Quintessential Kination and Thermal Production of SUSY e-WIMPs

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Abstract. The impact of a kination-dominated (KD) phase generated by a quintessential exponential model on the thermal abundance of Supersymmetric (SUSY) extremely Weakly Interacting Massive Particles (e-WIMPs) is investigated. For values of the quintessential energy-density parameter at the eve of nucleosynthesis close to its upper bound, we find that (i) the gravitino ($\tilde{G}$) constraint is totally evaded for unstable $\tilde{G}$’s; (ii) the thermal abundance of stable $\tilde{G}$ is not sufficient to account for the cold dark matter (CDM) of the universe; (iii) the thermal abundance of axinos ($\tilde{a}$) can satisfy the CDM constraint for values of the initial (“reheating”) temperature well above those required in the standard cosmology (SC).

Keywords: Cosmology, Dark Energy, Dark Matter
PACS: 98.80.Cq,98.80.-k, 95.35.+d

INTRODUCTION

A plethora of data [1] indicates that the two major components of the universe are CDM and Dark Energy (DE). The DE component can be explained with the introduction of a slowly evolving scalar field, $q$, called quintessence [2]. An open possibility in this scenario is the existence of an early KD era [3], where the universe is dominated by the kinetic energy of $q$. During this era, the expansion rate of the universe increases with respect to (w.r.t.) its value in SC. As a consequence, the relic abundance of WIMPs (e.g., the lightest neutralino) is also significantly enhanced [4, 5].

WIMPs are the most natural candidates for the second major component of the universe, CDM. In addition, supersymmetric theories predict the existence of even more weakly interacting massive particles, known as e-WIMPs [6]. These are the gravitino and the axino (SUSY partners of the graviton and the axion respectively). Their interaction rates are suppressed by the reduced Planck scale, $m_p = M_P/\sqrt{8\pi}$ (where $M_P = 1.22 \times 10^{19}$ GeV is the Planck mass) in the case of $\tilde{G}$ and by the axion decay constant, $f_a \sim (10^{10} - 10^{12})$ GeV in the case of $\tilde{a}$. Due to the weakness of their interactions, e-WIMPs depart from chemical equilibrium very early and their relic density is diluted by primordial inflation. However, they can be reproduced in two ways: (i) in the thermal bath, through scatterings and decays involving superpartners [7, 8, 9], and (ii) non-thermally from the decay of the next-to-lightest supersymmetric particle; in this case the results are highly model dependent.

In this talk, which is based on Ref. [10], we reconsider the creation of a quintessential kination scenario (QKS) in the context of the exponential quintessential model [5, 11], and investigate its impact on the thermal production of e-WIMPs.
THE QUINTESSENTIAL EXPONENTIAL MODEL

The quintessence field, $q$, of our model satisfies the equation:

$$\dot{q} + 3Hq + dV/dq = 0, \quad \text{where } V = V_0 e^{-\lambda q/m_P}$$  \hspace{1cm} (1)

is the adopted potential, dot denotes derivative w.r.t the cosmic time $t$ and $H$ is the Hubble parameter, $H \simeq \sqrt{\rho_q + \rho_R}/\sqrt{3}m_p$ where $\rho_q = \dot{q}^2/2 + V$ and $\rho_R = \pi^2 g_* T^4 / 30$ the radiation energy density with $g_*$ the effective number of massless degrees of freedom.

We impose on our quintessential model the following constraints:

1. **Initial Domination of Kination.** We focus our attention in the range of parameters with $\Omega_q^I = \Omega_q(T_I) \gtrsim 0.5$ where $\Omega_q \simeq \rho_q/(\rho_q + \rho_R)$ is the quintessential energy-density parameter.

2. **Nucleosynthesis (NS) Constraint.** At the onset of NS, $T_{NS} = 1$ MeV, $\rho_q$ is to be sufficiently suppressed w.r.t $\rho_R$, i.e., $\Omega_q^{NS} = \Omega_q(T_{NS}) \leq 0.21$ at 95% c.l.

3. **Inflationary Constraint.** Assuming that the power spectrum of the curvature perturbations is generated by an early inflationary scale, an upper bound on the initial value of $H, H_I$, can be obtained, namely $H_I \lesssim 2.65 \times 10^{14}$ GeV.

4. **Cosmic Coincidence Constraint.** The present value of $\rho_q$, $\rho_q^0$, must be compatible with the preferred range for DE, implying $\Omega_q^0 = 0.74$.

5. **Acceleration Constraint.** Successful quintessence has to account for the present-day acceleration of the universe, i.e. $-1 \leq w_q(0) \leq -0.86$ (95% c.l.), where $w_q = (\dot{q}^2/2 - V)/(\dot{q}^2/2 + V)$ is the barotropic index of the $q$-field.

