Comment on Vacuum Stability and Electroweak Baryogenesis in the MSSM with Light Stops.

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Abstract

We show that, for all values of $Tan\beta$ and the light right-handed stop mass for which the electroweak phase transition is strong enough to avoid washout following electroweak baryogenesis, the electroweak vacuum is stable over the lifetime of the observed Universe. Cosmological stability of the electroweak vacuum is violated only if the light right-handed stop is lighter than 100-115GeV.
The possibility of producing the baryon asymmetry of the Universe at the electroweak phase transition \[1, 2\] in the minimal supersymmetric standard model (MSSM) has been the subject of much study in recent times \[3, 4, 5\]. It has become clear that although the MSSM can readily provide the CP violation necessary for baryon asymmetry generation \[4\], it has more difficulty producing a sufficiently strong first-order phase transition to prevent subsequent wash-out of the asymmetry \[5\]. One possibility for having a Higgs mass consistent with electroweak baryogenesis is associated with the rather special range of parameters for which the soft SUSY breaking mass squared parameter of the mostly right-handed (r.h.) stop partially cancels against the finite-temperature contribution to its mass, in order that the contribution from the Higgs expectation value dominates the r.h. stop mass during the electroweak phase transition \[6, 7\]. (It has also been recently suggested that the inclusion of two-loop QCD thermal corrections to the effective potential of the Higgs field may sufficiently increase the strength of the electroweak phase transition to evade wash-out, even with a positive mass squared for the r.h. stops \[8, 9\]). Since this requires a negative soft SUSY breaking mass squared for the r.h. stop, there will typically be a second minimum of the effective potential with a non-zero stop expectation value. For the values of the parameters of the model which give the largest possible Higgs mass consistent with electroweak baryogenesis, this vacuum is generally of lower energy than the electroweak vacuum. In this case it is important to consider the cosmological stability of the electroweak vacuum i.e. whether it is longer lived than the observed Universe. In a previous analysis \[7\] it was shown that the upper bound obtained on the Higgs mass in the MSSM is strongly dependent on whether or not it is possible to have a sufficiently long-lived metastable electroweak vacuum. If it is not possible, then the upper bound obtained from the 1-loop resummed finite-temperature effective potential is around 80GeV \[7\]. On the other hand, if we can have a metastable vacuum over the whole range of parameters for which the electroweak phase transition is consistent with electroweak baryogenesis, then the Higgs mass could be as large as 100GeV. The actual lifetime of the
metastable electroweak vacuum was not, however, calculated. In this short letter we will consider the lifetime of the metastable electroweak vacuum and show that it is indeed cosmologically stable over the range of parameters for which electroweak baryogenesis is possible.

We consider the effective theory consisting of the Standard Model (SM) fields plus light r.h. stops. All other fields will be considered to be too heavy to contribute to the finite-temperature effective potential. This corresponds to the limit of large pseudo-scalar Higgs mass \( m_A \) and heavy left-handed (l.h.) stop with negligible left-right stop mixing, which is known to give the electroweak baryogenesis upper bound on the Higgs mass in the MSSM in this case [4]. The effect of supersymmetry is then to fix the lightest Higgs mass in terms of \( \tan \beta \). At 1-loop the Higgs mass is given by [4]

\[
m_H^2 = m_Z^2 \cos^2 2\beta + \frac{3m_t^4}{4\pi^2 v^2} \ln \left( \frac{m_H^2}{m_T^2} \right) \quad (1),
\]

where \( m_T \) is the mass of the heavy l.h. stop, \( m_t \) is the top quark mass, \( m_t \) is the mass of the light r.h. stop \( (m_t = (-m_u^2 + \lambda_t^2 v^2/2)^{1/2}) \), \( \lambda_t \approx 1 \) is the SM top quark Yukawa coupling and \( v \) is the Higgs vacuum expectation value \( (v = 250\text{GeV}) \). The potential of the Higgs \( (\phi) \) and r.h. stop \( (U) \) field in this effective theory is given by

\[
V(\phi, U) = -\frac{m_\phi^2}{2} \phi^2 + \frac{\lambda}{4} \phi^4 - \frac{m_U^2}{2} U^2 + \frac{\lambda_U}{4} \phi^2 U^2 + \frac{g_3^2}{24} U^4 \quad (2).
\]

