Curvaton dynamics in brane-worlds

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Abstract. We study the curvaton dynamics in brane-world cosmologies. Assuming that the inflaton field survives without decay after the end of inflation, we apply the curvaton reheating mechanism to the Randall–Sundrum model and to its curvature corrections: Gauss–Bonnet, induced gravity and combined Gauss–Bonnet and induced gravity cosmological models. In the case of chaotic inflation and requiring suppression of possible short-wavelength generated gravitational waves, we constrain the parameters of a successful curvaton brane-world cosmological model. If density perturbations are also generated by the curvaton field, then the fundamental five-dimensional mass could be much lower than the Planck mass.

Keywords: cosmology with extra dimensions, inflation, cosmological applications of theories with extra dimensions

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1. Introduction

Inflation is an indispensable part of the hot big bang cosmology, solving long standing problems of standard cosmology, such as homogeneity, isotropy and flatness of the universe [1]. Moreover, it generates superhorizon fluctuations which become classical after crossing out the Hubble horizon and seed the matter and radiation fluctuations observed in the universe after re-entry of the horizon at the end of the inflation. The microwave anisotropy encodes information from this early inflationary phase of the cosmological evolution. The three-year WMAP result as well as other astronomical data [2] supports the inflation scenario of the hot big bang cosmology (for a review see [3]).

The results of WMAP three-year data presented in [2] do not favour a scale-invariant spectrum of fluctuations giving the value of the index of the power spectrum as \( n_s = 0.951^{+0.015}_{-0.019} \). Also, these results favour the simple chaotic inflation model with potential \( m^2 \phi^2 \) which fits the observations very well. The WMAP three-year data provides then significant constraints on the inflation models and has ruled out some of them [4].

In spite of many phenomenological successes of inflation, there are nevertheless many serious problems that remain to be understood, such as the initial cosmological singularity problem, the trans-Planckian problem, and problems concerning the very existence of the scalar field, the inflaton, driving inflation. The general belief is that these problems can only be addressed in a more general framework than effective field theory, such as string theory, the only self-consistent theory until now.

Recently effort was spent in addressing the problem of inflation in brane-world models. The most successful model that incorporates the idea that our universe lies in a three-dimensional brane within a higher dimensional bulk spacetime is the Randall–Sundrum model of a single brane in an AdS bulk [5]. There are also other brane cosmological models which are merely curvature generalizations of the Randall–Sundrum model and give novel features compared to standard cosmology. The induced gravity cosmological model [6] arises when we add to the brane action a four-dimensional scalar curvature term, while the Gauss–Bonnet model [7] arises when we include a Gauss–Bonnet term in the five-dimensional action. Finally, if both terms are included in the action, the combined cosmological model [8] describes their cosmological evolution (for reviews on the brane-world cosmological models see [9]).

All these brane-world inflationary models have in the high energy limit correction terms in their Friedmann equations. These terms have important consequences in the inflationary dynamics. In the case of steep inflation [10], the exponential potential is so steep that it would not drive inflation in the standard cosmology, but it may do so in the presence of these corrections terms. In the light of the WMAP three-year data, the chaotic inflation brane-world models with simple quadratic inflaton potentials are more favoured, giving a value for the index of the power spectrum close to observations [11]. In these models the correction terms assist inflation damping the kinetic energy of the inflaton field. However, as the energy density decreases, these corrections become unimportant, and the inflaton field enters a kinetic energy dominated regime, bringing inflation to an end. As the inflaton may survive this process without decay, an alternative reheating mechanism is required.

The curvaton reheating mechanism [12] was proposed as an alternative mechanism to complement the other two known mechanisms, the conventional decay of the inflaton
energy density into ordinary matter [13] and the mechanism of gravitational particle production at the end of inflation [14]. The curvaton scenario was firstly suggested as an alternative mechanism for generating the primordial scalar perturbation which is responsible for the structure formation. In this scenario the primordial density perturbation originates from the vacuum fluctuation of some ‘curvaton’ field $\sigma$, different from the inflaton field [15]. In brane-worlds the curvaton reheating mechanism was employed in [16], where it was shown that it can overcome the problems encountered in the reheating process via gravitational particle production [10,14,17] in steep inflation, allowing a high reheat temperature and preventing short-wavelength gravitational wave dominance [18].

If the inflaton field is to survive without decay at the end of inflation, then it may play the role of the quintessence field [19]. In this case the inflaton field enters a long kinetic epoch and an alternative mechanism for the reheating is required. Thus in all quintessential inflation models [20] the need of the curvaton field proves to be essential. Moreover, the quintessential inflation can also be applied to brane-worlds [21].

In this work we will study in a systematic way the curvaton dynamics in brane-world cosmological models. The curvaton dynamics can be introduced in brane-worlds in various ways. In their high energy limit, depending on the parameters of the model, we have transitions, as the energy density decreases, from one dimensionality spacetime to another of different dimensionality. If inflation occurs in one spacetime and survives without decay the curvaton reheating can occur in the same dimensionality spacetime or in a different dimensionality one. For example, in the induced gravity model there is a choice of parameters for which the universe can start in four dimensions pass from a five-dimensional phase and then again end up in four dimensions before nucleosynthesis [22,23].

From the recent observation data there is no compelling reason for deciding whether the primordial density perturbations originate from the vacuum fluctuations of an inflaton or curvaton field. If the curvaton field is employed for both reheating and primordial density perturbations, since it is not directly linked with the high energy scales of the inflaton field, it can be associated with lower energy particles of the TeV region [24]. We will show that in brane-worlds if the density perturbations are generated by the curvaton field then the energy scale of the fifth dimension can be much below the $M_{Pl}$ scale.

Another important constraint that all the curvaton models should satisfy in the case the inflaton survives without decay is that short-wavelength gravitational waves generated during the kinetic epoch should not dominate over radiation. In all the cases we consider, we will derive constraints which the parameters should satisfy in order to suppress the gravitational radiation dominance during the kinetic period.

To capture the curvaton dynamics, we will develop a general curvaton formalism for two different cosmological regimes followed one another and which are characterized by different Friedmann equations, corresponding to two different dimensionality spacetimes in brane-world cosmologies. Constraints on the curvaton parameters in these two regimes will be derived and also constrained relations on the parameters will be extracted from the requirement of not having gravitational waves dominance. Then, this general formalism for quadratic potentials for both the inflaton and curvaton field will be applied to Randall–Sundrum, Gauss–Bonnet, induced gravity and combined Gauss–Bonnet and induced gravity cosmological models. We will get bounds on the various parameters first with the assumption that the inflaton field is responsible for the primordial density perturbations
and the curvaton field for the reheating and second with the assumption that both density perturbations and reheating are generated by the curvaton field.

2. General curvaton formalism

We assume that the cosmological evolution of the universe is described by two different cosmological regimes characterized by the following general Friedmann equations:

\[ H_1^2 = \beta_1 \rho_1^{\alpha_1}, \]
\[ H_2^2 = \beta_2 \rho_2^{\alpha_2}, \]

where \( \beta_1 \) and \( \beta_2 \) are proportional to Newton’s constants in the two regimes and the powers \( \alpha_1 \) and \( \alpha_2 \) may take values that give conventional or unconventional energy densities. The transition from the one regime to the other occurs at

\[ \rho_{1,2} = \left( \frac{\beta_2}{\beta_1} \right)^{1/(\alpha_1 - \alpha_2)}, \]
\[ H_{1,2} = \beta_2^{1/2} \left( \frac{\beta_2}{\beta_1} \right)^{\alpha_2/(2(\alpha_1 - \alpha_2))}. \]

We consider on the brane two scalar fields, the inflaton field \( \phi \) and a curvaton field \( \sigma \) with no interactions between them. Their energy densities are given by

\[ \rho_{\phi} = \frac{\dot{\phi}^2}{2} + V(\phi), \]
\[ \rho_{\sigma} = \frac{\dot{\sigma}^2}{2} + U(\sigma), \]

with their potentials of the form

\[ V(\phi) = \delta \phi^7, \]
\[ U(\sigma) = \frac{1}{2} m^2 \sigma^2. \]

The equations of motion of the inflaton and curvaton fields are

\[ \ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \]
\[ \ddot{\sigma} + 3H\dot{\sigma} + U'(\sigma) = 0. \]

Providing that the curvaton field is responsible for the reheating of the universe, the curvaton reheating mechanism can proceed in three different ways:

1st case. The curvaton oscillates and decays into radiation during the regime 1. Nucleosynthesis takes place during the regime 2.

2nd case. The curvaton oscillates during the regime 1 but decays into radiation in the regime 2.

3rd case. The curvaton oscillation and decay take place during the regime 2.
Depending on whether the curvaton becomes the dominant component before or after decay we will have to consider two subcases for each of the above three cases:

1a. \( m > H_{eq1} > \Gamma > H_{1,2} > H_{nucl} \)
1b. \( m > \Gamma > H_{eq1} > H_{1,2} > H_{nucl} \)
2a. \( m > H_{1,2} > H_{eq2} > \Gamma > H_{nucl} \)
2b. \( m > H_{1,2} > \Gamma > H_{eq2} > H_{nucl} \)
3a. \( H_{1,2} > m > H_{eq2} > \Gamma > H_{nucl} \)
3b. \( H_{1,2} > m > \Gamma > H_{eq2} > H_{nucl} \)

where \( m \) denotes the mass of the curvaton at the moment when it starts to oscillate, \( H_{eq1} \) and \( H_{eq2} \) are the Hubble parameters at the moment when the curvaton field or its decay products starts to dominate over the inflaton field during the regime 1 or 2 respectively, \( \Gamma \) is the decay parameter of the curvaton field and \( H_{nucl} \) the Hubble parameter at the moment when nucleosynthesis starts. We have to consider two subcases depending on whether the curvaton field comes to dominate in the universe before or after it decays as radiation. In both subcases the crucial moment is when the energy density of the inflaton field becomes equal to the energy density of the curvaton field \( \rho_{\sigma} = \rho_{\phi} \) at the moment of curvaton domination \( a = a_{eq1,2} \). In the following we will distinguish these two different subcases and we will find constrained relations between the three parameters \( m, \Gamma, \sigma_i \), with \( \sigma_i \) the initial value of the curvaton field, for each of the above cases separately.

2.1. Curvaton domination before decay

2.1.1. First case: oscillation and decay in the first regime. After the inflationary period begins there is a kinetic period (labelled by ‘kin’) where the universe is still dominated by the inflaton field where \( \frac{1}{2} \dot{\phi}^2 \gg V(\phi) \) and behaves as stiff matter \((w = 1)\rho_{\phi} \propto a^{-6}\). Therefore the energy density of the inflaton field evolves as

\[
\rho_{\phi} = P_{\phi}^{(kin)} \left( \frac{a_{kin}}{a} \right)^6,
\]

and using (2.1) the Hubble parameter evolves as

\[
H = H_{kin} \left( \frac{a_{kin}}{a} \right)^{3\alpha_1},
\]

where

\[
H_{kin}^2 = \beta_1 P_{\phi}^{(kin)\alpha_1}.
\]

During this stage the curvaton field is effectively a constant keeping its initial value \( \sigma_i \).

