Cusp anomalous dimension in maximally supersymmetric Yang-Mills theory at strong coupling

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We construct an analytical solution to the integral equation which is believed to describe logarithmic growth of the anomalous dimensions of high spin operators in planar $N = 4$ super Yang-Mills theory and use it to determine the strong coupling expansion of the cusp anomalous dimension.

1. Introduction: The cusp anomalous dimension is an important observable in four dimensional gauge theories ranging from QCD to maximally supersymmetric $N = 4$ Yang-Mills theory (SYM) since it governs the scaling behavior of various gauge invariant quantities like logarithmic growth of the anomalous dimensions of high-spin Wilson operators, Sudakov asymptotics of elastic form factors, the gluon Regge trajectory, infrared singularities of on-shell scattering amplitudes etc. By definition [1, 2], $\Gamma_{\text{cusp}}(g)$ measures the anomalous dimension of a Wilson loop evaluated over a closed contour with a light-like cusp in Minkowski space-time. It is a function of the gauge coupling only and its expansion at weak coupling is known in QCD to three loops [3] and in $N = 4$ SYM to four loops [4]. Recently, a significant progress has been achieved in determining $\Gamma_{\text{cusp}}(g)$ at strong coupling in planar $N = 4$ SYM. Within the AdS/CFT correspondence [5, 6], $\Gamma_{\text{cusp}}(g)$ at strong coupling is related to the semiclassical expansion of the energy of folded string rotating in AdS$_3$ part of the target space [6, 7] (see also [8])

$$\Gamma_{\text{cusp}}(g) = 2g - \frac{3\ln 2}{2\pi} + O(1/g), \quad g = \frac{\sqrt{\lambda}}{4\pi},$$

with $\lambda = g_Y^2 N_c$ being $t'$-Hooft coupling. On the gauge theory side, the Bethe ansatz approach to calculating $\Gamma_{\text{cusp}}(g)$ in the weak coupling limit was developed in [9] based on integrability symmetry of planar Yang-Mills theory and use it to determine the strong coupling expansion of the cusp anomalous dimension [10] to any desired order.

Let us introduce two even/odd functions $\gamma_{\pm}(t) = \pm \gamma_{\pm}(t)$

$$\frac{e^t - 1}{t} \tilde{\gamma}(t) = \frac{\gamma_+ (2gt)}{2gt} + \frac{\gamma_- (2gt)}{2gt} = \gamma_{\pm}(2gt).$$

Following [11], we expand $\gamma_{\pm}(t)$ into the Bessel function Neumann series

$$\gamma_{\pm}(t) = \sum_{k \geq 1} (-1)^{k+1} (2k) J_{2k}(t) \gamma_{2k},$$

with the expansion coefficients $\gamma_k \sim \int_0^\infty dt' t' J_k(t') \gamma_\sigma(t') (\sigma = +/−$ for $k =$ even/odd) and sign factors introduced for the later convenience. Substituting [8] into equation [2] and separating even/odd in $t$ parts, we find after some algebra that [2] is equivalent to the (infinite) system of equations

$$\int_0^\infty \frac{dt}{t} \left[ \frac{\gamma_+(t)}{1 - e^{-t/2g}} - \frac{\gamma_-(t)}{e^{t/2g} - 1} \right] J_{2n}(t) = 0,$$

$$\int_0^\infty \frac{dt}{t} \left[ \frac{\gamma_-(t)}{1 - e^{-t/2g}} + \frac{\gamma_+(t)}{e^{t/2g} - 1} \right] J_{2n-1}(t) = \frac{1}{2^{n_1}}.$$
At weak coupling, one finds from (5) that \( \gamma_+ = J_1(t) + O(g^2) \) leading to \( \Gamma_{\text{cusp}}(g) = 4g^2 + O(g^4) \) in agreement with the known one-loop result (1).

2. Exact solution: The system (3) has the following remarkable property. Introducing two even/odd functions \( \Gamma_\pm(t) = \gamma_\pm(t) + \gamma_\mp(t) \coth \frac{t}{2g} \), or equivalently

\[
2\gamma_\pm(t) = \left[1 - \text{sech} \frac{t}{2g}\right] \Gamma_\pm(t) \pm \tanh \frac{t}{2g} \Gamma_\mp(t),
\]

we find from (5)

\[
\int_0^\infty \frac{dt}{t} \left[ \Gamma_+(t) + \Gamma_-(t) \right] J_{2n}(t) = 0 \tag{7}
\]

\[
\int_0^\infty \frac{dt}{t} \left[ \Gamma_-(t) - \Gamma_+(t) \right] J_{2n-1}(t) = \delta_{n,1},
\]

(with \( n \geq 1 \)) and the cusp anomalous dimension is now given by \( \Gamma_{\text{cusp}}(g) = -2g\Gamma_{1}(0) \).

