A THEORETICAL INTERPRETATION OF THE BLACK HOLE FUNDAMENTAL PLANE

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Received 2007 January 11; accepted 2007 June 20

ABSTRACT

We examine the origin and evolution of correlations between properties of supermassive BHs and their host galaxies using simulations of major galaxy mergers, including the effects of gas dissipation, cooling, star formation, and BH accretion and feedback. We demonstrate that the simulations predict the existence of a BH “fundamental plane” (BHFP), of the form $M_{\text{BH}} \propto \sigma^{3.0\pm0.3} R_{e}^{0.43\pm0.19}$ or $M_{\text{BH}} \propto M_{*}^{0.34\pm0.17} \sigma^{2.2\pm0.5}$, similar to relations found observationally. The simulations indicate that the BHFP can be understood roughly as a tilted intrinsic correlation between BH mass and spheroid binding energy, or the condition for feedback coupling to power a pressure-driven outflow. While changes in halo circular velocity, merger orbital parameters, progenitor disk redshifts and gas fractions, ISM gas pressurization, and other parameters can drive changes in, e.g., $\sigma$ at fixed $M_*$, and therefore changes in the $M_{\text{BH}}-\sigma$ or $M_{\text{BH}}-M_*$ relations, the BHFP is robust. Given the empirical trend of decreasing $R_e$ for a given $M_*$ at high redshift (i.e., increasingly deep potential wells), the BHFP predicts that BHs will be more massive at fixed $M_*$ in good agreement with recent observations. This evolution in the structural properties of merger remnants, to smaller $R_e$ and larger $\sigma$ (and therefore larger $M_{\text{BH}}$, conserving the BHFP) at a given $M_*$, is driven by the fact that disks (merger progenitors) have characteristically larger gas fractions at high redshifts. Adopting the observed evolution of disk gas fractions with redshift, our simulations predict the observed trends in both $R_e(M_*)$ and $M_{\text{BH}}(M_*)$. The existence of this BHFP also has important implications for the masses of the very largest black holes and immediately resolves several apparent conflicts between the BH masses expected and measured for outliers in both the $M_{\text{BH}}-\sigma$ and $M_{\text{BH}}-M_*$ relations.

Subject headings: cosmology: theory — galaxies: active — galaxies: evolution — quasars: general

Online material: color figures

1. INTRODUCTION

Correlations between the masses of supermassive black holes (BHs) in the centers of galaxies and the properties of their host spheroids (e.g., Kormendy & Richstone 1995) imply a fundamental bond between the growth of BHs and galaxy formation. A variety of such correlations have been identified, linking BH mass to host luminosity (Kormendy & Richstone 1995), mass (Magorrian et al. 1998), velocity dispersion (Ferrarese & Merritt 2000; Gebhardt et al. 2000), concentration or Sérsic index (Graham et al. 2001; Graham & Driver 2007), and binding energy (Aller & Richstone 2007). However, the connection between these relationships is obscured by the fact that the properties of the host spheroids are themselves correlated (for a comparison of the observed relations see, e.g., Novak et al. 2006). The lack of a clear motivation for favoring one correlation over another has led to considerable debate over the interpretation of systems deviating from the mean correlation between host properties and over the demographics of the most massive BHs (e.g., Bernardi et al. 2007; Lauer et al. 2007a; Batcheldor et al. 2007; Wyithe 2006). Analytical estimates (e.g., Silk & Rees 1998; Burkert & Silk 2001) and studies using simulations (Cox et al. 2006b; Robertson et al. 2006b) have demonstrated that these correlations, in particular the $M_{\text{BH}}-\sigma$ (Di Matteo et al. 2005) and $M_{\text{BH}}-M_*$ (Robertson et al. 2006c) relations, can be reproduced in feedback-regulated models of BH growth. However, determining the fundamental character and evolution of these correlations with redshift is critical for informing analytical models (e.g., Croton 2006) and simulations (Di Matteo et al. 2005; Robertson et al. 2006c; Hopkins et al. 2005) that follow the coformation of BHs and bulges, as well as theories that relate the evolution and statistics of BH formation and quasar activity to galaxy mergers (e.g., Hopkins et al. 2006a, 2006b, 2007c, 2007f) and to the remnant spheroid population (Hopkins et al. 2006c, 2007b). Likewise, the significance of observations tracing the buildup of spheroid populations (e.g., Cowie et al. 1996) and associations between spheroids in formation, mergers, and quasar hosts (Hopkins et al. 2007a) depends on understanding the evolution of BH/host correlations.

Unfortunately, efforts to directly infer these correlations at redshifts $z \gg 0$ are difficult and still limited by the small number of observable hosts. Furthermore, without understanding the fundamental nature of the correlations between BH and host properties, it is difficult to interpret these observations, as they do not all probe the same correlations. Consequently, different groups have arrived at seemingly contradictory conclusions. Velocity dispersion measurements have favored both no evolution (Shields et al. 2003; from O vi velocity dispersions) and substantial evolution (Shields et al. 2006; Woo et al. 2006; from CO dispersions and spectral template fitting). BH clustering measurements (Adelberger & Steidel 2005; Wyithe & Loeb 2005; Hopkins et al. 2007e; Lidz et al. 2006) suggest moderate evolution in the ratio of BH to host halo mass at redshifts $z \sim 1$–3. Direct host R-band luminosity measurements (Peng et al. 2006) and indirect comparison of quasar luminosity and stellar mass densities (Merloni et al. 2004) or BH and stellar mass functions (Hopkins et al. 2006d) similarly favor moderate evolution in the ratio of BH to host spheroid stellar mass occurring at $z \gtrsim 1$, and dynamical masses from CO measurements suggest that this evolution may extend to $z \sim 6$ (Walter et al. 2004). Better understanding the dependence of BH mass on host properties can provide both a self-consistent paradigm in which to interpret these observations and (potentially) a physically and observationally motivated prediction of their evolution.

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One possibility is that these different correlations are projections of the same “fundamental plane” (FP) relating BH mass with two or more spheroid properties such as stellar mass, velocity dispersion, or effective radius, in analogy to the well-established FP of spheroids. For the case of spheroids, it is now understood that various correlations, including the Faber-Jackson relation (Faber & Jackson 1976) between luminosity (or effectively stellar mass $M_*$) and velocity dispersion $\sigma$, the Kormendy (1977) relation between effective radius $R_e$ and surface brightness $I_e$, and the size-luminosity or size-mass relations (e.g., Shen et al. 2003) between $R_e$ and $M_*$, are all projections of an FP relating $R_e \propto \sigma^\alpha I_e^\beta$ (Dressler et al. 1987; Djorgovski & Davis 1987).

In their analysis of the relation between BH mass and host luminosity or dynamical mass, $M_\text{dyn}$, Marconi & Hunt (2003; see also de Francesco et al. 2006) noted that the residuals of the $M_\text{BH}-\sigma$ relation (effectively $M_\text{BH}/\sigma^4$; Tremaine et al. 2002) were significantly correlated with the effective radii of the systems in their sample. A more detailed study by Hopkins et al. (2007d, hereafter Paper I) found that the observations indeed favor an FP-type relation for $M_\text{BH}$ and any two of $R_e$, $\sigma$, or $M_*$, at $>3\sigma$ confidence. This observed, low-redshift BHFP has the form $M_\text{BH} \propto \sigma^{3.0 \pm 0.3} R_e^{4.4 \pm 0.19}$ or $M_\text{BH} \propto M_*^{0.54 \pm 0.17} \sigma^{2.7 \pm 0.5}$, with the early-type FP providing a tight mapping between $M_\ast$, $R_e$, and $\sigma$. Given the mean correlations between, e.g., $R_e$ and $M_\ast$ or $M_*$, and $\sigma$, the previously recognized correlations between BH mass and spheroid mass, luminosity, velocity dispersion, and binding energy can all naturally be explained as projections of this intrinsic BHFP. However, these observations remain limited in the range of systems they probe and are restricted to passive, local spheroids. We therefore wish to understand both the origin of this BHFP and how it should (or should not) evolve with redshift.

In this paper we investigate the nature of the correlation between BH mass and host properties and the existence of an FP relating BH mass and spheroid mass, velocity dispersion, and effective radius. In §3 we describe a large set of numerical simulations that we use to study and predict the nature of the BH-host correlations under a wide variety of conditions, and in §2 we describe the observational data sets we compile to study the observed correlations. In §4 we describe the correlations determined from both simulations and observations and then analyze the correlations between residuals in, e.g., the $M_\text{BH}-\sigma$ relation and secondary properties such as $R_e$ and $M_\ast$, which leads us in §4.2 to discuss the FP relating BH mass and $\sigma$, $R_e$, and $M_\ast$. In §5 we discuss the implications of this relation for predicting BH masses and demographics, and in §6 we consider the physical origin of the BHFP relation. In §7 we study how various theoretical quantities or initial conditions drive systems along the BHFP relation and, as a consequence, drive evolution in the various projections of the BHFP, and in §8 we apply this to understand the observed evolution with redshift in the $M_\text{BH}-M_\ast$ and $M_\text{BH}-\sigma$ relations. We summarize our conclusions and discuss future tests of our proposed relations in §9.

Throughout we adopt an $\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$, $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$ cosmology (and correct all observations accordingly), but note that this choice has little effect on our conclusions.

2. THE DATA

The observational data set with which we compare is described in detail in Paper I, but we summarize it here. We consider the sample of local BHs for which masses have been reliably determined via either kinematic or maser measurements. Specifically, we adopt the sample of 38 local systems for which values of $M_\text{BH}$, $\sigma$, $R_e$, $M_\text{dyn}$, and bulge luminosities are compiled in Marconi & Hunt (2003) and Häring & Rix (2004; see also Magorrian et al. 1998; Merritt & Ferrarese 2001; Tremaine et al. 2002). We adopt the dynamical masses from the more detailed Jeans modeling in Häring & Rix (2004). We estimate the total stellar mass $M_*$ from the total K-band luminosity given in Marconi & Hunt (2003) using the K-band mass-to-light ratios as a function of luminosity from Bell et al. (2003) (specifically assuming a “diet” Salpeter initial mass function, although this only affects the absolute normalization of the relevant relations). In Paper I it is also noted that the choice of these mass-to-light ratios as opposed to those determined from, e.g., photometric fitting makes little difference. We take measurements of the Sérsic index $n_s$ from Graham & Driver (2007). Where possible, we update measurements of $R_e$, $\sigma$, and $n_s$ with more recent values from Lauer et al. (2005, 2007b), McDermdid et al. (2006), and Kormendy et al. (2007), which extends the baseline of surface brightness measurements, allowing more robust estimates of $n_s$ and $R_e$.

Although it should only affect the normalization of the relations herein, we note that our adopted cosmology is identical to that used to determine all quoted values in these works. The concentration index $R_{30}/R_{50}$ for the observed systems is calculated assuming a Sérsic profile with the best-fit $n_s$. When we fit the observations to, e.g., the mean $M_\text{BH}-\sigma$ relation and other BH-host relations, we consider only the subsample of 27 objects in Marconi & Hunt (2003), which are deemed to have “secure” BH and bulge measurements (i.e., for which the BH sphere of influence is clearly resolved, the bulge profile can be well measured, and maser spots [where used to measure $M_\text{BH}$] are in Keplerian orbits). Our results are not qualitatively changed if we consider the entire sample in these fits, but their statistical significance is somewhat reduced.

