THE STRUCTURAL DISTRIBUTION OF THE ELEMENTS AND THE ORE – METAL DEPENDENCE OF A RESERVOIR

REPARTIȚIA STRUCTURALĂ A ELEMENTELOR ȘI DEPENDENȚA MINEREU-METAL DINTR-UN ZĂCĂMÂNT

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Abstract: The distribution laws for copper content and thickness of a copper mineralization investigated by exploratory have been studied in this paper. Knowing these distribution laws, content and thickness mean values, main parameters in the economical and industrial evaluation of deposit can be determined more precisely. Further on, the analytical relations between copper contents and more resources are established. These relations can be used for studying the optimization of the technological process based on mathematical models.

Keywords: reservoir, exploration, metamorphic rock, ore – metal correlation, algorithm, distribution.

Rezumat: Au fost studiate legile de distribuție pentru conținutul și grosimea dintr-o mineralizare a cuprului cercetat prin investigarea zăcământului. Cunoscând aceste legi de distribuție, conținutul și grosimea valorilor medii, principali parametrii în evaluarea economică și industrial a zăcământului se vor determina mai precis. Mai departe se stabilesc relațiile analitice dintre conținutul de cupru și celelalte resurse. Aceste relații pot fi utilizate pentru studierea optimizării procesului tehnologic bazat pe modelele matematice.

Cuvinte cheie: rezervor, explorare, rocă metamorfică, corelația metal – minereu, algoritm, distribuție.

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1. Introduction

The development of the economic yield regarding the exploitation of a reservoir is achieved through a detailed research of the results and also the implementation of appropriate measures. Thus, the last periodic investigation of the information was imposed in practice by means of the mathematical statistical. These offer the possibility of a step-by-step control of the density of the exploration works, while also allowing the rationalization of the works in order to achieve a high degree of the reservoir knowledge.

In this research, two aspects are analyzed regarding the processing of the information of the reservoir exploration as follows: determining the law on structural distribution of the elements, the correlations in the ore reserves as well as the contents in useful components. This knowledge of these aspects leads to the determination of the average values of the calculation parameters regarding the reserves of useful mineral substances and the rational delimitation of the reserve blocks, on the basis of a marginal limit it is content in useful components, which can ensure the optimal use of the reservoir. It was considered that the reservoir is located in the skarn, a metasomatic contact that can be used mainly for copper.

The skarn represents a metamorphic rock that was born through metasomatic contact metamorphism in the haloes of granodioritic, dioritic, monzonitic or sienitic intrusions on account of carbonate rocks. This rock is part of the Cornish family. The chemical composition of this rock highly varies and depends on the nature and quantity of mineralizers released from the magma and is reactivated with the host rock [2]. In the composition of the rock, the following accessory minerals are identified: iron oxides (magnetite, oligist), sulfides of Fe, Cu, Mo, Bi, Zn, Pb, native gold and gold tellurium, casiterite, native bismuth.

The elongated areas are composed by metalliferous, irregular, overlapping and discontinuous concentrations in the direction of contacts between the apophyses of granodioritic rocks and mesozoic limestone [4]. These areas are highlighted with the help of chemical analyzes of the collected samples. Initially the reservoir was exploited with the help of the drills arranged in the square network with the side of 150 m. On a narrower area, the drillings were thickened to 50 m, which is why they went to the final stage of exploration through mining works and underground drillings. Also, analysis was the
situation of the continued exploitation of the rich mineralized areas, with special regard to the drilling network.

On the other hand, the problem of rational contouring of ore reserves blocks was studied, depending on the minimum marginal content that can be determined in the optimization and recovery of the reservoir.

Also, gangue minerals are represented by cobalt silicates, iron associated with magnesium, quartz, calcite, anhydrite and zeolited, so that the ore content rarely exceeds 2% copper.

On the other hand, the reservoir are associated with granitic rocks granodiorite, monzonite, quartz monzonite. Examples of reservoirs can be mentioned: The Simon Iuda from Ocna de Fier, the mineralizations from Ciclova, Sasca and reservoirs in the Moldova Nouă area genetically associated with banatite (granodiorite) and considered to belong to the type of transition to hydrothermal.

