Optical microcavities are typically microscale dielectric structures that trap light to exploit total internal reflection. As resonators, they support whispering gallery modes (WGMs) with a high quality ($Q$) factor and small mode volume.\textsuperscript{1,2} Whispering gallery microcavities have garnered considerable interest owing to their practical applications as filters, modulators, sensors, and lasers.\textsuperscript{3-5} In recent decades, WGM resonators have been molded into a wide variety of geometries, ranging from the simplest ones, i.e., the microspheres,\textsuperscript{6} microdisks,\textsuperscript{7} and microrings,\textsuperscript{8} to more exotic shapes, such as microbottles\textsuperscript{9-11} and microbubbles,\textsuperscript{12-14} each with their own specific advantages and disadvantages.

Recent years have seen the design and fabrication of many innovative sensors based on WGM microcavities for measuring the physical, chemical, and biological properties of the surrounding environment. Compared with other WGM resonators, such as microspheres, microdisks, and microrings, the main advantage of hollow WGM microcavities is that the samples can be confined inside the resonator, rather than outside, which ensures that the outer surface — where total internal reflection occurs — always remains in air. This markedly extends the tuning range of the sensor.\textsuperscript{15,16} Hollow whispering gallery microcavities have already exhibited remarkable potential for optical component integration and cost-effective mass manufacturing.\textsuperscript{17,18} Many researchers have applied hollow microcavities experimentally, but very few have explored the phenomenon whereby the core refractive index (from $n_{\text{core}} < n_{\text{wall}}$ to $n_{\text{core}} > n_{\text{wall}}$) can be adjusted to change the WGM resonance characteristics.

This paper reports a hollow glass microsphere (HGM) with nanoscale wall thickness for use as an optical whispering gallery microcavity. We numerically simulated the core refractive-index cross-sectional electric-field distributions of the HGM and theoretically analyzed the transfer of the WGM resonant position from the wall to the core at the jump point. We opened a microhole on the surface of the HGM, which is filled with liquids of varying refractive index, and observed whispering gallery microcavity resonances as the jump point ($n_j$), the refractive-index sensitivity, $Q$ value, and free spectral range (FSR) of the WGM resonance spectrum of the HGM filled with liquids of different refractive indices differed markedly after the jump point. WGM resonance is formed by continuous total internal reflections at the curved boundary between the cavity and surrounding medium.\textsuperscript{15} When the HGM core has a higher or lower refractive index than the wall (that is, from $n_{\text{core}} < n_{\text{wall}}$ to $n_{\text{core}} > n_{\text{wall}}$), total internal reflection may occur in different positions. Additionally, in the case of a thin wall, the WGM resonances induced by the wall surface and the WGM resonances induced by the core surface differ; thus, the WGM resonant properties are changed. The core refractive index has a certain value ($n_j$), which forms the jump point for WGM resonance transfer. We performed a series of numerical simulations and experiments to verify the transfer of the WGM resonant properties inside the HGM with cores of different refractive indices at the jump point.

The numerical simulations were performed using COMSOL Multiphysics. The model consists of a hollow spherical shell surrounded by air, and the core was assigned different refractive indices. To ensure the accuracy of the numerical simulation, we examined the WGM electric-field distributions for cores of different refractive indices in the HGM by tracking the same angular mode number. Because the resonant wavelength of the same angular mode number changed with respect to the core refractive index, the resonance wavelengths for different refractive indices at the same angular mode number must be calculated. For simplicity, we consider transverse-magnetic (TM) modes only. The resonant wavelength of a TM WGM with the mode number $l$ is determined by the following system of equations:\textsuperscript{19,20}

