Resonance $X(5568)$ as an exotic axial-vector state

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(Received: January 23, 2017)

The mass and meson-current coupling constant of the resonance $X(5568)$, as well as the width of the decay $X(5568) \rightarrow B^+_s \pi^+$ are calculated by modeling the exotic $X(5568)$ resonance as a diquark-antidiquark state $X_b = [su][bd]$ with quantum numbers $J^P = 1^+$. The calculations are made employing QCD two-point sum rule method, where the quark, gluon and mixed vacuum condensates up to dimension eight are taken into account. The sum rule approach on the light-cone in its soft-meson approximation is used to explore the vertex $X_b B^+_s \pi$ and extract the strong coupling $g_{X_b B^+_s \pi}$, which is a necessary ingredient to find the width of the $X_b \rightarrow B^+_s \pi^+$ decay process. The obtained predictions are compared with the experimental data of the D0 Collaboration, and results of other theoretical works.

PACS numbers: 12.39.Mk, 14.40.Rt, 14.40.Pq

I. INTRODUCTION

Investigation of the ”exotic” hadrons, which cannot be described as usual $q\bar{q}$ and $qqq$ structures, and are composed of more than three valence quarks is now one of the intriguing and developing branches of high energy physics. Existence of such particles, their possible properties, and experiments suitable for their observation were among interests of physicists during last three decades. Starting from discovery of the theory of strong interactions, i.e. the Quantum Chromodynamics (QCD), a qualitative analysis of the exotic states were replaced by the quantitative calculations of their parameters in the strong context of the quantum field theory. Some of parameters of the exotic states were computed in eighties using namely the nonperturbative tools of QCD in Refs. [1, 2]. But theoretical studies of the exotic particles then were not supported by a reliable experimental information, which slowed the growth of the field.

Situation changed when the various collaborations, such as Belle, BaBar, BESIII, LHCb, CDF, and D0 started to supply the hadron physics by the valuable experimental data on the quantum numbers, masses, and decay widths of new heavy resonances, some of which were interpreted as four-quark (i.e., as tetraquark) exotic states [3]. Experimental studies, naturally, boosted suggestion of new theoretical models to explain an internal structure of the observed resonances, as well as led to invention of new or adapting existing nonperturbative approaches to meet problems of the new emerging field of hadron physics. It is worth noting that during last years a considerable success was achieved both in the experimental and theoretical studies of the exotic states (see, the reviews [4] and references therein).

Among the family of the tetraquarks the narrow resonance $X(5568)$ has a unique position. The evidence for existence of these resonance was announced recently by the D0 Collaboration [13], which was founded on the $pp$ collision data at $\sqrt{s} = 1.96$ TeV. It was observed in the channel $X(5568) \rightarrow B^0_s \pi^\pm$ through chain of decays $B^0_s \rightarrow J/\psi \phi, J/\psi \rightarrow \mu^+ \mu^-$, and $\phi \rightarrow K^+ K^-$. It is not difficult to conclude that the state $X(5568)$ consists of valence $b, s, u$ quarks, whereas its decay width remains unchanged [13]. In this case the resonance has the quantum numbers $J^P = 1^+$. But if the resonance $X(5568)$ decays as $X(5568) \rightarrow B^+_s \pi^+ \rightarrow B^0_s \pi^\pm$ with unseen soft $\gamma$, then the difference $m(B^+_s) - m(B^0_s)$ should be added to the mass of the resonance $X(5568)$, whereas its decay width remains clear.

The decay channel $B^0_s \pi^\pm$ was investigated by the LHCb Collaboration utilizing the $pp$ collision data at energies 7 TeV and 8 TeV collected at CERN [14]. The aim was to confirm the existence of the $X(5568)$ state and measure its spectroscopic parameters. But the LHCb Collaboration could not fix the resonance structure in the $B^0_s \pi^\pm$ invariant mass distribution at the energies less than 6000 MeV. Very similar conclusions were drawn by the CMS collaboration [15], in which a mass range up to 5900 MeV was searched for a possible structure, setting an upper limit. Nevertheless, the D0 Collaboration from analysis of the semileptonic decays of $B^0_s$ meson recently confirmed in Ref. [10] the observation of the $X(5568)$ resonance. As is seen, experimental situation with the existence of the exotic state $X(5568)$, supposedly built of four different quark flavors, remains intriguing and unclear.

Namely these conditions make the theoretical studies of the $X(5568)$ state even more urgent than just after first announcement of its observation. Suggestions about inner organization of this particle as the bound state of
a diquark and antidiquark, or as a meson molecule compound were already made in Ref. [13]. Theoretical works, appeared afterwards to determine the mass and decay width of the $X(5568)$ state followed mainly these suggestions. Thus, in Ref. [17] it was considered as the diquark-antidiquark structure $X_b = [su][bd]$ with the quantum numbers $0^{++}$, where its mass $m_{X_b}$ and meson-current coupling (hereafter, the meson coupling) $f_{X_b}$ were calculated. In the framework of the diquark-antidiquark model the mass and other parameters of $X(5568)$ were also explored in Refs. [18–21]. The values for $m_{X_b}$ found in these works agree with each other, and are consistent with the experimental data of the D0 Collaboration.

The width of the $X_b \to B^0_s \pi^+$ decay channel was calculated in Ref. [22] employing for $X_b$ the same structure and interpolating current as in Ref. [17]. To this end, authors applied QCD light-cone sum rule method and soft-meson approximation adjusted for studying of the tetraquark states in Ref. [23], where relevant explanations and technical details can be found. The result for $\Gamma(X^\pm \to B^0_s \pi^\pm)$ derived in Ref. [22] describes correctly the experimental data. The width of the decay channels $X^\pm(5568) \to B_s \pi^\pm$ was also analyzed in Refs. [24–26], employing the three-point QCD sum rule approach. In these works authors found a nice agreement between the theoretical predictions for $\Gamma(X^\pm \to B^0_s \pi^\pm)$ and data, as well.

