Zero Zeros After All These (20) Years*

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Abstract

We celebrate two birthdays connected with the radiation zero phenomenon. First, a striking dip in the theoretical angular distributions of radiative weak-boson production was discovered twenty years ago. The key experimental interest is that this will not occur in any deviation from the standard model. Second, the classical training of Stanley Brodsky began sixty years ago, which was instrumental in understanding why theoretical spin-independent radiation zeros appear in almost all Born amplitudes for the radiation of photons and gluons and other massless gauge bosons (but rarely in physical kinematic regions). And there are approximate zeros for massive bosons and “Type II” zeros that can also be studied. We discuss how the difficulties in observing the original Mikaelian-Samuel-Sahdev zero finally may be surmounted next year.

ORIGINAL ZERO

The first radiation zeros were discovered as a consequence of a general investigation 1,2 of the production of electroweak pairs in hadronic collisions

\[ p\bar{p}, pp \rightarrow WW, ZZ, WZ, W\gamma + X \]  

and neutrino reactions

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addressed to very-high-energy cosmic-ray physics. The investigation probed the sensitivity of these reactions to the trilinear gauge boson couplings and, at the present time, useful limits on their deviations from gauge theory predictions have been obtained in $e^+e^-$ and hadron collider experiments [3]. Pronounced dips were found [2] in the angular distributions for the production of $W\gamma$ and $WZ$ in the two-body parton and lepton reactions. Subsequent work [4] by Mikaelian, Samuel, and Sahdev proved that the $W\gamma$ dips were in fact exact zeros at particular angles, which would be ruined by non-gauge couplings. The Wisconsin brain trust [5] followed with a general demonstration of the implied amplitude factorization.

**SIMPLE ZERO**

Since it is so easy to do so, we show how a radiation zero arises in lowest-order radiative charged-scalar fusion

$$\text{scalar 1} + \text{scalar 2} \rightarrow \text{scalar 3} + \text{photon}$$

whose Feynman diagrams lead to the Born amplitude

$$M_{\gamma}^{sc} = \frac{Q_3}{p_3 \cdot q} p_3 \cdot \epsilon - \frac{Q_1}{p_1 \cdot q} p_1 \cdot \epsilon - \frac{Q_2}{p_2 \cdot q} p_2 \cdot \epsilon$$

with charges $Q_i$, four-momenta $p_i$, and photon momentum $q$ and polarization $\epsilon$. (The trilinear scalar coupling is taken to be unity.) Using momentum conservation, we observe that $M_{\gamma}^{sc} = 0$ if all $Q_i/p_i \cdot q$ are the same. This is exactly the same condition found for the $W\gamma$ amplitude, independent of the spins. It leads to a zero when

$$\cos \theta^* = \frac{Q_1 - Q_2}{Q_1 + Q_2}$$

for the center-of-mass angle $\theta^*$ of particle 3 relative to the direction of particle 1 (or between the photon and particle 2). This reduces to $\cos \theta^* = 1/3 \ (-1/3)$ for $u\bar{d} \rightarrow W^+\gamma$ ($d\bar{u} \rightarrow W^-\gamma$).

**ZEROS EVERYWHERE: THEOREMS**

Faced with the vanishing of the above amplitudes, Brodsky asserted that the way to look at these zeros was as the complete destructive interference of classical radiation patterns. Following this lead, and making a long story short (see the longer story with better referencing in [4]), an arbitrary number $n$ of external charged particles was considered and a general set of radiation interference theorems were found [7]. Again considering the emission of a photon with momentum $q$, the tree amplitude approximation vanishes, independent of any particle’s spin $\leq 1$, for common charge-to-light-cone-energy ratios (the radiation null zone),

$$M_{\gamma}(\text{tree}) = 0$$

if $\frac{Q_i}{p_i \cdot q} =$ same, all $i$
where the $i^{th}$ particle has electric charge $Q_i$ and four-momentum $p_i$. All couplings must be prescribed by local gauge theory. We see why it took so long to discover radiation zeros since the first null zone requirement is that all charges must have the same sign. Fractionally charged quarks and weak bosons are needed in order to get three things: Same-sign charges, a process well-approximated by a Born amplitude, and a four-particle reaction so the null zone was simple. While there are zeros associated with any gauge group when the corresponding massless gauge bosons are emitted, in QCD, color charges are averaged or summed over in hadronic reactions. In thinking of the weak bosons themselves, electroweak symmetry is broken and nonzero weak-boson masses ruin radiation interference.

