An Erdős-Ko-Rado Theorem for unions of length 2 paths

Carl Feghali∗ Glenn Hurlbert †
Vikram Kamat ‡

Abstract

A family of sets is intersecting if any two sets in the family intersect. Given a graph $G$ and an integer $r \geq 1$, let $I^{(r)}(G)$ denote the family of independent sets of size $r$ of $G$. For a vertex $v$ of $G$, the family of independent sets of size $r$ that contain $v$ is called an $r$-star. Then $G$ is said to be $r$-EKR if no intersecting subfamily of $I^{(r)}(G)$ is bigger than the largest $r$-star. Let $n$ be a positive integer, and let $G$ consist of the disjoint union of $n$ paths each of length 2. We prove that if $1 \leq r \leq n/2$, then $G$ is $r$-EKR. This affirms a longstanding conjecture of Holroyd and Talbot for this class of graphs and can be seen as an analogue of a well-known theorem on signed sets, proved using different methods, by Deza and Frankl and by Bollobás and Leader.

Our main approach is a novel probabilistic extension of Katona’s elegant cycle method, which might be of independent interest.

1 Introduction

The set $\{i \in \mathbb{N} : m \leq i \leq n\}$ is denoted by $[m, n]$, $[1, n]$ is abbreviated to $[n]$, and $[0]$ is taken to be the empty set $\emptyset$. For a set $X$, the power set of $X$
(that is, \{A: A \subseteq X\}) is denoted by \(2^X\). The family of \(r\)-element subsets of \(X\) is denoted by \(\binom{X}{r}\). The family of \(r\)-element sets in a family \(\mathcal{F}\) is denoted by \(\mathcal{F}(r)\). If \(\mathcal{F} \subseteq 2^X\) and \(x \in X\), then the family \(\{A \in \mathcal{F}: x \in A\}\) is denoted by \(\mathcal{F}(x)\) and called a star of \(\mathcal{F}\) with centre \(x\). A family \(\mathcal{F}\) is intersecting if \(F, F' \in \mathcal{F}\) implies \(F \cap F' \neq \emptyset\).

How large can an intersecting family \(\mathcal{F} \subseteq \binom{[n]}{k}\) be? If \(2k > n\) then \(|\mathcal{F}| = \binom{n}{k}\) is obvious, while if \(2k \leq n\) the classical Erdős–Ko–Rado (EKR) Theorem [9] states that \(\mathcal{F}\) can be no larger than a star.

**EKR Theorem** (Erdős, Ko and Rado [9]). Let \(n, k \geq 0\) be integers, \(n \geq 2k\). Let \(\mathcal{F} \subseteq \binom{[n]}{k}\) be intersecting. Then

\[
|\mathcal{F}| \leq \binom{n-1}{k-1} = |\mathcal{F}(1)|. \tag{1}
\]

When \(n = 2k\), the proof of the EKR Theorem is easy: simply partition \(\binom{[2k]}{k}\) into complementary pairs. Since \(\mathcal{F}\) can contain at most one set from each pair, \(|\mathcal{F}| \leq \frac{1}{2} \binom{2k}{k} = \binom{2k-1}{k-1}\). To deal with the case \(n > 2k\) Erdős, Ko and Rado [9] introduced an important operation on families called shifting.

Let \(I_G\) denote the family of independent sets of \(G\). The size of a maximum independent set of \(G\) is denoted \(\alpha(G)\). Holroyd and Talbot [12] introduced the problem of determining whether \(I_G^{(r)}\) has the star property for a given graph \(G\) and an integer \(r \geq 1\). Following their terminology, graph \(G\) is said to be \(r\)-EKR if no intersecting family of \(I_G^{(r)}\) is bigger than the largest star of \(I_G^{(r)}\).

