On the human inability to process inverse variables in intuitive judgments: different cognitive processes leading to the time loss bias

Ola Svenson\textsuperscript{a,b} and Anna Borg\textsuperscript{b}

\textsuperscript{a}Decision Research, Eugene, OR, USA; \textsuperscript{b}Risk Analysis, Social and Decision Research Unit, Department of Psychology, Stockholm University, Stockholm, Sweden

\section*{ABSTRACT}

The time loss bias describes overestimation of time lost after speed decreases from high speeds and underestimations after decreases from low driving speeds. Participants judged the speed decrease from one speed (e.g. 130 km/h) that would give the same time loss as a decrease from another speed (e.g. from 40 to 30 km/h). We carried out descriptive spectral analyses of distributions of judgments for each problem. Each distribution peak was associated with a judgment rule. The first study found two different judgment processes both leading to the time loss bias: a Difference process rule used for 20\% and a Ratio rule used for 31\% of the judgments. The correct rule applied to 10\% of the judgments. The second study added verbal protocols. The results showed that the Ratio rule was most common (41\%) followed by the Difference (12\%) and correct (8\%) rules. Verbal reports supported these results.

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Spectral analysis; time judgments; time loss bias; inverse variables; response distribution analysis

\section*{Introduction}

The purpose of this study is twofold. First, we aim at describing the cognitive processes that produce the time loss bias (Svenson & Treurniet, 2017). Second, we will show the usefulness of analysis of judgment distributions for detecting and describing different cognitive processes that produce a human judgment. People often use simplifying cognitive processes and make bounded rationality or satisficing decisions rather than more elaborate decisions because of restrictions of cognitive processing capacity or ignorance (Kahneman et al., 1982; Shah & Oppenheimer, 2008; Simon, 1959, 2000). Cognitive heuristics or simplifying cognitive rules enable people to judge relationships which they understand very poorly or not at all, but think that they can judge. Even if they are often valuable (Gigerenzer & Todd, 1999a, 1999b), heuristics can also lead to systematic biases (Cohen et al., 1956; Gilovich et al., 2002; Johnson-Laird, 1999; Kahneman & Tversky, 1982; Montibeller & Winterfeldt, 2015; 1979; Wikipedia, 2019).

There is a large literature on heuristics and rules in decision making (Bettman et al., 1990; Brandstätter et al., 2006; Gigerenzer & Brighton, 2009; Gigerenzer, 2008; Gigerenzer & Gaissmaier, 2011; Gigerenzer & Todd, 1999a, 1999b; Marewski et al., 2018, 1993; Maule, 1989; Price et al., 2013; Rieskamp & Hoffrage, 2008; Svenson, 1979, 1998). Stanovich and West (2000), summarised a number of studies and suggested that biases in human cognition can depend on (1) performance errors, (2) computational limitations, (3) the wrong norm being applied by the experimenter and (4) a different interpretation of the task by the participant than intended by the researcher. Computational limitations were evident in studies of, e.g. the time saving bias (Svenson et al., 2018) and different interpretations of a task by participants was found in another study of time savings by Svenson et al. (2018). There is a more limited literature on rules in judgments than rules in choice, but in connection with computational limitations it may be worth to mention that mental arithmetics has attracted much research with theoretically interesting and practically applicable results (Ashcraft, 1982; Price et al., 2013; Svenson, 1985).
Svenson (2016) provided a theoretical framework, the Numerical Judgment Process Model, NJP for describing cognitive processes activated when a judge gives a numerical response to a numerical problem. A judgment process typically includes the following cognitive process stages: problem reading, recognition, association, similarity assessment and problem interpretation. These processes precede three main strategies, associative, computational and analogue strategies that can create a judgment. Both conscious and non-conscious processes are active in a judgment process. A single problem solution process may use a rule that employs two or more of these main strategies and it may include loops repeating earlier stages. Associative strategies are exemplified by the immediate recall from long term memory of the response 8 when the problem 4 + 4 appears. Computational strategies are illustrated by non-Bayesian algorithms or rules used by participants in Bayesian problems. To illustrate, Gigerenzer and Hoffrage (1995) found that a number of participants who were asked to judge p(H|D) calculated p(H) × p(D|H) instead of the correct formula. To illustrate further, the time saving bias describes how people overestimate time savings when high speeds are increased and underestimate the time savings when low speeds are increased (Peer & Gamliel, 2012; Herberz et al., 2019; Peer, 2010; Peer & Gamliel, 2012; Svenson, 1970, 2008). The bias is a result of frequent use of two incorrect computational judgment processes (Peer, 2010; Svenson, 2008) and in some cases incorrect problem interpretations of the problem (Svenson et al., 2019). Analogue strategies involve cognitive processes that are analogous to perceptual-motor representations, e.g. movements on a number line or adjustments of an initial preliminary anchor value on a scale of possible responses.

