Control of dynamical localization in atom-optics kicked rotor

S. Sagar Maurya, J. Bharathi Kannan, Kushal Patel, Pranab Dutta, Korak Biswas, Jay Mangaonkar, M. S. Santhanam, and Umakant D. Rapol
Department of Physics, Indian Institute of Science Education and Research, Pune 411008, Maharashtra, India

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Atom-optics kicked rotor represents an experimentally realizable version of the paradigmatic quantum kicked rotor system. After a short initial diffusive phase the cloud settles down to a stationary state due to the onset of dynamical localization. In this work we realise an enhancement of localization by modification of the kick sequence. We experimentally implement the modification to this system in which the sign of the kick sequence is flipped by allowing for a free evolution of the wavepackets for half the Talbot time after every $M$ kicks. Depending on the value of $M$, this modified system displays a combination of enhanced diffusion followed by asymptotic localization. This is explained as resulting from two competing processes – localization induced by standard kicked rotor type kicks, and diffusion induced by half Talbot time evolution. The evolving states display a localized but non-exponential wave function profiles. This provides another route to quantum control in kicked rotor class of systems. The numerical simulations agree well with the experimental results.

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The kicked rotor (KR) has been extensively investigated as a paradigmatic model of both classical and quantum chaos [1]. The atom-optic kicked rotor (AOKR) is an experimentally realisable analog of the KR model in which an ensemble of cold atoms are periodically kicked by sinusoidal potentials formed by a standing wave of light [2][6]. The classical limit of AOKR is chaotic for sufficiently strong kick strengths and exhibits intrinsic stochasticity and diffusive growth of mean energy. In contrast, the quantum regime entirely suppresses the classical diffusive growth beyond a short break-time due to destructive quantum interferences analogous to Anderson localization in real space. This shows up as localization of wavefunctions in momentum space, $\psi_p \sim e^{-p/\xi}$, where $p$ labels discrete momentum states and $\xi$ is the localization length. Further, AOKR and its variants are studied in other fields – condensed matter physics [6][7], molecular physics [8][11], quantum information [12][13], exploration of quantum correlations and many-body effects.

In the context of AOKR, the ability to control localization length and the saturated energy of the localized states can be useful [14] and has direct implications in the emerging area of quantum technologies. In the context of dynamical localization it has been shown that the addition of stationary noise to system parameters or spontaneous emission [3][15][21], coupling with rotor [22], and quantum measurements [12] can destroy the delicate nature of dynamical localisation. However, surprisingly, it was shown experimentally that Lévy noise added to kick sequences of AOKR could control the decoherence rate and even the mean energy of localization, the Lévy parameter in the noise distribution acting as the control parameter [23]. More conventional routes to exercise control is by manipulating the phases of the initial wavefunctions, as shown in refs. [24][25] for suppressing or enhancing quantum-chaotic behaviour. Control of localization requires the unitary evolution operator, that evolves the initial state to change, which is not easily achievable in experiments by just controlling the phases. In one such control scheme, introduced by Gong and Brumer [26], the phase of the kicking field is flipped periodically. This variant of AOKR is different from the amplitude-modulated kicked rotors in which the kick strength is a stochastic variable drawn from a suitable distribution [18][20].

In this work, we experimentally realize a protocol of quantum control (similar in spirit to the one introduced in Ref. [26]) of diffusive and localized phases by appropriate modulation of the perturbations. Effectively, the sign of the kick strength in the kicked rotor system is periodically flipped. This is achieved using periodic, time delayed kicks after a certain number of standard kicks that induce dynamical localization. Quite remarkably, this simple modification of kick sequences in AOKR does not destroy localization but leads to an enhancement of quantum energy at which localization takes place. In contrast to the earlier work [26] that depends on presence of classical transporting islands in phase space for energy enhancements, in this work we show that enhancements arise from a competition between two periodic kick sequences – one that supports localization and the other that supports diffusion – in the AOKR system.

The system of interest is a modification of kicked rotor given by [26],

$$H = \frac{p^2}{2} + K \cos(x) \sum_n f_M(n) \delta(t-n),$$

(1)

where $p$ and $x$ denote the dimensionless momentum and position respectively, $K$ is the chaos parameter, and time $t$ is scaled by the pulse period $T$ such that $t \rightarrow t/T$. If $f_M(n) = 1$, then Eq. (1) is just the standard KR. In this
work, \(|f_M(n)| = 1\), and \(f_M(n)\) changes sign after every \(M\) kicks. In rest of this paper, Eq. [3] will be referred to as the Modified Kicked Rotor (MKR) model, and it can be thought of as a special realisation of a generalised KR model in Ref. [27]. The classical stroboscopic map for the phase space variables for MKR [26] is shown in Fig. 1.