What about other photonic zeros? The zero in electron-electron bremsstrahlung is less interesting. Zeros in electron-quark and quark-antiquark bremsstrahlung require jet identification along with the more complicated phase space \cite{8}. While we shall say more about other tests, we find ourselves returning again and again to the original $W \gamma$ zero.

**ZERO $\neq$ ZERO**

There are various corrections that turn the $W \gamma$ zero into a dip. Theoretically, higher-order (closed-loop) corrections will not vanish in the null zone, since the internal loop momenta cannot be fixed. Structure function effects, higher order QCD corrections, finite $W$ width effects, and photon radiation from the final state lepton line all tend to fill in the dip.

The main complication in the extraction of the $\cos \theta^*$ distribution in $W \gamma$ production, however, originates from the finite resolution of the detector and ambiguities in reconstructing the parton center of mass frame. The ambiguities are associated with the nonobservation of the neutrino arising from $W$ decay. Identifying the missing transverse momentum, $p_T$, with the transverse momentum of the neutrino of a given $W \gamma$ event, the unobservable longitudinal neutrino momentum, $p_L(\nu)$, and thus the parton center of mass frame, can be reconstructed by imposing the constraint that the neutrino and charged lepton four momenta combine to form the $W$ rest mass. The resulting quadratic equation, in general, has two solutions. In the approximation of a zero $W$-decay width, one of the two solutions coincides with the true $p_L(\nu)$. On an event by event basis, however, it is impossible to tell which of the two solutions is the correct one. This ambiguity considerably smears out the dip caused by the amplitude zero. Problems associated with the reconstruction of the parton center of mass frame could be avoided by considering hadronic $W$ decays. The horrendous QCD background, however, renders this channel useless.

**ZERO PROGRESS**

At present there is only a preliminary study by the CDF collaboration of the $\cos \theta^*$ distribution \cite{9} from a partial data sample of the 1992-95 Tevatron run. The event rate is still insufficient to make a statistically significant statement about the existence of the radiation zero. One can at best say that “there is a hint of gauge zero” in Fig. \cite{9}.
FIG. 1. Preliminary CDF results for the $\cos \theta^*$ distribution obtained from a partial data set of the 1992-95 Tevatron run (from Ref. [9]). The points are the data and the open histogram is the sum of the SM prediction and the background. The shaded histogram is the background estimate.

**ZERO HELP**

Instead of trying to reconstruct the parton center of mass frame and measure the $\cos \theta^*$ or the equivalent rapidity distribution in the center of mass frame, one can study rapidity correlations between the observable final state particles in the laboratory frame. Knowledge of the neutrino longitudinal momentum is not required in determining this distribution. Event mis-reconstruction problems originating from the two possible solutions for $p_L(\nu)$ are thus automatically avoided. In $2 \to 2$ reactions differences of rapidities are invariant under boosts. One therefore expects that the double differential distribution of the rapidities, $d^2\sigma/dy(\gamma)dy(W)$, where $y(W)$ and $y(\gamma)$ are the $W$ and photon rapidity, respectively, in the laboratory frame, exhibits a ‘valley,’ signaling the SM amplitude zero [10]. In $W^\pm \gamma$ production, the dominant $W$ helicity is $\lambda_W = \pm 1$ [11], implying that the charged lepton, $\ell = e, \mu$, from $W \to \ell \nu$ tends to be emitted in the direction of the parent $W$, and thus reflects most of its kinematic properties. As a result, the valley signaling the SM radiation zero should manifest itself also in the $d^2\sigma/dy(\gamma)dy(\ell)$ distribution of the photon and lepton rapidities. The theoretical prediction of the $d^2\sigma/dy(\gamma)dy(\ell)$ distribution in the Born approximation for $p\bar{p}$ collisions at 1.8 TeV is shown in Fig. 2 and indeed exhibits a pronounced valley for rapidities satisfying $\Delta y(\gamma, \ell) = y(\gamma) - y(\ell) \approx -0.3$. The location of the valley can be easily understood from the value of $\cos \theta^*$ for which the zero occurs and the average difference between the $W$ rapidity and the rapidity of the $W$ decay lepton [10].

