Features of high energy \( pp \)
elastic scattering at small \( t \)

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Abstract  

A method of determination of the real part of the elastic scattering amplitude is examined for high energy proton-proton elastic scattering at small momentum transfer. The method allows to decrease the number of model assumptions, to obtain the real parts of the spin non-flip and spin-flip amplitudes in the narrow region of momentum transfer.
A large number of experimental and theoretical studies of the high energy elastic proton-proton and proton-antiproton scattering at small angles gives a rich information about this processes, which allows to narrow the circle of examined models and to solve a number of difficult problems. Especially this concerns the energy dependence of some of characteristics of these reactions and the contribution of the odderon.

Some of these questions are connected with the $s$ and $t$ dependence of the spin-non-flip phase of hadron-hadron scattering. The majority of the models define the real part of the scattering amplitude phenomenologically. Some models use local dispersion relations and the hypothesis of the geometrical scaling. As it is known, using some simplifying assumption, the information about the phase of the scattering amplitude can be obtained from the experimental data at small momentum transfers where the interference of the electromagnetic and hadron amplitudes takes place. On the whole, the obtained information confirms the local dispersion relations. The knowledge of the structure of the elastic proton-nuclei and nuclear-nuclear scattering is needed to distinguish different models describing high energy nuclei interactions. This is important for the QCD approach of the high energy nuclei interaction [1].

The standard procedure to extract the magnitude of the real part includes the fit of the experimental data taking the magnitude of total cross section, of the slope, of $\rho$, and, sometimes of the normalization coefficient as free parameters.

$$\sum_j \chi_j^2 = \sum_{j=1}^{N} \frac{(d\sigma/dt)_j^{\text{Exp.}} - (d\sigma/dt)_j^{\text{Theor.}})^2}{(\Delta_j^{\text{Exp.}})^2}. \tag{1}$$

This procedure requires a sufficiently wide interval of $t$ and a large number of experimental points.

The spin-independent amplitude can be written as a sum of nuclear $\Phi^h(s, t)$ and electromagnetic $\Phi^e(s, t)$ amplitudes:

$$\Phi(s, t) = \Phi^h(s, t) + e^{i\alpha\varphi} \Phi^e(s, t). \tag{2}$$

where $\Phi^e(s, t)$ are the leading-terms at high energies of the one-photon amplitudes as defined, for example, in [2] and the common phase $\varphi$ is

$$\varphi = -[\gamma + \log (B(s, t)|t|/2) + \nu_1 + \nu_2] \tag{3}$$
where $B(s,t)$ is the slope of the nuclear amplitude, $\gamma = 0.577$, and $\nu_1$ and $\nu_2$ are small correcting terms define the behavior of the Coulomb-hadron phase at small momentum transfers (see [3]). At very small $t$ and fixed $s$, these electromagnetic amplitudes are such that $\Phi_1(s,t) = \Phi_3(s,t) \sim \alpha/t$, $\Phi_2(s,t) = -\Phi_4(s,t) \sim \alpha \cdot \text{const.}$, $\Phi_5(s,t) \sim -\alpha/|t|$, where $\alpha$ is the fine-structure constant. We assume, as usual, that at high energies and small angles the double-flip amplitudes are small with respect to the spin-nonflip one and that spin-nonflip amplitudes are approximately equal. Consequently, the observables are determined by two amplitudes:

$$F(s,t) = \Phi_1(s,t) + \Phi_3(s,t) = F_N + F_C \exp(rm_i \alpha \varphi).$$

In the standard fitting procedure

$$d\sigma/dt = \pi[(F_C(t))^2 + (\rho(s,t))^2 + 1)(ImF_N(s,t))^2]$$

$$+ 2(\rho(s,t) + \alpha \varphi(t))F_C(t)ImF_N(s,t)].$$

(4)

$F_C(t) = \mp 2\alpha G^2/|t|$ is the Coulomb amplitude; and $G^2(t)$ is the proton electromagnetic form factor squared. $Re F_N(s,t)$ and $Im F_N(s,t)$ are the real and imaginary parts of the nuclear amplitude and $\rho(s,t) = Re F_N(s,t)/Im F_N(s,t)$. The formula (3) is used for the fit of experimental data determining the Coulomb and hadron amplitudes and the Coulomb-hadron phase in order to obtain the value of $\rho(s,t)$.

$ReF_N(s,t)$ is obtained by fitting the differential cross sections either taking into account the value of $\sigma_{tot}$ from another experiment, as made by the UA4/2 Collaboration, or taking $\sigma_{tot}$ as a free parameter, as made in [4]. If one does not take the normalization coefficient as a free parameter in the fitting procedure, its definition requires the knowledge of the behavior of imaginary and real parts of the scattering amplitude in the range of small transfer momenta and the magnitude of $\sigma_{tot}(s)$ and $\rho(s,t)$.

