MECHANICAL ENGINEERING | RESEARCH ARTICLE

Safety analysis of a railway car under the periodic excitation from the track

G.F.M. dos Santos* and R.S. Barbosa

Abstract: A railway derailment is usually an unpredictable event where the damage could range from a simple train delay to a big loss of goods, assets (rolling stock and permanent way) and, in extreme cases, even human lives. Therefore, Brazil’s railways are always implementing actions that minimize the risk of a derailment. One of the main actions is to establish tight maintenance criteria for rolling stock and for track geometry. However, these criteria usually do not consider the vehicle dynamics and the geometry of the permanent way together. Thus, this paper develops a mathematical model of a high capacity railcar together with the flexible support of the track to determine a method of assessing bounce, roll and pitch motions from instrumented wagons, so this method can be used in routine inspections. The mathematical model was satisfactorily validated by comparing the natural frequencies obtained from the model and from a real vehicle tested in the field. In addition, the model results were compared against field measurements from an instrumented wagon and a track geometric vehicle. A strategic method for operational safety analysis was suggested and used, which was able to determine the wavelength of permanent way anomalies that should be given priority for maintenance actions as well as considered when adopting speed restrictions.

Subjects: Dynamics & Kinematics; Transport & Vehicle Engineering; Railway Technology & Engineering

Keywords: railway system; vehicle-track interaction; derailment

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PUBLIC INTEREST STATEMENT

Railroad business in Brazil are increasing over the years. A particularly railroad at the North of Brazil operates a 330 wagons as a typical train consists, split in three blocks with distributed traction and the maximum authorized speed of 80 km/h. This railroad uses a gondola wagon designed to carry 130 gross tons of a commodity, but recently it was acquired new wagons with a higher capacity. The use of this larger cars has caused safety concern (derailment) and questioning whether the actual safety standards are still valid for this new situation. It is known that these recommendations generally do not consider the vehicle’s response to determine the limits for the irregularities of the permanent way. Using mathematical models validated experimentally, this work presents a procedure to address this issue, that is to analyze operational safety of the higher capacity wagons considering its dynamic response and a real track geometric.
1. Introduction
A railway derailment usually has an unpredictable damage varying from a simple delay of the train schedule up to a millionaire loss of assets (rolling stock and permanent way) and, in extreme cases, human lives. However, as the volume of goods transported increases, the reliability of the track system must be increased as well. One of the biggest railroad in Brazil is nowadays carrying about 120 Millions Gross Tons (MGT) per year and planning to reach about 350 MGT very soon. Considering their cycle time, it may require a train every hour. So, the track maintenance time will be very short that will force its strategy to be changed. This paper gives an alternative.

One of the most common way to analyse the safety in railway transport is to analyze the relationship between the contact forces between wheel and rail, particularly the ratio of lateral load (transverse) and vertical forces. The best-known criterion for defining a limit to this is established by Nadal Equation (Equation (1)), widely used by railway engineers (Barbosa, 2005; Li, Berggren, & Berg, 2007; Li, Berggren, Berg, & Persson, 2008; Santos, Lopes, Kina, & Tunna, 2010; Spinola Barbosa, 2004):

\[
\frac{L}{V} = \frac{\tan(\alpha) - \mu}{1 + \mu \tan(\alpha)}
\]

where \(L\) = lateral force; \(V\) = vertical force; \(\alpha\) = wheel-rail contact angle; \(\mu\) = friction coefficient.

Nadal criterion is generally used for wheel flange climb on curves (Spinola Barbosa, 2004) but derailments can also occur in tangent due to a resonance excitation under a track geometry characteristic which is the focus of this work.

2. Contribution to the state of the art
Track geometry cars are usually used to measure the geometry variation of the track gage, alignments and cross-level. These parameters are analyzed against those that are recommended by International Standards, for instance the Federal Railroad Administration (FRA-Standard. Office of Railroad Safety, 2012) or the European Standard (EN 13848-1, 2003). However, it is used only statics data of the geometry without any consideration of the vehicle dynamic.

