Spatial oscillations in the spontaneous emission rate of an atom inside a metallic wedge

H J Zhao¹ and M L Du²

¹ School of Physics and Information Science, Shanxi Normal University, Linfen 041004, People’s Republic of China
² Institute of Theoretical Physics, Chinese Academy of Sciences, PO Box 2735, Beijing 100190, People’s Republic of China

E-mail: duml@itp.ac.cn

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Abstract
A method of images is applied to study the spontaneous emission of an atom inside a metallic wedge with an opening angle of $\pi/N$, where $N$ is an arbitrary positive integer. We show that the method of images gives a rate formula consistent with that from quantum electrodynamics. Using the method of images, we show the correspondence between the oscillations in the spontaneous emission rate and the closed orbits of emitted photon going away and returning to the atom inside the wedge. The closed orbits can be readily constructed using the method of images, and they are also extracted from the spontaneous emission rate.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The effect of boundaries and environments on the spontaneous emission of atoms is an interesting subject in the physics of radiation with a long history [1]. Our understanding of the subject has been greatly improved by the theoretical and experimental developments over the years [2–6]. For example, Hulet et al. in an experiment demonstrated clearly that the lifetime of an atomic state can be greatly modified by the two metal plates [7]. From a practical point of view, once the boundaries are given and specified for a system, we would like to know how the spontaneous emission rate is modified. In particular, how does the spontaneous emission rate depend upon the position and polarization of the emitting atom in the system. Quantum electrodynamics (QED) relates the spontaneous emission rate of an atom to the quantized electromagnetic field [8, 9]. In principle, the calculations of the spontaneous emission rate of an atom in a cavity, therefore, become the search for the quantized electromagnetic field of the specified system, the evaluation of the matrix elements and the summation of the matrix elements squared. In practice, however, the above process can be complicated and difficult to apply even for a simple system such as a metallic wedge [10, 11]. Therefore, there is a need for alternative methods.

In this paper we will apply a theoretical method of images to study the spontaneous emission rate in a metallic wedge. In an interesting application, the method of images has recently been used to study quantum interference at corners [12]. We will show that the method works for a wedge with an angle of $\pi/N$, where $N$ is an arbitrary positive integer. As examples, we have worked out the details for wedges of $\pi/3$ and $\pi/4$ corresponding to $N = 3$ and $N = 4$. We find that the method of images gives a formula for the spontaneous emission rate which is consistent with the one obtained by Rosa et al. involving long derivations using QED. The method of images is not only easy to apply, it also provides a way to understand the rate formula by associating each term with an image and a closed orbit. For other systems, we have shown that the large-scale oscillations in the spontaneous emission rate [13, 14] are similar to the oscillations for photo-ionization and photo-detachment in the presence of external fields [15–19]. Using scaling variables, we have demonstrated the direct correspondence between the oscillations in the spontaneous emission rate and the closed orbits of photon [13, 14]. Here we will use the same method to analyse the
oscillations in the spontaneous emission rate in a wedge with an angle of $\pi/3$ and $\pi/4$ respectively. We will show that there are four distinct oscillations for a $\pi/3$ wedge, and there are six distinct oscillations for a $\pi/4$ wedge.

The paper is organized as follows. In the next section, we discuss the construction of the images and the calculation of the emission rate. The method of images has been applied to discuss the spontaneous emission rate of an atom near a mirror and for an emitting atom near a plane mirror, it has been demonstrated that the spontaneous emission rate can be inferred from the power radiation rate of an antenna [9]. Consider the radiation damping of a dipole antenna with the dipole moment $\vec{d}e^{-i\omega t}$, where $\omega_0$ is the frequency. The radiation damping rate is defined as $W_d = P/U$, where $P$ is the average radiation power of the antenna and $U$ is the mechanical energy of the antenna. Let $\vec{r}$ denote a vector of a point relative to the dipole position; then the radiation damping rate can be written as [9]

$$W_d = \frac{\omega_0}{2U} \text{Im}[\vec{d}\cdot \vec{E}(\vec{r} = 0)] = \frac{\omega_0}{2U} [\text{Im}[\vec{d}\cdot \vec{E}_0(\vec{r} = 0)] + \text{Im}[\vec{d}^*\cdot \vec{E}_\text{ref}(\vec{r} = 0)]] \quad (1)$$

