1. INTRODUCTION

Turck-Chièze et al. (2001b) have published neutrino predictions deduced from a solar model in which the difference with the helioseismic observations in the sound speed in the radiative interior was minimized. This procedure required us to change a small number of ingredients of the model, typically within their known error bars. The objective was to obtain emitted neutrino fluxes seismically representative. The boron neutrino flux is dramatically dependent on the central conditions, hence the need for a solar model close to the real Sun in this region. The flux we proposed is in perfect agreement with the recent results of Ahmad et al. (2001, 2002). This strengthened the idea that the solar neutrino problem found a solution through the neutrino flavor oscillations. In the astrophysical domain, it is the result of improvements on both the theory and observations (e.g., Turck-Chièze 2001).

In this paper we carry on the work. We introduce quantities derived from our first seismic model (other than the neutrino capture rates), like the $g$-mode frequencies and electron and neutron number densities. We test the stability of the neutrino fluxes by computing new seismic models, which differ in the modified physical ingredients, to point out that our seismic neutrino predictions are robust. We also discuss how the magnetic fields may impact the production and transport, through the RSFP process, of these neutrinos. Section 2 is dedicated to the observational data, the improvement of the extracted frequencies, and the inversion of the sound speed profile. Section 3 presents the physical inputs of the models and introduces the seismic models. Section 4 introduces the acoustic and gravity mode predictions and other astrophysical results. Section 5 gives the variables related to the neutrino oscillations: emitted fluxes, electron and neutron densities. Section 6 discusses the effect of a mean magnetic field in different parts of the Sun and its consequences on the neutrino emission and transport.

2. THE HELIOSEISMIC OBSERVATIONS AND THE INVERSION OF THE SOLAR SOUND SPEED

Compared to ground-based networks, the Solar and Heliospheric Observatory (SOHO) spacecraft provides unperturbed continuous measurements leading to detection of low-amplitude oscillations (down to 3 mm s$^{-1}$ after 4 yr). This is particularly important for studying the solar core because the low-frequency part of the oscillation spectrum is accessible and not influenced by turbulent and magnetic solar cycle effects at the surface.

2.1. The Seismic Data and Systematic Errors

To derive the sound speed ($c_s$) of the Sun, we use the data from the Global Oscillation at Low Frequency (GOLF; see Gabriel et al. 1995) and Michelson Doppler Interferometer (SOI/MDI; see Scherrer et al. 1995) instruments onboard SOHO, as well as the inversion procedure described in the next section. For the $p$-modes with $l \leq 3$, we refer to Bertello et al. (2000a, 2000b). These frequencies are derived from GOLF. For the $p$- and $f$-modes with $l > 3$, we use Rhodes et al. (1997), who utilized MDI. The combination of the GOLF and MDI data is necessary because the MDI instrument designed to observe medium-$l$ modes has significant shutter noise at low $l$, while the GOLF instrument specifically targets the low-$l$ modes observing the Sun as a...
star. However, systematic errors in the inversions might arise from the combination of two different data sets (see, e.g., Gough & Kosovichev 1993). Such errors should be mainly due to the following:

1. Intrinsic differences in the data sets, if they do not have the same length and are related to different phases of the solar cycle (it is known that the mode frequencies drift with the solar activity), or if the instrument characteristics are not the same (e.g., integrated disk or resolved measurements).

2. Differences in the data processing (as pointed out by Schou et al. 2002), especially if the mode fitting procedures are not the same (e.g., use of asymmetric or symmetric line profile).

The systematics are very difficult to detect and estimate. Moreover, even a single data set may contain systematic errors due to the data processing pipeline. Errors in the inversion procedure itself may also arise that depend on the inversion method applied.

First, the comparison between GOLF and MDI data for 759 days of nearly uninterrupted observations pointed out the consistency between these two data sets for modes between 1.4 and 3.7 mHz (Bertello et al. 2000b). The frequencies agree within the 1σ uncertainty. A similar comparison between MDI and GOLF frequencies has also been performed at Saclay (R. A. García 2001, private communication): 760 days of GOLF and MDI data starting on 1996 May 25 have been used; to avoid possible biases, the same window function, power spectral estimator, and maximum allowed us to definitely reject some nonstandard physical factors (e.g., Kosovichev 1999). We separate the modes in order modes for these inversions. On Figure 1 we draw the observed frequencies as a function of the inner turning time, here 759 days (Fig. 1, middle and bottom panels), but it is clear that the agreement between computed and observed frequencies is better when we access the low-frequency range (n < 15 or υ < 2.2 mHz). In this case, the modes have a longer lifetime, and an insight into the very central core (important for the boron neutrino flux) can be given with a greater sensitivity.

Therefore, the inversion of the solar sound speed is more accurate (even if the use of the sole low-order modes reduces the radial accuracy). Moreover, the use of these modes avoids a residual contamination of the core coming from the variability with the solar cycle. In Figure 1 the inner...
The sound speed profile is inferred by using the optimally localized averages (OLA) method (see Kosovichev 1999 for mathematical details). In this method, the differences between the observed and model frequencies are expressed as a sum of two linear integrals for the corresponding relative differences in the sound speed and density (see eq. [52] of Kosovichev 1999). The sensitivity kernels in these integrals are calculated by using a variational principle for Legendre polynomials of degree less than 5.

The estimates of the localized averages for the sound speed and density corrections to the solar models are obtained by considering linear combinations of the integral relations for the frequency differences. We proceed such that the corresponding linear combinations of the sensitivity kernels form narrow, localized, Gaussian-type kernels at various target positions along the solar radius for one of the variables (sound speed or density) and are negligible for the other variable. The inversion procedure includes additional constraints to eliminate the surface term and also to minimize the errors on the sound speed and density corrections. The latter constraint is based on observational error estimates of the mode frequencies and includes a regularization parameter that controls the trade-off between the spatial resolution of the inversions (measured as a “spread” of the averaging kernels) and the error magnification. The regularization parameter is chosen to provide a sufficiently smooth radial dependence of the sound speed and density corrections.

The results are presented in the form of the horizontal and vertical bars centered at the center of gravity of the localized averaging kernels (see some values of the spatial uncertainties in Turck-Chieze et al. 2001b). The size of the horizontal bars corresponds to a characteristic width (spread) of the averaging kernels and provides an estimate of the spatial resolution, and the vertical bars correspond to the 1 $\sigma$ formal error of the sound speed correction.

The standard solar model of Brun, Turck-Chieze, & Zahn (1999) was used as a reference. The inversion procedure was tested by using various other solar models as the Sun’s proxy and adding random Gaussian noise to frequencies we calculated on these models, to simulate the observational errors.

2.4. The Results of the Inversions with Some Low-Order Modes

Figure 2 shows the results obtained for the sound speed and density. This figure demonstrates that the presence of low-order modes in our data set slightly changes the characteristics of the $c_s$ profile in the solar core, compared to previous inversions in which these low-order modes were absent. Our profile is close to the one derived by Basu et al. (2000) using an asymmetric Lorentzian profile to fit the GOLF data. We also introduce the averaging kernels of the inversion in the core. The comparison of these kernels, with and without the three low-$n$ modes, underlines the interest of these modes to better describe the core (especially visible on the density kernels) and more specifically the region of $^8$B neutrino emission (see Fig. 5): the averaging kernels

3 We use the standard definitions of the center and width of the averaging kernels introduced by Backus & Gilbert (1968), which provide a robust measure of localization of the averaging kernels. Some authors (e.g., Basu et al. 1997) have attempted to replace these definitions with local properties of the averaging kernels such as the location of the maximum and the distance between quartile points. However, these may lead to misrepresentation of the inversion results and give an impression that the localization closer to the solar center is achieved because in their definition the contributions of remote sidelobes of the averaging kernels are not properly taken into account.
obtained with the low-\( n \) modes are slightly better localized in the deepest layers. Future data analysis efforts should focus on getting more robust measurements of low-degree modes.

