Low-mass dileptons from nonequilibrium QGP

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Abstract
The rate of the emission of the high energy low-mass dileptons from the QGP is found in the first nonvanishing order with respect to strong coupling. We base on the real-time kinetic approach [2] without an explicit assumption about a complete thermal equilibrium in the emitting system. For the class of the partons distributions which may simulate that of the ”hot glue scenario”[1] the rate of emission is found analytically.
1. Most of the results concerning the rate of emission of low-mass dileptons from quark-gluon plasma implicitly assumed that the emitting plasma is in thermal equilibrium. More general consideration is needed in connection with some modern scenario of the QGP formation which predict not complete thermalization of QGP but suppression of the quark component against the background of a hot glue [1]. For this reason we can not use any of standard thermal field approaches and base on the field kinetic technique developed in paper [2].

2. Contrary to other electromagnetic probes of the QGP as heavy dileptons and real photons, the very notion of a low-mass dileptons should be carefully specified. It is not enough to chose the kinematic region as \( E >> M, T \) where the energy \( E \) of the dilepton is much larger than its invariant mass \( M \) and the temperature \( T \). Emission of the virtual photon from the hot nuclear matter is a complicate dynamical phenomenon. The inclusive rate of dilepton emission includes many exclusive channels. Some of them have a threshold behavior. For instance, the first Born’s term. For the Drell-Yan prototype of the dilepton emission the threshold is defined by the minimum \( 2m_q \) of the invariant mass of the annihilating particles. Indeed, the rate of the dilepton emission is given by[2]

\[
\frac{dN_{e^+e^-}}{d^4k d^4x} = \frac{i e_0^2}{2(2\pi)^6} \frac{\pi}{M^2} \sqrt{1 - \frac{4m_l^2}{M^2} \left[ 1 - \frac{1}{3} \left( 1 - \frac{4m_l^2}{M^2} \right) \right]} \theta(M^2 - 4m_l^2) g_{\mu\nu} \Pi_{10}^{\mu\nu}(-k) \tag{1}
\]

where \( m_l \) is a lepton mass. If we denote \( g_{\mu\nu} \Pi_{10}^{\mu\nu}(-k) \) as \( \pi(k) \) and confine calculations to the lowest perturbation order we easily get

\[
\pi_{\text{Born}}(k) = -\frac{3i e_0^2}{4\pi} \left( 1 + \frac{2m^2}{M^2} \right) \sqrt{1 - \frac{4m^2}{M^2}} M^2 e^{-k/\theta} \theta(M^2 - 4m^2) \tag{2}
\]

The square root here is typical for many thresholds in quantum mechanics. If \( M^2 < 4m^2 \) then initial quarks are too heavy and the emission of the dilepton is possible only due to the processes of the next perturbative orders. How shall we proceed if we want to deal only with small masses: low-mass virtual photons and massless quarks? For real photons the answer is evident. Two polarizations of a real photon are consistent only with \( M = 0 \) and the Born’s term is absent.

3. To find out if the Born’s term is present in the dilepton production by the massless quarks we are to compare the dilepton mass with some scale inherent to the emitting system.

In the previous papers [3,4] we considered emission of the heavy dileptons and photons and found that the zero quark masses are potentially dangerous only in collinear geometry of the emission process. Indeed, for the hard quarks the influence of the rest mass onto the energy balance is negligible. The finite mass works as a geometric parameter, the Compton length \( l_c = 1/m_i \) thus restricting the domain of
the interaction. In the media this role is played by the minimal length $l_{fs}$ defined by the amplitude of the forward scattering. If we demand that $E \gg M, T$ then the initial quarks should be really hard and the estimate $l_{fs} \sim 1/gT \sim 1/m_{\text{therm}}$ is very reliable. A potential danger hidden in the collinear geometry of annihilation emerges from the low relative momentum of the $q\bar{q}$-pair because in this case the Coulomb interaction is not a perturbative effect. But it does not seem to be important in neutral plasma at high temperatures.

