Lorentz violation from cosmological objects with very high energy photon emissions

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Abstract

Lorentz violation (LV) is predicted by some quantum gravity theories, where photon dispersion relation is modified, and the speed of light becomes energy-dependent. Consequently, it results in a tiny time delay between high energy photons and low energy ones. Very high energy (VHE) photon emissions from cosmological distance can amplify these tiny LV effects into observable quantities. Here we analyze four VHE $\gamma$-ray bursts (GRBs) from Fermi observations, and briefly review the constraints from three TeV flares of active galactic nuclei (AGNs) as well. One step further, we present a first robust analysis of VHE GRBs taking the intrinsic time lag caused by sources into account, and give an estimate to quantum gravity energy $\sim 2 \times 10^{17}$ GeV for linear energy dependence, and $\sim 5 \times 10^9$ GeV for quadratic dependence. However, the statistics is not sufficient due to the lack of data, and further observational results are desired to constrain LV effects better.

Keywords: Lorentz violation, $\gamma$-ray bursts, active galactic nuclei

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1. Introduction

The unification of standard model and general relativity is the most intriguing and desirous goal of modern physics, and it stimulates the development of many theoretical ideas such as string theory and loop gravity in the past decades. It is interesting that some of them predict Lorentz symmetry violation (LV) at low energies or being realized in a highly nonlinear form. This has arisen in the space-time foam [1, 2, 3, 4], loop gravity [5, 6], torsion in general gravity [7], vacuum condensate of antisymmetric tensor fields in string theory [8, 9, 10, 11, 12], and the so-called double special relativity [13, 14, 15].

In most of LV theories, the photon dispersion relation is modified and the correction is believed to be suppressed by the large Planck scale, $E_P \equiv \sqrt{\hbar c^5/G} \simeq 1.22 \times 10^{19}$ GeV [16, 17]. These theories are mainly classified into two categories. One is effective field theory (EFT), which provides an excellent framework where the tiny LV effects are introduced through LV operators: renormalizable ones with dimension three and/or four, see, e.g., standard model extension (SME) [10, 11], and the further extended non-renormalizable ones with dimension five and/or six [12, 18]. However, not all quantum gravity theories can be embedded into the EFT framework, such as the quantum space-time foam model [1, 2, 3, 4] and double special relativity [13, 14, 15], and they also introduce modifications to the canonical dispersion relation. Consequently, they can result in an energy dependence of the speed of light in the vacuum, owing to the propagation through an effective gravitational medium containing space-time quantum fluctuations. Fortunately, this scenario of lower energy “relic probe” of Planck scale events can be tested carefully through laboratory experiments [19, 20] or astronomical observations [21, 22, 23, 24, 25, 26, 27, 28].

Generally, the most model-independent photon dispersion relation reads in the context of Taylor series as,

$$v(E) = c_0 \left(1 - \xi \frac{E}{E_P} - \zeta \frac{E^2}{E_P^2}\right), \quad (1)$$

where $v(E)$ is the speed of photons with energy $E$, $c_0$ is the speed of low energy photons, and $\xi$, $\zeta$ are model-dependent parameters, characterizing the energy where LV occurs. Because of the suppression of Planck energy, the terms of higher orders are negligible, and the quadratic term takes effect only when the linear term vanishes. It could be possible that a more concrete
form of dispersion relation may contain more specific terms of energy dependence, but we may take Eq. (1) as a general estimate in a model-independent manner.

2. High energy γ-ray bursts

Amelino-Camelia et al. [1, 2] first suggested using cosmological γ-ray bursts (GRBs) to test LV. Due to the large cosmological distance and the fine time structure of GRBs, tiny LV effects can be amplified into observable quantities. By taking into account cosmological expansion of the universe, the time lag induced by LV modified dispersion relation, i.e., Eq. (1), between photons with high energy $E_h$, and those with low energy $E_l$, is [29],

$$\Delta t_{LV} = \frac{1+n}{2H_0} \left( \frac{E^n_h - E^n_l}{E^n_{QG}} \right) \int_0^z (1+z')^n dz' h(z'), \quad (2)$$

where $n = 1$ and $n = 2$ stand for linear and quadratic energy dependence, with quantum gravity energy $E_{QG,L} = |\xi|^{-1}E_P$ and $E_{QG,Q} = |\zeta|^{-1/2}E_P$, respectively; $H_0 \approx 71$ km s$^{-1}$ Mpc$^{-1}$ is the Hubble constant; $z$ is the redshift of the source, and $h(z)$ is defined as

$$h(z) = \sqrt{\Omega_\Lambda + \Omega_M(1+z)^3}, \quad (3)$$

where $\Omega_\Lambda \approx 0.73$ is the vacuum energy density, and $\Omega_M \approx 0.27$ is the matter energy density in current universe.

