Non-Magnetic Impurity induced in-gap bound states in two band $s_{\pm}$ superconductors

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In this paper we study the effect of single non-magnetic impurity in two-band $s_{\pm}$ superconductors where the two $s_{\pm}$-wave order parameters have relative phase $\delta \neq 0$ between them. We show that in-gap bound states are always induced by non-magnetic impurities when $\delta = \pi$ ($s_{\pm}$-wave superconductors). The bound state is a consequence of the topological nature of the corresponding Bogoliubov-de Gennes equation.

We start with the Path integral formulation of BCS theory. The system we consider is characterized by the BCS action, $S = S_0 + S_I$, where

$$S_0 = -\sum_{i,k} \Psi_i^+(k) \left( \frac{i\omega_n - \epsilon_{ik} + \mu}{\Delta_{0i}} \right) \Psi_j(k),$$

(1)

$$\Psi_i(k) = \begin{bmatrix} c_{i1}(k) \\ c_{i2}^\dagger(-k) \end{bmatrix}, \quad i = 1, 2, \text{ is the band index and } k = (\mathbf{k}, \omega_n). \quad \epsilon_{ik} \text{ is the energy dispersion for electrons in band } i \text{ and } c_{\sigma}, c_{\sigma}^\dagger \text{ are spin-} \sigma \text{ electron annihilation(create) operators}. \quad S_0 \text{ is a sum of two bulk BCS mean-field actions describing two superconducting bands coupled only by Josephson interaction. } \Delta_{0i} \text{ is the superconducting gap when impurities are absent. The effect of a single non-magnetic impurity is represented by } S_I, \text{ where}

$$S_I = \frac{1}{\Omega} \sum_{i,j=1,2} \Psi_i(i\omega_n) \left( \frac{U_{ij}}{\Delta_{ij}} \right) \Psi_j(i\omega_n),$$

(2)

where $\Omega = \text{volume of system and } \Psi_i(i\omega_n) = \sum_k \Psi_i(k). \quad S_I \text{ describes the effects of an impurity scattering potential } U(\mathbf{r}) \sim \delta^d(\mathbf{r}) \sum_{i,j=1,2} U_{ij} c_{i\sigma}(\mathbf{r})^c c_{j\sigma}(\mathbf{r}), \text{ where } U_{ij} \text{ are the scattering matrix element between bands } i \text{ and } j \text{ and } \Delta_{ij} = \delta_{ij}\Delta_1 \text{ is the induced change in local superconducting gap as a result of the impurity scattering potential. We have approximated the induced change in gap to be of form } \Delta_i(\mathbf{r}) \sim \delta^d(\mathbf{r})\Delta_1, \text{ here, consistent with our simplified form of impurity scattering potential.}$

The superconducting order parameters are determined by the mean-field equation

$$\Delta_i(\mathbf{r}) = \Delta_{0i} + \Delta_i(\mathbf{r}) = -\sum_{j=1,2} V_{ij} \langle c_{i\uparrow}^{(j)}(\mathbf{r}) c_{j\uparrow}(\mathbf{r}) \rangle,$$

(3)

where $\langle c_{i\uparrow}^{(j)}(\mathbf{r}) c_{j\uparrow}(\mathbf{r}) \rangle$ is the pairing amplitude between electrons in $j^{th}$ band, $V_{ii}$ represents the pairing interaction between electrons in band $i$ and $V_{12} = V_{21}$ is the Josephson coupling between the pairing order parameters in the two bands.
The fermion fields in $S$ can be integrated out to obtain an effective action $S_{eff}$ in terms of $U_{ij}$ and $\Delta_i$, we obtain

$$S_{eff} = \ln \det(M_0) + \ln \det(1 + G_0 M_1(U, \tilde{\Delta})),$$

where $\ln \det(M_0)$ is coming from the the mean-field BCS action in the absence of impurity and

$$G_0(\omega_n) = M_0(\omega_n)^{-1} = \begin{pmatrix} g_{01}(\omega_n) & 0 \\ 0 & g_{02}(\omega_n) \end{pmatrix}$$

where

$$g_{0i}(\omega_n) = \frac{\pi N_i(0)}{\sqrt{\Delta_{0i}^2 - (\omega_n)^2}} \begin{pmatrix} -\omega_n & \Delta_{0i} \\ \Delta_{0i}^* & -\omega_n \end{pmatrix} \theta(\omega_D - |\omega_n|)$$

