Supersymmetric Large Extra Dimensions and the Cosmological Constant Problem

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Abstract: This article briefly summarizes and reviews the motivations for — and the present status of — the proposal that the small size of the observed Dark Energy density can be understood in terms of the dynamical relaxation of two large extra dimensions within a supersymmetric higher-dimensional theory.

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Résumen : French version of abstract (supplied by CJP if necessary)
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1. Reading the Tea Leaves

The start of the new millennium finds the fundamental sciences at an odd cross-roads. On one hand we have in the Standard Model (which I take to include General Relativity and neutrino masses) an exquisitely accurate description (at least in principle) of all experiments that have ever been done. On the other hand we also understand the limitations of this theory, which tells us it must break down at sufficiently high energies. What might be the new theory which replaces the Standard Model at higher energies, and are we likely to discover it in the near future?

Much in our science depends on the answer to these questions, and the nature of the answers depends crucially on precisely at which energies the Standard Model fails. We know that the physics of the longitudinal modes of the $W$ and $Z$ bosons — i.e. the physics of electro-weak symmetry breaking — must lie just beyond our present experimental reach (at a few TeV or less), and this is the ultimate rationale behind the construction of the Large Hadron Collider at CERN. But what happens beyond this depends on more of the details, depending on what is found. Various clues have been proposed over the years, motivated by understanding the relative strength of the weak and gravitational interactions (the electro-weak hierarchy); the nature of CP violation in the strong interactions; the patterns of masses and mixings of the known elementary particles; or the small observed gravitational response to the energy of the vacuum.

All of the above clues share the feature that they demand an explanation for one of the Standard Model’s small dimensionless numbers. The reason why such small numbers provide a useful clue for the nature of new physics is based in how we understand small numbers in modern quantum field theories. Suppose a parameter $\lambda$ is found to be small when measured in an experiment which is performed at an energy scale $\mu$. We wish to understand this in terms of a microscopic theory of physics which is...
defined at energies $\Lambda \gg \mu$, in terms of which the prediction for $\lambda$ is given by

$$\lambda(\mu) = \lambda(\Lambda) + \delta \lambda(\mu, \Lambda).$$

(1)

Here $\lambda(\Lambda)$ represents the direct contribution to $\lambda$ due to the parameters in the microscopic theory, and $\delta \lambda$ represents the contributions to $\lambda$ which are obtained as we integrate out all of the physics in the energy range $\mu < E < \Lambda$.

Science has encountered an enormous number of examples of small parameters like this — ranging from the ratio of the size of the nucleus to the size of atoms in atomic physics, to the ratio of the small superconducting gap to the typical electron energy in condensed-matter physics. In every single case (so far) we understand the smallness of $\lambda(\mu)$ in two steps: (i) the quantity $\lambda(\Lambda)$ is understood to be small in the microscopic theory, and (ii) the process of integrating out from $\Lambda$ to $\mu$ gives an equally small contribution $\delta \lambda$. In this way the smallness of $\lambda(\mu)$ can be understood for any choice we may choose to make for the microscopic scale $\Lambda$. The ubiquity of this kind of explanation for small quantities has earned it a name: ‘technical naturalness’.

Red flags are raised when both steps (i) and (ii) cannot be understood, and a naturalness problem is declared for the corresponding small quantity, $\lambda(\mu)$. The significance of such naturalness problems comes from the clues they provide about the existence of new physics. In particular, although we may not be able to understand why $\lambda(\Lambda)$ is small until we have the ultimate microscopic theory in our hands, we do expect to be able to understand why the ordinary physics which we believe we already understand at energies $\mu < E < \Lambda$ should not make $\delta \lambda(\mu, \Lambda)$ unacceptably large. If we find $\delta \lambda$ to be many orders of magnitude larger than the measured value $\lambda(\mu)$ then we suspect that we do not actually understand the energies $\mu < E < \Lambda$ as well as we thought - and progress can be made because this suggests changes are required in an energy range which is experimentally accessible.

