The influence of load distribution along a line on power and place of connection of a photovoltaic system

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Abstract. Distributed generation in the electrical network causes a redistribution of power and leads to a reduction in technological losses and a decrease in voltage deviations for consumers. The installation of additional generators of electric energy based on the use of wind generators and photovoltaic systems has become widespread in the world energy industry, and has now received development in the Russian Federation. It is shown that the connection of an additional source based on photovoltaic systems can reduce losses in a supply line. According to results of the analysis of a possible location of the connection of an additional power supply on a line with a uniformly decreasing or increasing load, it is proved that the greatest reduction in the level of losses in a line is achieved when additional power supply is connected to it in the case of increase in the load from the beginning to the end. Analytical expressions for determination of economically expedient capacities and a place of connection of an additional source for linearly decreasing or increasing specific load of a supply line are presented. There was determined the optimal power of an additional power source in the form of a photovoltaic system and the optimal point of its connection on a line to reduce power losses during power transmission.

1. Introduction
Power losses in power supply and distribution lines are one of the important criteria used to assess the efficiency of a network. Traditionally, to reduce energy losses, attention must be paid to the rational construction of a network circuit, increasing the cross section of conductors, reactive power compensation, etc. [1, 2]. Nevertheless, modern conditions dictate new tasks for design and operation of power supply networks, for which the government stimulates the development of distributed generation, placing ever higher demands on environmental and energy security [3].

Distributed generation, using a variety of energy sources, including photovoltaic systems, is one of the ways of energy development [4–6].

When connecting photovoltaic systems to the electrical network, it is necessary to solve problems with the parameters of the lines, such as determining a rational point of connection, voltage fluctuations, and power and energy losses. The photovoltaic system is usually connected to the low voltage side of the supply substation or to the consumer's supply input. In this case, there may be difficulties with the placement of a photovoltaic system or when it is split into a larger number of individual installations.
The source of stabilized supply is frequently used on the basis of the inverter with electric current output for connection to power distribution networks of a photoelectric system and [7]. It generates energy into the network, the parameters of the photovoltaic system are coordinated with the distribution network [8, 9]. The option of connecting a photovoltaic generating unit to power the load is shown in Figure 1.

**Figure 1.** Block diagram for connecting a photovoltaic system to power the load.

In the diagram proposed in Figure 1, an additional source (photovoltaic system) is connected to one of the loads (Load, $i$), and not to the beginning of the power supply line for the loads Load, 1, Load, 2, ... Load, $N$. Such connection provides advantages: reduction of losses in the supply line and equalization of voltages at consumers connected to the line.

To determine the optimal power and the connection point of the additional source with the uniform distribution of the synchronously changing load along the line, a technique was proposed [3].

But the distribution of loads along the line can be uneven. In this paper we analyze the effect of load distribution, linearly decreasing or increasing, on power and place of connection of the photovoltaic system $P_{Aac}$, as shown in Figure 2.

**Figure 2.** Rated scheme of a feeder with linear distribution of active power of consumers: a – decreasing load, b – increasing load.

We believe that the photovoltaic system is an additional source of energy of the active power $P_{Aac}$ replaced by a current source and it needs to be installed at a distance of $L_{Aac}$ from the beginning of the line. Let’s determine the optimal value of the additional source of active power $P_{Aac}$ and distance $L_{Aac}$ if the efficiency of the installation of an additional source to estimate the amount of power loss reduction in the line.
2. Method

We have (Figure 2) for distribution of active loads on the line with connected additional \( P_{\text{Aac}} \) at a distance of \( L_{\text{Aac}} \) from the beginning of the line. We believe that the total load of the line is \( P_{\text{lin}} \). Then the average specific load of the line is \( P_{\text{ud.sr}} = \frac{P_{\text{lin}}}{L_{\text{lin}}} \). When distributing the specific load as shown in Figure 2, the law of its change along the line will be defined as:

- For decreasing load \( P_{\text{ud,1}} = 2P_{\text{ud.sr}} \left(1 - \frac{L}{L_{\text{lin}}^2}\right) \),
- For increasing load \( P_{\text{ud,2}} = 2P_{\text{ud.sr}} \frac{L}{L_{\text{lin}}} \).

