Maximum-Likelihood Power-Distortion Monitoring for GNSS Signal Authentication

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Abstract—We propose an extension to the so-called PD detector, a GNSS signal authentication technique. The PD detector jointly monitors received power and correlation profile distortion to detect the presence of GNSS spoofing, jamming, or multipath. We show that classification performance can be significantly improved by replacing the PD detector’s symmetric-difference-based distortion measurement with one based on the post-fit residuals of the maximum-likelihood estimate of a single-signal correlation function model. We call the improved technique the PD-ML detector. In direct comparison with the PD detector, the PD-ML detector exhibits improved classification accuracy when tested against an extensive library of recorded field data. In particular, it is (1) significantly more accurate at distinguishing a spoofing attack from a jamming attack, (2) better at distinguishing multipath-afflicted data from interference-free data, and (3) less likely to issue a false alarm by classifying multipath as spoofing. The PD-ML detector achieves this improved performance at the expense of additional computational complexity.

Index Terms—satellite navigation systems, Global Positioning System, navigation security, GNSS spoofing, GNSS jamming

I. INTRODUCTION

Civil GNSS receivers are susceptible to counterfeit spoofing signals [1]–[4]. To defend against spoofing attacks, several algorithms have been developed and reported in the literature. A comprehensive review of GNSS spoofing detection methods is provided in [5], where methods are broadly categorized as (1) cryptographic techniques, (2) geometric techniques that exploit angle-of-arrival diversity, or (3) signal processing techniques that do not fall into categories (1) or (2).

Within the latter category, the subset of techniques which do not require additional hardware and can be implemented via a firmware update are particularly attractive for widespread adoption. Of these, the PD detector has been shown to reliably alarm in the presence of spoofing or jamming attacks while maintaining a low multi-channel false alarm rate [6].

An appealing attribute of the PD detector is simplicity: its received power and correlation profile distortion measurements are computationally lightweight and amenable to analysis. However, when tested in [6] against an extensive set of empirical data, the PD detector was shown to incorrectly classify most instances of spoofing as jamming, and its 0.57% single-channel false alarm rate (false declaration of interference-free or multipath-afflicted signals as spoofing or jamming), is too high for some applications of practical interest. This paper extends the PD detector to address these shortcomings.

The PD detector’s shortcomings stem from two undesirable features of the symmetric difference (SD) as a measure of distortion. First, the standard SD involves only a pair of complex correlation function taps and is not particularly sensitive to correlation function distortion not aligned with these taps. Thus, if the SD taps are closely spaced around the prompt correlation tap, then the SD can be a good distortion measure during the initial stage of a carry-off-type spoofing attack, but quickly becomes insensitive as the attack proceeds. Second, the SD depends on the receiver’s code tracking loop to align its pair of taps symmetrically about the authentic correlation peak. But even in the absence of spoofing or significant multipath, thermal noise prevents the code tracking loop from achieving perfectly symmetric alignment. As a consequence, the SD tends to exaggerate the actual distortion under the null (thermal noise only) and jamming hypotheses.

The PD-ML detector’s distortion measure avoids both of these limitations. It begins by measuring the correlation function at many more than two taps, which offers two advantages: (1) the PD-ML detector can be made sensitive to distortion far from the prompt tap, thus extending the range of spoofing-to-authentic code offsets over which it can detect spoofing, and (2) the PD-ML detector’s measure of distortion can be made independent of the receiver’s code and carrier tracking loops in the sense that small tracking errors in no way affect the reported distortion. This is done by exploiting data from the additional taps to obtain the maximum-likelihood estimates of the amplitude, code phase, and carrier phase of the correlation peak in a single-signal correlation function model. After removing this estimated correlation function model from each of the correlation taps, the PD-ML detector takes as its distortion measure the squared magnitude of the normalized post-fit residuals. As will be shown, distortion measured in this way permits more accurate classification of jamming, multipath, and spoofing than distortion based on a simple symmetric difference.

For compactness, this correspondence article is intended to be read alongside the original PD detector paper [6]. Herein, we adopt, without alteration, the signal models, probability distributions, and the Monte-Carlo-type Bayes-optimal decision rule design strategy developed in [6]. This correspondence, a significant extension of our conference paper [7], exploits multi-tap maximum-likelihood multipath estimation and demonstrates the proposed PD-ML detector’s advantages through a direct comparison with the original PD detector on the same experimental data recordings and using the same Bayes-optimal detector design strategy as presented in [6]. Details of the PD-ML detector are presented in Section III followed by simulated (Section IV) and experimental (Section VI) evaluation of its classification performance. All source
code required to generate decision rules using the PD-ML detector is publicly available at https://github.com/navSecurity/P-D-defense.

