A SCHEMATIC MODEL FOR DENSITY DEPENDENT VECTOR MESON MASSES

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Abstract

A schematic two-level model consisting of a “collective” bosonic state and an “elementary” meson is constructed that provides interpolation from a hadronic description (a la Rapp/Wambach) to B/R scaling for the description of properties of vector mesons in dense medium. The development is based on a close analogy to the degenerate schematic model of Brown for giant resonances in nuclei.

\footnote{Talk given by G.E. Brown at the AIP/KKG Memorial Meeting, 3 October 1998, who dedicated this work to Klaus Kinder–Geiger, long-time friend and stimulating colleague.}
1 Introduction

The density dependence of vector-meson masses suggested by Brown/Rho (B/R) scaling [1] stimulated a lot of interest. In particular, the CERES dilepton experiments [2] provided strong evidence that the properties of the $\rho$ mesons are nontrivially modified in hadronic matter. An excess of the dilepton with low invariant mass, as well as strength missing from the region of the free $\rho$-mass, are found in the experiments, although these determinations are not very quantitative up to now. However, experiments now underway with TPC should determine with good accuracy just how much strength is left at the free $\rho$-meson pole during the time of overlap of the heavy nuclei up until freezeout, that is, during the fireball.

The simplest and most economical explanation for the observed low-mass dileptons is given in terms of quasiparticles (both fermions and bosons) whose masses drop according to B/R scaling, thereby making an appealing link to the chiral structure of the hadronic vacuum. In an alternative view to this description, Rapp, Chanfray and Wambach (R/W) [3] claimed that the excess of low-mass dileptons can also follow from conventional many-body physics. On a rather general ground, this “alternative” description was in a sense anticipated as discussed by one of the authors [4]. In analogy to the quark-hadron duality in heavy-light meson decay processes, one may view B/R scaling as a “partonic” picture while R/W as a hadronic one. One way of succinctly summarizing the situation is that the former is a top-down approach and the latter a bottom-up one. The link between B/R scaling and the Landau quasiparticle interaction $F_1$ established in [5] is one specific indication for this “duality.” Indeed, in [5], Brown et al argued that the R/W explanation could be interpreted as a density-dependent $\rho$-meson mass, calculated in a hadron language (in contrast to that of constituent quarks used by Brown and Rho). In particular it was suggested in ref.[5] that if one replaced the $\rho$-meson mass $m_\rho$ by the mass $m_\rho^\ast(\rho)$ at the density being considered, one would arrive at a description, in hadron language, which at high densities appeared dual to that of the Brown/Rho one in terms of constituent quarks. These developments involved the interpretation of a collective isobar-hole excitation as an effective vector meson field operating
on the ground state of the nucleus; i.e.,

\[
\frac{1}{\sqrt{A}} \sum_i [N^*(1520)N^{-1}_i]^{1-} \simeq \sum_i [\rho(x_i) \text{ or } \omega(x_i)]|\Psi_0 >, \tag{1}
\]

with the antisymmetrical (symmetrical) sum over neutrons and protons giving the \( \rho \)-like (\( \omega \)-like) nuclear excitation. The dropping vector meson masses could then be calculated in terms of mixing of the nuclear collective state, eq.(1), with the elementary vector meson through the mixing matrix elements of fig. 1. Now building up the collective nuclear mode, the latter can be identified as an analog to the state in the degenerate schematic model of Brown for giant dipole resonance [7]. An important development which leads to the assumption eq.(1) was furnished by Friman, Lutz and Wolf[9]. From empirical values of the amplitudes such as \( \pi^+ N \rightarrow \rho^+ N \), etc. they constructed the \( \rho \)-like or \( \omega \)-like states encouraged by our assumption eq.(1). Thus our input assumption receives substantial empirical support. Furthermore, one can obtain the coupling constants of the nucleon to three collective states from their work, that to the \( \rho \)-like excitation being close to the one used by Brown et al in [8].

In this paper, we reformulate the heuristic idea described in [8] in a more specific form by clearly stating the set of assumptions we make in implementing the strategy. Our principal aim is to construct a model that interpolates the R/W theory valid near zero density to the B/R theory valid near the chiral phase transition density. Within the schematic two-level (“collective” field and “elementary” field) model defined by the coupling matrix element \( M_{ij} \) of fig.1, we assume that the self-energy \( \Sigma \), eq.(8), that enters the dispersion formula (to be given below) encodes the mechanism to interpolate
between the two regimes. In \[6\], it was suggested that going to B/R scaling from R/W theory corresponds to replacing the \(m_\rho^2\) appearing in the denominator of (8) by \(m_\rho^*^2\). In this paper, we shall show that this is indeed consistent with what is expected at \(\rho \sim 0\) and \(\rho \sim \rho_c\). Specifically, with the present construction, \(m_\rho^*\) goes to zero at \(\rho_c \sim 2.75\rho_0\) as in the Nambu-Jona-Lasinio calculation \([10]\). Without this replacement in R/W, however, the \(\rho\) mass can never go to zero at any density. This is because the self-energy is prefixed by \(q_0^2 \rightarrow m_\rho^*^2\), so that it would vanish if \(m_\rho^* \rightarrow 0\), resulting in a contradiction. Furthermore at any density, there will always be two states of the \(\rho\) quantum number in R/W whereas in B/R, all of the strength (\(A(\omega)\) defined later) goes into the lower one as \(m_\rho^* \rightarrow 0\) with the width going to zero as well since the phase space for decay goes to zero. We identify this state as the effective \(\rho\) degree of freedom as one approaches the critical density. At lower densities, we cannot make this identification, because of the two different \(\rho\)-states, so our model has a clear interpretation only at \(\rho \approx 0\) and \(\rho \approx \rho_c\).

