How to Improve the Stability and Durability of Sand Castles

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Abstract. On the beach in midsummer, one of the favorite recreational activities for adults and children is to build the castle of the sea sand. However, the castles carefully built by the artists are often ruined by the inflow of waves or rising tides. We built the best 3D model of the sandcastle foundation. Moreover, we found the optimal essential moisture content. Due to the frequent precipitation in summer, our model also provides some reinforcement schemes. we compared the bearing capacity coefficients of several common foundation shapes and obtained the optimal shape on a plane. Then we compared the various indexes of the round table with different stacking angles in the vertical direction. We constructed the evaluation function using hierarchical analysis and then found the optimal stacking angle at the point where the first derivative of the function was zero. When calculating the thickness of the foundation based on a sandcastle with specific gravity, we chose the ordinary column foundation bearing capacity formula as the bearing capacity formula of the circular table foundation. Furthermore, we use this formula to find the corresponding thickness of the foundation in combination with the optimal stacking angle previously obtained.

Keywords: Optimal round table stacking angle, Bearing capacity correction model, Differential equation model, Particle swarm optimization.

1. Introduction
In midsummer, many people yearn for surfing on the seashore, surfing the sea breeze, and assembling the sandcastles walls. On the beach, it is adults or children, and one of the favorite recreational projects is to build the castle of the sea sand. Everyone knows that before making a sandcastle, an initial foundation is usually made. The sandcastle foundation consists of a single, invisible wet dune, and then it is cut and shaped into a recognizable three-dimensional geometry. Based on this, the building that builds the features of the castle. It is irritating that well-built castles are often ruined by the inflow of waves or rising tides.

Basic concept: The slope of the stockpile (that is, the angle between the stockpile and the ground) when the material (ore) is naturally stacked is called the stacking angle, also known as the repose angle. Chu Shizhang, Dong Jiahao, and others introduced the concept of accumulation angle into the field of
particulate matter and defined it as the angle between the cone-shaped stack formed by the accumulation of particulate matter and the bottom surface.

Reason for selection: The process of building the sandcastle foundation is similar to the material stacking process, and its practical process mechanism is similar. Therefore, the angle between the sandcastle foundation and the ground can be similarly called the accumulation angle.

2. Literature Review

There is an interesting fact: even if the dunes are constructed in the same shape, the same number/volume/mass, the same water-sand mixing ratio, and the same distance from the sea, the sandcastle's response to waves and tides is different. The phenomenon similar to the formation and collapse of sand piles has also attracted the interest of many scholars and even derived the famous "sandpile model" [Per Bak, Chao Tang, and Kurt Weidenfeld, 1987]. J.-P. Bouchaud et al. Studied the dynamic system of the sandpile surface. A new continuity description was proposed; TG Mason et al. Carried out a calculation study on the maximum stability angle of the sand pile to improve the stability of the sandpile model.

To this day, people are still exploring whether there is the best sandcastle foundation in three-dimensional geometry, which can resist the invasion of sea waves and tides, making the sea sand castle challenging to fall.

3. Methodology

3.1. Assumptions

- There are no impurities in the foundation, and the compacted sand is uniform in the medium.
- The foundation sand will not collapse when it is pressed.
- A sandy beach is a horizontal plane, so the weight of the castle is vertical downwards, and all of its acts on the contact surface.
- The size of the angle of repose of the same granular sand at the same water content may be different, but in this article, we believe that it is the same.

Notations

Let us first define the list of notations used in this article:

| symbol | Description | Unit       |
|--------|-------------|------------|
| $f_u$  | The function of bearing capacity | ——— |
| $N_c$  | Bearing capacity coefficient | ——— |
| $q$    | Unit flow   | m$^3$/min  |
| $L$    | Round Slope Length | m |
| $I$    | Unit precipitation | mm/min |
| $m$    | Changes in soil and water flow | ——— |
| $H$    | Round platform slope height | m |
| $\mu$  | Slope roughness coefficient | ——— |
| $\alpha$ | The angle between the horizontal plane and the slope | $(^\circ)$ |
| $g$    | Gravitational acceleration | m$/s^2$ |
| $\tau_f$ | Shear strength | kpa |
| $w$    | Water content | % |
| $c$    | Soil cohesion | Kpa |
4. Findings and Analysis

4.1. Dependency Theory

In order to build the same raw materials (that is, use the same type, the same quantity/volume/mass, and the same water-sand mix ratio), the construction location and the distance from the seaside (that is, the same salinity and humidity on the seashore) Next, determine the basis of the best three-dimensional geometry that can best resist the invasion of the tides. We establish a three-dimensional model at the three-dimensional level, that is, a rectangular coordinate system is established on the basis of which the length of the foundation is represented by $X$, the width of the foundation is represented by $Y$, the height of the foundation is represented by $Z$, and $O$, represents the center of the bottom surface of the foundation. The XOY plane is a two-dimensional horizontal plane. The combination of XYZ is a three-dimensional space.

Terzaghi on this basis proposed the formula for the vertical bearing capacity of the shallow foundation center in consideration of the soil's weight:

$$f_u = cN_c + qN_q + \frac{\gamma b}{2} N_y$$

(1)

Which, $c$ is the cohesion of the soil, $q$ is overloaded on both sides of the foundation, $N_c$, $N_q$, $N_y$ is bearing capacity coefficients, $\gamma$ is the severity of the land; $b$ is the base width; Zheng Shuaiqun et al. Established a three-dimensional numerical analysis model by finite difference method to analyze the theory of ultimate bearing capacity and failure mode of clayey heterogeneous foundations with linearly increasing strength with depth under rectangular and circular foundations. The ultimate bearing capacity of pure viscous soil with an internal friction angle equal to $0$ is defined:

$$P_u = cN_c$$

(2)

Then calculate the bearing capacity coefficient $N_c$ of the heterogeneous foundation at this time. Therefore, we have sufficient reasons to believe that when the cohesion of the land is a particular value, $N_c$ reflects the bearing capacity of a particular volume foundation, so we will determine the sandcastle on the XOY plane by comparing several conventional shape foundations Optimal shape.

