Importance of the Mechanism of Resonance Enhancement of Neutrino Oscillations in Matter for the Precise Testing of the Electroweak Interaction Model. Present Experimental Status of This Resonance Mechanism.

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Abstract
The mechanism of resonance enhancement of neutrino oscillations in matter and some critical remarks to this mechanism are considered. Using of this resonance mechanism is very important to examine the model of electroweak interactions since the processes induced by this mechanism grow multiply. In contrast to the electromagnetic and strong interactions in weak interactions, $P$-parity is violated therefore a problem of mass generations in the weak interactions is considered (the interaction must be left-right symmetric for mass generations). It is concluded that a possibility of mass generation in the framework of the weak interactions is not proved.

The present experimental status of this resonance mechanism is considered and it is done conclusion that this effect has no clear experimental confirmation. For this purpose it is necessary to fulfil precision experiments with solar neutrinos and the neutrinos passed through the Earth matter.

PACS numbers: 14.60.Pq; 14.60.Lm

Keywords: neutrino, mixings, oscillations, angle mixings, matter, resonance effect, Cherenkov effect, solar and terrestrial neutrinos.
1 Introduction

The suggestion that, in analogy with $K^o$, $\bar{K}^o$ oscillations, there could be neutrino-antineutrino oscillations ($\nu \to \bar{\nu}$), was made by Pontecorvo [1] in 1957. It was subsequently considered by Maki et al. [2] and Pontecorvo [3] that there could be mixings (and oscillations) of neutrinos of different flavors (i.e., $\nu_e \to \nu_\mu$ transitions).

The first experiment [4] on the solar neutrinos has shown that there is a deficit of neutrinos, i.e., the solar neutrinos flux detected in the experiment was few times smaller than the flux computed in the framework of the Sun Standard Model [5]. The subsequent experiments and theoretical computation have confirmed the deficit of the solar neutrinos [6].

The short base reactor and accelerator experiments [7] have shown that there is no neutrino deficit, then this result was interpreted as an indication that neutrino vacuum angle mixing is very small. Then the question arises: what is the deficit of the solar neutrinos related? In 1978 the work by L. Wolfenstein [8] appeared where an equation describing neutrino passing through the matter was formulated (afterwards that equation was named Wolfenstein’s). In the framework of this equation the enhancement of neutrino oscillations in matter arises via weak interactions. This mechanism of neutrino oscillations enhancement in the matter attracted attention of neutrino physicists after publications [9] by S. Mikheyev and A. Smirnov where it was shown that in the framework of this equation the resonance enhancement of neutrino oscillations in matter will take place. Also it is clear that there adiabatic neutrino transitions can arise in matter if effective masses of neutrinos change in matter [10]. After that an enormous number of works appeared where the deficit of the solar neutrinos was explained by this mechanism. It is supposed that neutrino vacuum angle mixing is very small [11] and at resonance enhancement of neutrino oscillations in the solar matter this angle becomes maximal ($\pi/4$). This mechanism was recognized as the only mechanism to explain the origin of the Sun neutrino deficit and it is supposed that
the vacuum angle mixing is very small. The situation changes after detection that the atmospheric neutrinos angle mixing \cite{12} is big and close to the maximal one \(\pi/4\). The \(\bar{\nu}_e \rightarrow \bar{\nu}_\mu\) angle mixing obtained in KAMLand detector \cite{13} appears to be big and near to the maximal one. Then the Day-Night effect does not obtain a confirmation \cite{14}. Also the Sun neutrino energy spectrum has no distortion in the energy region of \(E_{\nu_e} = 0.816 \div 13\) MeV., which cannot be in the case if the resonance mechanism is realized. However some authors insisted and continue to insist that this mechanism has already been confirmed at present time.

In the author’s works \cite{15} two remarks were done: 1) the Wolfenstein’s equation is a left-right symmetrical one while the weak interactions are left-handed interactions (then this equation has no connection with the weak interactions), 2) Since the weak interactions with the charged current are the left side ones, then these interactions cannot generate masses (masses can be generated only in the left-right symmetric interactions), then neutrino effective masses cannot change in matter and resonance conversion will be absent.

Below we show that the problem of resonance enhancement of neutrino oscillations in matter has a fundamental sense to verify the weak interactions theory and therefore we must obtain a strict and direct proof of this effect (the used \(\chi^2\) method \cite{16} is not sufficient for this purpose).

2 Why is This Resonance Mechanism Important for Verification of the Weak Interactions Theory?

It is necessary to remark that changing of couple constants (running coupling constants) of the strong, electromagnetic and weak interactions arises due to vacuum polarization.

