The paper presents late time cosmology in \( f(Q, T) \) gravity where the dark energy is purely geometric in nature. We start by employing a well motivated \( f(Q, T) \) gravity model, \( f(Q, T) = mQ^n + bT \) where \( m, n \) and \( b \) are model parameters. Additionally we also assume the universe to be dominated by pressure-less matter which yields a power law type scale factor of the form 
\[
a(t) = c_2 (At + c_1)^{\frac{1}{A}},
\]
where 
\[
A = \frac{3(8\pi + b)}{n(16\pi + 3b)}
\]
and \( c_1 \) & \( c_2 \) are just integration constants. To investigate the cosmological viability of the model, constraints on the model parameters were imposed from the updated 57 points of Hubble data sets and 580 points of union 2.1 compilation supernovae data sets. We have thoroughly investigated the nature of geometrical dark energy mimicked by the parametrization of \( f(Q, T) = mQ^n + bT \) with the assistance of statefinder diagnostic in \( \{s, r\} \) and \( \{q, r\} \) planes and also performed the Om -diagnostic analysis. The present analysis makes it clear-cut that \( f(Q, T) \) gravity can be promising in addressing the current cosmic acceleration and therefore a suitable alternative to the dark energy problem. Further studies in other cosmological areas are therefore encouraging to further investigate the viability of \( f(Q, T) \) gravity.

PACS numbers: 04.50.Kd

I. INTRODUCTION

Cosmological observations indicate that our Universe is going through a phase of accelerated expansion [1, 2]. With regard to the theoretical predictions and some leading surveys in observational perspective indicates the presence of a curious form of energy with high negative pressure and increasing density. The cosmological entity responsible for the acceleration should account for almost three-quarters of the such curious energy budget of the universe. In addition to that, this entity should also be able to create an anti-gravity effect permeating throughout the entire observable universe. Since normal baryonic matter do not posses such an equation of state, let alone to account for such alion-share of the energy budget of the universe, several alternate scenarios have been proposed and investigated [3].

Interestingly, such an accelerated expansion is possible with normal baryonic matter if the gravitational forces felt by cosmic objects turns out to be different than that proposed by general relativity [4]. Such scenarios are built upon modified theories of gravity in which the geometrical sector of the field equations undergoes a subtle modifications without altering the matter sector. Extended theories of gravity such as \( f(R) \) gravity, \( f(G) \) gravity, \( f(R, T) \) gravity, etc. are widely employed in modern cosmology (For a recent review on modified gravity see [5]. Also see [6] for some interesting cosmological applications of modified gravity) to address the late-time acceleration and other shortcomings of \( \Lambda CDM \) cosmologies.

\( f(Q, T) \) gravity is a recently proposed extended theory of gravity in which the Lagrangian density of the gravitational field is a function of the non-metricity \( Q \) and trace of energy momentum tensor \( T \) [7]. In [8], the authors studied

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*Electronic address: dawrasimran27@gmail.com
†Electronic address: shibesh.math@gmail.com
‡Electronic address: snehasish.bhattacharjee.666@gmail.com
§Electronic address: pksahoo@hyderabad.bits-pilani.ac.in
inflationary and late time cosmology in \( f(T, T) \) gravity where \( T \) and \( T \) are torsion and trace of energy momentum tensor. A very recent study of Yixin \([9]\) shows the non-minimal coupling between \( Q \) and \( T \) taking into account some different class of cosmological models with some specific functional forms of \( f(Q, T) \). Different forms of \( f(Q, T) \) can result in obtaining a large variety of cosmological evolution including the deceleration and acceleration expansions. Also the study by Bhattacharjee \([10]\), \( f(Q, T) \) gravity was found to yield excellent theoretical estimates of baryon-to-entropy ratios and therefore could solve the puzzle of over-abundance of matter over anti-matter. With that in mind, we seek out to investigate the cosmological viability of \( f(Q, T) \) gravity in sufficing the conundrum of late-time acceleration without incorporating dark energy.

The manuscript is organized as follows: The present cosmological scenario is studied and introduced in section I. In Section II we present an overview of \( f(Q, T) \) gravity. In Section III we present the cosmological model employed in the work with some model parameters and derived some physical parameters. In accordance, the non-parametric methods are sometimes more beneficial as the evolution of our universe can be found directly from the observational data. Therefore, in section IV we constrain the model parameters using Hubble (\( H(z) \)) data-sets and Supernovae (\( SN \)) data-sets. In Section V we present some geometrical diagnostics which results in distinction between various dark energy models and \( \Lambda \)CDM. We present the behavior of energy density in Section VI. In Section VII, a special case is discussed. Finally, in Section VIII we present our results and conclusions.

