‘c’ is the speed of light, isn’t it?

George F.R. Ellis
Department of Mathematics and Applied Mathematics,
University of Cape Town,
Rondebosch 7700, Capetown, South Africa

Jean-Philippe Uzan
Institut d’Astrophysique de Paris, GRG-CNRS, 98bis boulevard Arago, 75014 Paris, France
Laboratoire de Physique Théorique, CNRS-UMR 8627,
Université Paris Sud, Bâtiment 210, F-91405 Orsay cédex, France

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Theories proposing a varying speed of light have recently been widely promoted under the claim that they offer an alternative way of solving the standard cosmological problems. Recent observational hints that the fine structure constant may have varied during over cosmological scales also has given impetus to these models. In theoretical physics the speed of light, c, is hidden in almost all equations but with different facets that we try to distinguish. Together with a reminder on scalar-tensor theories of gravity, this sheds some light on these proposed varying speed of light theories.

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I. INTRODUCTION

Recent observational claims, based on the measurement of distant quasar absorption spectra, that the fine structure constant may have been smaller in the past have restarted the debate on the nature of the constants of nature. On the experimental side, many new (and sharper) constraints from a large variety of physical systems on different time scales have been determined (see Ref. [2, 3] for recent reviews). On the theoretical side, tests of the constancy of the constants of Nature extend the testing of the Einstein equivalence principle to astrophysical scales. The proof that some constants have varied during the history of the universe will be a sign of the existence of a new force that will most probably be composition dependent [2, 4]. Many theoretical motivations for such variation, mostly in the framework of higher dimensional theories such as string theory, have been put forward [5]. Among these models, the varying speed of light (VSL) models have been argued to be an alternative way (compared to inflation) to solve the standard cosmological problems [6, 7, 8, 9, 10] and a recent study on black holes suggested [11] that a variation of the speed of light can be discriminated from a variation of the elementary charge (see however Ref. [12] for a clear refutation of this claim).

It is however well known that only the variation of dimensionless quantities can be determined [2, 15], mainly because measuring a physical quantity reduces to comparison with a physical system that is chosen as reference. Clearly, the values of the standard units depend on historical definitions and the numerical values of the constants of nature are somehow dependent on these choices. What is independent of the definition of the units is dimensionless ratios that e.g. characterize the relative size, strength etc. of two objects or forces (see Section II.B of Ref. [2] for a detailed discussion of metrology and measurements and the link with the constants of nature). However, while only variation of dimensionless constants is meaningful, one can implement a theory in which such dimensionless constants are varying by assuming that a dimensional constant is varying while specifying clearly what other quantities are being kept fixed. An example is Dirac theory [13] in which the gravitational constant varies as the inverse of the cosmic time but in atomic units, so that in particular the electron mass, $m_e$, is fixed. It follows that this corresponds to a theory in which the dimensionless quantity $Gm_e^2/\hbar c$ is varying. This theory can be considered as resulting from atomic clocks varying relative to gravitational clocks [14].

Before we turn to the specific case of the speed of light, let us recall that the role and status of the fundamental constants of physics have been widely debated. Here we shall define these constants as all the physical parameters that are not determined by our theory at hand. They are fundamental in the sense put forward by Steven Weinberg [16]:

*Electronic address: ellis@maths.uct.ac.za
†Electronic address: uzan@iap.fr
“we cannot calculate [them] with precision in terms of more fundamental constants, not just because the computation is too complicated [...] but because we do not know anything more fundamental”. The constants that can be included in such a list can be split (in a non-unique way) into two sets: a set of dimensionless ratios (that can be called fundamental parameters and are pure numbers) and a list of dimensional constants (that can be called, following a proposal by Okun [15, 16], fundamental units). How many such fundamental units are needed is still debated. To build on this debate (see Ref. [15] for different views), let us recall a property of the fundamental units of physics that seems central to us: each of these constants has acted as a “concept synthesizer” [17, 18], i.e. it unified concepts that were previously disconnected into a new concept. This for instance happens in the case of the Planck constant and the relation \( E = \hbar \omega \), that can be interpreted not as a link between two classical concepts (energy and pulsation, or in fact matter and wave) but rather as creating a new concept with broader scope, of which energy and pulsation are just two facets. The speed of light also played such a synthesizing role by leading to the concept of space-time, as well as (with Newton’s constant) creating the link, through the Einstein equations, between spacetime geometry and matter (see Refs. [17, 18] for further discussion). These considerations, as well as facts on the number of independent fundamental parameters and are pure numbers) and a list of dimensional constants (that can be called, following a adapted natural units system, but their “concept synthesizer” role remains. This has the consequence that these constants relate different concepts and thus play an intricate role in physical laws. If one were to relax their being constants then one would also need to relax the synthesis they underpinned. It follows that one would need to develop a careful conceptual framework to implement their possible variation. Our goal is to discuss, within this understanding, a class of theories that have recently received a lot of attention: the varying speed of light theories.

To examine the role of the speed of light, \( c \), in physics, we start as a warm-up by recalling briefly its biography in Section II.A (further details and references can be found in Ref. [18]) and then try, in Section II.B to describe the various facets of the speed of light in physical theory. In Section II. we recall some well-known results concerning scalar-tensor theories of gravity in order to emphasize the importance of writing and varying a Lagrangian in a consistent way. In Sections IV and V we then discuss, in the light of the previous facts, some theoretical frameworks in which the speed of light is supposed to vary, finding a number of problems in their implementation. We conclude in Section VI.

II. WHAT IS THE SPEED OF LIGHT?

A. A short biography of \( c \)

During Antiquity, it was believed that our eyes were at the origin of light and that its speed was infinite. One had to wait for Galileo for this view to start changing. He was the first to try to measure the speed of light experimentally, but his experiment was unsuccessful. Ironically, by discovering the satellites of Jupiter in 1610, he opened the door to the first determination of the speed of light by the Danish astronomer Ole Roemer in 1676. This measurement was then sharpened by James Bradley in 1728 who utilized his discovery of the aberration of light.

