Abstract. We report on the extension of the data fitting considering the QCD inspired model based on the summation of gluon ladders applied to the ep scattering. In lines of a two Pomeron approach, the structure function $F_2$ has a hard piece given by the model and the remaining soft contribution: a soft Pomeron and non-singlet content. In this paper, we carefully estimate the relative role of the hard and the soft pieces from a global fit in a large span of $x$ and $Q^2$.

1. Introduction

The HERA small-$x$ data [1] have introduced a challenge to the phenomenologists in order to describe the strong growth of the inclusive structure function $F_2$ as the Bjorken scale $x$ decreases, supplemented by the scaling violations on the hard scale given by the photon virtuality $Q^2$. Concerning the Regge approach, at high energy the ep scattering process is dominated by the exchange of the Pomeron trajectory in the $t$-channel. From the hadronic phenomenology, this implies that the structure function would present a mild increasing on energy $(s \simeq Q^2/x)$ since the soft Pomeron intercept ranges around $\alpha_{\text{P}}(0) \approx 1.08$. Such behavior is in contrast with high energy ep data, where the effective intercept takes values $\lambda_{\text{eff}} \simeq 1.3 - 1.4$. In the Regge language this situation can be solved by introducing the idea of new poles in the complex angular momentum plane, for instance rendered in the multipoles models [2, 3, 4], producing quite successful data description. Other proposition is the two Pomeron model [5], introducing an additional hard intercept and corresponding residue. However, a shortcoming from these approaches is the poor knowledge about the behavior
On virtuality, in general modeled in an empirical way through the vertex functions.

On the other hand, the high photon virtuality allows applicability of the QCD perturbative methods. The DGLAP formalism [6] is quite successful in describing most of the measurements on structure functions at HERA and hard processes in the hadronic colliders. This feature is even intriguing, since its theoretical limitations at high energy are well known [7]. Other perturbative approach is the BFKL formalism [8], well established at LO level but not yet completely understood at NLO accuracy. The main issue in the NLO BFKL effects is the correct account of the sub-leading corrections in the all orders resummation [9]. The LO BFKL approach can describe HERA structure function in a limited kinematical range, i.e. at not so large $Q^2$ and small-$x$. A consistent treatment considering higher order resummations is currently being available and applications should be allowed in a near future.

In this contribution we report on the extension of the data fitting to the HERA structure function $F_2$ using as a model for the hard Pomeron the finite sum of gluon ladders [10]. The model is based on the truncation of the BFKL series considering only the first few orders in the strong coupling perturbative expansion, where subleading contributions can be absorbed in the adjustable parameters. From the phenomenology on hadronic collisions [11], just three orders, $\sim [\alpha_s \ln(1/x)]^2$, are enough to describe current accelerator data. The hard Pomeron model should be supplemented by a soft piece accounting for the non-perturbative contributions to the process. The description therefore turns out similar to the two Pomeron model [5], with the advantage of a complete knowledge of the behaviors on $x$ and $Q^2$. The original model contains a reduced number of adjustable parameters: the normalization $\mathcal{N}_p$ and the non-perturbative scale $\mu^2$ from the proton impact factor and the parameter $x_0$ scaling the logarithms on energy.

In spirit of a global analysis, in the Ref. [12] two distinct choices for the soft Pomeron were analysed. The resulting two Pomeron model was successful in describing data on structure function $F_2$ and its derivatives (slopes on $Q^2$ and $x$) for $x \leq 2.5 \times 10^{-2}$ and $0.045 \leq Q^2 \leq 1500$ GeV$^2$. Here, we extend the description to the whole $x$ range and analyse the relative role of the hard and soft pieces in the results. This work is organized as follows. In the next section we shortly review the main expressions for the hard piece given by the summation up to the two-rung ladder contribution. In Sec. (3), an overall fit to the recent deep inelastic data is performed based on the hard contribution referred above supplemented by the remaining soft Pomeron and non-singlet contributions. In the last section we draw up our conclusions.
2. The hard contribution: summing gluon ladders

Here we review the elements needed to calculate the structure function using the finite sum of gluon ladders in the ep collision, with center of mass energy $W$. The proton inclusive structure function, written in terms of the cross sections for the scattering of transverse or longitudinal polarized photons, reads as [13]

$$F_2(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}} \left[ \sigma_T(x, Q^2) + \sigma_L(x, Q^2) \right], \quad (1)$$

$$\sigma_{T,L}(x, Q^2) = \frac{G}{(2\pi)^4} \int \frac{d^2k}{k^+} \int \frac{d^2k'}{k'^+} \Phi_{\gamma^* T,L}^{\gamma^* T,L}(k) F(x, k, k') \Phi_p(k'), \quad (2)$$

where $G$ is the color factor for the color singlet exchange and $k$ and $k'$ are the transverse momenta of the exchanged reggeized gluons in the $t$-channel. The $\Phi_{\gamma^* T,L}^{\gamma^* T,L}(k)$ is the virtual photon impact factor and $\Phi_p(k')$ is the proton impact factor. The first one is well known in perturbation theory at leading order, while the latter is modeled since in the proton vertex there is no hard scale to allow pQCD calculations. The kernel $F(x, k, k')$ contains the dynamics of the process, for instance the BFKL kernel.

