An Ionospheric Anomaly Monitor Based on the One Class Support Vector Algorithm for the Ground-Based Augmentation System

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Abstract: An ionospheric anomaly is the irregular change of the ionosphere. It may result in potential threats for the ground-based augmentation system (GBAS) supporting the high-level precision approach. To counter the hazardous anomalies caused by the steep gradient in ionospheric delays, customized monitors are equipped in GBAS architectures. A major challenge is to rapidly detect the ionospheric gradient anomaly from environmental noise to meet the safety-critical requirements. A one-class support vector machine (OCSVM)-based monitor is developed to clearly detect ionospheric anomalies and to improve the robust detection speed. An offline-online framework based on the OCSVM is proposed to extract useful information related to anomalous characteristics in the presence of noise. To validate the effectiveness of the proposed framework, the influence of noise is fully considered and analyzed based on synthetic, semi-simulated, and real data from a typical ionospheric anomaly event. Synthetic results show that the OCSVM-based monitor can identify the anomaly that cannot be detected by other commonly-used monitors, such as the CCD-1OF, CCD-2OF and KLD-1OF. Semi-simulation results show that compared with other monitors, the newly proposed monitor can improve the average detection speed by more than 40% and decrease the minimum detectable gradient change rate to 0.002 m/s. Furthermore, in the real ionospheric anomaly event experiment, compared with other monitors, the OCSVM-based monitor can improve the detection speed by 16%. The result indicates that the proposed monitor has encouraging potential to ensure integrity of the GBAS.

Keywords: GNSS; GBAS; ionospheric gradient anomaly; one class support vector machine

1. Introduction

The Ground-Based Augmentation System (GBAS) is a short-baseline, airport-based augmentation of the Global Navigation Satellite Systems (GNSS). It can provide advanced civil-aviation services concerning accuracy, integrity, continuity, and availability. Integrity is one of the key aspects to indicate the safety of precision approach at airports around the world. It refers to the capability to alert when system outputs cannot be trusted. The GBAS ensures integrity in three ways [1], i.e., providing local-area differential corrections and removing common-mode errors; providing the authorized users with customized monitors for rare system faults; allowing users to establish a bound on residual errors to perform navigation operations.

Among all the identified hazards, the severest one is the undetected ionospheric anomaly [2]. Ionospheric delays because of free electrons along the path of the GNSS signal are always uniformly distributed under normal conditions. However, when GNSS signals travel through the disturbed ionosphere, severe extreme errors have been observed, which...
potentially compromise integrity of the GBAS. A model is thus developed to predict the maximum errors that GBAS users can suffer from this threat. The threat of ionospheric anomaly in mid-latitude regions is modeled as a spatially linear semi-infinite and wave-front wedge moving at a constant speed. A predigested model of the ionospheric anomaly is shown in Figure 1a [3]. This piecewise and linear curve is parameterized by the slope of the ramp, gradient width and front speed. In the presence of the ionospheric anomaly, the ionospheric delay changes monotonously. The spatial change rate of the ionospheric delay is the ionospheric gradient. It is the “slope” with the dimension of mm/km in Figure 1a measured by ground stations, namely the maximum ionospheric delays (with the dimension of mm) divided by the gradient width (with the dimension of km). We can directly estimate the ionospheric correction residual through the distance between the user and the reference station. The ionospheric gradient is simplified to a constant in small regions, although seasonal variances play a significant role on the dynamics of the ionosphere [4]. This model is consistent with the study of ionospheric delays in the case of ionospheric anomalies in the mid-latitude region. Compared with the ionospheric delay anomaly, an extreme steep gradient in the ionosphere caused by intense ionospheric storms is generally considered as a dominant ionospheric threat. A predigested model of the ionospheric gradient anomaly is shown in Figure 1b.

![Figure 1](image.png)

**Figure 1.** Illustrations of predigested ionospheric anomaly (a) and ionospheric gradient anomaly (b) models (Figure based on [3]).

Ionospheric gradients can be estimated using code and carrier measurements in practice, including the long-term ionospheric anomaly monitoring and the single-frequency carrier-based and code-aided method [5,6]. Severe extreme gradients have been identified on rare occasions [7]. The largest slant gradient observed in the ionosphere was up to 850 mm/km in Brazil [8], which means measurement errors as large as 850 mm per kilometer were generated between the airborne user and the ground reference station. These errors cannot be accurately corrected using the precise ionospheric model, thus causing inaccurate positioning results and catastrophic consequences for civil aviation. This is unacceptable for the safety-critical GNSS application. An ideal way is to apply a combination of dual-frequency measurements, which can theoretically eliminate the first-order effect of the ionosphere [9,10]. However, this way is not yet available for civil aviation applications, because the correlative GBAS Approach Service Type F (GAST F) is still in the stage of standardization. Therefore, rapid and accurate detection of ionospheric anomalies for single-frequency-based GBAS is still crucial for its performance.

Monitors deployed in the GBAS to detect the ionospheric anomaly mainly refer to the code-carrier divergence (CCD) monitor [11], the dual solution pseudo-range ionospheric gradient monitor [12], and the ground ionospheric gradient monitor [2,13]. Among all the monitors, the CCD monitor is the only one capable of detecting ionospheric gradients observable to both the user and ground subsystem for GAST D. CCD is the anomalous phenomenon caused by the GNSS signal propagation in the disturbed ionosphere, where code phase measurements are delayed and carrier phase measurements are advanced. The CCD monitor is a filter operating on the difference of the code-minus-carrier (CMC) to
estimate ionospheric divergences, i.e., the change rate of ionospheric delay over time. If anomalies occur, the divergence increases and can be tested by the customized monitor. The normal ionospheric data with well-defined patterns and the data with unexpected profiles can be separated. In addition, the divergence can also be triggered by potential payloads occurring at the ranging source. Thus, the monitor performs dual tasks, deployed to counter both satellite-induced faults and ionospheric anomalies by detecting divergences.

