Phase Transitions in the Early Universe
(Is there a Strongly First Order Electroweak Phase Transition?)

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Abstract. After some introductory remarks about the prospects of first order phase transitions in the early universe, we discuss in some detail the electroweak phase transition. In the standard model case a clear picture is arising including perturbative and nonperturbative effects. Since in this case the phase transition is not strongly first order as needed for baryogenesis, we discuss supersymmetric variants of the standard model, the MSSM with a light stop $R$ and a NMSSM model where this can be achieved. We conclude with some remarks about the technical procedure and about possible effects of a strongly first order electroweak phase transition including baryogenesis.

1. Introduction: Phase Transitions at High Temperature

As also witnessed by this conference, there is a very fruitful connection between cosmology and elementary particle physics: Accelerator experiments and our theoretical understanding of elementary particle processes

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constitute a solid basis for detailed calculations of effects in the early universe up to temperatures $T \sim 100$ GeV - 1 TeV of the order of the weak scale, which is the borderline of our present understanding. On the other hand, cosmological considerations restrict possible extensions beyond the standard model which most of us feel to be necessary to finally obtain a theory including Planck scale physics.

The Universe expands and cools down, and just like in terrestrial experiments with alloys, vapor-liquid systems, superconductivity etc. we expect phase transitions to occur: relativistic kinematics would be involved and they could be very rapid. It is a most interesting question whether such phase transitions specific for a certain set of fields and couplings have left traces in our universe and whether we perhaps can select between the various “beyond the standard” models.

Phase transitions can be discussed in a simple way by inspection of a temperature $T$ dependent (Ginzburg-Landau type) effective potential in some order parameter (field) $\varphi$: a change in the minimum of $\varphi$ due to some thermal mass term $\sim T^2 \varphi^2$ from $<\varphi>=0$ at large $T$ to $<\varphi>\neq0$ at $T<T_c$ leads to a spontaneous symmetry breaking if $\varphi$ carries some charge, and signals a phase transition.

The standard model (SM) of elementary particle physics divides into Quantumchromodynamics (QCD) for the interaction of colored quarks and gluons with a scale of $\sim 100$MeV and into the Electroweak Theory (EWT) for flavored quarks, $W$ – $Z$ vector bosons and the Higgs boson(s) with a scale $\sim 100$ GeV. The convergence of running gauge couplings and perhaps the need for baryon number violation point towards some Grand Unified Theory (GUT) at a scale $\geq 10^{15}$ GeV which should also be brought into some connection with a model of inflation to fulfill the demands of cosmologists. In table 1 we have listed these three gauge theories together with the relevant symmetries and order parameters.

| QCD | Electroweak Theory | GUT/Inflation |
|-----|--------------------|---------------|
| Scale | 100 MeV | 100 GeV | $10^{15}$-GeV |
| Symmetry | chiral symmetry | $SU_L(2) \times U_Y(1)$ | $SU(5), SO(10), E_6$ |
| Order parameter | $<\bar{\psi}_q \psi_q>$ | $<H>= \left( \begin{array}{c} \varphi \\ 0 \end{array} \right)/\sqrt{2}$ | $<X_{GU-repr.}>$ |
|           |                   |               | $<\text{Inflaton}>$ |
There may be also some further intermediate scales connected to $U(1)'$ 
$SU(2)_R$ gauge interactions.

What could be the observable effects of such phase transitions? We can think of

- Primordial Density Fluctuations (PDF) leading to fluctuations in the microwave background and in the galaxy distribution
- Deformations of PDF
- Gravitational waves
- Generation of the baryon asymmetry $(n_B - n_B)/n_\gamma \sim 10^{-10}$ after inflation. According to Sakharov this requires besides baryon number violation C, CP violation and nonequilibrium. Thus only GUT’s and the electroweak theory are to be discussed in this point.
- Large scale magnetic fields
- $(n_B - n_B)$ fluctuations.

In the following we will concentrate on the most violent first order PT’s involving the condensation of critical bubbles of the new phase after some supercooling- similar to the condensation of bubbles of liquid from vapor, the bubble expansion- and the coalescence of bubbles.

Thus we do not dwell on the also very interesting phenomenon of defect formation, in particular of cosmic strings. For this a first order PT is not mandatory: The expanding universe provides a particle horizon and the size of topological defects (Kibble mechanism), and also some nonequilibrium. Of course, quasi-stable configurations (topological configurations, Q-balls are the most interesting possibilities.

