Abstract: We propose a novel single-electron (SE) circuit with unique information-processing that mimics the behavior of bubble film, i.e., a “bubble-inspired SE circuit.” It is known that the behavior of bubble film can be assumed to solve the shortest Steiner problem. In this study, we focus on the behavior to design a novel SE circuit. For this, there are three important points, we believe, for mimicking the behavior for the circuit: 1) the film gradually shrinking, 2) the film stopping shrinking when reaching points like pillars, and 3) films stopping shrinking when they collide with each other. We here designed and tested our SE circuit. By computer simulation, we confirmed that the designed circuit displayed the three processes correctly as desired. Therefore, the “bubble-inspired SE circuit” we designed has the potential for novel information processing.

Key Words: single-electron circuit, bubble film, natural phenomenon, Steiner problem

1. Introduction

In recent years, the necessity of processing vast amounts of information has increased because of advances in today’s information society. For this, technology for miniaturizing CMOS LSI devices has advanced, improving their information processing performance. In addition, due to this technological innovation (advances in nanotechnology), nano-scale devices (e.g., single-electron devices [1]) have gained focus due to their unique properties, and the design, construction, and fabrication of their unit elements is more possible than ever before. In this study, we focus on single-electron (SE) devices as the targeted devices. Generally, SE devices that can operate using individual electrons by controlling a quantum effect (Coulomb blockade) have tunneling junctions as a main component [2]. It is known that this has many advantages, including extremely low power consumption and high integrability [3]. However, there is a problem in that no optimal way for information-processing has been established, although many applications for SE devices have been proposed up to now [4–6]. As a candidate, we draw inspiration from a natural phenomenon, i.e., behaviors shown in soap bubble films, that can be regarded as a form of information processing [7]. Bubble film forms a special structure caused by the surface tension of the film having minimum energy. From the perspective of engineering, it can...
be assumed that a certain nonlinear problem, that is, the shortest Steiner problem, can be solved by using this property. For example, when two plastic boards are prepared, placed face to face, connected to each other by using a few plastic pillars, immersed in a bubble liquid, and pulled up, the bubble film shrinks depending on where the pillars were placed. It is known that the shrunken form of this film shows the solution to the shortest Steiner problem [8]. In this study, we aim to design a novel information-processing system with SE devices that can solve the shortest Steiner problem by mimicking the behavior of bubble film.

2. Targeted behavior of bubble film
As described above, the behavior of bubble film can be assumed to solve the shortest Steiner problem. Therefore, if an information-processing device were to have special behaviors or functions like bubble film, it could solve the shortest Steiner problem. To develop such a device, mimicking the behavior, i.e., considering how to express the behavior, is very important. For this study, while we have not yet focused on the surface tension of bubble film, we here focus on the behavior of bubble film caused by the surface tension as a first step. That is, we mimic not the surface tension but the behavior of the bubble film as we assume that three behaviors result from the tension. Therefore, we here focus on these three behaviors as follows.

0) (A swollen bubble that is large enough to envelop a targeted area, e.g., two plastic boards and a few pillars are prepared as described above.)
1) The bubble shrinks gradually.
2) When the shrinking bubble film touches a point, e.g., a pillar, then the film stops shrinking at that point.
3) When two or more shrinking films collide or merge with each other, then the film stops shrinking at that point.

Figure 1 shows a sample demonstration of the bubble film behavior. In this case, three pillars, A, B, and C, are prepared. The bubble film finally forms special line segments, i.e., the solution to the shortest Steiner problem, by following the above process from 0 to 3.

3. Single-electron circuit
To design our bubble-inspired SE circuit, an SE oscillator (SEO) [9] and SE memory (SEM) [1] should be prepared, as we considered. In the following sections, both the SEO and the SEM are described. As a first step of this study, thermal noise is omitted. That is, we simulated all SE circuits under a temperature condition of 0 [K]. SE circuits are generally very sensitive to thermal noise. If the circuits are placed under a thermal noise condition, electrons display unexpected behavior that causes errors in operation. As a solution to the thermal noise problem, approaches have been reported. The typical way is to use capacitors and tunneling junctions with a smaller capacitance. In addition, some unique approaches have been reported. For example, it is reported that SE circuits operate correctly even
under the thermal noise condition when the circuits are constructed to mimic stochastic resonance (SR) behavior in neural networks (e.g., [10]). A system with SR behavior can be considered as a certain type of noise-energy-harnessing system, and it will operate correctly under the thermal noise condition. We expect our circuit to operate correctly under the condition when the above ways are applied in the near future. However, we here omitted thermal noise for simplicity.