During this period the status of the speed of light was somehow not different from that of the speed of sound: it was simply a property of light. Huygens proposed a description in terms of waves, contrary to Newton’s corpuscular description. This wave description was backed up experimentally by Foucault in 1850 who checked that the speed of light was smaller in a refractive medium than in vacuum. Quite naturally, this lead almost all physicists to believe that light requires a medium to propagate in, which was named ether. The speed of light was thus a property of the ether itself, in the same way as the speed of sound can be computed in terms of the temperature, pressure, and properties of the gas it propagates in. Clearly, it was not thought to be fundamental.

In 1855, Kirchhoff realized that \( (\varepsilon_0 \mu_0)^{-1/2} \) has the dimension of a speed, where \( \varepsilon_0 \) and \( \mu_0 \) are two constants entering the laws of electricity and magnetism. Weber and Kohlrausch measured this constant in 1856, using only electrostatic and magnetostatic experiments. Within the experimental accuracy, it agreed with the speed of light. This remained a coincidence until Maxwell formulated his theory of electromagnetism in 1865 where he concluded that “light is an electromagnetic disturbance propagated through the field according to electromagnetic laws”. At that stage, the status of \( c \) increased tremendously since it became a characteristics of all electromagnetic phenomena. Note that it is not only related to a velocity of propagation since it can be measured by some electrostatic and magnetostatic experiments.

The next mutation of \( c \) arose from the incompatibility of Maxwell’s equations with Galilean invariance: either one sticks to Galilean invariance and Maxwell’s equations only hold in a preferred frame, so that measurements of the velocity of light should allow one to determine this preferred frame, or one has to improve on Galilean invariance. Michelson–Morley experiments, under Einstein’s interpretation, implied one should follow the second road: Galilean
invariance was replaced by Lorentz invariance, and $c$ triggered the (special) relativity revolution. It was becoming the link between space and time and so entered most of the laws of physics, mainly because it enters the notion of causality. Thus for example it became later apparent that it also is the speed of propagation of gravitational waves, and indeed that of any massless particle.

The last, but not least, change happened in 1983 and was driven by experimental needs. The accuracy of the experimental determination of $c$ was limited by the accuracy and reproducibility of the realization of the meter. The BIPM thus decided to redefine this unit in terms of a value of $c$ that was fixed by law. This mirrored J.L. Synge’s emphasis that the theoretically best way to measure distance on macro-scales is by radar or equivalent methods dependent on the speed of light, so that the best units for distance were light-seconds or light-years; and indeed that realization was then turned into technological reality through surveying instruments such as the Tellurometer and more recent developments such as the GPS system widely used for navigation.

We see from this summary that the speed of light moved from a simple property of light to the role of a fundamental constant that enters many laws of physics that are a priori disconnected from the notion of light itself.

B. One constant with many facets

It follows that the speed of light is a complex quantity that turns out to have different origins that lead to coincident values. It may be of some use to try to distinguish its different facets. As the previous section tends to show, it seems clear that $c$ is not only the speed of light and more interestingly is probably not even always the speed of light.

1. $c_{\text{EM}}$: the electromagnetism constant

To start with, let us remember the Maxwell equations in MKSA units

$\nabla \cdot \mathbf{D} = \rho, \quad \nabla \cdot \mathbf{H} = 0, \quad \nabla \times \mathbf{H} = \mathbf{J} + \partial_t \mathbf{D}, \quad \nabla \times \mathbf{E} = -\partial_t \mathbf{B}$, \hfill (1)

where $\rho$ is the density of free charges and $\mathbf{J}$ the current density. The displacement, $\mathbf{D}$, is related to the electric field, $\mathbf{E}$, and the polarization, $\mathbf{P}$, by $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$ while the magnetic field, $\mathbf{H}$, is related to the magnetic induction (magnetic flux density), $\mathbf{B}$, and the magnetization, $\mathbf{M}$, by $\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})$, $\varepsilon_0$ and $\mu_0$ being respectively the permittivity and permeability of the vacuum. Introduce the potential by the standard definition $\mathbf{E} = -\nabla \phi - \partial_t \mathbf{A}$ and $\mathbf{B} = \nabla \times \mathbf{A}$ (which is unique up to a gauge transformation). In vacuum, it is an easy exercise to show that the wave equation for an electromagnetic wave is

$$(\partial_t^2 - c_{\text{EM}}^2 \Delta) (\phi, \mathbf{A}) = 0$$ \hfill (2)

where $\Delta$ is the Laplacian and with

$$c_{\text{EM}}^2 \equiv \frac{1}{\varepsilon_0 \mu_0}. \hfill (3)$$

Here $c_{\text{EM}}$ appears as the velocity of any electromagnetic wave, and thus of light, in vacuum.

Calling $c_{\text{EM}}$ the speed of light is somewhat too restrictive even at that level since it is characteristic of any electromagnetic phenomena. It should be referred to as the electromagnetism constant.

Formally, by setting $x^0 = c_{\text{EM}} t$, the Maxwell equations can be recast in the form

$$\partial_\mu F^{\mu \nu} = j^\nu \quad \text{with} \quad A^\mu = (\phi, \mathbf{A}/c_{\text{EM}}), \quad j^\mu = (\rho, \mathbf{J}/c_{\text{EM}}), \quad F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \hfill (4)$$

with $\mu, \nu = 0..3$. Imposing the Lorentz gauge ($\partial_\mu A^\mu = 0$) we recover the propagation equation for $A^\mu$. Indeed, here the wave equation constant is embodied in the notation since $x^0$ is defined in terms of $c_{\text{EM}}$. Of course in a relativistic framework, one would get the same equations since Maxwell electrodynamics is Lorentz invariant by construction.

2. $c_{\text{ST}}$: the spacetime constant

The next role of the speed of light that we alluded to, is the synthesizer between space and time. This constant enters the Lorentz transformation and the spacetime description of special relativity. There are many ways to derive the Lorentz transformations (see e.g. Ref. 22 for an extensive discussion).
Following the proposal of Ref. [22], let us call the constant that appears in the Lorentz transformations the spacetime structure constant, $c_{\text{ST}}$, so that the Minkowski line element takes the form

$$ds^2 = -(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2$$

(5)

$$= -(c_{\text{ST}}dt)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2. $$

(6)

It can be shown that this constant can be defined (completely independently of electromagnetism) as the universal invariant limit speed $c_{\text{ST}}$ so that speeds are not additive, even though they may be approximately so at low speeds.

