PULSE PROFILES FROM SPINNING NEUTRON STARS IN THE HARTLE–THORNE APPROXIMATION

DIMITRIOS PSALTIS 1 AND FERYAL ÖZEL 1,2
Astronomy Department, University of Arizona, 933 North Cherry Avenue, Tucson, AZ 85721, USA;
dpsaltis@email.arizone.edu, fozel@email.arizone.edu
Received 2013 May 22; accepted 2014 June 13; published 2014 August 20

ABSTRACT

We present a new numerical algorithm for the calculation of pulse profiles from spinning neutron stars in the Hartle–Thorne approximation. Our approach allows us to formally take into account the effects of Doppler shifts and aberration, of frame dragging, and on the magnitude of strong-field gravitational lensing experienced by the photons as they propagate through the neutron-star spacetime (Pechenick et al. 1983). Given a model for the emerging radiation, the properties of the brightness oscillation can, therefore, be used in mapping the neutron-star surface and spacetime, as well as in measuring its mass and radius.

Pulse-profile modeling techniques have been used to explore the surface emission properties in many types of neutron stars, such as slow pulsars (Page 1995), magnetars (DeDeo et al. 2001; Özel et al. 2001), rotation-powered millisecond pulsars (Pavlov & Zavlin 1997; Bogdanov et al. 2007), X-ray bursters (Weinberg et al. 2001; Nath et al. 2002; Muno et al. 2002, 2003), and accretion-powered millisecond pulsars (Poutanen & Gierliński 2003; Bhattacharyya et al. 2005; Leahy et al. 2008, 2009, 2011; Lamb et al. 2009; Morsink & Leahy 2011). This technique also defines the key scientific objectives of several upcoming or proposed X-ray missions, such as NASA’s NICER and ESA’s LOFT.

Key words: gravitation – stars: neutron
Online-only material: color figures

1. INTRODUCTION

A temperature anisotropy on the surface of a spinning neutron star leads to a periodic oscillation of its brightness at the stellar spin frequency, as viewed by an observer at infinity. The amplitude and spectrum of this oscillation depends on the temperature profile on the stellar surface, on the beaming of the emerging radiation, and on the magnitude of strong-field gravitational lensing experienced by the photons as they propagate through the neutron-star spacetime (Pechenick et al. 1983). Given a model for the emerging radiation, the properties of the brightness oscillation can, therefore, be used in mapping the neutron-star surface and spacetime, as well as in measuring its mass and radius.

Pulse-profile modeling techniques have been used to explore the surface emission properties in many types of neutron stars, such as slow pulsars (Page 1995), magnetars (DeDeo et al. 2001; Özel et al. 2001), rotation-powered millisecond pulsars (Pavlov & Zavlin 1997; Bogdanov et al. 2007), X-ray bursters (Weinberg et al. 2001; Nath et al. 2002; Muno et al. 2002, 2003), and accretion-powered millisecond pulsars (Poutanen & Gierliński 2003; Bhattacharyya et al. 2005; Leahy et al. 2008, 2009, 2011; Lamb et al. 2009; Morsink & Leahy 2011). This technique also defines the key scientific objectives of several upcoming or proposed X-ray missions, such as NASA’s NICER (Gendreau et al. 2012), ESA’s LOFT (Feroci et al. 2012), and ISRO’s Astrosat (Agrawal 2006), which aim to measure the masses and radii of several millisecond rotation-powered pulsars and X-ray bursters with high precision.

The effects of gravitational lensing on the surface photons have been explored with a variety of techniques and approximations since the original work on non-spinning neutron stars (Pechenick et al. 1983). At relatively low spin frequencies (i.e., \( \lesssim 300 \text{ Hz} \)), the spacetime of the neutron star is accurately described by the Schwarzschild metric and its spin primarily causes Doppler shifts and aberration (Miller & Lamb 1998; Muno et al. 2002; Poutanen & Beloborodov 2006). At similar spin frequencies, the effects of frame dragging are marginal (Braje et al. 2000). At moderate spin frequencies (i.e., \( \lesssim 800 \text{ Hz} \)), the neutron-star spacetime acquires a quadrupole moment and its surface becomes oblate. Finally, at spin frequencies near breakup (i.e., \( \gtrsim 1 \text{ kHz} \)), higher order multipoles become important and the neutron-star spacetimes can only be calculated by numerically solving the field equations for particular equations of state (Cook et al. 1994; Stergioulas & Friedman 1995; Stergioulas 2003; see also Cadeau et al. 2007).

