van Dam-Veltman-Zakharov discontinuity in
topologically new massive gravity

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Abstract

We study van Dam-Veltman-Zakharov discontinuity in the topologically new massive gravity (TNMG). The reduction from 2 degrees of freedom to one is interpreted as van Dam-Veltman-Zakharov discontinuity appeared when going from anti-de Sitter spacetime to Minkowski spacetime in the linearized TNMG.

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1 Introduction

Recently, there is a debate on the local degrees of freedom (DOF) of graviton on the topologically new massive gravity (TNMG) \[1\]. It turned out that the linearized TNMG provides a single spin-2 mode with mass $m^2/\mu$ in the Minkowski spacetime \[1, 2\]. However, it was shown that when using the Hamiltonian formulation where non-linear effect is not neglected, its DOF is 2 \[3\]. Thus, the reduction (2 → 1) of local DOF is considered as an artefact of the linearized approximation in the Minkowski spacetime. It implies that the TNMG suffers from a linearization instability. This reduction is related to the emergence of a linearized Weyl (conformal) symmetry at the linearized TNMG \[1\].

On the other hand, there were debates in the massless limit of the massive graviton propagator in four dimensional Einstein gravity \[4, 5, 6, 7, 8\]. An important issue of this limit indicates that van Dam-Veltman-Zakharov (vDVZ) discontinuity \[9\] is peculiar to Minkowski spacetime, but it seems unlikely to arise in (anti) de Sitter space. The vDVZ discontinuity implies that the limit of $M^2 \to 0$ does not yield a massless graviton at the linearized level. For this purpose, one may use the Fierz-Pauli mass term with mass squared $M_{FP}^2$ \[10\].

If the cosmological constant ($\Lambda$) was introduced \[6\], a smooth $M_{FP}^2/\Lambda \to 0$ limit exists. We note that $M_{FP}^2 \to 0$ and $\Lambda \to 0$ limits do not commute. Taking the $M_{FP}^2 \to 0$ limit first and then the $\Lambda \to 0$ limit recovers a massless graviton with 2 DOF, leading to no vDVZ discontinuity in the Einstein gravity. Taking the $\Lambda \to 0$ limit first, one encounters the vDVZ discontinuity with 3 DOF. If one-loop graviton vacuum amplitude is computed for a massive graviton \[11\], the discontinuity appears again in anti-de Sitter spacetime. It may imply that the absence of the vDVZ discontinuity may be considered as an artefact of the linearized approximation. Also, there was the Boulware-Deser instability which states that at the non-linearized level, a ghost appears again in the massive gravity theory \[12\].

In this Report, we show that the reduction in the number of local DOF is considered as the van Dam-Veltman-Zakharov discontinuity in the linearized TNMG which appears when going from anti-de Sitter to Minkowski spacetime. Here we will not consider the issues arising when interactions are included.

We start with the cosmological generalized massive gravity (GMG) action \[13, 14\]

$$S_{cGMG} = \frac{1}{16\pi G} \int d^3x \sqrt{-g} \left[ \sigma R - 2\Lambda_0 + \frac{1}{m^2} K + \frac{1}{\mu} L_{CS} \right],$$ \[1\]
where $K$ ($L_{\text{CS}}$) is the new massive gravity (NMG) term (the gravitational Chern-Simons term) given by

$$K = R_{\mu\nu}R^{\mu\nu} - \frac{3}{8}R^2,$$

(2)

$$L_{\text{CS}} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\Gamma^\alpha_{\mu\rho} \left[ \partial_\nu \Gamma^\beta_{\alpha\rho} + \frac{2}{3} \Gamma^\beta_{\nu\gamma} \Gamma^\gamma_{\rho\alpha} \right].$$

(3)

Here $m$ and $\mu$ are the two mass parameters, while $\sigma$ is a dimensionless sign parameter. Also $\Lambda_0$ is the cosmological constant, and (1) leads to GMG without it. The pure-$K$ is the massless new massive gravity (NMG) [15] and the pure-$L_{\text{CS}}$ is the conformal Chern-Simons gravity (CSG) [16]. The topologically massive gravity (TMG) is a combination of $\sigma R + L_{\text{CS}}$ [17]. In the limits of $\sigma \to 0$ and $\Lambda_0 \to 0$, one recovers the TNMG action [1, 18]. The limit of $\sigma \to 0$ is called the cosmological TNMG.