3. SIMULATIONS

3.1. Methodology

Our simulations are taken from Robertson et al. (2006b), who utilize a set of several hundred simulations of major mergers to study the properties of remnants on the early-type galaxy FP. The properties of the models are discussed in detail therein, but we briefly review them here. The simulations were performed with the parallel TreeSPH code GADGET-2 (Springel 2005), based on a fully conservative formulation (Springel & Hernquist 2002) of smoothed particle hydrodynamics (SPH), which conserves energy and entropy simultaneously even when smoothing lengths evolve adaptively (see, e.g., Hernquist 1993; O’Shea et al. 2005). Our simulations account for radiative cooling, heating by a UV background (as in Katz et al. 1996; Davé et al. 1999), and incorporate a subresolution model of a multiphase interstellar medium (ISM) to describe star formation and supernova feedback (Springel & Hernquist 2003; Springel et al. 2005). Feedback from supernovae is captured in this subresolution model through an effective equation of state for star-forming gas, enabling us to stably evolve disks with arbitrary gas fractions (see Springel & Hernquist 2005; Robertson et al. 2006a). This feedback prescription can be adjusted between an isothermal gas with effective temperature of $10^4$ K and our full multiphase model with an effective temperature of $\sim 10^5$ K.

Supermassive BHs are represented by “sink” particles that accrete gas at a rate $\dot{M}$ estimated from the local gas density and sound speed using an Eddington-limited prescription based on Bondi-Hoyle-Lyttleton accretion theory. The bolometric luminosity of the BH is taken to be $L_{\text{bol}} = c_\text{e} M \dot{M}^2$, where $c_\text{e} = 0.1$ is the radiative efficiency. We assume that a small fraction (typically $\approx 5\%$) of $L_{\text{bol}}$ couples dynamically to the surrounding gas and that this feedback is injected into the gas as thermal energy, weighted by the SPH smoothing kernel. This fraction is a free
parameter: we adjust it to match the normalization of the local $M_{\text{BH}}-\sigma$ relation as in Di Matteo et al. (2005, 2007) and Sijacki et al. (2007). We emphasize that this controls only the normalization of this relation; i.e., inefficient feedback coupling means that a BH must grow proportionally larger in order to couple the same energy to the ISM and self-regulate, but the scalings of BH mass with $\sigma$ and host properties (i.e., slopes of the BH-host relations and correlations between residuals in these relations) are not changed. Because our comparisons throughout are based on the relative scalings of BH mass with host properties, this normalization choice is simply a matter of convenience. For now, we do not resolve the small-scale dynamics of the gas in the immediate vicinity of the BH, but we assume that the time-averaged accretion rate can be estimated from the gas properties on the scale of our spatial resolution (roughly $\approx$20 pc, in the best cases).

The progenitor galaxies in the mergers are constructed following Springel et al. (2005). For each simulation, we generate two stable, isolated disk galaxies, each with an extended dark matter halo with a Hernquist (1990) profile, motivated by cosmological simulations (e.g., Navarro et al. 1996; Busha et al. 2005), an exponential disk of gas and stars, and (optionally) a bulge. The galaxies have total masses $M_{\text{vir}} = V_{\text{vir}}^2/(1000h_0)$ for $z = 0$, with the baryonic disk having a mass fraction $m_d = 0.041$, the bulge (when present) having $m_b = 0.0136$, and the rest of the mass in dark matter. The dark matter halos are assigned a concentration parameter scaled as in Robertson et al. (2006c) appropriately for the galaxy mass and redshift following Bullock et al. (2001). The disk scale length is computed based on an assumed spin parameter $\lambda = 0.033$, chosen to be near the mode in the $\lambda$ distribution measured in simulations (Vivitska et al. 2002), and the scale length of the bulge is set to 0.2 times this.

Typically, each galaxy initially consists of 600,000 dark matter halo particles, 20,000 bulge particles (when present), 40,000 gas and 40,000 stellar disk particles, and 1 BH particle. We vary the numerical resolution, with many simulations using twice, and a subset up to 128 times, as many particles. We choose the initial seed mass of the BH either in accord with the observed $M_{\text{BH}}-\sigma$ relation or to be sufficiently small that its presence will not have an immediate dynamical effect, but we have varied the seed mass to identify any systematic dependencies. Given the particle numbers employed, the dark matter, gas, and star particles are all of roughly equal mass, and central cusps in the dark matter and bulge are reasonably well resolved (see Fig. 2 in Springel et al. 2005).

We consider the set of several hundred simulations from Robertson et al. (2006b) in which we vary the numerical resolution, the orbit of the encounter (disk inclinations, pericenter separation), the masses and structural properties of the merging galaxies, initial gas fractions, halo concentrations, the parameters describing star formation and feedback from supernovae and BH growth, and initial BH masses. The detailed list of varied properties is given in Tables 1 and 2 of Robertson et al. (2006b). For example, the progenitor galaxies have virial velocities $V_{\text{vir}} = 50, 80, 115, 160, 226, 320,$ and 500 km s$^{-1}$ and are constructed to match disks at redshifts $z = 0, 2, 3,$ and 6, and our simulations span a range in final BH mass $M_{\text{BH}} \sim 10^7-10^{10} M_\odot$. The extensive range of conditions probed provides a large dynamic range, with final spheroid masses spanning $M_s \sim 10^8-10^{13} M_\odot$, covering the entire range of the observations we consider at all redshifts, and allows us to identify any systematic dependencies in our models. We consider initial disk gas fractions (by mass) of $f_{\text{gas}} = 0.05, 0.2, 0.4, 0.6, 0.8,$ and 1.0 for several choices of virial velocities, redshifts, and ISM equations of state.

The results described in this paper are based primarily on simulations of equal-mass mergers; however, by examining a small set of simulations of unequal-mass mergers, we find that the behavior does not change significantly for mass ratios down to about 1:3 or 1:4, below which mass ratio the mergers produce neither substantial BH nor bulge growth, and therefore they are no longer appropriate to compare to local relations between BHs and massive spheroids.

3.2. Analysis

Each simulation is evolved until the merger is complete and the remnants are fully relaxed, typically $\sim$1–2 Gyr after the final merger and coalescence of the BHs. We then measure kinematic properties of the remnants following Robertson et al. (2006b) and Cox et al. (2006b). The effective radius $R_e$ is the projected half-mass stellar effective radius, and the velocity dispersion $\sigma$ is the average one-dimensional velocity dispersion within a circular aperture of radius $R_e$. Projected quantities such as $R_e$, $\sigma$, and the stellar surface mass density $I_e = M_e(r < R_e)/\pi R_e^2$ are averaged over 100 random lines of sight to the remnant. Throughout, the stellar mass $M_*$ refers to the total stellar mass of the galaxy, and the dynamical mass $M_{\text{dyn}}$ refers to the traditional dynamical mass estimator

$$M_{\text{dyn}} \equiv k \sigma^2 R_e / G,$$

where we adopt $k = 8/3$ (although this choice is irrelevant as long as we apply it uniformly to both observations and simulations). We define $\phi_*$, as the gravitational potential from the galaxy (excluding the BH itself) at the location of the BH (given the typical resolution over which this quantity is smoothed, this is effectively the converged bulge potential at $r = 0$, in the absence of the BH; see Springel 2005). As a concentration index we adopt the ratio of half-mass radius $R_e = R_{30}$ to 30% mass radius $R_{30}$ and measure $n_e$ from projected mock images following E. Krause et al. (2008, in preparation). We note that, for convenience, $f_{\text{gas}}$ typically refers to the gas fraction in the merging disks when the simulations are initialized, but we show in §7 that our results are unchanged (although $f_{\text{gas}}$ itself systematically shifts) regardless of the time before the merger at which we choose to define the gas fraction of the systems.

4. THE LOCAL BH-Host Correlations

4.1. One-to-One Relationships

Figure 1 shows the location of our simulation remnants on the $M_{\text{BH}}-\sigma$ and $M_{\text{BH}}-M_*$ relations. As demonstrated by Di Matteo et al. (2005), they agree well with the observed relations over a large dynamic range. Critically, although modifying our feedback prescriptions and, as we show below, adjusting the kinematic properties of the remnants by changing, e.g., orbital parameters and gas fractions of the merging systems can shift the normalization of the relations, the slopes are not adjustable or tunable but are a natural consequence of self-regulated BH growth. We note that there is a slight offset between the normalization (but not slope) of our predicted $M_{\text{BH}}-M_*$ relation and that measured by Häring & Rix (2004), but a weaker offset in $M_{\text{BH}}-\sigma$. This owes to the fact that, at fixed $\sigma$, our simulated systems typically have slightly larger (mean offset $\approx$0.16 dex) stellar masses than the observed systems on the $M_{\text{BH}}-\sigma$ relation. There are a number of possible explanations for this offset in the Faber-Jackson [$M_* (\sigma)$] relation. As pointed out by Bernardi et al. (2007), the systems with measured BH masses in Häring & Rix (2004) actually lie above the
Faber-Jackson relation observed for typical early-type galaxies, perhaps owing to a selection bias. Their estimate of the magnitude of this bias is quite similar to the offset here and can completely account for our results.

Furthermore, we show below that at fixed $M_*$, changing the gas fractions of the merging systems, their orbits, or their structural properties can systematically drive changes in $\sigma$. This means that the precise normalization of the observed Faber-Jackson relation depends, in detail, on the exact star formation and merger histories of the systems observed. Since we do not model these cosmological histories, but rather isolate different mergers in order to study how these changes are driven, it is not surprising that the normalizations of the relations do not happen to match perfectly. In any case, we are interested in how offsets or evolution from one relation or the other are produced and how such residuals may scale with other host properties, for which the actual normalization of the relation factors out completely. It therefore makes no difference to our analysis (although we have considered both cases) whether we compare to our full suite or select only a subset of simulations that reproduce the (mean) normalization in the observed Faber-Jackson relation.

Figure 2 shows the correlation between BH mass and a wide variety of host properties, from both our simulations and the observed sample. The slopes of the simulated correlations are essentially identical to those observed in every case. Note that many of the correlations are similarly tight, including the correlations with velocity dispersion $\sigma$, stellar mass $M_*$, dynamical mass $M_{\text{dyn}}$, effective bulge binding energy $\sigma^2$, and central potential $\phi_c$. The best-fit correlations are listed in Table 1, along with the intrinsic scatter in $M_{\text{BH}}$ estimated about each correlation from the simulations. We do not list the correlation with $M_{\text{halo}}$, as it is clear in our simulations that BH mass is correlated with small-scale bulge properties (unsurprising, given that the central potential of the bulge is strongly baryon dominated). Therefore, while there is an indirect correlation with $M_{\text{halo}}$ through, e.g., the $M_{\text{halo}}^2 M_*$ and $M_{\text{BH}} M_*$ relations, its nature depends systematically on the exact $M_{\text{halo}} M_*$ relation.

We also note that the (relatively large) scatter in the correlations between $M_{\text{BH}}$ and concentration or Sérsic index in Figure 2 appears contrary to the conclusions of Graham et al. (2001), who argue for very small intrinsic scatter in these correlations. However, Novak et al. (2006) point out that uncertainty in this correlation, unlike for the $M_{\text{BH}} \sigma$ or $M_{\text{BH}} - M_{\text{dyn}}$ relations, is dominated by the measurement errors in concentration index or $n_\text{s}$, which means that improved observations are needed to determine whether the relation is actually consistent with small intrinsic scatter. In fact, when we update the measurements from Graham et al. (2001) and Graham & Driver (2007) with the $n_\text{s}$ measurements from Kormendy et al. (2007), which typically reduce the measurement error in $n_\text{s}$ from $\approx 20\%$ to $\approx 5\%$ (and in at least two cases $\approx 4$ change $n_\text{s}$ by $>3\sigma$ relative to the Graham & Driver [2007] fit), the quality of the correlation is substantially degraded, and a significantly larger intrinsic scatter is implied.

