COMPUTATIONAL AND NUMERICAL SIMULATIONS FOR THE DEOXYRIBONUCLEIC ACID (DNA) MODEL

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Abstract. In this research paper, the modified Khater method, the Adomian decomposition method, and B-spline techniques (cubic, quintic, and septic) are applied to the deoxyribonucleic acid (DNA) model to get the analytical, semi-analytical, and numerical solutions. These solutions comprise much information about the dynamical behavior of the homogenous long elastic rods with a circular section. These rods constitute a pair of the polynucleotide rods of the DNA molecule which are plugged by an elastic diaphragm that demonstrates the hydrogen bond's role in this communication. The stability property is checked for some solutions to show more effective and powerful of obtained solutions. Based on the role of analytical and semi-analytical techniques in the motivation of the numerical techniques to be more accurate, the B-spline numerical techniques are applied by using the obtained exact solutions on the DNA model to show which one of them is more accurate than other, to explain more of the dynamic behavior of the homogenous long elastic rods, and to show the coincidence between the different types of obtained solutions. The obtained solutions verified with Maple 16 & Mathematica 12 by placing them back into the original equations. The performance of these methods shows the power and effectiveness of them for applying to many different forms of the nonlinear evolution equations with an integer and fractional order.

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1. Introduction. The modern biophysics is considered as one of the most exciting fields in studying as it is based on the delay life. This branch of science contains many interested models such as the model of the electric field in the human brain, Human cortical folding across regions model, the DNA model, and so forth. This paper selected one of these models to study which is the DNA model. The main purpose of this research is studying the dynamics of DNA molecules in mathematical form. It is considered as one of the most difficult models to study because it accompanies a lot of different movements like the longitudinal, transverse, and torsional motions. That distinct motions enable studying this model in a simple way which depends on the following two items:

- Choosing some of these motions and neglect the other one. These selected motions must be dominated in known range of the time scale.
- Modelling these motions in different equations.

The modeling process depends on using the properties of the nonlinear wave in the biological processes at the molecular level. However, many models such as Barkley and Zimm (BZ.) model [3] and Yakushevich model [18] were derived but the question of which one of them is preferable still opening. The BZ model is considered as the simplest model of DNA that can be used in the electronic microphotographs, where it describes the motion in the range of time scale. The DNA model is considered as a one long elastic homogenous rod with a circular section. While the Yakushevich model consists of two long flexible and weakly interacting rods. These rods are lapped around each other that behavior is responsible for the configure of the double helix.

In this paper, we study one of these models that can be used as the mechanical model of the $\varphi^4$-field equation and the nonlinear cubic Klein-Gorden equation and proposed in the neglect of the discreteness localization of DNA and the continuum-limit approximation. Also, this model is an admissible and affordable model of DNA as its length (10^6 base pair). So, it is logically used to describe the nonlinearity behavior of the DNA’s molecules in the colon bacillus. In the way of deriving a the DNA model, the neglect of the double helical structure of DNA model and replace it with two parallel rods which have a straight line property and consider just two motions (longitudinal and transverse motions) and neglect the third motion (torsional motion), are considered as the two fundamental assumptions [5]. Although there exists another model of DNA that study only one motion (torsional motion), it is considered a difficult model for mathematical study [6]- [20].

The Hamiltonian form of first model is given by

$$\Xi = \Gamma + \Lambda^{(1)} + \Lambda^{(2)}, \quad (1.1)$$

where $\Gamma$, $\Lambda^{(1)}$, $\Lambda^{(2)}$ constitute the kinetic energy of the longitudinal vibrations of the elastic rods, the potential energy of the elastic rods, and potential energy of the elastic membrane, respectively. For $\Gamma$, $\Lambda^{(1)}$, $\Lambda^{(2)}$, we have

$$\begin{align*}
\Gamma &= \frac{1}{2} \int \left[ \varrho_1 \Psi_1 \left( \frac{d}{dx} \Upsilon_1 \right)^2 + \left( \frac{d}{dx} \Omega_1 \right)^2 \right] dx, \\
\Lambda^{(1)} &= \frac{1}{2} \int \left[ \Theta_1 \Psi_1 \left( \frac{d}{dx} \Upsilon_1 \right)^2 + \Theta_2 \Psi_2 \left( \frac{d}{dx} \Upsilon_2 \right)^2 \right] dx, \\
\Lambda^{(2)} &= \frac{1}{2} \int \varsigma (\Delta \chi (x))^2 dx, \text{since } \Delta \chi = \sqrt{\left( h + \Omega_1 - \Omega_2 \right)^2 + \left( \Theta_2 - \Theta_1 \right)^2 - l_0},
\end{align*} \quad (1.2)$$
where $[\psi_i, \Omega_i, \varpi_i, \chi_i, \phi_i, \rho_i, \Gamma_i, \Delta \chi(x), k, l_0, \text{where } (i = 1, 2)]$ represent the longitudinal displacements of the top and bottom rods, the transverse displacements of the top and bottom rods, the mass density, the area of transverse, the young modulus, the tension density of the $i$-th strand, the rigidity of the elastic membrane, the stretching of the elastic membrane, the distance of the two strands, and the height of the membrane in the equilibrium position, respectively. According to Eqs. (1.1), (1.2), the dynamical models of DNA with two basic motion is given as

$$
\begin{align*}
\Psi_1 \left[ \varphi_1 \left( \frac{d^2}{dx^2} \psi_1 \right)^2 - \Theta_1 \left( \frac{d^2}{dx^2} \psi_1 \right)^2 \right] &= \varsigma \vartheta (\psi_2 - \psi_1), \\
\Psi_2 \left[ \varphi_2 \left( \frac{d^2}{dx^2} \psi_2 \right)^2 - \Theta_2 \left( \frac{d^2}{dx^2} \psi_2 \right)^2 \right] &= \varsigma \vartheta (\psi_1 - \psi_2), \\
\Psi_1 \left[ \varphi_1 \left( \frac{d^2}{dx^2} \Omega_1 \right)^2 - \Gamma_2 \left( \frac{d^2}{dx^2} \Omega_1 \right)^2 \right] &= \varsigma \vartheta (\Omega_2 - \Omega_1 - h), \\
\Psi_2 \left[ \varphi_2 \left( \frac{d^2}{dx^2} \Omega_2 \right)^2 - \Gamma_2 \left( \frac{d^2}{dx^2} \Omega_2 \right)^2 \right] &= \varsigma \vartheta (\Omega_1 - \Omega_2 + h),
\end{align*}
$$

