Radiative corrections to the semileptonic Dalitz plot with angular correlation between polarized decaying hyperons and emitted charged leptons

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Abstract

We obtain a model-independent expression for the Dalitz plot of semileptonic decays of polarized hyperons including radiative corrections to order α and neglecting terms of order αq/πM1, where q is the four-momentum transfer and M1 is the mass of the decaying hyperon. We specialize our results to exhibit the correlation between the charged-lepton momentum and the spin of the decaying hyperon. We present results for the three-body region of the Dalitz plot and for the complete Dalitz plot (which includes the four-body region). From these results we also obtain the corresponding radiative corrections to the integrated lepton spin-asymmetry coefficient. Our formulas are valid for charged as well as for neutral decaying hyperons and are appropriate for model-independent experimental analysis whether the real photon is discriminated or not.

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I. INTRODUCTION

The form factors of hyperon semileptonic decays, $A \rightarrow B\ell\nu_\ell$, contain important information about the low-energy strong interactions of spin-1/2 baryons ($A$ and $B$ are such baryons and $\ell$ and $\nu_\ell$ are the accompanying charged lepton and neutrino). Their experimental determination requires the use of accurate formulas in the analysis of the measurements of several observables. An important one of these observables is the angular correlation between the spin $\hat{s}_1$ of $A$ and the direction $\hat{l}$ of the momentum of $\ell$. It is the purpose of this paper to calculate the radiative corrections (RC) to the Dalitz plot (DP) with the spin correlation $\hat{s}_1 \cdot \hat{l}$ exhibited explicitly.

We shall obtain expressions that are suitable for a model-independent experimental analysis. The model dependence of the virtual RC is handled following the approach of Sirlin [1] to the RC of neutron beta decay whereas the model dependence of the bremsstrahlung is controlled by the theorem of Low [2]. In previous work we have discussed the unpolarized DP [3] and the DP with the $\hat{s}_1 \cdot \hat{p}_2$ spin correlation kept explicitly [4] ($\hat{p}_2$ is the direction of the momentum of the emitted baryon $B$). It is not possible to derive the result for the spin correlation $\hat{s}_1 \cdot \hat{l}$ from the final result of Ref. [4], because all kinematical integrations, except for the $\ell$ and $B$ energies $E$ and $E_2$, respectively, were already performed. However, since we are going to follow the same approach of this reference, much of the work has already been advanced.

The bremsstrahlung RC is a four-body decay whose DP covers entirely the DP of the three-body decay $A \rightarrow B\ell\nu_\ell$. We shall refer to the latter as the three-body region (TBR) and to the non-overlap of the former and the latter as the four-body region (FBR). Even when no experimental arrangement has been made to detect and discriminate real photons, it is possible to eliminate the photons that belong to the FBR by energy-momentum conservation. Therefore, in calculating the bremsstrahlung RC we shall keep a clear distinction between these two regions.

We shall also obtain the radiative corrections to the integrated lepton spin-asymmetry coefficient $\alpha_{\ell}$. As we shall see the distinction between the TBR and the FBR leads to a perceptible change in the RC to this asymmetry coefficient.

Our results will be presented in two final forms. One where the triple integration over the real photon three-momentum $k$ is left indicated and ready to be performed numerically. And another one, an analytical form, where such a triple integration has been performed. Both forms can be used to numerically cross-check on one another. However, the analytical result, although tedious to feed into a Monte Carlo program, leads to a considerable saving of computer time, because the triple integration does not have to be performed within the Monte Carlo every time $E$ and $E_2$ or the form factors are changed.

For the use of our results it is important that this paper be as self-contained as possible. In Sec. II we introduce our notation and conventions and we review the virtual RC; also the infrared divergence of this part is clearly separated. In Sec. III the real photon emission is calculated and separated into the contributions from the TBR and FBR. The calculation of Ref. [4] is adapted to the present case. The infrared divergence is extracted following Ref. [3] and its cancellation with the one of Sec. II is discussed in detail. Our first main result is established, allowing for the elimination (or not) of real photons from the experimental analysis. In Sec. IV we proceed to the analytical evaluation of the triple integration over
the bremsstrahlung three-momentum. Our second main result is established, also allowing for the experimental discrimination (or not) of real photons. In Sec. V we use the analytical result to obtain the RC to the asymmetry coefficient \( \alpha\ell \). In Sec. VI we make numerical evaluations for several HSD and also for the \( \Lambda_e^+ \to \Lambda e^+\nu \) decay. We compare with other results available in the literature. Section VII is reserved to discuss and summarize our results. To make this paper self-contained we introduce three appendices. In Appendix A we give the amplitudes for virtual and bremsstrahlung RC, emphasizing how the model dependence is kept under control. In Appendix B we give the analytical expressions of all the coefficients, both new and of Refs. \[3\] and \[4\] required to compute the RC to the \( \hat{s}_1 \cdot \hat{l} \) correlation. Finally, in Appendix C we review briefly the procedure of Ref. \[5\] to extract the infrared divergences.

Our results have been obtained neglecting contributions of order \( \alpha q/\pi M_1 \) and higher (\( q \) is the momentum transfer and \( M_1 \) is the mass of \( A \)). They cover both neutral and charged \( A \) and are reliable up to 0.5\% or better in HSD. Furthermore, they provide a useful result for charm decay experiments with several thousands of events. For higher statistics experiments it will be necessary to incorporate \( \alpha q/\pi M_1 \) contributions.

II. VIRTUAL RADIATIVE CORRECTIONS

In this section we shall discuss the virtual radiative corrections, up to order \( \alpha \) and neglecting terms of order \( \alpha q/\pi M_1 \), to the DP of the hyperon semileptonic decay

\[
A \to B + \ell + \nu_\ell,
\]

with \( A \) polarized. Our results will be specialized to exhibit the angular correlation \( \hat{s}_1 \cdot \hat{l} \).

A convenient procedure to get such results is that of Ref. \[3\]. So, in this paper we adopt the same approach, the same approximations and the same conventions of this reference. In this way, \( p_1 = (E_1, \mathbf{p}_1) \), \( p_2 = (E_2, \mathbf{p}_2) \), \( l = (E, \mathbf{l}) \), and \( p_\nu = (E_\nu, \mathbf{p}_\nu) \) will be the four-momenta of \( A \), \( B \), \( \ell \), and \( \nu_\ell \), respectively. \( M_1 \), \( M_2 \), and \( m \) will denote the masses of the first three particles. We shall assume throughout this paper that \( m_\nu = 0 \). \( \hat{p}_2 \) will denote a unit vector along \( \mathbf{p}_2 \), etc. We shall make our calculations in the center-of-mass frame of \( A \). In this case, \( p_2 \), \( l \), and \( p_\nu \) will denote the magnitudes of the corresponding three-momenta. There will be no confusion because the expressions obtained will not be manifestly covariant. Because we want our results to exhibit the correlation \( \hat{s}_1 \cdot \hat{l} \), it is convenient to choose the \( z \)-axis along \( \mathbf{l} \) and not along \( \mathbf{p}_2 \) as in Ref. \[3\].

At this point, it is convenient to mention that it is not possible to obtain the virtual RC of our present case by using the final virtual RC given by \( d\Gamma_V \) of Eq. (15) of Ref. \[3\]. This is because in that equation the correlation \( \hat{s}_1 \cdot \hat{p}_2 \) was singled out after the integration over the azimuthal angle \( \phi_\ell \) of \( \mathbf{l} \) was performed [in such Eq. (15) \( d\phi_\ell \) is still present in the phase space. As it appears and because of the choice of \( z \)-axis its integration amounts a 2\( \pi \) factor]. Therefore, all the terms in \( d\Gamma_V \) containing the product \( \hat{s}_1 \cdot \hat{p}_2 \) and which appear before the integration over \( d\phi_\ell \) is performed, have been transformed leaving the correlation \( \hat{s}_1 \cdot \hat{p}_2 \) only. There is no way to recover the \( \hat{s}_1 \cdot \hat{l} \) terms from this Eq. (15) of Ref. \[3\]. So, for obtaining our \( d\Gamma_V \), exhibiting the correlation \( \hat{s}_1 \cdot \hat{l} \) only, we have to take a few steps back before that Eq. (15).
Our calculation now starts at the point where the scalar products \( \mathbf{s}_1 \cdot \mathbf{l} \) and \( \mathbf{s}_1 \cdot \mathbf{p}_2 \) appear for the first time, that is, at Eq. (13) of Ref. \[3\], namely,

\[
\sum_{\text{spins}} |M_V|^2 = \frac{1}{2} \sum_{\text{spins}} |M'_V|^2 - \frac{1}{2} \sum_{\text{spins}} |M''_V|^2.
\]

Here \( M_V \) is the sum of the order zero amplitude \( M'_0 \), corrected by the model-dependent part of the virtual radiative corrections through the modified form factors \( f'_1 \) and \( g'_1 \), and the model-independent amplitude \( M_v \) of such RC. \( M'_0 \) and \( M_v \) are given explicitly in Appendix A [see Eqs. (A1) and (A3)].

The two terms on the right-hand side of Eq. (2) arise after the spinor \( u_A(p_1) \) of the polarized hyperon \( A \) is replaced by \( \Sigma(s_1)u_A(p_1) \) in the amplitude \( M_V \). \( \Sigma(s_1) \) is the spin projection operator of \( A \) given in Eq. (4) of Ref. \[4\].

With Eq. (2) we can express the differential decay rate \( d\Gamma_V \) as

\[
d\Gamma_V = \frac{dE_2\, dE\, d\Omega_l\, d\phi_2}{(2\pi)^5} M_{2mmv} \left[ \frac{1}{2} \sum_{\text{spins}} |M'_V|^2 - \frac{1}{2} \sum_{\text{spins}} |M''_V|^2 \right]
= d\Gamma'_V - d\Gamma^{(s)}_V.
\]

Notice the variables in the phase space of this equation, they correspond to our new choice of the \( z \)-axis along \( \mathbf{l} \).

In Eq. (3), \( d\Gamma'_V \) corresponds to the first term within the square brackets. It can be identified with the differential decay rate with virtual radiative corrections of unpolarized \( A \) given in Eq. (10) of Ref. \[3\] and, therefore, there is no need to recalculate it now. \( d\Gamma^{(s)}_V \) corresponds to the second term of the square brackets of Eq. (3) and it contains the scalar products \( \mathbf{s}_1 \cdot \mathbf{l}, \mathbf{s}_1 \cdot \mathbf{p}_\nu \), and \( \mathbf{s}_1 \cdot \mathbf{p}_2 \). The scalar product \( \mathbf{s}_1 \cdot \mathbf{p}_\nu \) can be expressed in terms of \( \mathbf{s}_1 \cdot \mathbf{l} \) and \( \mathbf{s}_1 \cdot \mathbf{p}_2 \) by three-momentum conservation. In this way, only \( \mathbf{s}_1 \cdot \mathbf{l} \) and \( \mathbf{s}_1 \cdot \mathbf{p}_2 \) will appear in \( d\Gamma^{(s)}_V \). Now, we require that \( \mathbf{s}_1 \cdot \mathbf{l} \) be the only scalar product present in \( d\Gamma^{(s)}_V \). This can be accomplished by noting that the most general form of \( \mathbf{s}_1 \cdot \mathbf{p}_2 \) depends on \( \mathbf{s}_1 \cdot \mathbf{l}, \mathbf{l} \cdot \mathbf{p}_2 \), and \( \cos \phi_2 \). The terms directly proportional to this cosine drop out after integration over \( \phi_2 \) from 0 to 2\( \pi \). This fact allows us to use the replacement

\[
\mathbf{s}_1 \cdot \mathbf{p}_2 \rightarrow (\mathbf{s}_1 \cdot \mathbf{l})(\mathbf{l} \cdot \mathbf{p}_2) = \mathbf{s}_1 \cdot \mathbf{l} y_0,
\]

in \( d\Gamma^{(s)}_V \), leaving us with an expression which only contains the correlation \( \mathbf{s}_1 \cdot \mathbf{l} \). In Eq. (4) \( y_0 \) is the cosine of the angle between \( \mathbf{p}_2 \) and \( \mathbf{l} \) when the real photon is not present. In the CM of \( A \), \( y_0 \) is given by

\[
y_0 = \frac{(E^0_\nu)^2 - p^2_2 - l^2}{2p_2 l},
\]

where

\[
E^0_\nu = M_1 - E_2 - E,
\]

is the neutrino energy (in this three-body decay).

