Model-independent analysis for determining mass splittings of heavy baryons

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Abstract

We study the hyperfine mass differences of heavy hadrons in the heavy quark effect theory (HQET). The effects of one-gluon exchange interaction are considered for the heavy mesons and baryons. Based on the known experimental data, we predict the masses of some heavy baryons in a model-independent way.

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I. INTRODUCTION

It is widely accepted that Quantum Chromodynamics (QCD) is the correct theory for strong interactions. QCD is a renormalizable quantum field theory which is closely modeled after quantum electrodynamics (QED), the most accurate physical theory we have to date. However, in the low-energy regime, QCD tells us that the interactions between quarks and gluons are strong, so that quark-gluon dynamics becomes non-perturbative in nature. Understanding the structures of hadrons directly from QCD remains an outstanding problem, and there is no indication that it will be solved in the foreseeable future. In 1989, it was realized that, in low energy situations where the typical gluon momenta are small compared with the heavy quark mass \( m_Q \), QCD dynamics becomes independent of the flavor and spin of the heavy quark \([1, 2]\). For the heavy flavors, this new symmetry called heavy quark symmetry (HQS). Of course, even in this infinite heavy quark mass limit, low energy QCD dynamics remains non-perturbative, and what HQS can do for us is to relate otherwise unrelated static and transition properties of heavy hadrons, and hence enormously reduces the complexity of theoretical analysis. In other words, HQS allows us to factorize the complicated light quark and gluon dynamics from that of the heavy one, and thus provides a clearer physical picture in the study of heavy quark physics. Beyond the symmetry limit, a heavy quark effective theory (HQET) can be developed by systematically expanding the QCD Lagrangian in powers of 1/\( m_Q \), with which HQS breaking effects can be studied order by order \([2, 3, 4]\).

In the experimental area, all masses of \( s \)-wave charmed hadrons and bottomed mesons which containing one heavy quark are found at present. However, except the particle \( \Lambda_0^b \) was already found in the early 1980’s, there has not been significant progress in searching \( s \)-wave bottomed baryons until these months. Recently some bottomed baryons were discovered at Fermilab. They are the exotic relatives of the proton and neutron \( \Sigma_b^{(s)+} \) and \( \Sigma_b^{(s)-} \) by CDF collaboration \([5]\) and the triple-scoop baryon \( \Xi_b^- \) by D0 and CDF collaborations \([6, 7]\). It is reasonable that the remainder particles, which include \( \Xi_b^{'}, \Xi_b^*, \Omega_b, \) and \( \Omega_b^* \), will be observed in the foreseeable future. All these heavy hadrons provide a testing ground for HQET with the phenomenological models to the low-energy regime of QCD. In this paper we focus on one static property, that is, the mass spectrum of heavy hadrons and combine HQET with the known experimental data to predict the mass splitting of some heavy baryons. The
phenomenological models are not needed here.

The paper is organized as follows. In Sec. II brief introductory notes are given for HQET. In Sec. III we formulate the hyperfine mass splitting for heavy mesons and baryons. In Sec. IV we evaluate the numerical results and predict some mass differences between heavy baryons. Finally, the conclusion is given in Sec. V.

II. HEAVY QUARK EFFECT THEORY

The full QCD Lagrangian for a heavy quark (c, b, or t) is given by

$$\mathcal{L}_Q = \bar{Q} \left( i\gamma_\mu D^\mu - m_Q \right) Q,$$

(1)

where $D^\mu \equiv \partial^\mu - ig_s T^a A^a$, with $T^a = \lambda^a/2$. Inside a hadronic bound state containing a heavy quark, the heavy quark $Q$ interacts with the light degrees of freedom by exchanging gluons with momenta of order $\Lambda_{QCD}$, which is much smaller than its mass $m_Q$. Consequently, the heavy quark is close to its mass shell, and its velocity does not deviate much from the hadron’s four-velocity $v$. In other words, the heavy quark’s momentum $p_Q$ is close to the “kinetic” momentum $m_Q v$ resulting from the hadron’s motion

$$p_Q^\mu = m_Q v^\mu + k^\mu,$$

(2)

