Simulation of Phase Noise Estimation and Compensation in Coherent FSOC System

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Abstract. We study the phase characteristics of weak turbulence and analyze the estimation and compensation of phase noise caused by atmospheric turbulence. It is found that the traditional compensation algorithm is not very suitable in Free Space Optical Communication (FSOC). We propose to introduce the Frequency Domain Equalization (FDE) of single carrier into the FSOC with coherent reception, and the system performance can be greatly improved by the combination of FDE and phase compensation.

1. Introduction

In recent years, with the development of various optical technologies, free space optical communication (FSOC) technology has come into people's field of vision again and developed at an amazing speed. With the advent of the Internet era, people have higher requirements on communication capacity and quality. In the atmospheric turbulence channel, the random change of refractive index makes the original propagating beam change greatly. Therefore, after coherent reception, the signal must be processed to make up for all kinds of damage caused by the channel in the communication process. Nowadays, with the rapid development of devices, people choose to process the received signals in the electrical domain, which is not only simple and fast, but also very low cost. These digital signal processing algorithms include row orthogonalization recovery, clock recovery, polarization equalization, frequency difference estimation and phase estimation. At present, these compensation algorithms have been well developed in optical fiber communication, including orthonormal recovery [1], clock recovery [2], phase recovery [3] and so on. Such as inter-satellite optical communication and satellite-to-ground optical communication, coherent receiving technology [4] is often used to process the received digital signal. However, these two scenarios are still different from the current low-altitude free-space optical channels, and there are few studies on the effects of atmospheric turbulence channels on the phase of optical signals. At present, in FSOC system with atmospheric turbulence, some studies directly do transmission experiments in atmospheric space to verify the performance of digital signal processing algorithms at the receiver [5], and some use different spatial spectral density functions of atmospheric refractive index. Numerical simulation is carried out to simulate atmospheric turbulence channels, so as to verify different digital signal processing algorithms [6]. From the existing research results, we can see that both theoretical research and numerical simulation have their own limitations.

This paper mainly studies the characteristics of phase fluctuation under weak turbulence and estimates and compensates the phase noise caused by atmospheric turbulence. At present, the main phase compensation algorithms used in FSOC are M-th power algorithm and DAML (Decision-aided...
maximum likelihood) algorithm [7]. There are some nonlinear operations in these algorithms, which are not conducive to hardware implementation, and the estimation range is also limited. Therefore, we introduce single-carrier frequency domain equalization (SC-FDE) into coherent FSOC for the first time, and improve the system performance to the greatest extent through the combination of frequency domain equalization and phase compensation. Although this idea has been applied in coherent optical fiber communication [8], the application in free space optical communication is still rare. Simulation results show that this combination not only reduces the computational complexity, but also greatly improves the system performance when meeting a certain SNR, and there is no limit to the estimation range.

2. Study Of Weak Atmospheric Turbulence Channel

We add Gaussian noise and phase noise to the channel. Gaussian noise can include the ubiquitous Gaussian thermal noise in optoelectronic devices and the Gaussian noise in the channel. The phase noise is mainly caused by atmospheric turbulence. At present, the related studies of moderate turbulence and strong turbulence are not very mature, and there is still a big gap between the experimental results and the theoretical analysis results. It is mentioned in Kolmogorov's theory that for fully developed turbulence with high Re number, a scale range can always be found [12]. In this range, the structure function and spectral density satisfy the scaling law. This scale range is generally called the inertia region. In order to simplify it, we also limit it to the inertia zone.

When considering large-scale turbulence fluctuations, the Von-Karman spectral model is commonly used in the study.

$$\phi_n(K) = 0.033C_n^2(K^2 + L^2_0)^{-11/6}$$

The refractive index structure constant $C_n^2$ is usually used to describe the fluctuation intensity of optical turbulence, and depends on height and wind speed. $C_n^2$ is approximately equal to $1.05 \times 10^{-15} m^{-2/3}$, which is simulated at a height of 50 meters. The coherence function of phase fluctuation in atmospheric turbulence is

$$B(\rho, L) = (2\pi k)^2 \int_0^L dz \int_0^L J_0(k\gamma \rho) \cos^2 \{P(\gamma, \kappa, z)\} \phi_n(\kappa) |_{\kappa d\kappa}$$