(1.3)

where $\vartheta = \frac{\Delta \chi}{\Delta \chi + \chi_0}$. By neglecting the high order more than two and assuming $|\psi_1 - \psi_2| \ll h$, $|\Omega_1 - \Omega_2| \ll h$, we obtain

$$
\begin{align*}
\Psi_1 \left[ \varphi_1 \left( \frac{d^2}{dx^2} \psi_1 \right)^2 - \Theta_1 \left( \frac{d^2}{dx^2} \psi_1 \right)^2 \right] &= \varsigma \varpi (\psi_2 - \psi_1), \\
\Psi_2 \left[ \varphi_2 \left( \frac{d^2}{dx^2} \psi_2 \right)^2 - \Theta_2 \left( \frac{d^2}{dx^2} \psi_2 \right)^2 \right] &= \varsigma \varpi (\psi_1 - \psi_2), \\
\Psi_1 \left[ \varphi_1 \left( \frac{d^2}{dx^2} \Omega_1 \right)^2 - \Gamma_2 \left( \frac{d^2}{dx^2} \Omega_1 \right)^2 \right] &= \varsigma \varpi (\Omega_2 - \Omega_1 - h), \\
\Psi_2 \left[ \varphi_2 \left( \frac{d^2}{dx^2} \Omega_2 \right)^2 - \Gamma_2 \left( \frac{d^2}{dx^2} \Omega_2 \right)^2 \right] &= \varsigma \varpi (\Omega_1 - \Omega_2 + h),
\end{align*}
$$

(1.4)

where $\varpi = \varpi(\psi_1, \psi_2, \Omega_1, \Omega_2) = 1 - \frac{\psi_0}{h} + \frac{\psi_0}{h^2} (\Omega_1 - \Omega_2) + \frac{2 \psi_0}{h^3} (\psi_2 - \psi_1)^2 + (\Omega_2 - \Omega_1)^2$.

Considering the symmetric case on Eq. (1.4), change the variables as following

$$
\begin{align*}
\psi = \frac{\Omega_1 + \Omega_2}{\sqrt{2}}, \varphi = \frac{\psi_1 - \psi_2}{\sqrt{2}}, u = \frac{\Omega_1 + \Omega_2}{\sqrt{2}}, \phi = \frac{\Omega_1 - \Omega_2}{\sqrt{2}}
\end{align*}
$$

and discrete the Hamiltonian into two different of phase (in/out-stage). The out phase motion is used to find the DNA model that is given by

$$
\begin{align*}
\varphi_{tt} - c_1^2 \varphi_{xx} &= \lambda_1 \varphi + \gamma_1 \varphi \phi + \mu_1 \varphi^3 + \beta_1 \varphi \phi^2, \\
\phi_{tt} - c_2^2 \phi_{xx} &= \lambda_2 \phi + \gamma_2 \varphi^2 + \mu_2 \varphi^2 \phi + \beta_2 \phi^3 + c_0,
\end{align*}
$$

(1.5)

where $c_1 = \pm \sqrt{\frac{\mu_0}{\vartheta}}$, $c_2 = \pm \sqrt{\frac{\lambda_1}{\vartheta}}$; $\lambda_1 = \frac{2 \varphi \phi}{\vartheta \vartheta \varphi} (h - \chi_0)$; $\lambda_2 = \frac{2 \varphi \phi}{\vartheta \vartheta \varphi}$; $\gamma_1 = \frac{2 \varphi \phi}{\vartheta \vartheta \varphi}$; $2 \varphi \phi = \frac{2 \varphi \phi}{\vartheta \vartheta \varphi}$; $\mu_1 = \mu_2 \chi_0 = \frac{\chi_0}{\vartheta \vartheta \varphi}$; $\beta_1 = \beta_2 = \frac{4 \varphi \phi}{\vartheta \vartheta \varphi}$; $c_0 = \frac{\sqrt{2} \varphi \phi (h - \chi_0)}{\vartheta \vartheta \varphi}$. Using the following transformation $\varphi = a \varphi + b$ on Eq. (1.5), where $a, b$ are arbitrary constants,
yields
\[
\begin{align*}
\varphi_{tt} - c_1^2 \varphi_{xx} &= \varphi_3' (\mu_1 + \beta_1 a^2) + u^2 (2 \beta_1 a b + a \gamma_1) + \rho (\lambda_1 + b \gamma_1 + \beta_1 b^2), \\
\varphi_{tt} - c_2^2 \varphi_{xx} &= \varphi_3' (\mu_2 + \beta_2 a^2) + \varphi_2' (\frac{2a}{a} + \frac{a \beta_2 b}{b} + 3 \beta_2 a b) \\
&+ \varphi (\lambda_2 + 3 \beta_2 b^2) + \frac{\lambda b}{a} + \frac{\beta_2 b^2}{a} + \frac{3a}{a}.
\end{align*}
\] (1.6)

By reducing and comparing two equations of Eq. (1.6) with the following conditions
\[
\Theta = \Gamma, \ b = \frac{a \sqrt{2}}{2},
\]
Eq. (1.6) transforms to the following equation
\[
\varphi_{tt} - c_1^2 \varphi_{xx} - \lambda \varphi_3 - \mu \varphi_2 - \rho \varphi = 0,
\] (1.7)

where
\[
\lambda = \mu_1 + \beta_1 a^2, \ \mu = 2 \beta_1 a b + a \gamma_1, \ \rho = \lambda_1 + b \gamma_1 + \beta_1 b^2.
\]

In the context of the mathematical and physical studying of this kind of equations, many analytical, semi-analytical, and numerical schemes have been being derived to get the exact and numerical solutions of such this kind of equations [12]-[15].

The order of this research paper is summed up as follows: In section 2, we apply the modified Khater method [7]-[10], Adomain decomposition method [16]-[13], and B-spline techniques [14]-[19] to the double chain model of DNA. In section 3, We give more discussion about our obtained solutions. In section 4, we give conclusions.

2. Application. This section applies three different schemes on the DNA model to get the computational and numerical solutions of this model. Using the next wave transformation
\[
\varphi(x,t) = \varphi(\xi), \ \xi = \delta x + \omega t,
\]
reduces Eq.(1.7) to the following ODE:
\[
\kappa \varphi'' - \lambda \varphi^3 - \mu \varphi^2 - \rho \varphi = 0,
\] (2.1)

where \(\kappa = \omega^2 - \delta^2 c_1^2 \neq 0\). Balancing \(u''\) and \(u^3\) leads to, \(N + 2 = 3N \rightarrow N = 1\). Therefore, we have the formal solution:

2.1. Analytical solution. Applying the modified Khater method to Eq. (2.1) enables putting the general form of solution of the DNA model
\[
\varphi(\xi) = \sum_{i=1}^{n} a_i K^{i_f(\xi)} + \sum_{i=1}^{n} b_i K^{-i_f(\xi)} + a_0,
\] (2.2)

where \(a_i, b_i, K, (i = 1, 2, 3, \ldots)\) are arbitrary constant and \(f(\xi)\) is a workaround function of the following auxiliary equation
\[
f'(\xi) = \beta + \alpha K^{-f(\xi)} + \sigma K^{f(\xi)}
\]
\[b_n(K),
\] (2.3)

where \(\beta, \alpha, \sigma\) are arbitrary constants will be determine later. Substituting Eq. (2.2) along (2.3) into Eq. (2.1) and gathering all coefficients of the same power of \(K^{i_f(\xi)}\), \(i = 0, 1, 2, \ldots\), obtain a system of algebraic equations. Solving this system of equation, yields

