Adaptive sliding mode control for multi-UAV systems with parametric uncertainties

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Abstract. This paper proposes a parametric uncertain adaptive sliding mode control method for multi-UAVs formation. The tracking effect is well under the condition of uncertain inertial matrix parameters and external interference. An adaptive law is designed to estimate the inertial matrix parameters of each UAV in the formation on-line, and the estimated values are used in the sliding mode controllers, which guarantee that the tracking error is uniformly asymptotically converged to zero. Then the adaptive sliding mode control the stability performance of is proved by Lyapunov stability theory. Finally, simulation illustrated that under the uncertain conditions and external disturbances in the system, the method can achieve the Multi-UAVs formation with a stable attitude and speed synchronization.

1. Introduction

Due to vertical take-off, fixed point hover and ultra-low altitude flight, etc. series of advantages, quadrotors are widely used in military and civilian fields, from search military applications such as rescue, remote monitoring, etc. and are extended to disaster monitoring and maps drawing and other civilian directions[1, 2]. But when performing reconnaissance or attack missions, a single UAV is often limited by its functions and capabilities[3]. Compared to a single UAV, the UAV team has formed a formation system with many advantages: (1). It can improve the overall efficiency of UAVs. (2). It can increase the hit rate. (3). Formation flight can reduce overall flight resistance in terms of aerodynamic efficiency and structural strength.

In order two or more agents connected to each other to complete a given task through coordinated motion, coordinated control design of the control system is required. And consensus algorithms for multi-agent systems have been extensively studied in the literature[4-8]. However, in practical situations, the mass of quadrotors is not fixed when equipped with different sensors for different missions. Hence, intrinsic parameter uncertainties should be taken into consideration. But the commonly used method for obtaining the moment of inertia is through the three-wire pendulum experiment or Solid works software mapping. Its disadvantage is that the results are not accurate enough.

In terms of control, sliding mode control and adaptive control has been widely used due to its rapid response, insensitivity to parameter variations and external disturbances, and simple physical implementation. A novel decentralized adaptive full-order sliding mode control framework is proposed
for the robust synchronized formation motion of multiple-UAVs subject to system uncertainty in [9]. An innovative method is proposed for formation control of UAVs in [10], the scheme is based on the sliding mode reference conditioning technique in a sort of supervisory level. An adaptive sliding backstepping control law for quadcopter attitude control proposed in [11], by employing adaptive elements in the sliding mode control formulation the proposed control law avoids a priori knowledge of the upper bounds on the uncertainty.

The main contribution of this paper is to design an adaptive sliding mode control method suitable for multiple UAVs to overcome the problem of external disturbance and uncertainty of the inertial matrix parameters, according to Lyapunov stability theory proved the system is stable. Finally, through mathematical proofs and simulation results.

The remainder of the paper is organized as follows. In Section 2 the model and its coupling project of multiple unmanned systems are proposed. In Section 3 proposed an adaptive sliding mode controller based on sliding mode control. In Section 4, a simulation experiment was constructed to verify the effectiveness of the algorithm. Finally, in Section 5, this article is summarized.

2. Problem Statement
The problem in this paper is how to realize the formation flight of multiple UAVs with uncertainties of the inertial matrix parameters and external disturbances. A new type of controller needs to be designed so that the formation can track the expected trajectory well.

2.1. Multi-UAVs dynamic model
In this paper, for convenience to design the controllers, we decouple the system into two subsystems: $\Sigma^1_i$ is the rotational subsystem and $\Sigma^2_i$ is the translational subsystem. The model differential Eqs. (1) and (2) of the $i$th quadrotor are expressed as

$$
\begin{align*}
(\Sigma^1_i) : & \quad \dot{\mathbf{Q}}_i = \frac{1}{2} T(\mathbf{Q}_i) \mathbf{\omega}_i \\
& \quad \mathbf{J}_i \dot{\mathbf{\omega}}_i = \tau_i - S(\mathbf{\omega}_i) J_i \mathbf{\omega}_i + \mathbf{d} - \sum_{j=1}^n k_{ij}(\mathbf{q}_{ij} + \gamma(\mathbf{\omega}_j - \mathbf{\omega}_i))
\end{align*}
$$

