Observer based sliding mode control for subsonic piezo-composite plate involving time varying measurement delay

Na Qi1, Chen Zhang1 and Jiaxin Yuan2

Abstract
In this study, an observer based sliding mode control (SMC) scheme is proposed for vibration suppression of subsonic piezo-composite plate in the presence of time varying measurement delay by using the piezoelectric patch (PZT) actuator. Firstly, the state space form of the subsonic piezo-composite plate model is derived by Hamilton’s principle with the assumed mode method. Then an state observer involving time varying delay is constructed and the sufficient condition of the asymptotic stability is derived by using the Lyapunov-Krasovskii function, descriptor method and linear matrix inequalities (LMIs) for the state estimation error dynamical system. Subsequently, a sliding manifold is constructed on the estimation space. Then an observer-based controller is synthesized by using the SMC theory. The proposed SMC strategy ensures the reachability of the sliding manifold in the state estimate space. Finally, the simulation results are presented to demonstrate that the proposed observer-based controller strategy is effective in active aeroelastic control of subsonic piezo-composite plate involving time varying measurement delay.

Keywords
Piezo-composite plate, active vibration control, piezoelectric patch, time varying delay, observer-based sliding mode control

Introduction
Aeroelastic phenomenon is a kind of harmful vibration, which results from the interaction between aerodynamics, inertial force, and structural dynamics. When experiencing aerodynamic load, flexible structures such as wing, helicopter/wind turbine blade, beam, plate, shell may vibrate strongly, resulting in structural fatigue failure. To suppress the adverse vibrations, various kinds of control theories have been proposed or developed including PID control,1 robust control,2,3 adaptive control,4,5 sliding mode control,6,7 LQG regulator,8–10 adaptive nonlinear optimal control.11 Instead of full state measurements or observer design, Singh et al.12 proposed a feedback controller based on partially available measurements to achieve flutter suppression.

However, in the above studies, time delay (TD) is ignored. In practice, time delay is objective, which may result from data collection system and actuation system.13 TD may make the controller fail and even cause the aeroelastic system to switch from a stable state to an unstable state.14,15 In Ramesh and Narayanan,16 the dynamics of a two-dimensional airfoil with a constant TD is investigated by PID control strategy using a single state feedback signal. Yuan et al.17 focused on the nonlinear dynamical character of a two-dimensional supersonic lifting surface with constant TD. They found that TD has a significant effect on the bifurcating motion. For example, it could transfer subcritical Hopf bifurcations to supercritical. Similarly, Zhao18,19 proved that TD has a huge impact on flutter boundary of the controlled aeroelastic system. Thus, TD should not be ignored in the design of active aeroelastic control system.

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Almost all of the above studies are focused on the effect of TD on the dynamic characteristics of the system, few researchers are devoted to the design of active control aeroelastic systems involving TD. By using the Lyapunov-Krasovskii function, free-weighting matrix and LMIs, Zhao et al.20 presented $H_\infty$ control of a flexible plate with control input delay theoretically and experimentally. Luo et al.13 adopted the model transformations method to deal with the flutter of a 2D airfoil using SMC considering the control input delay. Ming-Zhou and Guo-Ping21 adopted the finite-time adaptive fault-tolerant control technique to depress the vibration of the 2D wing airfoil. Recently, a robust passive adaptive fault control for a 2D airfoil model with control input delay was studied by Li et al.22. In their study, a fault tolerant observer was designed to estimate the wing flutter states for control system.

However, all these studies are about 2D airfoil aeroelastic system with wing flap as the control surface. As best as the authors know, there is no literature report on the controller design for the high-dimensional aeroelastic system in the presence of time varying delay. Furthermore, piezoelectric materials can replace the unmeasurable states. A disturbance observer-based SMC for the system with time varying measurement delay, using the piezoelectric actuator.

