Performance analysis of independent suspension coil spring

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Abstract. Compared to ordinary springs, the unique advantages of variable-parameter coil springs are explained. Their main characteristics and evaluation indexes are analyzed. And focus on the analysis of their load-deformation characteristics and stress characteristics in combination with the parameter characteristics of variable-parameter springs to provide a certain theory for vehicle suspension spring selection support.

1. Introduction
Today, military vehicles are developing towards high mobility and light weight, which puts stricter requirements on vehicle suspension systems. In the case of ensuring sufficient strength, the mass of the assembly should be reduced as much as possible to improve the mobility of the vehicle. The emergence of variable-parameter coil springs not only greatly reduces the weight and occupied space of the vehicle suspension, but also improves the ride comfort and safety of the vehicle [1].

2. The structure and characteristics of coil spring
2.1. Ordinary coil spring
The geometry of the ordinary coil spring is spiral, and the basic parameters of the spiral formed by the spring centerline are shown in Figure 1[2].

![Figure 1. Geometrical parameters of coil spring.](image-url)
D—the diameter of helical cylinder, which is the middle diameter of the spring;  

d—the diameter of coil spring material;  

H0—Effective height of coil spring;  

t—the rising height of the coil, which is the pitch of the spring;  

Coil springs with constant diameters of wire diameter d, spring diameter D, pitch t and helix angle α are the most simple cylindrical coil springs.

2.2. Variable parameter coil spring

The coil spring has four main structural parameters: wire diameter d, coil diameter D, pitch t, and helix angle α. It can be said that the ordinary fixed-parameter spring is a special case of a variable-parameter coil spring, that is, the four parameters are constant, as shown in Figure 1. If any of these four parameters, or more than one of them changes with the length of the wire l, it is called a variable-parameter coil spring.

In the design, the most common variable-parameter coil spring is a three-variable spring, that is, the wire diameter d, the coil diameter D and the pitch t or helix angle α change simultaneously. The various performance of the four-variable coil spring is excellent, but it is rarely used in practice.

Table 1 shows the performance requirements of modern cars and the performance that can be achieved by the elastic components used. This table briefly explains the reasons for using variable parameter coil springs.

| Vehicle | Aims | Requirements | Spring | Features |
|---------|------|--------------|--------|----------|
| Ride comfort | Body vibration frequency is independent of load | The spring deformation fe is constant |
| Ride safety | Sufficient ground adhesion | Lower spring stiffness |
| Body stance | Low altitude change at no load and full load | Larger spring rate stiffness |
| Headlight position | The deformation of the front and rear axle suspensions is equal under no load and full load | $R_f/R_r=\Delta P_f/\Delta P_r$, The ratio of front and rear suspension stiffness to front and rear load is equal |
| Energy consumption | With large internal space and small weight | Small weight, small size |

Not only the calculation method of the variable parameter coil spring needs to be established and perfected, but its main performance also needs to be explored and understood. In this paper, we studies the influence of parameter changes on its main performance, clarifies its advantages and disadvantages, and puts forward performance evaluation indicators, in order to better guide the design of variable parameter coil springs.

3. Advantages of variable parameter spring

In the design of modern off-road vehicles, the engine is generally placed above the front axle, and the power is transmitted to the front and rear axles through the transfer case. This arrangement makes most of the load of the occupants and supplies carried by the rear suspension spring, and the load changes larger. Thus, the design of the rear spring is of great significance. It can be seen from Table 1 that springs with equal stiffness can only meet limited performance requirements.

