Role of acoustic phonons in exotic conductivity of two-dimensional Dirac electrons

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We examine the effect of acoustic phonon scattering on the conductivity of two-dimensional Dirac electrons. The temperature ($T$) dependence of the conductivity ($\sigma$) is calculated using the Green’s function with damping by both the impurity ($\Gamma_0$) and phonon ($\Gamma_{ph}$). For zero or small doping, on which the present Rapid Communication focuses, $\sigma(T)$ increases and becomes almost constant due to the competition between the Dirac electrons and the phonon scattering. Such strange behavior of $\sigma(T)$ is ascribed to an exotic mechanism of phonon scattering, whose momentum space is strongly reduced in the presence of a Dirac cone. For large doping, $\sigma$ decreases due to the interplay of the Fermi surface and the phonon. The unconventional $T$ dependence of the resistivity $\rho (= 1/\sigma)$ for small doping is compared with that of the experiment of Dirac electrons in an organic conductor.

Two-dimensional massless Dirac fermions are well known as a system exhibiting topological properties, e.g., the quantum Hall effect in graphene monolayers. The linear dispersion of the energy spectrum with a Dirac cone gives a topological anomaly associated with the Berry phase, which can be shown using a spin rotation operator of the wave function. When the chemical potential is located at the energy of the Dirac point, i.e., zero doping, unconventional properties appear in the transport phenomena. While the density of states vanishes linearly at the energy of the Dirac point, the conductivity even in the limit of weak impurities displays not a zero-gap semiconductor but a metallic behavior. In fact, the conductivity at zero doping is known as a minimum conductivity with a universal constant. This quantum conductivity comes from the singularity of the Dirac cone, while the conductivity treated classically by the Boltzmann equation is reduced to zero at zero doping.

Thus, the conductivity is one of the most fundamental phenomena and has been shown to have very peculiar behaviors in bulk systems of two molecular conductors. One is the two-dimensional organic conductor [BEDT-TTF=bis(ethylenedithio)tetrathiafulvalene], in which a zero-gap state with a tilted Dirac cone was found using a tight-binding model. The other is a single-component molecular conductor under high pressures, which shows a three-dimensional Dirac electron with a nodal line semimetal. These conductors share a common feature for the temperature dependence of the conductivity. It is known theoretically that the conductivity of two-dimensional Dirac electrons at absolute zero temperature becomes a constant even within the self-consistent Born approximation and increases with increasing temperature. However, the calculation of the temperature dependence was performed using the damping only by impurities and without taking account of a mechanism of suppressing the conductivity by phonons at finite temperature. Indeed, the resistivity observed in these conductors is almost constant with increasing temperature. In spite of the strong temperature dependence of the Hall coefficient, the constant resistivity has been regarded as evidence of the presence of a Dirac electron.

In this Rapid Communication, the constant resistivity of the two-dimensional Dirac electrons of the organic conductor is studied by examining the scattering by acoustic phonons in addition to impurity scattering. The temperature dependence of resistivity of the Dirac electrons with phonon scattering has been studied for the doped graphene using the Boltzmann equation. They show that the scattering rate depends linearly on the temperature ($T$) at high temperatures and is proportional to $T^4$ at low temperatures. This $T$ dependence for large doping is reasonable due to the existence of the Fermi surface. However, such mechanism of scattering is invalid for the zero or small doping due to the absence of a robust Fermi surface. Here, we demonstrate that the unconventional $T$ dependence of resistivity originates from the fact that the momentum space of the phonon scattering is strongly reduced at zero or small doping. For such cases, it is important to carry out the linear response calculation based on the Kubo formula, not in the Boltzmann equation. We will show that the interplay of such phonon scattering and the Dirac cone is crucial to comprehend the unconventional temperature dependence of the conductivity and resistivity of Dirac electrons.

