Article
Thermal Analysis of the Medium Voltage Cable

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Abstract: The use of high voltage power cables in distribution and transmission networks is still increasing. As a result, the research on the electrical performance of cable lines is still up to date. In the paper, an analytical method of determining the power losses and the temperature distribution in the medium voltage cable was proposed. The main feature of the method is direct including the skin and proximity effects. Then the Joule law is used to express the power losses in the conductor and screen, and the Fourier-Kirchhoff equation is applied to find out the temperature distribution in the cable. The research was focused on a cable with isolated screen and return current in the screen taken into account. The proposed method was tested by using the commercial COMSOL software (5.6/COMSOL AB, Stockholm, Sweden) as well as by carrying out laboratory measurements. Furthermore, the results obtained via the proposed method were compared with those given in literature. The differences between the temperature values calculated by the analytical method, numerical computations and obtained experimentally do not exceed 10%. The proposed analytical method is suitable in prediction the temperature of the power cables with good accuracy.

Keywords: medium voltage cable; temperature; power losses; cable ampacity; dielectric losses

1. Introduction

An increasing demand for electric energy, among others due to growth in population density, results in wide usage of power cables [1–5]. In such places as city centers, the installation of overhead lines not always is desirable or even possible; therefore, underground cables for the electric energy transmission are used. The first underground cable line was installed in the United Kingdom in 1890. Over 100 years experience in electric power infrastructure make high voltage cables a good solution for using in underground high-power transmission system [6–11]. Compared with overhead transmission lines, power cable lines are safer and more reliable. On the other hand, malfunctions of cable lines are harder to detect and their maintenance or repair is time-consuming and more expensive. Increasing requirements for reliability in power transmission are the challenges in optimizing the design of the power cable lines [12–16].

The crucial factors defining the cable ampacity, i.e., the largest current passing safely in the cable, are the conductor cross-sectional area, its electrical conductivity and the operating temperature. In case of DC cables, the bigger the cross-sectional area and conductivity the higher the ampacity. For AC cables, this rule applies for small enough cross-sections, because the current density distribution across the cable is not uniform due to electromagnetic induction (skin and proximity effects). The passage of current generates heat in the conductor, which rises the temperature and affects the insulating layers of the cable. If the insulation temperature exceeds the permissible one for a longer time, the insulation material (usually polyethylene) can be melted leading to permanent damage of the cable [17–24].

A typical power cable includes of a copper or aluminum conductor and a number of auxiliary layers that perform insulating, shielding and protective functions. The structure of
the medium voltage cable is presented in Figure 1. There are two types of losses generated in a cable: current-dependent losses and voltage-dependent losses. Current-dependent losses refer to the heat generated in conductor and screen. In turn the voltage-dependent losses refer to the heat generated in insulation layers. The heat generated by the above-mentioned losses tends to increase the temperatures of the associated and surrounding cable parts.

Figure 1. The medium voltage cable cross section: 1—braided conductor, 2, 4—semi-conductive layer, 3—XLPE insulation, 5, 7—semi-conductive swelling tape, 6—wire screen, 8—aluminum laminated sheath, 9—PE oversheath.

Heating of cable lines was a subject of several dozen papers in recent years. The thermal properties of high and medium voltage cables have been studied both analytically [7,8] as well as numerically [3,6,15,19]. Numerical methods such as finite elements or finite differences have been applied in thermal analysis of power cables. Several papers concern power cables buried in the ground or placed in tunnels [2,6,10]. In the calculations reported in literature, the power losses generated in cable conductor are calculated using Joule’s law for cable conductor AC resistance, without taking into account skin and proximity effects [2]. The skin effect and proximity effects cause non-uniform distribution of the current density in the conductor and the screen. As a result, the effective cross-section of the current path decreases, and the cable resistance increases. Consequently, the power losses and temperature in conductor increases, too. Thus, in the precise thermal analysis of the power cables the skin and proximity effects should be taken into account. Usually, these effects are taken into account through introducing specific coefficients into the computational model. This is the easiest way to include the above-mentioned effects in thermal considerations, and is widely used [3,6,8]. In contrast, the method proposed in this paper is based on the use of analytical solutions of the current densities in a cylindrical conductor and in tubular screen. Even though the current densities are expressed by Bessel functions, it is possible to calculate the integrals of these functions and thus determine the power losses in the cable parts. It is therefore possible to determine the volume heat sources and the temperature distribution in the cable.

