Buckling analysis of composite laminates with a circular hole

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Abstract. The stress concentration will appear at the edge of the hole of composite laminate after the openings, which will affect the buckling load and bearing capacity of the composite laminate. Generally, a local reinforcement near the hole is an effective method to alleviate the concentration near the hole and improve the bearing capacity of the structure. In addition, the curvilinear fiber laying scheme is tried to solve the stress concentration problem in this paper. Therefore, the finite element models of ordinary composite laminates with a circular opening, variable angle composite laminates with an open-hole and composite laminates with reinforcement near the circular hole were established by ABAQUS software, the buckling analysis and progressive damage analysis of the above models under shear load were carried out considering the Hashin damage failure criterion and Camanho stiffness degradation criterion. By comparing the buckling load and ultimate load of three different kinds of composite laminates with a circular hole, it is found that besides composite laminates with reinforcement near the circular hole, the curvilinear laying laminates can also increase the stiffness of the laminates and improve the buckling load and ultimate bearing capacity of the structure.

1. Introduction
Variable angle tow composite laminates can make full use of the fiber performance advantages by adjusting the fiber placement angle, and then improve the bearing capacity of composite laminates. The concept of variable angle tow composite laminates was first proposed by Hyer et al. [1] and guradal et al. [2], the stress distribution of laminates can be changed by introducing curved fiber with variable angle, and then improve the bearing capacity of structure. Wu et al. [3] verified the superiority of mechanical properties of variable angle laminates through experiments, and the final mechanical properties of variable stiffness laminates were better than the theoretical prediction value. Beyond that, there are many research results on the mechanical properties of composite laminates with holes. Hyer and Lee [4] used the finite element method to study the plane stress of a plate with a circular hole under uniaxial tension, the results show that the curvilinear fiber placement can greatly improve the strength and buckling capacity of the plate with openings compared with the traditional linear fiber placement. In addition, fiber paths with flow functions have also been used to determine the optimal fiber direction for plates with openings [5-8].

Meanwhile, for composite laminates with open-holes, intercalation reinforcement and patch reinforcement are usually used to ensure that the stress in the area around the hole is kept within the range of material limitations. Eiblmeier and Loughlan [9] carried out the numerical analysis of buckling of a shear loaded square composite plate with an open-hole which was reinforced by a composite ring, the results show that the reinforcement rings bonded on both sides near the hole can significantly increase critical buckling load. Zhu et al. [10] studied the reinforcement types of plates
with openings through numerical simulation and experimental, and found that the ultimate tensile load of the plates with openings can be greatly increased by using VAT reinforced materials, especially the elliptical VAT reinforced material.

In this paper, buckling and nonlinear post buckling progressive damage analysis of composite laminates with a circular hole under shear load was carried out by finite element software ABAQUS. By comparing the buckling and post buckling behaviors of open-hole composite laminates with different enhancement methods, the influence of curvilinear laying track and reinforcement near the hole on buckling load and ultimate load of composite laminates with openings is studied.

2. Analysis methods

2.1. Define of laying path of variable angle composite laminates

At present, the fiber orientation varies linearly along the length of the plate is usually adopted for the variable angle composite laminated plate. The reference path is shown in Figure 1. In the rectangular coordinate system x-y, the fiber orientation changes linearly with the laying direction, the angle between the geometric path of any point (x, y) on the central line and the x-axis, that is, the fiber orientation angle θ (x), can be expressed as

$$\theta(x) = \frac{2(T_1 - T_0)}{a} |x| + T_0$$  \hspace{1cm} (1)

Fig.1 The schematic diagram of the <45|90> placement

Where $T_0$ is the fiber orientation angle at the center of the plate, which is called the starting angle, $T_1$ is the fiber orientation angle at both ends of the plate, called the end angle, and $a$ is the length of the rectangular plate. After the reference path is determined, the parallel method is used to move the reference path so that the fiber can cover the whole plane, as shown in Figure 2. In formula (1), when $T_0 = T_1$, it is the traditional straight fiber lay-up laminates, as shown in Figure 3.