Since the average phase difference is 0, the correlation function of phase fluctuation at 0 is the phase fluctuation variance of a single point.

$$\sigma_{\rho n}^2(L) = (2\pi k)^2 \int_0^L dz \int_0^L \cos^2 \{P(\gamma, \kappa, z)\} \phi_n(\kappa) |_{\kappa d\kappa}$$

We consider the diffraction factor and propagation factor under the plane wave, among which $\gamma$ is the transmission factor. For plane wave, the propagation factor is 1. $\kappa$ is the spatial wavenumber. $P(\gamma, \kappa, z)$ the diffraction factor. For plane waves,

$$P(\gamma, \kappa, z) = \frac{(L-z)}{2k} \kappa^2$$

$Z$ is the transmission path, $\phi_n(\kappa)$ is the spectral density function of atmospheric refractive index. $\kappa$ is the plane wavenumber. The refractive index spectral density function is expressed by formula (1). The refractive index spectral density function adopts the Von Karman turbulence spectrum expressed by formula (1). Formula (3) is reduced to

$$\sigma_{\rho n}^2(L) = (2\pi k)^2 \times 0.033 \int_0^L C_n^2(z) dz \int_0^L \cos^2 \{\frac{(L-z)}{2k} \kappa^2\} |\kappa^{-8/3} d\kappa$$
For a uniform propagation path, the integral can be obtained as follows:

$$\sigma_{\text{pm}}^2 (L) = 0.782 L k^2 C_n^2 \frac{\pi}{\lambda_0}$$

(6)

where \( L \) is the propagation distance, \( k \) is the plane wavenumber and \( L_0 \) is the outer scale of turbulence. When the wavelength \( \lambda_0 \) is 1550nm, \( C_n^2 \) is \( 1.05 \times 10^{-15} \text{m}^{-2/3} \), \( L_0 \) is 0.5m and \( L \) is 10m, the simulation results are as follows: \( \sigma_{\text{pm}}^2 \) is 0.043rad\(^2\) and the standard deviation of phase noise is 11.79 degrees. Only when the transmission distance \( L \) is changed to 100m, \( \sigma_{\text{pm}}^2 \) becomes 0.433rad\(^2\) and the standard deviation of phase noise is 37.70 degrees.

**Table 1.** Simulation results of phase noise under different atmospheric channel conditions

| Model | \( l/\text{nm} \) | \( L_0/\text{m} \) | \( L/\text{m} \) | \( \sigma_{\text{pm}}^2/\text{rad}^2 \) | \( \sigma_{\text{pm}}^2/\text{degrees} \) |
|-------|-----------------|-----------------|-----------------|----------------|------------------|
| Model-1 | 1550 | 0.5 | 10 | 0.043 | 11.79 |
| Model-2 | 1550 | 0.5 | 100 | 0.433 | 37.70 |
| Model-3 | 1550 | 1 | 10 | 0.135 | 21.05 |
| Model-4 | 1550 | 1 | 100 | 1.349 | 66.55 |

From table 1, the phase fluctuation of the signal caused by atmospheric turbulence is much larger than that in the traditional coherent optical fiber communication. If the effective phase estimation and compensation algorithm is not used, the performance and reliability of the communication system will be seriously deteriorated. At the outer scale of 1m turbulence, the standard deviation of the phase noise of 100m transmission will be more than 66 degrees. In addition, the ubiquitous complex Gaussian white noise will bring not only the amplitude fluctuation of the transmitted optical signal, but also the phase fluctuation noise, especially when the signal-to-noise ratio is small, the phase noise caused by complex Gaussian white noise cannot be ignored.

**3. Analysis Of Phase Compensation Method**

M-th power algorithm and DAML algorithm are widely used in phase compensation in optical fiber communication. However, it is necessary to analyze whether it is suitable for weak atmospheric turbulence channels. When the phase noise is weak, these algorithms have a good compensation effect. Figure 1 shows the comparison of system performance before and after using M-th Power algorithm. We can see that when the standard deviation of phase noise is small, that is, the noise fluctuation range is small, the performance of the algorithm is obviously improved. When the standard deviation of phase noise is less than 15 degrees, the bit error rate (BER) can reach below \( 10^{-3} \). When the phase noise is large, although the algorithm can reduce the bit error rate to a certain extent, it cannot achieve better system performance. This is the limitation of M-th Power algorithm (can only correct the phase noise in the range of \((-\pi/4,\pi/4)\)). At the same time, it can be seen that with the gradual increase of the standard deviation of phase noise, the difference of system performance before and after using the algorithm is gradually smaller.

