Research Article

The Effect of Heat Transfer on MHD Marangoni Boundary Layer Flow Past a Flat Plate in Nanofluid

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The problem of heat transfer on the Marangoni convection boundary layer flow in an electrically conducting nanofluid is studied. Similarity transformations are used to transform the set of governing partial differential equations of the flow into a set of nonlinear ordinary differential equations. Numerical solutions of the similarity equations are then solved through the MATLAB “bvp4c” function. Different nanoparticles like Cu, Al₂O₃, and TiO₂ are taken into consideration with water as base fluid. The velocity and temperature profiles are shown in graphs. Also the effects of the Prandtl number and solid volume fraction on heat transfer are discussed.

1. Introduction

The convection induced by the variations of the surface tension gradients is known as the Marangoni convection. This convection has received great consideration in view of its application in the fields of welding and crystal growth. Also this convection is necessary to stabilize the soap films and drying silicon wafers. During the study of the existence of the steady dissipative layers which occur along the liquid-liquid or liquid-gas interfaces, Napolitano [1] first called the boundary layer as the Marangoni boundary layer. Many researchers such as Okano et al. [2], Christopher and Wang [3], Pop et al. [4] and Magyari and Chamkha [5] have investigated the Marangoni convection in various geometries. Al-Mudhaf and Chamkha [6] obtained the similarity solution for the MHD thermosolutal Marangoni convection over a flat surface in the presence of heat generation or absorption with fluid suction and injection. Chen [7] investigated the flow and the heat transfer characteristics on the forced convection in a power law liquid film under an applied Marangoni convection over a stretching sheet. In recent years, the study on convective transport of nanofluids has become one of the popular topics of interest. Nanotechnology takes an important part for the development of high performance, compact, and cost-effective liquid cooling systems. Moreover, nanofluids have effective applications in many industries such as electronics, transportation, biomedical, and many more. Nanotechnology has been an ongoing topic of discussion in public health as some of the researchers claimed that nanoparticles could present possible dangers in health and environment. Jang and Choi [8] have introduced nanosized particle in a base fluid, which is also termed nanofluid, for the first time. Arifin et al. [9] have examined the influence of nanoparticles on the Marangoni boundary layer flow using a model proposed by Tiwari and Das [10]. An extended work was done by Buongiorno [11], Daungthongsuk and Wongwises [12], Trisaksri and Wongwises [13], Wang and Mujumdar [14], and Kakaç and Pramuanjaroenkij [15]. Recently Hamid et al. [16] studied the radiation effects on the Marangoni boundary layer flow past a flat plate in nanofluid. In the present paper, we study a numerical solution of MHD heat transfer problem in nanofluid with nanoparticles Cu, Al₂O₃, and TiO₂. We also observed the effects of the Prandtl number and solid volume fraction on the Nusselt number. The results are shown graphically.
2. Mathematical Formulation

Consider a steady two-dimensional Marangoni boundary layer flow past a permeable flat plate in a water-based nanofluid containing different types of nanoparticles like Cu (Copper), Al₂O₃ (Aluminium Oxide), and TiO₂ (Titanium dioxide). Assume that the fluid is incompressible and the flow is laminar. Also it is assumed that the base fluid and the particles are in thermal equilibrium and no slip occurs between them. The thermophysical properties of nanoparticles are given in the Table I. Further, we consider a Cartesian coordinate system (x, y), where x and y are the coordinates measured along the plate and normal to it, respectively, and the flow takes place at \( y \geq 0 \). Assume that the temperature of the plate is \( T_w(x) \) and that of the ambient fluid is \( T_\infty \).

