Type I seesaw mechanism for quasi degenerate neutrinos

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Abstract

We discuss symmetries and scenarios leading to quasi-degenerate neutrinos in type-I seesaw models. The existence of degeneracy in the present approach is not linked to any specific structure for the Dirac neutrino Yukawa coupling matrix $y_D$ and holds in general. Basic input is the application of the minimal flavour violation principle to the leptonic sector. Generalizing this principle, we assume that the structure of the right handed neutrino mass matrix is determined by $y_D$ and the charged lepton Yukawa coupling matrix $y_l$ in an effective theory invariant under specific groups $G_F$ contained in the full symmetry group of the kinetic energy terms. $G_F$ invariance also leads to specific structure for the departure from degeneracy. The neutrino mass matrix (with degenerate mass $m_0$) resulting after seesaw mechanism has a simple form $\mathcal{M}_\nu \approx m_0(I - p y_l y_l^T)$ in one particular scenario based on supersymmetry. This form is shown to lead to correct description of neutrino masses and mixing angles. The thermal leptogenesis after inclusion of flavour effects can account for the observed baryon asymmetry of the universe within the present scenario. Rates for lepton flavour violating processes can occur at observable levels in the supersymmetric version of the scenario.

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I. INTRODUCTION

Our present knowledge on neutrino parameters is based on information obtained from (a) positive results of neutrino oscillation experiments (b) negative results of the neutrinoless double beta decay searches and (c) neutrino mass bounds from the cosmological observations. Combinations of these allow two qualitatively different patterns for neutrino masses: One in which the neutrino masses follow some hierarchy, normal or inverted while in the other all three neutrino masses are (nearly) degenerate. The stringent constraints on the degenerate mass $m_0$ comes from cosmology. Depending on which data set one uses and method of analysis, $3m_0$ can vary between 0.9-1.7 eV or 2-3 eV, the latter limit is based solely on the information from the cosmological microwave background studies. All the neutrinos having a quasi degenerate mass in the range 0.3-1 eV is thus an allowed possibility. It is non-trivial to accommodate this possibility within the conventional pictures of neutrino mass generation. Indeed, unified treatment of all fermion masses tend to generate hierarchical masses for neutrinos as well. For example, the light neutrino masses are related to the Dirac neutrino mass matrix $m_D$ in type I seesaw model and generically follow the hierarchical patterns. The purpose of this letter is to identify symmetries and scenarios based on them which lead to quasi degenerate neutrinos in type I seesaw model which is by far the most popular mechanism for neutrino mass generation. Unlike all the previous models of degenerate neutrino masses, the existence of degeneracy in the present approach is insensitive to the detailed structure of $m_D$ and $M_R$ both of which can hierarchical and it is not linked to a type II contribution as is the case with some of the models.

Our basic formalism derives ideas from the minimal flavour violation (MFV) hypothesis which uses symmetries of the standard model Lagrangian to construct effective theories of flavour violations in frameworks going beyond it. While the structure of flavour violations is not our primary concern we use similar ideas to constrain structure of neutrino masses. If quarks are massless then the SM Lagrangian is invariant under the flavour group $G_q = U(3)^3$ corresponding to independent unitary rotations on three flavours of quark doublets $q_L$, and singlets $u_R$ and $d_R$. The Yukawa couplings violate this symmetry preserving the baryon number and hypercharge symmetries. The basic assumption of MFV hypothesis is that these Yukawa couplings are the sole source of flavour violations and they determine the structure of flavour violations in theories which go beyond SM.

MFV principle has also been used in the lepton sector as well in several works. Its implementation depends crucially on the source of neutrino masses and how lepton number gets violated. Straightforward possibility assumes that the charged lepton Yukawa couplings and the neutrino mass matrix appearing as coefficient of dimension 5 lepton number violating operator are the irreducible sources of flavour violations and an effective theory of flavour is constructed using them. The basic Yukawa couplings and the flavour symmetry would be different in more fundamental theory of neutrino masses. Consider for example the seesaw model with three right handed (RH) neutrinos $\nu_R$. The Yukawa couplings are
\[ -\mathcal{L}_y = \bar{l}_L y_l e_R \phi + \bar{l}_L y_D \nu_R \tilde{\phi} + \text{H.C.} \quad (1) \]

\( \phi \) is the standard Higgs doublet and \( \tilde{\phi} = i\sigma_2 \phi^* \). \( y_l \) and \( y_D \) are the Yukawa coupling matrices. In the absence of these couplings, the Lagrangian is invariant under the symmetry group \( G_l \equiv U(3)_l \times U(3)_e \times U(3)_\nu \), where each \( U(3)_f \) \( (f = l, e, \nu) \), corresponds to independent rotations on \( l_L, e_R, \nu_R \). Breaking of this flavour symmetry is governed by the Yukawa couplings as well as the explicit Majorana mass term for the RH neutrinos

\[ \frac{1}{2} \nu_R^T C^{-1} M_R \nu_R + \text{H.C.} \quad (2) \]

