Critical Zeeman Splitting of Fermi Superfluidity at Infinite Scattering Length

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We determine the critical Zeeman energy splitting for Fermi superfluidity at infinite s-wave scattering length according to the Monte Carlo and experimental results of the equations of state. Based on the universality hypothesis, we show that there exist two critical fields \( H_{c1} \) and \( H_{c2} \), between which a superfluid-normal mixed phase is energetically favored, and model-independent formulae for \( H_{c1} \), \( H_{c2} \) and the critical population imbalance \( P_c \) are derived. Using recent Monte Carlo and experimental results of \( P_c \), \( H_{c1} \) and \( H_{c2} \) are determined. It is found \( H_{c1} = 0.41\epsilon_F \) and \( H_{c2} = 0.50\epsilon_F \), with \( \epsilon_F \) being the Fermi energy of non-interacting gas.

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While Bardeen-Cooper-Schiffer (BCS) superconductivity/superfluidity in Fermi systems has been investigated many years ago, the main scientific interest in recent experiments of two-component ultracold Fermi gas is to create Fermi superfluids in the BCS–Bose-Einstein condensation (BEC) crossover. At a wide Feshbach resonance point where the s-wave scattering length \( a_s \) diverges, a novel type of Fermi superfluid has been observed. In the dilute gas limit, there exists only a single distance \( n^{-1/3} \) which reflects the inter-particle distance \( n \). In this so-called unitary Fermi gas, any physical quantity can be expressed in terms of its value in the non-interacting case multiplied by a universal constant \( \xi \). For example, the energy density can be written as \( E = \xi E_0 \), where \( \xi \) is a universal constant and \( E_0 \) the energy density of non-interacting Fermi gas.

In addition to the idealized case where fermion pairing happens on a uniform Fermi surface, the effect of Zeeman energy splitting \( E_Z = \mu B \) between spin-up and -down electrons on BCS superconductivity was known many years ago (in the following we absorb the magnetic moment \( \mu \) in to the definition of “magnetic field” \( H \)). At a critical Zeeman field or the so-called Chandrasekhar-Clogston (CC) limit \( H_c = 0.7072\Delta_0 \) where \( \Delta_0 \) is the zero temperature gap, a first order phase transition from the gapped BCS state to the normal state occurs. Further theoretical studies showed that the inhomogeneous Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state may survive in a narrow window between \( H_c \) and \( H_{FFLO} = 0.754\Delta_0 \). However, since the thermodynamic critical field is much smaller than the CC limit due to strong orbit effect, it is hard to observe the CC limit and the FFLO state in ordinary superconductors.

Recent experiments for strongly interacting ultracold Fermi gases give an alternative way to study the pure Zeeman effect on Fermi superfluidity [3]. The atom numbers of the two lowest hyperfine states of \(^6\)Li atom, denoted by \( N_\uparrow \) and \( N_\downarrow \), are adjusted to create a population imbalance or polarization \( P = (N_\uparrow - N_\downarrow)/(N_\uparrow + N_\downarrow) \), which simulates effectively the Zeeman field \( H \) in a superconductor. At unitary, phase separation between unpolarized superfluid and polarized normal gas, predicted by early theoretical works [9, 10] and Monte Carlo (MC) simulations [11], is observed, but the evidence for the FFLO and the breached pairing [7, 8] states is lacking.

An important scientific problem is to determine the ratio between the critical Zeeman energy splitting and the Fermi energy for a homogeneous unitary Fermi gas, \( H_c/\epsilon_F \), which should be a universal constant at unitary. In an early work [11], the CC limit at unitary was predicted to be

\[
\frac{H_c}{\Delta(\mu)} = \frac{1}{\beta} \left( \frac{2^{2/5}}{\xi^{3/5}} - 1 \right),
\]