(4b)

is the (on-site) Nambu matrix Green’s function for band $i$ electrons in the absence of the impurity. $\omega_D >> \max(|\Delta_0|^2)$ when $\omega$ is a cutoff frequency for phonon superconductors) and $N_i(0) \sim E_F^{-1}$ is the band $i$ density of states at the Fermi level. We have assumed $E_F >> \omega_D$, $\Delta_{0(2)}, U_{ij}, \omega_n$, etc. to justify using a constant density of states in Eq. (4a). We also have

$$M_1 = \begin{pmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{pmatrix}, \quad W_{ij} = \begin{pmatrix} U_{ij} & \delta_i \tilde{\Delta}_j \\ \delta_j \tilde{\Delta}_j^* & -U_{ij} \end{pmatrix}$$

(5a)

The free energy associated with the impurity is

$$F_I = -\frac{1}{\beta} \sum_{|\omega_n|<\omega_D} \ln \det(1 + G_0 M_1(U, \tilde{\Delta}))$$

$$+ \sum_{ij} \left( \Delta_{0i} + \tilde{\Delta}_i^* \right)(V^{-1})_{ij} \left( \Delta_{0j} + \tilde{\Delta}_j \right)$$

and the mean-field equation for $\tilde{\Delta}_i$ can be obtained by minimizing the free energy with respect to $\tilde{\Delta}_i$. We obtain

$$\frac{1}{\beta} \sum_{|\omega_n|,i} V_{ij} \left[ (1 + G_0(\omega_n) M_1)^{-1} G_0(\omega_n) \right]_{j_1,j_2} = \tilde{\Delta}_i + \Delta_{0i}$$

(7)

where $(j_1,j_2) = (2j-1,2j)$ for $j = 1,2$.

**single-band case**

It is helpful to first consider the situation of single band superconductor. In this case the mean-field equation for $\Delta$ is

$$\frac{1}{\beta} \sum_{|\omega_n|<\omega_D} \frac{\pi N(0) (\Delta' + \frac{\Delta_0}{\sqrt{|\Delta_0|^2 - (\omega_n)^2}}) + \Delta^*}{1 + |U'|^2 + |\Delta'|^2 + \frac{\delta_{\Delta_0}^2}{\sqrt{|\Delta_0|^2 - (\omega_n)^2}}} = \Delta_0 + \Delta'$$

(8)

where $X' = \pi N(0) X$, where $X = U, V, \tilde{\Delta}, \Delta_0, \omega_D$. The BCS mean-field equation in the absence of impurity is recovered if we set $U' = \Delta' = 0$. Eq. (5) can be solved analytically in the limit $V', |U'|, |\Delta_0|$, and $\omega_{D}' = \pi N(0) \omega_D' << 1$, which is the case for weakly-coupled BCS superconductors. In this case it is straightforward to show that

$$\Delta \sim -\Delta_0 |U'|^2 + O((U', V', \omega_{D}')^4),$$

and the effect of impurity is to reduce the gap amplitude at the impurity site. Correspondingly a bound state is induced at the impurity site which is determined by the equation $1 + G_0(\omega) M_{1/2} = 0$, or

$$\sqrt{|\Delta_0|^2 - \omega^2 (1 + |U'|^2 + |\Delta'|^2) + 2 \Rec(\Delta_0 \tilde{\Delta})} = 0.$$

We see that a solution $\omega < |\Delta_0|$ exists when $\Delta_0 \tilde{\Delta} < 0$. The solution has energy $\omega \sim \Delta_0 (1 - 2|\Delta_0|^2 |U'|^4) >> \Delta_0 - |\Delta|$ in the limit $|U'|, V', \omega_{D}' << 1$. The bound state solution is a direct consequence of local suppression of superconducting order-parameter by the impurity which creates a local “potential well” in the system. The bound state has energy $\omega > local$ gap magnitude $= \Delta_0 - |\Delta|$ and is therefore not a true “in-gap” state.

