Effective theory for real-time dynamics in hot gauge theories

Edmond Iancu

*Service de Physique Théorique, CE-Saclay, 91191 Gif-sur-Yvette, France*

Abstract

For a high temperature non-Abelian plasma, we reformulate the hard thermal loop approximation as an effective classical thermal field theory for the soft modes. The effective theory is written in local Hamiltonian form, and the thermal partition function is explicitly constructed. It involves an ultraviolet cutoff which separates between hard and soft degrees of freedom in a gauge-invariant way, together with counterterms which cancel the cutoff dependence in the soft correlation functions. The effective theory is well suited for numerical studies of the non-perturbative dynamics in real time, in particular, for the computation of the baryon number violation rate at high temperature.
The violation of the baryon number in the high-temperature, symmetric phase of the electroweak theory is an important example of a physical process which is sensitive to the non-perturbative real-time dynamics of hot gauge theories [1]. Unlike static characteristics, like the free energy, which can be computed on the lattice, the non-perturbative evolution in real-time cannot be studied through the standard lattice simulations formulated in imaginary-time. However, it has long been recognized that a fully quantum calculation is actually not necessary [1]: the non-perturbative phenomena are associated with long wavelength (λ ≳ 1/g²T) magnetic fields, which, because of Bose enhancement, have large occupation numbers,

\[ N_0(E) \equiv \frac{1}{e^{\beta E} - 1} \approx \frac{T}{E} \quad \text{for} \quad E \ll T, \quad (1) \]

and should therefore exhibit a classical behaviour. Based on this observation, there has been attempts to compute the baryon number violation rate \( \Gamma \) through lattice simulations of the classical thermal Yang-Mills theory [2, 3].

However, it has been recently observed [4, 5] that the baryon violating processes are actually sensitive to the hard thermal modes with momenta \( \sim T \), for which the classical approximation is well-known to fail: the hard modes cause the damping of the soft field configurations, an effect which is predicted to reduce \( \Gamma \) by a factor of \( g^2 \) as compared with its classical estimate in Refs. [1, 2]. In order to verify this prediction and eventually compute \( \Gamma \), one has to properly take into account the effects of the hard modes on the dynamics of the soft fields. To leading order in \( g \), these effects are encompassed by the so-called “hard thermal loops” (HTL) [6–11], which are non-local one-loop corrections to the soft (\( k \ll gT \ll T \)) field propagator and vertices due to the hard (\( k \sim T \)) thermal modes.

It has been first suggested in Ref. [12] to use the local version of the HTL effective theory [8–11] in classical lattice simulations in order to compute the baryon number violation rate. However, in order to transpose this idea into practice, one first needs a precise, and gauge-invariant, separation between hard and soft degrees of freedom (to avoid overcounting, and to provide an ultraviolet cutoff to the effective theory for the soft modes). Loosely speaking, this requires an intermediate scale \( \mu \), with \( gT \ll \mu \ll T \), which should act as an infrared (IR) cutoff for the hard modes and as an ultraviolet (UV) cutoff.

\[ T \] denotes the temperature, assumed to be large enough for the coupling constant \( g(T) \) to be small: \( g \ll 1 \).
for the soft ones, and which should cancel in the calculation of physical quantities. But the practical implementation of such a separation of scales meets with technical difficulties: (i) In the hard sector, one cannot simply introduce $\mu$ as an infrared cutoff in the one-loop diagrams for the HTL's since this would break gauge symmetry \cite{12}. (ii) In the soft sector, one cannot use the lattice spacing to provide the necessary upper cutoff $\sim \mu$: indeed, a finite (and relatively large: $a \sim \mu^{-1} \gg T^{-1}$) lattice spacing introduces lattice artifacts which make impossible the matching with the hard sector \cite{12,13}.

Another problem is the proper definition of classical thermal expectation values within the effective theory. In order to compute such expectation values, one has to solve first the equations of motion for given initial conditions, and then average over the classical phase space with the Boltzmann weight $\exp(-\beta H)$. Since the local formulations of the HTL theory which are currently available \cite{8,9,11} are not in canonical form, it is a non-trivial task to identify the independent degrees of freedom and construct the classical phase space.

