Extending the eikonal approximation at low energy

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Reactions with exotic nuclei

Exotic nuclei, like halo nuclei, are studied through reactions

Elastic scattering
Breakup $\equiv$ dissociation of halo from core
by interaction with target

Need a good understanding of the reaction mechanism
i.e. an accurate theoretical description of reaction
coupled to a realistic model of projectile:
- Continuum Discretised Coupled Channel (CDCC)
- Eikonal approximation
- ...

Experimental facilities will be able to provide RIBs at $\sim 10$ AMeV
(HIE-ISOLDE @ CERN, ReA12 @ MSU,...)
CDCC being expensive, can we use eikonal at these energies?
Reaction modelling
- CDCC
- Dynamical eikonal approximation
- Usual eikonal

Coulomb breakup of $^{15}$C
- 68 AMeV
- 20 AMeV

Nuclear breakup of $^{11}$Be at 20 AMeV
- Wallace's nuclear correction

Summary
Framework

Projectile \((P)\) modelled as a two-body system: core \((c)\)+loosely bound nucleon \((f)\) described by

\[ H_0 = T_r + V_{cf}(r) \]

\(V_{cf}\) adjusted to reproduce bound state \(\Phi_0\) and resonances

Target \(T\) seen as structureless particle

\(P-T\) interaction simulated by optical potentials

⇒ breakup reduces to three-body scattering problem:

\[
\left[ T_R + H_0 + V_{cT} + V_{fT} \right] \Psi(r, R) = E_T \Psi(r, R)
\]

with initial condition \(\Psi(r, R) \xrightarrow{Z \to -\infty} e^{iKZ}\ldots \Phi_0(r)\)
Continuum Discretised Coupled Channel (CDCC)

Solve the three-body scattering problem:

\[
\left[T_R + H_0 + V_{cT} + V_{fT}\right]\Psi(r, R) = E_T\Psi(r, R)
\]

by expanding \( \Psi \) on eigenstates of \( H_0 \)

\[
\Psi(r, R) = \sum_i \chi_i(R)\Phi_i(r) \quad \text{with} \quad H_0\Phi_i = \epsilon_i\Phi_i
\]

Leads to set of coupled-channel equations:

\[
\left[T_R + \epsilon_i + V_{ii}\right]\chi_i + \sum_{j\neq i} V_{ij}\chi_j = E_T\chi_i,
\]

with \( V_{ij} = \langle \Phi_i | V_{cT} + V_{fT} | \Phi_j \rangle \)

The continuum has to be discretised:

[Austern et al., Phys. Rep. 154, 125 (1987)]
[Tostevin, Nunes, Thompson, PRC 63, 024617 (2001)]

Fully quantal approximation
No approximation on \( P-T \) motion, nor restriction on energy
But expensive computationally
Dynamical eikonal approximation (DEA)

Three-body scattering problem:

\[
\left[T_R + H_0 + V_{cT} + V_{fT}\right] \Psi(r, R) = E_T \Psi(r, R)
\]

with condition \( \Psi \rightarrow e^{iKZ} \Phi_0 \)

Eikonal approximation: factorise \( \Psi = e^{iKZ} \hat{\Psi} \)

\[
T_R \Psi = e^{iKZ} \left[T_R + vP_Z + \frac{\mu_{PT} v^2}{2}\right] \hat{\Psi}
\]

Neglecting \( T_R \) vs \( P_Z \) and using \( E_T = \frac{1}{2} \mu_{PT} v^2 + \epsilon_0 \)

\[
i\hbar v \frac{\partial}{\partial Z} \hat{\Psi}(r, b, Z) = [H_0 - \epsilon_0 + V_{cT} + V_{fT}] \hat{\Psi}(r, b, Z)
\]

solved for each \( b \) with condition \( \hat{\Psi} \rightarrow \Phi_0(r) \)

This is the dynamical eikonal approximation (DEA)

[Baye, P. C., Goldstein, PRL 95, 082502 (2005)]

(usual) eikonal includes the adiabatic approximation: \( (H_0 - \epsilon_0) \approx 0 \)
Usual eikonal model

Eikonal solution: \( \Psi^{\text{eik}}(b, Z) = e^{iKZ} e^{i\chi_0(b, Z)} \Phi_0, \)
with \( \chi_0(b, Z) = -\frac{1}{\hbar v} \int_{-\infty}^{Z} V_{PT}(b, Z') dZ' \)

- Easy to interpret and implement
- Fast calculations

Valid in nuclear-dominated reactions

\(^{11}\text{Be}+\text{C@50A}\text{MeV} \quad [\text{Baye, Goldstein, P. C. PRC 73, 024602 (2006)}]\)
Usual eikonal model

