Space shuttle’s liftoff: a didactical model*

Riccardo Borghi and Turi Maria Spinozzi

Dipartimento di Ingegneria, Università degli Studi ‘Roma tre’ Via Vito Volterra 62, I-00146 Rome, Italy

E-mail: riccardo.borghi@uniroma3.it

Received 27 March 2017, revised 10 May 2017
Accepted for publication 25 May 2017
Published 15 June 2017

Abstract

The pedagogical aim of the present paper, thought for an undergraduate audience, is to help students to appreciate how the development of elementary models based on physics first principles is a fundamental and necessary preliminary step for the behaviour of complex real systems to be grasped with minimal amounts of math. In some particularly fortunate cases, such models also show reasonably good results when are compared to reality. The speed behaviour of the Space Shuttle during its first two minutes of flight from liftoff is here analysed from such a didactical point of view. Only the momentum conservation law is employed to develop the model, which is eventually applied to quantitatively interpret the telemetry of the 2011 last launches of Shuttle Discovery and Shuttle Endeavour.

Keywords: general physics, education, Newtonian mechanics

1. Introduction

During the final minutes of the third extravehicular activity, a short demonstration experiment was conducted. A heavy object (a 1.32-kg aluminium geological hammer) and a light object (a 0.03-kg falcon feather) were released simultaneously from approximately the same height (about 1.6 m) and were allowed to fall to the surface. Within the accuracy of the simultaneous release, the objects were observed to undergo the same acceleration and strike the lunar surface simultaneously, which was a result predicted by well-established theory, but a result nonetheless reassuring considering both

* To the STS-51-L and STS-107 astronauts, in memoriam.
the number of viewers that witnessed the experiment and the fact that the homeward journey was based critically on the validity of the particular theory being tested.

We decided to start with the words of Joseph P. Allen, capsule communicator and scientist of the 1971 Apollo 15 mission [1]. These words are, in our opinion, well representative of the message we intend to convey to readers: even the most challenging, technologically sound, human ventures are eventually based on a few ‘simple’ physical laws. The fact that the ‘Moon conquest’ was made possible ultimately by Newton’s laws of dynamics and gravitation should represent, from a didactical point of view, a really amazing and intriguing argument for most undergraduate students of first-year physics courses. But what, in our opinion, could render such a topic much more interesting is to realise how elementary physical models can be devised in order for the ‘real’ behaviour of very complex systems to be adequately accounted for. This is what we are going to do in the present paper, which has been conceived as a possible didactical unit for a first-year general physics course mainly oriented to physics or engineering students.

In the first part of the lecture the impulse conservation law will be employed for an elementary, one-dimensional model of rocket propulsion to be devised. To this end the current literature is inspiring, being it rich of several introductory physics textbooks well exploring the role of mechanics’ first principles in enlightening rocket propulsion. Among them we quote the beautiful book by French [2], the fourth volume of the Feynman Lectures [3], the first volume of Sommerfeld’s Physics course [4]. In the second part of the lecture, the ‘toy’ model so developed will be used to analyse the first two minutes of flight of a celebrated space transportation system: the Space Shuttle. It should be noted that Stinner and Metz have also presented a detailed analysis of the launch of a real Space Shuttle mission, precisely the STS-119 mission, in the past [5]. Their analysis, aimed at giving a detailed model of the whole 8 min dynamics of the Space Shuttle flight (i.e., from liftoff to orbit’s insertion) to high school teachers, was based on authentic values of the kinematic and dynamic parameters, through a mass point analysis.

The aim of the present paper is rather different. We wish to focus our attention on the momentum conservation law, which represents the first principle behind a rocket’s, and thus Shuttle’s propulsion. To this end, only the first two minutes into the flight, during which the three main engines (SSMEs henceforth) and the two solid rocket boosters (SRBs henceforth) are fully active, will be analysed. Our model is built up on simpler assumptions, with respect to what was done in [5]. In particular, the Shuttle’s trajectory will be assumed to be rectilinear and vertical, and the drag pressure exerted by the atmosphere will be neglected. In this way, a remarkably simple analytical model of the temporal behaviour of the Shuttle’s speed can be devised. Such an elementary model will reveal to be able in giving a good agreement with experimental data that have been extracted from two official NASA videos related to the last launches of Shuttle Discovery (STS-133 mission) [6] and Endeavour (STS-134 mission) [7]. Students could then be persuaded that devising elementary models must be considered not only a didactical divertimento but also a mandatory preliminary step for the real behaviour of complex systems to be grasped.

