LARGE-ANGLE Bhabha Scattering at LEP 1

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Abstract

A critical assessment is given of the theoretical uncertainty in the predicted cross-sections for large-angle Bhabha scattering at LEP 1, with or without $t$-channel subtraction. To this end a detailed comparison is presented of the results obtained with the programs ALIBABA and TOPAZ0. Differences in the implementation of the radiative corrections and the effect of missing higher-order terms are critically discussed.

¹Research supported by a fellowship of the Royal Dutch Academy of Arts and Sciences.
1 Introduction

In 1995 the Z phase of LEP has come to an end and at present the ultimate analysis of the data is imminent. This involves in particular the completion of the line-shape analysis, including the final LEP energy calibration. Consequently, the safest possible estimate of the theoretical accuracy is of the upmost importance. It should be noted that the LEP 1 data (1990–1995) have been taken in the energy ($\sqrt{s}$) range $|\sqrt{s} - M_Z| < 3$ GeV and consist of the hadronic and leptonic cross-sections, the leptonic forward–backward asymmetries, the various polarization asymmetries, the partial widths, and the quark forward–backward asymmetries. All this makes it mandatory to assess the theoretical precision of the available programs for different channels and for energies up to $\sim 3$ GeV away from the resonance.

In this note we focus on the electron (Bhabha) channel. Bhabha scattering is measured with remarkable precision at LEP 1/SLC in two complementary kinematical regions: small and large scattering angles. The former plays a crucial role in determining the luminosity. The latter is essential for extracting the Z-boson properties. The main message of this note concerns an update of the theoretical precision in the large-angle regime. This precision depends on the beam energy and on the event selection used, the worst case being at a few GeV above the Z resonance. There the different programs are found to deviate by as much as 1%. This should be contrasted with the expected experimental systematic errors [1], displayed in Table 1 (an example of the statistical errors [2] is given in Table 2). In this note we present a detailed analysis of the observed deviations and translate this into estimates for the theoretical precision.

\begin{table}[!h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
 & ALEPH & DELPHI & L3 & OPAL \\
\hline $\sigma_e$ & 0.30/0.30/0.31\% & 0.59/0.54/0.75\% & 0.30/0.23/1.0\% & 0.23/0.24/\% \\
$A_{FB}$ & 0.0025/0.0022/0.0021 & 0.003/0.003/0.01 & 0.0016/0.0016/0.002 & \\
\hline
\end{tabular}
\caption{The (preliminary) experimental systematic errors for the years 1993/1994/1995 at the Z peak, not including the common uncertainty due to the LEP energy calibration.}
\end{table}

2 Comparison of ALIBABA and TOPAZ0

In order to assess the theoretical uncertainties in the predictions for large-angle Bhabha scattering (LABS), we first perform a detailed numerical comparison of the programs ALIBABA [3] and TOPAZ0 [4]. Previous comparisons can be found in [5] and also in [6]. The input parameters chosen for the comparison are: $M_Z = 91.1863$ GeV, $m_t = 175.6$ GeV, $M_H = 300$ GeV, and $\alpha_s(M_Z^2) = 0.118$. The other Standard Model parameters we take from Ref. [7]. The acceptance cuts for the final-state particles consist of a minimum energy for both particles (1 GeV), an angular acceptance for the electron ($40^\circ < \vartheta_e < 140^\circ$), and a maximum acollinearity angle ($10^\circ$ or $25^\circ$). For the energy we take the characteristic LEP 1 energies: $\sqrt{s} = 88.45, 89.45, 90.20, 91.19, 91.30, 91.95, 93.00$, and $93.70$ GeV.

In Table 3 we present the comparison for the cross-sections ($\sigma$) and forward–backward asymmetries ($A_{FB}$). Besides the results for the full Bhabha process, we also show the pure s-channel contributions. The reason for that is twofold. First, the s-channel branch of TOPAZ0 allows a systematic inclusion of the full $O(\alpha^2)$ QED corrections [8], which are only present in leading-log approximation in ALIBABA.
| Year | √s [GeV] | 1990 | 1991 | 1992 | 1993 | 1994 | 1995 |
|------|---------|------|------|------|------|------|------|
|      | stat. error | 3.8% | 2.5% | 3.1% | 1.2% | 1.6% | 0.7% |
|      | stat. error | 3.3% | 2.2% | 3.1% | 1.6% | 1.6% | 0.7% |
|      | stat. error | 3.8% | 2.5% | 3.1% | 1.2% | 1.6% | 0.7% |
|      | stat. error | 3.3% | 2.2% | 3.1% | 1.6% | 1.6% | 0.7% |
|      | stat. error | 3.8% | 2.5% | 3.1% | 1.2% | 1.6% | 0.7% |
|      | stat. error | 3.3% | 2.2% | 3.1% | 1.6% | 1.6% | 0.7% |

Table 2: Sample of statistical errors for the electron cross-section from ALEPH. For the various runs the highest and lowest energies are shown, as well as the energy closest to the $Z$ peak.

