Planet Migration and Disk Destruction due to Magneto-Centrifugal Stellar Winds

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ABSTRACT

This paper investigates the influence of magneto-centrifugally driven or simply magnetic winds of rapidly-rotating, strongly-magnetized T Tauri stars in causing the inward or outward migration of close-in giant planets. The azimuthal ram pressure of the magnetized wind acting on the planet tends to increase the planet’s angular momentum and cause outward migration if the star’s rotation period \( P_* \) is less than the planet’s orbital period \( P_p \). In the opposite case, \( P_* > P_p \), the planet migrates inward. Thus, planets orbiting at distances larger (smaller) than \( 0.06\text{AU}(P_*/5\text{d})^{2/3} \) tend to be pushed outward (inward), where \( P_* \) is the rotation period of the star assumed to have the mass of the sun. The magnetic winds are likely to occur in T Tauri stars where the thermal speed of the gas close to the star is small, where the star’s magnetic field is strong, and where the star rotates rapidly. The time-scale for appreciable radial motion of the planet is estimated as \( \sim 2 – 20 \text{Myr} \). A sufficiently massive close-in planet may cause tidal locking and once this happens the radial migration due to the magnetic wind ceases. The magnetic winds are expected to be important for planet migration for the case of a multipolar magnetic field rather than a dipolar field where the wind is directed away from the equatorial plane and where a magnetospheric cavity forms. The influence of the magnetic wind in eroding and eventually destroying the accretion disk is analyzed. A momentum integral is derived for the turbulent wind/disk boundary layer and this is used to estimate the disk erosion time-scale as \( \sim 1 – 10^2 \text{Myr} \), with the lower value favored.

Key words: stars: pre-main sequence: stars — magnetic fields: planets and satellites: general: accretion, accretion disks: stars: winds, outflows

1 INTRODUCTION

More than 250 planets have been discovered around solar-type stars. About 20\% of them are located close to the star, at \( R < 0.1 \text{AU} \), and a significant number are located as close as at \( R < 0.05 \text{AU} \). According to the presently favored interpretation, planets form far away from the star either through core accretion (Mizuno 1980; Pollack et al. 1996), or through instabilities in the disk (Boss 2001). Subsequently, they migrate inward due to their gravitational interaction with the disk (reviewed by Papaloizou & Terquem 2006). For typical conditions planets migrate inward as a result of interaction of the planet with the disk matter. The planet loses part of its orbital angular momentum by overtaking collisions with the disk outside its orbit and it gains a smaller part by overtaking collisions of the disk matter inside its orbit.

Some planets are expected to migrate close to the star where the disk properties are strongly influenced by the star and its rotating magnetic field. In particular, the inner regions of the disk may be dispersed as a result of heating by the star (Kuchner & Lecar 2002) and photo-evaporation (Matsuyama, Johnstone & Murray 2003). Alternatively, the equatorial-plane density of the accretion disk may be enormously reduced inside of what is termed the “magnetospheric gap” \( (R < R_m) \) owing to to the strong magnetic field of the rotating protostar (Lin, Bodenheimer & Richardson 1996; Romanova & Lovelace 2006). In this situation inward migration of the planet stops once the planet is some distance inside the gap \( (R < 0.63R_m) \) because there is no matter for it to transfer its angular momentum to. On the other hand, the magnetic interaction between a planet and the star may also influence a planet’s migration (Papaloizou 2007; Fleck 2008). Indications of such interactions have been observed by Shkolnik et al. (2005). In these models, the star’s magnetic field is assumed to be dipolar with sufficient strength that a low density cavity forms near the star.
In contrast, if the star’s magnetic field is dominantly multipolar rather than dipolar (e.g., Saifer 1998; Donati et al. 2007; Johns-Krull 2007), then there may be no cavity near the star. Instead the magnetic field consists of closed field regions and multiple open field line regions where the field extends to large distances. A wind can flow outward along the open field lines (Jardine et al. 2006; Gregory et al. 2006). Spectral measurements provide evidence of strong stellar winds from young stars (e.g., Kwan, Edwards, & Fischer 2007).

