Splitting of quantum information in travelling wave fields using only linear optical elements

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Abstract

In this paper we present a feasible post-selection scheme to split quantum information in the realm of travelling waves with success probability of 50%. Taking advantage of this scheme we have also proposed the generation of a class of W states useful for perfect teleportation and superdense coding. The scheme employs only linear optical elements as beam splitters (BS) and phase shifters, plus two photon counters and a source of two spontaneous parametric down-conversion photons. It is shown that splitting of quantum information with high fidelity is possible, even when using inefficient detectors and photoabsorption BS.

(Some figures in this article are in colour only in the electronic version)

Entanglement is a commonplace of remarkable applications of quantum mechanics, such as quantum computation [1], superdense code [2], quantum teleportation [3], one-way quantum computation [4], quantum metrology [5], etc. A state describing N subsystems is entangled when it cannot be factorized into a product of N states, each one concerning a subsystem. In this respect, in spite of being spatially separated the N subsystems are no longer independent. As a consequence, a measurement upon one of them not only gives information about the other, but also provides possibilities of manipulating it [6].

There are various types of entangled states and classifying them is an arduous task, specially for multi-partite systems [7]. However, Bennett et al [8] shed light on this question through the use of local operations and classical communication (LOCC) to define classes of equivalence in the set of entangled states; they showed that two entangled states belong to the same class of equivalence if one of them can be obtained from the other certainly, by means of the LOCC. According to this criterion all bipartite pure-state entanglements are equivalent to that of the EPR type ($\langle 00 | + | 11 \rangle / \sqrt{2}$) [9]. Concerning tripartite states, [10] used stochastic LOCC to show that there are two genuine entanglements of tripartite systems: GHZ ($\langle 000 | + | 111 \rangle / \sqrt{2}$) and W ($\langle 100 | + | 010 | + | 001 \rangle / \sqrt{3}$) states, i.e. GHZ (W) state cannot be converted into a W (GHZ) state under the stochastic LOCC. With respect to the W states, although they cannot be used for perfect teleportation and superdense coding; [11] introduced the following class of W states, suitable for these tasks:

$$\frac{1}{\sqrt{2+2\xi}}(|001\rangle+\sqrt{\xi}e^{i\gamma}|010\rangle+\sqrt{\xi+1}e^{i\delta}|100\rangle), \quad (1)$$

where $\xi$ is a real number and $\gamma$ and $\delta$ are phases. A scheme to generate this class of W states in the cavity-QED context was proposed in [12]. Also, the W states were generalized to multi-qubit and multi-particle systems with a higher dimension [13].

Another interesting application of the quantum entanglement is to split quantum information, required to implement the quantum secret sharing [14], which consists in the splitting of secret quantum information between various parties, so that the original information can be recovered if and only if the partners cooperate with each other. The splitting of the quantum information has been discussed in various scenarios in EPR [15], GHZ [16], pseudo-GHZ [17], multi-qubit GHZ [18] and W [19] states. In particular, for the W states the procedure relies on the following steps: (i) the use of a three-qubit W state previously prepared and shared by Alice, Bob and Charlie; (ii) a fourth qubit prepared in an unknown state (whose information one wants to split) in Alice’s ownership; (iii) a Bell-state measurement done by Alice and informing her result to Bob and Charlie; (iv) the agreement between Bob and Charlie to send their particles to each other. After these steps, the split quantum state shared...
by Bob and Charlie can be reconstructed after an appropriate rotation.

Here we show how to split quantum information in the domain of running wave fields via a convenient class of W states $|W\rangle$ shared by Alice, Bob and Charlie. A very attractive scheme from the experimental point of view for the generation of this class of W states is also presented. Our scheme makes use of only linear optical elements, as beam splitters (BS) and phase shifters (PS), plus photodetectors (D) and a source of two spontaneous parametric down-conversion (SPDC) photons [20].

In our scheme, the quantum information to be split is physically represented by the entangled states of the two electromagnetic field modes ($a$ and $\tilde{a}$), given by

$$|\psi\rangle_{ab} = C_0 |0\rangle_a |1\rangle_b + C_1 |1\rangle_a |0\rangle_b,$$

where $C_0$ and $C_1$ are the coefficients that obey $|C_0|^2 + |C_1|^2 = 1$. This choice is similar to the superposition of vacuum and one-photon states of a specific mode and does not suffer from any control of relative phase between the vacuum and one-photon states [21]. This state is easily generated by a single photon impinging on a BS with transmissivity $T = C_0$ and reflectivity $R = C_1$. As detailed in [22], the action of BS on the modes $a$ and $\tilde{a}$ is represented by the unitary operator $U_{BS} = \exp[i\theta(a'\tilde{a} + a\tilde{a}')]$, which leads to the initial state $|C_0\rangle_{a} |\tilde{a}\rangle_{b}$ to that given in equation (2), where $T = \cos(\theta)$ and $R = i \sin(\theta)$, and $a'$ and $\tilde{a}'$ are the creation and annihilation operators for the mode $a$ ($\tilde{a}$).