SMC is a typical variable structure control method, which is famous for its robustness and insensitivity to uncertainty. SMC has been used to achieve the trajectory tracking of nonlinear robotic manipulator under varying loads problem,23 with uncertainties and external disturbance problem,24 or with backlash hysteresis problem.25 An adaptive fractional-order non-singular fast terminal SMC law was designed for a lower-limb exoskeleton system.26 Using SMC, a hybrid robust tracking control scheme was proposed for an underwater vehicle in dive plane.27 Researchers have also studied the observer-based SMC for the system with unmeasurable states. A disturbance observer-based super-twisting SMC was proposed for formation maneuvers of multiple robots.28 An extended state observer-based SMC strategy was proposed for an under-actuated quadcopter UAV.29 However, the literature concerning observer-based SMC for the aeroelastic system is very few.30

In our recent work, by using the PZT actuator, we have studied the observer-based SMC scheme for suppressing bending-torsion coupling flutter motions of a wing aeroelastic system with constant time measurement delay.31 In this paper, the model of the subsonic plate is chosen as the high-dimensional aeroelastic system, and we focus on observer-based controller design for the system with time varying measurement delay. Hamilton’s principle with the assumed mode method is applied to establish the aeroelastic model. Then, by using the Lyapunov-Krasovskii function, descriptor method, an observer is designed and the sufficient condition for the asymptotic stability of the observer is guaranteed in terms of linear matrix inequalities (LMIs). Then, the sliding mode control is employed on the estimation space to achieve the observer-based controller design. Lastly, the controller performance is verified by numerical simulation.

### Aeroelastic model and solution methodology

#### Mathematical model of the piezoelectric plate subjected to subsonic aerodynamics

A uniform simply-supported rectangular plate with piezoelectric patch actuator bonded on its top surface subjected to subsonic aerodynamics is considered. As shown in Figure 1, the plate has thickness $t_p$ and its dimensions along $x$ and $y$ directions are $a$ and $b$, respectively. Also the piezoelectric layer possesses thickness $t_p$ and its location coordinates along along $x$ and $y$ directions are $l_{1x}$, $l_{2x}$ and $l_{1y}$, $l_{2y}$, respectively.

The stress-strain relations of the base plate is presented as follows (refer to page 64 in Jalili22).

$$
\begin{align*}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_6
\end{bmatrix} =
\begin{bmatrix}
c_{11} & c_{12} & 0 \\
0 & c_{11} & 0 \\
0 & 0 & c_{66}
\end{bmatrix}
\begin{bmatrix}
S_1 \\
S_2 \\
S_6
\end{bmatrix}
\end{align*}
$$

where $\sigma_1$, $\sigma_2$, $\tau_6$, and $S_1$, $S_2$, $S_6$ are the normal and shear stresses and strains, $c_{ij}$ represent the stiffness coefficients, $S_i = -\frac{\partial \phi_i}{\partial x_j}$, $S_2 = -\frac{\partial \phi_2}{\partial y}$, $S_6 = -2z z \frac{\partial^2 w}{\partial x \partial y}$, and $w$ represents the transverse displacements of the plate.

It is noted that where the piezoelectric patch is attached ($l_{1x} < x < l_{2x}$, $l_{1y} < y < l_{2y}$), the neutral surface is changed to $z_n = \frac{E_p t_p}{2(E_p + E_b)t_p}$. $E_p$ and $E_b$ are the respective Youngs modulus of elasticity for plate and piezoelectric materials. Then the corresponding strain equations is modified to $S'_1 = -(z - z_n) \frac{\partial w}{\partial x}$, $S'_2 = -(z - z_n) \frac{\partial w}{\partial y}$, $S'_6 = -2(z - z_n) \frac{\partial^2 w}{\partial x \partial y}$, and the constitutive equations are modified as follows.
The total potential energy can be given as follows:

\[ U = \frac{1}{2} \int \sigma^T \sigma \, dV + \frac{1}{2} \int \left\{ \sigma^p \right\}^T \left\{ \sigma^p \right\} \, dV - \frac{1}{2} \int \sigma^p \, dV \]  

The total kinetic energy can be given by

\[ T = \int \rho(x, y) \left( \frac{\partial w(x, y, t)}{\partial t} \right)^2 \, dx \, dy \]  

where \( \rho(x, y) \) is the variable density of the combined piezoelectric and plate materials defined as

\[ \rho(x, y) = \rho_p t_p + \rho_p t_p G(x, y) \]  

with the piezoelectric/plate section indicator function \( G(x, y) \) given by

\[ G(x, y) = \begin{bmatrix} H(x - l_{1x}) - H(x - l_{2x}) & \vphantom{H(x - l_{1x}) - H(x - l_{2x})} \\
H(y - l_{1y}) - H(y - l_{2y}) & \vphantom{H(x - l_{1x}) - H(x - l_{2x})} \end{bmatrix} \]  

and \( H(x) \) is the Heaviside function, \( \rho_p \) and \( \rho_p \) are the respective plate and piezoelectric volumetric densities.

