Semileptonic decays of $B_c$ mesons into charmonium states

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Abstract. In this work we study the semileptonic decays of $B_c$ meson. We evaluated $B_c \to D(D^*)$, $B_c \to D_s(D^*_s)$ and $B_c \to \eta(J/\psi)$ transitions form factors in the full kinematical region within the covariant quark model. The calculated form factors are used to evaluate the semileptonic decays of $B_c$ meson and it was defined ratios ($R_{\eta c}$, $R_{J/\psi}$, $R_D$, $R_{D^*}$) of the branching ratios, which will be hopefully tested on LHC experiments. We compare the obtained results with the results from other theoretical approaches.

1 Model

The covariant quark model was developed by G.V. Efimov and M.A. Ivanov [1–3].

The effective Lagrangian describing the transition of a meson $M(q_1 \bar{q}_2)$ to its constituent quarks $q_1$ and $\bar{q}_2$ in model looks like

$$
\mathcal{L}_{\text{int}}(x) = g_M M(x) \cdot J_M(x) + \text{h.c.},
$$

$$
J_M(x) = \int dx_1 \int dx_2 F_M(x, x_1, x_2) \bar{q}_2(x_2) \Gamma_M q_1(x_1),
$$

(1)

with $\Gamma_M$ a Dirac matrix which projects onto the spin quantum number of the meson field $M(x)$. The vertex function $F_M$ characterizes the finite size of the meson. Translational invariance requires the function $F_M$ to fulfill the identity $F_M(x + a, x_1 + a, x_2 + a) = F_M(x, x_1, x_2)$ for any four-vector $a$. A specific form for the vertex function is adopted

$$
F_M(x, x_1, x_2) = \delta(x - w_1 x_1 - w_2 x_2) \Phi_M((x_1 - x_2)^2),
$$

(2)

where $\Phi_M$ is the correlation function of the two constituent quarks with masses $m_{q_1}$, $m_{q_2}$ and the mass ratios $w_i = m_{q_i}/(m_{q_1} + m_{q_2})$.

A simple Gaussian form of the vertex function $\Phi_M(-k^2)$ is selected

$$
\Phi_M(-k^2) = \exp\left(k^2/\Lambda_M^2\right)
$$

(3)

with the parameter $\Lambda_M$ linked to the size of the meson. The minus sign in the argument is chosen to indicate that we are working in the Minkowski space. Since $k^2$ turns into $-k^2_E$ in the Euclidean space, the form (3) has the appropriate fall-off behavior in the Euclidean region. Any choice for $\Phi_M$...
is appropriate as long as it falls off sufficiently fast in the ultraviolet region of the Euclidean space to render the corresponding Feynman diagrams ultraviolet finite. We choose a Gaussian form for calculational convenience.

The fermion propagators for the quarks are given by

\[ S_i(k) = \frac{1}{m_{q_i} - k} \]  

with an effective constituent quark mass \( m_{q_i} \).

The so-called compositeness condition \([4, 5]\) is used to determine the value of the coupling constants \( g_M \). It means that the renormalization constant \( Z_M \) of the elementary meson field \( M(x) \) is to be set to zero, i.e.,

\[ Z_M = 1 - \frac{3g_M^2}{4\pi^2} \bar{\Pi}'_M(m_M^2) = 0, \]  

where \( \bar{\Pi}'_M \) is the derivative of the meson mass operator. Its physical meaning in Eq. (5) becomes clear when interpreted as the matrix element between the physical and the corresponding bare state: \( Z_M = 0 \) implies that the physical state does not contain the bare state and is appropriately described as a bound state. The interaction makes the physical particle dressed, i.e. its mass and wave function have to be renormalized. The condition \( Z_M = 0 \) also effectively excludes the constituent degrees of freedom from the space of physical states. It thereby guarantees the absence of double counting for the physical observable under consideration, the constituents exist only in virtual states. The tree-level diagram together with the diagrams containing self-energy insertions into the external legs (i.e. the tree-level diagram times \( Z_M - 1 \)) give a common factor \( Z_M \) which is equal to zero.

