MECHANISM DESIGN IN A SUPPLY CHAIN WITH AMBIGUITY IN PRIVATE INFORMATION

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(Communicated by Changzhi Wu)

Abstract. This paper considers a two-echelon supply chain with one supplier and one retailer. The retailer holds private information on the stochastic demand, and the supplier knows neither the market information nor its prior distribution. Two decision making criteria based on the generalized lexicographic preference are proposed to the supplier: the profit criterion and the regret criterion. Our analyses show that the profit criterion always coordinates the supply chain, while the regret criterion does not. If the market state is low, the profit criterion generates higher profit for the supplier than the regret criterion does, and vice versa. In both criteria, we show that the supplier’s regret is bounded by both the range of the market state and the volatility of the demand. We further show that, the supplier should cooperate with the retailer with full demand information if the differences between market states are large enough, while selling the product by himself is better if the differences between market states are small enough.

1. Introduction. Achieving higher system performance through supply chain interactions is a key issue in supply chain management. Contracting is popular in practice, and various contracts are proposed to coordinate the system (e.g., buyback contract and revenue-sharing contract). Some of these contracts can allocate system profit with full flexibility under the full information assumption (see, [5]). However, such full information assumption is rare in practice (see, [5, 26]). For example, personal computer manufacturers often over submit their orders to their suppliers, due to the lack of demand information, the suppliers may build more capacities than needs (see, [16]). And Solectron, a major electronics supplier had build $4.7 billion more capacity due to the misleading of its retailers (see, [8]). And in 1997, Boeing’s large orders were not fulfilled because its suppliers did not believe Boeing’s demand forecast (see, [7]). Therefore, information asymmetry affects supply chain performance crucially, and there is a need for a mechanism design to reveal private information credibly.

To the best of our knowledge, most existing works on supply chain models with incomplete information assume that the less informed member knows a credible

2010 Mathematics Subject Classification. Primary: 90B50, 91A10; Secondary: 91A80.

Key words and phrases. Supply chain management, information asymmetry, information ambiguity, incentive compatible.

* Corresponding author. This work has been supported by the the National Social Science Foundation of China under Project No. 12CGJ023. The authors are grateful to the anonymous referees for their constructive comments and suggestions.
prior distribution of the unknown information. However, when a supplier enters a new market and needs to cooperate with a retailer who has existed in the market for a long time, the supplier can hardly obtain such distribution information. And after the data scandal of Facebook, firms pay more attention in protecting their private data and information. Hence, acquiring credible distribution of other firms’ private information becomes even more difficult. We believe most existing works can’t guide firms and provide efficient mechanisms in such situation.

For example, “I Diy Robotec Corp” (http://www.rtec.cc/) was first founded in Taiwan, and came to mainland China in 2009. This company provides classes through robot DIY (do it yourself) to the teenagers in the areas of mechanics, engineering, computer programming, automatic control, and so on. Until now, this company has built its campus in most capital cities in China, and cooperates with some cram schools in second-tier cities (mostly in south-east of China). If the company decides to expand its businesses to the west of China, then there are two possible ways. The company can build schools and provide advanced courses by itself (e.g., in Wuhan city), but the lack of demand information may lead to a larger waste or shortage in the teaching resources. Or the company can cooperate with a local cram school who cannot provide such advanced courses but is familiar with the local demand (e.g., in Zhuzhou city). The company can design different course packs and provide them to the local cram school in different prices. But the local cram school may share a large part of channel profit through his advanced demand knowledge. Hence, the company may face the following problems: how to sell the product, by myself or cooperating with a local retailer; if cooperating with a local retailer is better, which one to choose (with more or less information), and how to contract with the retailer.

To explore the answers to these problems, we consider a two-stage supply chain model, in which the retailer faces a stochastic demand, and the mean of the demand (average demand or market state) is privately known by the retailer. The supplier knows neither the market state nor its distribution. However, it is unlikely for the supplier to know nothing about the demand information. For example, the demand of daily consumer goods (or luxury goods) is often bounded by the total (or high income) population, which is relatively easier to obtain. And in the “I DIY Robotec Corp” case, the market state of two cities with similar population may have similar potential demand. However, the distributions of the demand may be very different, since the willingness of going to a cram school varies with the culture of the city (i.e., education-oriented or not). Therefore, we assume the supplier has some knowledge about the range of the demand information (e.g., low, medium or high). Since the prior distribution information is missing, we provide two decision making rules (the profit criterion and the regret criterion) other than maximizing the expected profit for the supplier. We show that the profit criterion always coordinates the supply chain, and the regret criterion can nearly coordinate the supply chain with low market volatility. The losses caused by information asymmetry are bounded by both the market range and the market volatility. We also discuss the choice of the supplier to sell the product by himself or cooperate with a local retailer. Our results suggest that, if the demand ambiguity level is low, the supplier should sell the product by himself; if the demand ambiguity level is high, the supplier should cooperate with a retailer who fully understands the market.

The remainder of our paper is organized as follows. Section 2 reviews existing works in supply chain models that are related to our work. In Section 3, our supply
chain model is formulated, two decision making criteria are proposed and the retailer choice problem of the supplier is discussed. Finally, our work is concluded in Section 4, and some future directions are given.

2. Literature review. Our work relates to two research topics in supply chain models: information asymmetry and ambiguity. We review papers in these two topics separately.

The literature on supply chain model with asymmetric information can be generally divided into two streams. One stream deals with the signaling problems, in which the more informed member offers a contract to signal his private information. The other stream deals with the screening problems, in which the less informed member offers a contract to screen the private information. Our work belongs to the second stream. In screening problems, various types of private information are studied, e.g., demand information, cost, risk aversion degree and consumer return rate. Our work focus on the information asymmetry on demand. In the following, we only review works that consider demand information to be asymmetric in supply chain models, more comprehensive reviews are given by Shen et al. [30].

A few works assume the upstream supplier has private demand information, and can invest in advertising to increase demand (see, [6, 12]). Chu [6] assumes the demand to be deterministic while Geng and Minutolo [12] assume the demand to be stochastic. They all show that the high demand type supplier can earn more profit than the low demand type supplier. Different to these two work, we assume the retailer holds the private demand information. But we obtain a similar result, that is, the high demand type information holder earns more than the low demand type information holder.

Another small amount of works considers competition of suppliers in a two-echelon supply chain (see, [20, 19]). These two works both assume one supplier is flexible (high unit production cost and low fixed cost) and another supplier is efficient (low unit production cost and high fixed cost). However, Li et al. [20] use probability theory to characterize the private information while Liu et al. [19] apply uncertain theory. They all show that, comparing with only the flexible supplier is available, competition only affect the high demand type retailer, and the such retailer can extract some of the supply chain profit.

The remaining works mainly focus on two-echelon supply chains without competition and the retailer holds the private demand information. These works mainly consider three different issues.

The first issue is how to screen private information and achieve higher system profit (see, [26, 11, 2]). The two works [26] and [2] are related to our work closely. Our assumption of demand function is similar to Özer and Wei [26], but they do not consider regret as objective function. Özer and Wei [26] assume the supplier needs to build capacity before contracting, and show that the supplier’s “cost of screening” increases as the range of the private information increases. We obtain a similar result that the bounds of the “cost of screening” increases as the range of the private information increases, and further show that the supplier’s “cost of screening” is also bounded by the market volatility. Babich et al. [2] consider more general demand function than we do, but they assume the private demand information has only two different types. They apply buy-back contract and show that system coordination is not achievable, but the solution of the buy-back contracts can be arbitrarily close to the coordinated solution. Different from this result, we show that the profit criterion always coordinates the system. A drop-shipping situation,
that is, the supplier has to build inventory and bears the demand uncertainty is considered by Gan et al. [11]. The retailer may send an over-estimated demand forecast to the supplier. Hence, the authors propose a menu of commitment-penalty contracts to screen the retailer's private information, and show that the system is not coordinated when the demand type of the retailer is low.

The second issue compares different contracts in the presence of information asymmetry (see, [4, 31, 18, 9]). Burnetas et al. [4] compare principle agent model, all-unit quantity discount contract, and incremental quantity discount contract. They show that principle-agent model generates the highest profit for the supplier while incremental quantity discount contract generates the lowest profit. The rebate and return contracts are discussed by Taylor and Xiao [31]. The rebate contract gives more incentives to the retailer to forecast the demand while the return contract provides lower screening cost to the supplier. Li et al. [18] assume that the retailer also holds a private retail price information, and show that the forward contract has larger impact on both supply chain members profits than the option contract does. Buy-back, fixed payment and revenue sharing contracts are compared by Fang and Wang [9]. They show that only the revenue sharing contract can coordinate the system, and fixed payment contract provides higher system efficiency than the buy-back contract does. Finally, Lv et al. [21] discuss price-quantity, price-only and the hybrid contracts in an assembly supply chain with two suppliers that produce complementary components and one retailer. They show that the supplier who makes decision first prefers the hybrid contract.

The third issue studies the impacts of information asymmetry on supply chain members’ decisions (see, [17, 32, 33]). Comparing to the symmetric information situation, information asymmetry may reduce the production quantity ([17]), reduce the retail price and increase store assistance level ([33]). When the private demand information of the retailer may not always be true, but have a certain level of accuracy, Wang et al. [32] show that it is optimal for the supplier's to provide contract manu in a threshold structure of the unit production cost. And high precision level of the retailer’s information is not always preferred by the supplier. We confirm this result with regret criterion, but the supplier always prefers the retailer to have high precision level of demand information with profit criterion.

Although supply chain models with private demand information is extensively studied in literature, existing works all assume that the less informed member knows the prior distribution of the demand information. Different from those works, we assume the less informed member does not know the prior distribution but only knows the range information. That is, we assume information ambiguity exists in the supply chain.

The existing researches of demand ambiguity in supply chain models mainly focus on newsvendor problem, in which the newsvendor need to make the order quantity decision when the distribution of the demand is unknown. Most existing works assume that, the newsvendor knows mean and variance of the demand and applies the max-min preference to maximize the worst case expected profit (see, [29, 10, 22, 24, 1, 15, 23]). Scarf et al. [29] first derive the optimal order quantity decision of the max-min problem. Following their work, a number of works incorporate different features into the model and study the optimal order quantity decision: a second ordering chance after observing the demand [10], demand is sensitive to the inventory [22], consumers may return the product [24], shortage penalty [1], advertising effect [15], and remaining inventory can be sold with a discount [23].
Besides maximizing the worst case expected profit, a few works apply the max-
min preference to maximize the worst case expected regret: with only mean and
variance of the demand [34], and with moments information of the demand [27].
And there are few works make restrictions on the demand distribution: the support
of the demand is known [13] and the demand is non-skewed [14]. Finally, Saghafian
and Tomlin [28] assume the least information of the demand, that is the demand is
positive, but they consider a multi-period newsvendor model. Instead of analyzing
the worst case, the authors use the demand data in the past periods to make the
order quantity decision in the current period. They provide a closed form order
decisions and updating mechanisms of the demand distribution at each period, and
the convergence rate of such updating mechanism is discussed.

Our work different from these works in two ways. On the one hand, all these
works apply the max-min preference when the unknown information is character-
ized by a continuous distribution; while we apply the generalized lexicographic
preference when the unknown private information is characterized by a discrete dis-
tribution. Hence we face a lexicographic problem (see, [3]). On the other hand, all
these works assume that the decision maker face the demand directly; while we con-
sider the decision maker have a chance to cooperate with a retailer with advanced
information.