3.1 Single-electron oscillator

The SEO consists of a bias voltage source $V_d$, a high resistance $R$, and a tunneling junction $C_j$ in series. Figure 2 shows a schematic of the SEO. Because of the contained tunneling junction, the SEO operates as a threshold element. That is, the tunneling junction has a threshold voltage for electrons to tunnel through it. When a $V_d$ that is set to higher than the threshold value is applied to the SEO, the electron tunneling probabilistically occurs. Then, the node voltage $V_n$ changes sharply. An example of the operation of the SEO is shown in Fig. 3. In this case, the $V_d$ was set to the sub-

![Fig. 2. Single-electron oscillator.](image)

![Fig. 3. Example operation of single-electron oscillator ($V_d = 7 \text{ mV}$, $R = 1 \text{ G}\Omega$, $C_j = 10 \text{ aF}$).](image)

![Fig. 4. One-dimensional arrayed single-electron oscillator system.](image)
threshold value of the SEO. When an external trigger voltage was inputted, then $V_n$ exceeded the threshold voltage. After that, electron tunneling occurred, and the $V_n$ changed sharply. From this, it is possible to control the electron tunneling by inputting a trigger or not. Moreover, the voltage change can be propagated by connecting SEOs because changing $V_n$ in an SEO can be a trigger input for the adjacent ones. For example, when SEOs are arrayed and connected one-dimensionally as shown in Fig. 4, the voltage change can propagate in the circuit because of continuous electron tunneling. The result of simulating the circuit in Fig. 4 is shown in Fig. 5. In addition, when the SEOs are arrayed and connected two-dimensionally as shown in Fig. 6, the voltage change can propagate...
in the circuit like a wave because of continuous electron tunneling [9]. The result of simulating the circuit in Fig. 6 is shown in Fig. 7, where the voltage changes of the SEO are shown in gray scale. The color black represents a low voltage, and white represents a high voltage. In the gray scale, the elements to which a minus bias voltage was added are displayed with an inverted node voltage for ease of viewing. As a whole, “high” means a positive voltage and “low” a negative one. This wave-like propagation is caused by the chain reaction of the electron tunneling with probability. We thought that the movement of gradually shrinking bubble film could correspond to the movement of travelling voltage waves on two-dimensional SEOs. To mimic the shrinking behavior of the film, which is one of the most important behaviors to mimic, the movement speed should be constant, as we considered. Therefore, we here used multiple tunneling junctions for each SEO instead of a single $C_j$ as shown in Fig. 8. An example of the operation of an SEO using multiple tunneling junctions is shown in Fig. 9. A circuit in which SEOs that use multiple tunneling junctions are arrayed and connected one-dimensionally is shown in Fig. 10. The result of simulating this circuit is shown in Fig. 11. A circuit in which SEOs that use multiple tunneling junctions are arrayed and connected two-dimensionally is shown in Fig. 12, and the result of simulating this circuit is shown in Fig. 13, where the multiple tunneling junctions show average operation compared with the probabilistic operation of a normal SEO [11]. In an individual tunneling junction, the waiting time randomly appears before electron tunneling when the voltage across the junction goes over the threshold. Therefore, the timing of the electron tunneling in a SEO that consists of a single junction shows randomness when its node
Fig. 9. Example operation of single-electron oscillator using multiple tunneling junctions ($V_d = 9.2 \text{ mV}$, $R = 200 \text{ M}\Omega$, $C_j = 500 \text{ aF}$, number of multiple tunneling junctions in oscillator is 50).

Fig. 10. One-dimensional arrayed single-electron oscillator system using multiple tunneling junctions.

Fig. 11. Example operation of system (Fig. 10) ($V_d = 7.88 \text{ mV}$, $R = 200 \text{ M}\Omega$, $C_j = 200 \text{ aF}$, $C = 2 \text{ aF}$, number of multiple tunneling junctions in oscillator is 20).
Fig. 12. Two-dimensional arrayed single-electron oscillator system using multiple tunneling junctions.

Fig. 13. Example operation of system (Fig. 12) \((V_d = 7.5 \text{ mV}, R = 200 \text{ M}\Omega, C_j = 100 \text{ aF}, C = 2 \text{ aF}, \text{number of multiple tunneling junctions in oscillator is } 10, 30 \times 30 \text{ SEOs are arrayed})\).