Although not phrased in the language of graphs, one of the earliest results in the area, proved using different methods by Deza and Frankl [7] and by Bollobás and Leader [3], was to show that if \(G\) is the vertex-disjoint union of \(n\) complete graphs each of size \(k \geq 2\), then \(G\) is \(r\)-EKR (\(1 \leq r \leq n\)). This result was extended in various ways [1, 2, 4, 8, 6]. One such extension that is directly relevant to us is given by Hilton and Spencer [10, 11], showing that if \(G\) is the vertex-disjoint union of powers of cycles or of a power of a path and powers of cycles, then \(G\) is \(r\)-EKR (\(1 \leq r \leq \alpha(G)\)), provided some condition on the clique number is satisfied (see [5] for short proofs with somewhat weaker bounds). The problem, however, of obtaining an EKR result for vertex-disjoint unions of (powers of) paths remained elusive. In this note, we make the first step towards this problem in the following theorem.
Theorem 1. Let $2r \leq n$, and let $G$ be the vertex-disjoint union of $n$ paths each of length 2. Then $G$ is $r$-EKR.

We remark that Theorem 1 verifies a conjecture of Holroyd and Talbot [13] for vertex-disjoint unions of length 2 paths.

Although (we believe) Theorem 1 is interesting in itself, our most important message is the technique used to establish Theorem 1. Namely, we shall use the cycle method, a technique first introduced by Katona [14] in his beautiful proof of the EKR Theorem; however, some difficulties which are not present in [14] must be dealt with.

To be slightly more precise, the main cause of difficulty is the lack of ‘symmetry’ of the independent sets in our graph; in particular, some independent sets may or may not contain both leaves of a path – a situation which does not, for example, occur when considering unions of complete graphs. To overcome this, we shall essentially ‘force’ symmetry by partitioning the intersecting family of independent sets into subfamilies of independent sets which are ‘sufficiently similar’ to one another. We then follow a probabilistic approach akin to Katona’s but essentially further condition on those subfamilies. Having considered each subfamily independently, we are able to exploit its symmetric properties, which, in turn, also allows us to extract enough information about the original intersecting family.

2 Preliminaries

Throughout the rest of the paper, let $n$ be a positive integer, and let $G$ be the vertex-disjoint union of $n$ paths $P_1, \ldots, P_n$ each of length 2. For $i \in [n]$, the vertex set and edge set of $P_i$ are, respectively, $\{x_i, y_i, z_i\}$ and $\{x_iy_i, y_iz_i\}$. Define $X = \{x_i : 1 \leq i \leq n\}$, $Y = \{y_i : 1 \leq i \leq n\}$, $Z = \{z_i : 1 \leq i \leq n\}$, and $L = X \cup Z$. For a vertex $\ell \in L$, we let $\zeta(\ell) \in Y$ denote the unique neighbour of $\ell$ in $G$. We say that $x_i$ and $z_i$ are siblings. We say that an independent set $A$ of $G$ is

- of type I if whenever $A \cap L$ contains an element, then it does not contain its sibling,
- of type II if whenever $A \cap L$ contains an element, then it contains its sibling, and
- of type III in all other cases.
To prove the theorem, we will use the powerful shifting technique, first introduced by Erdős, Ko and Rado [9]. A family \( \mathcal{F} \subseteq \mathcal{I}_G^{(r)} \) is said to be shifted if \( F \in \mathcal{F} \) and \( y_i \in F \cap Y \neq \emptyset \) implies \( (F \setminus \{y_i\}) \cup \{x_i\} \in \mathcal{F} \).

Let \( \phi : V(G) \to V(G) \) be the function given by

\[
\phi(y_i) = x_i \text{ for } i \in [n], \quad \phi(v) = v \text{ otherwise.}
\]

For \( A \subseteq V(G) \), let

\[
\phi_i(A) = \{ \phi_i(a) : a \in A \},
\]

and note that if \( A \) is independent then so is \( \phi(A) \).

Before we can concisely use the method in the next section, here we only make a few definitions and prove a lemma. We shall find it convenient to think of \( L \) as \([2n]\) by setting \( x_i = i \) and \( z_i = n + i \) for \( i \in [n] \). We use the permutation notation \( \sigma = (a_1, \ldots, a_{2n}) \) to mean that the elements of \( L \) are

Lemma 1. Let \( \mathcal{F} \subseteq \mathcal{I}_G^{(r)} \) be an intersecting family. Then \( |\Phi(\mathcal{F})| = |\mathcal{F}| \) and \( A \cap B \cap L \neq \emptyset \) for all \( A, B \in \Phi(\mathcal{F}) \).