where $k^\mu$ is the so-called “residual” momentum and is of order $\Lambda_{QCD}$. To describe the properties of such a system which contains a very heavy quark, it is appropriate to consider the limit $m_Q \to \infty$ with $v$ and $k$ being kept fixed. In this limit, it is evident that the quantity $m_Q v$ is “frozen out” from the QCD dynamics, so it is appropriate to introduce the “large” and “small” component fields $h_v$ and $H_v$, which are related to the original field $Q(x)$ by

$$h_v(x) = e^{im_Q v \cdot x} P_+ Q(x),$$

$$H_v(x) = e^{im_Q v \cdot x} P_- Q(x),$$

(3)

where $P_+$ and $P_-$ are the positive and negative energy projection operators respectively: $P_\pm = (1 \pm \gamma^0)/2$. so that

$$Q(x) = e^{-im_Q v \cdot x} \left[ h_v(x) + H_v(x) \right].$$

(4)

It is clear that $h_v$ annihilates a heavy quark with velocity $v$, while $H_v$ creates a heavy antiquark with velocity $v$. In the heavy hadron’s rest frame $v = (1, 0)$, $h_v(H_v)$ correspond to the upper (lower) two components of $Q(x)$. 

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In terms of the new fields, the QCD Lagrangian for a heavy quark given by Eq. (1) takes the following form

\[ \mathcal{L}_Q = \bar{h}_v i v \cdot D h_v - \bar{H}_v (i v \cdot D + 2 m_Q) H_v + \bar{h}_v i \not{n}_H H_v + \bar{H}_v i \not{n}_H h_v \]

(5)

where \( D_\perp^\mu = D^\mu - v^\mu v \cdot D \) is orthogonal to the heavy quark velocity, \( v \cdot D_\perp = 0 \). From Eq. (5), we see that \( h_v \) describes massless degrees of freedom, whereas \( H_v \) corresponds to fluctuations with twice the heavy quark mass. The heavy degrees of freedom represented by \( H_v \) can be eliminated using the equations of motion of QCD. Substituting Eq. (4) into \((i \not{n} - m_Q)Q(x) = 0\) gives

\[ i \not{n} h_v + (i \not{n} - 2 m_Q) H_v = 0. \]

(6)

Multiplying this equation by \( P_\pm \), one obtains

\[ - i v \cdot D h_v = i \not{n}_H H_v, \]

(7)

\[ (i v \cdot D + 2 m_Q) H_v = i \not{n}_H h_v. \]

(8)

The second equation can be solved schematically to give

\[ H_v = \frac{1}{(i v \cdot D + 2 m_Q - i \epsilon)} i \not{n}_H h_v, \]

(9)

which shows that the small component field \( H_v \) is indeed of order \( 1/m_Q \). One can insert this solution back into Eq. (7) to obtain the equation of motion for \( h_v \). It is easy to check that the resulting equation follows from the effective Lagrangian

\[ \mathcal{L}_{Q,\text{eff}} = \bar{h}_v i v \cdot D h_v + \bar{h}_v i \not{n}_H \frac{1}{(i v \cdot D + 2 m_Q - i \epsilon)} i \not{n}_H h_v. \]

(10)

\( \mathcal{L}_{Q,\text{eff}} \) is the Lagrangian of the heavy quark effective theory (HQET), and the second term of Eq. (10) allows for a systematic expansion in terms of \( iD/m_Q \). Taking into account that \( P_+ h_v = h_v \), and using the identity

\[ P_+ i \not{n}_H i \not{n}_H P_+ = P_+ \left[ (i D_\perp)^2 + \frac{g_s}{2} \sigma_{\alpha \beta} G^{\alpha \beta} \right] P_+, \]

(11)

where

\[ G^{\alpha \beta} = T_a G_a^{\alpha \beta} = \frac{i}{g_s} [D^\alpha, D^\beta] \]