The amplitudes can be calculated order by order: for instance the Born contribution coming from the two gluon exchange and the one-rung ladder contribution read as,

$$\mathcal{A}^{(0)} = \frac{2\alpha_s W^2}{\pi^2} \sum_f e_f^2 \int \frac{d^2k}{k^+} \Phi_{\gamma^* T,L}^{\gamma^* T,L}(k) \Phi_p(k),$$

$$\mathcal{A}^{(1)} = \frac{6\alpha_s^2 W^2}{8\pi^4} \sum_f e_f^2 \ln \left( \frac{W^2}{W_0^2} \right) \int \frac{d^2k}{k^+} \int \frac{d^2k'}{k'^+} \Phi_{\gamma^* T,L}^{\gamma^* T,L}(k) K(k, k') \Phi_p(k'),$$

where $\alpha_s$ is considered fixed since we are in the framework of the LO BFKL approach. The perturbative kernel $K(k, k')$ can be calculated order by order in the perturbative expansion [13]. The Pomeron is attached to the off-shell incoming photon through the quark loop diagrams, where the Reggeized gluons are attached to the same and to different quarks in the loop. The virtual photon impact factor averaged over the transverse polarizations reads as [14],

$$\Phi_{\gamma^* T,L}^{\gamma^* T,L}(k) = \frac{1}{2} \int_0^1 \frac{d\tau}{2\pi} \int_0^1 \frac{d\rho}{2\pi} \frac{k^2(1-2\tau\tau')(1-2\rho\rho')}{k^2\rho\rho' + Q^2\rho\tau\tau'}, \quad (3)$$

where $\rho$, $\tau$ are the Sudakov variables associated to the momenta in the photon vertex and $\tau' \equiv (1-\tau)$ and $\rho' \equiv (1-\rho)$. 
Gauge invariance requires the proton impact factor vanishing at $k$ going to zero and is modeled in a simple way,

$$\Phi_p(k') = N_p \frac{k'^2}{k'^2 + \mu^2}, \quad (4)$$

where $N_p$ is the unknown normalization of the proton impact factor and $\mu^2$ is a scale which is typical of the non-perturbative dynamics. These scales will be considered adjustable parameters in the analysis. Considering the electroproduction process, summing the first orders in perturbation theory we can write the expression for the inclusive structure function,

$$F_{2,\text{hard}}(x, Q^2) = \frac{8}{3} \alpha_s^2 \sum_f e_f^2 N_p \left[ I^{(0)}(Q^2, \mu^2) + \frac{3}{\pi} \ln \frac{x_0}{x} I^{(1)}(Q^2, \mu^2) \right. \left. + \frac{1}{2} \left( \frac{3}{\pi} \ln \frac{x_0}{x} \right)^2 I^{(2)}(Q^2, \mu^2) \right], \quad (5)$$

where the functions $I^{(n)}(Q^2, \mu^2)$ correspond to the $n$-rung gluon ladder contribution. The quantity $x_0$ gives the scale normalizing the logarithms on energy for the LLA BFKL approach, which is arbitrary and enters as an additional parameter. The contributions are written explicitly as,

$$I^{(0)} = \frac{1}{2} \ln^2 \left( \frac{Q^2}{\mu^2} \right) + \frac{7}{6} \ln \left( \frac{Q^2}{\mu^2} \right) + \frac{77}{18}, \quad (6)$$

$$I^{(1)} = \frac{1}{6} \ln^3 \left( \frac{Q^2}{\mu^2} \right) + \frac{7}{12} \ln^2 \left( \frac{Q^2}{\mu^2} \right) + \frac{77}{18} \ln \left( \frac{Q^2}{\mu^2} \right) + \frac{131}{27} + 2 \zeta(3), \quad (7)$$

$$I^{(2)} = \frac{1}{24} \ln^4 \left( \frac{Q^2}{\mu^2} \right) + \frac{7}{36} \ln^3 \left( \frac{Q^2}{\mu^2} \right) + \frac{77}{36} \ln^2 \left( \frac{Q^2}{\mu^2} \right) \left( \frac{131}{27} + 4 \zeta(3) \right) \ln \left( \frac{Q^2}{\mu^2} \right) + \frac{1396}{81} - \frac{\pi^4}{15} + \frac{14}{3} \zeta(3), \quad (8)$$

where $\zeta(3) \approx 1.202$. The main result in Ref. [12] is in a good agreement, in terms of a $\chi^2$/dof test, for the inclusive structure function in the range $0.045 \leq Q^2 \leq 1500$ GeV$^2$, once considering the restricted kinematical constraint $x \leq 0.025$. The non-perturbative contribution (from the soft dynamics), initially considered as a background, was found to be not negligible. We have estimated that such effects introduce a correction of the same order in the overall normalization. In the next section we perform a global analysis in lines of our previous work [12], extending the range on $x$ fitted by adding the non-singlet contribution modeled through the usual Regge parameterizations.
3. Fitting results and conclusions