The gravitational response of the vacuum energy is the case which most egregiously resists such a technically-natural explanation [1]. In this case cosmological observations indicate that the vacuum’s energy density is at present at most $\rho \sim (10^{-3} \text{ eV})^4$ (in units with $\hbar = c = 1$) [2]. But since a particle of mass $m$ typically contributes an amount $\delta \rho \sim m^4$ when it is integrated out, such a small value for $\rho$ can only be understood in a technically natural way if $\Lambda \sim 10^{-3} \text{ eV}$ or less. Since essentially all of the elementary particles we know (including the electron, for which $m_e \sim 5 \times 10^5 \text{ eV}$) have $m \gg 10^{-3} \text{ eV}$, it has proven impossible to understand why our understanding of ordinary particles agrees with experiments so well, and yet so badly predicts the vacuum’s observed gravitational response.

2. SLED

2.1. Motivation

A technically natural explanation for the vacuum energy density requires two things. First, it requires a fundamental modification of how gravity responds to physics at scales $E > \mu \sim 10^{-3} \text{ eV}$. Second, whatever provides this modification must not ruin the excellent agreement with all of the many non-gravitational experiments which have been performed over the years for the energies $\mu < E < \Lambda$, with $\Lambda \sim 10^{11} \text{ eV}$, to which we have experimental access. It is the mutual contradictions of these two conditions which has made it so difficult to make progress.

Remarkably, a framework now exists within which both of these conditions may be able to coexist: the framework of Large Extra Dimensions [3]. According to this picture — which is motivated by the discovery of $D$-branes within string theory — all of the observed particles apart from the graviton may be trapped on a (3+1)-dimensional surface within an extra-dimensional space. In such a world the presence of the extra dimensions would only make a difference for gravitational observables, since only gravitational probes could be used to probe for their existence. Remarkably, the present upper limit for the size of such extra dimensions is $r < 100 \mu$m, or $1/r > 10^{-3} \text{ eV}$ [4] — very close to the scale $\mu$ above which our natural understanding of the vacuum energy breaks down! Furthermore, extra dimensions can only be this large if there are precisely two of them, and if the fundamental scale, $M_9$,
of the extra-dimensional physics is around 10 TeV, due to the relation $M_p = M_g^2 r$ which relates $M_g$ and $r$ to the observed Planck mass: $M_p = (8\pi G)^{-1/2} \sim 10^{27}$ eV (where here $G$ denotes Newton’s constant). Other constraints from astrophysics can also be satisfied for these choices of scales [5, 6, 7].

From this we see that it is logically possible that the gravitational response of the vacuum could depart dramatically from our 4-dimensional expectations at precisely the scales $E > 10^{-3}$ eV where these expectations cannot account for the vacuum energy in a technically natural way. This observation leads one to ask what the gravitational response of the vacuum might be in such a framework, and to ask whether the gravitational response of the 6D theory can be much smaller than would be expected from the 4D perspective.

2.2. 6D Gravitational Response

This line of thought leads to the proposal of Supersymmetric Large Extra Dimensions (SLED), which posits there two large (i.e. $r \sim 10 \mu m$) extra dimensions, arising within a supersymmetric theory [8, 9, 10, 11, 12, 13, 14, 15]. The motivation for the large extra dimensions is as above — having extra dimensions this large provides a loophole for the usual (four-dimensional) arguments which say that the vacuum energy must be of order $m^4$, and so be too large. Supersymmetry is motivated partially because our best high-energy theories are supersymmetric, and partially because of the cancellations between bosons and fermions which appears in the vacuum energy. This proposal is briefly summarized here, following the discussion of ref. [15].

In the SLED picture supersymmetry must be badly broken on our brane, since we know that there are no super-partners for the observed particles having masses which are much smaller than $M_g$. Given this scale for supersymmetry breaking on the brane there is also a trickle-down of supersymmetry breaking to the ‘bulk’ between the branes, whose size is set by the bulk’s Kaluza-Klein scale, $m_{sb} \sim M_{KK}$, and so which for unwarped geometries can be as low as $m_{sb} \sim 1/r \sim 10^{-2}$ eV [16, 8]. Much of the success of the SLED proposal relies crucially on the ability to maintain this hierarchy between the scales of supersymmetry breaking on the brane and in the bulk (for other approaches to separating the supersymmetry breaking scale see ref. [17]).