**two-band situation**

Next we consider the two-band situation. To see the new physics associated with the appearance of multiple bands we first consider bound states assuming $\tilde{\Delta} = 0$, i.e. there is no induced local change in the gap amplitudes.

In this case we obtain after some algebra

$$\det(1 + G_0(\omega) M_2) = 1 + \frac{|U_{12}'|^2 (|U_{12}'|^2 - 2 U_{11}' U_{22}')}{(1 + |U_{11}'|^2)(1 + |U_{12}'|^2)}$$

$$\frac{2 (\omega^2 - |\Delta_0| |\Delta_0| \cos \delta)}{|\Delta_0|^2 (1 + |U_{11}'|^2)(1 + |U_{12}'|^2)} \sqrt{(\Delta_0 + |\Delta_0|)^2 - \omega^2}$$

(9)

$$|\Delta_0|^2 - \omega^2 = \frac{2 \Rec(\Delta_0 \tilde{\Delta})}{|\Delta_0|^2 (1 + |U_{11}'|^2)(1 + |U_{12}'|^2)} \sqrt{(\Delta_0 + |\Delta_0|)^2 - \omega^2}$$

(10)

where $U_{ij}' = \pi \sqrt{N_i(0) N_j(0)} U_{ij}$ and $\delta$ is the relative phase between the two order parameters $\Delta_0$ and $\Delta_{02}$. The solutions to the equation $\det(1 + G_0(\omega) M_2) = 0$ can be obtained easily since the equation is quadratic in $\omega^2$. We are interested at the bound state solution with $\omega < \min(|\Delta_0|, |\Delta_{02}|)$. Notice that the equation has no solution when $U_{12} = 0$, consistent with what we observe in the single-band case.

Assuming that $|\Delta_{02}| > |\Delta_0|$, it is easy to see from Eq. (10) that bound state solution with energy $\omega < |\Delta_0|$ exists only when $|\Delta_0| > \Delta_0 |\Delta_0| \cos \delta$. In particular, no bound state solution exists when $\delta = 0$. However bound state exists when $\delta = \pi$, when the two superconducting order parameters are out of phase, even when there is no induced local changes in the order parameters. Solving the equation we obtain

$$\omega^2 = \frac{1}{2 (1 - 4 r^2)} \left( |\Delta_0|^2 + \Delta_{02}^2 + 8 r^2 |\Delta_0||\Delta_{02}| \right)$$

$$-(|\Delta_0| + |\Delta_{02}|) \sqrt{(\Delta_{02})^2 + 16 r^2 |\Delta_0||\Delta_{02}|}$$
and the corresponding bound state energy depends on $U$ in the strong scattering (Unitary) limit with tight-binding calculations\[11, 12, 13\] where shallow.

\[ \omega - |\Delta_0| \sim -2\omega^2 \frac{(|\Delta_0| + |\Delta_{02}|)}{(|\Delta_{02}|-|\Delta_0|)}|\Delta_0| + O(r^4), \]

in the limit $|\Delta_{02}| - |\Delta_0| >> 4\sqrt{|\Delta_0||\Delta_{02}|}$ and becomes $\omega - |\Delta_0| \sim -2|\Delta_0|r$ in the opposite limit $|\Delta_{02}| - |\Delta_0| << 4\sqrt{|\Delta_0||\Delta_{02}|}$. More generally, it is straightforward to show that solutions with $\omega \geq 0$ exists when $r^2 \leq 1/4$ and no solution exists at $r^2 > 1/4$.