In the simulations, it is possible, given appropriate gas fractions or orbital parameters, to substantially change $n_\text{s}$ or $R_{\text{S}}/R_{\text{S}0}$ at fixed stellar mass without driving a corresponding change in $M_{\text{BH}}$ ($n_\text{s}$ also appears to be more variable sight line to sight line than other projected quantities such as $\sigma$ or $R_\text{e}$). As will be discussed in detail in E. Krause et al. (2008, in preparation), this has the consequence that the values of $n_\text{s}$ predicted by a uniform set of simulations are not as tightly correlated with spheroid mass as is observed (and, as a secondary consequence, $M_{\text{BH}}$, which is tightly correlated with $M_*$, is less well correlated with $n_\text{s}$). Furthermore, the weaker correlation in the simulations should not be
surprising: unlike the $M_\bullet$-$R_e$ or $M_\bullet$-$\sigma$ correlations, which are produced fundamentally (at least to lowest order) by basic dynamical effects (and are not especially sensitive to the exact type of spheroid-producing mergers), it is commonly believed (e.g., Kormendy et al. 2007) that the correlation between $n_s$ and $M_\bullet$ is driven by an increasing prevalence of dissipationless (spheroid-spheroid) mergers at higher masses. Such mergers will, by definition, conserve the $M_\bullet$-$M_*/C3$ relation, but simulations have shown that they dramatically change $n_s$ (e.g., Boylan-Kolchin et al. 2006) and therefore the $M_\bullet$-$n_s$ relation. If this is true, then the $M_\bullet$-$n_s$ correlation, unlike the $M_\bullet$-$M_*$ relation, must fundamentally be driven by cosmological effects (e.g., the differential contribution of different merger types as a function of mass and redshift), which our simulations do not represent. A more accurate modeling is outside the scope of this paper, as it would require a fully cosmological prediction for merger rates (as a function of type, mass, and redshift) with high-resolution cosmological simulations such that the central galaxy structure and BH feedback effects could be modeled. The $M_\bullet$-$M_*$ and BHFP relations on which we focus, on the other hand, should be relatively robust to these effects (see also § 9).

![Image of correlations between BH mass and a variety of host spheroid properties. Gray symbols with error bars are the observations; the gray dashed line shows the least-squares best fit to each relation. Note that the quality of the correlation between $M_\bullet$ and concentration or Sérsic index observed is significantly reduced relative to that in Graham et al. (2001) and Graham & Driver (2007) when we adopt the Sérsic index measurements from Kormendy et al. (2007). Black symbols and lines are the results of hydrodynamic simulations (see § 3). The slopes of the simulated and observed relations are statistically identical in every case, with all normalization offsets owing to the small Faber-Jackson [$M_\bullet(\sigma)$] offset in Fig. 1. [See the electronic edition of the Journal for a color version of this figure.]


4.2. A Black Hole Fundamental Plane

We wish to determine whether or not a simple one-to-one correlation between, e.g., $M_{\text{BH}}$ and $\sigma$ is a sufficient description of the simulations, or whether there is evidence for additional dependence on a second parameter such as $R_e$ or $M_\ast$. The most efficient way to determine such a dependence is by looking for correlations between the residuals of the various projections of such a potential BHFP relation. Following Paper I, Figure 3 plots the correlation between BH mass $M_{\text{BH}}$ and host bulge effective radius $R_e$ at fixed $\sigma$. Specifically, we determine the residual with respect to the $M_{\text{BH}}$-$\sigma$ relation by fitting $M_{\text{BH}}(\sigma)$ to an arbitrary log-polynomial

$$\log(M_{\text{BH}}) = \Sigma [a_n \log(\sigma)^n],$$

allowing as many terms as the data favor (i.e., until $\Delta \chi^2$ with respect to the fitted relation is <1), and then take

$$\Delta \log(M_{\text{BH}}|\sigma) \equiv \log M_{\text{BH}} - \langle \log M_{\text{BH}} \rangle(\sigma).$$

We determine the residual $\Delta \log (R_e|\sigma)$ [or $\Delta \log (R_e|\sigma)$, for the stellar mass] in identical fashion and plot the correlation between the two. We allow arbitrarily high terms in $\log$ $\sigma$ to avoid introducing bias by assuming, e.g., a simple power-law correlation between $M_{\text{BH}}$ and $\sigma$, but we find in practice that such terms are not needed: as discussed in Paper I, there is no significant evidence for a log-quadratic (or higher order) dependence of $M_{\text{BH}}$ on $\sigma$, $R_e$, or $M_\ast$, so allowing for these terms changes the residual best-fit solutions by $\ll 1$ $\sigma$. Of course, even this approach could in principle introduce a bias via our assumption of some functional form, and so we have also considered a nonparametric approach where we take the mean of $\log M_{\text{BH}}$ in bins of $\log$ $\sigma$. Our large set of simulations allows us to do this with very narrow binning, and we recover a nearly identical answer.

The simulations show a highly significant correlation between $M_{\text{BH}}$ and $R_e$ at fixed $\sigma$, similar to the observed trend in residuals. We therefore introduce an FP-like relation of the form

$$M_{\text{BH}} \propto \sigma^\alpha R_e^\beta,$$

which can account for these dependencies. Formally, we determine the combination of ($\alpha$, $\beta$) that simultaneously minimizes the $\chi^2/\nu$ of the fit and the significance of the correlations between the residuals in $\sigma$ and $M_{\text{BH}}$ (or $R_e$ and $M_{\text{BH}}$). This yields similar results to the direct fitting method of Bernardi et al. (2003b) from the spheroid FP, which minimizes the quantity

$$\Delta^2 = (\log M_{\text{BH}} - \alpha \log \sigma - \beta \log R_e - \delta)^2,$$

and corresponds exactly to the method used in fitting the observations in Paper I, to which we compare. This yields a best-fit BHFP relation

$$\log M_{\text{BH}} = 8.16 + 2.90(\pm 0.38)\log(\sigma/200~\text{km}~\text{s}^{-1})$$

$$+ 0.54(\pm 0.11)\log(R_e/5~\text{kpc}).$$

The BHFP relation is close to those formally determined for the residuals in Figure 3. Figure 4 plots the residuals of $M_{\text{BH}}$ with respect to these FP relations, at fixed $R_e$ and fixed $\sigma$. The introduction of a BHFP eliminates the strong systematic correlations between the residuals, yielding flat errors as a function of $\sigma$ and $R_e$.

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TABLE 1

BH-HOST CORRELATIONS

| VARIABLES | OBSERVED | SIMULATED |
|-----------|----------|-----------|
| $\sigma^2 R_e^0$ | $8.24 \pm 0.06$ | $8.16 \pm 0.05$ |
| $M_\ast^2$ | $3.64 \pm 0.07$ | $3.96 \pm 0.06$ |
| $M_\ast^2 R_e^0$ | $2.82 \pm 0.10$ | $3.33 \pm 0.10$ |
| $\sigma^2$ | $8.28 \pm 0.08$ | $8.16 \pm 0.06$ |
| $\sigma^2 R_e^0$ | $9.89 \pm 0.10$ | $9.72 \pm 0.04$ |
| $M_\ast$ | $18.28 \pm 0.06$ | $18.84 \pm 0.05$ |
| $R_e$ | $8.44 \pm 0.10$ | $8.48 \pm 0.12$ |
| $\sigma$ | $0.40 \pm 0.06$ | $0.28 \pm 0.06$ |

a For the variables $(x, y)$, a correlation of the form $\log M_{\text{BH}} = \alpha \log x + \beta \log y + \delta$ is assumed, where the normalization is $\delta$ and $\alpha$, $\beta$ are the logarithmic slopes.

b Central potential $\phi_0$, normalization at $\phi_0 = 10^5$ km$^2$ s$^{-2}$. There are no observational measurements presently available to compare with this correlation.

c The normalization gives $\log (M_{\text{BH}}/M_\ast) = 10^5$ km$^2$ s$^{-2}$, where $\phi_0$ is the combination of ($\phi_0$, $\phi_0^{\text{sim}}$). Specifically, we determine the residual with respect to the given best-fit relation.
relation at high significance. We find a nearly identical result using the dynamical mass, \( M_{\text{dyn}} \), instead of stellar mass \( M_* \), as expected from the BHFP in terms of \( r_e \) and \( R_e \). The exact values of the best-fit coefficients of this BHFP determined from the observations are given (along with those of various other BHFP projections) in Table 1.

Because the simulation merger remnant spheroids lie on a stellar mass FP very similar to observed elliptical galaxies (Robertson et al. 2006b), which tightly relates \( M_* \) and \( R_e \) at fixed \( \sigma \) for the observed systems is shown (gray \( P_{\text{null}} \)); the observations imply a secondary FP-type correlation at 3 \( \sigma \). Bottom: Same as the top panel, but considering the correlation between \( M_{\text{BH}} \) and \( \sigma \) at fixed effective radius \( R_e \). [See the electronic edition of the Journal for a color version of this figure.]

\[ \Delta \log(M_{\text{BH}} | R_e) \]

\[ \Delta \log(\sigma | R_e) \]

\[ \Delta \log(M_{\text{BH}} | R_e) \]

\[ \Delta \log(\sigma | R_e) \]

![Fig. 3.—Top: Correlation between the residuals in the \( M_{\text{BH}}-\sigma \) relation and \( R_e-\sigma \) relation, from our simulations (black symbols) and observed sample (gray symbols with errors). At fixed \( \sigma \), systems with larger effective radii \( R_e \) also have larger BH masses \( M_{\text{BH}} \). The fit to this residual correlation is shown with the black lines (±1 \( \sigma \) range in the best-fit correlation shown as dashed lines; note that they are strongly inconsistent with zero correlation), with the slope shown. The probability of the null hypothesis of no correlation in the residuals (i.e., no systematic dependence of \( M_{\text{BH}} \) on \( R_e \) at fixed \( \sigma \)) for the observed systems is shown (gray \( P_{\text{null}} \)); the observations imply a secondary FP-type correlation at 3 \( \sigma \). Bottom: Same as the top panel, but considering the correlation between \( M_{\text{BH}} \) and \( \sigma \) at fixed effective radius \( R_e \). [See the electronic edition of the Journal for a color version of this figure.]

\[ \Delta \log(M_{\text{BH}} | R_e) \]

\[ \Delta \log(\sigma | R_e) \]

\[ \Delta \log(M_{\text{BH}} | R_e) \]

\[ \Delta \log(\sigma | R_e) \]

![Fig. 4.—Correlation between the residuals in our BHFP relation \( M_{\text{BH}} \propto \sigma^2 \) and effective radius \( R_e \) at fixed \( \sigma \) (top) or \( \sigma \) at fixed \( R_e \) (bottom). Accounting for the joint dependence of \( M_{\text{BH}} \) on \( \sigma \) and \( R_e \) removes the strong systematic dependencies in the residuals from Fig. 3 (\( P_{\text{null}} \) is large, meaning that there is no further residual dependence). [See the electronic edition of the Journal for a color version of this figure.]

\[ \Delta \log(M_{\text{BH}} | R_e) \]

\[ \Delta \log(\sigma | R_e) \]

\[ \Delta \log(M_{\text{BH}} | R_e) \]

\[ \Delta \log(\sigma | R_e) \]

5. IMPLICATIONS OF THE BHFP FOR LOCAL BH MASSES AND DEMOGRAPHICS

Given that the “true” correlation between \( M_{\text{BH}} \) and host properties appears to follow an FP-like relation, it is natural to ask how adopting such a relation affects the estimation of BH masses from observed host properties. Figure 7 shows the observed and simulated systems in the FP. The relation appears to be a good predictor of \( M_{\text{BH}} \) over a large dynamic range, and there is no evidence for any curvature or higher order terms in the relation (fitting, e.g., a log-quadratic relation in this space yields \( \Delta \chi^2 < 1 \)). As detailed in Table 1, the intrinsic scatter in the BHFP is small, typically ~0.2 dex, and in all cases smaller than the scatter in, e.g., the \( M_{\text{BH}}-\sigma \) or \( M_{\text{BH}}-M_* \) relations.

