} m_{S_b}^0 + m_{\Xi^-})^2 - m_{B_u^0}^2] \text{Ph}(m_{S_b^0}, m_{\Xi^-}, m_{B_u^0})}, \\
\mathcal{C}_{S_b}^2 & = \frac{16\pi m_{\Theta_b^+} \Gamma(\Theta_b^+ \to p\bar{B}_d)}{[(\hat{\pi} m_{\Theta_b^+} + m_p)^2 - m_{B_d^0}^2] \text{Ph}(m_{\Theta_b^+}, m_p, m_{B_d^0})},
\end{align*}
\]

where \( \text{Ph}(a, b, c) = \sqrt{1 - (b + c)^2/a^2}(1 - (b - c)^2/a^2) \). \( \hat{\pi} \) is the eigenvalue of parity of the heavy pentaquark.

For decays with \( B^* \) in the final states, we have

\[
\begin{align*}
\mathcal{C}_{R_b}^{2, B^*} & = \frac{16\pi m_{S_b}^0 \Gamma(S_b^0 \to \Xi^- B_u^*)}{f(m_{S_b^0}, m_{\Xi^-}, m_{B_u^*}) \text{Ph}(m_{S_b^0}, m_{\Xi^-}, m_{B_u^*})}, \\
\mathcal{C}_{S_b}^{2, B^*} & = \frac{16\pi m_{\Theta_b^+} \Gamma(\Theta_b^+ \to p\bar{B}_d^*)}{f(m_{\Theta_b^+}, m_p, m_{B_d^*}) \text{Ph}(m_{\Theta_b^+}, m_p, m_{B_d^*})},
\end{align*}
\]

where \( f(a, b, c) = a^2 + b^2 - c^2 - \hat{\pi} 6ab + ((a^2 - b^2)^2 - c^4)/c^2 \).

Similarly one can obtain \( \mathcal{C}_{R_c(S_c^0)ND(D^*)}^{2} \) by considering \( S_c^- \to D_u^0(D_u^0)\Xi^- \) and \( \Theta_c^0 \to p\bar{D}_d^*(\bar{D}_d^{*+}) \) decays, respectively.

At present there is only some information on the width of \( \Gamma(\Theta_c \to p\bar{D}_d^*) \). Assuming the narrow resonant state of width 12 \( \pm 3 \) MeV at H1 is the \( \Theta_c \) particle, we obtain

\[
\begin{align*}
\mathcal{C}_{S_c, ND^*}^2 & \approx \begin{cases} 
1.712 & \hat{\pi} = 1 \\
0.167 & \hat{\pi} = -1.
\end{cases}
\end{align*}
\]

Using these numbers, the decay widths for decay modes in Table II involving \( D^* \) can be predicted. These predictions can be tested.

### III. WEAK HADRONIC DECAYS OF HEAVY PENTAQUARKS

Heavy pentaquark can also decay through weak interactions. If kinematically the strong decays discussed in the previous section are not allowed, weak interaction will dominate heavy pentaquark decays. These decays can be semi-leptonic or purely hadronic ones. Analysis on some of the heavy pentaquark properties have been carried out \[11, 12, 13, 14, 15, 16, 17, 18\]. Here we will concentrate on some two body hadronic heavy pentaquark decays.
A.  \( R_b(S_b) \rightarrow R_c(S_c) + \Pi \) decays

In this subsection we study pentaquark decays of the type \( R_b(S_b) \rightarrow R_c(S_c)\Pi \). Here \( \Pi \) represents a meson in the pseudoscalar octet which is given by

\[
\Pi = \left( \Pi_i^j \right) = \begin{pmatrix}
\frac{\pi_0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\
\frac{\pi^-}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 & \frac{2\eta}{\sqrt{6}} \\
\bar{K}^0 & \frac{\pi^-}{\sqrt{2}} & \frac{\bar{K}_0}{\sqrt{6}} + \frac{\eta}{\sqrt{6}} \\
\end{pmatrix}.
\]  

(11)

The quark level effective Hamiltonian for \( R_b(S_b) \rightarrow R_c(S_c)\Pi \) is given by

\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ V_{cb}^* V_{uq} (c_1 O_1 + c_2 O_2) + V_{ub} V_{cq}^* (c_1 \tilde{O}_1 + c_2 \tilde{O}_2) \right],
\]  

(12)

\[
O_1 = \bar{b} \gamma_\mu (1 - \gamma_5) c \bar{u} \gamma^\mu (1 - \gamma_5) q, \quad O_2 = \bar{b} \gamma_\mu (1 - \gamma_5) q \bar{u} \gamma^\mu (1 - \gamma_5) c,
\]  

(13)

\[
\tilde{O}_1 = \bar{b} \gamma_\mu (1 - \gamma_5) u \bar{c} \gamma^\mu (1 - \gamma_5) q, \quad \tilde{O}_2 = \bar{b} \gamma_\mu (1 - \gamma_5) q \bar{c} \gamma^\mu (1 - \gamma_5) u.
\]  

(14)

The two operators \( O_{1,2} \) in the above can induce decays of the type, \( R_b(S_b) \rightarrow R_c(S_c) + \Pi \), while the operators \( \tilde{O}_{1,2} \) will not cause beauty heavy pentaquark to charmed pentaquark transitions. We write it down here for later discussions.