The weak interactions theory demands a strong and precise check up and the resonance mechanism gives us these possibilities. The
phenomena are: distortion of the Sun neutrino spectrum, Day-Night effect and resonance effect in the Earth matter at appropriate neutrino energies since they are direct consequences of the weak interactions. In the accelerator experiments with neutrino we cannot avoid influence of the strong and electromagnetic interactions in order to separate the contribution of weak interaction running coupling constant from the contribution of running coupling constant of these interactions. Since $W, Z^0$ masses are a very big consequently deposit of the weak interactions running coupling constant in accelerator processes is very small in comparison with the above mentioned interactions \[17\].

The physics of weak interactions have bad luck from the very beginning. After discovery of the nuclear beta decay it has been supposed that in this processes the law of energy momentum conservation does not fulfil. The situation was corrected after V. Pauli’s letter to the Tubingen Physical Society \[18\]. Afterwards E. Fermi offered a famous beta decay theory \[19\]. For the first time the interaction generated by neutrino was observed in experiment of F. Reines and C. L. Cowan \[20\].

It is necessary to remark that in the standard theory of neutrino oscillations it is supposed that $\nu_e, \nu_\mu, \nu_\tau$ neutrinos have no definite masses \[3, 21\] (definite masses have only $\nu_1, \nu_2, \nu_3$ neutrinos). Since $\nu_e, \nu_\mu, \nu_\tau$ neutrinos have no definite masses, then we cannot formulate the law of energy momentum conservation in the processes with neutrino participation. Work \[22\] suggested a scheme of neutrino oscillations where the law of energy momentum conservation is fulfilled.

It is necessary to stress that at neutrino passing through matter there can be two types of processes (we neglect inelastic interactions): a) elastic scattering and b) polarization matter by neutrino.

With a naive point of view, in analogy with the strong and electromagnetic interactions, we can suppose that the both processes will take place at neutrino passing through matter via weak interactions. It is a usual practice. However, we know that the weak interactions with the charged current are left-handed type interactions therefore it
is needed to prove that there is analogy with the above interactions. It is necessary to remark that if the weak interactions cannot generate masses, then resonance enhancement of neutrino oscillations in matter can be realized only at violations of the law of energy momentum conservation (see ref. [23] and also below).

3 Elements of Theory (Mechanism) of Resonance Enhancement of Neutrino Oscillations in Matter and Some Critical Remarks

Before consideration of the resonance mechanism it is necessary to gain an understanding of the physical nature origin of this mechanism. As stressed above, at neutrino passing through matter there can be two processes- neutrino scattering and polarization of the matter by neutrino. Obviously resonance enhancement of neutrino oscillations in matter will arise due to polarization of the matter by neutrino. If the weak interaction can generate not only neutrino scattering but also polarization of matter, then the resonance effect will exist, otherwise this effect cannot exist.

In the ultrarelativistic limit, the evolution equation for the neutrino wave function $\nu_{\Phi}$ in matter has the following form [8]:

$$i\frac{d\nu_{\Phi}}{dt} = (p\hat{I} + \frac{\hat{M}^2}{2p} + \hat{W})\nu_{\Phi}, \quad (1)$$

where $p, \hat{M}^2, \hat{W}$ are, respectively, the momentum, the (nondiagonal) square mass matrix in vacuum, and the matrix, taking into account neutrino interactions in matter,

$\nu_{\Phi} = \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$, \quad $\hat{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$,

$\hat{M}^2 = \begin{pmatrix} m_{\nu_e\nu_e}^2 & m_{\nu_e\nu_\mu}^2 \\ m_{\nu_\mu\nu_e}^2 & m_{\nu_\mu\nu_\mu}^2 \end{pmatrix}$. 


If we suppose that neutrinos in matter behave analogously to the photon in matter (i.e., the polarization at neutrino passing through matter arises) and the neutrino refraction indices are defined by the expression

\[ n_i = 1 + \frac{2\pi N}{p^2} f_i(0) = 1 + 2\frac{\pi W_i}{p}, \]

(2)

where \( i \) is a type of neutrinos (\( e, \mu, \tau \)), \( N \) is density of matter, \( f_i(0) \) is a real part of the forward scattering amplitude, then \( W_i \) characterizes polarization of matter by neutrinos (i.e. it is the energy of matter polarization).

The electron neutrino (\( \nu_e \)) in matter interacts via \( W^\pm, Z^0 \) bosons and \( \nu_\mu, \nu_\tau \) interact only via \( Z^0 \) boson. These differences in interactions lead to the following differences in the refraction coefficients of \( \nu_e \) and \( \nu_\mu, \nu_\tau \)

\[ \Delta n = \frac{2\pi N}{p^2} \Delta f(0), \]

\[ \Delta f(0) = \sqrt{2} \frac{G_F}{2\pi} p, \]

(3)

where \( G_F \) is the Fermi constant.