## II. OVERVIEW OF \( f(Q, T) \) GRAVITY

The action in \( f(Q, T) \) Gravity is given as \([7]\),

\[
S = \int \left( \frac{1}{16\pi} f(Q, T) + L_m \right) d^4x \sqrt{-g}. \tag{1}
\]

where \( f \) is an arbitrary function of the non-metricity \( Q \) and the trace of the matter-energy-momentum tensor \( T \), \( L_m \) represents the matter Lagrangian and \( g = det(g_{\mu\nu}) \) and

\[
Q = -g^{\mu\nu}(L^\alpha_{\beta\mu}L^\beta_{\nu\alpha} - L^\alpha_{\beta\nu}L^\beta_{\mu\alpha}). \tag{2}
\]

where \( L^\alpha_{\beta\gamma} \) is the deformation tensor given by,

\[
L^\alpha_{\beta\gamma} = -\frac{1}{2} g^{\alpha\lambda}(\nabla_\gamma g_{\beta\lambda} + \nabla_\beta g_{\lambda\gamma} - \nabla_\lambda g_{\beta\gamma}). \tag{3}
\]

The non-metricity \( Q \) and trace of energy momentum tensor \( T \) are defined respectively as

\[
Q_\alpha = Q^\mu_{\alpha\mu}, \quad T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}L_m)}{\delta g^{\mu\nu}} \tag{4}
\]

and

\[
\Theta_{\mu\nu} = g^{\alpha\beta}\frac{\delta T_{\alpha\beta}}{\delta g^{\mu\nu}}. \tag{5}
\]

The variation of the gravitational action \([1]\) leads to the following field equation

\[
8\pi T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \nabla_\alpha(f_Q\sqrt{-g}P^\alpha_{\mu\nu} - \frac{1}{2} f g_{\mu\nu} + f_T(T_{\mu\nu} + \Theta_{\mu\nu}) - f_Q(P_{\mu\alpha\beta}Q^\alpha_{\nu\beta} - 2Q^\alpha_{\nu\beta}P_{\alpha\beta\nu}). \tag{6}
\]

where \( P^\alpha_{\mu\nu} \) is the super-potential of the model as mentioned in \([7]\).

We now assume a flat FLRW metric as,

\[
ds^2 = -N^2(t)dt^2 + a(t)^2(dx^2 + dy^2 + dz^2), \tag{7}
\]

where \( a(t) \) is the scale factor and \( N(t) \) the Lapse function. We consider the case when \( N(t) = 1 \) i.e. the case of the standard FLRW geometry. Thus we have \( Q = 6H^2 \) and the generalized Friedmann equations are,

\[
8\pi \rho = \frac{f}{2} - 6FH^2 - \frac{2G}{1 + G} (\dot{F}H + F\dot{H}). \tag{8}
\]

and

\[
8\pi p = -\frac{f}{2} + 6FH^2 + 2(\dot{F}H + F\dot{H}). \tag{9}
\]
where dot represents derivative with respect to time and $F = f_Q$ and $8\pi G = f_T$ represent differentiation with respect to $Q$ and $T$ respectively. Also note that $F = f_Q = mnQ^{n-1}$ and $8\pi G = f_T = b$.

Combining equations (8) & (9) we can get

$$\dot{H} + \frac{\dot{F}}{F} = \frac{4\pi}{(1 + G)}(\rho + p).$$

(10)

For generality we assume that the cosmological matter satisfies an equation of state of the form $p = (\gamma - 1)\rho$, where $\gamma$ is a constant, and $1 \leq \gamma \leq 2$. Solving Eqs. (8) and (10), we obtained the energy density $\rho$ as

$$\rho = \frac{f - 12FH^2}{16\pi(1 + \gamma G)}.$$ 

(11)

III. COSMOLOGICAL MODEL WITH $f(Q, T) = mQ^n + bT$

In this section, we shall consider the functional form of $f(Q, T)$ as

$$f(Q, T) = mQ^n + bT,$$

(12)

where $m$, $b$ and $n$ are model parameters.

