Mass of Galactic Lenses in Modified Newtonian Dynamics

Mu-Chen Chiu,¹,² Chung-Ming Ko,¹,³,⁴ Yong Tian,³ and HongSheng Zhao⁵

¹Institute of Astronomy, National Central University, Jhongli, Taiwan 320, R.O.C.
²Scottish University Physics Alliance, Institute for Astronomy, the Royal Observatory, University of Edinburgh, Blackford Hill, Edinburgh, EH9 3HJ, UK
³Department of Physics, National Central University, Jhongli, Taiwan 320, R.O.C.
⁴Center for Complex Systems, National Central University, Jhongli, Taiwan 320, R.O.C.
⁵Scottish University Physics Alliance, University of St. Andrews, KY16 9SS, UK

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Abstract

Using strong lensing data Milgrom’s MOdified Newtonian Dynamics (MOND) or its covariant TeVeS (Tensor-Vector-Scalar Theory) is being examined here as an alternative to the conventional ΛCDM paradigm. We examine 10 double-image gravitational lensing systems, in which the lens masses have been estimated by stellar population synthesis models. While mild deviations exist, we do not find out that strong cases for outliers to the TeVeS theory.

* corresponding author: cmko@astro.ncu.edu.tw
† corresponding author: hz4@st-andrews.ac.uk
I. INTRODUCTION

The concordant ΛCDM paradigm [see e.g., 1] has been accepted as a successful framework to understand the evolution of the universe. However, in this standard model there are two unexplained dark sectors, dubbed dark matter and dark energy, still need more fundamental understandings. One of various endeavors on this explanation is the approach of modified gravity, including Bekenstein’s TeVeS [2]. Unlike most theories of modified gravity that are proposed for explaining dark energy, TeVeS was originally proposed to explain dark matter instead. Indeed, TeVeS was built up by Bekenstein for a viable relativistic version of Milgrom’s MOdified Newtonian Dynamics (MOND) [3]. Other many recent theories which recovers MOND in it’s limit include the bimetric theory of [4] and the dark fluid theory of [5].

In MOND, the demand of the exotic dark matter is replaced by the modification of Newton’s second law:

\[ \tilde{\mu}(\frac{|a|}{a_0})a = -\nabla \Phi_N, \]  

with \( a_0 \approx 1.2 \times 10^{10} \text{ m s}^{-2} \). \( \tilde{\mu}(x) \) is called interpolation function in literature sometimes. \( \tilde{\mu}(x) \approx x \) for \( x \ll 1 \) (the deep MOND regime), and \( \tilde{\mu}(x) \to 1 \) when \( x \to 1 \) (the Newtonian regime). Here \( x = |a|/a_0 \), the ratio of the acceleration to \( a_0 \), is a measure of modified gravity.

MOND –in its relativistic version TeVeS [2]–is not only able to explain CMB successfully with 2 eV neutrino mass [6, 7] but also even more successful than the ΛCDM paradigm on the dynamics of spiral galaxies [8, 9].

Unlike cold dark matter, however, massive neutrino cannot aggregate at galactic scales and below, so any evidence for the demand of dark matter at these scales is devastating to MOND. Beside dynamical analysis on rotation curve (of spiral galaxies), gravitational lensing offers us another way to investigate MOND and TeVeS at these scales. Indeed, the requirement of dark matter in the bullet cluster makes gravitational lensing even more important [10].

The earliest works about gravitational lensing in MOND were studied by [11] and [12]. Although their works were performed even before the appearance of a viable general covariant MOND, and their calculations on the angle of deflection were artificially forced into the deep MOND regime when the acceleration is less than \( a_0 \), i.e., \( \tilde{\mu}(x) = x \) for \( x \leq 1 \) (not \( x \ll 1 \)), [11] and [12] could show a good guess on how to calculate lensing in MOND. After
the advent of TeVeS, a first calculation on light bending in TeVeS was done by Bekenstein himself, and was followed up by [13]. Based on a point mass model, [13] apply the formalism of gravitational lensing in TeVeS to the theoretical discussion on angle of deflection, magnification, as well as time delay.

