More D-brane bound states

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Abstract

The low-energy background field solutions corresponding to D-brane bound states which possess a difference in dimension of two are presented. These solutions are constructed using the T-duality map between the type IIA and IIB superstring theories. Since supersymmetry is preserved by T-duality, the bound state solutions retain the supersymmetric properties of the initial (single) D-brane states from which they are produced, \textit{i.e.}, they preserve one half of the supersymmetries.

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1 Introduction

The past two years have seen remarkable developments in our understanding of non-perturbative aspects of string theory\[1\]. In particular, all five consistent superstring theories can now be connected using various string dualities. This has been interpreted as evidence that these theories are in fact perturbative expansions about different points in the phase space of a more fundamental framework, christened M-theory. With the discovery of these string dualities has come the realization that extended objects beyond just strings play a crucial role in these theories. Of particular interest for the Type II (and I) superstrings are Dirichlet branes (D-branes) which carry charges of the Ramond-Ramond (RR) potentials\[2\].

D-branes have also proven to be a valuable tool from a calculational standpoint. For example, bound states of D-branes have recently been used to compute, for the first time, the entropy of black holes from a counting of the underlying microscopic degrees of freedom\[3\]. In this analysis, the bound states were required to be supersymmetric in order that the counting, which can only be done at weak coupling, is protected from loop corrections by BPS saturation as the coupling is increased to where the bound state forms a black hole. Thus supersymmetric D-brane bound states are of particular interest. Up to now attention has been focussed on examples where the difference in the dimension of the D-branes is a multiple of four. This preference arises because it is the well-known requirement for supersymmetry in a configuration of two separated D-branes\[4\].

This feature is also revealed by an examination of the static (long-range) potential between separated D-branes, where supersymmetry implies stability or a precise cancellation of the inter-brane forces. For example, consider a D0-brane separated a distance $r$ from a D$p$-brane, where we will allow $p = 0, 2, 4, \text{or } 6$. There are three contributions to the static potential: gravitational, dilatonic and vector:

\[
U_{\text{grav}} = -\frac{\kappa^2}{8A_{8-p}} \frac{m_0 m_p}{r^{7-p}} \\
U_{\text{dila}} = -\frac{1}{2(7-p)A_{8-p}} \frac{\alpha_0 \alpha_p}{r^{7-p}} \\
U_{\text{vect}} = +\frac{1}{(7-p)A_{8-p}} \frac{q_0 q_p \delta_{0,p}}{r^{7-p}}
\]

The Kronecker delta appears in the gauge field potential because only D0-branes carry electric charge under the RR vector. Using the relations relating the various charges – which may be determined by examining the explicit low-energy solutions (see, e.g., \[5\] and \[6\])

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1 The normalization of the mass and charge densities (i.e., $m_p$ and $q_p$) in these potentials will be discussed in section 3.1. The ‘charge’ density for dilaton is chosen such that the asymptotic field around a $p$-brane takes the form: $\phi \simeq \frac{1}{(r-p)A_{8-p}} \frac{\alpha_p}{r^p}$. In these formulae, $A_n$ is the area of a unit $n$-sphere.
below) – i.e., $q_0 = \sqrt{2} \kappa m_0$ and $\alpha_p = \frac{3 - p}{2} \kappa m_p$, we may sum these potentials to find

$$U_{\text{total}} = -\frac{\kappa^2}{2(7 - p) A_{8-p}} \frac{m_0 m_p}{r^{7-p}} (4 - p - 4\delta_{0,p}) \, .$$

(2)

Hence we see that the three forces precisely balance for two D0-branes, resulting in a constant (vanishing) potential. Even in the absence of the gauge potential, however there is a similar cancellation for the D0- and D4-brane system. In this case, the two branes carry dilaton charges of opposite signs so that the dilatonic repulsion precisely balances the gravitational attraction. The vanishing potential or stability of these two configurations is a reflection of the supersymmetry which is preserved. In the former, 1/2 of the supersymmetries are preserved, while 1/4 are preserved in the latter.

If we consider the case of a D0-brane with a D2-brane, we see that total potential is attractive and so this configuration is unstable. Hence at the same time, it fails to preserve any supersymmetries. However, since the potential is attractive (i.e., $U_{\text{total}} < 0$), the D0-brane would presumably be drawn into the Dirichlet membrane and eventually the combined system would settle into a stable bound state configuration. While supersymmetry implies stability, the converse is not necessarily true. However we will be able to show by an explicit construction that in fact the stable ground state configuration is supersymmetric, preserving 1/2 of the supersymmetries. In general, our construction allows for the construction of supersymmetric bound states involving D-branes with dimensions differing by two.

An outline of the paper is as follows: We start by establishing our conventions in section 2 by presenting the low-energy actions for the Type II theories. As well, some low energy solutions representing individual D-branes are given. Section 3 begins by reminding the reader of some aspects of the stringy description of D-branes. We use this to motivate our construction in which we consider the T-dual of a ‘tilted’ D$p$-brane. The result is a supersymmetric bound state of a D($p+1$)-brane and a D($p-1$)-brane. We consider in detail the construction yielding the D2- and D0-brane bound state. In section 4, we provide solutions for bound states of D($p+1$)- and D($p-1$)-branes for $p = 2, 3, 4, 5$. The last section provides a brief discussion of our results.

2 Some preliminaries

The bosonic part of the low-energy action for type IIA string theory in ten dimensions is (see e.g., [4])

$$I_{\text{IIA}} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-G} \left\{ e^{-2\phi_a} \left[ R + 4(\nabla\phi_a)^2 - \frac{1}{12} (H^{(a)})^2 \right] - \frac{1}{4} (F^{(2)})^2 \right. $$

$$\left. - \frac{1}{48} (F^{(4)})^2 \right\} - \frac{1}{4\kappa^2} \int B^{(a)} dA^{(3)} dA^{(3)}$$

(3)

This mechanism was also observed for the multicenter solutions constructed in ref. [4].
where $G_{\mu\nu}$ is the string-frame metric, $H^{(a)} = dB^{(a)}$ is the field strength of the Kalb-Ramond field, $F^{(2)} = dA^{(1)}$ and $F^{(4)} = dA^{(3)} - H^{(a)}A^{(1)}$ are the Ramond-Ramond field strengths, and finally $\phi_a$ is the dilaton. Assuming the dilaton vanishes asymptotically, Newton’s constant is given by $\kappa^2 = 8\pi G_N$. In the type IIB case, we write the action as

$$I_{IIB} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-J} \left\{ e^{-2\phi_b} \left[ R + 4(\nabla \phi_b)^2 - \frac{1}{12}(H^{(b)})^2 - \frac{1}{2}(\partial \chi)^2 - \frac{1}{12}(F^{(3)} + \chi H^{(b)})^2 - \frac{1}{480}(F^{(5)})^2 \right] + \frac{1}{4\kappa^2} \int A^{(4)} F^{(3)} H^{(b)} \right\}$$