Family I
\[
\begin{align*}
\frac{a_1}{\beta} &\rightarrow a_0 \frac{\sigma}{\beta}, \ b_1 &\rightarrow a_0 \frac{a_0 \lambda}{2\beta^2}, \ \kappa &\rightarrow \frac{a_0^2 \lambda}{2\beta^2}, \ \mu &\rightarrow - \frac{1}{2} (3a_0 \lambda), \ \rho &\rightarrow \frac{a_0^2 \beta^2 \lambda - 4 \alpha a_0^2 \lambda \sigma}{2\beta^2}.
\end{align*}
\]
Thus, the solitary wave solutions of Eq. (1.7) are given by:

When \( \beta^2 - 4\alpha\sigma < 0 \& \sigma \neq 0 \)

\[
\varphi_1(x,t) = \frac{a_0 \left( \beta^2 - 4\alpha\sigma \right) \sec^2 \left( \frac{1}{2} \sqrt{4\alpha\sigma - \beta^2} (t\omega + \delta x) \right)}{2\beta \left( \beta - \sqrt{4\alpha\sigma - \beta^2} \tan \left( \frac{1}{2} \sqrt{4\alpha\sigma - \beta^2} (t\omega + \delta x) \right) \right)},
\]

(2.4)

\[
\varphi_2(x,t) = \frac{a_0 \left( \beta^2 - 4\alpha\sigma \right) \csc^2 \left( \frac{1}{2} \sqrt{4\alpha\sigma - \beta^2} (t\omega + \delta x) \right)}{2\beta \left( \beta - \sqrt{4\alpha\sigma - \beta^2} \cot \left( \frac{1}{2} \sqrt{4\alpha\sigma - \beta^2} (t\omega + \delta x) \right) \right)}.
\]

(2.5)

When \( \beta^2 - 4\alpha\sigma > 0 \& \sigma \neq 0 \)

\[
\varphi_3(x,t) = \frac{a_0 \left( \beta^2 - 4\alpha\sigma \right) \sech^2 \left( \frac{1}{2} \sqrt{\beta^2 - 4\alpha\sigma} (t\omega + \delta x) \right)}{2\beta \left( \beta + \sqrt{\beta^2 - 4\alpha\sigma} \tanh \left( \frac{1}{2} \sqrt{\beta^2 - 4\alpha\sigma} (t\omega + \delta x) \right) \right)}.
\]

(2.6)

\[
\varphi_4(x,t) = -\frac{a_0 \left( \beta^2 - 4\alpha\sigma \right) \csch^2 \left( \frac{1}{2} \sqrt{\beta^2 - 4\alpha\sigma} (t\omega + \delta x) \right)}{2\beta \left( \beta + \sqrt{\beta^2 - 4\alpha\sigma} \coth \left( \frac{1}{2} \sqrt{\beta^2 - 4\alpha\sigma} (t\omega + \delta x) \right) \right)}.
\]

(2.7)

When \( \beta = \frac{a}{2} = \kappa \& \sigma = 0 \)

\[
\varphi_5(x,t) = a_0 \left( \frac{2}{e^\kappa(t\omega+\delta x)} - 2 \right).
\]

(2.8)

When \( \beta = \sigma = \kappa \& \alpha = 0 \)

\[
\varphi_6(x,t) = \frac{a_0}{1 - e^\kappa(t\omega+\delta x)}.
\]

(2.9)

When \( \alpha = 0 \& \beta \neq 0 \& \sigma \neq 0 \)

\[
\varphi_7(x,t) = -\frac{2a_0}{\sigma e^{\beta(t\omega+\delta x)} - 2}.
\]

(2.10)

When \( \sigma = 0 \& \beta \neq 0 \& \alpha \neq 0 \)

\[
\varphi_8(x,t) = a_0 \left( \frac{\alpha}{\beta e^{\beta(t\omega+\delta x)} - \alpha} + 1 \right).
\]

(2.11)

When \( \beta^2 - 4\alpha\sigma = 0 \)

\[
\varphi_9(x,t) = \frac{1}{2} a_0 \left( \frac{4\alpha\sigma (-\beta - 2}{\beta^3 \omega + \delta x x} + \frac{2}{\beta t \omega + \beta \delta x + 2} + 1 \right).
\]

(2.12)

Family II

\[
a_1 \to \frac{2a_0\sigma}{\beta}, b_1 \to 0, \kappa \to \frac{2a_0^2\lambda}{\beta^2}, \mu \to 0, \rho \to -\frac{a_0^2\lambda \left( \beta^2 - 4\alpha\sigma \right)}{\beta^2}.
\]

Thus, the solitary wave solutions of Eq. (1.7) are given by:

When \( \beta^2 - 4\alpha\sigma < 0 \& \sigma \neq 0 \)

\[
\varphi_{10}(x,t) = a_0 \sqrt{4\alpha\sigma - \beta^2} \tan \left( \frac{1}{2} \sqrt{4\alpha\sigma - \beta^2} (t\omega + \delta x) \right),
\]

(2.13)

\[
\varphi_{11}(x,t) = a_0 \sqrt{4\alpha\sigma - \beta^2} \cot \left( \frac{1}{2} \sqrt{4\alpha\sigma - \beta^2} (t\omega + \delta x) \right).
\]

(2.14)
When $\beta^2 - 4\alpha \sigma > 0$ & $\sigma \neq 0$

$$\varphi_{12}(x, t) = -\frac{a_0 \sqrt{\beta^2 - 4\alpha \sigma} \tanh \left( \frac{1}{2} \sqrt{\beta^2 - 4\alpha \sigma} (t\omega + \delta x) \right)}{\beta},$$

$$\varphi_{13}(x, t) = -\frac{a_0 \sqrt{\beta^2 - 4\alpha \sigma} \coth \left( \frac{1}{2} \sqrt{\beta^2 - 4\alpha \sigma} (t\omega + \delta x) \right)}{\beta}.$$

(2.15)

(2.16)

When $\beta = \sigma = \kappa$ & $\alpha = 0$

$$\varphi_{14}(x, t) = u = -\coth \left( \frac{1}{2} \kappa (x\delta + t\omega) \right) a_0.$$  

(2.17)

When $\alpha = 0$ & $\beta \neq 0$ & $\sigma \neq 0$

$$\varphi_{15}(x, t) = a_0 \left( -\frac{4}{\sigma e^{\beta (t\omega + \delta x)} - 2} - 1 \right).$$

(2.18)

When $\beta^2 - 4\alpha \sigma = 0$

$$\varphi_{16}(x, t) = a_0 \left( \frac{4\alpha \sigma (-\beta - \frac{2}{t\omega + \delta x})}{\beta^3} + 1 \right).$$

(2.19)

