Latency and Reliability Limits with Decoding Complexity Constraints

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Abstract—One of the most important application scenarios in next-generation wireless networks is ultra-reliable low-latency communication (URLLC) where stringent constraints on both reliability and latency must be guaranteed. In most existing communication-theoretic studies, latency is defined as the time required for the transmission of a message over the communication channel. This implies that other operations necessary for the successful delivery of the information, e.g., the time required for the decoding of the transmitted message, are assumed to happen instantaneously. However, with strict latency requirements, the decoding time cannot be neglected. In contrast, a more refined modeling of the decoding complexity and the delay that this incurs must be considered. In this paper, we study the performance of state-of-the-art channel codes as a function of the decoding complexity and propose an empirical model that accurately quantifies the corresponding trade-off. Based on the proposed model, several optimization problems, relevant to the design of URLLC systems, are introduced and solved. It is shown that the decoding time has a drastic effect on the aggregate latency when decoding complexity constraints are considered.

Index Terms—5G mobile communication, URLLC, internet-of-things, low-latency communication, ultra-reliable communication, low-complexity receivers, channel coding, ordered-statistics decoder.

I. INTRODUCTION

Ultra-Reliable Low-Latency Communication (URLLC) is one of the three main service categories that have been defined in 5G with the other two being enhanced Mobile Broadband and massive Machine-Type Communication [1]. URLLC provides communication support with stringent constraints on reliability and end-to-end latency and has attracted extensive attention and significant research interest, since information transmission with low-latency and high reliability is crucial for enabling various mission-critical services, such as machine-to-machine communication, remote surgery, augmented reality, vehicle automation, industrial robotics, factory automation, and smart-grid [2].

Capacity of a channel represents the asymptotic limit of the transmission rate where reliable communication is guaranteed for unbounded codeword length [3] and is mostly associated with latency tolerant communication systems. In existing literature, the performance of a latency constrained communication system is often evaluated on the basis of the outage capacity [4], which is the maximal transmission rate under a given positive probability that the instantaneous mutual information falls below a desired rate. Similar to channel capacity, limits in the outage analysis are also derived for the asymptote of infinite codeword length [5]. However, this assumption does not hold for URLLC, since stringent constraints on aggregate latency yield transmission of codewords with short blocklengths [6]. Although research on maximal achievable transmission rates for finite blocklengths has a history going back to 1960s [7], significant amount of progress has been achieved in the context of non-asymptotic information theory in the recent years (see [8] and references therein). Non-asymptotic achievability and converse bounds for the finite blocklength regime are derived in [9]. It is shown that compared to the asymptotic limits, a rate penalty needs to be paid when transmitting in the finite blocklength regime. This study attracted huge interest from the research community and several studies on the non-asymptotic achievable bounds for various channels with different fading environments are published [10]–[12].

Although the non-asymptotic achievable bounds reveal the theoretical limits, achieving them is still an open problem. Therefore, the selection of a channel encoding and decoding scheme that can perform close to the limit is significant in terms of increasing the transmission efficiency of the communication system. Several coding schemes that are suitable for URLLC are introduced in [13]–[21]. Their performances in the finite blocklength regime are also shown therein where performance of a decoder, in general, is identified according to its gap to the non-asymptotic limits. However, although it is observed that some channel coding schemes can perform very close to the limits, cost of computational complexity is neither taken into account in the comparisons of the coding schemes nor in the derivation of the theoretical limits.

In general, it is assumed that decoding happens instantaneously and the latency due to the decoding of a packet is negligible [22]. However, the cost of computational complexity, in terms of latency, is inversely proportional to the computation power [23], [24]. A computationally intensive decoding process takes relatively longer duration in a complexity constrained receiver, such as low-budget IoT receiver. In such applications, latency due to decoding is a significant determinant of decoder cost [6], [25]–[27]. In order to decrease the aggregate latency, one may select a relatively lower complex decoder, which in turn may compromise the error
probability of the decoder. This, therefore, reveals trade-offs between latency, reliability, and computational complexity.

Although there is no generally accepted model for the computational complexity of a typical channel decoder, the total number of operations per-information-bit is often selected as a metric for the computational complexity. In [13], per-information-bit computational complexity, i.e. total number of binary operations per information bit, of several decoding algorithms is presented. It is shown in [14] and [15] that coding schemes that perform close to the limits have relatively complex decoding algorithms. Further, performance comparisons of various practical codes are studied as a function of per-information-bit complexity in [14] and [19]. It is observed that complexities of the coding schemes exponentially increase as they approach to the theoretical limits in [28]. It is also further shown that an excess power with respect to the theoretical limits must be spent to achieve a fixed allowed error rate at a fixed transmission rate, when a particular code is chosen. Recently, studies in which decoding duration is taken into account are published in [29] and [30]. It is shown that decoding complexity has a considerable effect on the maximal limits of the short block-length codes.

Contributions: This work extends the authors’ previous work on analyses of low-latency communication with computational complexity constrained receivers [29]. In this paper the following contributions are presented.

- First, a consistent way to compute the aggregate latency due to the decoding process for complexity constrained receivers is presented.
- A mathematically tractable model that can accurately show the trade-off between the computational complexity of a decoder, in number of binary operations per information bit, versus the excess power to the non-asymptotic achievability bounds, derived in [9], is introduced.
- With the help of this model, we address non-trivial optimization problems that are related to URLLC systems with computational complexity constraints. The following optimization problems are answered
  - Given that a fixed number of information bits are intended to be transmitted under reliability and power constraints, what is the optimum selection of transmission parameters that leads to the minimum aggregate latency?
  - Given that a fixed number of information bits are intended to be transmitted under reliability, power, and latency constraints, what is the optimum selection of transmission parameters that leads to the minimum energy-per-bit?
  - Under reliability, power, and latency constraints, what is the optimum selection of transmission parameters that leads to the maximum number of information bits to be transmitted?

It is shown that optimum solutions to these problems are directly associated with the constraints. Thus, the optimal design of a URLLC system is substantially influenced with the latency and complexity constraints when decoding latency is taken into consideration. The rest of this paper is organized as follows. The system model is introduced in Section II. Section III deals with modeling the decoding complexity. Latency analysis with complexity constraints is provided in Section IV. Finally, optimization problems are formulated and their optimum solutions are addressed in Section V. Conclusions are drawn in Section VI.

