Defining a Service Angle for Planar Mechanisms of Manipulators based on the Instantaneous States Analysis

F Pritykin and O Gordeev
Department of Engineering Geometry and CAD, Omsk State Technical University, 11, Mira ave., Omsk, 644050, Russia
E-mail: pritykin@mail.ru

Abstract. The paper provides an algorithm of defining a service angle for planar mechanisms of manipulators having different structures of kinematic chains. The algorithm is based on the motion synthesis in the vector of velocities. The functions defining specified angle determination accuracy by the developed method are obtained.

1. Introduction
The development of universal algorithms that enable us to explore the functionality of manipulators mechanisms is still a crucial task [1-4]. A virtual construction of the manipulator motion implementing the method of instantaneous states allows one to predict fine precision motion of a real mechanism and its interference with known in advance objects of the environment. The method is based on the division of the planned path of the output link into segments and finding a configuration relevant to a motion volume minimization criterion (under motion redundancy). If there are forbidden zones in the service area of a manipulator the use of the configuration relevant to the motion minimization criterion is unlikely to be practical. If that is the case, it is necessary for the intelligent control system to search for different configurations that provide output link shifting to the target point with the required positional accuracy and choose those ones providing avoidance of the forbidden zones, minimum power losses being desirable. A method of searching arrays of configurations defining the service angle based on the instantaneous states analysis of planar mechanisms of manipulators is proposed.

2. The analysis of instantaneous states implementation
The convention is that the value of the generalized velocities vector relevant to the volume minimization criterion is defined as point $M^Q$. Index $Q$ means pertaining to multidimensional space of generalized velocities $Q_n$. Figure 1 shows two configurations of a planar seven-link mechanism of the manipulator, links lengths being equal to 120 mm. Figure 1 also represents the results of the instantaneous states implementation under the specified velocity vector of output link $V(V_x, V_y)$, $V_x = 1$ mm/s, $V_y = 0$ and position accuracy $\delta \leq 1$ mm. Parameter $r = 2$ assigns vector length $V$, the orientation of the output link not being considered. The sets of configurations relevant to $\delta \leq 1$ mm are obtained by applying AutoCAD and algorithmic programming language AutoLISP. The arrays of configurations are constructed as follows. Point $M^Q$ in the space of generalized velocities $Q_n$ is known to be found with the linear system of equations reflecting dependence of $V$ and $Q$, where $Q$ is the vector of generalized velocities of length $n$. In addition to the equations expressing individual components of vector $V$ (output link linear motions) the system (in case $n > r$) uses the equation of
complementary mutually perpendicular planes, their amount being equal to the length of \( p \)-plane, determining motion redundancy. Normals of the complementary planes reduced to unit vectors \( Q_{ll}, Q_{l2}, \ldots, Q_{ln} \) specify an orthonormal basis in coordinate system \( O^p \), defining point position \( N^0 \) in \( p \)-plane about point \( M^0 \). System \( O^p \), coordinates are determined by means of parameters \( k_1, k_2, \ldots, k_n \) defining point \( N^0 \) motion length towards vectors \( Q_{ll}, Q_{l2}, \ldots, Q_{ln} \) accordingly. The vector equation defining point \( N^0 \) position in space \( Q_6 \) is as follows:

\[
Q_N = Q_M + k_1 Q_{ll} + k_2 Q_{l2} + \ldots + k_n Q_{ln},
\]

(1)

Where \( Q_M \) and \( Q_N \) vectors defining \( M^0 \) and \( N^0 \) points position in space \( Q_6 \). Thus, implementation of vector \( Q_N \) values with parameter \( \delta \) allows constructing zone of \( A^0 \) configurations. Figure 1a, b shows that the angle formed by a gripping device for two different configurations corresponds to zones \( A^0 \) and equals one and twenty four degrees and it does not detect the service angle.

![Figure 1. Zone \( A^0 \) defining manipulator configuration arrays.](image)

