Economy and embedded exhaustification

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Abstract Building on previous works which argued that scalar implicatures can be computed in embedded positions, this paper proposes a constraint on exhaustification (an economy condition) which restricts the conditions under which an exhaustivity operator can be licensed. We show that this economy condition allows us to derive a number of generalizations, such as, in particular, the ‘Implicature Focus Generalization’: scalar implicatures can be embedded under a downward-entailing operator only if the (relevant) scalar term bears pitch accent. Our economy condition also derives specific predictions regarding the licensing of so-called Hurford disjunctions.

Keywords Exhaustivity · Scalar implicatures · Hurford disjunctions · Redundancy · Focus

1 Introduction

The origin of scalar implicatures (SIs) is a topic of much contention. Recent literature, in particular, has been concerned with whether SIs follow directly from principles of language use, as envisioned by Paul Grice, or whether novel grammatical mechanisms need to be postulated. In order to understand the debate, it

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is useful to define an operator \textit{exh} (for \textit{exhaustification}) which, when applied to a sentence \textit{S} and a set of alternatives ALT, returns the conjunction of \textit{S} and its scalar implicatures relative to ALT (see, among others, Spector 2003, 2007; van Rooij and Schulz 2004, 2006; Schulz and van Rooij 2006). With the help of \textit{exh}, one can characterize, in a compact way, the predictions of various competing theories; more specifically, one can try to compare a theory that incorporates this operator directly into the grammar, henceforth the Grammatical Theory (as in Chierchia 2006; Chierchia et al. 2012; Fox 2007a; Fox and Hackl 2006; Groenendijk and Stokhof 1984a, b; Krifka 1993; Landman 1998; Sevi 2006), with one that derives the results with the aid of postulated principles of language use (henceforth the Neo-Gricean Theory; e.g., Horn 1972, 1989; Gazdar 1979; Gamut 1991; Sauerland 2004; Spector 2003, 2007; van Rooij and Schulz 2004, 2006; Schulz and van Rooij 2006; Russell 2006; Geurts 2011). Various challenges have been recently presented to the Neo-Gricean Theory and have been argued to favor the grammatical alternative (see in particular Chierchia et al. 2012, Sect. 4, and references therein; Magri 2009; Chemla and Spector 2011; Katzir 2013; Fox 2014; Spector 2014; and objections to some of these arguments in Geurts 2009, 2011; Russell 2011; Ippolito 2011; Geurts and Pouscoulous 2009; Geurts and van Tiel 2013; a.o.)

We would like to focus here on one challenge that has been voiced already in the 1970s (Cohen 1971 and much subsequent work) on the basis of the claim that implicatures sometimes need to be computed in embedded positions in order to derive the correct meaning of complex sentences.\footnote{Additional challenges involve environments where SIs are obligatory (Chierchia 2006; Magri 2009; Spector 2014), the connection between SIs and NPI licensing (Chierchia 2004, 2006), Modularity (Fox 2004; Fox and Hackl 2006; Magri 2009; Singh 2008a), the SIs of sentences in which numerals receive cumulative interpretations (Landman 1998), Free Choice Phenomena (Fox 2007a, Chemla 2009; Franke 2011; Klinedinst 2006), and the relationship between SIs and so-called ignorance inferences (Fox 2014). For a review of some of the material, see Chierchia et al. (2012, Sect. 4).
It has been often argued that the Neo-Gricean Theory is conceptually superior to the grammatical alternative (Horn 2009; Geurts 2011; Sauerland 2012). In our opinion, however, this evaluation is not obvious. It is true that the neo-Gricean approach allows for a simpler grammar (in that it avoids the grammatical mechanisms postulated by the grammatical alternative). However, as pointed out by Fox (2007a, 2014), the cost is a more complicated pragmatics than what can be assumed the moment the grammatical mechanism is introduced.} It seems to be a consensus that if this claim is correct, it is problematic for the Neo-Gricean Theory, and the reason is fairly straightforward. Under the Neo-Gricean Theory, SIs are computed on the basis of principles that regulate the choice of communicative acts, and therefore do not apply to sub-constituents of a sentence. By contrast, under the Grammatical Theory there is—within grammar—an implicature-computing operator, and, if no further stipulations are introduced, there should be no ban on embedding this operator in any position in which it would be interpretable.

Various arguments have been given for the existence of embedded implicatures, but there are considerations that go the other way, namely cases where this kind of embedding seems to be impossible. On the basis of these considerations, Horn has famously argued that embedded implicatures should be derived by the postulation of various meta-linguistic operators that would leave the theory of SIs unaffected (see...
in particular Horn 1989, Sect. 6.2). So, the relevance of the phenomenon for the theory of SIs is still much under debate.

The goal of the current paper is to contribute towards a resolution of this dilemma. Specifically, we will suggest that embedded implicatures are real and follow from the Grammatical Theory, i.e., from inserting the operator $exh$ in embedded positions. We will use a more transparent term for this phenomenon, namely *embedded exhaustification*, EE. However, we would also like to deal with counter-arguments of the sort discussed by Horn, i.e., limitations on the availability of EE that are compatible with the Grammatical Theory only if application of $exh$ could somehow be constrained. These limitations, one might think, should argue for the Neo-Gricean Theory. But that, of course, is true only if this theory could account for the full paradigm, which, at least to us, seems unlikely. So we will propose to limit the application of exhaustification by a constraint that we would like to think of as an economy condition. This economy condition will allow $exh$ to be inserted in a given position only if it has a particular overall effect on the meaning of the sentence in which it is embedded, namely if it does not lead to a meaning that is entailed (overall weaker than, or equivalent to) the meaning that would have resulted without it. However, this global requirement will be computed incrementally, at a particular point in sentence processing, based on a technique developed by Philippe Schlenker in another context (Schlenker 2008), and will be stated in terms of a constraint on the choice of an alternative set for the exhaustivity operator. 2

But before we get started, we should say a few words about the meaning of $exh$. The operator takes a sentence $S$ and a set of alternative sentences $C$ and returns a sentence which is stronger than $S$—a sentence which asserts $S$ and denies/excludes certain members of $C$. Which members of $C$ are excluded by $exh$ is a rather complicated matter, but for now we will make the simplifying (though we believe false) assumption that it is the set of sentences not entailed by $S$:

$$exh_C(S) \text{ is true iff } S \text{ is true and } \forall S' \in C [S' \text{ is not entailed by } S \rightarrow S' \text{ is false}].$$

So in order to determine the meaning that results from applying $exh$ to a sentence $S$, one has to know the identity of the set of alternatives, $C$. This set is determined by an interaction of grammar and context, much like the alternatives of focus-sensitive operators, e.g., *only* or *even*. Specifically, $C$ is the intersection of formal alternatives determined by grammar and a set of contextually relevant alternatives. For now we will assume that the formal alternatives are the scalar alternatives of $S$, defined as follows: 3

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2 The overall idea bears obvious resemblance to Chierchia (2004), who claims that SIs are computed locally, but get removed in DE contexts because of a preference for stronger meanings. Our economy condition will retain this idea, with two differences. We do not want to claim that SIs have to be computed locally, and we do not want to entirely rule out EE under DE operators, else we would lose important evidence for EE (see Chierchia et al. 2012).

3 If this is correct, it means that there is a special definition of $C$ that does not follow from the theory of focus—not an optimal assumption, and one we will revise once we discuss the effects of pitch accent on $C$.  

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(2) $\text{ALT}(S) = \{S': S' \text{ can be derived from } S \text{ by replacing scalar items by members of their Horn set}\}$

(3) Examples of Horn sets (usually called ‘Horn scales’):\(^4\)
   a. Connectives: \{or, and\}
   b. Quantifiers: \{some, all\}\(^5\)
   c. Numerals: \{one, two, three, four,…\}

So—ignoring the role of context—we can determine the nature of C based on S and a full specification of Horn sets. For this reason, we will very often omit C from the representation, using sloppy notations such as the following:\(^6\)

(4) a. exh(John talked to Mary or Sue)
   Equivalent to ‘John talked to Mary or Sue but not to both’.
   b. exh(some boys came)
   Equivalent to ‘Some but not all boys came’.
   c. exh(John introduced 3 people to Mary)
   Equivalent to ‘John introduced 3 but not 4 people to Mary’, i.e., to ‘John introduced exactly 3 people to Mary’.

2 Three problems pertaining to embedded exhaustification

One of the most straightforward arguments for EE comes from downward-entailing operators (DE operators). As we’ve seen, $\text{exh}$ strengthens the meaning of a sentence with which it combines.\(^7\) If the output of this combination is embedded under the scope of a DE operator, the overall result is entailed by the representation we get without $\text{exh}$. ($\text{OP}[$exh$(S)$] is entailed by $\text{OP}[S]$ if $\text{OP}$ is a DE operator.) Hence, simple investigation of judgments of assent might tell us whether application of $\text{exh}$ below a DE operator is possible.\(^8\)

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\(^4\) For the most part we will use the two terms (‘Horn set’ and ‘Horn scale’) interchangeably. The term ‘Horn scale’ is somewhat misleading since (a), as we will see, scalar alternatives need not be totally ordered (i.e., need not form a scale), and (b) the order of the members of the set is irrelevant for SI; what is relevant is the ordering of the members of $\text{ALT}(S)$ (see Sauerland 2004). We will be careful to use the term ‘Horn scale’ when we make specific reference to the ordering of the elements in the ‘Horn set’, e.g., when we explain (in Sect. 3.1) why we label certain disjunctions DEDs.

\(^5\) The set probably has additional members, e.g., many, most, which we will, for now, ignore.

\(^6\) As we progress, we will say more about the way context and grammar interact to determine alternatives, and at that stage we will be unable to omit the parameter C.

\(^7\) We use strengthen in the weak sense here, meaning simply that $\text{exh}(S)$ always entails $S$. The two can be equivalent when there are no alternatives to C that get to be excluded. By the simplified definition of $\text{exh}$ in (1), equivalence would hold only when S entails all alternatives in C, in which case the application of $\text{exh}$ is vacuous.

\(^8\) As we will discuss in Sects. 7–10, if $\text{exh}$ is inserted above a DE operator, the overall environment for yet another $\text{exh}$ below the DE operator need not be DE (it could be non-monotonic instead). This will not affect the methodological point about judgments of assent, for which the environment needs to be non-upward monotone, but need not be downward monotone.
Consider from this perspective the acceptability of (5). If \( exh \) were not embedded under negation, the second sentence in (5) would contradict the first.

(5) John didn’t do the reading OR the homework. He did both.  
(based on Cohen 1971)

This is because the negation of a disjunction is tantamount to the claim that each disjunct is false, which of course entails the negation of the conjunction:  
\[ \neg(p \lor q) \iff \neg p \land \neg q \Rightarrow \neg(p \land q). \]
If, however, \( exh \) could be embedded under negation, we will have a straightforward explanation for the fact that the sentences are not judged to be contradictory (i.e., that one might assent to (5), or assert it in the first place). The negation of the exhaustified meaning of a disjunction (of the exclusive disjunction; see (4a)) is not equivalent to the claim that each of the disjuncts is false. Instead, it is compatible with two possibilities—either each of the disjuncts is false or they are both true:  
\[ \neg \text{exh}(p \lor q) \iff \neg((p \lor q) \land \neg(p \land q)) \iff \neg(p \lor q) \lor (p \land q). \]

So the acceptability of (5) can be taken to argue in favor of EE. However, it has been noted in many places, most notably in Horn (1989), that (5) is acceptable only on a “marked” pronunciation, namely pronunciation with a pitch accent on the scalar item or. Without this pitch accent, the sentence is unacceptable:

(6) #John didn’t do the reading or the homework. He did both.

This allows us to state our first problem.

(7) Problem #1
   a. If EE is possible, why is (6) bad?  
      (a question raised most forcefully by Horn 1989)
   b. If EE is impossible, why is (5) good?  
      (a question raised by many: Cohen 1971; Levinson 2000, i.a.)

More specifically, we will argue for the following generalization, and our problem will be to understand why it holds.

(8) The Implicature Focus Generalization:
   Implicatures can be embedded under a downward entailing (DE) operator only if the (relevant) scalar term bears pitch accent.\(^9\)

In Chierchia et al. (2012, henceforth CF&S), we argued that EE is subject to an economy condition that bars any occurrence of an exhaustivity operator if it leads to a reading that is logically (strictly) weaker than (i.e., asymmetrically entailed by) the reading that would have resulted in its absence. This economy condition predicts

\(^9\) Our main argument for this generalization is based on the rejection of an alternative generalization that suggests itself, the one which is presupposed in Horn’s work, namely that any embedding of implicatures requires pitch accent on scalar items.
that an exhaustivity operator cannot occur in a DE environment, which captures the judgment given in (6). In order to account for the fact that EE appears to be possible in the scope of DE operators if the relevant scalar item bears pitch accent (cf. (5)), one could adopt Horn’s view that cases of this sort involves a special use of negation (or other DE operators), commonly labeled *metalinguistic*.

In this paper, we will develop a modified version of the economy condition proposed in CF&S, and this modification will still be compatible with the view that cases like (5) involve a metalinguistic use of negation. However, the arguments we will develop in connection with other cases of EE will suggest a different perspective on the generalization given in (8), one that relates the requirement of pitch accent to independently needed properties of the theory of focus.

Our second problem is based on an observation due to Gajewski and Sharvit (2012) (henceforth G&S), when combined with an argument for EE made in CF&S. The latter argue that (9a) involves EE, specifically that it has the parse in (9b).

(9) a. John talked to Mary or Sue, or both.
   b. exh(John talked to Mary or Sue) or John talked to both

One of the arguments, which we will review below, is based on the fact that this parse is needed in order for the sentence to comply with a general condition that rules out disjunctive sentences in which one of the disjuncts entails another. The condition is known as *Hurford’s Constraint* (henceforth HC; Hurford 1974). We will call such disjunctive sentences—namely those that would violate HC without EE—‘Hurford disjunctions’.

If EE is possible, why is a Hurford disjunction bad when embedded under negation, for example in (10a) (as observed by G&S)? After all, it should be possible for this sentence to receive the parse in (10b), which does not violate HC.10

(10) a. *John didn’t talk to Mary or Sue, or both.  (G&S)
   b. Not [exh(John talked to Mary or Sue) or John talked to both]

(11) Problem #211
   a. If EE is possible, why can’t a Hurford disjunction be embedded under negation? E.g., why is (10a) bad?
   b. If EE is impossible, why are Hurford disjunctions in general good? E.g., why is (9a) good?

Our third problem is similar to the second problem and pertains to a constraint on Hurford disjunctions discovered by Singh (2008b). Consider the contrast in (12) below. (12a) is a repetition of the acceptable Hurford disjunction we have seen—(9a)—and, as mentioned, we assume that it is acceptable due to EE. But why should

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10 Note that (10b) does not violate CF&S’s economy condition mentioned above (below example (8)), since in this case the exhaustivity operator is semantically vacuous.

11 The account of Hurford’s Constraint developed in Meyer (2013, 2014) leads to a different solution to this problem (though not to Problems #1 and #3). We do not have the space to discuss this solution here, but hope to return to it on some other occasion.
the Hurford disjunction be bad when the order of the disjuncts is reversed, as in (12b)?

(12) a. John talked to Mary or Sue, or to both Mary and Sue.
    b. #John talked to both Mary and Sue, or to Mary or Sue.

(13) **Problem #3**

If EE is possible, why can’t the disjuncts in a Hurford disjunction such as (9a) be reversed?

So, we have seen three cases where EE needs to be constrained: it cannot apply under DE operators, unless there is pitch accent on the relevant scalar item (Problem #1, the Implicature Focus Generalization); it is unable to obviate HC under negation (Problem #2); and it also cannot obviate HC on a second disjunct of a Hurford disjunction (Problem #3). Our goal for this paper is to develop an economy condition which will solve these three problems and to present a few arguments in favor of our solution.

