Teachers’ involvement in children’s mathematizing – beyond dichotomization between play and teaching

Camilla Björklund, Maria Magnusson and Hanna Palmér

ABSTRACT
The focus of this article is on mathematics teaching in a play-based and goal-oriented practice, such as preschool, and on how different lines of actions may impact children’s learning opportunities. Video recordings of authentic play activities involving children and nine teachers from different preschools were analyzed qualitatively to answer the following research questions: (1) What lines of action do teachers use when they teach mathematics in play? and (2) What implications may different ways of teaching have for children’s learning opportunities? The analysis revealed four different categories: confirming direction of interest; providing strategies; situating known concepts; and challenging concept meaning. As these differ regarding both the mathematics content focused on and the kind of knowledge emphasized, they have implications for children’s learning opportunities.

KEYWORDS
Developmental pedagogy; early childhood education; mathematics; play; teaching

Introduction
There is a growing consensus in policy and research that early mathematics is important and bears relevance for children’s development in the short and long term. Young children can possess deep and rich mathematical competencies (English and Mulligan 2013; Newton and Alexander 2013), and several studies show that early mathematical competencies have positive effects on later school performance (Duncan et al. 2007; Perry and Dockett 2008; Ginsburg 2009).

Despite the solid view that early mathematics is important, there is no agreement as to how preschool mathematics education should be conducted. Differences in opinion are visible both within and between countries, resulting in a plural view on preschool mathematics (Palmér and Björklund 2016). One of the most prominent differences regards the relation between play and teaching: is teaching to be integrated with or separated from children’s play? On the one hand, there are paradigms emphasizing children’s right to play, undisturbed by adults, for the sake of play itself (Sundsdal and Øksnes 2015); and on the other hand there is contemporary Nordic research developed within theoretical
frameworks that emphasizes a consolidation of the two (Pramling and Pramling Samuelsson 2011; Pramling, Doverborg, and Pramling Samuelsson 2017). The former paradigms often have philosophical underpinnings, as opposed to the latter paradigms’ embracing preschool as part of the education system. We find such dichotomies (between play and teaching) unfruitful and contradictory to the fact that many countries around the world include preschool children in the education system and that both teaching and play are central features of this practice.

Sweden is one example whereby early childhood education is available to all children aged one to six years, with a national curriculum that clearly states that the preschool practice is commissioned to ensure that children develop their competencies to their full potential. However, experiences from the Swedish context show that early childhood education is a delicate issue that needs further study (Swedish Schools Inspectorate 2016). As part of the education system, teaching is to be conducted in the Swedish preschool (Education Act 2010:800). At the same time, ‘a conscious use of play’ (6) is emphasized as important in relation to children’s learning (National Agency of Education 2011). What is left for the teachers and researchers within this field to determine is how to teach in a play-based and goal-oriented practice. We address this question in this article by studying how teaching (mathematics) in a play-based and goal-oriented practice can be conducted, and how different lines of action may impact children’s learning opportunities.

The empirical material used is part of a larger research project aiming to investigate the teaching–play relation in Swedish preschool practice. Mathematics as a content of learning is of special interest in this context, since while teachers in general claim that they teach mathematics in preschool (Björklund and Barendregt 2016), an evaluation by the Swedish Schools Inspectorate (2017) shows that mathematics teaching needs to be further developed in a majority of Swedish preschools.

The specific research questions focused on in the article are: (1) What lines of action do teachers use when they teach mathematics in play? and (2) What implications may different ways of teaching have for children’s learning opportunities?