We shall call “seismic model” a solar model that reproduces as well as possible the inverted sound speed profile in the solar core and in the radiative zone: we primarily seek an agreement with the Sun in the regions where neutrinos are produced. This agreement is obtained by varying a few physical parameters of the solar models within their error bars. This is possible now, thanks to years of improvements in solar modeling with the introduction of updated physics, microscopic diffusion, and turbulence at the base of the convection zone (e.g., Turck-Chièze et al. 1988; Turck-Chièze & Lopes 1993; Dzitko et al. 1995; Brun, Turck-Chièze, & Morel 1998; Brun et al. 1999).

3. THE SEISMIC SOLAR MODELS

The solar models are computed with the CESAM code (Code d’Evolution Stellaire Adaptatif et Modulaire; see Morel 1997). This is a one-dimensional quasi-static stellar
evolution code that solves the stellar structure equations by a spline collocation method. We always start the modeling of the evolution of the Sun from the pre-main sequence (PMS). The basic physical characteristics of the models are as follows:

1. The nuclear reaction rates are taken from Adelberger et al. (1998), with Miller intermediate screening (Miller 1977). For the $^7\text{Be}(p, \gamma)^8\text{Be}$ reaction we use the $S(0)_{17}$ value derived by Hamecke et al. (1998). For the $^7\text{Li}(p, ^4\text{He})^4\text{He}$ astrophysical factor we use Engstler et al. (1992).

2. The opacities are derived from the OPAL95 opacity tables (Iglesias & Rogers 1996) for temperatures larger than 5600 K. For lower temperatures we use the Alexander opacities (Alexander & Ferguson 1994).

3. The equation of state (EOS) is OPAL (Rogers, Swenson, & Iglesias 1996). For the $\text{seismic}_1$ model we use the “EOSplus” data tables. For the $\text{seismic}_2$ and $\text{seismic}_3$ models introduced in this paper we use the more recent “EOS2001” data tables released on the Web site of the Livermore group in 2001 December. These tables take into account the relativistic effect on electrons as recommended by Elliot & Kosovichev (1998).

4. The microscopic diffusion is taken into account with the prescription of Michaud & Profitt (1993).

5. The turbulent mixing at the base of the convection zone (BCZ) is treated following Brun et al. (1999). In order to properly compute the lithium burning on the PMS, we adjust the time step and the rotation law according to Piau & Turk-Chie`ze (2002).

6. We take the most recent capture rate of $^8\text{B}$ neutrinos by chlorine atoms ($1.14 \times 10^{-42} \text{ cm}^2$; see Bahcall et al. 1996).

In this study our goal is to minimize the discrepancy in the sound speed between the solar models and the real Sun, especially in the core. To achieve this, we modify a few physical parameters used in CESAM. Of course, we derive seismic models that are not unique. We adjust the physical quantities to which the sound speed profile is sensitive enough and modify them within their error bars. We decided to change the parameters as little as possible.

Our last updated model (Btz; Brun et al. 1999) was designed to take into account the horizontal motion produced by the sharp transition in the tachocline from the differential rotation of the convective envelope to almost solid rotation of the radiative core. By introducing a turbulent term at the BCZ, we obtained for the first time a correct $^7\text{Li}$ abundance at the solar surface ($A_{11} = 1.16 \pm 0.10$ dex according to Grevesse & Noels 1993). This is the only element abundance that was not predicted before. This was a real advancement because lithium is an important indicator of the internal structure for numerous stars, and its abundance is very difficult to predict in the classical stellar evolution theory. However, the adopted tachocline prescription is purely hydrodynamic and does not account for any magnetic field, even though the magnetic dynamo process is thought to occur in the thin tachocline. The main impacts of this prescription are a reduced influence of microscopic diffusion by almost 25% for helium (diffusion of heavy elements toward the center of the Sun is slowed down) and enhanced burning of some $^7\text{Li}$ on the main sequence. Three main parameters define the tachocline structure: its current width, taken as $d = 0.05 \ R_{\odot}$; the Brünt-Väisälä frequency at the BCZ, $N = 25 \ \mu\text{Hz}$; and the present rotation rate at the BCZ, $\Omega_0 = 415 \ \text{nHz}$. The turbulent diffusion coefficient in the tachocline is time dependent since it is related to the rotation rate. In the Btz model the rotation as a function of time follows Skumanich’s law (Skumanich 1972).

For the seismic models, we start from Btz and include recent progresses in the stellar evolution theory, like the change in the rotation law on the PMS. We utilize the law proposed by Bouvier, Forestini, & Allain (1997), following Piau & Turk-Chie`ze (2002): the rotation speed of the Sun slightly slows down during the early evolution phase and then experiences a rapid acceleration after 10 million years when the circumstellar accretion disk separates from the young Sun. Finally, the rotation rate decreases following Skumanich’s law because of magnetic breaking. Even though the impact of such changes of the rotation law on the solar evolution is rather weak, taking these into account makes the model more realistic. To reach a correct $^7\text{Li}$ content at 4.6 Gyr, we suppose that the Sun was a slow rotator on the PMS (this is why we choose the disk separation at 10 million years). Moreover, we adjust the parameters defining the tachocline:

1. Now its width is $0.025 \ R_{\odot}$, in accordance with the recent helioseismic results (e.g., Elliott, Gough, & Sekii 1998 announced $d = 0.02 \ R_{\odot}$, and Corbard et al. 1999 obtained $d < 0.05 \ R_{\odot}$).

2. The present rotation rate $\Omega_0$ is 430 nHz according to Corbard et al. (1999).

3. We increase the Brünt-Väisälä frequency, $N$, in the tachocline up to 105 (for the $\text{seismic}_1$ model) and 55 $\mu\text{Hz}$ (for the $\text{seismic}_2$ model). This frequency undergoes a dramatic change when approaching the convection zone. An increase in $N$ reduces the efficiency of the turbulent mixing and diminishes the $^7\text{Li}$ depletion. Thus, this change is related to the lithium destruction on the PMS.

Table 1 lists all the main features of the models seismically compatible with the observed sound speed in the deep interior, together with our reference model.

3.1. The First Solar Model: Seismic$_1$

This model was introduced in Turk-Chie`ze et al. (2001b) with its neutrino capture predictions. In the present paper we introduce related physical quantities. We improve the $\text{seismic}_1$ with minor changes that do not affect the neutrino predictions: a better atmospheric model and a slightly different initial metallicity. We briefly recall the specific modifications made to Btz to obtain this model:

1. We increase the $p-p$ cross section [hereafter $S(0)_{\mu\text{p}}$] by 1%. The change of $S(0)_{\mu\text{p}}$ has a rather large impact on $c_s$. An increase in this cross section induces a decrease of the core temperature, since the model is calibrated to obtain the solar luminosity.

2. We adjust the initial metallicity ($Z_0$) of the Sun. Compared to Btz ($Z_0 = 0.01959$), we increase it by 3.9%.

3. The OPAL EOS is tabulated and depends on $Z_0$. Instead of $Z_0 = 0.01959$ used to compute the EOS table for Btz, we use a table computed with $Z_0 = 0.0203$.

