The effect of quark exchange in $A = 3$ mirror nuclei and neutron/proton structure functions ratio

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Abstract

By using the quark-exchange formalism, realistic Faddeev wave functions and Fermi motion effect, we investigate deep inelastic electron scattering from $A = 3$ mirror nuclei in the deep-valence region. The initial valence quarks in put are taken from the GRV’s next-to-leading order calculations on $F^p(x, Q^2)$ which give very good fit to the available data in the $(x, Q^2)$-plane. It is shown that the free neutron to proton structure functions ratio can be extracted from corresponding EMC ratios for $^3\text{He}$ and $^3\text{H}$ mirror nuclei by using self-consistent iteration procedure and the results are in good agreement with other theoretical models as well as present available experimental data, especially the one expected form the proposed 11 GeV Jefferson Laboratory.

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I. INTRODUCTION

In the framework of Standard Model, the hadrons are composed of valence-quarks, sea-quarks and gluons [1]. In recent years, in order to test the perturbative and non-perturbative nature of quantum chromodynamics, most of the experiments and theoretical works have been focused on studies of the hadrons (mainly proton and neutron) structure functions at small $x$ ($x$ is Bjorken scaling variable) regions [2] where the sea-quarks and gluons play important roles. While the middle $x$ region i.e. $0.3 < x < 0.7$ at moderate $Q^2$ (the photon 4-momentum), have been assumed to be understood and can be explained by the valence-quarks dynamics, there is lack of information about the nucleons structure functions in the deep-valence region, $x > 0.7$ at any $Q^2$. There are a few reasons to study the quark distributions in nucleons at large $x$. The $d/u$ quark distribution function ratio near $x \simeq 1$ can give information about (1) the spin-flavor symmetry breaking in the nucleons (2) the onset of perturbation behavior [3] and (3) search for new physics beyond Standard Model [4] in the high energy colliders at high $Q^2$ e.g. the uncertainty in the gluon distributions.

While the proton structure functions is quite well known both experimentally and theoretically [1-3], the neutron structure functions usually should be extracted from deuterium and because of the large nuclear corrections, there could be uncertainties as large as %50 in the $d/u$ or $F_2^n/F_2^p \simeq \frac{1+4d}{4+u}$ ratios [5]. The bounds value of $\frac{1}{4} \leq \frac{F_2^n}{F_2^p} \leq 4$ has been proposed by Nachtmann [6] for the all $x$ regions. On the other hand, based on different models the values of $\frac{2}{3}$ (SU(6) symmetry)[7], $\frac{1}{4}$ (phenomenological and Regge considerations)[8] and $\frac{3}{7}$ (quarks counting roles and perturbative QCD) [9] have been predicted as $x \rightarrow 1$.

Recently, the possible use of an unpolarized tritium target has been proposed, by using the 11GeV upgraded beam of Jefferson Laboratory [10], and aimed to measure $F_2^n$, using the ratio of structure functions of helium-3 ($F_2^3He$) and tritium ($F_2^3H$) in order to reduce the systematic errors both in the experimental measurements and theoretical calculations (which are model dependent).

Several years ago the quark-exchange formalism was originally introduced by Hoodbhoy
and Jaffe [HJ] to investigate the quarks distributions in nuclear systems [11,12]. This formalism was applied by one of the authors (MM) to light nuclei [13] and nuclear matter [14] and was reformulated by us to derive the spin structure function of the three-nucleon systems as well as the proton and neutron [15] and finally it was used as the initial conditions for the QCD evolution equations [16] to calculate the sea-quarks and gluons contributions to the proton structure function at the leading and next-to-leading order (NLO) levels. In the most of the above calculations we found satisfactory agreements with the available experimental data. So we think the quark-exchange formalism is good motivation for looking at $F_2^n/F_2^p$ ratios at deep-valence region i.e. $x \geq 0.7$.