A literature review shows that researchers have been working on the correlation of the track excitation and the vehicle dynamics (Barbosa, 2016; Wilson, 2012). Some of them (Czop, Mendrok, & Uhl, 2011; Kawasaki & Youcef-Toumi, 2002; Sun, Cole, & Spiryam, 2013; Tsunashima, Naganuma, & Kobayashi, 2014), are trying to identify the track irregularity by solving an inverse problem, i.e. given the vehicle response, what was the possible input that caused that. As shown in (Ketchum & Wilson, 2012), not all location that was out of the standard recommendation was considered unsafe when taking the vehicle behavior in count. On the other hand, some place initially within the limits, have been reclassified as unsafe. Thus, the conclusion is: the vehicle response must be considered every time.

This paper presents an alternative procedure to manage the situation described in Section 2. This procedure is detailed in Section 6. It also can be graphically seen in the Figure 8.

3. Methodology
The methodology adopted is divided into three steps: mathematical modeling, validation with field tests and computer simulation/analysis.

The step of modeling was divided into the following phases:

- Construction of the vehicle model using previous survey data from a railway.
- Track modeling using geometric measurements of the track.

The modeling validation step was performed with field tests using sensors for measuring the car body movement (natural frequency).
The last step—computer simulations with the validated models and analysis of results allowed development of inferences about the quality of the permanent way from the point of view of periodic irregularities. These inferences were used to recommend new maintenance criteria to be used to improve the safety of rail transport.

4. Mathematical Modeling

A typical gondola car, type GDU (Figure 1), was chosen to be modeled because it is the top Brazilian vehicle in capacity with a total maximum load of 150,000 kg (dead load of 22,000 kg).

The Equations of motion for the wagon were written using Newton’s second law and can be shown in matrix form as following:

\[ [M] \{ \ddot{U} \} + [C] \{ \dot{U} \} + [K] \{ U \} = \{ F \} \tag{2} \]

where \( M = \) mass, \( C = \) damping, \( K = \) stiffness; \( F = \) excitation, \( U = \) displacement and its time derivation.

The idea to use a mathematical model with a reduced number of degree of freedom was to build a representative model to be used when a commercial software license (usually it is very expensive) is not available.

Table 1 shows the mechanical parameters of the GDU vehicle.

| Item                          | Parameter     | Value   | Unit   |
|-------------------------------|---------------|---------|--------|
| Car body and truck bolsters   | Mass          | 111,253 | kg     |
|                               | Roll inertia  | 640,000 | kg m²  |
|                               | Distance between trucks | 5.410 | m     |
|                               | Bounce frequency | 13.06 | rad/s |
|                               | Pitch frequency | 14.76  | rad/s  |
| Truck suspension              | Stiffness     | 9,501   | kN/m   |
|                               | Damping       | 175,127 | Ns/m   |
| Unsprung mass (side frames)   | Mass          | 8,992   | kg     |
| Shear pads                    | Stiffness     | 175,118 | kN/m   |
|                               | Damping       | 3,502   | Ns/m   |

Figure 1. Typical railway car (GDU).
The structure of the permanent way as modeled consists of rail supported by sleepers and ballast (Correa, 2003). The subgrade below the ballast was considered as the base and does not have any degree of freedom.

In this model, the sleepers are considered a spring/damper system with parallel connections to the rail and the subgrade. Reference (Lei & Noda, 2002) shows this approach. Thus, given the discretization of rail as a finite element type bar on discrete supports (spring-damper) representing the ballast and sleeper, the equation of motion of the rails can be written as following:

\[ M_\text{t} \ddot{U}_\text{t} + C_\text{t} \dot{U}_\text{t} + K_\text{t} U_\text{t} = F(t) \]  

where \( M_\text{t} \), \( C_\text{t} \) and \( K_\text{t} \) comprise the matrix of mass, damping and stiffness at the global coordinate system, respectively; \( \ddot{U}_\text{t} \), \( \dot{U}_\text{t} \) and \( U_\text{t} \) are the acceleration, velocity and displacement of the rail, respectively; \( F(t) \) are the wheel rail contact forces.

