MSSM Higgs sector CP violation at photon colliders: Revisited

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Abstract

We present a comprehensive analysis on the MSSM Higgs sector CP violation at photon colliders including the chargino contributions as well as the contributions of other charged particles. The chargino loop contributions can be important for the would-be CP odd Higgs production at photon colliders. Polarization asymmetries are indispensable in determining the CP properties of neutral Higgs bosons.

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I. INTRODUCTION

The discovery of Higgs boson(s) at the current/future colliders is one of the most important goals of high energy particle physics experiments. Its (non)discovery would be crucial for testing our present understanding of the origin of electroweak symmetry breaking (EWSB) and the subsequent generation of masses of electroweak (EW) gauge bosons and chiral fermions in the Standard Model (SM). This would be also true of the Minimal Supersymmetric Standard Model (MSSM), which is the most popular candidate for the new physics beyond the SM.

The Higgs sector in the MSSM possesses three neutral Higgs particles: two CP-even neutral scalars \( h \) and \( H \), one CP-odd neutral scalar \( A \), and a pair of charged Higgs scalars \( H^\pm \) \[1\]. The tree-level MSSM Higgs potential does not allow spontaneous CP violation unlike general two-Higgs doublet model. Even if one include the one-loop corrected effective potential for the Higgs sector, the spontaneous CP violation \[2\] can not be realistic, because the resulting lightest neutral Higgs boson should be far less than the current lower limit on the Higgs boson \[3\]. Still, there are many new explicitly CP violating complex parameters in the soft supersymmetry (SUSY) breaking sector of the MSSM Lagrangian, and some of them can have large phases (without conflict with the electron/neutron electric dipole moment (EDM) constraints), and thus can lead to some observable consequences in various CP violating phenomena in \( K \) and \( B \) decays \[4\] and electroweak baryogenesis \[5\], etc. Especially, the complex phases of the stop and sbottom trilinear couplings \( A_{t,b} \) and the Higgsino mass parameter \( \mu \) can cause the mixing between CP-odd and CP-even neutral Higgs bosons in the neutral Higgs sector via loop corrections in the MSSM, namely, the Higgs sector CP violation \[6\].

In most phenomenological studies of the MSSM, the large SUSY CP violating (CPV) phases were usually neglected, since they may lead to large EDMs of electron and neutron, or \( \epsilon_K \), depending on whether they are flavor preserving (FP) or flavor changing (FC). The SUSY CPV phases are assumed to be very small, so that the only source of CP violation would be the single Kobayashi-Maskawa (KM) phase in the CKM mixing matrix in the charged weak current of down type quarks. In this case, the SUSY effects on \( K \) and \( B \) phenomenology are minimal in the sense that deviations from the SM predictions are quite small. However one can consider large FP SUSY CPV phases, since one can avoid the EDM constraints in basically three different ways:

- **Decoupling (Effective SUSY Model):** The 1st/2nd generation sfermions are heavy (and degenerate to some extent) enough, so that the SUSY CP and \( \epsilon_K \) problems are evaded. Only third generation sfermions and gauginos have to be lighter than \( O(1) \) TeV in order that one solves the gauge hierarchy problem by SUSY \[7\]. In this case, the SUSY CPV phases need not be zero, and they can lead to substantial deviations from the SM cases, especially for the third generation. In this scenario, \( B \) factories may be able to probe the SUSY CPV phases from direct asymmetry in \( B \to X_s \gamma \) and the lepton forward-backward asymmetry in \( B \to X_s l^+l^- \), etc.

- **Cancellation:** Various contributions to electron/neutron EDMs may cancel one another, leading to the net results which are consistent with experimental lower bounds \[8\]. In this case, many of the SUSY CPV phases can be \( O(1) \) as in the decoupling
scenario. However this scenario is tightly constrained when the data on the mercury
\(^{199}\text{Hg}\) atom EDM is included [9].

- **Non-universal Scenario:** \(|A_e|, |A_{u,c}|, |A_{d,s}| \lesssim 10^{-3}|\mu|\) to evade \(e/n\) EDM’s, but \(A_t, A_b, A_\tau\) can have large CP violating phases [10]. However there is a strong two-loop Barr-Zee type constraint for large \(\tan \beta\). Therefore large CPV phases can be allowed in this scenario and decoupling scenario only for \(\tan \beta \lesssim 20 - 30\).

The reliable determinations of the neutral Higgs sector CP-violation in the MSSM can be achieved by observing the CP-properties of all the three neutral Higgs particles directly. Higgs bosons can be produced in \(\gamma\gamma\) collisions via one-loop diagrams in which all the possible charged particles participate. The \(s\)-channel resonance productions of neutral Higgs bosons in \(\gamma\gamma\) collisions have been considered as crucial tools of studying the CP properties of Higgs particles [11,12]. Because the polarizations of the colliding photons can strongly govern both the \(\gamma\gamma\) luminosity spectrum and the cross sections, obtaining the highly polarized photon beams is important to Higgs boson detections. This is possible by Compton backscattering of laser photons off the linear collider electron and positron beams which can produce high luminosity \(\gamma\gamma\) collisions with a wide spectrum of \(\gamma\gamma\) center of mass energy [13].

In particular, one can observe CP violating effects through the \(s\)-channel resonance for CP-odd neutral Higgs particle production in the linear collider. Due to the mixing effect between the CP-odd and CP-even neutral Higgs bosons, there are the additional loop contributions of charged scalars and vectors to the would-be CP-odd neutral Higgs \(H_2\) production in \(\gamma\gamma\) collision, resulting in the enhanced production cross section. In Ref. [14], the CP violation of the neutral Higgs sector at a photon collider was studied using the \(s\)-channel resonance production cross sections and the polarization asymmetries of Higgs particles for \(3 \leq \tan \beta \leq 10\). In the loop diagrams relevant to \(\gamma\gamma \rightarrow \) neutral Higgs bosons, the contributions of charginos were neglected by assuming that they were heavy enough to be decoupled from the productions of the Higgs bosons. However, charginos are not much heavier than the lighter stop in many SUSY breaking scenarios, and their effects should be included in a realistic analysis. The current lower limit on the lighter chargino mass from LEP II experiment is only \(M_{\tilde{\chi}^-_1} > 103\) (83.6) GeV for \(m_{\tilde{\nu}} > (<)\) 300 GeV in the minimal supergravity scenario [13]. It is even less stringent in the AMSB scenario: \(M_{\tilde{\chi}^-_1} > 45\) GeV. Therefore we include the chargino contributions to \(\gamma\gamma \rightarrow H_{k=1,2,3}\), and investigated their effects when other parameters are fixed as Ref. [14].

In this work, we investigate the neutral Higgs productions at \(\gamma\gamma\) collisions, including the chargino loop contributions as well as other charged particles in the MSSM, and study the CP properties of the MSSM Higgs sector. This paper is organized as follows. In Section II, we review briefly the loop-induced CP violation and the mixing of CP-even and CP-odd Higgs bosons in the neutral Higgs sector of the MSSM. In Section III, we derive the cross sections for the Higgs productions in \(\gamma\gamma\) collisions and the polarization asymmetries in terms of two form factors appearing in the \(\gamma\gamma \rightarrow H_{k=1,2,3}\) amplitudes. In Section IV, we present detailed numerical analyses and discuss the potential importance of chargino loop contributions to the CP violation in \(\gamma\gamma \rightarrow H_k\). The formulae for the chargino and stop mass matrices, their eigenvalues and the corresponding mixing matrices are given in Appendix A. The interaction Lagrangians relevant to \(\gamma\gamma \rightarrow H_k\) are recapitulated for both convenience and completeness.
II. THE NEUTRAL HIGGS SECTOR IN THE MSSM