Since we have a simple schematic model which can describe (roughly) the Rapp/Wambach or Brown/Rho regimes, depending on whether one scales with \(m_\rho\) or \(m_\rho^*\), we can easily calculate the general amount of strength to be found in low-mass dileptons, and the strength removed from the free-\(\rho\) pole. We adopt the following strategy.

We calculate the weighting factor \(Z\) for the two states, nuclear collective and elementary vector, which mix. The large imaginary part of the energy of the former state makes it difficult to show in detail exactly how its strength is distributed, but we know that the amount of the strength in that state – in our two-level model – must be just the strength removed from the higher state by the mixing. This strength will be formed at low invariant masses. We make rough estimates, by including or not including various widths, of the energies at which this lower strength will be formed.

\footnote{We have no convincing argument for the validity of this procedure. Our conjecture is as follows. To zeroth order in density, the \(\rho N^* N\) coupling is of the form \(\frac{F q_0}{m}\) with a dimensionless constant \(f\) and \(q_0\) is the fourth component of the four-vector of the \(\rho\) meson. If one writes this as \(F q_0\) with \(F = f/m\), then one should compute the medium renormalization of the constant \(F\) which will then depend on density \(\rho\). In order for the vector meson mass to go to zero at some high density so as to match B/R scaling, it is required that \(F(\rho) q_0 \rightarrow \text{constant} \neq 0\). For \(q = |\vec{q}| \approx 0\) which we are considering, this can be satisfied if \(F(\rho) \sim m^*-1\), modulo an overall constant. This is essentially the essence of the proposal of ref.\[6\].}
We note here that in our two-level model, the state originating from the elementary $\rho$ (or $\omega$) is pushed up substantially in energy. We believe much of this displacement to be an artefact of our two-level model, because there is substantial strength with $\rho$ (or $\omega$) quantum numbers lying above the single $\rho$ (or $\omega$) excitation we have chosen and the strength above will push down the upper $\rho$ (or $\omega$). Whereas some shift upwards of the strength originally in the elementary $\rho$ (or $\omega$) may be formed, our two-level model certainly will overdo the shift. We do not believe this defect to greatly change the amount of strength shifted to lower invariant mass, however.

Of course the total strength is conserved, so the amount of strength shifted to lower energies must be that missing from the higher state. However we note that the spectral strength $A(\omega)$ is related to $Z(\omega)$ by a factor

$$A(\omega) = \frac{Z(\omega)}{2\omega}. \quad (2)$$

This is clear because the sum rule on the $A(\omega)$, essentially oscillator strength, must be just that of the (energy weighted) Thomas-Reiche-Kuhn sum rule. It is the quantity $A(\omega)$ which enters into the rate equation for the dilepton production. Thus if the nuclear collective state is pushed down to an energy $\omega \simeq m_{\rho}/2$ with 25% of the strength being removed from the elementary $\rho$ pole, then one finds roughly equal spectral weights in the low-energy region and in the region of the elementary $\rho$. Because of the much larger Boltzmann factor in the low-energy region, a factor of several more dileptons will come from it than from the $\rho$-pole, given temperature $T \sim 150$ MeV.

## 2 The $\rho$-Meson in Nuclear Matter

The in-medium $\rho$-meson propagator is given by,

$$D_\rho(q_0, \vec{q}) = 1/[q_0^2 - \vec{q}^2 - (m_\rho^0)^2 - \Sigma_{\pi\pi}(q_0, \vec{q}) - \Sigma_{\rho N N}(q_0, \vec{q})] \quad (3)$$

where $m_\rho^0$ is a bare mass. The real part of $\Sigma_{\pi\pi}$ is taken into account approximately by defining $m_\rho^2 = (m_\rho^0)^2 + \text{Re}\Sigma_{\pi\pi} = (770)^2$ MeV$^2$. The imaginary part is taken to be

$$\text{Im}\Sigma_{\pi\pi}(q_0, \vec{q}) = -m_\rho \Gamma_{\pi\pi}(q_0, \vec{q}). \quad (4)$$
Then we get

$$D_\rho(q_0, \vec{q}) = 1/[q_0^2 - \vec{q}^2 - m_\rho^2(q_0) + im_\rho \Gamma_\pi \pi(q_0) - \Sigma_{\rho N^* N}]$$  \hspace{1cm} (6)

where $m_\rho(q_0)$ is the energy-dependent mass with the energy dependence lodged in the self-energy. The $\rho$-meson dispersion relation (at $\vec{q} = 0$) is given by

$$q_0^2 = m_\rho^2 + \text{Re} \Sigma_{\rho N^* N}(q_0).$$  \hspace{1cm} (7)

