Constraints on the Leading-Twist Pion Distribution Amplitude
from A QCD Light-Cone Sum Rule with Chiral Current

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Abstract

We present an improved analysis of the constraints on the first two Gegenbauer moments, $a_2^\pi$ and $a_4^\pi$, of the pion’s leading-twist distribution amplitude from a QCD light-cone sum rule analysis of $B \to \pi$ weak transition form factor $f_+(q^2)$. Proper chiral current is adopted in QCD light-cone sum rule so as to eliminate the most uncertain twist-3 contributions to $f_+(q^2)$, and then we concentrate our attention on the properties of the leading-twist pion DA. A nearly model-independent $f_+(q^2)$ as shown in Ref. [14] that is based on the spectrum of $B \to \pi l\nu$ decays from BaBar, together with their uncertainties, are adopted as the standard shape for $f_+(q^2)$ to do our discussion. From a minimum $\chi^2$-fit and by taking the theoretical uncertainties into account, we obtain $a_2^\pi(1 GeV) = 0.17^{+0.15}_{-0.17}$ and $a_4^\pi(1 GeV) = -0.06^{+0.20}_{-0.22}$ at the 1σ confidence level for $m_b^* \in [4.7, 4.8] GeV$.

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The pion distribution amplitude (DA) that enters into the exclusive processes via the factorization theorem at high momentum transfer is an important factor in perturbative QCD. The leading twist pion DA is usually expressed in terms of its conformal expansion

$$\phi_{\pi}(x, \mu) = 6x(1-x) \left( 1 + \sum_{n=1}^{\infty} a_{2n}^{\pi}(\mu) C_{2n}^{3/2} (2x - 1) \right),$$  

(1)

where \(x \in [0,1]\) is the momentum fraction of the quark in the pion. \(C_{2n}^{3/2}(2x - 1)\) are Gegenbauer polynomials and \(a_{2n}^{\pi}(\mu)\), the so-called Gegenbauer moments, are hadronic parameters that depend on the factorization scale \(\mu\). Many works are presented to provide precise values for these Gegenbauer moments, but till now, whether the pion’s leading twist DA is asymptotic like [1] or CZ-like [2] is still an open question, a simple review of this issue can be found in Ref.[3]. Calculations of the second Gegenbauer moment \(a_{2}^{\pi}\) of pion DA have attracted quite a bit of attentions and has been discussed through different approaches, a summary of them can be found in Ref.[4] and references therein. Recently, through a comprehensive analysis of the pion-photon transition from factor \(F_{\pi\gamma}\) involving the transverse momentum corrections with the CLEO experimental data [5], in which the contributions beyond the leading Fock state have been taken into consideration, Ref.[6] shows that \(a_{2}(4GeV^{2}) = 0.002^{+0.063}_{-0.054}\) and \(a_{4}(4GeV^{2}) = -0.022^{+0.026}_{-0.012}\) that are closed to the asymptotic-like behavior of the pion DA.

The process \(B \to \pi\ell\nu\) provides a good platform for studying the pionic distributions. The QCD light-cone sum rule (LCSR) provides a useful way to study its key factor, i.e. the \(B \to \pi\) transition form factor, in the large and intermediate energy regions \((q^{2} \lesssim 16GeV^{2})\). By taking the conventional correlation function for the \(B \to \pi\) transition form factors [8, 9], it is found that the main uncertainties in estimation of the \(B \to \pi\) transition form factors come from the different twist structures of the pion wave functions, e.g. the twist-2 and twist-3 contributions have the same importance. So to extract more reliable information of the leading-twist DA, one needs a better understanding of the twist-3 contribution. A comprehensive analysis calculated from QCD sum rules on the light-cone to \(O(\alpha_s)\) accuracy for twist-2 and the dominant twist-3 contributions has been presented in Refs.[8, 9], and it

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1 A consistent analysis of the \(B \to \pi\) form factor in its whole physical region by analyzing the perturbative QCD, LCSR and Lattice QCD results can be found in Ref.[7].
is found at the 1σ confidence level that

$$a_2^\pi(1\text{GeV}) = 0.19 \pm 0.19, \quad a_4^\pi(1\text{GeV}) \geq -0.7.$$ (2)

On the other hand, it has been found that by choosing proper chiral currents in the LCSR approach, the contributions from the most uncertain twist-3 structures to the form factor can be directly eliminated [11, 12]. In Ref. [13] we have shown that these two treatments to deal with the twist-3 contributions of the $B \to \pi$ or $B \to K$ form factors are equivalent to each other. Since the pollution from the twist-3 structures are eliminated and the even higher twist structures provide small contributions (less than 5%), so the LCSR with chiral current may derive more precise information on the leading twist-2 DA. This is the purpose of the present letter. Furthermore, our present analysis shall also provide a meaningful cross check of $a_2^\pi$ and $a_4^\pi$ derived in Ref. [10] through the conventional LCSR calculation.