On the other hand, taking from a more phenomenal approach, [14] showed that TeVeS is consistent with a sample of double-image lenses in CASTLES catalogue by modeling lenses as Hernquist model. [15] studied the effects of asymmetric systems on gravitational lensing. [16] then applied non-spherical model to investigate strong lensing in TeVeS, and found 10 out of 15 systems are consistent with TeVeS. All of other 5 systems are found to reside in or close to clusters of galaxy, where external field could have significant influence. Furthermore, the effect of large filaments on gravitational lensing was studied by [17]. They argued that filamentary structures might have complex but significant contribution to the system such as bullet cluster, so the need of dark matter in bullet cluster might be spared again in MOND. However, all of these work above are non-relativistic approximation of TeVeS. In a conference paper, [18] also showed the lens data from CASTLES and SLACS catalog are consistent with TeVeS, but they did not show the details.

The first effort to calculate gravitational lensing from first principle was given by [32]. Along with their other works [19, 20], they showed that TeVeS might still need dark matter to explain the lensing systems from CASTLES, and is lack of consistent results between dynamical and gravitational lensing analysis [19, 20].

Due to the importance of studying gravitational lensing in TeVeS, and the inconsistency of the results on this field, in this paper, we are going to apply the relativistic approach developed by 02Bekenstein [2] and 04Chiu et al. [13], and the Hernquist model (used for TeVeS firstly by 34Zhao et al. [14]) to the 10 double-image systems from a total of 18 objects studied in 12Ferreras et al. [21]. In this paper we will also clarify the underlying assumptions of lensing calculation in TeVeS. The structure of the paper is organized as below. In Sec. II we will briefly outline the formalism of gravitational lensing in TeVeS. We will also discuss how gravitational lensing in TeVeS will depend on different choice of $\hat{\mu}(|a|/a_0)$, and its application to double-image systems. We then will give our result in Sec. III and discussion in Sec. IV.
II. FORMALISM OF GRAVITATIONAL LENSING IN TEVES

A. Lensing in TeVeS

While considering strong lensing in TeVeS, we take the assumption that in the weak field limit, the physical metric of a static spherically symmetric system can be expressed in the isotropic form (c=1):

$$\tilde{g}_{\alpha\beta} dx^\alpha dx^\beta = -(1 + 2\Phi) dt^2 + (1 - 2\Phi) [d\varrho^2 + \varrho^2 (d\theta^2 + \sin^2 \theta d\varphi^2)] ,$$

(2)

where $\Phi = \Xi \Phi_N + \phi$ and $\Xi \equiv e^{-2\phi_c} (1 + K/2)^{-1}$ with $\phi_c$ as the asymptotic boundary value of $\phi$ and $K < 10^{-3}$ from the constrain of PPN parameters [2]. Since in TeVeS all kinds of matter are coupled to physical metric rather than Einstein’s metric, the connection in the geodesic equation,

$$\frac{d^2 x^\mu}{dp^2} + \Gamma^\mu_{\nu\lambda} \frac{dx^\nu}{dp} \frac{dx^\lambda}{dp} = 0 ,$$

(3)

has to be constructed from physical metric. Here $p$ is some proper affine parameter. In the case of light bending by a static spherically symmetric lens, we can apply Eq.(2) to the geodesic equation, Eq.(3). Consider a photon propagates on a null geodesic, and moves in the equatorial plane, i.e., $\theta = \pi/2$, the geodesic equation or equations of motion can be written as

$$(1 + 2\Phi) \dot{t} = E ,$$

(4)

$$(1 - 2\Phi) \varrho^2 \dot{\varphi} = L ,$$

(5)

$$(1 - 2\Phi) \dot{\varrho}^2 + (1 + 2\Phi) \varrho^{-2} L^2 - (1 - 2\Phi) E^2 = 0 ,$$

(6)

where over dot denotes the derivative with respect to $p$. Recall the fact that at the closest approach $\dot{\varrho} = 0$, $\varrho = \varrho_0$, we can express the ratio of the angular momentum $L$ to energy $E$ as

$$b^2 \equiv \frac{L^2}{E^2} = \frac{\varrho_0^2 (1 - 2\Phi_0)}{(1 + 2\Phi_0)} ,$$

