Reliability Analysis a Two-Unit Cold Redundant System Working in a Pharmaceutical Agency with Preventive Maintenance

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Abstract: The present paper examines the working and reliability of a pair of a two-unit cold redundant system in a pharmaceutical agency. Initially, out of two units, one unit is operative and the other unit acts as a cold redundant. On the failure of the working unit, the redundant unit is put to work. As the demand varies, the manufacturing of the product is also very much affected and so the machines may need to put into the rest period when the vast range of manufacturing is in distinction to those demanded. It leads to a decline in the warranty period of the product. Additionally, another factor of the preventive/corrective maintenance (PM/CM) is also taken into the consideration. The main aim is to an evaluation of the expected rest period and expected PM/CM period and to determine the maximum profit gained by the company using the Markov process and regenerative technique. The authors have calculated various measures of the system's effectiveness. Numerical consequences and graphs of a precise case are additionally included.

Keywords: Reliability, Cold Redundant System, Production, Variation in Demand, PM/CM.

1. Introduction

The sphere of human life is strongly surrounded by science and technology. It has given us new, complex, and sophisticated structures. A good machine is one that meets human needs and desires and performs flawlessly over a long period. In other words, a machine is reliable if functions for the expected period. To meet the demand of the customers, components with a high degree of reliability are used in the designing and development of any system. But nothing is permanent in this world and the same concept is true for human-designed systems [25]-[26] as well. They may get smashed down after some period which affects the reliability, availability, and hence the cost analysis of the system. Many researchers have done a lot of work on reliability analysis. A lot of research papers and studies using different concepts are taken into consideration while writing this paper. In the middle of the 1930s, Swedish engineer and mathematician suggested a simple and convenient mathematical model for the description known as Weibull distribution. It was the first significant result in the future development of reliability theory. The probabilistic theory of reliability has grown out of the demands of modern technology and particularly out of the experiences in World War II with complex military systems as high failure rates were observed especially in vacuum tubes just after World War II. In 1950, the U.S. Air Force shaped an ad hoc group to improve time-honored gear reliability, and in 1952, the Defense Department hooked up the Advisory Group on Reliability of Electronic Equipment (AGREE). The requirement of reliability attempting out and demonstration of new structures emerged from this advisory group. Work posted at some stage in the Nineteen Fifties located on the use of the exponential distribution to signify failure times. An association for engineers working with reliability questions was soon established. The first journal in the subject, “IEEE Transactions on Reliability” came out in 1963, and several textbooks in the subject were published in the 1960s. Davis [1] mentioned failure records
and goodness of fit for competing failure distributions. Epstein [2], [3], [4] labored in the concern of a way of existence checking out with the useful resource of wondering about exponential distribution. After these studies, the exponential failure time distribution acquired a distinct characteristic in reliability analysis. To achieve monetary gains from systems, the aspect of profit along with various costs involved was important to be studied and hence this aspect was taken into consideration by a good number of researchers from the 1980s onwards in the field of reliability. Goel et al. [5] determined the profit analysis of a two-unit cold and warm redundant systems by introducing different types of operation and repair. Taneja et al. [6], Tuteja and Taneja [7] developed reliability models for a redundant system with instructions and types of failure. The master-slave concept was introduced by Prashar and Taneja [8].

All these researches were based on fixed demand. Since the demand is not fixed, there is a need to study the cost-benefit and reliability analysis of a machine with various demands. Malhotra and Taneja [9]-[13] have studied the cost-benefit assessment of a single and 2-unit redundant unit in a cable manufacturing plant with a variant in demand by applying scheduled maintenance, the concept of inspection. A similar situation has been shown in a Pharmaceutical Company situated at Nalagarh (Himachal Pradesh), India, the area the demand for the produced gadgets is no longer consistent and consequently the current paper. Thus the current paper investigates the reliability of a two-unit redundant gadget in a pharmaceutical company, relying on demand variations, need a rest period, and PM/CM. The measure of the system effects such as expected rest duration and expected PM/CM duration is analyzed by using semi-Markov techniques and regenerative factor technique. The concept of a redundant unit/gadget.

