Quantum and classical correlated imaging

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Abstract

We outline the potential gains of quantum correlated imaging and compare it to classical correlated imaging. As shown earlier by A. Gatti, E. Bambilla, M. Bache, and L. A. Lugiato, ArXive:quant-ph/0405056, classical correlated imaging can mimic most features of quantum imaging but at lower signal-to-noise ratio for a given mean photon number (or intensity). In this paper we specifically investigate coherent correlated imaging, and show that while it is possible to perform such imaging using a thermal source, a coherent light-source provides a less demanding experimental setup. We also compare the performance to what can be obtained by using non-classical light.

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I. INTRODUCTION

Quantum imaging has attracted much attention in recent years [1, 2, 3, 4, 5, 6, 7, 8, 9]. Typically, this technique exploits the quantum entanglement of the state generated by spontaneous parametric down-conversion (PDC) in a two-beam setup. The object to be imaged is located in one of the beams and the information about the spatial distribution of the object is obtained by registering the coincidence counts as a function of the transverse position of the photon in the reference beam, which holds a known reference object [1, 2, 3, 4, 5, 6]. There has been a lively debate whether or not quantum entanglement is a necessary ingredient to perform correlated imaging [5, 7, 8, 9, 10]. The topic became hotly debated after the correlated image experiment of Ref. [3] was successfully reproduced using classically correlated beams [8]. Subsequent analysis [7, 11, 12], has shown that correlated imaging can be done to some extent also by classical beams, and experiments are underway to demonstrate correlated imaging using a light-source with a thermal photo-count distribution. It has been pointed out that one could obtain more information about an imaged object if one performs coherent correlated imaging [12]. That is, instead of using only intensity correlation, one could use the correlations between the object and reference beams’ field quadrature amplitudes, and one could then sequentially obtain information from two (or more) non-compatible field quadratures. We will show that, perhaps counterintuitively, coherent correlated imaging can be done using light either from a spontaneous parametric down-conversion source, or from a thermal source, or better yet, using coherent light. Only the signal-to-noise ratio scaling, with the respect to the number of photons used, differ. The coherent light source offers practical advantages over both other sources and gives a better performance than a thermal source.

II. ANALYSIS

In order to bring out the essence of correlated imaging, one should carefully examine where the difference between different imaging techniques emerges. Correlated imaging entails using two set of modes (these sets are often referred to as beams) passing through an object characterized by the impulse function $h_1(x, x')$ and through a reference object, characterized by the impulse response function $h_2(x, x')$. Computing the final correlation
signal usually amounts to evaluating a multiple integral (or a sum) with respect to the mode functions, usually denoted by an index $q$, cf. [12]. However, in our opinion, the multi-mode treatment is not needed in order to understand the essence of quantum imaging.

In this paper we shall deal with the imaging problem by only considering two modes (at a time). The justification for this simplification of the problem is that, in the plane pump-wave approximation, the transverse momentum in the parametric process is strictly conserved, so the object (signal) $q_i$ and reference (idler) modes $q'_i$ (in a plane-wave mode expansion), illustrated in Fig. [11] are strictly pair-wise correlated. Therefore, there is no first-order correlation between different object spatial modes, nor between an object mode with transverse wave vector $\vec{q}_i$ (with respect to the pump wave vector) and the reference modes with wave vectors $\vec{q}'_i \neq -\vec{q}_i$. Typically, this condition is described by stating that the (spontaneously), parametrically generated signal and idler beams have no transverse coherence. Of course, it is possible to obtain any wanted transverse coherence length (at least in principle) by appropriate filtering. However, this is also true for any of the other two light sources considered in this paper, and this fact does not change our results in any substantial way. If the PDC object and reference beams are filtered so that they have a finite transverse correlation length, then it is possible to find another set of modes (than plane-wave modes) so that in this new set, the modes are only pair-wise correlated.

We shall also assume that the detection system is arranged so that the detection modes coincide with the set of modes where the correlations are manifested (as common sense dictates). In an experiment, where the pump both has a finite spatial extent because it is focused, this entails an imaging system between the source, the object and the (array) detector, and likewise, between the source, reference object and its (array) detector.