The novelty of the paper is the development of an analytical method for determining power losses and temperature of a power cable, taking into account skin and proximity effects. In addition, this paper presents the analytical method of cable temperature determination with simultaneous taking into account of the return current in the screen and losses in insulation layers. The proposed method can be used therefore to determine the cable temperature in non-nominal and short-circuit conditions. Moreover, an additional advantage of the presented method is the determination of the temperature distribution along the radius of the cable. Therefore, the proposed method can be used for cable structure optimization.
The outline of the method is as follows: first we determine the current-dependent losses which are generated in the braided conductor and wire screen. In the calculations we assume that the braided conductor is a cylindrical conductor and the wire screen is a tubular conductor. Power losses generated in these parts are functions of load current and they are determined from Joule’s law with the skin and proximity effects taken into account. Next, from the Fourier-Kirchhoff equation the distribution of the temperature in the cable is determined. Two configurations are considered: cable with isolated screen and cable with return current in the screen. The proposed analytical method is validated through finite element method and laboratory measurements. Moreover, in Section 5 the results of calculations taking into account the voltage-dependent losses are presented.

2. Power Losses and Temperature in the Cable with Isolated Screen

Let us consider the power losses in the medium voltage cable of cross-section presented in the Figure 2. Assuming the braided conductor is a cylindrical conductor with outer radius \( R_1 \) one obtains the current density in the following form [25]:

\[
I(r) = \frac{1}{2} \frac{l}{\pi R_1} \frac{l_0(\Gamma r)}{l_1(\Gamma R_1)}
\]  

(1)

where \( I \) means complex rms (root mean square) value of current, \( l_0(\Gamma r) \), \( l_1(\Gamma R_1) \) are the modified Bessel functions of the first kind of order 0 and 1 [26], respectively, \( \Gamma = \sqrt{j\omega\mu_0\gamma} \) is the complex propagation constant, \( \omega \) is the angular frequency, \( \mu_0 = 4\pi \times 10^{-7} \, \text{H} \cdot \text{m}^{-1} \) is the magnetic permeability of the vacuum, and \( \gamma \) is the electrical conductivity of the conductor.

![Figure 2. Simplified model of the medium voltage cable with isolated screen.](image)

In turn, assuming the screen is a tubular conductor with the internal and external radii \( R_2 \) and \( R_3 \), respectively, the following equation for eddy current density inside the screen is obtained:

\[
J_{e0}(r) = \frac{1}{2} J_{e0}(r) = \frac{L}{2\pi R_2} j_0 r = 1 \cdot j_{e0}(r) \exp[j\varphi_{e0}(r)]
\]

(2)

where:

\[
j_{e0}(r) = \frac{j_0}{L} l_0(\Gamma r) + \frac{j_0}{L} K_0(\Gamma r)
\]

(3)

In addition:

\[
d_0 = l_1(\Gamma R_3) K_1(\Gamma R_2) - l_1(\Gamma R_2) K_1(\Gamma R_3)
\]

(4)
where power $P$ is expressed by Equation (10), and the volume of the screen is:

$$V_e = \pi \left( R_s^2 - R_e^2 \right) l$$

In the next step the temperature is determined. Assuming the cable length, $l$, is much larger than its transverse dimensions (Figure 3), then in steady state the temperature in the conductor ($0 < r < R_1$) fulfills the following equation [28]:

$$\frac{d^2 T}{dr^2} + \frac{1}{r} \frac{dT}{dr} = -\frac{q_V}{\lambda_l}$$
In the insulation layers ($R_1 < r < R_2$) and ($R_3 < r < R_4$) the temperature is described by the equation as follows:

$$\frac{d^2 T}{dr^2} + \frac{1}{r} \frac{dT}{dr} = 0$$

(19)