(4)

where $J_{\mu\nu}$ is the string-frame metric, $H^{(b)} = dB^{(b)}$ is the field strength of the Kalb-Ramond field, $F^{(3)} = dA^{(2)}$ and $F^{(5)} = dA^{(4)} - \frac{1}{2}(B^{(b)} F^{(3)} - A^{(2)} H^{(b)})$ are RR field strengths, while $\chi = A^{(0)}$ is the RR scalar, and $\phi_b$ is the dilaton. We are following the convention that the self duality constraint $F^{(5)} = *F^{(5)}$ is imposed by hand at the level of the equations of motion. All of the solutions in the following will be presented in terms of the string-frame metric, however, conversion to the Einstein-frame metric would be accomplished using:

$$g_{\mu\nu} = e^{-\phi_a/2}G_{\mu\nu}, \quad j_{\mu\nu} = e^{-\phi_b/2}J_{\mu\nu}.$$  

(5)

Note that $A^{(n)}$ and $F^{(n)}$ will always denote Ramond-Ramond potentials and field strengths, while $B$ and $H$ are reserved for the Neveu-Schwarz two-form and its field strength.

The low energy background field solutions\cite{3, 8} describing a single $Dp$-brane contain only a nontrivial metric, dilaton and a single RR potential, $A^{(p+1)}$:

$$ds^2 = \sqrt{\mathcal{H}(\vec{x})} \left( -dt^2 + \frac{dy^2}{\mathcal{H}(\vec{x})} + dx^2 \right)$$

$$A^{(p+1)} = \pm \left( \frac{1}{\mathcal{H}(\vec{x})} - 1 \right) dt \wedge dy^1 \wedge \cdots \wedge dy^p$$

$$e^{2\phi} = \mathcal{H}(\vec{x})^{3-p}.\quad (6)$$

Here, the $p$ spatial coordinates $y^a$ run parallel to the worldvolume of the brane, while the orthogonal subspace is covered by the $9-p$ coordinates $x^i$. Thus the solution is completely specified by a single function which may be written as

$$\mathcal{H} = 1 + \frac{\mu}{r^{p-1}} \left( \frac{\ell}{r} \right)^{7-p}.\quad (7)$$

for $p = 0, 1, \ldots, 6$.\footnote{This solution is also valid for $p = 8$, while $\mathcal{H} = 1 - \mu \log(r/\ell)$ for $p = 7$. These solutions can also be extended to the D-instanton with $p = -1$, for which the metric becomes euclidean without $t$ or $y^9$.} Here, $\mu$ is some dimensionless constant, $\ell$ is an arbitrary length scale and $r^2 = \sum_{i=1}^{9-p} (x^i)^2$. The RR field strength for this configuration is

$$F^{(p+2)} = \mp \mathcal{H}^{-2} \partial_j \mathcal{H} \ dx^j \wedge dt \wedge dy^1 \wedge \cdots \wedge dy^p.\quad (8)$$
For $p > 3$, the D-branes are actually magnetically charged in terms of the RR fields appearing in the above low energy actions, (3) and (4). In this case, eq. (8) describes the Hodge dual of the magnetic field

$$F^{(8-p)} = \pm \partial_j \mathcal{H} \ i_{\hat{x}^j} (dx^1 \wedge \cdots \wedge dx^{9-p})$$

where $i_{\hat{x}^j}$ denotes the interior product with a unit vector pointing in the $x^j$ direction. For $p = 3$, the five-form field strength should be self-dual. In this case, the correct solution may be constructed by replacing the electric five-form (8) by $(F^{(5)} + \ast F^{(5)})/2$ to produce

$$F^{(5)} = \pm \frac{\partial_j \mathcal{H}}{2} \left( \frac{1}{\mathcal{H}^2} dx^j \wedge dt \wedge dy^1 \wedge dy^2 \wedge dy^3 - i_{\hat{x}^j} (dx^1 \wedge \cdots \wedge dx^6) \right)$$

while the dilaton remains constant (i.e., $e^\phi = 1$) in accord with eq. (6).

### 3 Bound state of $p = 0, 2$ D-branes

At the world-sheet level, a $Dp$-brane is described by imposing a combination of Neumann and Dirichlet boundary conditions on the string world-sheet boundaries (see e.g., [4]). Neumann conditions are imposed on the coordinate fields associated with the $p + 1$ directions parallel to the D-brane’s world-volume, i.e., $\partial_{\text{normal}} X^\mu = 0$. The fields associated with the remaining $9 - p$ coordinates orthogonal to the D-brane satisfy Dirichlet boundary conditions, i.e., $X^\mu = \text{constant}$, which fixes the world-sheet boundaries to the brane.

These objects were originally discovered by considering the action of T-duality in the toroidal compactification of open superstring theories [11]. In this context, T-duality trades the standard Neumann condition for a Dirichlet-like boundary condition, i.e., $\partial_{\text{tangent}} X^\mu = 0$. Imposing the latter condition does not fix the zero-mode $X^\mu_0$, which is then still integrated over in the Polyakov path integral. Hence the Dirichlet-like boundary condition yields a D-brane which is not localized in a particular direction (as it must if T-duality is to leave the string amplitudes unchanged). This is in contrast to the original Dirichlet boundary condition which fixes the coordinate zero-mode and produces a D-brane with a specific position.

Hence if T-duality is implemented along one of the world-volume coordinates of a $Dp$-brane, one of the Neumann boundary conditions is replaced by a Dirichlet-like condition to produce a (delocalized) $D(p-1)$-brane [12]. Alternatively applying T-duality to a coordinate in the transverse space will replace a Dirichlet-like condition with a Neumann condition extending the $Dp$-brane to a $D(p+1)$-brane. For the present purposes, we wish to consider a $Dp$-brane which is oriented at an angle with respect to some orthogonal coordinate axes,

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4This is not quite a duality rotation because the kinetic term for $F^{(5)}$ in the IIB action (3) has the unconventional normalization $1/(4 \cdot 5!)$, which simplifies the T-duality transformation — rather than $1/(2 \cdot 5!)$ which is implicit in producing eq. (8).
e.g., tilted in the \((X^1, X^2)\)-plane. This would require imposing Neumann and Dirichlet-like boundary conditions on linear combinations of these coordinates

\[
\partial_n (X^1 + \tan \varphi X^2) = 0 \\
\partial_t (X^1 - \cot \varphi X^2) = 0
\]  \hspace{1cm} (11)

Now consider implementing the T-duality on \(X^2\) in this example. The interchange of the Neumann and Dirichlet-like conditions results in mixed boundary conditions which may be expressed as

\[
\partial_n X^1 + i \tan \varphi \partial_t X^2 = 0 \\
\partial_n X^2 - i \tan \varphi \partial_t X^1 = 0
\]  \hspace{1cm} (12)

Here the factor of \(i\) appears since we are considering a euclidean world-sheet. Now these mixed boundary conditions can be recognized as an example of the compatible boundary conditions arising when the Kalb-Ramond potential \(B_{\mu \nu}\) and/or the world-volume gauge field strength \(F_{\mu \nu}\) acquire a nonvanishing expectation value\(^{[13]}\), i.e.,

\[
\partial_n X^\mu - i F^\mu_\nu \partial_t X^\nu = 0
\]  \hspace{1cm} (13)

where \(F_{\mu \nu} = B_{\mu \nu} + 2\pi \alpha' F_{\mu \nu}\). In the present situation then, T-duality has induced \(F_{12} = - \tan \varphi\).