(We use canonically normalized real scalar fields throughout). This potential has minima at \( (\phi_o, 0) \) and \( (0, U_o) \), separated by a "ridge" which has a minimum height at a saddle point given by \( (\phi_m, U_m) \), where [4]

\[
\phi_o^2 = \frac{m_\phi^2}{\lambda} \quad (3),
\]

\[
U_o^2 = \frac{6m_U^2}{g_3^2} \quad (4),
\]

\[
\phi_m^2 = \frac{2(m_\phi^2 - \frac{3m_H^2 \lambda_t^2}{g_3^2})}{2\lambda - \frac{3\lambda_t^2}{g_3^2}} \quad (5)
\]

and

\[
U_m^2 = \frac{2(m_u^2 - \frac{m_H^2 \lambda_t^2}{g_3^2})}{g_3^2 - \frac{\lambda_t^2}{\lambda}} \quad (6).
\]
In order to discuss vacuum stability in this model we would generally have to consider vacuum tunnelling with two real scalar fields. However, this is a difficult problem in general, since the simple bounce action approach \cite{10, 11} cannot be applied here \cite{12}. A recent analysis of charge and colour breaking minima in the MSSM approached the problem via a lattice minimization of a modified Euclidean action \cite{12}. In the present paper we will adopt a more direct approach. We will first derive upper and lower bounds on the decay rate using the one scalar field bounce action approach and then argue that, so long as the constraints imposed on the model parameters by these upper and lower bounds are close, we can safely use them to determine whether or not the electroweak vacuum is cosmologically stable.

An upper bound may be found by simply considering the one-dimensional "straight line" potential connecting the electroweak minimum and the stop minimum and then calculating the bounce action for this potential; this amounts to an Ansatz for the minimum action solution, obtained by fixing the value of the orthogonal component of the scalar field and minimizing the remaining action. Since this will not be a true solution of the Euclidean equations of motion, the resulting tunnelling action will be higher than that of the true solution and so we will obtain a lower bound on the vacuum decay rate. In order to obtain an upper bound on the vacuum decay rate, we will adopt the following procedure. In general, the barrier at the saddle point will not be on the straight line connecting the minima and will be lower than the barrier of the straight line potential. Thus if we were to consider a second straight line potential, with the same distance in field space between the electroweak and stop minima and with the same energy splitting between the vacua as for the true straight line potential but with a barrier height equal to that of the saddle point, then we would expect the bounce action for this potential to be smaller than the true bounce action (which would have at least as large a barrier and a longer minimum action path). Thus by considering the bounce action along these two straight line potentials we will obtain upper and lower bounds on the vacuum decay rate. If these upper and lower bounds turn out to be close, then this procedure will give
us a simple and accurate method for estimating the vacuum decay rate.

The true straight line potential is obtained by first defining \( \phi' = \phi_o - \phi \) and then defining a real scalar field \( \rho \) such that \( \phi' = \rho \cos \theta \) and \( U = \rho \sin \theta \), where \( \tan \theta = U_o/\phi_o \). The straight-line potential (which we denote by the subscript 1) is then given by

\[
V_1 = \frac{\alpha_1}{2} \rho^2 - \frac{\beta_1}{2\sqrt{2}} \rho^3 + \frac{\gamma_1}{4} \rho^4 \tag{7}
\]

where

\[
\alpha_1 = 3\lambda \phi_o^2 \phi_\theta^2 + \frac{\lambda^2}{2} \phi_o^2 \phi_\theta^2 - (m_\phi^2 c_\theta^2 + m_\phi^2 s_\theta^2) \tag{8}
\]

\[
\beta_1 = \sqrt{2} \phi_o \phi_\theta (2\lambda \phi_o^2 + \lambda^2 s_\theta^2) \tag{9}
\]

and

\[
\gamma_1 = \lambda c_\theta^4 + \phi_o^2 \phi_\theta^2 \phi_\theta^2 + \frac{g_2^2}{6} s_\theta^4 \tag{10}
\]

\( \Delta V \), the splitting in the potential energy, and \( \rho_o \), the minimum of the potential, are given by

\[
\Delta V = -\left( \frac{\alpha_1}{2} \rho_o^2 - \frac{\beta_1}{2\sqrt{2}} \rho_o^3 + \frac{\gamma_1}{4} \rho_o^4 \right) \tag{11}
\]

and

\[
\rho_o = \frac{1}{4\sqrt{2}\gamma_1} \left( 3\beta_1 + \sqrt{9\beta_1^2 - 32\alpha_1\gamma_1} \right) \tag{12}
\]