The winds from magnetized stars may be thermally driven as the Solar wind is as proposed by Matt and Pudritz (2008a,b). In this case, the magnetic field still has an important role in the outward transport of angular momentum (Weber & Davis 1967; Matt & Pudritz 2008a,b). In contrast, for conditions where the thermal speed of the gas close to the star is small (compared with the escape velocity), where the star’s magnetic field is strong, and where the star rotates rapidly, there are magneto-centrifugally driven winds (or more simply magnetic winds) (Michel 1969, M69 hereafter; Belcher & MacGregor 1976. BM76 hereafter). The rapid rotation of the stars results of course from the accretion of high specific angular momentum from a disk. The inner region of the disk may be dispersed by heating by the thermal energy in the disk (Zahn 1977; Zahn & Vandenbosch 1977). In this limit the radial flow velocity at very large distances is the Parker velocity

\[ v_R^{\ast} = \frac{v_s}{\sqrt{1 + \frac{\Omega^2}{\Omega^2_s}}} \]

which is assumed larger than the flow speed at this point. Here, \( \Omega_s \) is the angular rate of the star, \( v_s \) is the sound speed. The ideal MHD equations can be integrated from the surface of the star to very large distances for appropriately chosen parameters (BM76). Specifically, the parameters are chosen so as to allow the smooth integration of \( v_R^{\ast} \) through the slow and fast magnetosonic singular points.

If the star is not rotating, the radial flow velocity at very large distances is the Parker velocity.

\[ v_R = \frac{v_s}{\sqrt{1 + \frac{\Omega^2}{\Omega^2_s}}} \]

This paper first analyzes the torque due to magnetic stellar winds on close-in giant planets. This torque can cause the planet to migrate either inward or outward depending on the star’s rotation period and the planet’s orbital period. We go on to develop a model for the erosion of disks by magnetic winds. It is thought that close-in giant planets formed at distances greatly reduced so that the inward migration stops. The inner region of the disk may be dispersed by heating by the star as mentioned, and it may be eroded by a magnetic wind. There may be a strong tidal interaction which acts to lock the star’s surface layer to the planets motion (Zahn 1994; Marcy et al. 1997; Donati et al. 2007). Once locking occurs the migration due to a magnetic wind stops.

Section 2 of this paper discusses magneto-centrifugally driven winds (or simply magnetic winds). Section 3 analyzes the influence of magnetic winds in pushing close-in planets outward or inward. Section 4 comments on the influence of the tidal interaction on close-in planet migration. Section 5 investigates the role of magnetic winds in eroding and eventually destroying the accretion disk. Section 6 gives the conclusions of this work.

### 2 MAGNETO-CENTRIFUGALLY DRIVEN WINDS

We adopt the Weber and Davis (1976) model of an ideal magnetohydrodynamic (MHD) stellar wind, where the wind is assumed stationary and axisymmetric, and where attention is focused on the equatorial region of the flow. In spherical \( (R, \theta, \phi) \) inertial coordinates, the density, pressure, and flow velocity of the wind are \( \rho(R), \rho^\ast \), and \( \mathbf{v} = v_R \mathbf{R} + v_\phi \mathbf{\phi} \). The magnetic field, \( \mathbf{B} = B_R \mathbf{R} + B_\phi \mathbf{\phi} \), emanates from a perfectly conducting star of radius \( R_\ast \) and mass \( M_\ast \), that rotates at the angular rate \( \Omega_\ast \). It is assumed that \( \gamma = \text{const} \) and \( \rho^\ast \) where \( \gamma = \text{const} \) is the polytropic index. In the equatorial region the mass flux of the wind per unit solid angle is conserved, \( \dot{M}_w = R^2 \rho v_R = \text{const} \), and the radial magnetic flux per unit solid angle is also conserved, \( \Phi_R / \pi = R^2 B_R = \text{const} \).

There are a number of conserved quantities of the flow, one of these gives \( B_\phi / B_R = (v_\phi - \Omega_\ast R_\ast) / v_R \), another gives \( L = R [\rho v_R - B_R B_\phi (4 \pi v_R)] = \text{const} \), the angular momentum per unit mass, and a third gives the Bernoulli constant \( B = (v_R^2 + v_\phi^2) / 2 + \Phi_\phi^2 / \gamma - GM_\ast / R - \Omega_\ast R_\ast B_R B_\phi / (4 \pi v_R) = \text{const} \), where \( G \) is the gravitational constant and \( c_s \equiv \sqrt{(\gamma P / \rho)^{1/2}} \) is the sound speed. The ideal MHD equations can be combined to give an equation for \( dB_R / dR \) which can be integrated from the surface of the star to very large distances for appropriately chosen parameters (BM76). Specifically, the parameters are chosen so as to allow the smooth integration of \( v_R \) through the slow and fast magnetosonic singular points.

If the star is not rotating, the radial flow velocity at very large distances is the Parker velocity

\[ v_R = \frac{v_s}{\sqrt{1 + \frac{\Omega^2}{\Omega^2_s}}} \]

where \( v_s \equiv v_s / \Omega_\ast R_\ast \) is the sound speed at the initial radius \( R_\ast \), which is assumed larger than the flow speed at this distance. For a rotating star, one can define the reference velocity,

\[ v_M \equiv \left( \frac{\Omega_\ast^2 (R_\ast R^2) \dot{M}_w}{v_s^2} \right)^{1/3} \]

approximated by Michel (1969). Here, \( P_s = 2 \pi / \Omega_\ast \) is the rotation period of the star, \( B_{Rs} \equiv B_R(R_\ast) \), and the second line of equation (1) the star’s radius is assumed to be twice the radius of the Sun, \( R_\ast = 1.4 \times 10^{11} \) cm (Armitage & Clarke 1996).