The conditions that the composition of speed (denoted $\oplus$) satisfies are

(i) that it has an identity element, $O$ (i.e., $O \oplus u = u \oplus O = u$ for all $u$),

(ii) that there is a universal element, in our case $c_{\text{ST}}$, such that $c_{\text{ST}} \oplus u = u \oplus c_{\text{ST}} = u$ for all $u$,

(iii) the associative rule $u \oplus (v \oplus w) = (u \oplus v) \oplus w$,

(iv) that the differentials $d(u \oplus v)/du$ and $d(u \oplus v)/dv$ should exist and be continuous in $u$ and $v$, and

(v) that $d(u \oplus v)/du > 0$ and $d(u \oplus v)/dv > 0$ provided $u, v \neq 0$.

These lead to the standard velocity combination rules of Special Relativity with $c_{\text{ST}}$ as the limiting speed; see Ref. [22] for details and other ways to derive the Lorentz transformations (interestingly most of these constructions are based on axioms that are universal principles, and are not determined by the properties of any specific interaction of nature).

The limiting speed is then given by the metric (4) through the equation $ds^2 = 0$, corresponding to motion at the speed $c_{\text{ST}}$.

Finally let us stress that the special relativity conversion factor between energy and mass is given by $E = mc_{\text{ST}}^2$.

Let us now come to the case of the curved spacetime of general relativity and the corresponding measurement of distances, and consider for that purpose a space-time with line element $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$. Any observer can define their proper time by the relation

$$ds^2 = -c_{\text{ST}}^2d\tau^2,$$

(7)

which is the most natural choice since this implies the limiting speed given by $ds^2 = 0$ is the same for all observers independently of their position in space and time and of their motion; it also gives the standard time dilation effect as experimentally measured for example in the life times of cosmic-ray decay products. Clearly, we have that

$$-c_{\text{ST}}^2d\tau^2 = g_{00}(dx^0)^2$$

(8)

for any observer at rest relative to the coordinate system $(x^\alpha)$. In the standard case (where $c_{\text{EM}} = c_{\text{ST}}$), we can define the spatial distance, $d\ell$, between two points with coordinates $x^i$ and $x^i + dx^i$ as the radar distance given by $c_{\text{ST}}/2$ times the proper time measured by an observer located at $x^i$ for a signal to go out and come back between $x^i$ and $x^i + dx^i$. It will follow that (see e.g. Ref. [23])

$$d\ell^2 = \gamma_{ij}dx^i dx^j \quad \text{with} \quad \gamma_{ij} = g_{ij} - \frac{g_{0i}g_{0j}}{g_{00}},$$

(9)

with $i, j = 1, 2, 3$. Thus the determination of distance requires both the measurement of time and the use of a signal exchanged between the distant points. In the standard case, there is no ambiguity because one can use light that propagates at the universal speed ($c_{\text{ST}} = c_{\text{EM}}$). In a case where these velocities do not coincide the determination of distance will become more involved because light will follow a timelike (and not a null) geodesic of the metric. Furthermore, in any situation where this standard matter-geometry coupling does not hold, we need to be told what geometric meaning, if any, the metric tensor has, and by what alternative method times and spatial distances are to be determined.

3. Agreement of $c_{\text{EM}}$ and $c_{\text{ST}}$

The historical path was from electrodynamics to the demonstration that the speed of light was constant (Michelson-Morley experiments) to the Lorentz transformation and the group structure of spacetime. Then it was realized from the study of relativistic dynamics that any particle with vanishing mass will propagate with the speed of light. But clearly, the speed of light $c_{\text{EM}}$ agrees with the universal speed, $c_{\text{ST}}$, only to within the experimental precision of Michelson-Morley type experiments (or put differently, the photon has zero mass only within some accuracy) and the causal cone need not coincide with the light cone. If one were to prove experimentally that the photon is massive then the standard derivation of relative from electromagnetism would have to be abandoned.
It is thus clear that one can keep the basic principles of a metric theory of gravity and obtain a variable speed of light \(c_{\text{EM}}\) by modifying Maxwell equations [24]. Many ways can be followed and many terms can be added to the electromagnetic Lagrangian. In the Proca theory, one modifies Maxwell equation to get a massive spin-1 state
\[
\partial_\mu F^{\mu\nu} + m^2 A^\nu = 0. \tag{10}
\]
It follows that the Lorentz gauge condition is automatically satisfied if the photon is massive but we have lost the freedom of gauge transformation. Teyssandier [25] considered terms that do not violate gauge invariance and couple to the curvature as
\[
L_{\text{EM}} = \frac{1}{4}(1 + \xi R)F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\eta R_{\mu\nu}F^{\mu\nu}F_{\rho\sigma} + \frac{1}{4}\zeta R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}, \tag{11}
\]
where \(R\) is the Ricci scalar. From this he concluded that
\[
c_{\text{EM}} = 1 + \frac{(\eta + \zeta)(4U - R)}{3 - (3\xi + \eta)R - 2(\eta + \zeta)U} \tag{12}
\]
when the Ricci tensor takes the form \(3R_{\mu\nu} = (4U - R)u_\mu u_\nu + (U - R)g_{\mu\nu}, u^\mu\) being a unit timelike vector field. Let us also emphasize that in quantum electrodynamics, the photon gets an effective mass due to vacuum polarization [26] usually described by adding the Euler-Heisenberg Lagrangian [27] to the standard electromagnetic Lagrangian.

As we see, on the one hand, if the universal speed is fixed by electromagnetism it is difficult to understand why other interactions are locally Lorentz invariant, and on the other hand, the invariance of the laws of physics under the Poincaré group allows the existence of massless particles but does not imply that every gauge boson must be massless. It is therefore important conceptually to carefully distinguish the speed of light \(c_{\text{EM}}\) from the universal speed \(c_{\text{ST}}\) that dictates the properties of spacetime.

