Research Article

Jun Qiu, Yiping Luo*, Yuqing Li, Jiao Luo, Zhibin Su, and Ying Wang

Research on a mechanical model of magnetorheological fluid different diameter particles

https://doi.org/10.1515/ntrev-2022-0015
received October 7, 2021; accepted November 28, 2021

Abstract: In this paper, the chain structure of magnetorheological fluid (MRF) magnetic particles was studied and analyzed, the mechanical model of MRF with different diameter ferromagnetic particles was established, silicone oil-based MRF with different particle volume fractions was prepared, the shear properties of the MRF were tested, and the theoretical and experimental data were compared. The experimental results show that the shear stress is stable with the increase of shear strain rate under the action of the magnetic field, and it has a shear thinning effect. The shear stress increases linearly with the increase of particle volume fraction. The shear stress increases with the increase of magnetic induction intensity. After data analysis and in the case of control variables, the average error of improved theoretical data and experimental data is lower than that of previous theoretical data and experimental data, which verifies that the improved theory (mechanical model) has a certain accuracy.

Keywords: magnetorheological fluid, shear stress, mechanical model, data analysis

1 Introduction

Magnetorheological fluid (MRF) is a suspension liquid formed by micron (nanometer) magnitude magnetic particles dispersed in a non-magnetic liquid. In the absence of a magnetic field, the MR fluid is a free-flowing Newtonian fluid. Under the action of an external magnetic field, the state of the MR fluid will change rapidly from the free-flowing state at the beginning to a solid-like non-Newtonian fluid with a certain shear yield stress. Because of its rheological effect and mechanical properties, MRF also has the advantages of low energy consumption, good controllability, simple preparation, wide temperature adaptation range, low pollution, strong safety, and reliability. It is widely used in automotive, aerospace, mechanical engineering, construction engineering, medical engineering, and other fields and is honored as one of the most promising intelligent materials [1–5].

The shear yield stress of MRF is one of the indexes to evaluate the properties of MR fluid and the practical engineering application of MR devices. How to calculate and analyze and establish the macroscopic mechanical model of MRF from the microscopic structure has become the focus of researchers all over the world. Lemaire and Bossis [6] used a controlled stress rheometer to measure the variation of the static yield stress of magnetic colloidal suspension with the applied magnetic field. The magnetization curves of different volume fractions were used to analyze the experimental results. The calculation formula of shear yield stress is derived by combining the restoring force formula in the literature [7]. Rodriguez-López et al. [8] studied the effect of particle volume fraction on the microstructure of MRF by using ultrasonic technology. As the volume fraction increases, the particles are observed to rearrange, thereby reducing the compressibility of the system. Zhang et al. [9] proposed an MRF constitutive model based on a six-sided tightly arranged structure. Yang et al. [10] introduced the microstructure model of MR fluid into an MR damper. To more accurately use the shear yield stress model of MRF, Li et al. [11] compared and analyzed the influence of various parameters on the single-chain dipole model error based on the finite element model and optimized the single-chain dipole model. Xu et al. [12] proposed the hypothesis that the double-chain tightly arranged structure and the Angle between the particle chain and the magnetic field direction followed the exponential distribution and deduced the expression that could describe the shear stress of MRF.

* Corresponding author: Yiping Luo, School of Mechanical and Automotive Engineering, Shanghai University of Engineering Science, Shanghai 201620, China, e-mail: lyp777@sina.com, tel: +86-13801933171

Jun Qiu, Yuqing Li, Jiao Luo, Zhibin Su, Ying Wang: School of Mechanical and Automotive Engineering, Shanghai University of Engineering Science, Shanghai 201620, China

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and the shear stress of each influencing parameter. Zhao et al. [13] compared the simplified dipole model, accurate dipole model, and Maxwell stress tensor based on three theories in their paper. Based on a more accurate dipole model and statistical method, Yi et al. [14] established a micro-macro description of the constitutive behavior of MR fluid under shear deformation. The accuracy of the above-mentioned models has been verified by experiments.

