Is the apparent period-doubling in Blazhko stars actually an illusion?

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ABSTRACT

It is known that the light curves of many Blazhko stars exhibit intervals in which successive pulsation maxima alternate between two levels in a way that is characteristic of period-doubling. In addition, hydrodynamical models of these stars have clearly demonstrated period-doubling bifurcations. As a result, it is now generally accepted that these stars do indeed exhibit period-doubling. Here we present strong evidence that this assumption is incorrect. The alternating peak heights likely result from the presence of one or more near-resonant modes which appear in the stellar spectra and are significantly offset from 3/2 times the fundamental frequency. A previous explanation for the presence of these peaks is shown to be inadequate. The phase-slip of the dominant near-resonant peak in RR Lyr is shown to be fully correlated with the parity of the observed alternations, providing further strong evidence that the process is nonresonant and cannot be characterized as period-doubling. The dominant near-resonant peak in V808 Cyg has side-peaks spaced at twice the Blazhko frequency. This indicates that it corresponds to a vibrational mode and also adds strong support to the beating-modes model of the Blazhko effect which can account for the doubled frequency.

Key words: instabilities – stars: oscillations (including pulsations) – stars: variables: RR Lyrae – stars: individual: RR Lyr – stars: individual: V808 Cyg

1 INTRODUCTION

It is well known that some nonlinear dynamical systems can exhibit a period-doubling bifurcation (see, e.g., Strogatz 1994). For parameter values below the bifurcation point, the system is oscillating periodically at frequency $f$ and may also have spectral peaks at the harmonics of this frequency. In passing the bifurcation point, a small peak at $f/2$ emerges and grows in size. The oscillations exhibited by the system alternate between two slightly different paths in the phase space, so that the period has suddenly become twice as long as it was originally. Typically this is observed in the successive peak heights of the oscillation which alternate between two slightly different levels. Odd harmonics of $f/2$ typically appear in the spectrum as well. We will refer to these as half-integer frequencies. Often additional period-doublings occur as the parameter is further advanced resulting in the appearance of $f/4$, $f/8$, etc. This cascade typically goes all the way to $f/\infty$ over a finite change of the parameter and beyond this point the dynamics do not repeat and the system is called “chaotic”.

Kolenberg et al. (2010) first reported seeing alternating maxima in the light curves of some Blazhko stars and Szabo et al. (2010) demonstrated period-doubling in a hydrodynamical model of RR Lyr. Since then it has been generally accepted that the observed alternation effect was indeed caused by a period-doubling bifurcation (see, e.g., Buchler & Kollath 2011; Kollath et al. 2011; Guggenberger et al. 2012; Molnar et al. 2012; Kolenberg 2012; Benko et al. 2014; Szabo et al. 2014; Le Borgne et al. 2014). In this paper we show the serious problems with this claim, and present strong evidence to support an alternate explanation, namely that the alternation is caused by the presence of one or more excited modes whose frequencies are close to 3/2 times the fundamental frequency and (in most cases at least) are not in resonance with the fundamental mode. The largest of these peaks have frequencies that can be accurately determined and can be observed over the entire Kepler data set of about four years. Other studies have noticed that these peaks are off resonance, but a seemingly plausible explanation has been offered (Szabo et al. 2014, Section 3.2) and as a result the idea persists that presence of these near-resonant peaks indicates that a period-doubling bifurcation has occurred. This belief is strengthened by the fact that actual period-doubling has been observed in hydro-

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dynamical models [Szabo et al. 2010; Kollath et al. 2011; Smolec et al. 2011], and it is assumed that therefore the same must be occurring in the actual RR Lyrae stars. But as is shown below, these hydrodynamical results deviate quite substantially from the results obtained from actual stellar data and thus they do not prove that period-doubling is occurring in the real stars.