Under \( SU(3)_f \) symmetry \( O_{1,2} \) transforms as an octet \( H_j^i \). With proper normalization the non-zero entries of \( H_j^i \) can be written as

\[
q = d, \quad H_2^1 = 1; \quad q = s \quad H_3^1 = 1.
\]  

(15)

The \( SU(3)_f \) invariant decay amplitudes are

\[
H(R_b \rightarrow R_c\Pi) = V_{cb}^* V_{uq} [r_{s1} \bar{R}_{bi} R_{cij} \Pi_k^j H_k^j + r_{s2} \bar{R}_{bi} R_{cij} \Pi_k^j H_k^j + r_{s3} \bar{R}_{bi} R_{cij} \Pi_k^j H_k^j];
\]

\[
H(R_b \rightarrow S_c\Pi) = V_{cb}^* V_{uq} [s_{s1} \bar{R}_{bi} S_{cjl} \Pi_m^j H_m^l \epsilon^{ijk} + s_{s2} \bar{R}_{bi} S_{cjl} \Pi_m^j H_m^l \epsilon^{ikm} + s_{s3} \bar{R}_{bi} S_{cjl} \Pi_m^j H_m^l \epsilon^{ikm}].
\]

(16)

The amplitudes for \( S_b \rightarrow R_c\Pi \) can be obtained by interchanging the indices \( b \) and \( c \), and treating the processes as the charge conjugated ones in the second equation of eq.(16).

The amplitudes for \( S_b \rightarrow S_c\Pi \) can be written as
\[
H(S_b \to S_c \Pi) = V_{cb}^* V_{cq} [s_1 \bar{S}_{b}^{ij} S_{c,ij} \Pi_k^i H_k^j + s_2 \bar{S}_{b}^{ij} S_{c,kl} \Pi_i^k H_l^j ] \\
+ s_3 \bar{S}_{b}^{ij} S_{c,kl} \Pi_i^k H_l^j + s_4 \bar{S}_{b}^{ij} S_{c,kl} \Pi_i^k H_l^j ].
\] (17)

We list the results in Tables III, IV and V. One can easily generalize the above formulation to the case with the vector meson nonet (\(\rho^{0,\pm}, K^{*0,\pm}, \omega, \phi\)).

**B.** \(R_b(S_b) \to R_c(S_c)D(D^*)\)

The effective Hamiltonian for these processes is given by

\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{cq} (c_1 O_1^c + c_2 O_2^c).
\] (18)

Here \(O_1^c = \bar{b} \gamma^\mu (1 - \gamma_5) c \bar{c} \gamma_\mu (1 - \gamma_5) q\) and \(O_2^c = \bar{c} \gamma^\mu (1 - \gamma_5) \bar{b} \gamma_\mu (1 - \gamma_5) q\). In the above we have neglected small penguin contributions. This effective Hamiltonian transforms as a triplet 3 with non-zero entries,

\[
q = d, \quad H_2 = 1; \quad q = s, \quad H_3 = 1.
\] (19)

The \(SU(3)_f\) invariant decay amplitudes can be written as

\[
H(R_b(S_b) \to R_c(S_c)D) = V_{cb}^* V_{cq} [r_{31} \bar{R}_i^c R_{c,i} N_j^k H_k^j + r_{32} \bar{R}_i^c R_{c,j} D_j^i + r_{33} \bar{R}_i^c S_{c,jl} N_k^l D_k^j H_l^j ] \\
+ s_{31} \bar{S}_{b,ij} S_{c}^i H_k^j + s_{32} \bar{S}_{b,ij} S_{c}^k H_k^j D_j^i ].
\] (20)

The results for individual processes are given in Table VI.

**C.** \(B \to N \bar{R}_c(S_c)\) and \(B \to \bar{N} R_c(S_c)\)

The conjugate operators of \(O_{1,2}\) in eq. (14) is of the form \(\bar{c} \bar{b} \bar{q} u\) and can induce decays of the type, \(B \to N + \bar{R}_c(S_c)\).

The \(SU(3)_f\) invariant decay amplitudes are

\[
H(R_c) = V_{cb} V_{uq}^* [\tilde{r}_{81} \bar{B}_i \bar{R}_i^c N_j^k H_k^j + \tilde{r}_{82} \bar{B}_i \bar{R}_i^c N_j^k H_k^j + \tilde{r}_{83} \bar{B}_i \bar{R}_i^c N_j^k H_k^j ];
\]

\[
H(S_c) = V_{cb} V_{uq}^* [\tilde{s}_{81} \bar{B}_i \bar{S}_{c,jl} N_m^l H_k^j e^{ijk} + \tilde{s}_{82} \bar{B}_i \bar{S}_{c,jl} N_m^l H_k^j e^{jkm}],
\] (21)
The conjugate operators in the second term of eq. (14) is of the form $\bar{q}b\bar{u}c$. This operator can induce $B \to \bar{N} + R_c(S_c)$. It contains a $SU(3)$ triplet and an anti-sixtet. The non-zero entries are:

$$q = d, \ H(3c)_3 = 1, \ H(6c)^{12} = H(6c)^{21} = 1;$$

$$q = s, \ H(3c)_2 = -1, \ H(6c)^{13} = H(6c)^{31} = 1. \ (22)$$

One can write down $SU(3)_f$ decay amplitudes for $B \to \bar{N}R_c(S_c)$ as the following

$$H(R_c) = V_{ub}V_{cq}\bar{r}_{31}\bar{B}_iR_{cj}\bar{N}_k^iH_k^j\epsilon^{ijk} + \bar{r}_{32}\bar{B}_iR_{cj}\bar{N}_k^jH_k^i;$$

$$H(S_c) = V_{ub}V_{cq}^*\bar{s}_{31}\bar{B}_iS_{cj}^i\bar{N}_k^jH_k^j + \bar{s}_{32}\bar{B}_iS_{cj}^j\bar{N}_k^jH_k^i + \bar{s}_{61}\bar{B}_iS_{cj}^{ij}\bar{N}_k^kH_k^lm\epsilon_{jkm} + \bar{s}_{62}\bar{B}_iS_{cj}^{jk}\bar{N}_k^lH_k^im\epsilon_{jlm}. \ (23)$$

The hadronic parameters $\bar{r}_{ij}$ are expected to be similar, $\Gamma(B \to \bar{N}R_c(S_c))$ would be smaller by a factor of $|V_{ub}V_{cq}|^2/|V_{ub}V_{cq}^*|^2$ compared with $\Gamma(B \to \bar{N}\bar{R}_c, (\bar{S}_c))$. The details are listed in Tables VII, VIII and IX.