Research on mathematics teaching and play

The relation between teaching, mathematics and play can be seen as either ‘mathematics made playful’, such as games in which counting, sorting and different mathematical operations are prominent, or ‘mathematizing elements of play’ whereby the primary act is play and a teacher might try to introduce mathematical concepts or operations to the child’s play activities (van Oers 1996, 74). The notion of mathematizing is often used when the emphasis is on children (including preschool children) trying to understand different phenomena in their surrounding world and mathematics becomes a part of this exploration (Freudenthal 1968). According to this perspective, the mathematics teaching of young children should necessarily be based in the children’s own lived experiences (for example play) and involve extending these experiences through mathematical inquiry (Gravemeijer and Terwel 2000). Anghileri (2006, 49) suggests that the teaching process involves the teacher ‘initiating reflective shifts such that what is said and done in action subsequently becomes an explicit topic of discussion’. In other words, the actions that children and teacher are involved in become the topic that will be discussed from a mathematical point of view and situated in the activity. This can be related to the act of playing,
whereby participants often enter and exit the play context to negotiate the meaning and progress of the ongoing play.

Play activities, and particularly role play, may serve as teaching opportunities if the teacher participates and is able to make use of occurring mathematical phenomena (van Oers 1996). The role of the teacher is then to extend children’s encounters with mathematics, in addition to organizing a rich environment that offers opportunities to explore new as well as familiar notions (Wager and Parks 2016). In this approach, it is not enough that mathematical representations and notions are present in the play; the reflection on and enhancement of mathematical thinking in the children depends on the teacher’s ability to seize the moment. This comes down to the idea of how children learn; Wager and Parks (2016), as well as van Oers (1996), embrace the idea of children’s initiatives and explorations as essential, but emphasize that teachers need to bring in new content and perspectives that will extend the children’s experiences, including the children’s own play (e.g. Magnusson and Pramling 2017).

This means that teaching mathematics is not merely about promoting counting, adding, naming, or using measures; it is rather about expanding the play and helping the children to understand the surrounding world, thus to mathematize. A key, according to van Oers (1996), can be found in questions from the teacher that encourage the children to discern a problem that emerges in the play activity, helping them mathematize their play content, and furthermore to solve the problem through mathematical operations or representations. There are significant findings on the relation between children’s learning outcomes and the frequency and duration of play-based mathematics activities they are engaged in (Cohrsen, Tayler, and Cloney 2015), but less is known about the efficacy of different ways of teaching in relation to play.

**Theoretical framing**

In our study, we base our understanding of teaching on the framework of developmental pedagogy (Pramling and Pramling Samuelsson 2011; Pramling, Doverborg, and Pramling Samuelsson 2017). Similar to the previously described mathematizing (Gravemeijer and Terwel 2000), developmental pedagogy takes its point of departure from children’s own lived experiences, whereby teaching implies enabling a child to experience familiar phenomena in new ways or widen his/her experiences, resulting in an extended repertoire of ways to encounter the surrounding world. The teaching triad – consisting of the teacher (who facilitates this extension of experiences), the learner, and the content for learning – is relational. This means that there is a delicate balance between the learner’s concept knowledge and the (teacher’s) intended way of understanding a concept that relies on their coordinating their perspectives.

*Intersubjectivity*, in the sense of coordinating perspectives between teacher and learner, is considered necessary in both play and teaching. In play situations, the participants agree on the play rules and boundaries, whereby certain codes are known by the participants and promote the progression of the play. In teaching situations, teacher and learner have to establish a common temporary view, some kind of joint understanding of what they are talking about and that there might be different ways of seeing the object of learning. Thus, a situation in which intersubjectivity is temporarily sufficiently established is a pre-condition for both play and teaching.
In accordance with developmental pedagogy, teaching in early childhood education is understood as supporting children’s awareness and their making sense of the surrounding world, which include both getting acquainted with new concepts and experiencing and exploring familiar phenomena in new ways. Support for children’s awareness and understanding can be achieved by establishing sustained shared thinking. This pedagogical interaction is characterized by ‘two or more individuals who “work together” in an intellectual way to solve a problem, clarify a concept, evaluate activities, or extend a narrative’ (Siraj-Blatchford 2010, 157). In an interaction in which sustained shared thinking is established, both parties (teacher and child/ren) contribute to, develop, and extend their thinking. This kind of interaction has shown to have positive outcomes for children’s learning (Siraj-Blatchford 2010), and will work as one guiding principle in our interpretation of implications of teachers’ different lines of action.