4. \(c_{\text{GW}}\): the speed of gravitational waves in vacuum

As long as we are in a vacuum, the Einstein equations derived from the action
\[
S = \int R\sqrt{-g}d^4x \tag{13}
\]
can be linearized around Minkowski spacetime. It can be shown that the only degrees of freedom are massless states of spin 2 that are gravitational waves. With the line element
\[
ds^2 = -(c_{\text{ST}}^2 dt)^2 + (\eta_{ij} + h_{ij})dx^i dx^j, \quad h_{ij}\eta^{ij} = 0, \quad \partial_i h^{ij} = 0, \tag{14}
\]
the linearised Einstein vacuum field equations reduce to the propagation equation
\[
(\partial_t^2 - c_{\text{ST}}^2 \Delta) h_{ij} = 0 \tag{15}
\]
so that the speed of propagation of these gravitational waves is given by the universal speed of our spacetime, \(c_{\text{GW}} = c_{\text{ST}}\). As long as we assume that General Relativity is valid this conclusion cannot be avoided but, were we able to formulate a theory with light massive gravitons in their spectra (see e.g. Ref. [29] for some attempts in the braneworld context) then the speed of propagation of gravity might be different from the universal speed and would lead to modification of the Newton law of gravitation on astrophysical scales [30]. Let us emphasize that there are dangerous issues associated with the presence of extra-polarization states of massive gravitons [31]. Among other problems [31], these massive gravitons have a scalar polarization state whose coupling to matter does not depend on the mass of the graviton and that is, at the linear level, analogous to a Jordan-Brans-Dicke coupling with \(\omega = 0\). That coupling will modify the standard relation of interaction between matter and light by a factor 3/4 so that one will expect a 25% discrepancy in the tests of gravitation in the Solar System which are actually at a level of 10^{-5} [32].

According to the previous discussion, it seems difficult to consider a speed of gravity that differs from \(c_{\text{ST}}\), nevertheless, it may be good to keep an open mind and not forget that these two speeds may differ.

5. \(c_{E}\): the (Einstein) spacetime-matter constant

Let us now consider gravity coupled to matter. The Einstein field equations
\[
G_{\mu\nu} = \frac{\alpha}{2} T_{\mu\nu} \tag{16}
\]
involve a coupling constant $\alpha/2 = 8\pi G/c^4$ between the Einstein tensor $G_{\mu\nu}$ and the stress-energy tensor $T_{\mu\nu}$, where $c$ has the dimensions of speed; let us call it $c_e$. To interpret this constant, one needs to consider the weak field limit of the field equations (10) in which the spacetime metric is given by

$$ds^2 = -(1 + h_{00})(c_{ST} dt)^2 + \eta_{ij} dx^i dx^j.$$  \hspace{1cm} (17)

The geodesic equation of a massive particle ($u^\mu \partial_\mu u^\nu = 0$ with $u^\mu \equiv dx^\mu/ds = (1, v/c_{ST})$) then reduces to

$$\partial_t v^i = -c_{ST}^2 \eta^{ij} \partial_i h_{00}/2.$$  \hspace{1cm} (18)

To get the Newtonian law, one needs to identify the metric perturbation $h_{00}$ with the Newtonian gravitational potential, $\phi$, giving

$$h_{00} = 2\phi/c_{ST}^2.$$  \hspace{1cm} (19)

The second step is to compare the Einstein equation to the Poisson equation. Since $R_{\mu0} \sim \partial_i \Gamma_{00}^i \sim -g_{ij} \partial_j g_{00}$ we get that

$$2 \Delta \phi/c_{ST}^2 = \alpha \left(T_{00} - \frac{1}{2} g_{00} T\right)/2.$$  \hspace{1cm} (20)

For a fluid, $T_{00} = \rho c_{ST}^2$ so that we need

$$\alpha = 16\pi G/c_{ST}^4$$  \hspace{1cm} (21)

to get the correct Newtonian limit for gravity. In the context of general relativity, it is thus clear that $c_e = c_{ST}$. It is however important to distinguish these concepts, as will become clearer when discussing the varying speed of light models.

6. Conclusion

If one wants to formulate a theory in which the speed of light is varying, the first step is to specify unambiguously which of the speeds identified here is varying and then to propose a theoretical formulation (i.e. a Lagrangian) to achieve this task. There is no reason why after relaxing the property of constancy of the speed of light, the different facets of $c$ described in this section will still coincide. Besides this, it is important to clearly state which are the varying constants, this may not be the case anymore (see below).

III. INTERLUDE ON SCALAR TENSOR THEORY OF GRAVITY

Before we proceed further with the speed of light, let us make some general comments about scalar-tensor theories of gravity. This section follows the notation and the main results about such theories detailed in Ref. [34], see also Ref. [35]. In such theories, one considers a Lagrangian of the form

$$S = \frac{1}{16\pi G} \int d^4 x \sqrt{-g} \left[ F(\psi) R - Z(\psi) g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi - 2U(\psi) \right] + S_m[\phi_m; g_{\mu\nu}]$$  \hspace{1cm} (22)

where $G$ is the bare gravitational constant, which differs from the measured one, and $R$ is the Ricci scalar of the metric $g_{\mu\nu}$. The function $F$ is dimensionless and needs to be positive for the graviton to carry positive energy. The dynamics of $\psi$ depends on the functions $F$, $Z$ and $U$ but note that $Z$ can always be set equal to 1 by a redefinition of the field $\psi$ so that there remain only two arbitrary functions. The matter action, $S_m[\phi_m; g_{\mu\nu}]$, depends only on the matter fields and the metric. Such a form implies that the weak equivalence principle holds. In models describing varying constants, this may not be the case anymore (see below).

The variation of this action gives

$$F(\psi) G_{\mu\nu} = 8\pi G T_{\mu\nu} + Z(\psi) \left( \partial_\alpha \psi \partial_\beta \psi - \frac{1}{2} g_{\mu\nu} (\partial_\alpha \psi \partial_\beta \psi) \right)$$

$$+ \nabla_\mu \partial_\nu F(\psi) - g_{\mu\nu} \nabla_\alpha \nabla^\alpha F(\psi) - g_{\mu\nu} U(\psi)$$

$$2Z(\psi) \nabla_\alpha \nabla^\alpha \psi = -\frac{dF}{d\psi} R - \frac{dZ}{d\psi} (\partial_\alpha \psi)^2 + 2 \frac{dU}{d\psi},$$  \hspace{1cm} (23)

$$\nabla_\mu T^\mu = 0,$$  \hspace{1cm} (24)

$$\nabla_\mu T^\mu = 0.$$  \hspace{1cm} (25)
where the matter stress-energy tensor is defined as $T^\mu\nu \sqrt{-g} = 2\delta S_m/\delta g_{\mu\nu}$. All the previous equations are written in the Jordan frame. In the action (22), matter is universally coupled to the metric tensor $g_{\mu\nu}$ so that this metric is in fact the one that defines lengths and times as measured by laboratory rods and clocks, with associated speed of light $c_{ex}$ (which we have here set equal to unity by appropriate choice of units). All experimental and observational data have their standard interpretation in this frame.

