Umklapp scattering at reconstructed quantum-Hall edges

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Abstract

We study the low-lying excitations of a quantum-Hall sample that has undergone edge reconstruction such that there exist three branches of chiral edge excitations. Among the interaction processes that involve electrons close to the three Fermi points is a new type of Umklapp-scattering process which has not been discussed before. Using bosonization and a refermionization technique, we obtain exact results for electronic correlation functions and discuss the effect Umklapp scattering has on the Luttinger-liquid properties of quantum-Hall edges.

Keywords: Quantum Hall effect, Edge reconstruction, Umklapp scattering

1. Introduction

The electronic structure at the edge of quantum-Hall (QH) systems depends sensitively on the interplay between the external potential confining the electrons to the finite sample, electrostatic repulsion, as well as exchange and correlation effects. For an ultimately sharp edge \cite{1}, a single branch of chiral one-dimensional (1D) excitations is predicted to exist when the filling factor $\nu = 1/m$ where $m$ is an odd integer \cite{2}. In that case, the dynamics of edge excitations can be described \cite{3} using a Tomonaga-Luttinger (TL) model \cite{4} with only the right-moving \cite{5} degrees of freedom present. However, for a confining potential that is just not sharp enough to stabilize a single-branch edge, a different configuration is realized where a lump of electron charge is separated from the bulk of the QH sample \cite{6,7}. Such a reconstructed edge supports three branches of chiral 1D edge excitations, two right-moving and one left-moving. For even weaker confining potential, further reconstructions occur, leading to a proliferation of edge-excitation modes \cite{8}. The microscopic structure of a very smooth edge is dominantly determined by electrostatics, which favors a phase separation of the 2D electron system at the edge into a series of alternating compressible and incompressible strips \cite{9}.

Effective TL theories \cite{3} describing single-branch and multi-branch QH edges predict Luttinger-liquid behavior, i.e., power laws governing the energy dependence of electronic correlation functions. The characteristic exponents of these power laws depend, in general, on details of the microscopic edge structure. However, in the absence of coupling between different chiral edge branches or, in some cases, due to disorder effects \cite{10}, power-law exponents turn out to be universally dependent on the bulk filling factor. At present,
microscopic details of the edge structure that is realized in experiment [11] are not fully known. To facilitate a realistic comparison between theory and experiment, it is necessary to study the low-lying edge excitations of reconstructed and smooth edges and investigate interaction effects on the Luttinger-liquid power-law exponents when more than one branch of edge excitations is present.

2. Derivation of the effective edge theory

We focus on the edge of a spin-polarized [12] QH sample at $\nu = 1$ that has undergone reconstruction such that three branches of edge excitations are present. To be specific, we choose the Landau gauge where lowest-Landau-level (LLL) basis states $\chi_k(x,y) = \Phi_k(y) \exp\{ikx\}/\sqrt{L}$ are labeled by a 1D wave vector $k$. Here, $L = \sqrt{\hbar c/eB}$ denotes the magnetic length, $L$ is the edge perimeter, and $\Phi_k(y) = \exp\{-y(k^2/2\ell^2)/(2\ell^2)\}/\sqrt{\pi^2/2\ell}$. In the absence of interactions between different edge branches, the ground state would be a generalized Fermi-sea state that is a Slater determinant of LLL basis states whose wave-vector label satisfies $k \leq k_F(R)$ or $k_F(W) \leq k \leq k_F(B)$. The Fermi ‘surface’ consists of three (Fermi) points $k_F(R) < k_F(W) < k_F(B)$. As in Tomonaga’s approach to interacting 1D electron systems [4], long-wave-length electronic excitations at the reconstructed edge can be identified according to which Fermi point they belong to. This makes it possible to rewrite the long-wave-length part of the electron operator as follows:

$$\psi(r) = \Phi_{k_F}^{(R)}(y) e^{ik_F(R)x} \chi^{(R)}(x) + \Phi_{k_F}^{(W)}(y) e^{ik_F(W)x} \chi^{(W)}(x) + \Phi_{k_F}^{(B)}(y) e^{ik_F(B)x} \chi^{(B)}(x) .$$  \hspace{1cm} (1)

