4D construction of bulk supersymmetry breaking

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Abstract

In this letter, we discuss a four-dimensional model with modulus fields which are responsible for supersymmetry breaking. Given non-trivial moduli dependence of the action, the model is found to give a proper description of higher-dimensional supersymmetry breaking. We explicitly calculate gaugino and scalar mass spectrum and show that several classes of scenarios proposed in the literature are described in certain regions of the parameter space of the moduli vacuum expectation values. The model in other generic regions of the moduli space gives unexplored scenarios (mass spectra) of supersymmetry breaking in four dimensions.

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1 Introduction

Supersymmetry is one of the most interesting ideas which have been introduced to overcome some unsatisfactory points of the Standard Model. For example, the gauge coupling unification from the precise electroweak measurements [1] and the stability of the Planck/weak mass hierarchy [2] are great successes of phenomenological applications of supersymmetry. It is, however, experimentally certain that supersymmetry is broken above the weak scale, while a variety of mechanisms for supersymmetry breaking have been proposed so far.

Among these, the mechanisms which are involved in higher-dimensional physics have been extensively studied in various ways. The existence of extra spatial dimensions provides novel ways to break supersymmetry and to communicate it to our four-dimensional world, which is the low-energy effective theory of the models. A well-known framework is the string-inspired four-dimensional supergravity [3]. In large classes of these models, there are two modulus fields concerned with the compactified extra dimensions, called dilaton and overall modulus, which are assumed to develop non-vanishing vacuum expectation values (VEV) in their auxiliary components. The supersymmetry-breaking effect is transmitted to our low-energy degrees of freedom via (super)gravity interactions. There have been other interesting mechanisms for supersymmetry breaking with extra dimensions [4]. These approaches provide phenomenologically viable particle spectra due to intrinsic nature of higher-dimensional theories.

In this letter, we present a purely four-dimensional framework which can describe supersymmetry breaking in the bulk. To this end, it is convenient to regard extra dimensions as being latticized [5, 6]. With this method, it is possible to revisit many interesting feature of higher-dimensional effects from the four-dimensional point of view [7]. Thus, models can incorporate various four-dimensional mechanisms such as for flavor problems, and at the same time utilize five-dimensional nature stated above. We study a model with two types of modulus fields which are supposed to have supersymmetry-breaking VEVs. Given non-trivial moduli dependences of the action, it is found that certain limits in this two-dimensional parameter space of VEVs reproduce the mass spectra of the bulk scenarios in the literature. Other generic regions of the moduli space give unexplored scenarios for supersymmetry breaking.

In Section 2, we explain our setup and briefly touch on the spectrum of vector fields. Supersymmetry breaking (non-zero $F$-terms) in this model is discussed in Section 3, where we identify various modulus fields and reveal their connections in the light of the construction of model. In Section 4, we calculate the mass spectrum of vector multiplets with the non-vanishing moduli $F$-terms, and show typical mass splitting in the limits that correspond to various supersymmetry-breaking mechanisms in higher
dimensions. Section 5 is devoted to the summary of our results.

2 Model

We consider a four-dimensional supersymmetric gauge theory with the \( N \) copies of gauge groups \( G^N = G_1 \times G_2 \cdots \times G_N \). We assume that, for simplicity, all the gauge theories \( G_i \)'s have the same structure and particularly have the common gauge coupling \( g \). The \( N = 1 \) vector multiplet \( V_i \) of the \( G_i \) gauge theory contains a gauge field \( A^i_\mu \) and a gaugino \( \lambda^i \). In addition, there are \( N = 1 \) chiral multiplets \( Q_i \) \((i = 1, \cdots, N)\) in bifundamental representation, that is, \( Q_i \) transforms as \( (\Box \Box) \) under the \((G_i, G_{i+1})\) gauge symmetries. The fields \( Q_i \) are referred to as link variables in that they link up two neighboring gauge theories. The field content of the theory is summarized in Table 1. It is shown that this simple model can imitate a five-dimensional theory with

|         | \( G_1 \) | \( G_2 \) | \( G_3 \) | \( \cdots \) | \( G_N \) |
|---------|---------|---------|---------|---------|---------|
| \( Q_1 \) | \( \Box \) | \( \Box \) | 1       | \( \cdots \) | 1       |
| \( Q_2 \) | 1       | \( \Box \) | \( \Box \) | \( \cdots \) | 1       |
| \vdots  | \vdots  | \vdots  | \vdots  | \( \ddots \) | \vdots  |
| \( Q_N \) | \( \Box \) | 1       | 1       | \( \cdots \) | \( \Box \) |

