Abstract

As it is well known, the Minkowski vacuum appears thermally populated to a quantum mechanical detector on a uniformly accelerating course. We investigate how this thermal radiation may contribute to the classical nature of the detector’s trajectory through the criteria of decoherence. An uncertainty-type relation is obtained for the detector involving the fluctuation in temperature, the time of flight and the coupling to the bath.

1 Introduction

In the Copenhagen interpretation of the measurement process, the existence of classical systems is postulated from the start. In quantum cosmology however, the whole universe receives a quantum mechanical description and classicality must be somehow derived. An important mechanism by which some degrees of freedom may behave classically is the environment-induced decoherence. In that case, partial diagonalization of the density matrix in a given basis is obtained by integrating out some unobserved degrees of freedom. A simple illustration is given by a system and its environment having each a single degree of freedom described by the actions $S_{sys}[x]$ and $S_{env}[y]$ respectively, with interaction described by the action $S_{int}[x,y]$. Starting with an uncorrelated density matrix at $t=0$,

$$\rho(x_1,y_1,x_2,y_2; t=0) = \rho_{sys}(x_1,x_2; t=0) \times \rho_{env}(y_1,y_2; t=0),$$ (1)

then at a time $t$ later, the reduced density matrix (where the environment is integrated out) will be given by:

$$\rho_{red}(x_1^f,x_2^f;t) = \int dx_1^i \int dx_2^i P(x_1^f,x_2^f,t|x_1^i,x_2^i,0) \rho_{sys}(x_1^i,x_2^i;0),$$ (2)

with the kernel

$$P(x_1^f,x_2^f,t|x_1^i,x_2^i,0) = \int_{x_1^i}^{x_1^f} Dx_1 \int_{x_2^i}^{x_2^f} Dx_2 e^{i(S_{sys}[x_1] - S_{sys}[x_2])} e^{i\Gamma[x_1,x_2]},$$ (3)

where the boundary conditions on the functional integrals are given for times 0 and $t$ (our units are such that $\hbar = c = 1$). $\Gamma[x_1,x_2]$ is the influence functional (IF). It is a property of the environment and the way it is coupled to the system. For an environment in a pure state at $t = 0$, it takes the conceptually simple form

$$e^{i\Gamma[x_1,x_2]} = \langle \psi_2(t)|\psi_1(t) \rangle.$$ (4)
Here, $|\psi_1(t)\rangle$ is the state that evolved from the initial state at $t = 0$ under the dynamics dictated by $S_{\text{env}}[y_1] + S_{\text{int}}[x_1, y_1]$ in which $x_1$ acts as a c-number, time dependent source; likewise $|\psi_2(t)\rangle$ is governed by $S_{\text{env}}[y_2] + S_{\text{int}}[x_2, y_2]$. For paths $[x_1(t)]$ and $[x_2(t)]$ such that the states in (4) are not adiabatically disturbed, $\Gamma$ will have a (positive) imaginary part $\Gamma_I$. It is clear that if $\Gamma_I$ is large for pair of paths $(x_1(t), x_2(t))$ that are far apart from one another in spacetime, then the contribution of these pairs will be suppressed in (3). As a result, $\rho_{\text{red}}(x_1^f, x_2^f; t)$ will be more diagonal. Decoherence can thus be studied through $\Gamma_I$.

In recent times, the study of black holes received renewed interest, in part due to the introduction of more tractable two dimensional models. An important issue arising in that context is the limit of the semiclassical approximation, in which the Hawking radiation is derived. As part of the groundwork to that problem, we report here on the related issue of the decoherence of a detector in uniform acceleration. In the next section, we review how the IF predicts the heating of the scalar field forming the environment of a detector on a uniformly accelerating course. In Section 3 the spacetime trajectory is taken to be a decohered spacetime path, with a spread around the uniformly accelerating trajectory, and taking now the detector’s excitation to be fixed. We then obtain an uncertainty-type relation involving the spread in acceleration involved in constructing the approximately classical path, the time of flight and the coupling to the thermal bath.