Two kinds of traditional moving-average-based filters are implemented in the GBAS to monitor the divergence [14,15], i.e., one first-order low-pass auto-regressive and moving average (ARMA) filter, named CCD-1OF, and two first-order cascade ARMA filters, named CCD-2OF. Generally, the CCD-2OF outperforms the CCD-1OF in monitoring divergences due to the smaller smoothing constant. However, both monitors cannot detect divergence rapidly and suffer from a serious delay [16]. These monitors are statistical based, which extract information in the form of simple one-dimensional time series. We cannot understand which process can generate such phase time series. Additionally, the inherent averaging characteristics of filters lead to the weak anomaly-related components being easily overwhelmed by normal noise. Consequently, it is often difficult for traditional divergence monitors to quickly detect the ionospheric anomaly. These monitors even fail to alert in noisy environment. These limitations increase the probability of missed detection, thus leading to misleading information [1]. Therefore, high-level precision approaching requirements are difficult to meet. Developing a novel monitor is highly necessary for future GBAS constructions.

To improve detection performance, a variety of optimization strategies have been studied. Instances of creative research include the monitors based on generalized least squares [17], a two-step method based on an adaptive Kalman filter [3], and a Kullback–Leibler divergence metric [18]. These approaches prove the feasibility of capturing abnormal ionospheric behaviors. However, further work needs to be carried out to overcome the shortcomings, such as the insufficient estimation accuracy, low generalization ability caused by miscellaneous assumptions and poor performance under weak noise.

To overcome these issues, an option is to exploit machine learning algorithms rather than purely statistical-based algorithms. The support vector machine (SVM) is a supervised algorithm designed for binary classification problems [19]. It is famous and appealing, as it can simplify the complex task of classification in an explainable way. It has been extensively utilized in the GNSS community such as ionospheric scintillation detection [20], GBAS availability prediction [21], and high precision positioning [22]. A one class support vector machine (OCSVM) algorithm extends the SVM algorithm to solve a single classification problem [23]. It learns the underlying characteristics of the existing normal samples to judge whether the new samples come from this distribution, thus well suitable for anomaly detection. It has complete mathematical derivation and a meaningful interpretation. As anomalies occur rarely, it is reasonable to assume that they only occur in the tails of normal distributions, thus the goal of anomaly detection is to estimate the density level sets of the normal distributions. This assumption is typical for the OCSVM algorithm. When only a few anomalies occur and the knowledge of new information is limited or unavailable, the OCSVM is suitable, as it does not require any anomalous data. The OCSVM has been found successful in accurately sensing the abnormal information in many domains, such as materials science [24], computer vision [25], power system [26], and agriculture [27]. However, it has not been applied in the GBAS. It is of great significance to investigate the feasibility of using the OCSVM for ionospheric anomaly detection in the GBAS.

The remaining sections are arranged as follows. Section 2 introduces the principle of the OCSVM, followed by the methodology proposed in this work, i.e., an offline-online ionospheric anomaly monitor based on the OCSVM. In Section 3, the synthetic, semi-simulated and real anomalous events are discussed in detail. Finally, conclusions and perspectives of this work are summarized in Section 4.
2. Ionospheric Anomalies Detection Based on the OCSVM

As previously described, a new monitor is required to meet the requirements of the high-level precision approach. The major goal is to perform ionospheric anomaly detection in a time-efficient way. To achieve this purpose, we present an analysis of the statistics of OCSVM metrics and establish a framework of the monitor in this section.

2.1. The OCSVM Algorithm

The principles of the OCSVM are described in this section. As a special form of the SVM, the OCSVM is a domain-based detection algorithm and can tackle the one-class classification problem. It learns the training set with identical labels and determines whether the samples from the new test set are subordinate to the training set [23].

Suppose that we have vectors \(X = \{x_i, i = 1, \ldots, K\}\). \(K\) is the number of vectors in training sets and \(x_i\) comes from the input space \(R\). \(\phi(x_i)\) represents the projection of the original training sample \(x_i\) in \(R\) to the huge dimensional feature space \(F\). A hyper-plane \(f\) is used to separate the projected vectors from the original which is assigned as the only anomaly in \(F\). The hyper-plane is given by

\[
f(x) = \omega^T \phi(x) - \rho \tag{1}\]

where \(\omega\) and \(\rho\) are the normal phase vectors and compensation values of the hyperplane in \(F\), respectively. These parameters can be calculated by the following quadratic programming issues

\[
\min_{\omega, \phi, \xi} \frac{1}{2} \|\omega\|^2 + \frac{1}{\gamma K} \sum_{i=1}^{K} \xi_i - \rho \\
\text{s.t.} \quad \omega \cdot \phi(x_i) \geq \rho - \xi_i \\
\quad \xi_i \geq 0, i = 1, \ldots, K 
\tag{2}\]

where \(\gamma \in (0, 1]\) is the trade-off constant value used to determine the proportion of the normal and anomalous data. \(\xi_i\) is the slack variable, referring to the extent where the samples are misclassified. \(\|\Delta\|\) denotes the 2-norm of the vector \(\Delta\). The Lagrange factor \([\alpha_1, \ldots, \alpha_K]^T\) is introduced for each vector \(x_i\) to obtain the dual problem of (2). Solving the dual problem gives rise to

\[
\omega = \sum_{i=1}^{K} \alpha_i \phi(x_i) \tag{3}\]

where \(0 \leq \alpha_i \leq 1/\gamma K\). The famous kernel trick can replace the inner product of two vectors in \(F\) with a kernel operation in \(R\). This projection transforms a non-linearly separable problem into a linearly separable problem. Thus (1) combined with (3) becomes a function as follows

