The GUP effect on Hawking Radiation of the 2+1 dimensional Black Hole

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Abstract. We investigate the Generalized Uncertainty Principle (GUP) effect on the Hawking radiation of the 2+1 dimensional Martinez-Zanelli black hole by using the Hamilton-Jacobi method. In this connection, we discuss the tunnelling probabilities and Hawking temperature of the spin-1/2 and spin-0 particles for the black hole. Therefore, we use the modified Klein-Gordon and Dirac equations based on the GUP. Then, we observe that the Hawking temperature of the scalar and Dirac particles depend on not only the black hole properties, but also the properties of the tunnelling particle, such as angular momentum, energy and mass. And, in this situation, we see that the tunnelling probability and the Hawking radiation of the Dirac particle is different from that of the scalar particle.
1. Introduction

The discovery of the black hole radiation, known as Hawking radiation in the literature, is one of the milestones to construct a consistent connection between the relativity theory, the statistical mechanics and the quantum mechanics. The nature of a black hole has been started to be investigated in the framework of the thermodynamical and the quantum mechanical concepts since 1970 [1, 2, 3, 4, 5, 6]. Hawking investigated the thermodynamical properties of a black hole in the frame of quantum field theory based on the Heisenberg uncertainty principle on a curved spacetime. Since then, the Hawking radiation has been investigated as a quantum tunnelling effect of the relativistic particles from a black hole [7, 8, 9, 10, 11, 12, 13, 14]. Also, the Hawking radiation as a tunnelling process of the particles from various black holes has been studied, extensively, in the literature in both 3+1 and 2+1 dimensional [13, 14, 15, 16, 17, 18, 19, 20, 21, 22].

On the other hand, the suitable candidate quantum gravity theories, such as string theory and loop quantum gravity theory, indicate the presence of a minimal observable length in Planck scale [23, 24, 25, 26]. The existence of such a minimal length leads to the generalized Heisenberg uncertainty principle (GUP). The GUP can be expressed as [27, 28],

\[ \Delta x \Delta p \geq \frac{\hbar}{2} \left[ 1 + \beta (\Delta p)^2 \right] \]  

where, \( \beta = \frac{\beta_0}{M_p^2} \), the \( M_p^2 \) is the Planck mass and \( \beta_0 \) is the dimensionless parameter. Then, the modified commutation relation becomes,

\[ [x_\mu, p_\nu] = i\hbar \delta_{\mu\nu} \left[ 1 + \beta p^2 \right], \]  

where, \( x_\mu \) and \( p_\mu \) are the modified position and the momentum operators, respectively, defined by

\[ x_\mu = x_{0\mu}, \]
\[ p_\mu = p_{0\mu}(1 + \beta p_{0\mu}^2), \]

where, the \( x_{0\mu} \) and \( p_{0\mu} \) are the standard position and momentum operators, respectively, and they satisfy the usually commutation relation \([x_{0\mu}, p_{0\nu}] = i\hbar \delta_{\mu\nu}\). These modified relations play an important role in physics. For example, in recent years, using the GUP, the thermodynamics properties of the black holes were investigated via a particle tunnelling from the black holes. To include the quantum gravity effect, the Klein-Gordon and Dirac equations are modified by the GUP framework [29]. With these modified relativistic wave equations, the corrected Hawking temperature of various 3+1 and higher dimensional black holes computed via a particle tunnelling process [30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40]. In this motivation, we will investigate the Hawking radiation of the 2+1 dimensional Martinez-Zanelli black hole by the scalar and Dirac particles tunnelling process under the effect of the GUP. The metric of the Martinez-Zanelli black hole is given by [41]

\[ ds^2 = F(r)dt^2 - \frac{1}{F(r)}dr^2 - r^2d\theta^2 \]
where $F(r)$,

$$F(r) = \frac{1}{l^2} \left[ r^2 - 3B^2 - \frac{2B^3}{r} \right] = \frac{(r + B)^2 (r - 2B)}{rl^2},$$

and $l^2 = -1/\Lambda$ is the cosmological constant and $B$ is the mass parameter related to the black hole mass $M$, as $B = \sqrt{Ml^2/3}$ [42]. Hence, the black hole has a singularity at $r = 0$ surrounded by horizon located at $r_h = 2B$ under the condition $B \neq 0$.