The hadronic matrix element relevant for $B \to \pi \ell \nu$ is given by

$$\langle \pi(p_\pi) | \bar{u} \gamma_\mu b | B(p_B) \rangle = \left( p_{B\mu} + p_{\pi\mu} - \frac{m_B^2 - m_\pi^2}{q^2} q_\mu \right) f_+(q^2) + \frac{m_B^2 - m_\pi^2}{q^2} q_\mu f_0(q^2),$$ (3)

where the form factors $f_+,0$ depend on $q^2 \equiv (p_B - p_\pi)^2$, the invariant mass of the lepton-pair, with $0 \leq q^2 \leq (m_B - m_\pi)^2 \approx 26.4\text{GeV}^2$. Only $f_+(q^2)$ is needed for calculating the spectrum, i.e.

$$\frac{d\Gamma}{dq^2}(B^0 \to \pi^- \ell^+ \nu_\ell) = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_B^3} \lambda^{3/2}(q^2) |f_+(q^2)|^2$$ (4)

for massless leptons, where $\lambda(q^2) = (m_B^2 + m_\pi^2 - q^2)^2 - 4m_B^2 m_\pi^2$ is the usual phase-space factor. By taking the LCSR with chiral current, it is found that the main theoretical uncertainty comes from the pion’s leading-twist light-cone DA $\phi_\pi$, and other smaller uncertainty sources include the $b$ quark mass, the quark condensate, sum rule specific parameters (Borel parameter and continuum threshold) and etc. Numerically, it can be found that the $q^2$-dependence of the form factor $f_+(q^2)$ is mostly sensitive to $a_2^\pi$ and only to a lesser extent to higher Gegenbauer-moments. We hence decide to use the $\phi_\pi$ proposed in Eq.(1), which we truncate after the contribution in $a_4^\pi$.

A nearly model-independent analysis for $f_+(q^2)$ based on the experimental data has been given in Ref. [14], in which the value of $V_{ub}$ from the UTfit Collaboration [15] and the CKMfitter Collaboration [16], e.g. $|V_{ub}| = (3.50 \pm 0.18) \times 10^{-3}$, and the spectrum of $B \to \pi \ell \nu$ decays from BaBar [17] have been adopted. The best fits obtained by using five parameterizations of $f_+(q^2)$, i.e. Becirevic/Kaidalov (BK) [18], Ball/Zwicky(BZ) [19],
FIG. 1: Best fits for $f_+(q^2)$ that are derived from the fitting of the BaBar experimental data [17], where the solid line is for BK parameterization [14]. The shaded band shows the total uncertainties that include the errors of the five parameterizations.

Boyd/Grinstein/Lebed (BGL) [20] with two choices for its free parameter $q_0^2$ (called as BGLa or BGLb parameterization respectively), and the Omnes representation of Ref. [21] (AFHNV), are very close to each other in the low and intermediate energy regions (all best-fit form factors agree within 2% [14]) and noticeable differences occur only for large $q^2$ region. Since the QCD LCSR are reliable only in low and intermediate energy regions that is less than $16 GeV^2$, so we shall adopt the fitted results of these five parameterizations with their possible errors within the region of $q^2 < 16 GeV^2$ as the standard shape for $f_+(q^2)$ to do our following discussion. We shall not extrapolate our LCSR result to even higher energy regions in order to minimize any uncertainty from extrapolating in $q^2$. More explicitly, the best fits obtained by using those five parameterizations of $f_+(q^2)$ shown in Fig. (1), together with the additional ±3% error from the total branching ratio of $B \to \pi$, shall be used as the experimentally determined shape of the form factor. Further more, since the center value of the above listed five parameterizations are very close to each other in the region of $q^2 < 16 GeV^2$, so we take the simpler BK-parameterization to be the center value of $f_+(q^2)$ as shown by the solid line of Fig. (1), i.e.

$$f_+(q^2) \bigg|_{BK} = \frac{f_+(0)}{(1 - q^2/m_{B^*}^2)(1 - \alpha q^2/m_{B^*}^2)}$$  \hspace{1cm} (5)$$

with $m_{B^*} = 5.325 GeV$, $f_+(0) = 0.26$ and $\alpha = 0.53$ [14]. Then by fitting our LCSR result with chiral current to the experimentally determined shape for $f_+(q^2)$, we can determine the
TABLE I: Parameters for $f_B$, where $m_b^*$ and $f_B$ are given in GeV, $s_0$ and $M^2$ in GeV$^2$. The first direct measurement of $f_B$ by Belle experiment shows $f_B = 229^{+36}_{-31}(\text{stat.})^{+34}_{-37}(\text{syst.})$ MeV [23].