With these considerations in mind, we can express the \( d\Gamma_V \) of Eq. (3) as
\[ d\Gamma_V = \frac{G_V^2}{2} \frac{dE_2 \, dE \, d\Omega_t \, d\phi_2}{(2\pi)^4} 2M_1 \left\{ A_0' + \frac{\alpha}{\pi} (A_1' \phi + A_1'' \phi') \right\} \\
- \hat{s}_1 \, \hat{l} \left[ B_0'' + \frac{\alpha}{\pi} (B_2' \phi + B_2'' \phi') \right] . \tag{7} \]

This \( d\Gamma_V \) is the DP with virtual radiative corrections up to order \( \alpha \) (and neglecting terms of order \( aq/\pi M_1 \)), leaving \( E_2 \) and \( E \) as the relevant variables and with only the angular correlation \( \hat{s}_1 \, \hat{l} \) explicitly exhibited. The integration over \( \phi_2 \) only amounts the factor \( 2\pi \). Now, \( A_0', A_1', A_1'' \) are given in Eqs. (B1)-(B3) of Appendix B. The new terms \( B_0'', B_2', B_2'' \) are
\[
\begin{align*}
B_0'' &= Q_6 E p_2 y_0 + Q_7 E l , \\
B_2' &= -D_3 E p_0 l + D_4 E (p_2 y_0 + l) , \\
B_2'' &= D_4 E (p_2 y_0 + l) . 
\end{align*} \tag{8-10} \]

In these equations the coefficients \( Q_6 \) and \( Q_7 \) are long quadratic functions of the form factors. They are given in Eqs. (B6) and (B7), respectively, of Ref. [4]. For completeness we repeat them too [see Eqs. (B9) and (B10)]. \( D_3 \) and \( D_4 \) depend on the leading form factors \( f'_1 \) and \( g'_1 \) and they are given in Eqs. (B13) and (B14), respectively. The primes on \( A_0', A_1', A_1'', B_0'', B_2', B_2'' \) indicate that these terms contain the model-dependence of the virtual radiative corrections through the leading form factors. The model-independent functions \( \phi \) and \( \phi' \) are
\[
\begin{align*}
\phi (E) &= 2 \left( \frac{1}{\beta} \text{arctanh}\beta - 1 \right) \ln \frac{\lambda}{m} - \frac{1}{\beta} (\text{arctanh}\beta)^2 + \frac{1}{\beta} L \left( \frac{2\beta}{1 + \beta} \right) \\
&\quad + \frac{1}{\beta} \text{arctanh}\beta - \frac{11}{8} + \left\{ \frac{\pi^2}{\beta} + \frac{3}{2} \ln (M_2/m) \text{ NDH} \right. \\
&\quad \left. + \frac{1}{2} \ln (M_1/m) \text{ CDH} \right\} \\
\phi' (E) &= \left( \beta - \frac{1}{\beta} \right) \text{arctanh}\beta . \tag{11-12} \end{align*} \]

NDH corresponds to neutral decaying hyperons and CDH corresponds to charged decaying hyperons. The infrared divergence appears in the function \( \phi (E) \) through the parameter \( \lambda \). Let us mention that \( d\Gamma_V \) of Eq. (7) contains two infrared divergent terms. The first one appears in \( A_1' \phi \) of the spin-independent part and the other one appears in \( B_2' \phi \) of the spin-dependent part. Both terms will be canceled by their counterparts in the bremsstrahlung contribution.

Let us close this section by comparing the \( d\Gamma_V \) of Eq. (7) with the \( d\Gamma_V \) of Eq. (15) of Ref. [4]. In spite of minor differences in their phase space factors, we can see that their spin-independent parts are the same. This is not the case for their spin-dependent parts. We can appreciate that the coefficients \( A_0'', A_2', A_2'' \) which appear in the spin-dependent part of \( d\Gamma_V \) of Ref. [4], have changed to the coefficients \( B_0'', B_2', B_2'' \), respectively, of \( d\Gamma_V \) of Eq. (7). We observe in Eqs. (8)-(10) that in the \( B \) coefficients \( y_0 \) always appears as a factor of \( p_2 \), while for the \( A \) coefficients of Ref. [4] \( y_0 \) always appears as a factor of \( l \). This latter observation may induce us to think into the possibility of obtaining the \( d\Gamma_V \) of Eq. (7) from the \( d\Gamma_V \) of Ref. [4] by simply interchanging \( p_2 \) with \( l \). Unfortunately this rule does not work because under such an interchange the \( A \) coefficients do not lead to the \( B \) coefficients and, thus, we can not obtain Eq. (7) directly from the final \( d\Gamma_V \) of Ref. [4].
III. BREMSSTRAHLUNG RADIATIVE CORRECTIONS

In addition to the virtual RC the bremsstrahlung contributions must be calculated to get the complete RC to the DP of polarized decaying hyperons. In this section, we shall obtain them, in the same order of approximation as the virtual RC, both for the TBR and for the FBR. First, we shall define those regions and next, we shall proceed to the calculations. As is discussed in Appendix A, these corrections are model-independent by virtue of the theorem of Low [2].

A. Kinematics, TBR, and FBR

The DP in the variables $E$ and $E_2$ is the kinematically allowed region of the four-body decay

$$A \rightarrow B + \ell + \nu_\ell + \gamma,$$

where $\gamma$ represents a real photon with four-momentum $k = (\omega, k)$. The DP can be seen as the union of the TBR and FBR, each one defined presently.

The TBR of the DP is the region where the three-body decay (1) and the four-body decay (13) overlap completely. The energies $E$ and $E_2$ satisfy the bounds

$$E_2^- \leq E_2 \leq E_2^+,$$

and

$$m \leq E \leq E_m.$$  \hspace{1cm} (14)

Here, $E_2^+$ ($E_2^-$) is the upper (lower) boundary of the TBR given by

$$E_2^\pm = \frac{1}{2} (M_1 - E \pm l) + \frac{M_2^2}{2 (M_1 - E \pm l)},$$

where $E_m$ is the maximum energy of the charged lepton

$$E_m = \frac{M_1^2 - M_2^2 + m^2}{2M_1}.$$ \hspace{1cm} (16)

The FBR of DP is the region where only the four-body decay (13) can take place. The energies $E$ and $E_2$ in this region satisfy the bounds

$$M_2 \leq E_2 \leq E_2^-,$$

and

$$m \leq E \leq E_B,$$ \hspace{1cm} (18)

where

$$E_B = \frac{(M_1 - M_2)^2 + m^2}{2 (M_1 - M_2)}.$$  \hspace{1cm} (20)

Both regions TBR and FBR are depicted in Fig. 1 of Ref. [3] where more details about these regions can be found.

We can now proceed to the calculation of the bremsstrahlung RC to the DP. First, we shall do this for the TBR and next for the FBR. The complete bremsstrahlung RC to the DP are obtained by simply adding the results of the TBR to those of the FBR. To obtain the complete RC of each region, we must also add the virtual RC of Eq. (7).
B. TBR bremsstrahlung RC

Here we shall obtain the bremsstrahlung RC restricted to the TBR and with the angular correlation $\hat{s}_1 \cdot \hat{l}$ explicitly shown. We shall follow the procedure introduced in Ref. [4]. Therefore, we start from Eq. (31) of this reference,

$$\sum_{\text{spins}} |M_B|^2 = \frac{1}{2} \sum_{\text{spins}} |M'_B|^2 - \frac{1}{2} \sum_{\text{spins}} |M^{(s)}_B|^2. \quad (21)$$

This equation is the square, summed over spins, of the bremsstrahlung transition amplitude $M_B$ of the four-body process (13) with the spinor $u_A(p_1)$ replaced by $\Sigma(s_1)u_A(p_1)$. The explicit form of $M_B$ is given in Eq. (A5) of Appendix A. Equation (21) enables us to express the bremsstrahlung differential decay rate as

$$d\Gamma_{TBR}^B = \frac{M^2 m m}{2 \pi^5} \frac{d^3 p_2}{E_2} \frac{d^3 k}{E} \frac{d^3 \rho_\nu}{E_\nu} \sum_{\text{spins}} |M_B|^2 \delta^4(p_1 - p_2 - l - \rho_\nu - k) \equiv d\Gamma_B' - d\Gamma_B^{(s)}, \quad (22)$$

where $d\Gamma_B'$ contains the first term of Eq. (21) and is independent of $\hat{s}_1$, while $d\Gamma_B^{(s)}$ contains the second term of this equation and is spin-dependent. $d\Gamma_B'$ is readily identified with the bremsstrahlung differential decay rate for unpolarized hyperons given in Eq. (33) of Ref. [4] as

$$d\Gamma_B' = d\Gamma_{ir}^B + d\Gamma_a^B + d\Gamma_b^B, \quad (23)$$

where, in accordance to Eq. (34) of this reference,

$$d\Gamma_{ir}^B = \frac{\alpha}{\pi} d\Omega' \left\{ A_1' \left[ \hat{I}_0(k/\lambda) + C + C_1 \right] + C_2 \right\}. \quad (24)$$

The phase space factor in Eq. (24) is now

$$\frac{\alpha}{\pi} d\Omega' = \frac{\alpha}{\pi} \frac{G^2}{2} \frac{dE_2}{2} \frac{dE}{2} \frac{d\Omega_\ell}{2} \frac{d\phi_2}{2} 2M_1, \quad (25)$$

instead of the $d\Omega$ of Eq. (35) of Ref. [4]. The term $\hat{I}_0(k/\lambda)$ fully contains the infrared divergence while $C$ and $C_1$ are the finite terms that come along with it. An alternative approach to extract the infrared divergence and the terms $C$ and $C_1$ is the one of Ref. [3], which was applied to the RC of DP of $K^\pm_{e3}$ decays. This procedure was followed in Ref. [4] to extract the infrared divergence of the spin-dependent part $d\Gamma_B^{(s)}$ of its Eq. (32), and it can also be employed to extract the infrared divergence of the spin-independent part $d\Gamma_B'$ without any difficulty. In Appendix C we give a brief review of this, here we only need the result of Eq. (56) of Ref. [4], which is

$$I_0(E, E_2) = \hat{I}_0(k/\lambda) + C + C_1. \quad (26)$$

Here $I_0(E, E_2)$ is the infrared-divergent integral given by Eq. (C6) of Appendix C. With this equation and the Eqs. (36)-(38) of Ref. [4] for $C_2$, $d\Gamma_B^b$, and $d\Gamma_B^b$, respectively, we can express the $d\Gamma_B'$ of Eq. (23), with some minor rearrangements, in the compact form
\[ d \Gamma'_B = \frac{\alpha}{\pi} d\Omega \left\{ A'_1 I_0 (E, E_2) + \frac{p_2 l}{4\pi} \int_0^{\pi} dx \int_0^{\pi} dy \int_0^{2\pi} d\phi_\kappa \left[ |M'|^2 + |M''|^2 \right] \right\}, \]  
\tag{27}\]

where
\[ |M'|^2 = \frac{\beta^2 (1 - x^2)}{(1 - \beta x)^2} \left[ D_2 - D_1 E + D_2 l x \right], \]  
\tag{28}\]
and
\[ |M''|^2 = \frac{E_\nu}{DE (1 - \beta x)} \left[ D_1 \left( \omega + E (1 + \beta x) - \frac{m^2}{E (1 - \beta x)} \right) \right. \]
\[ + D_2 \hat{p}_\nu \cdot \left( 1 + \hat{k} (E + \omega) - \hat{k} \frac{m^2}{E (1 - \beta x)} \right). \]  
\tag{29}\]

In these last three equations, \( x = \hat{l} \cdot \hat{k} \) and \( y = \hat{l} \cdot \hat{p}_2 \) are the cosines of the polar angles of \( \hat{k} \) and \( \hat{p}_2 \), respectively whereas \( \phi_\kappa \) is the azimuthal angle of \( \hat{k} \). Furthermore, \( E_\nu = E_\nu^0 - \omega \) and \( D = E_\nu^0 + (p_2 + 1) \cdot \hat{k} \). The coefficients \( D_1 \) and \( D_2 \) depend on the leading form factors and they are given in Eqs. (B11) and (B12) of Appendix B. Equation (27) is also the explicit substitutions of Ref. [6], given in Eqs. (B13) and (B14) of Appendix B.

In order to express \( d \Gamma'_B \) of Eq. (22), we can turn our attention to the spin-dependent part \( d \Gamma''_B \) of this equation. In order to compute it we shall start from Eq. (43) of Ref. [4], namely,
\[ d \Gamma''_B = d \Gamma^I_B + d \Gamma''^I_B, \]  
\tag{30}\]
where \( d \Gamma^I_B \) contains \( \sum_{\text{spins}} |M^{(s)}|^2 \) and \( d \Gamma''^I_B \) contains \( \sum_{\text{spins}} \left( |M^{(s)}|^2 + 2 \text{Re} \left[ M^{(s)} \right] \left[ M^{(s)} \right]^\dagger \right) \).