The organization of this work are as follows: In the Section 2, we modified the Klein-Gordon equation by using the GUP. Subsequently, from the modified Klein-Gordon equation written in the 2 + 1 dimensional Martinez-Zanelli Black hole background, we calculate the tunnelling possibility of the scalar particle by using the semi-classical method, and then, we find the Hawking temperature. In the Section 3, the modified Dirac equation is written in the 2 + 1 dimensional Martinez-Zanelli black hole, and then, the tunnelling probability of the Dirac particle from the black hole and its Hawking temperature is also calculated. Finally, in conclusion, we evaluate and summarize the results.

2. The Modified Klein-Gordon Equation and the Scalar particle tunnelling

To investigate the quantum gravity effect on the tunnelling process of the scalar particles from the black hole and on its Hawking temperature, we will discuss the modified Klein-Gordon equation under the GUP relations. The standard Klein-Gordon equation can be written as [43],

$$p_\mu p^\mu \phi = m_0^2 \phi, \quad (5)$$

or its explicit form is

$$-(i\hbar)^2 \partial_t \partial_t \phi = \left[ (-i\hbar)^2 \partial_i \partial^i - m_0^2 \right] \phi, \quad (6)$$

where $\phi$ is the wave function of the scalar particles. On the other hand, in the context of the GUP, the modified energy relation is given by

$$\tilde{E} = E \left( 1 - \beta E^2 \right) = E \left[ 1 - \beta \left( p^2 + m_0^2 \right) \right], \quad (7)$$

where $E^2 = p^2 + m_0^2$. Then, the square of the momentum operator can obtained by using the Eq.(3) as follows;

$$p^2 \simeq -\hbar^2 \left[ \partial_i \partial^i - 2\beta \left( \partial_i \partial^i \right) \left( \partial_j \partial^j \right) \right], \quad (8)$$

where the higher order terms of the $\beta$ parameter are neglected. Then, using the Eq.(7) and Eq.(8) in the standard Klein-Gordon equation, the modified Klein-Gordon equation is written as follows;

$$-(i\hbar)^2 \partial_t \partial_t \Phi = \left[ (-i\hbar)^2 \partial_i \partial^i - m_0^2 \right] \left[ 1 - 2\beta \left( -\beta \partial_i \partial^i + m_0^2 \right) \right] \Phi, \quad (9)$$

where $\Phi$ is the generalized wave function of the scalar particles. Hence, the modified Klein-Gordon equation in the Martinez-Zanelli black hole background is
\[
\frac{\hbar^2}{F(r)} \frac{\partial^2 \Phi}{\partial t^2} - \hbar^2 F(r) \frac{\partial^2 \Phi}{\partial r^2} - \frac{\hbar^2}{r^2} \frac{\partial^2 \Phi}{\partial \phi^2} + 2\beta F(r) \hbar^4 \frac{\partial^2}{\partial t^2} \left[ F(r) \frac{\partial^2 \Phi}{\partial t^2} \right] \\
+ \frac{2\beta \hbar^4}{r^2} \frac{\partial^2}{\partial \phi^2} \left[ \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \phi^2} \right] + m_0^2 \left( 1 - 2\beta m_0^2 \right) \Phi = 0.
\]

