Simulation Study on State Estimation for a Quadrotor Generalized Attitude Model

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Abstract. This paper studies the state estimation of a quadrotor yaw-channel generalized attitude model (GAM) using the subspace identification based state estimation method under the conditions that the measurements for the control quantity and the output attitude angle are corrupted with disturbing noises and that there are no available models. The quadrotor attitude control loop for the yaw-channel is first designed by MATLAB using an observer based state feedback law. A high-order coprime factor (CF) model for the GAM is then identified from simulation data by the subspace identification method. Next, the estimation states for the GAM can be given by the similarity transform and the relations between the GAM and its CF model. Finally, the efficacy of the presented state estimation procedure and the accuracy of the state estimation are verified.

1. Introduction
Quadrotors have a wide range of applications in all walks of life in recent decades, so their attitude control accuracy is of high demand. Due to their good applicability to nonlinear multivariable complex control objectives, numerous advanced state-feedback controller design methods have been applied in the quadrotor attitude control. A quadrotor is a highly linear, multi-variable, strongly coupled, and underactuated system since it has three degrees of freedom (pitch, roll, and yaw) and only three control inputs (pitching, rolling, and yaw moments). It is a type of unmanned aerial vehicle which is lifted by a set of four rotors. It has two pairs of identical fixed propellers when two of them rotate clockwise and the other two propellers rotate counter clockwise [1]. State estimation is one of the fundamental problems in control systems in which the true states of the system are estimated based on the measured data and knowledge about the system.

Geng et al studies for a closed-loop errors-in-variables (EIV) system with a state observer-based state feedback, in terms of the available input and output measurements in addition to the reference, a coprime factor (CF) model for the true plant can first estimate a model by employing the subspace identification method [2]. Liu et al use the subspace identification method to identify the quadrotor attitude model of a quadrotor in consideration of input and output noise in the true environment, use the PRBS as the reference input and collect input and output data and estimate a linear model of the three quadrotor attitude angles, roll yaw and pitch and estimate the quadrotor EIV system attitude model using the new subspace identification method. Then depending on the collected PRBS as the reference input to collect input and output data, a linear model of the three attitude angles of the quadrotors is estimated [3]. Chao et al studies the subspace identification algorithm and identify open or closed-loop multivariable finite-dimensional linear time-invariant EIV systems whose output was additionally corrupted with colored...
noise apart from the input and output measurement white noises [4]. Van et al study a comprehensive overview of closed-loop subspace identification methods [5]. Tohru et al studies subspace methods and first construct the state estimates from given input-output data by using a simple procedure based on tools of numerical linear algebra and obtain the state-space model by solving a least-squares problem, from which can easily compute a transfer matrix [6]. Verhaegen developed a subspace identification algorithm to estimate open or closed-loop multivariable finite dimensional linear time-invariant EIV systems whose output was corrupted with colored noise apart from input and output measurement white noises [7].

The main contribution of this paper is to estimate the states of the quadrotor generalized attitude model (GAM) using a subspace identification based state estimation method in the presence of input and output noises. The quadrotor yaw-channel attitude control system with an observer-based state feedback is first designed and built by MATLAB according to a certain control performance. Then, the simulation data are collected and used for the study of the state estimation of the quadrotor GAM.

2. Problem Formulation

The quadrotor studied in this paper is shown in Figure 1 [8], from which it is known that an X-shaped configuration is used to provide driving forces and rotational torques. The state estimation framework for the quadrotor yaw-channel GAM is shown in Figure 2 [9], in which the GAM is stabilized by a discrete-time-observer based state feedback scheme.

![Figure 1. The quadrotor configuration](image1)

![Figure 2. The state estimation framework](image2)

As can be seen from the quadrotor attitude control loop, the available experimental data are \( \psi_r(k) \), \( u_r(k) \) and \( \psi_m(k) \), which denote the reference attitude angle, the control quantity measurement and the output attitude angle measurement for the quadrotor yaw channel, respectively; \( n_{\psi_r}(k) \) and \( n_{\psi_m}(k) \) are the measurement noises contaminating on the control quantity \( u_4(k) \) and the real attitude angle \( \psi(k) \), respectively; \( \{A,B,C,D\} \) is a continuous state-space realization of the quadrotor GAM while \( \{F,G,C,D\} \) is its discrete state-space realization from \( \{A,B,C,D\} \) via the zero-order hold method; \( K \) is a state feedback gain and \( \hat{x}(k) \) being the discrete estimation for the state \( x \) of \( \{A,B,C,D\} \) is given by the state observer designed in terms of \( \{F,G,C,D\} \). It is obvious that this framework is a closed-loop EIV with a state feedback.

According to the related researches and analyses given by Liu [8], the quadrotor mechanical GAM is the dynamics from \( u_4 \) to \( \psi \) and can be approximated by two integrals in the following way

\[
G(s) \triangleq \frac{\psi(s)}{u_4(s)} = \frac{0.1}{s^2}
\]

whose realization is given by the quadruple \( \{A, B, C, D\} \).
\[
A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D = 0
\]

The main objective of this paper is to estimate the state \( x \) of the realization \( \{A, B, C, D\} \) for the quadrotor GAM using the available experimental data \( \psi_r(k), u_r(k) \) and \( \psi_m(k) \).