To simulate detector response, transverse momentum cuts of $p_T(\gamma) > 5$ GeV, $p_T(\ell) > 20$ GeV and $p_T(\bar{\ell}) > 20$ GeV, rapidity cuts of $|y(\gamma)| < 3$ and $|y(\ell)| < 3.5$, a cluster transverse mass cut of $m_T(\ell; p_T) > 90$ GeV and a lepton-photon separation cut of $\Delta R(\gamma, \ell) > 0.7$ have been imposed in the Figure. Here, $\Delta R(\gamma, \ell)$ is the separation between the lepton and
FIG. 2. The double differential distribution $d^2\sigma/dy(\gamma)dy(\ell)$ for $p\bar{p} \rightarrow W^+\gamma \rightarrow \ell\nu_\gamma$ at the Tevatron.

The photon in the azimuthal angle-pseudorapidity plane,

$$\Delta R(\gamma, \ell) = \sqrt{\Delta \Phi(\gamma, \ell)^2 + \Delta \eta(\gamma, \ell)^2}. \quad (7)$$

The cluster transverse mass cut suppresses final state photon radiation which tends to obscure the dip caused by the radiation zero. For 10 fb$^{-1}$, a sufficient number of events should be available to map out $d^2\sigma/dy(\gamma)dy(\ell)$ in future Tevatron experiments.

For smaller data sets, the rapidity difference distribution, $d\sigma/d\Delta y(\gamma, \ell)$, is a more useful variable. In the photon lepton rapidity difference distribution, the SM radiation zero leads to a strong dip located at $\Delta y(\gamma, \ell) \approx -0.3$ [10]. The LO and NLO predictions of the SM $\Delta y(\gamma, \ell)$ differential cross section for $p\bar{p} \rightarrow \ell^+\ell^-\gamma$ at the Tevatron are shown in Fig. 3a. Next-to-leading QCD corrections leave a reasonably visible dip.

In $pp$ collisions, the dip signaling the amplitude zero is shifted to $\Delta y(\gamma, \ell) = 0$. Because of the increased $qg$ luminosity, the inclusive QCD corrections are very large for $W\gamma$ production at multi-TeV hadron colliders [12]. At the LHC, they enhance the cross section by a factor $2-3$. The rapidity difference distribution for $W^+\gamma$ production in the SM for $pp$ collisions at $\sqrt{s} = 14$ TeV is shown in Fig. 3b. Here we have imposed the following lepton and photon detection cuts:

$$p_T(\gamma) > 100 \text{ GeV}/c, \quad |\eta(\gamma)| < 2.5, \quad (8)$$
$$p_T(\ell) > 25 \text{ GeV}/c, \quad |\eta(\ell)| < 3, \quad (9)$$
$$\not{p}_T > 50 \text{ GeV}/c, \quad \Delta R(\gamma, \ell) > 0.7. \quad (10)$$

The inclusive NLO QCD corrections are seen to considerably obscure the amplitude zero. The bulk of the corrections at LHC energies originates from quark-gluon fusion and the
kinematical region where the photon is produced at large $p_T$ and recoils against a quark, which radiates a soft $W$ boson which is almost collinear to the quark. Events which originate from this phase space region usually contain a high $p_T$ jet. A jet veto therefore helps to reduce the QCD corrections. Nevertheless, the remaining QCD corrections still substantially blur the visibility of the radiation zero in $W\gamma$ production at the LHC \cite{10}.

Given a sufficiently large integrated luminosity, experiments at the Tevatron studying lepton-photon rapidity correlations therefore offer a unique chance to observe the SM radiation zero in $W\gamma$ production. Nonstandard $WW\gamma$ couplings tend to fill in the dip in the $\Delta y(\gamma, \ell)$ distribution caused by the radiation zero.

Indirectly, the radiation zero can also be observed in the $Z\gamma$ to $W\gamma$ cross section ratio \cite{13}. Many theoretical and experimental uncertainties at least partially cancel in the cross section ratio. On the other hand, in searching for the effects of the SM radiation zero in the $Z\gamma$ to $W\gamma$ cross section ratio, one has to assume that the SM is valid for $Z\gamma$ production. Since the radiation zero occurs at a large scattering angle, the photon $E_T$ distribution in $W\gamma$ production falls much more rapidly than that of photons in $Z\gamma$ production. As a result, the SM $W\gamma$ to $Z\gamma$ event ratio as a function of the photon transverse energy, $E_T^\gamma$, drops rapidly.

