A New Description of the Superstring

Nathan Berkovits

Dept. de Física Matemática, Univ. de São Paulo
CP 20516, São Paulo, SP 01498, BRASIL
and
IMECC, Univ. de Campinas
CP 1170, Campinas, SP 13100, BRASIL

e-mail: nberkovi@snfma2.if.usp.br

This is a review of the new manifestly spacetime-supersymmetric description of the superstring. The new description contains \(N=2\) worldsheet supersymmetry, and is related by a field redefinition to the standard RNS description. It is especially convenient for four-dimensional compactifications since \(SO(3,1)\) super-Poincaré invariance can be made manifest. Parts of this work have been done in collaboration with Warren Siegel and Cumrun Vafa.

This review is based on lectures given at the VIII J.A. Swieca summer school and should be easily accessible to anyone familiar with the RNS superstring description.

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1. Introduction

This review is based on five lectures given at the VIII J.A. Swieca summer school in Rio de Janeiro in 1995. The first half of the lectures consisted of an introduction to string theory, while the second half concentrated on the new spacetime-supersymmetric description of the superstring. Since much better introductions to string theory are available, this review will only describe the second half of the lectures.

There are two ways to look for a manifestly spacetime-supersymmetric description of the superstring. One can either try to spacetime-supersymmetrize the Ramond-Neveu-Schwarz description, or one can try to covariantize the light-cone Green-Schwarz description. As will be shown in this review, both of these approaches lead to the same answer. Although historically, the GS approach was used earlier, I shall start with the RNS approach since it is familiar to a wider audience.

After reviewing the RNS superstring in section 2, I discuss the new spacetime-supersymmetric description in section 3. This new description has critical N=2 worldsheet superconformal invariance and is related to the RNS description by a field redefinition. For compactifications to four dimensions, it allows the full SO(3,1) super-Poincaré invariance to be made manifest.[1]

In section 4, I introduce the N=4 topological method, which was developed together with Vafa. This method can be used to calculate scattering amplitudes for any string with critical N=2 superconformal invariance, and is much simpler than conventional N=2 techniques. [2]. In section 5, I discuss the relationship with conventional and twistor-like Green-Schwarz descriptions, and in section 6, I conclude with a list of applications for the new superstring description.

2. Review of the RNS Description

2.1. Worldsheet fields and N=1 superconformal generators

The standard RNS description of the superstring is a critical c=15 representation of N=1 worldsheet superconformal invariance.[3] The simplest such representation (corresponding to an uncompactified ten-dimensional manifold) consists of ten bosonic worldsheet scalars, \( x^\mu \) (\( \mu = 0 \) to 9), and ten fermionic worldsheet spinors, \( \psi^\mu_L, \psi^\mu_R \). (\( \psi^\mu_L \) and \( \psi^\mu_R \) are left and right-moving components of the worldsheet spinor. When the \( L/R \) index is suppressed, we shall always mean the left-moving component. Although this review will
only discuss the open and closed superstring, all methods can easily be extended to the heterotic superstring.)

The action and left-moving c=15 N=1 superconformal generators for this representation are

\[
S = \frac{1}{2\pi} \int d^2 z \left( \frac{1}{2} \partial x^\mu \partial x_\mu + i\psi_L^{\mu} \bar{\partial} \psi_{L\mu} + i\psi_R^{\mu} \partial \psi_{R\mu} \right)
\]

\[
T = \frac{1}{2} \partial x^\mu \partial x_\mu + \frac{i}{2} \psi^\mu \partial \psi_\mu,
\]

\[
G = \psi^\mu \partial x_\mu
\]

where \(\partial = \frac{\partial}{\partial z}\) and \(\bar{\partial} = \frac{\partial}{\partial \bar{z}}\). Because the action is quadratic, the worldsheet fields have the free-field OPE’s:

\[
x^\mu(y)x^\nu(z) \rightarrow \eta^{\mu\nu} \log |y - z|^2, \quad \psi^\mu(y)\psi^\nu(z) \rightarrow i\eta^{\mu\nu}(y - z)^{-1}
\]

as \(y \rightarrow z\) (\(\eta^{\mu\nu} = (+, -, ..., -)\)). Using these OPE’s, it is straightforward to show that the above generators satisfy the c=15 N=1 super-Virasoro algebra:

\[
G(y)G(z) \rightarrow \frac{10i}{(y - z)^3} + \frac{2iT}{y - z}, \quad T(y)T(z) \rightarrow \frac{15}{2(y - z)^4} + \frac{2T}{(y - z)^2} + \frac{\partial T}{y - z}.
\]

To gauge-fix the N=1 superconformal invariance, one needs a c=−15 ghost system consisting of a fermionic pair \((b, c)\) with weights 2 and −1, and a bosonic pair \((\beta, \gamma)\) with weights \(\frac{3}{2}\) and −\(\frac{1}{2}\). For constructing Ramond states and spacetime-supersymmetry generators, it is convenient to fermionize the bosonic ghosts as

\[
\beta = -i\partial \xi e^{-\phi}, \quad \gamma = i\eta e^\phi
\]

where \((\eta, \xi)\) are fermionic with weights (1,0), and \(\phi\) is a chiral boson with background charge +2. The OPE’s for these fields are

\[
b(y)c(z) \rightarrow i(y - z)^{-1}, \quad \xi(y)\eta(z) \rightarrow i(y - z)^{-1}, \quad \phi(y)\phi(z) \rightarrow -\log(y - z) + \frac{i\pi}{2}
\]

as \(y \rightarrow z\).

Because the \(\xi\) zero mode can not be expressed in terms of \(\beta\) and \(\gamma\), this fermionization procedure is only invertible on the “small” Hilbert space, defined as the space of states \(\Phi\) satisfying \(\oint d\bar{z}\eta, \Phi\) = 0. The “large” Hilbert space, on the other hand, is defined to also include states which are proportional to the \(\xi\) zero mode.
2.2. Physical states

A nilpotent BRST operator for this critical N=1 system is defined by

\[ Q = \frac{1}{2\pi} \oint dz \left[ e(T + i b \partial c + \frac{1}{2} \partial \phi \partial \phi + \partial^2 \phi + i \eta \partial \xi) + i \eta e^\phi G + i b \eta \partial \eta e^{2\phi} \right] \]  

(2.6)

where \( T \) and \( G \) are defined in (2.1). Physical states are described by vertex operators \( V \) which are in the BRST cohomology of \( Q \), i.e. \( \{Q, V\} = 0 \) and \( V \neq [Q, \Lambda] \) for any \( \Lambda \) in the small Hilbert space. Note that if \( \Lambda \) were allowed to live in the large Hilbert space, the cohomology would be trivial since \( \{Q, V\} = 0 \) implies \( V = [Q, ic \xi \partial \xi e^{-2\phi} V] \). Depending on the boundary conditions of \( \psi^\mu \), the vertex operator can represent either a spacetime boson (Neveu-Schwarz boundary conditions) or a spacetime fermion (Ramond boundary conditions).

Besides being in the BRST cohomology, a vertex operator must also satisfy the following four conditions in order to uniquely represent a physical state:

1) The first condition is that the vertex operator is hermitian, i.e. \( V = V^\dagger \). This type of reality condition is expected since \( V \) is interpreted as a second-quantized field, rather than a first-quantized wave-function.

2) The second condition on the physical vertex operator is that it is fermionic and GSO projected, i.e. has no square-root cuts with the spacetime-supersymmetry generator

\[ q_a = \frac{1}{2\pi i} \oint dz e^{\frac{1}{2} (i \phi \pm \sigma_0 \pm \sigma_1 \pm \sigma_2 \pm \sigma_3 \pm \sigma_4)} \]  

(2.7)

where \( \psi^9 \pm \psi^0 = e^{\pm i \sigma_0} \), \( \psi^j \pm i \psi^{j+4} = e^{\pm i \sigma_j} \) for \( j = 1 \) to 4, and there are an even number of + signs in the exponent (\( a = 1 \) to 16). The spacetime-supersymmetry generator of (2.7) transforms Neveu-Schwarz boundary conditions into Ramond boundary conditions, and this second condition ensures that the physical spectrum is spacetime-supersymmetric.

3) The third condition comes from the fact that each physical state is represented by infinitely many vertex operators in the BRST cohomology. This is because of the existence of the picture-changing operator,

\[ Z = \{Q, \xi\} = ie^\phi \psi_\mu \partial x^\mu + i b \partial \eta e^{2\phi} + i \partial (b \eta e^{2\phi}) + ic \partial \xi, \]  

(2.8)

and the inverse picture-changing operator,

\[ Y = ic \partial \xi e^{-2\phi}. \]
Since \([Q, Z] = 0, [Q, V] = 0\) implies that \([Q, ZV] = 0\). Also \(ZV = \{Q, \Lambda\}\) implies that \(ZV\) is in the BRST cohomology if \(V\) is. Similarly \(Z^nV\) and \(Y^nV\) are also in the BRST cohomology for arbitrary positive integer \(n\).

In order to choose a unique vertex operator for each physical state, it is convenient to define the “picture-counting” operator

\[
P = \frac{1}{2\pi} \oint dz (i\xi \eta - \partial \phi).
\]

If \([P, V] = ipV\) where \(p\) is the “picture” of \(V\), then \([P, Z^nV] = i(p + n)V\) and \([P, Y^nV] = i(p - n)V\). One can therefore fix this overcounting by demanding that \(V\) sits in a certain picture (for example, one could demand that all Neveu-Schwarz vertex operators have picture \(-1\) and all Ramond vertex operators have picture \(-\frac{1}{2}\)). However, since the spacetime-supersymmetry generators of (2.7) carry picture, such a restriction breaks manifest spacetime supersymmetry.