where \( \beta = \Delta(\mu)/\mu \) with \( \mu \) being the fermion chemical potential is another universal constant at \( H = 0 \). With the MC data of \( \xi \) and \( \beta \), \( H_c/\Delta \simeq 1 \) is found. However, this result denotes only the first order superfluid-normal phase transition point in the grand canonical ensemble with the chemical potential \( \mu \) fixed, and hence not the wanted result \( H_c/\epsilon_F \). On the other hand, this result is obtained assuming the normal phase is fully polarized, but recent MC work [12] and experiment [13] show that the normal phase at the phase transition is partially polarized. In addition, in Ref. [14, 15] it is shown that there exist two critical fields \( H_{c1} \) and \( H_{c2} \) in the BCS-BEC crossover and phase separation is the energetically favored ground state in the region of \( H_{c1} < H < H_{c2} \), like the type-II superconductors. Since the particle numbers \( N_\uparrow \) and \( N_\downarrow \) are used as tunable parameters in MC calculations and experiments, only the first order phase transition point \( (H/\mu)_c \) and the critical spin population \( P_c \) are directly determined, and the two CC limits \( H_{c1}, H_{c2} \) for homogeneous Fermi gas has not yet been measured in MC calculations [12] and experiments [13].

In this paper, we will determine the two CC limits \( H_{c1} \) and \( H_{c2} \) in terms of the Fermi energy \( \epsilon_F \) for a homogeneous Fermi gas at infinite scattering length, based on the universal property of the thermodynamics. While a similar approach was considered in [16, 17], the main purpose of our work is to determine the two critical fields for homogeneous Fermi gas, and our study should be exact, since we do not assume the normal phase is fully polarized.
polarized.

At unitary, we can construct the exact equation of state (EOS) in the grand canonical ensemble from the universality hypothesis. The pressure in the polarized normal phase \((N)\) as a function of averaged chemical potential \(\mu = (\mu_1 + \mu_2)/2\) and the Zeeman field \(H = (\mu_1 - \mu_2)/2\) takes the form

\[
P_N(\mu, H) = \frac{2}{5} c \mu^{5/2} G \left( \frac{H}{\mu} \right), \quad c = \frac{(2M)^{3/2}}{3\pi^2},
\]

where \(\mu_1\) and \(\mu_2\) are the effective chemical potentials for the two spin components, \(M\) is the fermion mass. \(G(x)\) is an unknown universal scaling function, and only \(G(0) = \xi_N^{-3/2}\) is known to be \(\xi_N \simeq 0.56\). The total number density and magnetization are, respectively, \(n_N(\mu, H) = \partial P_N/\partial \mu\) and \(m_N(\mu, H) = \partial P_N/\partial H\). There exists a universal critical value \(\delta_0\) of \(H/\mu\), below which the gas is in the partially polarized state \((\text{NFP})\) with \(m_N < n_N\). In the fully polarized normal state \((\text{NFP})\) with \(m_N = n_N\), the gas should be non-interacting with \(G(x) = \frac{1}{2}(1 + x)^{5/2}\). While mean field theory predicts \(\delta_0 = 1\) \([14]\), it is 3.78 \([12]\) from recent MC simulations.

Monte Carlo studies \([12]\) and experiments \([13]\) show that the stable superfluid phase \((\text{SF})\) at unitary should be unpolarized. The pressure does not depend on \(H\) explicitly, and takes the well-known form

\[
P_{\text{SF}}(\mu, H) = \frac{2}{5} c \mu^{5/2} \xi^{-3/2}.\]

The total density reads \(n_{\text{SF}}(\mu, H) = c \mu^{3/2} \xi^{-3/2}\).

In the grand canonical ensemble, a first order SF-N phase transition occurs when the pressure of the superfluid and the normal phase equalize, i.e., when \(H/\mu\) reaches another universal critical value \(\gamma\), which is determined by the algebra equation

\[
G(\gamma) = \xi^{-3/2}.
\]

If the normal phase at the phase transition is \(\text{NFP}\), we have \(\frac{1}{2}(1 + H/\mu)^{5/2} = \xi^{-3/2}\). Combining the relation \(\Delta(\mu) = \beta \mu\) at unitary, we immediately recover the result \([11]\) derived in \([11]\).

To determine the critical fields \(H_{c1}, H_{c2}\) for homogeneous Fermi gas, we turn to the canonical ensemble with fixed total particle number density \(n = (2M\epsilon_F)^{3/2}/(3\pi^2)\). The Zeeman splitting \(H\) is treated as a real external field, and the conversion between particles in the states \(\uparrow\) and \(\downarrow\) is allowed, but the chemical potential \(\mu\) is then not a free parameter.