QCD in itself is a perfect theory, its field content and couplings are fixed. Thus the question what happens if a QCD plasma is heated up, is well posed and should have a clear answer. Unfortunately, this is not the case up to
now. The reason is of course that QCD is mostly nonperturbative and that this is even more pronounced in thermal QCD. Lattice calculations are an appropriate tool. But particularly because fermion fields cause problems on a lattice, many things still have to be clarified [11]. In fig. 1 we sketch the phase diagram [12] according to our present understanding, for the case of 2+1 flavors ($m_u = m_d, m_s$). For very small $m_u = m_d$, the critical temperature $T_c$ turns out to be of the order of the physical strange mass $m_s$. Thus it is not clear at all [13] if there is a first order PT and, in case there is one, whether it is strong. In phenomenological models (MIT bag model, chiral $\sigma$-model, Nambu-Jona-Lasinio-type models) there seems to be a tendency to overestimate the strength.

If there would really be a first order PT, interesting effects could be spelled out: The speed of sound would go to zero in course of the transition, and the primordial density fluctuations would be deformed [14] with very dangerous consequences for primordial nucleosynthesis. However peaks in the hadron-photon-lepton fluid are argued [14] to be wiped out during neutrino decoupling. Also black holes of about a solar mass could be formed [15] but this is controversial [14]. Presumably the PT with a chemical potential (not included in fig. 2) is much more important [16], not so much in cosmology, but in dense nuclear matter physics discussed for heavy ion collisions and neutron stars.

Grand Unified Theories (GUTs) and inflation should be dealt with together [17] because there may be no separate GUT-PT after inflation: The reheating temperature after inflation might be too small for the production of GUT-particles, and they might only be produced in a preheating period [18]. Inflation in its different scenarios, besides being the remedy of the well-known problems of standard cosmology, is also most interesting as a source for primordial density fluctuations leading to structure formation. The field content of such theories (except the postulate for a heavy neutrino?) is rather unclear: the nature of the inflaton field, the GUT-gauge symmetry and models of a hybrid inflation. We expect extreme supercooling for the inflation transition, an extremely relativistic situation, perhaps no thermal state before the PT and a complicated pre-heating process. Thus this would be anything else but a conventional PT. If the nonequilibrium situation during and after the phase transition (out of equilibrium decay...) is supposed to generate the baryon asymmetry, which is the well-known textbook scenario [19], one has to keep in mind that the $B + L$ violating electroweak interaction with an unsuppressed transition rate $\Gamma \sim (\alpha_w T)^4$ in the symmetric phase of electroweak matter in the equilibrium period at temperatures $T \gg 100$ GeV washes out a previously generated baryon asymmetry if $B - L = 0$. Thus one needs a $(B-L)$-violating GUT gauge interaction (e.g. $SO(10)$ [20], [21]).

Let us then consider in detail a PT in electroweak matter. We will argue
that it has a reliable theoretical description. As it turns out, the standard
electroweak theory does not provide a strongly first order PT but we will
see, that it can be achieved in supersymmetric extensions of the SM.

2. The Electroweak Phase Transition

The Electroweak Standard Model (SM) is so successful in explaining high
energy elementary particle processes because the gauge couplings $g_w, g'_w$
of standard $SU(2)_W \times U(1)_Y$ are small and infrared problems are tamed
by the Higgs mechanism, and because therefore perturbation theory works
very well. Still there are nonperturbative features predicted: the instanton-
induced B+L-violating interaction. However, this is an unmeasurable small
effect $\sim e^{-8\pi^2/g_w^4}$ unless it is amplified in multi-gauge boson production
[22]. This is not true anymore for large temperatures $T > T_c$, where the
B+L-violating thermal transition rate $T \sim (\alpha_w T)^4$ (recently the prefac-
tor was discussed intensively [23]) is unsuppressed. Thus for
B-L=0 an existing baryon asymmetry of the universe would be erased in
the equilibrium situation before the electroweak PT. The last chance to
create the observed baryon asymmetry would then be in course of a first
order electroweak PT [26, 27]. In the Higgs phase with $\langle \varphi \rangle = v(T)$,
the asymmetry should “freeze out”, i.e. the thermal sphaleron transition rate
should be sufficiently Boltzmann-suppressed $\sim e^{-\text{const} \, v(T)/T}$. Thus
one has to study the PT carefully. In particular, infrared (IR) effects are
important if the Higgs vev is reduced. They have to be properly taken
into account.

A (naive) 1-gauge boson (+ghost) loop calculation in a Higgs field back-
ground of the $T \neq 0$ effective potential $V(\varphi^2, T)$ (free energy) is easily
performed [24]. Substitute Matsubara frequencies $2\pi n T$ in the log

\[ \int \frac{d^4p}{(2\pi)^4} \log(p^2_0 + \vec{p}^2 + \frac{1}{4}g_w^2 \varphi^2) \]

\[ \Rightarrow T \sum_n \int \frac{d^3p}{(2\pi)^3} \log((2\pi n T)^2 + \vec{p}^2 + \frac{1}{4}g_w^2 \varphi^2). \quad (2.1) \]