4) The fourth condition on the physical vertex operator is that it has ghost-number +1. Although the ghost-number operator is usually defined by \(\frac{1}{2\pi} \oint dz (ibc + \beta \gamma) = \frac{1}{2\pi} \oint dz (ibc + \partial \phi)\), we shall instead define it by

\[
J_g = \frac{i}{2\pi} \oint dz (bc + \xi \eta)
\]

so that ghost-number commutes with picture-changing (i.e. \(V\) has the same ghost-number as \(Z^nV\)). Note that at zero picture, the two definitions of ghost-number coincide.

2.3. Scattering amplitudes

Superstring scattering amplitudes are calculated by evaluating correlation functions of BRST-invariant vertex operators on N=1 super-Riemann surfaces. Although the details of multiloop amplitude calculations are complicated, it will be useful to sketch the multiloop expression before concentrating on the tree-level amplitude. For the closed-string scattering of \(n\) states described by the vertex operators \(V_r (r = 1 \text{ to } n)\), the RNS expression for the \(g\)-loop scattering amplitude is

\[
\lambda^{2g-2} \sum_{l=1}^{2^g} \prod_{i=1}^{3g-3+n} \int d^2\tau_i < |\xi(z_0)\prod_{j=1}^g \oint_{A_j} dz_j \eta(z_j) \delta(\oint_{A_j} dz_j (\partial \phi - i\xi \eta)) > \sum_{r=1}^n \prod_{s=1}^{2g-2+n-p} Z(y_s)^2 \prod_{r=1}^n V_r(\tau_r) >
\]
where $I$ labels the $2^{2g}$ spin-structures for the worldsheet spinors (for each of the $2g$ non-trivial loops on a genus $g$ surface, one can choose periodicity or anti-periodicity conditions for the worldsheet spinors), $\tau_i$ are Teichmuller parameters used to describe the genus $g$ Riemann surface with $n$ punctures (the first $n$ of these $3g-3+n$ parameters are the locations of the punctures), $\mu_i$ are the corresponding Beltrami differentials, $\oint_{A_j}$ is a contour integral around the $j^{th}$ $A$-cycle of the surface, and the sum of the pictures of the external vertex operators is equal to $p$.

The term
$$\xi(z_0) \prod_{j=1}^{g} \oint_{A_j} dz_j \eta(z_j) \delta\left( \oint_{A_j} dz_j(\partial \phi - i \xi \eta) \right)$$

(2.11)
comes from the need to restrict the picture of states propagating in internal loops. Without this restriction, each propagating state would be represented infinitely many times, violating unitarity. Although not obvious, it can be checked that this term does not violate modular invariance (i.e. the choice of $A$ versus $B$-cycles is irrelevant). Note that the location of $z_0$ is arbitrary since only the $\xi$ zero mode contributes.

The term $\prod_{s=1}^{2g-2+n-p} Z(y_s)$ comes from integrating over the fermionic supermoduli of the N=1 super-Riemann surface where the $y_s$ locations depend on the parameterization of the supermoduli. One would expect that the scattering amplitude should be independent of the parameterization of the supermoduli, and therefore, independent of the choice of the $y_s$ locations. Indeed, it is straightforward to show that changing the locations of the $y_s$'s shifts the correlation function by a total derivative in the $\tau_i$ parameters, which can be ignored if the region of integration for the $\tau'_i$s can be compactified (i.e. if there are no singularities near the boundary of moduli space).

Note that the above expression for the scattering amplitude is only spacetime-supersymmetric after summing over the $2^{2g}$ spin-structures since, before summing, the correlation functions contain unphysical square-root cuts. This makes it difficult to check finiteness properties in the RNS formalism since, before summing over spin-structures, the amplitude contains many unphysical divergences. As will be later shown, the new description of the superstring does not suffer from this problem since all fields are automatically GSO-projected, and there are therefore no square-root cuts and no need to sum over spin structures.

The tree-level scattering amplitude is much simpler and is given by the expression
$$\lambda^{-2} < V_1(z_1)V_2(z_2)V_3(z_3)Z_L(z_3)Z_R(\bar{z}_3)$$

(2.12)
\[
\prod_{r=4}^{n} \int d^2z_r \langle b_L, \{ b_R, V_r(z_r) \} \rangle Z_L(z_r) Z_R(\bar{z}_r) >
\]

where \( \{ b, V \} \) means the contour integral of \( b \) around \( V \), \( V_r \) is in the \(-1\) picture, and the amplitude is independent of \( z_1, z_2, \) and \( z_3 \).

Despite the remarkable simplicity of this expression, it has two problems which will be partially cured in the new superstring description. Firstly, although the amplitude is spacetime-supersymmetric (i.e., \( A(\{ q_\alpha, V_1 \}, ..., V_n \) + ... + \( A(V_1, ..., \{ q_\alpha, V_n \} ) = 0 \), the spacetime-supersymmetry is far from manifest since Ramond vertex operators look very different from Neveu-Schwarz vertex operators. A second disadvantage is that explicit calculations require SO(9,1) Poincaré invariance to be broken to SU(5) (actually, a Wick rotated version of SU(5)) in order that Ramond fields can be expressed in terms of the bosonized \( \sigma_j \)'s of (2.7).

### 2.4. Four-dimensional compactifications

The N=1 c=15 superconformal representation with ten \( x \)'s and ten \( \psi \)'s contains sixteen spacetime-supersymmetries, and is the most symmetric representation for the superstring. However one can also construct consistent superstring representations with fewer spacetime supersymmetries.

One such representation is the “four-dimensional” superstring, which contains four bosonic worldsheet scalars \( x^m \) (\( m = 0 \) to 3), four fermionic worldsheet spinors \( \psi^m_{L,R} \), and a c=9 N=2 superconformal field theory which is described by the N=2 generators \([T_C, G^+_C, G^-_C, J_C]\). For different choices of the c=9 N=2 superconformal field theory, this representation can be used to describe any four-dimensional compactification of the ten-dimensional superstring which preserves at least N=1 4D spacetime-supersymmetry.[6]

The action and c=15 N=1 generators for this superconformal representation are:

\[
S = \frac{1}{2\pi} \int d^2z (\frac{1}{2} \partial x^m \partial x_m + i \psi^m_L \partial \psi_{Lm} + i \psi^m_R \partial \psi_{Rm} ) + S_C \quad (2.13)
\]

\[
T = \frac{1}{2} \partial x^m \partial x_m + \frac{i}{2} \psi^m_L \partial \psi_{Lm} + T_C
\]

\[
G = \psi^m \partial x_m + G^+_C + G^-_C
\]

where \( S_C \) is chosen such that \([T_C, G_C, \bar{G}_C, J_C]\) satisfy the following c=9 N=2 OPE’s:

\[
G^+_C(y) G^-_C(z) \rightarrow \frac{3i}{(y-z)^3} + \frac{J_C(z)}{(y-z)^2} + \frac{iT_C + \frac{1}{2} \partial J_C}{y-z}, \quad (2.14)
\]
\[ T_C(y)T_C(z) \rightarrow \frac{9}{2(y-z)^4} + \frac{2T_C}{(y-z)^2} + \frac{\partial T_C}{y-z}. \]

The simplest example of such an \( S_C \) is

\[ S_C = \frac{1}{2\pi\alpha'} \int d^2 z (\partial x^j \bar{\partial} \bar{x}_j + 2i\psi^i_L \bar{\partial} \bar{\psi}_{Lj} + 2i\psi^i_R \partial \bar{\psi}_{Rj}) \tag{2.15} \]

where \( j = 1 \) to 3. It is easy to check that the action and N=1 generators of (2.1) are the same as those of (2.13) where \( x^j \) is identified with \( 2^{-\frac{1}{2}}(x^3+j + ix^6+j) \), \( \bar{x}_j \) is identified with \( 2^{-\frac{1}{2}}(x^3+j - ix^6+j) \), \( \psi^j \) is identified with \( 2^{-\frac{1}{2}}(\psi^3+j + i\psi^6+j) \), and \( \bar{\psi}^j \) is identified with \( 2^{-\frac{1}{2}}(\psi^3+j - i\psi^6+j) \). This representation therefore corresponds to an uncompactified superstring (if \( x^j \) takes values in \( R^6 \)) or a toroidally-compactified superstring (if \( x^j \) takes values in \( T^6 \)).

As before, one can add an N=1 ghost system and construct a nilpotent BRST operator. Physical vertex operators must satisfy the same conditions as before, however there are now only four spacetime-supersymmetry generators defined by:

\[ q_a = \frac{1}{2\pi i} \int dze^{\frac{1}{2}(i\phi^+\sigma_0 \pm \sigma_1 \pm H_C)} \tag{2.16} \]

where \( \psi^3 \pm \psi^0 = e^{\pm i\sigma_0} \), \( \psi^1 \pm i\psi^2 = e^{\pm i\sigma_1} \), \( \partial H_C = J_C \), and there are an even number of + signs in the exponent. Note that \( H_C(y)H_C(z) \rightarrow -3\log(y-z) \) as \( y \rightarrow z \), so the integrand of \( q_a \) has weight 1.[3]

3. The New Spacetime-Supersymmetric Description

3.1. Off-shell spacetime-supersymmetry generators

In the RNS description, the spacetime-supersymmetry generators of (2.7) satisfy the anti-commutation relations:

\[ \{q_a, q_b\} = \frac{1}{2\pi i} \int dze^{-\phi} \psi_\mu \gamma^\mu_{ab} \tag{3.1} \]

which is not the usual supersymmetry algebra

\[ \{q_a, q_b\} = \frac{1}{2\pi} \int dz \partial x_\mu \gamma^\mu_{ab} \tag{3.2} \]
(note that $\frac{1}{2\pi} \oint dz \partial x_\mu$ is the string momentum).