However, as Novak et al. (2006) note, minimizing the intrinsic scatter does not necessarily maximize the observational ability to predict BH masses. The BHFP relations depend on measuring two of either \( \sigma \), \( R_e \), or \( M_* \) and therefore introduce additional errors from the measurements of two (as opposed to just one) of
these quantities. At low redshifts, it may be possible to obtain accurate measurements of both $C_27$ and $R_e$ and therefore still obtain more accurate mass estimates (although we caution that several of the literature sources from which we compile observations differ by $>2$ in some $R_e$ measurements, owing to various systematic issues such as the choice of observed bands). However, at high redshifts $M_*$ remains the most easily applicable proxy for $M_{\text{BH}}$, and it is not clear that the additional accuracy gained by introducing the $R_e$ term substantially improves the predictive power of the relation.

Ultimately, the $M_{\text{BH}}-\sigma$ and $M_{\text{BH}}-M_*$ relations are not much worse in a mean sense around $M_{\text{BH}} \sim 10^6-10^9 M_\odot$, with relatively small intrinsic scatter (see Table 1). The reason these relations work as well as they do is that they are both nearly edge-on projections of the BHFP. Given a relation $M_{\text{BH}} \propto \sigma^{0.5} R_e^{0.5}$, it is not surprising that $\sigma$ is an acceptable proxy for $M_{\text{BH}}$ in many situations (whereas the $M_{\text{BH}}-R_e$ correlation has quite large scatter, as $R_e$ enters with a relatively weak dependence in the BHFP).

However, given that there is a systematic dependence on, e.g., $M_*$ or $R_e$ at fixed $\sigma$, which is captured only by the BHFP relations, we expect that the importance of estimating BH masses from the BHFP will be enhanced at the extremes of observed distributions. Figure 8 compares the locations of outliers in the simulated and observed $M_{\text{BH}}-\sigma$ and $M_{\text{BH}}-M_*$ relations with their locations on the BHFP relations. Indeed, several systems that appear as outliers in one of the projections of the BHFP are no longer significant outliers in the BHFP relation. This is true for a number of systems on both the $M_{\text{BH}}-M_*$ relation and the $M_{\text{BH}}-\sigma$ relation. These systems typically have abnormally high or low velocity dispersions given their stellar mass and therefore appear deviant in the BHFP projections, just as such systems typically appear to deviate from the spheroid FP in projections such as the $R_e-M_*$ or Faber-Jackson ($M_*-\sigma$) relations. Therefore, while the typical scatter about the mean relation is not dramatically different for the BHFP ($\sim 0.2$ dex) compared to the $M_{\text{BH}}-\sigma$ relation ($\sim 0.3$ dex), the tails of this distribution are substantially suppressed when we adopt the BHFP as a BH mass estimator.

Given the potential importance of the BHFP for predicting the masses of BHs, especially in extreme systems, we should examine

5 NGC 1023, NGC 3384, NGC 4697, NGC 5252, and Cygnus A have BH mass measurement errors of $<0.1$ dex and measured masses that are 0.32, 0.96, 0.81, 0.53, and 0.34 dex discrepant with the expectation from the Tremaine et al. (2002) relation but only 0.10, 0.70, 0.51, 0.44, and 0.23 dex discrepant with the BHFP expectation, respectively.
the implications that the relation has for the local demographics of BHs. There has been substantial debate recently about whether or not high-M_s systems begin to deviate from the low-M_s Faber-Jackson relation (see, e.g., Boylan-Kolchin et al. 2006; Bernardi et al. 2007; Batcheldor et al. 2007). If so, this implies that either the M_BH-M_s or M_BH-σ relation must change slope at the highest masses, with one of the two or some other relation being the conserved relation. Because the distribution of spheroid velocity dispersions (Sheth et al. 2003) declines more steeply at high σ than the galaxy mass or luminosity functions do at high M_s (when BCGs are included), the assumption that the M_BH-M_s relation remains unchanged (the simplest expectation from gas-free or “dry” mergers) naively predicts a much higher abundance of very high mass (∼10^9 M_☉) BHs (for an extended discussion see Lauer et al. 2007a).

Figure 9 compares the expected BH mass function (BHMF) from the observed distribution of spheroid velocity dispersions and the M_BH-M_s relation with that expected from the early-type galaxy stellar mass function and the M_BH-M_s relation. We adopt the distribution of velocity dispersions σ from Sheth et al. (2003) and the early-type galaxy stellar mass function from Bell et al. (2003) with the addition of BCGs at high masses from Lin & Mohr (2004) (yielding a shallower power law—like falloff at high masses, as opposed to an exponential cutoff). Note that a similar result is obtained estimating the BCG mass function from Lauer et al. (2007a) or using the Cole et al. (2001) or Jones et al. (2006) K-band luminosity functions from the 2dF and 6dF, respectively (converted to mass functions following Bell et al. 2003), which include BCGs and extend to the low space densities of interest. (In any case we are simply attempting to highlight the key qualitative behavior, that with the inclusion of BCGs the stellar mass function falls off more slowly than the velocity dispersion function.)

For now, we assume a one-to-one correlation between M_BH and σ or M_s. Given this, there would also be a one-to-one correlation between M_s and σ [i.e., σ(M_s) is given by matching the two distributions at fixed number density], so we can use the BHFP relation in the form M_BH ∝ M_s σ^2 to estimate the BHMF from this relation. Unsurprisingly, we find that the BHMF from M_BH-σ cuts off rapidly compared to that from M_BH-M_s. With a mixed dependence on both σ and M_s, the BHFP relation predicts a somewhat intermediate case at high masses, although it is closer to the expectation from M_BH-M_s.

However, this treatment ignores the important fact that there is scatter in these correlations. To predict the BHMF from the distribution of velocity dispersions, we should properly convolve over the mean relation broadened by some (approximately lognormal) dispersion with width ∼0.3 dex. The intrinsic scatter is difficult to determine from the observations, if errors are not completely understood, so we adopt the intrinsic scatter in each correlation estimated from our simulations (Table 1). Given this scatter, we recalculate the expected BHMFs, shown in Figure 9. Interestingly, the three predicted BHMFs are now almost identical, even at very high BH masses (∼5 × 10^9). This is expected: if we completely understand the correlation (and its scatter) between M_BH and either σ or M_s over the entire mass range of interest, then any projected version of the same BHMF must yield a similar BHMF: the fact that the distribution of σ cuts off more steeply than M_s is compensated by the fact that the intrinsic scatter in M_BH-σ is slightly larger than in M_BH-M_s (see also Marconi et al. 2004). This is roughly equivalent to the statement that at large masses, where the relation between σ and M_s may change, the M_BH-σ relation must change correspondingly (following the true BHFP relation; for a more detailed comparison see Lauer et al. 2007a). Because the distribution of σ falls rapidly at high σ, there is little contribution at low M_BH from high-σ systems, so a change in slope and increase in scatter in M_BH-σ both have the primary effect of increasing the expected number of high-mass BHs, reconciling the BHMFs.
The scatter is of critical importance at these masses: we consider the BHMF derived from \( \frac{M_{\text{BH}}}{C_{27}} \) if we change the estimated intrinsic scatter by just 25% (i.e., within the range 0.27–0.36 dex), all within the range allowed by the present observations (Tremaine et al. 2002), and find that this relatively small difference in the intrinsic scatter estimate makes a larger difference at high \( M_{\text{BH}} \) than the choice of correlation (\( M_{\text{BH}}, M_{\text{BH}}, \text{or BHFP} \)) adopted. This reinforces the point emphasized by Yu & Tremaine (2002), Yu & Lu (2004), and Tundo et al. (2007) that the estimated intrinsic scatter can dominate the demographics of high-mass BHs; accounting for this, the BHFP does not substantially change these estimates.

6. THE PHYSICAL ORIGIN OF THE FUNDAMENTAL PLANE

If BH growth terminates because of self-regulation, the fundamental requirement is that sufficient energy be released to unbind the surrounding galactic gas. Given a radiative efficiency \( \eta_r \) and feedback coupling efficiency \( \eta_f \), the energy coupled to the intergalactic medium from accretion onto the BH over its lifetime is simply

\[
E_{\text{BH}} = \eta_f \sigma_{\text{obs}}, \quad \eta_f = \frac{\dot{E}_{\text{gas}}}{\dot{E}_{\text{acc}}},
\]

and the binding energy of the gas in the center of the galaxy is

\[
E_{\text{gas}} = \sigma_{\text{obs}} M_{\text{BH}} \sigma_{\text{gas}}^2,
\]

where \( \sigma_{\text{obs}} \) is a constant that depends on the shape of the bulge profile (\( \sigma_{\text{obs}} = 10.1 \) for a Hernquist [1990] profile). In detail, the energy in equation (8) is proportional to the accreted (as opposed to the total) BH mass, and the binding energy of the bulge changes as a function of time during the merger. However, this simple estimate is actually a reasonable approximation to what occurs in the simulations, for two reasons.

First, as is also demanded by numerous empirical constraints (e.g., Soltan 1982; Hopkins et al. 2007f), the majority (\( \gtrsim 70\% \)) of the BH growth occurs in the observed “quasar” phase, which is constrained to be both short lived (e.g., Martini 2004) and near Eddington (e.g., Kollmeier et al. 2006), as occurs in the final growth phase near the end of the merger. Therefore, most of the final \( M_{\text{BH}} \) is accreted in the final \( \epsilon \)-folding of BH growth, over a Salpeter time \( t_\epsilon \sim 4.2 \times 10^7 \) yr. This is small compared to the cooling time of the galactic gas at \( \sim R_{\text{g}} \), so the approximation that the energy released \( \sim M_{\text{BH}} \) is reasonable. Second, by the time of the quasar phase, the merging galaxies have coalesced, and the bulge is largely formed and in place, so the BH growth occurs in the relatively fixed potential of the remnant.
Equating the energy needed to unbind the surrounding gas and terminate accretion yields the expected scaling

\[ M_{\text{BH}} \approx 10^8 M_\odot \frac{0.005}{\eta \epsilon_r^{1/2} f_{\text{gas}}} \left( \frac{M_*}{10^{11} M_\odot} \right) \left( \frac{\sigma}{200 \text{ km s}^{-1}} \right)^2, \]

where we adopt a Hernquist (1990) profile and a typical \( \epsilon_r = 0.1 \), \( \eta \sim 0.05 \) (similar to the values adopted in our simulations), which roughly reproduces the observed \( M_{\text{BH}}-M_* \) normalization.

