Noise correlation-induced splitting of Kramers’ escape rate from a metastable state

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Abstract

A correlation between two noise processes driving the thermally activated particles in a symmetric triple well potential, may cause a symmetry breaking and a difference in relative stability of the two side wells with respect to the middle one. This leads to an asymmetric localization of population and splitting of Kramers’ rate of escape from the middle well, ensuring a preferential distribution of the products in the course of a parallel reaction.

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I. INTRODUCTION

The escape of particles from a metastable well has long been the focal theme of activated rate processes in chemical kinetics and condensed matter physics. When a stochastic system is thermodynamically made open by the action of external periodic or random forces significant changes in the dynamics take place which reflect the constructive role of noise in dynamical systems. The well-known examples include stochastic resonance, resonant activation, noise-induced transition, ratchet and rectification of noise etc to mention a few. In the overwhelming majority of these cases the essential physics relies on double-well as a model potential. The present work concentrates on the motion of Brownian particles in the middle well of a symmetric triple well potential which diffuses symmetrically to the left and the right well. At a finite temperature and in absence of any bias force the particles are activated only by inherent thermal fluctuations resulting in equalization of population in the two side wells. Application of an additive noise can not lead to any change of time averaged relative population of the two wells. The situation is expected to remain unaltered even if, in addition, one introduces a multiplicative noise which makes the diffusion state dependent. The aim of the present work is to look for a scheme which leads to a symmetry breaking resulting in a preferential population distribution in one of the two side wells. In what follows we show that the presence of correlation between the applied additive and multiplicative noises may cause a change in relative stability of the side wells with respect to the middle one. This correlation induced interference of the two noises leads to a splitting of Kramers’ rate of escape from the metastable well.

The correlation between noise processes has been the subject of study in a number of issues. For example, it has been shown that correlation strongly influences the noise-induced phase transitions from unimodal to bimodal distribution. Fox has investigated the correlation between multi-component, Markovian and Gaussian stochastic processes. The effect of correlation between quantum noises in laser modes and in the description of hydrodynamic modes are of interest in the related context. Our proposal in this work concentrates on altering the relative stability of the two side wells with respect to the middle one of a triple-well potential under the influence of correlation between the noises, where the underlying idea rests on controlling the pathways of a parallel reaction. A prototypical example may be set by considering a nucleophilic attack by $X^-$ (a halid ion of $HX$) at the carboxyl
group of a ketone say $R_1(R_2)C = 0$ which produces $D$ and $L$ forms of $R_1(R_2)C(OH)X$ - the two optical isomers. They are of same energy but differ in optical properties. The middle well represents the reactant state while the two side wells refer to the product states of the parallel reaction. To realize an experimental situation we introduce a light field of fluctuating intensity which polarizes the photosensitive carbonyl group rendering planarity of the polarized states and causing symmetric oscillation of the barrier height. If an electric field of fluctuating intensity and of common origin is now imposed, in addition, on the polarized system, then depending on the cross-correlation of the fluctuations of light and electric fields, the nucleophilic attack of an anion $X^-$ will be asymmetric. This is because the electric field rocks the side wells (similar to what one observes in stochastic resonance) implying the differential relative stability of the transition states of the complex comprising polarized system plus anion. Based on a Langevin and an associated Fokker-Planck description for the dynamics where the additive and multiplicative noise processes are independently Gaussian and $\delta-$correlated in character but the cross-correlation between them is exponential, we derive an analytical expression for the splitting of Kramers’ rate due to correlation induced asymmetry in the state dependent diffusion. This results in preferential distribution of reaction products of the parallel reaction due to differential relative stability of the two wells.

