Evaluation on Accuracies of Phenomenological Yield Criteria for Automotive Al sheets

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Abstract. The anisotropic behavior of two typical cold rolled Al alloy sheets, AA5182-O and AA6016-T4, were investigated and used to predictive the yield surface by the well-known yield criteria: Hill90, Barlat89 and Barlat2000. Uniaxial tensile tests with axial and wide extensometers were carried out at different angles to the rolling direction (RD). Then the yield stress $\sigma_\theta$ and anisotropic coefficient $r_\theta$ were obtained. Biaxial tensile tests of cross-shaped specimens under certain liner loading paths were conducted on the well-designed biaxial tensile testing machine. The loading paths includes $F_x:F_y=1:4, 2:4, 3:4, 4:4, 4:3, 4:2, 4:1$. Yield stresses points ($\sigma_x, \sigma_y$) and their strain ratios in the yield locus plane can be obtained from the strain-stress curves of the biaxial tensile tests. Parts of tested values were used to determinate the yield locus according to different yield functions. All experimental data was used to evaluate the precisions of the yield functions by taking into account of the accuracy of $\sigma_\theta$ and $r_\theta$ and the yield stress and strain ratios of biaxial tensile tests.

1. Introduction
The sheet metals generally exhibit a significant anisotropy due to the crystal texture after the most common production rolling process [1]. The most apparent anisotropy behavior of rolled plate is the vastly different properties, such as the yield stress, between the rolling and transverse directions. In the automotive industry, sheet forming and die design processes are so important that are essential to analysis by numerical method, which is widely used recently. As being the initial property, the anisotropic mechanical properties of the sheet metals should be described accurate enough in the modeling of sheet forming processes to make sure that the simulation result is accuracy and acceptable. For now, the phenomenological approach is considered an adequate tool and widely applied in the commercial finite element software. In the stress space the yield stresses of different stress states constitute a closed, smooth and convex surface, also called yield surface [2].

During the last decades, a series of reliable formulations of yield criteria were proposed to describe the anisotropic behavior of the materials. The tendency is that to improve predictions more material parameters will be used in the expression of the yield function. The first yield criterion for anisotropic materials was proposed by von Mises in the form of a quadratic function [3]. In 1948 Hill proposed an anisotropic yield criterion as a generalization of the Huber-Mises-Hencky criterion [4]. The material is supposed to be anisotropy with three orthogonal symmetry planes. And the quadratic description has been improved by Hill himself in 1990 and 1993[5, 6]. In 1972[7] Hosford ‘rediscovered’ Hershey’s model and used it for the development of an anisotropic yield criterion. Barlat et al.[8] and Banabic et al.[9] as well as some other researchers proposed further extensions of the model. In this article, the
most popular yield functions which have been embedded into commercial finite element software are investigated: Hill90, Barlat89 and Barlat2000.

2. Materials and experiments

Two typical cold rolled Al alloy sheets widely used in the automotive industry are used in this paper: AA5182-O and AA6016-T4. The uniaxial tensile tests were carried out on the MTS tensile testing machine with a width extensometer. The true stress and true strain were obtained from the load force F and displacement Δl according to Eq.1-2. And the r-value was calculated according to Eq.4.

\[ \sigma = \frac{F}{S_0} \left(1 + \frac{\Delta l}{l_0}\right) \]  
\[ \varepsilon_x = \ln \left(1 + \frac{\Delta l}{l_0}\right) \]  
\[ \varepsilon_y = \ln \left(1 + \frac{\Delta w}{lw_0}\right) \]  
\[ r = -\frac{\varepsilon_y}{\varepsilon_x} \]

The biaxial tensile testing system used in this study was composed of a loading unit, a control unit and related software which could perform biaxial loading test under given loading path and record the displacement of both rolling and transverse direction with a pair of extensometers [10]. The geometry of cruciform specimen was designed with slots on each arm which was first proposed by Kuwabara’s [11] for the purpose of lowering the shear stress in the concerned central area. For the purpose of obtaining the yield loci in the normal stress space, the biaxial tensile tests were conducted under linear loading path: \( F_x:F_y = 4:0, 4:1, 4:2, 4:3, 4:4, 3:4, 2:4, 1:4, 0:4 \). The normal strain components, \( \varepsilon_x \) and \( \varepsilon_y \), were calculated by the displacement measured by extensometers.