The chemical potential \(\mu_N(H)\) in the polarized normal phase is solved from the number equation \(n_N(\mu_N, h) = n\), and the energy density \(E_N(h) = \mu_N(H)n - P_N(\mu_N(H), H)\) reads

\[
E_N(H) = \frac{5}{3} \left( \frac{\mu_N}{\epsilon_F} - \frac{2}{5} G \left( \frac{H}{\mu_N} \right) \left( \frac{\mu_N}{\epsilon_F} \right)^{5/2} \right) \epsilon_0
\]

with \(\epsilon_0 = \frac{3}{5} \epsilon_F^{5/2}\). The well-known relation \(E = 3P/2\) breaks down here, since the interacting energy with external field \(H\) is included. In the phase \(\text{NFP}\), we have \(\mu_N(H) = 2^{2/3} \epsilon_F - H\). The \(\text{NFP}-\text{NFP}\) transition occurs at \(H_0 = 2^{2/3} \epsilon_F \delta_0/(1 + \delta_0)\). While mean field theory predicts \(H_0 = 2^{-1/3} \epsilon_F \simeq 0.794\epsilon_F\), we find \(H_0 \simeq 1.26\epsilon_F\) from the recent MC simulations. Solving the number equation \(n_{\text{SF}}(\mu, H) = n\) for the superfluid phase, the chemical potential and energy density are given by \(\mu_{\text{SF}}(H) = \xi \epsilon_F\) and \(E_{\text{SF}}(H) = \xi \epsilon_0\). At \(H = 0\), there is the BCS instability \(\xi \epsilon_F(0) < \xi \epsilon_0(0)\) which is numerically supported by the fact \(\xi_N > \xi\).

While \(E_{\text{SF}}(H)\) keeps independent of \(H\), \(E_N(H)\) should be a monotonously decreasing function. If there exists no heterogeneous mixed phase, a phase transition occurs at \(\xi_{\text{N}}(H_c) = E_{\text{SF}}\). Once \(G(x)\) is known, one can determine \(H_c\). If we assume the normal state at \(H_c\) is fully polarized, i.e., \(H_c \geq H_0\), we find

\[
\frac{H_c}{\epsilon_F} = 2^{2/3} - \xi, \quad \frac{H_c}{\xi} = \frac{1}{\beta} \left( \frac{2^{2/3}}{\xi} - 1 \right).
\]

Here \(\Delta_0 = \beta \xi \epsilon_F\) is the energy gap at fixed density. From \(H_c \geq H_0\), there is \(\xi \leq 2^{2/3} / (1 + \delta_0) \simeq 0.33\), which is inconsistent with recent MC result \(\xi \simeq 0.42\) or \(\xi \simeq 0.44\). On the other hand, taking \(\xi \simeq 0.4 - 0.5\) and \(\beta \simeq 1.2\) from the recent MC data, we have \(H_c/\Delta_0 \geq 1.5\), which is in contrast with the constraint \(H_c < \Delta_0\) \([12]\) to ensure the superfluid phase is unpolarized. Therefore, the MC data on \(\xi\) and \(\beta\) indicates that the phase transition is not directly into the state \(\text{NFP}\).

The absence of heterogeneous SF-N mixed phase in above analysis is not adequate since the first order phase transition is often associated with the phase separation phenomenon at fixed density. We then take the heterogeneous mixed phase into account. In this case, the critical field \(H = H_c(\mu) = \gamma \mu\) for the first order phase transition in the grand canonical ensemble splits into a lower and a upper critical fields \(H_{c1}\) and \(H_{c2}\), and the SF-N mixed phase appears in the region \(H_{c1} < H < H_{c2}\). \(H_{c1}\) and \(H_{c2}\) can be determined by equating the chemical potential \(\mu\) to its value in the superfluid and the normal phase respectively, \(H_{c1} = \gamma_{\text{HFS}}\) and \(H_{c2} = \gamma_{\text{HNN}}(H_{c2})\), where \(\mu_N(H_{c2})\) is obtained by the number equation in the normal state, \(\mu_N(H_{c2}) = \xi \epsilon_F \left[ 1 - \frac{2}{5} \xi^{3/2} G(\gamma) \right]^{-2/3}\). Thus we find the following model-independent expression for the lower and upper critical Zeeman fields

\[
H_{c1} = \gamma \xi \epsilon_F, \quad H_{c2} = \gamma \xi \epsilon_F \left[ 1 - \frac{2}{5} \xi^{3/2} G(\gamma) \right]^{-2/3}.
\]

From the phase equilibrium condition \(P_{\text{SF}}(\mu, H) = P_\parallel(\mu, H)\), in the mixed phase the ratio \(H/\mu\) keeps a constant \(\gamma\) and the chemical potential reads \(\mu_{\text{FS}}(H) = H/\gamma\).