In this work we consider magneto-centrifugally driven winds where \( v_M \gg v_R \) and \( v_M \gg c_s \) as discussed by M69 and BM76. In this limit the radial wind velocity at large
The radial flow velocity increases from a small value at the star to a value \( \frac{2}{3} \) at distances \( R \approx R_A \). The azimuthal wind velocity \( v_\phi \approx \frac{1.8 R}{1 + 0.8 R/R_A} \) increases to \( \frac{2}{3} v_M \), which is larger than the planet’s azimuthal velocity.

These functions are shown in Figure 1. Because \( B_\phi/B_R \approx -1.55 \), the ratio \( \beta \equiv 4\pi v^2/B^2 \) also depends only on \( R/R_A \) and increases with \( R \) monotonically to \( \approx 0.319 \) at \( R = R_A \).

Figure 3. Sketch of a magnetic field line for conditions where the azimuth wind velocity \( v_\phi \) is larger than the planet’s azimuthal velocity.

\[ R_A, B_\phi/B_R \approx -1.55 \]

The geometry is sketched in Figure 3. The azimuthal drag force on the planet’s motion through the wind leads to a gradual change of the planet’s angular momentum \( L_p = M_p(\Omega M R_p)^{1/2} \):

\[
\frac{dL_p}{dt} = \frac{1}{2} M_p \left( \frac{GM_*}{R_p} \right)^{1/2} \frac{dR_p}{dt} = \frac{M_p(\Omega M R_p)^{1/2}}{2T_w} \]

\[
= \text{sign}(v_\phi - v_{Kp}) R_p A_{\text{eff}} P_{\text{ram}}. \tag{4}
\]

Here, \( T_w \) is the timescale for the wind to significantly change the planet’s orbit, \( A_{\text{eff}} \) is the effective cross section of the planet discussed below,

\[
P_{\text{ram}} = \frac{1}{4\pi} \rho (v_\phi - v_{Kp})^2 + \frac{1}{4\pi} B^2. \tag{5}
\]

is the ram pressure, and \( v_{Kp} = (GM_*/R_p)^{1/2} \) is the Keplerian velocity of the planet.

We consider that the effective cross-section of the planet \( A_{\text{eff}} \) may be larger than its projected area \( \pi R_p^2 \) because the planet may have its own magnetic field or the planet may have an evaporatively driven wind (Vidal-Madjar, et al. 2003). We consider the case where the planet has a dipole magnetic field with surface strength \( B_{p0} \) (Sánchez-Lavega 2004). Then, the effective area is

\[
A_{\text{eff}} = \pi R_p^2 \left( \frac{B_{p0}^2}{4\pi P_{\text{ram}}(R_p)} \right)^{1/3} \tag{6}
\]

if this quantity is larger than \( \pi R_p^2 \); otherwise \( A_{\text{eff}} = \pi R_p^2 \).

The stellar wind causes outward (inward) migration of the planet when the azimuthal speed of the wind \( v_\phi \) is larger (smaller) than the planet’s azimuthal speed \( v_{Kp} \). We find that equality \( v_\phi = v_{Kp} \) holds for \( R_p \) equal to a critical value, \( R_c \), that depends on the parameters of the wind, \( M_w, B_{w*}, v_M \), and the star’s rotation rate \( \Omega_* = 2\pi/P_* \). For \( R_c \ll R_A \) we find
torque on the planet acting to move it outward is transmitted inward). For this figure the planet’s mass and radius have been taken to be equal to Jupiter’s while the planet’s magnetic field is taken to be 100 G.

\[ R_{cr} \sim \left( \frac{GM_p}{\Omega_p^2} \right)^{1/3} \approx 0.87 \times 10^{12} \text{cm} \left( \frac{P_{\text{sd}}}{5 \text{ d}} \right)^{2/3}, \] (7)

which is the corotation radius of the star. A planet in a circular orbit of radius \( R_{cr} \) has an orbital period of \( P_c \). This expression for \( R_{cr} \) follows directly using equation (3) which gives \( v_p \approx \Omega_c R \) (for \( R \ll R_A \)) and setting this equal to the planet’s Keplerian velocity \( v_{Kp} \).

