On Joining Graphs

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Figure 2: View of the data graph as a relational database.

ABSTRACT

In the graph database literature the term “join” does not refer to an operator used to merge two graphs. In particular, a counterpart of the relational join is not present in existing graph query languages, and consequently no efficient algorithms have been developed for this operator.

This paper provides two main contributions. First, we define a binary graph join operator that acts on the vertices as a standard relational join and combines the edges according to a user-defined semantics. Then we propose the “CoGrouped Graph Conjunctive \( \theta \)-Join” algorithm running over data indexed in secondary memory. Our implementation outperforms the execution of the same operation in Cypher and SPARQL on major existing graph database management systems by at least one order of magnitude, also including indexing and loading time.

1. INTRODUCTION

Despite the term “join” appearing in the literature on GDBMSs, there is no counterpart of the relational join operator over two distinct graphs. Recalling relational algebra’s joins, those are defined as a composition of a selection predicate and a cartesian product between two tables. In literature “graph join” describes an operation which is neither binary (that is, involving two distinct graphs) nor involving graphs as a whole (that is, graph paths obtained in traversal operations over one single graph are considered instead). In current literature “join” expresses a “path join” \[30, 23, 16\] over path queries of arbitrary length, where specifically joins are performed over adjacent vertices \[3, 11, 15\].

In order to reinforce such statement, we should see that the so-called graph joins in graph database literature are always defined over a single graph \[2\] even if there are some graph query languages, like SPARQL, that allow accessing multiple graph resources in a same query. In the particular case of SPARQL, the join operation is expressed as a join between graph paths through relational join operators \[1, 2\] through triple composition.

The introduction of an explicit graph join operator allows to better delineate the problem, and hence helps finding a tailored algorithm that performs the join operation profitably. By doing so we obtain a specific join algorithm that has a running time that is lower than the time required to perform the same operation over current graph database query languages.

In this paper we outline two contributions:

- **Definition of the Graph Join operator**: this operation joins the vertices and combines the edges with different possible semantics. In particular we propose two different semantics, that are the conjunctive join and the disjunctive join. The first one forms an edge between two joined vertices iff such vertices were connected by an edge in both graphs; the second one forms an edge if the vertices were connected in at least one of the two graphs.

- **CoGrouped Graph Conjunctive \( \theta \)-Join**: this algorithm implements the graph join with the conjunctive semantics. First, we index and load into secondary memory the graph data structure by associating each vertex to a specific hash value. As a last step, we perform a join over the operands which are stored in secondary memory by both (i) joining only the vertices which have the same hash value, and (ii) linking such joined vertices according to the conjunctive semantics.

As a secondary outcome of the flexibility of the graph join definition, we implement specific graph operations even though those are not literary specified as graph joins. Subgraph extractions from two graphs over user communities \[4, 5\] and (Unweighted) Ontology RollUp \[31\] over a same graph are just a few examples.

Section \[2\] outlines the details of our proposed graph data model and defines the graph join operation (Section \[2.1\]). Section \[3\] develops both the basic version of the join definitions and the proposed CoGrouped Graph Conjunctive \( \theta \)-Join, specifically designed for the graph conjunctive join. Section \[4\] exposes the graph data structures that are used both to store the result in primary memory and the join operands.

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in secondary memory. Section 5 describes the experiment’s set-up where we generalize the subgraph extraction problem in order to increase the results’ multiplicity. In Section 6 we draw our conclusions and outline our future works. Section 7 provides more details on the state of the art of current graph database languages and on the usage of the term “join” on current graph database literature.

2. GRAPH DATA MODEL

We now define a simplified Property Graph data model, which uses only the basic desired feature that are required to develop a graph join operation over graphs.

Definition 1 (Graph Data Model). A graph is defined as a triple \((V, E, A)\), where \(V\) is a set of vertices, that are represented as tuples having a schema \(A\). \(E\) is a set of (unlabelled) edges defined as a pair of vertices in \(V^2\).

Using the former definition, we can represent any graph through an ER diagram (Figure 2). This simplified model could be even stored in a relational database (Figure 1), because each vertex is an entity and the edges are the binary relations among the vertices. Similar attempts have been carried out for the Property Graph model [13, 38].

Example 1. Figure 1a and 1d represent two communication patterns between the two vertices, where each vertex represents a post created by an user and each edge \((u, v)\) represents that \(u\) receives a reply from \(v\). Consequently, we have:

\[
G_1 = \{(v_2, v_3, v_4), (v_2, v_4, v_3), \{User, MsgTime1\}\}
\]

\[
G_2 = \{(w_1, w_2, w_3, w_4), (w_1, w_2, w_3, w_4, w_1, w_2), \{User, MsgTime2\}\}
\]

Each graph can be represented by two tables, one for the vertices (Figure 1b and 1e) and the other for the edges (Figure 1c and 1f).

The evidence of a viable mapping between graphs and relational databases is used in the following subsection to outline the graph join operation.

2.1 Graph Joins

We conceive graph joins as an extension of the relational join operator. We propose a graph join between two graphs \(G_1 \bowtie^\theta\text{op} G_2\), where the vertices are considered as to-be-joined relational tuples \((V_1 \bowtie\theta V_2)\), and the resulting edges are given by combining the graph operand’s edges \(E_1\) and \(E_2\) with a specific \text{op} semantics. The graph join operator has two parameters: the \(\theta\) binary predicate over the vertices and the \text{op} semantics that combines the edges from both graphs. This modularity is similar to the graph products defined in graph theory literature [19, 25], where instead of a join between vertices we have a cross product.