Notation: Vectors and matrices are denoted by bold face lower and upper case letters, respectively. The Euclidean norm of a vector \( \mathbf{x} \) is denoted by \( \| \mathbf{x} \|_2 \) and \( \mathcal{N}(\mu, \sigma^2) \) represents a real Gaussian random variable with mean \( \mu \) and \( \sigma^2 \) variance. All logarithms in this paper are with base 2 and \( \oplus \) and \( \otimes \) represent the binary addition and multiplication, respectively.

II. System Model

We consider communication over a discrete-time, binary-input AWGN (BI-AWGN) channel. A sequence of \( n \) symbols
\[
\mathbf{x} = [x_1, x_2, \ldots, x_n], \quad x_i \in \{-1, +1\},
\]
which is termed as codeword, is transmitted over the channel. The observed sequence at the receiver is
\[
y = \sqrt{\rho x} + z,
\]
where \( z \sim \mathcal{N}(0, I_n) \) and \( \rho \) denotes the signal-to-noise ratio.

Without latency constraints, it is known that there exists a codebook, i.e., collection of codewords, with size \( 2^{nC} \) codewords, such that the codeword error probability (CEP) vanishes as \( n \to \infty \). The quantity \( C \) is called channel capacity [3] and for the channel in (2) is given, as a function of \( \rho \), by
\[
C = \frac{1}{2\pi} \int e^{-\frac{z^2}{2}} \left( 1 - \log \left( 1 + e^{-2\rho^{1/2}z}\right) \right) dz.
\]

With strict latency constraints, however, i.e., when \( n \) is not allowed to take arbitrarily large values, the CEP is strictly positive and \( C \) overestimates the rate of information transmission through (2). Recently, more refined upper and lower bounds on the maximal codebook size have been proposed for finite \( n \) and a non-zero CEP, \( \varepsilon > 0 \) [9], [28]. Based on these bounds the maximal codebook size with codewords of length \( n \) and CEP \( \varepsilon \) can be well approximated for a wide range of \( n \) and \( \varepsilon \) by \( 2^{nR(n, \rho, \varepsilon)} \), where
\[
R(n, \rho, \varepsilon) = C - \sqrt{\frac{V}{n}}Q^{-1}(\varepsilon) \log e + O \left( \frac{1}{n} \right).
\]

The quantity \( V \) is the channel dispersion, and for (2) it is given by [28]
\[
V = \frac{1}{2\pi} \int e^{-\frac{z^2}{2}} \left( 1 - \log \left( 1 + e^{-2\rho^{1/2}z}\right) - C \right)^2 dz.
\]

and \( Q^{-1}(\cdot) \) is the inverse of the Gaussian \( Q \)-function
\[
Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2} dt.
\]

That is the probability that the receiver decides in favor of a codeword that is different from the one actually sent.
The expression in (4) is termed as the normal approximation to the maximal coding rate. In our analysis, we take the first two terms of (4) into account and neglect the Ordo-term. Although this makes (4) an approximation, for the purpose of this paper we assume that it is exact.

The transmission latency of a codeword is \( L_T = n T_s \) seconds, where \( T_s \) is the symbol duration. It is often encountered in the literature of latency-constrained communication that the aggregate latency equals the transmission latency, which implies that the latency attributed to decoding of the observed sequence, \( y \), at the receiver is ignored. However, in this paper the aggregate latency \( L_A \) is considered, where

\[
L_A = L_T + L_D, \quad (7)
\]

with \( L_D \) denoting the decoding latency. It is one of the aims of this work to propose a model that describes in a general, accurate and tractable way the latency introduced due to decoding based on existing, state-of-the-art, decoding algorithms. In the following, we introduce this model.

### III. Decoding Complexity Modeling

The decoding complexity model that we propose in the present paper is based on linear block codes with ordered statistics (OS) decoders. In prior works [13, 32], it has been shown that these codes come very close to the information-theoretic bounds for finite \( n \). Also, their decoding performance can be easily parameterized by a single parameter, i.e., the order of the decoder, \( s \in \mathbb{Q} \). Finally, the operations that are executed during decoding can be accurately tracked and the decoding complexity can be efficiently and intuitively described.

An uncoded binary sequence

\[
u = [u_1, u_2, \ldots, u_k], \quad u_i \in \{0, 1\} \quad (8)
\]

of \( k \leq n \) bits is mapped to an encoded binary sequence

\[
b = G \otimes u, \quad (9)
\]

\( G \in \{0, 1\}^{n \times k} \), of \( n \) bits which are then mapped to the transmitted codeword \( x \) using the rule \( x_i = 2u_i - 1 \). At the decoder, we consider the use of an OS decoder with order \( s \). The components \( \{y_i\}_{i=1}^n \) of the observed sequence \( y \) are sorted in order of descending amplitudes and the hard-decoded \( k \) most-reliable bit sequence, \( r \), is obtained. We denote the resulting permutation by \( \kappa(\cdot) \). The columns of \( G \) are reordered by the same permutation, \( \kappa(\cdot) \), and Gauss-Jordan elimination is applied to form the corresponding systematic generator matrix \( G_s \). Associated with \( s \), a list, \( \mathcal{L}_{\text{TDP}} \), of

\[
|\mathcal{L}_{\text{TDP}}| = \sum_{i=0}^{[s]} \binom{k}{i} + (s - [s]) \left[ \binom{k}{s} + 1 \right] \quad (10)
\]

test error patterns (TEPs), i.e., bit sequences of length \( k \), is formed. This list includes all the TEPs with Hamming weight \( \leq [s] \) and the most probable TEPs with Hamming weight \( [s] + 1 \). The permuted codebook is then formed by mapping

\[
G_s \otimes (r + l_i), \quad l_i \in \mathcal{L}_{\text{TDP}}. \quad (11)
\]

The codeword that minimizes the distance between the permuted sequence, \( \kappa(y) \), and the permuted codebook is selected as the most probable codeword. The decoded information sequence is produced by performing the inverse permutation, \( \kappa^{-1}(\cdot) \), and selecting the first \( k \) bits.