Recently, several groups have paid attention to the above matter [17-22] and have used different models (mostly based on impulse formalism and different spectral function approximation) to calculate $F_2^n/F_2^p$ ratios at deep-valence region. We will present their results in this work and compare them with ours.

So the paper will be organized as follows: In section II we briefly explain the quark-exchange model and we calculate the valence quark momentum distributions for proton and neutron in $^3He$ and $^3H$. In section III, we calculate the structure functions of helium 3 and tritium by including the Fermi motion effect. The self-consistent calculation (iteration procedure) of the ratio $F_2^n/F_2^p$ will be explained in section IV. Finally in section V we present our numerical results, discussion and conclusion.

II. THE QUARK-EXCHANGE FORMALISM

Let us start with a brief summary of the quark-exchange formalism. We take the nucleon states to be composed of three valence quarks [11,15],

$$|\alpha\rangle = N^{\alpha\dagger}|0\rangle = \frac{1}{\sqrt{3!}} N_{\mu_1\mu_2\mu_3}^{\alpha} q_{\mu_1}^\dagger q_{\mu_2}^\dagger q_{\mu_3}^\dagger |0\rangle$$  \hspace{1cm} (1)

where $\alpha$ ($\mu_i$) describe the nucleon (quark) states $\{\vec{P}, M_S, M_T\}$ ( $\{\vec{k}, m_s, m_t, c\}$)(note that $M_T(m_t) = +\frac{1}{2}$ and $-\frac{1}{2}$ for the proton (up-quark) and neutron (down-quark), respectively).
As usual $q^\mu_i (\mathcal{N}^{\alpha_i})$ denote the creation operators for the quarks (nucleons) with the state index $\mu$ ($\alpha$). With this convention that a repeated index means a summation as well as integration over $\vec{k}$. The totally antisymmetric nucleon wave function $\mathcal{N}^{\alpha}_{\mu_1\mu_2\mu_3}$ are written as,

$$\mathcal{N}^{\alpha}_{\mu_1\mu_2\mu_3} = D(\mu_1, \mu_2, \mu_3; \alpha_i) \times \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 - \vec{P}) \phi(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{P})$$

(2)

where $\phi(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{P})$ is the three nucleon wave function which is approximated by a Gaussian form ($b \simeq$ nucleons radius) :

$$\phi(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{P}) = \left(\frac{3b^4}{\pi^2}\right)^{\frac{3}{4}} \exp\left[ -b^2 \left( \frac{k_1^2 + k_2^2 + k_3^2}{2} + \frac{b^2P^2}{6} \right) \right]$$

(3)

$D(\mu_1, \mu_2, \mu_3; \alpha_i)$ are the product of four Clebsch-Gordon coefficients, $C_{m_{S1}m_{S2}m_{T1}, m_{T2}m_{T3}}^{\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}}(\epsilon_{c_1c_2c_3}$ are the color factors) which are defined as,

$$D(\mu_1, \mu_2, \mu_3; \alpha_i) = \frac{1}{\sqrt{3!}} \epsilon_{c_1c_2c_3} \sum_{s,t=0,1} C_{m_{S1}m_{S2}m_{T1}, m_{T2}m_{T3}}^{\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}} C_{m_{S1}m_{S2}m_{T1}, m_{T2}m_{T3}}^{\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}}$$

(4)

Now, based on the nucleon creation operators, we can define the nucleus states as,

$$|A_i = 3\rangle = (3!)^{-\frac{3}{4}} \chi^{\alpha_1\alpha_2\alpha_3} \mathcal{N}^{\alpha_1\dagger} \mathcal{N}^{\alpha_2\dagger} \mathcal{N}^{\alpha_3\dagger} |0\rangle$$

