Determination of pattern allowances for a steel casting using an inverse elastoplastic deformation analysis

D Galles¹, J Lu², and C Beckermann²

¹Oak Ridge Associated Universities, Oak Ridge, TN 37830, USA
²Department of Mechanical Engineering, University of Iowa, Iowa City, IA 52242, USA

Corresponding author: daniel.j.galles.ctr@mail.mil

Abstract. The determination of pattern dimensions using simulation is an inefficient trial-and-error process that requires several design iterations. In this study, the finite element inverse elastoplastic analysis is utilized to calculate the pattern geometry in a single iteration for a plastically deformed body. A simplified casting system is simulated to demonstrate the feasibility of the inverse method. An inverse simulation is performed first to calculate the pattern shape. This configuration is then used as the input geometry for a forward simulation, which is shown to successfully recover the original as-cast shape used for the inverse analysis. Through this sequence, the inverse deformation method is shown to be a viable technique for the determination of pattern allowances in production castings.

1. Introduction

Casting distortions are unintended deformations that occur during casting. Their presence creates dimensional inaccuracies in the as-cast part, which may lead to feature dimensions that do not adhere to customer specifications. For this reason, pattern design is inherently an iterative process for which the patternmaker alters the pattern through a trial-and-error process until the desired dimensions are achieved. For each iteration, a casting is produced and then evaluated through a dimensional analysis. Such a design strategy is not only expensive but also time-consuming. Furthermore, the efficiency of the process heavily relies on the patternmaker’s expertise, which can vary considerably.

In recent years, the remarkable increase in computer speeds has coincided with advancements in computational codes suitable for predicting stresses and distortions during casting. Such developments have paved the way for a paradigm shift from physically to digitally-based design within industry. Today’s casting simulations can be performed with good accuracy and in a fraction of the time needed to produce and analyze a physical casting. As a result, considerable time and cost savings are realized.

Despite the benefits, digitally-based pattern design is also an iterative procedure. This is due to the fact that the kinematics of FEM codes are implemented in a forward framework. In other words, the simulations input the pattern shape and calculate the as-cast shape. Hence, digitally-based pattern design remains a trial-and-error approach whose efficiency relies on the expertise of the design engineer.

Pattern design belongs to a class of engineering applications in which the unknown reference geometry must be determined. To this end, Yamada [1] and Govindjee et al. [2] pioneered an inverse method that directly solves the equilibrium boundary condition for an elastic material. These studies spurred the development of inverse finite element implementations capable of determining the reference
configuration firstly for elastic deformations and later for elastoplastic deformations. A review of studies concerning the elastic formulation is given by Lu and Li [3]. Unfortunately, deformations for the elastoplastic problem are history dependent and thus, the inverse deformation problem is generally ill-posed. A unique solution is only achieved when the loading history or the plastic strain in the deformed state is known, as demonstrated by Germain et al. [4]. However, Lu and Li [5] demonstrated that an inverse boundary value problem gives reasonably accurate results for both displacement and plastic variables for the case of moderately large deformations and nearly monotonic loading.

In this study, the direct approach of inverse analysis developed by Lu and Li [5] is utilized for plastically-deformed material bodies to determine pattern dimensions for the desired as-cast geometry shown Figure 1. The mold is excluded from the stress analyses to prevent numerical difficulties encountered with contact interactions. Consequently, deformations are induced exclusively by thermal stresses. A sequential thermal-mechanical analysis is performed in which the calculated transient temperature fields are inputted into the stress analyses. Loop tests are conducted, in which the inverse problem is solved first to determine the pattern shape, which is subsequently used as the input geometry for the forward analysis. The inverse technique is evaluated by the forward problem’s ability to recover the original geometry used for the inverse analysis. Finally, the accuracy of the inverse elastoplastic solution is evaluated through an error analysis.

2. Inverse elastoplastic problem
The inverse elastoplastic problem presented here calculates the pattern geometry from the desired as-cast shape. The model reviewed here is the general quasi-static formulation presented by Lu and Li [5] with the following simplifications. Due to the relatively small casting sizes, body forces will have a negligible impact on distortions and therefore, are not considered. Also, the exclusion of the mold from the mechanical problem precludes the need to include surface tractions in the formulation. These simplifications reduce the boundary value problem to

\[ \sigma_{ij} = 0 \text{ in } \Omega \]
\[ \Phi = \overrightarrow{\Phi} \text{ on } \partial \Omega_e \]

where \( \sigma_{ij} \) is the Cauchy stress and \( \overrightarrow{\Phi} \) is a prescribed displacement on boundary \( \partial \Omega_e \). Equation 1 determines an inverse motion \( \Phi : \Omega \rightarrow B \in R^3 \) for the sought reference configuration, \( B \), when starting from the given current configuration, \( \Omega \). Such a relation is the kinematic inverse of the usual forward deformation \( \phi(X, t) \) of a material point that progresses from the reference coordinate to the current coordinate. Additional details of the formulation can be found elsewhere [5].