As we have mentioned above, the $X(5568)$ state can also be considered as a molecule composed of $B$ and $K$ mesons, which was realized in Refs. [26–28]. But in this scenario, in accordance with Ref. [27], the mass of such molecule and width of the decay $X^\pm(5568) \to B_s \pi^\pm$ exceed the experimental data of D0, and predictions of the diquark-antidiquark model. These facts were interpreted in favor of the diquark-antidiquark organization of the $X(5568)$ state.

The experimental data of the D0 and LHCb collaborations generated an appearance of interesting theoretical works, where the structure and spectroscopic parameters, production mechanisms of the $X(5568)$ state were considered. The details of used methods, necessary explanations, and conclusions made there concerning a nature of the resonance $X(5568)$ can be found in the original papers [29–41].

In the present work we calculate the mass, meson coupling of the $X_b = [su][bd]$ diquark-antidiquark state treating it as an axial-vector particle with the quantum numbers $J^P = 1^+$. To this end, we apply QCD two-point sum rule approach and include into our analysis the quark, gluon and mixed condensates up to eight dimensions. We are also going to determine the width of the decay $X_b \to B^*_s \pi^+$ using the soft-meson version of QCD light-cone sum rule method. Performed investigation should allow us to answer a question is the $X(5568)$ resonance a state with $J^P = 0^+$ or $J^P = 1^+$.

This work is organized in the following form. Section II is devoted to the sum rule calculations of the mass and meson coupling of the axial vector $X_b$ state. Here we derive also the light-cone sum rule expression for the strong coupling $g_{X_b B^*_s \pi}$ necessary to compute the decay width of the process $X_b \to B^*_s \pi^+$. In Sect. III we conduct numerical calculations and extract values of the mass, meson coupling and decay width under consideration. Here we compare our results with the experimental data and predictions of other theoretical works. Section IV contains our brief conclusions on the nature of the $X_b$ state. After this, we consider the correlation function given below as

$$\Pi_{\mu\nu}(q) = i \int d^4x e^{iq\cdot x} \langle 0 | \{ J_\mu(x), J_\nu^\dagger(0) \} | 0 \rangle,$$

where $J_\mu(x)$ is the interpolating current with the quantum numbers of the $X_b$ state. We consider $X_b$ as a particle with the quantum numbers $J^P = 1^+$. Then in the diquark-antidiquark model one of the acceptable interpolating currents $J_\mu(x)$ is defined by the expression [20]

$$J_\mu(x) = \bar{s}_a^T(x) C \gamma_\mu u_b(x) \left\{ \bar{b}_a(x) \gamma_\mu C \tilde{d}_b^T (x) - \bar{b}_b(x) \gamma_\mu C \tilde{d}_a^T (x) \right\},$$

where $a$ and $b$ are color indices and $C$ is the charge conjugation matrix.

In order to derive QCD sum rule expression we first have to calculate the correlation function in terms of the physical parameters of the $X_b$ state. Saturating the correlation function with a complete set of the $X_b$ state and performing integral over $x$ in Eq. (1), we get

$$\Pi_{\mu\nu}^{\text{Phys}}(q) = \frac{\langle 0 | \{ J_\mu(x_b(q)), J_\nu^\dagger(q) \} | x_b(q) \rangle | J^\dagger_0 | 0 \rangle}{m_{X_b}^2 - q^2} + \ldots$$

with $m_{X_b}$ being the mass of the $X_b$ state. Here the dots indicate contributions to the correlation function arising from...
from the higher resonances and continuum states. We define the meson coupling \( f_{X_b} \) using the matrix element

\[
\langle 0|J_\mu|X_b(q)\rangle = f_{X_b}m_{X_b}\varepsilon_\mu,
\]

where \( \varepsilon_\mu \) is the polarization vector of the \( X_b \) state. Then in terms of \( m_{X_b} \) and \( f_{X_b} \), the correlation function can be written in the form

\[
\Pi^{\text{phys}}_{\mu\nu}(q) = \frac{m_{X_b}^2 f_{X_b}^2}{m_{X_b}^2 - q^2} \left( -g_{\mu\nu} + \frac{q_\mu q_\nu}{m_{X_b}^2} \right) + \ldots
\]

(5)

Equation (5) has been derived within the single pole assumption, which in the case of a multiquark state requires additional justification. The reason is that for such systems in the physical side of the sum rule one has to take into account also two-hadron reducible contributions, which for the tetraquarks however are negligible \[^{42} \]. Therefore, we can apply the Borel transformation directly to Eq. (5), which yields

\[
\mathcal{B}_{q\mu} \Pi^{\text{phys}}_{\mu\nu}(q) = \frac{m_{X_b}^2 f_{X_b}^2}{m_{X_b}^2 - m_{X_b}^2} \left( -g_{\mu\nu} + \frac{q_\mu q_\nu}{m_{X_b}^2} \right) + \ldots
\]

(6)

From the QCD side the same function must be calculated employing of the quark-gluon degrees of freedom. To this end, we contract the heavy and light quark fields and find for the correlation function \( \Pi^{\text{phys}}_{\mu\nu}(q) \) the following expression:

\[
\Pi^{\text{QCD}}_{\mu\nu}(q) = i \int d^4xe^{iqx} \left\{ \frac{\gamma_5 S_5}{2} \delta_{\mu\nu} S_5^a(x) \right\} - T \left[ \gamma_\mu S_b^a(x) \right] T \left[ \gamma_\nu S_b^a(x) \right] - T \left[ \gamma_\mu S_b^a(x) \right] T \left[ \gamma_\nu S_b^a(x) \right] + \ldots
\]

(7)

