Mapping relativistic to ultra/non-relativistic conformal symmetries in 2D and finite $\sqrt{TT}$ deformations

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ABSTRACT: The conformal symmetry algebra in 2D (\text{Diff}(S^1) \oplus \text{Diff}(S^1)) is shown to be related to its ultra/non-relativistic version (BMS\textsubscript{3} \cong \text{GCA}\textsubscript{2}) through a nonlinear map of the generators, without any sort of limiting process. For a generic classical CFT\textsubscript{2}, the BMS\textsubscript{3} generators then emerge as composites built out from the chiral (holomorphic) components of the stress-energy tensor, $T$ and $\bar{T}$, closing in the Poisson brackets at equal time slices. Nevertheless, supertranslation generators do not span Noetherian symmetries. BMS\textsubscript{3} becomes a bona fide symmetry once the CFT\textsubscript{2} is marginally deformed by the addition of a $\sqrt{TT}$ term to the Hamiltonian. The generic deformed theory is manifestly invariant under diffeomorphisms and local scalings, but it is no longer a CFT\textsubscript{2} because its energy and momentum densities fulfill the BMS\textsubscript{3} algebra. The deformation can also be described through the original CFT\textsubscript{2} on a curved metric whose Beltrami differentials are determined by the variation of the deformed Hamiltonian with respect to $T$ and $\bar{T}$. BMS\textsubscript{3} symmetries then arise from deformed conformal Killing equations, corresponding to diffeomorphisms that preserve the deformed metric and stress-energy tensor up to local scalings. As an example, we briefly address the deformation of N free bosons, which coincides with ultra-relativistic limits only for $N = 1$. Furthermore, Cardy formula and the S-modular transformation of the torus become mapped to their corresponding BMS\textsubscript{3} (or flat) versions.
1 Introduction

Conformal symmetries enhance those of special relativity, and become pivotal in the description of generic relativistic systems enjoying scale invariance; see e.g., [1, 2]. Conformal field theories (CFT’s), built in terms of these extended symmetries, are well-known to play a fundamental role in a broad variety of subjects [3–8]. Their power turns out to be particularly impressive in two spacetime dimensions, as a direct consequence of the fact that the conformal group exceptionally becomes infinite-dimensional. The conformal algebra in 2D is described by two copies of the Witt or centerless Virasoro algebra, being isomorphic to two copies of the algebra of diffeomorphisms on the circle \((\text{Diff}(S^1) \oplus \text{Diff}(S^1))\), spanned by

\[
[L_m, L_n] = (m - n) L_{m+n} \ , \quad [\bar{L}_m, \bar{L}_n] = (m - n) \bar{L}_{m+n} ,
\]

with \([L_m, \bar{L}_n] = 0\) and \(m, n \in \mathbb{Z}\).

Another interesting accident that occurs in 2D is that the ultra and non-relativistic limits of the conformal algebra are isomorphic (see e.g., [9]). Intuitively, ultra/non-relativistic limits are such that the light cone tends to shrink towards the vertical/horizontal axis, and so one limit can be attained from the other by swapping the role of time and space coordinates. As an additional curiosity, the ultra/non-relativistic algebra also becomes isomorphic to the Bondi-Metzner-Sachs one in 3D without central extensions [10–12]. In other words, the so-called Galilean Conformal and Conformal Carrollian algebras in 2D turn out to be isomorphic to the BMS\(_3\) algebra (GCA\(_2\) \(\cong\) CCA\(_2\) \(\cong\) BMS\(_3\)), given by the semidirect sum of the Witt algebra and supertranslations:

\[
[J_m, J_n] = (m - n) J_{m+n} \ , \quad [J_m, P_n] = (m - n) P_{m+n} ,
\]

where \([P_m, P_n] = 0\). The algebra (1.2) and its centrally-extended version appeared long ago in the context of the tensionless limit of string theory [13–21], and more recently in the “flat” analog of Liouville theory [22, 23] as well as in fluid dynamics and integrable systems in 2D [24–26]. It also plays a leading role for nonrelativistic and flat holography [27–40], and it emerges from the spacetime structure near generic horizons [41–55]. Induced, coadjoint and unitary representations have also been developed in [56–59].