Family III

$$a_1 \rightarrow -\frac{a_0 \left( \sqrt{32\alpha \sigma + \beta^2 + \beta} \right)}{8\alpha}, b_1 \rightarrow -\frac{a_0 \left( \sqrt{32\alpha \sigma + \beta^2 + \beta} \right)}{8\sigma},$$

$$\kappa \rightarrow \frac{a_0^2 \lambda \left( \beta \left( \sqrt{32\alpha \sigma + \beta^2 + \beta} + 16\alpha \sigma \right) \right)}{64\alpha^2 \sigma^2},$$

$$\mu \rightarrow -\frac{3a_0 \lambda \left( \beta \left( \sqrt{32\alpha \sigma + \beta^2 + \beta} + 16\alpha \sigma \right) \right)}{16\alpha \sigma},$$

$$\rho \rightarrow \frac{a_0^2 \lambda \left( 128\alpha^2 \sigma^2 + 36\alpha \beta^2 \sigma + 20\alpha \beta \sigma \sqrt{32\alpha \sigma + \beta^2 + \beta^3} \sqrt{32\alpha \sigma + \beta^2 + \beta^4} \right)}{64\alpha^2 \sigma^2}.$$ 

Thus, the solitary wave solutions of Eq. (1.7) are given by:

When $\beta^2 - 4\alpha \sigma < 0$ & $\sigma \neq 0$

$$\varphi_{17}(x, t) = \frac{1}{8} a_0 \left[ \frac{\left( \sqrt{32\alpha \sigma + \beta^2 + \beta} \right) \left( \beta - \sqrt{4\alpha \sigma - \beta^2} \tanh \left( \frac{1}{2} \sqrt{4\alpha \sigma - \beta^2} (t\omega + \delta x) \right) \right)}{2\alpha \sigma} \right.$$  

$$+ \frac{2 \left( \sqrt{32\alpha \sigma + \beta^2 + \beta} \right)}{\beta - \sqrt{4\alpha \sigma - \beta^2} \tanh \left( \frac{1}{2} \sqrt{4\alpha \sigma - \beta^2} (t\omega + \delta x) \right) + 8}],$$

$$\varphi_{18}(x, t) = \frac{1}{8} a_0 \left[ \frac{\left( \sqrt{32\alpha \sigma + \beta^2 + \beta} \right) \left( \beta - \sqrt{4\alpha \sigma - \beta^2} \cot \left( \frac{1}{2} \sqrt{4\alpha \sigma - \beta^2} (t\omega + \delta x) \right) \right)}{2\alpha \sigma} \right.$$  

$$+ \frac{2 \left( \sqrt{32\alpha \sigma + \beta^2 + \beta} \right)}{\beta - \sqrt{4\alpha \sigma - \beta^2} \cot \left( \frac{1}{2} \sqrt{4\alpha \sigma - \beta^2} (t\omega + \delta x) \right) + 8}],$$

(2.20)

(2.21)

When $\beta^2 - 4\alpha \sigma > 0$ & $\sigma \neq 0$

$$\varphi_{19}(x, t) = \frac{1}{8} a_0 \left[ \frac{\left( \sqrt{32\alpha \sigma + \beta^2 + \beta} \right) \left( \beta + \sqrt{\beta^2 - 4\alpha \sigma} \tanh \left( \frac{1}{2} \sqrt{\beta^2 - 4\alpha \sigma} (t\omega + \delta x) \right) \right)}{2\alpha \sigma} \right.$$  

$$+ \frac{2 \left( \sqrt{32\alpha \sigma + \beta^2 + \beta} \right)}{\beta + \sqrt{\beta^2 - 4\alpha \sigma} \tanh \left( \frac{1}{2} \sqrt{\beta^2 - 4\alpha \sigma} (t\omega + \delta x) \right) + 8}].$$
When \( \sigma > 0 \) & \( \alpha \neq 0 \) & \( \sigma \neq 0 \) & \( \beta = 0 \)

\[
\varphi_{21}(x, t) = a_0 \left( 1 - \sqrt{2} \csc \left( 2\sqrt{\alpha \omega} (t \omega + \delta x) \right) \right),
\]

(2.24)

\[
\varphi_{22}(x, t) = a_0 \left( \sqrt{2} \csc \left( 2\sqrt{\alpha \omega} (t \omega + \delta x) \right) + 1 \right).
\]

(2.25)

When \( \sigma < 0 \) & \( \alpha \neq 0 \) & \( \sigma \neq 0 \) & \( \beta = 0 \)

\[
\varphi_{23}(x, t) = a_0 \left( \frac{\sqrt{2} \alpha \sigma \csc \left( 2\sqrt{-\alpha \omega} (t \omega + \delta x) \right)}{-\alpha^2 \sigma^2} + 1 \right),
\]

(2.26)

\[
\varphi_{24}(x, t) = a_0 \left( 1 - \frac{\sqrt{2} \alpha \sigma \csc \left( 2\sqrt{-\alpha \omega} (t \omega + \delta x) \right)}{-\alpha^2 \sigma^2} \right).
\]

(2.27)

When \( \beta = 0 \) & \( \alpha = -\sigma \)

\[
\varphi_{25}(x, t) = a_0 \left( \frac{\sqrt{2} \alpha \sigma \csc \left( 2\alpha (t \omega + \delta x) \right)}{\sqrt{-\alpha}} + 1 \right).
\]

(2.28)

When \( \beta = 0 \) & \( \alpha = \sigma \)

\[
\varphi_{26}(x, t) = a_0 \left( 1 - \frac{\sqrt{2} \alpha \sigma \csc \left( 2(C + \alpha t \omega + \alpha \delta x) \right)}{\sqrt{\alpha^2}} \right).
\]

(2.29)

When \( \beta^2 - 4\alpha \sigma = 0 \)

\[
\varphi_{27}(x, t) = \frac{1}{16} a_0 \left( \frac{\beta^2 \left( \sqrt{32 \alpha \sigma + \beta^2 + \beta} \right) (t \omega + \delta x)}{\alpha \sigma (32 \alpha \omega + 4 \beta t \omega + \beta \delta x + 2)} + \frac{4 \left( \sqrt{32 \alpha \sigma + \beta^2 + \beta} \right) (32 \alpha \sigma + 4 \beta t \omega + \beta \delta x + 2)}{\beta^2 (t \omega + \delta x)} + 16 \right).
\]

(2.30)

**Family IV**

\[
\begin{align*}
\kappa & \to \frac{a_0 \beta - \sqrt{a_0^2 (\beta^2 - 4\alpha \sigma)}}{2\alpha}, \quad b_1 \to 0, \\
\mu & \to \frac{3a_0 \lambda \left( a_0 (\beta^2 - 2\alpha \sigma) - 2\sqrt{a_0^2 (\beta^2 - 4\alpha \sigma)} \right)}{4\alpha^2 \sigma^2}, \\
\rho & \to \frac{a_0 \lambda \left( a_0 (\beta^2 - 4\alpha \sigma) \right) \left( a_0 (\beta^2 - 2\alpha \sigma) - 2\sqrt{a_0^2 (\beta^2 - 4\alpha \sigma)} \right)}{4\alpha^2 \sigma^2}.
\end{align*}
\]