In this way some of the statements that will be made for the full Bhabha process can be checked in the s-channel mode. Secondly, the so-called $t$-channel subtraction procedure for extracting the pure s-channel part is still very popular among the experimental Collaborations. As such comparisons of all components of LABS are well deserving the effort.

The numerical precision of the TOPAZ0 numbers is 0.00001 in $A_{FB}$ and 0.001% (relative precision) in $\sigma$. For ALIBABA the numerical precision is much less in view of the five-dimensional VEGAS integration, i.e. 0.0006 in $A_{FB}$ and 0.05% in $\sigma$. The technical precision of the results, however, will not affect our conclusions. Whereas the observed LABS differences between TOPAZ0 and ALIBABA are largely acceptable for a maximum acollinearity angle of 25°, this is certainly not true for 10°. In that case large deviations up to 1% show up for energies above the $Z$ resonance, as already discussed in [5].

In Table 4 we try to quantify where this difference is coming from. In this context it is important to note that the main conceptual difference between TOPAZ0 and ALIBABA in the Bhabha channel lies in the implementation of the non-leading-log QED corrections. Both programs are based on the structure-function method for calculating the (dominant) leading-log corrections $[\propto (\alpha L/\pi)^n]$, with $n = 1, 2$ and $L = \log(s/m_e^2)$. When it comes to the non-leading-log corrections both programs use quite different approaches. TOPAZ0 was designed to be an efficient fitter of realistic observables around the $Z$ resonance, where LABS is effectively dominated by the annihilation channel. The structure functions in TOPAZ0 are based on the iterative solution of the master evolution equation, which accounts for the well-known second-order electron form factor, and therefore they reproduce the sub-leading terms $[\propto (\alpha/\pi)^n L^m]$, with $n = 1, 2$ and $n > m$ for the $s$ channel. This procedure, however, does not reproduce the (unknown) correct answer for the sub-leading terms in the $t$ channel.

In TOPAZ0 a part of the sub-leading terms in the $t$ channel enters through a final-state-radiation factor, which is subsequently convoluted with the structure functions. In this way part of the $O(\alpha)$

1In a realistic (i.e. not fully-extrapolated) set-up some $O(\alpha)$ hard-photonic non-log corrections are missing in the $s$ channel. However, their effect is tiny around the $Z$ resonance  [8].
of the QED corrections in both programs can be estimated by Bhabha results are indicated by the superscript "NL"

Table 3: Comparison of TOPAZ0 (T) and ALIBABA (A) for the cross-section (in pb) and the forward–backward asymmetry. The TOPAZ0 results do not include initial-state pair production. The full Bhabha results are indicated by the superscript “s + \bar{t}”, the s-channel contributions by “s”. The quantity \( \delta \) stands for the relative deviation 100\% \((T − A)/T\). The input parameters can be found in the text.

non-log terms are implemented, as well as a subset of sub-leading second-order terms \( \propto a^2 L/\pi^2 \). These sub-leading higher-order terms are not present in ALIBABA. However, in the ALIBABA approach the full set of non-log \( \mathcal{O}(a) \) corrections are determined by means of the five-dimensional VEGAS integration, needed for handling the radiative process. These non-log corrections are listed in the NL\(_A\) entries in Table 3. The entries \( \Delta_{NL} \) in Table 3 quantify the deviations caused by considering non-leading terms only from final-state radiation. In conclusion, the main difference in the treatment of the QED corrections in both programs can be estimated by \( C_F (\sigma^{s+\bar{t}} - \text{NL}_A) - \text{NL}_A \). Here the \( \mathcal{O}(a) \) final-state-radiation factor \( C_F \) is given by \(-0.0165 \) \((-0.0052)\) for a maximum acollinearity angle