Let us remember that there exists another interesting frame, the Einstein frame, defined by performing the conformal transformation

$$\bar{g}_{\mu\nu} = F(\psi)g_{\mu\nu}; \quad \left(\frac{d\varphi}{d\psi}\right)^2 = \frac{3}{4} \left(\frac{d\ln F}{d\psi}\right)^2, \quad A(\varphi) = F^{-1/2}, \quad 2\bar{V}(\varphi) = U/F^2$$

so that the action (22) takes the form

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[ \bar{R} - 2\bar{g}^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - 4\bar{V}(\varphi) \right] + S_m[\phi_m; A^2(\varphi)\bar{g}_{\mu\nu}].$$

With this form, the action looks like the action of general relativity but the matter fields are now explicitly coupled to the metric, so that for instance $\bar{T}_{\mu\nu} = A^2(\varphi)T_{\mu\nu}$. The main difference between the two frames arises from the fact that the spin-2 degrees of freedom are perturbations of $\bar{g}^{\mu\nu}$ and that $\varphi$ is a spin-0 scalar. The perturbation of $g^{\mu\nu}$ actually mix the spin-2 and spin-0 excitations.

An easy application is to derive the cosmological equations:

$$3F \left( \dot{H}^2 + \frac{K}{a^2} \right) = 8\pi G\rho + \frac{1}{2}Z\dot{\psi}^2 - 3H\dot{F} + U,$$

$$-2F \left( \dot{H} - \frac{K}{a^2} \right) = 8\pi G(\rho + P) + Z\dot{\psi}^2 + \ddot{F} - HF,$$

$$\dot{\rho} + 3(H(\rho + 3P)) = 0,$$

$$Z \left( \ddot{\psi} + 3H\dot{\psi} \right) = 3F' \left( \dot{H} + 2H^2 + \frac{K}{a^2} \right) - Z\dot{\psi}^2/2 - U'$$

where $F' = dF/d\psi$ and $a(t)$ is the scale factor of the Robertson-Walker metric assumed, $K = \pm 1, 0$ is the curvature index, a dot refers to a derivative with respect to the cosmic time $t$, and $H \equiv \dot{a}/a$, while $\rho$ is the matter density and $P$ its pressure.

In the preceding framework, the universality of free-fall is not violated and all constants but the gravitational constant are constant. It can be further generalized by allowing different couplings in $S_m$, e.g.

$$S_m = \int d^4x \sqrt{-g} \left( \frac{B}{4}(\psi)F^2 + \ldots \right).$$

which will described a theory in which both the gravitational constant and fine structure constant are varying. In that case the field couple differently to different particles so that one expects a violation of the universality of free fall (see also Ref. 37 for a very general scalar-tensor theory in which the speed of electromagnetic waves is computed).

IV. SINGLE-METRIC FORMULATION OF THE VSL MODELS

Single-metric varying speed of light models have been formulated in different ways. Moffat proposed the idea in 1992 [62] as a solution to the same cosmological puzzles as inflation, and somewhat different versions have more recently been widely popularized under the claim that they were able to solve these problems [7,8,9]. Here we take a fresh look at the latter papers, taking into account what we learned from the former sections.

Briefly, the claim in [7,8,9] to solving the standard cosmological problems was built on the assumption that the Friedmann equations remain valid even when $\dot{c} \neq 0$, so that

$$\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho - \frac{Kc^2}{a^2},$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left( \rho + 3\frac{P}{c^2} \right).$$
Usually, due to the Bianchi identity, one can deduce from these two equations the equation of energy conservation. In this new setting, since $\dot{c} \neq 0$, this becomes

$$\dot{\rho} + 3H \left( \rho + \frac{P}{c^2} \right) = \frac{3Kc^2}{4\pi Ga^2} \dot{c}.$$ \hspace{1cm} (35)

At this stage, let us say that the set of equations (33-35) are a consistent set of equations. But these equations are just postulated and are not sufficient to define clearly the theory they derive from. In particular Eqs. (33-35) do not provide an equation for the evolution of $c$. Let us recall, as Jordan [37] first pointed out, that it is usually not consistent to allow a constant to vary in an equation that has been derived from a variational principle under the hypothesis of this quantity being constant. He stressed that one needs to go back to the Lagrangian and derive new equations after having replaced the constant by a dynamical field.

A. The VSL variational principle

It was argued [7, 8, 9, 10] that Eqs. (33-35) can be derived from an action involving a scalar field $\psi \equiv c^4$ given by

$$S = \int d^4x \left[ \sqrt{-g} \left( \frac{\psi(x^4)}{16\pi G} (R + 2\Lambda) + \mathcal{L}_M \right) + \mathcal{L}_\psi \right]$$ \hspace{1cm} (36)

where $\Lambda$ is the cosmological constant. $\mathcal{L}_M$ is the matter Lagrangian and $\mathcal{L}_\psi$ controls the dynamics of $\psi$. The authors of Refs. [7, 8, 9, 10] explain that the Riemann tensor (and Ricci scalar) is to be computed in one frame (or at constant $\psi$, as in the usual derivation); additional terms in $\partial_\mu \psi$ must be present in other frames. According to this interpretation of the variational principle, it is argued that the field equations resulting from (36) are

$$G_{\mu\nu} - g_{\mu\nu}\Lambda = \frac{8\pi G}{\psi} T_{\mu\nu}.$$ \hspace{1cm} (37)