In turn, in the screen ($R_2 < r < R_3$) the temperature satisfies the following equation:

$$\frac{d^2 T}{dr^2} + \frac{1}{r} \frac{dT}{dr} = - \frac{q_e}{\lambda_{III}}$$

(20)

The general solutions of the above equations can be expressed as follows:

$$T_I(r) = A \ln r - \frac{q V r^2}{4 \lambda_I} + B$$

(21)

$$T_{II}(r) = C \ln r + D$$

(22)

$$T_{III}(r) = E \ln r - \frac{q V r^2}{4 \lambda_{III}} + F$$

(23)

$$T_{IV}(r) = G \ln r + H$$

(24)

where $A, B, \ldots, H$ are constants that can be determined from interface and boundary conditions. In the considered case they yield:

$$\left. \frac{dT_I}{dr} \right|_{r=0} = 0$$

(25)

$$T_I(R_1) = T_{II}(R_1)$$

(26)

$$\lambda_I \left( \frac{dT_I}{dr} \right)_{r=R_1} = \lambda_{II} \left( \frac{dT_{II}}{dr} \right)_{r=R_1}$$

(27)

$$T_{II}(R_2) = T_{III}(R_2)$$

(28)

$$\lambda_{II} \left( \frac{dT_{II}}{dr} \right)_{r=R_2} = \lambda_{III} \left( \frac{dT_{III}}{dr} \right)_{r=R_2}$$

(29)

$$T_{III}(R_3) = T_{IV}(R_3)$$

(30)
\[ \lambda_{III} \left( \frac{dT_{III}}{dr} \right)_{r=R_3} = \lambda_{IV} \left( \frac{dT_{IV}}{dr} \right)_{r=R_3} \]  
\[ \lambda_{IV} \left( \frac{dT_{IV}}{dr} \right)_{r=R_4} = -\alpha (T_{IV} - T_0) \]  

where \( T_0 \) is the ambient temperature of the space around the cable, \( \alpha \) is the heat transfer coefficient between the cable and the surrounding space, and \( \lambda_i \) is the thermal conductivity of the \( i \)-th region. Using conditions (25)–(32) in Equations (21)–(24) leads to the following results:

\[ A = 0 \]  
\[ B = \frac{qV}{4\lambda_{III}} R_1^2 + C \ln R_1 + D \]  
\[ C = -\frac{qV}{2\lambda_{III}} R_1^2 \]  
\[ D = -\frac{qVe}{4\lambda_{III}} R_2^2 + E \ln R_2 - C \ln R_2 + F \]  
\[ E = -\frac{qV}{2\lambda_{III}} R_2^2 + \frac{qVe}{2\lambda_{III}} R_2^2 \]  
\[ F = \frac{qVe}{4\lambda_{III}} R_3^2 - E \ln R_3 + G \ln R_3 + H \]  
\[ G = -\frac{qVe}{2\lambda_{IV}} R_3^2 - \frac{qV}{2\lambda_{IV}} R_1^2 + \frac{qVe}{2\lambda_{IV}} R_2^2 \]  
\[ H = T_0 - \frac{\lambda_{IV} G}{\alpha R_4} - G \ln R_4 \]  

Thus, the temperature of the external surface of the cable (\( r = R_4 \)) will be given by the equation:

\[ T(r = R_4) = T_0 - \frac{\lambda_{IV}}{\alpha} \frac{1}{R_4} \left[ -\frac{qVe}{2\lambda_{IV}} R_3^2 - \frac{qV}{2\lambda_{IV}} R_1^2 + \frac{qVe}{2\lambda_{IV}} R_2^2 \right] \]  

The above presented model allows for determining the operation temperature of the medium voltage cable. Since the material properties such as electrical and thermal conductivity change with temperature, nonlinear thermal analysis should be performed. For the considered range of temperature, the electrical conductivity can be assumed as follows [6]:

\[ \gamma = \frac{\gamma_{20}}{1 + k(T - 20)} \]  

where: \( \gamma_{20} \) is an electrical conductivity at 20 °C, \( k \) is the temperature coefficient for electrical resistivity. In turn, the thermal conductivity of individual layers of the cable should be determined iteratively using physical tables.