In this analysis we also omit to include the impulse response functions $h_1(x, x')$ and $h_2(x, x')$ of the object and of the reference object. Since these are assumed to be linear, any difference between classical and quantum imaging must stem from the light sources, and not from the impulse response functions. For the same reason the detectors are also omitted from the discussion as the detectors used in the classical and the quantum imaging are assumed to have the same characteristics.

In order to compare different sources of correlated light, the quantity of interest in most
cases is the correlation between the intensity fluctuations (or between the field quadratures).

\[ G(x, x') = \langle I_1(x)I_2(x') \rangle - \langle I_1(x) \rangle \langle I_2(x') \rangle. \tag{1} \]

However, it is customary to normalize the correlation function \[13\], so in the following we will use

\[ C(x, x') = \frac{\langle I_1(x)I_2(x') \rangle - \langle I_1(x) \rangle \langle I_2(x') \rangle}{\sqrt{\langle \langle I_1(x)^2 \rangle - \langle I_1(x) \rangle^2 \rangle \langle \langle I_2(x')^2 \rangle - \langle I_2(x') \rangle^2 \rangle}}. \tag{2} \]

(Such second order correlation functions are often denoted \( g(x, x') \) in the literature.) The normalized correlation function between field quadratures is expressed in a similar way. It is also worth to note that the “coordinates” \( x \) and \( x' \) do not necessarily denote spatial modes, but can refer to any set of orthogonal modes, e.g., plane-wave modes that has no transverse spatial dependence at all.

To formalize the discussion, we use the input-output relations for a linear four-port device \[14\], such as a beam splitter or a parametric amplifier. We denote the input modes \( \hat{a} \) and \( \hat{b} \) and the output modes \( \hat{c} \) and \( \hat{d} \). The output modes correspond to any pair of modes \( q_i \) and \( q'_i \) in Fig. 1. A beam splitter of (power) transmittance \( T \) then has the following operator relations:

\[ \hat{c} = \sqrt{T} \hat{a} - \sqrt{1-T} \hat{b} \tag{3} \]

and

\[ \hat{d} = \sqrt{T} \hat{b} + \sqrt{1-T} \hat{a}, \tag{4} \]

where \( 0 \leq T \leq 1 \). In the following we shall always assume that \( T = 1/2 \). An ideal parametric down-conversion amplifier obey similar relations:

\[ \hat{c} = \sqrt{G} \hat{a} + \sqrt{G-1} \hat{b}^\dagger \tag{5} \]

and

\[ \hat{d} = \sqrt{G} \hat{b} + \sqrt{G-1} \hat{a}^\dagger, \tag{6} \]

where \( G \) is the (power) gain of the amplifier. The last two equations are, in a different notation, identical to Eq. (1) in \[12\]. The difference in the intensity and quadrature correlations stems from a) what type of input states are used, and b) the appearance of the creation operator in the parametric four-port. Let us now use these unitary transformations to compute the correlation functions. We will consider three cases:
1) A PDC with vacuum input $|\psi_{in}\rangle = |0\rangle \otimes |0\rangle = |0,0\rangle$

2) A beam splitter with a coherent state input in one port and a vacuum state incident on the other port: $|\psi_{in}\rangle = |\alpha\rangle \otimes |0\rangle = |\alpha,0\rangle$

3) A beam splitter with a thermal state input on one port and a vacuum state incident on the other port, where we use the corresponding density operator: $\hat{\rho}_{in} = \hat{\rho}_{\text{therm}} \otimes |0\rangle\langle 0|$. The relevant fluctuation correlation for intensity correlation imaging is

$$C_I = \frac{\langle \psi_{in}|\hat{c}^\dagger \hat{d}^\dagger \hat{d} |\psi_{in}\rangle - \langle \psi_{in}|\hat{c}^\dagger \hat{c} |\psi_{in}\rangle \langle \psi_{in}|\hat{d}^\dagger \hat{d} |\psi_{in}\rangle}{\sqrt{\langle \psi_{in}|(\hat{c}^\dagger \hat{c})^2 |\psi_{in}\rangle - \langle \psi_{in}|\hat{c}^\dagger \hat{c} |\psi_{in}\rangle^2}} \frac{\langle \psi_{in}|(\hat{d}^\dagger \hat{d})^2 |\psi_{in}\rangle - \langle \psi_{in}|\hat{d}^\dagger \hat{d} |\psi_{in}\rangle^2}{\langle \psi_{in}|(\hat{d}^\dagger \hat{d})^2 |\psi_{in}\rangle - \langle \psi_{in}|\hat{d}^\dagger \hat{d} |\psi_{in}\rangle^2}}^{1/2}. \quad (7)$$