At some later time the curvaton field starts to oscillate (labelled by ‘osc’). Using (2.10) we can see that this happens when \( H \simeq m \) while in this case the universe still evolves in the regime 1. In order to avoid a stage of curvaton driven inflation the universe should still be dominated by the inflaton field which means \( \rho_{\sigma}^{(osc)} \ll \rho_{\phi}^{(osc)} \). At this moment we have

\[
\rho_{\sigma}^{(osc)} = \frac{1}{2} m^2 \sigma_i^2,
\]

\[
\rho_{\phi}^{(osc)} = \beta_1^{-1/\alpha_1} m^{2/\alpha_1},
\]
which implies the following constraint on the initial value of the curvaton field:
\[
\sigma_i^2 \ll 2 \beta_1^{-1/\alpha_1} m^{(2-2\alpha_1)/\alpha_1}.
\] (2.16)

The inflaton still decays as stiff matter
\[
\rho_{\phi} = \rho_{\phi}^{(\text{kin})} a_{\text{kin}}^6 \frac{a_{\text{osc}}^6}{a^6}
\] (2.17)
and using (2.1) the Hubble parameter evolves as
\[
H = H_{\text{kin}} a_{\text{kin}}^{3/\alpha_1} a_{\text{osc}}^{3/\alpha_1},
\] (2.18)
where \(H_{\text{kin}}^2\) is given by (2.13). The energy of the curvaton field decays as non-relativistic matter \((w = 0)\):
\[
\rho_{\sigma} = \rho_{\sigma i} a_{\text{osc}}^3 a_{\text{kin}}^3.
\] (2.19)

Then using (2.17), (2.19) and (2.13) we find that
\[
\frac{1}{2} m^2 \sigma_i^2 \beta_1^{1/\alpha_1} H_{\text{kin}}^{-2/\alpha_1} a_{\text{osc}}^3 a_{\text{kin}}^3 = a_{\text{kin}}^3 a_{\text{eq1}}^3.
\] (2.20)

Moreover from (2.12) we have
\[
\frac{a_{\text{osc}}^3}{a_{\text{kin}}^3} = \frac{H_{\text{kin}}^{1/\alpha_1}}{m^{1/\alpha_1}},
\] (2.21)
obtaining
\[
\frac{1}{2} m^{2-1/\alpha_1} \sigma_i^2 \beta_1^{1/\alpha_1} H_{\text{kin}}^{-1/\alpha_1} = a_{\text{kin}}^3 a_{\text{eq1}}^3.
\] (2.22)

From (2.18) we finally conclude that
\[
H_{\text{eq1}} = \frac{\beta_1}{2 \alpha_1} \sigma_1^{2 \alpha_1} m^{2 \alpha_1 - 1}.
\] (2.23)

We can easily see that if in the regime 1 we have the conventional four-dimensional cosmological evolution, which corresponds to \(\alpha_1 = 1\) and \(\beta_1 = 1/3M_{\text{Pl}}^2\), we find
\[
H_{\text{eq1}} = \frac{\sigma_1^2}{6 M_{\text{Pl}}^2} m.
\] (2.24)

2.1.2. Second case: oscillation in the first regime and decay in the second regime. In this case, during the oscillation of the curvaton field the universe undergoes a transition period from the regime 1 to the regime 2. Therefore the energy density of the inflaton field evolves as
\[
\rho_{\phi} = \rho_{\phi}^{(\text{kin})} a_{\text{kin}}^6 \frac{a_{\text{osc}}^6}{a^6} a_{1,2}^6
\] (2.25)
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and using (2.1) and (2.2) the Hubble parameter evolves as

$$H = H_{\text{kin}} a_{\text{kin}}^{3(\alpha_1 - \alpha_2)} a_{\text{osc}}^{3\alpha_2} a_{1.2}^{3\alpha_2},$$  \hspace{1cm} (2.26)

where (2.13) still holds. The energy of the curvaton field decays as non-relativistic matter ($w = 0$):

$$\rho_\sigma = \rho_\sigma a_{\text{osc}}^3 a_{1.2}^3 a_3^3.$$  \hspace{1cm} (2.27)

Then for subcase 2a we need the value of $H_{eq2}$ when $\rho_\sigma = \rho_\phi$ at $a = a_{eq2}$. Here we can rewrite equation (2.26) as

$$H = H_{\text{kin}} a_{\text{kin}}^{3(\alpha_1 - \alpha_2)} a_{\text{osc}}^{3\alpha_2} a_{eq2}^{3\alpha_2}.$$  \hspace{1cm} (2.28)

Using (2.25), (2.27) and (2.13) we find as in case 1a that

$$\frac{1}{2} a_{\text{kin}}^2 a_{1.2}^2 a_3^2 \beta_1^{1/\alpha_1} H_{kin}^{-1/\alpha_1} = \frac{a_3^3}{a_{eq2}^3}.$$  \hspace{1cm} (2.29)

Moreover from (2.26) we have

$$\frac{a_{\text{kin}}^3}{a_{1.2}^3} = \frac{H_{k_1}^{1/\alpha_1}}{H_{k_2}^{1/\alpha_1}}.$$  \hspace{1cm} (2.30)

and we find that

$$H_{eq2} = \frac{\sigma_1^{2\alpha_2}}{2\sigma_2} m^{2\alpha_2 - (\alpha_2/\alpha_1)} \beta_1^{2\alpha_2/\alpha_1} \beta_2^{1/2}.$$  \hspace{1cm} (2.31)

2.1.3. Third case: oscillation and decay in the second regime. The results are the same as in the first case replacing all indices 1 by 2. Thus we have

$$H_{eq2} = \frac{\sigma_1^{2\alpha_2}}{2\sigma_2} m^{2\alpha_2 - (\alpha_2/\alpha_1)} \beta_1^{2\alpha_2/\alpha_1} \beta_2^{1/2}.$$  \hspace{1cm} (2.32)

2.2. Curvaton decay before domination

2.2.1. First case: oscillation and decay in the first regime. In this section 2.2 we consider the case where the curvaton field starts decaying as radiation before it comes to dominate the inflaton field. Therefore the universe is still dominated by the inflaton field and for subcase 1b its energy density evolves as

$$\rho_\phi = \rho_\phi^{(\text{kin})} a_{\text{kin}}^6 a_{\text{osc}}^6.$$  \hspace{1cm} (2.33)
and using (2.1) the Hubble parameter evolves as

\[ H = H_{\text{kin}} \frac{a_{\text{kin}}^{3/3\alpha_1} a_{d}^{3/3\alpha_1} a_{\text{osc}}^{3/3\alpha_1}}{a_{d}^{3/3\alpha_1} a_{d}^{3/3\alpha_1}}, \tag{2.34} \]

where ‘d’ labels the quantities at the time of curvaton decay. Here (2.13) still holds. The energy of the curvaton field decays as radiation (\(w = 1/3\)):

\[ \rho_\sigma = \rho_{\sigma_1} a_{\text{osc}}^{4/3} a_{d}^{4/3} a_{d}^{4/3}. \tag{2.35} \]

At decay (\(H = \Gamma\)) we have

\[ \frac{\Gamma}{H_{\text{kin}}} = \frac{a_{\text{kin}}^{3/3\alpha_1}}{a_{d}^{3/3\alpha_1}} \tag{2.36} \]

and using (2.33), (2.35) and (2.13) we find that

\[ \frac{1}{2} m^{2-1/\alpha_1} \sigma_1^{2/\alpha_1} \beta_1^{1/3\alpha_1} H_{\text{kin}}^{2/3\alpha_1} \Gamma^{-1/3\alpha_1} = \frac{a_{\text{kin}}^{2/3\alpha_1}}{a_{\text{eq}}^{1/3\alpha_1}}. \tag{2.37} \]

Then from (2.34) we finally conclude that

\[ H_{eq_1} = \left( \frac{\beta_1}{2\alpha_1} \right)^{3/2} \sigma_1^{3\alpha_1} m^{3\alpha_1-3/2} \Gamma^{-1/2}. \tag{2.38} \]

2.2.2. Second case: oscillation in the first regime and decay in the second regime. In this case, one has to pay attention to the fact that at decay relation (2.36) does not hold but

\[ \frac{a_d}{a_{\text{kin}}} = \frac{\beta_2^{1/6\alpha_2} H_{\text{kin}}^{1/3\alpha_1}}{\beta_1^{1/6\alpha_2} \Gamma^{1/3\alpha_2}}. \tag{2.39} \]

As previously we obtain

\[ \frac{1}{2} m^{2-1/\alpha_1} \sigma_1^{2/\alpha_1} \beta_1^{5/6\alpha_1} H_{\text{kin}}^{-2/3\alpha_1} \beta_2^{1/6\alpha_2} \Gamma^{-1/3\alpha_2} = \frac{a_{\text{kin}}^{2/3\alpha_1}}{a_{\text{eq}}^{1/3\alpha_1}}. \tag{2.40} \]

and finally

\[ H_{eq_2} = \frac{a_{\text{kin}}^{3/2\alpha_2} m^{3\alpha_2-3\alpha_2/2\alpha_1}}{2^{3/2\alpha_2}} \beta_1^{3\alpha_2/4\alpha_1} \beta_2^{3/4} \Gamma^{-1/2}. \tag{2.41} \]

2.2.3. Third case: oscillation and decay in the second regime. The results are the same as in the first case replacing all indices 1 by 2. Thus we have

\[ H_{eq_2} = \left( \frac{\beta_2}{2\alpha_2} \right)^{3/2} \sigma_1^{3\alpha_2} m^{3\alpha_2-3/2} \Gamma^{-1/2}. \tag{2.42} \]
2.3. Gravitational waves and curvaton dynamics

Gravitational waves behave as massless scalar fields and their amplitude remains constant during inflation; therefore we can identify the amplitude of the gravitational waves $h_{GW}^2$ with the amplitude of the scalar–tensor perturbations $A_T^2$ in the weak field approximation. During the kinetic epoch, the background is dominated by the inflaton field, and thus the energy density of the background is that of the inflaton scalar field which decays as stiff matter ($\propto a^{-6}$) and the gravitational waves ($\propto a^{-4}$) evolve as:

$$\rho_g = \frac{32}{3\pi} h_{GW}^2 \rho_{\phi} \left( \frac{a}{a_{\text{kin}}} \right)^2.$$  \hfill (2.43)

We will discuss first the case of curvaton domination before it decays. At the epoch of stiff scalar matter equality with curvaton matter ($\rho_\sigma = \rho_{\phi}$), we have $\rho_g \ll \rho_\sigma$. From equation (2.43) with the use of equation (2.22) for cases 1a, 2a, 3a, we obtain the following constraint:

$$\frac{\rho_g}{\rho_\sigma} \bigg|_{a=a_{\text{eq}}} = \frac{32}{3\pi} h_{GW}^2 \left( \frac{2 H_{\text{kin}}^{1/\alpha_1}}{m^{2-1/\alpha_1} \beta_1^{1/\alpha_1} \sigma_1^2} \right)^{2/3} \ll 1,$$ \hfill (2.44)

where for the case 3a the replacement of indices 1 with 2 is understood. Then the curvaton decays before nucleosynthesis and we have the following constraint for the decay parameter $\Gamma$:

$$H_{\text{nucl}} = 10^{-40} M_{\text{Pl}} < \Gamma < H_{eq}.$$  \hfill (2.45)

Next we consider the case of the decay of the curvaton before it dominates the universe. We will first discuss the case 2b where kination and oscillation occur during the 1 regime while the curvaton decays during the 2 regime. The curvaton field decays producing radiation at the time when $\Gamma = H$ and we have (2.39). The radiation produced evolves as:

$$\rho^{(\sigma)} = \rho_{\phi} a_{\text{osc}}^3 \frac{a_\text{d}^3}{a^4}.$$ \hfill (2.46)

Then, using the expression (2.25) for the evolution of the inflaton field we find that:

$$\frac{a_{\text{kin}}^2}{a_\text{d}^2} = \frac{1}{2} m^{2-1/\alpha_1} \sigma_1^2 \beta_1^{5/6\alpha_1} H_{\text{kin}}^{-2/3\alpha_1} \Gamma^{-1/3\alpha_2} \beta_2^{1/6\alpha_2}.$$ \hfill (2.47)

The radiative regime is reached when $a = a_\text{d}$ where the energy density of the background is $\rho_{\phi} + \rho_{\text{rad}} \simeq 2\rho_{\text{rad}}$. We used $\rho_{\text{rad}}$ to point out that during the decay phase the curvaton decays as radiation. Then the evolution of the gravitational waves is given by:

$$\rho_g = \frac{64}{3\pi} h_{GW}^2 \rho_{\text{rad}} \left( \frac{a}{a_{\text{kin}}} \right)^2,$$ \hfill (2.48)

Note that the formula (2.43) describes the gravitational radiation in four-dimensional gravity. In brane-worlds the brane is embedded in five-dimensional spacetime. However, we can use the above formula because as was shown in [25] the gravitational waves are localized on the world volume of the four-dimensional brane and the five-dimensional effects are negligible.
where with the use of (2.47) we finally obtain the following constraint from gravitational waves:

$$\rho_k \bigg|_{a = a_{eq}} = \frac{64}{3\pi} \frac{H_{GW}^2}{m^{2-1/\alpha_1}} \frac{2H_{\text{kin}}^{2/3\alpha_1}}{m^{1-1/\alpha_1} \sigma_1^2 \beta_1^{1/3\alpha_2}} \ll 1.$$ (2.49)

For the case 1b this constraint becomes

$$\rho_k \bigg|_{a = a_{eq}} = \frac{64}{3\pi} \frac{H_{GW}^2}{m^{2-1/\alpha_1}} \frac{2H_{\text{kin}}^{2/3\alpha_1}}{m^{1-1/\alpha_1} \sigma_1^2 \beta_1^{1/3\alpha_2}} \ll 1,$$ (2.50)

while for case 3b it is

$$\rho_k \bigg|_{a = a_{eq}} = \frac{64}{3\pi} \frac{H_{GW}^2}{m^{2-1/\alpha_2}} \frac{2H_{\text{kin}}^{2/3\alpha_2}}{m^{1-1/\alpha_2} \sigma_1^2 \beta_1^{1/3\alpha_2}} \ll 1.$$ (2.51)

The constraint on decay parameter $\Gamma$ is now given by

$$H_{eq} < \Gamma < m.$$ (2.52)

3. Curvaton dynamics in the Randall–Sundrum brane-world model

We will apply the general formalism developed in section 2 in the Randall–Sundrum brane-world model [26]. In this model we have two regimes before nucleosynthesis: a five-dimensional regime and a four-dimensional one.

