Advanced Plasmadynamic Modelling for Fluid Plasmas

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Abstract. High-fidelity plasma simulations can be accomplished using microscopic models. However, this is often infeasible for dense plasma problems, in which case continuum models are more economical. Our work has investigated construction of high-fidelity fluid plasma models that retain effects such as displacement current, charge separation effects and plasma wave behaviour.

1. Introduction
Plasmadynamic modelling and simulation plays a significant role in furthering the understanding of the basic underlying physics, and in predicting the behaviour of the plasma under certain conditions. Broadly, two classes of models may be employed to simulate the behaviour of plasma: microscopic models, which include kinetic, Boltzmann and particle methods, and continuum approaches, characterized by statistical averaging. The former more naturally captures anisotropic and tensorial behaviours as well as micro-physics; however, for dense plasmas characterized by a very low Knudsen number, these models may be intractable on contemporary hardware, and the latter approaches are then far more economical for simulation of the overall plasma evolution.

Our work has focused on the exploration of higher-fidelity physical models and the development of numerical methods to construct approximate solutions to such models with only relatively modest computational effort. This paper briefly describes some of these approaches and outlines essential results. First, we present the physical model used in these investigations. We outline the numerical methods developed, and then present some typical results that show excellent agreement with theory.

2. Physical model
The union of the Navier-Stokes and Maxwell equations can be compactly written as a coupled, nonlinear system of first-order hyperbolic partial differential equations,

\[
\begin{bmatrix}
\frac{\partial}{\partial t} (\rho u + R)
\rho e + U
B
E
\end{bmatrix}
+ \partial_k
\begin{bmatrix}
\rho u
\rho uu + p1 - T
\rho l/u + S
1 \times B
- c^2 (1 \times E)
\end{bmatrix}
= \begin{bmatrix}
0
0
0
0
\frac{i}{\epsilon_0}
\end{bmatrix},
\tag{1}
\]
where $\rho$ is the mass density, $u$ is the fluid velocity, $e$ is the total specific energy, $H$ is the total enthalpy, $p$ is the fluid pressure, $E$ and $B$ are the electromagnetic fields, and $j$ is the current density. Here, $R = S/c^2$, where $S$ is the Poynting vector, $U$ is the electromagnetic energy density, and $T$ is the Maxwell stress tensor. $I$ is the identity matrix. The current density is calculated using Ohm’s law, and the ideal gas relation is used for relating the pressure and specific energy.

3. Numerical methods
The numerical solver is a parallelized, high-performance research code that supports fully unstructured meshing in one, two or three dimensions. It is capable of discretizing and solving any number of coupled, nonlinear, first-order hyperbolic partial differential equations using an advanced treatment of the finite volume method.

An important issue is the treatment of the divergence constraints (scalar Maxwell’s equations). These equations are constraints placed on the initial conditions; analytically, if the initial conditions satisfy these constraints, then the solution will for all time. However, numerically, this is not the case; divergence error frequently accumulates unchecked, and pollutes the solution into an unphysical one. To avoid this, the divergence constraints were cast into a hyperbolic form by adding a first-order time derivative in a numerical potential. These hyperbolic equations were driven to a pseudo-steady-state each time step, eliminating the divergence error.

The generic system of equations can be written in a generic compact vector form as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot F = H,$$  \hspace{1cm} (2)

and can be solved implicitly using preconditioned pseudo-time schemes to accelerate the convergence.

4. Results
Typical results for basic science simulations have included investigations of plasma turbulence and instability effects [1-4]. Several investigations have characterized Kelvin-Helmholtz plasma instability under the influence of a magnetic field. The magnetic field can be shown to suppress the smaller-wavelength instabilities (see Figure 1).

Simulations have also investigated compressible plasma turbulence behaviour. A common benchmark problem for this case is the Orszag-Tang problem, which has been tested under the new solution approach, and the solution is in excellent agreement with the typical magnetohydrodynamic solution (see Figure 2).

**Figure 1.** Results of a Kelvin-Helmholtz instability simulation with identical initial conditions and at an identical simulation time [1]. (Left) Without an applied magnetic field. (Right) With a weak applied magnetic field.
5. Conclusions and future work
The physical model has already been extended now to fully relativistic, multiple-species plasma dynamics. Future work will continue this trend of extending this approach for high-fidelity fluid modelling of dense plasmas, and applying it to explore cases of both basic plasma science and engineering device applications.

References
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