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of 10° (25°). In Table 3 we present this estimate in the form of a relative correction factor ($\Delta_{FSR}$) with respect to the full Bhabha cross-section. Comparing the estimate with the actual deviation $\delta$ in Table 3, a systematic shift ($\delta - \Delta_{FSR}$) is observed that is roughly independent of the maximum acollinearity angle. The resulting shift can be attributed to the fact that the ALIBABA weak library is not up to date. This is supported by a direct comparison with the Bhabha branch of the program ZFITTER [9]. Another thing to note is that for a maximum acollinearity angle of 10° both TOPAZ0 and ALIBABA seem to miss terms of the order of 0.3–0.6% (cf. $\Delta_{NL}$ and $\Delta_{NL} - \Delta_{FSR}$ in Table 3).

Below and near the Z resonance both effects go in the same direction, compensating each other in the difference. This leads to a somewhat misleading agreement between the programs. Above the resonance the effects have different signs, leading to an enhancement of the deviations. The relative deviation in the non $s$-channel components ($\sigma^{s+t} - \sigma^{s}$), which is practically negligible below the resonance, grows up to $\sim 10\%$ on the high-energy side. There, however, the non $s$-channel cross-sections are rapidly decreasing (e.g. roughly 23 pb at $\sqrt{s} = 93.70$ GeV, i.e. 6.5% of full LABS, versus 284 pb at $\sqrt{s} = 88.45$ GeV, i.e. 62% of full LABS).

### 3 Theoretical error estimates for large-angle Bhabha scattering

Having established the main sources of the differences between TOPAZ0 and ALIBABA, we can now address the question of the error in the theoretical predictions for large-angle Bhabha scattering. In the experimental analyses either the full Bhabha cross-section is used or merely the $s$-channel contributions. The latter are obtained through $t$-channel subtraction, i.e. by subtracting the non $s$-channel contributions that involve $t$-channel gauge-boson exchange. The subtraction procedure is aimed at reducing the full LABS to a simple annihilation process to be subsequently analyzed by some up-to-date (from the point of view of the WEAK/QCD library) program.

Correspondingly we will present the theoretical errors for both procedures. The bulk of the theoretical errors are due to missing QED corrections: missing non-log $\mathcal{O}(\alpha)$ corrections in TOPAZ0,
missing higher-order sub-leading effects from the structure-function convolution of the non-log $\mathcal{O}(\alpha)$ corrections in ALIBABA, missing effects in both programs from the (unknown) sub-leading terms in the structure functions, and missing initial-state pair-production effects in ALIBABA. In addition there is the usual calorimetric measurement problem, which arises if the cross-section is not inclusive in the energy of the outgoing fermions. While with the present cuts the effect is estimated to be negligible by TOPAZ0 ($\ll 0.1\%$), for high-energy thresholds the contribution will grow considerably.

In general one should remember that an absolute test of the QED corrections does not exist. QED corrections are convoluted with – in principle – different kernel cross-sections, so that the shift in the absolute QED corrections also depends on the differences in the non-QED parts.

Before coming to all these QED effects, we first estimate the uncertainty in the weak sector. From a detailed study with TOPAZ0 it follows that different weak options influence the cross-sections by at most 0.06% and the forward–backward asymmetry by at most 0.0002. In this respect the quality of the estimate has not changed since the work done in [3], since the upgrading in TOPAZ0 and ZFITTER have been constantly cross-checked. The uncertainty in ALIBABA roughly equals the $\Delta F_{SR} - \delta$ shifts displayed in Table 4: varying $M_H$ between 60 and 1000 GeV, and $M_Z$, $m_t$, and $\alpha_s(M_Z^2)$ within one experimental standard deviation, leads to variations of the cross-sections and asymmetries that closely resemble the ones observed with TOPAZ0. The ALIBABA variations have the same sign as the TOPAZ0 ones and do not differ by more than 0.06% for the cross-section and 0.0006 for the forward–backward asymmetry, which is of the same size as the numerical error in the ALIBABA results.

In the context of the QED errors, the ALIBABA error owing to the absence of initial-state pair-production (ISPP) effects can be estimated in a straightforward way with the help of TOPAZ0. For the eight LEP 1 energy points we find for the full LABS cross-section: $-0.23\%$, $-0.25\%$, $-0.26\%$, $-0.25\%$, $-0.24\%$, $-0.19\%$, $-0.04\%$, $+0.07\%$. Here the final-state $e^+e^-$ pair is required to have an invariant mass that exceeds 50% of the total energy $\sqrt{s}$ (i.e. $z_{\text{min}} = 0.25$). It should be noted that among all radiative corrections the ISPP appears as the most questionable. While relatively safe around the resonance, different implementations of ISPP start to register some disagreement at higher energies [10] and strongly depend on kinematical cuts, in other words on the precise separation between 2-fermion and 4-fermion physics.