Methodology and data sources

The study reported on in this article was conducted in collaboration with preschool teachers from different preschools as well as researchers from three universities in Sweden participating in a joint research project. The research relies on authentic documentation from the preschools where children and teachers are engaged in play in different ways and with different content. The legal guardians of the preschool children gave their written consent for their children to participate in video-recorded activities in the regular preschool practice. Information and written forms for participation were distributed by the preschool teachers. Children who were not allowed to be video recorded for the project’s purpose have not been observed by the research group or included in the analysis. The names of the participants are fictive in the publications, in order to ensure their integrity.

Data for the current analysis consist of 42 video documentations made by 9 participating preschool teachers. The length of these documentations varies from 3 minutes to approximately 20 minutes. The instructions given to the teachers were to document play activities in which they interact with the preschool children and take part in the children’s play to determine what learning objects were possible to develop knowledge about. In the project, these documentations were first analyzed at joint meetings between the researchers and preschool teachers. After these meetings, the observations were transcribed and analyzed in more detail by the researchers, using interaction analysis methods focusing on the dialogue between teacher and child/ren and their joint construction of the activity (Bryman 2008).

The guiding question in this analysis was how teaching is framed in the play activities and what became possible for the participating children to learn. In this article, we direct specific attention to play in which the children mathematize and the teachers act to support the children’s awareness and understanding, in accordance with developmental pedagogy principles. The results presented in the next section are based on 42 video documentations, of which 30 contained mathematical content. We analyzed the activities, focusing on the ways the teachers established sufficient intersubjectivity in order to find the different ways in which teachers participate and teach in play. We found four different lines of action used by the teachers. The unit of analysis is the dialogical interaction about mathematical content, rather than the teachers themselves. This means
that several actions were found within the same video recording, and the preschool teachers varied their lines of action on different occasions. Thus, the categorization should not be understood quantitatively; instead; the purpose has been to explore different ways of teaching mathematics in relation to play. Our analysis shows that no single line of action dominates over another in our sample; they occur fairly evenly in the play activities. In the next section, we will present the four lines of actions, illustrated with examples from two play activities with two different preschool teachers. Our choice to use these two activities derives from the finding that different lines of action are used within the same play by the same preschool teacher. Nevertheless, together these examples illustrate the four re-occurring lines of action found in the video documentations.

Results

Four different lines of action emerge in our analysis: confirming direction of interest; providing strategies; situating known concepts; and challenging concept meaning. In the following presentation, these are illustrated with examples from the analyzed observations. Our analysis reveals how the teacher and children initiate and respond to the mathematical content present in their dialogue, and which opportunities for mathematics learning emerge.

Confirming direction of interest

The first line of action – confirming direction of interest – implies that the teacher shows interest in what the children are engaged in by asking questions or repeating a child’s utterance. Confirming or repeating what a child says or does is a common way for teachers to interact with children. In this way, they express that they are aware of the child’s directed interest, and usually that they support the ongoing activity.

Below is an example of confirming direction of interest. In the example, we find three children (aged four and five) and one teacher sitting around a table. On the table is a large box of building material. The children are making Beyblades (spin toys connected to a popular game and TV series in which the players compete with their spinning Beyblades) with the material. At the start, one child is spinning one of his Beyblades. The other children and the teacher observe how the color of the Beyblade changes as it spins.