Note that, in detail, the BH feedback need not be radiative: kinetic wind or jet feedback could inject comparable energy (e.g., Maraschi 2004; Tavecchio et al. 2004; Merloni et al. 2005), with some efficiency \( E = \tilde{\eta} M_{\text{BH}} c^2 \). This yields an identical equation (10), as \( \tilde{\eta} \) is functionally equivalent to the previous \( \eta \epsilon_r \); it does not matter in this derivation whether the net feedback efficiency \( \tilde{\eta} \epsilon_r \) represents a radiative or kinetic mode (or some sum of the two), or exactly what fraction of the total BH mass is accreted in a given growth phase, as the coupling efficiency simply serves to set the normalization of the \( M_{\text{BH}} \)-host correlations. It should also be noted that \( f_{\text{gas}} \) changes by a large amount during the evolution of typical galaxies or (especially) mergers; the value relevant for equation (10) is some effective value in the central regions near the BH during the final \( e \)-folding(s) of growth (hence the notation \( f_{\text{gas}}^* \), which does not necessarily trace the global or pre–active-phase gas fraction of the system (see \S 7).

We therefore naively expect that the BH mass should scale with \( M_*, \sigma^2 \). In Figure 10 we examine the residuals of the best-fit correlation between \( M_{\text{BH}} \) and this binding energy \( M_* \sigma^2 \), in the manner of Figure 3. In this space, there does not appear to be any strong \( \gtrsim 2 \sigma \) evidence for a correlation of the residuals in...
It seems that the correlation between BH mass and bulge binding energy is in some sense more basic than the correlation between BH mass and, e.g., $M_*$ or $\sigma$. However, when we fit $M_{\text{BH}}$ to a function of $M_*, \sigma^2$, we do not recover equation (10); in fact, a linear proportionality between $M_{\text{BH}}$ and $M_*, \sigma^2$ is ruled out at $\sim 5\sigma$ in the observations (and $\sim 10\sigma$ in our simulations). Instead, BH mass follows a “tilted” relation of the form

$$M_{\text{BH}} \propto (M_*, \sigma^2)^{\alpha},$$

with $\alpha \approx 0.71$. Furthermore, this relation is not exactly the same as the BHFP we recover from the observations, which is closer to $M_{\text{BH}} \propto M_*^{0.5} \sigma^2$.

If we revisit our argument, we note that we have naively assumed that the accretion energy from the BH is coupled in an (effectively) infinitely short period of time and unbinds the surrounding gas. More properly, what occurs in our simulations is a pressure-driven outflow from the central regions (e.g., Hopkins et al. 2006b; Hopkins & Hernquist 2006). This implies that the necessary condition for self-regulation is the injection of sufficient pressure-driven outflow from the central regions (e.g., Hopkins et al. 2006b; Hopkins & Hernquist 2006). This implies that the mechanism for self-regulation is the injection of sufficient pressure-driven outflow from the central regions (e.g., Hopkins et al. 2006b; Hopkins & Hernquist 2006).

When the rate is below this threshold, it can drive material from the central regions where it is initially bound or infalling onto the BH, but the momentum coupled is insufficient to entrain the larger scale material and the outflow fails to halt accretion (typical of the early-stage weak winds seen in our simulations in earlier merger stages; see Fig. 1 in Cox et al. 2006a). This condition gives us the requirement

$$\dot{p} \Delta t \propto M_{\text{BH}}^2/\sigma^3 \propto M_*, \sigma,$$

or

$$M_{\text{BH}} \propto M_*/2^{1/2} \sigma^2,$$

similar to the observed BHFP.

Ultimately, the qualitative conclusions of these derivations are similar. Empirical constraints (e.g., Soltan 1982) demand that most BH mass is accumulated in high Eddington ratio quasar phases, so the details of accretion at lower Eddington ratios (whether set by, e.g., the Bondi-Hoyle rate or other accretion mechanisms) are not important. This also implies that the BH mass is accumulated in a short period of time $\sim 10^7-10^8$ yr, such that whether growth is driven at the end of galaxy mergers or in secular disk or bar instabilities, the environment (background potential) local to the BH is relatively fixed. The BH then self-regulates when it is sufficiently massive that its feedback energy can unbind infalling gas and halt accretion. Effects that deepen the local potential at the galactic center and increase the binding energy of gas near the BH will prevent gas from being unbound until the BH has grown more massive than it would otherwise. For example, as we show in detail in §7, if spheroid progenitors are more gas-rich, there is more gas that can cool and form stars on small scales in the center of the spheroid, deepening the potential well there (i.e., yielding a more concentrated remnant with smaller $R_c$ and larger $\sigma$, while having little effect on the total stellar mass). This requires that any BH grow larger in order to unbind nearby gas and halt accretion, in a manner similar to the scaling of equation (14).

We might also ask how sensitively this BHFP scaling or “tilt” is related to dissipational processes, as Robertson et al. (2006b) demonstrate that the origin of the spheroid FP tilt lies essentially in the scale dependence of dissipational processes such as gas cooling and star formation. From the derivation above, we would expect that although the processes from Robertson et al. (2006b) might affect the structure of the merger remnants themselves, they should not change how, fundamentally, the central BH self-regulates. Unfortunately, BH accretion is, itself, naturally a dissipational process, and therefore we cannot simply test this theory in our case by running simulations where gas dissipation is turned off.

However, Robertson et al. (2006b) further show that it actually requires substantial initial gas fractions for these dissipational effects to act: in mergers with very low initial gas fractions ($\leq 20\%$) no “tilt” (in the spheroid FP) is induced, consistent with requirements from the observed phase-space densities of ellipticals (e.g., Hernquist et al. 1993). We therefore briefly consider just a set of simulations with initial $f_{\text{gas}} = 0.05$. This is sufficiently low that the remnants act dissipationlessly and lie on the virial relation as opposed to the spheroid FP (see Fig. 10 of Robertson et al. 2006b), but sufficiently large that we do not need to worry about artificially “strangling” the BH by giving it insufficient gas to accrete (given typical $M_{\text{BH}} \approx 0.001M_*$, the BH need only have access to $\sim 2\%$ of this gas to grow normally).

We find that these merger remnants obey a similar BHFP relation to their high-$f_{\text{gas}}$ counterparts, implying that so long as feedback-driven self-regulation (as opposed to, e.g., gas starvation) determines the final BH mass, these scalings are robust. However, the significance of the preference for, e.g., a BHFP relation as opposed to a simpler $M_{\text{BH}} \propto M_*$ or $M_{\text{BH}} \propto \sigma^2$ relation is greatly reduced. This is because, without significant effects of dissipation to change the central phase-space structure and potential depth of the remnant, the velocity dispersion $\sigma$ and effective radius $R_e$ are simply set by the violent relaxation of the scattered stellar disks. The general scalings of the BHFP are not, then, unique to gas-rich progenitors, but their significance and the importance of accounting for the observed dependencies are so.

Indeed, regardless of prescriptions for BH accretion, feedback, and star formation, the above derivations (and qualitative dependence on galaxy properties) depend on only three relatively robust assumptions: that BH growth is dominated by bright, high Eddington ratio phases; that feedback from accretion affects the gas on small scales around the BH (sensitive to the central potential of the galaxy); and that some sort of heating, momentum coupling, or effective pressure eventually halts accretion. This does place some constraints on scenarios for BH growth, however. In models where, for example, the BH accretion rate is a pure function of the galactic star formation rate (see, e.g., Kawakatu et al. 2006; Monaco & Fontanot 2005; Granato et al. 2004; Cattaneo et al. 2005; although in some of these cases the Eddington limit is still preserved as an upper limit), it is difficult to explain how the BH mass would be sensitive to the central potential in the manner of the observed BHFP and not simply trace the galactic stellar mass (i.e., yield a pure $M_{\text{BH}}M_*$ relation, which the observations disfavor at $\sim 3\sigma$).

7. DRIVING SYSTEMS ALONG THE FUNDAMENTAL PLANE

We have thus far considered systems in terms of the observable properties of the remnants, specifically quantities like $M_*$, $\sigma$, and $R_c$. We now turn to the “theorist’s question,” namely, how are the positions of systems on the $M_{\text{BH}}M_*$, $M_{\text{BH}}\sigma$, and BHFP relations affected by theoretical quantities or initial conditions?
Figure 11 considers the residuals in the $M_{\text{BH}}-M_*$ and BHFP relation for merger remnants with different virial velocities, redshifts, and orbital parameters. Note that “redshift” in this context simply refers to the characteristics of the progenitor disks (which are initialized to resemble disks at low redshifts $z \approx 0$, or higher, $z = 2, 3, 6$). We select each set of simulations in which all parameters are identical except that plotted in the figure, then consider the residual with respect to the mean relation for just those simulations (each set of points of different shade plotted in the figure represents one such set of simulations).

In terms of $M_{\text{BH}}/M_*$, there are weak ($\sim 0.2$–$0.3$ dex over the maximal range spanned by the simulations) trends toward lower $M_{\text{BH}}/M_*$ for higher redshift and larger angular momentum mergers, but the systems lie on the same BHFP regardless of $V_{\text{vir}}$, $z$, orbital parameters, or any other initial quantities we vary. The trends in $M_{\text{BH}}/M_*$ simply reflect changes in the structural properties of the remnants: for example, particular orbital parameters produce smaller bulges and lower values of $\sigma$ at fixed $M_*$. This is expected: because most of the BH growth occurs over a short period of time at the end of a merger or other phase of activity (the last one to two
e-folding times) during which the environment is relatively dynamically settled, changing initial conditions should only affect $M_{\text{BH}}$ indirectly by altering the central structure of the remnant. The lack of a strong trend in $V_{\text{vir}}$ simply reflects the fact that the central regions that set the potential depth of relevance for determining $M_{\text{BH}}$ are very much baryon dominated. This is not to say that $M_{\text{BH}}$ does not scale with $V_{\text{vir}}$ in a mean sense, but simply that the dependence on $V_{\text{vir}}$ is subsumed in the more direct dependence on $M_*$ and $\sigma$, which themselves can depend on halo mass or $V_{\text{vir}}$.