For notational consistency, given a fixed codebook containing \( 2^k \) codewords of length \( n \), we denote a decoder as \( d(n, k, \rho, s, \varepsilon) \), where it is meant that the decoder of order \( s \) operates on the given codebook at a received SNR, \( \rho \), and has a CEP, \( \varepsilon \). The number of binary operations per-information-bit of an observed sequence, \( y \), when the decoder \( d(n, k, \rho, s, \varepsilon) \) is used, is given by [29, 35]

\[
K(d(n, k, \rho, s, \varepsilon)) = \frac{k^2}{8} + \frac{n}{2} |\mathcal{L}_{\text{TDP}}|, \quad (12)
\]

where the first term is due to the Gaussian-Jordan elimination of the permuted \( G \) matrix and the second term is due to the total codeword comparisons. When \( s < 2 \), (12) is dominated by the Gaussian-Jordan elimination and therefore \( K(d(n, k, \rho, s, \varepsilon)) = \mathcal{O}(k^2) \). However, for \( s \geq 2 \), the second part dominates the complexity and the complexity order can be expressed as \( K(d(n, k, \rho, s, \varepsilon)) = \mathcal{O}(nk^s) \).

The choice of the order, \( s \), limits the search space for the most probable TEP by limiting the size of the list, \( \mathcal{L}_{\text{TDP}} \), in (10). In comparison to the ML decoder, that performs in general an exhaustive search over the \( 2^k \) codewords of the codebook, a choice of a moderate \( s \) leads to substantial reduction in decoding complexity. As a side comment, it is shown in [33] that the required order, \( s_r \), to achieve the ML decoder performance is approximately

\[
s_r = \min \left\{ \frac{d_{\min}}{4} - 1, k \right\}, \quad (13)
\]

where \( d_{\min} \) denotes the smallest Hamming distance of the selected codebook.

Some easily verifiable properties hold for the relative performance of two decoders operating on the same codebook follows:

**Property 1.** Let two decoders, \( d(n, k, \rho, s_1, \varepsilon_1) \) and \( d(n, k, \rho, s_2, \varepsilon_2) \), operating on the same codebook with \( s_1 \leq s_2 \). It follows immediately by the selection of the TEP lists that \( \mathcal{L}_{\text{TDP},1} \subseteq \mathcal{L}_{\text{TDP},2} \), which implies \( \varepsilon_1 \geq \varepsilon_2 \) and \( K(d(n, k, \rho, s_1, \varepsilon_1)) \leq K(d(n, k, \rho, s_2, \varepsilon_2)) \). Intuitively, more complex decoder leads to lower CEP.

**Property 2.** In addition, let two decoders \( d(n, k, \rho_1, s, \varepsilon_1) \) and \( d(n, k, \rho_2, s, \varepsilon_2) \), operating on the same codebook with \( \rho_1 \leq \rho_2 \). Then, it must be true that \( \varepsilon_1 \geq \varepsilon_2 \). Intuitively, higher operating SNR leads to lower CEP.

Numerical performance results for OS decoders with orders \( s = \{0, 1, 2, 3, 4, 5\} \) for \( n = 128 \) and \( k = 64 \) are shown in
suppose that a latency constraint such as

\[ L_A \leq L_M \]

is applied, where \( L_M \) represents the maximum allowed latency. Notice that this constraint imposes an upper bound on the per-information-bit decoder complexity such that

\[ K(d(n, k, \rho, s, \varepsilon)) \leq \frac{L_M - nT_s}{kT_b}, \]

as long as \( L_M \geq nT_s \). Furthermore, for some fixed \( n \) and \( k \), (15) restricts the order \( s \) as follows

\[ s \leq s_m \]

where \( s_m \) denotes the maximum allowed order. However, since \(|\mathcal{E}_{\text{TEP}}|\) is formed in the sum of binomial coefficients, which is mathematically intractable for large \( s \), an upper bound on the per-information-bit complexity that gets tighter with larger \( s \) is derived (see Appendix)

\[ K(d(n, k, \rho, s_m, \varepsilon)) \leq \frac{k^2}{8} + 2^{kh(\frac{k}{4})}, \]

where \( h(z) = -z \log(z) - (1-z) \log(1-z) \) is the binary entropy function. (19) introduces the following inequality on \( s_m \)

\[ h \left( \frac{s_m}{k} \right) \geq \frac{\log \tau}{k}, \]

where \( \tau = \frac{1}{n} \left( \frac{2(L_M - nT_s)}{kT_b} - \frac{k^2}{4} \right) \). A lower bound on \( s_m \) can be numerically evaluated from (20) since the binary entropy function is monotonically increasing for \( \frac{s_m}{k} \leq \frac{1}{2} \). Although a closed form expression cannot be achieved, using the tight approximation for binary entropy function, given as \( h(z) \approx (4z(1-z))^{3/4} \), an approximation can be derived as

\[ s_m \approx k \left( 1 - \sqrt{1 - \left( \frac{\log \tau}{k} \right)^{4/3}} \right). \]

However, notice that a constraint on order \( s \) may lead to a degradation in the CEP performance of the OS decoder. In particular, if \( s_m < s_r \), the CEP of the most complex allowable decoder will be appreciably higher in comparison to the ML CEP bound.

A. Power Penalty

It is shown in (18) that the selection of an order \( s \) for a particular code of fixed \( n, k \) and \( \rho \) can be used to control the aggregate latency \( L_A \) of the communication, albeit at the expense of reduced reliability. In Fig. 1 for a fixed SNR the lowest CEP is given by the \( \varepsilon_m \) curve. Constraining the order \( s \) of the decoder, though, incurs a CEP degradation that is a vertical upwards step to the curve with corresponding \( s \). One way to satisfy a desired target reliability, is that some amount of excess power, named as the power penalty, has to be paid. Visually, this can be represented as a horizontal rightward step. Hence, an interesting, yet complex, relation between power, aggregate latency, decoding complexity arises.