The integration between both systems—vehicle and track—was done as suggested by (Lei & Noda, 2002), using the Newmark Method (please see Lei & Noda, 2002 for details).

5. Modeling validation
A mathematical model needs to be validated in order to give realistic results compared with a real wagon. In this work, the model was validated by comparing the following three field measurements with model results:

- Natural frequency from modal testing.
- Truck suspension travel from a instrumented wagon.
- Vertical contact forces measured with an instrumented wheel-set.

5.1. Natural frequency from modal testing
A modal test according to (Wilson, Urban, & Burnett, 1997) was performed on the wagon shown in Figure 1 using specific sensors to collect the free motion of the main car body due to an impulsive excitation. From this procedure it was possible to estimate the natural frequency (eigenvalue) of each modal movement. Table 2 compares the measured natural frequencies to the model results and shows good agreement between the model and real wagon.

5.2. Truck suspension travel from an instrumented wagon
An instrumented iron ore car (Darby, Alvarez, McLeod, Tew, & Crew, 2005) was used to measure suspension travel from the wagon over a specific stretch of a railway in Brazil. The suspension was measured using special sensor as shown in Figure 2.
A track geometry car was used to measure the vertical alignment of the rails that is the excitation input to the wagon. This track excitation was used as an input to the mathematical model. The model's output was then compared to the suspension travel measured in field by the instrumented wagon (Figure 3) and checked against one another. Results are shown in Figure 3.

**Table 2. Wagon natural frequencies from modal testing and the mathematical model**

|                         | Modal testing (field) (Hz) | Math model (Hz) | Error (%) |
|-------------------------|-----------------------------|-----------------|-----------|
| Bounce                  | 1.85                        | 1.84            | -0.5      |
| Pitch                   | 2.30                        | 2.29            | -0.4      |
| Yaw                     | 1.55                        | 1.41            | -9.0      |
| Upper center roll       | 3.09                        | 2.90            | -6.1      |
| Lower center roll       | 0.78                        | 0.77            | -1.3      |

Figure 3. Suspension springs vertical displacement.

Notes: Red: model; blue: field test.

Figure 4. Vertical contact forces between the leading right wheel and rail.

Notes: Red: instrumented, Black: math. model.
Figure 3 shows a fair agreement between the model and the instrumented wagon. Some of the differences can be explained due to track ballast stiffness and damping that may require a deeper evaluation.

5.3. Vertical contact forces measured with an instrumented wheel-set

The third method used to validate the mathematical model was to directly measure the vertical contact forces at the leading wheel set. An instrumented wheel set was used to measure the forces while the vehicle was travelling along the same track path where the geometry car measured the vertical alignment.

As one can see from Figures 4 and 5, the results have a fair agreement. Figure 6 shows the Power Spectral Density of this signal and also show very close agreement. Again, some difference is expected mainly because of the uncertainly of the track ballast characteristics.

6. Computer simulation

To carry out the computer simulations it was necessary to define what track geometry should be considered. The literature (Grando, 2012) has already conducted an extensive analysis considering transverse periodic twist out of phase between the right and left rails, because one of its objectives was to analyze rail vehicle lateral rocking motion (roll). Therefore, in this work, only the transverse periodic twist IN PHASE was analyzed because this condition was observed experimentally. Any amplitude differences between the rails over the cross section, which could also excite the lateral mode, were not considered. The Figure 7 reproduces two examples of the periodic excitation considered. The wave in blue has a wavelength of 11.88 m and the red one 5.3 m.
Figure 8 shows the simulation strategy. The innovation of this methodology is to not only consider the amplitude of irregularities as maintenance triggers (generally recommended in practical railway field manuals and recommendations of regulatory agencies such as the U. S. Federal Railroad Administration), but to also consider the wavelength of such irregularities.