It is worth emphasizing that the conformal algebra in 2D (1.1) and the BMS\(_3\) algebra (1.2) are not isomorphic. Nevertheless, the latter can be obtained from the former through suitable In\(\text{o}n\-Wigner contractions. Indeed, changing the basis of the conformal algebra (1.1) according to

\[
P_m = \ell^{-1} (L_m + \bar{L}_{-m}) \ , \quad J_m = L_m - \bar{L}_{-m} ,
\]

one recovers the BMS\(_3\) algebra (1.2) in the limit \(\ell \to \infty\) (see e.g., [23, 82]). Alternatively, the following change of basis: \(P_m = \ell (L_m - \bar{L}_m)\), \(J_m = L_m + \bar{L}_m\) yields the same result provided that \(\ell \to 0\) [27, 83]. The parameter \(\ell\) can then be naturally identified with the inverse of the speed of light.

\(^1\)The algebra (1.2) also manifests as nonlocal symmetries of a massless Klein-Gordon field in 3D [60]. Different extensions of the BMS\(_3\) algebra have been constructed in [61–81].
2 Map between relativistic and ultra/non-relativistic conformal algebras in 2D

Intriguingly, the conformal symmetry algebra in 2D (1.1) can be shown to be related to its ultra/non-relativistic version (1.2) by means of a precise nonlinear map of the generators, without the need of performing any sort of limiting process.

In order to explicitly see the map it is useful to work in the continuum, so that the generators of the conformal algebra (1.1) can be traded by two arbitrary periodic functions defined on the circle, according to

\[ L_m = \int d\phi \bar{T} (\phi) e^{-im\phi}, \quad \bar{L}_m = \int d\phi T (\phi) e^{im\phi}. \]

Thus, the conformal algebra is equivalently expressed as

\[
\{ T (\phi), T (\theta) \} = (2 T (\phi) \partial_\phi + \partial_\phi T (\phi)) \delta (\phi - \theta), \\
\{ \bar{T} (\phi), \bar{T} (\theta) \} = - (2 \bar{T} (\phi) \partial_\phi + \partial_\phi \bar{T} (\phi)) \delta (\phi - \theta),
\]

with \( \{ T (\phi), \bar{T} (\theta) \} = 0 \), and \( [\cdot, \cdot] = i \{\cdot, \cdot\} \). Note that the continuous version of the conformal algebra (2.1) can be naturally interpreted as a Poisson structure.

The searched for mapping is then defined as follows

\[
P = T + \bar{T} + 2 \sqrt{T \bar{T}} , \quad J = T - \bar{T},
\]

so that the corresponding brackets involving \( J \) and \( P \) can be readily found by virtue of the “fundamental” ones in (2.1), which exactly reproduce the continuous version of the BMS3 algebra, given by

\[
\{ J (\phi), J (\theta) \} = (2 J (\phi) \partial_\phi + \partial_\phi J (\phi)) \delta (\phi - \theta), \\
\{ J (\phi), P (\theta) \} = (2 P (\phi) \partial_\phi + \partial_\phi P (\phi)) \delta (\phi - \theta),
\]

with \( \{ P (\phi), P (\theta) \} = 0 \). In Fourier modes, \( J_m = \int d\phi J (\phi) e^{im\phi}, \quad P_m = \int d\phi P (\phi) e^{im\phi}, \) the algebra (2.3) then reduces to (1.2).

Bearing in mind that supertranslation generators are defined up to a global scale factor, making \( P \rightarrow \alpha P \) with constant \( \alpha \) in the map (2.2), yields the same result. Thus, for simplicity and later convenience, we keep assuming \( \alpha = 1 \) afterwards.

In sum, the nonisomorphic conformal and BMS3 algebras, in (2.1) and (2.3) respectively, are nonlinearly related by virtue of the map defined through (2.2), and it is worth highlighting that that no limiting process is involved in the mapping.

3 BMS3 generators within CFT2

The mapping in (2.2) naturally makes one wondering about how precisely the BMS3 algebra manifests itself for a generic (nonanomalous) classical CFT2. Indeed, the mapping directly prescribes a way in which BMS3 generators emerge as composites of those of the conformal symmetries. Nonetheless, it can be shown that the composite generators do not span a Noetherian symmetry of the CFT2.