Therefore the velocities (or effective masses) of \( \nu_e \) and \( \nu_\mu, \nu_\tau \) in matter are different. And at the suitable density of matter this difference can lead to a resonance enhancement of neutrino oscillations in matter [8], [9]

\[ \sin^2 2\theta_m = \sin^2 2\theta \cdot \left[ (\cos 2\theta - \frac{L_0}{L^0})^2 + \sin^2 2\theta \right]^{-1}, \]

(4)

where \( \sin^2 2\theta_m \) and \( \sin^2 2\theta \) characterize neutrino mixings in matter and vacuum, \( L_0 \) and \( L^0 \) are length of oscillations in vacuum and matter

\[ L_0 = \frac{4\pi E_\nu \hbar}{\Delta m^2 c^3}, \quad L^0 = \frac{\sqrt{2}\pi \hbar c}{G_F n_e}, \]

(5)

where \( E_\nu \) is neutrino energy, \( \Delta m^2 \) - difference between squared neutrino masses, \( c \) is light velocity, \( \hbar \) is Plank constant, \( G_F \) is fermi constant and \( n_e \) is electron density of matter.
At resonance

\[ \cos 2\theta \simeq \frac{L_0}{L^0} \quad \sin^2 2\theta_m \simeq 1 \quad \theta_m \simeq \frac{\pi}{4}. \quad (6) \]

It is necessary to stress that this resonance enhancement of neutrino oscillation in matter is realized when neutrino velocity is less than the light velocity in matter (i.e. \( v_i < \frac{c}{n_i} \)).

As we can see from the form of Eq. (1), this equation holds the left-right symmetric neutrinos wave function \( \Psi(x) = \Psi_L(x) + \Psi_R(x) \). This equation contains the term \( W \), which arises from the weak interaction (contribution of \( W \) boson) and which contains only a left-handed interaction of the neutrinos, and is substituted in the left-right symmetric equation (1) without indication of its left-handed origin. Then we see that equation (1) is an equation that includes term \( W \) which arises not from the weak interaction but from a hypothetical left-right symmetric interaction (see also works [24], [25], [26]. Therefore this equation is not the one for neutrinos passing through real matter. The problem of neutrinos passing through real matter has been considered in [24], [25], [26], [23].

Then there is a question: Can this resonance effect exist if the weak interactions do not generate masses (i.e. do not change neutrino masses in matter)?

4 What problem arises in Weak Interactions Theory in Contrast to the Strong and Electromagnetic Interactions Theory?

In strong and electromagnetic interactions the left-handed and right-handed components of spinors participate in interactions in symmetric manner. In contrast to these interactions only the left-handed components of spinors participate in the weak interactions with charged current (it is also necessary to remark that in the weak neutral current the left-handed and right-handed components of spinors participate in
non symmetric manner). This is a distinctive feature of the weak interactions.

4.1 Elements of the Electroweak Interactions Model

Electroweak interaction lagrangian includes the following lepton and quark doublets

\[ \Psi_{iL} = \left( \begin{array}{c} \nu_l \\ l \end{array} \right)_L, \Psi_{iR}, l = e, \mu, \tau \]

\[ \Psi_{iL} = \left( \begin{array}{c} u \\ d \end{array} \right)_L, \left( \begin{array}{c} c \\ s \end{array} \right)_L, \left( \begin{array}{c} t \\ b \end{array} \right)_L \]

and left components of charged leptons and quarks

\[ \Psi_{iR} = u_R, d_R; \quad c_R, s_R; \quad t_R, b_R. \]

And this lagrangian has the following form [27]:

\[ \mathcal{L}_I = ig j^{K, \alpha} A_{\alpha}^K + ig' \frac{1}{2} j^{\Upsilon, \alpha} B_{\alpha}, \]  

where

\[ j^{K, \alpha} = \sum_{i=1}^{3} \bar{\Psi}_{i,L} \gamma^\alpha \tau^K \Psi_{i,L} + \]

\[ \sum_{l=e,\mu,\tau} \bar{\Psi}_{l,L} \gamma^\alpha \tau^K \Psi_{l,L}, \]

and

\[ \frac{1}{2} j^{\Upsilon, \alpha} = j^{em, \alpha} - j^{3, \alpha}, \]