The purpose of this Letter is to give an explicit solution to the above mentioned problems by constructing a local effective theory for the soft modes with a $\mu$-dependent Hamiltonian. The classical partition function will involve an explicit UV cutoff, chosen so as to cancel the $\mu$-dependence of the Hamiltonian in the calculation of IR-sensitive correlation functions, like the baryon number violation rate $\Gamma$.

Our construction relies in an essential way on the local formulation of the HTL theory presented in Ref. \cite{8}, which we briefly review now. It involves a set of coupled equations for the soft fields and their induced current, namely eqs. (2)–(4) below. The soft fields $A_\mu^a(x)$ satisfy the Yang-Mills equations with an induced current in the right hand side:

\[(D_\nu F^{\nu\mu})_a = j^\mu_a, \quad (2)\]

where $D_\mu = \partial_\mu + ig[A_\mu, \cdot ]$, $A_\mu = A_\mu^a T^a$, and $F_{\mu\nu} = [D_\mu, D_\nu]/(ig)$. (The generators of the colour group in the adjoint representation are denoted by $T^a$; they satisfy $[T^a, T^b] = if^{abc} T^c$ and $\text{Tr}(T^a T^b) = C_A \delta^{ab}$, with $C_A = N$ for SU($N$).) The induced current $j_\mu = j_\mu^a T^a$ is related to the colour fluctuations of the hard thermal modes:

\[j^\mu_a(x) = 2gC_A \int \frac{d^3k}{(2\pi)^3} v^\mu \delta N_a(k, x). \quad (3)\]

In this equation, $\delta N(k, x) = \delta N_a(k, x) T^a$ is a phase-space colour density matrix for hard gluons ($|k| \sim T$), which describes long-wavelength colour correlations as induced by the
soft fields $A^a_\mu$. Furthermore, $\nu^\mu = (1, \mathbf{v})$ and $\mathbf{v} = \mathbf{k}/k$ is the velocity of the hard particle ($k = |\mathbf{k}|$, and $|\mathbf{v}| = 1$). The system is closed by the kinetic equation for the density matrix, which is a non-Abelian generalization of the Vlasov equation:

\[
 (v \cdot D_x)\delta N(\mathbf{k}, x) = -g \mathbf{v} \cdot \mathbf{E}(x) \frac{dN_0}{dk},
\]

where $E^i_a \equiv F^0_a$ and $N_0(k) = 1/(e^{\beta k} - 1)$. The dynamics described by the above equations is gauge invariant, and the current $j^\mu_a$ is covariantly conserved: $D^\mu j_\mu = 0$.

From eq. (4), we note that the $\mathbf{v}$ and $k$-dependence can be factorized in $\delta N^a(\mathbf{k}, x)$ by writing:

\[
 \delta N^a(\mathbf{k}, x) \equiv -gW^a(x, \mathbf{v}) \left(dN_0/dk\right).
\]

The new functions $W^a(x, \mathbf{v})$ satisfy the equation:

\[
 (v \cdot D_x)W(x, \mathbf{v}) = \mathbf{v} \cdot \mathbf{E}(x),
\]

which is independent of $k$ since the hard particles move at the speed of light: $|\mathbf{v}| = 1$.

Eqs. (2)–(6) above provide a local description of the soft field dynamics in the HTL approximation. In order to use these equations for classical thermal calculations, one needs to (i) introduce an IR cutoff $\mu$ in the hard sector, (ii) perform a Hamiltonian analysis (to identify the independent degrees of freedom and the corresponding Hamiltonian), (iii) write down the classical partition function, and (iv) supply the effective theory with an UV cutoff $\sim \mu$. We shall address these problems in this order:

(i) By inspection of egs. (2)–(4), it is quite obvious how to introduce the intermediate scale $\mu$: since $\mathbf{k}$ is the momentum carried by the hard particles, it is sufficient to integrate in eq. (3) with a lower cutoff equal to $\mu$. With eq. (3), the radial integration in eq. (3) can be worked out, with the result

\[
 j^\mu_a(x) = m^2_H(\mu) \int \frac{d\Omega}{4\pi} v^\mu W_a(x, \mathbf{v}),
\]

where the angular integral $\int d\Omega$ runs over the unit sphere spanned by $\mathbf{v}$, and

\[
 m^2_H(\mu) \equiv -\frac{g^2CA}{\pi^2} \int_\mu^\infty dkk^2 \frac{dN_0}{dk} \sim \frac{g^2CA}{3} \left(T^2 - \frac{3}{\pi^2}\mu T\right).
\]

The quantity $m_D \equiv m_H(\mu = 0)$ is the physical Debye mass to leading order in $g$. For $gT << \mu << T$, $m_H(\mu)$ is the hard sector contribution to $m_D$. 