\( d \Gamma^I_B \) contains the infrared-divergent terms as well as many infrared-convergent ones. \( d \Gamma''^I_B \) is infrared-convergent only. To compute \( d \Gamma''^I_B \) we follow the procedure of Ref. [4] to extract the infrared divergence (see Appendix C). According to this and using the explicit form of \( \sum_{\text{spins}} |M^{(s)}|^2 \) given in Eq. (44) of Ref. [4], we can write \( d \Gamma^I_B \) as
\[ d \Gamma^I_B = \frac{\alpha}{8\pi \eta} \frac{1}{\lambda^2} \lim_{\lambda \to 0} \int^{\eta_{\text{max}}}_{\lambda} d\eta \frac{d^3 k}{\omega} \frac{d^3 p_\nu}{\nu} \frac{d^3 E_\nu}{E_\nu} \delta^4 (p_1 - p_2 - l - p_\nu - k) \]
\[ \times \left[ -D_3 \hat{s}_1 \cdot \hat{l} E_\nu^0 + D_4 \hat{s}_1 \cdot \hat{p}_2 E + D_5 \hat{s}_1 \cdot \hat{k} E \right] \times \sum_\epsilon \left( \frac{l \cdot \epsilon}{l \cdot k} - \frac{p_1 \cdot \epsilon}{p_1 \cdot k} \right)^2. \]  
\tag{31}\]

The infrared divergence is contained in the first three terms within the square brackets, the remaining two terms are infrared-convergent. \( \eta = (p_\nu + k)^2 \) is the invariant mass, which in the CM of \( A \) is given by \( \eta = 2 p_\nu l (y_0 - y) \). In the TBR, \( \eta_{\text{max}} = 2 p_\nu l (y_0 + 1) \) and \( \eta_{\text{min}} = \lambda^2 \), with \( \lambda^2 \to 0 \). The coefficients \( D_3 \) and \( D_4 \) depend on the leading form factors and they are given in Eqs. (B13) and (B14) of Appendix B. \( \epsilon_\mu \) is the polarization four-vector of the real photon.

In order to express \( d \Gamma^I_B \) in terms of the correlation \( \hat{s}_1 \cdot \hat{l} \) we have to transform the scalar products \( \hat{s}_1 \cdot \hat{p}_2 \) and \( \hat{s}_1 \cdot \hat{k} \) of Eq. (31) in terms of \( \hat{s}_1 \cdot \hat{l} \). This can be achieved by using the substitutions of Ref. [5].
In Eq. (31), with these substitutions, we can separate the infrared-divergent terms from the infrared-convergent ones as

\[
\Gamma_{B} = \frac{\alpha}{\pi} d\Omega \bar{s}_1 \cdot \hat{l} \left\{ B_2' \frac{1}{8\pi} \lim_{\lambda \to 0} \int_{\lambda^2}^{m_{\text{max}}} d\eta \frac{d^3 k \, d^3 p_{\nu}}{E_{\nu}} \delta^4 (p_1 - p_2 - l - p_{\nu} - k) \right.
\]

\[
x \left[ \frac{2p_1 \cdot l}{p_1 \cdot k \cdot l \cdot k} - \frac{m^2}{(l \cdot k)^2} - \frac{M^2_1}{(p_1 \cdot k)^2} \right]
\]

\[
+ \frac{1}{8\pi} \lim_{\lambda \to 0} \int_{\lambda^2}^{m_{\text{max}}} d\eta \frac{d^3 k \, d^3 p_{\nu}}{E_{\nu}} \delta^4 (p_1 - p_2 - l - p_{\nu} - k)
\]

\[
\times \left[ D_3 \omega l + D_4 \hat{l} \cdot \hat{k} \omega E - D_4 \frac{\eta}{2\beta} \right] \frac{\beta^2 (1 - \hat{l} \cdot \hat{k})^2}{\omega^2 (1 - \beta \hat{l} \cdot \hat{k})^2} \right\}.
\]

Here \( B_2' \) is given in Eq. (4). In the first integral the sum over polarizations indicated in Eq. (31) was performed in covariant form, while in the second one it was performed by using the Coester representation \([7]\).

The first integral in Eq. (33) can be identified with the divergent integral \( I_0 \) of Eq. (34). The second one can be put in the convenient form of Eq. (38) of Ref. [3] by performing the integration over the \( \delta \) function and leaving \( y \) as the integration variable instead of \( \eta \). In this way, we can express \( d\Gamma_B \) finally as

\[
d\Gamma_B = \frac{\alpha}{\pi} d\Omega \bar{s}_1 \cdot \hat{l} \left\{ B_2' I_0 (E, E_2) + \frac{p_2 E}{4\pi} \int_{-1}^{1} dx \int_{-1}^{\eta_0} dy \int_{0}^{2\pi} d\phi_k \right.
\]

\[
\times \left[ D_3 \frac{\beta l}{D} - D_4 \left(1 - \frac{lx}{D}\right) \right] \frac{\beta^2 (1 - x^2)}{(1 - \beta x)^2} \right\}.
\]

The other term of Eq. (33), \( d\Gamma_B^\Pi \), does not need any calculation. We can take the result of Eq. (57) of Ref. [4] and, with only minor changes in the phase space factor, we can adapt it to our case. After the application of rule (32) in such Eq. (57) is performed, we obtain

\[
d\Gamma_B^\Pi = \frac{\alpha}{\pi} d\Omega \bar{s}_1 \cdot \hat{l} \frac{p_2 \beta}{4\pi} \int_{-1}^{1} \frac{dx}{1 - \beta x} \int_{-1}^{\eta_0} dy \int_{0}^{2\pi} d\phi_k \frac{1}{D}
\]

\[
x \left\{ D_3 E_{\nu} \left[ -l - (E + \omega) x + \frac{m^2 x}{E 1 - \beta x} \right]
\]

\[
+ D_4 \left[ \omega + (1 + \beta x) E - \frac{m^2}{E 1 - \beta x} \right] (p_{2} y + l + \omega x) \right\}.
\]

At this point we can collect our partial results to get our first main result, namely the DP of polarized decaying hyperons with radiative corrections up to order \( \alpha \), neglecting terms of order \( \alpha q/\pi M_1 \), and restricted to the TBR. It can be set as

\[
d\Gamma_{\text{TBR}} = d\Gamma_V + d\Gamma_B^\Pi
\]

\[
= d\Gamma_V + \left[ d\Gamma_B^\prime - \left( d\Gamma_B^\prime + d\Gamma_B^\Pi \right) \right],
\]
where $d\Gamma_V$ is given in Eq. (7), $d\Gamma_{\text{TBR}}^B$ is given in Eq. (22), and $d\Gamma_B^\prime$, $d\Gamma_B^\prime\prime$, and $d\Gamma_B^{\prime\prime}$ are given in Eqs. (27), (34), and (35), respectively. The integrations over the three-momentum of the real photon in these last three equations are ready to be performed numerically (but they can be performed analytically also, as we shall see in the next section).

The result of Eq. (36) can be compared with the corresponding result of Ref. [4] [the sum of Eqs. (15), (33), (51), and (57) of this reference]. We observe that the spin-independent parts $d\Gamma_B^\prime$ are the same, although our present $d\Gamma_B^\prime$ is expressed in a more compact form, while the spin-dependent parts are different. The results of Ref. [4] show that the correlation $\hat{s}_1 \cdot \hat{l}$ is mixed with two others, $\hat{s}_1 \cdot \hat{l}$ and $\hat{s}_1 \cdot \hat{k}$ [see Eqs. (51) and (57) of this reference], while our present results exhibit only the correlation $\hat{s}_1 \cdot \hat{l}$. If we insisted in transforming the correlations $\hat{s}_1 \cdot \hat{l}$ and $\hat{s}_1 \cdot \hat{k}$ of Ref. [4] as functions of $\hat{s}_1 \cdot \hat{l}$ by using the substitutions of Eq. (D6) of this reference, the result would not give the same coefficient of $\hat{s}_1 \cdot \hat{l}$ as right here above.

**C. FBR bremsstrahlung RC and complete RC**

We shall now calculate the bremsstrahlung contribution of the FBR and afterwards we shall obtain the complete RC to the DP, with the addition of the TBR and virtual contributions.

The calculation of bremsstrahlung in the FBR is relatively simple because the events in this region have the same amplitude $M_B$ of Eq. (A5) and it is infrared-convergent. We have to change only the upper limit of the integrals over the variable $y$ of Eqs. (27), (34), and (35), which becomes one now, and to change the previously infrared-divergent integral $I_0$ of these equations with $I_{0F}$,

$$I_{0F} = \frac{\theta_{0F}}{2} \ln\left(\frac{y_0 + 1}{y_0 - 1}\right), \quad (37)$$

with

$$\theta_{0F} = 4 \left(\frac{1}{\beta} \arctanh\beta - 1\right). \quad (38)$$

$I_{0F}$ is no longer infrared-divergent because in the FBR the photon has a minimum energy which is nonzero. It can be easily calculated from Eq. (C4). The invariant mass $\eta$ of this equation must be now integrated from a minimum value $\eta_{\text{min}} = 2p_2l(y_0 - 1)$ to a maximum value $\eta_{\text{max}} = 2p_2l(y_0 + 1)$.

With these changes we can write the differential decay rate corresponding to the FBR as

$$d\Gamma_B^{\text{FBR}} = d\Gamma_B^{\text{FBR}} - d\Gamma_B^{(s)\text{FBR}}, \quad (39)$$

with

$$d\Gamma_B^{\text{FBR}} = \frac{\alpha}{\pi} d\Omega' \left\{ A_1 I_{0F}(E, E_2) + \frac{p_2 l}{4\pi} \int_{-1}^{1} dx \int_{-1}^{1} dy \int_{0}^{2\pi} d\phi_k \left[ |M'|^2 + |M''|^2 \right] \right\} \quad (40)$$

and
\[ d\Gamma^{(s)FBR}_B = \frac{\alpha}{\pi} d\Omega'_s \hat{s}_1 \cdot \hat{1} \left\{ B'_2 I_{0F}(E, E_2) + \frac{p_2 l}{4 \pi} \int_{-1}^{1} dx \int_{-1}^{1} dy \int_{0}^{2\pi} d\phi_k \left[ |M''|^2 + |M^{IV}|^2 \right] \right\}. \] (41)

In Eq. (40), \( |M'|^2 \) and \( |M''|^2 \) are given in Eqs. (28) and (29), respectively. Equation (41) is the sum of Eqs. (34) and (35), after the above changes are performed. \( |M''|^2 \) and \( |M^{IV}|^2 \) are

\[ |M''|^2 = \frac{\beta^2 (1 - x^2)}{(1 - \beta x)^2} \left[ \frac{D_3 l + D_4 E x}{D} - D_5 \frac{1}{\beta} \right], \]

\[ |M^{IV}|^2 = \frac{1}{DE} \left[ D_3 E_\nu \left( -l - E x - \omega x + \frac{m^2}{E} \frac{x}{1 - \beta x} \right) \right. \]

\[ + D_4 \left( \omega - \frac{m^2}{E} \frac{1}{1 - \beta x} + (1 + \beta x) E \right) (p_2 y + l + \omega) \]. \]

Equation (42) is the FBR-contribution to the RC of the DP. It can be added to the TBR-contribution to obtain the complete RC to DP of polarized hyperons within the approximations mentioned before. This completes our first main result of Eq. (36) by including the emission of all real photons allowed by energy-momentum conservation. It is displayed as

\[ d\Gamma = d\Gamma^{TBR} + d\Gamma^{FBR}_B, \] (44)

with \( d\Gamma^{TBR} \) and \( d\Gamma^{FBR}_B \) given in Eqs. (36) and (39), respectively. The integrations over the photon variables are ready to be performed numerically.

Let us close this section by mentioning that all the integrals which arise in the two regions of the DP can be performed analytically. Because of this, we can obtain completely analytical results for the RC of DP. We shall do this in the next section.

**IV. ANALYTICAL INTEGRATIONS**

In this section we shall perform analytically the photon three-momentum integrals contained in Eqs. (27), (34), (35), (40), and (41) to obtain an analytical expression for the RC to the DP restricted to the TBR first and for the total DP later.

