Simultaneous Retrieval of PWV and VTEC by Low-Cost Multi-GNSS Single-Frequency Receivers

Chuanbao Zhao1,2, Baocheng Zhang1, Wei Li1, Yunbin Yuan1, and Min Li3

1Institute of Geodesy and Geophysics, Chinese Academy of Sciences, Wuhan, China, 2University of Chinese Academy of Sciences, Beijing, China

Abstract Precipitable water vapor (PWV) and ionospheric vertical total electron content (VTEC) are two essential components of space-atmosphere parameters. The zenith troposphere delay can be converted into PWV, which plays a crucial role in meteorological studies. In the meantime, the importance of the VTEC lies in providing ionospheric corrections for single-frequency (SF) positioning, navigation, and timing users. Currently, the global navigation satellite system (GNSS) has become one of the most commonly used tools for retrieving PWV and VTEC and normally relies on dual-frequency, geodetic-grade receivers and antennas. However, this reliance also implies high hardware costs. In this paper, we propose a single-frequency ionosphere and troposphere retrieval approach that enables the simultaneous retrieval of PWV and VTEC from multi-GNSS data collected by low-cost SF receivers. The use of SF receivers can greatly reduce the cost of hardware. Furthermore, the simultaneous provision of PWV and VTEC also has a positive effect on studying the coupling mechanisms of the ionosphere and troposphere. The accuracy of the estimated zenith troposphere delay can be better than 10 mm compared with the troposphere products published by the International GNSS Service, and the PWV is no more than 3 mm compared with radiosonde-derived results. Referring to the final International GNSS Service global ionosphere map products and the Jason altimeter data, the VTEC retrieved from the single-frequency ionosphere and troposphere retrieval method can perform at roughly equal levels compared to the customary dual-frequency method.

1. Introduction

Tropospheric and ionospheric delays are the two most crucial atmospheric errors in the global navigation satellite system (GNSS). With the help of surface pressures and weighted mean temperatures, the zenith tropospheric delay (ZTD) can be converted into precipitable water vapor (PWV), which is generally considered as the crucial indicator of the water content in the atmosphere (Bevis et al., 1992; Krietemeyer et al., 2018; Zhang, Yuan, Li, et al., 2017). PWV is closely associated with atmospheric processes, including the hydrological cycle, atmospheric radiation, and climate change (Gutman & Benjamin, 2001; Wang et al., 2007). Furthermore, the ionospheric delay error also needs to be handled quickly in GNSS applications, especially for common single-frequency (SF) users, because of the refraction of transionospheric radio waves. The ionospheric delay can reach several or even hundreds of meters if it is disregarded, which is unacceptable for GNSS users.

Since 1994, when the Global Positioning System (GPS) became fully operational, the rapid development of space-atmosphere studies has benefitted from the GNSS technique due to its high spatial and temporal resolution. In recent years, the GNSS has been developing rapidly, with the BeiDou Navigation Satellite System (BDS) and the Galileo satellite system offering position, navigation, and timing services and the continuous improvement to GPS and GLobal NAVigation Satellite System. The provision of abundant GNSS data provides a rare opportunity for geodetic and, especially, atmospheric research. Therefore, studies of the two atmospheric parameters, ZTD and ionospheric delay, have become increasingly dependent on GNSS data.

The customary method used to measure PWV in the air is the deployment of radiosonde balloons, which is performed only a few times per day and is strictly constrained by the weather. For the advantages of high-accuracy, all-weather capability and high spatial and temporal resolutions provided by the GNSS, researchers have begun to focus on dual-frequency (DF) GNSS approaches to derive the PWV (Alshawaf et al., 2015; Bevis et al., 1994; Bevis et al., 1992; Jin & Luo, 2009; Li et al., 2015; Wang et al., 2007; Zhang, Yuan, Li, et al., 2017) since the 1990s. Furthermore, there are also many researchers that have used SF GNSS observations to
retrieve the ZTD as well as PWV (Krietemeyer et al., 2018; Wang et al., 2018). However, those methods, for instance, the satellite-specific epoch-differenced ionospheric delay (SEID) proposed by Deng et al. (2010), still rely on a station network equipped with DF receivers, and the precision of the estimated ZTD is related to the distance between SF stations and DF stations.

For the ionosphere, the simplest and most widely used DF method for ionospheric vertical total electron content (VTEC) retrieval consists of two main steps (Banville et al., 2014; Ciraolo et al., 2007; Stephens et al., 2011): First, ionospheric observables are obtained by the so-called carrier-to-code leveling (CCL) method, which is mainly based on geometry-free linear combinations of the GNSS code and carrier-phase measurements (Brunini et al., 2008; Brunini & Azpilicueta, 2010); then, the VTEC and satellite differential code biases (SDCBs) are isolated by the use of the thin-layer ionospheric model (Brunini, 2011). However, there is a systematic arc-dependent error (i.e., leveling error) that is induced by the code delay noise and multipath effects in the CCL-based ionospheric observables. In this regard, some researchers have solved the leveling error by the precise point positioning (PPP) technique based on raw DF GNSS observations (Li et al., 2013; Wei et al., 2012; Zhang et al., 2012) and have obtained various noteworthy achievements. Zhang et al. proposed a novel uncombined method based on SF PPP that enables the joint estimation of VTEC and SDCBs from raw GNSS observations (Li et al., 2019; Zhang, Teunissen, et al., 2017). This method realizes ionosphere sensing and SDCB determination with low-cost receivers. However, only GPS or BDS systems are considered in their work, and they did not apply it to PWV and ZTD studies.

Generally, the existing globally distributed International GNSS Service (IGS) reference stations were constructed mainly for geodetic purposes, such as crustal change detection, satellite orbit determinations and clock offset estimations, or for some other geoscience applications. The current number of GNSS reference stations has already met the needs of those studies but is far from sufficient for space-atmosphere studies, such as tropospheric and ionospheric sensing. Additionally, the large number of stations equipped with DF geodetic-grade receivers for space-atmosphere studies leads to higher hardware costs. According to our investigation, a Trimble geodetic-grade receiver like NETR9 is approximately fifteen thousand dollars, and a UBLOX SF receiver, such as EVA-M8T, is only approximately U.S. $250. Thus, SF receivers are obviously competitive with DF receivers in terms of cost. Based on cost considerations and the deficiencies in the existing studies mentioned above, a single-frequency ionosphere and troposphere retrieval method (SFITR) is proposed for the joint retrieval of the PWV and VTEC in this work. Two sequential steps are included in our SF approach: In the first step, the ionospheric observables, which consist of ionospheric slant total electron content (STEC), SDCBs, and the receiver clock offset at the first epoch and the ZTD parameters, are simultaneously estimated; then, in the second step, the ZTD is converted into PWV by the surface pressure and weighted mean temperature, and the process of VTEC isolation is the same as that of the customary DF method described above. Once the performances of the VTEC, ZTD, and PWV estimated by the method proposed in this paper are validated to be consistent with the DF approaches or some other external high-precision reference data, we can vigorously promote the use of low-cost SF receivers for space-atmosphere research, for example, PWV and VTEC.

