Decaying vacuum cosmology and its scalar field description

F Andrade-Oliveira$^{1,3}$, F E M Costa$^{1,2}$ and J A S Lima$^1$

$^1$ Departamento de Astronomia, Universidade de São Paulo, Rua do Matão 1226, 05508-900, São Paulo, SP, Brazil
$^2$ Universidade Federal Rural do Semi-Árido, 59900-000, Pau dos Ferros, RN, Brazil

E-mail: foliveira@astro.iag.usp.br, ernandesmc@usp.br and limajas@astro.iag.usp.br

Received 7 February 2013, revised 11 November 2013
Accepted for publication 2 January 2014
Published 22 January 2014

Abstract
We discuss the cosmological consequences of an interacting model in the dark sector in which the $\Lambda$ component evolves as a truncated power series of the Hubble parameter. We also discuss some of the thermodynamic features of this model. In order to constrain the free parameters of the model we carry out a joint statistical analysis involving observational data from current type Ia supernovae, recent estimates of the cosmic microwave background shift parameter and baryon acoustic oscillations measurements. Finally, we adopt a theoretical method to derive the coupled scalar field version for this non-equilibrium decaying vacuum accelerating cosmology.

Keywords: cosmology, supernovae, scalar field
PACS numbers: 98.80.−k, 97.60.Bw, 11.10.−z

(Some figures may appear in colour only in the online journal)

1. Introduction

In the last decade, the analysis and interpretation of observational data are strongly indicating that the present universe is accelerating and that its geometry is spatially flat [1–4]. These observations gave rise to a resurgence of interest in a non-zero cosmological constant $\Lambda$ that dominates the current composition of the universe [5]. The main motivation behind this idea is that a very small value for $\Lambda$ ($\simeq 70\%$ of the cosmic composition), not only would explain the observed late-time acceleration of the universe (as indicated by type Ia supernovae observations but would also reconcile the inflationary flatness prediction, corroborated by cosmic microwave background data, ($\Omega_{\text{Total}} \simeq 1$) with the current clustering estimates that point systematically to $\Omega_m \simeq 0.3$ [3, 6].

$^3$ Author to whom any correspondence should be addressed.
Although flat models with a very small cosmological term are in good agreement with almost all sets of observational data, the present day value of $\Lambda$ amounts to nearly 50–122 orders of magnitude smaller than the value predicted by quantum field theories. Such an extreme discrepancy is usually referred to as the cosmological constant problem, and requires a complete cancellation from an unknown physical mechanism [7–11] or some vacuum decaying process in the course of the evolution.

Another intriguing puzzle with the cosmological constant (or the vacuum energy density) is usually dubbed the coincidence problem. In brief, although a very small (but non-zero) value of $\Lambda$ could conceivably be explained by some unknown physical symmetry being broken by a small amount, one should be able to explain not only why it is so small but also why it has exactly the right value to drive the expansion of the universe only at very late stages (low redshifts). Since both components (dark matter and the vacuum energy density-$\Lambda$-term) are usually assumed to be independent and, therefore, scale in different ways, this would require a remarkable coincidence which still lacks a definite causal connection.

Many phenomenological models with variable cosmological term (decaying vacuum) have been proposed in literature as an attempt to alleviate the cosmological constant problem [10–12] and more recently the coincidence problem [13–15, 17] or both [16]. In the context of the general relativity theory a cosmological term that varies in space or time requires a coupling with some other cosmic component, so that the total energy–momentum tensor is conserved. Thus, $\Lambda(t)$ models provide either a process of particle production or an increasing time-varying mass of the dark matter particles [18].

In this paper, we focus our attention on the theoretical and observational aspects of a $\Lambda(t)$ cold dark matter (CDM) model in which the cosmological term evolves as a truncated power series of the Hubble parameter. The decaying vacuum component is assumed to be coupled only with the dark matter (interacting dark sector) thereby producing dark matter particles. The work is structured as follows. In section 2, we discuss the general aspects and solve the basic equations of the decaying vacuum model. The second law of thermodynamics applied to this model is discussed in the section 3. The observational constraints on parameters of the model are discussed in section 4. In section 5, by following an approach first proposed by Maia and Lima [19], we also derive a coupled scalar field version for this vacuum decay scenario. Finally, the basic results are summarized in the conclusion section 6.

2. The $\Lambda(t)$ model

Let us first consider a homogeneous, isotropic and spatially flat universe described by the Friedmann–Robertson–Walker (FRW) line element. In such a background, the Einstein field equations are given by

$$8\pi G \rho_f + \Lambda = 3H^2, \tag{1}$$

and

$$8\pi G \rho_f - \Lambda = -2\dot{H} - 3H^2, \tag{2}$$

where $\rho_f$ and $p_f$ are, respectively, the energy density and pressure of the cosmic fluid (radiation, baryons and dark matter) and the dot denotes the derivative with respect to time.