Thus, the solitary wave solutions of Eq. (1.7) are given by:

When \( \beta^2 - 4\alpha \sigma < 0 \) & \( \sigma \neq 0 \)

\[
\varphi_{27}(x, t) = \frac{1}{16} a_0 \left( \frac{\beta^2 \left( \sqrt{32 \alpha \sigma + \beta^2 + \beta} \right) (t \omega + \delta x)}{\alpha \sigma (32 \alpha \omega + 4 \beta t \omega + \beta \delta x + 2)} + \frac{4 \left( \sqrt{32 \alpha \sigma + \beta^2 + \beta} \right) (32 \alpha \sigma + 4 \beta t \omega + \beta \delta x + 2)}{\beta^2 (t \omega + \delta x)} + 16 \right).
\]

(2.30)
\[ \phi_{28}(x,t) = \left( \sqrt{a_0^2 (\beta^2 - 4\alpha\sigma) - a_0 \beta} \right) \left( \beta - \sqrt{4\alpha\sigma - \beta^2} \tan \left( \frac{1}{2} \sqrt{4\alpha\sigma - \beta^2} (t\omega + \delta x) \right) \right) + a_0, \tag{2.31} \]

\[ \phi_{29}(x,t) = \left( \sqrt{a_0^2 (\beta^2 - 4\alpha\sigma) - a_0 \beta} \right) \left( \beta + \sqrt{4\alpha\sigma - \beta^2} \tanh \left( \frac{1}{2} \sqrt{4\alpha\sigma - \beta^2} (t\omega + \delta x) \right) \right) + a_0. \tag{2.32} \]

When \( \beta^2 - 4\alpha\sigma > 0 \) & \( \sigma \neq 0 \)
\[ \phi_{30}(x,t) = \left( \sqrt{a_0^2 (\beta^2 - 4\alpha\sigma) - a_0 \beta} \right) \left( \beta + \sqrt{4\alpha\sigma - \beta^2} \coth \left( \frac{1}{2} \sqrt{4\alpha\sigma - \beta^2} (t\omega + \delta x) \right) \right) + a_0. \tag{2.33} \]

When \( \sigma \alpha > 0 \) & \( \sigma \neq 0 \) & \( \alpha \neq 0 \) & \( \beta = 0 \)
\[ \phi_{31}(x,t) = a_0 - \frac{\sqrt{-\alpha a_0^2 \sigma} \tan \left( \sqrt{-\alpha\sigma} (t\omega + \delta x) \right)}{\sqrt{-\alpha\sigma}}. \tag{2.34} \]

When \( \sigma \alpha < 0 \) & \( \sigma \neq 0 \) & \( \alpha \neq 0 \) & \( \beta = 0 \)
\[ \phi_{32}(x,t) = a_0 - \frac{\sqrt{-\alpha a_0^2 \sigma} \coth \left( \sqrt{-\alpha\sigma} (t\omega + \delta x) \right)}{\sqrt{-\alpha\sigma}}. \tag{2.35} \]

When \( \beta = 0 \) & \( \alpha = -\sigma \)
\[ \phi_{33}(x,t) = a_0 - \frac{\sqrt{-\alpha a_0^2 \sigma} \coth \left( \sqrt{-\alpha\sigma} (t\omega + \delta x) \right)}{\alpha}. \tag{2.36} \]

When \( \beta = \frac{\alpha}{2} = \kappa \) & \( \sigma = 0 \)
\[ \phi_{34}(x,t) = \frac{a_0 \kappa - \sqrt{a_0^2 \kappa^2} \left( e^{\kappa (t\omega + \delta x)} - 2 \right)}{4\kappa} + a_0. \tag{2.37} \]

When \( \beta = 0 \) & \( \alpha = \sigma \)
\[ \phi_{35}(x,t) = a_0 - \frac{\sqrt{-\alpha^2 a_0^2 \sigma} \tan \left( \sqrt{-\alpha\sigma} (t\omega + \delta x) \right)}{\alpha}. \tag{2.38} \]

When \( \sigma = 0 \) & \( \beta \neq 0 \) & \( \alpha \neq 0 \)
\[ \phi_{36}(x,t) = \frac{a_0 \beta - \sqrt{a_0^2 \beta^2} \left( e^{\beta (t\omega + \delta x)} - \frac{a}{2} \right)}{2\alpha} + a_0. \tag{2.39} \]
When $\beta^2 - 4\alpha\sigma = 0$

$$\varphi_{39}(x, t) = \frac{-2a_0}{\beta t\omega + \beta \delta x}.$$  

**(Family V)**

$$a_1 \to 0, b_1 \to \frac{2\alpha a_0}{\beta}, \kappa \to \frac{2a_0^2\lambda}{\beta^2}, \mu \to 0, \rho \to -\frac{a_0^2\lambda (\beta^2 - 4\alpha\sigma)}{\beta^2}.$$  

Thus, the solitary wave solutions of Eq. (1.7) are given by:

When $\beta^2 - 4\alpha\sigma < 0 \& \sigma \neq 0$

$$\varphi_{40}(x, t) = a_0 \left(1 - \frac{4\alpha\sigma}{\beta^2 - \beta^2\sqrt{4\alpha\sigma - \beta^2} \tan \left(\frac{1}{2}\sqrt{4\alpha\sigma - \beta^2}(t\omega + \delta x)\right)}\right),$$  

(2.44)

$$\varphi_{41}(x, t) = a_0 \left(1 - \frac{4\alpha\sigma}{\beta^2 - \beta^2\sqrt{4\alpha\sigma - \beta^2} \cot \left(\frac{1}{2}\sqrt{4\alpha\sigma - \beta^2}(t\omega + \delta x)\right)}\right).$$  

(2.45)

When $\beta^2 - 4\alpha\sigma > 0 \& \sigma \neq 0$

$$\varphi_{42}(x, t) = a_0 \left(1 - \frac{4\alpha\sigma}{\beta^2 + \beta^2\sqrt{4\alpha\sigma - \beta^2} \tanh \left(\frac{1}{2}\sqrt{4\alpha\sigma - \beta^2}(t\omega + \delta x)\right)}\right),$$  

(2.46)

$$\varphi_{43}(x, t) = a_0 \left(1 - \frac{4\alpha\sigma}{\beta^2 + \beta^2\sqrt{4\alpha\sigma - \beta^2} \coth \left(\frac{1}{2}\sqrt{4\alpha\sigma - \beta^2}(t\omega + \delta x)\right)}\right).$$  

(2.47)

When $\beta = \frac{a}{2} = \kappa \& \sigma = 0$

$$\varphi_{44}(x, t) = a_0 \left(\frac{4}{e^{\kappa(t\omega + \delta x)} - 2} + 1\right).$$  

(2.48)