Figure 4 shows the timescale for the wind to significantly change the planet’s orbit \( T_w \) as a function of the mass loss rate of the wind, \( \dot{M}_w \). The bracketed quantities represent the star’s surface magnetic field in kG and its rotation period in days. The planet’s mass and radius are taken to be equal to Jupiter’s while the planet’s magnetic field is taken to be 100 G.

\[ T_w \sim \frac{2.06 M_p}{r_p^2} \left( \frac{GM_p}{\Omega_p^2} \right)^{11/9} \frac{B_{13/9}}{B_{\text{sd}}^{2/3}} R_{13/3}^{8/3}, \]

\[ \approx 17.5 \text{ Myr} \left( \frac{P_{\text{sd}}}{5 \text{ d}} \right)^{13/9} \left( \frac{10^3 \text{ G}}{B_{\text{sd}}} \right)^{4/3}, \] (8)

where it is assumed that \( A_{eff} \geq \pi r_p^2 \). In the second line the planet’s mass and radius is assumed equal to Jupiter’s and its surface magnetic field is 100G. The mass of the star is \( M_\odot \) and its radius is 2R_\odot.

Alternatively, the torque on the planet can be thought of in terms of a DC current circuit following the model of Io’s interaction with Jupiter’s magnetoosphere by Goldreich and Lynden-Bell (1969). This is applicable if the Alfvén wave transit time from the planet to the star and back is less than the time for the planet to move a distance of the order of its diameter relative to the wind (Drell, Foley, & Ruderman 1965). The torque is \( T = -R_p I_o B_R (R_p/2r_{eff})/c \), where \( I_o \) is the poloidal current flowing in the \( \theta \)-direction through planet’s magnetopause. This current then flows down the wind field line intersecting the top of the planet at \( \theta = r_{eff}/R_p \); it then flows through the star’s atmosphere; and finally it flows outward on the wind field line which intersects the bottom of the planet at \( \theta = -r_{eff}/R_p \). The voltage drop across the planet in its reference frame is \( \Delta V = -(2r_{eff})(v_o - v_{Kp}) B_R(R_p)/c \), which acts as a battery. For representative values of \( R_p = 10^{12} \) cm, \( B_R = 20 \) G, \( r_{eff} = 7.15 \times 10^3 \) cm (Jupiter’s radius), and \( v_o - v_{Kp} = 10^6 \) cm/s, one finds \( \Delta V \approx 3 \times 10^3 \) V. The parts of the circuit along the field lines have negligible resistance. Thus the total current \( I_o = \Delta V/R \), where \( R \) is the resistance of the parts of the circuit in the star’s turbulent atmosphere and in the magnetopause of the planet. Setting the magnetic field torque (equation 4) from the MHD model equal to the circuit model torque \( T \) gives the total poloidal current flow, \( I_o \approx 3.5 \times 10^{10} \) A for the mentioned conditions. This implies that the circuit resistance is \( R \approx 0.09 \) Ohm. The circuit picture is helpful for understanding the interaction but a consistent treatment requires a treatment using the MHD equations (Parker 1969).

Observations and theory relevant to the magnetic coupling of hot Jupiters to their stars is discussed by Cuntz, Saar, & Musielak (2000), Shkolnik et al. (2005), McIvor, Jardine, & Holzwarth (2006), and Preusse et al. (2006).

The stellar wind acts to spin-down the star giving a torque on the star \( -(R_\odot^3 \sin^2(\theta)/\Omega_w)M_w \) (Weber & Davis 1967) as discussed by Matt and Pudritz (2008a, b) for the case of a thermally driven wind. Here, \( (R_\odot^3 \sin^2(\theta))^{1/2} \) is the mass-weighted, angle averaged Alfvén radius for the angular momentum outflow in the wind. This average Alfvén radius may be significantly smaller than the planets considered above. The angular momentum of the star is \( kM_r R_{\odot}^2 \Omega \), with \( k \approx 0.2 \) for stars younger than \( 10^7 \) yr (Armitage & Clarke 1996) so that the spin-down time-scale of the star is

\[ T_{sd} \sim \frac{0.2 M_r R_{\odot}^2}{\dot{M}_w (R_\odot^3 \sin^2 \theta)} \] (9)

For the reference parameters of equation (8) and \( \dot{M}_w \sim 10^{-9} \)–\( 10^{-13} \)M_\odot/yr, the spin-down time \( T_{sd} \) is less than the wind time \( T_w \). Under such conditions the planet would at first have \( R_p > R_{cr} \) and be pushed outward by the wind, and subsequently it would have \( R_p < R_{cr} \) and be pushed inward.

## 4 Influence of Tidal Interaction

We consider that initially the planet is at a relatively large distance with \( R_p > R_{cr} = (GM_p/\Omega_p^2)^{1/3} \) but with \( R_p \ll R_A \). We assume that the inner region of the disk (< 1 AU) has been expelled so that there is no migration due to the planet’s interaction with the disk.

