Exploring the relationship between teachers’ instructional and students’ geometrical thinking levels based on van Hiele theory

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Abstract. Geometry is one of the mathematics topics learned in school. Therefore, it is important to explore how the geometry was taught. Lines and angles material at grade VII Junior High School were chosen to meet this purpose. This study investigated the teacher instructional practice and students' geometrical thinking levels based on van Hiele theory. The correlation between teachers' instructional and students' thinking levels was determined. The teacher instructional practices were observed and analyzed, then students’ levels of geometry thinking were accessed using the van Hiele test. The results of this study were descriptions and explanations of the relationship between the learning phases and students' geometrical thinking level based on van Hiele theory.

1. Introduction
Geometry is an important mathematics topic in school curriculum [1][2][3]. Usiskin [4] states that there are several reasons why geometry needs to be learned in school, i.e: geometry generally studies the visual form, geometry is related to the physical realm, and geometry is a mathematical system. Although geometry is an integral part of the curriculum, many students fail to develop an in-depth understanding of basic geometry concepts [5]. Furthermore, according to the National Council of Teachers of Mathematics [1], the learning of geometry up to the 12th grade should be enriched by the ability to analyze the properties of geometry and basic arguments in understanding the relationships between that properties. Referring to the curriculum and the problem, it is necessary for the teacher to conduct an evaluation in order to choose the right learning method to enhance students outcome in learning geometry.

One of the learning theories which can improve students' thinking ability levels in studying geometry is the van Hiele theory [3]. This theory can be used as an instructional teacher in the learning process and at the same time to assess the ability of students [5][6]. The choice of using van Hiele theory is believed as the best way for the teacher in teaching geometry. Therefore, the purpose of this study is to explore the relationship between the teachers’ instructional practice and the thinking ability level of junior high school students in studying geometry, especially lines and angles material. The main results of this study are prior knowledge for conducting learning activities and can be used as a basis for building students' knowledge and skills [7]. Further, an overview of the van Hiele theory presented in the next section.
2. Literature review and theoretical framework

According to Pierre van Hiele and Dina van Hiele-Geldof [8], students in geometry study will go through five levels of hierarchical thinking ability. Students cannot get one level of thinking (n level) without passing the previous level (n-1 level). The five levels are level 1 (visualization), level 2 (analysis), level 3 (informal deduction), level 4 (deduction), and level 5 (rigor). More will be explained as follows:

- Level 1 (visualization). At this level, students use their thoughts in visual form. Students recognize geometrical shapes based on their "overall" form and compare the shapes based on a given figure or everyday objects. They use simple language. But it cannot identify the properties of geometrical shapes [9].
- Level 2 (analysis). At this level, already seen the existence of student analysis to the concept and properties of geometrical shapes. Students can determine the properties of a figure through observation, measurement, drawing, and modeling. However, students have not been able to fully explain the relationship between the properties of a shape [6].
- Level 3 (informal deduction). At this level, students already understood the relationship between the properties of a shape (e.g., in a quadrilateral, opposite sides being parallel necessitates opposite angles being equal) and students can establish the interrelationships of properties both within shapes (e.g., a square is a rectangle because it has all the properties of a rectangle). It shows students can give informal arguments. Thus they can deduce properties of a shape and recognize classes of shapes [6].
- Level 4 (Deduction). At this level, students can present deductive geometric proofs. They have been able to distinguish between necessary and sufficient conditions. They recognize which properties are implied by others. They have understood the role of definitions, theorems, axioms, and proofs [9].
- Level 5 (Rigor). At this level, the student can work in a class of axiomatic systems, that is, non-Euclidean geometries can be studied, and many systems can be compared [6]. The student can compare systems based on different axioms and can study multiple geometries in the absence of concrete model [10].

According to some experts, besides to these five levels of thinking, there is a level of geometric thinking of students who stated not yet to the category level 1 base on van Hiele's theory. The level is called level 0 or the pre-visualization level [7]. In relation to these five levels of geometric thinking, there are also five phases of sequential geometry study according to van Hiele's theory, namely phase of inquiry/information, directed orientation, explication, free orientation, and phase of integration [5][6][8]. The five phases of learning will be explained as follows:

- Phase 1: Inquiry/Information. Teacher and students do asking, then students observe the examples and not examples of a concept through existing information [9]. The purpose of this activity are: (1) the teacher learns the first knowledge of the students on the topic discussed. (2) the teacher learns the instructions will do to determine to be taken in the next lesson [6].
- Phase 2: Directed Orientation. Instructions are designed to explore problems or objects (by rotating, folding, measuring, drawing) to obtain the implicit nature of an example/concept with teacher guidance [8].
- Phase 3: Explication. Students make a temporary conclusion with their own language and communicate the results of the discussion then do discuss between student and student as well as between teacher and students [6].
- Phase 4: Free orientation. At this phase, instructions are designed to explore complex tasks independently to find relationships (for example, knowing the characteristics of one type of shape, investigating these properties for a new form, such as a kite) [8].
- Phase 5: Integration. At this phase, students summarize all the lessons learned and then reflect for gain new knowledge [8].
3. Method
This research type is descriptive qualitative research which sees the framework of geometry learning in one of the Junior High Schools in Lombok, Indonesia. This research did conduct when the instruction activity of lines and angles material. When the instruction it, every data correlated including this research did collect. Subjects in this study are the teacher of mathematics and students Grade VII numbered 22 students with age range 12 to 13 years. So, the practice of teaching and learning geometry seen in this study is a description of the class. In this research, the instruction was designed by the researcher and then discussed with the teacher to get the same perception of the planned learning. Thus teacher prior do learning activities positively have understood the lesson plan well.

In this research, data collection techniques are using tests and observations. The steps, students are beginning given a test that measures the levels of students' geometry thinking abilities based on Van Hiele's theory. The test consists of 4 item questions where per item contains concerning level 1 (visualization), level 2 (analysis), level 3 (informal deduction), and level 4 (deduction). Why the test made to level 4? because according to some experts, students of the junior high school have a level of thinking only up to level 3 (informal deduction) [11]. The results of this test are used to study the suitability between teacher's instructional practice and the geometrical thinking levels of students display. The occasional classroom observation uses the video recorder, then analyzed using the indicator of thinking level of van Hiele theory geometry developed by Crowley in 1987 [6].

4. Results and Discussion
The test was given to students of grade VII Junior High School of 22 students who have been selected as research subjects. The test consists of 4 item questions where per item contains concerning level 1 (visualization), level 2 (analysis), level 3 (informal deduction), and level 4 (deduction). The recapitulation of the test results of the students' geometrical thinking levels did display in table 1.

| No | Van Hiele Levels        | Percentage (N=22) |
|----|-------------------------|-------------------|
| 1  | Level 0: Pre-visualisation | 2 (9.1%)          |
| 2  | Level 1: Visualisation  | 8 (36.4%)         |
| 3  | Level 2: Analysis       | 7 (31.8%)         |
| 4  | Level 3: Informal Deduction | 4 (18.2%)       |
| 5  | Level 4: Deduction      | 1 (4.5%)          |

From table 1, we can see that there are 9.1% of students who fall into the category level 0 (pre-visualisation). This level is the level of thinking where students are declared not yet reached to level 1 van Hiele theory. Students at level 0 are students who cannot answer a single test given correctly. Furthermore, there are 36.4% of students who fall into the level 1 (visualization) category. That means there a total of 8 students who can only answer item number 1 correctly. Then there a total of 7 students who can answer number 1 and number 2 correctly, so the percentage toward level 2 (analysis) is 31.8%. Next, the percentage of level 3 (informal deduction) is 18.2%. That shows there a total of 4 students who can answer number 1, number 2, and number 3 correctly. In this study, from the test result was found that there a total of 1 student who could answer the test question until number 4 correctly. Consequently, that findings then cross with De Walle's statement [11] which states that students of grade VII up to grade VIII of junior high school can only include in the level 3 (informal deduction) category.

Here are examples of the questions and student answers on each of the test items that represent the students' geometric thinking levels. The geometrical shape used in the test can be seen in figure 1.
Figure 1. The geometrical shape used in the test.

Item question of number 1 asked, "Based on figure 1, (parallelogram, square, rectangle, and rhomb) are shown on the number?". The overview of the student's answer at level 1 (visualization) for question number 1 can be seen in figure 2.

From figure 2, student responses at level 1 can only answer that "parallelogram shown by number 4, square = number 1, rectangle = number 2, and rhombus = number 3". Students at level 1 can not answer that "parallelogram can be indicated by number 4, number 3, or number 2" or "rectangle can also be indicated by number 1". The description of this student's answer corresponds to the opinion that "students at level 1 only see the form of the shape through what is seen, not analyze the properties of the shape" [9].

At item test of number 2, students are given the properties of the parallelogram "a) opposite sides are congruent, b) opposite sides are parallel, c) opposite angles are congruent, and d) diagonals bisect each other". Then from that properties, the students were asked: "what is the same and different between the properties of the rectangle with the parallelogram and properties of the rhombus with the parallelogram?". The overview of sample student answers at level 2 (analysis) for question number 2 can be seen in figure 3.