**A. TBR Analytical form**

The \( \mathbf{k} \)-integrals corresponding to the TBR of the DP are those of Eqs. (27), (34), and (35). They can be performed analytically by following the procedure of Sec. V of Ref. [3], where the RC of the DP of unpolarized hyperons were obtained. Fortunately much of the work has already been advanced. The integrals that concern us now can be expressed in terms of the functions \( \theta_i, i = 2, \ldots, 9 \) given by Eq. (99) of that reference, and in terms of the \( \theta_j, j = 10, \ldots, 16 \) given by Eq. (46) of Ref. [8]. In this last reference the RC include all the terms of order \( \alpha q/\pi M_1 \), which are dropped here. We can express the analytical form of Eq. (27) as
Eqs. (85), (86), (90), and (91) of Ref. [3], respectively. The functions Eqs. (46)-(47) can be expressed as
\[ d \rho = \frac{\alpha}{\pi} d\Omega \left[ A'_1 I_0 + (D_1 + D_2) (\theta' + \theta'') + D_2 \left( \theta'' + \theta^{IV} \right) \right]. \] (45)

This equation is equivalent to \( dw_B \) of Eq. (92) of Ref. [3]. Similarly, the expressions for \( d\Gamma_B^I \) and \( d\Gamma_B^{II} \) of Eqs. (53) and (55) become
\[ d\Gamma_B^I = \frac{\alpha}{\pi} d\Omega \, \hat{s}_1 \cdot \hat{r} \left[ B'_2 I_0 + D_3 \rho_1^f + D_4 \rho_2^f \right], \] (46)
\[ d\Gamma_B^{II} = \frac{\alpha}{\pi} d\Omega \, \hat{s}_1 \cdot \hat{r} \left[ D_3 \rho_3^f + D_4 \rho_4^f \right]. \] (47)

In Eq. (45) the \( \theta_i \) functions are contained in the functions \( \theta', \theta'', \theta''' \) and \( \theta^{IV} \) given by Eqs. (85), (86), (90), and (91) of Ref. [3], respectively. The functions \( \rho_i^f, i = 1, \ldots, 4 \) in Eqs. (16)-(17) can be expressed as
\[ \rho_1^f = \frac{p_2 l^2}{2} \left[ (\beta^2 - 1) \theta_2 + 2 \theta_3 - \theta_4 \right], \] (48)
\[ \rho_2^f = \frac{p_2 E^2}{2} \left[ - \frac{2}{\beta} \theta_0 + (\beta^2 - 1) \theta_2 - (\beta^2 - 3) \theta_3 - 2 \theta_4 - 2 \theta_5 \right], \] (49)
\[ \rho_3^f = \frac{p_2}{2} \left\{ E \left( E + E_\nu \right) \left( 1 - \beta^2 \right) \theta_2 - \left[ (3 - \beta^2) \frac{E^2}{2} + EE_\nu \right] \theta_3 \right. \\
+ \frac{1}{2} E \left( 1 + \beta^2 \right) \theta_4 - \frac{1}{2} \left( E + 2 E_\nu \right) \theta_5 - \frac{m^2}{2 E} \theta_6 \\
+ \left. \frac{1}{2} \left( E - E_\nu \right) \theta_7 - \frac{1}{2} \left( E - E_\nu \right) \theta_8 + \frac{1}{4} \theta_9 - \frac{3}{2} l^2 \theta_{10} - \frac{1}{4} \theta_{11} \right\}. \] (50)
\[ \rho_4^f = \frac{p_2}{2} \left\{ m^2 \left[ 2 - \beta^2 + \frac{E_\nu}{E} \right] \theta_2 + \left[ - \frac{7}{2} m^2 + p_2 l y_0 \right] \theta_3 \right. \\
+ \left[ (3 - \beta^2) \frac{E^2}{2} - EE_\nu - p_2 l y_0 \right] \theta_4 - \left[ \frac{1}{2} E^2 + 2 E_\nu \right] \theta_5 \\
- \frac{m^2}{2 E} \theta_6 + \frac{1}{2} \left( 3 E + p_2 \beta y_0 \right) \theta_7 - E \theta_8 + \frac{1}{4} \theta_9 - \frac{5}{2} l^2 \theta_{10} \\
- p_2 l \left( 1 - \beta^2 \right) \theta_{11} + 2 p_2 l \theta_{12} - p_2 l \theta_{13} - \frac{l}{2} \theta_{14} - \frac{1}{4} \theta_{15} - \frac{1}{4} \theta_{16} \right\}. \] (51)

With Eqs. (45)-(47) all the integrals over \( k \) in Eqs. (27), (34), and (35) have been expressed in an analytical form. We can obtain now the bremsstrahlung differential decay rate \( d\Gamma_B^{TBR} \) of decaying polarized hyperons with the photon integrals expressed analytically. Substituting in Eq. (22) the analytical forms of \( d\Gamma_B' \) Eq. (13) and of \( d\Gamma_B^{(s)} \), which is the sum of Eqs. (16) and (17), we obtain the analytical form of \( d\Gamma_B^{TBR} \)
\[ d\Gamma_B^{TBR} = \frac{\alpha}{\pi} d\Omega \left\{ A'_1 I_0 + (D_1 + D_2) (\theta' + \theta'') + D_2 \left( \theta'' + \theta^{IV} \right) \\
- \hat{s}_1 \cdot \hat{r} \left[ B'_2 I_0 + D_3 \left( \rho_1^f + \rho_3^f \right) + D_4 \left( \rho_2^f + \rho_4^f \right) \right] \right\}. \] (52)
We are now in a position to obtain our second main result in this paper: the analytical RC to the DP of decaying polarized hyperons to order \( \alpha \) and neglecting terms of order \( \alpha q / \pi M_1 \). This result comes from the addition of the virtual RC, \( d\Gamma_V \) of Eq. (4), and of \( d\Gamma_B^{TBR} \) of Eq. (22). It can be put compactly as

\[
d\Gamma_B^{TBR} = \frac{G_F^2}{2} \frac{dE_2 dE d\Omega_2}{(2\pi)^4} M_1 \left\{ A_0' + \frac{\alpha}{\pi} \Phi_1 - \hat{s}_1 \cdot \hat{p}_1 \left[ B_0'' + \frac{\alpha}{\pi} \Phi_2 \right] \right\}.
\]  

(53)

Here \( A_0' \) and \( B_0'' \) are the same as Eqs. (B1) and (8), respectively. \( \Phi_1 \) and \( \Phi_2^\ell \) are

\[
\Phi_1 = A_1' (\phi + I_0) + A_1'' \phi' + (D_1 + D_2) (\theta' + \theta''') + D_2 (\theta'' + \theta^{IV})
\]

\[
(54)
\]

\[
\Phi_2^\ell = B_2' (\phi + I_0) + B_2'' \phi' + D_3 (\rho_1^\ell + \rho_3^\ell) + D_4 (\rho_2^\ell + \rho_4^\ell).
\]  

(55)

The coefficients \( A_1' \), \( A_1'' \), \( B_2' \), and \( B_2'' \) are given in Eqs. (B2), (B3), (1) and (10), respectively. \( D_i \), \( i = 1, \ldots, 4 \) are given in Eqs. (B11)-(B14). The functions \( \phi, \phi', \) and \( I_0 \) appear in Eqs. (11), (12), and (C4), respectively. The new model-independent functions \( \rho_i^\ell, i = 1, \ldots, 4 \) were given in Eqs. (B11)-(B14). The sums \( \theta' + \theta''' \) and \( \theta'' + \theta^{IV} \) appear explicitly in Eqs. (93) and (94) of Ref. [3], respectively. We have corrected a misprint in that Eq. (93). Its fourth term has to be \( \frac{1}{2} \theta_5 \) rather than \( -\frac{1}{2} \theta_5 \). For completeness, we reproduce these two sums in Appendix B [see Eqs. (B33) and (B40)].

Because the infrared divergence, which appears in the virtual part \( d\Gamma_V \) through the function \( \phi \), cancels out with its bremsstrahlung counterpart, which appears in \( I_0 \), the sum \( \phi + I_0 \) is no longer infrared-divergent and, therefore, \( d\Gamma_B^{TBR} \) of Eq. (53) is infrared-convergent.

\( d\Gamma_B^{TBR} \) is the result corresponding to \( d\Gamma \) of Eq. (101) in Ref. [4]. Comparing both results, we can observe that the spin-independent parts are the same, but the spin-dependent parts show important differences. In Eq. (101) of Ref. [4] we have, within the square brackets that accompany the correlation \( \hat{s}_1 \cdot \hat{p}_2 \), the terms \( A_0'' \) and \( \Phi_2^\ell \), while in Eq. (53) the corresponding terms accompanying \( \hat{s}_1 \cdot \hat{p}_2 \) are \( B_0'' \) and \( \Phi_2^\ell \). They are different. In fact, we may notice that in \( \Phi_2^\ell \) of Eq. (53) only the \( \theta_i \)-functions, \( i = 2, \ldots, 16 \) appear, while in \( \Phi_2 \) of Eq. (103) of Ref. [4] also the \( \eta \)-functions appear [see Eqs. (91)-(95) of this reference]. Because of this, the RC to \( \hat{s}_1 \cdot \hat{p}_2 \) and \( \hat{s}_1 \cdot \hat{p}_2 \) correlations are quite different.

### B. FBR analytical form

The \( k \)-integrals of the FBR are those contained in Eqs. (40) and (41) and they can be performed analytically, too. Because \( d\Gamma_B^{FBR} \) of Eq. (40) has the same form as the corresponding \( d\Gamma_B \) of Eq. (27) of the TBR, we can follow the same procedure of the Sec. V of Ref. [3] to calculate the analytical form of this \( d\Gamma_B^{FBR} \). The result has the same structure as \( d\Gamma_B \) of Eq. (15),

\[
d\Gamma_B^{FBR} = \frac{\alpha}{\pi} d\Omega \left[ A_1 I_{0F} + (D_1 + D_2) (\theta' + \theta''') + D_2 (\theta'' + \theta^{IV}) \right].
\]  

(56)

with
\[ \theta_F' + \theta_F'' = \frac{p_2 l}{2} \left[ -E_\nu^0 (1 - \beta^2) \theta_{2F} + \left( E_\nu^0 - \frac{1 + \beta^2}{2E} \right) \theta_{3F} + \frac{E}{2} \theta_{4F} ight. \\
+ \frac{l}{2} \theta_{5F} + \frac{1 - \beta^2}{2} \theta_{6F} - \frac{2E - E_\nu^0}{2E} \theta_{7F} + \frac{1}{2} \theta_{8F} - \frac{1}{4E} \theta_{9F} \right], \tag{57} \]

d and

\[ \theta_F' + \theta_F'' = \frac{p_2 l}{2} \left[ \theta_{10F} - \left( E + E_\nu^0 + \beta p_2 y_0 \right) \theta_{3F} + \left( E_\nu^0 + E \right) \theta_{4F} + l \theta_{5F} \right]. \tag{58} \]

These last two equations also have the same structure as \( \theta' + \theta'' \) and \( \theta'' + \theta^{IV} \), which appear in the analytical form of \( d\Gamma_B' \) of Eq. \((45)\). The difference between \( d\Gamma_B'^{FBR} \) and \( d\Gamma_B' \) of the TBR lies in the integrals \( I_0, I_{0F} \) and in the functions \( \theta_i \). In the FBR these functions change to \( \theta_i \) because the variable \( y \) must be integrated from \(-1\) to \(1\) and not from \(-1\) to \(y_0\) as in the TBR. The new set \{\( \theta_{iF} \), \( i = 2, \ldots, 16 \)\} is explicitly given in Appendix B and \( \theta_{0F} \) is given in Eq. \((48)\). We can compare with the \( \theta_i^T \) of Ref. [9] where the RC of DP for the FBR were calculated up to order \( \alpha q/\pi M_1 \). We cannot take readily the result of this reference because, according to our approximations, we would have to neglect all the terms of order \( \alpha q/\pi M_1 \) in that result to obtain ours. We find the procedure of Ref. [9] more adequate and straightforward for our purposes. However, in order to check our results we have reproduced the Table I of Ref. [9]. Our numerical evaluations coincide very well within the approximation of neglecting terms of order \( \alpha q/\pi M_1 \); we shall not display here this numerical evaluation.

In a similar way, we can see that the spin-dependent part of \( d\Gamma_B^{FBR} \) has the same structure as the corresponding spin-dependent part of \( d\Gamma_B^{TBR} \). Thus, we get

\[ d\Gamma_{B}^{(s)FBR} = \frac{\alpha}{\pi} d\Omega' \hat{s}_1 \cdot \hat{I} \left[ B_2^2 I_0 + D_3 \left( \rho_{1F}^\ell + \rho_{3F}^\ell \right) + D_4 \left( \rho_{2F}^\ell + \rho_{4F}^\ell \right) \right], \tag{59} \]

with the functions \( \rho_i^\ell, \ i = 1, \ldots, 4 \) having the same structure as the previous \( \rho_i^\ell \) of Eqs. \((48)-(51)\). Explicitly, they are

\[ \rho_{1F} = \frac{p_2 l^2}{2} \left[ (\beta^2 - 3) \theta_{2F} - 2 \theta_{3F} - \theta_{4F} \right], \tag{60} \]

\[ \rho_{2F} = \frac{p_2 E^2}{2} \left[ -\frac{2}{E} \theta_{0F} + (\beta^2 - 1) \theta_{2F} - (\beta^2 - 3) \theta_{3F} - 2 \theta_{4F} - \beta \theta_{5F} \right], \tag{61} \]

\[ \rho_{3F} = \frac{p_2}{2} \left\{ E \left( E + E_\nu^0 \right) (1 - \beta^2) \theta_{2F} - \left( 3 - \beta^2 \right) \frac{E^2}{2} + EE_\nu^0 \right\} \theta_{3F} + \frac{1}{2} E^2 (1 + \beta^2) \theta_{4F} - \frac{l}{2} \left( E + 2E_\nu^0 \right) \theta_{5F} - \frac{m^2}{2E} \theta_{6F} \\
+ \frac{1}{2} \left( 2E - E_\nu^0 \right) \theta_{7F} - \frac{1}{2} \left( E - E_\nu^0 \right) \theta_{8F} + \frac{1}{4} \theta_{9F} - \frac{3}{2} p_2 \theta_{10F} - \frac{1}{4} \theta_{15F} \right\}; \tag{62} \]

\[ \rho_{4F} = \frac{p_2}{2} \left\{ m^2 \left( 2 - \beta^2 + \frac{E_\nu^0}{E} \right) \theta_{2F} + \left[ -\frac{7}{2} m^2 + p_2 y_0 \right] \theta_{3F} + \left( 3 - \beta^2 \right) \frac{E^2}{2} - EE_\nu^0 - p_2 y_0 \right\} \theta_{4F} - \frac{1}{2} \left( E^2 + 2\epsilon E_\nu^0 \right) \theta_{5F} \]
\[ -\frac{m^2}{2E}\theta_0 + \frac{1}{2}(3E + p_2\beta y_0)\theta_7 - E\theta_8 + \frac{1}{4}\theta_9 - \frac{5}{2}\theta_{10} + p_2\left(1 - \beta^2\right)\theta_{11} + 2p_2l\theta_{12} - p_2l\theta_{13} - \frac{l}{2}\theta_{14} - \frac{1}{4}\theta_{15} - \frac{1}{4E}\theta_{16}. \]  

