Multiplexing Gain of Millimeter-Wave Massive MIMO Systems with Distributed Antenna Arrays

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Abstract. This paper is concerned with multiplexing gain analysis for millimeter-wave (mmWave) massive MIMO systems with distributed antenna deployment. For a point-to-point mmWave system in which the transmitter and receiver consist of \(K_t\) and \(K_r\) subarrays, respectively, an asymptotic multiplexing gain formula is derived when the numbers of antennas at subarrays go to infinity. Specifically, considering that all subchannels have the same average number of propagation paths \(L\), the formula shows that by employing the distributed antenna architecture, an exact average maximum multiplexing gain of \(K_tK_rL\) can be achieved. This result means that compared to the co-located antenna architecture, using the distributed antenna architecture can scale up the multiplexing gain proportionally to \(K_rK_t\).

1 Introduction

Recently, millimeter-wave (mmWave) communication has gained considerable attention as a candidate technology for 5G mobile communication systems and beyond [1]. The main reason for this is the availability of vast spectrum in the mmWave band that is very attractive for high data rate communications. However, compared to communication systems operating at lower microwave frequencies, propagation loss in mmWave frequencies is much higher. Fortunately, given the much smaller carrier wavelengths, mmWave communication systems can make use of compact massive antenna arrays to compensate for the increased propagation loss. Nevertheless, the large-scale antenna arrays together with high cost and large power consumption of the mixed analog/digital signal components makes it uneconomical to equip a separate radio-frequency (RF) chain for each antenna and perform all the signal processing in the baseband. Therefore, research on hybrid analog-digital processing of precoder and combiner for mmWave communication systems has attracted very strong interests from both academia and industry [2-4]. In particular, a lot of work has been performed to address challenges in using a limited number of RF chains. For example, the authors in [3] consider single-user precoding in mmWave massive MIMO systems and establish the optimality of beam steering for both single-stream and multi-stream transmission scenarios. In [4], the authors show that hybrid processing can realize any fully-digital processing if the number of RF chains is twice the number of data streams.

It should be pointed out, however, that mmWave signal propagation exhibits multipath sparsity in both the temporal and spatial domains, which means that the true multiplexing benefit might be limited if the deployment of the antenna arrays is co-located. In general, it is known that deploying distributed antennas can help to increase spectral efficiency and expand coverage of microwave wireless...
communication networks. As such, the authors in [5] and [6] investigate the use of distributed antennas in mmWave massive MIMO systems, which includes co-located array architecture as a special case. This paper analyzes the multiplexing performance of mmWave massive MIMO systems with the distributed antenna architecture. The obtained analysis clearly quantifies multiplexing advantage provided by multiple distributed antenna arrays and can be used conveniently to compare various mmWave massive MIMO systems with different distributed antenna array structures.

2 System Model
Consider a point-to-point mmWave massive MIMO system that employs the distributed antenna architecture as shown in Fig. 1. The transmitter is equipped with a distributed antenna array to send $N_t$ data streams to a receiver, which is also equipped with a distributed antenna array. Here, a distributed antenna array means an array consisting of several remote antenna units (RAUs) (i.e., antenna subarrays) that are distributively located. Specifically, the antenna array at the transmitter consists of $K_t$ RAUs, each of which has $N_t$ antennas and is connected to a baseband processing unit (BPU) by fiber. Likewise, the distributed antenna array at the receiver consists of $K_r$ RAUs, each having $N_r$ antennas and also being connected to a BPU by fibers.

The transmitter accepts as its input $N_t$ data streams and is equipped with $N_t^{\text{rf}}$ RF chains, where $N_t \leq N_t^{\text{rf}} \leq N_t K_t$. Given $N_t^{\text{rf}}$ transmit RF chains, the transmitter can apply a low-dimension $N_t^{\text{rf}} \times N_t$ baseband precoder, $W_t$, followed by a high-dimension $K_t N_t \times N_t^{\text{rf}}$ RF precoder, $F_t$. Note that amplitude and phase modifications are feasible for the baseband precoder $W_t$, while only phase changes can be made by the RF precoder $F_t$ through the use of variable phase shifters and combiners.