We find the solution for zero pressure (dust matter) for which, we have $\gamma = 1$ and Eq. (11) together with the $f(Q, T)$ function given in (12), will reduce to,

$$\rho = \frac{m6^nH^{2n}(1 - 2n)}{16\pi + 3b}.$$ 

(13)

Now, the dynamical equation Eq. (10) describing the dynamics of the model reads as,

$$\dot{H} + \frac{3(8\pi + b)}{n(16\pi + 3b)}H^2 = 0,$$

which readily integrated to give the time evolution of the Hubble parameter $H(t)$ as,

$$H(t) = \frac{1}{At + c_1},$$

where $A = \frac{3(8\pi + b)}{n(16\pi + 3b)}$.

(15)

and $c_1$ is a constant of integration. From equation (15), we obtain the explicit form of scale factor as a simple power law type solution given by,

$$a(t) = c_2(At + c_1)^{\frac{1}{A}}.$$ 

(16)

where $c_2$ is another constant of integration. As, we are dealing with zero pressure matter and trying to explain the present cosmic acceleration, we shall express all the above cosmological parameters in terms of redshift $z$, defined by $z = \frac{a_0}{a} - 1$, where $a_0$ is the present value (at time $t = t_0$) of the scale factor. Also, we will consider the normalized value $a_0 = c_2(At_0 + c_1)^{\frac{1}{A}} = 1$ for which the $t$-$z$ relationship will be established as,

$$t(z) = -\frac{c_1}{A} + \frac{1}{A}[c_2(1 + z)]^{-A}.$$ 

(17)

Using the $t$-$z$ relation in Eq. (17), the Hubble parameter in terms of $z$ can be written as,

$$H(z) = H_0(1 + z)^A = H_0(1 + z)^{\frac{3(8\pi + b)}{n(16\pi + 3b)}},$$

containing only two model parameters $n$ and $b$. The deceleration parameter $q = - \frac{\ddot{H}}{H^2}$ comes out to be,

$$q(t) = -1 + \frac{3(8\pi + b)}{n(16\pi + 3b)},$$

(19)

which is a constant as expected due to power-law type expansion of the model.
IV. PARAMETERS OF THE MODEL & OBSERVATIONAL CONSTRAINTS

In the above expressions, we can see that the Eqs. (18) and (19) have two model parameters regulating the dynamics of the model. The model parameter \( n \) is more significant than \( b \) which can be analyzed in the term \( \frac{24\pi + 3b}{16\pi + 3b} \) containing homogeneous term \( 3b \) in both numerator and denominator. The choice of model parameters \( n \) & \( b \) must be in such a way that the deceleration parameter must attain a negative value at present and consistent with the observational value of \( q_0 \simeq -0.54 \) \[11\][13]. This require \( n = \frac{25.142 + b}{7.542 + 0.45b} \) and for a model consistent with these observations and the \( q_0 \) value in the neighborhood of \(-0.54\), the model parameters \( n \) and \( b \) must satisfy the relation,

\[
n = \frac{25.142 + b}{7.542 + 0.45b}.
\]

The following plot is an illustration for the choice of these two model parameters shown in Fig. 1.

![Plot showing variation of model parameters](image)

**FIG. 1:** The plot shows the variation of model parameters \( n \) and \( b \) in the shown ranges to have a \( q_0 \) value consistent with observations.

From the above Fig. 1 we can make a rough estimate for the range of the model parameters. The best estimate of these could be in the range \( n \in (1, 3) \) and with \( b \in (0, 2) \). In order to obtain better constrained values of these model parameters \( n \) & \( b \), we have here considered two observational data sets with 57 points of Hubble data sets and 580 points of union 2.1 compilation supernovae data sets. The methodology to this is explained below.

**A. Hubble Datasets**

In a recent paper, Sharov and Vasiliev \[14\] has compiled a list of 57 points of \( H(z) \) values in the redshift range \( 0.07 \leq z \leq 2.42 \) (see the Table-1 with 31 points from DA method and 26 points from BAO & other methods with errors).
The mean values of the model parameters $n \& b$ are determined by minimizing the chi-square value by maximum likelihood analysis reads as,

$$ \chi^2_{OHD}(p_s) = \sum_{i=1}^{28} \frac{[H_{th}(p_s,z_i) - H_{obs}(z_i)]^2}{\sigma_{H(z_i)}^2}, $$

(21)

where, $H_{th}$ and $H_{obs}$ respectively refer to the theoretical and observed value of Hubble parameter $H$, $p_s$ refers to the parameter space of the model to be constrained. Also $\sigma_{H(z_i)}$ stands for the standard error in the observed value of $H$.

