On Estimating the Autoregressive Coefficients of Time-Varying Fading Channels

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Abstract—As several previous works have pointed out, the evolution of the wireless channels in multiple input multiple output systems can be advantageously modeled as an autoregressive process. Therefore, estimating the coefficients, and, in particular, the state transition matrix of this autoregressive process is a key to accurate channel estimation, tracking, and prediction in fast fading environments. In this paper we assume the time varying spatially uncorrelated channel which is approximately the case with proper antenna spacing at the base station in rich scattering environments. We propose a method for autoregressive parameter estimation for a single input multiple output (SIMO) channel. We show an almost sure convergence of the estimated coefficients to the true autoregressive coefficients in large dimensions. We apply the proposed method to the SIMO channel tracking.

Index Terms—Time-varying channels, multiple antenna systems, autoregressive models, parameter estimation

I. INTRODUCTION

It is well-known that the temporal variations of wireless channels due to changes in the propagation environment or mobility are advantageously modeled by autoregressive (AR) processes. When the parameters of the AR process are accurately estimated, estimating and predicting the process states, and thereby the wireless channel coefficients become feasible by Kalman filters [1], [2] Therefore, a large body of works related to the estimation of the parameters of AR processes as well as the application of such processes to channel estimation, prediction, equalization and detection exists [2]–[10]. Approximating mobile channels in single input single output systems with an AR model has been studied in e.g. [3] and [5]. While in [5] a first order AR model is used for data-aided signal-to-noise ratio (SNR) estimation, the works reported in [2] and [3] use higher order AR models for developing channel estimation and data detection algorithms. In contrast, papers [1] and [4] study multiple-input multiple-output (MIMO) systems in fast Rayleigh fading environments and use AR processes to characterize the temporal variations of the channels, and evaluate their effects on the receiver structures and performance. More recently, paper [8] developed algorithms for tracking the angles of departures and arrivals in multi-antenna systems using extended Kalman filter.

In the context of large-scale MIMO systems, a series of recent works have focused on combatting the negative effects of channel aging [6], [7], [9], [10]. These papers also make use of the characteristics of AR models for channel estimation and prediction purposes, since high quality channel state information is needed for various MIMO algorithms, including data reception in the uplink and spatial precoding in the downlink. Recognizing the importance of properly mapping the AR process to the measured wireless channel variations, papers [10], [11] use the Yule-Walker and Levinson-Durbin equations to identify the AR system parameters. In a recent work reported in [12] several algorithms to estimate the AR coefficients of p-order processes, denoted by AR(p), are developed. The proposed algorithms in [12] are useful in practice, because they not only estimate the AR coefficients, but also the variance of the observation and process noise, based only on measurements that are feasible in practice.

In this paper, we propose an estimation method of AR coefficients of a single-input multiple-output (SIMO) channel vector following AR(p) process making use of measurements over the time and spatial dimensions. We assume the signals arriving at multiple antennas are uncorrelated and the variances or the underlying noise processes can be estimated by existing noise variance estimation schemes [12]. The main contribution consists in deriving concentration inequalities that are useful for evaluating the consistency of the proposed estimators. To illustrate the operation of the proposed technique, we apply it to channel tracking in a SIMO system.

The rest of this paper is structured as follows. The next section presents our system model. Next, Section III proposes an estimation of the AR(p) parameters, while Section IV shows a specific application of the AR model in the context of wireless channel tracking. Section V discusses numerical results obtained by the proposed estimation scheme and compares the results to relevant benchmarks. Finally, Section VI concludes the paper.