After renormalization (as at $T = 0$) the $n \neq 0$ modes produce a function $\sim
T^4 f \left( \frac{m^2}{T^2} \right)$ with $m^2 = \frac{1}{4}g_w^2 \varphi^2$ whose expansion gives the well-known positive
thermal mass term $\sim \varphi^2 T^2$. The $n = 0$ mode belongs to a 3-dimensional
theory without time. After renormalization it contributes $-ET(\varphi^2)^{3/2}$ to
the effective potential; this is the famous $\varphi^3$ term. This term would make
the PT first order (fig. 2), i.e. we would have two degenerate minima of the
potential at the critical temperature $T = T_c$. However, we should not trust
this result because it completely neglects IR effects. Its gauge dependence
A proper way to discuss IR effects \[28\] - \[31\] is to first integrate out all nonzero Matsubara frequencies of the theory including all fermions (with \( n = 1/2, \ldots \)). One thus reduces the action to a 3-dimensional one. In a second step also the longitudinal gauge bosons with Debye mass \( m_D \sim g_w T \) can be integrated out. Here “integrating out” means matching a set of static 4-dimensional amplitudes containing the above modes in the loop to a 3-dimensional truncated Lagrangian for the Higgs and transversal gauge boson zero modes:

\[
L_{\text{eff}}^{3-\text{dim.}} = \frac{1}{4} (F_{ik}^a)^2 + (D_i \phi_3) (D_i \phi_3) + m_3^2(T) \phi_3^\dagger \phi_3 + \lambda_3(T)(\phi_3^\dagger \phi_3)^2. \tag{2.2}
\]

This Lagrangian then contains all the IR problems whereas the first step can be done in (two-loop) perturbation theory without problems. The truncation of, e.g., terms like \((\phi_3^\dagger \phi_3)^3\) gives only an error of a few percent (\(0(g_3^2)\)) \[29\]. The theory (2.2) is superrenormalizable. Its dimensionful variables, the gauge coupling \( g_3^2 \) (\(T\)), and \( m_3^2 \) (\(T\)), \( \lambda_3 \) (\(T\)) can be reduced to the dimensionless quantities

\[
y = \frac{m_3^2(T)}{g_3^2(T)} \quad x = \frac{\lambda_3}{g_3^2} \tag{2.3}
\]

\(y\) is \(\sim (T - T_c)\) and fixes the temperature, whereas \(x\) determines the nature of the phase transition. Of course, \(x, y\) characterize a whole class of models obtained by dimensional reduction. The most secure way to handle (2.2) is to put it on a lattice \[32\] \[33\]. There is only one scale \(g_3^2\), we have only three dimensions, and there are no fermions. Thus this is particularly safe. An alternative treatment would be in the framework of Wilsonian renormalization \[34\]. The results of such lattice calculations \[32\] \[33\] are:

Figure 2. First order phase transition effective potential

is strongly reduced in 2-loop order \[25\] but of course this does not dispense us from the search for possible nonperturbative effects.
Figure 3. The perturbatively calculated interface tension $\sigma$ (including $Z$-factor effect and gauge variations) vs. $x$ compared to lattice data (squares, triangles and circles) from references in the text.

(i) There is a first order PT for $x \lesssim 0.11$ and there is a second-order PT at the endpoint $\boxed{35}$. Above $x = 0.11$ one has a crossover – there is no PT anymore.

(ii) Comparing the 2-loop perturbative expressions obtained from (2.2) with lattice results, there are deviations for $x \gtrsim 0.05$ especially for the interface tension (fig. 3) (from [36], lattice points from ref. [32, 37, 38]).

(iii) $v(T_c)/T_c = \varphi_{\min}(T_c)/T_c \gtrsim 1$ for $x \gtrsim 0.04$.

To protect a previously generated baryon asymmetry in a universe with $B - L = 0$ from erasure by sphaleron transitions $\sim \exp(-A\nu(T)/T)$ in a thermodynamic equilibrium period inside the Higgs phase one needs $v(T_c)/T_c \gtrsim 1$. With $x = \frac{1}{8}m_H^2/m_W^2 + c_{P\text{os}}m_W^2/m_W^4$ where the second term alone is $>0.04$ for the observed top mass $m_t$, this can never be achieved in the SM, independent of the Higgs mass. Together with its CP-violating effects being smaller than needed for an asymmetry production, this prevents the SM from explaining the baryon asymmetry of the universe.

Lattice results give a clear picture for the phase diagram for the Lagrangian (2.2). However, for some questions like sphaleron action, shape and action of the critical bubble – an explicit effective (coarse-grained) action would still be useful. It is also very important to have some (semi)analytic picture which tells us where one can trust perturbation theory and where not. This will be particularly true in the case of more complicated effective actions where lattice results may not be available.
Figure 4. 1-loop graph contributing to the potential $V(\varphi^2, <g_3^2 F^2>)$.

Thus we shortly discuss such a model \[29\].