The SFITR approach proposed in this contribution greatly reduces hardware costs compared to the customary approaches that are generally based on radiosonde balloons or geodetic-grade DF receivers. The joint use of multi-GNSS observations makes the GNSS-related applications, such as the estimation of tropospheric and ionospheric delays, more reliable and accurate (Montenbruck et al., 2013; Rizos et al., 2013), compared to the single system. Furthermore, there are many researchers focusing on the relationships among different space-atmosphere parameters (Jian et al., 2017; Wang et al., 2016). With the fact that the SEID method can only obtain the tropospheric delay and only ionospheric sensing was focused on in Li et al.’s or Zhang et al.’s studies (Li et al., 2019; Zhang, Teunissen, Yuan, et al., 2017), the SFITR method proposed in this paper solves the rank deficiency problem in uncombined SF PPP models and thus makes the ionospheric delays estimable. The synchronous retrieval of the PWV and VTEC in the SFITR approach provides convenience for additional studies of the interrelation between the troposphere and ionosphere, especially during unique weather events. Note that the method proposed in this paper relies on the satellite orbits and clock offsets from an external organization. This reliance is because the multi-GNSS SF PPP method is actually a geometrically fixed method, as the receiver positions, the satellite positions and clocks are fixed to a priori known values or to externally provided values (Zhao et al., 2019). Additionally, the orbital and/or clock errors, if left unmanaged, can degrade the performance of
the PWV and VTEC. However, with the continuous improvement in satellite orbit and clock products, this limitation can be reasonably overcome in the future.

2. Methodology

2.1. Full-Rank Multi-GNSS SF PPP Model Based on Raw Observations

The raw code and carrier-phase observation equations for the first frequency data can be expressed as follows:

\[
\begin{align*}
P_r^{T,S}(k) &= \rho_r^{T,S}(k) + \left( d_{r1}^{T} - d_{r1}^{T,S} + b_{r1}^{T} - b_{r1}^{T,S} \right) \\
L_r^{T,S}(k) &= \rho_r^{T,S}(k) + \left( d_{r1}^{T} - d_{r1}^{T,S} \right) - I_{r1}^{T,S}(k) \\
&\quad + I_{r1}^{T}(k) + I_{r1}^{T,S}(k) + \epsilon_{r1}^{T}(k) \\
&\quad + \frac{f_1^2}{f_1-f_2} b_{r1}^{T} - \frac{f_2^2}{f_1-f_2} b_{r1}^{T,S} \\
&\quad + \lambda_{r1}^{T} N_{r1}^{T,S} + \epsilon_{r1}^{T}(k)
\end{align*}
\]

where \( P_r^{T,S}(k) \) and \( L_r^{T,S}(k) \) denote the raw code and carrier-phase data, respectively, from the satellite \( S \) and system \( T \) to the receiver \( r \) at the first frequency in epoch \( k \); \( \rho_r^{T,S} \) is the geometric distance of the receiver-to-satellite; \( d_{r1}^{T} \) and \( d_{r1}^{T,S} \) refer to the receiver and satellite clock offsets, respectively, where the units are in meters; \( b_{r1}^{T} \) and \( b_{r1}^{T,S} \) are the receiver and satellite code biases, respectively, for which the units are in meters and both are considered as time-constant parameters (Teng et al., 2016; Teng et al., 2018); \( I_{r1}^{T,S} \) denotes the slant ionospheric delay from the receiver \( r \) to the satellite \( S \) at the first frequency; \( T_{r1}^{T,S} \) refers to the slant tropospheric delay; \( N_{r1}^{T,S} \) is the float ambiguity absorbing receiver and satellite phase bias at the first frequency; \( \lambda_{r1}^{T} \) is the corresponding carrier-phase wavelength; and \( \epsilon_{r1}^{T} \) are the combined observational noise and unmodeled multipath effects of the raw code and carrier-phase observations, respectively. The time-constant parameters do not have an epoch index \( k \). Note that only the estimated parameters are listed in the observation equations for a clearer deduction of the functional model.

Precise satellite orbit and clock offset products are indispensable in the PPP process. Generally, the precise satellite clock products published by the IGS are referred to an ionosphere-free combination of satellite code hardware bias of the ionosphere-free combined function model used in the precise satellite clock offset estimation (Wei et al., 2012; Zhang, Teunissen, et al., 2017). The biased satellite clock offset can be expressed as follows:

\[
d_t^{T,S} = d_t^{T} + \frac{f_1^2}{f_1-f_2} b_{t1}^{T} - \frac{f_2^2}{f_1-f_2} b_{t1}^{T,S} \tag{2}
\]

We can obtain the following expression based on equations (1) and (2) after reparameterization and linearization:

\[
\begin{align*}
\tilde{P}_r^{T,S}(k) &= \tilde{e}_r^{T,S} \Delta x + T_r^{T,S}(k) + [d_{r1}^{T}(k) + b_{r1}] + \left[ I_r^{T,S}(k) + \frac{f_2^2}{f_1-f_2} (b_{t1}^{T,S} - b_{t2}^{T,S}) \right] \\
\tilde{L}_r^{T,S}(k) &= \tilde{e}_r^{T,S} \Delta x + T_r^{T,S}(k) + [d_{r1}^{T}(k) + b_{r1}] - \left[ I_r^{T,S}(k) + \frac{f_2^2}{f_1-f_2} (b_{t1}^{T,S} - b_{t2}^{T,S}) \right] \\
&\quad + \lambda_{r1}^{T} N_{r1}^{T,S} - b_{r1} + \frac{f_2^2}{f_1-f_2} (b_{t1}^{T,S} - b_{t2}^{T,S})
\end{align*}
\]

where \( \tilde{P}_r^{T,S} \) and \( \tilde{L}_r^{T,S} \) are the observed minus computed (O-C) code and carrier-phase observations after linearization of the equation system (1), respectively; \( \tilde{e}_r^{T,S} \) is the unit vector from the satellite \( S \) and system \( T \) to the receiver \( r \); \( \Delta x \) is the increment in a priori receiver coordinates; \( f_1 \) and \( f_2 \) are the phase frequencies; and the other parameters are consistent with the previous expressions.