And laws of change of active current along a line from its beginning to the point \( K \) at connection of an additional source

\[
I_{\text{lin},L_{\text{Aac}}} = \frac{P_{\text{ud.sr}}}{U_{\text{grid}}} \left(\frac{L_{\text{lin}}^2 - L}{2}\right),
I_{\text{lin},L_{\text{Aac}}} = \frac{P_{\text{ud.sr}}}{U_{\text{grid}}} \left(\frac{L}{2}\right)
\]

By integrating the entire length of the line for such changes in the value of current along the line we can obtain expressions for losses in the entire line with distributed load according to laws shown in Figure 2, in the absence of additional power supply

\[
\Delta P_{0.1} = \frac{R_{\text{den}}}{U_{\text{grid}}^2} \int_0^{L_{\text{lin}}} \left[I_{\text{lin},L_{\text{Aac}}}^2 \right] dL = \frac{P_{\text{ud.sr}} R_{\text{den}}}{5U_{\text{grid}}^2} \left[\frac{L_{\text{lin}}^3}{2}\right]
\]
\[
\Delta P_{0.2} = \frac{R_{\text{den}}}{U_{\text{grid}}^2} \int_0^{L_{\text{lin}}} \left[I_{\text{lin},L_{\text{Aac}}}^2 \right] dL = \frac{8P_{\text{ud.sr}} R_{\text{den}}}{15U_{\text{grid}}^2} \left[\frac{L_{\text{lin}}^3}{2}\right]
\]

where \( R_{\text{den}} \) – specific active resistance to the line.

If there is an additional source of active power \( P_{\text{Aac}} \) attached in the point \( K \), the laws of current change along the line from the point to its beginning will have the form

\[
I_{\text{lin},0K,1} = \frac{P_{\text{ud.sr}}}{U_{\text{grid}}} \left(\frac{L_{\text{lin}}^2 - L}{2}\right) - \frac{P_{\text{Aac}}}{U_{\text{grid}}},
I_{\text{lin},0K,2} = \frac{P_{\text{ud.sr}} L}{U_{\text{grid}}^2} - \frac{P_{\text{Aac}}}{U_{\text{grid}}}
\]

Losses in the line from active current generated by the load and additional source of active power are determined by the integration of its two sections with different current distribution along the line

\[
\Delta P = \frac{R_{\text{den}}}{U_{\text{grid}}^2} \left[\int_0^{L_{\text{lin}}} \left[I_{\text{lin},0K}^2 \right] dL + \int_{L_{\text{Aac}}}^{L_{\text{lin}}} \left[I_{\text{lin},L_{\text{Aac}}}^2 \right] dL\right]
\]

Take the keys for transition to relative units

\[
P_{\text{lin}} = P_{\text{den}}/L_{\text{lin}} \quad \text{general active load of the line;}
P_{\text{pu}} = P_{\text{Aac}}/P_{\text{lin}} \quad \text{relative power of the additional source;}
L_{\text{pu}} = L_{\text{Aac}}/L_{\text{lin}} \quad \text{relative length of a line to the place of connection of the additional source;}
\]

Relative power of losses

\[
\Delta P_{\text{pu}} = \frac{\Delta P}{\Delta P_0}
\]

where \( \Delta P_0 = \frac{P_{\text{ud.sr}} R_{\text{den}}}{U_{\text{grid}}^2} \left[\frac{L_{\text{lin}}^3}{2}\right] \) rated losses in the line from total active power of load connected to the end of the line at absence of the additional source of active power.
Then, in relative units for designation of losses in the line at different distributions of loads we obtain

\[ \Delta P_{pu0,1} = \frac{1}{5}, \Delta P_{pu0,2} = \frac{8}{15}. \]

The calculation of the integrals in (1) allows one to obtain expressions for the losses in the line with the distribution of its loads according to Figure 2

\[ \Delta P_{pu1} = \left[ P_{pu} L_{pu} \left( 2L_{pu} - \frac{2}{3} L_{pu}^2 + P_{pu} - 2 \right) + \frac{1}{5} \right], \]  

\[ \Delta P_{pu2} = \left[ P_{pu} L_{pu} \left( \frac{2}{3} L_{pu}^2 + P_{pu} - 2 \right) + \frac{8}{15} \right]. \]

We define the extremum of functions of losses from equality to a zero of their derivate on \( L_{pu} \).