But before we get there, we will have to review the arguments presented in CF&S in favor of EE in Hurford disjunctions. We will then move to our solution for Problems #1–3, which will be easiest to understand in reverse order. The discussion of Singh’s observation (Problem #3), with which we will start (Sect. 5), will be based on a simplified version of our economy condition. Our goal will be to show that this simplified condition derives Singh’s basic observation and makes a host of new predictions, which we will try to corroborate. We will then discuss how the condition might be extended to predict that EE is impossible in DE contexts, thus accounting for Gajewski and Sharvit’s problem (Problem #2), with an additional new prediction (Sect. 6). This discussion will seem to be at odds with the Implicature Focus Generalization (Problem #1), but we will show that it need not be (Sect. 7). Specifically, we will discuss certain assumptions about the nature of scalar alternatives and their relationship to focus that will allow us to extend the proposal to a solution for this problem as well. As we develop our proposal we will be presenting a variety of very detailed predictions, some of which will involve rather subtle contrasts in acceptability judgments. We are not always as confident about these judgments as we would like to be. Nevertheless we think that stating the predictions explicitly would be useful in understanding the nature of our proposal.

3 **Hurford’s constraint and embedded exhaustification**

In both sentences in (14), the second disjunct entails the first one. Hurford uses the unacceptability of sentences of this sort to argue that entailment between disjuncts is, in general, impossible—that is, for the constraint in (15), to which we alluded above.

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12 Two notes are in order. First, the relevant notion of entailment for HC is that of entailment given background presuppositions (e.g., that Paris is in France or that a German Shepherd is a dog), so-called 'contextual entailment'. Second, to simplify the discussion, we will treat all our disjuncts as sentential. This choice is not crucial for our purpose: it would be simple enough to assume non-sentential coordinates, but this, as usual, would require irrelevant complications: i.e., a new flexible type for $\textbf{exh}$, along with the familiar flexible type for $\textbf{or}$ and a generalized notion of entailment.
(14) a. #John was born in France or Paris.
   b. #I have a dog or a German Shepherd.

(15) **Hurford’s Constraint (HC):**

A disjunction \( p \) or \( q \) is unacceptable when \( p \) entails \( q \)
or \( q \) entails \( p \).\(^{13}\)

But the acceptability of the sentences in (16) now appears to be problematic.

(16) a. John talked to [Mary or Sue] or both. (Hurford 1974)
   b. John did some or all of the homework.
   c. John read three books or more. (Gazdar 1979)

There are two possible reactions to (16). HC might be modified, as suggested by Gazdar (1979), or the disjuncts might receive a special analysis making them compatible with HC despite surface appearances. We will not go over Gazdar’s suggestion, but instead review some of CF&S’s arguments in favor of the second possibility, specifically in favor of the claim that the sentences in (16) are compatible with HC due to EE, and that, more generally, (17) holds.

(17) **EE is the culprit:** HC is correct, and wherever it appears to be false, EE is involved.

CF&S, in particular, argue that the sentences in (16) must receive the parse in (16’), which doesn’t violate HC. In this parse the first disjunct is exhaustified, and the exhaustified meaning involves the denial of the second disjunct. Thus, the second disjunct does not entail the first.

(16’) a. \([\text{exh}(p \lor q)] \lor (p \land q)\)
   b. \([\text{exh}(\text{John did some of the homework})] \lor (\text{John did all of the homework})\)
   c. \([\text{exh}(\text{John read 3 books})] \lor (\text{John read more than 3 books})\)

Some have suggested that local implicatures (what we analyze as EE) always involve a marked intonation (e.g., Horn), but the sentences in (16) do not require a marked intonation.\(^{14}\) So, if the parse in (16’) is correct, EE does not always require marked intonation. In other words, if the analysis is correct, Hurford disjunctions

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\(^{13}\) It is tempting to think of this condition as one that bans redundancy (in the offending disjunctions, the stronger disjunct is redundant). See Katzir and Singh (2013) and Meyer (2013, 2014) for a discussion of how this might be achieved. See Singh (2008b) for arguments that the constraint should be strengthened—arguments, which as Singh mentions, do not bear on our conclusions in this section.

\(^{14}\) Geurts (2009, 2011) argues that cases of this sort involve pragmatic strengthening of the scalar item, but views this strengthening as completely distinct from scalar implicature generation. He assumes that a pragmatic ‘reconstrual’ mechanism can affect the interpretation of lexical items. Sauerland (2012) and footnote 20 of this paper contain arguments that a purely lexical mechanism is not sufficient to cover all the relevant data. Geurts (2011) seems to envision in passing a non-purely lexical mechanism that would also be distinct from scalar implicature generation (footnote 6, pp. 184–185), but this mechanism is left entirely unspecified in the case of embedded scalar items. If a non-lexical strengthening mechanism is posited for cases of apparently embedded SIs, it is unclear to us that the resulting theory, once made explicit, will be clearly distinct from one that allows for embedded exhaustification.
support our statement of the generalization pertaining to pitch accent, namely that pitch accent is needed only when \textit{exh} is embedded under DE operators (see note 9).

But what can tell us whether the parse is correct? After all, \textit{exh} in (16') has no consequences for the overall meaning of the construction. However, CF&S argue for this parse based on constructions where this is not the case. The arguments, which we will review, will be relevant for two reasons. First they will provide support for EE, without which there will be no point in postulating our economy constraint. Second, they will be directly relevant for our arguments in favor of the economy condition, since this condition will be sensitive to the overall semantic consequences of EE, and the manipulations needed to show that EE can have semantic consequences in Hurford disjunctions will turn out to be directly relevant in determining whether EE can apply.

3.1 Distant entailing disjunctions

The reason EE has no overall semantic consequences in (16') is that the states of affairs excluded by \textit{exh} in the first disjunct verify the second disjunct. Subsequently, the result of the exclusion is rendered vacuous. To see this, consider (16'c). \textit{Exh} on the first disjunct excludes worlds where John read more than three books. But in such worlds the second disjunct is true, and the sentence as a whole is, consequently, true. In other words, everything excluded by \textit{exh} in the first disjunct is allowed to “sneak in” by the second disjunct.

One way to ensure that exhaustifying the first disjunct has overall effects on meaning is to construct disjunctive sentences where this is not the case: where there are states of affairs that are excluded by \textit{exh} in the first disjunct that do not get to sneak in thanks to the second disjunct. If EE is required, the sentence will be false in those states of affairs.

The disjunctive sentences in (18) have the specified property.

(18) a. John has three or six children.\footnote{Recent literature has suggested that numerals might always have an \textit{exactly} meaning (Breheny 2008; Kennedy 2013, 2015; see Spector 2013 for discussion). If these claims are correct, then (18a) would be an irrelevant piece of our argument. It is our impression, however, that there is no escape from the \textit{at-least} meaning being one of the possible meanings for numerals (see Kennedy 2015’s use of Partee shifters to weaken the \textit{exactly} meaning to form an \textit{at-least} meaning). If this impression is correct, it is hard to see how to derive the effect in (18a) on Gazdar’s approach.}

- Horn scale <…3, 4, 5, 6, …>

b. The water is (somewhat) warm or absolutely boiling.

- Horn scale <\textit{warm}, \textit{hot}, \textit{absolutely boiling}>

To see this, consider first (18a). Exhaustifying the first disjunct excludes any world where John has more than three children, and there are, of course, worlds of this sort where the second disjunct is false, i.e., where John doesn’t have six children. In those worlds, i.e., worlds where John has exactly four or exactly five children, exhaustifying the first disjunct will make the sentence false. Similarly, exhaustifying the first disjunct in (18b) excludes not only worlds that are allowed to sneak in by the second disjunct— worlds where the water is absolutely boiling—but also worlds
that are excluded by the second disjunct, namely worlds where the water is hot but not yet absolutely boiling. Exhaustifying the first disjunct will make the sentence false in these worlds.

The combination of HC and EE thus makes new predictions for these sentences. They should be limited to interpretations that are false in the states of affairs we just characterized (worlds that are excluded by the first disjunct and do not verify the second disjunct). This prediction seems to be correct. Thus (18a) seems to be false if John has exactly four or five children, and (18b) seems to be false if the water is hot but not absolutely boiling.

What was special about the examples in (18) is that the two disjuncts contained scalar items that are non-adjacent (distant) members of a Horn scale. This is why we will call disjunctive constructions of this sort *Distant Entailing Disjunctions* (DEDs). So in (18a) the scalar items 3 and 6 are separated on the Horn scale by two other numerals (4 and 5). Similarly in (18b) (*somewhat*) warm and *absolutely boiling* are separated on their Horn scale by *hot*. Therefore, the exhaustified meaning of the first disjunct excludes weaker sentences than the second disjunct, i.e., sentences consistent with situations that the second disjunct excludes.\(^{16}\) These situations are predicted to falsify the sentence.

But the same effect can also be generated without distant members of a Horn scale. We can thus generalize the notion of a Distant Entailing Disjunction in the following way.

*(19)* A *Distant Entailing Disjunction* (DED) is a disjunctive phrase \(p \text{ or } q\) with the following properties:

a. \(q\) entails \(p\)

b. This entailment can be obviated by exhaustification: there is a way to strengthen \(p\) by \(\text{exh, } p^*\) such that \(q\) doesn’t entail \(p^*\).\(^{17}\)

c. \((p \text{ or } q)\) is not equivalent to \(p^* \text{ or } q\) (it’s strictly weaker) where \(p^*\) is the relevant strengthening of \(p\).\(^{18}\)

Since \(q\) entails \(p\) in a DED, HC predicts that a DED should be bad as is. EE allows the DED to receive the meaning of \(p^* \text{ or } q\), rather than that of \(p \text{ or } q\) (which is equivalent to \(p\)). This prediction seems to be corroborated for (18), as mentioned. But there are consequences for other construction types discussed in CF&S.

One prediction pertains to the strengthened meaning of sentences that contain operators with a universal force. Consider the construction in (20).

\(^{16}\) This effect is observed only when the intermediate member of the scale, namely *hot*, is taken into account—a highly context-dependent condition. What is important is that the reading we describe here is clearly a possible reading. See Sect. 9, in which we discuss constraints on the choice of alternatives for \(\text{exh}\).

\(^{17}\) We write \(p^*\) instead of \(\text{exh}(p)\) because in some cases the exhaustivity operator that must be introduced in order to break the entailment relation will actually not take scope over \(p\), but over a sub-constituent of \(p\) (see Chierchia et al. 2009, ex. 17). \(p^*\) thus stands for an expression identical to \(p\) yet stronger due to the fact that at least one occurrence of \(\text{exh}\) has been added.

\(^{18}\) The disjunctive phrase in the reversed order, \(q \text{ or } p\), is also going to be called a DED, once we get to Singh’s problem. But for now we will focus on the order \(p \text{ or } q\).
Either the students are required to do the reading or the homework, or they are required to do both.

There should, in principle, be two ways of strengthening the first disjunct, both of which will have overall semantic consequences. Specifically, \( \text{exh} \) can be embedded within the first disjunct either above or below the universal modal \( \text{required} \): 
\[
\text{exh}[\Box(p \lor q)], \text{ or } \Box[\text{exh}(p \lor q)].
\]
For reasons that are not entirely clear to us (see CF&S for some discussion), the former possibility is very much preferred when the sentence is uttered in isolation, so we will focus on it.

To compute the meaning of 
\[
\text{exh}[\Box(p \lor q)]
\]
we need to know the scalar alternatives of 
\[
\Box(p \lor q).
\]
But independently of our purpose here, we know what the result has to be, namely that the strengthened-meaning/implicature of \textit{the students are required to do the reading or the homework} should involve the claim that it is up to the students to decide whether to do the reading or the homework. For now, we will derive this under Sauerland’s (2004) assumption that the Horn set for disjunction contains each of the disjuncts in addition to the conjunction. From this, it follows that
\[
\text{exh} \Box(p \lor q)
\]
involves the exclusion of the two alternatives \( \Box p \) and \( \Box q \).
That is: 
\[
\text{exh} \Box(p \lor q) \leftrightarrow \Box(p \lor q) \land \neg \Box p \land \neg \Box q \Rightarrow \Box q \lor \Box p.19
\]

If this is the strengthening that is used to obviate HC, the overall result is not vacuous.20 The sentence ends up meaning that \textit{either} the students are required to do the reading or the homework and have a choice as to which one to do, \textit{or} they are required to do both.21 It is thus incompatible with \( \Box \neg p \) and with \( \Box \neg q \). This is not equivalent to the reading corresponding to a structure with no \text{exh}, \((\Box(p \lor q) \lor \Box(p \land q))\), equivalent to \( \Box(p \lor q) \), since the latter parse is compatible with \( \Box \neg p \) (and likewise with \( \Box \neg q \)).22 This prediction seems to us to be correct. That is, (20) is judged false in a situation where students are required to do the reading but are not required to do the homework, or the other way around, but not in a situation where they are required to do both.

In the next subsection we will discuss the predicted effects of EE on the implicatures of sentences that embed Hurford disjunctions—effects that follow from the consequences of EE for the scalar alternatives of a Hurford disjunction. But first

---

19 See also Spector (2003, 2006), Sauerland (2004), Fox (2007b), among others.

20 Note that in this case exhaustification takes place at an intermediate scope site, i.e. within the scope of the matrix disjunction but above \textit{required}. For this reason, attempts to salvage the neo-Gricean approach by assuming a process of local \textit{lexical} enrichment in order to account for apparent cases of embedded SIs fail to account for such cases. Sauerland (2012) mentions additional examples. Consider:

(i) We are either required to write more than three papers or more than four (I don’t remember).

This sentence is easily understood as a disjunctive statement about the minimal writing requirement, a meaning that can only result from exhaustifying each disjunct above \textit{required}.

21 As noted by a reviewer, (20) also triggers an \textit{ignorance implicature} whereby one understands that the speaker does not know whether the students have a choice between doing the reading and doing the homework, or whether they have to do both. We assume here that such ignorance implicatures are \textit{not} computed within the grammar. See Meyer (2013, 2014) for an alternative view; see also footnote 33 below.

22 It is not equivalent either to what would result if \text{exh} applied to the whole sentence (‘\text{exh}[\Box(p \lor q) \lor \Box(p \land q)]’), which, given the definition in (1), would implicate (among others) \( \Box \neg(p \land q) \), i.e. ‘We are not required to do both the reading and the homework’. (This is not an available parse; see the discussion in Sect. 3.2.1).
we need to provide a bit more background about the idea that the disjuncts are alternatives of a disjunctive expression. This idea, which seems to be needed for a variety of purposes in addition to the one mentioned above (see Sauerland 2004; Fox 2007a, b), is hard to implement given the view of scalar alternatives we are adopting; in particular, it doesn’t follow from the Horn set for disjunction given in (3a).

To deal with this, we could follow Sauerland (2004) and expand the set to include the abstract elements $L$ and $R$, where $p L q$ is equivalent to $p$ and $p R q$ is equivalent to $q$:

(21) Sauerland’s alternatives for disjunction (Sauerland 2004):

\{or, $L$, $R$, and\}

It would obviously be better to derive the same alternatives without appeal to $L$ and $R$ (see Katzir 2007; Alonso-Ovalle 2006, 2008), and ultimately we will move to a system in which we derive the right alternatives without resorting to this undesirable stipulation. But for now, let us adopt Sauerland’s implementation—the only implementation, as far as we know, compatible with the idea that scalar alternatives are derived based on Horn sets.

The obvious problem with the proposal that the disjuncts are among the alternatives of a disjunctive statement needed to account for the free choice effect is that our operator $exh$, defined in (1), now yields a contradiction when it applies to a simple disjunctive sentence: if (1) is correct, $exh(p or q)$ states that the disjunctive sentence is true and each of its alternatives (none of which it entails) is false. In particular, it would now follow that each of the disjuncts is false. The problem arises whenever the set of alternatives contains what we might call symmetry, i.e., whenever it contains two or more alternatives that can be excluded separately but not together (e.g., if $p or q$ is consistent with the exclusion of $p$ or with the exclusion of $q$, but the moment both are excluded the result is contradictory).