Therefore there is an intermediate range of parameters $U$, where bound state solution does not exist. At around the resonance point $r^2 = 1/4 - \delta$ we obtain

\[ \omega^2 = 4\delta |\Delta_0|^2 |\Delta_{02}|^2 |\Delta_{01}|^2 + O(\delta^2). \]

Our result indicates that the existence of in-gap state and the corresponding bound state energy depends on the particular form of impurity scattering potential which determines the parameters $U_1$, $U_2$, and $U_1$. Assuming the $U$'s are all proportional to each other we find that in the strong scattering (Unitary) limit $|U_0|^2 \to \infty$, $r^2 \to 0$ and the bound state energy approaches zero asymptotically. This result is in qualitative agreement with tight-binding calculations\[11, 12, 13\] where shallow in-gap states are found to exist easily in two-band superconductors with order parameters of opposite sign. Our non-model-specific result suggests that $\omega \to 0$ bound states are in general allowed in both two band superconductors with frustrated sign between order parameters.

To examine whether the in-gap state is robust against changes in the superconducting order parameters we consider the case of symmetric bands with $g_0(\omega) = g_0(\omega)$ and $\Delta_0 = \Delta_0 e^{i\beta}$, i.e. the two bands differ only in the phase of the order parameters. To simplify the problem further we set $U_{11} = U_{22} = U$ so that

\[ M_1 = \begin{pmatrix} 0 & \tilde{\Delta} e^{i\beta_1} & U & 0 \\ \Delta e^{-i\beta_1} & 0 & 0 & -U \\ 0 & 0 & 0 & \Delta e^{i\beta_2} \\ U & 0 & 0 & \tilde{\Delta} e^{-i\beta_2} \\ \tilde{\Delta} e^{-i\beta_1} & 0 & 0 & \Delta e^{i\beta_2} \end{pmatrix}, \quad (11) \]

where $\tilde{\Delta}$ and $\beta_1$ are to be solved self-consistently from the mean-field equation. The determinant of $(1 + G_0 M_1)$ can be computed analytically in this case. We obtain after lengthy algebra

\[ \det(1 + G_0(\omega) M_1) = A(\omega) + B(\omega) [\cos(\theta_1 - \phi_1) + \cos(\theta_2 - \phi_2) + C(\omega)[\cos(\theta_2 - \phi_1) + \cos(\theta_1 - \phi_2)] + \frac{2|U_{12}|^2|\tilde{\Delta}^2| \cos(\theta_1 - \theta_2) + E(\omega) \cos(\theta_1 - \phi_1) \cos(\theta_2 - \phi_2)}{1 + |U'|^2 + |\tilde{\Delta}|^2}, \]

where

\[ A(\omega) = (1 + |\tilde{\Delta}'|)^2 + |U_1|^2 + 2(\frac{i\omega^2}{(\omega^2 - |\Delta_0|^2)}^2, \]

\[ B(\omega) = -2\Delta_0 \tilde{\Delta}' \sqrt{|\Delta_0|^2 - (i\omega)^2} [1 + |\tilde{\Delta}'|^2], \]

\[ C(\omega) = -2\Delta_0 \tilde{\Delta}' \sqrt{|\Delta_0|^2 - (i\omega)^2} |U_1|^2, \]

\[ D(\omega) = - \frac{2\Delta_0 |U_1|^2}{(\omega^2 - |\Delta_0|^2)}, E(\omega) = - \frac{2\Delta_0 |\tilde{\Delta}'|^2}{(\omega^2 - |\Delta_0|^2)}, \]

where $X' = \pi N(0)X$ as before.

The mean-field equation can be solved in the weak impurity scattering limit $|U'|, |\Delta_0|, |V_{ij}|, \omega_2^2 << 1$. Keeping only terms to order $|U'|^2$ and $\tilde{\Delta}'$ in the mean-field equation, we obtain $\theta_i = \phi_i (i = 1, 2)$ and

\[ \tilde{\Delta} \sim |U'|^2 \Delta_0 \cos(\phi_1 - \phi_2). \]

Notice that the superconducting order parameter is \textit{enhanced} by scattering between the two bands if $\phi_1 = \phi_2$ but is \textit{suppressed} by scattering if $\phi_1 - \phi_2 = \pm \pi$, suggesting that non-magnetic impurity induces a local ferromagnetic Josephson coupling between the superconducting order parameters which disfavors a s-wave state\[9\].