3. The supply chain model. In this section, we specify the decision problems of
the supplier in a two-echelon supply chain. The supplier sells a product in a new
region at unit production cost $c$ and unit retail price $r$. The stochastic demand
at the region is assumed to be $D = t + \eta$, in which $t \in T$ represents the market
state (with distribution $\mu$ on $T$), and $\eta$ is a random variable (with CDF $F$ and
PDF $f$) that represents the market volatility. Hence, without loss of generality, we
assume $\eta$ supports on $[-\tau_1, \tau_2]$ and its mean is zero. The supplier does not observe
t nor the distribution $\mu$, but he knows that $t$ comes from the set $T = \{t_1, \ldots, t_n\}$
($\tau_1 \leq t_1 < \cdots < t_n$). And the market volatility is common knowledge, that is, the
distribution $(F, f)$ of $\eta$ is common knowledge. The supplier can sell the products
to the market by himself, or contract with a local retailer who understands the
market state better, that is, has further information about $t$. If the supplier sells
the products by himself, then he determines the production quantity and sells the
products at the selling season. If the supplier contracts with the retailer, then he
determines the wholesale price. After observing the wholesale price, the retailer
determines the order quantity and sells the products at the selling season. At the
end of the selling season, unsold products have zero salvage value and unmet demand
is lost with zero penalty.

Remark 1. Considering salvage value and penalty to be positive will not affect our
main results. The reason is that, a model with the unit salvage value $s_1$ and the unit
unmet penalty $s_2$ is equivalent to a model with production cost $c - s_1$ and retail price
$r + s_2 - s_1$. Hence, we assume unsold products have zero salvage value and unmet demand is lost with zero penalty. 2. We assume that the unknown information
of the supplier has a discrete distribution and use the generalized lexicographic
preference, while some previous works assume a continuous distribution and some
statistics (e.g., mean and variance) are known and use max-min preference. We
do not make such assumptions for the following reasons. On the one hand, we are considering the case that the supplier is entering a new market without any
historical data, hence it is not appropriate to assume the supplier has some statistic
information. On the other hand, we are interested in the mechanism design problem
of the supplier, while previous works mainly focus on the max-min criterion with the given statistics information. If the statistics information is not known, the max-min criterion will degenerate to a trivial decision making problem. Hence, in order to avoid such trivialness, we assume the information to be discrete and focus on the generalized lexicographic preference.

3.1. Selling the product without retailer. We begin by considering the situation that the supplier sells the products by himself. For any quantity decision $q$ and a market state $t_i$, the expected profit of the supplier is $rE(q \land (t_i + \eta)) - cq$. The supplier does not know $t$ nor its distribution, therefore, for any quantity decision $q$, there are $n$ possible outcomes of the profit (we call it profit vector):

$$\{rE(q \land (t_i + \eta)) - cq\}_{i=1}^{n}.$$  

(1)

Thus, the outcomes of the supplier is a subset of vector space $\mathbb{R}^n$. Lexicographic order is a natural preference on $\mathbb{R}^n$ that can be used for the supplier to make decisions. However, the lexicographic order compares the first coordinate to the last coordinate sequentially, which means the importance of the coordinates are decreasing from the first to the last. But for the supplier, each coordinate represents a possible profit. Hence, all coordinates of the profit vector are equally important to the supplier. Therefore, we consider the generalized lexicographic preference, in which the supplier first reformulates the profit vectors in the increasing manner, then applies the lexicographic order. In other words, the supplier first maximizes the minimum possible profit, then maximizes the second minimum possible profit, and so on. The formal definition of the generalized lexicographic preference is provided below.

**Definition 3.1.** Let $v$ and $u$ be two vectors in $\mathbb{R}^n$, reformulates the coordinates of $v$ and $u$ in the increasing manner, then two other vectors $\tilde{v} = (\tilde{v}_1, \ldots, \tilde{v}_n)$ and $\tilde{u} = (\tilde{u}_1, \ldots, \tilde{u}_n)$ are obtained ($\tilde{v}_1 \leq \cdots \leq \tilde{v}_n$ and $\tilde{u}_1 \leq \cdots \leq \tilde{u}_n$). If there exists a $k \in \{1, \ldots, n\}$, such that $\tilde{v}_i = \tilde{u}_i$ for $i = 1, \ldots, k - 1$ and $\tilde{v}_k > \tilde{u}_k$, then we say $v$ is larger than $u$ with respect to the generalized lexicographic order and write $v >_L u$. Further, $v \geq_L u$ means either $v >_L u$ or $v = u$.

The following lemma characterizes the optimal decision of the supplier with respect to the generalized lexicographic preference. In our paper, all the proofs are provided in the appendices.

**Lemma 3.2.** If the supplier sells the products by himself, the optimal quantity is $q^* + t_1$, in which $q^*$ maximizes $U(x) = rE(x \land \eta) - cx$, i.e., $F(q^*) = (r - c)/r$. And the supplier’s profit vector is

$$\{U(q^* + t_1 - t_i) + (r - c)t_i\}_{i=1}^{n}.$$  

(2)

3.2. Contracting with a fully informed retailer. Next, we consider the situation that the supplier cooperates with a retailer who fully understands the market state (i.e., the retailer knows the true value of $t$). The timing of the events is illustrated as follows: firstly, the supplier designs a list of contracts, and gives it to the retailer; secondly, nature generates $t$ with respect to $\mu$, and privately reveals $t$ to the retailer; then the retailer chooses to sign one contract or does not sign any one; finally, decisions are made according to the chosen contract if the retailer has signed one.
3.2.1. Bayesian incentive compatible mechanisms. We assume both supply chain members to be risk-neutral and try to maximize their expected profits. Although the supplier can not maximize his expected profit directly (since $\mu$ is unknown), any decision made by the supplier is related to a hidden expected profit. According to Myerson [25], any possible hidden expected profit can be achieved by a Bayesian incentive compatible mechanism (IC mechanism). Hence, it suffices to only consider IC mechanisms.

We assume the contracts designed by the supplier includes two items: the order quantities $q$ and the total transfer payment $P$. In the IC mechanism, the supplier will provide $n$ contracts $\{q_i, P_i\}|i=1,...,n$ to the retailer. And it is optimal for a retailer with private information $t_i$ to sign the contract $(q_i, P_i)$. In another words, the profit of a retailer with private information $t_i$ and contract $(q_i, P_i)$ is larger than the profit of a retailer with private information $t_i$ and contract $(q_j, P_j)$ ($i \neq j$). And additionally, it is necessary for the retailer to earn positive expected profit. Thus, the following constraints (IC constraints) are needed:

$$
\begin{align*}
    rE(q_i \land (t_i + \eta)) - P_i &\geq rE(q_j \land (t_i + \eta)) - P_j, & 1 \leq i, j \leq n; \\
rE(q_i \land (t_i + \eta)) - P_i &\geq 0, & 1 \leq i \leq n.
\end{align*}
$$

Substitute $U(x)$ into (3), we have

$$
\begin{align*}
    U(q_i - t_i) + cq_i - P_i &\geq U(q_j - t_i) + cq_j - P_j, & 1 \leq i, j \leq n; \\
    U(q_i - t_i) + (r - c)t_i + cq_i - P_i &\geq 0, & 1 \leq i \leq n.
\end{align*}
$$

3.2.2. Two decision making criteria. As we have mentioned before, the supplier cannot maximize his expected profit directly, but he faces a profit vector. Hence, on the one hand, the supplier can maximize the profit vector with respect to $\geq_L$ (the profit criterion). For a given IC mechanism $\{q_i, P_i\}|i=1,...,n$, the profit vector faced by the supplier is $(P_i - cq_i)_{i=1}^n$. Hence, the supplier solve the following problem with the profit criterion:

$$
\begin{align*}
    \max & \quad (P_i - cq_i)_{i=1}^n, \\
    \text{with respect to order } & \geq_L, \\
    \text{subject to } & (4) \text{ and } q_i \geq 0.
\end{align*}
$$

The profit criterion indicates that, the supplier first maximizes his worst case profit (i.e., optimizes the worst case), then maximizes his second worst case profit (i.e., optimizes the second worst case), and so on.

On the other hand, the IC mechanism reveals the private information of the retailer. As soon as the retailer have signed a contract, the supplier will know the true value of $t$. Hence, the supplier may realize that, he can design a better contract to gain more profit. Such profit gap between knowing $t$ and not is called regret. Therefore, any IC mechanism designed by the supplier is related to a regret vector. If the supplier fully knows $t$, then he can fully extract all channel profit, which is $\max_q rE(q \land (t_i + \eta)) - cq = U(q^*) + (r - c)t_i$ if $t = t_i$. Hence, for a given IC mechanism $\{q_i, P_i\}|i=1,...,n$, the regret vector faced by the supplier is $(U(q^*) + (r - c)t_i - P_i + cq_i)_{i=1}^n$. Following the same logic in the profit criterion, the supplier can first optimizes the worst case (i.e., minimizes his worst case regret), then optimizes the second worst case (i.e., minimizes his second worst case regret), and so on. That is, the supplier maximizes the inverse of the regret vector $-(U(q^*) + (r - c)t_i - P_i + cq_i)_{i=1}^n$ with respect to the generalized lexicographic order $\geq_L$. We
refer this procedure to regret criterion, which is equivalent to solve the following problem:

\[
\max (P_i - cq_i - U(q^*) - (r - c)t_i)_{i=1}^n,
\]

with respect to order \( \geq L \),

subject to (4) and \( q_i \geq 0 \).

3.2.3. Model analysis. The following two theorems characterize the solution of (5) and (6).

**Theorem 3.3.** The solution of (5) is \( q^*_i = q^* + t_i \) and \( P^*_i = U(q^*) + \sum_{j=1}^{i-1} [U(q^*) - U(q^* + t_j - t_{j+1})] + (r - c)t_i + cq^*_i \) for \( i = 1, ..., n \). And the following properties hold:

1. the supplier’s profit vector is

\[
\left\{ \Pi^s_i \right\}_{i=1}^n = \left\{ U(q^*) + \sum_{j=1}^{i-1} [U(q^*) - U(q^* + t_j - t_{j+1})] + (r - c)t_i \right\}_{i=1}^n,
\]

and \( \Pi^s_i \) is increasing in \( i \);

2. the retailer’s profit vector is

\[
\left\{ \Pi^r_i \right\}_{i=1}^n = \left\{ (r - c)(t_i - t_1) - \sum_{j=1}^{i-1} [U(q^*) - U(q^* + t_j - t_{j+1})] \right\}_{i=1}^n,
\]

and \( \Pi^r_i \) is increasing in \( i \);

3. the supplier’s regret vector equals to the retailer’s profit vector, i.e.,

\[
\left\{ \text{Regret}_i^s \right\}_{i=1}^n = \left\{ \Pi^r_i \right\}_{i=1}^n,
\]

and the supply chain is coordinated.

Theorem 3.3 indicates that, the supply chain is coordinated if the supplier adopts the profit criterion, while coordination is impossible in the works of Gan et al. [11] and Babich et al. [2]. As the market gets larger, on the one hand, the supplier earns more profit; on the other hand, the supplier experience more regrets. For the retailer, the higher the market state, the more the profit, which coincides with the works of Chu [6], Geng and Minutolo [12].