To investigate the tunnelling radiation of the Martinez-Zanelli black hole with the Eq. (10), we employ the wave function of the scalar particle as,

\[
\Phi(t, r, \phi) = Ae^{i \frac{\hbar}{2} S(t, r, \phi)}
\]

where \(A\) is a constant and \(S(t, r, \phi)\) is the classical action term for the outgoing particle trajectory. Substituting the Eq. (11) into the Eq. (10) and neglecting the higher order terms of \(\hbar\), we get the equation of motion of the scalar particle as

\[
\left( \frac{\partial S}{\partial t} \right)^2 - F^2(r) \left( \frac{\partial S}{\partial r} \right)^2 - \frac{F(r)}{r^2} \left( \frac{\partial S}{\partial \phi} \right)^2 - 2\beta F^3(r) \left( \frac{\partial S}{\partial r} \right)^4 \\
- \frac{2\beta F(r)}{r^4} \left( \frac{\partial S}{\partial \phi} \right)^4 - m_0^2 \left( 1 - 2\beta m_0^2 \right) F(r) = 0.
\]

Due to the commuting Killing vectors \((\partial_t)\) and \((\partial_\phi)\) we can separate the \(S(t, r, \phi)\), in terms of the variables \(t, r, \phi\), such as \(S(t, r, \phi) = -Et + j\phi + K(r)\), where \(E\) and \(j\) are the energy and angular momentum of the particle, respectively, and \(K(r) = K_0(r) + \beta K_1(r) [37]\). And, from the Eq. (12), the radial integral, \(K(r)\), becomes as follows;

\[
K_\pm(r) = \pm \int \sqrt{\frac{E^2 - F(r) \left( m_0^2 + \frac{j^2}{r^2} \right) F(r)}{1 + \beta \Sigma}} \, dr
\]

and it is computed as,

\[
K_\pm(r_h) = \pm \frac{i\pi l^2 E}{972 \beta^3} \left[ 216 B^2 + \beta \left( 324 m_0^2 B^2 + 16 E^2 l^2 + 81 j^2 \right) \right]
\]

where, \(\Sigma\) is

\[
\Sigma = \left( \frac{F(r) \left( m_0^2 - \frac{j^2}{r^2} \right)}{E^2 - F(r) \left( m_0^2 + \frac{j^2}{r^2} \right)} - \frac{E^2 - F(r) \left( m_0^2 + \frac{j^2}{r^2} \right)}{F(r)} \right)
\]

and \(K_+(r_h)\) is outgoing and \(K_-(r_h)\) is incoming solutions of the radial part. The total imaginary part of the action is \(ImS(t, r, \phi) = ImK_+(r) = ImK_+(r) - ImK_-(r)\) . Hence, the two kind probabilities of the crossing from the outer horizon, from outside to inside and from inside to outside, are given by \([13, 18, 44]\)

\[
P_{out} = \exp \left( -\frac{2}{\hbar} ImK_+(r_h) \right)
\]

and

\[
P_{in} = \exp \left( -\frac{2}{\hbar} ImK_-(r_h) \right),
\]
respectively. Then, the tunneling probability of the scalar particle is written as
\[
\Gamma = \frac{P_{\text{out}}}{P_{\text{in}}} = \exp \left\{ -\frac{\pi l^2 E}{243\hbar B^3} \left[ 216B^2 + \beta \left( 324m_0^2B^2 + 16E^2l^2 + 81j^2 \right) \right] \right\}.
\]

Hence, the modified Hawking temperature is obtained from the lowest order in the expansion of the classical action in terms of the particle energy,
\[
\Gamma = \exp \left( -\frac{2\bar{\hbar} I\text{m}S}{T_H'} \right) = \exp \left( -\frac{E}{T_H'} \right)
\]
(16)
where \( T_H' \) is the modified Hawking temperature of the outer horizon, and it is given by
\[
T_H' = \frac{9\hbar B}{8\pi l^2} \left[ 1 + \beta \left( \frac{324m_0^2B^2 + 16E^2l^2 + 81j^2}{216B^2} \right)^{-1} \right].
\]

If, at first, we expand the \( T_H' \) in terms of the \( \beta \) powers, and, second, neglect the higher order of the \( \beta \) terms, then we get the modified Hawking temperature of the Martinez-Zanelli black hole as follows;
\[
T_H' = T_H \left[ 1 - \beta \frac{324m_0^2B^2 + 16E^2l^2 + 81j^2}{216B^2} \right]
\]
(17)
where the \( T_H = \frac{9\hbar B}{8\pi l^2} \) is the standard Hawking temperature of the black hole. From the \( T_H' \) expression, we see that the modified Hawking temperature is related to not only the mass parameter of the black hole, but also the angular momentum, energy and mass of the emitted scalar particle from the black hole, and it is lower than the standard Hawking temperature.