One may also be interested in performing correlated imaging using homodyne detection. In that case, the relevant correlations are found in the field quadrature observables:

$$\hat{c}_i = \frac{1}{2}(\hat{c} + \hat{c}^\dagger) \quad (8)$$

and

$$\hat{c}_o = \frac{1}{2i}(\hat{c} - \hat{c}^\dagger). \quad (9)$$

The pertinent correlation becomes (e.g., between the in-phase quadratures):

$$C_i = \frac{\langle \psi_{in}|\hat{c}_i^\dagger \hat{d}_i |\psi_{in}\rangle - \langle \psi_{in}|\hat{c}_i |\psi_{in}\rangle \langle \psi_{in}|\hat{d}_i |\psi_{in}\rangle}{\sqrt{\langle \psi_{in}|(\hat{c}_i^\dagger \hat{c}_i)^2 |\psi_{in}\rangle - \langle \psi_{in}|\hat{c}_i^\dagger \hat{c}_i |\psi_{in}\rangle^2}} \frac{\langle \psi_{in}|(\hat{d}_i^\dagger \hat{d}_i)^2 |\psi_{in}\rangle - \langle \psi_{in}|\hat{d}_i^\dagger \hat{d}_i |\psi_{in}\rangle^2}{\langle \psi_{in}|(\hat{d}_i^\dagger \hat{d}_i)^2 |\psi_{in}\rangle - \langle \psi_{in}|\hat{d}_i^\dagger \hat{d}_i |\psi_{in}\rangle^2}}^{1/2}. \quad (10)$$

Please note that, due to symmetry, the out-of-phase field quadrature fluctuation correlation will be identical to the in-phase fluctuation correlations for all cases considered.

Now, using any standard textbook in quantum optics, e.g., [15], we may compute the correlations. First, we compute the intensity and quadrature phase correlations for PDC:

$$\hat{C}_{I,PDC} = 1 \quad (11)$$

and

$$\hat{C}_{i,PDC} = \frac{2\sqrt{G(G-1)}}{2G-1} = \frac{2\sqrt{\langle \hat{n} \rangle (\langle \hat{n} \rangle + 1)}}{2\langle \hat{n} \rangle + 1}, \quad (12)$$

where $\langle \hat{n} \rangle = G - 1$ is the average photon number per mode. The result $\hat{C}_{I,PDC} = 1$ is of course expected as we have considered an ideal two-photon creation process.

Next, for the coherent state we obtain

$$C_{I,coh} = 0 \quad (13)$$
and

\[ C_{i,\text{coh}} = 0. \] (14)

This result is intuitive, too. As a coherent state split into two in a beam splitter evolves into a product state of two coherent states. Therefore, we cannot expect any fluctuation correlations. However, please note that this does not imply that the coherent state cannot be used for correlated imaging. It only implies a noise limit for the imaging.

Finally, for the thermal state we get

\[ C_{i,\text{th}} = \frac{\langle \hat{n} \rangle}{\langle \hat{n} \rangle + 1}, \] (15)

whereas the quadrature field correlations for the split thermal mode is

\[ C_{i,\text{th}} = \frac{2\langle \hat{n} \rangle}{2\langle \hat{n} \rangle + 1}, \] (16)

where, again, \( \langle \hat{n} \rangle \) is the average photon number in the object and in the reference modes.

In Fig. 2, we have plotted the intensity correlations, and in Fig. 3 the (in-phase) quadrature correlations as a function of the photon number in the one of the two output beams. Note that for the coherent state and the thermal state the input average photon number is twice this number since the beam is split in half. We see that the normalized fluctuation correlations are almost as strong for the thermal state as for the PDC. However, this correlation is deceiving, because the fluctuations of a thermal state are well above the standard quantum limits, so that the difference between the (correlated) fluctuations in the two modes is at the standard quantum limit, as we shall see below.