II. SIGNAL MODELS

For a thorough discussion of the GNSS signal models adopted in this work, starting with the pre–correlation signal model, the reader is directed to the paper on the PD detector [6]. However, to provide the context necessary to understand this paper’s proposed extension to the PD detector, a brief review of the single-interferer post-correlation model assumed in [6], which is adapted from [8], is presented here. This model gives the complex-valued receiver correlation function at some arbitrary code offset, or lag, τ, as

\[
\xi_k(\tau) = \beta_k[\xi_{Ak}(\tau) + \xi_{ik}(\tau) + \xi_{Nk}(\tau)]
\]

where \(\beta_k\) is the average value of the automatic gain control scaling factor over the \(k\)th accumulation interval, and \(\xi_{Ak}(\tau), \xi_{ik}(\tau), \xi_{Nk}(\tau)\) are the complex correlation function components corresponding to the authentic signal, the interference signal, and thermal noise, respectively.

Just as in [6], the correlation components for the authentic and interference signal, \(\xi_{Ak}(\tau)\) and \(\xi_{ik}(\tau)\), are assumed to be modeled as

\[
\xi_{Ak}(\tau) = \sqrt{P_A}R(-\Delta_\tau_{Ak} + \tau) \exp(j\Delta_\theta_{Ak})
\]

\[
\xi_{ik}(\tau) = \sqrt{\eta_kP_A}R(-\Delta_\tau_{ik} + \tau) \exp(j\Delta_\theta_{ik})
\]

where \(R(\tau)\) is the GNSS auto-correlation function, and, at the \(k\)th accumulation interval, \(P_A\) is the average power of the authentic signal’s power, \(\eta_k\) is the average interference signal’s power advantage over the authentic signal (i.e., \(\eta_k = P_{Ik}/P_{Ak}\)), and \(\Delta_\tau_{Ak}\) is the average value of the code offset \(\tau_A - \hat{\tau}\), with \(\tau_A\) being the true code phase of the authentic signal and \(\hat{\tau}\) being the receiver’s estimate of the same. Similar definitions follow for \(\Delta_\tau_{ik}, \Delta_\theta_{Ak}, \text{ and } \Delta_\theta_{ik}\) [6].

The thermal noise component of the correlation function, \(\xi_{Nk}(\tau)\), is modeled as having independent in-phase (real) and quadrature (imaginary) components, each modeled as a zero-mean Gaussian white discrete-time process:

\[
E[R\{\xi_{Nk}(\rho)\}E\{\xi_{Nj}(\nu)\}] = 0 \quad \forall \rho, \nu, k \neq j
\]

As discussed in [9], thermal noise is correlated in the lag domain. For samples within \(2\tau_c\) of each other, where \(\tau_c\) is the chip-width of the GNSS signal, correlation in \(\xi_{Nk}(\tau)\) is modeled as

\[
E[\xi_{Nk}(\rho)\xi_{Nk}(\nu)] = \begin{cases} 2\sigma_N^2(1-|\rho - \nu|/\tau_c) & |\rho - \nu| \leq 2\tau_c \\ 0 & |\rho - \nu| > 2\tau_c \end{cases}
\]

Here, * denotes the complex conjugate and \(\sigma_N^2\) is the variance of the in-phase and quadrature components of \(\xi_{Nk}(\tau)\), which is related to the spectral density of a white noise process that is modeled as the sum of two independent components associated with thermal noise and multi-access noise. The spectral density of the thermal noise, \(N_0\), is assumed to be constant whereas the spectral density of the multi-access noise, \(M_0\), is assumed to be variable, as detailed in [6]. When averaging over an accumulation interval \(T\) the variance is given as \(\sigma^2_N = (N_0 + M_0)/2T\).

III. MEASUREMENT MODELS

Like the PD detector, the PD-ML detector simultaneously monitors received power and correlation function distortion. Its received power monitor is identical to the PD detector’s, as described in [6], but its distortion monitor differs considerably. This section develops the measurement model for the PD-ML detector’s distortion monitor.

The PD-ML detector models the in-phase and quadrature (IQ) samples of the correlation function \(\xi_k\) as a function of only the authentic signal and thermal noise, neglecting \(\xi_{ik}\) in [1]. It employs a maximum-likelihood estimator to estimate the authentic signal’s amplitude, code phase, and carrier phase, and takes squared the magnitude of the normalized post-fit residuals as its measure of correlation profile distortion.

Maximum-likelihood estimation based on GNSS correlation data is well-established in the literature. It is routinely employed within high-end GNSS receivers for multipath mitigation. This paper’s distortion monitor adapts the particular maximum-likelihood estimator developed in [10], as described subsequently.