We next examine the solution(s) to the equation

\[ \det(1 + G_0(\omega) M_1) = 0 \quad \text{with} \quad \omega^2 < |\Delta_0|^2. \]

Defining $y^2 = \Delta_0^2 - \omega^2$ we find that a zero at some $0 < y_0 < \Delta_0$ implies an in-gap bound state. Solving the equation (with $\theta_i = \phi_i (i = 1, 2)$) we obtain

\[ y_0 = -(\beta + \gamma) / \alpha, \]

where

\[ \beta = 2\Delta_0 \tilde{\Delta}' (1 + |\tilde{\Delta}'|^2 + |U'|^2) \cos(\phi_1 - \phi_2), \]

\[ \alpha = 1 + 2(|U'|^2 + |\tilde{\Delta}'|^2) + |U'|^4 + |\tilde{\Delta}'|^4 + 2|U_1|^2 + |\tilde{\Delta}'|^2 \cos(\phi_1 - \phi_2), \]

and

\[ \gamma^2 = 2|U_1|^2 \Delta_0^2 (1 + |U'|^2 + |\tilde{\Delta}'|^2)^2 [1 - \cos(\phi_1 - \phi_2)]. \]

It is easy to see that

\[ y_0 = - \frac{2\Delta_0 \tilde{\Delta}'}{1 + |U'|^2 + |\tilde{\Delta}'|^2}, \]

for $\phi_1 = \phi_2$ and a $y > 0$ solution does not exist for small $|U_{12}|^2$ where $\Delta_0 \tilde{\Delta}' > 0$. The situation is very different for $\phi_1 - \phi_2 = \pm \pi$. In this case

\[ y_0 = 2\Delta_0 \frac{|U'|^2 ((1 + |\tilde{\Delta}'|^2 + |U'|^2) - \tilde{\Delta}' (1 + |\tilde{\Delta}'|^2 - |U'|^2))}{1 + (|\tilde{\Delta}'|^2 + |U'|^2) + (|\tilde{\Delta}'|^2 - |U'|^2)^2}, \]

~ $2\Delta_0 |U'|$

and $\omega \sim \Delta_0 - 2\Delta_0^2 |U'|$ in the limit $|U'|, |\tilde{\Delta}'| << 1$. Notice that the bound state has energy below the “local” gap magnitude $\Delta_0 + \tilde{\Delta} \sim \Delta_0 (1 - |U'|^2)$, indicating that the formation of in-gap bound state is robust to local gap distortion.
Impurity averaged single particle density of states

To have a more quantitative feeling of the effect of impurities we compute the single-particle density of states in our model with \( U_{11} = U_{22} = \alpha U_{12} = \alpha U \) and \( \Delta = 0 \). In this case the matrix Green’s function \( G(\omega; U) = (1 + G_0(\omega)M_1(U))^{-1}G_0(\omega) \) can be evaluated exactly and the impurity averaged Green’s function

\[
\langle G(\omega) \rangle = \int dU G(\omega; U) P(U)
\]

(14)
can be evaluated for given impurity potential distribution \( P(U) \). Contrary to self-consistent Born type calculations the calculation here is valid only in the limit of low concentration of impurities where interference effects between different impurity scattering events are negligible. We find that \( \langle G_{12}(\omega) \rangle = \langle G_{21}(\omega) \rangle = 0 \) for even distribution \( P(U) = P(-U) \) and only intra-band Green’s function survives impurity average. (Inter-band Green’s function will contribute when evaluating two-particle correlation functions.) The trace of the total electron Green’s function is

\[
\text{Tr}G(\omega) = -2\Gamma\omega[1 + U'/(1 + \alpha^2)]
\]

\[
\frac{S_1(\omega) + S_2(\omega)}{a(\omega)U'^2 + b(\omega)U'^2 + c(\omega)},
\]

with

\[
a(\omega) = (1 - \alpha^2)^2S_1(\omega)S_2(\omega),
\]

\[
b(\omega) = -2 \left[ \omega^2 - |\Delta_0|^2 \cos \delta - \alpha^2S_1(\omega)S_2(\omega) \right],
\]

\[
c(\omega) = S_1(\omega)S_2(\omega).
\]