and

$$
\begin{align*}
(\Sigma^2_i) : \quad & \dot{\mathbf{p}}_i = \mathbf{v}_i \\
& \dot{\mathbf{v}}_i = g \mathbf{e}_z - \frac{T_i}{m_i} R(\mathbf{Q}_i)^T \mathbf{\dot{e}} + \mathbf{d} - \sum_{j=1}^n k_{ij}((\mathbf{p}_i - \mathbf{p}_j - \mathbf{\delta}_{ij}) + \gamma(\mathbf{v}_i - \mathbf{v}_j))
\end{align*}
$$

which $\mathbf{Q}_i, \mathbf{\omega}_i, \mathbf{p}_i$ and $\mathbf{v}_i$ are the attitude, angular velocity, position and velocity respectively; $m_i$ and $g$ denote respectively the mass of the $i$th UAV and the acceleration due to gravity; $\mathbf{J}_i$ are the symmetric positive definite constant moment of inertia matrix of the $i$th UAV; $\tau_i$ and $T_i$ respectively represents the control torque and the magnitude of the rotor thrust for each UAV; $R(\mathbf{Q})$ denote the rotation matrix. $\mathbf{d}$ represent the external disturbances and $\gamma > 0$, the vector $\mathbf{\delta}_{ij} \in \mathbb{R}^3$ satisfied $\mathbf{\delta}_{ij} = -\mathbf{\delta}_{ji}$ defines the desired constant offset between the $i$th and $j$th UAV. For ease of writing later, the coupling terms $\sum_{j=1}^n k_{ij}(\mathbf{q}_{ij} + \gamma(\mathbf{\omega}_j - \mathbf{\omega}_i))$, $\sum_{j=1}^n k_{ij}((\mathbf{p}_i - \mathbf{p}_j - \mathbf{\delta}_{ij}) + \gamma(\mathbf{v}_i - \mathbf{v}_j))$ are abbreviated as $\Delta_1, \Delta_2$.

2.2. UAV error dynamic model
In UAV attitude tracking task, the attitude of the body coordinate system and earth inertial coordinate system is changed, the desired attitude $\mathbf{Q}_d$ can be expressed in the desired unit quaternion
According to the rotational subsystem, multiply both sides of the Equation by \( \mathbf{J}_i \), can get the error dynamics attitude model of UAVs as follows

\[
\hat{\mathbf{Q}} = \frac{1}{2} \mathbf{T}(\bar{\mathbf{Q}}) \hat{\mathbf{a}}.
\]

(3)

As before, the error dynamics translational model of UAVs as follows

\[
\hat{\mathbf{p}}_i = \mathbf{v}_i - \mathbf{T} \mathbf{w}_d, \\
\hat{\mathbf{v}}_i = g \hat{\mathbf{e}}_3 - \frac{T}{m_i} \mathbf{R}(\bar{\mathbf{Q}})^T \hat{\mathbf{e}}_3 - \mathbf{v}_d - \hat{\mathbf{d}}_i.
\]

(4)

**Remark:** In order to ensure that \( \mathbf{w}_d, \mathbf{\dot{w}}_d \) and \( \mathbf{\dot{w}}_d \) are all bounded, so suppose \( \mathbf{q}_i, \mathbf{\eta}_i \) and their three derivatives are bounded. The inertia matrix of each UAV is an unknown, positive-definite symmetric constant matrix. The disturbance torque vector \( \mathbf{d} \) is bounded and satisfied.

\[
|\mathbf{d}_i| \leq D, \quad i = 1, 2, 3.
\]

where \( \mathbf{d} = [d_1, d_2, d_3]^T \), and \( D \) is an unknown normal number.

### 2.3. Adaptive sliding model controller design

In this section, under the inertial matrix parameters of each quadrotor are unknown, an adaptive sliding mode controller is designed to achieve attitude tracking while approximating real inertial parameters.

#### 2.3.1. Control design for rotational subsystem

First, we design the controller for the rotational subsystem, define the rotational sliding surface as following equation

\[
\mathbf{S}_i = \mathbf{\lambda}_i \mathbf{\bar{q}}_i + \mathbf{\bar{w}}_i,
\]

(5)

where \( \mathbf{\lambda}_i = \text{diag}(\lambda_{i1}, \lambda_{i2}, \lambda_{i3}), \lambda_{ij} > 0 \) \( (i = 1, 2, 3) \), \( \mathbf{S}_i = [S_{i1}, S_{i2}, S_{i3}]^T \).