II. THE MODEL

Consider an overdamped Brownian particle in a triple-well potential $V(x)$ kept in a thermal bath at temperature $T$ and subjected to two stochastic forces $\epsilon_1(t)$ and $\epsilon_2(t)$. The governing Langevin equation is given by

$$\gamma \dot{x} = -V'(x) + \epsilon_1(t) + x\epsilon_2(t) + \Gamma(t) \quad (1)$$

where $V(x) = x^2(bx^2-c)^2$ symmetric triple-well potential (Fig.1); $b$ and $c$ are the parameters of the potential and $\gamma$ is the dissipation constant. Thermal fluctuation $\Gamma(t)$ of the bath is modeled by Gaussian, zero mean and delta correlated noise

$$\langle \Gamma(t) \rangle = 0 \quad (2a)$$

$$\langle \Gamma(t)\Gamma(t') \rangle = 2D\delta(t-t') \quad (2b)$$
$D$ being the strength of thermal fluctuation and is given by $D = kT/\gamma$. Here the additive random force $\epsilon_1(t)$ rocks the potential wells sidewise randomly, whereas the multiplicative force $\epsilon_2(t)$ sets random fluctuation of the barrier height around $\Delta V_0 = 4c^3/27b$, in a symmetric manner. The system as described by (1) is associated with both the thermal and non-thermal environments. To achieve our desired asymmetry in the dynamics of the particle in a symmetric triple-well potential, we apply an electric field and a radiation field simultaneously. The interaction with the radiation field is relatively stronger than that with the applied electric field. The multiplicative and additive noises in Eq.(1) correspond to fluctuating amplitude of the radiation field and the electric field, respectively. To keep the treatment on a general footing one may assume, $\epsilon_1$ and $\epsilon_2$ to be colored. This may cause a serious difficulty for an analytical approach. On the other hand to capture the essential physics we may assume $\epsilon_1(t)$ and $\epsilon_2(t)$ to be Gaussian white noises. This does not change the inherent feature of the proposed model problem. The characteristics of the noise processes can be summarized as follows

\[
\langle \epsilon_1(t) \rangle = \langle \epsilon_2(t) \rangle = 0 \tag{3a}
\]
\[
\langle \epsilon_1(t)\epsilon_1(t') \rangle = 2Q_1\delta(t - t') \tag{3b}
\]
\[
\langle \epsilon_2(t)\epsilon_2(t') \rangle = 2Q_2\delta(t - t') \tag{3c}
\]

$Q_1, Q_2$ are the strength of $\epsilon_1(t)$ and $\epsilon_2(t)$. Now if the simultaneous action of fluctuating electric and radiation fields, is due to a common origin then the statistical properties of the noises are not expected to differ widely and may be correlated. We characterize the correlation of $\zeta(t)$ and $\eta(t)$ as follows

\[
\langle \epsilon_1(t)\epsilon_2(t') \rangle = \langle \epsilon_1(t')\epsilon_2(t) \rangle = \frac{\lambda\sqrt{Q_1Q_2}}{\tau} \exp \left[ -\frac{(t - t')}{\tau} \right] \tag{4}
\]
\[
= 2\lambda\sqrt{Q_1Q_2} \delta(t - t') \quad \text{as} \quad \tau \rightarrow 0
\]

$\lambda$ is strength of cross-correlation and $\tau$ is the cross-correlation time. By colored correlation between the white noises we mean that both the external fluctuations are affected by each other for certain ranges frequencies. Again if we assume that cross-correlation is a $\delta$–function, then the gross feature of our model problem will remain unchanged. However for generality, we have assumed colored cross-correlation.

We now proceed with a probabilistic description corresponding to Langevin equation (1) with the prescriptions (2,3,4) for internal thermal noise and external forces, respectively.
Following the time evolution equation for the probability density is given by
\[
\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial x} V'(x)P(x,t) - \frac{\partial}{\partial x} \langle \epsilon_1(t) \delta(x(t) - x) \rangle - \langle x \epsilon_2(t) \delta(x(t) - x) \rangle + D \frac{\partial^2 P(x,t)}{\partial x^2}
\] (5)
where \( P(x,t) = \langle \delta(x(t) - x) \rangle \); the averages \( \langle ... \rangle \) in Eq.\( \text{(5)} \) can be calculated for Gaussian noise by the Novikov theorem. The resulting equation is the Fokker-Planck description as given by.