2. An analytically solvable rocket model

The present section briefly resumes how the impulse conservation law leads to rocket’s equation. Consider an isolated rocket (later the presence of gravity will be included). We
denote by $m$ its total mass, including the fuel mass, and by $u$ the speed of the gases produced by burning the fuel, measured with respect to the rocket itself. Figure 1 shows a sketch of the acceleration mechanism of the rocket via fuel burning. To grasp such mechanism it is sufficient to apply the impulse conservation law within an infinitesimal temporal interval, say $[t, t + dt]$. Since the rocket is supposed to be isolated, i.e., no external forces are acting on it, on denoting by $-dm$ the infinitesimal mass of fuel ejected during the infinitesimal time $dt$ and by $dv$ the corresponding (infinitesimal) increment of rocket’s speed $v$, from figure 1 the impulse conservation law gives at once
\[ mv = (m + dm)(v + dv) - dm(v - u), \]  
from which, after rearranging and simplifying, we obtain
\[ m dv = -u dm, \]  
where the higher-order infinitesimal quantity $dm dv$ has been neglected. Equation (2) is the celebrated rocket equation, first conceived in 1903 by the Russian scientist Konstantin E Tsiolkovsky. In order to apply equation (2) it is sufficient to divide its left and right side of equation (1) by $dt$,
\[ m \frac{dv}{dt} = -u \frac{dm}{dt}, \]  
where it must kept in mind that the rocket’s total mass $m$ is a decreasing function of $t$. From equation (3) it clearly appears that the product $-um$, with dot denoting derivation with respect to time, can be interpreted as the rocket’s thrust, say $F_{th}$, by writing
\[ F_{th} = u (-m). \]  
In astronautics, technical data sheets of rocket engines often provide the value of the thrust together with, in place of speed $u$, a parameter called specific impulse. The specific impulse is customarily defined as the period (expressed in seconds) for which a 1 pound (0.45 kg) mass of propellant (total of fuel and oxidiser) will produce a thrust of 1 pound (0.45 kg) of force.\footnote{In the technical language the thrust is often expressed in lbs or in kg, so that the student must pay attention to its conversion in Newton.}

Within our notation, the specific impulse, say $I_{sp}$, is related to thrust and mass decay by
\[ I_{sp} = \frac{F_{th}}{g (-m)}, \]  
where $g$ denotes gravity’s acceleration. In particular, equations (4) and (5) give
\[ u = I_{sp} g. \]
Although the specific impulse is a characteristic of the propellant system, its exact value is to some extent function of the operating conditions and design of the rocket engine. For this reason, different numbers are often quoted for a given propellant or combination of propellants. To give a practical example, the specific impulse of one of the three SSMEs of the Space Shuttle Orbiter is, in vacuum, about 450 s, corresponding to a speed $u$ of the order of 4.4 km s$^{-1}$. The same engine, at a sea level, is characterised by a specific impulse of about 370 s ($u \sim 3.6$ km s$^{-1}$). Similarly, the specific impulse for a single SRB is about 270 s ($u \sim 2.6$ km s$^{-1}$) in vacuum and about 240 s ($u \sim 2.4$ km s$^{-1}$) at sea level. In the following, we shall use specific impulses at sea level for both SSMEs and SRBs.

