STRINGS AND DISSIPATIVE MECHANICS.

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Abstract:
Noncritical strings in the "coupling constant" phase space and bosonic string in the affine-metric curved space are dissipative systems. But the quantum descriptions of the dissipative systems have well known ambiguities. We suggest some approach to solve the problems of this description. The generalized Poisson algebra for dissipative systems is considered.

1 Introduction.

The dissipative models in string theory are expected to have more broad range of application:
1) Noncritical strings are dissipative systems in the "coupling constant" phase space [14, 13]. In this case, dissipative forces are defined by non-vanishing beta-functions of corresponding coupling constant and by Zamolodchikov metric [14, 13].
2) Problems of quantum description of black holes on the two-dimensional string surface lead to the necessity of generalization of von Neumann equation for dissipative systems [5, 14, 13].
3) The motion of a string (particle) in affine-metric curved space is equivalent to the motion of the string (particle) subjected to dissipative forces on Riemannian manifold [15, 17]. So the consistent theory of the string in the affine-metric curved space is a quantum dissipative theory [15, 16, 11, 17].

But the quantum descriptions of the dissipative systems (particles, strings,...) have well known problems [2, 25, 26, 3, 4, 6, 7, 8]. The simple example of these problems is following. It is easy to see that quantum equation of motion for dissipative systems are not compatible with Heisenberg algebra. Let us consider the quantum Langevin equation [10]

\[ \dot{a} = (-i\alpha - \beta) a + f(t), \quad \dot{a}^\dagger = (-i\alpha - \beta) a^\dagger + f(t)^\dagger \]

We have \([\dot{a}, a^\dagger] + [a, \dot{a}^\dagger] = -2\beta\) On the other hand, the total time derivative of the Heisenberg algebra

\[ [a, a] = [a^\dagger, a^\dagger] = 0, \quad [a, a^\dagger] = 1 \]

and Leibnitz rule lead to the following \([\dot{a}, a^\dagger] + [a, \dot{a}^\dagger] = 0\) So quantum dissipative equation of motion are not compatible with canonical commutation relations and Heisenberg algebra.

Let us describe briefly some approaches to the quantum description of the dissipative systems.

1.1. Problems of canonical quantization.
A) Lemos [2] proved that canonical quantization relations is not compatible with equation of motion for dissipative systems. Note that Lemos considered the total time derivatives of the commutation relations for the coordinates and momentums, used the Jacobi identity and the dissipative equations of motion of Heisenberg operator.

B) As is known that the equation of motion is the Euler-Lagrange equation based on local a Lagrangian function when the Helmholtz conditions are satisfied. Havas [24] considered a general theory of multipliers which allows (by using the Helmholtz conditions) a Lagrangian formulation for a broad class of the equation of motion of the dissipative systems, which can not fit into Lagrangian mechanics by usual approach. Havas hence noted that the quantization of systems described by Lagrangian of the above type is either impossible or ambiguous [25]. This is follows from the fact that in classical mechanics of the dissipative systems there are many quite different Lagrangians and Hamiltonians leading to the same equations of motion [2]. So we do not know which of the possible Lagrangians is corrected and one to choose for quantization procedure.
C) Edwards [3] showed that, although classical Hamiltonians are necessary for canonical quantization, their existence is not sufficient for it. The quantization of the Hamiltonian which is not canonically related to the energy is ambiguous and therefore the results are conflicting with physical interpretations. It is not sufficient for the Hamiltonian to generate the equation of motion, but Hamiltonian must also be necessarily related via canonical transformation to the total energy of the system. However, this condition can only be met by conservative systems, thus excluding dissipative systems from possible canonical quantization [3].

D) Hojman and Shepley [4] started with classical equation of motion and set very general quantization conditions (relation that the coordinate operators commute). The total time derivative of this commutation relation was considered and the commutator of the coordinate operator and the velocity operator form a symmetric tensor operator was showed. They proved that classical analog of this tensor operator is a matrix which inverse matrix satisfies the Helmholtz conditions. Using the Jacobi identity for the coordinate and the velocity, Hojman and Shepley conclude the following: the general quantization condition implies that the equation of motions are equivalent to the Euler-Lagrange equations with some Lagrangian.