### 3. State Estimation Method

When the state estimation \( \hat{x}(k) \) is equal to \( x(k) \), one can derive the following single-input and two-output state space model [9]

\[
\begin{aligned}
&x(k+1) = A_0 x(k) + B_0 \psi_r(k) \\
&\psi(k) = C_0 x(k) + D_0 \psi_r(k) \\
u_4(k) = C_1 x(k) + D_1 \psi_r(k) + \begin{bmatrix} n_{\psi_r}(k) \\ n_{\psi_m}(k) \end{bmatrix}
\end{aligned}
\]

with

\[
A_0 = F - G K, \quad B_0 = G, \quad C_0 = \begin{bmatrix} C - D K \\ -K \end{bmatrix}, \quad D_0 = \begin{bmatrix} I \\ 0 \end{bmatrix}
\]

In light of Zhou et al. [10], the transfer functions from \( \psi_r \) to \( \psi \) and \( u_4 \) being defined as \( N(z) \) and \( M(z) \), respectively are just a pair of coprime factors for the quadrotor discrete GAM. The CF model consisting of such coprime factors can be identified from Figure 2 by using the available experimental measurements as follows [9]

\[
\begin{aligned}
x(k+1) &= A_0 x(k) + B_0 \psi_r(k) + e(k) \\
u_4(k) &= C_1 x(k) + D_1 \psi_r(k) + \begin{bmatrix} n_{\psi_r}(k) \\ n_{\psi_m}(k) \end{bmatrix}
\end{aligned}
\]

where \( e(k) \) is caused by replacing the ideal state feedback law \( u_4(k) = \psi_r(k) - K\hat{x}(k) \) with the practical one \( u_4(k) = \psi_r(k) - K\hat{x}(k) \) and by other possible unknown process noises; \([n_{\psi_r}(k), n_{\psi_m}(k)]\) denotes the measurement noises. According to the similarity transform relating two minimal realizations of a transfer function, a pivotal non-singular transformation matrix \( P \) can be constructed as [9]

\[
P = C_0^{-1} C_1
\]

which is capable of realizing the transformation from the data driven state space model in (5) to the ideal one in (3) in practice. In terms of the state estimation idea [2][9], the continuous generalized attitude estimation model \( \{ \hat{A}, \hat{B}, \hat{C}, \hat{D} \} \) can thus be obtained.

### 4. State Estimation procedure

The state estimation procedure for the quadrotor GAM from Figure 2 is eventually summarized as a flow chart in Figure 3 [9].

As can be seen from the flow chart, persistently excited PRBS identification experiments are necessary to be elaborately designed at the beginning. Then, apply the subspace identification method to acquire a discrete CF state space model, and then obtain the estimated GAM and noise model. Next, the estimate states of the quadrotor can be extracted from the identification models. Finally, the estimated states are compared with the true states. If the state estimation accuracy is satisfactory, then the state estimation procedure comes to an end for the time being. Otherwise, another state estimation cycle will restart.
Excite the quadrotor attitude control system and collect identification experimental data

Apply the subspace identification method to acquire a discrete coprime factor state space model

Obtain estimated GAM and noise model

Extraction of the estimated states

Comparison of estimated states and true states

Is state estimation accuracy satisfactory?

Yes

End

No

Start

Figure 3. The state estimation procedure

5. State estimation results

Given the continuous state-space model of the quadrotor GAM for the yaw channel in (2), its discrete realization can be obtained via the zero-order hold method. According to the time-domain performance specifications of the zero overshoot and the settling time being 1 second, the desired poles for control and observer design are selected as \{-3, -15\} and \{-60, -61\} in s domain, respectively, from which the state feedback gain and the observer gain can be designed as \(K = [201.3505, 93.5376]\) and \(L = [1.9953, 9.9528]^T\). In the simulation, \(n_v(k)\) and \(n_p(k)\) are assumed to obey independent identically-distributed Gaussian normal noises with zero mean and the standard deviation 0.01. According to the settling time and the maximum operation frequency of the observer-based state feedback control loop, a persistently excited reference input can be designed as a three-period PRBS with the clock time being 0.1 seconds. Figure 4 shows the first period of the reference attitude input, the control quantity measurement and the output attitude measurement, which are used for the GAM state estimation [9].

After the implementation of the state estimation procedure, the state comparison results are shown in Figure 5 [9], where \(x_{r1}, x_{r2}\) are the true states, \(x_{e1}, x_{e2}\) are the estimation states. As can be seen from the comparison curves, the estimated state \(x_{e1}, x_{e2}\) are very near to the true state \(x_{r1}, x_{r2}\), respectively and are satisfactory.

Therefore, it can be concluded that the subspace identification-based state estimation method is efficient for the quadrotor GAM state estimation in the presence of measurement noises and in the absence of unknown models.
Figure 4. The simulation experimental data

Figure 5. The state comparisons

6. Conclusions

This paper has studied the quadrotor GAM state estimation using the subspace identification based state estimation method. A high-order CF model for the GAM is first identified by using the subspace identification method. The estimation states for the GAM can then be given by similarity transform and the relations between the GAM and its CF model. From the comparison between the true and estimation states, the state estimation accuracy is quite satisfactory.

7. References

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