At large \( g \), we expect from (4) that the functions \( \Gamma_{\pm}(t) \) admit expansion in the Bessel function Neumann series

\[
\Gamma_+(t) = \sum_{k \geq 0} (-1)^{k+1} J_{2k}(t) \Gamma_{2k},
\]

\[
\Gamma_-(t) = \sum_{k \geq 0} (-1)^{k+1} J_{2k-1}(t) \Gamma_{2k-1}.
\]

In distinction to (5), the first series involves \( J_0(t) \) term which ensures that \( \Gamma_+(0) \neq 0 \). Indeed, for \( t \to 0 \) one gets from (4) and (6) that \( \gamma_+(t) \sim t^0 \) and \( \gamma_-(t) \sim t \) and, therefore, \( \Gamma_+(t) \sim t^0 \) and \( \Gamma_-(t) \sim t \). Also, in virtue of \( J_1(t) = -J_0(t) \), the coefficient in front of \( J_1(t) \) is given by \( (\Gamma_+ + \Gamma_-) \) so that it is only the sum of two coefficients that is uniquely defined. We make use of this ambiguity to choose \( \Gamma_- = 1 \).

Substitution of (6) into (5) yields an infinite system of finite-difference equations for the coefficients \( \Gamma_k \). Applying standard methods, we were able to construct its solution for \( \Gamma_k \) (with \( k \geq -1 \)) in the following form (detailed analysis will be published elsewhere)

\[
\Gamma_k = -\frac{1}{2} \Gamma_k^{(0)} + \sum_{p=1} g^p \left[ c_{p} \Gamma_{k}^{(2p-1)} + c_{p}^{+} \Gamma_{k}^{(2p)} \right],
\]

where \( \Gamma_k^{(p)} \) are basis functions independent on \( g \)

\[
\Gamma_{2m} = \frac{\Gamma(m + p - \frac{1}{2})}{\Gamma(m + 1) \Gamma(\frac{1}{2})}, \quad \Gamma_{2m-1}^{(p)} = \frac{(-1)^p \Gamma(m + 1 - p) \Gamma(\frac{1}{2})}{\Gamma(m + 1 - p) \Gamma(\frac{1}{2})},
\]

and the expansion coefficients \( c_{p}^{\pm} \) given by series in inverse powers of the coupling, \( c_{p}^{\pm} = \sum_{r \geq 0} g^{-r} c_{p}^{\pm,r} \). The sum over \( p \) in the r.h.s. of (9) describes the contribution of zero modes of (7). Their dependence on \( g \) is fixed by the additional condition of scaling behavior of \( \gamma_\pm(t) \) (see Eqs. (15) and (19) below). Knowing the \( c_{p}^{\pm} \)—coefficients we can determine the cusp anomalous dimension \( \Gamma_{\text{cusp}}(g) = -2g\Gamma_{1}(0) = 2g\Gamma_{0} \).

\[
\Gamma_{\text{cusp}}(g) = 2g + \sum_{p=1} \frac{1}{g^{p-1}} \left[ \frac{2g}{\sqrt{\pi}} \Gamma(2p - \frac{3}{2}) + \frac{2c_{p}^{+}}{\sqrt{\pi}} \Gamma(2p - \frac{1}{2}) \right].
\]

(11)

Let us now establish the relation between the coefficients \( \Gamma_n \) and \( \gamma_n \). To this end, we return to the relation (6) and apply the identities

\[
\text{sech} t - 1 = \sum_{n \geq 1} (-1)^n a_{2n} t^{2n},
\]

\[
\tanh t = \sum_{n \geq 1} (-1)^n a_{2n-1} t^{2n-1},
\]

where \( a \)—coefficients with even/odd indices are related to the Euler/Bernoulli numbers, respectively. This leads to

\[
2\gamma_\pm(t) = \sum_{n \geq 1} (-1)^{n+1} \times \left[ \frac{a_{2n}}{g^{2n}} (t/2)^{2n} \Gamma_{\pm}(t) \mp \frac{a_{2n-1}}{g^{2n-1}} (t/2)^{2n-1} \Gamma_{\mp}(t) \right].
\]