4. We also look at the outer part of the $\delta c_1^2/c^2$ profile: it is still far from flat, since no appropriate model exists for the upper layers with a one-dimensional code. However, it is possible to obtain a better agreement with the Sun by calibrating the seismic model at a radius $R_1$ different from the standard one $R_1 = 6.9599 \times 10^{10} \ \text{cm}$ deduced from
TABLE 1
Comparison of the Seismic Models to the Biz Model for Astrophysical Quantities and Neutrino Predictions

| Parameter | Btz  | Seismic$_1$ | Seismic$_2$ | Seismic$_3$ | Seismic$_3$ Biz$_{12}$ |
|-----------|------|-------------|-------------|-------------|------------------------|
| Age (Gyr) | 4.6  | 4.6         | 4.6         | 4.654       | 4.6                    |
| Radius ($10^{10}$ cm) | 6.9599 | 6.95936 | 6.9599 | 6.9599 | 6.95866 |
| $X_0$ | 0.70817 | 0.70377 | 0.70642 | 0.70857 | 0.70270 |
| $Z_0$ | 0.01959 | 0.02035 | 0.01890 | 0.01959 | 0.02035 |
| $(Z/X)$ | 0.02766 | 0.02892 | 0.02675 | 0.02765 | 0.02896 |
| $s_0$ | 1.755 | 1.934 | 2.040 | 2.000 | 1.774 |
| $(Z/X)_d$ | 0.0255 | 0.02628 | 0.02447 | 0.02527 | 0.02647 |
| $Y_f$ | 0.2508 | 0.2508 | 0.2510 | 0.2481 | 0.2531 |
| $\gamma$ (dex) | 1.14 | 1.10 | 1.08 | NR | NR |
| $d(R_c)$ | 0.05 | 0.025 | 0.025 | 0.025 | 0.025 |
| $N(\mu\text{Hz})$ | 25 | 105 | 55 | 45 | 45 |
| $\Omega_0(\text{Hz})$ | 415 | 430 | 430 | 430 | 430 |
| BCZ$^a$ ($R_c$) | 0.7142 | 0.7115 | 0.7113 | 0.7132 | 0.7118 |
| $T_i$ ($10^8$ K) | 1.5706 | 1.5739 | 1.5712 | 1.5720 | 1.5754 |
| $\rho_i$ (g cm$^{-3}$) | 153.13 | 153.022 | 153.686 | 154.053 | 153.567 |
| $X_i$ | 0.3385 | 0.3338 | 0.3371 | 0.3356 | 0.3284 |
| $Y_i$ | 0.6405 | 0.6445 | 0.6428 | 0.6434 | 0.6461 |
| $^{32}$Ga (SNU) | 127.1 ± 8.9 | 128.1 ± 8.9 | 126.8 ± 8.9 | 127.9 ± 8.9 | 128.7 ± 9.0 |
| $^{37}$Cl (SNU) | 7.25 ± 0.94 | 7.48 ± 0.97 | 6.90 ± 0.90 | 7.47 ± 0.97 | 7.60 ± 0.99 |
| $^{9}$B ($10^6$ cm$^{-2}$ s$^{-1}$) | 4.82 ± 0.72 | 4.98 ± 0.73 | 4.85 ± 0.72 | 4.88 ± 0.73 | 5.07 ± 0.76 |
| Atm$^b$ | H | K | K | K | H |
| EOS (at low T) | CEFF | H | MHD | OPAL | OPAL | CEFF |

- Initial hydrogen and metal mass fraction.
- Initial metal-to-hydrogen mass fraction ratio.
- Mixing length parameter.
- Surface metal-to-hydrogen mass fraction ratio at the solar age.
- Surface helium mass fraction at the solar age.
- Lithium surface abundance at the solar age.
- This model was run with a time step larger than the one required to derive a realistic $^7$Li abundance. Therefore, this quantity is nonrelevant here.
- Base of the convection zone.
- Central temperature and density at the solar age.
- Central hydrogen and helium mass fraction at the solar age.
- The atmosphere model: H for Hopf, K for Kurucz5777.

changes $\delta c^2_{\text{p}}/c^2_{\text{p}}$ by only $\approx 0.1\% - 0.2\%$ and thus does not improve significantly the agreement with the Sun. On the other side, the sound speed is quite sensitive to the $p$-$p$ reaction rate. Therefore, it seems rather appropriate to adjust only $S(0)_{pp}$. Thus, we build the best model with the simplest assumptions.

We also notice that while the solar models are primarily sensitive to $S(0)_{pp}$, the opacities, and the heavy-element abundances, they can undergo changes under secondary parameters such as the EOS or the solar age. Two examples are given in the next sections through new solar models.

3.2. The Seismic$_2$ Solar Model

The agreement of the first model with the observed $(Z/X)_0$ surface ratio is only marginal, since Grevesse & Noels (1993) find $(Z/X)_0 = 0.0245 \pm 10\%$. Moreover, the recent improvement of the EOS tables reduces the difference between measured and calculated sound speed in the solar core. Consequently, in the second seismic model the following applies:

1. We introduce the most recent OPAL tabulated EOS: the EOS2001 tables that take into account the relativistic effects on the electrons. We worked out that this reduces the first adiabatic exponent $\Gamma_1$ by about $3\%$ below $0.1 R_c$ as predicted by Elliott & Kosovichev (1998). The new OPAL EOS

photometric observations (e.g., Allen 1976). Analyses based on $f$-mode frequencies (e.g., Schou et al. 1997; Antia 1998) worked out seismic radii $R = 695.78$ and 695.68 Mm, respectively, slightly smaller than the photometric one. The latter is greater than the optical determination $R = 695.51$ Mm by Brown & Christensen-Dalsgaard (1998). There is neither a firm answer explaining the origin of these discrepancies nor an understanding of the effect of the solar cycle on the radius. Thus, we calibrate the seismic$_1$ model with $R_1 = 6.95936 \times 10^{10}$ cm, with a consequent ad hoc improvement of the sound speed profile in the convective zone.

5. For this paper we use a Kurucz’s atmosphere model instead of the basic Hopf’s model.

The adjustments made increase the overall agreement between the Sun and the solar model on the $c_s$ profile (see Fig. 3, top panel). To reach this improvement in the core and the radiative zone, we restricted the modifications to two parameters: $S(0)_{pp}$ and $Z_0$.

The seismic$_1$ model is not unique, but we have experienced that it is not easy to reach a good agreement in the solar core by modifying other parameters. Turck-Chièze et al. (2001a) highlight the lack of sensitivity of the sound speed to several nuclear reactions, like $^4$He($^3$He, $2p$)$^7$He, $^3$He($^3$He, $\gamma$)$^7$Be, or the CNO bi-cycle. For instance, a 25% increase of the cross section of the first two reactions
is tabulated for temperatures as low as 2000 K. Therefore, we do not use a second EOS at low temperature anymore. This has an impact on the upper solar layers, and we no longer need to calibrate the model at a radius different from the standard one.

2. We reduce \( Z_0 \) by 3.5% to reach a \( (Z/X) \) ratio equal to the one proposed by Grevesse & Noels (1993). By lowering \( Z_0 \), we reduce the mean molecular weight in the central part of the Sun and the Rosseland opacities \( \kappa_{\text{Ross}} \) as well. The sound speed profile suffers from this change.