Excerpt 1: Spinning Beyblades

|   |   |
|---|---|
| 1. Teacher: It [referring to the color] changes a little bit, anyway. How did that happen? That it turned blue? |
| 2. Mark: I don’t know. But. Because I have magic fingers. |
| 3. Teacher: You have magic fingers. Yes, that’s an idea. |
| 4. Tom: Now I’m going to make a panda. |
| 5. Teacher: You’re going to make a panda. |

The question of why the Beyblade changes color as it spins can be regarded as possible content for learning within the frames of the ongoing play. An open question like this can become the basis for reasoning, as in using arguments to motivate one’s answer. Thus, with her question ‘How did that happen?’ the teacher introduces two possible learning objects, the phenomenon of the changed color and the competence of reasoning. Reasoning is
emphasized as an important mathematical competence (Kilpatrick, Swafford, and Findell 2001), even in early mathematics (National Agency for Education 2011) and is commonly found in similar kind of interactions in early childhood education. The child answers that the color changes because he has ‘magic fingers’ and thus produces a plausible explanation for the phenomenon. According to Lithner (2008), reasoning does not have to be grounded in formal logic and the arguments used can even be considered incorrect from an outsider’s perspective, as long as they make sense and are reasonable to the individual in the situation.

Questions like the one posed by the teacher in Excerpt 1 make it possible to explore reasoning, which can then gradually be developed into mathematical reasoning. The open character of the posed question promotes creative reasoning whereby the child has to develop new (to him) arguments as he does not know the formal answer to the question (Lithner 2008). However, for this to occur the child’s argument (magic fingers) would need to be challenged by the teacher, encouraging the child to develop his argument. However, the continuation of the dialogue does not focus on the change of color; instead, the teacher starts interacting with another child. In the continuation of the activity the children build and spin Beyblades, and the teacher confirms what the children are saying and asks new questions.

A common characteristic of this way of confirming children’s directed interest is the teacher repeating the children’s initiatives and thereby confirming the notions they use, whether it is a search for the smallest pig that has suddenly gone missing in a fairytale told with props, or the sorting of colors to make houses for play figures. The confirmation of notions used as proper for the current situation is one important aspect of mastering known mathematical notions. Nevertheless, in this strategy, these notions are not challenged or extended to support further concept development.

Providing strategies

The next line of action found is the teacher providing children with strategies. Many teachers seem to focus their pedagogical efforts on supporting children in mastering counting skills, which in our study seems to become a challenging skill to master for many of the participating children. When a teacher provides a strategy it is done by supporting the children in pointing while counting, directing attention to what has been counted or reciting the counting rhyme out loud along with the children. These are considered necessary strategies for solving a counting task, and direct the children’s actions towards accessible paths.

The example below of providing strategies is from the same play activity as the previous example. Later in the same building activity one of the children is counting her Beyblades. The girl counts out loud, touching one Beyblade at a time. However, as she counts she does not match her counting to each individual item, mixing which Beyblades she has and has not counted.

Excerpt 2: Counting slowly

10. Teacher: If you count slowly Sarah. Then, then. Do it from the beginning and then count, yes.
11. Sarah: One, two, three, four, five, six, seven. (She counts out loud as she moves one Beyblade at a time to the side on the table)
12. Teacher: How nice. Yes, it’s always good to count slowly.
The teacher has a clear goal for which skill she wants the girl to develop: counting. The coordination of number words and objects, whereby one and only one number word is assigned to each object, is one necessary how-to-count principle (Gelman and Gallistel 1978). At the same time, not being able to coordinate verbal counting words with actions such as pointing at objects is common among young children (Clements and Sarama 2009). Thus, counting slowly and moving one item at a time are reasonable strategies for supporting this child’s counting act. In this situation, the teacher makes visible and further develops a mathematical procedure occurring in the play. However, while this is being done, the activity moves out of the play frame towards an instructive frame whereby the teacher provides strategies to facilitate her counting. It is not obvious how counting slowly – or counting in the first place – is part of the building of or playing with the Beyblades. Other counting acts found in the analyzed observations also include the counting of objects as part of the play, for instance when the teacher and children go to Antarctica with their many animals and all of them have to be on board. These counting tasks are most often introduced by the teachers, but provide the children with opportunities to mathematize the content of their play.

