Abstract
Classification of time series is a topical issue in machine learning. While accuracy stands for the most important evaluation criterion, some applications require decisions to be made as early as possible. Optimization should then target a compromise between earliness, i.e., a capacity of providing a decision early in the sequence, and accuracy. In this work, we propose a generic, end-to-end trainable framework for early classification of time series. This framework embeds a learnable decision mechanism that can be plugged into a wide range of already existing models. We present results obtained with deep neural networks on a diverse set of time series classification problems. Our approach compares well to state-of-the-art competitors while being easily adaptable by any existing neural network topology that evaluates a hidden state at each time step.

1. Introduction
Classification of time series is a common problem in machine learning. Learning methods, such as deep networks, are usually trained to solely reach the highest possible accuracy when predicting on a test set. However, there are some contexts where the accuracy is not the single goal. For instance, a doctor might want to make an accurate diagnosis early enough to treat the patient with the most appropriate medicine (Ghalwash et al., 2012). Similarly, identifying droughts as early as possible by space-born imagery analysis allows local authorities to distribute resources efficiently and prevent famine and ultimately loss of life. This discipline in machine learning is known as early classification (Xing et al., 2012).

Early classification methods generally rely on i) learning a classifier per timestamp using a classification-only objective and ii) designing an early stopping criterion that utilizes predictions from these classifiers.

Conversely to existing works, we propose an end-to-end learning framework that is optimized jointly on accuracy and earliness by estimating a stopping probability \( \delta_t \) based on a dual loss function. To ensure that the stopping probability is parametrized according to our loss function, we design an attention-like mechanism \( P(t) = \delta_t B_t \). This is used to focus the loss-penalty to specific times in the sequence and is subtracted from a monotonically decreasing attention budget \( B_0 \). A qualitative example in Fig. 1 illustrates these components. Note that the stopping probability \( \delta_t \) increases after processing a classification characteris-

Figure 1. Qualitative sample of the learnable decision mechanism produced by a 1d convolutional model. The model evaluates a hidden feature vector \( h_t \) at each time step \( t \) based on which classification confidences \( \hat{y}_t \) and a stopping probability \( \delta_t \) are derived. To parameterize this stopping probability at training time, an attention-like score \( P(t) \) is evaluated based on a given budget \( B_t \), as shown in the bottom two figures.
tic feature in the time series. This indicates that enough classification-relevant information has been observed to make a decision at this time.

Our stopping probability is broadly applicable, since any differentiable model that estimates a hidden state $h_t$ at time instance $t$ can be modified with this mechanism. We demonstrate this quantitatively and qualitatively on recurrent and convolutional model implementations.

2. Related Work

In the following, we will address early classification of time series. Time series are ordered sequences of observations of the form:

$$x = (x_0, x_1, \ldots, x_T).$$

We denote $x_{\rightarrow t}$ the partial observation of $x$ up to time $t$. Early classification consists in predicting as early as possible the class of an incoming time series $x$. In the literature, this translates as deciding, for any timestamp $t$, whether $x_{\rightarrow t}$ contains sufficient information to make a decision or if more data should be collected. Hence, the early classification setting is related to the missing data problem. Yet, it has its own peculiarities, due to the structure of the missing future data and the question whether more data should be waited for.

In a seminal paper, Xing et al. (2012) suggest to delay decision-making for a nearest-neighbor classifier until neighborhood of the time series to be classified gets stable over time. The method is named Early Classification on Time Series (ECTS). This idea of delaying the decision until the classifier is reliable enough has been investigated in several other works. (Hatami & Chira, 2013) base their approach on an ensemble of classifiers for which a certain level of consensus has to be reached for the decision to be considered reliable. RELCLASS (Parrish et al., 2013) waits until it is highly probable that the decision made based on $x_{\rightarrow t}$ is the same as the decision that would be made based on the entire time series $x$. Another example of this family of methods is EDSC (Xing et al., 2011) in which local features called shapelets (Ye & Keogh, 2009) are extracted based on their predictive power. A later feature selection scheme is then used to select those features that can predict both early and accurately.

One strong limitation of these methods is that they do not explicitly model the trade-off between earliness and accuracy.