(63)

From Eqs. (56) and (59) we obtain the analytical bremsstrahlung differential decay rate \( d\Gamma_{FB} \) of decaying polarized hyperons corresponding to Eq. (39),

\[ d\Gamma_{FB} = \frac{\alpha}{\pi} d\Omega \left[ \Phi_1 - \hat{s}_1 \cdot \hat{l} \Phi_2 \right], \]  

(64)

with

\[ \Phi_1 = A_1 I_1 + (D_1 + D_2) (\theta_{11} + \theta_{12}) + D_2 \left(\theta_{14} + \theta_{15}\right), \]  

(65)

\[ \Phi_2 = B_2 I_1 + D_3 \left(\rho_{14} + \rho_{15}\right) + D_4 \left(\rho_{24} + \rho_{25}\right). \]  

(66)

At this point we complete our second main result. The addition of \( d\Gamma_{FB} \) of Eq. (64) and \( d\Gamma_{TBR} \) of Eq. (53) gives us the complete analytical RC to the DP of decaying polarized hyperons to order \( \alpha \) and neglecting terms of order \( \alpha q/\pi M_1 \). This complete result can be expressed compactly as

\[ d\Gamma_{TOT} = \frac{G_0^2}{2} \frac{dE d\Omega}{(2\pi)^4} 2M_1 \left\{ A'_0 + \frac{\alpha}{\pi} (\Phi_1 + \Phi_1) - \hat{s}_1 \cdot \hat{l} \right\}. \]  

(67)

Here \( A'_0, B''_0, \Phi_1, \Phi_{1F}, \Phi_{2F}, \) and \( \Phi_{2F} \) are given in Eqs. (B1), (5), (54), (65), (55), and (66), respectively.

From Eq. (67) we can obtain easily Eq. (53) to the RC of DP with the TBR only by dropping \( \Phi_{1F} \) and \( \Phi_{2F} \). It is this Eq. (67) which must be used to obtain, in HSD, totally integrated observables, such as the spin-asymmetry coefficient of the charged lepton. We shall calculate this asymmetry coefficient in the next section, allowing for the possibility that real photon emission be discriminated experimentally via energy-momentum conservation.

**V. SPIN ASYMMETRY COEFFICIENT \( \alpha_\ell \)**

In this section we shall obtain the RC to the spin-asymmetry coefficient of the charged lepton \( \alpha_\ell \). We shall consider the two cases discussed all along, namely, that bremsstrahlung photons not be discriminated at all or that directly or indirectly the photons belonging to the FBR be eliminated from the experimental analysis. We will discuss the former case first and afterwards we will discuss the latter case. As we shall see in the numerical evaluation of the next section, an appreciable difference can be observed between these two cases.

\( \alpha_\ell \) can be calculated from the total DP of Eq. (67). This equation can be used to get the quantities \( N^\pm \) which appear in the definition of \( \alpha_\ell \),

\[ \alpha_\ell = 2 \frac{N^+ - N^-}{N^+ + N^-}. \]  

(68)
Here $N^+ (N^-)$ denotes the number of the emitted charged leptons with momenta in the forward (backward) hemisphere with respect to the polarization of the decaying hyperon. With those numbers calculated, we may express $\alpha_\ell$ as

$$\alpha_{\ell}^T = -\frac{B_2^\ell + (\alpha / \pi) \left( a_2^\ell + a_{2F}^\ell \right)}{B_1 + (\alpha / \pi) (a_1 + a_{1F})}. \quad (69)$$

Here

$$B_2^\ell = \int_{m}^{E_m} \int_{E_2^-}^{E_2^+} B_0^\ell dE_2 dE, \quad (70)$$

$$a_2^\ell = \int_{m}^{E_m} \int_{E_2^-}^{E_2^+} \Phi_2^\ell dE_2 dE, \quad (71)$$

$$a_{2F}^\ell = \int_{m}^{E_0} \int_{M_2}^{E_2^-} \Phi_{2F}^\ell dE_2 dE, \quad (72)$$

$$B_1 = \int_{m}^{E_m} \int_{E_2^-}^{E_2^+} A_0^\ell dE_2 dE, \quad (73)$$

$$a_1 = \int_{m}^{E_m} \int_{E_2^-}^{E_2^+} \Phi_1^\ell dE_2 dE, \quad (74)$$

$$a_{1F} = \int_{m}^{E_0} \int_{M_2}^{E_2^-} \Phi_{1F}^\ell dE_2 dE. \quad (75)$$

In these integrals, $B_0^\ell$, $\Phi_2^\ell$, $\Phi_{2F}^\ell$, $A_0^\ell$, $\Phi_1^\ell$, and $\Phi_{1F}^\ell$ are given in Eqs. (8), (55), (66), (B1), (54), and (65), respectively.

In Eq. (69) we have attached an upper index T to denote that the asymmetry coefficient includes the total DP of the real photons. The contributions of the TBR to the RC of $\alpha_\ell$ are given by the terms $a_2^\ell$ and $a_1$, while the contributions of the FBR are given by the terms $a_{2F}^\ell$ and $a_{1F}$. We can now rewrite $\alpha_{\ell}^T$ to comply with our approximations, i.e., in such a way that only the terms of order $\alpha$, neglecting terms of order $aq / \pi M_1$, appear. The corresponding expression is

$$\alpha_{\ell}^T = \alpha_0^\ell \left[ 1 + \frac{\alpha}{\pi} \left( \frac{a_2^\ell + a_{2F}^\ell}{B_2^\ell (0)} - \frac{a_1 + a_{1F}}{B_1 (0)} \right) \right], \quad (76)$$

where $\alpha_0^\ell$ is the spin-asymmetry coefficient of the charged lepton without RC. It is obtained from Eq. (69) by dropping the terms proportional to $\alpha$, namely,

$$\alpha_0^\ell = -\frac{B_2^\ell}{B_1}. \quad (77)$$

$B_2^\ell (0)$ and $B_1 (0)$ in the denominators of Eq. (76) are the zero-order $q/M_1$ approximations of the $B_2^\ell$ of Eq. (70) and of the $B_1$ of Eq. (73), respectively. Explicitly they are,
B_2^\ell (0) = \int_{E_m}^{E_m^+} \int_{E_2^-}^{E_2^+} B_2' dE_2 dE, \quad (78)

B_1 (0) = \int_{E_m}^{E_m^+} \int_{E_2^-}^{E_2^+} A_1' dE_2 dE, \quad (79)

with B_2' and A_1' given in Eqs. (9) and (B2), respectively.

The coefficient for \( \alpha \) when only the TBR of the DP is allowed can be easily obtained now. All that has to be done is to drop \( a_2^\ell f \) and \( a_1 f \) from Eq. (76) so that

\[
\alpha_R^\ell = \alpha_0^\ell \left[ 1 + \frac{\alpha}{\pi} \left( \frac{a_2^\ell}{B_2^\ell (0)} - \frac{a_1}{B_1 (0)} \right) \right].
\]

We attached an upper index R to denote that the bremsstrahlung correction is restricted to the TBR.

In Ref. [4] we only calculated the emitted baryon asymmetry-coefficient \( \alpha_B \) corresponding to \( \alpha_R^\ell \). In this reference it was assumed that the FBR photons were always discriminated. The contribution of these photons should be calculated and added to the results of this reference in order to get an \( \alpha_B \) corresponding to the above \( \alpha_T^\ell \).

In the next section we shall display numerical evaluations that will allow us to compare our results with others available in the literature and, also, to appreciate the relevance of discriminating or not FBR photons.

VI. NUMERICAL RESULT

In order to compare the coefficients \( \alpha_R^\ell \) of the TBR of the DP and \( \alpha_T^\ell \) of the total DP, we shall make numerical evaluations of them for several decays. These results will enable us to establish the relevance of the difference between \( \alpha_R^\ell \) and \( \alpha_T^\ell \) in the study of HSD. We shall also compare them with other results reported in the literature. In Table I we give the values of the form factors used in the numerical evaluation of the coefficients \( \alpha_R^\ell \) and \( \alpha_T^\ell \) for the decays \( n \rightarrow p e\bar{\nu}, \Lambda \rightarrow p e\bar{\nu}, \Sigma^- \rightarrow n e\bar{\nu}, \Sigma^- \rightarrow \Lambda e\bar{\nu}, \Sigma^+ \rightarrow \Lambda e^+\nu, \Xi^- \rightarrow \Lambda e\bar{\nu}, \Xi^- \rightarrow \Sigma^0 e\bar{\nu}, \Xi^0 \rightarrow \Sigma^+ e\bar{\nu}, \text{and } \Lambda^+_c \rightarrow \Lambda e^+\nu \). For this last decay we take the form factors of Ref. [10]. The sign of the form factor \( g_1 \) must be changed when the charged lepton is positive [11]. In the radiatively uncorrected amplitudes the \( q^2 \)-dependence of the form factor was neglected along with the contributions arising from \( f_3, g_2, \text{and } g_3 \) as was done in other calculations in the literature.

To evaluate \( \alpha_R^\ell \) we use Eq. (80) and for \( \alpha_T^\ell \) we use Eq. (76). These equations involve the double integration over the energies \( E \) and \( E_2 \). At this point is convenient to mention a technical aspect that we have to deal with in calculating the integrals

\[
\int_{E_2^-}^{E_2^+} \ln (y_0 + 1) dE_2, \quad (81)
\]

and

\[
\int_{M_2}^{E_2^+} \ln \left( \frac{y_0 + 1}{y_0 - 1} \right) dE_2, \quad (82)
\]
which are contained in the infrared-divergent integral $I_0$ of Eq. (C6) and in the integral $I_{0F}$ of Eq. (37), respectively. The integrals of Eqs. (81) and (82) diverge when $y_0 \to \pm 1$, that is when $E_2 = E_2^\pm$. However, it turns out that the result is finite as was shown in Ref. [12]. To calculate it we can either follow this approach and implement it in the program to evaluate the $\alpha_\ell$’s or we may neglect the points where $y_0 \to \pm 1$. This last is equivalent to leaving out the boundaries of the TBR and the FBR of the DP. The numerical difference between these two alternatives is negligible. Here we follow the second alternative.

In Table II we display our numerical results for the radiative corrections of the asymmetry coefficients $\alpha_\ell^R$ and $\alpha_\ell^T$. We compute them by taking the percentage differences (that is, we multiply by 100)

$$\delta \alpha_\ell^{R,T} = \alpha_\ell^{R,T} - \alpha_\ell^0,$$

where $\alpha_\ell^0$ is the uncorrected spin-asymmetry coefficient of the charged lepton, Eq. (77).

In the second column of Table II we display the $\delta \alpha^R$ corresponding to the TBR of the DP, in the third column the $\delta \alpha^T$ corresponding to the complete DP are given, in the fourth column we give the results for $\delta \alpha$ obtained from Eq. (23) and Table I of page 58 of Ref. [13] and, finally, in the fifth column we give the two values reported in Ref. [6].

From this Table II we see that there is a very good agreement between our $\delta \alpha^T$ and the $\delta \alpha$ of Refs. [4] and [5]. In both these references the FBR was included. The inclusion or exclusion of the FBR is appreciable, as can be seen by comparing the second and third columns, except for the decays $n \to pe\bar{\nu}$ and $\Sigma^- \to ne\bar{\nu}$. In several instances the inclusion of the FBR contributions reduces the total radiative corrections, even to the point of making them negligibly small. It may be that the values in the second column are one order of magnitude larger than the corresponding ones in the third column. Therefore, in general, there is an important difference between $\alpha_\ell^R$ and $\alpha_\ell^T$. Experiments of HSD where no provision to detect the photon has been made must use $\alpha_\ell^T$ to study such decays. But, if the photon is discriminated eliminating the FBR of the DP then the adequate coefficient to study HSD is $\alpha_\ell^R$.