II. SYSTEM MODEL

A. AR p-deep model

We consider the vector $h(t) = [h_0(t), \ldots, h_{N_r-1}(t)]^T \in \mathbb{C}^{N_r \times 1}$ with independent and identically distributed (i.i.d.) elements for $n \in 0, \ldots, N_r - 1$. We assume each element $(h_n(t))_{t \in \mathbb{Z}}$, for $n \in \{0, \ldots, N_r - 1\}$, is a complex Gaussian stationary process following the model AR(p) in time. Assum-
ing the size of the observation window is equal to \( T \), at time instant \( t \in 0, \ldots, T - 1 \), we have

\[
\hat{h}(t) = A_1\hat{h}(t-1) + \cdots + A_p\hat{h}(t-p) + x(t)
\]

(1)

where the matrices \( A_i \in \mathbb{C}^{N_r \times N_r} \) for \( i \in \{1, \ldots, p\} \) are assumed to be constant in time, and \( x(t) = [x_0(t), \ldots, x_{N_r-1}(t)]^T \in \mathbb{C}^{N_r \times 1} \) is the process white noise with i.i.d. elements \( x_n(t) \sim \mathcal{CN}(0, \sigma_n^2) \) where the notation \( \mathcal{CN}(0, \sigma^2) \) represents the complex circular Gaussian distributions with mean \( 0 \) and variance \( \sigma^2 \). Note that as \( h \) has i.i.d. elements in space, the matrices \( A_i \) are diagonal and equal to \( A_i = a_i I_N \), for \( i \in \{1, \ldots, p\} \) with \( a_i \) referring to the AR coefficients and the notation \( I_N \) denoting the identity matrix of dimension \( n \times n \). In the time-variant system framework (such as Kalman filter), Equation (1) refers to the state transition equation, and the matrices \( A_i \) refer to the state transition matrices and are usually assumed to be known. However, in realistic scenarios, the matrices \( A_i \) are not known and need to be estimated. In time-variant systems, it is assumed that there is an observation model and we consider the observation equation at time instant \( t \in 0, \ldots, T - 1 \) is given by

\[
\hat{h}(t) = h(t) + w(t)
\]

(2)

where \( w(t) = [w_0(t), \ldots, w_{N_r-1}(t)]^T \in \mathbb{C}^{N_r \times 1} \) is the observation white noise with i.i.d. elements \( w_n(t) \sim \mathcal{CN}(0, \sigma_w^2) \).

In the following, we propose a method for estimating the AR coefficients \( a_i \) of the above model for \( i \in \{1, \ldots, p\} \) from the \( T \) observations given in (2). Before presenting the proposed method, we need to describe the time covariance matrix (including the covariance coefficients) of the vector \( h \), the estimation of which is the main step of the proposed algorithm.

In the following, we assume that \( N_r \) and \( T \) are of the same order and converge to the infinity such that \( N_r/T \to c > 0 \). In practice, as we will see in the simulation part, \( N_r \) and \( T \) can take finite values in order to achieve a reasonable performance.

**B. Covariance matrix of \( \hat{h} \)**

The covariance function of \( h_n(t) \) process, for \( k = 1 - T, \ldots, T - 1 \), is defined as

\[
r(k) = \mathbb{E} [h_n(t)h_n(t-k)^*]
\]

for each \( n \)th element of the vector \( h(t) \in \mathbb{C}^{N_r \times 1} \). We assume in this paper the absolute summability of the covariance coefficients resulting in a bounded sum \( \sum_{k=1-T}^{T-1} |r(k)| \leq K \) where \( K \) is a positive fixed constant as \( T, N_r \to \infty \). This assumption is not restrictive in general and holds in a large variety of the practical cases.

The covariance matrix of \( h_n = [h_n(0), \ldots, h_n(T-1)] \in \mathbb{C}^{1 \times T} \) is given by

\[
R_n \triangleq \mathbb{E} [\hat{h}_n^H \hat{h}_n] = \mathcal{T} (r(1-T), \ldots, r(T-1))
\]

where \( \mathcal{T} (r(1-T), \ldots, r(T-1)) \) refers to the Toeplitz matrix [13] formed from the coefficients \( r(1-T), \ldots, r(T-1) \).