When computational strategies apply to simple numerical judgments based on more than one variable, the Numerical Judgment Process Model predicts that additive and linear functions are attempted first before curvilinear functions (Svenson, 2016). NJP assumes that the reason for this is the great power of addition and linear functions to approximate also curvilinear functions in the environment and people learn about this and have additive and linear functions readily available when a problem appears. Also, NJP asserts that the cognitive effort needed for additive strategies is generally smaller than the effort needed for, for example, multiplicative strategies.

Relationships that include inverse variables are notoriously difficult to estimate and often lead to systematic judgment biases. As an illustration, assume that you are driving a 30 km long road at a mean speed of 100 km/h and then continue driving another road 60 km long at a speed of 50 km/h.

What is the mean speed over the trip? Some people are tempted to answer with the mean 75 km/h because they do not use the (correct) inverse measures of the speeds (e.g. 1/50 km/h). Others may understand that it is not the simple average and weigh the speeds by the distances driven 60 and 30 km, which is also wrong (Svenson & Salo, 2010). However, there are more serious problems than biased mean speed judgments, because improper use of inverse variables appear also in intuitive judgments of industry production efficiency (Svenson, 2011), health care efficiency (Svenson, 2008), well fare costs (Tscharaktschiew, 2016), consumer products (De Langhe & Puntoni, 2016), fuel efficiency of cars (Larrick & Soll, 2008) as well as time savings by driving faster (Svenson, 2008).

In most judgment studies mean/median judgments are the dependent measures in the search for rules that can explain a set of judgments. However, distributions of response frequencies have been used in some studies. Gigerenzer and Hoffrage (1995) investigated judgments in Bayesian judgments as described above. Svenson et al. (2018) investigated biased judgments of mean speed, time saving, braking capacity, fatal accident risks and braking distance in a driving context. They all found different rules within each problem area.

In the present paper, we will call the analysis of response frequencies over a judgment continuum Spectral analysis because of its roots in the natural sciences. To illustrate, the term is used in chemistry and physics, gas chromatography–mass spectrometry analysis of a gas and in neuropsychology in analyses of regularities of brainwaves (van Vugt et al., 2006). Spectral analysis as used here is a kind of cluster analysis, but it uses only one dimension in its analyses. To illustrate the difference, Configural frequency analysis (Von Eye, 2003) also uses response frequencies, but it needs more than one response variable because it builds on contingencies between variables in its analyses. In the present context, the spectral analysis will be applied in a descriptive approach in a search for judgment rules. When a response frequency distribution of judgments for a problem has a peak, the magnitude on the judgment scale that corresponds to the peak
is associated with a rule that could have produced that judgment.

**Time loss**

Svenson and Treurniet (2017) studied judgments of time lost after a speed decrease. They reported that the judgments were biased, but were unable to describe in detail the particular heuristics or rules that were used by the judges or how often they were used. The present studies were conducted in order to find out which judgment rules are most likely to lead to the time saving bias.

The objective formula for computing time loss after a decrease of speed is given in equation (1).

\[ \text{Time loss} = cD(1/V_2 - 1/V_1) \] (1)

In Equation (1), \( D \) denotes distance or amount of work, \( V_1 \) the original average speed, \( V_2 \) the lower average speed and \( c \) is a dimensionless constant depending on the units of the variables.

In the Svenson and Treurniet study (2017), participants were asked to match the time loss of a given speed decrease of alternative A from \( VA_1 \) to \( VA_2 \) with a corresponding decrease of alternative B from \( VB_1 \). Hence, the time loss following a change of speed from \( VB_1 \) to the judged speed, \( JVB_2 \) should be the same as for the A alternative when the same distances are driven. \( JVB_2 \) is called the matching judgment.