As evident from Fig. 1(a), the phase space is largely chaotic with a few regular islands. An ensemble of initial conditions launched from these islands will remain bound to these islands. In the case of MKR with \(M = 2\), special non-chaotic structures called transporting trajectories exist in phase space, as seen in the inset of Fig. 1(b). They are present in many periodically driven dynamical systems, e.g. standard map [28], Hamiltonian ratchets [29], atomic KR [30] [31] and are usually referred to as the accelerator modes because they support anomalous classical diffusion, i.e., \(\langle E \rangle_n \propto n^2\). The quantum dynamics of MKR is obtained through the use of split operator technique [28].

To build the quantum dynamics of MKR, let us consider the period-1 Floquet operator corresponding to the standard kicked rotor \(\hat{F}_{KR}^+\) for which \(f_M(n)\) is a constant, i.e., \(f_M(n)\) is either +1 or −1 for all \(n\). Thus, the required operator is

\[
\hat{F}_{KR}^\pm(T) = \exp\left[\mp i \frac{\hbar \text{eff} T}{2} \frac{\partial^2}{\partial x^2}\right] \exp\left[\mp i \frac{K}{\hbar \text{eff}} \cos(x)\right],
\]

where \(\hbar \text{eff} = T \hbar\) is the scaled Planck’s constant, \(k = K/\hbar \text{eff}\) denotes the strength of phase modulation imparted by the kicks. Using this as the building block, the Floquet operator for MKR can be constructed as \(M\) applications of \(\hat{F}_{KR}^+\) followed by \(M\) application of \(\hat{F}_{KR}^−\). Thus, for MKR, we obtain

\[
\hat{F}_{MKR}(T) = \left[\hat{F}_{KR}^−(T)\right]^M \left[\hat{F}_{KR}^+(T)\right]^M.
\]

![FIG. 1. Poincaré sections of the standard \((M = 0)\) and modified kicked rotor for kick strength \(K = 5\) and (a) \(M = 0\), (b) \(M = 2\) and (c) \(M = 3\). The regular islands in the chaotic sea in (a) are non-transporting (b) transporting in nature, albeit much smaller in size. Inset in (b) shows an enlarged view of one of these islands. No regular structures are visible in (c).](image)

KR and MKR with \(M = 2\). The inset of Fig. 2(b) shows the momentum distribution. Since abrupt phase changes are technically challenging, an alternative method to realize MKR is by introducing controlled time delays after every \(M\) kicks [26]. Consider a wave-function of the system \(\Psi(x,t)\) at any time \(t\). This can be expanded in the momentum basis as \(\Psi(x,t) = \sum_m A_m(x|m)\) with \(A_m\) being the expansion coefficients. The flipping of sign of \(K\) can also be thought of as shift in spatial coordinate by \(\pi\), since \(K \cos(x + \pi) = -K \cos x\). Hence, it is convenient to use \(x \rightarrow x + \pi\) instead of \(K \rightarrow -K\). Through simple manipulations, it can be shown that the change of sign of kicking strength effectively introduces a phase difference of \(\pi\) between the neighbouring states. This phase difference can also be generated by introducing time delays in the system. To see this, consider the action of a free-evolution operator of MKR for \(t = T\) acting on a momentum state \(\exp\left(i p^2 T / (2 \hbar)\right) |m\rangle = \exp\left(i m^2 h T / 2\right) |m\rangle\). From this, we can estimate the duration of free evolution \(T_d\) (called delay time) required to obtain a phase-difference of \(\pi\) between neighbouring momentum states. For a phase difference of \(\pi\), the condition to be satisfied is \(\exp\left\{ i \hbar d (m+1)^2 - m^2 \hbar^2 / 2\right\} = \exp\{i \pi\}\), and from this we get the time duration to be \(T_d = 2 \pi / \hbar = 2 \pi T / \hbar \text{eff}\). This delay time \(T_d\) corresponds to half Talbot time and has the same effect as flipping the sign of the kick strength between the pulses [26]. Figure 3(a) shows the kicking scheme for KR (blue), and MKR with \(M = 2\) (red), and MAKR with \(M = 2\). The kick period \(T\) and delay time \(T_d\) are also indicated in this figure. Due to the presence of two periods, the system is no longer periodic with time period \(T\), but has an effective period of \(T(M - 1) + T_d\). Thus, the Floquet operator \(\hat{F}_{MAKE}\) for the Modified Atom Optic Kicked Rotor (MAKR) is given by:

\[
\hat{F}_{MAKE} = \hat{F}_{KR}^−(T_d) \left[\hat{F}_{KR}^+(T)\right]^{M-1} = \hat{F}_{KR} \hat{F}_{KR}^M. \tag{4}
\]

In this, \(\hat{F}_{KR}^M\) denotes \(M - 1\) applications of \(\hat{F}_{KR}\) for time duration \(T\), and \(\hat{F}_{KR}\) denotes the application of kicked rotor Floquet operator with a free propagation time \(T_d\).