The MFV hypothesis can be implemented by assigning the Yukawa couplings \( y_l \) and \( y_D \) appropriate transformation property under \( G_l \) in such a way that eq. (1) becomes \( G_l \) invariant. The RH mass term is not invariant under \( G_l \). It is made invariant in [6] by assuming a smaller flavour symmetry group \( U(3)_l \times U(3)_e \times O(3)_\nu \) and assuming that \( M_R \) is proportional to identity. We wish to consider here an alternative possibility which assumes that \( M_R \) also arises from presumably small \( y_l, y_D \) in an effective theory from physics at high scale with broken lepton number. This can be realized if it is assumed that the relevant effective symmetry is a sub-group of \( G_l \). Specifically, we assume that

(1) The effective flavour symmetry group \( G_F \equiv O(3)_l \times O(3)_e \times O(3)_\nu \times U(1)_R \) of the Yukawa Lagrangian is a subgroup of the full flavour symmetry \( G_l \) of the SM Lagrangian without the Yukawa couplings. The last \( U(1)_R \) corresponds to the lepton number transformation on the RH neutrinos. \( y_l \) and \( y_D \) are assumed to transform under \( G_F \) to make eq. (1) invariant under \( G_F \). Specifically,

\[ l_L \rightarrow O_l l_L \quad , \quad e_R \rightarrow O_e e_R \quad , \quad \nu_R \rightarrow O_\nu \nu_R \quad , \]

\[ y_l \rightarrow O_l y_l O_e^T \quad , \quad y_D \rightarrow O_l y_D O_\nu^T \quad . \quad (3) \]

\( O_{l,e,\nu} \) are three orthogonal matrices. In addition, \( y_D \) and \( \nu_R \) are assigned opposite charges under \( U(1)_R \). We shall comment subsequently on other choices of \( G_F \).

(2) \( y_l \) and \( y_D \) are the only irreducible couplings which not only determine flavour violations but also the structure of \( M_R \) and hence of the neutrino mass matrix. \( M_R \) is determined using \( y_{l,D} \) and the above transformation properties.

**II. EFFECTIVE \( M_R \)**

For orientation, we consider an explicit scheme to realize above assumptions. We introduce two complex fields \( \eta_l \) and \( \eta_D \). They are singlets with respect to SM but transform under \( G_F \) respectively as \((3,3,1)_0\) and \((3,1,3)_{-1}\) where the suffix corresponds to \( U(1)_R \) values. \( \eta_l \)
and $\eta_D$ are $3 \times 3$ matrices in flavour space. The flavour symmetry forbids renormalizable Yukawa couplings but allows the following non-renormalizable operators as in Froggatt Nielsen proposal [9]:

$$- L_Y = \frac{1}{2\Lambda} \nu_R^T C^{-1} \eta_D^T \left( c_0 + \frac{c_1}{\Lambda^2} \eta_l \eta_l^T + \frac{d_1}{\Lambda^2} \eta_l^\dagger \eta_l^\dagger + \frac{d_2}{\Lambda^2} (\eta_D \eta_D^\dagger + \eta_D^\dagger \eta_D^\dagger) + \frac{d_3}{\Lambda^2} (\eta_l \eta_l^\dagger + \eta_l^\dagger \eta_l^\dagger) + \ldots \right) \eta_D \nu_R,$$

$$+ \frac{1}{\Lambda} \left( \bar{\nu}_L \eta_l e_R \phi + \bar{\nu}_L \eta_D \nu_R \phi^\dagger \right) + \text{H.C.} .$$

(4)

Here $c_0, c_1, d_1, d_2, d_3$ are coefficients of $\mathcal{O}(1)$. Several comments are in order in connection with the above equation.

- Bare mass term for the RH neutrinos is not allowed by the $U(1)_R$ symmetry.

- The RH neutrino mass term (first line of eq.(4)) has to be symmetric in flavour space and its structure is completely determined by $G_F$ and the transformation rule given in eq.(3). In particular, terms proportional to $d_1, d_2, d_3$ would be absent from the superpotential of the supersymmetric generalization of the model. They may arise at higher order from the D-terms.

- The total lepton number is explicitly broken in eq.(4) while the RH lepton number gets spontaneously broken by the vacuum expectation value of $\eta_D$. Lepton number conservation is restored in the limit $\Lambda \rightarrow \infty$. $\Lambda$ therefore sets the scale of lepton number violation. Flavour violations are determined by the scale $\equiv \Lambda_{FV}$ set by the vevs of $\eta_{l,D}$ and $\Lambda_{FV} \lesssim \Lambda$. $\langle \eta_{l,D} \rangle$ in fact play dual role here. On one hand, they determine the structure of the Yukawa couplings:

$$y_l = \frac{\langle \eta_l \rangle}{\Lambda}, \quad y_D = \frac{\langle \eta_D \rangle}{\Lambda}. \quad (5)$$

On the other, $\langle \eta_D \rangle$ also determines the RH handed neutrino masses:

$$M_R \approx c_0 \frac{\langle \eta_D \rangle^T \langle \eta_D \rangle}{\Lambda}. \quad (6)$$

This shows that the RH neutrino masses are suppressed compared to $\Lambda$ indicating the seesaw origin for these masses as well.