3. Therefore, we raise \( \kappa_{\text{Ross}} \) to compensate for the decrease induced by the modification of \( Z_0 \). To change \( \kappa_{\text{Ross}} \), we follow Brun et al. (1998) by simulating an increase in the C, N, and O opacities. This raise could be attributed to uncertainties in the bound-bound and bound-free processes in the opacity calculations. The change in the opacities of the CNO elements produces an increase of about 1.5% in \( \kappa_{\text{Ross}} \) in the solar core and about 5.9% at 0.7 \( R_\odot \) and above. For the opacity error estimates we can refer to Neuforge-Verheecke et al. (2001). By comparing the Los Alamos LEDCOP opacities with the OPAL ones, they found a difference of about 1.5%–2% at 0.1 \( R_\odot \) and \( \geq6\% \) at the BCZ. Our modifications stay within these uncertainties. The OPAL95 tables, which we used to interpolate the \( \kappa_{\text{Ross}} \) values, were calculated assuming a Grevesse & Noels (1993) mixture. During the solar evolution the metal composition in the core changes: because of the CNO polycycle and the microscopic diffusion, the relative number fractions of the metals are modified. This should have an impact on the opacities: we may have to correct them when the solar core composition is “far” from Grevesse & Noels (1993) (this occurs at the Sun’s age of, roughly speaking, 100 million years). From the seismic1 model we compute a very similar model but with a correction for \( \kappa_{\text{Ross}} \) below 0.15 \( R_\odot \), to take into account the change in the composition. Actually, this correction is very small: from \(-0.88\% \) at 0 \( R_\odot \) to \(-0.5\% \) at 0.15 \( R_\odot \), for the Sun at 4.6 Gyr. It has only a minor impact on the sound speed profile: it just increases \( \delta c_1^2 / c_1^2 \) by 0.0005 in the core! Thus, we do not need to take this correction into account in the solar models. Instead of an increase in \( S(0)_{\text{p-p}} \) by 1% for seismic1, an increase by only 0.75% is enough with this modification.

4. All these changes yield a residual difference on \( c_s \) in the very core and below the convection zone. Since a change in \( S(0)_{\text{p-p}} \) is known to act in a symmetrical way in the core and at the BCZ, we modify it (increase of 0.25%). The other nuclear reactions have a larger impact in the core than anywhere else; thus, a change in their rates is not appropriate.

The seismic3 model produces a sound speed profile similar to the one of seismic1 and very close to the Sun below 0.65 \( R_\odot \) (see Fig. 3, middle panel).

### 3.3. The Impact of the Sun’s Age: The Seismic3 Model

We check the sensitivity of \( c_s \) to the solar age \( t_\odot \). Dziembowski et al. (1999) used helioseismology to determine \( t_\odot \). Depending on the method, they derived different results. However, using the small frequency separation that they claim to be the most accurate measure of the Sun’s age, they conclude that \( t_\odot = 4.66 \pm 0.11 \) Gyr (the lifetime on the PMS must be added). We compute the seismic3 model as follows:

1. We apply the new EOS as previously.
2. We increase the age by only 1.2%, i.e., \( t_\odot = 4.654 \) Gyr including PMS.

This produces a significant change in the \( c_s \) profile and reduces the discrepancy with the helioseismic profile, even though the reduction is less significant compared to the other seismic models and affects only the region below 0.3 \( R_\odot \) (see Fig. 3, bottom panel). The seismic3 model shows that, if we underestimate the solar age, a lesser increase in

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**Fig. 3.**—Difference in the square of the sound speed between the Sun and some models. The three panels are for the seismic1 (top), seismic2 (middle), and seismic3 (bottom) models (curves with error bars). The plain curves with no error bar are for the Saclay standard solar model, while the dashed curves are for the Btz model.
$S(0)_{p,p}$ and $Z_0$ is needed to reproduce the flat $\delta c_2^2/c_1^2$ profile of seismic$_2$. Even though this does not change the need to raise both $S(0)_{p,p}$ and $Z_0$, this reduces the amplitude of the increase. On the contrary, a solar age less than 4.6 Gyr would favor a larger increase of these two parameters.

4. THE ASTROPHYSICAL RESULTS OF THE MODELS

4.1. The Oscillation Frequencies

An important result of these solar models is the computation of the oscillation frequencies of $p$- and $g$-modes, using an updated version of the adiabatic oscillation code (Christensen-Dalsgaard 1982). The $p$-mode frequencies of seismic$_1$ and seismic$_2$ differ mainly because of the differences in the EOS at low temperature (see above). This yields a difference of $\approx 1$ $\mu$Hz. Figure 4 shows the differences for $l = 0$–5 for the two models. We may observe that the agreement with the Sun is within 0.5 $\mu$Hz for $\nu$ smaller than 1500 $\mu$Hz. The discrepancy increases to 16 $\mu$Hz at 4500 $\mu$Hz, as a result of the difficulty to model the turbulent surface. Table 2 gives the frequencies of the low-degree $g$-modes for the seismic$_1$ model. They differ by less than 0.5 $\mu$Hz with the frequencies of the seismic$_2$ model, with a more significant effect for higher frequencies. The sensitivity of the $g$-modes to the very central part of the Sun could lead to a discrepancy with the observed $g$-modes greater than the one measured for the $p$-modes, provided that we detect some $g$-modes. We observe a difference up to 2 $\mu$Hz with the previous predictions of Brun et al. (1998) and Provost, Berthomieu, & Morel (2000). The progress in the description of the solar core impacts greatly on the $g$-mode predictions and could help in their detection (Turck-Chièze et al. 2002a).

4.2. Comments on Some Physical Quantities

We have presented different solar models that are close to the observed sound speed in the solar core. These models are not expected to be better (in the sense of the physics) than the Bt model because some modifications that we applied might be explained by inaccuracies in modeling of some physical phenomena with the CESAM code (especially the dynamical phenomena like the Eddington-Sweet circulation that are not taken into account). However, the sound speed profile of these models is closer to the real Sun, and we need this to derive reliable neutrino fluxes (e.g., the boron flux is very sensitive to the thermodynamics of the core as it varies as $T^2$, with $T$ the central temperature).

These models systematically favor a slight increase in $S(0)_{p,p}$ (less than 1%). The strong influence of the $p$-$p$ reaction was known for a long time (Turck-Chièze & Lopes 1993), and a raise was also proposed by Antia & Chitre (1998). However, the increase we work out is less than theirs and could be even less if the solar age turned out to be slightly larger than 4.6 Gyr [on the contrary, a lower age implies a larger increase in $S(0)_{p,p}$]. Incidentally, this result confirms that the calculations of $S(0)_{p,p}$ are very good despite their purely theoretical basis.

Concerning $Z_0$, an increase by 3.9% is favored by seismic$_1$, while seismic$_2$ shows that an increase of $\delta g_{\text{Ross}}$ combined with a decrease of $Z_0$ yields the same result in terms of

![FIG. 4.—Weighted difference between the $p$-mode frequencies of the models and the ones we observed. We used the GOLF low frequencies for $l \leq 3$ (Garcia et al. 2001; Bertello et al. 2000b) and the MDI frequencies for $l = 4$ and 5 (Rhodes et al. 1997). The solid lines correspond to the seismic$_2$ model, the dashed lines to seismic$_1$.](image)

| $l$ | $n$ | $\nu$ (MHz) | $l$ | $n$ | $\nu$ (MHz) | $l$ | $n$ | $\nu$ (MHz) | $l$ | $n$ | $\nu$ (MHz) |
|-----|-----|-------------|-----|-----|-------------|-----|-----|-------------|-----|-----|-------------|
| 1.... | 10 | 95.47       | 2.... | 10 | 104.21      | 3.... | 10 | 136.49      | 4.... | 10 | 145.31      | 5.... | 10 | 159.55      |
| 1.... | 10 | 109.28      | 2.... | 10 | 136.58      | 3.... | 10 | 165.27      | 4.... | 10 | 180.14      | 5.... | 10 | 192.39      | 6.... | 10 | 201.94      |
| 1.... | 10 | 122.63      | 2.... | 10 | 153.25      | 3.... | 10 | 171.92      | 4.... | 10 | 189.21      | 5.... | 10 | 200.19      | 6.... | 10 | 214.69      |
| 1.... | 10 | 145.31      | 2.... | 10 | 165.27      | 3.... | 10 | 189.21      | 4.... | 10 | 189.21      | 5.... | 10 | 200.19      | 6.... | 10 | 214.69      |