To overcome this limitation, Dachraoui et al. (2015) proposed a framework for early classification which included the cost of delaying the decision in the loss function. Clustering and cross-validation schemes are used to predict future classification costs. From that information, a decision is made only if it is unlikely that the cost will decrease in the future. Tavenard & Malinowski (2016) elaborated on this method by formulating the decision to delay or not classification as a binary classification problem itself and removing the need for data clustering in the process. In Mori et al. (2017), the authors also consider to explicitly optimize trade-off between earliness and accuracy. This is done in a two-step process. First, classifiers are learned based on a classification-only criterion. Then, based on these classifiers, a heuristic parametric stopping rule is optimized using a genetic algorithm to minimize the target trade-off between earliness and accuracy.

In this work we will primarily compare accuracy and earliness by our method with Mori et al. (2017). In order to easily compare performances on an equal footing, we will use in this paper a similar cost formulation:

$$C = \alpha I_{\hat{y} \neq y} + (1 - \alpha) \frac{t}{T},$$

where $t$ is the decision time and $\hat{y}$ is the classification decision itself. This loss is the $\alpha$-weighted sum of a classification component and an earliness term $\frac{t}{T}$ as linear ratio relative to the sequence length $T$. As they designed a non-differentiable stopping rule, they trained their method using a genetic algorithm. With this framework, they opted for a straightforward indicator loss $I_{\hat{y} \neq y}$ that counts the number of mis-classifications. The trade-off parameter $\alpha$ is meant to be tuned based on the adopter’s willingness to trade accuracy for an earlier classification.

We emphasize that these state-of-the-art methods require a separate classifier for each timestamp $t$. In contrast, our stopping mechanism can be implemented on a single model that classifies time series data of arbitrary lengths. Finally, in these methods, classifiers and stopping criteria are optimized sequentially, while we propose an approach in which a pre-trained model can be fine-tuned end-to-end using standard gradient back-propagation.

3. Methodology

In this section, we first describe our proposed differentiable decision-making mechanism that, after training, produces a stopping decision probability $\delta_t$. The key feature of our approach is how we encourage the network to parameterize the weights $\theta_\delta$ of this stopping probability. We start with designing a loss function that simultaneously optimizes for accuracy and earliness similar to Mori et al. (2017). Based on $\delta_t$, we introduce a mechanism $P(t)$ that focuses the loss penalty on selected observations.

3.1. Mechanism

Let us assume we have a time series classification model that is able to compute, at each time step $t$, a hidden feature representation $h_t$ from which it derives class probability
To utilize this information, we add a second linear output layer \( \delta_t = \sigma(\theta_\delta h_t) \) with a sigmoidal non-linear activation function \( \sigma(\cdot) \). We interpret the output of this layer as the probability of stopping at time \( t \). At inference, we sample a hard stopping decision based on this probability. A schematic illustration of the mechanism is shown in Fig. 2.

Given \( \delta_t \), one can derive the probability of making a decision at time \( t \) as

\[
P(t) = \delta_t \cdot \prod_{\tau=0}^{t-1} 1 - \delta_{\tau_t}. \tag{3}
\]

In order to ensure that these \( P(t) \) probabilities sum to one, we set \( \delta_T \) to 1 regardless of \( h_T \).

These decision probabilities \( P(t) \) will be used to weight losses computed at all timestamps during training, such that decisions that are made at highly probable decision timestamps will induce larger gradients, as graphically illustrated in Fig. 2.

Since \( P(t) \) is computed based on \( \delta_t \) (as detailed later), gradients can now be back-propagated to \( \theta_\delta \) and the stopping probability \( \delta_t \) can be parametrized end-to-end.

With this mechanism, we see a certain analogy to attention scores (Bahdanau et al., 2014) and, thus, propose a second interpretation. If we interpret \( P(t) \) as attention-like scores, one can see the product of all previous inverse probabilities as a budget \( B_{t-1} \) that is initialized with one. At each time, \( P(t) = \delta_t \cdot B_{t-1} \) is calculated as a fraction of the remaining budget. At the last timestamp \( T \), we consume all the remaining budget \( B_{T-1} \) whatever the observations.

Fig. 1 presents an example of the evolution of these quantities over time.

### 3.2. Loss function

Once equipped with these quantities, we can now turn our focus to the actual definition of a loss function that approximates the cost function defined in Eq. (2).