However without any additional specification, the action (36) is nothing but a scalar-tensor theory with $F = \psi$ (cf. the discussion in the previous section) and should be varied in the usual way, i.e. using the standard variational methods as given e.g. in [48] (p.84) or [33] (p.300). Note that the resulting Euler-Lagrange equations are the conditions for the action to be stationary, and the proof that this is so is independent of the frame chosen for the calculation. This standard variational principle leads to the results recalled in Section III. Setting $Z = U = 0$ in those equations to match the VSL equations as closely as possible gives

$$\psi G_{\mu\nu} = 8\pi G T_{\mu\nu} + \nabla_\mu \partial_\nu \psi - g_{\mu\nu}(g^{\alpha\beta} \nabla_\alpha \nabla_\beta)\psi.$$ \hspace{1cm} (38)

Furthermore, as can be now easily seen, Eq. (37) does not agree with the Lagrangian (36) varied in the standard way. To be specific about this: equations (37) do not result from the action given by (36) being stationary under arbitrary variations when $\psi$ is allowed to be a space-time function. Indeed, the cosmological equations resulting from this action (and with $c_{ST}$ set equal to 1) take the form

$$H^2 = \frac{8\pi G}{3\psi} \rho + \frac{1}{6} Z \frac{\dot{\psi}^2}{\psi^2} - H \frac{\dot{\psi}}{\psi} + \frac{U}{3\psi} - \frac{K}{a^2}.$$ \hspace{1cm} (39)

$$\frac{\dot{a}}{a} = -\frac{4\pi G}{3\psi} (\rho + 3P) - \frac{1}{3} Z \frac{\dot{\psi}^2}{\psi^2} - \frac{H}{2\psi} \frac{\dot{\psi}}{\psi} + \frac{U}{2\psi}.$$ \hspace{1cm} (40)

$$\dot{\rho} + 3H(\rho + 3P) = 0.$$ \hspace{1cm} (41)

$$Z \left( \ddot{\psi} + 3H \dot{\psi} \right) = 3 \left( \dot{H} + 2H^2 + \frac{K}{a^2} \right) - \frac{\dot{Z}}{2} \dot{\psi}^2 - U.$$ \hspace{1cm} (42)

which differ from Eqs. (33-35).

Thus the claimed variational principle used in the VSL models [7, 8, 9, 10], leading from (36) to (37), is not a standard variational principle; how this implication can be deduced is unclear. Let us also add that even in the case in which $c$ is not varying but $G$ is, the equations (3-6) in which $\dot{c} = 0$ of Ref. [10] do not reduce to the scalar-tensor equations, which are the archetype of varying $G$ theories.
B. VSL Field equations

Let us have an open minded attitude and forget about the VSL action \( (36) \), which is useless since it implies a non-standard and non-defined variational principle. Let us assume rather that one can postulate the VSL-Einstein field equation \( (37) \), even if we know this is a dangerous approach, as explained by Jordan [37]. Then the Bianchi identities imply that the stress-energy tensor satisfies the conservation equation

\[
\nabla_\mu T^{\mu\nu} = -T^{\mu\nu} \nabla_\mu \psi. \tag{43}
\]

Postulating the equations \( (37) \) and \( (43) \) drives a set of questions and comments:

1. Can it be demonstrated that these equations can be derived from some kind of Lagrangian varied in the standard way?
2. From the equations \( (37) \) and \( (43) \), one cannot derive a propagation equation for \( \psi \). It is thus impossible to determine the degrees of freedom of the theory just on the basis of this set of equations (is \( \psi \) a spin-0 field or just a function?). This step requires in general writing a Lagrangian (recall the discussion between Jordan and Einstein frames of Section III). In particular, without a Lagrangian, it seems impossible to decide whether the theory is well defined (e.g. has no negative energy etc.).
3. How are we sure that \( T^{\mu\nu} \) in Eq. \( (37) \) is in fact the stress energy tensor? Let us recall that in general, if one has a matter action of the form

\[
S_m = \int \sqrt{-g} L(\phi, \partial_\mu \phi, g_{\mu\nu}) d^4x \tag{44}
\]

that is a scalar, then invariance under any reparameterization \( \xi^\mu \) implies that if the Euler-Lagrange equations are satisfied then

\[
\int \frac{\delta \sqrt{-g} L}{\delta g_{\mu\nu}} \mathcal{L}_\xi g_{\mu\nu} d^4x = 0 \tag{45}
\]

where \( \mathcal{L}_\xi g_{\mu\nu} \) is the Lie derivative of the metric. It follows that, for all \( \xi^\mu \),

\[
\int \xi_\mu \nabla_\rho T^{\mu\rho} d^4x = 0 \quad \text{with the definition} \quad T^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta \sqrt{-g} L}{\delta g_{\mu\nu}} \tag{46}
\]

which yields the stress-tensor conservation equation. Thus, what we call the stress-energy tensor is closely related with the invariance under coordinate transformations. This definition agrees with the weak field limit of what we call energy, etc.

When postulating Eq. \( (37) \), how can one be sure that the \( T^{\mu\nu} \) that appears there agrees with the standard notion of stress-energy tensor? In particular, one can redefine the stress-energy tensor as \( T^{\mu\nu}/\psi \) in order to make \( \psi \) disappear. This new stress-energy tensor will be conserved as usual, and it would seem natural to identify it as the physical stress-energy tensor. What would need to be demonstrated to justify the proposed alternative identification in the VSL papers, in some sense equivalent to measuring energy and momentum in different units than usual, is that \( \psi \) and \( T^{\mu\nu} \) can be measured independently, and that the latter agrees with the usual notion of mass etc.

In conclusion, we see that even if one can always postulate equations, their interpretation is somewhat easier when one has a Lagrangian, but a Lagrangian relates as usual to standard physics only when it is varied in the usual way. Besides, the preceding remarks seem to show that in fact equations \( (37) \) and \( (48) \) describe nothing more than standard general relativity in weird (spacetime dependent) units. Another hint that this may be the case is given by the cosmological equations: Eq. \( (35) \) is the only one to depend on \( \dot{c} \) and then only if \( K \neq 0 \). We know that if \( K = 0 \), the Euclidean space is scale-free, so that a homogeneous change of units will not affect it, while spherical and hyperbolic spaces have a preferred scale set by their curvature that is affected by a change of units.