(5)

where $\chi^{\alpha_1\alpha_2\alpha_3}$ are a complete antisymmetric nuclear wave functions (they are taken from the Faddeev calculation with Reid soft core potential [15]. According to Afnan et al. [20], the choice of nucleon-nucleon potential does not affect the EMC result. However we will investigate this matter in our future works) and it should be interpreted as the center of mass motion of the three nucleons. By using the same definition as the one we did for C-G coefficients in equation (4) i.e.,

$$D(\alpha_1, \alpha_2, \alpha_3; A_i) = \frac{1}{\sqrt{2}} \sum_{S,T=0,1} C_{M_{S1}M_{S2}M_{S3}, M_{T1}M_{T2}M_{T3}}^{\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}} C_{M_{S1}M_{S2}M_{S3}, M_{T1}M_{T2}M_{T3}}^{\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}}$$

(6)

Then we can write the nuclear wave functions as

$$\chi^{\alpha_1\alpha_2\alpha_3} = \chi(\vec{P}, \vec{q}) D(\alpha_1, \alpha_2, \alpha_3; A_i)$$

(7)

Relevant information comes from the momentum distribution of the constituent quarks, which can be defined for valence quarks with the fixed flavor and nucleon iso-spin projection in the three nucleon system as,
The sign bar means no summation on $M_T$, $m_t$ and integration over $\vec{k}$ on the repeated index $\mu$. The calculation of $\langle A_i = 3 | A_i = 3 \rangle$ would become straightforward by doing summation over $\bar{\mu}$,

$$\langle A_i = 3 | A_i = 3 \rangle = \frac{1}{9} \langle A_i = 3 | q_\mu^\dagger q_\mu | A_i = 3 \rangle = \chi^{* \alpha_1 \alpha_2 \alpha_3} (\delta^\alpha_1 \delta^\alpha_2 \delta^\alpha_3 - \mathcal{E}_{\mu \bar{\mu}}^{\alpha_1 \alpha_2 \alpha_3, \beta_1 \beta_2 \beta_3}) \chi^{\beta_1 \beta_2 \beta_3}$$

where

$$\mathcal{E}_{\mu \bar{\mu}}^{\alpha_1 \alpha_2 \alpha_3, \beta_1 \beta_2 \beta_3} = N^{\alpha_2}_{\mu_1 \mu_2 \mu_3} N^{\beta_2}_{\bar{\mu}_2 \bar{\mu}_3 \bar{\mu}_1} N^{\alpha_3}_{\mu_2 \mu_3 \mu_1} N^{\beta_3}_{\bar{\mu}_1 \bar{\mu}_2 \bar{\mu}_3} \delta^{\alpha_1 \beta_1}$$

After performing some algebra, one could derive the following equation for the expectation value of $q^\dagger q$,

$$\langle A_i = 3 | q^\dagger q | A_i = 3 \rangle = 9 \chi^{* \alpha_1 \alpha_2 \alpha_3} (U_{\mu \bar{\mu}}^{\alpha_1 \alpha_2 \alpha_3, \beta_1 \beta_2 \beta_3} - V_{\mu \bar{\mu}}^{\alpha_1 \alpha_2 \alpha_3, \beta_1 \beta_2 \beta_3}) \chi^{\beta_1 \beta_2 \beta_3}$$

where

$$U_{\mu \bar{\mu}}^{\alpha_1 \alpha_2 \alpha_3, \beta_1 \beta_2 \beta_3} = N^{\alpha_1}_{\bar{\mu} \bar{\sigma}_2 \bar{\sigma}_3} N^{\beta_1}_{\bar{\mu} \bar{\sigma}_2 \bar{\sigma}_3} \delta^{\alpha_2 \beta_2} \delta^{\alpha_3 \beta_3}$$

and

$$V_{\mu \bar{\mu}}^{\alpha_1 \alpha_2 \alpha_3, \beta_1 \beta_2 \beta_3} = 3 N^{\alpha_1}_{\bar{\mu} \bar{\sigma}_2 \bar{\sigma}_3} N^{\beta_1}_{\bar{\mu} \bar{\sigma}_2 \bar{\sigma}_3} N^{\alpha_2}_{\mu_1 \mu_2 \mu_3} N^{\beta_2}_{\mu_1 \mu_2 \mu_3} N^{\alpha_3}_{\mu_1 \mu_2 \mu_3} N^{\beta_3}_{\mu_1 \mu_2 \mu_3}$$