In order to see that, let us consider a generic CFT2 on a cylinder. In the conformal gauge, using null coordinates \( x = t + \phi \) and \( \bar{x} = t - \phi \), the canonical generators of the conformal symmetries are given by (see e.g., [1, 2])
\[ Q_{\text{CFT}} [\epsilon, \bar{\epsilon}] = \int d\phi (\epsilon T + \bar{\epsilon} \bar{T}) \quad , \]  
(3.1)

being conserved (\( \dot{Q}_{\text{CFT}} = 0 \)) by virtue of the (anti-)chirality of the components of the stress-energy tensor and the parameters (\( \partial \bar{T} = \dot{\partial} \bar{T} = \partial \bar{\epsilon} = \dot{\partial} \bar{\epsilon} = 0 \)). The transformation laws of \( T \) and \( \bar{T} \) then read from the conformal algebra (2.1), since \( \delta \eta_i Q_{\text{CFT}} [\eta_2] = \{ Q_{\text{CFT}} [\eta_2], Q_{\text{CFT}} [\eta_1] \} \) with \( \eta_i = (\epsilon_i, \bar{\epsilon}_i) \), so that

\[ \delta T = 2T \partial \epsilon + \partial T \epsilon \quad , \quad \delta \bar{T} = 2\bar{T} \partial \bar{\epsilon} + \partial \bar{T} \bar{\epsilon} \quad . \]  
(3.2)

The nonlinear map (2.2) implies that the generators (3.1) and transformation laws (3.2), can be expressed as

\[ Q_{\text{CFT}} [\epsilon, \bar{\epsilon}] = \int d\phi (\epsilon J + \epsilon P) \quad , \]  
(3.3)

\[ \delta P = 2P \epsilon'_J + P' \epsilon_J \quad , \quad \delta J = 2P \epsilon'_P + P' \epsilon_P + 2J \epsilon'_J + J' \epsilon_J \quad , \]  
(3.4)

where prime stands for \( \partial \phi \), while the parameters \( \epsilon_J, \epsilon_P \), relate to \( \epsilon \) and \( \bar{\epsilon} \) through

\[ \epsilon = \epsilon_J + \left( 1 + \sqrt{\frac{T}{\bar{T}}} \right) \epsilon_P \quad , \quad \bar{\epsilon} = -\epsilon_J + \left( 1 + \sqrt{\frac{T}{\bar{T}}} \right) \epsilon_P \quad . \]  
(3.5)

Thus, the generators and transformation laws in (3.3), (3.4), acquire the expected form of those for the BMS\(_3\) algebra (see e.g., [22, 25])\(^2\).

Legitimate BMS\(_3\) generators are obtained when the parameters \( \epsilon, \bar{\epsilon} \) are no longer chiral, but instead, being manifestly state-dependent according to (3.5). Hence, at fixed time slices, the parameters \( \epsilon_J, \epsilon_P \) can be consistently assumed to be state-independent arbitrary functions, so that the Poisson brackets of the generators

\[ \dot{Q} [\epsilon, \epsilon_P] = \int d\phi (\epsilon J + \epsilon_P P) \quad , \]  
(3.6)

clearly close according to the BMS\(_3\) algebra by virtue of (2.3).

It is worth emphasizing that since \( J \) stands for the momentum density, superrotation generators yield the corresponding conserved charges. Nevertheless, supertranslation generators are not conserved, as it can be seen from the time evolution of \( P \), that can be obtained from that of the (anti-)chiral \( T \) and \( \bar{T} \) by virtue of the map (2.2), given by

\[ \dot{P} = 2J' - J (\log P)' \quad . \]  
(3.7)

Therefore, supertranslations do not correspond to Noetherian symmetries of the CFT\(_2\).

\(^2\) A warning note is in order: if the parameters \( \epsilon \) and \( \bar{\epsilon} \) were still assumed to be chiral, this would be just a mirage; because in that case, the new ones, \( \epsilon_J, \epsilon_P \), would become state-dependent, and hence, this would only amount to an alternative way of expressing the original conformal algebra generators and transformation laws in (3.1), (3.2), in terms of different variables.
4 BMS$_3$ symmetries from $\sqrt{TT}$ deformations

According to the map (2.2), the supertranslation density $P$ can be seen as a finite nontrivial marginal deformation of the CFT$_2$ energy density $H = T + \bar{T}$. Hence, a simple way to achieve conservation of supertranslations consists in deforming the original Hamiltonian of the CFT$_2$ to coincide with the supertranslation generator. Thus, starting from the CFT$_2$ in the conformal gauge, the simplest deformation is implemented through the Hamiltonian density $\tilde{H} = H + 2\sqrt{TT} = P$, so that the deformed action reads

$$\tilde{I} = I_{CFT} - \int dx d\bar{x} \sqrt{TT} .$$

(4.1)