Fig.2 Linear variation of fiber orientation  
Fig.3 Traditional straight fiber placement
2.2. Damage failure criterion and stiffness degradation criterion
Compared with other strength criterion, Hashin strength criterion [11] is simple in form, can predict various failure modes, and has better prediction accuracy. Therefore, this criterion is used as the damage failure criterion of composite laminates, the expression is as follows:

Fiber tensile failure ($\sigma_{11} \geq 0$):

$$\left( \frac{\sigma_{11}}{X_T} \right)^2 \geq 1$$

Fiber compressive failure ($\sigma_{11} < 0$):

$$\left( \frac{\sigma_{11}}{X_C} \right)^2 \geq 1$$

Matrix tensile failure ($\sigma_{22} \geq 0$):

$$\left( \frac{\sigma_{22}}{Y_T} \right)^2 + \left( \frac{\sigma_{12}}{S_{12}} \right)^2 + \left( \frac{\sigma_{11}}{S_{13}} \right)^2 + \left( \frac{\sigma_{23}}{S_{23}} \right)^2 \geq 1$$

Matrix compressive failure ($\sigma_{22} < 0$):

$$\left( \frac{\sigma_{22}}{2S_{23}} \right)^2 + \left( \frac{\sigma_{12}}{S_{12}} \right)^2 + \left( \frac{\sigma_{11}}{S_{13}} \right)^2 + \left( \frac{\sigma_{23}}{S_{23}} \right)^2 + \frac{\sigma_{22}}{2S_{23}} \left[ \left( \frac{Y_C}{Y_T} \right)^2 - 1 \right] \geq 1$$

In this paper, camacho [12] degradation model is used to degrade the stiffness of composites. The degradation coefficients are shown in Table 1.

| Degradation Coefficient                      | $E_1$ | $E_2$ | $E_3$ | $\nu_{12}$ | $G_{12}$ | $G_{13}$ | $G_{23}$ |
|---------------------------------------------|-------|-------|-------|------------|----------|----------|----------|
| Fiber Tensile Failure                        | 1     | 0.2   | 1     | 0.15       | 0.2      | 1        | 0.2      |
| Fiber Compressive Failure                    | 1     | 0.4   | 1     | 0.15       | 0.4      | 1        | 0.4      |
| Matrix Tensile Failure                        | 1     | 0.2   | 1     | 0.15       | 0.2      | 1        | 0.2      |
| Matrix Compressive Failure                   | 0.07  | 0.07  | 0.07  | 0.07       | 0.07     | 0.07     | 0.07     |

2.3. Numerical analysis based on finite element method
The buckling and post buckling analysis of composite laminates with a circular hole were carried out by using ABAQUS commercial software. The geometrical dimension of the laminate is shown in Figure 4. There are 12 layers in total, and the thickness of each layer is 0.185mm. The mechanical parameters of the composite are shown in Table 2. The four shear plate areas of the laminates are coupled to two reference points on the diagonal, and the displacement and rotation constraints are applied to the lower corner and the displacement load is applied to the upper corner.

| $E_1$ | $E_2$ | $\mu_{12}$ | $G_{12}$ | $X_T$ | $X_C$ | $Y_T$ | $Y_C$ | $S$ |
|-------|-------|------------|----------|-------|-------|-------|-------|-----|
| MPa   | MPa   | /MPa       | /MPa     | /MPa  | /MPa  | /MPa  | /MPa  | /MPa|
| 162000| 9140  | 0.331      | 4570     | 2610  | 1450  | 55    | 285   | 105 |

For the straight fiber laminates with a circular hole (denoted by S), the fiber laying scheme of each ply is shown in Fig. 3; For the linear variable angle laminates with a circular hole (denoted by V), the fiber laying scheme of each ply is shown in Fig. 2; For the straight fiber laminates with reinforcement near the hole (denoted by R) shown in Fig. 5, the fiber laying scheme of each ply is shown in Fig. 2.

