Abstract

The detected anomalous frequency drift acceleration in Pioneer’s radar data finds its explanation in a Berry phase that obtains the quantum state of a photon that propagates within an expanding space-time. $a_t$ is just the adiabatic expansion rate and an analogy between the effect and Foucault’s experiment is fully suggested. In this sense, light rays play a similar rôle in the expanding space than Foucault’s Pendulum does while determining Earth’s rotation. On the other hand, one could speculate about a suitable future experimental arrangement at ”laboratory” scales able to measure the local cosmological expansion rate $\dot{R}(0)$ using the procedure outlined in this paper.

Introduction. 

It is not unlikely that the anomalous Doppler drift reported on the Pioneer’s echo signals [1], [2] had a cosmological origin [3], [4] [5][6]. The fact that the figure of the clock acceleration, $a_t$ almost exactly coincides with that of Hubble’s constant requires a theoretical explanation. By this time, however, a commonly held opinion was that such a kind of coincidences can not be assigned to any real cosmological effect since the expansion should only affect to bigger scales than that of the solar system (galaxies or cluster of galaxies, for instance). In General Relativity, as a local theory, holds Birkhoff’s theorem that tells us that in a homogeneous zero-pressure cosmological model we can evacuate a spherical region and replace the material with a compact mass $M$ at the center, without affecting space-time outside the region. The standard view is that derived from the Einstein and Strauss Schwarzschild vacuole in a FLRW background [7],[8] - which, indeed, is unstable by construction.- The inverse of this theorem, however, does not hold, i.e., we can not build a global metric from the ”averaged” set of local metrics - see, for instance, [9]. - It means that Einstein’s field equations can not hold on all scales simultaneously (at least we still do not know how could that ever be done), remaining open the question of the possible influence of the global averaged solution on the compatible local metric. Of course, those influences could never be dynamical -for the same reason that there is no gravitational acceleration inside a hollow of spherical mass distribution. On the other hand, the physical metric can only be

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\footnote{To my wife Rufina and my daughter Ana}
determined empirically upon the measurement of space and time coordinates for non simultaneous events. This requires synchronization of remote clocks, i.e., transmissions of light signals and the actual reception of the reflected ray from a remote "mirror" - in a way entirely similar to the determination of the spacecraft position in Pioneer’s experiment.- Therefore, experiments set up positively, after the precise and beautiful Pioneer’s orbital determination, the possibility to bring into the experimental arena the important question of the physical influence of the expanding space-time background on local scales. The ansatz for our analysis will be

\[ ds^2 = c^2 dt^2 - R(t)^2 (dr^2 + r^2 d\Omega^2) , \]

where, for arbitrary closed light path of total flight time \( T \)

\[ R(0) = R(T) = 1 . \]

The meaning of this constraint is that of a physical re-scaling, i.e., the statement that the speed of light is a local physical constant. Thus, from the measurement of the local speed of light, \( R(t) \) is always in classical accordance with Birkhoff’s theorem, in other words, an observer could never, in principle, determine (locally) the time variation of the scale factor. This statement holds for the classical theory but have to be re-examined on the light of the quantum theory.

The quantum state of the photon. The emitted photon requires a representation for its state in Hilbert space because its trajectory is not an observable. In other words, the photon, does not really evolve in space-time but it is just its quantum state what changes in time. Given that Eq. (2) only states that at the observer’s position there exists no classically observable cosmological gravitational field, we have no right to determine that \( R(t) \equiv 1 \) for all time at different not observable positions of space-time - since these are not empirically known.- From this perspective one would expect to obtain non local effects on the final state of a photon that was emitted and observed in a delayed time. The position representation for the state of a photon of energy \( E \) and momentum \( \vec{p} = (E/c)R(t)\vec{l} \), with \( |\vec{l}| = 1 \), that propagates in the metric given by Eq. (1) is

\[ < r|\phi(t); R(t) > = \frac{1}{2\pi} \exp[i\Psi] , \]

where \( \Psi(r, t) \) is the Eiconal given by

\[ \Psi(r, t) = -\frac{1}{\hbar}(Et - \frac{E}{c}\vec{l} \cdot \vec{r}R(t)) . \]

Evidently, this gives for the group velocity of the waves

\[ \vec{v} = \frac{\partial E}{\partial \vec{p}} = \frac{c}{R} \vec{l} . \]
This is consistent with the values of the speed of light obtained from solving $ds^2 = 0$ from Equation (1). Now, Equation (2) imposes $\dot{r}(0) = \dot{r}(T) = c$ as required from classical theory.