3.1. The Randall–Sundrum brane-world model

The gravitational action of the Randall–Sundrum brane-world model is

$$S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-(5)g} \left( (5)R - 2\Lambda_5 \right) + \frac{1}{2\kappa_4^2} \int_{y=0} d^4x \sqrt{-(4)g} \left( -2\Lambda_4 \right),$$ (3.1)

where $\kappa_5^2 = M_5^{-3} = 8\pi G_5$ is the 5D fundamental gravitational constant while $\kappa_4^2 = M_4^{-2} = 8\pi G_4$ is the effective 4D gravitational constant. $\Lambda_5$ is the cosmological constant of the AdS bulk which is related to its characteristic length scale $l$ by $\Lambda_5 = -6/l^2$ and $\Lambda_4$ is the 4D cosmological constant, while the brane tension is $\lambda = \Lambda_4/\kappa_4^2$. The brane tension is related to the AdS length scale through the fine-tuning relation

$$\lambda = \frac{6}{\kappa_5^2} l,$$ (3.2)

which implies that $l = \kappa_5^2/\kappa_4^2$ indicating that $l$ plays the role of the crossover scale between the 5D and the 4D regimes. The Friedmann equation is given by

$$H^2 = \frac{\kappa_4^2}{3} \rho \left( 1 + \frac{\rho}{2\lambda} \right),$$ (3.3)

which gives rise to two different regimes for the dynamical evolution of the brane-universe:

- the RS regime, when $\rho \gg \lambda$ ($HL \gg 1$),

$$H^2 \simeq \frac{\kappa_4^2}{36} \rho^2,$$ (3.4)

- the 4D regime, when $\rho \ll \lambda$ ($HL \ll 1$),

$$H^2 \simeq \frac{\kappa_4^2}{3} \rho.$$ (3.5)
3.2. Slow-roll inflation in the Randall–Sundrum brane-world model

We will now review the results of the slow-roll inflationary dynamics for the Randall–Sundrum brane-world model \[11\]. We consider an inflaton field $\phi$ with energy density given by (2.5) and obeying the Klein–Gordon equation (2.9). Then the slow-roll conditions for the inflaton field are

$$\frac{1}{2} \dot{\phi}^2 \ll V(\phi),$$

$$\ddot{\phi} \ll 3H \dot{\phi}$$

(3.6)

and the slow-roll parameters, at high energies, are defined as

$$\varepsilon = \frac{1}{2 \kappa_4^2} \left( \frac{V'}{V} \right)^2 \left[ \frac{4\lambda}{V} \right],$$

$$\eta = \frac{1}{\kappa_4^2} \left( \frac{V''}{V} \right) \left[ \frac{2\lambda}{V} \right],$$

(3.7, 3.8)

where terms in brackets express the 5D corrections to general relativity. The number of e-folds is given by

$$N = -\kappa_4^2 \int_{\phi_i}^{\phi} \frac{V}{V'} \left[ \frac{V}{2\lambda} \right] \, d\phi.$$ 

(3.9)

The scalar amplitude and the scalar spectral index generated during inflation have been computed \[11\] (see also \[28\]) and are respectively given by

$$A_s^2 = \frac{\kappa_4^6}{75 \pi^2} \left[ G_{RS}^2(\rho) \right]_{\tilde{k}=aH},$$

$$n_s - 1 = \frac{d \ln A_s^2}{d \ln \tilde{k}} = 2\eta - 6\varepsilon,$$

(3.10, 3.11)

where

$$G_{RS}^2(x) = \left[ \frac{\sqrt{1 + x^2} + 1}{2} \right]^3,$$

(3.12)

is the RS correction and $\tilde{k}$ is the comoving wavenumber. We find at the high energy limit ($\rho \gg \lambda$)

$$A_s^2 = \frac{\kappa_4^6}{75 \pi^2} \frac{V^3}{V'^2} \left[ 1 + \frac{V}{2\lambda} \right]^3.$$ 

(3.13)

Moreover the tensor amplitude and tensor spectral index are respectively given by \[27\]

$$A_T^2 = \frac{32 \kappa_4^4}{75 \pi^2} V \left[ F_{RS}^2(\rho) \right]_{\tilde{k}=aH},$$

$$n_T = \frac{d \ln A_T^2}{d \ln \tilde{k}} \simeq -2\varepsilon,$$

(3.14, 3.15)

with

$$F_{RS}^2(x) = \left\{ \sqrt{1 + x^2} - x^2 \ln \left[ \frac{1}{x} + \sqrt{1 + \frac{1}{x^2}} \right] \right\}^{-1/2}.$$ 

(3.16)
The tensor amplitude (3.14) in the high energy limit becomes

\[ A_T^2 = \frac{32 \kappa_4^4}{75 \pi^2} \left( \frac{V}{2\lambda} \right) \bigg|_{k=aH}, \]  

(3.17)

giving the consistency relation

\[ \frac{A_T^2}{A_s^2} \simeq -\frac{n_T}{2}. \]  

(3.18)

For a chaotic inflation with inflaton potential \( V(\phi) = \frac{1}{2} m_\phi^2 \phi^2 \), \( N = 55 \) and assuming that the density perturbations are generated by the inflaton field using \( A_s \simeq 2 \times 10^{-5} \) the above relations give

\[ m_\phi \simeq \frac{0.32}{(192N + 80)^{5/6}} M_5 \simeq 1.4 \times 10^{-4} M_5, \]  

(3.19)

\[ \lambda \simeq 5.6 \times 10^3 (192N + 80)^5 \frac{m_6^6}{M_4^2} \simeq 4 \times 10^{-38} M_5^6, \]  

(3.20)

\[ \phi_i \simeq \frac{(192N + 80)^{2/3}}{0.57} M_5 \simeq 8.5 \times 10^2 M_5, \]  

(3.21)

\[ \phi_f \simeq 3.8 \times 10^3 M_5, \]  

(3.22)

\[ n_s \simeq 1 - \frac{5}{2N} \simeq 0.9545. \]  

(3.23)

All the above parameters of the RS model except the scalar spectral index are functions of the five-dimensional fundamental mass. To keep the quantum gravity corrections under control we require \( M_5 < 10^{17} \text{ GeV} \). Also note that the value of the scalar spectral index is within the limits of the three-year WMAP data.

### 3.3. Curvaton reheating in the Randall–Sundrum brane-world model

Inflation occurs in the five-dimensional RS regime and depending on which regime the curvaton reheating occurs we have six different cases for the curvaton evolution:

- **Case 1a**: \( H_{\text{RS}} > H_f > m > H_{\text{eq1}} > \Gamma > H_{\text{RS,GR}} > H_{\text{nucl}} \)
- **Case 1b**: \( H_{\text{RS}} > H_f > m > \Gamma > H_{\text{eq1}} > H_{\text{RS,GR}} > H_{\text{nucl}} \)
- **Case 2a**: \( H_{\text{RS}} > H_f > m > H_{\text{RS,GR}} > H_{\text{eq2}} > \Gamma > H_{\text{nucl}} \)
- **Case 2b**: \( H_{\text{RS}} > H_f > m > H_{\text{RS,GR}} > \Gamma > H_{\text{eq2}} > H_{\text{nucl}} \)
- **Case 3a**: \( H_{\text{RS}} > H_f > H_{\text{RS,GR}} > m > H_{\text{eq2}} > \Gamma > H_{\text{nucl}} \)
- **Case 3b**: \( H_{\text{RS}} > H_f > H_{\text{RS,GR}} > m > \Gamma > H_{\text{eq2}} > H_{\text{nucl}} \).

If we apply the general results of section 2 we obtain

\[ H_{\text{RS,GR}} = \frac{\lambda \kappa_5^2}{3} \]  

(3.24)
and for the various cases we have

- **Case 1a:**
  \[ H_{eq1} = \frac{\sigma_1^4}{4} \frac{m^3}{36 M_5^6}, \tag{3.26} \]

- **Case 2a:**
  \[ H_{eq2} = \frac{1}{12} \frac{m \sigma_1^2}{M_5^3} \sqrt{\frac{\lambda m}{3 M_5^3}}, \tag{3.27} \]

- **Case 3a:**
  \[ H_{eq2} = \frac{\kappa_2^2}{6} \sigma_2^2 m, \tag{3.28} \]

- **Case 1b:**
  \[ H_{eq1} = \frac{\sigma_1^6}{8} \frac{m^{9/2}}{216 M_5^6 \Gamma_{1/2}}, \tag{3.29} \]

- **Case 2b:**
  \[ H_{eq2} = \frac{3^{3/4} m^2 \sigma_1^3 \lambda}{216} \left( \frac{m}{\lambda M_5^3} \right)^{1/4} \Gamma_{-1/2}, \tag{3.30} \]

- **Case 3b:**
  \[ H_{eq2} = \frac{\sigma_1^3 m^{3/2}}{6^{3/2} M_5^3} \Gamma_{-1/2}. \tag{3.31} \]

As for the gravitational waves, using (2.44) we have the following constraint for the cases 1a and 2a:

\[
\left. \frac{\rho_g}{\rho_\sigma} \right|_{a=a_{eq}} = \frac{32}{3\pi} \frac{h_{GW}^2}{m^2 \frac{6 H_{\text{kin}}}{m \sigma_1^2 \kappa_2^2}} \frac{2/3}{\Gamma_{-1/2}} \ll 1 \tag{3.32}
\]

and for case 3a the constraint

\[
\left. \frac{\rho_g}{\rho_\sigma} \right|_{a=a_{eq}} = \frac{32}{3\pi} \frac{h_{GW}^2}{m^2 \frac{12}{\sigma_1^2 \kappa_2^2}} \frac{2/3}{\Gamma_{-1/2}} \ll 1. \tag{3.33}
\]

Similarly, using (2.50), (2.49) and (2.51) we obtain respectively for cases 1b, 2b and 3b the constraints

\[
\left. \frac{\rho_g}{\rho_{\text{rad}}} \right|_{a=a_{eq}} = \frac{128}{\pi} \frac{h_{GW}^2}{m \frac{12}{\sigma_1^2 \kappa_2^2}} \frac{\Gamma_{1/3}^2}{H_{\text{kin}}^{2/3}} \ll 1, \tag{3.34}
\]

\[
\left. \frac{\rho_g}{\rho_{\text{rad}}} \right|_{a=a_{eq}} = \frac{128 \times 2^{5/6}}{\pi} \frac{h_{GW}^2}{m^{3/2} \frac{\Gamma_{1/3}^2}{\sigma_1^2 \kappa_2^2 \kappa_3^2 \kappa_4^2}} \ll 1, \tag{3.35}
\]

\[
\left. \frac{\rho_g}{\rho_{\text{rad}}} \right|_{a=a_{eq}} = \frac{512}{\pi} \frac{h_{GW}^2}{m^{3/2} \frac{\Gamma_{1/3}^2}{\sigma_1^2 \kappa_2^2}} \ll 1. \tag{3.36}
\]
3.4. Constraints on the parameters in the Randall–Sundrum brane-world model

In all cases the transition from five to four dimensions using (3.25) happens at

\[ H_{\text{RS,GR}} = 7.5 \times 10^{-39} M_5^3. \]  

(3.37)

Also from the nucleosynthesis constraint \( \Gamma > H_{\text{nucl}} = 10^{-40} M_{\text{Pl}} \) we obtain

\[ \Gamma > 1.2 \times 10^{-21} \, \text{GeV}, \]

(3.38)

for \( M_{\text{Pl}} = 1.2 \times 10^{19} \, \text{GeV} \), while inflation ends at

\[ H_{f} = \frac{\kappa_5^2}{12} m_{\phi}^2 \phi_f^2 = 2.4 \times 10^{-4} \, M_5. \]  

(3.39)

We will analyse in more detail case 3 which is more interesting. The other two cases give similar bounds for the parameters. For case 3 the universe inflates in 5D and the primordial density perturbations are generated by the inflaton field. Looking at the values of the inflaton field (3.21) and (3.22) at the beginning and at the end of inflation we see that during inflation the inflaton field does not change much. Therefore we can assume that it survives without decay when the universe enters the 4D regime in which the curvaton field reheats the universe. In this picture, inflation and the generation of density perturbations are high energy effects, while reheating can occur at lower energy scales.