At present, most magnetorheological hydrodynamic models are established based on the homogeneous and spherical shape of magnetic particles (carbonyl iron powder). Moreover, the establishment of the mechanical model under double particle size has not been mentioned, which leads to the lack of accuracy of the model (Figure 1). How to calculate the magnetic force between magnetic dipoles and derive the mechanical model of magnetic particles in reducer MRF under different chained structures is a key problem. Researchers from all over the world have proposed various hypotheses on the mechanism of chain formation, among which the field dipole moment theory is the most representative one.

To improve the accuracy of the MRF mechanical model, in this paper, authors calculated the interaction between magnetic particles based on the dipole theory, analyzed the chain process of magnetic particles, deduced the mechanical model of the MRF reducer ferromagnetic particles, and then prepared the MRF samples for experimental verification.

2 Establishment of the mechanical model of MRF with different diameter particles

2.1 The interaction force of magnetic particles

According to the theory of field-induced dipole moment, magnetic particles are magnetized into magnetic dipoles under the action of a magnetic field, and the magnetic dipoles attract each other to form an ordered chain and column structure. Then, according to the knowledge of magnetism and mechanics, the interaction force between magnetic particles can be calculated by the theory of field-induced dipole moment and Maxwell stress tensor.

Magnetization is a physical quantity that describes the magnetization (direction and degree of magnetization) of the magnetic medium. It represents the sum of the magnetic moment vectors of the magnetic dipole in the unit volume magnetic medium. If an integral element $\Delta V$ is taken in the magnetic medium, and there is a large number of magnetic dipoles in the volume element, the magnetic moment of each magnetic dipole is $\mathbf{m}$, and then the magnetization intensity can be defined as

$$M = \frac{\sum \mathbf{m}}{\Delta V}. \quad (1)$$

According to the dipole theory [15], each dipole has two magnetic poles $Q^m_1$ and $Q^m_2$. According to Coulomb’s law, the magnetic field intensity $\mathbf{H}$ generated by the magnetic pole of the magnetic dipole at $r$ is

$$\mathbf{H} = \frac{Q^m}{4\pi\mu_0} \left( \mathbf{\hat{r}}_1 - \mathbf{\hat{r}}_2 \right) = \frac{Q^m}{4\pi\mu_0} \left( \mathbf{r}_1 \mathbf{\hat{r}}_1 - \mathbf{r}_2 \mathbf{\hat{r}}_2 \right), \quad (2)$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ are, respectively, the displacement vectors of two magnetic poles to point P; $\mathbf{\hat{r}}_1$ and $\mathbf{\hat{r}}_2$ are unit vectors in the $\mathbf{r}_1$ and $\mathbf{r}_2$ directions; and $\mathbf{a} = a\mathbf{n}$, $\mathbf{n}$ are unit vectors in the direction of the external magnetic field, as shown in Figure 2.

Assuming that the size of magnetic particles $a$ is much smaller than the spherical center distance $r$ between magnetic particles, and considering the definition of magnetic dipole moment $j$ and the relationship between magnetic moment and magnetic dipole moment, i.e.

$$j = Q^m a, \quad j = \mu_0 \mathbf{m}, \quad (3)$$

where $\mu_0$ is the vacuum permeability.

When the particle size $a$ is much smaller than $r_1$ and $r_2$, $r$ is approximate to $r_1$ and $r_2$, thus obtaining the magnetic field generated by magnetized of a magnetic particle.

