Dynamical behaviors of two electrons confined in a line shape three quantum dot molecules driven by an ac-field

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Using the three-site Hubbard model and Floquet theorem, we investigate the dynamical behaviors of two electrons confined in a line-shape three quantum dot molecule driven by an ac electric field. Since the Hamiltonian contains no spin-flip terms, the 6 dimension singlet state and 9 dimension triplet state sub-spaces are decoupled and can be discussed respectively. In particular the 9 dimension triplet state sub-spaces can also be divided into 3 three-dimensional state sub-space which are fully decoupled. The analysis shows that the Hamiltonian in each three-dimensional triplet state sub-space as well as the singlet state sub-space in the no double-occupancy case has the same form which is similar to that of the driven two electron in two quantum dot molecule. By solving the time-dependent Schrödinger equation, we investigate the dynamical properties in the singlet state sub-space, and find that the two electrons can maintain its initial localized state when driving by an appropriately ac-field. In special, we find the electron interaction enhances the dynamical localization effect. Using both perturbation analytic and numerical approach to solve the Floquet function leads to a detail understanding of this effect.

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I. INTRODUCTION

Quantum state engineering via optical or electrical manipulation over the coherent dynamical of suitable quantum mechanical systems is a subject of great current interest because of a growing number of possible experimental applications. The development of laser and masers also open the doorway for creation of novel effects in nonlinear quantum systems which interact with strong electromagnetic field. In particular, recent experimental suc-
cesses in detecting Rabi oscillations in quantum dot (QD) systems driven by ac-field has also spurred interest in the use of driven ac-field to coherently manipulate the time development of electronic states. Achieving an understanding of the dynamics of the QD system is extremely desirable, as the ability to rapidly control the localization of electrons suggests possible application to quantum computation and quantum information processing, in which the coherent manipulation of entangled quantum states is essential component. An exciting possibility is to make use the phenomenon of coherent destruction of tunnelling (CDT), in which the tunnelling dynamics of a quantum system become suppressed at certain parameters of the field. Tuning the driving field thus provides a simple mechanism to localize or move charge within the quantum dot on a rapid time-scale by destroying or restoring the tunnelling between regions of the devices, so allowing the ac-field to be used as ”electron tweezers”.

In this paper, we address the dynamical localization of two interacting electrons confined in three QD molecule with linear arrangement driven by an ac electric field. We use an three-site model of Hubbard-type to describe the dynamical system, which gives a considerable computational advantage over standard numerical approaches, and also allows us to easily include the important effects of the electron correlations produced by the Coulomb interaction. The two electron distribution states are taken as basis vectors to write the Hamiltonian matrix. We show that the system can be analyzed in singlet state and triplet state sub-space respectively, and the triplet state sub-space can also be divided into 3 three-dimensional syate sub-space. By solving the time-dependent Schrödinger equation, we investigate the dynamical properties in the singlet state and triplet state subspace, and explain these finding by Floquet theorem. These exposition are useful to understand how the ac electronic field can affect the charge distribution inside a QD molecule, and how they can be used for quantum control.

II. MODEL AND METHODS

We consider a highly simplified model in which each quantum dot is replaced by a single site. The electron can tunnel between in the sites, and importantly, we include interactions by means of a Hubbard $U$-term. We do not take the energy level detuning between the three quantum dots into account. Assuming that the AC field frequency is much lower that
single-particle level spacing, the higher-lying single-particle states are also ignored. The Hamiltonian defining the system reads:

\[
H(t) = \sum_{(\sigma=\uparrow,\downarrow)(k=1,2,3)} \epsilon_{\sigma k}(t) d_{\sigma k}^\dagger d_{\sigma k} + \sum_{(\sigma=\uparrow,\downarrow)(k=1,2)} W(d_{\sigma k}^\dagger d_{\sigma k+1} + h.c.) + \sum_{(k=1,2,3)} U_1(n_{\uparrow k} n_{\downarrow k} + n_{\downarrow k} n_{\uparrow k}) + \sum_{(\sigma,\sigma')(k=1,2)} U_2(n_{\sigma k} n_{\sigma'(k+1)}).
\]  

