Test Sequencing Optimization Method based on MDP under Multi-value Tests

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Abstract. For the optimization of multi-value test in complex systems, this paper proposes a novel method based on Markov decision process, since the fault diagnosis process is a typical Markov process through analysing. The optimal test sequencing can be obtained as the solution of MDP is got. And the example also illustrates that the proposed method is efficient and scientific to design the optimal test sequence.

1. Introduction
As an important technique of fault diagnosis, the test sequence can achieve the fault detection and isolation with consuming the expected test cost as little as possible. The current test sequence optimization algorithms, such as Greedy algorithm, AND/OR graph (AO*) search algorithm and Rollout algorithm, are based on the fault-test dependency matrix. And the outcome of test is assumed to be binary which can only represent the pass or fail of the test. However, the outcome of binary test is Boolean values, so the system state information described is very limited.

Actually, in engineering practice, the state information of the system is multiple and the outcomes of tests are always multi-valued. More state information can be obtained with multi-value tests in comparison with binary tests, and the diagnosis efficiency and accuracy can be effectively improved, as well as reducing the test cost and time. [1-2] designed the optimal diagnosis strategy under multi-value tests based on the information entropy algorithm, which is easy to fall into local search. [3] proposed a search pruning techniques for strategy optimization method of multi-value test based on artificial intelligence, but it is for small systems only. [4-5] combined the Rollout algorithm and information entropy algorithm, the search result of which is superior to information entropy algorithm only, but it can’t ensure to obtain the global optimal solution.

Information entropy algorithm is a general heuristic search algorithm. Markov decision process (MDP) is an effective theory for sequential decision problems under uncertainty [6]. So in this paper, the fault diagnosis model is constructed based on MDP considering the uncertainty of test result, with the information entropy. The following parts give the details of the proposed method and the simulation of an example is verified for scientific validity.

2. Problem statement
In this work, the diagnosis strategy optimization problem of multivalued tests is under a single or none fault assumption. The problem can be further defined as follows.

1) $F = \{f_0, f_1, f_2, \cdots, f_m\} (m \geq 0)$ is a finite set of failure sources in a system, where $f_i (0 \leq i \leq m)$ denotes the $i$-th failure source and $f_0$ denotes the fault-free condition.
2) The a priori probability of each failure source, \( p(f) \) is known and \( \sum_{i=0}^{k} p(f_i) = 1 \).

3) \( T = \{t_1, t_2, \ldots, t_n\} \) (\( 1 \leq n \)) is a finite set of tests, where each test \( t_j (1 \leq j \leq n) \) can check a subset of failure sources.

4) The test cost set \( C = \{c_1, c_2, \ldots, c_m\} \) measured in terms of time or other economic factors, which corresponds to test set \( T \).

5) The fault-test dependency matrix \( D = [d_{ij}]_{m \times n} \), and \( d_{ij} \) represents the test attribute of the test to the fault. Also \( d_{ij} \) is an arbitrary integer and \( d_{ij} \in [0, k] \), which means the dimension of the outcomes is \( k+1 \).

Based on the above, the problem is to obtain an optimal test sequence with minimum test cost in long run to isolate the failure source to a definite LRU or SRU.

3. Fault diagnosis model based on MDP under multi-value tests

The fault diagnosis can be regarded as a process of reducing the ambiguity of system fault state [7], and outcomes of tests at each step are used to induce the possible fault state, which can be expressed as

\[
F_{jz} = \{f_i | d_y = z, \forall f_i \in F\}, 0 \leq z \leq k, t_j \in T
\]  

where \( F \) is the fault ambiguity state of the system before executing test \( t_j \), and it contains all possible failure sources before test. So the fault ambiguity set \( F \) is divided into \( k+1 \) subsets. And the next fault ambiguity state would be \( F_{jz} \), one of the \( k \) subsets, judging from the outcome of \( t_j \). Also it’s easy to find that the next fault state \( F_{jz} \) of the system has no relation to the fault states and tests before, but with current fault state and test being chosen. It is a typical Markov process and sequential decision problem, so the MDP is applied which is a general formalism for sequential decision problems to obtain the optimal test sequence in fault diagnosis.

Generally, MDP can be defined by a five-tuple

\[
\{S, A(i), K, p(j | i, a), r(i, a)\} \quad \{i, j \in S, a \in A\}
\]

where \( S \) is a finite set of states, which contains all the possible situations of the system at each time step; \( A(i) \) is a finite set of available actions at state \( i \) and \( A \) is the set of all possible actions; \( K \) is the set of time steps where decisions need to be made; \( p(j | i, a) \) is the probability function of system state \( i \) transfers to \( j \) after taking action \( a \), and \( \sum_{j=1}^{n} p(j | i, a) = 1 \); \( r(i, a) \) denotes the reward or payoff of taking action \( a \) as system is at state \( i \).