Neutrino masses follow from eqs.(4,5):

$$m_D = v y_D,$$

$$M_R = \Lambda y^T D \left( c_0 + c_1 y_l y_l^T + d_1 y_l^\dagger y_l^\dagger + d_2 (y_D y_D^\dagger + y_D^\dagger y_D^\dagger) + d_3 (y_l y_l^\dagger + y_l^\dagger y_l^\dagger) \right) y_D. \quad (7)$$

where $v \sim 174$ GeV denotes the Higgs vacuum value. The light neutrino mass term is then given by:

$$\frac{1}{2} \nu_L^T C^{-1} M^* \nu_L + \text{H.C.} ,$$
with
\[
\mathcal{M}_\nu \equiv m_D M_R^{-1} m_D^T,
\]
\[
\approx m_0 \left( 1 - \frac{c_1}{c_0} y_D y_l^T - \frac{d_1}{c_0} y_l^* y_l^\dagger - \frac{d_2}{c_0} (y_D y_D^\dagger + y_D^* y_D^T) - \frac{d_3}{c_0} (y_l y_l^T + y_l^* y_l^T) + ... \right). \tag{8}
\]

Seesaw mechanism together with \(\mathcal{G}_F\) invariance has resulted in an effective neutrino mass matrix with three almost degenerate neutrinos having a common mass
\[
m_0 \equiv \frac{v^2}{c_0 \Lambda}.
\]

This is contrary to the standard expectations in the type-I seesaw model where hierarchical \(m_D\) leads to hierarchical neutrino masses. The lepton number violation scale is restricted to be \(\Lambda \gtrsim 10^{14}\) GeV for \(c_0 \sim 1\) and \(m_0 \lesssim 0.3\) eV. This scale could even be higher if \(c_0\) is suppressed. Note that \(m_0\) is independent of the scale of the flavour symmetry breaking and the RH neutrino masses. This happens because of the seesaw origin of the RH neutrino masses, see eq.(6). The role of the RH neutrinos is to give a quasi degenerate spectrum through this double seesaw mechanism. Moreover, the \(\mathcal{G}_F\) invariance also results in a very specific structure of departures from degeneracy.

The present scheme differs from all other previous models \([3, 4]\) of the degenerate neutrinos in an important way. These models need to have some restrictions on the structure of the Dirac mass matrix and/or require degenerate spectrum for the RH neutrino masses. In contrast, \(m_D\) here can be arbitrary (as long as the Yukawa couplings \(y_D < 1\)) and it determines the structure of \(M_R\). Both \(m_D\) and \(M_R\) can be simultaneously hierarchical yet result into (almost) degenerate spectrum after the seesaw mechanism once \(\mathcal{G}_F\) invariance is imposed.

The type I seesaw model and \(\mathcal{G}_F\) invariance together led to quasi degeneracy. The quasi degeneracy follows on a more general ground from the \(\mathcal{G}_F\) invariance alone. Consider SM model without RH neutrinos. Dirac Yukawa couplings \(y_D\) are absent and appropriate symmetry would be \(O(3)_l \times O(3)_e\) in this case. Neutrino masses can be understood as arising from an effective dimension five operator. Requiring invariance of this operator under \(O(3)_l \times O(3)_e\), the transformation rules eq.(3) imply the same neutrino mass matrix as in eq.(5) but without the \(y_D\) terms.

We close this section with a comment on a possible origin of eq.(4) which was written down in an effective theory approach using the MFV. Let us add three sterile neutrinos \(s_R\) transforming under \(\mathcal{G}_F\) as \((3, 1, 1)_0\). This allows the following renormalizable \(\mathcal{G}_F\) invariant interactions
\[
\sim \bar{s}_R \eta_D \nu_R + \frac{\Lambda}{2} s_R^T C^{-1} s_R + \text{H.C.}.
\]

The bare mass term for \(s_R\) sets the scale of lepton number violation. Integration of \(s_R\) after the seesaw mechanism generates the \(\eta_D\)-dependent terms given in the right handed mass matrix, eq.(7).
III. NEUTRINO Masses AND MIXING