So the boundary between low-mass and heavy dileptons lies at $M \sim 2m_{\text{therm}}$. Above this threshold the Born’s term (the direct $q\bar{q}$-annihilation) very quickly grows up and dominates over the next order contributions until $M/T$ reaches value about 10 $3$.4. The threshold $k^2 = 4m^2$ of the Born’s term is that for the one-loop radiative corrections. Below it they have nothing to interfere with. So only “real” processes contribute the rate of emission. In this case the trace of the electromagnetic polarization tensor reads as $3$

$$\pi_{\text{real}} = -\frac{ie^2 g^2 N_c C_F}{2\pi^5} \int d^4p d^4q \delta(q^2) \delta[(p-k)^2 - m^2] \delta[(p+q)^2] (SW_a + SW_c) \times$$

$$\times \left\{ 1 + \frac{A}{(p^2 - m^2)^2} + \frac{B}{p^2 - m^2} + \frac{C}{p^2 - m^2 + 2kq} \right\}$$

where we denoted:

$$A = 2m^2(k^2 + 2m^2), \quad B = 3m^2 + k^2 + 2(kq) - \frac{4m^4 - (k^2)^2}{2(kq)}$$

$$C = (k^2 + m^2) + \frac{4m^4 - (k^2)^2}{2(kq)}$$

The expression in the curly brackets is a sum of the squared moduli of the matrix elements of the annihilation process, $q\bar{q} \to g\gamma$, or Compton process, $qg \to q\gamma$ and $\bar{q}g \to \bar{q}\gamma$. Specification of the process is due to statistical weights $SW_{a,c}$.

As in the previous papers $2,3,4$ we will not assume that quarks and gluons are in thermal equilibrium. Instead we shall use the Boltzmann-like distributions with fugacities $\zeta_Q$ and $\zeta_G$ which all together will be considered as the measure of the chemical and kinetic nonequilibrium in the quark-gluon system. Then statistical weight of the annihilation process with the emission of a gluon looks as

$$SW_{em} = \theta(k_0 - p_0)\theta(q_0 + p_0)\theta(q_0)n_F(k_0 - p_0)n_F(q_0 + p_0)[1 + n_B(q_0)] \approx \theta(k_0 - p_0)\theta(q_0 + p_0)\theta(q_0)\zeta_Q^2 e^{-(ku)/T} e^{-(qu)/T} + \zeta_G e^{-2(qu)/T}$$

$$= e^{-(ku)/T} e^{-(qu)/T} + \zeta_G e^{-2(qu)/T}$$
For the Compton rate of the dilepton emission the statistical weight equals to
\[
SW_{\text{com}} = -\theta(p_0 + q_0)\theta(p_0 - k_0)\theta(-q_0)n_F(p_0 + q_0)[1 - n_F(p_0 - k_0)]n_B(-q_0) - \\
-\theta(-p_0 - q_0)\theta(k_0 - p_0)\theta(-q_0)n_F(k_0 - p_0)[1 - n_F(-p_0 - q_0)]n_B(-q_0) - \approx \\
\approx -\zeta Q\zeta_G\{\theta(p_0 + q_0)\theta(p_0 - k_0)\theta(-q_0)e^{-\frac{p(T)}{M}}[1 - \zeta Qe^{-(p_0-k_0)/2T}] + \\
+\theta(-p_0 - q_0)\theta(k_0 - p_0)\theta(-q_0)e^{-\frac{k(T)}{M}}e^{(q_0-p_0)/2T}[1 - \zeta Qe^{(q_0-p_0)/T}]\} \quad (6)
\]

Then we can perform an exact integration over \(p\) using the Breit reference system where \(k + q = 0\), c.m.s. of the reaction \(q\overline{q} \rightarrow g\gamma\). This immediately leads to
\[
\pi_{\text{ann}} = -\frac{i\epsilon^2 g^2 N_C F}{4\pi^4} \zeta Q e^{-\frac{k(T)}{M}} \int d^4q \delta(q^2)\theta(q_0)\theta[(k+q)^2 - 4m^2]\{e^{\frac{q(T)}{M}} + \xi_G e^{-2\frac{q(T)}{M}}\} F_a(kq; M^2)
\]
\[\quad \text{where} \]
\[
F_a(x; M^2) = (1 + \frac{M^2 + 2m^2}{x} + \frac{M^4 - 4m^4}{2x^2}) \ln \left(1 - \sqrt{1 - 4m^2/(M^2 + 2x)} \right) \left(1 + \sqrt{1 - 4m^2/(M^2 + 2x)} \right)
+ (1 + \frac{M^2 + 2m^2}{x} + \frac{M^2(M^2 + 2m^2)}{2x^2}) \sqrt{1 - \frac{4m^2}{M^2 + 2x}} \quad (8)
\]