For this reason, at fixed $M^{1/2} \sigma^2$, we expect BH mass to be independent of halo mass and redshift. Of course, $M_*$ and $\sigma$ will, in the mean, scale with $M_{\text{halo}}$, which could yield evolution in the $M_{\text{BH}}$-$M_{\text{halo}}$ relation. For a bulge-dominated system (i.e., ignoring the complication that the same mass halo could host a disk-dominated system with a much smaller BH), $M_* = f_*(\Omega_0/\Omega_m)M_{\text{halo}}$, where $f_*(M_{\text{halo}}, \sigma)$ is the typical fraction of baryons incorporated into the galaxy. In the simulations, we find a rough correlation $\sigma \propto v_{\text{max}}$ (similarly, $t_c \propto v_{\text{max}}$ for our progenitor disks, although that is by construction), where $v_{\text{max}}$ is the maximum halo circular velocity [$\sqrt{V_{\text{vir}}^2 + c^2}$, where $c(M_{\text{halo}}, \sigma)$ is the halo concentration], modulo the effects of, e.g., gas fraction and orbital parameters changing $\sigma$ at fixed $M_{\text{halo}}$. Observationally, a similar mean correlation $\langle \sigma \rangle \approx 0.6v_{\text{max}}$, nearly identical to the best-fit normalization in our simulations; e.g., Kronawitter et al. 2000; Gerhard et al. 2001) is found. Given $c \propto M_{\text{halo}}^{0.3}(1 + z)^{-1}$ (Bullock et al. 2001), this implies $M_{\text{BH}} \propto \alpha(z)f_{\text{gas}}^{0.9}M_{\text{halo}}^{1.04}$, where $\alpha(z)$ represents the weak remaining redshift evolution term, $\alpha \equiv \Omega_0\Delta_0(z)/\Omega_m(z)18\pi^2$ $\frac{1}{3}$, which changes by only $\sim 20\%$ from $z = 0$ to 6. We therefore expect that evolution in the $M_{\text{BH}}$-$M_{\text{halo}}$ relation will be dominated by the effects of evolution in typical gas fractions and remnant structural properties on $\sigma$ (changing $\sigma$ at fixed $M_{\text{halo}}$ in a systematic sense), as well as cosmological evolution in typical baryon incorporation fractions $f_*$ and/or bulge-to-disc ratios in galaxies hosted by halos of a given mass. To the extent that such effects occur, they of course should be also traced in some mean evolution in, e.g., the $\sigma$-$v_{\text{max}}$ relations.

We have also studied a large subset of simulations of different masses and gas fractions with a variety of prescriptions for feedback and winds from star formation. These results will be discussed in detail in Cox et al. (2007), but we briefly summarize their relevant conclusions. The inclusion of massive stellar winds (with high mass loading efficiencies $\eta_w \gtrsim 1$, where $M_{\text{wind}} = \eta_w M_*$) can affect the structure of merger remnants, although not necessarily in a monotonic or easily predictable fashion. For example, strong winds can remove gas from the central regions of the galaxy and yield a lower effective gas fraction $f_{\text{gas}}$, but they can also cycle such gas at earlier stages, preventing it from immediately being turned into stars before the final merger and actually raising the effective $f_{\text{gas}}$ at the final coalescence. In any case, the effects are usually small (especially in the most massive galaxies of interest here) or comparable to those owing to the choice of orbital parameters.

More importantly, regardless of the stellar feedback efficiency, star formation alone cannot prevent gas from accreting onto the central BH (especially given that only $\sim 0.1\%$ of the galaxy mass in gas needs to reach the BH to affect the $M_{\text{BH}}$-host correlations), and in fact strong stellar winds can (either by cycling gas as above or by shocking and increasing the supply of low angular momentum material) greatly increase the fuel supply for accretion at the galactic center. The correlation between BH mass and host properties is therefore, regardless of the prescription for stellar feedback, still set by the local self-regulation of the BH and obeys (in all our simulations) an identical BHFP relation. Stellar winds can, in principle, influence the final BH mass, but only indirectly by affecting the structure of the remnant, in the same manner as changing progenitor structural parameters, orbital parameters, and gas fractions.

Figure 12 considers the trend in $M_{\text{BH}}/M_*$ as a function of the initial gas fraction of our simulations. In contrast to trends with $V_{\text{vir}}$, orbital parameters, or the evolution of disk structural parameters with redshift, the dependence of $M_{\text{BH}}/M_*$ on $f_{\text{gas}}$ is quite strong, varying by nearly an order of magnitude from low ($f_{\text{gas}} \lesssim 0.2$) to high ($f_{\text{gas}} \gtrsim 0.8$) initial gas fractions. We consider this in detail by examining a small case study set of simulations. We construct a fiducial set of simulations of Milky Way-like initial disks ($V_{\text{vir}} = 160$ km s$^{-1}$) and collide them in otherwise identical mergers except for varying the initial disk gas fractions with values $f_{\text{gas}} = 0.05, 0.2, 0.4, 0.6, 0.8, 1.0$. We construct three such suites, each with a different orbit (roughly bracketing the extremes

![Figure 12](image-url). Trend in $M_{\text{BH}}/M_*$ as a function of simulation initial disk gas fraction. Top: $M_{\text{BH}}/M_*$ for three suites of simulations (solid, dashed, and dot-dashed lines). Each of the three is a set of otherwise identical mergers of Milky Way–like systems, varying only the gas fraction with values $f_{\text{gas}} = 0.05, 0.2, 0.4, 0.6, 0.8, 1.0$. The three suites consider three different orbital configurations for the merger. Bottom: Consideration of these residuals and those from our other simulations vs. $f_{\text{gas}}$ as in Fig. 11. The black solid line is a loglinear fit (dashed lines show ±1σ), while the gray solid line is the more accurate fit log ($M_{\text{BH}}/M_*) = -3.27 \pm 0.36$ erf $([f_{\text{gas}} - 0.4]/0.28)$. High-$f_{\text{gas}}$ mergers produce much larger BHs than low-$f_{\text{gas}}$ systems. [See the electronic edition of the Journal for a color version of this figure.]
of possible merger configurations). Figure 12 also shows the trend of $M_{\text{BH}}/M_*$ in these simulations: it is clear that, all else being equal, larger values of $f_{\text{gas}}$ drive the systems to larger $M_{\text{BH}}/M_*$. However, the trend here does not resemble the simple $M_{\text{BH}} \propto f_{\text{gas}}$ scaling that we naively predicted in equation (10) by demanding that the BH be able to unbind the entire initial gas content of the galaxy. In fact, Figure 13 shows the correlation between $M_{\text{BH}}$ and $M_*\sigma^2$ for simulations of different gas fractions, and there is no systematic trend with $f_{\text{gas}}$. Likewise, the remnants lie on the BHFP regardless of their gas fractions; i.e., the change in $M_{\text{BH}}/M_*$ can be entirely accounted for by the change in $\sigma$ at fixed $M_*$.

We can understand this behavior with a simple toy model. Assume that the stars formed in the disk(s) before and during the merger (i.e., those that will act dissipationlessly) are scattered into a typical bulge with a Hernquist (1990) profile with scale length $R_e(\sigma_{\text{gas}} = 0)$, independent of the central gas content, and that a fraction $\mu$ of the initial gas mass ($M_{\text{gas}} = f_{\text{gas}}M_{\text{gal}}$) survives to the late stages of the merger and, via dissipation, falls to the center of the galactic potential. There, it will form a highly concentrated central stellar component [scale length $\ll R_e(\sigma_{\text{gas}} = 0)$; for simplicity we take it to be effectively a point concentration]. The total enclosed mass as a function of radius is then

$$M_{\text{gal}}(< r) = M_{\text{gal}}\left\{\mu f_{\text{gas}} + \frac{(1 - \mu f_{\text{gas}})r^2}{[r + R_e(\sigma_{\text{gas}} = 0)]^2}\right\}.$$ (15)

This yields a half-mass effective radius $R_e$ of

$$\frac{R_e}{R_e(\sigma_{\text{gas}} = 0)} = \frac{x}{(\sqrt{2} + 1)(1 - x)},$$

$$x = \left(\frac{1/2 - \mu f_{\text{gas}}}{1 - \mu f_{\text{gas}}}\right)^{1/2}.$$ (16)

Figure 14 compares this simple expectation for $R_e$ and $\sigma^2 \propto M_*/R_e$ with that from the simulations as a function of $f_{\text{gas}}$: for a representative $\mu = 0.5$, our toy model describes the simulations quite well (until $f_{\text{gas}} \rightarrow 1$, where our assumption that the inner stellar component is infinitely concentrated breaks down; this “saturation” in $\sigma$ reflects the fact that even extremely gas-rich systems will still turn much of that gas into stars that scatter into some large orbits, and may be important for, e.g., the steep observed cutoff in the observed $\sigma$ distribution). Note that this scaling is not very sensitive to our assumption about the exact profile shape: assuming that the bulge follows an exact $r^{1/4}$ law or adopting a different inner power-law slope changes the predicted scaling by only $\sim 10\%$--$20\%$. Given this change in $R_e$ at fixed $M_*$, owing to increasing $f_{\text{gas}}$, the FP implies that $M_{\text{BH}}$ should increase, roughly as $\sim R_e^{-1}$. Figure 15 plots the dependence of $M_{\text{BH}}/M_*$ on $f_{\text{gas}}$, compared with the expectation from this simple model. From the agreement here, and the fact that Figure 14 finds no change in
Fig. 14.—Structural properties of the merger remnants from our set of gas fraction case studies from Fig. 12. Changing $f_{\text{gas}}$ has almost no effect on $M_*/C_3$, but by increasing the amount of dissipation and fraction of stellar material formed in the final, central starburst, increasing $f_{\text{gas}}$ produces more concentrated remnants with smaller effective radii $R_e$ and larger central velocity dispersions $\sigma$. Gray lines show the expectation of a toy model in which a fraction $\mu$ of the gas participates in a central starburst (eq. [16]). [See the electronic edition of the Journal for a color version of this figure.]

Fig. 15.—Dependence of $M_{\text{ BH}}/M_*$ on $f_{\text{gas}}$, as in Fig. 12. Panels plot this as a function of $f_{\text{gas}}$ measured at the initial time in each simulation (left) and at times $\Delta t = 1.0, 0.5,$ and $0.2$ Gyr before the final merger and quasar phase. Black lines in each plot the best-fit trend (solid lines; with $\pm 1 \sigma$ range in dashed lines), and gray lines plot the expectation from our simple model (eq. [16]), assuming that the BHFP is always preserved and that a fraction $\mu$ of the gas mass measured at each time will participate in the final, central starburst ($\mu$ is smaller at early times because much of this gas will form stars in the two disks well before they merge). Regardless of when $f_{\text{gas}}$ is defined, the trend is similar, and evolution in $M_{\text{ BH}}/M_*$ is driven by preserving the BHFP and increasing $\sigma$ at fixed $M_*$ in the same manner. [See the electronic edition of the Journal for a color version of this figure.]
the BHFP with \( f_{\text{gas}} \), we conclude that more gas-rich mergers drive evolution in \( M_{\text{BH}} \) by producing more concentrated remnants with smaller \( R_e \) and larger \( \sigma \) at fixed \( M_* \), and therefore larger \( M_*, \sigma^2 \).

There is one important caveat to our discussion of disk gas fractions \( f_{\text{gas}} \). Lacking a full cosmological simulation in which to determine how gas continuously accretes onto the disks, we have simply referred to \( f_{\text{gas}} \) as the initial gas fraction in our simulations. Of course, during a simulation, \( f_{\text{gas}} \) will decrease as gas is turned into stars, so that the actual gas fractions by the time the systems merge may be substantially lower than the numbers we quote. In Figure 15 we reproduce our plot from Figure 12 of the residuals in \( M_{\text{BH}}/M_* \), as a function of the initial gas fraction of the simulations. However, we also return to the simulations and measure the gas fraction for each at a set of uniform times \( \Delta t = 1.0, 0.5, \) and 0.2 Gyr before the final merger (defined for convenience as the coalescence of the two BHs). As the simulations approach the final merger event, it is clear that the trend of residuals with \( f_{\text{gas}} \) is qualitatively unchanged; however, the absolute values of \( f_{\text{gas}} \) systematically decrease. By \( \Delta t = 0.2 \) Gyr, the gas fractions are systematically lower by a factor of \( \sim 2–3 \). Therefore, the exact values of \( f_{\text{gas}} \) that we quote should not be taken too literally: if gas is accreted in the real universe such that \( f_{\text{gas}} \) changes less rapidly in earlier stages of a merger, then the initial gas fraction need only be as large as \( f_{\text{gas}} \sim 0.3 \) to be equivalent to our most extreme \( f_{\text{gas}} = 1 \) cases (with our more typical \( f_{\text{gas}} = 0.4–0.5 \) cases corresponding to rather moderate premerger or \( \Delta t = 0.2 \) Gyr gas fractions of \( \sim 0.2 \)).