(1)

Where \(\sigma\) is the electron spin, and \(\uparrow\) and \(\downarrow\) are used to indicated the up-spin and down-spin; \(k = 1, 2, 3\) is the serial number of the QD; \(d_{\sigma k}^\dagger/d_{\sigma k}\) are the creation/annihilation for an electron of spin \(\sigma\) on dot \(k\); The quantity \(W\) denotes the hopping between adjacent dot; \(U_1\) is the standard Hubbard-\(U\) terms, giving the energy cost for the double-occupation of a dot; \(U_2\) presents the Coulomb repulsion between electrons occupying neighboring dots; \(n_{\sigma k} = d_{\sigma k}^\dagger d_{\sigma k}\) is the total charge occupation of dot \(k\). For convenience, we consider the three QD molecule is in a standing wave field, and the centra QD is just in the wave node. We also let the QD molecule align with propagation direction of the standing wave field. So the energy level can be parameterized as: \(\epsilon_{\sigma k}(t) = \pm(k-2)V\cos(\omega t)\), where \(\omega\) is the frequency of driving field, and \(V\) is the amplitude of energy level. Since \(V\) is proportionate to the amplitude of the ac-field, we still take \(V\) as the amplitude of the ac-field conveniently.

There are 15 kinds of two electron distribution in the three QD molecule which are shown in Table I, where \(A, B, C\) indicate the site of the three QDs. We replace the states \(|5\rangle, |6\rangle, |9\rangle, |10\rangle, |13\rangle, |14\rangle\) by \(|5'\rangle = \frac{1}{\sqrt{2}}(|5\rangle + |6\rangle), |6'\rangle = \frac{1}{\sqrt{2}}(|5\rangle - |6\rangle), |9'\rangle = \frac{1}{\sqrt{2}}(|9\rangle + |10\rangle), |10'\rangle = \frac{1}{\sqrt{2}}(|9\rangle - |10\rangle), |13'\rangle = \frac{1}{\sqrt{2}}(|13\rangle + |14\rangle), |14'\rangle = \frac{1}{\sqrt{2}}(|13\rangle - |14\rangle)\), and take them as basis vectors. A major advantage of these is that they manifest the two-electron spin and space distribution clearly. We can write the Hamiltonian(Eq.1) in the space spanned by the basis vectors. Therefore the Hamiltonian(Eq.1) can be described by \(15 \times 15\) matrix. The Hamiltonian(Eq.1) contains no spin-flip terms since measurements on semiconductor QD show that the spin-flip relaxation time is typically extremely long, and so the singlet state and triplet state sub-space are completely decoupled. Thus we can investigate the dynamical behaviors of two electrons in two sub-space respectively. Therefore, if the initial state posses a definite party this will be retained throughout its time evolution, and we only need to include states of the same party in the basis.

The Hamiltonian(Eq.1) also contains no spin-orbital coupled terms, and so the wave function can be written as the direct product of spin and orbital wave function. A many-
particle basis can then be constructed by taking Slater determinants of single particle states defined on the QD. The basis vectors (|4⟩, |12⟩, |8⟩) constitute the up-spin sub-space, and the basis vectors (|7⟩, |15⟩, |11⟩) constitute the down-spin sub-space. Their spin wave function are symmetric under particle exchange. The orbital wave function of basis vector (|6′⟩, |14′⟩, |10′⟩) are antisymmetric under particle exchange, so their spin wave function are symmetric. Therefore, the basis vectors (|4⟩, |12⟩, |8⟩, |7⟩, |15⟩, |11⟩, |6′⟩, |14′⟩, |10′⟩) constitute 9-dimension spin-triplet state sub-space. The calculation also shows that the three-dimensional spin symmetric sub-space are completely decoupled, and their Hamiltonian spanned by three spin-symmetric basis vectors hold the same form which is

\[
H(t) = \begin{pmatrix}
U_2 - V \cos(\omega t) & W & 0 \\
W & 0 & W \\
0 & W & U_2 + V \cos(\omega t)
\end{pmatrix}.
\] (2)

So the dynamical behaviors of the two electrons are identical in the three-dimensional sub-space. For simple, we only investigate the dynamical characters of two electrons in up-spin sub-space (|4⟩, |12⟩, |8⟩).