Note that since only the Hamiltonian was deformed, the Poisson brackets remain the same as those of the original CFT$_2$ in (2.1). Hence, the time evolution of supertranslation and superrotation densities can be readily obtained from (2.3)

$$\dot{\tilde{P}} = \{P, \tilde{\mathcal{H}}\} = 0 , \quad \dot{\tilde{J}} = \{J, \tilde{\mathcal{H}}\} = P',$$

with $\tilde{\mathcal{H}} = \int d\phi P$; so that the canonical BMS$_3$ generators (3.6) are now manifestly conserved ($\dot{\tilde{Q}} = 0$) provided that the parameters fulfill $\dot{\epsilon}_P = \epsilon'_J$ and $\dot{\epsilon}_J = 0$, being apparently state independent.

Therefore, the BMS$_3$ generators (3.6) span a bona fide Noetherian symmetry of the deformed action (4.1).

It is also worth pointing out that the deformed theory (4.1) retains the integrability properties of the original CFT$_2$, since the universal enveloping algebra of BMS$_3$ also contains an infinite number of independent commuting (KdV-like) charges [25].

For a generic gauge choice, the deformation (4.1) can be written as

$$\tilde{I} = I_{CFT} - \int d^2x \sqrt{\det T_{\mu\nu}} .$$

(4.2)

where it is implicitly assumed that $I_{CFT}$ is written in Hamiltonian form, and $T_{\mu\nu}$ stands for the stress-energy tensor of the undeformed CFT$_2$. Remarkably, the action (4.2) keeps being invariant under diffeomorphisms and local scalings, but it is no longer a CFT$_2$ because the energy and momentum densities of the deformed theory yield to generators that fulfill the BMS$_3$ algebra (2.3) instead of the conformal one in (2.1).

In order to see that, let us consider the original CFT$_2$ in a generic (non-conformal) gauge, so that in a local patch, the two-dimensional metric can be brought to the same conformal class as the following one c.f., [108]

$$ds^2 = -N^2 dt^2 + \left( d\phi + N^0 dt \right)^2 ,$$

(4.3)

\footnote{This last property resembles that of the $TT$ deformation [84–86] (being widely studied in e.g., [87–107]); nonetheless, some differences must be stressed. Indeed, in that case the conformal weight of the deformation implies that it is an irrelevant one, also depending on a single continuous parameter, and where $\bar{T}$ and $T$ stand for those of the deformed theory; while in our case, they correspond to those of the original CFT$_2$ and yield to a rigid finite marginal deformation.}
where \( N \) and \( N^\phi \) stand for the lapse and shift functions, respectively\(^4\). The total Hamiltonian of the CFT\(_2\) then reads

\[
\mathcal{H}_{CFT} = \int d\phi \left[ N (T + \overline{T}) + N^\phi (T - \overline{T}) \right] = \int d\phi \left( N H + N^\phi J \right). \tag{4.4}
\]

The deformation in (4.2) has the net effect of deforming the energy density of the CFT\(_2\) to be that of a supertranslation, i.e., \( H \rightarrow P \), so that the total Hamiltonian deforms as \( \mathcal{H}_{CFT} \rightarrow \tilde{\mathcal{H}} \), with

\[
\tilde{\mathcal{H}} = \int d\phi \left( NP + N^\phi J \right). \tag{4.5}
\]

Supertranslation and superrotation densities evolution is then spanned by the deformed Hamiltonian \( \tilde{\mathcal{H}} \), which by virtue of (2.3) reads

\[
\dot{P} = \{ P, \tilde{\mathcal{H}} \} = 2PN^\phi + P'N^\phi , \\
\dot{J} = \{ J, \tilde{\mathcal{H}} \} = 2PN' + P'N + 2JN^\phi + J'N^\phi . \tag{4.6}
\]

In absence of global obstructions, the canonical generators become expressed as an integral over the spatial circle precisely as in (3.6), but now being conserved provided that the state-independent parameters fulfill

\[
\dot{\epsilon}_P = N \epsilon'_J - N' \epsilon_J + N^\phi \epsilon'_P - N^{\phi'} \epsilon_P , \quad \dot{\epsilon}_J = N^\phi \epsilon'_J - N^{\phi'} \epsilon_J . \tag{4.7}
\]

Thus, the transformation law of supertranslation and superrotation densities is given by (3.4), corresponding to Noetherian BMS\(_3\) symmetries.