The effective diffusion constant is an asymmetric function of position, which means it is dissimilar in the three wells of the triple well potential. Around the minimum of the middle well diffusion is almost independent of space and has a constant value, while towards the right well diffusion strength quadratically increases with position and decreases towards the left well. So the implications for the simultaneous action of the two forces \( \epsilon_1(t) \) and \( \epsilon_2(t) \) is that, the diffusive nature of the Brownian motion in the three wells differs and therefore one

III. STATIONARY DISTRIBUTION AND ASYMMETRIC LOCALIZATION

We now return to the Fokker-Planck Eq.\( \text{(6)} \). We can recast it in the more simpler form as follows
\[
\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial x} f(x)P(x,t) + Q_2 \frac{\partial}{\partial x} x \frac{\partial}{\partial x} x P(x,t) + \frac{\lambda \sqrt{Q_1 Q_2}}{1 + 8c^2 \tau} \frac{\partial}{\partial x} x \frac{\partial}{\partial x} x P(x,t)
\]
\[
+ \frac{\lambda \sqrt{Q_1 Q_2}}{1 + 8c^2 \tau} \frac{\partial^2}{\partial x^2} x P(x,t) + (Q_1 + D) \frac{\partial^2}{\partial x^2} x P(x,t)
\] (6)
The only constraint on \( \tau \) is that, \( 1 + 8c^2 \tau > 0 \) (7)
c is a potential parameter and a real positive number. Thus practically there exists no restriction on \( \tau \) in this case.

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can expect an asymmetric distribution of the particles in the two side wells and different transition rates from middle well to right and left well.

As \( t \to \infty \), the system reaches the stationary state \( \left( \frac{\partial P}{\partial t} = 0 \right) \) with the current attaining a constant or zero value. As the process \( x(t) \) is bound to the triple-well potential it is expected that in the stationary state there will have no net flow of particles, and hence one may assume a zero current stationary state. The solution of the Eq.(8) is given by

\[
P(x) = D(x, \tau)^{-1/2} \exp \left[ \int_{x}^{\infty} dy \frac{f(y)}{D(y, \tau)} \right]
\]  

(10)

It is apparent from the expressions (9) and (10) that as a result of interplay of two stochastic driving forces the distribution function is asymmetric in space. The asymmetry in the distribution function arises due to asymmetric diffusion of the particles. To illustrate the asymmetric localization of the particles we have plotted the distribution function as a function of position in Fig.2. The solid line of Fig.2 presents a symmetric distribution in absence the external driving forces. The dotted line presents the same plot in presence of additive and multiplicative noise but for no cross-correlation \( (\lambda = 0) \). In this case the distribution function still remains symmetric but the population of the middle well is much higher than that of the side wells due to the fact that in presence of multiplicative noise the effective diffusion becomes space dependent, \( D(x) = Q_2 x^2 + Q_1 + D \). As a result the particles in the side wells diffuse to the middle more quickly as compared to the particles of the middle well which diffuse to the terminal wells at a relatively slower rate and the particles spend most of the time in the middle well. This implies that higher diffusion destabilizes the side wells compared to the middle well. When the two stochastic forces are correlated \( (\lambda \neq 0) \), interestingly, because of the spatial asymmetry in diffusion the Brownian particles are preferentially localized in the left well compared to the right. This has been presented by the dashed line in Fig.2.