In order to correct for this problem, we have to revisit the question of which alternatives get to be excluded. (1) stated that entailed alternatives don’t get excluded. But why? The reason seems rather obvious. Exclusion of weaker alternatives would lead to an automatic contradiction. As we’ve just seen, though, there are other exclusions that would lead to a contradiction, namely exclusions of symmetric alternatives. We might therefore suggest that $exh$ be modified to eliminate contradiction in situations of symmetry. In particular, given a sentence $S$ and a set of alternatives $C$, we would like to define a set of innocently excludable alternatives, $IE(S,C)$—a set of sentences that can all be false while $S$ is true. 23

(22) a. $exh(S)$ is true iff $S$ is true and $\forall S' \in IE(S,C)$: $S'$ is false.

b. $IE(S,C)$ is the intersection of maximal excludable alternatives of $C$ given $S$.

c. $M \subseteq C$ is an excludable alternative of $C$ given $S$, if the conjunction of $S$ and the negation of all members of $M$ is consistent.

23 This is the definition of Fox (2007a), which is very much inspired by Sauerland’s algorithm for the computation of SLs. There is a stronger definition of $exh$, inspired by Groenendijk and Stokhof (1984b), which also does not derive a contradiction in the relevant cases; for details see Spector (2003, 2007), van Rooij and Schulz (2004, 2006), and Schulz and van Rooij (2006). See Spector (2016) for discussion.
d. \( M \) is a maximal excludable alternative of \( C \) given \( S \) if \( M \) is an excludable alternative of \( C \) given \( S \) and there is no superset of \( M \) in \( C \) which is an excludable alternative of \( C \) given \( S \).

The result of applying \( exh \) could be rather difficult to compute at times, but the notion itself is rather simple. All alternatives that are not entailed by the prejacent are excluded unless there is symmetry, in which case as many alternatives are excluded as possible (without making any arbitrary choices among symmetric alternatives). However, in most cases we will not need to consider (22). Specifically, whenever (1) is non-contradictory, we can continue to use it. The reason is simple. If (1) is non-contradictory, the set of innocently excludable alternatives is precisely the set of sentences in \( C \) not entailed by \( S \). We will, thus, continue to use (1) whenever the result is consistent.

However, one important consequence of symmetry should be noted. Whenever a sentence \( S \) has symmetric alternatives \( S_1 \) and \( S_2 \) (such that \( S \land \neg S_1 \) is consistent and \( S \land \neg S_2 \) is consistent but \( S \land \neg S_1 \land \neg S_2 \) is inconsistent), two effects are expected. First, \( S \) will not generate SIs based on \( S_1 \) or \( S_2 \); that is, neither \( \neg S_1 \) nor \( \neg S_2 \) will be SIs of \( S \). Second, if \( S \) is embedded under a universal operator, symmetry will be eliminated and the SIs should re-emerge, since, e.g., \( \Box S \land \neg \Box S_1 \land \neg \Box S_2 \) is consistent (see Fox 2007b for details). These two consequences can be used to argue for the existence of symmetry in cases where the nature of the alternatives is not obvious (see Fox and Katzir 2011). This is what we turn to next. We will see that Hurford disjunctions have symmetric alternatives only if the first disjunct is exhaustified, as assumed by CF&S, and that the two consequences of symmetry are indeed attested.

### 3.2 Embedding Hurford disjunctions under matrix \( exh \)

(distinct implicatures)

Another prediction discussed by CF&S pertains to the (non-embedded) implicatures of sentences that themselves embed Hurford disjunctions. Since we are deriving implicatures by \( exh \), be they embedded or global/matrix, we describe the relevant environments as environments where a Hurford disjunction is embedded under a matrix \( exh \). If HC is correct, the environments can be schematized as follows:

\[(23) \text{Structure for implicatures of sentences that embed Hurford disjunctions:} \]

\[exh(...[exh(p) \ or \ q]...)

#### 3.2.1 Vacuous embedding

The first case to consider is a simple Hurford disjunction, as in (24b).

\[\text{(24) a. John bought some of the furniture.} \]

\[\text{b. John bought some or all of the furniture.} \]

Since the early days of theorizing on the nature of SIs, the fact that (24b) is not associated with the same SIs as the simple sentence in (24a) has been viewed as a
problem. We will see that the problem is eliminated the moment the role of EE is understood.

The two sentences in (24) are equivalent. Under the Neo-Gricean Theory, as well as the grammatical alternative we are considering, the only way two equivalent sentences can be systematically associated with different SIs is if they have different scalar alternatives. The problem is that, without EE, (24a) and (24b) have equivalent scalar alternatives.

To see this, consider the scalar items in (24b): *some*, *all*, and *or*, with the following Horn sets.

(25) a. Connectives: \{or, L, R, and\}
b. Quantifiers: \{some, all\}

We thus get the following sentential alternatives:

(26) a. Alternatives for (24a):
{John bought some of the furniture, John bought all of the furniture}

b. Alternatives for (24b):
{John bought some of the furniture, John bought all of the furniture,
John bought some or all of the furniture, John bought all or all of the furniture,
John bought some or some of the furniture, John bought some and all of the
furniture, John bought some and some of the furniture, John bought all and all of the
furniture}

Although there are more alternatives in (26b) than in (26a), the alternatives in (26b) divide into two sets, each of which consists of equivalent sentences (separated by a vertical line). One set (on the left) is equivalent to the *some* alternative in (26a), and the other (on the right) to the *all* alternative. So it is easy to see that the same SI is predicted for (24a) and (24b).

This problem is obviated the moment EE applies to the first disjunct. With EE, we have another alternative, namely the first disjunct \(exh(John \ bought \ some \ of \ the \ furniture)\), which states that John bought some but not all of the furniture. This alternative is symmetric relative to the *all* alternative. One cannot exclude both, and hence neither is innocently excludable. There are thus no innocently excludable alternatives, and matrix application of \(exh\) is vacuous—precisely the result we want.
3.2.2 Non-vacuous embedding

However, as discussed above, symmetry is eliminated the moment we embed the relevant sentence under a universal operator (e.g., a universal quantifier or a necessity modal). We thus predict that (27b) (under the reading where either...or takes scope below required) will contrast with (27a) in its implicatures.

(27) a. You’re required to buy some of the furniture.
   Alternative: □[you buy all of the furniture].
   b. You’re required to either buy some or all of the furniture.
      Alternatives (predicted by HC and EE):
      1. □[you buy all of the furniture].
      2. □[exh (you buy some of the furniture)].
         ([I.e., □[you buy some but not all]).

Example (27b) has an alternative that (27a) does not have (alternative 2). As discussed abstractly at the end of Sect. 3.1, one can deny both alternatives of (27b) without contradiction, hence both are innocently excludable. It follows that (27b) can implicate the negation of both alternative 1 and alternative 2, as expressed schematically in (28a), which, together with the literal meaning of (27b), yields the strengthened reading expressed in (28b):

(28) a. ¬□(you buy some but not all of the furniture) and
   ¬□(you buy all of the furniture)
   b. You are required to buy some of the furniture,
      you are not required to buy all of it, but you are allowed to buy all of it.

EE is crucial for deriving the inference ‘You are allowed to buy all of the furniture’ since it gives us alternative 2. We thus predict (27b) to contrast with (27a) precisely in this respect—a prediction that seems to be corroborated by the contrast between (29b) and (29c) below:

(29) a. You’re required to buy some of the furniture.
   No! We have to buy all of it.
   (Good objection: denies the implicature ‘We don’t have to buy all of it’)
   b. You’re required to buy some of the furniture.
      # No! We are not allowed to buy all of it.
      (Bad objection: ‘You are allowed to buy all of it’ is not an implicature)
   c. You’re required to either buy either some or all of the furniture.
      No! we are not allowed to buy all of it.
      (Good objection: denies the implicature ‘We are allowed to buy all of it’)

If we are right, the inferences in (28) are entailments of (27b) under the following parse, where matrix exh is responsible for the negation of the alternatives mentioned below (27b):
(30) \( \text{exh}(\square [\text{exh(you do some of the homework) or you do all of the homework}]) \)

The point that we have just made can in fact be generalized to all cases where a Hurford disjunction is embedded within the scope of an operator with universal force. We expect completely similar facts when a Hurford disjunction is embedded within the scope of a universal quantifier over individuals (instead of a universal quantifier over worlds, i.e., a necessity modal). That is, in the case of (31) below, both alternatives 1 and 2 can be negated without contradiction.

(31) Every student either solved most or all of the problems.
    Alternatives (predicted by HC and EE):
    1. Every student solved all of the problems.
    2. Every student \( \text{exh(solved most of the problems)} = \)
       Every student solved most of the problems but not all of them.

So applying \( \text{exh} \) at the matrix level to (31) yields (32a), which is equivalent to (32b):

(32) a. Every student solved most of the problems, and not every student solved all of the problems, and not every student solved only most of the problems.
    b. Every student solved most of the problems; and some, but not all, students solved all of the problems.

The embedded exhaustivity operator is responsible for the inference that some students solved all of the problems (since it is responsible for the presence of alternative 2).

As a result, the following contrast is predicted:

(33) a. Every student solved most of the problems.
    #No! No student solved all of the problems.
    b. Every student solved most or all of the problems.
    No! No student solved all of the problems.

3.3 Interim summary

By assuming that apparent violations of Hurford’s Constraint involve parses where an exhaustivity operator applies to the first disjunct, we can explain a number of complex facts in a simple way. First, we account for these apparent violations. Second, we predict very specific readings in cases which involve Distant Entailing Disjunctions, which turn out to be the perceived readings (Sect. 3.1). Third, we also predict the fact that the relevant sentences themselves trigger different implicatures from their corresponding, non-Hurford-violating counterparts (Sect. 3.2). These two types of prediction will play an important role in motivating our account for Singh’s asymmetry (Problem #3).
4 Basic strategy

In the rest of the paper, we provide a solution to the three problems that we presented in Sect. 2. As mentioned at the outset, our basic strategy for addressing the constraints on the distribution of Hurford disjunctions is to introduce an economy condition that will prevent \textit{exh} from appearing in the position required for HC to be obviated.

Specifically, our economy condition will be designed to block the schematic representations in (36) for (10b) and (12), which are repeated below as (34) and (35).

(34) Gajewski and Sharvit’s restriction (Gajewski and Sharvit 2012):
*John didn’t talk to Mary or Sue or both.

(35) Singh’s Asymmetry (Singh 2008b):
*John either talked to both Mary and Sue or to Mary or Sue.

(36) a. ¬[exh(p or q) or (p & q)]
    b. (p and q) or exh(p or q)

As a result, the only available parses for (34) and (35) will be the ones below, which, of course, violate HC.

(37) a. (p and q) or (p or q)
    b. ¬[(p or q) or (p & q)]

Finally, our solution to Problem #1 (the account of the Implicatures Focus Generalization) will be based on a generalization of the economy condition. It will thus make sense for us to address our problems in reverse order.

5 An economy condition on \textit{exh} insertion (first version)

In this section, we will introduce a preliminary version of our economy condition which is able to solve Problem #3, and we will derive from it a number of new predictions directly connected to the data discussed in the previous section. Specifically, in precisely those cases where the obligatory presence of \textit{exh} in a Hurford disjunction had a detectable semantic effect, our economy condition will predict that the disjuncts \textit{can} be reversed.

5.1 Addressing Problem #3: first version of the economy condition

Let us first present our idea in a rather informal way. As we observed, sentences of the form ‘\textit{p or q or (p and q)}’ and ‘\textit{(p and q) or p or q}’ violate HC, but this is not so for the schematic sentences in (38).

(38) a. exh(p or q) or (p and q)
    b. (p and q) or exh(p or q)
Our economy condition must thus allow \( \text{exh} \) to appear in (38a) but not in (38b). Consider first a rather natural yet incorrect principle, one that bars any occurrence of \( \text{exh} \) which is semantically vacuous:

\[
(39) \quad \text{if } S \text{ is equivalent to } S', \text{ where } S' \text{ is obtained from } S \text{ by deleting an occurrence of } \text{exh},
\]

This condition is clearly too strong, as it would rule out both (38a) and (38b): in both cases \( \text{exh} \) is semantically vacuous, since both (38a) and (38b) are equivalent to \( (p \text{ or } q) \), which is equivalent to \( (p \text{ or } q) \) or \( (p \text{ and } q) \) (and to \( (p \text{ and } q) \) or \( (p \text{ or } q) \)).

But can we find a natural modification of this principle that would distinguish the two parses? In other words, one that would rule out (38b) but not (38a)? Indeed, we can, building on a technique developed in Schlenker (2008) in the context of an argument for a somewhat similar economy condition aimed at predicting how presuppositions project. Consider again (38a), repeated as (40):

\[
(40) \quad \text{exh}(p \text{ or } q) \text{ or } (p \text{ and } q)
\]

In this construction \( \text{exh} \) is semantically vacuous. Note, however, that at the point where \( \text{exh}(p \text{ or } q) \) has been encountered by a hearer (i.e., before the second disjunct has been reached), there is no way for the hearer to know that \( \text{exh} \) will turn out to be vacuous. Assume that just after \( \text{exh}(p \text{ or } q) \) has been parsed, the hearer entertains the correct hypothesis about the overall structure of the sentence, i.e., one that would not lead to a garden path. In other words, assume that the hearer assigns the overall structure \([\text{exh}(p \text{ or } q) \ X \ Y]\), where \(X\) stands for a binary connective and \(Y\) stands for an arbitrary constituent of the appropriate type. There are choices for \(X\) and \(Y\) that would not make \( \text{exh} \) semantically vacuous (take for instance \(X = \text{or}\) and \(Y = j\), with \(j\) logically independent of \(p\) and of \(q\), yielding \(\text{exh}(p \text{ or } q) \text{ or } j\)). When such choices exist, we will say that \( \text{exh} \), though globally vacuous, is not incrementally vacuous.

For \( \text{exh} \) to be incrementally vacuous in a given sentence \(S\), it must be the case that, given a correct parse for \(S\), \( \text{exh} \) can be determined to be globally vacuous independently of the meaning of the constituents that follow \( \text{exh} \) and its argument (i.e., if it can be determined to be vacuous right at the point after \( \text{exh} \) and its argument have been encountered). More precisely, for \( \text{exh} \) to be incrementally vacuous in a sentence \(S\), it should be globally vacuous not only in \(S\), but also in every sentence \(S'\) that can be obtained from \(S\) by replacing any constituent that follows \( \text{exh} \) and its argument with an arbitrary constituent.\(^{24}\)

---

\(^{24}\) Incremental vacuity is defined in terms of both linear ordering and constituent structure: that is, the only sentences that have to be considered in order for us to know whether a certain occurrence of \( \text{exh} \) in a sentence \(S\) is incrementally vacuous are those in which some material following \( \text{exh} \) and its argument is replaced by some other material in a way that respects \(S\)’s constituent structure. So, for instance, in the case of a sentence of the form \([X [\text{exh}(A) Y] Z]\) (where \(X\), \(Y\), and \(Z\) are constituents), one does not have to consider, in order to determine whether \( \text{exh} \) is incrementally vacuous, structures like \([ [X \text{exh}(A)] [Y' Z'] ]\), even though the linear ordering of the occurring lexical items is the same in both structures up to \( \text{exh}(A) \). Because the human parser is assumed to make hypotheses about the final constituent structure of a sentence before the sentence has been entirely parsed, it is natural to define the set of “possible continuations” of a sentence at a certain point in sentence processing as the set of sentences consistent with the structure that the parser has assumed at that point (cf. Fox 2008).
Our first version of the economy condition thus states that an occurrence of \( \text{exh} \) in a sentence \( S \) is ruled out if \( \text{exh} \) is incrementally vacuous in \( S \) (see below for an explicit statement of the condition). This condition ensures that (40) is licensed, because \( \text{exh} \) is not incrementally vacuous in this sentence (though it is globally vacuous).

Consider now (38b), repeated below as (41):

\[
(41) \ [(p \text{ and } q) \text{ or } \text{exh}(p \text{ or } q)]
\]

As before, \( \text{exh} \) is globally vacuous in (41). But in this case, \( \text{exh} \) is also \textit{incrementally vacuous}. Indeed, \( \text{exh}(p \text{ or } q) \) is not followed by any constituent. So it is trivially the case that \( \text{exh} \) is globally vacuous in every sentence obtained from (41) by replacing any constituent following \( \text{exh}(p \text{ or } q) \) in (41) with an arbitrary constituent.\(^{25}\)

Our first version of the economy condition is, thus, the following:

\[
(42) \ \text{Economy Condition on Exhaustification (first version)} \nonumber
\]

An occurrence of \( \text{exh} \) in a sentence \( S \) is not licensed if this occurrence of \( \text{exh} \) is incrementally vacuous in \( S \).

