Research on acoustic emission characteristics of marble damaged by pre-peak unloading

Zihan Zhu1,2, Liyuan Yu1, Jinglong Li2, Haijian Su1, Zhanqun Zhang1, Jing Zhang1

1State Key Laboratory for Geomechanics and Deep Underground Engineering, China University of Mining and Technology, Xuzhou, 221116, China;
2School of Civil Engineering, Shandong University, Jinan 250061, China

Liyuan Yu: yuliyuan@cumt.edu.cn

Abstract. Excavation of the tunnel in high geo-stress environment induces strong stress redistribution, resulting in unloading damage of surrounding rock. The damaged surrounding rock is the main load bearing body for the tunnel in the running period. Therefore, it is necessary to study the acoustic emission (AE) characteristics during the reloading of damaged rock. The marble was chosen to manufacture specimens in this paper, and we designed pre-peak unloading tests. Confining pressure of pre-peak unloading test is 15MPa and three types of unloading point (70MPa, 80MPa and 90MPa).

The laboratory tests are made for studying AE characteristics of damaged marble samples by using MTS816 and acoustic emission instrument AE21C. The test results showed that as the unloading point increases, the peak strength of the rock decreases with increased corresponding peak strain. Rock failure mode gradually changed from brittle failure to quasi-brittle failure; Accumulated AE count is negatively correlated with the unloading point, and the AE quiet period is positively correlated with the unloading point. The evolution of $b$-value is divided into three phases: slow increase, sharp increase and rapid decrease. The findings of this research can provide the significance for selecting mechanical parameters of the surrounding rock and optimizing support scheme.

1. Introduction

With the rapid development of the world economy, the demand for energy mining and underground space development required for human survival and development has exploded, forcing human engineering activities to develop into deep, high-stress areas on a large scale. Deep underground projects inevitably encounter the stability problems of underground rocks in deep buried or high ground stress environments. Compared with shallow buried engineering or rock mass under general stress conditions, deep rock mass is a carrier for direct action of deep resources and underground space development. The exploitation of deep underground resources is accompanied by strong unloading disturbances, which poses a huge threat to the stability of the surrounding rocks of the underground works and the personal safety of site personnel. Therefore, it is not only of great theoretical value to study the deformation and failure mechanism of rock mass under unloading conditions, but also has significant engineering guiding significance.

In recent years, many scholars have conducted in-depth studies on the mechanical properties of rock masses under unloading conditions, which provide an important foundation for improving the stability of deep surrounding rocks and ensuring the safe mining and utilization of underground resources and spaces.
Wu et al. [1] found that during and after excavation of rock masses, rock masses close to the surface of the excavated rock often undergo unloading and stress relaxation, which may lead to severe rock failure, such as rock fracture, spalling and Collapse. Huang et al. [2] explored that the damage evolution of rock during loading and unloading is a non-equilibrium and non-linear process. The origin and nature of damage evolution characteristics in rock may be significantly different during loading and unloading. The damage evolution behavior directly affects the stability of the rock structure, which is essential for any safety considerations of the rock structure. Dai et al. [3] conducted an unloading test on granite and found that the failure strain during unloading is smaller than the failure strain during loading. As the confining pressure increases, the difference between the two strains becomes larger and larger. Xing Huang et al. [4] conducted a triaxial unloading confining pressure test on sandy mudstone specimens, and obtained the expansion and expansion characteristics of soft rock, the effect of unloading rate and the evolution law of rock damage, and revealed the process of deep soft soil excavation Deformation mechanism. Manouchehrian et al. [5] proposed a modeling method that can be used to predict the failure of unstable rocks and estimate the release of kinetic energy, simulating the failure of unstable rocks under multiaxial unloading conditions. Zhao et al. [6] based on various studies on rock mechanical behavior, rock sample size and rockburst simulation test, changed the height of granite rock sample and used an improved true triaxial unloading test machine to perform the size effect of unloading test on granite specimens the study. On the basis of mechanical theory, Dong et al. [7] designed a test system that considered initial in-situ stress and dynamic unloading, carried out a similar simulation test of cyclic excavation of deep roadway, and studied the unloading of surrounding rock under cyclic dynamic excavation effect. Zhao XG et al. [8] studied the influencing factors of rockburst based on the true triaxial strain bursting system, and the rockburst test results showed that the unloading rate largely caused the occurrence of rockburst.