Here, $r = (x, y)$ denotes the coordinate vector in the 2D plane, and the operator $\psi^{(R,W,B)}(x)$ creates an electron belonging to the chiral 1D edge branch labeled R, W, B, respectively. The interaction part of the 2D Hamiltonian for electrons in the LLL is

$$H_{\text{int}} = \frac{1}{2} \int d^2r \int d^2r' \ V(r - r') \phi(r) \phi(r') ,$$  \hspace{1cm} (2)

where $\phi(r) = \psi^\dagger(r)\psi(r)$ is the electron density. We consider the case when electrons interact via unscreened Coulomb interaction,

$$V(r - r') = \frac{e^2/\epsilon}{\sqrt{(x - x')^2 + (y - y')^2}} .$$  \hspace{1cm} (3)

We obtain the low-energy part of $H_{\text{int}}$ by inserting Eq. (1) into Eq. (2); it is effectively 1D and comprises various scattering processes of electrons that are close to one of the three Fermi points. Terms corresponding to forward scattering and backscattering [13] have been discussed before [7]. Together with the one-body part of the original 2D Hamiltonian, $H_{\text{TL}}$, which is quadratic in the Fourier components $\phi_k^{(\alpha)}$ of the chiral 1D densities $\psi_k^{(\alpha)}(x)$ (here, $\alpha \in \{R, W, B\}$). In the long-wave-length limit, where Coulomb matrix elements dominate the bare Fermi velocities, the three normal modes of $H_{\text{TL}}$ are [7] a) the edge-magneto-plasmon mode, $\rho^{(\text{emp})} = \rho^{(B)} + \rho^{(R)} + \rho^{(W)}$, which is right-moving, and b) two linearly dispersing neutral modes, a right-moving one given by $\rho^{(\text{nm})} = (\rho^{(B)} - \rho^{(R)})/\sqrt{2}$, and the left-moving neutral mode $\rho^{(\text{ln})} = (\rho^{(B)} + \rho^{(R)} + 2\rho^{(W)})/\sqrt{2}$.

3. Umklapp scattering

In addition to forward and backscattering, the following term occurs in the effective 1D Hamiltonian describing the low-energy excitations of a reconstructed QH edge:

$$H_U = \int dx \int dx' \ V_U(x - x') \left\{ [\psi^{(R)}(x)]^\dagger [\psi^{(B)}(x')]^\dagger \times \psi^{(W)}(x') \psi^{(W)}(x) e^{iD\frac{\delta}{\ell^2} - i\delta \frac{\ell}{\delta} + \text{H.c.}} \right\} ,$$  \hspace{1cm} (4)

Here we introduced the parameters $\delta = k_F^{(B)} + k_F^{(W)} - 2k_F^{(R)}$ and $D = k_F^{(B)} - k_F^{(R)}$. The distance $D\ell^2$ corresponds to the width of the edge. Note that $H_U$ represents interaction processes and their time-reversed version where two electrons from the left-moving W-brancho are scattering off each other such that one of them ends up in the right-moving R-
Therefore, the particle number in each edge branch separately. We linearized the non-interacting single-particle dispersion close to intersection points with the chemical potential $\mu$ that define the three fermi points. Shown is one interaction process represented by $H_U$ where two left-moving electrons from the W-branch are scattered into the R and B-branches.

branch and the other one in the right-moving B-branch. (See Fig. 1.) Interaction processes converting two left-movers into two right-movers (and vice versa) are familiar from lattice models for conventional interacting 1D electron systems; there they are called Umklapp processes [14]. Based on that analogy, we adopt the term Umklapp scattering for the interaction processes given by $H_U$. Note that momentum conservation implies a commensuration issue for Umklapp scattering. In the 1D Hubbard model, low-energy properties are only affected by Umklapp processes if the large momentum transfer accrued ($4k_F$) is close to a reciprocal-lattice vector [14] which is the case, e.g., at half-filling. Similarly, Umklapp scattering at a reconstructed edge is most relevant in the symmetric case when $\delta = 0$. Note also that the matrix element $V_U(x - x')$ gets small rapidly with increasing $D$ and $\delta$; it is given by

$$V_U(x - x') = \frac{e^2}{\epsilon \ell} \exp \left\{ -\frac{e^2}{\epsilon} \left[ \frac{\delta^2 + D^2}{2} \right] \right\} \sqrt{\frac{2}{\pi \ell}} \times \int d\kappa \frac{\exp \left\{ -\frac{1}{2} \kappa - \ell D/2 \right\}}{\sqrt{(x - x')^2 + \ell^2 + \kappa^2}}. \tag{5}$$