For short: \( \ast S(\text{exh}(A)) \), if \( \text{exh} \) is incrementally vacuous in \( S \).

\[
(43) \ \text{Definition of incremental vacuity} \nonumber
\]

a. An occurrence of \( \text{exh} \) is globally vacuous in a sentence \( S \) if eliminating it doesn’t change truth conditions (for short: if \( S(\text{exh}(A)) \) is equivalent to \( S(A) \)).

b. An occurrence of \( \text{exh} \) which takes \( A \) as argument is incrementally vacuous in a sentence \( S \) if it is globally vacuous for every continuation of \( S \) at point \( A \).

c. \( S' \) is a \textit{continuation} of \( S \) at point \( A \) if \( S' \) can be derived from \( S \) by replacement of constituents that follow \( A \).

d. \( Y \) follows \( A \) if all the terminals of \( Y \) are pronounced after all the terminals of \( A \).

Footnote 24 continued

Schlenker’s notion of incremental vacuity makes reference only to strings of terminal symbols. Viewing sentences as strings, a continuation of a sentence \( S \) at point \( A \) can be defined, in Schlenker’s approach, as any well-formed sentence \( S' \) which is identical to \( S \) up to \( A \). This notion may seem very different from ours. Note, however, that Schlenker applies this notion to a language that includes parentheses, so that strings actually encode constituent structure. In other words, a continuation \( S' \) at point \( A \) is such that all opening parentheses before \( A \) have a corresponding closing parenthesis after \( A \). This makes Schlenker’s definition very close to ours (see Schlenker 2009 for discussion). While there is no general equivalence between the definition in terms of strings for a language with parentheses and ours in terms of constituent structure, in practice we have not found a case where it makes a difference, and in any case it does not make any difference for the example sentences we discuss in this paper.

\(^{25}\) Recall that \( \text{exh} \) is incrementally vacuous in \( [S… \text{exh}(A)…] \) if and only if it is globally vacuous in every sentence \( S' \) obtained by replacing a \textit{constituent} following \( \text{exh}(A) \) in \( S \) with an arbitrary constituent. Thus if \textit{no} constituent follows \( \text{exh}(A) \), incremental vacuity and global vacuity coincide. See the previous footnote.
As we have shown, (42) together with HC predicts Singh’s Asymmetry. Acceptable Hurford disjunctions of the form ‘p or q or (p and q)’ are licensed because they can receive the parse in (44a2), which satisfies both HC and Economy. Bad Hurford disjunctions of the form ‘(p and q) or (p or q)’ have no parse that jointly satisfies HC and Economy. Either HC is satisfied but Economy is not (cf. (44b1) or the other way around (cf. (44b2)):

(44) The Hurford case:
   a1. [exh(p or q)] or (p and q) Economy: exh is not incrementally vacuous
   a2. * (p or q) or (p and q) HC

The Singh case:
   b1. *(p and q) or exh[(p or q)] Economy: exh is incrementally vacuous
   b2. *(p and q) or (p or q) HC

5.2 Further predictions

In this subsection, we will show that our economy condition does not actually rule out every Hurford disjunction in which the disjuncts appear in ‘reverse order’ (i.e., where the exhaustivity operator has to be introduced in the second disjunct). In certain circumstances, Singh’s Asymmetry is predicted to disappear.

Preliminary evidence that this is a desirable outcome comes from a cursory look at the distribution of Hurford disjunctions in the Corpus of Contemporary American English (http://corpus.byu.edu/coca/): it seems that even though Singh’s Asymmetry describes a clear statistical tendency (namely, examples in the ‘good’ order are much more frequent than examples in the ‘wrong’ order), it does not hold as an absolute constraint. Below are the numbers returned by a search of the Corpus of Contemporary American English for various Hurford disjunctions (‘canonical order’ refers to the case where the weaker scalar item occurs in the first disjunct, as in the first column of the table):26

26 This table reflects all our searches (we did not test any other Hurford disjunction). The numbers we report are those that were returned on December 22, 2014 (we had slightly different numbers in 2012, which we replaced because we did not record the date of our initial queries—the pattern was essentially the same as the one we report here). Examples based on the ‘canonical order’ are much more frequent than those using the ‘reverse order’, and the difference we found between the canonical and the reverse order is statistically significant according to a simple sign-test, since in 15 out of 17 cases we found the predicted asymmetry and the two other cases were ties (two-tail p value = 0.0023, counting the two ties—as negative). A binomial test comparing the number of observed ‘canonical’ (785) and ‘non-canonical’ (247) cases yields a two-tail p value smaller than 10^{-15}.
So there is a clear asymmetry in Singh’s direction, but it doesn’t seem to be absolute. We want to argue that this tendency is to be accounted for by an absolute constraint (Economy) which rules out Hurford disjunctions in the ‘wrong’ order in the most basic cases but turns out to have no consequence in more complex environments.

5.2.1 Preview

The basic logic can be summarized as follows. For an occurrence of \( \text{exh} \) to be incrementally vacuous, it has to be globally vacuous (\textit{incremental vacuity entails global vacuity}). In the most simple cases of Hurford disjunctions, \( \text{exh} \) is globally vacuous; Singh’s Asymmetry follows from the fact that in such cases, \( \text{exh} \) is furthermore \textit{incrementally} vacuous when and only when the disjuncts are reversed. This is so because when (and only when) \( \text{exh} \) scopes over the final disjunct, global vacuity and incremental vacuity are equivalent. However, we have seen in Sect. 3 that in certain Hurford disjunctions, the presence of \( \text{exh} \) has global consequences for meaning; in such cases, \( \text{exh} \) is not globally vacuous. Since reversing the order of the disjuncts does not modify the truth conditions of the relevant sentences, \( \text{exh} \) is not globally vacuous either when the disjuncts are reversed and hence not incrementally vacuous. In such examples, Singh’s Asymmetry is, therefore, predicted to disappear.

The relevant examples can be built on the basis of the very sentences we used in order to show that \( \text{exh} \) can have global consequences for meaning (i.e., that \( \text{exh} \) is not necessarily globally vacuous in Hurford disjunctions), namely Distant Entailing Disjunctions and non-vacuous cases in which a Hurford disjunction is embedded.

|                  | Canonical order | Reverse order |
|------------------|----------------|--------------|
| some or all      | 396            | 53           |
| some or many     | 7              | 0            |
| some or most     | 8              | 1            |
| most or all      | 164            | 152          |
| many or all      | 14             | 2            |
| can or must      | 1              | 0            |
| may or must      | 0              | 0            |
| sometimes or always | 3       | 2            |
| sometimes or often | 19        | 7            |
| often or always  | 16             | 14           |
| possible or certain | 1           | 0            |
| might or must    | 0              | 0            |
| allowed or required | 2           | 0            |
| few or none      | 19             | 4            |
| rarely or never  | 55             | 12           |
| right or obligation | 1           | 0            |
| good or excellent| 79             | 34           |
| TOTAL            | 785            | 247          |
under another exhaustivity operator. We predict that in such cases, reversing the disjuncts should yield an acceptable Reverse Hurford Disjunction (in contrast to Singh 2008a, who designed a system where the asymmetry is a primitive).

5.2.2 Distant Entailing Disjuncts

It should now be clear that the following is one way to construct a Reverse Hurford Disjunction that does not violate our economy condition on exhaustification:

(45) **Recipe to construct a Reverse Hurford Disjunction:**

Take two Distant Entailing Disjuncts (DEDs) and form a disjunction in the reverse order of that discussed in (19); in other words, take two sentences $p$ and $q$ such that:

a. $q$ entails $p$;

b. this entailment can be obviated by exhaustification: there is a way to strengthen $p$ by $exh\, p^*$, such that $q$ doesn’t entail $p^*$;\(^{27}\)

c. $q$ or $p^*$ is logically stronger than ($q$ or $p$).

In such a disjunction, strengthening of $p$ by $exh$ will be licensed by Economy (since it is not vacuous), and $q$ or $p$ will receive the parse $q$ or $p^*$, which will not violate HC. We can apply this recipe to (18b), which involved a Hurford disjunction where the relevant scalar items were non-adjacent members of a scale. This results in (46) below, which we correctly predict to be acceptable:\(^{28}\)

(46) The water is absolutely boiling or (somewhat) warm.

To investigate further the predictions of our economy condition, let us examine various other cases where the same logic is at play.

5.2.2.1 Non-adjacent members of a scale  It is standardly assumed that the Horn scale for $some$ contains the scalar item $all$ as well as an intermediate member, e.g., is something like $<some, most, all>$. Since $some$ and $all$ are non-adjacent members of this scale, they should qualify as DEDs, and the Reverse Hurford Disjunction $all$ or $some$ should be acceptable whenever the intermediate member of the scale, i.e., $most$, is sufficiently salient. For then the strengthened meaning of $some$ is going to be $some$ but not $most$, and as a result $exh$ is not globally vacuous in $all$ or $exh$(some) (meaning $all$ or (some but not most)). Let us consider a concrete example:

(47) Did John do most of the homework?

No. He did all of it or some of it.

In order to meet HC, the answer in (47) must have the following parse:

(48) He did all of the homework or $exh_<(he did some of the homework)$.

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27 See footnote 17.

28 Applying the same recipe to (18a) yields ‘John has six or three children’, which is also acceptable. But this example is irrelevant, because exhaustification with numerals appears to be insensitive to the economy condition. See Spector (2013).
In this context, the set of alternatives C for \( exh \) can be assumed to include not only ‘he did some of the homework’ and ‘he did all of the homework’, but also the sentence based on the intermediate member of the scale, i.e., ‘he did most of the homework’. As a result, the two disjuncts are Distant Entailing Disjuncts, and therefore our economy condition should be satisfied. More precisely, the second disjunct ends up meaning ‘he did some of the homework but not most of it’, with the result that (48), as a whole, means ‘He either did all of the homework or some but not most of it’. Note that (48) is false in a situation where John did most of the homework but not all of it, contrary to what would have resulted if \( exh \) were deleted. So \( exh \) is not globally vacuous, and hence not incrementally vacuous either. We thus predict that the answer in (47) should be felicitous, under the interpretation corresponding to (48)—a prediction that we believe is borne out. 29

5.2.2.2 Free choice effect in the second disjunct Another example involving DEDs was given in (20), repeated below as (49):

(49) Either the students are required to do the reading or the homework, or they are required to do both.

Recall that the meaning of the first disjunct after exhaustification (needed in order to meet HC) entails the following “free choice inference”:

(50) The students are allowed to do the reading without doing the homework and they are allowed to do the homework without doing the reading.

As we observed, the overall meaning of (49) is thus predicted to exclude situations where the students are required to do the reading and are not required to do the homework (and vice versa), and this is what made the exhaustification of the first disjunct non-vacuous globally.

Since \( exh \) is not globally vacuous in (49), reversing the disjuncts is predicted to yield an acceptable Reverse Hurford Disjunction. The following contrast is therefore predicted:

(51) a. # He did both the reading and the homework or he did one of them.
   b. The students are required to do both the reading and the homework or they’re required to do one of them.

29 Out of the blue, the pragmatically strengthened meaning of \( some \) seems to be \( some \) but not all, rather than \( some \) but not most, which is why Horn (1989), among others, assumes that it is possible to ignore intermediate members of a scale. But the parse \( all \) or \( exh(some) \), interpreted as \( all \) or \( (some \) but not all) \), violates our economy condition.

Strictly speaking, what we therefore predict is a subtle asymmetry between the order \( exh(some) \) or \( all \) and the order \( all \) or \( exh(some) \). Both should be acceptable, but in the case of \( exh(some) \) or \( all \), \( exh \) can be globally vacuous and still meet the economy condition, and the sentence is therefore predicted to be ambiguous between \( (some \) but not all) or \( all \) (i.e. \( some \)) and \( (some \) but not most) or \( all \). In the case of \( all \) or \( exh(some) \), only the reading \( all \) or \( (some \) but not most) should be possible. It seems, however, that intermediate alternatives need to be sufficiently ‘salient’ in order to be active, so that accessing this reading is relatively hard, unless the context makes the intermediate member salient, as in (47). Given this, ‘all or some’ is predicted to sound less felicitous in a context in which, unlike (47), the alternative with ‘most’ is clearly not relevant—a subtle prediction that might need to be tested in a controlled setting. See Fox and Katzir (2011) on the role played by relevance in constraining sets of alternatives.
5.2.2.3 Universally quantified contexts  Other universally quantified contexts allow us to construct similar pairs:

(52)  
    a. *Either John did both the reading and the homework or he did the reading or the homework.
    b. Either everyone did both the reading and the homework or everyone did the reading or the homework.

Sentence (52b) is expected to be acceptable for the following reason. A possible
parse for the second disjunct of (52b), one that insures that HC is obeyed, is the following:

(53) exh(everyone did the reading or the homework)

Now, following our assumption from Sect. 3.1 that the alternatives induced by a
disjunctive phrase include each disjunct separately, the alternatives for the prejacent
in (53) will include ‘everyone did the reading’ and ‘everyone did the homework’.  As a result, (53) entails the negation of these very alternatives and is equivalent to
(54a), which in turn entails (54b)—these inferences are the counterpart of the ‘free-
choice inference’ triggered by disjunction under a necessity modal (cf. (50)).

(54)  
    a. Everyone did the reading or the homework, not everyone did the reading,
    and not everyone did the homework.
    b. Someone did the reading without doing the homework, someone did the
    homework without doing the reading.

Under this analysis, (52b) meets HC since (54b) contradicts the first disjunct
(‘everyone did the reading and the homework’). Furthermore, (52b), under this
parse, excludes a situation where everybody did the homework and some people,
but not all, also did the reading: for in such a situation the first disjunct—everyone
did both the reading and the homework—is false and the second one, which is
equivalent to (54b), is false as well, since nobody did the reading without doing the
homework. This interpretation would not have arisen in the absence of exh on the
second disjunct. Exh is therefore not globally vacuous in (52b), hence is not
incrementally vacuous, and (52b) is predicted to be acceptable.

5.2.3 Embedding under matrix exh

The other type of construction which enabled us to show that the presence of exh in
a Hurford disjunction can have a detectable semantic effect was one in which the
Hurford disjunction (hence an embedded exhaustivity operator) was under the scope
of a matrix exhaustivity operator (Sect. 3.2). In such a construction, the embedded
exhaustivity operator was shown to be globally non-vacuous. This will obviously
remain the case if the relevant disjuncts are reversed and will thus lead us to expect
Singh’s Asymmetry once again to disappear.

Let us thus consider again one such example, in the canonical order:

(55) You are required to either buy some or all of the furniture.
As we argued above (Sect. 5.2.2.2), (55) triggers the free-choice inference that it is up to the addressee to decide whether to buy some but not all of the furniture or all of it. And we showed that this inference is an entailment of (55) under the following parse:

(56) exh[You are required to 
    either exh(PRO buy some of the furniture) or 
    PRO buy all of the furniture]

In this structure, neither of the two occurrences of exh is globally vacuous, since the two of them are jointly responsible for the free-choice inference. And this will remain true if we reverse the order of the two disjuncts. Therefore, (57a) below, under the parse given in (57b), meets the economy condition and is predicted to be acceptable—a prediction which we believe is correct.

(57) a. You are required to either buy all or some of the furniture.
    b. exh[You are required to 
           either PRO buy all of the furniture or 
           exh(PRO buy some of the furniture)]

As observed in Sect. 3.2.2, examples where a Hurford disjunction is embedded under a necessity modal are special cases of embedding under a universal quantifier. So what we see in (57) should extend to other universal quantifiers: we expect that reversing the disjuncts should be acceptable for all Hurford disjunctions embedded under a universal quantifier. In other words, we predict the following contrast:

(58) a. *The student solved all or some of the problems.
    b. Every student solved all or some of the problems.

Example (58a) is a simple, unembedded Reverse Hurford Disjunction, and therefore violates the economy condition (unless an intermediate alternative, e.g., most, is made salient). But (58b), under the schematic parse given in (59), does not.