In this paper, we will work with the $\Lambda(t)$ function written as a power series of the Hubble parameter up to the second order, i.e.,

$$\Lambda(t) = \lambda + \sigma H + 3\beta H^2, \tag{3}$$

where $\lambda$ and $\sigma$ are constants with dimensions of $H^2$ and $H$, respectively, while $\beta$ is a dimensionless constant. The term proportional to $H^2$ was proposed long ago by
Carvalho et al [10] based on dimensional arguments. Later on, it was also justified by using techniques from the renormalization group approach [20]. The linear term was first discussed by Carneiro and collaborators [21]. The interest in the extended model represented by the above truncated power series was also suggested in the appendix C of a paper by Basilakos et al [22]. As remarked before, here we discuss the observational potentialities of this model, and, for completeness, we also derive the associated scalar field description.

Now by combining the above equations and considering that \( p_f = (\gamma - 1) \rho_f \) where \( \gamma \) is the adiabatic index, we find

\[
2\dot{H} + 3\gamma (1 - \beta)H^2 - \sigma \gamma H - \gamma \lambda = 0. \tag{4}
\]

The integration of above equation leads the following \( H \) parameter

\[
H \equiv \frac{\dot{a}}{a} = \frac{1}{3(1 - \beta)} \left[ \frac{\sigma}{2} + \alpha \frac{e^{\alpha t} + 1}{e^{\alpha t} - 1} \right], \tag{5}
\]

where \( \alpha = (\gamma/2) \sqrt{\sigma^2 + 12\lambda (1 - \beta)} \) and \( a \) is the cosmic scale factor. In what follows, we will restrict our analysis to the case \( \lambda = 0 \). The practical reason for this choice is that the linear term in equation (3) causes a transition decelerating/accelerating phase. So that, we can obtain the following solution for the scale factor

\[
a(t) = C (e^{\sigma t/2} - 1)^{2/3(1 - \beta)}, \tag{6}
\]

where \( C \) is a integration constant. Combining equations (5) and (6) one finds an expression for \( H(a) \), i.e.,

\[
H = \frac{\sigma}{3(1 - \beta)} \left[ 1 + \left( \frac{C}{\alpha} \right)^{3(1 - \beta)/2} \right], \tag{7}
\]

and we have used \( \gamma = 1 \) in the last equation. It is worth notice that in our analysis \( \Lambda(t) \) is coupled only with the cold dark matter component. Naturally, in a more complete scenario, several coupling constants should be introduced, say, dark matter (\( \beta_{dm} \)), photons (\( \beta_{\gamma} \)) and baryons (\( \beta_{b} \)). In principle, this happens because the coupling constants of the vacuum are not the same for the different decay channels. Here we are assuming an interaction internal to the dark sector, so that \( \beta_{dm} \equiv \beta \) and \( \beta_{\gamma} = \beta_{b} = 0 \). In particular, this means that the recombination and big-bang nucleosynthesis proceeded in the standard way. Of course, by adopting different choices, the corresponding coupling constants should be constrained by the mentioned phenomena.

Now, by taking equations (1), (3) and (7) at current time it is possible to show that \( C = \Omega_m/(1 - \beta - \Omega_m)^{2/3(1 - \beta)} \) and \( \sigma = 3H_0(1 - \beta - \Omega_m) \), where \( \Omega_m = 8\pi G \rho_m,0/3H_0^2 \). Now, by considering that \( a = (1 + z)^{-1} \), the Hubble parameter can be rewritten as

\[
H = H_0 \left[ \frac{\Omega_m}{1 - \beta} + \frac{\Omega_m}{1 - \beta} (1 + z)^{3(1 - \beta)/2} \right]. \tag{8}
\]

It should be noticed that for \( \beta = 0 \), the above expression reduces to the dynamical \( \Lambda \) solution derived in [21].

It is also straightforward to show from (8) that the deceleration parameter, defined as \( q(a) = -\dot{a}/a^2 \), now takes the following form:

\[
q(z) = \frac{3}{2} \left[ \frac{\Omega_m (1 + z)^{3(1 - \beta)/2}}{1 - \frac{\Omega_m}{1 - \beta} + \frac{\Omega_m}{1 - \beta} (1 + z)^{3(1 - \beta)/2}} \right] - 1. \tag{9}
\]

Note that for \( z = 0 \), one finds \( q_0 = 1.5\Omega_m - 1 \), and, therefore, the present value of the deceleration parameter is negative (see table 1) and does not depend on the values of \( \beta \). It is
Table 1. The results from joint statistical analysis.