Yusuf [14] evaluated an expression using the probabilistic approach and determined failure effect, repair rate on system availability. He considered that no system meets a sudden failure, rather they go under slow deterioration over some time. Such systems may be repaired at the failure, or after or even before the failure. In later researches, Yusuf [15] developed various relationships between reliability measures and performance with built-in redundancy. The corresponding characteristic expressions were evaluated for steady-state availability, repair period, and profit function. Bala and Yusuf [16] did profit modeling and comparison between two dissimilar units, considering two types of failures based on the Markovian Birth-Death process. Both types are categorized based on minor and major failures respectively. Minor failures reduce work capacity whereas major failure leads to whole system failure. Yusuf and Ramatu [17] estimated the failure time of a system of a consecutive three-stage deterioration system. Taj et al. [17]-[20] determined the reliability measures by taking the concept of preventive maintenance, rest period in the subsystem of the manufacturing plant. The redundant optimization of cold redundant systems with failure mechanisms, periodic inspection, and using reusable elements was evaluated by the researchers[21]-[24]

Model Description and Assumptions

Initially state $S_0$, one of the units (gadgets) is operative and the other acts as a cold standby. It is assumed that demand $\geq$ production. The system goes to state $S_1$ when the operative unit fails and the standby unit becomes operative. The system moves to state $S_2$ when demand reduces and the system may require putting into the rest period. The system may go to state $S_5$ of preventive maintenance/corrective maintenance. After doing PM/CM, the system may go to the state $S_6$. If demand increases, one unit again starts functioning. It is assumed that only one unit can be operative at a time. The following assumptions have been made:
1. The gadgets are statistically independent.
2. The two-units are working alternatively: At first, one unit is working whilst the other act as cold redundant.
3. The repairman takes minimum time to repair the system.
4. Each system is assumed new after repair.
5. If a system fails, the redundant system takes over it automatically.
6. The system cannot go to a leisure state when it has completely failed.
7. Switching is ideal and instantaneous.
8. It is assumed that in a working state when demand \( \geq \) production.

2. Notations

| Symbol | Description |
|--------|-------------|
| \( \lambda \) | The failure rate of the serviceable unit |
| \( \gamma_1 \) | Rate of going to rest state from an operative state |
| \( \gamma_2, \gamma_3 \) | Rate of going/doing for PM/CM |
| \( \gamma_4 \) | The rate with which the demand becomes greater than the production and system starts after the completion of PM/CM |
| \( \gamma_5 \) | Rate of going to state of rest from the state where one unit is under repair and the other is operable |
| \( A_i(t) \) | The probability that the system is in upstate when demand \( \geq \) production at instant \( t \) given that it achieved the state \( i \) at \( t=0 \) |
| \( B_i(t) \) | The probability that the mechanist is busy in repairing the failed unit at instant \( t \) given that achieved the state \( i \) at \( t=0 \) |
| \( m_{ij} \) | Contribution to mean sojourn time in regenerative state \( i \) before transiting to regenerative state \( j \) without visiting any other state |
| \( q_{ij}(t) \) | p.d.f. and c.d.f. of first passage time from a regenerative state \( i \) to a regenerative state \( j \) or to a failed state \( j \) without visiting any other regenerative state ‘\( k \)’ in \( 0, t \] |
| \( Q_{ij}(t) \) | p.d.f. and c.d.f. of first passage time from a regenerative state \( i \) to a regenerative state \( j \) or to a failed state \( j \) with visiting a regenerative state ‘\( k \)’ once in \( 0, t \] |
| \( \circledcirc \) | The symbol for Laplace convolution |
| \( g(t), G(t) \) | p.d.f. and c.d.f. of repair time of the unit |
| \( P_d(t) \) | Expected PM/CM duration |
| \( DT_i(t) \) | Expected rest period in \( 0, t \] |

3. Symbols for the States of System

- \( O \) serviceable state
- \( r \) rest state
- \( F_r \) Failed unit under repair
- \( F_R \) Repair of failed unit continuing from the previous state
- \( F_{wr} \) Failed unit waiting for repair
- \( cs \) Cold Standby
- \( r_0 \) Ready to become viable as the demand \( \geq \) production
- \( r_s \) Read Statistical Analysis of Study habits in Mathematics
Achievement to become standby as the demand \( \geq \) production