Voltage goes over the threshold, and this randomness causes the random movement speed of the wave-like propagation as shown in Fig. 5. In contrast, if multiple tunneling junctions are used for the SEO instead of a single junction (Fig. 8), the waiting times are averaged as a whole. That is, if the multiple tunneling junctions contain \(N\) junctions, the capacitance of the individual junctions is set to \(C_j/N\) compared with the original one. Here, the first electron tunneling occurs in the one \(N\) junction that shows the shortest waiting time among all when the node voltage of the multiple-tunneling-junction SEO goes over the threshold. After that, the second electron tunneling occurs in the next junction, among the \((N-1)\) junctions, that shows the shortest waiting time after the first one. For adjacent SEOs in an arrayed SEO network, the sum of the voltage change caused by the electron tunneling in
some of the junctions, e.g., about 5 junctions if \( N = 20 \), becomes the trigger. Electron tunneling in all of the multiple tunneling junctions is not required for the trigger. The time span caused by the electron tunneling for the trigger shows almost the same (averaged) value. As a result, the movement of the wave-like propagation in the arrayed SEO network had an average speed as shown in Fig. 11. Therefore, a system consisting of SEOs using multiple tunneling junctions can generate smooth waves having a constant propagation speed. The difference in the colors between Figs. 7 and 13 is caused by changes in the node voltage (charging of each tunneling junction after the electron tunneling) of each SEO depending on the RC time constant. However, this difference does not influence the circuit operation. The important point is only the movement of the wave front.

3.2 Single-electron memory

The SEM consists of a bias voltage source \( V_d \), a capacitor \( C_L \), and two tunneling junctions \( C_j \)'s. A schematic of the SEM is shown in Fig. 14. The SEM operates as a threshold element with a hysteresis characteristic as a function of the bias voltage \( V_d \) as shown in Fig. 15. Therefore, it is possible to hold two states of voltage as a memory element. Aiming for the bubble-inspired circuit, we considered that SEOS and the SEMs must be connected, i.e., we used the SEOS to mimic the shrinking behavior of

![Fig. 14. Single-electron memory.](image)

![Fig. 15. Example operation of SEM \((C_L = 4 \text{ aF}, C_j = 40 \text{ aF})\).](image)
the bubble and the SEMs for output. An example circuit that has an SEO and SEM connected by using a coupling capacitor $C$ is shown in Fig. 16. Figure 17 shows example operation of the SEO and SEM connected circuit. The fine line represents the operation of the SEO. The heavy line represents that of the SEM. The bias voltage for the SEM was set to the sub-threshold value on the negative side as indicated by the black line in Fig. 15. In this case, when the electron tunneling occurs in the SEO, the sudden voltage change of the SEO induces the electrons to tunnel in the SEM as a trigger. Then, the electron tunneling that occurred in the SEO is memorized by the connected SEM.

**Fig. 16.** Example construction of connecting SEO with SEM.

**Fig. 17.** Example operation of demonstrated circuit shown in Fig. 16 ($V_{d1} = 5 \text{ mV}, -V_{d2} = -20 \text{ mV}, R = 1 \text{ G}\Omega, C_{j1} = 10 \text{ aF}, C_{j2} = 40 \text{ aF}, C_L = 4 \text{ aF}, C = 2 \text{ aF}$).
4. Design of bubble-inspired single-electron circuit

If the processes (behaviors 1 to 3) as described in Sec. 2 can be mimicked on an electronic device, the device must operate as a “bubble-inspired device.” We here express the bubble film behavior on an SE circuit through a combination of SEO and SEM. To design our bubble-inspired SE circuit, we express the shrinking behavior of the bubble by using a two-dimensional array of SEOs. We considered that processes 0 to 3 as described in Sec. 2 can be expressed by the arrayed SEOs. This is because the traveling voltage wave can correspond to the behavior of shrinking bubble film. Therefore, by setting the trigger on the circle, the changes in voltage gradually propagate to the inside. However, the original arrayed SEOs cannot hold the information of the collision points of waves [9]. Therefore, we here use the SEO and SEM connected circuit as described above. The configuration of the device proposed here has three layers for operation, as shown in Fig. 18. In the first layer, a two-dimensional array of SEOs using multiple-tunneling-junctions is used. The second and third layers are for sensing collision points and for output with SEMs. The second layer senses the collision points of the waves on the first layer. For this, each SEM in the second layer is given a bias voltage of $-V_{d1}$ and placed