(7)

where $\Phi_0 \equiv \Phi(\varrho_0)$. Here $b$ is called the impact parameter. Combining Eqs. (5)-(7) gives the orbit of the photon,

$$- (1 - 4\Phi) + (1 - 4\Phi_0) \left( \frac{\varrho_0}{\varrho} \right)^2 \left[ \frac{1}{\varrho^2} \left( \frac{d\varrho}{d\varphi} \right)^2 + 1 \right] = 0 ,$$

(8)
of which the solution in quadrature is

$$\varphi = \int^\varrho \left\{ \left( \frac{\varrho}{\varrho_0} \right)^2 [1 - 4(\Phi - \Phi_0)] - 1 \right\}^{-1/2} \frac{d\varrho}{\varrho}. \quad (9)$$

If we take the Taylor expansion of Eq. (9) to the first order of $\Phi$, we get

$$\varphi \simeq \int^\varrho \frac{2\varrho_0 d\varrho}{\varrho (\varrho^2 - \varrho_0^2)^{1/2}} + \varrho_0 \int^\varrho \frac{4\varrho(\Phi - \Phi_0)}{(\varrho^2 - \varrho_0^2)^{3/2}} d\varrho. \quad (10)$$

The first term is the orbit without gravity, i.e., a straight line ($\varphi \to \pi$ as $\varrho \to \infty$). The second term is the angle of the deviation due to the gravity. Hence, to first order of $\Phi$, the deflection angle is

$$\Delta \varphi = \varrho_0 \int^\varrho \frac{4\varrho(\Phi - \Phi_0)}{(\varrho^2 - \varrho_0^2)^{3/2}} d\varrho = \varrho_0 \int^\varrho \frac{4|\nabla \Phi|}{(\varrho^2 - \varrho_0^2)^{1/2}} d\varrho - \frac{4\varrho_0 \Phi}{(\varrho^2 - \varrho_0^2)^{1/2}}, \quad (11)$$

where we have made use of integration by parts. Since for very large $\varrho$, $\Phi$ behaves as $\ln \varrho$, it is legitimate to ignore the second term of Eq. (11) for strong lensing systems [13]. We then have the kernel equation of strong gravitational lensing in TeVeS (up to first order of $\nabla \Phi$),

$$\Delta \varphi = 4\varrho_0 \int^\infty_{\varrho_0} \frac{|\nabla \Phi|}{(\varrho^2 - \varrho_0^2)^{1/2}} d\varrho. \quad (12)$$

Recall that $\varrho_0$ is the distance of the closest approach, and $\nabla \Phi$ is the MONDian gravity.

**B. Gravity in TeVeS**

In last subsection we have shown that, under the assumption of the validity of Eq. (2), the difference between the angle of deflection in GR and that in TeVeS only arises from $\nabla \Phi$. In this subsection, we are going to discuss the relation between Newtonian gravity, $\nabla \Phi_N$, and MONDian gravity, $\nabla \Phi$, and its application to strong lensing.

It has been shown in quasi-static limit, the MONDian potential in TeVeS can be expressed as the combination of Newtonian potential and a scalar field [2],

$$\Phi = \Xi \Phi_N + \phi, \quad (13)$$

where $\Xi$ is a parameter in TeVeS and is approximately 1, and $\phi$ represents the strength of a scalar field. Moreover, the scalar field itself is linked to the Newtonian potential via a free function $\mu$ [2],

$$\nabla \phi = (k/4\pi \mu) \nabla \Phi_N. \quad (14)$$
where $\mu$ is a function of $|\nabla \phi|$. This free function $\mu$ should be chosen carefully in order to reproduce Newtonian or MOND behavior at quasi-static limits. In fact, Eq. (13), Eq. (14), and Eq. (1) give the relation between $\mu$ and $\tilde{\mu}$ of the modified Poisson equation or Milgarom’s law Eq. (1)

$$
\frac{1}{\tilde{\mu}} = \Xi + \frac{k}{4\pi \mu}.
$$

(15)

TeVeS has only two parameters ($k$ and $\Xi$) and one free function ($\mu$). Therefore, the MONDian behavior controlled by $\tilde{\mu}$ in Milgrom’s law Eq. (1) could be understood via the parameters and free function in TeVeS. On the other hand, we are able to express the modified gravity $\nabla \Phi$ (even in TeVeS) as a function of Newtonian gravity $\nabla \Phi_N$ via $\tilde{\mu}$. In the following, we are going to discuss three commonly used interpolation functions, $\tilde{\mu}$.