Based on the properties of the parallelogram are given, figure 3 shows the student answered "the rectangle has equal properties as the parallelogram, but the rectangle has right-angles (90°). Then the rhombus has the equal properties as the parallelogram, but the rhombus has all sides are congruent". From that answer, students can already analyze and mention properties of the geometrical shapes. The students imply included in the students' level 2. This is in line with the statement that "students at level 2 can determine the properties of a figure through observation, measure, draw, and modeling. However, students have not been able to explain the relationship between the properties" [6].

Then item question of number 3 asked: "is a rectangle including a parallelogram? Why?" and "is a rhombus including a parallelogram? Why?". Question number 3 is extended of question number 2.
Example of student answers at the level 3 (informal deduction) to question number 3 can be viewed in figure 4.

![Jawab: a. Ya, karena... b. Ya, karena...]

**Figure 4.** The overview of the student's answer at level 3.

From figure 4, the student answers "a) Yes, a rectangle is a parallelogram. Since the rectangle has the properties of the parallelogram and b) Yes, a rhombus is a parallelogram because the rhombus has the properties of the parallelogram". This student's response is a conclusion given based on the answer in number 2. If students able to answer like that, then students imply included in level 3 category. The students have been able to see the relationship between the two geometric shapes. This is in line with the opinion that "students at level 3 can already understand the relationship of properties between two or more shapes" [5].

Meanwhile, the item question of number 4 asks the students to make a conclusion about the definition of rectangles and rhombus based on the student's answers at number 2 and number 3. This question item used to measure whether there are students who belong to the level 4 (deduction) category. The example student answers at level 4 (deduction) can be seen in figure 5.

![Belah ketupat adalah jajar genjang... a. Persegi panjang adalah jajar genjang...]

**Figure 5.** The overview of the student's answer at level 4.

From figure 5, the student concludes that "a) the rhombus is a parallelogram that has the same side length and b) the rectangle is a parallelogram having a right-angle (90°)". Based on these answers, in this research, there exist student who is already in the category level 4 (deduction), although only a student (see table 1). Students at this level can already understand the role of definition or theorem [6].

Based on the result test progress of the students' geometrical thinking level, this can be a guide to see if there is any difference between the teachers' instructional practice and the students' thinking levels or not. Overview of the relationship can be seen in the following description.

4.1. **Overview of level 1 (visualisation)**

From table 1 there are 8 of 22 students included level 1 category. Students at level 0 (pre-visualization) also included in the treatment of the same learning activity as level 1. This means that in this research subject most of the students are at level 1 (visualization). Observation of learning starts with the introduction of the concept of point, lines, and flat (two-dimension). Then learning continued with differentiating the concept of lines and curves. The teacher asks the students to show which are lines and which are curves with showing them on the board. A teacher said, "what is the equal of those examples and what is different from those examples?". This activity is the focus of learning at level 1, where students are directed to distinguish the forms of shapes [6][7]. This activity does include in the first phase of learning based on van Hiele's theory (inquiry or information). Teacher and students do question and answer to know the ability of students before doing learning activities. Here the teacher also investigates the accuracy of the language used by the students [7]. Then in the second phase (directed orientation), students are directed to explore the form of "cube" used as a tool in learning. It is used to observe whether there is an enhancement in students' thinking ability at level 1 or not. Here is an overview of learning activities:
Teacher: Look at the cube (figure 6)! Which includes two parallel lines?
(Use one of the flat you know!)
Student: Line segment AB is parallel to the line segment CD.
Teacher: Which two line segments intersect?
Student: Line segment AB intersected by line segment BC.
Teacher: What is formed if two line segments intersect?
(What is his name if you find it in an example of everyday life?)
Student: There are corners and points of intersection.
Teacher: What happens at a corner?
Student: Two sides join.

The use of a combination of two-dimensional (2D) and three-dimensional (3D) shapes make it easy for teachers to introduce the concept and efficiency for students to understand the concepts presented. In addition, the use of 2D and 3D also show that concepts in mathematics have a connection between ideas in mathematics. This is in line with the opinion that "the importance of mathematical connections in learning mathematics is because it supports students to understand a concept substantially and helps them to improve their knowledge" [12][13]. Then in the next phase, students are given the freedom to explore the task of learning (free orientation phase) and the end of the learning process summarizes the day's lesson material (integration phase). Here are the attributes of the learning activities used at this level.

