Spin polarization inducing diamagnetism, inverse magnetic catalysis and saturation behavior of charged pion spectra

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We offer a unified picture to understand the novel properties of magnetized QCD matter revealed by lattice calculations, including diamagnetism at low temperature, the inverse magnetic catalysis around the critical temperature and the saturation behavior of charged pion spectra under magnetic fields. Motivated by the diamagnetic material caused by paired electrons in the atoms with zero net magnetic moment, we propose that the diamagnetism in magnetized QCD matter is caused by spin polarization condensation of quark antiquark with parallel spin pairing, which carries zero net magnetic moment and reduces the total net magnetic moment of the system, thus the system with spin polarization exhibits diamagnetism. The parallel spin pairing condensation naturally breaks chiral condensate, and induces the inverse magnetic catalysis around the critical temperature. It is noticed that the spin polarization operator is the same as the dynamical anomalous magnetic moment of quarks, its coupling with magnetic field causes Zeeman splitting of the dispersion relation of quarks thus changes the neutral pion and charged pion mass spectra under magnetic fields. The neutral pion mass decreases with magnetic field and charged pion mass show a nonlinear behavior, i.e., firstly linearly increases with the magnetic field and then saturates at strong magnetic fields, which resembles with the lattice result.

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Introduction: Understanding properties of QCD matter under strong magnetic field is of vital importance to further explore the interior of magnetar, neutron-star mergers, non-central heavy-ion collisions and the evolution of the early universe. The study of the QCD vacuum and strongly interacting matter under external magnetic fields has attracted much attention, see reviews e.g. Refs. [7–11]. With the presence of a magnetic field background, the strongly interacting matter shows a large number of exotic phenomena, for example, Chiral Magnetic Effect (CME) [12–15], Magnetic Catalysis (MC) in the vacuum [16–18], Inverse Magnetic Catalysis (IMC) around the critical temperature [19–21].

The catalysis of the chiral symmetry breaking induced by the magnetic field, i.e., the MC effect can be easily understood from the dimension reduciton. On the other hand, the IMC effect, the critical temperature of chiral phase transition decreases with the magnetic field, which intuitively contradicts with the MC effect and still remains as a puzzle, though there have been many literatures trying to explain the IMC by considering neutral pion fluctuations [22], chirality imbalance [23], running coupling constant [24]. Recently, lattice calculations show more interesting and novel properties of magnetized QCD matter: The charged pion mass shows a saturation magnetic field dependent behavior [25], and magnetized matter exhibits diamagnetism (negative susceptibility) at low temperature and paramagnetism (positive susceptibility) at high temperature [26–27].

Magnetic fields modify the spectrum of charged particles. The point particle approximation gives the charged pion mass $m_{\pi^\pm}(B) = m_{\pi^\pm}(B = 0) + eB$ linearly with the magnetic field, and neutral pion mass keeps as a constant. Calculation in the effective quark model, e.g., the Nanu–Jona-Lasinio (NJL) model which takes into account of quark magnetization, modifies the linear slope of charged pion and shows similar results for pion mass spectra as point-approximation results [28–38]. Recent lattice calculation in Ref. [29] shows that the neutral pion mass decreases with the magnetic field and the charged $\pi^\pm$ and $K^\pm$ shows a non-monotonic behavior, it firstly increases linearly and then decreases as magnetic field increases, which are quite different from point-particle approximation and previous results from effective models. Such non-monotonic behavior of charged pion mass is firstly pointed out by [25]. It is worthy of mentioning that lattice results in [19, 39] show a nonlinear and saturation behavior for charged pion, no decreasing with magnetic field is observed at strong magnetic field.