We find for both cases 3a and 3b the following constraints on the parameters:

\[ \begin{align*}
5.98 \times 10^5 \, \text{GeV} < M_5 &< 10^{17} \, \text{GeV} \\
1.6 \times 10^{-21} \, \text{GeV} < \Gamma &< m < 7.5 \times 10^{12} \, \text{GeV} \\
5.73 \times 10^2 \, \text{GeV} < \sigma_1 &< 3.92 \times 10^{19} \, \text{GeV} \\
2.46 \times 10^{18} \, \text{GeV}^3 &< m\sigma_1^2 < 1.15 \times 10^{52} \, \text{GeV}^3.
\end{align*} \]

(3.40)

The lower bounds of the mass of curvaton field and its initial value indicate that the curvaton field could be identified with a low energy scalar particle.

If the density perturbations are generated by the inflaton field then relations (3.33) and (3.36) give respectively for cases 3a and 3b the following extra constraints on the different parameters:

\[ \begin{align*}
m\sigma_1^{4/3} &> 1.80 \times 10^{-44} M_5^{13/3} \\

m^{3/2} \sigma_1^2 &> 2.85 \times 10^{-43} M_5^{16/3} \Gamma^{1/6}.
\end{align*} \]

(3.41)

(3.42)

One obtains similar constraints on the parameters \( m, \sigma_1, M_5 \) and \( \Gamma \) for cases 1 and 2.

If the density perturbations are not generated by the inflaton then, the mass, the initial and final values of the inflaton fields as a function of \( A_s \) and \( M_5 \) are

\[ \begin{align*}
m_{\phi} & = 9.44 \times 10^{-2} A_s^{2/3} M_5, \\
\phi_i & = 3.31 \times 10 \, A_s^{-1/3} M_5, \\
\phi_f & = 1.02 \times 10 \, A_s^{-1/3} M_5.
\end{align*} \]

(3.43)

If the amplitude of density perturbations \( A_s \) generated by the inflaton field is much lower than its observed value and for fixed values of the inflaton field (in order to keep the successful prediction of the spectral index (3.23)) the fundamental mass \( M_5 \) could get
values much lower than the four-dimensional Planck mass\(^2\). However if the curvaton is to generate the density perturbations additional constraints should be satisfied by the curvaton parameters \([15]\). In the case where curvaton decays after domination, the following constraint should be satisfied:

\[
P_\zeta \simeq \frac{1}{9\pi^2} \frac{H_i^2}{\sigma_i^2}, \tag{3.44}
\]

where \(P_\zeta\) is the Bardeen parameter\(^3\) whose observed value is about \(2 \times 10^{-9}\) and \(H_i\) is the value of the Hubble parameter at curvaton oscillation. Hence for case 3a we find that

\[
m = 3\sqrt{6} \pi M_{Pl} P_\zeta^{1/2}, \tag{3.45}
\]

which gives \(m = 1.24 \times 10^{16} GeV\) for \(P_\zeta = 2 \times 10^{-9}\) which is incompatible with (3.40). Thus for case 3a density perturbations could not have been generated by the curvaton field. In contrast, for cases 1a and 2a density perturbations can be generated by the curvaton field, because the extra constrained relation involving the parameters \(m, \sigma_i\) and \(M_5\) is compatible with the corresponding bounds of the parameters for the cases 1a and 2a. If the curvaton decays when subdominant, the Bardeen parameter is given by [15]

\[
P_\zeta \simeq \frac{r_d^2 H_i^2}{36\pi^2 \sigma_i^2}. \tag{3.46}
\]

Here \(r_d\) is the ratio of curvaton energy density to stiff scalar matter at curvaton decay, which is given for case 3b by [16]

\[
r_d = \left. \frac{\rho_\sigma}{\rho_\phi} \right|_{a=a_d} = \frac{m \sigma_i^2}{6 \Gamma M_{Pl}^2}. \tag{3.47}
\]

This gives the following constraint:

\[
m \sigma_i = 9.22 \times 10^{26} \Gamma^{1/2} GeV^2, \tag{3.48}
\]

which is compatible with the results of (3.40). For cases 1b and 2b we respectively have

\[
r_d = \frac{m^{3/2} \sigma_i^2}{12 \Gamma^{1/2} M_5^{3}}, \tag{3.49}
\]

\[
r_d = \frac{m^{3/2} \sigma_i^2}{12 \Gamma M_5^{3/2}} \frac{H_{RS,GR}^2}{M_{Pl}^2}, \tag{3.50}
\]

which in each case gives an extra constraint that is compatible with the various bounds for cases 1b and 2b. Therefore in all cases b, the scalar perturbations can be generated by the curvaton field.

\(^2\) This opens up the possibility that inflation could occur in a low energy regime [29].

\(^3\) The Bardeen parameter is used to describe the curvature perturbations. If the perturbations are adiabatic, which is usually the case for a single scalar field, then the curvature perturbations are identified with the scalar perturbations through the formula \(A_s^2 = 4P_\zeta/25\).
4. Curvaton dynamics in the Gauss–Bonnet brane-world model

4.1. The Gauss–Bonnet brane-world model

In this section we will study the curvaton dynamics in the Gauss–Bonnet brane-world model which has the following gravitational action:

\[
S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g} \left( \Lambda_5 + \alpha \left( \mathcal{R}^2 - 4 \mathcal{R}^a_{\ b} \mathcal{R}^b_{\ a} + \mathcal{R}^{abcd} \mathcal{R}_{abcd} \right) \right) \\
+ \frac{1}{2\kappa_4^2} \int_{y=0} d^4x \sqrt{-g} (-2\Lambda_4),
\]

(4.1)

where \( \alpha > 0 \) is the Gauss–Bonnet coupling constant. At first order in \( \alpha \), the relation between the 5D cosmological constant of the AdS bulk and its characteristic length scale \( l \) has the form

\[
\Lambda_5 = -\frac{6}{l^2} + \frac{12\alpha}{l^4} \left( 1 - \frac{1}{2\beta} \right),
\]

(4.2)

where \( \beta \equiv 4\alpha/l^2 \). The brane tension \( \lambda = \Lambda_4/\kappa_4^2 \) is related to the AdS length scale through the fine-tuning relation

\[
\lambda = \frac{6}{\kappa_5^2 l} \left( 1 - \frac{1}{3\beta} \right),
\]

(4.3)

implying that \( l = \kappa_5^2 \beta / \kappa_4^2 (1 + \beta) \) which can be considered as a scale of crossover between the 5D and the 4D regimes.

The Friedmann equation of the Gauss–Bonnet brane model is given by

\[
\kappa_5^2 (\rho + \lambda) = \frac{2}{l} \sqrt{1 + H^2 l^2} (3 + \beta (2H^2 l^2 - 1)),
\]

(4.4)

which, assuming that the GB term represents correction to the Einstein–Hilbert term, i.e. \( \beta \ll 1 \), gives rise to three different regimes for the dynamical evolution of the brane-universe. We use a characteristic Gauss–Bonnet energy scale

\[
m_\beta = \left[ \frac{8(1 - \beta)^3}{l^2 \beta \kappa_5^2} \right]^{1/8}
\]

(4.5)

and we find that:

- at the GB high energy regime, \( \rho \gg m_\beta^4 \) \((HI \gg \beta^{-1} \gg 1)\),

\[
H^2 \approx \left[ \frac{\kappa_5^2}{4\beta l^2} \rho \right]^{2/3},
\]

(4.6)

- at an intermediate RS regime, \( m_\beta^4 \gg \rho \gg \lambda \) \((\beta^{-1} \gg HI \gg 1)\),

\[
H^2 \approx \frac{\kappa_5^2}{6\lambda \rho^2},
\]

(4.7)

- at the GB low energy regime (GR limit), \( \rho \ll \lambda \) \((\beta^{-1} \gg 1 \gg HI)\),

\[
H^2 \approx \frac{\kappa_5^2}{3} \rho.
\]

(4.8)
4.2. Slow-roll inflation in the Gauss–Bonnet brane-world model

We will now review the results of the slow-roll inflationary dynamics for the Gauss–Bonnet brane-world model [30]. If we define a new variable \( \chi \) as

\[
\kappa_5^2 (\rho + \lambda) = \frac{2}{l} \left[ \frac{2(1 - \beta)^3}{\beta} \right] \sinh \chi, \tag{4.9}
\]

then the Friedmann equation (4.4) can be written in the particularly simple form [31]

\[
H^2 = \frac{1}{\beta l^2} \left[ (1 - \beta) \cosh \left( \frac{2 \chi}{3} \right) - 1 \right]. \tag{4.10}
\]

As previously we consider an inflaton field \( \phi \) with energy density given by (2.5) and obeying the Klein–Gordon equation (2.9). Since during slow-roll inflation \( V \approx \rho \gg \lambda \) then from (4.5) and (4.9) we have that

\[
V = m_4^4 \sinh \chi. \tag{4.11}
\]

The slow-roll parameters, at high energies, for the inflaton field satisfying the slow-roll conditions (3.6) are

\[
\varepsilon_{\text{GB}} = \varepsilon_{\text{RS}} \frac{2(1 - \beta)^4 \sinh(2/3) \chi \tanh \chi \sinh^2 \chi}{9(1 + \beta)(3 - \beta)[(1 - \beta) \cosh(2/3) \chi - 1]^2}, \tag{4.12}
\]

\[
\eta_{\text{GB}} = \eta_{\text{RS}} \frac{2(1 - \beta)^3 \sinh^2 \chi}{3(1 + \beta)(3 - \beta)[(1 - \beta) \cosh(2/3) \chi - 1]}, \tag{4.13}
\]

where \( \varepsilon_{\text{RS}} \) and \( \eta_{\text{RS}} \) are the RS slow-roll parameters (3.7) and (3.8). In the limit \( \chi \ll 1 \) we have \( \varepsilon_{\text{GB}} \rightarrow \varepsilon_{\text{RS}} \) and \( \eta_{\text{GB}} \rightarrow \eta_{\text{RS}} \) recovering the RS results. The number of e-folds is

\[
N \approx -3 \int_{\chi_i}^{\chi_f} \frac{H^2}{dV/d\chi} \left( \frac{d\phi}{d\chi} \right)^2 d\chi, \tag{4.14}
\]

where \( \chi_f \) is evaluated at the end of inflation (\( \varepsilon = 1 \)) and \( \chi_i \) is evaluated when cosmological scales leave the horizon. This latter parameter is constrained by a quantum gravity upper limit which requires that \( V < (8\pi M_5)^4 \) thus yielding

\[
\sinh \chi_i \left( \frac{8\pi M_5}{m_\beta} \right)^4. \tag{4.15}
\]

The scalar amplitude and the scalar spectral index generated during inflation have been computed [32] and are respectively given by

\[
A_s = \frac{\kappa_6^4}{75\pi^2} \frac{V^3}{V''} \left[ G_{\text{GB}}^2 (H l) \right]_{k = a H}, \tag{4.16}
\]

\[
n_s - 1 = \frac{d \ln A_s^2}{d \ln k} = 2\eta - 6\varepsilon; \tag{4.17}
\]

where

\[
G_{\text{GB}}^2(x) = \left[ \frac{3(1 + \beta) x^2}{2\sqrt{1 + x^2(3 - \beta + 2\beta x^2) + 2(\beta - 3)}} \right]^3. \tag{4.18}
\]
is the GB correction and \( \tilde{k} \) is the comoving wavenumber. We then find at the GB regime
\[
A_s^2 = \frac{\kappa_5^6}{75\pi^2} \frac{V^3}{V''} \left[ \frac{27}{64} \left( \frac{1 + \beta}{\beta} \right)^3 \frac{1}{(Hl)^3} \right].
\]
(4.19)