\section*{MULTIZERO}

Adding more external photons to a reaction with a Born-amplitude radiation zero will still leave us with a null zone which demands, however, that all photons be collinear \cite{7,14}. In view of the fact that the quadrilinear coupling $WW\gamma\gamma$ contributes, it is of interest to consider the radiation zero in $W\gamma\gamma$ production. The $\Delta y(\gamma\gamma, W) = y_{\gamma\gamma} - y_W$ distribution is expected
FIG. 4. Rapidity difference distributions for $p\bar{p} \rightarrow e^-\bar{\nu}\gamma\gamma$ at $\sqrt{s} = 2$ TeV. Part (a) shows the $y_\gamma - y_W$ spectrum, while part (b) displays the $y_\gamma - y_e$ distribution. The solid (dashed) curves are for $\cos\theta_{\gamma\gamma} > 0$ ($\cos\theta_{\gamma\gamma} < 0$).

to display a clear dip for photons with a small opening angle, $\theta_{\gamma\gamma}$, in the laboratory frame, i.e. at $\cos\theta_{\gamma\gamma} \approx 1$. Calculations show [15] that requiring $\cos\theta_{\gamma\gamma} > 0$ is already sufficient. Figure 4a displays a pronounced dip in $d\sigma/d\Delta y(\gamma\gamma, W)$ for $\cos\theta_{\gamma\gamma} > 0$ at the Tevatron, located at $\Delta y(\gamma\gamma, W) \approx 0.7$ (solid line) for $e^-\bar{\nu}\gamma\gamma$ production at the Tevatron. In contrast, for $\cos\theta_{\gamma\gamma} < 0$, the $\Delta y(\gamma\gamma, W)$ distribution does not exhibit a dip (dashed line).

In the dip region, the differential cross section for $\cos\theta_{\gamma\gamma} < 0$ is about one order of magnitude larger than for $\cos\theta_{\gamma\gamma} > 0$. In addition, the $\Delta y(\gamma\gamma, W)$ distribution extends to significantly higher $y_\gamma - y_W$ values if one requires $\cos\theta_{\gamma\gamma} > 0$. This reflects the narrower rapidity distribution of the two-photon system for $\cos\theta_{\gamma\gamma} < 0$, due to the larger invariant mass of the system when the two photons are well separated.

Exactly as in the $W\gamma$ case, the dominant helicity of the $W$ boson in $W^{\pm}\gamma\gamma$ production is $\lambda_W = \pm 1$. One therefore expects that the distribution of the rapidity difference of the $\gamma\gamma$ system and the charged lepton is very similar to the $y_\gamma - y_W$ distribution and would show a clear signal of the radiation zero for positive values of $\cos\theta_{\gamma\gamma}$. The $y_\gamma - y_e$ distribution, shown in Fig. 4b, indeed clearly displays these features. Due to the finite difference between the electron and the $W$ rapidities, the location of the minimum is slightly shifted.

To simulate the finite acceptance of detectors, we have imposed the following cuts in Fig. 4:

$$
\begin{align*}
   p_T(\gamma) &> 10 \text{ GeV}, & |y_\gamma| < 2.5, & \Delta R(\gamma\gamma) > 0.3 & \text{ for photons}, \\
   p_T(e) &> 15 \text{ GeV}, & |y_e| < 2.5, & \Delta R(e\gamma) > 0.7 & \text{ for charged leptons},
\end{align*}
$$

(11)
and
\[ p_T > 15 \text{ GeV}. \] (12)

In addition, to suppress the contributions from final photon radiation, we have required that
\[ M_T(e^-\nu) > 70 \text{ GeV}. \] (13)

The characteristic differences between the \[ \Delta y(\gamma\gamma,e) = y_{\gamma\gamma} - y_e \] distribution for \( \cos \theta_{\gamma\gamma} > 0 \) and \( \cos \theta_{\gamma\gamma} < 0 \) are also reflected in the cross section ratio
\[ R = \frac{\int_{\Delta y(\gamma\gamma,e) > -1} d\sigma}{\int_{\Delta y(\gamma\gamma,e) < -1} d\sigma}, \] (14)

which may be useful for small event samples. Many experimental uncertainties cancel in \( R \). For \( \cos \theta_{\gamma\gamma} > 0 \) one finds \( R \approx 0.25 \), whereas for \( \cos \theta_{\gamma\gamma} < 0 \) \( R \approx 1.06 \).