(59) exh(∀x (ALL…x…) or exh(SOME…x…))

This is so because the two exhaustivity operators are jointly responsible for the entailment that some students solved all of the problems (cf. Sect. 3.2). One may indeed typically use a sentence such as (58b) in order to convey the information that the students can be partitioned into two non-empty classes, namely those who solved all of the problems and those who solved most but not all of the problems.

This prediction seems to be corroborated by a Google search for the strings all or most and all or some, which revealed examples such as the following, in which the disjunction is embedded under a modal or a quantifier, as in (60), or under a performative, as in (61):

(60) A new Harris Poll finds a plurality of Americans want all or most abortions to be illegal.
(61) What are **all or some** of the differences and similarities between Roman Architecture and Egyptian Architecture?

Similar to: ‘Tell me **all or some** of the differences and similarities between Roman Architecture and Egyptian Architecture.’

Both examples illustrate the predicted free choice effect. In particular, (60) supports the inference that some Americans want all abortions to be illegal, while others want most but not all of them to be illegal (and that there is no majority for either position). Similarly (61), which is a question in a homework assignment, is understood as giving the addressee the option of listing all of the relevant differences or just some of them.30

6 Gajewski and Sharvit and version 2 of the economy condition

Let us now return to Problem #2, which arises from the observation that a simple Hurford disjunction cannot be embedded immediately below negation. As mentioned, we would address this problem if we could find a way to rule out structures such as the following:

(62) \( \neg [\text{exh}(p \text{ or } q) \text{ or } (p \text{ and } q)] \)

Our economy condition on exhaustification, as it stands, does not rule out (62): even though \( \text{exh} \) is globally vacuous in (62), it is clearly not incrementally vacuous, for replacing the second disjunct \((p \text{ and } q)\) with, e.g., any expression which is logically independent of \( p \text{ or } q \) gives rise to a structure in which \( \text{exh} \) is not globally vacuous.31

Since our economy condition in its current formulation is too weak to rule out (62), we would like to strengthen it. Now, note that \( \text{exh} \) in (62), though not incrementally vacuous, is incrementally weakening in the following sense: any sentence of the form \( \neg [\text{exh}(p \text{ or } q) \Gamma r] \), where \( \Gamma \) stands for either \( \text{or} \) or \( \text{and} \), is

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30 Singh (2008a) pointed out a problem for our proposal (presented at the time in an MIT talk) that we haven’t been able to resolve. Our approach predicts that Singh’s Asymmetry should be obviated whenever the relevant disjunction involves Distant Entailing Disjuncts (DEDs). Yet this does not seem to be the case with Hurford disjunctions that do not contain scalar items but rather referential expressions that can give rise to an exhaustive interpretation. That is, the following contrast is unexpected from our point of view:

(i) a. #John solved all the problems or Problem #1.
   b. John solved Problem #1 or all of the problems.

Example (ib), at least in certain contexts, behaves like a DED, and is then interpreted as ‘John solved Problem #1 and no other problem or solved all the problems’. And yet, (1a), where the ordering of the disjuncts is reversed, seems to us to be odd. We don’t have an account for this fact. See Singh (2008a) for relevant discussion.

31 Generally speaking, if an occurrence of \( \text{exh} \) is not incrementally vacuous in a sentence \( S \), then it is not incrementally vacuous in \( \neg S \) either. Since \( \text{exh} \) is not incrementally vacuous in \( \text{exh}(p \text{ or } q) \text{ or } (p \text{ and } q) \), it is also not incrementally vacuous in \( \neg [\text{exh}(p \text{ or } q) \text{ or } (p \text{ and } q)] \).
necessarily entailed by \(-((p \lor q) \Gamma r)\).\(^{32}\) Therefore, at the point where \(exh(p \lor q)\) has been encountered in (62), it can be determined that whatever the meaning of the following constituents will be, the resulting meaning will be either logically weaker than, or equivalent to, what would have resulted in the absence of \(exh\). If we strengthen the economy condition into the condition that no occurrence of \(exh\) can be incrementally weakening in this sense, we predict the impossibility of structures such as (62). Such a principle is reminiscent of various versions of the so-called \textit{Strongest Meaning Hypothesis} found in the literature (cf. the work on reciprocals and plurality by Dalrymple et al. 1998, Winter 2001, among others), according to which disambiguation between various readings of a given sentence tends to favor the strongest possible reading.\(^{33,34}\) In the domain of scalar implicatures, a somewhat similar condition was already proposed by Chierchia (2004), in order to rule out embedded implicatures in DE contexts, an issue which we will return to shortly.\(^{35}\) The main innovation of the current proposal is the embedding of this general idea as an incremental principle of the sort we advocated in the previous section, following work by Schlenker.

\(^{32}\) \(exh(p \lor q) \Gamma r\) entails \((p \lor q) \Gamma r\) because \(exh(p \lor q)\) entails \((p \lor q)\), and \(\Gamma\), which is either \(or\) or \(and\), is monotone increasing with respect to both its right and left argument. So \(-((p \lor q) \Gamma r)\) entails \(-((exh(p \lor q) \Gamma r)\).

\(^{33}\) Note that the principle according to which an occurrence of \(exh\) should not be incrementally weakening never forces the presence of an embedded \(exh\). We do not adopt a version of the Stronger Meaning Hypothesis which would force embedded exhaustification when the resulting meaning is stronger than what would have resulted without it. Our economy condition can only disallow certain insertion sites for \(exh\), in cases where inserting \(exh\) leads to an overall weaker (or equivalent) reading. Hence our principle never predicts that embedded exhaustification should be the default. More concretely, consider the three following possible logical forms for a sentence such as (i):

\[(i) \quad \text{Every student read the Russian books or the French books.}\]
\[(a) \quad (\text{Every student})x \; exh(x \; \text{read the French books or the Russian books})\]
\[(b) \quad exh[(\text{Every student})x \; (x \; \text{read the French books or the Russian books})]\]
\[(c) \quad (\text{Every student})x \; (x \; \text{read the French books or the Russian books})\]

Form (ia) asymmetrically entails (ib), which asymmetrically entails (ic). Yet all of these three LFs satisfy the new economy condition: in both (ia) and (ib), \(exh\) is not incrementally weakening, and in (ic) the principle does not apply because no \(exh\) is present. Any version of the Strongest Meaning Hypothesis states that a given reading \(R\) for a certain sentence is dispreferred if it is logically weaker than some competing reading in a certain fixed class of competitors. In our implementation, while (ic) is a competitor for both (ia) and (ib), (ia) is not a competitor for either (ib) or (ic), and (ib) is not a competitor for either (ia) or (ic). (See CF&S, Sect. 5.6.)

\(^{34}\) A possible functional motivation for our principle is the following (from Fox 2007a): The role of \(exh\) is to eliminate unwanted ignorance inferences derived by Gricean reasoning. If \(exh\) is weakening, it cannot eliminate ignorance inferences.

\(^{35}\) Chierchia (2004) took the view that embedded implicatures are possible in monotone increasing and non-monotonic contexts, but not in monotone decreasing contexts. In this paper, we argue that embedded implicatures are in principle possible in every context, but require pitch accent on the relevant scalar item in DE contexts (cf. our generalization (8)). We’ll show in Sec.10 how this generalization can be made to follow from a refined version of our economy condition.
So we propose to modify our economy condition as follows:

(63) **Economy Condition on Exhaustification** (second version):
    An occurrence of \( exh \) in a sentence \( S \) is not licensed if this occurrence of \( exh \) is incrementally weakening in \( S \), with ‘incrementally weakening’ defined as in (64). For short: \( *S(exh(A)) \), if \( exh \) is incrementally weakening in \( S \).

(64) a. An occurrence of \( exh \) is globally weakening in a sentence \( S \) if eliminating it does not alter or strengthens truth conditions, i.e., if \( S(A) \) entails \( S(exh(A)) \) (a special case is when \( S \) is equivalent to \( S(exh(A)) \), i.e., when \( exh \) is vacuous).
    b. An occurrence of \( exh \) which takes \( A \) as argument is incrementally weakening in a sentence \( S \) if \( exh \) is globally weakening for every continuation of \( S \) at point \( A \).
    c.–d. As Before (cf. (43)).

Note that this modification does not affect the results reported in the previous section. The reason is obvious: we did not consider downward-entailing environments, hence all the cases we discussed in which the first version of the economy condition was satisfied were cases in which the relevant occurrence of \( exh \) was not only incrementally non-vacuous, but in fact incrementally non-weakening as well (a fact that we invite the reader to check).

The second version of the economy condition on exhaustification makes an interesting additional prediction: we can construct cases where a Hurford disjunction appears in the immediate scope of a DE operator but is nevertheless not (incrementally) weakening because this DE operator is itself under the scope of another DE operator, to the effect that the overall context for \( exh \) is upward-entailing. Here are some relevant contrasts:

(65) a. *John didn’t hand in the first or second assignment or both.
    b. Everyone who didn’t hand in the first or second assignment or both failed the class.

(66) a. #I would go to the movies without John or Bill or both.
    b. I wouldn’t go to the movies without John or Bill or both.

36 Note however that the following is also acceptable, which might be considered problematic since here a Hurford disjunction occurs in the restrictor of a universal quantifier, hence in a DE context:

(i) Every student who handed in the first or the second assignment or both failed the class.

We believe that (i) is not in fact a counterexample to our proposal, because quantifiers tend to trigger a non-vacuity inference according to which their restrictor does not have an empty denotation. If this inference is taken into account, the restrictor of a universal quantifier (or of any other quantifier) no longer qualifies as a DE context. In fact, a sentence such as (i) strongly suggests that at least one student handed in the first or the second assignment but not both, and at least one student handed in both. Given our assumption that the embedded Hurford disjunction contains an exhaustivity operator applying to the first disjunct, this is a specific instance of the following generalization: a sentence of the form ‘Every (\( A \) or \( B \)) is \( P \)’ triggers the inference that there are some \( A \)s and there are some \( B \)s. Assuming we can explain how \( exh \) can yield this generalization, it will be non-vacuous in (i), since it will be responsible for the fact that the sentence triggers the inference that some students handed in the first or the second assignment but not both.
But we will have additional predictions, closer in spirit to those we discussed in relation with Singh’s Asymmetry, which we will be able to discuss once we propose a solution for Problem #1.

7 Towards a solution of Problem #1

Let us now return to the Implicature Focus Generalization (IFG)—the generalization that EE is possible under a DE operator only if the relevant scalar item receives pitch accent. At first sight, our economy condition appears to rule out EE under a DE operator altogether, since an exhaustivity operator appears to be weakening in such a context.\(^\text{37}\)

One might suggest that no more needs to be said. Our current economy condition, as it stands, manages to account for Problems #3 and #2, and appears to make the further prediction that EE is impossible in DE contexts. True, we would still need an account for the fact that embedded implicatures appear to be possible in DE contexts if the relevant scalar item is prosodically marked. But, as mentioned at the outset, there is already a proposal in the literature which is designed to account for this apparent exception, namely the idea that metalinguistic negation yields an effect tantamount to EE (Horn 1989). By supplementing our economy condition on exhaustification with Horn’s theory of metalinguistic negation, we might be able to predict all the known facts (along lines similar to those suggested in Chierchia 2004).

Although this is a possible position, we would like to investigate an alternative. One of our motivations has been already acknowledged by Horn, namely that resort to metalinguistic negation is not sufficient, since, under the relevant prosodic pattern, EE can arise under the scope of other DE operators. Horn’s (1989: 379–382) response to this problem consists in generalizing the notion of metalinguistic negation to other operators, i.e., in assuming that many (perhaps all) operators are ambiguous between an ordinary meaning and a metalinguistic meaning. But we think that this is a costly assumption which should motivate the search for alternatives.

Another motivation is analytic. Our proposal, as it stands, turns out to predict that, under very specific circumstances, EE should be possible even under DE operators. Capitalizing on this observation, we will see that it is possible to construct an alternative to Horn’s ambiguity hypothesis which is consistent with our economy condition. Under this alternative, the so-called metalinguistic meaning would result from embedding \textit{exh} below a DE operator. Furthermore, we will see that the

\(^{37}\) The relevant notion is of course that of incremental weakening, which is achieved if the DE operator scoping over the relevant scalar item occurs to its left, as is usually the case in a head-initial language. It would be very interesting to investigate head-final languages in this context. Note also that a structure such as 

\[
\text{That exh(}\text{Jack talked to Mary or Sue}) \text{ is false}\]

\text{is expected to meet our incremental economy condition. Even though exh occurs in a DE context, this fact cannot be ‘known’ at the point where exh is encountered, but only after the last word has been parsed. However, there does not seem to be a clear contrast between this sentence and }

\[
\text{It is false that Jack talked to Mary or Sue. This potential problem might be solved by assuming that the incremental component of the economy condition does not only depend on surface linear order, but also on more abstract aspects of linguistic structure which play a role for the order in which constituents are parsed. Schlenker’s (2008) account of left-right asymmetries in presupposition projection runs into a somewhat similar problem in cases involving postponed if-clauses (see Mandelkern and Romoli 2017).} \]
requirement for pitch accent on the scalar item could follow if we adopt yet another modification of the economy condition, which we think will remain faithful to its original motivation.

More specifically, our task can be divided into two. First, we need to understand how embedding $exh$ under a DE operator could satisfy an economy condition which bans (incremental) weakening. Second, we need to understand how pitch accent could end up being relevant. Let’s start with the first task. Needless to say, if $exh$ is embedded in a DE context, the overall result is weaker than the alternative (derived without the presence of $exh$):

(67) $exh$ is necessarily weakening in a DE context:

If $S(\ldots)$ is a DE environment, then $S(exh(A))$ will necessarily be equivalent to or weaker than—i.e., entailed by—$S(A)$, since $exh(A)$ entails $A$.

However, as we’ve already seen (in the context of our discussion of (65) and (66)), elements in the scope of a DE operator need not be in a DE context (globally). Specifically, if the DE operator is itself embedded under yet another operator, properties of the other operator might affect the monotonicity of the overall context.

(68) The scope of a DE operator is not necessarily a DE context:

If $A$ is in the scope of a DE operator, dominated by a sentence $S$, then the context, $S(A)$, need not be a DE context.

So in (65b), for example, the Hurford disjunction is in the scope of a DE operator, but is nevertheless not in a DE context in the matrix sentence, since in the matrix sentence the DE operator is further embedded in the restrictor of a universal quantifier, thus reversing monotonicity once more.

To see something similar, with an example relevant for Problem #1, consider an occurrence of $exh$ below negation with another occurrence of $exh$ above negation, as in (69):

(69) $exh(\neg exh(p \lor q))$

Since $exh$ is a non-monotonic operator, the most embedded occurrence of $exh$ in (69) is not in a DE context, and therefore might be able to meet the economy condition. To see one circumstance under which the economy condition would indeed be satisfied, assume that the alternative for the higher occurrence of $exh$ could be a sentence just like the prejacent, differing only in that it lacks the exhaustivity operator:

(70) ALT (–$exh(p \lor q)$) = {–$exh(p \lor q)$, –$(p \lor q)$}

Since $–(p \lor q)$ is strictly stronger than $–exh(p \lor q)$, applying $exh$ to $–exh(p \lor q)$ yields the following:

(71) $exh_{ALT}(–exh(p \lor q))$

$= –exh(p \lor q)$ and $–(p \lor q)$

$= –exh(p \lor q)$ and $(p \lor q)$

$= –[(p \lor q)$ and $–[p \land q)]$ and $(p \lor q)$

$= [–(p \lor q)$ or $(p \land q)]$ and $(p \lor q)$

$= (p \land q)$
In the absence of the most embedded $exh$, the resulting structure $exh(\neg(p \lor q))$ would be equivalent to $\neg(p \lor q)$. Therefore, the most embedded occurrence of $exh$ in (69) is not globally weakening: $(p \land q)$ is not entailed by $\neg(p \lor q)$. Hence it is not incrementally weakening, and the economy condition is met for this occurrence of $exh$. Likewise for the higher occurrence of $exh$: without this occurrence, the resulting structure, $\neg(exh(p \lor q))$, would be weaker than $\neg(p \lor q)$, and hence, again, would not entail the actual meaning, $p \land q$.  

