Study of $qqqc\bar{c}$ five quark system with three kinds of quark-quark hyperfine interaction

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Abstract. The low-lying energy spectra of five quark systems $uudc\bar{c}$ ($I = 1/2, S = 0$) and $udsc\bar{c}$ ($I = 0, S = -1$) are investigated with three kinds of schematic interaction: the chromomagnetic interaction, the flavor-spin–dependent interaction and the instanton-induced interaction. In all the three models, the lowest five-quark state ($uudc\bar{c}$ or $udsc\bar{c}$) has an orbital angular momentum $L = 0$ and the spin-parity $J^P = 1/2^-$; the mass of the lowest $udsc\bar{c}$ state is heavier than the lowest $uudc\bar{c}$ state.

1 introduction

The conventional picture of the proton and the corresponding excited states are a bound state of three light quarks $uud$ in the constituent quark model (CQM). Recently, a new measurement about parity-violating electron scattering (PVES) at JLab affords new information about the contributions of strange quarks to the charge and magnetization distributions of the proton, which provides a direct evidence of the presence of the multiquark components in the proton [1]. The importance of the sea quarks in the proton is also found in the measurement of the $d/\bar{u}$ asymmetry in the nucleon [2].

Theoretically, the systematic investigation of baryon mass spectra and decay properties in CQM shows large deviations of theoretical values from the experimental data [3], such as the large $N\eta$ decay branching ratio of $N^+(1535)$ and the strong coupling of $\Lambda(1405)$ to the $KN$ channel. Riska and co-authors suggested that the mixtures of three-quark components $qqq$ and the multiquark components $qqqqq$ reduce these discrepancies [4–11]. In a recent unquenched quark model, by taking into account the effects of multiquark components via $^3P_0$ pair creation mechanism, it is also very encouraging to understand the proton spin problem and flavor asymmetry [12–14]. The $qqqqq$ components could also be in the form of meson-baryon configurations, such as $N^+(1535)$ as a $K\Sigma^+$ bound state [15] and $\Lambda(1405)$ as a $KN$ bound state [16].

In the early 1980s, Brodsky et al. proposed that there are non-negligible intrinsic $uudc\bar{c}$ components ($\sim 1\%$) in the proton [17]. Later the study of Shuryak and Zhitnitsky show a significant charm component in $\eta'$ also [18]. It is natural to expect the high-excited baryons contain a large hidden charm five-quark components too. Recently, some narrow hidden charm $N^*_{cc}$ and $\Lambda^*_{cc}$ resonances were predicted to be dynamically generated in the $PB$ and $VB$ channels with mass above 4 GeV and width smaller than 100 MeV [19–22]. These resonances, if observed, definitely cannot be accommodated into the frame of conventional $qqq$ quark models. An interesting question is whether these dynamically generated $N^*_{cc}$ and $\Lambda^*_{cc}$ resonances can be distinguished from penta-quark configuration states [4–11]. To distinguish between the two hadron structure pictures, it should be worthwhile to explore the mass spectrum of the $qqqc\bar{c}$ consisting of the colored quark cluster $qqqc$ and $\bar{c}$.

The five-quark configuration $qqqq\bar{s}$ and $qqqc\bar{c}$ with exotic quantum numbers have been extensively studied in the chiral quark model [23–26], colormagnetic interaction model [27,28] and instanton-induced interaction model [29]. In this work, we study the mass spectra of the hidden-charm systems $uudc\bar{c}$ and $udsc\bar{c}$ with three types of hyperfine interactions, color-magnetic interaction (CM) based on one-gluon exchange, chiral interaction (FS) based on meson exchange, and instanton-induced interaction (Inst.) based on the non-perturbative QCD vacuum structure.
This paper is organized as follows. In sect. 2, we show the wave functions of five-quark states and Hamiltonians for the three types of interactions. In sect. 3, the mass spectra in the positive and negative sectors are presented. The paper ends with a brief summary.

2 The wave function and Hamiltonian

As dealing with the conventional three-quark model we need the wave functions and Hamiltonian to study the spectrum.

2.1 Wave functions of five-quark systems

Before going to hidden-charm five-quark uudc\bar{c} and udsc\bar{c} systems with isospin and strangeness as \((I,S) = (1/2,0)\) and \((I,S) = (0,-1)\), respectively, we first consider the four-quark subsystem, which can be coupled to an antiquark to form a hidden-charm five-quark system. We use the eigenvector function as given in ref. \[30\] to derive the udsc wave functions of the flavor symmetry \([211]_F, [22]_F, [4]_F, [31]_F\) and \([1111]_F\), which correspond to the SU(4) flavor representation \(15, 20, 35, 45, 1\), respectively. For these flavor multiplets combined with \(\bar{c}\), the following decomposition of the SU(4) representation into SU(3) representations can be found:

\[
15 \times \bar{c} = 8^0 + 1^0 + 8^0 + 1^0 + 3^1 + 6^1 + 1^1 + 15^1 \\
+ 3^1 + 3^2 + 6^2 + 3^1 + 3^{-1}, \quad (1)
\]

\[
20 \times \bar{c} = 8^0 + 10^0 + 3^1 + 6^1 \\
+ 1^1 + 3^1 + 15^1 + 6^1 - 8^0 + 6^0, \quad (2)
\]

\[
35 \times \bar{c} = 10^0 + 35^0 + 24^1 + 6^1 + 15^2 + 3^2 \\
+ 3^4 + 15^1 + 10^0 + 6^2 + 3^2 + 8^3 + 1^3 + 1^3, \quad (3)
\]

\[
45 \times \bar{c} = 8^0 + 10^0 + 27^0 + 24^1 + 6^1 + 15^1 \\
+ 3^1 + 6^1 + 15^2 + 3^2 + 6^2 + 3^2 + 8^3 + 1^3 + 1^3 \\
+ 15^1 + 10^0 + 8^0 + 6^1 + 3^1 + 3^2, \quad (4)
\]

\[
1 \times \bar{c} = 1^0 + 3^1, \quad (5)
\]

where the upper indexes denote the charm number. The decomposition notations in ref. \[31\] are adopted. In the current work we only consider the hidden-charm five quark system, which means charm number \(C = 0\). The states lying in the octet \(8^0\) and singlet \(1^0\) of the qqqc\bar{c} states carry the isospin and strangeness as \((I,S) = (1/2,0)\) and \((I,S) = (0,-1)\) for the octet, and \((I,S) = (0,-1)\) for the singlet, respectively, which are the states we need. The octet can be derived from \([211]_F, [22]_F\) and \([31]_F\). The singlet can be derived from \([211]_F\) and \([1111]_F\). The symmetry \([4]_F\) can form a decuplet when combined with the antiquark \(\bar{c}\), but this does not contain the isospin and strangeness quantum numbers we want. The udsc wave function can be constructed directly by replacement rules mentioned in ref. \[32\]. The explicit form of uudc and udsc wave functions are relegated to appendix A. The phase convention is same as in refs. \[5–11,30\].