### B. Supernovae datasets

The other data sets, we use for our analysis is the Union 2.1 compilation supernovae data sets [34] with 580 points. The chi-square formula for the supernovae data sets is given as,

$$ \chi^2_{SN}(\mu_0, p_s) = \sum_{i=1}^{580} \frac{[\mu_{th}(\mu_0, p_s, z_i) - \mu_{obs}(z_i)]^2}{\sigma_{\mu(z)}^2}, $$

(22)

where, $\mu_{th}$ and $\mu_{obs}$ are respectively, the theoretical and observed distance modulus with the standard error in the observed value denoted by $\sigma_{\mu(z)}$. The distance modulus $\mu(z)$ is defined by $\mu(z) = m - M = 5\log D_l(z) + \mu_0$, where $m$ and $M$ are respectively, the apparent and absolute magnitudes of a standard candle and the luminosity distance $D_l(z)$ and the nuisance parameter $\mu_0$ are defined by $D_l(z) = (1 + z)H_0 \int_0^z \frac{1}{H(z')}dz'$ and $\mu_0 = 5\log H_0^{-1} + 25$ respectively. In order to calculate luminosity distance, we have restricted the series of $H(z)$ up to tenth term then integrate the approximate series to obtain the luminosity distance.

The following plots in Fig. 2 shows the contour plots with 57 points of $H(z)$ data sets and 580 points of SN data sets showing the likelihood values of $n \& H_0$ at $1\sigma$, $2\sigma$ & $3\sigma$ level in the $n$-$H_0$ plane together with the constrained values shown in black dots.

| $z$  | $H(z)$ | $\sigma_H$ | Ref. | $z$  | $H(z)$ | $\sigma_H$ | Ref. | $z$  | $H(z)$ | $\sigma_H$ | Ref. |
|------|--------|------------|------|------|--------|------------|------|------|--------|------------|------|
| 0.070 | 69     | 19.6       | [15] | 0.4783 | 80.9   | 9          | [19] | 0.24 | 79.69  | 2.99       | [22] |
| 0.90  | 69     | 12         | [16] | 0.4800 | 97      | 62         | [15] | 0.30 | 81.7   | 6.22       | [23] |
| 0.120 | 68.6   | 26.2       | [15] | 0.5938 | 104     | 13         | [17] | 0.31 | 78.18  | 4.74       | [24] |
| 0.170 | 83     | 8          | [16] | 0.6797 | 92      | 8          | [17] | 0.34 | 83.8   | 3.66       | [22] |
| 0.1791| 75     | 4          | [17] | 0.7812 | 105     | 12         | [17] | 0.35 | 82.7   | 9.1        | [25] |
| 0.1993| 75     | 5          | [17] | 0.8754 | 125     | 17         | [17] | 0.36 | 79.94  | 3.38       | [24] |
| 0.200 | 72.9   | 29.6       | [18] | 0.8800 | 90      | 40         | [15] | 0.38 | 81.5   | 1.9        | [26] |
| 0.270 | 77     | 14         | [16] | 0.9000 | 117     | 23         | [16] | 0.40 | 82.04  | 2.03       | [24] |
| 0.280 | 88.8   | 36.6       | [18] | 1.0378 | 154     | 20         | [17] | 0.43 | 86.45  | 3.97       | [22] |
| 0.3519| 83     | 14         | [17] | 1.3000 | 168     | 17         | [18] | 0.44 | 82.6   | 7.8        | [27] |
| 0.3802| 83     | 13.5       | [19] | 1.3634 | 160     | 33.6       | [21] | 0.44 | 84.81  | 1.83       | [24] |
| 0.400 | 95     | 17         | [16] | 1.4300 | 177     | 18         | [16] | 0.48 | 87.79  | 2.03       | [24] |
| 0.4004| 77     | 10.2       | [19] | 1.5300 | 140     | 14         | [16] | 0.51 | 90.4   | 1.9        | [26] |
| 0.4247| 87.1   | 11.2       | [19] | 1.7500 | 202     | 40         | [16] |      |        |            |      |
| 0.4497| 92.8   | 12.9       | [19] | 1.9655 | 186.5   | 50.4       | [21] |      |        |            |      |
FIG. 2: The left panel shows the contour plot due to the 57 points of Hubble data sets showing likelihood values of the model parameters $n$ and $b$ in $n$-$b$ plane at $1\sigma$, $2\sigma$ & $3\sigma$ level and the black dot shows the constrained values of model parameters found as $n = 1.7763$ and $b = 0.8491$ with $\chi^2_{\text{min}} = 194.3194$. The right panel shows the contour plot due to the 580 points of union 2.1 compilation supernovae data sets showing likelihood values of the model parameters $n$ and $b$ in $n$-$b$ plane at $1\sigma$, $2\sigma$ & $3\sigma$ level and the black dot shows the constrained values of model parameters found as $n = 1.7769$ and $b = 2.4889$ with $\chi^2_{\text{min}} = 790.922$.