In the hot symmetric phase with background $\varphi = 0$ the Lagrangian \[22\] describes a 3-dimensional QCD-type theory with scalar Higgs “quarks”. Lattice calculations \[33\] show that indeed in this phase static “quarks” experience a constant string tension which furthermore is approximately equal to that of pure SU(2)-Yang-Mills theory. This hints to a nonperturbative dynamics dominated by “W-gluons”. Also a spectrum of correlation masses of gauge-invariant $H\bar{H}$ bound states and of $W$-glueballs has been calculated on the lattice \[41\]. The former is compatible with a linearly rising potential in a relativistic bound state model \[42\] (like that of Simonov in 4-dimensional QCD \[43\]). There is only a small mixing with the W-gluons \[41\] in agreement with the suggestion above that we have pure “W-gluon” dynamics.

An interesting phenomenological description of the QCD vacuum is the “stochastic vacuum model” of Dosch and Simonov \[44, 45\]. Its main virtue is that it leads very naturally to the area law of confinement. We have applied it to the 3-dimensional theory \[2.1\] with an $SU(2)_W$ gauge group. Its main ingredient is a correlated gauge field background with a purely Gaussian correlation

$$\langle g_3^2 F_{ia}^\alpha(x') F_{ia}^\alpha(x) \rangle = \langle g_3^2 F^2 \rangle D\left(\frac{(x-x')^2}{a^2}\right)$$ \hspace{1cm} (2.4)

This correlator is already simplified by choice of a coordinate gauge and by averaging over the tensor structure. $\langle g_3^2 F^2 \rangle$ is the normalization by the usual local gauge field condensate and $D(D(0) = 1)$ is a form factor containing the correlation length $a$. The correlator has been tested in 3-dimensional lattice calculations \[46\] and the correlation length was obtained as $a \sim 1/0.73g_3^2 \sim 2/m_{\text{glueball}}$. In ref. \[39\] we presented strong indications that the $<g_3^2 F^2>$ ground state is unstable (similar to the Savvidy instability of QCD) for small Higgs vevs. Thus one obtains nonperturbative effects by a fluctuating gauge field background of the type \[2.4\].
Figure 5. Sketch of the potential $V(\varphi^2, <g_3^2 F^2>)$ in $F^2$-direction for two different values of $\varphi^2$.

One can estimate the effect of such a background on the W-boson (and ghost) loop leading to the 1-loop effective potential $V(\varphi^2, <g_3^2 F^2>)$ (fig. 3). We found two contributions to a momentum-dependent effective ("magnetic") mass:

(i) an IR regulator mass $m_{\text{conf}}^2(p^2, \varphi^2, <g_3^2 F^2>)$ of gauge bosons and ghosts due to the string tension (area law) which cures the IR problems of perturbation theory.

(ii) a negative effective (mass) $-\tilde{S}_F(p^2, \varphi^2, <g_3^2 F^2>)$ due to spin-spin forces which becomes important for larger $p^2$ ("paramagnetism") and does not spoil the nice IR properties of $m_{\text{conf}}^2$. If we introduce these masses in the 1-loop action (gauge boson loop) it has roughly the form

$$V(\varphi^2, <g_3^2 F^2>) \sim ... \int \frac{d^3p}{(2\pi)^3} \log[p^2 + \frac{1}{4}g_3^2\varphi^2 + m_{\text{conf}}^2(p^2, \varphi^2, <g_3^2 F^2>)] - \tilde{S}_F(p^2, \varphi^2, <g_3^2 F^2>)].$$

(2.5)

(This has to be corrected for combinatorics and also has to be renormalized). Both masses depend on $<g_3^2 F^2>$. Expanding (2.5) up to first order in $<g_3^2 F^2>$, the spin-spin force in $-\tilde{S}_F$ produces the well-known negative $F^2$-term destabilizing the $F^2 = 0$ vacuum. Adding the tree $\frac{1}{4}F^2$ we can obtain an effective potential sketched in fig. 5 stabilized at some value $F^2 \neq 0$ by confinement forces. This is a 1-loop calculation and the masses $m_{\text{conf}}^2$ and $-\tilde{S}_F$ are determined only roughly (in lack of lattice data support). Thus we have only a qualitative picture. To proceed, we fixed $<g_3^2 F^2>$ at the minimum by a relation to the lattice string tension.
Figure 6. $m_{\text{conf}}^2(p^2, m^2)$ and $\tilde{S}_F(p^2, m^2)$ in units of $(g_3^2)^2$ plotted for $m^2 = 0$.