To proceed further we require a quantifier which measures the asymmetry in localization in the two wells. To this end we choose the mean position of the particle as its measure. For a symmetric distribution mean position \( \langle x \rangle = 0 \) and for the localization of the particles in left or right well, the value of mean position is negative or positive, respectively. To this end an expression for \( \langle x \rangle \) from a direct steady state solution of Fokker-Planck equation (10) can be formally obtained. However, since this involves a complicated form of space dependent diffusion coefficient, the final expression, which, in principle, contains all the
information regarding the interwell transitions, is a lumped expression and it is difficult to figure out the details of transition from one well to another. To have a closer look into this aspect we examine the variation of mean position $\langle x \rangle$ with temperature, $Q_1$, $Q_2$ and correlation time with the help of a discrete three-state model for the continuous triple well potential. Three states are denoted by $x_0$, $\pm x_m$ for the symmetric unperturbed system corresponding to three minima. The diffusional motion causes transitions between them and it is schematically presented by the following kinetic model

$$
k_L \quad k^R_M
$$

$$L \iff M \iff R$$

$$k^L_M \quad k_R$$

$k_L$, $k_R$, $k^R_M$, $k^L_M$ denote the time averaged rate of transition from left to middle well, right to middle well, middle to right and middle to left well, respectively. The number of particles in the three states at time $t$ are denoted by $n_L$, $n_R$ and $n_M$. The governing master equations for $n_i$ ($i = L, R, M$) read as

$$\frac{dn_L}{dt} = -k_L n_L + k^L_M n_M$$

$$\frac{dn_R}{dt} = -k_R n_R + k^R_M n_M$$

$$\frac{dn_M}{dt} = k_L n_L + k_R n_R - (k^L_M + k^R_M) n_M$$

At the steady state ($\dot{n}_L = \dot{n}_M = \dot{n}_R = 0$) the probability of finding the particles at the three wells $P_i$ ($i = L, R, M$) are

$$P_L = k^L_M k_R / P, \quad P_R = k^R_M k_L / P, \quad P_M = k_L k_R / P$$

where $P = k_R k^L_M + k_R k_L + k_L k^R_M$. The expression for the mean position is then given by

$$\langle x \rangle = \int_{-\infty}^{+\infty} x P(x) \, dx = x_m P_R + x_0 P_M - x_m P_L$$

$$= \left( \frac{\sqrt{27\Delta V_0}}{2c} \right) \frac{k^R_M k_L - k^L_M k_R}{k_R k^L_M + k_R k_L + k_L k^R_M}$$

The above expression clearly expresses the dependence of mean position and probability on four rate constants. All the individual rate constants have a typical dependence on the system parameters, such as, temperature, intensity of the external noises and cross-correlation
time. We therefore anticipate a distinct signature of asymmetric localization of the particles with the variation of system parameters. As revealed by Eq. (15) the mean position is directly proportional to \( (k^R_M k^L_R - k^L_M k^R_M) \), that is, the difference involving the product of right hand directed transition rates and the product of left hand directed transition rates.

Our numerical illustration shows that the mean position \( \langle x \rangle \neq 0 \) only if \( \lambda \neq 0 \). This means that the particles are asymmetrically localized in the triple-well potential only when there is a finite correlation between two external stochastic drives \( \epsilon_1(t) \) and \( \epsilon_2(t) \). This type of behaviour can be physically explained as follows: As both the external forces \( \epsilon_1(t) \) and \( \epsilon_2(t) \), act independently to rock the potential well randomly in an asymmetric manner and randomly modulate the barrier heights in a symmetric way, respectively, then the symmetry of the triple-well system remains intact as the individual action of \( \epsilon_1(t) \) and \( \epsilon_2(t) \) are not able to break the symmetry of the system. If there is some correlation between \( \epsilon_1(t) \) and \( \epsilon_2(t) \), which means that for a number of stochastic realizations, both forces have the same sign at a particular instant of time. A qualitative interpretation of the localization may be given as follows: as long as the force \( \epsilon_2(t) \) causing symmetric fluctuation of the barrier height attains its lower value, the force \( \epsilon_1(t) \) points to the right well so that the particle in the middle well move towards the right well very quickly. On the other hand as the tilting force points to the left \( \epsilon_2(t) \) sets the barrier height at a larger value and consequently the particle in the middle well takes relatively larger time to speed up from middle to left well for the simultaneous action of the synchronized forces. The particles in the middle well therefore have a greater chance to cross the right-hand barrier and the particles in the right well have a greater chance to move to the middle well as the fluctuation of barrier height in the right well occurs with a higher amplitude. Therefore the relative stability of the two wells with respect to the middle differs and an asymmetric localization is observed.