(12)
is the gluon field strength tensor, one finds that

$$L_{Q, \text{eff}} = \bar{h}_v iv \cdot Dh_v + \frac{1}{2m_Q} \bar{h}_v (iD_\perp)^2 h_v + \frac{g}{4m_Q} \bar{h}_v \sigma_{\alpha\beta} G^{\alpha\beta} h_v + \mathcal{O}(\frac{1}{m_Q^2}).$$  \hspace{1cm} (13)

The new operators at order $1/m_Q$ are

$$\mathcal{O}_1 = \frac{1}{2m_Q} \bar{h}_v (iD_\perp)^2 h_v,$$  \hspace{1cm} (14)

$$\mathcal{O}_2 = \frac{g}{4m_Q} \bar{h}_v \sigma^{\mu\nu} G_{\mu\nu} h_v,$$  \hspace{1cm} (15)

where $\mathcal{O}_1$ is the gauge invariant extension of the kinetic energy arising from the off-shell residual motion of the heavy quark, and $\mathcal{O}_2$ describes the color magnetic interaction of the heavy quark spin with the gluon field. It is clear that both $\mathcal{O}_1$ and $\mathcal{O}_2$ break the flavor symmetry, while $\mathcal{O}_2$ breaks the spin symmetry as well. For instance, $\mathcal{O}_1$ would introduce a common shift to the masses of pseudoscalar and vector heavy mesons, and $\mathcal{O}_2$ is responsible for the color hyperfine mass splittings $\delta m_{HF}$.

The full expansion of $L_{Q, \text{eff}}$ in $iD/m_Q$ can be organized as follow

$$L_{Q, \text{eff}} = \sum_{n=0} \left( \frac{1}{2m_Q} \right)^n \mathcal{L}_n$$  \hspace{1cm} (16)

where the first few terms are given by

$$\mathcal{L}_0 = \bar{h}_v (iv \cdot D) h_v,$$

$$\mathcal{L}_1 = \bar{h}_v (iD_\perp)^2 h_v + \frac{g}{2} \bar{h}_v \sigma_{\alpha\beta} G^{\alpha\beta} h_v,$$

$$\mathcal{L}_2 = g\bar{h}_v \sigma_{\alpha\beta} v_\gamma iD^\alpha G^{\beta\gamma} h_v + g\bar{h}_v v_\alpha iD_\beta G^{\alpha\beta} h_v.$$

We must emphasize that this effective theory comes from first principle directly, and it is in terms of the power of $1/m_Q$, which is small enough, to calculate the physical quantities which concerning heavy quarks perturbatively, i.e., order by order. If one chooses the appropriate frame and phenomenological model, then one can handle many physical processes systematically. We also note that, as shown in Eq. (16), the information of heavy quark flavor is only involved in the factor $(1/2m_Q)^n$. In other words, all $\mathcal{L}_n$’s are independent of the heavy quark flavor. We will use this property to evaluate the mass splittings of some heavy hadrons.
III. HYPERFINE MASS SPLITTING

First, we consider the hyperfine mass splitting between pseudoscalar and vector heavy mesons. The operators that break HQS to order $1/m_Q$ are $O_1$ and $O_2$ given in Eqs. (14) and (15) respectively. $O_1$ can be separated into a kinetic energy piece and a one-gluon exchange piece:

$$O_1 = O_{1k} + O_{1g}$$  \hspace{1cm} (17)

where

$$O_{1k} = \frac{-1}{2m_Q} \bar{h}_v [\partial_\mu \partial^\mu + (v \cdot \partial)^2] h_v,$$  \hspace{1cm} (18)

$$O_{1g} = \frac{-g_s}{2m_Q} \bar{h}_v [(p + p')_\mu - v \cdot (p + p') v_\mu] A^\mu h_v,$$  \hspace{1cm} (19)

Also, $O_2$ can be reexpressed as

$$O_2 = -g_s T^a \sigma_{\mu \nu} A^{a \mu}.$$

(20)

With the $1/m_Q$ corrections included, the heavy meson masses can be expressed as

$$M_M = m_Q + \bar{\Lambda}^q - \frac{1}{2m_Q} (\lambda_1^q + d_M \lambda_2^q),$$