In Eq. (7) \( S^{ab}_{s(u,d)}(x) \) and \( S^{ab}_b(x) \) are the light s, u, d quarks and b-quark propagators, respectively. Here we also use the notation

\[
\tilde{S}_{s(d)}(x) = C S^T_{s(d)}(x).C.
\]

We work with the light quark propagator \( S^{ab}_q(x) \) defined in the form

\[
S^{ab}_q(x) = i \delta^{ab} \frac{f}{2\pi^2 x^2} - \frac{m_q}{4\pi^2 x^2} - \delta^{ab} \frac{\langle \overline{q}q \rangle}{12} + i\alpha \frac{f m_q}{48} \frac{x^2}{192} - \frac{\langle \overline{q}q \rangle G_{\sigma G}}{1152} + \ldots
\]

(8)

Let us emphasize that in calculations we set the light quark masses \( m_u \) and \( m_d \) equal to zero, preserving at the same time dependence of the propagator \( S^{ab}_b(x) \) on the \( m_s \). For the b-quark propagator \( S^{ab}_b(x) \) we employ the formula given in Ref. \(^{43} \):

\[
S^{ab}_b(x) = i \int \frac{dk}{(2\pi)^4} e^{-ikx} \left\{ \frac{\delta^{ab} (k + m_b)}{k^2 - m_b^2} - \frac{g_\alpha G^{ab\beta}}{4} \frac{\sigma_{\alpha\beta} (k + m_b)}{k^2 - m_b^2} \right\} + \ldots
\]

(9)

In Eqs. (8) and (9) we use the notations

\[
G^{ab\beta} = G^{ab\beta}_{\lambda A}, \quad G^2 = G^{A\beta} G^{A\beta},
\]

\[
G^3 = f^{ABC} G^{AB}_{\nu\mu} G^{BC}_{\nu\mu},
\]

(10)

where \( a, b = 1, 2, 3 \) and \( A, B, C = 1, 2 \ldots 8 \) are the color indices. Here \( \epsilon^{A} = \lambda^{A}/2 \), and \( \lambda^{A} \) are the Gell-Mann matrices. In the nonperturbative terms the gluon field strength tensor \( G_{\alpha\beta} \equiv G_{\alpha\beta}(0) \) is fixed at \( x = 0 \).

The correlation function \( \Pi^{\text{phys}}_{\mu\nu}(q) \) can be decomposed over the Lorentz structures \( \sim g_{\mu\nu} \) and \( \sim q_\mu q_\nu \). The QCD sum rule expressions can be obtained after fixing the same Lorentz structures in the both \( \Pi^{\text{phys}}_{\mu\nu}(q) \) and \( \Pi^{\text{QCD}}_{\mu\nu}(q) \). We choose the term \( \sim g_{\mu\nu} \), which receives a contribution from only spin-1 states, whereas the invariant amplitude corresponding to the structure \( \sim q_\mu q_\nu \) forms due to contributions of both spin-0 and spin-1 states.

The chosen invariant amplitude \( \Pi^{\text{QCD}}(q^2) \) can be written down as the dispersion integral

\[
\Pi^{\text{QCD}}(q^2) = \int_{(m_b + m_s)^2}^{\infty} \frac{\rho^{\text{QCD}}(s)}{s - q^2} ds + \ldots
\]

(11)

where \( \rho^{\text{QCD}}(s) \) is the two-point spectral density. Now utilizing the Borel transformation to \( \Pi^{\text{QCD}}(q^2) \), equating the obtained expression with the relevant part of the function \( \mathcal{B}_{q\mu} \Pi^{\text{phys}}_{\mu\nu}(q) \), and subtracting the continuum contribution we, as a result, get the required sum rule. Then the mass of the \( X_b \) state can be evaluated from the sum rule

\[
m_{X_b}^2 = \int_{(m_b + m_s)^2}^{\infty} ds \rho^{\text{QCD}}(s)e^{-s/M^2}. \]

(12)

To extract the meson coupling \( f_{X_b} \) we can employ the sum rule formula

\[
m_{X_b}^2 f_{X_b}^2 e^{-m_{X_b}^2/M^2} = \int_{(m_b + m_s)^2}^{\infty} ds \rho^{\text{QCD}}(s)e^{-s/M^2}. \]

(13)
The key component of these expressions is the two-point spectral density \( \rho^{QCD}(s) \). Because the methods for its derivation were presented in the literature (see, Ref. [23]), here we omit technical details of calculations moving the final expressions of the spectral density and its components to Appendix.

### B. Strong coupling \( g_{X_bB^*_\pi} \) and width of the decay \( X_b \to B^*_\pi \)

In this subsection we outline the soft-meson approximation of QCD light-cone sum rule method used here to explore the strong vertex \( X_bB^*_\pi \) and find the sum rule expression for the coupling \( g_{X_bB^*_\pi} \). The latter will be used to calculate the width of the decay \( X_b \to B^*_\pi \) decay process.

To this end, we start from the correlation function

\[
\Pi_{\mu\nu}(p, q) = i \int \frac{d^4xe^{ipx}}{M^2} \langle 0 | J_{\mu}^{B^*_\pi}(x) J_{\nu}^B(0) | 0 \rangle,
\]

where the interpolating current for the \( B^*_\pi \) meson has the form

\[
J_{\mu}^{B^*_\pi}(x) = \bar{b}(x)\gamma_\mu s_1(x),
\]

where \( J_{\mu}(x) \) is defined by Eq. (2). Here \( p, q \) and \( p' = p + q \) are the momenta of the mesons \( B^*_\pi \) and \( \pi \), and the \( X_b \) state, respectively.

