ON DETERMINING THE HOMOLOGICAL CONLEY INDEX OF POINCARÉ MAPS IN AUTONOMOUS SYSTEMS

ROMAN SRZEDNICKI

Dedicated to the memory of Professor Andrzej Granas

ABSTRACT. A theorem on computation of the homological Conley index of an isolated invariant set of the Poincaré map associated to a section in a rotating local dynamical system $\phi$ is proved. Let $(N,L)$ be an index pair for a discretization $\phi^h$ of $\phi$, where $h > 0$, and let $S$ denote the invariant part of $N \setminus L$; it follows that the section $S_0$ of $S$ is an isolated invariant set of the Poincaré map. The theorem asserts that if the sections $N_0$ of $N$ and $L_0$ of $L$ are ANRs, the homology classes $[u_j]$ of some cycles $u_j$ form a basis of $H(N_0,L_0)$, and for some scalars $a_{ij}$, the cycles $u_j$ and $\sum a_{ij} u_i$ are homologous in the covering pair $(\tilde{N},\tilde{L})$ of $(N,L)$ and the homology relation is preserved in $(\tilde{N},\tilde{L})$ under the transformation induced by $\phi^t$ for $t \in [0,h]$ then the homological Conley index of $S_0$ is equal to the Leray reduction of the matrix $[a_{ij}]$. In particular, no information on the values of the Poincaré map or its approximations is required. In a special case of the system generated by a $T$-periodic non-autonomous ordinary differential equation with rational $T/h > 1$, the theorem was proved in the paper M. Mrozek, R. Srzednicki, and F. Weilandt, SIAM J. Appl. Dyn. Syst. 14 (2015), 1348–1386, and it motivated a construction of an algorithm for determining the index.

2020 Mathematics Subject Classification. Primary: 37B30; Secondary: 37B35.

Key words and phrases. Poincaré map; isolated invariant set; index pair; Conley index.

This research is partially supported by the Polish National Science Center under Grant No. 2014/14/A/ST1/00453.