$b \rightarrow s\ell^+\ell^-$ in the high $q^2$ region at two-loops

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We report on the first analytic NNLL calculation for the matrix elements of the operators $O_1$ and $O_2$ for the inclusive process $b \rightarrow X\ell^+\ell^-$ in the kinematical region $q^2 > 4m^2_c$, where $q^2$ is the invariant mass squared of the lepton-pair.
1. Introduction

In the Standard Model, the flavor-changing neutral current process $b \rightarrow X_s l^+ l^-$ only occurs at the one-loop level and is therefore sensitive to new physics. In the kinematical region where the lepton invariant mass squared $q^2$ is far away from the $c\bar{c}$-resonances, the dilepton invariant mass spectrum and the forward-backward asymmetry can be precisely predicted using large $m_b$ expansion, where the leading term is given by the partonic matrix element of the effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu) O_i(\mu).$$

(1.1)

We neglect the CKM combination $V_{us}^* V_{ub}$ and the operator basis is defined as in [1]. In [2] we published the first analytic NNLL calculation of the high $q^2$ region of the matrix elements of the operators

$$O_1 = (\bar{s}_L \gamma^\mu c_L)(\bar{c}_L \gamma^\mu b_L), \quad O_2 = (\bar{s}_L \gamma^\mu c_L)(\bar{c}_L \gamma^\mu b_L),$$

(1.2)

which dominate the NNLL amplitude numerically. Earlier these results were only available analytically in the region of low $q^2$ [3, 4]. Using equations of motion the NNLL matrix elements of the effective operators take the form

$$\langle s^+\ell^- | O_i | b \rangle_{2\text{-loops}} = -\left(\frac{\alpha_s}{4\pi}\right)^2 \left[ F_1^{(7)} \langle O_7 \rangle_{\text{tree}} + F_1^{(9)} \langle O_9 \rangle_{\text{tree}} \right],$$

(1.3)

where $O_7 = e g_5^2 m_b (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}$ and $O_9 = e^2 g_5^2 (\bar{s}_L \gamma^\mu b_L) \sum l (\bar{l} \gamma^\mu l)$.

2. Calculations

Figure 1: Diagrams that have to be taken into account at order $\alpha_s$. The circle-crosses denote the possible locations where the virtual photon is emitted (see text).

The diagrams contributing at order $\alpha_s$ are shown in Figure [1]. We set $m_s = 0$ and define

$$\hat{s} = \frac{q^2}{m_b^2} \quad \text{and} \quad z = \frac{m_c^2}{m_b^2},$$

(2.1)
where $q$ is the momentum of the virtual photon. After reducing occurring tensor-like Feynman integrals \([5]\) the remaining scalar integrals can be further reduced to master integrals using integration by parts (IBP) identities \([3]\). Considering the region $\delta > 4z$, we expanded the master integrals in $z$ and kept the full analytic dependence in $\delta$.

For power expanding Feynman integrals we use a combination of method of regions \([6, 7]\) and differential equation techniques \([8, 9]\): Consider a set of Feynman integrals $I_1, \ldots, I_n$ depending on the expansion parameter $z$ and related by a system of differential equations obtained by differentiating $I_\alpha$ with respect to $z$ and applying IBP identities:

$$\frac{d}{dz} I_\alpha = \sum_\beta h_{\alpha \beta} I_\beta + g_\alpha,$$

(2.2)

where $g_\alpha$ contains simpler integrals which pose no serious problems. Expanding both sides of (2.2) in $\varepsilon$, $z$ and $\ln z$

$$I_\alpha = \sum_{i,j,k} I_{\alpha, i}^{(j,k)} e^{i k} (\ln z)^k,$$

(2.3)

$$h_{\alpha \beta} = \sum_{i,j} h_{\alpha \beta, i} e^{i j},$$

(2.3)

$$g_\alpha = \sum_{i,j,k} g_{\alpha, i}^{(j,k)} e^{i k} (\ln z)^k,$$

(2.3)

and inserting (2.3) into (2.2) we obtain algebraic equations for the coefficients $I_{\alpha, i}^{(j,k)}$

$$0 = (j + 1) I_{\alpha, i}^{(j+1,k)} + (k + 1) I_{\alpha, i}^{(j+1,k+1)} - \sum_\beta \sum_\gamma h_{\alpha \beta, \gamma} I_{\beta, i}^{(j-\gamma,k)} - \delta_{\alpha, i}^{(j,k)}.$$

(2.4)

This enables us to recursively calculate higher powers of $z$ once the leading powers are known. In practice this means that we need the $I_{\alpha, i}^{(1,0)}$ and sometimes also the $I_{\alpha, i}^{(1,0)}$ as initial condition to (2.4). These initial conditions can be computed using method of regions. A non trivial check is provided by the fact that the leading terms containing logarithms of $z$ can be calculated by both method of regions and the recurrence relation (2.4).