However after hitting the left-hand side of (3.1) with the picture-changing operator \( Z \), it becomes

$$\frac{1}{2\pi i} \oint dz Ze^{-\phi \psi_\mu \gamma^\mu_{ab}} = \frac{1}{2\pi} \oint dz \partial x_\mu \gamma^\mu_{ab}.$$ 

So up to picture-changing, the \( q_a \)'s form a supersymmetry algebra.

But to make spacetime-supersymmetry manifest, one needs the \( q_a \)'s to form a supersymmetry algebra without applying picture-changing operations. This is because manifest spacetime-supersymmetry requires the generators to form an off-shell supersymmetry algebra, but picture-changing is only well-defined when the states are on-shell (otherwise, the states are not independent of the locations of the picture-changing operators).

So manifest spacetime-supersymmetry requires modification of the \( q_a \)'s. Note that \( q_a \) has picture $-\frac{1}{2}$ and the momentum $\frac{1}{2\pi} \oint dz \partial x_\mu$ has picture 0, so we need generators with picture $+\frac{1}{2}$. The obvious guess is

$$\bar{q}_a = Z q_a = \frac{1}{2\pi i} \oint dz \left[ b \eta e^{\frac{i}{2}(-3i\phi \pm \sigma_0 \pm \sigma_1 \pm \sigma_2 \pm \sigma_3 \pm \sigma_4)} + i : (e^\phi \psi_\mu \partial x^\mu) e^{\frac{i}{2}(i\phi \pm \sigma_0 \pm \sigma_1 \pm \sigma_2 \pm \sigma_3 \pm \sigma_4)} : \right].$$

It is easy to check that \( \{ q_\alpha, \bar{q}_\beta \} = \frac{1}{2} \oint dz \partial x_\mu \gamma^\mu_{ab} \), so we now have a supersymmetry algebra, but we also have twice too many supersymmetry generators! In ten dimensions, it is not possible to keep half of the 32 generators in an SO(9,1) Lorentz-covariant way. But for compactifications to four or six dimensions, it is possible to covariantly keep half of the generators. Although we shall only discuss the four-dimensional case in this review, the six-dimensional case has been discussed in reference [2].

For compactifications to four dimensions, one can choose to keep the chiral part of \( q_a \) and the anti-chiral part of \( \bar{q}_a \):

$$q_\alpha = \frac{1}{2\pi i} \oint dz \left[ e^{\frac{i}{2}(i\phi (\sigma_0 + \sigma_1) + H_C)} \right]$$

$$\bar{q}_\dot{\alpha} = \frac{1}{2\pi i} \oint dz \left[ b \eta e^{\frac{i}{2}(-3i\phi \pm (\sigma_0 - \sigma_1) - H_C)} + i : (e^\phi \psi_\mu \partial x^\mu) e^{\frac{i}{2}(i\phi \pm (\sigma_0 - \sigma_1) - H_C)} : \right].$$

These satisfy off-shell the 4D N=1 supersymmetry algebra

$$\{ q_\alpha, \bar{q}_\beta \} = \frac{1}{2\pi} \oint dz x_\alpha \beta$$

where we are using the standard notation \( x_\alpha \beta = x_m \sigma^m_{\alpha \beta} \).
3.2. Hermiticity

Although we have solved the problem of finding off-shell super-Poincaré generators, we now have a new problem: using the standard RNS definition of hermiticity where all fundamental fields are hermitian or anti-hermitian (the anti-hermitian field is $\sigma_0$), the hermitian conjugate of $q_\alpha$ is no longer $\bar{q}_\dot{\alpha}$. Fortunately, this new problem can be solved by modifying the definition of hermiticity.\[7\]

To find the appropriate hermiticity definition, one first writes $\bar{q}_\dot{\alpha}$ of (3.4) in the form

$$\bar{q}_\dot{\alpha} = e^R \left( \frac{1}{2\pi i} \oint dz \bar{\eta} e^{\frac{i}{2}(-3i\phi \pm (\sigma_0 - \sigma_1) - H)} \right) e^{-R}$$

where

$$R = \frac{1}{2\pi} \oint dz c \xi e^{-\phi}(\psi^m \partial x_m + G'^C + G'^{-C})$$

and $e^R Fe^{-R} = F + [R, F] + \frac{1}{2}[R, [R, F]] + \ldots$ (the expansion usually stops after two terms).

One can then define hermiticity as:

$$(x_m)^\dagger = e^R x_m e^{-R}, \quad (\psi_m)^\dagger = e^R \psi_m e^{-R}, \quad (F_C)^\dagger = e^R \bar{F}_C e^{-R},$$

$$(e^\frac{\mathcal{F}}{2})^\dagger = e^R (c \xi e^{-\frac{i}{2}\phi}) e^{-R}, \quad (e^{-\frac{\mathcal{F}}{2}})^\dagger = e^R (b \xi e^{\frac{i}{2}\phi}) e^{-R},$$

$$(b)^\dagger = e^R (ib \eta \partial e^{2\phi}) e^{-R}, \quad (c)^\dagger = e^R (-ic \xi \partial e^{-2\phi}) e^{-R},$$

$$(\eta)^\dagger = e^R (i\eta \partial e^{2\phi}) e^{-R}, \quad (\xi)^\dagger = e^R (-i\xi \partial e^{-2\phi}) e^{-R},$$

where $F_C$ are the worldsheet fields in the $c=9$ $N=2$ superconformal fields theory.

Since $R^\dagger = R$,

$$\left( (F)^\dagger \right)^\dagger = (e^R \bar{F} e^{-R})^\dagger = (e^{-R})^\dagger \bar{F}^\dagger (e^R)^\dagger = e^{-R} (e^R F e^{-R}) e^R = F$$

as desired. It is straightforward to check that the new hermiticity definition preserves all OPE’s and implies that $(q_\alpha)^\dagger = \bar{q}_\dot{\alpha}$.

One strange feature of the definition is that a field may have a different conformal weight from its hermitian conjugate since $(T)^\dagger = T + \partial (bc + \xi \eta)$. This will be explained in subsection 3.4 where it will be related to the “twisting” of an $N=2$ superconformal field theory.
3.3. Field redefinition

The above definition of hermiticity appears to be much more complicated than the standard RNS definition. However, one can now define a new set of worldsheet fields which are simple with respect to the new hermiticity definition. Besides simplifying the hermiticity definition, this new set of fields will also allow spacetime-supersymmetry to be made manifest.

Since $q_\alpha$ is now the hermitian conjugate of $\bar{q}_\dot{\alpha}$, it is natural to define fermionic superspace coordinates, $\theta^\alpha$ and $(\theta^\alpha)\dagger = \bar{\theta}^{\dot{\alpha}}$, which should satisfy $\{q_\alpha, \theta^\beta\} = \delta^\beta_\alpha$ and $\{\bar{q}_{\dot{\alpha}}, \bar{\theta}^{\dot{\beta}}\} = \delta^{\dot{\beta}}_{\dot{\alpha}}$. From the definitions of $q_\alpha$ and $\bar{q}_{\dot{\alpha}}$, the natural candidates are

$$\theta^\alpha = e^{\frac{i}{2}(-i\phi \pm (\sigma_0 + \sigma_1) - H)} \quad \bar{\theta}^{\dot{\alpha}} = c\xi e^{\frac{i}{2}(3i\phi \pm (\sigma_0 - \sigma_1) + H)}.$$ (3.10)

These fermionic coordinates have no singularities among themselves, and are conjugate to

$$p_\alpha = e^U \theta^\alpha e^{\frac{i}{2}(i\phi \pm (\sigma_0 + \sigma_1) + H)} e^{-U},$$

$$\bar{p}_{\dot{\alpha}} = (\bar{\theta}^{\dot{\alpha}})^\dagger = e^U (b\eta e^{\frac{i}{2}(3i\phi \pm (\sigma_0 - \sigma_1) - H)}) e^{-U}$$

where

$$U = \frac{1}{2\pi} \int dz\xi e^{-\phi} \left( \frac{1}{2} \psi_m \partial x^m + G_C^{-}\right)$$ (3.12)

(note that $U + U\dagger = R$).

In order that $x^m$ has no singularities with $p_\alpha$ or $\bar{p}_{\dot{\alpha}}$, we shall redefine

$$x^m_{\text{new}} = e^U x^m_{\text{old}} e^{-U} = x^m_{\text{old}} + \frac{i}{2} \theta^\alpha \sigma^m_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}}.$$ (3.13)

Note that $(x^m_{\text{new}})^\dagger = x^m_{\text{new}}$ with respect to the new hermiticity definition.

We shall also redefine the c=9 N=2 superconformal field theory so that it has no singularities with $p_\alpha$ or $\bar{p}_{\dot{\alpha}}$. This is done by redefining all the fields in the c=9 theory as

$$F_{C_{\text{new}}} = e^U (i\eta e^{\phi})^n F_{C_{\text{old}}} e^{-U}$$

where $n$ is the U(1) charge of $F_{C_{\text{old}}}$. (For example, for the c=9 N=2 superconformal field theory of (2.13), $x_j^\text{new} = x_j^\text{old}$, $x_j^\text{new} = \bar{x}_j^\text{old} + ic\xi e^{-\phi} \bar{\psi}_j^\text{old}$, $\psi_j^\text{new} = i\eta e^{\phi} \psi_j^\text{old} + ic\partial x_j^\text{old}$, $\bar{\psi}_j^\text{new} = -i\xi e^{-\phi} \bar{\psi}_j^\text{old}$.)