The transmitted signal vector can be written as $x = F_t W_t P_t^{1/2} s$, where $s$ is the $N_t \times 1$ symbol vector such that $\mathbb{E} \{ s s^H \} = I_{N_t}$, and $P_t = \text{diag} \{ p_1, p_2, \ldots, p_{N_t} \}$ is a diagonal power allocation matrix with $\sum_{n=1}^{N_t} p_n = P$. Thus $P$ represents the average total input power. Considering a narrowband block fading channel, the $K_t N_t \times 1$ received signal vector is

$$y = H F_t W_t P_t^{1/2} s + n$$

(1)

where $H$ is $K_t N_t \times K_t N_t$ channel matrix and $n$ is a $K_t N_t \times 1$ vector consisting of i.i.d. $\text{CN}(0,1)$ noise samples, where $\text{CN}(0,1)$ denotes a circularly symmetric complex Gaussian random variable with zero mean and unit variance. Throughout this paper, $H$ is assumed known to both the transmitter and receiver. Given that $N_t^{\text{rf}}$ RF chains (where $N_t \leq N_t^{\text{rf}} \leq N_t K_t$) are used at the receiver to detect the $N_t$ data streams, the processed signal is given by

$$z = W_r F_r H F_t W_t P_t^{1/2} s + W_r F_r H n$$

(2)

where $F_r$ is the $K_r N_r \times N_t^{\text{rf}}$ RF combining matrix, and $W_r$ is the $N_t^{\text{rf}} \times N_t$ baseband combining matrix.

![Fig. 1. Block diagram of a mmWave massive MIMO system with distributed antenna arrays.](image-url)
Furthermore, according to the architecture of RAUs at the transmitting and receiving ends, $H$ can be written as

$$
H = \begin{bmatrix}
\sqrt{g_{11}} H_{11} & \cdots & \sqrt{g_{1K}} H_{1K} \\
\vdots & \ddots & \vdots \\
\sqrt{g_{K1}} H_{K1} & \cdots & \sqrt{g_{KK}} H_{KK}
\end{bmatrix}
$$

(3)

In the above expression, $g_{ij}$ represents the large scale fading effect between the $i$th RAU at the receiver and the $j$th RAU at the transmitter, which is assumed to be constant over many coherence-time intervals. The normalized subchannel matrix $H_{ij}$ represents the MIMO channel between the $j$th RAU at the transmitter and the $i$th RAU at the receiver. Given that the antenna subarrays are sufficiently separated in distance at both the transmitter and receiver, all subchannel matrices $\{H_{ij}\}$ are mutually independent. A clustered channel model based on the extended Saleh-Valenzuela model is often used in mmWave channel modeling and standardization [3], and it is also adopted in this paper. For simplicity of exposition, each scattering cluster is assumed to contribute a single propagation path. Using this model, the subchannel matrix $H_{ij}$ is given by

$$
H_{ij} = \sqrt{N} \sum_{l=1}^{L_y} \alpha_{ijl} a_r(\phi_{ij}^r, \theta_{ij}^r) a_t^H(\phi_{ij}^t, \theta_{ij}^t)
$$

(4)

where $L_y$ is the number of propagation paths, $\alpha_{ijl}$ is the complex gain of the $l$th ray, and $\phi_{ij}^r(\theta_{ij}^r)$ and $\phi_{ij}^t(\theta_{ij}^t)$ are its random azimuth (elevation) angles of arrival and departure, respectively. Following previous studies of hybrid precoding for mmWave-MIMO systems [3], the complex gains $\alpha_{ijl}$ are assumed to be $\CN(0,1)$, i.e., their magnitude is Rayleigh distributed. The vectors $a_r(\phi_{ij}^r, \theta_{ij}^r)$ and $a_t(\phi_{ij}^t, \theta_{ij}^t)$ are the normalized receive/transmit array response vectors at the corresponding angles of arrival/departure. For an $N$-element uniform linear array (ULA), the array response vector is

$$
a^{\text{ULA}}(\phi) = \frac{1}{\sqrt{N}} \begin{bmatrix} 1, e^{j2\pi d_x \sin(\phi)}, e^{j2\pi(N-1)d_x \sin(\phi)} \end{bmatrix}^T
$$

(5)

where $\lambda$ is the wavelength of the carrier and $d_x$ is the inter-element spacing. It is pointed out that the angle $\theta$ is not included in the argument of $a^{\text{ULA}}(\phi)$ since the response for an ULA is independent of the elevation angle. In this paper, we suppose that the antenna configurations at all RAUs are ULA.

### 3 Analysis of Multiplexing Gain

From the structure and definition of the channel matrix $H$ in Section 2, there is a total of $L_y = \sum_{i=1}^{K_r} \sum_{j=1}^{K_t} L_y$ propagation paths. Naturally, $H$ can be decomposed into a sum of $L_y$ rank-one matrices, each corresponding to one propagation path. Specifically, $H$ can be rewritten as

$$
H = \sum_{i=1}^{K_r} \sum_{j=1}^{K_t} \sum_{l=1}^{L_y} \tilde{\alpha}_{ijl} a_r(\phi_{ijl}^r, \theta_{ijl}^r) a_t^H(\phi_{ijl}^t, \theta_{ijl}^t)
$$