The general expression in the flavor-spin coupling scheme for these five-quark wave functions is constructed as

\[
\psi^{(i)}(J,J_z) = \sum_{a,b,c,d,f} \sum_{L_z,S_z,S_z} C^{[X(i)]}_a[F^{(i)}]_{a,T_z} [S^{(i)}]_b \psi^{[211]_d}_{C(a,b,c,d,e,f)}
\]

where \(J\) is the total angular momentum of four quarks and \(S\) the total spin of four quarks, \(i\) is the number of the qqqc\bar{c} configuration in both positive- and negative-parity sectors, which will be given explicitly later. \(\psi^C\), \(\psi^S\) and \(\xi_z\) represent the color, flavor and spinor wave functions of the antiquark, respectively. \(\varphi(r_c)\) represents the space wave function for antiquark. The symbols \(C^{[1]}_{a,b,c,d}\) are \(S_A\) Clebsch-Gordan coefficients for the indicated color-flavor-spin \([(FCS)]\), color \(\psi^C\), flavor-spin \([(FS)]\), flavor \([(F)]\), spin \([(S)]\), and orbital \([(X)]\) wave functions of the qqqc system.

2.2 Hamiltonians

To investigate the mass spectrum of the five-quark system, the non-relativistic harmonic oscillator Hamiltonian is introduced as in the light-flavor case \[33\],

\[
H = \sum_{i=1}^{5} \left( m_i + \frac{\vec{p}_i^2}{2m_i} \right) - \frac{\vec{P}_{cm}^2}{2M} \\
+ \frac{1}{2} \sum_{i<j}^5 C[r_i - r_j]^2 + V_0 + H_{hyp}, \quad (7)
\]

where \(m_i\) denotes the constituent masses of quarks \(u, d, s\) (and the antiquark \(\bar{c}\), and \(P_{cm}\) and \(M\) are the total momentum and total mass \(\sum_{i=1}^{5} m_i\) of the five-quark system. \(C\) and \(V_0\) are constants. As pointed out by Glazman and Riska \[32\], one may treat the heavy-light quark mass difference by including a flavor-dependent perturbation term \(H''_0\)

\[
H' = \sum_{i=1}^{5} \left( m_i + \frac{\vec{p}_i^2}{2m} \right) - \frac{\vec{P}_{cm}^2}{10m} \\
+ \frac{1}{2} \sum_{i<j}^5 C[r_i - r_j]^2 + V_0 + H''_0 + H_{hyp}, \quad (8)
\]

with \(m\) denoting the \(u, d, s\) quark mass. The Hamiltonian may be rewritten as a sum of 4 separated Hamiltonians.
in Jacobi coordinates. The perturbation term $H_0''$ has the following form:

$$
H_0'' = -\sum_{i=1}^{4} \left(1 - \frac{m}{m_c}\right) \left(\frac{\vec{p}_i^2}{2m} - \frac{m_c\vec{p}_{cm}^2}{5m(3m + 2m_c)}\right) \delta_{ic} - \left(1 - \frac{m}{m_c}\right) \left(\frac{\vec{p}_i^2}{2m}\right) \delta_{5i},
$$

where the Kronecker symbol $\delta_{ic}$ means that the flavor-dependent term is non-zero when the $i$-th quark of four quarks is charm quark. If the center-of-mass term is dropped, the matrix element of perturbation term on the harmonic oscillator state in the negative-parity sector ($L = 0$) will be

$$
\langle H_0'' \rangle_{[^{4}S_{1/2},^{1}P_{1/2}]c[211]c[31]c} = -\frac{3}{4} \delta, \tag{10}
$$

where $\delta = (1 - m/m_c)\omega_5$ with the oscillator frequency $\omega_5 = \sqrt{5}/m$. For other states considered in this work the matrix elements can also be written in such simple forms.

The term $H_{hyp}$ reflects the hyperfine interaction between quarks in the hadrons. In this work we consider three types of the hyperfine interactions, i.e., flavor-spin interaction ($FS$) based on meson exchange, color-magnetic interaction ($CM$) based on one-gluon exchange, and instanton-induced interaction ($Inst.$) based on the non-perturbative QCD vacuum structure.

The flavor-spin-dependent interaction reproduces well the light-quark baryon spectrum, especially the correct ordering of positive- and negative-parity states in all the considered spectrum [34]. The flavor-dependent interaction has been extended to heavy baryons in sector in ref. [32]. Given that $SU(4)$ flavor symmetry is broken mainly through the quark mass differences and the ratio of the spatial matrix element of the meson exchange potential is very close to the quark-mass ratio $[32,34]$, the hyperfine Hamiltonian can be written in the following form as in ref. [35]:

$$
H_{FS} = -C_{\chi} \sum_{i,j}^{4} \sum_{m_i,m_j}^{m} \sum_{F=1}^{14} \vec{\lambda}_{iF} \cdot \vec{\lambda}_{jF} \bar{\sigma}_i \cdot \bar{\sigma}_j, \tag{11}
$$

where $\sigma_i$ and $\vec{\lambda}_F$ are Pauli spin matrices and Gell-Mann $SU(4)_F$ flavor matrices, respectively, and $C_{\chi}$ a constant phenomenologically 20 $\sim$ 30 MeV. In the chiral quark model [34], only the hyperfine interactions between quarks are considered while the interactions between the quarks and the heavy antiquark $\bar{c}$ are neglected.

The chromomagnetic interaction, which have achieved considerable empirical success in describing the splitting in baryon spectra [36–39], are intensively used in the study of multiquark configurations [31, 40–44]. A commonly used hyperfine interaction is as the following [42, 43]:

$$
H_{CM} = -\sum_{i,j} C_{i,j} \vec{\chi}_i \cdot \vec{\chi}_j \bar{\sigma}_i \cdot \bar{\sigma}_j, \tag{12}
$$

where $\sigma_i$ is the Pauli spin matrix, $\vec{\chi}_i$ is the Gell-Mann $SU(3)_C$ color matrices, and $C_{i,j}$ the color-magnetic interaction strength. The quark-antiquark strength factors are fixed by the hyperfine splittings of the mesons. For an antiquark the following replacement should be applied [45]: $\vec{\lambda}_c \rightarrow -\vec{\lambda}_c$.