where the electric field $\vec{E}$ at the position of dipole antenna has been decomposed into a direct part $\vec{E}_0$ and a reflected part $\vec{E}_\text{ref}$. The direct electric field is

$$\vec{E}_0 = \frac{d k^3}{4 \pi \varepsilon_0} \left( \hat{r} \times \hat{d} \right) \times \hat{r} \left( \frac{1}{kr} \right) + [\hat{d} - 3\hat{r}(\hat{r} \cdot \hat{d})] \times \left( \frac{i}{(kr)^3} - \frac{1}{(kr)^5} \right) e^{i(kr-\omega t)} \quad (2)$$

where $\hat{d}$ is a unit vector in the dipole direction and $\varepsilon_0$ is the vacuum permittivity. If the dipole antenna is at a distance of $l/2$ from a plane metal boundary and is parallel to the metal surface, the effect of the conducting boundary can be replaced by an image dipole and the reflected electric field $\vec{E}_\text{ref}$ of the dipole can be calculated directly from the imaging dipole. The imaging dipole direction is opposite to the original dipole direction and it is located at a distance of $l/2$ behind the metal surface. The reflected field at the position of the antenna is

$$\vec{E}_\text{ref} = -\frac{d k^3}{4 \pi \varepsilon_0} \hat{d} \left( \frac{1}{kl} + \frac{i}{(kl)^2} - \frac{1}{(kl)^4} \right) e^{i(kl-\omega t)} \quad (3)$$

where $l$ is the distance between the image and the original antenna.

When a dipole antenna is placed in a metallic wedge as shown in figure 1, the reflected field can be calculated by using a set of images, and the reflected field is a sum of contributions from each image of the set. We find that when the opening angle of the wedge is $\theta = \pi/N$, where $N$ is an arbitrary positive integer, the number of images is finite. The images can be constructed using a technique derived by Robinett [20] from a wave-packet evolution study inside a wedge. If the emitting atom’s cylindrical coordinate system is $(\rho, \phi, z)$, the first image with a dipole $-\hat{d}$ will appear at $(\rho, -\phi, z)$. The original source and the first image can be rotated by $2\theta$ clockwise to give the second image located at $(\rho, \phi - 2\theta, z)$ with a dipole $-\hat{d}$ and the third image at $(\rho, -\phi - 2\theta, z)$ with a dipole $-\hat{d}$. The original source and the first image are rotated by $4\theta$ clockwise to give image 4 located at $(\rho, \phi - 4\theta, z)$ with a dipole $-\hat{d}$ and image 5 at $(\rho, -\phi - 4\theta, z)$ with a dipole $-\hat{d}$. This process can be continued until the last two images are obtained by rotating the original source and the first image by $2(N-1)\theta$ clockwise. For $\theta = \pi/N$, where $N$ is an arbitrary positive integer, this rotation process closes on itself. In the end, we have one original source and $(2N-1)$ images. $(N-1)$ of those images with the same dipole $\hat{d}$ as that of the original are located at $(\rho, \phi - 2n\pi/N, z)$, where $n = 1, 2, \ldots, (N-1)$. The other $N$ images are located at $(\rho, -\phi + 2n\pi/N, z)$, where...
n = 0, 1, ..., (N − 1), and they have the opposite dipole −d to that of the original. Figure 2 shows the five images and the original source for a θ = π/3 wedge.

The reflected electric field at the position of the source antenna in the wedge with an opening angle θ = π/N can be obtained from the electric fields of all the images using equation (3). The distance between the source at (ρ, φ, z) and the image at (ρ, −φ, z) is l = 2ρ sin(φ); the distance between the source at (ρ, φ, z) and the images at (ρ, φ − 2nπ/N, z), where n = 1, 2, ..., (N − 1), is l = 2ρ sin(nπ/N); the distance between the source at (ρ, φ, z) and (ρ, −φ + 2nπ/N, z), where n = 1, 2, ..., (N − 1), is l = 2ρ sin(nπ/N − φ).