**Theorem 3.4.** The solution of (6) is \( q^*_i = q^* + t_i \) and \( P^*_i = U(q^*) + \sum_{j=1}^{i-1} [U(q^*) - U(q^* + t_j - t_{j+1})] + (r - c)t_i + cq^*_i \) for \( i = 1, ..., n \), in which \( q^*_n = q^* + t_n \), \( t_i - \tau_1 \leq q^*_i \leq q^* + t_i \) and

\[
U(q^*_i - t_{i+1}) + (r - c)t_{i+1} = U(q^*_i - t_{i+1}) + (r - c)t_i \text{ for } i = 1, ..., n - 1.
\]

Further, the following properties hold:

1. the supplier’s profit vector is

\[
\left\{ \Pi^s_i \right\}_{i=1}^n = \left\{ U(q^*_1 - t_1) + (r - c)t_1 \right\}_{i=1}^n,
\]

and \( \Pi^s_i \) is increasing in \( i \);

2. the retailer’s profit vector is

\[
\left\{ \Pi^r_i \right\}_{i=1}^n = \left\{ U(q^*_1 - t_1) - U(q^*_1 - t_1) \right\}_{i=1}^n,
\]

and \( \Pi^r_i \) is increasing in \( i \);
and Regret considering the following simple example:

The criterion is defined inductively. To show the results of Theorem 3.4 more clearly, we
consider the following simple example: $r = 2, c = 1, t_i = i$, and $\eta$ is uniformly
distributed on $[-1, 1]$. Then, the optimal contract matrix satisfies

$$ q^o_i = n, q^o_{i-1} - (i - 1) = -0.5(q^o_i - i)^2 - 0.5. $$

Since $q^o_i - n = 0 \in [-1, 0]$, we know that $q^o_i$ is decreasing in $i$ and $q^o_i \to 0$ as
$n \to +\infty$. Further, we can calculate the limitations of $\Pi_1^{R,s}, \Pi_n^{R,s}, \Pi_1^{R,r}, \Pi_n^{R,r}$,
$\text{Regret}_{i}^{R,s}, \text{Loss}_1$ and $\text{Loss}_n$ as $n \to +\infty$:

$$
\begin{align*}
\Pi_1^{R,s} &\to 0, \Pi_n^{R,s} = n - 0.5 \to +\infty, \Pi_1^{R,r} = 0, \Pi_n^{R,r} \to 0.5, \\
\text{Regret}_{i}^{R,s} &\to 0.5, \text{Loss}_1 \to 0.5, \text{Loss}_n = 0.
\end{align*}
$$

For $n = 10$, the optimal decisions and the related vectors are

$$
\begin{pmatrix}
q^o \\
P^o \\
\Pi^{R,s}_n \\
\Pi^{R,r}_n \\
\text{Regret}^{R,s}
\end{pmatrix}
= \begin{pmatrix}
0.15 & 1.16 & 2.18 & 3.2 & 4.22 & 5.26 & 6.31 & 7.38 & 8.5 & 10 \\
0.29 & 2.3 & 4.32 & 6.34 & 8.36 & 10.4 & 12.44 & 14.51 & 16.64 & 19.14 \\
0.14 & 2.14 & 3.14 & 4.14 & 4.14 & 5.14 & 6.14 & 7.14 & 8.14 & 9.14 \\
0 & 0.01 & 0.02 & 0.04 & 0.06 & 0.09 & 0.12 & 0.17 & 0.24 & 0.36 \\
0.36 & 0.36 & 0.36 & 0.36 & 0.36 & 0.36 & 0.36 & 0.36 & 0.36 & 0.36 \\
0.36 & 0.35 & 0.34 & 0.32 & 0.3 & 0.28 & 0.24 & 0.2 & 0.13 & 0
\end{pmatrix}.
$$

Theorem 3.4 indicates that, the supply chain is no longer coordinated with the
regret criterion. The higher the market state, the higher the supply chain efficiency.
The main reason is that, the supply chain loss equals to the supplier’s regret minus
the retailer’s profit. When $t$ gets larger, more incentives are needed for the retailer
to reveal his private information. Hence, the retailer’s profit increases while the
regret of the supplier keeps in a certain level. As a result, the supply chain loss
decreases when market state gets higher. Our results also indicate that the supply
chain loss is bounded by the volatility of the demand.

**Remark.** Different to Theorem 3.3 that the optimal contract matrix under the profit
criterion has closed form expression, the optimal contract matrix under the regret
criterion is defined inductively. To show the results of Theorem 3.4 more clearly, we
considering the following simple example: $r = 2, c = 1, t_i = i$, and $\eta$ is uniformly
distributed on $[-1, 1]$. Then, the optimal contract matrix satisfies

$$ q^o_i = n, q^o_{i-1} - (i - 1) = -0.5(q^o_i - i)^2 - 0.5. $$

Since $q^o_i - n = 0 \in [-1, 0]$, we know that $q^o_i$ is decreasing in $i$ and $q^o_i \to 0$ as
$n \to +\infty$. Further, we can calculate the limitations of $\Pi_1^{R,s}, \Pi_n^{R,s}, \Pi_1^{R,r}, \Pi_n^{R,r}$,
$\text{Regret}_{i}^{R,s}, \text{Loss}_1$ and $\text{Loss}_n$ as $n \to +\infty$:

$$
\begin{align*}
\Pi_1^{R,s} &\to 0, \Pi_n^{R,s} = n - 0.5 \to +\infty, \Pi_1^{R,r} = 0, \Pi_n^{R,r} \to 0.5, \\
\text{Regret}_{i}^{R,s} &\to 0.5, \text{Loss}_1 \to 0.5, \text{Loss}_n = 0.
\end{align*}
$$

For $n = 10$, the optimal decisions and the related vectors are

$$
\begin{pmatrix}
q^o \\
P^o \\
\Pi^{R,s}_n \\
\Pi^{R,r}_n \\
\text{Regret}^{R,s}
\end{pmatrix}
= \begin{pmatrix}
0.15 & 1.16 & 2.18 & 3.2 & 4.22 & 5.26 & 6.31 & 7.38 & 8.5 & 10 \\
0.29 & 2.3 & 4.32 & 6.34 & 8.36 & 10.4 & 12.44 & 14.51 & 16.64 & 19.14 \\
0.14 & 2.14 & 3.14 & 4.14 & 4.14 & 5.14 & 6.14 & 7.14 & 8.14 & 9.14 \\
0 & 0.01 & 0.02 & 0.04 & 0.06 & 0.09 & 0.12 & 0.17 & 0.24 & 0.36 \\
0.36 & 0.36 & 0.36 & 0.36 & 0.36 & 0.36 & 0.36 & 0.36 & 0.36 & 0.36 \\
0.36 & 0.35 & 0.34 & 0.32 & 0.3 & 0.28 & 0.24 & 0.2 & 0.13 & 0
\end{pmatrix}.
$$
Comparing the profit vectors of the two criteria, we obtain the following theorem.

**Theorem 3.5.** There exists \( k \in \{1, \ldots, n - 1\} \), such that \( \Pi_i^{P,s} - \Pi_i^{R,s} \geq 0 \) for \( i \leq k \) and \( \Pi_i^{P,s} - \Pi_i^{R,s} \leq 0 \) for \( i \geq k + 1 \).

Theorem 3.5 indicates that the profit (regret) criterion generates higher profit for the supplier than the regret (profit) criterion does if the market state is low (high).

The regret faced by the supplier is mainly caused by information asymmetry, hence represents the value of the information. The following lemma gives upper bounds to the value of the information.

**Lemma 3.6.** The regrets of the supplier satisfy

\[
\text{Regret}_i^{R,s} \leq \min\{(r - c)(t_n - t_1), (r - c)(\tau_1 + \tau_2)\},
\]

\[
\text{Regret}_i^{P,s} \leq \min\{(r - c)(t_n - t_1), (n - 1)r(\tau_2 + \tau_1)\},
\]

for all \( i = 1, \ldots, n \).

Lemma 3.6 indicates that the supplier’s regret (or “cost of screening”) is bounded by \( t_n - t_1 \) (the range of the private information) and \( \tau_1 + \tau_2 \) (the volatility of the demand). This result is similar to the result of Özer and Wei [26], which shows that the “cost of screening” increases as the range of the private information increases. If the volatility of the demand (or the range of the private information) tends to zero, the supplier will experience zero regret. Note that, the supplier can coordinate the supply chain and obtain all channel profit if \( t \) is known. Hence, both criteria can coordinate the supply chain and help the supplier obtain all supply chain profit if the market volatility (or the range of the private information) tends to zero.

### 3.3. Contracting with a partially informed retailer.

Finally, we consider a situation that the supplier contracts with a retailer who does not perfectly understand the market state. For each realization of the market state \( t \), we assume that the retailer is informed about a subset of \( T \), such that the market state must come from the subset. In more detail, there is a partition \( \Delta S = \{s_1, \ldots, s_m\} \) of \( \{1, 2, \ldots, n\} \) \((1 = s_1 < \ldots < s_m = n + 1)\). For each realized \( t_i \), the retailer is informed about the set \( \{s_j, s_j + 1, \ldots, s_{j+1} - 1\} \) such that \( s_j \leq i \leq s_{j+1} - 1 \). And the retailer knows that \( t_i \in \{t_{s_j}, t_{s_j+1}, \ldots, t_{s_{j+1}-1}\} \).

The partially informed retailer faces a similar situation as the supplier, i.e., he faces a vector of profits. Since no further information is obtained, the retailer can hardly experience regret. Hence, we assume that the retailer applies the profit criterion to select a contract in the market. Be aware of this, the supplier needs to design \( m - 1 \) contracts \((q_{s_j}, P_{s_j})_{j=1}^{m-1}\) to screen the retailer’s private information. The incentive compatible constraints can be characterized by the following terms,

\[
\{rE(q_{s_j} \wedge (t_i + \eta)) - P_{s_j}\}_{i=s_j}^{s_{j+1}-1} \geq_L \{rE(q_{s_k} \wedge (t_i + \eta)) - P_{s_k}\}_{i=s_k}^{s_{j+1}-1},
\]

\[
\{rE(q_{s_j} \wedge (t_i + \eta)) - P_{s_j}\}_{i=s_j}^{s_{j+1}-1} \geq_L 0,
\]

or equivalently

\[
\{U(q_{s_j} - t_i) + (r - c)t_i + cq_{s_j} - P_{s_j}\}_{i=s_j}^{s_{j+1}-1} \geq_L \{U(q_{s_k} - t_i) + (r - c)t_i + cq_{s_k} - P_{s_k}\}_{i=s_k}^{s_{j+1}-1},
\]

\[
\{U(q_{s_j} - t_i) + (r - c)t_i + cq_{s_j} - P_{s_j}\}_{i=s_j}^{s_{j+1}-1} \geq_L 0.
\]
Hence, the problem faced by the supplier with profit criterion is

\[
\max \ (P_{s_j} - cq_{s_j})_{j = 1}^{m-1},
\]

with respect to order \(\geq L\),

subject to \((7)\) and \(q_{s_j} \geq 0\).

(8)

Similar to the full information case, as soon as a contract is chosen by the retailer, the supplier may experience a regret. For any given private information \(\{s_j, s_j + 1, \ldots, s_{j+1} - 1\}\) of the retailer, the supplier only need to provide a single contract \((q, P)\) to the retailer if such private information is also known by the supplier. And the only constraint is that, accepting the contract is better than not doing so for the retailer, i.e., the profit vector of the retailer is better than zero vector with respect to the generalized lexicographic preference. Hence, the supplier needs to solve the following problem if he knows the private information of the retailer:

\[
\max \ P - cq,
\]

subject to \(\{rE(q \land (t_1 + \eta)) - P\}_{t_i = s_j}^{s_{j+1} - 1} \geq L\) 0.

(9)

**Lemma 3.7.** The solution of the problem \((9)\) is

\[
(q, P) = \left(q^* + ts_j, U(q^*) + (r - c)ts_j + c(q^* + ts_j) \right).
\]

According to Lemma 3.7, the supplier’s regret is \(U(q^*) + (r - c)ts_j - (P_{s_j} - cq_{s_j})\) if the private information of the retailer is \(\{s_j, s_j + 1, \ldots, s_{j+1} - 1\}\). Thus, the problem faced by the supplier with regret criterion is

\[
\max \ (P_{s_j} - cq_{s_j} - U(q^*) - (r - c)ts_j)_{j = 1}^{m-1},
\]

with respect to order \(\geq L\),

subject to \((7)\) and \(q_{s_j} \geq 0\).