Moreover the tensor amplitude and tensor spectral index are respectively given by [32]
\[
A_T^2 = \frac{32\kappa_4^4}{75\pi^2} V[F_{GB}(Hl)]_{\tilde{k}=aH},
\]
(4.20)
\[
n_T \equiv \frac{d \ln A_T^2}{d \ln \tilde{k}} \simeq -2\varepsilon \left[ 1 - \frac{(Hl)^2 F_{GB}^2(Hl) \{1 - (1 - \beta) \sqrt{1 + x^2} \sinh^{-1} (Hl)^{-1}\}}{(1 + \beta) \sqrt{1 + x^2}} \right],
\]
(4.21)
with
\[
F_{GB}^{-2}(x) = \sqrt{1 + x^2} - \left( \frac{1 - \beta}{1 + \beta} \right) x^2 \sinh^{-1} \frac{1}{x}.
\]
(4.22)

We then find that at the GB regime
\[
A_T^2 = \frac{32\kappa_4^4}{75\pi^2} V \left[ \frac{1 + \beta}{2\beta} \right] (Hl)^{-1},
\]
(4.23)
and the consistency relation becomes
\[
\frac{A_T^2}{A_s^2} \simeq -\frac{Q_{GB}}{2} n_T,
\]
(4.24)
where
\[
Q_{GB} = \frac{1 + \beta + 2\beta x^2}{1 + \beta + \beta x^2}.
\]
(4.25)

Inflation can end either in the GB regime (\( \chi_f \gg 1 \)) or in the RS regime (\( \chi_f \ll 1 \)). We consider first the case of a chaotic inflation with potential \( V(\phi) = \frac{1}{2} m_\phi^2 \phi^2 \) ending in the RS regime. We have assumed that \( \beta \ll 1 \) and from the definitions of section 4.1 we can write the following relations:
\[
\kappa_5^4 = \frac{6\kappa_4^2}{\lambda},
\]
(4.26)
\[
m_\phi^8 = \frac{2\lambda^2}{9\beta},
\]
(4.27)
\[
\ell^2 \kappa_5^4 = \frac{36}{\lambda^2}.
\]
(4.28)

The slow-roll parameters (4.12) and (4.13) are then
\[
\varepsilon = \frac{4\beta m_\phi^2}{3\kappa_4^2 \lambda} f(\chi),
\]
(4.29)
\[
\eta = \frac{2\beta m_\phi^2}{\kappa_4^2 \lambda} g(\chi),
\]
(4.30)
where
\[ f(\chi) = \frac{\sinh(2\chi/3) \tanh \chi}{[\cosh(2\chi/3) - 1]^2}, \quad (4.31) \]
\[ g(\chi) = \frac{1}{\cosh(2\chi/3) - 1}. \quad (4.32) \]

The number of e-folds (4.14) is
\[ N = \frac{\kappa_4^2 \lambda}{4\beta m_\phi^2} [I(\chi_i) - I(\chi_f)], \quad (4.33) \]
with
\[ I(\chi) = \frac{3}{2} \left[ \cosh\left(\frac{2\chi}{3}\right) - 1 - \ln\left(\frac{1}{3} + \frac{2}{3} \cosh\left(\frac{2\chi}{3}\right)\right) \right], \quad (4.34) \]
whereas the amplitude of scalar perturbations (4.19) is given by
\[ A_s^2 = \frac{\kappa_4^6 \lambda^2}{32 \sqrt{2} \pi^2 \beta^{5/2} m_\phi^2} \left[\cosh(2\chi/3) - 1\right]^3 \sinh \chi. \quad (4.35) \]

From (4.29) the condition for the end of inflation \( \varepsilon_f = 1 \) together with (4.33) and with \( \chi_f \ll 1 \) gives the following relation:
\[ \frac{6\beta m_\phi^2}{\kappa_4^2 \lambda} = \frac{I(\chi_i)}{1 + (2/3) N}, \quad (4.36) \]
while (4.35) gives
\[ \frac{\beta^{3/2}}{\kappa_4^4 \lambda} = \frac{3}{16 \sqrt{2} \pi^2} \frac{A_s^2 I(\chi_i)}{A_s^2} \frac{[\cosh(2\chi/3) - 1]^3}{\sinh \chi}. \quad (4.37) \]

Moreover using (4.36) with the slow-roll parameters (4.29) and (4.30) we get the following expression for the scalar spectral index:
\[ n_s = 1 - \frac{2}{3} \frac{I(\chi_i)}{1 + (2/3) N} \left[ 2f(\chi_i) - g(\chi_i) \right] \]
and the condition (4.15) becomes
\[ \sinh^6 \chi < \frac{9 \beta^3}{128 \kappa_4^3 \lambda^2}. \quad (4.39) \]

We thus have three equations (4.36), (4.37) and (4.38) with six parameters, \( n_s, N, \chi_i, \beta, \lambda \) and \( m_\phi \). Moreover using
\[ \lambda = 6 \frac{M_5^6}{M_4^2}, \quad (4.40) \]
we get the following final expressions for \( \beta, m_\phi, m_\beta \) and \( \phi_i \) as functions of \( N, \chi_i \) and \( M_5^4 \):
\[ \beta = \left[ \frac{9}{8 \sqrt{2} \pi^2} \frac{[\cosh(2\chi_i/3) - 1]^2}{A_s^2 I(\chi_i)} \right]^{2/3} \frac{M_5^4}{\sinh \chi_i^{2/3} M_4^4}. \quad (4.41) \]
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Figure 1. Scalar spectral index as a function of $\chi_1$ for $N = 55$.

$$m_\phi = \left[ \frac{8 \sqrt{2} \pi^2 A_s^2 I(\chi_1)^{5/2}}{9 (1 + (2/3)N)^{5/2}} \right]^{1/3} \frac{\sinh \chi_1^{1/3}}{\cosh (2\chi_1/3) - 1} M_5, \quad (4.42)$$

$$m_\beta = \left[ \frac{256 \pi^2 A_s^2 I(\chi_1)}{9 (1 + (2/3)N)} \right]^{1/12} \frac{\sinh \chi_1^{1/12}}{[\cosh (2\chi_1/3) - 1]^{1/4} M_5}, \quad (4.43)$$

$$\phi_i = \left[ \frac{144 (1 + (2/3)N)^4}{\pi^4 A_s^2 I(\chi_1)^4} \right]^{1/6} \left[ \cosh \left( \frac{2\chi_1}{3} \right) - 1 \right]^{1/2} \sinh \chi_1^{1/3} M_5. \quad (4.44)$$

The constraint relation (4.39) with $M_4 = M_{Pl} = 1.2 \times 10^{19}$ GeV, $A_s^2 = 4 \times 10^{-10}$ and $N = 55$ gives $\chi_1 < 12.57$.

In figure 1 we plot the scalar spectral index $n_s$ as a function of $\chi_1$. Motivated from the WMAP three-year results [2] we take $n_s < 0.966$ therefore we get $\chi_1 < 1.42$. As a consequence inflation starts either at a RS regime for $\chi_1 \ll 1$ (where we recover the results of section 3) or during a mixed GB–RS regime for $\chi_1 \sim 1$. We can thus fix $\chi_1 = 0.8$ giving $n_s = 0.9604$. Moreover we have in our case $\phi_i \simeq 3.34 \phi_f$. Therefore we obtain the following values for the different parameters as functions of $M_5$:

$$\beta = 2.31 \times 10^{-71} M_5^4, \quad (4.45)$$

$$m_\beta = 2.53 \times 10^{-1} M_5, \quad (4.46)$$

$$m_\phi = 6.63 \times 10^{-5} M_5, \quad (4.47)$$

$$\lambda = 4.16 \times 10^{-38} M_5^6, \quad (4.48)$$

$$\phi_i = 1.28 \times 10^{3} M_5, \quad (4.49)$$

$$\phi_f = 3.83 \times 10^{2} M_5. \quad (4.50)$$
The parameter $\beta \ll 1$; therefore for various values of $\beta$ we get

| $\beta = 10^{-3}$ | $\lambda^{1/4} = 1.04 \times 10^{16}$ GeV | $m_\phi = 5.40 \times 10^{12}$ GeV | $m_\beta = 2.05 \times 10^{16}$ GeV |
|-----------------|-----------------|-----------------|-----------------|
| $\phi_i = 1.04 \times 10^{20}$ GeV | $\phi_f = 3.12 \times 10^{19}$ GeV | $M_5 = 8.11 \times 10^{16}$ GeV |
| $\beta = 10^{-5}$ | $\lambda^{1/4} = 1.86 \times 10^{15}$ GeV | $m_\phi = 1.71 \times 10^{12}$ GeV | $m_\beta = 6.49 \times 10^{15}$ GeV |
| $\phi_i = 3.29 \times 10^{19}$ GeV | $\phi_f = 9.85 \times 10^{18}$ GeV | $M_5 = 2.57 \times 10^{16}$ GeV |
| $\beta = 10^{-7}$ | $\lambda^{1/4} = 3.30 \times 10^{14}$ GeV | $m_\phi = 5.40 \times 10^{11}$ GeV | $m_\beta = 2.05 \times 10^{15}$ GeV |
| $\phi_i = 1.04 \times 10^{19}$ GeV | $\phi_f = 3.12 \times 10^{18}$ GeV | $M_5 = 8.11 \times 10^{15}$ GeV |
| $\beta = 10^{-12}$ | $\lambda^{1/4} = 4.40 \times 10^{12}$ GeV | $m_\phi = 3.03 \times 10^{10}$ GeV | $m_\beta = 1.15 \times 10^{14}$ GeV |
| $\phi_i = 5.84 \times 10^{17}$ GeV | $\phi_f = 1.75 \times 10^{17}$ GeV | $M_5 = 4.56 \times 10^{14}$ GeV |

We can see from the above table that in order to keep the value of the inflaton field below the Planck scale, the $\beta$ parameter should be $<10^{-7}$ constraining in this way the value of the fundamental mass to $M_5 < 10^{16}$ GeV.

We consider also the case of a chaotic inflation with potential $V(\phi) = \frac{1}{2} m_\phi^2 \phi^2$ ending in the GB regime where $\chi_I \gg 1$. From (4.29) the condition for the end of inflation $\varepsilon_I = 1$ together with (4.33) gives the following relation:

$$\frac{6 \beta m_\phi^2}{\kappa_4^2 \lambda} = \frac{3 I(\chi_i)}{2(N + 1/2)}.$$ \hspace{1cm} (4.51)

while (4.35) gives

$$\frac{\beta^{3/2}}{\kappa_4^4 \lambda} = \frac{2(N + 1/2)}{16 \sqrt{2} \pi^2 \lambda A_2^2 I(\chi_i)} \frac{[\cosh(2\chi/3) - 1]^3}{\sinh \chi}.$$ \hspace{1cm} (4.52)

Moreover using (4.51) with the slow-roll parameters (4.29) and (4.30) we get the following expression for the scalar spectral index:

$$n_s = 1 - \frac{I(\chi_i)}{N + (1/2)} [2f(\chi_i) - g(\chi_i)]$$ \hspace{1cm} (4.53)

and the condition (4.15) becomes

$$\sinh^6 \chi < \frac{9 \beta^3}{128 \kappa_4^8 \lambda^2}.$$ \hspace{1cm} (4.54)

We thus have three equations (4.51), (4.52) and (4.53) with six parameters, $n_s, N, \chi_I, \beta, \lambda$ and $m_\phi$. Taking again $N = 55$ the constraint relation (4.54) gives $\chi_I < 12.56$. The plot of the scalar spectral index $n_s$ as a function of $\chi_I$ has the same shape as in figure 1. In this case however we have $\chi_I \gg 1$ which means that the value of the spectral index should be greater than the largest value given by the WMAP three-year results [2]. Therefore the case of an inflationary period ending in a GB regime is ruled out by observational data. Inflation should end in a RS regime$^4$.