IV. DISCUSSIONS AND CONCLUSIONS

If the recently discovered state $\Theta^+$ is interpreted as pentaquark bound state with an $\bar{s}$ and four light quarks, heavy pentaquarks with the $\bar{s}$ replaced by a $\bar{b}$ or a $\bar{c}$ should exist. H1 experiment at HERA-B has obtained some evidences for $\Theta_c$. They form $SU(3)_f$ triplets $R_{c,b}$ and anti-sixtets $S_{c,b}$. These states can be further studied at future collider experiments. $R_c$ and $S_c$ can also be produced from $B$ decays at $B$-factories. If pentaquarks $R_{c,b}$ and $S_{c,b}$ are kinematically allowed to decay through strong interactions, one can use Tables I and II to relate different decay widths, and test the model.

At present there is only some evidence for $\Theta_c \to pD^{*-}p$. Using the decay width $\Gamma = 12$MeV obtained from H1, we determine $c_{S_{ND}}^2$ to be 1.712 and 0.167 for $\Theta_c$ with positive and negative parities, respectively. One expects that $c_{S_{NB}}^2$ to be similar to $c_{S_{ND}}^2$. Using Table III we can obtain other decay widths which can be tested in the future. We have nothing much to say about the size of the couplings except that we expect the couplings
c_{R_b N_b} and c_{S_b N_D} are about the same as c_{R_c N_D} and c_{S_c N_D}, respectively. If one extends SU(3)_f to SU(4)_f, the strong couplings can be related in principle to the ones involving just light pentaquarks \[12\]. We will not consider this possibility here.

The heavy pentaquarks can decay through weak interaction. We have parameterized some of the two body hadronic decays in terms of SU(3)_f invariant amplitudes. From the Tables obtained we see that there are several relations among different decay modes. For example, for $\Delta S = 0$ processes of the type $S_b \rightarrow S_c \Pi$, from Table V we obtain

$$\Gamma(T_b^- \rightarrow T_c^0 \pi^-) = 2\Gamma(T_b^- \rightarrow N_c^0 K^-) = 4\Gamma(T_b^0 \rightarrow T_c^0 \pi^0) = 12\Gamma(T_b^0 \rightarrow T_c^0 \eta)$$

$$= 4\Gamma(T_b^0 \rightarrow N_c^0 K^0) = 2\Gamma(N_b^0 \rightarrow T_c^0 K^0),$$

$$\Gamma(N_b^+ \rightarrow T_c^0 K^+) = \Gamma(\Theta_b^+ \rightarrow N_c^0 K^+).$$

(24)

More relations can be read off from the Tables. These relations can be used to study the properties of heavy pentaquarks and test the model provided that the decays have substantial branching ratios which requires knowledge about the size of the SU(3)_f invariant amplitudes.

Theoretical calculations of the decay amplitudes are very difficult since multi-quarks are involved. However for certain decays, the structure is very simple and can be related to experimentally measured modes, such as $\Theta_b^+ \rightarrow \Theta_c^0 \pi^+$ can be related to $\Lambda_b \rightarrow \Lambda_c^+ \pi^-$. In $\Theta_b^+ \rightarrow \Theta_c^0 \pi^+$ decay, the main contributions is due to factorized matrix elements where the $\pi$ is emitted from two light quarks in the effective Hamiltonian, and the transition of $\Theta_b^+$ to $\Theta_c^0$ is due to the transition of a $\bar{b}$ quark to a $\bar{c}$ quark in the Hamiltonian, and the structure of the rest of the four light quarks in the pentaquarks are basically preserved. Based on this intuitive picture, Ref.\[18\] relates $\Theta_b^+ \rightarrow \Theta_c^0 \pi^+$ to $\Lambda_b \rightarrow \Lambda_c^+ \pi^-$ using heavy quark effective theory, and concluded that the branching ratios for these two processes are similar. The decay rate $\Gamma(\Theta_b^+ \rightarrow \Theta_c^0 \pi^+)$ is estimated to be about $2.5\Gamma(B^0 \rightarrow D^- \pi^-)$. This prediction can be tested at future collider experiments.

From Table IV we see that $\Theta_b^+ \rightarrow \Theta_c^0 \pi^+$ is proportional to the SU(3)_f invariant amplitude $s_1$, one therefore can use the estimate in Ref.\[18\] to obtain an estimate for $s_1$. The invariant amplitudes $s_{2,3,4}$ involves more complicated topology and is much harder to estimate. Although it is difficult to know all decay amplitudes, knowing $s_1$ one can make some useful predictions. The branching ratios for processes in Tables IV and V which just depend on $s_1$ are therefore known. Up to mass splitting corrections in phase space, we obtain

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\[ \Gamma(T_b^0 \rightarrow T_c^- K^+) \sim \frac{1}{4} \frac{|V_{us}|^2}{|V_{ud}|^2} \Gamma(\Theta^+_b \rightarrow \Theta^0_c \pi^+), \]
\[ \Gamma(T_b^+ \rightarrow T_c^0 K^+) \sim \frac{|V_{ud}|^2}{|V_{us}|^2} \Gamma(\Theta^+_b \rightarrow \Theta^0_c \pi^+). \]  

(25)

Using heavy quark effective theory, one can also relate several other \( SU(3)_f \) invariant amplitudes to \( \Lambda_b \rightarrow \Lambda_c \pi \) by realizing the fact that all amplitudes for the heavy pentaquark transitions of the form \( R_{b,j}^i, S_{b,ij}^i S_{c}^{ij} \) have similar factorization structure, and their strength should also be similar. We therefore expect that \( s_1 \sim r_{s1} \sim r_{31} \sim s_{31} \).