Definition 2 (General Graph \(\theta\)-Join). Given two data graphs \(G_1 = (V_1, E_1, A_1)\) and \(G_2 = (V_2, E_2, A_2)\), a general graph \(\theta\)-join is defined as follows:

\[
G_1 \bowtie^\theta\text{op} G_2 = (V_1 \bowtie\theta V_2, E_{\text{op}}, A_1 \cup A_2)
\]

where \(\theta\) is a binary predicate over the vertices and \(\bowtie\theta\) the \(\theta\)-join among the vertices (as tables), and \(E_{\text{op}}\) is a subset of all the possible edges linking the vertices in \(V_1 \bowtie\theta V_2\) expressed with the \text{op} semantics. \(\bowtie\theta\) is defined as the relational join among relations as follows:

\[
\{v \bowtie\theta v' \mid v, v' \in V_1 \bowtie\theta V_2, \theta(v, v')\}
\]

Moreover, \(\oplus\) is the operation of merging two vertices (and hence, two tuples).

![Figure 3: Given two graph with vertices with same id, hence sharing the same value, the graph conjunctive join extracts the common pattern, while the disjunctive join retrieves at least one edge shared among the matched vertices.](image-url)
If we impose that \( E_{\text{op}} \) contains an edge iff an edge between the merged vertices appears in both original graphs (and hence in both \( G_1 \) and \( G_2 \)), we obtain a **Conjunctive Join**, that in graph theory is known as *Kronecker graph product* \([39, 19]\). In this case \( E_{\text{op}} \) is defined with the "\( \land \)" semantics as follows:

\[
E_{\land} = \{(v \oplus v', v' \oplus v'') \in (V_1 \times V_2)^2 \mid (v, v') \in E_1 \land (v'', v''') \in E_2\}
\]

Such edge semantics extracts all the shared edge patterns between the two graphs among equivalent vertices, where \( \theta \) determines whether the two vertices are equivalent or not. Given this definition, we could provide an implementation for the Unweighted Ontology Rollup \([31] \):

**Example 2.** Given a graph \( G = (V, E) \) where each edge represents a "is-a" relation, then \( G \upharpoonright (v, v') \in E \) produces the rollup for each ontology object \( o \in V \) over which "is-a" relations are formulated. E.g., if camera "is-a" device and device "is-a" hardware, then in the resulting graph camera-device "is-a" device-hardware.

This graph join semantics is implemented in our proposed algorithm in Section 3.2. As an alternative, we would like to grasp from both graphs all the edges that are shared among the equivalent edges (see Figure 5): this is possible with the **Disjunctive Join**, where \( E_{\text{op}} \) is defined with the "\( \lor \)" semantics as follows:

\[
E_{\lor} = \{(v \oplus v', v' \oplus v'') \in V_1 \times V_2 \mid (v, v') \in E_1 \lor (v'', v''') \in E_2\}
\]

In order to differentiate the two proposed graph joins, that is when the conjunctive edge semantics is used instead of the disjunctive one, we use \( G_1 \upharpoonright \theta \) \( G_2 \) for the graph conjunctive join, and \( G_1 \upharpoonright \theta' \) \( G_2 \) for the graph disjunctive join.

**Example 3.** Let us now consider the two graphs in Figure 4 and try to define the join \( G_1 \upharpoonright \text{MsgTime1} \leq \text{MsgTime2} \) \( G_2 \), where each vertex value is provided inside a table. We want to select the communication patterns that are shared at increasing times: the resulting graph is given in Figure 4a, where each \( z_i \) is defined as \( v_i \oplus w_i \). Observe that the graph conjunctive join operation is symmetric as the one defined in relational algebra, and hence:

\[
G_1 \upharpoonright \text{MsgTime1} \leq \text{MsgTime2} \leq \text{MsgTime1} G_2 = G_2 \upharpoonright \text{MsgTime1} \leq \text{MsgTime2} \leq \text{MsgTime1} G_1
\]

On the other hand, the result of performing the symmetrical operation, that is \( G_1 \upharpoonright \text{MsgTime1} \leq \text{MsgTime2} \leq \text{MsgTime1} G_2 \), is provided in Figure 4b. Observe that the graph disjunctive join operation is symmetric as the one defined in relational algebra, and hence:

\[
G_1 \upharpoonright \text{MsgTime1} \leq \text{MsgTime2} \leq \text{MsgTime1} G_2 = G_2 \upharpoonright \text{MsgTime1} \leq \text{MsgTime2} \leq \text{MsgTime1} G_1
\]

Let us now consider the same examples provided previously but performed over the Disjunctive Join: as we can see from Figure 4b, such implementation allows obtaining the edges from both graphs and behaves as a missing data operator for the edges. As we can see from Figure 4b whether both graphs share the same edge, the final edge is not duplicated.

### 3. ALGORITHMS

In this section we design both a basic approach implementing the graph join for both conjunctive (\( \land \)) and disjunctive (\( \lor \)) semantics (Section 3.1), and then we outline the proposed join algorithm for the conjunctive semantics (Section 3.2), focusing more specifically on the equijoin case (the discussion on the less-equal join is postponed to Appendix A).

#### 3.1 Basic implementation

A basic implementation of both conjunctive and disjunctive join is provided in Algorithm 1 on the following page. This simple algorithm can be immediately translated in common graph query languages such as Cypher and SPARQL. To the
best of our knowledge, no graph language exists where such graph join operator is designed as a specific operator and hence, the computation of such kind of query is not optimized, although all the graph database indexing techniques are used.