The virtual work by the aerodynamic pressure can be written as

\[ \delta W = \int_a^b \int_0^b (\Delta p \delta w) \, dx \, dy \]  

For low Mach number subsonic flow, the aerodynamic pressure is approximately given by Dowell and Ashley

\[ \Delta p = A_0 \rho_p U^2 \left[ \frac{\partial n_1^2}{\partial x^2} + 2 \frac{\partial n_2^2}{\partial x \partial t} + \frac{1}{U_x} \frac{\partial w^2}{\partial t} \right] \]  

Lemma 1 (Jensen’s inequality \(^{34}\) page 87) For any \( n \times n \) matrix \( R > 0 \), scalars \( a, b \) with \( 0 < a < b \), and a vector function \( \phi : [a, b] \rightarrow \mathbb{R}^n \) such that the integrations concerned are well defined, the following matrix inequality holds:

\[ \left[ \int_a^b \phi^T(s)R\phi(s) \, ds \right] \geq \frac{1}{b-a} \int_a^b \phi^T(s) \, ds \int_a^b \phi(s) \, ds \]  

Lemma 2 (page \(^{97}\) \( \) ) Let \( R_i \in \mathbb{R}^{n_i \times n_i} \), \( S_{ij} \in \mathbb{R}^{n_j \times n_j} \), for \( i = 1, \ldots, N \) be positive matrices. Then for all \( e_i \in \mathbb{R}^{n_i} \), \( e_n \in \mathbb{R}^{n_N} \), for all \( \alpha_i > 0 \) with \( \sum \alpha_i = 1 \) and for all \( S_{ij} \in \mathbb{R}^{n_i \times n_j} \), \( i = 1, \ldots, N \), \( j = 1, \ldots, i-1 \) such that:

\[ \left[ \frac{R_i}{S_{ij}} \right] \geq 0 \]  

the following inequality holds:
\begin{equation}
\sum_{i=1}^{N} \frac{1}{\alpha_i} e_i^T R e_i \geq 0
\end{equation}

\[ \sum_{i=1}^{N} \frac{1}{\alpha_i} e_i^T R e_i \geq 0 \]

**Observer-based sliding mode controller design and analysis**

**Observer design and stability analysis**

In this section, an observer-based sliding mode controller is designed, and the sufficient condition for the asymptotic stability of the observer and controller systems are derived in terms of LMIs.

The state observer is constructed of the following form for the delayed output system (14)

\[ \dot{x}(t) = A \hat{x}(t) + Bu + BL(y - C \hat{x}(t - \tau(t))) \]

where \( \hat{x} \) represents the estimate of the system states \( x \), and \( L \) is the observer feedback matrix to be designed later.

In view of (13) and (18), define \( e(t) = x(t) - \hat{x}(t) \), the state estimation error dynamics is given by:

\[ \dot{e}(t) = Ae(t) - BLCe(t - \tau(t)) \]

The following theorem gives the sufficient condition for the asymptotically stability of the state estimation error dynamical system (19).

**Theorem 1** If there exist scalar \( \epsilon \), matrices \( P_1 \geq 0 \), \( R_2 > 0 \), \( Q_3 > 0 \), and \( Q_4 > 0 \), \( Z \), \( S_{12} \), \( P_2 \) satisfying the following two LMIs:

\[ \begin{bmatrix}
\phi_{11} & \phi_{12} & S_{12} & R_2 - S_{12} - ZC \\
\phi_{21} & \phi_{22} & 0 & -\epsilon C \\
* & * & -(Q_3 + R_2) & -2R_2 + S_{12} - (1 - \delta)Q_4 \\
* & * & * & -2R_2 + S_{12} - (1 - \delta)Q_4
\end{bmatrix} \leq 0 \]

with

\[ \phi_{11} = A^T P_2 + P_2^T A + Q_3 + Q_4 - R_2, \]

\[ \phi_{12} = P_1 - P_2^T + \epsilon A^T P_2, \]

\[ \phi_{22} = -\epsilon P_2 - \epsilon P_2^T + \epsilon \hat{A} R_2 \]

And

\[ \begin{bmatrix}
R_2 & S_{12} \\
S_{12} & R_2
\end{bmatrix} \geq 0 \]

And * denotes the symmetric terms in a symmetric matrix, then the error dynamical system is asymptotically stable.