We consider a two-dimensional electron-phonon ($e$-$p$) system given by

$$H = H_0 + H_{int} + H_{imp},$$

which is the Fröhlich Hamiltonian applied to the Dirac electron system. $H_0$ is expressed as

$$H_0 = \sum_k \sum_{\gamma = \pm} \epsilon_{k,\gamma} a_{k,\gamma}^\dagger a_{k,\gamma} + \sum_q \omega_q b_q^\dagger b_q,$$  \hspace{1cm} (2)

where $\epsilon_{k,\gamma} = \gamma v |k|$, $\omega_q = v_s q$, with $k = |k|$ and $q = |q|$, and $v \gg v_s$. The lattice constant is taken as unity. $\gamma = + (=-)$ denotes a conduction (valence) band with a Dirac cone. $a_{k,\gamma}$ and $b_q$ are annihilation operators of the electron with a wave vector $k$ of the $\gamma$ band and acoustic phonon with a vector $q$. The first term of Eq. (2) is obtained by diagonalizing a $2 \times 2$ isotropic model which is simplified compared with the effective model for the
Dirac electron in the organic conductor, but contains a minimum ingredient to comprehend the effect of phonon on the Dirac electron. The electron-phonon (e-p) interaction given by $H_{int}$ is expressed as

$$H_{int} = \sum_{k,\gamma} \sum_q \alpha_q a_{k+q,\gamma} a_{k,\gamma} \phi_q,$$  \hspace{1cm} (3)

where $\phi_q = b_q + b^\dagger_{-q}$. We introduce a coupling constant $\lambda = |\alpha_q|^2/\omega_q$ which becomes independent of $|q|$ for small $|q|$. The e-p scattering is considered in the same band (i.e., intraband) due to the energy conservation with $v \gg v_\gamma$. The third term of Eq. (1), $H_{imp}$, denotes a normal impurity scattering, which is introduced to avoid infinite conductivity in the presence of only the e-p interaction. We take $\hbar = 1$.

First, we note a relation between the chemical potential $\mu$ and $\mu_0$ where $n = \mu_0^2/2\pi v^2$ and the doping rate $n$ is calculated from the density of states (DOS), $D(\omega) = \sum_k \delta(\omega - \epsilon_k) = |\omega|/(2\pi v^2)$. Using $y = (\omega - \mu)/T$ and $\Theta(x) = 1$ for $x > 0$ and 0 otherwise, the self-consistency equation for $\mu$ is given by $\int_{0}^{\infty} d\omega D(\omega)[1/(e^{\omega y} + 1) - \Theta(\mu_0 - \omega)] = 0$, which is rewritten as $(\mu/\mu_0)^2 = 1 - B(\mu, \mu_0, T)$ with

$$B = 2T/\mu_0) \int_{0}^{\infty} d\omega \frac{y + \mu/T - |y - \mu/T|}{e^{\omega y} + 1}. \hspace{1cm} (4)$$

When the doping is zero ($\mu_0 = 0$), $\mu$ is determined only by $T/\mu_0$. The asymptotic form is given by $\mu/\mu_0 \simeq 1 - (\pi^2/6)(T/\mu_0)^2$ for $T/\mu_0 < 0.3$, and $\mu/\mu_0 \simeq (\mu_0/T)/(4 \ln 2)$ for $1 < T/\mu_0$. This form reproduces well the exact result within the visible scale except for the intermediate region, e.g., $\mu/\mu_0 = 0.760, 0.575$, and 0.444 for $T/\mu_0 = 0.4, 0.6$, and 0.8, respectively.

Using a self-energy of the electron Green’s function, the damping of an electron by a phonon is obtained as

$$\Sigma_\gamma(k, n) = T \sum_m \sum_q |\alpha_q|^2 \frac{1}{i\omega_{n+m} - \xi_{k+q,\gamma}} \frac{2\omega_q}{\omega_{n+m}^2 + \omega_q^2}, \hspace{1cm} (5)$$

which is a product of the electron and phonon Green’s functions. $\omega_n = (2n + 1)\pi T$, $\omega_m = 2\pi m T$, with $n$ and $m$ being integers. $\xi_{k,\gamma} = \epsilon_{k,\gamma} - \mu$, where $\mu$ is a chemical potential measured from the energy of the Dirac point. In Eq. 5, the Green’s function without interactions is used in the sense of the perturbational method. After the analytical continuation $i\omega_n \rightarrow \omega + i\delta$ with $\delta = +0$, the imaginary part is calculated as