In Bekenstein’s TeVeS paper [2] he proposed the following interpolation function

$$
\tilde{\mu}(x) = \frac{-1 + \sqrt{1 + 4x}}{1 + \sqrt{1 + 4x}},
$$

(16)

where $x \equiv |a|/a_0$. The Bekenstein’s form above fails to fit the rotation curves of spiral galaxies [9, 22]. In order to apply the MONDian lens equations to observational data, we also study the simple form and the standard form which fit the rotation curves better. The simple form is [22]

$$
\tilde{\mu}(x) = \frac{x}{1 + x},
$$

(17)

and the standard form is [3]

$$
\tilde{\mu}(x) = \frac{x}{\sqrt{1 + x^2}}.
$$

(18)

Even though the standard form will lead to bi-values problem, so is supposed to be unphysical under the framework of TeVeS [23, 24], here we still compare the standard form with the other two by treating it as empirical function and for the sake of curiosity.

In fact, all these three forms can be included in the following two-parameter form

$$
\tilde{\mu}(x) = \left[1 - \frac{2}{(1 + \eta x^\alpha) + \sqrt{(1 - \eta x^\alpha)^2 + 4x^\alpha}}\right]^{1/\alpha}.
$$

(19)

Here, $(\alpha, \eta) = (1, 0), (1, 1), (2, 1)$ and $(\infty, 1)$ correspond to Bekenstein form, simple form, standard form and the naive sharp-break form $\tilde{\mu} = \min(1, x)$, respectively. Recall that we have set $c = 1$. We note that Zhao & Famaey [24] have proposed a similar expression in which they combined Bekenstein form and simple form. We may call Eq. (19) the invertible canonical interpolation function which goes from the naive sharp-break to the smooth
Bekenstein form. The corresponding $\mu$ can be found by Eq. (15). The nicest thing of our invertible canonical interpolation function is that it has a very simple counterpart in the recent Quasi-MOND theory or its relativistic version called Bi-metric MOND [4, 25]. Inverting Eq. (1) with $\tilde{\mu}$ given by Eq. (19) gives

$$-a = \nabla \Phi = \nu \nabla \Phi_N,$$

(20)

where

$$\nu(x_N) \equiv \left[ 1 - \frac{\eta}{2} + \sqrt{x_N^{\alpha} + \left(\frac{\eta}{2}\right)^2} \right]^{1/\alpha},$$

(21)

and $x_N \equiv |\nabla \Phi_N|/a_0$. This analytical result can greatly simplify the calculations in MOND.

Note that some earlier [11, 12] and recent [19] physics literature formulated Milgrom’s law as

$$\nabla \Phi = \tilde{\mu}^{-1/2}(x_N) \nabla \Phi_N.$$

(22)

However, this formulation is incorrect except for a sharp-break function $\tilde{\mu} = \min(1, a/a_0)$, so $a = \max(a_N, \sqrt{a_0a_N})$. At this particular situation,

$$\tilde{\mu}^{-1}(a/a_0) = a_0/a = \sqrt{a_0a_N} = \tilde{\mu}^{-1/2}(a_N/a_0),$$

(23)

which in general does not hold for other choices of $\tilde{\mu}(x)$.

In Fig. 1 we compare $|\nabla \Phi|$ of different forms. We also plot the results of the standard form and simple form of the formalism adopted in 13Ferreras et al. [19]. For the same form and mass, the MONDian gravity of ours is always stronger than that of 13Ferreras et al. [19].

C. Doubled Image systems

In realistic strong lensing systems, light rays often penetrate the mass distribution of lens such that point mass model is not appropriate. Since MOND is supposed to have little dark matter, we adopt the Hernquist model of elliptical galaxies [26] for our lenses. The Newtonian gravity is given by $|\nabla \Phi_N| = GM/(\rho + r_h)^2$. In the following calculations we assume $r_h = 0.551r_e$, where $r_e$ is the effective radius from surface brightness observation [26].