From now onwards we specialize to supersymmetric model and consider the following simpler version of eq.(8):

\[ M_{\nu} \approx m_0 (1 - p y_l y_l^T) , \]  

with \( p \equiv \frac{\alpha}{\cos \beta} \) and now \( m_0 = \frac{v^2 \sin^2 \beta}{\cos \beta} \). In this case, it is the charged lepton Yukawa couplings rather than \( y_D \) which determine both mixing among neutrinos and their (mass)\(^2\) differences. This remarkably simple structure is capable of explaining all the features of the neutrino spectrum. Necessary condition for this to happen is that either \( p \) and/or \( y_l \) are complex indicating the presence of CP violation in general. \( p \) can be chosen real without loss of generality. \( y_l \) also has fairly general structure in the absence of further assumptions in spite of some freedom offered by the \( G_F \) in the choice of flavour basis. The structure of \( y_{l,D} \) in the outlined model is determined by the vacuum structure of \( \eta_{l,D} \) which together represent four independent real \( 3 \times 3 \) matrices. \( G_F \) invariance can be used to make one of them (say \( \mathrm{Re} \langle \eta_D \rangle \)) diagonal and the remaining freedom can be used to make three of the elements in \( \langle \eta_l \rangle \) real or purely imaginary. This still allows fairly general forms for \( y_{l,D} \). Each choice of these couplings correspond to a specific direction in \( G_F \) space and implies a definite form for neutrino masses. Below we give specific but fairly general forms for \( y_l \) leading to a successful description of the neutrino spectrum.

A general \( y_l \) can be written as

\[ y_l = V_{lL} d_i V_{lR}^\dagger . \]  

Here, \( V_{lL,IR} \) are \( 3 \times 3 \) unitary matrices and \( d_i = \text{diag.}(y_{e}, y_{\mu}, y_{\tau}) \) is the known diagonal matrix of the charged lepton Yukawa couplings, e.g. \( y_{\tau} = \frac{m_{\tau}}{v \cos \beta} \) in the MSSM. Neutrino mixing is determined in the flavour basis defined as

\[ M_{\nu f} \equiv V_{lL}^\dagger M_{\nu} V_{lL}^* \approx m_0 \left( U_{IL} - p d_i U_{IR} d_i \right) , \]  

where \( U_{IL,IR} \equiv V_{lL,IR}^\dagger V_{lL,IR}^\ast \) are symmetric unitary matrices. In particular, if \( y_l \) are real then \( V_{lL,IR} \) are orthogonal and \( U_{IL} = U_{IR} = 1 \). \( M_{\nu f} \) becomes diagonal in this case and there is no mixing among neutrinos although they are now non-degenerate. Thus we need to assume \( V_{IR} \) and/or \( V_{IL} \) to be complex. The first term corresponds to the most general mass matrix for the degenerate neutrinos studied in [10]. \( U_{lL,IR} \) are both symmetric and unitary and can be parametrized [10] as

\[ U_{lL,IR} = P_{L,R} R_{23}^T(\phi_{L,R}) U_{12}(\theta_{L,R}, \alpha_{L,R}) R_{23}(\phi_{L,R}) P_{L,R}^\dagger , \]  

where

\[ R_{23}(\phi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & \sin \phi & -\cos \phi \end{pmatrix} ; U_{12}(\theta, \alpha) = \begin{pmatrix} \cos \theta & \sin \theta & 0 \\ \sin \theta & -\cos \theta & 0 \\ 0 & 0 & e^{i\alpha} \end{pmatrix} \]

and \( P_{L,R} \) are diagonal phase matrices. Phases in one of these can be removed by redefining the phases of the charge leptons, see eq.(11) and we choose to make \( P_L = I \).
Notice that $U_L$ becomes invariant under $\nu_\mu \leftrightarrow \nu_\tau$ interchange if $\phi_L = \frac{\pi}{4}$. This makes $M_{\nu f}$ $\mu$-$\tau$ symmetric to leading order and leads to prediction [1] of the maximal atmospheric mixing angle $\theta_{23}$ and vanishing $\theta_{13}$. The second term in eq. (11) generically violates $\mu$-$\tau$ symmetry by a small amount since $y_\mu \neq y_\tau \ll 1$. But even in this case, there is a unique choice for $U_{IR}$ which allows $\mu$-$\tau$ symmetric perturbations as well. This is given by

$$U_{IR} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & e^{i\beta_3} \\ 0 & e^{i\beta_3} & 0 \end{pmatrix}.$$  \hspace{1cm} (14)

For this special case, the departure from degeneracy are characterized by a single parameter $\epsilon_1 \equiv p y_\mu y_\tau$ and the solar scale $\Delta_\odot$, the atmospheric scale $\Delta_A$ and the solar angle $\theta_{12}$ get determined as $\epsilon_1$ as:

$$\tan 2\theta_{12} \approx \frac{\tan \theta_L}{\cos \beta_3} \left(1 - \frac{\epsilon_1 \sin^2 \beta_3 \cos \beta_3}{2 \cos \theta_L} + \mathcal{O}(\epsilon_1^2)\right),$$

$$\Delta_\odot \cos 2\theta_{12} \approx 2m^2_\odot \epsilon_1 \left(\cos \theta_L \cos \beta_3 + \mathcal{O}(\epsilon_1)\right),$$

$$\Delta_A \approx 2m^2_\odot \epsilon_1 \left(\cos(\alpha_L - \beta_3) + \frac{\cos \beta_3 \cos \theta_L \sin^2 \theta_{12}}{\cos 2\theta_{12}} + \mathcal{O}(\epsilon_1)\right).$$  \hspace{1cm} (15)