The MSSM Lagrangian possesses many new CP violating phases in the soft SUSY breaking terms in addition to the KM phase in the CKM matrix element. Using Peccei-Quinn $U(1)_{PQ}$ and $U(1)_{R}$ symmetries, we can redefine some parameters to be real. We will work in the basis where $B\mu$ and the wino mass parameter $M_{2}$ are real. In the MSSM, the Higgs potential is CP-conserving at the tree level and only the soft terms (and the usual CKM mixing matrix) can have CP violating phases. However, CPV phases in soft terms can induce CP violation in the effective potential of Higgs bosons through quantum corrections involving squarks and other SUSY particles in the loop. The effective potential of the Higgs fields at the one-loop level\(^1\) can be written as

\[
\mathcal{V}_{\text{Higgs}}^{\text{eff}} = \mu_{1}^{2}\Phi_{1}^{\dagger}\Phi_{1} + \mu_{2}^{2}\Phi_{2}^{\dagger}\Phi_{2} + (m_{12}^{2}\Phi_{1}^{\dagger}\Phi_{2} + \text{h.c.}) + \lambda_{1}(\Phi_{1}^{\dagger}\Phi_{1})^{2} + \lambda_{2}(\Phi_{2}^{\dagger}\Phi_{2})^{2} + \lambda_{3}(\Phi_{1}^{\dagger}\Phi_{1})(\Phi_{2}^{\dagger}\Phi_{2}) + \lambda_{4}(\Phi_{1}^{\dagger}\Phi_{2})(\Phi_{2}^{\dagger}\Phi_{1}) + \lambda_{5}(\Phi_{2}^{\dagger}\Phi_{2})^{2} + \lambda_{6}(\Phi_{2}^{\dagger}\Phi_{1})(\Phi_{1}^{\dagger}\Phi_{2}) + \lambda_{7}(\Phi_{1}^{\dagger}\Phi_{1})(\Phi_{2}^{\dagger}\Phi_{2}) + \lambda_{8}(\Phi_{2}^{\dagger}\Phi_{2})(\Phi_{1}^{\dagger}\Phi_{1}).
\]  

The fields $\Phi_{i}$ ($i = 1, 2$) are the scalar components of the Higgs superfields, with $\Phi_{2}(\Phi_{1})$ giving masses to the up-type (down-type) fermions. In the MSSM, one has $\lambda_{i} = 0$ ($i = 5, 6, 7$) at tree level so that there is not Higgs sector CP violations in the MSSM. But these couplings are generated at one loop level and can be complex if $\mu, A_{i}$ possess CPV phases. Also the Higgs bilinear couplings $m_{12}^{2}$ \[^{10}\,\] can be complex by quantum corrections. For small $\tan\beta \sim O(1)$, where the stop contributions are dominant over sbottom or chargino contributions to the Higgs sector CP violations \[^{3,13,17}\,\] one has, for example \[^{18}\,\]

\[
m_{12}^{2} \simeq B\mu + \frac{1}{16\pi^{2}}h_{t}^{2}A_{t}\mu \left[ \frac{m_{12}^{2} + m_{11}^{2}}{4(m_{12}^{2} - m_{11}^{2})} \ln \frac{m_{12}^{2}}{m_{11}^{2}} - \frac{1}{2} \right],
\]  

where the top Yukawa coupling $h_{t} = \sqrt{2m_{t}(m_{t})}/v\sin\beta$, and $m_{ti}$ ($i = 1, 2$) are the masses of the lighter and heavier stops. The contributions of the 1st and 2nd generation squarks are negligible because of their small Yukawa couplings. The mixing of two CP-even Higgs bosons is denoted by the real parameter $B\mu$, whereas the $h_{t}^{2}A_{t}\mu$ term with the complex $A_{t}$ trilinear coupling generates the mixings among all two CP-even and one CP-odd neutral Higgs bosons. Therefore, the quadratic term of the Higgs fields with the coefficient $m_{12}^{2}$ plays an important role in the Higgs mixing. If $h_{t}^{2}A_{t}\mu$ terms are much less than $B\mu$, and $\mu_{1}^{2} \sim \mu_{2}^{2} \sim B\mu$, we can expect that the scalar-scalar mixing is much larger than the scalar-pseudoscalar mixing. For large $\tan\beta \gtrsim 30$, the contribution of the chargino sector can dominate those of the stop and sbottom sectors in the mixing between the CP-even and CP-odd Higgs bosons \[^{14}\,\] The same is true of other quartic couplings $\lambda_{5,6,7}$, whose imaginary parts vanish in the CP conserving limit (or at tree level) in the MSSM. One has to keep in mind that there is a strong constraint from two-loop Barr-Zee type $e/n$ EDM constraints

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\[^{1}\]We follow the notations of the recent third paper of Ref. \[^{4}\,\].
for large $\tan \beta$ ($40 \lesssim \tan \beta \lesssim 60$). Therefore, we will choose rather low $\tan \beta \lesssim 20$ and allow maximal CPV phase in the $\mu$ and $A_t$ parameters.

Since the electroweak gauge symmetry is broken spontaneously into $U(1)_{\text{em}}$, two Higgs doublets can be written as

$$\Phi_1 = \left( \frac{\phi_1^1}{(v_1 + \phi_1 + ia_1)/\sqrt{2}} \right), \quad \Phi_2 = e^{i\xi} \left( \frac{\phi_2^2}{(v_2 + \phi_2 + ia_2)/\sqrt{2}} \right),$$

where the VEVs $v_i$ are real. The relative phase $\xi$, which is renormalization-scheme dependent, is determined from the minimum energy conditions of the Higgs potential: $T_\phi = \partial V_{\text{Higgs}}^{\text{eff}}/\partial \phi = 0$. It turns out $\xi$ is very small in the $\overline{\text{MS}}$ scheme, and will be ignored in the numerical analysis. Because the electroweak symmetry is spontaneously broken to $U(1)_{\text{em}}$, three Goldstone bosons are eaten by $W^\pm, Z^0$ gauge bosons, and one ends up with two charged Higgs and three neutral Higgs bosons. The $3 \times 3$ (mass)$^2$ mass matrix $M_N^2$ for three neutral Higgs bosons is a real symmetric matrix, and is diagonalized by a $3 \times 3$ orthogonal matrix $O$:

$$O^T M_N^2 O = \text{diag}(M_{H_1}^2, M_{H_2}^2, M_{H_3}^2),$$

where $M_{H_3} \geq M_{H_2} \geq M_{H_1}$. The corresponding mass eigenstates, $H_i$ ($i = 1, 2, 3$), are defined from the weak eigenstates as

$$(a, \phi_1, \phi_2)^T = O(H_1, H_2, H_3)^T.$$

III. NEUTRAL HIGGS BOSON PRODUCTIONS AT PHOTON COLLIDERS

Both within the SM and the MSSM, the neutral Higgs decays into two gluons ($gg$) or two photons ($\gamma\gamma$) have been interesting subjects. The inverse of the former process is a main production mechanism for the neutral Higgs bosons at hadron colliders if the Higgs bosons have intermediate masses. The latter is an important mode for tagging the neutral Higgs bosons at hadron colliders. Its inverse process is the mechanism for neutral Higgs productions in the $\gamma\gamma$ collision which can be run at next linear colliders (NLC).

The reactions $gg \rightarrow H_k$ ($k = 1, 2, 3$) are generated by the $(s)quark$ loops, and have been already discussed by two groups in the presence of the MSSM Higgs sector CP violation\[19\]. We have calculated these processes and confirmed their results, although we do not reproduce them here. The case for $\gamma\gamma \rightarrow H_k$ is more complicated than the previous case ($gg \rightarrow H_k$), since one has to include all the charged particle ($W^\pm, H^\pm$ and charginos) contributions as well as the $(s)quark$ loop contributions. It is straightforward to perform the loop integrations. The only thing to take into account is the various mixing components for charginos and neutral Higgs bosons. We present the chargino mass matrix $M_C$, its mass eigenvalues $M_{\tilde{\chi}^-_1}, M_{\tilde{\chi}^-_2}$ and two mixing matrices $U$ and $V$: $U^* M_C V^\dagger = \text{diag}(M_{\tilde{\chi}^-_1}, M_{\tilde{\chi}^-_2})$ (see Appendix A for explicit expressions).