5 Geometric aspects

Since the deformed action is manifestly invariant under diffeomorphisms \( \xi = \xi^\mu \partial_\mu \), it is reassuring to verify that the Noether current \( j^\mu = \tilde{T}^\mu_\nu \xi^\nu \), with

\[
\tilde{T}^\mu_\nu = \begin{pmatrix}
NP + N^\phi J \\
-N^\phi (N^\phi J + 2NP) - (NP + N^\phi J)
\end{pmatrix}, \tag{5.1}
\]

is conserved (\( \partial_\mu j^\mu = 0 \)) provided that the evolution equations of the energy and momentum densities (4.6), as well as those of the parameters in (4.7) hold. The precise form of the diffeomorphisms is then identified as

\[
\xi^\mu = N^{-1} \left( \epsilon_P, N \epsilon_J - N^\phi \epsilon_P \right) , \tag{5.2}
\]

which close in the Lie brackets, \([\xi_1, \xi_2] = \xi_3\), with

\[
\epsilon^3_P = \epsilon^1_j \left( \epsilon^2_P \right)' + \epsilon^1_P \left( \epsilon^2_j \right)' - (1 \leftrightarrow 2) , \quad \epsilon^3_J = \epsilon^1_j \left( \epsilon^2_j \right)' - (1 \leftrightarrow 2) , \tag{5.3}
\]

according to the BMS\(_3\) algebra when the parameters \( \epsilon^i_P, \epsilon^i_J \) obey (4.7).

\(^4\)In null (holomorphic) coordinates, this would amount to switch on the Beltrami differentials.
Note that one might be tempted to extract an stress-energy tensor $\Theta^\mu_\nu$ from the corresponding density in (5.1) by making use of the metric of the undeformed theory $g^\mu_\nu$ in (4.3), according to $\tilde{T}^\mu_\nu = \sqrt{-\tilde{g}} \Theta^\mu_\nu$. However, this tensor is not conserved ($\nabla_\mu \Theta^\mu_\nu \neq 0$), reflecting the fact that the metric the of CFT$_2$ is not preserved under BMS$_3$ diffeomorphisms $\xi^\mu$ up to a local scaling, i.e.,

$$\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu - \lambda g^\mu_\nu \neq 0 .$$  

Hence, the metric of the undeformed CFT$_2$ is not a suitable object to describe the geometric properties of the deformed theory.

An appropriate Riemannian metric for the geometric description of the deformation is obtained as follows. Note that the total deformed Hamiltonian (4.5) is a homogeneous functional of $\tilde{T}$ and $\bar{T}$ of degree one, so that it fulfills the following identity

$$\tilde{\mathcal{H}} = \int d\phi \left( \frac{\delta \tilde{\mathcal{H}}}{\delta T} T + \frac{\delta \tilde{\mathcal{H}}}{\delta \bar{T}} \bar{T} \right) = \int d\phi \left( \frac{\delta \tilde{\mathcal{H}}}{\delta H} H + \frac{\delta \tilde{\mathcal{H}}}{\delta J} J \right) .$$  

Therefore, the deformed theory can be equivalently described by placing the original CFT$_2$ on a state-dependent curved metric, whose lapse and shift functions, $\tilde{N}$ and $\tilde{N}^\phi$, are respectively given by the variation of the deformed Hamiltonian with respect to the energy and momentum densities of the undeformed theory, i.e.,

$$d\tilde{s}^2 = - \left( \frac{\delta \mathcal{H}}{\delta H} \right)^2 dt^2 + \left( d\phi + \frac{\delta \mathcal{H}}{\delta J} dt \right)^2 .$$

The mapping (2.2) allows to express the deformed metric (5.6) in terms of the supertranslation and superrotation densities, so that it reads

$$d\tilde{s}^2 = - N^2 \left( \frac{2P^2}{J^2 - P^2} \right)^2 dt^2 + \left( d\phi + \left( N^\phi + N \frac{2JP}{J^2 - P^2} \right) dt \right)^2 ,$$

where $N$ and $N^\phi$ correspond to the (state-independent) lapse and shift functions of the original undeformed metric in (4.3), respectively.