C. Dynamics of \( \psi \)

Let us come back to the dynamics of \( \psi \). As we said, if it is not dynamical then it is not a true degree of freedom of the theory and can be eliminated.
It was proposed in the papers referred to that its dynamics is driven by the Lagrangian
\[ \mathcal{L}_\psi = -\frac{\omega}{16\pi G \psi^4} \dot{\psi}^2 \] (47)
in the preferred rest-frame. Note that it is important that this is not multiplied by \( \sqrt{-g} \) in the action (36).

Unfortunately, since (as shown above) the standard variational principle does not apply and we were not given the proposed new variational principle, one cannot guess the equation of evolution for \( \psi \) from this Lagrangian so that, at this stage, this new piece of information is useless.

D. Is it possible to formalize the VSL variational principle?

As emphasized in the original works on VSL, it must be emphasized that a spacetime dependent speed of light will imply that local Lorentz invariance is broken. The action (36) seems at first glance Lorentz invariant, but it was argued in the VSL papers that it holds only in a favored frame, backing up our claim that VSL theory uses a non-standard variational principle.

There are other cases in which local Lorentz invariance is broken that are known in physics, e.g. trans-Planckian physics both for black-holes and cosmology. In those cases, it is well known that one can write down a Lagrangian (40) that can be varied in the standard way. This requires the introduction of an auxiliary timelike vector field \( u^\mu \) (with \( g_{\mu\nu}u^\mu u^\nu = -1 \)) that specifies the preferred frame and according to which an observer moving with \( u^\mu \) (i.e. \( dx^\mu / ds = u^\mu \)) can speak of space and time.

Proceeding in this way, the Lagrangian for the scalar field \( \psi \) corresponding to Eq. (47) will take the form
\[ \mathcal{L}_\psi = -\frac{\omega}{16\pi G \psi^4/2} (u^\mu \partial_\mu \psi)^2 \] (48)
(assuming that the dot in Eq. (47) referred to a derivative with respect to the proper time of an observer moving with \( u^\mu \)). Then, to express the Einstein Lagrangian in term of \( u^\mu \), one would need to use the Gauss equation and introduce the metric of the 3-space as defined by \( u^\mu \)
\[ \gamma_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu. \] (49)

We will not go further, since it will imply trying to define for instance what we mean by speed of light, causality etc. in this context and many choices are (in principle) possible. In any case, one would get an action of the type
\[ S = \int d^4x \sqrt{-g} \left( \frac{\psi}{16\pi G} (R[\gamma_{\mu\nu}, \psi, u^\mu] + 2\Lambda) + \mathcal{L}_M + \lambda (u^\mu u_\mu + 1) + \mathcal{L}_u \right) \]
\[ - \int d^4x \left[ \frac{\omega}{16\pi G \psi^4/2} (u^\mu \partial_\mu \psi)^2 \right] \] (50)
where \( \lambda \) is a Lagrange multiplier to ensure the normalization of \( u^\mu \) during the variation and \( \mathcal{L}_u \) is the Lagrangian describing the dynamics of \( u^\mu \). Such a Lagrangian is covariant but involves a preferred frame so that it can be a starting point to formulate a varying speed of light theory. Clearly, it is much more complicated and involved than the naive version (36) and will certainly not lead to field equations of the simple form (37).

E. Conclusions

The variational principle used in the VSL theories is clearly non-standard. The field equations postulated seem in fact not to describe anything but general relativity itself (written with strange units), while the action proposed (if varied in the usual way) would correspond to a usual scalar-tensor theory. We stress that it is possible to write variational theories in which local Lorentz invariance is broken, but than this is much more involved that what is presented in Refs. [7, 8, 9, 10].

Some other comments are needed. It seems that before calling a theory ‘varying speed of light’, it would have been necessary to make explicit which speed of light was meant, and indeed it should have been related to both major meanings (\( c_{\text{EM}} \) and \( c_{\text{ST}} \)) via the relevant equations. This was not done; for instance, no study of electrodynamics was presented in these papers. Most of the advertisement for the theory relies on the cosmological equations. It is indeed odd to discuss the cosmology of a claimed varying-c theory that has not been defined properly in relation
to the usual meanings of the speed of light, and related to standard physics of measurement and electromagnetism. Thus, it cannot be taken seriously, according to usual theoretical standards, as a proposal for the speed of light to vary. According to what we explained before, it also seems that the resolution of the e.g. horizon problem arises from the fact that one compared clocks with different dials, rather than providing either a mechanism for the speed $c_{\text{EM}}$ of electromagnetic waves to vary or for the limiting causal speed $c_{\text{ST}}$ to vary (cf. the discussion of Moffat’s bimetric theory below; see also Ref. [49] for earlier arguments).

As to Moffat’s original theory [8], it has a much more sophisticated variational principle than the papers discussed above, but also gives us no reason as to either why the physical speed of light (as determined by Maxwell’s equations) should vary, nor any reason as to why an arbitrarily introduced time coordinate time $t$ should be regarded as having physical meaning, rather than proper time $\tau = \int \sqrt{-g} \, dx^{\mu} dx_{\mu}$ determined by the metric tensor. Indeed no physical characterisation is given whereby the coordinate time $t$ in Moffat’s equations (26) and (46) can be determined by some measurement process.

A last comment. If the speed of light is dynamical and is self-consistently replaced by a dynamical field (or in fact many, relaxing the standard concordance of all the facets of $c$), then these fields will couple to ordinary matter. To discuss the effect of the new forces that will appear and in particular the possible violation of the universality of free fall, one requires to study also the implications of the theory in the Solar System. It was assumed that the preferred frame is the cosmological frame in which the Friedmann equations hold (indeed expansion breaks Lorentz invariance); this does not easily allow one to perform such a Solar System study.

