Showers and all that

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Accidents happen :-(
Last-minute recycled slides —
think of this as complementing Stefan Hoeche’s summary at HP2 …
Shower & Parton Branching Paradigms

Parton branchings order in angle.

- Driven by QCD coherence
- Recoil global
- Links to analytic use of coherent branching

Dipole branchings order in transverse momentum.

- Driven by large-N dipole pattern and colour flows
- Momentum conservation for each emission
- Advantageous for matching & merging

Sequences of emission scales and momentum fractions as Markov process.
Restore momentum conservation per emissions or at end of evolution.

\[ dS = \frac{\alpha_s}{2\pi} \frac{d\tilde{q}_i^2}{\tilde{q}_i^2} \int dz P(z_i) \exp \left( - \int \frac{Q^2}{\tilde{q}_i^2} d\frac{q^2}{q^2} \int_{z_- (k^2)}^{z_+ (k^2)} d\xi \frac{\alpha_s}{2\pi} P(z) \right) \]

\[ \sigma(n \text{ jets}, \tau) \sim \sum_{k} \sum_{l \leq 2k} c_{nkl} \alpha_s^k(Q) \ln \frac{1}{\tau} \]
Matching & Merging

Matching determined by expanding shower to fixed order, and subtracting it from fixed-order cross section.

- De-facto standard in multi-purpose event generators
- Tweaks still possible

[Nason, Salam — JHEP 01 (2022) 067]

Herwig 7/Matchbox & KrkNLO, MG5_aMC@NLO, PowhegBox, Sherpa

Unitartized merging algorithms are state of the art.

[Plätzer — JHEP 08 (2013) 114] [Lönnblad, Prestel — JHEP 02 (2013) 049]
[Bellm, Gieseke, Plätzer — EPJ C78 (2018) 244]

Allow for combination with higher orders
e.g. [Prestel — JHEP 11 (2021) 041]

Matching at NNLO explored, but requires better showers.

[Nason, Salam — JHEP 01 (2022) 067]

[eXiv:2108.07133]
# LHC-age Working Horses

| Current release series | Hard matrix elements | Shower algorithms | NLO Matching       | Multijet merging     | MPI          | Hadronization | Shower variations |
|------------------------|----------------------|-------------------|--------------------|----------------------|--------------|---------------|-------------------|
| Herwig 7               | Internal, libraries, event files | QTilde, Dipoles   | Internally automated | Internally automated | Eikonal      | Clusters, (Strings) | Yes               |
| Pythia 8               | Internal, event files | Pt ordered, DIRE, VINCIA | External           | Internal, ME via event files | Interleaved | Strings        | Yes               |
| Sherpa 2               | Internal, libraries  | CSShower, DIRE    | Internally automated | Internally automated | Eikonal      | Clusters, Strings | Yes               |
Accuracy of Parton Showers

Global event shapes from coherent branching

\[ H(\alpha_s) \times \exp \left( L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots \right) \]

\( \alpha_s L \sim 1 \)

LL — qualitative
NLL — quantitative
NNLL — precision
Accuracy of Parton Showers

Global event shapes from coherent branching

\[
\sum_\iint \mathcal{Q}_i = \int \mathcal{Q}_i + \mathcal{O}\left(\frac{\alpha^2}{\alpha_s}\right)
\]

Non-global observables in the large-N limit from dipole branching

\[
\frac{\partial G_{ab}(t)}{\partial t} = -\int_{\text{in}} \frac{d\Omega}{4\pi} \omega_{ab}(k)G_{ab}(t) + \int_{\text{out}} \frac{d\Omega}{4\pi} \omega_{ab}(k) \left[ G_{ak}(t)G_{kb}(t) - G_{ab}(t) \right]
\]

[Catani, Trentadue, Webber, Marchesini ...]