**Situating known concepts**

A third line of teachers’ action in play appears when the children, or the teacher, use familiar concepts to act in the play, but a joint exploration of the meaning of the concept occurs when someone (the teacher or a child) takes initiatives to situate the concept within the play. The concept then adds meaning *to the play*, and the concept meaning is situated in the actual play.

In the example (Excerpt 3) of *situating known concepts*, we find two children and one teacher engaged in role play in a separate room at the preschool. The teacher plays the character of a younger sister. The two children, Lisa and Alice, act as older sisters. Together they are planning for the younger sister’s birthday party, and of particular importance is the concept of space, which is situated in the play frame.

Excerpt 3: Planning a birthday party

1. Teacher: I would like to invite Kalle and Lisa and Olle. (holds up one hand, extending one finger for each name she says) How many can I invite? Ten, twenty?
2. Lisa: As many as you’d like.
3. Teacher: Twenty, Thirty, ah! Where should we be then? In my room?
4. Lisa: At home.
5. Teacher: At home? How big is it then?
6. Lisa: Big!
7. Teacher: Ha!
8. Alice: This big! (Alice stretches her arms to each side)

Both counting and space are present in this example, but as the dialogue turns out, ‘space’ can be understood as the primary content. While acting in her role as little sister, the teacher makes space visible as a mathematical phenomenon. By the teacher asking ‘how big is it then’ (referring to the size of the imagined house), space is discussed in relation to how many to invite, which can be regarded as an example of functional measurement (Clements and Sarama 2009). However, since the notion of space is explored in relation to an imagined
home, the space is not physically present and cannot be measured in a normative way. This is an example of a teacher’s questions encouraging children to discern a problem emerging from the play activity. Her question invites the children to mathematize their play content, and furthermore to solve the problem through mathematical operations and representations (van Oers 1996). They share a common interest in figuring out how many guests to invite in relation to the available space, which is visualized through different expressions such as finger counting and stretched arms. The reasoning by Lisa and Alice is mathematical, referring to space in both wording (big) and gestures (arms).

As seen in the example above, the reasoning about known concepts is not necessarily creative but rather imitative (Lithner 2008), as the children use notions and procedures they are already familiar with and no new mathematics are explored in the situation. From this line of action, what the children are offered to learn is an extension of the known concept, which is of importance in preschool where many mathematical phenomena are necessarily situated in concrete situations (see Björklund 2018) and in meaningful use (such as play).

**Challenging concept meaning**

The fourth line of action we found is when teachers use play activities to become engaged in exploring concept meaning together with the children. What stands out in this way of teaching mathematics in play is that the teacher does not provide an answer, or strategies, but rather contrast meanings to what the children express. In this way, a concept emerging in the play is highlighted and the children are challenged and inspired to explain their view and elaborate their expressions of meaning, in order to establish a shared (and more advanced) understanding.

The example below of *challenging concept meaning* is from the same role play as the previous example (Excerpt 3). In this, the teacher and the two children are talking about their ages in the role play. Talking about the ages can be considered meta-communication about the play, but the teacher continues to act as the little sister while they are talking. (Important in relation to the dialogue below is that, in Sweden, the year before compulsory school begins is commonly called ‘the zero’. This refers to the grade before ‘first grade’/‘the one’, when children start compulsory school at the age of seven.)

**Excerpt 4: The meaning of zero**

44. Alice: Aa … I’m seven years and I’m in zero … in first grade.
45. Teacher: So you’re in first grade?
46. Alice: You’ll start in the zero.
47. Teacher: Zero! But that’s nothing! She says that I’ll start in the zero.
48. Lisa: They’re next to us.
49. Alice: The zero in school?!
50. Teacher: Is there a zero in school?
51. Alice: Yes, there’s a zero.
52. Teacher: Aha.
53. Alice: Yes, that’s zero and I’m in the one.
54. Teacher: Yes, and you’re in the one.