In this flowchart (Figure 8) is carried out a speed scanning of the wagon starting with a standardized or track geometry measured in the field. If a resonance event of any degree of freedom occurs at a certain speed within limits range tested, the track geometry wavelength should be recalculated for this amplified frequency and also tuned to an operating speed or to the maximum authorized speed.

So, new simulations are performed looking for an unsafe conditions (derailment risk). If it is found, corrective maintenance or operational action must be adopted (a speed restriction, for instance).

Following this methodology (Figure 8), the starting point was to build the periodic excitation of the track, which can be related to the length of the rail bars (in Brazil—12 m and 24 m, the latter being more common). These bars are welded together forming a long welded rail (LWR) which can reach up to 360 m long. In this work, wavelengths of 6, 11.88 (US standard length), 12, 24 and 30 m were adopted, besides those explained below.

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Figure 8. Methodology flowchart.
Another way to build the theoretical track is to do a spectral analysis of track geometry. Applying the Fast Fourier Transform (FFT) to measured track geometry yields the frequency spectrum of the spatial transverse alignment (Figure 9). Due to the total length of the railway analyzed (50 m) in this paper, the recoverable frequency range is up to 10 m. However, as explained in the preceding paragraph, other wavelengths were purposely added in the analyses.

The results shown in Figure 9 indicate that there are four main wavelengths: 4.3, 5.3, 5.8, 7.1 m. These wavelengths were also used in the construction of irregularities in this work.

The travel speed of the vehicle was incremented from 5 to 100 km/h, in increments of 5 km/h.

The safety criteria found in the literature (Equation (1)) is the measure of the ratio of the lateral and vertical forces on the wagon wheels. However, it is difficult to measure this parameter in regular service operation. The most common direct measurement method is the use of instrumented wheel sets. However, this tool is very expensive, difficult calibration procedure, and expensive operation and maintenance prevents their continuous use.

On the other hand, the use of instrumented wagons (Darby et al., 2005) for monitoring and safety assessment has been demonstrated to be a good tool (Santos & Reichl, 2014) since this equipment monitors the movements of the main body of the car.
The results of the simulations are summarized in Table 3. An important observation is that as the wavelength increases, the speed at which the amplitude of the motion is amplified also increases. This is explained by the fact that there is a direct and linear dependence between speed and wavelength. In this case, as can be seen in Figure 10, the ratio is approximately the natural frequency (Table 2) of the respective wagon mode.

Following the proposed method (Figure 8), the rightmost column of Table 3 shows the wavelength that would excite the wagon with defined frequency (shown in the column of Frequency) at the maximum operational speed. However, this analysis can only be justified for those respective lengths whose speed was less than 80 km/h.

Thus, the virtual car was again subjected to irregularities for this new wavelength, but at maximum operating speed in order to check whether the mode remains excited or not. These results are shown in Figures 11 and 12. Due to the similarity of the results, only two examples illustrating the phenomenon are shown.

Figure 11 shows the occurrence of beat in the vertical movement while Figure 12 the pitch movement is continuously amplified. There is no excitement of lateral movement, as expected. Figures 11 and 12 show peak-to-peak amplitude of the wagon's carbody movement.