Table 1: The matter content

bulk gauge multiplets [5, 6]. Consider the link variables \( Q_i \) develop vacuum expectation values proportional to the identity, \( \langle Q_1 \rangle = \cdots = \langle Q_N \rangle = v \). Below the scale \( \sim g v \), the gauge symmetries are reduced to a diagonal subgroup \( G \) and the other gauge multiplets become massive with discrete mass spectrum. This just looks like a five-dimensional \( G \) gauge theory compactified on a circle \( S^1 \), resulting Kaluza-Klein mass spectra. Note here that it is a simple assumption for the bulk theory being five-dimensional Lorentz invariant that the gauge couplings and VEVs of \( Q_i \) take the common values. This way of deconstructing or latticized dimensions is useful in that one can study higher-dimensional theories from a familiar four-dimensional point of view.

We here briefly review the mass spectrum of gauge bosons in this model [5, 6]. The complete Lagrangian and mass spectra are given later. The mass matrix is derived from the Kähler term of \( Q_i \) fields, which gives

\[
\mathcal{L} = \frac{1}{2} k^2 g^2 v^2 A^i_\mu M_{ij} A^{\mu j},
\]

\[ (2.1) \]

*The diagonal form of VEVs is provided, for example, by a superpotential introduced in [5, 8], which gives (supersymmetry-breaking) masses only to the trace part of \( Q_i \), and does not affect the discussion below.
where we have not written the implicit dependence of gauge indices, and $k$ is the normalization factor of link variables that could depend on the gauge coupling $g$ (see the Lagrangian (4.1)). The matrix $M$ is

$$M = \begin{pmatrix}
2 & -1 & -1 \\
-1 & 2 & -1 \\
& & \ddots & \ddots & \ddots \\
& & -1 & 2 & -1 \\
& & & -1 & 2
\end{pmatrix}.$$ (2.2)

From this, one obtains the mass eigenvalues $m_n^2$ and the corresponding eigenstates $\tilde{A}_n$, labelled by an integer $n$ (the Kaluza-Klein level),

$$m_n^2 = 4k^2 g^2 v^2 \sin^2 \frac{n\pi}{N},$$ (2.3)

$$\tilde{A}_n^\mu = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} (\omega_n)^j A_j^\mu, \quad (n = 0, \cdots, N - 1)$$ (2.4)

where $\omega_n = e^{2\pi i n/N}$. One can see that there is a massless gauge boson and in addition the Kaluza-Klein tower of massive gauge fields, the low-lying modes ($n \ll N$) of which gauge bosons have masses approximately written as

$$m_n \simeq 2k g v \frac{n\pi}{N} = \frac{n}{R},$$ (2.5)

where we identify the compactification radius as $2\pi R = N/k gv$. The mass term (2.1) becomes the kinetic energy transverse to the four-dimensions in the continuum limit ($N \to \infty$).

3 Moduli and supersymmetry-breaking scenarios

3.1 Moduli

A supersymmetry-breaking scenario in this type of models was examined in [8, 9] assuming that supersymmetry-breaking dynamics is on one endpoint of the lattice sites. From a five-dimensional viewpoint, that corresponds to supersymmetry being broken only on a four-dimensional space like the gaugino mediation [10].

In this work, we study supersymmetry breaking in the above four-dimensional model. To have insights into bulk symmetry breaking, there need to be some modulus fields which are commonly coupled to any multiplet in the theory. Here we consider two candidates of these moduli. One is the dilaton field $S$. One may define a modulus $S_i$ for each gauge group whose scalar component gives a gauge coupling constant $g_i$. As noted before, however, they have to interact with a universal strength in order for
this model to describe a proper five-dimensional theory (on the flat background). In
what follows, we therefore assume \( S \equiv S_1 = \cdots = S_N \). We have also assumed the
universal value \( v \) for the VEVs of link variables. Another modulus we will consider is
referred to as \( Q \) which gives this universal VEV. The modulus \( Q \) may be a normalized
composite field of \( Q_i \)'s. We take the modulus forms which are invariant under
a translation transverse to the four dimensions, for simplicity. Non-universal values
of couplings and VEVs may be interpreted as the presence of brane-like interactions
and/or curved backgrounds, and that issue will be studied elsewhere.

These modulus fields may have some connections with spacetime symmetries since
the modulus \( S \) corresponds to dilatation and \( Q \) relates to a size of compactification ra-
dius. It should be noticed that, exactly speaking, \( S \) is neither four-, nor five-dimensional
dilaton, and \( Q \) might not be the radion correctly. In our model, all of these are not
independent variables as seen below.