The remaining IMC puzzle and recently discovered novel phenomena attract our renewed interest to revisit QCD vacuum and matter under external magnetic field and try to find a selfconsistent explanation for all these properties. In this work, we offer a unified picture to reveal the secret behind the novel properties of magnetized QCD matter, including diamagnetism at low temperature, the inverse magnetic catalysis around the critical temperature and saturation behavior of charged pion spectra under magnetic fields. Remind that the diamagnetic material is due to paired electrons in the atoms with zero net magnetic moment(MM), we propose that...
the diamagnetism in magnetized QCD matter is caused by spin polarization(SP) condensation of quark antiquark with parallel spin pairing, which has zero net MM. The magnetic field catalyzes the chiral condensate of quark antiquark with antiparallel spin pairing which carries net MM and triggers a dynamical anomalous magnetic moment(AMM) of quarks. Except the chiral condensate, the magnetic field also induces SP [43], i.e. the condensation of quark antiquark with parallel spin pairing. This parallel spin pairing condensation naturally breaks chiral condensate, and induces the inverse magnetic catalysis around the critical temperature. Also because the parallel spin pairing does not carry net MM, it reduces the total net MM of the system, thus the system with SP exhibits diamagnetism. It is noticed that the SP operator is the same as the dynamical anomalous magnetic moment of quarks, its coupling with magnetic field causes Zeeman splitting of the dispersion relation of quarks thus changes the neutral pion and charged pion meson mass spectra under magnetic fields. The neutral pion mass decreases with magnetic field and charged pion mass shows a saturation behavior, i.e., firstly linearly increases with the magnetic field and then saturates at strong magnetic field, which is consistent with the lattice result.

\[ \langle \bar{\psi} \sigma^{12} \psi \rangle = N_c \frac{m_f |q_f B|}{\pi} \int \frac{dp_v dp_z}{(2\pi)^2} \sum_{n,s} \bar{\psi} \sigma^{12} \psi \times n, s \]

where \( q = \text{sign}(q_f B) \), and it is not difficult to find that the contributions \( \{ n = k, s_f = 1 \} \) and \( \{ n = k + 1, s_f = -1 \} \) cancel with each other and the Lowest Landau Level (LLL) \( \{ n = 0, s_f = -1 \} \) contributes, then we can see that the tensor polarization condensation is proportional to the chiral condensate as in Refs. [26, 40, 41]

\[ \langle \bar{\psi} \sigma^{12} \psi \rangle \times \langle \bar{\psi} \psi \rangle. \]  

(2)

It is noticed that the tensor polarization operator \( \bar{\psi} \sigma^{12} \psi \) is also called the SP operator or spin density because \( \bar{\psi} \sigma^{12} \psi = \psi^1 \gamma^0 \Sigma^3 \psi \) with \( \Sigma^3 = \frac{1}{2} [\gamma^3, \gamma^0] \) and \( \sigma^3 = -i a^3 \sigma^2 \). The SP condensation is for quark anti-quark with parallel spin pairing

\[ \bar{\psi} \sigma^{12} \psi \sim \bar{q}_f \gamma^3 q_f - \bar{q}_f \gamma^3 q_f, \]  

(3)

and the SP condensation in dense quark matter as an origin of magnetic field in compact stars has been investigated in [42].

In Fig.1 we compare the contribution of chiral condensate and SP condensation to the magnetism of the system in a simple constituent quark picture. Every charged q particle with mass \( m \) and spin \( \hbar \) carries the MM \( \mu = \frac{q \hbar}{2m} \), for quark anti-quark with anti-parallel spin pairing, it exhibits a net MM, thus the chiral condensate triggers a dynamical AMM [43-46]. Under the magnetic field, the net MM turns to align along the direction of the magnetic field. On the other hand, for quark anti-quark with parallel spin pairing, the MM of the spin aligned quark and anti-quark cancel with each other and the pairing does not exhibit net MM. Therefore, the total net MM of the system with only chiral condensate. Therefore the system exhibits relatively diamagnetism. This is similar to the diamagnetic materials with paired electrons in the atoms with zero net MM, here the electron pairs are anti-parallel spin pairing. At high temperature, all charged quarks are single small magnets, and turns align along the magnetic field, thus QCD matter at high temperature shows paramagnetism. Also if the system develops SP condensation, it naturally breaks the chiral condensation. In the vacuum, the chiral condensation still dominates and in total we find the MC effect. However, around the critical temperature, the chiral condensate almost vanishes and the effect of spin alignment will show up and the critical temperature for the chiral phase transition decreases, i.e. we find the IMC.