Now a nonvanishing \(F_{\mu \nu}\) will induce new couplings of the D-brane to the RR form potentials\(^{[14]}\). The full coupling of the RR fields to a \(Dp\)-brane is given by the following integral over the world-volume

\[
\int \text{Tr} \left[ e^{\mathcal{F}} \sum A^{(n)} \right]
\]  \hspace{1cm} (14)

Hence in the above example if we begin with a \(Dp\)-brane angled in the \((X^1, X^2)\)-plane, the result is a \(D(p+1)\)-brane with a nonvanishing flux \(F_{12}\). This final brane would then couple to both \(A^{(p+2)}\) and \(A^{(p)}\), and so should be regarded as a bound state of a \(D(p-1)\)-brane with a \(D(p+1)\)-brane.

While the above description is formulated at the level of the string world-sheet, we can easily lift the discussion to one of background fields. We begin by constructing the solution for a (delocalized) \(Dp\)-brane oriented at an angle in the \((X^1, X^2)\)-plane, and apply T-duality on \(X^2\) to find a solution describing the bound state of a \(D(p-1)\)-brane and a \(D(p+1)\)-brane. This will be our approach to building the background field solutions for these bound states. We illustrate the procedure in this section by considering in detail the construction of a bound state solution for \(p = 0\) and 2 branes.

We begin with the low energy Type IIB solution describing a D-string

\[
ds^2 = \sqrt{\mathcal{H}} \left( -dt^2 + \frac{dy^2}{\mathcal{H}} + dx^2 + \sum_{i=2}^{8} (dx^i)^2 \right) \\
A^{(2)} = \pm \left( \frac{1}{\mathcal{H}} - 1 \right) dt \wedge dy \\
e^{2\phi_b} = \mathcal{H}
\]  \hspace{1cm} (15)
where \( y \) is the coordinate parallel to the D-string, and we have singled out one of the transverse coordinates as \( x = x^1 \), for later convenience. Now \( \mathcal{H} \) is a harmonic function in the transverse coordinates, and normally, we would choose \( \mathcal{H} = 1 + \frac{\mu}{6}(\ell/r)^6 \) as in eq. (1).

For our present purposes, however, we need a slightly different harmonic function in that we want to delocalize the D-string in one of the transverse directions, \( i.e., x \), as would be appropriate for the Dirichlet-like boundary condition discussed above.

This can be done in at least two different ways. The harmonic function \( \mathcal{H} \) is a solution of (the flat-space) Poisson’s equation in the transverse coordinates, with some delta-function source. For example in eq. (7), the source is chosen so that \( \partial^i \partial_i \mathcal{H} = -\mu \ell^6 A_7 \prod_{i=1}^{8} \delta(x^i) \). The first way to accomplish a delocalization of the string is to follow the ‘vertical reduction’ approach\[15\]: One adds an infinite number of identical sources in a periodic array along the \( x \)-axis. Then a smeared solution may be extracted from the long range fields, for which the \( x \)-dependence is exponentially suppressed. An easier approach, which might be termed ‘vertical oxidation’, is to simply replace the above eight-dimensional \( \delta \)-function source by that of a line source extending along \( x \), \( i.e., \partial^i \partial_i \mathcal{H} = -\mu \ell^5 A_6 \prod_{i=2}^{8} \delta(x^i) \). This construction produces one of the anisotropic \( (p, q) \)-branes considered in ref. \[16\]. This approach also seems more in keeping with the delocalized description which arises in the string amplitudes, discussed above.

In any event, the number of dimensions transverse to our smeared-out D-string is effectively only 7, rather than 8, and the solution may be taken as in eq. (7) with \( p = 2 \):

\[
\mathcal{H} = 1 + \frac{\mu}{5} \left( \frac{\ell}{r} \right)^5
\]

where here \( r^2 = \sum_{i=2}^{8}(x^i)^2 \). Note that the form of the RR potential in eq. (13) tells us that we have a D-string oriented along \( y \) and smeared out in \( x \), rather than the other way around.

Now we perform a rotation on our delocalized D-string, in the \( x \)-\( y \) plane:

\[
\begin{pmatrix}
x \\
y
\end{pmatrix}
= \begin{pmatrix}
\cos \varphi & -\sin \varphi \\
\sin \varphi & \cos \varphi
\end{pmatrix}
\begin{pmatrix}
\tilde{x} \\
\tilde{y}
\end{pmatrix}
\]

where \( \varphi \) will be the angle between the \( \tilde{y} \)-axis and axis of the D-string, \( i.e., \) the \( y \)-axis. We then have,

\[
\begin{align*}
x & = \cos \varphi \, d\tilde{x} - \sin \varphi \, d\tilde{y} \\
y & = \cos \varphi \, d\tilde{y} + \sin \varphi \, d\tilde{x}
\end{align*}
\]

and after the rotation, the solution (13) becomes

\[
ds^2 = \sqrt{\mathcal{H}} \left\{ \frac{-dt^2}{\mathcal{H}} + \left( \frac{\cos^2 \varphi}{\mathcal{H}} + \sin^2 \varphi \right) d\tilde{y}^2 + \left( \frac{\sin^2 \varphi}{\mathcal{H}} + \cos^2 \varphi \right) d\tilde{x}^2 \\
+ 2 \cos \varphi \sin \varphi \left( \frac{1}{\mathcal{H}} - 1 \right) d\tilde{y} d\tilde{x} + \sum_{i=2}^{8}(dx^i)^2 \right\}
\]
\( A^{(2)} = \pm \left( \frac{1}{\mathcal{H}} - 1 \right) dt \wedge (\cos \varphi \, d\tilde{y} + \sin \varphi \, d\tilde{x}) \)
\( e^{2\varphi_a} = \mathcal{H}. \)  \hspace{1cm} (19)

