A sub-target approach to the kinodynamic motion control of a wheeled mobile robot

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Abstract. A mobile robot with two independently driven wheels is popular, but it is difficult to stabilize it by a continuous controller with a constant gain, due to its nonholonomic property. It is guaranteed that a nonholonomic controlled object can always be converged to an arbitrary point using a switching control method or a quasi-continuous control method based on an invariant manifold in a chained form. From this, the authors already proposed a kinodynamic controller to converge the states of such a two-wheeled mobile robot to the arbitrary target position while avoiding obstacles, by combining the control based on the invariant manifold and the harmonic potential field (HPF). On the other hand, it was confirmed in the previous research that there is a case that the robot cannot avoid the obstacle because there is no enough space to converge the current state to the target state. In this paper, we propose a method that divides the final target position into some sub-target positions and moves the robot step by step, and it is confirmed by the simulation that the robot can converge to the target position while avoiding obstacles using the proposed method.

1. Introduction
In these days, many mobile robots with two independently driven wheels are developed for transportation or giving service. These robots need an autonomous locomotion, and there are many researches about an obstacle avoidance and a path planning for the two-wheeled mobile robot. For the smooth moving of the robot in an environment including obstacles, it is desirable to consider about the kinematic constraint and the dynamic constraints. Therefore, the method called "kinodynamic motion planning" is proposed[1]. The kinodynamic motion planning aims to design the control input so that the kinematic constraints generated from the environment and the dynamic constraints generated from the dynamic characteristics of the controlled object are both considered simultaneously. Masoud realized the kinodynamic motion planning for a point mass by combining two control inputs: one is generated by considering the dynamics of the controlled object and the other is based on the gradient information of the potential field called harmonic potential field (HPF) which is generated from the boundary information of the environment[2]. In this research, it is aimed at applying the kinodynamic motion planning to the two-wheeled mobile robot by extending the method proposed by Masoud.

The kinodynamic motion planning using the HPF is achieved by adding the gradient information of an HPF to the control input that has considered the dynamic characteristics of the controlled object[3]. To apply the kinodynamic motion planning based on an HPF, at first, it needs the control input that considers the dynamic model of the controlled object. Here, the stabilizing control to the
origin using an invariant manifold is given as one of the methods to control the two-wheeled mobile robot. By using the control based on the invariant manifold, the controller can reliably stabilize the controlled object to the origin kinematically and dynamically[4]. In the previous research, the authors confirmed that the proposed method was able to guide the robot to the target position while avoiding obstacles to a certain degree. However, it was also confirmed that there was the case that the robot was not able to avoid the obstacle because there was no enough space to converge the state to the target state. In this paper, we propose the method that dividing the target position into some pieces and moving step by step, and it is confirmed by the simulation that the robot can converge to the target position while avoiding obstacles using the proposed method.

2. A model of two-wheeled mobile robot

\[ \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} v \cos x \\ v \sin x \\ \omega \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/M \\ 0 \end{bmatrix} F + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tau \]  

Figure 1 shows the schematic structure of the two-wheeled drive mobile robot when assuming that the world coordinate system is the X-Y coordinate system. Note that, 2L, R, and \( \theta \) in Fig. 1 mean the tread, the radius of the wheel, and the direction angle from the X-axis respectively. The controlled point of the two-wheeled drive mobile robot, \( O(x, y) \), exists at the center of the tread. \( v \) means the forward velocity, and omega means the angular velocity around the point O. Hereafter, the authors call the two-wheeled drive mobile robot “the robot.” The dynamic model of the robot with two-inputs and five-states is shown as:

\[ \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} v \cos x \\ v \sin x \\ \omega \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/M \\ 0 \end{bmatrix} F + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tau \]  

Here, \( M \), \( I \), \( F \) and \( \tau \) mean the mass of the robot, the moment of inertia, the force and the torque, respectively.

3. Control using the invariant manifold

The invariant manifold means a manifold constructed by the scalar functions whose differential value is 0 with respect to time. There are two types of control using the invariant manifold. One is the
switching control that is able to divide the first-step (until on the invariant manifold) and the second step (on the invariant manifold), and the other is the quasi-continuous exponential stabilization control that excludes the switching control by combining the first and second steps. It is known that the robot can converge to the origin by using these control methods. Generally, a nonholonomic system includes several complex nonlinear terms, and it is difficult to design the controller from the original model. On the other hand, by using the canonical model, such as the chained form, it becomes easy to make a control system, which assures a globally asymptotic stability.