One then has, up to mass splitting corrections in phase space, the following relations

\[ \Gamma(\Theta^+_b \rightarrow \Theta^0_c \pi^+) \sim \Gamma(S^0_b \rightarrow S^- \pi^+) \sim \frac{|V_{ud}|^2}{|V_{us}|^2} \Gamma(D^0_b \rightarrow D^- K^+) \]
\[ \sim \frac{|V_{ud}|^2}{|V_{cd}|^2} \Gamma(U^+_b \rightarrow U^0_c D_d), \Gamma(S^0_b \rightarrow S^- D_d) \]
\[ \sim \frac{|V_{ud}|^2}{|V_{cs}|^2} \Gamma(U^+_b \rightarrow U^0_c D_s), \Gamma(D^+_b \rightarrow \Sigma^- D_s), \Gamma(S^0_b \rightarrow \Sigma^- D_s) \]
\[ \sim \frac{|V_{ud}|^2}{|V_{cd}|^2} \Gamma(T^+_b \rightarrow T^- D_d), \Gamma(N^0_b \rightarrow N^- D_d), \Gamma(\Theta^+_b \rightarrow \Theta^0 D_d) \]
\[ \sim \frac{|V_{ud}|^2}{|V_{cs}|^2} \Gamma(T^+_b \rightarrow T^- D_s), \Gamma(T^0_b \rightarrow \Sigma^0 D_s), \Gamma(T^+_b \rightarrow T^- D_s). \]  

(26)

The above relations also hold for processes with \( D \) replaced by \( D^* \).

Pentaquark properties can also be studied at \( B \) factories. We have studied \( B \rightarrow N R_c(S_c) \) and \( B \rightarrow \bar{N} R_c(S_c) \) decays. From Tables VII, VIII and IX we see that there are several relations. For example

\[ \Gamma(B_u \rightarrow \bar{U}^0_c \Sigma^-) = \frac{|V_{ud}|^2}{|V_{us}|^2} \Gamma(B_u \rightarrow \bar{U} \Sigma^-), \]
\[ \Gamma(B_u \rightarrow \bar{p} U^0_c) = \frac{|V_{ud}|^2}{|V_{us}|^2} \Gamma(B_u \rightarrow \Sigma^+ U^0_c). \]  

(27)

Should the heavy pentaquarks be discovered, these relations can also provide important information. The decay amplitudes for \( B \rightarrow N R_c(S_c) \) and \( B \rightarrow \bar{N} R_c(S_c) \) are difficult to estimate. We are not able to provide any reliable estimate, except that we expect them to be smaller than \( B \rightarrow N \Lambda_c \) amplitudes.
Using the same formulation, one can also study $B$ decays into an ordinary baryon $N$ and a light pentaquark, such as $\Theta^+$ in the anti-decuplet. We however expect that the branching ratios to be smaller than $B \to N\bar{N}$. Since $B \to N\bar{N}$ have small branching ratios, it may be difficult to study $B \to N\bar{\Theta}^+$ experimentally. This situation may change if one studies light pentaquark decays of $B$ by three body decays, such as $B \to Dp(n)\Theta$. $B \to DK$ decay has a branching ratio of order a few times $10^{-4}$. The $K$ has a strong coupling to $p(n)\Theta^+$ which can be determined from $\Theta^+$ decay. With $\Gamma_\Theta$ of $\Theta^+$ width to be about a MeV, one can obtain a branching ratio as large as $10^{-6}$ which is within the reach of near future B factories.

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TABLE III: $SU(3)$ decay amplitudes for $R_b \rightarrow R_c(S_c)\Pi$. The Lorentz structure of the bi-spiner product is of the form $\bar{R}_c(\bar{S}_c)(1 + b\gamma_5)R_b$. Here $b$ is a parameter.