(10)

Solving the problems \((8)\) and \((10)\), we obtain the following two theorems.

**Theorem 3.8.** The solution of \((8)\) is

\[
\left(q^*_{s_j}, P^*_{s_j}\right) = \left(q^* + ts_j, U(q^*) + \sum_{k=1}^{j-1} [U(q^*) - U(q^* + ts_k - ts_{k+1})] + (r - c)ts_j + cq^* + cts_j\right).
\]

And the following properties hold:

1. **the supplier’s profit vector is**

\[
\left\{\Pi_{s_j}^{P, s, \Delta S}\right\}_{j = 1}^{m-1} = \left\{U(q^*) + \sum_{k=1}^{j-1} [U(q^*) - U(q^* + ts_k - ts_{k+1})] + (r - c)ts_j\right\}_{j = 1}^{m-1},
\]

and \(\Pi_{s_j}^{P, s, \Delta S}\) is increasing in \(j\):

2. **the supplier’s regret vector is**

\[
\left\{Regret_{s_j}^{P, s, \Delta S}\right\}_{j = 1}^{m-1} = \left\{(r - c)(ts_j - ts_1) - \sum_{k=1}^{j-1} [U(q^*) - U(q^* + ts_k - ts_{k+1})]\right\}_{j = 1}^{m-1},
\]

and \(Regret_{s_j}^{P, s, \Delta S}\) is increasing in \(j\).
Theorem 3.9. The solution of (10) is
\[
\left( q_{s_j}^{o,\Delta S}, p_{s_j}^{o,\Delta S} \right) = \left( q_{s_j}^{o,\Delta S}, U(q_{s_j}^{o,\Delta S} - t_{s_j}) + (r - c)t_{s_j} + cq_{s_j}^{o,\Delta S} \right),
\]
in which \( q_{s_j}^{o,\Delta S} = q^* + t_{s_m-1} \),
\[
U(q_{s_j}^{o,\Delta S} - t_{s_j+1}) + (r - c)t_{s_j} = U(q_{s_j+1}^{o,\Delta S} - t_{s_j+1}) + (r - c)t_{s_j}, \quad j \in \{1, ..., m-2 \}
\]
and \( t_{s_j} - \tau_i \leq q_{s_j}^{o,\Delta S} \leq q^* + t_{s_j} \). Further, the following properties hold:
(1) the supplier’s profit vector is
\[
\left\{ \Pi_{s_j}^{R,s,\Delta S} \right\}^{m-1}_{j=1} = \left\{ U(q_{s_j}^{o,\Delta S} - t_{s_j}) + (r - c)t_{s_j} \right\}^{m-1}_{j=1},
\]
and \( \Pi_{s_j}^{R,s,\Delta S} \) is increasing in \( j \);
(2) the supplier’s regret vector is
\[
\left\{ \text{Regret}_{s_j}^{R,s,\Delta S} \right\}^{m-1}_{j=1} = \left\{ U(q^*) - U(q_{s_j}^{o,\Delta S} - t_{s_j}) \right\}^{m-1}_{j=1},
\]
and \( \text{Regret}_{s_j}^{R,s,\Delta S} \) is constant for all \( j \).

Theorem 3.8 and Theorem 3.9 show that the optimal contracts under both criteria are similar to that in the full information case. Hence, comparing the two criteria leads to similar results as previous subsection. Therefore, we do not repeat the results. In the following subsection, we consider the retailer choice problem of the supplier.

Remark: The length of the profit and regret vectors are \( m \), i.e., the numbers of possible information received by the retailer. Hence retailers with different level of partial information may lead to different length of the profit and regret vectors. The generalized lexicographic preference can not compare the profit (or the regret) vectors with different length, thus we need to expand these vectors to be of length \( n \), i.e., the number of possible market states. In more detail, we define the following profit and regret vectors:

Profit vectors: \[ \left\{ \Pi_i^{P,s,\Delta S} \right\}^{n}_{i=1} \]
Regret vectors: \[ \left\{ \text{Regret}_i^{P,s,\Delta S} \right\}^{n}_{i=1} \]
\( i = 1 \), \[ \left\{ \Pi_i^{R,s,\Delta S} \right\}^{n}_{i=1} \]
\( i = 1 \), \[ \left\{ \text{Regret}_i^{R,s,\Delta S} \right\}^{n}_{i=1} \]
in which \( \Pi_i^{P,s,\Delta S} = \Pi_i^{P,s,\Delta S} \), \( \Pi_i^{R,s,\Delta S} = \Pi_i^{R,s,\Delta S} \), \( \text{Regret}_i^{P,s,\Delta S} = \text{Regret}_i^{P,s,\Delta S} \), \( \text{Regret}_i^{R,s,\Delta S} = \text{Regret}_i^{R,s,\Delta S} \).

3.4. Supplier’s choice of the retailer. Let \( \Delta \) be the collection of all partitions \( \Delta S \); then there is a nature partial order \( \succ \) on \( \Delta \), that is, \( \Delta S \succ \Delta S \) if \( \{ s_1, ..., s_m \} \subseteq \{ \bar{s}_1, ..., \bar{s}_m \} \). The following theorem characterizes the monotone properties of the profit and regret vectors with respect to the partial order \( \succ \).

Theorem 3.10. If \( \Delta S \succ \Delta S \), then
\[
\left\{ \Pi_i^{P,s,\Delta S} \right\}^{n}_{i=1} \geq_L \left\{ \Pi_i^{P,s,\Delta S} \right\}^{n}_{i=1} ; \left\{ \Pi_i^{R,s,\Delta S} \right\}^{n}_{i=1} \geq_L \left\{ \Pi_i^{R,s,\Delta S} \right\}^{n}_{i=1} ;
\]
\[
- \left\{ \text{Regret}_i^{P,s,\Delta S} \right\}^{n}_{i=1} \geq_L - \left\{ \text{Regret}_i^{P,s,\Delta S} \right\}^{n}_{i=1} ;
\]
\[
- \left\{ \text{Regret}_i^{R,s,\Delta S} \right\}^{n}_{i=1} \geq_L - \left\{ \text{Regret}_i^{R,s,\Delta S} \right\}^{n}_{i=1} .
\]
Theorem 3.10 indicates that, the supplier prefers the retailer with full (non) information under profit (regret) criterion. The reason is that, a retailer with more information can help the whole supply chain achieve higher total profit, hence may increase both supply chain members’ profits; while such retailer also requires more information rents from the supplier, which increases the regret of the supplier.

If the supplier chooses to cooperate with a retailer and adopts the regret criterion, then he should choose the retailer with the least information (i.e., the partition \( \Delta S = \{1, n+1\} \)). According to Theorem 3.9, the profit vector of the supplier is \( \{U(q^*) + (r - c)t_i\}_{i=1}^n \). According to Lemma 3.2, if the supplier sell the product by himself, his profit vector is \( \{U(q^* - t_1 + t_i) + (r - c)t_i\}_{i=1}^n \). Since \( U(q^* - t_1 + t_i) + (r - c)t_i - [U(q^*) + (r - c)t_1] \geq 0 \) for \( i \in \{1, ..., n\} \), the supplier should sell the product by himself rather than adopting the regret criterion with a retailer. This result is rather intuitive, because no regret is experienced if the supplier sells the product by himself. While for the profit criterion, whether the supplier should sell the product by himself is discussed in the following theorem.

**Theorem 3.11.** (1) Let \( x_1 \) be the solution of \( 2F(x_1) = (r - c)/r \). If \( t_n - t_1 \leq q^* - x_1 \), then

\[
U(q^* + t_1 - t_i) + (r - c)t_i \geq \Pi_i^{P,s} \Delta S, \text{ for all } i \in \{1, ..., n\} \text{ and } \Delta S \in \Delta.
\]

Which means that the supplier should sell the product by himself.

(2) If \( t_i - t_i \geq \max (2q^* - 2U(q^*)/(r - c), q^*) \) for \( i \in \{1, ..., n-1\} \), then

\[
U(q^* + t_1 - t_k) + (r - c)t_k \leq \Pi_k^{P,s}, \text{ for } k \in \{1, ..., n\}.
\]

Which means that the supplier should contract with the retailer with full demand state information.

Theorem 3.11 indicates that, whether the supplier should sell the product by himself depends on the diversity of all possible market states crucially. If the differences of all market states are small, then screening retailers with different private information becomes more difficult. Hence, the retailers will benefit more from the information rent, and it is optimal for the supplier to save the information rent and sell the product by himself. If the differences of all market states are sufficiently large, then screening retailers with different private information becomes easier. Hence, it is optimal for the supplier to pay little information rent and contracting with a retailer with full information.

4. **Conclusion.** This paper mainly discusses two decision making criteria (the profit criterion and the regret criterion) in a supply chain model with one supplier and one retailer. The retailer holds a private market state information and the supplier has limited prior information. Our analyses show that the profit criterion can coordinate the whole supply chain while the regret criterion can not. However, such coordination does not mean that the supplier always gains higher profit with profit criterion. The regret criterion generates higher profit for the supplier if the market state is high, while the profit criterion is better for the supplier if the market state is low. Information asymmetry provides information rent for the retailer and such rent increases as the market state gets higher. Information asymmetry also causes regret for the supplier, which can also be regarded as the value of the information. We show that the values of the information in both criteria are bounded by the market volatility and the range of the market state. That is, if the market
Appendix

A. research direction.

To study the strategic interactions between the retailers is another interesting future direction. Applying cooperative game theory, finally, we do not consider the possibility that multiple local retailers can form a union and negotiate with the supplier together. Discussing the supplier’s optimal decisions with respect to the generalized lexicographic preference. Secondly, we do not consider competitions in the model, discussing the interactions between competing suppliers would be an interesting future direction. Finally, we do not consider the possibility that multiple local retailers can form a union and negotiate with the supplier together. Applying cooperative game theory to study the strategic interactions between the retailers is another interesting future research direction.

Appendix A. Proof of Lemma 3.2. Note that, \( rE(q \wedge (t_i + \eta)) - cq \leq rE(q \wedge (t_j + \eta)) - cq \) for any \( i \leq j \). Therefore, the supplier will choose a proper \( q \) to maximize \( rE(q \wedge (t_1 + \eta)) - cq = rE((q - t_1) \wedge \eta) - c(q - t_1) + (r - c)t_1 = U(q - t_1) + (r - c)t_1. \) Since

\[
\frac{dU}{dx}(x) = r(1 - F(x)) - c,
\]

\[
\frac{dU^2}{dx^2}(x) = -rf(x) \leq 0,
\]

\( U(x) \) is concave, and achieves its maximum at \( q^* \), in which \( F(q^*) = (r - c)/r. \) Hence, the optimal production quantity of the supplier is \( q^* + t_1 \), and the expected profit vector is

\[
\{ rE((q^* + t_1) \wedge (t_i + \eta)) - c(q^* + t_1) \}_{i=1}^{n} = \{ rE((q^* + t_1 - t_i) \wedge \eta) - c(q^* + t_1 - t_i) + (r - c)t_i \}_{i=1}^{n} = \{ U(q^* + t_1 - t_i) + (r - c)t_i \}_{i=1}^{n}. \]

\( \square \)

Appendix B. Proof of Theorem 3.3. We first show that, two inequalities in (4) can be shortened.