The reflected field at the position of the original dipole antenna can be written as

\[
\vec{E}_{\text{ref}} = \frac{dk^3}{4\pi e_0} d \left[ \frac{1}{k l_\perp} + i \left( \frac{k l_\perp}{(k l_\perp)^2} - \frac{1}{(k l_\perp)^3} \right) e^{i(k l_\perp - o_0 t)} \right]
\]

where

\[
E_{\text{ref}} = \frac{dk^3}{4\pi e_0} \sum_{n=1}^{N-1} \left[ \frac{1}{k l_{\perp n}} + i \left( \frac{k l_{\perp n}}{(k l_{\perp n})^2} - \frac{1}{(k l_{\perp n})^3} \right) e^{i(k l_{\perp n} - o_0 t)} \right] - \left[ \frac{1}{k l_{\perp n}} + i \left( \frac{k l_{\perp n}}{(k l_{\perp n})^2} - \frac{1}{(k l_{\perp n})^3} \right) e^{i(k l_{\perp n} - o_0 t)} \right].
\]

Inserting equations (4) and (2) into equation (1) and after some manipulations, we get the damping rate inside the wedge:

\[
W_d = W_0 \left( 1 - \frac{3}{2} G_1[2k\rho \sin \phi] - \frac{3}{2} \sum_{n=1}^{N-1} \left[ G_1 \left[ 2k\rho \sin \left( \frac{n\pi}{N} - \phi \right) \right] - G_1 \left[ 2k\rho \sin \left( \frac{n\pi}{N} \right) \right] \right] \right),
\]

where we have defined

\[
G_1(S) = \frac{\sin S}{S} + \frac{\cos S}{S^2} - \frac{\sin S}{S^3}
\]

and \( W_0 = \frac{2\gamma_l S_0^2}{34\pi e_0} \) is the damping rate in free space [9].

The spontaneous emission rate in a wedge γ can be inferred from equation (5) as

\[
\gamma = \gamma_0 \left( 1 - \frac{3}{2} G_1[2k\rho \sin \phi] - \frac{3}{2} \sum_{n=1}^{N-1} \left[ G_1 \left[ 2k\rho \sin \left( \frac{n\pi}{N} - \phi \right) \right] - G_1 \left[ 2k\rho \sin \left( \frac{n\pi}{N} \right) \right] \right] \right),
\]

where \( \gamma_0 \) is the spontaneous emission rate without the wedge. It should be noted that the method of images has its limitation, and the correct value of \( \gamma_0 \) from QED will have to be used in equation (7) [9, 21].

Rosa et al. studied the emitted power of an atom in a perfect wedge with an arbitrary opening angle using QED. The emitted power is proportional to the spontaneous emission rate. The formula derived by Rosa et al. using QED involves special functions and appears quite complicated. But when the opening angle of the wedge is restricted to \( \theta = \pi/N \), where \( N \) is an arbitrary positive integer, the QED formula can be simplified considerably (see equation (46) in [10]). We find that equation (7), using the method of images, is consistent with the result of Rosa et al. using QED for a \( \theta = \pi/N \) wedge. It is not easy to see why the complicated QED formula involving special functions obtained by Rosa et al. can be reduced to a simple form similar to equation (7) when the opening angle of the wedge is \( \theta = \pi/N \), where \( N \) is an arbitrary positive integer. The method of images seems to suggest that the origin of the simplification is in the symmetries of the finite number of images. When the opening angle is \( \pi/N \), where \( N \) is an arbitrary positive integer, the 2N − 1 images and the original source form a set; this set is invariant when it is rotated by a multiple of 2π/N either clockwise or anti-clockwise. Furthermore, the set can be changed to another set by an inversion through either the upper plane or the lower plane of the wedge. The effect of the inversion is equivalent to reversing the dipole direction of the original set. However, the rate formula in equation (7) is invariant under the above operations as is apparent from the derivation of the method of images. We believe that the reduction of the QED formulæ to the simple form should be related to the above-mentioned symmetries.