| decay          | $\Delta S = 0$ | $\Delta S = 1$ |
|----------------|----------------|----------------|
| $U_b^+ \rightarrow S_1$ | $r_{s1} + r_{s2}$ | $U_c^0 K^+ \frac{1}{\sqrt{2}}(s_{s1} - s_{s2})$ |
| $T_c^0 K^+$ | $s_{s1} - s_{s2}$ | $N_c^0 K^+ \frac{1}{\sqrt{2}}(s_{s1} - s_{s2})$ |
| $N_c^0 \pi^+$ | $-\frac{1}{\sqrt{2}}(s_{s1} - s_{s2})$ | $\Theta_c^0 \pi^+ -(s_{s1} - s_{s2})$ |
| $D_b^0$ decay | $U_c^0 K^0 \frac{1}{\sqrt{2}}(r_{s2} - r_{s3})$ | $U_c^0 \pi^0 r_{s3}$ |
|              | $U_c^0 \eta \frac{1}{\sqrt{2}}(r_{s2} + r_{s3})$ | $D_c^- K^+ r_{s1}$ |
|              | $D_c^- \pi^+ r_{s1} + r_{s2}$ | $N_c^- K^+ \frac{1}{\sqrt{2}}(s_{s1} - s_{s2})$ |
|              | $S_c^- K^+ r_{s2}$ | $N_c^0 K^0 -\frac{1}{\sqrt{2}}s_{s2}$ |
|              | $T_c^- K^+ \frac{1}{\sqrt{2}}(s_{s1} + s_{s3})$ | $\Theta_c^0 \pi^0 \frac{1}{\sqrt{2}}(s_{s1} - s_{s2})$ |
|              | $T_c^0 K^0 -(s_{s2} + s_{s3})$ | $\Theta_c^0 \eta \frac{1}{\sqrt{2}}(s_{s1} + s_{s2})$ |
|              | $N_c^- \pi^0 \frac{1}{\sqrt{2}}s_{s3}$ | $N_c^0 \pi^0 \frac{1}{\sqrt{2}}(s_{s1} - s_{s2} - s_{s3})$ |
|              | $N_c^0 \pi^0 \frac{1}{\sqrt{2}}(s_{s1} + s_{s2} + 3s_{s3})$ | $N_c^- \pi^0 \frac{1}{\sqrt{2}}s_{s3}$ |
|              | $\Theta_c^0 K^0 s_{s3}$ | $\Theta_c^0 K^0 s_{s3}$ |
| $S_b^0$ decay | $U_c^0 K^0 r_{s3}$ | $U_c^0 \pi^0 \frac{1}{\sqrt{2}}(2r_{s2} - r_{s3})$ |
|              | $S_c^- \pi^+ r_{s1}$ | $U_c^0 \eta \frac{1}{\sqrt{2}}(2r_{s2} - r_{s3})$ |
|              | $T_c^- \pi^+ \frac{1}{\sqrt{2}}s_{s1}$ | $D_c^+ \pi^+ r_{s2}$ |
|              | $T_c^0 \pi^0 \frac{1}{\sqrt{2}}s_{s1}$ | $S_c^- K^+ r_{s1} + r_{s2}$ |
|              | $T_c^0 \eta \frac{1}{\sqrt{2}}(s_{s1} - 2s_{s2})$ | $T_c^- K^+ \frac{1}{\sqrt{2}}s_{s3}$ |
|              | $N_c^0 \bar{K}^0 \frac{1}{\sqrt{2}}s_{s2}$ | $T_c^0 \bar{K}^0 -s_{s3}$ |
|              | $N_c^- \pi^+ \frac{1}{\sqrt{2}}(s_{s1} + s_{s3})$ | $N_c^0 \pi^0 \frac{1}{\sqrt{2}}(s_{s1} + s_{s3})$ |
|              | $N_c^0 \pi^0 -\frac{1}{\sqrt{2}}(s_{s1} + s_{s3})$ | $N_c^- \eta \frac{1}{\sqrt{2}}(s_{s1} - 2s_{s2} - 3s_{s3})$ |
|              | $\Theta_c^0 \bar{K}^0 s_{s2} + s_{s3}$ | $\Theta_c^0 \bar{K}^0 s_{s2} + s_{s3}$ |
TABLE IV: $SU(3)$ decay amplitudes for $S_b \to S_c \Pi$ with $\Delta S = 0$. The Lorentz structure is similar to Table III

| $T^{-}_b$ | $T^{-}_c \pi^+$ | $T^{-}_c \pi^0$ | $T^{-}_c \eta$ | $T^0_-\pi^-$ | $N^{-}_c \bar{K}^0$ | $N^0_-K^0$ |
|-----------|----------------|----------------|---------------|-------------|----------------|-------------|
| $s_1 + s_3$ | $\frac{1}{\sqrt{2}}(s_2 - s_3 + s_4)$ | $\frac{1}{2\sqrt{3}}(s_2 + s_3 + s_4)$ | $s_2$ | $\frac{1}{\sqrt{2}}s_3$ | $\frac{1}{\sqrt{2}}s_2$ | $s_2$ |

| $T^0_-\pi^+$ | $T^0_-\pi^0$ | $T^0_-\eta$ | $N^0_-\bar{K}^0$ |
|-------------|---------------|--------------|----------------|
| $\frac{1}{3}(s_1 + s_2 + s_4)$ | $\frac{1}{2}s_2$ | $\frac{1}{2\sqrt{3}}s_2$ | $\frac{1}{s_2}$ |

| $T^+_b$ | $T^0_+\pi^+$ |
|---------|---------------|
| $s_1 + s_4$ |

| $N^0_-\pi^+$ | $N^0_-K^0$ | $N^0_-\pi^0$ | $N^0_-\eta$ | $N^0_-\bar{K}^0$ |
|-------------|---------------|--------------|--------------|----------------|
| $\frac{1}{3}(s_2 + s_4)$ | $\frac{1}{\sqrt{2}}s_2$ | $\frac{1}{2}(s_1 + s_3)$ | $-\frac{1}{2\sqrt{2}}(s_3 - s_4)$ | $-\frac{1}{2\sqrt{2}}(s_2 - s_3 - s_4)$ | $\frac{1}{\sqrt{2}}s_3$ |

| $N^+_b$ | $N^+_c K^+$ | $N^+_c \pi^+$ | $N^+_c \bar{K}^0$ | $N^+_c \eta$ | $N^+_c \bar{K}^0$ |
|---------|---------------|--------------|--------------|--------------|----------------|
| $\frac{1}{\sqrt{2}}s_4$ | $\frac{1}{2}(2s_1 + s_4)$ | $\frac{1}{\sqrt{2}}s_4$ | $s_1$ |

| $\Theta^+_b$ | $\Theta^+_c K^+$ | $\Theta^+_c \pi^+$ |
|--------------|----------------|----------------|
| $\frac{1}{\sqrt{2}}s_4$ | $s_1$ |

TABLE V: $SU(3)$ decay amplitudes for $S_b \to S_c \Pi$ with $\Delta S = 1$. The Lorentz structure is similar to that in Table III

| $T^-_b$ | $T^-_c K^+$ | $T^-_c K^0$ | $N^-_c \pi^0$ | $N^-_c \eta$ | $N^-_c \pi^-$ | $\Theta^-_c K^0$ |
|---------|---------------|--------------|--------------|--------------|-------------|----------------|
| $s_1 + s_3$ | $\frac{1}{\sqrt{2}}s_3$ | $\frac{1}{2}(s_2 + s_4)$ | $\frac{1}{2\sqrt{3}}(s_2 - s_3 + s_4)$ | $\frac{1}{\sqrt{2}}s_2$ | $s_2$ |

| $T^0_-K^+$ | $N^-_c \pi^+$ | $N^-_c \pi^0$ | $N^-_c \eta$ | $\Theta^-_c K^0$ |
|-------------|---------------|--------------|--------------|----------------|
| $\frac{1}{2}s_1$ | $\frac{1}{2}(s_2 + s_4)$ | $-\frac{1}{2\sqrt{2}}s_2$ | $\frac{1}{2\sqrt{3}}s_2$ | $\frac{1}{s_2}$ |