Rotation powered millisecond pulsars and thermonuclear X-ray bursters, which make up the majority of neutron star targets for NICER and LOFT, have fast spin frequencies in the \( 300–800 \text{ Hz} \) range. Stars with these spin frequencies, especially if they also possess larger radii, acquire notably oblate shapes and large quadrupole moments. Morsink et al. (2007) demonstrated that, for this relevant range of parameters, the oblateness of the neutron-star surface significantly affects the resulting pulse profiles. In a similar range of frequencies, Bauböck et al. (2013) showed that the quadrupole moment of a neutron-star spacetime also significantly alters the spectroscopic properties of its surface emission.

In this article, we describe a new algorithm for calculating the light curves of spinning neutron stars in the Hartle–Thorne approximation, which formally takes into account the oblateness and quadrupole moment of the star, as well as the effects of frame dragging, Doppler shift, and aberration. Contrary to calculations that are based on numerical spacetimes, our approach allows us to simulate light curves based only on the macroscopic properties of the neutron stars, without the need to assume a particular equation of state.

In Section 2, we describe our calculations, which are based on the ray-tracing algorithm of Psaltis & Johannsen (2012) and Bauböck et al. (2012), and compare them to earlier results obtained under different assumptions and approximations. In Section 3, we present our results and, in Section 4, we discuss their implications for the mass and radius measurements with upcoming X-ray missions.

2. CALCULATING LIGHT CURVES OF SPINNING NEUTRON STAR

Our goal is to calculate the brightness and the spectrum of emission from the surface of a spinning neutron star at different
rotational phases and for different viewing geometries. To this end, we define a coordinate system with its origin at the center of the neutron star and the z-axis aligned with the rotation axis of the star. We then set up an image (or detector) plane at a distance $d$ from the center and at an angle $\theta_0$ with respect to the rotational axis of the star. At $\theta_0 = 0$, the image plane is parallel to the x-y plane, while for $\theta_0 = \pi/2$, the vector $d$ lies along the x-axis and the image plane is parallel to the y-z plane. On the image plane, we also define a two-dimensional Cartesian coordinate system $(\alpha_0, \beta_0)$, with the $\beta_0$-axis on the x-z plane and the $\alpha_0$-axis pointing toward the y-axis (see Figure 1).

The radiation flux at photon energy $E$ received on the image plane at time $t$ is given by the integral

$$F_E(t) = \frac{1}{d^2} \int_{\alpha_0, \beta_0} I(\alpha_0, \beta_0; E, t) d\alpha_0 d\beta_0,$$

where $I(\alpha_0, \beta_0; E, t)$ is the specific intensity on the image plane of a point with coordinates $(\alpha_0, \beta_0)$. Of all the photon rays that reach the image plane, only those that originate on the neutron-star surface have a non-zero intensity. Because of the strong gravitational field of the neutron star, these photon rays are not straight but are rather curved due to gravitational lensing.

The radii of all realistic neutron stars are larger than the radius of the photon orbit in their spacetimes. Under these conditions, photons arriving at position $(\alpha_0, \beta_0)$ on the image plane are uniquely connected to a position and direction on the stellar surface. As a result, we can define a number of one-to-one maps between the coordinates $(\alpha_0, \beta_0)$ at which a photon ray crosses the image plane and various other properties of the photons that travel along this ray. In particular, we will use the five maps

$$\phi = \phi(\alpha_0, \beta_0)$$

$$\theta = \theta(\alpha_0, \beta_0)$$

$$\delta = \delta(\alpha_0, \beta_0)$$

$$\epsilon = \frac{E(\alpha_0, \beta_0)}{E_0(\phi, \theta)}$$

$$t_d = t_d(\alpha_0, \beta_0)$$

that connect the image-plane coordinates $(\alpha_0, \beta_0)$ of a photon ray to the longitude $\phi$ and latitude $\theta$ on the neutron star surface where the ray originates, to the angle $\delta$ between the photon momentum on the stellar surface and the normal to the surface, to the ratio $\epsilon$ between the observed and emitted energies $E$ and $E_0$, and to the time of flight between the neutron star surface and the image plane.