**Proof 1** Choose the Lyapunov-Krasovskii functional candidate as:

\[ V_2(t) = e^T(t) P_1 e(t) + d \int_{-d}^{0} \dot{e}^T(s) R_2 \dot{e}(s) dsd \theta + \int_{t-d}^{t} e^T(\tau) Q_3 e(\tau) d\tau + \int_{t-\tau(t)}^{t} e^T(s) Q_4 e(s) ds \]

Taking the time derivative, it follows that

\[ \dot{V}_2 = 2e^T(t) P_1 e(t) + \dot{e}^T(t) R_2 \dot{e}(t) - d \int_{t-d}^{t} e^T(s) R_2 \dot{e}(s) ds + e^T(t)(Q_3 + Q_4)e(t) - e^T(t - d)Q_4 e(t - d) - (1 - \delta(t)) e^T(t - \tau(t)) Q_4 e(t - \tau(t)) \]

By employing the following representation, we have

\[ d \int_{t-d}^{t} \dot{e}^T(s) R_2 \dot{e}(s) ds = d \int_{t-\tau(t)}^{t} \dot{e}^T(s) R_2 \dot{e}(s) ds \]

By lemma 1, applying Jensen’s inequality to both terms in equation (24), we get

\[ d \int_{t-\tau(t)}^{t} \dot{e}^T(s) R_2 \dot{e}(s) ds \geq \frac{d}{\tau(t)} \]

In view of equation (25) and lemma 2, we have

\[ -d \int_{t-\tau(t)}^{t} \dot{e}^T(s) R_2 \dot{e}(s) ds = -d \int_{t-d}^{t} \dot{e}^T(s) R_2 \dot{e}(s) ds \]

\[ -d \int_{t-\tau(t)}^{t} \dot{e}^T(s) R_2 \dot{e}(s) ds \]

\[ \leq -\frac{d}{\tau(t)} \int_{t-d}^{t} \dot{e}^T(s) R_2 \dot{e}(s) ds \]

\[ \leq -\frac{1}{\tau(t)} \int_{t-\tau(t)}^{t} \dot{e}^T(s) R_2 \dot{e}(s) ds \]

And

\[ \begin{bmatrix}
R_2 & S_{12} \\
S_{12} & R_2
\end{bmatrix} \geq 0 \]
can rewrite equation (27) into matrix form yields equation (20).

Asymptotically stable. By defining $Z = P_2^T \mathbf{B}L$, and $P_3 = \varepsilon P_2$, we can obtain the linear matrix inequality

$$
\dot{V}_2 \leq 2e^T(t)P_1 \dot{e}(t) + d^T \tilde{e}^T(t)R_2 \tilde{e}(t) + e^T(t)(Q_3 + Q_4)e(t) + e^T(t)(t)Q_4e(t) - (e(t) - e(t))^T R_2(e(t) - e(t))
$$

Substitute equation (26) into equation (23), we obtain

$$
\dot{V}_2 \leq 2\left[e^T(t)P_1 \dot{e}(t) + d^T \tilde{e}^T(t)R_2 \tilde{e}(t)
\right.
\left. + e^T(t)(Q_3 + Q_4)e(t)
\right]
$$

Then, by adopting the descriptor method, where the right-hand side of the following expression

$$
0 = 2[e^T(t)P_2^T + \tilde{e}^T(t)P_3^T]
\left[\dot{e}(t) - BLCe(t) - \tilde{e}(t)\right]
$$

with proper dimensions matrices $P_2, P_3$ is added to the right-hand side of equation (27).

Define $\chi = [e(t) \dot{e}(t) e(t - d) e(t - \tau(t))]^T$, we can rewrite equation (27) into matrix form yields

$$
\dot{V}_2 \leq \chi^T(t)\Omega\chi(t)
$$

where

$$
\Omega = \begin{bmatrix}
\phi_{11} & \phi_{12} & S_{12} & R_2 - S_{12} - P_2^T BLC \\
0 & \phi_{22} & \ast & -(Q_3 + R_2)
\end{bmatrix}
$$

Hence, for $\chi(t) \neq 0$ it follows from (29) that $\dot{V}_2 < 0$ if $\Omega < 0$. This implies that the error dynamical system is asymptotically stable. By defining $Z = P_2^T \mathbf{B}L$, and $P_3 = \varepsilon P_2$, we can obtain the linear matrix inequality equation (20).