3. The Modified Dirac Equation and Fermion tunnelling

The Dirac equation in a (2+1) dimensional spacetime is given by the following representation [45],
\[
\{ i\sigma^\mu(x) [\partial_\mu - \Gamma_\mu(x)] \} \Psi(x) = \frac{m_0}{\hbar} \Psi(x).
\]
(18)
In this representation; the Dirac spinor, \( \Psi(x) \), has only two components corresponding positive and negative energy eigenstates, which the each one has only one spin polarization. \( \sigma^\mu(x) \) are the spacetime dependent Dirac matrices and they are written in terms of the constant Dirac matrices, \( \bar{\sigma}^i \), by using triads, \( e^\mu_{(i)}(x) \), as follows;
\[
\sigma^\mu(x) = e^\mu_{(i)}(x)\bar{\sigma}^i,
\]
(19)
where \( \bar{\sigma}^i \) are the Dirac matrices in flat spacetime and they are given by
\[
\bar{\sigma}^i = (\bar{\sigma}^0, \bar{\sigma}^1, \bar{\sigma}^2)
\]
(20)
with
\[
\bar{\sigma}^0 = \sigma^3, \bar{\sigma}^1 = i\sigma^1, \bar{\sigma}^2 = i\sigma^2,
\]
where \( \sigma^1, \sigma^2 \) and \( \sigma^3 \) Pauli matrices, and \( \Gamma_\mu(x) \) are the spin affine connection by the following definition,
\[
\Gamma_\mu(x) = \frac{1}{4} g_{\lambda\alpha} (e^{i}_{\nu\mu} e^{\alpha}_{i} - \Gamma^\alpha_{\nu\mu}) s^{\lambda\nu}(x).
\]
(22)
Here, $\Gamma^\alpha_{\nu\mu}$ is the Christoffel symbol, and $g_{\mu\nu}(x)$ is the metric tensor that is given in terms of the triads as follows,

$$g_{\mu\nu}(x) = e^{(i)}_{\mu}(x) e^{(j)}_{\nu}(x) \eta_{(i)(j)}, \quad (23)$$

where $\mu$ and $\nu$ are a curved spacetime indices running from 0 to 2. $i$ and $j$ are flat spacetime indices running from 0 to 2 and $\eta_{(i)(j)}$ is the metric of the (2+1) dimensional Minkowski spacetime, with signature (1,-1,-1), and $s^{\lambda\nu}(x)$ is a spin operator defined as

$$s^{\lambda\nu}(x) = \frac{1}{2} [\sigma^\lambda(x), \sigma^\nu(x)]. \quad (24)$$

Using the Eq.(3), Eq.(7) and Eq.(8) in the Dirac equation, the generalized Dirac equation becomes

$$-i\sigma^0(x) \partial_0 \Psi = \left( i\sigma^i(x)\partial_i - i\sigma^\mu(x)\Gamma_\mu - \frac{m_0}{\hbar} \right) \left( 1 + \beta \hbar^2 \partial_0 \partial^0 - \beta m_0^2 \right) \Psi, \quad (25)$$

and it is rewritten as

$$[i\sigma^i(x)\partial_0 + i\sigma^\mu(x) \left( 1 - \beta m_0^2 \right) \partial_0 + i\beta \hbar^2 \sigma^\mu(x)\partial_i \left( \partial_j \partial^i \right) - \frac{m_0}{\hbar} \left( 1 + \beta \hbar^2 \partial_0 \partial^0 - \beta m_0^2 \right)$$

$$-i\sigma^\mu(x)\Gamma_\mu \left( 1 + \beta \hbar^2 \partial_0 \partial^0 - \beta m_0^2 \right) ]\Psi = 0, \quad (26)$$

where the $\Psi$ is the generalized Dirac spinor.