Following the discussion at the beginning of this section, we apply T-duality in the \( \tilde{x} \) direction on our delocalized and rotated D-string. The resulting solution should then describe a bound state of a D-point (\( p = 0 \)) and a D-membrane (\( p = 2 \)). The ten-dimensional T-duality map between the type IIA and the type IIB string theories was given in ref. [7]. Using our notation and conventions, the map from the IIB to the IIA theory reads as

\[
\begin{align*}
G_{\tilde{x}\tilde{x}} &= \frac{1}{J_{\tilde{x}\tilde{x}}} \\
G_{\mu\nu} &= J_{\mu\nu} - J_{\tilde{x}\mu}J_{\tilde{x}\nu} - B_{\tilde{x}\mu}^{(b)}B_{\tilde{x}\nu}^{(b)} \\
& \hspace{1cm} \text{where the fields are as described in section 2. Here } \tilde{x} \text{ denotes the Killing coordinate with respect to which the T-dualization is applied, while } \mu, \nu, \rho \text{ denote any coordinates other than } \tilde{x}. \\
A_{\mu}^{(3)} &= A_{\mu}^{(2)} + 2 A_{[\mu}^{(2)} J_{\nu]\tilde{x}] / J_{\tilde{x}\tilde{x}} \\
A_{\mu\nu\rho}^{(3)} &= A_{\mu\nu\rho}^{(4)} + \frac{3}{2} \left( A_{\mu}[B_{\nu\rho]}^{(b)} - B_{\mu}[A_{\nu\rho}]^{(b)} - 4 B_{\mu}[A_{\nu\rho}]^{(2)} / J_{\tilde{x}\tilde{x}} \right) \\
\end{align*}
\]  \hspace{1cm} (20)

A straightforward application of the T-duality map (20) to the solution (19) yields

\[
\begin{align*}
ds^2 &= \sqrt{\mathcal{H}} \left\{ -\frac{dt^2}{\mathcal{H}} + \frac{d\tilde{x}^2 + d\tilde{y}^2}{1 + (\mathcal{H} - 1) \cos^2 \varphi} + \sum_{i=2}^{8} (dx^i)^2 \right\} \\
A^{(3)} &= \pm \frac{(\mathcal{H} - 1) \cos \varphi}{1 + (\mathcal{H} - 1) \cos^2 \varphi} \, dt \wedge d\tilde{x} \wedge d\tilde{y} \\
A^{(1)} &= \pm \frac{\mathcal{H} - 1}{\mathcal{H}} \sin \varphi \, dt \\
B^{(a)} &= \frac{(\mathcal{H} - 1) \cos \varphi \sin \varphi}{1 + (\mathcal{H} - 1) \cos^2 \varphi} \, d\tilde{x} \wedge d\tilde{y}. \\
e^{2\varphi_a} &= \frac{\mathcal{H}^2}{1 + (\mathcal{H} - 1) \cos^2 \varphi} \\
\end{align*}
\]  \hspace{1cm} (21)

Hence as expected this solution involves both \( A^{(3)} \) and \( A^{(1)} \) indicating the presence of a D2-brane and a D0-brane, respectively, in the \((\tilde{x}, \tilde{y})\)-plane. Since the bound state solution only
depends on \( r^2 = \sum_{i=2}^{8} (x^i)^2 \) as in eq. (10), the D0-brane is delocalized in world-volume of the D-membrane. Remarkably T-duality has produced \( G_{\tilde{y}\tilde{y}} = G_{\tilde{x}\tilde{x}} \) so that the bound state is spatially isotropic, even though it has lost the usual world-volume Lorentz invariance which characterizes the single D-brane solutions \( (3) \). Note that the off-diagonal term in the metric \( (15) \), which was produced by the rotation \( (17) \), has disappeared. Instead a Kalb-Ramond field has been generated, as is required by the Kalb-Ramond coupling appearing in \( F^{(4)} \) and by the presence of both \( A^{(3)} \) and \( A^{(1)} \) in this solution. One can verify that with \( \varphi = 0 \), the T-dual solution reduces to a D-membrane with \( A^{(1)} = 0 = B^{(a)} \), as expected. Similarly with \( \varphi = \pi/2 \), \( A^{(3)} \) and \( B^{(a)} \) vanish leaving a single D0-brane delocalized in the \((\tilde{x}, \tilde{y})\)-plane. We should also note that this solution \( (21) \) for a bound state of D0- and D2-branes appears in ref. \( [17] \).

### 3.1 Mass and Charge Relations

In this section, we consider some of the physical characteristics of the above bound state solution \( (21) \). The physical charge densities associated with the various RR fields are given by\([5]\)

\[
q^e = \frac{1}{\sqrt{2\kappa}} \oint F^{(n)} , \quad q^m = \frac{1}{\sqrt{2\kappa}} \oint F^{(n)}
\]

where the integrals are evaluated in the asymptotic region, and Hodge duality in the \( q_e \) formula is performed with respect to the string-frame metric. We arrange that in our solutions the form potentials vanish asymptotically so that the above formulae yield the correct results while ignoring the interactions between the different potentials. The D-particle and D-membrane carry charges for \( A^{(1)} \) and \( A^{(3)} \), respectively, which for the above solution yields

\[
q_0 = \pm \frac{(2\pi)^2 R_{\tilde{x}} R_{\tilde{y}}}{\sqrt{2\kappa}} \mu \ell^5 \sin \varphi A_6
\]

\[
q_2 = \pm \frac{1}{\sqrt{2\kappa}} \mu \ell^5 \cos \varphi A_6
\]

(23)

where in calculating \( q_0 \) we have set \( \tilde{x} \) \((\tilde{y})\) to have a range of \( 2\pi R_{\tilde{x}} \) \((2\pi R_{\tilde{y}})\). Here \( q_2 \) is a charge per unit area while \( q_0 \) is the total charge. The corresponding charge density associated with the delocalized D0-branes is then

\[
\tilde{q}_0 = \frac{q_0}{(2\pi)^2 R_{\tilde{x}} R_{\tilde{y}}} = \pm \frac{1}{\sqrt{2\kappa}} \mu \ell^5 \sin \varphi A_6 .
\]

(24)

For a \( p \)-brane, the ADM mass per unit \( p \)-volume is defined as\([13]\):

\[
m = \frac{1}{2\kappa^2} \oint \sum_{i=1}^{9-p} n^i \left[ \sum_{j=1}^{9-p} (\partial_j h_{ij} - \partial_i h_{jj}) - \sum_{a=1}^{p} \partial_i h_{aa} \right] r^{8-p} d\Omega
\]

(25)
where $n^i$ is a radial unit vector in the transverse space and $h_{\mu\nu}$ is deformation of the Einstein-frame metric

$$h_{\mu\nu} = g^{E}_{\mu\nu} - \eta_{\mu\nu}$$

from flat space in the asymptotic region. In eq. (25), the indices $i$ and $j$ denote the $9 - p$ transverse coordinates, while $a$ labels the $p$ spatial coordinates parallel to the world-volume. The ADM mass density of the bound state (21), which for the present purposes is effectively a membrane with $p = 2$, is then

$$m_{0,2} = \frac{1}{2\kappa^2} \mu_5 A_6 .$$

Therefore we have

$$(m_{0,2})^2 = \frac{1}{2\kappa^2} \left(q_0^2 + q_2^2\right).$$

This relation indicates that this bound state saturates the BPS bound for this system [4].