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2Refer to the third paper of Ref. \[6\].
The interaction Lagrangian between the charginos and three neutral Higgs bosons is

$$\mathcal{L}(H_j \tilde{\chi}_k^+ \tilde{\chi}_l^-) = H_j \overline{\tilde{\chi}}_k \left[ \text{Re}(\kappa_{kl}^j) + i\gamma^5 \text{Im}(\kappa_{kl}^j) \right] \tilde{\chi}_l^-, \quad (6)$$

(with \( j = 1, 2, 3 \) and \( k, l = 1, 2 \)) where

$$\kappa_{kl}^j = -\frac{g}{\sqrt{2}} \left[ e^{i\xi_j} U_{k1} V_{l2}(O_{3,j} + i\cos\beta O_{1,j}) + U_{k2} V_{l1}(O_{2,j} + i\sin\beta O_{1,j}) \right]. \quad (7)$$

In this work, it suffices to keep \( \kappa_{kk}^j \) only, since we consider the chargino loop contribution to \( \gamma\gamma \to H_i \). Note that there are two CP violating phases (\( \xi \) and \( \theta_\mu = \arg(\mu) \)) in the couplings \( \kappa_{kk}^j \). Also note that the \( H_j - \tilde{\chi}_k^+ - \tilde{\chi}_k^- \) couplings arise from the Higgs-gaugino-Higgsino couplings in the current basis. Thus the chargino loop effects will be maximized if the wino-Higgsino mixing is large. This requires \( \mu \approx M_2 \). In our study, however, we are interested in large \( \mu \) parameter (which we fix to \( \mu = 1.2 \text{ TeV} \)) in order to have large CP mixing between CP-even and CP-odd Higgs bosons from the stop loop. Then the charginos become too heavy to be relevant to \( \gamma\gamma \to H_i \). For a smaller wino mass parameter \( M_2 = 150 \text{ GeV} \), the wino-Higgsino mixing becomes smaller, but the lighter chargino mass becomes also very light, and the loop function will be enhanced. The net result turns out that the light chargino loop effects are important for the reaction \( \gamma\gamma \to H_i \) even if the lighter chargino is dominantly a wino state (\( M_2 \ll |\mu| \)).

The amplitudes for \( \gamma(k_1, \epsilon_1) + \gamma(k_2, \epsilon_2) \to H_i(q) \) (with \( i = 1, 2, 3 \)) can be defined in terms of two form factors \( A_i(s) \) and \( B_i(s) \) as follows in a model independent way (we closely follow the convention of Ref. [14] in the following):

$$\mathcal{M}(\gamma\gamma \to H_i) = M_{H_i} \frac{\alpha}{4\pi} \left\{ A_i(s) \left[ \epsilon_1 \cdot \epsilon_2 - \frac{2}{s} (\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_1) \right] - B_i(s) \frac{2}{s} \epsilon_{\mu\nu\alpha\beta} \epsilon_1^\mu \epsilon_2^\nu \kappa_{k1}^i \kappa_{k2}^i \right\}, \quad (8)$$

where \( s \equiv (k_1 + k_2)^2 = M_{H_i}^2 \). Including the chargino loop contributions, the CP-even form factors \( A_i \) at \( s = M_{H_i}^2 \) are

$$A_i(s = M_{H_i}^2) = \sum_{f=t,b} A_i^f + \sum_{j=t_1,2, b_1,2} A_i^j + A_i^{H^\pm} + A_i^{W^\pm} + \sum_{j=1,2} A_i^{\chi_j^\pm}, \quad (9)$$

The CP-even functions \( A_i^f, A_i^j, A_i^{H^\pm}, \) and \( A_i^{W^\pm} \) are given in Ref. [14]. We confirmed their results and reproduced them and the related form factor loop functions in Tables 1 and 2 for completeness. The chargino contribution to \( A \) form factor is

$$A_i^{\chi_j^\pm} = 2\text{Re}(\kappa_{jj}^i) \frac{M_{H_i}}{M_{\chi_j^\pm}} F_{sf}(\tau_{i\chi_j^\pm})$$

where \( \tau_{i\chi} = M_{H_i}^2/4M_{\chi_j^\pm}^2 \). The form factor \( F_{sf}(\tau) = \tau^{-1} [1 + (1 - \tau^{-1}) f(\tau)] \) (and other loop functions defined in Table II) depends on the scaling function \( f(\tau) \):\n
$$f(\tau) = -\frac{1}{2} \int_0^1 \frac{dy}{y} \log[1 - 4\tau y(1 - y)] = \begin{cases} \arcsin^2(\sqrt{\tau}) & \text{for } \tau \leq 1 \\ \frac{1}{4} \left[ \log \left( \frac{\sqrt{\tau} + \sqrt{1 - \tau}}{\sqrt{\tau} - \sqrt{1 - \tau}} \right) - i\pi \right]^2 & \text{for } \tau \geq 1. \end{cases} \quad (11)$$
On the other hand, the CP-odd form factor $B_i$ have contributions only from the fermion loops and not from the boson loops:

$$B_i(s = M_{H_i}^2) = \sum_{f=t,b} B_i^f + \sum_{j=1,2} B_i^{\tilde{\chi}^\pm_j},$$

(12)

where $B_i^f$ are given in [14] (see also Tables 1 and 2), and the chargino contributions are

$$B_i^{\tilde{\chi}^\pm_j} = -2\text{Im}(\kappa^i_{jj}) \frac{M_{H_i}}{M_{\tilde{\chi}^\pm_j}} F_{pf}(\tau_{\tilde{\chi}^\pm_j}),$$

(13)

where $F_{pf}(\tau) = \tau^{-1} f(\tau)$. Therefore, when a CP-odd Higgs boson $A$ is produced in $\gamma\gamma$ collision in the CP-conserving limit, only fermion loops (not boson loops) contributes to the production reaction.

It is also convenient to define two helicity amplitudes $\mathcal{M}_{\pm\pm}$ by

$$\mathcal{M}_{\lambda_1\lambda_2} = -M_{H_i} \frac{\alpha}{4\pi} \{A_k(s)\delta_{\lambda_1\lambda_2} + i\lambda_1 B_k(s)\delta_{\lambda_1\lambda_2}\},$$

(14)

where $\lambda_{1,2} = \pm$ are photon helicities. Then, in the narrow-width approximation, the partonic cross sections of the $s$-channel Higgs productions [14] are

$$\sigma(\gamma\gamma \to H_i) = \frac{\pi}{4M_{H_i}^4} \left( |\mathcal{M}_{++}|^2 + |\mathcal{M}_{--}|^2 \right) \delta(1 - M_{H_i}^2/s) \equiv \hat{\sigma}(H_i) \delta(1 - M_{H_i}^2/s).$$

(15)

By using the amplitudes of $\gamma\gamma \to H_i$ at $s = M_{H_i}^2$, we can also obtain the unpolarized decay rates of the neutral Higgs bosons into two photons,

$$\Gamma(H_i \to \gamma\gamma) = \frac{\alpha^2}{256\pi^3} M_{H_i} \left( |A_i(s = M_{H_i}^2)|^2 + |B_i(s = M_{H_i}^2)|^2 \right).$$

(16)

The Higgs sector CP violation can be measured in the following three polarization asymmetries $A_a$ ($a = 1, 2, 3$) [12] which are defined in terms of two independent helicity amplitudes:

$$A_1 = \frac{|\mathcal{M}_{++}|^2 - |\mathcal{M}_{--}|^2}{|\mathcal{M}_{++}|^2 + |\mathcal{M}_{--}|^2} = \frac{2\text{Im}(A_i(s)B_i(s)^*)}{|A_i(s)|^2 + |B_i(s)|^2},$$