In this talk, we briefly describe some new procedures of simplifying the determination of elastic scattering amplitude parameters.

From equation (4) one can obtain the equation for $ReF_N(s,t)$ for every experimental point $i$

$$ReF_N(s,t_i) = -ReF_C(s,t_i)$$
\[
\pm[(1 + \delta)/\pi d\sigma/dt(t = t_i) - (\alpha\phi F_C(t_i) + Im F_N(t_i))^2]^{1/2}.
\] (5)

where \(\delta\) is the corrections coefficient which reflect the accuracy of the normalization parameter \(n = 1 + \delta\). As the imaginary part of scattering amplitude is defined by

\[
Im F_N(s, t) = \sigma_{tot}/(0.389 \cdot 4\pi) exp(B/2t),
\] (6)

it is obvious from (5) that the determination of the real part depends on \(n, \sigma_{tot}, B\). The magnitude of \(\sigma_{tot}\) determined from experimental data depends on the normalization parameter \(n = 1 + \delta\) which reflects the experimental error in determining \(d\sigma/dt\) from \(dN/dt\). The equation (5) shows the possibility to calculate the real part in every separate point of \(t_i\) if the imaginary part of scattering amplitude and \(n\) are determined and to check up the form of the obtained real part of the scattering amplitude or vice versa (see [5]). This form shows also a minimum value of \(n\), as the expression situated under the square root cannot be less then zero.

Let us define the sum of the real part hadron and Coulomb amplitudes as \(\Delta_R\), so we can write:

\[
\Delta_R(t_i) = [Re F_N(s, t_i) + Re F_C(s, t)]^2 =
\]

\[
[(1 + \delta)/\pi d\sigma/dt(t = t_i) - (\alpha\phi F_C(t_i) + Im F_N(t_i))^2].
\] (7)

This formula shows a significant property for the proton-proton cross section at a very high energy and proton-antiproton scattering at low energy, where the real part of the hadronic amplitude is sufficiently large and is opposite in sign relative to the Coulombic amplitude. We therefore get

\[
\Delta_R(t_i) = (1 + \delta)(Re F_N(s, t_i) + Re F_C(s, t))^2
\]

\[-\delta(\alpha\phi F_C(t_i) + Im F_N(t_i))^2].
\] (8)

Let us examine this expressions for the pp-scattering at energies above \(\sqrt{s} = 540\, GeV\) where the real part of the hadron spin-non-flip amplitude is positive and non-negligible. For this aim, let us make a gedanken experiment and calculate \(d\sigma/dt\) with definite parameters (\(\rho = 0.15\) and \(\sigma_{tot} = 63\) taking them as experimental points. For the pp-scattering at high energies, the equation (5) has a remarkable property.
The real part of the Coulomb scattering amplitude of $pp$-scattering is negative and exceeds the size of $F_h(s,t)$ at $t \to 0$, but has a large slope. As the real part of the hadronic amplitude is positive at high energies, it results that $\Delta_R$ has a minimum situated in $t$ independent of $n$ and $\sigma_{tot}$ as shown in Fig. 1.

The position of the minimum gives us the value $t_R$ where $\text{Re} F_N = -\text{Re} F_C$. As we know the Coulomb amplitude, we estimate the real part of the $pp$-scattering amplitude at this point. Note that all other methods give us the real part only in a sufficiently wide interval of the transfer momenta. If we choose the right normalization coefficient and $\sigma_{tot}$ our minimum will be equal zero. But if the normalization coefficient is not right one the minimum will be or above or lower then zero, but practically it is located at the same point $t_R$. So, the size of $\Delta_R$ shows us the valid determination of the normalization coefficient and $\sigma_{tot}$.

This method works only in the case of the positive real part of the nucleon amplitude, and it is especially good in the case of large $\rho$. So, it is interesting for the experiment $PP2PP$ at RHIC and the future TOTEM experiment at LHC.

Though in the range of ISR we have small $\rho(s,t \approx 0)$ and few experimental points, let us try to examine one experiment, for example, at $\sqrt{s} = 52.8$ GeV. This analysis is shown in Fig. 2. One can see that in this case the minimum is sufficiently large, and $-t_{\text{min}} = (3.3 \pm 0.1)10^{-2}$ GeV$^2$. The corresponding real part equals $0.442 \pm 0.014$ GeV.

Our analysis gives $\rho = 0.063 \pm 0.003$, while the paper [6] gives $\rho = 0.077 \pm 0.08$.