Throughout the example, the teacher is acting in her role as little sister and the children in their roles as big sisters. In the dialogue, ‘zero’ becomes an object of exploration with the
teacher highlighting the possibility to interpret the expression in different ways. Numbers imply different things in different situations, and by saying ‘Zero! But that’s nothing!’ the teacher makes visible two dimensions of how the numeral zero can be used, as the identification of a grade in school and as a quantity. Preschool children who lack formal mathematics education often have a limited understanding of zero, since at this age counting is often connected to quantity and it seldom makes sense to count zero objects (Clements and Sarama 2009). In this example, the teacher and children are working together in an intellectual way to solve the problem of what zero implies, and the children contribute to the explanation by saying ‘The zero in school’ and ‘Yes, that’s zero and I’m in the one’. The teacher’s role is of certain importance, since the two dimensions of ‘zero’ would not have been challenged if the teacher had not brought in the contrasting meaning. This way of teaching concept meaning appears quite often in the role play in our study, with the teacher introducing new concepts or a different meaning of a concept in a natural way.

Conclusions

The starting point for this article was the ongoing debate regarding the relation between play and teaching, and the question of how to teach (mathematics) in a play-based and goal-oriented practice (Pramling and Pramling Samuelsson 2011; Sundsdal and Øksnes 2015; Pramling, Doverborg, and Pramling Samuelsson 2017). In the results, through the use of examples, we have illustrated four different lines of action in teaching mathematics in play: confirming direction of interest; providing strategies; situating known concepts; and challenging concept meaning. In the examples, the relation between mathematics and play is what van Oers (1996) refers to as mathematizing elements of play; implying that mathematical concepts or operations that can contribute to the play are explored and/or introduced. Our interest is directed towards the teachers’ involvement in the children’s mathematizing within the frames of play. Different ways to respond to children’s mathematizing initiatives constitute different learning opportunities, in which the teacher’s responsiveness to the children’s acts and understanding is a key feature. This includes both retaining intersubjectivity and handling the delicate balance between remaining within the play frame and extending the children’s experiences. In this section, we will conclude our results and note their similarities and differences.

Our analysis shows that the outcome of the teaching act turns out differently depending on the teachers’ responsiveness (how they interact/respond) to the children’s mathematizing. The four lines of action have in common that opportunities occur in which it is possible to challenge the conceptual meaning of mathematical notions and in different ways extend the children’s experiences. While some possible mathematical content for learning is thereby a necessary starting point, this possible mathematical content for learning can be dealt with in different ways. Different lines of action will open up for different opportunities for the children to deal with mathematical questions. To confirm direction of interest means that the teacher is responsive to the children’s ideas; however, this confirmative approach seldom seems to benefit an extension of knowledge or inquiry of the phenomenon that was initiated. Providing strategies, on the other hand, is a more goal-oriented act of directing the child towards skills or tools that will help him/her master a challenge. This strategy can be associated with
the notion of guided participation (Rogoff 1990), which includes interpersonal actions whereby the teacher and learner go side-by-side in a culturally organized activity. The key to the adult’s role here is being responsive to the child’s perspective, rather than directing it. Thus, mathematics teaching in line with this can be understood as the teacher providing the child with new strategies to facilitate the activity the child is involved in. **Situating known concepts** implies mathematical content being applied to a current play context. The content may come to be of importance for the progression of the play – and indeed, making connections between and generalizing the use of mathematical concepts is part of what mathematics is about (Anghileri 2006). Finally, **challenging concept meaning** includes sustained shared thinking, whereby the teacher and child/ren work together in an intellectual way to solve a problem or clarify a concept (Siraj-Blatchford 2010). The key in this line of action is the teacher introducing some new meaning or perspective that extends the child/ren’s experiences and brings forth a new way of understanding or adds value to the play.