But, of course, we still need to understand how the alternatives in (70) are determined. We will entertain an answer to this question that also meets our second task, in that it accounts for the correlation with pitch accent. Our starting point will be a modification of our theory of $exh$ in favor of an alternative which we think might be conceptually superior on independent grounds, namely a theory according to which the choice of alternatives for any occurrence of $exh$ correlates with focus marking—that is, a theory in which $exh$ is literally a focus-sensitive operator (see Fox and Katzir 2011).

The requirement of pitch accent on the scalar item will be translated with the aid of such a theory to a requirement that $exh$ have a particular set of alternatives as its first argument. In order to explain why this latter requirement holds in the scope of a DE operator, we will provide a more general statement of our economy condition on exhaustification. The new condition will ban an occurrence of $exh$ in a sentence $S$, associated with a certain set of alternatives, if the overall result is weaker than what would have resulted if $exh$ had been associated with a smaller set of alternatives.

8 $exh$ as a focus-sensitive operator

Before we present any of this in detail, we would like to explain how $exh$ could be viewed as a focus-sensitive operator. Our discussion will be centered on the ingredients needed for our specific purposes, but can be embedded within a more general theory of focus (based on Fox and Katzir 2011).

8.1 Narrow focus, broad focus, and minimize focus

Consider first the sentence in (72), which contains the paradigmatic focus-sensitive operator $only$.

(72) John only talked to $[Mary]$$_F$

\[LF: \text{Only}_{ALT} [\text{prejacent John talked to } [Mary]_F]\n\[AF: \text{ALT must be a contextually salient subset of } F(\text{prejacent}).\]

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38 In Sect. 11.1 we argue that this conjunctive interpretation is indeed the only possible interpretation when $or$ is interpreted as exclusive under negation.

39 There is a debate regarding the nature of association with focus, which we do not address here; while Schwarzschild (1997) argued that association with focus is always a pragmatic phenomenon, Rooth (1992), Beaver and Clark (2000), and much subsequent literature has argued that it is encoded in the semantics of at least some focus-sensitive expressions ($only$). In this paper we adopt the latter view, but we think that our proposal could be rephrased in terms of a pragmatic theory of association with focus.
\(F(\text{prejacent}) = \{\text{John talked to Mary, John talked to Fred, John talked to Sue,} \ldots\}\)

The alternatives for \textit{only} are constrained by the theory of association with focus (AF) to be a (contextually salient) subset of the focus value of the “prejacent”, and this focus value is determined to be the set of sentences derived from the prejacent by replacing focused constituents with their alternatives—in the case of (72), the set of sentences of the form \textit{John talked to x} where \textit{x} names an individual.

\textit{Only}, just like \textit{exh}, states that the prejacent is true and that every member of ALT which is true is entailed by the prejacent.\(^\text{40}\) (Though there are reasons to believe that both need to be modified along lines discussed in (22), a modification which—as mentioned—can be ignored whenever the result of the simpler formulation is consistent.) Since in (72) the members of ALT must all be sentences of the form \textit{John talked to x}, the sentence as a whole ends up entailing that John did not talk to various individuals. But the actual meaning depends on the contribution of context, i.e., on what is taken to be contextually salient/relevant.

Consider next the SI of a simple disjunctive sentence such as \textit{John talked to Mary or Sue}. Under our proposal, deriving this SI requires a structure in which \textit{exh} appears above disjunction. But if \textit{exh} is a focus-sensitive operator, we also have to ask what constituent is focused. Let’s consider first the relatively easy case in which the disjunctive coordinator \textit{or} is focused:

\[\text{exh}_{\text{ALT}[\text{prejacent John talked to Mary OR F Sue}]}
\]

\[\text{AF: ALT must be a contextually salient subset of } F(\text{prejacent}).\]

\[F(\text{prejacent}) = \{\text{John talked to Mary or Sue, John talked to Mary and Sue}\}\]

Under this focus structure, we can say that the alternatives for \textit{exh} are determined in exactly the same way as the alternatives for \textit{only} in (72). Specifically, they are determined by AF to be a (contextually salient) subset of the focus value of the prejacent, and this focus value is determined to be the set of sentences derived from the prejacent by replacing focused constituents with their alternatives. If we think that the conjunctive word \textit{and} is an alternative of \textit{or}, we get precisely the right result.\(^\text{41}\)

But wouldn’t we predict, incorrectly, that focus—hence pitch accent—on \textit{or} would be required for the SI to be derived? Well, actually no. The only thing that might be required for the SI to be derived is that the conjunctive alternative (‘John talked to Mary and Sue’) be a member of ALT. This requirement will be met if \textit{or} is a focus-marked constituent, but it will also be met in other ways. Under standard theories of focus, it will be met as long as \textit{or} is contained within some focus-marked constituent. This would be sufficient to guarantee that the sentence that can be

\(^\text{40}\) We ignore the fact that \textit{only}, unlike \textit{exh}, triggers complex presuppositions (whose exact nature is disputed), as discussed in Horn (1969) and much subsequent work (van Rooij and Schulz 2006; Ippolito 2008, a.o.).

\(^\text{41}\) Here we depart from Rooth’s own framework, in which the focus value for any expression \textit{E} consists of the set of all objects that belong to the semantic type of \textit{E}. In the case of \textit{or}, the predicted focus value would have been the set of all binary Boolean functions, which is clearly inadequate. There is agreement that the focus value of a scalar term must be further constrained: for instance, it might consist in its Horn scale. See Katzir (2007) and Fox and Katzir (2011), in which an alternative proposal is presented that can dispense with Horn scales.
derived from the prejacent by replacing or with and will be a focus alternative of the prejacent, as exemplified in (74).  

\[(74)\]  
exh_{\text{ALT}[\text{CP John talked to [Mary or SUE]}]}  
AF: ALT must be a contextually salient subset of F(prejacent).  
F(prejacent) = \{John talked to Mary or Sue, John talked to Mary and Sue, John talked to Fred, John talked to Mary and Fred,…\}  

What we’ve seen is an instance of a much broader observation. Whenever we consider two focus structures for a given sentence, one in which a constituent, x, is focused (narrow focus) and another in which a larger constituent, y (i.e., one that dominates x), is focused (broad focus), we observe that the focus value of the sentence under narrow focus is a subset of the focus value of the sentence under broad focus. This means that any alternative of the sentence under narrow focus will also be an alternative under broad focus. Hence, we do not predict that deriving SIs requires scalar items to be focused. Instead, we predict that SIs require the relevant scalar items to be either focused or dominated by a focused constituent, an assumption that we think is not problematic. (For evidence in favor of this assumption, see Rooth 1985: 42–43; Rooth 1992, Sect. 3.3; and Fox and Katzir 2011.)  

However, there is another observation that we need to consider, namely that gratuitous focus is, in general, not allowed (Schwarzschild 1999). Under the relevant condition, dubbed Minimize Focus (MF), broad focus in particular is allowed only if among the contextually selected alternatives there is one which is not a member of the focus value under narrow focus. It is thus not sufficient to assume AF, as stated in (75a); MF, as stated in (75b), must be considered as well:  

\[(75)\]  
a. Association with Focus (AF): The set of alternatives for a focus-sensitive operator must be a subset of the focus value of the prejacent.  
b. Minimize Focus (MF): A sentence can’t have a focus value F if it would satisfy AF with another focus value F’ (derivable by a different distribution of focus marking) and F’ ⊂ F.  

Once these two principles are adopted, we predict a contrast between (73) and (74). In both sentences, AF allows the conjunctive alternative, John talked to Mary and Sue (M&S for short), to be a member of ALT, hence both should allow the exclusive SI to be computed. But MF treats the two sentences differently: in (74), if M&S ∈ ALT, there must be at least one alternative in ALT which is not a member of the focus value of (73) (else focus on or would be sufficient to satisfy AF). Consequently, if (74) yields the ‘not-and’ inference, it must yield an additional
exclusive inference that would make it stronger than (73), e.g., that John did not talk to Jane.

The facts are hard to determine—an expected state of affairs given that the additional exclusive inference could be close to a tautology when the whole prejacent is focused, since it will amount to the negation of some other relevant sentence, i.e., a contextual tautology if the sentence in question is already known to be false. But if we consider a specific background question that would fix the focus structure appropriately, e.g., focus on the DP that dominates disjunction as in (74), we think the facts go in the right direction. In particular, when (74) is uttered as an answer to the question *Who did John talk to?*, an inference that John did not talk to Jane would definitely arise if Jane is taken to be in the domain that the question quantifies over (cf. Spector 2006 for an argument that the exclusive interpretation of disjunction in such a case is tied to additional exhaustivity effects, whether this is achieved by a covert exhaustivity operator or by only). To make the point clearer, consider e.g. *Who among these four girls did John talk to?*. More generally, we make the following prediction:

\[(76) \textit{A restriction on broad focus}: \text{Let } S_{\text{narrow}} \text{ be a sentence that contains one scalar item with narrow focus on the scalar item, and let } \textit{not } S_1 \text{ be its SI. Let } S_{\text{broad}} \text{ be identical to } S_{\text{narrow}} \text{ with the sole exception that focus is on a constituent that properly dominates the scalar item. Predictions:}
\]

\[a. \text{ AF allows } S_{\text{broad}} \text{ to have } \textit{not } S_1 \text{ as an SI.}
\]

\[b. \text{ But then it must have an extra SI, } \textit{not } S_2, \text{ where } S_2 \text{ is not a member of the focus value of } S_{\text{narrow}}.\]

44 Once the potential role of focus is taken at face value, some arguments against the grammatical approach to SIs lose their force. For instance, Ippolito (2011) points out that, in a sentence such as *John wishes that Mary had eaten some of the cookies*, it is hard to understand ‘some’ as equivalent to ‘some but not all’—unless ‘some’ is stressed, as is for instance the case in *Mary ate all of the cookies. John wishes she had eaten SOME of the cookies*. This, however, provides an argument against embedded implicatures only to the extent that focus on ‘some’ would not be expected on independent grounds. It turns out that, in the very same context, adding an overt only under the scope of *wish* also makes it obligatory to stress ‘some’: *Mary ate all of the cookies. John wishes she had only eaten SOME of the cookies*. In fact, with or without an overt only, stress on ‘some’ is expected here to result from the interplay of AF and givenness considerations (Schwarz schild 1999).

45 In the present formulation, broad focus would satisfy MF if the set of active alternatives consisted of members of the focus value under narrow focus along with non-excludable alternatives (that would be in the focus value only under broad focus). We haven’t come up with any concrete case where this possibility is realized in a way that threatens our predictions. But we suspect that this is a wrong prediction and would like to suggest the possibility of strengthening MF (in (75b)) by requiring that broader focus be semantically motivated. More specifically, the suggested modification of MF is the following:

\[(75b)^* \textit{Minimize Focus (strengthened): Let } O(C)(S) \text{ be a sentence where } O \text{ is a focus-sensitive operator, C is its restrictor, and } S \text{ its prejacent. Then } S \text{ cannot have a focus value } F \text{ if}
\]

\[(i) \text{ there is another sentence } O(C')(S') \text{ that satisfies AF such that } S' \text{ differs from } S \text{ only in that the focus value of } S' \text{ is } F' \text{ and } F' \subseteq F \text{ and}
\]

\[(ii) \text{ the meaning of } O(C)(S) \text{ is identical to that of } O(C')(S').\]
8.2 Alternatives derived by deletion

Our eventual goal is to explain why EE under a DE operator requires narrow focus on the scalar item. Once this is explained the IFG would follow from basic observations about the phonological realization of focus. Our explanation is going to be based on the observation we just made that broad focus requires an extra inference ‘not S₂’. As we will see, this extra inference leads to overall weakening of the meaning, something which is going to be blocked by a generalization of our economy condition.

But before we get there, we have to say something about the parse we are going to assume for EE under DE operators, e.g., the one in (71), repeated below, with two subscripts ALT1 and ALT2 that represent the restrictors for the two occurrences of exh.

(77) exh_{ALT1}(¬exh_{ALT2}(p \lor q))

We’ve already said something about the identity of ALT2, namely that it is going to contain the single conjunctive alternative p \land q when \lor receives narrow focus and that it will contain at least one additional alternative under broad focus. But we still have to say something about the identity of ALT1.

In order to know what ALT1 can be, we have to know the focus value of its prejacent, ¬exh_{ALT2}(p \lor q). For the sake of this discussion, we will assume (following Rooth 1985, 1992) that the focus value of p \lor q is irrelevant; namely, we assume that the focus value of a constituent does not percolate beyond a focus-sensitive operator which associates with this focus value. Furthermore, we will assume that the lower occurrence of exh is either focused or (more plausibly) dominated by a focus-marked constituent and that this is sufficient for generating the alternative we identified in Sect. 7, namely ¬(p \lor q). Following Katzir (2007) and Fox and Katzir (2011), we will call this alternative, an alternative generated by deletion. There are also alternatives we could consider in which the lower occurrence of exh is replaced by other focus-sensitive operators (e.g., only or even), but we can think of none that will affect our overall result. Hence we assume the structure in (78), which we generalize to DE operators other than negation in (79).

(78) exh_{\{¬(p \lor q)\}}(¬exh_{ALT2}(p \lor q))

(79) exh_{[OP(S)]}[OP [exh_{ALT2}(S)]]

8.3 Computing embedded exhaustification under DE operators

So now we would like to compute the meaning of representations such as (78) for both narrow and broad focus on the embedded S. Under narrow focus, we’ve already gone over the computation in (71), though at the time we did not put too much thought into the way alternatives are determined:

(80) exh_{\{¬(p \lor q)\}}(¬exh_{[p \land q]}(p \lor q)) \land

\neg¬(p \lor q) = p \land q
Now the choice of alternatives is understood, and with it, the compatibility of this particular form of EE with our economy condition. As stated before, neither occurrence of $exh$ is (incrementally) weakening, and the economy condition is thus satisfied. But—to repeat—in order to address Problem #1, we still need to understand the correlation with pitch accent.

Given what we said in Sect. 8, we need to block the representation in which the prejacent of the embedded occurrence of $exh$ receives broad focus. There are two cases to consider. The first case, presented in (81a), is identical in all respects (other than F-marking) to (80). This representation, as discussed in Sect. 8.1, is blocked by MF.

$$\text{(81) a. } exh_{[-(p \lor q)]} \neg[exh_{[p \land q]} (p \lor q)_F]_F \text{ blocked by MF}$$

$$\text{b. } exh_{[-(p \lor q)]} \neg[exh_{[p \land q, d]} (p \lor q)_F]_F \text{ Hope: the lower } exh \text{ is blocked}$$

Next we need to consider representations such as that in (81b), in which an additional sentence $d$ is added as an alternative to the embedded $exh$—as required by MF. If the IFG is to follow, representations of this sort must be ruled out as well. To see how this might be achieved, we compute the resulting meaning in (82).

$$\text{(82) Interpretation of (81b):}$$

$$exh_{[-(p \lor q)]} \neg[exh_{[p \land q, d]} (p \lor q)]$$

$$= \neg[exh_{[p \land q, d]} (p \lor q)] \land \neg(p \lor q)$$

$$= (p \lor q) \land \neg[exh_{[p \land q, d]} (p \lor q)]$$

$$= (p \lor q) \land \neg((p \lor q) \land \neg(p \land q) \land \neg d)$$

$$= (p \lor q) \text{ and either } [(p \land q) \text{ or } d]$$

$$= [p \land q] \text{ or } [(p \lor q) \text{ and } d]$$

It is easy to see that this meaning is weaker than what we get in (80), and it is this observation which we will build on in our explanation of the IMF in the next section.

9 Comparison class for economy

The previous version of our economy condition on exhaustification looked at a constituent $exh(\phi)$ in a given syntactic context $S(exh(\phi))$ and checked how it fared relative to its competitor $\phi$. Specifically, $S(exh(\phi))$ could not be (incrementally) weaker than $S(\phi)$. This formulation ignored the set of alternatives of $exh$, ALT. There is an equivalent way of stating the same constraint which does not ignore ALT and suggests on obvious generalization.