In order to obtain the dynamical characters of the three-level system, it is necessary to examine the system of two electrons confined by two QD molecule driven by ac-field, which has been fully investigated in Ref.4,5,6,9. In this system, the basis vectors (|I, I⟩, (|I, 1⟩ - |1, I⟩)/√2, |1, 1⟩) constitute the triplet state sub-space, and the basis vectors (|2, 0⟩, (|I, 1⟩ + |1, I⟩)/√2, |0, 2⟩) constitute the singlet state sub-space. The two sub-space are also decoupled. In the triplet state sub-space, the two-particle basis vectors (|I, I⟩, (|I, 1⟩ - |1, I⟩)/√2, |1, 1⟩) are the eigenvectors of its Hamiltonian and constitute the trivial triplet state sub-space in which the electron number on each QD is invariably one, and the time-dependent term does not influence this characteristics. In the singlet state sub-space, the Hamiltonian is similar to that of Formula (Eq.2). The relationships of the parameters and the basis vectors between the two system are: \( U_2 \rightarrow U_1 - U_2, \ W \rightarrow \sqrt{2}W, \ |4⟩ \rightarrow |2⟩, \ |8⟩ \rightarrow |0⟩, \ |12⟩ \rightarrow (|I, 1⟩ + |1, I⟩)/\sqrt{2}. \)
III. RESULTS

A. The two-electron dynamical properties in the triplet state subspace

We now investigate the dynamical properties of the three-dimensional triplet subspace and take the subspace \((|4\rangle, |12\rangle, |8\rangle)\) for example. The time periodicity of Hamiltonian (Eq. 2) enables us to describe the dynamics within the Floquet formalism. In addition, since the Hamiltonian is invariant under the combined dynamic parity operation \(z \rightarrow -z; t \rightarrow t + \pi/\omega\), each Floquet state is either odd or even. Quasi-energies of different parity may cross, otherwise an avoided crossing may occur. We calculate the quasi-energies and \(P_{\text{min}}\) as function of \(V\) with the parameters \(U_2 = n\omega\). We used two methods to obtain quasi-energies: (1) the numerical method which is to diagonalize the unitary time-evolution operator for one period of the driving field \(U(t+T, t)\). It may be easily shown that the eigenvectors of this operator are equal to the Floquet states, and its eigenvalues are related to the quasi-energies via \(\lambda_j = \exp(-i\varepsilon_j T)\). This method is particularly well-suited to our approach, as \(U(T, 0)\) can be obtained by integrating the unit matrix in time over one period of the field using the Runge-Kutta method. (2) the perturbation approach\(^6,7\). In this method, the quasi-energies are obtained by first solving the Floquet equation in the absence of tunnelling terms, and then performing perturbation theory with the tunnelling terms as the "perturbation". The perturbation solution are: \(\varepsilon_{1,2} = \pm \sqrt{2}J_n(V/\omega)W, \varepsilon_3 = 0\), where, \(J_n\) is the bessel function of the \(n\) order. We now define the probability \(P_{\text{min}}\). In this singlet state sub-space, the wave function can be written as a superposition of basis vectors: \(|\psi(t)\rangle = \sum_\alpha C_\alpha(t)|\alpha\rangle\), \(|\alpha\rangle\) is the basis vector and \(\alpha = 4, 12, 8\). Substituting this expansion into the time-dependent Schrödinger equation yields a first-order differential equation for the expansion coefficients \(C_\alpha\) (\(\hbar = 1\)):

\[
\begin{bmatrix}
C_4 \\
C_{12} \\
C_4 \\
C_4
\end{bmatrix}
\frac{\partial}{\partial t}
= H(t)
\begin{bmatrix}
C_4 \\
C_{12} \\
C_4 \\
C_4
\end{bmatrix}
\] (3)