The mass functions for the pseudoscalar meson (spin \( S = 0 \)) and vector meson (spin \( S = 1 \)) are defined as

\[ \Pi_p(x - y) = + i \langle T\{J_p(x)J_p(y)\}\rangle_0, \]  

\[ \Pi^\mu_\nu(x - y) = - i \langle T\{J^\mu_\nu(x)J^\nu_\nu(y)\}\rangle_0. \]  

Herein we use the updated values of the model parameters from \([6]\) which are shown in Eq. (8,9).

\[
\begin{array}{cccccc}
 m_{u/d} & m_s & m_c & m_b & \lambda & \\
 0.241 & 0.428 & 1.67 & 5.05 & 0.181 & \text{GeV} \\
\end{array}
\]  

\[
\begin{array}{cccccccc}
 \Lambda_{B_s} & \Lambda_{b_s} & \Lambda_{J/\psi} & \Lambda_D & \Lambda_{D^*} & \Lambda_{D_s} & \Lambda_{D_{s*}} & \Lambda_B & \Lambda_{B_s} \\
 2.73 & 3.97 & 1.74 & 1.6 & 1.53 & 1.75 & 1.56 & 1.96 & 1.8 & 2.05 & \text{GeV} \\
\end{array}
\]
2 Semileptonic decays

We give the necessary definitions of the leptonic decay constants, invariant form factors and helicity amplitudes. The leptonic decay constants are defined by

\[ M(H_{12} \to \bar{\nu}) = \frac{G_F}{\sqrt{2}} V_{q_1 q_2} \mathcal{M}_{12}^\mu(p) \bar{u}(k_i) O^\mu u_\nu(k_\nu), \]

\[ \mathcal{M}_{12}^\mu(p) = -3 g_{12} \int \frac{d^4k}{(2\pi)^4 i} \Phi_{12}(-k^2) \mathrm{tr} \left[ \Gamma_H \bar{S}_2(k - c_{12}^2 p) O^\mu \bar{S}_1(k + c_{12}^2 p) \right], \]

\[ \Gamma_H = i \gamma^5, \quad \Gamma_V = \varepsilon_V \cdot \gamma, \]

\[ \mathcal{M}_{12}^\mu(p) = -if_p p^\mu, \quad \mathcal{M}_V^\mu(p) = f_V m_V \varepsilon_V. \] (10)

The semileptonic decays of the $B_c$-meson may be induced by a $b$-quark transition.

\[ M(H_{13} \to H_{23} + \bar{\nu}) = \frac{G_F}{\sqrt{2}} V_{q_1 q_2} \mathcal{M}_{13}^\mu(p_1, p_2) \bar{u}(k_i) O^\mu u_\nu(k_\nu), \]

\[ \mathcal{M}_{13}^\mu = -3 g_{13} g_{23} \int \frac{d^4k}{(2\pi)^4 i} \Phi_{13}(-k^2) \Phi_{23}(-k + c_{23}^2 p_2^2) \]

\[ \times \mathrm{tr} \left[ i \gamma^5 \bar{S}_3(k) \Gamma_{23} \bar{S}_2(k + p_2) O^\mu \bar{S}_1(k + p_1) \right], \]

\[ \times \mathrm{tr} \left[ i \gamma^5 \bar{S}_3(k - p_1) O^\mu \bar{S}_2(k - p_2) \Gamma_{21} \bar{S}_1(k) \right], \] (11)

where $q_1 \equiv b$ and $q_3 \equiv c$ whereas $q_2$ denotes either of $c, u, d, s$.