It needs to consider the error from the origin to an arbitrary point to converge the robot state value to the arbitrary point. The chained form transformation, which takes account of the error, is called an “error model.” The error model is shown as:

$$\dot{e} = \begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \\ \dot{e}_4 \\ \dot{e}_5 \end{bmatrix} = \begin{bmatrix} e_4 \\ e_5 \\ e_2e_4 \\ u_1 \\ u_2 \end{bmatrix}$$

(2)

Here, $u_1$ and $u_2$ denote the control inputs on the chained form.

$$u_1 = \frac{\tau}{I}$$

$$u_2 = -z_4^2z_2 - \frac{\tau}{I}z_3 + \frac{F}{M}$$

(3)

Then, the error state $[e_1 \, e_2 \, e_3 \, e_4 \, e_5]^T$ is represented by

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{bmatrix} = \begin{bmatrix} z_1 - z_{1d} \\ z_2 - z_{2d} \\ z_2 - z_{2d} - \frac{1}{2}e_2^2z_2d - z_{2d}e_1 \\ z_4 - z_{4d} \\ z_5 - z_{5d} \end{bmatrix}$$

(4)

Note that, the $[z_1 \, z_2 \, z_3 \, z_4 \, z_5]^T$ is

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \\ \dot{z}_5 \end{bmatrix} = \begin{bmatrix} \theta \\ x \cos \theta + y \sin \theta \\ x \sin \theta - y \cos \theta \\ \omega \\ \nu - (x \sin \theta - y \cos \theta)\omega \end{bmatrix}$$

(5)

4. The quasi-continuous exponential stabilization control

The quasi-continuous exponential stabilization control is the method that is realized in one unified manner, instead of conducting the control one by one in the first and second steps described previously. This control excludes the switching, and it can be controlled without a sudden change of the input. The control input of the quasi-continuous exponential stabilization control is given by

$$u_1 = -2ke_4 - k^2e_1$$

$$u_2 = \frac{fS(e)}{e_1} - k^2e_2 - 2ke_5$$

(6)

Here, the time response of $S(e)$ becomes
\[ S(t) = S(0)e^{-\frac{f}{4k}t} \] (7)

5. Combining of the control using invariant manifold with the HPF

The aim of this method is to achieve the kinodynamic motion planning for a two-wheeled drive mobile robot by combining the quasi-continuous exponential stabilization control with the HPF. Concretely, the kinodynamic motion planning is achieved by adding the directional force in gradient to the control inputs, \( u_1 \) and \( u_2 \) in the chained form. However, if the control input with the information from the HPF is used in the whole area of the environment, the differential value of the invariant manifold does not become 0, then it is considered that the state does not reach the target point. In other words, if always adding the directional force from the HPF to the controller, the stability of the manifold is broken. Therefore, in this chapter, basically the quasi-continuous exponential stabilization control is used, and added the information of the HPF to its control input locally. In this method, the convergence to the target position can be achieved finally, because only the quasi-continuous exponential stabilization control is used in the area without the obstacles and then the stability of the manifold is not broken. In what follows, it is described a force in the gradient of the HPF and how to design the input torque for the robot by introducing its information as a force, and state the control that combines the quasi-continuous exponential stabilization control with the HPF.

From the HPF, the x-directional and y-directional gradient vector of each position of the environment can be calculated. These gradient vectors show the direction that the robot should move. By using the x-directional gradient \( \Delta x \) and the y-directional gradient \( \Delta y \), the gradient direction \( \theta_k \), which is the direction that the robot should move, can be written as follows:

\[ \theta_k = \tan^{-1} \frac{\Delta y}{\Delta x} \] (8)

Assuming that the input torque with the gradient directional force is \( \tau' \), it can be written as the following equation:

\[ \tau' = \frac{L}{R}(\tau_1 - \tau_2) + k_1 \theta_k \] (9)

with the gain \( k_1 > 0 \). Therefore, the kinodynamic motion planning should be able to be achieved by adding the gradient directional force of the HPF \( \theta_k \) to \( u_1 \). Assuming the gain \( k_1 > 0 \), the control inputs of the quasi-continuous exponential stabilization control can be written by

\[
\begin{align*}
    u_1 &= \frac{-2ke_1 - ke_1 + k_1 \theta_k}{e_1} \\
    u_2 &= \frac{fS(e_1) - ke_2 - ke_5}{e_1}
\end{align*}
\] (10)