TABLE 2
$g$-Mode Frequencies with the Seismic$_1$ Model

\begin{tabular}{ll|ll|ll|ll|ll|ll|ll}
\hline
$l$ & $n$ & $\nu$ (MHz) & $l$ & $n$ & $\nu$ (MHz) & $l$ & $n$ & $\nu$ (MHz) & $l$ & $n$ & $\nu$ (MHz) \\
1 &  &  & 3 &  &  &  &  &  &  &  &  \\
1 &  &  & 4 &  &  &  &  &  &  &  &  \\
1 &  &  & 5 &  &  &  &  &  &  &  &  \\
1 &  &  & 6 &  &  &  &  &  &  &  &  \\
2 &  &  & 7 &  &  &  &  &  &  &  &  \\
2 &  &  & 8 &  &  &  &  &  &  &  &  \\
2 &  &  & 9 &  &  &  &  &  &  &  &  \\
2 &  &  & 10 &  &  &  &  &  &  &  &  \\
2 &  &  & 11 &  &  &  &  &  &  &  &  \\
2 &  &  & 12 &  &  &  &  &  &  &  &  \\
2 &  &  & 13 &  &  &  &  &  &  &  &  \\
2 &  &  & 14 &  &  &  &  &  &  &  &  \\
2 &  &  & 15 &  &  &  &  &  &  &  &  \\
2 &  &  & 16 &  &  &  &  &  &  &  &  \\
\hline
\end{tabular}
Since either of these solutions is acceptable, we cannot favor the increase or decrease of $Z_0$. The effects of opacities and heavy-element abundances are closely related, and part of the error bars on $R_\text{Ross}$ are due to the uncertainty on $Z$. However, a change by almost 4% in $Z$ for seismic$_1$ is to be considered as important since the microscopic diffusion may change $Z$ by only 10% since the onset of the hydrogen burning. Actually, this need to increase $Z_0$ could be the manifestation of some “forgotten” hydrodynamic phenomenon. In the present approach we only considered variation of the classical ingredients of the solar models, but the need to modify the composition and/or opacities may be due to dynamical effects that are not taken into account in the classical framework, like the meridional circulation (Vauclair 1999), the transport of angular momentum by internal waves (e.g., Talon, Kumar, & Zahn 2002), or the magnetic field. The use of the density and rotation profiles in the core and/or the detection of $g$-modes may help to solve this point by giving us access to the angular momentum evolution in the Sun. The present analysis tends to show that the poor modeling of the dynamical phenomena in the core has a rather weak impact on the neutrino prediction (see Table 1 and next section).

Concerning the Sun’s age, it seems that $t_0 = 4.6$ Gyr is a correct value for the solar age: the value needed to best reduce the discrepancy on $c_s$ below 0.6 $R_\odot$ is 4.654 Gyr. Even if this is outside the 1 $\sigma$ uncertainty on the solar age ($t_0 = 4.49 \pm 0.04$ Gyr plus 50 million years for the PMS, according to Guenther 1989), this is less than the $t_0$ value proposed by Dziembowski et al. (1999).

5. RESULTS ON THE PREDICTED NEUTRINO FLUXES

5.1. The Solar Neutrino Predictions with the Seismic Models

The main goal of our seismic models is to determine and stabilize the neutrino fluxes from the observation of the acoustic modes, especially for the $^8$B neutrino flux. Its theoretical prediction greatly varied with time and solar models between $\simeq 3.8 \times 10^{56}$ and $\simeq 6.5 \times 10^{56}$ cm$^{-2}$ s$^{-1}$, depending on the authors. The related error estimate has varied from $\simeq 20\%$ to $\simeq 60\%$. If we take into account nonstandard solar models, the change in the boron flux is even more dramatic. Of course, some neutrino fluxes are better constrained, like the $p$-$p$ flux set by the solar luminosity (see Turck-Chieze 2001 for a review on the progress made to stabilize these flux predictions).

The neutrino production is distributed over the solar core. For the calculation of the expected emitted electron-type neutrino ($\nu_e$) flux, it is necessary to know where the neutrinos are produced. Figure 5 shows the production zones for the $p$-$p$, pep, $^8$B, $^7$Be, $^{13}$N, $^{15}$O, and $^{17}$F neutrinos obtained with seismic$_1$. It appears that most of these fluxes depend on the very central part of the Sun. In this paper we use helioseismology to better determine the predicted neutrino fluxes. However, we must stress the fact that the sound speed profile does not go deeper than 0.07 $R_\odot$ $\pm$ 3.6% (Table I of Turck-Chieze et al. 2001b). Only the detection of $g$-modes may significantly improve this situation.

The neutrino fluxes have been calculated for the seismic$_1$, seismic$_2$, and seismic$_3$ models (see Table 1), and detailed neutrino capture predictions are introduced for the seismic$_1$ model in Table 3. As an important result, we may note that these fluxes do not vary significantly among the models, despite the differences in the building of the latter. This confirms the robustness of the model calculation and that we stabilized the neutrino flux estimates. This result is not trivial, since the seismic models were derived in different ways, and some neutrino fluxes (beryllium, boron, and the CNO cycle) are very sensitive to the physical parameters of the core. It emphasizes that the neutrino fluxes do not depend strongly on the solar model if this model is consistent with the helioseismic observations in the solar core. Since no seismic model can be exact as a result of the potential presence of systematic errors in the inversions (as discussed in § 2.1), we also computed the neutrino fluxes for another seismic model based on Figure 2a (i.e., based on the $\delta c^2/c^2$ profile derived without the three low-order modes). The predicted neutrino capture rates for this model are $127.6 \pm 8.9$ SNU for the $^{37}$Ga, $7.36 \pm 0.96$ SNU for the $^{37}$Cl, and $(4.89 \pm 0.73) \times 10^6$ cm$^{-2}$ s$^{-1}$ for the $^8$B. This model is close to seismic$_1$, except that we increased $S(0)_{pp}^p$ by 0.5%. The difference between the $^8$B flux of this model and the one of seismic$_1$ is well within the uncertainty we quote in Table 1. This confirms that our neutrino predictions are rather robust and should not be changed in a significant way by the presence of systematic errors in the inversion process.

Moreover, the predicted neutrino fluxes are not very different from other recent solar models (e.g., Brun et al. 1999; Bahcall, Pinsonneault, & Basu 2001; Watanabe &
Shibahashi 2001). This proves that the flux predictions converge now thanks to years of helioseismic research.

We point out that the present predictions are of great interest (in comparison with those obtained 10 years ago) because they are based now on accurate helioseismic data that validate the updated physics at the required precision level (see, e.g., Table 2 of Turk-Chieze et al. 2001a). The emitted neutrino fluxes we predict are no longer purely theoretical but deduced from precise seismic “observations” of the solar core.

5.2. Comparison of the $^8$B Emitted Neutrino Flux with the SNO Results

The Sudbury Neutrino Observatory (SNO) collaboration has published the neutral current (NC), charged current (CC), and elastic scattering off electrons (ES) reaction results for the boron neutrino capture (Ahmad et al. 2002). The CC reaction is only sensitive to $\nu_e$, and the NC and ES reactions are sensitive to all leptons flavors. SNO can derive the fluxes of neutrino with the different flavors. They also use the more precise Super-Kamiokande results for the ES reaction (Fukuda et al. 2001). Even though these measurements are extremely difficult to carry out and need the rejection of a large background noise, the coherence of the results strongly favors the existence of neutrino oscillations. With this assumption they have estimated the total $^8$B neutrino flux emitted by the Sun:

$$\Phi(\nu_e) = (4.98 \pm 0.73) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}.$$  

This flux is very close to the seismic1 $^8$B neutrino prediction (see Table 3):

$$\Phi(\nu_e) = (4.98 \pm 0.73) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}.$$  

The seismic models introduced in this paper confirm this agreement (see Table 1). All our predictions include the recent rejection of several astrophysical solutions to the neutrino puzzle proposed in the past. The agreement prediction/observation definitely establishes the origin of the neutrino puzzle: the minimum standard model of particle physics is not complete and neutrinos have masses.