Proof. The proof is completely standard, but we include the details for completeness. We first demonstrate that \( \Phi_1(\mathcal{F}) \) is intersecting for \( i \in [n] \).

Let \( A, B \in \Phi_1(\mathcal{F}) \). If \( A, B \in \mathcal{F} \), then \( A \cap B \neq \emptyset \) since \( \mathcal{F} \) is intersecting. If \( A, B \in \Phi_1(\mathcal{F}) \) \( - \mathcal{F} \), then by definition \( x_i \in A \cap B \). So we can assume that \( A \in \Phi_1(\mathcal{F}) \cap \mathcal{F} \) and \( B \in \Phi_1(\mathcal{F}) \cap \mathcal{F} \). Then \( B = (C - \{y_i\}) \cup \{x_i\} \) for some \( C \in \mathcal{F} \) and either \( y_i \notin A \) or \( D = (A - \{y_i\}) \cup \{x_i\} \) for some \( D \in \mathcal{F} \). If \( y_i \notin A \), then \( A \cap B = A \cap ((C - \{y_i\}) \cup \{x_i\}) \supseteq A \cap C \neq \emptyset \) since \( A, C \in \mathcal{F} \). Finally, if \( D \in \mathcal{F} \), then \( \emptyset \neq C \cap D = ((B - \{x_i\}) \cup \{y_i\}) \cap ((A - \{y_i\}) \cup \{x_i\}) = A \cap B \).

This shows that \( \Phi_i(\mathcal{F}) \) is intersecting for \( i \in [n] \).

By definition, \( |\Phi(\mathcal{F})| = |\mathcal{F}| \). Since each \( \Phi_1(\mathcal{F}) \) is intersecting, \( \Phi(\mathcal{F}) \) is intersecting. We are left to show that \( A \cap B \cap L \neq \emptyset \) for all \( A, B \in \Phi(\mathcal{F}) \). Suppose that \( A \cap B = \{y_{j_1}, \ldots, y_{j_t}\} \subseteq G \setminus L \) for some \( 1 \leq t \leq n \). By definition, \( A' = (A \setminus \{y_{j_1}, \ldots, y_{j_t}\}) \cup \{x_{j_1}, \ldots, x_{j_t}\} \) is a member of \( \Phi(\mathcal{F}) \). But then \( A' \cap B = \emptyset \), contradicting that \( \Phi(\mathcal{F}) \) is intersecting. \( \square \)

Another tool in our proof of the theorem is the cycle method of Katona. Before we can concisely use the method in the next section, here we only make a few definitions and prove a lemma. We shall find it convenient to think of \( L \) as \([2n]\) by setting \( x_i = i \) and \( z_i = n + i \) for \( i \in [n] \). We use the permutation notation \( \sigma = (a_1, \ldots, a_{2n}) \) to mean that the elements of \( L \) are
listed in the order $a_1, \ldots, a_{2n}$ around the circle. In this case we write $\sigma(i) = a_i$.
For $j \geq 1$, $M \subseteq L$ and a permutation $\sigma$ of $L$, let

$$\sigma^j = \{\sigma(c + j) : c \in M\}.$$ 

**Example 1.** Let $L = \{1, 2, 3, 4, 5, 6\}$, $\sigma = (3, 5, 6, 1, 2, 4)$ and $M = \{1, 4, 5\}$. Then one has, for example, $\sigma^1 = \{5, 2, 4\}$ and $\sigma^3 = \{1, 3, 5\}$.