For \( R_p > R_{cr} \), the rotating magnetic wind tends push the planet outward with a time-scale \( T_{sd} \) given by equation (8). At the same time, the magnetic wind causes the star to spin-down with a time-scale \( T_{sd} \) given by equation (9). If the spin-down time is longer than \( T_{sd} \) the planet will be gradually pushed outward. This case is not considered further. In the opposite case where \( T_{sd} < T_{sd} \), the planet and star will evolve to have \( R_p < R_{cr} \), and the wind will tend to push the planet inward, again with the time-scale of equation (8). However, the spin-down of the star acts to increase \( T_w \) and...
very likely $T_w/T_{ed}$ with the result that the inward motion of the planet will cease.

As the planet moves inward it raises an increasing tide on the star. We assume that the planet itself is tidally locked and in a circular orbit (Marcy et al. 1997). The time-scale for the establishment of corotation of the entire star is estimated by Zahn (1994), Marcy et al. (1997), and Donati et al. (2008) as

$$T_{tdl} \sim 1.2 \times 10^{12} \text{yr} \left(\frac{M_*}{10^3 M_\odot}\right)^2 \left(\frac{R_p}{10 R_\odot}\right)^6$$,

(10)

where we have assumed a solar mass star of radius $2 R_\odot$.

The values of $T_{tdl}$ are probably longer than the age of the systems except for very massive planets.

Marcy et al. (1997) and Donati et al. (2008) suggest that only the convective envelope of the star of mass $\delta M_* \ll M_*$ is brought into corotation or “locked” to the planet. The time-scale for this to occur is reduced from $T_{tdl}$ by a factor $\sim (\delta M_*/M)^{1/2}$. The core of the star then rotates slower than its surface. For a Jupiter mass planet, $R_p = 10^2 \text{cm}$, and $\delta M_* / M_* < 0.005$, we have $T_{tdl} < T_w$. Of course, once the star’s surface becomes locked to the planet, the effect of the magnetic wind on the planet’s orbit vanishes. With the planet/stellar surface locked, the loss of angular momentum from the planet/stellar surface system due to a magnetic wind acts to reduce the planet’s period if it is initially relatively large. However, the reduction of the angular momentum is limited for a fixed $\delta M_*$. This limit may such that a sufficiently massive planet will fall into the star. For a less massive planet the system behavior for further angular momentum loss is unclear.

Note that the influence of the wind on the planet’s orbit decreases as $M_p$ increases and as the star’s rotation period increases (equation 8). On the other hand the tidal interaction increases as $M_p$ increases, and it increases strongly as $R_p$ decreases (equation 10). The remarkable object τ-Bootis has a planet mass $M_p \approx 7.5 M_J$ and period $P_p = 3.3$ d and is locked to the rotation of the star’s surface (Donati et al. 2008; Catala et al. 2007; Butler et al. 1997). On the other hand the famous object 51 Pegasi (Mayor & Queloz 1995) has a planet of mass $0.7 M_J$ and a period of 4.23 d which orbits 51 Peg which has a rotation period $P_* \approx 37$ d.

Consider what may happen to a giant planet after its fast inward migration ends at a distance say $R_p \lesssim 0.1$ AU owing to the planet entering a magneto-spheric gap or the region of the disk which has been blown away by a stellar wind. The planet’s distance $R_p$ may be unaffected by the star’s wind owing to the star’s rapid spin down. If the planet’s mass is large enough relative to the mass of the convective envelope, then it can cause tidal locking of the star’s surface as observed in τ-Boo. For a smaller mass planet there is no locking as is the case of 51 Pegasus. Another possibility is that $R_p$ changes due to the influence of the magnetic wind. The spin-down of the star is expected to give $P_\star < P_p$, in which case the wind causes a slow inward motion of the planet or decrease of $R_p$. The planet moves inward until there is tidal locking with the star’s surface (the time-scale for which is $\sim R_p^6$). The tidal locking eliminates the wind effect on the planet’s orbit. The migration due to the stellar wind allows smaller mass giant planets to produce tidal locking.

For tidal locking in a given period $T_{tdl}$, the distance where this occurs is $R_p(t_{dl}) \propto R_\star (M_p/\delta M_*)^{1/3}$. For stars of decreasing mass $M_* < M_\odot$ (but $M_* > 0.1 M_\odot$), the star’s radius decreases strongly, and the mass of the convective envelope $\delta M_*$ increases rapidly (the star is fully convective for $M_* < (0.3 - 0.4) M_\odot$) (Chabrier & Baraffe 2000). Thus $R_p(t_{dl})$ decreases rapidly as $M_p$ decreases from $M_\odot$.