**Sliding mode controller design and stability analysis**

For the state estimate system (18), a sliding manifold can be constructed as:

\[ s(t) = B^T \tilde{p}(t) \]  

where $P \in \mathbb{R}^{\times 8}$ is an symmetrical positive determined matrix.

According sliding mode control theory, setting $\dot{s}(t) = 0$, we can obtain the equivalent control law

\[ u = - (B^T \mathbf{PB})^{-1} B^T P \left( A \tilde{s}(t) + BL(y - C \tilde{s}(t - \tau(t))) \right) \]

Combining the robust term, our proposed terminal sliding mode controller is designed as:

\[ u = - (B^T \mathbf{PB})^{-1} B^T P \left( A \tilde{s}(t) + BL(y - C \tilde{s}(t - \tau(t))) \right) - (B^T \mathbf{PB})^{-1} \left[k_s(t) + \eta \|s(t)\|^{q/p} s^\#(t) \right] \]

where $k, \eta > 0$, lpt $p, q$ are positive odd integers and $p > q$.

Substituting equation (33) into equation (18) yields

\[ \dot{s}(t) = - \left[k_s(t) + \eta \|s(t)\|^{q/p} s^\#(t) \right] \]

The finite time reachability and stability analysis of the sliding mode control has been given in Appendix.

Similar to our previous work, the auxiliary state feedback matrix $\mathbf{P}$ can be obtained by solving the inequality on the variables $X$ and $F$ as follows

\[ AX - BF + XAT - F^T B^T < 0 \]

where $X = P^{-1}$. See the Appendix for the detailed derivation process.

**Numerical simulations**

In this section, numerical simulations are carried out to illustrate the effectiveness of our proposed control strategy. The lay-up configuration of the plate and material properties of the piezo-actuator are displayed in Table 1. In the simulation, choose $M = 4, N = 1$ in equation (11), then the modal variable $x(t)$ is a eight-dimensional vector. Adequate structural modes only affect the precise properties of the piezo-actuator are displayed in Table 1.

![Table 1. Model parameters.](image)

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| $a$       | 0.2 m | $t_b$     | 0.0008125 m |
| $b$       | 0.8 m | $\rho_o$  | 3960 kg/m$^3$ |
| $sp$      | 0.0002032 m | $\rho_p$  | 7750 kg/m$^3$ |
| $f_{31}$  | $5 \times 10^7$/m | $\rho_a$  | 1.225 kg/m$^3$ |
| $J_s$     | $66.47 \times 10^7$/m | $E_0$     | $69 \times 10^7$/m$^2$ |
| $f_{21}$  | $-1.8 \times 10^{-9}$/m | $\eta_0$  | 0.25 |
| $I_{x,t}$ | 0.3 | $\beta_{13}^2$ | $-4.179 \times 10^7$ |
| $I_{z,t}$ | 0.1 m | $I_{yr}$   | 0.1 m |
| $I_{z,t}$ | 0.2 m | $I_{yr}$   | 0.2 m |
divergence speed $U_a = 89.82m/s$ is calculated by stability analysis. Divergence speed is located at the point where one of the real parts of the roots of the characteristic equation of the matrix $A$ in (13) become positive. Figure 2 depicts the time histories of the forth order mode state variables of the open-loop system under $U_a = 89.82m/s$. It can be seen from the figure that the trajectory of mode $x_4$ diverges while other mode states converge after an initial transient. Figure 3 studies sensitivity of vibration response to $x$-direction with fixed $y/b = 0.5$. From Figure 3, we can see that except for the convergence of the vibration response at coordinate $(x/a = 0.5, y/b = 0.5)$, the responses at other coordinates are divergent. Figure 4 studies sensitivity of vibration response to $y$-direction with fixed $x/a = 0.5$. As can be seen from Figure 4, the vibration responses at all coordinates have converged. Increasing or decreasing $y/b$ with coordinate $(x/a = 0.5, y/b = 0.5)$ as the reference, the response amplitude will gradually decrease as evidenced in Figure 4. Comparing Figure 3 with Figure 4, we can find that the vibration response is sensitive to both $x/a$ and $y/b$ coordinates, while the $x/a$ coordinate has the more significant effects on the system instability.