(15)
The single-particle density of states given by \( \rho(\omega) = \Im \text{Tr}G(\omega) \) is evaluated numerically for \( s_\pm \) superconductors (\( \delta = \pi \)) with \( P(U) = \sqrt{\frac{\alpha}{\pi}} e^{-\alpha U'^2} \) for two different values of \( a = 0.5 \) to 3 and \( \alpha = 0.4 \) with \( |\Delta_0'| = 0.5, |\Delta_0'| = 1 \) and \( N_1(0) = N_2(0) \). The results are shown in fig.1. The density of states for \( \alpha = 0.95, 1.05 \) and find that nonzero density of state \( \rho(\omega) \) is induced inside the gap with nonzero spectral weight on the Fermi surface in general. The precise form of \( \rho(\omega) \) depends also on \( U_{11}, U_{22} \) and the distribution of impurity scattering strength \( P(U) \). The in-gap spectral weight increases with decreasing \( a \) and shifts to lower energy with decreasing \( \alpha \), indicating that in-gap states are strengthened by strong inter-band scattering but suppressed by intra-band scattering. Notice also that for \( \alpha > 1, r < 1/2 \) and no \( \omega = 0 \) bound state exists! The in-gap states have energy \( \omega > \omega_c \sim 0.2 \) for \( \alpha = 1.05 \) as shown in fig.2. We cautioned that we haven’t included the induced changes in gap functions \( \Delta’s \) in our calculation. Thus we expect that the near gap edge behavior in our calculation may not be reliable but the deep in-gap behavior should be qualitatively correct.

![Figure 2](image)  
*Figure 2: The averaged density of states \( \rho(\omega) \) for \( a = 1 \) and \( \alpha = 0.95, 1.05 \) with \( |\Delta_0'| = 0.5, |\Delta_0'| = 1 \) and \( N_1(0) = N_2(0) \).*

Our result indicates that non-magnetic impurities is a relevant perturbation to the physics of multi-band superconductors with frustrated sign between order parameters. They introduce a “ferromagnetic” Josephson coupling between the two superconducting order parameters which disfavors the \( s_\pm \) state and introduce in-gap bound states in the quasi-particle spectrum. The energy spectrum of the in-gap states depend on the nature of the impurity potential and is “non-universal”.

Origin of the in-gap states

It is important to understand why in-gap bound states exist so easily when the relative phase between the two superconducting order parameters is \( \pi \), independent of the microscopic details of the system. The robustness of the in-gap bound states can be understood if we notice that the approximate mean-field Green’s functions we used in our calculation is the exact Green’s function of a corresponding one-dimensional superconductor problem, if we linearize the fermion spectrum \( \epsilon_{ik} - \mu \rightarrow \pm v_f(\epsilon - k_Fi) \) in action 1. In this case the
density of state $N_i(\epsilon) \sim dk/d\epsilon$ becomes constant and Eq. \ref{eq:} becomes exact. In this representation, the impurity introduces finite tunnelling probability between two one-dimensional superconductors, one located on the left of the impurity, and the other one on the right when $U_{12} \neq 0$. This problem has been studied in Refs.\cite{14} and \cite{15}, where bound states are found to exist when the phase difference between the two superconductors is $\delta = \pi$.

The existence of in-gap bound state can be understood by noting that the one-dimensional Bogoliubov equation with a linearized electron spectrum around the Fermi surface is essentially a Dirac equation for spinless fermions at one-dimension\cite{14}. In particular, the gap function $\Delta$ becomes the mass term in the Dirac fermion representation. Thus the problem is mathematically equivalent to a tunnelling problem between two species of Dirac fermions with masses of opposite sign. For perfect tunnelling, it is known that a $\omega = 0$ mid-gap state exists if the masses of the Dirac fermions have opposite sign at the two sides of the tunnelling barrier because of the topological structure of the problem\cite{16}. The bound states split into two with energies $\pm \omega > 0$ when a tunnelling barrier exists\cite{13,15} and eventually merge into the continuum spectrum when the tunnelling barrier is high enough. This is in qualitative agreement with what we find here when $\Delta_i = 0$. In addition, we also observed that the induced $\Delta_i$ is small and does not affect the in-gap bound states in the limit $|U_{ij}|, |\Delta_{ij}|, |V_{ij}| << 1$ in the case $|\Delta_{01}| = |\Delta_{92}|$ and $U_{11} = U_{22} = 0$.