A. Multi–Tap Maximum–Likelihood Multipath Estimator

Let \(l\) be the number of signal taps devoted to maximum-likelihood estimation. We assume that \(l\) is odd and that taps are distributed so the centermost tap is approximately aligned with the receiver’s estimated correlation function peak and the remaining taps are spaced evenly across the range \(\pm \tau_c\). The uniform tap interval is

\[
\Delta_\delta = \frac{2\tau_c}{(l - 1)}
\]

and the \(l \times 1\) vector of tap locations is given by

\[
\delta = [-\tau_c, -\tau_c + \Delta_\delta, \ldots, -\tau_c - (l - 1)\Delta_\delta, \tau_c]^T
\]

with \(\delta_i = -\tau_c + (i - 1)\Delta_\delta\) representing the \(i\)th tap location, \(i = 1, \ldots, l\).

Modeling the correlation function \(\xi_k(\tau)\) as interference-free (i.e., \(\xi_{ik}(\tau) = 0\) for all \(\tau\)), the complex-valued \(i\)th tap at time index \(k\) is expressed in terms of the authentic signal’s gain-controlled amplitude \(a_{Ak}\), carrier phase \(\phi_{Ak}\), and code phase \(\tau_{Ak}\) as

\[
\xi_k(\delta_i) = \beta_k[\xi_{Ak}(\delta_i) + \xi_{Nk}(\delta_i)] = a_{Ak}\exp(j\phi_{Ak})R(\delta_i - \tau_{Ak}) + \xi_{Nk}(\delta_i)
\]

(2)

The parameters \(\{a_{Ak}, \tau_{Ak}, \phi_{Ak}\}\) are estimated according to this model by an adaptation of the maximum likelihood technique in [10]. This approach separates estimation of the code phase from estimation of amplitude and carrier phase by exploiting the linear relationship

\[
\xi_k = H(\tau_{Ak}, \delta)a_{Ak}\exp(j\phi_{Ak})
\]

(3)

where \(\xi_k = [\xi_k(\delta_1), \ldots, \xi_k(\delta_l)]^T\) and the observation matrix \(H(\tau_{Ak}, \delta)\) is

\[
H(\tau_{Ak}, \delta) = \begin{bmatrix} R(\delta_1 - \tau_{Ak}) \\ \vdots \\ R(\delta_l - \tau_{Ak}) \end{bmatrix}
\]

(4)
First, a coarse search is performed by setting the code phase estimate $\hat{\tau}_{Ak} = \delta_i$ for $i = 1, \ldots, l$ and, for each candidate $\hat{\tau}_{Ak}$, solving for the maximum-likelihood estimate $a_{Ak} \exp(j\phi_{Ak})$:

$$\hat{a}_{Ak} \exp(j\hat{\phi}_{Ak}) = \left[ H^T(\hat{\tau}_{Ak}, \delta) Q^{-1} H(\hat{\tau}_{Ak}, \delta) \right]^{-1} H^T(\hat{\tau}_{Ak}, \delta) Q^{-1} \xi_k $$

(5)

where $Q$ is the $l \times l$ Toeplitz matrix that accounts for the correlation of the complex Gaussian thermal noise among the taps [10]. The $(a, b)$th element of $Q$ is $Q_{a,b} = R(|a - b| \Delta \delta)$, where $\Delta \delta$ is the tap spacing.

The cost $J_k$ corresponding to each set of estimates $\{\hat{a}_{Ak}, \hat{\tau}_{Ak}, \hat{\phi}_{Ak}\}$, is calculated as

$$J_k = \left\| \xi_k - H(\hat{\tau}_{Ak}, \delta) \hat{a}_{Ak} \exp(j\hat{\phi}_{Ak}) \right\|^2_Q$$

(6)

where the norm is defined such that $\|x\|^2_Q = x^T Q^{-1} x$.

The two sets of estimates yielding the smallest cost $J_k$ are extracted. Because the cost $J_k$ is proportional to the negative log likelihood function, the set with the minimum cost is the maximum likelihood estimate.

In a second step, a refined code phase estimate is obtained by a bisecting search between the two lowest-cost code phase estimates. At each bisection point, new amplitude and carrier phase estimates are determined by re-evaluating (5). The process is repeated until $J_k$ is no longer significantly reduced. Upon convergence, the resulting estimates are accepted as the maximum-likelihood estimates $\{\hat{a}_{Ak}, \hat{\tau}_{Ak}, \hat{\phi}_{Ak}\}$, and the corresponding $J_k$ is taken as the distortion measurement $D_k$.