VII. CONCLUSIONS

In this paper we have obtained the radiative corrections to order $\alpha$ to the Dalitz plot of the semileptonic decays of polarized spin-$1/2$ baryons, neglecting terms of order $\alpha q/\pi M_1$ and higher. Our main result, which is compactly given by

$$d\Gamma = d\Gamma_{TBR} + d\Gamma_{FBR},$$

is presented in two forms: one in which the triple $k$-integration is ready to be performed numerically, Eqs. (36) and (44), and another one in which such integration has been analytically performed, Eqs. (53) and (67). Since real photons may be discriminated either directly (by detection) or indirectly (by energy-momentum conservation) we have split the above two forms to cover this possibility. In case this photon discrimination takes place, Eq. (84) becomes

$$d\Gamma_{TBR} = d\Gamma_V + \left[d\Gamma'_B - \left(d\Gamma''_B + d\Gamma''''_B\right)\right],$$

18
where \( d\Gamma_V \) is given in Eq. (7) and it corresponds to the virtual RC. \( d\Gamma'_B, d\Gamma_B^T, \) and \( d\Gamma_B^{II} \) correspond to the bremsstrahlung RC and are given by Eqs. (27), (34), and (35), respectively, with the photon integrals remaining to be performed numerically. The infrared divergence and the finite terms that accompany it have been explicitly and analytically extracted, however. They appear in the \( I_0 \) of \( d\Gamma'_B \) and of \( d\Gamma_B^T \). The analytical result for Eq. (85) is given in Eq. (53).

When real photons are not discriminated what-so-ever, then the DP with the union of the TBR and the FBR must be used, as in Eq. (84). \( d\Gamma_B^{FBR} \) is given in Eq. (39), where the \( k \)-integration remains to be evaluated numerically, and the analytical result for Eq. (84) is given in Eq. (67).

An important integrated observable is the charged-lepton spin-asymmetry coefficient \( \alpha_\ell \). Using the analytical form we obtained the radiative corrections, through Eq. (67), to this observable. The integrations over \( E \) and \( E_2 \) were performed numerically and the results are displayed in Table II. A systematic behavior of the RC to \( \alpha_\ell \) is observed. The contribution of the FBR bremsstrahlung may be as important as the RC from the TBR and even of opposite sign, in such a way that when no photon discrimination takes place the complete RC to \( \alpha_\ell \) may become almost negligible. In this Table we also compare with results reported in other references. This comparison is satisfactory.

To our knowledge Eqs. (53) and (77) for the RC to the DP allowing for the TBR only and including the FBR are the only analytical expressions available in the literature. In Ref. [6] the relevant variables of the DP are also \( E \) and \( E_2 \), but there only numerical results are given, which are compromised to particular values of the form factors. Our numerical evaluations coincide very well with the numbers reported there for the percent RC of the asymmetry coefficient \( \alpha_\ell^T \) for the decays \( \Sigma^- \to n\nu\ell \) and \( \Lambda \to p\nu\ell \). We have also evaluated the RC for the process \( \Lambda_c^+ \to \Lambda e^+\nu \). The numbers obtained are presented in Table II and in this decay they are a good first approximation, which is already useful given that the experimental statistics are not yet too high.

Our results are model-independent and are not compromised to any particular value of the form factors. All the model dependence of radiative corrections has been absorbed into the \( f_1 \) and \( g_1 \) form factors in our approximation of neglecting contributions of order \( \alpha q/\pi M_1 \). This is indicated by putting a prime on them. For hyperons our results are reliable up to a precision of around 0.5%. This precision is useful for experiments involving several thousands of events. For high statistics experiments involving several hundreds of thousands of events or for decays involving charm as \( \Lambda_c^+ \to \Lambda e^+\nu \) or even heavier quarks our equations provide a good first approximation. To improve the precision of our formulas it becomes necessary to include \( \alpha q/\pi M_1 \) contributions. Our results are valid both for neutral or charged decaying hyperons and whether the emitted positively or negatively charged lepton is either electron-type or muon-type. To conclude let us remark that in a Monte Carlo analysis the advantage of the analytical form is that the triple \( k \)-integration does not have to be repeated every time the values of \( f_1 \) and \( g_1 \), or of \( E \) and \( E_2 \), are changed. This leads to a considerable saving of computer time.
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APPENDIX A:

In this Appendix we give for completeness the amplitudes for the RC of the decay (1). All of them are also given in Ref. [4]. The uncorrected transition amplitude $M_0$ for process (1) is

$$M_0 = \frac{G_V}{\sqrt{2}} [\bar{u}_B (p_2) W_\mu (p_1, p_2) u_A (p_1)] [\bar{u}_\ell (l) O_\mu v_\nu (p_\nu)] \, ,$$  \hspace{1cm} (A1)

where

$$W_\mu (p_1, p_2) = f_1 (q^2) \gamma_\mu + \frac{f_2 (q^2)}{M_1} \sigma_{\mu \nu} q^\nu + \frac{f_3 (q^2)}{M_1} q^\mu \gamma_5 + \left[ g_1 (q^2) \gamma_\mu + \frac{g_2 (q^2)}{M_1} \sigma_{\mu \nu} q^\nu + \frac{g_3 (q^2)}{M_1} q^\mu \right] \gamma_5. \hspace{1cm} (A2)$$

Here $O_\mu = \gamma_\mu (1 + \gamma_5)$ and $q$ is the four-momentum transfer.

The model-independent part of the virtual radiative corrections has the amplitude

$$M_v = \frac{\alpha}{2\pi} \left[ M_0 \phi (E) + M_{p_1} \phi' (E) \right] ,$$  \hspace{1cm} (A3)

where $\phi (E)$ and $\phi' (E)$ were given in Eqs. (11) and (12), respectively. $M_{p_1}$ is

$$M_{p_1} = \left( \frac{E}{m M_1} \right) \frac{G_V}{\sqrt{2}} [\bar{u}_B W_\lambda u_A] [\bar{u}_\ell \not{k} O_\lambda v_\nu] \ .$$  \hspace{1cm} (A4)

The model-dependent part of the virtual radiative corrections is absorbed into $M_0$ through the definition of effective form factors $f_1'$ and $g_1'$. This fact is denoted by putting a prime on $M_0$. The bremsstrahlung transition amplitude $M_B$ is obtained following the Low theorem [2],

$$M_B = \frac{e G_V}{\sqrt{2}} [\bar{u}_B W_\lambda u_A] [\bar{u}_\ell O_\lambda v_\nu] \left[ \frac{l \cdot \epsilon}{l \cdot k} - \frac{p_1 \cdot \epsilon}{p_1 \cdot k} \right] + \frac{e G_V}{\sqrt{2}} [\bar{u}_B W_\lambda u_A] \left[ \frac{\bar{u}_\ell \not{k} O_\lambda v_\nu}{2 l \cdot k} \right] \equiv M_a + M_b .$$  \hspace{1cm} (A5)

Within our approximation the Low theorem guarantees that no model-dependence appears here.
APPENDIX B:

In order to make this paper self-contained, we reproduce here all the coefficients which appear in our final results. For Eq. (7) they come from Ref. [4],

\[ A' = Q_1 E E_\nu^0 - Q_2 E p_2 (p_2 + l y_0) - Q_3 l (p_2 y_0 + l) + Q_4 E_\nu^0 p_2 (p_2 y_0) - Q_5 p_2^2 y_0 (p_2 + l y_0), \]  
(B1)

\[ A'_1 = D_1 E E_\nu^0 - D_2 l (p_2 y_0 + l), \]  
(B2)

\[ A''_1 = D_1 E E_\nu^0. \]  
(B3)

The coefficients \( Q_i, i = 1, \ldots, 7 \) are given in Eqs. (B1)-(B7) of Ref. [4], respectively. For completeness we reproduce them here.

\[ Q_1 = F_1^2 \left[ \frac{2E_2 - M_2}{M_1} \right] + \frac{1}{2} F_2^2 \left[ \frac{M_2 + E_2}{M_1} \right] + F_1 F_2 \left[ \frac{M_2 + E_2}{M_1} \right] + F_1 F_3 \left[ 1 + \frac{M_2}{M_1} \right] \times \left[ 1 - \frac{E_2}{M_1} \right] + F_2 F_3 \left[ \frac{M_2 + E_2}{M_1} \right] \left[ 1 - \frac{E_2}{M_1} \right] + G_1^2 \left[ \frac{2E_2 + M_2}{M_1} \right] - \frac{1}{2} G_2^2 \left[ \frac{M_2 - E_2}{M_1} \right] + G_1 G_2 \left[ \frac{M_2 - E_2}{M_1} \right] + G_1 G_3 \left[ \frac{M_2}{M_1} - 1 \right] \left[ 1 - \frac{E_2}{M_1} \right] \right], \]  
(B4)

\[ Q_2 = -\frac{G_1^2}{M_1} \left[ \frac{M_2}{M_1} - 1 \right] - G_2 G_3 \left[ \frac{M_2}{M_1} - 1 \right] + \frac{2 F_1 G_1}{M_1} + M_1^2 Q_5 \left[ \frac{M_2 - E_2}{M_1} \right], \]  
(B5)

\[ Q_3 = Q_1 - 2 F_1^2 \left[ \frac{E_2 - M_2}{M_1} \right] - 2 G_1^2 \left[ \frac{E_2 + M_2}{M_1} \right] - M_1^2 Q_5 \left[ \frac{M_1 - E_2}{M_1} \right], \]  
(B6)

\[ Q_4 = Q_2 - 4 \frac{F_1 G_1}{M_1}. \]  
(B7)

\[ Q_5 = \frac{F_1^2}{M_1^2} \left[ \frac{M_2 + E_2}{M_1} \right] - G_1^2 \left[ \frac{M_2 - E_2}{M_1} \right] - 2 F_1 F_3 \left[ \frac{M_2 + E_2}{M_1} \right] + 2 G_1 G_3 \left[ \frac{M_2 - E_2}{M_1} \right], \]  
(B8)

\[ Q_6 = F_1^2 \left[ \frac{E_2 - M_2}{M_1} - \frac{p_2 \beta y_0}{M_1} \right] + G_1^2 \left[ \frac{E_2 + M_2}{M_1} - \frac{p_2 \beta y_0}{M_1} \right] + 2 F_1 G_1 \left[ \frac{E_2 - p_2 \beta y_0}{M_1} \right] \times \left[ \frac{p_2 \beta y_0}{M_1} \right] + F_2 G_2 \left[ -1 + \left( 1 + \beta^2 \right) \frac{E}{M_1} + \frac{E_2}{M_1} + \frac{p_2 \beta y_0}{M_1} \right] \right] + \left[ \frac{G_1 G_2 - F_1 F_2}{M_1} \right] + F_2 G_2 \left[ -1 + \left( 1 + \beta^2 \right) \frac{E}{M_1} + \frac{E_2}{M_1} + \frac{p_2 \beta y_0}{M_1} \right] \right] - G_1 F_2 \left[ -1 - \frac{M_2}{M_1} + \left( 1 + \beta^2 \right) \frac{E}{M_1} + \frac{p_2 \beta y_0}{M_1} \right] \right] - \frac{M_2}{M_1} + \left( 1 + \beta^2 \right) \frac{E}{M_1} + \frac{p_2 \beta y_0}{M_1} \right] \right] - F_3 G_3 \left[ \frac{m_e^2}{M_1^2} \left( \frac{E}{M_1} - \frac{1 - \beta^2}{M_1} \frac{E}{M_1} + \frac{p_2 \beta y_0}{M_1} \right) \right]. \]
\[ + F_1 G_3 \left[ \frac{m^2}{M_1 E} \left( -1 + \frac{M_2}{M_1} + \frac{E}{M_1} \right) \right] - F_3 G_1 \left[ \frac{m^2}{M_1 E} \left( -1 - \frac{M_2}{M_1} + \frac{E}{M_1} \right) \right] \]

\[- (F_2 G_3 + F_3 G_2) \left\{ \frac{m^2}{M_1 E} \left( \frac{M_1 - E_2 - E}{M_1} \right) \right\}, \quad \text{(B9)}\]

\[ Q_7 = F_1^2 \left[ \frac{(M_1 + M_2) (E_2 - M_2)}{M_1 E} \right] + G_1^2 \left[ \frac{(M_1 - M_2) (E_2 + M_2)}{M_1 E} \right] \]

\[ + 2F_1 G_1 \left[ \frac{M_1 (-M_1 + E_2 + 2E) - m^2}{M_1 E} \right] + F_1 G_2 \left( \frac{E_2 - M_2}{M_1} \right) \]

\[ \times \left( \frac{M_1 - 2E - E_2}{E} \right) - G_1 F_2 \left( \frac{E_2 + M_2}{M_1} \right) \left( \frac{M_1 - 2E - E_2}{E} \right) \]

\[ + F_3 G_1 \left( \frac{E_2 + M_2}{M_1} \right) \left( \frac{m^2}{M_1 E} \right) - G_3 F_1 \left( \frac{E_2 - M_2}{M_1} \right) \left( \frac{m^2}{M_1 E} \right) \]

\[ + (F_1 F_2 - G_1 G_2) \left( \frac{E_2^2 - M_2^2}{M_1 E} \right). \quad \text{(B10)}\]

The coefficients \( D_j, j = 1, \ldots, 4 \) are

\[ D_1 = f_1'^2 + 3g_1'^2, \quad \text{(B11)}\]

\[ D_2 = f_1'^2 - g_1'^2, \quad \text{(B12)}\]

\[ D_3 = 2(f_1' g_1' - g_1'^2), \quad \text{(B13)}\]

\[ D_4 = 2(f_1' g_1' + g_1'^2). \quad \text{(B14)}\]

\( E^0_\nu \) and \( y_0 \) were defined in Eqs. (3) and (4), respectively.