4. Probability of Transition and Mean Sojourn Times

In Fig. 1, the transition layout displaying different system states is shown. The epochs of entry into states \( S_0, S_1, S_2, S_3, \) and \( S_6 \) are regeneration points, and as a consequence states are regenerative states. State \( S_4 \) is the failed state. States \( S_2, S_3, S_5, \) and \( S_6 \) represent the rest state.

\[
\begin{align*}
\gamma_2 & \quad \rightarrow \quad \gamma_5 \\
S_2 & \quad (r, r) \\
g(t) \\
\gamma_1 & \quad \rightarrow \quad \gamma_1 \\
S_0 & \quad (O, C) \\
g(t) \\
S_1 & \quad (F, O) \\
\gamma_4 & \quad \rightarrow \quad \gamma_4 \\
S_6 & \quad (PM, CM) \\
\end{align*}
\]

Fig. 1 State Transition Diagram

The probabilities of transition are as follows:
\[
\begin{align*}
q_{01}(t) & = \lambda e^{-(\lambda+\gamma_1)t} \\
q_{02}(t) & = e^{-(\lambda+\gamma_1)t}\gamma_1 \\
q_{10}(t) & = e^{-(\lambda+\gamma_3)t}h(t) \\
q_{12}(t) & = (\gamma_1 e^{-(\lambda+\gamma_3)t}\gamma_3 h(t)) \\
q_{14}(t) & = \lambda e^{-(\lambda+\gamma_3)t}\int_{0}^{t}H(\tau)\,d\tau \\
q_{11}(t) & = (\lambda e^{-(\lambda+\gamma_3)t}\gamma_3 h(t)) \\
q_{25}(t) & = \gamma_2 e^{-\gamma_2 t} \\
q_{56}(t) & = \gamma_3 e^{-\gamma_3 t}
\end{align*}
\]
\[
q_0(t) = \gamma_4 e^{-\gamma_4 t}
\]

The non-zero elements \( p_{ij} \) are obtained as \( p_{ij} = \lim_{s \to 0} q_{ij}'(s) \). The calculations are not shown here just to shorten the length of paper.

The mean sojourn time (\( \mu_i \)) at different values of \( i \), are

\[
\begin{align*}
\mu_0 &= \frac{1}{(\lambda + \gamma_1)} \\
\mu_1 &= \frac{(1 - h^*(\lambda + \gamma_5))/(\lambda + \gamma_5)}{\lambda}
\end{align*}
\]

Now as \( m_0 = \int_{0}^{\infty} t q_0(t) \, dt = -q_0'(0) \)

Thus,

\[
\begin{align*}
m_{10} + m_{02} &= \mu_0 \\
m_{10} + m_{12} + m_{11} &= k_1 \quad \text{where } k_1 = \int_{0}^{\infty} t g(t) \, dt \\
m_{10} + m_{12} + m_{14} &= (\lambda \mu_1 + \gamma_5 k_1)/(\lambda + \gamma_5) \\
m_{25} &= \mu_2 \\
m_{56} &= \mu_5 \\
m_{60} &= \mu_6
\end{align*}
\]

5. Measures of System Effectiveness:

System effectiveness is measured and is given by:

**Mean Time To System Failure (MTSF)**

\[
\mu_0 + p_{01}(\lambda \mu_1 + \gamma_1 k_1)\mu_0 + (\mu_2 + \mu_5 + \mu_6)(p_{12}^2 + p_{02})
\]

**Steady State Availability (A_i)**

\[
\frac{\mu_1 p_{01} + (1 - p_{11}^2)\mu_0}{D_1}
\]

**Busy Period of the repairman**

\[
\frac{k_1 p_{01}}{D_1}
\]

**PM/CM Period of the repairman**

\[
\frac{\mu_5 [p_{01} p_{12}^3 + p_{02} (1 - p_{11}^4)]}{D_1}
\]

**Expected Rest Period**

\[
\frac{\mu_0 p_{01} (-\mu_1 + k_1)\gamma_1 + (\mu_2 + \mu_5 + \mu_6)(p_{01} p_{12}^2 + (1 - p_{11}^4) p_{02})}{D_1}
\]

where \( D_1 = k_1 p_{01} + (1 - p_{11}^4)\mu_0 + (\mu_2 + \mu_5 + \mu_6)(p_{01} p_{12}^2 + p_{02} (1 - p_{11}^4)) \)