Note that the solar mixing angle arises at zeroth order and perturbation makes only a small change. The $\Delta_\odot, \Delta_A$ are generated at the first order in $\epsilon_1$. Above equations serve to determine $\beta_3, m^2_\odot \epsilon_1$ in terms of $\theta_L$ and the known quantities. $\theta_L$ is required to be small and $\beta_3$ close to $\frac{\pi}{2}$ for obtaining the correct $\theta_{12}$ and $\frac{\Delta}{\Delta_A}$. Similarly, $\alpha_L$ is also required to be non-zero. Both these phases violate CP and appear as phases in the Majorana mass for light neutrinos. Underlying CP violation is however not manifested through the Dirac phase because of the $\mu$-$\tau$ symmetry which makes $\theta_{13}$ zero. We discuss below a generalization which does not impose $\mu$-$\tau$ symmetry and leads to observable CP violating phase.

In the most general situation and neglecting $y_e$, perturbations to degeneracy are governed by two parameters $\epsilon_1$ defined above and $\epsilon_2 \equiv py_\mu^2$. It follows from eq. (11) that $\epsilon_2$ would dominate except when $(U_{IR})_{33} \ll \frac{y_e}{y_\mu}(U_{IR})_{22}$. Specific choice $(U_{IR})_{33} \approx 0$ (corresponding to $\cos^2 \phi_{RE^{\alpha R}} \approx \sin^2 \phi_R \cos \theta_R$) in eq. (12) is quite interesting. In this case, the lower $2 \times 2$ block of $M_{\nu f}, M_{\nu f}^\dagger$ becomes $\mu$-$\tau$ symmetric (up to terms of order $\frac{y_e}{y_\mu}$) for arbitrary values of other angles and phases in $U_{IL,IR}$. Thus this choice naturally leads to a large atmospheric mixing angle. Moreover, perturbations are essentially controlled in this case by the single parameter $\epsilon_1$.

We have numerically studied implications of the specific choice $(U_{IR})_{33} = 0$ using the general parametrization in eq. (12). We randomly vary the independent parameters $\theta_L, \phi_L, \beta_3, \alpha_L$ in the full range. $\beta_3$ here represents the phase of $(U_{IR})_{23}$ in eq. (12). $\epsilon_1$ is redefined as $\epsilon_1 \equiv p y_\mu y_\tau |(U_{IR})_{23}|$. Since entire mixing and mass differences are determined by the perturbations in eq. (11), $m^2_\odot \epsilon_1$ acts as a normalization constraint but we have imposed the condition that $\epsilon_1 < 0.5$ and $m_0 < 0.3$ eV. We require that each choice of random inputs correctly reproduce the $\Delta_\odot, \Delta_A, \sin^2 2\theta_{23}, \tan^2 \theta_{12}$ within $1\sigma$. Two specific outcomes of this random
FIG. 1: Allowed ranges of $\sin^2 \theta_{23}$ versus $\sin^2 \theta_{13}$ in model implied by eq. (11) and the assumption $(U_{lR})_{33} = 0$, see text for details.

FIG. 2: Allowed ranges of $\sin^2 \theta_{13}$ versus the Jarlskog invariant $J$ in model implied by eq. (11) and the assumption $(U_{lR})_{33} = 0$, see text for details.

Analysis are displayed in Fig.(1) and Fig.(2). Fig.(1) displays the variation of $\sin^2 \theta_{13}$ with $\sin^2 \theta_{23}$. Interestingly, the atmospheric mixing stays close to maximal for all of the random allowed choices mentioned above but the $\theta_{13}$ can span the entire 3$\sigma$ range. CP violation is an essential ingredient in this analysis since its absence implies no mixing as argued above. Fig.(2) displays the allowed values of the Jarlskog invariant $J$ versus $\sin^2 \theta_{13}$. There is a clear correlation between $J$ and the $\sin^2 \theta_{13}$ which can be tested. The above results hold for strictly zero $(U_{lR})_{33}$. We find that one could obtain sizable deviation from the maximality by allowing $(U_{lR})_{33} \sim \frac{y_u}{y_t} (U_{lR})_{22}$. Even in this case, large values of $\theta_{13}$ is possible and prediction
of the Jarlskog invariant shown in Fig.(2) does not change appreciably.

The above discussion was based on the specific choice $G_F = O(3)_l \times O(3)_e \times O(3)\nu \times U(1)_R$. One could consider a smaller symmetry by replacing $O(3)_l \times O(3)_e$ by its vectorial sub-group under which both $l, e_R$ transform as triplets. Quasi degeneracy is still maintained but now eq.(9) gets replaced by

\[ M_\nu \approx m_0(1 - p(y_l + y_T^l) + ....) \quad , \]

(16)

Now the departure from degeneracy occurs at first order in the Yukawa couplings. Just as in the previous case this case too is capable of reproducing the neutrino spectrum.