Nevertheless, some specific issues must be stressed. First, we cannot rule out that some $g$-modes may be detected and reveal new solar features below 0.07 $R_\odot$. Second, the $S(0)_{17}$ factor has no impact on the solar structure and was poorly determined more than 4 years ago (see, e.g., Turk-Chieze 2001). For instance, Adelberger et al. (1998) propose $S(0)_{17} = 19^{+2}_{-1}$ eV barns. The experimental measurements have been largely improved. Here we use the value of Hammache et al. (1998) of $18.5 \pm 1$ eV barns. Unfortunately, the recent result of Junghans et al. (2002) of $22.3 \pm 0.7$ eV barns only marginally agrees with the previous measurement. This new value may increase the boron flux by 17%. This does not really invalidate the previous conclusions as the global error on the $^8$B flux is currently of the same order of magnitude. Another point under debate is the framework we assumed to compute our models: we consider a static solar core with only "classical" phenomena. This representation is compatible with the current seismic results, but we still need to properly take into account the rotation and magnetic field in the radiative zone to definitely check this assumption.

5.3. Neutrino Oscillations and Related Quantities

5.3.1. The Electron Number Density

A part of the neutrino oscillations could be explained by the Mikheyev-Smirnov-Wolfenstein (MSW) effect (see, e.g., Mikheyev & Smirnov 1986): an electron-type neutrino may undergo a resonant oscillation in the Sun and then be converted into a muon- or tau-type neutrino. This effect assumes that the neutrinos have masses and that the flavor eigenstates are different from the mass eigenstates.

The conversion probability depends on whether the oscillation is adiabatic or not. In both cases, the electron number density ($n_e$) is needed with a high accuracy all along the solar radius to compute this probability.

Therefore, we derive $n_e$ for the seismic1 model. We use the mass fractions returned by CESAM as a function of the fractional radius, for the $^1$H, $^2$H, $^3$He, $^4$He, $^7$Li, $^7$Be, $^9$Be, $^{12}$C, $^{13}$C, $^{14}$N, $^{15}$N, $^{16}$O, and $^{17}$O atoms, plus an extra element (with $A = 28$ and $Z = 13$). We convert these mass fractions into number fractions and multiply them by the electron number of the related chemical element. By adding all the quantities obtained this way, we determine the electron number density as a function of the radius. The result is shown in the left-hand panel of Figure 6. The values of $n_e$ are very similar for seismic2, even near the solar surface, despite the use of a different atmosphere model and EOS. However, these values are different in the upper solar layers from the ones announced by Bahcall et al. (2001), probably as a result of their use of a Krishna-Swamy relationship for the atmosphere: this atmosphere model is less precise than the one we use.

5.3.2. The Neutron Number Density

If the neutrino has a magnetic moment (either a dipole or/and transition moments), it might interact with the solar magnetic field. Provided that this magnetic moment and the magnetic field are large enough, this field could flip the spin...
of the neutrino: a left-handed neutrino could become right-handed. Moreover, the possible flavor transition magnetic moments could result in a spin-flavor precession: the neutrino could change both its chirality and its flavor. This double precession could be matter-enhanced through the interactions of neutrino with the electrons, protons, and neutrons. This is the resonant spin flavor precession (RSFP) process (e.g., Lim & Marciano 1988). To compute the conversion probabilities for the RSFP, the neutron number density \( n_n \) is required. We derive it the same way as \( n_e \) (see the right-hand panel of Fig. 6), but instead of using the electron number of each chemical element, we use its neutron number. The \( n_n \) profile is rarely shown, and the one we introduce here takes into account the presence of the tachocline and the microscopic diffusion: this impacts significantly the neutron density above 0.65 \( R_\odot \). The seismic1 and seismic2 models have very close neutron density values.

6. THE ADDITION OF MAGNETIC PRESSURE TO THE SOLAR MODELS

Despite the overall agreement in the sound speed between the Sun and our solar models below 0.65 \( R_\odot \), three regions of our star remain poorly described: the very central core, the tachocline region, and the upper layers. Concerning the core, the central rotation law is not taken into account in the present analysis (Turck-Chieze et al. 2002b). A one-dimensional stellar evolution code cannot provide an efficient treatment of the dynamic regions. Neither the rotation of the Sun nor its magnetic field is modeled. The neutrino transport may depend on this physics. For instance, if the central core does not rotate like the rest of the radiative zone (e.g., Gough 2001), this may create a discontinuity in the central electron density profile. In this section we investigate how the solar large-scale magnetic field \( \mathbf{B} \) may influence the neutrinos. We add magnetic pressure and derive new solar models. This puts upper bounds on the field strength (due to its influence on the sound speed) and shows its impact on the neutrino flux predictions and transport.

With this analysis we also test the sensitivity of \( c_s(r) \) to the magnetic field. Any magnetic field should be imprinted in the “magnetoacoustic” wave velocity. Unfortunately, we cannot account for the field structure with a one-dimensional code, and so we can only add a magnetic pressure term \( P_{\text{mag}} \) in the stellar structure equations: \( P_{\text{mag}} = B^2/(8\pi) \) (in cgs units).

The main problem is to choose an appropriate \( B(r) \) for the solar interior, since little is known about the inner field. Moreover, it is unclear whether or not the toroidal part of the field prevails in the solar interior: the surface activity proves that the toroidal field is larger than the poloidal one in the upper layers, but only a few clues of what happens in the deep interior are known. In this paper we consider just toroidal fields.

6.1. Simulated Magnetic Profiles

Following Gough & Thompson (1990), we simulate fields as

\[
B_\phi = a(r) \frac{d}{d\theta} P_k(\cos \theta) e_\phi ,
\]

with the spherical coordinates \((r, \theta, \phi)\). \( P_k(\cos \theta) \) is a Legendre polynomial of degree \( k \). We assume \( k = 2 \), meaning that the field is quadrupolar (in accordance with the surface magnetism manifestations). When computing the wave velocity, we take into account the Alfven speed \( v_\Lambda \).

Since we are interested in the radial velocity and a toroidal field is perpendicular to the radial direction, the new wave velocity is \((c_s^2 + v_\Lambda^2)^{1/2}\) (in the figures we continue to note this quantity as “\( c_s \)” for convenience, even though it is no longer the sound speed, strictly speaking). For the function \( a(r) \) two profiles are considered.