The profit of the proposed model is given by:

\[
P = C_0 A_0 - C_1 B_0 - C_2 P_0 - C_3 D T_0
\]

Where, \( C_0 = \text{Revenue per unit uptime of system when demand} \geq \text{production} \)

\( C_1 = \text{Cost per unit time of engaging the mechanist for repair} \)

\( C_2 = \text{Cost per unit time for doing PM/CM} \)
C₃ = Loss per unit time during the system go to the rest period

7. Graphical Analysis

Let us take measure of failure is taken into account as exponentially distributed, i.e. g(t) = α e⁻αt

Using the above particular case and the values of other parameters are taken as:
γ₁ = 0.00013, γ₂ = .000121, γ₃=0.0021, γ₄= 0.5, γ₅=0.00011,
C₀ = 1000, C₁=400, C₂=100, C₃ = 700 (all rates are in per hour and costs are in Indian rupees.)

Various graphs are plotted which helps to analyze the system. Some of the graphs have been shown as follow:

Fig. 2 indicates the conduct of MTSF concerning failure rate (λ) for different values of repair rate (α).

It can be concluded from the graph that MTSF gets decreased for higher values of failure rate and increased for higher values of repair rate.

![Fig. 2 MTSF versus Failure rate for Different values of Repair Rates](image)

Fig. 3 suggests the conduct of availability concerning failure rate (λ) for specific values of repair rate (α).

The graph states that the availability gets decreased with higher values of failure rate. whereas, its values get increased for greater values of repair rate (α).
Fig. 3 Relationship between Availability and Failure rate

Fig. 4 Relationship between Profit ($P$) versus Revenue per unit uptime ($C_0$) for different Values of ($C_1$)

Fig. 4 suggests the variation of profit concerning income per unit uptime ($C_0$) for specific values of payment given per visit of the mechanist ($C_1$). The graph concludes that the income will increase with the expansion in $C_0$ and has decrease values for greater $C_1$. Additionally, it has located from the format that for $C_1 = 5100$, the income is high-quality or zero or poor in accordance as $C_1 > or = < 1780.3$. So, the machine is worthwhile solely if $C_1 > 1780.3$. For $C_1 = 5200$, the earnings is effective or zero or bad in accordance as $C_1 > or = < 1992$. So, the device is worthwhile solely if $C_1 > 1992$. For $C_1 = 5400$, the income is effective or zero or poor in accordance as $C_1 > or = < 2279$. So, the gadget is worthwhile solely if $C_1 > 2279$. 
Fig. 5 shows the graph profit versus income per unit uptime ($C_0$) for specific values of charges required for doing PM/CM ($C_2$). It can be concluded from the layout that the income will increase with the expansion in values of $C_0$ and has decrease values for greater values of $C_2$. It is additionally located from the format that for $C_2 = 800$, the income is high-quality or zero or poor in accordance as $C_2 > or < 293.5$. So, the machine is worthwhile solely if $C_2 > 293.5$. For $C_2 = 1400$, the earnings is effective or zero or bad in accordance as $C_2 > or < 448.7$. So, the device is worthwhile solely if $C_2 > 448.7$. For $C_2 = 2000$, the income is effective or zero or poor in accordance as $C_2 > or < 594.6$. So, the gadget is worthwhile solely if $C_2 > 594.6$.

8. Conclusion

The study deals with the reliability of a two-unit cold redundant system in a pharmaceutical agency with preventive maintenance. When the demand is less than the production, the system may be required to put it to the rest period. Thus, the model is developed by introducing the concept of the rest period and PM/CM. The system is analyzed by using semi-Markov processes and regenerative point technique. The MTSF, availability expected rest period, and expected PM/CM duration are calculated by solving recursive relations. Graphical analysis is done by taking a particular case. The cut-off points have been calculated for the income per unit uptime, and payment for repairmen’s visit. These cut-off points help to get fixed the least revenue on the price of the unit produced by the company. As this model is general, can be useful for any company where this model can be fitted. Thus, the paper helps in making decisions beneficial to the companies.

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