| $m_b^*$ | $s_0$ | $M^2$ | $f_B$ | $s_0$ | $M^2$ | $f_B$ |
|--------|-------|-------|-------|-------|-------|-------|
| 4.7    | 33.5  | 2.80  | 0.165 | 33.5  | 2.80  | 0.219 |
| 4.8    | 33.2  | 2.39  | 0.131 | 33.2  | 2.31  | 0.174 |

Before a comparison of our LCSR result with the fitted shape for $f_+(q^2)$, we make some comments on the treatment of $f_B$. To be consistent, $f_B$ should be varied accordingly and be determined by using the two-point sum rule with the chiral currents. The sum rule for $f_B$ up to NLO can be obtained from Ref. [22] through a proper combination of the scalar and pseudo-scalar results shown there, which can be schematically written as

$$f_B^2 M_B^2 e^{-M_B^2/M^2} = \int_{m_b^2}^{s_0} \rho_{\text{tot}}(s) e^{-s/M^2} ds,$$

(6)

where the spectral density $\rho_{\text{tot}}(s)$ can be read from Ref. [22]. The Borel parameter $M^2$ and the continuum threshold $s_0$ are determined such that the resulting form factor does not depend too much on the precise values of these parameters; in addition, 1) the continuum contribution, that is the part of the dispersive integral from $s_0$ to $\infty$, should not be too large, e.g. less than 20% of the total dispersive integral; 2) the contributions from the dimension-six condensate terms shall not exceed 15% for $f_B$; 3) the derivative of the logarithm of Eq.(6) with respect to $1/M^2$ gives the B-meson mass $M_B$ [19],

$$M_B^2 = \int_{m_b^2}^{s_0} \rho_{\text{tot}}(s) e^{-s/M^2} s ds \int_{m_b^2}^{s_0} \rho_{\text{tot}}(s) e^{-s/M^2} ds,$$

and we require its value to be full-filled with high accuracy $\sim 0.1\%$. These criteria define a set of parameters for each value of the effective $b$-quark $m_b^*$ and some typical values are listed in Tab.II, where $f_B$ is taken as the extremum within reasonable region of $(M^2, s_0)$.

The LCSR with chiral current for $f_+(q^2)$ including twist-2 contributions to $\mathcal{O}(\alpha_s)$ accuracy and twist-4 contributions at tree-level can be found in Refs. [13, 24, 25]. The interesting reader may turn to these references for more detailed technology, especially the $B \rightarrow \pi$ form factor can be directly obtained from Ref. [13] by properly ignoring the $SU_f(3)$-breaking effect.
FIG. 2: $f_+(q^2)$ of $a_2^\pi$ for the case of $a_4^\pi = 0$, $m_b^* = 4.7 GeV$ (Left) and $m_b^* = 4.8 GeV$ (Right). $f_+(q^2)$ increases with the increment of $a_2^\pi$ in the lower energy region but decreases with the increment of $a_2^\pi$ in the higher energy region.

in the $B \to K$ form factor. As a comparison, we obtain values for $f_+(q^2)$ in dependence of $a_2^\pi$, $a_4^\pi$ and $m_b^*$ using the same criteria as suggested in Ref. [10] for the evolution of the LCSRs. Further more, for each value of $f_+(q^2)$ we calculate the theoretical uncertainty by varying: 1) the Borel parameter $M^2$ in the LCSR for $f_+(q^2)$ within the region of $[10,18]$ GeV$^2$; 2) $s_0$ by ±1 GeV$^2$; 3) the central value 20% of the continuum contribution between 15% and 25%. The above ranges of sum rule parameters are rather conservative and account for the “systematic” uncertainty of QCD sum rule calculations.

Next, we require the LCSR result to be compatible with the BaBar experimental data, i.e. the LCSR result of $f_+(q^2)$ should be within the shaded band of Fig.(1) with $q^2 \leq 16$ GeV$^2$, then we can derive the reasonable ranges for $a_2^\pi$, $a_4^\pi$ and $m_b^*$. Further more, we adopt $a_2^\pi(1 GeV) \geq 0$ that is favored in the literature [27], and $m_b^* \in [4.7,4.8]$ GeV [10]$^2$ to do our discussions.