Let us first define the loss attached to a possible decision made at time \( t \) for a trade-off parameter \( \alpha \) value and a time series \( x_{\rightarrow t} \) from class \( y \) as

\[
\mathcal{L}_t(x_{\rightarrow t}, y; \alpha) = \alpha \mathcal{L}_c(x_{\rightarrow t}, y) + (1 - \alpha) \mathcal{L}_c(t). \tag{4}
\]

The commonly used choice for \( \mathcal{L}_c \) would be the logistic or cross entropy loss \( \mathcal{L}_c(x_{\rightarrow t}, y) = -y \log(\hat{y}) \). This loss is fully compatible with our proposed method and exponentially penalizes wrong classifications, as illustrated in Fig. 3. However, we would like to be able to compare our model with the best state-of-the-art early classification model of Mori et al. (2017) using the identical trade-off parameters \( \alpha \).

When approximating the cost function defined in Eq. (2), we will use

\[
\mathcal{L}_c(t) = \frac{t}{T} \tag{5}
\]

and

\[
\mathcal{L}_c(x_{\rightarrow t}, y) = 1 - \hat{y}^+, \tag{6}
\]

where \( \hat{y}^+ \) is the probability output by the model for class \( y \) when fed with input \( x_{\rightarrow t} \). As shown in Fig. 3, our \( \mathcal{L}_c \) formulation is closer to the \( 0 - 1 \) loss on average, hence allowing to better model the target trade-off between earliness and accuracy for a given \( \alpha \) value.
Note however that Fig. 3 suggests our loss function has lower gradients for bad solutions (low $\hat{y}^+$), which means it will be weaker at preventing from bad classification decisions. To find a compromise between optimization efficiency and comparability to Mori et al. (2017), we decided to adopt a two-phase training procedure, as described in Section 4.1. In our experiments, we first pre-train the model purely for accuracy with a logistic regression loss and then subsequently fine-tune it end-to-end for accuracy and earliness using this loss formulation.

Then, we can compute a per-time-series overall loss as the expectation of $\mathcal{L}_t$ over decision timings

$$
\mathcal{L}(x, y, \alpha) = \mathbb{E}_t [\mathcal{L}_t(x \rightarrow t, y; \alpha)]
$$

$$
= \sum_{t=0}^{T} P(t) \cdot [\alpha \mathcal{L}_c(x \rightarrow t, y) + (1 - \alpha) \mathcal{L}_e(t)]
$$

Finally, at classification time, for each timestamp $t$, the decision about performing classification or waiting for more samples is drawn from a Bernoulli distribution of parameter $\delta_t$.

3.3. Sequence classification models

The previous section described the loss function that we use to parameterize the probabilistic stopping decision $\delta_t$. In this part we provide details on the classification models used throughout this paper.

We want to emphasize that the stopping output $\delta_t$ can be implemented to any differentiable model that evaluates a high-level information of the input data. However, these networks are often difficult to optimize on smaller datasets. For this work, we employ a straightforward classification strategy using stacked LSTM layers, denoted by a bracketed, raised index. Each recurrent layer $l$ produces a hidden state $h^l_t \leftarrow \text{LSTM}(h^{l-1}_t, h^{l-1}_{t-1})$ based on the current output of the previous layer $h^{l-1}_t$ and time $h^{l-1}_{t-1}$. The input observation is used for the first layer while the last hidden state is employed to produce classification probabilities. We implement our approach by considering a second linear output from the last layer’s hidden state and fine-tune the model end-to-end, as described in the previous section. We will present qualitative results using this network topology in Section 4.3.

3.3.2. 1D Convolutional shapelet Model

For a quantitative evaluation, we chose a single-layer convolutional model that is similar in spirit to shapelet models (Ye & Keogh, 2009; Hills et al., 2014; Grabocka et al., 2014), as illustrated in Fig. 4. An input time series $x$ is shown at the top that is known up to time $t$ indicated by the vertical line. Mono-dimensional convolution blocks $f^l = x \otimes k$ produce feature maps of varying temporal context using increasingly
large kernels \( k \in \mathbb{R}^{w \times d} \).