This has the effect of redefining the c=9 N=2 generators to be:

$$T_C^{\text{new}} = e^U (T_C^{\text{old}} + \frac{3}{2}(\partial \phi + i\eta \xi)^2 - i(\partial \phi + i\eta \xi) J_C^{\text{old}}) e^{-U},$$ (3.14)
\[ G_{C_{\text{new}}}^+ = e^U(i\eta e^\phi G_{C_{\text{old}}}^+) e^{-U}, \quad G_{C_{\text{new}}}^- = e^U(-i\xi e^{-\phi} G_{C_{\text{old}}}^-) e^{-U} \]

\[ J_{C_{\text{new}}} = J_{C_{\text{old}}} + 3i(\partial \phi + i\eta). \]

It is straightforward to check that these generators still form a c=9 N=2 algebra with the standard hermiticity properties (i.e. \( (T_{C_{\text{new}}}^\dagger) = T_{C_{\text{new}}}, \quad (G_{C_{\text{new}}}^+)^\dagger = G_{C_{\text{new}}}, \quad (J_{C_{\text{new}}}^\dagger) = J_{C_{\text{new}}}). \)

Besides the c=9 N=2 superconformal field theory, the original RNS system had five bosons \( (x^m, \phi) \) and eight fermions \( (\psi^m, b, c, \eta, \xi) \). Since we so far have four bosons \( (x^m) \) and eight fermions \( (\theta^\alpha, \bar{\theta}^\dot{\alpha}, p_\alpha, \bar{p}_{\dot{\alpha}}) \), there is one remaining boson which has no singularities in its OPE’s with the other fields. This boson, which will be called \( \rho \), is chiral (like \( \phi \)) and is defined by:

\[ \partial \rho = -3i\partial \phi + cb + 2\xi \eta - J_{C_{\text{RNS}}}. \] (3.15)

Under hermitian conjugation, \( (\rho)^\dagger = \rho. \)

Besides simplifying the hermiticity properties, the new worldsheet fields \( [x^m_{\text{new}}, \theta^\alpha, \theta^\dot{\alpha}, p_\alpha, \bar{p}_{\dot{\alpha}}, \rho, F_{C_{\text{new}}}^\dagger] \) have an important advantage over the original RNS fields. Any operator constructed out of integer powers of the new fields (or \( e^{in\rho} \) where \( n \) is an integer) is automatically GSO-projected, i.e. it has no branch-cuts with the spacetime-supersymmetry generators. This property makes it possible to eliminate the sum over spin structures, which is crucial for manifest spacetime-supersymmetry.

The RNS worldsheet action is simple to translate into the new fields since the new fields have only free-field OPE’s with each other. The action is

\[ S = \frac{1}{2\pi} \int d^2z \left( \frac{1}{2} \partial x^m_{\text{new}} \bar{\partial} x^m_{\text{new}} + p_{L\alpha} \tilde{\partial} \theta^\alpha L + \bar{p}_{L\dot{\alpha}} \tilde{\partial} \tilde{\theta}^\dot{\alpha} + p_{R\alpha} \partial \theta^\alpha R + \bar{p}_{R\dot{\alpha}} \partial \tilde{\theta}^\dot{\alpha} \right) + \frac{1}{2} \partial \rho L \bar{\partial} \rho L + \frac{1}{2} \partial \rho R \bar{\partial} \rho R + S_{C_{\text{new}}}^\dagger \] (3.16)

with free-field OPE’s as \( y \rightarrow z \):

\[ x^m(y)x^n(z) \rightarrow \eta^{mn} \log |y - z|^2, \quad \rho(y)\rho(z) \rightarrow \log(y - z), \]

\[ p_\alpha(y)\theta^\beta(z) \rightarrow \frac{\delta^\beta_\alpha}{y - z}, \quad \bar{p}_{\dot{\alpha}}(y)\tilde{\theta}^\dot{\beta}(z) \rightarrow \frac{\delta_{\dot{\alpha}}^\dot{\beta}}{y - z}. \] (3.17)

For completeness, we have temporarily unsuppressed the right-moving degrees of freedom in the action. Note that \( \rho_L \) and \( \rho_R \) are independent left and right-moving chiral bosons.
3.4. Twisted N=2 structure

Since $Q$ of equation (2.6) is a useful operator in the RNS formalism, it is natural to ask what is $(Q)^\dagger$? Writing

$$Q = \frac{1}{2\pi} \oint dz e^R (ib\eta \partial \eta e^{2\phi}) e^{-R},$$

it is easy to see that $Q^\dagger = \frac{1}{2\pi} \oint db$. (In this way of writing $Q$, it is trivial to check that $Q$ is nilpotent.) If we define $j_{BRST} = e^R (ib\eta \partial \eta e^{2\phi}) e^{-R}$, then $(j_{BRST})^\dagger = b$.

This hermiticity condition looks strange in an N=1 superconformal system, but it is natural if we interpret $j_{BRST}$ and $b$ as twisted fermionic N=2 superconformal generators, $G^+$ and $G^-$. Such an interpretation is possible since $j_{BRST}$ and $b$ satisfy the OPE’s of twisted c=6 N=2 superconformal generators:

$$G^+(y)G^-(z) \rightarrow \frac{2i}{(y-z)^3} + \frac{J(z)}{(y-z)^2} + \frac{iT}{y-z},$$

$$T(y)T(z) \rightarrow \frac{2T}{(y-z)^2} + \frac{\partial T}{y-z},$$

where

$$T = T_{RNS}, \quad G^+ = j_{BRST}, \quad G^- = b, \quad J = bc + \eta \xi.$$ (3.20)

In terms of the new fields defined in the previous subsection, it is straightforward to calculate that these twisted c=6 N=2 superconformal generators are:

$$T = T_{RNS}$$

$$= \frac{1}{2} \partial x^m \partial x_m + p_\alpha \partial \theta^\alpha + \bar{p}_{\dot{\alpha}} \partial \bar{\theta}^{\dot{\alpha}} + \frac{1}{2} \rho \partial \rho + \frac{i}{2} \partial^2 \rho + T_C + \frac{i}{2} \partial J_C,$$

$$G^+ = j_{BRST} = e^{i\rho} d_\alpha d^\alpha + G_C^+,$$

$$\bar{G} = b = e^{-i\rho} \bar{d}_{\dot{\alpha}} \bar{d}^{\dot{\alpha}} + G_C^-,$$

$$J = bc + \eta \xi = -\partial \rho + J_C,$$

where

$$d_\alpha = p_\alpha + \frac{i}{2} \bar{\theta}^{\dot{\alpha}} \partial x_{\alpha \dot{\alpha}} - \frac{1}{4} (\bar{\theta})^2 \partial \theta_\alpha + \frac{1}{8} \theta_\alpha \partial (\bar{\theta})^2,$$

$$\bar{d}_{\dot{\alpha}} = \bar{p}_{\dot{\alpha}} + \frac{i}{2} \theta^\alpha \partial x_{\alpha \dot{\alpha}} - \frac{1}{4} (\theta)^2 \partial \bar{\theta}_{\dot{\alpha}} + \frac{1}{8} \bar{\theta}_{\dot{\alpha}} \partial (\theta)^2,$$ (3.22)
and \((\theta)^2 = \frac{1}{2} \epsilon_{\alpha\beta} \theta^\alpha \theta^\beta\), \((\bar{\theta})^2 = \frac{1}{2} \epsilon_{\dot{\alpha}\dot{\beta}} \theta^{\dot{\alpha}} \theta^{\dot{\beta}}\). (In (3.21) and for the rest of this paper, we will suppress the “new” label on the worldsheet fields.)

Similarly, the spacetime-supersymmetry generators of (3.4) can be calculated to be

\[
q_\alpha = \frac{1}{2\pi i} \int dz (p_\alpha - \frac{i}{2} \bar{\theta}^{\dot{\alpha}} \partial x_{\alpha\dot{\alpha}} - \frac{1}{8} \theta_\alpha \partial (\bar{\theta})^2),
\]

\[
\bar{q}_{\dot{\alpha}} = \frac{1}{2\pi i} \int dz (\bar{p}_{\dot{\alpha}} - \frac{i}{2} \bar{\theta}^{\dot{\alpha}} \partial x_{\alpha\dot{\alpha}} - \frac{1}{8} \bar{\theta}_{\dot{\alpha}} \partial (\theta)^2).
\]

As was shown by Siegel,\[10\] \(d_\alpha\) and \(\bar{d}_{\dot{\alpha}}\) anti-commute with these spacetime-supersymmetry generators and satisfy the OPE that

\[
d_\alpha (y) d_\beta (z) \rightarrow \frac{i \Pi_{\alpha\dot{\alpha}}}{y - z}, \quad d_\alpha (y) \Pi_{\beta\dot{\beta}} (z) = \frac{-i \epsilon_{\alpha\beta} \partial \bar{\theta}_{\dot{\beta}}}{y - z}
\]

(3.24)

where \(\Pi_{\alpha\dot{\alpha}} = \partial x_{\alpha\dot{\alpha}} - \frac{i}{2} (\theta_\alpha \partial \bar{\theta}_{\dot{\alpha}} + \bar{\theta}_{\dot{\alpha}} \partial \theta_\alpha)\).