Another important question on the asymmetric localization concerns in which well the particles will be preferentially localized, out of two side wells of the triple-well potential. Secondly, what will be the sign of \( \langle x \rangle \). An answer to this question may be obtained as follows. As the diffusive motion of the particles in the right well is greater than that in the left well (as given by the Eq. (15)), the particles will prefer to be localized in the left well, since higher diffusion makes the right well relatively less stable. So the sign of \( \langle x \rangle \) will be always negative. The sign of \( \langle x \rangle \) and hence asymmetric localization can be inverted by reversing the sign of \( \epsilon_1(t) \) in the Eq. (1).
We now proceed to analyze the behavior of localization (mean position) with the variation of system parameters. The effect of temperature in asymmetric localization and in the product distribution of the parallel reaction at the steady state is intimately related to the manipulation of inherent condition rather than coherence in selecting and controlling the reaction pathways. Keeping in view of the Arrhenius temperature dependence of the individual rate constants, the variation of $\langle x \rangle$ with temperature is expected to show a bell-shaped curve. The departure of $\langle x \rangle$ from zero towards negative direction indicates the preferential distribution of the product in the left well. The variation of mean position $\langle x \rangle$ as a function of temperature for several values of strength of correlation for the input forces as shown in Fig.3 corroborates this assertion. Fig.3 also reveals that for fixed values of temperature and other parameter set the mean position increases with increase of the strength of cross-correlation. It clearly indicates that, for an appropriate correlation between additive and multiplicative noises, the particles will be preferentially localized more asymmetrically. In Fig.4 we show the mean position as a function of intensity of the multiplicative noise. With the increase of values of $Q_2$ the mean position gradually moves to a maximum negative value followed by a return to zero at high intensity ($Q_2^2$). This sort of behavior can be understood from the expression for the effective diffusion constant(9). In this expression $Q_2^2$ appears as the symmetric contribution in the first term and as the asymmetric contribution (as $\sqrt{Q_2^2}$) in the second term, so that if one starts from the very low value of $Q_2$, the asymmetry in diffusion first increases then at a relatively high value it starts decreasing. In Fig.5 we present the variation of mean position as a function of intensity of the additive driving force. With increase of $Q_1$ the value of mean position starts departing from zero to reach finally a limiting value. Finally, in Fig.6 we have examined the effect of the cross-correlation time in the asymmetric localization by plotting the mean position as a function of $\tau$ for different values of the strength of cross-correlation. As the asymmetry in diffusion decreases with increase of $\tau$, the departure of the mean position decreases with increasing values of cross-correlation time.
IV. TRANSITION RATE FROM MIDDLE WELL TO SIDE WELLS: SPLITTING OF KRAMERS’ RATE

To what extent the correlation induced asymmetry in the effective diffusion coefficient is reflected in the kinetics of activated processes. An answer to this question lies in examining the transition rates of the particles from the middle well to the side wells. The differential behavior of the transition rates signifies the scope of controlling the path of a parallel reaction. We approach the problem by calculating the mean escape time. The expression for MPFT (mean first passage time) for a particle to reach the final point \( \pm x_b \), starting from an initial point \( x_0 \) is given by

\[
T_R = \int_{x_0}^{+x_b} \frac{dx}{D(x,\tau)P(x)} \int_{-\infty}^{x} P(y)dy
\]

\[
T_L = \int_{x_0}^{-x_b} \frac{dx}{D(x,\tau)P(x)} \int_{-\infty}^{x} P(y)dy
\]