Lastly, we comment that the effect of magnetic impurity can also be included straightforwardly by introducing a magnetic scattering term $V_{ij}^{\sigma\sigma'}(\vec{r}) \sim \delta(\vec{r})\sigma V_{ij}\delta_{\sigma\sigma'}$ in our calculation. It is known that magnetic scattering introduces in-gap bound states even for single-band superconductors and this feature remains for two-band superconductors without phase frustration. Therefore magnetic impurities do not induce qualitative differences between two-band $s$-wave and $s_{\pm}$-wave superconductors\cite{13} which is why we concentrate only on non-magnetic impurities in the present paper.

Summarizing, by studying the one-impurity problem carefully, we show in this paper that (non-magnetic) impurity is a relevant perturbation in two-band $s_{\pm}$-wave superconductors where the order parameters are of opposite signs. The suppression of $s_{\pm}$ state and generation of in-gap bound states in quasi-particle spectrum are natural phenomena associated with non-magnetic impurities and should be taken into account carefully in understanding the electronic properties of Iron-base superconductors.

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\begin{thebibliography}{99}
\bibitem{1} K. Seo, B.A. Bernevig and J. Hu, Phys. Rev. Lett. \textbf{101}, 206404 (2008).
\bibitem{2} I.I. Mazin \textit{et.al.}, Phys. Rev. Lett. \textbf{101}, 057003 (2008).
\bibitem{3} F. Wang, H. Zhai, Y. Ran, A. Vishwanath and D.-H. Lee, Phys. Rev. Lett. \textbf{102}, 047005 (2009).
\bibitem{4} K. Matano \textit{et.al}, Europhys. Lett. \textbf{83}, 57001 (2008).
\bibitem{5} Cong Ren, \textit{et.al} \texttt{arXiv:0804.1726} (2008).
\bibitem{6} Lin Zhao \textit{et.al}, Chin. phys. Lett. \textbf{25}, 4402 (2008).
\bibitem{7} H. Ding \textit{et.al}, Europhys. Lett. \textbf{83}, 47001 (2008).
\bibitem{8} Y. Bang, H.-Y. Choi and H. Won, Phys. Rev. B \textbf{79}, 054529 (2009).
\bibitem{9} D. Parker, O.V. Dolgov, M.M. Korshunov, A.A. Golubov and I.I. Mazin, Phys. Rev. B \textbf{78}, 134524 (2008).
\bibitem{10} G. Preosti and P. Muzikar, Phys. Rev. B \textbf{54}, 3489 (1995).
\bibitem{11} D. Zhang, T. Zhou and C. Ting, \texttt{arXiv:0904.3708} (2009).
\bibitem{12} T. Zhou, D. Zhang, X. Hu, J.-X. Zhu and C. Ting, arXiv: 0904.4173 (2009).
\bibitem{13} W.-F. Tsai, Y.-Y. Zhang, C. Fang and J. Hu, arXiv:0905.0734 (2009).
\bibitem{14} T.K. Ng and N. Nagaosa, \texttt{arXiv:0809.3343}.
\bibitem{15} X.Y. Feng and T.K. Ng, Phys. Rev. B \textbf{bf 79}, 184503 (2009).
\bibitem{16} Atiyah M. and Singer I., \textit{Ann. Math.} \textbf{87} 484, 1968.
\bibitem{17} W.-F. Tsai, D.-X. Yao, B.A. Nernveg and J. Hu, arXiv: 0812.0661 (2008).
\end{thebibliography}