We further assume that the surface tension \( \sigma \) is to vary linearly with temperature as

\[
\sigma = \sigma_0 \left[ 1 - \gamma (T - T_\infty) \right],
\]

where \( \sigma_0 \) is the surface tension at the interface and \( \gamma \) is the rate of change of surface tension with temperature (a positive fluid property). It is also assumed that a uniform magnetic field, \( H_0 \) is imposed in the direction normal to the surface (Figure 1). Then, the steady state boundary layer equations for a nanofluid in the Cartesian coordinates are given by

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2}
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma}{\rho_{nf}} H_0^2 u, \tag{3}
\]

\[
\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = \alpha_{nf} \frac{\partial^2 T}{\partial y^2}, \tag{4}
\]

together with the boundary conditions

\[
v = 0, \quad T = T_\infty + ax^2, \tag{5}
\]

\[
\frac{\partial u}{\partial y} = \frac{\sigma}{\rho_{nf}} \frac{\partial T}{\partial x} \quad \text{at} \quad y = 0,
\]

\[
u = 0, \quad T = T_\infty \quad \text{as} \quad y \rightarrow \infty.
\]

Here \( u \) and \( v \) are the components of velocity along the \( x \)- and \( y \)-axes, respectively. \( T \) is the temperature, \( \alpha_{nf} \) is the thermal diffusivity, \( \rho_{nf} \) is the effective density, \( k_{nf} \) is the effective thermal conductivity, and \( \mu_{nf} \) is the effective viscosity of the nanofluid. Moreover, \( \alpha \) is the coefficient of temperature gradient. Consider the following:

\[
\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}},
\]

\[
\rho_{nf} = (1 - \phi) \rho_f + \phi \rho_s,
\]

\[
\mu_{nf} = \frac{\mu_f}{(1 - \phi)^2},
\]

Further, the dimensionless temperature \( \theta \) is given by

\[
\theta(\eta) = \frac{T - T_\infty}{ax^2}. \tag{9}
\]
Substituting (6), (7), (8), and (9) into (3) and (4), we obtain a set of nonlinear ordinary differential equations:

\[
f'' = (1 - \phi)^{2.5} \left[ (1 - \phi) + \phi \frac{\rho_s}{\rho_f} \right] \left( f'^2 - ff'' \right) + M(P_r)^{1/3} \left( k_f \right)^{1/3} \left( C_p_f \right)^{-1/3} \left( \rho_f \right)^{-2/3} f', \tag{10}
\]

\[
\theta'' = \frac{\left[ (1 - \phi) + \phi \left( \frac{\rho_s C_p_s}{\rho_f C_p_f} \right) \right]}{k_{nl}/k_f} \left( 2f' \theta - f\theta' \right) P_r,
\]

and the boundary conditions become

\[
f(0) = 0, \quad \theta(0) = 1, \quad \frac{1}{(1 - \phi)^{2.5}} f''(0) = -2,
\]

\[
f'(\infty) = 0, \quad \theta'(\infty) = 0,
\]

where the magnetic field parameter \( M = \sigma^{1/3} H_0^2 / (\gamma a)^{2/3} \).

Also one can define the surface velocity and the local Nussel number, respectively, as

\[
\eta_w(x) = \frac{\left( \frac{\sigma_0}{\gamma a} \right)^2}{\rho_f \mu_f} x f'(0), \tag{12}
\]

\[
\text{Nu}_x = \frac{x q_w(x)}{k_f [T(x,0) - T(x, \infty)]}, \tag{13}
\]

where \( q_w(x) \) is the heat flux from the surface of the plate and is given by

\[
q_w(x) = -k_{nl} \frac{\partial T}{\partial y} \bigg|_{y=0}. \tag{14}
\]

Using the above nondimension quantities, one can obtain the local Nussel number as

\[
\text{Nu}_x = -\frac{k_{nl} C_2 \theta'(0)}{k_f}. \tag{15}
\]

Based on the average temperature difference between the surface and the ambient fluid temperature we define

\[
\text{Nu}_L = -\frac{k_{nl}}{k_f} \left( \frac{\text{Ma}_L}{Pr} \right)^{1/3} \theta'(0), \tag{16}
\]

where \( \text{Ma}_L \) is the Marangoni based on \( L \) and is defined as

\[
\text{Ma}_L = \frac{\left( \frac{\partial \sigma}{\partial T} \right) (\Delta T) L}{\alpha_f \mu_f}. \tag{17}
\]