Set $\sigma^0 = \sigma$ and $(1, \ldots, 2n)M = M$. Following Bollobás and Leader [3], call a permutation $\sigma$ of $L$ good if any $n$ elements $a_1, \ldots, a_n$ in $L$ appearing consecutively in $\sigma$ do not contain both $x_i$ and $z_i$ for each $i \in [n]$. (For instance, the permutation $(1, \ldots, 2n)$ is a good permutation of $L$.) Let $\mathcal{L}$ denote the set of all good permutations of $L$. Given a good permutation $\sigma = (a_1, \ldots, a_{2n})$ of $L$, we let $\sigma' = (\zeta(a_1), \ldots, \zeta(a_{2n}))$.

We now prove a lemma analogous to Katona’s lemma in his proof of the EKR Theorem.

For integers $s \geq 0$ and $t, u \geq 1$, define the intervals

$$S = [1, \ldots, s],$$
$$T_1 = [s + 1, \ldots, s + t],$$
$$U_1 = [s + 1 + n, \ldots, s + u + n],$$
$$T_2 = [s + u + n + 1, \ldots, s + u + n + t],$$
$$U_2 = [s + t + 1, \ldots, s + t + u].$$

Let $C_i(t, u) = T_i \cup U_i$ for $i \in \{1, 2\}$. We also occasionally write $(C_{3-i}(t, u))^*$ to denote $C_i(t, u)$ for $i \in \{1, 2\}$. For $\sigma \in \mathcal{L}$, define

$$\sigma C(t, u) = \{\sigma^j(t, u) : 1 \leq j \leq 2n\} \cup \{\sigma^j(t, u) : 1 \leq j \leq 2n\}.$$

When $\sigma$ is understood and fixed, we drop its use notationally.

**Example 2.** Consider the parameter values in Figure 1, and let

$$\sigma = (5, 8, 21, 6, 20, 1, 11, 14, 36, 22, 10, 34, 30, 27, 7, 15, 31, 17, 23, 26, 3, 24, 2, 19, 29, 32, 18, 4, 28, 16, 12, 9, 25, 33, 13, 35).$$
Figure 1: The sets $S$, $T_1$, $U_1$, $T_2$, and $U_2$, with $s = 2$, $t = 6$, and $u = 3$, where $n = 18$. $C_1(6, 3)$ is in blue and $C_2(6, 3)$ is in red.

Then $C(6, 3)$ consists of the following sets.

$$
C_1^1 = \{6, 20, 1, 11, 14, 36, 24, 2, 19\} \quad C_2^1 = \{29, 32, 4, 28, 16, 22, 10, 34\}
$$

$$
C_1^2 = \{20, 1, 11, 14, 36, 22, 2, 19, 29\} \quad C_2^2 = \{32, 18, 4, 28, 16, 12, 10, 34, 30\}
$$

$$
C_1^3 = \{1, 11, 14, 36, 22, 10, 19, 29, 32\} \quad C_2^3 = \{18, 4, 28, 16, 12, 9, 34, 30, 27\}
$$

| $C_1^{36}$ | $C_2^{36}$ |
|----------|----------|
| \{21, 6, 20, 1, 11, 14, 3, 24, 2\} | \{19, 29, 32, 18, 4, 28, 36, 22, 10\} |

**Lemma 2.** Let $n, t, u \geq 0$ be integers such that $t \geq u$. Let $\sigma$ be a good permutation of $L$. For any intersecting family $\mathcal{B} \subseteq C(t, u)$,

(i) $|\mathcal{B}| \leq t$ and $|C(t, u)| = 2n$ if $u = 0$ and $n \geq t$;

(ii) $|\mathcal{B}| \leq t$ and $|C(t, u)| = n$ if $t = u$ and $n \geq 2t$;

(iii) $|\mathcal{B}| \leq 2(t + u)$ and $|C(t, u)| = 4n$ if $t > u \geq 1$ and $n \geq 2(t + u)$. 