First assuming that a participant interprets the information correctly and gives a correct judgment of \( JVB_2 \), then Equation (2) describes the formula for matching the two time losses.

\[ 1/VA_2 - 1/VA_1 = 1/JVB_2 - 1/VB_1. \] (2)

This equation\(^1\) can be transformed into.

\[ (VA_1 - VA_2)/(VA_1 \cdot VA_2) = (JVB_2 - VB_1)/(VB_1 \cdot JVB_2) \] (3)

In Equation (3), all variables except \( JVB_2 \) represent information defining a problem. If judgments of \( JVB_2 \) deviate from the correct value we have a bias and we will use a sign for subjective equality \( \equiv \). In Equation (4) all variables represent subjective interpretations (or judgments) of speed.

\[ (VA_1 - VA_2)/(VA_1 \cdot VA_2) \equiv (JVB_2 - VB_1)/(VB_1 \cdot JVB_2). \] (4)

The use of subjective variables in Equation (4) is not trivial because it describes what information a participant uses and some assumptions about operations. To illustrate, a participant may use the closest 10 multiple as an approximation of the highest speed (e.g. 100 instead of 97 km/h), when processing the information and making a matching speed judgment. This illustrates a slightly “different interpretation of the task by the participant than the experimenter” bias listed by Stanovich and West (2000).

Svenson and Treurniet (2017) found that participants’ judgments seemed to simplify Equation (4) in different ways, which produced the time loss bias. They assumed correct interpretations of the speeds given and did not find strong statistical evidence for any particular rule used to judge time loss. However, some evidence indicated a Ratio rule which means that the ratios between the high and low speeds were made equal for both alternatives.\(^2\) Another potential rule was called the Difference rule, which made the differences between the speeds equal. In other words, the variability of the denominators in Equation (3) were ignored and set to 1.0. The present work will start from scratch in an exploratory search for judgment rules behind the time loss bias and perhaps find evidence for these or other rules.

It is interesting to note that the present study of the time loss bias has similarities with the Bayesian inference task used by Gigerenzer and Hoffrage (1995) and others.\(^3\) In both tasks there are correct solutions. Three numbers are given and a participant is asked to integrate them (e.g. estimate the posterior odds, speed) and provide a judgment. Both tasks are difficult and invite biased judgments.

We will return to this comparison later in the paper.

In the first study of this contribution, we will use spectral analysis in an exploratory reanalysis of the Svenson and Treurniet (2017) study to find cognitive process rules that could have produced the biased judgments. In principle, this descriptive exploratory method needs no prior hypotheses but because of earlier findings from time savings and the study by Svenson and Treurniet, we will pay particular attention to Difference and Ratio rules. In a second follow

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\(^1\) We did not solve this equation for \( JVB_2 \) because the results showed us that this would not assist in understanding the rules used by the participants.

\(^2\) Note, a Ratio rule may reflect different judgment rules, but all include the ratios between the speeds (Svenson, 1970).

\(^3\) We thank a reviewer who made us aware of this similarity.
up study, we will collect new data to check the results from the first study.

Study 1

Method

Participants
In the original study, 126 participants made time loss judgments, 63 students of engineering (14 female) and 63 students of social sciences (44 female). The participants’ ages varied between 19 and 47 years (M = 25.08, SD = 5.44) and 95 participants reported having a driving licence (51 in the engineering group and 44 in the social sciences group). The participants were students at KTH, Royal Institute of Technology in Stockholm and at the Department of Psychology, Stockholm University. They were approached when they were out of class and given a questionnaire to fill out at the site. Participants who studied psychology received course credits if they so wished and students of engineering participated without any compensation. The questionnaires ended with a problem checking whether a participant had read the problem instructions or not and 15 participants who failed this test were excluded.

Design
Two of the groups of participants A and B (N = 83) solved one set of problems 1–8 (Table 1) and one group, B (N = 41) also another set of problems (9–16) without any instruction about how to improve the judgments. After having made time loss judgments, the participants answered a set of questions including driving licence, age, gender, nationality.4

Materials
The participants were instructed to compare two speed decrease situations. The instruction included the following.

Sometimes, because of road works or other circumstances the speed limit has to be decreased on shorter or longer distances. A lower speed limit means that drivers drive more slowly, and loose time compared with the normal driving speed. Below, you will find examples of speed decreases from different original speeds and we will ask you to consider the time lost on two different road sections, both 25 km long. Specifically, you will find mean speed reductions on road A and road B situations. For road A the new lower speed is given and for B it is not.

