Heat flux during boiling of superfluid helium inside a porous body based on a two-fluid model

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Abstract The calculation of the recovery heat flux density is considered for superfluid helium boiling on the cylindrical heater inside porous structure. System of equations is based on the methods of continuum mechanics and molecular kinetic theory. The new type of boundary condition on the vapor-liquid interface based on the two-fluid model is formulated. Heat transfer in a free liquid is described by the Gorter-Mellink semi-empirical theory. Inside the porous structure the processes is discussed by the two-fluid model and filtration equation. Experimental data on the boiling of superfluid helium inside the porous structure are interpreted based on the formulated mathematical model. The qualitative and in some cases quantitative agreement between the calculated and experimental values of the recovery heat flux were obtained in the considered range of parameters.

1. Introduction
The high efficiency of energy and mass transfer in superfluid helium determines the features of the processes description during the movement of liquid in channels, during boiling in a large volume, and the boundary conditions on the liquid-vapor interface. With its unique ability to fill the entire working space, helium-II is an excellent cryoagent for cryostating high-load power equipment [1]. In addition, having the lowest temperature of liquids, it serves as a working medium for studying the behavior of various metals and alloys, including for obtaining and using the magneto-caloric effect [2].

However, the using of He-II as a heat-removing working fluid entails the need to assess the probability of an exit from the nominal mode of heat removal from various elements of heat-generating assemblies, including superconducting windings of electromagnetic systems. As a result of the possible local release of high thermal pulses, vapor is formed in the volume of helium-II, and, consequently, the transition from direct liquid – solid wall contact to film boiling (as is known, there is no bubble boiling mode in superfluid helium) with low heat transfer efficiency.

One of the defining parameters of the boiling of superfluid helium is the recovery heat flux value. The minimum heat flux at which the non-boiling regime is restored is usually referred to as the recovery heat flux. The method for calculating this value was developed in [3]. In the absence of heat flux, the vapor phase collapses (condenses) and the vapor-liquid interface ceases to exist. This process was experimentally observed in [4], and the calculation method based on the solution of the Rayleigh equation using the results of molecular kinetic analysis is presented in [5].

At the same time, superfluid helium is an excellent medium for studying the dynamics of the phase interface during film boiling – the evolution of the vapor cavity on the heater. The solution of the dynamic problem for boiling superfluid helium on a cylindrical heater is presented in [6] on the basis of...
the Rayleigh equation in cylindrical coordinates, the nonequilibrium boundary condition [7], and various models for describing heat transfer in a liquid.

Later, the solution for a spherical heater located in a volume of superfluid helium was improved on the basis of the Landau-Khalatnikov two-fluid hydrodynamics equations [8]. As a result, a new equation describing the motion of the vapor—liquid interface is obtained, which differs significantly from the classical Rayleigh equation and includes additional dissipative terms due to the interaction of normal and superfluid motion. Numerical solutions show differences in the rate of attenuation of the interfacial surface oscillations.

This work is a combination and development of the ideas given in [8] and an approach to the description of heat and mass transfer processes using the presentation of components of the tensor of momentum flux density and the mass flux density in superfluid helium on the two-fluid hydrodynamics base [9].

In addition, it is necessary to pay attention to the experimental data. Paper [10] presents the investigations results of superfluid helium boiling on wires with porous insulation. Measurements of peak and recovery heat fluxes are performed for coated for bare RhFe and Cu wire. The temperature dependence of these quantities is measured for saturated and pressurized He-II. It was shown, that for all coatings the heat flux increases monotonically with decreasing temperature.

At last in this part, it should be mentioned the new type of superfluid helium boiling inside porous structure [11, 12]. Experimental studies of the He-II boiling on the surfaces of the cylindrical heaters in a bulk of He-II and under constrained conditions are described. Visualization of helium-II film boiling was first performed when observed from the end of a cylindrical heater. The experimental results show that at different depths of immersion of the experimental cell into the He-II, the boiling pattern characteristic of helium-II was not usually observed. Instead, the vapor film remained open in the major part of experiments, and the vapor accumulated in the upper part of the internal cavity of the experimental cell.

2. Problem and method

The problem of determining the recovery heat flux $q_R$ at the moment of collapse of a vapor film during silent film boiling on a cylindrical heater with a radius $R_w$ placed inside a coaxial porous shell with an internal radius $R_0$ and a thickness $L$, which is immersed in superfluid helium to a certain depth $h_w$, is considered (Figure 1). Above the free surface of the liquid, a constant pressure $P_b$ is maintained, the liquid in the volume of the vessel is in equilibrium with the vapor at the temperature $T_b$, so that $P_S(T_b) = P_b$, the saturation line $P_S(T)$ is known.

![Figure 1. Main parameters of the problem statement](image-url)
The stationary equation system is formulated. The first step is boundary conditions on the vapor-liquid interface.