Replacing \( \gamma_\pm(t) \) and \( \Gamma_\pm(t) \) by the series (4) and (6), respectively, we make use of the Bessel function Neumann series for \( (t/2)^n J_n(t) \) in the r.h.s. of (13) to obtain

\[
\gamma_{2m} = \sum_{n=1} \sum_{j=0}^{m-n} \left[ \Gamma_{2j-1} K_{2m,2j-1} + \Gamma_{2j} K_{2m,2j} \right],
\]

\[
\gamma_{2m-1} = \sum_{n=1} \sum_{j=0}^{m-n} \left[ \Gamma_{2j-1} K_{2m-1,2j-1} + \Gamma_{2j} K_{2m-1,2j} \right].
\]

Here the notation was introduced for the coefficients

\[
K_{m,j} = -\frac{a_n/g^n}{2\Gamma(n)} \times \frac{\Gamma(\frac{1}{2}(m + j + n)) \Gamma(\frac{1}{2}(m - j + n))}{\Gamma(\frac{1}{2}(m + j - n) + 1) \Gamma(\frac{1}{2}(m - j - n) + 1)},
\]

which are given by a product of two identical even/odd polynomials of degree \( n \) in variables \( (m \pm j) \)

\[
K_{m,j} = -\frac{2a_n/g^n}{4^n \Gamma(n)} \left( (m + j)^{n+1} + \ldots \right) \left( (m - j)^{n-1} + \ldots \right),
\]

(16)

with ellipses denoting terms with smaller nonnegative power of \( (m \pm j) \). Replacing \( \gamma_j \) in (13) by their explicit expressions, Eqs. (9) and (10), we express \( \gamma_{2m} \) and \( \gamma_{2m-1} \) in terms of yet unknown coefficients \( c_{p}^{\pm} \). In particular, the first two terms of their large-\( g \) expansion look as

\[
\gamma_k = \frac{1}{g^k} \Gamma_k^{(0)} + \frac{1}{g^k} \left[ c_+ \gamma_k^{(1),+} + c_- \gamma_k^{(1),-} \right] + O(1/g^3),
\]

(17)

with \( c_+ = c_+^{+} + \frac{1}{2} c_+^{++} \), \( c_- = c_-^{+} + \frac{1}{2} c_-^{+-} \) and \( \gamma_k^{(0),i} \) given in terms of \( \Gamma_k^{(p)} \), Eq. (10), \( \gamma_k^{(0)} = \gamma_k^{(0)} + \gamma_k^{(1),i} = \frac{1}{2} \Gamma_k^{(1),+}, \gamma_k^{(2),-} = \frac{1}{2} \Gamma_k^{(2),-} \).
The solutions to (2) have the following remarkable scaling behavior.

3. Quantization conditions: In our approach, the coefficients $c_p^\pm$ are determined from the behavior of $\gamma_{2m}$ and $\gamma_{2m-1}$, Eq. (13), at large $m$. To this end, we introduce the functions $z_{\pm}(x) \equiv \gamma_{2m} - \gamma_{2m-1}$ and examine their asymptotic behavior in the double-scaling limit

$$m, g \to \infty, \quad x = (m - \frac{1}{2})^2/g = \text{fixed}. \quad (18)$$

Employing the approach of [17] and going through numerical analysis of $z_{\pm}(x)$, we found that in the limit (18) the solutions to (2) have the following remarkable scaling behavior

$$z_+(x) = \left(\frac{gx}{g^2}\right)^{-1/4} \left[ z^{(0)}_+(x) + \frac{z^{(1)}_+(x)}{gx} + O(1/g^2) \right], \quad (19)$$

$$z_-(x) = \left(\frac{4gx}{g^2}\right)^{-3/4} \left[ z^{(0)}_-(x) + \frac{z^{(1)}_-(x)}{gx} + O(1/g^2) \right],$$

where the functions $z^{(r)}_{\pm}(x)$ with $r \geq 0$ do not depend on $g$ and have faster-than-power decrease at large $x$. For $x \to 0$, small-$x$ expansion of $z^{(r)}_{\pm}(x)$ runs in integer positive powers of $x$ only. Indeed, matching (17) into (19) we find $z^{(0)}_+(x) = 1 + c_0 x + O(x^2)$ and $z^{(0)}_-(x) = 1 + (8c_0 - 3c_1) x + O(x^2)$. For $x \to \infty$, asymptotic behavior of $z^{(r)}_{\pm}(x)$ is controlled by the coefficients $c_p^\pm$. The quantization conditions for $c_p^\pm$ follow from the requirement $\int_0^\infty dx x^p z^{(r)}_+(x) = \text{finite}$ for any given $p, r \geq 0$.