When the initial condition \(C_\alpha(0)\) are given, we use the Runge-kutta method to solve the differential equation (Eq. 3) and obtain the expansion coefficients \(C_\alpha(t)\). When \(C_4(0) = 1, C_{12}(0) = 0, C_8(0) = 0\), We term the minimum value of \(|C_4(t)|^2\) of evolving 25 driving period \(LP_{\text{min}}\). When \(C_4(0) = 0, C_{12}(0) = 1, C_8(0) = 0\), We term the minimum value of \(|C_{12}(t)|^2\)
of evolving 25 driving period $CP_{\text{min}}$. $LP_{\text{min}}$ and $CP_{\text{min}}$ are used to quantify the degree of quantum tunnelling. If $LP_{\text{min}} = 1$ and $CP_{\text{min}} = 1$, it means that the two electrons can maintain its initial localization state in a short time span, and the tunnelling between different electron distribution is suppressed completely. If $P_{\text{min}} = 0$ (or $CP_{\text{min}} = 0$), it means that the two electrons can not maintain its initial state.

We calculate the quasi-energies $\varepsilon$ and the quantity $LP_{\text{min}}$ and $CP_{\text{min}}$ in Fig.1 as a function of $V$ with the parameters $U_2 = 8$, $W = 1$, $\omega = 1$. The calculation show that the excellent agreement between the quasi-energies exact solution and perturbation solution for strong and moderate fields. For weak fields, however, the driving terms do not dominate the tunnelling terms and the perturbation theory breaks down. Figure 1 also shows that the dynamical properties can be divided into two regime: the strong field regime and the weak field regime. In the strong field regime, the peaks in $LP_{\text{min}}$ and $CP_{\text{min}}$ is found by locating the roots of $J_8(V/\omega)$. So the initial states $|4\rangle$, $|12\rangle$ and $|8\rangle$ can be maintained. Hence the tunnelling between different electric distributions is depressed. In the weak field, $CP_{\text{min}}$ decays smoothly to zero, and a series of peak of $LP_{\text{min}}$ is found at the quasi-energy crossing. So the initial state $|12\rangle$ fail to hold on when increasing the driving ac field. However the initial state $|4\rangle$ and $|8\rangle$ keep its initial state even though the strong interaction between them.

To interpret the dynamical behaviors for the strong field regime, we seek the perturbation approach. We treat the tunnelling terms in Hamiltonian (Eq.2) as a perturbation. Solving the Floquet equation, the degeneracy Floquet states of the zeroth-order approximation are given by ($U_2 = n\omega$):

\[
\begin{align*}
|u_1(t)\rangle &= |\exp[-iU_2t + i\frac{V}{\omega}\sin\omega t], 0, 0\rangle \\
|u_2(t)\rangle &= |0, 1, 0\rangle \\
|u_3(t)\rangle &= |0, 0, \exp[-iU_2t + i\frac{V}{\omega}\sin\omega t]\rangle
\end{align*}
\]

With the degeneracy Floquet states, we obtained the perturbing operator $P_{ij} = \langle\langle u_i(t)|H_t|u_j(t)\rangle\rangle$, where $\langle\langle \cdot \cdot \cdot \rangle\rangle$ denotes the inner product in the extended Hilbert space. By using the identity: $\exp[-i\beta\sin\omega t] = \sum_{m=-\infty}^{\infty} J_m(\beta) \exp[-im\omega t]$ to rewrite the form of $|u_i(t)\rangle$, the matrix element of $P$ can be found to be $P_{12} = P_{23} = W J_n(\frac{V}{\omega})$. When the ratio of the field strength to the frequency is a root of Bessel function $J_n$, the transition between two Floquet states $|4\rangle$ and $|12\rangle$ as well as $|12\rangle$ and $|8\rangle$ is forbidden. Therefore, the tunnelling be-
tween different electrical distribution is depressed intensively, and the phenomenon of CDT occurs.