If \( (q_i, P_i)_{i=1}^{n} \) satisfy (4), then the first inequality in (4) is equivalent to

\[
P_i - P_j \leq U(q_i - t_i) + cq_i - U(q_j - t_i) - cq_j.
\]

(11)

Interchanging \( i \) and \( j \),

\[
P_j - P_i \leq U(q_j - t_j) + cq_j - U(q_i - t_j) - cq_i.
\]

(12)

Therefore

\[
U(q_i - t_j) + cq_i - U(q_j - t_j) - cq_j \leq U(q_i - t_i) + cq_i - U(q_j - t_i) - cq_j.
\]
Which is
\[
U(q_j - t_i) - U(q_j - t_j) \leq U(q_i - t_i) - U(q_i - t_j).
\]
Choosing \(i_0 < j_0\), then \(t_{i_0} < t_{j_0}\). Note that \(\frac{dU}{dx}(x - t_{i_0}) - \frac{dU}{dx}(x - t_{j_0}) = r(F(x - t_{i_0}) - F(x - t_{j_0})) \leq 0\), therefore \(U(x - t_{i_0}) - U(x - t_{j_0})\) is decreasing in \(x\). If \(q_{i_0} > q_{j_0}\), then \(U(x - t_{i_0}) - U(x - t_{j_0})\) must be constant for \(x \in [q_{j_0}, q_{i_0}]\). (11) and (12) must be equalities for \(i = i_0\) and \(j = j_0\), \(\begin{equation}
P_{i_0} - P_{j_0} = U(q_{i_0} - t_{i_0}) + cq_{i_0} - U(q_{j_0} - t_{i_0}) - cq_{j_0}
= U(q_{i_0} - t_{j_0}) + cq_{i_0} - U(q_{j_0} - t_{j_0}) - cq_{j_0}.
\end{equation}\)
Consider \((\bar{q}_i, \bar{P}_i)_{i=1}^n\) such that \((\bar{q}_i, \bar{P}_i) = (q_i, P_i)\) for \(i \neq i_0, j_0\), \((\bar{q}_{i_0}, \bar{P}_{i_0}) = (q_{j_0}, P_{j_0})\) and \((\bar{q}_{j_0}, \bar{P}_{j_0}) = (q_{i_0}, P_{i_0})\). In the following, we show that, \((\bar{q}_i, \bar{P}_i)_{i=1}^n\) satisfies (4), and generates the same profit vector for the supplier as \((q_i, P_i)_{i=1}^n\) does. We only need to consider the inequalities for two coordinates \(i = i_0\) and \(i = j_0\).

For \(i = i_0\), we have \(\begin{align*}
U(\bar{q}_{i_0} - t_{i_0}) + c\bar{q}_{i_0} - \bar{P}_{i_0} \\
= U(q_{j_0} - t_{i_0}) + cq_{j_0} - P_{j_0} \\
= U(q_{i_0} - t_{i_0}) + cq_{i_0} - P_{i_0} \\
= \max_{j \in \{1, \ldots, n\}} U(q_j - t_{i_0}) + cq_j - P_j \\
= \max_{j \in \{1, \ldots, n\}} U(\bar{q}_j - t_{i_0}) + c\bar{q}_j - \bar{P}_j,
\end{align*}\)
and
\[
U(\bar{q}_{i_0} - t_{i_0}) + (r - c)t_{i_0} + cq_{i_0} - \bar{P}_{i_0}
= U(q_{j_0} - t_{i_0}) + (r - c)t_{i_0} + cq_{j_0} - P_{j_0}
= U(q_{i_0} - t_{i_0}) + (r - c)t_{i_0} + cq_{i_0} - P_{i_0} \geq 0.
\]
Hence, retailer with type \(t_{i_0}\) will choose \((\bar{q}_{i_0}, \bar{P}_{i_0})\), and his profit equals to choosing contract \((q_{i_0}, P_{i_0})\).

Case \(i = j_0\) is similar to the case \(i = i_0\). Hence, \((\bar{q}_i, \bar{P}_i)_{i=1}^n\) is equivalent to \((q_i, P_i)_{i=1}^n\). Hence, it suffices to consider contract menus with \(q_i\) increasing in \(i\).

Take \(i < j < k\) in a Bayesian incentive compatible mechanism, we know that \(q_i \leq q_j \leq q_k\). Since \((i, j)\) and \((j, k)\) both satisfy (11), then \(\begin{align*}
P_i - P_k & = (P_i - P_j) + (P_j - P_k) \\
& \leq U(q_i - t_i) - U(q_j - t_j) + U(q_j - t_j) - U(q_k - t_j) + c(q_i - q_k).
\end{align*}\)
Note that \(U(x - t_i) - U(x - t_j)\) is decreasing in \(x\), and \(q_j \leq q_k\), then \(\begin{align*}
P_i - P_k & \leq U(q_i - t_i) - U(q_k - t_i) + U(q_k - t_j) - U(q_k - t_j) + c(q_i - q_k) \\
& = U(q_i - t_i) - U(q_k - t_i) + c(q_i - q_k).
\end{align*}\)
Therefore, \((i, k)\) will automatically satisfy (11), if \(q_i \leq q_j \leq q_k\), \((i, j)\) and \((j, k)\) both satisfy (11).

Similarly, since \((i, j)\) and \((j, k)\) both satisfy (12), then \(\begin{align*}
P_k - P_i & = (P_k - P_j) + (P_j - P_i) \\
& \leq U(q_k - t_k) - U(q_j - t_k) + U(q_j - t_j) - U(q_i - t_j) + c(q_k - q_i) \\
& \leq U(q_k - t_k) - U(q_i - t_k) + U(q_i - t_j) - U(q_i - t_j) + c(q_k - q_i) \\
& = U(q_k - t_k) - U(q_i - t_k) + c(q_k - q_i).
\end{align*}\)
Hence, \((i, k)\) will automatically satisfy (12), if \(q_i \leq q_j \leq q_k\), \((i, j)\) and \((j, k)\) both satisfy (12).

The above discussions indicate that (4) is equivalent to

\[
U(q_{i+1} - t_i) - U(q_i - t_i) + c(q_{i+1} - q_i) \leq P_{i+1} - P_i, \\
P_{i+1} - P_i \leq U(q_{i+1} - t_{i+1}) - U(q_i - t_{i+1}) + c(q_{i+1} - q_i), \quad 1 \leq i \leq n - 1;
\]

\[
P_i \leq U(q_i - t_i) + (r - c)t_i + cq_i, \quad 1 \leq i \leq n;
\]

\[
0 \leq q_1 \leq \cdots \leq q_n.
\]

Next, we will simplify the constraint \(P_i \leq U(q_i - t_i) + (r - c)t_i + cq_i\). Note that \(\frac{dU}{dx}(x) = r(1 - F(x)) - c \leq r - c\), thus

\[
P_{i+1} = P_{i+1} - P_i + P_i
\]

\[
\leq U(q_{i+1} - t_{i+1}) - U(q_i - t_i) + c(q_{i+1} - q_i) + U(q_i - t_i) + (r - c)t_i + cq_i
\]

\[
= U(q_{i+1} - t_{i+1}) + cq_{i+1} + (U(q_i - t_i) - U(q_i - t_{i+1})) + (r - c)t_i
\]

\[
\leq U(q_{i+1} - t_{i+1}) + cq_{i+1} + (r - c)(q_i - t_i - (q_i - t_{i+1})) + (r - c)t_i
\]

\[
= U(q_{i+1} - t_{i+1}) + cq_{i+1} + (r - c)t_{i+1}.
\]

The above calculation indicates that constraints \(P_i \leq U(q_i - t_i) + (r - c)t_i + cq_i\) for \(i \geq 2\) can be deleted. Hence, (4) is equivalent to

\[
U(q_{i+1} - t_i) - U(q_i - t_i) + c(q_{i+1} - q_i) \leq P_{i+1} - P_i, \quad 1 \leq i \leq n - 1;
\]

\[
P_{i+1} - P_i \leq U(q_{i+1} - t_{i+1}) - U(q_i - t_{i+1}) + c(q_{i+1} - q_i), \quad 1 \leq i \leq n - 1;
\]

\[
P_1 \leq U(q_1 - t_1) + (r - c)t_1 + cq_1;
\]

\[
0 \leq q_1 \leq \cdots \leq q_n. \tag{13}
\]

The objective of the supplier is \((P_i - cq_i)_{i=1}^n\), hence, for fixed \(q_1 \leq \cdots \leq q_n\), the optimal \(\{P_i\}_{i=1}^n\) will satisfy \(P_i = U(q_1 - t_1) + (r - c)t_1 + cq_1\) and \(P_{i+1} - P_i = U(q_{i+1} - t_{i+1}) - U(q_i - t_{i+1}) + c(q_{i+1} - q_i)\) for \(i \geq 1\). Therefore, the objective of the supplier becomes

\[
\max \left\{ U(q_i - t_i) + (r - c)t_i + \sum_{j=1}^{i-1} [U(q_j - t_j) - U(q_j - t_{j+1})] \right\}_{i=1}^n \tag{14}
\]

with respect to order \(\geq L\),

s.t. \(0 \leq q_1 \leq \cdots \leq q_n\).

In the following, we prove that, \(q^*_i = q^* + t_i\) is the only maximizer of the above problem. If not, assume \(\{\tilde{q}_i\}_{i=1}^n\) (different from \(\{q^*_i\}_{i=1}^n\) gives a larger profit vector with respect to \(\geq L\). The profit vector generate by \(q^*_i\) is

\[
\left\{ U(q^*) + (r - c)t_1 + \sum_{j=1}^{i-1} [U(q^*) - U(q^* + t_j - t_{j+1})] \right\}_{i=1}^n
\]

Which is increasing in \(i\) \((q^*\) maximizes \(U\)). Since \(\{\tilde{q}_i\}_{i=1}^n\) gives a larger profit vector with respect to \(\geq L\), \(\{\tilde{q}_i\}_{i=1}^n\) must satisfy

\[
U(\tilde{q}_1 - t_1) + (r - c)t_1 \geq U(q^*) + (r - c)t_1.
\]

Therefore \(\tilde{q}_1 = q^* + t_1 = q^*_1\). Now assume \(\tilde{q}_i = q^*_i\) for \(i = 1, \ldots, k - 1\), then we consider \(i = k\). Since \(\{\tilde{q}_i\}_{i=1}^n\) gives a larger profit vector with respect to \(\geq L\), \(\{\tilde{q}_i\}_{i=1}^n\)
Theorem 3.3, we know that, the optimal properties in the theorem.

\[ U(q_k - t_k) + (r - c)t_1 + \sum_{j=1}^{k-1} [U(q_j - t_j) - U(q_j - t_{j+1})] \]

\[ \geq U(q^*) + (r - c)t_1 + \sum_{j=1}^{k-1} [U(q^*) - U(q^* + t_j - t_{j+1})]. \]

Since \( \bar{q}_i = q_i^* \) for \( i = 1, \ldots, k - 1 \), \( \{\bar{q}_i\}_{i=1}^n \) must satisfy \( q_k = q^* + t_k = q_k^* \). By induction, we know \( \bar{q}_i = q_i^* \) for \( i = 1, \ldots, n \), this contradicts with the assumption that \( \{\bar{q}_i\}_{i=1}^n \) is different from \( \{q_i^*\}_{i=1}^n \). This contradiction indicates that \( q_i^* = q^* + t_i \) is the only maximizer. Hence,

\[ P_i^* = U(q^*) + (r - c)t_1 + \sum_{j=1}^{k-1} [U(q^*) - U(q^* + t_j - t_{j+1})] + cq^* + ct_i \]

and the main part of the theorem has been proved. Finally, we prove the three properties in the theorem.

(1) When \( t = t_i \), the profit of the supplier is

\[ P_i^* - cq_i^* = U(q^*) + (r - c)(t_i - t_1) - \sum_{j=1}^{i-1} [U(q^*) - U(q^* + t_j - t_{j+1})] \]

\[ = \sum_{j=1}^{i-1} \{(r - c)(t_{j+1} - t_j) - [U(q^*) - U(q^* + t_j - t_{j+1})]\}. \]

Since \( \frac{dU}{dx}(x) = r(1 - F(x)) - c \leq r - c \), we know that \( (r - c)(t_{j+1} - t_j) - [U(q^*) - U(q^* + t_j - t_{j+1})] \geq 0 \). Hence, the profit vector of the retailer is increasing in \( i \).