Definition (Power penalty). Fix a codebook of \( 2^k \) codewords of blocklength \( n \). For a reference SNR, \( \rho_r \), consider the ML
decoder that achieves a CEP of \( \varepsilon \) and the suboptimal decoder \( d(n, k, \rho, s, \varepsilon) \) that achieves \( \varepsilon \) at SNR \( \rho \). The quantity

\[
\Delta \rho = \rho - \rho_r
\]  

(22)
is the power penalty required, such that the suboptimal decoder can achieve the same CEP as the ML decoder.

For a fixed rate \( r = \frac{k}{n} \) the reference SNR \( \rho_r \) can be computed by taking the inverse of (4),

\[
\rho_r = R^{-1}(n, r, \varepsilon).
\]  

(23)

Notice that \( R(n, \rho, \varepsilon) \) is strictly increasing in \( \rho \) and therefore invertible. Although there is no closed form expression of \( \rho_r \) for BL-AWGN channels, it can be numerically evaluated.

Since no coding scheme can transmit above the theoretical bound, \( \Delta \rho \) is positive, \( \Delta \rho \geq 0 \). Having \( \Delta \rho = 0 \) yields operating at rate \( r \) with SNR \( \rho_r \) which indicates that the transmission occurs right on the bound. Although this power-rate selection is theoretically possible, it requires very complex decoder to achieve \( \varepsilon \) since (12) and (13) show that computational complexities of decoders exponentially increase as their performance approach to the bound. Similar empirical results are also presented in [14] and [29]. Extensive studies on OS decoders reveal that this exponential increase is similar at all rates for fixed \( n \) [29].

It is clear from the above that a model is required to quantify the power penalty. Bounds on the performance of OS decoders are available [33], [39], however, these bounds are mathematically intractable for further analytical analysis. In Fig. 2a we plot the required SNR values so that an OS decoder of order \( s \) achieves CEP \( \varepsilon = 10^{-5} \) for a codebook of blocklength \( n = 128 \) and various rates. These values were computed via extensive simulations of the respective codes. For the purpose of comparison, we also show the ergodic capacity in the asymptotic regime with the dashed line and the normal approximation with the solid line. Fig. 2a illustrates the following remarks:

- The performance of OS decoders closely approach \( R(n, \rho, \varepsilon) \) at any rate if \( s \) is sufficiently high.
- As the decoding complexity increases with increasing \( s \), the power penalty required for the desired CEP decreases.
- Conversely, an aggregate latency constraint, which implies a decoding complexity constraint, i.e., an upper bound on the order \( s \), leads to a corresponding power penalty, if a desired CEP is to be guaranteed.

In Fig. 2b the total number of binary operations per information-bit is plotted as a function of the power penalty for order \( s = \{0, 1, 2, 3, 4, 5\} \) where \( n = 128, k = 64 \) and \( n = 64, k = 36 \) codes. Similar numerical results have also been produced for various \( n \) and \( k \) values for fixed \( \varepsilon \) and it has been observed that for all cases, the relation between computational complexity and power penalty can be modeled by a law of the type

\[
F(\Delta \rho) = \log K \left( d(n, k, \rho_r + \Delta \rho, s, \varepsilon) \right)
\]  

(24)

\[
\Delta \frac{1}{a\sqrt{\Delta \rho} + b}
\]  

(25)

with appropriate choices of the constants \( a > 0 \) and \( b > 0 \). This model describes in an accurate and tractable way the trade-off between decoding complexity and power penalty for practical finite-length codes.

Based on extensive numerical simulations, it is observed that for fixed \( n \), the values of \( a \) and \( b \) do not appreciably change as \( k \) varies. Therefore, for simplicity, we assume that \( a \) and \( b \) are fixed for fixed \( n \) and needed to be updated for different \( n \). As \( \Delta \rho \to 0 \), \( F(\Delta \rho) = 1/b \) and therefore \( K(\Delta \rho) = 2/a \) gives the ultimate complexity of an optimum decoder that can achieve the benchmark. Given that \( a \) and \( b \) are strictly positive, \( F(\Delta \rho) \) is a monotonically decreasing function in \( \Delta \rho \).
since
\[ F' = \frac{a}{2\sqrt{\Delta \rho} \left( a\sqrt{\Delta \rho} + b \right)^2} < 0, \] (26)

The monotonicity of \( F(\Delta \rho) \), which follows from (26), is not a convenient choice of the authors. It is an intuitive choice based on Fig. 3b and is a direct consequence of the decoder’s operation. See also the remarks in Section III. Further, (26) reveals that a desired CEP can be achieved with a lower complexity decoder as long as sufficient excess power is available, and vice-versa.

**Lemma 1.** Let a constraint \( L_A < L_M \) with \( L_M > nT_s \) is imposed to a complexity constrained receiver, where the aggregate latency is expressed as (14), the minimum amount of power penalty that is required to guarantee a desired CEP is
\[
\Delta \rho_m = \left( \frac{1}{a} \left[ \left( \log \frac{L_M - nT_s}{kT_b} \right)^{-1} - b \right]^+ \right)^2, \tag{27}
\]
where \([z]^+ = \max\{0, z\}\).

**Remark 1.** Lemma 7 reveals the minimum amount of excess power that is needed in order to fulfill the latency and reliability constraints for a complexity constrained URLLC receiver. From (27), it is clear that for fixed \( n \) and \( T_s \), as \( T_b \) decreases, i.e. more powerful processor is implemented at the receiver, \( \Delta \rho_m \) decreases and hence the gap to the normal approximation shrinks and disappears if \( T_b \leq \frac{L_M - nT_s}{kT_b} \). On the other hand, for fixed \( n \), if the transmission rate, \( r \), increases, \( \Delta \rho_m \) also increases and the gap to the normal approximation widens.

The upper bound on per-information-bit decoder complexity is given in (16). For some fixed \( n \), as \( k \) increases, i.e., at higher rates, the upper bound decreases. Thus, in order to assure this inequality, as \( k \) increases, a simpler decoder, with smaller \( s \), is required, which eventually leads to higher power penalty. Notice that selecting the maximum allowed \( K(d(n, k, \rho, s, \varepsilon)) \) leads to the minimum amount of power penalty, which is introduced in Lemma 1.