Let us discuss the relations between these combinations of modulus fields. First we
have the dilaton \( S \) and the modulus \( Q \) whose VEVs are assumed to be

\[
S = \frac{1}{4g^2} + F_S \theta^2, \quad (3.1)
\]
\[
Q = v + F_Q \theta^2. \quad (3.2)
\]

In addition to these, we define the (combinations of) moduli fields that give the fol-
lowing VEVs;

\[
S_4 = \frac{1}{4g_4^2} + F_{S4} \theta^2, \quad (3.3)
\]
\[
S_5 = \frac{1}{4g_5^2} + F_{S5} \theta^2, \quad (3.4)
\]
\[
T = \frac{1}{R} + F_T \theta^2, \quad (3.5)
\]

where \( g_4, g_5 \) are the effective four- and five-dimensional gauge couplings, and \( R \) is the
compactification radius of extra dimensions. By comparing the low-energy description
of the model (at the energy below \( \sim v \)) with Kaluza-Klein theory, the following tree-
level relations among the parameters are found [5, 6]†,

\[
\frac{1}{2\pi R} \frac{1}{N} = \frac{kgv}{N}, \quad g_4^2 = g_5^2. \quad (3.6)
\]

The first equation is required to match the spectrum to that of Kaluza-Klein theory, and
the second equation can be regarded as a volume suppression of bulk gauge coupling.

†The 1PI and holomorphic gauge couplings differ only at higher-loop level in perturbation theory.
In addition, the five-dimensional gauge coupling is defined as (irrespectively of how to get a five-dimensional model)

\[ g_5^2 = 2\pi R g_4^2, \quad (3.7) \]

which comes from the normalization of gauge kinetic terms. These relations among the couplings suggest that the modulus fields satisfy the relations

\[ S_4 = NS, \quad (3.8) \]
\[ S_5 = \frac{1}{2} Q S^{1/2} k(S) , \quad (3.9) \]
\[ T = \frac{\pi}{N} Q S^{-1/2} k(S) . \quad (3.10) \]

The appropriate form of the factor \( k(S) \) will be fixed in the next section by holomorphy and other phenomenological arguments. From these, we see that \( S_4, S_5 \) and \( T \) are expressed in terms of two moduli \( S \) and \( Q \). Of course every choice of two independent moduli such as \((S, S_5), (S_4, T)\), etc., can describe the same physics, and in the present four-dimensional model, a natural choice is \((S, Q)\). Each set of non-vanishing \( F \)-terms corresponds to one supersymmetry-breaking scenario.

Extracting the \( \theta^2 \) terms, we obtain the \( F \)-components of moduli

\[ F_{S_4} = NF_S, \quad (3.11) \]
\[ F_{S_5} = \frac{k v}{4 g} \left( \frac{F_Q}{v} + 2g^2 \left( 1 + 2 \left< \frac{\partial \ln k(S)}{\partial \ln S} \right> F_S \right) \right), \quad (3.12) \]
\[ F_T = \frac{2\pi g v}{N} \left( \frac{F_Q}{v} - 2g^2 \left( 1 - 2 \left< \frac{\partial \ln k(S)}{\partial \ln S} \right> F_S \right) \right). \quad (3.13) \]

It is emphasized that the four-dimensional dilaton \( S_4 \) is almost close to the dilaton \( S \), but its \( F \)-term satisfies the relation

\[ \frac{F_{S_4}}{\langle S_4 \rangle} = \frac{F_{S_5}}{\langle S_5 \rangle} - \frac{F_T}{\langle T \rangle}, \quad (3.14) \]

independently of the detailed form of \( k(S) \). Notice that this relation comes out through the equation \((3.7)\), which implies \( S_4 \) depends on the radion \( T \) and the five-dimensional dilaton \( S_5 \).

In the next section, we will discuss supersymmetry-breaking effects of these moduli and calculate sparticle mass spectrum of the model. When introducing appropriate potentials for the modulus fields, their VEVs are fixed to some region or point in the moduli space of vacua. For example, since \( S \) is the dilaton for each gauge group, dilaton stabilization mechanisms proposed in the literature are easily incorporated in our
framework. The situation is similar for the modulus $Q$, corresponding to the radion. Moreover in describing five-dimensional theory, $Q$ is actually assumed to be stabilized by relevant superpotential terms as in Refs. [5, 8]. It is therefore understood that deformation of (superpotential) terms could also induce a supersymmetry-breaking VEV of $Q$. However, instead of doing that, we study a more generic case. That is, in this letter we investigate the whole parameter space of the moduli $F$-terms, and then focus on several limits corresponding to bulk supersymmetry-breaking scenarios. We do not try to construct specific dynamics for modulus fields to have five-dimensional nature by tuning potential couplings, since our aim here is not to present five-dimensional theories. It is only the specific region of moduli space where our model reproduces the known bulk supersymmetry-breaking scenarios. In other words, the present framework contains unexplored four-dimensional phenomena of supersymmetry breaking. It should be noted that the tree-level mass formulae given in the next section are not modified by the existence of moduli potentials. The only possible case where the mass formula might be affected is that potentials for moduli stabilization contain the multiplets for which one wants to calculate their spectrum. We do not consider such a peculiar case in this letter.