(2) When \( t = t_i \), the profit of the retailer is

\[ rE(q_i^* \land (t_i + \eta_t)) - P_i^* = (r - c)(t_i - t_1) - \sum_{j=1}^{i-1} [U(q^*) - U(q^* + t_j - t_{j+1})] \]

(3) When \( t = t_i \), the regret of the supplier is

\[ U(q^*) + (r - c)t_i - (P_i^* - cq_i^*) \]

\[ =(r - c)(t_i - t_1) - \sum_{j=1}^{i-1} [U(q^*) - U(q^* + t_j - t_{j+1})]. \]

The sum of the supplier’s profit and the retailer’s profit is \( U(q^*) + (r - c)t_i \), which is the maximum supply chain profit. Hence, the supply chain is coordinated.

**Appendix C. Proof of Theorem 3.4.** Follow the same logic in the proof of Theorem 3.3, we know that, the optimal \( \{P_i\}_{i=1}^n \) will satisfy \( P_1 = U(q_1 - t_1) + (r - c)t_1 + cq_1 \) and \( P_i - P_i = U(q_{i+1} - t_{i+1}) - U(q_i - t_{i+1}) + c(q_{i+1} - q_i) \) for \( i \geq 1 \).

And the goal of the supplier becomes

\[
\max \left\{ U(q_i - t_i) - U^*(q^*) - (r - c)(t_i - t_1) + \sum_{j=1}^{i-1} [U(q_j - t_j) - U(q_j - t_{j+1})] \right\}_{i=1}^n
\]

with respect to order \( \geq L \),

\[ 0 \leq q_1 \leq \cdots \leq q_n. \]
Solving the above optimization problem is a little bit difficult, in the following, we will relax the problem (15) to

\[
\begin{align*}
\max \{ & R_i(q_1, \ldots, q_i) - U^*(q^*) \} \\
\text{s.t.} \quad & 0 \leq q_i \quad i = 1, \ldots, n, \\
& R_i(q_1, \ldots, q_i) = U(q_i - t_i) - (r - c)(t_i - t_1) + \sum_{j=1}^{i-1} [U(q_j - t_j) - U(q_j - t_{j+1})].
\end{align*}
\]

(16)

And we will show that (16) and (15) have the same maximizer. The main idea of this proof is that, we will find a maximizer of (16). Then we will prove that, such maximizer is a feasible solution of (15).

Since only the \(n\)th coordinate \(R_n(q_1, \ldots, q_n)\) is related to \(q_n\), we know that the maximizer of (16) satisfies \(q^*_n = q^* + t_n\). Now we consider \(q_{n-1}\), only the \(n\)th and \((n-1)\)th coordinates are related to \(q_{n-1}\). Taking derivatives with respect to \(q_{n-1}\)

\[
\begin{align*}
\frac{\partial R_n}{\partial q_{n-1}} (q_1, \ldots, q_{n-1}, q^*_n) &= r(F(q_{n-1} - t_n) - F(q_{n-1} - t_{n-1})) \\
\frac{\partial R_{n-1}}{\partial q_{n-1}} (q_1, \ldots, q_{n-1}) &= r(1 - F(q_{n-1} - t_{n-1})) - c.
\end{align*}
\]

Hence, \(R_n\) is decreasing in \(q_{n-1}\), \(R_{n-1}\) is increasing in \(q_{n-1}\) if \(q_{n-1} \leq q^* + t_{n-1}\) and decreasing in \(q_{n-1}\) if \(q_{n-1} \geq q^* + t_{n-1}\). Thus, the maximizer of (16) satisfies \(q^*_{n-1} \leq q^* + t_{n-1}\). Note that \(dU/dx \leq r - c\), if \(q_{n-1} = q^* + t_{n-1}\) we have

\[
\begin{align*}
R_n(q_1, \ldots, q_{n-1}, q^*_n) - R_{n-1}(q_1, \ldots, q_{n-1}) &= U(q^*_n - t_n) - U(q_{n-1} - t_n) - (r - c)(t_n - t_{n-1}) \\
&= U(q^*_n - t_n) - U(q^*_n - t_n) - (r - c)(t_n - t_{n-1}) \\
&
\end{align*}
\]

Hence, (16) is maximized if and only if \(R_n(q_1, \ldots, q_{n-1}, q^*_n) - R_{n-1}(q_1, \ldots, q_{n-1}) = 0\)

\((t_{n-1} - \tau_1 \leq q_{n-1} \leq q^* + t_{n-1})\). Therefore, the maximizer of (16) satisfies

\[
U(q^*_{n-1} - t_n) + (r - c)t_n
\]

Now we assume that, for \(i = k + 1, \ldots, n - 1\) the maximizer of (16) satisfies

\[
\begin{align*}
U(q^*_i - t_{i+1}) + (r - c)t_{i+1} &= U(q^*_i - t_{i+1}) + (r - c)t_i; \\
q^*_n &= q^* + t_n \quad t_i - \tau_1 \leq q^*_i \leq q^* + t_i.
\end{align*}
\]
Then, we consider \( i = k \). It is easy to see that only the coordinates from the \( k \)th to the \( n \)th are related to \( q_k \). Hence, for \( i = k+1, \ldots, n-1 \) we have

\[
\begin{align*}
R_{i+1}(q_1, \ldots, q_k, q_{k+1}^i, \ldots, q_{n+1}^i) - R_i(q_1, \ldots, q_k, q_{k+1}^i, \ldots, q_{n+1}^i) &= U(q_{k+1}^i - t_{k+1}) - U(q_k^i - t_{k+1}) - (r - c)(t_{k+1} - t_i) \\ &\leq U(q_{k+1}^i + t_{k+1}) - U(q_k^i + t_{k+1}) - (r - c)(t_{k+1} - t_i) \\ &\leq U(q^* - U(q^* + t_k - t_{k+1}) - (r - c)(t_{k+1} - t_i) \\ &\leq 0.
\end{align*}
\]

Therefore, for any \( q_k \geq 0 \), \( R_{k+1}(q_1, \ldots, q_k, q_{k+1}^i, \ldots, q_{n+1}^i) = \cdots = R_n(q_1, \ldots, q_k, q_{k+1}^i, \ldots, q_{n+1}^i) \).

Taking derivatives with respect to \( q_k \) we have

\[
\begin{align*}
\frac{\partial R_{k+1}}{\partial q_k}(q_1, \ldots, q_k) &= r(F(q_k - t_{k+1}) - F(q_k - t_k)) \\ &= r(1 - F(q_k - t_k)) - c.
\end{align*}
\]

Hence, \( R_{k+1} \) is decreasing in \( q_k \). \( R_k \) is increasing in \( q_k \) if \( q_k \leq q^* + t_k \) and decreasing in \( q_k \) if \( q_k \geq q^* + t_k \). Thus the maximizer of (16) satisfies \( q_k^* \leq q^* + t_k \). Note that \( dU/dx \leq r - c \), if \( q_k = q^* + t_k \) we have

\[
\begin{align*}
R_{k+1}(q_1, \ldots, q_k) &= U(q_{k+1}^i - t_{k+1}) - U(q_k^i - t_{k+1}) - (r - c)(t_{k+1} - t_k) \\ &\leq U(q^* - U(q^* + t_k - t_{k+1}) - (r - c)(t_{k+1} - t_k) \\ &\leq 0.
\end{align*}
\]

If \( q_k = t_k - \tau_1 \), since \( q_{k+1}^i - t_{k+1} \in [-\tau_1, q^*] \), we have

\[
\begin{align*}
R_{k+1}(q_1, \ldots, q_k, q_{k+1}^i) - R_k(q_1, \ldots, q_k) &= U(q_{k+1}^i - t_{k+1}) - U(q_k^i - t_{k+1}) - (r - c)(t_{k+1} - t_k) \\ &\leq U(q^* - U(q^* + t_k - t_{k+1}) - (r - c)(t_{k+1} - t_k) \\ &\leq 0.
\end{align*}
\]

Hence, (16) is maximized if and only if \( R_{k+1}(q_1, \ldots, q_k, q_{k+1}^i) - R_k(q_1, \ldots, q_k) = 0 \) \( (t_k - \tau_1 \leq q_k \leq q^* + t_k) \). Therefore, the maximizer of (16) satisfies

\[
U(q_{k+1}^i - t_{k+1}) + (r - c)t_{k+1} = U(q_{k+1}^i - t_{k+1}) + (r - c)t_k, \quad t_k - \tau_1 \leq q_{k+1}^i \leq q^* + t_k.
\]

By induction, we know the maximizer of (16) is \( (q_{i+1}^n)_{i=1}^n \), such that

\[
q_n^* = q^* + t_n, \quad t_i - \tau_1 \leq q_i^* \leq q^* + t_i
\]

\[
U(q_{i+1}^n - t_{i+1}) + (r - c)t_{i+1} = U(q_{i+1}^n - t_{i+1}) + (r - c)t_i
\]

From the above equation, we can easily see that \( q_i^* \leq q_i^t \). Hence, \( (q_{i+1}^n)_{i=1}^n \) is also a maximizer of (15), which means \( q_i^o = q_i^t \) and

\[
\begin{align*}
P_i^o &= U(q_i^o - t_i) + (r - c)t_1 + \sum_{j=1}^{i-1} [U(q_j^o - t_j) - U(q_j^o - t_{j+1})] + cq_i^o \\ &= U(q_i^o - t_i) + (r - c)t_1 + cq_i^o
\end{align*}
\]
Finally, we prove the properties in the theorem.

1. When $t = t_i$, the profit of the supplier is
   \[ P_i^o - cq_i^o = U(q_i^o - t_1) + (r - c)t_i, \]
   which is increasing in $i$.

2. When $t = t_i$, the profit of the retailer is
   \[ \begin{align*}
   & rE(q_i^o \land (t_i + \eta)) - P_i^o \\
   = & U(q_i^o - t_i) + (r - c)t_i + cq_i^o - P_i^o \\
   = & U(q_i^o - t_i) - U(q_i^o - t_1).
   \end{align*} \]
   Note that, $U(q_i^o - t_1) - U(q_{i+1}^o - t_1) = U(q_i^o - t_i) - U(q_i^o - t_{i+1}) + (r - c)(t_{i+1} - t_i) \geq 0$.
   Hence, the profit vector of the retailer is increasing in $i$.

3. When $t = t_i$, the regret of the supplier is
   \[ \text{Regret}_i = U(q^*) + (r - c)t_i - (P_i^o - cq_i^o) = U(q^*) - U(q_i^o - t_1), \]
   which is constant for all $i$.

4. The sum of the supplier’s profit and the retailer’s profit is $U(q_i^o - t_i) + (r - c)t_i$, while the maximum supply chain profit is $U(q^*) + (r - c)t_i$. Therefore, the supply chain loss is
   \[ \text{Loss}_i = U(q^*) - U(q_i^o - t_i), \]
   which is decreasing in $i$. Since $dU/dx \leq r - c$ and $q_i^o - t_i \in [-\tau_1, q^*]$, we have $U(q^*) - U(q_i^o - t_i) \leq (r - c)(q^* - q_i^o + t_i) \leq (r - c)(\tau_1 + q^*) \leq (r - c)(\tau_1 + \tau_2)$.