$$+ 4 N^{\alpha_2}_{\mu_1 \mu_2 \mu_3} N^{\beta_2}_{\mu_1 \mu_2 \mu_3} N^{\alpha_3}_{\mu_1 \mu_2 \mu_3} N^{\beta_3}_{\mu_1 \mu_2 \mu_3} \delta^{\alpha_1 \beta_1} + 2 N^{\alpha_2}_{\mu_1 \mu_2 \mu_3} N^{\beta_2}_{\mu_1 \mu_2 \mu_3} N^{\alpha_3}_{\mu_1 \mu_2 \mu_3} N^{\beta_2}_{\mu_1 \mu_2 \mu_3} \delta^{\alpha_1 \beta_1}$$

Then by assuming the nucleus to be in the rest frame and defining the Fourier transform of $\chi(\vec{P}, \vec{q})$, we can calculate the expectation values of different terms in equations (10) and (11)

$$\chi^{* \alpha_1 \alpha_2 \alpha_3} N^{\alpha_1}_{\bar{\mu} \bar{\sigma}_2 \bar{\sigma}_3} N^{\beta_1}_{\bar{\mu} \bar{\sigma}_2 \bar{\sigma}_3} \delta^{\alpha_2 \beta_2} \delta^{\alpha_3 \beta_3} \chi^{\beta_1 \beta_2 \beta_3} = \left( \frac{3b^2}{2\pi^2} \right)^{\frac{3}{2}} \exp \left[ -\frac{3}{2} \mu^2 \vec{k}^2 \right] D(\bar{\mu}, \sigma_2, \sigma_3; \alpha_1)$$
\[ D(\mu, \sigma_2, \sigma_3; \beta_1)D(\alpha_1, \alpha_2, \alpha_3; A_i) D(\beta_1, \beta_2, \beta_3; A_i) \delta^{\alpha_2 \beta_2} \delta^{\alpha_3 \beta_3} \]  
\[ (12) \]

\[ I \left( \frac{27b^2}{8\pi^2} \right)^{\frac{3}{2}} \exp\left[ -\frac{3}{2} b^2k^2 \right] D(\mu, \sigma_2, \sigma_3; \alpha_1) D(\mu, \sigma_2, \sigma_3; \beta_1) D(\mu_1, \mu_2, \mu_3; \alpha_2) D(\rho_1, \rho_2, \rho_3; \beta_2) \]
\[ D(\rho_1, \rho_2, \rho_3; \beta_3) D(\alpha_1, \alpha_2, \alpha_3; A_i) D(\beta_1, \beta_2, \beta_3; A_i) \delta^{\alpha_1 \beta_1} \]
\[ (13) \]

\[ I \left( \frac{27b^2}{7\pi^2} \right)^{\frac{3}{2}} \exp\left[ -\frac{12}{7} b^2k^2 \right] D(\mu_1, \mu_2, \mu_3; \alpha_2) D(\mu_1, \mu_2, \mu_3; \beta_2) D(\rho_1, \rho_2, \rho_3; \alpha_3) D(\rho_1, \rho_2, \rho_3; \beta_3) D(\alpha_1, \alpha_2, \alpha_3; A_i) D(\beta_1, \beta_2, \beta_3; A_i) \delta^{\alpha_1 \beta_1} \]
\[ (14) \]

\[ I \left( \frac{27b^2}{4\pi^2} \right)^{\frac{3}{2}} \exp\left[ -3b^2k^2 \right] D(\mu_1, \mu_2, \mu_3; \alpha_2) D(\mu_2, \mu_3; \beta_2) D(\rho_1, \rho_2, \rho_3; \alpha_3) D(\rho_1, \rho_2, \rho_3; \beta_3) D(\alpha_1, \alpha_2, \alpha_3; A_i) D(\beta_1, \beta_2, \beta_3; A_i) \delta^{\alpha_1 \beta_1} \]
\[ (15) \]