The number \( l \) of 1d convolution blocks, as well as the number \( d^l \) of kernels per kernel length \( w^l \) are model-specific hyper-parameters. We pad the input time sequence with zeros for the respective kernel size \( w^l \) to ensure that the observation time \( t \) aligns with the right end of the convolution kernel. This padding guarantees that in our model no input data after \( t \) is convolved. To gather features for a classification decision up to a specific time \( t \), sub-sequence max-pooling is used \( \max(f_{l,t}) \).

The resulting max-pooled features of all convolutions are then concatenated to one combined feature vector \( h_t \). Additionally, we employ dropout and batch normalization to this feature vector to improve convergence and counteract overfitting. Note that sub-sequence level max-pooling allows the model to produce a hidden representation \( h_t \) whatever the length of the input time series. Based on \( h_t \), probabilities for each class are evaluated by a fully connected layer with softmax activation. Similarly, a second fully connected layer (with sigmoid activation) is employed for the probabilistic stopping rule \( \delta_t \).

4. Results

Now that we have defined our model, we assess its validity through extensive experiments on a diverse set of 46 publicly available time series classification datasets. These 46 datasets are used in (Mori et al., 2017) who, fortunately, also report classification accuracy and earliness of compared approaches. They are made available on the UCR repository (Dau et al., 2018; Chen et al., 2015) and cover a wide range of data modalities, ranging from motion capture through audio to Electrocardiography (ECG) sensor data. The competitors considered here are the best method from (Mori et al., 2017) (denoted SR2-CF2), the pioneering work of (Xing et al., 2012) (ECTS), as well as EDSC (Xing et al., 2011), and RELCLASS (Parrish et al., 2013) methods.

4.1. Training Procedure

As presented in Section 3, our approach is designed to be easily plugged on a pre-trained shapelet-like model. To simulate this in our experiments, we train our network in two distinct phases. The classification phase is the pre-training based on a standard cross-entropy loss function.

After this pre-training step, we branch our earliness module parametrized by \( \theta_e \) and fine-tune all model parameters using the loss function defined in Section 3.2. These two learning phases differ in three main points. First, a term that penalizes for late classifications is introduced in the second phase. Second, the way we train our model using time series of varying lengths during phase 1 is equivalent to considering a uniform law for \( P(t) \), while, during phase 2, this is adapted to lower the delay penalization term in the loss. Third, in phase 2, the classification-based part of the loss is no longer a cross-entropy loss but rather a linear one so as to better match the trade-off between earliness and accuracy as defined by the target cost function.

4.2. Model selection

As stated above, the 46 datasets that are used in these experiments are diverse in nature, which makes the selection of hyper-parameters a critical task. We select the network configuration through three-fold cross validation after 30 epochs of training on pure accuracy. The parameter space was evaluated using grid search. The tuned hyper-parameters for the convolutional model include the number of convolution blocks \( l \in \{4,6,8\} \), the number of kernels per convolution \( d^l \in \{50,75,100\} \), the increase rate of kernel width per layer \( \delta_w \in \{30,50,70\} \). As for the hyper-parameters for the recurrent model, we varied the number of recurrent layers \( n \in \{2,3,4,5,6\} \) and the number of hidden features per layer \( r \in \{64,128,256,512\} \). For both models, we searched with two learning rates \( \eta \in \{0.1,0.01\} \) for our Adam optimizer (Kingma & Ba, 2014).

To counteract over-fitting we choose a dropout rate of 50.

We decided to perform the model-specific hyper-parameter tuning solely based on the pure classification phase, to separate model-specific hyper-parameters related to classification performance from parameters related to earliness. Note that this separate tuning strategy conveniently follows the spirit of fine-tuning an existing model with our proposed stopping rule.

4.3. Qualitative Results

In this section we present qualitative results obtained from the recurrent model described in Section 3.3.1.