3
In contrast to the usual one-loop calculations [12], the above implementation of \( \mu \) has preserved gauge symmetry automatically: indeed, the kinetic equation (4) is gauge covariant for any value of \( k \), so that the \( \mu \)-dependent current in eq. (7) is covariantly conserved.

(ii) A Hamiltonian analysis of the HTL theory has been given by Nair [9], in terms of some new auxiliary fields. Here, we shall rather follow Refs. [11] and propose a simpler Hamiltonian formulation which involves the fields \( W_a(x,v) \) introduced above. In the gauge \( A_{a0} = 0 \), the independent degrees of freedom are \( E^a_i, A^a_i \) and \( W^a \), and the corresponding equations of motion follow from eqs. (2), (6) and (7) above:

\[
E^a_i = -\partial_0 A^a_i, \\
-\partial_0 E^a_i + \epsilon_{ijk}(D_j B_k)^a = m_H^2(\mu) \int \frac{d\Omega}{4\pi} v_i W^a(x,v), \\
(\partial_0 + v \cdot D)^{ab} W_b = v \cdot E^a, \\
\]

(9)

together with Gauss' law which in this gauge must be imposed as a constraint:

\[
(D \cdot E)^a + m_H^2(\mu) \int \frac{d\Omega}{4\pi} W^a(x,v) = 0.
\]

(10)

Note that eqs. (9) are not in canonical form: this is already obvious from the fact that we have an odd number of equations. Accordingly, it is not a priori clear that these equations are Hamiltonian in any sense. It has been shown in Ref. [11] that eqs. (9) are conservative: the associated, conserved energy can be computed in any gauge as:

\[
H = \frac{1}{2} \int d^3x \left\{ E_a \cdot E_a + B_a \cdot B_a + m_H^2(\mu) \int \frac{d\Omega}{4\pi} W_a(x,v) W_a(x,v) \right\}.
\]

(11)

Remarkably, we show now that, in the gauge \( A_{a0} = 0 \), the functional (11) also acts as a Hamiltonian, that is, as a generator of the time evolution. To this aim, we introduce the following Poisson brackets (see also Ref. [9]):

\[
\{ E^a_i(x), A^b_j(y) \} = -\delta^{ab}\delta_{ij}\delta^{(3)}(x-y), \\
\{ E^a_i(x), W^b(y,v) \} = v_i \delta^{ab}\delta^{(3)}(x-y), \\
m_H^2 \{ W^a(x,v), W^b(y,v') \} = \left( g f^{abc} W^c + (v \cdot D_x)^{ab} \right) \delta^{(3)}(x-y)\delta(v,v').
\]

(12)

\[\dagger\] Recall that the Hamiltonian structure is a non-trivial issue already for the Abelian Maxwell-Vlasov equations [14], of which eqs. (9) can be seen as a non-Abelian generalization.

\[\S\] More precisely, these are generalized Lie-Poisson brackets, according to the terminology in Ref. [14].
Here, $\delta(v, v')$ is the delta function on the unit sphere, normalized such that
\[
\int \frac{d\Omega}{4\pi} \delta(v, v') f(v) = f(v'),
\] (13)
and all the other Poisson brackets are assumed to vanish. We also assume standard properties for such brackets, namely antisymmetry, bilinearity and Leibniz identity. It is then straightforward to verify that (a) the Poisson brackets (12) satisfy the Jacobi identity (as necessary for consistency) and (b) the equations of motion (9) follow as canonical equations for the Hamiltonian (11). For instance, $\partial_0 W^a = \{H, W^a\}$, and similarly for $E_i^a$ and $A_a^i$.