3. Power Losses in the Cable with Return Current in the Screen

Now let us consider the power losses in the medium voltage cable with return current in the screen—see Figure 4.

In the case of cable shown in Figure 4, the current density in the screen equals [25]:

\[ J_e(r) = J_{e0}(r) + J_{ew}(r) \]  

where current density \( J_{e0}(r) \) takes into account the internal proximity effect and is given by Equation (2), whereas current density \( J_{ew}(r) \) can be expressed as follows:

\[ J_{ew}(r) = -\frac{1}{2\pi R_3} \int J_{ew}(r) \exp\{j \varphi_{ew}(r)\} \]
where:

\[ j_{ew}(r) = \frac{K_1(R_2) I_0(R_r) + I_1(R_r) K_0(R_r)}{d_0} \]  

(45)

In addition:

\[ d_0 = I_1(R_r) K_1(R_r) - I_1(R_r) K_1(R_r) \]  

(46)

The power losses in the conductor of the cable with the return current in the screen are expressed by Equation (9). In turn, the power losses in the screen can be calculated from the equation:

\[ P_c = P_{c0} + P_{ew} \]  

(47)

where \( P_{c0} \) is given by Equation (10) and power \( P_{ew} \) can be expressed as follows:

\[ P_{ew} = \frac{\Gamma I^2}{4 \pi \gamma R_3 b_0 b_r^*} \]  

(48)

where:

\[ a_x = K_1(R_2) K_1^*(R_2) [I_0(R_3) I_1^*(R_3) - j I_1(R_3) I_0^*(R_3)] - I_1(R_2) I_1^*(R_2) [K_0(R_3) K_1^*(R_3) - j K_1(R_3) K_0^*(R_3)] + I_1(R_2) K_1^*(R_2) [K_0(R_3) I_1^*(R_3) + j K_1(R_3) I_0^*(R_3)] - K_1(R_2) I_1^*(R_2) [I_0(R_3) K_1^*(R_3) + j I_1(R_3) K_0^*(R_3)] \]  

(49)

In addition:

\[ b_x = I_1(R_3) K_1(R_2) - I_1(R_2) K_1(R_3) \]  

(50)

\[ b_r^* = I_1^*(R_3) K_1^*(R_2) - I_1^*(R_2) K_1^*(R_3) \]  

(51)

Figure 4. The simplified model of medium voltage cable with return current in the screen.

4. Numerical Example

Based on presented model, the temperature distribution in an XRUHAKXS 1 \times 120/50 12/20 kV cable [29] was calculated. Table 1 lists the materials and thermal properties of the individual cable layers shown in Figure 3. Current in the aluminum conductor was \( I = 200 \text{ A} \). The frequency was 50 Hz. The ambient temperature was \( T_o = 20 \text{ °C} \). The heat transfer coefficient was assumed to be \( \alpha = 5 \). The electric conductivity was \( \gamma = 35 \text{ MS} \cdot \text{m}^{-1} \) for the aluminum conductor and \( \gamma = 56 \text{ MS} \cdot \text{m}^{-1} \) for the copper screen.
Table 1. Thermal properties and thicknesses of XRUHAKXS $1 \times 120/50$ 12/20 kV cable.

| Layer No | Cable Layer       | Radius  | Thermal Conductivity W/(mK) |
|----------|-------------------|---------|-----------------------------|
| I        | Aluminum Conductor | $R_1 = 6.2$ mm | 280                         |
| II       | XLPE Insulation   | $R_2 = 12.2$ mm | 0.4                         |
| III      | Copper Screen     | $R_3 = 14.9$ mm | 400                         |
| IV       | PE Oversheath     | $R_4 = 17.9$ mm | 0.42                        |