To calculate the tunnelling probability of a Dirac particle from the black hole, we use the following ansatz for the wave function;

$$\Psi(x) = \exp \left( \frac{i}{\hbar} S(t, r, \phi) \right) \left( \begin{array}{c} A(t, r, \phi) \\ B(t, r, \phi) \end{array} \right) \quad (27)$$

where the $A(t, r, \phi)$ and $B(t, r, \phi)$ are the functions of space-time. Inserting the Eq.27 in Eq.26, we get the resulting equations to leading order in $\hbar$ and $\beta$ as follows;

$$A \left[ \frac{1}{\sqrt{F(r)}} \frac{\partial S}{\partial t} + m_0 \left( 1 - \beta m_0^2 \right) + \frac{\beta m_0}{r^2} \left( \frac{\partial S}{\partial \phi} \right)^2 + \beta m_0 F(r) \left( \frac{\partial S}{\partial r} \right)^2 \right]$$

$$+ B \left[ i \sqrt{F(r)} \left( 1 - \beta m_0^2 \right) \frac{\partial S}{\partial t} + 1 - \beta m_0^2 \frac{\partial S}{\partial \phi} + i \beta F(r) \sqrt{F(r)} \left( \frac{\partial S}{\partial r} \right)^3 \right]$$

$$+ B \left[ i \frac{\beta}{r^2} \frac{\sqrt{F(r)}}{r} \left( \frac{\partial S}{\partial t} \right)^2 + \beta F(r) \left( \frac{\partial S}{\partial r} \right)^2 + \beta \left( \frac{\partial S}{\partial \phi} \right)^3 \right] = 0$$

$$A \left[ -i \sqrt{F(r)} \left( 1 - \beta m_0^2 \right) \frac{\partial S}{\partial r} + 1 - \beta m_0^2 \frac{\partial S}{\partial \phi} - i \beta F(r) \sqrt{F(r)} \left( \frac{\partial S}{\partial \phi} \right)^3 \right]$$

$$+ A \left[ -i \frac{\beta}{r^2} \frac{\sqrt{F(r)}}{r} \left( \frac{\partial S}{\partial t} \right) \left( \frac{\partial S}{\partial \phi} \right)^2 + \beta F(r) \left( \frac{\partial S}{\partial r} \right)^2 + \frac{\beta}{r^3} \left( \frac{\partial S}{\partial \phi} \right)^3 \right]$$

$$+ B \left[ \frac{1}{\sqrt{F(r)}} \frac{\partial S}{\partial t} - m_0 \left( 1 - \beta m_0^2 \right) - \frac{\beta m_0}{r^2} \left( \frac{\partial S}{\partial \phi} \right)^2 - \beta m_0 F(r) \left( \frac{\partial S}{\partial r} \right)^2 \right] = 0 \quad (28)$$

These two equations have nontrivial solutions for the $A(t, r, \phi)$ and $B(t, r, \phi)$ in case the determinant of the coefficient matrix is vanished. Accordingly, when neglecting the
terms containing higher order of the $\beta$, then we get
\[
\frac{1}{F(r)} \left( \frac{\partial S}{\partial t} \right)^2 - F(r) \left( \frac{\partial S}{\partial r} \right)^2 - \frac{1}{r^2} \left( \frac{\partial S}{\partial \phi} \right)^2 - m_0^2 \\
+ \beta \left[ - \frac{2}{r^4} \left( \frac{\partial S}{\partial \phi} \right)^4 - 2F^2(r) \left( \frac{\partial S}{\partial r} \right)^4 - \frac{4F(r)}{r^2} \left( \frac{\partial S}{\partial r} \right)^2 \left( \frac{\partial S}{\partial \phi} \right)^2 + 2m_0^4 \right] = 0.
\]