Einstein equation of motion takes the form

$$\sigma G_{\mu\nu} + \Lambda_0 g_{\mu\nu} + \frac{1}{2m^2}K_{\mu\nu} + \frac{1}{\mu}C_{\mu\nu} = 0,$$

(4)

where

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R,$$

(5)

$$K_{\mu\nu} = -\frac{1}{2}\nabla^2 Rg_{\mu\nu} - \frac{1}{2} \nabla_\mu \nabla_\nu R + 2\nabla^2 R_{\mu\nu} + \frac{3}{2} R^2 R_{\mu\nu} - R_{\alpha\beta} R^{\alpha\beta} g_{\mu\nu} + \frac{3}{8} R^2 g_{\mu\nu},$$

(6)

and the Cotton tensor is given by

$$C_{\mu\nu} = \epsilon_\mu{}^{\alpha\beta} \nabla_\alpha \left( R_{\beta\nu} - \frac{1}{4} g_{\beta\nu} R \right).$$

(7)

In this work, we consider three dimensional anti de Sitter (AdS$_3$) spacetimes

$$ds^2_{\text{AdS}} = \bar{g}_{\mu\nu}dx^\mu dx^\nu = \ell^2 \left( - \cosh^2 \rho d\tau^2 + d\rho^2 + \sinh^2 \rho d\phi^2 \right).$$

(8)

In this case, one finds a relation among $m^2$, $\Lambda_0$, and $\Lambda$ as

$$\Lambda_0 = \frac{\Lambda^2}{4m^2} + \sigma \Lambda, \quad \Lambda = -\frac{1}{\ell^2},$$

(9)

where $\ell$ is the AdS$_3$ curvature radius. Now, we consider the perturbation

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

(10)
around the AdS$_3$ spacetime. Then, the linearized Einstein equation takes the form

$$\sigma G^{(1)}_{\mu\nu} + \Lambda_0 h_{\mu\nu} + \frac{1}{2m^2} K^{(1)}_{\mu\nu} + \frac{1}{\mu} C^{(1)}_{\mu\nu} = 0,$$

(11)

where their linearized tensors were given in Ref. [19]. Let us choose the transverse-traceless (TT) gauge

$$\bar{\nabla}_\mu h^{\mu\nu} = 0, \ h^\mu_\mu = 0.$$  

(12)

Upon choosing the TT gauge, the linearized Einstein equation becomes the fourth-order differential equation

$$\left(\bar{\nabla}^2 - 2\Lambda\right)\left[\bar{\nabla}^2 h_{\mu\nu} + \frac{m^2}{\mu} \epsilon^a_\mu \bar{\nabla}_a h_{\beta\nu} + \left(\sigma m^2 - \frac{5}{2}\Lambda\right) h_{\mu\nu}\right] = 0.$$  

(13)

Introducing four mutually commuting operators of

$$(D^{L/R})^\beta_\mu = \delta^\beta_\mu \pm \ell \epsilon^\alpha_\mu \bar{\nabla}_\alpha, \quad (D^{m_i})^\beta_\mu = \delta^\beta_\mu + \frac{1}{m_i} \epsilon^\alpha_\mu \bar{\nabla}_\alpha, \quad (i = 1, 2),$$

(14)  

(15)

the linearized equation of motion (13) can be written to be compactly

$$\left(D^R D^L D^{m_1} D^{m_2} h\right)_{\mu\nu} = 0.$$  

(16)

Here, two masses are given by

$$m_1 = \frac{m^2}{2\mu} + \sqrt{\frac{m^4}{4\mu^2} - \sigma m^2 - \frac{\Lambda}{2}},$$

$$m_2 = \frac{m^2}{2\mu} - \sqrt{\frac{m^4}{4\mu^2} - \sigma m^2 - \frac{\Lambda}{2}},$$

(17)

where their combinations are given by

$$m_1 + m_2 = \frac{m^2}{\mu}, \quad m_1 m_2 = \sigma m^2 + \frac{\Lambda}{2}.$$  

(18)

Firstly, we consider the case of $\Lambda = 0$. This corresponds the GMG, where two massive modes are propagating on the Minkowski spacetime. Their masses $m_1$ and $m_2$ all are positive for $\sigma = 1$ and $4 < m^2/\mu^2$ as they take the forms

$$m_1 = \frac{m^2}{2\mu} \left[1 + \sqrt{1 - \frac{4\mu^2}{m^2}}\right], \quad m_2 = \frac{m^2}{2\mu} \left[1 - \sqrt{1 - \frac{4\mu^2}{m^2}}\right].$$  

(19)
This means that there is no reduction of local DOF when going from AdS$_3$ to Minkowski spacetime unless the Einstein-Hilbert action is omitted.

Secondly, let us consider the case without the Einstein-Hilbert action by choosing $\sigma = 0$, leading to the cosmological TNMG. If $\Lambda \neq 0$, there is no linearized Weyl symmetry. This means that the linearized equation (11) does not exhibit an additional gauge symmetry acting on $h_{\mu\nu} : h_{\mu\nu} \rightarrow h_{\mu\nu} + 2\Omega\bar{g}_{\mu\nu} - 2\ell^2 \nabla_\mu \nabla_\nu \Omega$. Here $\Omega$ is the linearized Weyl factor. Therefore, two masses take the forms

$$
m_1 = \frac{m^2}{2\mu} \left[ 1 + \sqrt{1 + \frac{2\mu^2|\Lambda|}{m^4}} \right],
$$

$$
m_2 = \frac{m^2}{2\mu} \left[ 1 - \sqrt{1 + \frac{2\mu^2|\Lambda|}{m^4}} \right]
$$

with $|\Lambda| = 1/\ell^2$. It is observed from $(D^{mn}h)_{\mu\nu} = 0$ that for $2|\Lambda| < m^4/\mu^2$, one has two massive modes with masses