As shown in last subsection, seeking solution $a = \nabla \Phi$ of Eq. (11) depends on the choice of $\tilde{\mu}$. In general we can express $|\nabla \Phi| = g(\nabla \Phi_N)$, and the deflection angle produced by a
FIG. 1. The strength of the MONDian gravity in different forms of $\tilde{\mu}(x)$.

A spherical lens (Eq. (12)) can be written as

$$\Delta \varphi = \frac{4GM}{\varrho_0} f,$$

where

$$f \equiv \int_{\varrho_0}^{\infty} \frac{g(\nabla \Phi_N)\varrho_0^2}{GM(\varrho^2 - \varrho_0^2)^{1/2}} \, d\varrho.$$  \hspace{1cm} (25)

Moreover, since what we can measure in sky are not deflection angles but positions of the projected images, it is useful to define $\theta = \varrho_0/D_L$ and $\theta_E = \sqrt{4GM D_{LS}/D_L D_S}$, where $D_L, D_S$ and $D_{LS}$ are distances from the observer to lens, observer to source and lens to source, respectively. Here $\theta_E$ is called the Einstein radius. For a spherical strong lens, two images are located on both sides of the lens ($\theta_+$ and $\theta_-$). The corresponding lens equations
where $\beta$ is the source position. These two lens equations can be combined into

$$\theta_E^2 = \frac{\theta_+ \theta_- (\theta_+ + \theta_-)}{(\theta_+ f_+ + \theta_- f_-)}.$$  

Therefore, with the observed positions of the two images, $\theta_+$ and $\theta_-$, we are able to infer the total mass of the lens by computing $\theta_E$.

### III. RESULTS

We apply our formalism to 10 double-image lenses from the CASTLES Catalogue \[27\], of which the stellar masses have been estimated by stellar population synthesis model with different initial mass functions (IMFs) \[21\]. Table 1 lists our result in conventional $\Lambda$CDM cosmology ($\Omega_m = 0.3$, $\Omega_{\Lambda} = 0.7$, $h = 0.7$). The lensing mass $M_L$ estimated from the Bekenstein’s form is the smallest, then the simple form and then the standard form, on top of them is the standard form by 13Ferreras et al. \[19\]. Furthermore, comparison of $M_L$ from GR and TeVeS supports the idea that the mass discrepancy between the Newtonian and the MONDian paradigm can be quite significant.

For a more consistent analysis, lensing in TeVeS should be studied in the MONDian cosmology - $\nu$HDM ($\Omega_b = 0.05$, $\Omega_\nu = 0.17$ $\Omega_{\Lambda} = 0.78$, $h = 0.7$) \[7\]. We list the total mass of the 10 systems for the three forms of $\tilde{\mu}(x)$ in the bracket of columns 3 to 5 (in bracket) of Table 2. It appears that in TeVeS, lensing mass in the $\nu$HDM cosmology is about 5% less massive than in $\Lambda$CDM.

12Ferreras et al. \[21\] estimated the aperture stellar mass by two IMFs: 03Chabrier \[28\] and 23Salpeter \[29\], and the results are listed in columns 6 & 7 of Table 2, respectively. For comparison, we compute the mass enclosed inside the truncated radius given in 12Ferreras et al. \[21\] from the lens total mass. The result is listed in columns 3 to 5.

When comparing with the stellar mass ($M_{\text{Salp}}$) from Salpeter’s IMF, we find that, except for BRI0952 – 0115 (where lensing gives smaller mass), all masses derived from the simple form are within the uncertainty of $M_{\text{Salp}}$. However, when comparing with stellar mass
| Lens               | Bekenstein | Simple | Standard | FSY08\textsuperscript{a} | FSY08 |
|--------------------|------------|--------|----------|--------------------------|-------|
| Q0142 – 100        | 19.39      | 24.32  | 28.34    | 29.28                    | 32.29 |
| HS0818 + 1227      | 29.59      | 37.64  | 44.68    | 46.31                    | 50.80 |
| FBQ0951 + 2635     | 2.20       | 2.72   | 3.04     | 3.82                     | 3.30  |
| BRI0952 – 0115      | 2.59       | 3.35   | 4.05     | 6.62                     | 4.60  |
| Q1017 – 207        | 6.41       | 8.17   | 9.69     | 9.04                     | 10.95 |
| HE1104 – 1805      | 64.53      | 82.99  | 99.76    | 103.17                   | 112.67|
| LBQ1009 – 025      | 11.65      | 14.65  | 17.02    | 19.28                    | 19.28 |
| B1030 + 071        | 17.55      | 21.54  | 24.40    | 27.08                    | 27.08 |
| SBS1520 + 530      | 17.92      | 22.68  | 26.51    | 29.57                    | 29.57 |
| HE2149 – 274       | 14.33      | 18.14  | 21.36    | 24.14                    | 24.14 |