The instanton-induced interaction, introduced first by ’t Hooft for [ud]-quarks [46, 47] and then extended to the three-flavor case [48], is also quite successful in generating the hyperfine structure of the light baryon spectrum [49–52]. The instanton-induced charm-light quark interaction is phenomenologically introduced and reproduces the single charm baryon hyperfine splittings [53, 54]. The non-relativistic limit of the unregularized quark-quark ’t Hooft interaction has the form

$$
H_{Inst.} = -4P_{S_{0}} \otimes [W_{nn} P_{A}^F (mn) + W_{ns} P_{A}^F (ns) + W_{nc} P_{A}^F (nc)] \otimes P_{A}^C \times \bar{c},
$$

where $P_{S_{0}}$ is the radial matrix element of the contact interaction between a quark pair with flavors $f_1$ and $f_2$, $P_{A}^F (f_1, f_2)$ the projector onto flavor-antisymmetric quark pairs; $P_{A}^C$ and $P_{A}^S$ the projectors onto color antitriplet and color sextet pairs, respectively; $P_{S_{0}}$ and $P_{S_{0}}$ the projectors onto antisymmetric spin-singlet and symmetric spin-triplet states, respectively. For a three-quark system, only two quarks $qq$ interaction in a spin singlet state with the flavor antisymmetry can contribute to the baryon spectrum. Here, we phenomenologically consider the instanton-induced interaction of the $nc$ and $sc$ quark pairs, although some authors [55, 56] assume that the heavy flavor decouples when the quark gets heavier than the $A_{QCD}$.

### 3 Mass spectra of uudc$\bar{c}$ and udsc$\bar{c}$ systems

In this section, we present the numerical results for the low-lying spectra of the five-quark systems of uudc$\bar{c}$ and udsc$\bar{c}$ with the hyperfine interaction given by the color-magnetic interaction, the flavor-spin interaction, and the instanton-induced interaction, respectively. For the kinetic part and the confinement potential part of the Hamiltonian, we take the parameters of refs. [32, 33], i.e., $m_u = m_d = 340$ MeV, $m_s = 460$ MeV, $m_c = 1652$ MeV and $C = m_u \omega_5^2/5$ with $\omega_5 = 228$ MeV.

All other parameters for three different hyperfine interactions are listed in table 1. For the $FS$ model, the $C_{\chi}$ and $V_0$ parameters are taken from ref. [33]. For the $CM$ model, we take the $C_{i,j}$ parameters of refs. [42, 43], determined by a fit to the charged ground states. For the $Inst.$ model, the parameters are determined by a fit to the splittings between the baryon ground states $N(938)$, $\Lambda(1232)$, $\Lambda(1116)$, $\Sigma^0(1193)$, $\Omega(1672)$, $\Lambda_c(2286)$, $\Sigma_c(2455)$, $\Xi_c^0(2471)$, $\Xi_c^+(2578)$ and $\Xi_c^{++}(2645)$. The fit yields a ratio of about $W_{ns}/W_{nn} \approx 2/3$, which is the
same as in ref. [50]. The parameter $V_0$ for each model is adjusted to reproduce the mass of $N^*(1535)$ as the lowest $J^P = 1/2^-N^*$ resonance of penta-quark nature. In our concrete calculations, the $[31]_{FS}[22]_{FS}[31]_S$ flavor-spin configuration is used for the $N^*(1535)$ since it gives the lowest energy state for all three hyperfine interactions. The $V_0$ here is found to be different from the case for the ground-state baryons which are assumed to be 3-quark states. This may be due to the fact that 3-quark states and penta-quark states may have different mean fields.

With all these Hamiltonian parameters fixed and the wave functions of five quark system outlined in the sect. 2, the matrix elements of Hamiltonian for various five-quark wave functions of five quark system outlined in the sect. 2, the convenient coupling schemes for the $FS$ and $CM$ models are different, i.e., $[1111]_{CF}[2111]_{FS}[2111]_{FS}[211]_S$ and $[1111]_{CF}[2111]_{FS}[2111]_S[22]_{FS}[2111]_S$, respectively. The flavored configurations for the $udsc$ and $uudc$ systems of spacial ground state for the $FS$ and CM models are listed in table 2, where the configurations $|1^+\rangle$ and $|3^-\rangle$ are only for the $uudc$ system. For the $udsc$ system, the $|2^-\rangle$ and $|3^-\rangle$ are computed with three kinds of hyperfine interactions.

Table 1. The parameters (in units of MeV) for three kinds of hyperfine interactions.

| CM [42,43] | $C_{qq}$ | 20 | $C_{qs}$ | 14 | $C_{qc}$ | 4 | $C_{sc}$ | 5 | $C_{cci}$ | 6.6 | $C_{cc}$ | 6.7 | $C_{csc}$ | 5.5 | $V_0$ | –208 |
|---------------|----------|---|----------|---|----------|---|----------|---|----------|-------|----------|-------|----------|-------|------|
| $FS$ [33]     | $C_S$    | 21 | $V_{hn}$ | 315 | $W_{hn}$ | 200 | $W_{hsc}$ | 70 | $W_{sc}$ | 52   | $V_{0}$  | –213  |           |       |       |

Table 2. The flavor-spin configurations for the $uudc$ and $udsc$ systems of spacial ground state $|4\rangle_X$ for the $FS$ and $CM$ models.

| Conf. | $|1^+\rangle$ | $|3^-\rangle$ | $|1^+\rangle$ | $|3^-\rangle$ | $|J^P\rangle$ |
|-------|---------------|---------------|---------------|---------------|----------------|
| $FS$  | $[31]_{FS}[211]_{FS}[22]_S$ | $[211]_{FS}[31]_{FS}[211]_{FS}[22]_S$ | $[211]_{FS}[31]_{FS}[211]_{FS}[22]_S$ | $[211]_{FS}[31]_{FS}[211]_{FS}[22]_S$ | $|J^P\rangle$ |
| $CM$  | $[31]_{FS}[211]_{FS}[22]_S$ | $[211]_{FS}[31]_{FS}[211]_{FS}[22]_S$ | $[211]_{FS}[31]_{FS}[211]_{FS}[22]_S$ | $[211]_{FS}[31]_{FS}[211]_{FS}[22]_S$ | $|J^P\rangle$ |

Table 3. Energies (in units of MeV) of the $udsc$ and $uudc$ system of the spacial ground state with three kinds of hyperfine interactions for different flavor-spin configurations.