Apart from analytical calculations with use of the proposed method, also numerical simulations with use of finite elements were performed with the aid of the commercial COMSOL software. The Electromagnetic Heating interface and two modules: Magnetic Fields and Heat Transfer in Solids were used. In the computations the radiation and natural convection were taken into account. The natural convection was considered by applying the Heat Transfer in Fluid condition in the external region and setting the Velocity field option at 0.01 m/s along the y axis. The radiation was considered by setting the boundary condition called Diffuse Surface on the external surface of cable and by introducing the emissivity coefficient $\varepsilon = 0.5$. The mesh was set as: Physics-controlled mesh. The complete mesh consisted of 9974 domain elements and 304 boundary elements. Figures 5 and 6 show the temperature distribution in the considered cable with isolated screen, whereas Figures 7 and 8 present the temperature distribution in the cable with return current in the screen.

Figure 5. Temperature distribution in the XRUHAKXS $1 \times 120/50$ 12/20 kV cable—analytical method.

Figure 6. Temperature distribution in the XRUHAKXS $1 \times 120/50$ 12/20 kV cable—Comsol computation.
The plots presented in Figures 5–8 indicate the temperature in the aluminum conductor and copper screen is nearly constant. The constancy of the temperature distribution is due to the very good thermal conductivity of the material (aluminum or copper) from which the conductor and screen is made. It means that in the steady state the temperature distribution inside conductor and screen is constant despite non-uniform distribution of the current density and power losses. This is a valuable remark that can inspire engineers involved in the analysis, design and testing of power cables, because it is possible to determine the conductor temperature versus the load current.

The calculations of the temperature of the aluminum conductor and copper screen depending on the load current value were also performed. The dependence of the conductor temperature on the current value is shown in Figure 9. In turn, the dependence of the screen temperature on the current value is shown in Figure 10. The calculations presented in Figures 9 and 10 were made for the XRUHAKXS 1 × 120/50 12/20 kV cable with isolated screen.
To verify the proposed analytical method as well as the COMSOL computations, suitable measurements were also carried out. The laboratory setup used in the temperature measurements of the XRUHAKXS 1 × 120/50 12/20 kV cable is shown in Figure 11. A current of 200 A was excited in the cable by an AC current source. In order to avoid additional heating of the XRUHAKXS cable (disturbing the measurements), the connection between the generator and the tested cable was made using two copper cables with a cross-section of 120 mm² each. The ambient temperature was \( T_a = 20 \, ^\circ \text{C} \). Temperature measurements were made with a 9-channel temperature recorder and by TP-202J-1b-200-2.0 thermocouples. The temperature recorder was connected to a computer via a USB connector, which allowed for saving the measurement results (Figure 12). Three temperature sensors (J type) were placed in the phase conductor, another three sensors in the screen, next two on the surface of the cable and one sensor measured the ambient temperature. The temperature sensors were installed in properly drilled holes in the middle of the cable length (Figure 13). The sensors were connected to the temperature recorder. Ambient temperature was measured by a probe immersed in oil to eliminate possible short-term sudden changes in the ambient temperature. The measurements were carried out for 5 h until reaching steady state. The results of the measurements and calculations are shown in Table 2.
Figure 10. The dependence of the screen temperature on the current value.