1.2. Generalization of von Neumann equation.

The generalized von Neumann equations proposed until now [6, 7, 5, 8], which should describe dissipative and irreversible processes are derived by the addition the superoperator which acts on the statistical operator and describes dissipative part of time evolution. Note that proposed generalizations of the von Neumann equation are derived heuristically. For a given set of generalized equations different requirements for superoperator exist. Most of the requirements proposed until now, which should determine superoperator uniquely are not unique themselves and so one has to deal with the problems arising from these ambiguities. The superoperator form is not determine uniquely. Moreover, the generalizations of the von Neumann equation are not connected with classical Liouville equation for dissipative systems [29, 30, 28]. Hence the quantum description of the dissipative system dynamics used the generalizations of von Neumann equation proposed until now is ambiguous.

1.3. Nonassociative Lie-admissible quantization.

The generalization of the canonical quantization of the dissipative systems was proposed by Santilli [9]. Santilli showed that the time evolution law of dissipative equation not only violates Lie algebra law but actually does not characterize an algebra. Therefore Santilli suggested, as a necessary condition to preserve the algebraic structure, that the quantum dynamics of the dissipative systems should be constructed within the framework of the nonassociative algebras. This is exactly the case of the noncanonical quantization at the level of the nonassociative Lie-admissible (Lie-isotopic) enveloping algebra worked out by Santilli.

The quantization of the dissipative systems was proposed by Santilli [9] as an operator image of the Hamiltonian-admissible and Birkhoffian generalization of the classical Hamiltonian mechanics. The generalized variations used by Santilli [9] to consider the dissipative processes in the field of the holonomic variational principles are connected with the generalized multipliers suggested by Havas [26] and therefore lead to an ambiguity in generalized variations.

1.4. Nonholonomic variational principle.

Sedov [20]- [24] suggested the variational principle which is the generalization of the least action principle for the dissipative and irreversible processes. The holonomic and nonholonomic functionals are used to include the dissipative processes in the field of the variational principle.

Nonholonomic principle was suggested in [11, 12, 18] to generalize classical mechanics in phase space. The suggested form of classical mechanics of the dissipative systems in the phase space can be used to consider the generalizations of canonical quantization for dissipative systems and von Neumann equation.

1.5. We can conclude the following:

1. Canonical quantization of the dissipative systems are impossible if all of operators in quantum
theory are associative [4, 2].

2. Equation of motion of dissipative systems are not compatible with canonical commutation relations (with Heisenberg algebra).

3. Coordinate and momentum operators must be satisfy the canonical commutation relations. The generalized operator algebra must not violate Heisenberg algebra.

4. Generalization of the von Neumann equation must be connected with classical Liouville equation for dissipative systems [12, 13].

5. Hamiltonian must be canonically related to the physical energy \((T + U)\) of the dissipative system [3].

6. Total time derivative of the dissipative system operator does not satisfy the Leibnitz rule [31].

7. Canonical quantization of the dissipative systems described within framework of holonomic variational principles (least action principle) is either impossible or ambiguous [25, 26].

8. Quantum description of dissipative systems within framework of quantum kinetics is very popular and successful, but it is not valid in the fundamental theories such as string theory.

9. Dissipative systems can be described within framework of the nonholonomic variational principle [20] - [24].

In this paper we consider the some main points of the quantum description of the dissipative systems which take into account these conclusions.

1.6.

In order to solve the problems of the quantum description of dissipative systems we suggest to introduce an operator \(W\) in addition to usual (associative) operators. The suggested operator algebra does not violate Heisenberg algebra because we extend the canonical commutation relations by introducing an operators of the nonholonomic quantity in addition to the usual (associative) operators of usual (holonomic) coordinate-momentum functions. That is the coordinate and momentum satisfy the canonical commutation relations. To satisfy the generalized commutation relations the operator \(W\) of nonholonomic quantity must be nonassociative non-Lieble (does not satisfied the Jacobi identity) operator [11, 16]. As the result of these properties the total time derivative of the multiplication and commutator of the operators does not satisfies the Leibnitz rule. This lead to compatibility of quantum equations of motion for dissipative systems and canonical commutation relations. The suggested generalization of the von Neumann equation is connected with classical Liouville equation for dissipative systems.