With this implementation we first perform the $\theta$ join over the graphs' vertices (line 2), then we visit the graph searching for the neighbours (line 4) and then we check if the selected vertices satisfy the specific semantics through the $\text{op}$ semantics.

3.2 Implementing the Conjunctive $\theta$-Join through hashing

We propose an implementation of the Conjunctive $\theta$-join through hashing that uses the CoGrouping\cite{9} that is a proposed generalization for relational tables' join, combining the Sort-Merge Join approach and the Partition-based Hash Join. With the former approach the tuples are sorted by and the latter the input is divided in small co-partitions\cite{32}. In particular, (partition) hash joins associate all the vertices with a same hash value to a same given bucket and can be only applied over equi-join predicates\cite{42}. On the other hand, by ordering the buckets by hash value, we can both decrease the time required to search the buckets with the same hash value, and use the hash join even when the predicate is a “$\leq$” over at most one single attribute per graph, containing ordinal values. Since we would like to have an algorithm supporting both equijoins and “$\leq$” predicates, we use the CoGrouped (hash) join technique; hereby we call such algorithm “CoGrouped Graph Conjunctive $\theta$-Join”.

As a first step, for each vertex of the left graph $G_1 = (V_1, E_1, A)$ we have to reduce the number of the vertices that have to be scanned in the right graph $G_2 = (V_2, E_2, B)$ while searching for a possible match. Our aim is to prune the combination of visited pair of distinct vertices, jointly from searching for a possible match. Our aim is to prune the vertices having the same hash value are stored contiguously. As a result, we can directly access to the vertices with hash code $h_c$ ($u \in V_1$ and $v \in V_2$ having $h(u) = h(v)$). In order to visit only the vertices having hash values that are shared between the two indexed graphs, we preventively evaluate the set $HI$ storing the hash values shared on both graphs ($HI := h(V_1) \cap h(V_2)$). We still have to check if the predicate $\theta$ is satisfied for each pair of such vertices ($\theta(u, v)$) as long as the join condition holds (that is $(u \oplus v)[A] = v \land (u \oplus v)[B] = v$). If all those tests are passed, then the joined vertex $u \oplus v$ is the next vertex candidate for the resulting graph.

The last step of the algorithm involves the creation of the edges between the joined vertices, $u \oplus v$, and their neighbours. Since the neighbours must be elements of $V_1 \oplus B_2$ by graph join definition we choose, among all the possible neighbours, those that have a hash value $h_d$ appearing in both graphs ($h_d \in HI$). In particular even in this case, the vertices have to pass the previously described tests.

Algorithm\cite{2} implements the CoGrouped Graph Conjunctive EquiJoin described idea in pseudocode with no assumptions regarding both the operands' and the result's data structure. Better performances can be achieved by using ad hoc graph data representations, as described in the following section.

This technique also applies whether the predicate $\theta$ is defined as a total order over at most one attribute of a vertex (e.g. $\theta(u, v) = u.A \leq v.B$) and in this case the hashing function should be monotone w.r.t. the ordered values (if $u.A \leq v.B$, then the hashes match with $h(u) \leq h(v)$).

In our case study we examine the algorithm when the join predicate is expressed in an equivalence form such as $\theta = 1$. On the other hand, Appendix\cite{A} provides some details on the case of the less-equal ($\leq$) predicate, both providing the algorithm and the experimental results.

Algorithm 2 CoGrouped Graph Conjunctive EquiJoin

1: procedure CoGroupedJOIN($G_1, G_2, \theta$) $\rightarrow G_1 \bowtie^\theta G_2$
2: $V \leftarrow V_1 \bowtie^\theta V_2; E \leftarrow \emptyset$
3: $HI \leftarrow h(V_1) \cap h(V_2)$
4: for each $h_c \in HI$ do
5: for each $u \in V_1, v \in V_2$ s.t. $h(u) = h_c = h(v)$ do
6: if $\theta(u, v), (u \oplus v)[A] = u, (u \oplus v)[B] = v$ then
7: $V \leftarrow V \cup \{u \oplus v\}$
8: for each $nu \in out_{V_1}(u)$ do
9: if $h(nu) \notin HI$ then continue
10: for each $nv \in out_{V_2}(v)$ do
11: if $h(nu) \neq h(nv)$ then continue
12: if $\theta(nu, nv)$,
13: $(nu \oplus nv)[A] = nu,$
14: $(nu \oplus nv)[B] = nv$ then
15: $V \leftarrow V \cup \{nu \oplus nv\}$
16: $E \leftarrow E \cup \{(u \oplus v, nu \oplus nv)\}$
17: return $(V, E, A_1 \cup A_2)$
4. GRAPH DATA STRUCTURES

In this section we show the two different strategies that our algorithm uses in order to compute the previously described algorithm efficiently. We first describe the result graph data structure that has to allow a fast element insertion and exiguous memory occupation (Section 4.1), and then the secondary memory data structure that allows a quick scan of the graph’s vertices (Section 4.2).

4.1 Bulk Graph

The graph resulting from a join graph query could be too large to fit in main memory. For this reason some implementations like RDF4J do not explicitly store the result unless explicitly required, but actually evaluate the query step by step. On the other hand Neo4J sometimes fails to accomplish in providing the final result due to the employment of all the available RAM memory (Table 2). In order to allow the storage of the whole result in main memory we chose to implement an ad-hoc graph data structure represented as an adjacency list, where each entry represents a result’s vertex and where each vertex’s neighbour is only represented by its id.