**The Berry Phase.** In terms of the period of the waves $R(0)/\dot{R}(0) \gg \tau = 2\pi \hbar/E$. The state then, adiabatically evolves in a path of the slow parameter $R(t)$ space where $R(0) = R(T)$ holds. The general formulation of the adiabatic evolution in parameter space was given by M. V. Berry [10][11]. The quantum state change, in a closed path, under the influence of the adiabatic parameter obtaining a phase (the "Berry phase") given by

$$< r|\phi(T); R(T) > =< r|\phi(T); R(0) > \exp[i\gamma(r, T)]$$ , (6)

where

$$\gamma = i \int dR < \phi_R|\nabla_R|\phi_R >$$ , (7)

or, if the reflection takes place at $t = T/2$,

$$\gamma = i(\frac{iE}{\hbar c}) \{ \int_0^{T/2} dR \vec{l}_+ \cdot \vec{r} + \int_{T/2}^{T} dR \vec{l}_- \cdot \vec{r} \}$$ , (8)

Now, $\vec{l}_\pm \cdot \vec{r} = \pm r$ and the path integral gives

$$\gamma = \frac{E_{r}}{c\hbar} \{ -R_{0}^{T/2} + R_{T}^{T/2} \} = 2\frac{E_{r}}{c\hbar} \{ \frac{1}{2}(R(0) + R(T)) - R(T/2) \}$$ . (9)

Eq. (2) taken into account we finally obtain

$$\gamma = \frac{2E_{r}}{c\hbar} \{ R(0) - R(T/2) \} \simeq \frac{2E_{r}}{c\hbar} \{ R(0) - [R(0) + \frac{T}{2} \dot{R}(0)] \} = - \frac{E_{r}T \dot{R}(0)}{c\hbar}$$ . (10)

Which is our main result. It depends on the total flight time $T$. The Berry phase is a geometric object, i.e., it is invariant under the election of the path. This can be seen, for instance, if we select, before observation, a light path corresponding to three reflections on mirrors at the space-time coordinates points $P_1 = (T/4, cT/4)$; $P_2 = (T/2, 0)$ and $P_3 = (3T/4, cT/4)$

$$\gamma = i(\frac{iE}{\hbar c}) \{ \int_0^{T/4} dR \vec{l}_+ \cdot \vec{r} + \int_{T/4}^{T/2} dR \vec{l}_- \cdot \vec{r} \} + i(\frac{iE}{\hbar c}) \{ \int_{T/2}^{3T/4} dR \vec{l}_+ \cdot \vec{r} + \int_{3T/4}^{T} dR \vec{l}_- \cdot \vec{r} \}$$ , (11)

this gives

$$\gamma = \frac{E_{r}}{\hbar c} \{ -R_{0}^{T/4} + R_{T/4}^{T/2} - R_{T/2}^{3T/4} + R_{3T/4}^{T} \}$$ , (12)

or

$$\gamma = \frac{2E_{r}}{\hbar c} \{ \frac{1}{2}(R(0) + R(T)) - R(T/4) + R(T/2) - R(3T/4) \}$$ . (13)
which, after (2), leads to
\[
\gamma \simeq \frac{2Er}{\hbar c} \left\{ R(0) - (R(0) + \dot{R}(0) - \frac{T}{4}) + R(0) + \dot{R}(0) - \frac{T}{2} - (R(0) + \dot{R}(0) - \frac{3T}{4}) \right\} = -\frac{ErT \dot{R}(0)}{\hbar c},
\]
that coincides with (10). Of course, one could make indefinitely many additional subdivisions of the path without changing the result. It means that we could speculate about a suitable future experimental arrangement at "laboratory" scales able to measure the local cosmological expansion rate \(\dot{R}(0)\) using the procedure of this example. One does not really need a spacecraft as the classical apparatus (mirror) for the reflection of the photon. Let us now define the constant with dimensions of an acceleration
\[
a_P \equiv \dot{R}(0)c, \tag{15}
\]
so as to write Eq. (10) as
\[
\gamma = -\left(\frac{E}{c^2}\right) \frac{a_P r}{\hbar} T . \tag{16}
\]
Hence, the final state of the photon is given by
\[
<r|\phi(T); R(T) > = \frac{1}{2\pi} \exp\left\{ \frac{i}{\hbar} [(E + \frac{E}{c^2} a_P r)T - \frac{E}{c} r] \right\} . \tag{17}
\]
Now, the measurable Hamiltonian at time \(T\) is
\[
i\hbar \frac{\partial}{\partial T} < r|\phi(T) > = H < r|\phi(T) > , \tag{18}
\]
for
\[
H(r) = E + \frac{E}{c^2} a_P r ; \tag{19}
\]
while, for the measurable momentum we get
\[
-ih\nabla_r < r|\phi(T) > = \vec{p} < r|\phi(T) > , \tag{20}
\]
where
\[
\vec{p} = \left(\frac{E}{c} - \frac{E}{c^2} a_P T\right) \vec{l}. \tag{21}
\]
**Physical interpretation.** Given that the Berry phase is geometric, no new force is involved; nevertheless, we could try to wrongly translate into the language of classical physics our results. Let us consider, from Eq. (21)
\[
E \simeq pc(1 + \frac{a_P T}{c}) + O(\frac{a_P T}{c})^2 , \tag{22}
\]
so that the Hamiltonian in Eq. (19) be written as

\[ H(r, p; T) = pc(1 + \frac{a_p T}{c}) + pr \frac{a_p}{c} + O\left(\frac{a_p T}{c}\right)^2 . \]  