4. Bosonization

The new Umklapp process does not conserve particle number in each edge branch separately. Therefore, $H_U$ cannot be written in terms of a Tomonaga-Luttinger model. However, using the bosonization identity[15] for the 1D fermionic operators,

$$\psi^{(R)}(x) = 1/\sqrt{L} \exp[i \phi^{(R)}(x)], \tag{6}$$

$$\psi^{(W)}(x) = 1/\sqrt{L} \exp[-i \phi^{(W)}(x)], \tag{7}$$

$$\psi^{(B)}(x) = 1/\sqrt{L} \exp[i \phi^{(B)}(x)] \tag{8}$$

where $\ldots$ symbolizes normal ordering, and

$$\phi^{(\alpha)}(x) = i \frac{2\pi}{L} \sum_{q \neq 0} \frac{e^{-iqx}}{q} \hat{q}^{(\alpha)} \tag{9}$$

it is possible to rewrite $H_U$ entirely in terms of bosonic degrees of freedom:

$$H_U = 2\Lambda^2 g_U \int dx \cos \left[ \phi^{(in)}(x) + \delta x \right], \tag{10}$$

$$\phi^{(in)}(x) = \phi^{(R)}(x) + \phi^{(B)}(x) + 2\phi^{(W)}(x). \tag{11}$$

Expressions of the form (10) are routinely obtained within the bosonized description of Umklapp scattering in conventional 1D electron systems [14]. The parameter $\Lambda$ is a physical ultraviolet cut-off; for the situation considered in this work, we have $\Lambda < \sim (D\ell^2)^{-1}$. The effective Umklapp coupling constant, $g_U$, is derived from the original Coulomb interaction between the electrons. It is given by

$$g_U = e^2 \exp \left\{ -\frac{\ell \delta^2 + D^2}{8} \right\} \sqrt{\frac{2}{\pi}} \times \int d\kappa e^{-\frac{1}{2} \kappa} \int_{0}^{\infty} d\eta \frac{\sin[\ell D\eta/2]}{\sqrt{\eta^2 + \kappa^2}}. \tag{12}$$

Note that $\phi^{(in)}(x)$ is a chiral boson field given in terms of the Fourier components $\hat{q}_q^{(in)}$ as expressed in Eq. (9). The Umklapp part of the Hamiltonian, $H_U$, introduces a self-interaction of the left-moving neutral normal mode of $H_{TL}$. The edge-magnetoplasmon mode and the right-moving neutral mode are unaffected by Umklapp scattering and remain free.

5. Exact solution: Summary of results

We have been able to solve the theory including Umklapp exactly for arbitrary $\delta$ and $g_U$ us-
ing a refermionization technique [16] whereby the Hamiltonian for the chiral 1D bosonic field $\phi_{\text{in}}(x)$ with interaction $H_U$ is mapped onto that of a chiral 1D pseudo-spin-$1/2$ fermion in an external magnetic field that is perpendicular to the pseudo-spin quantization axis. In addition to the characteristic energy scale for Umklapp scattering, given by $\Delta_0 = 2\Lambda g_U$, the parameter $\xi = h\nu_{\text{in}}|\delta|/(2\Lambda g_U)$ emerges from the calculation that measures the ineffectiveness of Umklapp scattering due to deviation from perfect commensuration. We obtain the spectral functions for tunneling into the chiral edge branches. Due to Umklapp scattering, a crossover occurs in their energy dependence between different power laws. With $\alpha \in \{R, W, B\}$, we find

$$A^{(\alpha)}(\varepsilon) \propto \begin{cases} \varepsilon^{[\lambda_{\text{emp}}(\alpha)]^2 + [\lambda_{\text{emp}}(\alpha)]^2 - 1} & \text{for } \varepsilon < \Delta \\ \varepsilon^{[\lambda_{\text{emp}}(\alpha)]^2 + [\lambda_{\text{emp}}(\alpha)]^2 + [\lambda_{\text{emp}}(\alpha)]^2 - 1} & \text{for } \varepsilon > \Delta \end{cases},$$

(13)

where the crossover energy scale is $\Delta = \Delta_0 \left[ \sqrt{1 + \xi^2} - \xi \right]$, and the $\lambda_{\beta}^{(\alpha)}$ are the Bogoliubov coefficients relating the chiral density fluctuations localized at the R,W,B branches to the normal modes of $H_{\text{TL}}$. Based on a realistic model for a reconstructed edge, we obtained the order-of-magnitude estimate $\Delta \sim 10 \ldots 100 \mu\text{eV}$. As the central result of our study, we find that Umklapp scattering diminishes the value of the power-law exponent in the tunneling density of states below the characteristic energy scale $\Delta$.

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