To derive the sum rule for the strong coupling \( g_{X_bB^*_\pi} \), we take the standard prescriptions of the QCD sum rule approach and calculate \( \Pi_{\mu\nu}(p, q) \) in terms of the physical parameters of the involving particles. Then we obtain

\[
\Pi^{\text{Phys}}_{\mu\nu}(p, q) = \frac{\langle 0 | J_{\mu}^{B^*_\pi}(p') | B^*_\pi(p) \rangle}{p^2 - m_{B^*_\pi}^2} \langle B^*_\pi(p) \pi(q) | X_b(p') \rangle
\]

\[
\times \frac{\langle X_b(p') | J_{\nu}^B(0) \rangle}{p'^2 - m_{X_b}^2} + \ldots,
\]

where the dots denote, as usual, contributions of the higher resonances and continuum states. The expression of the function \( \Pi^{\text{Phys}}_{\mu\nu}(p, q) \) can be further simplified and expressed in terms of the particle parameters if we introduce the new matrix elements

\[
\langle 0 | J_{\mu}^{B^*_\pi} | B^*_\pi(p) \rangle = f_{B^*_\pi} m_{B^*_\pi} \varepsilon_\mu,
\]

\[
\langle B^*_\pi(p) \pi(q) | X_b(p') \rangle = \langle [p \cdot p'](\varepsilon^* \cdot \varepsilon') \rangle | X_b(p') \rangle,
\]

where \( f_{B^*_\pi}, m_{B^*_\pi}, \varepsilon_\mu \) are the decay constant, mass and polarization vector of the \( B^*_\pi(p) \) meson, and \( \varepsilon' \) is the polarization vector of the \( X_b \) state.

Using these matrix elements, as well as one given by Eq. (11), one can rewrite the correlation function as

\[
\Pi^{\text{Phys}}_{\mu\nu}(p, q) = \frac{f_{B^*_\pi} f_{X_b} m_{X_b} m_{B^*_\pi}}{(p^2 - m_{X_b}^2) (p'^2 - m_{B^*_\pi}^2)} g_{X_b B^*_\pi}
\]

\[
\times \left( \frac{m_{X_b}^2 + m_{B^*_\pi}^2}{2} g_{\mu\nu} - p'_\mu p'_\nu \right) + \ldots
\]

It evidently contains two Lorentz structures \( g_{\mu\nu} \) and \( p'_\mu p'_\nu \). We are going to use both of them to derive the sum rules for the coupling \( g_{X_b B^*_\pi} \) and compare our results with each other in order to estimate a sensitivity of the obtained predictions to the chosen structures. Below we consider explicitly the correlation function corresponding to \( g_{\mu\nu} \), and provide only final expression for the spectral density \( \rho^{QCD}_2(s) \) derived using the terms \( \sim p'_\mu p'_\nu \).

In the soft-meson limit, i.e. in the limit \( q = 0 \), the Borel transformation of the invariant amplitude corresponding to \( g_{\mu\nu} \) has the form [Ref. [23]],

\[
\Pi^{\text{Phys}}(M^2) = f_{B^*_\pi} f_{X_b} m_{X_b} m_{B^*_\pi} g_{X_b B^*_\pi} m^2
\]

\[
\times \frac{1}{M^2} e^{-m^2/M^2} + \ldots,
\]

where \( m^2 = (m_{X_b}^2 + m_{B^*_\pi}^2)/2 \).

It is known, that to obtain the sum rule expression for the strong coupling of the three-hadron vertex, in the soft-meson approximation one has to apply the one-variable Borel transformation instead of the two-variable Borel transformation accepted in the standard light-cone sum rule procedures [23]. As a result, the hadronic part of the sum rule becomes more complicated and contains contributions of transitions from exited states to the ground level, shown in Eq. (19) as the dots, which in the soft-meson approximation are unsuppressed even after Borel transformation. In order to remove these terms from the hadronic side one has to use some technical tools. One of such methods was suggested in Ref. [44], which implies acting by the operator

\[
\left( 1 - \frac{M^2}{d M^2} \right) \frac{M^2 e^{m^2/M^2}}{d M^2}
\]

to both the hadronic and QCD sides of the sum rule. It was successfully used in Ref. [23], and is adopted in the present work, as well.

But before that we have to calculate the correlation function Eq. (14) in terms of the quark propagators and find QCD side of the sum rule. Contractions of \( s \) and \( b \)-quark fields in Eq. (14) give

\[
\Pi_{\mu\nu}^{QCD}(p, q) = i \int d^4xe^{ipx} \left\{ \left[ \gamma_5 S^\mu_\alpha(x) \gamma_\mu \right] \phi^\alpha_b(-x) \psi^\beta_b \langle \pi(q) | \bar{b}_\alpha \bar{b}_\beta | 0 \rangle \right. \]

\[
- \left[ \gamma_5 S^\mu_\alpha(x) \gamma_\mu \bar{b}^\alpha_b(-x) \gamma_\nu \right] \phi^\beta_b \langle \pi(q) | \bar{b}_\alpha \bar{b}_\beta | 0 \rangle \right\},
\]

where \( \alpha \) and \( \beta \) are the spinor indices, the quark fields are defined at \( x = 0 \).