## 2 Unruh Effect

Consider an environment made of a massless scalar field in two spacetime dimensions. In a finite box of spatial dimension $L$, the mode expansion reads

$$\phi(x, t) = \sqrt{\frac{2}{L}} \sum_k [y_k^+ \cos kx + y_k^- \sin kx],$$

with $k = 2\pi n/L$ with $n = 1, 2, \ldots$. The kinetic term is

$$L_{\text{env}} = \int dx \left( \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - \frac{1}{2} \mu \sum_{\sigma=+,-} \sum_k [(\dot{y}_k^\sigma)^2 - k^2 (y_k^\sigma)^2] \right),$$

where dots denote time derivative. The system is formed by a detector that is linearly and locally sensitive to the matter field with an interacting Lagrangian density

$$L_{\text{int}}(x) = -\varepsilon Q \phi(x, t) \delta(x - X(t)), $$

where $\varepsilon$ is a coupling constant while $Q(t)$ is the DeWitt monopole moment and plays the role of $x(t)$ in Section 1 while $X(t)$ is the trajectory of the detector. For a uniform acceleration $a$, $X(\tau) = t_0 \cosh(a\tau)$ and $t(\tau) = \frac{1}{a} \sinh(a\tau)$ where $\tau$ is the proper time and the resulting IF is given by

$$\Gamma_R[Q_1, Q_2] = -\int_0^t ds_1 \int_0^{s_1} ds_2 [Q_1(s_1) - Q_2(s_1)] \mu(s_1, s_2) [Q_1(s_2) + Q_2(s_2)],$$

$$\Gamma_I[Q_1, Q_2] = \int_0^t ds_1 \int_0^{s_1} ds_2 [Q_1(s_1) - Q_2(s_1)] \nu(s_1, s_2) [Q_1(s_2) - Q_2(s_2)],$$

(8)
\[
\mu(\tau_1, \tau_2) = -\int_0^\infty d\omega \frac{\varepsilon^2}{2\pi\omega} \sin(\omega(\tau_1 - \tau_2)),
\]
\[
\nu(\tau_1, \tau_2) = \int_0^\infty d\omega \frac{\varepsilon^2}{2\pi\omega} \coth \frac{\pi\omega}{a} \cos(\omega(\tau_1 - \tau_2))
\]
are respectively the noise and dissipation kernels arising for the same detector at rest in a bath of thermal oscillators at temperature \( T = a/2\pi \).

3 Fluctuation in acceleration

In displaying the Unruh effect in the last section, it was assumed that the position degree of freedom of the detector was not quantum mechanical, as the detector followed a determined trajectory. From a path integral point of view, this treatment would require that the uniformly accelerating trajectory sufficiently decohere from other neighboring paths to form a consistent history. But of course the decoherence is always finite at a given time. We now consider the uncertainty associated with the fluctuations in the detector’s paths around the uniformly accelerating trajectory. For simplicity, we assume that the value of the monopole \( Q \) is given by its quantum mechanical average, supposed to be constant in time. To study the extent to which the acceleration is well defined, we consider two uniformly accelerating trajectories with slightly different accelerations: \( x_i = \sqrt{a_i - t^2} \) with \( i = 1, 2 \). For the scalar field in (5) with interaction in (7), the sum over modes can be converted into an integral, and we then find

\[
\Gamma_I(a_1, a_2, t) = \frac{\varepsilon^2 Q^2}{8\pi} \int_0^t ds_1 \int_0^{s_1} ds_2 P(s_1, s_2, a_1, a_2),
\]

where

\[
P(s_1, s_2, a_1, a_2) \equiv \ln \left( \frac{C^2 - F^2}{A^2 - F^2} \frac{D^2 - F^2}{B^2 - F^2} \right)^2.
\]

and

\[
A \equiv \sqrt{a_1^{-2} + s_1^2} - \sqrt{a_1^{-2} + s_2^2}, \quad B \equiv \sqrt{a_2^{-2} + s_1^2} - \sqrt{a_2^{-2} + s_2^2},
\]
\[
C \equiv \sqrt{a_1^{-2} + s_1^2} - \sqrt{a_2^{-2} + s_2^2}, \quad D \equiv \sqrt{a_2^{-2} + s_1^2} - \sqrt{a_1^{-2} + s_2^2},
\]
\[
F \equiv s_1 - s_2.
\]