The property of the normal bubble in the mixed phase depends on the value of \(\gamma\). The MC calculations \([12]\) predict \(\gamma < \delta_0\), i.e., the normal bubble is partially polarized.
The volume fractions of the superfluid and normal phase, denoted by $x$ and $1-x$ respectively, are determined by $n = x(H)\mu_{SF}(\mu_{PS}, H) + [1-x(H)]\mu_N(\mu_{PS}, H)$. Using the EOS for the phases SF and N, we find

$$x(H) = \frac{(H/H_{c1})^{-3/2} - (H_{c2}/H_{c1})^{-3/2}}{\frac{2}{5} \xi^{3/2} \gamma G'(\gamma)}.$$ (8)

The energy density of the mixed phase, $E_{PS}(H) = \mu_{PS}n - P_{PS}(\mu_{PS}, H)$, can be evaluated as

$$E_{PS}(H) = \frac{5}{3} \frac{H}{H_{c1}} \left[ 1 - \frac{2}{5} \left( \frac{H}{H_{c1}} \right)^{3/2} \right] \xi E_0.$$ (9)

Since the normal bubble is polarized, there is a nonzero global polarization $P$ in the mixed phase, i.e., the system becomes “spontaneously magnetized” when $H > H_{c1}$. From the definition $P = (N_F - N_1) / (N_F + N_1)$, we find

$$P(H) = \frac{1}{\gamma} \left[ (H/H_{c1})^{3/2} - 1 \right].$$ (10)

The mixed phase continuously link the superfluid and normal state. We have $\mu_{PS} = \mu_{SF}$ at $H = H_{c1}$ and $\mu_{PS} = \mu_N$ at $H = H_{c2}$, which ensures $0 \leq x \leq 1$ with $x(H_{c1}) = 1$ and $x(H_{c2}) = 0$.

There are three constraints on the universal constants and scaling function. The appearance of the mixed phase requires $H_{c1} < H_{c2}$, which gives rise to $0 < \gamma G'(\gamma) < \frac{5}{2} \xi^{-3/2}$. Since the superfluid phase is unpolarized, there should be $H_{c1} < \Delta_0$ or $\beta > \gamma$, which plays the same role as the lower bound for the ratio $\mu_1/\mu_1$ proposed in [16]. Finally, the normal state at $H = H_{c1}$ is partially polarized, there is $H_{c2} < H_0$.

For the discussions above, the mixed phase is assumed to be the ground state in the region $H_{c1} < H < H_{c2}$. As a complete study, we have to prove that the mixed phase has the lowest energy in this region. While the information on the scaling function $G(x)$ is still lacking, the requirement of the lowest energy can tell us some constraints on it.

Firstly, the energy density of the mixed phase can be written as $E_{PS}(H) = \frac{5}{3} \xi E_0 f(H/H_{c1})$ with $f(z) = z - \frac{2}{5} z^{5/2}$. Since $f'(z) = 1 - z^{3/2}$, $E_{PS}(H)$ is a monotonously decreasing function of $H$ in the region $H_{c1} < H < H_{c2}$. Combining with the fact that $E_{PS} = E_{SF}$ at $H = H_{c1}$ and $E_{SF}$ is $H$-independent, there is always $E_{PS}(H) < E_{SF}(H)$ for $H_{c1} < H < H_{c2}$.