We next modify \( \alpha, \beta \) and \( \gamma \) in order to obtain a second potential, \( V_2 \), with the same values of \( \Delta V \) and \( \rho_o \) but with a maximum of the potential equal to the height of the saddle point on the ridge separating the two minima. The saddle point barrier height is given by

\[
V_b = -\frac{3(\lambda_f^2 m_\phi^2 - 2\lambda m_u^2)^2}{4\lambda(2\lambda g_\theta^2 - 3\lambda^2)} \tag{13}
\]

In this case we find that the potential parameters are given by

\[
\alpha_2 = \frac{2}{\rho_o^2} \left( \frac{\gamma_2}{4} \rho_o^4 - 3\Delta V \right) \tag{14}
\]

\[
\beta_2 = \frac{4\sqrt{2}}{\rho_o^3} \left( \frac{\gamma_2}{4} \rho_o^4 - \Delta V \right) \tag{15}
\]

and

\[
\frac{64V_b\gamma_2^3}{\rho_o^4} = (\gamma_2 + y)(\gamma_2 - 3y)^3 \tag{16}
\]
where

$$y = \frac{4\Delta V}{\rho_0^4} \quad (17).$$

We next consider the bounce action for these two potentials. The bounce action is given by

$$S_4 = \frac{2\alpha}{\beta^2} \tilde{S}_4(\bar{\lambda}) \quad (18),$$

where $\tilde{S}_4(\bar{\lambda})$ is the rescaled bounce action \[11\] and

$$\bar{\lambda} = \frac{4\alpha}{\beta^2} \gamma \quad (19)$$

is the scalar self-coupling in the rescaled theory. We find that we can accurately fit $\tilde{S}_4(\bar{\lambda})$ for $\bar{\lambda} \leq 0.8$ by

$$\tilde{S}_4(\bar{\lambda}) \approx a_1 + a_2 e^x + a_3 e^{2x} + a_4 x + a_5 x e^x + a_6 x e^{2x} + a_7 x^2 + a_8 x^2 e^x + a_9 x^2 e^{2x} \quad (20),$$

where the values of the $a_i$ are given in Table 1. In practice, for the cosmologically unstable vacuum of interest we find that $\bar{\lambda} \approx 0.6$. (The field at the centre of the corresponding true vacuum bubble, from which the bounce solution may be readily obtained \[11\], is given by $\psi_0(\bar{\lambda}) \approx 5.78 - 2.77\bar{\lambda} - 0.91\bar{\lambda}^2 - 0.24\bar{\lambda}^3$).

Cosmological stability of the electroweak vacuum requires that $S_4 \gtrsim 400$ \[14\]. This constraint is then compared with the bounce action obtained for the case where (i) the electroweak phase transition is consistent with electroweak baryogenesis and (ii) we arrive in correct final vacuum state. The first condition requires that the value of the scalar field at the end of the phase transition is large enough to suppress the sphaleron rate and so prevent washout of the asymmetry (we refer to this as the "no washout" constraint) \[1, 2, 13, 15\]. This requires that at the end of the electroweak phase transition we have

$$\frac{\phi_+(T_1)}{T_1} \gtrsim 1 \quad (21),$$

where $\phi_+(T_1)$ is the value of $\phi(T)$ at the non-zero minimum of the effective potential, with the phase transition essentially occurring as soon as the minima of the finite-temperature effective potential become degenerate at $T_1$ \[13\]. The second condition
requires that the finite-temperature effective potential along the light stop direction is stable at the temperature of the electroweak phase transition, in order that there is no phase transition to the squark minimum (we refer to this as the "no squark transition" constraint) [7].