Because of the Lorentz invariance of the photon occupation number along a photon ray, we can now relate the specific intensity $I(\alpha_0, \beta_0; E, t)$ on the image plane to the specific intensity $I_{NS}(\theta, \phi, \delta; E, t)$ on the neutron star surface as

$$I(\alpha_0, \beta_0; E, t) = \epsilon^3 I_{NS}(\phi(\alpha_0, \beta_0), \theta(\alpha_0, \beta_0),$$

$$\times \delta(\alpha_0, \beta_0); \frac{E}{\epsilon}, t - t_d(\alpha_0, \beta_0)).$$

Inserting this last expression into Equation (1) leads to an integral expression for the time- and energy-dependent radiation flux that flows through the image plane.

2.1. Ray Tracing

We calculate the five mapping relations (2)–(6) using the ray-tracing algorithm described in Psaltis & Johannsen (2012) and Bauböck et al. (2012). In this algorithm, we describe the exterior spacetime of a neutron star spinning at a moderate rate using the variant of the Hartle–Thorne metric (Hartle & Thorne 1968) developed by Glimpke & Babak (2006).

The metric coefficients depend on the mass $M$ of the neutron star, on its specific angular momentum $a$, and on the mass quadrupole moment $q$ of the spacetime. We choose to write the quadrupole moment as

$$q = -a^2 (1 + \eta)$$

for two reasons. First, when $\eta = 0$, the quadrupole moment of the spacetime reduces to that of the Kerr metric. Second, calculations of the quadrupole moments of spinning neutron stars using numerical algorithms (Laarakkers & Poisson 1999; Pappas & Apostolatos 2012) showed that Equation (8) remains valid even for neutron stars that are spinning near their breakup points. Typical values of the quadrupole moment require $\eta \sim 1–6$, depending on the equation of state, as well as on the stellar mass and radius.

The shape of a neutron star spinning at moderate rates deviates from spherical. In the Hartle–Thorne approximation, the dependence of the neutron-star radius on the polar angle $\theta$ can be written in terms of two parameters, which we choose to be the equatorial and the polar radius of the star, $R_p$ and $R_{eq}$, as

$$\frac{R(\theta)}{R_{eq}} = \sin^2 \theta + \frac{R_p}{R_{eq}} \cos^2 \theta.$$

For the purposes of the calculations reported in this paper, we use the analytic fitting formula for the ratio of the polar to equatorial radius devised by Morsink et al. (2007) as discussed in Bauböck et al. (2012). Moreover, the equatorial radii that we are reporting are defined such that the proper circumference of a circle in the equator at radius $R_{eq}$ is equal to $2\pi R_{eq}$. In all the simulations presented hereafter, the value of the neutron star radius quoted represents $R_{eq}$ at the specified spin frequency.

Finally, we need to specify the intensity of the radiation on the stellar surface. In this paper, we will assume that radiation emerges only from a small circular spot of angular radius $\rho$ that is fixed at a colatitude $\theta_i$ with respect to the rotational pole of the neutron star. The spectrum of the emerging radiation is that of a blackbody of temperature $T_{NS}$, and the emission is isotropic. Because we will be reporting our results with the photon energy...
expressed in units of $T_{NS}$, the latter quantity will not represent an additional parameter in our calculations.

In summary, the light curve from a spinning neutron star in the Hartle–Thorne approximation and with the approximations discussed above depends on the following parameters: (1) the mass $M$ of the neutron star, (2) the equatorial radius $R_{\text{eq}}$ of the neutron star, (3) its specific angular momentum $a$, (4) the quadrupole moment of its spacetime as measured by the parameter $\eta$, (5) the observer inclination $\theta_0$, (6) the colatitude of the spot $\theta_\phi$, and (7) the angular radius of the spot $\rho$.