$$\text{Im} \Sigma_\gamma(k, \omega + i\delta) = -\pi \sum_q |\alpha_q|^2 \times \{\delta(\omega + \omega_q - \xi_{k+q,\gamma}) \times [n_q + f(\xi_{k+q,\gamma})]
$$

$$+ \delta(\omega - \omega_q - \xi_{k+q,\gamma}) \times [n_q + 1 - f(\xi_{k+q,\gamma})]\}, \hspace{1cm} (6)$$

where $n_q = 1/(\exp[\omega_q/T] - 1)$ and $f(x) = 1/(e^{x/T} + 1)$. Note that $\Sigma_\gamma(k, \omega)$ reduces to zero at $T = 0$, since the phonon is absent and the excitation between the electron and hole becomes zero due to $n_q$ and $f(\xi_{k+q,\gamma})$, respectively.

The Green’s function with the damping of both the impurity and phonon scattering is given as

$$G_\gamma(k, i\omega_n)^{-1} = i\omega_n + \epsilon_{k,\gamma} - \mu + i\Gamma_\gamma, \hspace{1cm} (7)$$

where $\Gamma_\gamma = \Gamma_0 + \Gamma_{\gamma, ph}$ and $\Gamma_{\gamma, ph} = \Sigma_\gamma^0(k, \gamma v k - \mu)$. $\Gamma_0$, which is the damping by impurity scattering, is taken as a parameter to scale the energy. Note that Eq. (4) can be improved by introducing $\Gamma_0$ in Eq. (5) for the electron Green’s function. This gives the Lorentzian function instead of the $\delta$ function in Eq. (6) and the resultant reduction of $\Sigma_\gamma^0(k, \omega)$ is large (small) for $T < \Gamma_0$ ($T > \Gamma_0$).

Using a linear response theory and Eq. (7), the conductivity $\sigma_{xx}$ along the $x$ direction is calculated from

$$\sigma_{xx} = \sigma(T) = i \lim_{\omega \rightarrow 0} \frac{Q(\omega) - Q(0)}{\omega}, \hspace{1cm} (8)$$

$$Q(i\omega_m) = 2e^2 T \sum_n \sum_k \sum_{\gamma,\gamma'} |v_{\gamma,\gamma'}(k)|^2 \Pi_{\gamma,\gamma'}(n, m, k), \hspace{1cm} (9)$$

where $\Pi_{\gamma,\gamma'}(n, m, k) = G_{\gamma,\gamma'}(k, i\omega_{n+m})G_{\gamma,\gamma'}(k, i\omega_n)$, and $Q(\omega) = Q(i\omega_m)$ at $i\omega_m \rightarrow \omega + i\delta$. The factor 2 comes from the spin. The velocity matrices are given by $|v_{\gamma,\gamma'}(k)|^2 = v^2(k_{\gamma}^*/k)^2$ for $\gamma = \gamma'$ (intra-band) and $v^2(k_{\gamma}^*/k)^2$ for $\gamma = \gamma'$ (inter-band) where $k = (k_x, k_y)$.

From Eq. (6) with a vertex correction included in $\Gamma_{\pm}$, the conductivity normalized by $\sigma_{xx}^0 = e^2/2\pi^2$ per spin is given as follows,

$$\frac{\sigma(T)}{\sigma_{xx}^0} = \int_{-\infty}^{\infty} d\omega \left( \frac{\partial f(z)}{\partial z} \right) F(z), \hspace{1cm} (10)$$

$$F(z) = \int_{-\infty}^{\infty} d\eta \left( \frac{\Gamma_+/\Gamma_0}{(z/\Gamma_0 - \eta + \mu/\Gamma_0)^2 + (\Gamma_+/\Gamma_0)^2}ight) \frac{(\Gamma_-/\Gamma_0)^2}{(z/\Gamma_0 + \eta + \mu/\Gamma_0)^2 + (\Gamma_-/\Gamma_0)^2}, \hspace{1cm} (11)$$