\[
f(x) = \sum_{i=1}^{K} \alpha_i G(x_i, x) - \rho \tag{4}\]

where \(G(x_i, x)\) is the inner product of two projecting functions, i.e., \(G(x_i, x) = (\phi(x_i), \phi(x))\). A popular radial basis function (RBF) kernel function is utilized here to expand vectors into the high dimension. The RBF is given by

\[
G(x_i, x) = \exp \left( -\frac{\|x_i-x\|^2}{\sigma} \right) \tag{5}\]

where \(\sigma\) is the hyper-parameter controlling the width of the kernel. The kernel provides the OCSVM the ability to detect abnormal samples in the high dimension feature space \(F\). Moreover, it can be proved that only for the vector on the boundary, namely the support vector, the non-zero \(\alpha\) imposes effects on the construction of the boundary.

Based on the derivation above, the OCSVM can be constructed and used to detect new samples. If the new samples fit well with the trained model, they are labeled as normal samples. Otherwise, they are adjudged as anomaly samples. \(f(x)\) serves as a score function.
to determine the classification quantitatively. A vector $x$ is classified to be normal with $f(x) > 0$ and anomalous with $f(x) < 0$. The smaller the $f(x)$ is, the more abnormal the vector is viewed.

### 2.2. Ionospheric Anomaly Detection with the OCSVM-Based Monitor

This section applies the OCSVM algorithm to the monitor. Based on the analysis above, a new offline-modeling-online-monitoring monitor is proposed. It has the potential to meet the high-level approaching requirements. Figure 2 shows the overall schematic diagram of the proposed integrity monitoring algorithm illustrating the process. Statistical information can be extracted to characterize the normal states offline. A classifier can then be trained to detect the potential ionospheric anomaly. The GBAS can monitor the ionospheric anomaly more effectively through the OCSVM-based detection strategy. Detailed analyses are given next.

**Figure 2.** Overall schematic diagram of the ionospheric anomaly monitor based on the OCSVM.

In data collection, we collected both the code-phase and carrier-phase measurements. To eliminate the same item in the code and carrier phase measurements and emphasize the differences, a CMC is introduced, given by

$$
CMC = P - \Phi = 2I + N\lambda + M
$$

where $P$ and $\Phi$ represent the code-phase measurement and carrier-phase measurement between the receivers and satellites, respectively. $I$ is the ambiguity of whole cycles. $\lambda$ is the carrier wavelength. $M$ includes noise including multipath effects, thermal noise and other error sources.

Before the introduction of the monitor design, it is necessary to understand which original measurements can be dealt with. A pre-processing is performed to process GNSS raw measurements and to perform various corrections to get the true CMC as in [28]. Recall that our aim is to detect the ionospheric anomaly and separate it from the other errors. The first term in the right side of (6) is what we are interested in. The presence of the double ionospheric delay errors is induced by the dispersive nature of the ionosphere. Although ionospheric delays are time-variant during normal ionospheric days, we can use a neural network and single-frequency GNSS signals to estimate the delays [29]. Therefore, the delays during normal ionospheric days can be considered as a time-variant but known bias. The second term in the right side of (6) is the ambiguity term. It does not directly affect the results when cycle slips are absent in measurements. It is always fixed or easily repaired under the circumstance of relatively weak ionospheric perturbations using the Kalman filter [30]. Therefore, the cycle-relevant term is treated as a pure bias. The third term in the right side of (6) indicates ground-related noise. The noise cannot be simply modeled by a zero-mean Gaussian distribution. It is considered the major obstacle in the ionospheric anomaly detection, as the ionospheric anomaly-related components are easily
overwhelmed. Based on the above analysis, we understand what we can cope with before stepping into the monitor design. Compared with traditional CCD monitors, we focus on the original CMC instead of the difference of the CMC. Namely, we concern about the accumulation errors induced by the ionospheric gradient anomalies over a period of time.

Due to the fact that only vector inputs can be imported into the OCSVM, one-dimension time series cannot be applied directly [26]. Therefore, the CMC sequence must be converted to a high-dimension vector first. We need to ensure that we can construct a sound mathematical model from a single time series. The method of nonlinear analysis can be used, provided that the CMC is a dynamic, chaotic, and irregular sequence. The CMC is assumed as an observable variable of a multivariate system. Additional knowledge obtained from the collection of real data is required to establish the system. The state phase space of the original system can be reconstructed by the time-delay embedding to maintain the system dynamics. Specifically, let \( CMC_1, \ldots, CMC_k \) be a time sequence, it can be unfolded into the phase space \( Q \), where \( Q \subseteq \mathbb{R}^L \) and \( L \) refers to embedding dimension (ED).

We set the embedding delay as the sampling interval to consider more information [31]. The reconstructed vector can be then obtained by

\[
CMC_{L,k} = [CMC_{k-L+1} \quad CMC_{k-L+2} \ldots \quad CMC_k]
\]

(7)

where \( CMC_k \subseteq Q \). Thus, a one-dimensional time series can be transferred to a string of vector \( V_L(N) \), given by

\[
V_L(N) = \{CMC_{L,k}, k = L, \ldots, N\}
\]

(8)

A continuous track formed by vectors connected by lines in the reconstructed phase space is called a phase diagram, whose geometry reflects the characteristics of unknown systems. The reconstructed phase space is topologically identical to the original system, provided that the embedding dimension is sufficiently large.