3.2 Supersymmetry breaking in the bulk

So far various supersymmetry breaking models have been discussed in the literature within the frameworks of being concerned with higher dimensional physics, and several examples are mentioned in the Introduction. In the following, we will particularly focus on the dilaton and moduli dominated supersymmetry breaking in the string-inspired four-dimensional supergravity [3] and supersymmetry breaking by the radion $F$-term [11, 12]. Here one should pay attention to relevant choices of modulus $F$-terms in examining supersymmetry-breaking models. That is, four-dimensional (low-energy effective) theories know $F_{S4}$ and $F_T$ as fundamental quantities, but on the other hand, five-dimensional ones $F_{S5}$ and $F_T$. This point is important to the following discussion.

The dilaton dominance scenario is the four-dimensional supergravity specified by a non-vanishing $F$-term of the four-dimensional dilaton $S_4$ and negligible contribution from the overall modulus $T$. We find from the result in the previous section that in the model where the appropriate modulus fields are $S$ and $Q$, the scenario is described by $F_S \neq 0$ and $F_Q = 2g^2 v (1 - 2 \langle \frac{\partial \ln k(S)}{\partial \ln S} \rangle) F_S$. The VEVs of the four and five-dimensional dilaton $F$-terms are then found to be $F_{S4}/\langle S_4 \rangle = F_{S5}/\langle S_5 \rangle = F_S/\langle S \rangle$.

On the other hand, the moduli domination is also the four-dimensional model characterized by the opposite limit of $F$-terms; a non-zero $F_T$ and a vanishing dilaton $F$-term, $F_{S4} = 0$. As a typical spectrum of this scenario, gauginos are massless at tree level. This is because the string perturbation theory shows that the gauge kinetic
function, which induces gaugino masses, depends only on $S_4$ at tree level. This limit of $F$-terms is translated into the present model as $F_S = 0$ and $F_Q \neq 0$. The other modulus $F$-components are then given by $F_{S5} = (k/4g)F_Q$ and $F_T = (2\pi kg/N)F_Q$.

The field-theoretical model similar to the moduli dominated supersymmetry breaking is discussed in [13]. This Kaluza-Klein mediation model is a four-dimensional effective theory and has the identical $F$-term VEVs with those in the moduli domination. Sparticle mass spectra in this case are easily calculated from renormalization-group functions in four dimensions, and the mechanism has a wide variety of realistic model construction.

A related idea utilizing $F_T$ supersymmetry breaking is suggested in the radion mediation model. It is a five (or higher) dimensional model, and therefore a reasonable choice of two independent moduli is $T$ and $S_5$. The radion mediation is thus defined by $F_T \neq 0$ and $F_{S5} = 0$. In turn, this corresponds to $F_S \neq 0$ and $F_Q = -2g^2v(1 + 2(\partial \ln k(S))/\partial \ln S))F_S$ in the present model. As a result, the four-dimensional dilaton $F$-term becomes

$$F_{S4} = \frac{-N}{2g^2v} \left(1 + 2\left(\frac{\partial \ln k(S)}{\partial \ln S}\right)\right)^{-1} F_Q = \frac{-N^2}{8\pi kg^3v} F_T.$$  (3.15)

This means the four-dimensional gaugino mass $m_\lambda = -F_{S4}/2\langle S_4 \rangle = F_T/2\langle T \rangle$, which agrees with the result of zero-mode gaugino mass in [12].