### 2.2. Discretization, Convergence, and Performance

In order to calculate efficiently the flux of radiation through the image plane, especially when the size of the emitting region is very small, we evaluate the integral (1) using a set of two nested grids, as shown in Figure 2. We initially use the ray-tracing algorithm to calculate and store the five mapping relations (2)–(3) on a low-resolution grid of $N_l \times N_l$ points, with a typical value of $N_l = 65$. For each rotational phase of the neutron star, we use these results in order to outline a rectangular region in the low-resolution grid that surrounds the projection of the emitting region on the image plane (shown as red rectangles in Figure 2 for three rotational phases). We then set up a high-resolution grid within each of these rectangular regions of $N_h \times N_h$ points, with a typical value of $N_l = 256$. Finally, we use the ray-tracing algorithm to calculate the mapping relations on the high-resolution grid and evaluate numerically the integral (1) only within this grid using a trapezoid integration.

Attaining a sufficiently high resolution for the first three of five mapping relations (2)–(6) is what drives the requirement for a high-resolution grid. This is shown in Figure 2, where contours of constant latitude and longitude on the neutron star surface (in intervals of $10^\circ$, which is the same magnitude as the angular size of the spot in this example) are projected on the image plane. The separation of nearby contours decreases rapidly from the center of the stellar image to its edge; therefore, the number of image plane points falling within a spot near the edges also declines rapidly. The outline of even a large spot of an angular radius of $10^\circ$, as in this example, would be barely resolvable in the low-resolution grid shown in the figure as the spot makes its way toward the edge of the stellar image. In contrast, the last two mapping relations (5)–(6) can be well approximated even on the low-resolution grid. This is shown in Figure 3, in which contours of constant energy ratios $\epsilon$ and time delays $\tau_d$ are plotted on the image plane. As expected, the contours of constant redshift are nearly vertical curves spanning a narrow range, whereas the contours of constant time delay are nearly concentric circles, centered at the origin of the $\alpha_0$, $\beta_0$ plane. In more detail, the waviness of the redshift contours arises from two competing effects: the frame-dragging correction to the magnitude of the surface velocity as measured by a zero-angular-momentum observer and the quadrupole correction to the gravitational redshift. Note that the redshift contours are further distorted and develop the island and saddle shapes shown, e.g., in the right panel of Figure 5 of Bauböck et al. (2013), when the neutron star is viewed from a smaller inclination angle and has a larger quadrupole moment. The frame-dragging and the quadrupole corrections to the time-delay contours are typically less than 1/1000 of the neutron star spin period and are, therefore, negligible.

The convergence of the ray-tracing algorithm was demonstrated in Psaltis & Johannsen (2012), where the results were also compared to other algorithms for calculating the profiles of relativistically broadened fluorescent lines around black holes. In Figure 4, we derive numerically the convergence rate of our integration algorithm for the calculation of the radiation flux. We have taken the calculation shown in Figure 2 with a high-resolution grid of 512 × 512 points nested in a low-resolution
grid of 65 × 65 points as the fiducial one and calculated the time-dependent flux at a photon energy of $E = 3k_B T_{NS}$ at 32 phase bins. We then repeated the same calculation with $N_h = 16, 32, 64, 128, \text{ and } 256$ grid points along each dimension of the high-resolution grid and compared these lower-resolution runs with the fiducial calculation. We plot in Figure 4 the rms fractional difference between each of the lower-resolution runs and the fiducial run, as a function of the number of grid points used. The blue line shows the best-fit power-law relation between the two quantities plotted, with the fractional difference scaling approximately as $\sim N^{-1.56}$. This convergence rate is determined predominantly by the ability of the two-dimensional grid to trace the shape of the deformed circular spot, as the latter is projected onto the image plane.

Figure 4 demonstrates that a high-resolution grid of $128 \times 128$ points leads to an accuracy of one part in a thousand, which is adequate for most applications. A typical calculation takes approximately 1 s per phase bin on a fast workstation. This time can be reduced by up to two orders of magnitude if the ray-tracing part of the algorithm is performed on a GPU card (see Chan et al. 2013).