8. IMPLICATIONS FOR THE REDSHIFT EVOLUTION IN BH-HOST RELATIONS

8.1. Empirical Predictions

Given that the BHFP appears to be robust against all varied quantities in our simulations, we expect that it should be preserved at all redshifts. However, this implies that, at fixed \( M_* \), evolution with redshift in the typical velocity dispersions and/or effective radii of spheroids will also manifest as evolution in the typical \( M_{\text{BH}}/M_* \) relation.

It is observed empirically (Trujillo et al. 2006) and expected theoretically (Kochfahr & Silk 2006) that high-redshift spheroids will be more compact at a given stellar mass \( M_* \) than their low-redshift analogs. Specifically, Trujillo et al. (2006) compile a number of measurements of the evolution, relative to \( z = 0, \) of the effective radii of spheroids (defined as systems with Sérsic indices \( n_s > 2.5 \)) at fixed stellar mass \( M_* > 3 \times 10^{10} \) and \( > 6 \times 10^{10} \) \( M_\odot \), corresponding to typical \( L_\odot \) galaxies at most redshifts. If the BHFP is preserved, this necessarily implies evolution in the \( M_{\text{BH}}/M_* \) relation, of the form \( M_{\text{BH}}/M_* \propto R_e(M_*)^{-1} \) (see Table 1 for exact values).

Figure 16 plots this expected evolution in \( M_{\text{BH}}/M_* \) from the Trujillo et al. (2006) measurements, normalized to the value observed at the same stellar mass by Häring & Rix (2004) at \( z \approx 0. \) For comparison, we plot the estimated evolution in \( M_{\text{BH}}/M_* \) from Peng et al. (2006) (specifically, we adopt their early-type template to convert their measured luminosities to stellar masses) and limits on this evolution from Hopkins et al. (2006d) normalized to the same local value.

We can also attempt to empirically infer the evolution in \( M_{\text{BH}}/M_* \) by considering the clustering of quasars as a function of redshift. Essentially, this expands on the measurement in Adelberger & Steidel (2005). A more detailed discussion of this (and the samples we consider) is given in Hopkins et al. (2007e) and Fine et al. (2006), but we briefly review it here.

We consider a quasar sample (typically observed near \( \sim L_\odot \) in the quasar luminosity function) for which BH masses or typical Eddington ratios have been directly measured (for 2dF quasars, we directly adopt the observed BH mass distributions from Fine et al. [2006]; otherwise, we use those determined as a function of quasar luminosity and redshift in Kollmeier et al. [2006]). We then use the observed clustering properties of that sample to infer a characteristic host halo mass; in other words, we match the observed large-scale bias of the quasar population to the average (number density weighted) large-scale bias for halos in some mass range (at the same redshift). We calculate the expected large-scale bias of halos as a function of mass following Mo & White (1996) with the improved fitting formulae from Sheth et al. (2001). If we assume that the ratio of galaxy to halo mass for \( \sim L_\odot \) galaxies does not evolve much with redshift (which appears to be observationally confirmed to at least \( z \sim 1 \); see, e.g., Heymans et al. 2006; Conroy et al. 2007; Zheng et al. 2007), then we have (implicitly) obtained the average host galaxy mass of these quasars, for which we know \( M_{\text{BH}} \), i.e., an estimate of the mean \( M_{\text{BH}}/M_*(z) \). Of course, the bias as a function of halo mass will depend on the cosmology adopted (specifically the value of \( \sigma_8 \)); this and, e.g., the BH mass measurements from quasar spectral energy distributions all introduce fairly large systematic uncertainties (at least a factor of \( \sim 2 \)) in the absolute implied value of \( M_{\text{BH}}/M_* \). However, to the extent that we are interested only in the relative evolution of this with redshift, these uncertainties are much smaller.

Compiling a number of measurements of quasar clustering as a function of redshift, the inferred evolution in \( M_{\text{BH}}/M_* \) is shown in Figure 17. This provides a completely independent measurement of \( M_{\text{BH}}/M_* \) from that of Peng et al. (2006) with entirely different systematics, but nevertheless it is in reasonable agreement with their estimates and with our simple expectation from the
BHFP relation. Of course, the current observations are not sufficiently robust to distinguish between evolution by a factor of ~2 and evolution by a factor of ~3 at high redshifts, but different probes seem to suggest a roughly comparable effect to that predicted.

8.2. A Dissipation-driven Explanation

Given the BHFP, we have the empirical expectation that, at high redshift as spheroids become more concentrated, $M_{\text{BH}}$ must be larger at fixed $M_*$. However, this does not explain what physically drives these trends. In § 7, we showed that increasing the gas fractions of merger progenitors has both of these effects, namely, that by increasing the amount of dissipation, more centrally concentrated remnants with smaller $R_d$, higher $\sigma$, and larger $M_{\text{BH}}$ at fixed $M_*$ are produced. It is both expected and observed that high-redshift disks are characteristically more gas-rich, as star formation histories of local disks are fitted to $\text{SDSS}$. [See the electronic edition of the Journal for a color version of this figure.]

If we demand that the normalization of the Schmidt-Kennicutt law agree with the normalization from the fitted $\tau$ model, we then arrive at an equation for the implied evolution in $f_{\text{gas}}$,

$$f_{\text{gas}}^{1.4} = f_{\text{gas}}^{0.1} \left( \frac{1 - \exp[(t - t_0)/\tau]}{1 - \exp[(t - t'_0)/\tau]} \right) \exp \left[ \frac{(t - t'_0)/\tau}{(t - t_0)/\tau} \right],$$

where $f_0 = f_{\text{gas}}(z = 0)$. Adopting the measured best-fit $\tau$ and $z = 0$ gas fraction $f_0$ as a function of $z = 0$ stellar mass $M_*$ from Bell & de Jong (2000) and Kannappan (2004), respectively, we finally obtain an expected typical $f_{\text{gas}}$ and SFR as a function of disk stellar mass at any cosmic time $t$.

Figure 18 compares the expected $f_{\text{gas}}(M_*(t))$ from this parameterization with that observed at a number of different redshifts. We also compare this estimate to another, even simpler parameterization we could adopt: assuming an isolated disk obeying a $\tau$ model, with $f_{\text{gas}} = 1$ at $t = 0$ and $f_{\text{gas}} \approx 0.1$ (appropriate for a Milky Way–like $\sim L_*$ disk today) at $z = 0$. This implies an exponential growth of $f_{\text{gas}}$ with look-back time, with $\varepsilon$-folding time $\sim 6$ Gyr. The results are similar and appear to describe the observed $f_{\text{gas}}$ evolution reasonably well. We have also checked that the expectation from equation (18) is consistent with observed specific star formation rates as a function of $M_*$ from $z = 0$ to 3 (see, e.g., Bauer et al. 2005; Feulner et al. 2005; Papovich et al. 2006) and find good agreement even up to $z \sim 3$ (which should not be surprising, as this essentially just says that the $\tau$ model is indeed a reasonable description of the mean star formation history).

Given this evolution in $f_{\text{gas}}$, we are now in a position to estimate how this should change the effective radii and velocity dispersions of merger remnants, as well as (given the BHFP) the average value of $M_{\text{BH}}$ at fixed stellar mass $M_*$. Figure 19 shows the evolution of the $M_* - \sigma$ and $R_\sigma - M_*$ relations with $f_{\text{gas}}$, from our simulations, where the behavior is similar to that we discussed in § 7. Combining the trend in $R_\sigma(M_*)$ as a function of $f_{\text{gas}}$ from our
simulations and the trend in \( f_{\text{gas}}(z) \) above, we also then predict how \( R_e(z) \) should evolve, at fixed \( M_s \). Comparing this to the results from Trujillo et al. (2006) shows excellent agreement given our simple estimate of \( f_{\text{gas}}(z) \).

Likewise, either directly adopting our fitted trend of \( M_{\text{BH}}/M_s \) as a function of \( f_{\text{gas}} \) from Figure 12 or using our estimate of how \( R_e \) (and, correspondingly, \( \sigma \)) evolves at fixed \( M_s \) with \( f_{\text{gas}} \) and applying them to the BHFP relation \( M_{\text{BH}} \propto M^{0.7} \sigma^{1.5} \propto M^{1.5} R_e^{-1} \), this predicts that the mean \( M_{\text{BH}}/M_s \) should increase with redshift. Figures 16 and 17 show the evolution in \( M_{\text{BH}}/M_s \) expected from this simple derivation. Again, the agreement with the observationally estimated rate of evolution in \( M_{\text{BH}}/M_s \) is good.

We do caution that our estimate of the mean evolution in \( f_{\text{gas}} \), while consistent with the observational constraints on gas fractions and specific star formation rates at all redshifts with which we compare (\( z \leq 3 \)), is only intended as a rough lowest order approximation to realistic disk evolution. It is of course possible that other disk properties, such as the Tully-Fisher relation, begin to evolve at high redshift (see, e.g., Genzel et al. 2006), or that elliptical formation at high redshift might proceed in more chaotic multiple mergers for which our simple approximations are not valid, especially in the most massive systems at very high redshifts \( z \sim 6 \). However, we have seen in Figure 11 that evolution in properties such as virial velocities, disk structure, and orbital parameters does not drive much additional evolution in \( M_{\text{BH}}/M_s \). Furthermore, some reassurance comes from Li et al. (2007), who consider simulations that adopt cosmologically derived merger histories for the
mergers, and find that the remnant obeys a similar $M_{\text{BH}}/M_*$ relation to our idealized high-$f_{\text{gas}}$ simulations; i.e., it is consistent with our estimated evolution in $M_{\text{BH}}/M_*$ (and $R_e/M_*$) as a function of redshift. Despite a chaotic, rapid merger history, the remnant properties can approximately be predicted as a function of stellar mass and gas fraction from our simple prescription. The most important (at lowest order) element of our estimate is simply the qualitative statement that the progenitors of high-redshift ellipticals should be characteristically more gas-rich, which, given the rapid star formation timescales estimated for massive disks (Bell & de Jong 2000) and observationally inferred brief, potentially burst-dominated star formation histories of the most massive ellipticals (e.g., Thomas et al. 2005), is naturally expected in theories of hierarchical growth. In other words, so long as the total stellar mass being violently scattered and total gas supply reaching the spheroid center (i.e., total mass and gas fraction) are similar, the details of the merger history and distinction between binary or multiple mergers should not significantly change the simple gravitational physics that determine the most basic elements of the remnant structure. We therefore suggest that evolution in $f_{\text{gas}}$ is indeed the dominant (although not the only) physical agent driving evolution in the $M_{\text{BH}}/M_*$ relation.