It is interesting to consider the ratio of the charge densities

$$\frac{q_0}{q_2} = -\tan \varphi.$$  

We also know that the source for $\tilde{q}_0$ is spread over the $(\tilde{x}, \tilde{y})$-plane, and so in the stringy discussion surrounding eq. (14), we would expect that the D-membrane carries a flux $F_{\tilde{y}\tilde{x}} = -\tan \varphi$. In fact, this flux precisely agrees with that arising in the preceding discussion given the identification: $X^1 = \tilde{y}$, $X^2 = \tilde{x}$. Further, we might consider the limit

$$\lim_{r \to 0} B_{\tilde{y}\tilde{x}} = -\tan \varphi.$$  

This suggests that the Kalb-Ramond field accounts for the total flux in $F$, and so the world-volume gauge field should vanish, i.e., $F_{\mu\nu} = 0$. Of course, $B_{\tilde{y}\tilde{x}}$ can be shifted by a constant via a gauge transformation, which at the same time would induce a nonvanishing $F_{\tilde{y}\tilde{x}}$. This has no physical consequences for the bound state solution, but it is amusing to show that in this case the T-dual solution is a rotated D-string in a background where the $\tilde{x}$ and $\tilde{y}$ axes are also tilted.

It is also interesting to see that the results for the charges are consistent with the appropriate charge quantization rules [2], namely

$$q_p = n_p \mu_p = n_p \frac{(2\pi)^{\frac{7-p}{2}}}{\sqrt{2\kappa}} (\alpha')^{\frac{1}{2}(3-p)}$$

where $\mu_p$ is the charge density of a fundamental $D_p$-brane and $n_p$ is an integer. If one begins with a D-string with $q_1 = n_1 \mu_1$, then the charges in the T-dual bound state satisfy $q_0 = n_0 \mu_0$ and $q_2 = n_2 \mu_2$ with $n_1 = -(n_0 + n_2)$. This requires taking into account that the range of $\tilde{x}$ in the original solution before T-duality solution is $R_{\tilde{x}} = \alpha'/R_{\tilde{x}}$, and similarly the gravitational couplings of the T-dual theories are related by $\kappa' = \kappa \sqrt{\alpha'/R_{\tilde{x}}}$. Further, one notes that the rotation angle is quantized as $\tan \varphi = \frac{m_{\tilde{x}} R_{\tilde{y}}}{n_{\tilde{x}} R_{\tilde{y}}}$.  

\footnote{The orientation for $F$ is in keeping with that used to calculate $q_0$.}
4 More bound state solutions

In the preceding section, we presented in detail the procedure for constructing the solution for a D0-brane bound to a D-membrane by beginning with a D-string. It is now a simple matter to construct other bound state solutions by simply changing the starting point of the construction. In general if we begin with a D\(_p\)-brane, the resulting solution describes a D\((p-1)\)-brane bound to a D\((p+1)\)-brane. In the following, we present the results for \(p = 2, 3, 4\) and 5. We also give a solution describing a bound state of a D4-brane, D0-brane, and two different D2-branes, which results from applying our procedure twice on a certain D-membrane solution.

In general, the resulting bound state solutions are anisotropic in that the full Lorentz invariance in the world-volume of the D\((p+1)\)-brane is lost. The invariance that remains is Euclidean invariance in the plane in which the D\((p-1)\)-brane is delocalized, i.e., \((\tilde{x}, \tilde{y})\)-plane in eq. (21), and Lorentz invariance in the remaining world-volume directions of the D\((p+1)\)-brane.

As \(p\) is varied in these examples, the relevant T-duality alternates between mapping IIB fields to IIA fields, and vice versa. The former transformation is given in eq. (20). Using our conventions, the T-duality map from type IIA theory to the type IIB theory is explicitly:

\[
\begin{align*}
J_{\tilde{x}\tilde{x}} &= \frac{1}{G_{\tilde{x}\tilde{x}}} \\
J_{\mu\nu} &= G_{\mu\nu} - \frac{G_{\tilde{x}\mu}G_{\tilde{x}\nu} - B_{[\tilde{x}\mu}B_{\tilde{x}\nu]}^a}{G_{\tilde{x}\tilde{x}}} \\
B_{\mu\nu}^{(b)} &= B_{\mu\nu}^{(a)} + 2\frac{G_{\tilde{x}[\mu}B_{\nu]\tilde{x}]}{G_{\tilde{x}\tilde{x}}} \\
A_{\mu\nu}^{(2)} &= A_{\mu\nu\tilde{x}}^{(3)} - 2A_{[\mu}^{(1)}B_{\nu]\tilde{x}]^a + 2\frac{G_{\tilde{x}[\mu}B_{\nu]\tilde{x}]^a}{G_{\tilde{x}\tilde{x}}}A_{\tilde{x}}^{(1)} \\
A_{\mu\nu\rho\tilde{x}}^{(4)} &= A_{\mu\nu\rho\tilde{x}}^{(3)} - 3\left( A_{[\mu}^{(1)}B_{\nu]\rho]\tilde{x}]^a + \frac{G_{\tilde{x}[\mu}B_{\nu]\rho]}{G_{\tilde{x}\tilde{x}}}A_{\tilde{x}}^{(1)} + \frac{G_{\tilde{x}[\mu}A_{\nu]\rho]\tilde{x}}{G_{\tilde{x}\tilde{x}}} \right) \\
\chi &= -A_{\tilde{x}}^{(1)}
\end{align*}
\]

The field definitions are again given in section 2, and \(\tilde{x}\) is the Killing coordinate which is T-dualized (while \(\mu, \nu, \rho \neq \tilde{x}\)). Note that in this map only the elements of the four-form RR potential involving \(\tilde{x}\) are given. The remaining components are determined by requiring that the corresponding five-form field strength is self-dual.

i) \(p = 3, 1\) branes:

Here our approach is to begin with the D-membrane solution carrying electric charge from \(A^{(3)}\). We single out \(x = x^1\) and delocalize the solution in this transverse direction.
Then we rotate by an angle $\phi$ as in eq. (17) where we set $y = y^1$. The resulting solution is

$$ds^2 = \sqrt{H} \left\{ \frac{-dt^2 + (dy^2)^2}{H} + \frac{(\cos^2 \varphi + \sin^2 \varphi)dy^2}{H} + \frac{(\sin^2 \varphi + \cos^2 \varphi)d\tilde{x}^2}{H} + 2 \cos \varphi \sin \varphi \left( \frac{1}{H} \right) d\tilde{y} d\tilde{x} + dr^2 + r^2 \left( d\theta^2 + \sin^2 \theta (d\phi_1^2 + \sin^2 \phi_1 (d\phi_2^2 + \sin^2 \phi_2 (d\phi_3^2 + \sin^2 \phi_3 d\phi_4^2))) \right) \right\}$$

$$A^{(3)} = \pm \left( \frac{1}{H} - 1 \right) dt \wedge (\cos \varphi \, d\tilde{y} + \sin \varphi \, d\tilde{x}) \wedge dy^2$$

$$e^{2\phi_{a}} = \sqrt{H}.$$  \hspace{1cm} (33)

where $H = 1 + \frac{4}{\ell} (\ell/r)^4$. We have also introduced polar coordinates on the effective transverse space (originally described by $x^i$ with $i = 2, \ldots, 7$). This facilitates writing the magnetic contribution to the four-form RR potential which appears after T-dualizing.

Now applying T-duality with respect to $\tilde{x}$ as in eq. (32), we obtain the following solution:

$$ds^2 = \sqrt{H} \left\{ \frac{-dt^2 + (dy^2)^2}{H} + \frac{dy^2 + d\tilde{x}^2}{1 + (H - 1) \cos^2 \varphi} \right\}$$

$$A^{(4)} = \pm \cos \varphi \, \frac{H - 1}{2} \left( \frac{\frac{H}{1 + (H - 1) \cos^2 \varphi}}{1 + (H - 1) \cos^2 \varphi} \right) dt \wedge d\tilde{y} \wedge dy^2 \wedge d\tilde{x}$$

$$A^{(2)} = \pm \frac{H - 1}{H} \sin \varphi \, dt \wedge dy^2$$

$$B^{(b)} = \frac{(H - 1) \cos \varphi \, \sin \varphi \, d\tilde{x} \wedge d\tilde{y}}{1 + (H - 1) \cos^2 \varphi}$$

$$e^{2\phi_{a}} = \frac{H}{1 + (H - 1) \cos^2 \varphi}.$$  \hspace{1cm} (34)

Note that the T-duality map (32) explicitly produced the electric component of the potential $A^{(4)}$, and the magnetic component was determined by demanding that $F^{(5)}$ be self-dual. As evidenced by the presence of the four-form and two-form RR potentials, we have a bound state of a D-three-brane and a D-string.

ii) $p = 4, 2$ branes

Once again we apply the same procedure of delocalization and rotation on a D3-brane, followed by T-duality. This case is slightly more complicated, as the D3-brane is charged by the self-dual five-form field strength. Thus one must use the linear combination of electric and magnetic fields given in eq. (34).
The rotated solution is
\[ ds^2 = \sqrt{\mathcal{H}} \left\{ \frac{-dt^2 + (dy^2)^2 + (dy^3)^2}{\mathcal{H}} + \left( \frac{\cos^2 \varphi + \sin^2 \varphi}{\mathcal{H}} + \frac{\sin^2 \varphi + \cos^2 \varphi}{\mathcal{H}} \right) d\bar{\gamma}^2 + \frac{2 \cos \varphi \sin \varphi (\frac{1}{\mathcal{H}} - 1)d\bar{\gamma}d\bar{x}}{\mathcal{H}} + d\bar{r}^2 + r^2 (d\theta^2 + \sin^2 \theta (d\phi_1^2 + \sin^2 \phi_1 (d\phi_2^2 + \sin^2 \phi_2 d\phi_3^2))) \right\} \]

\[ A^{(4)} = \pm \frac{1}{2} \left( \frac{1}{\mathcal{H}} - 1 \right) dt \wedge (\cos \varphi d\bar{y} + \sin \varphi d\bar{x}) \wedge dy^2 \wedge dy^3 \]

\[ e^{2\phi_b} = 1 \]

where \( \mathcal{H} = 1 + \frac{\ell}{3}(\ell/r)^3 \). Note also that the dilaton here is a constant which has been set equal to zero.

Applying the duality map (20) gives us the result:
\[ ds^2 = \sqrt{\mathcal{H}} \left\{ \frac{-dt^2 + (dy^2)^2 + (dy^3)^2}{\mathcal{H}} + \frac{d\bar{y}^2 + d\bar{x}^2}{1 + (\mathcal{H} - 1) \cos^2 \varphi} + d\bar{r}^2 + r^2 (d\theta^2 + \sin^2 \theta (d\phi_1^2 + \sin^2 \phi_1 (d\phi_2^2 + \sin^2 \phi_2 d\phi_3^2))) \right\} \]

\[ A^{(3)} = \mp \frac{1}{2} \frac{\mathcal{H} - 1}{\mathcal{H}} \sin \varphi dt \wedge dy^2 \wedge dy^3 \]

\[ B^{(a)} = \frac{(\mathcal{H} - 1) \cos \varphi \sin \varphi}{1 + (\mathcal{H} - 1) \cos^2 \varphi} d\bar{x} \wedge d\bar{y} \]

\[ e^{2\phi_a} = \frac{\sqrt{\mathcal{H}}}{1 + (\mathcal{H} - 1) \cos^2 \varphi} \]

Here the interpretation is that of a D-membrane, associated with the electric component of the three-form potential, \( A^{(3)}_{\theta \phi_1 \phi_3} \), in a bound state with a D4-brane carrying a magnetic field with \( A^{(3)}_{\theta \phi_1 \phi_3} \). This is consistent with the dyonic nature of the initial five-form self-dual field strength.