| CM | $FS$ | $Inst.$ | $|J^P\rangle$ |
|----|-----|--------|----------------|
| $|1^+\rangle$ | $4404$ | $4169$ | $4211$ | $|\frac{1}{2}^-\rangle$ |
| $|3^-\rangle$ | $4325$ | $4169$ | $4222$ | $|\frac{1}{2}^-\rangle$ |
| $|1^+\rangle$ | $4432$ | $4169$ | $4222$ | $|\frac{1}{2}^-\rangle$ |
| $|3^-\rangle$ | $4480$ | $4372$ | $4287$ | $4125$ | $|\frac{1}{2}^-\rangle$ |
| $|1^+\rangle$ | $4441$ | $4333$ | $4200$ | $4059$ | $4322$ | $4167$ | $|\frac{1}{2}^-\rangle$ |
| $|3^-\rangle$ | $4538$ | $4430$ | $4200$ | $4059$ | $4322$ | $4167$ | $|\frac{1}{2}^-\rangle$ |
| $|1^+\rangle$ | $4552$ | $4436$ | $4182$ | $4052$ | $4347$ | $4195$ | $|\frac{1}{2}^-\rangle$ |
| $|3^-\rangle$ | $4471$ | $4368$ | $4229$ | $4096$ | $4360$ | $4202$ | $|\frac{1}{2}^-\rangle$ |
| $|1^+\rangle$ | $4572$ | $4468$ | $4229$ | $4096$ | $4360$ | $4202$ | $|\frac{1}{2}^-\rangle$ |
| $|3^-\rangle$ | $4617$ | $4508$ | $4258$ | $4133$ | $4386$ | $4237$ | $|\frac{1}{2}^-\rangle$ |
| $|1^+\rangle$ | $4585$ | $4478$ | $4258$ | $4133$ | $4386$ | $4237$ | $|\frac{1}{2}^-\rangle$ |
| $|3^-\rangle$ | $4629$ | $4526$ | $4362$ | $4236$ | $4461$ | $4322$ | $|\frac{1}{2}^-\rangle$ |
| $|1^+\rangle$ | $4719$ | $4616$ | $4362$ | $4236$ | $4461$ | $4322$ | $|\frac{1}{2}^-\rangle$ |

The corresponding seven $udsc$ wave functions with spin-parity $1/2^-$ are $|1^+,1/2^-,1/2^-,1/2^-,21/2^-,31/2^-,41/2^-,51/2^-,...$, respectively. The one wave function with spin-parity $5/2^-$ is $|6,5/2^-angle$. They form three subspace of $J^P = 1/2^-$, $3/2^-$ and $5/2^-$, respectively.

The energies for these different configurations have been calculated with three kinds of hyperfine interactions and are listed in table 3. For subspace of $J^P = 1/2^-$ and $3/2^-$, some non-diagonal matrix elements of Hamiltonians are not zero and lead to the mixture of the configurations with the same spin-parity. After considering the configuration mixing, the eigenvalues of the Hamiltonians of the five-quark $udsc$ and $uudc$ systems in the spatial ground state are listed in table 4. The corresponding mixing coefficients of the states with spin-parity $1/2^-$ for three different models are listed in tables 5-7. The spin symmetry $[4]_S$ is orthogonal to the spin symmetry $[31]_S$ and $[22]_S$. There is no mixing between the configuration $[31]_{FS}[31]_S[4]_S$ and other 7 configurations.

For the lowest spatial excited states, one quark should be in the $P$-wave, which results in a positive parity for the five-quark system. For the $udsc$ system, there are thirty-four wave functions with spin-parity $1/2^+$ and $3/2^+$, twenty-two with $5/2^+$ and four with $7/2^+$. Similarly, there are too many states for the $uudc$ system. Here, ten of all the states with spin-parity $1/2^+$, five of the lowest states with spin-parity $3/2^+$, five of the lowest states with $5/2^+$, and all the states with spin-parity $7/2^+$ are listed in table 8 in terms of energy.

While in the negative-parity sector there are three subspaces for $1/2^-$, $3/2^-$ and $5/2^-$, respectively, for the
positive-parity sector, there are four subspaces for $1/2^+$, $3/2^+$, $5/2^+$ and $7/2^+$, respectively. In the process of the calculation, we take the $L S$ coupling scheme with standard Clebsch-Gordan coefficients of the angular momentum [57]. For the flavor-spin and instanton-induced interactions, due to the ignoring of the quark-antiquark interaction, the $1/2^+$ and $3/2^+$ states of the same configuration $[f]_{FS}[f]_{PS}$ degenerate. In the $CM$ model, the two states of the same configuration but different four-quark angular momentum $J$ have a small splitting magnitude of several MeV as shown in table 8. Here only the masses of several lower-energy states, which are more interesting to us, are listed in table 8.

Table 4. Energies (in units of MeV) of the $udsc\bar{c}$ and $uudc\bar{c}$ systems in the spatial ground state under three kinds of hyperfine interactions (i.e., with configuration mixing considered).

| $J^P$ | CM | FS | Inst. |
|-------|-----|-----|-------|
| 1/2^- | 4273 | 4267 | 4084 | 3933 | 4209 | 4114 |
| 1/2^- | 4377 | 4363 | 4154 | 4013 | 4216 | 4131 |
| 1/2^- | 4453 | 4377 | 4160 | 4119 | 4277 | 4204 |
| 3/2^- | 4469 | 4471 | 4171 | 4136 | 4295 | 4207 |
| 3/2^- | 4494 | 4541 | 4253 | 4156 | 4360 | 4272 |
| 5/2^- | 4576 | 4263 | - | 4362 |
| 7/2^- | 4649 | 4278 | - | 4416 |
| 9/2^- | 4431 | 4389 | 4154 | 4013 | 4216 | 4131 |
| 11/2^- | 4503 | 4445 | 4171 | 4119 | 4295 | 4204 |
| 13/2^- | 4549 | 4476 | 4263 | 4136 | 4362 | 4272 |
| 15/2^- | 4577 | 4526 | 4278 | 4236 | 4416 | 4322 |
| 17/2^- | 4629 | 4362 | - | 4461 |

Table 5. The mixing coefficients of the states with spin-parity $1/2^-$ under the $CM$ interaction including the $q\bar{q}$ interaction.

| $udsc\bar{c}$ | $|1\rangle$ | $|2\rangle$ | $|3\rangle$ | $|4\rangle$ | $|5\rangle$ |
|-------------|-------------|-------------|-------------|-------------|-------------|
| $1/2^-$     | 0.54 0.06 0.02 0.84 0.05 0.01 | 4273         |
| $3/2^-$     | 0.05 0.61 0.08 0.12 0.77 0.15 0.11 |
| $5/2^-$     | 0.83 0.03 0.10 0.52 0.15 0.09 0.03 |
| $7/2^-$     | 0.07 0.17 0.20 0.05 0.11 0.95 0.09 |
| $9/2^-$     | 0.02 0.46 0.64 0.02 0.40 0.30 0.36 |
| $11/2^-$    | 0.14 0.61 0.55 0.06 0.45 0.03 0.31 |
| $13/2^-$    | 0.03 0.08 0.48 0.02 0.11 0.02 0.87 |