A larger choice for $G_F$ can also lead to degenerate spectrum with additional assumptions. Consider the group $G_F = U(3)_l \times U(3)_e \times O(3)\nu \times U(1)_R$. This is the group chosen in ref \[6\] except for the additional $U(1)_R$. Using the same arguments as in the above cases, leading order expression of $M_R$ is given by

\[ M_R = c_0 \Lambda \left( y_D^\dagger y_D + y_D^T y^*_D \right) + ..... \]

This $M_R$ also leads to degenerate neutrinos provided $y_D^\dagger y_D$ is real.

\section{IV. LEPTOGENESIS}

The baryon asymmetry $Y_B$ in the universe can be generated through leptogenesis [11] in the present approach after due considerations of flavour effects [11, 12, 13]. The latter become important in MSSM for temperature below $(1 + \tan^2 \beta)10^{12}$ GeV when the tau interactions start equilibrating and tend to wash out asymmetry in the tau flavour. In a general seesaw framework, this asymmetry depends on two distinct set of parameters: The Dirac Yukawa couplings and the right handed neutrino masses. Here, the single relevant quantity is the Dirac Yukawa coupling matrix $\tilde{y}_D$ in basis with diagonal charged leptons and the RH neutrinos. Let $V_R$ be a unitary matrix which diagonalizes $M_R$ in eq.(7).

\[ V_R^T M_R V_R = D_R \]

where $D_R$ is diagonal matrix of the RH neutrino masses. $\tilde{y}_D$ is then given by

\[ \tilde{y}_D = V_L^\dagger y_D V_R . \]

The lepton asymmetry generated in flavour $\alpha = (e, \mu, \tau)$ by the out of equilibrium decay of the lightest right handed neutrino is given in the MSSM by [11]

\[ \epsilon_{aa} \approx - \frac{3M_1}{8\pi M_k} \frac{Im[(\tilde{y}_D^\dagger \tilde{y}_D)_{1k} (\tilde{y}_D)_{1\alpha} (\tilde{y}_D)_{\alpha k}]}{(\tilde{y}_D^\dagger \tilde{y}_D)_{11}} , \]

(17)

where $M_k, k = 1, 2, 3$ represent the RH neutrino masses and $M_1$ corresponds to the lightest one. The $Y_B$ generated through $\epsilon_{aa}$ also depend on the wash out parameters

\[ \bar{m}_\alpha = \frac{v^2 \sin^2 \beta (\tilde{y}_D^\dagger)_{1\alpha} (\tilde{y}_D)_{\alpha 1}}{M_1} . \]

(18)
Eq. (17) can be written in terms of \( \tilde{y}_D \) as

\[
D_R = c_0 \Lambda \tilde{y}_D^T V_{IL}^T (1 + p \ y_i y_i^T) V_{IL} \tilde{y}_D ,
\]

We can invert above equation to obtain a parametrization for \( \tilde{y}_D \):

\[
\tilde{y}_D \approx V_{IL}^\dagger (1 - \frac{1}{2} p \ y_i y_i^T) R \left( \frac{D_R}{c_0 \Lambda} \right)^{1/2}
\]

(20)

to leading order in \( p \ y_i y_i^T \). Here \( R \) is a complex orthogonal matrix.

The expressions for \( \epsilon_{\alpha\alpha} \) get simplified in the \( \mu-\tau \) symmetric limit, \( \phi_L = \frac{\pi}{4} \), in eq. (12) In this limit, \( V_{IL} \) which reproduces \( U_{IL} \) in eq. (12) is given by

\[
V_{IL} \approx \begin{pmatrix}
1 & 0 & 0 \\
0 & -\frac{i}{\sqrt{2}} & -\frac{i}{\sqrt{2}} \\
0 & \frac{1}{\sqrt{2}} e^{-i\alpha L/2} & -\frac{1}{\sqrt{2}} e^{-i\alpha L/2}
\end{pmatrix} .
\]

(21)

It is now straightforward to work out the lepton asymmetries and wash out parameters:

\[
\epsilon_e \approx -\frac{3 M_1 m_0}{8 \pi v^2 \sin^2 \beta} \frac{\text{Im}[R_{11}^2]}{(R^\dagger R)_{11}} ,
\]

\[
\epsilon_{\mu} \approx -\frac{3 M_1 m_0}{16 \pi v^2 \sin^2 \beta} \frac{\text{Im}[(R_{12}^\dagger - i R_{13}^\dagger e^{-i\alpha L/2})(R_{12}^\dagger + i R_{13}^\dagger e^{i\alpha L/2})]}{(R^\dagger R)_{11}} ,
\]

\[
\epsilon_{\tau} \approx -\frac{3 M_1 m_0}{16 \pi v^2 \sin^2 \beta} \frac{\text{Im}[(R_{12}^\dagger + i R_{13}^\dagger e^{-i\alpha L/2})(R_{12}^\dagger - i R_{13}^\dagger e^{i\alpha L/2})]}{(R^\dagger R)_{11}} .
\]