First, we want to get an impression on the training process of a model that was previously trained on pure accuracy and is now additionally optimized for earliness. To demonstrate our approach, we selectively chose the TWOPATTERNS synthetic dataset from the UCR datasets. We present quantitative results on all UCR datasets in the next section. Here, we show the accuracy and loss functions on the test dataset during the training process in Fig. 5. Following Eq. (4), the overall loss, shown in Fig. 5(a) is a \( \alpha = 0.8 \) weighted sum of classification loss in Fig. 5(b) and earliness loss Fig. 5(c). By observing the \( P(t) \) scores of a single input sample at selected epochs in Fig. 6 we gain a complete picture of the visible dynamics in the training process. As stated before, we initialized the parametrization of the stopping probability \( \theta_e \) such that late classifications are favored in the first training steps. This can be observed in Fig. 6(b) where \( P(t) \), and thus \( \delta_t \) focus solely on the last observation. This is confirmed by the earliness loss during training in Fig. 5(c) that...
indicates a large loss in the initial epochs. After few training epochs on earliness and accuracy, one can observe how the earliness loss falls to a lower, stable level. When cross-referencing the example in Fig. 6(c) at epoch 55, one can see that the $P(t)$ has reached a stable position at time $t = 85$. This time corresponds to a classification-characteristic feature in the input time series in Fig. 6(a). The classification loss in Fig. 5(b) and accuracy Fig. 5(d) during the entire training process show that the classification accuracy remained on a stable level. Overall, this qualitative example has demonstrated how a recurrent neural network can be retroactively optimized for performing earlier classifications without loss in accuracy.

4.4. Quantitative state-of-the-art comparison

In this section we quantitatively compare our early classification method with the other early classification approaches presented in Section 2. As stated above, Mori et al. (2017) provide accuracy and earliness results for their and comparative methods in their supplementary material. We can hence compare our results on a diverse collection of datasets with a large number of early classification approaches. However, we would like to emphasize that a comparison of these early classification methods requires care in terms of evaluation criteria and choice of earliness parameters. The trade-off between earliness and accuracy is parametrized differently in each approach. We followed the parameters evaluated by Mori et al. (2017), as shown in Table 1. In contrast to the other methods, we carefully designed our loss function and trade-off parameter $\alpha$ to be numerically comparable with Mori et al. (2017). To enable this, we avoided the commonly used cross entropy loss and employed a more comparable linear loss, as shown in Eq. (6) and illustrated in Fig. 3.

Also, we compare methods based on two criteria, earliness
We further analyze the differences between our approach, we decided to use win when comparing to SR2-CF2 is a fair and natural way to (2017) and Parrish et al. (2013), respectively. Each dot the vertical axis is based on scores reported by Mori et al. (2017) achieve a better score on many datasets with their non-ated trade-off factors. However, it seems that (Mori et al., 2017) method in Fig. 8. Here we show accuracy, and accuracy, that should be jointly evaluated. Considering both objectives separately is not sufficient for a fair evaluation, since the overall goal is to minimize the mixed cost function defined in Eq. (2). Evaluating on this cost function when comparing to SR2-CF2 is a fair and natural way to compare performance since this cost is the explicit quantity that both our method and its baseline aim at minimizing. Hence we calculate an \( \alpha \)-weighted score similar analogous to the loss function in Section 3.2. This score is closest to the objective defined to the respective approaches and should be the primary evaluation focus.

Since we evaluate this score on a large number of datasets, we decided to use winloose plots for a first quantitative evaluation. Datasets where our method achieved a higher scores than the competitor are counted to the win category and vice-versa. We illustrate the results in Table 2.

| \( \alpha \) | SR2-CF2 | RELCLASS | EDSC | ECTS |
|---------|--------|----------|------|------|
| 0.6     | 7 / 38 | 31 / 14  | 34 / 8 | 40 / 5 |
| 0.7     | 3 / 42 | 28 / 17  | 28 / 14 | 35 / 10 |
| 0.8     | 5 / 40 | 23 / 22  | 30 / 12 | 34 / 11 |
| 0.9     | 12 / 33 | 19 / 26  | 33 / 9  | 26 / 19 |

Our approach clearly outperforms the early classification models EDSC, RELCLASS, and ECTS on almost all evaluated trade-off factors. However, it seems that (Mori et al., 2017) achieve a better score on many datasets with their non-differentiable early stopping heuristic.

We further analyze the differences between our approach, SR2-CF2 of Mori et al. (2017) and the RELCLASS (Parrish et al., 2013) method in Fig. 8. Here we show accuracy, earliness and \( \alpha \) weighted scores as scatter plots. In this figure, the horizontal axis shows our performance, while the vertical axis is based on scores reported by Mori et al. (2017) and Parrish et al. (2013), respectively. Each dot in these plots corresponds to a single dataset for a given \( \alpha \) value. Colors indicate different target trade-offs (i.e. \( \alpha \) values). The diagonal marks equal performance between the approaches. It seems that our approach achieves similar accuracies compared to SR2-CF2, as shown in Fig. 5(d), but performs the classifications later. This results in an overall score that is slightly worse, as shown in Fig. 8(c).