The covariance matrix of the observation model \( \hat{h}_n = [h_n(0), \ldots, h_n(T-1)] \in \mathbb{C}^{1 \times T} \), with using (2), is given by:

\[
R_n \triangleq \mathbb{E} [\hat{h}_n^H \hat{h}_n] = E \left[ (\hat{h}_n + w_n)^H (\hat{h}_n + w_n) \right] = E [\hat{h}_n^H \hat{h}_n] + E [w_n^H w_n] = \sigma_w^2 R + \sigma_w^2 I_T
\]

(3)

where \( w_n = [w_n(0), \ldots, w_n(T-1)] \in \mathbb{C}^{1 \times T} \).

**C. Definition of the AR coefficients**

For \( k = 1, \ldots, p \), the Yule-Walker equations [14] are given by

\[
r(k) = \sum_{i=1}^{p} a_i r(k-i).
\]

From these equations and defining \( R_p \triangleq \mathcal{T} (r(1-p), \ldots, r(p-1)) \) as a \( p \)-truncated version of \( R \) for any \( p \in \{1, \ldots, T\} \), we can write a linear system of equations in the matrix form as:

\[
r_p = R_p a_p
\]

where \( r_p = [r(1), \ldots, r(p)]^T \in \mathbb{R}^{p \times 1} \) and \( a_p = [a_1, a_2, \ldots, a_p]^T \in \mathbb{R}^{p \times 1} \).

As the matrix \( R_p \) is of full rank, it is invertible and we can express \( a_p \) as:

\[
a_p = R_p^{-1} r_p.
\]

The remaining of this paper deals with the estimation of the vector of AR coefficients \( a_p \).

**III. Estimation of the AR Coefficients**

The estimation of the AR coefficient vector \( a_p \) is based on the estimation of \( r_p \) and \( R_p \) whose estimates are provided in the following two subsections. The main result is presented in the third subsection.

**A. Estimation of \( r_p \)**

Concatenating the observation vector \( \hat{h}_n(t) \in \mathbb{C}^{N_r \times 1} \) from (2) over \( T \) observations, we can write

\[
\hat{H} = \begin{bmatrix} \hat{h}_n(t) \end{bmatrix}_{n,t=0}^{N_r-1,T-1} = \begin{bmatrix} \hat{h}_n(0), \ldots, \hat{h}_n(T-1) \end{bmatrix} = X R^{1/2} + W
\]

(4)

where \( X = [x(0), \ldots, x(T-1)] \in \mathbb{C}^{N_r \times T} \) with \( x_n(t) \sim \mathcal{CN}(0, \sigma_x^2) \) and \( W = [w(0), \ldots, w(T-1)] \in \mathbb{C}^{N_r \times T} \) with \( w_n(t) \sim \mathcal{CN}(0, \sigma_w^2) \), as previously defined.

The following Lemma, which is an adapted version of the estimates proposed in [15], provides the estimates for the \( r(k) \) coefficients.

**Lemma 1.** Let the observation matrix \( \hat{H} = \begin{bmatrix} \hat{h}_n(t) \end{bmatrix}_{n,t=0}^{N_r-1,T-1} \) be defined as in (4). The biased and unbiased estimates of \( r(k) \) are given, respectively, for \( k = 0 \) by