For RHIC energies we can simulate the ”experimental” data taking the calculated theoretical curve with certain parameters $\sigma_{tot}, B, \rho$ and the magnitude of errors which are expected in the future experiment. Then we calculate the deviation from the theoretical curve in units of errors using a Gaussian random procedure in order to calculate the probability of the deviation by a number of errors. As a result, we obtain the differential cross sections modeling the future experimental data, for example, with the posible size of $\rho = 0.135$ and $\rho = 0.175$. Then we can determing the value of $\Delta_R$ from these “gedanken” experimental data, which are shown in Fig.3 (a,b) correspondingly. The difference between these two modeling data rep-
resentations is obvious. The pure theoretical representation of $\Delta_R$ with the same values of $\rho$ and with $\rho = 0$ are shown also.

Our predictions for the LHC energies are shown in Fig.4 and 5. for the value of $\rho = 0.15$.

Note that the point $t_R$ is important for the determination of the real part of spin-flip amplitude also [7]. At high energies and small angles the analyzing power can written in form

$$A_N \frac{d\sigma}{dt}/2 = ImF^h_{n,f}(ReF^c_{s,f} + ReF^h_{s,f}) + ImF^c_{n,f}(ReF^c_{s,f} + ReF^h_{s,f})$$

$$-ImF^c_{s,f}(ReF^c_{n,f} + ReF^h_{n,f}) - ImF^h_{s,f}(ReF^c_{n,f} + ReF^h_{n,f}).$$

We obtain for proton-proton scattering at high energies at the point $t_R$ where $ReF^h_{n,f} = -ReF^c_{c,f}$,

$$ReF^h_{s,f}(s,t) = \frac{-1}{2(ImF^h_{n,f}(s,t) + ImF^c_{n,f}(t))} A_N(s,t) \frac{d\sigma}{dt} - ReF^c_{s,f}(t).$$

At this point some terms in the definition of analyzing power will be canceled. Such a representation can be used for the determination of the real part of the hadron spin-flip amplitude at high energy and small angles.

It is interesting to apply this new method to the proton-nuclear scattering at high energies. The size of the hadron amplitude grows only slightly less then proportional to $A$. If $\sigma_{tot}(pp) = 38$ mb in the region of hundred GeV, the $\sigma_{tot}(p^{12}C) = 335$ mb. The most important difference with $pp$-scattering is that the slope is very high, near 70 $mb/GeV^2$. The electromagnetic amplitude grows like $Z$ and its slope also grows. It is interesting that the simple calculations which take the hadron amplitude at small momentum transfer in the usual exponential form with large slope leads the practically the same results as for the proton-proton scattering.

The precise experimental measurements of $dN/dt$ and $A_N$ at RHIC, as well as, if possible, at the Tevatron, will therefore give us unavailable information on the hadron elastic scattering at small $t$. New phenomena at high energies could be therefore detected without going through the usual arbitrary assumptions (such as
the exponential form) concerning the hadron elastic scattering amplitudes. It is interesting to apply this method to the proton-nuclei scattering at high energies, especially at RHIC energies. This method offers a unique possibility search for the behavior of the real part of the hadronic amplitude in nuclear reactions.
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Figure captions

FIG.1. The model calculations of $\Delta R$ for the $pp$-scattering at RHIC energy $\sqrt{s} = 540$ GeV and with $\sigma_{\text{tot}} = 63$ mb and different $n$.

FIG.2. The calculation of $\Delta R$ for the $pp$-scattering using the experimental data of $d\sigma/dt$ at $\sqrt{s} = 52.8$ GeV [6]. The lines are the polynomial fit of the points calculated with experimental data and with different $n$.

FIG.3. The calculation of $\Delta R$ for the $pp$-scattering at RHIC with a) $\rho_1 = 0.135$ and b) $\rho_2 = 0.175$ The solid, short-dashed, and dotted lines are the theoretical curves for $\rho_2 = 0.175$, $\rho_1 = 0.135$, and $\rho_0 = 0$ respectively.

FIG.4. The calculation of $\Delta R$ for the $p^{12}C$-scattering with $\rho = 0.1$ and $\rho = 0.075$ (the solid and dashed lines, correspondingly) using of the exponential behavior of the hadronic amplitude.

FIG.5. The calculation of $\Delta R$ for the $pp$-scattering with $\rho = 0.15$ at $\sqrt{s} = 4$ TeV

FIG.6 The calculation of $\Delta R$ for the $pp$-scattering with $\rho = 0.15$ at $\sqrt{s} = 14$ TeV
Fig. 1

Fig. 2
Fig. 3 a

Fig. 3 b
Fig. 4

\[ \Delta R \text{ (GeV}^{-4}) \]

\[ -t \text{ (GeV}^2) \]

Fig. 5

\[ \Delta R \text{ (GeV}^{-4}) \]

\[ -t \text{ (GeV}^2) \]