| Parameter | Best fit | Error bars $^{+\sigma}_{-\sigma}$ |
|-----------|----------|---------------------------------|
| $\Omega_m$ | 0.325    | $^{+0.035}_{-0.035}$            |
| $\beta$   | $-0.185$ | $^{+0.045}_{-0.055}$            |
| $\sigma/H_0$ | 2.58    | $^{+0.24}_{-0.27}$              |
| $q_0$     | $-0.513$ | $^{+0.053}_{-0.053}$            |
| $z_t$     | 0.99     | $^{+0.19}_{-0.20}$              |
| $t_0H_0$  | 1.003    | $^{+0.049}_{-0.051}$            |

easy to check that the transition redshift ($z_t$), defined to be the zero point of the deceleration parameter, is given by

$$z_t = \left[ \frac{2(1 - \beta - \Omega_m)}{(1 - 3\beta)\Omega_m} \right]^{2/3(1-\beta)} - 1. \quad (10)$$

In this connection, we recall that some authors have recently suggested a picture based on theoretical and observational evidences where the cosmic acceleration may have just peaked and started to slow down [23]. From expression (9) for the deceleration parameter, we see that in the future (negative redshifts) the value of $q(z)$ approaches $-1$.

For the sake of completeness, we derive the age-redshift relation for this class of $\Lambda(t)$ models. We find

$$t(z) = \frac{2}{\sigma} \ln \left[ 1 + \frac{1 - \beta - \Omega_m}{\Omega_m} (1 + z)^{-3(1-\beta)/2} \right]. \quad (11)$$

Note also that the value of the present age of the universe in this decaying vacuum model is given by

$$t_0 = \frac{2H_0^{-1}}{3(1 - \beta - \Omega_m)} \ln \left( \frac{1 - \beta}{\Omega_m} \right), \quad (12)$$

so that $H_0t_0 \sim 1$ in agreement with the current observations (see table 1).

3. Entropy evolution of decaying vacuum model

In this section, we address some of the thermodynamical properties of the decaying vacuum model discussed in this work. For this purpose, we investigate how the second law of thermodynamics constrains the behavior of the entropy of the universe, i.e., if the entropy of the universe never decreases, $\dot{S} \geq 0$, and if it is a concave function, $\ddot{S} < 0$, at least in the last stage of evolution of the system. The latter condition guarantees that the irreversible change by which the system can pass takes it to a new stage of equilibrium [28].

The total entropy $S$ is the sum the entropy of apparent horizon $S_H$ and the entropy of dust matter $S_m$. The entropy of apparent horizon is given by [29]

$$S_H = \frac{k_B A}{l_p^2}, \quad (13)$$

where $A = 4\pi r_A^2$ is the area of apparent horizon, $r_A = c/\sqrt{H^2 + ka^2}$ is the radius of apparent horizon in physical units [30], $l_p$ is the Planck length and $k_B$ is the Boltzmann constant. For the spatially flat model considered here, we have $r_A = cH^{-1}$. 


On the other hand, the entropy of dust particles can be estimated by considering that each single particle inside the horizon adds a unit of $kB$ for the entropy. Therefore, the total entropy of the $N$ particles inside the horizon is $NkB$, or,

$$S_m = kB \frac{4 \pi \tilde{r}_h^3}{3 m_d} \rho,$$

where $m_d$ is the mass of dust particle and $\rho$ is the density of dust matter.

Except in the very beginning of the expansion of the universe, i.e., $a \ll 1$, the entropy of matter is negligible in comparison to the entropy related to the horizon. This can be explicitly shown by taking the ratio between both sources of entropy

$$\frac{S_m}{S_H} = \frac{4 \pi c_l^2}{3 m_{dm}} \frac{H}{\rho},$$

or, using that $\rho/H = 3H_0\Omega_m a^{-3(1-\beta)/2}/(8\pi G)$,

$$\frac{S_m}{S_H} = \frac{4 \pi c_l^2}{3 m_{dm}} \frac{3H_0\Omega_m}{8\pi G a^{3(1-\beta)/2}},$$

which has the limit

$$\lim_{a \to \infty} \frac{S_m}{S_H} = 0.$$

In other words, this means that the term $S_m$ starts dominating the term $S_H$ but, after $a \sim [l_p^2/m_d]^2/(3(1-\beta))$ (in SI units), it is quickly suppressed by $S_H$. Once this happens much before the period covered by the model, namely, the matter and vacuum domination eras, we can simply consider that the entropy evolution is driven by $S_H$.

Given those considerations, we now go further in our investigations about the second law of thermodynamics. By taking the first derivative of (13) with respect to time, we have

$$\dot{S} = -\frac{3 \pi k_B c^2 H}{l_p^2} \frac{H}{H^3},$$

which is always positive, since $\dot{H} \leq 0$ and $H > 0$.