2 Classical Dissipative Mechanics.

2.1. Generalization of the Poisson brackets.

Let the coordinates \(z^k, (k = 1, \ldots, 2n)\), where \(z^i = q^i, z^{n+i} = p_i \) \((i = 1, \ldots, n)\) and \(w, t\) of the \((2n+2)\)-dimensional extended phase space be connected by the equations

\[ dw - a_k(z,t) \, dz^k = 0 \]

(1)

where \(a_k (k = 1, \ldots, 2n)\) are the vector functions in phase space. Let us call the dependence \(w\) on the coordinate \(q\) and momentum \(p\) the holonomic-nonholonomic function and denote \(w = w(z) \in F^*(M)\) (generalization of the potential for closed and nonclosed 1-forms). If the vector functions satisfy the relation

\[ \frac{\partial a_k(z)}{\partial z^l} = \frac{\partial a^l(z)}{\partial z^k} \]

(2)

where \(k,l= 1,\ldots,2n\), the coordinate \(w\) is the holonomic function \((w \in F(M) = \Lambda^0(M))\), i.e. potential for closed 1-form. By definition, if these vector functions don’t satisfy the relation (2) the object \(w(z)\) we call the nonholonomic function \((w \in F^*/F)\) – generalized potential of the nonclosed 1-form and \(dw(z) \neq 0\).
Let us define the generalized Poisson brackets for the generalized potentials \( f, g, s \in F^* \) of the (closed and nonclosed) 1-forms \( \alpha = df, \beta = dg, \gamma = ds \) \((d^2 f \neq 0, f \in F^*)\) on the symplectic manifold \((M, \omega)\)

\[
[f, g] \equiv \Psi(\alpha, \beta) = \omega(X_\alpha, X_\beta) = \Psi^{kl} a_k b_l
\]

where \( X_\alpha : i(X_\alpha) \omega = \alpha \); \( \omega \) - closed \((d\omega = 0)\) 2-form \([33, 34]\), called symplectic form; \( i \) - internal multiplication of the vector fields and the form \([33, 34]\), \( \Psi^{kl} \) - contrvariant 2-tensor, which is the matrix inverse to matrix of the symplectic form and satisfies \([?, 34]\):

a) Skew-symmetry:

\[
\Psi^{kl} = -\Psi^{lk}
\]

b) Zero Schouten brackets:

\[
\Psi([\alpha, \beta], \gamma) = [\Psi^{km} \partial_m \Psi^{lk} + \Psi^{lm} \partial_m \Psi^{ks} + \Psi^{km} \partial_m \Psi^{st} = 0
\]

The generalized Poisson brackets can be represented by

\[
[f, g] = \frac{\delta f}{\delta q^i} \frac{\delta g}{\delta p_i} - \frac{\delta f}{\delta p_i} \frac{\delta g}{\delta q^i} = a_0 b_{n+i} - a_{n+i} b_i
\]

where

\[
\alpha = a_k(z) dz^k = a_1 dq^1 + a_{n+i} dp^i, \quad \beta = b_k(z) dz^k = b_1 dq^1 + b_{n+i} dp^i
\]

The basic properties of the generalized Poisson brackets:

1) Skew-symmetry: \( \forall f, g \in F^* \) \( [f, g] = -[g, f] \in F; \)

2) Jacobi identity: \( \forall f, g, s \in F \) \( J[f, g, s] = 0; \)

3) NonLiebility: \( \forall f, g, s \in F^*: f \vee g \vee s \in F^*/F; J[f, g, s] \neq 0; \)

4) Distributive rule: \( \forall f, g, s \in F^* \) \( [\alpha f + \beta g, s] = \alpha [f, s] + \beta [g, s] \)

5) Leibnitz rule: \( \forall f, g \in F^* \) \( \partial_f \Psi^{kj} = \frac{\partial}{\partial f} \) \( \frac{\partial}{\partial g} = \frac{\partial}{\partial f} \) \( + \frac{\partial}{\partial g} \)

where \( J[f, g, s] = [f, [g, s]] + [g, [s, f]] + [s, [f, g]] \);

\( \alpha \) and \( \beta \) are the real numbers.