Preschool mathematics is not (only) about pre-prepared activities but also entails teachers being involved in children’s mathematizing activities, for example in play (Siraj-Blatchford and Sylva 2004). Based on our study, we cannot see a dichotomy between play and teaching as outlined by, for example, Sundsdal and Øksnes (2015). Their interpretation, based on a narrow interpretation of teaching, results in a false dichotomy between play and teaching, which is not fruitful for our understanding of either play or teaching in preschool. Our results, presented in this article, show that the activity of play continues even when the teacher clearly teaches mathematical content. The teachers can offer extensions without destroying, disrupting or controlling the play. Instead, the teachers more or less extend the mathematical content within the play in different ways. The three lines of action **providing strategies**, **situating known concepts** and **challenging concept meaning** are in line with developmental pedagogy (Pramling Samuelsson and Asplund Carlsson 2008), in that the teacher supports the children’s awareness and exploration of meaning, either by exploring familiar phenomena in new ways or by helping them get acquainted with a new or contrasting meaning of a phenomenon. In the case of **challenging concept meaning**, a key feature of teaching in a play-oriented practice is highlighted: balancing between the challenging contrasts while still maintaining the intersubjectivity and play frame that have been established. Regardless of strategy, the teachers’ responsiveness to the children’s ideas is likely the most essential feature in the play and teaching activities (cf. Wager and Parks 2016); but in order for learning to occur (in the sense of extending one’s knowledge and understanding) there are also other features that seem necessary, such as a joint problem to solve or stated differences in the perception of a common phenomenon. In line with van Oers (1996), we also find that teachers’ questions form a key feature of preschool mathematics teaching, in that they frame the opportunities children are given to discern such problems that emerge in the play activity, helping them mathematize their play content.

Based on the results presented, we do not claim that mathematics teaching in preschool should occur only in play, but we do claim that the mathematizing of elements of play is both a possible and desirable part of mathematics teaching in preschool. These results are important as a contribution to the current debate about the policy and practice of teaching in early childhood education. A false dichotomy between play and teaching carries the risk that the focus of discussion will be on the wrong question, asking *if* instead of *how*.
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ORCID
Maria Magnusson http://orcid.org/0000-0002-5806-4475

References
Anghileri, J. 2006. “Scaffolding Practices that Enhance Mathematics Learning.” Journal of Mathematics Teacher Education 9 (1): 33–52. doi:10.1007/s10857-006-9005-9.

Björklund, C. 2018. “Learning About the Notion ‘Half’: Critical Aspects and Pedagogical Strategies in Preschool.” Scandinavian Journal of Educational Research 62 (2): 245–263. doi:10.1080/00313831.2016.1212264.

Björklund, C., and W. Barendregt. 2016. “Teachers’ Mathematical Awareness in Swedish Early Childhood Education.” Scandinavian Journal of Educational Research 60 (3): 359–377. doi:10.1080/00313831.2015.1066426.

Bryman, A. 2008. Social Research Methods. 3rd ed. Oxford: Oxford University Press.

Clements, D. H., and J. Sarama. 2009. Learning and Teaching Early Math. The Learning Trajectory Approach. New York: Routledge.

Cohrsen, C., C. Tayler, and D. Cloney. 2015. “Playing with Maths: Implications for Early Childhood Mathematics Teaching from an Implementation Study in Melbourne, Australia.” Education 43 (6): 641–652. doi:10.1080/03004279.2013.848916.

Duncan, G. J., C. J. Dowsett, A. Claessens, K. Magnuson, A. C. Huston, P. Klebanov, L. S. Pagani, et al. 2007. “School Readiness and Later Achievement.” Developmental Psychology 43 (6): 1428–1446.

English, L. D., and J. T. Muligian. 2013. “Perspectives on Reconceptualizing Early Mathematics Learning: Introduction.” In Perspectives on Reconceptualizing Early Mathematics Learning, edited by L. D. English and J. T. Mulligan, 1–4. Dordrecht: Springer.

Freudenthal, H. 1968. “Why to Teach Mathematics so as to Be Useful” Educational Studies in Mathematics 1 (1): 3–8.