A small $J_k$ indicates that the single-signal model (2) accurately fits the correlation function data at time $k$; a large $J_k$ indicates the opposite, suggesting that multipath or spoofing interference is present. This distortion metric is more informative than the symmetric difference metric used in [6] because it (1) exploits data from $l$ taps, whereas the symmetric difference only uses two, and (2) is insensitive to noise- or dynamics-induced misalignment of the prompt tap (located at $\delta_i = 0$) with the authentic signal peak, whereas the symmetric difference falsely reports distortion in this circumstance.

IV. DECISION RULE

Design of the PD-ML’s decision rule follows the same procedure outlined in the PD detector [6], but with the new distortion metric replacing the symmetric difference. First, we simulate the post-correlation model (1) using the same model priors, parameter probability distributions, and probability transition mechanisms for each hypothesis as in [6]. The hypotheses are denoted $H_i$, $i \in I$ for $I = \{0, 1, 2, 3\}$, where the null hypothesis $H_0$ corresponds to the interference-free case, and $H_i$, $i = 1, 2, 3$ correspond respectively to multipath, spoofing, and jamming. For each Monte-Carlo sample under each hypothesis, a power measurement is made as in [6] and a distortion measurement $D_k = J_k$ is made as described above.

Fig. 1 shows the power-distortion measurements from a Monte-Carlo simulation with $l = 11$ taps. One can observe the following behavior of the new distortion metric $D_k$:

- Under $H_0$ (interference-free): $D_k$ is small, as only thermal noise is present.
- Under $H_1$ (multipath-afflicted): $D_k$ is similar to that of the $H_0$ case when multipath is weak but attains a significant magnitude as multipath severity increases.
- Under $H_2$ (spoofing): $D_k$ overlaps with that of multipath ($H_1$), but exhibits a much wider range.
- Under $H_3$ (jamming): $D_k$ is nearly indistinguishable from distortion under $H_0$, as jamming has little effect on the gain-controlled correlation function.

Based on the simulated data shown in Fig. 1 decision regions corresponding to a Bayes-optimal decision rule for a parameter-dependent cost $C[i, \theta]$ were found as described in [6]. Fig. 2 shows the optimum decision regions $\Gamma_i$ for $i \in I$.

Table I shows classification statistics as evaluated by applying the Bayes-optimal rule to an independent set of Monte-Carlo-generated measurements like the one shown in Fig. 1. This table reveals that the PD-ML detector exhibits similar theoretical performance to the PD detector. Both tend to misclassify multipath ($H_1$) as interference free ($H_0$) because of the low cost assigned to this error when multipath is mild. Otherwise, the PD-ML detector, like the PD detector, exhibits high detection power and a low false alarm rate.

The sensitivity of the PD-ML detector’s performance to the number of taps $l$ was explored. Starting from $l = 41$ taps, the entire design process was repeated for several choices of $l$ of decreasing value. Performance generally decreased with reduced $l$. It was found that at least $l = 11$ taps were needed to maintain a level of theoretical detector performance.
Fig. 2: Optimum decision regions for the $\theta$-dependent cost $C[i, \theta]$: $\Gamma_0$ (no interference, green), $\Gamma_1$ (multipath, black), $\Gamma_2$ (spoofing, red), and $\Gamma_3$ (jamming, blue).

TABLE I: Simulation-evaluated classification for the decision regions in Fig. 2. The table’s $(i, j)$th element is the relative frequency with which the detector chose $i$ when $j$ was the true scenario.

| Decision True Scenario | $H_0$ | $H_1$ | $H_2$ | $H_3$ |
|------------------------|-------|-------|-------|-------|
| $H_0$                  | 0.9947| 0.9083| 0.0670| 0.0039|
| $H_1$                  | 0     | 0.0698| 0.0117| 0     |
| $H_2$                  | 0.0043| 0.2214| 0.8463| 0.0155|
| $H_3$                  | 0.0010| 0.0005| 0.0750| 0.9806|

TABLE II: As Table I, but for the detector applied to the experimental recordings (sans Table II).

| Decision True Scenario | $H_0$ | $H_1$ | $H_2$ | $H_3$ |
|------------------------|-------|-------|-------|-------|
| $H_0$                  | 0.8500| 0.4337| 0.0001| 0     |
| $H_1$                  | 0.1500| 0.5632| 0     | 0     |
| $H_2$                  | 0     | 0.0031| 0.8289| 0.2176|
| $H_3$                  | 0     | 0     | 0.1676| 0.7824|

V. EXPERIMENTAL DATA

An independent evaluation of PD-ML was carried out against 27 empirical GNSS data recordings, including 6 recordings of various spoofing scenarios, 14 multipath-dense scenarios, 4 jamming scenarios of different power levels, and 3 scenarios exhibiting negligible interference beyond thermal and multi-access noise. The details of the data sets, including their source, are given in [6]. To ensure a fair and direct comparison with the PD detector, these data were pre-processed using the same approach as in [6].