After converting the time series to a set of vectors in the phase space, the OCSVM can be used to monitor the ionospheric anomaly. We can model the collected data offline and monitor the real-time data online [32]. An offline modeling needs to be built by training tests. We first pay attention to the parameter selection. The RBF is chosen as the kernel function of the OCSVM. The selection of the hyper-parameter \( \sigma \) affects the final detection results. A larger \( \sigma \) results in a greater generalization capability but a lower discernibility and vice versa. Note that both \( \sigma \) and \( L \) affect the final test results. They will be discussed in the next section. Because the probability of serious ionospheric anomaly in the actual environment is extraordinarily low, it is advisable to set the trade-off parameter \( \gamma \) to 1, indicating that there is no outlier in the training set. Thus, the OCSVM model can be obtained by the offline data. Additionally, the OCSVM operating on the RBF kernel performs poorly with the number of samples in training sets. It does not perform well in the case of big data because of the increased computational load, while hardware auxiliaries can be used to overcome this difficulty [33]. Considering the actual situation, the offline OCSVM is feasible to be trained quickly. At the same time, the offline modeling part is not required to be real-time, and the appropriate computational load is acceptable in practice.

Once a model reflecting the normal state is established, detecting any departure from the normal operation is vital. Because of the complexity of the real environment, we cannot directly determine whether the new sample is an ionospheric anomaly by the sign of the score value. A threshold is set to determine the decision result. A validation test is required to predict new scores. It acts as cross-validation to judge the extent of how the trained OCSVM matches the underlying distribution. Then, the threshold needs to be obtained by the scores. Because the distribution of the OCSVM scores does not follow the Gaussian distribution, the traditional Gaussian overbounding to construct a threshold is a little conservative. Instead, we use non-parametric methods to estimate the threshold. A famous kernel density estimation (KDE) is employed to approach the probability density function in a non-parametric approach [34]. A simple Gaussian kernel is chosen in this work. Since KDE is to estimate the values of unknown random variables
from known samples, it is reasonable to assume that the weight of known samples on unknown variables decreases as a Gaussian distribution gradually for the central limit theorem. The Gaussian kernel is widely employed to estimate the underlying distribution of the scores [35]. Note that different from the traditional monitors, the OCSVM monitor uses test statistics to describe the degree of misclassification. When the value is very large, the distribution of the test sample is very close to the real sample, and there is no abnormality. The ionospheric anomaly occurs only when the statistic is negative and its absolute value is large. Therefore, the threshold is unilateral. Then the threshold $T_{KDE}$ can be determined as the unilaterally lower bound of the estimated cumulative distribution function with respect to the corresponding false-alarm risk. As a result, the real threshold $T$ is the minimum of zero and $T_{KDE}$. For real samples, the errors are strongly correlated with the elevation angle due to the multipath, thus the threshold is determined related to the elevation bin [36]. It is also noted that because the ambiguity term is included, each satellite channel in each elevation bin needs to set a different threshold, which makes a higher request for the GNSS signal channel.

For the online-monitoring part, the parameter is the same as the offline model. The calculated test statistic is the score function as $D$. This statistic is compared with the threshold values calculated offline to find whether the sample patterns deviate from the majority. If the test statistics are less than the detection threshold $T$, timely warnings are provided and broadcast. Note that the OCSVM only parse the information in normal conditions without any presupposition for any ionospheric anomaly points, thus any ionospheric anomaly causing CMC changes will be detected. Although this unpredictable and unclassified fault mode may cause troubles in areas with high fault probabilities, any threat identified should be informed for the GBAS. Moreover, the linear characteristics of the online-monitoring ensure that the computational cost is comparable to that of the traditional monitors.

The OCSVM-based monitor has three benefits in monitoring ionospheric anomalies: (1) Using the phase-space reconstruction technique, one-dimensional CMC time series can be transformed into high-dimensional vectors. The full dynamics of the CMC system accessible in the phase space can be passed through the OCSVM; (2) The OCSVM is a domain-based detection algorithm that can distinguish between normal and anomaly classes with a redefined boundary or “domain” as the test statistics. It offers better flexibility compared with the Gaussian statistics; (3) The OCSVM has certain robustness to noise and generalization ability to ionospheric anomalies. Therefore, superior performance can be expected with the proposed OCSVM-based monitor to exhibit high-level precision approach.

3. Experiment Analysis

In this section, three examples are illustrated to show the anomalous monitoring capability of the proposed OCSVM-based monitor. In the first example, synthetic data are used to simulate the ionospheric gradient-free and non-negligible ionospheric gradient anomaly CMC, respectively. In the second example, we verified performance of ionospheric anomaly detection with semi-simulation data. The raw BeiDou Navigation Satellite System (BDS) data collected at Dongying Shengli Airport with an artificial anomaly are used to demonstrate the effectiveness of the proposed monitor. In the third example, a real anomaly event are taken into account to evaluate the performance under real circumstance. The OCSVM-based monitor is compared with traditional CCD monitors and KLD-1OF monitor with respect to the ionospheric anomaly detection performance.

3.1. Ionospheric Anomaly Detection with Synthetic Data

The aim of the synthetic data analysis is to show the effect of the different noise levels on the required time of detecting anomalous events, which cannot be realized using real data. Because the CMC depends solely on the difference in code and carrier phase rather than individual values in the single-frequency mode, code phase and carrier
phase anomalies are equivalent [10]. Thus, only one fault mode analysis is required. To demonstrate the ionospheric anomaly detection performance, 2000 samples are generated to simulate the gradient changes induced by the ionosphere. The first 1000 samples are ionospheric gradient-free. To simulate the ionospheric anomaly, a constant steep gradient change, as shown in Figure 1, is added. The ramp-type change starts at the 1001st sample at a velocity of 0.01 m per epoch [3,18]. The parameters of the samples are written as

\[
l_k = \begin{cases} 
4 + n_k & 0 < k \leq 1000 \\
4 + 0.01(k - 1000) + n_k & 1000 < k \leq 2000 
\end{cases}
\]  