(21)

where $\lambda_1^q$ comes from $O_1$ and $\lambda_2^q$ comes from $O_2$. $\lambda_1^q$ receive two different contributions, one from $O_{1k}$ and the other from $O_{1g}$, thus

$$\lambda_1^q = \lambda_{1k}^q + \lambda_{1g}^q.$$  \hspace{1cm} (22)

The parameter $\bar{\Lambda}^q$ in Eq. (21) is the residual mass of heavy mesons in the heavy quark limit. In other words, $\bar{\Lambda}^q$ is independent of heavy quark flavor. $\lambda_{1k}^q$ comes from the heavy quark kinetic energy, $\lambda_{1g}^q$ and $\lambda_2^q$ are respectively the chromoelectric and chromomagnetic contributions. $\lambda_1^q$ parameterizes the common mass shift for the pseudoscalar and vector mesons, and $\lambda_2^q$ accounts for the hyperfine mass splitting. In both the non-relativistic and relativistic quark models, the hyperfine mass splitting comes from a spin-spin interaction of the form

$$H_{HF} \sim \vec{S}_Q \cdot \vec{J}_l.$$  \hspace{1cm} (23)
TABLE I: The $s$-wave heavy baryons and their quantum number, where the subscript $l$ stands for the quantum number of the two light quarks.

| state | $\Lambda_Q$ | $\Sigma_Q$ | $\Sigma^*_Q$ | $\Xi_Q$ | $\Xi'_Q$ | $\Xi^*_Q$ | $\Omega_Q$ | $\Omega^*_Q$ |
|-------|-------------|-------------|--------------|--------|--------|--------|--------|--------|
| $J^P$ | $\frac{1}{2}^+$ | $\frac{1}{2}^+$ | $\frac{3}{2}^+$ | $\frac{1}{2}^+$ | $\frac{1}{2}^+$ | $\frac{3}{2}^+$ | $\frac{1}{2}^+$ | $\frac{3}{2}^+$ |
| $J_l$ | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |

where $\vec{S}_Q$ is the spin operator of the heavy quark and $\vec{J}_l$ is the angular momentum operator of the light degree of freedom. Thus

$$d_M = -\langle M(v)|4\vec{S}_Q \cdot \vec{J}_l|M(v)\rangle$$

$$= -2[S_M(S_M + 1) - S_Q(S_Q + 1) - J_l(J_l + 1)], \quad (24)$$

where $S_M$ is spin quantum number of the meson $M$. Consequently, $d_M = -1$ for a vector meson, and $d_M = 3$ for a pseudoscalar meson. Therefore, we obtain the hyperfine mass splitting,

$$\Delta M_{VP} \equiv M_V - M_P = \frac{2\lambda^q_2}{m_Q}. \quad (25)$$

We next consider the hyperfine mass splitting among the baryons containing one heavy quark ($Q$) and two light quarks ($q_1, q_2$). Each light quark is in a triplet $q = (u, d, s)$ of the flavor $SU(3)$. Since $3 \otimes 3 = 6 \oplus 3$ and the lowest lying light quark state has $n = 1$ and $L = 0$ ($S$-wave), there are two different diquarks: a symmetric sextet ($\vec{J}_l = 1$) and an antisymmetric antitriplet ($\vec{J}_l = 0$). When the diquark combines with a heavy quark, the sextet contains both spin-$\frac{1}{2}$ ($\mathcal{B}_6$) and spin-$\frac{3}{2}$ ($\mathcal{B}^*_6$) baryons, and the antitriplet contains only spin-$\frac{1}{2}$ ($\mathcal{B}_3$) baryons. The multiplets $\mathcal{B}_3$ and $\mathcal{B}^*_6$ are illustrated in Fig. 1 (a) and (b), respectively, and their quantum numbers are listed in TABLE I.