Since, the values of the model parameter $b$ ranges from $-\infty$ to $+\infty$, the $b$-axis in the above contour plots are unbounded but the model parameter $n$ is in its fixed range ($n \in (1, 4)$) in both the contour plots. We have obtained best fitting pair $(n, b)$ of model parameter values ($1.7763, 0.8491$) due to $H(z)$ data sets and ($1.7769, 2.4889$) due to Union 2.1 supernovae data sets. With these values, we have shown the Error bar plots of 57 points of $H(z)$ data sets and 580 points of supernovae data sets and compared our model with ΛCDM model in the following Fig. 3.

FIG. 3: The left panel shows the Error bar plot of 57 points of Hubble datasets together with the presented model shown in solid red line with $n = 1.7767$ & $b = 0.8491$ compared with the ΛCDM model shown in black dashed line showing a poor fit at higher redshift but better at lower redshift. A blue line shown is a fiducial model just for comparison with the values $n = 1.5$ & $b = 0.8491$ which lies outside the contour. The right panel shows the Error bar plot of 580 points of union 2.1 compilation supernovae data sets together with the presented model shown in solid red line with $n = 1.7765$ & $b = 2.4889$ compared with the ΛCDM model shown in black dashed line.

V. DIAGNOSTIC ANALYSIS

A. Statefinder diagnostic

The well-known geometrical parameters in cosmology are Hubble parameter $H = \frac{\dot{a}}{a}$ and deceleration parameter $q = \frac{-\ddot{a}}{aH^2}$ which are useful in describing the expansion history of the Universe. Also, various dark energy models have
been proposed to explain the accelerated expansion of the Universe. Another parameters proposed are known as statefinder parameters written in pairs as \( \{ r, s \} \) and \( \{ r, q \} \). These are geometrical quantities engaged to identify the various dark energy models \([35-37]\). The \( r \) and \( s \) parameters are defined as,

\[
r = \frac{\ddot{a}}{a H^3},
\]

\[
s = \frac{r - 1}{3(q - \frac{1}{2})}.
\]

The plot of \( s-r \) and \( q-r \) plane is shown below in Fig. 4.

![Diagram of s-r and q-r planes](image)

**FIG. 4:** The left panel shows the \( s-r \) plane for our model with \( b = 0.8491 \) & \( b = 2.4889 \) and varying \( n \). The right panel shows the \( q-r \) plane for the model with \( b = 0.8491 \) & \( b = 2.4889 \) and varying \( n \).

Various dark energy models in \( s-r \) plane illustrate some different growing trajectories. The point \((s, r) = (0, 1)\) correspond to the \( \Lambda \)CDM model in flat FLRW background. It is observed in Fig. 4 that the point \((s, r) = (0, 1)\) represent the \( \Lambda \)CDM while \((q, r) = (-1, 1)\) the de Sitter(dS) point. The red line divides the plane into two parts denoting the Quintessence phase as a lower half. The statefinder plots have been done for the values of \( n \) and \( b \) constrained by the \( H(z) \) and \( SN \) data sets. The similar behavior can be seen in \([37, 38]\). We note that for the Hubble data sets, the \( r \) and \( s \) parameters at the present epoch are \( r_0 = -0.111918 \) and \( s_0 = 0.553917 \) while for the \( SN \) data sets, \( r_0 = -0.118324 \) and \( s_0 = 0.538516 \). Currently observations are not sensitive enough to measure these parameters. However, Ref \([39]\) claims that these parameters can be deduced from future observations which would greatly help to constrain the nature of dark energy.