Solving this equation is equivalent to determining the zeros in the real part of the inverse $\rho$-meson propagator, eq.(6).

### 2.1 The Rapp/Wambach approach

We start with the crucial ingredients in R/W (Rapp/Wambach) theory [6]. The $\rho$-meson self-energy coming from the particle-hole excitation $N^*(1520)N^{-1}$ is

$$\Sigma_{\rho N^* N}(q_0) = f_{\rho N^* N}^2 \frac{8 q_0^2}{3 m_\rho^2} \frac{\rho_0}{4} \left( \frac{2(\Delta E)}{(q_0 + i\Gamma_{\text{tot}}/2)^2 - (\Delta E)^2} \right)$$  \hspace{1cm} (8)

where $\Delta E = M_{N^*} - M_N \simeq 1520 - 940 = 580$ MeV and $\Gamma_{\text{tot}} = \Gamma_0 + \Gamma_{\text{med}}$ where $\Gamma_0$ is the full width of $N^*(1520)$ in free space, $\sim 120$ MeV. The $\Gamma_{\text{med}}$ represents medium corrections to the width of $N^*(1520)$ [8]. In this calculation, we shall just replace integration over fermi-momentum by nuclear density $\rho_0$, an approximation presumably good at low density. The real part of $\Sigma_{\rho N^* N}(q_0)$ then takes the form

$$\text{Re} \Sigma_{\rho N^* N} = f_{\rho N^* N}^2 \frac{4 q_0^2}{3 m_\rho^2} \rho_0 \frac{\Delta E (q_0^2 - (\Delta E)^2 - \frac{1}{4} \Gamma_{\text{tot}}^2)}{(q_0^2 - (\Delta E)^2 - \frac{1}{4} \Gamma_{\text{tot}}^2)^2 + \Gamma_{\text{tot}}^2 q_0^2}$$  \hspace{1cm} (9)

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2From the particle data book, we find $\Gamma_{\pi \pi} = 150$ MeV (full width). In ref.[3] (compare this with that in ref.[8]), the authors used the following form for $\Gamma_{\pi \pi}$

$$\Gamma_{\pi \pi} = \frac{p(q_0)^3}{p_0^3} \left( \frac{2 \Lambda_\rho^2 + m_\rho^2}{2 \Lambda_\rho^2 + q_0^2} \right) \Gamma_0^{\pi \pi}$$  \hspace{1cm} (5)

with $\Gamma_0^{\pi \pi} = 120$ MeV and $p_0 = p(q_0 = m_\rho)$. Here $p$ refers to the pion momentum ($p = |\vec{p}|$) and $q_0$ the energy of the $\rho$-meson.
This leads to the $\rho$-meson dispersion relation (for $\vec{q} = 0$)

\[
q_0^2 = m_\rho^2 + \text{Re} \Sigma_{\rho N^*N}(q_0) = m_\rho^2 + f_{\rho N^*N}^2 \frac{4}{3} \frac{q_0^2}{m_\rho^2} \rho_0 \frac{\Delta E(q_0^2) - (\Delta E)^2 - \frac{1}{4} \Gamma_{\text{tot}}^2}{(q_0^2 - (\Delta E)^2 - \frac{1}{4} \Gamma_{\text{tot}}^2)^2 + \Gamma_{\text{tot}}^2 q_0^2}.
\] (10)

The $Z$-factor that represents the spectral weight of the upper state is, in general, defined by

\[
Z = (1 - \frac{\partial \Sigma}{\partial q_0^2})^{-1}.
\] (11)

To get this quantity, we first evaluate $\frac{\partial \Sigma_{\rho N^*N}}{\partial q_0^2}$. Due to the width of $N^*(1520)$, $\Sigma_{\rho N^*N}$ has an imaginary part. We shall define

\[
Z = (1 - \frac{\partial}{\partial q_0^2} \text{Re} \Sigma_{\rho N^*N})^{-1}
\] (12)

taking the real part of $\Sigma_{\rho N^*N}$ so as to make the $Z$-factor real.\footnote{There is a point which should be clarified. If we solve the equation of the $\rho$-meson dispersion relation (eq.(13)), we will possibly get one real and two complex valued solutions for $q_0$. For the real solution, our definition of $Z$-factor (eq.(11)) could be correct. But in the case of the complex solution, we are not sure whether this definition still makes sense. Of course, we can use a sum rule for $Z$-factor to estimate the $Z$-factor corresponding to complex solutions. This point needs further study.}