It must be emphasized that the manifest state dependence of the lapse and shift functions (or Beltrami differentials) of the deformed metric (5.7) provides a local obstruction to gauge them away, preventing the possibility of choosing the standard conformal gauge once the theory is deformed.

A proper stress-energy tensor $\tilde{\Theta}^\mu_\nu$, consistent with invariance under diffeomorphisms and local scalings of the deformed action (4.2), is then readily obtained from $\tilde{T}^\mu_\nu = \sqrt{-\tilde{g}} \tilde{\Theta}^\mu_\nu$, where $\tilde{g}_{\mu\nu}$ stands for the state-dependent metric in (5.7). Indeed, the deformed stress-energy tensor fulfills

$$\tilde{\Theta}_{\mu\nu} = \tilde{\Theta}_{\nu\mu} , \quad \tilde{\Theta}^\mu_\mu = 0 , \quad \nabla_\mu \tilde{\Theta}^\mu_\nu = 0 ,$$

Beltrami differentials are determined by the variation of the deformed Hamiltonian with respect to $T$ and $\bar{T}$.

Note that the Ricci scalar of the deformed metric $\tilde{g}_{\mu\nu}$ differs from that of the undeformed one $g_{\mu\nu}$ ($\tilde{R} \neq R$). In contradistinction, the corresponding metrics in the geometric interpretation of the $TT$ deformation [109–112] are related through state-dependent diffeomorphisms.
being automatically symmetric and traceless, while its conservation implies the evolution equations of supertranslation and superrotation densities (4.6). Therefore, the canonical BMS$_3$ generators (3.6) can be written in manifestly covariant way as

$$\tilde{Q} [\epsilon_J, \epsilon_P] = \int d\phi \sqrt{\tilde{\gamma}} \tilde{n}_\mu \tilde{\Theta}^\mu \xi^\nu,$$

with $\xi^\mu$ given by (5.2), and according to the deformed metric $\tilde{g}_{\mu\nu}$ in (5.7), the unit timelike normal is given by $\tilde{n}_\mu = (\tilde{N}, 0)$, and $\tilde{\gamma} = 1$.

The geometric description of the deformed theory is then suitably carried out in terms of the two relevant structures, $\tilde{g}_{\mu\nu}$ and $\tilde{\Theta}^\mu \nu$, being inextricably intertwined. In fact, since both objects are state dependent, they acquire nontrivial functional variations when acting on them under diffeomorphisms, given by

$$\delta \xi \tilde{g}_{\mu\nu} = \frac{\delta \tilde{g}_{\mu\nu}}{\delta P} \delta P + \frac{\delta \tilde{g}_{\mu\nu}}{\delta J} \delta J,$$

$$\delta \xi \tilde{\Theta}^\mu \nu = \frac{\delta \tilde{\Theta}^\mu \nu}{\delta P} \delta P + \frac{\delta \tilde{\Theta}^\mu \nu}{\delta J} \delta J.$$

Therefore, since the functional variations (5.10) must be taken into account, BMS$_3$ symmetries geometrically arise from diffeomorphisms $\xi^\mu$ that preserve the form of both relevant structures up to a local scaling, i.e., from the solutions of the following deformed conformal Killing equations

$$\tilde{\nabla}_\mu \xi^\nu + \tilde{\nabla}_\nu \xi^\mu - \lambda \tilde{g}_{\mu\nu} = \delta \xi \tilde{g}_{\mu\nu},$$

$$\mathcal{L}_\xi \tilde{\Theta}_{\mu\nu} = \delta \xi \tilde{\Theta}_{\mu\nu},$$

(5.11)

where $\mathcal{L}_\xi$ stands for the Lie derivative.

It is amusing to verify that starting from scratch with the deformed metric and stress-energy tensor, $\tilde{g}_{\mu\nu}$ and $\tilde{\Theta}^\mu \nu$, the deformed conformal Killing equations (5.11) can be exactly solved. Indeed, the solution is precisely given by the BMS$_3$ diffeomorphisms $\xi^\mu$ in (5.2) with parameters $\epsilon_P, \epsilon_J$ fulfilling (4.7), where the transformation law of supertranslation and superrotation densities is also found to be given by (3.4).

Note that the geometric interpretation also allows to find the transformation law of the fields in the deformed theory from those of the original undeformed (primary) fields, collectively denoted by $\chi$, by writing them in a manifestly covariant way, and then acting with the Lie derivative along BMS$_3$ symmetries spanned by $\xi$, i.e., $\delta \chi = \mathcal{L}_\xi \chi$.