The invariant form factors for the semileptonic $B_c$-decay into the hadron with spin $S = 0, 1$ are defined by

\[ \mathcal{M}_{S=0}^\mu = P^\mu F_+(q^2) + q^\mu F_-(q^2), \] (12)

\[ \mathcal{M}_{S=1}^\mu = \frac{1}{m_1 + m_2} c_5^\nu \left\{ -g^{\mu \nu} P q A_0(q^2) + P^\mu P^\nu A_+(q^2) + q^\mu P^\nu A_-(q^2) + i \varepsilon^{\mu \nu \rho \sigma} P_\rho q_\sigma V(q^2) \right\}, \]

\[ P = p_1 + p_2, \quad q = p_1 - p_2. \]

It is convenient to express all physical observables through the helicity form factors $H_m$. The helicity form factors $H_m$ can be expressed in terms of the invariant form factors in the following way [7]:

(a) Spin $S = 0$:

\[ H_t = \frac{1}{\sqrt{q^2}} \left\{ (m_1^2 - m_2^2) F_+ + q^2 F_- \right\}, \]

\[ H_\pm = 0, \]

\[ H_0 = \frac{2 m_1 |p_2|}{\sqrt{q^2}} F_+. \] (14)
The calculation of the semileptonic decay widths is straightforward. For the CKM-matrix elements we use

\[
\begin{align*}
|V_{ud}| & = 0.974 \\
|V_{us}| & = 0.225 \\
|V_{cd}| & = 0.220 \\
|V_{cs}| & = 0.995 \\
|V_{cb}| & = 0.0405 \\
|V_{ub}| & = 0.00409
\end{align*}
\]

(b) Spin \( S = 1 \):

\[
H_t = \frac{1}{m_1 + m_2} \frac{m_1 |p_2|}{2 m_2 \sqrt{q^2}} \left( (m_1^2 - m_2^2)(A_+ - A_0) + q^2 A_- \right),
\]

\[
H_\pm = \frac{1}{m_1 + m_2} \left\{ -(m_1^2 - m_2^2) A_0 \pm 2 m_1 |p_2| V \right\},
\]

\[
H_0 = \frac{1}{m_1 + m_2} \frac{1}{2 m_2 \sqrt{q^2}} \left\{ -(m_1^2 - m_2^2) (m_1^2 - m_2^2 - q^2) A_0 + 4 m_1^2 |p_2|^2 A_+ \right\}.
\]

where \(|p_2| = A^{1/2}(m_1^2, m_2^2, q^2)/(2 m_1)\) is the momentum of the outgoing particles in the \( B_c \) rest frame. The semileptonic \( B_c \)-decay widths are given by

\[
\Gamma(B_c^+ \to M_{cc} l\bar{\nu}) = \frac{G_F^2}{(2 \pi)^3} |V_{cb}|^2 \int_{m_t^2}^{q^2} dq^2 \frac{(q^2 - m_1^2)^2 |p_2|}{12 m_1^2 q^2} \left\{ 1 + \frac{m_1^2}{2 q^2} \right\} \sum_{i=\pm,0} \left( H_i^{B_c \to M_{cc}} (q^2) \right)^2 \left( H_i^{B_c \to M_{cc}} (q^2) \right)^2 \right\},
\]

\[
\Gamma(B_c^- \to D^0 l\bar{\nu}) = \frac{G_F^2}{(2 \pi)^3} |V_{ub}|^2 \int_{m_t^2}^{q^2} dq^2 \frac{(q^2 - m_1^2)^2 |p_2|}{12 m_1^2 q^2} \left\{ 1 + \frac{m_1^2}{2 q^2} \right\} \sum_{i=\pm,0} \left( H_i^{B_c \to D^0} (q^2) \right)^2 \left( H_i^{B_c \to D^0} (q^2) \right)^2 \right\},
\]

where \( q^2 = (m_1 - m_2)^2 \), \( m_1 \equiv m_{B_c} \), and \( m_2 \equiv m_f \). Note that \( M_{cc} \) and \( D^0 \) denote both the pseudoscalar and vector cases.