### 3. Results and Discussion

Numerical solutions were obtained for the effect of the Prandtl number and solid volume fraction on the Marangoni heat transfer in a nanofluid. In this paper, we considered three different nanoparticles whose thermophysical properties were given in Table 1. The nonlinear ordinary differential equations (10) subject to the boundary conditions (11) were solved numerically using the MATLAB “bvp4c” routine. We considered the range of nanoparticles volume fraction \( \phi \) as \( 0 \leq \phi \leq 0.3 \) and the Prandtl number \( Pr \) as \( 2 \leq Pr \leq 8 \) (for the base fluid (water) \( Pr = 6.2 \)). The influences of the magnetic field parameter \( M \), the nanoparticles volume fraction \( \phi \) on velocity and, temperature and also the influence of the Prandtl number \( Pr \) and solid volume fraction \( \phi \) on the Nussel number are presented in graphs.

Figures 2, 3, and 4 display the velocity profiles, and Figures 5, 6, and 7 display the temperature profiles of Cu-water, Al\(_2\)O\(_3\)-water, and TiO\(_2\)-water, respectively, for different values of magnetic field parameter \( M \). It is observed from the figures that the velocity in the boundary layer decreases and temperature increases as the Magnetic field parameter increases; this is due to the resistive force, called the Lorentz force, which is produced by the induced magnetic field within the boundary layer.
Figure 4: Velocity profile for TiO$_2$ nanoparticles for various $M$.

Figure 5: Temperature profile for Cu nanoparticles for various $M$.

Figure 6: Temperature profile for Al$_2$O$_3$ nanoparticles for various $M$.

Figure 7: Temperature profile for TiO$_2$ nanoparticles for various $M$.

Figure 8: Velocity profile for different $\phi$.

Figure 8 depicts the influence of volume fraction on the velocity profile of the nanofluid particles. It is observed near the wall that velocity decreases with an increase in the volume fraction $\phi$. Also, it is observed that the velocity of TiO$_2$ nanoparticles is higher than that of Cu nanoparticles. From Figure 9, it is clear that an increase in the value of volume fraction enhances the temperature profile, and Cu nanoparticles exhibit more temperature than that of the other nanoparticles. It is also known from Figure 10 that temperature decreases with an increase in the Prandtl number. This is because of a decrease in thermal diffusivity with an increase in the Prandtl number (Pr).

Figures 11 and 12 depict the influence of the Prandtl number and volume fraction on heat transfer, respectively. It is observed that the Nusselt number increases with an
increase in Prandtl number and decreases with an increase in the volume fraction.

From Table 2, it is observed that skin friction decreases with the increase in the volume fraction.

4. Conclusion

In the present paper, we studied the effect of heat transfer on the Marangoni boundary layer flow past a flat plate in nanofluid in presence of transverse magnetic field. With the similarity transformation, the governing equations of motion together with boundary conditions were transformed to a set of nonlinear ordinary differential equations. The numerical solutions are then obtained for these equations by the help of MATLAB “bvp4c” programming tool. Different types of nanoparticles like Cu, Al₂O₃, and TiO₂ were taken into consideration with H₂O as base fluid. The effects of magnetic field parameter $M$, solid volume fraction of the nanofluid $\phi$ on the velocity and temperature fields for different nanoparticles, and the Prandtl number $Pr$ on temperature field were plotted and analyzed. Also the effects of the Prandtl number $Pr$ and solid volume fraction $\phi$ on local the Nusselt number for the different nanoparticles were discussed for a fixed value of magnetic field parameter $M$. It is found that the inclusion of the magnetic field parameter on the flow
increased the temperature and decreased the velocity fields in all types of nanofluids. A similar profile was observed on the inclusion of solid volume fraction of the nanoparticles. It was noted that presence of the Prandtl number reduced the temperature field. Also it was observed that for a fixed Prandtl number and other parameters, the rate of heat transfer is more in TiO$_2$-H$_2$O.

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