The functions \( \theta_i \) which appear in Eqs. (48)-(51) corresponding to the TBR of the DP are given by

\[ \theta_i = \frac{1}{p_2} \left( T_i^+ + T_i^- \right), \quad \text{(B15)}\]

where \( i = 2, \ldots, 16, \) and

\[ T_2^\pm = \pm \frac{1 \mp a^\pm}{(1 \mp \beta)(1 \mp \beta a^\pm)} \ln \left\{ \frac{1 \mp \beta}{1 - \beta x_0} \right\} \pm \frac{(1 \pm x_0) \ln(1 \pm x_0)}{(1 \pm \beta)(1 - \beta x_0)} \]

\[ \pm \frac{1 \mp a^\pm}{(1 \mp \beta)(1 \mp \beta a^\pm)} \ln(1 \pm a^\pm) - \frac{(x_0 + a^\pm) \ln(\pm x_0 \pm a^\pm)}{(1 \mp \beta a^\pm)(1 - \beta x_0)}, \quad \text{(B16)}\]

\[ T_3^+ = T_3^- = \frac{1}{2\beta} \left\{ L \left[ \frac{1 - \beta}{1 - \beta x_0} \right] - L \left[ \frac{1 - \beta x_0}{1 + \beta} \right] - L \left[ \frac{1 + \beta a^-}{1 - \beta x_0} \right] + L \left[ \frac{1 + \beta a^-}{1 + \beta} \right] \right\} \]

\[ + L \left[ \frac{1 - \beta x_0}{1 + \beta a^+} \right] - L \left[ \frac{1 - \beta}{1 + \beta a^+} \right] + \ln \left[ \frac{1 - \beta x_0}{1 - \beta} \right] \ln \left[ \frac{1 + \beta a^+}{1 + \beta} \right] \}, \quad \text{(B17)}\]
\[ T_4^\pm = (x_0 \pm 1) \ln(1 \mp x_0) \pm (1 \pm a^\pm) \ln(1 \pm a^\pm) - (x_0 + a^\pm) \ln(x_0 \pm a^\pm), \]  

(B18)

\[ T_5^\pm = -\frac{1}{2} \left\{ (1 - x_0^2) \ln(1 \pm x_0) + (x_0 \mp 1)a^\pm + 1 - (1 - a^\pm^2) \ln(1 \pm a^\pm) + (x_0^2 - a^\pm^2) \ln[\pm(x_0 + a^\pm)] \right\}, \]  

(B19)

\[ T_6^\pm = \left[ -l + p_2 \pm \frac{\beta E_0^0(x_0 + a^\mp)}{1 + \beta a^\pm} \right] I_4 \pm \frac{\beta E_0^0(x_0 + a^\mp)}{(1 + \beta a^\pm)^2} I_1 \pm \left[ E_0^0 - \frac{\beta E_0^0(x_0 + a^\mp)}{1 + \beta a^\pm} \right] J_4 \]  

\[ = \frac{\beta E_0^0(x_0 + a^\mp)}{(1 + \beta a^\pm)^2} J_1 \pm \frac{E_0^0(x_0 + a^\mp)}{(1 + \beta a^\pm)^2} J_2^\mp, \]  

(B20)

\[ T_7^\pm = \left[ p_2 - l \pm \frac{\beta E_0^0(x_0 + a^\pm)}{1 + \beta a^\pm} \right] I_1 \pm \frac{E_0^0(x_0 + a^\pm)}{1 + \beta a^\pm} I_2^\pm \pm \left[ E_0^0 - \frac{\beta E_0^0(x_0 + a^\pm)}{1 + \beta a^\pm} \right] J_1 \]  

\[ - \frac{E_0^0(x_0 + a^\pm)}{1 + \beta a^\pm} J_2^\pm, \]  

(B21)

\[ T_8^\pm = -2(l - p_2 + E_0^0x_0) \mp E_0^0(x_0 + a^\pm)I_2^\pm - E_0^0(x_0 + a^\pm)J_2^\pm, \]  

(B22)

\[ \frac{T_9^\pm}{4l^2} = -\frac{3E}{2l^2} (l - p_2 + E_0^0x_0) + \frac{3(l - p_2)}{4\beta l} + \frac{3E_0^0p_2}{4l^2} + \beta G^\pm \left[ I_1 \mp \frac{(E_0^0)^2(x_0 + a^\pm)^2}{4l^2(1 + \beta a^\pm)} I_3^\pm \right] \]  

\[ - \frac{(E_0^0)^2(x_0 + a^\pm)^2}{4l^2(1 + \beta a^\pm)} J_3 ^\pm + G^\pm J_2^\pm \pm \frac{3E_0^0(E_0^0 + l x_0)}{4l^2 \pm} \beta G^\pm \left[ J_1 \pm G^\pm J_2^\pm \right], \]  

(B23)

\[ T_{10}^\pm = \frac{1}{3} (x_0^3 \mp 1) \ln(1 \mp x_0) + \frac{1}{3} \left[ (a^\mp)^3 \mp 1 \right] \ln(1 \mp a^\mp) - \frac{1}{3} \left[ x_0^3 + (a^\mp)^3 \right] \ln[\mp(x_0 + a^\mp)] \]  

\[ + \frac{1}{6} (1 - x_0^2)(a^\mp \mp 1) \mp \frac{1}{3} (x_0 \pm 1) \left[ 1 - (a^\mp)^2 \right], \]  

(B24)

\[ T_{11}^\pm = T_{11}^\pm = \frac{1}{2p_2\beta} \left\{ E_0^0 \left[ (1 - \beta x_0)J_4 - J_1 \right] - (\beta E_0^0 + l - p_2)I_4 + (l - p_2)I_1 \right\}, \]  

(B25)

\[ T_{12}^\pm = T_{12}^\pm = \frac{1}{2p_2\beta} \left[ E_0^0(1 - \beta x_0)J_1 + 2E_0^0x_0 + 2(l - p_2) - (\beta E_0^0 + l - p_2)I_1 \right], \]  

(B26)

\[ T_{13}^\pm = T_{13}^\pm = -\frac{1}{2p_2} E_0^0(1 - x_0^2), \]  

(B27)

\[ T_{14}^\pm = E_0^0 \left[ 1 + x_0^2 + 2a^\pm(x_0 \mp 1) \pm a^\pm(x_0 + a^\mp)(I_2^\pm \pm J_2^\pm) \right], \]  

(B28)

\[ T_{15}^\pm = 3E_0^0 \left[ 2p_2(1 + y_0) + l(1 - x_0^2) \right] - (E_0^0)^2(x_0 + a^\pm)^2(J_3^\pm \pm I_3^\pm) \]  

\[ - 2lE_0^0(x_0 + a^\pm)a^\pm(J_2^\pm \pm I_2^\pm), \]  

(B29)
The explicit form of the photon integrals corresponding to the FBR of the DP is

\[ T_{16}^\pm = 4l^2 \left[ \frac{3}{2\beta^2} \left[ 2(l - p_2 + E_{\nu}^0 x_0) + \beta E_{\nu}^0(1 - x_0^2) \right] + \left( -\frac{3(l - p_2 + \beta E_{\nu}^0)}{2\beta^2} \right) 
- p_2(1 + y_0) + \frac{p_2(E_{\nu}^0)^2}{2l^2} \right] I_1 - \frac{(E_{\nu}^0)^2(x_0 + a^\pm)^2}{2l(1 + \beta a^\pm)} (\beta J_1 + J_2^\pm \pm \beta I_1 \pm I_2^\pm) 
+ \left( \frac{3E_{\nu}^0(1 - \beta x_0)}{2\beta^2} + \frac{(E_{\nu}^0)^2(E_{\nu}^0 + l x_0)}{2l^2} \right) J_1 \right]. \] (B30)

The following definitions are used in the above expressions

\[ x_0 = -\frac{p_2 y_0 + l}{E_{\nu}^0}, \quad a^\pm = \frac{E_{\nu}^0 \pm p_2}{l}, \] (B31)

\[ I_1 = \frac{2}{\beta} \arctanh \beta, \quad I_2^\pm = \ln \left| \frac{a^\pm + 1}{a^\pm - 1} \right|, \] (B32)

\[ I_3^\pm = \frac{2}{a^\pm^2 - 1}, \quad I_4 = \frac{2}{1 - \beta^2}, \] (B33)

\[ J_1 = -\frac{1}{\beta} \left\{ \ln \left| \frac{1 + \beta}{1 - \beta x_0} \right| + \ln \left| \frac{1 - \beta}{1 - \beta x_0} \right| \right\}, \] (B34)

\[ J_2^\pm = \ln \left| \frac{a^\pm - 1}{a^\pm + x_0} \right| + \ln \left| \frac{a^\pm + 1}{a^\pm + x_0} \right|, \] (B35)

\[ J_3^\pm = -2 \left[ \frac{a^\pm}{a^\pm^2 - 1} - \frac{1}{a^\pm + x_0} \right], \] (B36)

\[ J_4 = \frac{2}{\beta} \left[ \frac{1}{1 - \beta^2} - \frac{1}{1 - \beta x_0} \right], \] (B37)

\[ G^\pm = \pm \frac{\beta(E_{\nu}^0)^2(x_0 + a^\pm)^2}{4l^2(1 + \beta a^\pm)^2} \pm \frac{a^\pm(a^\pm^2 - 1)}{4(1 + \beta a^\pm)}. \] (B38)

The sums \( \theta' + \theta'' \) and \( \theta'' + \theta''' \) which appear in Eq. (54) are

\[ \theta' + \theta'' = \frac{p_2 l}{2} \left[ -E_{\nu}^0(1 - \beta^2) \theta_2 + \left( E_{\nu}^0 - \frac{1 + \beta^2}{2} E \right) \theta_3 + \frac{E}{2} \theta_4 \right. \]
\[ + \left. \frac{l}{2} \theta_5 + \frac{1 - \beta^2}{2} \theta_6 - \frac{2E - E_{\nu}^0}{2E} \theta_7 + \frac{1}{2} \theta_8 - \frac{1}{4E} \theta_9 \right], \] (B39)

\[ \theta'' + \theta''' = \frac{p_2 l}{2} \left[ \theta_0 - \left( E + E_{\nu}^0 + \beta p_2 y_0 \right) \theta_3 + \left( E_{\nu}^0 + E \right) \theta_4 + l \theta_5 \right]. \] (B40)

The explicit form of the photon integrals corresponding to the FBR of the DP is

\[ \theta_{2F} = \frac{1}{\beta p_2} \left[ \frac{I_2^-}{b^-} - \frac{I_2^+}{b^+} + \frac{E^2}{m^2} \left( I_2^+ - I_2^- + \beta \ln \left| \frac{I_3^-}{I_3^+} \right| \right) \right] + \frac{2l_1}{Eb-b^+}, \] (B41)
\[ \theta_{3F} = \frac{I_1}{p_2} \ln \left| \frac{b^+}{b^-} \right| + \frac{1}{\beta p_2} \left[ L \left( \frac{1 - \beta}{b^-} \right) - L \left( \frac{1 - \beta}{b^+} \right) + L \left( \frac{1 + \beta}{b^-} \right) - L \left( \frac{1 + \beta}{b^+} \right) \right], \]  
\[ \theta_{4F} = \frac{1}{p_2} \left[ a^+ I_2^+ - a^- I_2^- + \ln \left| \frac{I_3^-}{I_3^+} \right| \right], \]  
\[ \theta_{5F} = \frac{1}{2p_2} \left[ (1 - a^+ \beta) I_2^+ - (1 - a^- \beta) I_2^- + \frac{4p_2}{I} \right], \]  
\[ \theta_{6F} = 2 \frac{y_0}{(b^-)^2} (I_2^+ - \beta I_1) - 2 \frac{y_0}{(b^+)^2} (I_2^- + \beta I_1) + 2 \left[ 2 + \beta \left( \frac{y_0^-}{b^-} - \frac{y_0^+}{b^+} \right) \right] I_4, \]  
\[ \theta_{7F} = 2 \left[ 2I_1 + \frac{y_0^-}{b^-} (\beta I_1 + I_2^-) - \frac{y_0^+}{b^+} (\beta I_1 + I_2^+) \right], \]  
\[ \theta_{8F} = 2 \left[ 4 + (y_0^-) I_2^- - (y_0^+) I_2^+ \right], \]  
\[ \theta_{9F} = 24E + 2 \left[ 6(E_0^0 - E) + \beta (G_F^- + G_F^+) \right] I_1 + 2(G_F^- I_2^- + G_F^+ I_2^+) + 2p_2 \left[ \frac{(y_0^-)^2}{b^-} I_3^- - \frac{(y_0^+)^2}{b^+} I_3^+ \right], \]  
\[ \theta_{10F} = \frac{1}{3p_2} \left[ 2(a^2 - a^+ a^2) - a^- I_2^- + a^+ I_2^+ + \ln \left| \frac{I_3^-}{I_3^+} \right| \right], \]  
\[ \theta_{11F} = \frac{2(I_4 - I_1)}{\beta p_2}, \]  
\[ \theta_{12F} = \frac{2(I_1 - 2)}{\beta p_2}, \]  
\[ \theta_{13F} = 0, \]  
\[ \theta_{14F} = 2 \left[ (2 - a^- I_2^-) (y_0^-) - (2 - a^+ I_2^+) (y_0^+) \right], \]  
\[ \theta_{15F} = 24E_0 + 4l \left[ a^- y_0^- I_2^- - a^+ y_0^+ I_2^+ \right] + 2p_2 \left[ (y_0^-)^2 I_3^- - (y_0^+)^2 I_3^+ \right], \]  
\[ \theta_{16F} = 24E^2 (I_1 - 2) + 8 [(E_0^0)^2 - 2E^2 \beta^2] I_1 \]  
\[ + 4lp_2 \left[ \frac{(y_0^-)^2}{b^-} (\beta I_1 + I_2^-) - \frac{(y_0^+)^2}{b^+} (\beta I_1 + I_2^+) \right], \]  
where \( a^\pm, I_1, I_2^\pm, I_3^\pm, I_4 \) are given in Eqs. (B31)-(B33) and \( b^\pm = 1 + \beta a^\pm, \quad y_0^\pm = y_0 \pm a^\pm, \) 
\[ G_F^\pm = \mp \beta \left( 2Ea^\pm + p_2 \frac{y_0^\pm}{b^\pm} \right) \frac{y_0^\pm}{b^\pm}. \]  