3. Defining a service angle implementing instantaneous states

The accuracy of detection of angle \( U_p \), which with certain assumptions detects service angle \( U_p \) will be examined with the use of the developed algorithm based on instantaneous states implementation. To configure the mechanism shown in Figure 2a, the service angle specified by the position of segments \( B_6^1 B_7 \) and \( B_6 B_7 \) for the point of service zone \( A_2 = B_7 \) is detected by an intersection of two circles. The centers of the circles specified coincide with points \( O_o \) and \( A_2 \), their radii being equal to \( 120 \cdot 5 = 600 \) mm and \( 120 \) mm, respectively. To configure the datum of the gripper unit center position determined by \( A_2 (q_1 = 11.5^\circ, q_2 = 0, q_6 = 90^\circ) \) the service angle specified equals to \( 169^\circ \), where parameters \( q_i \) determine the values of generalized coordinates. Points \( B_1 \), \( B_7 \) in Figure 2 define the node points of mechanisms. Thus, to configure the mechanism in Figure 2b the service angle is determined by the position of segments \( B_6 B_7 \) and \( B_6^1 B_7 \). For the mechanism given the limited values of the generalized coordinates prescribing the translational motions of joints are predetermined with inequality \( 50 \) mm \( \geq \varphi_1,2 \geq 0 \), values of generalized coordinates being \( \varphi_1 = 11.5^\circ, \varphi_4,5 = 0^\circ, \varphi_6 = 90^\circ \), the service angle of the mechanism equals \( 151^\circ \). Point \( B_6^1 \) is determined as the tangency point of line \( l \) with circle \( a_o \), its radius being defined by segment \( B-B_6 \) and it equals \( 120 \) mm. Here \( B_3 = \epsilon l, B_3^1 = l \cap a_1 \), where \( \epsilon \) and \( \cap \) denote the relation of the membership and intersection, respectively. In Figure 2b the distance between vertical line \( l_1 \) and origin of the fixed coordinate system is \( 50 \) mm, \( a_1 \) is a circle with the centre at point \( B_5 \) its radius being defined by segment \( B_5 B_7 \).

Let us define the aggregate of angle \( U_p \) values implementing instantaneous states for the configurations of the mechanism, whereby point \( A_1 \), (the gripper centre) is in segment \( A_1 A_3 \). The length of the segment equals the double length of the link \( (A_1A_3 = 240 \) mm, wherein \( \varphi_2,5 = 0^\circ \) for the configuration under study).
We shall consider seven different positions of point $A_i$ in segment $A_1A_3$. Neighboring points $A_i$ are at a distance of 30 mm. The output links of two configurations calculated during the start and k iterations (Figure 2a) determine angle $U_m^k$. The angle between segments $B_6B_7$ and $B_3B_7$ determines the angle specified, wherein $U_m^k = U_2 - U_1$. We shall construct the aggregate of configurations implementing instantaneous states, the value of angle $U_m^k$ being changed counterclockwise. This value is more than value $U_m^{k-1}$, calculated during the previous iteration. The starting position of the mechanism configuration (Figure 2a) for point $A_2$ corresponds to the values of generalized coordinates $q_1 = 11.5^\circ$, $q_{2-5} = 0$, $q_6 = 90^\circ$.

![Figure 2a](image1.png)

**Figure 2.** Determination of service angle $U$ using graphic constructions.

Angle $U_m^k = 0$ for the configuration given. An important point is that at each calculation step the maximum value of angle $(U_m^k)^{\max}$ is defined based on values $U_m^k$ and $U_m^{k+1}$:

$$(U_m^k)^{\max} = \max(U_m^k, U_m^{k+1}).$$

As soon as the configuration satisfying the conditions is defined as

$$U_m^{k+1} > (U_m^k)^{\max},$$

$$\delta < \delta_{\min},$$

(2)

(3)
the next \((k+2)\) configuration is determined, the list of generalized coordinates \(q_{\pi} = q_i\) being retained. Parameter \(\delta_{\text{min}}\) imposes the prescribed positioning accuracy. Here the \((k+1)\) configuration is counted as the initially prescribed configuration, conditions (1, 2) being satisfied for that configuration. The case considered corresponds to the synthesis of link \(B_6B_7\) motions rotating about point \(B_7\) counterclockwise.

If we consider the motion of link \(B_6B_7\) rotating clockwise, condition (1) \(B_6B_7\) is substituted by the following condition:

\[
(U_m^k)_{\text{min}} = \min(U_m^k, U_m^{k+1}),
U_m^{k+1} < (U_m^k)_{\text{min}}.
\]  

(4)

We use variable \(kk\) to define the rotation variant: when \(kk = 0\), the counterclockwise rotation is considered; when \(kk = 1\), clockwise rotation is viewed.

The algorithm of constructing configurations satisfying conditions (2-4) is shown in Figure 3.