Eikonal solution: \( \Psi^{\text{eik}}(b, Z) = e^{iKZ} e^{i\chi_0(b, Z)} \Phi_0 \),

with \( \chi_0(b, Z) = -\frac{1}{\hbar v} \int_{-\infty}^{Z} V_{PT}(b, Z')dZ' \)

- Easy to interpret and implement
- Fast calculations

\text{BUT} diverges for Coulomb breakup:

\(^{11}\text{Be} + \text{Pb} @ 69\text{AMeV}\) [Baye, Goldstein, P. C. PRC 73, 024602 (2006)]
Coulomb breakup of $^{15}$C

$^{15}$C+Pb @ 68.4 MeV: energy distribution

- Excellent agreement between CDCC and DEA
  
  [P.C., Esbensen and Nunes, PRC 85, 044604 (2012)]

- Excellent agreement with experiment
  
  [Nakamura et al. PRC 79, 035805 (2009)]

$\Rightarrow$ Confirms the validity of the approximations

... and the two-body structure of $^{15}$C
\(^{15}\text{C} + \text{Pb} @ 68\text{AMeV} : \text{angular distribution}\)

- DEA agrees well with CDCC
- Though a slight shift compared to CDCC...
$^{15}$C+Pb @ 20A MeV

DEA too high and too forward due to lack of Coulomb deflection

[P.C., Esbensen and Nunes, PRC 85, 044604 (2012)]
Coulomb breakup of $^{15}\text{C}$

$^{15}\text{C} + \text{Pb} \ @ \ 20\text{AMeV}$

DEA too high and too forward due to lack of **Coulomb deflection**

[P.C., Esbensen and Nunes, PRC 85, 044604 (2012)]

Could **Eikonal-CDCC** solve the problem?
Eikonal-CDCC (E-CDCC)

Solving the eikonal problem expanding $\Psi$ upon $H_0$ eigenstates $\Phi_i(r)$ assuming discretised continuum [Ogata et al. PRC 68, 064609 (2003)]

$$\Psi(r, R) = \sum_i \xi_i(b, Z)\Phi_i(r)e^{i[K_iZ+\eta_i\ln[K_iR-K_iZ]}}$$

⇒ set of coupled equations

$$\frac{\partial}{\partial Z}\xi_i(b, Z) = \frac{1}{i\hbar v_i(R)} \sum_{i'} F_{ii'}(b, Z)\xi_i'(b, Z)e^{i(K_i'-K_i)Z}\mathcal{R}_{ii'}(b, Z),$$

with coupling potential

$$F_{ii'}(b, Z) = \left\langle \Phi_i \left| V_{cT} + V_{fT} - V_C \right| \Phi_{i'} \right\rangle_r.$$  

E-CDCC takes proper account of energy conservation: $v_i(R)$

$$\mathcal{R}_{ii'}(b, Z) = \frac{(K_i' - K_i Z)^{n_{ii'}}}{(K_i R - K_i Z)^{n_i}}$$ accounts for part of the Coulomb distortion.
Coulomb breakup of $^{15}\text{C}$

$^{15}\text{C} + \text{Pb} \ @ \ 20\text{A}\text{MeV}$

The shift in $\theta$ translates into a shift in $L \leftrightarrow b \Rightarrow$ semi-classical correction. $b \rightarrow b'$ (classical closest approach) corrects $\sigma_{\text{bu}} (L)$ and hence $d\sigma_{\text{bu}}/d\Omega$ [Fukui, Ogata, P. C. PRC 90, 034617 (2014)] $\Rightarrow$ Improves eikonal model to the level of CDCC. . . at least for Coulomb-dominated reactions.
Coulomb breakup of $^{15}$C

$^{15}$C+Pb @ 20AMeV

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$^{15}\text{C}+\text{Pb} @ 20\text{A}_{\text{MeV}}$

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[Fukui, Ogata, P.C. PRC 90, 034617 (2014)]

$\Rightarrow$ Improves eikonal model to the level of CDCC... at least for Coulomb-dominated reactions
Nuclear breakup of $^{11}$Be at 20AMeV

$^{11}$Be+C @ 20AMeV

[calculations by F. Colomer]

On **nuclear-dominated** reactions correction $b \rightarrow b'$ not sufficient:

- **DEA** overestimates cross sections at large angles (breakup and elastic scattering)
- **DEA** leads to damped oscillations