Although we have restricted the discussion above to \( e\nu\gamma\gamma \) production, our results also apply to \( p\bar{p} \rightarrow \mu\nu\gamma\gamma \). NLO QCD corrections are not expected to obscure the dip signaling the radiation zero at the Tevatron, but may significantly reduce its observability at the LHC. Given a sufficiently large integrated luminosity, experiments at the Tevatron studying correlations between the rapidity of the photon pair and the charged lepton therefore offer an excellent opportunity to search for the SM radiation zero in hadronic \( W\gamma\gamma \) production. Unfortunately, for the cuts listed above, the \( W\gamma\gamma \) production cross section at the Tevatron is only about 2 fb. Thus, in order to observe the radiation zero in \( W\gamma\gamma \) production, an integrated luminosity of at least \( 20 - 30 \text{ fb}^{-1} \) is needed.

**APPROXIMATELY ZERO**

At energies much larger than the \( Z \) boson mass, one naively expects that the \( Z \) boson in the process
\[ q_1 \bar{q}_2 \rightarrow W^\pm Z \] (15)
behaves essentially like a photon with unusual couplings to the fermions. One therefore might suspect that an approximate radiation zero is present in \( WZ \) production. In Ref. \[ \text{[17]} \] it was demonstrated that the process \( q_1 \bar{q}_2 \rightarrow W^\pm Z \) indeed exhibits an approximate zero located at
\[ \cos \Theta^* \approx \pm \frac{1}{3} \tan^2 \theta_W \approx \pm 0.1, \] (16)

where \( \Theta^* \) is the scattering angle of the \( Z \) boson relative to the quark direction in the \( WZ \) center of mass frame. The approximate zero is the combined result of an exact zero in the dominant helicity amplitudes \( \mathcal{M}(\pm,\mp) \), and strong gauge cancellations in the remaining amplitudes. At high energies, only the \((\pm,\mp)\) and \((0,0)\) amplitudes remain nonzero:

\[ \mathcal{M}(\pm,\mp) \rightarrow \frac{F}{\sin \theta^*} (\lambda_w - \cos \Theta^*) \left[ (g_\pm^{q_1} - g_\pm^{q_2}) \cos \Theta^* - (g_\pm^{q_1} + g_\pm^{q_2}) \right], \] (17)

\[ \mathcal{M}(0,0) \rightarrow \frac{F}{2} \sin \theta^* \frac{M_Z}{M_W} (g_0^{q_2} - g_0^{q_1}). \] (18)
Here, $\lambda_w$ denotes the $W$ boson polarization ($\lambda = \pm 1, 0$ for transverse and longitudinal polarizations, respectively), and

$$F = C \frac{e^2}{\sqrt{2} \sin \theta_w},$$  \hspace{1cm} (19)$$

where $C = \delta_{i_1 i_2} V_{q_1 q_2}$ and $\theta^* = \pi - \Theta^*$. $i_1$ ($i_2$) is the color index of the incoming quark (antiquark) and $V_{q_1 q_2}$ is the quark mixing matrix element.

$$g^f = \frac{T_3^f - Q_f \sin^2 \theta_w}{\sin \theta_w \cos \theta_w}$$  \hspace{1cm} (20)$$

is the coupling of the $Z$-boson to left-handed fermions with $T_3^f = \pm \frac{1}{2}$ representing the third component of the weak isospin. $Q_f$ is the electric charge of the fermion $f$.

The existence of the zero in $\mathcal{M}(\pm, \mp)$ at $\cos \Theta^* \approx \pm 0.1$ is a direct consequence of the contributing $u$- and $t$-channel fermion exchange diagrams and the left-handed coupling of the $W$ boson to fermions. Unlike the $W^\pm \gamma$ case with its massless photon kinematics, the zero has an energy dependence which is, however, rather weak for energies sufficiently above the $WZ$ mass threshold.