First, to simulate a magnetic field in the radiative zone, we choose

\[
a(r) = \begin{cases} 
K_\lambda \left( \frac{r}{r_0} \right)^2 \left[ 1 - \left( \frac{r}{r_0} \right)^2 \right]^\lambda & \text{if } r \leq r_0 , \\
0 & \text{otherwise} , 
\end{cases}
\]

where \( K_\lambda = (1 + \lambda)(1 + 1/\lambda)^{1/2}B_\theta, r_0 = 0.712 R_\odot \) is approximately the BCZ, and \( \lambda = 10r_0 + 1 \). \( B_\theta \), the highest intensity of the field, is set to different values (see Table 4). The addition of the magnetic pressure is made when the Sun enters the zero-age main sequence (ZAMS). The maximum values of the \( P_{\text{mag}}/P_{\text{gas}} \) ratio (hereafter \( \beta^{-1} \)) are available in
TABLE 4

| Name          | $B_0$ (T) | Center* ($R_\odot$) | $(P_{\text{mag}}/P_{\text{gas}})_{\text{max}}$ |
|---------------|----------|---------------------|-----------------------------------------------|
| Seismic$_{\text{B1}}$ | $10^4$   | 0.236               | $2.85 \times 10^{-2}$                         |
| Seismic$_{\text{B11}}$ | $5 \times 10^3$ | 0.236               | $6.96 \times 10^{-3}$                         |
| Seismic$_{\text{B12}}$ | $3 \times 10^3$ | 0.236               | $2.49 \times 10^{-3}$                         |
| Seismic$_{\text{B13}}$ | $1 \times 10^3$ | 0.236               | $2.80 \times 10^{-4}$                         |
| Seismic$_{\text{B2}}$ | 30       | 0.712               | $6.15 \times 10^{-4}$                         |
| Seismic$_{\text{B3}}$ | 50       | 0.712               | $1.71 \times 10^{-4}$                         |
| Seismic$_{\text{B11}}$ | 2        | 0.96                | $1.34 \times 10^{-4}$                         |
| Seismic$_{\text{B13}}$ | 3        | 0.96                | $3.02 \times 10^{-4}$                         |

* Radius at which $P_{\text{mag}}$ is maximum.

Table 4, for all the models discussed here. We could also give the $v_A/c_s$ ratio, but $\beta$ and this ratio are closely related: $v_A/c_s = [2/(\beta T)]^{1/2}$. Both contributions of the magnetic field to the change in $c_s$—the indirect change through the modification of the solar structure, and the direct change through the addition of $v_A$—depend on the $\beta$-values. $\beta^{-1}$ is maximum around $0.25 \ R_\odot$. This can be justified by the rotation profile we inferred in the radiative interior with the GOLF data (Turck-Chièze et al. 2002b).

Second, we also utilize an $a(r)$ profile:

$$a(r) = \begin{cases} B_0 \left[1 - \left(\frac{r - r_0}{d}\right)^2\right] & \text{if } |r - r_0| \leq d, \\ 0 & \text{otherwise}, \end{cases}$$

where $d$ is the half-width of the zone of a magnetic field and $r_0$ is the center of this zone. To simulate a magnetic field in the tachocline, parameters are set to $d = 0.02 \ R_\odot$ and the radius $r_0$ of transition between radiative and convective zones varies along the solar evolution. We set $B_0$ to 30 T according to Antia, Chitre, & Thompson (2000) for the seismic$_{\text{B2}}$ model and 50 T for the seismic$_{\text{B22}}$ model. The same profile is used to simulate a possible field in the upper solar layers. Such a field was hinted at by Antia et al. (2000) from an analysis of the Global Oscillation Network Group (GONG) and MDI data. To test a field anchored at 0.96 $R_\odot$ according to Antia et al. (2000) from an analysis of the Global Oscillation Network Group (GONG) and MDI data. To test a field anchored at 0.96 $R_\odot$, parameters are set to $r_0 = 0.96 \ R_\odot$ and $d = 0.035 \ R_\odot$. $B_0$ is set to 2 (Antia et al. 2000) and 3 T. The fields at the BCZ and in the upper layers are both added when the Sun is 85 million years old and the solar core is no longer convective.

Since all the physical quantities considered here are radial quantities, we average the magnetic pressure $P_{\text{mag}}(r, \theta)$ over $\theta$.

6.2. Impact of the Magnetic Pressure on the Solar Models and on the Neutrino Production

The three different magnetic pressure profiles are drawn on Figure 7. The addition of magnetic pressure in the radiative zone following seismic$_{\text{B1}}$ induces a great change in the thermodynamic quantities, especially in the $c_p$ profile (see Figs. 8 and 9). On the contrary, the impact of a field 10 times smaller following seismic$_{\text{B13}}$ is minuscule (see Figs. 8 and 9). The precision we have on the solar sound speed rules out a magnetic field with such a profile and an intensity as large as $B_0 = 10^4$ T. Actually, we can put an upper limit for a (toroidal) magnetic field in the radiative zone of about $3 \times 10^3$ T (seismic$_{\text{B12}}$ model). If $B_0 \leq 10^3$ T, then $c_p$ is not sensitive enough to $P_{\text{mag}}$, and we cannot draw any conclusion about the likelihood of a field like the one of the seismic$_{\text{B13}}$ model. We show in Table 1 the impact on the neutrino production of the seismic$_{\text{B12}}$ model. The fluxes are slightly larger than the ones deduced from the adjustment in the physics of the seismic models. The result obtained for the $^8$B neutrinos is $5.07 \times 10^6$ cm$^{-2}$ s$^{-1}$. It remains in agreement with the present SNO results.

We face a problem when adding $P_{\text{mag}}$ at the BCZ and in the upper layers (see Fig. 8): it is difficult to draw any conclusions because the sound speed is not sensitive enough to such modifications. With the current accuracy that we have for $c_p$, it is only possible to state that a field in the tachocline can reach an amplitude as large as 50 T without perturbing the sound speed profile. We conclude the same for a toroidal field anchored at 0.96 $R_\odot$ and as large as 3 T. Given the minuscule impact on $c_p$, we did not draw the $\delta c_p^2/c_p^2$ profile obtained with these fields.

This section on the magnetic field confirms that the sound speed is only sensitive to the $\beta$ ratio. With the different models we computed, each one with a different magnetic field profile and/or intensity, we can conclude that only the large-scale fields with $\beta^{-1}$ larger than, at least, $\approx 3 \times 10^{-4}$ impact the $c_p$ profile. An interesting result is that a field strength greater than $3 \times 10^3$ T can be ruled out for a toroidal field inside the radiative zone. Despite the great accuracy we reached on this quantity, the sound speed is not suited to the determination of the large-scale magnetic features of the Sun. Many physical processes, to which $c_p$ is quite sensitive, are still affected by large uncertainties, and, thus, a potential magnetic field looks like "background noise" compared to these processes. Yet the use of the sound speed might be more promising for constraining an upper layer field, since the "weakness" of $P_{\text{gas}}$ near the solar surface makes $c_p$ more sensitive to weaker fields.

Concerning the neutrino puzzle, none of the reasonable models including a central magnetic field (as models seismic$_{\text{B11}}$, seismic$_{\text{B12}}$, and seismic$_{\text{B13}}$) greatly modify the neutrino flux predictions, unlike the ruled-out seismic$_{\text{B1}}$ model (it increases the $^8$B neutrino flux by more than 20%). With the upper bounds we have for a central large-scale magnetic field we conclude that $B$ has only a very slight impact on the neutrino emission: $\approx 2\%$. We must notice
that the magnetic field intensities we applied in the radiative interior are much larger than the upper bound of \( \approx 30 \) G proposed by Boruta (1996). Therefore, the upper limit we derived for the core magnetic field seems not stringent, and a field of only 30 G should have no impact on the neutrino production.

6.3. Connection between Magnetic Field and Neutrino Transport

To briefly comment on the impact of a field on the transport of \( \nu_e \) in the radiative and convective zones, we consider the following transition for a Majorana neutrino: 

\[
|\nu_e\rangle \rightarrow |\bar{\nu}_e\rangle \quad \text{(we assume the standard two-neutrino case)}.
\]

The transition from an electron-type left-handed neutrino to a muon-type right-handed antineutrino is a specific case of the RSFP theory and may occur in the solar plasma if \( B \) and the magnetic moment (\( \mu_e \)) of the neutrinos are large enough. We note that (Lim & Marciano 1988)

\[
|\nu_e\rangle = \cos \theta_e |\nu_1\rangle + \sin \theta_e |\nu_2\rangle, \quad (5)
\]

\[
|\nu_\mu\rangle = -\sin \theta_e |\nu_1\rangle + \cos \theta_e |\nu_2\rangle, \quad (6)
\]

where \( |\nu_1\rangle \) and \( |\nu_2\rangle \) are the two neutrino mass eigenstates and \( \theta_e \) is the vacuum mixing angle.