\textsuperscript{a} Ferreras et al.(2008)

\((M_{\text{Chab}})\) from Chabrier IMF, Bekenstein’s form is a better choice: 8 objects agree with \(M_{\text{Chab}}\).

Figs. 2 & 3 show the mass ratio of TeVeS to GR for the 10 lensing systems. For comparison of mass differences between the three forms, all ratios are plotted against \(|\nabla \Phi|/a_0\) of the simple form. We found the ratio increases slightly with \(|\nabla \Phi|/a_0\). This does make sense because a smaller \(|\nabla \Phi|/a_0\) means the system is closer to the MOND regime, and a larger mass discrepancy is expected.

IV. DISCUSSION

In our analysis, the simple form of \(\tilde{\mu}(x)\) (i.e., \(\alpha = 1, \eta = 1\) in the canonical form (19)) yields the most reasonable lensing mass in TeVeS: in 9 of 10 systems the lens mass \(M_L\) is within the uncertainty of \(M_{\text{Salp}}\), the mass estimated from population synthesis model with Salpeter’s IMF. When compare with result estimated from Chabrier’s IMF, 4 systems are
TABLE II. Aperture mass (and total mass) of lenses ($10^{10} M_\odot$) in \(\nu\)HDM

| Lens         | \(\nabla \Phi_s / a_0\) | Bekenstein \(M_\odot\) | Simple \(M_\odot\) | Standard \(M_\odot\) | Chabrier \(M_\odot\) | Salpeter \(M_\odot\) |
|--------------|--------------------------|--------------------------|---------------------|---------------------|---------------------|---------------------|
| Q0142 – 100  | 5.85                     | 10.79 (18.36)            | 13.66 (23.20)       | 16.05 (27.31)       | 20.9^{30.8}_{13.0} | 18.3^{32.2}_{13.2} |
| HS0818 + 1227| 5.67                     | 18.14 (28.30)            | 23.39 (36.17)       | 27.79 (43.35)       | 16.2^{21.2}_{12.6} | 20.8^{28.1}_{13.4} |
| FBQ0951 + 2635| 10.23                  | 1.54 (2.16)              | 1.91 (2.67)         | 2.15 (3.01)         | 1.1^{2.1}_{0.5}    | 1.5^{3.0}_{0.8}    |
| BRI0952 – 0115| 5.28                    | 2.01 (2.48)              | 2.59 (3.21)         | 3.19 (3.93)         | 3.5^{4.0}_{2.7}    | 4.4^{5.2}_{3.5}    |
| Q1017 – 207  | 5.16                     | 2.45 (5.89)              | 3.22 (7.55)         | 3.81 (9.15)         | 4.3^{13.0}_{1.4}   | 6.4^{19.0}_{2.3}   |
| HE1104 – 1805| 4.98                     | 45.17 (59.58)            | 58.44 (77.08)       | 71.78 (94.68)       | 22.8^{51.2}_{12.7} | 36.6^{63.7}_{23.1} |
| LBQ1009 – 025| 5.40                     | 7.71 (10.79)             | 9.76 (13.67)        | 11.53 (16.15)       | 5.5^{7.9}_{12}     | 7.4^{9.8}_{5.0}    |
| B1030 + 071  | 8.06                     | 9.76 (16.61)             | 12.06 (20.51)       | 13.80 (23.47)       | 10.6^{15.3}_{6.5}  | 14.5^{21.3}_{8.3}  |
| SBS1520 + 530| 6.03                     | 11.91 (16.67)            | 15.20 (21.28)       | 18.08 (25.31)       | 18.5^{30.9}_{11.2} | 21.8^{34.1}_{11.9} |
| HE2149 – 274 | 5.86                     | 7.04 (13.58)             | 8.96 (17.28)        | 10.67 (20.58)       | 4.6^{6.7}_{3.0}    | 6.9^{9.5}_{5.0}    |