The argument by Szabo et al. (2010) is that the half-integer frequencies appear to be off-resonance because they are effectively modulated, periodically and/or randomly, and that this produces a cluster of side-peaks in the spectrum. It does not explain however, how the central peak can be shifted to one side of the exact half-integer frequency. Even for a completely random modulation the result should be a cluster of peaks that is centered on the base frequency. This is in fact what happens in their simulation of the effect as seen in their Figure 10, where the cluster of peaks in part (b) does line up with the main peak in part (a), and if the image is magnified it can be seen that the main peak in part (a) is in precise alignment with one of the peaks in part (b), and therefore this peak is exactly on-resonance. (Of the two tallest peaks in the cluster, it is the one on the left.)

We will show that the dominant peak near \((3/2)f_0\) in RR Lyr is substantially off-resonance and persists over the entire Kepler data set and is therefore not an artifact of some random or chaotic process. We will also show that the observed half-integer oscillation is not phase locked to the fundamental but is constantly slipping in phase at the rate predicted by the offset of the peak location from exact resonance. We also show that a similar off-resonance peak exists in the spectrum for V808 Cyg, another star which is believed to exhibit a strong period-doubling effect [Szabo et al. 2010]. This peak has side-peaks consistent with modulation at double the Blazhko frequency. One explanation for his strange double oscillation (described in detail below) is that this peak does in fact correspond to a nonresonant mode and that the side peaks result from interactions between this mode and the modes that generate the Blazhko effect as described by the “beating-modes model” Bryant (2015). It thus simultaneously presents evidence against period-doubling and evidence in favor of the beating-modes Blazhko model (since it would appear that other models cannot provide an explanation for the doubled frequency).

2 RESULTS AND DISCUSSION

In Figure 1 we show spectra for RR Lyr. The first four curves from the top down were calculated for groups of three consecutive quarters of Kepler data as indicated in the caption. The bottom curve was calculated using all available Kepler data, quarters 1 through 17, and thus has much higher frequency resolution. Note the consistent pattern in this set of curves with a dominant peak at 2.6645, henceforth called \(f_c\), while the expected location for resonance would be at 2.6664 (marked with a dashed vertical line in the figure). The difference being 0.0181 means that this oscillation will be shifted to one side of the exact half-integer frequency.

The bottom curve with a dominant peak at 2.6645, henceforth called \(f_c\), while the expected location for resonance would be at 2.6664 (marked with a dashed vertical line in the figure). The difference being 0.018 means that this oscillation will be shifted to one side of the exact half-integer frequency.

Figure 1. Spectra for RR Lyr in the vicinity of \((3/2)f_0\) computed from Kepler project KIC 7198959, long cadence corrected flux data using Period04 software. Average (or zero point) flux for this data is about \(1.2222 \times 10^7\). Data was pre-whitened, removing the fundamental and its harmonics through tenth. The upper curve is computed from quarters Q5, Q6 and Q7; it is offset by 12 units vertically for clarity. The second curve uses Q7, Q8 and Q9 with offset 9. The third curve uses Q11, Q12 and Q13 with offset 6. The fourth curve uses Q14, Q15 and Q16 with offset 3. The fifth curve uses all available data: Q1 through Q17 with no offset. Note that Q1 and Q17 are partial quarters and Q3, Q4 and Q10 are not available. The dashed vertical line marks the location of \((3/2)f_0\). Note that none of the five curves have a peak at this location, while they all have one at about 2.6645 which is clearly nonresonant.

We also found this frequency (0.018) in their spectral analysis of the alternation effect (see their Figure 3). We verify this phase slip effect in Table 1 which shows a full correlation in the data for RR Lyr between the high cycles being odd or even and the number of half cycle slips being odd or even. Most, if not all, of the easily visible alternation sequences of length longer than 10 are included in the table. Note that this continuous phase slip means that there is no phase-locking between the fundamental and this other mode and therefore there is no resonance and no period-doubling.

An interesting thing to note in Figure 1 is that the peak \(f_c\) is diminishing in height over time. The amplitude of the Blazhko effect is also decreasing simultaneously. This might be taken as evidence of a possible connection between them. Bryant (2015) has shown that the Blazhko effect may also be an artifact of a near-resonant nonradial mode. So one simple explanation for the correlation could be that the excitation levels of all of these nonradial modes are decreasing.