During the inverse solution procedure, the material response obeys a Hencky constitutive relation, which is very close to a linear elastic response at small strains, provided the stress levels do not exceed the yield strength of the casting. For the elastic properties, a constant value of 0.3 was specified for Poisson’s ratio and a temperature-dependent Young’s modulus was taken from Koric and Thomas [6].

![Figure 1. Desired as-cast plate casting geometry. Dimensions in mm.](image-url)
For the case of yielding, the following elasto-plastic constitutive relation is invoked:

\[ \sigma = \sigma_0 \left(1 + \varepsilon_p^p \right)^n \]  \hspace{1cm} (2)

where \( \sigma \) is the von Mises stress, \( \sigma_0 \) is the initial yield stress, \( \varepsilon_p \) is the equivalent plastic strain, and \( n \) is the hardening exponent. Temperature-dependent model parameters \( (\sigma_0 \text{ and } n) \) were determined using a Levenberg-Marquardt non-linear least squares algorithm. Thermal strains were calculated from [7].

3. Numerical example

3.1. Introduction

During casting, cooling induces thermal strains that may lead to considerable stresses and associated distortions. For simple casting geometries without internal features (e.g., holes), minimal mechanically-induced distortions can be expected after solidification, as the casting quickly gains the necessary strength to prevent any distortions created by mold-metal interactions. This rationale motivated the design shown in Figure 1. Dimensions are shown in mm. The inclusion of simplified risers and a gating system produced the varying section thicknesses that naturally led to uneven cooling.

The numerical examples presented in this section were conducted utilizing a sequential thermal-mechanical coupling. Heat transfer simulations were performed first to calculate the transient temperature fields, which were then used as inputs for the forward and inverse stress analyses. A loop test served to evaluate the inverse method. For this test, the inverse analysis was performed first to determine the pattern shape, which was subsequently used as the input geometry for the forward analysis. Validation of the inverse technique is based on the loop test’s ability to recover the initial geometry used for the inverse analysis.

3.2. Heat transfer simulations

For this study, spatial temperature gradients drive the thermal loading that generates distortions. Temperatures were calculated using the casting simulation software MAGMASOFT® [8]. In order to generate realistic casting temperatures, the bonded sand mold was included in the simulation. MAGMASOFT® uses the finite volume method to calculate casting and mold temperatures during filling and cooling. Simulation inputs include temperature-dependent thermophysical properties for the casting and mold. In addition, the temperature-dependent solid volume fraction during solidification and latent heat of solidification are specified in order to calculate heat transfer due to latent heat effects. Finally, an interfacial heat transfer coefficient is inputted to simulate the formation of an air gap at the mold-metal interface. This parameter allows for the partial decoupling of the thermal-mechanical problem. The entire calibration procedure for the heat transfer model can be found elsewhere [9].

Temperature fields at various times are shown at the casting mid-plane of the plate casting in Figure 3. Spatial temperature gradients, caused by variations in section thicknesses throughout the geometry, can be seen throughout the casting, most notably at \( t = 5 \) min and \( t = 21 \) min. After these times, the plate temperature fields are nearly isothermal. The calculated temperature fields were written at a sufficient number of time steps to ensure a smooth profile and copied onto the finite element mesh.
3.3. Stress simulations

3D Stress simulations were performed using the general purpose finite element code FEAP. In order to prevent rigid body translations and rotations, minimal boundary conditions were specified. Due to symmetry, only $\frac{1}{4}$ of the geometry was modelled.

Displacement differences between the forward and inverse analyses were quantified through a configuration error, $\text{err}(\text{conf})$, which was computed by normalizing differences in the current position $x$ by the forward displacement $u$:

$$\text{err}(\text{conf})[\%] = \frac{\int_{\Omega} \| x_{\text{Inverse}} - x_{\text{Forward}} \| dv}{\int_{\Omega} \| u \| dv} \times 100$$

(3)

In Equation 3, $x_{\text{Inverse}}$ refers to the input geometry for the inverse simulation, whereas $x_{\text{Forward}}$ relates to the calculated coordinates of the forward simulation.

In addition to the computed errors, the final vertical profile of the casting’s top edge is plotted to help the reader visualize the deformations. Also, the temporal evolution of vertical displacement for a single point located at the upper left corner of each casting is plotted.

For the loop test, the inverse analysis was performed first. The uneven cooling calculated by the heat transfer analysis resulted in the calculated configuration shown in Figure 3(a), which reveals 1) the ends of the plate are distorted upward and 2) the casting length increases. Keep in mind that this deformation was calculated within the inverse framework and is therefore reverted. In reality, the plate ends will distort downward. Similarly, the usual thermal contractions that occur during casting are manifested as expansions for the inverse problem. The shape calculated by the forward simulation (see Figure 3(b))
reverses the input geometry used for the inverse analysis. The final vertical displacement at the top edge for both configurations is plotted as a function of axial position in Figure 3(c). Thermal and plastic strains cause the vertical position of the top edge to evolve from a constant vertical height of 25 mm to the profile shown by the blue curve for the inverse simulation. The forward simulation subsequently recovers the constant vertical profile of 25 mm, as demonstrated by the red curve in Figure 3(c).