Note that the effective theory in eqs. (9)–(12) involves the infrared cutoff $\mu$ (and also the temperature $T$) only through a single mass parameter, namely the “hard” Debye mass $m^2_H(\mu)$ of eq. (8).

(iii) We are now in position to construct (generally time-dependent) thermal expectation values within the classical field theory defined by eqs. (9)–(11). The thermal phase-space is defined by the initial conditions for eqs. (9), and the canonical weight is given by the effective Hamiltonian (11). Thus, the thermal correlation functions of the fields $A_a^i$ can be obtained from the following generating functional:
\[
Z_{cl}[J^a_i] = \int D\mathcal{E}_i^a D\mathcal{A}_i^a D\mathcal{W}^a \delta(G^a) \exp \left\{ -\beta H + \int d^4x J^a_i(x) A^a_i(x) \right\},
\] (14)
where $A^a_i(x)$ is the solution to eqs. (9) with the initial conditions $\{E^a_i, A^a_i, W^a\}$ (that is, $E^a_i(t_0, \mathbf{x}) = \mathcal{E}^a_i(\mathbf{x})$, etc., with arbitrary $t_0$), and $H$ is expressed in terms of the initial fields. Since the dynamics is gauge-invariant, it is sufficient to enforce Gauss’ law at $t = t_0$:
\[
G^a \equiv (\mathcal{D}_i \mathcal{E}_i)^a + m^2_H(\mu) \int \frac{d\Omega}{4\pi} \mathcal{W}^a = 0.
\] (15)

The only subtle point in eq. (14) is the definition of the measure in the phase-space\footnote{I am grateful to Tanmoy Bhattacharya for an illuminating discussion on this point.}: since we are not using canonical variables, we still have to verify that the naïve measure $D\mathcal{E}_i^a D\mathcal{A}_i^a D\mathcal{W}^a$ is indeed the correct one. A necessary condition is that this measure be invariant under the time evolution described by eqs. (9), so that $Z_{cl}[J]$ be independent of $t_0$, as it should. This condition can be most easily verified by considering an infinitesimal time evolution of the form $\Phi_\alpha \rightarrow \Phi'_\alpha \equiv \Phi_\alpha + \{H, \Phi_\alpha\} dt$, where $\Phi_\alpha$ refers to any of the field variables $\{\mathcal{E}_i^a, \mathcal{A}_i^a, \mathcal{W}^a\}$. Then, by using eqs. (9) — or, equivalently, the Poisson brackets
it is straightforward to verify that the Jacobian for this transformation is equal to one, to linear order in $dt$:

$$J \equiv \left| \frac{\delta (\mathcal{E}_i', \mathcal{A}_i', \mathcal{W}')}{\delta (\mathcal{E}_i, \mathcal{A}_i, \mathcal{W})} \right| = 1 + O((dt)^2).$$  (16)

As a further check, one can verify that eq. (14) reduces to standard results in some particular cases. For instance, for $J^a_i = 0$ this equation yields the result expected from dimensional reduction \[15\], that is,

$$Z_{cl} = \int \mathcal{D}A_0^a \mathcal{D}A_i^a \exp \left\{ -\frac{\beta}{2} \int d^3x \left( B_i^a B_i^a + (\mathcal{D}_i A_0)^a (\mathcal{D}_i A_0)^a + m_H^2 (\mu) A_0^a A_0^a \right) \right\},$$  (17)

where the $A_0^a$ components of the gauge fields have been reintroduced as Lagrange multipliers to enforce Gauss' law, and the functional integrals over $\mathcal{E}_i^a$ and $\mathcal{W}^a$ have been explicitly performed. Conversely, the general formula (14) can be seen as a generalization of the dimensional reduction method to include dynamical (i.e., time-dependent) phenomena. Another check is provided by the Abelian limit, where eq. (14) yields, after a straightforward calculation:

$$Z_{cl}[J_i] = \exp \left\{-\frac{1}{2} \int d^4x \int d^4y J_i(x) * D_{ij}(x-y) J_j(y) \right\},$$

$$*D_{ij}(x-y) = \int \frac{d^4q}{(2\pi)^4} e^{-iq(x-y)} * \rho_{ij}(q) N_{cl}(q_0),$$  (18)

where $*\rho_{ij}(q)$ is the magnetic photon spectral density in the HTL approximation \[6\] and $N_{cl}(q_0) \equiv T/q_0$ is the classical thermal distribution function, which coincides with the low energy limit of the quantum distribution (cf. eq. (1)). In the second line of eq. (18) we recognize, as expected, the classical limit of the soft photon 2-point function in the HTL approximation.