Since \(T\) can be written as

\[
T = \frac{1}{2} \Pi^m \Pi_m + d_\alpha \partial \theta^\alpha + \bar{d}_{\dot{\alpha}} \partial \bar{\theta}^{\dot{\alpha}} + \frac{1}{2} \partial \rho \partial \rho + \frac{i}{2} \partial^2 \rho + T_C + \frac{i}{2} \partial J_C,
\]

the above twisted \(c=6\) \(N=2\) generators are manifestly spacetime-supersymmetric (note that four-dimensional spacetime-supersymmetry now commutes with all compactification-dependent variables). Another nice feature of these \(c=6\) \(N=2\) generators is that they split cleanly into a set of \(c=-3\) \(N=2\) generators (which depend on the “four-dimensional” fields) and a set of \(c=9\) \(N=2\) generators (which depend on the compactification fields).

So using RNS matter and ghost fields, we have constructed a set of twisted \(c=6\) \(N=2\) generators which, when expressed in terms of new variables, satisfy standard hermiticity properties and are manifestly spacetime-supersymmetric.

Since \(c=6\) is the critical central charge for an \(N=2\) matter system, we can now forget about its \(N=1\) origin, untwist the \(N=2\) generators by shifting \(T \rightarrow T - \frac{i}{2} J\), introduce a set of \(c=-6\) \(N=2\) ghosts, construct an \(N=2\) BRST operator and vertex operators, and calculate scattering amplitudes using standard \(N=2\) techniques. This was the method used in reference \[11\]. Although it is not obvious that the resulting \(N=2\) prescription produces vertex operators and scattering amplitudes which coincide with those produced by the standard \(N=1\) RNS prescription, it was proven in reference \[9\] that they indeed do coincide.

However there is a simpler method to calculate scattering amplitudes. Since the above \(N=2\) matter system was constructed out of RNS matter and ghost fields, there should be
no need to introduce additional N=2 ghosts. Note that $T(y)T(z)$ has no central charge in the twisted N=2 system, and all bosonic and fermionic worldsheet fields have integer spin. So although there are certainly physical propagating states, the system appears to be topological.

Indeed, scattering amplitudes for this system can be calculated in a manner which is a direct N=4 generalization of N=2 topological techniques. This topological method of calculation can be generalized to any c=6 N=2 system, and has been used to prove vanishing theorems and calculate multiloop amplitudes for the c=6 N=2 system corresponding to 4D self-dual gravity.

4. N=4 topological method

4.1. N=4 generators

To formulate the N=4 topological method, one first constructs a set of twisted c=6 N=4 generators out of the original twisted c=6 N=2 generators. These are defined by

$$
T_{N=4} = T_{N=2}, \quad G^+_{N=4} = G^+_{N=2}, \quad G^-_{N=4} = G^-_{N=2};
$$

$$
\tilde{G}^+_{N=4} = iG^-_{N=2}(e^{iH}), \quad \tilde{G}^-_{N=4} = -iG^+_{N=2}(e^{-iH}),
$$

$$
J_{N=4} = J_{N=2}, \quad J^+_{N=4} = e^{iH}, \quad J^-_{N=4} = e^{-iH}
$$

where $J_{N=2} = \partial H$ and $G^\pm_{N=2}(e^{\mp iH})$ means the single pole in the OPE of $e^{\mp iH}$ and $G^\pm_{N=2}$.

For a worldsheet primary field $f$ with U(1) charge $k$ (i.e. $J(f) = ikf$), we shall define $\tilde{f}$ to be the pole of order $k^2$ in the OPE of $e^{ikH}$ and $\tilde{f}$. Note that $\tilde{f}$ carries the same U(1) charge and conformal weight as $f$, and $\tilde{f} = f$.

So $T$, $G^-$, $\tilde{G}^-$ and $J^-$ carry conformal weight 2, $G^+$, $\tilde{G}^+$ and $J$ carry conformal weight 1, and $J^{++}$ carries conformal weight 0. It is straightforward to check that these generators satisfy the “small” N=4 OPE’s:

$$
G^+ \tilde{G}^- \sim G^- \tilde{G}^+ \sim 0
$$

$$
G^+(z)G^-(0) \sim \frac{2i}{z^3} + \frac{J(0)}{z^2} + \frac{iT(0)}{z}
$$

$$
\tilde{G}^+(z)\tilde{G}^-(0) \sim -\frac{2i}{z^3} - \frac{J(0)}{z^2} - \frac{iT(0)}{z}
$$
\[
G^+(z)\tilde{G}^+(0) \sim \frac{J^{++}(0)}{z^2} + \frac{\partial J^{++}(0)}{2z}
\]
\[
G^-(z)\tilde{G}^-(0) \sim \frac{J^{--}(0)}{z^2} + \frac{\partial J^{--}(0)}{2z}
\]

In terms of the RNS and supersymmetric variables, the new generators are given by

\[
\tilde{G}^+ = \eta = e^{i(-2\rho + HC)}(\bar{d})^2 + G_C^-(e^{i(-\rho + HC)}),
\]

\[
\tilde{G}^- = \{Q_{RNS}, b\xi\} = bZ + \xi T_{RNS} = e^{i(2\rho - HC)}(d)^2 + G_C^+(e^{i(\rho - HC)}),
\]

\[
J^{++} = c\eta = e^{-i(\rho - HC)}, \quad J^{--} = b\xi = e^{i(\rho - HC)}.
\]

### 4.2. Physical vertex operators

Since \(\frac{1}{2\pi} \oint dz G^+ = Q\) and \(\frac{1}{2\pi} \oint dz \tilde{G}^+ = \frac{1}{2\pi} \oint dz \eta\), it is straightforward to translate the RNS language of section 2.2 into N=4 topological language. We shall find that by combining N=4 topological language with the new supersymmetric variables, the conditions on physical vertex operators can be greatly simplified.

Firstly, one needs to translate the requirement that the vertex operator \(V\) is in the BRST cohomology. This is simply

\[
V : \quad G^+(V) = \tilde{G}^+(V) = 0, \quad V \neq G^+(\Lambda)
\]

for any \(\Lambda\) satisfying \(\tilde{G}^+(\Lambda) = 0\).

Note that \(\tilde{G}^+(h) = 0\) implies that \(h = \tilde{G}^+(f)\) where \(f = \xi h\). Similarly, \(G^+(h) = 0\) implies that \(h = G^+(f)\) where \(f = c\xi \partial \xi e^{-2\phi} h\). So \(\oint dz G^+\) and \(\oint dz \tilde{G}^+\) have trivial cohomologies. Also note from (4.2) that \(G^+(y)\tilde{G}^+(z) \to (-\partial_y + \partial_z)(J(z)/2(y - z))\) as \(y \to z\), so \(\oint dz G^+, \oint dz \tilde{G}^+\) = 0.

So the BRST cohomology condition of (4.4) can be written in a more symmetric form as

\[
V : \quad G^+(V) = \tilde{G}^+(V) = 0, \quad V \neq G^+(\tilde{G}^+(\Lambda))
\]

for any \(\Lambda\).

Furthermore, since \(\tilde{G}^+(V) = 0\) implies \(V = \tilde{G}^+(\Phi)\) for \(\Phi = \xi V\), the BRST cohomology can also be defined as

\[
\Phi : \quad G^+(\tilde{G}^+(\Phi)) = 0, \quad \Phi \neq G^+(\Lambda) + \tilde{G}^+(\tilde{\Lambda})
\]
The $\tilde{\Lambda}$ gauge invariance comes from the ambiguity in defining $\Phi$, since $\tilde{G}^+(\Phi + \tilde{G}^+(\tilde{\Lambda})) = \tilde{G}^+(\Phi)$.

We shall now translate the additional four physical conditions of section 2.2 into the new language:

1) The reality condition that $V = V^\dagger$ can not be directly translated into the new language since $V^\dagger$ no longer has the same conformal weight as $V$. Instead, the new reality condition will be that $\Phi = \tilde{\Phi}$ where the $\tilde{}$ operation is defined below (4.1) and $V = G^+(\Phi)$.

2) The condition of being GSO-projected is trivially satisfied if the vertex operator is a single-valued function of the new supersymmetric variable $s$. This is because integer powers of the worldsheet variables are automatically GSO-projected. Note that $V$ is fermionic and $\Phi$ is bosonic.

3) Although the picture-changing operator $Z$ does not play a role in the N=4 topological method, there is a natural way to understand picture-changing. As was just shown in (4.6), any BRST-invariant vertex operator $V$ can be written as $V = G^+(\Phi)$ where $G^+(\tilde{G}^+(\Phi)) = 0$.

Now consider the vertex operator $V^{(1)} = G^+(\Phi)$. Since $\tilde{G}^+(G^+(\Phi)) = G^+(\tilde{G}^+(\Phi)) = 0$, $G^+(V^{(1)}) = \tilde{G}^+(V^{(1)}) = 0$, so $V^{(1)}$ is also BRST-invariant. One can continue the procedure to obtain $V^{(n)}$ for arbitrary positive $n$. It is easy to check that $V^{(n)} = Z^n V + G^+(\tilde{G}^+(\Lambda^{(n)}))$ for some $\Lambda^{(n)}$.

Similarly, one can write $V = G^+(\Phi')$ where $\tilde{G}^+(G^+(\Phi')) = 0$, and consider the operator $V^{(-1)} = G^+(\Phi')$. Once again, $V^{(-1)}$ is BRST-invariant, and the procedure can be continued to give $V^{(-n)} = Y^n V + G^+(\tilde{G}^+(\Lambda^{(-n)}))$ for some $\Lambda^{(-n)}$.

So for any N=4 topological theory, there is an infinite family of BRST-invariant vertex operators for each physical state, and one has to choose a unique representative. In the RNS section, we learned that the picture eigenvalue $p$ could be used to select a unique representative, but this breaks manifest spacetime-supersymmetry since $[q_\alpha, P] \neq 0$.