By analogy with Eq. (21), the heavy baryon masses can be expressed as

$$M_B = m_Q + \bar{\Lambda}^{q_1q_2}_{J_l} - \frac{1}{2m_Q}(\lambda_1^{q_1q_2} + d_B\lambda_2^{q_1q_2}), \quad (26)$$

where $\bar{\Lambda}^{q_1q_2}_{J_l}$ is the residual mass of heavy baryons in the heavy quark limit. The proportions of $\lambda_1^{q_1q_2}$ (for meson) to $\lambda_1^{q_1q_2}$ (for baryon) are

$$\lambda_1^q \sim \lambda_1^{q_1q_2},$$

$$\lambda_2^q \sim N_c\lambda_2^{q_1q_2}, \quad (27)$$
where $N_c$ is the color number. Thus

$$d_B = -\langle B(v)|4(\bar{S}_Q \cdot \vec{J}_l)|B(v)\rangle$$

$$= -2[S_B(S_B + 1) - S_Q(S_Q + 1) - J_l(J_l + 1)], \quad (28)$$

where $S_B$ is spin quantum number of the baryon $B$. Consequently, $d_B = 0$ for a $B_3$ baryon, $d_B = 4$ for a $B_6$ baryon, and $d_B = -2$ for a $B'_6$ baryon. Therefore, the hyperfine mass splittings of $\Lambda_Q$, $\Sigma_Q$, and $\Sigma^*_Q$ are

$$\Delta M_{\Sigma^*_Q \Sigma_Q} = \frac{3\lambda_{q\bar{q}_2}}{m_Q}, \quad (29)$$

$$\Delta M_{\Sigma_Q \Lambda_Q} = \frac{-2\lambda_{q\bar{q}_2}}{m_Q} + \delta\bar{\Lambda}q^2, \quad (30)$$

$$\Delta M_{\Sigma^*_Q \Lambda_Q} = \frac{\lambda_{q\bar{q}_2}}{m_Q} + \delta\bar{\Lambda}q^2, \quad (31)$$

where $q$ is the $u$ or $d$ quark and $\delta\bar{\Lambda}q^2 = \bar{\Lambda}_1q^2 - \bar{\Lambda}_0q^2$. For the $\Xi_Q$ baryons, however, the complexity is increased because of the flavor $SU(3)$ symmetry breaking. We consider the flavor $SU(3)$ symmetry breaking and write down the hyperfine mass splittings of $\Xi_Q$, $\Xi'_Q$, and $\Xi^*_Q$ as

$$\Delta M_{\Xi'_Q \Xi_Q} = \frac{3\lambda_{q\bar{q}}}{m_Q}, \quad (32)$$

$$\Delta M_{\Xi^*_Q \Xi_Q} = \frac{-2\lambda_{q\bar{q}}}{m_Q} + \delta\bar{\Lambda}q^2, \quad (33)$$

$$\Delta M_{\Xi^*_Q \Xi_Q} = \frac{\lambda_{q\bar{q}}}{m_Q} + \delta\bar{\Lambda}q^2. \quad (34)$$

It is worth to mention that, from Eqs. (30), (31), (33), and (34), $\delta\bar{\Lambda}q^2$ are

$$\delta\bar{\Lambda}q^2 = \frac{M_{\Sigma_Q} + 2M_{\Sigma^*_Q}}{3} - M_{\Lambda_Q}, \quad (35)$$

$$\delta\bar{\Lambda}q^2 = \frac{M_{\Xi_Q} + 2M_{\Xi^*_Q}}{3} - M_{\Xi_Q}. \quad (36)$$

Finally, the hyperfine mass splitting of $\Omega_Q$ and $\Omega^*_Q$ is

$$\Delta M_{\Omega^*_Q \Omega_Q} = \frac{3\lambda_{ss}}{m_Q}. \quad (37)$$

We can use the hyperfine mass differences which are experimentally known for charmed baryons to calculate the ones for bottomed baryons.
IV. NUMERICAL RESULTS

Now we consider the numerical results of the hyperfine mass splitting for heavy mesons. As mentioned above, \( \lambda_{2q} \) just relates to the light degrees of freedom and independent of the heavy quark mass \( m_Q \). Thus, from Eq. (25), we obtain

\[
\frac{\Delta M_{B^*B}}{\Delta M_{D^*D}} = \frac{m_c}{m_b} = \frac{\Delta M_{B^*B_s}}{\Delta M_{D^*_sD_s}},
\]