The concept of plastic work contour was applied in this paper for investigating the initial and subsequent yield behavior of the materials. Yield stresses \( \sigma_0 \) were taken at 0.2% plastic strain from RD tensile experiments. The other yield stresses were calculated according to the equivalent plastic work energy. Different states were considered to be on the same yield locus only if their plastic work per unit volume was equal. As a result, the yield stresses of biaxial tensile with the same equivalent plastic to the uniaxial stress could calculate from:

\[ \int \sigma_1 d\varepsilon_1^p + \int \sigma_2 d\varepsilon_2^p = \int \bar{\sigma} d\varepsilon^p \]  

The ratio of the principal strains \( \eta_b \) which define the coefficient of biaxial anisotropy is calculated from the two normal strain components [2]:

\[ \eta_b = \frac{\varepsilon_y}{\varepsilon_x} \]  

3. Theoretical

3.1. Hill90 yield criterion

A generalized criterion which expressed in general coordinate system was proposed by Hill in 1990[5]:

\[ \varphi = |\sigma_{11} + \sigma_{22}|^m + \sigma_{12}^m |(\sigma_{11} - \sigma_{22})^2 + 4\sigma_{12}^2|^m/2 + |\sigma_{11} + \sigma_{22} + 2\sigma_{12}^2|^m/2 - 1 \cdot 
\[ -2a(\sigma_{11}^2 - \sigma_{22}^2) + b(\sigma_{11} - \sigma_{22})^2 \] = (2\sigma_b)^2 \]  

The value of the exponent m is calculated from:

\[ m = \frac{n[2(r_{ab}+1)]}{ln(2\sigma_b/\sigma_{45})} \]  

I this paper the yield functions were determined by two group of experimental date: yield stresses, denoted by (\( \sigma \)) and Lankford coefficient, denotes by (r).

3.2. Barlat89 yield criterion

Barlat and Lian[13,14] proposed a generalization formulation. In the particular case of thin sheets under plane stress conditions, Barlat 1989 yield criterion has the following formulation:

\[ \bar{\sigma} = [a|k_1 + k_2|^M + a|k_1 - k_2|^M + (1-a)|2k_2|M]^{1/M} \]  
\[ k_1 = \frac{\sigma_{11} + h\sigma_{22}}{2}; \quad k_2 = \sqrt{\left(\frac{\sigma_{11} + h\sigma_{22}}{2}\right)^2 + p^2\sigma_{12}} \]  

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To identify the material parameters a, p and h, three experimental values are needed using numerical methods. In this work, two group of experimental date are applied for the identification procedure: yield stresses ($\sigma_0, \sigma_{90, \sigma_b}$), denotes by ($\sigma$) and Lankford coefficient ($r_0, r_{45, r_{90}}$), denotes by (r).

3.3. Barlat2000 yield criterion

The Barlat2000 yield function is expressed by the relationship:

$$\Phi = \Phi' + \Phi'' = 2\sigma^m$$

where $m$ is the material coefficient based on the crystallographic structure of the material.

$$\Phi' = |Y'_{11} - Y''_{22}|^m$$

$$\Phi'' = |2Y''_{11} + Y''_{22}|^m + |Y''_{11} + 2Y''_{22}|^m$$

$$Y'_{11} = \frac{1}{2} \left( X'_{11} + X'_{22} \pm \sqrt{(X'_{11} - X'_{22})^2 + 4X'_{12}^2} \right)$$

$$Y''_{11} = \frac{1}{2} \left( X''_{11} + X''_{22} \pm \sqrt{(X''_{11} - X''_{22})^2 + 4X''_{12}^2} \right)$$

$\sigma$ is Cauchy stress tensor and $a_1$ to $a_8$ are 8 anisotropy coefficients, which could be determined by 7 material properties of the sheet metal [8], such as $\sigma_0, \sigma_{45}, \sigma_{90}, r_0, r_{45}, r_{90}$ and $\sigma_b$.

3.4. Evaluating the accuracy

The performances of those yield criteria are evaluated by comparing to the experimental data. These comparisons include the planar distributions of the uniaxial yield stresses and r-values, the yield locus shape, and the coefficient of biaxial anisotropy. The accuracy can be defined as:

$$E = a_1 e(\sigma_0) + a_2 e(r_0) + a_3 e(\sigma_{90}) + a_4 e(r_{90})$$

$e(\sigma_0)$ is the accuracy of the uniaxial yield stresses and $e(r_0)$ is the accuracy of the uniaxial r-values:

$$e(\sigma_0) = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{\sigma_{0i}^{exp} - \sigma_{0i}^{pri}}{\sigma_{0i}^{exp}} \right|$$

$$e(r_0) = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{r_{0i}^{exp} - r_{0i}^{pri}}{r_{0i}^{exp}} \right|$$

$e(\sigma_b)$ is the accuracy of the yield locus shape in the plane of the principal stresses.

$$e(\sigma_b) = \frac{1}{n} \sum_{i=1}^{n} \frac{d(P_i^{exp}, P_i^{pri})}{Y_{ref}}$$

Where $d(P_i^{exp}, P_i^{pri})$ is the distance from an experimental point $P_i$ to the same load point on the predicted yield locus.