$$S_{23} \frac{\chi'(z_3)}{\chi'(z_4)} = C \frac{\psi(z_3)}{\psi(z_4)}$$

and

$$C_l = S_{12} \psi(z_1) \chi'(z_2) - \psi(z_1) \chi'(z_3)$$

where $z_1 = n_1 k_r 1$, $z_2 = n_2 k_r 2$, $z_3 = n_3 k_r 3$, and $z_4 = n_4 k_r 4$, $S_{12}$ is the refractive indices of the core, wall, and air, respectively. $r_1$ and $r_2$ are the inner and outer diameters of the HGM, $\psi(z) = j_l(z)$ and $\chi(z) = n_l(z)$, where $j_l$ and $n_l$ are the spherical Bessel and Neumann functions, respectively. There is a series of $\lambda$ values that satisfy Eqs. (1) and (2) for a specific $l$. In the increasing order of $\lambda$, which are known as the 1st, 2nd, $\ldots$, and $l$th orders of the WGM.
To ensure the accuracy of the numerical simulation, we used different concentrations of the glycerol aqueous solution in the HGM core to change the core refractive index. \(^{21,22}\) Figure 1(a) shows a section view of the space electric-field distributions inside the HGM with outer diameters and wall thicknesses of 90 \(\mu\)m and 892 nm, respectively (angular mode number 255). We set the refractive indices of the air, wall, and core as 1, 1.45, and 1.43 (glycerol concentration 67.58\%), respectively. WGM resonance formed at the equatorial plane of the spherical shell (resonant wavelength of 1,536.90 nm). We set the wall and air refractive indices to remain unchanged and changed the refractive index of the core from 1.43 to 1.47 (corresponding glycerol concentrations of 67.58, 74.54, 81.51, 88.48, and 95.45\%) to obtain a series of section views of the electric-field distributions inside the HGM [Fig. 1(b)]. We found that as the core refractive index increases, the main energy in the HGM inside the HGM \[\text{Fig. 1(b)}\]. We investigated the influence of liquids of various refractive indices filling the HGM on the WGM resonance. A schematic of our experimental setup is shown in Fig. 3(a). The light output from the amplified spontaneous emission (ASE) light source (wavelength range: 1,525 to 1,570 nm) and the tunable laser source (resolution of 1 pm, centered at 1,550 nm) was coupled to the optical fiber through a optical fiber couple (OFC) and connected to a 1–2-\(\mu\)m tapered fiber \[\text{Fig. 3(a), inset}\] via a polarization controller (PC). The PC was used to effectively excite the TM mode. The transmission signal at the output ports of the fiber taper was connected to a 50/50 inline beam splitter (BS). One output of the BS was sent to a digital oscilloscope (DSO) via a photodiode (PD), and the other one was connected to an optical spectrum analyzer (OSA) with a minimum resolution of 0.02 nm. The tapered fiber was fabricated using a flame-heated drawing technique. The positions of the liquid-filled HGM and the fiber taper were controlled using a three-dimensional \(X-Y-Z\) stage to ensure a high coupling efficiency. A 9-\(\mu\)m-diameter capillary \[\text{Fig. 3(a), inset}\], which was also prepared via the flame-heated drawing, was connected to a micro-injection pump, which we used to control the air pressure inside the capillary and achieve liquid injection and extraction, as well as other operations.

Figure 3(c) shows a micrograph of the HGM filled with liquid that was injected through a capillary. The liquid did not leak, owing to its surface tension and the size of the microhole. The inside of the HGM was completely filled with the liquid, and there were no microbubbles of air inside the hollow cavity. Figure 3(b) shows the typical WGM resonance spectrum of (diameter \(\approx 90.4 \mu\)m) the liquid-filled HGMs \((n_{\text{core}} = 1.434 \text{ at } 25^\circ C\), refractive-index matching liquids, Cargille Lab) received by an oscilloscope. The corre-
sponding Q-factor (defined as \( Q = \lambda / \Delta \lambda \), where \( \Delta \lambda \) is the linewidth of the peak) was 0.565 \( \times 10^5 \).

We needed to repeatedly fill liquids into the same HGM for the purposes of our experiment. First, we siphoned the liquid form the HGM by simply manipulating the micro-injection pump status. We used alcohol to flush the residual liquid inside the HGM and then allowed the inside of the HGM to dry. We investigated the shift in the WGM spectra as the HGM was cleaned repeatedly, as shown in Table I. We found that the WGM resonance wavelength and FSR recovered to the state with no injected liquid (that is, the effect of the residual liquid was eliminated) once the liquid-filled HGM was cleaned three times.