Obviously, equation (3) is a rank-deficient system for a linear dependency between the columns of the design matrix for some variables, such as the receiver clock offset, code hardware delay, and phase ambiguities. We can solve this issue by reparameterization as follows (Odijk et al., 2016; Teunissen, 1985):
\[
\begin{aligned}
\begin{cases}
\bar{P}_r^S(k) = \bar{e}_r^T \Delta x + T_r^S(k) + \hat{d}_r^T(k) + \bar{I}_r^S(k) \\
\bar{L}_r^S(k) = \bar{e}_r^T \Delta x + T_r^S(k) + \hat{d}_r^T(k) - \bar{I}_r^S(k) + \bar{N}_{r,1}^S
\end{cases}
\end{aligned}
\]

(4)

with

\[
\begin{aligned}
\hat{d}_r^T(k) &= \hat{d}_r^T(k) + b_r, \\
\bar{I}_r^S(k) &= I_r^S(k) + \frac{f_2^2}{f_1^2-f_2^2}(b_r^S-b_r^T) \\
\bar{N}_{r,1}^S &= \lambda_1^S \cdot N_{r,1}^S - b_{r,1} + \frac{f_2^2}{f_1^2-f_2^2}(b_r^S-b_r^T)
\end{aligned}
\]

(5)

We find that there is still a rank deficiency among \(\hat{d}_r^T, \bar{I}_r^S\), and \(\bar{N}_{r,1}^S\) in equation (4). In view of this problem, we begin estimating the biased receiver clocks at the second epoch, and the estimable receiver clock offsets are reparameterized as the drifts of the original receiver clocks relative to the first or “pivot” epoch. Therefore, we can obtain the full-rank functional model for epoch \(k=2, 3, \ldots, l\):

\[
\begin{aligned}
\begin{cases}
\bar{P}_r^S(1) = \bar{e}_r^T \Delta x + T_r^S(1) + \bar{I}_r^S(1) \\
\bar{L}_r^S(1) = \bar{e}_r^T \Delta x + T_r^S(1) - \bar{I}_r^S(1) + \bar{N}_{r,1}^S \\
\bar{P}_r^S(k) = \bar{e}_r^T \Delta x + T_r^S(k) + \hat{d}_r^T(k) + \bar{I}_r^S(k) \\
\bar{L}_r^S(k) = \bar{e}_r^T \Delta x + T_r^S(k) + \hat{d}_r^T(k) - \bar{I}_r^S(k) + \bar{N}_{r,1}^S
\end{cases}
\end{aligned}
\]

(6)

with

\[
\begin{aligned}
\Delta \hat{d}_r^T(k) &= \hat{d}_r^T(k) - \hat{d}_r^T(1) \\
\bar{I}_r^S(k) &= I_r^S(k) + \hat{d}_r^T(1) + \frac{f_2^2}{f_1^2-f_2^2}(b_r^S-b_r^T)
\end{aligned}
\]

(7)

The receiver clock offset in the first epoch enters into the \(\bar{I}_r^S\) value estimated in each epoch. Therefore, the estimated \(\bar{I}_r^S\) is biased by the first epoch receiver clock offset and SDCB, as shown in equation (7). We assume that the combined value of the first epoch receiver clock offset and the SDCB is a constant during the entire process. Notably, if the data are interrupted for some reason, there will be a gap between the subsequent and preceding results, as a new receiver clock offset is introduced as the first epoch receiver clock offset.

### 2.2. PWV Retrieval by ZTD

The ZTD generally consists of two parts, the zenith hydrostatic delay (ZHD) and zenith wet delay (ZWD), as expressed in equation (8). Because the ZWD is a function of atmospheric water vapor and temperature, the PWV can be determined from the ZWD if the weighted mean temperature of the atmosphere is known (Bevis et al., 1992; Wang et al., 2018; Zhang, Yuan, Li, et al., 2017). Thus, the weighted mean temperature \((T_m)\) and surface pressure \((P_s)\) are the two key parameters of great relevance to PWV retrieval from GNSS data. Typically, the ZHD can be calculated by the Saastamoinen model with surface pressure data, as shown in equation (9):

\[
ZTD = ZHD + ZWD
\]

(8)

\[
ZHD = (2.2779 \pm 0.0024)P_s/(1-0.00266cos2\varphi-0.00028h)
\]

(9)

where \(h\) is the ellipsoid height (km) of the station and \(\varphi\) is the latitude of the station. The ZWD can be obtained by subtracting ZHD from ZTD. Then, the PWV (mm) can be retrieved from the ZWD with the conversion factor, \(\Pi\), as follows:
where $\rho_w$ is the density of liquid water, $R_v$ is the gas constant of water vapor, and $k_2 = (17 \pm 10)$ K/hPa and
$k_3 = (377600 \pm 400)$ K$^2$/hPa are physical constants (Bevis et al., 1994). $T_m$ is the weighted mean temperature of the atmosphere (K), which is defined as follows:

$$T_m = \frac{\int e(z) T(z) \, dz}{\int e(z) \, dz}$$

where $e(z)$ and $T(z)$ are the water vapor pressure (hPa) and temperature (K) at the $z$ height level, respectively (Zhang, Yuan, Li, et al., 2017). $T_m$ in equation (12) is based on numerical weather data, which need to be obtained from the meteorological organizations.