Due to the launch of high quality satellites, e.g., Swift and Fermi, our understanding of GRBs has accomplished evolutionary improvements. Especially, Fermi Large Area Telescope (LAT), which is sensitive up to energy of photons $\sim 300$ GeV, has led us to the very high energy (VHE) domain and inaugurates a new era. LAT discovered that high energy photons have a tendency to arrive later relative to low energy ones, which might present potential evidence for LV [27, 30, 31].

However, the determination of time lag from observational data is highly nontrivial and affected by many facets, both artificial and instrumental. As a case study of GRB 090510, Ref. [31] discussed several choices, e.g., the time lag between the arrival of the highest energy photon and the Gamma-ray Burst Monitor (GBM) trigger, the onset of the main GBM emission, the onset of $> 0.1$ GeV emission, and the onset of $> 1$ GeV emission. For simplicity, we here choose the observed time lag $\Delta t_{\text{obs}}$ as the difference between the arrival
Table 1: The time lag of the highest energy photons of Fermi LAT GRBs, relative to the Fermi GBM trigger time. The possible Lorentz violation energies, $E_{QG,L}$ and $E_{QG,Q}$, are listed for linear and quadratic energy dependence respectively, without astrophysical effects taking into account.

| GRBs       | $z$   | $E$ (GeV) | $\Delta t_{\text{obs}}$ (s) | $E_{QG,L}$ (GeV) | $E_{QG,Q}$ (GeV) |
|------------|-------|-----------|-----------------------------|------------------|------------------|
| 080916C    | 30    | 4.35      | 13.22                       | 1.5 × 10^{18}    | 9.7 × 10^{9}     |
| 090510     | 31    | 0.903     | 31                          | 1.7 × 10^{19}    | 3.4 × 10^{10}    |
| 090902B    | 34    | 1.822     | 33.4                        | 3.7 × 10^{17}    | 5.9 × 10^{9}     |
| 090926A    | 36    | 2.1062    | 19.6                        | 7.8 × 10^{17}    | 6.8 × 10^{9}     |

The time lag of the highest energy photon and the GBM trigger time. The energy of GBM trigger photons is about 0.1 MeV, therefore it is negligible in Eq. (2), compared to the highest energy photons, whose energies are significantly larger than $\sim 1$ GeV.

Four delayed GRBs with known redshifts, observed by the LAT instrument, are listed in Table 1. Their indicated LV scales are given as well, relying on the assumption $\Delta t_{\text{obs}} = \Delta t_{\text{LV}}$. As the central engines and emission mechanism of GRBs are not totally understood yet, and the lags have been detected explicitly in several events, we boldly treat the results as possible indicators of LV effects instead of lower boundaries.

The values derived in the last two columns of Table 1 differ more than tenfold between each other, with average values $E_{QG,L} \sim (4.9 \pm 8.1) \times 10^{18}$ GeV and $E_{QG,Q} \sim (1.4 \pm 1.3) \times 10^{10}$ GeV utilizing the least square method. We attribute the large deviations to the fact that all source effects are neglected here.

The primary uncertainty comes from the unknown effects from source activities, mainly due to our imperfect knowledge of radiation mechanism of GRBs. However, we can separate the source effects if we can achieve a survey of GRBs at different redshifts. The time lag induced by LV accumulates with propagation distance, as it is a gravitational medium effect. On the contrary, the intrinsic source induced time lag is likely to be a distance-independent quantity, which can be regarded as a constant for a particular class of sources in the leading order approximation.

Ellis et al. [38, 39] have led a robust analysis of sets of GRBs from BATSE, HETE, and Swift, utilizing the wavelet technique [40] to search for potential
lags. However, due to the scarcity of the VHE observational data then, no survey of VHE GRBs with explicit time delay has been treated in a robust way yet. We here make a first coarse attempt to include available LAT GRBs and give a global estimate to LV parameters.

On assuming that the intrinsic time lag $\Delta t_{\text{in}}$, originated from astrophysical effects, is independent of redshift and constant for objects of a particular class, which depends only on the type of sources, then the observed delay is

$$\Delta t_{\text{obs}} = \Delta t_{\text{LV}} + \Delta t_{\text{in}}(1 + z).$$

(4)

Inspired by Refs. [38, 39], after a few steps from Eq. (2) and Eq. (4), we can get a linear formula with an intercept $\Delta t_{\text{in}}$, and the slope of the line equals to $1/E_{\text{QG,L}}$ for the linear energy dependence and $1/E_{\text{QG,Q}}^2$ for the quadratic dependence,

$$\Delta t_{\text{obs}}/(1 + z) = K/E_{\text{QG}}^n + \Delta t_{\text{in}},$$

(5)

where $K$ is defined as

$$K = \frac{1 + n}{2H_0} \frac{E_{b}^n - E_{l}^n}{1 + z} \int_0^z (1 + z')^n dz'/h(z').$$