The participants were asked to fill in the lower new speed in situation B that would give the same time loss as the speed decrease of A, the reference alternative. The distance driven was always 25 km. One example of a problem is the following.

| A (reference alternative) | B (judgment alternative) |
|---------------------------|--------------------------|
| Old speed 110 km/h        | Old speed 45 km/h        |
| New speed 70 km/h         | New speed __ km/h        |

There were two different sets of problems with no overlapping problems. The study by Svenson and Treurniet (2017) investigated the effects on intuitive judgments of information about the correct relation between speeds and time loss and the interested reader is referred to that publication. Because the present purpose was to investigate cognitive rules/heuristics that produce the time loss bias, we did not analyse judgments made after information about the correct rule. There were 83 participants who judged problems without such prior information and their judgments were analysed in Study 1. Of these participants, there were two groups who judged problems 1–8 (N = 83) and one of these groups (N = 41) also judged problems 9–16 in that order. Data from the participants who judged problems 1–8 were merged into one group and Table 1 shows the problems.

Procedure
The participants were randomly assigned to one of the groups. There were 21 engineering students and 21 social science students in group A and 20 (21) in group B. A consent form was added to inform about the rights as a participant and it was signed by each participant. Participants were not allowed to use calculators, phones, other external aids or to discuss the content of the questionnaire with other participants. They completed the questionnaire in different environments in the KTH or Stockholm University buildings in 15–45 min.

Results
We will first give an overview of the results with mean judgments and lower speed predictions following the correct rule and the Difference and

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4Svenson and Treurniet (2017) describe details of their design. Here, we have omitted details that are not relevant in the present context. The groups who solved the first 8 problems were labeled condition I and III in the Svenson and Treurniet paper and the last 8 problems were judged by condition III participants.
Table 1. Study 1 problems with correct solutions, Difference and Ratio rules. All measures are in km/h and x the speed to be judged in a problem. All differences between judgments and correct values follow the predicted time loss bias and are significant at the 0.001 level.

| Problem | Average judgment(SD) | Correct | Judgment—Correct (bias) | Difference rule | Ratio rule | N |
|---------|-----------------------|---------|-------------------------|-----------------|-----------|---|
| 1       | 27.39 (10.18)         | 25.20–29.58 | 36.47 | –9.08 | 5.00 | 28.64 | 83 |
| 2       | 90.00 (18.79)         | 88.95–97.03 | 62.40 | 30.59 | 120.00 | 97.50 | 83 |
| 3       | 92.07 (8.59)          | 90.22–93.92 | 95.33 | –3.26 | 90.00 | 93.08 | 83 |
| 4       | 62.14 (10.66)         | 59.85–64.43 | 46.51 | 15.63 | 70.00 | 57.27 | 83 |
| 5       | 34.95 (8.08)          | 32.21–36.69 | 44.83 | –9.88 | 20.00 | 38.46 | 83 |
| 6       | 91.93 (9.49)          | 89.89–93.97 | 75.13 | 16.80 | 100.00 | 88.46 | 83 |
| 7       | 34.36 (14.17)         | 34.36–40.46 | 13.46 | 23.95 | 55.00 | 28.00 | 83 |
| 8       | 28.64 (8.05)          | 26.11–29.57 | 36.94 | 9.10 | 15.00 | 30.56 | 83 |
| 9       | 42.12 (12.91)         | 38.12–46.07 | 21.82 | 20.30 | 65.00 | 40.00 | 41 |
| 10      | 34.80 (11.90)         | 31.16–38.44 | 44.93 | –10.13 | 15.00 | 35.62 | 41 |
| 11      | 79.40 (16.46)         | 74.36–84.44 | 68.28 | 11.12 | 95.00 | 82.50 | 41 |
| 12      | 28.15 (12.62)         | 24.29–32.01 | 42.19 | –14.04 | 5.00 | 33.32 | 41 |
| 13      | 69.27 (15.78)         | 64.44–74.10 | 43.75 | 25.52 | 85.00 | 63.00 | 41 |
| 14      | 21.40 (8.80)          | 18.71–24.09 | 28.80 | –7.40 | 5.00 | 22.50 | 41 |
| 15      | 81.32 (12.03)         | 77.64–85.00 | 85.71 | –4.39 | 80.00 | 83.33 | 41 |
| 16      | 99.02 (14.93)         | 94.45–103.59 | 69.69 | 29.33 | 115.00 | 97.22 | 41 |

Note: Standard deviations in parentheses. Difference predictions by the Difference rule and Ratio predictions by the Ratio rule. Correct new speed is speed resulting in physically equal time losses. The t values are mainly descriptive, but as inferences they would correspond to deviation of prediction from judged mean: $p < 0.05$ numbers in italics; $p < 0.01$ bold italics; $p < 0.001$ in bold, two tailed t-tests.