2.3. Comparison with Earlier Calculations

Figure 5 compares three light curves of a slowly spinning (1 Hz) neutron star calculated with the current algorithm to the results presented by Pechenick et al. (1983) in their Figure 10. For this particular comparison, the beaming of the emission that emerges from the stellar surface was taken to be proportional to $\sin \delta$. The three different light curves correspond to neutron stars with increasing compactness, from $2GM/Rc^2 = 1/4$ to $2GM/Rc^2 = 1/1.7$. As expected, at this low spin frequency, the Doppler effects, as well as those of frame dragging, oblateness, and of the spacetime quadrupole moment, are negligible.

Figure 6 compares two additional light curves with the results shown in Figure 2 of Poutanen & Beloborodov (2006), who used the Schwarzschild+Doppler approximation. Because we consider this to be a verification comparison, we artificially set the frame dragging, the stellar oblateness, and the spacetime quadrupole to zero in our calculations. As discussed in, e.g., Braje et al. (2000) and Poutanen & Beloborodov (2006), Doppler shifts and aberration have three main effects on the pulse profiles: they increase their amplitudes, they shift the location of their maxima to earlier phases, and they introduce an asymmetry to the profile. Even though in our approach the effects of Doppler shifts and of aberration are calculated automatically and cannot be separated from those of the gravitational lensing and of the gravitational redshift, our results agree, as expected, with the earlier approximate methods.

3. RESULTS

We now explore the effects of the stellar oblateness and the spacetime quadrupole moment on the pulse profiles. Figure 7
Figure 7. Flux of the radiation received from a spinning neutron star (left) for a photon energy equal to three times the neutron-star temperature but observed at different inclinations and (right) at an observer inclination of $30^\circ$ but for three different photon energies. In both panels, the solid red line shows the calculation with the Hartle–Thorne metric, the dashed blue line shows the calculation in the Schwarzschild+Doppler approximation with the oblateness of the neutron star taken into account, and the dotted green line shows the calculation in the Schwarzschild+Doppler approximation. All calculations correspond to a $1.8M_{\odot}$ neutron star, with an equatorial radius $R_{\text{eq}}$ fixed to 15 km and spinning at 600 Hz, and with values for the remaining parameters that are typical for the L equation of state. The emitting region is taken to be a circular hot spot with a semi-angular size of $10^\circ$ and positioned at a colatitude of $40^\circ$ from the stellar rotational pole. The emission is that of a blackbody with isotropic beaming. All fluxes have been multiplied by the same constant factor for clarity.

(A color version of this figure is available in the online journal.)

Figure 8. Same as in the left panel of Figure 7 but for a neutron star spinning at 300 Hz.

(A color version of this figure is available in the online journal.)

Figure 9. Same as in the left panel of Figure 7 but for a $1.8M_{\odot}$, 10 km neutron star, spinning at 600 Hz, with parameters that are typical for the FPS equation of state.

(A color version of this figure is available in the online journal.)
less important and the redshift experienced by the photons is dominated by the spacetime quadrupole.

Figures 8 and 9 show pulse profiles with the same setup and observer inclinations, but for a neutron star spinning at 300 Hz and for a 10 km neutron star, respectively. These figures show that, as expected, the effects of the stellar oblateness and of the spacetime quadrupole become less important as the spin frequency of the neutron star or its equatorial radius are reduced. The correction introduced by the oblateness of the neutron star, albeit small, still dominates the correction introduced by the spacetime quadrupole.

Figure 10 quantifies the magnitudes of the errors introduced in the calculation of the pulse profiles by neglecting the stellar oblateness and the spacetime quadrupole. As expected from the above discussion, the effects of the stellar oblateness become large as the observer inclination becomes very different from the colatitude of the emitting region, but they have a weak dependence on photon energy. On the other hand, the effects of the spacetime quadrupole have a stronger energy dependence and become more significant (in absolute value) as the inclination of the observer is reduced. This is consistent with the fact that quadrupole effects arise primarily from the influence of the spacetime quadrupole on the gravitational redshift experienced by the photons, and their relative contribution to the observed flux increases as the Doppler effects become less pronounced.

The overall effects of the stellar oblateness and of the spacetime quadrupole for a 600 Hz spin frequency are of ~10%–30% and ~1%–5%, respectively, depending on the radius of the neutron star and the observer inclination. These are comparable to the stated 5% target uncertainty in the measurement of neutron-star masses and radii using observations of pulse profiles with NICER and LOFT. As a consequence, achieving the goals of these two missions requires calculating pulse profiles with both of these effects taken into account.

