Power-Maxwell holographic superconductors probed by entanglement entropy

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Abstract. We study the behaviors of entanglement entropy (EE) in the Power-Maxwell holographic superconductors. We observe that the EE is a useful tool to study the critical point of the phase transition. The phase transition is less likely with the Power-Maxwell field. In the condensed phase, the EE gets larger as the strength of the Power-Maxwell field and the width increases. Furthermore, the width modifies the EE more significantly than the Power-Maxwell factor.

1. Introduction
One of the unexplained mysteries in modern condensed-matter physics is the core mechanism governing the high temperature superconductors. Based on the anti-de Sitter/conformal field theory (AdS/CFT) duality\cite{1-3}, the authors stated that some properties of high temperature superconductors in d-dimension can be potentially depicted by classical general relativity living in d+1-dimension\cite{4}\cite{5}, which is called as holographic superconductor. Since this novel idea provides a new way to explore the pairing mechanism in some materials that have the high temperatures of the phase transitions, a large number of research have been carried out for calculating the properties of phase transition in multifarious holographic models\cite{6-13}.

On the other side, the EE is considered as a vital tool to measure the degrees of freedom of the strongly coupled systems. The calculation of the EE is found to be difficult except for the 1+1 dimensional case. In the light of the AdS/CFT duality, fortunately, the EE for the strongly coupled systems can be computed by the Ryu-Takayanagi minimal surface\cite{14}\cite{15}. The authors in Ref. \cite{16} found that the EE can be devoted to explore the property of the metal/superconductor phase transition. Taking the higher derivative correction to the electromagnetic action into consideration, we studied the behavior of the EE with Born-Infeld electromagnetic field and found that the slope of the EE at the phase transition point has a jumping behavior, which means the order of the phase transition is second. When the Born-Infeld coupling parameter gets bigger, the EE at the critical point has a non-monotonic behavior\cite{17}. In the present paper, we want to extend our work to the EE with the Power-Maxwell electromagnetic field, and investigate the effect of the Power-Maxwell field on the EE in the strip geometry.

The paper is organized as follows. In section 2, we briefly review the nonlinear Power-Maxwell holographic superconductors in AdS black hole space-time and solve the equations of motion in this physical model. In section 3, we explore the behavior of EE with Power-Maxwell electromagnetic. Section 4 is our conclusion.
2. Power-Maxwell holographic superconductors in AdS black hole space-time

The action for the Power-Maxwell holographic superconductors in four dimensions is [18]

\[ S = \int d^4 x \sqrt{-g} \left[ \frac{1}{16\pi G_4} (R + \frac{6}{L^2}) - \left[ \nabla \psi - iA \psi \right]^2 - m^2 |\psi|^2 - b F^{\|q} \right] \]  

(1)

Where \( G_4 \) is the gravitational constant, \( R \) is the Ricci scalar, and \( L \) is the radius of AdS space-time. \( \psi \) is complex scalar field, \( A \) is the gauge field, and \( m \) is the mass. \( q \) and \( b \) are respectively the power parameter and coupling constant. Taking the full back reaction into consideration, the metric in 4-dimensional spacetime is as follows

\[ ds^2 = \frac{dr^2}{f(r)} - f(r) e^{-\chi(r)} dr^2 + r^2 (dx^2 + dy^2) \]  

(2)

Where \( \chi(r) \) is the back reaction effect and \( f(r) \) is the metric function. The Hawking temperature of background is

\[ T = \frac{e^{-\chi(r)/2}}{4\pi} f'(r_c) \]  

(3)

Where \( r_c \) is the event horizon. The form of matter fields are

\[ A_\mu = \varphi(r), \quad \psi = \psi(r) \]  

(4)

The independent equation of motion can be obtained as

\[ \psi' + \left( \frac{2}{r} + \frac{\dot{\chi}}{f} - \frac{\dot{\varphi}}{f^2} \right) \psi - \frac{e^{2\psi} \varphi^2}{f^2} - m^2 \psi = 0 \]  

(5)

\[ \varphi' + \frac{2}{(2q-1) r} \varphi - \frac{e^{2\psi} \varphi^2 \psi^{(2-2q)}}{b q (2q-1) f} = 0 \]  

(6)

\[ \chi' + r \left( \frac{e^{2\psi} \varphi^2}{f} + \psi^2 \right) = 0 \]  

(7)

\[ f' + \frac{f}{r} - \frac{3r}{L^2} + \frac{e^{2\psi} \varphi^2 \psi^2}{f} + m^2 \psi^2 + f \psi^2 - b (-2)^q e^{\psi q} (2q - 1) \varphi^{2q} = 0 \]  

(8)

In order to get the superconductor solution, we have to take the regularity conditions into consideration. The conditions at \( r_c \) are

\[ f(r_c) = 0, \quad \varphi(r_c) = 0 \]  

(9)

In the asymptotic AdS region, we have

\[ f(r) = 0, \quad \chi(r) = 0, \quad \psi(r) = \frac{\psi_+}{r^{\Delta_+}} + \frac{\psi_-}{r^{\Delta_-}}, \quad \varphi(r) = \mu - \frac{\rho^{1/2q-1}}{r^{3-2q/2q-1}} \]  

(10)

Where \( \rho \) is the charge density and \( \mu \) is the chemical potential in the CFT boundary. The conform dimensions of the operator are

\[ \Delta_\pm = \frac{1}{2} \left[ 3 \pm (9 + 4m^2)^{1/2} \right] \]  

(11)