Writing the picture operator $P$ in terms of the new variables,

$$P = \frac{1}{2\pi} \int dz (i\xi \eta - \partial \phi) = \frac{1}{2\pi} \int dz (i\partial \rho - \frac{1}{2}p_\alpha \theta^\alpha + \frac{1}{2}\bar{p}_\dot{\alpha} \bar{\theta}^{\dot{\alpha}}),$$

it is obvious that $[P, q_\alpha] \neq 0$ since $P$ contains $\theta^\alpha$ zero modes. However, if we use $\rho$ charge instead of picture to select a unique representative, spacetime-supersymmetry is preserved since $\oint dz \partial \rho$ commutes with $q_\alpha$ and $\bar{q}_{\dot{\alpha}}$.

1 For the N=4 topological theory representing self-dual gravity, these vertex operators are related by $V^{(n)} = e^{i\alpha n} V$ where $\alpha$ depends on the particle’s momentum.
As elaborated in reference [13], it is possible to restrict all compactification-independent states to have zero ρ-charge, and all compactification-dependent states to have −1, 0, or +1 ρ-charge. It was proven in reference [13] that this restriction selects a unique representative for each physical state in a manner that preserves manifest SO(3,1) super-Poincaré invariance.

4) The final condition of +1 ghost-number is easily translated into N=4 topological language since the ghost-number current,

\[ J_g = \frac{i}{2\pi} \oint dz (bc + \xi \eta) = \frac{1}{2\pi} \oint dz (-\partial \rho + J_G), \quad (4.7) \]

is also the U(1) current. So \( V \) must have +1 U(1)-charge, which means that \( \Phi \) is U(1)-neutral. Note that \( \Phi = \bar{\Phi} \) implies \( \Phi = \Phi^\dagger \) when \( \Phi \) is U(1)-neutral.

So for compactification-independent states, \( \Phi \) uniquely represents a physical state if \( \Phi \) is a real single-valued bosonic function of four-dimensional fields, is U(1)-neutral, satisfies \( G^+ (\bar{G}^+(\Phi)) = 0 \), and cannot be written as \( \Phi = G(\Lambda) + \bar{G}^+ (\bar{\Lambda}) \) for any \( \Lambda \) and \( \bar{\Lambda} \). Note that U(1)-neutrality implies zero ρ-charge since \( \Phi \) is independent of the compactification fields. (For simplicity, only compactification-independent states will be discussed in this review. Details on compactification-dependent states can be found in references [1], [13] and [14]).

This N=4 topological definition of physical vertex operators was obtained by comparing with the N=1 RNS definition, and naively appears to be unrelated to the standard definition coming from a critical N=2 superconformal field theory. In critical N=2 superconformal field theories, physical vertex operators correspond to bosonic primary fields \( \Phi \) of zero conformal weight and zero U(1)-charge (i.e., \( \Phi \) has no double poles with the N=2 generators \( T \), \( G^+ \), and \( G^- \), and has no single pole with \( J \)). In integrated form, the vertex operator for open N=2 strings is given by

\[ W = \int dz G^- (G^+(\Phi)) \]

where \( G^- (X) \) means the single pole in the OPE of \( G^- \) and \( X \).

Although not obvious, it was proven in references [9] and [15] that the N=1 and N=2 definitions of BRST cohomology coincide, and there is therefore a one-to-one correspondence.

---

2 For the N=2 open string representing 4D self-dual Yang-Mills, \( \Phi(x) \) is the Yang scalar and

\[ W = \int dz (\partial x^j \partial_j \Phi + i \psi^j \bar{\psi}_k \partial_j \bar{\partial}^k \Phi). \]
between $\Phi$’s satisfying the N=4 topological definition and $\Phi$’s satisfying the standard N=2 definition. Note that
\[ W = \int dz b(V) = \int dz G^-(V) = \int dz G^- (G^+ (\Phi)) \] (4.8)
is the integrated form of the RNS vertex operator, so the integrated vertex operators also agree in the two definitions.

As an example, we shall now describe the vertex operators for all massless states of the open superstring which are independent of the compactification. It will be seen that the new spacetime-supersymmetric variables allow these vertex operators to be expressed in much more compact form than when using the RNS variables.

4.3. Massless compactification-independent states

For the open superstring, the massless compactification-independent states are those of 4D N=1 super-Yang-Mills, where the gauge group comes from Chan-Paton factors. The vertex operator $\Phi$ for these fields only depends on the $[x^m, \theta^\alpha, \bar{\theta}^{\dot{\alpha}}]$ zero modes and is simply the super-Yang-Mills prepotential $\Phi^I (x, \theta, \bar{\theta})$, which contains the gluon field in its $\theta \bar{\theta}$ component and the gluino fields in its $(\theta)^2 \bar{\theta}$ and $\theta (\bar{\theta})^2$ components ($I$ labels the gauge group generators).

Using the N=4 topological definition of (4.6), the on-shell condition is
\[ \tilde{G}^+ (G^+ (\Phi^I)) = e^{iH_C - i\rho} D_\alpha (D) D^\alpha \Phi^I = 0, \] (4.9)
and the gauge-invariances are
\[ \delta \Phi^I = G^+ (\Lambda^I) + \tilde{G}^+ (\tilde{\Lambda}^I) = (D)^2 \lambda^I + (\tilde{D})^2 \tilde{\lambda}^I \] (4.10)
where $\Lambda^I = e^{-i\rho} \lambda^I$ and $\tilde{\Lambda}^I = e^{-iH_C + 2i\rho} \tilde{\lambda}^I$. These imply the usual linearized equations of motion and gauge-invariances for the super-Yang-Mills prepotential.

Using the standard N=2 definition, $\Phi^I$ is a physical field if it has no double poles with $G^+$, $G^-$, or $T$, i.e. if $\Phi^I$ satisfies the conditions
\[ (D)^2 \Phi^I = (\tilde{D})^2 \Phi^I = \partial_m \partial^m \Phi^I = 0. \]
Note that $(D)^2 \Phi^I = (\tilde{D})^2 \Phi^I = 0$ is the supersymmetric version of the Lorentz gauge condition $\partial^m A^I_m = 0$, and in this gauge, $\partial^m \partial_m \Phi^I = 0$ is the linearized equation of motion.

In integrated form, the open superstring vertex operator is
\[ W = \int dz (d^\alpha (\tilde{D})^2 D_\alpha \Phi^I + \bar{d}^{\dot{\alpha}} (D)^2 \bar{D}_{\dot{\alpha}} \Phi^I \]
\[ + \partial \theta^\alpha D_\alpha \Phi^I - \partial_{\bar{\theta}} \bar{D}_{\dot{\alpha}} \Phi^I - i\Pi^{\alpha \dot{\alpha}} [D_\alpha, \bar{D}_{\dot{\alpha}}] \Phi^I. \] (4.11)
4.4. Scattering amplitudes

One way to compute scattering amplitudes in the new supersymmetric description is using standard N=2 techniques, where one integrates correlation functions of BRST-invariant vertex operators on N=2 super-Riemann surfaces. Although this N=2 prescription was successfully used in reference [11], it was later shown in reference [2] that the N=4 topological method provides a simpler prescription and produces equivalent amplitudes. For this reason, only the N=4 topological prescription will be described in this review. As in section 2.3, we shall begin by briefly describing multiloop amplitudes, and then concentrate on the simpler case of tree amplitudes.

The simplest derivation of the N=4 topological prescription is to start with the N=1 prescription of (2.10), and translate it into N=4 topological language. This is done in three steps:

1) Choose the picture-changing operators to be located at the insertion. 2) Choose the \( \xi(z_0) \) insertion to be located at one of the remaining \( b \) insertions. 3) Change the picture restriction, \( \delta(\oint A_j dz_j(\partial \phi - i \xi \eta)) \), to a restriction on the \( \rho \) charge, \( \delta(\oint A_j dz_j \partial \rho) \).

The first two steps are allowed since the amplitude is independent of the locations of the picture-changing operators and of the \( \xi \) zero mode. The third step is allowed since, just like picture, the \( \rho \) charge can be used to select a unique representative for propagating states.