In the soft-meson approximation the QCD side of the sum rule is considerably simpler than one in the standard approach. Indeed, in the standard light-cone sum rule the non-local matrix elements of the hadrons (in the case under analysis, of the pion) are expressed in terms of its various distribution amplitudes, whereas in the soft-meson limit we get only few local matrix elements.
The following stages of calculations include applying of the expansion
\[ \overline{\tau}_a d_a^j = \frac{1}{4} \Gamma^j_{\alpha\beta} (\overline{\tau}^\alpha \Gamma^j d^\beta), \] (22)
where \( \Gamma^j \) is the full set of Dirac matrixes, performing the color summation and inserting into the quark matrix elements the gluon field strength tensor \( G \). These operations lead to the correlation function \( \Pi_{\mu
u}^{\text{QCD}}(p, q) \), which depends on the two and three-particle matrix elements of the pion: Let us note that in the present work we neglect terms \( \sim G^2 \) and \( \sim G^3 \). The final step in this way is computation of the imaginary part of the obtained correlation function to extract the desired spectral density. Because these manipulation are described in a clear form in Ref. [23], we refrain from providing further details and write down the final expression for \( \rho_{\mu}^{p, QCD}(s) \) obtained in this way as the sum its the perturbative and nonperturbative parts
\[ \rho_{\mu}^{QCD}(s) = \rho_{\mu}^{\text{pert.}}(s) + \rho_{\mu}^{n, \text{pert.}}(s), \] (23)
where
\[ \rho_{\mu}^{\text{pert.}}(s) = \frac{f_{\pi}\mu_{\pi}}{48\pi^2s^2}(m_b^2-s)(m_b^2+s-6m_b s - 2s^2) \] (24)
and
\[ \rho_{\mu}^{n, \text{pert.}}(s) = \frac{f_{\pi}\mu_{\pi}}{12}(\overline{s} s) \left[ s m_b \delta^{(1)}(s-m_b^2) \right. \]
\[ + (m_b - 2m_b) \delta(s-m_b^2) + \left. \frac{f_{\pi}\mu_{\pi}}{72}(\overline{g}_s \sigma G s) \right. \]
\[ \times \left\{ 3(2m_b-m_a) \delta^{(1)}(s-m_a^2) + s(3m_b-5m_a) \right. \]
\[ \times \delta^{(2)}(s-m_b^2) - s^2 m_a \delta^{(3)}(s-m_b^2) \}. \] (25)
The contributions \( \sim \delta^{(n)}(s-m_b^2) = (d/ds)^n \delta(s-m_b^2) \) arise from the imaginary part of the pole terms.

Calculations of the spectral density \( \rho_{\mu}^{QCD}(s) \) performed employing the terms, which in the soft limit transform to ones \( \rho_{\mu}^{p, QCD}(s) \), yield:
\[ \overline{\rho}_{\mu}^{\text{pert.}}(s) = \frac{f_{\pi}\mu_{\pi}}{24\pi^2 s^3}(s^3 - 3sm_b^2 + 2m_b^4) \] (26)
and
\[ \overline{\rho}_{\mu}^{n, \text{pert.}}(s) = -\frac{f_{\pi}\mu_{\pi}}{6} m_s (\overline{s} s) \delta^{(1)}(s-m_b^2) \]
\[ + \frac{f_{\pi}\mu_{\pi}}{36} m_s (\overline{g}_s \sigma G s) \left( 4 \delta^{(2)}(s-m_b^2) \right. \]
\[ + s \delta^{(3)}(s-m_b^2) \}. \] (27)

As is seen, the spectral densities depend on the parameters \( f_{\pi} \) and \( \mu_{\pi} \) of the pion through the local matrix element
\[ \langle 0|\overline{d} \gamma_5 u|\pi(q) \rangle = f_{\pi}\mu_{\pi}, \] (28)
where
\[ \mu_{\pi} = \frac{m_{\pi}}{m_u + m_d} = - \frac{2(\overline{q}q)}{f_{\pi}^2}. \] (29)

Performing the continuum subtraction in the standard manner and applying the operator Eq. (20), we get the sum rule to evaluate the strong coupling
\[ g_{X_b^{\prime}B_{\pi}^+} = \frac{1}{f_{B_{\pi}^* f_{X_b^{\prime}B_{\pi}^+}}} \left( 1 - M^2 \frac{d}{dM^2} \right) M^2 \]
\[ \times \int_{s_0}^{s_{\text{top}}} \text{d} s \frac{12}{(m_b + m_s)^2} \rho_{\mu}^{QCD}(s). \] (30)
The similar computations for the structure \( p_{\mu}^{p, QCD}(s) \) give:
\[ g_{X_b^{\prime}B_{\pi}^+} = \frac{1}{f_{B_{\pi}^* f_{X_b^{\prime}B_{\pi}^+}}} \left( 1 - M^2 \frac{d}{dM^2} \right) M^2 \]
\[ \times \int_{s_0}^{s_{\text{top}}} \text{d} s \frac{12}{(m_b + m_s)^2} \rho_{\mu}^{QCD}(s). \] (31)

The width of the decay \( X_b \rightarrow B_{\pi}^+ \) can be borrowed from Ref. [23], and is identical for both the strong couplings \( g_{X_b^{\prime}B_{\pi}^+} \) and \( g_{X_b^{\prime}B_{\pi}^+} \)
\[ \Gamma(X_b \rightarrow B_{\pi}^+) = \frac{g_{X_b^{\prime}B_{\pi}^+}}{24\pi} \lambda(m_{X_b}, m_{B_{\pi}^*}, m_{\pi}) \]
\[ \times \left\{ 3 + \frac{2\lambda^2}{m_{B_{\pi}^*}} \left( m_{X_b}, m_{B_{\pi}^*}, m_{\pi} \right) \right\}, \] (32)
where
\[ \lambda(a, b, c) = \sqrt{a^4 + b^4 + c^4 - 2(a^2b^2 + a^2c^2 + b^2c^2)} \]
\[ \times s_0 \] (26)
The final expressions (30), (31) and (32) will be used for numerical analysis of the decay channel \( X_b \rightarrow B_{\pi}^+ \).

III. NUMERICAL RESULTS

The QCD sum rules contain numerous parameters, i.e. quark, gluon and mixed condensates, masses of the \( b \) and \( s \) quarks, and \( B_{\pi}^* \) meson’s mass and decay constant \( f_{B_{\pi}^*} \). Values of these parameters are written down in Table I. Let us note, that we use the well known values for the quark-gluon condensates and fix them within usual procedures. The gluon condensate \( \langle g_s^2 G^3 \rangle \), which is not commonly employed in the sum rule calculations, is borrowed from Ref. [43]. Further we choose the mass of the \( b \) quark in the \( \overline{M}S \) scheme at the scale \( \mu = m_b \), whereas for the decay constant \( f_{B_{\pi}^*} \) invoke the result presented in Ref. [46]. Other parameters are taken from Ref. [47].