(6)

where
\[ \hat{\alpha}_y = \sqrt{\frac{N_s N_t}{L_y}} \alpha'_y, \]  

(7)

\( \hat{a}_{ij}(\phi'_y, \theta'_y) \) is a \( K_N \times 1 \) vector whose \( b \) th entry is

\[ \left[ \hat{a}_{ij}(\phi'_y, \theta'_y) \right]_b = \begin{cases} a_{ij}(\phi'_y, \theta'_y), & b \in Q'_i \\ 0, & b \notin Q'_i \end{cases} \]  

(8)

with \( Q'_i = ((i-1)N_r, iN_r) \), and \( \hat{a}_{ij}(\phi'_y, \theta'_y) \) is a \( K_N \times 1 \) vector whose \( b \) th entry is

\[ \left[ \hat{a}_{ij}(\phi'_y, \theta'_y) \right]_b = \begin{cases} a_{ij}(\phi'_y, \theta'_y), & b \in Q'_i \\ 0, & b \notin Q'_i \end{cases} \]  

(9)

with \( Q'_j = ((j-1)N_r, jN_r) \).

**Lemma 1** When \( N_r \to \infty \), all \( L_s \) vectors \( \{ \hat{a}_{ij}(\phi'_y, \theta'_y) \} \) are orthogonal to each other. Likewise, when \( N_t \to \infty \), all \( L_s \) vectors \( \{ \hat{a}_{ij}(\phi'_y, \theta'_y) \} \) are orthogonal to each other.

**Proof** It follows immediately from (8) and (9) that if \( u \neq y \), then vectors \( \{ \hat{a}_{uj}(\phi'_u, \theta'_u) \} \) and \( \{ \hat{a}_{iy}(\phi'_y, \theta'_y) \} \) are orthogonal. On the other hand, when \( u = y \) and \( p = q \), it is known from Lemma 1 and Corollary 2 in [3] that vectors \( \{ \hat{a}_{uj}(\phi'_u, \theta'_u) \} \) and \( \{ \hat{a}_{iy}(\phi'_y, \theta'_y) \} \) are orthogonal. The proof that \( \{ \hat{a}_{ij}(\phi'_y, \theta'_y) \} \) is a set of orthogonal vectors can be shown similarly.

**Lemma 2** Suppose that \( N_r = N_r^{(t)} = N_t^{(t)} = L_s \). Then in the limit of large \( N_r \) and \( N_t \), the system's achievable rate is given by

\[ R = \sum_{j=1}^{K_r} \sum_{l=1}^{K_t} \sum_{s=1}^{L_s} \log_2 (1 + p_{ij}^{(s)} |\hat{\alpha}_y|^2) \]  

(10)

where

\[ \left\{ p_{ij}^{(s)} \mid 1 \leq i \leq K_r, 1 \leq j \leq K_t, 1 \leq l \leq L_s \right\} = \left\{ p_1, p_2, K, p_N \right\} \]  

(11)

By using the optimal power allocation (i.e., the well-known water-filling power allocation), the system achieves the maximum rate, which is denoted as \( R_{\text{opt}} \).

**Definition 3** The distributed MIMO system is said to achieve spatial multiplexing gain \( G_m \) if its achievable rate with optimal power allocation satisfies

\[ G_m(R_{\text{opt}}) = \lim_{P \to \infty} \frac{R_{\text{opt}}(P)}{\log_2(P)} \]  

(12)

With the help of Lemma 1 and Lemma 2, we can derive the following main result.

**Theorem 4** When \( N_r \) and \( N_t \) are very large, the maximum spatial multiplexing gain is equal to

\[ G_m = \min \left\{ L_s, \frac{N_r^{(t)}}{N_r}, \frac{N_t^{(t)}}{N_r} \right\} \]  

(13)

**Corollary 5** Let \( N_r^{(t)} \geq L_s \) and \( N_t^{(t)} \geq L_s \). Under the assumption of \( \bar{L}_y = \bar{L} \) for any \( i \) and \( j \), the distributed massive MIMO system can obtain an average maximum spatial multiplexing gain of \( K_K \).
while the corresponding co-located massive MIMO system can only obtain an average maximum spatial multiplexing gain of $L$.

4 Conclusion

This paper has investigated the use of distributed antenna architecture for mmWave massive MIMO systems and analyzed the asymptotic multiplexing gain when the number of antennas at each subarray goes to infinity. In particular, a simple formula of the asymptotic multiplexing gain under the scenario that the subchannel matrices corresponding to different pairs of antenna subarrays are independent, thanks to the distributed antenna architecture, was derived. The formula shows that the mmWave system with distributed antenna architecture can achieve a much larger multiplexing gain than the system having co-located antenna architecture.

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6 References

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