$$\Gamma_{\pm}/\Gamma_0 - 1 = g_{\pm}(\mu/\Gamma_0, \tilde{v}_s, T/\Gamma_0, \eta) = \lambda K' h_{\pm}, \hspace{1cm} (12)$$

where $g_{\pm} = \Gamma_{\pm}/\Gamma_0, \eta = \xi/\Gamma_0, \xi = \nu k$ and $K = (\nu k/\Gamma_0)(2/\pi v)^2$. As typical values, we will take $\lambda = 0.031$ eV, $v_s/\nu = 0.05$, and $v \sim 0.05$ eV for organic conductors, which lead to $\lambda_0 K = 10^{-3}$ for $\Gamma_0 = 0.001$ eV. The dimensionless e-p coupling constant is given by $\lambda_0/(2\pi v)^2 \simeq 0.1$. Note that $g_{\pm}$ at high temperature of $T \gg v_s q$ is almost independent of $v_s$ for fixed $\lambda$. The
quantity $h_\pm$ is given by

$$h_\pm \equiv \int_0^{2/(1+\varepsilon_0)} d y \frac{y}{y^2 + 2y \cos \theta^0_\pm} \times (1 - \cos \theta^0_\pm) \left[ (1 - f^0_\pm \tilde{n} + f^0_1 (1 + \tilde{n}) \right]$$

$$+ \int_0^{2/(1+\varepsilon_0)} d y \frac{y}{y^2 + 2y \cos \theta^0_\pm} \times (1 - \cos \theta^0_\pm) \left[ (1 - f^0_\pm \tilde{n} + f^0_1 (1 + \tilde{n}) \right],$$

with $\tilde{n} = (e^{\varepsilon_0 y/T} - 1)^{-1}$, $y = q/k$, $\tilde{T} = T/\xi$, $\tilde{v}_s = v_s/\xi$, $\tilde{\mu} = \mu/\xi$, $f^0_1 = 1/(e^{(\pm 1 + \varepsilon_0 y - \tilde{\mu})/T} + 1)$, $f^0_2 = 1/(e^{(\pm 1 - \varepsilon_0 y - \tilde{\mu})/T} + 1)$, $\cos \theta^0_\pm = (\tilde{v}_s - 1)g/2\tilde{v}_s$, and $\theta^0_\pm > 0$. The condition of $q/k < 2/(1 \pm \tilde{\nu})$ in Eq. (13), which originates from the energy conservation with the Dirac cone in Eq. (10) gives $\nuq$ (not $\nuq$) bounded by $T$ in the calculation of $\sigma(T)$. Thus $g_\pm$ for small doping is reduced by a factor $\tilde{v}_s (\ll 1)$ compared with the conventional case with a robust Fermi surface. In Eq. (13), a factor $(1 - \cos \theta^0_\pm)$ has been introduced as a vertex correction, where $\theta^0_\pm = (1 + y \cos \theta^0_{\pm}) / \sqrt{1 + y^2 + 2y \cos \theta^0_{\pm}}$. Note that $\sigma$ is an even function with respect to $\mu$.

Using scaled quantities $T/\Gamma_0$ and $\mu/\Gamma_0$, we examine numerically the normalized conductivity $\sigma(T)$ of Eq. (10). Figure 1 shows the normalized damping $g = g_s$ as a function of $\eta = v_0 k/\Gamma_0$ for $(\mu/\Gamma_0, v_0/v, T/\Gamma_0) = (0, 0.05, 1)$ (I), $(0, 0.05, 10)$ (II), $(0, 0.1, 1)$ (III), and $(20, 0.05, 1)$ (IV), respectively. The inset denotes the schematic energy dispersion of Dirac cones for the conduction $(\epsilon_{k,1})$ and valence $(\epsilon_{k,2})$ bands, where $\Gamma_0$ denotes the chemical potential close to zero doping.