For superfluid helium the momentum flux density tensor in the dissipative-free approximation according to [9] is written:

$$\Pi_{ik} = \rho_n \bar{u}_n \bar{u}_{nk} + \rho_s \bar{u}_s \bar{u}_{sk} + p \delta_{ik}, \quad (1)$$

where \(\bar{u}_n\) is the velocity of the normal motion, \(\bar{u}_s\) is the velocity of the superfluid motion, \(\rho_n\) is the density of the normal component, \(\rho_s\) is the density of the superfluid component, \(p\) – pressure, \(\delta_{ik}\) is the Kronecker symbol. This approach is presented in [8] to derive an equation describing the motion of the interfacial surface.

Let’s investigate the situation when the mass flow is equal to zero at the helium II-vapor phase interface:

$$j_x = \rho_n \bar{u}_{nx} + \rho_s \bar{u}_{sx} = 0. \quad (2)$$

In the presence of heat transfer through the helium II-vapor phase interface, the velocity of normal motion \(\bar{u}_n\) perpendicular to this boundary in its own coordinate system has a well-defined, non-zero value, and the expression for the component of the viscous tensor \(\tau_{xx}\) for the superfluid liquid will take the form:

$$\tau_{xx} = 2\mu \frac{\partial \bar{u}_{nx}}{\partial x}, \quad (3)$$

where \(\mu'\) - dynamic viscosity.

The normal component of the momentum flux density tensor is transferred in the next form with the expression (1)-(3):

$$\Pi_{xx} = \rho_n \left[1 + \frac{\rho_n}{\rho_s} \right] \bar{u}_{nx}^2 + P' - 2\mu' \frac{\partial \bar{u}_{nx}}{\partial x}, \quad (4)$$

For the vapor side the component of the viscous tensor \(\tau_{xx}\) due to the Newton's law of viscous friction:

$$\tau_{xx} = \frac{4}{3} \mu' \frac{\partial \bar{u}_s''}{\partial x}. \quad (5)$$

Boundary condition for the normal component of the momentum flux density tensor:

$$(P'' - P') - \left(\frac{4}{3} \mu'' \frac{\partial \bar{u}_s''}{\partial x} - 2\mu \frac{\partial \bar{u}_{nx}}{\partial x}\right) - \rho_n \left[1 + \frac{\rho_n}{\rho_s} \right] \bar{u}_{nx}^2 = 2\sigma H, \quad (6)$$

where \(P''\) – vapor pressure, \(P'\) – liquid pressure near interface, \(\sigma\) – surface tension, \(H\) – curvature of the interface. Since the cylindrical problem is considered, then \(H = \frac{1}{2Rw}\). The vapour in the film is considered as stationary, then \(\bar{u}_x'' = 0\).

In conjugate problems with superfluid helium – vapor interface the closed system of conservation equations under condition (6) must be used. This correlation takes into account the two velocity motion in He-II. The boundary condition (6) is written for a proper reference frame for the vapor-liquid interfaces at rest.

The vapor pressure \(P_1''\) is determined on the basis of the equation obtained by solving the Boltzmann kinetic equation by the moment method for evaporation-condensation problems in the linear formulation [7] under the condition that the mass flow from the interfacial surface is equal to zero:

$$P_1'' = P_s(T_1) + 0.44 \frac{q_R}{\sqrt{2RT_1}}, \quad (7)$$

where \(P_s(T_1)\) is the pressure corresponding along the saturation line to the temperature of the liquid at the interfacial surface \(T_1\), \(R\) is the individual gas constant.

Since, according to the problem statement, the liquid as a whole does not move (\(u = 0\)), the pressure difference in the inner cavity of the porous structure is zero:
\[ P'_1 = P_0, \]  
where \( P_0 \) is the pressure of the liquid on the inner surface of the porous body.

As a description of heat transfer in a liquid, it is proposed to use the stationary Gorter-Mellink heat transfer equation:

\[ \frac{dT}{dr} = -f(T)q^n, \]  

The result of integration in cylindrical coordinates has the form:

\[ q_R^n = \frac{(n-1)}{R_w f(T)} (T_i - T_0) \left( 1 - \left( \frac{R_w}{R_0} \right)^2 \right)^{-1}, \]  

where \( f(T) \) is the Gorter-Mellink constant, \( T_0 \) is the temperature of the liquid on the inner surface of the porous body.

A special feature of superfluid helium is the connection of the heat flux \( q(r) \) with the velocity of the normal component \( u_n(r) \), which follows from the equations of two-velocity hydrodynamics of L. D. Landau. The expression taking into account the zero velocity of the fluid as a whole \( (u = 0) \) is obtained in [9]:

\[ q(r) = \rho' ST' \cdot u_n(r), \]  

where \( S \) is the entropy of the liquid.