When $\sigma = 0 \& \beta \neq 0 \& \alpha \neq 0$

$$\varphi_{45}(x, t) = a_0 \left(1 - \frac{2\alpha}{\alpha - \beta e^{3(t\omega + \delta x)}}\right).$$  

(2.49)

When $\beta^2 - 4\alpha\sigma = 0$

$$\varphi_{46}(x, t) = \frac{2a_0}{\beta t\omega + \beta \delta x + 2}.$$  

(2.50)

2.2. Semi-Analytical solution. Implement of the Adomian decomposition method enables rewriting Eq. (2.1) to be in the following form:

$$L\varphi(\xi) + R\varphi(\xi) + N\varphi(\xi) = 0,$$  

(2.51)

where $L, R, N$ represent a differential operator, a linear operator and nonlinear term, respectively. Using the inverse operator $L^{-1}$ on (2.51), we get

$$\sum_{i=0}^{\infty} \varphi_i(\xi) = \varphi(0) + \varphi'(0)\xi - \frac{\rho}{\kappa} L^{-1} \left(\sum_{i=0}^{\infty} \varphi_i\right) - \frac{\lambda}{\kappa} L^{-1} \left(\sum_{i=0}^{\infty} A_i\right) - \frac{\mu}{\kappa} L^{-1} \left(\sum_{i=0}^{\infty} A_i\right).$$  

(2.52)
Figure 1. Periodic soliton wave of the longitudinal displacements of the top and down strands by using Eq. (2.6) in three(a), two(c)-dimensional and contour plot(b), when \(\alpha = 2, a_0 = 6, \beta = 3, \delta = 4, \sigma = 1, \omega = 5, x \in [-1, 5], t \in [-2, 9]\)

Under the following condition \([\beta = 3, \alpha = 2, \sigma = 1, \lambda = 4, a_0 = 5, \mu = 0, a_1 = \frac{10}{3}, b_1 = 0, \kappa = \frac{200}{9}, \rho = \frac{-100}{9}\] on Eq. (2.15), we get:

\[
\varphi_{\text{exact}} = -\frac{5}{3} \tanh \left( \frac{\xi}{2} \right).
\] (2.53)

So that, we obtain:

\[
\varphi_0 = -\frac{1}{6}(5\xi),
\] (2.54)

\[
\varphi_1 = \frac{5\xi^3}{72} - \frac{\xi^5}{192},
\] (2.55)

\[
\varphi_2 = \frac{\xi^{10}}{55296} - \frac{25\xi^8}{64512} + \frac{\xi^7}{16128} - \frac{\xi^5}{576}.
\] (2.56)

Eqs. (2.54)-(2.57), yield the following form of an approximate solution of Eq. (2.1)

\[
v_{\text{approximate}} = -\frac{\xi^{13}}{12779520} - \frac{\xi^{12}}{14598144} + \frac{\xi^{11}}{337920} + \frac{\xi^{10}}{2322432} - \frac{67\xi^9}{1161216} + \frac{31\xi^7}{48384}.
\]
2.3. Numerical solutions. This section studies the numerical solutions of the DNA model by applying the B-spline techniques to which one of them is considered as the most suitable method of this model. Using the exact solution Eq. (2.53) with the following initial conditions \[
\varphi(0) = \frac{2}{5}, \varphi'(0) = \frac{2e}{1+2e}, \varphi'(1) = \frac{2e}{1+2e} - \frac{4e^2}{(1+2e)^2}, \varphi''(0) = -\frac{2}{27}, \varphi''(1) = 2e \left( \frac{8e^2}{(1+2e)^3} - \frac{2e}{(1+2e)^2} \right) + \frac{2e}{1+2e} - \frac{8e^2}{(1+2e)^2},
\]
allows applying the following B-spline schemes, as follows:

2.3.1. Cubic-Spline. Based on the cubic B-spline, the suggested solution of the ordinary differential DNA model is given by
\[
\varphi(\xi) = \sum_{i=-1}^{n+1} c_i B_i, \tag{2.58}
\]
Figure 3. Periodic kink traveling wave of the longitudinal displacements of the top and down strands by using Eq. (2.15) in three(a), two(c)-dimensional and contour plot(b), when \( \alpha = 2 \), \( a_0 = 6 \), \( \beta = 3 \), \( \delta = 4 \), \( \sigma = 1 \), \( \omega = 5 \), \( x \in [-1, 5] \), \( t \in [-2, 9] \).

Table 1. Analytical and semi-analytical solution of Eq. (2.1) at different point \( 0.1 \leq \xi \leq 1 \).

| Value of \( \xi \) | Approximate Solutions | Exact Solutions | Absolute Value of Error |
|-------------------|-----------------------|----------------|------------------------|
| 0.1               | 0.0832639583          | 0.0832639583   | 3.85979 \times 10^{-12}|
| 0.2               | 0.1661133254          | 0.1661133244   | 9.8317 \times 10^{-10}  |
| 0.3               | 0.2481417478          | 0.2481417227   | 2.50479 \times 10^{-8}  |
| 0.4               | 0.3289591155          | 0.3289588670   | 2.48485 \times 10^{-7}  |
| 0.5               | 0.4081992403          | 0.4081977707   | 1.46967 \times 10^{-6}  |
| 0.6               | 0.4855272861          | 0.4855210208   | 6.26536 \times 10^{-6}  |
| 0.7               | 0.5606472102          | 0.5606259072   | 0.0000213030             |
| 0.8               | 0.6333096392          | 0.6332482704   | 0.0000613687             |
| 0.9               | 0.7033207446          | 0.7031650088   | 0.0001557360             |
| 1                 | 0.7705527947          | 0.7701952621   | 0.0003575330             |
Figure 4. Periodic cuspon traveling wave of the longitudinal displacements of the top and down strands by using Eq. (2.57) in three(a), two(c)-dimensional and contour plot(b), when $\delta = 4$, $\omega = 5$, $x \in [-1, 5]$, $t \in [-2, 9]$

where $c_i, B_i$ fulfill the next conditions:

$$ L \varphi(\xi) = f(\xi_i, \varphi(\xi_i)) \quad \text{where} \quad (i = 0, 1, ..., n) $$

and

$$ B_i(\xi) = \frac{1}{6 \, h^3} \left\{ \begin{array}{ll}
(\xi - \xi_{i+2})^3, & \xi \in [\xi_{i-2}, \xi_{i-1}], \\
-3(\xi - \xi_{i-1})^3 + 3h(\xi - \xi_{i-1})^2 + 3h^2(\xi - \xi_{i-1}) + h^3, & \xi \in [\xi_{i-1}, \xi_i], \\
-3(\xi_{i+1} - \xi)^3 + 3h(\xi_{i+1} - \xi)^2 + 3h^2(\xi_{i+1} - \xi) + h^3, & \xi \in [\xi_i, \xi_{i+1}], \\
(\xi_{i+2} - \xi)^3, & \xi \in [\xi_{i+1}, \xi_{i+2}], \\
0, & \text{otherwise},
\end{array} \right. $$

where $i \in [-2, n + 2]$. Thus, the approximate solution is given as

$$ \varphi_i(\xi) = c_{i-1} + 4 \, c_i + c_{i+1}. \quad (2.60) $$

Substituting Eq. (2.60) and its derivatives into Eq. (2.1), lead to a system of equations. Solving this system of equations, gives the value of $c_i$. Substituting the values of $c_i, B_i$ into Eq. (2.58), gives
Figure 5. Two dimensional plot of exact and approximate solution in combined, separate, and radar plots of Eqs. (2.53), (2.57).