6 Deformed free bosons

Let us see how the deformation works in a simple and concrete example, given by the action of N free bosons with flat target metric,

$$I [\Phi^I] = -\frac{1}{2} \int d^2 x \sqrt{-g} \delta_{IK} \partial_\mu \Phi^I \partial^\mu \Phi^K.$$

Before implementing the generic deformation (4.2), it is useful to express the background metric $g_{\mu\nu}$ in the gauge choice (4.3), so that the Hamiltonian action reads

$$I [\Phi^I, \Pi_J] = \int dx^2 \left( \Pi_I \Phi^I - NH - N^\phi J \right),$$

(6.2)
where $\Pi_I = \frac{\delta L}{\delta \dot{\Phi}^I}$, and $H = \frac{1}{2} (\Pi^I \Pi_I + \Phi^I \dot{\Phi}^I)$. The deformed Hamiltonian action is then given by

$$I [\Phi^I, \Pi_I] = \int dx^2 \left( \Pi_I \dot{\Phi}^I - NP - N^0 J \right),$$

(6.3)

with $P = H + \sqrt{H^2 - J^2}$. The transformation law of the fields and their momenta under BMS$_3$ symmetries spanned by $\xi$ in (5.2) are then found to be

$$\delta \xi \Phi^I = \{ \Phi^I, \tilde{Q} \} = \epsilon_J \Phi^I + \epsilon_P \left( \Pi^I + \frac{H \Pi^I - J \Phi^I}{\sqrt{H^2 - J^2}} \right),$$

(6.4)

$$\delta \xi \Pi_I = \{ \Pi_I, \tilde{Q} \} = \left[ \epsilon_J \Pi_I + \epsilon_P \left( \Phi^I + \frac{H \Phi^I - J \Pi_I}{\sqrt{H^2 - J^2}} \right) \right]^\prime,$$

where $\tilde{Q}$ reads as in (3.6). The field equations $\dot{\Phi}^I = \{ \Phi^I, \tilde{H} \}$, $\dot{\Pi}_I = \{ \Pi_I, \tilde{H} \}$, with $\tilde{H}$ given by (4.5), then follow from (6.4) by the replacement $\epsilon_P \to N$ and $\epsilon_J \to N^0$. Note that the transformation law of $\Phi^I$ in the deformed theory also reads from $\delta \xi \Phi^I = L_\xi \Phi^I$. The transformation of supertranslation and superrotation densities in (3.4) is then recovered from those in (6.4), which goes hand in hand with the fact that $P$ and $J$ now fulfill the BMS$_3$ algebra (2.3) by virtue of the canonical Poisson bracket $\{ \Phi^I (\phi), \Pi_K (\varphi) \} = \delta^I_K \delta (\phi - \varphi)$. Moreover, the stress-energy tensor of the deformed theory is obtained from $\tilde{T}^\mu_\nu = \sqrt{-\tilde{g}} \tilde{\Theta}^\mu_\nu$ with $\tilde{T}^\mu_\nu$ and $\tilde{g}_{\mu\nu}$ respectively given by (5.1) and (5.7).

It is worth highlighting that the deformed action (6.3) clearly cannot be obtained from any standard limiting process of the undeformed one for $N > 1$. The peculiarity of the deformed single free boson ($N = 1$) stems from the fact that the supertranslation density simplifies as $P = \Pi^2$, so that the momentum can be eliminated from its own field equation, and the deformed action (6.3) can be written in Lagrangian form as

$$I [\Phi] = \frac{1}{4} \int d^2 x \left( \varphi^\mu \partial_\mu \Phi \right)^2,$$

(6.5)

where $\varphi^\mu = (\sqrt{-g})^{1/2} n^\mu$ stands for a vector density of weight $1/2$, constructed out from the metric $g_{\mu\nu}$ in (4.3) of the undeformed theory. Noteworthy, this vector density is invariant under the BMS$_3$ symmetries spanned by $\xi$ in (5.2), since $L_\xi \varphi^\mu = 0$. Therefore, the deformed action of a single free boson (6.5) coincides with the ultra-relativistic limit of the undeformed theory (6.1) for $N = 1$, when the Carrollian limit is taken in a similar way as for the tensionless string [16, 113–116].