For the Compton rates we start with the chain of changes of variables, \(p \rightarrow -p + k - q\), in the first term and \(q \rightarrow -q - p\) in both terms we may also easily perform the integration over \(p\) using the same Breit reference system. It gives
\[
\pi_{\text{compt}} = -\frac{i\epsilon^2 g^2 N_C F}{4\pi^4} \zeta Q \zeta_G e^{-\frac{k(T)}{M}} \int d^4q \delta(q^2 - m^2)\theta(q_0)[e^{-\frac{q(T)}{M}} - \zeta Q e^{-2\frac{q(T)}{M}}] F_c(kq; M^2)
\]
\[\quad \text{where q is the momentum of the (anti)quark in the final state and} \]
\[
F_c(x; M^2) = -\frac{4m^2 + M^2 - 2x - 2(M^4 - 4m^4)/(M^2 + 2x)}{2\sqrt{x^2 - M^2m^2}} \ln \frac{m^2 + x + \sqrt{x^2 - M^2m^2}}{m^2 + x - \sqrt{x^2 - M^2m^2}}
+ \frac{4(M^2 + 2m^2)}{M^2 + 2x} + \frac{(M^2 + 2x)(m^2 + x)}{(M^2 + m^2 + 2x)^2} \quad (10)
\]

5. Further integration, first angular, is easily performed in the rest frame of the dilepton were \(k = 0\). Having expressed the result in the invariant form and using \((k_0, k)\) as the components of 4-vector \(k\) in the rest frame of the emitting media we get
\[
\pi_a = \frac{i\epsilon^2 g^2 N_C F}{2\pi^3} \zeta Q e^{-\frac{k(T)}{M}} M^2 \int_{(4m^2/M^2-1)/2}^{\infty} [S(k, y) + \zeta_G S(k, 2y)] F_a(M^2y) y dy \quad (11)
\]
The nonzero lower limit has appeared because a dilepton production without emission of a gluon is impossible in this kinematic region. In sequence, we need do radiative corrections to eliminate the IR singularity at zero momentum of the emitted gluon. The terms with the extra factor $\zeta_G$ relate to the induced emission of a gluon and we denoted,

$$S(k, y) = e^{-k_0/T} \frac{\sinh(|k|y/T)}{|k|y/T}$$  \hspace{1cm} (12)

The term with the extra factor $\xi_G$ relates to the induced emission of a gluon.

If we are interested only in the distribution of the dileptons over their invariant masses we are to follow Eq. (1) and get function

$$U_a(y) = \int \frac{d^3k}{2k_0} e^{-k_0/T} S(k, y) = 2\pi M^2 \frac{K_1(M\sqrt{1+2y/T})}{M\sqrt{1+2y/T}} \approx 2^{-1}(2\pi)^{3/2} M^{1/2} T^{3/2} (1 + 2y)^{-3/4} e^{-M\sqrt{1+2y/T}}$$  \hspace{1cm} (13)

It can be first integrated over the history of the system (if necessary, with fugacities $\zeta$) and only then substituted into the integrands of Eq.(11).

For the Compton rate, proceeding as above, we get

$$\pi_{compt} = -\frac{ie^2 g^2 N_c C_F}{2\pi^3} \xi_Q \zeta_Q e^{-k_0/T} m^2 \int_0^\infty \frac{x^2 dx}{x_0} \mathcal{F}_c(x)[C(x, k, T) - \xi_Q C(x, k, T^2)]$$  \hspace{1cm} (14)

where $x_0 = \sqrt{1+x^2}$ and

$$C(x, k, T) = e^{-k_0 m x_0} \frac{\sinh(x)}{x} \left(\frac{|k|m}{MT} x\right)$$  \hspace{1cm} (15)

The term with the extra factor $\xi_Q$ reflects the Pauli suppression of the quark in the final state of the Compton process.

Again, if we want to get only the spectrum of invariant masses we should use a function

$$U_c(x, T/b) = \int \frac{d^3k}{2k_0} e^{-k_0/T} C(x, T/b) = 2\pi M^2 \frac{K_1(Z)}{Z}$$

$$Z = \frac{1}{T} \sqrt{M^2 + m^2 b^2 + 2m Mb x_0}$$  \hspace{1cm} (16)

which may be separately integrated over the history of the emitting plasma.