Combined the fault diagnosis process with the above MDP model, the MDP-based fault diagnosis model is constructed. Firstly, following Eq. (1), all the possible fault states \( F_{j0}, F_{j1}, \ldots, F_{jw} \) after test \( t_j \) is got. Then \( F_{j0} \) can be divided into two or more subsets by test \( t_l \in T (l \neq j) \), also \( F_{j1} \) can be divided similarly, and so on with others, as shown in Fig.1. And they are labelled as \( F_{j1}, F_{j2}, \ldots, F_{jw} (w \geq 1) \) in sequence. So \( S = \{F_1, F_2, \ldots, F_w\} \), where \( F_1=F \), and \( A=T \).

And the probability of state transferring from \( F \) to \( F_{jz} \) after taking test \( t_j \in T \) can be calculated as

\[
P(F_{jz} | F, t_j) = P(F_{jz}) = \sum_{f_i \in F_j} p(f_i)
\]

where \( z = 0, 1, \ldots, k \). Then the probability of failure sources in \( F_{jz} \) should be updated for the next division by test \( t_l \in T (l \neq j) \) or diagnosis, which is expressed by

\[
p'(f_j) = \frac{p(f_j)}{\sum_{f_i \in F_j} p(f_i)}
\]

Moreover, the bigger reward are expected in long-run and the information gain of each step is introduced, so the reward function can be confirmed by test cost and given by
\[ r(F, t_j) = -\sum_{j=0}^{k} P(F_{j'}) \log_2 P(F_{j'}) - c_j \] (4)

\[
V^\pi(F_i) = E^\pi \left[ r_{k} + \sum_{k=1}^{\gamma} P(s_k = F_i) \right] \\
= \sum_{t_k \in \pi} \pi(F_i, t_j)[r(F_i, t_j) + \gamma \sum_{F_q \in S} P(F_q | F_i) V^\pi(F_q)], \pi \in \Pi
\] (5)

where \( \gamma \) is the discounter factor, \( E^\pi[\ast] \) is the expectation operation, \( \Pi \) is the strategy space, and \( s_k \) is the state at time step \( k \). Also the optimal value function can be obtained as

\[
V^{\pi^*}(F_i) = \sup_{x \in \Omega} V^{\pi}(F_i) = \max_{t_k \in \pi^*} \left[ r(F_i, t_k) + \gamma \sum_{t_n \in s} P(F_q | F_i, t_n) V^{\pi^*}(F_q) \right]
\] (6)

where \( F_1 \) could be other fault state. And the optimal strategy or test sequence \( \pi^* \) is obtained as

\[
\pi^* = \arg \max_{x \in \Omega} V^{\pi}(F_i)
\] (7)

That’s to say, \( \pi^* \) is obtained by solving Eqs. (6) and (5), and the policy iteration is powerful to do that.

4. Case study

The example in [5] is analyze to verify the proposed method based on MDP. There are 6 failure sources and 4 tests in this example. And each test has 4 outcomes at most. This is multi-value test system with dimension of 4. Then the fault states in the diagnosis process can deduced according to Eq. (1) as shown in Table 2. So \( S = \{F_1, F_2, \ldots, F_6\} \) and \( A = \{t_1 \sim t_4\} \).

Table 1. Dependency matrix of the system.

| FT  | \( t_1 \) | \( t_2 \) | \( t_3 \) | \( t_4 \) | \( p(f) \) |
|-----|--------|--------|--------|--------|--------|
| \( f_0 \) | 1      | 1      | 1      | 0      | 0.70   |
| \( f_1 \) | 2      | 1      | 2      | 1      | 0.01   |
| \( f_2 \) | 3      | 0      | 0      | 0      | 0.02   |
| \( f_3 \) | 1      | 0      | 0      | 0      | 0.10   |
Table 2. Dependency matrix of the system.

| State | Failure | State | Failure | State | Failure | State | Failure |
|-------|---------|-------|---------|-------|---------|-------|---------|
| $F_1$ | $f_0$   | $F_2$ | $f_0$   | $F_3$ | $f_0$   | $F_4$ | $f_0$   |
| $F_5$ | $f_3$   | $F_6$ | $f_3$   | $F_7$ | $f_3$   | $F_8$ | $f_3$   |
| $F_9$ | $f_1$   | $F_{10}$ | $f_4$   | $F_{11}$ | $f_3$   | $F_{12}$ | $f_4$   |
| $F_{13}$ | $f_0$ | $F_{14}$ | $f_3$   | $F_{15}$ | $f_5$   | $F_{16}$ | $f_3$   |

The probabilities of state transitions can be obtained following Eqs. (2) and (3). For example, $P(F_2 | F_1, t_1) = 0.7 + 0.1 + 0.05 = 0.85$ , $P(F_3 | F_1, t_1) = 0.01 + 0.12 = 0.13$ , and $P(F_4 | F_1, t_1) = 0.02$ . The other probabilities will not be list one by one for limited space. The MDP toolbox is used to get the optimal solution in this paper with policy iteration algorithm, with the value of $V$-function for each state, and the results are given in Figure 2.

Figure 2. Optimal test sequencings and utility values.

Figure 3. Optimal diagnosis tree.

5. Conclusion
This paper provides a novel method to get the optimal test sequence under multi-value test, and the diagnosis process is regarded as a MDP. The information theory is introduced as well as the test cost to get the optimal test sequence. And the example also demonstrates the validity and efficiency of the proposed method which can guide the fault detection and isolation fast.

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