where \(n_k\) is the independent measured noise at the \(k\)th epoch. Because the noise follows a heavy-tailed distribution, we choose distributions that can characterize upper and lower bounds of the real distribution to conduct the simulation. The tail distribution of real noise lies between Gaussian and Laplacian distributions [36]. Thus, the noise is assumed to be zero-mean, and it includes components following the Gaussian distribution with the variance ranging from 0.1 to 2.5, and components following the Laplacian distribution with the scale ranging from 0.1 to 2.5, respectively [3,18]. We choose them to simulate the noise of different levels. When the variance and scale are 0.1, the noise is weak. With the increase of the variance and scale, the noise amplitude in the simulated data increases gradually. When the variance and scale approach 2.5, the noise is strong enough to conservatively simulate hostile environment [37]. According to our experience, noise beyond this range seldom appears in the real environment. Figure 3 shows the variation of the simulated data amplitude when both the variance of Gaussian noise and the scale of the Laplacian noise are set to 0.5 and 2.5, respectively.

![Figure 3](image)

**Figure 3.** Variation of the amplitude of synthetic data simulating ionospheric anomalies with Gaussian noise (a) and Laplacian noise (b). The ionospheric gradient anomalies are simulated from the 1001st epoch by adding ramp-type errors. The absolute maximum values of noise in (a) are 1.7 m (variance = 0.5) and 6.9 m (variance = 2.5). The absolute maximum values in (b) are 1.9 m (scale = 0.5) and 9.6 m (scale = 2.5).

To detect whether anomalies occur, (9) is first reconstructed to the form of (7). Figure 4 takes the \(L = 3\) as an example to show the constructed phase state of Gaussian and Laplacian noise. The distributions in the constructed phase state are spindle-shaped, indicating that the distributions of normal and anomalous vectors are different. The thick middle part where points are brought together represents normal vectors, while the sharp head
part where points spread far away represents abnormal vectors. These two parts are easily separated.

![Figure 4](image)

**Figure 4.** Vector sets of the time series in a phase space ($L = 3$) under (a) Gaussian noise and (b) Laplacian noise. Both distributions are spindle-shaped, indicating that ionospheric anomalies, denoted by the sharp head part, can be easily distinguished from the normal points denoted by the thick middle part.

The first 500 epochs of the time sequences are considered as the training set to construct the OCSVM, while the 501st to 1000th epochs are the validation set to obtain the threshold [3,18]. Figure 5 is the visualization of the 3D trained OCSVM decision bound and the corresponding support vectors in the input space. The vectors are constructed under Gaussian noise with the variance set to 2.5 in Figure 5a, and under Laplacian noise with the scale set to 2.5 in Figure 5b. The red plane is the 3D decision boundary. The corresponding hyperplane maximizes the distance between the original points and vectors in the high-dimensional space. Thus, the plane separates the normal vectors from the anomaly ones. The case when the variance and scale parameters are 0.5 is basically the same as that of 2.5, thus it is not shown.

During the monitoring process, the parameter determination of the dimension and the RBF kernel is important for the determination of an appropriate OCSVM-based monitor. It is noticeable that both of them have an impact on the OCSVM-based monitor; thus, they need to be estimated simultaneously. Optimal parameters are chosen to get the lowest average detection time under different levels of noise. By adjusting the amplitude of noise from 0.5 to 2.5 at a step of 0.5 to characterize the main changes in noise [18], the corresponding parameter is obtained. Taking Gaussian noise as an example, the average detection time with different dimensions and kernel scales is shown in Figure 6. There is an obvious variation of the detection time with the increase of the dimension $L$. The minimum average time of 86 epochs occurs when $L = 10$. As for the RBF kernel scale $\sigma$, a larger scale generally results in a lower average time. When it reaches a critical point, the average detection time tends to converge and maintains unchanged even keeping increasing the kernel scale continuously. When $L = 10$, the critical point appears when $\sigma$ is 20, which is considered the optimal parameter of the RBF kernel scale. As a result, $\sigma = 20$ and $L = 10$ are chosen as the optimal values. A similar method is used to determine the optimal parameter under the Laplacian noise. In this case, $\sigma = 18$ and $L = 60$ are chosen as the optimal values.
Figure 5. Trained OCSVM and the vectors in an input phase space \((L = 3)\) with (a) Gaussian noise and (b) Laplacian noise. Red plane is the 3D contour plane.

Figure 6. Variation of the average detection time in relation to \(L\) and \(\sigma\) with synthetic data. The minimum time of 86 epochs occurs when \(\sigma = 20\) and \(L = 10\).

To compare the ionospheric anomaly detection performance, three monitors are used as the control group, i.e., the traditional CCD-1OF with \(\tau_1 = 100\) epochs [14], the traditional CCD-2OF with \(\tau_1 = \tau_2 = 30\) epochs [15], the Kullback-Leibler metric-based monitor KLD-1OF with \(\tau = 150\) epochs and \(L = 50\) [18]. The proposed OCSVM-based monitor is selected with the parameters and threshold estimated through the process described previously. The probability of fault detection allocated for the monitor is \(10^{-8}\).

Figure 7 illustrates the test statistics and the corresponding thresholds of the CCD-1OF, CCD-2OF, KLD-1OF and the OCSVM-based monitors, under Gaussian noise with the variance set as 0.5 and 2.5, respectively. In the presence of weak noise when the noise variance is set to 0.5 in Figure 7a, the test statistic of the OCSVM-based monitor takes...
the shortest time to cross the threshold, thus the OCSVM-based monitor can perform faster than others. In the presence of strong noise when the noise variance is set to 2.5 in Figure 7b, traditional CCD monitors cannot detect any divergence, thus the detection of the ionospheric anomaly is missed. Therefore, such noise, which may be significant for high-level precision approach, can be missed without timely alarming with CCD-1OF and CCD-2OF. Although the KLD-1OF can also detect anomalies, the OCSVM-based monitor outperforms it in the detection speed.