Ratio rules. Then, we will use spectral analyses of judgments to find out what cognitive process rules can explain the time loss judgment.

**Average judgments**

Each problem was analysed and a mean judgment was compared with the correct time loss value. First, Table 1 shows that all correct values fall outside the 95% confidence limits around the mean judgments as predicted by the time loss bias (Svenson & Treurniet, 2017). That is, the effects on time from changes of lower speeds were always underestimated relative to changes from higher speeds. Second, all mean judgments deviated significantly from the values predicted by the Difference rule. Third, the Ratio rule could be rejected for half of the problems. In summary, these results based within problem group data show that on average the biased judgments were not described by the Difference rule and there was no clear support for the use of the Ratio rule. Therefore, more refined analyses are needed and they will follow next.

**Spectral analysis**

Spectral analysis (Svenson et al., 2018) localises peaks in a frequency distribution of judgments over the response continuum for each problem. Each peak is used to infer a process rule that would give the judgments at or close to the peak value.

As an example, Figure 1 shows the distribution of judgments for problem 4 enabling a closer look at an application of the method. To locate the peaks, we first identified the mode. Then, we defined a cluster of judgments around the mode within an uncertainty interval. This first tentative interval was chosen so that the response frequencies outside the cluster decreased above and/or below the mode. We then inferred a rule that could produce a judgment in the cluster. The judgment predicted by a rule was the mode or close to the mode but within the interval. After this, we removed all judgments in the cluster around the mode. When the cluster of judgments around the first mode was removed, we found a new mode in the remaining set of judgments and identified a new cluster of responses and repeated the procedure in search for a second process rule. This was done also a third and a fourth time if the data indicated several clusters. The first tentative interval around a mode was later adjusted so that it gave the best descriptions across problems and rules in terms of differentiation between rules across problems. It

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We used conventional t-values at this stage as descriptive tools because, as we will show next, most of the distributions of the judgments were not normal. Hence, the confidence intervals and t-tests are only approximate descriptions of the dispersions of judgments and illustrate what a researcher would get without checking the distributions. The reader can compare these approximate results with those of the spectral analyses in the next section of the paper.
should be noted that the method is fundamentally exploratory and descriptive even if it can be used to check hypothetical judgment rules. In principle, there is no guarantee that a rule coupled with a specific node is the correct one. Another rule could have generated the same result, e.g. an anchoring and adjustment rule interpreted as a simple computational rule. However, it seems likely that different modes have their origins in different rules. In the present context, a cluster was defined to include judgments around the mode (±3). If a rule predicted a value in the interval around the mode all judgments, then ±3 around the rule prediction value were classified as produced by that rule.

If two rules, e.g. the Ratio (the ratios between the high and low speeds in each alternative made equal for both alternatives) and the Difference rule (the differences between the high and low speeds made equal) predicted judgments closer than 6 units apart, we used the midpoint between the predictions to separate the clusters and corresponding process rules. The midpoint was classified to belong to the process generating the highest value on the response scale.

Each cluster of responses that did not include predictions by the Difference, Ratio or correct rules and contained at least 10% of the responses within ±3 units from the mode was categorised as a “10 mode cluster rule”. Judgments that did not fall into any of the clusters were classified as unknown. Table 2 shows the distributions across rules.

Table 2 shows that the Ratio rule was used most often followed by the Difference rule and the correct rule in that order. About one-quarter of the judgments could not be included in a cluster. There were 8 clusters with about 10% of all judgments of a problem that could not be coupled with any of the above rules and we decided to take a closer look at these clusters.

We identified the modes defining the clusters in Table 2 and found that all modes were multiples of 10. Assuming that rounding to the nearest multiple of 10 could be one judgment strategy, we noted the closest rule prediction for each problem. There was 1 cluster closest to the Difference rule prediction, 4 closest to the Ratio prediction and 4 closest to the correct prediction. Moving judgments of multiples of 10 into the clusters of the other rules changes the distribution over rules slightly as shown in Table 2. All rules increase in applicability and the correct rule increases more than the others. Then, about 15% percent of the judgments did not follow the time loss bias.

**Study 2**

In this study, we focused on those problems among the 16 in Study 1 for which the Ratio, Difference and correct rule predictions are maximally separated. This will make it easier to couple a judgment with only one process rule. We also added verbal reports to validate the results of the spectral analysis.