Let us analyze the dynamical properties in the weak field regime. In order to show it clearly, we show in Fig.2(a) the magnified view of Fig.1(a) for the weak field regime. The Fig.2(a) shows that the dynamical localization occurs at the crossing of quasi-energies $\varepsilon_2$ and $\varepsilon_3$. In the absent of an external driving field, the eigenvalues and eigenvectors are

$$
\begin{align*}
|\varphi_1\rangle &= |1, -a/(\sqrt{2}W), 1\rangle, & E_1 = b &= (-\sqrt{U_2^2 + 16W^2 + U_2})/2 \\
|\varphi_2\rangle &= |-1, 0, 1\rangle, & E_2 &= U_2 \\
|\varphi_3\rangle &= |1, -b/(\sqrt{2}W), 1\rangle, & E_3 &= a = (\sqrt{U_2^2 + 16W^2 + U_2})/2
\end{align*}
$$

(5)

When driving by weak field, the two-electron Floquet states are approximated by their eigen-states. We note $U_2 \gg W$, and the eigen-state $|\varphi_3\rangle$ has small component of two-electron state $|12\rangle$, so the eigen-state $|\varphi_2\rangle$ and $|\varphi_3\rangle$ are similar to the excited state (asymmetry) and the ground state (symmetry) of a electron in two QD. Thus it expects the initial two-electron state $|4\rangle$ and $|8\rangle$ remains forever at the crossing of quasi-energies $\varepsilon_2$ and $\varepsilon_3$. The rate $V/\omega$ at the first crossing is about 1.2, a root of the zero-order Bessel function, suggesting that this kind of dynamical properties can be approximates by a two-level model. When increasing the interaction $U_2$, the component of state $|12\rangle$ in $|\varphi_3\rangle$ will decrease. So the Coulomb repulsion may help to maintain the initial states $|4\rangle$ and $|8\rangle$. We present the calculation in Fig.2(b) with $U_2 = 16$, and the other parameters is the same as that of Fig.2(a). The calculation in Fig.2(b) shows that a series peaks of $LP_{min}$ emerge, and its height increase obviously with the increasing of $U_2$.

**B. The two-electron dynamical properties in the singlet state subspace**

We now investigate the dynamical behaviors for the singlet state sub-space. The orbital wave function of the basis vectors ($|1\rangle$, $|2\rangle$, $|3\rangle$) are symmetric under the electron exchange, so their spin wave function are antisymmetric. The spin wave function of the basis vectors ($|9'\rangle$, $|5'\rangle$, $|13'\rangle$) are antisymmetric under the electron exchange, so their orbital wave function are symmetric. Therefore, the basis vectors ($|9'\rangle$, $|2\rangle$, $|5'\rangle$, $|3\rangle$, $|13'\rangle$, $|1\rangle$) constitute the singlet sub-space. The Hamiltonian spanned by the six spin-antisymmetric basis vectors can be written as $6 \times 6$ matrix, which is
Due to the diversity of the size and couple between the QDs, the dynamical behaviors of the system show multiplicity. We begin our investigation by first considering the simplest case, that of the Hubbard $U_1$-term to be infinitely large - that is, we work in the sub-space of states with no double occupation. Our Hilbert space is thus three-dimensional spanned by the basis vectors $(|5\rangle, |13\rangle, |9\rangle)$. Therefore, the Hamiltonian can be written as:

$$H(t) = \begin{pmatrix}
U_2 + V \cos(\omega t) & \sqrt{2}W & 0 & \sqrt{2}W & W & 0 \\
\sqrt{2}W & U_1 & \sqrt{2}W & 0 & 0 & 0 \\
0 & \sqrt{2}W & U_2 + V \cos(\omega t) & 0 & W & \sqrt{2}W \\
\sqrt{2}W & 0 & 0 & U_1 + 2V \cos(\omega t) & 0 & 0 \\
W & 0 & W & 0 & 0 & 0 \\
0 & 0 & \sqrt{2}W & 0 & 0 & U_1 - 2V \cos(\omega t)
\end{pmatrix}.$$ (6)

This form of Hamiltonian(Eq.6) is the same as that of Hamiltonian(Eq.2). Thus the six-level system can be simplified to the three-level system, and the dynamical characters in this special case is also similar to two electrons in two QD molecule.