In correlated imaging, the measurement limit is set by the relative difference-fluctuations between the (in this case) pair-wise correlated modes. If the difference in the measured signal through the object and the reference (e.g., the difference in transmitted photon number) is smaller that the statistical fluctuations between the two modes, it will be difficult to detect the difference between the object and the reference. Therefore, the uncorrelated fluctuations set a limit to the resolution of the correlation measurement, e.g., to how small an absorption difference that can be detected between the object and the reference. Because, in all considered cases we have assumed a symmetrical generation setup with respect of the object and reference mode (\( \langle \hat{c}^\dagger \hat{c} \rangle = \langle \hat{d}^\dagger \hat{d} \rangle \)), the uncorrelated intensity fluctuation variance is given by the expectation value of

\[ (\hat{c}^\dagger \hat{c} - \hat{d}^\dagger \hat{d})^2. \] (17)
The quadrature field difference fluctuation variance is given by the expectation value of

\[(\hat{c}_1 - \hat{d}_1)^2.\]  \hspace{1cm} (18)

Calculating the uncorrelated fluctuations in terms of mean photon number per mode for the six cases we get

\[\hat{V}_{I, PDC} = 0,\]  \hspace{1cm} (19)

\[\hat{V}_{i, PDC} = \frac{1}{2}(2\langle \hat{n} \rangle + 1 - 2\sqrt{\langle \hat{n} \rangle(\langle \hat{n} \rangle + 1)}),\]  \hspace{1cm} (20)

\[\hat{V}_{I, coh} = 2\langle \hat{n} \rangle,\]  \hspace{1cm} (21)

\[\hat{V}_{i, coh} = \frac{1}{2},\]  \hspace{1cm} (22)

\[\hat{V}_{I, th} = 2\langle \hat{n} \rangle, \quad \text{and}\]  \hspace{1cm} (23)

\[\hat{V}_{i, th} = \frac{1}{2}.\]  \hspace{1cm} (24)

From these equations it is clear that the PDC offers superior performance both for correlated intensity imaging and for correlated coherent imaging. However, in the latter case the imaging will be beset by practical difficulties. While the field quadratures are strongly correlated, they have a vanishing expectation value, so that they will randomly jump between positive and negative values (in a correlated fashion). This will take place on a time-scale of the (first-order) coherence time of the thermal source. That is, unless the thermal modes are very narrowly spectrally filtered, the fluctuations will occur on a time-scale that is too short for a typical homodyne detector to follow.

In Fig. 4 we have plotted the uncorrelated intensity fluctuation floor \(V_{I, PDC}\), etc., normalized to the standard quantum limit for two-mode intensity fluctuation difference, \(2\langle \hat{n} \rangle\). In Fig. 5 the uncorrelated (in-phase) quadrature fluctuation floor is plotted. In both plots the abscissa is chosen to be the mean photon number \(\langle \hat{n} \rangle\) in the one of the two output modes. As expected, the coherent light will show uncorrelated fluctuations just at the standard quantum limit, both for the intensity and for the field quadrature fluctuations. The great advantage in this case is that the fluctuations occur around a mean, so that using, e.g., the same laser to produce the local oscillator needed at the detection side of the coherent correlated imaging setup and the light used to produce the object and reference modes, one will minimize the problems associated with coherent imaging such as the frequency- and the phase-stability of the local oscillators.
The thermal source, finally, will also reach an uncorrelated fluctuation floor at the standard quantum limit. The disadvantage from the practical point of view is that this floor is far below the intensity fluctuation variance for each mode separately. Hence, any small imperfection in the cancellation of the correlated fluctuations between the object and reference mode will lead to a large fluctuation penalty that translates to a poor measurement resolution. Note that this is also the case for an imperfect PDC. However, from the fundamental point of view, it is possible to reach the same performance with a thermal source as with a coherent source. If one aims to make coherent correlated imaging, the field quadrature fluctuations of the thermal source have a vanishing mean, so just like the PDC, correlated coherent imaging with this source will present technical difficulties.