Table 3. Simulation results

| Wavelength (m) | Seed (km/h) | Seed (m/s) | Motion amplified | Frequency (Hz) | Wavelength to the maximum authorized speed (80 km/h) |
|----------------|-------------|------------|------------------|----------------|----------------------------------------------------|
| 4.3            | 30          | 8.3        | Bounce           | 1.94           | 11.5                                               |
| 4.3            | 35          | 9.7        | Pitch            | 2.26           | 9.8                                                |
| 5.3            | 35          | 9.7        | Bounce           | 1.83           | 12.1                                               |
| 5.3            | 45          | 12.5       | Pitch            | 2.36           | 9.4                                                |
| 5.8            | 40          | 11.1       | Bounce           | 1.92           | 11.6                                               |
| 5.8            | 50          | 13.9       | Pitch            | 2.39           | 9.3                                                |
| 6.0            | 40          | 11.1       | Bounce           | 1.85           | 12.0                                               |
| 6.0            | 50          | 13.9       | Pitch            | 2.31           | 9.6                                                |
| 7.1            | 45          | 12.5       | Bounce           | 1.76           | 12.6                                               |
| 7.1            | 60          | 16.7       | Pitch            | 2.35           | 9.5                                                |
| 11.9           | 80          | 22.2       | Bounce           | 1.87           | 11.9                                               |
| 11.9           | 80          | 22.2       | Pitch            | 1.87           | 11.9                                               |
| 12.0           | 80          | 22.2       | Bounce           | 1.85           | 12.0                                               |
| 12.0           | 80          | 22.2       | Pitch            | 1.85           | 12.0                                               |
| 18.0           | 80          | 22.2       | Bounce           | 1.23           | 18.0                                               |
| 18.0           | 80          | 22.2       | Pitch            | 1.23           | 18.0                                               |
| 24.0           | 80          | 22.2       | Bounce           | 0.93           | 24.0                                               |
| 24.0           | 80          | 22.2       | Pitch            | 0.93           | 24.0                                               |
| 30.0           | 80          | 22.2       | Bounce           | 0.74           | 30.0                                               |
| 30.0           | 80          | 22.2       | Pitch            | 0.74           | 30.0                                               |
Although the practice of rail operators is to generally take corrective action based on the only the amplitude of geometric irregularity, the results show that analysis of the spectral content of the permanent way is critical to the determination of operational safety. New technologies for permanent way quality assessment have been developed in parallel with this finding (Barbosa, 2016).

For this particular case considered, the biggest iron ore wagon capacity operating in Brazil has critical points when operating at speeds between 30 and 50 km/h in geometry perturbations with wavelengths close to 5.410 m (equivalent to its distance between trucks). The results show that irregularities of the permanent way between 4 and 7 m should be smoothed to the fullest because it amplifies the car body movements at speeds between 30 and 50 km/h (Table 3). If it is not possible, the operator should not apply speed restrictions in the range of 30 to 50 km/h while the track maintenance does not occur. The results also suggest that attention should also be given for the wavelength around 10 at speed of 80 km/h.

7. Conclusions
This study aimed to develop a mathematical model, validate it with experimental measurements and study the response of a rail vehicle to periodic geometrical irregularities of the permanent way. With this study it was possible to determine with certainty what kind of geometry irregularity should

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![Figure 11](image1.png)

**Figure 11.** Peak to peak amplitude of the vertical wagon balance for periodic irregularity with a wavelength of 9.30 m at speed of 80 km/h.

![Figure 12](image2.png)

**Figure 12.** Peak to peak amplitude of lateral balance and wagon pitch for periodic irregularity with a wavelength of 9.30 m at speed of 80 km/h.
be considered as a priority in the maintenance strategy of permanent way in order to also include the vehicle dynamics.

The combination of the track geometry with the response (output) of the vehicle proved to be fundamental to the study of operational safety, and the natural frequencies of the vehicle to determine its critical speeds must be known before any definition of authorized speed of traffic and mainly when determining speed restrictions.

The 15-degree-of-freedom model, was satisfactorily validated, from the comparison of natural frequencies obtained in the real wagon, and from comparison of results produced from specific input from a track geometry car with measured vertical contact forces taken from an instrumented wheel set traveling over the same track.

The procedure adopted, as illustrated by the flowchart in Figure 8, has been shown to be sufficient to determine the wavelength of permanent way irregularities to be prioritized in maintenance, as well as in car safety analysis when adopting speed restrictions. Specifically for the wagon type GDU, the analysis identified attention points at speeds between 30 and 50 km/h. It was also noted that at this speed range, the critical wavelength is between 4 and 7 m. Finally, the results also suggest that attention should also be given for the wavelength around 10 at speed of 80 km/h.

Next steps of this work suggest to analyze the influence of the maintenance variation of the car’s design and to set up alarms to the suspension travel.

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