**Remark 2.** The trade-off between computational complexity and power penalty for a fixed reliability constraint is modeled in (25) in a simple way. Although (25) was derived based on linear block encoders and OS decoders, results in the literature, [14] [24] [19] [26] Fig. 6.1 to Fig. 6.9, reveal that when it comes to the relation between computational complexity and power penalty in the short block-length regime, other families of codes, such as polar codes, convolutional codes, etc., follow a similar pattern. Hence, it can be advocated that (25) is a useful proxy for the study of URLLC systems with computational complexity constraints.

**B. Maximal Information Rate with Latency Constraints**

From (4), \( r \) is theoretically achievable if \( r \leq R(n, \rho, \varepsilon) \). However, as stated in the previous section, the normal approximation gives the maximum achievable information rate without taking decoding constraints into account. Here, the maximal information rate that can be achievable while having latency, reliability, and complexity constraints, denoted as \( M(n, \rho, \varepsilon) \), is presented.

**Lemma 2.** For a complexity constrained receiver with aggregate latency expressed in (14), the maximal achievable information rate subject to latency, \( L_A < L_M \) with \( L_M > nT_s \), and reliability constraints can be expressed as
\[
M(n, \rho + \Delta \rho_m, \varepsilon) = R(n, \rho, \varepsilon). \tag{28}
\]

**Proof.** For fixed rate and blocklength \( n \) the maximum allowable decoding time can be calculated using (14). This in turn yields the required power penalty \( \Delta \rho \) via (25) which eventually leads to \( \Delta \rho_m \). Finally, according to (27), \( M(n, \rho, \varepsilon) \) can be determined by shifting the normal approximation by \( \Delta \rho_m \) to the right.

**Lemma 3.** \( M(n, \rho, \varepsilon) \) is monotonically increasing in \( \rho \).

**Proof.** Let us introduce the following two maximal rates: \( R(n, \rho^1, \varepsilon) \) and \( R(n, \rho^2, \varepsilon) \). Suppose that \( \rho^2 \geq \rho^1 \), then using the monotonic structure of the channel capacity [40], \( R(n, \rho^2, \varepsilon) \geq R(n, \rho^1, \varepsilon) \), and therefore, using Remark 2, \( \Delta \rho_{m}^2 \geq \Delta \rho_{m}^1 \geq 0 \). Hence, \( M(n, \rho^2, \varepsilon) \geq M(n, \rho^1, \varepsilon) \).

In Fig. 3 the information rate is plotted as a function of the SNR in dB. The dashed line corresponds to the ergodic capacity and the solid line to the normal approximation for \( n = 128 \) and \( \varepsilon = 10^{-5} \). The remaining three plots in the figure correspond to maximal information rate when latency constraints \( L_M = \{10, 1, 0.3\} \) ms are imposed. It is assumed that the symbol interval is \( T_s = 1 \mu s \) and the time required for a binary operation is \( T_b = 1 \) ns. One can see that the achievability bound shifts to the right more as the constraint on time shrinks. It can be also observed that the gap between...
Lemma 4 and Lemma 2 reveal that constraints on aggregate latency and decoding complexity limits the maximal information rate. These results are crucial to understand the capabilities of the communication system and to increase the efficiency. Next, we will discuss some non-trivial optimization problems which affect the efficiency of the communication systems.

V. OPTIMAL COMMUNICATION WITH LATENCY AND DECODING CONSTRAINTS

A. Minimization of Aggregate Latency

We consider the transmission of a packet that contains a fixed number of information bits, $k$, and we are interested in minimizing the aggregate latency, $L_A$, subject to a reliability constraint, i.e., $\varepsilon \leq \varepsilon_m$, and a transmit power constraint, $\rho \leq \rho_m$. Such an optimization problem can be encountered in scenarios of industrial control, where, e.g., a sensor transmits a fixed-precision measurement or a control message out of a list of $2^k$ possible messages. The formulation of the problem follows

$$
\begin{align*}
&\text{minimize } L_A, \\
&s.t. \quad \varepsilon \leq \varepsilon_m, \\
&\quad \rho_r + \Delta \rho \leq \rho_m, \\
&\quad k/n \leq R(n, \rho_r, \varepsilon), \\
&\quad \rho_r \geq 0, \Delta \rho \geq 0, \quad 0 \leq s \leq k, \quad k \leq n.
\end{align*}
$$

Here, it is assumed that the hardware platform is fixed and therefore $T_b$ and $T_s$ are fixed. The optimization variables are $n, \varepsilon, \rho_r, \Delta \rho$, and $s$. (29b) and (29c) represent error rate and power budget constraints, respectively. Lastly, (29d) indicates the maximal achievable rate without decoding complexity constraints, as given by the normal approximation.

**Lemma 4.** The optimum point of (29) is achieved with equality in (29b).

**Proof.** We prove the lemma by contradiction. First of all, for fixed $\rho_r$ and given that $R(n, \rho_r, \varepsilon) \leq R(n, \rho_r, \varepsilon_m)$, the feasible set for $n$ becomes the largest for $\varepsilon = \varepsilon_m$. Also assume that the optimal decoder is $d(n^*, k, \rho_r^*, \Delta \rho^*, s^*, \varepsilon^*)$ with $\varepsilon^* < \varepsilon_m$. For some $\sigma > 0$ small enough we can find a decoder $d(n^*, k, \rho_r^*, \Delta \rho^*, s^* - \sigma, \varepsilon_m)$. However, the complexity of this decoder is smaller than the optimal one and hence achieves a smaller aggregate latency without violating the CEP constraint.

The problem now reduces to

$$
\begin{align*}
&\text{minimize } L_A, \\
&s.t. \quad \rho_r + \Delta \rho \leq \rho_m, \\
&\quad k/n \leq R(n, \rho, \varepsilon), \\
&\quad \rho_r \geq 0, \Delta \rho \geq 0, \quad 0 \leq s \leq k, \quad k \leq n,
\end{align*}
$$

which can be further split into a countable sequence of problems, one for every feasible $n$. Fixing $n$ implies that the rate is also fixed, i.e., $r = k/n$. Hence, the reference SNR, $\rho_r$, follows by solving $r = R(n, \rho, \varepsilon_m)$. It must be noted that a solution to the problem for fixed $n$ can be found only if

$$
\rho_r \leq \rho_m, \quad (31)
$$

else the problem is infeasible for the particular $n$.