Traditionally, there is a strong evanescent electromagnetic field on the surface of the WGM microcavity. Any slight variation in the refractive index near the WGM microcavity introduces either a change in the linewidth or a resonant-wavelength shift.\(^2\) Any variations in the refractive index of the HGM core also lead to a change in the FSR and a resonant-wavelength shift. We performed a systematic study on the changes in the WGM resonance after the jump point via experimental analysis and theoretical calculation.

We designed an experiment based on the HGM filled with liquids of different refractive indices to study the characteristics of the WGM resonance at the jump point. During the experimental process, for real-time monitoring of the transmission spectra, the input port was connected to the ASE, and the output ports of the fiber taper were connected to an OSA. Figure 4(a) shows the transmission spectra of the HGM (diameter \( \approx 84.0 \mu m \)) filled with liquids of different refractive indices. The angular mode number \( l \) of the WGM can be expressed as \( l = \lambda_{l+1}/(\lambda_l - \lambda_{l+1}) + 1 \), where \( \lambda_l \) and \( \lambda_{l+1} \) are the wavelengths of two successive modes with the same polarization.\(^2\) The \( l \) values are calculated as 253–255. By tracking the same angular mode number (for modes TM\(_{254}^1\) and TM\(_{255}^1\)), we found that the WGM resonance wavelength spectra redshifts as the liquid-core refractive index increases. Figure 5(a) shows the transmission spectra of an HGM filled with liquids of different refractive indices. (b) Resonant wavelengths (for modes TM\(_{254}^1\) and TM\(_{255}^1\)) of the liquid-filled HGM with respect to the liquid-core refractive index.

We also found that the FSR of the WGM spectrum of the HGM filled with liquids of different refractive indices exhibited larger differences at the jump point. Figure 5(a) shows the transmission spectra of the HGM filled with liquids of three different refractive indices; there are considerable FSR variations in the WGM resonance among them. Figure 5(b) shows the FSR and \( Q \) values of the WGM resonance with respect to the core refractive index; the FSR remains stable as the core refractive index increases before the jump point but significantly decreases after the jump point. In addition, the \( Q \) values change significantly at the jump point: when the WGM resonance moves from the HGM wall to the liquid-core surface, the increase of the scattering loss and liquid absorption loss reduces the \( Q \) value.

### Table I. WGM resonance with respect to the HGM cleaning time.

|            | Resonance wavelength (nm) | FSR (nm) | \( Q \)-value |
|------------|---------------------------|----------|---------------|
| Air-filled HGM | 1546.07 (TM\(_{257}^1\)) | 5.81     | \( 8.48 \times 10^5 \) |
| Liquid-filled HGM | 1549.87 (TM\(_{257}^1\)) | 5.85     | \( 5.42 \times 10^5 \) |
| First cleaning HGM | 1546.09 (TM\(_{257}^1\)) | 5.82     | \( 8.13 \times 10^5 \) |
| Second cleaning HGM | 1546.07 (TM\(_{257}^1\)) | 5.81     | \( 8.42 \times 10^5 \) |
| Third cleaning HGM | 1546.07 (TM\(_{257}^1\)) | 5.81     | \( 8.45 \times 10^5 \) |

![Figure 4](image1.png)

**Fig. 4.** (a) Transmission spectra of an HGM filled with liquids of different refractive indices. (b) Resonant wavelengths (for modes TM\(_{254}^1\) and TM\(_{255}^1\)) of the liquid-filled HGM with respect to the liquid-core refractive index.

![Figure 5](image2.png)

**Fig. 5.** (a) Transmission spectra of the HGM filled with liquids of three different refractive indices. (b) FSR and \( Q \) values of the WGM resonance with respect to the core refractive index.
We then verify the variation of the WGM resonance of hollow spheres with cores of different refractive indices via theoretical calculation. The resonant wavelength \( \lambda \) for the cores of different refractive indices was calculated using Eq. (1), as shown in Fig. 6(a). We found that the refractive-index sensitivity of the theoretical calculation is higher than that of the experiment [Fig. 4(b)]. This is mainly because some liquid remains inside the HGM during the cleaning process. In addition, the HGM wall thickness is not fixed, which contributes to the sensitivity difference.