In Eq. (13) $\theta^0_\pm$ is calculated self-consistently for $\nu_0 = 0.05$. Equation (13) comes from the classical phonon (i.e., $v_0 q < T$) since $\nu_0(= v_0 k/\Gamma_0) < 1$ in Fig. 1 corresponds to $v_0 q/\Gamma_0 < v_0 k/\Gamma_0 < 1 = T/\Gamma_0$. The validity of Eq. (13) is shown in the following calculation of the conductivity. For $T < v_0 k$, the deviation of $g$ from the linear dependence of $\eta$ is seen but does not contribute to $\sigma$, which is obtained for $v_0 k \approx T$. For $\mu/\Gamma_0 \neq 0$, $g_s$ exhibits a dip around $\eta \approx \mu/\Gamma_0$ as shown by the line (IV). Such an effect becomes noticeable with a further increase of $\mu/\Gamma_0$, where the behavior of the quantum phonon is expected at low temperatures.

In Fig. 2 $\sigma(T)$ is examined for $\mu = 0$ (i.e., zero doping) and $v_0/v = 0.05$ with some choices of $\lambda$. The case of $\lambda = 0$ is also shown by the dashed line where $\sigma(T) \sim 2T/\Gamma_0$ for large $T/\Gamma_0$ is understood from $\sigma(T) \propto D(T)$ with $D(T) = T/(2\pi v^2)$. At low temperature, where Eq. (12) is smaller than 1, the dominant contribution in Eq. (11) comes from $\xi \sim \Gamma_0$ for small $T/\Gamma_0$ and $\xi \sim T$ for $T > \Gamma_0$ due to a factor $-\partial f(z)/\partial z$ in Eq. (11), leading to the increase of $\sigma(T)$. However, $g_s$ in Eq. (12) becomes larger than 1 for $T$ above a crossover temperature at which $\sigma(T)$ takes a maximum. At higher temperatures, the dominant contribution in Eq. (11) is obtained for $\xi/\Gamma_0 \approx 0.1K\eta^2 g_s$. Furthermore, the maximum of $\sigma(T)$ is large for small $\lambda/\lambda_0$. In fact, the former exhibits a maximum while the latter stays almost constant with increasing $T$.

In Fig. 3 $\sigma(T)$ is shown with several choices of $\mu_0/\Gamma_0$ where the corresponding $\mu$ is calculated self-consistently using Eq. (13). At $T = 0$, $\sigma(0)$ increases monotonically with increasing $\mu$ due to $D(\mu)$. For small $\mu_0/\Gamma_0$, $\sigma(T)$ takes a minimum and a maximum while $\sigma(T)$ decreases monotonically for large $\mu_0/\Gamma_0$. A minimum of $\sigma$ at lower temperatures for $\mu_0/\Gamma_0 = 1, 2, 4$, and 6 comes from the $T$ dependence of $\mu/\mu_0$ which decreases rapidly for $\mu_0/\Gamma_0 > 0.5$. Such a minimum due to the chemical potential was also shown for ballistic graphene. The maximum at higher temperatures is ascribed to the following competition. The conductivity at low temperatures is enhanced by a thermaly excited carrier due to the DOS.
of the Dirac cone. At high temperatures, the effect of the damping $\Gamma_{\text{ph}}$ by the phonon scattering becomes compatible with that of the DOS ($\propto T$) since the average of $\Gamma$ in $\sigma(T)$ gives $\langle \Gamma \rangle \propto T^2$ from Eq. (13). Thus the maximum of $\sigma(T)$ at small doping originates from the competition between the Dirac cone and the phonon. At high temperatures, the difference in $\sigma(T)$ between $\mu_0/\Gamma_0 = 0$ and $\mu_0/\Gamma_0 \neq 0$ becomes small, since the effect of $T$ becomes larger than that of $\mu$. The asymptotic behavior of $\sigma(T)$ for large $T(\gg \Gamma_0)$ is given by 2, corresponding to a universal conductance. In Fig. 3 we also show the dashed line which is obtained using a formula of Eq. (14). A good coincidence between two results suggests that the $e-p$ scattering is determined by the classical phonon not only for large $T/\Gamma_0$ but also for small $T/\Gamma_0$.