[Banfi, Marchesini, Smye — JHEP 08 (2002) 006]

even at NLL: [Banfi, Dreyer, Monti — JHEP 10 (2021) 006]
Accuracy of Parton Showers
Understand and decide on accuracy of (existing) parton shower algorithms, take as a starting point for incremental improvements.

\[ H(\alpha_s) \times \exp \left( Lg_1(\alpha_sL) + g_2(\alpha_sL) + \alpha_s g_3(\alpha_sL) + \ldots \right) \]

\[ \alpha_sL \sim 1 \quad \text{LL} \quad \text{NLL} \quad \text{NNLL} \]
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Spin & Colour Correlations Matter

Spin correlations building on Collins-Knowles algorithm

Dynamic colour factors in dipole showers

\[ C_{ij}(\theta_{iq}, \theta_{LJ}) = \left( C_F \delta_i^{(q)} + \frac{C_A}{2} \delta_i^{(g)} \right) \theta(\theta_{iq} < \theta_{LJ}) + \left( \frac{C_A}{2} \delta_j^{(g)} + C_F \delta_j^{(q)} \right) \theta(\theta_{iq} > \theta_{LJ}) \]

Track angular extent of evolution to reproduce colour factors as dictated by coherence.

[Forshaw, Holguin, Plätzer — EPJ C81 (2021) 4]
[Hamilton, Medves, Salam, Scyboz, Soyez — JHEP 03 (2021) 041]
Spin correlations building on Collins-Knowles algorithm

Some further colour correlations can be restored

Dynamic colour factors

$$C_{ij}(\theta_{iq}, \theta_{LJ}) = \left(C_F \delta^a_i \delta^a_j \right) + \left(\frac{C_A}{2}\right) \delta^a_i \delta^a_j \theta_{LJ}$$

[Plätzer, Sjödahl — JHEP 1207 (2012) 042]
[Plätzer, Sjödahl, Thoren — JHEP 11 (2018) 009]
[Höche, Reichelt — Phys.Rev.D 104 (2021) 3, 034006]

[Forshaw, Holguin, Plätzler — EPJ C81 (2021) 4]
[Hamilton, Medves, Salam, Scyboz, Soyez — JHEP 03 (2021) 041]
Improving Shower Accuracy

Demonstrate NLL accurate evolution:

- PanScales — numerical
  [Dasgupta, Monni, Salam, Soyez + ….]
- Deductor — numerical/analytical
  [Nagy, Soper]
- Forshaw/Holguin/Plätzer — analytical
  [aim at improving Herwig 7 dipole shower]
- Sherpa — numerical/analytical
  [Herren, Höche, Krauss, Reichelt, Schönherr]

Provide higher order building blocks beyond single emissions

Towards second-order showers: unordered contributions

- sector showers allow to include direct $2 ightarrow 4$ branchings in a simple way
- divide phase space into strongly-ordered and unordered region
  - s.o. region: only single-unresolved limits
  - u.o. region: only double-unresolved limits
- $2 \rightarrow 4$ branchings important ingredient to NNLO+PS (and virtual corrections to $2 \rightarrow 3$)

[PanScales] (Based on amplitude evolution)

$$\begin{align*}
M_{\text{ih}} &= M \\
p_1 &= \text{Sp} \\
p_2 &= \text{Sp} \\
p_3 &= \text{Sp} \\
p_4 &= \text{Sp} \\
z_1 &= (1 - z)p_1 \\
z_2 &= (1 - z)(1 - z_1) \\
z_3 &= z_2
\end{align*}$$

[Herren, Höche, Krauss, Reichelt, Schönherr]

[C. Preuss for Vincia — PSR 21]

[PanScales] (Relative deviation from NLL for $\alpha_s \rightarrow 0$)

[D. Preuss for Vincia — PSR 21]

[Dasgupta, El-Menoufi — JHEP 12 (2021) 158]

[Dasgupta, El-Menoufi — JHEP 12 (2021) 158]

[B. Plätzer, Ruffa — JHEP 06 (2021) 007]

[Löschner, Plätzer, Simpson — arXiv:2112.14454]

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    (+ virtual corrections to $2 \rightarrow 3$)

Higher accuracy/order of building blocks is necessary but not sufficient to achieve more accurate algorithms.

[C. Preuss for Vincia — PSR 21]

[Plätzer, Ruffa — JHEP 06 (2021) 007]
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Based on amplitude evolution.

[PanScales]
Accuracy for massive event shapes

Coherent branching jet mass
distribution including mass effects

\[ z (1 - z) \tilde{q}^2 = -m_{ij}^2 + \frac{m_i^2}{z} + \frac{m_j^2}{1-z} - \frac{p_\perp^2}{z (1-z)} \]

\[ P_{q\rightarrow qg} = \frac{C_F}{1-z} \left[ 1 + z^2 - \frac{2m_q^2}{z\tilde{q}^2} \right] \]

[Gieseke, Stephens, Webber – JHEP 0312 (2003) 045]

NLL accurate for global observables with massive quarks.