3. Oscillations in the emission rate and closed orbits

We now analyse the spontaneous emission rate of an atom inside a π/N wedge. In figure 3 we show the pattern of the spontaneous emission rate described by equation (7) for a \( \theta = \pi/3 \) wedge. The overall spontaneous emission rate pattern resembles ‘an egg tray’ turned upside down. There are two main features in the pattern in figure 3. They are the suppression of the emission rate near the wedge surface and the regular oscillations of the rate inside the wedge. For the suppression of the rate, we note in the small S limit, \( G_1(S) \) can
Figure 3. The pattern of the spontaneous emission rate is formed as the position of the emitting atom is varied inside a wedge with an opening angle $\theta = \pi/3$. $\gamma_0$ is the emission rate without the wedge. The pattern is like 'an egg tray' upside down.

Figure 4. The spontaneous emission rate for the emitting atom at (a) $kx = 15$ and along the $y$-axis, (b) $y = 0$ and along the $x$-axis. The dotted line corresponds to equation (7) with $G_\parallel$ from equation (6), while the solid line corresponds to equation (7) with $G_\parallel$ approximated by $\sin S$. Therefore the rate goes to zero quadratically as the emitting atom approaches the wedge surface. The quadratical nature of the rate suppression can be seen in figure 4 in which the emission rate is shown along two straight lines parallel to the $x$-axis and $y$-axis respectively.

Figure 5. (a) The spontaneous emission rate for the $\theta = \pi/3$ wedge as a function of the scaling parameter $\alpha = \rho/\lambda_0$ with $\phi = \pi/9$ for the emitting atom; (b) FT (absolute value) of the above emission rate as defined in equation (8). The crosses near the peaks mark the peak positions and heights obtained from the analytic formulæ discussed in the text. The closed orbits corresponding to the peaks are also displayed and illustrated with the same colours (online).

As we will explain next, the oscillations are related to some special photon closed orbits along which the emitted photon travels away from and returns to the emitting atom inside the wedge. The oscillations in the cross sections for photo-ionization and photo-detachment in the presence of external fields are successfully related to the closed orbits of the photo-electron going away and returning to the nucleus [15–19]. Recently the oscillations in the spontaneous emission rate have also been related to the closed orbits of the photon going away from and returning to the emitting atom in two systems [13, 14]. In the following, we will first analyse the oscillations in the emission rate for a $\pi/3$ wedge and will demonstrate the correspondence between the oscillations and some photon closed orbits. A similar analysis is then performed for a $\pi/4$ wedge which confirms our understanding.

We define a scaling variable $\alpha$ by $\rho = \alpha \lambda_0$, where $\lambda_0 = 2\pi/k$ is the wavelength in vacuum. We now examine the spontaneous emission rate in the wedge as a function of $\alpha$ with an arbitrary but fixed $\phi$ of the emitting atom. We define the modified Fourier transform (FT)

$$\tilde{\gamma}(S) = \int_{\alpha_1}^{\alpha_2} [\gamma(\alpha) - \gamma_0] \alpha \, e^{iS\alpha} \, d\alpha,$$

where $\gamma(\alpha)$ represents the dependence of the emission rate as a function of the scaling variable $\alpha$. In our numerical calculations, we took $\alpha_1 = 1.0$, $\alpha_2 = 26.0$ and the integration step size $\Delta \alpha = 0.01$. As an example, for $\phi = \pi/9$, the emission rate as a function of the scaling variable $\alpha$ for the $\theta = \pi/3$ wedge is shown in figure 5(a). One can see that the rate in the wedge oscillates around the free space rate and the oscillation amplitude is reduced as $\alpha$ increases or the emitting atom is farther away from the wedge axis. The absolute value of the calculated modified FT is presented...
There are clearly four large peaks in figure 5(b). These four peaks imply that the emission rate in figure 5(a) contains four main frequencies. We now provide an explanation of the physical meaning for the peaks and give a quantitative description for the peak positions and peak heights.