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APPENDIX C:

In this appendix we give a brief review of the procedure of Ref. [3] to extract the infrared divergence and the finite terms that come along with it. This procedure can be adapted to our case by expressing the differential decay rate of the bremsstrahlung radiative corrections \( d\Gamma_B \) in terms of the invariant mass \( \eta = (p_\nu + k)^2 \) as follows

\[
d\Gamma_B = \frac{M_2 m_{m_\nu} dE dE_2 d\Omega_\ell d\phi_2}{4(2\pi)^8} \int_{\eta_{\text{min}}}^{\eta_{\text{max}}} d\eta \frac{d^3 k d^3 p_\nu}{\omega_E} \sum_{\text{spins}} |M_B|^2 \delta^4(p_1 - p_2 - l - p_\nu - k). \tag{C1}
\]

For the TBR and in the CM-frame of the decaying particle, \( \eta = 2p_2 l (y_0 - y) \), and \( \eta_{\text{max}} = 2p_2 l (y_0 + 1) \) and \( \eta_{\text{min}} = \lambda^2 \), with \( \lambda \to 0 \). Here \( \lambda \) is a small mass assigned to the polarization here) the divergent part of \( \sum_{\text{spins}} |M_B|^2 \) is contained in the first two terms of the square brackets of

\[
\sum_{\text{spins}} |M_B|^2 = \frac{e^2 G_F^2}{2} \frac{2M_1}{M_2 m_{m_\nu}} \sum_\epsilon \left( \frac{l + \epsilon}{l - \epsilon} - \frac{p_1 + \epsilon}{p_1 - \epsilon} \right)^2 \times \left[ D_1 E E_\nu^0 - D_2 l \cdot (p_2 + 1) - D_1 E \omega - D_2 l \cdot k \right]. \tag{C2}
\]

Because \( l \cdot (p_2 + 1) = l p_2 y_0 + 1^2 - \eta/2 \) (choosing the \( z \)-axis along \( l \)), we can split \( d\Gamma_B \) as

\[
d\Gamma_B = \frac{1}{2} G_F^2 \frac{dE dE_2 d\Omega_\ell d\phi_2}{(2\pi)^8} \frac{4 M_1}{\alpha} \left\{ A'_1 \lim_{\lambda \to 0} \int_{\lambda^2}^{\eta_{\text{max}}} d\eta \frac{1}{2\pi} \frac{d^3 k d^3 p_\nu}{\omega_E} \delta^4(p_1 - p_2 - l - p_\nu - k) \times \left[ \frac{2l + p_1}{(p_1 + k) (l + k)} - \frac{m^2}{w (p_1 + k)^2} - \frac{M_1^2}{w (p_1 + k)^2} \right] \right.
\]
\[
+ \left. \frac{1}{4} \int_{\lambda^2}^{\eta_{\text{max}}} d\eta \frac{1}{2\pi} \frac{d^3 k d^3 p_\nu}{\omega_E} \delta^4(p_1 - p_2 - l - p_\nu - k) \sum_\epsilon \left( \frac{l + \epsilon}{l - \epsilon} - \frac{p_1 + \epsilon}{p_1 - \epsilon} \right)^2 \times \left[ -D_1 E \omega - D_2 \left( l \cdot k + \frac{1}{2} \eta \right) \right] \right\} \sum_{\epsilon, \beta} [ |M_b|^2 + 2 \text{Re} [M_1 E [M_b] \beta] ], \tag{C3}
\]

where \( A'_1 \) is given in Eq. (B2). In the first integral we perform the sum over polarizations indicated in Eq. (B2) in covariant form. This allows us to identify this integral with the divergent integral \( I_0 \) of Ref. [3], namely,

\[
I_0 (E, E_2) = \frac{1}{4} \lim_{\lambda \to 0} \int_{\lambda^2}^{\eta_{\text{max}}} d\eta \left[ 2(l + p_1) I_{11} (l, p_1) - m^2 I_{20} (l, p_1) - M_1^2 I_{02} (l, p_1) \right], \tag{C4}
\]

with

\[
I_{mn} (l, p_1) = \frac{1}{2\pi} \int \frac{d^3 p_\nu d^3 k}{E_\nu \omega} \delta^4(p_1 - p_2 - l - p_\nu - k) \frac{m^2}{(l + k)^m (p_1 + k)^n}, \tag{C5}
\]

which are invariant integrals. The final form of \( I_0 \) is

\[
I_0 (E, E_2) = \frac{1}{\beta} \text{arctanh} \beta \left[ 2 \ln \left( \frac{2l}{\lambda} \right) + \ln \left( \frac{m^2}{4 (E + l) r_+} \right) \right] - \frac{1}{2} \frac{\ln \left( \frac{a^2}{4 r_+} \right)}{\eta_{m_\nu}^2} \left[ \frac{m^2}{m} \ln \left( \frac{2 m E_\nu (q^2 - m^2)}{2 m E_\nu (q^2 - m^2)} \right) \right] - \frac{1}{\beta} \left[ \ln \left( \frac{r_+}{a^2} \right) \right] - \frac{1}{\beta} \left[ \ln \left( \frac{m^2}{E_\nu (q^2 - m^2)} \right) \right], \tag{C6}
\]

with

\[
f_{\alpha} = \eta_{\nu_\alpha} \eta_{\nu_\beta} \eta_{\nu_\gamma} \eta_{\nu_\delta} \delta^4(p_1 - p_2 - l - p_\nu - k) \frac{m^2}{(l + k)^m (p_1 + k)^n}.
\]
where
\[
(E + l) r_\pm = \left[ E_{\nu} l^2 \left( q^2 - m^2 \right) - a^2 E/4 \right] \\
\pm \left\{ \left[ E_{\nu} l^2 \left( q^2 - m^2 \right) - a^2 E/4 \right]^2 - m^2 a^4/16 \right\}^{1/2},
\]
(C7)

\[
a^2 = \eta \left( 4p_2 l - \eta_m \right),
\]
(C8)
and
\[
q^2 = M_1^2 - 2M_1 E_2 + M_2^2.
\]
(C9)

For the FBR of DP the lower limit of the \(d\Gamma_B\) of Eq. (C1) is \(\eta_{\min} = 2lp_2 (y_0 - 1)\), and then \(I_0\) is no more infrared-divergent. We can calculate it from Eq. (C4) by changing \(\lambda^2\) by \(\eta_{\min}\). The second part of \(d\Gamma_B\) in Eq. (C3) is infrared-convergent. It is more convenient to transform it to the form of Eq. (38) of Ref. [3]. We can accomplish this by performing the \(\delta\)-integration and by changing the \(\eta\) variable to the \(y\) variable using \(\eta = 2p_2 l (y_0 - y)\). We do not reproduce the result here because it is very long. It is given in our Eq. (27) for \(d\Gamma_B'\).
REFERENCES

[1] A. Sirlin, Phys. Rev. 164, 1767 (1967).
[2] F. E. Low, Phys. Rev. 110, 974 (1958).
[3] D.M. Tun, S.R. Juárez W., and A. García, Phys. Rev. D 40, 2967 (1989).
[4] R. Flores-Mendieta, A. García, A. Martínez, and J.J. Torres, Phys. Rev. D 55, 5702 (1997).
[5] E. S. Ginsberg, Phys. Rev. 162, 1570 (1967).
[6] F. Glück and K. Toth, Phys. Rev. D 46, 2090 (1992).
[7] M. Jauch and F. Rohrlich, The Theory of Photons and Electrons (Addison-Wesley, Reading, MA, 1955).
[8] D.M. Tun, S.R. Juárez W., and A. García, Phys. Rev. D 44, 3589 (1991).
[9] S. R. Juárez W., Phys. Rev. D 53, 3746 (1996).
[10] R. Pérez-Marcial et al., Phys. Rev. D 40, 2955 (1989).
[11] A. Martínez, D. M. Tun, A. García, and G. Sánchez-Colón, Phys. Rev. D 50, 2325 (1994).
[12] J. Martínez-Castro (unpublished).
[13] A. García and P. Kielanoswki, The Beta Decay of Hyperons, Lectures Notes in Physics Vol. 222 (Springer-Verlag, Berlin, 1985).
TABLE I. Values of the form factors used in our numerical calculations. For the first three decays we take the form factor ratios of Ref. [6] while for the other decays we use Ref. [13]. For \( \Lambda^+_c \rightarrow \Lambda \) we take the values of Ref. [10]. For convenience we include in the last column the uncorrected \( \alpha_0^\ell \) of Eq. (77) corresponding to this choice of form factors.

| Decay        | \( f_1 \) | \( f_2 \) | \( g_1 \) | \( \alpha_0^\ell \) |
|--------------|----------|----------|----------|-------------------|
| \( n \rightarrow P \) | 1.000    | 1.970    | 1.261    | -0.0850           |
| \( \Lambda \rightarrow P \) | 1.236    | 1.199    | 0.890    | 0.0200            |
| \( \Sigma^- \rightarrow n \) | 1.000    | -0.970   | -0.340   | -0.6319           |
| \( \Sigma^- \rightarrow \Lambda \) | 0.000    | 1.172    | 0.601    | -0.7030           |
| \( \Sigma^+ \rightarrow \Lambda \) | 0.000    | 1.172    | 0.601    | -0.6474           |
| \( \Xi^- \rightarrow \Lambda \) | 1.225    | -0.074   | 0.354    | 0.2579            |
| \( \Xi^- \rightarrow \Sigma^0 \) | 0.707    | 1.310    | 0.899    | -0.1989           |
| \( \Xi^0 \rightarrow \Sigma^+ \) | 1.000    | 1.853    | 1.267    | -0.1913           |
| \( \Lambda^+_c \rightarrow \Lambda \) | 0.350    | 0.090    | 0.610    | -0.9513           |

TABLE II. Percentage radiative corrections [that is, Eq. (83) multiplied by 100] of the spin-asymmetry coefficient of the charged lepton in hyperon semileptonic decays. The prediction in the fourth column for \( \Lambda^+_c \rightarrow \Lambda \) uses the approach of Ref. [13], but it was not actually given there.

| Decay        | \( \delta \alpha^R = \alpha^R_\ell - \alpha_0^\ell \) | \( \delta \alpha^T = \alpha^T_\ell - \alpha_0^\ell \) | \( \delta \alpha \) Ref. [13] | \( \delta \alpha \) Ref. [6] |
|--------------|--------------------------------------------------|--------------------------------------------------|-----------------------------|-----------------------------|
| \( n \rightarrow P \) | 0.0119                                           | 0.0095                                           | 0.0101                      |                             |
| \( \Lambda \rightarrow P \) | 0.0813                                           | 0.0014                                           | -0.0023                     | -0.0                         |
| \( \Sigma^- \rightarrow n \) | 0.0832                                           | 0.0815                                           | 0.0758                      | 0.1                         |
| \( \Sigma^- \rightarrow \Lambda \) | 0.1432                                           | 0.0836                                           | 0.0770                      |                             |
| \( \Sigma^+ \rightarrow \Lambda \) | 0.1287                                           | 0.0755                                           | 0.0911                      |                             |
| \( \Xi^- \rightarrow \Lambda \) | 0.1024                                           | -0.0246                                          | -0.0310                     |                             |
| \( \Xi^- \rightarrow \Sigma^0 \) | 0.3327                                           | 0.0371                                           | 0.0212                      |                             |
| \( \Xi^0 \rightarrow \Sigma^+ \) | 0.3312                                           | 0.0350                                           | 0.0208                      |                             |
| \( \Lambda^+_c \rightarrow \Lambda \) | 0.0757                                           | 0.1294                                           | 0.1098                      |                             |