6. Having reached our goal to present the expression for the rates of emission in the form of simple quadratures we can easily find them numerically. We can analyze the reliability of different analytic approximations and compare them with those known in the literature.
At Fig. 1 we present dependence (14) and (16) of the rate of emission of the low mass dileptons as a function of total momentum $|k|$ at a given mass $M$. We consider the annihilation (solid lines) and Compton (dashed lines) channels separately and apart from the common factor $10^{-9}\exp(-E/T) \approx 10^{-9}\exp(-|k|/T)$. The fugacities are taken $\zeta_Q = \zeta_G = 1$. (The actual values of the fugacities which are the measure of the chemical equilibrium are about $\zeta_Q \sim 0.5$ and $\zeta_G \sim 0.75$ [1].)

The limit of the very low masses, is of a special interest. The formal limit of $M \rightarrow 0$ immediately reproduces polarization loop contribution to the real photon emission [4],

$$
\pi_{\text{ann}} = \frac{ie^2 g^2 N_c F}{2\pi^3} T^2 e^{-ku/T} \zeta_Q^2 \int_0^\infty dy \left[ e^{-y} + \frac{\zeta_G}{2} e^{-2y} \right] \times
\left[ (1 + \frac{\xi}{y} - \frac{\xi^2}{2y^2}) \cosh^{-1} \sqrt{\frac{y}{\xi}} + (1 + \frac{\xi}{y}) \sqrt{1 - \frac{\xi}{y}} \right] (17)
$$

$$
\pi_{\text{compt}} = -\frac{ie^2 g^2 N_c F}{2\pi^3} e^{-ku/T} T^2 \zeta_Q \zeta_G \int_0^\infty dy \left[ e^{-(y+\frac{\lambda^2}{4y})} - \frac{\zeta_Q}{2} e^{-2(y+\frac{\lambda^2}{4y})} \right] \times
\left[ (1 - \frac{\xi}{y} - \frac{\xi^2}{2y^2}) \ln(1 + \frac{4y}{\xi}) + \frac{2\xi}{y} + \frac{4y(\xi + 2y)}{(\xi + 4y)^2} \right] (18)
$$

where $\xi = m^2/(ku)T$, $\lambda = m/T$. For small but finite dilepton mass these equations are exact up to the terms of the order $M/m \sim M/gT << 1$.

Because of the pre-factor from Eq.(1) which is strongly contributed by the longitudinal mode and has a threshold at $M = 2m_t$ we can not reproduce a literal transition to the photon rate. Moreover, for the very polarization tensor this transition is nonanalytic and one needs a care when approaching this limit in numerical calculations.

There is some mystery in regularization of the collinear singularity by means of the very dilepton mass [5]. Physically it would have meant that the virtuality of the intermediate photon restricts the domain of coherence of quark-gluon and quark-photon interaction, which is hardly in line with the electromagnetic transparency of the quark-gluon plasma.

Mathematically, to achieve this regularization, the virtual mass and vertex radiative corrections were taken into account. But they unavoidably contain the trivial IR singularities at low gluon momenta (some of them with the extra powers of fugacities) [2]. These should be cancelled out by the emission and even absorption of the real soft gluons but for low-mass dileptons neither the emitted gluons can be too soft nor they can be absorbed.

7. Conclusion. In the limit of the low masses, the dilepton rate of emission can be expressed as a trace of the electromagnetic polarization tensor at $M = 0$ (Eqs.(17)
and (18)) times kinematic factor from the Eq.(1). In the approximation of the leading logarithm and at $M >> 2m_t$ it looks as

$$\frac{dN_{e^+e^-}}{d^4k d^4x} = \frac{5\alpha^2 \alpha_s}{27\pi^5} \frac{T^2}{M^2} \left(\zeta_Q + \zeta_G\right) e^{-ku/T} \ln \frac{4(ku)T}{m^2_{therm}}.$$  \hspace{1cm} (19)

and there is no Born’s term which accompany it (compare [5]). In virtue of the analysis of the accuracy of this approximation [4] which was performed for the real photons this formula essentially underestimate the rate of emission. We recommend to use a simple integral representations (11) and (13) instead of it.

We consciously do not integrate the rates over the history of the system which can be easily done for a simple form of the hydrodynamic background. The very approach of nonequilibrium field kinetics [2,3] was designed in order to use the nonequilibrium partons distributions generated in a ”realistic” cascade.

I am indebted to G. Brown, E. Shuryak and the Nuclear Theory group at SUNY at Stony Brook for continuous support.
I am grateful to E. Shuryak and I.Zahed for many fruitful and helpful discussions.

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