Most of the previous experimental studies have considered the damage evolution of rocks under load, and very few studies have been carried out under unloading conditions. The application of acoustic emission to the investigation of rock damage has also caused many scholars to explore its characteristics. The accumulated AE energy can better reflect the true energy of the AE event, which can be used to evaluate the energy release characteristics of the rock during the entire destruction process [9]. Thomas et al. [10] obtained the evolution characteristics of rock damage and acoustic emission parameters during loading and unloading. Zhao et al. [11] explored the strain characteristics and AE characteristics of granite under various triaxial unloading rates; Du [12] and Hu [13] discussed the evolution characteristics of AE parameters when rock burst occurred in tunnel excavation.

Although there is sufficient evidence to prove the failure and evolution of rock under load, the unloading of loads has not been satisfactorily explored. Damaged and cracked surrounding rock mass is the main carrier to ensure the long-term safety and stability of the project, which seriously threatens the safety of life and property of the underground project. At present, there are few reports on the energy characteristics and failure morphology of this type of damaged rock mass during reloading. In this paper, the MTS815.3 rock mechanics test system of ShanDong University was used to carry out constant axial pressure unloading confining pressure damage test, and unloading damage fracture samples were obtained. The uniaxial compression and
reloading tests were carried out to obtain the mechanical properties and acoustic emission characteristics of damaged and fractured marble.

2. Test methodology

2.1. Basic properties of marble and test apparatus

A marble rock was used for the experiments. Marble cores were drilled and drew out with a diameter of 50 mm. Then, these cores were cut into cylinders with a height of 50 mm for the SHPB test, as recommended by the ISRM. These marble specimens were manufactured with parallelism controlled within ±0.05 mm and surface flatness within ±0.02 mm, in accordance with the standard requirements of the ISRM. The P-wave velocity of the sample ranges from 4410 to 5320 m/s. The average density of marble is 2.867 g/cm³, and the uniaxial compressive strength of marble is 83.7 MPa. The X-ray fluorescence (XRF) results show that the marble is mainly composed of CaO (33.4%), MgO (19.1%) and SiO₂ (11.5%). In addition, other mineral impurities are also detected such as Fe₂O₃ (0.16%) and Al₂O₃ (0.16%). The test instrument used MTS 815 system of Shandong University. The maximum axial load of the equipment is up to 2600 kN and the confining pressure can be up to 140 MPa. During the triaxial test, the axial and circumferential strains were recorded by extensometers. The marble samples and test instruments as illustrated in Fig.1.

2.2. Test scheme

To investigate the impact of unloading damage on the mechanical properties during the marble reloading process, the unloading points were designed in three levels, including 70, 80, and 90 MPa. Three types of unloading confining pressure rates of 0.1, 0.5 and 1.0 MPa/s are set under each unloading point. Therefore, 9 sets of 27 specimens were prepared, as shown in Tab.1. The test scheme is shown in Fig.2. It can be seen from Fig.2 that the unloading point is determined by performing uniaxial compression test, and the unloading confining pressure rate of 0.1, 0.5 and 1.0 MPa/s is set under the same unloading point. In order to reduce the influence of dispersion, 3 samples were taken under each test condition, and a total of 27 unloading damaged samples were obtained. Finally, the uniaxial reloading test of unloading damaged specimens was conducted to obtain the mechanical properties of unloading damaged marble during the reloading process. Through the reloading failure test of unloading confining damaged marble, the macro-mechanical characteristics, acoustic emission characteristics and energy dissipation characteristics during the reloading process were obtained.