$^4$ An alert reader will notice that if we relax the upper bound of the spectral index obtained from the three-year WMAP results and consider larger values of $n_s$, figure 1 shows that $\chi_I$ can be much larger than 1, and thus we also have the interesting case of inflation ending in the GB regime.
4.3. Curvaton reheating in the Gauss–Bonnet brane-world model

For the Gauss–Bonnet brane-world model we have three different regimes of cosmological evolution and the use of the curvaton reheating mechanism gives the following cases:

**Case 1:** $H_{\text{GB}} > H_{\text{GB,RS}} > H_t$,  \hspace{1cm} (4.55)

where $H_t$ is the Hubble parameter at the end of inflation. We consider that the GB regime dominates at very early time and is followed by the intermediate RS regime where inflation takes place. Thus inflation ends at a RS regime and we recover the case of inflation results and curvaton reheating in the RS model (section 3).

**Case 2:** $H_{\text{GB}} > H_t > H_{\text{GB,RS}}$.  \hspace{1cm} (4.56)

In this case inflation occurs during the GB regime which is immediately followed by a transition to the intermediate RS regime. Thus in this case inflationary parameters are determined by the results of section 4.2 while the curvaton reheating mechanism gives the same results as in section 3.3.

We can also consider that inflation and oscillation of the curvaton occur at the GB regime; thus we can use the results of section 4.2 for the inflationary parameters and we have three different cases for the curvaton evolution:

**Case 3a:** $H_{\text{GB}} > H_t > m > H_{\text{eq1}} > \Gamma > H_{\text{GB,RS}} > H_{\text{RS,GR}} > H_{\text{nucl}}$

**Case 3b:** $H_{\text{GB}} > H_t > m > \Gamma > H_{\text{eq1}} > H_{\text{GB,RS}} > H_{\text{RS,GR}} > H_{\text{nucl}}$

**Case 4a:** $H_{\text{GB}} > H_t > m > H_{\text{GB,RS}} > H_{\text{eq2}} > \Gamma > H_{\text{RS,GR}} > H_{\text{nucl}}$

**Case 4b:** $H_{\text{GB}} > H_t > m > H_{\text{GB,RS}} > \Gamma > H_{\text{eq2}} > H_{\text{RS,GR}} > H_{\text{nucl}}$

**Case 5a:** $H_{\text{GB}} > H_t > m > H_{\text{GB,RS}} > H_{\text{RS,GR}} > H_{\text{eq3}} > \Gamma > H_{\text{nucl}}$

**Case 5b:** $H_{\text{GB}} > H_t > m > H_{\text{GB,RS}} > H_{\text{RS,GR}} > \Gamma > H_{\text{eq3}} > H_{\text{nucl}}$.

If we apply the general results of section 2 we obtain

\[ H_{\text{GB,RS}} = \frac{\sqrt{6}}{2\sqrt{\beta l}} \]  \hspace{1cm} (4.58)

\[ H_{\text{RS,GR}} = \frac{2}{l(1 + \beta)} \]  \hspace{1cm} (4.59)

Then for the various cases we have:

- **Case 3a:**
  \[ H_{\text{eq1}} = \left( \frac{\kappa_5^2}{8\beta^2 l^2} \right)^{2/3} \sigma_4^{4/3} m^{1/3}. \]  \hspace{1cm} (4.60)

- **Case 4a:**
  \[ H_{\text{eq2}} = \frac{\sigma_4^4 \kappa_5^4}{64 \beta l^2} m. \]  \hspace{1cm} (4.61)

- **Case 5a.** In these cases we perform calculations similar to those in section 2.1.2 but with two transitions before the equivalence, and we find
  \[ H_{\text{eq3}} = \frac{\sqrt{3} \sigma_2^2 m^{1/2} \kappa_5 \kappa_4}{12 \beta^{1/2} l}. \]  \hspace{1cm} (4.62)
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• **Case 3b:**

\[ H_{eq1} = \frac{\kappa_5^2}{8\beta l^2} \sigma_1^2 m^{1/2} \Gamma^{-1/2}. \]  

(4.63)

• **Case 4b:**

\[ H_{eq2} = \frac{\sigma_1^6}{64\sqrt{6} \beta^{3/2} l^5} \kappa_5^6 m^{3/2} \Gamma^{-1/2}. \]  

(4.64)

• **Case 5b.** In these cases we perform calculations similar to those in section 2.2.2 but with two transitions before the equivalence, and we find

\[ H_{eq3} = \frac{\sigma_1^3 m^{3/4}}{24\beta^{3/4} l^{5/2}} \kappa_5^{3/2} \Gamma^{-1/2}. \]  

(4.65)

As for the gravitational waves we have the following constrained relations. For cases 3a, 4a and 5a, the kination and oscillation epochs are in the GB regime; thus using (2.44) we obtain

\[ \rho_k \rho_\sigma |_{a=a_{eq}} = \frac{128}{3\pi} h_{GW}^2 H_{kin} \left( \frac{\beta l^2}{m^{1/2} \sigma_1^2 \kappa_5^2} \right)^{2/3} \ll 1. \]  

(4.66)

For cases 3b, 4b and 5b we need the regimes when kination and oscillation take place and in which regime the curvaton starts decaying. Using equation (2.49) we obtain:

**Case 3b:**

\[ \frac{\rho_k}{\rho_{rad}} |_{a=a_{eq}} = \frac{512}{3\pi} h_{GW}^2 H_{kin} \frac{\beta l^2}{m^{1/2} \sigma_1^2 \kappa_5^2} \Gamma^{1/2} \ll 1, \]  

(4.67)

**Case 4b:**

\[ \frac{\rho_k}{\rho_{rad}} |_{a=a_{eq}} = \frac{128}{3\pi} \times 6144^{1/6} h_{GW}^2 H_{kin} \frac{\beta^{5/6} l^{5/3}}{m^{1/2} \sigma_1^2 \kappa_5^2} \Gamma^{1/6} \ll 1, \]  

(4.68)

**Case 5b:**

\[ \frac{\rho_k}{\rho_{rad}} |_{a=a_{eq}} = \frac{128}{3\pi} \times 3072^{1/6} h_{GW}^2 H_{kin} \frac{\beta^{5/6} l^{5/3}}{m^{1/2} \sigma_1^2 \kappa_5^2} \Gamma^{1/6} \Gamma^{1/3} \ll 1. \]  

(4.69)

### 4.4. Constraints on the parameters in the Gauss–Bonnet brane-world model

In all cases the transition from GB to RS and from RS to GR using (4.58) and (4.59) happens at

\[ H_{GB,RS} = \frac{8.51}{\sqrt{\beta}} \times 10^{-39} M_5^3, \]  

(4.70)

\[ H_{RS,GR} = 1.39 \times 10^{-38} M_5^3. \]  

(4.71)

Also from the nucleosynthesis constraint \( \Gamma > H_{nucl} = 10^{-40} M_{Pl} \) we obtain

\[ \Gamma > 1.2 \times 10^{-21} \text{ GeV}, \]  

(4.72)

for \( M_{Pl} = 1.2 \times 10^{19} \text{ GeV} \). From the results obtained in section 4.2 cases 2, 3, 4 and 5 are ruled out and the only valid cases are 1a and 1b studied in section 3 where inflation ends in a RS regime with

\[ H_f = \frac{\kappa_5}{12} m_\phi^2 \phi_1^2 \simeq 5.37 \times 10^{-8} M_5, \]  

(4.73)

for \( \chi_i = 0.8 \).
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If the density perturbations are not generated by the inflaton then the mass and the initial and final values of the inflaton fields as a function of $A_s$ and $M_5$ are

$$m_{\phi} = 9.00 \times 10^{-2} A_s^{2/3} M_5,$$
$$\phi_i = 3.47 \times 10^{-1} A_s^{-1/3} M_5,$$
$$\phi_f = 1.04 \times 10^{-1} A_s^{-1/3} M_5,$$

and for the gravitational waves we get similar constraints as in section 3.4.

As in the RS case, if the density perturbations are generated by the fluctuations of the curvaton field then we get low energy values for the fundamental mass $M_5$. It can be also shown that the constraints (3.44) and (3.46) give results similar to those in section 3.4.

5. Curvaton dynamics in the induced gravity (DGP) brane-world model

5.1. The induced gravity brane-world model

In this section we will apply the curvaton reheating mechanism to the induced gravity (DGP) brane-world scenario which has the following gravitational action:

$$S = \frac{1}{2 \kappa_5^2} \int d^5x \sqrt{\langle 5 \rangle} g \left( \langle 5 \rangle R - 2 \Lambda_5 \right) + \frac{r}{2 \kappa_5^2} \int_{y=0} d^4x \sqrt{\langle 4 \rangle} g \left( \langle 4 \rangle R - 2 \Lambda_4 \right),$$

where $r = \kappa_5^2 / \kappa_4^2 > 0$ is the induced gravity crossover scale, one of the two characteristic length scales of the model, while the second characteristic length scale, the AdS length scale, is the one related to the Planck coupling constant as $l = \kappa_5^2 / \kappa_{\text{Pl}}^2$. As in the Randall–Sundrum model, the relation between the 5D cosmological constant of the AdS bulk and its characteristic length scale $l$ is

$$\Lambda_5 = -\frac{6}{l^2}.$$  

The brane tension $\lambda = \Lambda_4 / \kappa_4^2$ is related to the AdS length scale through the fine-tuning relation

$$\lambda = \frac{6}{\kappa_5^2 l^2}.$$  

Moreover, in order to recover GR at low energies the two 4D coupling constants are related by $\kappa_{\text{Pl}}^2 = \mu \kappa_4^2$, where $\mu = \kappa_4^2 r^2 \lambda / 6$ [22]. Two different cosmological evolutions can be distinguished. The first, for $\mu \gg 1$, is a pure 4D evolution at all energies which we will not discuss here. The second, for $\mu \ll 1$, is giving an interesting cosmological evolution that we will consider. The Friedmann equation is given by

$$H^2 = \frac{\kappa_5^2}{3} \left\{ 1 + \frac{6}{\kappa_4^2 r^2 \rho} + \frac{\lambda}{\rho} + \frac{\epsilon}{2} \frac{2 \sqrt{3}}{\kappa_4 r \rho^{1/2}} \left( 1 + \frac{3}{\kappa_4^2 r^2 \rho} + \frac{\lambda}{\rho} + \frac{\lambda \kappa_4^2 r^2 \rho^{1/2}}{12} \right)^{1/2} \right\},$$

where $\epsilon = \pm 1$. However it was shown in [22] that only for $\epsilon = -1$ do we get an inflationary phase. Using the Friedmann equation (5.4) we can distinguish three different regimes for the dynamical evolution of the brane-universe:
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- the IND regime, when \( Hl \gg Hr \gg 1 \),
  \[
  H^2 \approx \frac{\kappa_4^2 \rho}{3} \left( 1 - \frac{2\sqrt{3}}{\kappa_4 r \rho^{1/2}} \right),
  \]
  (5.5)

- the intermediate RS regime, when \( Hl \gg 1 \gg Hr \),
  \[
  H^2 \approx \frac{\kappa_4^2}{6\lambda} \rho^2,
  \]
  (5.6)

- the GR regime at low energies, when \( Hr \ll Hl \ll 1 \),
  \[
  H^2 \approx \frac{\kappa_4^2}{3} \rho.
  \]
  (5.7)

Notice that in the case of \( r \to 0 \) we recover the RS model, whereas if we also have \( l \to 0 \) we recover GR cosmology.