### 9. DISCUSSION

Using a large set of numerical simulations of major galaxy-galaxy mergers, which include the effects of gas dissipation, cooling, star formation, and BH accretion, we find that a feedback-driven model of BH growth and self-regulation predicts the existence of a BHFP, of the form $M_{\text{BH}} \propto \sigma^3 R_{\text{eff}}^5$ or $M_{\text{BH}} \propto M_*^{0.5} \sigma^{1.5} R_{\text{eff}}^{3.0}$, analogous to the FP of spheroids. Comparing with existing BH mass measurements, the observed systems appear to follow a nearly identical BHFP relation. Specifically, there are significant (at >99.9% confidence) trends in the residuals of the $M_{\text{BH}}$ relation with $M_*$ and $R_e$ at fixed $M_*$, and likewise in the $M_{\text{BH}}-M_*$ relation (with $\sigma$ or $R_e$ fixed). While changes in halo circular velocity, merger orbital parameters, progenitor disk redshifts and gas fractions, ISM gas pressurization, and other parameters can drive changes in, e.g., $\sigma$ at fixed $M_*$, and therefore change in the $M_{\text{BH}}-\sigma$ or $M_{\text{BH}}-M_*$ relations, the BHFP is preserved.

This provides a new paradigm for understanding the traditional relations between BH mass and either bulge velocity dispersion or mass. These correlations (as well as those with other bulge properties such as effective radius, central potential, dynamical mass, concentration, Sérsic index, and bulge binding energy) are all projections of the same FP relation. Just as the Faber-Jackson relation between, e.g., stellar mass or luminosity and velocity dispersion ($M_*-\sigma$) is understood as a projection of the more fundamental relation between $M_*$, $\sigma$, and $R_e$, so too is the $M_{\text{BH}}-\sigma$ relation ($M_{\text{BH}} \propto \sigma^3$) a projection of the more fundamental relation $M_{\text{BH}} \propto \sigma^3 R_{\text{eff}}^3$. Recognizing this resolves the nature of several apparent outliers in the $M_{\text{BH}}-\sigma$ relation, which simply have unusual $\sigma$-values for their stellar masses or effective radii, and eliminates the strong correlations between residuals (in both observations and simulations). While the various changes above in merger properties can and do bias the various projections of the BHFP to different values, they simply move remnants along the BHFP relation.

Given the empirical tendency toward more compact, smaller $R_e$ spheroids at fixed stellar mass $M_*$ at high redshift, the BHFP predicts that BHs should be more massive at fixed $M_*$. Trujillo et al. (2006) compile a number of observations of the sizes of early-type galaxies at fixed stellar mass (for typical ~$L_*$ galaxies) and find a best-fit trend $R_e \propto (1+z)^{-0.45}$. The observed BHFP predicts that BH mass scales roughly as $M_*^{1.5} R_e^{-1.0}$, which yields the prediction that the typical hosted BH mass at fixed stellar mass (or ratio of $M_{\text{BH}}/M_*$) should increase as $(1+z)^{0.5}$. This agrees well with recent direct estimates of the BH-to-host stellar mass ratio at high redshift (Peng et al. 2006), as well as indirect estimates of the evolution in the mean $M_{\text{BH}}/M_*$ from comparisons of quasar luminosity functions and early-type mass density measurements (Merloni et al. 2004), BH and spheroid mass functions (Hopkins et al. 2006d), and quasar clustering as a function of redshift (Adelberger & Steidel 2005; Wyithe & Loeb 2005; Hopkins et al. 2007e; Fine et al. 2006). Interestingly, if we consider this in greater detail, observations suggest that the evolution in spheroid sizes is relatively weak to $z \sim 0.8$ (McIntosh et al. 2005) and stronger from $z \sim 1$ to 2. Our BHFP analysis argues that the same should be true for the ratio of BH mass to stellar mass, and indeed Peng et al. (2006) note that there is no significant evolution at lower redshifts $z \lesssim 1$ in their sample, compared to the substantial evolution they observe at $z \sim 1$–3.

We have also developed a physically motivated model for this evolution. Based on the empirical and theoretical expectation that the progenitor disks in typical mergers should be more gas-rich at higher redshifts, we expect mergers to be more dissipational, yielding more concentrated remnants and driving the evolution in $M_{\text{BH}}/M_*$ along the BHFP. Indeed, adopting an empirical estimate for the mean $f_{\text{gas}}$ as a function of stellar mass and redshift, we predict a trend with redshift in the size-mass relation of our merger remnants that is similar to the observations compiled in Trujillo et al. (2006) and consequently a trend in $M_{\text{BH}}/M_*$ like that observed by Peng et al. (2006). Our simulations thus provide critical support to arguments from a semianalytic context, such as those made by Khochfar & Silk (2006) that the observed evolution in $R_e(M_*)$ can be explained by the increasingly gas-rich, dissipational nature of merger progenitors at high redshifts. It is worth noting that, although it does not rule out such mergers occurring, the trend in $R_e(M_*)$ can be explained entirely by changing gas fractions in gas-rich, dissipational mergers, without invoking subsequent “dry” mergers at low redshifts to increase $R_e$ (see also E. Krause et al. 2008, in preparation).

We also emphasize that our results are entirely consistent with the previous study of Robertson et al. (2006c). However, in that case, the authors considered only the effects of the (relatively weak) scaling of disk sizes at fixed $M_*$ with redshift and found that this introduced a weak evolution in the $M_{\text{BH}}-\sigma$ relation. Allowing for $f_{\text{gas}}$ to scale systematically with redshift drives the evolution in $M_{\text{BH}}/M_*$ that is analyzed herein, and placing both simulations and observations in the context of the FP relation reconciles the apparent disagreement between the predictions of Robertson et al. (2006c) for the $M_{\text{BH}}-\sigma$ relation and observations by, e.g., Peng et al. (2006) of the high-redshift $M_{\text{BH}}-M_*$ relation.

There are a number of direct, testable predictions of this FP model for the correlations between BH and host properties. At both low and high redshifts, systems should lie on the same BHFP. Therefore, measurements of the effective radii or velocity dispersions of the Peng et al. (2006) objects should find that they are more compact (smaller $R_e$, larger $\sigma$) than their $z = 0$ counterparts of the same stellar mass, in a manner consistent with the BHFP in Table 1. If it is really the BHFP driving the apparent evolution in $M_{\text{BH}}/M_*$ with redshift, i.e., the fact that at fixed $M_*$, higher redshift systems are more compact, then this also predicts different evolution for BH mass relative to $\sigma$ (the $M_{\text{BH}}-\sigma$ relation) than for BH mass relative to $M_*$. Adopting the Trujillo et al. (2006) estimate for how $R_e$ scales with redshift and the near-IR spheroid FP of Pahre et al. (1998) to relate $\sigma$ and $R_e$ at fixed $M_*$, along with our BHFP in terms of $M_*$ and $\sigma$, this predicts a trend of the form $M_{\text{BH}}/\sigma^4 \propto (1+z)^{-0.25}$, i.e., weaker and inverse evolution...
in $M_{\text{BH}}/\sigma$ at fixed stellar mass, quite similar to the predictions made by Robertson et al. (2006c) and consistent with the observations of Shields et al. (2003).

At low redshifts, improved measurements of the host properties of systems with well-measured BHs can significantly improve constraints on the BHFP. As noted in Table 1, the present observations demand a correlation of the form $M_{\text{BH}} \propto \sigma^{2} M_{*}^{\alpha}$ over a simple correlation with either $\sigma$ or $M_{*}$ at $\gtrsim 3 \sigma$ confidence. Already, this puts strong constraints on theoretical models of BH growth and evolution: BH mass does not simply scale with the star formation (stellar mass) or virial velocity of the host galaxy. However, there is still a substantial degeneracy between the slopes $\alpha$ and $\beta$ (roughly along the axis $\beta \approx 1 - \alpha/4$). For example, the current data do not allow us to significantly distinguish a pure correlation with spheroid binding energy $M_{\text{BH}} \propto (M_{*} \sigma^{2})^{2/3}$ from the marginally favored relation $M_{\text{BH}} \propto M_{*}^{1/2} \sigma^{2}$; both suggest that the ability of BHs to self-regulate their growth must be sensitive to the potential well at the center of the galaxy (and therefore to galactic structure), but the difference could reveal variations in the means by which BH feedback couples to the gas on these scales.

Increasing the observed sample sizes and, in particular, extending the observed baselines in mass and $\sigma$ will substantially improve the lever arm on these correlations. In particular, the addition of stellar mass $M_{*}$ information to the significant number of objects that have measurements of $\sigma$ and indirect measurements of $M_{\text{BH}}$ from reverberation mapping would enable considerably stronger tests of our proposed BHFP relation. Furthermore, to the extent that the evolution in the $M_{\text{BH}}/M_{*}$ and $R_{e}(M_{*})$ relations is driven by the relatively gas-rich nature of the merger progenitors, the residuals in, e.g., $M_{\text{BH}}/M_{*}$ should also be correlated with other tracers of the amount of dissipation in the spheroid-forming merger. These include quantities such as $R_{e}$ and $\sigma$, of course, in the context we have discussed, but also, e.g., the central phase-space density, kinematic properties such as rotation and kinematically decoupled components (see Hernquist & Barnes 1991; Cox et al. 2006b), and potentially the presence of central cusps in the stellar light profiles of the remnants (Mihos & Hernquist 1994b). We do note the caveat from §7, however, that care should still be taken to consider only bulge properties and remove, e.g., rotationally supported contributions to the velocity dispersion.

Given the robust nature of the BHFP, we might also ask if there are processes that we might expect to drive systems away from the BHFP. For example, what are the effects of subsequent gas-poor (spheroid-spheroid or dry) mergers on the BHFP? Such mergers, by definition, conserve total stellar mass and BH mass (simply adding the $M_{*}$ and $M_{\text{BH}}$ of the two merged systems). However, simple energetic arguments imply that $\sigma$ is not dramatically changed (e.g., Hernquist et al. 1993; Nipoti et al. 2003; Boylan-Kolchin et al. 2006). If we assume that $\sigma$ is unchanged in a dry merger, then the BHFP relations in Table 1 imply a $\sim 0.08-0.12$ dex offset (in the sense of $M_{\text{BH}}$ being too large) from a major (equal mass) dry merger (the offset being $\sim 0.03-0.05$ dex for a more probable $1:3$ mass-ratio merger). This is also supported by a small subset of full numerical remeasurement simulations from Robertson et al. (2006b). Although this implies that the BHFP will not survive a large number ($\gtrsim 3$) of successive dry mergers, the observationally estimated rate of $\sim 0.5-1$ major dry mergers for a typical massive elliptical (Bell et al. 2006; van Dokkum 2005) implies that the realistic resulting deviations from the BHFP are small (smaller than the internal scatter in the relation itself). This may be important, however, for explaining how the compact, high-redshift spheroids observed and predicted herein increase in size to lie on the local $R_{e}/M_{*}$ relations (since each major dry merger will approximately double $R_{e}$), a possibility that will be investigated in detail in future work.

Therefore, as appears to be borne out by the local observations we consider, the BHFP appears to be a robust correlation, which provides an improved context in which to understand the nature and evolution of the numerous observed correlations between BH and host spheroid properties. In particular, the results described here provide new, important constraints for models of BH growth, feedback, and self-regulation and support the proposal developed by Hopkins et al. (2006a) that major mergers between gas-rich galaxies represent the principle mechanism for triggering intense starbursts (e.g., Barnes & Hernquist 1991; Mihos & Hernquist 1994a, 1996) that evolve into quasars (e.g., Sanders et al. 1988) and that eventually leave remnants satisfying the same structural correlations observed for elliptical galaxies (e.g., Robertson et al. 2006b).

We thank Chien Peng for illuminating discussion and comments. This work was supported in part by NSF grant AST 03-07690 and NASA ATP grants NAG5-12140, NAG5-13292, and NAG5-13381.