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Table 1. Correlation between the phase slip of the near resonant mode and the phase of the peak height alternation pattern for RR Lyr. A single point is chosen near the middle of each low-high alternation sequence. It is required to be the point of maximum flux in a “high” cycle. In some cases the sequence may include a few bad or non-alternating points provided the alternation continues with the proper phase. The first point in the table has truncated Barycentric Julian Date 54992.569. Columns: time is the time in days relative to the first point; len is the approximate length in cycles of the alternation sequence; cyc is the cycle number, calculated by multiplying time by \( T_0 (1.76429291) \); par1 is the parity of cyc after rounding to the nearest integer; slip is the phase slip in half cycles, calculated by multiplying time by twice the frequency offset of the near resonant mode (2 \( \times 0.0181 \)) and adding a correction of -0.3 (chosen to make the best overall fit to the entire set); par2 is the parity of slip after rounding to the nearest integer; cor is the correlation of par1 and par2, indicated as “yes” if they match and “no” if they do not.

| time   | len | cyc | par1 | slip | par2 | cor |
|--------|-----|-----|------|------|------|-----|
| 0      | 12  | 0   | even | -0.3 | even | yes |
| 36.843 | 24  | 65.001 | odd  | 1.034 | odd  | yes |
| 65.715 | 16  | 115.941 | even | 2.079 | even | yes |
| 285.658 | 12 | 503.985 | even | 10.041 | even | yes |
| 306.644 | 54 | 541.011 | odd  | 10.801 | odd  | yes |
| 338.931 | 24 | 597.973 | even | 11.909 | even | yes |
| 392.244 | 32 | 692.031 | even | 13.899 | even | yes |
| 424.55 | 30  | 749.03 | odd  | 15.069 | odd  | yes |
| 456.814 | 32 | 805.954 | even | 16.237 | even | yes |
| 540.159 | 18 | 953 | odd  | 19.254 | even | yes |
| 587.789 | 11 | 1037.032 | odd | 20.978 | odd | yes |
| 618.95 | 35 | 1092.009 | even | 22.106 | even | yes |
| 653.504 | 26 | 1152.972 | odd | 23.357 | odd | yes |
| 669.974 | 17 | 1182.03 | even | 23.953 | even | yes |
| 701.136 | 20 | 1237.01 | odd | 25.081 | odd | yes |
| 731.175 | 43 | 1290.007 | even | 26.169 | even | yes |
| 857.722 | 30 | 1479.03 | odd | 33.247 | odd | yes |
| 1036.675 | 20 | 1828.998 | odd | 37.228 | odd | yes |

for some reason, e.g. perhaps a slow increase in turbulence is occurring, causing increased damping of the modes.

The spectrum also shows additional peaks at frequencies that are mixing products of this peak with the fundamental and its harmonics, i.e. frequencies of the form \( f_C + n f_D \) where \( n \) is an integer. These peaks are all near, but not equal to, the half-integer frequencies that are expected for period-doubling. The amplitudes of these peaks are determined, based on all available data, and the results compared in Figure 2 with the corresponding spectrum for period-doubling in a hydrodynamical model of RR Lyr (taken from M8 in Figure 11 in Smolec et al. 2011). Note that their spectrum has the dominant peak at (1/2) \( f_D \) rather than at (3/2) \( f_D \). The ratio of the peak heights (the 3/2 peak height over the 1/2 peak height) changes rather drastically between the two cases. This lack of agreement in the spectrum can be taken as fairly strong evidence that the model is not presenting a correct representation of the dynamics of the actual star. Having the maximum half-integer peak near 3/2 in the observational spectrum is of course consistent with the source for this peak being an excited mode with that frequency. The other half-integer peaks are then nonlinear mixing peaks, which would typically be expected to be smaller in amplitude than the one near 3/2 in agreement with observations. Other observational results also have the maximum at 3/2 rather than 1/2 (Szabo et al. 2010, Figure 8). The Smolec et al. (2011) results also show an exact correlation between the Blazhko phase of their model and the appearance and disappearance of the period-doubling, something that is entirely missing in the actual stellar data. This is of course only relevant if we assume that the Blazhko effect involves actual modulation as opposed to a beating-modes process (Bryant 2013).