**Appendix D. Proof of Theorem 3.5.** According to Theorem 3.3 and Theorem 3.4, we have

\[ \begin{align*}
(\Pi_i^{P,s} - \Pi_i^{R,s}) & = U(q^*) - U(q_i^o - t_i) - (r - c)(t_{i+1} - t_i) \\
& \leq 0.
\end{align*} \]

Hence, $\Pi_i^{P,s} - \Pi_i^{R,s}$ is decreasing in $i$. Note that, $(\Pi_i^{P,s})_{i=1}^n$ is the profit vector under profit criterion. Thus $(\Pi_i^{P,s})_{i=1}^n \geq L (\Pi_i^{R,s})_{i=1}^n$, which indicates $\Pi_i^{P,s} - \Pi_i^{R,s} \geq 0$.

Similar, $(U(q^*) + (r - c)t_i - \Pi_i^{R,s})_{i=1}^n$ is the regret vector under regret criterion. Thus $-(U(q^*) + (r - c)t_i - \Pi_i^{R,s})_{i=1}^n \geq L -(U(q^*) + (r - c)t_i - \Pi_i^{P,s})_{i=1}^n$, which indicates $-(U(q^*) + (r - c)t_n - \Pi_n^{R,s}) \geq -(U(q^*) + (r - c)t_n - \Pi_n^{P,s})$, i.e., $\Pi_n^{P,s} - \Pi_n^{R,s} \leq 0$.

Hence, there exists a $k \in \{1, ..., n - 1\}$, such that $\Pi_i^{P,s} - \Pi_i^{R,s} \geq 0$ for $i \leq k$ and $\Pi_i^{P,s} - \Pi_i^{R,s} \leq 0$ for $i \geq k + 1$.

**Appendix E. Proof of Lemma 3.6.** According to Theorem 3.4, we have

\[ \begin{align*}
\text{Regret}_i^{R,s} & = \text{Regret}_i^{R,s} \\
& = (r - c)(t_n - t_1) - \sum_{j=1}^{n-1} [U(q_j^o - t_j) - U(q_j^o - t_{j+1})] \leq (r - c)(t_n - t_1),
\end{align*} \]

\[ \begin{align*}
\text{Regret}_i^{R,s} & = \text{Regret}_i^{R,s} = U(q^*) - U(q_i^o - t_1) = \text{Loss}_i \leq (r - c)(\tau_1 + \tau_2).
\end{align*} \]
According to Theorem 3.3, on the one hand, we have
\[ \text{Regret}^n = \Pi^n \]
\[ = (r - c)(t_n - t_1) - \sum_{j=1}^{n-1} [U(q^*) - U(q^* + t_j - t_{j+1})] \leq (r - c)(t_n - t_1). \]

On the other hand, we have
\[ \text{Regret}^n = \Pi^n = (r - c)(t_n - t_1) - \sum_{j=1}^{n-1} [U(q^*) - U(q^* + t_j - t_{j+1})] \]
\[ = \sum_{j=1}^{n-1} \int_{t_j - t_{j+1}}^{t_j} \left[ (r - c) - \frac{dU}{dx}(q^* + y) \right] dy = \sum_{j=1}^{n-1} \int_{t_j - t_{j+1}}^{t_j} rF(q^* + y) dy. \]

Since \( \eta \) is supported on \([-\tau_1, \tau_2]\), then \( q^* \in [-\tau_1, \tau_2] \) and we have
\[ \int_{t_j - t_{j+1}}^{t_j} rF(q^* + y) dy \leq \int_{-\tau_1 - q^*}^{0} rF(q^* + y) dy \leq r(q^* + \tau_1) \leq r(\tau_2 + \tau_1). \]

Therefore we have
\[ \text{Regret}^n \leq \Pi^n \leq (n - 1)r(\tau_2 + \tau_1). \]

In summary, we have shown that
\[ \text{Regret}^n \leq \text{Regret}^n \leq \min\{(r - c)(t_n - t_1), (n - 1)r(\tau_2 + \tau_1)\}. \]

\[ \square \]

Appendix F. Proof of Lemma 3.7. Note that \( rE(q \land (t_i + \eta)) - P \) is increasing in \( i \), hence the problem (9) becomes,
\[ \max \quad P - cq, \]
\[ \text{subject to} \quad rE(q \land (t_{s_j} + \eta)) - P \geq 0. \]
Hence \( P - cq \leq rE(q \land (t_{s_j} + \eta)) - cq = U(q - t_{s_j}) + (r - c)t_{s_j} \leq U(q^*) + (r - c)t_{s_j}. \)

And the upper bound is achieved at
\[ (q, P) = \left( q^* + t_{s_j}, U(q^*) + (r - c)t_{s_j} + c(q^* + t_{s_j}) \right). \]

\[ \square \]

Appendix G. Proof of Theorem 3.8. Constraints (7) indicates that
\[ U(q_{s_j} - t_{s_j}) + cq_{s_j} - P_{s_j} \geq U(q_{s_{k}} - t_{s_{k}}) + cq_{s_{k}} - P_{s_{k}} \]
\[ U(q_{s_j} - t_{s_j}) + (r - c)t_{s_j} + cq_{s_j} - P_{s_j} \geq 0. \]
(17)

Replace the constraints (7) in (8) with (17), and apply Theorem 3.3, we may obtain the optimal solution \( (q^*_{s_j}, \Delta S, P_{s_j}', \Delta S') \), which is an upper bound of (8). Thus, the theorem is proved if \( (q^*_{s_j}, \Delta S, P_{s_j}', \Delta S') \) satisfies (7), and we will prove it in the following.

If \( k > j \), then
\[ U(q^*_{s_j} - t_{s_j}) + cq^*_{s_j} - P^*_{s_j} - U(q^*_{s_k} - t_{s_k}) - cq^*_{s_k} - P^*_{s_k} \]
\[ = U(q^*) - U(q^* + t_{s_k} - t_{s_j}) + \sum_{i=j}^{k-1} [U(q^*) - U(q^* + t_{s_i} - t_{s_{i+1}})] > 0. \]

Note that, \( U(q - x) + (r - c)x+ \) is increasing in \( x \). Therefore,
\[ \min \left\{ U(q^*_{s_j} - t_{i}) + (r - c)t_{i} + cq^*_{s_j} - P^*_{s_j} \right\}_{i=s_j}^{s_{j+1} - 1} \]
Which indicates,

\[ \left\{ U(q^s_{s_j} - t_i) + (r-c)t_i + cq^s_{s_j} - P^s_{s_j} \right\}^{s_{j+1}-1}_{i=s_j} \geq L \left\{ U(q^s_{s_j} - t_i) + (r-c)t_i + cq^s_{s_j} - P^s_{s_j} \right\}^{s_{j+1}-1}_{i=s_j}. \]

If \( k < j \), then

\[
\frac{d}{dx} \left\{ U(q^s_{s_j} - x) - U(q^s_{s_k} - x) \right\} = r(F(q^s_{s_j} - x) - F(q^s_{s_k} - x)) \geq 0.
\]

Therefore, for \( i \in \{s_j, s_j + 1, ..., s_{j+1} - 1\} \), we have

\[
U(q^s_{s_j} - t_i) + cq^s_{s_j} - P^s_{s_j} - [U(q^s_{s_k} - t_i) + cq^s_{s_k} - P^s_{s_k}] \geq 0.
\]

Which indicates,

\[ \left\{ U(q^s_{s_j} - t_i) + (r-c)t_i + cq^s_{s_j} - P^s_{s_j} \right\}^{s_{j+1}-1}_{i=s_j} \geq L \left\{ U(q^s_{s_j} - t_i) + (r-c)t_i + cq^s_{s_j} - P^s_{s_j} \right\}^{s_{j+1}-1}_{i=s_j}. \]

Finally, since \((d/dx)U \leq r - c\), it is easy to check for \( i \in \{s_j, s_j + 1, ..., s_{j+1} - 1\} \) it holds,

\[ U(q^s_{s_j} - t_i) + (r-c)t_i + cq^s_{s_j} - P^s_{s_j} \geq 0. \]

Hence, the theorem is proved. \(\square\)

**Appendix H. Proof of Theorem 3.9.** Similar to the proof of Theorem 3.8, we only need to check \((q^o_{s_j}, \Delta S^o, P^o_{s_j}, \Delta S)\) satisfies (7).

If \( k > j \), then

\[
U(q^o_{s_j} - t_s) + cq^o_{s_j} - P^o_{s_j} - [U(q^o_{s_k} - t_s) + cq^o_{s_k} - P^o_{s_k}] \geq 0.
\]

Note that for \( a > b \)

\[
\frac{d}{dx} \left\{ U(x - a) - U(x - b) \right\} = r \{ F(x - b) - F(x - a) \} \geq 0.
\]
Hence,
\[
U(q_{t,s}^{0,\Delta S} - t_{s_j}) + cq_{t,s}^{0,\Delta S} - P_{s_j}^{0}\Delta S - [U(q_{s_j}^{0,\Delta S} - t_{s_j}) + cq_{s_j}^{0,\Delta S} - P_{s_j}^{0}\Delta S] = U(q_{t,s}^{0,\Delta S} - t_{s_k}) - U(q_{s_k}^{0,\Delta S} - t_{s_k}) + \sum_{i=j}^{k-1} \left[ U(q_{s_i}^{0,\Delta S} - t_{s_i}) - U(q_{s_i}^{0,\Delta S} - t_{s_i+1}) \right] \\
\geq U(q_{t,s}^{0,\Delta S} - t_{s_k}) - U(q_{s_k}^{0,\Delta S} - t_{s_k}) + \sum_{i=j}^{k-1} \left[ U(q_{s_i}^{0,\Delta S} - t_{s_i}) - U(q_{s_i}^{0,\Delta S} - t_{s_i+1}) \right] \\
\geq \ldots \\
\geq U(q_{t,s}^{0,\Delta S} - t_{s_{j+1}}) - U(q_{s_{j+1}}^{0,\Delta S} - t_{s_{j+1}}) + \sum_{i=j_{s_{j+1}}}^{s_{j+1} - 1} \left[ U(q_{s_i}^{0,\Delta S} - t_{s_i}) - U(q_{s_i}^{0,\Delta S} - t_{s_{j+1}}) \right] \\
\geq 0.
\]

If the above inequality becomes equality, then \( F(q_{s_{j+1}}^{0,\Delta S} - t_{s_{j+1}}) = F(q_{s_j}^{0,\Delta S} - t_{s_j}) \).