\( j^{em, \alpha} \) - electromagnetic current of quarks and leptons, where \( A_{\alpha}^l, B_{\alpha} \) - are gauge fields associated with \( SU(2)_L \) \( U(1) \) - groups; \( \Upsilon \) - hypercharges of quarks and leptons.
At transition from $A^3_{\alpha}, B_{\alpha}$ fields to $Z_{\alpha}, A_{\alpha}$ fields

$$Z_{\alpha} = A^3_{\alpha} \cos \theta_W - B_{\alpha} \sin \theta_W,$$

$$A_{\alpha} = A^3_{\alpha} \sin \theta_W + B_{\alpha} \cos \theta_W,$$

interaction lagrangian for $Z_{\alpha}, A_{\alpha}$ fields gets the following form:

$$\mathcal{L}_I^0 = i \frac{g}{2 \cos \theta_W} j^{\alpha,\alpha} Z_{\alpha} + i e j^{e m,\alpha} A_{\alpha},$$

where $j^{\alpha,\alpha} = 2 j^{3,\alpha} - 2 \sin^2 \theta_W j^{e m,\alpha}$ - is neutral current of the standard model.

### 4.2 Running Coupling Constant in the Standard Weak Interactions Model

It is supposed that in the electroweak model the coupling constants $g, g'$ depend on transfer momenta \[28\] and equation for $g(Q^2)$ has the following form:

$$\frac{dg^{-1}}{d(lnQ^2)} = \frac{1}{4\pi} \left[ \frac{22}{3} - \frac{4F}{3} \right],$$

where $F$ is family numbers ($F = 3$) (here we consider only a weak part of the electroweak model since in the electromagnetic interactions there is renormalization of the coupling constant). It means that in the weak interactions the vacuum polarization takes place as in the strong and electromagnetic interactions. It is necessary to remember that in the weak interactions in contrast to these interactions only the left components of fermion participate in the weak interactions of charged current (as stressed above in the weak neutral current the left-handed and right-handed components of spinors also participate in non symmetric manner).

If the coupling constant of the weak interaction is renormalized then the effective masses of fermions in matter also change, i.e. the standard weak interaction can generate effective masses. It means that at the weak interactions the resonance enhancement of neutrino oscillations in matter will take place \[8-10\]. It is necessary to keep in mind that our consideration refers only to the weak interaction with charged current.
4.3 Remarks About the Coupling Constant of the Standard Weak Interactions Model

As we have stressed above, a distinctive feature of the weak interactions is violation of $P$ parity. Now let us consider the consequences of the distinctive feature for coupling constant of the weak interactions.

The simplest method to prove the absence of the polarization in vacuum and matter is [29]:

If we put an electrical (or strong) charged particle in vacuum, polarization of vacuum will appear. Since the field around the particle is spherically symmetrical, the polarization must be also spherically symmetrical. Then the particle will be left at rest and the law of energy and momentum conservation is fulfilled.

If we put a weakly interacting particle (a neutrino) in vacuum then, since the field around the particle has a left-right asymmetry (weak interactions are left interactions with respect to the spin direction), polarization of vacuum must be nonsymmetric, i.e., on the left side there will be maximal polarization and on the right side there will be zero polarization. Since polarization of the vacuum is asymmetrical, there arises an asymmetrical interaction of the particle (the neutrino) with vacuum and the particle cannot be at rest and will be accelerated. Then neutrino will get energy-momentum from the vacuum and the law of energy momentum conservation will be violated. The only way to fulfil the law of energy-momentum conservation is to require that polarization of vacuum be absent at the weak interactions. The same situation will take place in matter (do not mix up it with particle acceleration at the weak interactions!).

It is interesting to remark that in the gravitational interaction the polarization does not exist either [30].
4.4 Is Mass Generation Possible in Weak Interactions?

It is well known that masses are generated in the strong and electromagnetic interactions. Is mass generation possible in the weak interactions? This question arises for the left-handed character charged current of the weak interactions. Let us consider consequences of this feature.

We will show that the lepton masses cannot be generated in the framework of the electroweak interactions model [26] or in the weak interactions model included in this electroweak interactions model as a component. Consideration will be carried out for $U(1)$ theory and then it will be generalized for $SU(2)$ theory.

Dirac equation for the lepton (spinor) wave function $\psi = \psi_R + \psi_L$ in external field $A_\mu$ has the following form:

\[
(E + \sigma_i H_i)\psi_L - M\psi_R = 0, \quad i = 1 - 3, \quad (13)
\]

\[
(E - \sigma_i H_i)\psi_R - M\psi_L = 0, \quad E = \epsilon - eA_4,
\]

where $H_i = P_i - eA_i$, $\sigma_i$ - Pauli matrices.