From the point of view of genesis, the native copper is formed under conditions of a reducing environment during various geological processes. It is identified in two types of reservoir, as follow:

- **Primary reservoirs** that are related to basis eruptions or in geological formations adjacent to them. Depending on the pattern of occurrence, several types of mineralization are distinguished:
  
  a) *in the form of veins and geodes in basalts*, which form a characteristic association with zeolites;

  b) *in the form of microscopic separations*, disseminated in basic rocks, associated with minerals such as chalcopyrite, bornite, calcosite;

  c) *in the form of impregnations in detritical sedimentary rocks* (sandstone), copper cements sand granules.

- **The reservoirs in which secondary native copper** (supergen) is identify, they came from the decomposition of copper sulphides in the upper parts of the sulfide reservoirs (oxidantion zones).

In Romania the native copper mineralizations are numerous, except for higher concentrations (for example in the are of Dobrogea Altân – Tepe) these occurrences have only a mineralogical importance.
2. Component distribution

Given that copper has previously been shown to be the main element in the reservoir, only its distribution has been studied based on data on the content and thickness of the exploration wells. Subsequently, this content was separated on mineralized areas on the average drilling data. The breakdown by classes as well as the determination of the statistical parameters of the copper content (on all the mineralized areas highlighted), is shown in (Table 1).

**Table 1 – The parameters of the copper content statistical distribution on mineralized areas**

| \(x_i,\%\) | Absolute frequency \((n_i)\) | Relative frequency \((f_i = n_i/n)\) | \(x_i - x_0\) | \(n_i (x_i - x_0)\) | \(n_i (x_i - x_0)^2\) |
|----------------|-----------------------------|----------------------------------|----------------|----------------------|----------------------|
| 0,2             | 5                           | 0,0175                           | - 1            | - 5                  | 5                    |
| 0,4             | 123                         | 0,4316                           | - 0,8          | - 98,4               | 78,72                |
| 0,6             | 54                          | 0,1895                           | - 0,6          | - 32,4               | 19,44                |
| 0,8             | 44                          | 0,1544                           | - 0,4          | - 17,6               | 7,04                 |
| 1               | 24                          | 0,0842                           | - 0,2          | - 4,8                | 0,96                 |
| 1,2             | 16                          | 0,0562                           | 0              | 0                    | 0                    |
| 1,4             | 8                           | 0,0281                           | 0,2            | 1,6                  | 0,32                 |
| 1,6             | 5                           | 0,0175                           | 0,4            | 2                    | 0,80                 |
| 1,8             | 3                           | 0,0105                           | 0,6            | 1,8                  | 1,08                 |
| 2               | 2                           | 0,0070                           | 0,8            | 1,6                  | 1,28                 |
| 2,2             | 1                           | 0,0035                           | 1              | 1                    | 1                    |
| 285             | 1                           | 150,2                            | 115,64         |                      |                      |

2.1 Calculation of statistical distribution parameters of the copper content by mineralized area

Given the copper content distributed over mineralized areas, the absolute frequency dependence of the interval was simulated graphically in Figure 1. Given the fact that the relative frequency has very low values, this cannot be represented graphically.
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Figure 1. Absolute frequency dependence

The following calculation algorithm is applied [3]:

The average arithmetic:
\[
\bar{x} = x_0 + \frac{1}{n} \sum n_i (x_i - x_0) = 1,2 - \frac{1}{285} \cdot 150,2 = 0,68 \%
\] (1)

The statistical dispersion:
\[
s^2 = \frac{1}{n} \sum n_i (x_i - x_0)^2 - (\bar{x} - x_0)^2 = \frac{115,64}{285} - (-0,52)^2 = 0,1353 \%
\] (2)

The real dispersion:
\[
\sigma^2 = \frac{n}{n-1} s^2 = \frac{285}{284} \cdot 0,1353 = 0,1357
\] (3)

The standard deviation:
\[
\sigma = \sqrt{\sigma^2} = \sqrt{0,1357} = 0,368
\] (4)

The average deviation:
\[
\sigma_m = \frac{\sigma}{\sqrt{n}} = \frac{0,368}{\sqrt{285}} = 0,02 \%
\] (5)

The average relative error:
\[
\varepsilon = \frac{\sigma_m}{\bar{x}} \cdot 100 = 2,94 \%
\] (6)
The variation coefficient:

\[ k_x = \frac{\sigma}{\bar{x}} \times 100 = \frac{0.368}{0.68} \times 100 = 54\% \]  

(7)

This variation content of the coefficient determines that the reservoir is classified in the second group \((k_x = 54\%)\).