Figure 7. Variation of the test statistics and thresholds of the CCD-1OF, CCD-2OF, KLD-1OF and the proposed OCSVM-based monitors under Gaussian noise, with the noise variance set to 0.5 in (a) and 2.5 in (b). If the ionospheric anomaly is successfully detected, the detection time is included. Note that the ionospheric anomaly is inserted from the 1000th epoch.

Figure 8 illustrates the test statistics and the corresponding thresholds under Laplacian noise with the scale set to 0.5 and 2.5, respectively. When the amplitude of the noise is small, the OCSVM-based monitor outperforms all the other monitors. When the noise variance is 2.5, only the test statistic of the OCSVM-based monitor crosses the threshold, which means that only this monitor successfully detects anomalies. This result again indicates the effectiveness of the OCSVM-based monitor under all noise levels.
Figure 8 illustrates the test statistics and the corresponding thresholds under Laplacian noise with the scale set to 0.5 and 2.5, respectively. When the amplitude of the noise is small, the OCSVM-based monitor outperforms all the other monitors. When the noise variance is 2.5, only the test statistic of the OCSVM-based monitor crosses the threshold, which means that only this monitor successfully detects anomalies. This result again indicates the effectiveness of the OCSVM-based monitor under all noise levels.

Figure 8. Variation of the test statistics and thresholds of the CCD-1OF, CCD-2OF, KLD-1OF and the proposed OCSVM-based monitors under Laplacian noise, with the noise variance set to 0.5 in (a) and 2.5 in (b). If the ionospheric anomaly is successfully detected, the detection time is included. Note that the ionospheric anomaly is inserted from the 1000th epoch.
Figure 9 presents the average detection time of the four monitors in the presence of varied noise from 50 repeated tests. The results are not shown when monitors fail to detect any ionospheric anomaly. The proposed OCSVM-based monitor is always faster than the others in detecting the ionospheric anomaly. When the noise amplitude is getting larger, the CCD-1OF and CCD-2OF monitors fail to sense the anomaly and the KLD-1OF detects the anomaly at a lower speed. Only the OCSVM-based monitor can detect the anomaly at a relatively high speed. In conclusion, the OCSVM-based monitor is more suitable for ionospheric anomalous detection compared with the traditional CCD monitors in the GBAS regardless of the noise levels and types.

![Figure 9](image_url)

Figure 9. Variations of the ionospheric anomaly time in relation to Gaussian (a) and Laplacian (b) noise levels when the CCD-1OF, CCD-2OF, KLD-1OF and OCSVM-based monitors are applied. Ionospheric anomalies that cannot be detected are not shown in the figure.

### 3.2. Ionospheric Anomaly Detection with Semi-Simulation Data

To further evaluate the performance of the proposed ionospheric anomaly monitor, the real observed data are processed. In this work, the real data are collected from the GBAS deployed at the Dongying Airport in Shandong, China [38]. The B1I signals of BDS-2 are captured by four reference receivers, each equipped with the same choke ring antenna as shown in Figure 10. The BDS code and carrier phases measurements are recorded continuously at a sampling rate of 1 Hz. Previous literature shows that one day’s data is sufficient to construct a model to perform an evaluation of the detection time [3,5,18]. The antenna of this receiver is installed beside the runway of the airport. The elevation mask is set to 15°. Thus, 24-h data collected by the 1st receiver on 2 September 2020 are used in this analysis. In Figure 11, the ground-related ranging noise of PRN 7 is drawn as an example. It is obtained as shown in [29]. The amplitude lies between $-1.5$ m and 1.5 m. The data are then binned at an interval of 5° to illustrate the performance of monitors under different noise levels. In addition, because the probability of ionospheric gradient anomaly in practice is very small, which is in the magnitude of about $10^{-7}$, we artificially add the simulated anomalies rather than process the real anomalies. As the airport is located in the mid-latitude region, we use the ramp-type error model to simulate the anomaly as shown in Figure 1.
Figure 10. BDS GBAS antenna mounted on the ground near the runway at the Dongying Airport in Shandong, China.

Figure 11. Processed BDS ranging noise of PRN 7.

To test the performance of the OCSVM-based monitor, a parameter determination process for real data is required first, which is the same as that for the synthetic data. $L$ and $\sigma$ are estimated in the light of the average detection time of the satellite PRN 7 with the ionospheric gradient changing at a rate of 0.018 m/s [3,15]. The satellite is chosen as it covers a larger range of elevations. According to Figure 12, values of $L = 20$ and $\sigma = 9.6$ are chosen as the optimal parameters.

Figure 12. Variation of the average detection time for determining the optimal $L$ and $\sigma$ with real data. The minimum time of 14.75 s occurs when $\sigma = 9.6$ and $L = 20$. 
After determining the parameters of the proposed monitor, the threshold in every elevation bin is calculated with the similar method described in Section 2.2. Figure 13 shows the estimated metric and the determined thresholds as a function of elevations.