For our simplest rocket model the mass rate $\dot{m}$ and speed $u$ are supposed to be constant. This, in turn, implies thrust’s values to be constant too. Under such assumptions, equation (3) can be integrated in an elementary way to provide a rocket speed logarithmic law [4].

$$v(t) = -u \log \left(1 - \frac{t}{\tau}\right).$$

(7)

Here the temporal constant $\tau = m_0/\mu$ is defined as the ratio between the initial value of the total mass $m_0$ and the mass decrease rate $\mu = -\dot{m}$. For a rocket leaving the Earth’s surface along a radial trajectory, the gravitational force $mg$ has to be added to the rocket thrust, as sketched in figure 2. In this way equation (7) becomes [4]

$$v(t) = -u \log \left(1 - \frac{t}{\tau}\right) - gt.$$  

(8)

In particular, immediately after the liftoff, i.e., for $t \ll \tau$, the logarithmic function can be approximated by $\log(1 - t/\tau) \simeq -t/\tau$, so that equation (8) gives at once

$$v(t) \simeq \left(\frac{u}{\tau} - g\right)t = \left(\frac{F_{th}}{m_0} - g\right)t,$$

(9)

corresponding to a uniformly accelerated motion.

2 https://rocket.com/rs-25-engine
3. Quantitative analysis of STS-133 and STS-134 launches

The aim of the present section is to employ the analytical results above found in order to quantitatively describe the temporal behaviour of the Shuttle speed during the first two minutes of flight. We shall analyse the launch phases of two of the last missions of the Space Shuttle, namely the STS-133 *Discovery* (24 February 2011) and the STS-134 *Endeavour* (16 May 2011). Beautiful videos provided by NASA and currently available on internet contain the Space Shuttle telemetry of the whole launches (about 8.5 min length) of both STS-133 [6] and STS-134 [7] missions. From such videos the data concerning the *Discovery* and the *Endeavour* speeds have been retrieved as functions of the official time of flight. To this end, a sequence of frames is, for each NASA video file, built up by using a batch extraction feature of the free software FFmpeg [8]. In doing so, the temporal position of the initial frame has to be chosen to match as best as possible the liftoff time $t = 0$. The subsequent frames are then extracted every second and stored into separate .jpg files. Each image is then analysed and the corresponding speed value, after conversion from mph into metric units ($\text{ms}^{-1}$), is eventually transcribed. The experimental data so obtained are plotted in figure 3, where the open and the black circles represent *Endeavour*’s and *Discovery*’s speed, respectively. From figure 3, the high degree of reproducibility of the speed temporal behaviour should also be appreciated.

In order to employ the elementary model developed in the previous section, we shall assume (i) a rectilinear vertical Shuttle trajectory and (ii) a constant mass decrease rate. Of course, some information about the STS parameters are needed. At liftoff the total mass of the entire system is estimated as $m_0 \approx 2.05 \times 10^6$ kg. During the first two minutes into the flight, the Shuttle’s thrust comes from the two SRBs and the three SSMEs. From NASA official information a thrust value of about 13 MN for each SRB and about 2.2 MN for each SSME will be chosen. Accordingly, the full thrust of the STS at liftoff can be reasonably estimated as $F_{\text{th}} \approx 32.6$ MN. Then, by using the

---

3. https://en.wikipedia.org/wiki/STS-133, https://en.wikipedia.org/wiki/STS-134.
4. See for instance: http://science.ksc.nasa.gov/shuttle/technology/sts-newsref/srb.html.
5. See for example https://nasa.gov/pdf/491387main_STS-133%20Press%20Kit.pdf and https://nasa.gov/pdf/538352main_sts134_presskit_508.pdf.
above estimate of the initial mass, equation (9) gives an initial (i.e., at \( t = 0 \)) acceleration of \( 0.6g \). In figure 4 the corresponding uniformly accelerated motion given by equation (9) has been superimposed on the NASA data through a dashed line, which accounts for the real data only near liftoff.