Because the relative error of the average is small \((\varepsilon = 2.94\%)\) through drilling the average copper content can be determined. In figure (figure 2), the histogram is shown respectively the frequency curve of the statistical distribution of the copper content, thus suggesting a lognormal theoretical distribution.

![Figure 2. Frequency curve histogram](image)

2.2 Calculation of the parameters of the statistical distribution on the content in copper – average values per drillings

The values taken into account are presented in (Table 2).

| \((x_i, \%)\) | Absolute frequency \((n_i)\) | Relative frequency \((f_i = n_i/n)\) | \(x_i - x_0\) | \(n_i \ (x_i - x_0)\) | \(n_i \ (x_i - x_0)^2\) |
|---------------|-----------------------------|----------------|---------------|----------------|----------------|
|               |                             |                |               |                |                |
Using mathematical statistics, the computation algorithm is reintegrated [5]:

The average arithmetic:
\[ \bar{x} = x_0 + \frac{1}{n} \sum n_i (x_i - x_0) = 0.7 + \frac{1}{69} = 0.71 \% \]  
(8)

The statistical dispersion:
\[ s^2 = \frac{1}{n} \sum n_i (x_i - x_0)^2 - (\bar{x} - x_0)^2 = \frac{123.98}{69} - (-0.01)^2 = 0.0576 \% \]  
(9)

The real dispersion:
\[ \sigma^2 = \frac{n}{n-1} s^2 = 1.01 * 0.0576 = 0.0581 \]  
(10)

The standard deviation:
\[ \sigma = \sqrt{\sigma^2} = \sqrt{0.0581} = 0.27 \]  
(11)

The average deviation:
\[ \sigma_m = \frac{\sigma}{\sqrt{n}} = \frac{0.27}{\sqrt{69}} = 0.03 \% \]  
(12)

The average relative error:
\[ \varepsilon = \frac{\sigma_m}{\bar{x}} 100 = \frac{0.032}{0.71} 100 = 4.5 \% \]  
(13)

The variation coefficient:
\[ k_x = \frac{\sigma}{\bar{x}} 100 = \frac{0.27}{0.71} 100 = 38 \% \]  
(14)
The histogram of the distribution of the copper content is shown in (figure 3).

![Histogram of the statistical distribution of the average copper content on drillings and the theoretical frequency curve](image)

**Figure 3. Histogram of the statistical distribution of the average copper content on drillings and the theoretical frequency curve**

2.3 **Checking the normality of the content distribution (average values per drillings)**

The values taken into account are presented in *(Table 3).*

Where: $\bar{x} = 0,71\%$, $\sigma = 0,27$

*Table 3 – Checking of copper content the normal distribution (as average values on boreholes) according to the established criteria*

| $x_i$, % | Relative frequency cumulative $F_n(x)$ | $x_i-\bar{x}$ | $z_i=(x_i-\bar{x})/\sigma$ | $\phi(z)$ | $F(x)=0.5+\phi(z)$ | $F_n(x)-F(x)$ |
|----------|-----------------------------------------|---------------|--------------------------|-----------|------------------|----------------|
| 0,4      | 0,218                                   | -0,21         | -0,78                    | -0,3      | 0,21             | 0              |
| 0,6      | 0,566                                   | -0,01         | -0,04                    | -0,02     | 0,48             | 0,08           |
| 0,8      | 0,784                                   | 0,19          | 0,70                     | 0,25      | 0,75             | 0,03           |
| 1        | 0,929                                   | 0,39          | 1,44                     | 0,42      | 0,92             | 0,004          |
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| 1.2 | 1 | 0.59 | 2.18 | 0.48 | 0.98 | 0.015 |

Will be used the relationship:

\[ dn = \max | F_n(x) - F(x) | = 0.082 \] \quad (15)

\[ p = 1 - q = 0.95 = K(\lambda) \] \quad (16)

It was considered:

\[ \lambda = 1.36 \text{ and } q = 0.05 \]

\[ d = \frac{\lambda}{\sqrt{n}} = \frac{1.36}{\sqrt{69}} = \frac{1.36}{8.3} = 0.164 \] \quad (17)

The normal distribution is allowed, given that \( dn < d \).