According to the equivalent statement, the competitor of $exh_{\text{ALT}}(\phi)$ would not be $\phi$ itself but rather the vacuous exhaustification of $\phi$, $exh_\phi(\phi)$—the exhaustification of $\phi$ with no alternatives whatsoever. The vacuous exhaustification of $\phi$ is, of course, equivalent to $\phi$ itself, hence $exh_{\text{ALT}}(\phi)$ would be weakening relative to $\phi$ iff it is weakening relative to $exh_\phi(\phi)$. 
(83) **Economy Condition on Exhaustification** (equivalent to (63)):
\[ S(\text{exh}_C(A)), \text{if } \text{exh}_C \text{ is incrementally weakening in } S. \]

(84) a. An occurrence of \( \text{exh}_C \) is globally weakening in a sentence \( S \) if \( S(\text{exh}_S(A)) \) entails \( S(\text{exh}_C(A)) \).

b.–d. As before (cf. (43))

Although equivalent, the conceptions suggested by (83) and (63) are somewhat different. Unlike (63), what (83) cares about is not the presence or absence of exhaustification, but rather the precise nature of exhaustification: (83) says that the presence of alternatives should not be weakening. But if this suggested conception is correct, there is an obvious generalization to consider, namely that every innocently excludable single alternative (or subset of innocently excludable alternatives) must be non-weakening. That is, it should never be the case that by adding to its prejacent the negation of an alternative, \( \text{exh} \) yields a global result that is overall weaker than or equivalent to what would have resulted if this alternative had not been excluded. In other words, given a certain set of alternatives \( C \) associated with an occurrence of \( \text{exh} \), the comparison class for Economy is every set of alternatives such that their innocently excludable members are a proper subset of the innocently excludable alternatives of \( C \).

(85) **Economy Condition on Exhaustification:**
\[ S(\text{exh}_C(A)), \text{if } \text{exh}_C \text{ is incrementally weakening in } S. \]

(86) a. An occurrence of \( \text{exh}_C \) is globally weakening in a sentence \( S(\text{exh}_C(A)) \) if there is a set \( C' \) such that \( IE(A,C') \) is a proper subset of \( IE(A,C) \) and \( S(\text{exh}_C(A)) \) entails \( S(\text{exh}_C(A)) \).

b.–d. As before (cf. (43))

It is easy to see that if \( \text{exh}_C \) is (incrementally) weakening by (63) (= (83)), then it is also (incrementally) weakening by (86) (Just let the empty set be \( C' \)).

---

46 In a previous version of this paper, the comparison class included all subsets of \( C \), with no reference to innocently excludable alternatives. As noted by a reviewer, our original constraint would generally rule out any alternative set that contains non-excludable alternatives. For instance, exhaustifying \( A \) or \( B \) with respect to the alternative set \( \{ A, B, A \text{ and } B \} \) would be disallowed since the alternative set consisting simply of \( \{ A \text{ and } B \} \) would yield the same result—and the alternative set \( \{ A, A \text{ and } B \} \) would yield a strictly stronger result (see Fox and Katzir 2011 for an independent constraint ruling out such an alternative set, and also footnote 42). It is not clear that our previous definition creates any problem (after all, exhaustifying just with respect to \( \{ A \text{ and } B \} \) would be allowed and would give rise to the desired outcome), but the version we adopt here, though more complex to state, is in many respects more simple to apply and just as adequate, as far as we can see.

47 There is, however, a complication regarding the treatment of basic Hurford disjunctions and Singh’s Asymmetry. Given our approach to exhaustification, a sentence such as ‘\( \text{John did some or all of the homework} \)’ could satisfy Hurford’s Constraint if it received the following parse: ‘\( \text{exh}_{R}(\text{SOME}) \) or \( \text{ALL} \)’, where \( R \) is some proposition that is logically independent of both disjuncts. In this case, the first disjunct is interpreted as ‘\( \text{SOME} \) & \( \lnot R \)’, and is thus not entailed by \( \text{ALL} \). Note that in such a case, \( \text{exh} \) is neither incrementally nor globally vacuous, and we thus expect the reverse order (‘\( \text{John did all or some of the homework} \)’) to be fine, under the parse ‘\( \text{ALL} \) or \( \text{exh}_{R}(\text{SOME}) \)’. In order to rule out such cases, we think that we could rely on certain reasonable assumptions regarding the way alternative sets are constrained by relevance. The idea would be that an utterance of ‘\( \text{exh}_C (\text{SOME}) \) or \( \text{ALL} \)’ indicates that ‘\( \text{ALL} \)’ is relevant, with the result that ‘\( \text{ALL} \)’ should be included in the alternative set \( C \). More specifically, in order to allow for Distant Entailing Disjunctions (e.g. for the parse ‘\( \text{exh}_{\text{MANY}}(\text{SOME}) \) or \( \text{ALL} \)’), we could state that when a certain potential alternative \( X \) is contextually relevant, then \( X \) can be pruned from the restrictor \( C \) of \( \text{exh} \) only if the resulting
10 Deriving the implicature focus generalization

We now have the necessary ingredients to derive the IFG. First note that if $exh$ is inserted in a DE context, the result would violate Economy for reasons mentioned in Sect. 7, so we need to postulate representations with two instances of $exh$, as in (79), repeated below.

(79) $exh_{OP(S)} [OP [exh_{ALT2}(S)]]$ (where $OP$ is a DE operator).

Our task is to explain why $S$ must receive narrow focus on the scalar item.

For starters, let’s focus on a specific case—let’s explain how the violation of the IFG in (81) (repeated below) is blocked.

(81) a. $exh_{\neg(p \lor q)} ([\neg exh_{[p \land q]} (p \lor q)]_F)_F)$ blocked by MF
b. $exh_{\neg(p \lor q)} ([\neg exh_{[p \land q, d]} (p \lor q)]_F)_F)$ lower $exh$ is incrementally weakening

The structure in (81a) was already blocked by MF. Our problem was to explain why (81b) is blocked, and now the answer is clear. The meaning of (81b) is weaker than what we would have gotten if the lower $exh$ excludes a proper subset of alternatives, namely without $d$ as an alternative (see our discussion of (80)). Hence Economy rules out this representation.

Now let’s see how this explanation can be generalized to other scalar items and other DE operators. Consider (87), where the embedded sentence $S$ contains a scalar item which (when replaced with a scalar alternative) yields a stronger sentence $S^+$. It is clear that (87a), like (81a), is blocked by MF. Our goal is to show that (87b), like (81b), is blocked by Economy, whenever $A$ contains innocently excludable members other than $S^+$ (the scalar alternative of $S$). If this could be achieved, we will have an explanation for the necessity for narrow focus.48

Footnote 47 continued
reading of $exh_{C(S)}$ settles $X$, i.e. entails $X$ or its negation. In the above case, this would allow $C$ to contain the alternative MANY instead of ALL, since ‘SOME & \neg-ALL’ entails ‘\neg-ALL’.

48 Our economy condition does not impose narrow focus on disjunction for all structures of the form ‘$exh_{C (\neg exh_{C^'} (p \lor q))}$’. Consider indeed the following:

(i) $exh_{\neg(p \lor q)} ([\neg exh_{[p \lor q]} (p \lor q)]_F)_F) = \neg [exh_{[p \lor q]} (p \lor q)]_F \land \neg(p \lor q) = (p \lor q)$ and $r$

But note that this fact is not obviously relevant to the IFG, which might be stated as a constraint on the embedding of scalar implicatures, and the lower $exh$ in (i) does not trigger an exclusive construal of disjunction. Whether examples such as (i) are in fact available is something that we have not examined in any detail, but the following might suggest that they are:

(ii) Context: Student comes in late to class.
Teacher: What do you need to do to get a grade?
Student: To hand in some of the homework.
Teacher. No. You don’t need to hand in some of the homework. You need to hand in some of the homework and attend every single class.
a. \( \text{exh}_{\{\text{OP}(S)\}} (\text{OP}[\text{exh}_{S+} (S) F]F) \) blocked by MF

b. \( \text{exh}_{\{\text{OP}(S)\}} (\text{OP} [\text{exh}_{C = \{S+, A_1, \ldots, A_k\}} (S) F]F) \) lower \( \text{exh} \) is incrementally weakening

[with \( A_1, \ldots, A_k \) innocently excludable] 49

But this is an automatic consequence of the following fact:

(88) Let:
1. \( \text{OP} \) be a DE operator;
2. \( C, C' \) be two sets of sentences such that \( \text{IE} (S, C') \subset \text{IE} (S, C) \);
3. \( \text{exh}_{C}(S) \) and \( \text{exh}_{C'}(S) \) both asymmetrically entail \( S \).

Then:
\( \text{exh}_{\{\text{OP}(S)\}} (\text{OP}[\text{exh}_{C}(S)]) \) entails \( \text{exh}_{\{\text{OP}(S)\}} (\text{OP}[\text{exh}_{C'}(S)]) \)

**Proof**

(a) \[ \text{exh}_{\{\text{OP}(S)\}} (\text{OP}[\text{exh}_{C}(S)]) \Leftrightarrow [\text{OP}[\text{exh}_{C}(S)] \land \neg \text{OP}([S])] \]
(because \( \text{exh}_{C}(S) \) asymmetrically entails \( S \) and \( \text{OP} \) is DE) 50

\[ \text{exh}_{\{\text{OP}(S)\}} (\text{OP}[\text{exh}_{C}(S)]) \Leftrightarrow \text{OP}[\text{exh}_{C}(S)] \land \neg \text{OP}([S]) \]
(because \( \text{exh}_{C}(S) \) asymmetrically entails \( S \) and \( \text{OP} \) is DE)

Since \( \text{IE} (S, C') \subset \text{IE} (S, C) \), \( \text{exh}_{C}(S) \) entails \( \text{exh}_{C'}(S) \). Since \( \text{OP} \) is DE, \( \text{OP} [\text{exh}_{C}(S)] \land \neg \text{OP}([S]) \) entails \( \text{OP}[\text{exh}_{C'}(S)] \land \neg \text{OP}([S]) \). Given (a) and (b),
\( \text{exh}_{\{\text{OP}(S)\}} (\text{OP}[\text{exh}_{C}(S)]) \) entails \( \text{exh}_{\{\text{OP}(S)\}} (\text{OP}[\text{exh}_{C'}(S)]) \).

11 Further predictions

What we’ve seen in Sect. 10 is that there is a natural generalization of our economy condition ((85) and (86)) which (together with certain auxiliary assumption motivated independently for the theory of focus) derives the IFG. In this section, we would like to draw additional predictions that follow from this derivation. The predictions, unfortunately, are not always very easy to test. So there is a lot of work that still needs to be done in order to properly assess our proposal in this domain. It might be useful to state the obvious here, namely that this paper (like any other) should not be taken as a “package deal”. It is possible, for example, that we are right in our approach to Problems #2 and #3 and that Horn is right in his account of the IFG (see Sect. 7 above). And there are of course many other choice points worth exploring (see note 51).

49 See footnote 46.

50 Strictly speaking, this will hold in full generality only if the DE operator is ‘strongly DE’, in the following sense: an operator \( \text{Op} \) is strongly DE if, whenever \( x \) asymmetrically entails \( y \), \( \text{Op} (y) \) asymmetrically entails \( \text{Op}(x) \). All the DE operators we consider (and maybe all DE operators in natural languages) happen to be strongly DE.
11.1 **exh-exh**

The combination of *Minimize Focus, Association with Focus*, and our economy condition ensures that whenever an exhaustivity operator occurs under the scope of an unembedded DE operator, another exhaustivity operator must be present. We thus predict that whenever an embedded SI is computed under the scope of a non-embedded DE operator, some additional, non-embedded SI should be computed as well.

In particular, as discussed in Sects. 7 and 8.3, it is predicted that an exclusive construal of disjunction just under the scope of an unembedded negation should always yield a *conjunctive* interpretation:

\[(89)\] \[\text{exh} \left[ \neg \text{exh}(p \lor q)\right] = p \land q\]

A similar prediction is made for structures that result from replacing disjunction in (89) with other scalar items that have stronger alternatives. Namely, if S is a sentence with alternative S+ (where S+ asymmetrically entails S), we have:

\[(90)\] \[\text{exh}_{[\neg S]} \left[ \neg \text{exh}_{[S^+]}(S)\right] = S^+\]

To be more concrete, we predict that a sentence such as (91) below cannot be interpreted as equivalent to (92a), even if an embedded scalar implicature is present. Rather, an exclusive construal of the embedded disjunction should always lead to a conjunctive interpretation, as in (92b):

(91) Jack didn’t talk to Mary OR Sue.

(92) a. Jack didn’t [talk to Mary or Sue but not both]

   = Jack talked to either both Mary and Sue or neither Mary nor Sue.

   b. Jack talked to both Mary and Sue.

Before testing this prediction, let us contrast this case with a similar case, in which an exhaustivity operator also occurs in the scope of a DE operator, but the DE operator is not simply negation but instead a negative quantifier. Thus consider the following kind of structure, where S is a one-place predicate and S+ is an alternative of S stronger than S:

\[(93)\] \[\text{EXH}_{[\neg S]}[\neg x (\text{EXH}_{[S^+]}(S(x)))]\]

A concrete instantiation of (93) is the following:

(94) \[\text{EXH}_{[\neg \text{student studies phonology or morphology}]}[\neg \text{student studies phonology and morphology}]\]

Let us compute the meaning of (93):

\[(95)\] \[\text{EXH}_{[\neg S]}[\neg x (\text{EXH}_{[S^+]}(S(x)))] = \text{EXH}_{[\neg S]}[\neg x (S(x) \land \neg S(x))] = \]

\[\text{For no } x (S(x) \land \neg S^+(x)) \land \text{for some } x S(x) = \text{For no } x (S(x) \land \neg S^+(x)) \land \text{for some } S(x) +\]

Applied to (94), this yields the following reading:
(96) ‘No student studies phonology or morphology but not both, and at least one student studies both.’

Importantly, then, (94) (contrary to the previous case) is not expected to trigger a ‘conjunctive interpretation’; more precisely, it is not expected to entail that every student studies both phonology and morphology.

What are the facts? Consider first the following contrast:

(97) Nurse: The boy looks bad. Why is that?
    Doctor: He didn’t take Tylenol OR Advil as he was supposed to. He took both pills.
    Doctor’: He didn’t take Tylenol OR Advil as he was supposed to. #He took both pills or none of them.

Both replies (from Doctor and Doctor’) in (97) are expected to force an exclusive construal of the embedded disjunction, in order for the overall discourse to be coherent. But the last sentence in Doctor’’s reply is pragmatically incompatible with the conjunctive interpretation that we predict (because the last sentence—He took both pills or none of them—suggests that the speaker does not know which of the disjuncts is true). Given our prediction that an exclusive construal of disjunction forces a conjunctive reading, Doctor’’s reply is expected to be odd. In contrast with this, we do not predict that the following should be odd:51

(98) Nurse: All of the patients look bad. What happened?
    Doctor: None of them took Tylenol OR Advil as they were supposed to. Every single patient took both of the pills or none of them.

While the facts are pretty subtle, the speakers we consulted detect a contrast in the predicted direction both between Doctor and Doctor’’s replies in (97), on the one hand, and between Doctor’’s reply and (98), on the other hand.

11.2 Gajewski and Sharvit (2012)

We also make relatively complex predictions pertaining to Problem #2, namely Gajewski and Sharvit’s observation that a Hurford disjunction is infelicitous in DE contexts. In Sect. 6, we pointed out that our economy principle prevents a Hurford disjunction from occurring in a DE context. But we ignored the possibility that DE-ness can be disrupted by an additional matrix exh, as it is in our account of the IFG.

As we will see shortly, the predictions are rather nuanced. Our current economy condition on exhaustification turns out to allow Hurford disjunctions to occur in such structures, but only under very special circumstances. To a certain extent, this will prove to be a good feature of our account, since we will see systematic exceptions to the generalization that are predicted. However, our account will nevertheless appear to be too liberal, licensing sentences which are in fact deviant. We will start with what we think are good predictions, pertaining to Distant

51 While our informants report a contrast between (97) and (98), some report that (97) is not as bad as it should be. It might be relevant to point out that the conjunctive meaning is not predicted if speakers can parse the sentence with the matrix K operator posited in Meyer (2013, 2014).
Entailing Disjuncts. Then, we will consider problematic consequences: it will turn out that all Hurford disjunctions which appear to occur in DE contexts should in principle be able to be ‘saved’ by the presence of a matrix $exh$, if this matrix $exh$ is associated with a very specific set of alternatives. We will not have a solution for this problem but will sketch a possible strategy for addressing it.