![Figure 13. Variation of the metric and threshold of the proposed OCSVM-based monitor in terms of satellite elevations.](image)

The impact of severe ionospheric gradients is then simulated. Two different ionospheric gradients, with the change rate respectively set to 0.01 and 0.02 m/s, are inserted to illustrate the effectiveness of the proposed monitor. The duration of the inserted gradients is 200 s. The experiment is repeated 50 times to compare the stability of different monitors. Figure 14 shows the average detection time in each elevation bin when the four different monitors are implemented in Figure 14a,c. Standard deviations of the detection time are also shown in Figure 14b,d. A greater deviation indicates a lower stability of the detection performance. Because the signal at a lower elevation angle tends to suffer more noise, such as multipath, the detection time decreases as the elevation increases. The standard deviation at lower elevation bin is missing in the figure due to the detection failure. Compared with the other three monitors, the OCSVM-based monitor has the shortest detection time and the greatest detection speed. Especially, compared with the KLD-1OF, the OCSVM-based monitor can decrease the average detection time respectively by about 55% and 44% when the change rate is 0.01 m/s and 0.02 m/s. Additionally, the CCD-2OF and KLD-1OF perform more steadily than the OCSVM-based monitor. The reason is that the averaging process is absent in the OCSVM-based monitor, thus the OCSVM-based monitor has more instabilities in detecting the ionospheric anomalies. However, the shortest detection time of the traditional monitor is mostly greater than the longest detection time of the OCSVM-based monitor. Thus, the instability of the OCSVM-based monitor can be tolerated. In addition, there is no false alarm event in the simulation.

Because the GBAS involves the field of life safety, small ionospheric anomalies are non-negligible for the reliable navigation [18]. Being able to detect small gradient changes, such as at a rate of 0.001 m/s is also a clear advantage compared with the traditional monitors. The capabilities of the four different monitors are compared for detecting the ionospheric anomaly occurring on PRN 7. The detection capability refers to the minimum gradient change rate that can be detected during the ionospheric anomaly occurrence. In this example, the rate of the gradient change is varied from 0.001 to 0.025 m/s. Figure 15 shows the minimum gradient changes that can be detected for the four monitors. As it can be seen, the OCSVM-based monitor presents a better detection capability than the other monitors. When it is applied, the minimum gradient changes detected are lower than...
0.005 m/s over all elevations. Thus, the OCSVM-based monitor can reduce the integrity risk by alarming quickly and accurately.

Figure 14. Comparison of the average and the standard deviations of the detection time when applying the CCD-1OF, CCD-2OF, KLD-1OF and the proposed OCSVM-based monitors to detect the ionospheric anomaly on PRN7. The ionospheric gradients, with the change rate set to 0.01 m/s in (a, b) and 0.02 m/s in (c, d), are inserted.

To further verify the effectiveness of the OCSVM-based monitor, all visible satellites are considered in the ionospheric anomaly detection. The ionospheric gradient change, with the rate of 0.02 m/s, is added to all the satellites. The detection time of ionospheric anomaly on each satellite in each elevation bin is counted through 50 experiments when the traditional and proposed monitors are implemented. The average and the standard deviation of the detection time in bin are calculated. Additionally, because the signal qualities of satellites are different, the signals with large noise may be excluded when using the traditional monitors [12]. By contrast, the signal quality does not affect the OCSVM-based monitor, since the OCSVM-based monitor can detect ionospheric anomalies in the environment with large noise. The detection time is shown in Figure 16. It can be seen that the detection time maintains the lowest when the OCSVM-based monitor is used. The OCSVM-based monitor decreases the average detection time by about 43% compared with the KLD-1OF monitor. When the elevations are high, the CCD-2OF and KLD-1OF monitors slightly outperform in terms of the standard deviations, i.e., the stability. However, considering the significant contribution of the OCSVM-based monitor to the detection speed, it is acceptable with more instability for the proposed monitor. It can be seen that the test results of all visible satellites are quite similar to the results of PRN7 alone, regardless of the average or the standard deviation of the detection time. This proves that the OCSVM-based monitor performs well under all circumstances. Therefore, the proposed monitor can help to ensure the integrity of the GBAS by accelerating the detection. Thus, the proposed monitor can help to ensure integrity of the GBAS by accelerating the detection.
Figure 15. Variations of the minimum gradient changes that can be detected by different monitors in terms of elevations.

Figure 16. Comparison of the average (a) and the standard deviations (b) of the detection time of all BDS-2 satellites in relation to satellite elevations using the CCD-1OF, CCD-2OF, KLD-1OF and the proposed OCSVM-based monitors. The ionospheric gradient change with a rate of 0.02 m/s is added to all visible satellites.

3.3. Real Ionospheric Anomaly Event Experiment

The performance of the monitors is evaluated in the presence of real ionospheric anomaly in this section. Global Positioning System (GPS) data on L1 measurements collected from the Crustal Movement Observation Network of China from 2011 to 2014 were processed in [39]. During this period, the largest detected gradient as 71 mm/km in northern China occurred on 23 May 2012, which was identified using the data from two stations, i.e., BJSH (Shisanling, Beijing, 40.3° N, 116.2° E) and BJYQ (Yanqing, Beijing, 40.4° N, 116.0° E), located in middle latitude region with a baseline of 25.5 km. The satellite affected by the ionospheric anomaly was PRN 10. The geomagnetic storm class of this day is moderate (Kp = 4.7, Dst = −30 nT). When the GPS signal passed through the ionosphere, it was affected by the irregular geomagnetic storm, resulting in an ionospheric anomaly on this day [40].

The ionospheric slant delays for PRN 10 captured at the BJSH and BJYQ stations from 15 to 21 Universal Time (UT) are shown in the Figure 17a. It can be seen that ionospheric...
delay calculated at the two stations shares a similar tendency, while it is slightly larger from 15.6 to 15.7 UT at BJSH. This difference is caused by a spatial ionospheric gradient. Figure 17b shows the gradient levels measured along the BJSH-BJYQ baseline. The maximum value of 71 mm/km occurs at 15.7 UT. This spatial gradient is not as extreme as those reported in [7], but it is still a severe threat for the safety of civil aviation. Figure 17c shows the elevation of PRN 10 viewed at BJSH and BJYQ stations. It can be seen that the anomaly occurred when the satellite was about 30°. Then we pay attention to the measurements at BJSH station. In Figure 17d, the observation ranging noise and ionospheric delay of PRN 10 calculated at BJSH station are shown. During the period of ionospheric anomalies lasting for 420 s, the anomalous ionospheric gradient increased by about 2.1 m; thus, the average change rate is approximately 0.005 m/s. Additionally, the predicted normal ionospheric delay is opposite to the actual abnormal ionospheric delay [29], so the net ionospheric delay change rate exceeds 0.005 m/s and is up to a constant as 0.008 m/s.