$$
m_1 \simeq \frac{m^2}{\mu}, \quad m_2 \simeq -\frac{\mu|\Lambda|}{2m^2}
$$

in AdS$_3$ spacetime. Even though $m_2$ seems to be a tachyonic mass in the Minkowski spacetime, it might not be a tachyonic mass in the AdS$_3$ spacetime. For AdS vacua, it is well-known that unitarity allows scalar fields to have a negative mass squared if the Breitenlohner-Freedman (BF) bound is satisfied [20]. For a massive scalar propagation in AdS$_3$ spacetime, this bound is given by

$$
M^2 \geq m_{BF}^2 = -|\Lambda| = -\frac{1}{\ell^2}. \tag{22}
$$

It has been argued that the same bound could apply to spin-2 propagation [21]. However, this bound is not applicable to the tensor propagation which satisfies the first-order equation because of its mass squared form. Observing the right-tensor gauge mode satisfying $(D^{R}h)_{\mu\nu} = 0$ in (16) [22], we might choose the mass bound for the tensor propagation as

$$
M \geq -\frac{1}{\ell} = -\sqrt{|\Lambda|}. \tag{23}
$$

Hence, if the negative mass $m_2$ satisfies the inequality

$$
-\frac{1}{\ell} < m_2 < 0, \tag{24}
$$

the massive tensor mode with $m_2$ is not a tachyonic mode in the AdS$_3$ spacetime.
We are in a position to discuss the following three limiting cases:

i) Taking $\Lambda \to 0$ for keeping $m^4/\mu^2$ fixed leads to one DOF with mass

$$m_1 = \frac{m^2}{\mu},$$

(25)

which corresponds to a single massive graviton propagating on the Minkowski spacetime [1]. The $m_2 = 0$ case corresponds to a massless gauge degree of freedom.

ii) Taking $\Lambda \to 0$ limit first (and then, the $m^4/\mu^2 \to 0$ limit) recovers the massless NMG with $m_1 = 0$ and $m_2 = 0$ [15], leading to no massive DOF.

iii) Taking $m^4/\mu^2 \to 0$ limit first (and then, the $\Lambda \to 0$ limit) recovers the massless NMG with $m_1 = 0$ and $m_2 = 0$, leading to no massive DOF. Actually, there is no distinction between ii) and iii).

For 4D Einstein gravity with cosmological constant, one has 2 DOF for a massless graviton, while one has 5 DOF for a massive graviton. The reduction of DOF is from 5 to 3 when taking $\Lambda \to 0$ limit first and then the $M_\text{FP}^2 \to 0$ limit, leading to the vDVZ discontinuity. Another reduction (5 $\to$ 2) is done by taking $M_\text{FP}^2 \to 0$ limit first and then the $\Lambda \to 0$ limit, leading to no vDVZ discontinuity.

At this stage, we have to define the vDVZ discontinuity differently in three dimensions. Although a single spin-2 mode of mass $m_1$ becomes the massless mode of massless NMG in the limit of $\mu \to \infty$ limit [15], we consider the massive mode as the physically propagating mode in the Minkowski spacetime. This is because there is no massless DOF for graviton as is shown $D(D - 3)/2|_{D=3} = 0$ DOF. In general, the massless modes are pure gauge, whereas the massive modes constitute physical degrees of freedom. In this sense, it would be say that the massless NMG (3D Einstein gravity, CSG) has no physical DOF in the linearized approximation. However, TMG has a single massive DOF, while GMG has two massive DOF. This means that the massless limit of $m^4/\mu^2 \to 0$ is not necessary to define the vDVZ discontinuity in the linearized approximation. We do not need to introduce the propagator approach to define the vDVZ discontinuity in three dimensions. What we want to do is to check what happens for masses in the limit of $\Lambda \to 0$. The cosmological TNMG has 2 DOF with different masses [21]. Also, we note that the cosmological term “$\Lambda_0 h_{\mu\nu}$” in (11) breaks a linearized Weyl symmetry which exists at the linearized TNMG. This explains a reason why the cosmological TNMG has two massive modes. This two massive DOF is reduced to one ($m_1 = m^2/\mu, \ m_2 = 0$) when going from AdS$_3$ spacetime to Minkowski spacetime ($\Lambda \to 0$). Hence, it is reasonable to insist that this reduction is called the vDVZ discontinuity in three dimensions. That is, the reduction of 2 $\to$ 1 happens only when changing from AdS$_3$ to Minkowski
spactime. In this case, the linearized Weyl symmetry is restored and, the partial massless is achieved at the linearized level.

Consequently, the reduction from 2 degrees of freedom to one could be interpreted as van DVZ discontinuity in the linearized TNMG. This discontinuity appeared when going from anti-de Sitter to Minkowski spacetime (cosmological TNMG $\rightarrow$ TNMG). Partial massless of $m_1 = m^2/\mu$ and $m_2 = 0$ appeared in the limit of $\Lambda = 0$ is dubbed the vDVZ discontinuity.

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