Solving Eq. (1) with $q(T_I) = 0$ and $\dot{q}(T_I)$ such that the condition 1 is satisfied, we find that, during its evolution, $q$ undergoes three phases (see Fig. 1-(a), where we plot $\log \bar{\rho}_i$ with $i = q$ and $R$ versus $T$ for $T_I = 10^9$ GeV, $\Omega_q^{NS} = 0.01$ and $\lambda = 0.5$):

- The **KD phase** [3], where $\rho_q \simeq \dot{q}^2/2$, implying $\omega_q \simeq 1$ and thus, $\rho_q \propto T^6$. The transition from kination to radiation occurs at $T_{KR}$ such that $\rho_q(T_{KR}) = \rho_R(T_{KR})$ – see Fig. 1-(a). Solving this we find $T_{KR} = T_{NS} \left( g_*^{NS} / g_*^{KR} \right)^{1/2} \left( (1 - \Omega_q^{NS}) / \Omega_q^{NS} \right)^{1/2}$.

- The **frozen-field dominated phase**, where the universe becomes radiation dominated and $\rho_q$ is dominated initially by $\dot{q}/2$ and then by $V$.

- The **late-time attractor dominated phase** during which $\rho_q \simeq V$ dominates the universal evolution, with $w_q \simeq w_q^{fp} = \lambda^2/3 - 1$ for $\lambda < \sqrt{3}$.

Today we obtain a transition from the frozen-field to the attractor dominated phase. Although this does not provide a satisfactory resolution of the coincidence problem, the observational data can be reproduced. In particular, satisfying the condition 5 implies $\lambda \leq 0.9$ whereas the condition 4 can be fulfilled by conveniently adjusting $V_0$ [5, 11]. These two constraints are independent of the parameters $T_I$ and $\Omega_q^{NS}$, which can be restricted by the requirements 1 – 3. In Fig. 1-(b), we present the allowed region of our model in the log $T_I$ – log $\Omega_q^{NS}$ plane (shaded in gray and light gray). We observe that for a reasonable set of the parameters $(\lambda, T_I, \Omega_q^{NS})$, our model can become consistent [5, 11] with the observational data.
THERMAL PRODUCTION OF SUSY e-WIMPS

The number density $n_{\chi}$ of a SUSY e-WIMP $\chi$ (where $\chi$ stands for $\tilde{G}$ or $\tilde{a}$) satisfies the Boltzmann equation [8, 9, 13] which can be written as

$$\dot{n}_{\chi} + 3Hn_{\chi} = C_{\chi}n_{eq}^2 + \sum_{i} \frac{g_i}{2\pi^2} m_i^2 T K_1(m_i/T) \Gamma_i.$$  (2)

Here $n_{eq} = \xi(3)T^3/\pi^2$ is the equilibrium number density of the bosonic relativistic species, $m_i [g_i]$ is the mass [number of degrees of freedom] of the particle $i$ and $K_n$ is the modified Bessel function of the 2nd kind. In the relativistic regime ($T \gg m_i$) $C_{\chi}$ has been calculated [7, 9] using the Hard Thermal Loop Approximation, resulting to $C_{\chi} = C_{\chi}^{HT}$, where

$$C_{\chi}^{HT} = \begin{cases} \left(\frac{3\pi}{16\xi(3)}m_{\tilde{G}}^2\right) \sum_{\alpha=1}^{3} \left(1 + M_{\alpha}^2/3m_{\chi}^2\right) c_{\alpha}g_{\alpha}^2 \ln(k_{\alpha}/g_{\alpha}) & \text{for} \quad \chi = \tilde{G}, \\ \left(27g_3^4/\pi^3f_\alpha^2\xi(3)\right) \ln(1.108/g_3) & \text{for} \quad \chi = \tilde{a}. \end{cases}$$  (3)

Here, $g_{\alpha}$ and $M_{\alpha}$ (with $\alpha = 1,2,3$) are the gauge coupling constants and gaugino masses respectively, associated with the gauge groups $U(1)_Y$, $SU(2)_L$, and $SU(3)_C$, $(k_{\alpha}) = (1.634, 1.312, 1.271)$ and $(c_{\alpha}) = (33/5, 27, 72)$. Throughout our analysis we impose universal initial conditions for the gaugino masses, $M_{\alpha}(M_{\text{GUT}}) = M_{1/2}$ and gauge coupling constant unification, i.e., $g_{\alpha}(M_{\text{GUT}}) = g_{\text{GUT}}$. Eq. (3) gives meaningful results only for $T > T_C = 10^4$ GeV. Towards lower values of $T$, non-relativistic ($T \ll m_i$) contributions start playing an important role. In the case of $\tilde{a}$, where $\Omega_{\tilde{a}}h^2$ takes cosmologically interesting values for $T \ll m_i$, $C_{\tilde{a}}$ has been calculated numerically in Ref. [10]. In the latter case, $\Gamma_i$ with $i = \tilde{g}, \tilde{q}$ and $\tilde{B}$ are taken into account, also, using the formulas of Refs. [13, 14].