| $T^+_b$ | $T^0_+K^+$ |
|---------|---------------|
| $s_1$ | $\frac{1}{\sqrt{2}}s_4$ |

| $N^0_-K^+$ | $N^0_-K^0$ | $\Theta^0_-\pi^0$ | $\Theta^0_-\eta$ |
|-------------|---------------|--------------|--------------|
| $\frac{1}{2}(2s_1 + s_2 + s_3 + s_4)$ | $\frac{1}{2}(s_2 + s_3)$ | $\frac{1}{\sqrt{2}}s_4$ | $-\frac{1}{\sqrt{2}}(s_2 - s_3 - s_4)$ |

| $N^+_b$ | $N^+_c K^+$ | $\Theta^+_c \pi^+$ |
|---------|---------------|----------------|
| $\frac{1}{\sqrt{2}}(2s_1 + s_4)$ | $\frac{1}{\sqrt{2}}s_4$ |

| $\Theta^+_b$ | $\Theta^+_c K^+$ |
|--------------|----------------|
| $s_1 + s_4$ |

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TABLE VI: SU(3) decay amplitudes for $S_b \rightarrow S_c D$. The Lorentz structure is similar to that in Table II. For $S_b \rightarrow S_c D^*$, the Lorentz structure of the bi-spinor product is of the form $\bar{S}_c \gamma_\mu (1 + b \gamma_5) S_b$.

| $\Delta S = 0$ | $\Delta S = 1$ |
|-----------------|-----------------|
| $U^+_b$ | $r_{31} U^0_c D_d$ | $r_{31} U^0_c D_s$ |
| $D^+_b$ | $(r_{31} + r_{32}) D^+_c D_d + r_{32} U^0_c D_u + r_{32} S^-_c D_s$ | $r_{31} D^-_c D_s$ |
| $S^+_b$ | $r_{31} S^-_c D_d$ | $(r_{31} + r_{32}) S^-_c D_d + r_{32} U^0_c D_u + r_{31} D^-_c D_d$ |
| $U^+_b$ | $r T^0_c D_s - \frac{1}{\sqrt{2}} r N^0_c D_d$ | $\frac{1}{\sqrt{2}} r N^0_c D_s - r \Theta^0_c D_d$ |
| $D^+_b$ | $\frac{1}{\sqrt{2}} r N^0_c D_u - \frac{1}{\sqrt{2}} r T^-_c D_s$ | $-\frac{1}{\sqrt{2}} r N^-_c D_u + r \Theta^0_c D_u$ |
| $S^+_b$ | $\frac{1}{\sqrt{2}} r T^-_c D_d - r T^0_c D_u$ | $\frac{1}{\sqrt{2}} r N^-_c D_d - \frac{1}{\sqrt{2}} r N^0_c D_u$ |
| $T^-_b$ | $s_{31} T^-_c D_d + \frac{1}{\sqrt{2}} s_{32} T^-_c D_u$ | $s_{31} T^-_c D_d + \frac{1}{\sqrt{2}} s_{32} N^-_c D_u$ |
| $T^+_b$ | $(s_{31} + \frac{1}{\sqrt{2}} s_{32}) T^0_c D_d$ | $s_{31} T^0_c D_d + \frac{1}{\sqrt{2}} s_{32} N^0_c D_d$ |
| $N^+_b$ | $s_{31} N^0_c D_d + \frac{1}{\sqrt{2}} s_{32} N^0_c D_u + \frac{1}{\sqrt{2}} s_{32} T^-_c D_s$ | $(s_{31} + \frac{1}{\sqrt{2}} s_{32}) N^-_c D_d + \frac{1}{\sqrt{2}} s_{32} N^0_c D_u$ |
| $N^+_b$ | $(s_{31} + \frac{1}{\sqrt{2}} s_{32}) N^0_c D_d + \frac{1}{\sqrt{2}} s_{32} T^0_c D_s$ | $(s_{31} + \frac{1}{\sqrt{2}} s_{32}) N^0_c D_d + \frac{1}{\sqrt{2}} s_{32} \Theta^0_c D_d$ |
| $\Theta^+_b$ | $s_{31} \Theta^0_c D_d + \frac{1}{\sqrt{2}} s_{32} N^0_c D_d$ | $(s_{31} + s_{32}) \Theta^0_c D_d$ |
TABLE VII: $SU(3)$ decay amplitudes for $B \rightarrow N\bar{R}_c(\bar{S}_c)$. The Lorentz structure should be understood to be $\bar{N}(1 + b\gamma_5)R_c(S_c)$ since weak interaction can have S- and P-wave amplitudes.

| $B_u$ decay | $\Delta S = 0$ | $\Delta S = -1$ |
|-------------|----------------|----------------|
| $\bar{U}_c^0\Sigma^-$ | $\bar{r}_{81} + \bar{r}_{83}$ | $\bar{r}_{81} + \bar{r}_{83}$ |
| $\bar{T}_c^0\Xi^-$ | $\bar{s}_{81} - \bar{s}_{82}$ | $\bar{s}_{81} - \bar{s}_{82}$ |
| $\bar{N}_c^0\Sigma^-$ | $-\frac{1}{\sqrt{2}}(\bar{s}_{81} - \bar{s}_{82})$ | $-\frac{1}{\sqrt{2}}(\bar{s}_{81} - \bar{s}_{82})$ |