Gelman, R., and C. R. Gallistel. 1978. The Child’s Understanding of Number. London: Harvard UP.

Ginsburg, H. P. 2009. “Early Mathematics Education and How to Do It.” In Handbook of Child Development and Early Education: Research to Practice, edited by O. A. Barbarin and B. H. Wasik, 403–428. New York: Guilford Press.

Gravemeijer, K., and J. Terwel. 2000. “Hans Freudenthal: A Mathematician on Didactics and Curriculum Theory.” Journal of Curriculum Studies 32 (6): 777–796.

Kilpatrick, J., J. Swafford, and B. Findell. 2001. Adding It Up: Helping Children Learn Mathematics. Washington, DC: National Academies Press.

Lithner, J. 2008. “A Research Framework for Creative and Imitative Reasoning.” Educational Studies in Mathematics 67 (3): 255–276.

Magnusson, M., and N. Pramling. 2017. “In ‘Numberland’: Play-Based Pedagogy in Response to Imaginative Numeracy.” International Journal of Early Years Education. doi:10.1080/09697602017.1368369.

National Agency for Education. 2011. Curriculum for the Preschool Lpfö98. Revised 2010. Stockholm: National Agency for Education.

Newton, K. J., and P. A. Alexander. 2013. “Early Mathematics Learning in Perspective: Eras and Forces of Change.” In Perspectives on Reconceptualizing Early Mathematics Learning, edited by L. D. English and J. T. Mulligan, 5–28. Dordrecht: Springer.
Palmér, H., and C. Björklund. 2016. “Different Perspectives on Possible – Desirable – Plausible Mathematics Learning in Preschool.” Nordic Studies in Mathematics Education 21 (4): 177–191.
Perry, B., and S. Dockett. 2008. “Young Children’s Access to Powerful Mathematical Ideas.” In Handbook of International Research in Mathematics Education, edited by L. D. English, 75–108. New York: Routledge.

Pramling, N., E. Doverborg, and I. Pramling Samuelsson. 2017. “Re-metaphorizing Teaching and Learning in Early Childhood Education Beyond the Instruction – Social Fostering Divide. International Perspectives on Early Childhood Education and Development.” In Nordic Social Pedagogical Approach to Early Years, edited by C. Ringsmose and G. Kragh-Müller, 205–218. Switzerland: Springer.
Pramling, N., and I. Pramling Samuelsson, eds. 2011. Educational Encounters: Nordic Studies in Early Childhood Didactics. Dordrecht: Springer.
Pramling Samuelsson, I., and M. Asplund Carlsson. 2008. “The Playing Learning Child: Towards a Pedagogy of Early Childhood.” Scandinavian Journal of Educational Research 52 (6): 623–641.

Rogoff, B. 1990. Apprenticeship in Thinking: Cognitive Development in Social Context. Oxford: Oxford University Press.
Siraj-Blatchford, I. 2010. “A Focus on Pedagogy: Case Studies of Effective Practice.” In Early Childhood Matters. Evidence from the Effective Pre-school and Primary Education Project, edited by K. Sylva, E. Melhuish, P. Sammons, I. Siraj-Blatchford, and B. Taggart, 149–165. London: Routledge.

Swedish Schools Inspectorate. 2016. Förskolans pedagogiska uppdrag [Preschool's Pedagogical Mission]. Stockholm: Swedish Schools Inspectorate.
Swedish Schools Inspectorate. 2017. Förskolans kvalitet och måluppfyllelse [Preschool Quality and Aims Fulfilled]. Stockholm: Swedish Schools Inspectorate.

van Oers, B. 1996. “Are You Sure? Stimulating Mathematical Thinking During Young Children’s Play.” European Early Childhood Education Research Journal 4 (1): 71–87. doi:10.1080/13502939685207851.

Wager, A., and A. N. Parks. 2016. “Assessing Early Number Learning in Play.” ZDM Mathematics Education 48 (7): 991–1002. doi:10.1007/s11858-016-0806-8.