\[ I \left( \frac{3}{2} \right)^{\frac{3}{2}} D(\mu_1, \mu_2, \mu_3; \alpha_2) D(\rho_1, \rho_2, \rho_3; \beta_2) D(\rho_1, \rho_2, \rho_3; \alpha_3) \]
\[ D(\mu_1, \mu_2, \mu_3; \beta_3) D(\alpha_1, \alpha_2, \alpha_3; A_i) D(\beta_1, \beta_2, \beta_3; A_i) \delta^{\alpha_1 \beta_1} \]
\[ (16) \]

where
\[ I = 8\pi^2 \int_0^\infty x^2 \, dx \int_0^\infty y^2 \, dy \int_{-1}^1 d(cos \theta) \exp\left[ -\frac{3x^2}{4b^2} \right] |\chi(x, y, \cos \theta)|^2 \]
\[ (17) \]

The above equations have been calculated with the same approximation as the one used in the references [11,16], specially a leading order expansion for \( \chi(\vec{P}, \vec{q}) \) [16]. As we pointed out before according to references [20,23], the other choice of nucleon-nucleon potentials do not change the \( A = 3 \) nuclear wave function and the EMC effect very much.
III. NUCLEUS STRUCTURE FUNCTION

The structure function measures the distribution of quarks as a function of $k^+$ (the light-cone momentum of initial quark) in the target rest frame which is equivalent to boosting the nucleus to an infinite momentum frame. This is usually done by using an ad hoc prescription for $k^0$ as a function of $|\vec k| (k^0 = [(\vec k^2 + m^2)^{1/2} - \epsilon_0])$. It has been shown that the resulting structure functions are not sensitive to this assumption \[11,24\]. So the valence-quark distribution at each $Q^2$, can be related to momentum distribution for each flavor in the nucleons of nucleus $A_i$ according to the following equation ($j = p, n$ ($a = u, d$) for protons (up-quarks) and neutrons (down-quarks), respectively),

$$ q_j^a(x, Q^2; A_i) = \int \rho_a^j(\vec k; A_i) \delta(x - \frac{k^+}{M}) \, d\vec k $$

After doing the angular integration, we get,

$$ q_j^a(x, Q^2; A_i) = 2\pi M \int_{k_{\text{min}}}^\infty \rho_a^j(\vec k; A_i) k \, dk $$

with

$$ k_{\text{min}}(x) = \frac{(xM + \epsilon_0)^2 - m^2}{2(xM + \epsilon_0)} $$

where $m$ ($M$) is the quark (nucleon) mass and $\epsilon_0$ is the quark binding energy. For each $Q^2$ value, it is possible (by using the fitting procedure as it will be described later on) to calculate the corresponding values of $m$ and $\epsilon_0$. Finally, the target structure function $F_{2A_i}^z(x, Q^2)$ can be expressed in terms of the valence quark distributions as following :

$$ F_{2,\text{ex}}^{A_i}(x, Q^2) = x \sum_{a=u,d; j=p,n} Q_a^2 q_j^a(x, Q^2; A_i) $$