However, \( \eta \) is supported on \([-\tau_1, -\tau_2]\), which means that \( F \) is strictly increasing in \([-\tau_1, q^*]\). Thus \( F(q_{s_{j+1}}^{0,\Delta S} - t_{s_{j+1}}) < F(q_{s_j}^{0,\Delta S} - t_{s_j}) \), contradiction! Therefore,

\[
\min \left\{ U(q_{s_i}^{0,\Delta S} - t_i) + (r-c)t_i + cq_{s_i}^{0,\Delta S} - P_{s_i}^{0}\Delta S \right\}_{i=s_{j+1}}^{s_{j+1}-1} = U(q_{t,s}^{0,\Delta S} - t_{s_j}) - U(q_{s_j}^{0,\Delta S} - t_{s_j}) + \sum_{i=j}^{k-1} \left[ U(q_{s_i}^{0,\Delta S} - t_{s_i}) - U(q_{s_i}^{0,\Delta S} - t_{s_i+1}) \right] \\
\geq \min \left\{ U(q_{s_i}^{0,\Delta S} - t_i) + (r-c)t_i + cq_{s_i}^{0,\Delta S} - P_{s_i}^{0}\Delta S \right\}_{i=s_{j+1}}^{s_{j+1}-1}.
\]

Which indicates,
\[
\left\{ U(q_{s_i}^{0,\Delta S} - t_i) + (r-c)t_i + cq_{s_i}^{0,\Delta S} - P_{s_i}^{0}\Delta S \right\}_{i=s_{j+1}}^{s_{j+1}-1} \\
\geq L \left\{ U(q_{s_i}^{0,\Delta S} - t_i) + (r-c)t_i + cq_{s_i}^{0,\Delta S} - P_{s_i}^{0}\Delta S \right\}_{i=s_{j+1}}^{s_{j+1}-1}.
\]

If \( k < j \), then
\[
\frac{d}{dx} \left\{ U(q_{s_j}^{0,\Delta S} - x) - U(q_{s_k}^{0,\Delta S} - x) \right\} = r(F(q_{s_j}^{0,\Delta S} - x) - F(q_{s_k}^{0,\Delta S} - x)) \geq 0.
\]

Therefore, for \( i \in \{s_j, s_j + 1, \ldots, s_{j+1} - 1\} \), we have
\[
U(q_{s_i}^{0,\Delta S} - t_i) + cq_{s_i}^{0,\Delta S} - P_{s_i}^{0}\Delta S - [U(q_{s_i}^{0,\Delta S} - t_i) + cq_{s_i}^{0,\Delta S} - P_{s_i}^{0}\Delta S] \\
\geq U(q_{s_i}^{0,\Delta S} - t_i) + cq_{s_i}^{0,\Delta S} - P_{s_i}^{0}\Delta S - [U(q_{s_i}^{0,\Delta S} - t_i) + cq_{s_i}^{0,\Delta S} - P_{s_i}^{0}\Delta S] \geq 0.
\]

Which indicates,
\[
\left\{ U(q_{s_i}^{0,\Delta S} - t_i) + (r-c)t_i + cq_{s_i}^{0,\Delta S} - P_{s_i}^{0}\Delta S \right\}_{i=s_{j+1}}^{s_{j+1}-1} \\
\geq L \left\{ U(q_{s_i}^{0,\Delta S} - t_i) + (r-c)t_i + cq_{s_i}^{0,\Delta S} - P_{s_i}^{0}\Delta S \right\}_{i=s_{j+1}}^{s_{j+1}-1}.
\]

Finally, it is easy to check for \( i \in \{s_j, s_j + 1, \ldots, s_{j+1} - 1\} \) it holds,
\[
U(q_{s_i}^{0,\Delta S} - t_i) + (r-c)t_i + cq_{s_i}^{0,\Delta S} - P_{s_i}^{0}\Delta S \\
\geq U(q_{s_i}^{0,\Delta S} - t_i) + (r-c)t_i + cq_{s_i}^{0,\Delta S} - P_{s_i}^{0}\Delta S \geq 0.
\]

Hence, the theorem is proved.
Appendix I. Proof of Theorem 3.10. It suffices to prove the theorem for a one-step refinement $\Delta \bar{S}$. That is, $\Delta \bar{S}$ is obtained by adding exactly one more cut point to $\Delta S$. Assume another cut point $s_0$ is added to $\{s_1, \ldots, s_m\}$ and $s_k < s_0 < s_{k+1}$.

It is easy to check that $\Pi_{i}^{P_s, \Delta \bar{S}} = \Pi_{i}^{P_s, \Delta S}$ for $i < s_0$, and

$$\Pi_{s_0}^{P_s, \Delta \bar{S}} = U(q^*) + \sum_{i=1}^{k-1} [U(q^*) - U(q^* + t_{s_i} - t_{s_{i+1}})] + [U(q^*) - U(q^* + t_{s_k} - t_{s_0})] + (r - c)t_{s_1}$$

$$> U(q^*) + \sum_{i=1}^{k-1} [U(q^*) - U(q^* + t_{s_i} - t_{s_{i+1}})] + (r - c)t_{s_1}$$

$$= \Pi_{s_k}^{P_s, \Delta \bar{S}} = \Pi_{s_0}^{P_s, \Delta S}$$

Hence,

\[ \left\{ \Pi_{i}^{P_s, \Delta \bar{S}} \right\}_{i=1}^{n} \geq L \left\{ \Pi_{i}^{P_s, \Delta S} \right\}_{i=1}^{n} . \]

Also, through directly computing we have

$$\text{Regret}_{n}^{P_s, \Delta \bar{S}} - \text{Regret}_{n}^{P_s, \Delta S}$$

$$= U(q^* + t_{s_k} - t_{s_0}) - U(q^* + t_{s_k} - t_{s_{k+1}}) - [U(q^*) - U(q^* + t_{s_0} - t_{s_{k+1}})]$$

Note that

$$\frac{d}{dx} [U(x) - U(x + t_{s_0} - t_{s_{k+1}})] = r [F(x + t_{s_0} - t_{s_{k+1}}) - F(x)] \leq 0;$$

$$r [F(q^* + t_{s_0} - t_{s_{k+1}}) - F(q^*)] < 0.$$ 

Hence

$$\text{Regret}_{n}^{P_s, \Delta \bar{S}} - \text{Regret}_{n}^{P_s, \Delta S} > 0,$$

which means that

$$\left\{ \text{Regret}_{i}^{P_s, \Delta \bar{S}} \right\}_{i=1}^{n} \geq L \left\{ \text{Regret}_{i}^{P_s, \Delta S} \right\}_{i=1}^{n} .$$

Finally, it is easy to see that $q_{s_j}^{o, \Delta \bar{S}} = q_{s_j}^{o, \Delta S}$ for $j > k$, and

$$U(q_{s_k}^{o, \Delta \bar{S}} - t_{s_{k+1}}) + (r - c)t_{s_{k+1}} = U(q_{s_k}^{o, \Delta S} - t_{s_{k+1}}) + (r - c)t_{s_k};$$

$$U(q_{s_k}^{o, \Delta \bar{S}} - t_{s_k}) + (r - c)t_{s_k} = U(q_{s_k}^{o, \Delta S} - t_{s_{k+1}}) + (r - c)t_{s_0};$$

$$U(q_{s_k}^{o, \Delta \bar{S}} - t_{s_0}) + (r - c)t_{s_0} = U(q_{s_k}^{o, \Delta S} - t_{s_0}) + (r - c)t_{s_k}.$$ 

The first equality minus other two equalities leads to

$$U(q_{s_k}^{o, \Delta \bar{S}} - t_{s_{k+1}}) - U(q_{s_k}^{o, \Delta S} - t_{s_0})$$

$$= U(q_{s_k}^{o, \Delta \bar{S}} - t_{s_{k+1}}) - U(q_{s_k}^{o, \Delta S} - t_{s_{k+1}}).$$

Note that $q_{s_0}^{o, \Delta \bar{S}} > q_{s_k}^{o, \Delta \bar{S}}$ and

$$\frac{d}{dx} \{U(x - t_{s_{k+1}}) - U(x - t_{s_0})\} = r \{F(x - t_{s_0}) - F(x - t_{s_{k+1}})\} \geq 0,$$

$$r \{F(q_{s_0}^{o, \Delta \bar{S}} - t_{s_0}) - F(q_{s_k}^{o, \Delta \bar{S}} - t_{s_{k+1}})\} > 0.$$ 

Hence,

$$U(q_{s_k}^{o, \Delta \bar{S}} - t_{s_{k+1}}) - U(q_{s_k}^{o, \Delta S} - t_{s_0})$$

$$> U(q_{s_k}^{o, \Delta \bar{S}} - t_{s_{k+1}}) - U(q_{s_k}^{o, \Delta S} - t_{s_0}).$$
Which indicates \( q_{sk}^{a,S} > q_{sk}^{b,S} \). Note that \( q_{si}^{a,S} \) and \( q_{si}^{b,S} \) (for \( i = 1 \) to \( k \)) satisfy

\[
U(q_{sj} - t_{sj+1}) + (r - c)t_{sj+1} = U(q_{sj+1} - t_{sj+1}) + (r - c)t_{sj},
\]

Therefore, \( q_{si}^{a,S} > q_{si}^{b,S} \) for \( i = 1 \) to \( k \), and

\[
\Pi_{i}^{R,s,\Delta S} = U(q_{si}^{a,S} - t_{s_i}) + (r - c)t_{s_i},
\]

\[
< U(q_{si}^{b,S} - t_{s_i}) + (r - c)t_{s_i} = \Pi_{i}^{R,s,\Delta S};
\]

\[
\text{Regret}_{i}^{R,s,\Delta S} = U(q^{\ast}) - U(q_{si}^{a,S} - t_{s_i})
\]

\[
< U(q^{\ast}) - U(q_{si}^{b,S} - t_{s_i}) = \text{Regret}_{i}^{R,s,\Delta S}.
\]

Which indicates

\[
\left\{ \Pi_{i}^{R,s,\Delta S} \right\}_{i=1}^{n} \geq L \left\{ \Pi_{i}^{R,s,\Delta S} \right\}_{i=1}^{n};
\]

\[
- \left\{ \text{Regret}_{i}^{R,s,\Delta S} \right\}_{i=1}^{n} \geq L - \left\{ \text{Regret}_{i}^{R,s,\Delta S} \right\}_{i=1}^{n}.
\]

Hence, the theorem is proved. \( \square \)

**Appendix J. Proof of Theorem 3.11.** (1) Since \( t_{i} - t_{j} \leq q^{\ast} - x_{1} \) for \( i > j \), we have

\[
U(q^{\ast}) - U(q^{\ast} - t_{i} + t_{j}) = \int_{0}^{t_{i}-t_{j}} \frac{dU}{dq}(q^{\ast} - t_{i} + t_{j} + x)dx
\]

\[
= \int_{0}^{t_{i}-t_{j}} \{r[1 - F(q^{\ast} - t_{i} + t_{j} + x)] - c\}dx
\]

\[
\leq \int_{0}^{t_{i}-t_{j}} \{r[1 - F(x_{1})] - c\}dx = \frac{r - c}{2}(t_{i} - t_{j}).
\]

Assume \( s_{j} \leq i \leq s_{j+1} - 1 \), then

\[
U(q^{\ast} + t_{i} - t_{i}) + (r - c)t_{i} - \Pi_{i}^{P,s,\Delta S}
\]

\[
= (r - c)(t_{i} - t_{i}) + [U(q^{\ast}) - U(q^{\ast} + t_{i} - t_{i})]
\]

\[
- \Sigma_{k=1}^{j-1} [U(q^{\ast}) - U(q^{\ast} + t_{sk} - t_{sk+1})]
\]

\[
\geq (r - c)(t_{i} - t_{i}) - \frac{r - c}{2}(t_{i} - t_{i}) - \frac{r - c}{2} \Sigma_{k=1}^{j-1}(t_{sk+1} - t_{sk})
\]

\[
= \frac{r - c}{2}(t_{i} - t_{i}) \geq 0.
\]

(2) Since \( t_{i+1} - t_{i} \geq q^{\ast} \), thus \( U(q^{\ast} - t_{i+1} + t_{i}) = (r - c)(t_{i} - t_{i+1}) \), and for \( i \geq 2 \) it holds

\[
\Pi_{i}^{P,s} - U(q^{\ast} + t_{i} - t_{i}) - (r - c)t_{i}
\]

\[
= iU(q^{\ast}) + (r - c)(t_{i} - t_{i}) - (r - c)iq^{\ast}
\]

\[
\geq iU(q^{\ast}) + (r - c)(i - 1) \left\{ 2q^{\ast} - \frac{2U(q^{\ast})}{r - c} \right\} - (r - c)iq^{\ast}
\]

\[
= (i - 2) \{(r - c)q^{\ast} - U(q^{\ast})\} \geq 0.
\]

And it is easy to check \( \Pi_{i}^{P,s} - U(q^{\ast} + t_{1} - t_{1}) - (r - c)t_{1} = 0 \). Hence, the theorem is proved. \( \square \)
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Received October 2017; revised May 2018.

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