In this way, we show via deconstruction that various known supersymmetry-breaking scenarios can be seen by the difference in the choices of non-zero modulus $F$-terms (as summarized in Table 2). The parameter space spanned by two independent $F$-terms is therefore the space of supersymmetry breaking in the bulk, and several special limits in this parameter space correspond to the scenarios which have been discussed in the literature.

| moduli | $F_S$ | $F_Q$ | $F_{S4}$ | $F_{S5}$ | $F_T$ |
|--------|-------|-------|----------|----------|-------|
| dilaton ($F_T = 0$) | $F_S$ | $2g^2v(1-x)F_S$ | $NF_S$ | $kgvF_S$ | 0 |
| moduli ($F_{S4} = 0$) | 0 | $F_Q$ | 0 | $k/4gF_Q$ | $2\pi kgF_Q$ |
| radion ($F_{S5} = 0$) | $-F_Q$ | $F_Q$ | $-NF_Q$ | $0$ | $4\pi kgF_Q$ |

Table 2: The moduli $F$-terms and the typical supersymmetry-breaking models in the bulk. The parameter $x$ is defined by $x \equiv 2(\partial \ln k(S))/\partial \ln S)$. The holomorphy and some phenomenological arguments suggest $x = 1$ and $k = 1/g$. 

7
4 Spectrum

In this section, we explicitly show the resulting supersymmetry-breaking spectrum of Kaluza-Klein modes. We here focus on the vector multiplets, but the quantitative aspects discussed below are completely the same for bulk hypermultiplets.

Since we consider broken gauge symmetries and massive gauge fields, it is convenient to use the unitary gauge for vector multiplets. In this gauge, the goldstone chiral multiplets (the fluctuations around the VEVs (3.2)) are absorbed into the vector multiplets with suitable gauge transformations. Consequently each vector multiplet contains a massive vector field and two spinors, gaugino and goldstone fermion. In addition, other dynamical and auxiliary bosonic components are introduced. The link variables \( Q_i \)'s are then treated as background fields with non-zero VEVs.

First it is easily found that the gauge fields do not get supersymmetry-breaking contribution, and the mass spectrum is just given by that calculated in Section 2; one massless gauge multiplet corresponding to the diagonal subgroup \( G \) and the Kaluza-Klein tower with discrete mass spectrum (2.3).

The gauge fermion masses with supersymmetry breaking are calculated as follows. The relevant piece of Lagrangian is

\[
\mathcal{L} = \sum_i \left[ \int d^2 \theta W_i W_i + \text{h.c.} + \int d^2 \theta d^2 \bar{\theta} K(S, S^\dagger) Q_i^\dagger \sigma^V Q_i \right].
\]

We have included the universal couplings of the dilaton \( S \). The relevant field to appear here is \( S \), and not the effective four or five-dimensional dilaton \( S_4, S_5 \). The real function \( K(S, S^\dagger) \) fixes the overall scale of discrete mass spectra of this model (\( k = \langle K |_{\theta=0} \rangle^{1/2} \)) and its form will be determined later. Inserting the VEVs of (3.1) and (3.2), the mass terms take the following form;

\[
\mathcal{L}_{\text{mass}} = -F_S \lambda_i \lambda_i - k^2 v^2 \chi_i M_{ij} \lambda_j + \frac{1}{2} k^2 v (F_Q + v \langle \frac{\partial \ln K(S, S^\dagger)}{\partial S} \rangle F_S) \chi_i M_{ij} \chi_j + \text{h.c.},
\]

where \( \chi_i \) is the goldstone fermion now included in the vector multiplet \( V_i \). The first term comes from the gauge kinetic term and the last two are induced by the tree-level Kähler term of \( Q_i \), so the flavor structure is the same as that of the gauge fields, which is explained by the matrix \( M \) (2.2). Since \( M \) also defines the kinetic terms of \( \chi_i \)'s, the canonically normalized fields are obtained by the redefinition \( k v P \chi \rightarrow \chi \) where \( P \) is a square-root of \( M \) (\( M = P^t P \)) and written by

\[
P = \begin{pmatrix}
1 & -1 & \cdots & \cdots & -1 \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
-1 & -1 & \cdots & \cdots & 1
\end{pmatrix}.
\]
With this redefinition and a rescaling $\lambda \to g\lambda$, the mass matrix of the normalized spinors becomes

$$
\mathcal{L}_{\text{mass}} = \frac{1}{2} \left( \begin{array}{c} \chi \\ \lambda \end{array} \right) \left( \begin{array}{cc} 2g^2F_S & k gvP \langle \frac{\partial \ln K(S,S^\dagger)}{\partial S} \rangle F_S \\ k gvP & -F_Q \end{array} \right) \left( \begin{array}{c} \chi \\ \lambda \end{array} \right) \ + \ h.c. \ . \ \ (4.4)
$$