Within the above framework gravitational physics is effectively 6-dimensional for any energies above the scale, $1/r \sim 10^{-2}$ eV, and so the cosmological constant problem must be posed within this new context. In order to see how the cosmological constant problem is phrased in 6 dimensions, one must integrate out the degrees of freedom between the scales $M_g \sim 10$ TeV and $1/r \sim 10^{-2}$ eV. We seek the cosmological constant within the effective 4D theory obtained after performing this integration, which describes gravitational physics (like present-day cosmology) on scales much larger than $r$. Imagine, therefore, performing the integration over modes having energies $1/r < E < M_g$ in the following three steps [8]:

1. First, integrate out (exactly) all of the degrees of freedom on the branes, to obtain the low-energy brane dependence on the massless 4D graviton mode. In so doing we obtain (among other things) a large effective brane tension, $T \sim M_g^4$ for each of the 3-branes which might be present, which includes the vacuum energies of all of the presently-observed elementary particles.

2. Next, perform the classical part of the integration over the bulk degrees of freedom. This amounts to solving the classical supergravity equations to determine how the extra dimensions curve in response to the brane sources which are scattered throughout the extra dimensions. It will be argued that it is this classical response which cancels the contributions from the branes obtained in Step 1, above.

3. Finally, perform the quantum part of the integration over the bulk degrees of freedom. Given the cancellation of the previous two contributions, it is this contribution which is responsible for the fact that the present-day Dark Energy density is nonzero. It is argued below that in some
In a 4D world, the only contribution we would have is that of Step 1, above, and the problem is that this is much too large. But in a 6D world, because all of the observed particles are localized on our brane their vacuum energy should be thought of as a localized energy source in the extra dimensions, to which the bulk geometry must respond. We next argue that the classical part of this bulk response (Step 2, above) is of the same order as the direct contribution of Step 1, and precisely cancels it in a way which does not depend on the details of the supergravity involved or of the precise extra-dimensional geometry which lies between the various branes. In this way it provides a 6 dimensional realization of self-tuning, whereby the effective 4D cosmological constant is automatically adjusted to zero by the classical response of the 2D bulk to the brane sources. The final nonzero result finally comes from Step 3, due to the quantum bulk contributions. But because the supersymmetry breaking scale in the bulk is so small, these bulk loops are arguably the proper size to agree with the recently-observed Dark Energy density. (It is this part of the argument which non-supersymmetric proposals crucially miss [18].)

Step 1 consists of exactly integrating over all brane fields having masses larger than \(1/r\), and this produces a variety of local interactions in the effective theory for energies \(E < \sim 1/r\) on the brane. Since our interest is in the dependence of the effective theory on the 4D metric, and we assume a large volume for the extra 2 dimensions – \(M_g r \gg 1\) – we may expand these effective interactions in powers of the curvature:

\[
L_b = -\sqrt{-g} \left[ T_b + \frac{1}{2} \mu_b^2 R + \cdots \right],
\]

where on dimensional grounds we expect \(T_b \sim M_g^4\), \(\mu_b^2 \sim M_g^2\) etc..

Step 2 consists of the classical part of the bulk integration, and so is equivalent to substituting into the classical action the bulk field configurations which are found by solving the classical field equations using the above effective brane action as a source. It happens that for many situations this classical response of the extra dimensions can be computed explicitly within the approximation that the branes are regarded as delta-function tension sources.

In the absence of nontrivial brane couplings to bulk fields like the dilaton or bulk gauge fields (this assumption is relaxed below), co-dimension 2 objects the extra-dimensional curvature tensor typically acquire a delta-function singularity at the position of the branes, corresponding geometrically to the presence of a conical defect at the brane position. Einstein’s equations require that the singular contribution to the two-dimensional curvature is given by

\[
\sqrt{g_2} R_2 = -2 \sum_b T_b \delta^2(y - y_b) + (\text{smooth contributions}),
\]

Here \(y_b\) denotes the position of the ‘i’th brane in the transverse 2 dimensions, and the ‘smooth contributions’ are all of those which do not involve a delta-function at the brane positions.