The variables and parameters of kicked rotor system and AOKR are related as: \(x \rightarrow 2k_L x, p \rightarrow 2k_L T p / m\), and \(\hbar \text{eff} = 8 \omega T\) where \(k_L, m\) and \(\omega\) are the wavenumber of the lattice laser, mass of the atoms and recoil frequency respectively. The kick strength is \(k = h \Omega^2 \tau / 8 \Delta\), where \(\Omega, \Delta, \tau\) are the resonant Rabi frequency, the detuning of the light used to create the optical lattice potential and the pulse duration respectively. The experimental setup and the implementation sequence is the same as in Ref. 23. To synthesise the MKR Hamiltonian, the off-time between the pulses is adjusted. The on-time \(\tau\) of the standing wave is kept as 100 ns.

To keep \(K\) fixed, we adjust \(\hbar \text{eff}\) for different \(\hbar \text{eff}\). To calibrate the kick strength \(k\), we use Raman-Nath diffraction on an almost ideal zero-momentum state i.e., a Bose-Einstein Condensate. By fitting the number of atoms in
FIG. 2. (a) Pulse sequence for the standard and the modified kicked rotor with \( k = 5 \) and \( \hbar_{\text{eff}} = 1 \) (KR). For \( M = 2 \), the sign of \( K \) is flipped at every kick. (b) Simulated energy evolution of KR (blue solid line) and MKR with \( M = 2 \) (red solid line). Inset shows the momentum distribution for the same at \( t = 1000 \). Open squares are for KR and open circles are for MKR with \( M = 2 \).

FIG. 3. a) The kick sequence for different values of \( M \). b) Mean energy vs. \( M \) at \( k = 5 \) and \( \hbar_{\text{eff}} = 1 \).

In the other limit, as \( M \to 1 \), the mean energy corresponding to MKR is enhanced with respect to the standard KR. Further, the width of momentum distribution (inset in Fig. 2(b,c)) is also significantly enhanced for MKR.

Enhancement is largely due to the presence of two time scales \( T \) and \( T_d \) in the MAKR system. The time evolution with period \( T \) and kick strength \( k \) is arranged such that it induces localization in the system. For an initial state with significant momentum spread \( (w \gtrsim 1) \) a pulse period corresponding to half Talbot time \( T_d \) leads to a linear diffusive growth in energy \( (E \propto t) \). The dynamics of MAKR is governed by the competition between these contrasting behaviours of localization and diffusion. For a finite number of total kicks \( N \) applied to the atomic cloud, the number of (diffusion inducing) free evolution phase with time period \( T_d \) is \( N_d = N/M - 1 \). Number \( N_l \) of localization-inducing evolution phases with time period \( T \) is \( N_l = (M - 1)N_d \). Thus, if \( N_d \gg 1 \), then \( N_l \gg N_d \). In this scenario, localization effects dominate and diffusion is suppressed (The standard KR limit denoted by the red curve in Fig. 3(b)). In the other limit, as \( M \to 1 \), diffusive growth of energy is strongly favoured over localization. The competition between these processes determines the enhancement of
maximum energy enhancement and momentum distribution will have $I_D \sim 1$, while for a completely delocalized wavefunction $I_D \propto 1/D$. As evident in Fig. 5 the IPR for $M = 2, 3, 4$ have a much lower value compared to the KR, implying the spread in the momentum distribution and energy enhancement compared to KR. But $I_D$ for $M = 2, 3, 4$ are very close to each other, indicating that the spread is almost the same for all of them. As transporting islands of significant size are only present in the case of MKR with $M = 2$ ($k = 5$), one would expect to have a maximum energy enhancement and momentum distribution spread for $M = 2$ \cite{26}. But in our case, we observe the momentum distribution to be very close to each other for all the $M = 2, 3, 4$. This implies that the enhancement in the localization length or momentum distribution spread arising from transporting islands for $M = 2$ is less significant in MAKR. Indeed, in this experiment, the bound in the fluctuations in the parameters are sufficiently large enough that in the corresponding classical phase space transporting islands do not exist for $M = 2$. The dynamics is largely determined through the interplay of two time periods in the system. It has been shown that very small changes to the system parameters ($k$ and $\hbar_{eff}$) can lead to the destruction of transporting islands present in the classical phase space and non-exponential shape of quantum momentum distribution \cite{26}. This change in the line shape for dynamical localization even occurs without causing an obvious difference in energy absorption behaviour. A similar behaviour is observed for various values of $M$ and for a range of parameters $k$ and $h_{eff}$ in the MKR model. The only requirement being that $k$ should be sufficiently large for dynamical localization to take place in the system. As in the standard KR, the experimental data in Fig. 5 shows that for a fixed $M$, a decrease in $h_{eff}$ is associated with broadening of the wavefunction profile. Hence, $I_D$ increases as $h_{eff} \to 0$.