Therefore, the entropy of the universe always grows, i.e., from the thermodynamics frame, the cosmological expansion can be understood as an irreversible process.

Now, we want to investigated the second derivative of entropy. Using the relation

$$\dot{H} = -\frac{3}{2} H_0 \Omega_m a^{-3(1-\beta)/2},$$

the first derivative of entropy can be rewritten as

$$\dot{S} = -\frac{3 \pi k_B c^2 H_0 \Omega_m a^{-3(1-\beta)/2}}{l_p^2 H^3}.$$

By derivation the above equation with respect to time, we have:

$$\ddot{S} = -\frac{3}{2} \frac{\pi k_B c^2 \Omega_m H_0 a^{-3(1-\beta)/2}}{l_p^2 H^3} \left[ \frac{3H^2}{2} + 2\dot{H} \right],$$

or, by using again (18),

$$\ddot{S} = -\frac{3}{2} \frac{\pi k_B c^2 \Omega_m H_0 a^{-3(1-\beta)/2}}{l_p^2 H^3} \left[ \frac{3}{2} H - 3\Omega_m H_0 a^{-3(1-\beta)/2} \right].$$

From the expressions (21) and (8), we conclude that the $\ddot{S} < 0$ when $a_S > [\Omega_m/(1 - \beta - \Omega_m)]^{2/3(1-\beta)}$. Once the density of vacuum energy starts to dominated at
\[ a = \left[ (1 - 2\beta)\Omega_m/(1 - \beta - \Omega_m) \right]^{2/(1 - \beta)}, \]

which means that from the transition from matter to vacuum era until the final expansion of the universe, the entropy will be a concave function.

Furthermore, the entropy evolution is bounded to a maximum value:

\[ S_{\text{max}} = \lim_{a \to \infty} S = \frac{\pi k_B c^2}{T_p^2 H_0^2} \left( \frac{(1 - \beta)^2}{(1 - \beta - \Omega_m)^2} \right) \approx 10^{122} k_B, \]

for the best-fit values. This proves that in the scenario adopted here the universe asymptotically approaches to a thermodynamic equilibrium state.

Thus, we have shown that the model vacuum decay model considered here has an increasing entropy during the past and future expansion as well as the second derivative of entropy is negative for the final stage of cosmic expansion. As a result, we conclude that the accelerating expansion of the Universe can be thermodynamically understood as a non-equilibrium process evolving to a final state of thermodynamic equilibrium, when \( a \to \infty \).

4. Observational analysis

In this section we will discuss bounds on the free parameters \( \Omega_m \) and \( \beta \). To this end we will use different observational sets of data, as described below.

4.1. Constraints from type Ia supernovae

In this test we use one of the most recent SNe Ia compilation, the so-called Union 2.1 sample compiled in [24] which includes 580 data points after selection cuts.

The best fit to the set of parameters \( s = (\Omega_m, \beta) \) is found by using a \( \chi^2 \) statistics, i.e.,

\[ \chi^2 = \sum_{i=1}^N \left[ \frac{\mu^i_{p}(z_s)(z_i) - \mu^i_{o}(z_i)}{\sigma^2_i} \right]^2, \]

where \( \mu^i_{p}(z_s)(z_i) = 5 \log d_L + 25 \) is the predicted distance modulus for a supernova at redshift \( z_i \), \( d_L \) is the luminosity distance, \( \mu^i_{o}(z_s) \) is the extinction corrected distance modulus for a given SNe Ia at \( z_i \) and \( \sigma_i \) is the uncertainty in the individual distance moduli.

4.2. Constraints from CMB/BAO ratio

Additionally, we also use measurements derived from the product of the cosmic microwave background (CMB) acoustic scale \( \xi_A = \pi d_A(z_r)/r_s(z_r) \) and from the ratio of the sound horizon scale at the drag epoch to the baryon acoustic oscillations (BAO) dilation scale, \( r_s(z_d)/D_V(z_{\text{BAO}}) \), where \( d_A(z_r) \) is the comoving angular-diameter distance to recombination \( (z_r = 1089) \) and \( r_s(z_r) \) is the comoving sound horizon at photon decoupling. In the above expressions, \( z_d \approx 1020 \) is the redshift of the drag epoch (at which the acoustic oscillations are frozen in) and the dilation scale, \( D_V \), is given by \( D_V(z) = \left[ z^2 (z)/H(z) \right]^{1/3} \). By combining the ratio \( r_s(z_d = 1020)/r_s(z_r = 1090) = 1.044 \pm 0.019 \) [4, 25] with the measurements of \( r_s(z_d)/D_V(z_{\text{BAO}}) \) at \( z_{\text{BAO}} = 0.20, 0.35 \) and \( 0.6 \), one finds [26, 27]