Without the $F$-term contributions, the mass eigenstates take the same form as the gauge fields. This is an indication of $N = 2$ supersymmetry, equivalently $N = 1$ supersymmetry in five dimensions, which is expected to appear in the infrared. In this mass basis of $\tilde{\lambda}_n$ and $\tilde{\chi}_n$, the mass matrix is rewritten as follows

$$
\mathcal{L}_{\text{mass}} = \frac{1}{2} \left( \begin{array}{c} \tilde{\lambda} \\ \tilde{\chi} \end{array} \right) \left( \begin{array}{cc} 2g^2F_S & \frac{B}{v} - \langle \frac{\partial \ln K(S,S^\dagger)}{\partial S} \rangle F_S \\ \frac{B}{v} - \langle \frac{\partial \ln K(S,S^\dagger)}{\partial S} \rangle F_S & -F_Q \end{array} \right) \left( \begin{array}{c} \tilde{\lambda} \\ \tilde{\chi} \end{array} \right) \ + \ h.c. \ , \ \ (4.5)
$$

where the elements of the diagonal matrix $B_{ij} = 2k gv \sin \frac{j\pi}{N} \delta_{ij}$ are the Kaluza-Klein Dirac masses. The irrelevant phase factors have been absorbed with field redefinitions. We finally obtain the mass eigenvalue of the level–$n$ Kaluza-Klein gauge fermions ($n = 0, \cdots, N - 1$);

$$
m_{\lambda_n} = \frac{1}{2} \left[ \pm \sqrt{4m_n^2 + \left( \frac{F_Q}{v} + 2g^2 \left( 1 + 2 \langle \frac{\partial \ln K(S,S^\dagger)}{\partial S} \rangle \right) F_S \right)^2} - \frac{F_Q}{v} + 2g^2 \left( 1 - 2 \langle \frac{\partial \ln K(S,S^\dagger)}{\partial S} \rangle \right) F_S \right], \ \ (4.6)
$$

where $m_n$ is the Kaluza-Klein mass of gauge fields (2.3), which is supersymmetric contribution. The positive (negative) sign in the bracket corresponds to the gaugino (the goldstone fermion) mass. Here the states which are equal to $\tilde{\lambda}, \tilde{\chi}$ in the supersymmetric limit are referred to as gauginos and goldstone fermions, respectively. It is interesting to note that the gauge fermion mass (4.6) can be more simply expressed with only the five-dimensional quantities:

$$
m_{\lambda_n} = \frac{1}{2} \left[ \pm \sqrt{4m_n^2 + \left( \frac{F_{S^n}}{\langle S_S \rangle} \right)^2} - \frac{F_T}{\langle T \rangle} \right]. \ \ (4.7)
$$

This result implies that higher-dimensional effects, even including supersymmetry breaking, are properly reproduced in our model.

We now examine our result for the supersymmetry-breaking models discussed in the previous section.

• Dilaton dominated supersymmetry breaking
This scenario is characterized by the limit $F_T = 0$. We then obtain the Kaluza-Klein masses with the supersymmetry-breaking effect

$$m_{\lambda_n} (\text{dilaton}) = \pm \sqrt{m_n^2 + (2g^2F_S)^2}. \quad (4.8)$$

The spectrum is just as expected in the dilaton dominant case in supergravity models. The first term in the square-root is the Kaluza-Klein Dirac mass, and the second one is a supersymmetry-breaking part that is certainly provided by the four-dimensional dilaton coupling ($2g^2F_S = F_{S4}/2\langle S_4 \rangle$). Note that all the Kaluza-Klein states including zero modes receive the universal supersymmetry-breaking contribution. The two level–$n$ spinors are degenerate in mass, and the mass splitting between bosons and fermions are equal for all Kaluza-Klein modes. This fact is regarded as a reflection that the dilaton field (the action of dilatation) commonly couples to any field in the theory. The universal mass spectrum is one of the major motivations to investigate the dilaton dominant limit in supergravity models. The universality in our model is more clearly seen for scalar components in hypermultiplets. In that case, taking the $F_T = 0$ limit washes away the bulk mass dependence of supersymmetry-breaking scalar masses [14].

- **Moduli dominated supersymmetry breaking**

  With the definition of $F_{S4} = 0$, the gauge fermion mass spectrum becomes

  $$m_{\lambda_n} (\text{moduli, KK}) = \pm \sqrt{m_n^2 + \left(\frac{F_Q}{2v}\right)^2} - \frac{F_Q}{2v}. \quad (4.9)$$