4.2 Secondary Storage

The proposed algorithm shows that we can achieve the advantages of a CoGrouped join if the vertices are all sorted by hash value. If such hash-sorted vertices are stored in a linear data structure as in Figure 5a (e.g. an array, VA[]), then we could create an hash index (e.g. Hash[]) where each record stores the hash value value and the offset VAOffset of VA where to retrieve the vertices with the same hash.

Let us now focus on VA’s vertices. Each vertex in VA is stored by omitting the vertices’ attributes and storing only the values (val[1]...val[M]) and, in order to avoid data replication, each incoming vertex in[i] and outgoing vertex out[j] is stored only by its id.

Given that VA’s vertices have variable data size, we need another linear data structure (e.g. VertexIndex[]) for accessing efficiently such vertices. The Id vertex is stored at the record with number Id in VertexIndex[], in such record VAOffset points out where the current vertex is stored in VA. Consequently, this indexing data structure allows accessing each vertex in O(1) time by its Id. Further details on how to implement such data structure are given in Appendix B, where Algorithm 2 is rewritten using these data structures.

Example 4. The graph depicted in Figure 5b could be represented in Figure 5c and hence the graph data model implementation can be presented as follows:

\[ G_1 = \{ \text{VertexIndex}, VA, Hash, \{ User, MsgTime1 \} \} \]

where the first three arrays refer to the graph implementation previously described (Figure 5a) and \{ User, MsgTime1 \} refers to the attribute schema associated to the graph.

While modern Relational DBMSs uses variants of the B-Tree data structure to store tuples in main memory, we decided to use a linear data structure, since the most frequent operation for the Join algorithm is the linear scan of all the vertices that share the same hashing value. More precisely, a visit of a balanced binary search tree with \( N \) vertices takes \( 2N \) (because some vertices are visited two times), while the visit of a linear data structure with \( N \) records takes exactly \( N \).

We can access to our data structure using memory mapping techniques: by doing so we delegate the Operating System to handle which pages have to be either cached or retrieved from secondary memory. As a consequence, no cache has to handle which pages have to be either cached or retrieved from secondary memory. As a consequence, no cache has to handle which pages have to be either cached or retrieved from secondary memory. As a consequence, no cache has to handle which pages have to be either cached or retrieved from secondary memory. As a consequence, no cache has to handle which pages have to be either cached or retrieved from secondary memory. As a consequence, no cache has to handle which pages have to be either cached or retrieved from secondary memory. As a consequence, no cache has to handle which pages have to be either cached or retrieved from secondary memory. As a consequence, no cache has to handle which pages have to be either cached or retrieved from secondary memory. As a consequence, no cache has to handle which pages have to be either cached or retrieved from secondary memory. As a consequence, no cache has to handle which pages have to be either cached or retrieved from secondary memory.
Going more specifically on our implementation, the whole code was written in Java. In order to perform memory mapping I/O over files greater of MAX_INT size, we had to use the JNA library, through which it is possible to directly interact with the mmap system call. In this way we can potentially address at least 64TB of data in virtual memory without explicitly allocating any data value: as a result we avoid creating objects in Java, thus also reducing the amount of work of the Java Garbage Collector, since all the comparisons are done over the data’s in-memory representation.

5. DATASET
We tested our algorithm using the LiveJournal [29] and the YouTube [40] Social Network Graphs. The former contains 4,847,571 unlabelled vertices and 68,993,773 edges, while the latter contains 1,134,890 unlabelled vertices and 2,987,624 edges. In both graphs each vertex represents a user which is connected to one if its friends by an edge. For both graphs we followed the same procedure for obtaining our graph join operands through random walk sampling. Such procedure is described below.

Since no data values are given within the datasets, we enriched each graph: we used the guidelines of the LDBC Social Network Benchmark protocol [12], and hence associated to each user an IP address, an Organization and the year of employment. We choose to not use the whole LDBC model, both because we prefer to test our dataset with real world graph data, and because those guidelines are specifically built for generating RDF graphs.

For each social network graph, the left and right graph operands were obtained by starting the random walk from a same vertex but using a different seed for the graph traversal. Moreover, such operands were obtained incrementally by visiting each time a number of vertices that is a power of 10. The operands were stored in secondary memory as Neo4J property graphs, after each run we have to completely delete the database in order to perform with no result stored in the graph database. As a consequence we decided to sequentially perform the benchmarks over the different graph data sizes for each GDBMS.

Before discussing the(indexing) implementation choices for Neo4J, RDF4J and our proposed implementation, we must discuss the join operation that will be benchmarked. We perform the algorithm using as operands the two distinct subgraphs with a same vertex size, where the \( \theta \) predicate is the following one:

\[
\theta(u, v) \equiv u.Year1 = v.Year2 \land \\
u.Organization1 = v.Organization2
\]

In this way we generalize the “subgraph extraction from two graphs” problem by matching not only one user per time, but also all the users with the same employment year and work for the same organization. The resulting graph has an edge between two vertices iff there was a friendship relation among the two users in the original graph.