In ref. [19], the authors give a solution of a bound state of a D-membrane with a D4-brane. Their solution, obtained from compactification of \( D = 11 \) supergravity, agrees precisely with the solution eq. (36) given above.

iii) \( p = 5, 3 \) branes

Here the starting point is a D4-brane which would carry an electric six-form field strength according to eq. (3), so we must Hodge dualize to the magnetic four-form field.
strength (9). The magnetic potential is again most easily expressed using polar coordinates in the transverse space around the delocalized D4-brane. Applying our standard construction, the final solution, as the reader can easily verify, is

\[
\begin{align*}
\text{ds}^2 &= \sqrt{\mathcal{H}} \left\{ -dt^2 + \sum_{i=2}^{4} (dy^i)^2 + \frac{dy^2 + dx^2}{1 + (\mathcal{H} - 1) \cos^2 \varphi} + dr^2 + r^2 (d\theta^2 + \sin^2 \theta (d\phi_1^2 + \sin^2 \phi_1 d\phi_2^2)) \right\} \\
A^{(4)} &= \pm \mu \ell^2 \sin \varphi \left( 1 + \frac{\mathcal{H} - 1}{2} \cos^2 \varphi \right) \sin^2 \theta \cos \phi_1 d\tilde{y} \wedge d\tilde{x} \wedge d\theta \wedge d\phi_2 \\
A^{(2)} &= \pm \mu \ell^2 \cos \varphi \sin^2 \theta \cos \phi_1 \theta d\phi_2 \\
B^{(b)} &= \frac{(\mathcal{H} - 1) \cos \varphi \sin \varphi}{1 + (\mathcal{H} - 1) \cos^2 \varphi} d\tilde{x} \wedge d\tilde{y} \\
e^{2\phi_b} &= \frac{1}{1 + (\mathcal{H} - 1) \cos^2 \varphi}
\end{align*}
\]

(37)

where \( \mathcal{H} = 1 + \frac{\mu}{2} (\ell/r)^2 \). In this case the bound state is made up of dyonic D3-branes and magnetically charged D5-branes.

iv) \( p = 6, 4 \) branes

Beginning with a D5-brane, we dualize the associated electric seven-form field strength to a magnetic three-form field strength and compute the two-form magnetic potential in polar coordinates. After repeating the usual steps once again, the final result is

\[
\begin{align*}
\text{ds}^2 &= \sqrt{\mathcal{H}} \left\{ -dt^2 + \sum_{i=2}^{5} (dy^i)^2 + \frac{dy^2 + dx^2}{1 + (\mathcal{H} - 1) \cos^2 \varphi} + dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi_1^2) \right\} \\
A^{(3)} &= \pm \frac{\mu \ell \sin \varphi}{1 + (\mathcal{H} - 1) \cos^2 \varphi} \cos \theta d\tilde{y} \wedge d\tilde{x} \wedge d\phi_1 \\
A^{(1)} &= \mp \mu \ell \cos \varphi \cos \theta d\phi_1 \\
B^{(a)} &= \frac{(\mathcal{H} - 1) \cos \varphi \sin \varphi}{1 + (\mathcal{H} - 1) \cos^2 \varphi} d\tilde{x} \wedge d\tilde{y} \\
e^{2\phi_a} &= \frac{1}{\sqrt{\mathcal{H}} (1 + (\mathcal{H} - 1) \cos^2 \varphi)}
\end{align*}
\]

(38)

where \( \mathcal{H} = 1 + \mu \ell/r \). The bound state here contains a D4-brane and a D6-brane, which are both magnetically charged.
v) $p = 4, 2, 2, 0$ branes

It is a simple exercise to apply our procedure involving delocalization, rotation and T-duality with respect to more than just one of the transverse coordinates of the original D-brane solutions. The resulting solution describes a bound state involving more than just two types of D-branes. To illustrate this idea, we considered the following example: Beginning with the D-membrane solution (3), we singled out two orthogonal planes: $(x^1, y^1)$ and $(x^2, y^2)$. Applying the procedure in the $(x^1, y^1)$-plane – with a rotation angle $\varphi$ to $(\tilde{x}, \tilde{y})$ – produces a bound state of $p = 3$ and 1 D-branes, as in part (i) above. Repeating the procedure a second time in the $(x^2, y^2)$-plane – rotating by $\psi$ to $(\hat{x}, \hat{y})$ – yields the following solution

$$A^{(3)} = \pm \frac{(H - 1) \cos \varphi \sin \psi}{1 + (H - 1) \cos^2 \varphi} \, dt \wedge d\tilde{y} \wedge d\tilde{x} \pm \frac{(H - 1) \cos \psi \sin \varphi}{1 + (H - 1) \cos^2 \psi} \, dt \wedge d\hat{y} \wedge d\hat{x}$$

$$A^{(1)} = \pm \frac{H - 1}{H} \sin \varphi \sin \psi \, dt$$

$$B^{(a)} = \frac{(H - 1) \cos \varphi \sin \varphi}{1 + (1 - H) \cos^2 \varphi} \, d\tilde{x} \wedge d\tilde{y}$$

$$+ \frac{(H - 1) \cos \psi \sin \psi}{1 + (H - 1) \cos^2 \psi} \, d\hat{x} \wedge d\hat{y}$$

$$e^{2\phi_a} = \frac{H^2}{1 + (H - 1) \cos^2 \varphi} (1 + (H - 1) \cos^2 \psi)$$

where $H = 1 + \frac{\mu}{3}(\ell/r)^3$. The electric potential $A^{(1)}$ indicates the presence of D0-branes, while the magnetic component of $A^{(3)}$ arises from D4-branes. Meanwhile the two electric components of $A^{(3)}$ indicates that there are two kinds of D-membranes, one in the $(\tilde{x}, \tilde{y})$-plane and another in the $(\hat{x}, \hat{y})$-plane.

5 Discussion

Using T-duality, we have provided a straightforward construction of low-energy background field solutions corresponding to D-brane bound states for which the difference in dimension is two. We have also presented a number of explicit examples of such solutions. Since supersymmetry is preserved by T-duality, the bound state solutions retain the supersymmetric properties of the initial configuration which involves a single D-brane. Hence these bound states preserve one half of the supersymmetries. Our discussion of the background fields complements that of Polchinski, who recently gave a string world-sheet description of these
bound states. Indeed eq. (28) explicitly shows that the bound state of $p = 0, 2$ branes saturates the BPS bound given there. Similarly extending the calculations of section 3.1 to the other examples, we find

$$( m_{p-1,p+1} )^2 = \frac{1}{2\kappa^2} \left( \tilde{q}_p^2 + q_{p+1}^2 \right)$$

(40)

with $m_{p-1,p+1} = \frac{\mu^{6-p}}{2\kappa} A_{7-p}$. In close analogy to eq. (24), we defined the charge density of the D($p-1$)-brane as $\tilde{q}_{p-1} = ((2\pi)^2 R_3 R_y)^{-1} q_{p-1}$. For the dyonic D3-branes, the charge density that enters this formula can be written as the sum of the electric and magnetic contributions:

$$q_3 = \frac{1}{2}(q_e^3 + q_m^3).$$

(41)

Note, of course, that $q_e^3 = q_m^3$. In the last example with a bound state of four kinds of branes, this relation extends in the obvious way with a sum of squares of all of the charge densities.