In Figure 3 we show spectra for V808 Cyg. The first three curves from the top down were calculated for groups of three consecutive quarters of Kepler data as indicated in the caption. The bottom curve was calculated using all available Kepler data, and thus has much higher frequency resolution. Note the consistent pattern in this set of curves, with a dominant peak at 2.6983, henceforth called \( f_D \), while the expected location for resonance would be at 2.7379 (marked with a dashed vertical line in the figure). This time the dominant peak is found on the opposite side of the resonance compared to Figure 1. An interesting feature of the bottom curve is that the \( f_D \) has split into two peaks: \( f_{D1} = 2.6977 \) and \( f_{D2} = 2.6999 \). One explanation would be that the noisy turbulent environment of the star is causing the amplitude to fluctuate in a way that happens to look like the sum of these two frequencies. Another explanation is that there really are two closely spaced modes that generate these peaks. In favor of the second explanation is the fact that the difference between the peak frequencies is 0.0022 and from this one can determine that the two modes slipped in phase by about 3.2 cycles over the time span of the full data set. This seems like a lot consistent cycles to be accidentally generated by a random process. A second interesting feature is the fact that the pair of peaks appears to have pairs of side peaks. A total of 8 side peaks (four pairs) are visible (although one is barely visible above the noise). The fact that it appears modulated is a strong indication that this pair of peaks does correspond to an actual vibrational mode (or...
axial symmetry, i.e. a mode with indices $l = 1$ and $m = 0$ (see, e.g. Unno et al 1989, Section 4). The dipolar character will cause the radial component of the oscillations in the northern hemisphere to be be 180 degrees out of phase with those in the southern hemisphere. If, at one point in time, this oscillation has a particular phase relationship with the fundamental mode in the northern hemisphere, then one half of a Blazhko period later it have the identical phase relationship with the fundamental mode in the southern hemisphere. So for the mode at $f_D$ these two points in time present an equivalent “environment” for that mode to reside in. As a result, one might expect that the amplitude and frequency of that mode will be the same for any two points separated in time by one half the Blazhko period. Thus if there is any modulatory effect induced by the Blazhko effect on the mode at $f_D$ it will have twice the Blazhko frequency. This would not be the case if the Blazhko effect involved an actual modulation of the fundamental at the Blazhko frequency. One would then expect that any modulation of the mode at $f_D$ would also be at the Blazhko frequency. Thus this unusual side-peak spectrum represents a strong piece of evidence in favor of beating-modes model of the Blazhko effect, as well as strong evidence that a mode at $f_D$ exists and is responsible for the observed peak height alternations in V808 Cyg.

Figure 3. Spectra for V808 Cyg in the vicinity of $(3/2)f_0$ computed from Kepler project KIC 4484128, long cadence corrected flux data using Period04 software. Average (or zero point) flux for this data is about 8.8363 × 10^3. Data was pre-whitened, removing the fundamental and its harmonics through tenth. The upper curve is computed from quarters Q2, Q3 and Q4; it is offset by 15 units vertically for clarity. The second curve uses Q7, Q8 and Q9 with offset 10. The third curve uses Q11, Q12 and Q13 with offset 5. The fourth curve uses all available data: Q1 through Q17 with no offset. Note that Q1 and Q17 are partial quarters and Q6, Q10 and Q14 are not available. The dashed vertical line marks the location of $(3/2)f_0$. Note that none of the five curves have a peak at this location, while they all have one at about 2.6983 which is clearly nonresonant. See text for discussion of the split peak in the bottom curve.

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1 Extension to other indices is possible: for $m = 0$, odd values of $l$ will work since they have the same symmetry about the equator. Nonzero even values of $l$ have the opposite symmetry, but may still work approximately since they do have zones of opposite phase. Modes with nonzero $m$ will work exactly but must be excited in pairs whose $m$ values add to zero.