To compute the probability for the transition to exist, we considered that \( \mu_e = 3 \times 10^{-12} \mu_B \), which is a rather large value (Raffelt 1999). Here \( \mu_B \) is the Bohr magneton. Depending on the difference in the mass square of the two neutrino mass eigenstates (\( \Delta m^2 \)), the RSFP resonance occurs at a different depth in the Sun. The recent results from the KamLAND experiment (Kamioka Liquid scintillator Anti-Neutrino Detector; see, e.g., Fogli et al. 2003) favor the MSW-LMA solution (MSW effect, Large Mixing Angle) to the solar neutrino puzzle. That means \( \Delta m^2 \geq 5.5 \times 10^{-5} \) eV\(^2\). Therefore, we first assumed \( \Delta m^2 = 5 \times 10^{-5} \) eV\(^2\) to test how the RSFP may act. The left-hand panel of Figure 10 shows the probability for the transition to occur as a function of the magnetic field amplitude, if \( \Delta m^2 = 5 \times 10^{-5} \) eV\(^2\), and for \( \cos \theta_e = 1 \). We also set the energy of the neutrino to 15 MeV (boron neutrino). In this case the resonance is located at about 0.2 \( R_\odot \). The probability computation is based on the work of Petcov (1997).

As a result of the resonance location with our assumptions, the magnetic field in the very solar core may affect the neutrino transport. On the figure, it appears that the amplitude of this field in the radiative zone must be everywhere larger than \( 10^5 \) G; otherwise, the transition probability is zero. For instance, an amplitude of \( 10^6 \) G gives a probability of 34.7\% for the transition to occur. This amplitude is smaller than the upper limit previously derived from the seismic data. This limit is very likely conservative but means that if the \( \Delta m^2 \) value of the Majorana neutrinos is larger than \( 5 \times 10^{-3} \) eV\(^2\), then it is possible that the deep magnetic field impacts the neutrino transport.

A similar analysis with a smaller \( \Delta m^2 \) shows that the resonance occurs closer to the solar surface when the difference in the mass square decreases. As an example, we set \( \Delta m^2 = 10^{-8} \) eV\(^2\). The resonance occurs beyond 0.9 \( R_\odot \).
this case, the right-hand panel of Figure 10 shows that a field larger than about $10^3 \, \text{G}$ is enough for the transition to occur with a probability of a few percent (but the field must maintain this amplitude all along the solar radius).

However, the magnetic field of the convective zone is not a large-scale mean magnetic field, as was assumed in our calculations. To accurately compute the impact of the in-convective-zone field on the neutrino transport, we need to run three-dimensional magnetohydrodynamic simulations of this solar region. Thus, we will also get access to the way the electron and neutron densities vary inside the flux tubes and the magnetic profile of these tubes.

The short analysis we carried out shows that if the difference in the mass square of a Majorana neutrino is larger than or equal to the one favored by the LMA solution, then large magnetic fields in the solar core might act on the neutrino transport. Such fields are not excluded by the seismic data. On the other hand, if the $\Delta m^2$ of the neutrinos is small enough, then those are the magnetic fields in the convective zone that play some role. The recent KamLAND results favor a rather large $\Delta m^2$ value. However, Sturrock & Scargle (2001), Sturrock & Weber (2002), and Sturrock & Caldwell (2002) found that the neutrino flux emitted by the Sun is modulated by the solar rotation. This modulation seems to occur mainly in the convective zone and is probably due to magnetic fields whose flux tubes are frozen in the solar plasma. In the absence of firm conclusion about $\Delta m^2$ it is still difficult to know exactly where, in the Sun, the magnetic fields are more likely to impact the neutrino transport. However, the upper limits derived with the seismic data on the magnetic field intensity do not rule out such an impact.

7. CONCLUSION

Thanks to the recent seismic data from the GOLF and MDI instruments aboard SOHO, a precise sound speed profile was derived down to $0.07 \, \text{R}_\odot$. With this profile, we have derived solar models that are very close to the Sun regarding the sound speed below $0.65 \, \text{R}_\odot$. We wanted to obtain the agreement in this part of the Sun to deduce neutrino (and g-mode) predictions. Neutrino fluxes are very sensitive to the temperature profile in the neutrino production regions (especially for the $^8\text{B}$), hence the need to get a closer agreement to the Sun in these regions. The first three seismic models we propose fulfill this requirement. We have also shown that the different assumptions to reproduce the solar sound speed do not change the $^8\text{B}$ neutrino fluxes by more than $\pm4.5\%$ (including the presence of a magnetic field in the core). The results of this paper confirm the excellent agreement between seismic predictions and observations of the neutrino fluxes with the SNO detector. The neutrino predictions are known with a better precision, and here we prove their stability and convergence, using seismic data.

Further improvements in solar modeling require three-dimensional codes to correctly account for the dynamic processes in the Sun and the detection of some g-modes for a study of the very center of our Sun.

With our models we derive frequencies for the acoustic and gravity modes. These frequency predictions, together with the internal rotation profile, should help in the detection of g-modes.

The solar models considered so far are the result of years of improvements in both the seismic data and the solar physics. The sound speed profile is quite sensitive to the $p$-$p$ reaction rate, the metallicity, the opacities, the solar age, and even some magnetic fields. This is a powerful tool that might be useful to constrain the large-scale solar magnetism, provided that some further improvements are realized on the other physical parameters that have a large impact on $c_s$.

Another result of our solar models is to favor a slight increase of the $p$-$p$ reaction rate by $\approx1\%$. They also support the solar age of 4.6 Gyr (even if the sound speed favors a slightly larger value). We also find that the presence of a large-scale magnetic field in the radiative interior does not seem to change significantly the neutrino emission: with the field constrained by the seismic data, the boron flux is only changed by 2%. We discuss the impact of the magnetic fields on the neutrino transport, considering the RSFP theory: the recent KamLAND results favor a $\Delta m^2$ range of values implying that the magnetic fields of the radiative interior might affect the neutrino transport (provided that their intensity is large enough). Always to deal with the neutrino puzzle, we compute the different quantities necessary to deduce the neutrino oscillation parameters.
Many advances are still needed in the solar models, especially for the upper layers, but the high quality of the seismic data combined with the improvement in the physics have allowed us already to achieve a good result in solar modeling: the seismic$^1$ and seismic$^2$ models are very close to the real Sun in the regions of concern, and the seismic$^3$B$^1$1 or B$^2$2 models are also interesting to consider as a result of the presence of magnetic pressure. Contrary to the usual seismic models, our solar models were obtained with a stellar evolution code. However, as previously mentioned, as far as the very internal rotation profile is not included in such a study, new surprises may appear that invalidate the “classical” approach as it does for the solar convective zone.

The local helioseismology methods that are currently under development should provide invaluable information about the solar surface and the upper part of the convection zone. These techniques, combined with some hydrodynamical simulations of the upper solar layers, should improve the solar modeling.

The seismic$^1$ model is available with the detailed values of many parameters for a large number of shells, including the electron and neutron number densities, on the World Wide Web.$^4$

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4 See http://www-dapnia.cea.fr/Phys/Sap/Documents/soilei/solarmodel.html.