![Fig. 18. Bubble-inspired circuit. (a) Schematic of layers and (b) ways of connecting elements.](image-url)
just under (same coordinate) the SEOs of the first layer. The SEMs also connect to eight SEOs that are neighbors of the upper SEOs for the SEMs in the first layer. That is, the SEMs receive eight input signals from eight SEOs. The third layer memorizes the information of the collision points. Here, each SEM in the third layer is given a bias voltage of \(-V_{d2}\) and connects to SEO #9 in the first layer (the upper SEO of the SEM) and the SEM in the second layer (the SEM under the SEO), i.e., the same coordinates as the SEO and the SEM. That is, the SEM receives signals that are inputted from SEO #9 in the first layer and the SEM in the second layer. With these, the collision points (merged bubble films) are displayed on the third layer, which will be the solution to the shortest Steiner problem. Here, for the first layer, \(15 \times 15 + V_d\)-biased SEOs and \(15 \times 15 - V_d\)-biased SEOs were prepared, and they were placed alternately like a checkerboard. For the second layer, \(-V_{d1}\)-biased SEMs and \(15 \times 15 + V_{d1}\)-biased SEMs were prepared. For the third layer, the same as the second layer, \(-V_{d2}\)-biased SEMs and \(15 \times 15 + V_{d2}\)-biased SEMs were prepared. The minus-biased SEMs were connected to the plus-biased SEOs in the first layer, and the plus-biased SEMs were connected to the minus-biased SEOs in the first layer for operation, as shown in Fig. 18. For operation, we set an appropriate threshold value for the SEMs of the second and third layers. Concretely, the SEMs are used as the threshold element. For the second layer, the threshold value is set to 4.5 for the eight inputs. This is because the travelling wave front in the first layer has no more than four SEOs in which the electron tunneling occurs. The SEMs will receive more than five inputs, when more than two waves exist just before collision in the first layer as shown in Fig. 19. With this, our circuit can detect the collision points of waves on the first layer. For the third layer, the threshold is set to 1.5 for two inputs. As a result, the voltage change of the second layer propagates slightly later than the wave of the voltage change of the first layer. When waves collide (merge) in the first layer, electron tunneling occurs at the same-coordinate SEM of the second layer. In this situation, electron tunneling in the third layer does not occur. On wave collision parts, electron tunneling in the SEMs of the second layer occurs in advance before electron tunneling in the SEOs in the first layer. Therefore, electron tunneling does not occur in the SEMs of the third layer. As a result, a difference appears in the node voltage between

![Fig. 19. Bubble-inspired single-electron circuit operation.](image-url)
the overlapping parts and the other parts. That is, the third layer corresponds to the result of the circuit to mimic the behavior of the bubble film.

In this circuit, we used an SEM using four multiple tunneling junctions in the third layer. In our designed circuit, the SEMs in the second layer were placed at the same coordinates as one of the SEOs in first layer, i.e., SEO #9 as shown in Fig. 18. The SEM and SEO #9 must receive input signals from neighboring SEOs #1 to #8 in the first layer at the same time. Also, the SEMs in the

![Image of the circuit]

**Fig. 20.** (a) Node voltage change in 1st layer. (b) Node voltage change in 2nd layer. (c) Node voltage change in 3rd layer \[V_d = 7.9 \text{ mV}, -V_{d1} = -27.3 \text{ mV}, -V_{d2} = -25.7 \text{ mV}, R = 20 \text{ G}\Omega, C = 2.2 \text{ aF}, C_j = 500 \text{ aF}, \text{ number of multiple tunneling junctions in oscillator is 50, 30} \times 30 \text{ elements (SEO}s \text{ for 1st and SEMs for 2nd and 3rd layers) are arrayed}.\]
third layer were placed at the same coordinates as the SEOs in the first layer as described above. When the electron tunneling occurs in multiple SEOs in the first layer at the same moment, there is a case in which the SEMs of both the second layer and the third layer exceed the threshold voltage. In this case, there is a possibility that electron tunneling in the SEMs of the second layer could occur in advance. For this, we cannot get the desired result because electron tunneling in the SEMs of the third layer other than those of the collision parts does not occur. Therefore, we apply multiple tunneling junctions in the third layer. It is highly possible that electron tunneling could occur in the third layer in advance because the tunneling junctions in the third layer have a high possibility of having the minimal waiting time. Therefore, we applied an SEM using four multiple tunneling junction in the third layer.