(38)

whatever the values of light quark masses \((m_u, m_d, m_s)\) and other parameters appearing in any phenomenological model are. Experimentally, the ratio of hyperfine mass splitting is given by [9]

\[
\left\| \frac{\Delta M_{B^*B}}{\Delta M_{D^*D}} \right\|_{\text{expt}} = 45.78 \pm 0.35 \quad 141.38 \pm 0.12 = 0.3238 \pm 0.0028,
\]

(39)

\[
\left\| \frac{\Delta M_{B^*_sB_s}}{\Delta M_{D^*_sD_s}} \right\|_{\text{expt}} = 46.1 \pm 1.5 \quad 143.9 \pm 0.4 = 0.3204 \pm 0.0113,
\]

(40)

where we take the masses of \( D^* \) and \( D \) mesons to the average ones of their charged and neutral mesons. This agreement is not only a triumph of HQET, but also reveals that the \( 1/m_Q \) corrections are enough here. In addition, from the experimental data shown in Eqs. (39) and (40), we also find that

\[
\frac{\lambda_{2s} - \lambda_{2\bar{q}}}{\lambda_{2s} + \lambda_{2\bar{q}}} \bigg|_{D_sD} = (0.88 \pm 0.15) \%
\]

(41)

\[
\frac{\lambda_{2s} - \lambda_{2\bar{q}}}{\lambda_{2s} + \lambda_{2\bar{q}}} \bigg|_{B_sB} = (0.35 \pm 1.67) \%
\]

(42)

This means the \( SU(3) \) breaking effect of the hyperfine mass splitting in heavy mesons is very small.

Next we consider the numerical results of mass difference between the heavy baryons \( B^*_6 \) and \( B_6 \). From Eq. (29) and the ratio in Eq. (39), we predict

\[
\Delta M_{\Sigma^*_c \Sigma^*_b} = \frac{m_c}{m_b} \Delta M_{\Sigma^*_c \Sigma^*_s}^{\text{expt}} = 20.9 \pm 1.0 \text{ MeV},
\]

(43)

where

\[
\Delta M_{\Sigma^*_c \Sigma^*_s}^{\text{expt}} = 64.4 \pm 2.4 \text{ MeV}
\]

(44)
The result in Eq. (43) is in agreement with the experimental data [5]: \(\Delta M_{\Sigma_b^0 \Sigma_c^0}^{\text{expt}} = 21.5 \pm 2.0\) MeV. This provides a strong vote of confidence for the predictions of the other hyperfine mass differences. From Eq. (32) and the ratio in Eq. (39), we predict

\[
\Delta M_{\Xi_b^* \Xi_c^*} = \frac{m_c}{m_b} \Delta M_{\Xi_c^* \Xi_c^0}^{\text{expt}} = 22.5 \pm 1.3\text{ MeV},
\]

where

\[
\Delta M_{\Xi_c^* \Xi_c^0}^{\text{expt}} = 69.5 \pm 3.3\text{ MeV}
\]

From Eq. (37) and the ratio in Eq. (39), we also predict

\[
\Delta M_{\Omega_b^* \Omega_b} = \frac{m_c}{m_b} \Delta M_{\Omega_c^* \Omega_c}^{\text{expt}} = 22.9 \pm 0.7\text{ MeV},
\]

where the value

\[
\Delta M_{\Omega_c^* \Omega_c}^{\text{expt}} = 70.8 \pm 1.5\text{ MeV}
\]

is taken from Ref. [9]. In addition, from Eqs. (44), (46), and (48), we obtain

\[
\frac{\lambda_2^{\bar{q}q} - \lambda_2^{\bar{q}2}}{\lambda_2^{\bar{q}q} + \lambda_2^{\bar{q}2}} = (3.8 \pm 3.1)\%
\]

\[
\frac{\lambda_2^{s-s} - \lambda_2^{s\bar{q}}}{\lambda_2^{s-s} + \lambda_2^{s\bar{q}}} = (0.9 \pm 2.6)\%
\]

These results reveal that the flavor \(SU(3)\) breaking effect of the hyperfine mass splitting is very small in heavy baryons, as well as in heavy mesons.