Analogously to the radiation zero in $q_1 \bar{q}_2 \rightarrow W\gamma$, one can search for the approximate zero in $WZ$ production in the rapidity difference distribution $d\sigma/d\Delta y(Z, \ell_1)$ \[16\], where

$$\Delta y(Z, \ell_1) = y(Z) - y(\ell_1)$$  \hspace{1cm} (21)$$

is the difference between the rapidity of the $Z$ boson, $y(Z)$ and the rapidity of the lepton, $\ell_1$ originating from the decay of the $W$ boson, $W \rightarrow \ell_1 \nu$. The $y(Z) - y(\ell_1)$ distribution for $W^\pm Z$ production in the Born approximation is shown in Fig. 3. The approximate zero in the $WZ$ amplitude leads to a dip in the $y(Z) - y(W)$ distribution, which is located at $y(Z) - y(W) \approx \pm 0.12$ $(= 0)$ for $W^\pm Z$ production in $pp$ ($pp$) collisions. However, in contrast to $W\gamma$ production, none of the $W$ helicities dominates in $WZ$ production \[14\]. The charged lepton, $\ell_1$, thus only partly reflects the kinematical properties of the parent $W$ boson. As a result, a significant part of the correlation present in the $y(Z) - y(W)$ spectrum is lost, and only a slight dip survives in the SM $y(Z) - y(\ell_1)$ distribution. This, and the much smaller number of $WZ \rightarrow \ell_1 \nu \ell_2^+ \ell_2^-$ events, make the approximate radiation zero in $WZ$ production much more difficult to find at the Tevatron or LHC than the radiation amplitude zero in $W\gamma$ production.

Due to the nonzero average rapidity difference between the lepton $\ell_1$ and the parent $W$ boson, the location of the minimum of the $y(Z) - y(\ell_1)$ distribution in $pp$ collisions is slightly shifted to $y(Z) - y(\ell_1) \approx 0.5$. In Fig. 5a a rapidity cut of $|\eta(\ell)| < 2.5$ has been imposed, instead of the cut used in Fig. 3a. All other rapidity and transverse momentum cuts are as described before. Furthermore, $\Delta R(\ell, \ell) > 0.4$ is required for leptons of equal electric charge in Fig. 3. The significance of the dip in the $y(Z) - y(\ell_1)$ distribution depends to some extent on the cut imposed on $p_T(\ell_1)$ and the missing transverse momentum. Increasing (decreasing) the cut on $p_T(\ell_1)$ $(p_T)$ tends to increase the probability that $\ell_1$ is emitted in the flight direction of the $W$ boson, and thus enhances the significance of the dip. If the $p_T > 50$ GeV cut at the LHC could be reduced to 20 GeV, the dip signaling the approximate zero in the $WZ$ production amplitude would be strengthened considerably. In contrast to
FIG. 5. $Z$ lepton rapidity difference distribution for $WZ$ production in the SM at a) the Tevatron and b) the LHC.

the situation encountered in $W\gamma$ production, nonstandard $WWZ$ couplings do not always 
tend to fill in the dip caused by the approximate radiation zero. This is due to the relatively 
strong interference between standard and anomalous contributions to the helicity amplitudes 
for certain anomalous couplings. As a result, the dip may even become more pronounced in 
some cases.

Before we turn to the prospects of observing radiation zeros in the near future, we would 
like to mention a new development in the general question of radiation zeros. Different 
kinds of null zones have been found (“Type II” radiation zeros [18]) in the important process 
$q\bar{q} \rightarrow W^+W^−\gamma$, for which there are no regular (also called Type I) zeros. The Type II zeros 
require soft photons and certain coplanarity, but dips survive for harder photons that are 
sensitive to the trilinear and quadrilinear gauge boson couplings. It will be interesting to 
see how visible they are in an analysis incorporating acceptance cuts, detector resolution 
effects, finite $W$ width effects, and decay-lepton radiation.

NONZERO ZEROS IN ZERO ZERO?

How long will we wait for a real dip to appear? A sufficient rapidity coverage is essential 
to observe the radiation zero in $d^2\sigma/dy(\gamma)dy(\ell)$ and/or the $\Delta y(\gamma, \ell)$ distribution [19]. This 
is demonstrated in Fig. 6, which displays simulations of the rapidity difference distribution 
for 1 fb$^{-1}$ in the electron channel at the Tevatron. If both central ($|y| < 1.1$) and endcap 
($1.5 < |y| < 2.5$) electrons and photons can be used (Fig. 6a), the simulations indicate that 
with integrated luminosities $\geq 1$ fb$^{-1}$ it will be possible to conclusively establish the dip in 
the photon lepton rapidity difference distribution which signals the presence of the radiation
**ZEROING IN ON BRODSKY**

The radiation zeros are the generalization of the well-known vanishing of classical non-relativistic electric and magnetic dipole radiation occurring for equal charge/mass ratios (indeed, the low-energy limit of the null zone conditions) and equal gyro-magnetic g-factors. The null zone is exactly the same as that for the completely destructive interference of radiation by charge lines (a classical convection current calculation [7]) and is preserved by the fully relativistic quantum Born approximation for gauge theories.