(22)

\[
\tilde{m}_e \approx m_0 |R_{11}|^2 ; \quad \tilde{m}_\mu \approx \frac{m_0}{2} |R_{21} - i R_{31} e^{i\alpha L/2}|^2 ; \quad \tilde{m}_\tau \approx \frac{m_0}{2} |R_{21} + i R_{31} e^{i\alpha L/2}|^2 .
\]

(23)

In writing above equations, we have retained only zeroth order terms in \( p \). Note that the scale of the individual lepton asymmetries is set by the degenerate mass \( m_0 \) and not the atmospheric or solar scale as in models with hierarchical neutrinos. Also it is easy to check from the leading order expression given above that the sum \( \epsilon_i \equiv \sum_{\alpha} \epsilon_{\alpha\alpha} \) vanishes. By including the non-leading terms and using eq. (15), we find

\[
\epsilon_i \approx \frac{3 M_1^2}{8 \pi v^2} \Delta_\odot \cos 2\theta_{12} \frac{\text{Im}[R_{12}^2]}{(R^\dagger R)_{11}} - 2 \sin \frac{\alpha_i}{2} \frac{\text{Im}[R_{21}^* R_{31}]}{(R^\dagger R)_{11}} .
\]

(24)

This is much smaller than the individual asymmetries. The latter can be quite large \( \sim \mathcal{O}(10^{-5}) \) for \( M_1 \sim 10^{10} \text{ GeV} \). We use the approximate approximation for the \( Y_B \) given in [13]:

\[
Y_B \approx -\frac{10}{31 g_*} \left[ \epsilon_e \eta \left( \frac{93}{110} \tilde{m}_e \right) + \epsilon_{\mu} \eta \left( \frac{19}{30} \tilde{m}_\mu \right) + \epsilon_{\tau} \eta \left( \frac{19}{30} \tilde{m}_\tau \right) \right]
\]

(25)

valid for the temperature range \( (1 + \tan^2 \beta)10^5 \text{ GeV} \leq T \leq (1 + \tan^2 \beta)10^9 \text{ GeV} \). The washout function \( \eta(x) \) is given by

\[
\eta(x) \approx \left( \frac{x}{8.25 \times 10^{-3} \text{ eV}} \right)^{-1} + \left( \frac{0.2 \times 10^{-3} \text{ eV}}{x} \right)^{-1.16}^{-1}
\]
and \( g_s = 228.75 \). Note that the matrix \( R \) is independent of the light neutrino mass parameters and there exists large ranges of three complex angles parametrized \( R \) for which \( Y_B \) can be significant. We give one set for illustrative purpose. Let us define \( R_{11} = \cos z_{13} \cos z_{12}, \ R_{21} = \cos z_{13} \sin z_{12} \) and \( R_{31} = \sin z_{13} \) where \( z_{ij} \) are complex angles. Then the choice \( z_{13} = 0, z_{12} = 0.30 + 0.13 i, m_0 = 0.3 \) eV, \( M_1 = 7.9 \times 10^{10} \) GeV, \( \tan \beta = 10 \) leads to \( Y_B \approx 8.6 \times 10^{-11} \). Individual lepton asymmetries are quite large for this choice \( \epsilon_\mu = \epsilon_\tau = \frac{-1}{2} \epsilon_e \approx 7.4 \times 10^{-6} \) but there sum vanishes emphasizing the role played by flavour effects.

V. LEPTON FLAVOUR VIOLATION

As in other MFV approaches, the structure of the leptonic flavour violations is coded in \( y_l, y_D \). The ratio of the scale of lepton number violation, \( \Lambda \) to lepton flavour violation \( < \eta_D > \) is not very large \( \sim (1 - 100) \). Consequently, if these are the only scales in the theory, then the the rates of lepton flavour violating (LFV) processes like \( l_i \rightarrow l_j + \gamma \) are highly suppressed (see for example, the discussion in Ref. \([2]\)).

In the supersymmetric version of the theory however, there is a new scale at low energy in terms of the slepton and sneutrino masses at the weak scale. These soft masses continue to carry the memory of high scale flavour violation due to the presence of the seesaw mechanism, leading to large flavour violating effects at the weak scale \([15]\]. In our present scheme, we assume that the soft masses are universal below the high scale \( \Lambda \). This sets the following hierarchy of scales

\[
\Lambda \gtrsim \Lambda_{\tilde{m}_0} \gtrsim < \eta_D > \gtrsim M_R,
\]

where \( \Lambda_{\tilde{m}_0} \) determines the scale where soft masses are universal. At the weak scale, the sleptons receive corrections proportional to \( y_D, y_l \) due to renormalization group (RG) effects, which are roughly given as

\[
m^2_{\tilde{L}} \approx \tilde{m}_0^2 \left( k_0 I - y_D y_D^T t^0_1 - y_l y_l^T t^0_2 \right),
\]

\[
m^2_{\tilde{e}_c} \approx \tilde{m}_0^2 \left( k'_0 I - y_l y_l^T t^0_3 \right),
\]