![Figure 1: Photo of iron powder sample with nominal diameter of 10 µm carbonyl.](image)
\[ H = \frac{1}{4\pi r^2} \left[ \frac{3(r \cdot m)}{r^2} \times r - m \right]. \]  

In free space, \( \vec{B} = \mu_0 \vec{H} \) is satisfied between magnetic field intensity \( \vec{H} \) and magnetic induction intensity \( \vec{B} \), so the magnetic induction intensity generated by magnetic dipole \( j \) at magnetic dipole \( i \) is
\[ \vec{B}_{ij} = \frac{\mu_0}{4\pi r_{ij}} \left[ 3(\vec{r}_{ij} \cdot \vec{m}_i)\vec{r}_{ij} - \vec{m}_i \right], \]
where \( \vec{r}_{ij} \) is the displacement vector from the \( i \) magnetic particle to the \( j \) particle and \( \vec{r}_{ij} \) is the unit vector of the displacement vector \( r_{ij} \).

### 2.2 Mechanical model establishment

In this paper, based on the analysis of the microstructure and morphology of MRFs, it is necessary to conduct in-depth research on the macro characteristics of MRFs and establish MR mechanical models with different particle sizes \((R_i, R_j)\) (as shown in Figure 3).

For the single-chain structure of MRF magnetic particles as shown in Figure 3, the total magnetic energy of magnetic dipole \( i \) under the action of an external magnetic field and additional magnetic fields generated by other particles in the particle chain can be expressed as
\[ W_i = -m_i \cdot \left( B_0 + \sum_{j \neq i} B_{ij} \right) = -\left( m_i \cdot B_0 + m_i \cdot \sum_{j \neq i} \vec{B}_{ij} \right). \]

Because the added magnetic field in the area of the MRF is uniform (can approximate to uniform magnetic field), the magnetic energy contribution to the shear stress is zero; so, the magnetic dipole \( i \) generated in other magnetic dipole chains’ additional magnetic field of the energy change is the magnetic dipole \( i \) in other magnetic dipole chain that produce the disturbance of magnetic field of magnetic energy changes as \( \tilde{W}_i \)
\[ \tilde{W}_i = m_i \sum_{j \neq i} \vec{B}_{ij} = \mu_0 \sum_{j \neq i} \frac{m_i m_j}{4\pi r_{ij}^3} (3 \cos^2 \theta - 1). \]

Due to the action of the base liquid, an oil film is formed on the surface of the magnetic particles. Let the thickness of the oil film be \( t \), and there is \( \delta \) gap between the adjacent particles in the chain. Assuming that the external magnetic field is vertically upward, in the shear process, the particle chain is elongated. Then, the particle chain is tilted and the included Angle with the direction of the magnetic field is \( \theta \). Therefore, the distance between two magnetic dipoles in the same chain with \( A \) distance of \( k \) particles apart is
\[ \rho_k = \frac{k(R_i + R_j + 2t + \delta)}{\cos \theta}, \quad k = 1, 2, \ldots, 2n. \]

According to equations (1), (7) and (8), it can be obtained as
\[ \tilde{W}_i = \mu_0 \sum_{k=1}^{2n} \frac{m_i m_j}{4\pi \rho_k^3} (3 \cos^2 \theta - 1), \]
\[ = \mu_0 \sum_{k=1}^{2n} \frac{4\pi R_i R_j^2 \rho_k^2 (3 \cos^2 \theta - 1) \sin^2 \theta}{9k^3 (R_i + R_j + 2t + \delta)^3}. \]

In the formula, \( 2n \) is the number of particles of a particle chain between two plates, \( A = \sum_{k=1}^{2n} 1 \). It is easy to get using Matlab \( A \mid_{n=3} = 1.1903 \), \( A \mid_{n=8} = 1.202 \), and \( A \mid_{n=\infty} = 1.202 \). Therefore, \( A = 1.2 \) can satisfy the analysis of this study.