![Fig. 21.](image)

(a) Node voltage change in 1st layer. (b) Node voltage change in 2nd layer. (c) Node voltage change in 3rd layer. (d) Final simulation result for 3rd layer [$V_d = 7.89$ mV, $-V_{d1} = -26.8$ mV, $-V_{d2} = -25.37$ mV, $R = 20$ GΩ, $C = 2.2$ aF, $C_j = 500$ aF, number of multiple tunneling junctions in oscillator is 50, $30 \times 30$ elements (SEO for 1st and SEM for 2nd and 3rd layers) are arrayed].
To confirm our circuit operation, we tested it by Monte Carlo simulation. As a first test, we added two triggers on only the lower left and upper right of the first layer. Figure 20 shows the simulation result. The node voltage $V_n$ is expressed in gray scale. The color white represents high voltage, and black represents low voltage. In the gray scale, the elements to which a minus bias voltage was added are displayed with an inverted node voltage for ease of viewing. As a whole, “high” means a positive voltage, and “low” means a negative one as same as explained in Sec. 3.

In Fig. 20, when the triggers were added on the lower left and upper right in the first layer, the voltage waves propagated gradually in the first layer. In the second layer, it could be confirmed that the propagation of the waves was delayed compared with the first layer, but electron tunneling occurred in advance on only the collision parts. In the SEMs of the third layer except for the wave-collision parts, electron tunneling occurred immediately after electron tunneling in the SEOs of the first layer occurred. In the SEMs of the wave collision parts of the third layer, electron tunneling did not occur even if electron tunneling in the connected SEOs of the first layer occurred since electron tunneling in the connected SEMs of the second layer occurred in advance. That is, electron tunneling in the SEMs of the third layer did not occur for only the wave collision parts. Therefore, the “bubble-inspired SE circuit” we designed was confirmed to be as desired. As the second test, triggers on the circle parts were added in the first layer. Figure 21 shows the simulation result. Here, the pillars were expressed by turning off the bias voltage in the SEOs in the first layer. That is, the pillars did not respond even if electron tunneling in the neighboring SEOs occurred. Therefore, when the wave reached the pillars, it spread and propagated, bypassing the pillars. The waves that spread met after passing the pillars, i.e., they collided with each other. Then, the second and the third layers detected the collision points. As a result, a network of line segments was created.

In Fig. 21, we confirmed that the final display in the third layer formed a network through all pillars. The obtained network shown in Fig. 21 was not the perfect shortest network so far. However, the total distance among the three pillars was estimated to differ 12% compared with ideal solution [12]. This was not that bad of a result, we thought. Focusing on the first layer, the wave propagation of the first layer did not shrink gradually because the pillars where the bias voltage was turned off were set. By setting the pillars, the node voltage in the connected SEOs changed. The changing of the node voltage caused fluctuations in the connected SEMs in the second and the third layer. This fluctuation was propagated throughout the whole circuit, so the wave did not shrink gradually. Therefore, we think that setting pillars another way and controlling parameters are necessary to improve. If these problems are solved, it would be possible to shrink the wave on the circle gradually. In addition, we believe that it would be possible to obtain networks with other pillar patterns in the near future. For this, we are designing additional and modified circuits. In this study, we have found the possibility of creating a novel device to seek the shortest network through all pillars, although there are some problems that should be overcome.

5. Conclusion

In this study, we aimed to design a unique SE circuit that mimicked the behavior of bubble film. It is known that the behavior of bubble film can be considered to be a certain type of information processing that solves nonlinear problems. Therefore, we considered that mimicking the behavior on an SE circuit would lead to a novel, functional information-processing device that has the ability to solve nonlinear problems. By relating the behavior of bubble film with the behavior of the SE circuit, we designed our bubble-inspired circuit. From a first simulation of our designed bubble-inspired single-electron circuit, we confirmed that wave collision parts appeared as we desired. From a second simulation, we confirmed that the network appeared through all of the pillars, but many errors were output by causing fluctuations throughout the whole circuit due to the setting of the pillars. Although some problems still remain, we confirmed that our “bubble-inspired single-electron circuit” displayed its own information processing, and it is expected to be a candidate for the next-generation of novel information-processing circuits.
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