\[
\begin{align*}
0.20 &= d_A(z_r)/D_V(0.2) = 18.32 \pm 0.59, \\
0.35 &= d_A(z_r)/D_V(0.35) = 10.55 \pm 0.35, \\
0.60 &= d_A(z_r)/D_V(0.60) = 6.65 \pm 0.32.
\end{align*}
\]
Figure 1. The results of our statistical analysis. The constraints from SNe Ia and CMB/BAO ratio data are shown by shaded contours for 3σ confidence level, while the contours from combining observational data are represented with black lines for (68.3% CL, 95.4% and 99.73%) confidence intervals.

4.3. Combining observational sets data

Let us now discuss the constraints given by the above different sets of data. In our statistical analysis, we minimize the function $\chi^2_T = \chi^2_{SN} + \chi^2_{CMB/BAO}$, where $\chi^2_{SN}$ and $\chi^2_{CMB/BAO}$ corresponding to the SNe Ia and CMB/BAO $\chi^2$ functions, respectively.

In figure 1, we display the space parameter ($\Omega_m, \beta$). The supernova data set fitted with Union 2.1 are shown by the bluish contours while the orange contours are derived from CMB/BAO ratio data alone. The results from our joint analysis are shown by dashed lines. By marginalizing on the nuisance parameter $h (H_0 = 100 h \text{km s}^{-1}\text{Mpc}^{-1})$ we find $\beta = -0.185^{+0.045+0.075+0.095}_{-0.055-0.085-0.135}$ and $\Omega_m = 0.325^{+0.035+0.060+0.080}_{-0.033-0.060-0.080}$ at 68.3%, 95.4% and 99.7% of confidence level, respectively, with $\chi^2_{\text{min}} = 564$ and $\nu = 581$ degrees of freedom. The reduced $\chi^2_r = 0.97$ where ($\chi^2_r = \chi^2_{\text{min}}/\nu$), thereby showing that the model provides a very good fit to these data. We also note that, due to the process of matter production resulting from the vacuum decay, the current observational bounds on $\Omega_m$ provide a value slightly higher than the $\Lambda$CDM model.

By using the best-fit values of $\Omega_m$ and $\beta$ it follows that $q_0 = -0.513^{+0.053}_{-0.053} \pm 0.049$ and $\Omega_m H_0 = 1.003^{+0.049}_{-0.049}$. The main results of our joint analysis are shown in table 1.

It is interesting to compare our results with other from literature. For example, [31] has found constraints on $(\Omega_m, \beta)$ very similar to the our results. In the analysis of the SNIa data of [28] was used a sample of 307 supernovae (Union 08) [32], while we use a sample of 580 data points [24]. Therefore, from statistical view point, our analysis is much more robust. Furthermore, [28] uses the BAO parameter ($A$), only for $z_{\text{BAO}} = 0.35$, and separately from CMB shift parameter ($R$). As discussed in [33], these observables were obtained in the context of the $\Lambda$CDM model, and can be considered a good approximation only for some classes of dark energy models. For BAO parameter, for instance, it is implicitly assumed that the evolution of matter density perturbations during the matter-dominated era must be similar to the $\Lambda$CDM case (at least until the characteristic redshift $z_{\text{BAO}}$) and also that the comoving distance to the horizon at the time of equilibrium between matter and radiation must scale with $(\Omega_m H_0^2)^{-1}$. In model with decaying vacuum the production of matter leads to a different
dependence of the horizon at the time of matter-radiation equality on the present matter density, 
thereby, these observables cannot be used directly in the analysis decaying vacuum models. 
In fact, we do not use these observables separately in our analysis. We use the ratio \( r_s(z_d)/D_V \) 
which is very weakly model-dependent.

By assuming a coupling between the cosmological term and the radiation fluid (\( \beta_r \neq 0 \)) 
[34] have found from big bang nucleosynthesis (BBN) \( \beta_r < 0.13 \). Later on, [35] rediscussed 
such a bound inferred from BBN thereby obtaining \( \beta_r \leq 0.16 \). We also notice that restrictive 
bounds on the \( \beta_r \) parameter have recently been obtained based on the Sunyaev–Zeldovich effect 
(see, for instance, [36]). Such results comes from a modification of the FRW temperature law 
due to a coupling of the vacuum (or dark energy) with the thermal bath (see [37]). However, as 
we have argued, these results cannot be applied to our model because our discussion focused 
only the \( \Lambda(t) \)CDM dominated phase (and the corresponding coupling). In other words, we 
are still completely free to assume any hypothesis concerning the coupling with the thermal 
component.