(23)

We can now derive a measurable blue shift in the frequency of the vector state very easy; to see how, we notice that the energy changes in time according to

\[ \dot{H} = \frac{\partial}{\partial T} H(r, p; T) = pa_p , \]  

(24)
on the other hand, we get, for the momentum

\[ \dot{p} = -\nabla_r H(r, p; T) = -p \frac{a_p}{c} . \]  

(25)

Now, recall \( H = \hbar \omega \) and \( p = \hbar k \), so that, Eq. (24) and (25) obtain equivalently

\[ \dot{\omega} = a_p k , \]  

(26)

\[ \dot{k} = -\frac{a_p}{c} k . \]  

(27)

That is

\[ \dot{\omega} + c \dot{k} = 0 , \]  

(28)
or

\[ \omega(T) + ck(T) = \omega(0) + ck(0) . \]  

(29)

Solving together Eq. (26) to Eq. (29), we get

\[ k(T) = k(0)(1 - \frac{a_p T}{c}) , \]  

(30)

and, after Eq. (29)

\[ \omega(T) = \omega(0) + ck(0) \frac{a_p T}{c} = \omega(0)(1 + a_t T) . \]  

(31)

Where \( \omega(0) = ck(0) \) was used. This is the reported Pioneer blue shift anomaly, correspondig to an apparent clock acceleration given by

\[ a_t = a_p / c = \dot{R}(0) . \]  

(32)

An important result can also be derived with respect to the effective velocity of light. Notice that

\[ \dot{r} = \frac{\partial}{\partial p} H(r, p; T) = c(1 + \frac{a_p T}{c}) + \frac{a_p}{c} r , \]  

(33)

whose solution is

\[ \dot{r}(T) = c(1 + 2 \frac{a_p T}{c}) . \]  

(34)
obtaining
\[ \omega(T) = \dot{r}(T)k(T) \] .

These expressions are compatible with other models that derive the Pioneer anomaly like Ranada’s (see [12] and references therein); this author claims that, owing to the existence of quantum vacuum fluctuations and the cosmological expansion of space, the velocity of light increases with time in a way analogously to the expression given above for \( \dot{r} \). Ranada uses such a kind of dependence in time to solve Maxwell wave equation and derives the following compatibility condition between \( k, \dot{r}(T) \) and the frequency \( \omega \) (in our notation)
\[ k = \frac{\omega}{\dot{r}}(1 + \frac{\dot{\omega}T}{\omega}) \] .

We might see that our conclusions are compatible with Ranada’s empirical interpretation of the effect since our derived conclusions are particular solutions from his compatibility condition. Nonetheless, Ranada physical solution is different from the one given here in that it implies that observations of the wavelength fail to find any effect, contrary to the solution given in this paper that predicts a time variation given by
\[ \lambda(T) = \lambda(0)(1 + \frac{a_rT}{c}) \] .

**Conclusions.** From the point of view of this paper, the "Pioneer effect" detected in radar signals [1], [2], [4] should have nothing to do with the probe but only with the fact that the spacecraft is acting as a "mirror" for light signals, thus, being the classical apparatus of a quantum system that is locally being monitored by the global expanding space-time metric. Of course, this can only be obtained for a photon so there is not physical acceleration of any kind. This important feature of the effect finally explains why planets are not sensible to that acceleration. The Doppler anomalous phase shift finds its explanation on a Berry phase, a geometrical effect. A non dynamical element of the quantum evolution in the expanding space background. This demonstrates entirely that the effect should not affect to the planets but only to light and that it is wrongly interpreted as a dynamical acceleration, being fully equivalent to a calibration effect similarly to the Foucault Pendulum angle defect in measuring Earth rotation (the Hannay’s angle which, indeed, is the classical analog to the Berry quantum phase, also a geometrical effect). In this sense, light rays play a similar rôle in the expanding space than Foucault’s Pendulum does while determining Earth’s rotation. On the other hand it also relates the effect to the very non-locality of quantum mechanics since it is just the geometry of the Universe what monitors the quantum state of the photon. The result has nothing to do with dynamics and, therefore, it does not violate Birkhoff’s theorem. Moreover, the measurable anomaly only depends on the "Time of Flight" of the photon, since
it has also been demonstrated that it does not really depends on the location of the spacecraft, for instance, a geostationary system of satellites would obtain the same result in case that the time of flight were to last enough (up to obtaining a similar optical resolution to that of the Pioneer experiment.) But, this is standard physics. No new physics is involved; the very result has nothing to do with the probe but only with non-local properties of the quantum state.

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