| $B_d$ decay | |
|-------------|----------------|
| $\bar{U}_c^0\Sigma^0$ | $-\frac{1}{\sqrt{2}}(\bar{r}_{81} - \bar{r}_{82})$ | $\bar{r}_{81}$ |
| $\bar{U}_c^0\Lambda$ | $\frac{1}{\sqrt{6}}(\bar{r}_{81} + \bar{r}_{82})$ | $\bar{s}_{82}$ |
| $\bar{D}_c^-\Sigma^-$ | $\bar{r}_{82} + \bar{r}_{83}$ | $\bar{r}_{82} - \frac{1}{\sqrt{2}}\bar{s}_{81}$ |
| $\bar{S}_c^-\Xi^-$ | $\bar{s}_{82}$ | $\bar{s}_{82}$ |
| $\bar{T}_c^-\Xi^-$ | $-\frac{1}{\sqrt{2}}(\bar{s}_{81} + \bar{s}_{83})$ | $\bar{s}_{81} - \frac{1}{\sqrt{2}}\bar{s}_{81}$ |
| $\bar{T}_c^0\Xi^0$ | $-(\bar{s}_{82} + \bar{s}_{83})$ | $\bar{s}_{81} + \bar{s}_{82}$ |
| $\bar{N}_c^-\Sigma^-$ | $\frac{1}{\sqrt{2}}\bar{s}_{83}$ | $\bar{s}_{81} + \bar{s}_{82}$ |
| $\bar{N}_c^0\Sigma^0$ | $\frac{1}{2}(\bar{s}_{81} - \bar{s}_{82} - \bar{s}_{83})$ | 
| $\bar{N}_c^0\Lambda$ | $\frac{1}{2\sqrt{3}}(\bar{s}_{81} + \bar{s}_{82} + 3\bar{s}_{83})$ | 
| $\bar{\Theta}_c^0n$ | $\bar{s}_{83}$ | 

| $B_s$ decay | |
|-------------|----------------|
| $\bar{U}_c^0n$ | $\bar{r}_{81}$ | $\bar{r}_{81}$ |
| $\bar{S}_c^-\Sigma^-$ | $\bar{r}_{83}$ | $\bar{s}_{82}$ |
| $\bar{T}_c^-\Sigma^-$ | $\frac{1}{\sqrt{2}}\bar{s}_{81}$ | $\bar{s}_{81}$ |
| $\bar{T}_c^0\Sigma^0$ | $-\frac{1}{\sqrt{2}}\bar{s}_{81}$ | $\bar{s}_{82} + \bar{r}_{83}$ |
| $\bar{T}_c^0\Lambda$ | $\frac{1}{\sqrt{6}}(\bar{s}_{81} - 2\bar{s}_{82})$ | $\bar{s}_{82} + \bar{r}_{83}$ |
| $\bar{N}_c^0n$ | $\frac{1}{\sqrt{2}}\bar{s}_{82}$ | $\bar{s}_{83}$ |
| $\bar{N}_c^-\Sigma^-$ | $\frac{1}{\sqrt{2}}(\bar{s}_{81} + \bar{s}_{83})$ | 
| $\bar{N}_c^0\Sigma^0$ | $\frac{1}{2}(\bar{s}_{81} + \bar{s}_{83})$ | 
| $\bar{N}_c^0\Lambda$ | $\bar{s}_{82} + \bar{s}_{83}$ | 
| $\bar{\Theta}_c^0n$ | $\bar{s}_{83}$ | 
TABLE VIII: $SU(3)$ decay amplitudes for $B \to \bar{N}(R_c,S_c)$ with $\Delta S = 0$. The Lorentz structure is of the form $\bar{R}_c(\bar{S}_c)(1 + b\gamma_5)N$.

| $B_d$ | $\Sigma^+_cU^0_c$ | $(\bar{r}_{31} + \bar{r}_{32} + \bar{r}_{61} + \bar{r}_{62})$ |
|-------|-------------------|-------------------------------------------------|
| $\Sigma^0 D^-_c$ | $-\frac{1}{\sqrt{2}}(\bar{r}_{31} + \bar{r}_{32} - \bar{r}_{61} + \bar{r}_{62})$ |
| $\bar{\Lambda}D^-_c$ | $-\frac{1}{\sqrt{6}}(\bar{r}_{31} - \bar{r}_{32} + 2\bar{r}_{33} - \bar{r}_{61} - \bar{r}_{62})$ |
| $\Xi^0 S^-_c$ | $\bar{r}_{32} - \bar{r}_{33} + \bar{r}_{62}$ |
| $\Xi^- T^-_-^c$ | $\bar{s}_{31} - \bar{s}_{61} - \bar{s}_{62}$ |
| $\Xi^0 T^-_c$ | $\frac{1}{\sqrt{2}}(\bar{s}_{31} + \bar{s}_{61} - \bar{s}_{62})$ |
| $\Sigma^0 N^-_c$ | $\frac{1}{2}(\bar{s}_{32} + 2\bar{s}_{61} + \bar{s}_{62})$ |
| $\bar{p}\Theta^0_c$ | $\bar{s}_{32} + \bar{s}_{62}$ |
| $B_d$ | $\Sigma^0 U^0_c$ | $-\frac{1}{\sqrt{2}}(\bar{r}_{31} + \bar{r}_{32} + \bar{r}_{61} - \bar{r}_{62})$ |
| $\bar{\Lambda}U^0_c$ | $\frac{1}{\sqrt{6}}(\bar{r}_{31} - \bar{r}_{32} + 2\bar{r}_{33} + \bar{r}_{61} + \bar{r}_{62})$ |
| $\Sigma^- D^-_c$ | $-(\bar{r}_{31} + \bar{r}_{32} - \bar{r}_{61} - \bar{r}_{62})$ |
| $\Xi^- S^-_c$ | $-(\bar{r}_{32} - \bar{r}_{33} + \bar{r}_{62})$ |
| $\Xi^- T^-_-^c$ | $\frac{1}{\sqrt{2}}(\bar{s}_{31} - \bar{s}_{61} + \bar{s}_{62})$ |
| $\Xi^0 T^-_c$ | $\bar{s}_{31} + \bar{s}_{61} + \bar{s}_{62}$ |
| $\Sigma^- N^-_c$ | $\frac{1}{\sqrt{2}}(\bar{s}_{32} - \bar{s}_{62})$ |
| $\bar{\Lambda}N^0_c$ | $-\frac{1}{2\sqrt{3}}(2\bar{s}_{31} - \bar{s}_{32} - 3\bar{s}_{62})$ |
| $\Sigma^0 N^0_c$ | $-\frac{1}{2}(\bar{s}_{32} - 2\bar{s}_{61} - \bar{s}_{62})$ |
| $\bar{n}\Theta^0_c$ | $\bar{s}_{32} - \bar{s}_{62}$ |
| $B_s$ | $\Xi^0 U^0_c$ | $\bar{r}_{31} + \bar{r}_{33} + \bar{r}_{61}$ |
| $\Xi^- D^-_c$ | $-(\bar{r}_{31} + \bar{r}_{33} - \bar{r}_{61})$ |
| $\Xi^- N^-_c$ | $\frac{1}{\sqrt{2}}(\bar{s}_{31} + \bar{s}_{32} - \bar{s}_{61})$ |
| $\Xi^0 N^0_c$ | $\frac{1}{\sqrt{2}}(\bar{s}_{31} + \bar{s}_{32} + \bar{s}_{61})$ |
| $\Sigma^0 \Theta^0_c$ | $\sqrt{2}\bar{s}_{61}$ |
| $\bar{\Lambda}\Theta^0_c$ | $-\sqrt{\frac{2}{3}}(\bar{s}_{31} + \bar{s}_{32})$ |
TABLE IX: $SU(3)$ decay amplitudes for $B \to \bar{N}(R_c, S_c)$ with $\Delta S = -1$. The Lorentz structure is the same as Table VIII.