So we have found the relation between quark momentum distribution and the target structure function. In order to fix the values of $m$ and $\epsilon_0$, we apply the above equations as well as the quark-exchange formalism to the proton ($A_i = 1, M_T = M_T = \frac{1}{2}$) as our target (obviously by considering the proton as our target there is no exchange term and we have just the direct term). In this case for each value of $b$ we find the pairs $m$ and $\epsilon_0$ such that we get the
best fit to the valence u and d quarks distribution functions of GRV’s partons structure function calculations [25]. The GRV’s partons structure functions fit the experimental proton structure function data over the whole range of \((x, Q^2)\) plane very well. Figure 1 shows our fitted proton structure function (obviously only for valence quarks) for \(b = 0.8\, fm\) with the \((m, \epsilon_0)\) pairs of \((165\, MeV, 150\, MeV)\) and \((165\, MeV, 250\, MeV)\) at \(Q^2 = 4\, GeV^2\). The dotted curve is those of GRV and other curves have been obtained by the same pairs of \((m, \epsilon_0)\), as above, but for different \(b\). The experimental data of SLAC [19] and NMC [29] as well as the full GRV’s NLO structure functions for proton (dash curve) and neutron (dash dotted curve) have been also given for comparison. It is seen that for \(x \geq 0.2\) we get very good fit to the data. With the same parameters we can calculate the corresponding neutron, \(^3He\) and \(^3H\) structure functions which are uncertain because of the lack of information about the neutron structure functions. Obviously in general our method fails for \(x \to 0\) (because of fitting procedure). It is also not good as \(x \to 1\) especially for the nucleus target, because we have ignored the Fermi motion effect by using the expansion in equation (17). In order to take into the account the Fermi motion effect we have worked in the convolutions approach and the harmonic oscillator basis for tritium and helium 3 with the procedure described in reference [1,26] as following. In the convolutions approach the nucleus structure function can be written as:

\[
F^A_{2i}(x, Q^2) = \sum_{j=p,n} \int_x^\infty dz f^A_j(z)F^j_2(x/z, Q^2) \tag{22}
\]

where \(f^A_j(z)\) and \(F^j_2(x, Q^2)\) are the proton or neutron distribution functions in the target nucleus and the corresponding structure functions, respectively. In the harmonic oscillator basis we have:

\[
f^A_j(z) = \sum_{n_j,l_j} G_{n_j,l_j} S_{n_j,l_j}(z, M, \hbar \omega, \varepsilon_{nl_j}) \tag{23}
\]

where \(G_{n,l}, S_{n,l}(z, M, \hbar \omega, \varepsilon_{nl})\), \(\hbar \omega = \frac{\hbar^2 \alpha^2}{M}\) and \(\varepsilon_{nl}\) are the occupation numbers, sum of harmonic oscillators polynomials (for present calculation it is just a Gaussian function), oscillator parameter and single particle energies [26]. For three-body system \(\alpha^2 = \frac{9}{2<r^2>}\) where
the rms radius, \( < r^2 >^{1/2} \), is 1.95\( fm \) and 1.7\( fm \) for helium 3 and tritium \([27]\), respectively. We also set \( \varepsilon_{nl} = 0 \), since we only intend to calculate the Fermi motion effect (the quark exchange is responsible for the binding effect). Figure 2(a) shows the ratio of Fermi motion effect for \(^3\)He to \(^3\)H i.e \( R_{\text{Fermi}}^{\text{He}}(x, Q^2)/R_{\text{Fermi}}^{\text{H}}(x, Q^2) \) where,

\[
R_{\text{EMC}}^{\text{He,Fermi}}(x, Q^2) = \frac{F_{\text{Fermi},2}^{\text{He}}(x, Q^2)}{2F_p^2(x, Q^2) + F_n^2(x, Q^2)}, \quad R_{\text{EMC}}^{\text{H,Fermi}}(x, Q^2) = \frac{F_{\text{Fermi},2}^{\text{H}}(x, Q^2)}{F_p^2(x, Q^2) + 2F_n^2(x, Q^2)}.
\]

(24)

It is seen from this figure that the Fermi motion is the same for helium 3 and tritium up to \( x \approx 0.55 \) and since the size of \(^3\)He is larger than \(^3\)H, the ratio start to decrease from this value i.e. the Fermi motion effect has larger size in tritium with respect to helium, as one expects.