$e(r_b)$ is the accuracy of the biaxial strain ratio.

$$e(r_b) = \frac{1}{n} \sum_{i=1}^{n} \left| \arctan \left( r_{bi}^{exp} \right) - \arctan \left( r_{bi}^{pri} \right) \right|$$

It should point out that each one of the four types of data, $\sigma_0, r_0, \sigma_b$ and $r_b$, may have different importance in different cases. Such as in earing of deep-drawing process, the $\sigma_0$ is the decisive factor, while in the in-plane stress process, the $\sigma_b$ and $r_b$ may more important. Therefore weight $a_i$ of each data is given to describe its importance.

4. Results and discussions

Experimental data from the uniaxial tensile test are listed in Table 1. The following parameters, $r_0, r_{45}, r_{90}, \sigma_0, \sigma_{45}, \sigma_{90}$ and $\sigma_b$, were used to determinate the coefficients of the yield functions. The material constants of the yield functions can be determined either from the yield stresses along particular directions or from the transverse-to-thickness strain-ratio values [13]. All yield stresses are obtained at initial yield correspond to 0.2% offset plastic strain for both uniaxial and biaxial cases.
Table 1. Experimental data for uniaxial yield stresses and r-values of AA5182-O and AA6016-T4

|         | AA5182-O | AA6016-T4 |
|---------|-----------|-----------|
| θ (°)   | 0 | 22.5 | 45 | 67.5 | 90 | 0 | 45 | 90 |
| σ₀ (MPa) | 107.2 | 106.8 | 106.3 | 108.1 | 109.0 | 159.9 | 157.4 | 158.8 |
| r₀      | 0.68 | 0.73 | 0.74 | 0.72 | 0.71 | 0.87 | 0.55 | 0.78 |

The experimental R-values and yield stresses of AA5182-O and AA6016-T4 are listed in Table 1. It shows that the in-plane yield stresses and r-values of AA5182-O are in narrow ranges, apparently near in-plane isotropic. While there are bigger differences among the r-values of AA6016-T4, that exhibits a significant anisotropy possibly resulted from the texture [1].

Figure 1. Predicted and experimental values, (a) r-values of AA5182-O, (b) yield stresses of AA5182-O, (c) r-values of AA6016-T4, (d) yield stresses of AA6016-T4

Table 2. Experimental data for biaxial yield stresses and strain routes

|         | 4:0 (uni axial) | 4:1 | 4:2 | 4:3 | 4:4 (equi-biaxial) | 3:4 | 2:4 | 1:4 | 0:4 (uni axial) |
|---------|-----------------|-----|-----|-----|-------------------|-----|-----|-----|----------------|
| AA5182-0 |                 |     |     |     |                   |     |     |     |                |
| σₓ(MPa) | 107.2           | 116.8 | 121.5 | 120.6 | 110.2             | 89.8 | 59.8 | 29.4 | 0.0            |
| σᵧ(MPa) | 0.0             | 28.5 | 59.2 | 88.7 | 110.2             | 120.6 | 120.3 | 118.9 | 109.0          |
| rₓ      | -0.40           | -0.16 | 0.00 | 0.26 | 1.79              | 4.40 | ∞     | -8.60 | -2.44          |
| AA6016-T4 |                |     |     |     |                   |     |     |     |                |
| σₓ(MPa) | 159.9           | 174.1 | 180.7 | 173.1 | 159.7             | 135.3 | 87.1 | 42.3 | 0.0            |
| σᵧ(MPa) | 0.0             | 42.7 | 93.1 | 129.2 | 158.8             | 168.3 | 174.3 | 169.2 | 158.8          |
| rₓ      | -0.47           | -0.17 | 0.02 | 0.23 | 1.63              | 4.56 | -66.56 | -5.27 | -2.29          |
The predicted and measured r-values and yield stresses as functions of angle, \( \theta \), to the RD are showed in Figure 1. It’s no surprise that the predicted r-values curves of the function determined based on the r-values through the points, \( r_0 \), \( r_{45} \) and \( r_90 \) because they are the input parameters for solving the constants. The same reason is for the stresses curves through the points, \( \sigma_0 \), \( \sigma_{45} \) and \( \sigma_{90} \). The predicted r-value curves using the yield function determined by the yield stresses do not go through the \( r_0 \), \( r_{45} \) and \( r_90 \), sometimes the predictions are very inaccurate, such as Barlat89(\( \sigma \)), which are much higher than expected. The same situation for the stresses predictions is similar. In short, the Barlat89 and Hill90 cannot both predict r-values and stresses well. The predicted r-values and stresses curves of Barlat2000 seem better agreement with the experiments.