Initially we consider the type of the decreasing load

\[ \frac{\partial \Delta P_{pu1}}{\partial L_{pu}} = P_{pu} (4L_{pu} - 2L_{pu}^2 + P_{pu} - 2) = 0 \]  

From (4) we obtain the best value of \( L_{pu} \) by increasing of losses – relative remote from the place of the additional source from the beginning of the line for present load distribution. Name it economically reasonable point \( L_{puE1} \).

\[ L_{puE1} = 1 - \frac{\sqrt{P_{pu}}}{2} \]  

When an additional source is connected to a point that is efficient for reducing losses, the dependence of the relative power losses in the line is obtained from (4) and (2)

\[ \Delta P_{puPE1} = P_{pu}^2 - \frac{2P_{pu}}{3} - \frac{2P_{pu}^2}{3} + \frac{1}{5}. \]  

We can define the economically rational value \( P_{puE1} \) plugging in anywhere in the \( L_{pu} \) line, calculating the derivative on \( P_{pu} \) equality (2) equaled to zero.

\[ \frac{\partial \Delta P_{pu}}{\partial P_{pu}} = \frac{2}{3} L_{pu} \left( 3L_{pu} - L_{pu}^2 + 3P_{pu} - 3 \right) = 0. \]

We obtain the expression that shows the dependence of the economically feasible power of an additional source \( P_{puE1} \) in its place of connection

\[ P_{puE1} = \frac{L_{pu}^2}{3} - L_{pu} + 1, \]

If we put (8) in (2), we will find the dependence of power losses in the line at switching of the additional source in the economically cost-effective point of the line

\[ \Delta P_{puLE1} = \frac{L_{pu}^2}{3} \left( 2L_{pu} - \frac{L_{pu}^2}{3} - 5 \right) + 2L_{pu}^2 - L_{pu} + \frac{1}{5}. \]

3. Results

If (2) the objective function of the value of losses in the line, then to determine the minimum (2) we find its se-ond partial derivatives with respect to \( P_{pu} \) and \( L_{pu} \)

\[ \frac{\partial^2 \Delta P_{pu1}}{\partial P_{pu}^2} = 2L_{pu} \]

\[ \frac{\partial^2 \Delta P_{pu1}}{\partial L_{pu}^2} = 4P_{pu} \left( 1 - L_{pu} \right) \]
and compound derivatives

\[ \frac{\partial^2 \Delta P_{pu1}}{\partial L_{pu} \partial P_{pu}} = \frac{\partial^2 \Delta P_{pu1}}{\partial P_{pu} \partial L_{pu}} = 2 \left( P_{pu} + 2L_{pu} - L_{pu}^2 - 1 \right) \]

Taking together (4) and (7) we obtain two vectors of real roots of this system of equations

\[
\begin{pmatrix}
    P_{pu} \\
    L_{pu}
\end{pmatrix} = \begin{pmatrix}
    \sqrt{21} + 11 & 11 - \sqrt{21} & 0 & 0 \\
    25 & 25 & 0 & 0 \\
    9 - \sqrt{21} & \sqrt{21} + 9 & 2 & 0 \\
    10 & 10 & 1 & 0
\end{pmatrix}
\]