### 2.3. Ionospheric VTEC Retrieval by a Thin-Layer Ionospheric Model

The thin-layer ionospheric model used to retrieve the ionospheric VTEC assumes that the electrons in the path of signal propagation are concentrated at a specified height. Additionally, the ionospheric effective height (IEH) used in the oblique-to-zenithal thin shell conversion is usually set to the average altitudinal position of the electron density profile peaks. On the basis of the study of Lanyi and Roth (1988), the IEH is usually set in the range of 350 to 400 km. According to the global ionosphere maps (GIMs) provided by the Ionospheric Associate Analysis Centers via the Crustal Dynamics Data Information System (ftp://cddis.gsfc.nasa.gov/gps/products/ionex/), the ionospheric shell heights are fixed at 450 km. Li et al. (2018) also validated that the optimal IEH ranges from 400 to 600 km and that a height of 450 km is the most frequent IEH for both high and low solar activities. Therefore, we select 450 km as the IEH in this contribution. In this model, the ionosphere TEC at the line of sight is compressed at a point called the ionospheric penetration point (IPP). The STEC and VTEC are related by a mapping function, MF($z$) (Li et al., 2012; Roma-Dollase et al., 2018; Schaer, 1999), which can be expressed as follows:
MF(z) = \left[1 - \sin^2 z \left(1 + \frac{H_{\text{ion}}}{R_E}\right)^2\right]^{-1/2} \tag{13}

with STEC_T^{T,S} = MF(z) \cdot \text{VTEC}_T^{T,S}, where R_E is the Earth's mean radius in kilometers, H_{\text{ion}} is the altitude of the thin-layer ionosphere and z is the elevation angle of the satellites as seen from the receiver, r.

The ionospheric VTEC is then individually modeled by a generalized triangular series function over each individual station based on a thin-layer approximation for isolating the ionospheric TEC and SDCBs from the processed ionospheric observables above. The biased ionospheric observables can be expressed as follows:

\[
\begin{align*}
\text{ION}^{T,S}_r & = I^T_r^{T,S} + \text{DCB}^{T,S}_r = \frac{A}{f_1^T} \cdot \text{MF}(z) \cdot \text{VTEC}_T^{T,S}(\varphi, h) + \text{DCB}^{T,S}_r \\
\text{VTEC}(\varphi, h) & = \sum_{n=0}^{2} \sum_{m=0}^{2} \left\{ E_{nm}(\varphi - \varphi_0)^n \cdot h^m \right\} + \sum_{k=1}^{4} \left\{ C_k \cos(k \cdot h) + S_k \sin(k \cdot h) \right\} \tag{14}
\end{align*}
\]

where ION^{T,S}_r is the ionospheric observables; A = 40.28 \times 10^{16}; \varphi and \varphi_0 are the geomagnetic latitudes of the IPP for a receiver-satellite pair and receiver, respectively; E_{nm}, C_k, and S_k are the coefficients to be estimated; f_1 is the first signal frequency; and h is the solar longitude of the IPP.

Table 1

| Station | Receiver type | Antenna type | System | Latitude   | Longitude   | Height (m) |
|---------|---------------|--------------|--------|------------|-------------|------------|
| SPU3    | UBLOX EVK-M8T | TRM9800.00   | GEC    | -32.007°   | 115.895°    | -0.10      |
| CUAU    | UBLOX EVK-M8T | TRM9800.00   | GEC    | -32.004°   | 115.895°    | 23.60      |
| SPA8    | SEPT POLARXS  | TRM9800.00   | GRC    | -32.007°   | 115.895°    | -0.10      |
| CUT1    | SEPT POLARX4  | TRM9800.00   | GREC   | -32.004°   | 115.895°    | 23.98      |
| BG02    | UBLOX LEA-M8T | TW3470       | GR     | 51.986°    | 4.388°      | 75.48      |
| UBX0    | UBLOX NEO-M8T | ANN-MS       | GRE    | 51.986°    | 4.388°      | 75.26      |
| DLF1    | TRIMBLE NETR9 | LE1AR25.R3   | GREC   | 51.986°    | 4.387°      | 75.83      |

Figure 2. Time series of the biases (a) in the dual-frequency precise point positioning-derived ZTD compared with the International Global Navigation Satellite System Service ZTD products and their boxplots (b) in January 2018 at 10 selected stations. ZTD = zenith tropospheric delay; DOY = day of year.
3. Data and Processing

3.1. Data Description

Since the number of SF stations is limited, we use two different sets of multi-GNSS data to assess the method proposed in this contribution. The first set of data is from 65 globally distributed IGS Multi-GNSS experiment (MGEX) stations (see Figure 1) equipped with geodetic-grade dual-frequency or multifrequency receivers, such that all major regions of the world are nearly covered. The second set of data is collected from the low-cost SF receivers from 122 to 213 day of year (DOY) in 2016 provided by Curtin University (Perth, Australia) and from 244 to 365 DOY in 2017 provided by the Delft University of Technology (Netherlands). All data in the second set are collected from real low-cost SF receivers, and the distances between receivers do not exceed 400 m. Basic information about the receivers for the second data set is listed in Table 1. The receivers used in the MGEX stations are mainly from five different manufacturers (TRIMBLE, JAVAD, SEPTENTRIO, LEICA, and TOPCOM) and are equipped with geodetic-grade antennas (Liu et al., 2018). It is noted that the two low-cost SF receivers (BG02 and UBX0) are both equipped with low-cost antennas. Since there are no barometers and thermometers used in our experimental stations, the surface pressure and weighted mean temperature data used in this work are from the European Centre for Medium-Range Weather Forecasts and the Global Geodetic Observing System, respectively.

To evaluate the performance of the VTEC results retrieved by our SF method, the IGS final GIM and Jason altimeter data are used as external references. For the ZTD, we use the troposphere product published by the IGS (ftp://cddis.gsfc.nasa.gov/pub/gps/products/troposphere/zpd/) and the DF PPP results as the reference. The validation of the GNSS-based PWV is performed by a comparison with radiosonde observation data at two nearby stations (94610, latitude: –31.93, longitude: 115.97; orthometric height: 20 m; 06260, latitude: 52.10, longitude: 5.18; orthometric height: 4 m), which are extracted from the Integrated Global Radiosonde Archive from the National Climatic Data Center of the National Oceanic and Atmospheric Administration (https://www.ncdc.noaa.gov/data-access/weather-balloon/integrated-global-radiosonde-

---

**Table 2**

| Station | SF Bias | SF RMS | DF Bias | DF RMS |
|---------|---------|--------|---------|--------|
| ALGO    | −0.2    | 4.3    | −0.8    | 3.0    |
| CAS1    | 0.1     | 4.5    | −0.4    | 2.7    |
| PICL    | −0.6    | 4.2    | 0.1     | 2.6    |

Note: RMS = root-mean-square; ZTD = zenith tropospheric delay; SF = single frequency; DF = dual frequency; IGS = International Global Navigation Satellite System Service.
The data contain 1–2 atmospheric observations per day at these two stations. The PWV is calculated from the raw atmospheric profiles by using the following formula:

$$\text{PWV} = \frac{1}{\rho_w} \int \rho_{wv} \, dz = \frac{1}{\rho_w R_v} \int \frac{e}{T} \, dz$$

(15)

where $\rho_{wv}$ is the water vapor density and the other parameters are consistent with the previous expression.