Figure 1 shows the experimental setup. A nonlinear crystal slab is pumped by a single mode UV laser and emits two SPDC photons, one of them (mode $a$) impinges on the BS$_1$ and produces in the output port the entangled state $|\psi\rangle_{ab}$; the other (mode $b$) on the BS$_2$ is used to create the W state as described below. These two photons are experimentally selected to have equal properties, such as pulse bandwidth, polarization, carrier frequency, and arrival time (time-jitter), which makes the two photons indistinguishable. The ‘ancillary’ mode $\tilde{a}$ provides a clock synchronization for the scheme.

**W state generation.** To engineer the desired W state we employ two 50/50 BS, as shown in the dashed region of figure 1. The initial state is given by $|\psi\rangle_{bcd} = |\tilde{a}\rangle_i |0\rangle_c |0\rangle_d$. After the interaction between modes $b$–$c$ in the BS$_2$ and that in the modes $c$–$d$ in the BS$_3$, the state of the three qubits is

$$|\psi\rangle_{bcd} = \frac{1}{\sqrt{2}} |1\rangle_b |0\rangle_c |0\rangle_d + i \frac{\sqrt{2}}{2} |0\rangle_b |1\rangle_c |1\rangle_d = \frac{1}{2} |0\rangle_b |0\rangle_c |1\rangle_d,$$

Note that this W state belongs to the class described by equation (1) with $\xi = 1$, $\gamma = 3\pi/2$, and $\delta = \pi/2$.

**Quantum information splitting.** To start with our protocol, the state $|\psi\rangle_{ab}$ is sent to Alice, who shares with Bob and Charlie the entangled W state $|\psi\rangle_{bcd}$. Hence, the state of the whole system reads

$$|\phi\rangle_{abcd} = (\sqrt{2}C_0 |0, 1, 1, 0\rangle_{abcd} + iC_0 |0, 1, 0, 1\rangle_{abcd} - C_0 |0, 1, 0, 1\rangle_{abcd} + \sqrt{2}C_1 |1, 0, 1, 0\rangle_{abcd} + iC_1 |1, 0, 0, 1\rangle_{abcd} - C_1 |1, 0, 0, 1\rangle_{abcd})/2.$$

We can rewrite this equation in an alternative form (up to normalization):

$$|\psi^{(\pm)}\rangle_{ab} = |\sqrt{2}C_0 |1, 0, 0\rangle_{abcd} + C_1 |1, 0, 1\rangle_{abcd} + i |0, 0, 1\rangle_{abcd} \rangle_{bcd},$$

$$+ |\psi^{(-)}\rangle_{ab} = |\sqrt{2}C_0 |1, 0, 0\rangle_{abcd} - C_1 |1, 0, 1\rangle_{abcd} + i |0, 0, 1\rangle_{abcd} \rangle_{bcd},$$

$$+ |\psi^{(+)}\rangle_{ab} = |\sqrt{2}C_0 |1, 0, 0\rangle_{abcd} + C_1 |1, 0, 1\rangle_{abcd} + i |0, 0, 1\rangle_{abcd} \rangle_{bcd},$$

$$+ |\psi^{(-)}\rangle_{ab} = |\sqrt{2}C_0 |1, 0, 0\rangle_{abcd} - C_1 |1, 0, 1\rangle_{abcd} + i |0, 0, 1\rangle_{abcd} \rangle_{bcd},$$

where $|\psi^{(\pm)}\rangle_{ab}$ and $|\Phi^{(\pm)}\rangle_{ab}$ are the EPR states of the Bell base:

$$|\Phi^{(\pm)}\rangle_{ab} = (|0, 1\rangle_{ab} \pm i |1, 0\rangle_{ab}) \rangle_{\sqrt{2}},$$

$$|\Phi^{(\pm)}\rangle_{ab} = (|1, 1\rangle_{ab} \pm i |0, 0\rangle_{ab}) \rangle_{\sqrt{2}}.$$

So, after Alice’s measurement, the state of the particles $c$ and $d$ on Charlie and Bob hands, respectively, plus the ‘ancillary’ mode $\tilde{a}$, collapses onto one of the four entangled states that appear inside the brackets of equation (5).