We mention behaviors for large doping. With increasing $\mu_0/\Gamma_0$, $\sigma(0)$ increases due to the increase of DOS, i.e., $D(\mu)$, and the maximum in $\sigma(T)$ disappears. In this case, the scattering at low temperatures is determined by the quantum phonon since the condition $\omega_q > T$ is allowed for $\mu_0/\Gamma_0 > 1$. The case of such doping is shown in the inset of Fig. 4 where the resistivity $\rho (= 1/\sigma)$ at low temperature is compared with that of the classical phonon calculated by Eq. (13) (dashed line). The $T$-linear dependence of the classical phonon is replaced by the $T^4$ dependence of the quantum phonon ($\mu_0/\Gamma_0 = 40$) which comes from the vertex correction with a factor $\Gamma^2(1 - \cos \theta)^2$ in Eq. (13). Such behavior for the large $\mu_0/\Gamma_0$ which comes from the reduction of $g_\eta$ in Eq. (13) at $\sqrt{\nu_k} \sim \mu/\Gamma_0$ [see the line (IV) in Fig. 4] leads to conventional phonon damping in the presence of a robust Fermi surface.

We estimate the maximum of $\sigma(T)$ obtained in Fig. 2 using Eq. (14) in which $\eta$ can be replaced by $\sim T/\Gamma_0$ from Eq. (11). Using $g = 0.02(T/\Gamma_0)^2$ for $\lambda/\lambda_0 = 1$ and $\sigma(T) \approx 1.1/(T/\Gamma_0)^2$ for the dashed line ($\lambda = 0$) except for small $T/\Gamma_0$ in Fig. 2 we obtain $\sigma(T)$ with $\mu = 0$ and $\Gamma_0 = 0.001$ as

$$
\sigma(T) \approx \frac{1.1 \times (T/\Gamma_0)}{1 + 0.02 \times (\lambda/\lambda_0)/(T/\Gamma_0)^2},
$$

which, within the visible scale, reproduces the line for $\lambda/\lambda_0 = 1$ and $T/\Gamma_0 > 4$ in Fig. 2. Note that $\Gamma_{\text{ph}} = g\Gamma_0(\sqrt{(\lambda/\lambda_0)^2} - 1)$ for the classical phonon is independent of $\Gamma_0$. Using Eq. (15), the conductivity for arbitrary $\Gamma_0$ is given by $\sigma(T) \approx 2.2 \times T/\Gamma_0 + \Gamma_{\text{ph}}$, where $\Gamma_{\text{ph}} = 20(\lambda/\lambda_0)T^2$. Thus, the maximum of $\sigma(T)$ is estimated as $\approx 7.7(\sqrt{(\lambda/\lambda_0)^2} \times 10^3$ at $T \approx 7 \times 10^4 \Gamma_0/(\lambda/\lambda_0)$.

Here, we discuss the $T$ dependence of $\mu$, which was used to analyze the experiment[22]. For simplicity, we calculate $\sigma(T)$ with a choice of $\mu/\Gamma_0 = 2 - T/\Gamma_0$, which is qualitatively similar to that taken to explain the $T$ dependence of the carrier density[13] and the Hall coefficient[14]. The result is shown by the dotted-dashed line in Fig. 3. With increasing $T$, the crossover from electron doping to hole doping occurs and the minimum of $\sigma(T)$ exists at lower temperature than $\mu(T) = 0$. Such $\mu(T)$ also exhibits a behavior that $\sigma(T)$ is almost $T$ independent at high temperatures.