In the unloading damage test, the confining pressure is first loaded to 15 MPa at a loading rate of 0.5 MPa/s, and the axial pressure is loaded to the unloading points 70, 80 and 90 MPa at a loading rate of 0.003 mm/s after the confining pressure is stabilized. Subsequently, the axial stress is kept constant, and the confining pressure is unloaded at a rate of 0.1, 0.5, and 1.0 MPa/s. In order to establish the relationship between the permeability of marble and the ratio of cumulative unloading confining pressure to initial confining pressure, 3 MPa water pressure is applied at both ends of the marble during unloading. When the confining pressure is unloaded to a preset value, the confining pressure is kept constant and the water pressure at one end of the sample is reduced to 1 MPa, forming a liquid osmotic pressure with a pressure difference of 2 MPa, and then the permeability of the marble when the initial confining pressure is discharged to this confining pressure is measured.

The MTS815 test system and AE test system were used to perform uniaxial reloading failure test on
unloaded damaged marble samples. Axial loading adopts displacement control, and the loading rate is 0.003mm/s until the specimen is destroyed.

3. Results

3.1. Results of unloading confining damage test

The damage variables expression is as follow[14]:

\[ D = 1 - \left(1 - \frac{\varepsilon_i^p}{\varepsilon_i} \right) \frac{E_u}{E_0} \]  

(1)

Where \( D \) is damage variables, \( \varepsilon_i^p \) is axial residual strain, \( \varepsilon_i \) is axial strain, \( E_u \) is the elastic modulus of unloading process, \( E_0 \) is initial elastic modulus.

The damage variable result is shown in the Fig.3. The damage variable increases with the unloading point and decreases with the unloading rate. When the unloading rate is 0.1 MPa/s, the average damage variable at the 70MPa unloading point is 0.078, and the average damage variable at the 90 MPa unloading point is 0.149. The damage variable increases by approximately 91.03% when the unloading point increases from 70MPa to 90MPa. When the unloading point is 90 MPa, the average damage variable at the unloading rate of 0.1 MPa/s is 0.149, and the average damage variable at the unloading rate of 1.0 MPa/s is 0.138, which decreases about 7.4%. Therefore, it can be seen that during the unloading damage test, the damage degree of the marble sample is greatly affected by the unloading point, and the influence of the unloading rate is relatively small.

The evolution relationship of residual strain with damage variables is shown in Fig.4. It can be seen from Fig.4 that when the damage variable is 0.004-0.016, the residual strain is linearly correlated with the damage variable. Damage variables are used to characterize the development of internal micro-cracks in the specimen during unloading damage. The larger the damage variable, the more internal micro-cracks caused by unloading will inevitably lead to an increase in residual strain, which also shows that the calculation results of the damage variable are reliable.

According to the definition of dissipated energy, residual strain is one of the external characteristics of dissipated energy. Therefore, the relationship between residual strain and dissipated energy is established. The evolution relationship is shown in Fig.5. It can be seen from Fig. 5 that the residual strain increases exponentially with the increase of the energy dissipation density. Dissipated energy is mainly used for internal micro-crack initiation, initial crack expansion and plastic deformation. The greater the dissipated energy density, the more energy used for internal micro-cracks. The internal micro-cracks and the resulting plastic deformation will inevitably increase. Therefore, the residual strain increases exponentially with the increase of the dissipated energy density, which also verifies the reliability of the calculated results of the dissipated energy density.

The relationship between the dissipated energy density and the damage variable is shown in Fig.6. It can be seen from Fig.6 that the dissipated energy density increases exponentially with the increase of the damage variable, and the calculation results of the dissipated energy density and the damage variable are consistent.