### 3 Numerical results

We take the following values of the meson masses and the \( B_c \)-meson’s lifetime from the PDG [8].

\[
\begin{array}{cccccccc}
| & m_{B_c} & m_{B_c} & m_{J/\psi} & m_D & m_{D^*} & m_{D_s} & m_{D_{s0}} & \tau_{B_c} \\
| & 6.275 & 2.984 & 3.097 & 1.869 & 2.010 & 1.968 & 2.112 & 0.507 ps \\
\end{array}
\]

The calculation of the semileptonic decay widths is straightforward. For the CKM-matrix elements we use

\[
\begin{align*}
|V_{ud}| & = 0.974 \\
|V_{us}| & = 0.225 \\
|V_{cd}| & = 0.220 \\
|V_{cs}| & = 0.995 \\
|V_{cb}| & = 0.0405 \\
|V_{ub}| & = 0.00409
\end{align*}
\]

\[\tag{17}\]
The value of the decay constant $f_{\eta_c}$ was calculated from the branching ratio for the $\eta_c$ meson decay into two photons using the last data [8]. The quality of the fit may be assessed from the entries in Table 1.

The form factors are calculated in the full kinematical region of momentum transfer squared and are shown in Table 2. The curves are depicted in Fig. 1 and Fig. 2.

**Figure 1.** The $F_+(q^2)$ and $F_-(q^2)$ form factors for $B_c \to D$, $B_c \to D_s$ and $B_c \to \eta_c$ transitions, respectively.

**Figure 2.** The $A_0, A_-, A_+, V$ form factors for $B_c \to D^*$, $B_c \to D_s^*$ and $B_c \to J/\psi$ transitions, respectively.
Table 1. Leptonic decay constants $f_H$ (MeV).

|       | This work | Other                     | Ref.          |
|-------|-----------|---------------------------|---------------|
| $f_{B_c}$ | 489       | 489 ± 4 ± 3               | LAT [9]       |
|       |           | 395 ± 15                  | [10]          |
| $f_D$  | 206       | 222.6 ± 16.7$^{+2.8}_{-3.4}$ | CLEO [11]     |
|       |           | 201 ± 3 ± 17              | MILC LAT [12] |
|       |           | 235 ± 8 ± 14              | LAT [13]      |
|       |           | 210 ± 10$^{+17}_{-16}$    | UKQCD LAT [14]|
|       |           | 211 ± 14$^{+2}_{-12}$     | LAT [15]      |
|       |           | 204.6±5.0                 | PDG [16]      |
| $f_{D^*}$ | 244       | 245 ± 20$^{+3}_{-2}$      | LAT [15]      |
|       |           | 278 ± 13 ± 10             | LAT [17]      |
|       |           | 252.2 ± 22.3 ± 4          | QCD SR [18]  |
| $f_{D_s}$ | 257       | 257.5±4.6                 | PDG [16]      |
|       |           | 249 ± 3 ± 16              | MILC LAT [12] |
|       |           | 266 ± 10 ± 18             | LAT [13]      |
|       |           | 290 ± 20 ± 29 ± 29 ± 6    | LAT [19]      |
|       |           | 236 ± 8$^{+17}_{-14}$     | UKQCD LAT [14]|
|       |           | 231 ± 12$^{+8}_{-1}$      | LAT [15]      |
|       |           | 311±9                      | LAT [17]      |
|       |           | 272(16)$^{+3}_{-20}$      | LAT [15]      |
|       |           | 305.5 ± 26.8 ± 5          | QCD SR [18]  |
| $\frac{f_{D_s}}{f_D}$ | 1.25       | 1.258±0.038               | PDG [16]      |
|       |           | 1.24 ± 0.01 ± 0.07        | MILC LAT [12] |
|       |           | 1.13 ± 0.03 ± 0.05        | LAT [13]      |
|       |           | 1.13 ± 0.02$^{+0.04}_{-0.02}$ | UKQCD LAT [14]|
|       |           | 1.10 ± 0.02               | LAT [15]      |
| $f_{\eta_c}$ | 628       | 420 ± 52                  | [20]          |
|       |           | 337.7 ± 18.2              | pQCD[21]      |
| $f_{J/\psi}$ | 415       | 405 ± 14                  | pQCD[22]      |
|       |           | 416.2 ± 7.4               | pQCD[21]      |
Table 2. Form factors for $B_c \to D(D^*)$, $B_c \to D_s(D^*_s)$ and $B_c \to \eta_c(J/\psi)$ transitions. Form factors are approximated by the form $F(q^2) = F(0)/(1 - a \hat{s} + b \hat{s}^2)$ with $\hat{s} = q^2/m_{B_c}^2$.