We note that when $S$ is larger than about $\pi$, $G_j(S) = \sin S/S$ is an accurate approximation of $G_j(S)$. When this approximation is made in equation (7), the emission rate inside the wedge takes the following form:

$$\gamma(\alpha) = \gamma_0 + \sum_j \left(\frac{3\gamma_0}{2S_j^0}\right) \sin \left(S_j^0\alpha + \phi_j\right),$$

(9)

which is in the form of closed-orbit theory for absorption spectra [15–18]. Equation (9) can be interpreted using closed-orbit theory. $\gamma_0$ is the background rate associated with the initial emission process in which the emitted photon leaves the atom and never returns. It is equal to the spontaneous emission rate in free space. The sum in equation (9) is over all closed orbits of the emitted photon going away from and returning back to the emitting atom; the emitted photon travels in straight paths in the wedge, and it follows the laws of reflection when hitting the metal boundary of the wedge. $S_j^0 = kl_j^0$ is the ‘action’ around the $j$th closed orbit, $k$ is the wave number, and $l_j^0$ is the geometric length for the $j$th closed orbit corresponding to the case $\alpha = 1$, or the distance between the emitting atom and the wedge axis is just equal to one wavelength; since each reflection by the wedge surface produces a $\pi$ phase shift, the phase $\phi_j = -m_j\pi$, where $m_j$ counts the number of reflections by the wedge surface; the amplitude $\left(\frac{3\gamma_0}{2S_j^0}\right)$ provides a measure of the intensity of the returning group of photon wave, and in the present case, is inversely proportional to the length of the orbit. The peak positions in the FT of equation (8) are therefore given by the action $S_j^0$ of photon closed orbits with corresponding peak heights proportional to $g_j/l_j^0$, where $g_j$ is a degeneracy factor counting closed orbits with identical contribution to the peak. Both $S_j^0$ and $l_j^0$ are evaluated for the system size corresponding to $\alpha = 1$ or $\rho = \lambda_0$.

For a general wedge, we must launch trajectories in all directions from the location of the emitting atom at $(\rho, \phi, z)$ to search for all closed orbits which leave from and return to the emitting atom. Each trajectory follows a straight line until it reaches the wedge and is reflected. When the trajectory returns back to the location of the emitting atom, a closed orbit is then formed. The closed orbits must be on the cross-sectional plane of the atom because of the propagation rules of trajectories. If a trajectory leaves the atom off the cross-sectional plane, it cannot return to the atom to form a closed orbit. For a wedge with an arbitrary opening angle, the number of closed orbits can be large. But for a $\pi/N$ wedge, we can find all the closed orbits from the images constructed earlier. We find that there is a one-to-one correspondence between the images and the closed orbits.

In figure 6, we show this correspondence for a $\theta = \pi/3$ wedge. There are five images labelled from 1 to 5 clockwise. There are also five closed orbits. The first segment of the closed orbit is given by the straight line connecting the corresponding image and the source. The geometrical length of the closed orbit is equal to the distance between the atom and the image. The first closed orbit leaves the emitting atom downward and returns to the atom after being reflected back by the lower wedge surface; the second closed orbit is first reflected by the lower wedge surface and then by the upper wedge surface before it returns to the emitting atom; the third closed orbit is first reflected by the lower wedge surface and then travels perpendicular to the upper wedge surface, finally it retraces back to the emitting atom after being reflected by the upper wedge surface. Similar descriptions can be given to the fourth closed orbit and the fifth closed orbit. We also find that the closed orbit two and four have the same shape but their propagation directions are opposite.

The scaled ‘actions’ of the five closed orbits depend only on the emitting atom’s cylindrical angle $\phi$ and they are

$$S_0^2 = 2k\lambda_0 \sin \left(\frac{\pi}{3}\right) = 4\pi \sin \left(\frac{\pi}{3}\right),$$

$$S_1^2 = 2k\lambda_0 \sin \left(\frac{\pi}{3} + \phi\right) = 4\pi \sin \left(\frac{\pi}{3} + \phi\right),$$

$$S_2^0 = 2k\lambda_0 \sin \left(\frac{\pi}{3} - \phi\right) = 4\pi \sin \left(\frac{\pi}{3} - \phi\right).$$

(10)

Since each reflection by the wedge induces a phase loss of $\pi$, the phases corresponding to the five closed orbits above are

$$\phi_1 = \psi_5 = -\pi,$$

$$\phi_2 = \psi_4 = -2\pi,$$

$$\phi_3 = -3\pi.$$

(11)

We have used the formula $hS_j/l_j^0$ to compute the height of the $i$th peak located at $S_i^0$ for the $\pi/3$ wedge. The results
from the analytic formulæ with $\phi = \pi/9$ are marked as crosses in figure 5(b). The degeneracy factor $g_i$ is equal to 1 for the first, second and fourth peaks because one closed orbit corresponds to each of the three peaks; for the third peak near 10.89, because there are two closed orbits making the same contributions in both the emission rate and the FT, the corresponding degeneracy factor is 2. For the purpose of comparing with the numerical results, the constant $h$ was determined by matching with the first peak extracted from FT. We can see that all the peak positions and heights are accurately described by the analytic formulæ. In figure 5(b), we also show the closed orbits correlated with the peaks. A detailed comparison is made in table 1 between the numerically extracted peak positions and heights and the analytic formulæ. We find that they are in good agreement.