The QCD sum rule expressions depend also on the continuum threshold \( s_0 \) and Borel parameter \( M^2 \). To extract values of the quantities under consideration we have to choose such regions for these parameters, where dependence of the physical quantities under consideration on
The QCD sum rule method, as we know, suffers from the theoretical uncertainties, which are its unavoidable property. The main sources of ambiguities in extracting the theoretical accuracy achieved in the present work in deriving the spectral density ρ\text{QCD}(s), including contributions of the terms \(\langle gG^2/s\rangle\) and \(\langle gG^2/s\rangle\), the spectral density ρ\text{QCD}(s) encompasses effects of the terms appearing due to more detailed formulas for the quark propagators accepted in the present work.

The working window for the Borel parameter is determined from joint requirement of the ground state dominance in the sum rule and convergence of the relevant operator product expansion. The latter implies suppression of the nonperturbative terms’ contributions to the sum rule within chosen interval for \(M^2\). As a result, for the mass and meson coupling calculations we fix the following range for \(M^2\)

\[
3 \text{ GeV}^2 \leq M^2 \leq 6 \text{ GeV}^2. \tag{33}
\]

The choice of the continuum threshold \(s_0\) depends on the energy of the first excited state with the same quantum numbers and content as the particle under consideration. It can be extracted from comparative analysis of the pole contribution and higher states-continuum contributions in the relevant operator product expansion: The former contribution should overcome the latter one. Results of the performed numerical computations are shown in Fig. 1.

The analysis performed on the basis of this \(ρ\text{QCD}(s)\) allows us to determine the range of \(s_0\) as

\[
43 \text{ GeV}^2 \leq s_0 \leq 45 \text{ GeV}^2. \tag{34}
\]

With these main parameters at hands we can proceed and carry out computations of the mass and meson coupling of the \(X_b\) state utilizing corresponding two-point sum rules. Our results for \(m_{X_b}\) and \(f_{X_b}\) are plotted in Figs. 2 and 3. As is seen, in the working regions of \(s_0\) and \(M^2\) the mass and meson coupling demonstrate dependencies on these parameters, which are, nevertheless, mild. Hence, considering \(X_b\) as the axial-vector diquark-antidiquark state, for its mass we obtain

\[
m_{X_b} = (5864 ± 158) \text{ MeV}, \tag{35}
\]
whereas for the meson-current coupling $f_{Xb}$ we get

$$f_{Xb} = (0.42 \pm 0.14) \times 10^{-2} \text{ GeV}^4. \quad (36)$$

To compare our result for the mass of the state $X_b$ with quantum numbers $J^P = 1^+$ with the experimental data of the D0 Collaboration, we first have to find this value. In accordance with explanations in Ref. [13], it is defined by the expression

$$m[X(1^+)] = m[X(0^+)] + m(B_{s}^*) - m(B_0^s). \quad (37)$$

This equality, when taking into account $m(B_{s}^*) - m(B_0^s) \simeq 48.7 \text{ MeV}$ (see, Refs. [47]), leads to the experimental value

$$m[X(1^+)] \simeq 5617 \text{ MeV}. \quad (38)$$

In other words, treating $X(5568)$ as an axial-vector particle, we have to consider Eq. (38) as the data of the D0 Collaboration. Then it becomes clear that, our prediction for the mass of $X_b$ differs from the result of the D0 Collaboration given by Eq. (38): Even after taking into account errors of calculations it overshoots corresponding experimental result.

In evaluation of the strong coupling $g_{X_b B_{s}^* \pi}$ the window for the Borel parameter shifts towards larger values

$$6 \text{ GeV}^2 \leq M^2 \leq 8 \text{ GeV}^2, \quad (39)$$

whereas the range of $s_0$ remains unchanged. Results of our computations of the strong coupling $g_{X_b B_{s}^* \pi}$ and its dependence on $M^2$ and $s_0$ are depicted in Figs. 4 and 5 respectively. One can see, that the strong coupling is sensitive to the choice of the Borel parameter and almost stable under varying of $s_0$.

The coupling $g_{X_b B_{s}^* \pi}$ extracted from the sum rule Eq. (30) reads

$$g_{X_b B_{s}^* \pi} = (0.20 \pm 0.05) \text{ GeV}^{-1}. \quad (40)$$

Carrying out similar numerical analysis for the coupling $\tilde{g}_{X_b B_{s}^* \pi}$ we find:

$$\tilde{g}_{X_b B_{s}^* \pi} = (0.21 \pm 0.06) \text{ GeV}^{-1}. \quad (41)$$

The difference between predictions for the couplings obtained using the different Lorentz structures is rather small. We use its mean value $g \Rightarrow (g + \tilde{g})/2$ to evaluate the width of the decay $X_b \to B_{s}^* \pi^+$ process and get

$$\Gamma(X_b \to B_{s}^* \pi^+) = (20.2 \pm 6.1) \text{ MeV}, \quad (42)$$

whereas the experiment gives $\Gamma_{\text{exp}}[X(5568) \to B_{s}^* \pi^+] = 21.9 \pm 6.4(\text{stat}) \pm 5.0(\text{syst}) \text{ MeV}$.