  It is interesting that even when supersymmetry breaking is turned on, the zero-mode gaugino is massless and does not get a mass splitting with the zero-mode gauge field. (The $n = 0$ spinor being affected by the non-zero $F$-terms is the goldstone fermion $\tilde{\chi}_0$.) This is exactly the spectrum predicted in this type of supersymmetry-breaking scenario [3, 13]. By definition, the scenario assumes a vanishing $F$-term of the four-dimensional dilaton. The zero-mode gaugino mass is then shifted at loop level by string threshold corrections or effects of bulk fields. In our model, the spectrum is easily read from the mass matrix (4.5). The gaugino $\tilde{\lambda}_0$ is massless due to the vanishing $F_S$ and Kaluza-Klein mixing mass. As for the excited modes, the supersymmetry-breaking contribution from $F_Q$ is transmitted to gauginos through the non-zero Kaluza-Klein masses. The situation is similar to the models where gauge multiplets behave as messengers, and sparticle soft masses at loop level are calculated from wave-function renormalization in four dimensions [15]. Therefore our approach is also likely to describe this limit well.

  There may be an intuitive explanation for this type of spectrum as was discussed in Ref. [13]. That is, a non-zero $F$-term of the modulus which gives Kaluza-Klein masses does not induce tree-level supersymmetry-breaking masses for zero modes, as these
two mass terms are proportional to Kaluza-Klein numbers. In the present case, such a modulus corresponds to the one whose scalar component obtains a VEV $\propto 1/R$, and is given by $T \propto Q$. This interpretation becomes manifest in examining mass spectra of bulk hypermultiplets with moduli fields [14].

- Radion $F$-term breaking

This scenario takes the $F$-term assumption $F_{S5} = 0$, that is converted into $F_Q = -2g^2v(1 + 2(\partial \ln k(S))F_S$. We find that the gaugino mass matrix (4.5) in this limit has the exactly same form as calculated in Ref. [16], where they use an $N = 1$ superfield formalism of the five-dimensional action with the radion superfield. The mass eigenvalues of the Kaluza-Klein spinors become

$$m_{\lambda_n}({\text{radion}}) = \pm m_n - \frac{F_Q}{v} - \left(\frac{\partial \ln k(S)}{\partial S}\right)F_S.$$ (4.10)

The scenario assumes a non-zero value of the radion $F$-term. However, compared to the moduli dominance scenario stated above, there is a difference in a contribution from the dilaton field $S$, resulting the non-zero $F$-term of the four-dimensional dilaton $S_4$. This gives a tree-level mass of the gaugino zero mode. In other words, if the moduli domination were seen from a five-dimensional viewpoint, there would appear to be an additional contribution from $S_5$ such that the definition $F_{S4} = 0$ is preserved (see Eq. (3.14)). On the other hand, the masses of the Kaluza-Klein excited modes are rather similar to each other. In particular, the low-lying modes have masses

$$m_{\lambda_n}({\text{moduli, KK}}) = m_{\lambda_n}({\text{radion}}) \simeq \pm \frac{n}{R} - \frac{R}{2}F_T.$$ (4.11)

where we have assumed that the supersymmetry-breaking part is smaller than the supersymmetric Kaluza-Klein mass (i.e., $RF_T \ll v$).

It has been shown [16, 17] that the radion mediation model has the same spectrum as that predicted by the Scherk-Schwarz mechanism [18]. The Scherk-Schwarz theory is essentially higher dimensional and adopts twisted boundary conditions for bulk fields along the extra dimensions. On the other hand, the moduli dominated supersymmetry breaking in four-dimensional supergravity (and the Kaluza-Klein mediation) is not a Scherk-Schwarz theory and does gives different soft terms, as explicitly shown in the above.

Let us finally discuss the normalization function $K(S, S^\dagger)$ in the Lagrangian (4.1). It should be mentioned that the form of gaugino masses (4.7) is not affected by any details of the factor $K(S, S^\dagger)$, and the above qualitative discussions about the gaugino mass spectrum are generic and still preserved. We propose the proper form of $K$ is given by

$$K(S, S^\dagger) = \frac{8}{1/S + 1/S^\dagger}.$$ (4.12)
The factors $k$ and $k(S)$ defined in Section 2 then become $k = 1/g$ and $k(S) = 2S^{1/2}$, respectively. Though the complete form of $K$ is not determined without referring to higher-dimensional physics, (4.12) is found to be certainly consistent with several non-trivial and independent requirements. First, notice that to have right results based on holomorphy, the normalization of the link variables $Q_i$’s is required to be $\langle K \rangle = 1/g^2$. With this choice, the gauge and adjoint chiral multiplets of the low-energy $G$ gauge theory have the same field normalization. Moreover, in this case, the radion superfield in our model becomes independent of the dilaton superfield (see the relation (3.10)), which result is plausible since, for example, it does not lead to an undesirable relation between the theta angle and the graviphoton field.