The estimation error \( \mathbf{\theta}_i \) for the inertia parameter of the UAV is defined as \( \hat{\mathbf{\theta}} = \mathbf{\theta}_i - \mathbf{\hat{\theta}}_i \), where \( \mathbf{\theta}_i = [J_{i1}, J_{i2}, J_{i3}, J_{i4}, J_{i5}, J_{i6}]^T \) is UAV inertia parameters real value, UAV inertia parameter estimation value to be \( \hat{\mathbf{\theta}}_i \). The linear operator \( \mathbb{R}^{3\times3} \rightarrow \mathbb{R}^{3\times6} \) is defined as follow

\[
\mathbf{L}(\mathbf{\xi}) = \begin{bmatrix}
\xi_1 & 0 & 0 & \xi_2 & \xi_3 & 0 \\
0 & \xi_2 & 0 & \xi_1 & 0 & \xi_3 \\
0 & 0 & \xi_3 & 0 & \xi_1 & \xi_2
\end{bmatrix}.
\]

For \( \forall \mathbf{\xi} = [\xi_1, \xi_2, \xi_3]^T \in \mathbb{R}^{3\times3} \), have \( \mathbf{J}_i \mathbf{\xi}_i = \mathbf{L}(\mathbf{\xi}_i) \mathbf{\theta}_i \).

Take the derivative of both sides of Eq. (5) with respect to time \( t \), multiply by \( \mathbf{J}_i \) on both sides can get

\[
\mathbf{J}_i \dot{\mathbf{S}}_i = \tau_i - S(w_i)\mathbf{J}_i w_i + J_i(S(\mathbf{\bar{a}}) R(\mathbf{\bar{Q}}))w_d - R(\mathbf{\bar{Q}})\mathbf{\dot{w}}_d + \frac{\mathbf{J} \mathbf{T}(\mathbf{\bar{Q}}) \mathbf{\dot{w}}_d}{2} + \hat{\mathbf{d}}_i.
\]

(6)

Utilize the linear operator \( \mathbf{L}(\mathbf{\xi}) \), the above equation can be rewritten as

\[
\mathbf{J}_i \dot{\mathbf{S}}_i = \tau_i + \mathbf{F}_i \mathbf{\theta}_i + \mathbf{d}
\]

(7)

where \( \mathbf{F}_i = -S(w_i)\mathbf{L}(w_i) + \mathbf{L}(S(\mathbf{\bar{a}}) R(\mathbf{\bar{Q}}))w_d - R(\mathbf{\bar{Q}})\mathbf{\dot{w}}_d + \frac{\mathbf{J} \mathbf{T}(\mathbf{\bar{Q}}) \mathbf{\dot{w}}_d}{2} \).
So, the controllers and adaptive laws designed as follows

\[
\begin{align*}
\epsilon_i &= -F_i \hat{\theta}_i - \eta_i \text{sign}(S_{i_\theta}) \\
\hat{\theta}_i &= \Gamma F_i^T S_{i_\theta}.
\end{align*}
\] (8)

**Theorem 3-1** Consider the UAVs rotational subsystem with unknown inertia matrix and disturbance torque described in Eq. (1), the control input and adaptive control law shown in Eq. (8). The adaptive regulation law ensures that \( \theta_i \) are uniformly ultimately bounded, and the error angular velocity and the component \( e \) of the error quaternion asymptotically converge to zero, i.e. \( \lim_{t \to \infty} \tilde{q}_i = 0 \) and \( \lim_{t \to \infty} \tilde{w}_i = 0 \).

**Proof.** Consider the Lyapunov function candidates as follow

\[
V_1 = \frac{1}{2} S_{i_\theta}^T J_i S_{i_\theta} + \frac{1}{2} \hat{\theta}_i^T \Gamma^{-1} \hat{\theta}_i.
\] (9)

then substituting Eq. (6) into the derivative of Eq. (9), yields

\[
\dot{V}_1 = -S_{i_\theta}^T (\eta_i \text{sign}(S_{i_\theta}) - d).
\] (10)

therefore, when \( \eta_i > D \), \( \dot{V}_1 \leq 0 \). That is, when \( t \to \infty \), \( S_{i_\theta} \to 0 \), so we can get that the rotational subsystem is asymptotically stable.