The missing sub-leading higher-order terms from the convolution of the non-log $\mathcal{O}(\alpha)$ corrections (MSL1) in ALIBABA is harder to estimate. Here we deploy two methods. As can be read off from the $\text{NL}_A$ entries in Table 4 the non-log $\mathcal{O}(\alpha)$ corrections in ALIBABA appear in two different shapes. For a maximum acollinearity angle of $10^\circ$ the $\text{NL}_A$ contribution exhibits a resonance-like behaviour, with a pronounced peak around $\sqrt{s} = M_Z$. In that case the leading-log corrections to this contribution should closely resemble the leading-log corrections to the lowest-order cross-section. A natural way of estimating these higher-order effects is by dividing the $\text{NL}_A$ contributions by the lowest-order cross-sections (in ‘dressed-Born’ form), taking the maximum value of this ratio, and subsequently multiplying the leading-log corrections in ALIBABA by this number. Note that in principle the sign of the missing terms will be fixed. For a maximum acollinearity angle of $25^\circ$ the $\text{NL}_A$ contribution is rather flat, with a sudden jump just above the $Z$ resonance. In that case the best way to estimate the higher-order effects is by simply multiplying the maximum value for $|\text{NL}_A|$ by the factor $(4\alpha L/\pi)$. Here an additional factor of two is added for safety. The sign of the missing terms is only determined for the three highest energy points, i.e. the ones after the jump. Since the structure-function convolution probes the variation in the $\text{NL}_A$ contributions at lower energies, a negative sign is expected for the higher-order corrections in these three energy points. Note that this second method also applies to
the non s-channel cross-sections needed for the t-channel subtraction, irrespective of the maximum acollinearity angle. Finally, the effect from the (unknown) sub-leading terms in the structure functions (MSL2) can be estimated by multiplying the leading-log corrections by the factor \((4\alpha/\pi)\). Again a factor of two is added for safety. Note that the sign of the MSL2 terms is not fixed.

In Table 5, we summarize the estimates for the theoretical errors of the ALIBABA program. In the total error, the various estimates are added in quadrature. For the unsubtracted cross-section we take the average of the two \(\delta - \Delta_{FSR}\) shifts, displayed in Table 4, as a measure of the error in the weak corrections. For the non s-channel cross-section a constant uncertainty of 0.2% is assumed. At this point one should bear in mind that the non s-channel cross-sections exhibit a strong cancellation

| LEP 1 energy in GeV | 88.45 | 89.45 | 90.20 | 91.19 | 91.30 | 91.95 | 93.00 | 93.70 |
|-------------------|-------|-------|-------|-------|-------|-------|-------|-------|
| weak              | 0.17% | 0.06% | 0.08% | 0.27% | 0.27% | 0.15% | 0.05% | 0.12% |
|                  | 0.57 pb | 0.63 pb | 0.64 pb | 0.39 pb | 0.33 pb | 0.33 pb | 0.33 pb | 0.33 pb |
| pairs             | 0.23% | 0.25% | 0.26% | 0.25% | 0.24% | 0.19% | 0.04% | 0.07% |
|                  | 0.65 pb | 0.78 pb | 0.83 pb | 0.48 pb | 0.40 pb | 0.32 pb | 0.07 pb | 0.12 pb |
| MSL1              | 0.34% | 0.42% | 0.47% | 0.39% | 0.36% | 0.13% | 0.22% | 0.35% |
|                  | 1.36 pb | 1.36 pb | 1.36 pb | 1.36 pb | 1.36 pb | 1.36 pb | 1.36 pb | 1.36 pb |
| MSL2              | 0.23% | 0.28% | 0.32% | 0.26% | 0.24% | 0.24% | 0.24% | 0.24% |
|                  | 0.46 pb | 0.54 pb | 0.50 pb | 0.31 pb | 0.42 pb | 0.76 pb | 0.52 pb | 0.35 pb |
| total             | 0.5 % | 0.6 % | 0.6 % | 0.6 % | 0.6 % | 0.4 % | 0.3 % | 0.4 % |
|                  | 1.7 pb | 1.8 pb | 1.8 pb | 1.5 pb | 1.5 pb | 1.6 pb | 1.5 pb | 1.4 pb |