Table 2. Analytical, numerical, and absolute values of error of obtained solutions of Eq. (2.1)

| Value of ξ | Values of Approximate Solutions | Values of Exact Solutions | Absolute Values of Error |
|----------|---------------------------------|--------------------------|--------------------------|
| 0        | 0.000000000000                 | 0.000000000000           | 0.000000000000           |
| 0.1      | -0.0832541541                 | -0.0832639583            | 9.8042 × 10^{-6}        |
| 0.2      | -0.1660944493                 | -0.1661133244            | 0.0000188750            |
| 0.3      | -0.2481152092                 | -0.2481417227            | 0.0000265135            |
| 0.4      | -0.3289267807                 | -0.3289588670            | 0.0000320863            |
| 0.5      | -0.4081627183                 | -0.4081977707            | 0.0000350524            |
| 0.6      | -0.4854860388                 | -0.4855210208            | 0.0000349819            |
| 0.7      | -0.5605943403                 | -0.5606259072            | 0.0000315669            |
| 0.8      | -0.6332236466                 | -0.6332482704            | 0.0000246238            |
| 0.9      | -0.7031509210                 | -0.7031650088            | 0.0000140878            |
| 1        | -0.7701952621                 | -0.7701952621            | 1.11022 × 10^{-16}     |

2.3.2. Quintic-spline. Based on the quintic B-spline, the suggested solution of the ordinary differential DNA model is given as follow

\[
\varphi(\xi) = \sum_{i=-1}^{n+1} c_i B_i, \quad (2.61)
\]
where $c_i, B_i$ meet the next conditions

$$L \varphi(\xi) = f(\xi_i, \varphi(\xi_i)) \text{ where } (i = 0, 1, ..., n)$$

and

$$B_i(\xi) = \frac{1}{h^5} \begin{cases} 
(\xi - \xi_{i-3})^5, & \xi \in [\xi_{i-3}, \xi_{i-2}], \\
(\xi - \xi_{i-3})^5 - 6(\xi - \xi_{i-2})^5, & \xi \in [\xi_{i-2}, \xi_{i-1}], \\
(\xi - \xi_{i-3})^5 - 6(\xi - \xi_{i-2})^5 + 15(\xi - \xi_{i-1})^5, & \xi \in [\xi_{i-1}, \xi_i], \\
(\xi_{i+3} - \xi)^5 - 6(\xi_{i+2} - \xi)^5 + 15(\xi_{i+1} - \xi)^5, & \xi \in [\xi_i, \xi_{i+1}], \\
(\xi_{i+3} - \xi)^5 - 6(\xi_{i+2} - \xi)^5, & \xi \in [\xi_{i+1}, \xi_{i+2}], \\
(\xi_{i+3} - \xi)^5, & \xi \in [\xi_{i+2}, \xi_{i+3}], \\
0, & \text{otherwise,}
\end{cases}$$

where $i \in [-2, n + 2]$. Thus, the approximate solution is given as

$$v_i(\xi) = c_{i-2} + 26 c_{i-1} + 66 c_i + 26 c_{i+1} + c_{i+2}.$$  \hspace{1cm} (2.63)

Substituting Eq. (2.63) and its derivatives into Eq. (2.1), yield a system of equations. Solving this system, gives the value of $c_i$. Substituting the values of $c_i, B_i$ into Eq. (2.61), obtains

$$v(\xi) = \sum_{i=-1}^{n+1} c_i B_i.$$  \hspace{1cm} (2.64)

2.3.3. Septic-Spline. Based on the septic B-spline, the suggested solution of the ordinary differential DNA model is given as follow

$$\varphi(\xi) = \sum_{i=-1}^{n+1} c_i B_i,$$
Table 3. Analytical, numerical, absolute values of error of obtained solution of Eq. (2.1)

| Value of $\xi$ | Approximate | Exact     | Absolute Error |
|----------------|-------------|-----------|----------------|
| 0              | $3.46945 \times 10^{-18}$ | $0.0000000000$ | $3.46945 \times 10^{-18}$ |
| 0.1            | -0.0832639540 | -0.0832639583 | 4.22639 $\times 10^{-9}$ |
| 0.2            | -0.1661133141 | -0.1661133244 | 1.02421 $\times 10^{-8}$ |
| 0.3            | -0.2481417081 | -0.2481417227 | 1.46466 $\times 10^{-8}$ |
| 0.4            | -0.3289588492 | -0.3289588670 | 1.78158 $\times 10^{-8}$ |
| 0.5            | -0.4081977514 | -0.4081977707 | 1.92278 $\times 10^{-8}$ |
| 0.6            | -0.4855210020 | -0.4855210208 | 1.87683 $\times 10^{-8}$ |
| 0.7            | -0.5606258910 | -0.5606259072 | 1.62749 $\times 10^{-8}$ |
| 0.8            | -0.6332482583 | -0.6332482704 | 1.20805 $\times 10^{-8}$ |
| 0.9            | -0.7031650035 | -0.7031650088 | 5.25365 $\times 10^{-9}$ |
| 1              | -0.7701952621 | -0.7701952621 | 0.0000000000 |

Figure 7. Two dimensional plot of exact and numerical solution that obtained by quintic spline technique in combined, separate, and radar plots

where $c_i$, $B_i$ fulfill the following condition

$L \varphi(\xi) = f(\xi_i, \varphi(x_i))$ where ($i = 0, 1, ..., n$)
and

\[
B_i(\xi) = \frac{1}{h^2} \begin{cases} 
(\xi - \xi_{i-4})^7, & \xi \in [\xi_{i-4}, \xi_{i-3}], \\
(\xi - \xi_{i-4})^7 - 8(\xi - \xi_{i-3})^7, & \xi \in [\xi_{i-3}, \xi_{i-2}], \\
(\xi - \xi_{i-4})^7 - 8(\xi - \xi_{i-3})^7 + 28(\xi - \xi_{i-2})^7, & \xi \in [\xi_{i-2}, \xi_{i-1}], \\
(\xi - \xi_{i-4})^7 - 8(\xi - \xi_{i-3})^7 + 28(\xi - \xi_{i-2})^7 + 56(\xi - \xi_{i-1})^7, & \xi \in [\xi_{i-1}, \xi_i], \\
(\xi_{i+4} - \xi)^7 - 8(\xi_{i+3} - \xi)^7 + 28(\xi_{i+2} - \xi)^7 + 56(\xi_{i+1} - \xi)^7, & \xi \in [\xi_i, \xi_{i+1}], \\
(\xi_{i+4} - \xi)^7 - 8(\xi_{i+3} - \xi)^7 + 28(\xi_{i+2} - \xi)^7, & \xi \in [\xi_{i+1}, \xi_{i+2}], \\
(\xi_{i+4} - \xi)^7 - 8(\xi_{i+3} - \xi)^7, & \xi \in [\xi_{i+2}, \xi_{i+3}], \\
(\xi_{i+4} - \xi)^7, & \xi \in [\xi_{i+3}, \xi_{i+4}], \\
0, & \text{otherwise}, 
\end{cases}
\]