The impact of the KD phase on the relic density $\Omega_{\chi}h^2 = 2.748 \times 10^8 Y_{\chi}^0 m_{\chi}/\text{GeV}$ can be found by solving Eqs. (1) and (2) numerically [10]. However, we can get a clear

![Diagram](image.png)
picture of the results via semi-analytical estimates [10]. In the high $T$ regime ($T_i \gg T_C$ and $T_{KR} \gg T_C$) we find, within a 10% accuracy, that:

$$Y_\chi^0 = \frac{n_\chi}{s} \simeq \begin{cases} y^\text{HT}_\alpha T_i / \sqrt{s^{KR}/s_{\alpha}} T_{KR} \ln (T_i/T_{KR}) + y^\text{HT}_\alpha T_{KR} & \text{in the SC,} \\ y^\text{HT}_\alpha T_i & \text{in the QKS,} \end{cases}$$

where $s$ is the entropy density and $y^\text{HT}_\alpha = \sqrt{8g_*/45\pi m_p y^\text{eq}2 C^\text{HT}_\chi}$ with $y^\text{eq} = n^{\text{eq}}/s$. From these expressions we can easily deduce that in QKS, $Y^0_\chi$ decreases with respect to its value in the SC (being proportional to $T_i$ in SC and to $T_{KR}$ – lower than $T_i$ – in QKS).

In the low $T$ regime ($T_i \ll T_C$ or $T_{KR} \ll T_C$), we find cosmologically interesting solutions only in the case of $\bar{a}$. Focusing on the most intriguing possibility, $T_i \gg T_{\text{SUSY}} = 1$ TeV but $T_{KR} \ll T_{\text{SUSY}}$, and for the benchmark values of $m_i$ used in our analysis:

$$m_\bar{q} = 1 \text{ TeV}, \quad m_\bar{g} = 1.5 \text{ TeV} \quad \text{and} \quad m_\beta = 0.3 \text{ TeV.}$$

we can write simple empirical relations which reproduce rather accurately the numerical results. In particular, in the SC, using fitting technics, we get a relation with a 15% accuracy, namely: $\Omega_{\bar{a}} h^2 = A m_\bar{a} (1 + CT_i) e^{-B/|T_i|} / f^2_{\alpha}$ with $A = 1.44 \times 10^{24}$ GeV, $B = 745.472$ GeV and $C = 0.001$/GeV. We observe that $\Omega_{\bar{a}} h^2$ decreases sharply as $T_i$ decreases due to the exponential factor. In the QKS this suppression is avoided since $\Omega_{\bar{a}} h^2 = D m_\bar{a} / f^2_{\alpha} \sqrt{\Omega^\text{NS}_q}$ with $D = 9.26 \times 10^{17}$ GeV. This relation reproduces the numerical results with excellent accuracy. We observe that $\Omega_{\bar{a}} h^2 \propto 1/\sqrt{\Omega^\text{NS}_q}$ or $\Omega_{\bar{a}} h^2 \propto T_{KR}$.

In Figs. 2-(a) [2-(b)], we display $\Omega_\chi h^2$ [$\Omega_\bar{a} h^2$] versus $m_\bar{G}$ [$m_\bar{a}$] for $M_{1/2} = 0.7$ TeV [$f_\alpha = 10^{11}$ GeV], $T_i = 10^9$ GeV and various $\Omega^\text{NS}_q$'s indicated in the curves. We observe that $\Omega_\chi h^2$ decreases as $\Omega^\text{NS}_q$ increases, $\Omega_\bar{G} h^2 \propto 1/m_\bar{G}$ and $\Omega_{\bar{a}} h^2 \propto m_\bar{a}$. 