(iv) The last step toward a well-defined classical effective theory is to supply the partition function (14) with an ultraviolet cutoff $\Lambda_{cl} \sim \mu$, chosen so as to cancel — in the calculation of the soft correlation functions — the explicit $\mu$-dependence of the effective Hamiltonian (11) (a cancellation to be subsequently referred to as matching). Since the effective theory is ultimately intended for non-perturbative calculations, it will be convenient to choose a regularization method which can be also implemented on a lattice. This cannot be the lattice spacing itself: indeed, if we choose to work with a finite lattice spacing $a \sim 1/\mu$, then we break rotational and dilatation symmetry, and the UV structure of the lattice theory gets so complicated that the matching cannot be performed anymore.
Lattice artifacts can be eliminated only by taking the continuum limit \( a \to 0 \), which requires \( a \) to be independent of \( \mu \).

The strategy that we propose here is thus the following: the effective theory will be formulated as a cutoff theory \textit{in the continuum}, and the matching will be performed in the continuum, rotationally-invariant theory. Then, for computational purposes, the resulting cutoff theory must be put on a lattice with small lattice spacing \( a \ll 1/\Lambda_{cl} \). Because of the explicit UV cutoff \( \Lambda_{cl} \), the continuum limit \( a \to 0 \) is well-defined.

Following Ref. \cite{13}, we introduce a smooth UV cutoff in the continuum theory by replacing, in the effective Hamiltonian \( (11) \),

\[
\text{Tr} \, B^i B^i \rightarrow \text{Tr} \, B^i f \left( \frac{D^2}{\Lambda_{cl}^2} \right) B^i, \tag{19}
\]

where \( f(z) = 1 + z^2 \) and the trace refers to color indices. Besides being gauge-invariant, the regularization prescription in eq. \( (19) \) has also the advantage that it can be carried out on the lattice (by using improved lattice Hamiltonians \cite{13,16}). However, this prescription breaks down dilatation symmetry and, as a consequence, the matching cannot be performed for all the soft correlation functions (see below), but only for the non-perturbative quantities which are infrared sensitive, like the baryon number violation rate \( \Gamma \). Let us explain this in more detail:

For matching purposes, we need the \( \Lambda_{cl} \)-dependent corrections to the soft correlation functions to one-loop order in the effective theory. Rather than computing loop diagrams, it is more convenient to rely on kinetic theory to describe the interactions between the genuinely soft fields, with momenta \( k \ll gT \), and the relatively “hard” classical modes, with momenta \( k \gtrsim \Lambda_{cl} \). The relevant kinetic equations can be derived in the same way \cite{8}, and look similarly, to our previous equations \( (2) - (6) \). The only differences refer to the replacement of \( N_0(k) \) by \( N_{cl}(E_k) \equiv T/E_k \), and of the unit vector \( \mathbf{v} \) by the group velocity \( \mathbf{v}_k \equiv \nabla_k E_k \). (These prescriptions can be readily verified by inspecting the derivation of the kinetic equations in Ref. \cite{8}; they have been also justified by a diagrammatic analysis in Ref. \cite{13}.) Here, \( E_k^2 = k^2 f(k^2 / \Lambda_{cl}^2) \) is the dispersion equation for the relatively “hard” \( (k \gtrsim \Lambda_{cl}) \) classical excitations, for which the HTL corrections are relatively small (since \( m_H \sim gT \ll \Lambda_{cl} \)) and can be neglected to the order of interest. It then follows