We now take the most general case, and consider the $U_1$ and $U_2$ to be a finite value which means that the three double-occupation are no longer energetically exclude from the dynamics, and accordingly we must take the full six-dimensional basis set. we calculate the quasi-energies and $P_{\text{min}}$ as function of $V$ with the parameters $U_1 = n_1\omega$ and $U_1 = n_2\omega$. We used two method to obtain quasi-energies: (1) the numerical method; (2) the perturbation approach, which are similar to the calculation of Figure 1. The perturbation solution of quasi-energies are

$$\begin{align*}
\varepsilon_{1,2} &= \pm \sqrt{2}J_{n_1-n_2}W \\
\varepsilon_{3,4} &= \pm \sqrt{2}J_{n_2}^2 + 6J_{n_1-n_2}^2W \\
\varepsilon_5 &= 0
\end{align*}$$ (8)

where, $J_{n_1-n_2}$ and $J_{n_2}$ are the bessel function of the $n_1 - n_2$ and $n_2$ order respectively.
We now define the probability $P_{\text{min}}$ in the singlet sub-space. The wave function can be written as a superposition of basis vectors: $|\psi(t)\rangle = \sum_{\alpha} C_{\alpha}(t)|\alpha\rangle$, $|\alpha\rangle$ is the basis vector and $\alpha = 9', \ldots, 1$. Substituting this expansion into the time-dependent Schrödinger equation yields a first-order differential equation for the expansion coefficients $C_{\alpha} (\hbar = 1)$:

\[
\frac{i}{\hbar} \frac{\partial}{\partial t} \begin{pmatrix} C_{9'} \\ \vdots \\ C_{1} \end{pmatrix} = H(t) \begin{pmatrix} C_{9'} \\ \vdots \\ C_{1} \end{pmatrix}.
\]

When the initial condition $C_{\alpha}(0)$ are given, we solve the differential equation (Eq. 9) by using the Runge-kutta method to obtain the expansion coefficients $C_{\alpha}(t)$. We term the minimum value of $|C_{\alpha}(t)|^2$ of evolving 25 driving period $P_{\text{min}}$, and use it to quantify the degree to which the ac-field bring about localization. In special, we term the minimum value of $|C_{1}(t)|^2$ of evolving 25 driving period $AP_{\text{min}}$ when $C_{1}(0) = 1, C_{\alpha}(0) = 0 \alpha \neq 1$. We also term the minimum value of $|C_{2}(t)|^2$ of evolving 25 driving period $BP_{\text{min}}$ with the initial condition $C_{2}(0) = 1, C_{\alpha}(0) = 0 \alpha \neq 2$. If $AP_{\text{min}} = 1$, it means that the two electrons can maintain its initial localization state (the left QD A) in a short time span, and the tunnelling between different electron distribution is suppressed completely, so the phenomenon occurs. If $AP_{\text{min}} = 0$, it means that the two electrons can not maintain its initial localization state (the left QD).

In order to show the dynamical properties in this general case, it is necessary to show the case of non-interacting and double-occupancy permitted firstly, and then investigate how the Coulomb interaction affects its dynamical behaviors. The three sites model in this non-interaction case is similar to that non-interaction electrons driven by ac-field in a superlattice, with the quasi-energy crossing corresponding to ”miniband collapse”. We term the phenomenon as ”quasi-energy collapse”. The electron tunnelling between QD is considerably suppressed with parameters at the roots of $J_0(V/\omega)$ ($J_0$ is the Bessel function of first kind). The calculation in Figure 3(a) verifies our assertion. We show in Figure 3(a) the quasi-energy spectrum and $AP_{\text{min}}$ obtained by sweeping over $V$. The dot and slimline present the exact solution and the perturbation solution of quasi-energies respectively, and the thick line presents $AP_{\text{min}}$. Figure 3(a) demonstrates the excellent agreement between
the exact solution and perturbation solution of the system quasi-energies for strong and moderate fields. For weak fields, however, the driving terms do not dominate the tunnelling terms and the perturbation theory breaks down. Figure 3(a) also shows that the peaks in $AP_{\min}$ by locating at the point of quasi-energies crossing. Hence the phenomenon of CDT occur at the point of quasi-crossing.