**FIGURE 2.** $\Omega_\chi h^2$ as a function of $m_\chi$ ($\chi = \bar{G}$ ($\chi = \bar{a}$)) for various $\Omega^\text{NS}_q$'s, indicated on the curves, $T_i = 10^9$ GeV and $M_{1/2} = 0.7$ TeV [$f_\alpha = 10^{11}$ GeV] (a [b]). For $\Omega^\text{NS}_q > 10^{-15}$, we take in our computation the values of $m_i$ indicated in Eq. (5). The CDM bounds are also, depicted by the two thin lines.
QUINTESSENTIAL KINATION AND $\tilde{G}$-CONSTRAINT

Unstable $\tilde{G}$ can decay after the onset of NS, affecting by unaccepted amounts the primordial abundances of light elements. To avoid this, an upper bound on $Y_{\tilde{G}}(T_{\text{NS}})$ can be extracted as a function of $m_{\tilde{G}}$ assuming that the hadronic branching ratio of $\tilde{G}$ is tiny. E.g., for $m_{\tilde{G}} \simeq 0.6$ TeV, we obtain [15] $Y_{\tilde{G}}(T_{\text{NS}}) \lesssim 10^{-14}$ which means that $T_I \lesssim 4 \times 10^7$ GeV in the SC. This very restrictive upper bound on $T_I$ can be avoided in the QKS, where we can set $T_I = 10^9$ GeV. The upper bound on $Y_{\tilde{G}}(T_{\text{NS}})$ can be satisfied for $\Omega_q^{\text{NS}} \gtrsim 10^{-21}$ or $T_{\text{KR}} \lesssim 6.8 \times 10^6$ GeV. The importance of a KD era in evading the $\tilde{G}$-constraint can be induced, also, by Fig. 3-(a), where we show the allowed (black lined) area by the $\tilde{G}$-constraint in the log $T_I - \log \Omega_q^{\text{NS}}$ plane, for $m_{\tilde{G}} = M_{1/2} = 500$ GeV. The (gray and lightly gray shaded) area allowed by the quintessential requirements 1–3 is also shown. We clearly see that the $\tilde{G}$-constraint can be totally eluded in the QKS even with tiny values of $\Omega_q^{\text{NS}}$ almost independently on $T_I$.

QUINTESSENTIAL KINATION AND $\tilde{G}$ OR $\tilde{a}$ CDM

Stable $\chi$’s constitute good CDM candidates provided that their relic density $\Omega_{\chi}h^2$ satisfies the CDM constraint [1]

$$0.097 \lesssim \Omega_{\chi}h^2 \lesssim 0.12.$$  (6)

This constraint can be satisfied by both the $\tilde{G}$ and $\tilde{a}$ thermal abundance. However, in the case of $\tilde{G}$, $\Omega_q^{\text{NS}}$ is to be tuned to extremely low values whereas in the case of $\tilde{a}$, $\Omega_q^{\text{NS}}$ may be even close to its upper bound posed by the condition 2. Indeed, in Fig. 3-(b),
we present the region in the log $T - \log \Omega_{q}^{NS}$ plane allowed by both the quintessential requirements, 1 – 3, (gray and lightly gray shaded area) and Eq. (6) for $\tilde{G}$-CDM (black lined region) with $m_{\tilde{G}} = 100$ GeV and $0.5 \leq M_{1/2}/\text{TeV} \leq 1$ or $\tilde{a}$-CDM (white lined region) with $m_{\tilde{a}} = 5$ GeV and $10^{10} \leq f_{a}/\text{GeV} \leq 10^{12}$. Obviously Eq. (6) is met for $\tilde{a}$-CDM with much more natural $\Omega_{q}^{NS}$’s than those required for $\tilde{G}$-CDM. Therefore, $\tilde{a}$ is more natural CDM candidate than $\tilde{G}$ in the QKS.

CONCLUSIONS

We examined the impact of a KD epoch, generated by an quintessential exponential model, to the thermal abundance of $\tilde{G}$ and $\tilde{a}$. The parameters of the quintessential model $(\lambda, T_{I}, \Omega_{q}^{NS})$ were confined so as $0.5 \leq \Omega_{q}(T_{I}) \leq 1$ and were constrained by using current observational data originating from NS, the acceleration of the universe, the inflationary scale and the DE density parameter. We found that $0 < \lambda < 0.9$ and studied the allowed region in the $(T_{I}, \Omega_{NS}^{q})$-plane. For unstable $\tilde{G}$, the $\tilde{G}$-constraint poses a lower bound on $\Omega_{NS}^{q}$ (almost independent of $T_{I}$). The CDM constraint can be satisfied by the $\tilde{G}$ thermal abundance for extremely low $\Omega_{q}^{NS}$’s. On the contrary, this constraint can be fulfilled by the $\tilde{a}$ thermal abundance with much larger $\Omega_{NS}^{q}$’s, making $\tilde{a}$ a very good CDM candidate.

ACKNOWLEDGMENTS

The research of S.L is funded by the FP6 Marie Curie Excellence Grant MEXT-CT-2004-014297. The work of M.E.G, C.P and J.R.Q is supported by the Spanish MEC projet FPA2006-13825 and the projet P07FQM02962 funded by “Junta de Andalucia”.

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