Before comparison, the RS-PWV was interpolated to the heights of the corresponding GNSS stations.

3.2. Processing Strategies

For our data processing, the use of low-elevation data is essential to improve the accuracy of GNSS analysis and to decorrelate station heights and tropospheric zenith delays. Typically, 3° and 7° are empirically selected as the satellite elevation thresholds in the Center for Orbit Determination in Europe and IGS troposphere products, respectively. However, the carrier-phase observations at excessively low satellite elevations may be susceptible to frequent cycle-slip, thus spoiling the PPP solutions. In this regard, we choose a moderate value of five degrees as the satellite elevation threshold in this contribution by trial and error. Elevation-dependent weighting ($\sin^2 (E)$) is used to decrease the effects of observation noise at low elevations. To strengthen the model, the receiver coordinates are fixed by the value in the IGS Solution INdependent EXchange product (ftp://cddis.gsfc.nasa.gov/pub/gps/products/). If the station coordinates are not listed in the Solution INdependent EXchange files, the coordinates that are fixed are taken as the mean value over all epochs' coordinates obtained by the relative Real Time Kinematic positioning technique, with a short baseline (<400 m). The satellite position and clock offsets are fixed by the GFZ multi-GNSS precise products (GBM). Given that we do not know exactly how the ionosphere changes in practice, for the sake of insurance, the slant ionospheric delays are estimated as white noise. The zenith troposphere delay is estimated as random-walk noise ($10^{-8} \text{m}^2/\text{s}$). We empirically set the zenith-referenced code standard deviation to 0.3 m and the carrier phase to 0.003 m. There are many receiver clock slips in the low-cost SF receivers; therefore, we detect and repair them in the SF PPP process. To eliminate the convergence process, we use smooth filtering for the parameter estimation. Moreover, the satellite and receiver phase center offsets and variations, phase windups, and

**Table 3**

| Item     | BG02 | UBX0 | DLF1_SF | DLF1_DF |
|----------|------|------|---------|---------|
| Bias     | −0.3 | −0.9 | −2.1    | −0.3    |
| RMS      | 4.6  | 4.9  | 4.3     | 3.3     |

Note. RMS = root-mean-square; ZTD = zenith tropospheric delay; IGS = International Global Navigation Satellite System Service.
solid Earth tide corrections are considered. The selected data are all processed on a daily basis.

4. Results and Analysis

4.1. Assessment of the SF-Derived ZTD Results

First, we assess if the ability of the ZTD estimated by DF PPP can compete with the reference in this work by contrasting it with the IGS-published ZTD products. Figure 2 shows the difference series between the DF PPP-derived ZTD and the IGS ZTD products at ten selected MGEX stations. As shown in Figure 2, the root-mean-square (RMS) of the DF PPP-derived ZTD is no more than 4 mm, and all other MGEX stations

Figure 5. Frequency histogram of the ZTD differences between the SF (SPU3 and CUAU) and DF (SPA8 and CUT1) stations from May to July 2016. RMS = root-mean-square; ZTD = zenith tropospheric delay; SF = single frequency; DF = dual frequency.

Figure 6. Time series of the PWV results derived from SPU3 (SF), SPA8 (DF), and radiosonde data during 122 to 213 DOY in 2016. PWV = precipitable water vapor; DOY = day of year.
also have consistent precisions. Therefore, the ZTD estimated by the DF PPP in this work is sufficiently precise as a reference for the SF results.

Due to spatial limitations, we only present some selections from the large number of consistent results. We selected three stations from the first set of data over seven consecutive days to illustrate the performance of the ZTD results estimated by the SF method. Figure 3 shows the ZTD series for the three selected stations (ALGO, CAS1, and PICL) and their corresponding bias series. The mean bias and RMS of the ZTD estimated by the SF/DF PPP method compared with the IGS ZTD products are listed in Table 2.

![Figure 7](image7.png)

**Figure 7.** Scatter plots of the radiosonde-based PWVs and GNSS-based PWVs from May to July 2016. The solid red lines depict the regression lines. PWV = precipitable water vapor; GNSS = global navigation satellite system; RMS = root-mean-square.

![Figure 8](image8.png)

**Figure 8.** Scatter plots of the radiosonde-based PWVs and GNSS-based PWVs from September to December 2017. The solid red lines depict the regression lines. PWV = precipitable water vapor; GNSS = global navigation satellite system; RMS = root-mean-square.
For the real SF data, we select 3 days in 2017 to show the details of the ZTD intraday changes and the consistency among ZTD series estimated by different methods. Figure 4 shows the ZTD daily series estimated by two low-cost SF receivers (BG02 and UBX0) and one geodetic-grade receiver (DLF1) on three selected days and the corresponding mean biases and RMS histograms. We find that the SF-derived ZTD series can also accurately reflect the tropospheric intraday variations. Table 3 summarizes the mean biases and RMSs of all ZTD results in 2017 compared to the IGS ZTD products. Figure 5 depicts the frequency histogram of the ZTD differences between the SF (SPU3 and CUAU) and DF (SPA8 and CUT1) stations from May to July 2016. We can see that some SF ZTD results may have precisions of up to tens of millimeters relative to the corresponding DF ZTD results. We infer that a possible factor for this precision is that the interruptions in observations on some days have negative effects on the quality of observations, which in turn affects the estimation of the ZTD. The mean biases and RMSs of the ZTD results estimated by the two low-cost SF receivers are 0.8/9.1 mm for SPU3 and 0.8/8.7 mm for CUAU, compared to the corresponding DF stations.

We find that the ZTD results are consistent with previous studies (Deng et al., 2010; Krietemeyer et al., 2018; Zhang, Yuan, Wei, et al., 2017). However, the SFITR approach is more flexible because it is independent of nearby DF stations, while the performance of ZTD derived by using the SEID method proposed by Deng relies on the distance between SF and DF stations.

Table 4

| Item             | Correlation coefficient | Item             | Correlation coefficient |
|------------------|------------------------|------------------|------------------------|
| SPU3-SPA8        | 0.95                   | BG02-DLF1        | 0.94                   |
| SPU3-RSPWV       | 0.91                   | BG02-RSPWV       | 0.92                   |
| SPA8-RSPWV       | 0.96                   | BG02-UBX0        | 0.93                   |
| CUAU-CUT1        | 0.95                   | UBX0-DLF1        | 0.91                   |
| CUAU-RSPWV       | 0.89                   | UBX0-RSPWV       | 0.91                   |
| CUT1-RSPWV       | 0.96                   | DLF1-RSPWV       | 0.97                   |

Note. PWV = precipitable water vapor.