The RNS expression of (2.10) can now be written as:

\[
\lambda^{2g-2} \sum_{I=1}^{2g} \prod_{i=1}^{3g-3+n} \int d^2 \tau_i < \prod_{j=1}^{g} \phi \left( \oint_{A_j} dz_j \eta(z_j) \delta(\oint_{A_j} dz_j \partial \rho) \right)
\]

\[
\int d^2 u_1 \xi b(u_1) \mu_1(u_1) \prod_{i=2}^{g-1+p} d^2 u_i b(u_i) \mu_i(u_i) \prod_{j=g+p}^{3g-3+n} d^2 u_j Z b(u_j) \mu_j(u_j) |^2
\]

\[
\prod_{r=1}^{n} \left[ \frac{1}{2\pi} \int d\bar{z} \eta_R, \frac{1}{2\pi} \int dz \eta_L, \xi V_r(\tau_r) \right]
\]

which easily translates into N=4 topological language as:

\[
\lambda^{2g-2} \sum_{I=1}^{2g} \prod_{i=1}^{3g-3+n} \int d^2 \tau_i < \prod_{j=1}^{g} \phi \left( \oint_{A_j} dz_j \tilde{G}^+(z_j) \delta(\oint_{A_j} dz_j \partial \rho) \right)
\]

\[
\int d^2 u_1 \xi b(u_1) \mu_1(u_1) \prod_{i=2}^{g-1+p} d^2 u_i b(u_i) \mu_i(u_i) \prod_{j=g+p}^{3g-3+n} d^2 u_j Z b(u_j) \mu_j(u_j) |^2
\]

\[
\prod_{r=1}^{n} \left[ \frac{1}{2\pi} \int d\bar{z} \eta_R, \frac{1}{2\pi} \int dz \eta_L, \xi V_r(\tau_r) \right]
\]

\[
\lambda^{2g-2} \sum_{I=1}^{2g} \prod_{i=1}^{3g-3+n} \int d^2 \tau_i < \prod_{j=1}^{g} \phi \left( \oint_{A_j} dz_j \tilde{G}^+(z_j) \delta(\oint_{A_j} dz_j \partial \rho) \right)
\]

After removing the \( \delta(\oint_{A_j} dz_j \partial \rho) \) term, the topological prescription can also be used to compute multiloop amplitudes for the N=2 string representing N=(2,2) self-dual gravity.\[3\] [12]
\[
\int d^2 u_1 J^{-}(u_1) \mu_1(u_1) \prod_{i=2}^{g-1+p} \int d^2 u_i G^{-}(u_i) \mu_i(u_i) \prod_{j=g+p}^{3g-3+n} \int d^2 u_j \tilde{G}^{-}(u_j) \mu_j(u_j) \right|^2
\]
\[
\prod_{r=1}^{n} G^+_R(G^+_L(\Phi_r(\tau_r))) > .
\]

Note that using the new spacetime-supersymmetric variables, there is no need to perform a sum over spin structures.

For tree-level amplitudes, (4.13) reduces for open strings to:
\[
< \Phi_1(z_1) \tilde{G}^+(\Phi_2(z_2)) G^+(\Phi_3(z_3)) \prod_{r=4}^{n} dz_r G^-(G^+(\Phi_r(z_r))). \tag{4.14}
\]

Plugging in the expression in section 4.3 for \( \Phi \), it is completely straightforward to compute massless tree amplitudes in a manifestly SO(3,1) super-Poincaré invariant manner. For example, the amplitude for three super-Yang-Mills particles is:
\[
f_{IJK} < \Phi_I(z_1) \tilde{G}^+(\Phi_J^J(z_2)) G^+(\Phi^K_3(z_3)) > \tag{4.15}
\]
\[
= f_{IJK} < \Phi_I(z_1) e^{-2ip+iHC} d^\alpha(z_2) \tilde{D}^\alpha \Phi_J^J(z_2) e^{ip} d^\alpha(z_3) D^\alpha \Phi^K_3(z_3) >
\]
\[
= f_{IJK} \int d^4 x \int d^2 \theta d^2 \tilde{\theta} \Phi_I^I(D^\alpha \tilde{D}^\alpha \Phi^J_2 \tilde{D}^\alpha D^\alpha \Phi^K_3 + (k^a_2 - k^a_3) D^\alpha \Phi^J_2 D^\alpha \Phi^K_3)
\]
where \( f_{IJK} \) is the structure constant and \( \int d^2 \theta d^2 \tilde{\theta} \) comes from the background charge condition \( < (\theta)^2(\tilde{\theta})^2 e^{-ip+iHC} > = 1. \)

5. Relationship with the Green-Schwarz formalism

In this section, we shall review the light-cone, conventional, and twistor-like Green-Schwarz descriptions of the superstring, and then show the relationship with the new description.

\[4 \] I would like to thank Konstantin Bobkov for pointing out an error in the original version of the three-point amplitude.
5.1. Light-cone Green-Schwarz formalism

All Green-Schwarz descriptions of the superstring reduce in light-cone gauge to the free-field action:

\[
\frac{1}{2\pi} \int d^2 z \left( \frac{1}{2} \partial x^i \partial x^i + i \theta^a \bar{\theta}^a + i \theta^a \partial \bar{\theta}^a \right) \tag{5.1}
\]

where \( x^i \) is an SO(D-2) vector and \( \theta^a \) is an SO(D-2) spinor. This light-cone gauge is defined by

\[
x^0 + x^{D-1} = \tau, \quad \partial (x^0 - x^{D-1}) = \partial x^i \partial x^i + i \theta^a \partial \bar{\theta}^a, \quad (\gamma^0 + \gamma^{D-1})_{\alpha\beta} = 0 \tag{5.2}
\]

where \( \tau \) is the worldsheet time, \( x^\mu \) is an SO(1,D-1) vector, \( \theta^\alpha \) is an SO(1,D-1) spinor, and \( (\gamma^0 - \gamma^{D-1})_{\beta\alpha} = \theta_\alpha \) is the light-cone spinor.

Although scattering amplitudes can be computed in light-cone gauge, the computations are complicated by the fact that light-cone diagrams have singular interaction points.\cite{17} \cite{18} \cite{19}. At these interaction points, non-trivial operators need to be inserted in order to preserve Lorentz invariance. (These insertions are also necessary in the light-cone RNS description, but not in the light-cone description of the bosonic superstring.\cite{20}) The non-trivial operators make it extremely difficult to write the amplitude in closed form, and for this reason, only four-point tree and one-loop superstring amplitudes have been computed using this light-cone method.\footnote{Although reference \cite{19} contains explicit expressions for multiloop amplitudes, these expressions contain unphysical divergences when interaction points coincide. It has not yet been determined how these expressions are affected by removing the unphysical divergences with contact terms. Also, Mandelstam has constructed an N-point tree amplitude which is spacetime-supersymmetric, but his proposal is based on unitarity arguments, rather than explicit light-cone calculations.\cite{21}}

5.2. Conventional Green-Schwarz formalism

Based on earlier work by Siegel on the superparticle,\cite{22} Green and Schwarz found a super-Poincaré invariant action for the superstring (in \( D=3,4,6 \) or 10) which reduces to (5.1) in light-cone gauge.\cite{23} In worldsheet conformal gauge, this action is

\[
S = \frac{1}{2\pi} \int d^2 z [\Pi^\mu \bar{\Pi}_\mu + i(\partial x_\mu + \frac{i}{2} \theta L \gamma_\mu \bar{\theta} L)(\theta L \gamma^\mu \bar{\theta} L - \theta R \gamma^\mu \bar{\theta} R) - i(\bar{\partial} x_\mu + \frac{i}{2} \theta L \gamma_\mu \bar{\theta} L)(\theta L \gamma^\mu \bar{\theta} L - \theta R \gamma^\mu \bar{\theta} R)] \tag{5.3}
\]
with Virasoro constraint $T = \Pi^\mu \Pi_\mu$ where

$$\Pi^\mu = \partial x^\mu - \frac{i}{2} \theta_L \gamma^m \partial \theta_L - \frac{i}{2} \theta_R \gamma^m \partial \theta_R.$$

Since the conjugate momentum for $\theta^\alpha$ is $P_{\theta^\alpha} = \partial S / \partial (\partial_0 \theta^\alpha)$, there is a spinor Dirac constraint:

$$d_\alpha = P_{\theta^\alpha} + \frac{i}{2} \Pi_\mu \gamma^\mu_{\alpha\beta} \theta^\beta = 0. \tag{5.4}$$

It is easy to compute that $\{d_\alpha, d_\beta\} = i \Pi_\mu \gamma^\mu_{\alpha\beta} \theta^\beta$, and since $\Pi_\mu \Pi^\mu$ is the Virasoro constraint, half of the $2D - 4$ components of $d_\alpha$ are first-class constraints and the other half are second-class. The first-class constraints in $d^\alpha$ generate the $D - 2$ fermionic $\kappa$-symmetries:

$$\delta \theta^\alpha = \Pi^\mu \gamma^\mu_{\alpha\beta} \kappa^\beta, \quad \delta x^\mu = \frac{i}{2} \theta^\gamma \gamma^\mu \delta \theta, \tag{5.5}$$

which allows half of the $\theta$’s to be gauge-fixed. (Note that the Virasoro constraint $\Pi_\mu \Pi^\mu$ implies that only half of the $2D - 4$ components of $\kappa_\beta$ contribute to $\delta \theta^\alpha$.)

Since (5.3) is not a quadratic action, quantization is not straightforward. Although (5.3) simplifies somewhat in the “semi-light-cone gauge” $(\gamma^0 + \gamma^{D-1})_{\alpha\beta} \theta^\alpha = 0$ to

$$\frac{1}{2\pi} \int d^2 z (\frac{1}{2} \partial x^\mu \partial x^\mu + \tag{5.6}$$

$$i[\theta_L (\gamma^0 - \gamma^{D-1}) \partial \theta_L] \partial (x^0 + x^{D-1}) + i[\theta_R (\gamma^0 - \gamma^{D-1}) \partial \theta_R] \bar{\partial}(x^0 + x^{D-1})),$$

even the semi-light-cone gauge action is difficult to quantize. Because of these quantization problems, neither (5.3) nor (5.6) has been successfully used to compute superstring scattering amplitudes.

5.3. Twistor-like Green-Schwarz formalism

In [26], Sorokin, Tkach, Volkov, and Zheltukhin discovered an alternative super-Poincaré invariant action which also reduces to (5.1) in light-cone gauge. These authors discovered that the fermionic $\kappa$-symmetries of (5.3) could be converted into superconformal invariances if one introduced bosonic spacetime spinor variables into the superstring

---

6 Actually, reference [26] only discusses the superparticle, but their work was soon generalized by other authors to the heterotic superstring. A simple twistor-like action for the Type II superstring is still lacking.
action. These new bosonic variables, $\lambda^\alpha$, are the worldsheet supersymmetric partners of $\theta^\alpha$.