\[
r_{b,u}(0) = \frac{1}{\sigma_x^2 N_r T} \sum_{n=0}^{N_r-1-T} \sum_{t=0}^{T-1} \hat{h}_n(t) \hat{h}_n(t)^* - \frac{\sigma_w^2}{\sigma_x^2}
\]
and for \( k = 1 - T, \ldots, -1 \) and \( k = 1, \ldots, T - 1 \) by
\[
\hat{r}^b(k) = \frac{1}{\sigma^2 N_r T} \sum_{n=0}^{N_r-1} \sum_{t=0}^{T-1} \hat{h}_n(t + k) \hat{h}_n(t)^* \\
\hat{r}^u(k) = \frac{1}{\sigma^2 N_r (T - |k|)} \sum_{n=0}^{N_r-1} \sum_{t=0}^{T-1} \hat{h}_n(t + k) \hat{h}_n(t)^*
\]
for \( 0 \leq t + k \leq T - 1 \). Then, for \( k = 1 - T, \ldots, T - 1 \), for any \( \epsilon > 0 \), we have
\[
\mathbb{P} \left( |\hat{r}^b(k) - r(k)| \geq \epsilon \right) \leq \frac{K'}{e^{\epsilon^2 N_r T}} \\
\mathbb{P} \left( |\hat{r}^u(k) - r(k)| \geq \epsilon \right) \leq \frac{K'}{e^{\epsilon^2 N_r (T - |k|)}}.
\]
where \( K' > 0 \) is a positive constant.

Proof. The proof is provided in Appendix VII-A.

We now provide the estimates of the covariance matrix based on the estimated coefficients and the results from [15].

**Lemma 2.** Let, for \( k = 1 - T, \ldots, T - 1 \), \( \hat{r}(k)^b \) and \( \hat{r}(k)^u \) be the biased and unbiased estimates of \( r(k) \), respectively, defined as in Lemma 1. Define the estimated covariance matrices as
\[
\hat{R}^b_p \triangleq T \left( \hat{r}^b(1-p), \ldots, \hat{r}^b(p-1) \right) \\
\hat{R}^u_p \triangleq T \left( \hat{r}^u(1-p), \ldots, \hat{r}^u(p-1) \right).
\]

Then, for any \( \epsilon > 0 \), we have
\[
\mathbb{P} \left[ \left\| \hat{R}^b_p - R_p \right\| > \epsilon \right] \leq \exp \left( -cT \left( \frac{\epsilon}{C} - \log \left( 1 + \frac{\epsilon}{C} \right) + o(1) \right) \right) \\
\mathbb{P} \left[ \left\| \hat{R}^u_p - R_p \right\| > \epsilon \right] \leq \exp \left( -C'T^2 \frac{\epsilon^2}{\log T} (1 + o(1)) \right)
\]
where \( o(1) \) is with respect to \( T \) and depends on \( \epsilon, c, C \), and \( C' \) are positive and bounded as \( T \to \infty \), and \( \| \cdot \| \) denotes the spectral norm.

Proof. The proof is provided in Appendix VII-B.

**C. Estimation of the AR coefficients \( \alpha_k \): main result**

Based on the results of the above subsections, we can now define the estimates of the AR coefficients in the following theorem.

**Theorem 1.** Let \( \hat{r}^{b,u}(k) \) be the biased or unbiased estimate defined as in Lemma 1. Define \( \hat{R}^{b,u}_p \triangleq [\hat{r}^{b,u}(1), \ldots, \hat{r}^{b,u}(p)]^\top \in \mathbb{R}^{p \times 1} \). We define
\[
\hat{R}^{b,u}_p \triangleq T \left( \hat{r}^{b,u}(1-p), \ldots, \hat{r}^{b,u}(p-1) \right).
\]
The biased and unbiased estimators are given by
\[
\hat{a}^{b,u}_p = \hat{R}^{b,u}_p^{-1} \hat{r}^{b,u}_p
\]
where \( \hat{a}^{b,u}_p = [\hat{a}^{b,u}_1, \ldots, \hat{a}^{b,u}_p]^\top \). Then, for \( i = 1, \ldots, p \), for any \( \epsilon > 0 \)
\[
\mathbb{P} \left[ |\hat{a}^b_i - a_i| > \epsilon \right] \leq \frac{K''}{e^{\epsilon^2 N_r T}} \\
\mathbb{P} \left[ |\hat{a}^u_i - a_i| > \epsilon \right] \leq \frac{K''}{e^{\epsilon^2 N_r (T - |i|)}},
\]
where \( K'' > 0 \) is a positive constant.