**Method**

This study used the same method as Study 1 with additions of a participant’s own free description of the process used to give a judgment and self-rate scales describing to what extent each of the rules characterised each person’s own judgment processes.

**Participants**

Participants were recruited by handing out a link to a Qualtrics questionnaire to students at KTH Royal Institute of Technology in Stockholm and the same link was also posted on the second author’s Facebook page. No compensation was given to participants. To be included in the study, participants were required to have a 100% response rate. A total of 92 of the invited participants fulfilled this criterion. Two of these participants completed the study in less than 1.5 min and they and another 6
participants who reported implausible answers (e.g. answered 3 on all questions), were excluded from further study. The responses from the remaining 84 participants were analysed further. The sample of participants consisted of 51 men and 33 women, aged between 16 and 63 years ($M = 31.86$, $SD = 9.21$). A total of 76 participants had a driver’s licence and 42 participants had a master’s degree, 33 a bachelor’s degree and 9 a high school degree.

**Procedure**

The participants were informed that no personal identifying information would be registered and that all responses would be treated according to the ethical and scientific guidelines at the department. Whenever a participant wanted she or he could leave the experiment. The participants were told that they were not allowed to use calculators, phones or other external aids because we wanted their unaided judgments. The questionnaire was filled out on a participant’s own computer or cellular phone.

**Materials**

The matching problems selected for this study are listed in Table 3. Because we selected problems with maximum separations of Ratio, Difference and corrects rule predictions, the participants judged only decreases from higher speeds matching decreases from lower speeds. We do not expect any systematic effects on the results of this choice of problems because the proportions of use of Ratio and Difference rules were not significantly different between the problems selected here and the remaining problems in Study 1 (Table 2). The questionnaire consisted of three sections. The first section contained the speed matching problems and questions about gender, nationality, age, possession of driver’s licence, level of education and field of study. The instruction was almost identical to the instruction of Study 1 and included the following.

Below, you will find examples of speed decreases from different original speeds and we will ask you to consider the time lost on two different road

### Table 2. Distribution of individual judgments across problems and participants for Difference (a), Ratio (b) and Correct (c) rules in Study 1. J stands for judgment. 10 mode clusters contained at least 10% of the judgments and were not associated with any rule initially. The last line gives numbers with associated 10 mode judgments. Unknown rules represent judgments that could not be linked to any particular rule or 10 mode cluster.

| Problem | No. of observations (percentage) | Matching ratios $VA_a/VA_b$ or $VA_a – VA_b = J – VB_b$ | 10 mode clusters with closest rule(s) | Correct rule $1/VA_a – 1/VA_b = 1/J – 1/VB_b$ |
|---------|---------------------------------|-------------------------------------------------|-------------------------------------|-------------------------------------|
| 1 A: 110/70, B: 45/x | 9 (10.8%) | 34 (41.0%) | – | 14 (16.9%) |
| 2 A: 40/30, B: 130/x | 12 (14.5%) | 19 (22.9%) | – | 1 (1.2%) |
| 3 A: 130/110, B: 110/x | 34 (41.0%) | 2 (2.4%) | 15 (18.1%) | 12 (14.5%) |
| 4 A: 55/35, B: 90/x | 24 (28.9) | 14 (16.9%) | 15 (18.1%) | 2 (2.4%) |
| 5 A: 130/100, B: 50/x | 14 (16.9%) | 36 (43.4%) | 13 (15.7%) | 8 (9.6%) |
| 6 A: 65/50, B: 115/x | 20 (24.1%) | 28 (33.7%) | – | 4 (4.8%) |
| 7 A: 25/10, B: 70/x | 14 (16.9%) | 38 (45.8%) | 11 (13.3%) | 7 (8.4%) |
| 8 A: 90/55, B: 50/x | 9 (10.8%) | 33 (39.8%) | – | 18 (21.7%) |
| 9 A: 30/15, B: 80/x | 5 (12.2%) | 24 (58.5%) | – | 3 (7.3%) |
| 10 A: 115/70, B: 60/x | 8 (19.5%) | 1 (2.4%) | 15 (36.6%) | 5 (12.2%) |
| 11 A: 60/45, B: 110/x | 8 (19.5%) | 16 (34.1%) | – | 4 (7.3%) |
| 12 A: 135/90, B: 50/x | 7 (17.1%) | 11 (26.8%) | 8 (19.5%) | 9 (22.0%) |
| 13 A: 50/30, B: 105/x | 8 (19.5%) | 14 (34.1%) | – | 1 (2.4%) |
| 14 A: 80/45, B: 40/x | 7 (17.0%) | 20 (48.8%) | – | 10 (24.4%) |
| 15 A: 120/100, B: 100/x | 14 (34.1%) | 2 (4.9%) | – | 5 (12.2%) |
| 16 A: 45/35, B: 125/x | 9 (22.0%) | 16 (39.0%) | – | 16 (39.0%) |
| Sum allowing for rounding to closest rule 10 multiple | 213 (52.5%) | 335 (33.8%) | – | 154 (15.5%) |