5. Scalar field description

\( \Lambda(t) \) cosmologies constitute a possible way to address the cosmological constant problem. 
However, in the absence of a natural guidance from fundamental physics, one needs to specify a 
phenomenological time-dependence for \( \Lambda \) in order to establish a definite model and study their 
observational and theoretical implications. A possible way to seek for physically motivated 
models is to represent them through a field theoretical language, the easiest way being through 
scalar fields. In this section, we discuss a possible derivation of a time-varying \( \Lambda \) models 
from fundamental physics based on the approach suggested in [19] (see also [38] for a general 
description with arbitrary curvature).

Following standard lines [19], let us define the function
\[
\gamma_* \equiv -\frac{2H}{3H^2} = 1 - \frac{\Lambda}{3H^2}.
\]
Each \( \Lambda(t) \) model is specified through the \( \gamma_* \) time-dependent parameter. Thus, for the vacuum 
decay scenario presented here, the function \( \gamma_* \) takes the following form:

\[
\gamma_* = \frac{3(1 - \beta)H^2 - \sigma H}{3H^2}.
\]  

(25)

As an intermediate step, it is convenient to replace the vacuum energy density and pressure 
as given by equations (1) and (2) by the corresponding scalar field expressions, i.e.,

\[
\Lambda/8\pi G \rightarrow \rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi),
\]

(26)

and

\[
-\frac{\Lambda}{8\pi G} \rightarrow p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi),
\]

(27)

where \( \rho_\phi \) and \( p_\phi \) are, respectively, the energy density and pressure associated to the coupled 
scalar field \( \phi \) whose potential is \( V(\phi) \).

Now, by defining a parameter \( x \equiv \dot{\phi}^2/(\dot{\phi}^2 + \rho_{\phi\text{m}}) \) with \( 0 \leq x \leq 1 \), we can manipulate 
equations (1), (2), (26) and (27) to separate the scalar field contributions (see [19] for more 
details), i.e.,

\[
\dot{\phi}^2 = \frac{3H^2}{8\pi G} \gamma_* x,
\]

(28)

and

\[
V(\phi) = \frac{3H^2}{8\pi G} \left[ 1 - \gamma_* \left( 1 - \frac{x}{2} \right) \right].
\]

(29)
Note that the above equations link directly the field and its potential with the related quantities of the dynamical $\Lambda(t)$ case. From equation (28), one can show that the field $\phi$ in terms of the Hubble parameter reads

$$\phi - \phi_0 = \pm \frac{1}{\sqrt{6\pi G}} \int_H^{H_0} \sqrt{\frac{x}{\gamma_*}} \frac{dx}{H}.$$  \hspace{1cm} \text{(30)}$$

Now, inserting equation (25) into (30) and integrating it, one obtains

$$\phi - \phi_0 = -k^{-1} \ln \left[ \frac{\sqrt{H - A + \sqrt{H}}}{\sqrt{H_0 - A + \sqrt{H_0}}} \right],$$  \hspace{1cm} \text{(31)}$$

where $k = \sqrt{3\pi G(1 - \beta)/2x}$, $A = \sigma/3(1 - \beta)$ and we have considered the positive sign in equation (30). By using equation (29) one finds

$$V(\phi) = \frac{D^2}{64\pi G} \left[ (2 - x)\sigma + Bf(\phi) \right] f(\phi),$$  \hspace{1cm} \text{(32)}$$

where

$$f(\phi) = \left[ e^{-2k(\phi - \phi_0)} + \left( \frac{A}{D^2} \right)^2 e^{2k(\phi - \phi_0)} + 2 \frac{A}{D^2} \right].$$  \hspace{1cm} \text{(33)}$$

with $B = 3D^2[2\beta + (1 - \beta)x]/4$ and $D = \sqrt{H_0 - A + \sqrt{H_0}}$. Note that by assuming $\sigma = 0 \implies A = 0$, and the potential takes the form of a simple exponential function thereby reducing to the potential initially found in [19]. It is worth mentioning that double exponential potentials of the type (32) have been considered in the literature as viable examples of quintessence scenarios (see, e.g., [39]). As discussed in [40], a scalar field potential given by the sum of two exponential terms is also motivated by dimensional reduction in M-theory with interesting implications for the late-time accelerating behavior of the cosmic expansion.

In figure 2, we display the evolution of the scalar field potential (in units of the present critical energy density $U(\phi) = V(\phi)/\rho_{c,0}$) for some selected values of the free parameters, namely: $\Omega_m = 0.29, 0.325, 0.36$ and $\beta = -0.24, -0.185, -0.14$. This choice correspond to the lower, best fit and upper limits on $\Omega_m$ and $\beta$ parameters at (1σ CL). We also consider
$x = 0.2$. This parameter indirectly quantifies the amount of energy that the potential $V(\phi)$ is delivering to each component along the universe evolution. It is a quantity dependent on the thermodynamic conditions underlying the decaying process of the scalar field. Note that all curves exhibit the same behavior differing only by a small shift.