Let us start with the leading term $z^{(0)}_+(x)$ in the expansion (19). From (14) and (9), we evaluate $z_+(x) = \gamma_{2m} - \gamma_{2m-1}$ in the scaling limit (18) and find that the sums in (14) receive dominant contribution from large $j$. This allows us to substitute the $K^{a,k}_{m,j}$-kernel in (14) by its leading asymptotic behavior (16) and evaluate sum over large $j$ in (14) by integration $\sum_j \to \int dj$, leading to

$$z_+(x) = -\frac{g^{-5/4}}{2\sqrt{\pi}} \sum_{p \geq 0} c_p^+ \Gamma(p - \frac{1}{4}) \sum_{n \geq 1} a_n x^{n+p-\frac{5}{2}} + \ldots \quad (20)$$

where ellipses denote terms suppressed by powers of $1/g$.

Then, taking the Laplace transform w.r.t. $x$ we obtain

$$\int_0^\infty dx x^p z_+(x) = -\frac{(gs)^{1/4}}{2g\sqrt{\pi}} \times \left( \sum_{p \geq 0} c_p^+ \Gamma(p - \frac{1}{4}) \right) \left( \sum_{n \geq 1} a_n s^n \right) + \ldots \quad (21)$$

The sum over $n$ can be evaluated with a help of [12] as

$$\sum_{n \geq 1} a_n s^n = -(1 - \sec s + \tan s) = -\frac{\sqrt{2} \sin(\frac{s}{2})}{\sin(\frac{3s}{4} + \frac{s}{2})}. \quad (22)$$

As a function of $s$, it contains an infinite number of both poles and zeros on the real $s$–axis. Requiring that the integrals $z_p = \int_0^\infty dx x^p z_+(x)$ should be finite for $p \geq 0$, we find that, firstly, $\int_0^\infty dx e^{-x/s} z_+(x)$ is an analytical function of $s$ for $\Re s > 0$ and, secondly, it scales at large $s$ as $\sim \ln(s) + O(1/s^2)$. To satisfy these conditions in the r.h.s. of (21), it proves sufficient to take

$$\sum_{p \geq 0} s^p c_p^+ \Gamma(p - \frac{1}{4}) = \xi_+ \frac{\Gamma(1 - \frac{s}{4})}{\Gamma(\frac{1}{4} - \frac{s}{4})} + O(1/g), \quad (23)$$

with $c_0^- = -\frac{1}{2}$ and $\xi_+$ the normalization factor. Putting $s = 0$ in both sides of (23), we get $\xi_+ = 2\Gamma(\frac{1}{4})^2$. Calculating the Laplace transform $\int_0^\infty dx e^{-x/s} z_+(x)$ in the similar manner and imposing the same conditions as for $z_+(x)$ we obtain the second quantization condition

$$\sum_{p \geq 0} s^p \left[ c_p^- \Gamma(p - \frac{3}{4}) + 2c_p^+ (p - \frac{1}{4}) \Gamma(p + \frac{1}{4}) \right] \quad (24)$$

$$= \xi_+ \frac{\Gamma(1 - \frac{s}{4})}{\Gamma(\frac{1}{4} - \frac{s}{4})} + O(1/g),$$

with $c_0^- = 0$ and $c_0^+ = -\frac{1}{4}$. In comparison with (24), the Laplace transform of $z_-(x)$ contains the factor $\sum_{n=1}^{\infty} a_n (-s)^n = \sqrt{2} \sin(\frac{\pi}{4})/\sin(\frac{3\pi}{4} + \frac{s}{4})$ that leads to (24). As before, putting $s = 0$ in both sides of (24), we fix the normalization factor $\xi_- = 4\Gamma(\frac{1}{4})^2$. Then, expanding both sides of the quantization conditions (23) and (24) around $s = 0$ and matching the coefficients in front of powers of $s$, we determine the coefficients $c_p^\pm$ (with $p \geq 1$) to the leading order in $1/g$. In this way, $c_1^- = \frac{3\ln 2}{4\pi^2} + 1 + O(1/g), \quad c_1^+ = \frac{3\ln 2}{4\pi^2} - \frac{1}{4} + O(1/g). \quad (25)$

Substituting these relations into (11), we obtain $\Gamma_{\text{cusp}}(g)$ which coincides with the string theory prediction (11).