Figure 7. Fading away of the first order phase transition with increasing $x = \frac{\Delta}{g_5}$, where $x_1 = 0.06$, $x_2 = 0.08$ and $x_3 = 0.11$. 
Fig. 6 [39] shows the qualitative form of \( m^2_{\text{conf}}(p^2, \varphi^2) \) and of \( \tilde{S}_F(p^2, \varphi^2) \). Fig. 7 [39] presents the new 1-loop potential at the critical temperature at various \( x \)-values, and one can see the first order PT fading away. One can also evaluate the interface tension (table) and roughly determine the endpoint of the PT by postulating that the effective \( \varphi^2 \) and \( \varphi^4 \) vanish at this (conformal) point with a second-order PT.

| \( x \) | \( \sigma \) | \( \sigma_{\text{perturbative}} \) |
|---|---|---|
| 0.06 | 0.016 | 0.013 |
| 0.08 | 0.004 | 0.007 |
| 0.11 | 0 | 0.004 |

We should stress again that this picture of nonperturbative effects is not really quantitative, in particular because 2-loop calculations in a correlated gauge-field background are (too) difficult. Still we might get an indication in which direction nonperturbative contributions go. In this context also the work [40] on subcritical bubbles should be mentioned. If the crossover can be described in this picture is an interesting question.

3. The MSSM with a “light” stop

Searching for modifications of the electroweak theory in order to obtain a strongly first order PT, one faces the by now sufficiently known situation that the success of the standard model is both blessing and burden. We do not have experimental hints which way to go. Supersymmetric theories have the well-known theoretical advantages. From a practical point of view, all one needs for a strongly first order PT is the strengthening of the “\( \varphi^3 \)”-term in the effective potential due to bosonic exchange in the loop. Thus one needs further bosons with a strong coupling to the Higgs. SUSY models have a host of new bosons in the superpartner sector. In particular, the \( s \)-top particles have a particularly strong Yukawa coupling \( h_t \) if the Higgs vev \( < v_2 > \) of the Higgs coupling to the top \( (m_{\text{top}} = h_t < v_2 >) \) is not very large, i.e., if \( \tan \beta = v_2/v_1 \) is not large. The superpartner of the right-handed top, the \( \text{stop}_R \), does not have \( SU(2)_W \) interactions, and thus is particularly flexible in its allowed mass (no \( \rho \)-parameter problem). As proposed in ref. [47, 48], its exchange (fig. 8) can enhance the PT significantly if its mass \( m_{\text{stop}} \) in the symmetric phase (including \( T^2 \)-plasma mass) is small:

\[
m_{\text{stop}}^2 = m_0^2 + cT^2, \tag{3.1}
\]

where \( m_0^2 \) is the SUSY-breaking scalar mass of the \( \text{stop}_R \). The \( T = 0 \) mass of the \( \text{stop}_R \) is

\[
m_{\text{stop}}^2 = m_0^2 + m_t^2 \tag{3.2}
\]
Figure 8. 1-loop stop contribution to the effective potential.

and is not much larger than the top mass for small positive $m_0^2$. A nonuniversal SUSY mass breaking at the GUT scale might be necessary for very small $m_0^2$ although the stop mass is naturally lowered by renormalization flow.

If the stop and one heavy combination of Higgses is integrated out, one is led again to a Lagrangian of the form (2.1), but now with an $x = \lambda_3/g_3^2$ value much smaller than in the SM (being bosonic, the stop contributes opposite to the top!) allowing for $m_H \lesssim 75$ GeV for a strongly first order PT with $v(T_c)/T_c \geq 1$ [50]-[53].

One can also ask [47] for stop masses smaller than the top mass taking $m_0^2 = -\tilde{m}_0^2$ negative in (3.1), (3.2). Then the stop than should not be fully (also zero modes) integrated out, but kept in the effective 3-dimensional theory together with the light Higgs fields. If one assumes that the CP-odd Higgs $A_0$ meson surviving spontaneous breaking is rather heavy ($\gtrsim 300$ GeV), there is a heavy Higgs sector to be integrated out, and just as above one Higgs field remains. Thus we have to consider a Lagrangian [51]

$$L_{eff}^{3-dim} = L_{eff}^{3-dim}(Higgs) + \frac{1}{4} G^A_{ij} G^A_{ij} + (D^i_i U)^+ (D^i_i U) + m_{U_3}^2 U^+ U + \lambda_{U_3} (U^+ U)^2 + \gamma_3 (\phi_3^+ \phi_3) (U^+ U) .$$

(3.3)

The $T$-dependent parameters are obtained by integrating out all non-zero modes and all heavy particles like in (2.2), which is the first part of the Lagrangian (3.3). Thus one has to specify the field content and the SUSY-breaking parameters of the model. The simplest choice is the minimal supersymmetric standard model (MSSM) [49] without universality for the top scalar SUSY-breaking masses. The partner of the left-handed top with a SUSY-breaking mass $m_{Q_3}^2$ should be heavy in order not to contribute too much to $\Delta \rho$.