Table 6. The mixing coefficients of the states with spin-parity $1/2^-$ under the $FS$ interaction.

| $udsc\bar{c}$ | $|1\rangle$ | $|2\rangle$ | $|3\rangle$ | $|4\rangle$ | $|5\rangle$ |
|-------------|-------------|-------------|-------------|-------------|-------------|
| $1/2^-$     | 0.03 0.75 0.66 | 4084         |
| $3/2^-$     | 0.95 0.22 0.21 | 4154         |
| $5/2^-$     | 0.02 0.09 0.35 0.18 0.06 |
| $7/2^-$     | 0.29 0.62 0.73 | 4253         |
| $9/2^-$     | 0.00 0.07 0.46 0.71 0.53 |
| $11/2^-$    | 0.76 0.65 0.00 |
| $13/2^-$    | 0.00 0.00 0.00 |
| $15/2^-$    | 0.00 0.00 0.00 |

Table 7. The mixing coefficients of the states with spin-parity $1/2^-$ under the $Inst.$ interaction.

| $udsc\bar{c}$ | $|1\rangle$ | $|2\rangle$ | $|3\rangle$ | $|4\rangle$ | $|5\rangle$ |
|-------------|-------------|-------------|-------------|-------------|-------------|
| $1/2^-$     | 0.99 0.07 0.08 | 4209         |
| $3/2^-$     | 0.97 0.12 0.02 0.19 |
| $5/2^-$     | 0.04 0.04 0.03 0.35 |
| $7/2^-$     | 0.00 0.00 0.00 0.00 |
| $9/2^-$     | 0.00 0.00 0.00 0.00 |
| $11/2^-$    | 0.00 0.00 0.00 0.00 |
| $13/2^-$    | 0.00 0.00 0.00 0.00 |
| $15/2^-$    | 0.00 0.00 0.00 0.00 |

The non-zero off-diagonal matrix elements introduce the mixture of the configurations with the same quantum number. The different hyperfine interactions give different admixture of configurations of certain state, which will result in different patterns of the electromagnetic and strong decays. The mixing effect has been explored in the light-quark sector, such as the decay of nucleon resonances $N^0(1440)$ [11] and $N^0(1535)$ [58].

For the $udsc\bar{c}$ system, in the $CM$ model without $q\bar{q}$ interaction, the $SU(3)$ flavor singlet with hidden charm, which has four quark configuration $[211]_T^{[31]}CS[211]_C^{[22]}s$, is dominant in the lowest-energy state, with a small admixture of $[211]_T^{[31]}CS[211]_C^{[22]}s$. The mixing of the two configurations is due to the flavor dependence of $C_{i,j}$. After considering the $q\bar{q}$ interaction in $CM$ model, the configuration $[211]_T^{[31]}CS[211]_C^{[31]}s$...
Table 8. Energies (in units of MeV) of positive-parity (L = 1) qqqc̄ states with quantum numbers of N* - and A* -resonances under three kinds of interaction, with configuration mixing considered.

| J^P | CM       | FS       | Inst.     |
|-----|----------|----------|-----------|
|     | udsc    | udsc    | udsc     |
| 2^+ | 4622    | 4456    | 4291     |
| 2^+ | 4636    | 4480    | 4297     |
| 2^+ | 4645    | 4557    | 4363     |
| 2^+ | 4658    | 4581    | 4439     |
| 2^+ | 4690    | 4593    | 4439     |
| 2^+ | 4696    | 4632    | 4467     |
| 2^+ | 4714    | 4654    | 4469     |
| 2^+ | 4728    | 4676    | 4486     |
| 2^+ | 4737    | 4714    | 4492     |
| 2^+ | 4766    | 4720    | 4510     | 4566
| 3^+ | 4623    | 4457    | 4291     |
| 3^+ | 4638    | 4515    | 4297     |
| 3^+ | 4680    | 4561    | 4363     |
| 3^+ | 4692    | 4582    | 4439     |
| 3^+ | 4695    | 4625    | 4439     | 4566
| 3^+ | 4705    | 4539    | 4297     | 4501
| 3^+ | 4719    | 4649    | 4439     | 4504
| 3^+ | 4773    | 4689    | 4467     | 4587
| 3^+ | 4793    | 4696    | 4486     | 4615
| 3^+ | 4821    | 4710    | 4492     | 4632
| 3^+ | 4945    | 4841    | 4638     | 4698
| 3^+ | 4955    | 4862    | 4671     | 4712
| 3^+ | 4974    | 4919    | 4705     | 4765
| 3^+ | 5017    | 4759    | 4759     | 4797

(∼ 72%) becomes the dominant wave function component, with a strong admixture of [211]_F [31]_C S [211]_C [22]_S (∼ 27%), as shown in table 5. The q̅ q̅ interaction leads to a further mixing of the two spin-symmetry configurations of [22]_S and [31]_S, besides the flavor symmetry breaking effects. In the FS model, the lowest state has a dominant four-quark configuration [31]_FS [211]_F [22]_S (∼ 42%), with a strong admixture of [211]_FS [31]_F [22]_S and [31]_FS [211]_F [22]_S, as shown in table 7. In the Inst. model, the lowest state predominantly has the configuration [211]_C [31]_FS [211]_F [22]_S, which is the same as the CM case without q̅ q̅ interaction.

For the udsc̄ system, there is no hidden charm SU(3) flavor singlet state. In the CM model after taking into account the q̅ q̅ interaction, the lowest-energy state is mainly the admixture of [211]_F [31]_C S [211]_C [31]_S (∼ 67%) and [211]_F [31]_C S [211]_C [22]_S (∼ 27%), as shown in table 5. In the FS model, the lowest state is the four-quark configuration [31]_FS [211]_F [22]_S (∼ 52%), with a strong admixture of [31]_FS [31]_F [22]_S (∼ 42%). In the present Inst. model, assuming phenomenologically that the 't Hooft’s force also operates between a light and a charm quark, the configuration [211]_F [31]_C S [211]_C [22]_S should be the lowest, as the spin [22]_S and flavor [211]_F contain more antisymmetrized quark pairs. In the Inst. model, if it is assumed that the light quark and charm quark decouples, the [211]_F [31]_C S [211]_C [22]_S and [31]_F [31]_C S [211]_C [22]_S states degenerate and should be the lowest.