Stan Brodsky has long emphasized the magic of the gyro-magnetic ratio value $g = 2$ predicted by gauge theory for spinor and vector particles. Only for this value will Born amplitudes have the same null zone as the classical radiation patterns for soft photons. And only for this value will Born amplitudes have good high-energy behavior. In this way we have a connection between the large and small distance scales, with the value $g = 2$ as a bridge.

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**FIG. 6.** Simulation of the photon lepton rapidity difference distribution for $W \gamma$ production at the Tevatron for 1 fb$^{-1}$, a) for central and endcap photons and electrons, b) for central electrons and photons only.

Zero in $W \gamma$ production. On the other hand, for central electrons and photons only, the dip is statistically not significant for 1 fb$^{-1}$. With the detector upgrades which are currently being implemented for the next Tevatron run, both CDF and DØ experiments should have the capability to analyze the $\Delta y(\gamma, \ell)$ distribution over the full rapidity range of $|y| < 2.5$. While the data analysis may take rather longer, we may hazard the guess that the year Y2K will have more than three zeros in it.
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REFERENCES

[1] R. W. Brown and K. O. Mikaelian, Phys. Rev. D19, 922 (1979).
[2] R. W. Brown, D. Sahdev, and K. O. Mikaelian, Phys. Rev. D20, 1164 (1979).
[3] J. Ellison and J. Wudka, Annu. Rev. Nucl. Part. Sci. 48, 33 (1998); The LEP Collaborations, CERN-EP/99-15 (report, February 1999).
[4] K. O. Mikaelian, M. A. Samuel, and D. Sahdev, Phys. Rev. Lett. 43, 746 (1979).
[5] D. Zhu, Phys. Rev. D22, 2266 (1980); C. J. Goebel, F. Halzen, and J. P. Leveille, Phys. Rev. D23, 2682 (1981).
[6] R. W. Brown, Understanding Something about Nothing: Radiation Zeros and T. Han, Exact and Approximate Radiation Amplitude Zeros - Phenomenological Aspects, February 1995, International Symposium on Vector Boson Self-Interactions, AIP Conference Proceedings 350, Eds. U. Baur, S. Errede, and T. Müller, pp. 261 and 224.
[7] S. J. Brodsky and R. W. Brown, Phys. Rev. Lett. 49, 966 (1982); R. W. Brown, K. L. Kowalski, and S. J. Brodsky, Phys. Rev. D28, 624 (1983); R. W. Brown and K. L. Kowalski, Phys. Rev. D29, 2100 (1984).
[8] C. L. Bilchak, J. Phys. G: Nucl. Phys. 11, 1117 (1985); M. A. Doncheski and F. Halzen, Z. Phys. C52, 673 (1992).
[9] D. Benjamin, Proceedings of the 10th Topical Workshop on Proton-Antiproton Collider Physics, Fermilab, May 1995, AIP Conference Proceedings 357, Eds. R. Raja and J. Yoh, p. 370.
[10] U. Baur, S. Errede, and G. Landsberg, Phys. Rev. D50, 1917 (1994).
[11] C. L. Bilchak, R. W. Brown, and J. D. Stroughair, Phys. Rev. D29, 375 (1984).
[12] U. Baur, T. Han, and J. Ohnemus, Phys. Rev. D48, 5140 (1993).
[13] U. Baur, S. Errede, and J. Ohnemus, Phys. Rev. D48, 4103 (1993).
[14] R. W. Brown, M. E. Convery and M. A. Samuel, Phys. Rev. D49, 2290 (1994).
[15] U. Baur, T. Han, N. Kauer, R. Sobey and D. Zeppenfeld, Phys. Rev. D56, 140 (1997).
[16] U. Baur, T. Han and J. Ohnemus, Phys. Rev. D51, 3381 (1995).
[17] U. Baur, T. Han and J. Ohnemus, Phys. Rev. Lett. 72, 3941 (1994).
[18] W. J. Stirling and A. Werthenbach, preprint [hep-ph/9905341]; M. Heyssler and W. J. Stirling, Eur. Phys. J. C4, 289 (1998).