V. OTHER IMPLEMENTATIONS: BIMETRIC THEORIES

A. Bekenstein type theories

Among the other implementations of a varying fine structure constant is a theory of varying electric charge formulated about 20 years ago by Bekenstein [41]. In that case, the speed of light and Planck constant are supposed to be fixed so that one gets an unambiguous varying fine structure constant theory in which a scalar field couples to the electromagnetic Lagrangian. This was rephrased in Ref. [42] after a field redefinition (hence leading to the same theory). In Ref. [43], a covariant form of the VSL theory was proposed, based on the introduction of a field $\chi$ as $c = c_0 \exp \chi$ and assuming that the full matter Lagrangian does not contain $\chi$, leading to an action of the type

$$S = \int d^4x \sqrt{-g} \left( \frac{R}{16\pi G} - \frac{\omega}{2} \partial_{\mu}\chi \partial^{\mu}\chi + {\mathcal{L}}_M e^{b\chi} \right).$$

(51)

Is this theory supposed to be a better version of the one discussed before, or a new theory? What facet of the speed of light is supposed to be varying? For instance, the latter Lagrangian is Lorentz invariant, contrary to all the discussions about the breakdown of this invariance in VSL [7, 8, 9, 10]. As correctly studied in Ref. [44], this theory has different observational implications than the Bekenstein theory [41] or its rephrasing [42]. It seems however exaggerated to conclude that either $c$ or $e$ is varying, at least as long as one has not defined properly what is meant by $c$. What is being examined in these papers, as far as we can judge from the Lagrangians, are different scalar-tensor theories with different kinds of coupling between standard matter and the scalar field. Although this is called a VSL theory, it does not seem to provide any way to solve the horizon problem in cosmology.

B. Bimetric theories

An alternative proposal is bimetric theories. Here we consider the geometric aspects of Moffat’s varying-$c$ theory as set out in [45]. In this paper a ‘bimetric form’ is proposed for the cosmological metric $g_{\mu\nu}$ as follows [equations (18)-(20)]:

$$g_{\mu\nu} = g_{0\mu\nu} + g_{m\mu\nu}$$

(52)

where

$$g_{0\mu\nu} dx^\mu dx^\nu = dt^2 c_0^2 \theta (t_c - t) - R^2(t) \left[ \frac{dr^2}{1 + kr^2} + r^2 d\Omega^2 \right]$$

(53)

$$g_{m\mu\nu} dx^\mu dx^\nu = dt^2 c_m^2 \theta (t - t_c) - R^2(t) \left[ \frac{dr^2}{1 + kr^2} + r^2 d\Omega^2 \right]$$

(54)
Here $d\Omega^2 = d\theta^2 + \sin^2 \theta d\varphi^2$ and $\theta(t)$ is the Heaviside stepfunction: $\theta(t) = 1$ for $t > 0$ and $\theta(t) = 0$ for $t < 0$. It is suggested that this leads to a phase transition in the speed of light at a time $t = t_c$:

$$c(t) = c_0\theta(t_c - t) + c_m\theta(t - t_c).$$

Now the metric (53) is degenerate for $t > t_c$; it has no time dimension then, and similarly, the metric (54) is degenerate for $t < t_c$. The final metric form (52) is a single metric with an apparent jump in the speed of light at time $t = t_c$. However this change does not represent a change in the physical speed of light. In the metric (53), define proper time $\tau$ by $2d\tau^2 = c_0^2 dt^2$ and similarly by $2d\tau^2 = c_m^2 dt^2$ in the metric (54). Then on using the proper time $\tau$ in each metric instead of the arbitrary coordinate time $t$, the metric form (52) will be

$$g_{\mu\nu}dx^\mu dx^\nu = 2d\tau^2 - 2R^2(\tau) \left[ \frac{dt^2}{1 + kr^2} + r^2 d\Omega^2 \right]$$

for all values of $\tau$. There is no jump in the final metric form (52), and indeed $\tau$ is just the standard physically measurable proper time in this continuous metric, up to a factor 2 (needed because the spatial part of the metric will occur there with a factor 2). As usual, the physical speed of light will take the constant value 1 at all times in these physical coordinates. We are given no physical reason to prefer the arbitrary coordinate time $t$ in these equations over proper time $\tau$ as determined in the standard way by the metric tensor.

Indeed, this is not really a bimetric form at all, i.e. there are not two separate metrics in use at each time with separate physical interpretations, rather in (54) we have a single metric written as the sum of two parts that vary in the two coordinate patches. A single coordinate patch can be found that eliminates the coordinate singularity and apparent change in the speed of light at the time $t = t_c$.

By contrast, the theories of Bekenstein [46], Clayton and Moffat [47], and Bassett et al. [48] are genuine bimetric theories: they have one metric governing gravitational phenomena and another governing matter [47] or explicitly electromagnetic effects [48]. Consequently these are much more complex than standard general relativity, needing field equations for both metrics (or equivalently, for one metric and for a vector or scalar that determines the difference between the metrics), hence raising many issues about gravitational lensing and the equivalence principle (see e.g. Ref. [32] for discussions). However this does provide a genuine physical basis for discussing the differences between the various facets of $c$ discussed in this paper.

VI. CONCLUSIONS

To conclude, we have tried to recall that the nature of the speed of light is complex and has many facets. These different facets have to be distinguished if one wants to construct a theory in which it is supposed to vary. In particular if it is the electromagnetic speed that is supposed to vary, we should be shown how Maxwell’s equations are to be changed; if it is the causal speed that is to vary we should be shown how the spacetime metric tensor structure and interpretation is altered.

As we emphasized, letting a constant vary implies replacing it by a dynamical field consistently. Letting it vary in equations derived under the assumption it is a constant leads to incorrect results, as the example of a scalar tensor theory clearly shows. One needs to go back to a Lagrangian that allows one to determine the degrees of freedom of the theory and to check if it is well defined.

Concerning the VSL theories in [5, 10, 11], let us recall that the variational principle used is clearly non-standard and that the field equations that are postulated seem in fact not to describe anything else but general relativity itself in unusual units. The emphasis must be put on what can be measured, and in that respect considering only the variation of dimensionless quantities makes sense.

The possibility that the fundamental constants may vary during the evolution of the universe offers an exceptional window onto higher dimensional theories and is probably linked with the nature of the dark energy that makes the universe accelerate today. Thus the topic is worth investigating. Physics is however about precise words and clear concepts, and it would be a pity to let the present confusion over the nature of varying- $c$ theories spread. It is also a pity that advocates of VSL theories choose to use propaganda methods, using phrases such as “religious fervour” and “risible” [50], rather than addressing the central scientific issues that arise, such as how distances may be measured with high accuracy independently of the speed of light. We hope that the basics recalled in this text will help in this direction.
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