Finally the total structure function for each nucleus can be written as the sum of the quark-exchange, equation (21) and Fermi motion, equation (22), effects :

\[
F_A^2(x, Q^2) = F_{2,\text{ex}}^{A}(x, Q^2) + F_{2,\text{Fermi}}^{A}(x, Q^2)
\]

(25)

We hope in our future works we omit this approximation by calculating the Fermi motion effect in the framework of quark-exchange formalism and the Faddeev wave functions for three-nucleon systems.

**IV. SELF-CONSISTENT TREATMENT OF** \( F_2^N/F_2^P \)

By defining the EMC-type ratios for the structure functions of helium 3 and tritium (as the one we did for the Fermi motion effect) i.e.:

\[
R_{\text{EMC}}^{\text{He}}(x, Q^2) = \frac{F_{2}^{3\text{He}}(x, Q^2)}{2F_p^2(x, Q^2) + F_n^2(x, Q^2)}
\]

(26)

and

\[
R_{\text{EMC}}^{\text{H}}(x, Q^2) = \frac{F_{2}^{3\text{H}}(x, Q^2)}{F_p^2(x, Q^2) + 2F_n^2(x, Q^2)}
\]

(27)
we can calculate the above EMC ratios by using the fitted GRV’s proton and neutron structure functions, the convolutions approach and the quark-exchange formalism. In figures 2(b) and 2(c) we have plotted these ratios for different values of b. The HERMES helium 3 data is taken from reference [28] at $Q^2 \simeq 7 GeV^2$. We should point out here that the quoted data is the combination of helium 3, deuterium and proton cross-sections (see Ackerstaff [28]) ($R = \frac{F^3_{He}}{F^3_{H} + F^2_{p}}$). We get good agreement with the present available data, especially for the deep-valence region. It is interesting that in this region the EMC ratios are not very sensitive to the different values of b.

Now by dividing the above two EMC ratios we find the following equation:

$$R_{EMC}^{He/3H}(x, Q^2) = \frac{R_{EMC}^{He}(x, Q^2)}{R_{EMC}(x, Q^2)} = \frac{R_{EMC}^{He/3H}(x, Q^2)[1 + 2F^n_2(x, Q^2)/F^n_2(x, Q^2)]}{2 + F^n_2(x, Q^2)/F^n_2(x, Q^2)}$$

with

$$R_{EMC}^{He/3H}(x, Q^2) = \frac{F^3_{He}(x, Q^2)}{F^3_{H}(x, Q^2)}$$

and by imposing the constrain,

$$R_{EMC}^{He/3H}(x, Q^2) \simeq 1$$

which will be discussed later on.

The above equation can be solved for the neutron to proton structure functions ratio, $F^n_2(x, Q^2)/F^n_2(x, Q^2)$, in terms of the EMC ratio, $R_{EMC}^{He/3H}(x, Q^2)$, which directly yields:

$$\frac{F^n_2(x, Q^2)}{F^n_2(x, Q^2)} = \frac{2R_{EMC}^{He/3H}(x, Q^2) - R_{EMC}^{He/3H}(x, Q^2)}{2R_{EMC}^{He/3H}(x, Q^2) - R_{EMC}^{He/3H}(x, Q^2)}$$

Equations (28) and (31) are coupled in terms of the neutron to proton structure function ratio and will be solved by a self-consistent iteration procedure i.e the output of equation (31) will be used as the input for equation (28) and so on, by using equation (30) as our constrain.

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V. RESULT, DISCUSSION AND CONCLUSION

In figure 3 the ratios of structure functions of helium 3 to tritium (equation (29)) have been plotted for different values of $b$ (full curves) by using GRV's structure functions. The sun-burst points are the expected ratio which have been estimated by using the kinematics of the proposed 11 GeV Jefferson Laboratory experiment [10,20]. It is seen that there is very good agreement between the estimated prediction [10,20] and the present calculation. This shows that the GRV’s proton and neutron structure functions and present model without any parameter can predict reasonably the structure functions of helium 3 and tritium.