Defining $x = \frac{q_0^2}{m_\rho^2}$, we get

\[
\frac{\partial}{\partial q_0^2} \text{Re} \Sigma_{\rho N^*N} = \frac{\partial}{\partial x} \left( \text{Re} \Sigma_{\rho N^*N} m_\rho^2 \right) = \frac{(2x - c_1 - c_2)((x - c_1 - c_2)^2 + 4c_2x)}{((x - c_1 - c_2)^2 + 4c_2x)^2} - \frac{x(x - c_1 - c_2)(2(x - c_1 - c_2) + 4c_2)}{((x - c_1 - c_2)^2 + 4c_2x)^2}
\] (13)

where $c = f_{\rho N^*N}^2 \frac{4}{3} \frac{\rho_0}{m_\rho^2} \frac{\Delta E}{m_\rho^2}$, $c_1 = \frac{(\Delta E)^2}{m_\rho^2} \simeq 0.567$ and $c_2 = \frac{1}{4} \frac{\Gamma_{\text{tot}}^2}{m_\rho^2}$. We can readily obtain the zeros in the real part of the $\rho$-propagator and the $Z$ factor by plotting figures like those of Fig.3 in ref.[6]. But here we shall get them by directly solving eq.(11) and calculating eq.(13).
Case of $\Gamma_{\text{tot}} = 0$

For simplicity, let us set $\Gamma_{\text{tot}} = 0$. The relevant equations simplify to

$$q_0^2 = m_\rho^2 + f_{\rho N^*}^2 \frac{4}{3} \frac{q_0^2 \rho_0}{m_\rho^2} \frac{\Delta E}{q_0^2 - (\Delta E)^2},$$

$$\frac{\partial \Sigma_{\rho N^* N}}{\partial q_0^2} = \frac{\partial}{\partial x} \left( \frac{\Sigma_{\rho N^* N}}{m_\rho^2} \right)$$

$$= f_{\rho N^*}^2 \frac{4}{3} \frac{\rho_0^2}{m_\rho^3} \frac{\Delta E}{m_\rho} \frac{-(\Delta E)^2}{(m_\rho^2)^2}.$$

(14)

Written in terms of the quantity $x \equiv q_0^2/m_\rho^2$, the dispersion relation reads

$$x = 1 + 0.208x \frac{1}{x - 0.567}$$

(15)

where we have used $f_{\rho N^*}^2 = 5.5$ from ref.[3] and $\rho_0 \simeq \frac{1}{2} m_\pi^3$. The solutions are

$$q_0^- \simeq 498 \text{ MeV}, \quad q_0^+ \simeq 897 \text{ MeV}.$$  

(16)

The formula for $Z$-factor

$$Z = (1 + 0.118 \frac{1}{(x - 0.567)^2})^{-1}$$

(17)

yields the corresponding $Z$-factors

$$Z(q_0^-) \simeq 0.16, \quad Z(q_0^+) \simeq 0.84.$$  

(18)

Naively extrapolated to a higher density, say, $\rho \simeq 2.5\rho_0$, the results come out to be

$$q_0^- \simeq 436 \text{ MeV}, \quad q_0^+ \simeq 1023 \text{ MeV}$$

(19)

and

$$Z(q_0^-) \simeq 0.17, \quad Z(q_0^+) \simeq 0.83.$$  

(20)

Case of $\Gamma_{\text{tot}} = \Gamma_0 = 120 \text{ MeV}$
Figure 2: \( \rho \)-meson dispersion relation for \( \Gamma_0 = 120 \) MeV at \( \rho = \rho_0 \) and \( \rho = 2.5\rho_0 \). The horizontal axis \( x \) represents \( \frac{q^2}{m^2_\rho} \) and the vertical axis \( f(x) = x - 1 - \text{Re} \Sigma_{\rho N^* N}(x) \).