Secondly, with an explicit form of $K(S, S^\dagger)$, one can evaluate tree-level masses of the scalar fields of $Q_i$’s. They are the adjoint scalar fields of the low-energy $G$ gauge theory, and are contained in vector multiplets of the enhanced $N = 2$ supersymmetry. The scalar mass $m^2_{c_n}$ generated by the Kähler term with (4.12) is

$$m^2_{c_n} = m_n^2 + 2 \text{Re} \left( \frac{F_{S5}^*}{\langle S_5 \rangle} \frac{F_T}{\langle T \rangle} \right). \quad (4.13)$$

Let us examine this mass formula in the limits discussed before. One can easily see that the radion mediation limit ($F_{S5} = 0$) does not give supersymmetry-breaking soft mass. This indeed agrees with the fact that the radion mediation is equivalent to the Scherk-Schwarz mechanism, which is now applied to the $SU(2)_R$ symmetry under which the adjoint scalars are singlet and hence do not get symmetry-breaking masses. If one first requires that the scalars $c_n$ do not have soft terms in the $F_{S5} = 0$ limit, $K(S, S^\dagger)$ has to satisfy $\langle \frac{\partial \ln K}{\partial S \partial S^\dagger} \rangle = -(2g^2)^2$. The most probable solution of this equation is $K = X(S)X(S^\dagger)/(S + S^\dagger)$, where $X$ is an arbitrary function. Then the holomorphy argument suggests $X(S) \propto S$ and thus (4.12). For completeness, we write down the scalar masses in the other limits;

$$m^2_{c_n} (\text{dilaton}) = m_n^2, \quad m^2_{c_n} (\text{moduli, KK}) = m_n^2 + 2 \left| \frac{F_T}{\langle T \rangle} \right|^2. \quad (4.14)$$

The third consistency is about the 5-5 component of the five-dimensional metric, $g_{55}$. In a continuum five-dimensional theory, the kinetic energy terms of bosonic fields along the fifth dimension have a dependence on $g_{55}$ as $\sqrt{g_{55}} g_{55} \propto 1/R$. In the model at hand, the second term in the Lagrangian (4.1) becomes this kinetic energy in the continuum limit, and its modulus dependence is given by $\langle K(S, S^\dagger) Q^\dagger Q \rangle$. The equation (4.12) then indicates $\langle KQ^2 \rangle \sim \langle SQ^2 \rangle \sim \langle S_5 T \rangle$. As a result, the desirable metric dependence appears, for a fixed value of the five-dimensional gauge coupling $g_5$.

We close this section by a comment on the model which turns into a five-dimensional theory compactified on an $S^1/Z_2$ orbifold. This can be formulated [6, 8] by getting rid
of a link variable, e.g. $Q_N$, from the $S^1$ model. In this case, additional fields may be introduced to cancel gauge anomalies on the orbifold fixed points. Examining a mass matrix, it is found that $Q_i$’s contain only massive modes, and the zero-mode state consists of an $N = 1$ vector multiplet without an associated adjoint chiral multiplet, which situation corresponds to the $Z_2$ orbifolding. In turn, this results in removing $\tilde{\chi}^0$ and $c_0$ in our analyses. The plus sign is chosen for the zero mode, and the gaugino masses in various limits discussed before are not altered. Results similar to those in the $S^1$ case hold for other quantities, for example, the Kaluza-Klein mass spectrum is unchanged except for a replacing $N \to 2N$ ($R \to 2R$).

5 Summary

In this paper, we have studied supersymmetry breaking in the four-dimensional model with two types of modulus fields. The model can describe five-dimensional physics in the infrared, and given the relations among the modulus fields, we have discussed supersymmetry breaking in the higher-dimensional bulk. The analysis is based on a four-dimensional model, that is renormalizable and calculable in a usual manner. We have made it clear that several specific limits in the two-dimensional parameter space of the modulus $F$-terms correspond to the bulk supersymmetry-breaking scenarios in the literature. We have shown this by examining the gaugino and adjoint scalar masses in the cases of the $S^1$ and $S^1/Z_2$ compactifications. It is non-trivial to establish such correspondences and indeed require a properly-fixed moduli dependence of the action. The moduli dependence will also be confirmed by detailed calculations of radiative corrections to mass spectrum [14]. Moreover it would be an interesting issue to study other choices of couplings and limits, which could describe unexplored supersymmetry breaking in four or higher dimensions, and we leave it to future work. Besides the issue of supersymmetry breaking, extra dimensions provides a new perspective for various subjects in particle physics. Realistic model construction along this line of using a purely four-dimensional one will deserve further investigations.

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