Two-loop calculations with (3.3) have shown that one can indeed obtain $v(T_c)/T_c \gtrsim 1$ even for lightest Higgs masses as big as 105 GeV [54]. The
Figure 9. Latent heat at $T_c$ as a function of mass parameter $\tilde{m}_U$ calculated on the lattice [54] compared to the analytic results [52].

Figure 10. Three dimensional latent heat as a function of $x$ calculated from the potential (2.5) (full line) compared to the result of ordinary 1-loop perturbation theory (dashed-dotted line).
parameter space is enlarged if one allows for $\text{stop}_L$-$\text{stop}_R$ mixing with a parameter $A_t = A_t + \mu...$. (Both parameters $\mu$ and $A_t$ are important in the discussion of CP-violations in the wall.) Ref. uses an improved 4-dimensional one-loop effective potential at high temperatures and still agrees well with the special case considered in 

For large enough negative $m_{\tilde{U}}^2 = -\tilde{m}_{\tilde{U}}^2$, one even obtains a two-stage phase transition with an intermediate stop condensate $\langle \tilde{U} \rangle$. This is only acceptable if the transition rate which is rapidly decreasing with increasing $\tilde{m}_{\tilde{U}}^2$, still allows to return from the stop phase to the Higgs phase. In the former phase one has a situation analogous to the Higgs phase, in particular, there are massive $SU(3)$ gauge bosons.

Recent lattice calculations confirm the perturbative results surprisingly well (fig. 9) – though there are also significant deviations. In particular the PT turned out to be more strongly first order – the latent heat and $v(T_c)/T_c$ are larger than in the perturbative result. We can understand this effect qualitatively with our model for nonperturbative contributions: The effective $x$-value in the Higgs part of (3.3) is much smaller than in the standard model and for these values (fig. 10) the latent heat and $v(T_c)/T_c$ are both increased compared to pure perturbation theory. The important additional graphs coming from Lagrangian (3.3) mostly involve $SU(3)$ gluons and the $\text{stop}_R$ both of which do not have $SU(2)_W$ interaction, and hence also no nonperturbative effects on this scale.

4. NMSSM with a strongly first order phase transition [55]

In the effective electroweak potential near the critical temperature a term of type $-\varphi^3$ triggers a first order PT. Up to now we discussed the generation of such terms in 1-loop order of perturbation theory. There is also the possibility to obtain it already on the tree level. An $SU(2)_W$-invariant third-order polynomial term in the potential cannot just contain the Higgs(es). Thus one has to enlarge the field content of the SM and also of the MSSM in the case of a supersymmetric theory. The simplest extension of the MSSM, the “next to minimal model” NMSSM [58, 59], contains a further superfield $S$, which is a gauge singlet, in an additional piece of the superpotential

$$g^S = \lambda S H_1 H_2 - \frac{k}{3} S^3. \quad (4.1)$$

The soft SUSY breaking term

$$V^S = A_\lambda \lambda S H_1 H_2 - \frac{k}{3} A_k S^3 \quad (4.2)$$
has the desired \( \phi^3 \) form if \( S \) can be treated similarly to the \( H_i \). The superpotential \( (4.1) \) has the virtue of avoiding the \( \mu \)-term \( g^\mu = \mu H_1 H_2 \) with its fine-tuning problem because this term automatically arises after the singlet field acquires a vev. However, because of its \( Z_3 \) symmetry it suffers from the well-known domain wall problem \( (4.1) \). It turns out that the NMSSM with just \( (4.1) \) and \( (4.2) \) besides having the domain wall problem also is unable to produce a phase transition in \( \langle S \rangle \) and \( \langle H \rangle \) simultaneously, which requires \( \langle S \rangle \) and \( \langle H \rangle \) to be of the same order of magnitude. With a very large \( \langle S \rangle \) one would first obtain a PT in \( \langle S \rangle \) and afterwards the ordinary MSSM PT in some Higgs field combination, which is not what we want. We thus as in ref. \( [62] \) choose the superpotential

\[
    g = g^S + \mu H_1 H_2 - rS. \tag{4.3}
\]

Unlike in ref. \( [42] \) more than a decade ago, we now keep the full parameter space of the model only restricted by universal SUSY breaking at the GUT scale. In the latter we differ from ref. \( [62] \) where the parameters were fixed at the electroweak scale without such a criterion. Besides the well-known gauge couplings in the \( D \)-terms we then have the parameters \( \lambda, k, \mu, r \) in the superpotential, and for the SUSY breaking a universal scalar mass squared \( m_0^2 \), a common gaugino mass \( M_0 \), as well as an analytic mass term \( B_0 \) for the Higgses and a universal trilinear scalar coupling \( A_0 \) corresponding to the second and third power terms in the superpotential, respectively.