Analytically calculate
perturbative correction to the
top mass as predicted by parton
branching algorithms

[Hoang, Plätzer, Samitz — JHEP 1810 (2018) 200]

\[ m_{t}^{MC} = m_{t}^{pole} + \Delta_{m}^{pert} + \Delta_{m}^{non-pert} + \Delta_{m}^{MC} \]

\[ m_{t}^{CB}(Q_0) = m_{t}^{pole} - \frac{2}{3} Q_0 \alpha_s(Q_0) + \mathcal{O}(\alpha_s^2) \]

See Silvia’s talk for related discussions.
Hadronization & perturbative variations

Start to gain control of perturbative variations in the shower, and since long on the impact of matching, e.g. by dependence on hard shower scale.

But how do these variations conspire with soft physics? What is the uncertainty budget of an event generator, comprehensively? E.g. check how variations of $\alpha_s$ are absorbed by re-tuning hadronization (more in a couple of minutes).
Amplitude evolution

\[
A_n(q) = \int_Q \frac{d k}{k'} P e^{-\int_q^{k'} \frac{d k'}{k'} \Gamma(k')} D_n(k) A_{n-1}(k) D_n^\dagger(k) \bar{P} e^{-\int_q^{k'} \frac{d k'}{k'} \Gamma^\dagger(k')}
\]

Markovian algorithm at the amplitude level:
Iterate gluon exchanges and emission.

Different histories in amplitude and conjugate amplitude needed to include interference.

[Angeles, De Angelis, Forshaw, Plätzer, Seymour – JHEP 05 (2018) 044]
[Forshaw, Holguin, Plätzer – JHEP 1908 (2019) 145]
Beyond Leading Colour

CVolver library implements numerical evolution in colour space.

Resummation of non-global logarithms at full colour:

- $\rho$ to $gg$
- $\Sigma(\rho) = \sum_n \int d\sigma(\{p_i\}) \prod_i \theta_{in}(\rho - E_i)$

Avoid complexity which grows with colour space dimensionality:

- Monte Carlo over colour flows,
- events at intermediate steps carry complex weights.
Would subleading-N matter?

Ave r a g e p a r t i c l e $p_T$, t r a c k $p_\perp > 500 \text{MeV}$, $|\eta| < 2.5$, $\tau > 300 \text{ps}$
Would subleading-N matter?

Approach colour reconnection from colour evolution: perturbative component?

Reconnection amplitude

\[ A_{\tau \rightarrow \sigma} = \langle \sigma | U \left( \{ p \}, \mu^2, \{ M_{ij}^2 \} \right) | \tau \rangle \]

\[ P e^{-\int_{q}^{k} \frac{d k'}{k'} \Gamma(k')} \]

[Gieseke, Kirchgaesser, Plätzer – EPJ C 78 (2018) 99]

[Gieseke, Kirchgaesser, Plätzer, Siodmok – JHEP 11 (2018) 149]
Generalize amplitude evolution paradigm to a fully exclusive observable.

Not a rigorously proven factorisation theorem, but certainly the form in which we expect a hadronization model to enter the calculation.

\[
\sigma[U] = \sum_n \int \alpha_s^n \text{Tr} \left[ M_n(Q; p_1, \ldots, p_n) U_n(Q; p_1, \ldots, p_n) \right] d\phi(Q) \prod_{i=1}^n (4\pi \mu^2) \epsilon [dp_i] \delta(p_i)
\]

Rely on factorisation properties of amplitudes to isolate divergent contributions. Physical cross section finite: resort to RGE methodology.

