A neutrino mass model with hidden $U(1)$ gauge symmetry

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Abstract

We propose a realisation of inverse seesaw model controlled by hidden $U(1)$ gauge symmetry, and discuss bosonic dark matter candidate by imposing a $Z_2$ parity symmetry. Then we consider the anomalous muon magnetic dipole moment (muon $g - 2$) and lepton flavor violating processes where the dark matter involves in them via new interactions allowed by the hidden gauge symmetry. In this paper we show the allowed region to explain muon $g - 2$ and relic density of dark matter, satisfying other leptonic constraints, and explore the possibility to detect our new particles at the large hadron collider (LHC).
I. INTRODUCTION

The discovering of the Higgs boson at the Large Hadron Collider in 2012 announced the success of the Standard Model (SM) and the data collected so far have affirmatively validated the high precision of this framework in explaining most of phenomenology. However the deviations from the expected SM prediction within the current experimental uncertainty can still accommodate the possibility of new particles existence motivated by those interesting scenarios of extra dimension, supersymmetry and composite Higgs model, etc. There are also several aspects such as non-zero masses of neutrinos and dark matter (DM) candidate which should involve physics beyond the SM. For neutrinos, several recent experiments observing the neutrino oscillations confirmed that the neutrino has a tiny mass at the order $< 0.1$ eV, which is much smaller compared with the SM quarks and leptons. One favourable neutrino model is supposed to account for the important features related to three active neutrinos with mixing angles of $\theta_{12,13,23}$ and the two neutrino mass square differences, $\Delta m_{12}^2$ and $|\Delta m_{23}^2|$ consistent with the observations [1]. Furthermore, several issues are very poorly understood, including whether neutrino is a Dirac fermion or Majorana one, in normal or inverted hierarchy pattern for the mass ordering, and the exact value of CP violation phase, and so on. In particular, the presence of Majorana field violating the lepton number in this type of models leads to the neutrinoless double beta decay detectable in experiments, as well as possibility to explain the Baryon Asymmetry of the Universe via ”leptogenesis” [2]. Thus it is important to explore and analyze viable neutrino mass models in order to reveal the nature and role of the neutrino sector.

The simplest idea to realize a tiny neutrino mass is seesaw mechanism by introducing heavier neutral fermions which can obtain Majorana mass at the GUT scale $\sim 10^{15}$ GeV. There are several types of seesaw models after a long time of evolving, such as type-I seesaw (aka canonical seesaw) or type-III seesaw involving either a $SU(2)_L$ singlet or triplet right-handed neutral fermions [3-6]. One alternative mechanism to obtain a small mass is the radiative seesaw provided the neutrino mass can only be generated at the one-loop level, such as the model proposed in the paper of [7], where the neutral $Z_2$ odd scalar interacting with neutrino could be the DM candidate. While an inverse seesaw is a promising scenario to reproduce neutrino masses and their mixings by introducing both left and right-handed neutral fermions so that the seesaw mechanism is proceeded via a two-step mediation and
this type of mechanism is often considered in extended gauge models such as the superstring inspired model or left-right models in unified gauge group [8, 9]. In this paper, we propose an inverse seesaw model with 2 Higgs doublets imposed by a hidden U(1) symmetry, which provides rather natural hierarchies among neutrinos and other heavier neutral fermions even at the tree level compared to another similar scenario of linear seesaw [10, 11]. In this model, due to the exotic U(1) symmetry, one of two Higgs doublets only couples to the right-handed neutrino so that large hierarchy in the Yukawa coupling is avoided.

This letter is organized as follows. In Section II, we present our model by showing new particle fields and symmetries, where the inverse seesaw mechanism is implemented in a framework of hidden U(1) gauge symmetry through introducing isospin singlet exotic fermions and additional scalar bosons. We add an inert boson that is expected to be a dark matter (DM) candidate, where a $Z_2$ symmetry is imposed to assure the stability of DM. The scalar potential formulated here is to trigger spontaneous symmetry breaking and generate the required mass hierarchy. We review the electroweak bounds from the lepton flavor violations and associated muon $g - 2$ operator, constraints of flavor changing leptonic Z boson decays and discuss the relic density contribution which comes dominantly from the Yukawa interaction of bosonic DM under some reasonable assumptions. In Section III we carry out the numerical analysis searching for allowed parameter region, and we also discuss the LHC collider physics in our model by exploring the pair production of vector-like charged leptons, which subsequently decay into the DM plus SM leptons. Finally we devote the Section IV to the summary and conclusion of our results.

II. MODEL SETUP AND CONSTRAINTS

In this section we formulate our model. First of all, we introduce three families of right(left)-handed vector-like fermions $U, D, E, N$ which are charged under $U(1)_H$ gauge symmetry; note that actually they are chiral under $U(1)_H$ and become vector-like fermions after its spontaneous symmetry breaking [19, 20]. To have gauge anomaly-free for $[U(1)_H]^2[U(1)_Y]$ and $[U(1)_H][U(1)_Y]^2$, the number of family has to be same for each fermion, another types of models are recently appeared on refs. [12, 13] for the canonical seesaw [14–16] for the inverse seesaw and for the linear seesaw [17, 18], which provide milder hierarchies among neutral fermion masses even at the tree level, by introducing large $SU(2)_L$ multiplet fields.
TABLE I: Charge assignments of the our fields under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_H \times Z_2$, where all the SM fields are zero charges under the $U(1)_H$ symmetry and even under the $Z_2$.