where \( t^0_1, t^0_2, t^0_3, k_0, k'_0 \) are coefficients generated by RG running with a typical size of the order \( 1/(16\pi^2) \ln \Lambda^2/\tilde{m}_0^2 \). In the basis where charged leptons and the RH neutrinos are diagonal, the flavour off-diagonal entries in the slepton mass matrices determine the amplitudes of the flavour violating processes. In this basis, these off-diagonal entries are proportional to \( \tilde{y}_D \tilde{y}_D^\dagger \) with \( \tilde{y}_D \) given by eq. (20). At the leading order this takes the form :

\[
\tilde{y}_D \tilde{y}_D^\dagger \approx V^\dagger_{lL} D_R R^1 V_{lL} + \mathcal{O}(p y_l y_l^T),
\]

where we have neglected \( \mathcal{O}(p y_l y_l^T) \) corrections. The strength of flavour violation is best judged by considering the ratio of the flavour violating off-diagonal entries to the flavour.
diagonal terms. Here we have
\[
(\delta^{(l)}_{LL})_{ij}|_{i \neq j} \approx \frac{m_0}{v^2} \left[ V_{iL}^L D_R t_0 R^\dagger V_{LL} \right]_{ij}
\]  
(28)

Here \(i, j\) are generation indices and \(t_0\) is a diagonal matrix containing the logarithmic terms for each of the right handed neutrinos, given as \(1/(16\pi^2) \ln \eta_D > 2 / M^2_l\). We have also exchanged \(c_0 \Lambda\) for the light neutrino mass scale \(m_0\). Note that when \(D_R\) is degenerate (or \(y_D y_D^T = I\) ) and \(R\) is real, there is no flavour violation in the theory. Assuming that the mass scale of the right handed neutrinos is roughly the same, we have the log factor to be \(\ln < \eta_D >^2 / M^2 \approx \ln \Lambda^2 / < \eta_D >^2\). Using eq. (28) one can estimate the branching fraction of the LFV process \(l_j \rightarrow l_i + \gamma\) as
\[
Br(l_j \rightarrow l_i + \gamma) \approx \frac{\alpha^3}{G^2_F} \frac{|(\delta^{(l)}_{LL})_{ij}|^2}{m^4_{susy}} \tan^2 \beta
\]  
(29)

Existing limits on \(\mu \rightarrow e + \gamma\) from the MEGA experiment constraint \(|(\delta^{(l)}_{LL})_{ij}| \lesssim 10^{-4}\) for slepton masses \(\sim m_{susy} \approx 400\) GeV and \(\tan \beta \sim 10\) [17]. Presently, the range between \(10^{-6} - 10^{-4}\) in \((\delta^{(l)}_{LL})_{21}\) is being probed. The explicit form of the relevant \((\delta^{(l)}_{LL})_{ij}\) in this case is given by
\[
(\delta^{(l)}_{LL})_{21} = \frac{m_0}{\sqrt{2} v^2} \left[ (-i R_{11} R_{21}^* + e^{-i \alpha L/2} R_{11} R_{31}^*) M_1(t_0)_{11} \right. \\
- \left. (i R_{12} R_{22}^* - e^{-i \alpha L/2} R_{12} R_{32}^*) M_2(t_0)_{22} - (i R_{13} R_{23}^* - e^{-i \alpha L/2} R_{13} R_{33}^*) M_3(t_0)_{33} \right]
\]  
(30)

From the above we see that for \(m_0 \sim 0.1\) eV and \(M_3 \sim 10^{14}\) GeV , \((\delta^{(l)}_{LL})_{21}\) is close to \(O(1)\). This will require additional suppression from the elements of \(R\) matrix, e.g. \(R_{13} \sim 10^{-3} - 10^{-4}\) can suppress the \((\delta^{(l)}_{LL})_{21}\). The leptogenesis can still work as exemplified above. Nevertheless both leptogenesis and LFV can together provide tight constraints on the model parameter space and it would be interesting to pursue this further.

VI. SUMMARY

We have discussed here a novel approach to obtaining quasi-degenerate neutrinos in the context of type-I seesaw models. This is based on generalization of the minimal flavour violation principle to the leptonic sector. Earlier attempts in this direction regarded \(M_R\) as an independent entity proportional to identity. Instead if it is assumed that \(M_R\) also arises from the Dirac Yukawa couplings in an effective theory then this can lead to quasi degenerate neutrino. As discussed here, this is possible by requiring invariance under \(G_F = O(3)_l \times O(3)_e \times O(3)_\nu \times U(1)_R\). Larger choice for \(G_F\) is also shown to lead to degeneracy with additional assumption. Consequences of the quasi degenerate structure implied by \(G_F\) were worked out. CP violation was found to be necessary in order to obtain non-trivial mixing in this approach. Allowing this, one can obtain correct mixing patterns and observable CP violation. Thermal leptogenesis aided by flavour effects is
shown to explain the observed baryon asymmetry. If CP violation is found to be small or nearly absent in future neutrino oscillation experiments then this approach will be strongly constrained if not ruled out entirely.

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