In order to perform the fitting procedure, for the hard piece one uses Eq. (5) and for the soft piece we have selected a model with the most economical number of parameters. For this purpose it was selected the latest version [15] of the CKMT model [16]:

\[ F_{2}^{\text{soft}}(x, Q^{2}) = A \left( \frac{x_{1}}{x} \right)^{\Delta(Q^{2})} \left( \frac{Q^{2}}{Q^{2} + a} \right)^{\Delta(Q^{2})+1} (1-x)^{n_{s}(Q^{2})}, \quad (9) \]

\[ \Delta(Q^{2}) = \Delta_{0} \left( 1 + \frac{Q^{2} \Delta_{1}}{\Delta_{2} + Q^{2}} \right), \quad n_{s}(Q^{2}) = \frac{7}{2} \left( 1 + \frac{Q^{2}}{Q^{2} + b} \right), \quad (10) \]

where \( \Delta(Q^{2}) \) is the Pomeron intercept. The non-singlet term is taken in a form,

\[ F_{2}^{\text{ns}}(x, Q^{2}) = A_{R} x^{1-\alpha_{R}} \left( \frac{Q^{2}}{Q^{2} + a_{R}} \right)^{\alpha_{R}} (1-x)^{n_{ns}(Q^{2})}, \quad (11) \]

\[ n_{ns}(Q^{2}) = \frac{3}{2} \left( 1 + \frac{Q^{2}}{Q^{2} + d} \right). \quad (12) \]

Concerning the hard piece, Eq. (5), we selected the overall normalization factor as a free parameter, defined as \( N = \frac{8}{3} a^{2} \pi^{2} \sum e_{f}^{2} N_{p} \), considering four active flavors. It was also supplemented by a factor \((1-x)^{n_{s}}\) accounting for the large \( x \) effects. For the fitting procedure we consider the data set containing all available HERA data for the proton structure function \( F_{2} \) [17], [18] - [25] adding only several data set of fixed target experiments [26]. For the fit we have used 900 experimental points for all available \( x \) and \( 0.045 \leq Q^{2} \leq 30000 \text{ GeV}^{2} \).

In the Figs. (1) (2) and (3) are shown the results considering three different fitting procedures: (I) overall fit using the hard piece and the soft pieces, Eqs. (9) and (11); (II) overall fit restricting the soft Pomeron intercept; (III) fitting only the hard piece plus non-singlet contribution. The best fit parameters for these procedures are presented in Table (1). In the following we discuss each one, pointing out the relative contribution from the hard and soft pieces. The procedure (I) provides the same quality description \( \chi^{2}/\text{dof} = 1.12 \) in the whole available interval of Bjorken variable and virtuality \((0 \leq x \leq 1 \text{ and } 0.045 \leq Q^{2} \leq 30000)\) as in the previous analysis [12]. It is worthy note that the relative contribution of soft Pomeron remains the same in comparison with this analysis, i.e. the hard and soft pieces are comparable in the considered experimental domain. This feature is related to the choice for a CKMT Pomeron, i.e. a \( Q^{2} \) dependent intercept.
Figure 1. The inclusive structure function at small $Q^2$ virtualities. The procedure (I) and (II) produce equivalent curves.

ranging from 1.07 up to 1.17. The fit is comparable with those using a two-Pomeron analysis [3, 5].

In the procedure (II), we have restricted the soft Pomeron intercept
through the following way,

\[ \Delta(Q^2) = \Delta_0 \left( \frac{Q^2}{\Delta_1 + Q^2} \right) + \Delta_2, \]

where a relatively good result was obtained in the restricted interval of\nvirtuality \(1.5 \leq Q^2 \leq 30000 \text{ GeV}^2\). The parameters obtained are shown in\nTable (1). The quality is slightly worst than the procedure (I).

Finally, we succeeded to perform a fit not considering the soft Pomeron\ncontribution, but keeping the non-singlet term. The procedure (III) gives\nreasonable results in the interval \(8.5 \leq Q^2 \leq 30000 \text{ GeV}^2\), as shown in\nTable (1). A shortcoming in these results is quite small value for the strong\ncoupling constant, suggesting it being kept fixed. For instance, one can uses\n\(\alpha_s(M_Z) = 0.119\). This analysis is not presented in the figures.

Concluding the analysis, in Figs. (2) and (3) we verify the relative role\nbetween the hard and soft Pomeron. We present it explicitly for the vir-
tualities 15, 150 and 8000 GeV\(^2\), where the contributions from fitting (I)
and (II) are shown. The general feature found is that in the procedure (I), the hard and soft pieces are almost equivalent at small $x$ and intermediate $Q^2$. The soft contribution strongly decreases as the virtualities are large. From the procedure (II), the soft piece corresponds to a small contribution.
in contrast with the analysis (I). In conclusion, we verify that the fitting procedure is equivalent to the model using a two-Pomeron approach [3, 5], with the advantage of clear understanding of the behaviors on \( x \) and \( Q^2 \) of the corresponding hard content and extending our previous analysis made in Ref. [12].

*Figure 3.* The results for the inclusive structure function at large \( Q^2 \) virtualities.
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