(6)

The plots of linear fits for two energy-dependent scenarios are illustrated in Fig. 1 and Fig. 2 individually. The dash-dotted lines represent linear fit to all four GRBs from Table 1, while the dashed lines only fit to three long GRBs (GRB 080916C, GRB 090902B, and GRB 091003A; duration $T_{90} > 2$ s). With the consideration that short GRB 090510 (duration $T_{90} < 2$ s) would have different intrinsic time delay from the long ones due to their distinct progenitor mechanism [41, 42], we expect that these two classes of GRBs should have different intercepts if more data of short GRBs are available. Actually, current prevailing paradigm regards that long GRBs come from the collapses of massive rapidly rotating stars, while short GRBs are believed to be originating from the coalescence of two neutron stars or a neutron star and a black hole [42]. Thus even with absence of more data, the apparent deviation of GRB 090510 from the red dashed line is expected.

Due to the lack of statistics, we do not give the errors of data as it is still too early to constrain LV parameters accurately without high enough statistics. Therefore, one should caution that the data are rather rough and strongly depend on artificial choices as mentioned. Nevertheless, from the figures, we can see that when only three long GRBs are included, the linear and quadratic fits are both very likely and give quantum gravity energy as
Figure 1: Linear fits to Fermi LAT GRBs where linear energy-dependent LV effects and constant intrinsic source effects are considered. The dash-dotted line stands for a fit to all four GRBs, while the dashed line only fits to three long GRBs (without the short GRB 090510). The intercept and slope of fitted line equal to $\Delta t_{\text{in}}$ and $1/E_{QG,L}$, respectively.
Figure 2: Linear fits to Fermi LAT GRBs where quadratic energy-dependent LV effects and constant intrinsic source effects are considered. The dash-dotted line stands for a fit to all four GRBs, while the dashed line only fits to three long GRBs (without the short GRB 090510). The intercept and slope of fitted line equal to $\Delta t_{\text{in}}$ and $1/E_{\text{QG},Q}^2$, respectively.

$E_{\text{QG,L}} = (2.2 \pm 0.2) \times 10^{17}$ GeV and $E_{\text{QG,Q}} = (5.4 \pm 0.2) \times 10^9$ GeV. After including the short GRB 090510, the fits are still reasonable and give $E_{\text{QG,L}} = (2.2\pm0.9) \times 10^{17}$ GeV and $E_{\text{QG,Q}} = (5.3\pm0.8) \times 10^9$ GeV. We can see that the mean values almost stay the same for two choices, whereas the fitted errors are somehow larger in the later fit due to inclusion of short GRB 090510. Our procedure avoids the inconsistently large deviations derived from different sources without taking astrophysical effects into account, as shown in the last two columns of Table 1.

Actually, the method to determine quantum gravity scale, when only one source is included without considering the intrinsic effects, is equivalent to only using the slope between a specific data point and the point of origin in our figures. Hence the lower right position of GRB 090510 in figures gives its slope extremely small compared to the other three sources, and hence the derived quantum gravity energy, inverse of the slope, is rather large [31]. From this viewpoint, the conclusion that dispersion relation with linear energy dependence is ruled out because of the early onset of high energy photons of GRB 090510 [31] seems too early to be solidified [43].
3. Active galactic nuclei

Active galactic nuclei (AGNs) locate nearer and their time structures are not so variable as those of GRBs. However, their VHE emissions are in the TeV range, which is significantly higher than the highest energy emissions observed in GRBs. Thus they represent another kind of VHE astronomical laboratories to probe possible evidence of LV.

Here we briefly review the three AGNs which were published previously to test LV effects. We recalculate the potential LV scales utilizing Eq. (2) and discuss the hints and limitations from AGNs as well.

**Markarian 421.** It was reported 10 years ago, that no time lag larger than 280 s was found between energy bands $< 1$ TeV and $> 2$ TeV during a TeV flare of Markarian 421 [44]. The AGN is known to locate at $z = 0.031$, thus it sets a lower boundary to LV with quantum gravity energy scales $E_{QG,L} > 4.9 \times 10^{16}$ GeV and $E_{QG,Q} > 1.5 \times 10^{10}$ GeV.

**Markarian 501.** MAGIC Collaboration found a time lag about 4 min for photons in the 1.2–10 TeV energy band relative to those in the range 0.25–0.6 TeV during a VHE flare of Markarian 501 [45], whose redshift equals to 0.034. The mean difference of the two bands is reported $\approx 2$ TeV, thus we can do an estimation and obtain $E_{QG,L} \sim 1.2 \times 10^{17}$ GeV, regardless of the intrinsic delay. This value is very close to our results from the global fits of GRBs.

