Particle Behavior in Curved Microchannels: Aspect Ratio Effects

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Abstract. The sorting devices of microfluidic systems operate by both passive and active mechanisms. The microchannels of these systems are often curved to cope with secondary flows, which allow sorting or mixing. Therefore, it is essential to characterize the secondary flows in microfluidic systems. In this study, we investigate the particle behavior in a spiral rectangular microchannel and clarify the effects of the aspect ratio and Dean number De on particle sorting by comparing experimental and numerical results. We fabricated Archimedean spiral microchannel models with the rectangular cross-section of millimeter-scale dimensions. Particle sorting in the microchannels was observed by the particle-tracking velocimetry (PTV) method. In addition, the particle-sorting mechanism was analyzed by the particle-tracking method in three-dimensional numerical simulations. When particle density exceeded water density, good sorting was observed when 20 < De < 40. The numerical particle-sorting behaviors agreed well with the experimental results.

1. Introduction

Recently, the development of a micro-chemical analysis device called the Micro Total Analysis System (μ-TAS) has attracted the interest of chemists and biotechnologists. Researchers have reported the plane vortex [1], secondary flow [2], viscoelastic effect [3, 4], and nonlinear inertial particle movements [5] in the microchannels of various μ-TASs. The working principle is a series of operations such as mixing, reaction, separation, and extraction.

The relationship between the flow path and particles during separation (arrangement) in μ-TAS can be clarified by bending the micro-flow path. In particular, the relationship between the hydraulic and particle diameters is revealed in the cross-section of a curved channel [6]. However, the relationship in channels with different cross-sections is less understood.

This study investigates the microchannel flow in a millimeter-scale model with a rectangular cross-section coiled into an Archimedean spiral. The magnified micro-scale model of the same system was numerically established. By comparing the experimental and numerical values, we investigated the separation factor and secondary flow characteristics of small, low-density particles in channels with different cross-section aspect ratios and particle densities.
2. Experimental Methods

2.1. Curved Channel and Dean Flow

Figure 1 shows Dean vortices in a curved channel. A flow through a curved channel experiences a radially outward centrifugal force and develops two counter-rotating vortices, called Dean vortices, one of each in the upper and lower halves of the channel.

![Figure 1. Dean vortex in a curved channel](image)

The magnitude of these secondary flows is quantified by a dimensionless number called the Dean number $De$ [7]. $De$ and the Reynolds number $Re$ are respectively given by

$$De = Re \frac{d_h}{2R},$$

$$Re = \frac{d_h V_f}{\nu},$$

where $d_h$ [m] is the hydraulic diameter of the channel, $V_f$ [m/s] is the mean flow velocity in the channel axis direction, and $\nu$ [m²/s] is the kinematic viscosity. $R$ [m] is the radius of the average curvature of the center line of the curved channel, calculated as

$$R = \frac{L}{2\pi n},$$

where $L$ [m] is the total length of the center line of the curved channel, and $n$ is the winding number. In a straight channel, $De = 0$, which indicates that no Dean vortices form in the secondary flow. In a curved channel, $De$ depends on $Re$, $R$, and $d_h$, and its magnitude increases with an increase in the size of Dean vortices. This relationship is defined in equation (4) as the ratio of the average Dean vortex velocity $V_{Dean}$ [m/s] to $De$ [8].

$$V_{Dean} = 1.8 \times 10^{-4} De^{1.63}.$$  

The solid particles in the curved flow channel receive a force from the Dean vortex according to the particle characteristics. Depending on the particle characteristics, the Dean vortices generate a drag force ($F_d$) that draws...
particles along their contours and are, thus, displaced toward the inner or outer channel wall. In this study, the particles were arranged by the Dean vortices in the curved channel. This force is called the Drag force \( F_D \); it acts on solid particles and is defined by equation (5) [9]. Here, \( \mu \) is the viscosity [Pa·s] of the working fluid, and \( d_p \) is the particle diameter [m] of the solid particles.

\[
F_D = 3\pi \mu V_{Dean} d_p = 5.4 \times 10^{-4} \pi De^{1.63} d_p .
\]

\( 5 \)

2.2. Experimental Setup

Figures 2 and 3 show the schematic diagrams of the experimental setup and the separation channel device, respectively. The flow channel is made of acrylic resin. If the channel is transparent, we can observe the flow of internally moving particles. The working fluid is dispensed by a microsyringe pump; then, the fluid passes through the channel device and reaches each outlet. The channel device is installed horizontally and is operated with a 293 K water to reduce temperature changes and the effects of image refraction during observation. The dimensions of the experimental curved channel are given in Table 1.

