2D and 3D simulations of the permanent magnet undulator for generation of MIR-FEL

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Abstract. This research focuses on computer modelling and magnetic field simulations of the permanent magnet undulator at the PBP-CMU Electron Linac Laboratory, Chiang Mai University. This undulator magnet will be used for generation of the mid infrared free electron laser (MIR-FEL). It has 40 periods with a period length of 40 mm. The magnetic field simulations were conducted by using computer programs PANDIRA and RADIA for 2D and 3D models, respectively. The simulation results obtained from both models are presented and discussed in this paper. The magnetic field can be varied by adjusting a gap of the undulator. A maximum magnetic field of 0.23 Tesla can be achieved for the undulator gap of 26 mm. The simulated magnetic field distributions along the undulator magnetic axis obtained from the 3D model were used to calculate the deflecting angle and trajectory of electron. The achievement of this study will be used as a significant information for further installation and operation of the undulator magnet at our facility.

1. Introduction

The MIR/THz Free-electron laser (FEL) facility is under the development at the PBP-CMU Electron Linac Laboratory of the Plasma and Beam Physics (PBP-CMU) Research Facility, Chiang Mai University (CMU). There are two insertion devices to generate the radiation, which are a mid-infrared (MIR) permanent magnet undulator and a terahertz (THz) electromagnet undulator. When electrons travelling in the undulator magnetic field, they are oscillating due to a vertical sinusoidal magnetic field

\[ B_y(z) = B_0 \sin(k_z z) , \]

where \(B_0\) is the peak magnetic field. Then, electromagnetic wave with a wavelength \(\lambda_e\) is emitted from electrons according to their energies and the magnetic field strength as \([1]\)

\[ \lambda_e = \frac{\lambda_u}{2n\gamma} \left(1 + \frac{K^2}{2} + \theta^2\gamma^2\right), \]  \(1\)

where \(n\) is the harmonic number, \(\gamma\) is the Lorentz factor and \(\theta\) is the angle of radiation observation. The quantity \(K\) is the undulator parameter, which is related to a peak magnetic \(B_0\) and an undulator period length \(\lambda_u\) as
\[
K = \frac{eB_0\lambda_u}{2\pi m_0 c} = 0.938B_0[T]\lambda_u[\text{cm}],
\]

(2)

where \(e\) is the electric charge of electron, \(m_0\) is the rest mass of electron and \(c\) is the speed of light. The deflecting angle of electron trajectory can be obtained by integrating the magnetic field over the length of the undulator as 
\[\chi' = \frac{eI}{\gamma m_0 c}.\]

The horizontal displacement of the electron trajectory is derived from the integral of the deflecting angle over the length of the undulator that is the second integral of the magnetic field as 
\[x = \frac{eI}{\gamma m_0 c}.\]

These integrals are called the first \((I_1)\) and the second \((I_2)\) field integrals, which are written as
\[
I_1 = \int_{-\infty}^{+\infty} B_y(z)dz \quad \text{and} \quad I_2 = \int_{-\infty}^{+\infty} dz \int_{-\infty}^{+\infty} B_y(z)dz.
\]

(3)

Both deflecting angle and horizontal displacement have significant influence on the produced radiation from the undulator magnet. They depend greatly on the magnetic field amplitude and distribution along the undulator length. Hence, investigation of the magnetic field in the undulator magnet is necessary. This study focuses on the numerical simulations of the MIR permanent magnet undulator, which was obtained from the collaboration with the Institute of Advanced Energy (IAE), Kyoto University. Both 2D and 3D simulations were done and the results are presented and discussed in this paper.

2. Two-dimensional (2D) simulations

The MIR permanent magnet undulator has 2 arrays of dipole magnets for generating a sinusoidal magnetic field. Each array has 161 magnetic blocks with Halbach arrangement as presented in fig. 1. The model in the figure was created with program PANDIRA [2]. For better illustration, a few magnet’s blocks at the two ends of the undulator are zoomed and also shown in the figure.

![Figure 1. PANDIRA model showing MIR permanent magnet undulator and the magnetic permanent blocks at the two ends.](image)

To obtain reliable magnetic field values from 2D simulations, we need to optimize the input parameters i.e. the simulation mesh size and the number of exporting data. Optimization of the mesh size was done by dividing each magnetic block to small pieces. The results are shown in fig. 2 (a). The quantity \(\Delta B_y\) represents the difference between simulated magnetic field values for each mesh size. The results show that \(\Delta B_y\) is almost zero or the magnetic field converges to the same value for the mesh size of \(\leq 0.14\) cm. Thus, we used this mesh size for all simulations with program PANDIRA. Moreover, the resolution of simulation results depends greatly on the number of the exporting data. The optimization of this number was done. The results are presented in fig. 2 (b) and shows that the optimal number of exporting data is 20,000.
After the optimization, magnetic field simulations with program PANDIRA were conducted. This permanent magnet undulator was built at another institute many years before it was transferred to Kyoto University. The information about the construction of the magnet was lost. Therefore, we do not have any information about the design parameters, the material type nor the construction procedure of this magnet. This is actually the main reason why we have to study the properties of this undulator for both numerical and experimental aspects. Since we do not have any information about the material of this magnet, simulations with two common permanent materials that are SmCo and NdFeB were performed to compare the results with the measured magnetic field values. The results in fig. 3 show that the simulated magnetic field distribution from PANDIRA model using SmCo gives similar values to the measurement results. Therefore, we used SmCo as the permanent magnetic material in all simulations with program PANDIRA and also for 3D simulations with program RADIA.

The gap of the undulator magnet was adjusted to investigate its influence on the magnetic field. The magnetic field at the 20th peak, which is the middle peak, was considered. The results in fig. 4 show that the magnetic field decreases as the gap size increases and the magnetic field stays at about 0.021 T when the gap is larger than 60 mm. An example section of the PANDIRA model with magnetic field lines and profiles is illustrated in fig. 5.

**Figure 2.** (a) The relationship between the vertical magnetic field and the mesh size. (b) The relationship between the horizontal displacement of 20 MeV electron and the number of exporting data.

**Figure 3.** Magnetic field distribution along the longitudinal distance for 2.5 periods of the undulator magnet.
3. **Three-dimensional (3D) simulations**

In this part, we used SmCo for 3D simulations with the computer program RADIA [3]. The sharp edge of each undulator block is cut for making it to be the same as the real undulator. The 3D RADIA model and the magnetic field distribution along the undulator length are shown in fig. 6.

When an electron travels in the undulator magnetic field, it will oscillate around the central axis. Its deflecting angle and trajectory can be obtained from the 1st and the 2nd integrals of the magnetic field distribution along the undulator axis and the results are presented in fig. 7. In this study we used an electron with energy of 20 MeV. The undulator gap was 26 mm.
Figure 6. RADIA model of the permanent magnet undulator (top). Simulated magnetic field along the undulator (bottom).

Figure 7. Deflection angle (left) and trajectory (right) of an electron along the undulator distance.

4. Conclusion
Simulations with 2D and 3D models of the permanent magnet undulator were performed. The results from 2D PANDIRRA model reveal that the material of the permanent magnet should be SmCo because it provides similar magnetic field values to the measurement results. The 3D RADIA model using the same material was also created. The simulation of the magnetic field for the 3D model was done. For the undulator gap of 26 mm, the peak magnetic field was 0.23 T. The simulated magnetic field distribution along the undulator magnetic axis obtained from the 3D model was used to calculate the deflecting angle and trajectory of electron with energy of 20 MeV.
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