Figure 17. A typical ionospheric anomaly caused by geomagnetic storms in Beijing, China on 23 May 2012. Panels are: (a) The ionospheric delay measured on GPS PRN 10 at BJSH and BJYQ stations; (b) the ionospheric gradient calculated along the baseline between the two stations; (c) the satellite elevations of PRN 10; (d) the observation ranging errors of PRN 10 at BJSH station. The amplitude of noise is briefly limited to 1.5 m.

Under the circumstance of the ionospheric anomaly shown in Figure 17, the performance of the proposed ionospheric anomaly monitor is evaluated. Parameters of traditional CCD monitors are the same as that presented in Section 3.2. $\tau = 30$ epochs and $L = 25$ are chosen for KLD-1OF [18], and $L = 21$ and $\sigma = 15$ are chosen for the OCSVM-based monitor. Due to the fact that the time of anomalies is only 420 s, the threshold is set to a fixed value during this period. Figure 18 illustrates the test statistics and the corresponding thresholds of four monitors. The CCD-1OF monitor fails to detect the ionospheric anomaly in this real case. On the other hand, the CCD-2OF, KLD-1OF and the OCSVM-based monitors can detect the ionospheric anomalies, while the OCSVM-based monitor is at a higher speed. Compared with the KLD-1OF monitor, the detection time can be decreased by 16%. There are two possible reasons to explain why the improvement in the real event is not as obvious as that in the simulation data. First, there may be cycle slips that are not fully repaired,
resulting in slightly different distributions between the training set and test set. In addition, the distribution of noise in the irregular environment may be more complex than that of normal noise. However, in any case, the OCSVM-based monitor still performs well in detecting the real anomalous ionospheric gradient.

Figure 18. Variation of the test statistics and thresholds of the CCD-1OF (a), CCD-2OF (b), KLD-1OF (c) and the proposed OCSVM-based (d) monitors in the presence of a real ionospheric anomaly event. If the ionospheric anomaly is successfully detected, the detection time is included. Note that the ionospheric anomaly starts at the 1000th epoch.

4. Conclusions and Perspectives

Anomalous ionospheric gradient is a challenging risk source during the high-level precision approach in the GBAS. Several traditional monitors such as CCD-1OF, CCD-2OF, KLD-1OF have been proposed to detect the ionospheric anomalies based on the statistics in terms of the divergence. However, they do not guarantee a rapid identification or a prompt warning to users in noisy environment. This issue may dissatisfy the requirements of the precision approach and result in potential risks.

To overcome this issue, a novel OCSVM-based monitor was proposed in this work to more accurately and rapidly detect ionospheric anomalies in the GBAS. By combining phase-space reconstitution, the one-dimensional CMC time series can be transformed into high-dimensional vectors. Then, the trained OCSVM model was determined using the normal vectors offline to effectively extract the characteristics of the normal measurements and the threshold. Then, current scores can be obtained on the basis of the online measurements passing through the trained model. The distinction between the online score and the offline threshold was sensed to determine whether the ionospheric anomalies occurred. On the basis of the synthetic and BDS real data, the detection speed for the single-frequency GBAS under different levels of noise was analyzed. In addition, the performance evaluation based on real ionospheric anomalous event environment is also carried out. The proposed monitor showed superiority over the CCD-1OF, CCD-2OF and
KLD-1OF monitors regardless of the noise levels. Results showed that the efficiency can be increased by more than 40% compared with the KLD-1OF monitor. Additionally, the OCSVM-based monitor mainly has the ability to detect small anomalies. In real abnormal environment, the OCSVM-based monitor performs better than the KLD-1OF monitor for a reduction of detection time by 16%. As a result, the OCSVM-based monitor can provide real-time protection against the ionospheric anomaly, which provides a better safeguard for future civil aviation.

Follow-up work can be carried out in three ways. First, the OCSVM-based monitor can only be utilized when the cycle slip is absent. Thus, this monitor is used under the implicit assumption that the ionospheric disturbance is slight. Under strong ionospheric disturbance, the possibility to improve the OCSVM-based monitor ability will be investigated in future work. Second, limited by hardware, only the ionospheric anomaly monitoring capability at Dongying airport is tested. Further verification of different scenarios is necessary before the OCSVM algorithm can be leveraged in practice in the safety-critical systems. Finally, the proposed OCSVM-based monitor could still be further applied in the other GNSS argumentation systems. A further analysis of how the proposed monitor can be utilized in other systems will be carried out.

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Abbreviations

| Acronym | Description |
|---------|-------------|
| ARMA    | Auto-regressive and moving average |
| BDS     | BeiDou Navigation Satellite System |
| CCD     | Code-carrier divergence |
| CCD-1OF | CCD monitor with one first-order low-pass ARMA filter |
| CCD-2OF | CCD monitor with two first-order cascade ARMA filters |
| GBAS    | Ground Based Augmentation System |
| CMC     | Code minus carrier |
| ED      | Embedding dimension |
| GAST    | GBAS Approach Service Type |
| GNSS    | Global Navigation Satellite System |
| GPS     | Global Positioning System |
| KDE     | Kernel density estimation |
| KLD-1OF | Kullback–Leibler divergence metric using one first order ARMA filter |
| OCSVM   | One class support vector machine |
| RBF     | Radial basis function |
| SVM     | Support vector machine |
| UT      | Universal time |
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