Although $[U(1)_H]^3$ and $[U(1)_H]$ are anomaly free between $U$ and $D$ or $E$ and $N$. In this sense, the minimal number is two generations that comes from neutrino oscillation data. In scalar sector, we add an isospin doublet boson $H'$ with charge 4 under the $U(1)_H$ symmetry that plays a role in having Dirac mass terms in the neutrino sector after spontaneous electroweak symmetry breaking. Also we require three isospin singlet bosons ($\varphi, \varphi', \chi$) with charges $(-3, -2, 1)$ under the $U(1)_H$ symmetry, where $\varphi, \varphi'$ have nonzero vacuum expectation values to induce masses for $U, D, E, N$, while $\chi$ is expected to be an inert boson that can be a DM candidate. Here, we denote that all the SM fields are neutral under $U(1)_H$ symmetry, and each of vacuum expectation value is symbolized by $\langle H'^i \rangle \equiv v_{H'^i}/\sqrt{2}$, and $\langle \varphi'^i \rangle \equiv v_{\varphi'^i}/\sqrt{2}$, where $H$ is the SM Higgs field. In addition we introduce $Z_2$ symmetry assigning odd parity to $\{\chi, U, D, E\}$ so that the stability of $\chi$ is guaranteed as a dark matter (DM) candidate. The $Z_2$ parity forbids additional interaction terms: $\lambda_0 \chi \varphi^* \varphi'^2$, $\lambda_0 \left( H'^i H \right) \varphi^* \chi$ and $\mu_0 \chi \varphi \varphi'^*$, which are permitted by the $U(1)_H$ symmetry but could lead to the decay of $\chi$ into SM particles. All the new field contents and their charge assignments are summarized in Table I. The valid new renormalizable Yukawa Lagrangian and Higgs potential under these symmetries are given by

\[
\mathcal{L}_Y = y_{Naa} L_{La} \tilde{H}' N_{Ra} + y_{N\varphi_a} N_{La} N_{Ra} \varphi + y_{N\varphi'_{ab}} N_{La} N_{La} \varphi' + y_{U\varphi_a} U_{Ra} U_{La} \varphi^* + y_{D\varphi_a} D_{Ra} D_{La} \varphi
\]

\[
y_{E\varphi_a} E_{Ra} E_{La} \varphi + (y_{u\chi})_{ia} \bar{u}_{Ri} U_{La} \chi + (y_{d\lambda})_{ia} \bar{d}_{Ri} D_{La} \chi + (y_{e\lambda})_{ia} \bar{e}_{Ri} E_{La} \chi + h.c.,
\]

\[
V = \sum_{\phi} \left[ \mu^2_{\phi} \phi^\dagger \phi + \lambda_0 |\phi^\dagger \phi|^2 \right] + \frac{1}{2} \sum_{\phi \neq \phi'} \lambda_{\phi \phi'} |\phi|^2 |\phi'|^2 + \lambda_{HH'} H^\dagger H' (H'^i H) + \mu \chi \varphi' + \lambda_0 (H'^i H') \varphi'^2 + h.c.,
\]
where $\tilde{H} \equiv i\sigma_2 H$, $\lambda_{\phi\phi}^{(i)} = \lambda_{\phi\phi}^{(j)}$, upper indices $(a,b,i) = 1,2,3$ are the number of families, and all the above Yukawa couplings are assumed to be diagonal except $y_{N\phi}$ for simplicity. Therefore, the neutrino mixings are induced via $y_{N\phi}$.

### A. Neutrino sector

After spontaneous symmetry breaking, one has neutral fermion masses which are defined by

$$m_D \equiv y_N v_H^{\prime} / \sqrt{2}, \quad M \equiv y_{N\phi} v_{\phi} / \sqrt{2}, \quad \mu_L \equiv y_{N\phi} v_{\phi} / \sqrt{2}.$$ 

Then, the neutral fermion mass matrix with $9 \times 9$ in the basis of $(\nu_i L, N_{aR}, N_{aL})$, $(i,a) = 1,2,3$, is given by

$$M_N = \begin{pmatrix}
0 & m_D & 0 \\
m_D^T & 0 & M^T \\
0 & M & \mu_L^* \\
\end{pmatrix}. \quad (3)$$

Then the active neutrino mass matrix can approximately be found as

$$m_\nu \approx m_D M^{-1} \mu_L^* (M^T)^{-1} m_D^T, \quad (4)$$

where $\mu \ll m_D \lesssim M$ is assumed. The neutrino mass matrix is diagonalized by unitary matrix $U_{MNS}$; $D_\nu = U_{MNS} m_\nu U_{MNS}^T$, where $D_\nu \equiv \text{diag}(m_1, m_2, m_3)$. One of the elegant ways to reproduce the current neutrino oscillation data [1] is to apply the Casas-Ibarra parametrization [21] without loss of generality, and find the following relation [16]

$$m_D = U_{MNS}^T \sqrt{D_\nu O_{\text{mix}}} R_N^{-1}. \quad (5)$$

Here $O_{\text{mix}}$ is an arbitrary $3 \times 3$ orthogonal matrix with complex values, and $R_N$ is a lower unit triangular [22], which can uniquely be decomposed to be $M^{-1} \mu_L^* (M^T)^{-1} = R_N R_N^T$, since it is symmetric. Note here that all the components of $m_D$ should not exceed 246 GeV, once perturbative limit of $y_N (\equiv \sqrt{2} m_D / v_{H^{\prime}})$ is taken to be 1.