The model in equation (8) will now be tested to analyse the first thirty seconds into the flight. In doing so, some preliminary considerations are needed. As recalled in section 2, the Shuttle’s propulsion is provided by two different engines, SRBs and SSMEs. Each of them is characterised by its own thrust parameters. This implies that the effective values of the mass decay rate \( \mu \) and of the gases relative speed \( u \) to be inserted into the elementary model developed in section 2 have to be suitably estimated, from the knowledge of all thrust parameters, in order for the real Shuttle behaviour to be correctly reproduced. In particular, on setting \( F_{\text{SRB}} \approx 26 \text{ MN}, F_{\text{SSME}} \approx 6.6 \text{ MN}, m_0 \approx 2.05 \times 10^6 \text{ kg}, \) and on using values of the specific impulses for SRBs and SSMEs of about \( 240 \text{ s} \) and \( 370 \text{ s} \), respectively, it is easy to prove that

\[
\mu = \frac{F_{\text{SRB}}}{g I_{\text{SRB}}} + \frac{F_{\text{SSME}}}{g I_{\text{SSME}}} \approx 12 \, 800 \text{ kg} \text{s}^{-1},
\]

for the effective mass decay rate and

\[
u = g \frac{F_{\text{SRB}}}{I_{\text{SRB}}} + \frac{F_{\text{SSME}}}{I_{\text{SSME}}} \approx 2.5 \text{ km} \text{s}^{-1},
\]

for the effective relative speed of gases.

Figure 5 shows the temporal behaviour of the speed predicted by our elementary model (8) (solid curve). According to it, the Space Shuttle can thus be thought of as a ‘simple’ rocket having an initial mass of about two thousand tons, firing back about thirteen tons of fuel per second at a relative speed of about two and half kilometres per second. In particular, it should also be observed that, in order to keep the theoretical model as simple as possible, the air drag has been neglected. From a purely pedagogical point of view, our task has been achieved: the behaviour of the actual speed of a Space Shuttle can be grasped in terms of the sole momentum conservation laws.

---

6 Students could be at first confused by watching NASA videos, where a liftoff acceleration (symbol \( G \)) of 1.6g is reported. However, students should be advised that \( G \) denotes the acceleration felt by astronauts inside the non-inertial reference frame, i.e., the accelerating Shuttle itself.
However, as can be clearly seen from figure 5, after thirty seconds into the flight the Shuttle dynamics becomes considerably more complicated, as clearly detailed in [5]. The Shuttle’s trajectory begins to bend with respect to the vertical direction (eventually the Shuttle must enter into a circular orbit). More importantly, its speed approaches the sound barrier (in the NASA video [6] the characteristic bang can be heard at about 38 s into the flight). During this phase the combination of the air density and the Shuttle speed results in a drag force (for unit surface) whose maximum value is reached at about 1 min into the flight [5, 9]. To prevent damages to the orbiter, the thrust is reduced for a while to about 70% of the full maximum values, as can be seen for instance by looking at table II of [5]. From the videos in [6, 7] it turns out that such a reduction approximately begins after about 32 s and ends after 38 s into the flight. The thrust is kept at the lower level for further about 10 s (48 s into the flight), then it increases and 6 s later it reaches again the higher level (54 s into the flight). Although this state of fact is, without doubt, quite complex, we are going to show how the experimental behaviour of the Shuttle speed could be mathematically described by using a very rough model for the thrust modulation, which is schematically depicted in figure 6.

Figure 5. The same as in figure 4, but using the elementary two-parameter model in equation (8) (solid curve).

Figure 6. Temporal behaviour of the full thrust to model the reduction of the dynamic pressure on the Shuttle. $T_1 = 35$ s, $T_2 = 51$ s.
In particular, the values of the transition times $T_1$ and $T_2$ have been chosen to be 35 s and 51 s, respectively, when the thrust of the Shuttle will pass instantaneously from the maximum value, 32.6 MN, to the 70% of it (about 22.8 MN) and vice versa. Such elementary thrust modulation allows a very simple analytical estimate of the Shuttle speed to be easily found. To this end, the whole temporal interval $[0, 120 \text{ s}]$ is divided into three subintervals. Within each of them a temporal behaviour of the rocket speed similar to that given into equation (8) is then implemented. In particular, when passing from an interval to the other, the gases effective relative speed $u$ does not change, as the ratio $F_{SRB}/F_{SSME}$ is supposed to be reasonably constant. As far as the ratio $\mu/m_0$ is concerned, we have that the total mass decay rate $\mu$ will display a temporal behaviour similar to that shown in figure 6. The values of the ‘initial’ mass within each of the three subintervals have to be properly adjusted in order to assure continuity to the function $v(t)$. It is not difficult to show (this of course could be offered as a nontrivial homework to students) that the corresponding speed temporal behaviour is described by the following function:

$$v(t) \simeq -gt - u \times \begin{cases} 
\log \left(1 - \frac{t}{\tau_1}\right), & 0 \leq t \leq T_1, \\
\log \left(1 - \frac{T_1}{\tau_1}\right) \left(1 - \frac{t - T_1}{\tau_2}\right), & T_1 \leq t \leq T_2, \\
\log \left(1 - \frac{T_1}{\tau_1}\right) \left(1 - \frac{T_2 - T_1}{\tau_2}\right) \left(1 - \frac{t - T_2}{\tau_3}\right), & t \geq T_2,
\end{cases}$$

(12)

where $\tau_1 = m_0/\mu$ and

$$\begin{align*}
\tau_2 &= \frac{\tau_1 - T_1}{\eta}, \\
\tau_3 &= (\tau_1 - T_1) \left(1 - \frac{T_2 - T_1}{\tau_2}\right),
\end{align*}$$

(13)

with $\eta \sim 70\%$ being the thrust reduction factor.

The result is shown in figure 7. Although the model of equations (12) and (13) seems more than able to interpret the speed behaviour within the first minute of flight, a bad
agreement with the experimental data is still displayed during the subsequent minute. The obvious reason is the fact that the actual SRB thrust temporal behaviour is far from being that (unrealistic) displayed in figure 6. Nevertheless, it is worth showing to students how the real speed behaviour during the second minute can be fitted by slightly reducing the total thrust level after the ‘throttle up’ command. This should be considered as a further evidence in favour of our elementary model. The final result is shown in figure 8, where the disagreement with the very last part of the temporal behaviour is due to the SRB cutoff before their detaching and staging.

4. Conclusions

The first two minutes of a Space Shuttle’s flight have been analysed, as far as the temporal behaviour of its speed is concerned, through a didactical model based ultimately only on the momentum conservation law. The theoretical predictions achievable by such a pedagogical description have been compared to the actual telemetry of the last launches of Space Shuttles Discovery and Endeavour. All experimental data have been collected from public sites that are currently available on the internet. In this way, students can be encouraged to reproduce the experiment as well as to test and/or improve the proposed model.

We believe that physics and engineering undergraduates would benefit from the present analysis to understand that, despite the current availability of high-tech simulation tools, analytical elementary models must always have priority for the real behaviour of complex systems to be grasped with minimal amounts of math.

Acknowledgments

We wish to thank the anonymous reviewers for their constructive criticism and suggestions.

7 An example of a real SRB thrust level behaviour is shown for instance at the following url: https://en.wikipedia.org/wiki/Space_Shuttle_Solid_Rocket_Booster.
References

[1] Apollo 15—Preliminary Science Report—NASA SP-289, Ch. 2 at the following internet address: http://hq.nasa.gov/alsj/a15/as15psr.pdf
[2] French A P 1971 Newtonian Mechanics (London: Clowes and Sons)
[3] Feynman R P, Leighton R B and Sands M 1963 The Feynman Lectures on Physics vol 4 (Reading, MA: Addison-Wesley) section 13-4
[4] Sommerfeld A 1970 Lectures on Theoretical Physics: Part I (New York: Academic)
[5] Stinner A and Metz D 2010 The flight of the Space Shuttle Discovery (STS 119) Phys. Teach. 48 162
[6] https://youtube.com/watch?v=Hy1Nc4YpAU1
[7] http://youtube.com/watch?v=ShRa2RG2KDI
[8] https://ffmpeg.org/
[9] Backman P 2012 Maximum aerodynamic force on an ascending space vehicle Phys. Teach. 50 167