If this bilinear operation "generalized Poisson bracket" is defined on the space of generalized potentials \( F^*(M) \), then the manifold \( M \) is called poisson manifold, and the space \( F^*(M) - \) generalized Poisson algebra \( P_0 \). Generalized Poisson algebra \( P_0 \) is not Lie algebra: Jacobi identity for \( F^*/F \) is not satisfied. But Jacobi identity is satisfied on the space \( F(M) \). For this reason we can define in the space \( F(M) \) a Lie algebra, which is the Poisson algebra \( P_0 \) \([24]\). It is easy to verify that this properties of the generalized Poisson brackets for the holonomic functions coincide with the properties of the usual Poisson brackets \([15]\).

That is generalized Poisson algebra \( P_0 \) contain a subalgebra which is the usual Poisson algebra \( P_0 \). Generalized Poisson bracket is the holonomic function so this subalgebra \( P_0 \) is the ideal of the algebra \( P_0^* \)

2.2. Let us consider now the characteristic properties of the physical quantities:

1) \( [p_i, p_j] = [q^i, q^j] = 0 \) and \( [q^i, p_j] = \delta^i_j \)

2) \( [w, p_i] = w^j_p \) and \( [w, q^i] = -w^j_p \) \( i \neq j \), \( [w, w] = 0 \)

3) \( [w, p_i] \neq [w, p_j], p_i \) or \( J[p_i, w, p_j] = \omega_{ij} \neq 0 \) \( i \neq j \)

4) \( [w, q^j], q^j \) \( \neq [w, q^j], q^j \) or \( J[q^i, w, q^j] = \omega_{ij} \neq 0 \) \( i \neq j \)

5) \( [w, q^j], p_j \neq [w, p_j], q^j \) or \( \omega_{ij} \neq 0 \)

where

\[
\omega_{ij} = -\frac{\partial w^j}{\partial p_i} - \frac{\partial w^i}{\partial q^j} = \frac{\delta^2 w}{\delta q^j \partial p_i} - \frac{\delta^2 w}{\delta q^j \partial p_i}
\]

This object \( \omega^{kl} \) \((k, l = 1, ..., 2n)\) characterizes deviation from the condition of integrability \([2]\) for the equation \([1]\) and by the Stokes theorem

\[
\int_\partial M \delta w = \int_M \omega^{kl} dz^k \wedge dz^l \neq 0
\]
Note that \( w \) is the nonholonomic object if one of \( \omega^{kl} \) is not trivial. Therefore some of the properties 3-5 can be not satisfied but one of it must be carry out if we consider the dissipative processes.

### 2.3. Equation of motion for dissipative systems.

Equations of motion for dissipative systems in the phase space is given by

\[
\frac{dq^i}{dt} = \frac{\delta h}{\delta p_i} - w_q^i - \delta \left( h - w \right) \frac{\delta}{\delta q^i} \quad ; \quad \frac{dp_i}{dt} = -\frac{\delta h}{\delta q^i} + w_p^i + \delta \left( h - w \right) \frac{\delta}{\delta p_i} \tag{7}
\]

where \( \delta w(q, p) = w_q^i \delta q^i + w_p^i \delta p_i \).

If we take into account generalized Poisson brackets the equation of motion in phase space for dissipative systems (7) takes the form

\[
\frac{dq^i}{dt} = \{ q^i, h - w \} \quad ; \quad \frac{dp_i}{dt} = \{ p_i, h - w \} \tag{8}
\]

The total time derivative of the physical quantity \( A = A(q, p, t) \in F \) is given by

\[
\frac{dA(q, p, t)}{dt} = \frac{\partial A(q, p, t)}{\partial t} + \{ A, h - w \} \tag{9}
\]

The equation of motion (8) can be derived from the equation (9) as a particular case. Note that any term which added to the Hamiltonian \( h \) and nonholonomic object \( w \) does not change the equations of motions (8), (9). This ambiguity in the definition of the Hamiltonian is easy to avoid by the requirement that Hamiltonian must be canonically related to the physical energy of the system \( [3] \): \( [w, q^i] = 0 \).