As shown in Figure 2, the single-chain length is \( L_s \), then its magnetic energy per unit length is
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\[
\dot{W}_i^0 = \frac{2nW_i}{L_y} = \frac{8A\mu_0\pi R_1^3 R_2^3 H^2}{9(R_1 + R_2 + 2t + \delta)\cos^3 \theta (3 \cos^2 \theta - 1)}.
\]  

Then, the shear resistance \( T_i \) of single chain of magnetic particles can be expressed as

\[
T_i = \frac{\partial \dot{W}_i^0}{\partial y}.
\]  

Thus, the single chain shear resistance can be obtained by combining equations (10) and (11)

\[
T_i = \frac{8A\mu_0\pi R_1^3 R_2^3 H^2 \cos \theta \sin \theta}{3(R_1 + R_2 + 2t + \delta)^3} (5 \cos^2 \theta - 1).
\]  

It is assumed that the volume of magnetic particles in the MRF is \( V \), the volume fraction of magnetic particles is \( \phi \), and the number of particles in each chain is \( 2n \). During the shear process, it is assumed that the number of particles in the particle chain is constant, and the change of particle chain length is mainly reflected in the change of particle spacing. Then, the number of particle chains in the unit cross-sectional area is

\[
P = \frac{x}{S} = \frac{L_y\phi}{4\pi mn(R_1^2 + R_2^2)} = \frac{3(R_1 + R_2 + 2t + \delta)\phi}{4\pi n(R_1^2 + R_2^2)},
\]  

where \( x \) is the total number of particle chains between two plates and \( S \) is the base area of the MRF perpendicular to the magnetic field direction.

Then, the sum of the single-chain shear resistance in a unit cross-sectional area can be expressed as the macroscopic shear yield stress of the MRF, that is, the shear yield stress of the MRF under the action of a magnetic field

\[
\bar{\tau} = \sum_{i=1}^{n/2} T_i.
\]  

And the sum of the shear yield stress \( \tau_0 \) (viscous damping force of the MRF under zero field) of the MRF in the absence of magnetic field, i.e

\[
\tau = \bar{\tau} + \tau_0.
\]  

In the non-free space, the magnetic induction intensity \( B \) and magnetic field intensity \( H \) satisfy \( H = \frac{B}{\mu_0(1 + \chi)} \), which can be obtained as

\[
\bar{\tau} = \sum_{i=1}^{n/2} 8A\mu_0 R_1^3 R_2^3 H^2 \sin \theta (5 \cos^2 \theta - 1).
\]  

MRF magnetic particle chains in the shear process are due to the influence of the additional magnetic field, external magnetic field, and macroscopic deformation. The chains deform and rearrange, part of the chains break, regenerate, and finally reach the dynamic equilibrium of mechanics, that is, the shear stress tends to stabilize. It is assumed that the included angle \( \theta \) between the magnetic dipole particle chain and the direction of the magnetic field in dynamic equilibrium obeys normal distribution, i.e.

\[
p(\theta) = P \cdot D(\theta), \quad \left( -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \right),
\]  

where

\[
D(\theta) = \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(\theta - \mu)^2}{2\sigma^2}},
\]  

\( \mu \) is the center of the curve distribution and \( \sigma \) is the degree of concentration relative to the center of the curve distribution.

Substituting equations (17) and (18) into equation (16) can be obtained as

\[
\bar{\tau} = \int_{-n/2}^{n/2} T_i(\theta)PD(\theta)d\theta
\]  

\[
= \int_{-n/2}^{n/2} 2PA\mu_0 R_1^3 R_2^3 H^2 \sin \theta (5 \cos^2 \theta - 1) \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(\theta - \mu)^2}{2\sigma^2}} d\theta.
\]  

Thus, the macroscopic shear yield stress of the MRF is

\[
\bar{\tau} = \tau_0 + \int_{-n/2}^{n/2} 2PA\mu_0 R_1^3 R_2^3 H^2 \sin \theta (5 \cos^2 \theta - 1) \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(\theta - \mu)^2}{2\sigma^2}} d\theta.
\]  

In the formula, both \( \mu \) and \( \sigma \) have specific physical meanings. \( \mu \) is the curve distribution center, and \( \sigma \) represents the concentration degree of the relative distribution center of the particle chain.