| LEP 1 energy in GeV | 88.45 | 89.45 | 90.20 | 91.19 | 91.30 | 91.95 | 93.00 | 93.70 |
|-------------------|-------|-------|-------|-------|-------|-------|-------|-------|
| weak              | 0.17% | 0.06% | 0.08% | 0.27% | 0.27% | 0.15% | 0.05% | 0.12% |
|                  | 0.61 pb | 0.67 pb | 0.69 pb | 0.42 pb | 0.38 pb | 0.38 pb | 0.38 pb | 0.38 pb |
| pairs             | 0.23% | 0.25% | 0.26% | 0.25% | 0.24% | 0.19% | 0.04% | 0.07% |
|                  | 0.71 pb | 0.84 pb | 0.89 pb | 0.53 pb | 0.45 pb | 0.36 pb | 0.08 pb | 0.13 pb |
| MSL1              | 0.21% | 0.15% | 0.11% | 0.08% | 0.09% | 0.11% | 0.20% | 0.27% |
|                  | 0.97 pb | 0.97 pb | 0.97 pb | 0.97 pb | 0.97 pb | 0.97 pb | 0.97 pb | 0.97 pb |
| MSL2              | 0.17% | 0.24% | 0.28% | 0.24% | 0.22% | 0.22% | 0.22% | 0.27% |
|                  | 0.26 pb | 0.35 pb | 0.30 pb | 0.50 pb | 0.62 pb | 0.95 pb | 0.71 pb | 0.53 pb |
| total             | 0.4 % | 0.4 % | 0.4 % | 0.4 % | 0.4 % | 0.3 % | 0.3 % | 0.4 % |
|                  | 1.4 pb | 1.5 pb | 1.5 pb | 1.3 pb | 1.3 pb | 1.5 pb | 1.3 pb | 1.2 pb |

Table 5: Error estimates for the large-angle Bhabha cross-section as predicted with ALIBABA. The numbers in the first row of every entry correspond to the uncorrected cross-sections and are given relative to \(\sigma^{s+t}\) (see Table 3). The numbers in the second row (given in pb) correspond to the non s-channel components needed for the t-channel subtraction.

We have also derived the non s-channel errors for the ALEPH angular acceptance, involving a 20° maximum acollinearity angle and \(45° < \angle(e^-, e_{beam}) < 155°\). The errors read: 1.1 pb, 1.2 pb, 1.2 pb, 1.1 pb, 1.1 pb, 1.2 pb, 1.1 pb, and 1.0 pb. These errors are smaller than the ones in Table 5, since also the non s-channel cross-sections themselves are smaller. The errors on the full LABS cross-section are given by: 0.5%, 0.4%, 0.4%, 0.5%, 0.5%, 0.4%, 0.4%, and 0.5%.

2We have also derived the non s-channel errors for the ALEPH angular acceptance, involving a 20° maximum acollinearity angle and \(45° < \angle(e^-, e_{beam}) < 155°\). The errors read: 1.1 pb, 1.2 pb, 1.2 pb, 1.1 pb, 1.1 pb, 1.2 pb, 1.1 pb, and 1.0 pb. These errors are smaller than the ones in Table 5, since also the non s-channel cross-sections themselves are smaller. The errors on the full LABS cross-section are given by: 0.5%, 0.4%, 0.4%, 0.5%, 0.5%, 0.4%, 0.4%, and 0.5%.
between the \( t \)-channel and \( s-t \) interference contributions at the three highest energy points (see Table 3). In order to avoid underestimating the weak and ISPP errors at these energies, we therefore use the cross-section at 91.30 GeV in the error estimates. The error estimates for the non \( s \)-channel cross-sections are given in pb, rather than relative to the full \( \sigma^{s+t} - \sigma^s \) results. In this way the errors are independent of the amount of cancellation between the \( t \)-channel and \( s-t \) interference contributions. Note that the non \( s \)-channel errors are not derived from the errors for the full Bhabha cross-section. They are derived directly from \( \sigma^{s+t} - \sigma^s \), eliminating in this way the correlation between the errors in \( \sigma^{s+t} \) and \( \sigma^s \).