5 EROSION OF THE DISK BY THE STELLAR WIND

Stellar winds have long been considered as a mechanism for the dispersal of accretion disks (Cameron 1973). Theoretical and simulation studies of this process have been out assuming an entirely hydrodynamic interaction (e.g., Cantó & Raga 1991; Richling & Yorke 1997; Hollenbach, Yorke, & Richstone 2000; Soker 2005). Here we discuss the interaction of a high velocity magnetized stellar wind with the accretion disk. The considered geometry is shown in Figure 5. The presence of the magnetic field leads to Reynolds numbers which are sufficiently large that the wind/disk boundary layer is strongly turbulent. In the hydrodynamic case the Reynolds numbers are much smaller so that the boundary layer is probably laminar (see Schlichting 1968, ch. 7). In the following we derive an estimate of the mass loss rate from the disk.

It is useful to consider the physical conditions in the wind at a distance $R = 1$ AU for fiducial conditions of wind density $n_w = 10^7$ cm$^{-3}$, wind speed $v_w = 500$ km/s, time averaged magnetic field $B_w = 0.1$ G (predominantly toroidal), and ion (proton) and electron temperatures $T_i = T_e = 10^5$ K. At this and larger distances note that (i) the wind velocity is predominantly radial, (ii) it is super fast magnetosonic, and (iii) it is much larger than the Keplerian velocity of the disk matter. The ion and electron gyro-frequencies are $\omega_i \approx 960$ s$^{-1}$ and $\omega_e \approx 1.8 \times 10^8$ s$^{-1}$, the ion and electron gyro-radii are $r_{i,1} \approx 3 \times 10^3$ cm and $r_{e,1} \approx 70$ cm; the ion and electron collision times ($\propto T_\nu^{1/2}/n$) are $\tau_i \approx 0.4$ s and $\tau_e \approx 10^{-3}$ s; and the ion and electron mean-free paths are $\ell_i = \ell_e \approx 1.4 \times 10^9$ cm (Braginskii 1965). Thus we have $\omega_i/T_i \approx 4.6 \times 10^{5}$ and $\omega_e/T_e \approx 2 \times 10^{6}$. In the absence of a magnetic field, the kinematic viscosities of ions and electrons are $\nu_{i,1} \approx v_{i,1} R_\theta \approx 4 \times 10^{15}$ cm$^2$ s$^{-1}$ and $\nu_{e,1} \approx v_{e,1} R_\theta \approx 1.7 \times 10^{17}$ cm$^2$ s$^{-1}$, where $v_{i,e} R_\theta$ are the ion or electron thermal speeds. Thus without a magnetic field the Reynolds number, $Re = R v_w/\nu_{i,1} \approx 4000$, is such that a boundary layer flow is laminar. With the magnetic field included, there are five different viscosity coefficients. However, for the considered problem the important viscosity coefficient is that for momentum transport across the magnetic field; for example the momentum flux-density component $\Pi_{\theta \phi} = -\nu_{i,1} R_\theta^2 \nu_{i,1} T_i/\theta$. For the ions it is $\nu_{i,1} \approx \nu_{0,1} (\omega_i/T_i) \approx 1 \times 10^9$ cm$^2$ s$^{-1}$ and for electrons $\nu_{e,1} \approx \nu_{0,1} (\omega_e/T_e) \approx 440$ cm$^2$ s$^{-1}$. Using the viscosity $\nu_{i,1}$, the effective Reynolds number for the wind is $Re_w = R v_w/\nu_{i,1} \sim 5 \times 10^{17}$ so that the boundary layer flow is strongly turbulent. This is in contrast with the non-magnetic case where the boundary layer flow would be laminar. The large reduction of the viscosity results from the particle step size between collisions being a gyro-radius rather
than a mean-free path. Thus the estimated Reynolds number also holds for a turbulent magnetic field.

Similarly, the important heat conductivity coefficient is that for the heat flux across the magnetic field, say \( q_B = -\kappa_\perp R^{-1} \partial (k_B T)/\partial \theta \), where \( \kappa_\perp \approx \kappa_{\perp,1} \approx 2 n v_{1,1}^2 \tau_{1,1} (\omega_{\perp,1} \tau_{\perp,1})^2 \approx 3.8 \times 10^9 \) (cm s\(^{-1}\)) and where \( k_B \) is Boltzman’s constant. For this heat conductivity, the heat flow from the wind into the disk is negligible compared with the energy flux per unit area from the disk \( 3GM/M_w/(8\pi R^3) \) for accretion rates \( M_a = 10^{-10} - 10^{-8} M_{\odot} \text{yr}^{-1} \).

At the place where the wind encounters the much denser disk, there is a weak oblique shock at a height \( H(R) \ll R \) above the equatorial plane and above the disk which has a half-thickness \( h(R) < H(R) \) as sketched in Figure 5.