The same equation without external field $A_\mu$ can be written in the following form:

\[
(E' + \sigma_i P'_i)\psi_L - M'\psi_R = 0, \quad (14)
\]

\[
(E' - \sigma_i P'_i)\psi_R - M'\psi_L = 0.
\]

From (13) (14) and using that $\Delta M = M - M'$ we obtain

\[
((E - E') + \sigma_i (H_i - P'_i))\psi_L = \Delta M \psi_R, \quad (15)
\]

\[
((E - E') - \sigma_i (H_i - P'_i))\psi_R = \Delta M \psi_L.
\]

that deposit of the interaction caused by the external field $A_\mu$ leads to appearance of masses difference $\Delta M$ which is symmetric to the left and right components of fermion. Now using expression (15) we can consider the case when in interaction there is only a left component.
of spinor (fermion) as it takes place in the weak interactions. Then expression (15) can be rewritten in the form:

\[(E - E') + \sigma_i (H_i - P'_i))\psi_L = 0, \tag{16}\]

\[0 = \Delta M \psi_L,\]

if in (16) \(\psi_R = 0\) and \(\psi_L\) differs from zero, then \(\Delta M = 0\).

So, if only the left-handed component of fermion participates in the interaction, then the fermion mass does not change. It is not difficult to generalize the above considered case \(U(1)\) theory to the case of \(SU(2)\) theory and then we come to a conclusion that since the right-handed components of fermions do not participate in the weak interactions, then these interactions cannot generate masses.

For obtaining the masses in the standard model of electroweak interactions founded on group \(SU(2)_L \times U(1)\) the Higgs mechanism \[31\] is used.

So, we come to a conclusion that the standard weak interactions by charged current cannot generate masses for their left-handed character.

If we consider neutrino oscillations in the scheme of mass mixings \[21\], \[28\]

\[M = \begin{pmatrix} m_{\nu_e} & m_{\nu_e\nu_\mu} \\ m_{\nu_\mu\nu_e} & m_{\nu_\mu} \end{pmatrix}, \tag{17}\]

where \(m_{\nu_e}, m_{\nu_\mu}\) are masses of \(\nu_e, \nu_\mu\) neutrinos and \(m_{\nu_e\nu_\mu}, m_{\nu_\mu\nu_e}\) are non-diagonal mass terms, then the expression for \(\sin^2(2\theta)\) has the following form:

\[\sin^2(2\theta) = \frac{(2m_{\nu_e\nu_\mu})^2}{(m_{\nu_e} - m_{\nu_\mu})^2 + (2m_{\nu_e\nu_\mu})^2}. \tag{18}\]

Since the weak interactions cannot generate masses, then masses \(m_{\nu_e}^{\text{matt}}, m_{\nu_\mu}^{\text{matt}}\) of \(\nu_e, \nu_\mu\) neutrinos in matter do not change

\[m_{\nu_e}^{\text{matt}} = m_{\nu_e}, \quad m_{\nu_\mu}^{\text{matt}} = m_{\nu_\mu},\]

and then

\[\sin^2_{\text{matt}}(2\theta) = \sin^2(2\theta) \tag{19},\]
i.e., the mixing angle $\theta$ does not change in matter and resonance effect must not exist.

4.5 How Can this Resonance Effect be Realized if the Weak Interactions do not Generate Masses?

The law of energy momentum conservation of particle (neutrino) has the following form (we take only matter polarization into account) \[32\]:

\[
\begin{align*}
  a) \quad & E_0 = E + W, \\
  b) \quad & p_0 = p + W\beta,
\end{align*}
\]

where $E_0, p_0, E, p$, are correspondingly, energy and momentum of neutrino in vacuum and matter, $W$ - energy polarization matter by neutrino, $c = 1, \quad \beta = \frac{v}{c} \rightarrow \nu, \quad \frac{W}{c} \rightarrow W$.

It is obvious that neutrino matter polarization (reaction) moves in matter with the velocity $\beta$ which is equal to neutrino velocity.

If to put expression b) in expression a), then we obtain the following:

\[
\sqrt{p_0^2 + m_0^2} = \sqrt{(p_0 - W\beta)^2 + m_0^2} + W \rightarrow \\
W[(1 - \beta^2)W - 2\sqrt{p_0^2 + m_0^2} + 2p_0\beta] = 0,
\]

what has a solution only if (obviously, if $m_0 = 0$ then $\beta = 1$):

\[
m_0 = 0,
\]

or

\[
W = 0.
\]

The demand of fulfilment of the law of energy momentum conservation for the weak interacting particle (neutrino) in matter will result in conclusion that if

\[
m_0 = 0, \quad \text{then} \quad W \neq 0,
\]

and if

\[
m_0 \neq 0, \quad \text{then} \quad W = 0.
\]
These decisions mean that for massive neutrino the matter polarization energy $W$ must be zero, i.e., $W = 0$ in contrast to energy polarization charged particle in electrodynamics.