The characteristic points of the normal curve, corresponding to the statistical distribution have the relations [6]:

\[ x_1 = \bar{x} = 0.71 \% \] \quad (18)

\[ y_1 = \frac{1}{\sqrt{2\pi \sigma}} \exp \left\{ \frac{(x-0.71)^2}{2(0.27)^2} \right\} \] \quad (19)

\[ x_{2,3} = \bar{x} \pm \sigma = 0.71 \pm 0.27 \] \quad (20)

\[ y_{2,3} = \frac{5}{8}y_1 = \frac{5}{8}0.3 = 0.187 \] \quad (21)

\[ x_{4,5} = \bar{x} \pm 2\sigma = 0.71 \pm 0.54 \] \quad (22)

\[ y_{4,5} = \frac{1}{8}y_1 = \frac{1}{8}0.3 = 0.037 \] \quad (23)

It can be written depending on the frequency:

\[ f(x) = \frac{1}{0.27\sqrt{2\pi}} \exp \left\{ \frac{(x-0.71)^2}{2(0.27)^2} \right\} \] \quad (24)

Also the distribution function can be determined:

\[ F(x) = \frac{1}{0.27\sqrt{2\pi}} \int_0^{1.3} \exp \left\{ \frac{(x-0.71)^2}{2(0.27)^2} \right\} dx \] \quad (25)

The histogram of the statistical distribution of the copper content suggests the existence of a normal theoretical distribution, (and) the hypothesis verified by the previously described calculations.

3. Results and discussions

3.1. Ore – metal correlations
In the present knowledge study of the reservoir, the correlations can be established with the indicative character between the quantities of ore reserves and the metal contents of respective reservoir.

### 3.1.1 The calculation of average contents (m) on reserve blocks summed up in descending order of selection content (x)

The data structure can be found in *(Table 4)*. These functions are only informative, given that the reserve blocks that can be delimited do not coincide with the actual exploitation blocks that will be shaped only by mining works of detailed exploration and preparation for exploration.

*Table 4* – The average contents (m) on reserve blocks summed successively in descending order of selection content (x) (0,3 %)

| Block | Average content per block (x_i, %) | Reserves on the block (R_i, t) (10^4) | ΣR_i, t (10^4) | R_i x_i | ΣR_i x_i | m = ΣR_i x_i / ΣR_i |
|-------|----------------------------------|--------------------------------------|----------------|---------|-----------|-----------------------|
| 0     | 0,82                             | 75,14                                | 751.4          | 616.148 | 616.148   | 0,82                  |
| 3     | 0,81                             | 132,48                               | 2.076.2        | 1.073.08 | 1.689.236 | 0,81                  |
| 2     | 0,80                             | 116,4                                | 32402          | 931.20  | 2.620.436 | 0,80                  |
| 1     | 0,76                             | 377,7                                | 70172          | 2.870.52 | 5.490.956 | 0,78                  |
| 4     | 0,74                             | 146,17                               | 84789          | 1.081.65 | 6.572.614 | 0,77                  |
| 7     | 0,71                             | 126,69                               | 97458          | 899.500 | 7.472.114 | 0,76                  |
| 5     | 0,66                             | 303,45                               | 127803         | 2.002.77 | 9.474.884 | 0,74                  |
| 9     | 0,64                             | 305,69                               | 158372         | 1.956.41 | 11.431.300 | 0,72               |
| 8     | 0,63                             | 259,54                               | 184236         | 1.635.10 | 13.066.402 | 0,70               |
| 6     | 0,62                             | 223,18                               | 206644         | 1.383.71 | 14.450.118 | 0,69                         |
| 10    | 0,54                             | 213,12                               | 227956         | 1.150.84 | 15.600.966 | 0,68                  |
| 11    | 0,37                             | 67,33                                | 234689         | 249.121  | 15.850.087 | 0,67                  |

### 3.1.2 The calculation of the coefficients a and b in the correlation equation between m and lnR:

\[ m = a + b \cdot \ln R; \quad y = \ln R \]  

The values are considered: \( m = 0,74; \ y = 15,9 \).

The data structure is presented in *Table 5*.