In order to compare our indexed implementation with other indexing features of both graph databases, for Neo4J we create the indices over the Year and Organization attributes and, for the RDF triple store where no properties are associated to the vertices, we create all the indices over the stored triples (subject, object and predicate). While Neo4J can directly store the data as described by our data model, RDF4J requires to translate such data since it uses the RDF data model. In order to do so, for each attribute \( A \), of our vertex \( v \) we create a triple \((v, A_i, v[A_i])\), representing two nodes, \( v \) and \( v[A_i] \), linked by an edge \( A_i \). Regarding our proposed data structure, we associate to each vertex the hashing value obtained from an automatically generated hashing composition between the hashes of the field Year and Organization. The hash function used for both fields are Java standard hashing functions.

6. RESULTS
We evaluated our proposed implementation against Neo4J and RDF4J with a computer with a 2.2 GHz Intel Core i7 processor and 16 GB of RAM at 1600 MHz, and an SSD Secondary Storage with an HFS file system. Moreover, Neo4J is already optimized for SSD and all triple stores have been tested with SSD secondary storage. All the queries (either in Cypher or SPARQL) have been evaluated using Java: this means that our solution has been implemented in Java using the JNA library for calling the OS memory mapping methods, and the Java APIs for both Neo4J and RDF4J. We chose those two graph databases because they implement two different graph query languages that are based upon distinct graph data model: Neo4J stores Property Graphs and queries the graphs with Cypher, while RDF4J is a triple store that queries the graph data with SPARQL. We chose not to test our graph joins over the SQLGraph model since there is no existing implementation of such model and, most importantly, the Gremlin query language allows only to perform graph traversal queries that usually return a bag of values (see Section 6 for further details).

While Neo4J explicitly produces the result in the graph database and returns it through the API interface, RDF4J explicitly evaluates the query either by creating a new graph in main memory or by iterating over an API object. Consequently, we choose to evaluate RDF4J while storing the result in a main-memory graph.

All the benchmarks are produced by performing a trimmed mean of 10 runs over the same dataset. In such trimmed mean the fastest and the slowest results are removed. Since Cypher stores the newly created vertices and edges into the secondary memory operands’ property graphs, after each run we have to completely delete the database in order to perform with no result stored in the graph database. As a consequence we decided to sequentially perform the benchmarks over the different graph data sizes for each GDBMS (i.e. we first perform the join over the \( 10 \times 10 \) operands, then over the \( 10^2 \times 10^2 \) operands and so on), and to perform the next run for Neo4J after re-creating the graph database in secondary memory. For the next run we recreate all the graph databases, and we recommence the mentioned process. Our solution outperforms the implementation over Neo4J an order of magnitude and at most two orders of magnitude.

3We presume you use a sensible machine, with a SSD (or enough IOPS) and decent amount of RAM.4

4 RDF4J is previously known as Sesame: http://neo4j.com/blog/neo4j-2-2-query-tuning/

5 See our code in https://bitbucket.org/unibogb/databasemappings/ released under GPLv3.
Table 1: Benchmarking the time (ς) required to load and index the LiveJournal graph join operands in secondary memory

| Operands Size (|V|) | Operands Storing Time (ms) | Speed-up |
|---|---|---|
| Left | Right | Neo4J (ς_L) | RDF4J (ς_R) | Our Storage Proposal (ς′) | Neo4J (ς/ς′) | RDF4J (ς/ς′) |
| 10 | 10 | 20 | 20 | 5.15x | 14.15x |
| 10^2 | 10^3 | 233 | 237 | 106 | 2.80x | 19.12x |
| 10^3 | 10^4 | 706 | 938 | 1093 | 1.42x | 17.00x |
| 10^4 | 10^5 | 3198 | 3598 | 16472 | 1.80x | 204.03x |
| 10^5 | 10^6 | 6245 | 424377 | 162348 | 3.60x | 26.99x |

Table 2: LiveJournal graph Join Running Time (τ). GDBMS are tested with different languages: Neo4J is tested with Cypher and RDF4J is tested with SPARQL

| Operands Size (|V|) | Result | Join Time (τ) | Speed-up |
|---|---|---|---|
| Left | Right | Neo4J-Cypher (τ_N) | RDF4J-SPARQL (τ_R) | Our CoGrouped Join (τ′) | Neo4J (τ/τ′) | RDF4J (τ/τ′) |
| 10 | 10 | 187.34 | 5.78 | 4.44 | 425.77x | 13.14x |
| 10^2 | 10^3 | 233.08 | 148.46 | 4.13 | 56.44x | 35.95x |
| 10^3 | 10^4 | 534.01 | 25393.86 | 11.29 | 42.61x | 2249.23x |
| 10^4 | 10^5 | 5466 | 2937.55 | 47.59 | 61.73x | 61942.05x |
| 10^5 | 10^6 | 87377 | 197872.78 | 443.15 | 456.82x | — |
| Block Size (KB) | Neo4J (ς_N) | RDF4J (ς_R) | Our Storage Proposal (ς′) | Neo4J (ς/ς′) | RDF4J (ς/ς′) |
| 10 | 10 | 308 | 76 | 24 | 12.83x | 3.16x |
| 10^2 | 10^3 | 408 | 204 | 32 | 12.75x | 6.37x |
| 10^3 | 10^4 | 1608 | 1616 | 216 | 7.44x | 7.48x |
| 10^4 | 10^5 | 141540 | 153876 | 20660 | 6.85x | 7.49x |
| 10^5 | 10^6 | 1458868 | 143748 | 193772 | 7.54x | 7.42x |

Figure 6: Plots for the benchmark tests.
while RDF4J performances degrades drastically when reaching graph operands with larger size (we outperform by at most five orders of magnitude with 10^4 vertices’ operand size). Moreover, we set a timeout of four hours (4H) while performing the benchmark tests.