To verify the proposed analytical method as well as the COMSOL computations, suitable measurements were also carried out. The laboratory setup used in the temperature measurements of the XRUHAKXS $1 \times 120/50$ $12/20$ kV cable is shown in Figure 11. A current of 200 A was excited in the cable by an AC current source. In order to avoid additional heating of the XRUHAKXS cable (disturbing the measurements), the connection between the generator and the tested cable was made using two copper cables with a cross-section of 120 mm$^2$ each. The ambient temperature was $T_o = 20$ °C. Temperature measurements were made with a 9-channel temperature recorder and by TP-202J-1b-200-2.0 thermocouples. The temperature recorder was connected to a computer via a USB connector, which allowed for saving the measurement results (Figure 12). Three temperature sensors (J type) were placed in the phase conductor, another three sensors in the screen, next two on the surface of the cable and one sensor measured the ambient temperature. The temperature sensors were installed in properly drilled holes in the middle of the cable length (Figure 13). The sensors were connected to the temperature recorder. Ambient temperature was measured by a probe immersed in oil to eliminate possible short-term sudden changes in the ambient temperature. The measurements were carried out for 5 h until reaching steady state. The results of the measurements and calculations are shown in Table 2.

Figure 11. Laboratory stand for temperature measurements in power cables: 1—supply, 2—temperature recorder, 3—XRUHAKXS $1 \times 120/50$ $12/20$ kV cable, 4—temperature sensors, 5—supply cables.

Figure 12. Schematic of the temperature sensors.
Figure 13. Temperature sensors installed in XRUHAKXS cable.

Table 2. Temperature in XRUHAKXS 1 × 120/50 12/20 kV cable [°C].

| Method             | Conductor | Screen |
|--------------------|-----------|--------|
| Analytical computations | 40.1 | 37.4   |
| COMSOL computations  | 40.2 | 38.8   |
| Measurements       | 38.8 | 37.6   |

The results presented in Table 2 indicate the proposed method can be used to determine the temperature of medium voltage cable conductive layers with good accuracy. The differences between the temperature values calculated by analytical method and the measured ones do not exceed 10%. It is noticeable that the measured values are slightly smaller than the computed ones. These differences are most likely due to the thermal conditions in the laboratory.

5. Power Losses in the Insulation Layers

In the cable temperature calculations, the dielectric losses in the insulation material should be taken into account, because the heat is generated not only as a result of active power losses in the conductor and cable screen, but also as a result of dielectric losses. The dielectric losses in the insulation material can be expressed as follows [23]:

\[ W_d = 2\pi f U C U^2 \tan \delta \]  \hspace{1cm} (52)

where \( f \) means frequency, \( U \) is an operating voltage, \( C \) is a capacitance of the insulation layer, \( \tan \delta \) is a loss tangent. The loss tangent equals:

\[ \tan \delta = \frac{\gamma}{\omega \varepsilon} \]  \hspace{1cm} (53)

where \( \omega \) is the angular frequency, \( \gamma \) is for the electrical conductivity of the insulation and \( \varepsilon \) is its electrical permittivity.

In the case of medium voltage cable (Figure 3), the capacitances of the insulating layers are as follows [25]:

\[ C_{II} = \frac{2\pi \varepsilon_{II} l}{\ln \frac{R_2}{R_1}} \]  \hspace{1cm} (54)

In addition:

\[ C_{IV} = \frac{2\pi \varepsilon_{IV} l}{\ln \frac{R_4}{R_3}} \]  \hspace{1cm} (55)
Taking into account the dielectric losses results in appearing the source term in Equation (19) so that it takes the following form:

\[
\frac{d^2T}{dr^2} + \frac{1}{r} \frac{dT}{dr} = -\frac{q_i}{\lambda_i}
\]

where \( i \) denotes the layer number. The general solutions of Equations (18), (56) and (20) become now:

\[
T_I(r) = A \ln r - \frac{q_{II}^2}{4\lambda_I} + B
\]

\[
T_{II}(r) = C \ln r - \frac{q_{II}^2}{4\lambda_{II}} + D
\]

\[
T_{III}(r) = E \ln r - \frac{q_{III}^2}{4\lambda_{III}} + F
\]

\[
T_{IV}(r) = G \ln r - \frac{q_{IV}^2}{4\lambda_{IV}} + H
\]