VI. EXPERIMENTAL RESULTS

This section presents the experimental performance assessment of the PD-ML detector based on the decision regions shown in Fig. 2 being applied to the observations, shown in Fig. 3 derived from the 27 experimental data recordings. Table II summarizes the PD-ML detector’s overall performance against the experimental data in terms of classification statistics.

When comparing this table to the corresponding one for the PD detector (Table III in [6]), the important result is that classification performance accuracy under a spoofing attack is improved more than four-fold. That is, with the PD detector, spoofing attacks were classified as jamming for 82% of experimental spoofing detection trials. This is reduced to only 17% with the PD-ML detector. All but 0.01% of spoofing or jamming attacks result in an alarm. Compared to the PD-ML detector, there is nearly a two-fold decrease in spoofing false alarms under $H_1$ (multipath). Finally, the PD-ML detector correctly classifies 56% of the multipath data, compared to only 12% with the PD detector.

Thus, although the PD-ML detector does not enjoy any significant advantage over the PD detector when tested against Monte-Carlo-simulated data (Table I), it significantly outperforms the PD detector when tested against the empirical dataset. A likely explanation is that the PD-ML detector’s greater number of correlation taps (for Table II $l = 11$) allow spoofer-induced distortion to be readily detected over a wider range of spoofer-to-authentic code phase offset.

The PD-ML detector was also applied to the specially stealthy $\text{tb7}$ attack, which mounts a nulling attack during the beginning of the attack before pull-off. This attack would be extremely difficult to realize outside the laboratory. Soon after pull-off begins, the PD-ML detector correctly classifies the attack as spoofing. Against the attack portion of the scenario, its decision rates were $H_0$: 17%, $H_1$: 17%, $H_2$: 66%, and $H_3$: 0%, as compared to $H_0$: 14%, $H_1$: 10%, $H_2$: 70%, and $H_3$: 6% for the PD detector. The PD-ML detector’s attack
Fig. 4: Cumulative time history of decisions for a single receiver tracking channel. Each trace represents the total number of times the corresponding hypothesis was chosen up to time $t_k$, normalized so that the final cumulative values sum to one. Both attacks begin at 120 seconds. Top: Jamming scenario $jdr$. Bottom: Spoofing scenario $tb4$ with the PD-ML detector applied to a mid-elevation satellite signal.

detection power is 10% lower than the PD detector’s on this attack, but it never mis-attributes spoofing as jamming.

The PD-ML detector’s superior spoofing vs. jamming classification is further illustrated in Fig. 4 which shows the single-channel cumulative time history of the detector’s decisions for example jamming and spoofing attack scenarios. This figure is directly comparable with Fig. 10 in [6]. Like the PD detector, in the jamming scenario (top panel), the attack is detected immediately at its onset, and continuously declared thereafter. In the spoofing scenario (bottom panel), the attack is detected immediately, and briefly classified as jamming because the spoofer’s near-perfect initial code-phase alignment causes little distortion (and, indeed, little harm to the receiver). However, as the spoofer begins its pull-off, the PD-ML detector correctly declares the attack to be spoofing and does so for the remainder of the attack. This is different from the PD detector, which declares the attack as an almost equal blend of jamming and spoofing as the pull-off proceeds.

VII. CONCLUSIONS

This paper presented the PD-ML detector, an extension to the recently-proposed PD detector. The PD-ML detector employs a maximum-likelihood multipath estimator and uses the magnitude of its post-fit residuals to monitor distortion, as opposed to the PD detector’s use of a symmetric difference. Like the PD detector, the PD-ML detector traps a would-be attacker between simultaneous monitoring of received power and complex correlation function distortion. In a head-to-head evaluation based on 27 high-quality experimental recordings of attack and non-attack scenarios, the PD-ML detector was shown to be significantly better at classifying spoofing vs. jamming, exhibiting a nearly four-fold improvement. In addition, the PD-ML detector was shown to significantly improve the classification of multipath data. The improved performance improvement comes at the expense of the additional computational complexity associated with producing correlation products from a larger number of taps and processing these with a maximum-likelihood estimator. Thus, depending on the application, the simpler PD detector may be more favorable despite the better classification performance offered by the PD-ML detector. The code for the PD-ML detector has been made available at the same repository as the PD detector: https://github.com/navSecurity/P-D-defense.

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