(17)

$$A_2 = \frac{2\text{Im}(\mathcal{M}^\dagger_{-\pm}\mathcal{M}_{++})}{|\mathcal{M}_{++}|^2 + |\mathcal{M}_{--}|^2} = \frac{2\text{Re}(A_i(s)B_i(s)^*)}{|A_i(s)|^2 + |B_i(s)|^2},$$

(18)

$$A_3 = \frac{2\text{Re}(\mathcal{M}^\dagger_{-\pm}\mathcal{M}_{++})}{|\mathcal{M}_{++}|^2 + |\mathcal{M}_{--}|^2} = \frac{|A_i(s)|^2 - |B_i(s)|^2}{|A_i(s)|^2 + |B_i(s)|^2},$$

(19)

In the CP-conserving limit, one of the form factors $A_i$ and $B_i$ must vanish, so that $A_1 = A_2 = 0$, and $A_3 = +1(-1)$ for a pure CP-even (CP-odd) Higgs scalar. From the definition of the function $f(\tau)$ in Eq. (14), we find that the form factors $A_i$ and $B_i$ may be complex, when the Higgs masses $M_{H_i}$ are two times larger than the particle mass in the loop. This will induce rich structures in the polarization asymmetries $A_a$ as functions of Higgs masses and other SUSY parameters in the presence of Higgs sector CP violation.
IV. NUMERICAL ANALYSES

The CP violation in the neutral Higgs sector through the stop loop with the complex \( A_t \) parameter always appear in the combination of \( \text{arg}(A_t \mu) \). In the following numerical analyses, we assume that the \( \mu \) parameter is real and positive, in order to simplify the discussions. For the complex \( \mu \) parameter, the chargino mass matrix will contain CPV phase, thereby there would be additional CP violating effects in the chargino loop contributions to \( \gamma \gamma \to H_i \). However, this CP violating effect is independent of the CP violation in the neutral Higgs sector through the mixing between the CP-even and the CP-odd Higgs bosons. Since our focus in this work is to examine the reaction \( \gamma \gamma \to H_i \) in the presence of Higgs sector CP violation through the mixing, we ignore complex phase in the chargino sector. We also assume \( A_t = A_b \) for simplicity even if these couplings are independent in general. The CP violating phase \( \text{arg}(A_t) \) is varied between 0 and 2\( \pi \). Also we choose the same parameters as Ref. [14] (except for the wino mass parameter \( M_2 \)) in our numerical analyses in order to investigate the chargino contributions more clearly;

\[
|A_t| = |A_b| = 0.4 \text{ TeV}, \quad \mu = 1.2 \text{ TeV}, \quad M_2 = 150 \text{ GeV}, \quad M_{\text{SUSY}} = 0.5 \text{ TeV}. \tag{20}
\]

Using these parameter set, we investigate in detail \( \hat{\sigma}_0(\gamma \gamma \to H_i) \) and \( A_a(H_i) \) for two different values of \( \tan \beta = 3 \) and \( \tan \beta = 10 \) as functions of each Higgs boson mass \( (M_{H_i}) \) and CP violating phase \( \text{arg}(A_t) \) with/without chargino loop contributions. As discussed in Section II, we do not consider a very large \( \tan \beta \) case, since the \( A_t \) phase is strongly constrained by the two-loop Barr-Zee type contributions to the EDMs of electron and neutron. Note that the chargino contributions to the Higgs mixing are negligible [17] for our choice of \( \tan \beta = 3 \) and \( \tan \beta = 10 \).

It turns out the Higgs sector CP violation is most prominent in the would-be CP-odd Higgs boson \( H_2 \) production at photon colliders. Therefore we first discuss the production of the would-be CP-odd Higgs scalar. In Fig. 1, we show the production cross section for \( \gamma \gamma \to H_2 \) as a function of \( M_2 \) in the CP conserving limit \( (\text{arg}(A_t) = 0^o) \) for \( \tan \beta = 3 \) (on the left side) and \( \tan \beta = 10 \) (on the right side), respectively. In both cases, we assumed \( \mu = 1.2 \text{ TeV} \), and we set \( M_{H^+} = 300 \text{ GeV} \) so that \( M_{H_2} = 291 \) (290) GeV for \( \tan \beta = 3 \) (10), respectively. The solid (dashed) curve represents the case with (without) chargino contributions. For \( \text{arg}(A_t) = 0^o \) (thick solid curve), \( H_2 \) will be the pure CP-odd state \( (A) \) for our parameter set [20], since we can neglect the effects of charginos on the Higgs mixing due to \( \tan \beta \lesssim 20 \) [17]. In this case, \( \hat{\sigma}_0(\gamma \gamma \to H_2) \) has only the fermion loop contributions, since the couplings of \( H_2 \) to the sfermion pairs, the charged Higgs-boson and \( W \)-boson pairs vanish in the CP conserving limit. The cross section for \( \gamma \gamma \to H_2 \) without chargino loop contributions is independent of \( M_2 \) (the horizontal dash-dotted lines), and are quite small \( (\lesssim 1 \text{ fb}) \). The bottom-quark contribution is negligible compared to the top-quark contribution for two reasons: (i) the small \( b \) quark mass and (ii) the smaller electric charge of \( b \) quark (note that the \( \gamma \gamma \to H_2 \) amplitude depends on \( e_q^2 \)). For our choice of parameters, the bottom quark contribution turns out to get significant only for \( \tan \beta \geq 10 \), and can be safely neglected for \( \tan \beta \lesssim 10 \). On the other hand, the cross section for \( \gamma \gamma \to H_2 \) is enhanced almost by an order of magnitude when the chargino loop contributions are included. The chargino loop contributions to \( \gamma \gamma \to H_2 \) cannot be ignored at all, if charginos are not very heavy. This is true even if we set \( M_2 \ll |\mu| \) so that the wino-Higgsino mixing is not large. Still
the lighter charginos are light enough ($M_2 = 150$ GeV for our parameter set) and the loop contribution is important. Also because of the $1/\tan\beta$ suppression factor for the top loop, the chargino loop contribution becomes more important for larger $\tan\beta$. Finally, as the $M_2$ increases, the lighter chargino becomes heavier and the chargino loop contribution decreases rather quickly due to the decoupling theorem. Since the chargino mass arises dominantly from SUSY breaking rather than from electroweak symmetry breaking, the decoupling of the chargino loop contribution is more effective than the top loop contribution. Also, the couplings $\text{Im}(\kappa_{jj}^2)$ decrease more quickly as functions of $\tan\beta$ compared to the loop functions as $M_2$ increases. Therefore, the difference between the cross sections for $\tan\beta = 3$ and $\tan\beta = 10$ increases as $M_2$ increases.

In Fig. 2, we show the cross section for $\gamma\gamma \to H_2$ as a function of $\arg(A_t)$ for $\tan\beta = 3$ (on the left side) and $\tan\beta = 10$ (on the right side), respectively. The solid (the dash-dotted) curves represents the case with (without) the chargino loop contributions. For $\arg(A_t) = 0^\circ$ (or $180^\circ$), the cross section is strongly enhanced by the chargino loop contributions as discussed in the previous paragraph. As $\arg(A_t)$ is turned on, the cross section is significantly enhanced even without the chargino loop contributions. This is because all the charged particles including bosons begin to contribute in the presence of CP violation in the Higgs sector. The dash-dotted curves strongly depend on $\arg(A_t)$ for the following reasons. First of all, the stop masses and the mixing angles depend on $\arg(A_t)$ very sensitively. Note that the stop masses have the $LR$ mixing term $m_{12}^{2,LR} = m_t(A_t^* e^{-i\xi} - \mu/\tan\beta)$, as shown in Eq. (A15) of the Appendix A. Since the mixing between CP-even and CP-odd neutral Higgs bosons arises from the stop loop [see Eq. (3)], the $A_t$ phase affects the CP mixing through $\text{Im}(A_t\mu)$ and the stop masses in Eq. (2). Also once CP is broken in the Higgs sector, all the charged particles including bosons as well as fermions contribute to $\gamma\gamma \to H_2$. Therefore the stop loop contribution will depend on $\arg(A_t)$. Still the dominant contribution comes from the chargino loops (see the solid curves in Fig. 2). The net result depends on $\arg(A_t)$ rather mildly, mainly through the $\arg(A_t)$ of the CP-odd and CP-even Higgs mixing.

