Learning Trajectory for Teaching Division using RME Approach at Elementary Schools

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Abstract. The research aimed to develop learning trajectory (LT) to teach the topic of integer division based on Realistic Mathematics Education (RME) which is valid, practical and effective through research on the development of Gravemeijer & Cobb types. The trial subjects were the third-grade of elementary school students in Padang. The research was supported by data collection techniques in the form of document analysis, observation, interviews, questionnaires, and tests. Data analysis was performed using descriptive statistics and parametric statistics. The results showed that the LT developed to teach the topic of integer division using RME helped the teacher in learning. LT describes the characteristics of RME and facilitates students in a class. Besides, LT can help students to find integer division algorithms. Students are more confident in using their strategies for solving the contextual problems so that students' mathematical problem-solving abilities also develop.

1. Introduction
Numeracy is one of the abilities that play a major role in learning mathematics. Mastery of basic arithmetic operations is very important because this operation will be the basis for those who want to learn mathematics [1,2]. Understanding the concept of arithmetic operations influences students' learning achievement in the subject of mathematics at elementary school [3]. Therefore, those who want to study mathematics need to comprehend the ability to count.

This ability often becomes a problem at elementary schools. Many elementary school graduates are not skilled enough to solve even simple calculation problems [4]. One of the issues of calculation is division. The division is the most difficult counting operation to learn among other arithmetic operations [1]. [5] stated that 12 out of 24 students have not achieved the target value of completeness or 50% of students have not yet completed learning the division of material. This happened because there are no teachers who use a learning approach that is appropriate to the characteristics of the students. [6] stated that the errors experienced by the students in the division of material are caused by learning the standard sharing algorithm that students learn mechanically in the classroom. Students only memorize it without applying the division procedure correctly. [7] stated that errors experienced by students occur in addition, subtraction, multiplication, and division especially if the two numbers do not belong to the same type and involve positive and negative integers.

The tendency of teachers to teach mathematics mechanically has caused of some problems in learning mathematics in Indonesia. First, the lack of interest and motivation of students towards mathematics [8,9,10]. Second, TIMSS study results reveal that Indonesian students are still weak in the aspect of giving a reason and solving a problem. [11]. The same condition was also found in the results...
of the PISA study which showed that Indonesia's ranking was always at the bottom 10%. Moreover, almost no Indonesian students reached the two highest levels (levels 5 and 6) [8, 12, 13, 14]. The ability to solve problems is the basic ability possessed by someone to identify and to solve the problems that include critical, logical, and systematic thinking [15]. One of the benefits of problem-solving is that the students learn that there are many ways to solve problems (different thoughts) and there is more than one possible solution and students are trained to explore, think, and reason comprehensively, and logically [16]. According to [17], problem-solving is one aspect of higher-level thinking starting from the process of accepting problems and trying to resolve the problems. The National Council of Teachers of Mathematics [18] places problem-solving as a long-term goal of mathematics education.

Observing the problems of learning mathematics as described above, schools should be the main actors in determining solutions to learning problems. [19] argued that the use of patterns proved to be quite effective for students because it is very easy and fast. This is because the students only memorize one pattern for all a matter of dividing up to 6 digits (even the infinite digits) by dividing the numbers from 2-99. If this pattern of division is used for diverging numbers (101-999) or even thousands (1001-9999), the same pattern can be used in the unit division [20].

To overcome the problems described above, it is necessary to design a learning trajectory (LT) for the topic of integer division using the RME approach. The RME approach, known in Indonesia as Indonesian Realistic Mathematics Education (PMRI), aims to change mathematics education. It is mostly used to make children enjoying doing mathematics, solving math problems, and developing mathematical skills and knowledge [21]. If we want the students rediscover mathematics by doing math, teachers need to find out the reason why the students do what they do and help them to build the mathematical concepts with their thinking [22].

Learning trajectory is a sequence of activities and tasks that can support the development of students' understanding of specific learning goals [23]. Learning trajectory is helpful to bridge the work of researchers and practitioners [24]. Learning trajectory also help teachers to evaluate and rethink teaching, which allows them to have a general vision of the class before they start teaching [25, 26]. In mathematics learning at elementary and secondary education [9, 27, 28], and also for teaching eyes particular college in college [29, 30, 31], many research results show that learning trajectory is helpful in building students' conceptual understanding [32, 33, 34]. Learning trajectory designed in this study is based on three main principles of RME for instructional design, namely, reinvention guided through progressive mathematization, didactic phenomenology, and emerging models [35, 36]. Meanwhile, the implementation of the learning trajectory in class is guided by the characteristics of RME [38, 39, 40, 41].

Based on the results of mathematics learning problems, which are supported by several research results as explained above, then through this research, a learning trajectory based on the RME approach was developed for the topic of Sharing. For this reason, the formulation of the problems is “How do the characteristics of LT based on a valid, practical and effective RME approach have a potential impact on students' mathematical solving abilities?” The results reported in this paper are the development of the results that have been published in the previous journal [42].