From this theorem we have the almost sure convergence of the proposed estimator of \( a_i \) to the true value for all \( i = 1, \ldots, p \).

Proof. The proof is provided in Appendix VII-C.

**IV. APPLICATION TO CHANNEL TRACKING**

We consider a communication system with \( N_t = 1 \) transmit antennas and \( N_r \) receive antennas. We assume that the random channel coefficients \( [h_n(t)]_{n,t=0}^{N_r-1,T-1} \) are i.i.d. in space, follow the AR(\( p \)) in time. Assuming an uplink transmission, the \( N_r \times 1 \) received signal at the base station is given by
\[
y(t) = h(t) s(t) + w(t) = \sum_{k=1}^{p} a_k h(t-k) s(t) + w(t)
\]
where \( s(t) \in \mathbb{C} \) is the transmitted signal, \( w(t) \) is the noise vector defined as in (2), and \( a_k \) for \( k \in \{1, \ldots, p\} \) are the coefficients of the AR(\( p \)) process model.

Concatenating over \( T \) time slots, the \( N_r \times T \) received signal matrix is given by
\[
Y = [y(0), \ldots, y(T-1)] = HS + W \tag{5}
\]
where \( S = \text{diag} \{ s(0), \ldots, s(T-1) \} \) is a unitary matrix such that \( SS^H = I_T \). With this assumption, it is clear that the covariance matrix in time of the received signal is equal to the covariance matrix of \( h \) from Section II-B. Hence, Theorem 1 can be directly applied with the observation model (5) in order to get the AR estimates \( \hat{a}_k \) for \( k = 1, \ldots, p \). The channel tracking equation at time \( t \) is then given by
\[
\hat{h}(t) = \sum_{i=1}^{p} \hat{a}_i \hat{h}(t-p)
\]
where \( \hat{a}_i \) are obtained from Theorem 1.

**V. SIMULATION RESULTS**

**A. AR coefficient estimation**

In this subsection, we show the performance of the proposed biased and unbiased estimators from Theorem 1 and compare it with the performance of an existing estimator referred here to the time-based method. The time-based estimator is similar to the one given in [12], for which there is no averaging over the spatial domain, i.e., assuming \( N_r = 1 \). We consider the AR(2) for which \( a_1 \in [0, 2] \) and \( a_2 \in (-1, 0) \). We assume here that \( a_1 = 1.8 \) and \( a_2 = -0.9 \). The choice of these values is motivated by the commonly used channel model corresponding to the so called Jakes’ model [16]. It has been shown in [17], that in order to approximate the
In this subsection, we consider the AR(2) Jakes’ channel model with \(a_1 = 1.8\) and \(a_2 = -0.9\). We apply a Kalman filter based channel estimation method from [18]. We assume \(N_r = 64\), the maximum size of the observation window is \(T = N_r = 64\) and the SNR is equal to 0 dB. At each time \(t\) the channel estimate is based on the \(t\) concatenated observations of the received signal using the estimates of the AR(2) coefficients based on those \(t\) observations. The estimates of the AR(2) coefficients are obtained using the same methods as in the previous section: the proposed approach and the time-based method. The genie method is referred to the channel estimation using the true values of the coefficients. The NMSEs of the different instantaneous (at time instant \(t < T\) based on \(t\) observations) channel estimation methods are compared in Figure 2. We observe that the proposed approach provides the best performance, especially for the unbiased case which is close to the one of the genie method. Moreover, we notice that a good performance is obtained with a quite small number of observations as compared to the size of the received signal. The same channel estimation methods are compared in Figure 3 in terms of the NMSE for \(T = N_r = 64\) versus SNR. We observe that the proposed method is more beneficial especially at a lower SNR for which the observation noise is higher. Finally, the above methods are compared in Figure 4 in terms of the NMSE for SNR= -5 dB versus \(N = T\). In this case we assume that the AR(2) coefficient estimates are based on \(T\) observations. Again, we observe that the proposed approach provides the best performance, close to the genie for the unbiased case.