If the flavor SU(3) symmetry is restored and the light quark and charm quark decouples, the udsc̄ is lower than the udsc̄. For the positive-parity udsc̄ states, under the CM interaction with the q̅ q̅ interaction, the lowest state has predominantly the four-quark configuration [31]_F [31]_C S [211]_C [22]_S, with a strong admixture of the configurations [22]_F [31]_C S [211]_C [22]_S and [31]_F [31]_C S [211]_C [22]_S. In the FS model, the lowest positive-parity state has predominantly the configuration [4]_FS [22]_F [22]_S. The Inst. model predicts that the lowest state is the configuration [1111]_F [31]_C S [211]_C [22]_S, which can form the SU(3) flavor singlet state when combined with the antiquark.

Different hyperfine interactions predict different configurations for the lowest five-quark states, which will result in different decay patterns and can be checked by future experiments.

4 Summary and discussions

In this work we have estimated the low-lying energy levels of the five-quark systems udsc̄ and udsc̄ with the hidden charm by using three kinds of hyperfine interactions. The hidden-charm states are obtained by diagonalizing the hyperfine interactions in each subspace with the same spin-parity. For the colormagnetic interaction, flavor-spin-dependent interaction and Inst.-induced interaction, all the models predict that the lowest states of the five-quark systems udsc̄ and udsc̄ have the spin-parity 1/2"-. The absolute value of the negative hyperfine energy for the configuration [4]_FS [22]_F [22]_S in the positive-parity sector is larger than the case of the [31]_FS [211]_F [22]_S in the negative-parity sector. But this difference cannot overcome the orbital excited energy of the P-wave five-quark system. This is in contrast with the situation in the light-flavor sector with the chiral hyperfine interaction [33], due to the fact that the hyperfine splitting depends on the quark masses and gets weak for heavy quarks. In addition, for the flavor-spin interaction, the lowest udsc̄ state has negative parity, which is opposite to the lowest positive-parity state of udsc̄ system containing only one heavy antiquark [24]. The four quarks udwd with colored quark cluster configuration [31]_X [4]_FS [22]_F [22]_S are strongly attractive due to the diquark structure [ud][ud]. However, the c quark in the diquarks [ud][uc] with the same flavor-spin symmetry reduces to a large extent the hyperfine interaction energy. The instanton-induced interaction only operates on the color sextet and antitriplet diquark, and thus favors as well the similar diquark structure. The P-wave diquark-triquark structure [ud][udc̄] is discussed under the colormagnetic interaction [28] and is almost as low
as the $|[ud]|$ and $|cc\rangle$. It would be of interests to study the configurations of $|[ud]|$ and $|cc\rangle$.

The coupled-channel unitary approach [19] predicted that the bound state $D_{s1}$ is $30 \sim 50$ MeV lower than the bound state $D_{s0}$. In the chiral quark model [21], there only exists the bound state $D_{s0}$. In the present model, for the colored-cluster picture with three kinds of the residual interactions, the lowest uudcc system is heavier than the uudcc system. So the meson-baryon picture and the penta-quark picture give different prediction on the mass order of the super-heavy $N^*$ and $\Lambda^*$ with hidden charm.

In the CM model, the lowest $1/2^-$ and $3/2^-$ states, corresponding to the same four-quark configuration, are split by the quark-antiquark interaction. And the $3/2^-$ state of the uudcc and uudcc system is about 150 MeV heavier than the corresponding $1/2^-$ state. In the FS and Inst. models, due to the lack of the quark-antiquark interaction, the two states degenerate.

In addition, we have also discussed the admixture pattern of the configurations with the same quantum numbers. The quark mass difference and the quark-antiquark interaction are the two sources of generating the configuration mixing, and the latter is more important for the configuration mixing and mass splitting of penta-quark states. Since various configurations will result in different electromagnetic and the strong decays, the study of the decay properties may provide a good test of the models.

Experimental observation of the super-heavy $N^*$ and $\Lambda^*$ with hidden charm and their decay properties from $p\bar{p}$ reaction at PANDA and $e^+e^-$ reaction at JLab 12 GeV upgrade are of great interests for our understanding dynamics of strong interaction.

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Appendix A. The wave functions for four-quark subsystems

Appendix A.1. Flavor and spin couplings

Take the decomposition of the flavor-spin configuration [31]$_F$[21]$_F$[22]$_S$ as an example,

$$\begin{align*}
|[31]_F S1\rangle &= \frac{1}{\sqrt{2}} \{ |[211]\rangle F1 [22]_S + |[211]\rangle F2 [22]_S \}, \\
|[31]_F S2\rangle &= \frac{1}{\sqrt{2}} \{ -\sqrt{2} |[211]\rangle F3 [22]_S + |[211]\rangle F2 [22]_S \} \\
&\quad - |[211]\rangle F1 [22]_S \}, \\
|[31]_F S3\rangle &= \frac{1}{\sqrt{2}} \{ |[211]\rangle F1 [22]_S + |[211]\rangle F2 [22]_S \} \\
&\quad + |[211]\rangle F3 [22]_S \}, \\
|[31]_F S4\rangle &= \frac{1}{\sqrt{2}} \{ -|[211]\rangle F1 [22]_S - |[211]\rangle F2 [22]_S \}.
\end{align*}$$

Appendix A.2. The flavor wave function of the four-quark subsystem uudc

The explicit forms of the flavor symmetry [21]$_F$

$$\begin{align*}
|[211]_{F1}\rangle &= \frac{1}{4} \{ 2\text{uduc} - 2\text{udcu} - \text{ducu} + \text{udcu} \\
&\quad - \text{ucud} + \text{ucdu} + \text{cduc} + \text{cuds} \}, \\
|[211]_{F2}\rangle &= \frac{1}{\sqrt{48}} \{ 3\text{uduc} - 3\text{ducu} + 3\text{cuds} \\
&\quad - 3\text{ucud} + 2\text{cduc} - 2\text{cdus} - \text{cuds} \}, \\
|[211]_{F3}\rangle &= \frac{1}{\sqrt{6}} \{ \text{ucud} + \text{udcu} + \text{ducu} \\
&\quad - \text{ucdu} - \text{cduc} - \text{cdus} \}.
\end{align*}$$

The explicit forms of the flavor symmetry [22]$_F$

$$\begin{align*}
|[22]_{F1}\rangle &= \frac{1}{\sqrt{24}} \{ 2\text{uduc} + 2\text{cduc} + 2\text{cuds} + 2\text{cdus} \\
&\quad - \text{ducu} - \text{udcu} - \text{cuds} - \text{cdus} \}, \\
|[22]_{F2}\rangle &= \frac{1}{\sqrt{8}} \{ \text{uduc} + \text{cduc} + \text{cuds} + \text{cdus} \\
&\quad - \text{ducu} - \text{udcu} - \text{cuds} - \text{cdus} \}.
\end{align*}$$