Since Wolfenstein’s equation describes massive neutrinos, then $W$ must be equal to zero. Therefore here an enhancement of neutrino oscillation in matter cannot arise.

Otherwise when $m_0 = 0$ the polarization energy $W$ can differ from zero, but as it is well known that massless neutrinos cannot oscillate. The intermediate case when some neutrinos have masses and the rest neutrinos are massless, is of no interest since it is clear how neutrinos which participating in the weak interactions (having a weak charge) can be massless.

So, we come to a conclusion that, if we demand fulfillment of the law of energy momentum conservation in matter, then the resonance enhancement of neutrino oscillations cannot appear. And it is a right consequence of the left-handed character of the weak interactions by charged current.

5 What is the Situation with Experimental Confirmation of this Resonance Mechanism?

At present the experimental data have been obtained on the accelerator, reactor, atmospheric and solar neutrinos. The data obtained in the reactor, accelerator and atmospheric neutrinos have shown that the $\theta_{12}, \theta_{23}$ have big values. The estimation of the value of this angle can be extracted from KamLAND \[33\] data and it is:

$$\sin^2(2\theta_{12}) \approx 1.0, \quad \theta \approx \frac{\pi}{4}, \quad \Delta m_{12}^2 = 6.9 \cdot 10^{-5} eV^2,$$

or

$$\sin^2(2\theta_{12}) \approx 0.83, \quad \theta_{12} = 32^\circ, \quad \Delta m_{12}^2 = 8.3 \cdot 10^{-5} eV^2.$$ 

The angle mixing for vacuum $\nu_\mu \rightarrow \nu_\tau$ transitions obtained on SuperKamiokande \[34\] for atmospheric neutrinos is:

$$\sin^2(2\gamma_{23}) \approx 1, \quad \gamma \approx \frac{\pi}{4} \quad \Delta m_{23}^2 \simeq 2.5 \cdot 10^{-3} eV^2.$$ 

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The value of the Solar neutrinos flow measured (through elastic scattering) on SNO \([35]\) is in a good agreement with the same value measured in SuperKamiokande \([36]\).

Ratio of \(\nu_e\) flow measured on SNO (CC) to the same flow computed in the framework of SSM \([37]\) \((E_\nu > 6.0\,\text{MeV})\) is:

\[
\frac{\phi^{\text{CC}}_{\text{SNO}}}{\phi_{\text{SSM}2000}} = 0.306 \pm 0.026(\text{stat.}) \pm 0.024(\text{syst.}). \tag{26}
\]

This value is in a good agreement with the same value of \(\nu_e\) relative neutrinos flow measured on Homestake (CC) \([38]\) for energy threshold \(E_\nu = 0,814\,\text{MeV}\)

\[
\frac{\Phi^{\text{exp}}}{\Phi_{\text{SSM}2000}} = 0.34 \pm 0.03. \tag{27}
\]

From these data we can come to a conclusion that the angle mixing for the Sun \(\nu_e\) neutrinos does not depend on neutrino energy thresholds \((0.8 \div 13 \,\text{MeV})\) and in this region the energy spectrum has no distortion.

\[\text{Figure 1:}\] The profile of the effect. Shown are the reconstructed values of the survival probability in different energy ranges. The lines correspond to the survival probability for the LMA and LOW solutions; (from \([41]\)).

The survival probability in different energy ranges of the solar neutrinos \([39]\) (see also ref. \([40]\)) was computed taking into account the
resonance effect. The profile of this effect is shown in Figure 1 (shown are the reconstructed values of the survival probability in different energy ranges. The lines correspond to the survival probability for the LMA and LOW solutions (from [41]).

From the above Figure 1 we see that the curves obtained from the computation in the framework of the resonance mechanism [39] are in clear discrepancy with the above given experimental data (also see below Figure 5). In spite of this fact some authors come to a conclusion that this mechanism has been proved in experiments (experimental errors given in this figure many times exceed the same published errors, it is necessary to suppose that these errors were smeared for obtaining small values for $\chi^2$ or better adjustment at smaller value of $\sigma$).

The same situation takes place in the last interpretations of the solar neutrino data [16], [42]. The energy profile of the solar $E_\nu$ survival probability $P_{ee}$ for best-fit LMA values ($\theta_{13} = 0$) is shown in Figure 2 (experimental data see in Figure 4 and 5 also in expressions (24)-(27)). Value for $\theta_{13} = 0$ was obtained from CHOOZ result analysis [43].