Although, in the coupled scalar field description there is also a freedom in the type of the interaction of the scalar field with the DM component, this field description presents some aspects that make it more attractive than models with decaying vacuum. In what follows we present some of these aspects. (i) Coupled scalar field models may avoid the cosmic coincidence problem, with the available data being used to fix the type of the interaction of the scalar field with the dark matter component and, consequently, the scalar field potential responsible for the present accelerating phase of the universe. (ii) If a coupled scalar field description for a given decaying vacuum law can be implemented, its associated Lagrangian can be used to extend the model to other spacetimes and other gravitational theories, like an effective high energy string cosmology, for example. For these reasons, we think that the scalar field description is more plausible.

6. Conclusions

In this paper we have investigated theoretical and observational features of a vacuum decay model in which the cosmological term is written as a power series of the Hubble parameter up to the second order. From the point of view of thermodynamics, we have shown that the cosmic expansion driven by the discussed model can be understood as a non-equilibrium process which asymptotically approaches the thermodynamic equilibrium. We have performed a statistical analysis involving the late observational measurements of SNe Ia, BAO peak and CMB shift parameter and found stronger constraints on the parametric space $\Omega_m - \beta$. By using the best-fit values for $\Omega_m = 0.325^{+0.035}_{-0.035}$ and $\beta = -0.185^{+0.045}_{-0.055}$ we have also derived the current values of $q_0$, $z_t$ and $t_0$, that are $q_0 = -0.513^{+0.053}_{-0.051}$, $z_t = 0.99^{+0.19}_{-0.20}$ and $t_0H_0 = 1.003^{+0.049}_{-0.051}$, respectively. These values are in excellent agreement with the predictions of the standard model (see table 1).

By following a theoretical method developed in [19] we have also derived the scalar field description for this cosmology and shown that the present $\Lambda(\phi)$CDM model is identified with a coupled quintessence field with the potential of equation (32). Since this $\Lambda(\phi)$CDM scenario is quite general thereby describing many of the previous proposals as a particular case, we have argued that a coupled quintessence field model whose potential is given by equation (32) is dynamically equivalent to a large number of decaying vacuum scenarios previously discussed in the literature.

Acknowledgments

The authors would like to thank VC Busti for helpful discussions. FEMC and FAO are supported by FAPESP and CNPq (Brazilian Research Agencies), respectively. JASL is partially supported by CNPq and FAPESP under grants 304792/2003-9 and 04/13668-0, respectively.