6
Proof. (i) Clearly, $C_{1}^{i}(t, 0) = C_{2}^{j+n}(t, 0)$ and hence $C(t, 0) = \{C_{1}^{i}(t, 0) : 1 \leq j \leq 2n\}$, which implies $|C(t, u)| = 2n$. Now assume without loss of generality that $C_{1}^{i}(t, 0) \in \mathcal{B}$. All the other sets $C_{1}^{i}(t, 0)$ that intersect $C_{1}^{i}(t, 0)$ can be partitioned into disjoint pairs $(C_{1}^{i}(t, 0), C_{1}^{i-t}(t, 0))$ for $i \in [2, t]$ since $n \geq t$. Since $\mathcal{B}$ is intersecting, it can contain at most one set from each pair. Hence $|\mathcal{B}| \leq t$.

(ii) Clearly, $C_{1}^{i}(t, t) = C_{2}^{j-i}(t, t) = C_{1}^{j+n}(t, t)$ and hence $C(t, t) = \{C_{1}^{i}(t, t) : 1 \leq j \leq n\}$, which implies $|C(t, u)| = n$. Since $n \geq 2t$, it follows as in (i) that $|\mathcal{B}| \leq t$.

(iii) Clearly, $C_{1}^{i}(t, u) \neq C_{2}^{j}(t, u)$ for any $1 \leq i, j \leq 2n$, implying $|C(t, u)| = 4n$. Now let $D^{i} = C_{1}^{i}(t, u) \cup C_{2}^{i}(t, u)$. Then

(2) $D^{i} = D^{j}$ if and only if $j = n + i$.

Define the family

$$\mathcal{D}' = \{D^{i} : C_{j}^{i}(t, u) \in \mathcal{B} \text{ for some } j \in \{1, 2\}\}.$$  

Then $\mathcal{D}'$ is intersecting since $\mathcal{B}$ is intersecting. Since $n \geq 2(t + u)$, it follows by (ii) that $|\mathcal{D}'| \leq t + u$. Now $\mathcal{B}$ can contain at most two of the sets in $\{C_{1}^{i}(t, u), C_{2}^{i}(t, u), C_{1}^{i}(t, u), C_{2}^{i}(t, u)\}$ since it is intersecting. By (2), $|\mathcal{B}| \leq 2|\mathcal{D}'| \leq 2(t + u)$ as required. \hfill \Box

3 The proof of Theorem 1

Let $\mathcal{F} \subseteq \mathcal{I}_{G}^{r}$ be an intersecting family. By Lemma 1, to prove the theorem we can assume that $\mathcal{F}$ is shifted.

For $0 \leq s \leq r$, define $\mathcal{I}_{G}^{(r)}(s) = \{A \in \mathcal{I}_{G}^{(r)} : |A \cap Y| = s\}$ and $\mathcal{F}_{s} = \{F \in \mathcal{F} : |F \cap Y| = s\}$. Since $\mathcal{F}$ is shifted, $\mathcal{F}_{r} = \emptyset$ and so $|\mathcal{F}| = \sum_{s=0}^{r-1} |\mathcal{F}_{s}|$. Let us now try to bound the size of each $\mathcal{F}_{s}$.

Define the families

$$\mathcal{A}_{s} = \bigcup_{t \in [\frac{s-s}{r}+1, \ldots, r-s-1]} \{C_{1}(t, r-s-t), C_{2}(t, r-s-t)\},$$

$$\mathcal{B}_{s}^{1} = \begin{cases} \{C_{1}(\frac{r-s}{2}, \frac{r-s}{2})\} & \text{if } r-s \text{ is even}, \\ \emptyset, & \text{otherwise}, \end{cases}$$

$$\mathcal{B}_{s}^{2} = \{C_{1}(r-s, 0)\}.$$
Each member of 

\[ D_s := A_s \cup B_s, \]

where \( B_s := B^1_s \cup B^2_s \), will be called *special*.