The angle between the incident flow and the shock is \( \beta \approx Rd(H/R)/dR \ll 1 \). In general we expect to have \( \beta > 0 \). In passing through the shock the flow is deflected through an angle \( \delta = 2\beta/(1 + \gamma) = 3\beta/4 \) for \( \gamma = 5/3 \) away from the equatorial plane, and the flow speed is reduced by a small fractional amount. The region between \( h \) and \( H \) is referred to as the boundary layer. The influx of wind matter into this layer is \(-dS \cdot (\rho_v v_w) \approx dS \rho_w v_w R [d(H/R)/dR] \), where \( dS \approx RdRd\phi \) is the area element on the shock side of the disk. The Keplerian velocity of the disk is small compared with the wind velocity for \( R \gg 1 \) AU, and it is neglected. The density in the boundary layer varies from \( \rho(R,h) \gg \rho_w \) at the surface of the disk to \( \rho_w \) at \( z = H \). The time-averaged radial flow velocity varies from \( |v_R(R,h)| \ll v_w \) to \( v_R = v_w \) at \( z = H \). Here, we neglect \( v_R(R,h) \).

For stationary conditions, the conservation of mass and radial momentum in the annular region \( abcd \) of the boundary layer on the top side of the disk - sketched in Figure 6 - gives

\[
\frac{\partial}{\partial R} (RF_w) = R \left[ \frac{d}{dR} \left( \frac{H/R}{R} \right) \right] \rho_w v_w + \frac{1}{2\pi} \frac{dM_d}{dR} ,
\]

\[
\frac{\partial}{\partial R} (RF_p) = R \left[ \frac{d}{dR} \left( \frac{H/R}{R} \right) \right] \rho_w v_w^2 ,
\]

where the term on the left-hand side is from the vertical sides of the region and the right-hand side is due to the sides \( ab \) and \( cd \). Here, \( F_w(R) \) is the mass flux and \( F_p(R) \) is the radial momentum flux both per unit circumference of the top side of the disk. That is,

\[
F_w = \int_h^H dz \langle \rho v_R \rangle , \quad F_p = \int_h^H dz \langle \rho v_w^2 \rangle ,
\]

where the averages are over the turbulent fluctuations. The mass loss rate of disk per unit radius due to entrainment is \( dM_d/dR \). The disk matter influx to the boundary layer brings in negligible radial momentum.

We now integrate equations (11) and (12) from an inner radius \( R_{in} \), where \( F_w \) and \( F_p \) are negligible, to distance \( R \). We define an average radial velocity in the boundary layer as \( u(R) \equiv \int dz \langle \rho v_R \rangle / \int dz \langle \rho v_w \rangle \). Thus we obtain an equation for the mass loss rate from the top and bottom sides of the disk between \( R = R_{in} \) and \( R \).

\[
M_d = 4\pi \left( \frac{v_w}{u} - 1 \right) \int_{R_{in}}^R RdR \left[ \frac{d(H/R)}{dR} \right] \rho_w v_w ,
\]

which is von Karman momentum integral for this problem (Schlichting 1968, ch. 8). Using the fact that \( \rho_w v_w = M_w/(4\pi R^2) \) we can evaluate this integral at the outer radius of the disk \( R_{out} \). This gives the mass loss rate of the disk,

\[
M_d = \left( \frac{v_w}{u} - 1 \right)_{R=\infty} \left( \frac{H}{R} \right)_{R=\infty} M_w .
\]

The main unknown in this equation is \( u \).

The average velocity \( u \) depends on the vertical profiles of density and radial velocity which are not known. We expect the profiles, for example \( \langle v_R(z) \rangle \) to be substantially different from those of laboratory turbulent boundary layers over solid surfaces (e.g., Schlichting 1968, ch. 23; Roy & Blottner 2006). The main reason for the difference is that the density at the surface of the disk \( \rho(R,h) \) is many orders of magnitude larger than the wind density \( \rho_w \). For a laboratory boundary layer, a mixing-length model of the momentum transport gives \( \langle z' \rangle^2 (d\langle v_R \rangle/dz')^2 = \text{const} \) (with \( z' \equiv z - h \)), and this gives the well-known logarithmic velocity profile (see e.g. Schlichting 1968). For this profile most of the change of velocity is quite close to the wall (\( z' = 0 \)). In contrast, for the disk boundary layer a mixing length model gives \( \langle z' \rangle^2 (d\langle v_R \rangle/dz')^2 = \text{const} \). Because of the density dependence, the change in the velocity occurs relatively far from the wall. An important consequence of this is that the average velocity \( u \) is much smaller than \( v_w \). If we make the weak assumption that \( u \) is less than \( v_w/2 \), then we have a lower limit on the disk mass loss rate \( M_d \geq (H/R)_{out} M_w \).