2. Method
This research used developmental research (developmental research approach) proposed by[22] The design of this research consisted of three phases, they are; preparing for the experiment, experimenting in the classroom, and conducting retrospective analysis [22]. This design was used to develop LT with the initial form of HLT. To make HLT, the activity begins with a thought experiment in which the researcher thought about the flow of learning that the students will go through and then reflecting it on the results of the experiments conducted. If the goal has not been reached, then the next thought experiment and instruction experiment will proceed. Thus, the LT guides the thought experiment and instruction experiment.
From the experiment of classroom phase, there are three schools that become the subject of the research, namely the third grade students at SDN 39 Tanjung Aur as a small group trial subject, third grade students at SDN 37 Sungai Bangek as the subject of field test or experimental class and third grade students SDN 21 Sungai Bangek as the subject of the control class. In collecting the data, document analysis, observation, interviews, tests, and questionnaires were used. This is based on the research phase and the research instruments.

3. Results and Discussion

In the initial stages of the research, a literature review is conducted to analyze the students’ needs (such as their hobbies, their preferences in learning, and their preference activities) to find the algorithm dividing integers from the informal to the formal stages. Based on the analysis, researchers designed HLT for teaching the topic. The HLT is designed to be discussed with a mathematics teacher or model teacher who is applied it on one-to-one evaluations and small groups before being tested in the field. After conducting a preliminary study, the researchers make revisions or improvement to the design that have been made which is based on the results obtained and discussions with the teacher. The following HLT design results:
Advice and suggestions obtained from students at the small group stage are used as revision material. After the revision, the field test is continued to assess the revised prototype. Following is the exposure of the field test results:

3.1. **Activity 1: Division with repeated substraction**

There are various alternative answers for students in completing this activity. Some students solve the problems by using pictures, some solve it by doing repetitive downward reduction, and the others solve it by repeating a reduction to the side. For the students who solve the problems by using pictures, it certainly is not by learning objectives expected at this first meeting, to direct students’ answers or thoughts as the researcher gives a probing question. The figures below are illustrated as a solution to the problems given in activity 1, students finish making drawings, such as figure 3a, 3b and 3c.

![Figure 3a. The Students’s strategy A for activity 1](image1)

![Figure 3b. The Students’s strategy B for activity 1](image2)

![Figure 3c. The Students’s strategy C for activity 1](image3)

3.2. **Activity 2: Division with substraction by 10**

From the activity 2, some students solve the problems by using repeated downward reduction and some of them use horizontal lines to complete the division. Meanwhile the rest of the students solve the problems by doing the whole distribution. This is illustrated as a solution to the problems given in activity 2, students finish making drawings, such as figure 4a, 4b and 4c.
3.3. Activity 3: Division with subtraction by multiplication of 10

Students still use the multiplication 10, whereas this was the solution of the previous meeting. Students divides 112 by 4 by making 10 as the result, later it is multiplied by the 4 as the divider number, its’ result is 40. Then, 112 is minus 40, the result is 72. Next, 72 is divided by 4, students still make the result at 10. Next, 72 minus 40 is 32. Then it is divided by 4, the result is 8. It has not accordance yet with the expected results. The solution made is the goal of the previous learning. It is illustrated as in solving the problem given in activity 2. Students finish making drawings, such as figure 5a and 5b.

Initial investigation phase carried out in order to determine the shape and characteristics of the learning tools that will be developed. This phase was planned several activities including needs analysis, curriculum analysis, analysis of the concept and analysis of learners.
3.4. Activity 4: Division with standard algorithms.

Students made the result of the division 249 divided by 3, it was 80. Then 80 is multiplied by 3, the result was 240. Next, 249 minus 240 is 9, then 9 is divided by 3, it was 3. Later, students added the results of the division so that, it is obtained 83. However, there were the students that still uses the multiplication of 10. Therefore, it has not accordance yet with the expected learning goals, they are students can complete the results of the division by using a standard algorithm. It is illustrated as in solving the problem given in activity 2. Students finish making drawings, such as figure 6a and 6b.

![Figure 6a](image)

**Figure 6a.** The Students’s strategy for activity 4

![Figure 6b](image)

**Figure 6b.** The Students’s strategy for activity 4

The result showed that HLT (with contextual problem solving activities at HLT) potentially to facilitates the students to recover the algorithm dividing integers. The activity is sequenced well. The HLT is accordance with the principles and the main characters of RME; and the HLT components are well designed and consistent one to others. LT was worked as it should during the trial. Students understand the contextual problems and they did the mathematics’ activity without any major obstacle. The probing’s questions that has been prepared as the anticipation of though and students’ solution were helped the students to achieve the goals of the activities. Moreover, the times provided to do the activity of solve contextual problems has been planned well. It is accordance with the previous findings that showed the LT and RME gave the positive impact in students’ problem solving. [26, 27].

4. Conclusion

Based on the research, it can be concluded that LT that has been developed to teach the division of integers uses RME topic are helpful the teachers in Teaching. LT describes the RME characteristics and facilitates the students in class. Moreover, LT help the students to solve the algorithm dividing integers. Students more confidence in using their own strategies in resolve the contextual problems. So the students' mathematical problem solving abilities are also developed.

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