Even though our evaluated scores are slightly worse than the numbers reported by Mori et al. (2017) we present a method that can be broadly employed, as we demonstrate on two separate classification models, as described in Section 3.3. In contrast to SR2-CF2, our method is scalable for large datasets, requires only a single model parametrization to process a variable length sequence, and is trained via standard back-propagation and gradient descent. In contrast, SR2-CF2 (Mori et al., 2017) trained separately a Gaussian process classifier for each possible sequence length and designed a binary, non-differentiable stopping decision that requires a parametrization using genetic algorithms.

Still, our approach clearly outperformed the baseline methods ECTS and EDSC and achieves better scores compared to RELCLASS in Table 2. Comparing to RELCLASS in particular in Fig. 8, one can observe the variability in terms of earliness of the classification between the approaches. Overall, our method is comparable and competes well with the state-of-the-art in early classification while being easier deployable and more applicable to the broader machine learning community by relying on a pure end-to-end optimization with standard gradient back-propagation.

### 4.5. Influence of the Earliness Accuracy trade-off factor

In our final experiment we qualitatively assess the effect of the accuracy-earliness trade-off parameter \( \alpha \) for our approach and the SR2-CF2 (Mori et al., 2017) method. As stated previously, we specifically designed our loss function, see Fig. 3, such that the effect of \( \alpha \) on earliness and accuracy is similar.

In Fig. 7, we show both approaches on the two datasets ECGFiveDays and Haptics. The SR2-CF2 method is illustrated in orange, while our approach is shown in blue. Accuracy and earliness for different values of \( \alpha \in \{0.6, 0.7, 0.8, 0.9\} \) are drawn on the axis. As intended, an increasing weight on the classification loss with \( \alpha \in \{0.8, 0.9\} \), improves the achieved accuracy, while simultaneously increasing the earliness, as can be observed consistently for both approaches. Comparing our approaches, it seems that our method tends towards late classifications at the end of the sequence when a large weight on accuracy \( \alpha = 0.9 \) is chosen. We suspect that the initialization of \( \delta_t \) for late classifications in the initial training epochs can introduce a bias towards later the overall classification times.
which could explain this behavior. Overall, even though we tailored our method towards comparability with (Mori et al., 2017), there seems to be a slightly different effect of $\alpha$ on overall accuracy and earliness. This demonstrates the difficulties involved in comparing different approaches on these two antagonistic objectives. Summarizing, these results show that the trade-off parametrization of earliness and accuracy is an important parameter that will have to be chosen based on the user’s preference.

5. Discussion

In this work, we propose a novel, end-to-end trainable mechanism that produces a probabilistic decision that can be used to prematurely stop evaluating a sequence of observations if enough classification-relevant characteristics have been observed. It is broadly applicable to any sequential model that evaluates a hidden representation at each time $t$. We demonstrated the applicability by implementing our mechanism on two different sequential classification models in Section 3.3. We were able to compare the accuracy and earliness of our method to other early classification approaches on a diverse set of 46 datasets that enclose various types of use-cases for time series classification. These results have shown that our method was better or comparable to methods in the state-of-the-art in early classification. Compared to the best early classification model of Mori et al. (2017) we achieved similar accuracies, but with later classifications, thus resulting in a worse overall score. Nevertheless, we argue that our approach is methodically simpler, more scalable to large datasets and easier to deploy in existing sequential classification methodologies. Further we do use a single model parametrization for a variable length sequence and train our stopping rule end-to-end. In contrast, Mori et al. (2017) used a different classification model parametrization at every time step, and were forced to employ genetic algorithms to train their models due to a non-differentiable cost function. Also, we would like to emphasize that we modified our training procedure and loss function to be able to compare to Mori et al. (2017) on an equal accuracy-earliness trade-off parameter $\alpha$. Potential adopters of our method can employ a more competitive loss function for classification, such as logistic regression, if they do not intend to strictly compare with the approach of Mori et al. (2017).
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