3 Experiment and analysis

3.1 Preparation of MRF samples

In this paper, carbonyl iron powder (3 and 6 µm) is used as magnetic particles, silicone oil is used as the base liquid.
of MRF, sodium dodecyl benzene sulfonate is used as a surfactant, nano-silica is used as a thixotropic agent, and graphite powder is used as a lubricant. According to the literature [16], MRF samples with volume fractions of 5, 10, 15, and 20% magnetic particles were prepared. The preparation process of MRF is as follows:

1) Magnetic particles, anhydrous ethanol, and grinding balls are added into the ball mill tank at a mass ratio of 1:1:1. Adding a certain amount of grinding balls into the ball grinding tank is divided into 5, 7, and 10 mm three kinds, and the mass ratio is 6:3:1. The ratio of grinding ball is determined according to the manual of the grinding machine.

2) The ball mill tank is placed in the ball mill to ball mill for a certain time, so that the surfactant and magnetic particles are fully mixed. When the ball milling is completed, the anhydrous ethanol in the upper layer of the ball milling tank is sucked out.

3) Dry the ball mill tank in a vacuum drying oven until the anhydrous ethanol in the ball mill tank evaporates completely.

4) The magnetic particles in the ball mill can be screened with a 50-mesh screen to get the coated magnetic particles.

5) Pour the coated magnetic particles, silicone oil, thixotropic agent, and graphite powder into the beaker in a certain proportion and use a mechanical agitator to stir for a certain time to make these materials fully and evenly mixed, so as to obtain the MRF sample needed for the experimental study (formula is shown in Table 1).

### 3.2 Experimental bench construction

In this study, a self-made lifting device was used to test the mechanical properties of the MRF. The experimental

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**Table 1: Formula of MRF after adding thixotropic agent, surfactant and graphite powder**

| Number | Volume fraction (%) | Milling time (h) | Mass fraction of thixotropic agent (%) | Mass fraction of graphite powder (%) | Silicone oil viscosity (mPa·s) |
|--------|---------------------|------------------|--------------------------------------|-------------------------------------|-----------------------------|
| 1      | 5                   | 12               | 0.5                                  | 0.5                                 | 973                         |
| 2      | 10                  | 12               | 0.5                                  | 0.5                                 | 973                         |
| 3      | 15                  | 12               | 0.5                                  | 0.5                                 | 973                         |
| 4      | 20                  | 12               | 0.5                                  | 0.5                                 | 973                         |

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**Figure 4: Test bench.**
bench includes a magnetic field generator, a shear device, a power source, a driving mechanism, and a force measuring device, as shown in Figure 4.

3.3 Experiment and data analysis

The prepared MRF was placed in the storage tank (made of the transparent acrylic plate), and the tank was clamped by a clamping device. Permanent magnets were placed on both sides of the clamping device. The power supply was turned on, the controller was started, and the experimental data of each sample were recorded. The data are fitted and compared with the improved theoretical data (equation (20)) and the previous theoretical data [14] (equation (21)).

\[
\bar{\tau} = \frac{4PA\mu_0 nR^2 \chi^2 H^2}{3(2R + 2t + \theta)^4}
\times \int_{-\pi/2}^{\pi/2} (5 \cos^2 \theta - 1) \cos^4 \theta \sin \theta \frac{1}{\sqrt{2\pi}} e^{-\frac{(\theta - \mu)^2}{2\sigma}} \, d\theta,
\]

where \(P\) is the number of chains in \(A\) unit cross-sectional area and \(A = 1.2\).