Figure 9. Time series of the SF-derived (in red) and dual-frequency-derived (in green) ionospheric VTEC and GIM TEC (in blue) at CAS1 and GMSD for the period 10–17 DOY in 2018. VTEC = vertical total electron content; SF = single frequency; TECU = TEC unit; CCL = carrier-code leveling; GIM = global ionosphere map; DOY = day of year.
Figure 10. Mean bias and RMS of the VTEC differences relative to the GIM TEC at CAS1 and GMSD for the period 10 to 17 DOY in 2018. VTEC = vertical total electron content; SF = single frequency; TECU = TEC unit; CCL = carrier-to-code leveling; GIM = global ionosphere map; DOY = day of year; RMS = root-mean-square.

Figure 11. Daily series of ionospheric VTECs for the four low-cost SF receivers with a 5-min resolution on four selected days. Each panel contains four lines. The red solid lines are the VTEC series estimated by the SF method by using low-cost receivers. The green solid lines are the VTEC series estimated by the SF method with geodetic-grade receivers. The blue dashed dotted lines are the VTEC series estimated by the dual-frequency CCL method, and the purple dotted lines are the reference VTEC series calculated by the GIM. VTEC = vertical total electron content; SF = single frequency; TECU = TEC unit; CCL = carrier-to-code leveling; GIM = global ionosphere map; DOY = day of year; IGS = International Global Navigation Satellite System Service.
4.2. Assessment of PWV Retrieval

The PWV can be retrieved from the ZTD by equations (8)–(11) with the help of two meteorological parameters: surface pressure and weighted mean temperature. Taking the stations SPU3 and SPA8 as examples, Figure 6 shows the time series of the PWV results derived from the SF station SPU3, DF station SPA8, and the radiosonde data during 122 to 213 DOY in 2016. Figure 7 shows the scatter plots of the PWV derived by the low-cost SF receivers (SPU3 and CUAU) and by the DF geodetic-grade receivers (SPA8 and CUT1) in 2016. The PWV results in 2017 (BG02, UBX0, and DLF1) are depicted in Figure 8. The mean bias and RMS of the PWV can be less than 1 and 3 mm, respectively, compared with the radiosonde-based PWV reference data, which meets the general accuracy requirements for PWV monitoring. Furthermore, we find that the DF PWV results are approximately 1 mm more accurate than the SF results. Table 4 lists the correlation coefficients among the SF, DF, and radiosonde PWV results. We find that most of the correlation coefficients are above 0.9; therefore, we can conclude that the PWV results derived from the SF, DF, and radiosonde-based methods are consistent with each other. Therefore, the SF-derived PWV values in this paper are equivalent to the results in Krietemeyer et al.’s study (Krietemeyer et al., 2018) without the limitation of distance between SF and DF stations; this approach is also cost-effective compared with those utilized in Zhang et al.’s study (Zhang, Yuan, Li, et al., 2017).

4.3. Assessment of Ionospheric VTEC Retrieval

To evaluate the performance of the ionospheric VTEC estimated by the SF method proposed in this work, two kinds of external high-precision TEC data sources are used as references: One is provided by the IGS final GIM product and the other is provided by the Jason altimeter.

4.3.1. Validation Against GIM TEC

First, we compare the SF-derived VTEC with the IGS final GIM product, whose accuracy is 2–8 TEC units (TECU; see http://www.igs.org/products). Figure 9 shows the ionospheric VTEC series calculated by the SF method, DF CCL method, and GIM at the two selected stations (CAS1 and GMSD) in the first data set. It is noted that we apply the local time/latitude quadratic interpolation recommended by Schaer et al. (1998) to the IONosphere map EXchange format. The ionospheric VTEC values calculated by these three methods are consistent with each other. The mean biases and RMSs of the VTEC results estimated by CAS1 and GMSD compared to those from GIM are shown in Figure 10.

| Period | SF mean bias | SF RMS | DF mean bias | DF RMS |
|--------|--------------|--------|--------------|--------|
| 2016   | 3.5          | 3.6    | 2.7          | 3.0    |
| 2017   | 0.6          | 1.5    | 0.7          | 0.9    |
| 2018   | 0.6          | 1.4    | 0.1          | 1.3    |

Note. RMS = root-mean-square; SF = single frequency; DF = dual frequency; VTEC = vertical total electron content; TECU = TEC unit; GIM = global ionosphere map.

Figure 12. Ionospheric VTEC estimates at each IPP from Jason data (in red), DF data (in green), and SF data (in blue) on three selected days in 2018. SF = single frequency; DF = dual frequency; VTEC = vertical total electron content; IPP = ionospheric penetration point; TECU = TEC unit; DOY = day of year.
The ionospheric VTEC daily series based on the real low-cost SF receivers are shown in Figure 11. We can find that, as commendable as the DF CCL method and IGS final GIM products are, the ionospheric VTEC estimated by the SF method can also effectively depict the ionospheric intraday variations. Table 5 summarizes the corresponding mean biases and RMSs of the ionospheric VTEC differences between the GIM and all selected receivers.

### 4.3.2. Validation Against Jason TEC Data

The ocean altimeter onboard the Jason satellite is another highly precise and reliable TEC data source, with a bias of approximately 2–5 TECU compared with real ionospheric TEC values. Therefore, this source can also be used as reference data for the SF-derived VTEC results.

Each panel in Figure 12 shows the Jason TECs and ionospheric VTECs estimated by the SF method and DF CCL method at the locations and times when the Jason satellite orbits on a selected day from the large amount of consistent results. Since there were not enough stations in 2016 and 2017, we only show the results of all selected stations in 2018, and the mean biases and RMSs of the VTEC results in 2016 and 2017 are listed in Table 6. We find that the variations in the SF-derived VTECs are consistent with those of the Jason TECs and the customary CCL DF-derived VTECs. Therefore, we can also conclude that the ionospheric VTECs calculated by our SF method has the ability to sense the spatial and temporal variations in the local ionosphere. The corresponding mean biases and RMSs of the ionospheric VTEC differences relative to the Jason TECs are summarized in Table 5.