![Figure 2. Experimental and predicted values: (a-c) 5182-O, (d-f) 6014-T4, (a) (d) Baralt89, (b) (e) Hill90, (c) (f) Baralt2000](image)

| Table 3. the errors of the predicted yield surfaces |
|-----------------------------------------------|
|                                | AA5182-O | AA6016-T4 |
|                                | Barlat  | Barlat  | Hill  | Hill  | Barlat  | Barlat  | Barlat  | Hill  | Hill  | Barlat  |
|                                | 89(\( r \)) | 89(\( \sigma \)) | 90(\( r \)) | 90(\( \sigma \)) | 2000 | 89(\( r \)) | 89(\( \sigma \)) | 90(\( r \)) | 90(\( \sigma \)) | 2000 |
| \( e(\sigma_0) \) (%)          | 0.38    | 0.17    | 0.31 | 0.18 | 0.18 | 2.56 | 0.00    | 3.45 | 0.00 | 0.00 |
| \( e(r_0) \) (%)              | 0.85    | 81.55   | 0.84 | 2.35 | 1.02 | 0.44 | 48.41   | 0.44 | 21.44 | 0.44 |
| \( e(\sigma_{45}) \) (%)      | 4.50    | 0.91    | 1.29 | 0.85 | 1.98 | 2.98 | 1.08    | 1.47 | 1.47 | 1.57 |
| \( e(r_{45}) \) (%)           | 1.49    | 1.81    | 1.41 | 1.37 | 1.81 | 0.90 | 1.14    | 1.31 | 1.34 | 1.11 |
| \( E \) (%)                   | 7.22    | 84.44   | 3.85 | 4.75 | 4.99 | 6.88 | 50.63   | 6.67 | 24.25 | 3.12 |

Experimental data from the biaxial tensile tests is list in Table 2 and showed in Figure 2. It shows that the Barlat89(\( r \)) does not predict the biaxial stresses accurately while the other criterions are much better to predict the biaxial stresses. As shown in Figure 2, the curvature near the equi-biaxial points of Hill90 yield surfaces are much small than Barlat89 and Barlat2000 due to the exponent \( m \) of Hill90 is
lower than the Barlat’s. The predicted strain ratios $r_\theta$ are agreed well with the experimental ones except the equi-biaxial case. The errors, according to Eq.16-20, between the predicted and measured data are list in the table 3. The weights of four groups of values are considered to be 1 in this work. It shows that the $e(r_\theta)$ values of Barlat89(σ) are for both metal are very big which consistent predictions are not agree with the experiments in the Figure 2 (a) (c). It results in the total errors value $E$ are very big that means the predictions are extremely inaccuracy. The Barlat89(r) predicts the yield surfaces much better than Barlat89(σ), and does Hill90. That means the yield surface determined by the r-values is more accuracy than the stresses. For Barlat2000, the both sheet’s yield surfaces are predicted well because the input parameters are more. The result indicates r-values determined yield loci as well as Barlat2000 are acceptable.Barlat2000 predicted well for yield surfaces both of sheets.

5. Conclusion
The performances of yield surface predicting with well-known yield criterions, Barlat89, Hill90 and Barlat2000, were investigated by comparing to the experimental data. The accuracy of the yield criterions were evaluated on basis of the uniaxial yield stresses and r-values, the yield locus shape, and the coefficient of biaxial anisotropy. Herewith the experimental data for AA5182-O and AA6016-T4 sheets were used. The results indicated that the Barlat2000 predicted well for yield surfaces both of AA5182-O and AA6016-T4. Hill90(r) and Barlat89(r) are acceptable and the Barlat89(σ) is the least accurate due to the big error for predicting r-values.

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