Due to the Killing vectors $\left( \partial_t \right)$ and $\left( \partial_\phi \right)$, we can separate the variables for $S(t, r, \phi)$ as $S(t, r, \phi) = -Et + j\phi + K(r)$, where, $E$ and $j$ are the energy and angular momentum of the particle, respectively, and $K(r) = K_0(r) + \beta K_1(r)$ \[37\]. Then, the integral of the radial equation, $K(r)$, becomes as follows;
\[
K_\pm(r) = \pm \int \sqrt{\frac{E^2 - F(r) \left( m_0^2 + \frac{j^2}{r^2} \right)}{F(r)}} \left[ 1 + \frac{\beta}{F(r)} \left( \frac{2E^2m_0^2F(r) - E^4}{E^2 - F(r) \left( m_0^2 + \frac{j^2}{r^2} \right)} \right) \right] \, dr
\]
and it is computed as
\[
K_\pm(r_h) = \pm i\pi \frac{l^2 E}{9\pi B^3} \left[ 216B^2 + \beta \left( 324m_0^2B^2 + 16E^2l^2 - 27j^2 \right) \right]
\]
Thus, from the Eq.(14) and Eq.(16), the tunneling probability of the Dirac particle is given by
\[
\Gamma = \frac{P_{\text{out}}}{P_{\text{in}}} = \exp \left\{ -\frac{\pi l^2 E}{243hB^3} \left[ 216B^2 + \beta \left( 324m_0^2B^2 + 16E^2l^2 - 27j^2 \right) \right] \right\}.
\]
Furthermore, from the Eq.(16), the modified Hawking temperature becomes as follows
\[
T_H^* = \frac{9hB}{8\pi l^2} \left[ 1 + \beta \frac{324m_0^2B^2 + 16E^2l^2 - 27j^2}{216B^2} \right]^{-1}
= T_H \left[ 1 - \beta \frac{324m_0^2B^2 + 16E^2l^2 - 27j^2}{216B^2} \right],
\]
where the $T_H = \frac{9hB}{8\pi l^2}$ is the standard Hawking temperature of the Martinez-Zanelli black hole. As in the case of the scalar particle tunnelling, the corrected Hawking temperature of the tunnelling Dirac particle is related to not only the mass parameter of the Martinez-Zanelli black hole, but also depends the angular momentum, energy and mass of the emitted Dirac particle, and it is lower than the standard Hawking temperature.

4. Concluding remarks

In this paper we have studied the issue of the quantum gravity effect on the Hawking radiation of the 2+1 dimensional Martinez-Zanelli black hole by using the particle tunnelling method. To take into account the quantum gravity effects, we modified the Dirac and Klein-Gordon equations by the generalized fundamental commutation relations to discuss the tunneling radiation of fermions and scalar particles, respectively. The results showed that the corrected Hawking temperature is not only determined by the mass parameter of the Martinez-Zanelli black hole, but also it is affected by the quantum properties (i.e., the angular momentum, energy and mass) of the emitted fermions and scalar particles. The other important results are given as follows:
• According to Eq. (17), the corrected Hawking temperature of the tunnelling scalar particle is lower than the standard temperature.

• In Eq. (29), when \(324m_0^2B^2 + 16E^2l^2 > 27j^2\), the corrected Hawking temperature of the tunnelling fermions is lower than the standard temperature. However, when \(324m_0^2B^2 + 16E^2l^2 < 27j^2\), the corrected temperature is higher than the standard temperature. If \(324m_0^2B^2 + 16E^2l^2 = 27j^2\), then the contribution of the GUP effect is canceled, and radiation temperature of the tunnelling fermions reduce to the standard temperature.

• By comparing the Eq. (29) with Eq. (17), we can say that the radiation temperature of the tunnelling fermions higher than the scalar particles temperature, even if their masses, energies, and angular momentums are same.

Finally, thanks to the GUP effect, we can determine whether the radiated particle from a black hole is the scalar particle or the Dirac particle.

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