While we have explicitly given all the bound state solutions with asymptotically flat Minkowski-signature geometries, one could also apply our procedure to constructing more exotic solutions involving instantons, strings, or domain walls — i.e., $Dp$-branes with $p = -1, 7$ and 8. For example, a euclidean $p = 0$ solution in the type IIA theory would correspond to an instantonic string. Applying our construction would lead to a ‘bound state’ solution with an instantonic membrane ($p = 1$) and a delocalized instanton ($p = -1$). One could also further explore the possibilities arising from multiple applications of our construction, as considered in example (v) of section 4. Another obvious extension would be to begin with multiple D-brane solutions. The harmonic function appearing in the original solutions was chosen to solve Poisson’s equation with a single delta-function source. It is straightforward to introduce more sources producing solutions which describe several separated parallel D-branes. Used as the starting point for the construction given here, these solutions would yield multiple bound states resting in static equilibrium — a possibility which arises due to their supersymmetric character.

It would also be of interest to examine in more detail the correspondance of our low energy background field solutions with the stringy description of these bound states. The charge and mass densities can in principle be extracted from a one-loop string amplitude describing the interaction of two D-branes (see e.g., [4]). This approach was in fact recently considered for the present D-brane bound states by Lifschytz [20]. Alternatively, by examining the scattering of closed strings from D-branes, one can also extract all of their long-range fields [21]. Applying this technique to the D-brane bound states, one again finds a precise agreement between these long-range fields and the corresponding low energy solutions [22].

Some work has been done on finding solutions corresponding to D-brane bound states for which the world-volume dimensions differ by four [23]. One might also look for solutions where the difference is six. Applying our method three times in orthogonal planes of a D3-brane solution produces a bound state with $p = 0$ and 6 branes, but also various branes
with \( p = 2 \) and 4. One might imagine that bound state of only D0- and D6-branes could be produced by inducing particular fluxes of non-abelian gauge fields in the world-volume of the D6-brane. As yet, we have been unable to find a ‘duality’ construction yielding such a bound state. A problem in our approach though is that we only considered beginning with configurations which were supersymmetric, a characteristic which would be preserved by the various duality transformations. However, Polchinski\(^4\) has recently shown that any such bound state can not saturate the BPS bound and so must not be supersymmetric. Further looking at the long-range potential \((2)\), we see that the total force between a D0-brane and a D6-brane is in fact repulsive. Hence, one is lead to conjecture that in fact such a bound state will not form.

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**References**

[1] see for example:
- M.J. Duff, “M-Theory (The Theory Formerly Known as Strings),” e-print [hep-th/9608117];
- J.H. Schwarz, “Lectures on Superstring and M-Theory Dualities,” e-print [hep-th/9607201];
- J. Polchinski, “String Duality: A Colloquium,” e-print [hep-th/9607050].

[2] J. Polchinski, *Phys. Rev. Lett.* **75** (1995) 4724 [hep-th/9510017].

[3] A. Strominger and C. Vafa, *Phys. Lett.* **B379** (1996) 99 [hep-th/9601029];
- G.T. Horowitz and A. Strominger, *Phys. Rev. Lett.* **77** (1996) 2368 [hep-th/9602051];
- G.T. Horowitz, J.M. Maldacena and A. Strominger, *Phys. Lett.* **B383** (1996) 151 [hep-th/9603109].

[4] J. Polchinski, “TASI Lectures on D-Branes,” e-print [hep-th/9611050];
- J. Polchinski, M. Chaudhuri and C.V. Johnson, “Notes on D-Branes,” e-print [hep-th/9602052].

[5] M.J. Duff, R.R. Khuri and J.X. Lu, *Phys. Rep.* **259** (1995) 213.

[6] J. Rahmfeld, *Phys. Lett.* **B372** (1996) 198 [hep-th/9512089];
- M.J. Duff and J. Rahmfeld, “Bound States of Black Holes and Other P-Branes,” e-Print [hep-th/9605085].

[7] E. Bergshoeff, C. Hull and T. Ortin, *Nucl. Phys.* **B451** (1995) 547 [hep-th/9504081].
[8] E. Bergshoeff, H.J. Boonstra and T. Ortín, *Phys. Rev.* **D53** (1996) 7206 [hep-th/9508091].

[9] G.T. Horowitz and A. Strominger, *Nucl. Phys.* **B360** (1991) 197.

[10] G.W. Gibbons, M.B. Green and M.J. Perry, *Phys. Lett.* **B370** (1996) 37 [hep-th/9511080].

[11] J. Dai, R.G. Leigh and J. Polchinski, *Mod. Phys. Lett.* **A4** (1989) 2073.

[12] E. Bergshoeff, M. de Roo, M.B. Green, G. Papadopoulos and P.K. Townsend, *Nucl. Phys.* **B470** (1996) 113 [hep-th/9601150];
E. Bergshoeff and M. de Roo, *Phys. Lett.* **B380** (1996) 265 [hep-th/9603123];
J. Borlaf and Y. Lozano, “Aspects of T-Duality in Open Strings,” e-print [hep-th/9607054].
E. Alvarez, J.L.F. Barbon and J. Borlaf, “T-Duality for Open Strings,” e-print [hep-th/9603089].
M.B. Green, C.M. Hull and P.K. Townsend, *Phys. Lett.* **B382** (1996) 65 [hep-th/9604119].

[13] R.G. Leigh, *Mod. Phys. Lett.* **A4** (1989) 2767.

[14] M. Li, *Nucl. Phys.* **B460** (1996) 351 [hep-th/9510161];
M.R. Douglas, “Branes within Branes,” e-print [hep-th/9512074].

[15] H. Lu, C.N. Pope, and K.S. Stelle, “Vertical versus Diagonal Reduction for P-Branes,” e-print [hep-th/9605082];
see also: R.C. Myers, *Phys. Rev.* **D35** (1987) 455.

[16] R.R. Khuri and R.C. Myers, *Nucl. Phys.* **B466** (1996) 60 [hep-th/9512061].

[17] J.G. Russo and A.A. Tseytlin, “Waves, Boosted Branes and BPS States in M-Theory,” e-print [hep-th/9611047].

[18] J.X. Lu, *Phys. Lett.* **B313** (1993) 29 [hep-th/9304159].

[19] M.B. Green, N.D. Lambert, G. Papadopoulos and P.K. Townsend, *Phys. Lett.* **B384** (1996) 86 [hep-th/9605146].

[20] G. Lifschytz, “Probing Bound States of D-Branes,” e-print [hep-th/9610125].

[21] M. Garousi and R.C. Myers, *Nucl. Phys.* **B475** (1996) 193 [hep-th/9603194].

[22] M. Garousi and R.C. Myers, in preparation.

[23] J.P. Gauntlett, D.A. Kastor and J. Traschen, “Overlapping Branes in M-theory,” e-print [hep-th/9604179].
A.A. Tseytlin, *Nucl. Phys.* **B475** (1996) 149 [hep-th/9604035].