Also note that the sensitivity of the cross section $\hat{\sigma}_0(\gamma\gamma \to H_2)$ to $\arg(A_t)$ decreases as $\tan\beta$ increases. This tendency can be understood by the strong phase dependences of stop masses, since stop loops contribute to (i) the mixing of the CP-even and the CP-odd Higgs bosons, and (ii) the loop diagrams. The scalar-pseudoscalar mixing is typically characterized by

$$\text{Im}(m_{12}^2) \propto h_t^2 \arg(A_t\mu),$$

whose $\tan\beta$ dependence is negligible for $3 \leq \tan\beta \leq 10$ [see Eq. (2)]. Also the stop mass eigenvalues are sensitive to the CP phase $\arg(A_t)$ when $|A_t| = |\mu|/\tan\beta$ due to the $LR$ mixing ($\tan\beta = 3$ for our parameter set $|A_t| = |\mu|/3 = 0.4$ TeV). The CP mixing would be a decreasing function of $\tan\beta$ for $\tan\beta \geq 3$, and the stop masses are less sensitive to CP phase $\arg(A_t)$ for the larger $\tan\beta = 10$. Therefore, the phase dependence of the mixing would be a decreasing function of $\tan\beta$ for $\tan\beta \geq 3$. Another dependence of the cross section on $\arg(A_t)$ originates from the stop masses in the loop, which is sensitive to the phase $\arg(A_t)$ in our choice of SUSY parameter set. In other words, $\tan\beta$-dependence of the phase sensitivity comes dominantly from the stop masses as in the CP-even and CP-odd Higgs mixing. Therefore, the cross section depends on the phase $\arg(A_t)$ less sensitively when $\tan\beta$ becomes larger for our parameter set. Finally, the heavier Higgs boson ($H$) is
also strongly affected by the CP mixing, since it can have a large mixing with the CP-odd scalar \( A \). The discussions for \( H \) will be similar to those for \( A \), and will not be repeated.

In Figs. 3 and 4, we show that the cross sections \( \hat{\sigma}_0(H_i) \) (\( i = 1, 2, 3 \)) in units of fb for five different \( A_i \) phases; \( \arg(A_i) = 0^\circ \) (thick solid curve), \( 40^\circ \) (dash-dotted curve), \( 80^\circ \) (dashed curve), \( 120^\circ \) (dotted curve) and \( 160^\circ \) (solid curve) for \( \tan \beta = 3 \) (Fig. 3) and \( \tan \beta = 10 \) (Fig. 4). We present two different cases: without the chargino loop contributions (the left column) as in Ref. [14] and with the chargino loop contributions (the right column) with \( M_2 = 150 \text{ GeV} \) for which \( M_{\tilde{\chi}_1^\pm} = 146 \) (148.2) GeV for \( \tan \beta = 3 \) (10). The chargino contributions to \( \gamma\gamma \to H_1 \) is negligible, since \( M_{H_1} \) is far below the chargino pair threshold \( 2M_{\tilde{\chi}_1^\pm} \) for our parameter set. On the other hand, two heavier Higgs productions are affected by chargino loops by significant amounts, and we can observe rich structures in the production cross sections due to the interference of all the charged particles’ contributions. For example, the production cross sections for \( \gamma\gamma \to H_2 \) without the chargino loop contributions (the left columns of Figs. 3 and 4) have only one single peak at the point \( M_{H_2} = 2m_t \) for \( \arg(A_t) = 0^\circ \). If the chargino loop contributions are included (the right columns), the production cross sections have two comparable peaks at the point \( M_{H_2} = 2M_{\tilde{\chi}_1^\pm} \) (lighter chargino) and \( M_{H_2} = 2m_t \) in the CP-conserving limit. As the CP violating phase \( \arg(A_t) \) increases, the cross section \( \hat{\sigma}_0(\gamma\gamma \to H_2) \) starts to get extra contributions from the charged boson loops (involving sfermions, the charged Higgs-boson and the W-boson pairs) due to the mixing between the CP-odd and the CP-even neutral Higgs bosons.

For a larger \( \tan \beta = 10 \) (Fig. 4), there appear three qualitative differences compared to the lower \( \tan \beta = 3 \); the effect of bottom quark loop contribution, the dominant chargino loop contributions, and the interchange of the CP-properties of the neutral Higgs bosons.

- Since the bottom quark Yukawa coupling (to the CP even Higgs boson) is proportional to \( 1/\cos \beta \), the bottom quark contribution can be significant in the region of large \( \tan \beta \). For \( \arg(A_t) = 0^\circ \), the CP-odd Higgs boson \( H_2 \) has pseudoscalar couplings to top and bottom quarks, where the coupling of \( H_2 \) to top (bottom) quark is proportional to \( \cot \beta \) (\( \tan \beta \)) [see Eqs. (B1) and (B2) of the Appendix B]. Furthermore, there are additional differences from different electric charges of top and bottom quarks, since the \( \gamma\gamma \to H_i \) amplitudes depend on \( e_i^2 \), which are \( (2/3)^2 \) vs. \( (-1/3)^2 \) for (s)top and (s)bottom, respectively. On the other hand, the loop functions have weaker \( \tan \beta \)-dependences. For our parameter set, it turns out that the bottom quark contribution begins to dominate the top quark contribution when \( \tan \beta \sim 10 \), and can be neglected for \( \tan \beta < 10 \).

- In the CP conserving limit, the chargino contribution to \( \gamma\gamma \to A \) is dominant over the top quark contribution, since the latter is suppressed by \( 1/\tan \beta \) relative to the former, even if we assume the mixing angles in the chargino sector are \( O(0.1) \). This is the reason why the top quark contribution decreases more quickly than the lighter chargino contributions as \( \tan \beta \) increases in Figs. 3 and 4.

- The final point is the interchange of the CP properties of the heavier Higgs bosons \( H_2 \) and \( H_3 \) for large \( \arg(A_t) \) and large \( \tan \beta = 10 \). Since there are only fermion contributions to the CP-odd Higgs production, i.e., two peaks at \( M_{H_i} = 2M_{\tilde{\chi}_1^\pm} \) and \( 2m_t \), we can find from Fig. 4 that \( H_3 \) for \( \arg(A_t) = 160^\circ \) has the same CP-odd property
as $H_2$ for $\arg(A_t) = 0^\circ$. This can be checked even more easily by using the polarization asymmetry $A_3$ which is $+1(-1)$ for a CP-even (CP-odd) Higgs boson, as discussed below in relation with polarization asymmetries (Figs. 5–7).

The importance of chargino loop contributions for $H_3$ production is also similar to the case of $H_2$ production as discussed above, and we will not repeat it again.