When the electron interactions involving, the difference from the non-interaction case arise. Using the same method as that of Figure 3(a), we calculate in Figure 3(b) the quasi-energy and $AP_{\min}$ as a function of $V$ with $U_1 = 12$ and $U_2 = 4$. It shows that the phenomena which is similar to ”miniband collapse” occurs in the strong-field regime of the quasi-energy, even though not all the quasi-energy levels cross at one point. At the weak-field regime, the feature of quasi-energy (dotted line) is different obviously from that of the non-interaction case, and a series of quasi-energy crossing appears. The calculation in Figure 3(b) also shows that $AP_{\min} - V$ can be divided apparently into two regime: the weak-field regime and the strong-field regime. At the strong-field regime, the character of $AP_{\min} - V$ is similar to that of the non-interaction case (Fig.3(a)). At the weak field regime, it is interesting to see that some sharp peaks of $P_{\min}$ occur at the quasi-energy crossing. The results indicate that two electrons localized in left QD initially can maintain its localization state (a 25 driving period) when driving by a weak-field, even though there is a strong Coulomb interaction. Therefore, the Coulomb interactions have little effect on strong-field regime, and they enhance the phenomena of CDT in the weak-field regime. This quantum phenomena can also occur in two electrons in two QD molecule, and have been found and analyzed in Ref 9.

We now analyze the system dynamical properties when driving by weak field. In order to show it clearly, we give in Fig 3(c) the magnified view of Fig 3(b) for the weak field regime. Fig 3(b) demonstrates that the position of the peak in $LP_{\min}$ to be found by locating at the crossing of quasi-energies $\varepsilon_1$ and $\varepsilon_2$. When driving by weak field, the Floquet states can be approximated by the corresponding unperturbed eigen-states. We diagonalize the Hamiltonian (Eq 6) when $V = 0$, and obtain $\varphi_1 = (-0.5774, 0.5774, -0.5774, 0, 0, 0)$ and $\varphi_2 = (-0.6969, 0, 0.6969, 0.1196, -0.1196, 0) \approx (-0.6969, 0, 0.6969, 0, 0, 0)$. The two eigenstates are similar to the symmetry (ground state) and asymmetry (the first excited state) states of a single electron in three QD. This is also like the case in the driven two-level model. Moreover, the value of the parameters $2V/\omega = 2.4$, which is the root of the zero-order Bessel function, suggesting that in this situation can be approximated by the driven
two-level model.

We apply the perturbation approach\cite{3,6} to analyze the dynamical property for the strong field regime. When driven by strong field, the Hamiltonian (Eq.6) can be divided into two parts: $H_t$ which contain all the tunnelling terms, and $H_I$ containing all interaction terms (these involving $U_1$ and $U_2$ and the electric field). We then find the eigen-system of the operator $H_I(t) = H_I - i(\partial/\partial t)$, and employ the tunnelling Hamiltonian as the perturbation. When $U_1 = n_1 \omega$ and $U_2 = n_2 \omega$, the eigenvalue of $H_I$ are 0. These represent the zeroth-order approximations to the quasi-energies in the perturbational expansion. The perturbation Hamiltonian $H_t$ induces the Floquet state transition. When the initial localization is $|1\rangle$, the nonzero transition matrix element is $P_{1,5'} = \langle 1 | H_t | 5' \rangle = \sqrt{2W} J_{n_1-n_2}(V/\omega)$. So the amplitude to frequency of driving field is the roots of Bessel function of $n_1 - n_2$ order, the transition of initial localization state $|1\rangle$ is strongly depressed. By using progressive formula of Bessel function: $J_m(x) \approx \sqrt{\frac{2}{\pi x}} \cos(x - \frac{\pi}{2} m - \frac{\pi}{4})$, we find the roots of $J_{n_1-n_2}$ are also the roots of $\sqrt{2J_{n_2}^2 + 6J_{n_1-n_2}^2}$ when $n_1 - n_2$ is even. Therefore, at the point of quasi-energies collapse, the two-electron dynamical localization is set up.