**PKS 2155-304.** HESS Collaboration published an interesting VHE flare of the BL Lacertae object PKS 2155-304 [46], which locates at $z = 0.116$ [47], more distant than Markarian 421 and Markarian 501. They made use of the modified cross correlation function and got a time lag $\sim 20$ s between lightcurves of two different energy bands. However, the delay is not sufficiently significant. They reported that the mean difference of two energy bands in this case is 1.0 TeV, while the mean quadratic difference is 2.0 TeV$^2$, hence we get the potential LV scales to be $E_{QG,L} \sim 2.6 \times 10^{18}$ GeV and $E_{QG,Q} \sim 9.1 \times 10^{10}$ GeV accordingly. They are both about one magnitude larger than our robust values.

It is worthy to mention that the estimations above are based on the assumption $\Delta t_{in} = 0$, and because of ignorance of the source mechanism, they remain controversial [27, 28]. And global fits for AGNs of different types are expected to present something different, like what happens in the GRBs case. However, current AGNs data are inadequate to carry out a robust analysis. Moreover, TeV flares from AGNs seem relatively rare and unpre-
dictable, and they are produced only occasionally by AGNs with restricted redshifts contrary to large and diverse redshifts of GRBs [28]. Furthermore, the sources of AGNs are very different, and we expect they have distinct astrophysical lags and intrinsic fluctuations. Therefore, it is not likely to present a robust analysis of AGNs for LV effects, at least in the recent future. However, they provide a complementary probe to LV effects due to the different observational method and distinct origins.

4. Discussions and summary

Nowadays, more very high energy (VHE) data become available and they provide a rich ground to look into cosmologically accumulated effects, rooted from quantum gravity scale. In this paper, we use cosmological objects such as γ-ray bursts (GRBs) and active galactic nuclei (AGNs) to estimate Lorentz violation (LV) effects from the time delay between photons with different energies. The quantum gravity scales from GRBs and AGNs are surprisingly compatible in some sense.

We give a crude attempt here to survey a set of VHE celestial events for LV hints and disentangle astrophysical origins from LV effects as well. Though the data are very limited in our study, even not sufficient to establish a good statistics, we get some potential clues of LV from these samples. Further, we notice that more observations are emerging and more experiment data are accumulating, e.g., Fermi LAT has detected GRB 090323, GRB 090328 and GRB 091003 with VHE emissions and their redshifts are attained from other observations as well, thus once these results are published, there will definitely be an opportunity to give a more stringent analysis. We suggest combining VHE GRBs at different redshifts to separate source effects, if data points approximately lay on two lines according to two different GRB types, it will be a supportive evidence to our approach.

Although the results in our analysis are very preliminary, they give hints for possible LV energy scales, $\sim 2 \times 10^{17}$ GeV for linear energy dependence and $\sim 5 \times 10^{9}$ GeV for quadratic dependence. However, the fits in Fig. 1 and Fig. 2 induce a negative intercept, $\Delta t_{\text{lin}} \sim -20$ s for the linear dependence and $-6$ to $-10$ s for the quadratic scenario, which means that at sources, high energy photons emit earlier than low energy ones. This conflicts with common expectations based on assumptions that low energy photons are produced by electrons while high energy ones are generated by protons, and because of lighter masses, electrons are accelerated earlier and hence low
energy photons come out first [28]. However, because that the last word about the emission mechanism of GRBs is not addressed yet, and the negative intercepts are not very significant in our fits, they may not trouble much.

Worthy to mention that, there are several stringent restrictions coming from various tests on LV effects in the content of given theories [12, 16, 23, 27, 28]. Synchrotron radiation measurements on electrons from the Crab Nebula [22, 48] place very strong constraints on the electron sector, making the linear dependence for electrons almost impossible. Most effective field theories (EFTs) predict birefringence for the photon sector [5, 18], and astrophysical observation leads to very tight restrictions on the linear suppression [49]. And the GZK cutoff originated from the ultra-high-energy cosmic rays (UHECRs) interacting with cosmic microwave background (CMB) photons also produces severe constraints for linear as well as quadratic energy dependence [24, 25, 26]. However, some scenarios can avoid the above restrictions [28], e.g., photons have different LV parameters as electrons and hadrons, thus our analysis above serves as a tentative estimate for some still surviving theories.

Finally, we stress that it would be premature to draw a rigorous conclusion on LV at the moment, and more theoretical considerations and practical data analysis are needed for a more stringent constraint and clarification on these issues.

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