5.2. Slow-roll inflation in the induced gravity brane-world model

We will review the results of the slow-roll inflationary dynamics for the induced gravity brane-world model [22]. As before we consider an inflaton field \( \phi \) with energy density given by (2.5) and obeying the Klein–Gordon equation (2.9). The slow-roll conditions for the inflaton field are

\[
\dot{\phi}^2 \left( 1 - \frac{\sqrt{3}}{\kappa_4 r V^{1/2}} \right) < V \left( 1 + \epsilon \frac{2\sqrt{3}}{\kappa_4 r V^{1/2}} \right),
\]

(5.8)

\[
\ddot{\phi} \ll 3 H \dot{\phi}
\]

(5.9)

and the slow-roll parameters, at high energies, are found to be

\[
\epsilon = \epsilon_{GR} \left( 1 + \frac{3\sqrt{3}}{\kappa_4 r V^{1/2}} \right),
\]

(5.10)

\[
\eta = \eta_{GR} \left( 1 + \frac{2\sqrt{3}}{\kappa_4 r V^{1/2}} \right),
\]

(5.11)

where \( \epsilon_{GR} \) and \( \eta_{GR} \) are the GR slow-roll parameters. The number of e-folds is

\[
N = -\frac{\kappa_4^2}{\mu} \int_{\phi_i}^{\phi_f} \frac{V}{V'} \left( 1 - \frac{2\sqrt{3}}{\kappa_4 r V^{1/2}} \right) d\phi,
\]

(5.12)

where \( \phi_f \) is evaluated at the end of inflation (\( \epsilon = 1 \)), and \( \phi_i \) is evaluated when cosmological scales leave the horizon. The scalar amplitude and the scalar spectral index generated during inflation have been computed [33] and are respectively given by

\[
A_s^2 = \frac{\kappa_4^6}{15\pi^2} \frac{V^3}{V'} \left[ C^2_{IND}(Hl) \right]_{k=aH},
\]

(5.13)

\[
n_s - 1 = \frac{d \ln A_s^2}{d \ln k} = 2\eta - 6\epsilon,
\]

(5.14)
where
\[ G_{\text{IND}}^2(x) = \frac{x^6}{\{\mu x^2 - 2(1 - \mu)[1 - \sqrt{1 + x^2}]\}^2}, \]  
(5.15)
is the IND correction and \( \tilde{k} \) is the comoving wavenumber. We then find at the IND regime
\[ A_s^2 = \frac{\kappa_4^6}{15\pi^2} \frac{V^3}{V''} \left[ \frac{1}{\mu^3} \right]. \]  
(5.16)
Moreover the tensor amplitude and tensor spectral index are respectively given by [33]
\[ A_T^2 = \frac{32\kappa_4^4}{15\pi^2} V \left[ F_{\text{IND}}^2(H_l) \right] \bigg|_{\tilde{k} = aH}, \]  
(5.17)
\[ n_T = \frac{d \ln A_T^2}{d \ln k}, \]  
(5.18)
with
\[ F_{\text{IND}}^2(x) = \mu + (1 - \mu) \left[ \sqrt{1 + x^2} - x^2 \arcsinh \frac{1}{x} \right]. \]  
(5.19)
We then find that at the IND regime
\[ A_T^2 = \frac{32\kappa_4^4}{15\pi^2} V \left[ \frac{1}{\mu} \right], \]  
(5.20)
so that the consistency relation becomes at the IND regime
\[ \frac{A_T^2}{A_s^2} \simeq -Q_{\text{IND}} \frac{n_T}{2}, \]  
(5.21)
where \( Q_{\text{IND}} \) is a complicated differential equation between \( V \) and \( F_{\text{IND}} \).

For a chaotic inflation with potential \( V(\phi) = \frac{1}{2} m_\phi^2 \phi^2 \) the following expressions can be obtained using relations (5.10)–(5.15):
\[ \phi_i = \frac{15 + 7c}{2(1 - n_s)} \kappa_4^4 \left( 1 + \sqrt{1 - \frac{112N(1 - n_s)}{(1 - c)(15 + 7c)^2}} \right), \]  
(5.22)
\[ V_0 = \frac{m_\phi^2}{2} = 3\pi^2 (1 - c), \]  
(5.23)
\[ r = \frac{M_5^2}{M_6^2} = -\frac{4\sqrt{3} \kappa_4 \phi_i (1 - c)}{\sqrt{V_0}(4N - \kappa_4^2 \phi_i^2 (1 - c^2))}. \]  
(5.24)
which are essentially functions of the spectral index \( n_s \), the parameter \( c = \phi_f/\phi_i \), the four-dimensional coupling constant \( \kappa_4 \) and the number of e-folds \( N \). Fixing the value of the scalar spectral index to \( n_s = 0.9645 \), the values of the various parameters using the above relations for \( N = 55, c = 0.1 \) and \( \mu = 0.001 \) are
\[ m_\phi \simeq 2.53 \times 10^{12} \text{GeV}, \]  
(5.25)
\[ M_5 \simeq 4.07 \times 10^{15} \text{GeV}, \]  
(5.26)
\[ \lambda^{1/4} \simeq 1.17 \times 10^{14}, \]  
(5.27)

\[ \phi_i \simeq 5.97 \times 10^{18} \text{ GeV}, \]  
(5.28)

\[ \phi_f \simeq 0.1 \phi_i = 5.97 \times 10^{17} \text{ GeV}. \]  
(5.29)

### 5.3. Curvaton reheating in the induced gravity brane-world model

In the induced gravity model we have three different cosmological regimes. We distinguish the following cases for the curvaton reheating mechanism:

**Case 1:** \( H_{\text{IND}} > H_{\text{IND,RS}} > H_f, \)  
(5.30)

where \( H_f \) is the Hubble parameter at the end of inflation. We consider that the IND regime dominates at very high energies and is followed by the intermediate RS regime where inflation takes place. Thus inflation ends at a RS regime and we recover the case of inflation results and curvaton reheating in the RS model (section 3).

**Case 2:** \( H_{\text{IND}} > H_f > H_{\text{IND,RS}}, \)  
(5.31)

where we consider that inflation occurs during the IND regime which is immediately followed by a transition to the intermediate RS regime. Thus in this case inflationary parameters are determined by the results of section 5.2 while for the curvaton reheating mechanism the results of section 3.3 are valid.

If inflation and oscillation of the curvaton occurs at the IND regime, results of section 5.2 are considered for the inflationary parameters and we have three different cases for the curvaton evolution:

**Case 3a:** \( H_{\text{IND}} > H_f > m > H_{\text{eq1}} > \Gamma > H_{\text{IND,RS}} > H_{\text{RS,GR}} > H_{\text{nucl}} \)

**Case 3b:** \( H_{\text{IND}} > H_f > m > \Gamma > H_{\text{eq1}} > H_{\text{IND,RS}} > H_{\text{RS,GR}} > H_{\text{nucl}} \)

**Case 4a:** \( H_{\text{IND}} > H_f > m > H_{\text{IND,RS}} > H_{\text{eq2}} > \Gamma > H_{\text{RS,GR}} > H_{\text{nucl}} \)  
(5.32)

**Case 4b:** \( H_{\text{IND}} > H_f > m > H_{\text{IND,RS}} > \Gamma > H_{\text{eq2}} > H_{\text{RS,GR}} > H_{\text{nucl}} \)

**Case 5a:** \( H_{\text{IND}} > H_f > m > H_{\text{IND,RS}} > H_{\text{RS,GR}} > H_{\text{eq3}} > \Gamma > H_{\text{nucl}} \)

**Case 5b:** \( H_{\text{IND}} > H_f > m > H_{\text{IND,RS}} > H_{\text{RS,GR}} > \Gamma > H_{\text{eq3}} > H_{\text{nucl}}. \)

If we apply the general results of section 2 we obtain

\[ H_{\text{IND,RS}} = 2r, \]  
(5.33)

\[ H_{\text{RS,GR}} = 2l = 2\mu r \]  
(5.34)

and for cases 3a and 3b respectively

\[ H_{\text{eq1}} = \frac{\kappa_{\text{Pl}}^2}{6\mu} \sigma_1^{4/3} m^{1/3}, \]  
(5.35)

\[ H_{\text{eq1}} = \frac{\kappa_{\text{Pl}}^3}{(6\mu)^{3/2}} \sigma_1^3 m^{3/2} \Gamma^{-1/2}, \]  
(5.36)

\footnote{Similar analysis is carried out in [34].}
while for cases 4a and 4b we obtain

\[ H_{eq} = \frac{k_{pl}^4}{72\mu^2} \sigma_1^4 m^2 r, \]  
(5.37)

\[ H_{eq} = \frac{k_{pl}^6}{432\sqrt{2}\mu^3} \sigma_1^6 m^3 r^{3/2} \Gamma^{-1/2}. \]  
(5.38)

For the cases 5a and 5b we perform analysis similar to that in section 2.2 but with two transitions before the equivalence, and we find respectively

\[ H_{eq} = \frac{k_{pl}^2}{6\mu} \sigma_1^2 m, \]  
(5.39)

\[ H_{eq} = \frac{k_{pl}^6}{6\sqrt{6}\mu^{3/4}} \sigma_1^3 m^{3/2} \Gamma^{-1/2}. \]  
(5.40)

For the gravitational waves the most interesting cases are cases 3a, 4a and 5a in which the kination and oscillation epochs are in the IND regime; thus using (2.44) we obtain

\[ \frac{\rho_g}{\rho_{\sigma}} \bigg|_{a=a_{eq}} = \frac{32}{3\pi} \frac{\mu}{m} \frac{H_{GW}}{m^{2/3} \sigma_1^{1/3}} \frac{\mu^{1/6}}{k_{pl}^{1/3}} \ll 1. \]  
(5.41)

For cases 3b, 4b and 5b we need the regimes when kination and oscillation take place and in which regimes the curvaton starts decaying. Using equation (2.49) we obtain:

**Case 3b:**

\[ \frac{\rho_g}{\rho_{rad}} \bigg|_{a=a_{eq}} = \frac{128}{\pi} \frac{\mu}{m} \frac{H_{GW}}{m^{2/3} \sigma_1^{1/3}} \frac{\mu^{1/6}}{k_{pl}^{1/3}} \Gamma^{1/3} \ll 1, \]  
(5.42)

**Case 4b:**

\[ \frac{\rho_g}{\rho_{rad}} \bigg|_{a=a_{eq}} = \frac{128}{\pi} \frac{\mu}{m} \frac{H_{GW}}{m^{2/3} \sigma_1^{1/3}} \frac{\mu^{1/6}}{k_{pl}^{1/3}} \left( \frac{2\Gamma}{r} \right)^{1/6} \ll 1, \]  
(5.43)

**Case 5b:**

\[ \frac{\rho_g}{\rho_{rad}} \bigg|_{a=a_{eq}} = \frac{256}{\pi} \frac{\mu}{m} \frac{H_{GW}}{m^{3/2} \sigma_1^{2/3} k_{pl}^{5/6}} \Gamma^{1/3} \ll 1. \]  
(5.44)

### 5.4. Constraints on the parameters in the induced gravity brane-world model

In all cases the transition from four to five dimensions and from five to four dimensions using respectively (5.33) and (5.34) happen at

\[ H_{\text{IND, RS}} = 2.88 \times 10^{38} M_5^{-3} \simeq 4.27 \times 10^{-12} \text{ GeV}, \]  
(5.45)

\[ H_{\text{RS, GR}} = 2.88 \times 10^{38} \mu M_6^{-3} \simeq 4.27 \times 10^{-15} \text{ GeV}, \]  
(5.46)

where the \( M_5 \) was determined for \( n_s = 0.9645, N = 55, c = 0.1 \) and \( \mu = 0.001 \). Also from the nucleosynthesis constraint \( \Gamma > H_{\text{nuc}l} = 10^{-40} M_{pl} \) we obtain

\[ \Gamma > 1.2 \times 10^{-21} \text{ GeV}, \]  
(5.47)

for \( M_{pl} = 1.2 \times 10^{19} \text{ GeV} \). Moreover inflation ends at

\[ H_f = \frac{k_4}{\sqrt{6}} m_\phi \phi_f \sqrt{\left( 1 - \frac{2\sqrt{6}}{k_4 r m_\phi \phi_f} \right)} = 5.09 \times 10^{13} \text{ GeV}, \]  
(5.48)
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thus $H_t > H_{\text{IND,RS}}$ which invalidates case 1 and because $H_t \gg H_{\text{IND,RS}}$ the transition to RS regime cannot occur immediately after the end of inflation, therefore case 2 is also excluded. Moreover, the low energy and close to each other values of $H_{\text{IND,RS}}$ and $H_{\text{RS,GR}}$ make case 3 the most probable case for the reheating mechanism. In case 3 the universe inflates in 4D induced gravity regime and the primordial density perturbations are generated by the inflaton field. Also the curvaton starts to oscillate in the four-dimensional induced gravity high energy regime. To have a better fitting of the parameters we fix $c = 0.1$ from which the values of the inflaton field (5.28) and (5.29) at the beginning and at the end of inflation do not change much.