![Figure 2. Diagram of the experimental setup](image1)

![Figure 3. Separation channel device](image2)

| \( \gamma \) [-] | \( a \) [mm] | \( b \) [mm] | \( d_p \) [mm] | \( n \) [-] | \( L \) [mm] | \( R \) [mm] |
|-----------------|------------|------------|------------|----------|------------|------------|
| 0.2             | 2.29       | 1.37       | 3.429      | 2.5      | 353.8      | 22.52      |
| 0.6             | 3.97       | 0.79       | 2.636      |          |            |            |

Here, \( \gamma \) is the aspect ratio of the channel cross-section, and \( \gamma \) is an important dimensionless parameter. \( a \) and \( b \) denote the width and height of the curved channel, respectively. To reduce the flow velocity and pressure change after branching, the sum of the outlet areas must equal the inlet area. Therefore, the outlet width was set to 0.5\( a \), and the outlet height was set to \( b \). The parameter \( \gamma \) is given by
\[ \gamma = \frac{b}{a} . \]  \hspace{1cm} (6)

In the flow channel, the drag force \( F_D \) is caused by the Dean vortex, and the lift force \( F_L \) is defined by equation (7) [10].

\[ F_L = 0.05 \frac{\mu d_p^2 V_{\text{max}}^2}{d_h^2} . \]  \hspace{1cm} (7)

Where \( \mu \) is the density of the working fluid \([\text{kg/m}^3]\), and \( V_{\text{max}} \) is the maximum flow velocity of the working fluid. This lift force considerably depends on the ratio between the solid particle diameter \( d_p \) and the hydraulic diameter \( d_h \). The condition under which the lift is dominant over the drag owing to this ratio is shown by equation (8) [11].

\[ \lambda = \frac{d_p}{d_h} \geq 0.07 . \]  \hspace{1cm} (8)

In this study, separation is performed under the condition that the drag due to the Dean vortex is dominant in each force applied to the solid particles flowing in the channel. Therefore, spherical glass beads with a solid particle diameter of \( d_p = 40 \mu m \) are adopted so that \( \lambda < 0.07 \). The particle density was varied as \( \rho_p = 2000 \text{ kg/m}^3 \) and \( 4200 \text{ kg/m}^3 \) (The Association of Powder Process Industry and Engineering, Japan).

3. Numerical Analysis Methods

To examine the secondary flow characteristics at different aspect ratios and particle densities, we also performed three-dimensional numerical simulations in the SCRYU/Tetra thermos-fluid simulation software (Software Cradle Co., Ltd.) using the particle-tracking method (PTM) to analyze particle sorting. The flow boundary conditions were set as follows. The inlet boundary was assigned a the uniform flow velocity \( V_f \) corresponding to each \( De \). The outlets were set at atmospheric pressure, and the walls were stationary. At the particle boundary conditions, the inlet was given an initial particle velocity \( V_p \) \([\text{m/s}]\). The followability of the fluid \([\text{given by Equation (9)}\] depended on the Stokes number \( St \) given by Equation (10), where \( \mu \) is the fluid viscosity \([\text{Pa s}]\).

\[ V_p = St V_f . \]  \hspace{1cm} (9)

\[ St = \frac{\rho_p d_p^2 V_f}{18 \mu d_h} . \]  \hspace{1cm} (10)

Figure 4 shows the schematic of the numerical analysis model. Here, we modeled only the channel part of the experimental model. The dimension of the curved channel model is given in Table 2. In addition to the experimentally tested aspect ratios \( (\gamma = 0.2 \text{ and } 0.6) \), we selected \( \gamma = 0.4 \).
Figure 4. Analysis of the channel model

Table 2. Dimensions of the curved channel model in the numerical analysis

| $\gamma$ [-] | $a$ [mm] | $b$ [mm] | $d_i$ [mm] | $n$ [-] | $L$ [mm] | $R$ [mm] |
|-------------|---------|---------|-----------|--------|--------|--------|
| 0.2         | 1.325   | 0.265   | 0.442     |        |        |        |
| 0.4         | 0.935   | 0.374   | 0.531     | 2.5    | 117.9  | 7.5    |
| 0.6         | 0.764   | 0.458   | 0.573     |        |        |        |

4. Results and Discussion

4.1. Experimental Results

In this experimental study, we examined the particle sorting in a solid–liquid two-phase flow through curved microchannel cross-sections with different aspect ratios. Figure 5 shows the effect of aspect ratio on the plots of separation factor versus the Dean number. With a change in the movement time of solid particles and the time to reach the outlet with the flow velocity, we adopted the dimensionless time. The separation factor and dimensionless time are respectively given by

$$\eta = \frac{n_1 - n_2}{n_1 + n_2},$$

$$t' = \frac{t V}{L},$$

where $n_1$ and $n_2$ are the numbers of particles in the outer and inner outlets, respectively, and $t$ is the real time.