**B. Om diagnostic**

\( Om \) diagnostic is another efficient tool emerging from Hubble parameter which provides the null test for the \( \Lambda \)CDM model. The diagnostic is adequate in refining various dark energy models from \( \Lambda \)CDM due to the variation in its slope. For flat Universe, the \( Om(z) \) is defined as \([40, 41]\)

\[
Om(z) = \frac{(H(z) / H_0)^2 - 1}{(1 + z)^3 - 1}.
\]
For the ΛCDM model, phantom and quintessence cosmological models, $O_m(z)$ have some different set of values. We can describe the behavior of dark energy as quintessence type which shows negative curvature, phantom type which shows positive curvature and ΛCDM the zero curvature. Fig. 5 depicts the behavior of $O_m(z)$ showing a decaying behavior for the constrained values of the model parameters obtained from $H(z)$ and SN data sets.

VI. EVOLUTION OF THE $\rho(z)$

The expression for the energy density in Eq. (13) can be written in terms of redshift $z$ as,

$$\rho(z) = \frac{6^n m(1 - 2n)}{16\pi + 3b} H_0^{2n}(1 + z)^2 \left(\frac{2b + 3b}{16\pi + 3b}\right).$$  \hspace{1cm} (26)

Now, defining the density parameter, $\Omega = \frac{8\pi G \rho}{3H^2}$ for which we have,

$$\Omega(z) = \frac{8\pi G}{3(16\pi + 3b)} [6^n m(1 - 2n)] H_0^{2n-2} (1 + z)^2 \left(\frac{2b + 3b}{16\pi + 3b}\right),$$  \hspace{1cm} (27)

with $\Omega(0) = \frac{8\pi G}{3(16\pi + 3b)} [6^n m(1 - 2n)] H_0^{2n-2}$. Since, the expressions in Eq. (26) and Eq. (27), there’s a term $(1 - 2n)$ which will be negative for the discussed range of values of $n$, we must take the adjustable free parameter $m$ in these expressions so that $\rho$ assumes positive value. We, have shown the evolution of the energy density with respect to redshift $z$ for the constrained numerical values of the model parameters $n$ and $b$ in the Fig. 6(a). Also, we have shown the evolution of the density parameter for our model with the constrained numerical values of the model parameters $n$ and $b$ in the Fig. 6(b) together with the evolution of density parameter of matter density $\Omega_m = 0.3089(1 + z)^3$ as in the ΛCDM model $H(z) = \sqrt{\Omega_m(1 + z)^3 + \Omega_A}$ for comparison (where $\Omega_{m0} = 0.3089$ and $\Omega_A = 0.6911$ as suggested by Planck 2015 results [42]).
VII. LINEAR FORM OF $f(Q, T) = mQ + bT$

We can see for $n = 1$, in the considered functional form of $f(Q, T) = mQ^n + bT$, the case reduce to the linear form of $f(Q, T)$ function as $f(Q, T) = mQ + bT$ which is similar to the linear functional form considered in [7] with $f(Q, T) = \alpha Q + \beta T$.

So, for $n = 1$, we can solve Eq. (11) which yield the expression of the density $\rho$ as,

$$\rho = \frac{-6mh^2}{16\pi + 3b}.$$  \hspace{1cm} (28)

Now from Eq. (10), we can obtain the dynamical equation in $H$ reads as,

$$\dot{H} + \frac{3(8\pi + b)}{16\pi + 3b}H^2 = 0,$$  \hspace{1cm} (29)

which yield a Hubble parameter in the form,

$$H = \frac{1}{Bt + k_1},$$  \hspace{1cm} (30)

where $B = \frac{24\pi + 3b}{16\pi + 3b}$ with $k_1$ a constant of integration. From Eq. (30), the scale factor can be obtained as

$$a(t) = k_2(Bt + k_1)^{\frac{1}{B}}.$$  \hspace{1cm} (31)

where $k_2$ is another constant of integration. As discussed above, we must express all the above cosmological parameters in terms of redshift $z$, for which the $t$-$z$ relationship will be established in this case as,

$$t(z) = -\frac{k_1}{B} + \frac{1}{B}[k_2(1+z)]^{-B}.$$  \hspace{1cm} (32)