\(^a\) Ferreras et al. (2005)

outside the uncertainty of \(M_{Chab}\), but the mass difference is within \(17^{+8.22}_{-6.64}\)%. Moreover, the simple form yields an average of 28% mass discrepancy between the inferred lensing mass of TeVeS and GR. This is close to the value 30% estimated by 24 Sanders [30]. For comparison, Bekenstein’s form gives an average of 44%, and provides a better fit to lower-mass IMFs. This consists with the conclusion of 20 Napolitano et al. [31].

We have to keep in mind that the uncertainty in estimating the mass of elliptical galaxies from IMF is still quite large. It is far from mature to claim that the MONDian paradigm favors which choice of \(\tilde{\mu}(x)\) in elliptical galaxies. However, our study shows that like spiral galaxies, elliptical galaxies can be used to distinguish different forms of \(\tilde{\mu}(x)\) as well. We believe that along side with an independently precise measurement of mass of elliptical galaxies is available, strong gravitational lensing can offer us a window to study the form of \(\tilde{\mu}(x)\). We explicitly write down a formalism for this application in the future.

We conclude that these systems do not show any apparent need of ad hoc dark matter in elliptical galaxies. On the other hand, with a simple spherically symmetric lens model, TeVeS
FIG. 2. Mass differences between $M_{\text{GR}}$ and $M_{\text{TeVeS}}$ in three different forms of $\tilde{\mu}(x)$. Estimated stellar mass using Salpeter’s IMFs [21] is shown for comparison.

is able to fit these system reasonably well. This differs from the conclusion of 13Ferreras et al. [19] but sides with 34Zhao et al. [14] and 26Shan et al. [16], where a constant mass-to-light ratio is assumed.

In 13Ferreras et al. [19], they concluded that MOND might have problems in explaining galactic lensing system. Ours analysis differs from their conclusion. The reason could be that they have applied the formalism of Eq.(22) in their calculation. Indeed, this might also explain why they have found a considerable discrepancy between MOND and TeVeS [32]. In 16Mavromatos et al. [32] and 14Ferreras et al. [20], the full relativistic equations with vector were solved for the first time. They discussed the result of two forms of $\mu$ proposed by 01Angus et al. [23]. Their choices of $\mu$ are identical to the simple form and Bekenstein’s
FIG. 3. Same as Fig. 2 except that the estimated stellar mass used Chabrier’s IMFs [21].

form in this paper, but a gap still exists between our result and 16Mavromatos et al. [32]. The disparity might be due to the fact that we choose Hernquist model rather than NFW model for the lens.

Our analysis also shows that the simple form seems to give the most reasonable angle of deflection in strong lensing, which coincides with the studies on the dynamics of spiral galaxies [9, 22, 24]. It is quite reasonable because the deviation of TeVeS from GR in strong lensing is only due to the change of gravity (Possion equation). So we do expect a consistent conclusion between gravitational lensing and dynamical analysis.

We should point out that our analysis is based heavily on the assumption that the lenses in these systems are spherical and quasi-static, so that we can use Eq. [2] to derive the lensing equation. However, the assumption of spherical distribution is obviously insufficient for most known lenses. Moreover, the choice of mass model for lens may also affect the inference of the total mass of the lens. Thus, it is worthwhile to study how the deviation from this assumption would affect our conclusion here. We note that while a spherical
Hernquist model is a reasonable assumption for some lenses, it is always a poor assumption for modeling rotation curves of spiral galaxies which have often exponential profiles in their disks, and the gravity is enhanced in the disk plane. In this regard the recent claim of inconsistency of lensing and galaxy rotation curve in MOND has yet to be corrected for these systematic effects. Systems in cluster environment which are not yet in full dynamical equilibrium it might be important to include non-trivial effects of the vector field.

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