The number of events is determined by the combination of the luminosity and the cross section for $\gamma\gamma \rightarrow H_2$. Although the photon beam luminosity depends on many parameters, if one only consider the high energy part of the generated photons, the 0.3 conversion factor and the comparable photon spot size to electron beam, the approximate luminosity of $\gamma\gamma$ collider \[ (21) \]

where $L_{\text{geom}}$ is the luminosity of $e^+e^-$ collider. Taking 100 fb$^{-1}$ as a nominal integrated luminosity in the $\gamma\gamma$ mode, we can infer from Figs. 3 and 4 that the maximum number of events for the CP-odd Higgs boson is approximately 100 (10) per a year for $\tan\beta = 3$ ($\tan\beta = 10$), when the unpolarized cross section does not contain chargino-loop contributions. However, the chargino-loop contributions enhance the maximum number of events as approximately 880 (710) for $\tan\beta = 3$ ($\tan\beta = 10$). Hence, the chargino loop contributions for the production of the would-be CP-odd Higgs boson can be significant at the $\gamma\gamma$ collider for larger $\tan\beta$.

In Fig. 5, we show three polarization asymmetries of $H_2$ as functions of $\arg(A_t)$ for $\tan\beta = 3$ (the left column) and $\tan\beta = 10$ (the right column). As in Figs. 1 and 2, we set $M_{H^+} = 300$ GeV so that $M_{H_2} = 291$ ($290$) GeV for $\tan\beta = 3$ ($\tan\beta = 10$), respectively. The case with (without) the chargino loop is represented by solid (dash-dotted) curves. We have fixed $M_2 = 150$ GeV as before. The polarization asymmetries $A_i(\Phi)$’s satisfy the following relations:

\[ A_{1,2}(\Phi) = -A_{1,2}(360^\circ - \Phi), \quad A_3(\Phi) = +A_3(360^\circ - \Phi), \]  

where $\Phi = \arg(A_t\mu) + \xi$ with $\xi = 0$. Namely, $A_{1,2}$ are CP-odd observables (antisymmetric about $\Phi = 180^\circ$) and $A_3$ is a CP-even observable (symmetric about $\Phi = 180^\circ$). Note that the chargino loops not only enhance the cross section but also affect the polarization asymmetries by significant amounts.

In Fig. 6, we show the polarization asymmetries $A_i(H_j)$ as functions of the neutral Higgs masses for $\arg(A_t) = 0^\circ$, $40^\circ$, $80^\circ$, $120^\circ$ and $160^\circ$ with $\tan\beta = 3$, including all the charged particles in the loops. The lightest Higgs boson $H_1$ still behaves like a CP-even scalar, since $-0.03\% \lesssim A_1 \leq 0$, $0 \leq A_2 \lesssim 0.4\%$, and $A_3 \simeq 1$. On the other hand, the heavier $H_2$ and $H_3$ are generically admixtures of CP-even and CP-odd states if the phase of $A_t$ does not vanish. For $H_2$ and $H_3$, chargino, top and stop loops give main contributions to the asymmetries above the chargino-pair threshold, but the chargino and $W^\pm$ loop contributions affect them below the chargino-pair threshold.

In Fig. 7, we present the polarization asymmetries for $\tan\beta = 10$. Again, the lightest Higgs boson $H_1$ behaves like a CP-even scalar for the larger $\tan\beta$, since $-0.1\% \lesssim A_1 \leq 0$, $0 \leq A_2 \lesssim 0.3\%$, and $A_3 \simeq 1$. If $\tan\beta$ becomes larger, the top (stop) loop contribution is accompanied by the bottom (sbottom) contribution to the polarization asymmetries of the
heavier Higgs bosons $H_2$ and $H_3$. Fig. 4 indicates that as the CP violating phase $\arg(A_t)$ increases for the case of large $\tan\beta$, the value of the asymmetry $A_3$ of $H_2$ approaches that of $H_3$ at $\arg(A_t) = 0^\circ$ and vice versa, i.e., the CP-properties of the heavier Higgs bosons $H_2$ and $H_3$ are interchanged.

From Figs. 6 and 7, the polarization asymmetry $A_2(H_1)$ is the most sensitive CP observable in detecting the CP violation of the lightest Higgs boson for both small and large $\tan\beta$, when the chargino contributions are included. This result is different from the first paper of Ref. [14], where charginos are neglected by assuming they are very heavy, and thus $A_2 (A_1)$ is the most powerful CP observable for $\tan\beta = 3$ ($\tan\beta = 10$). Unfortunately the asymmetry itself is very small so that it would not be easy to find nonzero $A_2(H_1)$. Still asymmetries for heavier neutral Higgs bosons can be sizable and thus be used as the probes of Higgs sector CP violation if they can be produced with high statistics at NLCs. Therefore, we need to prepare the colliding photon beams with large linear polarizations as well as high center of mass energy $\sqrt{s_{\gamma\gamma}}$ in order to produce neutral Higgs bosons and determine their CP properties in a model independent manner.

V. CONCLUSIONS

In this work, we presented a comprehensive analysis of the neutral Higgs boson productions through $\gamma\gamma \rightarrow H_{i=1,2,3}$ in the presence of the Higgs sector CP-violation of the MSSM. In particular, we have included the chargino loop contributions as well as the contributions from squarks, $W^\pm$ and charged Higgs particles. In many scenarios of SUSY breaking, charginos are not too heavy that their effects are generically important. First of all, the production of the would-be CP-odd $H_2$ boson is enhanced by an order of magnitude when chargino loop contributions are included even without the Higgs sector CP violation. If the phase of the $A_t$ parameter is turned on, CP violation in the Higgs sector become very rich in the structures. This is also true of the case of the heaviest Higgs boson $H_3$. Also the polarization asymmetries are affected by the Higgs sector CP violation.

If the $A_t$ parameter has a large CP violating phase, its effects can appear in various physical observables: the Higgs sector CP-violation as discussed in this work, and also the direct CP violation in $B \rightarrow X_s \gamma$ [22], for example. Since the latter is an indirect signature, it is important to probe SUSY CP violation in a direct way. Thus it is important to probe CP violation from the soft SUSY breaking sector such as $\arg(A_t)$ in the Higgs sector CP violation by using $\gamma\gamma$ colliders as discussed in this work. In this regards, the $\gamma\gamma$ mode at NLC with high $\sqrt{s_{\gamma\gamma}}$ and luminosity, and high quality beam polarizations will be indispensable for this purpose by measuring the cross sections of $\gamma\gamma \rightarrow H_i$ ($i = 1, 2, 3$) and three asymmetries $A_a(H_j)$ ($a, j = 1, 2, 3$) in the MSSM.

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APPENDIX A: CHARGINOS AND SCALAR TOPS

The chargino mass matrix in the \((\tilde{W}^+, \tilde{H}^+)\) basis \([20]\) is

\[
M_C = \begin{pmatrix}
M_2 & \sqrt{2}e^{-i\eta}m_W \cos \beta \\
\sqrt{2}m_W \sin \beta & \mu
\end{pmatrix},
\]

(A1)

where \(M_2 > 0\) and \(\mu\) are gaugino and Higgsino masses, and \(e^{+i\eta}\) is the the phase of the up-type Higgs VEV \([6]\). Since the mass matrix \(X\) is a general complex matrix, it is diagonalized by a biunitary transformation:

\[
U^*XV^{-1} \equiv \text{diag}(M_{\tilde{\chi}^1}, M_{\tilde{\chi}^2}),
\]

(A2)

with \(M_{\tilde{\chi}^2} \geq M_{\tilde{\chi}^1} \geq 0\). In order for \(M_{\tilde{\chi}^1} = 1\) to be positive, we define the unitary matrix \(U\) as a product of two unitary matrices

\[U \equiv HU'.\]

(A3)

The angles \(\theta_1\) and \(\phi_1\) of the unitary matrix

\[
U' = \begin{pmatrix}
\cos \frac{\theta_1}{2} & \sin \frac{\theta_1}{2}e^{+i\phi_1} \\
-\sin \frac{\theta_1}{2}e^{-i\phi_1} & \cos \frac{\theta_1}{2}
\end{pmatrix},
\]