| $B_u$ | $\bar{p}U_c^0$ | $\bar{r}_{31} + \bar{r}_{32} + \bar{r}_{61} + \bar{r}_{62}$ |
|-------|-----------------|--------------------------------------------------|
|       | $\bar{n}D_c^-$  | $\bar{r}_{32} - \bar{r}_{33} + \bar{r}_{62}$     |
|       | $\Sigma^0S_c^-$  | $-\frac{1}{\sqrt{2}}(\bar{r}_{31} + \bar{r}_{33} - \bar{r}_{61})$ |
|       | $\Lambda S_c^-  $ | $-\frac{1}{\sqrt{6}}(\bar{r}_{31} + 2\bar{r}_{32} - \bar{r}_{33} - \bar{r}_{61} + 2\bar{r}_{62})$ |
|       | $\Sigma^-T_c^-$  | $-(\bar{s}_{31} - \bar{s}_{61} - \bar{s}_{62})$ |
|       | $\Sigma^0T_c^-  $ | $-\frac{1}{\sqrt{2}}(\bar{s}_{31} - \bar{s}_{32} - \bar{s}_{61} - 2\bar{s}_{62})$ |
|       | $\Lambda T_c^-  $ | $-\frac{1}{2\sqrt{3}}(\bar{s}_{31} + \bar{s}_{32} + 3\bar{s}_{61})$ |
|       | $\bar{n}T_c^0$   | $-(\bar{s}_{32} + \bar{s}_{62})$ |
|       | $\bar{p}N_c^0$   | $-(\bar{s}_{32} + \bar{s}_{62})$ |

| $B_d$ | $\bar{n}U_c^0$ | $(\bar{r}_{31} + \bar{r}_{33} + \bar{r}_{61})$ |
|-------|-----------------|--------------------------------------------------|
|       | $\Sigma^-S_c^-$  | $-(\bar{r}_{31} + \bar{r}_{33} - \bar{r}_{61})$ |
|       | $\Sigma^-T_c^-$  | $-\frac{1}{\sqrt{2}}(\bar{s}_{31} + \bar{s}_{32} - \bar{s}_{61})$ |
|       | $\Sigma^0T_c^0$  | $\frac{1}{\sqrt{2}}(\bar{s}_{31} + \bar{s}_{32} - \bar{s}_{61})$ |
|       | $\Lambda T_c^0$  | $-\frac{1}{\sqrt{6}}(\bar{s}_{31} + \bar{s}_{32} + 3\bar{s}_{61})$ |
|       | $\bar{n}N_c^0$   | $-\frac{1}{\sqrt{2}}(\bar{s}_{31} + \bar{s}_{32} + \bar{s}_{61})$ |

| $B_s$ | $\bar{U}_c^0$ | $-(\bar{r}_{32} - \bar{r}_{33} - \bar{r}_{62})$ |
|-------|-----------------|--------------------------------------------------|
|       | $\Sigma^-D_c^-$  | $-(\bar{r}_{32} - \bar{r}_{33} - \bar{r}_{62})$ |
|       | $\Xi^-S_c^-$    | $-(\bar{r}_{31} + \bar{r}_{32} - \bar{r}_{61} - \bar{r}_{62})$ |
|       | $\Xi^-T_c^-$    | $-(\bar{s}_{32} + \bar{s}_{62})$ |
|       | $\Xi^0T_c^0$    | $-(\bar{s}_{32} - \bar{s}_{62})$ |
|       | $\Xi^0N_c^0$    | $\frac{1}{2}(\bar{s}_{31} - \bar{s}_{61} + \bar{s}_{62})$ |
|       | $\Lambda N_c^0$  | $-\frac{1}{2\sqrt{3}}(\bar{s}_{31} - 2\bar{s}_{32} + 3\bar{s}_{61} + 3\bar{s}_{62})$ |
|       | $\Sigma^-N_c^-$  | $-\frac{1}{2\sqrt{2}}(\bar{s}_{31} - \bar{s}_{61} + \bar{s}_{62})$ |
|       | $\bar{n}\Theta_c^0$ | $-(\bar{s}_{31} + \bar{s}_{61} + \bar{s}_{62})$ |