The permeability characteristics can reflect the micro-crack density inside the sample. The larger the
damage variable of the sample, the greater the internal micro-crack density compared to the smaller damage sample. Therefore, the damage variable is positively correlated with the permeability, which means that the greater the damage variable, the greater the permeability is. In order to verify the reliability of the damage variables calculated in this paper, the evolution relationship of permeability with damage variables is constructed, as shown in the Fig.7. The Fig.7 shows that the permeability of unloading damaged marble samples increases exponentially with the damage variable. The damage variable can be used to quantitatively describe the damage degree of marble samples in the unloading confining process. The larger the damage variable indicated that the unloading damage effect is significant, and the more micro-cracks generated by the unloading effect are perpendicular to the direction of the unloading confining pressure. Therefore, the permeability rises with the increase of the damage variable. When the average damage variable from 0.052 to 0.152, the marble permeability changed from $14.11 \times 10^{-19}$ to $17.39 \times 10^{-19} \text{m}^2$, increasing by about 23.25%. The higher the dissipation energy density, the higher the permeability, which is closely related to the number and distribution form of the internal micro-fractures in the sample. The development of the internal micro-fractures mainly depends on the dissipation energy density. Therefore, the permeability of marble samples gradually rises with the increase of damage variables in the unloading confining process.

3.2. Results of uniaxial compression reloading test

3.2.1 Macro mechanical parameters

The evolution of uniaxial compressive strength and elastic modulus with damage variable were shown in Fig.8 and Fig.9, respectively. It can be seen from Fig.8 and Fig.9 that the uniaxial compressive strength and elastic modulus of the damaged marble are divided into three stages with the increase of the damage variable.

Stage I: Stabilize stage, the damage variable at this stage is between 0 and 0.063. In the stabilize stage, the uniaxial compressive strength and elastic modulus of unloading damaged marble will tend to be stable with the increase of the damage variable, indicating that the number of internal micro-cracks caused by unloading confining pressure is small. Therefore, the uniaxial compressive strength and elastic modulus of the unloading damaged marble are similar to natural marble samples.

Stage II: Decrease slowly stage, the damage variable at this stage is between 0.063 and 0.10. In the decrease slowly stage, the uniaxial compressive strength and elastic modulus of the unloaded damaged marble gradually decrease with the damage variable. It shows that the micro-cracks of the marble samples gradually developed during the unloading confining pressure process. However, the density of the micro-cracks is still small. The unloading confining damage test begins to deteriorate the carrying capacity and resistance to deformation of marble. The uniaxial compressive strength and elastic modulus of the damaged marble began to decrease gradually compared with the natural specimen.

Stage III: Decrease rapidly stage, the damage variable at this stage is greater than 0.1. In the decrease rapidly stage, the uniaxial compressive strength and elastic modulus of the unloaded damaged marble rapidly decrease with the damage variable. It shows that the micro-cracks in the marble samples are more developed during the unloading confining pressure test, and the density of micro-cracks is larger. The unloading confining damage test significantly deteriorates the carrying capacity and resistance to deformation of marble. The uniaxial compressive strength and elastic modulus of unloaded damaged marble are sharply reduced.
compared with natural samples.

3.2.2 AE characteristics

The relationship between accumulative ring counts and time of marble samples with different damage degrees are shown in Fig.10. The accumulative ring counts shows a downward trend as the unloading point gradually increases, and the quiet period of the acoustic emission also increases as the unloading point increases. This is because with the increase of the unloading point, there are more micro-cracks in the sample than the sample under the action of the low unloading point. The micro-cracks inside the sample can degrade the mechanical properties of the sample, which is manifested by the weakening of the load-bearing capacity and resistance to deformation of the rock. Therefore, when the sample fails, the acoustic emission activity is relatively small, and the cumulative ring count is less than that of the non-damaged sample.