*Table 5* – The presentation of the a and b coefficients of the correlation equation between m and lnR (m = a + b ln R); (y = ln R)

| m, %  | m − m̄ | y=ln ΣR | y − ŷ     |
|-------|-------|--------|----------|
| 0,82  | 0,08  | 13,5   | -2,4     |
| 0,81  | 0,07  | 14,5   | -1,4     |
| 0,80  | 0,06  | 15     | -0,9     |
| 0,78  | 0,04  | 15,7   | -0,2     |
| 0,77  | 0,03  | 15,9   | 0        |
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|   |   |   |   |   |
|---|---|---|---|---|
| 0,76 | 0,02 | 16 | 0,1 |
| 0,74 | 0   | 16,3 | 0,4 |
| 0,72 | -0,02 | 16,5 | 0,6 |
| 0,70 | -0,04 | 16,7 | 0,8 |
| 0,69 | -0,05 | 16,8 | 0,9 |

Is calculated the correlation coefficients [1]

\[
r = \frac{\sum (m - \bar{m})(y - \bar{y})}{\sqrt{\sum (m - \bar{m})^2 \sum (y - \bar{y})^2}} = \frac{-0,576}{\sqrt{0,0308 \times 12,76}} = -0,92
\]  

(27)

\[
a = \frac{\sum y^2 \sum m - \sum y \sum my}{n \sum y^2 - (\sum y)^2} = \frac{3046,48 \times 8,94 - 190,8 \times 141,57}{12 \times 3046,48 - 190,8^2} = 1,43
\]  

(28)

\[
b = \frac{n \sum my - \sum m \sum y}{n \sum y^2 - (\sum y)^2} = \frac{12 \times 141,57 - 8,94 \times 190,8}{153} = -0,045
\]  

(29)

Is determined the correlation equation:

\[m = a + b \ln R = 1,43 - 0,045 \ln R\]

(30)

### 3.1.3 The calculation of the coefficients a’ and b’ in the correlation equation between x and \(\ln R\):

Was considered: \(x = a' + b' \ln R; y = 15,9\).

The data structure is presented in (Table 6).

**Table 6 – The presentation of the a’ and b’ coefficients of the correlation equation between \(x = a' + b' \ln R\); \(y = \ln R\)**

| \(x\), % | \(x - \bar{x}\) | \(y = \ln R\) | \(y - \bar{y}\) |
|---|---|---|---|
| 0,82 | 0,15 | 13,5 | -2,4 |
| 0,81 | 0,14 | 14,5 | -1,4 |
| 0,80 | 0,13 | 15 | -0,9 |
| 0,76 | 0,09 | 15,7 | -0,2 |
| 0,74 | 0,07 | 15,9 | 0 |
| 0,71 | 0,04 | 16 | 0,1 |
| 0,66 | -0,01 | 16,3 | 0,4 |
| 0,64 | -0,03 | 16,5 | 0,6 |
| 0,63 | -0,04 | 16,7 | 0,8 |
| 0,62 | -0,05 | 16,8 | 0,9 |
| 0,54 | -0,013 | 16,9 | 1 |
| 0,37 | -0,30 | 17 | 1,1 |
Can be determined the correlation coefficients, recalculate for new and terms, using previous relationships:

\[
r = \frac{\sum (x-\bar{x})(y-\bar{y})}{\sqrt{\sum (x-\bar{x})^2 \cdot \sum (y-\bar{y})^2}} = \frac{1.246}{\sqrt{0.185 \cdot 12.76}} = -0.81
\]

\[
a' = \frac{\sum y^2 \cdot \sum x - \sum y \cdot \sum xy}{n \sum y^2 - (\sum y)^2} = \frac{3046.48 + 8.10 - 190.8 \cdot 127.5}{153} = 2.17
\]

\[
b' = \frac{n \sum xy - \sum x \cdot \sum y}{n \sum y^2 - (\sum y)^2} = \frac{12 \cdot 127.54 - 8.10 \cdot 190.8}{153} = -0.010
\]

4. Conclusions and proposals

Metasomatic (contact) reservoir are of relatively low importance.

The statistical processing results of drillings operation of the analyzed the reservoir, allows some interpretations such:

- the copper content determined as an average for drillings is variable and is influenced by the normal distribution law, and the copper content on mineralized areas is much more variable and follows as lognormal law.

- as for the thickness of the mineralization, it is highly variable following a hyperbolic distribution law.

- the drilling operation has a preliminary character, it can be executed with drillings located at large distances given that their thickening doesn’t contribute to a substantial contribution.

- the outline of the reserve blocks for a future exploitation can be realized by mining works.

- the metal – ore correlations determine that the industrial reserve blocks can be shaped for a marginal content of 0.3 % Cu or even smaller allow a rational recovery of the analyzed reservoir.

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