The VTEC results derived by the SFITR approach in this paper are also consistent with the previous relevant results (Li et al., 2019; Zhang, Teunissen, et al., 2017). However, Zhang and Li only used a single system, GPS or BDS, which may be not reliable in poor observation conditions; further, they only paid attention to the VTEC parameters and did not focus on the ZTD and PWV parameters, which are also crucial atmospheric parameters. Therefore, in our SF approach, the simultaneous estimation of PWV and VTEC is considered, thus providing an efficient method for the study of coupling mechanisms of the troposphere and ionosphere (Jian et al., 2017; Wang et al., 2016).

### 5. Conclusions

In this research, we proposed a new SFITR method based on multi-GNSS SF receivers that can simultaneously retrieve two crucial space-atmosphere parameters (ionosphere and troposphere) and then obtain the VTEC and PWV. To evaluate the performance of the space-atmosphere parameters estimated by the SFITR method, we used the IGS-published ZTD products as the reference for the ZTD results and the radiosonde-derived PWV data as the reference for the PWV results. For the ionospheric VTEC, the final IGS GIM product and Jason TEC were used as the reference data. After the experiment and analysis, we found that the RMS values of the ZTD and those of the PWV estimated by SFITR were under 10 and 3 mm, respectively. For the ionospheric VTEC, the mean bias and RMS of the SF-derived ionospheric VTEC were 1.6/2.1 TECU compared with the IGS final GIM products and were −1.0/2.6 TECU compared with the Jason TEC data, respectively. The RMS of the differences between the SF-based ionospheric VTEC and those based on the CCL method is approximately 0.5 TECU. All results have accuracies similar to the previous relevant research. Therefore, the results calculated by SFITR and the customary DF approaches are at roughly equal levels. Hence, the VTEC and PWV obtained from our SF method can be applied for simultaneously monitoring the local ionosphere and troposphere with good spatial and temporal resolutions. Overall, the SF-derived PWV and VTEC in this contribution show consistent performance with those derived by the customary approaches, but the SFITR method proposed in this work has an obvious advantage in terms of hardware cost compared with the DF method or other approaches, such as radiosondes, used in previous studies. The abundance of multi-GNSS data makes the results more stable and accurate relative to the single system, especially in poor observation conditions. Unlike other methods, this method does not need any DF stations near the SF receivers, which makes the meteorological study simpler and more flexible. The simultaneous estimation of PWV (ZTD) and VTEC provides convenience for the study of coupling mechanisms of the
ionosphere and troposphere, especially during extreme weather. Furthermore, with the continuous improvement in satellite orbit and clock products, the reliance on satellite orbits and clocks, which is a potential limitation of the SF method, can be overcome in the future.

References

Alshawal, F., Fuhrmann, T., Knöppler, A., Luo, X., Mayer, M., Hinz, S., & Heck, B. (2015). Accurate estimation of atmospheric water vapor using GNSS observations and surface meteorological data. IEEE Transactions on Geoscience and Remote Sensing, 53(7), 3764–3771. https://doi.org/10.1109/TGRS.2014.2382713

Banville, S., Collins, P., Zhang, W., & Langley, R. B. (2014). Global and regional ionospheric corrections for faster PPP convergence. Navigation, 61(2), 115–124. https://doi.org/10.1002/navi.57

Bevis, M., Businger, S., Chiwesel, S., Herring, T. A., Anthes, R. A., Rocken, C., & Ware, R. H. (1994). GPS meteorology: Mapping zenith wet delays onto precipitable water. Journal of Applied Meteorology, 33(3), 379–386. https://doi.org/10.1175/1520-0450(1994)033<0379:GMPZW>2.0.CO;2

Bevis, M., Businger, S., Herring, T. A., Rocken, C., Anthes, R. A., & Ware, R. H. (1992). GPS meteorology: Remote sensing of atmospheric water vapor using the Global Positioning System. Journal of Geophysical Research, 97(D14), 15,787–15,801. https://doi.org/10.1029/92JD01517

Brunini, C. (2011). Simulation study of the influence of the ionospheric layer height in the thin layer ionospheric model. Journal of Geodesy, 85(9), 635–645. https://doi.org/10.1007/s00190-011-0470-2

Brunini, C., & Argüellicueta, F. (2010). GPS slant total electron content accuracy using the single layer model under different geomagnetic regions and ionospheric conditions. Journal of Geodesy, 84(5), 293–304. https://doi.org/10.1007/s00190-010-0367-5

Brunini, C., Meza, A., Gende, M., & Argüellicueta, F. (2008). South American regional ionospheric maps computed by GESA: A pilot service in the framework of SIRGAS. Advances in Space Research, 42(4), 737–744. https://doi.org/10.1016/j.asr.2007.08.041

Ciracolo, L., Argüellicueta, F., Brunini, C., Meza, A., & Radicella, S. M. (2007). Calibration errors on experimental slant total electron content (TEC) determined with GPS. Journal of Geodesy, 81(2), 111–120. https://doi.org/10.1007/s00190-006-0093-1

Deng, Z., Bender, M., Dick, G., Ge, M., & Wickett, J. (2010). Retrieving tropospheric delay using GPS single frequency receivers, EGU General Assembly Conference, pp. EGU2010-8256.

Gutman, S. I., & Benjamin, S. G. (2001). The role of ground-based GPS meteorological observations in numerical weather prediction. GPS Solutions, 4(4), 16–24. https://doi.org/10.1007/PL00012860

Jian, K., Yao, X., Xu, Y., Kuo, C., Liang, Z., Lei, L., & Zhai, C. (2017). A clear link connecting the troposphere and ionosphere: ionospheric responses to the 2015 Typhoon Dujuan. Journal of Geodesy, 91(9), 1067–1097.

Jin, S., & Luo, O. F. (2009). Variability and climatology of PWV from global 13-year GPS observations. IEEE Transactions on Geoscience and Remote Sensing, 47, 1918–1924.