Table 2 provides the time that is required to evaluate the join operation for the LiveJournal dataset. The operand size is expressed with the number of the graphs’ vertices. We can see that our solution outperforms the two benchmarked systems: while RDF4J behaves better than Neo4J on data with small operand size (10, 10^3), the opposite situation happens when operands have greater size (up to 10^6). A graphical representation is given in Figure 6a.

In particular, Figure 6b plots the same results for the YouTube social network graphs: as we can see, even if the speed-up in such solution is lower than in the previous dataset, our join implementation still outperforms the opponents’ graph join implementations.

Table 1 shows that our implementation is faster to store than both Neo4J’s property graph and RDF4J’s triple store. Moreover, our graph data structure requires less secondary memory blocks to be stored (Table 1). The constant gain of our implementation against Neo4J is supported by the fact that Neo4J has a secondary memory cache in order to speed up the graph traversal query process. On the other hand, RDF4J through the Sail storage represents the RDF triples into a B-Tree structure. Anyway, we must clarify that our data structures contain neither locks nor cache areas for other graph operations, that are usually implemented in all-round GDBMS solutions.

Table 3 shows that the time required for both loading and indexing the operands in secondary memory and join them is still lesser than the sole join query time for Neo4J and RDF4J (except for the case of the operand size 10 over RDF4J, when RDF4J outperforms our proposed solution).

7. CONCLUSIONS

We propose a new graph operation that, to the best of our knowledge, is not supported on current graph query languages, that is the graph join. This is a binary operation performing a relational join over the graphs’ vertices tables that, in a second step, creates the edges according to a specific semantics. The results highlight how our proposed graph data structure and join algorithm can be used for implementing the graph join operation over the state-of-the-art graph database management systems. This is due to the fact that the time that it takes to both store the proposed graph data structure in secondary memory and to join them is less than the sole query time of such systems (τ_N > τ’_v + τ’_E and τ_R > τ’_v + τ’_E).

Our graph join algorithm assumed that the graph operand’s edges are not labelled: further tests have to be carried out to both extend our data structure and our algorithm, in order to allow more of one type of edges at a time.

7.1 Future Works

The array data structure of our graph data model allows implementing the join algorithm with a parallel approach: we could easily assign to each process a subset of vertices with a specific hash code, store the result into a subset bulk graph and to append the partial results, by linking the end of one stack with the beginning of the next partial computation. As a next step we could see that it is possible to define a specific case of \( \theta \)-join when the relation \( \theta \) is defined over some edges that can link either two distinct graphs, or the edges of a same given graph. Consequently, given a set of edges \( E, R_E \) is defined as the relation \( v \in R_E (v' \Leftrightarrow (v, v') \in E). \) A specific application of such kind of join can be found in OrientDB’s (No)SQL language, where no joins, either explicit or implicit in the from clause (multiple tables) are allowed between vertices.

The Disjunctive Join acts as a missing data operator for the edges. The disjunctive join does not allow inheriting missing information from the two graph operands, since only the vertices that match over the two graphs are returned. Consequently, as done in the relational data model, we could provide more details on graph outer joins (such as left, right and full). While the graph conjunctive join acts as an intersection between the two graphs, the graph disjunctive full-outer join could act as a graph union operation.

8. RELATED WORK

Graph Query Languages can be categorised in two main classes; the first class of languages try to find a possible match for a specific traversal expression (Section 5.1) as a consequence such graph queries do not manipulate the graph data structure. The second class of queries use traversal expressions to extract a portion of the graph that has to be queried and altered in a next step.

8.1 Graph Traversal languages

GraphLOG. The GraphLOG [8] query language subserves a property graph data structure (direct labelled multigraph) where no properties are associated, neither to vertices nor to edges. Such query language is conceived to be visually representable, and hence path queries are represented as graphs, where simple regular expressions can be associated to the edges. The concept of visually representing graph traversal queries involving path regexes was later on adopted in [13], where some algorithms are showed for implementing such a query language in polynomial time. Such language does not support some path summarization queries that were introduced in GraphLOG [7].

NautilOD. The NautilOD [14] query language was conceived for performing path queries (defined through regular expressions) over RDF graphs with recursion operators (Kleene Star). The same paper shows that queries can be evaluated in polynomial time.

Gremlin. Another graph traversal language, Gremlin, have been proved to be Turing Complete [25]; by the way this is not a desired feature for query languages since it must guarantee that each query always returns an answer and that the evaluation of the query always terminates. Another problem with this query language is based on its semantics: while all the other graph traversal languages return the desired subgraph, Gremlin returns a bag of values (e.g., vertices, values, edges). This peculiarity does not allow the user to take advantage of partial query evaluations and to combine them in a final result.

8.2 (Proper) Graph Query Languages
All the following graph query languages offer only a limited support to pattern extraction from graphs, except from SPARQL, that has been recently extended in order to allow path traversal queries [28]. Consequently, such languages focus more on the graph data manipulation part.

**BiQL.** BiQL [10, 11] is a SQL-like query language that allows to (i) update the data graph with new vertices and edges, (ii) filter the desired vertices through a WHERE clause, (iii) extract desired subgraph through path expressions and (iv) provide some basics path and vertex summarization results. This language has not got a formal semantics yet and it is still under development, but has the aim to develop a closed language under query compositionality. The query patterns do not allow expressing regex-es over the paths.