**Non-unitarity:** At the end of this part, we should mention the possibility of non-unitarity matrix $U_{MNS}$. This is typically parametrized by the form

$$U_{MNS}' = \left(1 - \frac{1}{2} FF^\dagger\right) U_{MNS}, \quad (6)$$

where $F$ is a hermitian matrix that is determined by each of model, $U_{MNS}$ is the three by three unitarity matrix, while $U_{MNS}'$ represents the deviation from the unitarity. Then $F$ is
given by \[18, 23, 24\]

\[
F = (M^T)^{-1}m_D^T = (M^T)^{-1}R_N^{-1}O^T_{\text{mix}}\sqrt{\Delta}U_{\text{MNS}}^*.
\]

(7)

The global constraints are found via several experimental results such as the \textit{SM W boson mass} \(M_W\), \textit{the effective Weinberg angle} \(\theta_W\), several ratios of \(Z\) boson fermionic decays, \textit{invisible decay of} \(Z\), \textit{EW universality}, measured \(\text{CKM}\), and \(\text{LFVs}\). \[25\] The result can be given by \[28\]

\[
|FF^\dagger| \leq \begin{bmatrix}
2.5 \times 10^{-3} & 2.4 \times 10^{-5} & 2.7 \times 10^{-3} \\
2.4 \times 10^{-5} & 4.0 \times 10^{-4} & 1.2 \times 10^{-3} \\
2.7 \times 10^{-3} & 1.2 \times 10^{-3} & 5.6 \times 10^{-3}
\end{bmatrix}.
\]

(8)

\textit{The other exotic masses} are also given after the spontaneous symmetry breaking. Here, we just denote their masses as follows: \(M_U \equiv y_U v_\phi / \sqrt{2}\), \(M_D \equiv y_D v_\phi / \sqrt{2}\), \(M_E \equiv y_E v_\phi / \sqrt{2}\).

B. Scalar sector

The non-zero VEVs of scalar fields are obtained by the condition:

\[
\frac{\partial V}{\partial v_H} = \frac{\partial V}{\partial v_{H'}} = \frac{\partial V}{\partial v_\phi} = \frac{\partial V}{\partial v_{\phi'}} = 0,
\]

(9)

where we choose parameters to make VEV of \(\chi\) to be zero. Then we can simply obtain

\[
v_\phi \simeq \sqrt{-\mu_\phi^2 / \lambda_\phi}, \quad v_{\phi'} \simeq \sqrt{-\mu_{\phi'}^2 / \lambda_{\phi'}} \quad \text{and} \quad v_H \simeq \sqrt{-\mu_H^2 / \lambda_H}, \quad v_{H'} \simeq -\frac{\lambda_0 v_H v_{\phi'}^2}{2\mu_{H'}^2 + (\lambda_{HH'} + \lambda'_{HH'})v_H^2}.
\]

(10)

where we assumed couplings in the potential \(\{\lambda_{H\phi}, \lambda_{H\phi'}, \lambda_{H'\phi}, \lambda_{H'\phi'}, \lambda_{\phi\phi'}\}\) and \(v_{H'}\) to be sufficiently small, and we require \(\{-\mu_H^2, -\mu_{\phi'}^2, -\mu_{\phi'}^2, -\lambda_0\} > 0\) and \(2\mu_H^2 + (\lambda_{HH'} + \lambda'_{HH'})v_H^2 > 0\) to make all VEVs positive. The smallness of \(v_{H'}\) can be achieved by requiring \(\lambda_0\) coupling to be negligible, so that the \(v = \sqrt{v_H^2 + v_{H'}^2} = 246\ \text{GeV}\) is mainly determined by the \(v_H\) in the \textit{SM Higgs} doublet.

The Higgs doublet fields are parameterized as

\[
H = \begin{bmatrix}
w^+ \\
v_{H+h+i}\sqrt{2}
\end{bmatrix}, \quad H' = \begin{bmatrix}
w'^+ \\
v_{H'+h'+i'}\sqrt{2}
\end{bmatrix},
\]

(11)

where one massless combination of the charged scalar components after diagonalizing the mass matrix in basis of \((w^\pm, w'^\pm)\) is absorbed by the \textit{SM singly-charged gauge boson} \(W^\pm\),
and one degree of freedom in the CP-odd scalar sector $\eta$ and $\eta'$ is eaten by the neutral SM gauge boson $Z$. The other degrees of freedoms become massive charged and neutral scalar bosons which is the same as two-Higgs doublet models under our assumptions.