Applying the interface and boundary conditions (25)–(32) leads to the following results:

\[
A = 0
\]

\[
B = C \ln R_1 + D + \frac{R_1^2}{4} \left( \frac{q_{II}}{\lambda_I} - \frac{q_{II}}{\lambda_{II}} \right)
\]

\[
C = \frac{R_1^2}{2\lambda_{II}} (q_{II} - q_{I})
\]

\[
D = (E - C) \ln R_2 + F + \frac{R_2^2}{4} \left( \frac{q_{III}}{\lambda_{II}} - \frac{q_{III}}{\lambda_{III}} \right)
\]

\[
E = \frac{R_2^2}{2\lambda_{III}} (q_{III} - q_{II}) + \frac{R_2^2}{2\lambda_{III}} (q_{III} - q_{I})
\]

\[
F = (G - E) \ln R_3 + H + \frac{R_3^2}{4} \left( \frac{q_{IV}}{\lambda_{III}} - \frac{q_{IV}}{\lambda_{IV}} \right)
\]

\[
G = \frac{R_3^2}{2\lambda_{IV}} (q_{IV} - q_{III}) + \frac{R_3^2}{2\lambda_{IV}} (q_{IV} - q_{II}) + \frac{R_1^2}{2\lambda_{IV}} (q_{II} - q_{I})
\]

\[
H = T_0 - \frac{\lambda_{IV}}{\alpha} \frac{G}{R_4} - G \ln R_4 + \frac{q_{IV} R_4^2}{4\lambda_{IV}} + \frac{q_{IV} R_4}{2\alpha \lambda_{IV}}
\]

Exemplary temperature calculations taking into account dielectric losses were made for an XRUHAKXS 1 × 120/50 12/20 kV cable, assuming the geometrical and physical parameters as in Section 4 and: \( U = 12 \) kV, \( \varepsilon_0 = 8.85 \times 10^{-12} \) F/m, \( \varepsilon_{II} = \varepsilon_{III} = 2.25, \) \( \gamma_{II} = \gamma_{IV} = 10^{-12} \) S/m. This gave \( C_{II} = 0.185 \) nF, \( C_{IV} = 0.682 \) nF, \( q_{II} = 3.83 \) W/m\(^3\) and \( q_{IV} = 0 \) W/m\(^3\). The temperature distribution along the cable radius is shown in the Figure 14.

Comparing Figure 14 with Figure 5, it can be noticed that they are nearly the same, and this is due to the fact that the dielectric losses are much less than in the aluminum conductor. Thus, in the thermal analysis of the medium voltage cable the dielectric losses can be omitted because the temperature of the cable is determined by the power losses in the conductor.
6. Results Discussion

For a broader analysis, the results obtained via the analytical method proposed in the paper were compared with the simulation results with those for two types of XLPE cables given in [15] and [30]. Table 3 shows the materials and thermal properties of the individual XLPE-0 cable layers described in [15]. The current in the copper conductor was \( I = 1244 \) A. The frequency was 50 Hz. The ambient temperature was \( T_o = 17.7 \) °C. The electric conductivity was \( \gamma = 55 \) MS·m\(^{-1}\) for the copper conductor, and \( \gamma = 35 \) MS·m\(^{-1}\) for the aluminum screen. The cable conductor temperature calculation results are shown in Table 4.

Table 3. Thermal properties and geometrical parameters of XLPE-0 cable.

| Layer No. | Cable Layer            | Radius   | Thermal Conductivity W/(mK) |
|-----------|------------------------|----------|-----------------------------|
| I         | Copper Conductor       | \( R_1 = 13.3 \) mm | 400                          |
| II        | XLPE Insulation        | \( R_2 = 82 \) mm | 0.31                         |
| III       | Aluminum Screen        | \( R_3 = 86 \) mm | 280                          |
| IV        | PVC Oversheath         | \( R_4 = 92 \) mm | 0.42                         |

Table 4. Conductor temperature of XLPE-0 cable [°C].

| Method                              | Conductor Temperature |
|-------------------------------------|-----------------------|
| Analytical computations             | 90.7                  |
| Method presented in paper [15]      | 89.6                  |

In addition, the calculations of the temperature of a 110 kV XLPE cable from [30] were performed. The thermal and geometrical parameters of the cable layers are presented in Table 5. The current in the copper conductor was \( I = 850 \) A. The ambient temperature was assumed to be \( T_o = 40 \) °C. The results of the cable conductor temperature calculation are presented in Table 6.