In addition, figure 2 shows the comparison of system performance before and after using the DAML algorithm. The algorithm is only suitable for cases where the variance of phase noise is not very large. In the atmospheric turbulence channel, the variance of phase noise is very large, such as the results of our previous calculation of phase noise simulation table 1. The standard deviation can reach more than 30 degrees, and there is a large fluctuation of phase noise superimposed on the front and back symbols, so this algorithm is not very suitable for coherent free space optical communication scenarios. In coherent FSOC, the standard deviation of phase noise is more than 10 degrees, or even more than 30 degrees. In this scenario, DAML completely loses the performance of compensation, and even because of its feedback mechanism, there is a phenomenon of error transmission, which makes the performance of the system worse, even worse than that of the system without phase estimation.

You can see from figure 2 that when the standard deviation of phase noise \( \sigma_{\text{pm}} \) is less than 10 degrees,
the performance is better. When the standard deviation of phase noise $\sigma_{\text{pn}}$ is greater than 10 degrees, the performance of DAML algorithm deteriorates so rapidly that it is almost impossible to communicate.

Figure 1. Performance of M-th Power algorithm.

Figure 2. Performance of DAML algorithm under different phase noise standard deviation.

Considering that the free space optical communication is the same as the traditional wireless communication, the channel propagation environment is bad. In order to eliminate the impact of channel on the performance of the transmission system, people often carry out channel estimation and channel equalization, so that the whole system can meet certain transmission performance requirements. Next, we will focus on the method of using frequency domain equalization technology to compensate in free space optical communication.

4. Simulation Results Of SC-FDE

The basic idea of SC-FDE is to estimate the frequency domain response of each subchannel, and then calculate the equalization coefficient according to the corresponding equalization criterion to compensate each subchannel. We use a pilot-based nonlinear equalization technique, and compare the performance of ZF (zero-force equalization) and MMSE (minimum mean square error equalization).

Figure 3. Block diagram of SC-FDE system simulation.

The block diagram of SC-FDE simulation system is shown in Figure 3. The digital signal to be sent is mapped to the constellation by QPSK modulation, and then the modulated data is divided into blocks and added to the known UW (Unique Word) sequence. The data frame length of each frame is 256, including a 64-long UW sequence, which is used for channel estimation and frequency domain equalization. It is then loaded into the laser of 1550nm and enters the atmospheric turbulence channel.
After entering the baseband after ADC, the receiver first divides the data into blocks to remove the previous UW sequence, then transforms the signal into the frequency domain by FFT transform, equalizes the equalization coefficient obtained by the pilot sequence in the frequency domain, and then transforms the signal back to the time domain through IFFT, and then demodulates and decides. The simulation system does not consider the radio frequency part, and only describes the baseband system part of SC-FDE.

Figure 4 (a) and (b) show the signal constellation before and after equalization when the signal-to-noise ratio is 20dB. The phase noise caused by atmospheric turbulence is well eliminated by equalization, which makes the constellation map which cannot be judged become clearer and greatly reduces the bit error rate of the system. The outer circle in Figure 4 (a) is the deflection of the transmitted information signal after adding phase noise, and the inner circle is obtained by the deflection of the inserted UW sequence after adding phase noise. Figure 4 (b) shows the constellation before and after equalization under 20dB.

Due to the excessive Gaussian noise, the information signal is mixed with the pilot signal, but this does not affect the use of pilot signal for channel estimation and equalization. From Figure 4 (a) and (b), we can see that due to the excessive Gaussian noise, the information signal is mixed with the pilot signal, but this does not affect the use of pilot signal for channel estimation and equalization.

Figure 4 (c) (d) shows the signal constellation before and after equalization when the signal-to-noise ratio is 30dB.