Note: a, b and c stand for the Difference, Ratio and correct rules, C1 and C2 represent two different unidentified clusters of judgments.
Note: Standard deviations in parentheses. Di 7 A: 45/35, 6 A: 50/30, 5 A: 60/45, 4 A: 30/15, 3 A: 25/10, 2 A: 55/35, 1 A: 40/30, thought other factors a thoughts while solving the problems, (2) if they questions asking participants to report: (1) their of the questionnaire consisted of free text-entry time loss as A in this section. The second section the lower speed of B that would give the same predictions deviated significantly from the average judged time loss for all problems. **Spectral analysis**

As in Study 1, we identified clusters of judgments on the response continuum based on modes and judgments (±3.0) units from each mode. All judgments in the same cluster were classified as products of the same judgment rule. The results are shown in Table 4 and support the results from Study 1 with Difference and Ratio process rules and only a modest number of solutions correct.

The Difference rule was used in a minority of the judgments (12%) and the Ratio rule in 41% of the judgments. Most judgments that were multiples of 10 supported the Ratio rule. If multiples of 10 were included into the closest rule prediction cluster, the Ratio rule explained almost half of the judgments (48%). In the spectral analysis 68% of the judgments could be explained by one of the three rules. In summary, the results replicated the findings in Study 1: more judgments were described by the Ratio rule than by the Difference rule. Hence, a curvilinear rule was a better predictor of the time loss bias, than a linear rule.

**Consistency of rule across problems**

In order to compare spectral analysis and verbal protocols, we needed to focus only on participants who used the same rule consistently according to the spectral analysis. Participants were defined as consistent users of a specific process rule if their judgments were coupled with the same rule for 4 or
more of the 7 problems, which was the lax criterion allowing for initial adaptation to the task. A stricter criterion was also used: 6 or all 7 problems solved by the same rule process (Table 5). The table shows that following the lax criterion about half of the participants used a consistent rule across problems. In all 37% of the participants used the Ratio rule consistently and 8% the Difference rule.

Verbal protocols

After the participants had finished the matching speed problems, they were asked to answer the question: “What were your thoughts when you made your judgments? Please use the letters above (standing for the different speeds) if it helps you to explain your approach”. The categories used in the protocols were the following, Difference rule (e.g. “decreased speed with the same difference …”), Ratio rule (e.g. “simplified proportional …”, “proportional decrease in speed …”) Mention of speed and distance (Used 25 km/h as my benchmark for 1 h and 50 km/h as 0.5 h), Correct and unknown.

The 84 protocols were coded by two raters separately with a kappa = 0.92. When there was a disagreement, the two raters agreed on a category after a discussion. The Difference rule was indicated by 16 (19%) participants, the Ratio rule by 38 (45%) participants, the time/distance process by 8 (10%) participants and 1 (1%) participant followed the correct rule, while 21 (25%) participants followed unknown rules.

Spectral analysis, verbal protocols and rule scale assessments

The comparisons between the analyses of the spectral distributions and the verbal protocols concern only the 42 participants with consistent spectral rules for at least 4 problems because the results from verbal protocols concerned the overall strategy used by a participant. Table 6 presents a cross tabulation of the verbal and spectral rule classifications. Focusing on the Difference and Ratio rules, one finds that of the 7 participants who were classified as consistent Difference rule users in the spectral analysis, 6 (86%) also reported that this was the rule they had used. Of the 31 who were classified as Ratio rule users, 27 (87%) also gave the corresponding verbal descriptions. In conclusion, the verbal reports supported the results of the spectral analysis to a great extent.