Besides the tree potential and 1-loop Coleman-Weinberg corrections we include 1-loop plasma masses for the \( H_i \) and \( S \) fields and the 1-loop \( \phi^3 \) terms discussed in previous chapters which, however, now in general are small compared to the tree term \( (4.2) \). The most important finite temperature contributions come from the top quark and the gauge bosons, but in some parts of the parameter space the stops, charginos and neutralinos may become rather light and therefore are also included in the effective potential \( V_T(H_1, H_2, S) \).

Having at hand the potential we are interested in, a rather natural procedure would be as follows: (Randomly) choose a set of the GUT scale parameters listed above. Then use the (1-loop) renormalization group equations \( (4.5) \) to evolve the parameters down to the weak scale and minimize the \( T=0 \) effective potential in order to study the electroweak symmetry breaking. Of course, in order to reproduce the physical \( Z \)-boson mass \( M_Z \), a rescaling of all the (unknown) dimensional parameters is necessary. But after this rescaling in almost all cases there appear some unobserved light particles in the spectrum, so one has to try the next set of parameters and this whole “shot-gun” procedure is very inefficient.

Instead, we fix the \( T=0 \) electroweak minimum determined by \( M_Z, \tan \beta = v_2/v_1 \) and \( \langle S \rangle \) in addition to the parame-
tters $\lambda, k, m_{0}^{2}, M_{0}, A_{0}$ while $\mu, r, B_{0}$ remain unspecified. The important thing is that the latter do not enter the 1-loop renormalization group equations for $\lambda, k$ and the soft parameters except $B$. Thus we can calculate all parameters of the effective potential at the weak scale except $\mu, r$ and $B$ which we determine by applying the minimization conditions

$$\frac{\partial V}{\partial H_{i}} = 0, \quad \frac{\partial V}{\partial S} = 0.$$

Because of the complicated 1-loop corrections these equations cannot be solved analytically, but an iterative numerical solution taking the tree level solution as starting values is possible. Of course, whether the postulated minimum $(M_{Z}, \tan \beta, < S >)$ is indeed the global minimum has to be checked explicitly and constrains the parameter space of the model. Using this procedure we are left with the seven parameters

$$\tan \beta, < S >, \lambda, k, m_{0}^{2}, M_{0}, A_{0}$$

which still contain a lot of freedom. Fortunately, not all parameters are equally important with respect to the strength of the PT: Of most interest are the gaugino mass $M_{0}$ and the trilinear scalar coupling $A_{0}$, as they determine the coefficients $A_{\lambda}$ and $A_{k}$ of the “$\varphi^{3}$”-terms in eq. (4.2). Therefore we will study the plane of these parameters while keeping the others fixed. To maximize the lightest CP-even Higgs mass $M_{h}$ $\tan \beta$ should be taken large while $\lambda$ should be kept small. As stated above, a strong PT can only be expected, if $< S > \sim M_{Z}$ which requires $k$ to be not too small because of $< S > \sim \frac{A_{0}}{k}$. The remaining parameter $m_{0}^{2}$ only influences the masses of the additional Higgs bosons which we have chosen to be heavy.

An example of a scan in the $M_{0} - A_{0}$ plane is shown in fig. (11) where we fixed the remaining parameters according to the remarks before as $< S > = 100$ GeV, $\tan \beta = 5$, $\lambda = 0.05$, $k = 0.4$ and $m_{0} = 200$ GeV. There are several constraints on the parameter space: First of all, the minimum postulated in the elimination procedure discussed above has to be the global minimum which leads to the lower bound on $A_{0}$ in fig. (11). To prevent the appearance of a chargino with mass smaller than 80 GeV the gaugino mass $M_{0}$ has to be larger than 100 GeV corresponding to the vertical line in the plot. Finally, we require the lightest Higgs mass $M_{h}$ to be larger than 65 GeV which leads to the upper bound on $A_{0}$ in fig. (11). Compared with the current LEP data on SM-like Higgs bosons this may seem to be a rather low value, but one has to keep in mind that the lightest “Higgs” state in

3 Additionally, we require the top quark mass $M_{top} = 175$ GeV which allows us to fix the top Yukawa coupling as a function of $\tan \beta$. All the other Yukawa couplings are neglected which is only justified in the regime $\tan \beta \lesssim 10$.

4 Note that this also implies an upper bound on the gaugino mass depending on the remaining parameters.
Figure 11. Scan of the $M_0-A_0$ plane where the remaining parameters are fixed. The full line surrounds the phenomenologically viable part of the parameter space. The dotted lines are curves of constant lightest Higgs mass (75 and 85 GeV). The dashed line indicates the region where the lightest Higgs is predominantly a singlet. The dashed-dotted line separates the regions of strong ($v_c/T_c \gtrsim 1$) and weak PT.

this model always has some singlet component which even dominates in the region above the dashed line. Therefore the experimental constraints on $M_h$ are somewhat relaxed.