Substituting \( \Gamma_0 = 120 \) MeV and \( \Gamma_{med} = 0 \) into the dispersion relation at normal nuclear density \( \rho_0 \)

\[
x = 1 + 0.208x \frac{x - 0.567 - \frac{\Gamma^2_{tot}}{4m^2_\rho}}{(x - 0.567 - \frac{\Gamma^2_{tot}}{4m^2_\rho})^2 + \frac{\Gamma^2_{tot}}{m^2_\rho}x}
\]

we obtain the solutions

\[
x = \frac{q^2_0}{m^2_\rho} = 1.24, \ 0.49 + i0.267, \ 0.49 - i0.267
\]

where \( i \) refers to imaginary. Taking the real part of the solutions, we get\[4\]

\[
q_0^- \simeq \sqrt{0.49 * m^2_\rho} = 541 \text{ MeV}, \ q_0^+ \simeq 857 \text{ MeV}.
\]

The \( Z \)-factor for \( q_0^+ \) state is calculated to be

\[
Z(q_0^+) \simeq 0.86
\]

with the remaining strength going to the lower state. For \( \rho = 2.5\rho_0 \), we get

\[
x = 0.34, \ 0.55, \ 1.7
\]

\[4\]As stated, we do not know how to interpret physically the imaginary parts of the solution. Of course, the imaginary part tells us that the pole is located off the real axis.
corresponding to
\[ q_0 = 489 \text{ MeV}, \ 571 \text{ MeV}, \ 1003 \text{ MeV}. \] (26)

The corresponding Z-factors are
\[ Z(449) = 0.21 \]
\[ Z(571) = -0.056 \]
\[ Z(1004) = 0.83. \] (27)

The \( \rho \)-meson dispersion relation with \( \Gamma_0 = 120 \text{ MeV} \) at \( \rho = \rho_0 \) and \( \rho = 2.5\rho_0 \) is shown in fig.2.

2.2 The B/R approach

As stated in Introduction, we propose that approaching B/R-scaling from hadronic excitations is effected by replacing \( \frac{q_0^2}{m_{\rho}^2} \) by 1 in the \( \Sigma_{\rho N^* N} \) that enters in the dispersion relation [6]. Let us see how this ansatz works out in reproducing the structure of B/R scaling as density increases. From (10), we get a minimally modified dispersion relation for the \( \rho \)-meson in medium
\[ q_0^2 = m_{\rho}^2 + f_{\rho N^* N}^2 \frac{4}{3} \rho_0 \frac{\Delta E(q_0^2 - (\Delta E)^2 - \frac{1}{4} \Gamma_{tot}^2)}{(q_0^2 - (\Delta E)^2 - \frac{1}{4} \Gamma_{tot}^2)^2 + \Gamma_{tot}^2 q_0^2}. \] (28)

In this formula, we shall assume that in medium, \( \Delta E \) remains unchanged (assumption valid to the leading order in \( 1/N_c \)) while the width \( \Gamma_{tot} \) may be affected by density.

- **Case of \( \Gamma_{tot} = 0 \)**

In contrast to the R/W approach, this is a situation which is actually realizable as density approaches the chiral transition density \( \rho_c \) since the phase space for \( \rho \) decay goes to zero at that density. Let us consider what

\footnote{We interpret these three states of \( \rho \)-meson quantum number to be the “elementary” \( \rho, N^*(1520)N^{-1}\pi N \) and \( N^*(1520)N^{-1} \). See fig.3}
Figure 3: In-medium $\rho$-meson mass and $Z$ factor in B/R (solid line) and R/W (dashed line) theories for $\Gamma_{tot} = 0$.

happens at normal nuclear density ($\rho_0$) in the limit of zero width. The solutions are

$$q_0^- = 406.7 \text{ MeV}, \quad q_0^+ = 873.9 \text{ MeV}$$

(29)

with the corresponding $Z$-factors

$$Z(q_0^-) = 0.285, \quad Z(q_0^+) = 0.714$$

(30)

For $\rho = 2.5\rho_0$, we obtain

$$q_0^- = 136 \text{ MeV}, \quad q_0^+ = 956 \text{ MeV}$$

(31)

and

$$Z(q_0^-) = 0.356, \quad Z(q_0^+) = 0.646.$$  

(32)

Since the width should vanish near the critical density $\rho_c$, the dispersion formula with zero width should approach the correct one near it. Figure 3 shows indeed that $m^* \to 0$ as $\rho \to 2.75\rho_0$ as found in [10].

- Case of $\Gamma_{tot} = \Gamma_0 = 120 \text{ MeV}$

  The results at normal nuclear density are

$$\begin{align*}
q_0 & = 423 \text{ MeV}, \quad 567 \text{ MeV}, \quad 878 \text{ MeV} \\
Z(423) & = 0.34, \quad Z(567) = -0.0799, \quad Z(878) = 0.73.
\end{align*}$$

(33)
For \( \rho = 2.5 \rho_0 \), we get
\[
q_0 = 146 \text{ MeV}, \quad 577 \text{ MeV}, \quad 951 \text{ MeV}
\]
\[
Z(146) = 0.369, \quad Z(567) = -0.027, \quad Z(951) = 0.657
\]  
(34)

We compare in fig.3 the in-medium \( \rho \)-meson mass and \( Z \)-factors in B/R and R/W.