On the basis of the AdS/CFT duality, \( \phi_+ \) is the dual operator to the \( \psi \). In this paper, \( \psi_+ \) can be identified as the vacuum expectation value of the operator \( \phi_+ \). Using the numerical method, we can get the numerical solution of the above equations (5)-(8) and explore the effect of the coupling factor \( b \) on the scalar operator \( \phi_+ \). Concretely, we set \( L^2 = 1, m^2 = -2 \), and \( q = 1 \).
The scalar operator $\sigma_o$ versus the temperature $T$ is presented in Figure 1. It is shown that the scalar operator begins to condensate as the temperature below the critical temperature, which can be considered as superconductor phase. The value of the scalar operator tends to a constant as the temperature approaches zero, which is consistent with the traditional BCS theory. Interestingly, with the decrease of parameter $b$, that is, with the decrease of the intensity of the Power-Maxwell field, the critical value becomes lower. Namely, the scalar “hair” is more difficult to form with the nonlinear Power-Maxwell field.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{condensation_operator.pdf}
\caption{The condensation of the operator with different factor $b$.}
\end{figure}

3. The EE in Power-Maxwell holographic superconductor model

In terms of the AdS/CFT duality, the EE of CFT’s states living on the boundary can be investigated by the area of the minimal surface in bulk of that spacetime. The EE between the subsystem $A$ and its complement is defined as[14][15]

\[ S_A = \frac{\text{Area}(\gamma_A) - \text{Area}(\gamma_{\text{comp}})}{4G_4} \] (12)

Where $\gamma_A$ is the minimal surface in the bulk with the same boundary of the subsystem $A$. because the subsystem $A$ is arbitrary, we here focus on a geometry with a finite belt width $\ell$ along the $x$ direction. The range of subsystem $A$ is

\[ -\frac{\ell}{2} \leq x \leq \frac{\ell}{2}, \quad -\frac{w}{2} \leq y \leq \frac{w}{2} (w \to \infty) \] (13)

The surface $\gamma_A$ begin from $x = \ell/2$ where $r = 1/\zeta$, then, extends into the bulk until it arrives at $r = r_c$ where $\frac{dx}{dr} \big|_{r=r_c} \to \infty$. Finally, the surface $\gamma_A$ returns back to the AdS boundary $r = 1/\zeta$ where $x = -\ell/2$.

From the formula equation (12) in the paper, the EE is

\[ S_A[x] = \frac{w}{4G_4} \int_{-\ell/2}^{\ell/2} r \sqrt{\frac{r^2(x)^2}{f(r)} + r^2} dx \] (14)

From the equation (14), the equation for the minimal surface is

\[ r^2_c = \frac{\frac{r^2(x)^2}{f(r)} + r^2}{\frac{r^2(x)^2}{f(r)} + r^4} \] (15)
Then, the EE in $z$-coordinates is

$$S_h = \frac{\text{Area}(f_A)}{4G_4} = \frac{w}{2G_4} \left[ -\frac{z^2}{z^4} \left( z^4 - z^4 f(z) \right)^{1/2} + \frac{1}{z^4} \right]$$

(16)

Where $z = r_+/r$. The UV cutoff $1/z$ is taken into consideration and its value will not change. This is due to the fact that term $1/z$ is only sensitive to UV quantities[14][15]. The term $s$ in the equation (16) is physically important. In the following study, we intend to calculate the EE $s$ in the Power-Maxwell holographic superconductor phase transition. In Figure 2, we display the EE versus the temperature for different $b$ in the superconductor phase. The vertical dashed colored lines indicate the critical temperature $T_c$. The solid ones denote the EE in superconductor phases. For a given $b$, the $T_c$ is equal to the results obtained in the section 2. The temperature decreases as the coupling factor $b$ increases. This indicates that the EE is a powerful probe to explore the properties of the phase transition in Power-Maxwell holographic superconductor model. Moreover, the value of EE at the critical point becomes bigger as the coupling factor $b$ gets larger, which means the degrees of freedom at $T_c$ increase as the strength of the coupling parameter $b$ becomes stronger. In the condensed phase, similarly, the lower the parameter $b$ is, the smaller the EE will be.

![Figure 2](image-url)

**Figure 2.** The EE versus the temperature with different factor $b$.

To explore the influence of the width of the geometry on the EE, we present the behavior of EE versus the temperature for varying widths in Figures 3-5. Obviously, the critical temperature is independent of the width of the geometry. As the width increases, the value of the EE in the condensed phase becomes bigger, which is alike to the case of the factor $b$. Compare with the effects of Power-Maxwell factor on the EE in the`` hair`` phase, we note that the width modifies the EE more significantly than the parameter $b$. 
Figure 3. The EE versus the temperature with width $\ell \rho^{1/2} = 0.9$.

Figure 4. The EE versus the temperature with width $\ell \rho^{1/2} = 1.0$. 

b = 0.30 $\ell \rho^{1/2} = 0.9$

b = 0.3 $\ell \rho^{1/2} = 1.0$
4. Conclusion

By using the EE as a probe, we explored the property of the phase transition in the nonlinear Power-Maxwell holographic superconductor model. We found that the EE is a useful tool to explore the critical point of the phase transition. With the increase of the intensity of the nonlinear Power-Maxwell field, the value of the critical temperature becomes lower. Which suggests that the phase transition is less likely with the Power-Maxwell field. In the belt geometry, the EE at the critical temperature increases as the Power-Maxwell parameter becomes bigger. However, the width of geometry dose not effect on the critical temperature and the corresponding EE. In the superconductor phase, the EE gets larger as the Power-Maxwell parameter and width increases. Furthermore, the width modifies the EE more significantly than Power-Maxwell factor.

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