The temporal evolution of vertical displacement for a point at the top left corner of the plate is plotted on complete (80,000 s) and 5000 s time scales in Figure 4. At 100 s, the forward curve begins to decrease downward before reaching a minimum value at 600 s. This is caused by rapid cooling and in the gating, which triggers the onset of thermal contraction. Meanwhile, the plate is still mostly liquid and therefore, relatively weak. As a result, thermal contractions in the gating create a bending moment that distorts the plate ends downward. Eventually, the plate solidifies and begins to thermally contract, which counteracts the bending moment that caused the initial downward distortion. Consequently, the plate ends deflect upward until approximately 4500 s. Subsequent negative changes in forward displacement are due to additional thermal strains. The final casting shape is characterized by a slight downward distortion at its ends, which is manifested as a negative vertical displacement at 80,000 s for the forward curve. The forward curve increase from 7000-10,000 s is attributed to a volumetric expansion that accompanies the decomposition of austenite to ferrite and cementite. The final casting shape is characterized by a slight downward distortion at its ends, which is manifested as a negative vertical displacement at 80,000 s for the forward curve. This final shape was anticipated due to the generation of plastic strains at early times that increased the length at the top of the bar and prevented the bar ends from returning to their original vertical position. In general, the configuration error shown in Figure 4(c) is reasonably small (< 2%). The average configuration error for the entire plate casting system is 1.78%.

Figure 3. Plate configurations calculated by the inverse (a) and forward (b) simulations. The vertical position along the top edge of the plates is plotted as a function of axial position in (c). A 10x deformation factor is used in (a) and (b).
A comment is warranted regarding the vertical displacement plots in Figure 4, for which the displacement evolutions were equal but opposite. This observation may prompt the reader to suggest that one could simply subtract the forward displacement from the original configuration in Figure 1 to determine the pattern shape. Such a strategy will predict the pattern shape with good accuracy, provided that the displacements are small. However, for moderately large displacements, the initial configurations for the forward and inverse cases may differ considerably, for which case the calculated displacements will not mirror each other. Thus, the inverse deformation analysis is preferable, as it provides a robust and general method for accurate calculation of pattern dimensions over a wide range of deformations.

Finally, spatial limitations prevented the authors from adding additional stress and strain contour plots. It is notable, however, that the average equivalent plastic strain error (< 2.5%) was similar to the displacement error. Additionally, the simulations calculated monotonic plastic strain evolution, a necessary condition specified by Lu and Li [5].

4. Conclusions
This study explored the viability of the finite element inverse deformation analysis to calculate pattern allowances for steel castings. The basic concept involves finding, by solving a boundary value problem, a deformation that is the kinematic inverse of that for the usual forward analysis. As a result, the inverse analysis essentially works backward, starting from the desired as-cast configuration, to determine pattern dimensions in a single design iteration. For this study, the accuracy of the inverse analysis was demonstrated through a loop test, for which the average difference between inverse and forward configurations was found to be less than 2%.

The finite element inverse deformation analysis provides an efficient means to calculate pattern dimensions. Although this study represents an important step towards achieving this goal, further work
is needed before this technique can be applied to production castings. Primarily, mechanical interactions between the casting and mold must be considered. Currently, the authors only considered distortions caused by thermal stresses. Overcoming this issue is paramount to the success of the inverse deformation analysis. It should also be noted that accuracy of the inverse analysis was evaluated based on comparisons to the forward analysis. Thus, the importance of the forward simulation’s predictive capability cannot be overstated. The present material model is based one that was previously calibrated for a low alloy steel using in situ data. If the finite element inverse deformation analysis is used for other casting materials, care must be taken to properly calibrate the associated material model parameters for the forward problem. Despite these concerns, the present results lend confidence to the inverse deformation analysis for its ability to determine pattern allowances accurately and efficiently.

Acknowledgements
Research was sponsored by the Army Research Laboratory and was accomplished under Cooperative Agreement Number W911NF-18-2-0161. The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the Army Research Laboratory or the U.S. Government. The U.S. Government is authorized to reproduce and distribute reprints for Government purposes notwithstanding any copyright notation herein.

References
[1] Yamada T 1995 Technical Report No. 20
[2] Govindjee S and Mihalic PA 1996 Comput. Methods Appl. Mech. Eng. 136 pp 47-57
[3] Lu J and Li L 2016 Comput. Methods Appl. Mech. Eng. 310 pp 189–207
[4] Germain S, Steinmann P, and Menary G 2011 AIP Conf. Proc.-American Institute of Physics. 1353 pp 1169-74
[5] Lu J and Li L 2017 Finite Elements in Analysis and Design 130 pp 1-11
[6] Koric S and Thomas BG 2006 Int. J. Numer. Methods Eng. 66 pp 1955-89
[7] Galles D and Beckermann C 2016 Metall. Mater. Trans. A. 47 pp 811-29
[8] Magma GmbH, MAGMAsoft, Kackerstrasse 11, 52072. Aachen, Germany
[9] Galles D and Beckermann C 2011 Proc. of the 65th SFSA Tech. and Operating Conf. Chicago