(A4)

are given by

\[
tan \theta_1 = \frac{2\sqrt{2}m_W \left[M_2^2 \cos^2 \beta + |\mu|^2 \sin^2 \beta + M_2|\mu| \sin 2\beta \cos(\theta_\mu + \xi)\right]^{1/2}}{M_2^2 - |\mu|^2 - 2m_W^2 \cos 2\beta},
\]

(A5)

\[
tan \phi_1 = \frac{|\mu| \sin(\theta_\mu + \xi) \sin \beta}{M_2 \cos \beta + |\mu| \cos(\theta_\mu + \xi) \sin \beta},
\]

(A6)

where \(\theta_\mu = \text{arg}(\mu)\). The unitary mixing matrix \(V\) is

\[
V = \begin{pmatrix}
\cos \frac{\theta_2}{2} & \sin \frac{\theta_2}{2}e^{-i\phi_2} \\
-\sin \frac{\theta_2}{2}e^{i\phi_2} & \cos \frac{\theta_2}{2}
\end{pmatrix},
\]

(A7)

where

\[
tan \theta_2 = \frac{2\sqrt{2}m_W \left[M_2^2 \sin^2 \beta + |\mu|^2 \cos^2 \beta + M_2|\mu| \sin 2\beta \cos(\theta_\mu + \xi)\right]^{1/2}}{M_2^2 - |\mu|^2 + 2m_W^2 \cos 2\beta},
\]

(A8)

\[
tan \phi_2 = \frac{M_2 \sin \xi \sin \beta - |\mu| \sin \theta_\mu \cos \beta}{M_2 \cos \xi \sin \beta + |\mu| \cos \theta_\mu \cos \beta}.
\]

(A9)

By using the unitary matrix \(H = \text{diag}(e^{i\gamma_1}, e^{i\gamma_2})\), where \(\gamma_{1,2}\) are the phases of the diagonal elements of \(U^*XV^{-1}\), we finally obtain

\[
U^*XV^{-1} = \text{diag}(M_{\tilde{\chi}^1}, M_{\tilde{\chi}^2}).
\]

(A10)

And the mass eigenvalues of charginos are
\[ M^2_{\tilde{x}_1, \tilde{x}_2} = \frac{1}{2} (M_2^2 + |\mu|^2 + 2m_W^2) \pm \frac{1}{2} \left[ \left( M_2^2 - |\mu|^2 \right)^2 + 4m_W^4 \cos^2 2\beta \\
+ 4m_W^2 \left( M_2^2 + |\mu|^2 + 2M_2 |\mu| \cos \theta_\mu \sin 2\beta \right) \right]^{1/2}. \tag{A11} \]

Note that the mass eigenvalues and the mixing angles depend on the CP violating phases \( \xi \) and \( \theta_\mu \).

The stop (mass)\(^2\) matrix \( \mathcal{M}_t^2 \) \cite{18} is written as

\[
\mathcal{L}_{\text{mass}}^{\text{eff}} = - (\tilde{t}_L^* \tilde{t}_R) \mathcal{M}_t^2 (\tilde{t}_L \tilde{t}_R)
= - (\tilde{t}_L^* \tilde{t}_R)^* \left( \begin{array}{cc}
m^2_{\tilde{t}L} & m^2_{\tilde{t}LR} \\
m^2_{\tilde{t}R} & m^2_{\tilde{t}R}
\end{array} \right) \left( \begin{array}{c}
\tilde{t}_L \\
\tilde{t}_R
\end{array} \right), \tag{A12}
\]

where

\[
m^2_{\tilde{t}L} = M^2_{\tilde{t}L} + m^2_t + m^2_Z \cos 2\beta \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right), \tag{A13}
m^2_{\tilde{t}R} = M^2_{\tilde{t}R} + m^2_t + m^2_Z \cos 2\beta \cdot \frac{2}{3} \sin^2 \theta_W, \tag{A14}
m^2_{\tilde{t}LR} = m_t \left( A^*_t e^{-i\xi} - \mu \cot \beta \right). \tag{A15}
\]

The stop mixing angle \( \theta_{\tilde{t}} \) is

\[
\theta_{\tilde{t}} = \frac{1}{2} \arctan \left( \frac{2|m^2_{\tilde{t}LR}|}{m^2_{\tilde{t}L} - m^2_{\tilde{t}R}} \right). \tag{A16}
\]

The relations between the mass and the weak eigenstates of stops are given by

\[
\tilde{t}_1 = \tilde{t}_L \cos \theta_{\tilde{t}} + \tilde{t}_R e^{-i\beta_{\tilde{t}}} \sin \theta_{\tilde{t}},
\tilde{t}_2 = -\tilde{t}_L e^{i\beta_{\tilde{t}}} \sin \theta_{\tilde{t}} + \tilde{t}_R \cos \theta_{\tilde{t}}, \tag{A17}
\]

where \( \beta_{\tilde{t}} = -\arg(m^2_{\tilde{t}LR}) \). The mass eigenvalues of the lighter and heavier stops are

\[
m^2_{\tilde{t}_{1,2}} = \frac{m^2_{\tilde{t}L} + m^2_{\tilde{t}R} \mp \sqrt{(m^2_{\tilde{t}L} - m^2_{\tilde{t}R})^2 + 4|m^2_{\tilde{t}LR}|^2}}{2}. \tag{A18}
\]

Note that \( m^2_{\tilde{t}_{1,2}} \) is dependent on the CP violating phases, \( \arg(A_t) \) and \( \arg(\mu) \) due to \( m^2_{\tilde{t}LR} \) in Eq. (A15).

**APPENDIX B: RELEVANT COUPLINGS**

In this sections, we list the couplings relevant to \( \gamma\gamma \to H_i \) that appear in Table 1.

- Higgs-fermion-fermion couplings:

\[
\mathcal{L}_{Hff} = - \frac{gm_f}{2m_W} \bar{f} \left[ \left( \frac{\psi_f}{R_{\psi_f}} \right) - i\gamma_5 \left( \frac{R_{\psi_f} a_f}{R_{\psi_f}} \right) \right] f H_i, \tag{B1}
\]
where
\[
R^d_\beta = \bar{R}^u_\beta = \cos \beta \equiv c_\beta, \quad R^u_\beta = \bar{R}^d_\beta = \sin \beta \equiv s_\beta, \\
v^d_f = O_{2,i}, \quad v^u_f = O_{3,i}, \quad a^d_f = a^u_f = O_{1,i}.
\] (B2)

Here the matrix $O$ diagonalizes the Higgs mass matrix as in Eq. (5). In the presence of Higgs sector CP violation, the Higgs bosons couple with both CP-even and CP-odd bilinears, $\bar{f}f$ and $\bar{f}\gamma_5 f$, simultaneously.

- The Higgs-W-W couplings are determined by the gauge couplings:
\[
\mathcal{L}_{HW^+W^-} = g m_W (c_\beta O_{2,i} + s_\beta O_{3,i}) H_i W^+ W^- \mu.
\] (B3)

- The Higgs-sfermion-sfermion couplings:
\[
\mathcal{L}_{H_i \tilde{f}_j \tilde{f}_k} = g^i_{j,k} \tilde{f}_j^* \tilde{f}_k H_i,
\]
with
\[
g^i_{j,k} = \tilde{C}^{\alpha \beta \gamma}_{i,j,k} O_{\alpha,i}^* (U_f)^*_{\beta,j} (U_f)_{\gamma,k}.
\] (B4)

The matrix $U_f$ diagonalize the sfermion mass matrix:
\[
U_f^\dagger M_f^2 U_f = \text{diag}(m_{\tilde{f}_1}^2, m_{\tilde{f}_2}^2)
\]
with $m_{\tilde{f}_1} \leq m_{\tilde{f}_2}$. The indices $\alpha$ and $\{\beta, \gamma\}$ label the three neutral Higgs bosons $(a, \phi_1, \phi_2)$ and the sfermion chiralities $\{L, R\}$, respectively. The explicit expressions for $\tilde{C}^{\alpha \beta \gamma}_{i,j,k}$ can be found in Ref. [23].