Secondly, the condition $E_{PS}(H) < E_N(H)$ requires $g(\gamma) < g(\gamma')$ with $g(t) = h/t - \frac{2}{5} \xi G(t) (h/t)^{3/2}$, $h = H/\xi E_0$ and $\gamma' = H/\mu_N(H)$. Even though the full information of the scaling function $G(x)$ is lacked, it is sufficient to show $E_{PS}(H) < E_N(H)$ at $H \lesssim H_{c2}$, due to the continuity and the fact of $E_N(0) > E_{SF}(0)$. From the first order derivative of $g(t)$ at $t = \gamma$, $g'(\gamma) = \gamma^{-2} h \left[ (H/H_{c2})^{3/2} - 1 \right]$, $g(t)$ is a decreasing function near $t = \gamma$. Therefore, at $H \lesssim H_{c2}$ the condition $g(\gamma) < g(\gamma')$ requires $\gamma' < \gamma$ or $\mu_{PS} > \mu_{PS}$. Since $\mu_{PS}(0) > \mu_{SF}$ and $\mu_{PS}$ is an increasing function of $H$, we believe the relation $\mu_{SF} < \mu_{PS} < \mu_N$ holds in the region $H_{c1} < H < H_{c2}$. A schematic plot of the energy density for various phases is shown in Fig. 1.

![FIG. 1: The energy for the superfluid, normal and mixed phases in the region $H_{c1} < H < H_{c2}$.

While the universal constant $\xi$ and $\gamma$ has been determined in MC calculations and experiments, to determine the upper critical field $H_{c2}$ we need the value $G'(\gamma)$, which is not known so far. However, another important quantity, the critical population imbalance $P_c = P(H_{c2})$ is determined [12, 13]. We now consider systems with separately fixed $N_1$ and $N_1$ and without conversion between the up and down states. From $P(H_{c1}) = 0$ and $P(H_{c2}) = P_c$, the ground state is the unpolarized superfluid at $P = 0$ and the SF-N mixed phase for $0 < P < P_c$. In the mixed phase, the effective “magnetic field” is given by $H(P) = \gamma \xi E_0 (1 + P)^{2/3}$, and the critical population imbalance $P_c$ reads

$$P_c = \frac{2 \xi G'(\gamma)}{5 \xi^{-3/2} - 2 \gamma G'(\gamma)}.$$ (11)

It is very interesting that to determine the values of $H_{c1}, H_{c2}$, it is not necessary to know the full information of $G(x)$, we need only the value of $G'(x)$ at $x = \gamma$. On the other hand, we find there exists a simple relation between the CC limits in different cases,

$$\frac{H_{c2}}{H_{c1}} = (1 + \gamma P_c)^{3/2}.$$ (12)

To show the $P_c$ we theoretically obtained above is consistent with that obtained in MC calculations and experiments, we derive the energy density $E_{PS}$ as a function of the ratio $n_1/n_1$, [12]:

$$E_{PS}(n_1, n_1) = \frac{3}{5} \frac{n_1}{2M} \left( \frac{n_1}{n_1} \right)^{2/3}.$$ (13)
With fixed $N_f$ and $N_l$, the energy density defined as $\mathcal{E} = \mu_1 n_1 + \mu_2 n_2 - \mathcal{P}$ satisfies the relation $\mathcal{E} = 3P/2$ in all phases, since $H$ is now no longer treated as an external field. The function $I(z)$ can be shown to be

$$I(z) = 2^{-2/3} \xi [(1 + \gamma) + (1 - \gamma)z]^{5/3}, \quad (14)$$

which is consistent with the formula used in the MC calculation\textsuperscript{12} to obtain $P_c$.

We can now determine the critical Zeeman fields $H_{c1}$, $H_{c2}$ from the data of $\xi$, $\gamma$ and $P_c$, and compare them with that obtained with mean field $I$ and beyond mean field theories\textsuperscript{18}. The MC and experimental data quite close to each other. The MC calculation gives $\xi_{MC} = 0.42(1)$, $\gamma_{MC} = 0.967$, and $P_{cMC} = 0.281$, which is not measured in \textsuperscript{12}. We thus take the MC data to determine $H_{c1}$ and $H_{c2}$. Substituting them into equation \textsuperscript{11}, we find $\mathcal{G}'(\gamma_{MC}) = 2.587$. With equation \textsuperscript{11}, we obtain