Taking the sign of the determinant of the Hessian matrix for the system from equations (4) and (7), we find the required roots

\[ \begin{vmatrix}
    \frac{\partial^2 \Delta P_{pu1}}{\partial P_{pu}^2} & \frac{\partial^2 \Delta P_{pu1}}{\partial P_{pu} \partial L_{pu}} \\
    \frac{\partial^2 \Delta P_{pu1}}{\partial L_{pu} \partial P_{pu}} & \frac{\partial^2 \Delta P_{pu1}}{\partial L_{pu}^2}
\end{vmatrix} = \frac{26\sqrt{21} - 14}{125} > 0 \text{ only for the first pair of roots.}
\]

Thus, it has been proved that the global minimum of function (2) exists. Point coordinates

\[ P_{pu1} = \frac{\sqrt{21} + 11}{25} \approx 0.623, \quad L_{pu1} = \frac{9 - \sqrt{21}}{10} \approx 0.442. \quad (10) \]

Substitution of the optimal coordinates (10) into (2) gives the value of the minimum power losses in the line with an additional power source and load distributed according to Figure 2a.

\[ \Delta P_{pu1} \Delta P_{pu} = \frac{217 - 28\sqrt{21}}{3125} \approx 0.028, \quad (11) \]

or \[ \frac{\Delta P_{pu1}}{\Delta P_{pu.min1}} = \frac{1}{\frac{217 - 28\sqrt{21}}{3125}} = 7.047. \]

Thus, connecting an additional source of optimal power to the optimal point reduces power losses in the line by 7 times with the considered law of distribution of loads along the line. The distribution of the value \( \Delta P_{pu1} / \Delta P_{pu.min1} \) with changes in the values of \( L_{pu} \) and \( P_{pu} \) is shown in Figure 3.

Let’s consider the option of increasing load (Figure 2b). The derivative of the loss expression (3) for \( L_{pu} \) in relative units with the base \( \Delta P_0 \)

\[ \frac{\partial \Delta P_{pu2}}{\partial L_{pu}} = P_{pu} \left( 2L_{pu}^2 + P_{pu} - 2 \right) = 0. \quad (12) \]

From (12) we obtain the economically feasible value of the relative removal of the connection point of the additional source from the beginning of the line

\[ L_{pu.E2} = \sqrt{\frac{2 - P_{pu}}{2}}. \quad (13) \]

When an additional source is connected to a point that is expedient for reducing losses, the dependence of the relative power losses in the line is obtained from (13) and (3)

\[ \Delta P_{pu.E2} = P_{pu} \sqrt{\frac{2 - P_{pu}}{2}} \left( P_{pu} + \frac{2 - P_{pu}}{3} - 2 \right) + \frac{8}{15}. \quad (14) \]
Economically rational value \( P_{puE2} \), plug-in anywhere in the \( L_{pu} \) of the line, determine from the equality to zero of the derivative of the expression (3) on \( P_{pu} \).

\[
\frac{\partial \Delta P_{pu2}}{\partial P_{pu}} = 2L_{pu} \left( \frac{L_{pu}^2}{3} + P_{pu} + 1 \right) = 0. \tag{15}
\]

From (15) we obtain the expression

\[
P_{puE2} = 1 - \frac{L_{pu}^2}{3}, \tag{16}
\]

to determine the power, an additional source, which is advisable to reduce power losses, from the point of its connection.

If put (16) in (3), we will obtain the dependence of power losses in the line at connection of the additional source in economically rational place of the line

\[
\Delta P_{puLE2} = \frac{L_{pu}^2}{3} \left( 2 - \frac{L_{pu}^2}{3} \right) - L_{pu} + \frac{8}{15}. \tag{17}
\]

Expressions (14) and (17) for a given variant of load distribution along the line can be found economically feasible power or economically feasible place of connection if the power and place of connection of the additional source are known.