The effective 4D cosmological constant obtained after performing Steps 1 and 2 above is obtained by plugging the above expression into the classical bulk action. The effective 4D cosmological constant obtained at this order is then

\[
\rho_c = \sum_b T_b + \int_M d^2 y \, e_2 \left[ \frac{1}{2} R_2 + \cdots \right] = 0,
\]

where the sum on ‘b’ is over the various branes in the two extra dimensions and ‘…’ denotes all of the other terms besides the Einstein-Hilbert term in the supersymmetric bulk action. The final equality here
has two parts. First, the sum over brane tensions, $T_b$, precisely cancels the contribution of the singular part of the curvature, eq. (3), to which they give rise [19]. Second, for supersymmetric theories a similar cancellation also occurs amongst the various ‘smooth’ contributions in $\rho_c$ once these are evaluated for all of the bulk fields using the classical field equations [8]. Interestingly, this cancellation does not depend on the details of the bulk geometry, or on the number of branes, since it relies only on a classical scale invariance which all 6D supergravity actions enjoy [9]. Best of all, this cancellation does not depend at all on the value of the brane tension, $T_b$, and so applies equally well even if these tensions are large and include all of the quantum effects due to virtual particles localized on the branes.

We are left with the contribution of quantum effects in the bulk (Step 3), to which we return in more detail in subsequent sections. These must ruin the brane-bulk cancellation because the scale invariance of the classical supergravity equations is not a bona-fide quantum symmetry. However the bulk sector of the theory is also one which is almost supersymmetric, since the bulk supersymmetry-breaking scale is very small: $m_{sb} \sim 1/r \sim 10^{-2}$ eV. As a result we might expect standard supersymmetric cancellations to suppress the quantum part of the result by powers of $m_{sb}^2$, and if the leading term should be of order $m_{sb}^4$, this would be the right size to account for the observed Dark Energy density.

Under certain circumstances this is indeed what happens for some 6D supergravities [14]. Quantum corrections do lift the flat directions of the classical approximation, and those loops involving bulk fields do so by an amount which is of order $V(r) \sim m_{sb}^4$, leading to

$$V(r) \sim \frac{1}{r^4} (a + b \log r),$$

(5)

where $a$ and $b$ are calculable constants and the logarithmic corrections generically arise due to the renormalization of UV divergences in even dimensions [20]. The result is this small despite the fact that the bulk loops include an sum over Kaluza-Klein modes having 4D masses right up to the TeV scale, $M_g$, due to cancellations which the extra-dimensional supersymmetry enforces.

What is interesting about the potential, eq. (5), is that it falls into a category of potentials which can provide a phenomenologically viable description of the Dark Energy [21]. Furthermore, this remains true even though there are additional constraints which arise due to the extra-dimensional interpretation of the Dark Energy. In particular, although the cosmological evolution of the extra-dimensional volume can imply a potentially dangerous time-dependence of Newton’s constant over cosmological epochs [22], the existing bounds which constrain how much this can happen are fairly easily satisfied due to the effects of Hubble friction during the Universe’s expansion [23]. Furthermore, this potential predicts that scalar-potential domination occurs when $\log(M_g r)$ is of order $a/b$, which can easily be the required value given a modest hierarchy amongst the coefficients, $a/b \sim 70$.