In order to further analyze the relationship between the accumulative ring counts and the degree of damage, a graph of the relationship between accumulative ring counts and damage variable is established as shown in Fig. 11. It can be seen from Fig.11 that as the damage variable increases, the cumulative ringing count of the acoustic emission slowly decreases first, and then decreases rapidly, which is similar to the uniaxial compressive strength changing with the damage variable. Compared with the natural marble samples, the accumulative ring counts of the unloading damaged samples with damage variable of 0.078 was reduced by about $3.7 \times 10^4$ times, those with damage variable 0.112 by $8.5 \times 10^4$, and those with damage variable 0.149 by $12.7 \times 10^4$. When the damage variable is greater than 0.078, the cumulative ringing count decreases significantly, which indicates that the marble has obvious unloading damage, and the number of micro-cracks in the rock increases, so less elastic energy is released during the loading process, resulting in a significant decrease in the cumulative ringing count. During the reloading test, the failure form of marble begins to change from brittle failure to quasi-brittle failure.

AE $b$-value can be used to better understand the evolution law of internal micro-cracks in unloading damaged rocks during the reloading test, and to obtain the pre-warning information of the damage of unloading damaged marble. The increase of $b$ value means that the proportion of small-scale events increases, and the micro-fracture in the material is in a stable and extended state. The fluctuation of $b$ value in a small range indicates that the failure state of the micro-cracks inside the material is relatively slow. The decrease of $b$ value means that the proportion of large events increases, and the internal micro-cracks of the material are mainly caused by large-scale failure. $b$ value is constant, indicating that micro-cracks of different scales are relatively stable. Sudden change of $b$ value in a wide range means sudden change of micro-crack state, which is a fast unstable expansion.

In terms of AE technique, the Gutenberg–Richter formula therefore gets modified as[15]:

$$\text{Lg}N = a - b(A_m / 20)$$

(2)

Where $N$ is the number of AE hits with amplitude greater than the threshold $A_m$, $a$ an empirical constant and $b$ the $b$ value.

It can be seen from Fig.12(a) that the $b$ value of the acoustic emission of the natural sample exhibits a two-stage evolution trend during the loading process, increasing first and then decreasing. It can be seen from
Fig.12(b) that the acoustic emission $b$ value of the unloaded damaged marble samples during the loading process showed a three-stage evolution trend, first a small amplitude fluctuation, then gradually increased and then suddenly decreased.

The relationship between the acoustic emission $b$ value and the damage variable is shown in Fig.13. It can be seen from Fig.13 that as the damage variable increases (the larger the damage variable is, the more the micro-cracks in the marble sample are developed due to unloading) the acoustic emission $b$ value is divided into two stages, and the critical damage variable is about 0.08. In stage I, the stable stage is when $D$ is less than 0.08. At this time, the acoustic emission $b$ value increases only slightly with the increase of the damage variable. Combining the evolution graph of strength and elastic modulus with the damage variable, we can see that when the damage variable is 0.08, the strength and elastic modulus are less attenuated compared with natural samples, and the number of internal micro-cracks induced by unloading damage is less. In the reloading test, the micro-cracks induced by unloading are in a subordinate position, so the impact on the physical and mechanical properties of marble is relatively small, resulting in only a small increase in the $b$ value of acoustic emission. In phase II, when $D$ is greater than 0.08, the acoustic emission $b$ value increases with the increase of the damage variable. It can be seen from the foregoing that the increase in the $b$ value of acoustic emission means that the proportion of small-scale specimens increases. When the unloading damage of marble is large, the density of internal micro-cracks increases, and the micro-cracks are more likely to penetrate each other during the reloading process. The sample is damaged, so the small-scale events generated during the reloading process account for a relatively low damage. The marble sample is significantly improved, mainly with small-scale damage. The $b$ value of acoustic emission increases sharply with the increase of the damage variable.