Please note that for typical photon numbers, for instance in 1 mW of light power and one ns counting time, the photon number per measured mode is on the order of $10^7$, implying that for all practical purposes correlated imaging using a laser would work perfectly! What, to date, appears to be a bit unclear within the quantum physics community is that correlated imaging measuring the electromagnetic quadratures also works well with classical coherent state sources, and even, in principle, for a thermal source. From an engineering viewpoint this is perhaps less surprising.

III. DISCUSSION

Given these results, most already pointed out in [7, 8, 9, 11, 12], we see that if one only measures one observable at a time, e.g., we image an object and record the object and the reference mode intensities, everything quantum can also be done classically, using a coherent state generated by a laser or a thermal light source. The price to be paid is lower signal-to-noise ratio for a fixed detected photon number. In terms of engineering, the classical correlated imaging can be done with less effort, and in terms of overall energy efficiency (taking into account the low pumping efficiency in non-linear optical processes) with fewer photons than working with a parametric source. For such cases, we argue that the justification for using quantum imaging can be questioned. Gatti et al. argue that the imaging can be done with different wavelengths in the object and in the reference mode, which is undoubtedly true. This is one advantage we can see. However, it is also possible to do intensity correlation imaging using two coherent states (lasers) operation at different
wavelengths. Their difference fluctuations will of course be uncorrelated, but this is already
the case when the two beams originate from the same laser as shown by Eq. (13). The other
potential advantage with quantum correlated intensity imaging is seen in the few photon
regime, where the strong quantum correlations offer a clear advantage, c.f. [12], see also
[16]. This correlation has been proposed to be used in military ranging applications [17].

The main new result we report is that nothing is gained by using a thermal source rather
than a coherent source when doing classical correlated imaging. One may be lead to believe
otherwise by looking at the pairwise correlations between modes in a split thermal beam. The
correlations are almost as strong as the correlations between parametrically generated signal
and idler beams. However, this strong correlation is deceiving, because the fluctuations in
each of the thermal modes are much higher for a thermal source than for an equally intense
coherent source. The imaging signal-to-noise ratio for the thermal and coherent sources are
the same, but the coherent source will make an experimental implementation simpler, in
particular if coherent imaging is employed.

What then is the essence of correlated quantum imaging? In our opinion, this is to be
found in the nature of entanglement, and how much this is exploited. For instance, as
discussed in section IV of [12], and realized experimentally in [18, 19], when modifying the
experiment in order to utilize the correlations between the different observables, one also
needs to modify the classical states needed to mimic the quantum statistics. That is, a
classical state can have strong correlations between some pair of observables, but unlike
a quantum state it cannot simultaneously have strong correlations in a complementary
pair of observables [18, 19]. This is the essence of all Bell-inequalities, where a quantum
state’s summed correlations between incompatible observables exceed the limit set for any
locally realistic theory, such as classical physics. Another case of interest is if one images
true quantum objects, i.e., one “captures” more than a single observable from the object.
In this meaning, quantum imaging becomes very much related to (multi-mode) quantum
teleportation, a relationship that could be worthwhile to explore further. However, as soon
as the “capture” is collapsed by a readout of some observable, we are back to the imaging
described above.
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FIG. 1: Schematics of correlated imaging. In (a), the source with the object and the reference depicted. In (b), the different light sources considered in this paper are schematically illustrated.
Fig. 2: Intensity correlation function for parametric down-conversion (PDC), a beam splitter with a coherent state input, and a beam splitter with a thermal state input.
FIG. 3: In-phase quadrature correlation function for parametric down-conversion (PDC), a beam splitter with a coherent state input, and a beam splitter with a thermal state input.
FIG. 4: Normalized intensity fluctuation floor for parametric down-conversion (PDC), a beam splitter with a coherent state input, and a beam splitter with a thermal state input. Note that the coherent state defines the so-called standard quantum limit, abbreviated SQL.
FIG. 5: In-phase quadrature correlation fluctuation floor for parametric down-conversion (PDC), a beam splitter with a coherent state input, and a beam splitter with a thermal state input. Note that the coherent state defines the so called standard quantum limit-SQL.

Fig. 5, Bogdanski et al., Coherent quantum