2.3. The More General Case

The above arguments assume particularly simple couplings between the branes and the various bulk fields like the dilaton, and these assumptions appear to play an important role in the scale-invariance properties which underly the cancellation between brane and bulk contributions [9]. It is therefore natural to wonder what happens if these assumptions are relaxed. A good test of how these arguments generalize is based on a class of solutions to 6D chiral, gauged supergravity obtained in ref. [24] by Gibbons, Guvens and Pope (henceforth GGP). What makes these solutions so useful as a test of self-tuning is that these authors derive the most general solution to these field equations subject to two assumptions: (i) maximal symmetry in 4 dimensions (i.e. de Sitter, Minkowski or anti-de Sitter space), and (ii) axial symmetry in the internal 2 dimensions. That is, they find the most general solutions whose metric has the form

$$d\mathbf{s}^2 = W^2(\theta)g_{\mu\nu}(x)\,dx^\mu dx^\nu + d\theta^2 + a^2(\theta)d\varphi^2,$$

(6)

and for which $\phi = \phi(\theta)$ and $A_\theta = A_\phi(\theta)$. Here the intrinsic 4D metric, $g_{\mu\nu}$, satisfies $R_{\mu\nu\lambda\rho} = c(g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda})$, for some constant $c$. The assumed axial symmetry corresponds to shifts of the
coordinate $\varphi$, and the metric can have singularities at up to two positions, $\theta = \theta_{\pm}$, within the internal 2 dimensions corresponding to the positions of source branes, but these singularities are not required to be only conical in form. GGP find that there is a five-parameter family of solutions to the supergravity equations subject to these symmetry conditions. What is most remarkable about these solutions is that every single one of them has a flat intrinsic 4D geometry (i.e. $c = 0$), even though none of them is supersymmetric (except for a single solution containing no branes), as is consistent with the expectation that the effective 4D cosmological constant vanishes. This same intrinsic flatness also appears to apply to the known solutions which lie outside of the GGP assumptions [25].

Having the most general solutions, even subject to a symmetry ansatz, also allows some exploration of how generic is the classical cancellation of the effective 4D cosmological constant. In order to do so it is useful to keep track of the physical meaning of the parameters on which the general GGP solutions depend. There are 5 such parameters, but one of these simply parameterizes the flat direction whose existence is guaranteed by the classical scale invariance of the supergravity equations. A second parameter corresponds to another classical scaling property, under which a redefinition of the fields may be used to rescale the gauge coupling $g$ to any positive fixed value. The three remaining parameters are broadly related to the three physical quantities which characterize these geometries: the tensions, $T_{\pm}$, of the two branes which source the bulk geometry; and the overall magnetic flux of the background magnetic field which (marginally) stabilize it.

2.3.1. Topological Constraints

We now ask what may be said about the natural of self-tuning given the properties of these general solutions. There is a non-trivial constraint amongst the parameters of the model which hold quite generally for all of the GGP solutions, and it is natural to think that these constraints hide the fine-tunings which underlie the flatness of the 4D geometries. In fact, they do not because they have their origins in topology, as is now explained.

There are two topological conditions which the GGP solutions all share: one which expresses that the internal 2D geometry is topologically a sphere; and one which expresses the quantization (and so also conservation) of magnetic monopole flux [8, 26]. Since the first of these turns out to hold for all values of the parameters describing the classical solution, it is of less interest as a potential source of fine-tuning. It is the quantization of monopole number which directly imposes a relation between the brane tensions, the gauge couplings and one of the 5 parameters characterizing the various GGP solutions.

The resulting topological constraint can be written in the following way [12]:

$$\frac{g^2 e^{-\varphi_0/2}}{2} \left( T_+ - T_- \right) = N^2 \left( \frac{g^2}{\tilde{g}^2} \right),$$

where $N$ is the integer which labels the monopole number. Here $T_{\pm}$ are the two brane tensions, $g$ is the gauge coupling which appears explicitly in the 6D supergravity action, $\tilde{g}$ is the gauge coupling for the background magnetic field and $\varphi_0$ is an additive constant in the dilaton configuration (and so is one of the parameters describing the solution).

Although this looks like a hidden fine-tuning, first impressions deceive [9, 10]. Recall in this regard that the crucial issue for fine-tuning is whether or not the constraint is stable against renormalization. That is, if eq. (7) is imposed amongst the renormalized quantities at the TeV scale, does it automatically remain imposed as successive scales are integrated out down to the scales below 1 eV? If so, then the constraint is technically natural, in the sense described above, and so is not fine-tuned this (the most serious) notion of fine tuning. But topological constraints are always natural in this sense, because the integrating out of successive scales of physics is a continuous process, and since topological constraints involve quantization of quantities in terms of integers, they remain unchanged by any such continuous process. Topological constraints express global integrability conditions which must be satisfied in order.
for solutions to exist, rather than relations which select out a special class of (flat) solutions amongst a wider class which do not have this property.