The explicit forms of the flavor symmetry [31]$_F$

$$\begin{align*}
|[31]_{F1}\rangle &= \frac{1}{\sqrt{18}} \{ 2\text{cduc} - 2\text{cdus} + 2\text{cuds} - \text{cduc} - \text{cdus} - \text{cuds} \}, \\
|[31]_{F2}\rangle &= \frac{1}{\sqrt{12}} \{ 6\text{uduc} - 3\text{ducu} - 3\text{udcu} - 4\text{cdus} - 4\text{cuds} \\
&\quad + 5\text{ducu} + 5\text{cduc} + 2\text{cuds} - \text{cduc} - \text{cdus} - \text{cuds} \}, \\
|[31]_{F3}\rangle &= \frac{1}{\sqrt{48}} \{ -3\text{ducu} + 3\text{udcu} - 3\text{cdus} + 3\text{cuds} \\
&\quad - 2\text{cdus} + 2\text{cuds} - \text{cduc} - \text{cuds} + \text{cduc} + \text{cdus} \}.
\end{align*}$$

Appendix A.3. The flavor wave function of the four-quark subsystem udsc

The explicit forms of the flavor symmetry [111]$_F$

$$\begin{align*}
|[111]_{F}\rangle &= \frac{1}{\sqrt{24}} \{ \text{udsc} - \text{dusc} - \text{dusc} - \text{dusc} \\
&\quad + \text{udsc} - \text{udsc} - \text{dusc} - \text{udsc} \\
&\quad + \text{cuds} - \text{cuds} + \text{cuds} - \text{cuds} \\
&\quad + \text{cdus} - \text{cdus} - \text{cdus} - \text{cdus} \\
&\quad + \text{cuds} - \text{cuds} - \text{cuds} - \text{cuds} \\
&\quad + \text{udsc} - \text{dusc} + \text{dusc} - \text{dusc} \},
\end{align*}$$
The explicit forms of the flavor symmetry [22]$_F$, 
\[
|\{22\}_F\rangle = \frac{1}{4}\left\{ \text{usdc} + \text{cdsu} + \text{dcuds} + \text{dcsu} \right. \\
+ \text{uscd} + \text{sucd} + \text{sucd} + \text{dcus} \\
- \text{sduc} - \text{ucsd} - \text{ucds} - \text{sdcu} \\
- \text{cuds} - \text{cuds} - \text{dsuc} - \text{dsuc} \left. \right\}, \\
\text{(A.13)}
\]

The explicit forms of the flavor symmetry [31]$_F$, 
\[
|\{31\}_F\rangle = \frac{1}{\sqrt{12}}\left\{ \text{ucsd} - \text{cdsu} + \text{uscd} - \text{dcus} + \text{uscd} \\
- \text{sduc} + \text{ucsd} - \text{scdu} + \text{scdu} \\
+ \text{uscd} - \text{cdsu} \right\}, \\
\text{(A.15)}
\]

The explicit forms of the flavor symmetry [31]'$_F$, 
\[
|\{31\}'_F\rangle = \frac{1}{\sqrt{96}}\left\{ 3(\text{usdc} - \text{sduc} + \text{ucds} - \text{cdus}) \\
+ \text{sduc} - \text{dsuc} + \text{cdus} - \text{uscd} \\
+ 2(\text{scdu} - \text{scdu} + \text{cdsu} - \text{cdus}) \\
+ \text{ucsd} - \text{cdsu} + \text{uscd} - \text{dcus} + \text{uscd} \\
- \text{sduc} + \text{ucsd} - \text{scdu} - \text{dsuc} - \text{dcus} \right\}, \\
\text{(A.16)}
\]

The explicit forms of the flavor symmetry [22]'$_F$, 
\[
|\{22\}'_F\rangle = \frac{1}{4\sqrt{6}}\left\{ 3(\text{sduc} - \text{scdu} + \text{usdc} - \text{dsuc}) \\
+ \text{sduc} - \text{scdu} + \text{dsuc} - \text{uscd} \\
+ 2(\text{scdu} - \text{scdu} + \text{cdsu} - \text{cdus}) \\
+ \text{uscd} - \text{cdsu} + \text{uscd} - \text{dcus} + \text{uscd} \\
+ \text{dsuc} - \text{dcus} - \text{dsuc} - \text{dcus} - \text{dcus} \right\}, \\
\text{(A.17)}
\]

The explicit forms of the flavor symmetry [22]$_F$, 
\[
|\{22\}_F\rangle = \frac{1}{\sqrt{12}}\left\{ 2(\text{scdu} - \text{cdsu} + \text{usdc} - \text{dsuc}) \\
+ \text{scdu} - \text{cdsu} + \text{usdc} - \text{dsuc} \\
+ 2(\text{cdsu} - \text{scdu} + \text{uscd} - \text{dcus}) \\
+ \text{uscd} - \text{scdu} + \text{uscd} - \text{dcus} + \text{uscd} \\
+ \text{dsuc} - \text{scdu} - \text{scdu} - \text{dsuc} - \text{dcus} - \text{dcus} \right\}, \\
\text{(A.18)}
\]

The explicit forms of the flavor symmetry [211]$_F$, 
\[
|\{211\}_F\rangle = \frac{1}{2\sqrt{3}}\left\{ \text{cdsu} - \text{uscd} + \text{ucsd} - \text{dcus} \\
+ \text{cdus} - \text{scdu} + \text{scdu} - \text{cdus} \\
+ \text{scud} - \text{cdus} + \text{cdus} - \text{scud} \\
- \text{scus} - \text{scus} - \text{dsuc} - \text{dsuc} \right\}, \\
\text{(A.19)}
\]

The explicit forms of the flavor symmetry [211]'$_F$, 
\[
|\{211\}'_F\rangle = \frac{1}{2\sqrt{3}}\left\{ \text{cdsu} + \text{cdus} - \text{scdu} + \text{cdus} \\
+ \text{cdus} - \text{scdu} + \text{scdu} - \text{cdus} \\
+ \text{scud} - \text{cdus} + \text{cdus} - \text{scud} \\
- \text{scus} - \text{scus} - \text{dsuc} - \text{dsuc} \right\}, \\
\text{(A.20)}
\]

For the udsc system, there are in total, 24 flavor wave functions. Among them, 12 of the flavor states listed above have isospin 0 and are used for our study of $\Lambda^*$-like states; the other 12 flavor states have isospin 1, hence they are not relevant to our present study and not listed here. The explicit forms for all these flavor wave functions can be obtained by using the relevant formulae in chap. 4 of ref. [30].