Finally we consider the numerical results of mass difference which is related to the heavy baryons \(B_3\). Combining the experimental values [3, 9, 10] and the theoretical evaluation of \(m_{\Sigma_0^b} [11]\), we have

\[
\Delta M_{\Sigma_c^+ \Lambda_c^+}^{\text{expt}} = 166.4 \pm 0.4\text{ MeV},
\]

\[
\Delta M_{\Sigma_b^0 \Lambda_b^0} = 191.8 \pm 2.0\text{ MeV}.
\]

Then we get from Eq. (35)

\[
\delta\Lambda_{\Sigma_c^+ \Lambda_c^+} = 209.3 \pm 1.6\text{ MeV}
\]

\[
\delta\Lambda_{\Sigma_b^0 \Lambda_b} = 206.1 \pm 2.2\text{ MeV}.
\]
and

\[
\frac{\delta \tilde{\Lambda}_{\Sigma c}^\Delta - \delta \tilde{\Lambda}_{\Sigma c}^\Sigma}{\delta \tilde{\Lambda}_{\Sigma c}^\Delta + \delta \tilde{\Lambda}_{\Sigma c}^\Sigma} = (0.77 \pm 0.65) \% \tag{51}
\]

This result reveals that, as mentioned above, \(\delta \tilde{\Lambda}_{\Sigma c}^\Delta\) is just related to the light degrees of freedom and independent of the heavy quark flavors. Now we use the above argument to evaluate \(\delta \tilde{\Lambda}_{\Sigma c}^\Delta\). From the data \(\Delta M_{\Xi_c^0 \Xi_c^-}^{\text{exp}} = 176.9 \pm 0.9 \text{ MeV}\), we obtain

\[
\delta \tilde{\Lambda}_{\Sigma c}^\Delta = 154.4^{+3.8}_{-1.6} \text{ MeV for } \Xi_c^0 \Xi_c^- \text{ system.} \tag{52}
\]

This result can be used in the bottomed sector due to it is also independent of the heavy flavor. Thus, we predict

\[
\Delta M_{\Xi_b^0 \Xi_b^-} = 139.8^{+3.8}_{-2.0} \text{ MeV, } \quad \Delta M_{\Xi_b^0 \Xi_b^-} = 161.7^{+3.8}_{-2.0} \text{ MeV.}
\]

Combine the data \(M_{\Xi_b^0}^{\text{exp}} = 5792.9 \pm 3.0 \text{ MeV} \tag{7}\), we have

\[
M_{\Xi_b^0} = 5932.7 \pm 4.2 \text{ MeV, } \quad M_{\Xi_b^0} = 5954.6 \pm 4.2 \text{ MeV.} \tag{53}
\]

Furthermore, we may use the Gell-Mann/Okubo formula to obtain the equal mass difference equations

\[
M_{\Xi^{+}_c} - M_{\Sigma^{+}_c} = M_{\Omega^{+}_c} - M_{\Xi^{+}_c},
\]

\[
M_{\Xi^{-}_c} - M_{\Sigma^{-}_c} = M_{\Omega^{-}_c} - M_{\Xi^{-}_c}. \tag{54}
\]

The accuracy of Eq. (54) can be checked in charmed sector, the experimental data give

\[
M_{\Xi^{+}_c} - M_{\Sigma^{+}_c} = 123.3 \pm 2.1 \text{ MeV, } \quad M_{\Omega^{+}_c} - M_{\Xi^{+}_c} = 120.6 \pm 3.3 \text{ MeV,}
\]

\[
M_{\Xi^{-}_c} - M_{\Sigma^{-}_c} = 128.4 \pm 1.2 \text{ MeV, } \quad M_{\Omega^{-}_c} - M_{\Xi^{-}_c} = 121.9 \pm 3.1 \text{ MeV},
\]

where the values of \(M_{\Xi^{+}_c}\) and \(M_{\Sigma^{+}_c}\) are taken from the average masses in charged and neutral cases. Therefore, we have the confidence to use Eq. (54) in bottomed sector. From Eqs. (47) and (53), we predict the masses of \(\Omega_b\) and \(\Omega_b^*\)

\[
M_{\Omega_b} = 6053.9 \pm 8.9 \text{ MeV, } \quad M_{\Omega_b^*} = 6076.5 \pm 9.0 \text{ MeV.} \tag{55}
\]