![Figure 6.](image)

**Figure 6.** Level 1 learning attribute is "Cube ABCD.EFGH".

### 4.2. Overview of level 2 (analysis)

From table 1, there are 7 of 22 students who into the level 2 category. Unlike the practice of learning activities at level 1 focusing on identifying the form of shape, instructions teacher practice is designed so that students can understand the properties of a geometry concept [8][6][5]. Observations did when the teacher asked students to explore the parallelogram to identify properties of the two parallel lines. The teacher begins the lesson by asking "which pair of two lines parallel to the shape?". Here's an overview of learning activities at level 2:

Teacher: How is the distance between two line segments parallel to the shape if extended?
(Pay attention to the parallelogram (figure 7a))
Student: The distance between the two lines will constant.
Teacher: Will both intersect?
Student: No, the two lines will never meet.
Teacher: If two lines intersect, are the opposite angles the same size? How do we know that?
Student: Yes, because the form is the same.
Then to improve a student's level of thinking, the teacher asks students to measure the sizes of each inside angle on a parallelogram and triangle. The teacher can question "How total sizes of angles?". Imply that students can later get the conclusion that the total sizes angle of the quadrilateral = 360° and the cumulative sizes angle of the triangle = 180°. This activity uses to facilitate the students support the learning at the next level. In addition, according to NCTM [1] and Saminanto [14] "the becomes important to give because essentially the concept of inter-mathematics has a connection". Here are the attributes of the learning activities used at this level.

![Parallelogram and Triangle](image)

**Figure 7.** The level 2 learning attribute is "(a) parallelogram" and "(b) triangle".

4.3. **Overview of level 3 (informal deduction)**

From table 1 there are 4 of 22 students included in the level 3 category. Teaching and learning at level 3 is a continuous activity of learning at level 2 that is understanding the relation of the properties the shapes or geometry concepts, so that emerges informal deduction argument [6][3][6]. Observation of teacher instructional practices at this level starts with the teacher saying "try to recall the properties of two parallel lines!". Here is an example of an overview of learning activities at level 3:

**Example 1:**
- Look at figure 6! draw again the flat of ABFE!
- The point M is in the middle of line segment BF and the point N is in the middle of line segment AB.
- Draw a line of MN!
- If the line segment MN is extended, then will the line segment MN intersect with the extension of the line segment EA?

**Example 2:**
- From figure 8, make a justification conclusion using logical relationship!

![Figure 8](image)

**Figure 8.** Level 3 learning attribute.

From the learning activity "Example 1", one example of the student's response in this exploration activity is that students answer "yes, FA and MN will intersect because they are two parallel lines and both are in one flat of ABFE". Then the students' conclusions from "Example 2" are if angle α = angle β and angle γ = angle β, then angle α = angle γ because of they both equal angle β. This student
response indicates that the student has been able to think in an informal deduction. Where this is the focus of the learning at level 3.

Based on the learning activities at levels 1 to 3, there are different teacher instructional practices at every level of thinking geometry students. At level 1, the teacher guides students to differentiate the forms of shape, then learns the accuracy of the language used by the students. Then at level 2, the teacher guides students to identify properties of geometric shapes. While at level 3, learning activities guided the students able to make conclusions informal languages to see the relationship between concepts or geometric shapes that exist. This is in line with research conducted by C. Bleeker, et al [7] which states there is a relationship between teacher instructional practices and learners levels of geometrical thinking from grade 1 to grade 5 at a primary school in Pretoria, where the resulting relationship is not simple. In addition, making connections between concepts in learning activities can support students to improve their level of thinking ability. This is in line with the opinion that "mathematical connection supports students to comprehend a substantial concept and assists them to improve their understanding" [12][13]. Other opinions also support this statement, ie: "besides being sequential, geometrical thinking levels of students are also related to each other” [3][5][6][8].

In this research, there are limitations of research that researchers can only see the relationship between teacher instructional practices and students’ levels of geometrical thinking only on the material line and angle. This is due to the limited time given by the school as a research location.

5. Conclusion and recommendation
Based on the students' level of thinking test result there are: 9.1% students of level 0 category (pre-visualisation); 36.4% of students in level 1 (visualization) category; 31.8% of students of level 2 (analysis); 18.2% of students in level 3 (informal deduction) category; and 4.5% of students in level 4 (deduction) category. Then this result is correlated with teacher instructional practices and also learning phases based on van Hiele theory.

As a conclusion, (1) there is a relationship between teacher instructional practices and students levels of geometrical thinking only on material line and angle. (2) there is a difference in teacher instructional practices at each level of student thinking based on van Hiele theory. (3) to develop students levels of geometrical thinking in learning geometry, learning activities can be designed by linking learning topics at the end of each level to the next level. Thus, these findings can be used as recommendations for designing learning activities before they are implemented in the classroom.

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