$$H_{c1} = 0.407\xi_{EF}, \quad H_{c2} = 0.503\xi_{EF}. \quad (15)$$

Our model formulae \textsuperscript{11} and \textsuperscript{11} are model independent. In mean field theory, we have $\xi_{MF} = 0.5906$ and $\mathcal{G}(x) = (1 + x)^{5/2} \Theta(1 + x) + (1 - x)^{5/2} \Theta(1 - x))$. Numerical solution of equation \textsuperscript{11} leads to $\mathcal{G}(\gamma_{MF}) = 0.8071$ and $\mathcal{G}'(\gamma_{MF}) = 2.9307$. Thus we find $H_{c1}^{MF} = 0.477\xi_{EF}$, $H_{c2}^{MF} = 0.693\xi_{EF}$, and $P_{cMF} = 0.933$, which agree well with the numerical values obtained in \textsuperscript{11}. One finds the mean field value of $H_{c2}$ deviates significantly from our result. Also, the result $H_{c2} = 0.693 + 0.087/N + O(1/N^2)$ obtained by the large-N expansion method\textsuperscript{18} is also not consistent with our result.

In summary, we have presented a model independent calculation of the lower and upper Chandrasekhar-Clogston limits of a unitary Fermi superfluid. Future studies should focus on the calculation of the scaling function $\mathcal{G}(x)$ beyond-mean-field theories\textsuperscript{18, 19, 20}. Once the universal constant $\xi$ and function $\mathcal{G}(x)$ are known, one can directly obtain the critical polarization $P_c$ from our model independent formula \textsuperscript{11} and check the consistency between theory and experiment or MC simulation.

The above model independent approach can be generalized to finite temperature $T$, where both the superfluid and normal phase are polarized due to thermal excitations. From the universality, the EOS for the normal phase and superfluid reads\textsuperscript{3}

$$\mathcal{P}_{N, SF}(T, \mu, H) = \frac{2}{5} \xi^{5/2} \mathcal{G}_{N, SF}(H/\mu, T/\mu), \quad (16)$$

where we have set $k_B = 1$. The scaling functions for the superfluid and normal phase should be different.

In the grand canonical ensemble, one expects that the phase transition along the $T/\mu$ axis is of second order at small $H/\mu$ and first order at large $H/\mu$. The first order phase transition is determined by the equation

$$\mathcal{G}_N(H/\mu, T/\mu) = \mathcal{G}_{SF}(H/\mu, T/\mu), \quad \text{or explicitly} \quad H/\mu = \mathcal{W}(T/\mu) \text{ with known } \mathcal{W}(0) = \gamma. \quad \text{The first order phase transition should end at a so-called tricritical point } (H/\mu, T/\mu) = (a, b).$$

At fixed total particle number, $\mu$ is not a free variable, and the tricritical point is characterized by $(T_{TCP}, H_{TCP})$. Due to the continuity with the zero temperature case, for $T < T_{TCP}$, there exist two critical fields $H_{c1}(T) = \mu_1 \mathcal{W}(T/\mu_1)$ and $H_{c2}(T) = \mu_2 \mathcal{W}(T/\mu_2)$ with $\mu_1$ and $\mu_2$ being the chemical potentials corresponding to $H_{c1}$ and $H_{c2}$. The mixed phase region $H_{c1} < H < H_{c2}$ should decrease with increasing $T$, and finally disappears at the tricritical point with $H_{TCP} = aT_{TCP}/b$ and $\mu_1 = \mu_2 = T_{TCP}/b$.

When $N_l$ and $N_f$ are fixed, for $T < T_{TCP}$, the phase separation should be the ground state in the region $P_1 < P < P_2$ with $P_1 = P(H_{c1})$ and $P_2 = P(H_{c2})$. At $T \neq 0$, $P_1$ should be nonzero and increase with temperature, and $P_2 = P_{TCP}$ at the tricritical point. Once the scaling function $\mathcal{G}$ and the tricritical point $(a, b)$ are known, $P_{TCP}$ and $T_{TCP}$ can be calculated from the following model-independent formulae:

$$P_{TCP} = a \mathcal{G}'(a, b) - \frac{3}{2} a \mathcal{G}(a, b) - \frac{3}{2} b \mathcal{G}'(a, b),$$

$$\frac{T_{TCP}}{\xi_{EF}} = b \left[ \frac{5P_{TCP}}{2\mathcal{G}'(a, b)} \right]^{2/3} \quad \text{(17)}$$

with the definition $\mathcal{G}'(x, y) = \partial \mathcal{G}(x, y)/\partial x$ and $\mathcal{G}'(x, y) = \partial \mathcal{G}(x, y)/\partial y$, and $\mathcal{G}$ can be the scaling function of either the superfluid or the normal phase.

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