Now, choose an index \( i \in [2n] \) and a good permutation \( \sigma \) of \( L \) uniformly and independently at random and a special set \( C \) with probability

\[ h(C) := \frac{f(C)}{\sum_{D \in D_s} f(D)}, \]

where \( f(C) \) will be determined later on. We set

\[ I = \sigma \cdot S^i \cup \sigma \cdot C^n. \]

Clearly \( I \in \mathcal{I}_G^{(r)}(s) \). To ensure that \( I \) is uniformly chosen from \( \mathcal{I}_G^{(r)}(s) \), consider arbitrary \( K \in \mathcal{I}_G^{(r)}(s) \). What is the probability that \( I = K \)? It is the probability that \( \sigma \cdot C^i = K \cap L \) and \( \sigma \cdot S^i = K \cap Y \) and these probabilities are easily seen to be non-zero. Indeed, let \( k_1 = |K \cap L| = r - s \) and let \( k_2 \) be the number of pairs of siblings in \( K \cap L \). Then \( \sigma \cdot C^i = K \cap L \) only when \( k_2 \) is the number of pairs of siblings in \( C \): there is precisely one such \( C \) in the situation when \( k_1 = 2k_2 \) or \( k_2 = 0 \) and one such \( C \) together with its ‘complement’ \( C^* \) in all other cases. Hence conditioning on \( i \) the former probability is equal to

\[
\frac{(k_1 - k_2)!}{|L|} \cdot \frac{(n - k_1 + k_2)!}{|L|} \cdot h(C) \text{ if } K \text{ is of type I or II, and}
\]

\[
\frac{(k_1 - k_2)!}{|L|} \cdot \frac{(n - k_1 + k_2)!}{|L|} \cdot \left( h(C) + h(C^*) \right) \text{ if } K \text{ is of type III}
\]

while the latter probability is invariably equal to

\[
\frac{(n - s)!s!}{|L|}.
\]

Taking \( f(C) = \frac{1}{(k_1 - k_2)!(n - k_1 + k_2)!} \) if \( I \) is of type I or II and \( f(C) = f(C^*) = \frac{1}{2(k_1 - k_2)!(n - k_1 + k_2)!} \) if \( I \) is of type III, we have that \( I \) is uniformly chosen from \( \mathcal{I}_G^{(r)}(s) \). This means that

\[
(3) \quad \Pr[I \in \mathcal{F}_s] = \frac{|\mathcal{F}_s|}{|\mathcal{I}_G^{(r)}(s)|} = \frac{|\mathcal{F}_s|}{\binom{n}{s} \binom{2n - 2s}{r - s}}.
\]
We next turn to estimating \( \Pr[I \in F_s] \) in another way. For \( \sigma \in \mathcal{L} \) and special sets \( C_1, \ldots, C_t \), write

\[
\delta(I, \sigma, C_1, \ldots, C_t) := \Pr[I \in F_s \mid \sigma \land (C_1 \lor \cdots \lor C_t)] \cdot \Pr[\sigma \land (C_1 \lor \cdots \lor C_t)].
\]

Using Lemma 1 and either Lemma 2(i) or Lemma 2(ii), one has for each \( B \in \mathcal{B}_s \) and \( \sigma \in \mathcal{L} \)

\[
(4) \quad \Pr[I \in F_s \mid \sigma \land B] \leq \frac{r - s}{2n}.
\]

Similarly, using Lemma 1 in conjunction with Lemma 2(iii), one also has for each \( A \in \mathcal{A}_s \) and \( \sigma \in \mathcal{L} \)

\[
(5) \quad \Pr[I \in F_s \mid \sigma \land (A \lor A^*)] \leq \frac{2(r - s)}{4n} = \frac{r - s}{2n}.
\]

Define \( \mathcal{A}'_s = \{(A, A^*) : A, A^* \in \mathcal{A}_s\} \) and note that \(|\mathcal{A}_s| = 2|\mathcal{A}'_s|\). By (4), (5) and the law of total probability, we have that

\[
\Pr[I \in F_s] = \sum_{\sigma \in \mathcal{S}, B \in \mathcal{B}_s} \delta(I, \sigma, B) + \sum_{\sigma \in \mathcal{L}, (A, A^*) \in \mathcal{A}'_s} \delta(I, \sigma, A, A^*)
\]