So, the Hubble parameter can be written in this case as,

$$H(z) = H_0(1+z)^B = H_0(1+z)^{(\frac{24\pi + 3b}{16\pi + 3b})},$$  \hspace{1cm} (33)

containing only one model parameter $b$ with $q(t) = -1 + \frac{24\pi + 3b}{16\pi + 3b}$ which is also a constant with only one model parameter $b$. Since, the term $\frac{24\pi + 3b}{16\pi + 3b}$ assumes a constant value $\simeq \frac{3}{2}$ for any values of $b \in (-\infty, \infty)$, we have $q(t) = 0.5$ showing a constant deceleration and the model reduces to the standard lore with $H(z) = H_0(1+z)^{\frac{3}{2}}$ and $a(t) \propto (\frac{3}{2}t + k_1)^{\frac{1}{3}}$. We note that, in this case the energy density $\rho(z) = \frac{-6m}{16\pi + 3b}H_0^2(1+z)^3$ where the adjustable parameter $m$ is considered to be negative.
In this work, we have discussed late time cosmology employing a well motivated $f(Q,T)$ gravity model with the functional form $f(Q,T) = mQ^n + bT$, where $m, n$ and $b$ are model parameters proposed in [7]. By constraining the free parameters using observational data sets of the updated 57 points of Hubble data sets and 580 points of union 2.1 compilation supernovae data sets, we find the deceleration parameter to be negative and reads respectively $q_0 = -0.169125$ and $q_0 = -0.192226$ and therefore consistent with the present scenario of an accelerating universe. Previous works in power law cosmology also reported similar constraints (see for example [37, 38]). For the model considered in [7] with the $f(Q,T)$ function $f(Q,T) = -\gamma Q - \delta T^2$ ($\gamma$ and $\delta$ are model parameters), the solution and data analysis have already been discussed. For another model considered in the same paper [7] with the $f(Q,T)$ function $f(Q,T) = m Q^n + b T$ (where $m$ and $b$ are model parameters) which is similar to the linear case with $n = 1$ in our considered $f(Q,T) = m Q^n + b T$ form i.e. $m Q + b T$. In this linear case, the solution mimic the power law expansion model with $a(t) \propto (Bt + c_1)^{\frac{1}{2}}$ where $B = \frac{24\pi+36}{16\pi+36}$ containing only one model parameter $b$ wherein we can see the parameter $b$ contribute very less in the evolution because the term $B = \frac{24\pi+36}{16\pi+36} \approx \frac{3}{2}$ for $b < (\infty, \infty)$ implying the model behaves similar to the standard lore of $a(t) \sim t^\frac{1}{2}$ with a constant deceleration $q = 0.5$.

We have thoroughly investigated the nature of dark energy mimicked by the parametrization $f(Q,T) = m Q^n + b T$ with the assistance of statefinder diagnostic in $\{s, r\}$ and $\{q, r\}$ planes and also performed the $Om$-diagnostic analysis for the model. For the numerical values of the model parameters $n$ and $b$ obtained from constraining our model through 57 points of $H(z)$ data sets gives the statefinder parameters values as $r_0 = -0.111918$ and $s_0 = 0.553917$ while for the numerical values of the model parameters $n$ and $b$ obtained from constraining our model through 580 points for the Union 2.1 compilation data sets gives $r_0 = -0.118324$ and $s_0 = 0.538516$ as obtained earlier in [37, 38]. We can conclude that, the model considered here is good in explaining at present observations but may not explain the early evolution well (as the model is not consistent with the constraints coming from BAO data sets (not done here)). One of the significant observation is that the model parameter $b$ which is the coefficient of the trace $T$ in the $f(Q,T) = m Q^n + b T$ form considered contributes very very less in the evolution as it can be seen from the figures Fig. [3] which means linear trace $T$ do not affect the evolution. The behavior of energy density and the density parameter with respect to redshift $z$ for the constrained values of the model parameters $n$ and $b$ are depicted in Fig. [3] with $\Omega_{m0} = 0.3089$. Some more functional form of $f(Q,T)$ could be explored in the same way and is deferred to our future works.

Acknowledgments

S. A. acknowledges CSIR, Govt. of India, New Delhi, for awarding Junior Research Fellowship. PKS acknowledges CSIR, New Delhi, India for financial support to carry out the Research project [No.03(1454)/19/EMR-II Dt.02/08/2019]. We are very much grateful to the honorable referee and the editor for the illuminating suggestions that have significantly improved our work in terms of research quality and presentation.

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