- The $H_i - H^+ - H^-$ couplings are determined by the Higgs potential. If we define
\[
\mathcal{L}_{H_i H^+ H^-} = v C_i H_i H^+ H^-,
\]
then the couplings $C_i$ are given by [14]
\[
C_i = \sum_{\alpha = 1, 2, 3} O_{\alpha,i} c_\alpha
\]
with
\[
c_1 = 2 s_\beta c_\beta \text{Im}(\lambda_5 e^{2i\xi}) - s_\beta^2 \text{Im}(\lambda_6 e^{i\xi}) - c_\beta^2 \text{Im}(\lambda_7 e^{i\xi}),
\]
\[
c_2 = 2 s_\beta^2 c_\beta \lambda_1 + c_\beta^3 \lambda_3 - s_\beta^2 c_\beta \lambda_4 - 2 s_\beta c_\beta \text{Re}(\lambda_5 e^{2i\xi})
+ s_\beta (s_\beta^2 - c_\beta^2) \text{Re}(\lambda_6 e^{i\xi}) + s_\beta c_\beta^2 \text{Re}(\lambda_7 e^{i\xi}),
\]
\[
c_3 = 2 c_\beta^2 s_\beta \lambda_2 + s_\beta^3 \lambda_3 - c_\beta^2 s_\beta \lambda_4 - 2 c_\beta s_\beta \text{Re}(\lambda_5 e^{2i\xi})
+ c_\beta s_\beta^2 \text{Re}(\lambda_6 e^{i\xi}) + c_\beta (c_\beta^2 - 2 s_\beta^2) \text{Re}(\lambda_7 e^{i\xi}).
\] (B6)
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FIGURES

FIG. 1. The cross sections for $\gamma\gamma \to H_2$ with chargino loop contributions (solid curves) and without chargino loop contribution (dash-dotted curves) in unit of fb as functions of $M_2$ with $\tan \beta = 3$ and $\tan \beta = 10$. We choose $|A_t| = 0.4$ TeV, $M_{H^+} = 300$ GeV and $\arg(A_t) = 0^\circ$. The left (right) figure is for $\tan \beta = 3$ ($\tan \beta = 10$).

FIG. 2. The cross sections for $\gamma\gamma \to H_2$ with chargino loop contributions (solid curve) and without chargino loop contribution (dash-dotted curve) in unit of fb as functions of $\arg(A_t)$ for $M_2 = 150$ GeV. The left (right) figure is for $\tan \beta = 3$ ($\tan \beta = 10$).
FIG. 3. The unpolarized cross sections for $\gamma \gamma \rightarrow H_i \ (i = 1, 2, 3)$ without chargino loop contributions (left column) and with chargino loop contributions (right column) for $M_2 = 150$ GeV in units of fb as functions of each Higgs mass for five different values of the $A_t$ phase; arg($A_t$) = 0° (thick solid curve), 40° (dash-dotted curve), 80° (dashed curve), 120° (dotted curve) and 160° (solid curve).
FIG. 4. The unpolarized cross sections for $\gamma \gamma \rightarrow H_i$ ($i = 1, 2, 3$) without chargino loop contributions (left column) and with chargino loop contributions (right column) for $M_2 = 150$ GeV in units of fb as functions of each Higgs mass for five different values of the $A_t$ phase; $\arg(A_t) = 0^\circ$ (thick solid curve), $40^\circ$ (dash-dotted curve), $80^\circ$ (dashed curve), $120^\circ$ (dotted curve) and $160^\circ$ (solid curve).
FIG. 5. The polarization asymmetries $A_1$, $A_2$ and $A_3$ without (dash-dotted curve) and with (solid curve) chargino loop contributions as functions of $\arg(A_t)$. We take the parameter set (20) and $M_{H^+} = 300$ GeV. The left (right) figure is for $\tan\beta = 3$ (tan $\beta = 10$).
FIG. 6. The polarization asymmetries $A_1$, $A_2$ and $A_3$ with chargino loop contributions as functions of each Higgs mass for five different values of the $A_t$ phase with $\arg(\mu) = 0^\circ$; $\arg(A_t) = 0^\circ$ (thick solid curve), $40^\circ$ (dash-dotted curve), $80^\circ$ (dashed curve), $120^\circ$ (dotted curve) and $160^\circ$ (solid curve). We choose the parameter set [20] for $\tan \beta = 3$. 

[Graphs showing polarization asymmetries $A_1(H_1)$, $A_2(H_1)$, $A_3(H_1)$, $A_1(H_2)$, $A_2(H_2)$, $A_3(H_2)$, $A_1(H_3)$, $A_2(H_3)$, $A_3(H_3)$]
FIG. 7. The polarization asymmetries $A_1$, $A_2$ and $A_3$ with chargino loop contributions as functions of each Higgs mass for five different values of the $A_t$ phase; $\text{arg}(A_t) = 0^\circ$ (thick solid curve), $40^\circ$ (dash-dotted curve), $80^\circ$ (dashed curve), $120^\circ$ (dotted curve) and $160^\circ$ (solid curve). We take the parameter set (20) for $\tan \beta = 10$. 

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TABLE I. The amplitudes \(A_i^X\)'s and \(B_i^X\)'s, where \(i\) labels three neutral Higgs bosons, and \(X\) labels the species of charged particles in the triangle loop (with \(\tau_{iX} \equiv M_{H_i}^2/4m_X^2\)).

| A’s and B’s | Expressions |
|-------------|-------------|
| \(A_i^f\)   | \(-2(\sqrt2G_F)^{1/2}M_{H_i}N_e e_f^2 \left(\frac{v_i^2}{R_i^2}\right) F_{sf}(\tau_{if})\) |
| \(A_i^\tilde{f}\) | \(\frac{M_{H_i}N_e e_f^2 g_f^i}{2m_{\tilde{f}}^2} F_0(\tau_{i\tilde{f}})\) |
| \(A_i^{W\pm}\) | \((\sqrt2G_F)^{1/2}M_{H_i} (c_\beta O_{2,i} + s_\beta O_{3,i}) F_1(\tau_{iW})\) |
| \(A_i^{H\pm}\) | \(\frac{M_{H_i} c_{C_i}}{2m_{\tilde{H}_\pm}^2} F_0(\tau_{iH})\) |
| \(A_i^{\tilde{\chi}\pm}\) | \(2\text{Re}(\kappa_{jj}^i) \frac{M_{H_i}}{M_{\tilde{\chi}_j^\pm}} F_{sf}(\tau_{i\tilde{\chi}_j^\pm})\) |

| B’s          | Definitions |
|-------------|-------------|
| \(B_i^f\)   | \(2(\sqrt2G_F)^{1/2}M_{H_i}N_e e_f^2 \left(\frac{R_i^2}{R_f^2}\right) F_{pf}(\tau_{if})\) |
| \(B_i^{\tilde{\chi}\pm}\) | \(-2\text{Im}(\kappa_{jj}^i) \frac{M_{H_i}}{M_{\tilde{\chi}_j^\pm}} F_{pf}(\tau_{i\tilde{\chi}_j^\pm})\) |

TABLE II. Form factor loop functions \(F\)'s in terms of the scaling function \(f(\tau)\) defined in Eq. (12).

| F’s         | Definitions |
|-------------|-------------|
| \(F_{sf}(\tau)\) | \(\tau^{-1} \left[1 + (1 - \tau^{-1})f(\tau)\right]\) |
| \(F_{pf}(\tau)\) | \(\tau^{-1} f(\tau)\) |
| \(F_0(\tau)\) | \(\tau^{-1} \left[-1 + \tau^{-1} f(\tau)\right]\) |
| \(F_1(\tau)\) | \(2 + 3\tau^{-1} + 3\tau^{-1}(2 - \tau^{-1})f(\tau)\) |