**SPARQL.** At the time of writing, the most studied graph query language that has been studied both in terms of semantics and expressive power is SPARQL, as it is the most time-worn language among those that are both well-known and implemented. Some studies on the expressive power of SPARQL [1, 33] showed that such graph query language syntax allows to write very costly queries that can be computed more efficiently whether only a specific class of (equivalent) queries is allowed. As a result, the design flaws of a query language relapse on the computational cost of the allowed queries. These problems could be avoided from the very beginning whether the formal study had preceded the practical implementation of the language. However, such limitations do not preclude some interesting properties: the algebraic language used to formally represent SPARQL performs queries' incremental evaluations [37], and hence allows to boost the querying process while data undergoes updates (both incremental and decremental). Moreover, while SPARQL was originally designed to return tabular results, later extensions tried to overcome to such problem with the CONSTRUCT clause, that returns a new graph. Last but not least, the usage of so-called named graphs allows to perform queries over two different RDF graphs.

**LDQL.** The NautiLOD language was later on extended in LDQL [20, 21], where SPARQL patterns are added and different path union and concatenation are allowed. For this specific graph query language the time complexity of the query evaluation has not been studied in the time of the writing. Even if it is claimed that such language is more general than SPARQL, this language do not allow to create new graphs and to concatenate vertex values through the BIND clause, since most of the SPARQL operations are not matched by the sole RDF's triple matching.

**Cypher.** Cypher [32, 34, 27] is yet another SQL-like graph query language for property graphs. No formal semantics for this language were defined from the beginning as in GraphQL, but nervelessly some theoretic results have been carried out for a subset of Cypher path queries [21] by using an algebra adopting a path implementation over the relational data model. Similarly to BiQL, this language does not allow to express complex graph traversal patterns, but it allows to update a property graph and to produce a new graph as a result.

**GraphQL and GRAD.** GraphQL [22] is yet another graph query language with an SPARQL-like syntax, mainly conceived for pattern extraction from the data, called graph motifs, and their construction. The language allows graphs naming similarly to SPARQL named graphs. The most interesting scientific contribution of He [22] is the first attempt in defining a graph algebra for collection of graphs. This approach has been finally specialized for single graphs in the GRAD algebra [18, 17]. In this latter definition the cartesian product and join operations are still defined over graph collections and are still not specialized for the single graphs. Consequently, in both languages the cartesian product over two graph collections produces a graph containing two (possibly) disjoint graph components. The graph join over the two collections only merges the matched vertices and no considerations are made on the graphs’ edges structure.

In the end, GRAD propose an alternative graph data model that could be expressed as a specific implementation of the Property Graph model.

### 8.3 Proposed Graph Products and Joins

**Discrete Mathematics.** At the time of writing, the only field where graph joins where effectively discussed is Discrete Mathematics. In this field such operations are defined over either on finite graphs or on finite graphs with cycles, and are named graph products [19]. As the name suggests, every graph product of two graphs, e.g. $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, produces a graph whose vertex set is defined as $V_1 \times V_2$, while the edge set changes accordingly to the different graph product definition. Consequently the Kronecker Graph Product [39] is defined as follows:

$$G_1 \times G_2 = (V_1 \times V_2, \{(g, h), (g', h') \} \in V_1 \times V_2 \mid (g, g') \in E_1, (h, h') \in E_2 \})$$

while the cartesian graph product [26] is defined as follows:

$$G_1 \odot G_2 = (V_1 \times V_2, \{(g, h), (g', h') \} \in V_1 \times V_2 \mid (g = g', (h, h') \in E_2) \lor (h = h', (g, g') \in E_1) \})$$

Please observe that this definition creates a new vertex which is a pair of vertices: hereby such operation is defined differently from the relational algebra's cartesian product, where the two vertices are merged. As a consequence, such graph products admit commutativity and associativity properties only up to graph isomorphism. Other graph products are lexicographic product and strong product [19, 25].

**Graph Database.** Table 5 summarizes some graph database languages features that were previously described in Section 8. As we can see, no previously described graph query language has a join operand between graphs. Consequently, our paper proposes such operand and outlines an algorithm that can be used in order to implement some cases of the graph conjunctive join. As previously described in the introduction, the term “join” in graph databases has been used to indicate other types of non-binary graph operations: some papers [30, 23, 16, 43] use the term “path join” as path queries of arbitrary length, while others more specifically [3, 41, 15] refer to joins between adjacent vertices. While the first definition mainly focuses on path extraction from a single graph as a specific instance of a pattern extraction problem, the second problem focuses on the definition of new edges as a result of graph traversal queries.
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Algorithm 3 CoGrouped Graph Conjunctive LessEqual-Join
1: procedure LeqJoin(G1, G2, A1, B1) → G1 ∧ A1 ≤ B1, G2
2: V ← ∅; E ← ∅
3: HLeft ← iterate_from_min(V1)
4: HRight ← iterate_from_max(V2)
5: for each hL ∈ HLeft do
6: for each u ∈ V1 s.t. h(u) = hL do
7: for each hR ∈ HRight do
8: if hL > hR then break
9: for each v ∈ V2 s.t. h(v) = hR do
10: if u.A ≤ v.B, (u ⊕ v)[A] = u,
(σu.A ≤ v.B)[A] = v
then
11: for each neu ∈ outV1(u) do
12: for each neu ∈ outV2(v) do
13: if θ(neu),
(σneu)[A] = neu,
(σneu)[B] = neu
then
14: V ← V ∪ { neu ⊕ v, neu + neu }

APPENDIX

A. COGROUPED GRAPH CONJUNCTIVE LESSEQUAL-JOIN
Algorithm 3 shows the implementation of the graph join when the predicate is a less-equal over one attribute. Given a less-equal predicate h(u, v) = u.A ≤ v.B, and an hashing function h such that h(u) ≤ h(v) then u.A ≤ v.B, we join
all the vertices from the left graph that have a vertex hash code that is less or equal of the left one. Since the hashes can be scanned in advance through a linear data structure, we choose to scan the left operands’ hashes in ascending order: for each new hash value of such operand, we scan the right operands’ hashes in descending order. In this way, if the left operand’s current hash is greater than the one currently scanned over the right graph, we can break the whole internal iteration and proceed with the next hash for the left operand. Then, the algorithm behaves similarly when we have to join the neighbour vertices.