Before deriving the possible ranges of these parameters, we show the properties of $f_+(q^2)$ versus $a_2^\pi$ and $a_4^\pi$ respectively in Figs.(2, 3) by varying $a_2^\pi$ and $a_4^\pi$ independently. From Figs. (2, 3), it can be found that $f_+(q^2)$ increases (decreases) with the increment of $a_2^\pi$ ($a_4^\pi$) in the lower energy region but decreases (increases) with the increment of $a_2^\pi$ ($a_4^\pi$) in the higher energy region, so possible range of $a_2^\pi$ or $a_4^\pi$ can indeed be derived by demanding $f_+(q^2)$

$^2$ A review of Ref. [26] shows that $m_b^* \simeq 4.8 \pm 0.1$ GeV, which can also be adopted by allowing the discrepancy between the LCSR and the PQCD calculation in the low energy region to be less than 15% [13].
FIG. 3: $f_+(q^2)$ of $a_4^5$ for the case of $a_2^5 = 0.14$, $m_b^* = 4.7\,GeV$ (Left) and $m_b^* = 4.8\,GeV$ (Right). $f_+(q^2)$ decreases with the increment of $a_4^5$ in the lower energy region but increases with the increment of $a_4^5$ in the higher energy region.

FIG. 4: Allowed values of $a_2^5, a_4^5(1\,GeV)$ for $m_b \in [4.7, 4.8]\,GeV$ with $\chi^2 \leq \chi^2_{\alpha=68.27\%}(3) = 1.87$ for 3 d.o.f. at the 1σ C.L., within the fitted band of Fig.(1). Furthermore, since $f_+(q^2)$ increases with the increment of $m_b^*$, the value of $m_b^*$ should not be too large.

Now the task is to compare the form factor prediction to data and to determine best-fit values of $a_2^5$ and $a_4^5$, using the experimentally determined shape for $f_+(q^2)$ as shown in Fig.(1). More explicitly, we take one hundred $f_+(q^2)$ points uniformly within the range of $q^2 \in [0, 16]\,GeV^2$ respectively, together with their corresponding errors that can be derived from Ref.[14] to do the calculation. The resulting constraints are shown in Fig.(1). The minimum $\chi^2$ for $(a_2^5, a_4^5)$ is reached for $a_2^5 = 0.17$ and $a_4^5 = -0.06$, i.e. near the center
of the parameter space. The biggest counter include all \((a_2^\pi, a_4^\pi)\) for which the fit of the corresponding form factor to the data yields \(\chi_2^2 \leq \chi_\alpha^2(3) = 1.87\) for 3 degree of freedom (d.o.f.) at the 68.27% (1\(\sigma\)) confidence level (C.L.), where \(\chi_\alpha^2(n)\) is derived from \(\int_0^{\chi_2^2(n)} f(y; n)dy = \alpha\) with \(f(y; n) = \frac{1}{\Gamma(n/2)2^{n/2}y^{n/2-1}}e^{-y/2}\). One can immediately read off the following constraints of \(a_2^\pi, a_4^\pi\) at the 1\(\sigma\) confidence level

\[
a_2^\pi(1\text{GeV}) = 0.17^{+0.15}_{-0.17}, \quad a_4^\pi(1\text{GeV}) = -0.06^{+0.20}_{-0.22}.
\]

(7)

This result is consistent with that of Ref. [10], i.e. Eq. (2), but with less uncertainty, which shows that the two independent treatments of the pionic twist-3 contributions are consistent with each other. This also inversely implies that by properly taking the parameter values, the LCSR prediction can be compatible with the experimentally determined shape of the form factor.

To summarize: we have presented an improved analysis on the pionic leading twist DA posed by the recently derived fitted shape for the \(B \to \pi\) form factor \(f_+(q^2)\) from the BaBar experimental data and the value of \(V_{ub}\) from the UTfit and CKMfitter Collaborations. It is found that the present LCSR with chiral current are consistent with the conventional LCSR result [10], and we obtain \(a_2^\pi(1\text{GeV}) = 0.17^{+0.15}_{-0.17}\) and \(a_4^\pi(1\text{GeV}) = -0.06^{+0.20}_{-0.22}\) at the 1\(\sigma\) confidence level for \(m_b^* \in [4.7, 4.8] \text{ GeV}\). Since the twist-3 contribution is eliminated, the present LCSR result is less uncertain than that of the conventional LCSR analysis. The LCSR sum rule with chiral current provides a useful way to simply the conventional LCSR calculation, so it can be applied for other useful processes, a review of it shall be presented elsewhere [28].

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