In other words, the present investigation of the $X(5568)$ resonance and comparison of the obtained predictions with data of the D0 Collaboration does not confirm an axial-vector nature of this diquark-antidiquark state, the mass $m[X_b(1^+)]$ parameter being the decisive argument in making this conclusion.
Within QCD sum rule approach the mass of the $X(5568)$ state as an axial-vector particle was also investigated in Ref. [20]. The result obtained in this paper for the mass of the exotic state $X_b$ reads

$$m[X_b(1^+)] = 5.59 \pm 0.15 \text{ GeV.} \quad (43)$$

As is seen, there is a discrepancy between our prediction for $m[X_b(1^+)]$ and result of Ref. [20]. The possible source of the dissonance is the working regions for the parameters $s_0$ and $M^2$ used there, which were extracted by means of the two-point spectral density that differs from $\rho_{QCD}(s)$ derived in the present work, as it has been noted above.

**IV. CONCLUDING REMARKS**

In the current work we have done QCD sum rule analysis of the exotic $X_b$ state by considering it as an axial-vector particle built of a diquark and antidiquark. We have computed the mass $m_{X_b}$ and decay width of the process $X_b \rightarrow B_s^* \pi^+$, and compared our results with experimental data of the D0 Collaboration, as well as with theoretical prediction for $m_{X_b}$ made in Ref. [20]. Our result for the mass of the axial-vector exotic state $X_b$ exceeds the experimental data, whereas the decay width of the process $X_b \rightarrow B_s^* \pi^+$ calculated using two different structures in the correlation function is compatible the experimental data. Despite this last fact, we make our conclusion relying mainly on the mass calculation: If the $X(5568)$ resonance exists and its parameters measured by the D0 Collaboration are correct, then the present analysis with high level of confidence excludes, that it is an axial-vector diquark-antidiquark state with $J^P = 1^+$. It is interesting to note, that the same parameters were evaluated in Refs. [17, 22] using the diquark-antidiquark model for the $X_b$ state with quantum numbers $J^P = 0^+$. In these works a satisfactory agreement with the experimental data of the D0 Collaboration were found.

Further investigations are required to clarify the experimental situation and theoretical questions surrounding the $X(5568)$ resonance. But possible outputs of such studies justify efforts paid for their realization, because the $X(5568)$ state is very intriguing and interesting particle, supposedly composed of four-quarks of the different flavor, and its investigation may shed light on many problems of hadron physics.

**ACKNOWLEDGEMENTS**

This work was supported by TUBITAK under the grant no: 115F183.

**Appendix: The two-point spectral density**

This appendix contains the results obtained for the two-point spectral density

$$\rho_{QCD}(s) = \rho_{\text{pert}}(s) + \sum_{k=3}^8 \rho_k(s), \quad (A.1)$$

used to calculate from the QCD sum rules the mass and meson coupling of the $X_b$ state. In Eqs. (A.1) and (A.2) by $\rho_k(s)$ we denote the nonperturbative contributions to $\rho_{QCD}(s)$. The explicit expressions for $\rho_{\text{pert}}(s)$ and $\rho_k(s)$ are presented here as the integrals of the Feynman parameter $z$: 

A.1
\[ \rho_{\text{pert}}(s) = \frac{1}{1288\pi^6} \int_0^a \frac{dz z^4}{(z-1)^3} \left[ m_b^2 + s(z-1) \right]^3 \left[ m_b^2 + 5s(z-1) \right], \]

\[ \rho_3(s) = \frac{1}{128\pi^4} \int_0^a \frac{dz z^2}{(z-1)^2} \left[ m_b^2 + s(z-1) \right] \left\{ 2(\bar{d}d)m_b \left[ m_b^2 + s(z-1) \right] + m_s(z-1) \left( \bar{s}s - 2(\bar{u}u) \right) \left[ m_b^2 + 3s(z-1) \right] \right\}, \]

\[ \rho_4(s) = \frac{1}{9216\pi^4} \left( \alpha_s G^2 \right) \int_0^a \frac{dz z^2}{(z-1)^3} \left[ m_b^2(13z^2 - 21z + 9) + 2m_b^2 s(23z^3 - 63z^2 + 58z - 18) + s^2(z-1)^3(32z - 27) \right], \]

\[ \rho_5(s) = \frac{m_b^2}{192\pi^4} \int_0^a \frac{dz}{(1-z)} \left\{ 3m_b \langle \bar{d}d \rangle \left[ m_b^2 + s(z-1) \right] + m_s(z-1) \left( \bar{s}s - 3(\bar{u}u) \right) \left[ m_b^2 + 2s(z-1) \right] \right\}, \]

\[ \rho_6(s) = \frac{1}{6144\pi^6} m_b^2 \langle g_s^3 G^3 \rangle \int_d z \left\{ \frac{z^5}{(1-z)^3} + \frac{1}{1296\pi^4} \int_0^a dz \left\{ g_s^2(\bar{d}d) z \left[ m_b^2 + 2s(z-1) \right] + 54\pi^2 m_b m_s \langle \bar{d}d \rangle \left( \bar{s}s - 2(\bar{u}u) \right) \right\} + z \left\{ g_s^2(\bar{u}u)^2 + \bar{s}s \right\} \right\} + 108\pi^2 \langle \bar{s}s \rangle \langle \bar{u}u \rangle \left[ m_b^2 + 2s(z-1) \right], \]

\[ \rho_7(s) = \frac{1}{1152\pi^2} \left( \alpha_s G^2 \right) \int_0^a \int z \left\{ 8m_b \bar{d}d + m_s \left[ 3z\bar{s}s + 2(\bar{u}u)(1-4z) \right] \right\}, \]

\[ \rho_8(s) = \frac{11}{73728\pi^2} \left( \alpha_s G^2 \right) \int_0^a \frac{dz}{24\pi^2} m_b^2 \bar{s}s \langle \bar{u}u \rangle \int_0^a \int \left( z - 1 \right) dz, \quad (A.2) \]

where \( a = 1 - m_b^2/s. \)

[1] I. I. Balitsky, D. Diakonov and A. V. Yung, Phys. Lett. B 112, 71 (1982); Z. Phys. C 33, 265 (1986); V. M. Braun and A. V. KoLesnichenko, Phys. Lett. B 175, 485 (1986); V. M. Braun and Y. M. Shabelski, Sov. J. Nucl. Phys. 50, 306 (1989).