In order to compare our results with the EquiJoin ones, we performed our benchmarking over the same experimental conditions as described in Section 6.

The hash scanning choice is revealed to be poignant by the benchmarks that have been carried out. Table 6 provides the results for the less-equal test, where we kept the same subgraph samples that were used for the equi-join algorithm. In this case all the benchmarked implementations stopped with a graph input size of $10^4$ due to the high multiplicity of the result (with a $10^3$ dataset, we reach an average multiplicity of $\approx 509$). In this case, SPARQL evaluated with RDF4J has a better performance than Cypher evaluated over Neo4J: our algorithm is at most two orders of magnitude faster than the evaluation of both Cypher query with Neo4J and of the SPARQL query over RDF4J.

As a consequence, we confirm that our algorithm has better performances than current implementations when the multiplicity coefficient increases. We propose to store the graph results in secondary memory for further implementations. In this way we allow the algorithm to finish the join computation with no thwarts caused by the primary-memory size limitations.

### B. IMPLEMENTING THE COGROUPING GRAPH CONJUNCTIVE EQUIJOIN

Algorithm 4 assumes a hash-ordered vertex representation through which a CoGroup-join algorithm over the vertices is possible. This CoGroup-join technique (lines 15-22) is based on a previous evaluation of the hash value of each vertex, and hence this reduces the search cost of finding vertices with similar features (lines 13-14). Such technique is extended with the visit of the neighbour vertices (lines 23-29), where the neighbours’ hash value is used as a heuristic (line 25 and line 28) in order to skip a test of the equi-join predicate $\theta$ returning false. The result is stored using a Bulk Graph.
Table 6: Graph Join Running Time with a ≤ predicate (τ). GDBMS are tested with different languages: Neo4J is tested with Cypher and RDF4J is tested with SPARQL.

| Operands Size | Result Size | Avg. Multiplicity | Cypher-Neo4J (τ_N) | RDF4J-SPARQL (τ_R) | Our CoGrouped Join (τ') | Speed-up Cypher (τ_N/τ') | Speed-up SPARQL (τ_R/τ') |
|---------------|-------------|-------------------|-------------------|-------------------|------------------------|------------------------|------------------------|
| Left 10 | Right 10 | 55 | 5.5 | 194.94 | 10.49 | 0.40 | 487.35 | 26.22 |
| Left 10^2 | Right 10^2 | 5130 | 51.3 | 1082.58 | 556.95 | 3.83 | 282.66 | 145.42 |
| Left 10^3 | Right 10^3 | 510055 | 509.54 | 288619.91 | 88312.93 | 3.64 | 516.56 | 158.06 |

Algorithm 4 Hash Join algorithm using the vectorial data model in Secondary Memory and sequential reads

```plaintext
1: function Offset(Index, VA, i)
2: if i < |Index| - 1
3: return VA[Index[i].VAOffset, ... , Index[i+1].VAOffset]
4: else
5: return VA[Index[i].VAOffset, ... , |VA|]
6: function readVertex(Array);
7: function VX[i] := readVertex(Offset(VertexIndex X, VA X, i))
8: function GraphHashJoinVisit(GL, GR, θ)
9: bulkGraph := new Stack()
10: θ'(u, v) := θ(u, v) ∧ (u ⊕ v)[A1] = u ∧ (u ⊕ v)[A2] = v
11: HI := HashL ∩ HashR
12: for each h_c ∈ HI do
13: ArrayL := Offset(HashL, VA L, h_c)
14: while |ArrayL| > 0 do
15: u := readVertex(ArrayL)
16: ArrayR := Offset(HashR, VA R, h_c)
17: while |ArrayR| > 0 do
18: v := readVertex(ArrayR)
19: if !θ'(u, v) then continue
20: np := u ⊕ v
21: for each nup ∈ u.out do
22: nu := VX[nup]
23: if h(nu) ∉ HI then continue
24: for each npv ∈ v.out do
25: nv := VX[npv]
26: if h(nv) ∉ HI then continue
27: if !θ'(nu, nv) then continue
28: np.neighborus.add((nup, npv))
29: bulkGraph.push(np)
30: ArrayR := ArrayR[|v|, ... , |ArrayR|]
31: ArrayL := ArrayL[|u|, ... , |ArrayL|]
32: return bulkGraph
```

Return the first vertex stored inside Array
X is a placeholder for either left or right
Extends the θ equivalence predicate with the join test
Get the hash values in common to the two graphs
H_c returns the index of hash h_c in Hash_L
Checks if there are more vertices to read in Array_L
Checks if there are more vertices to read in Array_R
u.out is the array containing the indices of u’s outgoing neighbours
v.out is the array containing the indices of v’s outgoing neighbours
Prepare the array to read eventually the next vertex
Prepare the array to read eventually the next vertex