4.2. Numerical Analysis Results

The aspect ratio and particle density in the numerical analysis were set as described in the methods. The numerically determined separation factors are plotted as a function of the Dean number in Figure 6.

To investigate the causes of these trends, we plotted the secondary flows just before the channel branch (Figure 7). The numerically obtained secondary flows are shown in Table 3. The secondary flows immediately before branching are presented as vector diagrams. The lengths of the vectors indicate the magnitudes of the flow.
Figure 5. Experimentally determined separation factor versus the Dean number in channels with various aspect ratios $\gamma$ and particle densities $(a) \rho_p = 2000 \text{ kg/m}^3$ $(b) \rho_p = 4200 \text{ kg/m}^3$

Figure 6. Numerically determined separation factors versus the Dean number in channels with various aspect ratios and particle densities $(a) \rho_p = 2000 \text{ kg/m}^3$ $(b) \rho_p = 4200 \text{ kg/m}^3$ $(c) \rho_p = 500 \text{ kg/m}^3$ $(d) \rho_p = 3000 \text{ kg/m}^3$
4.3. Discussion

By comparing Figures 5 and 6, we determined that $\eta$–$De$ plots were equivalent at different particle densities ($\rho_p = 2000$ kg/m$^3$ and 4200 kg/m$^3$). Therefore, we could apply Reynolds’ similarity law.

Figure 6 shows that the solid particle density $\rho_p$ was lower than the water density $\rho_w$ ($\rho_p < \rho_w$), and $\eta$ was low for all $\gamma$ and $De$. This behavior is probably explained by the stirring of solid particles in the channel under the influence of the secondary flow. However, when the solid particle density $\rho_p$ exceeded the water density $\rho_w$ ($\rho_p > \rho_w$), $\eta$ was high for each $\gamma$. In the channel with the lowest aspect ratio ($\gamma = 0.2$), $\eta$ increased with an increase in $\rho_p$ for $De < 40$. At both aspect ratios ($\gamma = 0.2$ and 0.4), the solid particles were separated to the outside ($\eta > 0$). In the channel with the largest aspect ratio ($\gamma = 0.6$), $\eta$ was high only at high particle density ($\rho_p = 4200$ kg/m$^3$). The particles were separated outward ($\eta > 0$) at $De < 30$ and inward ($\eta < 0$) at $De > 60$.

As confirmed in Table 3, the secondary flow developed (formed vortices) when $De$ increased at the higher aspect ratio. Therefore, in the secondary flow of $\gamma = 0.2$, the outward component was large, and the component returning to the inner side was undeveloped. Thus, solid particles were transported outward along the flow. However, in the secondary flow of $\gamma = 0.4$, although the outward component was large as in $\gamma = 0.2$, the component returning to the inner side developed with an increase in $De$; thus, the separation factor $\eta$ should increase. In the secondary flow of $\gamma = 0.6$, the component returning to the inside developed at low $De$. Therefore, at particle densities $\rho_p$ below 4200 kg/m$^3$, it is

Table 3. Secondary flows at the cross-section shown in Figure 7

| $\gamma$ | $De = 24$ | $De = 48$ | $De = 80$ |
|----------|------------|------------|------------|
| 0.2      |            |            |            |
| 0.4      |            |            |            |
| 0.6      |            |            |            |

Figure 7. Position of the cross-section to show the secondary flow and its direction in the curved channel.
considered that the solid particles were stirred by the secondary flow in the channel. When \( \rho_p \) increased to 4200 kg/m\(^3\), the solid particles were transported to the outside at \( De < 30 \) and concentrated on the outer bottom surface and reached the outlet. When \( De \) exceeded 60, the solid particles transported to the outside returned to the inside. In this case, the solid particles were thought to be concentrated on the inner bottom surface and reached the outlet because their sedimentation velocity overcame the inward-carrying component that also rolled up from the inner bottom surface of the secondary flow.

5. Conclusions
In experiments and numerical analyses, we examined how particle sorting is influenced by the aspect ratio of a curved microchannel cross-section in a solid–liquid two-phase flow. Good sorting was obtained in the Dean number range \( 20 < De < 40 \) at the small aspect ratio of the microchannel. At high particle density (\( \rho_p = 4200 \) kg/m\(^3\)) and \( De > 60 \), the separation direction (outward or inward) can be controlled by selecting the desired aspect ratio.

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