Further, using Eq. (6) we estimate the real part of the self-energy, where $\Sigma_1(\k, \omega) = \Sigma_1 + i\Sigma_2$, $\Sigma_1 = \text{Re}\Sigma_1$, and $\Sigma_2 = \text{Im}\Sigma_1$. Using the Kramers-Kronig relation, $\Sigma_1$ is calculated from $\Sigma_2$ as

$$
\Sigma_1(\k, \omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{dz}{z - \omega} \Sigma_2(\k, z),
$$

Figure 2. (Color online) Normalized conductivity as a function of $T/\Gamma_0$ for $\lambda/\lambda_0 = 2$, 1, 0.5, and 0.25 with $\lambda_0 = 0.0625$ eV where the corresponding dimensionless coupling constant is given by $\lambda/(2\pi v) = 0.2$, 0.1, 0.05, and 0.025, respectively. The dashed line corresponds to $\lambda = 0$ (i.e., in the absence of $e-p$ scattering). The inset ($\lambda/\lambda_0 = 1$) denotes a behavior close to the maximum of $\sigma$, where the solid line corresponds to the main figure, the dotted line (A) is obtained without a vertex correction, the double-dotted-solid line (B) is calculated by substituting Eq. (14) into Eq. (12), and the double-dotted-solid line (C) is calculated using Eq. (15).

Figure 3. (Color online) Normalized conductivity (solid line) as a function of $T/\Gamma_0$ with $v_s/v = 0.05$ for $\mu_0/\Gamma_0 = 0$, 1, 2, 4, 6, and 10. The dashed line is obtained using Eq. (13), where the deviation from the solid line is invisible. The dotted-dashed line is shown as an example of $\sigma(T)$ when we assume that $\mu/\Gamma_0 = 2 - T/\Gamma_0$ (see details in the text). In the inset, the resistivity $\rho(T) = 1/\sigma(T)$ for $\mu_0/\Gamma_0 = 0$, 20, and 40 is compared with that of the classical approximation by Eq. (14) (dashed line).
with $\omega_0$ being a cutoff energy. Substituting Eq. (13) into Eq. (16) for the zero doping, we obtain $\Sigma_1(k, \omega) = b\omega$ with $b = (2C/\pi)( Tk / (\Gamma_0 \omega_0))$ and $C = 1.25 \times 10^{-2} \lambda / \lambda_0$ for $\omega / \omega_0 \ll 1$. Since $(i\omega_n \rightarrow \omega)$ in Eq. (7) is replaced by $(1 + b)\omega$ with $b \ll 1$, the real part $\Sigma_1$ can be neglected for small doping.

Finally, we compare the present resistivity $\rho(T)$ with that of the experimental one of the organic conductor showing almost $T$-independent $\rho(T)$, where $\rho(0) > \rho(T)$ at zero doping. There is a minimum $\rho(T_{\text{min}})/\rho(0) \approx 0.3$ at $T = T_{\text{min}} \approx 0.0007$. When $\Gamma_0 \approx 0.0002$ and $\lambda / \lambda_0 = 5$ (i.e., $\lambda / 2\pi v \approx 0.5$) are assumed, $\rho(0)/\rho(T_{\text{min}}) \approx 0.26$ and $T_{\text{min}} \approx 0.0014$, which agrees within a factor 2. We note tilting of the Dirac cone in the organic conductor, which is ignored in the first term of Eq. (2). The tilting gives anisotropic conductivity, but is irrelevant to the almost $T$-independent $\sigma(T)$, which is determined by the competition between the phonon and the DOS of the Dirac cone. Further, we comment on the effect of the short-range correlation, which gives rise to the fluctuation close to the boundary. Since the conductivity was measured at a pressure being much higher than that of the boundary, such a correlation effect is small and the phonon scattering becomes dominant at finite temperatures.

In summary, we examined the effect of phonon scattering on the $T$ dependence of $\sigma(T)$ for both zero ($\mu_0 = 0$) and finite ($\mu_0 \neq 0$) dopings. We found $\sigma(T)$ with almost constant behavior which arises from the competition between the enhancement by the Dirac cone and the suppression by the acoustic phonon. Such unconventional $T$ dependence comes from the fact that the momentum space of the phonon scattering is strongly reduced by the Dirac cone.

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