After spontaneous symmetry breaking we also have massless Nambu-Goldstone (NG) boson absorbed by $Z'$ and physical Goldstone boson from singlet scalar fields $\varphi$ and $\varphi'$. To discuss these massless bosons we write $\varphi$ and $\varphi'$ by

\[
\varphi = \frac{v_\varphi + \varphi_R}{\sqrt{2}} e^{i \frac{\alpha}{\sqrt{2}}} , \quad \varphi' = \frac{v_\varphi' + \varphi'_R}{\sqrt{2}} e^{i \frac{\alpha'}{\sqrt{2}}} .
\]

Then NG boson and physical Goldstone boson are written in terms of linear combination of $\alpha$ and $\alpha'$ where the mixing angle is determined by relative sizes of VEVs. We then obtain the expression for the NG boson $\alpha_{NG}$ and physical Goldstone boson $\alpha_G$:

\[
\alpha_{NG} = c_X \alpha + s_X \alpha', \quad \alpha_G = -s_X \alpha + c_X \alpha',
\]

\[
c_X = \cos X = \frac{3v_\varphi}{\sqrt{9v_\varphi^2 + 4v_{\varphi'}^2}}, \quad s_X = \sin X = \frac{2v_{\varphi'}}{\sqrt{9v_\varphi^2 + 4v_{\varphi'}^2}}.
\]

In our scenario, we choose $v_{\varphi'} \ll v_\varphi$ since the VEV of $\varphi$ generate heavy extra fermion masses while that of $\varphi'$ generate Majorana mass of $N_L$ which is preferred to be small in realizing tiny neutrino masses. Thus we simply write $\alpha_{NG} \simeq \alpha$ and $\alpha_G \simeq \alpha'$ in the following analysis.

The $Z_2$ odd scalar $\chi$ is written as $\chi = (\chi_R + i\chi_I)/\sqrt{2}$. The masses of the components are given by

\[
m_{\chi_R} = \mu^2_{\chi} + \frac{1}{2}(\lambda_{H\chi}v_H^2 + \lambda_{H'\chi}v_{H'}^2 + \lambda_{\varphi\chi}v_\varphi^2 + \lambda_{\varphi'\chi}v_{\varphi'}^2) + \sqrt{2}\mu v_{\varphi'}
\]

\[
m_{\chi_I} = \mu^2_{\chi} + \frac{1}{2}(\lambda_{H\chi}v_H^2 + \lambda_{H'\chi}v_{H'}^2 + \lambda_{\varphi\chi}v_\varphi^2 + \lambda_{\varphi'\chi}v_{\varphi'}^2) - \sqrt{2}\mu v_{\varphi'},
\]

where the last term in right-hand side provides the mass difference between $\chi_R$ and $\chi_I$. Depending on the sign of $\mu$ coupling, either the real or the imaginary part of the $\chi$ scalar can be the DM candidate.

Note that if we tune one of the quartic couplings, $\lambda_{H\varphi'}$, to be large enough in our model, this will result in a mixing term of $2\lambda_{H\varphi'}v_Hv_{\varphi'} h \text{Re}(\varphi')$. The mixing between those CP-even scalars will cause invisible Higgs decays $h \to \chi_R\chi_R$ depending on the mass spectrum as well as $h \to \alpha_G\alpha_G$ via the kinematic term. In such a case the Higgs coupling is universally rescaled by a mixing angle. Thus the upper limit of $\mu$ coupling in scalar potential can be constrained by the LHC precision Higgs measurement on the invisible branching ratio, for which we will omit further discussion since they are irrelevant for the neutrino mass.
C. Lepton flavor violations (LFVs)

LFVs arise from the term $y_{e\chi}$ at one-loop level, and its form can be given by

$$\text{BR}(\ell_i \to \ell_j\gamma) = \frac{48\pi^3\alpha_{\text{em}}C_{ij}}{G_F^2m_{\ell_i}^2} (|a_{R_{ij}}|^2 + |a_{L_{ij}}|^2), \quad \text{(17)}$$

$$a_{R_{ij}} \approx -\frac{m_{\ell_i}}{(4\pi)^2} \sum_{A=X,I} \sum_{a=1,2,3} (y_{e\chi})_a (y_{e\chi}^\dagger)_{ai} \int [dx]_3 \frac{xz}{xM_A^2 + (1-x)M_{Ea}^2}, \quad \text{(18)}$$

$$a_{L_{ij}} \approx -\frac{m_{\ell_i}}{(4\pi)^2} \sum_{A=X,I} \sum_{a=1,2,3} (y_{e\chi})_a (y_{e\chi}^\dagger)_{ai} \int [dx]_3 \frac{xy}{xM_A^2 + (1-x)M_{Ea}^2}, \quad \text{(19)}$$

where $\int [dx]_3 \equiv \int_0^1 dx \int_0^{1-x} dz$, $G_F \approx 1.17 \times 10^{-5}\text{[GeV]}^{-2}$ is the Fermi constant, $\alpha_{\text{em}} \approx 1/137$ is the fine structure constant, $C_{21} \approx 1$, $C_{31} \approx 0.1784$, and $C_{32} \approx 0.1736$. Experimental upper bounds are respectively given by $\text{BR}(\mu \to e\gamma) \lesssim 4.2 \times 10^{-13}$, $\text{BR}(\tau \to e\gamma) \lesssim 3.3 \times 10^{-8}$, and $\text{BR}(\tau \to \mu\gamma) \lesssim 4.4 \times 10^{-8}$ \cite{26, 27}.