We summarize the predictions of this work and list the other theoretical calculations and the experimental data in TABLE II.
TABLE II: Experimental data, the predictions of this work and the other theoretical calculation (in units of MeV).

|                  | Experiment | This work | [8] | [12] | [13] | [14] |
|------------------|------------|-----------|-----|------|------|------|
| $\Delta M_{\Sigma^c \Sigma_c}$ | 64.4 ± 2.4 | input     | 79.6 ± 5.3 | 86 ± 22 |
| $\Delta M_{\Xi^c \Xi_c}$    | 176.9 ± 0.9 | input     |          |       |      |
| $\Delta M_{\Xi^c \Xi'_c}$   | 69.5 ± 3.3  | input     | 61.1 ± 3.0 | 81 ± 19 |
| $\Delta M_{\Omega^c \Omega_c}$ | 70.8 ± 1.5 | input     | 42.6 ± 7.3 | 74 ± 16 |
| $\Delta M_{\Sigma^*_b \Sigma_b}$ | 21.5 ± 2.0 | 20.9 ± 1.0 | 23.8 ± 1.6 | $24_{-12}^{+13}$ |
| $\Delta M_{\Xi^*_b \Xi_b}$   | 161.7$^{+3.8}_{-2.0}$ | input     |          |       |      |
| $\Delta M_{\Xi^*_b \Xi'_b}$  | 21.9 ± 1.1  | 18.3 ± 0.9 | $23_{-12}^{+13}$ | | |
| $\Delta M_{\Omega^*_b \Omega_b}$ | 139.8$^{+3.8}_{-2.0}$ | 148$^{+35}_{-29}$ | | | |
| $\Delta M_{\Xi^*_b \Xi'_b}$  | 22.9 ± 0.7  | 12.8 ± 2.2 | 20 ± 9 | | |
| $M_{\Xi_b}$         | 5792.9 ± 3.0 | input     | 5812 | 5786.7 ± 3.0 |
| $M_{\Xi'_b}$        | 5932.7 ± 4.2 |          | 5937 |       |      |
| $M_{\Xi^*_b}$       | 5954.6 ± 4.2 |          | 5963 |       |      |
| $M_{\Omega_b}$      | 6053.9 ± 8.9 |          | 6065 | 6052.1 ± 5.6 |
| $M_{\Omega^*_b}$    | 6076.5 ± 9.0 |          | 6088 | 6082.8 ± 5.6 |

V. CONCLUSION

In this paper, based on HQET, we have presented a formalism to describe the hyperfine mass splittings of the heavy baryons. Furthermore, through the known experimental data in charmed sector, we predicted the hyperfine mass differences in bottomed sector. The parameters appearing in this analysis are the ratio $m_c/m_b$ and the residual mass of heavy baryons in the heavy quark limit $\Lambda^q_{c\Omega}$. On the one hand the ratio $m_c/m_b$ is fixed by the experimental values of heavy mesons, and on the other hand the residual mass difference $\delta \Lambda^q_{c\Omega}$, due to it is independent of heavy flavor, is obtained by the known mass differences of charmed baryons. The prediction of $\Delta M_{\Sigma^*_b \Sigma_b}$ is in agreement with the experimental values, we expect the deviations of the other predictive mass differences are all small for the future experimental data. In addition, in both heavy meson and baryon systems, we find that the flavor $SU(3)$ breaking effect of the hyperfine mass splitting is very small. Finally we also
estimated the masses of $\Xi_b^*$ and $\Xi_b^*$ and used the Gell-Mann/Okubo formula to calculate the masses of $\Omega_b$ and $\Omega_b^*$. The uncertainties of these four heavy baryon masses mainly come from the error of the measured value $M_{\Xi_b}^{\text{expt}}$. To get the more confidence in HQET, the more precise experimental data are needed.

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FIG. 1: The multiplets (a) $B_3$ and (b) $B_6^{(s)}$. 