We thank Michi Bauböck for many useful discussions on ray tracing in neutron-star spacetimes and the referee Sharon Morsink for enlightening discussions and comments. This work was supported in part by NSF grant AST-1108753, NSF CAREER award AST-0746549, and Chandra Theory grant TM2-13002X. F.Ö. gratefully acknowledges support from the Radcliffe Institute for Advanced Study at Harvard University.

REFERENCES
Agrawal, P. C. 2006, AdSpR, 38, 2989
Bauböck, M., Psaltis, D., & Özel, F. 2013, ApJ, 766, 87
Bauböck, M., Psaltis, D., Özel, F., & Johannsen, T. 2012, ApJ, 753, 175
Bhattacharyya, S., Strohmayer, T. E., Miller, M. C., & Markwardt, C. B. 2005, ApJ, 619, 483
Bogdanov, S., Rybicki, G. B., & Grindlay, J. E. 2007, ApJ, 670, 447
Braje, T. M., Romani, R. W., & Rauch, K. P. 2000, ApJ, 531, 483
Cadeau, C., Morsink, S. M., Leahy, D., & Campbell, S. S. 2007, ApJ, 654, 458
Chan, C.-K., Psaltis, D., & Özel, F. 2013, ApJ, submitted
Cook, G. B., Shapiro, S. L., & Teukolsky, S. A. 1994, ApJ, 424, 823
DeDeo, S., Psaltis, D., & Narayan, R. 2001, ApJ, 559, 346
Feroci, M., Stella, L., van der Klis, M., et al. 2012, ExA, 34, 3415
Gendreau, K. C., Arzoumanian, Z., & Okajima, T. 2012, Proc. SPIE, 8443, 844313
Glampedakis, K., & Babak, S. 2006, CQGra, 23, 4167
Hartle, J. B., & Thorne, K. S. 1968, ApJ, 153, 807
Laarakkers, W. G., & Poisson, E. 1999, ApJ, 512, 282
Lamb, F. K., Boutiloukos, S., Van Wassenhove, S., et al. 2009, ApJ, 706, 417
Leahy, D. A., Morsink, S. M., & Cadeau, C. 2008, ApJ, 672, 1119
Leahy, D. A., Morsink, S. M., & Chou, Y. 2011, ApJ, 742, 17
Leahy, D. A., Morsink, S. M., Chung, Y.-Y., & Chou, Y. 2009, ApJ, 691, 1235
Miller, M. C., & Lamb, F. K. 1998, ApJL, 499, L37
Morsink, S. M., & Leahy, D. A. 2011, ApJ, 726, 56
Morsink, S. M., Leahy, D. A., Cadeau, C., & Braga, J. 2007, ApJ, 663, 1244
Muno, M. P., Özel, F., & Chakrabarty, D. 2002, ApJ, 581, 550
Muno, M. P., Özel, F., & Chakrabarty, D. 2003, ApJ, 595, 1066
Nath, N. R., Strohmayer, T. E., & Swank, J. H. 2002, ApJ, 564, 353
Özel, F. 2013, RPPh, 76, 016901
Özel, F., Psaltis, D., & Kaspi, V. M. 2001, ApJ, 563, 255
Page, D. 1995, ApJ, 442, 273
Pappas, G., & Apostolatos, T. A. 2012, PhRvL, 108, 231104
Pavlov, G. G., & Zavlin, V. E. 1997, ApJ, 490, L91
Pechenick, K. R., Fuaclas, C., & Cohen, J. M. 1983, ApJ, 274, 846
Poutanen, J., & Beloborodov, A. M. 2006, MNRAS, 373, 836
Poutanen, J., & Gierlinski, M. 2003, MNRAS, 343, 1301
Psaltis, D., & Johannsen, T. 2012, ApJ, 745, 1
Stergioulas, N. 2003, LRR, 6, 3
Stergioulas, N., & Friedman, J. L. 1995, ApJ, 444, 306
Weinberg, N., Miller, M. C., & Lamb, D. Q. 2001, ApJ, 546, 1098