**Is the CHOOZ result analysis trustful (i.e., is it correct that $\theta_{13} = 0$)?**

The probability of $P_{\bar{\nu}_e\nu_e}$ transitions at three neutrino oscillations is:

$$P_{\bar{\nu}_e\nu_e}(R) = 1 - \cos^4(\theta_{13})\sin^2(2\theta_{12})\sin^2\left(\frac{R}{\mathit{L}_{12}}\right) -$$

$$\cos^2(\theta_{12})\sin^2(2\theta_{13})\sin^2\left(\frac{R}{\mathit{L}_{13}}\right) - \sin^2(\theta_{12})\sin^2(2\theta_{13})\sin^2\left(\frac{R}{\mathit{L}_{23}}\right)$$

(28),

where $L_{12}$, $L_{13}$ $L_{23}$, $R$, correspondingly, are lengths of neutrino oscillations and distance from neutrino source. Since $L_{13} \approx L_{23}$, we can rewrite expression (28) in the following form:

$$P_{\bar{\nu}_e\nu_e}(R) \approx 1 - \cos^4(\theta_{13})\sin^2(2\theta_{12})\sin^2\left(\frac{R}{\mathit{L}_{12}}\right) - \sin^2(\theta_{12})\sin^2(2\theta_{13})\sin^2\left(\frac{R}{\mathit{L}_{13}}\right)$$

(29)

if $L_{12} >> R$, and taking into account that $\frac{L_{12}}{L_{23}} \approx 30.5$, then the above
expression can be rewritten in the following form:

\[ P_{\bar{\nu}_e \to \bar{\nu}_e}(R) \approx 1 - \sin^2(2\theta_{13})\sin^2\left(\frac{R}{L_{13}}\right), \]  

(30)

Figure 2: The energy profile of the solar \( E_\nu \) survival probability \( P_{ee} \) for best-fit LMA values and \( \theta_{13} = 0 \). The function \( P_{ee}(E) \) shows a smooth transition from vacuum to the matter dominated regime as \( E \) increases, with some differences induced by averaging over different production regions (for 8B, 7Be and pp neutrinos) and, to a smaller extent, by nighttime (N) Earth effects with respect to daytime (D). Also shown are the corresponding solar neutrinos energy spectra (in arbitrary vertical scale).

since \( L_{12} \approx 160 \text{ km}, R_{CHOOZ} \approx 1 \text{ km}, \text{ then } \frac{R}{L_{13}} \approx 5.3, \ \sin^2\left(\frac{R}{L_{13}}\right) \approx \frac{1}{28} = 0.036. \) The expression for transition probability \( P_{\nu_e \to \nu_e}(R_{CHOOZ}) \) is

\[ P_{\nu_e \to \nu_e}(R_{CHOOZ}) \approx 1 - 0.036 \cdot \sin^2(2\theta_{13}), \]  

(31)

and then the value of \( 1 - P_{\nu_e \to \nu_e}(R_{CHOOZ}) \) cannot be larger than 0.036:

\[ 1 - P_{\nu_e \to \nu_e}(R_{CHOOZ}) \leq 0.036. \]

The precision of the CHOOZ experiment is \( \approx 5\% \), i.e. 0.05. It is clear that for obtaining a limitation on \( \sin^2(2\theta_{13}) \) the precision of this
experiment must be less than 0.036. So, we see that in this type of experiment a proper limitation on $\sin^2(2\theta_{13})$ is possible to obtain only if distances $R$ are $3 \div 5$ km or if the precision of the experiment is very big ($\approx 0.4 \div 0.5\%$).

Now there is a new mechanism of enhancement of neutrino oscillation which [44] is named as MaVaN (mass-varying neutrino oscillations) mechanism. The result of computation in the framework of this mechanism together with the profile of the MSW effect is given in Figure 3. We will not discuss this mechanism since at present a direct confirm of the dark matter existence is absent as well as its weak interactions with neutrinos.

![Figure 3](image)

**Figure 3:** $P(\mu_e \rightarrow \nu_e)$ vs. $E_\nu$ for MaVaN [44] oscillations (solid curve). The dashed curve corresponds to conventional oscillations with the best-fit solution to KamLAND data.