As a cross-check of the above error-estimate procedure, we have also derived the pure \( s \)-channel errors. For a maximum acollinearity angle of 10\( ^\circ \) we find 0.4\%, 0.4\%, 0.4\%, 0.5\%, 0.5\%, 0.3\%, 0.4\%, and 0.5\%; for 25\( ^\circ \) the errors are 0.7\%, 0.6\%, 0.5\%, 0.3\%, 0.3\%, 0.4\%, 0.4\%, and 0.4\%. This is in good agreement with the observed \( s \)-channel deviations in Table 3.

The main approximation for LABS in TOPAZ0 is the leading-logarithmic one in the non \( s \)-channel components. The QED theoretical error in TOPAZ0 is therefore growing when enlarging the angular acceptance to smaller angles, or in general in all situations where the non \( s \)-channel terms are increased.

In order to give an estimate for the theoretical error in TOPAZ0 we proceed in the following – conservative – way. First of all, we assign an overall 0.06\% error to the weak corrections. This is not the consequence of a mistreatment of the corrections, but a safe upper bound on the uncertainty arising from different implementation schemes.

Next, the ISPP as implemented in TOPAZ0 is strictly speaking only valid for \( s \)-channel processes. If \( \Delta_p \) is the relative ISPP effect, then \( \Delta_p \sigma^s \) is the correct \( s \)-channel contribution. The missing terms are proportional to the pure \( t \)-channel and \( s-t \) interference components. However, we refrain from assigning to LABS an overall uncertainty of \( \pm \Delta_p (\sigma^{s+t} - \sigma^s) \), since the difference \( \sigma^{s+t} - \sigma^s \) becomes very small above the peak, while the individual \( t \)-channel and \( s-t \) interference terms can be as large as below the peak. Clearly this cancellation cannot be transferred to the error estimates. We use two different procedures. First we proceed by assigning to the cross-section an error \( \pm \Delta_p \sigma_{\text{max}} \), where \( \sigma_{\text{max}} = \max(\sigma^s, \sigma^{s+t} - \sigma^s) \). The obtained error is roughly half of \( \Delta_p \sigma^{s+t} \) below the resonance and almost coincides with \( \Delta_p \sigma^{s+t} \) above it. Alternatively we compute \( \sigma_{\text{abs}} \), the sum of the \( t \)-channel term and the absolute value of the \( s-t \) interference, and assign an error \( \pm \Delta_p \sigma_{\text{abs}} \). In the two procedures the errors read: 0.14\%, 0.13\%, 0.17\%, 0.21\%, 0.21\%, 0.18\%, 0.04\%, 0.07\% and 0.17\%, 0.14\%, 0.11\%, 0.05\%, 0.05\%, 0.08\%, 0.03\%, 0.07\%. There are appreciable differences only around the peak. In the summary we will report the average of the two procedures.

Similarly the MSL2 effect in TOPAZ0 is only active for the \( t \)-channel and \( s-t \) interference contributions. For the associated uncertainty we take again the average of MSL2 \( \sigma_{\text{max}} \) and MSL2 \( \sigma_{\text{abs}} \), with MSL2 as estimated by ALIBABA. We have also performed a consistency check by computing the TOPAZ0 non \( s \)-channel cross-sections with the change \( L \to (1 \pm 4 \Delta \beta) L \) in the definition of \( \beta \), appearing in the structure functions. The resulting effect is roughly equal to the one obtained in the \( \sigma_{\text{abs}} \) method. Nevertheless we prefer to be more conservative and to average with the error from the \( \sigma_{\text{max}} \) method.

Another source of theoretical error remains in the missing non-log \( \mathcal{O}(\alpha) \) corrections (MNL) from QED radiation. TOPAZ0 has some of these non-log terms, which have been included into the structure functions through the exact second-order \( s \)-channel vertex. We have computed this contribution, \( \Delta_{NL}^T \), and found that it is largely due to the \( \pi^2/3 - 2 \) term in the K-factor. Next we assume that \( -\Delta_{NL} \), as taken from ALIBABA, is the exact contribution and we estimate the uncertainty to be \( -\Delta_{NL} - \Delta_{NL}^T \).
In Table 6 we summarize the estimates for the theoretical errors of the TOPAZ0 program. In the total error the various estimates are again added in quadrature.