**Figure 3.** A diagram of constructing the angle \(U_p\) algorithm implementing instantaneous states.

The following notations are used in Figure 3: 1 – start, determining the value of vector \(L_n (q_1, ..., q_6)\) and parameter \(kk\); 2 – calculation of point \(A_i\) of the service zone, the target point and vector \(V\); 3 – calculation of the matrix of particular gear ratios \(J\) and vector \(Q\) with the criterion of minimizing the motion range; 4 – calculation of the configuration \(L_n^{k+1}\) position corresponding to vector \(Q\), \(q_i := q_i + \dot{q}_i\) (assumption \(\Delta q_i \rightarrow q_i\) is admitted); 5 – calculation of implementations errors \(\delta\); 6 – conditional test \(\delta < \delta_{\text{min}}\); 7 – conditional tests \(q_i^{\text{min}} > q_i > q_i^{\text{max}}\), where \(q_i^{\text{min}}, q_i^{\text{max}}\) – prescribed limit values of generalized coordinates (calculating test cases \(q_i^{\text{min}} = -180^\circ, q_i^{\text{max}} = 180^\circ\); 8 \(- kk = 0\); 9 \(- U_m^{k+1} > (U_m^k)_{\text{max}}\); 10 \(- U_m^{k+1} < (U_m^k)_{\text{min}}\); 11 – configuration of image formation: \(kk=0\) value assignment \(\Rightarrow (U_m^k)_{\text{max}} := \max(U_m^k, U_m^{k+1})\); \(kk = 1\) \(\Rightarrow\) value assignment \((U_m^k)_{\text{min}} := \min(U_m^k, U_m^{k+1})\); \(q_{\pi} = q_i\), where \(q_{\pi}\) – list of generalized coordinates satisfying conditions (2), (3) or (3), (4), \(q_i\) – list of generalized coordinates of the current configuration; 12 – parameters \(k_1, k_2, ..., k_4\) values change.
defining coordinates of point \( N^Q \) in \( p \)-plane \( T \); 13 – \( k_1 > k_\text{max}, \ k_2 > k_\text{max}, \ldots, \ k_d > k_\text{max} \) (where \( k_\text{max} \) – limit – prescribed largest extremum experimentally determined in research process \( k_\text{max} = 20 \)); 14 – calculation of vector \( Q \) for \( k \neq 0 \); 16 – \( U_{m \ k+1} = (U_{m \ k})_\text{max} \); 17 – end.

In Figure 4 and Figure 5 the synthesis of motions to determine angle \( U_p \) using dependences (1-2) for three mechanism configurations studied is shown.

**Figure 4.** Determining angle \( U_p \) by implementation of instantaneous states: \( a \) – \( (q_1 = 11.5^\circ, \ q_2=5 = 0, \ q_6 = 90^\circ) \); \( b \) – \( (q_1 = 7.4^\circ, \ q_2=5 = 0, \ q_6 = 45.5^\circ) \).

**Figure 5.** Determining angle \( U_p \) by implementation of instantaneous states \( (\phi_{1,2}= 50 \text{ mm}, \ \phi_3 = 11.5^\circ, \ \phi_{4,5} = 0, \ \phi_6 = 90^\circ) \).
In Figures 6 and 7 the dependency graphs $U_{p3} = f_1(L)$, $U_{p4} = f_2(L)$, $U_{p5} = f_3(L)$, $U_{p6} = f_4(L)$ and $\mu_3 = f_1(L)$, $\mu_4 = f_2(L)$, $\mu_5 = f_3(L)$, $\mu_6 = f_4(L)$ are shown. These graphs reflect the angle $U_p$ variation and accuracy percentage $\mu$ of service angle determination for four-, five-, six- and seven–link planar mechanisms (Figure 2a) having link lengths equal to 120 mm.

Figure 6. Dependency graphs $U_{pn} = f_1(L)$. 

Figure 7. Dependency graphs $\mu_n = f_1(L)$. 

The accuracy percentage is calculated using dependence $\mu = \frac{U_p}{U} \cdot 100$, where the service angle or parameter $U$ was determined graphically. Parameter $L$ defines deproach of point $A_i$ from point $A_1$ (Figure 2a). Point $A_i$ for the mechanisms specified holds at distance $L_1 = 120 \cdot (n - 2)$ from origin $O_o$. The analysis of functions graphs shows the accuracy percentage of the service angle defined by the method developed for the mechanism in Figure 2a for $\delta_{min} \leq 1$ mm, where is from 94 to 100 percent depends on the position of the gripper center in segment $A_1A_3$. The accuracy percentage for $\delta \leq 3$ mm is from 80 to 85 percent for the mechanism shown in Figure 2b.

4. Conclusion
The proposed algorithm of determining angle $U_p$ with certain assumption defining the service angle is described as universal and can be used for manipulators having a random structure of planar kinematic chains, a random degree of motion redundancy and prescribed limited values of generalized coordinates. The construction of solid angles projections using the algorithm proposed is possible for spatial mechanisms of robots. The method of the output link reorientation can be applied for intelligent robot control systems to change the output link orientation relative to the prescribed objects of manipulation.

References
[1] Pratt J, Dilworth P and Pratt G 1997 Robotics and Automation: Proc. IEEE Int. Conf. 1 193–8
[2] You B, Zou Y, Xiao W and Wang J 2010 Telerobot control system based on dual-virtual model and virtual force Strategic Technology (IFOST) Int. Forum 246 – 50
[3] Hrr J, Pratt J, Chew C, Herr H and Pratt G 1998 Intelligence and Systems:Proc. IEEE Int. Joint Symp. 245–51
[4] Tsukamoto H, Takubo T, Ohara K, Mae Y and AraiT 2010 Information and Automation (ICIA): IEEE Int. Conf. 729–34