To calculate subleading strong coupling corrections to $\Gamma_{\text{cusp}}(g)$, or equivalently to determine the coefficients $c_p^\pm$, we expand further the Laplace transforms $\int_0^\infty dx e^{-x/s} z_+(x)$ in powers of $1/g$ and require each term of the expansion to verify the same analyticity conditions as the leading term. This can be done systematically by applying the Euler-Maclaurin formula to the sums over $j$ in the r.h.s. of (14). In this manner, we obtain the following all-order quantization conditions

$$\sum_{p \geq 0} s^p \left[ c_p^+ Q^+_p \left( \frac{1}{gs} \right) + \frac{1}{gs} c_p^- Q^-_{p-\frac{1}{4}} \left( \frac{1}{gs} \right) \right] \quad (26)$$

$$= \frac{\Gamma(1 - \frac{s}{4})}{\Gamma(\frac{1}{4} - \frac{s}{4})} \sum_{k=0}^{\infty} (gs)^{-k} \xi_+^k (1/g),$$

$$\sum_{p \geq 0} s^p \left[ c_p^- Q^-_{p-\frac{1}{4}} \left( \frac{1}{gs} \right) + c_p^+ Q^+_p \left( \frac{1}{gs} \right) \right]$$

$$= \frac{\Gamma(1 - \frac{s}{4})}{\Gamma(\frac{1}{4} - \frac{s}{4})} \sum_{k=0}^{\infty} (gs)^{-k} \xi_+^k (1/g),$$
where $\xi_k^+(1/g) = \sum_{n \geq 0} \xi_k^n g^{-n}$ and the $(g-$independent) functions $Q_{k,p}^+ (x)$ are of the form

$$Q_{k,p}^+(x) = \sum_{k,l \geq 0} x^{k+l} Q_{k,p}^{2l} Q_{k,l}^{2l+1}, \quad Q_{k,p}^-(x) = \sum_{k,l \geq 0} x^{k+l} Q_{k,p}^{2l+1}.$$  \hspace{1cm} (27)

Explicit expressions for the coefficients $Q_{k,p}^+$ follow univocally from the Euler-Maclaurin summation formula and the relations (26) coincide with (23) and (24) for $Q / g$ in 1.

We found that, up to order $O(1/g^{40})$, all expansion coefficients of $\Gamma_{\text{cusp}}(g)$ except the first one are negative. In addition, at large orders in $1/g$, they grow factorially and the asymptotic expansion is not Borel summable

$$\Gamma_{\text{cusp}}(g) \sim -g \sum_k \frac{\Gamma(k - \frac{1}{2})}{(2\pi g)^k} = g \int_0^\infty du u^{-1/2} e^{-u} \frac{1}{u - 2\pi g}, \hspace{1cm} (30)$$

with the Stieltjes integral having a pole at $u = 2\pi g$. This indicates that $\Gamma_{\text{cusp}}(g)$ receives nonperturbative correction $\sim g^{1/2} e^{-2\pi g}$ proportional to the residue at the pole.

Our prediction for the cusp anomalous dimension (28) relies on the strong coupling expansion of the solution to the BES equation (2). Eventual verification of (28) remains a challenge for the string theory. We would like to mention that our result for $c_2 = K / (4\pi)$ is in a structural agreement with the (revised) two-loop superstring result of [19] and in precise agreement with a new superstring computation (R. Roiban and A.A. Tseytlin, [arXiv:0709.0681] [hep-th]).

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As was shown in [22], $\Gamma_{\text{cusp}}(g)$, has the interpretation of an energy density of a certain flux configuration and, as such, it receives correction proportional to $m^2$ with $m \sim g^{1/4} e^{-\pi g}$ being the mass gap in the $O(6)$ sigma model.

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