B. Channel tracking

In this subsection, we consider the AR(2) Jakes’ channel models following the AR need to be further studied. This problem is motivated by the observation that when the parameters of the AR are properly set, the model can be used to develop channel estimators and predictors. We have shown the almost sure convergence of the proposed estimate to the true value. The proposed estimates have been used for channel tracking for a specific case of Jakes’ model. However, broader channel models following the AR need to be further studied.

VI. CONCLUSIONS

In this paper, we considered the problem of estimating the parameters of an AR process, which can model the evolution of time-varying wireless channel in SIMO systems. This problem is motivated by the observation that when the parameters of the AR are properly set, the model can be used to develop channel estimators and predictors. We have shown the almost sure convergence of the proposed estimate to the true value. The proposed estimates have been used for channel tracking for a specific case of Jakes’ model. However, broader channel models following the AR need to be further studied.

VII. Appendix

A. Proof of Lemma 1

The random variable given by \(\hat{b}(k)\) is integrable and has a finite variance \(\sigma_{\hat{b}}^2(k)\) and a finite mean \(\mathbb{E}(\hat{b}(k)) = (1 - |k|/T)r(k)\). Hence, from the Chebyshev’s inequality, for any \(\epsilon > 0\), we have

\[
\mathbb{P} \left[ |\hat{b}(k) - \mathbb{E}(\hat{b}(k))| \geq \epsilon \right] \leq \frac{\sigma_{\hat{b}}^2(k)}{\epsilon^2}.
\]

In the remaining of the proof, we consider the term \(\sigma_{\hat{b}}^2(k)\) and calculate its upper bound.
We define $\overline{H} \triangleq ZQ$ where $Z \in \mathbb{C}^{N \times T}$ has i.i.d. elements $z_n(t) \sim \mathcal{CN}(0, 1)$ with row denoted by $z_n$ and $Q = \sigma_R R_h^{1/2} + \sigma_p I_p = [q_0, \ldots, q_{T-1}]$ with $q_t \in \mathbb{C}^{T \times 1}$. The entries of the matrices $H$ from (4) and $\overline{H}$ have the same complex Gaussian distribution with independent rows and dependent columns with covariance matrices $R_h$ defined in (3). We can write, for $k = 1, \ldots, T - 1$:

$$\text{var}[\hat{r}^b(k)] = \text{var}\left[\frac{1}{\sigma_p^2 N_T} \sum_{n=0}^{N_r-1} \sum_{t=0}^{T-1} z_n q_{t+1} q_t^H z_n^H\right].$$

Using the Cauchy-Schwarz inequality, after some steps we get

$$\text{var}[\hat{r}^b(k)] \leq \frac{||R_h||}{\sigma_p^2 N_T^2 T^2} \sum_{n=0}^{N_r-1} \sum_{t=0}^{T-1} \text{var}[z_n(i)] = \frac{||R_h||}{\sigma_p^2 N_T},$$

where $z_n(i)$ are i.i.d. with a unit variance and $||Q||^2 = ||R_h||$ is bounded as the norm of the $R$ is bounded because of the absolute summability assumption of the covariance coefficients $\sum_{k=1}^{T-1} r(k) < \infty$. Hence, after some steps for any $\epsilon > 0$

$$P(\text{var}[\hat{r}^b(k)] - r(k) \geq \epsilon) \leq \frac{||R_h||}{\epsilon^2 \sigma_p^2 N_T}. $$

As $T, N_T$ converge to infinity, we get the almost sure convergence of the proposed estimator.