\footnote{Note that a similar strategy has been recently proposed by Arnold, in the context of purely classical Yang-Mills theory \cite{13}; of course, the matching was not an issue in Ref. \cite{13}, which was rather concerned with improving the rotational symmetry in classical lattice simulations.}

\[\parallel\]
that the colour current due to the “hard” classical modes — and which summarizes the \( \Lambda_{cl} \)-dependence of the effective theory to one loop order — has the form (compare to eqs. (7)–(8)):

\[
  j_{S}^{\mu}(x) = -2g^2C_{A} \int \frac{d^3k}{(2\pi)^3} \frac{dN_{cl}}{dE_{k}} v^{\mu}_{k} W^{a}(x, v_{k}),
\]

with the functions \( W^{a}(x, v_{k}) \) satisfying (in the temporal gauge \( A_{0} = 0 \)):

\[
  (\partial_{0} + v_{k} \cdot D)^{ab} W_{b}(x, v_{k}) = v_{k} \cdot E^{a}(x).
\]

An important difference with respect to eq. (7) is that the classical “hard” excitations do not move at the speed of light, but rather with a \( k \)-dependent velocity \( v_{k} \). Accordingly, the radial and angular integrations in eq. (20) cannot be disentangled anymore, and the current \( j_{S}^{\mu} \) will not be characterized, in general, by a single mass scale (in contrast to the HTL current in eq. (7)), which prevents us from performing a full matching.

However, as we show now, the matching can still be done in the calculation of non-perturbative quantities which are infrared sensitive. It is indeed well-known (see, e.g., Refs. [4, 5, 13, 17]) that the field configurations which are responsible for the non-perturbative phenomena (and also for the IR divergences of the perturbation theory) are very soft \( (k \equiv |k| \sim g^{2}T) \) magnetic fields of almost zero frequency: \( k_{0} \ll g^{4}T \ll k \).

Indeed, these are the only configurations which are not screened at the scale \( gT \) by the HTL’s. For such fields, time derivatives are suppressed with respect to spatial gradients, and eq. (21) reduces to:

\[
  (v \cdot D)W(x, v_{k}) = v \cdot E(x),
\]

where we have been able to simplify one factor of \( |v_{k}| \), so that \( v \) is an unit vector, as in eq. (8). Eqs. (22) shows that on the relevant, non-perturbative field configurations, the function \( W(x, v_{k}) \equiv W(x, v) \) is independent of the radial momentum \( k \). Then, the radial integral in eq. (21) factorizes and yields (for the relevant, magnetic piece of the current):

\[
  j^{ia}_{S}(x) \simeq m^{2}_{S}(\Lambda_{cl}) \int \frac{d\Omega}{4\pi} v^{i} W^{a}(x, v),
\]

with (compare to eq. (5)):

\[
  m^{2}_{S}(\Lambda_{cl}) = -\frac{g^{2}C_{A}}{\pi^{2}} \int_{0}^{\infty} dk \frac{k^{2}}{E_{k}} dN_{cl} \frac{dE_{k}}{dE_{k}} \frac{N_{cl}(E_{k})}{\kappa g^{2}C_{A} T \Lambda_{cl}}.
\]
The precise value of the numerical coefficient \( \kappa \) can be found in eq. (4.8) of Ref. [13].

Eq. (23) shows that, to one-loop order in the effective theory, the \( \Lambda_{cl} \)-dependent corrections to the amplitudes involving quasistatic and soft \( (k_0 \ll k \sim g^2 T) \) magnetic fields are characterized by a single mass scale, namely \( m_S^2(\Lambda_{cl}) \). This is similar to the HTL current in eq. (7), so it is now possible to perform the matching by requiring \( m_S^2(\Lambda_{cl}) \), eq. (24), to cancel the \( \mu \)-dependent piece of \( m_H^2(\mu) \), eq. (8). This is achieved by choosing \( \mu = \pi^2 \kappa \Lambda_{cl} \): with this matching condition, the IR-sensitive quantities computed in the effective theory (9)–(14) with the UV regularization (19) come out independent of \( \mu \) and \( \Lambda_{cl} \), for \( \mu \) in a large range of values: \( gT \ll \mu \ll T \).

In particular, the effective theory thus defined provides a \( \mu \)-independent value for the baryon number violation rate \( \Gamma \), to be ultimately computed on the lattice. The lattice implementation of the effective theory (which requires a lattice version of the new fields \( W^a(x, v) \) and of the corresponding equations of motion) is by itself a non-trivial issue, which remains beyond the scope of the present work.