### 2.3.2. Controller design for translational subsystem

It can be noticed from the second equation of Eq. (2) that the \( i \)th model of the translation subsystem is underactuated. As before, define the position of the sliding surface as follow

\[
S_{2_i} = \lambda_{2_i} \hat{p}_i + \hat{v}_i.
\] (11)

So, the sliding mode controllers given as follow:

\[
T_i = m_i \left[ \lambda_{2_i} \hat{v}_i + \kappa S_{2_i} + \eta_{2_i} \text{sign}(S_{2_i}) - v_{id} - \Delta_2 \right].
\] (12)

**Theorem 3-2** Consider dynamics (1) and (2) of the \( i \)th UAV. The reaching condition (11) of the sliding mode is satisfied, if the \( T_i \) given by Eq. (12). The translational subsystem is asymptotically stable.

**Proof.** Construct Lyapunov function \( V_{2_i} = \frac{1}{2} S_{2_i}^T S_{2_i} \), differentiate it with respect to time \( t \), yields

\[
\dot{V}_{2_i} \leq -\eta_{2_i} \left[ S_{2_i}^T - k_2 S_{2_i}^2 \right] < 0.
\] (13)

Therefore, the closed loop system of position is asymptotically stable. Finally, taking time derivative of \( V \) along the closed-loop system yields can get \( \dot{V} = \dot{V}_{2_i} + \dot{V}_{2_i} < 0 \). The proof is achieved completely.

### 3. Simulation Results

The control strategy discussed in section 3 is successfully implemented in simulation on a multi-UAVs consisting of four quadrotors. The initial inertia parameters of the four quadrotors and the true value of relevant parameters of each UAV are as the following table shows

| Quadrotor number | Initial \( J \left( \text{kg} \cdot \text{m}^2 \right) \) | True \( J \left( \text{kg} \cdot \text{m}^2 \right) \) |
|------------------|-----------------------------|-----------------------------|
| No.1             | [50 50 40 0.4 -0.2]        | [44 48 47 1 -0.2]          |
| No.2             | [50 52 42 0.2 -0.4]        | [48 48 51 0.3 -1.5]        |
| No.3             | [55 52 43 0.3 -0.4]        | [50 56 47 0.1 -0.3]        |
The initial state of quadrotors is randomly selected and the desired trajectory of each UAV is a trajectory with \( p_d(t) \) as the center and selected \( d \) as follows:

\[
\begin{align*}
\mathbf{p}_d(t) &= \begin{bmatrix} 20 \left(1 - \cos \left(\frac{\pi}{20} t\right)\right) \\ -14 \sin \left(\frac{\pi}{10} t\right) \end{bmatrix}, \\
\mathbf{d} &= \begin{bmatrix} 0.5 \sin \left(\frac{\pi t}{10}\right) \\ 0.6 \cos \left(\frac{\pi t}{5}\right) \\ 0.8 \sin \left(\frac{\pi t}{6}\right) \end{bmatrix}.
\end{align*}
\]

The numerical results are shown in figures 1~7. Figure 1 shows the error of the four quadrotors and their expected position in the \( x \), \( y \), and \( z \) directions. Similarly, figures 2~4 respectively show velocity error, attitude error and angular velocity error.

Figure 1. Position error of four quadrotors with their desired values.

Figure 2. Speed error of four quadrotors with their desired values.

Figure 3. Attitude error of four quadrotors with their desired values.

Figure 4. Angular speed error of four quadrotors with their desired values.

Figure 5 shows the process of self-adaptive inertial matrix parameters for each of the four quadrotors. Can see from the figures that around 10s, the parameters of the inertial matrix of each quadrotor are stable and approach a constant, and this value is not much different from the actual value. So, with this adaptive control method can solve the problem that the values of the inertial matrix parameter are indeterminate when the weight of the UAV is different or there is a change in actuality.
Figure 5. Estimated inertial parameter of UAVs

Besides, figures 6–7 illustrate the performance of the proposed adaptive laws with internal parametric uncertainties and external disturbances. Figure 6 illustrates the flight formation process and flying trajectories of each quadrotor UAV, in which all the four quadrotors finally converge to the desired square formation. The figure 7 shows the formation flying status of four quadrotors, they are taken as centered. The simulation results show that the trajectory of the quadrotors is consistent with the expected one.

4. Conclusions
In this paper, the problem of formation control of quadrotors with uncertain parameters of inertial matrix and external disturbances is studied. An adaptive law for the inertial parameters is proposed so that all quadrotors track the reference speed and maintain the desired flight pattern. Through this method can improve the attitude control performance of quadrotors. In addition, the algorithm can be extended to other systems with similar dynamic characteristics, and has a strong universality.

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