In order to investigate the strength of the PT we determine the critical temperature $T_c$ at which there exist two degenerate minima in $V_T(H_1,H_2,S)$, a broken minimum with $<H_i> \neq 0$ and a symmetric one with $<H_i> = 0$. For the previously discussed set of parameters the results are summarized in fig. (11). There the dashed-dotted line separates the region with a weak PT from the region where the baryon number washout criterion $v_c/T_c \gtrsim 1$ is fulfilled. One clearly sees that most of the parameter space is indeed compatible with electroweak baryogenesis. Interestingly enough, the region in which the Higgs mass is maximized ($M_h \sim 90$ GeV) is not excluded. Let us again stress that the situation drastically changes if we increase the singlet vev to e. g. $<S> = 300$ GeV while decreasing $k$ in order to obtain similar values of $M_h$. Then only a small range of val-

5 The singlet vev is different from zero even in the symmetric minimum.
ues of \( A_0 \) just above its lower bound allows a strong PT and most of the parameter space leads to erasure of the baryon asymmetry.

In the previous example the maximal value of the Higgs mass is 90 GeV but one can reach much higher values. By choosing \( \tan \beta = 10 \), \( M_h = 100 \) GeV can be obtained and still \( v_c/T_c > 1 \) can be fulfilled. Increasing the singlet vev to e.g. \( < S > = 250 \) GeV allows the even larger value of \( M_h = 115 \) GeV without violating the washout criterion. But with larger \( < S > \) the amount of fine-tuning of \( A_0 \) increases and there is the danger of metastability since the PT requires thermal tunneling over a rather high tree barrier.

5. Discussion

Having obtained some variant of the electroweak SM with a strongly first order PT, the way is free for the discussion of baryogenesis, of a lot of questions both on the conceptual, and on the technical side [66]-[68].

(i) The procedure of the PT can be worked out in detail:

- First one has to find the wall profile of the critical bubble (and later on of the stationary expanding bubble) in general in multidimensional field space (\( H_{1,2}, S, \text{stop}, \text{CP-violating angles...} \)). This is a very demanding numerical problem which has only been attacked recently.

- Given the wall profile, one can calculate the action, and one can calculate the transition rate using Langers’s formula, discuss supercooling, and obtain the nucleation temperature (1 bubble/universe).

- The interaction of the bubble wall with the hot plasma constitutes friction; this determines the stationary velocity of the wall in the heat bath [66]-[70]. Deflagration with velocity \( v_B \) smaller than the velocity of sound in the plasma seems to be favored. The particle mean free path usually turns out to be smaller than the thickness of the wall (“thick wall”).

- In front of the proceeding wall there is thermodynamic nonequilibrium and transport. Hydrodynamics and Boltzmann equations come into play.

- After some time many expanding bubbles have formed and collide. This finally leads to the new (Higgs) phase in the whole space and to some reheating. This is all beset by technical problems; but there are interesting proposals: \( v(T)/T \) might be lowered in the colliding bubbles [71], (seed) magnetic fields may be formed via turbulence [72, 80] or by the Kibble mechanism [81, 82]. The scale of the magnetic
fields seems to be too small to explain the observed magnetic fields, but there may be some enhancement.

(ii) If for some reason fluctuating primordial hypercolor magnetic fields ($U_y(1)$) are produced much before the electroweak PT via the chiral anomaly they could produce spatial fluctuations of $n_B - n_{\bar{B}} \[83\]$. These could be frozen in a first order electroweak PT and they would have effects on the early nucleosynthesis \[84\]. Primordial $Y$-magnetic fields also could strengthen the electroweak PT \[85\].

(iii) Last not least baryogenesis- the creation of a baryon-antibaryon asymmetry- can be discussed very concretely, favorably in some version of the charge transport mechanism \[77\]: the scattering of charginos, neutralinos, stops at the bubble wall creates some chiral current if CP is violated in the bubble wall \[72\]-\[76\]. This is then transformed into a baryon asymmetry by the $B+L$ violating “hot” sphaleron interaction in the hot phase. CP could be violated explicitly (in the MSSM by $A_t, \mu$) or spontaneously. If, in the latter case, this happens only for the temperatures of the PT, one does not have any problems with EDM-bounds \[76\].

In recent discussions \[86\]-\[90\] it is stressed that one should deal with (thick) wall scattering and diffusion simultaneously and that one should perhaps use quantum Boltzmann equations. This is a demanding program without agreement on most of the technicalities.

In conclusion one can say that for a first order electroweak PT we have an intriguing interplay of cosmology and elementary particle physics, and of non-equilibrium thermodynamics. Variants of the electroweak SM like the MSSM with a “light” stop and NMSSM models with $\mu \neq 0$ can give a strongly first order PT even for smallest Higgs masses of 100 GeV and perhaps even higher. A mixture of perturbation theory, lattice and (semi)analytic methods allows to produce reliable results for the PT. Fortunately perturbative results are more reliable for stronger first order PTs which are the most interesting ones.

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