⇒ look for a correction to the nuclear interaction

[Al-Khalili et al. PRC 55, R1018 (1997)]
Wallace’s correction

For one-body scattering, Wallace introduces an expansion of the $T$ matrix in terms of eikonal propagator $g$ [Ann. Phys. 78, 190 (1973)]

$$T = (V_N + V_N g V_N) + V_N g N g V_N + \ldots$$

At the $m^{\text{th}}$ order the scattering amplitude reads

$$f^{(m)}(\theta) = -\frac{iK}{2\pi} \int e^{i\mathbf{q} \cdot \mathbf{b}} \left\{ \exp[i\chi^{(m)}(b)] - 1 \right\} d^2b$$

with

$$
\begin{align*}
\chi^{(0)}(b) & = \chi_0(b) = -\frac{1}{\hbar v} \int_{-\infty}^{\infty} V_{PT}(b, Z) dZ \\
\chi^{(1)}(b) & = \chi_0(b) + \tau_1(b) \\
\chi^{(2)}(b) & = \chi_0(b) + \tau_1(b) + \tau_2(b) + i\omega_2(b) \\
\chi^{(3)}(b) & = \chi_0(b) + \tau_1(b) + \tau_2(b) + \tau_3(b) + \phi_3(b) + i [\omega_2(b) + \omega_3(b)]
\end{align*}
$$
where...

The corrective terms are obtained from derivatives of the potential
They affect only the nuclear part of the potential $V_N$

\[
\begin{align*}
\tau_1(b) &= -\frac{1}{\hbar v} \frac{\epsilon}{2} \int_{-\infty}^{\infty} \left( \frac{1}{R} \hat{D} \right) R^2 V_N^2(R) \ dZ \\
\tau_2(b) &= -\frac{1}{\hbar v} \frac{\epsilon^2}{6} \int_{-\infty}^{\infty} \left( \frac{1}{R} \hat{D} \right)^2 R^4 V_N^3(R) \ dZ - \frac{b}{3K^2} \left[ \frac{1}{2} \hat{\partial}^1 \chi_0(b) \right]^3 \\
\tau_3(b) &= -\frac{1}{\hbar v} \frac{\epsilon^3}{24} \int_{-\infty}^{\infty} \left( \frac{1}{R} \hat{D} \right)^3 R^6 V_N^4(R) \ dZ - \frac{b}{K^2} \left[ \frac{1}{2} \hat{\partial}^1 \tau_1(b) \right] \left[ \frac{1}{2} \hat{\partial}^1 \chi_0(b) \right]^2 \\
\phi_3(b) &= -\frac{1}{\hbar v} \frac{\epsilon}{2} \left[ 1 + \frac{5}{3} b \hat{\partial}^1 + \frac{1}{3} b^2 \hat{\partial}^2 \right] \int_{-\infty}^{\infty} \left[ \frac{1}{2K} \hat{D} V_N(R) \right]^2 \ dZ \\
\omega_2(b) &= \frac{b}{8K^2} \hat{\partial}^1 \chi_0(b) \left[ \hat{\partial}^2 \chi_0(b) + \frac{1}{b} \hat{\partial}^1 \chi_0(b) \right] \\
\omega_3(b) &= \frac{1}{8K^2} \left[ b \hat{\partial}^1 \chi_0(b) \hat{\partial}^2 \tau_1(b) + 2 \hat{\partial}^1 \chi_0(b) \hat{\partial}^1 \tau_1(b) + b \hat{\partial}^1 \tau_1(b) \hat{\partial}^2 \chi_0(b) \right]
\end{align*}
\]

We first test this correction for the elastic scattering of $^{10}\text{Be}$ on C
At high energy, the correction

- is **efficient** (up to 15°)
- **converges** quickly (order 1 provides most of the correction)
At lower energy, the correction
- still **converges** quickly
- **improves** the amplitude of the oscillations
- **not efficient** at large angle
- induces a **shift** towards forward angles
Including the Coulomb correction

Coulomb correction \((b \rightarrow b')\) provides correct phase of oscillations

Both corrections gives excellent agreement with exact calculation, although the cross section is overestimated for \(\theta > 20^\circ\)

\[\Rightarrow\] very promising to extend the eikonal approximation to low energy

[calculations by C. Hebborn]
**11^Be+C @ 10A MeV**

Study of correction for two-body projectile
Comparison of (usual) eikonal with CDCC

[calculations by C. Hebborn]

Correction not as efficient as for one-body projectile
Study of correction for two-body projectile
Comparison of (usual) eikonal with CDCC

Correction not as efficient as for one-body projectile
Due to dynamical effects? \(\Rightarrow\) test within DEA

What does it give on breakup observables?
Summary

- Halo nuclei studied mostly through reactions
  - elastic scattering
  - breakup
- DEA is an efficient model to describe reactions at 70A MeV
- At 20A MeV on Pb, DEA fails, due to Coulomb deflection
  But simple semiclassical correction works fine ($b \rightarrow b'$)
- This correction is not efficient on C
  ⇒ test Wallace’s nuclear correction in usual eikonal
  - very efficient for one-body projectile
    (when combined with Coulomb correction)
    - not so good for two-body projectile ⇒ use DEA ?
    - what will it give for breakup observables ?
- COntinuing effort to search for other corrections
  ⇒ Promising results : DEA might be used at lower energy (10A MeV)
Thanks to my collaborators