respectively, where \( T_R \) and \( T_L \) denote the mean escape time of the particle from \( x_0 \) to \( +x_b \) and \( -x_b \) respectively (\( +x_b \) and \( -x_b \) are the coordinates of barrier tops toward right and left well, respectively). Putting the expressions for \( P(x) \) and \( D(x,\tau) \), the integrals in the above Eqs(16a,16b) have been calculated using steepest-descent approximation to obtain the expressions for \( T_i \) (i = R, L) in the usual way

\[
T_R = \frac{2\pi}{\sqrt{\omega_R\omega_0}} \exp \left[ -\int_{x_0}^{+x_m} dx \frac{f(x)}{D(x,\tau)} \right]
\]

\[
T_L = \frac{2\pi}{\sqrt{\omega_L\omega_0}} \exp \left[ -\int_{x_0}^{-x_m} dx \frac{f(x)}{D(x,\tau)} \right]
\]

where \( \omega_0 \), \( \omega_R \), \( \omega_L \) are the frequencies corresponding to the potential minimum \( (x_0) \) and the barrier tops \( (\pm x_b) \), respectively. As \( D(x,\tau) \) is an asymmetric function, the integrals \( \int_{x_0}^{+x_b} dx \frac{f(x)}{D(x,\tau)} \) and \( \int_{x_0}^{-x_b} dx \frac{f(x)}{D(x,\tau)} \) are not same. It is thus apparent from the above expression that the transition rate from the middle well to the terminal wells splits up due to the interplay of two correlated stochastic forces. The ratio of the transition rates \( (k_{RM}/k_{LM}) \) deviates from unity \( (T_L/T_R = k_{RM}/k_{LM} \neq 1) \) only when the two external drives are correlated \( (\lambda \neq 0) \). If \( \lambda = 0 \), \( D(x,\tau) \) is a symmetric function of \( x \) and \( \int_{x_0}^{+x_b} dx \frac{f(x)}{D(x,\tau)} = \int_{x_0}^{-x_b} dx \frac{f(x)}{D(x,\tau)} \). So the ratio of the transition rates become unity. In Fig.7 we present the variation of the ratio of the transition rates \( (k_{RM}/k_{LM}) \) as a function of temperature for several values of the coupling strength. As revealed by Fig.7, the ratio of the transition rates at a very low temperature
significantly differs from unity and tends to equalize in the high temperature limit. For a fixed temperature the ratio of the transition rates deviates more from unity for an increase of the coupling strength ($\lambda$). In Fig.8(a,b) we plot the ratio of the transition rates as a function of the intensities ($Q_1, Q_2$) of the external noises. One observes that with increase of intensities of the external noises the magnitude of $k^R_M / k^L_M$ increases to maximum followed by a decrease. This resonance like behavior is observed due to the fact that with increase of intensities of the external noise the synchronization probability of the noise realizations from two noise processes($\epsilon_1(t), \epsilon_2(t)$) in a given time increases as a result of which the transition probability to the right increases. Further increase of noise intensities results in randomization of the system. In order to examine the influence of cross-correlation time of the nonthermal noises on the the ratio of transition rate we plot in Fig.9 the variation of $k^R_M / k^L_M$ as a function of $\tau$ for several values of $\lambda$. With increase of cross-correlation time the ratio of the transition rates monotonically decreases. This is again due to the asymmetry in the diffusion coefficient which decreases with increase of $\tau$.

V. CONCLUSION

We have considered the stochastic dynamics of the particles in a triple well potential driven simultaneously by two cross-correlated white noise processes. It has been shown that depending on the correlation between the two noise sources, one multiplicative and another additive, the relative stability of the two side wells with respect to the middle one may differ significantly. This originates from an asymmetry in diffusive motion of the interwell dynamics due to interference of the two noises. An offshoot of this symmetry breaking effect is the splitting of Kramers' rate of escape from the middle well to the sides wells. In a wider context the kinetic scheme may serve as a technique for preferentially selecting a pathway for a parallel chemical reaction.
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Figure Captions

Fig.1 A schematic illustration of a symmetric triple-well potential as an energy profile of the parallel reaction (specifically, optical isomerization reaction).