Table 5. Thermal properties and geometrical parameters of 110 kV XLPE cable.

| Layer No. | Cable Layer            | Radius   | Thermal Conductivity W/(mK) |
|-----------|------------------------|----------|-----------------------------|
| I         | Copper Conductor       | \( R_1 = 17 \) mm | 400                          |
| II        | XLPE Insulation        | \( R_2 = 37 \) mm | 0.2875                        |
| III       | Copper Screen          | \( R_3 = 39 \) mm | 280                          |
| IV        | XLPE Insulation        | \( R_4 = 44 \) mm | 0.2875                        |

Figure 14. Temperature distribution in the XRUHAKXS 1x120/50 12/20 kV cable—dielectric losses were taken into account.
Table 6. Conductor temperature of 110 kV XLPE cable [°C].

| Method                                | Conductor Temperature |
|---------------------------------------|-----------------------|
| Analytical computations               | 64.29                 |
| Method presented in paper [30]        | 66.27                 |

Based on the results presented in Tables 4 and 6, it can be concluded that the analytical method proposed in this paper can be successfully used to determine the temperature of power cables. The differences between the results presented in [15,30] and the values obtained with the use of the proposed approach do not exceed a few percent. The analysis of Tables 2, 4 and 6 confirm that the method presented in this paper is fairly accurate.

7. Conclusions

In the paper an analytical approach to the determination of the operation temperature of the medium voltage cable has been presented. The proposed method of determining the temperature allows for the derivation of not very complicated relationships that may be inspiring for engineers dealing with the analysis, design and testing of power cables. The most important advantage of the proposed method is its transparency and short calculation time. In the presented analytical model, the skin and proximity effects were considered. The results obtained via the proposed analytical method were compared with those by the finite element method, laboratory measurements and the results of simulations reported in other scientific papers. The differences between the temperature values calculated with use of the analytical method, computed by COMSOL and measured do not exceed 10% and the differences between the simulation results presented in [15] and those obtained with the use of the proposed analytical method do not exceed several percent. Thus, on the basis of the performed validation, it can be concluded that the analytical method proposed in the paper is fairly accurate.

The determination of the operation temperature of many power devices is made by numerical methods. However, they do not allow for generalization of the results and derivation of useful dependencies aiding the design of specific elements of power system. Analytical methods can be used for simple elements. Power cables are not complicated devices, so the analytical method to analyze them can be used. The adoption of certain simplifying assumptions allows for finding general relationships between the physical parameters and the operating temperature of the power cables. However, it should be emphasized that each extension of assumptions (e.g., taking into account nonlinearities) leads to significant difficulties in finding the analytical solution. In addition, an important problem is the knowledge of heat transfer coefficients. Attempts to determine analytically the value of the heat transfer coefficients usually lead to complex mathematical models even for not complicated devices. Therefore, the heat transfer coefficients are usually determined by semi-empirical methods based on the theory of similarity. No matter of the method used, the significance of the heat transfer coefficients in thermal calculations seems high.

In further research the proposed method will be developed and extended to more complex cable systems. Attempts will be made to determine the temperature of cables buried in the ground, placed in tunnels etc.

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29. TELE-FONIKA Company: High. Voltage Cables, 1st ed.; TELE-FONIKA Company: Cracow, Poland, 2012. (In Polish)
30. Chen, Y.; Duan, P.; Cheng, P.; Yang, F.; Yang, Y. Numerical Calculation of Ampacity of Cable Laying in the Ventilation Tunnel Based on Coupled Fields as well as the analysis on relevant factors. In Proceeding of the 11th World Congress on Intelligent Control and Automation, Shenyang, China, 29 June–4 July 2014; pp. 3534–3538.