The EMC ratios (equation (28)) has been given in figure 4. In general, since we have taken into the account all of the properties of the structure functions of proton, neutron, helium 3 and tritium, it is expected that $\mathcal{R}_{EMC}^{3\text{He}/3\text{H}}(x, Q^2) \simeq 1$, as we imposed it as a constrain in equation (30). However it is seen that the calculated EMC ratio has a very small variation from $\mathcal{R}_{EMC}^{3\text{He}/3\text{H}}(x, Q^2) = 1$. But this deviation increases as $x \to 1$. The similar behavior is also seen in the others calculations in which the impulse approximation have been used [20].

Finally, in figure 5 the calculated neutron to proton structure functions ratios have been plotted for different values of $b$. The curve without iteration is the one in which we have not imposed the constrain equation (30). The first and second iteration curves have been calculated by considering the constrain $\mathcal{R}_{EMC}^{3\text{He}/3\text{H}}(x, Q^2) = 1.01$. The third iteration is not distinguishable from the second one. The limit values of this ratio for $x = 1$ (as it was discussed in the introduction) from different models are shown in the left part of this figure by arrows. The data are from references [17,18,19,28]. The iterations approximately converge after the third iteration. Our results cover all of the range of present available data. This can be done by varying $\mathcal{R}_{EMC}^{3\text{He}/3\text{H}}(x, Q^2)$ between 0.99 and 1.01. Variation of $b$, i.e. the nucleon size, does not affect the results. But the size of EMC ratios is important as $x$ becomes closer to 1. Our calculated neutron to proton structure functions ratios are also in good agreement with present theoretical calculation in which different models and approximations have been used [17-22].
In conclusion, we have calculated the neutron to proton structure functions ratios in the framework of quark-exchange model. We have treated u and d quarks as well as proton and neutron explicitly in our formalism and we have found satisfactory results compare to present available data and others theoretical models. However, we can improve our calculation by (i) treating the Fermi motion effect explicitly in the quark-exchange framework i.e. calculating the expansion we made in equations (13)-(16) as well as using other choice of nuclear wave-functions which have been calculated with the new nucleon-nucleon potentials, (ii) evaluating the connected three-body diagram which we have been ignored in the present calculation [15] and finally (iii) taking into the account the nucleon-nucleon correlations. The later is interesting, because for nuclei one could have non-vanishing structure functions for x larger than one [29,30], which can be measured and calculated by considering the effect of short-range correlations in nuclei.

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FIGURES

FIG. 1. The proton structure function, only with valence quarks, for \((m, \epsilon_0)\) pairs of \((165 MeV, 150 MeV)\) and \((165 MeV, 250 MeV)\) at \(Q^2 = 4 GeV^2\) and various \(b\) values. The pairs \((165 MeV, 150 MeV)\) and \((165 MeV, 250 MeV)\) at \(Q^2 = 4 GeV^2\) have been chosen such that \(b = 0.8 fm\) gives the best fit to the GRV’s valence-quarks distributions (dotted curve)[25]. The dash and dotted dash curves are the GRV’s full NLO proton and neutron distribution functions. The NMC and SLAC data [19,28] for the structure function of proton have been also given for comparison.

FIG. 2. (a) The EMC ratios of the structure functions of helium 3 to tritium by considering only the Fermi motion effect with parameters given in the text. The EMC effect in helium 3 (b) and tritium (c) for different values of \(b\).

FIG. 3. (a) The ratios of structure functions of helium 3 to tritium (equation (29)) for different values of \(b\) (full curves). The dash curve is after second iteration with \(R_{EMC}^{3He/3H}(x, Q^2) = 1.01\). The sun-burst point are from references [10,20] (see also the text).

FIG. 4. The EMC ratios for different values of \(b\) without any constrain (equation (28)).

FIG. 5. The neutron to proton structure functions ratios (equation (31), see the text for more details). The data are from Whitlow et al. [19], Melnitchouk and Thomas [18], Bodek et al. [17] and NMC [28].
Fig. 1
Fig. 2
Fig. 3
Fig. 4
Fig. 5