Fig.2. Probability distribution function $P(x)$ vs position $x$ plot depicting the changes of distribution due to the addition of external noises for the parameter set: $\tau = 0$, $T = 0.44$, $b = 0.1$ and $c = 1.0$. (i) Solid line presents the distribution function in absence of external stochastic forces, (ii) dotted line presents the same plot in presence of external additive and multiplicative noise in absence of cross-correlation, (iii) dashed line presents also same plot in presence of external additive and multiplicative noise and their cross-correlation.

Fig.3. Variation of mean position $\langle x \rangle$ as a function of temperature $T$ for several values of coupling strength and for the parameter set: $\tau = 0.01$, $Q_1 = 0.02$, $Q_2 = 0.02$, $b = 0.1$ and $c = 1.0$.

Fig.4. Mean position($\langle x \rangle$) vs $Q_2$ (strength of the multiplicative noise) plot for several values of coupling strength and for the parameter set: $\tau = 0.01$, $Q_1 = 0.02$, $T = 0.34$, $b = 0.1$ and $c = 1.0$.

Fig.5. Mean position($\langle x \rangle$) vs $Q_1$ (strength of the multiplicative noise) plot for several values of coupling strength and for the parameter set: $\tau = 0.01$, $Q_2 = 0.02$, $T = 0.34$, $b = 0.1$ and $c = 1.0$.

Fig.6. Variation of $\langle x \rangle$ as function of cross-correlation time $\tau$ for different values of coupling strength and for the parameter set: $T = 0.34$, $Q_1 = 0.02$, $Q_2 = 0.02$, $b = 0.1$, $c = 1.0$.

Fig.7. Ratio of transition rates ($k_{RM}^R/k_{LM}^L$) vs temperature plot for several values of coupling strength and for the parameter set: $T = 0.05$, $Q_2 = 0.05$, $Q_1 = 0.05$, $b = 0.1$ and $c = 1.0$.

Fig.8. (a) Variation of the ratio of the transition rate ($k_{RM}^R/k_{LM}^L$) as function of the strength of additive noise and for several values of coupling strength and for the parameter set: $T = 0.05$, $\tau = 1.0$, $Q_2 = 0.02$, $b = 0.1$ and $c = 1.0$. (b) Variation of the ratio of the transition rate ($k_{RM}^R/k_{LM}^L$) as function of the strength of multiplicative noise for same parameter set as (a) but for $Q_1 = 0.05$.

Fig.9. Variation of the ratio of the transition rate ($k_{RM}^R/k_{LM}^L$) as function of cross-correlation time $\tau$ for different values of coupling strength and for the parameter set: $T = 0.05$, $Q_1 = 0.05$, $Q_2 = 0.05$, $b = 0.1$ and $c = 1.0$. 

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Fig. 1

Product state (L) Reactant state (M) +x_b -x_b +x_m -x_m

Reaction coordinate (x) Product state (R) ΔV_0 V(x)

V(x) ΔV_0

Product state (L) Reactant state (M) Product state (R)

-x_m -x_b x_0 +x_b +x_m

Reaction coordinate (x)

Fig. 1
Fig. 2

$P(x)$

$Q_1 = 0.0$, $Q_2 = 0.0$, $\lambda = 0.0$

$Q_1 = 0.01$, $Q_2 = 0.01$, $\lambda = 0.0$

$Q_1 = 0.01$, $Q_2 = 0.01$, $\lambda = 0.5$
Fig. 3
Fig. 4
Fig. 5
Fig. 6
Fig. 7

\[ \frac{k_M^R}{k_M^L} \]

- \( \lambda = 0.5 \)
- \( \lambda = 0.6 \)
- \( \lambda = 0.7 \)
Fig. 8
Fig. 9