Finally, the problem for fixed $n$, when feasible, can be written as

$$
\begin{align*}
&\text{minimize } K(d(n, k, \rho_r + \Delta \rho, s, \varepsilon_m)) \\
&s.t. \quad 0 \leq \Delta \rho \leq \rho_m - \rho_r, \\
&\quad 0 \leq s \leq k,
\end{align*}
$$

or equivalently

$$
\begin{align*}
&\text{maximize } a \sqrt{\Delta \rho} + b \\
&s.t. \quad 0 \leq \Delta \rho \leq \rho_m - \rho_r, \\
&\quad 0 \leq s \leq k,
\end{align*}
$$

which is maximized for $\Delta \rho = \rho_m - \rho_r$. The optimal $s$ is given by the following theorem.

**Theorem 5.** For a given $n$, such that the problem is feasible, the corresponding order $s$ that minimizes $t \_i$ can be closely approximated to

$$
\begin{align*}
\eta &\approx \frac{1}{2} \left( k - \sqrt{k^2 + 2\sqrt{k^2n^4}} \right),
\end{align*}
$$

where $\eta = F(\rho_m - \rho_r) + 1 - \log n$.

**Proof.** $F(\Delta \rho)$ is a monotonic decreasing function in $\Delta \rho$. The complexity of the simplest decoder that meets the constraints can be found while selecting the highest power that is $\rho_r + \Delta \rho = \rho_m$ and the complexity of this decoder is $2^{F(\rho_m - \rho_r)}$.

Finally, (34) can be achieved by using the Appendix and same analogy in (21).

The optimum selection can be found with exhaustive search over all $n$ values. A computationally efficient algorithm, linear in $n$, is proposed in Algorithm 1. A numerical realization of the set of feasible operating points, $S$, is illustrated in Fig. 4a where $k = 64$, $\rho_m = 5$ dB, $\varepsilon_m = 10^{-5}$, $T_b = 1 \mu$s, and $T_s = 1$ ns. Note that no feasible point is identified if the feasibility condition shown in (31) is not met. The optimum, that is shown with a red star, is found by searching along $\rho = \rho_m$.

In Fig. 4b the aggregate latency is plotted as a function of the codeword length, $n$. It can be seen that for small $n$ the codewrate of the selected codebook must be very high. Hence, either the transmission is not possible when the required codewrate exceeds (4) or the required decoder must operate close to the normal approximation, which yields high decoding complexity. This translates to very high aggregate latency. As $n$ increases, the required rate is decreasing, hence it is more likely that it can be supported by the power budget or a rate.
Algorithm 1 Minimization of $L_A$

1. for $n = n_{\min}, n_{\min+1}, \ldots, n_{\max}$ do
2. $r = k/n$
3. compute: $\rho_0$ from (23)
4. compute: $F(\Delta \rho)$ from (25)
5. if $\rho_m \geq \rho_r$ then
6. $K(d(n, k, \rho_m, s, \epsilon_m)) = 2F(\rho_m - \rho_r)$
7. else
8. $K(d(n, k, \rho_m, s, \epsilon_m)) = \emptyset$
9. end if
10. compute: $L_A(n) = nT_s + kK(d(n, k, \rho_m, s, \epsilon_m))T_b$
11. end for
12. Find minimum $L_A$ and select the optimum parameters

If $\rho_m$ is sufficiently far from the normal approximation can be selected. In this case, a decoder with low complexity can be selected and the aggregate latency is dominated by the codeword transmission latency. For power constraints and the aggregate latency is dominated by the codeword transmission rates or rates that are very close to the limits $r \leq \rho_m$, and latency constraints, $L_A \leq L_M$. This optimization problem is significant for communication scenarios where power efficiency is crucial, such as battery powered URLLC systems. A rough analysis may yield the following; minimization of per-information-bit energy is proportional to SNR minimization. However, given that a fixed number of $k$ information bits must be transmitted, low SNR values may either lead to theoretically unachievable transmission rates or rates that are very close to the limits and require very complex decoders which may eventually violate the latency constraint. The optimization problem can be formulated as

\begin{align}
\text{minimize} & \quad e_b \\
\text{s.t.} & \quad \epsilon \leq \epsilon_m, & (35a) \\
& \quad L_A \leq L_M & (35b) \\
& \quad \rho_r + \Delta \rho \leq \rho_m, & (35c) \\
& \quad k/n \leq R(n, \rho_r, \epsilon), & (35d) \\
& \quad \rho_r \geq 0, \quad \Delta \rho \geq 0, \quad 0 \leq s \leq k, \quad k \leq n. & (35e)
\end{align}

where $e_b = (\rho_r + \Delta \rho)/r$ represents the per-information-bit energy and $r = k/n$ is fixed for a selection of $n$. Similar to (29), it is assumed that the hardware platform is fixed and variables are same. In comparison to the problem in Section VA an additional aggregate latency constraint is imposed via (35c).

\[ \text{Algorithm 1 Minimization of } L_A \]

1. for $n = n_{\min}, n_{\min+1}, \ldots, n_{\max}$ do
2. $r = k/n$
3. compute: $\rho_0$ from (23)
4. compute: $F(\Delta \rho)$ from (25)
5. if $\rho_m \geq \rho_r$ then
6. $K(d(n, k, \rho_m, s, \epsilon_m)) = 2F(\rho_m - \rho_r)$
7. else
8. $K(d(n, k, \rho_m, s, \epsilon_m)) = \emptyset$
9. end if
10. compute: $L_A(n) = nT_s + kK(d(n, k, \rho_m, s, \epsilon_m))T_b$
11. end for
12. Find minimum $L_A$ and select the optimum parameters