References

[1] Perlmutter S et al 1998 Nature 391 51
   Riess A G et al 1998 Astron. J. 116 1009
[2] Kowalski M et al 2008 Astrophys. J. 686 749
   Amanullah R et al 2010 Astrophys. J. 716 712
[3] Spergel D N et al 2007 Astrophys. J. Suppl. Ser. 170 377
[4] Komatsu E et al 2011 Astrophys. J. 192 18
[5] Peebles P J E and Ratra B 2003 Rev. Mod. Phys. 75 559
  Padmanabhan T 2003 Phys. Rep. 380 235
  Lima J A S 2004 Braz. J. Phys. 34
  Copeland E J et al 2006 Int. J. Mod. Phys. D 15 1753
  Li M et al 2011 arXiv:1103.5870
[6] Carlb erg R, Yee H K C, Ellingson E, Abraham R, Gravel P, Morris S and Pritchec C J 1996
  Astrophys. J. 462 32
  Feldman H A et al 2003 Astrophys. J. 596 L131
[7] Zee A 1985 Stud. Nat. Sci. 20 211–30
[8] Weinberg S 1989 Rev. Mod. Phys. 61 1
[9] Sahni V and Starobinsky A A 2000 Int. J. Mod. Phys. D 9 373
[10] Carvalho J C, Lima J A S and Waga I 1992 Phys. Rev. D 46 2404
[11] Lima J A S and Maia J M F 1994 Phys. Rev. D 49 5597
  Lima J A S and Trodden M 1996 Phys. Rev. D 53 4280 (arXiv:astro-ph/9508049v1)
[12] Özer M and Taha M O 1986 Phys. Lett. B 171 363
  Özer M and Taha M O 1987 Nucl. Phys. B 287 776
  Freese K et al 1987 Nucl. Phys. B 287 797
  Chen W and Wu Y-S 1990 Phys. Rev. D 41 695
  Pavón D 1991 Phys. Rev. D 43 375
  Waga I 1993 Astrophys. J. 414 436
  Overduin J M and Cooperstock F I 1998 Phys. Rev. D 58 043506
  Vishwakarma R G 2001 Gen. Rel. Grav. 33 1973
  Cunha J V and Santos R C 2004 Int. J. Mod. Phys. D 13 1321 (arXiv:astro-ph/0402169)
  Shapiro I L et al 2005 J. Cosmol. Astropart. Phys. JCAP01(2005)012
  Elizalde E et al 2005 Phys. Rev. D 71 103504
  Grande J, Sola J and Stefancic H 2006 J. Cosmol. Astropart. Phys. JCAP08(2006)011
  Borges H A, Carneiro S and Fabris J C 2008 Phys. Rev. D 78 123522
[13] Wang P and Meng X 2005 Class. Quantum Grav. 22 283
[14] Jesus J F, Santos R C, Alcaniz J S and Lima J A S 2008 Phys. Rev. D 78 063514 (arXiv:0806.1366
  [astro-ph])
[15] Costa F E M, Alcaniz J S and Maia J M F 2008 Phys. Rev. D 77 083516
[16] Lima J A S, Basilakos S and Solá J 2012 arXiv:1209.2802 [gr-qc]
  Lima J A S, Basilakos S and Costa F E M 2012 Phys. Rev. D 86 103534 (arXiv:1205.0868
  [astro-ph.CO])
[17] Costa F E M and Alcaniz J S 2010 Phys. Rev. D 81 043506
[18] Alcaniz J S and Lima J A S 2005 Phys. Rev. D 72 063516 (arXiv:astro-ph/0507372)
[19] Maia J M F and Lima J A S 2002 Phys. Rev. D 65 083513 (arXiv:astro-ph/0112091)
[20] Shapiro I L and Sola J 2002 J. High Energy Phys. JHEP02(2002)006 (arXiv:hep-th/0112227)
  Sola J 2011 J. Phys.: Conf. Ser. 283 012033 For a recent review see (arXiv:1102.1815
  [astro-ph.CO])
[21] Carneiro S, Pigozzo C, Borges H A and Alcaniz J S 2006 Phys. Rev. D 74 023532
  See also Carneiro S and Lima J A S 2005 Int. J. Mod. Phys. A 20 2465 (arXiv:gr-qc/0405141)
  Borges H A and Carneiro S 2005 Gen. Rel. Grav. 37 1385
[22] Basilakos S, Plionis M and Sola J 2009 Phys. Rev. D 80 083511
[23] Carvalho F C et al 2006 Phys. Rev. Lett. 97 081301 (arXiv:astro-ph/0608439)
  Costa F E M 2010 Phys. Rev. D 82 103527
  Shafieloo A, Sahni V and Starobinsky A A 2009 Phys. Rev. D 80 103101
  Guimarães A C C and Lima J A S 2011 Class. Quantum Grav. 28 125026 (arXiv:1005.2986
  [astro-ph.CO])
[24] Suzuki N (The Supernova Cosmology Project) et al 2012 Astrophys. J. 746 85
  Percival W J et al 2009 arXiv:0907.1660
[25] Sollerman J et al 2009 Astrophys. J. 703 1374–85
[26] Blake C et al 2011 arXiv:1105.2862
[27] Mimoso J P and Pavón D 2013 Phys. Rev. D 87 047302
[28] Pavón D and Radicella N 2013 Gen. Rel. Grav. 45 63
[29] Bak D and Rey S J 2000 Class. Quantum Grav. 17 L83
[31] Basilakos S 2009 *Mon. Not. R. Astron. Soc.* **395** 1252
[32] Kowalski M et al 2008 *Astrophys. J.* **686** 749
[33] Doran M, Stern S and Thommes E 2007 *J. Cosmol. Astropart. Phys.* JCAP04(2007)015
[34] Birkel M and Sarkar S 1997 *Astropart. Phys.* **6** 197
[35] Lima J A S et al 2000 *IAU Symp.* **198** 111
[36] Luzzi G et al 2009 *Astrophys. J.* **705** 1122
[37] Noterdaeme P et al 2011 *Astron. Astrophys.* **526** L7
[38] Lima J A S 1996 *Phys. Rev.* D **54** 2571 (arXiv:gr-qc/9605055)
[39] Lima J A S, Silva A I and Viegas S M 2000 *Mon. Not. R. Astron. Soc.* **312** 747
[40] Alcaniz J S and Maia J M F 2003 *Phys. Rev.* D **67** 043502
[41] Sen A A and Sethi S 2002 *Phys. Lett.* B **532** 159
[42] Jarv L, Mohaupt T and Saueressig F 2004 *J. Cosmol. Astropart. Phys.* JCAP08(2004)016