Let us finally note a different proposal [18] for including the HTL’s, which is to treat the hard degrees of freedom as classical coloured particles [10]. This has been recently implemented in lattice simulations [19], with results which seem to confirm the predictions in Ref. [4]. By comparison, the method that we have proposed here, besides being derived from first principles, has also the advantages to give the hard modes the correct quantum statistics, to involve no free parameter, and to perform a precise matching between hard and soft degrees of freedom, thus allowing for the continuum limit to be taken in lattice simulations.

To conclude, we have provided an effective classical thermal field theory for the soft modes which includes the hard modes in the HTL approximation and which is well-suited for numerical studies of the real-time non-perturbative dynamics. Important applications include the high-\( T \) anomalous baryon number violation, the dynamics of the electroweak phase transition (which requires adding the Higgs field to the above theory), and non-perturbative properties of a hot quark-gluon plasma.

**Acknowledgements:** During the elaboration of this paper, I have benefited from discussions and useful remarks from a number of people. It is a pleasure to thank J. Ambjørn, T. Bhattacharya, J.P. Blaizot, S. Habib, A. Krasnitz, L. McLerran, E. Mottola, J.Y. Ollitrault, M.E. Shaposhnikov, A. Smilga and N. Turok.
References

[1] For a recent review, see V.A. Rubakov and M.E. Shaposhnikov, hep-ph/9603208, Usp. Fiz. Nauk. 166 (1996) 493.

[2] J. Ambjørn and A. Krasnitz, Phys. Lett. B362 (1995) 97; ibid. Nucl. Phys. B506 (1997) 387.

[3] G. D. Moore and N. G. Turok, Phys. Rev. D 55 (1997) 6538; ibid. Phys. Rev. D56 (1997) 6533.

[4] P. Arnold, D. Son, and L. Yaffe, Phys. Rev. D55 (1997) 6264.

[5] P. Huet and D. Son, Phys. Lett. B393 (1997) 94; D. Son, hep-ph/9707351.

[6] R.D. Pisarski, Phys. Rev. Lett.63 (1989) 1129; E. Braaten and R.D. Pisarski, Nucl. Phys. B337 (1990) 569.

[7] J. Frenkel and J.C. Taylor, Nucl. Phys. B334 (1990) 199; J.C. Taylor and S.M.H. Wong, ibid. B346 (1990) 115; R. Efraty and V.P. Nair, Phys. Rev. Lett.68 (1992) 2891; R. Jackiw and V.P. Nair, Phys. Rev. D48 (1993) 4991.

[8] J.P. Blaizot and E. Iancu, Phys. Rev. Lett.70 (1993) 3376; Nucl. Phys. B417 (1994) 608.

[9] V.P. Nair, Phys. Rev. D48 (1993) 3432; ibid. D50 (1994) 4201.

[10] P.F. Kelly, Q. Liu, C. Lucchesi and C. Manuel, Phys. Rev. Lett.72 (1994) 3461.

[11] J.P. Blaizot and E. Iancu, Nucl. Phys. B421 (1994) 565; ibid. B434 (1995) 662.

[12] D. Bödeker, L. McLerran, and A. Smilga, Phys. Rev. D52 (1995) 4675.

[13] P. Arnold, Phys. Rev. D55 (1997) 7781.

[14] J.E. Marsden and T.S. Ratiu, Introduction to Mechanics and Symmetry: A Basic Exposition of Classical Mechanical Systems (Springer Verlag, 1995).

[15] K. Farakos, K. Kajantie, K. Rummukainen and M.E. Shaposhnikov, Nucl. Phys. B425 (1994) 67; K. Kajantie, M. Laine, K. Rummukainen and M.E. Shaposhnikov, Nucl. Phys. B458 (1996) 90; E. Braaten, Phys. Rev. Lett.74 (1995) 2164.
[16] G.D. Moore, Nucl. Phys. B480 (1996) 689.

[17] J.P. Blaizot and E. Iancu, Phys. Rev. Lett. 76 (1996) 3080; Phys. Rev. D56 (1997) 7877.

[18] C. Hu and B. Müller, Phys. Lett. B409 (1997) 377.

[19] G.D. Moore, C. Hu and B. Müller, hep-ph/9710436.