(2.65)

where \( i \in [-3, n + 3] \). Thus, the approximate solution is given by

\[ v_i(\xi) = c_{i-3} + 120 c_{i-2} + 1191 c_{i-1} + 2416 c_i + 1191 c_{i+1} + 120 c_{i+2} + c_{i+3} \]  \hspace{1cm} (2.66)

Substituting Eq. (2.66), and its derivatives into Eq. (2.1), yield a system of equations. Solving this system, give the value of \( c_i \). Substituting the values of \( c_i \), \( B_i \) into Eq. (2.64), gives

| Value of \( \xi \) | Approximate Solution | Values of \( f(\xi) \) | Exact solution | Absolute value of Error |
|-------------------|----------------------|------------------------|----------------|------------------------|
| 0                 | 0.0000000000         | 0.0000000000          | 0              | 0                      |
| 0.1               | -0.0832592356        | -0.083263958          | 4.72267 \times 10^{-6} |
| 0.2               | -0.1661049996        | -0.1661133244         | 8.32474 \times 10^{-6} |
| 0.3               | -0.2481357283        | -0.2481417227         | 5.99441 \times 10^{-6} |
| 0.4               | -0.3289523831        | -0.3289588670         | 6.48396 \times 10^{-6} |
| 0.5               | -0.4081921510        | -0.4081977707         | 5.61967 \times 10^{-6} |
| 0.6               | -0.4855155144        | -0.4855210208         | 5.50631 \times 10^{-6} |
| 0.7               | -0.5606213388        | -0.5606259072         | 4.56842 \times 10^{-6} |
| 0.8               | -0.6332432277        | -0.6332482704         | 5.04269 \times 10^{-6} |
| 0.9               | -0.7031623445        | -0.7031650088         | 2.66428 \times 10^{-6} |
| 1                 | -0.7701952621        | -0.7701952621         | 1.11022 \times 10^{-16} |

3. Results and discussion. This section studies more details about our obtained solutions and gives a comparison between these different types of solutions. Moreover, it gives a comparison between the obtained analytical and other analytical solutions obtained by different analytical schemes.

- **Interpretation of obtained solution of Khater method**
  
  This method depends on auxiliary equation (2.3) that has a general solutions which is given by

  \[
  f(\xi) = \frac{\log \left( \frac{\sqrt{4a \sigma - \beta^2} \tan \left( \frac{1}{2} \left( c_1 \log(K) \sqrt{4a \sigma - \beta^2 + \xi \sqrt{4a \sigma - \beta^2}} \right) - \beta \right)}{2 \sigma} \right)}{\log(K)},
  \]

  (3.1)
where $c_1$ is arbitrary constant. Thus, all other solutions that are obtained and discussed in this paper are special forms of solutions which got by putting special conditions on Eq. (3.1). Moreover, we represent the equivalence between modified Khater method and some other recent methods.

**More details about obtained solutions**

We applied the modified Khater method to the double chain model of DNA and obtained some different forms of solutions and some similar solutions of previous research as follows:

In Ref [21], Zahran, E. H., & Khater, M. M., used an extended Jacobian elliptic function to find the analytical solutions of the same model. They gave all solutions by using $sech$-function and is completely different of our obtained solutions in this paper.

In Ref [1], Abdelrahman, M. A., Zahran, E. H., Khater, M. M., used the exp $(-\phi(\xi))$-expansion method to find the analytical solutions of the same model. Eqs. (3.18), (3.21) are similar to Eqs. (2.8), (2.9), (2.10) under the following conditions

$$a_0 = \frac{\lambda}{\alpha} \sqrt{-\frac{2(\omega^2 + c^2_1 \omega^2)}{\lambda}}, \quad \beta = \alpha = 1.$$

We plot some figures of our obtained solutions to show more physical properties of applied model Figs. 1, 2, 3.

**Analytical and semi-analytical solution**

We applied the Adomian decomposition method and our obtained solutions, Table 1, and Figs. 4, 5 show the convergence between the analytical and semi-analytical solutions and show the powerful of the Adomian decomposition
method and for every interval that much closer to zero, we get more accurate
solutions. That leads to the absolute error between exact and approximate
nears to zero.

- **Analytical and numerical solutions**
  Applying three different form of B-spline techniques to the double chain model
  of DNA, shows the accurate of quintic spline scheme is more accurate than
  other two schemes. That accurate is shown in Figs. 6, 7, 8 and Tables 2, 3, 4

- **Stability property**
  This item discusses the stability property of the obtained analytical solutions
  of the DNA model. Using the an Hamiltonian system property, leads to

\[
M|_{\delta=3} = \frac{1}{2} \int_{-\tau}^{\tau} \varphi(\xi) d\xi = \frac{50}{27\omega} \left( 75\omega + 2 \log \left( e^{-\frac{5\omega}{2}} (e^{5\omega} + e^{15}) \right) - 2 \log \left( e^{-\frac{5\omega}{2}} (e^{5(\omega+3)} + 1) \right) \right) \tag{3.2}
\]

where \( M, \omega \) represent the momentum \( M \) in the Hamiltonian system, and co-
efficient of \( t \). Thus,

\[
\frac{\partial M}{\partial \omega}|_{\omega=5} = 2.22204736 > 0. \tag{3.3}
\]

The relation Eq. (3.3) shows the obtained solution is stable and able to use
in the DNA’s applications.

4. **Conclusion.** In this paper, studying the numerical solutions of the DNA model
by using new exact solutions is the main goal. We applied three different techniques
to the DNA model. We successfully implement analytical, semi-analytical, and nu-
merical schemes to investigate the exact and approximate solutions of this model.
We obtained novel solitary solutions of that obtained in previous research papers.
Moreover, we compare the obtained analytical solutions with the approximate solutions
to be sure of the accuracy of our obtained solutions. The performance of the
modified Khater method shows the effective and ability of this method to apply to
other models with fractional order and integer order. We used one of the obtained
analytical solutions in semi-analytical and numerical schemes to get the initial condi-
tions of the model that allows obtaining the semi-analytical and numerical solutions
of it. Accurate studying of the obtained solutions shows the effectiveness of quintic
spline scheme on the other two used methods since the obtained values of absolute
error by using this method is smaller than its analogs using other methods. We plot
some of the figures to show and discuss all the obtained distinct solutions.

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