At this the liquid velocity can be expressed

\[ u_n \big|_{r=R_w} = \frac{q_R}{\rho' ST}, \]  

Corresponding derivative is

\[ \frac{\partial u_n}{\partial r} \bigg|_{r=R_w} = \frac{\partial}{\partial r} \left( \frac{q}{\rho' ST} \right) \bigg|_{r=R_w} = \frac{q_R}{\rho' ST} \left( f(T)q_R^n - \frac{T_i}{R_w} \right), \]  

To obtain a closed solution, it is necessary to take into account the heat and mass transfer during the flow of helium-II in the porous body. The relationship between the temperature difference and the pressure difference at the ends of the channel formed by particles in a porous medium is obtained from the equations of two-velocity hydrodynamics:

\[ P_0 - (P_b + \rho' gh_w) = \rho' S(T_0 - T_b), \]  

where \( g \) is the acceleration of gravity.

The hydraulic resistance of a porous tube depends on the structural and mechanical characteristics of the material. To describe the laminar flow of the normal component in the channels of a porous structure, the traditional filtration equation for the laminar regime is used:

\[ \frac{dP}{dr} = \frac{\mu' u_n(r) m}{k_p}, \]  

where the permeability coefficient \( k_p \) and the porosity \( m \) are assumed to be a constant value.

With the equation (11) the expression (15) can be transform:

\[ P_0 - (P_b + \rho' gh_w) = \frac{q_R R_w \mu'}{k_p \rho' ST} \ln \left( \frac{R_0 + L}{R_0} \right), \]  

Closure of the system of equations (heat balance):

\[ q(r) \big|_{r<R_w} = q_R \frac{R_w}{r}, \]  

Inside the porous structure, the porosity must be taken into account

\[ q(r) \big|_{r \geq R_w} = q_R \frac{R_w}{mr}, \]
In the future, it is necessary to solve the problem of the ambiguous influence of the term \( \rho'gh_w \) on the cross-section of the porous structure.

At last the system (6)-(18) transforms into three equations:

\[
\begin{align*}
P_0(T_1) + 0.44 \frac{q_R}{\sqrt{2RT_1}} - \rho' S(T_0 - T_b) - P_b - \rho'gh_w + \\
+ \frac{2\mu'q_R}{\rho'ST_1}\left(f(T)q_R n - \frac{T_1}{R_w}\right) - \rho_n \left[1 + \rho_n \left(\frac{q_R}{\rho'ST_1}\right)^2\right] = \frac{\sigma}{R_w},
\end{align*}
\]

(19)

\[
q_R^n = \frac{(n-1)}{R_w f(T)} \left(T_1 - T_0\right) \left(1 - \left(\frac{R_w}{R_0}\right)^2\right)^{-1},
\]

(20)

\[
\rho'S(T_0 - T_b) = \frac{\mu'q_R R_w}{k_{100}^p \rho'S\left(T_0\right)} \ln \frac{R_0 + L}{R_0},
\]

(21)

This system (19)-(21) contains three unknown quantities: \( T_0, T_1, q_R \). No further simplification is required due to the nonlinear equations and dependence of saturation pressure. It’s interesting that porosity is not appears in this system, only permeability effect on the common solution. Corresponding solution for the cylindrical heater in the free bulk is presented in [13].

3. Calculation results

The main results are presented in Figure 2. The recovery heat fluxes for different conditions were calculated and obtained results are compared with experimental data [10] for heater of diameter 38 \( \mu \text{m} \). As we can see from the dependencies, calculation line and experimental points are quite close to each other for free volume and they show qualitative changes when approaching the \( \lambda \)-point.

For confined inside porous body superfluid helium obtained results are the following. Qualitative agreement is observed in the all range of temperature and at low temperatures quantitative agreement is seen. It should be mentioned that temperature difference between bath and liquid near heater is reached 0.09K for 1.65K and decreases at 2K to 3mK.

![Figure 2](image)

**Figure 2.** Recovery heat flux dependence on temperature: 1, 2 – inside porous body; 3, 4 – free volume; lines – results of calculation, points – experimental data [10]

It’s interesting that as for free volume as for confined the thermophysical properties influence for the heat transfer, but the most highly variable is entropy, that included in (20) and in new approach [13] due to the two-velocity model.

4. Conclusion
The method of recovery heat flux calculation at the superfluid helium boiling inside porous structure is developed taking into account peculiarities of heat transfer in porous shell. At this the permeability of porous structure influences strongly on the value of recovery heat flux.

The calculation method for recovery heat flux at film boiling of superfluid helium is developed based on continuum mechanics and molecular kinetic theory. The experimental data were analyzed. Comparison of calculation and experimental data is good enough. At this, presented method is satisfactory described the known experimental data.

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