The summation index $j$ in (2.3) can take integer or half-integer values, depending on the specific set of integrals $I_\alpha$. In order to determine the possible powers of $z$ and $\ln(z)$ we used the algorithm described in \([9]\). A given $D$-dimensional $L$-loop Feynman integral $I(z)$ reads in Feynman parameterization

$$I(z) = (-1)^N \left(\frac{i}{4\pi} \right)^D \Gamma(N - LD/2) \int d^N x \, \delta(1 - \sum_{n=1}^N x_n) \, U^{N-(L+1)D/2} \left(\frac{z F_1 + F_2}{z F_1 + F_2} \right)^{N - LD/2},$$

(2.5)

where $U$, $F_1$ and $F_2$ are polynomials in $x_n$. Using Mellin-Barnes representation (2.5) can be cast into the following form

$$I(z) = (-1)^N \left(\frac{i}{4\pi} \right)^D \Gamma(s + N - LD/2) \int d^N x \, \delta(1 - \sum_{n=1}^N x_n) \, U^{N-(L+1)D/2} F_1^{s-N+LD/2} \left(\frac{z F_1 + F_2}{z F_1 + F_2} \right)^{N - LD/2}.$$

(2.6)

By closing the integration contour over $s$ to the right hand side the poles on the positive real axis turn into powers of $z$. If we apply the technique of sector decomposition \([10]\) to (2.6) we end up with terms of the following form

$$\sum_{l=1}^N \sum_k \int_0^d d^{N-1} t \left( \prod_{j=1}^{N-1} A_j - B_j e^{-C_j t} \right) U^{N-(L+1)D/2} F_1^{s-N+LD/2} F_2^{s-N+LD/2},$$

(2.7)
where $U_{lk}$, $F_{1, lk}$ and $F_{2, lk}$ contain terms that are constant in $\bar{t}$. From (2.7) we can read off that the poles in $s$ are located at:

$$s_{jn} = \frac{1 + n + A_j - B_j \varepsilon}{C_j},$$

(2.8)

where $n \in \mathbb{N}_0$.

Additionally, the procedure described above allows us to evaluate the coefficients of the expansion in $z$ numerically which we used to again test the initial conditions of the differential equations.

### 3. Results

In order to get accurate results we keep terms up to $z^{10}$. Our results agree with the previous numerical calculation [1] within less than 1% difference. To demonstrate the convergence of the power expansions, we show in Figure 3 the form factors defined in (1.3) as functions of $\hat{s}$, where we include all orders up to $z^6$, $z^8$ and $z^{10}$. We use as default value $z = 0.1$ such that the $c\bar{c}$-threshold is located at $\hat{s} = 0.4$. One sees from the figures that far away from the $c\bar{c}$-threshold, i.e. for $\hat{s} > 0.6$, the expansions for all form factors are well behaved.

The impact of our results on the perturbative part of the high $q^2$-spectrum [3]

$$R(\hat{s}) = \frac{1}{\Gamma(\bar{B} \to X_c e^- \bar{\nu}_e)} \frac{d\Gamma(\bar{B} \to X_c \ell^+ \ell^-)}{d\hat{s}},$$

(3.1)

is shown in Figure 3 (left), where we used the same parameters as in [2]. The finite bremsstrahlung corrections calculated in [4] are neglected. From Figure 3 (left) we conclude that for $\mu = m_b$ the contributions of our results lead to corrections of the order 10% - 15%. Integrating $R(\hat{s})$ over the high $\hat{s}$ region, we define

$$R_{\text{high}} = \int_{0.6}^{1} d\hat{s} R(\hat{s}).$$

(3.2)

Figure 3 (right) shows the dependence of the perturbative part of $R_{\text{high}}$ on the renormalization scale. We obtain

$$R_{\text{high, pert}} = (0.43 \pm 0.01(\mu)) \times 10^{-5},$$

(3.3)

where we determined the error by varying $\mu$ between 2 GeV and 10 GeV. The corrections due to our results lead to a decrease of the scale dependence to 2%.

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Volker Pilipp
Figure 2: Real and imaginary parts of the form factors $F_{1,2}^{(7,9)}$ as functions of $\hat{s}$. To demonstrate the convergence of the expansion in $z$ we included all orders up to $z^6$, $z^8$ and $z^{10}$ in the dotted, dashed and solid lines respectively. We put $\mu = m_b$ and used the default value $z = 0.1$. 

Volker Plipp
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Volker Pilipp

Figure 3: Perturbative part of $R(\hat{s})$ (left) and $R_{\text{high}}$ (right) at NNLL. The solid lines represents the NNLL result, whereas in the dotted lines the order $\alpha_s$ corrections to the matrix elements associated with $O_{1,2}$ are switched off. In the left figure we use $\mu = m_b$. See text for details.

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