2.3.2. Runaway Solutions

However one thing does emerge from an analysis of the properties of the general solutions [12] is the observation that not all initial brane configurations can give rise to classically stationary solutions. To see this notice that it turns out that only a subset of the general GGP solutions involve purely conical singularities, with the subset defined by the family of GGP solutions whose tensions satisfy the condition [9, 12]:

\[
\left(1 - \frac{T_+}{4\pi}\right) \left(1 - \frac{T_-}{4\pi}\right) = \frac{g^2 e^{-\phi_0/2}}{2} \left(\frac{T_+ - T_-}{4\pi}\right) = N^2 \left(\frac{g^2}{g^2}\right).
\]

(8)

Here the last equality follows from using the topological constraint, eq. (7). Purely conical singularities are only possible for a one-parameter locus of tensions within the \(T_+ - T_-\) plane.

Since brane solutions have conical singularities only in the absence of couplings to the dilaton (in the Einstein frame) [28] it should be possible to choose a configuration of branes whose tensions do not satisfy eq. (8), and so no bulk solution satisfying the GGP assumptions can exist for such a choice. One of the assumptions must fail for such a configuration and this is most likely the assumption of a static bulk geometry, most likely leading to a runaway configuration. Such a situation would be similar to what obtains if an arbitrary configuration of electric charges were assembled: generically the forces between them do not balance and so they move to zero or infinite separation.

However the existence of a runaway need not in itself be a problem. After all, we have seen that even if the classical solution is static, a runaway is generated by quantum corrections. Furthermore, this runaway can describe the Dark Energy density provided \(V \sim 1/r^4\), up to logarithmic corrections. If we suppose that such classical runaways exist, then the central question is: Is the classical runaway too steep to describe the Dark Energy? This need not be a problem even if so, since we need not demand that all solutions describe our world, and we know that some solutions do not admit classical runaways. In this case we must know if it is technically natural to demand that we find ourselves in a classically-static configuration.

A second question of central importance is: Are there hidden self-tunings, in particular amongst the brane couplings (for which supersymmetry cannot come to the rescue)? A proper understanding of this requires a detailed understanding of the matching conditions between brane properties and bulk solutions, and although the answer to this question is not yet clear it is at present under active study [27, 28].

3. Summary and Observational Consequences

The SLED proposal states that the world becomes six-dimensional at sub-eV energies, in such a way that the bulk gravitational physics is supersymmetric down to the sub-eV KK scale. The main motivation for this framework is that it dramatically changes the gravitational response of the energy of the vacuum in a way which appears to be technical natural, at least within the limits with which it has so far been checked [8, 9, 10, 11, 12].

Regardless as to how the naturalness arguments turn out, the SLED proposal dramatically changes how physics works at experimentally-accessible energies, and so it is falsifiable through the host of other robust phenomenological implications it makes beyond those it has for cosmology. These include:

- Deviations from the inverse square law for gravity, which more precise estimates show should arise for distances of order \(r/2\pi \sim 1 \mu\text{m}\) [29]:
• A particular scalar-tensor theory of gravity at large distances, with the scalar(s) being the moduli (like the volume) which describe the two large extra dimensions. This is the same scalar whose time-dependence now describes the Dark Energy [23].

• Distinctive missing-energy signals in collider experiments at the LHC due to the emission of particles into the extra dimensions [16, 13].

• Potential astrophysical signals (and bounds) due to the possibility of having too much energy loss into the extra dimensions by stars and supernovae [5, 6, 7, 16, 30].

If the SLED proposal is correct, it will be spectacularly so since it requires this entire suite of observational implications to be found. Indeed, it is this unprecedented connection between observables in cosmology and particle physics — which is driven by its addressing the fundamental naturalness issues described in previous sections — that sets the SLED proposal apart from other descriptions of Dark Energy.

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