Lay-up of three composite laminates with a circular hole:

- S: [45/−45/0/90/0/90]$_{2s}$
- V: [<45/0/>/<45/90/>/<0/45/>/<0/45/>/<90/45/>/<90/45>]$_{2s}$
R: the ply-up of the laminates and the reinforced patch are [45/-45/0/90/0/90]_2s and [45/0/90/]_2s respectively.

Fig. 4 Geometrical dimension of composite laminates with a circular hole

Fig. 5 Geometrical dimension of composite laminates with reinforcement near the hole

3. Results & Discussion

3.1. Buckling analysis
The subspace method of ABAQUS buckling solver is used to carry out the eigenvalue buckling analysis of composite laminates, the first-order modes and critical buckling loads of three composite laminates are obtained.

Fig. 6 The buckling models for three composite laminates
Table 3. The critical buckling load of composite laminates

| number | S     | V     | R     |
|--------|-------|-------|-------|
| Critical buckling load/KN | 18.23 | 19.38 | 30.52 |
| Comparison % | —     | ↑6.31 | ↑67.42 |

Under the shear loading, the buckling modes of the three kinds of laminates are almost the same, and all of them buckle near the center of the circular hole along the loading direction. The critical buckling loads of three kinds of laminates are given in Table 3, compared with the straight fiber laminates with a circular hole, the critical buckling load of linear variable angle laminates with a circular hole increased by 6.31%; Meanwhile, local reinforcement near the hole can effectively reduce the deformation around the hole, so the critical buckling load of the composite laminates with reinforcement near the circular hole increases significantly, reaching at 67.42%.

3.2. Post-buckling analysis

The displacement value of the first-order mode obtained from the buckling analysis is introduced into the nonlinear post buckling analysis as the initial disturbance. In the ABAQUS static analysis module, the Newton-Raphson iteration method is used to carry out the nonlinear static analysis.

![Fig.7 The displacement-load curves of post-buckling for different composite laminates](image)

Table 4. The ultimate load of composite laminates

| number | S     | V     | R     |
|--------|-------|-------|-------|
| Ultimate load/KN | 120.92 | 129.41 | 129.57 |
| Comparison % | —     | ↑7.02 | ↑7.15 |

Figure 8 shows the load-displacement curves of three different kinds of composite laminates with open-hole. The slope of the load-displacement curves of V and R is obviously larger than that of S, which indicates that the stiffness of the laminates can be effectively improved by laying fibers at variable angles or conducting reinforcement near the circular hole. At the same time, compared with S, the variable angle composite laminates can make the stress distribution more uniform, effectively reduce the stress concentration area of the laminates with a circular hole, and thus effectively improve the overall strength, therefore, the ultimate load of V is 7.02% higher than that of S. On the other hand, attaching a local reinforcement patch near the hole can keep the stress within the limitation of materials in the surrounding area, thus improving the bearing capacity of the structure, therefore, the
ultimate load of R is 7.15% higher than that of S. Both methods can improve the ultimate load of laminates, however, local reinforcement near the hole improve the bearing capacity of the structure at the cost of increasing volume and weight; on the contrary, adopting the curvilinear fiber laying scheme can increase the ultimate load under the same volume, obviously, the latter is a better choice to improve the bearing capacity of the structure.

4. Conclusions
Through the analysis of buckling and post buckling of three different composite laminates with a circular hole under shear load, it is found that the critical buckling load and the ultimate load of the composite laminates with a circular hole which adopt the curvilinear fiber laying scheme increased by 6.31% and 7.02% respectively; the critical buckling load and the ultimate load of the composite laminates with reinforcement near the hole increased by 7.02% and 7.15% respectively. The results can provide a reference for the structural design and optimization of composite laminates with circular holes.

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