Daniel Baye
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Tokuro Fukui

Filomena Nunes

Henning Esbensen
$^{15}$C+Pb @ 20A MeV

- **DEA**: too high and shifted to forward angles
- **TD**: absence of quantal interferences with Coulomb trajectories, reproduces global trend of CDCC
- **TD** with straight lines, reproduces global trend of **DEA**

⇒ influence of **Coulomb deflection**

[P.C., Esbensen and Nunes, PRC 85, 044604 (2012)]
DEA cross sections

Breakup transition matrix element:

\[ T_{fi}^{bu} = \langle e^{iK' \cdot R} \chi_k^{(-)} | V_{cT} + V_{fT} | e^{iKZ} \hat{\Psi} \rangle, \]

where \( H_0 \chi_k^{(-)} = \epsilon \chi_k^{(-)} \) (ingoing scattering wave)

\[ T_{fi}^{bu} = \langle e^{iK' \cdot R} \chi_k^{(-)} | H_0 - \epsilon + V_{cT} + V_{fT} | e^{iKZ} \hat{\Psi} \rangle \]

\[ = \langle e^{iK' \cdot R} \chi_k^{(-)} | i\hbar v \frac{\partial}{\partial Z} + \epsilon_0 - \epsilon | e^{iKZ} \hat{\Psi} \rangle \]

\[ \approx i\hbar v \int db e^{i\mathbf{q} \cdot \mathbf{b}} \int_{-\infty}^{\infty} dZ \frac{\partial}{\partial Z} \langle \chi_k^{(-)} | \hat{\Psi} \rangle, \]

assuming \( \mathbf{q} = K' - K\hat{Z} \) transverse

\[ T_{fi}^{bu} = i\hbar v \int db e^{i\mathbf{q} \cdot \mathbf{b}} \langle \chi_k^{(-)} | \hat{\Psi}(Z \to \infty) \rangle \]
Breakup cross section (2)

After integration over $\varphi_b$,

$$T_{fi}^{bu} \propto \sum_{lm} Y_l^m(\Omega_k) e^{i(m_0-m)\varphi} \int_0^\infty J_{|m_0-m|}(qb) S_{klm}(b)b \, db,$$

$S_{klm}(b) = \langle \Phi_{klm} | \Psi(Z \rightarrow \infty) \rangle$ are breakup amplitudes

Cross sections:

$$\frac{d\sigma_{bu}}{dKd\Omega} \propto |T_{fi}^{bu}|^2 \int d\Omega_k \int d\Omega \quad \Rightarrow \quad \frac{d\sigma_{bu}}{dEd\Omega} \quad \text{and} \quad \frac{d\sigma_{bu}}{dE}$$

⇒ Dynamical eikonal extends TD

takes into account interferences between trajectories (sum of breakup amplitudes)

How does DEA compares to CDCC and TD?
DEA, TD and eikonal

\[ i\hbar v \frac{\partial}{\partial Z} \widehat{\Psi}(r, b, Z) = [H_0 - \epsilon_0 + V_{cT} + V_{fT}] \widehat{\Psi}(r, b, Z) \]

DEA ≠ TD because \( b \) and \( Z \) are quantal
⇒ includes interference between trajectories

The usual eikonal uses adiabatic approx. \( H_0 - \epsilon_0 \sim 0 \)
⇒ neglects internal dynamics of projectile

\[ \widehat{\Psi}^{\text{eik}}(r, b, Z) = \exp \left\{ -\frac{i}{\hbar v} \int_{-\infty}^{Z} dZ' \left[ V_{cT}(r, b, Z') + V_{fT}(r, b, Z') \right] \right\} \Phi_0(r) \]

⇒ dynamical eikonal generalises TD and eikonal

- improves TD by including quantal interferences
- improves eikonal by including dynamical effects
- we know how to solve accurately TD

How does DEA compare to CDCC and TD?
Halo nuclei

Exotic nuclear structures are found far from stability. In particular, halo nuclei with peculiar quantal structure:

- Light, n-rich nuclei
- Low $S_n$ or $S_{2n}$

Exhibit large matter radius due to a strongly clusterised structure: neutrons tunnel far from the core and form a halo.

**One-neutron halo**

$^{11}\text{Be} \equiv ^{10}\text{Be} + n$

$^{15}\text{C} \equiv ^{14}\text{C} + n$

**Two-neutron halo**

$^{6}\text{He} \equiv ^{4}\text{He} + n + n$

$^{11}\text{Li} \equiv ^{9}\text{Li} + n + n$

Proton haloes are also possible, but less probable.