|               | $B_c \to D(D^*)$ | $B_c \to D_s(D^*_s)$ | $B_c \to \eta_c(J/\psi)$ |
|---------------|-------------------|----------------------|--------------------------|
| $F_+(0)$      | 0.186             | 0.254                | 0.74                     |
| $F_-(0)$      | -0.160            | -0.202               | -0.39                    |
| $A_0(0)$      | 0.276             | 0.365                | 1.65                     |
| $A_+(0)$      | 0.151             | 0.190                | 0.55                     |
| $A_-(0)$      | -0.236            | -0.293               | -0.87                    |
| $V(0)$        | 0.230             | 0.282                | 0.78                     |
Table 3. Branching ratios (in %) of semileptonic $B_c$ decays into ground state charmonium states.

| Mode | This work | [23] | [7] | [24, 25] | [26] | [27] | [28] |
|------|-----------|------|-----|----------|-----|-----|-----|
| $B_c^- \to \eta_c \ell \nu$ | 0.95 | 0.81 | 0.98 | 0.75 | 0.97 | 0.59 | 0.44 |
| $B_c^- \to \eta_c \tau \nu$ | 0.24 | 0.22 | 0.27 | 0.23 | 0.20 | 0.14 |
| $B_c^- \to J/\psi \ell \nu$ | 1.67 | 2.07 | 2.30 | 1.9 | 2.35 | 1.20 | 1.01 |
| $B_c^- \to J/\psi \tau \nu$ | 0.40 | 0.49 | 0.59 | 0.48 | 0.34 | 0.29 |
| $B_c^- \to D^- \ell \nu$ | 0.0033 | 0.0035 | 0.018 | | 0.004 | 0.006 | 0.0032 |
| $B_c^- \to D^- \tau \nu$ | 0.0021 | 0.0021 | 0.0094 | 0.002 | | 0.0022 |
| $B_c^- \to D^*- \ell \nu$ | 0.006 | 0.0038 | 0.034 | 0.018 | 0.018 | 0.011 |
| $B_c^- \to D^* \tau \nu$ | 0.0034 | 0.0022 | 0.019 | 0.008 | | 0.006 |

Table 4. Ratios of semileptonic decays of the $B_c$ meson

| Decay rate | This work | [23] | [28] |
|------------|-----------|------|-----|
| $R_{\eta_c} = \frac{B_c^- \to \eta_c \ell \nu}{B_c^- \to \eta_c \tau \nu}$ | 3.96 | 3.68 | 3.2 |
| $R_{J/\psi} = \frac{B_c^- \to J/\psi \ell \nu}{B_c^- \to J/\psi \tau \nu}$ | 4.18 | 4.22 | 3.4 |
| $R_D = \frac{B_c^- \to D^- \ell \nu}{B_c^- \to D^- \tau \nu}$ | 1.57 | 1.67 | 1.42 |
| $R_{D^*} = \frac{B_c^- \to D^* \ell \nu}{B_c^- \to D^* \tau \nu}$ | 1.76 | 1.72 | 1.66 |

The results of our evaluation of the branching ratios of the semileptonic $B_c$ decays appear in Table 3 which contains our predictions for the semileptonic $B_c$ decays into ground state charmonium states and charm meson states. We compare our ratios of semileptonic decays of the $B_c$ meson with those of other models in Table 4.

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