Figure 5 gives the profile of the MSW effect (i.e. the reconstructed values of the survival probability in different energy ranges for the LMA solution from [41]). The following experimental data are also shown:

1. From the Homestake experiment in 1970-1994y. [38] where the relation between the measured and calculated flux data is

$$\frac{\Phi^{exp}}{\Phi_{SSM2000}} = 0,34 \pm 0,03,$$

(32)

2. From the GALLEX (GNO) [45], [47] and SAGE [46], [47] experiments where the relation between measured and calculated BP04 [48]
flux data are
\[ \frac{\Phi_{\text{GALLEX}}^{\text{exp}}}{\Phi_{\text{BP04}}} = 0.53 \pm 0.04, \]  
\[ \frac{\Phi_{\text{SAGE}}^{\text{exp}}}{\Phi_{\text{BP04}}} = 0.51 \pm 0.04. \]  

The data from Ga-Ge experiments are placed higher than the data of other experiments. It is necessary especially to note that the value of these experimental data decreases with statistics increasing.

3. From the SNO \[35\] experiment where the relation between the measured and calculated SSM2000 \[37\] flux data are
\[ \frac{\phi_{\text{CC SNO}}}{\phi_{\text{SSM2000}}} = 0,35 \pm 0,02, \]  
and \[49\]
\[ \frac{\phi_{\text{CC SNO}}}{\phi_{\text{SSM2000}}} = 0,309 \pm 0,02, \]  

4. From the SuperKamiokande \[36\] experiment where the relation between the measured and calculated SSM2000 \[37\] flux data is
\[ \frac{\Phi_{\text{total}}^{\text{B SSM2000}}}{SSM2000} = 0.465 \pm 0,005(\text{stat}) + 0.016(-0.015)(\text{syst}), \]  
The data in Figure 5 above 5 MeV were obtained by subtraction of the neutral current \((Z^0 \text{ boson})\) deposit obtained in SNO from the SuperKamiokande data (see Figure 4) and this difference equals to \(\Delta = 0.156\) (it is the difference between the values of \(\frac{\Phi_{\text{total}}^{\text{B SSM2000}}}{SSM2000}\) in exp. (37) and \(\frac{\phi_{\text{CC SNO}}^{\text{exp}}}{\phi_{\text{SSM2000}}^{\text{exp}}}\) in exp. (36)). The theoretical value of \(\Delta\) is \(\Delta \approx 0.155\).

From Figure 5 we see that the data obtained in SuperKamiokande, Homestake do not coincide with the computation obtained on the resonance effect in matter, i.e., the resonance effect is not confirmed. Only one point obtained in GALLEX and SAGE comes out from the other neutrino experimental data. Therefore it is very important to study the solar neutrinos energy spectrum below 1 MeV to clarify the reason of this deviation.
Figure 4: The energy profile of the solar $E_{\nu_e}$ neutrinos flux from SuperKamoikande experiment ($P_{\nu_e}(E_{\nu})/P_{SSM2000}(E_{\nu})$).

The Day-Night effect is not confirmed. Usually it is claimed that this effect is very small. To avoid this argumentation it is necessary to carry out an experiment with the bigger statistics (for example, in SuperKamiokande). This problem also can be solved by using neutrinos passed through the Earth at resonance energies for the Earth densities

$$E_{res} = \frac{|\Delta m^2| \cos 2\theta_V}{2\sqrt{2}G_F n_{e,earth}},$$  \hspace{1cm} (38)

where $\theta_V$ is the vacuum angle mixing, $G_F$ is Fermi constant, $n_{e,earth}$ is electron density of the Earth.
Figure 5: The energy profile of the solar $E_\nu$ survival probability $P_{\nu_\mu,\nu_e}$. The point and circles are SAGE, GNO, Chlorine, SNO and SuperKamoi-kande experimental data. The dashed curve corresponds to the profile of MSW effect [39].
6 Conclusion

The mechanism of resonance enhancement of neutrino oscillations in matter and some critical remarks to this mechanism have been considered. Wolfenstein’s equation (1) contains term \( W \), which arises from the weak interaction (contribution of \( W \) boson) which is the only left-handed interaction of the neutrinos, and it is substituted in the left-right symmetric equation (1) without indication of its left-handed origin. Then we see that equation (1) is an equation that includes term \( W \) which arises not from the weak interaction but from a hypothetical left-right symmetric interaction. Therefore this equation is not the one for neutrinos passing through the real matter.

Using this resonance mechanism is very important to check the weak interaction part of the model of electroweak interactions since the processes induced by this mechanism grow multiply.

In contrast to the electromagnetic and strong interactions in the weak interactions, the \( P \)-parity is violated, therefore the problem of mass generations and charge renormalization in the weak interactions were considered (the interaction must be left-right symmetric for mass generations). It is concluded that the possibility of mass generation and charge renormalization in the weak interactions has been not proved.

The present status of this resonance effect by using the existence experimental data has been considered and it is concluded that this effect has no clear experimental confirmation. For this purpose it is necessary to fulfil precision experiments with solar neutrinos and the neutrinos which have passed through the Earth matter.
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