The no washout, no squark transition and vacuum stability constraints result in upper and lower bounds on the r.h. stop mass $m_{\tilde{t}}$ for a given $Tan\beta$, as given in Table 2. In calculating these bounds we considered the now standard 1-loop resummed finite temperature effective potential [13, 15], for the particular case of the SM plus light r.h. stops and for values of $Tan\beta$ corresponding to $m_H$ greater than the experimental lower bound, $m_H \gtrsim 65 GeV$ [16]. (We have fixed $m_{\tilde{t}} = 500 GeV$ in equation (1) throughout). The no squark transition constraint imposes a lower bound on $m_{\tilde{t}}$. In calculating this lower bound we considered the case where the $T^3$ term in the finite-temperature effective potential along the stop direction includes fully the contribution of the squarks and Higgs, corresponding to the case where their finite-temperature effective masses are dominated by the $U$ expectation value [7]. The no washout constraint imposes an upper bound on $m_{\tilde{t}}$. The upper bound on the Higgs mass then corresponds to the value of $Tan\beta$ at which these upper and lower bounds come together. We find that this occurs at $Tan\beta \approx 4.5$, corresponding to $m_H \approx 92 GeV$. (This is in broad agreement with reference [7]; they also allow for light gauginos, which we have not considered here). In addition, we give in Table 2 the lower bound on $m_{\tilde{t}}$ coming from the condition that the electroweak vacuum is an absolutely stable global minimum of the effective potential, as would be necessary if the metastable electroweak vacuum were not cosmologically stable. This imposes an upper bound on the Higgs mass, $m_H \approx 81 GeV$. We also give the lower bound on $m_{\tilde{t}}$ coming from the requirement that the electroweak vacuum is cosmologically stable. In fact we obtain two bounds, coming from $V_1$ and $V_2$, which turn out to be very close in practice. From Table 2 we see that the lower bound on $m_{\tilde{t}}$ coming from the requirement of cosmological stability of the vacuum is much smaller than the range permitted by the no washout and no squark transition conditions. Typically, the
electroweak vacuum in the effective theory consisting of the SM plus light r.h. stops becomes cosmologically unstable only for \( m_{\tilde{t}} \) less than 100-115GeV, depending on the value of \( \tan \beta \). (The present lower bound on the stop mass is \( m_{\tilde{t}} \gtrsim 70 \text{GeV} \) \[17\]).

Observation of a light, mostly r.h. stop of mass less than 100GeV together with a light Higgs of mass less than about 100GeV would therefore require new physics at the weak scale in order to stabilize the electroweak vacuum. (It is also possible to have a light stop without having a negative mass squared for the r.h. stop, but this would require a large mixing of left and right-handed stops).

From this we may conclude that the electroweak vacuum, although typically metastable for the case of light r.h. stops, is nevertheless cosmologically stable over the whole range of \( m_H \) and \( m_{\tilde{t}} \) consistent with electroweak baryogenesis. Therefore the upper bound on the Higgs mass should be taken to be that imposed by the no squark transition constraint and not that coming from the requirement of absolute stability of the electroweak vacuum. This should allow the light Higgs to have a mass of 90GeV or more without conflicting with electroweak baryogenesis.

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Table 1. Coefficients of the bounce action fit

| $a_1$ | $1.07645192933387 \times 10^{10}$ | $a_2$ | $-1.728837410146912 \times 10^{10}$ |
|-------|-------------------------------|-------|----------------------------------|
| $a_3$ | $6.523854899000699 \times 10^9$ | $a_4$ | $4.29228013770597 \times 10^9$   |
| $a_5$ | $2.250364456777935 \times 10^9$ | $a_6$ | $-2.302628340489138 \times 10^9$ |
| $a_7$ | $5.23330738163529 \times 10^8$  | $a_8$ | $-2.806865631829403 \times 10^9$ |
| $a_9$ | $2.349084983026382 \times 10^8$ |       |                                  |

Table 2. Upper and Lower Bounds on $m_\tilde{t}$ as a function of $\tan\beta$.

The upper bound is from the no washout constraint. The lower bounds are from the no squark transition constraint, from the condition of absolute electroweak vacuum stability ("stability") and from the condition for the electroweak vacuum to be cosmologically stable as calculated using $V_1$ and $V_2$ respectively. The 1-loop values of $m_H$ as a function of $\tan\beta$ are also given, using the no washout value of $m_H$. All masses are in GeV.

| $\tan\beta$ | $m_H$  | $m_\tilde{t}$ (no washout) | $m_\tilde{t}$ (no squark transition) | $m_\tilde{t}$ (stability) ($V_1$) | $m_\tilde{t}$ (stability) ($V_2$) | $m_\tilde{t}$ (cosmological stability) |
|-------------|--------|-----------------------------|--------------------------------------|-----------------------------------|-------------------------------------|---------------------------------------|
| 1.7         | 65.4   | 172.9                       | 155.1                                | 158.1                             | 113.1                               | 116.3                                 |
| 2.6         | 81.0   | 153.3                       | 147.0                                | 153.3                             | 107.4                               | 108.1                                 |
| 3.0         | 84.9   | 148.2                       | 145.0                                | 151.8                             | 105.8                               | 106.2                                 |
| 4.5         | 92.4   | 139.8                       | 139.8                                | 149.5                             | 102.7                               | 102.7                                 |
| 10.0        | 97.7   | 132.8                       | 136.1                                | 148.2                             | 100.3                               | 100.3                                 |