For an initial disk mass of \( M_d = 0.02 M_{\odot} \) (e.g., Kuchner 2004), \( (H/R)_{out} = 0.2 \), and \( M_w = 10^{-9} M_{\odot} \text{yr}^{-1} \), the disk loss time is \( T_d \leq M_d/M_d = 10^8 \) yr.

However, the above mentioned discussion suggests that \( u \) is much smaller than \( v_w \). Thus the above upper limit on \( T_d \) is a large over estimate of the erosion time. Because \( u \) is necessarily larger than the local escape velocity \( v_{\text{esc}} = (2GM_w/R)^{1/2} \) in order for the matter flow in the boundary layer to become unbound from the star. Thus we find an upper bound on the mass loss rate from the disk due to the wind

\[
M_d \lesssim \left( \frac{v_w H}{v_{\text{esc}} R} \right)_{out} M_w ,
\]

\[
\lesssim 17 \left( \frac{v_w}{500 \text{km/s}} \right) \left( \frac{R_{out}}{50 \text{AU}} \right) \left( \frac{(H/R)_{out}}{0.2} \right) M_w ,
\]

assuming \( v_{\text{esc}} \ll v_w \). For an initial disk mass of \( M_d = 0.02 M_{\odot} \), \( (H/R)_{out} = 0.2 \), \( R_{out} = 50 \) AU, and \( M_w = \)
$10^{-9}M_{\odot}\text{yr}^{-1}$, the disk loss time is $T_d \geq \frac{M_d}{\dot{M}_d} = 1.2 \times 10^6 \text{yr}$. Equation (16) has a different dependence on $H/R$ and gives different values compared with the hydrodynamic estimates (Hollenbach et al. 2000).

6 CONCLUSIONS

We conclude that magnetocentrifugally driven winds of strongly magnetized, rapidly rotating T Tauri stars may affect the orbits of close-in giant planets that are inside the Alfvén radius of the wind. The magnetic wind may of course disperse the inner part of the disk and thereby halt the inward migration of a giant planet. However, this work focuses on the affect of the azimuthal ram pressure of the magnetized wind on the planet. This gives a torque which tends to increase (decrease) the planet’s angular momentum thereby causing it to move outward (inward) if the planet is at a distance $R$ larger (smaller) than the corotation radius of the star, $R_{\text{cr}} = (GM_*/\Omega_*^2)^{1/3} \approx 0.07 AU(P_*/5d)^{1/3}$ for a solar mass star, where $\Omega_* = 2\pi/P_*$ is the star’s angular rotation rate. Such winds are likely to occur in T Tauri stars where the thermal speed of the gas close to the star is small compared with the escape velocity, where the star’s magnetic field is strong, and where the star rotates rapidly. The magnetic winds are expected to be important for giant planet migration for cases where the star’s magnetic field is a multipolar field rather dipolar. For an approximately aligned dipolar magnetic field the wind is directed away from the equatorial plane and a magnetospheric cavity forms. (Lin et al. 1996; Romanova & Lovelace 2006). We find that the time-scale for the magnetic wind to change the planet’s orbit $T_w$ ranges from 2–20 Myr for stellar rotation periods of 3–5 d and surface magnetic fields of 1–3 kG. This time-scale is of the order of or longer than the estimated spin down time of the star $T_{\text{sd}}$ due to the magnetic wind.

The migration of a close in giant planet due to the star’s wind may be strongly affected by the tidal interaction. The time-scale for the planet to establish tidal locking $T_{\text{tidal}}$ of the entire star is estimated to be longer than the age of most systems. However, locking of the convective envelope of the star may occur on a much shorter time-scale. This time scale varies as the sixth power of the distance to the planet. Once locking of the stars surface occurs, the influence of the wind on the planet’s orbit vanishes.

Compared with the magnetospheric cavity model proposed for halting the inward migration of planets (Lin et al. 1996; Romanova & Lovelace 2006), the influence of the magnetic wind may allow for a broader distribution of distances of exosolar planets (Pont 2007).

The influence of the magnetic wind in eroding and eventually destroying the accretion disk is analyzed. A momentum integral for the boundary layer is derived and used to estimate the disk erosion time-scale $T_d$. For the considered conditions we find $T_d \sim 1 - 10^5\text{ Myr}$ with the lower value favored.

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Figure 6. The right-hand part of the figure shows the control volume abcd discussed in the text. The left-hand side of the figure shows the vertical profiles of the density $\rho$ and radial velocity $v_R$ in the boundary layer. Here, $\rho_d$ is the density at the surface of the disk, and at $z = H$, $v_R = v_w$.  

Compared with the magnetospheric cavity model proposed for halting the inward migration of planets (Lin et al. 1996; Romanova & Lovelace 2006), the influence of the magnetic wind may allow for a broader distribution of distances of exosolar planets (Pont 2007).

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