**B. Minimization of per-Information-Bit Energy**

Here, we consider minimizing the per-information-bit energy consumption, where the transmission contains a fixed number of information bits, subject to reliability, i.e., $\epsilon \leq \epsilon_m$, transmit power, $\rho \leq \rho_m$, and latency constraints, $L_A \leq L_M$. This optimization problem is significant for communication scenarios where power efficiency is crucial, such as battery powered URLLC systems. A rough analysis may yield the following; minimization of per-information-bit energy is proportional to SNR minimization. However, given that a fixed number of $k$ information bits must be transmitted, low SNR values may either lead to theoretically unachievable transmission rates or rates that are very close to the limits and require very complex decoders which may eventually violate the latency constraint. The optimization problem can be formulated as

\[ \text{minimize} \quad e_b \]

\[ \text{s.t.} \quad \epsilon \leq \epsilon_m, \quad L_A \leq L_M, \quad \rho_r + \Delta \rho \leq \rho_m, k/n \leq R(n, \rho_r, \epsilon), \quad \rho_r \geq 0, \quad \Delta \rho \geq 0, \quad 0 \leq s \leq k, \quad k \leq n. \]

where $e_b = (\rho_r + \Delta \rho)/r$ represents the per-information-bit energy and $r = k/n$ is fixed for a selection of $n$. Similar to (29), it is assumed that the hardware platform is fixed and variables are same. In comparison to the problem in Section VA an additional aggregate latency constraint is imposed via (35c).

**Lemma 6.** The optimum point of (35) is achieved with equality in (35b).

**Proof.** Similar to Lemma 4 assume that the optimal decoder is $d(n^*, k, \rho_r^* + \Delta \rho^*, s^*, \epsilon^*)$ with $\epsilon^* < \epsilon_m$. However, for some $\Delta \rho^* \geq \sigma > 0$ small enough one can find a decoder $d(n^*, k, \rho_r^* + \Delta \rho^* - \sigma, s^*, \epsilon_m)$ which requires lower SNR than the optimal one and hence achieves a smaller per-information-bit energy consumption without violating the CEP constraint.

The power constraint in (35d) is directly proportional to $e_b$, and limits it such that $e_b \leq \rho_m/r$. Further, we fix $n$ and split the problem into countable sequence of problems. Now, the rate, $r$, and the reference SNR, $\rho_r$, are also fixed. For a
feasible $n$, that meets (35e) with $\rho_r \geq 0$, the problem (35) now reduces to

$$\begin{align*}
\text{minimize} & \quad \Delta \rho \\
\text{s.t.} & \quad L_A \leq L_M \\
& \quad 0 \leq \Delta \rho \leq \rho_m - \rho_r, \\
& \quad 0 \leq s \leq k.
\end{align*}$$

Without the latency constraint, given in (36b), (31) gives the feasibility condition. However, selecting $\Delta \rho$ closer to 0 corresponds to a decoder with high complexity, which may require longer $L_D$ for complexity constrained receivers and may violate the latency constraint.

Lemma 7. For a feasible $n$, there is a set of feasible solutions if

$$\Delta \rho_m \leq \Delta \rho \leq \rho_m - \rho_r$$

for $\Delta \rho \geq 0$. Thus, the feasibility condition is

$$\rho_r + \Delta \rho_m \leq \rho_m.$$ 

Proof. It is shown in (27) that $\Delta \rho_m$ gives the minimum amount of power penalty that needs to be paid due to the latency constraint for a fixed CEP. Therefore, selecting the minimum excess power as $\Delta \rho_m$, guarantees (37).

Finally, the optimization problem reduces to

$$\begin{align*}
\text{minimize} & \quad \Delta \rho \\
\text{s.t.} & \quad \Delta \rho_m \leq \Delta \rho \leq \rho_m - \rho_r, \\
& \quad 0 \leq s \leq k.
\end{align*}$$

Hence, the objective function is minimized when $\Delta \rho = \Delta \rho_m$ and it is worth noting that this operating point lies on $M(n, \rho, \epsilon_m)$. The corresponding order $s$ is given in (21). An efficient algorithm that solves (35) is shown in Algorithm 2.

Numerical realizations of the feasible sets, $S$, for various $n$ are demonstrated in Fig. 5a for $k = 64$, $\rho_m = 5$ dB. As seen, no feasible point can be identified unless (37) is satisfied. The optimum selection is also depicted with the red star. Notice that the optimum point lies on the $\rho_r + \Delta \rho_m$ line.

Minimum $e_b$ values for different $T_b$ are depicted in Fig. 5b where $L_M = 1$ ms, $T_s = 1$ $\mu$s, $\rho_m = 5$ dB, and $\epsilon_m = 10^{-5}$. The red dotted line represents the power constraint and a selection above that line is infeasible. Minimum $e_b$ values at each $n$ value are depicted for four different receivers such that $T_b = \{1, 0.1, 0.01, 0\}$ ns, where $T_b = 0$ ns represents infinite computation power. Notice that, due to the power constraint, for the receiver with $T_b = 1$ ns, feasible selections exist only in a small portion of $n$ and the minimum is located where $\rho_r + \Delta \rho_m = \rho_m$. For the rest, one can claim that as the hardware capability gets better, i.e. $T_b$ decreases, the optimum selection of $n$ increases whereas optimum $e_b$ decreases.

C. Maximization of Total Transmitted Information Bits

Next, we investigate the following optimization problem: What is the maximum $k$ that can be transmitted subject to
Selected so that the aggregate latency constraint is satisfied.

Similar to the previous optimization problems, here we show that optimum solution is achieved with equality in (40b). The proof is straightforward by using similar analogy that is shown in Lemma 4 and Lemma 6.

Let us first explain the solution to this problem where unlimited computational power is assumed. In this case, a codeword can be decoded instantly and therefore $L_D = 0$ and all the latency budget can be used for transmission of the codeword, i.e., $n_{\text{inf}} = L_M/T_S$ symbols can be transmitted at a rate that is determined by (4), which yields

$$k_{\text{inf}} = \left\lfloor n_{\text{inf}}R\left(n_{\text{inf}}, \rho_m, \varepsilon_m\right) \right\rfloor.$$  

Notice that SNR is chosen to be $\rho_m$ due to the monotonic structure of the channel (40).