Firstly, in combination with previous studies, we believe that the ultimate bearing capacity of a foundation is related to its bearing capacity coefficient, and the bearing capacity coefficient $N_c$ represents the bearing capacity per unit area of the foundation. Then we assume that the sandcastle is a regular geometry: On the one hand, the analytical formula of the XOY plane function is difficult and challenging to operate due to the calculation of the force. On the other hand, tourists have simple tools, simple motivation, distracting energy, and low technical skills when building sandcastles. It is impossible to perform quite complicated operations, so we only consider simple geometric shapes. Besides, since the sandcastle must have a regular shape, the foundation of the irregular shape must also be unreasonable in the contribution of bearing capacity. Based on the above factors, we choose one of the three shapes (i.e., rectangle, strip, and circle) that are widely used at present, and use it as the cross-sectional shape of the foundation on the XOY plane.

Considering that the sand and gravel on the beach are of different sizes, and there are substances such as stone shells, we believe that the medium of the sandcastle foundation is non-uniform. Based on the FLAC numerical analysis of Zhao Shaofei's and other non-homogeneous foundation bearing capacity and its failure mode, we studied the relationship between the strip and circular foundation bearing capacity and the soil unevenness coefficient:
We used the three-dimensional finite-difference calculation model established by Zheng Shuaiqun and others to solve the bearing capacity coefficient $N_{c0}$ of the heterogeneous foundation in the case of circular foundations and strip foundations, and reached the same conclusion:

From the analysis of the above figure, it is known that under non-homogeneous conditions, when the forcing non-uniformity coefficient $kB/c_0$ is the same, the bearing capacity of the strip foundation is above the circular foundation. It can be concluded that the bearing capacity of the strip foundation.

The Analytic Hierarchy Process (AHP) was proposed by American operations researcher Professor City at the University of Pittsburgh in the early 1970s. It is to decompose the elements that are always relevant to decision-making into goals, guidelines, plans, and other levels A decision-making method based on qualitative and quantitative analysis based on this, which is in line with the characteristics of our research.

Part 1: Build a hierarchical model. B represents the optimal Goals of stacked angle-angle:

Part 2: Constructed as a pair comparison matrix. To compare the importance levels of the four indicators, we must first determine their proportion (weight) in $Z$, so the method of pairwise comparison is used to quantify the importance of each factor. Take two factors $x_i$ and $y_j$ at a time, and use a positive number $a_{ij}$ to represent the ratio of the importance of $x_i$ and $y_j$. The pairwise comparison matrix is:

$$
A = \begin{bmatrix}
1 & 3 & 5 & 7 \\
\frac{1}{3} & 1 & 2 & 6 \\
\frac{1}{5} & \frac{1}{2} & 1 & 3 \\
\frac{1}{7} & \frac{1}{6} & \frac{1}{3} & 1 \\
\end{bmatrix}
$$

(3)

Part 3: Consistency check. We use the consistency index CI (consistency index) to measure the degree of inconsistency of CI, and find the corresponding average random consistency index RI:
Table 2. Random Consistency Index Table

| n      | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
|--------|----|----|----|----|----|----|----|----|----|
| RI     | 0  | 0  | 0.58 | 0.90 | 1.12 | 1.24 | 1.32 | 1.41 | 1.45 |

After inspection:

\[
CI = \frac{\lambda_{max} - n}{n-1} = 0.0319, \quad CR = \frac{CI}{RI} = 0.0358 < 0.1
\] (4)

Which \( \lambda_{max} \) is the maximum eigenvalue of the matrix, \( n \) represents the number of factors. Since \( CR < 0.1 \) we know that the matrix \( A \) is acceptable.

Part 4: Calculate the weight vector.
(1) Volume; (2) Individual bearing capacity;
(3) Ultimate destructive force, (4) Unit bearing capacity

Finally, we determined the specific weights of the weight vectors:

\[ A = (A_1, A_2, A_3, A_4)^T = (0.5728, 0.1236, 0.054, 0.2496)^T \] (5)

Part 5: Find the overall score of each plan after the total ranking of the levels. We calculate the weight of the final goal of each scheme and obtain the analytical formula of the target value. Let \( \frac{dAP}{d\theta} = 0 \), We can solve the optimal stacking angle, which is: 8.36°.

5. Conclusion
We built the best 3D model of the sandcastle foundation. Moreover, we found the optimal essential moisture content. Due to the frequent precipitation in summer, our model also provides some reinforcement schemes. We compared the bearing capacity coefficients of several common foundation shapes and obtained the optimal shape on a plane. Then we compared the various indexes of the round table with different stacking angles in the vertical direction. We constructed the evaluation function using hierarchical analysis and then found the optimal stacking angle at the point where the first derivative of the function was zero.

1) Research Limitations
The research exists imitation: we did not consider the horizontal forces on the foundation as a rigid body. In fact, due to many factors on the beach, there must be forces in this direction. In the process of constructing the model, a large number of formulas, unknowns, and logical concepts were introduced, which made the solution process complicated and complicated and increased the difficulty of the solution.

2) Further Research
We could come up with an optimal basic 3D model that contributed to individual carrying capacity, analyzed the optimal moisture content of the foundation, and then summarized the impact of rain on our model and made modifications.

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