**New contribution to the muon anomalous magnetic moment** (muon $g-2$: $\Delta a_\mu$) also arises from the same term as in LFVs, and it is given by\(^2\)

$$\Delta a_\mu = -m_\mu [a_R + a_L]_{22}. \quad \text{(20)}$$

To explain the current 3.3$\sigma$ deviation \cite{29}

$$\Delta a_\mu = (26.1 \pm 8.0) \times 10^{-10}. \quad \text{(21)}$$

It would be worth mentioning that we also have Barr-Zee type diagrams that contribute to the muon $g-2$. But they have small contribution due to the two loop diagrams and considering $H \to 2\gamma$ constraint \cite{30}.

D. Flavor-Changing Leptonic $Z$ Boson Decays

Here we study decays of the $Z$ boson to two charged leptons of different flavors at the one-loop level. \(^3\) The amplitudes of these decay modes involve the Yukawa coupling $f_{e\chi}$, some components of which can be of $O(1)$ in order to achieve a sizeable muon $g-2$. The form

\(^2\) For a comprehensive review on new physics models for the muon $g-2$ anomaly as well as lepton flavour violation, please see Ref. \cite{31}.

\(^3\) Although the quark pairs are also induced from the $y_{u\chi}$ and $y_{d\chi}$, we do not consider them because their experimental bounds are not so stringent.
factor for this flavor-changing operator is obtained through the vertex and wave-function renormalisation with the analytic expression calculated to be \[25, 30\]:

\[
\text{BR}(Z \to \ell_i^- \ell_j^+) \approx \frac{G_F}{12\sqrt{2}\pi} \frac{m_Z^3}{(16\pi^2)^2 \Gamma_{Z}^{\text{tot}}} \left( s_W^2 - \frac{1}{2} \right)^2 \times \left| \sum_{a=1}^{3} \sum_{J=R,I} (y_{e\chi})_{ia} (y_{e\chi})_{aj} [F_2(E_a, \chi J) + F_3(E_a, \chi J)] \right|^2 , \tag{22}
\]

where

\[
F_2(a, b) = \int_0^1 dx (1 - x) \ln \left[ (1 - x)m_a^2 + xm_b^2 \right] ,
\]

\[
F_3(a, b) = \int_0^1 dx \int_0^{1-x} dy (xy - 1)m_Z^2 + (m_a^2 - m_b^2)(1 - x - y) - \Delta \ln \Delta ,
\]

with \(\Delta \equiv -xym_Z^2 + (x + y)(m_a^2 - m_b^2) + m_b^2\) and the total \(Z\) decay width \(\Gamma_{Z}^{\text{tot}} = 2.4952 \pm 0.0023\) GeV. The current upper limit for the lepton flavor-changing \(Z\) boson decay branching ratios are published to be \[32\]:

\[
\begin{align*}
\text{BR}(Z \to e^\pm \mu^\mp) &< 1.7 \times 10^{-6} , \\
\text{BR}(Z \to e^\pm \tau^\mp) &< 9.8 \times 10^{-6} , \\
\text{BR}(Z \to \mu^\pm \tau^\mp) &< 1.2 \times 10^{-5} , \tag{23}
\end{align*}
\]

where the upper bounds are quoted at 95% CL. We have scanned the parameter space and found that all these constraints are less stringent than those from the LFV processes, as well as the flavor-conserving processes \(\text{BR}(Z \to \ell^- \ell^+)\) (\(\ell = e, \mu, \tau\)).

**E. Bosonic dark matter candidate**

We first fix DM to be the real part of \(\chi\); \(X \equiv \chi_R\), and assume that Higgs portal interaction is negligibly small by choosing couplings in the scalar potential associated with \(\chi\) to be small. This hypothesis could be reasonable because interactions from Higgs potential is strongly constrained by the spin independent DM-nucleon scattering cross section reported by several direct detection experiments such as LUX \[33\], XENON1T \[34\], and PandaX-II \[35\]. For completeness, we take into account the DM interactions among fermions via Yukawa couplings, with \(Z'\) gauge boson and physical Goldstone boson \(\alpha_G\). To derive the interaction with \(\alpha_G\), we rewrite \(\chi\) as \[36, 38\]

\[
\chi \rightarrow \chi e^{-i \frac{\alpha_G}{2v_G}}
\]
where we apply an approximation $\alpha' \simeq \alpha_G$ as we mentioned above. In such a way we can obtain the interactions from the kinetic term of $\chi$, instead from expanding the exponential phase factor in the term of $\chi\phi'$:

$$\langle D_\mu \chi \rangle (D^\mu \chi) \supset \frac{1}{2v_{\phi'}} \partial^\mu \alpha_G (\partial_\mu \chi_R \chi_I - \partial_\mu \chi_I \chi_R) + g_H Z'^\mu (\partial_\mu \chi_R \chi_I - \partial_\mu \chi_I \chi_R)$$

$$+ \frac{1}{4v_{\phi'}^2} \partial_\mu \alpha_G \partial^\mu \alpha_G (\chi_R^2 + \chi_I^2) - \frac{g_H}{v_{\phi'}} Z'^\mu \partial_\mu \alpha_G (\chi_R^2 + \chi_I^2) + \frac{g_H Z'_\mu}{v_{\phi'}} Z'^\mu (\chi_R^2 + \chi_I^2)$$