Using (5.35)–(5.40) together with (5.25)–(5.29) we find the following constraints on the parameters for

- **Case 3a**
  
  \[ 4.27 \times 10^{-12} \text{ GeV} < \Gamma < m < 5.09 \times 10^{13} \text{ GeV} \]
  \[ 3.68 \times 10^{24} \text{ GeV}^{5/3} < 8.62 \times 10^{35} \Gamma < \sigma_i^{4/3} m^{1/3} < 4.39 \times 10^{49} \text{ GeV}^{5/3}, \]

- **Case 3b**
  
  \[ 4.27 \times 10^{-12} \text{ GeV} < \Gamma < m < 5.09 \times 10^{13} \text{ GeV} \]
  \[ 3.58 \times 10^{-6} < 4.06 \times 10^{11} \Gamma^{3/2} < \sigma_i m^{1/2} < 1.50 \times 10^{16} \Gamma^{1/6} < 2.89 \times 10^{18}, \]

- **Case 4a**
  
  \[ 4.27 \times 10^{-15} \text{ GeV} < \Gamma < 4.27 \times 10^{-12} < m < 5.09 \times 10^{13} \text{ GeV} \]
  \[ 2.99 \times 10^{69} \text{ GeV}^6 < 2.21 \times 10^{84} \Gamma < \sigma_i^4 m^2 < 2.99 \times 10^{72} \text{ GeV}^6, \]

- **Case 4b**
  
  Incompatible

- **Case 5a**
  
  \[ 4.27 \times 10^{-12} \text{ GeV} < m < 5.09 \times 10^{13} \text{ GeV} \]
  \[ 1.20 \times 10^{-21} \text{ GeV} < \Gamma < 4.27 \times 10^{-15} \text{ GeV} \]
  \[ 1.04 \times 10^{15} \text{ GeV}^3 < 8.64 \times 10^{35} \Gamma < \sigma_i^2 m < 3.70 \times 10^{21} \text{ GeV}^3, \]

- **Case 5b**
  
  Incompatible

Moreover, for cases 3a, 4a and 5a the gravitational constraint (5.41) gives $m^2 \sigma_i > 8.00 \times 10^{27} \text{ GeV}^3$. For case 3b relation (5.42) gives $m \sigma_i^2 > 1.93 \times 10^{39} \Gamma^{1/3}$ while cases 4b and 5b are excluded from (5.52) and (5.54). If the density perturbations are not generated by the inflaton, then the different parameters expressed as a function of $A_s$ are

\[ m_\phi = 1.29 \times 10^{17} A_s, \]
\[ M_5 = 1.50 \times 10^{17} A_s^{1/3}, \]
\[ \lambda^{1/4} = 2.62 \times 10^{16} A_s^{1/2}. \]

As in the case of the RS or the GB model, if density perturbations are generated by the fluctuations of the curvaton field, the fundamental mass $M_5$ can get low energy
values. However, the constraints (3.44) and (3.46) should also be satisfied for the curvaton parameters of the induced gravity model. A detailed analysis shows, that only the case 3b can satisfy these constraints at the expense of introducing two extra constraints on the parameters \(m, \sigma_i\) and \(\Gamma\):

\[
\begin{align*}
    m \sigma_1 & > 8.70 \times 10^{20} \text{ GeV}^2, \\
    m \sigma_1 & > 4.84 \times 10^{25} \Gamma^{1/2} \text{ GeV}^2.
\end{align*}
\]

We note that case 5b is excluded in both cases, whether density perturbations are generated by the inflaton field or by the curvaton field.

6. Curvaton dynamics in the Gauss–Bonnet and induced gravity (GBIG) brane-world model

6.1. The GBIG brane-world model

In this section we will apply the curvaton reheating mechanism to the Gauss–Bonnet and induced gravity (GBIG) brane-world [8] scenario which has the following gravitational action:

\[
S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g} \left( (5)^R - 2\Lambda_5 + \alpha \left( (5)\mathcal{R} - 4 (5)^R_{ab} (5)^{ab} + (5)\mathcal{R}_{abcd} (5)^{abcd} \right) \right)
+ \frac{r}{2\kappa_5^2} \int_{y=0} d^4x \sqrt{-g} \left( (4)^R - 2\Lambda_4 \right),
\]

(6.1)

where \(\alpha > 0\) is the Gauss–Bonnet coupling constant, \(r = \kappa_5^2/\kappa_4^2 > 0\) the induced gravity crossover scale, where \(\kappa_4\) is the effective 4D coupling constant different from the 4D Planck coupling constant \(\kappa_{Pl}\). The AdS length scale is given by \(l = \kappa_5^2/\kappa_{Pl}^2\). Here the relation between the 5D cosmological constant of the AdS bulk and its characteristic length scale \(l\) is as in the GB model:

\[
\Lambda_5 = -\frac{6}{l^2} + \frac{12\alpha}{l^4},
\]

(6.2)

with the constraint \(\alpha \leq l^2/4\) which gives \(\Lambda_5 < 0\). The general form of the Friedmann equation as a cubic equation in \(H^2\) is given by

\[
4 \left[ 1 + \frac{8}{3} \alpha \left( H^2 + \frac{\Phi}{2} \right) \right]^2 \left( H^2 - \Phi \right) = \left[ rH^2 - \frac{\kappa_5^2}{3} (\rho + \lambda) \right]^2,
\]

(6.3)

where \(\Phi\) is a solution to

\[
\Phi + 2\alpha \Phi^2 = \frac{\Lambda_5}{6},
\]

(6.4)

which from (6.2) gives two solutions for \(\Phi\):

\[
\Phi_1 = -\frac{1}{l^2}, \quad \Phi_2 = \frac{1}{l^2} - \frac{1}{2\alpha}.
\]

(6.5)

These solutions can be combined together if we set \(\Phi = -\zeta/l^2\). Then, defining

\[
\tilde{l}^2 = \frac{1}{\Phi} = -\zeta,
\]

(6.6)
the cubic equation is written as
\[ \kappa_5^2 (\rho + \lambda) - 3 \tilde{r} H^2 = \frac{2}{\tilde{l}} \sqrt{1 + H^2 \tilde{l}^2} (3 + \beta (2H^2 \tilde{l}^2 - 1)), \] (6.7)
where \( \beta = 4\alpha/\tilde{l}^2 \). Moreover, in order to recover the late time GR cosmology we must also have \( \tilde{l} \gg r \). We thus find for this model four different regimes for the dynamical evolution of the brane-universe:

- at high energy, the GB regime, when \( H \tilde{l} \gg H r \gg \tilde{\beta}^{-1} \gg 1 \) or \( H \tilde{l} \gg \tilde{\beta}^{-1} \gg H r \gg 1 \)
  \[ H^2 \simeq \left[ \frac{\kappa_5^2 \rho}{4 \beta \tilde{l}^2} \right]^{2/3}, \] (6.8)
- a 4D IND regime, when \( \tilde{\beta}^{-1} \gg H \tilde{l} \gg H r \gg 1 \),
  \[ H^2 \simeq \frac{\kappa_4^2 \rho}{3} \left( 1 - \frac{2\sqrt{3}}{\kappa_4 r \rho^{1/2}} \right), \] (6.9)
- an intermediate 5D RS regime, when \( \tilde{\beta}^{-1} \gg H \tilde{l} \gg 1 \gg H r \),
  \[ H^2 \simeq \frac{\kappa_5^4}{36} (\rho + \lambda)^2, \] (6.10)
- the GR regime at low energy, when \( \tilde{\beta}^{-1} \gg 1 \gg H \tilde{l} \gg H r \),
  \[ H^2 \simeq \frac{\kappa_5^2 \rho}{3}. \] (6.11)

The brane tension obeys the same fine-tuning relation as in the GB model:
\[ \lambda = \frac{6}{\kappa_5 \tilde{l}} \left( 1 - \frac{1}{3} \tilde{\beta} \right) \] (6.12)
and the effective 4D Newton constant is
\[ \kappa_{\text{Pl}}^2 = \frac{\kappa_5^2}{\tilde{l} (1 + \tilde{\beta}) + r}. \] (6.13)

### 6.2. Inflation and reheating in the GBIG brane-world model

In this cosmological model inflation can start during a GB, a 4D IND or a RS regime and it can end in the same or in a different regime. Then the curvaton reheating follows. We can distinguish the following cases:

- **Inflation occurs in the GB regime**
  We show in section 4.2 that inflation cannot end in the GB region but it ends in the RS regime and then the results of section 4.4 apply. If inflation ends in the IND regime then the results of section 5.4 apply.

- **Inflation occurs in the ING regime**
  In this case the analysis of section 5.3 applies.

- **Inflation occurs in the RS regime**
  In this case the analysis of sections 5.3 and 5.4 applies.

---

6 We could have avoided the IND regime and kept a GB behaviour if we had instead used \( H \tilde{l} \gg \tilde{\beta}^{-1} \gg 1 \gg H r \). However this would finally give a pure GB behaviour which is not of any interest.
7. Conclusions and discussion

We studied the curvaton dynamics in brane-worlds. The curvaton reheating mechanism was applied to various stages of the cosmological evolution of the brane-world models. These models are introducing unconventional correction terms to the Friedmann equation of the standard cosmology. These terms have important consequences to the inflationary dynamics. They make the inflation easier because in most cases they act as friction terms. Also they enhance the scalar and tensor perturbations generated during inflation. However, these corrections terms are high energy effects and as the energy density is decreasing, soon they decouple from the cosmological dynamics. This leaves open the possibility that the inflaton field survives without decay after the end of inflation. Then, the curvaton field provides the mechanism for the reheating of the universe.

We developed a general curvaton formalism appropriate to brane-worlds. In all of the brane-world models there is a transition from one dimensionality spacetime to another as the energy density decreases. Thus the curvaton oscillates in one dimensionality space and may decay in the same or at a different dimensionality spacetime. Different dimensionalities spacetimes have different cosmological dynamics in brane-worlds and this is translated to a system of constraints that the curvaton parameters should respect.

We derived constrained relations which the curvaton parameters should also satisfy in order to suppress short-wavelength gravitational wave dominance over radiation. According to our hypothesis, the inflaton field survives without decay after the end of inflation. Then it enters a kinetic epoch until the curvaton takes over and dominates the cosmological evolution. If this epoch is long enough, there is a possibility of generation of large amplitude gravitational waves. In brane-worlds there are various interesting cases of competing or complementary effects. For example, the inflaton field can enter without decay a five-dimensional regime (induced gravity model). In this regime the kinetic epoch does not last long because the correction terms act as friction terms but at the same time the amplitude of gravitational perturbations is enhanced. The analysis of various such cases constrained further the curvaton parameters.

We investigated the possibility that density perturbations are also generated by the curvaton field. Then we found that this is not always possible. There are cases in which the curvaton parameters are so constrained such as curvaton fluctuations could not generate density perturbations. Nevertheless, in all brane-world models there are cases where density perturbations are indeed generated by the curvaton field. We showed that in these cases the upper bound of the fundamental $M_5$ mass decreases considerably and it can take values much lower than the Planck mass. Its final value depends on how much the inflaton fluctuations are suppressed compared to curvaton fluctuations.

For the Randall–Sundrum model we analysed in detail the case where inflation and density perturbations are generated by the inflaton field with a quadratic potential in the five-dimensional spacetime, while reheating is done by the curvaton field in four dimensions. This analysis indicates that inflation and primordial density perturbations are pure five-dimensional high energy effects, while curvaton dynamics decoupled from the high energy regime can lead to low energy values of its parameters. The analysis of the Gauss–Bonnet model, taking under consideration the latest three-year WMAP results, showed that the most probable case is inflation ending in the five-dimensional Randall–Sundrum regime, reproducing in this way all the results for the
reheating of the Randall–Sundrum model. In the induced gravity model we have the interesting possibility that inflation, density perturbations and reheating occur in the four-dimensional high energy regime. Finally, in the combined Gauss–Bonnet and induced gravity model with an appropriate choice of parameters we can reproduce the results obtained for the other three brane-world models.

The introduction of the curvaton field in the brane-worlds gave a better understanding of the early time cosmological dynamics of a brane-universe. This ‘hybrid’ inflaton–curvaton model gives more freedom to constrain the parameters and study physically interesting cases. However, a possible drawback for such a successful ‘hybrid’ model is that the parameters should satisfy a rather complex system of constraints.

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