Note in equation (5) that the two components are not symmetric: while one of them is a Bell state (particles $a$ and $b$) the other has the form of a W state (particles $c$, $d$ and $\tilde{a}$). The reconstruction of the state can be done provided that Bob and Charlie collaborate with each other. The BS$_5$, shown in figure 1, is used to decouple the states corresponding to modes $c$–$d$, as shown in the following. Figure 2 shows the schematic circuit to generate the W state and to split quantum information.

**Bell-state measurement.** The Bell-state measurement carried out by Alice occurs in the BS$_4$ and in the D on the modes $a$ and $b$. After this joint measurement the Bell-state evolves to

$$|\psi^{(\pm)}\rangle_{ab} \rightarrow \left\{\begin{array}{ll}
|1, 0\rangle_a, & \text{if } (+) \\
|0, 1\rangle_a, & \text{if } (-)
\end{array}\right.,$$

$$|\Phi^{(\pm)}\rangle_{ab} \rightarrow \left\{\begin{array}{ll}
|0, 0\rangle_a, & \text{if } (+) \\
|2, 0\rangle_a, & \text{if } (-)
\end{array}\right..$$
The ground-state expectation values for Langevin operators are written as

\[ \langle L_a \rangle = \langle L_a \rangle_{\text{vac}} + \langle L_a \rangle_{\text{int}} \]

and become

\[ \langle L_a \rangle_{\text{int}} = \langle L_a \rangle_{\text{vac}} + \langle L_a \rangle_{\text{int}} \]

where \( \langle L_a \rangle_{\text{vac}} \) is the expectation value for the vacuum state and \( \langle L_a \rangle_{\text{int}} \) is the expectation value for the interaction state. In what follows we study how the fidelity of this process is influenced by nonidealities of BS and D.

**Losses in BS and in D.** For nonideal symmetric BS, the input and the output operators can be written via a phenomenological approach [23], as

\[ a^{\dagger}_m \rightarrow t a^{\dagger}_m + i r b^{\dagger}_m + L^+_m, \]

\[ b^{\dagger}_m \rightarrow t b^{\dagger}_m + i r a^{\dagger}_m + L^+_m, \]

where \( a^{\dagger} \) and \( b^{\dagger} \) are the creation operators in the modes \( a \) and \( b \) of the BS; \( t = \sqrt{\kappa T} \) and \( r = \sqrt{\kappa R} \), where \( \kappa \) stands for a quality factor of the BS corresponding to the probability of photon nonabsorption; \( L^+_m \) and \( L^+_m \) stand for the Langevin operators, taking into account the deleterious effect of the dissipation in the BS; they satisfy the commutation relations \( [L^+_m, L^-_n] = [L^+_m, L^-_n] = 0 \) and \( [L^+_m, L^+_n] = [L^-_m, L^-_n] = 1 - \kappa \).

The ground-state expectation values for Langevin operators are

\[ \langle L^+_m \rangle = \langle L^+_m \rangle_{\text{vac}} = 0, \quad \langle L^+_m \rangle = \langle L^+_m \rangle_{\text{int}} = 1 - \kappa, \]

and \( \langle L^+_m \rangle = \langle L^+_m \rangle_{\text{int}} = 0 \). The inefficiency of the D can be treated in a similar way [24] by the relation

\[ a^{\dagger}_m \rightarrow \epsilon a^{\dagger}_m + L^+_m, \]

where \( \epsilon \) is the detector efficiency and \( L^+_m \) stands for the Langevin operators obeying the commutation relations: \( [L_m, L^+_m] = 1 - \epsilon \) and \( [L_m, L_n] = 0 \); so the ground-state expectation values of these pair products are \( \langle L_m L^+_n \rangle = 1 - \epsilon \) and \( \langle L_m L^+_n \rangle = 0 \).