The mass of $Z'$ is calculated to be $m_{Z'}^2 \simeq g_H^2 (9v_{\phi'}^2 + 4v_{\phi'}^2)$, where the mixing between $(Z, Z')$ is neglected due to the small value of $v_{H'}$. The valid terms for the relic density of $\chi$ are also given by Yukawa interactions

$$\frac{(y_{u\chi})_{ia}}{\sqrt{2}} \bar{u}_{R_i} U_{La} X + \frac{(y_{d\chi})_{ia}}{\sqrt{2}} \bar{d}_{R_i} D_{La} X + \frac{(y_{e\chi})_{ia}}{\sqrt{2}} \bar{e}_{R_i} E_{La} X + H.c.$$  \hspace{1cm} (26)

In the following analysis, we assume the Yukawa interaction provides dominant contribution to DM annihilation cross section in order to obtain sizable muon $g - 2$ simultaneously. It is realized by taking $v_H \ll v_{\phi'}$ and $g_H \ll 1$. Therefore the majority portion of required DM abundance is determined by the s-wave ($a_{eff}$) coefficient of the total cross section in powers of the relative velocity $v_{rel}$,

$$a_{eff} \approx \frac{3m_t^2 |(y_{u\chi})_{ia}(y_{u\chi}^\dagger)_{aj}|^2}{16\pi (M_X^2 + M_{Ua}^2)^2},$$

where we have assumed all the SM fermions except for top quark are massless. The resulting relic density is found to be $[39]$

$$\Omega h^2 \approx \frac{1.07 \times 10^9 x_f}{\sqrt{g_*(x_f)} M_{PL} a_{eff}},$$

where the present relic density is $0.1199 \pm 0.0054$ at the $2\sigma$ confidence level (CL) $[40]$, $g_*(x_f \approx 25) \approx 100$ counts the degrees of freedom for relativistic particles, and $M_{PL} \approx 1.22 \times 10^{19}$ GeV is the Planck mass.

In another extreme limit, if all the SM fermions are massless compared with DM mass, i.e. $m_t \ll M_X$, the dominant coefficient is the d-wave ($d_{eff}$) from the expansion of the total cross section in powers of the relative velocity $v_{rel}$ $[41] [42]$

$$d_{eff} \approx \frac{M_X^6}{80\pi} \sum_{a,i,j} \left[ \frac{|(y_{u\chi})_{ia}(y_{u\chi}^\dagger)_{aj}|^2}{(M_X^2 + M_{Ua}^2)^4} + \frac{|(y_{d\chi})_{ia}(y_{d\chi}^\dagger)_{aj}|^2}{(M_X^2 + M_{Da}^2)^4} + \frac{1}{3} \frac{|(y_{e\chi})_{ia}(y_{e\chi}^\dagger)_{aj}|^2}{(M_X^2 + M_{Ea}^2)^4} \right].$$  \hspace{1cm} (29)
FIG. 1: Allowed region between the lightest $M_E$ and $M_X$ in order to obtain $\Delta a_\mu = (26.1 \pm 8.0 x) \times 10^{-10}$, $\Omega h^2 = 0.1198 \pm 0.0027$ and satisfy various LFVs, where $x=1,2,3$, which is the confidence level, corresponding to green, blue, and red, respectively. The plot suggests that $M_E \lesssim 350$ GeV and $M_X \lesssim 300$ GeV.

assuming massless final-state of charged-leptons, up and bottom quarks. In this case the resulting relic density is given by:

$$\Omega h^2 \approx \frac{5.35 \times 10^7 x^3}{\sqrt{g^*(x_f)} M_{PL} d_{eff}}.$$  \hspace{1cm} (30)

III. NUMERICAL ANALYSIS AND COLLIDER PHYSICS

After obtaining formulas for some observables, we carry out numerical analysis searching for parameter space which can explain muon $g - 2$ and relic density of DM. Then we discuss collider physics focusing on vector-like charged lepton production at the LHC.

A. Numerical analysis

In this numerical analysis, we show the result for the correlation between $M_E$ and $M_X$ by recasting the bounds from the observed relic density and leptonic flavour constraints. We take the upper limit of Yukawa couplings as $\sqrt{4\pi}$, and the regions of $M_X$, $m_{\chi_I}$, $M_{U,D}$, and $M_E$ are scanned in the regions of $(10, 500)$ GeV, $(1.2 M_X, 550)$ GeV, $(1000, 2000)$ GeV, and $(100, 1000)$ GeV respectively. Here the lower bound of $m_{\chi_I}$ is set to forbid the coannihilation modes between $X$ and $\chi_I$ for simplicity, and we choose the lower bound of vector-like lepton to be $M_E = 100$ GeV, which simply comes from the LEP experiment, although the lower
limit from most recent LHC measurement can be more stringent. The Figure 1 represents the allowed regions of the lightest $M_E$ and $M_X$ which are consistent with precise observations of $\Delta a_\mu = (26.1 \pm 8.0) \times 10^{-10}$ and $\Omega h^2 = 0.1198 \pm 0.0027$, as well as satisfy various LFV bounds. We adopt different colours in the figure to emphasize the experimental constraint from the muon g-2 at the confidence level of 68% (green), 95% (blue), 99.7% (red). The LFV bounds specifically lead to the consequence that the most important Yukawa coupling is $(y_e\chi)_{22}$ with a typical value of 2~3, and the other parameters can be less than 1. However the other heavy quark masses are not so much restricted in this simplified model. As illustrated by the plot, the upper bounds for the DM and vector-like leptons masses are required to be $M_E \lesssim 350$ GeV and $M_X \lesssim 300$ GeV respectively, and the mass splitting between these two particles tends to be small, roughly in a scale of $\lesssim 50$ GeV.