However, with decoding complexity constraints the following trade-off arises: If $n$ is selected small, the available duration for decoding can be sufficient so that a high rate code can be used. As $n$ increases, the available duration for decoding shrinks and a code with decreasing coderate must be selected so that the aggregate latency constraint is satisfied. The solution of such a problem for complexity constrained receivers is not trivial and may need a comprehensive search with various parameters.

Without loss of generality, let us first fix $n$ and split the problem into a countable sequence of subproblems. It should be noted that $L_M/T_S$ is an upper bound of $n$. We set $\varepsilon = \varepsilon_m$ and rewrite the problem

maximize $k$  

s.t. $L_A \leq L_M$ \hspace{1cm} (40a)  

$\rho_r + \Delta \rho \leq \rho_m$, \hspace{1cm} (40b)  

$k/n \leq R(n, \rho_r, \varepsilon)$, \hspace{1cm} (40c)  

$\rho_r \geq 0$, $\Delta \rho \geq 0$, $0 \leq s \leq k$, $k \leq n$. \hspace{1cm} (40d)

It is shown in Lemma 7 that the latency constraint in (42b) can be converted to a power penalty constraint. Therefore, it is also shown that the feasibility constraint is $\rho_r + \Delta \rho \leq \rho_m$. Here, we further extend and instead of converting the latency constraint into a power constraint, using Lemma 11 we convert it to a rate constraint. Thus, the problem follows

maximize $k$  

s.t. $\Delta \rho \leq \rho_m - \rho_r$, \hspace{1cm} (43a)  

$k/n \leq M(n, \rho_r, \varepsilon_m)$, \hspace{1cm} (43b)  

$\rho_r \geq 0$, $\Delta \rho \geq 0$, $0 \leq s \leq k$, $k \leq n$. \hspace{1cm} (43c)

Numerical realization of such a problem is demonstrated in Fig. 6a where $n$ is fixed to 128 and $\varepsilon_m = 10^{-5}$, $\rho_m = 7$ dB, $L_M = 1$ ms, $T_S = 1 \mu$s, and $T_b = 1$ ns. The feasible set is shown with $S(n)$. Notice that, due to Lemma 3 the sub-optimum rate-power selection is the topmost point of the set $S(n)$, which is also the junction point of $M(n, \rho, \varepsilon_m)$ and $\rho = \rho_m$, that is $M(n, \rho_m, \varepsilon_m)$. Hence, the solution to the optimization problem in (40) follows

$$k_{\text{opt}} = \left\lfloor n_{\text{opt}} M(n_{\text{opt}}, \rho_m, \varepsilon_m) \right\rfloor.$$  

where $n_{\text{opt}}$, the optimum $n$ that maximizes $k$, follows

$$n_{\text{opt}} = \arg \max_{\{n \in \mathbb{N}\}} n M(n, \rho_m, \varepsilon_m).$$  

A computationally efficient algorithm, linear in $n$, is proposed in Algorithm 3.

Algorithm 3 Maximization of $k$

1: for $n = n_{\text{min}}, n_{\text{min}}+1, \ldots, n_{\text{max}}$ do  
2: compute: $R(n, \rho, \varepsilon_m)$ using (4)  
3: compute: $\Delta \rho_m$, using (28), $\forall n \in (0, 1]$  
4: compute: $M(n, \rho, \varepsilon_m)$ using (28)  
5: compute: $k(n) = \left\lfloor n M(n, \rho_m, \varepsilon_m) \right\rfloor$  
6: end for  

7: Find maximum $k$ and select the optimum parameters

In Fig. 6b numerical results that correspond to the investigated scenario are plotted for $L_M = 1$ ms, $\rho_m = 7$ dB, and $\varepsilon_m = 10^{-5}$. Four different choices for execution times for a binary operation are shown: $T_b = \{1, 0.1, 0.001, 0\}$ ms. The previously introduced trade-off is clear here and the maximums appear at $n_{\text{opt}} = \{227, 381, 734, 1000\}$, respectively. Corresponding $k_{\text{opt}}$ values are $k_{\text{opt}} = \{96, 169, 393, 901\}$. Ratios of $k_{\text{opt}}$ values found for complexity constrained receivers to the $k_{\text{opt}}$ of infinite computation power receiver are $\approx 0.1, 0.18, 0.43$, respectively. Thus, one can conclude that if complexity constraints and decoding duration are taken into account, one can see that, depending on the receiver capabilities, the maximum achievable values are much less than the theoretical limits.

VI. CONCLUSIONS

The aggregate latency caused by codeword transmission and decoding is the main focus in this study. A model that can accurately show the trade-off between complexity of OS decoders versus their power gap to the optimal decoder is proposed. Maximal achievable transmission rates under stringent latency and computational complexity constraints are
true that
\[
1 = \left( \frac{s}{k} + 1 - \frac{s}{k} \right)^k \\
\geq \sum_{i=0}^{s} \left( \frac{k}{i} \right) \left( \frac{s}{k} \right)^i \left( 1 - \frac{s}{k} \right)^{k-i} .
\]  
(47)

Define
\[
A_i = \left( \frac{s}{k} \right)^i \left( 1 - \frac{s}{k} \right)^{k-i} 
\]  
for \( i \in [0, s] \). Then
\[
\log A_i = i \log \left( \frac{s}{k} \right) + (k-i) \log \left( 1 - \frac{s}{k} \right) \\
\geq s \log \left( \frac{s}{k} \right) + (k-s) \log \left( 1 - \frac{s}{k} \right) \\
= -kh \left( \frac{s}{k} \right) ,
\]  
(51)
where the inequality in (50) is due to the fact that for \( \frac{s}{k} \leq \frac{1}{2} \), \( \log \left( \frac{s}{k} \right) \leq \log \left( 1 - \frac{s}{k} \right) \). Finally, the inequalities in (47) and (50) yield
\[
2kh \left( \frac{s}{k} \right) \geq \sum_{i=0}^{s} \left( \frac{k}{i} \right) .
\]  
(52)

Hence,
\[
K(d(n, k, \rho, s, \varepsilon)) \leq \frac{k^2}{8} + \frac{n}{2} 2kh \left( \frac{s}{k} \right) ,
\]  
(53)
and gets tighter with larger values of \( s \).

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