To achieve vibration suppression, active control strategies will be implemented. In the closed loop system, Figure 5 shows the time history of the time varying measurement delay $\tau(t)$ in equation (14) and $h = 0.7$. Output matrix $C$ is assumed as identity matrix. The observer gain matrix $BL$ and the controller auxiliary feedback matrix $P$ can be obtained by solving the LMIs (20) (35). Other control parameters are selected as $\mu = 0.01$, $\eta = 2$, $p = 5$, $q = 3$. The input saturation is also considered and assumed $u(t) \in [-600V, 600V]$. Furthermore, an LMI-based control method is used as a comparative study.\(^3\) (Remark: for clarity, we only drew the result curves with obvious differences.) Figures 6 and 7 shows the simulation results of the closed loop system. From these figures, it’s shown that
Figure 4. Time history of $w(x/a = 0.5, y/b)$. 

Figure 5. Time varying delay. 

Figure 6. System states $x_1, x_2, x_3, x_4$ and their estimation.
all of the system states can be stabilized in finite time, and all of their estimates can track their true values from the beginning. The settling time of mode state variables $x_4$ and $x_8$ are about 18 s, and other mode states gradually converge to 0 after about 2 s and the vibration amplitudes quickly decay to 0. Figure 8 shows the control signals of piezo-actuator, and the maximum input voltage is $\pm 600V$. From the figure, we can see that the chattering phenomenon is well suppressed, and the curve is much smoother compared with the LMI-based control method. Figure 9 shows the vibration response along the $x$-direction with fixed $y/b = 0.5$. It can be seen from the Figure 9, all of the vibration responses converge, and have the same settling time. Comparing Figure 9 with Figure 3, we can see that under the control law, the original divergent states (at $x/a = 0.2$, $x/a = 0.6$, $x/a = 0.9$) changes to the convergent states, and the original convergent state (at $x/a = 0.5$) remains convergent. From these simulation results, it can be concluded the proposed controller and observer are effective to deal with the active aeroelastic control problem with time varying delay output.

**Conclusion**

In this article, an observer based sliding mode controller was proposed for active aeroelastic control of subsonic piezo-composite plate subject to time varying measurement delay. The piezoelectric patch was bonded on the top surface of the plate as an actuator. Making using of simplified unsteady aerodynamics and adopting the two-dimensional piezoelectric actuation theory,
the coupled dynamical model of the aeroelastic system has been formulated by means of Hamilton’s principle with assumed mode method. The instability bound has been calculated by solving the system eigenvalues. The vibration responses at different coordinates are investigated. To achieve vibration suppression, the observer based sliding mode controller was designed, and the corresponding gain matrices are obtained by solving the LMIs. The asymptotic convergence was guaranteed by Lyapunov stability theory. The major conclusions can be drawn as follows

1. In the open loop system, the vibration response depends on both \( x/a \) and \( y/b \) coordinates of the plate, while the \( x/a \) coordinate has the more significant effects on the system instability.
2. The proposed observer based sliding mode controller is effective to eliminate unstable response and stabilize the system in finite time.

The results presented in this article indicate that our proposed control strategy can effectively deal with active aeroelastic control of subsonic piezo-composite plate problem with time varying measurement delay.

Authors’ Note

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Figure 9. Time history of \( w(x/a, y/b = 0.5) \) under control.
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**Appendix**

**Reachability and Stability Analysis**

*Theorem 2* When the following reaching law is adopted\(^{36}\)

\[
\dot{s}(t) = \left( -k s(t) - \eta \right) ||s(t)||^{q/p-1} s(t)
\]  

(36)

the state trajectories of the estimate space will reach the sliding surface \(s(t) = 0\) in a finite time \(t_s\)

\[
t_s = \frac{p}{k(p-q)} \ln \frac{k||s(0)||^{(p-q)/p} + \eta}{\eta}
\]  

(37)

*Proof 2* Right multiply equation (36) by \(s^T(t)\) and one can obtain

\[
\dot{s}(t)s^T(t) = \left( -k - \eta \right) ||s(t)||^{q/p-1} ||s(t)||^2
\]  

(38)

On the other hand,

\[
\dot{s}(t)s^T(t) = \frac{1}{2} \frac{d||s(t)||^2}{dt} = \frac{1}{2} \frac{d||s(t)||}{dt} - ||s|| \frac{d||s||}{dt}
\]  

(39)

The right hand side of equation (38) and equation (39) are equal, then we can obtain

\[
\frac{d||s(t)||}{dt} = -k - \eta \left[ ||s(t)||^{q/p-1} ||s(t)|| \right]
\]  