We now wish to extract the features of \( \Gamma_I \) for paths lasting a time large compared with the inverse accelerations \( (ta_1 \gg 1, ta_2 \gg 1) \). Consider the rate of increase of \( \Gamma_I \) with time. From (10),

\[
\dot{\Gamma}_I(t, a_1, a_2) = \frac{\varepsilon^2 Q^2}{8\pi} t R(t, a_m, a_d),
\]

with:

\[
R(t, a_m, a_d) = \int_0^1 dy P(s_1 = t, s_2 = yt, a_1, a_2) = R(\tilde{t}, \tilde{a}_d).
\]
In (14), we made the change of variable $s_2 = yt$ ($y$ is unitless), and worked with the average $a_m \equiv \frac{1}{2}(a_1^{-1} + a_2^{-1})$ and difference $a_d \equiv (a_1^{-1} - a_2^{-1})$ in inverse accelerations. In the last line of (14), $a_m$ was used for the scale factor in $P$, namely: $\tilde{t} \equiv t/a_m$ and $\tilde{a}_d \equiv a_d/a_m$. Now except for a transient time $\tilde{t} \lesssim 1$, we have $R(\tilde{t}, \tilde{a}_d) \to f(\tilde{a}_d)$ that is, $R$ is time independent to a good approximation. (Obviously, $f(\tilde{a}_d)$ is an even function.) This can be seen in Fig. 1, where numerical evaluations of $R$ are given as a function of $\tilde{t}$ for a few representative values of $\tilde{a}_d$. (The apparently large value for small $\tilde{t}$ is damped to a finite and narrow peak by the factor $t$ in (13).) The function $f(\tilde{a}_d)$ may be evaluated numerically, and for small $\tilde{a}_d$ is found to be nearly quadratic.

With (13), we thus have for $|\tilde{a}_d| \ll 1$ and $\tilde{t} \gg 1$ the excellent approximation

$$\Gamma_I(t, a_1, a_2) \approx \frac{N}{16\pi} \varepsilon^2 \langle Q \rangle^2 t^2 \left( \frac{a_1 - a_2}{(a_1 + a_2)/2} \right)^2,$$

(15)

where $Q$ was replaced by its quantum mechanical average (quantum fluctuations of $Q$ are neglected) and $N$ is a numerical constant. We now take the condition $\Gamma_I \approx 1$ as separating the decohering and non-decohering accelerations. Using (13) and the relation $T = a/2\pi$ between temperature and acceleration, we conclude that a detector on a decohered accelerating trajectory for a time $t$ will be subject to thermal fluctuations of the order:

$$\left| \frac{\Delta T}{T} \right| \approx \frac{1}{|\varepsilon| \langle Q \rangle t},$$

(16)

where we dropped a constant of order one.

4 Discussion

Eq. (16) can be understood as an uncertainty relation between the ‘time of flight’ of the detector and the spread in temperature, with the uncertainty
given by $\frac{1}{|\langle Q \rangle|}$ and valid for $t/T > 1$. In Eq. (15), the quadratic dependence of the decoherence on the coupling to the bath is general for a system coupled linearly to the bath (by construction, the amplitude of the detector’s excitations act to modify the effective coupling). It is interesting to see that for the class of paths considered here, the dependence in time is also quadratic. This shows that the time is also making the coupling constant stronger or inversely, that a larger coupling constant is identical to a longer time evolution. Qualitatively, this is in line with the result of Ref. [10]. As the coupling to the bath gets stronger and that one waits longer, clearly more particles are created and decoherence is increased. This can also be seen through (4). When particles are copiously produced, a slight difference in the two paths will easily make the two states orthogonal.

Issues of decoherence similar to the ones considered here could well prove to be relevant in the context of black holes. As it is well known, one of the distinguishing features of black holes is their ability to Hawking radiate, in close connection with the uniformly accelerating detector. The environment given by these radiated particles could then serve in the study of the validity of the semi-classical approximation. This issue is currently under investigation [11].

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**References**

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