B. Collider physics

In this sector, we proceed to provide an analysis for the LHC constraint by scanning over the mass region allowed by the bound from relic density and flavour violation processes. Due to the $Z_2$ parity presented in this model, the vector-like fermions $U$, $D$ and $L$, which are relevant to DM production via decay, can only be pair produced. In order to interpret the LHC measurement in this hidden $U(1)$ symmetry model, we consider the Drell-Yan production of vector-like lepton (VLL) pairs, with the VLL further decays into one DM particle $X$ plus one tau lepton. The final state of two hadronic tau leptons plus a large amount of $\not{E}_T$ has been adopted by the CMS collaboration to extract the upper limit of cross section for tau slepton pair productions [43]. For our analysis of the LHC dark matter signature, the insensitive tau efficiency from the measured data results in a very loose LHC constraint in the $(M_E, M_X)$ plane under the assumption of universal $E_a$-$\chi$-lepton couplings, i.e. $y_{E\chi}^{e,a} = y_{E\chi}^{\mu,a} = y_{E\chi}^{\tau,a}$, with the detail recast strategy described in the following.

In order to simulate the $2\tau_h + \not{E}_T$ signal in our model, we employ the MG5_AMC@NLO 44 to generate events for the production of $pp \rightarrow Z, \gamma \rightarrow E^+E^-$ at the leading order precision, with the decaying of heavy lepton $E$ into $X + \tau$ handled by the MadSpin module. The events are passed through Pythia 8 for parton shower and hadronization, where the tau lepton decays in both leptonic and hadronic modes are sophisticatedly processed. Event reconstruction is finally performed by Madanalysis 5 package [45], requiring the hadrons
| $M_X$ (GeV) | $M_E = 150$ (GeV) | $M_E = 200$ (GeV) | $M_E = 250$ (GeV) | SM BG | Observed |
|------------|-----------------|-----------------|-----------------|-------|---------|
| 40 < $M_{T2}$ < 90 GeV | 56.8 | 62.9 | 70.3 | 15.8 | 19.1 | 23.4 | 5.12 | 5.69 | 7.08 | - | - |
| $\Sigma M_T > 350$ GeV | 5.06 | 3.72 | 1.62 | 2.35 | 1.96 | 1.18 | 1.26 | 1.23 | 1.05 | - | - |
| $E_T^{\text{miss}} > 50$ GeV | 4.81 | 3.40 | 1.43 | 2.27 | 1.86 | 1.12 | 1.20 | 1.19 | 0.99 | 4.35$^{+1.75}_{-1.53}$ | 5 |

TABLE II: Number of events after each step of selection criterion in the one generation VLL mediation scenario with benchmark points $M_E = 150, 200, 250$ GeV, for an integrated luminosity of $\int L dt = 35.9$ fb$^{-1}$ at a $\sqrt{s} = 13$ TeV LHC.

are clustered into jets using the anti-$k_T$ algorithm implemented in FastJet, with a distance parameter of $R = 0.4$ and $p_T > 20$ GeV. The detector impact on the hadronic tau reconstruction is simulated by including a tagging efficiency of 60% for a $\tau_h$ candidature. The basic event selection demands that there are two hadronic taus in opposite signs, with a veto for electrons or muons in the final state. The CMS analysis further applied the discriminating kinematic cuts from $M_{T2}$ variable, sum of transverse mass $\Sigma M_T(\tau_i)$, missing energy $E_T^{\text{miss}}$ and the angle of di-tau in order to reduce SM background. The $M_{T2}$ variable is a generalization of transverse mass into the case with two invisible particles \cite{46, 47} and in this analysis we use the CMS interpretation by setting the trial mass $\mu_X$ of two missing particles to be zero so that our results can be directly compared with the CMS background simulation. We calculate the $M_{T2}$ as the minimum of all possible maximum of $(M_T(\tau_1), M_T(\tau_2))$, with the partition of missing momentum in two DMs add up to be $E_T^{\text{miss}}$ measured in the event:

$$M_{T2} = \min_{p_T^X_1 + p_T^X_2 = p_T} \left[ \max \left( M_T(p_T^\tau_1, p_T^{\tau_1}; \mu_X), M_T(p_T^\tau_2, p_T^{\tau_2}; \mu_X) \right) \right], \quad (31)$$

where the transverse mass in the case of massless particles is defined as:

$$M_T(p_T^\tau, p_T^{\tau i}) = \sqrt{2(E_T^{\tau i} E_T^{\tau i} - p_T^\tau \cdot p_T^{\tau i})}; \quad \text{with} \quad i = 1, 2. \quad (32)$$