[Plätzer – arXiv:2204.06956]
Evolution equations

We can now obtain the evolution equations we asked for:

\[
\partial_S A_n = \Gamma_{n,S} A_n + A_n \Gamma_{n,S}^\dagger - \sum_{s \geq 1} \alpha_s^S R_{S,n}^{(s)} A_{n-s} R_{S,n}^{(s)\dagger}
\]

\[
\partial_S S_n = -\tilde{\Gamma}_{S,n}^\dagger S_n - S_n \tilde{\Gamma}_{S,n} + \sum_{s \geq 1} \alpha_s^S \int \tilde{R}_{S,n+s}^{(s)\dagger} S_{n+s} \tilde{R}_{S,n+s}^{(s)} \prod_{i=n+1}^{n+s} [dp_i] \delta(p_i)
\]

Coupled system of evolution equations: For each resolution we have chosen, we get one. Directions of evolution are different in scale and multiplicity.

\[
\sigma[U_n] = \sum_n \int \alpha_S^n \text{Tr} [(A_n + \Delta_n) S_n] d\phi(Q) \prod_{i=1}^n \mu_R^{2\varepsilon} [dp_i] \tilde{\delta}(p_i)
\]

dressing of hard process ~ parton shower

\[
\text{soft evolution} \sim \text{hadronization model}
\]

\[
\alpha_s \text{ corrections to tower of logarithms present in } A
\]
We can now obtain the evolution equations we asked for:

\[
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\]

\[
\partial_S S_n = -\tilde{\Gamma}_{n,S}^\dagger S_n - S_n \tilde{\Gamma}_{S,n} + \sum_{s \geq 1} \alpha^S_{n,S} \int \tilde{R}_{s,n+S}^{(s)} S_{n+S} \tilde{R}_{S,n+S}^{(s)} \prod_{i=n+1}^{n+s} [dp_i] \tilde{\delta}(p_i)
\]

At second order subtract the iterated kernel from the order limit, and include un-ordered separately.

\[
R_n^{(2,0)} \circ R_n^{(2,0)^\dagger} = (\hat{D}_n^{(2,0)} \circ \hat{D}_n^{(2,0)^\dagger} \hat{\Theta}_{n,2} - \hat{D}_n^{(1,0)} \circ \hat{D}_n^{(1,0)^\dagger} \hat{\Theta}_{n,1})(1 - \Theta_{n-1,1}) \partial_S \Theta_{n,1}
\]
Hadronization models

We can now obtain the evolution equations we asked for:

\[ \partial_S A_n = \Gamma_{n,S} A_n + A_n \Gamma^\dagger_{n,S} - \sum_{s \geq 1} \alpha_S^s R_{S,n}^{(s)} A_{n-s} R_{S,n}^{(s)\dagger} \]

\[ \partial_S S_n = -\tilde{\Gamma}^\dagger_{S,n} S_n - S_n \Gamma_{S,n} + \sum_{s \geq 1} \alpha_S^s \int \tilde{R}_{S,n+s}^{(s)\dagger} S_{n+s} \tilde{R}_{S,n+s}^{(s)} \prod_{i=n+1}^{n+s} [dp_i] \delta(p_i) \]

Evolution equation for a hadronization model!

Features which relate to the high-energy dynamics of the Herwig cluster model.

[Gieseke, Kirchgaesser, Plätzer, Siodmok – JHEP 11 (2018) 149]
Sudakov-type densities central to Showers

\[ \frac{dS_P(q|Q, z, x)}{dq \, dz} = \Delta_P(Q_0|Q, x) \delta(q - Q_0) \delta(z - z_0) \]
\[ + \Delta_P(q|Q, x) P(q, z, x) \theta(Q - q) \theta(q - Q_0) \]

no emission

emission

Negative P or unknown overestimate requires weighted veto algorithm, with in principle arbitrary proposal kernel and veto probability.
Weighted branching algorithms exhibit prohibitive weight distributions & convergence issues.

Result without resampling

Result with resampling

Resampling algorithms can compress weight distributions at intermediate steps.

Different resampling method developed as event generator after-burner.

[Andersen, Gütschow, Maier, Prestel — EPJ C 80 (2020) 11]
Multi-purpose event generators well established for all FCC options. Matching & merging has been focus of last decade.

As we aim to use more and more of the complex structures, shower accuracy becomes the bottleneck. Also for matching to more than NNLO QCD.

The understanding of hadronization effects and models, and their interplay with parton showers will be one of the main topics in the future, not only in light of measuring fundamental parameters.

Electroweak being addressed, but only at the start to understand amplitude level effects and “soft” region.

[Hoang, Plätzer, Samitz — in progress]

[Masouminia, Richardson — arXiv:2108.10817]

[Plätzer, Sjödahl — arXiv:2204.03258]

[Christiansen, Sjöstrand, Bauer, Webber, Brooks, Verheyen, Skands …]
Thanks!