\[
\leq \frac{r - s}{2n} \left( \sum_{\sigma \in \mathcal{S}, B \in \mathcal{B}_s} \Pr[\sigma \land B] + \sum_{\sigma \in \mathcal{L}, (A, A^*) \in \mathcal{A}'_s} \Pr[\sigma \land (A \lor A^*)] \right)
\]

\[
= \frac{r - s}{2n} \left( \sum_{\sigma \in \mathcal{S}, B \in \mathcal{B}_s} \Pr[\sigma \land B] + \sum_{\sigma \in \mathcal{L}, A \in \mathcal{A}_s} \Pr[\sigma \land A] \right)
\]

\[
= \frac{r - s}{2n} \sum_{\sigma \in \mathcal{L}, D \in \mathcal{D}_s} \Pr[\sigma \land D]
\]

\[
(6) \quad = \frac{r - s}{2n}.
\]

Combining (3) and (6) yields the result, as follows.
\[ |\mathcal{F}| = \sum_{s=0}^{r-1} |\mathcal{F}_s| \leq \sum_{s=0}^{r-1} \frac{r - s}{2n} \binom{n}{s} \binom{2n - 2s}{r - s} = \sum_{s=0}^{r-1} \binom{n - 1}{s} \binom{2n - 2s - 1}{r - s - 1} = |\mathcal{F}(x_1)|. \]

4 Closing remark

Our bound on \( r \) in Theorem 1 is probably far from optimal. In view of the EKR Theorem, we make the following conjecture.

**Conjecture 1.** Let \( r \leq n \), and let \( G \) be the vertex-disjoint union of \( n \) paths each of length 2. Then \( G \) is \( r \)-EKR.

Acknowledgements

Carl Feghali was supported by grant 19-21082S of the Czech Science Foundation.

References

[1] R. Ahlswede and L. H. Khachatrian. The diametric theorem in Hamming spaces-optimal anticodes. *Advances in Applied Mathematics*, 20(4):429–449, 1998.

[2] C. Bey. An intersection theorem for weighted sets. *Discrete Mathematics*, 235(1-3):145–150, 2001.

[3] B. Bollobás and I. Leader. An Erdős-Ko-Rado theorem for signed sets. *Computers & Mathematics with Applications*, 34(11):9–13, 1997.
[4] P. Borg. Intersecting systems of signed sets. the electronic journal of combinatorics, 14(1):41, 2007.

[5] P. Borg and C. Feghali. On the Hilton–Spencer intersection theorems for unions of cycles. arXiv, 1908.08825, 2019.

[6] P. Borg and I. Leader. Multiple cross-intersecting families of signed sets. Journal of Combinatorial Theory, Series A, 117(5):583–588, 2010.

[7] M. Deza and P. Frankl. Erdős-Ko-Rado theorem–22 years later. SIAM Journal on Algebraic Discrete Methods, 4(4):419–431, 1983.

[8] K. Engel. An Erdős-Ko-Rado theorem for the subcubes of a cube. Combinatorica, 4(2-3):133–140, 1984.

[9] P. Erdős, C. Ko, and R. Rado. Intersection theorems for systems of finite sets. The Quarterly Journal of Mathematics, 12(1):313–320, 1961.

[10] A. Hilton and C. Spencer. A graph-theoretical generalization of Berges analogue of the Erdős-Ko-Rado theorem. In Graph Theory in Paris, pages 225–242. Springer, 2006.

[11] A. J. W. Hilton and C. L. Spencer. A generalization of Talbot’s theorem about King Arthur and his knights of the round table. Journal of Combinatorial Theory, Series A, 116(5):1023 – 1033, 2009.

[12] F. Holroyd, C. Spencer, and J. Talbot. Compression and Erdős-Ko-Rado graphs. Discrete Mathematics, 293(1-3):155 – 164, 2005.

[13] F. Holroyd and J. Talbot. Graphs with the Erdős–Ko–Rado property. Discrete mathematics, 293(1-3):165–176, 2005.

[14] G. O. Katona. A simple proof of the Erdős-Chao Ko-Rado theorem. Journal of Combinatorial Theory, Series B, 13(2):183–184, 1972.