Appendix A.4. The wave function of spin symmetry of the four-quark subsystems

The wave functions for spin symmetry $[22]_S$, 
\[
|\{22\}_S\rangle = \frac{1}{\sqrt{12}}\left\{ 2|\uparrow\uparrow\downarrow\downarrow\rangle + 2|\uparrow\downarrow\uparrow\downarrow\rangle - |\downarrow\uparrow\downarrow\downarrow\rangle - |\downarrow\uparrow\uparrow\downarrow\rangle \right\}, \\
\text{(A.24)}
\]
\[
|\{22\}'_S\rangle = \frac{1}{2}\left\{ |\uparrow\downarrow\downarrow\downarrow\rangle + |\downarrow\uparrow\downarrow\downarrow\rangle - |\downarrow\uparrow\uparrow\downarrow\rangle - |\uparrow\downarrow\uparrow\downarrow\rangle \right\}. \\
\text{(A.25)}
\]

More can be found in ref. [60].

References

1. A. Acha et al., Phys. Rev. Lett. 98, 032301 (2007).
2. G.T. Garvey, J.-C. Peng, Progr. Part. Nucl. Phys. 47, 203 (2001).
3. S. Capstick, W. Roberts, Progr. Part. Nucl. Phys. 45, 521 (2000).
4. B.S. Zou, D.O. Riska, Phys. Rev. Lett. 95, 072001 (2005).
5. C.S. An, D.O. Riska, B.S. Zou, Phys. Rev. C 73, 035207 (2006).
6. C.S. An, Q.B. Li, D.O. Riska, B.S. Zou, Phys. Rev. C 74, 055205 (2006).
7. B.S. Zou, Nucl. Phys. A 827, 333C (2009).
8. C.S. An, D.O. Riska, Eur. Phys. J. A 37, 263 (2008).
9. Q.B. Li, D.O. Riska, Nucl. Phys. A 766, 172 (2005).
10. Q.B. Li, D.O. Riska, Phys. Rev. C 73, 035201 (2006).
11. Q.B. Li, D.O. Riska, Phys. Rev. C 74, 015202 (2006).
12. R. Bijker, E. Santopinto, Phys. Rev. C 80, 065210 (2009).
13. E. Santopinto, R. Bijker, Phys. Rev. C 82, 062202 (2010).
14. Roelof Bijker, Elena Santopinto, AIP Conf. Proc. 1265, 240 (2010).
15. N. Kaiser, P.B. Siegel, W. Weise, Phys. Lett. B 362, 23 (1995).
16. E. Oset, A. Ramos, Nucl. Phys. A 635, 99 (1998).
17. S.J. Brodsky, P. Hoyer, C. Peterson, N. Sakat, Phys. Lett. B 93, 451 (1980).
18. E.V. Shuryak, A.R. Zhitnitsky, Nucl. Phys. B 57, 2001 (1998).
19. Roelof Bijker, Elena Santopinto, AIP Conf. Proc. 1265, 240 (2010).
20. W.L. Wang, F. Huang, Z.Y. Zhang, B.S. Zou, Phys. Rev. D 82, 117501 (2010).
21. J.J. Wu, R. Molina, E. Oset, B.S. Zou, arXiv:1011.2399 [nucl-th].
22. Z.C. Yang, J. He, X. Liu, S.L. Zhu, arXiv:1105.2901 [hep-ph].
23. Fl. Stancu, D.O. Riska, Phys. Lett. B 425, 171 (1998).
24. Marek Karliner, Harry J. Lipkin, hep-ph/0307243.
25. Marek Karliner, Harry J. Lipkin, hep-ph/0307343.
26. C. Semay, B. Silvestre-Brac, Eur. Phys. J. A 22, 1 (2004).
27. J.Q. Chen, Group Representation Theory for Physicists (World Scientific, 1989).
28. J.Q. Chen, J.L. Ping, F. Wang, Group Representation Theory for Physicists (World Scientific, 2002).
29. Fl. Stancu, Group theory in subnuclear physics (Clarendon Press, Oxford, 1996).
30. L.Ya. Glozman, D.O. Riska, Nucl. Phys. A 603, 326 (1996).
31. C. Helminen, D.O. Riska, Nucl. Phys. A 699, 624 (2002).
32. L.Ya. Glozman, D.O. Riska, Phys. Rep. 268, 263 (1996).
33. V. Borka Jovanović, S.R. Ignjatović, D. Borka, P. Jovanović, Phys. Rev. D 82, 117501 (2010).
34. A. De Rujula, H. Georgi, S.L. Glashow, Phys. Rev. D 12, 147 (1975).
35. N. Isgur, G. Karl, Phys. Rev. D 18, 4187 (1978).
36. N. Isgur, G. Karl, Phys. Rev. D 19, 2653 (1979).
37. S. Capstick, N. Isgur, Phys. Rev. D 34, 2809 (1986).
38. J. Leandri, B. Silvestre-Brac, Phys. Rev. D 40, 2340 (1989).
39. B. Silvestre-Brac, J. Leandri, Phys. Rev. D 45, 4221 (1992).
40. Franco Bucciella, Hallstein Hogassen, Jean-Marc Richard, Paul Sorba, Eur. Phys. J. C 49, 743 (2007).
41. M. Karliner, Harry J. Lipkin, hep-ph/0307243.
42. Marek Karliner, Harry J. Lipkin, hep-ph/0307343.
43. C. Semay, B. Silvestre-Brac, Eur. Phys. J. A 22, 1 (2004).
44. E.V. Shuryak, A.R. Zhitnitsky, Nucl. Phys. B 163, 45 (1980).
45. W.H. Blask, U. Bohn, M.G. Huber, B.Ch. Metsch, H.R. Petry, Z. Phys. A 337, 327 (1990).
46. U. Löring, B.C. Metsch, H.R. Petry, Eur. Phys. J. A 10, 395 (2001).
47. Jishnu Dey, Mira Dey, Peter Volkovitsky, Phys. Lett. B 261, 493 (1991).
48. Gerard ’t Hooft, arXiv:hep-th/9903189v3.
49. Sachiko Takeuchi, Nucl. Phys. A 642, 543 (1998).
50. Particle Data Group (K. Nakamura et al.), J. Phys. G 37, 075021 (2010).
51. C.S. An, B.S. Zou, Eur. Phys. J. A 39, 195 (2009).
52. Kingman Cheung, Phys. Rev. D 69, 094029 (2004).
53. C.S. An, B. Saghai, S.G. Yuan, Jun He, Phys. Rev. C 81, 045203 (2010).