It is easy to see that total time derivative of the generalized Poisson brackets does not satisfies the Leibnitz rule

\[
\frac{d}{dt}[f, g] = \left[ \frac{df}{dt}, g \right] + [f, \frac{dg}{dt}] + J[f, w, g] \tag{10}
\]

### 2.4. Generalized Poisson algebra of 1-forms.

It is known that Poisson brackets can be defined for nonclosed differential 1-forms \( \alpha = a_k(z)dz^k \) on the symplectic manifold \( (M, \omega) \) \([34]\), where \( \omega \)- closed \( (d\omega = 0) \) 2-form \([36]\). Poisson bracket for two 1-forms \( \alpha = a_k(z)dz^k \) and \( \beta = b_k(z)dz^k \) is 1-form \( (\alpha, \beta) \), defined by

\[
(\alpha, \beta) = d\Psi(\alpha, \beta) + \Psi(da_k, \beta) + \Psi(\alpha, db_k) \tag{11}
\]

that is the map of 1-forms \( \Lambda^1(M) \times \Lambda^2(M) \rightarrow \Lambda^1(M) \).

If this bilinear operation "Poisson bracket" is defined on the space of 1-forms \( \Lambda^1(M) \), then the manifold \( M \) is called poisson manifold, and the space \( \Lambda^1(M) \) - Poisson algebra \( P_1 \). Poisson algebra \( P_1 \) is a Lie algebra. It is caused by skew-symmetry \( (\alpha, \beta) = -(\beta, \alpha) \) and Jacobi identity:

\[
((\alpha, \beta), \gamma) + ((\beta, \gamma), \alpha) + ((\gamma, \alpha), \beta) = 0
\]

In order to describe dissipative systems we suggest to generalize the Poisson algebra \( P_1 \). Let us define the bilinear operation on \( \Lambda^1(M) \) : Generalized Poisson bracket of two 1-forms \( \alpha \) and \( \beta \) is 1-form \( (\alpha, \beta) \), defined by

\[
[\alpha, \beta] = \frac{d}{dt}(\Psi(\alpha, \beta)) \tag{12}
\]

It is easy to see the Jacobi identity for non-closed 1-forms is not satisfied. Therefore generalized Poisson algebra \( P_1^\ast \) is not Lie algebra. But Jacobi identity is satisfied for closed 1-forms. For this reason closed 1-forms define a Lie algebra, which is the Poisson algebra \( P_1 \). So generalized Poisson algebra \( P_1^\ast \) contain a subalgebra which is the usual Poisson algebra \( P_1 \). Generalized Poisson bracket is the closed 1-form so this subalgebra \( P_1 \) is the ideal of the algebra \( P_1^\ast \) and the exact algebraic diagram exists:

\[
0 \rightarrow P_1 \rightarrow P_1^\ast \rightarrow P_1^\ast / P_1 \rightarrow 0
\]

Generalized Poisson bracket for 1-forms satisfy the properties:
1) Skew-symmetry: \( \forall \alpha, \beta \in P_1^* \quad [\alpha, \beta] = -[\beta, \alpha] \in P_1 \);
2) Jacobi identity: \( \forall \alpha, \beta, \gamma \in P_1 \quad J[\alpha, \beta, \gamma] = 0 \);
3) Nonlineability: \( \forall \alpha, \beta, \gamma \in P_1^* : \alpha \lor \beta \lor \gamma \in P_1^*/P_1 \quad J[\alpha, \beta, \gamma] \neq 0 \);
4) Distributive rule: \( \forall \alpha, \beta, \gamma \in P_1^* \quad [\alpha b, \beta, \gamma] = a[\alpha, \beta, \gamma] + b[\beta, \gamma] \),

where \( J[\alpha, \beta, \gamma] \equiv [\alpha, [\beta, \gamma]] + [\beta, [\gamma, \alpha]] + [\gamma, [\alpha, \beta]] \), \( a \) and \( b \) are the real numbers.