Following the prescription in CMS analysis, we simply employ the event selection criteria in the search region 2 (SR2) for the $\tau_h\tau_h$ final states, and ignore the selection in the other two isolated regions of SR1 and SR3 due to their insensitivity and a larger number of the expected SM background than the LHC observed data. Therefore events should satisfy
these requirements: (1) $40 \text{ GeV} < M_{T2} < 90 \text{ GeV}$, (2) $\Sigma M_T > 350 \text{ GeV}$, (3) $E_T > 50 \text{ GeV}$, and (4) $\Delta \phi(l_1, l_2) > 1.5$. The number of events after each cut selection is reported in the table [II] for the case that only one lightest vector-like lepton effectively contributes to the signal. The assumption of universal coupling leads to the equal branching ratio of $Br(E^- \rightarrow e^- + X) = Br(E^- \rightarrow \mu^- + X) = Br(E^- \rightarrow \tau^- + X) = 1/3$, which can be consistent with the flavour constraint. As we can see from the cut table, for a fixed vector-like lepton mass $M_E$, the event number after the $\Sigma M_T$, $E_T$ and $\Delta \phi(l_1, l_2)$ cuts will decrease as the DM mass $M_X$ is increased. While the event number after the $M_{T2}$ cut will instead be enhanced with an increasing $M_X$. This is a reflection of the $M_{T2}$ quality as a function of the trial mass of missing particle. When the trial mass $\mu_X$ is set to the true mass of missing particle $M_X$, the end point of $M_{T2}$ gives the exact mass of the parent particle $M_E$. However when we increase the DM mass to significantly deviate from the input trial mass $\mu_X = 0$, the end point $M_{T2}$ will drop below the exact mass of vector-like lepton because there is less measured missing energy. In the left plot of Figure 2 we show the $M_{T2}$ distribution after the basic cut for $M_E = 200 \text{ GeV}$, $M_X = 100, 150 \text{ GeV}$. For a larger DM mass benchmark point, the events distribution shifts into the lower mass region due to a false trial mass and therefore leads to an increase for the $M_{T2}$ cut acceptance.

The current 13 TeV CMS search provides an analysis of the SM background contribution, which is $4.35^{+1.73}_{-1.53}$ in the SR2 signal region, and the observed event number is 5. This can be translated into a 68% upper limit exclusion region in the $(M_E, M_X)$ plan, as presented in the right plot of Figure 2. From the plot we can see, for the one vector-like lepton mediation scenario, the LHC constraint is not stringent, at most excluding small mass region $M_X \lesssim 110 \text{ GeV}$ for $M_E \lesssim 240 \text{ GeV}$. However for the three-generation case, the exclusion becomes much more relevant. In certain $M_E$ region, the upper exclusion limit for $M_X$ has reached $185 \text{ GeV}$, which possibly overlaps with the mass region permitted by the relic density and lepton flavour physics shown in Figure 1. Note that in this $U(1)$ gauge extended model, all three generations of vector-like leptons will contribute to this specific LHC signal with a mass hierarchy of $M_{E1} < M_{E2} < M_{E3}$. In particular the second generation of vector-like lepton is supposed to have a relatively larger coupling to muons than the electron and tau leptons, for the sake of the muon $g - 2$ anomaly. Thus under a simplified scenario that the universal coupling condition is loosely observed, the actual LHC exclusion region for $(M_E, M_X)$ should lie between the green and orange band.
FIG. 2: Left plot: The $M_{T2}$ distribution for $M_E = 200$ GeV after the baseline event selection (2 hadronic taus of opposite electric charge with additional veto of electrons and muons) at the $\sqrt{s} = 13$ TeV LHC with a luminosity of 35.9 fb$^{-1}$. Right plot: Exclusion region from the 2 hadronic taus plus MET measurement assuming that vector-like leptons universally couple to the three generations of SM leptons, with the $M_E$ denoting the lightest VLL mass. The green region corresponds to the LHC constraint in the case of one VLL effectively mediating in the production of $pp \rightarrow XX \tau^+\tau^-$; while the orange region corresponds to the three generations scenario with degenerate vector-like lepton masses of $M^1_E = M^2_E = M^3_E$.

IV. SUMMARY AND CONCLUSIONS

We have proposed an inverse seesaw scenario in a framework of hidden $U(1)_H$ gauge symmetry where extra scalars and vector-like neutrinos are introduced to assist the mass generation of neutrino, while vector-like quarks and leptons are required in order to cancel the $U(1)_H$ gauge anomaly. This model features a bosonic dark matter candidate by imposing a $Z_2$ symmetry to stabilise it. In particular, those vector-like quarks and leptons play an important role in realising the observed relic density via interactions controlled by the hidden symmetry. The scalar potential can be tuned so that there is minimal mixing among the SM Higgs and extra scalars. Under this assumption, we have shown the allowed region capable to accommodate the discrepancy in muon $g - 2$, consistent with relic density and other flavor constraints at the same time. However if we permit large mixing between the Higgs boson and the singlet scalar $\varphi'$, this could turn on the invisible Higgs decays and invoke the
Higgs portal DM annihilation channel. For this variation, we need to take into account the DM-nucleon interaction via Higgs exchange since the constraint from the direct detection will become stringent. A comprehensive exploration of this interesting possibility is beyond the current scope and can be considered in the future work.

Concerning the possibility to extract the DM mass bound at the LHC, we focus on the Drell-Yan pair production of vector-like charged lepton as it provides clear signal of charged leptons plus missing transverse energy involving DM. We recasted the recent CMS analysis for 2 hadronic taus plus MET signature based on the $M_{T2}$ variable selection, which shows that lower regions in $(M_E, M_X)$ are ruled out depending on the mass degeneracy among vector-like charged leptons $E_a$ and their branching ratio into tau leptons. However due to the current insensitivity to $\tau_h$, most of allowed parameter space from the relic density and flavor physics would survive even for anarchic Yukawa couplings in vector-like leptons.

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