Figure 5: Changes of particle volume fraction and shear stress under different magnetic induction intensities graph: (a) magnetic induction intensities is 0.08 T; (b) magnetic induction intensities is 0.26 T; (c) magnetic induction intensities is 0.3 T; and (d) magnetic induction intensities is 0.35 T.
The change of shear stress and volume fraction of MRF under the action of the magnetic field is shown in Figure 5. It can be seen from Figure 5(a)–(d) that under a certain magnetic induction intensity, the shear stress increases with the increase of particle volume fraction. It can be seen from Figure 5 that the improved theoretical data curve (fitting the curve of particle volume fraction and shear stress in MATLAB according to formula 21) is closer to the experimental data, so the improved theory (mechanical model) has a certain accuracy under different magnetic induction accuracy.

Under the action of the magnetic field, the change between magnetic induction and shear stress is shown in Figure 6. It can be seen from Figure 6(a)–(d) that the shear stress increases with the increase of magnetic induction intensity, and the increasing trend gradually increases. It is obvious from Figure 6(b)–(d) that the improved theoretical data (fitting the curve of magnetic induction and shear stress in MATLAB according to the formula (21)) is closer to the experimental data. For Figure 6(a) diagram, the error analysis between the improved theoretical data and the formula (21) 6µm data and the experimental data is carried out. The average errors between the shear stress corresponding to 0.08, 0.26, 0.3, and 0.35 T and the experimental data are 7.06 and 14.56%, respectively.

Figure 6: When the particle volume fraction is constant, the relationship between magnetic induction intensity and shear stress graph: (a) particle volume fraction is 5%; (b) particle volume fraction is 10%; (c) particle volume fraction is 15%; and (d) particle volume fraction is 20%.
Under the action of the magnetic field, the particle volume fraction is constant, and the shear strain rate and shear stress change under different magnetic induction intensities shown in Figure 7(a)–(d). At the same shear strain rate, it increases significantly with the increase of magnetic induction intensity. When the MRF is on the shear mode, the particle chains formed between the liquid storage tanks are elongated and broken, and new particle chains are formed under the action of the magnetic field. As the particle chain breaks and forms, it reaches equilibrium and the shear stress reaches a stable value. Therefore, the shear thinning phenomenon exists.

It can be seen from Figure 7 that under the same magnetic induction intensity, there is little difference between the improved theoretical data and the experimental data. According to this, the error analysis is carried out, and the average errors of the improved theoretical data, the formula (21) 6 µm data, and the experimental data are 6.78 and 15.46%, respectively, when the magnetic induction intensity is 0.08T. When the magnetic induction intensity is 0.26 T, the average errors of the improved theoretical data, the formula (21) 6 µm data and experimental data are 0.96 and 3.00%, respectively. When the magnetic induction intensity is 0.3 T, the average errors of the improved theoretical data, the formula (21) 6 µm data and experimental data are 0.79 and 2.42%, respectively; When the magnetic induction intensity is 0.35 T, the average errors of the improved theoretical data, the formula (21) 6 µm data and experimental data are 0.69 and 2.32%, respectively.

Figure 7: When the particle volume fraction is constant, the relationship between shear strain rate and shear stress under different magnetic induction intensities graph: (a) particle volume fraction is 5%; (b) particle volume fraction is 10%; (c) particle volume fraction is 15%; and (d) particle volume fraction is 20%.
4 Conclusion

In this paper, the chain analysis of MRF magnetic particles under the action of the magnetic field is studied, and the mechanical model of MRF different diameter ferromagnetic particles is established. The MRF samples were prepared and their mechanical properties were tested. The comparison between the tested data and the improved theory and the previous theory is analyzed. By studying the relationship among particle volume fraction, magnetic induction intensity, shear strain rate, and shear stress, it is verified that the improved theory is better than the previous theory, and the accuracy of the mechanical model is improved.

Acknowledgments: The authors thank Mr Luo’s guidance on the paper and the help of the lab teachers and brothers in the experiment.

Funding information: The authors state no funding involved.

Author contributions: All authors have accepted responsibility for the entire content of this manuscript and approved its submission.

Conflict of interest: The authors state no conflict of interest.

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