Next, we turn to the procedure for splitting the quantum information, now including the loss effects. Let us begin by considering the input state

\[ |\psi_1\rangle = (C_0 |0, 1\rangle_{ab} + C_1 |1, 0\rangle_{ab}) |1, 0, 0\rangle_{abcd} |0\rangle_R, \]

as shown in figure 1. Here \( |0\rangle_R \equiv \prod_k |0\rangle_k \) stands for the state of the environment composed of a huge number of vacuum-field states \( |0\rangle_k \). In sequence, the field in this state impinges on the other four BSs of the apparatus. In BS2 the modes \( a \) and \( b \) become entangled, the whole system being described as

\[ |\psi_2\rangle = (C_0 |0, 1\rangle_{ab} + C_1 |1, 0\rangle_{ab}) (t |1, 0, 0\rangle_{abcd} + i r |0, 1, 0\rangle_{abcd} + L^+_0 |0, 0, 0\rangle_{abcd} |0\rangle_R. \]

After the BS3, the state \( |\psi_2\rangle \) evolves to

\[ |\psi_3\rangle = (C_0 |0, 1\rangle_{ab} + C_1 |1, 0\rangle_{ab}) (t |1, 0, 0\rangle_{abcd} + i r |0, 1, 0\rangle_{abcd} + L^+_0 |0, 0, 0\rangle_{abcd} |0\rangle_R, \]

where \( |\phi_{\text{loss}}\rangle = |\chi^{(0)}\rangle_{abcd} R \) and \( |\chi^{(0)}\rangle_{abcd} R \) are the states corresponding to modes \( a, c, d \), and reservoirs when the modes \( a \) and \( b \) are described by \( |0\rangle \), \( |1\rangle \), \( |0\rangle \), \( |1\rangle \), respectively; \( \rho_{\text{res}} \) is the residual density operator, corresponding to the rejected terms in the detection by \( D_a \) and \( D_b \), namely

\[ ab \langle 0, 1 \rangle |\rho_{\text{res}}| 

where \( \langle 0, 1 \rangle \) \( \langle 1, 0 \rangle \), \( \langle 0, 1 \rangle \) \( \langle 1, 0 \rangle \), \( \langle 0, 1 \rangle \) \( \langle 1, 0 \rangle \), and \( \langle 0, 1 \rangle \) \( \langle 1, 0 \rangle \) are the unique possibilities allowing us to continue with the protocol.

Following the ideal protocol explained above, we assume that Charlie and Bob agree in collaborating for the reconstruction of the state. After they send their particles to interact through the BSs we have the following evolutions

\[ |\chi^{(0)}\rangle_{abcd} R \rightarrow |\eta^{(0)}\rangle_{abcd} R \] and \[ |\chi^{(0)}\rangle_{abcd} R \rightarrow |\eta^{(0)}\rangle_{abcd} R, \]

where

\[ |\eta^{(0)}\rangle_{abcd} R = \mathcal{N}_{01} (\langle C_0 \sqrt{r} \epsilon T^2 + 2 i C_0 r t \sqrt{\epsilon} \gamma_n^T \rangle + i C_1 r t \sqrt{\epsilon} \gamma_n^T - C_1 r t^2 \sqrt{\epsilon} \gamma_n^T - i C_1 r^2 \sqrt{\epsilon} \gamma_n^T - C_1 r^2 \sqrt{\epsilon} \gamma_n^T + i C_1 r^2 \sqrt{\epsilon} \gamma_n^T |0, 1, 0\rangle_R + i C_1 r^2 \sqrt{\epsilon} \gamma_n^T |0, 1, 0\rangle_R |0\rangle_R R - 2 i C_1 r^2 \sqrt{\epsilon} |0, 1, 0\rangle_R R) \]

Figure 2. Schematic diagram of the quantum circuit. \( \hat{U}_{\text{BS}} \) is the beam-splitter operator and \( \hat{U}_{\text{PS}} \) is the phase-shifter operator.
Figure 3. Fidelity of the reconstructed state considering photodetection \([0, 1]_{ab}\). (a) With fixed \(\kappa = 0.98\) and (b) with fixed \(\epsilon = 0.7\).

\[
|\eta(10)\rangle_{\text{out}} = N_0[|iC_0 t e^{\sqrt{\epsilon}} + 2iC_1 t e^{\sqrt{\epsilon}} L_1\rangle + C_1 t^2 e^{\sqrt{\epsilon}} L_1 L_3 + iC_1 t^2 e^{\sqrt{\epsilon}} L_2 L_3 - C_1 t^2 e^{\sqrt{\epsilon}} L_4
+ iC_1 t^2 e^{\sqrt{\epsilon}} L_2 L_3 + C_1 t^2 e^{\sqrt{\epsilon}} L_4 L_3 + 0, 1)_{\text{cd}} |0, 0, 0\rangle_R
+ \frac{1}{2} C_1 t^2 e^{\sqrt{\epsilon}} - iC_1 t^2 e^{\sqrt{\epsilon}} L_1 + 0, 1)_{\text{cd}} |0, 0, 1\rangle_R
- 2C_1 t^2 e^{\sqrt{\epsilon}} |0, 1, 1\rangle_{\text{cd}} |0, 0\rangle_R. \tag{20}
\]

\(N_0\) and \(N_1\) standing for normalization. Then, as done in the ideal protocol, a phase shift is applied on the mode \(d\) to change the phase according to the detected state, in a way that \(|0, 1\rangle_{ab}\) \(|1, 0\rangle_{ab}\) needs a phase shift of \(\pi/2 (\pi/2)\).