That is the structure of the anticommutative nonassociative algebra, which is not Lie algebra, is naturally defined in the space of all 1-forms \( \Lambda^1(M) \) on the symplectic manifold \( M \). This algebra is a generalization of anticommutative nonassociative Lie algebra of closed 1-forms and contain a subalgebra (ideal) which is this Lie algebra.

2.5. Liouville equation for dissipative systems.

It is easy to obtain the dissipative analogue of the Liouville equation \( \mathbf{28, 29, 30, 40, 48} \):

\[
\frac{d}{dt} \rho(q, p, t) = -\omega(t,q,p) \rho(q, p, t) \quad \text{or} \quad \frac{\partial}{\partial t} \rho(q, p, t) = \Phi L\rho(q, p, t) \quad (13)
\]

where \( \omega = \sum_{i=1}^n \omega_i^i = \sum_{i=1}^n J[q_i^i, W, p_i] \) and \( \Phi = i(\delta(h-w)/\delta p - \delta(h^2-w)/\delta q - \omega(t,q,p) \) is generalization of the Liouville operator \( \mathbf{1} \). In addition to the Poincare-Misra theorem \( \mathbf{1} \) can be obtained the statement: "There exists the Liapunov function of the coordinate and momentum in the dissipative Hamiltonian mechanics". Let us define the function \( \eta(q, p, t) = -n\rho(q, p, t) \) and assume \( \omega > 0 \). The equation \( \mathbf{13} \) shows that

\[
\frac{d\eta(q, p, t)}{dt} = \omega(t,q,p)
\]

and the function \( \eta \) satisfies the relations \( d\eta/dt > 0 \).

3 Quantum Dissipative Mechanics.

3.1. Generalization of the canonical commutation relations.

In order to solve the problems of the quantum description of dissipative systems we suggest to introduce an operator \( W \) in addition to usual (associative) operators. Let us use the usual rule of definition of the quantum physical quantities which have the classical analogues: If we consider the operators \( A, B, C \) of the physical quantities \( a, b, c \) which satisfy the classical Poisson brackets \( [a, b] = c \), then the operators must satisfy the relation: \( [A, B] \equiv (AB) - (BA) = i\hbar C \). If we take into account the characteristic properties the physical quantities operators are defined by the following relations:

1) \( [Q^l, Q^j] = [P_i, P_j] = 0 \quad [P_i, Q_j] = i\hbar \delta^j_i \)
2) \( [W, P_i] = i\hbar W^i \quad [W, Q^j] = -i\hbar P_j \quad [W, W] = 0 \)
3) \( [[W, P_i], P_j] \neq [[W, P_j], P_i] \quad i \neq j \quad \text{or} \quad J[P_i, W, P_j] = \Omega_{ij} \neq 0 \)
4) \( [[W, Q^j], Q^i] \neq [[W, Q^i], Q^j] \quad i \neq j \quad \text{or} \quad J[Q^i, W, Q^j] = \Omega_{ij} \neq 0 \)
5) \( [Q^l, [W, P_i]] \neq [P_j, [W, Q^j]] \quad \text{or} \quad J[Q^i, W, P_j] = \Omega_{ij} \neq 0 \)

where \( J[A, B, C] = -1/(\hbar^2) \left( [A[BC]] + [B[CA]] + [C[AB]] \right) \)

and \( Q^i = Q_i; \quad P^i = P_i; \quad W^i = W \). Let us require that the canonical quantum commutation rules be a part of this rules. To satisfy the commutation relations and canonical commutation rules for the operator of the holonomic function the operators of the nonholonomic quantities must be nonassociative. It is sufficient to require that the operator \( W \) satisfies the following conditions:

1) left and right associativity: \( (Z^k, Z^l, W) = (W, Z^k, Z^l) = 0 \)
2) left-right nonassociativity: \( (Z^k, W, Z^l) \neq 0 \quad \text{if} \quad k \neq l \)

where \( k, l = 1, ..., 2n; \quad Z^i = Q^i \quad \text{and}\quad Z^{n+i} = P_i; \quad i = 1, ..., n; \quad (A, B, C) \equiv (A(BC)) - ((AB)C) \) called associator.