From a more microscopic point of view, the MM of a single quark is seen by a photon probing the electromagnetic current, which to the first order has the form of

\[ J^\mu = e q_f \gamma^\mu q_f \]

\[ = e \bar{q}_f (\gamma^\mu + ia_\mu \sigma^{12} q_f) q_f \]  

(4)

Here \( a \) is the AMM [43, 46] and the MM of quarks is given by \( \mu = \mu_B (1 + a) \) with \( \mu_B = \frac{e \hbar}{2m} \) the Bohr magneton. Here it is noticed that the tensor structure \( \bar{\psi} \sigma^{12} \psi \) is the same as the spin density operator. It has been shown in Refs. [43, 45] that the breaking of chiral symmetry triggers dynamical AMM of quarks. Ref. [47] shows the possibility of IMC driven by AMM, and the equation of state, chiral condensate and meson mass spectra including the AMM under magnetic field have been investi-
in the two-flavor case, which is same for \( \kappa \). Nevertheless, to simplify the numerical calculation, we set \( \bar{\kappa} \) for quarks, and in principle, \( \kappa \) sation for vacuum properties as well as dynamical AMM
by:

\[
\mathcal{L} = \bar{\psi}(x)i\gamma^\mu D_\mu - m_0 + \kappa_f q_f B \sigma^{12}\psi(x) + G\{(\bar{\psi}(x)\psi(x))^2 + (\bar{\psi}(x)i\gamma^5 \tau \psi(x))^2\}, \tag{5}
\]

Here \( \psi \) are two-flavor quark filed \( \psi = (u, d)^T \), \( m_0 \) is current mass and we assume the current quark mass for both flavors are same: \( m_u = m_d = m_0 \). \( G \) is the coupling constant for (pseudo)scalar interaction channel and \( \tau \) is the Pauli matrix. The covariant derivative \( D_\mu = \partial_\mu - iq_f A_\mu \) with \( q_f \) quark charge, and \( A_\mu \) is Abelian gauge field and without loss of generality we choose external uniform magnetic field along \( z \)-direction, which leads to \( A_\mu = (0, 0, Bz, 0) \). \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \). The tensor term \( \bar{\psi} \kappa_f q_f B \sigma^{\mu\nu} \psi \) is to present the none zero SP condensation for vacuum properties as well as dynamical AMM for quarks, and in principle, \( \kappa_f \) is flavor-dependent, however, to simplify the numerical calculation, we set \( \kappa_f = \kappa \) in two-flavor case, which is same for \( u \) and \( d \) quark. Then the Lagrangian after mean-field approximation is given by:

\[
\mathcal{L} = -\frac{(M - m_0)^2}{4G} + \bar{\psi}(x)i\gamma^\mu D_\mu - M + \kappa q_f B \sigma^{12}\psi(x), \tag{6}
\]

The dynamical induced AMM as well as SP condensation has close relation with chiral condensate. For example, Ref.\[40\] consider dynamically generated AMM in NJL model, and they found that under Lowest Landau Level approximation, \( \kappa \) is proportional to quark condensates. To include the relation between AMM and chiral condensate, we made a simple assumption:

\[
\kappa = \nu \sigma, \tag{7}
\]

where \( \sigma = \langle \bar{\psi}\psi \rangle \) is the quark condensate, and we treat \( \nu \) as a free parameter, and \( M = m_0 + 2G\langle \bar{\psi}\psi \rangle \) is the dynamical quark mass. Following Refs.\[47\] \[50\] \[53\] we obtain the effective potential at zero baryon density potential:

\[
\Omega = \left(\frac{M - m_0}{4G}\right)^2 - N_c \sum_f \frac{|q_f B|^2}{2\pi} \sum_{n} \sum_{s=\pm 1} \int \frac{dp_z}{2\pi} E_{n,f,s} \tag{8}
\]

\[
- 2N_c T \sum_f \frac{|q_f B|^2}{2\pi} \sum_{n} \sum_{s=\pm 1} \int \frac{dp_z}{2\pi} \ln(1 + e^{-\frac{E_{n,f,s} - eB}{T}}),
\]

where \( E_{n,f,s} \) is the dispersion relation of quarks in the presence of external magnetic field with AMM and it can be solved from the Dirac equation and takes the form of:

\[
E_{n,f,s} = \left\{ p_z^2 + \sqrt{M^2 + (2n + 1 - s\xi_f)|q_f B| - s\kappa q_f B} \right\}^{\frac{1}{2}}. \tag{9}
\]

with \( n \) the Landau level index, \( s = \pm 1 \) and \( \xi_f = \text{sign}(q_f) \). The dispersion relation with AMM and without AMM is shown in Fig.\[2\], we can see that except the lowest Landau level, the AMM produces Zeeman splitting at nonzero energy levels.