When guiding the robot from the initial position to the target position by the proposed controller, the robot sometimes ignores the obstacles because the controller prioritizes the convergence of the orientation. Therefore, we propose a method based on using the potential of the environment which
has been already calculated, to calculate the ideal trajectory in the assumed environment and approach to the target state step by step. Concretely, the ideal trajectory is divided into an arbitrary number, whose divided points are used as the temporary target positions, i.e., sub-target positions. The controlled object moves to the nearest sub-target position, and removes to the next sub-target position if the current state would be converged.

6. Simulations

In this section, some simulations are conducted to verify the effectiveness of the kinodynamic control for the two-wheeled mobile robot. For the simulations, the MATLAB and PDE Toolbox (one of the toolbox for MATLAB) is used as the calculation tool. The conditions and the results of simulations are described below.

6.1. Conditions

In this simulation, it is confirmed that the initial state of the robot $x_0 = [x_0 \ y_0 \ \theta_0 \ v_0 \ \omega_0]^T$ can be converges to the target value $x_d = [x_d \ y_d \ \theta_d \ v_d \ \omega_d]^T$ by using the quasi-continuous exponential stabilization control and the HPF in the environment shown in Fig. 2. The physical parameters of the robot are set as $M = 5$ [kg], $I = 1$ [kg/m^2], $R = 0.03$ [m], and $L = 0.06$ [m], whereas the sampling interval and the threshold value are set to $\Delta t = 0.01$ [s] and $\varepsilon = 0.01$, respectively. The feedback gains are set to $k = 1$ and $f = 9$, and Eq. (10) is used as the control input. In the assumed environment, there are one rectangle obstacle shown in Fig. 2. Here, initial state $x_0 = [45 \ 45 \ 1.5 \ 0 \ 0]^T$ and the target value $x_d = [5 \ 5 \ 0 \ 0 \ 0]^T$. Note from the Eq. (10) that, the gain $k_1$ is set to $1$. The red line in Fig. 3 shows the ideal trajectory which is calculated from the gradient of the HPF. The dividing number is
3, and the divided points are shown by blue crosses in Fig. 3.

![Figure 4. The trajectory with the conventional method](image1)
![Figure 5. The trajectory with the proposed method](image2)

![Figure 6. The trajectory with the conventional method](image3)

6.2. Results
Figures 4 and 5 show the trajectories of the robot in the X-Y plane with the conventional method and the proposed method, respectively. In Fig. 5, the red solid line shows the trajectory connected from the initial state to the point where the robot reaches to the first sub-target position. The yellow solid line is similarly the trajectory connected from the first sub-target position to the second sub-target position, and the blue line is one connected from the second sub-target position to the final target position. Figure 6 shows the time response of the states x, y, and $\theta$ in the robot when using the proposed method.
6.3. Discussions

As shown in Fig. 4, the robot was not able to avoid the obstacle in the assumed situation when using the conventional method. On the other hand, it is confirmed from Fig. 5 that the robot can be converged to the target state while avoiding the obstacle using the proposed method, which is of having introduced some sub-target positions generated by dividing the ideal trajectory. It is found in Fig. 5 that the robot takes also a switchback route to adjust its orientation. Moreover, as shown in Fig. 6, it is confirmed that the position states $x$ and $y$ in the robot are finally converged to the target state $(5, 5)$ [m]. Thus, the usability of the proposed method is confirmed in this simulation. However, the number of divisions for the ideal trajectory, i.e., the number of sub-targets is decided using the empirical rule by a human, so that it needs to formulate it using any rule.

7. Conclusions

In this paper, a control method using the quasi-continuous exponential stabilization control and the HPF has been proposed for the two-wheeled drive mobile robot. In the proposed method, the control input was designed to reach the target point while avoiding obstacles by combining the quasi-continuous exponential stabilization control with the invariant manifold and the directional force in gradient of the HPF. It was confirmed in the simulation that the robot was able to reach the target position by approaching step by step to the final target position.

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