Additionally, the vector density can be reexpressed as $\varphi^\mu = \frac{1}{\sqrt{2}} e^{1/2} \tau^\mu$, where $e$ and $\tau^\mu$ correspond to the einbein and the dual of the “clock one-form” of a Carrollian geometry [117], respectively; so that action of the deformed free boson agrees with the Carrollian one found in [118].

Remarkably, the action (6.5) can be understood in terms of two inequivalent geometric structures. One of them is Riemannian and described through the state-dependent metric $\tilde{g}_{\mu\nu}$ in (5.7), while the remaining structure stands for a Carrollian manifold.
7 Ending Remarks

Since the map (2.2) possesses a square root, our results also carry out for its negative branch, i.e., when the supertranslation density is given by

\[ P_{(-)} = T + \bar{T} - 2\sqrt{T\bar{T}}. \]  

(7.1)

In particular, the deformed action of a single free boson for the negative branch reads

\[ \tilde{I}_{(-)} [\Phi, \Pi] = \int dx^2 \left( \Pi \dot{\Phi} - NP_{(-)} - N^\phi J \right), \]  

(7.2)

with \( P_{(-)} = \Phi'^2 \). Curiously, the deformed action \( \tilde{I}_{(-)} \) agrees with an inequivalent ultrarelativistic limit of the single free boson defined by \( \Phi \to \Phi/c, \Pi \to c\Pi \), when \( c \to 0 \). This limit coincides with that needed to pass from the standard Liouville theory to its “flat” version \[82\]. Indeed, starting from a single free boson in the conformal gauge (\( N = 1, N^\phi = 0 \)) the deformed free boson in the negative branch corresponds to the kinetic term of the flat Liouville theory.

It is also worth to pointing out that the centrally extended conformal algebra (given by two copies of the Virasoro algebra) can be shown to be related to BMS\(_3\) with central extensions, in terms of a map that is necessarily nonlocal. Nevertheless, if only zero modes are involved, the local nonlinear map in (2.2) still holds. Thus, blindly applying the map (2.2) for the zero modes, the Cardy formula once expressed in terms of left and right groundstate energies (\( \mathcal{L}_0, \bar{\mathcal{L}}_0 \)), given by

\[ S = 4\pi \sqrt{-\mathcal{L}_0 \bar{\mathcal{L}}} + 4\pi \sqrt{-\bar{\mathcal{L}}_0 \mathcal{L}}, \]  

(7.3)

reduces to its BMS\(_3\) (or flat) version

\[ \tilde{S} = 2\pi \frac{1}{\sqrt{-P_0 P}} [P J_0 + P_0 J], \]  

(7.4)

when the deformed energy and momentum of the groundstate \( P_0 \) and \( J_0 \) are expressed in terms of the BMS\(_3\) central charges \[119–122\].

Noteworthy, the hypotheses that ensure positivity of the Cardy formula (7.3) (\( \mathcal{L}_0 < 0, \bar{\mathcal{L}}_0 < 0, \mathcal{L} > 0, \bar{\mathcal{L}} > 0 \)), by virtue of both branches of the map, imply that the deformed entropy (2.2) is also positive (\( \tilde{S} > 0 \)).

Furthermore, the map between the chemical potentials follows the same rule as that of the parameters in (3.5), with \( (\epsilon, \bar{\epsilon}) \to (\beta, \bar{\beta}) \) and \( (\epsilon_P, \epsilon_J) \to (\beta, \bar{\theta}) \), where left and right temperatures relate to the modular parameter of the torus as \( \tau = \beta/2\pi \), and \( \beta, \bar{\theta} \) stand for the temperature and chemical potential of the deformed theory. Therefore, around equilibrium, the S-modular transformation \( \tau \to -1/\tau \) precisely maps into its BMS\(_3\) (flat) version \[119, 120\],

\[ \beta \to \frac{4\pi^2 \beta}{\bar{\theta}^2}, \quad \bar{\theta} \to -\frac{4\pi^2}{\bar{\theta}}. \]  

(7.5)

As a closing remark, since the uplift of the deformed action (4.2) to higher dimensions is clearly invariant under diffeomorphisms and local scalings, it would be worth exploring whether the \( D \)-dimensional deformed theories might be invariant under the conformal Carrollian algebra, which is known to be isomorphic to BMS\(_{D+1}\) \[12\].
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