To confirm our understanding, we also studied the emission rate in a $\pi/4$ wedge setting $\phi = \pi/12$. The results are summarized and presented in figures 7 and 8. The images are constructed in figure 7(a). The pattern of the emission rate inside the wedge calculated using equation (7) with $N = 4$ is shown in figure 7(b). This pattern also resembles ‘an egg tray’ turned upside down. The seven images and the seven corresponding closed orbits are illustrated in figure 7(c). The emission rate as a function of the scaling variable $\alpha$ is shown in figure 8(a). The numerically calculated FT is displayed in figure 8(b). We find that there are six main peaks in the FT. The crosses near the peaks mark the results of the analytic formulæ for the six peak positions and peak heights. The closed orbits corresponding to the peaks are illustrated by the inset figures. From left to right, each and every peak is associated with one closed orbit except the third one. The third peak corresponds to two closed orbits. For the $\pi/4$ wedge, the numerically extracted peak positions and peak heights are listed together with the results of the analytic formulæ in table 2. Again we find that the numerical results and the analytic formulæ agree well.

**Table 1.** The four peak positions and heights numerically extracted using FT are compared with the analytic predications discussed in the text for a $\pi/3$ wedge. The analytic results are represented by the crosses in figure 5(b). The peaks are labelled from left to right with increasing value of the scaled action.

| Position | Numerical | Analytic | Numerical | Analytic |
|----------|-----------|----------|-----------|----------|
| 1st      | 4.30      | 4.30     | 4.37      | 4.37     |
| 2nd      | 8.08      | 8.08     | 2.30      | 2.32     |
| 3rd      | 10.89     | 10.88    | 3.42      | 3.45     |
| 4th      | 12.40     | 12.38    | 1.50      | 1.52     |
4. Conclusions

In conclusion, we have studied the spontaneous emission of an atom in a $\pi/N$ wedge using a method of images. The method of images is particularly simple and the results for the emission rate are consistent with the results of QED. From the method of images, we can see that the rotation and inversion symmetries should play important roles in the reduction of complicated QED rate formulae to simple forms when the wedge opening angle is $\pi/N$, where $N$ is an arbitrary positive integer. For a wedge with an opening angle, which is not equal to $\pi/N$, the closed-orbit theory [13, 14, 16, 17] suggests the emission rate can still be expressed in a form similar to equation (7) although the number of oscillatory terms can be large, and the corresponding closed orbits will have to be searched numerically [17].

We analysed in detail the oscillations in the emission rate using scaling variables and Fourier transformation for the $N = 3$ and $N = 4$ cases as examples. We found only four oscillations in a $\pi/3$ wedge and only six oscillations in a $\pi/4$ wedge. We found a direct correspondence between the oscillations in the emission rate and closed orbits. We interpreted the oscillations as interferences between the outgoing and incoming photon waves travelling along closed orbits from atom to the atom. The closed orbits can be constructed readily from the method of images. The method of images provides a complementary perspective on the spontaneous emission process for an atom in a wedge, keeping in mind the difference between the method of images and the QED method [21]. It is interesting to note that in a recent experiment, Fleet et al have put a classical dipole radiating at wavelengths of the order of a few centimetres inside a wedge-shaped cavity and measured the radiated power [22]. They observed that the classical dipole also exhibits effects similar to the Purcell effects. However, the classical dipole antenna is not a point source and cannot yet simulate the emission of an atom in a wedge. On the other hand, the recent observation of the signature of photon periodic orbits in the spontaneous emission spectra in laterally confined vertically emitted cavities [23] provides a very encouraging sign that an observation of the photon closed orbits in a wedge is possible in the near future.

Acknowledgments

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