Then the dynamical quark mass can be obtained by solving the gap equation:

\[
M = m_0 + 2GN_c \sum_f |q_f B| \sum_{n} \sum_{s=\pm 1} \int \frac{dp_z}{2\pi} \left\{ 1 - 2(1 + e^{-\frac{E_{n,f,s} - eB}{T}})^{-1} \right\} \frac{M}{E_{n,f,s}} - \frac{s\kappa q_f B}{M_n}(10)
\]

where \( M_n = \sqrt{M^2 + (2n + 1 - s\xi_f)|q_f B|} \). The magnetic susceptibility is defined as:

\[
\chi = \left. -\frac{\partial^2 \Omega}{\partial (eB)^2} \right|_{eB=0} \tag{11}
\]

the relative magnetic susceptibility is \( \chi_0 = \chi(T) - \chi(T = 0) \). The numerical result of relative magnetic susceptibility is shown in Fig.\[3\]. It is found that with AMM or SP, \( \chi_0 \) is negative (diamagnetism) at low temperature and positive (paramagnetism) at high temperature, which is expected in our picture and is consistent with lattice results in \[20\] \[27\].

The quark mass and the critical temperature as functions of \( eB \) are shown in Fig.\[4\]. It is seen that nonzero AMM or nonzero SP plays the role of breaking chiral condensate, the vacuum still exhibits MC if SP/AMM is not big enough. At high temperature, one can observe the IMC due to nonzero SP/AMM.

Following the calculations in \[28\] \[30\], we use the modified dispersion relation of quarks with SP/AMM to calculate the neutral and charged pion mass and the results are shown in Fig.\[5\]. It is found that indeed, the neutral pion mass decreases with the magnetic field and the
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FIG. 3: Magnetic susceptibility as a function of temperature with \( \kappa = v \sigma \).

charged pion shows a saturation with the magnetic field and saturates at high magnetic field which is consistent with lattice results shown in \([39]\). It is observed that the neutral pion mass is much more sensitive to the nonzero SP/AMM.

Conclusion and discussion Recent lattice calculations reveal many interesting properties of QCD matter in the presence of magnetic field, e.g., the diamagnetism at low temperature, the inverse magnetic catalysis around the critical temperature and the saturation behavior of charged pion mass. These novel properties have not been fully understood yet. In this work, we offer a unified picture to understand novel properties of magnetized QCD matter. Motivated by the diamagnetic material caused by paired electrons in the atoms with zero net magnetic moment, we propose that the diamagnetism in magnetized QCD matter is due to spin polarization condensation of quark antiquark with parallel spin pairing with zero net magnetic moment. The parallel spin quark antiquark pairing carrying zero net magnetic moment and reduces the total net magnetic moment of the system, thus the system with spin polarization exhibits relative diamagnetism. The parallel spin pairing condensation naturally breaks chiral condensate, and induces the inverse magnetic catalysis around the critical temperature. It is noticed that the spin polarization operator is the same as the dynamical anomalous magnetic moment of quarks, its coupling with magnetic field causes Zeeman splitting of the dispersion relation of quarks thus changes the neutral pion and charged pion meson mass spectra under magnetic fields. The neutral pion mass decreases with magnetic field and charged pion mass show a saturation behavior, i.e., firstly linearly increases with the magnetic field and then saturate at strong magnetic field, which is consistent with the lattice results in \([20, 27]\), and no decreasing behavior of charged pion mass described in \([25]\) is observed in our calculation. It can be expected that in the spin polarized vacuum, the mass of charged rho meson will not decrease to zero \([54]\). This will be checked in the future.

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