In this work, we have investigated a modified kicked rotor model both experimentally and theoretically. The modified atomic kicked rotor is realized by introducing half Talbot time delays in the kicking sequence, which is equivalent to flipping the sign of the kick strength. It is shown that the modified atom-optics kicked rotor system with $M = 2, 3, 4$ shows enhanced quantum mean energies compared to the standard kicked rotor model. These results are not only of intrinsic interest in the quantum chaos of kicked rotor, but also in the broader context of quantum control of coherent phenomenon.

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[1] G. Casati and B. Chirikov, Cambridge University Press, New York, (1995).
[2] P. L. Moore, J. C. Robinson, C. F. Bharucha, B. Sundaram, and M. G. Raizen, Phys. Rev. Lett. 75, 4598 (1995).
[3] H. Ammann, R. Gray, I. Shvarchuck, and N. Christensen, Phys. Rev. Lett. 80, 4111 (1998).
[4] J. Ringot, P. Szriftgiser, J. C. Garreau, and D. Delande, Phys. Rev. Lett. 85, 2741 (2000).
[5] M. B. d’Arcy, R. M. Godun, M. K. Oberthaler, D. Castettnari, and G. S. Summy, Phys. Rev. Lett. 87, 074102 (2001).
[6] S. Fishman, D. R. Grempel, and R. E. Prange, Phys. Rev. Lett. 49, 509 (1982).
[7] G. Benenti, G. Casati, I. Guarneri, and M. Terraneo, Phys. Rev. Lett. 87, 014101 (2001).
[8] R. Blümel, S. Fishman, and U. Smilansky, The Journal of Chemical Physics 84, 2604 (1986).
[9] I. S. Averbukh and R. Arvieu, Phys. Rev. Lett. 87, 163601 (2001).
[10] (Academic Press, 2000) pp. 287–345.
[11] A. Tóth and A. Csehi, Phys. Rev. A 104, 063102 (2021).
[12] P. Facchi, S. Pascazio, and A. Scardicchio, Phys. Rev. Lett. 83, 61 (1999).
[13] B. Georgeot and D. L. Shepelyansky, Phys. Rev. Lett. 86, 2890 (2001).
[14] M. Bitter and V. Milner, Phys. Rev. Lett. 118, 034101 (2017).
[15] E. Ott, T. M. Antonsen, and J. D. Hanson, Phys. Rev. Lett. 53, 2187 (1984).
[16] D. Cohen, Phys. Rev. A 44, 2292 (1991).
[17] R. Graham and S. Miyazaki, Phys. Rev. A 53, 2683 (1996).
[18] B. G. Klappauf, W. H. Oskay, D. A. Steck, and M. G. Raizen, Phys. Rev. Lett. 81, 1203 (1998).
[19] D. A. Steck, V. Milner, W. H. Oskay, and M. G. Raizen, Phys. Rev. E 62, 3461 (2000).
[20] V. Milner, D. A. Steck, W. H. Oskay, and M. G. Raizen, Phys. Rev. E 61, 7223 (2000).
[21] Chaos, Solitons and Fractals 16, 409 (2003).
[22] S. Paul and A. Bäcker, Phys. Rev. E 102, 050102(R) (2020).
[23] S. Sarkar, S. Paul, C. Vishwakarma, S. Kumar, G. Verma, M. Sainath, U. D. Rapol, and M. S. Santhanam, Phys. Rev. Lett. 118, 174101 (2017).
[24] J. Gong and P. Brunner, The Journal of Chemical Physics 115, 3590 (2001).
[25] J. Gong and P. Brunner, Phys. Rev. Lett. 86, 1741 (2001).
[26] J. Gong, H. J. Wörner, and P. Brunner, Phys. Rev. E 68, 056202 (2003).
[27] I. Dana, E. Eisenberg, and N. Shnerb, Phys. Rev. E 54, 5948 (1996).
[28] Physics Reports 196, 299 (1990).
[29] H. Schanz, M.-F. Otto, R. Ketzmerick, and T. Dittrich, Phys. Rev. Lett. 87, 070601 (2001).
[30] D. A. Steck, W. H. Oskay, and M. G. Raizen, Science 293, 274 (2001).
[31] W. K. Hensinger, H. Häffner, and A. Browaeys, Nature 412, 52 (2001).
[32] J. Mangaonkar, C. Vishwakarma, S. S. Maurya, S. Sarkar, J. L. MacLennan, P. Dutta, and U. D. Rapol, Journal of Physics B 53, 235502 (2020).
[33] M. Saunders, P. L. Halliday, K. J. Challis, and S. A. Gardiner, Phys. Rev. A 76, 040415 (2007).