Let us consider the case when Alice detects the state \(|0, 1\rangle_{ab}\). Then the density operator is reduced to the subsystems \(c - d - \bar{a} - R\) in the form

\[
\rho_{\text{cdR}}^{(01)} = |\eta^{(01)}\rangle_{\text{cdR}} \langle \eta^{(01)}|.\tag{21}
\]

Next, we trace out the reservoir in the operator above to get the fidelity \(F_{01} = \text{Tr}_{\bar{a}} (\rho_{\text{cdR}}^{(01)} |\Phi\rangle_{\text{cd}} \langle \Phi|)_{\text{cd}}\), with \(\rho_{\text{cdR}}^{(01)} = \text{Tr}_{R} (\rho_{\text{cdR}}^{(01)})\) and \(|\Phi\rangle_{\text{cd}} = |0\rangle_{c} (|0, 0\rangle)_{\text{cd}} + C_1 |1, 1\rangle_{\text{cd}}\):

\[
F_{01} = N_0^2 \left[C_0^2 t^4 + 2C_0^2 C_1^2 t^2 e^{\sqrt{\epsilon}} + 4C_0^2 C_1^4 t^2 e^{\sqrt{\epsilon}} (1 - \epsilon) + C_0^2 C_1^4 t^2 e^{\sqrt{\epsilon}} - C_0^2 C_1^2 t^2 e^{\sqrt{\epsilon}} + C_0^2 C_1^4 t^2 e^{\sqrt{\epsilon}} (1 - \epsilon) + C_0^2 C_1^4 t^2 e^{\sqrt{\epsilon}} (1 - \epsilon) + C_0^2 C_1^2 t^2 e^{\sqrt{\epsilon}} (1 - \epsilon)
\]

\[
+ C_0^2 C_1^4 t^2 e^{\sqrt{\epsilon}} (1 - \epsilon)), \tag{22}
\]

where the normalization factor is

\[
N_0 = \left[C_0^2 t^4 + 4C_0^2 C_1^2 t^2 e^{\sqrt{\epsilon}} + C_0^2 C_1^4 t^2 e^{\sqrt{\epsilon}} (1 - \epsilon) + C_0^2 C_1^2 t^2 e^{\sqrt{\epsilon}} - C_0^2 C_1^4 t^2 e^{\sqrt{\epsilon}} (1 - \epsilon) + C_0^2 C_1^4 t^2 e^{\sqrt{\epsilon}} (1 - \epsilon) + C_0^2 C_1^2 t^2 e^{\sqrt{\epsilon}} (1 - \epsilon)
\]

\[
+ C_0^2 C_1^4 t^2 e^{\sqrt{\epsilon}} (1 - \epsilon))^{-1/2}. \tag{23}
\]

To clarify the relevance of the D efficiency, figure 4(b) shows the fidelity for \(\kappa = 0.98\) and \(\epsilon = 1\) (ideal detectors).

In conclusion, we have proposed a simple and feasible scheme to split the quantum information encoded in an entangled state of the vacuum and one-photon state of the two electromagnetic field modes. To this end, we have shown how to engineer a class of W states suitable for perfect quantum teleportation and superdense coding, with 100% success probability, making use of a simplified scheme that employs only one photon. Our scheme to split quantum information has 50% of success probability since it uses only linear optical elements: five BS, one PS and a couple of detectors. In addition, we need two SPDC photons, one of them prepares the entanglement of the vacuum- and one-photon states, the other photon is used to prepare the W state. The errors introduced by realistic BS and detectors were studied; in this case the fidelity of the state results better than that in the classical limit \((F = 0.5)\), which is the best achievable value without the use of entanglement for quantum teleportation protocols [27], similar to the value for quantum information splitting), even for the worst choice of experimental parameters concerned with the efficiency of detectors and losses in the BS.

Acknowledgments

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