The unbiased case is proved following similar steps with the variance bounded by

$$\frac{||R_h||}{\epsilon^2 \sigma_p^2 N_T, (T-1)|k|).}$$

### C. Proof of Theorem 1

The proof is based on the usage of Lemma 1 and Lemma 2. Let $\hat{r}_p$ be the biased or unbiased vector of estimated covariance

### B. Proof of Lemma 2

As in the above proof, the statistical behavior of the entries of the matrix $\overline{H}$ is equivalent to the statistics of the entries of $H$ defined in Appendix VII-A. We can apply Theorem 1 from [15] to get the almost sure convergence of $\hat{R}^b$ and $\hat{R}^u$ for both, biased and unbiased cases. From the fact that $R$ and $\overline{R}$ are Hermitian nonnegative Toeplitz (and so are $R_p$ and $\overline{R}_p$), we have $||R_p - \overline{R}_p|| \leq ||R - \overline{R}||$ for any $p \in \{1, \ldots, T\}$ for both cases, we get the result.

### References

[1] M. Yan and D. Rao, “Performance of an array receiver with a Kalman channel predictor for fast Rayleigh flat fading environments,” IEEE Journal on Selected Areas in Communications, vol. 6, no. 6, pp. 1164–1172, 2001.

[2] H. Hijazi and L. Ros, “Joint data QR-detection and Kalman estimation for OFDM time-varying Rayleigh channel complex gains,” IEEE Trans. on Communications, vol. 58, pp. 170–177, Jan. 2010.

[3] Y. Zhang, S. B. Gelfand, and M. P. Fitz, “Soft-output demodulation on frequency-selective Rayleigh fading channels using AR channel models,” IEEE Trans. on Communications, vol. 55, pp. 1929–1939, Oct. 2007.

[4] F. Lehmann, “Blind estimation and detection of space-time Trellis coded transmissions over the Rayleigh fading MIMO channel,” IEEE Trans. on Communications, vol. 56, pp. 334–338, Mar. 2008.

[5] H. Abeida, “Data-aided SNR estimation in time-variant Rayleigh fading channels,” IEEE Trans. on Signal Processing, vol. 58, pp. 5496–5507, Nov. 2010.

[6] K. T. Truong and R. W. Heath, “Effects of channel aging in massive MIMO systems,” Journal of Communications and Networks, vol. 15, no. 4, pp. 338–351, 2013.

[7] C. Kong, C. Zhong, A. K. Papazafeiropoulos, M. Matthaiou, and Z. Zhang, “Sum-rate and power scaling of massive MIMO systems with channel aging,” IEEE Trans. on Communications, vol. 63, no. 12, pp. 4879–4893, 2015.

[8] C. Zhang, D. Guo, and P. Fan, “Tracking angles of departure and arrival in a mobile millimeter wave channel,” in IEEE ICC, pp. 1–6, 2016.

[9] A. Papazafeiropoulos and T. Ratnarajah, “Modeling and performance of uplink cache-enabled massive MIMO heterogeneous networks,” IEEE Trans. on Wireless Communications, vol. 17, no. 12, pp. 8136–8149, 2018.

[10] H. Kim, S. Kim, H. Lee, C. Jang, Y. Choi, and J. Choi, “Massive MIMO channel prediction: Kalman filtering vs. machine learning,” IEEE Trans. on Communications, pp. 1–1, 2020. early access.

[11] J. Yuan, H. Q. Ngo, and M. Matthaiou, “Machine learning-based channel prediction in massive MIMO with channel aging,” IEEE Trans. on Wireless Communications, vol. 19, no. 5, pp. 2960–2973, 2020.

[12] M. Esfandiari, S. A. Vorobyov, and M. Karimi, “New estimation methods for autoregressive process in the presence of white observation noise,” Signal Processing (Elsevier), vol. 2020, no. 171, pp. 10780–10790, 2020.

[13] R. M. Gray, Toeplitz and circulant matrices: A review. Now Pub., 2006.

[14] S. M. Kay, Modern spectral estimation: Theory and application. Englewood Cliffs, N.J.: Prentice-Hall, 1988.

[15] J. Vinogradov, R. Couillet, and W. Hachem, “Estimation of Toeplitz and circulant matrices: A review,” Now Pub., 2006.