Fig.14 shows the uniaxial compression reloading test failure of unloaded damaged marble samples. With the increase of the damage variable, the failure pattern of the sample becomes more broken. When the damage variable is 0, shear failure is dominant. When the damage variable is 0.139, the failure pattern of the sample changes from shear failure to tensile failure. When the damage variable is small, the failure form of the sample is the mixed failure of the pull-shear. This is mainly due to the fact that the internal micro-cracks generated in the unloading damage test are perpendicular to the unloading direction, the damage variable is small, the internal micro-cracks are relatively few, that is, the sample is relatively complete, and the development of the internal micro-cracks is subordinate to the damage during the reloading process, whereas, the damage variable is large, the internal damage is serious, and the expansion of internal micro-cracks generated in unloading stage plays a critical role in the process of reloading failure. Therefore, with the increase of damage variable, the failure pattern of the sample changes from shear failure to mixed failure and tensioning failure.

4. Conclusions

This article takes marble as the research object, and uses the MTS815.03 test system to carry out confining pressure damage test on 27 marble samples. Uniaxial reloading test was carried out on the marble samples after unloading damage, and the following main conclusions were obtained.

1. In the unloading confining pressure damage test, the damage variable is positively correlated with
the unloading point, and the dissipated energy density and permeability increase exponentially as the damage variable increases.

2. In the reloading test, the uniaxial compressive strength and elastic modulus of the unloaded damaged marble samples are divided into three stages with the increase of the damage variable, which are the stable stage, the slowly decreasing stage, and the rapid decreasing stage.

3. In the reloading test, the accumulative ringing counts of acoustic emission is negatively correlated with the damage variable. As the damage variable increases, the cumulative ring count decreases slowly and then decreases rapidly. The acoustic emission b value is positively correlated with the damage variable. With the increase of the damage variable, the acoustic emission b value increases slowly and then increases rapidly.

4. With the increase of the damage variable, the failure pattern of the sample becomes more broken. When the damage variable is 0, shear failure is dominant. When the damage variable is 0.139, the failure pattern of the sample changes from shear failure to tensile failure.

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Fig. 1 Marble sample and test instruments
Preparation of 30 saturated marble samples

Pre-peak unloading damage test (27 samples)

Unloading confining pressure rates are 0.1, 0.5 and 1.0MPa / s

Unloading points are 70, 80 and 90MPa

Obtain pre-peak unloading damaged samples (27 samples)

Permeability characteristics of marble in unloading test

Uniaxial compression failure test of damaged specimen

AE characteristics

Macro mechanical parameters

Dissipation energy properties

Fig. 2 test scheme

![Diagram showing the test scheme](image)

Fig. 3 Relationship between damage variable and unloading point

![Graph showing the relationship between damage variable and unloading point](image)
Fig. 4 Relationship between residual strain and damage variable

\[ \varepsilon_r = 0.101D + 0.008 \]
\[ R^2 = 0.947 \]

Fig. 5 Relationship between residual strain and dissipated energy

\[ \varepsilon_r = 0.074 - 0.074 \times 0.99^{U_d} \]
\[ R^2 = 0.99 \]
**Fig. 6** Relationship between dissipated energy and damage variable

\[ U_d = 3.55e^{D/0.086} + 10.46 \]

**Fig. 7** Relationship between permeability and damage variable

\[ k = 1.281e^{(D/0.104)} + 11.905 \]

\[ R^2 = 0.910 \]
Fig. 8 Relationship between uniaxial compressive strength and damage variable

Fig. 9 Relationship between elastic modulus and damage variable
Fig. 10 The relationship between accumulative ringing counts and time of marble.

Fig. 11 Accumulative ring counts of marble at different damage variables.
Fig. 12 The relationship between AE $b$ value and time.

(a) $D=0$ 

(b) $D=0.069$
Fig. 13 The relationship between AE $b$ value and damage variable $D$

Test value

Average value

I: Stabilize

II: Increase

Damage variable $D$
Fig. 14 The failure form of different damage variables marble