| LEP 1 energy in GeV |
|---------------------|
| 88.45 | 89.45 | 90.20 | 91.19 | 91.30 | 91.95 | 93.00 | 93.70 |
| maximum acollinearity angle: 10° |
| weak | 0.06% | 0.06% | 0.06% | 0.06% | 0.06% | 0.06% | 0.06% |
| pairs | 0.16% | 0.14% | 0.14% | 0.13% | 0.13% | 0.13% | 0.13% |
| MSL2 | 0.16% | 0.15% | 0.17% | 0.14% | 0.13% | 0.17% | 0.21% |
| MNL | 0.00% | 0.07% | 0.23% | 0.33% | 0.29% | 0.28% | 0.03% |
| total | 0.2% | 0.2% | 0.3% | 0.4% | 0.4% | 0.2% | 0.3% |
| maximum acollinearity angle: 25° |
| weak | 0.06% | 0.06% | 0.06% | 0.06% | 0.06% | 0.06% |
| pairs | 0.17% | 0.14% | 0.14% | 0.14% | 0.13% | 0.04% |
| MSL2 | 0.12% | 0.13% | 0.15% | 0.13% | 0.12% | 0.15% |
| MNL | 0.65% | 0.41% | 0.12% | 0.00% | 0.00% | 0.01% |
| total | 0.7% | 0.5% | 0.3% | 0.2% | 0.2% | 0.5% |

Table 6: Error estimates for the large-angle Bhabha cross-section as predicted with TOPAZ0.

Finally we come to the theoretical errors on the forward–backward asymmetry. As can be seen from Table 3, the forward–backward asymmetry $A_{FB}$ shows the typical $t$-channel effect, which makes it unique among leptonic asymmetries. At the peak we observe a T–A absolute difference of $-0.0003$ ($+0.0017$) for an average asymmetry of $+0.1388$ ($+0.1493$) at $\theta_{acoll} = 10°$ ($25°$). This deteriorates up to a $-0.0030$ ($+0.0033$) difference for an average asymmetry of $+0.1803$ ($+0.3538$) at $\sqrt{s} = 93.70$ GeV ($89.45$ GeV) and $\theta_{acoll} = 10°$ ($25°$). The $s$-channel asymmetry, $A_{FB}$, is instead very small in the $Z$ peak region. For this quantity the disagreement is globally contained within an absolute difference of 0.0013. This is no surprise, since it has been shown in [5] that differences among programs are weak-dominated around the peak and QED-dominated far from the peak only. For $A_{FB}$ we assign to both codes an error $\pm \Delta_{FB}$, with $\Delta_{FB}$ given by the half-difference of the predictions (i.e. half of the T–A row of table 3).

4 Conclusions

The main emphasis of this note was on presenting the safest possible estimate for the theoretical accuracy of the large-angle Bhabha scattering calculations. As usual, in estimating theoretical uncertainties one proceeds by comparing the results of different programs starting from a common set of input parameters and kinematical cuts. In our case we have used the predictions of ALIBABA and TOPAZ0. The registered differences give a rough idea of the uncertainty associated with different implementations of radiative corrections. Successively one tries to estimate the internal accuracy of the programs by deriving bounds on the effects of missing parts of the calculations.

Our analysis shows a substantial agreement between the two programs for larger acollinearity cuts, with a deterioration towards small acollinearity cuts and energies on the high-energy side of the
resonance. For a maximum acollinearity angle of 10° we find deviations of $-0.14\%, +0.09\%, -1.00\%$
at $M_Z - 2.5$ GeV, $M_Z$, $M_Z + 2.5$ GeV for the full Bhabha cross-section ($-0.38\%, +0.24\%, -0.57\%$ for the $s$ channel alone). The main sources of these deviations seem understood.

As for the internal estimate, we find that for the tightest acollinearity cut both ALIBABA and TOPAZ0 miss terms of the order of 0.2–0.6%, depending on the c.m.s energy. For TOPAZ0 the bulk of the effect is in the non $s$-channel. From the point of view of TOPAZ0 the analysis of the full Bhabha cross-section is anyhow preferable to the $t$-channel subtraction procedure, provided that the $s$ channel dominates. The size of the uncertainty in ALIBABA resembles the one in TOPAZ0, although this is somehow accidental in view of the different origins of the contributing effects. The ALIBABA program is better suited for the $t$-channel subtraction procedure, with an uncertainty in the non $s$-channel components ranging from 1.0 pb to 1.8 pb for the various c.m.s. energies and angular acceptances. If subtraction is performed then the TOPAZ0 program is better suited for the $s$-channel analysis due to the high precision achieved there.

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