3.2. Quantum equation of motion for dissipative systems.
The state in the quantum dissipative mechanics can be represented by the "matrix-density" (statistical) operator \( \rho(t) \) which satisfy the condition \( \rho^\dagger(t) = \rho(t) \). The time variations of the operator of physical quantity \( A(t) \equiv A(Q, P, t) \) and of the operator of state \( \rho(t) \) are written in the form

\[
\frac{dA}{dt} = \frac{\partial A}{\partial t} + \frac{i}{\hbar}[H - W, A] \tag{14}
\]

\[
\frac{d\rho}{dt} = -\frac{1}{2}[\rho, \Omega]_+ + \frac{\partial}{\partial t} \rho = -\frac{i}{\hbar}[\rho, H] + \frac{i}{\hbar}[W, \rho] - \frac{1}{2}[\rho, \Omega]_+ \tag{15}
\]

where anticommutator \([,]_+\) is the consequence of the hermiticity for the density operator \( \rho \) and for the operator \( \Omega \), which is defined by

\[
\Omega = \sum_{i=1}^{n} \Omega_i = \sum_{i=1}^{n} J[Q^i, W, P_i]
\]

The solution of the first equation may be written in the form

\[
A(t) = S(t, t_0)A(t_0)S^\dagger(t, t_0) \quad \text{where} \quad S(t, t_0) = T \exp \frac{i}{\hbar} \int_{t_0}^{t} d\tau \ (H - W)(\tau) \tag{16}
\]

T-exponent is defined as usual, but we must take into account the following flow chart

\[
\exp A = 1 + A + \frac{1}{2}(AA) + \frac{1}{6}((AA)A) + \frac{1}{24}(((AA)A)A) + \ldots .
\]

The solution of the equation (15) is given by

\[
\rho(t) = U(t, t_0)\rho(t_0)U^\dagger(t, t_0) \quad \text{where} \quad U(t, t_0) = T \exp \frac{1}{2} \int_{t_0}^{t} d\tau \ \Omega(\tau) \tag{17}
\]

In this way the time evolution of the physical quantity operator is unitary and the evolution of the state operator is nonunitary. It is easy to verify that the pure state at the moment \( t = t_0 \) ( \( \rho^2(t_0) = \rho(t_0) \)) is not a pure state at the next time moment \( t \neq t_0 \). We can define the entropy operator \( \eta \) of the state \( \rho(t) \):

\[
\eta(t) = -\ln \rho(t).
\]

The entropy operator satisfies the equation \( d\eta(t)/dt = \Omega \).

3.3. Generalized Leibnitz rule.

It is easy to see that the commutator with nonassociative operator \( W \) and the total time derivative of both the quantum Poisson brackets and of the multiplication of the two operators do not satisfy the Leibnitz rule

\[
[A, B, W] = A[B, W] + [A, W]B + (A, W, B) \tag{18}
\]

\[
\frac{d}{dt}[A, B] = \left[ \frac{d}{dt} A, B \right] + [A, \frac{d}{dt} B] + J[A, W, B] \tag{19}
\]

\[
\frac{d}{dt}(AB) = ((\frac{d}{dt} A)B) + (A, \frac{d}{dt} B) + (A, W, B) \tag{20}
\]

where \( A \) and \( B \) are the associative operators (operators of the holonomic functions). This lead to compatibility of quantum equations of motion for dissipative systems and canonical commutation relations. We can formulate the path integration and generating functional in the quantum field theory which was considered in the papers [33, 15, 17].

3.4. Some applications.

The dissipative quantum scheme suggested in [11, 16] and considered in this paper allows to formulate the approach to the quantum dissipative field theory. As an example of the dissipative quantum field theory the sigma-model approach to the quantum string theory was considered in the recent papers [13, 17]. Conformal anomaly of the energy momentum tensor trace for closed bosonic string on the affine-metric manifold and two-loop metric beta-function for two-dimensional nonlinear dissipative sigma-model were calculated [11, 13, 17].
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