![Figure 4: In-medium \( \rho \)-meson mass in B/R theory. \( \Gamma_{\text{tot}} = \Gamma_{\text{tot}}(\rho) \) corresponds to the result of changing the value of \( \Gamma_{\text{tot}} \) from 260 MeV to 30 MeV as density increases.](image)

### 2.3 The \( m_\rho^* \) as an order parameter

In [10, 11], an argument was given that the in-medium mass of the \( \rho \)-meson can be taken, roughly, as an order parameter for the chiral phase transition. Figure 4 shows that our model described above predicts \( m_\rho^* \) dropping roughly linearly in density. This is consistent with the behavior of the quark condensate in medium,

\[
\frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle} \approx 1 - \frac{\sigma_{N\rho_N}}{f^2_\pi m^2_\rho}
\]

(35)

where the star denotes finite density (or temperature) and \( \rho_N \) the nuclear (vector) density. Indeed we would find from eq.(35) roughly the same \( \rho_c \) as in fig.3 for \( m_\rho^* \sim 0 \) by setting \( \langle \bar{q}q \rangle^* \sim 0 \). Thus our model has the quark condensate, on the average, dropping roughly linearly with density \( \rho \).
3 The $\omega$-meson in nuclear matter

In this section, we apply the same two-level model to the $\omega$-meson channel. We shall consider both R/W and B/R approaches.

3.1 The R/W approach

For this calculation, all we have to do is to replace $f_{\rho N^*N}(m_\rho)$ by $f_{\omega N^*N}(m_\omega)$ and $m_\rho$ by $m_\omega$ in eq.(8). A priori, we do not know how to relate $f_{\omega N^*N}$ to $f_{\rho N^*N}$. Assuming a generalized VDM would give the relation $f_{\omega N^*N} = 3 f_{\rho N^*N}$ but there is no reason, theoretical or empirical, to believe that such a relation should be reliable. We shall instead resort to the empirical result of Friman et al [9]. From their fig. 4, we find

$$f^2_{\omega N^*N} \approx 4.4 \times f^2_{\rho N^*N}$$

(36)

- Case of $\Gamma_{tot} = 0$

At normal nuclear matter density, the dispersion formula (corresponding to eq.(10) for the $\rho$ meson) is

$$x = 1 + 0.863 \frac{x}{x - 0.55}$$

(37)

where $x = q^2_0/m^2_\omega$. The solutions are

$$q_0^- \simeq 395 \text{ MeV}, \quad q_0^+ \simeq 1149 \text{ MeV}$$

(38)

with the corresponding $Z$ factors

$$Z(q_0^-) \simeq 0.155, \quad Z(q_0^+) \simeq 0.845.$$  

(39)

The behavior of the $\omega$ mass is compared with that of the $\rho$ mass in fig.5. Note that the stronger coupling makes the $\omega$ mass fall faster than the $\rho$ mass.

- Case of $\Gamma_0 = 120 \text{ MeV}$

In this case, we find

$$q_0^1 = 403 \text{ MeV}, \quad q_0^2 = 576 \text{ MeV}, \quad q_0^3 = 1145 \text{ MeV}$$

(40)
and

\[ Z(q_0^1) = 0.174, \quad Z(q_0^2) = -0.0293, \quad Z(q_0^3) = 0.856 \]  

(41)

at normal nuclear matter density. For comparison, we quote the values of Friman et al \[9\]:

\[ q_0^- \approx 328 \text{ MeV}, \quad q_0^+ \approx 1384 \text{ MeV} \]  

(42)

and

\[ Z(q_0^-) \approx 0.125 \]  

(43)

### 3.2 The B/R approach

Even if we can extract the \( \omega N^*N \) coupling constant from experiments at zero density, there is no reason to expect that that constant will remain unchanged in medium. Indeed we have reasons to believe that the ratio \( R \equiv (f_{\omega N^*N}/f_{\rho N^*N})^2 \) will decrease as density increases. For this reason, we shall consider two cases: (1) a density-independent constant; (2) a density-dependent constant.
With density-independent $f_{\omega N^*N}$

With B/R scaling, the factor 4.4 determined empirically in (36) for matter-free space turns out to give an unreasonably low critical density ($\rho_c \sim 0.7\rho_0$) at which the collective $\omega$ mass vanishes. While the $\omega$ mass is expected to drop faster than the $\rho$ mass as explained below, it does not seem reasonable that the $\omega$ mass vanish much before the $\rho$ mass does. To see what happens if one takes a constant coupling constant somewhat larger than the $\rho N^*N$ coupling, we take for illustration

$$f_{\omega N^*N}^2 \approx 1.6 \times f_{\rho N^*N}^2. \tag{44}$$

We have no particular reason to take this number but it gives a qualitative idea as to how things go. The results are summarized in fig. 6 for the case with $\Gamma_{\text{tot}} = 0$. Since $|f_{\omega N^*N}| > |f_{\rho N^*N}|$, the $\omega$ mass drops to zero faster than the $\rho$ mass: $m_{\omega}^* \rightarrow 0$ for $\rho \rightarrow \rho_c \approx 1.7\rho_0$. We shall suggest below that this feature of the $\omega$ properties may be interpreted in terms of an “induced symmetry breaking” (ISB) in the vector channel.