[2] J. Govaerts, L. J. Reinders, H. R. Rubinstein and J. Weyers, Nucl. Phys. B 258, 215 (1985); J. Govaerts, L. J. Reinders and J. Weyers, Nucl. Phys. B 262, 575 (1985).

[3] S.-K. Choi et al. (Belle Collaboration), Phys. Rev. Lett. 91, 262001 (2003); V. M. Abazov et al. (D0 Collaboration), Phys. Rev. Lett. 93, 162002 (2004); D. Acosta et al. (CDF II Collaboration) Phys. Rev. Lett. 93, 072001 (2004); B. Aubert et al. (BaBar Collaboration), Phys. Rev. D 71, 071103 (2005).

[4] R. L. Jaffe, Phys. Rept. 409, 1 (2005).

[5] E. S. Swanson, Phys. Rept. 429, 243 (2006).

[6] E. Klempt and A. Zaitsev, Phys. Rept. 454, 1 (2007).

[7] S. Godfrey and S. L. Olsen, Ann. Rev. Nucl. Part. Sci. 58, 51 (2008).

[8] M. B. Voloshin, Prog. Part. Nucl. Phys. 61, 455 (2008).

[9] M. Nielsen, F. S. Navarra, and S. H. Lee, Phys. Rep. 497, 41 (2010).

[10] R. Faccini, A. Pilloni and A. D. Polosa, Mod. Phys. Lett. A 27, 1230025 (2012).

[11] A. Esposito, A. L. Guerrieri, F. Piccinini, A. Pilloni and A. D. Polosa, Int. J. Mod. Phys. A 30, 1530002 (2014).

[12] H.-X. Chen, W. Chen, X. Liu, and S.-L. Zhu, Arxiv: 1601.02092 [hep-ph], 2016.

[13] V. M. Abazov et al. [D0 Collaboration], Phys. Rev. Lett. 117, 022003 (2016).

[14] R. Aaij et al. [LHCb Collaboration], arXiv:1608.00435 [hep-ex].

[15] The CMS Collaboration, CMS PAS BPH-16-002.

[16] The D0 Collaboration, D0 Note 6488-CONF, (2016).

[17] S. S. Agaev, K. Azizi and H. Sundu, Phys. Rev. D 93, 074024 (2016).

[18] Z. G. Wang, arXiv:1602.08711 [hep-ph].

[19] W. Wang and R. Zhu, arXiv:1602.08806 [hep-ph].

[20] W. Chen, H. X. Chen, X. Liu, T. G. Steele and S. L. Zhu, Phys. Rev. Lett. 117, 022002 (2016).

[21] C. M. Zanetti, M. Nielsen and K. P. Khemchandani, Phys. Rev. D 93, 096011 (2016).

[22] S. S. Agaev, K. Azizi and H. Sundu, Phys. Rev. D 93, 114007 (2016).

[23] S. S. Agaev, K. Azizi and H. Sundu, Phys. Rev. D 93, 074002 (2016).

[24] J. M. Dias, K. P. Khemchandani, A. Mart?nez Torres,
M. Nielsen and C. M. Zanetti, Phys. Lett. B 758, 235 (2016).
[25] Z. G. Wang, Eur. Phys. J. C 76, 279 (2016).
[26] C. J. Xiao and D. Y. Chen, arXiv:1603.00228 [hep-ph].
[27] S. S. Agaev, K. Azizi and H. Sundu, arXiv:1603.02708 [hep-ph].
[28] R. Chen and X. Liu, arXiv:1607.05566 [hep-ph].
[29] S. S. Agaev, K. Azizi and H. Sundu, Phys. Rev. D 93, 114036 (2016).
[30] X. G. He and P. Ko, arXiv:1603.02915 [hep-ph].
[31] Y. Jin and S. Y. Li, arXiv:1603.03250 [hep-ph].
[32] F. Stancu, arXiv:1603.03322 [hep-ph].
[33] T. J. Burns and E. S. Swanson, Phys. Lett. B 760, 627 (2016).
[34] L. Tang and C. F. Qiao, arXiv:1603.04761 [hep-ph].
[35] F. K. Guo, U. G. Meissner and B. S. Zou, arXiv:1603.06316 [hep-ph].
[36] Q. F. Lu and Y. B. Dong, arXiv:1603.06417 [hep-ph].
[37] A. Esposito, A. Pilloni and A. D. Polosa, Phys. Lett. B 758, 292 (2016).
[38] M. Albaladejo, J. Nieves, E. Oset, Z. F. Sun and X. Liu, Phys. Lett. B 757, 515 (2016).
[39] A. Ali, L. Maiani, A. D. Polosa and V. Riquer, arXiv:1604.01731 [hep-ph].
[40] X.-W. Kang and J. A. Oller, arXiv:1606.06665 [hep-ph].
[41] C. B. Lang, D. Mohler and S. Prelovsek, arXiv:1607.03185 [hep-lat].
[42] R. D. Matheus, F. S. Navarra, M. Nielsen and C. M. Zanetti, Phys. Rev. D 80, 056002 (2009).
[43] L. J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rept. 127, 1 (1985).
[44] B. L. Ioffe and A. V. Smilga, Nucl. Phys. B 232, 109 (1984).
[45] S. Narison, Nucl. Part. Phys. Proc. 270-272, 143 (2016).
[46] D. S. Huang and G.-H. Kim, Phys. Rev. D 55, 6944 (1997).
[47] K. A. Olive et al. [Particle Data Group Collaboration], Chin. Phys. C 38, 090001 (2014).