11.2.1 Distant Entailing Disjunctions

In this subsection, we show that EC (both on its incremental and its global version) predicts Hurford disjunctions to be able to occur in the scope of (apparently unembedded) negation if the disjunction involves Distant Entailing Disjuncts. That this is a correct prediction is illustrated, we think, by the coherent reply in (99b) (and its contrast with the original examples of Gajewski and Sharvit):

(99) a. I was told that at the beginning of each academic year you have a party for the first-year students alone, or one for all the students (first, second, and third year).

b. That’s not always true. This year, I will not invite the first-year students or all of the students. I will invite the first-year students and the second-year students.

This is to be compared with a minimally different dialogue with an embedded Hurford disjunction that does not involve DEDs, as in (100). Our approach predicts (100b) to sound less good than (99b).

(100) a. I was told that at the beginning of every year you have a party with your colleagues and just the first-year students, or one with your colleagues and the first- and second-year students.

b. # That’s not always true. This year, I will not invite the first-year students or the first- and second-year students. I will only invite my colleagues.

To see that (99b) can, indeed, meet our economy condition, consider the following parse:

(101) $exh \{¬(we will invite the 1st-year students or all of the students)\}(¬exh_{SECOND}$(FIRST) or ALL)

Let us schematize this structure as follows:

(102) $exh_{¬(FIRST OR ALL)}(¬exh_{SECOND}$(FIRST) or ALL)

The meaning of the prejacent of the matrix exhaustivity operator is the following:

(103) $(¬exh_{SECOND}$(FIRST) or ALL) = ¬(FIRST & ¬SECOND) or ALL

The only alternative to this prejacent is ¬(FIRST OR ALL), which is equivalent to ¬FIRST. Now, ¬FIRST is not entailed by the prejacent (i.e., by (103)), hence is innocently excludable. So the overall predicted meaning of (102) is as follows:

---

52 Note that even if the potential alternative ALL is taken to be relevant, there is no obligation to include it in the alternative set of either exhaustivity operator, given the constraint suggested in footnote 47—which states that relevant alternatives can be pruned if the overall meaning determines their truth value.
Given this result, it is clear that the matrix exhaustivity operator meets our economy condition, on both its global and its incremental version: its alternative set contains only one member, which is innocently excludable, and it is therefore not globally weakening, hence also not incrementally weakening (recall that if an occurrence of $exh$ is globally non-weakening, it is also incrementally non-weakening). We now only need to check that the embedded $exh$ is also globally non-weakening (hence incrementally non-weakening). So let us compute the meaning of the structure that results from (102) by replacing the alternative set of the embedded $exh$ with the empty set.

(105) Resulting reading if the lower $exh$ were associated with an empty alternative set:

$$exh_{\neg(FIRST \text{ or ALL})} \neg(exh_{\emptyset}(FIRST) \text{ or ALL})$$

$$= \neg(FIRST \text{ or ALL}) = \neg FIRST$$

$>>$ does not entail (104)

(The crucial point here is that the alternative for the matrix $exh$ is equivalent to its prejacent, hence not excludable.)

Since (105) does not entail (104), the lower $exh$ in (102) meets our economy condition. Since both occurrences of $exh$ in (102) meet the condition, (102) is licensed, which, by the judgments of the people we consulted with, is the desired result.

11.2.2 Non–distant entailing disjunctions: first pass

Consider now the feature of the structure in (102) that is responsible for our account of its acceptability. The reason why the higher $exh$ is not weakening is that its alternative is not entailed by its prejacent. And the reason why this is so is that the embedded $exh$ is itself not vacuous within the embedded Hurford disjunction, which involves DEDs; thanks to this feature, the embedded Hurford disjunction happens to be strictly stronger than its first disjunct, so that when negation is added, the result is now strictly weaker than the alternative of the matrix $exh$, and this alternative can be excluded consistently. The presence of DEDs is thus crucial to this result. To see this in greater detail, let us turn to cases involving non–distant entailing disjuncts (the cases that G&S focused on):

First consider the following structure:

(106) $exh_{\neg(\text{we will invite some or all of the students})} \neg[[exh_{\{\text{we will invite all of the students}\}} (\text{we will invite some of the students})] \text{ or we will invite all of the students}]$

The Hurford disjunction embedded under negation is equivalent to ‘we will invite some of the students’, i.e., to the negation of the alternative associated with the higher $exh$. It follows that the higher $exh$ is globally vacuous, hence globally...
weakening. The higher $exh$ is also incrementally weakening, since it applies to the whole sentence (hence nothing follows the argument of $exh$ in the sentence).

But to explain the unacceptability of the sentence, we need a more general result. Specifically, we need to show that there is no choice of alternatives for both exhaustivity operators satisfying both Hurford’s Constraint and our economy condition. That is, we need to rule out the structure in (107) for any choice of $C$ and $C'$ satisfying Hurford’s Constraint (as long as the two disjuncts are not DEDs, in which case we are back to the previous case).

(107) $exh_C \neg[[exh_{C'} (we will invite some of the students)] or we will invite all of the students]

We will start by showing that the global version of our economy condition delivers this result. In other words, we will see that the global version is able to account both for the cases that obey Gajewski and Sharvit’s generalization and for the exceptions that we have just noted. Let us assume that $C'$, the alternative set for the lower $exh$, includes the alternative ‘we will invite all of the students’, which ensures that the disjunctive phrase embedded under negation satisfies Hurford’s Constraint. (We also assume that no alternative using an intermediate scale-mate of ‘some’ and ‘all’, e.g., ‘many’, is present, since otherwise the two disjuncts would be DEDs.)

We can thus schematize (107) as follows:

(108) $exh_C \neg [[exh_{C'} = \{ALL, b_1, \ldots, b_n\} (SOME) or ALL]\]

Now, let us assume that the higher $exh$ is not weakening, i.e., that $C$ contains at least one excludable alternative. Let $a_1, \ldots, a_n$ be the excludable alternatives contained in $C$. The meaning of (108) is as follows, where $b_1, \ldots, b_j$, ALL are the innocently excludable members of $C'$:

(109) $\neg[(SOME \& \neg b_1 \& \ldots \& \neg b_j \& \neg ALL) or ALL ] & [\neg a_1 \& \ldots \& \neg a_n]$

Now, for the lower $exh$ to be globally non-weakening, each innocently excludable member of $C'$ should be such that removing it does not result in a stronger overall reading. We note that in the absence of the higher $exh$, each (excludable) member of $C'$ above would have a weakening effect (because the lower $exh$ would be in a DE context). The only way an excludable member of $C'$ can manage to be globally non-weakening is if it plays a role in making the higher $exh$ itself non-weakening. In other words, the only way the lower $exh$ could manage to be globally non-weakening would be a case where, for every excludable member $x$ of $C'$, at least one excludable member of $C$ would fail to be innocently excludable if $x$ were removed from $C$'.

As noticed in footnote 47, we independently need a constraint that ensures the alternative set $C$ in a Hurford disjunction such as ‘$exh_C (SOME) or ALL’ (be it embedded or not) includes either ALL or some member of the focus value of SOME that is entailed by ALL.

Otherwise, the only effect of the lower $exh$ would be to strengthen the negated constituent, i.e. weaken the overall meaning of the sentence. In all the cases we discussed where $exh$ was licensed under negation, this was because the relevant occurrence of $exh$ managed to meet the economy condition by ‘helping’ some other, higher occurrence of $exh$ to be itself non-weakening.

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has no effect on the meaning of the embedded Hurford disjunction. Hence it has also no effect on the meaning of the prejacent of the higher \( exh \): if we remove it, the set of excludable alternatives for the higher \( exh \) is bound to remain exactly the same as before. Therefore, the presence of \( \text{ALL} \) in \( C' \) violates the global version of our economy condition.

11.2.3 Non–distant entailing disjunctions and incrementality

We will now point out a problem for our proposal, namely that the incremental version of our economy condition does not predict Gajewski and Sharvit’s observations to hold for any choice of alternatives. For some (perhaps unnatural) choices of alternatives, a structure such as (107), even though it violates the global version of EC, meets the incremental version. Let us examine a case with this property, in a schematic form. Consider the following structure, where \( x \) is logically independent of both \( s \) and \( s^+ \), \( s \) and \( s^+ \) are (adjacent) members of a scale, and \( s^+ \) is stronger than \( s \) (for instance: \( s = \text{‘We will invite some of the students’}, s^+ = \text{‘We will invite all of the students’}, \) and \( x = \text{‘We will invite the professors’} \).

\[
\text{(110) } exh_{\{\text{NOT } (s \text{ or } x)\}} \text{NOT}(exh_{\{s^+\}}(s) \text{ or } s^+)
\]

In this structure the lower \( exh \) is not incrementally weakening.\(^{55}\) To show this, it is sufficient to show that there is at least one continuation of (110) at the point of occurrence of \( s \) (i.e., the prejacent) in which the lower \( exh \) is not globally weakening. Here is one such continuation:

\[
\text{(111) } exh_{\{\text{NOT } (s \text{ or } x)\}} \text{NOT}(exh_{\{s^+\}}(s) \text{ or } x) \\
= \text{NOT}((s \& \text{NOT}(s^+)) \text{ or } x) \& (s \text{ or } x) \\
= s^+ \& \text{NOT}(x)
\]

Compare (111) with what happens when you eliminate the lower \( exh \) in (110):

\[
\text{(112) } exh_{\{\text{NOT } (s \text{ or } x)\}} \text{NOT}(s \text{ or } x) \\
= \text{NOT}(s \text{ or } x) \\
= \text{NOT}(s) \& \text{NOT}(x)
\]

Since (112) does not entail (111), the lower ‘exh’ is not globally weakening in (111), hence is not incrementally weakening in (110).

What about the matrix \( exh \)? As mentioned above, for the matrix \( exh \) incremental weakening and global weakening are equivalent. We thus need to show that the higher \( exh \) is not globally weakening in (110). Now, (110) has the following meaning:

\[
\text{(113) } exh_{\{\text{NOT } (s \text{ or } x)\}} \text{NOT}(exh_{\{s^+\}}(s) \text{ or } s^+) \\
= \text{NOT}((s \& \text{NOT}-s^+) \text{ or } s^+) \& (s \text{ or } x) \\
= \text{NOT}(s) \& (s \text{ or } x) \\
= \text{NOT}(s) \& x
\]

\(^{55}\) Note that this structure does not violate either Association with Focus or Minimize Focus. The alternative for the higher \( exh \) meets both these conditions if the associate of the higher \( exh \) is the scope of negation, i.e. \( exh(s) \text{ or } s^+ \), with \( x \) an alternative of \( s^+ \).
Eliminating the higher ‘exh’ yields:

\[(114) \text{NOT}(\text{exh}_{s+})(s) \text{ or } s+) = \text{NOT}(s)\]

Since (114) does not entail (113), the higher exh is not globally weakening. We conclude that neither the lower nor the higher exh is incrementally weakening in (110), i.e., that (110) meets the incremental version of EC. (Note that the fact that x is logically independent of both s and s+ is crucial: if x were entailed by s+, whether or not it were also entailed by s, then s+ & NOT(x) would be contradictory, and thus would not be the meaning of (111). If x entailed s, whether or not it also entailed s+, then NOT(S) & X would be contradictory, hence could not be the meaning of (113) —the higher exh would be vacuous in (113).) One can wonder whether this prediction is right, i.e., whether Gajewski and Sharvit’s generalization is in fact obviated for some particular choices of alternatives. This is not easy to determine, because it is hard to come up with a context which would make the alternative for the higher exh (s or x) relevant.\(^{56}\) At this point, we could argue that we still predict that Gajewski and Sharvit’s observation holds when the choice of possible alternatives is restricted to ‘natural’ alternatives.

We thus find ourselves in a complicated situation. On the one hand, the global version of our economy condition (unlike the incremental version) does a good job at predicting when a Hurford disjunction is licensed under the scope of negation, as shown in the previous section. Specifically, it correctly predicts that a Hurford disjunction is not licensed under the scope of negation unless it involves Distant Entailing Disjuncts. On the other hand, the global economy condition makes wrong predictions for Hurford disjunctions that occur in upward-entailing contexts, since it rules out all Hurford disjunctions that do not involve DEDs, even in upward-entailing contexts. This is why we resorted to the incremental version to begin with. One might wonder whether there could be a reason why the global version is relevant for some cases but not others.

We would like to tentatively sketch a perspective that could help us make sense of this fact. Note that the reasoning whereby (110) was shown not to violate our economy condition on exhaustification is fairly complicated. In particular, in order to take into account the incremental aspect of our economy condition, one has to manage to find a continuation in which the lower exh is not globally weakening. In the cases we considered in Sect. 5 (e.g., unembedded Hurford disjunctions in the canonical order), it was easy to see that an occurrence of exh, though globally weakening, was not incrementally weakening, because there were many ‘continuations’ in which exh was not globally weakening and these continuations were not hard to find (it was sufficient to replace the second disjunct with an arbitrary disjunct that was logically independent of the first disjunct). In the case of (110), however,

\(^{56}\) We can nevertheless try to construct such a case. Here is our attempt (with x = ‘He did the reading’, s = ‘He did some of the homework’, s+ = ‘He did all of the homework’).

(i) a. Question: Did he do the reading, some of the homework, or all of the homework?
   b. Answer: He didn’t do some of the homework or all of the homework. He just did the reading.
things are different: in this type of case, only very specific continuations are such that neither occurrence of \textit{exh} is weakening. This in itself could be sufficient in order to explain why Gajewski and Sharvit’s observation seems to hold.

More generally, we should ask under which conditions the consequences of our economy condition, as we stated it, are tractable for speakers/hearers. Because of its incremental aspect, in order to make sure that an exhaustivity operator is ruled out, the economy condition instructs us to consider \textit{all possible relevant continuations}. But this, of course, is an impossible task in the literal sense, because there are infinitely many such continuations. In the cases we considered, it was possible to give a proof that the relevant occurrence of the exhaustivity operator was globally weakening in every continuation, without going through each individual continuation. But it is not obvious that individual grammars include an internalized proof system that could generate such a proof for all the relevant cases. Rather, we would like to suggest that the economy condition might be implemented in the following way. Whenever an exhaustivity operator is encountered, it needs a motivation. A motivation would be satisfied by a demonstration of \textit{some continuation} under which the relevant occurrence of \textit{exh} is not globally weakening (in the relevant sense). What is needed then is at least one continuation that would make \textit{exh} not globally weakening. If the relevant algorithm that implements Economy finds such a continuation, then \textit{exh} is licensed. If not, \textit{exh} is not licensed and the parser reanalyzes the structure being parsed as one in which \textit{exh} does not occur in the relevant position (or equivalently one in which \textit{exh} has an empty set of alternatives as a restrictor).

If this is how Economy is implemented, there may well be cases where there exists a continuation making a given occurrence of \textit{exh} non-weakening, but where the relevant parse is not available. This is particularly likely to happen when only very specific continuations would be able to meet Economy, such as the ones discussed in this section (Hurford disjunctions in the scope of negation and a matrix \textit{exh}). More generally, when an exhaustivity operator occurs in a downward-entailing context, the parser will have a harder time finding a continuation that would satisfy the global economy condition than when the exhaustivity operator occurs in an upward-entailing context (in which case most continuations in fact satisfy the global economy condition). We might thus expect to find an asymmetry between UE and DE environments, whereby people’s judgments will tend to correspond to the predictions made by the incremental version of our economy condition in UE environments, but would be closer to the predictions made by the global version in DE environments.

12 Conclusion

In earlier work we’ve argued that Hurford disjunctions make it rather clear that embedded implicatures exist. However, it is apparent that implicatures cannot be embedded in every possible position. We presented a condition which imposes very clear restrictions on the positions where implicatures can be embedded. Naturally,
we do not expect this to be to the complete story. We hope, however, that it is a reasonable starting point.

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