In order to preserve the number of physical degrees of freedom, $\lambda^\alpha$ must be constrained to satisfy

$$\lambda^{\alpha\gamma} \gamma^\mu_{\alpha\beta} \lambda^\beta = \Pi^\mu. \quad (5.7)$$

Since the zero mode of (5.7) resembles the twistor relation of Penrose, $\lambda^{\alpha\gamma} \gamma^\mu_{\alpha\beta} \lambda^\beta = P^\mu$ where $P^\mu$ is the particle momentum, the resulting string is called the twistor-like Green-Schwarz superstring.

There is also a fermionic worldsheet superpartner of (5.7),

$$\lambda^{\alpha\gamma} \gamma^\mu_{\alpha\beta} \theta^\beta = \psi^\mu, \quad (5.8)$$

which relates the RNS fermionic vector $\psi^\mu$ with the GS fermionic spinor $\theta^\alpha$. By combining the component fields into the worldsheet superfields,

$$X^\mu = x^\mu + i\kappa \psi^\mu, \quad \Theta^\alpha = \theta^\alpha + \kappa \lambda^\alpha, \quad (5.9)$$

the constraints of (5.7) and (5.8) can be expressed in worldsheet superconformally invariant notation as

$$D_\kappa X^\mu = i\Theta^\alpha \gamma^\mu_{\alpha\beta} D_\kappa \Theta^\beta, \quad (5.10)$$

where $D_\kappa = \partial_\kappa + i\kappa \partial_z$.

Although the invariances of the twistor-like approach are more geometrical than the $\kappa$-symmetries of the conventional approach, the twistor-like action is equally difficult to quantize. In $D = 3, 4, 6$ or 10, one has up to $D - 2$ superconformal invariances (which replace up to $D - 2$ $\kappa$-symmetries). So when $D = 10$, one can have N=8 super-Virasoro, which is not straightforward to quantize since the algebra is soft. $^{[29]}$ Another problem is that in addition to the fermionic second-class constraints of (5.4), one has bosonic second-class constraints coming from (5.7). $^{[30]}$

5.4. Relationship with the new description

Because the twistor-like approach contains worldsheet superconformal invariance, one would suspect it is closely related to the new formalism described in the previous sections. Indeed, it was shown in $^{[31]}$ that after partially gauge-fixing the $D = 10$ twistor-like action
(which fixes six of the eight fermionic worldsheet invariances and breaks SO(1,9) super-Poincaré invariance down to SU(4)×U(1)), one can obtain a free-field action with critical N=2 superconformal invariance. After a series of complicated field-redefinitions, this N=2 action can be written as (3.16), which was how (3.16) was originally found.

After gauge-fixing, four components of the twistor-like variables are related to the worldsheet variables of the previous sections by

\[ \lambda^\alpha = d^\alpha e^{i\rho}, \quad \tilde{\lambda}^{\dot{\alpha}} = \bar{d}^{\dot{\alpha}} e^{-i\rho}. \]  

(5.11)

Note that the OPE’s of \( d^\alpha \) and \( \rho \) imply that

\[ \lambda^\alpha(y)\tilde{\lambda}^{\dot{\alpha}}(z) \to \Pi^m \sigma^\alpha_{\dot{m}} \]

as \( y \to z \), which is the twistor constraint of (5.7). Also note that the four-dimensional part of the N=2 fermionic generators in (3.21) are

\[ G^+ = d_\alpha d^\alpha e^{i\rho} = d_\alpha \lambda^\alpha, \quad G^- = \bar{d}_\dot{\alpha} \bar{d}^{\dot{\alpha}} e^{-i\rho} = \bar{d}_\dot{\alpha} \tilde{\lambda}^{\dot{\alpha}}, \]

(5.12)

which implies that \( \lambda^\alpha \) and \( \tilde{\lambda}^{\dot{\alpha}} \) are the worldsheet superpartners of \( \theta^\alpha \) and \( \bar{\theta}^{\dot{\alpha}} \).

Finally, we shall show that in light-cone gauge, the action of (3.16) reduces to the standard light-cone GS action of (5.1). This proves the classical equivalence of the new description with the conventional and twistor-like GS descriptions.

The first step is to fermionize one component of \( \theta^\alpha \) and \( p_\alpha \) as

\[ \theta^1 = e^{i\sigma}, \quad p_1 = e^{-i\sigma} \]

(5.13)

where \( \sigma \) is a chiral boson. The second step is to use the U(1) invariance generated by \( J = -\partial \rho + J_C \) to gauge-fix \( \rho = \sigma \). The third step is to use the Virasoro constraint \( T \) to gauge-fix \( x^0 + x^3 = \tau \).

At this point, the fermionic N=2 generators are

\[ G^+ = d_\alpha d^\alpha e^{i\rho} + G^+_C = p_1 p_2 e^{i\rho} + ... = p_2 + ..., \]

\[ G^- = \bar{d}_\dot{\alpha} \bar{d}^{\dot{\alpha}} e^{-i\rho} + G^-_C = \partial(x^0 + x^3)\theta^1 \bar{p}_2 e^{-i\rho} + ... = \bar{p}_2 + ..., \]

(5.14)

where the \( \partial(x^0 + x^3)\theta^1 \) term comes from \( \bar{d}_1 \).

The next step is to use \( G^+ \) and \( G^- \) to gauge-fix \( \theta^2 = \bar{\theta}^2 = 0 \). Besides the compactification-dependent fields, this leaves \([x^0 - x^3, x^1, x^2, \theta^1, \bar{\theta}^1, p_2, \bar{p}_2, \rho]\). However \( x^0 - x^3, p_2, \bar{p}_2, \) and \( \rho \) are constrained by \( T = G^+ = G^- = J = 0 \).

So the only physical unconstrained variables are \( x^1, x^2, \theta^1, \bar{\theta}^1, p_1, \) and the compactification-dependent fields. In this gauge, the action for these fields is

\[ \frac{1}{2\pi} \int d^2z (\frac{1}{2} \partial x^1 \partial x^1 + \frac{1}{2} \partial x^2 \partial x^2 + \bar{p}_L \partial \bar{\theta}^1_L + \bar{p}_R \partial \bar{\theta}^1_R) + S_C, \]

(5.15)

which is equal to (3.1) if the compactification is flat and \([\bar{\theta}^1, \bar{p}_1, \psi^j, \bar{\psi}_j]\) are identified with the eight light-cone \( \theta^\alpha \)'s (\( \psi^j \) and \( \bar{\psi}_j \) are defined in (2.13) where \( j = 1 \) to 3).
6. Conclusions and Applications

In this review, we have introduced a new spacetime-supersymmetric description of the superstring which has \( \text{N}=2 \) worldsheet superconformal invariance. This description is manifestly SO\((3,1)\) super-Poincaré invariant for arbitrary compactifications to four dimensions which preserve \( \text{N}=1 \) 4D supersymmetry. It is related to the \( \text{N}=1 \) RNS description by embedding in an \( \text{N}=2 \) string and performing a field redefinition. It is related to the twistor-like GS description by gauge-fixing six of the eight fermionic worldsheet invariances.

There are three types of applications which have been developed for this new superstring description. One application is the explicit spacetime-supersymmetric computation of superstring scattering amplitudes. For \( \text{N} \)-point tree amplitudes, Koba-Nielsen formulas have been computed which are manifestly SO\((3,1)\) super-Poincaré invariant.\(^{32}\) These formulas are new and can be generalized to scattering in the presence of a \( D \)-brane. For certain multiloop amplitudes involving Ramond-Ramond states, explicit “topological” expressions have been computed to all loop-order for compactifications to four and six dimensions.\(^2\) The four-dimensional expressions reproduce the topological results of \(^{33}\), while the six-dimensional expressions are new.

It would be nice to have explicit spacetime-supersymmetric expressions for arbitrary multiloop amplitudes, and not just for “topological” ones. These expressions could be used for analyzing finiteness properties, which is difficult in the RNS formalism because of the need to sum over spin structures. The only obstacle to calculating multiloop amplitudes in the new description is evaluating the correlation function of the chiral boson \( \rho \), which may have unphysical poles (these unphysical poles are absent in topological amplitudes). Unphysical poles also occur for the RNS chiral boson \( \phi \), and hopefully, this obstacle can be overcome using methods similar to those of references \(^4\) and \(^5\).

A second type of application has been the construction of spacetime supersymmetric sigma models, which can be used to derive the low-energy equations of motion in superspace for the massless superstring fields. Unlike the standard GS sigma model,\(^{34}\) these sigma models contain a Fradkin-Tseytlin term which couples the spacetime dilaton to the worldsheet (super)curvature.\(^{35}\) \(^{14}\)

For 4D compactifications of the heterotic superstring, it has been verified to one-loop order in \( \alpha' \) that worldsheet \( \text{N}=(2,0) \) superconformal invariance of the sigma model implies the standard superspace equations of motion for the \( \text{N}=1 \) supergravity superfields.\(^{36}\) For 4D compactifications of the type II superstring, properties of the sigma model have been...
used to obtain new superspace actions for N=2 supergravity.\textsuperscript{14} It would be interesting to use worldsheet N=(2,2) superconformal invariance of the type II sigma model to check if the low-energy superstring equations of motion come from these new N=2 supergravity actions.

A third type of application has been the construction of an open superstring field theory action.\textsuperscript{13} Unlike the RNS field theory action,\textsuperscript{37}, this new action is manifestly SO(3,1) super-Poincaré invariant and does not suffer from contact-term divergences. Work is in progress on generalizing the construction for closed superstring field theory. Such an action might be very useful for studying perturbative and non-perturbative duality symmetries of the superstring.
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