(40)

*From simple calculation, one can get*

\[
dt = - \frac{d||s(t)||}{k||s(t)|| + \eta ||s(t)||^{q/p}}
\]  

\[
= - \frac{||s(t)||^{q/p} d||s(t)||}{k||s(t)||^{(p-q)/p} + \eta}
\]  

\[
= - \frac{p}{(p-q)k||s(t)||^{(p-q)/p} + \eta}
\]  

(41)
Integral equation (41) from 0 to \( t_s \), and let \( s(t_s) = 0 \), then one can obtain the reaching time \( t_s \) as follows

\[
\begin{align*}
    t_s &= -\frac{p}{(p-q)} \int_{0}^{s(t_s)} \frac{d[s(t)](s(t))^{\phi}}{k||s(t)||^{\phi+q/p} + \eta} \\
    &= -\frac{p}{(p-q)k} \ln(k||s(t)||^{\phi+q/p} + \eta) \\
    &= -\frac{p}{(p-q)\mu} \ln |s(0)|^{\phi+q/p} + \eta
\end{align*}
\]

(42)

Theorem 3 Consider the system (18) with the sliding function in (31). If the SMC law is designed as equation (32), the state trajectories of the estimate space can be driven onto the switching surface \( s(t) = 0 \) in finite time and remain there in subsequent time.

Proof 3 Choose the Lyapunov functional candidate as

\[
V_1 = \frac{1}{2}s^T(t)s(t)
\]

(43)

From equation (31) and equation (32), the derivative of equation (43) is given by

\[
\begin{align*}
    \dot{V}_1 &= s^T(t)\dot{s}(t) \\
    &= s^T(t)(B^TPAX(t) + Bu + BL(y - C\hat{x}(t - \tau(t)))) \\
    &= s^T(t)(B^TPA\hat{x}(t) + B^TPBu + B^TPBL(y - C\hat{x}(t - \tau(t)))) \\
    &= s^T(t)(B^TPA\hat{x}(t) - B^TP(A\hat{x} + BL(y - C\hat{x}(t - \tau(t)))) + B^TPBL(y - C\hat{x}(t - \tau(t)) - k\hat{x}(t) - \eta||s(t)||^{\phi+1}s(t)) \\
    &= s^T(t)[-k\hat{x}(t) - \eta||s(t)||^{\phi+1}s(t)] \\
    &= -k||s(t)||^{\phi+1} - \eta||s(t)||^{\phi+q/p}
\end{align*}
\]

(44)

Since \( p + q \) is even, therefore \( \dot{V}_1 \leq 0 \).

Combing the above two theorems, it can be concluded that the state trajectories of the estimate space will be driven onto the switching surface \( s(t) = 0 \) in finite time and remain there in subsequent time.

**Auxiliary state feedback Matrix P design**

The control law equation (32) is rewritten as:

\[
u = -K\hat{x} + u_1
\]

(45)

Where \( u_1 = K\hat{x} + u \) and \( K \) represents feedback collocation matrix, and will be decided later.

Consider the following Lyapunov function:

\[
V_2 = \hat{x}^TP\hat{x}
\]

(46)

Then the derivative of \( V_2 \) is given by:

\[
\dot{V}_2 = 2\hat{x}^TP\hat{x} = 2\hat{x}^T(A\hat{x} + Bu + BL(y - C\hat{x}(t - \tau(t))))
\]

(47)

where \( J = A - BK \).

According to the above stability analysis, when \( t > t_s \), the state trajectories reach the sliding surface, satisfying \( S = B^TPx = 0 \). Then we can obtain:

\[
\dot{V}_2 = 2\hat{x}^TPJ\hat{x} = \hat{x}^T(PJ + J^TP)\hat{x}
\]

(48)

In order to ensure \( \dot{V}_2 < 0 \), the following inequality should be satisfied:

\[
PJ + J^TP < 0
\]

(49)

The two side of the inequality are pre-and post-multiplied by \( P^{-1} \), we can obtain:

\[
JP^{-1} + P^{-1}J^T < 0
\]

(50)

Define \( X = P^{-1} \), then the following form can be obtained:

\[
(A - BK)X + X(A - BK)^T < 0
\]

(51)

Define \( F = XK \), we get the following inequality:

\[
AX - BF + XAT - FTB^T < 0
\]

(52)