![Figure 6: In-medium $\omega$-meson mass and $Z$ factor in B/R theory for $\Gamma_{\text{tot}} = 0$ for $R=1.6$ independent of density.](image)

3.2.2 Density-dependent $f_{\omega N^*N}$

The fact that empirically $R \approx 4$ [9] at zero density indicates that the vector dominance model (VDM) – which would give 9 – fails. This is not
surprising: there is no reason to expect that the VDM should work in the baryon sector, particularly where baryon resonances are involved. On the other hand, as density approaches the chiral phase transition point, we would expect that the system becomes a Fermi liquid of quasiquarks \[10\] to which the vector degrees of freedom corresponding to the \(\rho\) and the \(\omega\) would couple in an \(U(2)\) symmetric way. This would mean that the ratio \(R\) would go to 1 as \(\rho \to \rho_c\).

Here we shall implement this possibility in the dispersion formula for in-medium \(\omega\)'s assuming that the constant \(f_{\rho N^* N}\) depends little on density. It turns out that the simplest possible linear interpolation between \(\rho = 0\) and \(\rho = \rho_c\) gives too rapid a decrease of the \(\omega\) mass, vanishing at much too low a density than that of the \(\rho\). Different parameterizations have been tried and we report two of them which appear to be reasonable. One is the following log-type parametrization

\[
f_{\omega N^* N}^2 = f_{\rho N^* N}^2 \frac{4.4}{1 + 3.4 \log(1 + 1.72(c/2.8))}
\]  \hspace{1cm} (45)

where \(c\) is defined by \(\rho = c \rho_0\). In this parametrization, \(\rho_c\) is chosen as \(\rho_c = 2.8 \rho_0\). The results are given in fig.7.

![Figure 7: In-medium \(\omega\)-meson mass and \(Z\)-factor in B/R theory for \(\Gamma_{tot} = 0\) with the density dependence of \(f_{\omega N^* N}^2\) given by eq.(45).](#)

As an alternative parametrization, we take

\[
f_{\omega N^* N}^2 = f_{\rho N^* N}^2(4.4 - 3.4(c/2.8)\frac{1}{3}).
\]  \hspace{1cm} (46)
Figures 7 and 8 show that the two parameterizations (45) and (46) give qualitatively the same results. Because of the initially stronger coupling constant, the $\omega$ falls faster than the $\rho$ at the beginning, flattens in the middle and then approaches zero at the critical density together with the $\rho$.

![Figure 8: In-medium $\omega$-meson mass and $Z$ factor in B/R theory for $\Gamma_{tot} = 0$ with the density dependence of $f_{\omega N^* N}$ given by eq.(44).](image)

3.3 The dropping $\omega$ mass and a high-density “ISB phase”

The stronger $\omega N^* N$ coupling relative to the $\rho N^* N$ coupling leads naturally to the prediction in the present model that in medium the $\omega$ mass would fall faster than the $\rho$ mass as density increases. This is expected in both R/W and B/R approaches. In B/R, however, this leads to the additional prediction that the $\omega$ mass would go to zero (in the chiral limit) either before or at $\rho_c$ for the $\rho$ meson depending on whether the ratio $\mathcal{R}$ remains constant (of density) or goes to 1 at $\rho = \rho_c$. There is no theoretical reason known to favor one scenario over the other. However, that the $\omega$ mass falls faster than the $\rho$ mass – which is essentially dictated in the present formalism by the fact that $\mathcal{R} > 1$ – is consistent with the phase structure of dense matter previously arrived at in quark language by Langfeld et al [12, 13].

Briefly the scenario given by [12, 13] is as follows. If quark-quark interactions have a strength in vector channel comparable to what is found

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6The density at which the $\omega$ mass goes to zero differs slightly but we do not believe that any importance can be attached to this difference.
in one-gluon exchanges, then an induced Lorentz symmetry breaking could take place at a critical chemical potential $\mu_c$ at which chiral symmetry would be restored, i.e., $\langle \bar{q}q \rangle = 0$, and the baryon density $\langle \bar{B}\gamma_0 B \rangle$ would have a discontinuous increase as $\mu$ exceeds $\mu_c$ indicating a first-order transition. The consequence is that a low-energy collective state carrying the quantum number of an $\omega$ meson should emerge at $\mu_c$ as a pseudo-Goldstone vector boson.