We show above the phenomena of dynamical localization occurs at the quasi-energy collapse when the two electrons are localized in left QD. This quantum phenomena can also happen when the two electrons are localized initially in centra QD. We verify this assertion by calculating the quasi-energy and $BP_{\text{min}}$ in Figure 3(d) with the parameters $U_1 = 20$, $U_2 = 4$, $W = 1$, $\omega = 2$, the initial condition $C_2(0) = 1$, $C_{\alpha}(0) = 0$ when $\alpha \neq 2$. It shows that the features of the Figure 3(d) are similar to that of the Figure 3(b) for the strong field regime. The calculation indicates that the two electrons of initial localized in the centra QD can also maintain its localization at the quasi-energy collapse. For the weak field regime, the effects of Coulomb interaction emerge obviously. With the increase of $U_2$, a series of quasi-energy crossing, and the dynamical localization is set up at the quasi-energy crossing.

IV. SUMMARY

In summary, applying three-site Hubbard model and Floquet theorem, we have investigated the dynamic behaviors of two electrons confined in a line-shape three quantum dot molecule driven by an ac-field. We find the triplet state and triplet state sub-space are decoupled fully, and can be investigated respectively. In special, the triplet state subspace
can be divided into 3 three-dimensional triplet state sub-space which are also decoupled. The analysis indicates the dynamical properties in the 3 three-dimensional triplet state sub-space, as well as the singlet state sub-space of no double-occupancy, are similar to that driven two electrons in two QD molecule. The calculations and analysis indicate that the dynamical properties in every subspace can be divided into two regime: In the strong-field regime, the dynamical behaviors are similar to that of non-interacting electrons in superlattice, and in the weak-field regime, the Coulomb interaction enhance the localization effect. We believe the method and the results presented here are useful to explore the effects of ac-field to control of multi-electron and multi-QD system.

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TABLE I: The two electron distribution in three QD molecule

|   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|
| A | ↑↓| ↑ | ↑ | ↓ | ↓ |   |   |   |   | ↑↑ | ↑↑ | ↑↑ | ↑↑ | ↓↓ | ↓↓ |
| B | ↑↓| ↑ | ↓ | ↑ | ↓ | ↑ | ↓ | ↑ | ↓ | ↑↑ | ↑↑ | ↑↑ | ↓↓ | ↓↓ |   |
| C | ↑↓|   |   |   |   | ↑ | ↓ | ↑ | ↓ | ↑↑ | ↑↑ | ↑↑ | ↓↓ | ↓↓ |   |

FIG. 1: The dependence of quasi-energies and (a) $LP_{min}$, (b) $CP_{min}$ as function of $V$, with the parameters: $U_2 = 8$, $W = 1$, $\omega = 1$. dotted line = quasi-energy exact results, dotted line = quasi-energy exact results, Solid line = perturbation theory.
FIG. 2: (a) magnified view of the weak field regime in Fig. 1(a), (b) the quasi-energies and $L_{P_{\text{min}}}$ as function of $V$, with the parameters: $U_2 = 16$, $W = 1$, $\omega = 1$. Dotted line = quasi-energy exact results, dotted line = quasi-energy exact results, Solid line = perturbation theory.
FIG. 3: The dependence of $AP_{\text{min}}$ as function of $V$ with the parameters: (a) $U_1 = 0, U_2 = 0, W = 1, \omega = 2$; (b) $U_1 = 12, U_2 = 4, W = 1, \omega = 2$. (c) magnified view of Fig. 3(b) for the weak field regime. (d) The dependence of $BP_{\text{min}}$ as function of $V$ with the parameters $U_1 = 20, U_2 = 4, W = 1, \omega = 2$. 