It can be seen from Figure 4 (c) and (d) that through equalization, the phase noise introduced by atmospheric turbulence is well eliminated, the constellation which cannot be judged becomes clearer, and the bit error rate of the system is greatly reduced. It can be seen from Figure 5 that there is only phase noise in the system, and when the additive white Gaussian noise (complex noise) is not considered, the influence of phase noise can be eliminated by frequency domain equalization under the premise of matching the algorithm. In the case of imbalance, with the continuous increase of the standard deviation of phase noise, the bit error rate increases continuously, and it has reached the performance requirements of communication when the standard deviation of phase noise reaches 15 degrees. After using equalization, the BER obtained by simulation is 0. But the reality is that additive white Gaussian noise is everywhere, so we still need to consider the influence of this factor.

Next, we consider the impact of the decision mode of the receiver on the equalization performance. Because there are both Gaussian noise and phase noise in the channel, the constellation decision can improve the system performance relative to the threshold decision. Because it uses a two-dimensional
data (Euclidean distance) to determine the value of a data, and its fault tolerance range is much larger than the threshold decision. Sometimes, Gaussian noise and phase noise interact with each other through nonlinear interaction. We do not consider this special case here. Figure 6 (a) shows the BER performance of the system under different equalization criteria and different decision modes. It can be seen from the figure that when the constellation chart is used, the bit error rate of the system decreases a little faster with the increase of SNR. So this kind of decision method is used in the later simulation. And when using the threshold judgment, it is still troublesome to determine the best threshold.

Figure 6. BER performance with different equalization scheme

In Figure 6 (b), we can see the different performance of the system with and without equalization. The performance of the equalized system with low signal-to-noise ratio will be worse than that of an unbalanced system. At this time, the Gaussian noise is large, because it is complex Gaussian noise, it will not only cause the amplitude fluctuation but also cause the phase change. When equalization is used, the random phase noise caused by Gaussian noise will be compensated, but in fact, the phase change caused by Gaussian noise in each symbol is different, so the error of phase estimation will be increased under low signal-to-noise ratio. At the same time, when equalization is adopted, no matter which equalization criterion is adopted, the amplitude noise caused by Gaussian noise will be amplified to a certain extent. However, when the signal-to-noise ratio is greater than a certain value, the signal-to-noise ratio will decrease rapidly after equalization. At the same time, it can be seen that when using MMSE equalization, the SNR corresponding to the intersection of the unbalanced performance curve is smaller than that of zero-forcing equalization. The SNR corresponding to the intersection of the zero-forcing equilibrium is about 11dB and the corresponding SNR to the intersection of the MMSE equilibrium is about 8dB. Because the MMSE equalization criterion takes into account the influence of noise, although it still cannot improve the influence of phase noise caused by Gaussian noise, it can improve the influence of amplitude to some extent. Therefore, MMSE equalization can quickly improve the performance of the system with a lower signal-to-noise ratio than ZF equalization.

From figure 6 (b), we can see that when the standard deviation of phase noise is 15 degrees and 25 degrees, the performance of the two equalization criteria is similar, and the curves almost coincide, so it can be seen that there is no limitation on the range of phase noise when using frequency domain equalization to compensate phase noise. From the above analysis, we can see that as long as a certain signal-to-noise ratio is reached, the performance of the system can be well improved.

Compared with the standard deviation of 15 degrees and 25 degrees of phase noise, it can be seen that the performance of small phase noise is better than that of 25 degrees using constellation chart, and the equalization performance does not change with the standard deviation of phase noise. So at 15 degrees, the intersection with the unbalanced performance curve moves to the right relative to 25
degrees. The intersection of ZF equilibrium is 14dB and MMSE equilibrium is 13dB. Beyond this critical point, the equilibrium BER will decline rapidly.

5. Summary
In this paper, we have studied the phase noise caused by atmospheric turbulence and introduced three phase noise compensation algorithms. When the fluctuation of phase noise is not serious and the standard deviation of phase noise is less than 15 degrees, M-th power algorithm can be used to estimate and compensate the phase. The FDE technique is introduced and used to compensate the phase noise caused by atmospheric turbulence. On one hand, it reduces the computational complexity of the receiver, because it avoids some nonlinear operations, and uses mature FFT and IFFT fast operations. On the other hand, it can compensate for different degrees of phase noise. Through MATLAB simulation, it is found that there is almost no limitation of the compensation range of phase noise by FDE. In general, the method of FDE can be used to compensate the phase noise in the atmospheric turbulence channel.

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7. References
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