Krietemeyer, A., Ten Veldhuis, M. C., Hans, V. D. M., Realini, E., & Nick, V. D. G. (2018). Potential of cost-efficient single frequency GNSS receivers for water vapor monitoring. Remote Sensing, 10(9). https://doi.org/10.3390/rs10091493

Lanyi, G. E., & Roth, T. (1988). A comparison of mapped and measured total ionospheric electron content using global positioning system and beacon satellite observations. Radio Science, 23(4), 483–492. https://doi.org/10.1029/RS023i004p00483

Li, M., Yuan, B., Zhang, W., Wang, N., Li, Z., Liu, X., & Zhang, X. (2018). Determination of the optimized single-layer ionospheric height for electron content measurements over China. Journal of Geodesy, 92(2), 169–183. https://doi.org/10.1007/s00190-017-1054-6

Li, M., Zhang, B., Yuan, Y., & Zhao, C. (2019). Single-frequency precise point positioning (PPP) for retrieving ionospheric TEC from BDS B1 frequency observations. GPS Solutions, 23(1), 18. https://doi.org/10.1007/s10291-018-0810-2

Li, W., Teunissen, P., Zhang, B., & Verhagen, S. (2013). Precise point positioning with GPS and compass observations. In China Satellite Navigation Conference (CSNC) (pp. 367–378). Berlin, Heidelberg: Springer.

Li, X., Dick, G., Lu, C., Ge, M., Nilsson, T., Ning, T., et al. (2015). Multi-GNSS meteorology: Real-time retrieval of atmospheric water vapor from BeiDou, Galileo, GLONASS, and GPS observations. IEEE Transactions on Geoscience and Remote Sensing, 53(12), 6385–6393. https://doi.org/10.1109/TGRS.2015.2438395

Li, Z., Ou, J., & Huo, X. (2012). Two-step method for the determination of the differential code biases of COMPASS satellites. Journal of Geodesy, 86(11), 1059–1076. https://doi.org/10.1007/s00190-012-0565-4

Liu, T., Zhang, B., Yuan, Y., Li, Z., & Wang, N. (2018). Multi-GNSS triple-frequency differential code bias (DCB) determination with precise point positioning (PPP). Journal of Geodesy, 92(5), 765–784.

Montenbruck, O., Hugentobler, U., Khachikyan, R., Weber, G., Langley, R., & Mervart, L. (2013). IGS-MGEK: Preparing the ground for multi-constellation GNSs science. Int. Colloquium on Scientific and Fundamental Aspects of the Galileo System.

Odijk, D., Zhang, B., Khodabandeh, A., Odolinski, R., & Teunissen, P. J. G. (2016). On the estimability of parameters in undifferenced, uncompensated GNSS network and PPP-RTK user models by means of S-system theory. Journal of Geodesy, 90(1), 15–44. https://doi.org/10.1007/s00190-015-0854-9

Rizos, C., Montenbruck, O., Weber, R., Weber, G., Neielan, R., & Hugentobler, U. (2013). The IGS MGEK experiment as a milestone for a comprehensive multi-GNSS service, ION Pacific PNT Conference, pp. 289–295.

Roma-Dollase, D., Hernández-Pajares, M., Kranzowski, A., Kotulak, K., Ghodssi-Farid, R., Yuan, Y., et al. (2018). Consistency of seven different GNSS global ionospheric mapping techniques during one solar cycle. Journal of Geodesy, 92(6), 691–706. https://doi.org/10.1007/s00190-017-1088-9

Schaer, S. (1999). Mapping and predicting the Earth’s ionosphere using the Global Positioning System. Geod.-Geophys. Arb. Schweiz, Vol. 59, 59.

Schaer, S., Werner, G., & Joachim, F. (1998). IONEX: The ionosphere map exchange format version 1. Proceedings of the IGS workshop, Darmstadt, Germany.

Stephens, P., Komjathy, A., Wilson, B., & Mannucci, A. (2011). New leveling and bias estimation algorithms for processing COSMIC/FORMOSAT-3 data for slant total electron content measurements. Radio Science, 46, RS0D10. https://doi.org/10.1029/2010RS004588

Teng, L., Yuan, B., Zhang, B., Wang, N., Tan, B., & Chen, Y. (2016). Multi-GNSS precise point positioning (MGPPP) using raw observations. Journal of Geodesy, 91, 1–16.

Teng, L., Zhang, B., Yuan, Y., & Min, L. (2018). Real-time Precise Point Positioning (RTPPP) with raw observations and its application in real-time regional ionospheric VTEC modeling. Journal of Geodesy, 92(11), 1267–1283.
Teunissen, P. (1985). Zero order design: Generalized inverses, adjustment, the datum problem and S-transformations.

Wang, C., Rosen, I. G., Tsurutani, B. T., Verkhoglyadova, O. P., & Mannucci, A. J. (2016). Statistical characterization of ionosphere anomalies and their relationship to space weather events. *Journal of Space Weather and Space Climate, 6*, A5. https://doi.org/10.1051/swsc/2015046

Wang, J., Zhang, L., Dai, A., Hove, T. V., Baelen, J. V., & Wang, C. (2007). A near-global, 2-hourly data set of atmospheric precipitable water from ground-based GPS measurements. *Journal of Geophysical Research, 112*, D11107. https://doi.org/10.1029/2006JD007529

Wang, K., Khodabandeh, A., & Teunissen, P. J. G. (2018). Precision analysis of troposphere sensing using GPS single-frequency signals. *Advances in Space Research, 63*(1), 148–159.

Wei, L. I., Cheng, P., Bel, J., Wen, H., & Hua, W. (2012). Calibration of regional ionospheric delay with uncombined precise point positioning and accuracy assessment. *Journal of Earth System Science, 121*(4), 989–999. https://doi.org/10.1007/s12040-012-0206-6

Zhang, B., Teunissen, P. J. G., Yuan, Y., Zhang, H., & Li, M. (2017). Joint estimation of vertical total electron content (VTEC) and satellite differential code biases (SDCBs) using low-cost receivers. *Journal of Geodesy, 92*(4), 401–413.

Zhang, B. C., Yuan, Y. B., Li, Z. S., & Ou, J. K. (2012). Extraction of line-of-sight ionospheric observables from GPS data using precise point positioning. *Science China Earth Sciences, 55*(11), 1919–1928. https://doi.org/10.1007/s11430-012-4454-8

Zhang, B. C., Yuan, Y. B., Li, Z. S., & Ou, J. K. (2012). Joint estimation of vertical total electron content (VTEC) and satellite differential code biases (SDCBs) using low-cost receivers. *Journal of Geodesy, 92*(4), 401–413. https://doi.org/10.1002/jjgs.252

Zhao, C., Yuan, Y., Zhang, B., & Li, M. (2019). Ionosphere sensing with a low-cost, single-frequency, multi-GNSS receiver. *IEEE Transactions on Geoscience and Remote Sensing, 57*(2), 881–892. https://doi.org/10.1109/TGRS.2018.2862623