Next, we investigate the theoretical resonance transfer with cores of different refractive indices for different wall thicknesses \( t \) and HGM diameters \( D \), as shown in Figs. 6(c) and 6(d), respectively. In Fig. 6(c), it is observed that when the HGM diameter (84 \( \mu \)m) and angular mode number (254) are constant, the sensitivity decreases as the wall thickness increases. When the refractive index of the core is equal to that of the wall \( (n_{\text{core}} = 1.45) \), the liquid-filled HGM can be considered as solid microspheres with a uniform refractive index; thus, the WGM resonance wavelengths of hollow spheres with different wall thicknesses are the same at 1.45. In Fig. 6(d), it is observed that when the wall thickness (1.75 \( \mu \)m) is constant, the sensitivity decreases as the HGM diameter decreases. We found that when the wall thickness and HGM diameter are 1.75 and 83.96 \( \mu \)m, respectively, the theoretical results agree well with the experimental results. However, owing to the transfer of the WGM resonance characteristics at the jump point \( (n_{j}) \), the refractive-index sensitivities for Figs. 4(b) and 6 exhibit an obvious jump around 1.45.

The FSR of the WGM can be expressed as \( \text{FSR} = \frac{\lambda^2}{2n_{\text{eff}}r} \), where \( \lambda \) is the resonant wavelength, \( n_{\text{eff}} \) is the effective refractive index of the HGM, and \( r \) refers to the radius of the WGM resonance.\(^{2,24} \) The effective refractive index of the WGM resonance in the HGM is strongly dependent on the refractive index of the fluid in the core, as well as the thickness of the wall.\(^{25} \) In the experiment, we used the same HGM to ensure a consistent wall thickness; thus, \( n_{\text{eff}} \) is only related to the core refractive index. \( n_{\text{eff}} \) is defined as follows: \(^{1,26} \)

\[
  n_{\text{eff}} = \frac{\int n(r)E(r)^2 \, dr}{\int E(r)^2 \, dr}. \quad (3)
\]

The refractive index of the core and wall is a fixed value after the HGM is filled with liquid. After the above simplifications, Eq. (3) becomes

\[
  n_{\text{eff}} = \frac{E_{\text{core}}^2}{E_{\text{total}}^2} \cdot n_{\text{core}} + \frac{E_{\text{wall}}^2}{E_{\text{total}}^2} \cdot n_{\text{wall}} + \frac{E_{\text{air}}^2}{E_{\text{total}}^2} \cdot n_{\text{air}}, \quad (4)
\]

where \( E_{\text{core}}, E_{\text{wall}}, \) and \( E_{\text{air}} \) are the energy of the core, wall, and air. Figure 6(b) shows the results of the \( n_{\text{eff}} \) and the FSR formula calculation (angular mode number 255), which coincide with the experimental results. When \( n_{\text{core}} < n_{\text{wall}} \), the simulation results show that the WGM resonance occurs on the wall of the HGM. The effective refractive index \( n_{\text{eff}} \) and radius \( r \) (HGM radius) remain almost unchanged as the core refractive index increases; thus, the FSR remains unchanged. When \( n_{\text{core}} > n_{\text{eff}} \), as the core refractive index increases, \( n_{\text{eff}} \) increases, and \( r \) (liquid-core radius) remains unchanged; thus, the FSR decreases. Owing to the decrease in the FSR after the jump point, the refractive-index sensitivity of different angular mode numbers exhibits larger differences after the jump point.

In conclusion, the WGM resonance mechanism of the HGM was unambiguously verified via numerical simulation. The WGM resonance moves from the HGM wall to the core because of the changes in the core refractive index. Owing to the transfer of the WGM resonance characteristics at the jump point \( (n_{j}) \), the refractive-index sensitivity, \( Q \) value, and FSR of the WGM resonance spectrum exhibit an obvious jump, which is consistent with the theoretical calculation results. In principle, this mechanism can be extended to higher-quality hollow microcavity resonators. In addition, the liquid-filled HGM exhibits potential for highly sensitive sensing applications, and the maximum sensitivity reaches 259.5 nm/RIU.

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