If (3) the objective function of the value of losses in the line when distributing the load according to Figure 2, b, then to determine the minimum (3), we will find its second partial derivatives with respect to \( P_{pu} \) and \( L_{pu} \)

\[
\frac{\partial^2 \Delta P_{pu2}}{\partial P_{pu}^2} = 2L_{pu} \frac{\partial^2 \Delta P_{pu2}}{\partial P_{pu}^2} = 4P_{pu}L_{pu}
\]

and compound derivatives

\[
\frac{\partial^2 \Delta P_{pu2}}{\partial L_{pu} \partial P_{pu}} = \frac{\partial^2 \Delta P_{pu2}}{\partial P_{pu} \partial L_{pu}} = 2 \left( L_{pu}^2 + P_{pu} - 1 \right).
\]

Solving together (12) and (15) we will obtain two vectors of real roots of this system of equations.
\[
\begin{bmatrix}
P_{pu} \\
L_{pu}
\end{bmatrix} = \begin{bmatrix}
\sqrt{3} & -\sqrt{3} & \sqrt{3} & -\sqrt{3} & 0 & 0 \\
0 & 0 & 4 & 4 & 0 & 2
\end{bmatrix}
\]

The Hessian matrix determinant for the system of equations (12) and (15)
\[
\begin{vmatrix}
\frac{\partial^2 \Delta P_{pu2}}{\partial P_{pu2}^2} & \frac{\partial^2 \Delta P_{pu2}}{\partial L_{pu2} \partial P_{pu}} \\
\frac{\partial^2 \Delta P_{pu2}}{\partial L_{pu} \partial P_{pu}} & \frac{\partial^2 \Delta P_{pu2}}{\partial L_{pu2}^2}
\end{vmatrix} = \frac{16}{5} > 0 \text{ only for the third pair of roots.}
\]

Therefore, the target function (3) has a global minimum at the point with coordinates

\[P_{pu2} = \sqrt{3} \approx 0.775, \quad L_{pu2} = \frac{4}{5} = 0.8.\] (18)

Substitution of the optimal coordinates (18) into (3) gives the value of the minimum power losses in the line with an additional power source and load distributed according to Figure 2b.

\[\Delta P_{pu.min2} = \left( \frac{8}{15} - \frac{16\sqrt{15}}{125} \right) \approx 0.038,\] (19)

or
\[\frac{\Delta P_{pu2}}{\Delta P_{pu.min2}} = \frac{8}{15} \frac{16\sqrt{15}}{125} \approx 14.2.\]

From where, it can be concluded that the connection of an additional source to the line can reduce the loss of transmission of electrical energy by 14 times under the law of load distribution along the line (Figure 4). The distribution of the value \(\Delta P_{pu2} / \Delta P_{pu.min2}\) for the range \(L_{pu}\) and \(P_{pu}\) is shown in Figure 4.

**Figure 4.** Profile of dependences \(\Delta P_{pu2} / \Delta P_{pu.min2}\) of relative values of an additional source power the place of connection.

### 4. Discussion of results

As a result of the mathematical analysis of the possible location of an additional power supply on the line with unevenly distributed load (decreasing or increasing), it is found that the highest level of losses in the line will be with increasing load from the beginning to the end. Taking into account the results obtained by the authors earlier for the unevenly distributed load [10], the relative losses in the
supply line before and after installation of an optimal additional power supply can be characterized as
follows (table 1).

| Type of load          | Uniform distribution | Decreasing distribution (Figure 2a) | Increasing distribution (Figure 2b) |
|-----------------------|----------------------|--------------------------------------|-------------------------------------|
| additional source     | 1                    | 1                                    | 8                                   |
| absent                | 3                    | 5                                    | 15                                  |
| optimal               | 1                    | 1                                    | 8                                   |
| additional source     | 27                   | ≈ 35                                 | ≈ 212                               |

The optimal point of connection of an additional source for different load distributions is in the
range of 0.44 – 0.8 of the line length, and the optimal power of the additional source ranges from
0.66 – 0.77 of the total active power of consumers connected to the line.

5. Conclusions
There were obtained the analytical expressions for calculation of economically rational powers and
points of connection of the additional source for linearly decreasing or increasing specific load of the
supply line.
There were found the optimal points of connection of additional sources providing a minimum of
losses in the transmission of energy along the supply line with irregular distributed load.
It is shown that in a line with the considered distribution of loads, connecting an additional source of
optimal power to the optimal point of the line reduces losses by 7–14 times.

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