Our proposal is that the collective $N^*$-hole excitation of the $\omega$ quantum number built in our schematic degenerate model be identified with the low-mass ISB state described in quark language in refs.\cite{12, 13}. In this “dual” description, as the inverse $\omega$ propagator vanishes at the point where the mass of the lower $\omega$ branch goes to zero, the $\omega$ field develops an “induced VEV” $\delta \langle \omega_0 \rangle_{\rho_c}$ so that there would be a discontinuity at $\rho_c$ of the $\langle \omega_0 \rangle$. Translated into the baryon (or quark) density, this means that there will be a jump in density at the critical chemical potential $\mu_c$ if one looks at the density vs. $\mu$. One could think of this as a chiral symmetry restoration in dense matter in a way analogous to what was obtained in ERGF (exact renormalization group flow) by Berges et al.\cite{14}. A more appealing way of viewing the present scenario is that it provides a hadronic counterpart of the quark-model scenario of Langfeld et al.\cite{12, 13}. It is amusing to note that the ISB phenomenon in the quark sector is encoded in the empirical fact in the hadronic sector that $|f_{\omega N^*N}| > |f_{\rho N^*N}|$ for $\rho < \rho_c$.

4 Conclusion

We have constructed a schematic model of the Rapp/Wambach theory, emphasizing the role of the $[N^*(1520)N^{-1}]$ isobar-hole state. This model turns out to be essentially the same as the degenerate schematic model of Brown\cite{7} but for the $q_0$-dependence of the coupling of $\rho$ meson to $N^*N$. In fact when this coupling is cancelled by introducing $m_{\rho}^*$ as the mass scaling factor of the Lagrangian, the model becomes precisely that of Brown\cite{7}. We show that this latter model gives the same results as Brown/Rho scaling in the limit of $\rho \rightarrow \rho_c$, where $\rho_c$ is the chiral restoration density and propose to use it as an interpolation formula between R/W and B/R scaling. This then provides a possible mechanism to arrive at B/R scaling from the hadronic side, that is, in the bottom-up way.

New results from the TCP now in service with the CERES collaboration
should pin down the strength at the $\rho$-meson poles, or at least an average of this strength over the various densities encountered in this experiment. These will confirm or infirm our scenario: Since the expected strength entering into the dilepton rate is obtained by $A(\omega) = \frac{Z(\omega)}{Z(0)}$, lower-energy component(s) will be progressively more enhanced at higher densities. Furthermore, given temperature of $T \sim 150$ MeV, there will be larger Boltzmann factors at lower energies, so the net result will be that more leptons will come out of the lower-energy state(s).

We have also suggested that the $\omega$ mass should fall faster than the $\rho$ mass until one approaches the chiral phase transition and that the collective $N^*$-hole excitation of the $\omega$ quantum number in the schematic degenerate model is the hadronic ("Cheshire-cat") description of the pseudo-Goldstone vector boson generated by an induced symmetry breaking of Lorentz symmetry in dense medium.

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References

[1] G.E. Brown and M. Rho, Phys. Rev. Lett. 66, 2720 (1991)

[2] G. Agakichiev et al, Phys. Rev. Lett. 75, 1272 (1995)

[3] R. Rapp, G. Chanfray and J. Wambach, Nucl. Phys. A617, 472 (1997)

[4] M. Rho, Acta Phys. Pol. B29, 2297 (1998); “Chiral symmetry in nuclei,” nucl-th/9812012

[5] B. Friman and M. Rho, Nucl. Phys. A606, 303 (1996); Chaejun Song, G.E. Brown, D.-P. Min and M. Rho, Phys. Rev. C56, 2244 (1997); Chaejun Song, D.-P. Min and M. Rho, Phys. Lett. B424, 226 (1998); B. Friman, M. Rho and Chaejun Song, nucl-th/9809088

[6] G.E. Brown, C.Q. Li, R. Rapp, M. Rho and J. Wambach, Acta Phys. Pol. B29, 2309 (1998), nucl-th/9806026

[7] G.E. Brown, Chap. V. 3 in Unified Theory of Nuclear Models and Forces (North-Holland Pub. Co., Amsterdam, 1971)

[8] W. Peters, M. Post, H. Lenske, S. Leupold and U. Mosel, Nucl. Phys. A632, 109 (1998)

[9] B. Friman, M. Lutz and G. Wolf, “Masses of hadrons in nuclei,” nucl-th/9811040

[10] G.E. Brown, M. Buballa and M. Rho, Nucl. Phys. A609, 519 (1996)

[11] C. Adami and G. E. Brown, Phys. Rev. D46, 478 (1992)

[12] K. Langfeld, H. Reinhardt and M. Rho, Nucl. Phys. A622, 620 (1997); K. Langfeld, Nucl. Phys. A642, 96 (1998)

[13] K. Langfeld and M. Rho, “Quark condensation, induced symmetry breaking and color superconductivity at high density,” hep-ph/9811227.

[14] J. Berges, R.-U. Jungnickel and C. Wetterich, “The chiral phase transition at high baryon density from nonpertubative flow equations,” hep-ph/9811347.