Participants also assessed the correspondence between judgment strategy and the Ratio and Difference rules respectively, on 11-point scales (0 = does not correspond with my strategy at all and 10 = completely corresponds with my strategy).
Table 5. Consistent use of judgment process rule across problems according to the spectral analysis. Results for 84 participants in Study 2. Note, criterion 4 and 5 does not include participants who fulfilled the stricter criterion 6 and 7.

| Process rule | Criterion 4 and 5 | Criterion 6 and 7 | Sum |
|--------------|------------------|------------------|-----|
| Difference   | 1 (1%)           | 6 (7%)           | 7 (8%) |
| Ratio        | 23 (27%)         | 8 (10%)          | 31 (37%) |
| Correct      | 1 (1%)           | –                | 1 (1%) |
| Unknown      | 3 (4%)           | –                | 3 (4%) |
| Non-consistent use | –        | –                | 42 (50%) |

10 = corresponds extremely well with my strategy.

Participants were classified as users of one judgment rule over the other when one numerical response value was higher than the other (e.g. 8 on the Ratio rule scale and 2 on the Difference rule scale was classified as a Ratio rule user). Participants who judged both scales as equal (e.g. rated 5 on the Difference rule scale and 5 on the Ratio rule scale) were classified as unknown.

A comparison between the verbal protocols and the scale ratings showed that of the 8 verbal Difference rule classifications 5 (63%) gave the same result on the assessment scales. For the Ratio rule, the corresponding numbers were 27 and 16 (59%). The results may depend on a high number of ties 9 (35%) on the assessment scale for the verbal Ratio codings.8

Table 6. Cross tabulation of verbal protocol and spectral analysis rules for 42 participants who were consistent rule users according to the spectral analyses.

| Verbal analysis | Spectral analysis |  |  |  |
|----------------|------------------|---|---|---|
| Difference     | Ratio            | Correct | Unknown |  |
| Difference     | 6                | 2     | –        | –   |
| Ratio          | –                | 27    | –        | –   |
| Correct        | –                | –     | –        | –   |
| Unknown        | 1                | 2     | 1        | 3   |
| Sum            | 7                | 31    | 1        | 3   |

10 = corresponds extremely well with my strategy.

Analyses of response distributions in spectral analysis can be useful in studies in which one expects different judgment strategies, e.g. studies of industry production efficiency (Svenson, 2011), health care efficiency (Svenson, 2008), well fare costs (Tscharaktschiew, 2016), consumer products (De Langhe & Puntoni, 2016), fuel efficiency of cars (Larrick & Soll, 2008), mean speed judgments (Svenson et al., 2011) as well as time savings by driving faster (Svenson, 2008). All these problems include inverse variables that were more or less neglected by judges who gave the same kind of biased judgments.

In the present study, we identified different cognitive processes producing the same bias, in this case the time loss bias. Most of the participants used a ratio rule process to produce the solutions of the problems and a minority of the participants a difference rule. Gigerenzer and Hoffrage (1995) investigated a Bayesian inference task also with 3 numbers and asked the participants to integrate them (into posterior odds). In both the time loss and the Bayesian situation, judges have insufficient knowledge and yet they make judgments. The objective formulas include inverse variables in different positions and in both cases the variation of the denominators was neglected. This illustrates once more how difficult it is to use inverse relationships in intuitive judgments.

It was interesting to find that most of the participants used the presumably cognitively more demanding Ratio rule than the linear Difference rule that would have been predicted by the hypothesis of a hierarchy of rules (Svenson, 2016). However, we do not know if the participants first associated a problem with the less demanding Difference rule and then found it inadequate. Hence, it may be possible that the majority of participants who used the Ratio rule had first tried the Difference rule.

In Stanovich and West (2000) terms, the results of the studies gave no evidence for performance errors but computation errors. Neither could any wrong norms or wrong interpretations of the problems be found, which does not mean that there were no

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8 A test of the relationship between the measures across methods gives Chi square ($\chi^2$) = 20.57, $p < 0.01$. However, the fact that there are cells with less than 5 observations makes the statistics less reliable.
such norms or interpretations. We have no very detailed information about the fundamental cognitive processes active in the execution of the judgment rules in the same way as we know about, e.g. recognition, memory retrieval and reconstruction processes (Ashcraft, 1982; Svenson, 1975, 1985) or brain activity (Klein et al., 2019) in mental arithmetics. We know that participants produced time loss responses in computational strategies, but we do not know more details. A few verbal protocols indicated the use of anchoring and adjustment, but that is all we know. Other methods such as concurrent verbal protocols (we used retrospective protocols in the present study), solution time measures, special problem designs and brain scanning may shed some more light on the relevant cognitive processes. We look forward to future investigations using a multitude of methods and situations to find out how people process quantitative multi-variable information when they make judgments.

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