Figure 2 shows the load-deformation characteristic curve of the variable parameter (variable stiffness) spring and the ordinary (equal stiffness) spring. The following can be seen by comparison:

(1) $\Delta P$ Refers to the difference between the elastic force of the variable stiffness spring and the constant stiffness spring when the wheel rebounds from the equilibrium position Lno load to Lrebound. Obviously, the spring with variable stiffness has better grip when the wheel rebounds;

(2) $\Delta S$ refers to the difference in the amount of deformation between the variable stiffness spring and the equal stiffness spring when the spring force increases from the equilibrium position Pno load to...
the full load P_{full load} under full load. It can be seen that the spring with variable stiffness characteristics when subjected to a load, the amount of deformation is small, so the required space is reduced accordingly;

(3) ΔW represents the difference between the energy absorbed by the variable-stiffness spring and the equal-stiffness spring when the spring force is from full load L_{full load} to jump limit load L_{limit}. In general, the larger the absorbed energy, the larger the volume of the rubber damper block can be reduced accordingly;

(4) The use of variable stiffness springs can make the vibration frequency of the car body change less, thereby improving the smoothness of the car;

(5) Under the condition of not changing the equilibrium position of the spring deformation L_{no load}, the spring height can be reduced by replacing the constant stiffness spring with a variable stiffness spring;

(6) The diameter of the lateral stabilizer bar can be reduced accordingly, thereby reducing the weight.

In addition to the above-mentioned advantages, variable-stiffness springs can obtain the required characteristics in the entire deformation region of the spring or in the entire range of load-deformation.

![Figure 2](image_url)

**Figure 2.** Ordinary (equal stiffness) spring and variable parameter (variable stiffness) spring load-deformation curve.

4. The main performance of variable parameter coil spring

The core of the variable parameter coil spring is change. Excellent load-deformation characteristics and satisfactory stress distribution characteristics can be obtained through different changes of structural parameters.

4.1. Load-deformation characteristics

As shown in Figure 3, if the center O of the coil spring is the origin of coordinates, θ is the polar angle, and s is the projected length of the wire extension line, when it is assumed that the bending angle α is not greater than 9° [3], and the bending and the small deformation caused by pure shear stress is ignored, then the stiffness c of the ordinary coil spring with a circular cross section can be expressed as:

$$c = \frac{Gd^4}{64R^3n}$$  \hspace{1cm} (1)

In the formula, G—Torsional modulus of the material;
But for the coil spring with variable parameters, the coil radius $R$ and the wire diameter $d$ change with the polar angle $\theta$ or the projection length $s$, and the stiffness $c$ can be expressed by formula (2) [16]

$$\frac{1}{c} = \frac{32}{\pi G} \int_0^\theta R^3(\theta) d\theta = \frac{32}{\pi G} \int_0^s R^2(s) ds$$

(2)

Figure 3. Schematic diagram of spring coordinates.

After the material of the spring is determined, the spring stiffness can be determined by the geometric parameters of the spring (selecting the appropriate spring diameter $R(s)$ and wire diameter $d(s)$ and the spring projected length $s$). Changing the spring diameter $R(s)$ and wire diameter $d(s)$ and the projected spring length $s$ can obtain spring characteristics with variable stiffness.

Once the coil diameter function $R(s)$ and the wire diameter function $d(s)$ of a coil spring are determined, the stiffness $c$ of the coil spring is basically determined. When the research comes to this step, the only possibility to change the stiffness is to change the effective length or effective volume of the spring wire. The measure to change the length or volume of the wire is to change the effective number of turns.

The stiffness of each coil will be different, and the compression and load $P_{km}$ of the coil with different stiffness are also different, due to the change of the coil diameter function and the wire diameter function. Because the pitch $t$ of each coil is different, so the load of the coils with different pitches is also different. Different helix angle $\alpha$ will result in coils with different pitch $t$. Thus, the pressure and load of each coil are also different.

When the spring is working, as the load $P$ gradually increases, the deformation $f$ also gradually increases. When the load increases to a certain degree ($P = P_{km}$), the coil with the lowest stiffness or the
smallest pitch will be compressed first. In the process of merging the first coil, since the wires between the coils generally do not continuously contact point by point, the spring only maintains a linear relationship with a certain slope. As the load continues to increase, the remaining coils will be pressed one by one in order from weak to strong until the full springs are pressed together. At this step, the compressive load \( P_{km} \) is the compressive load of the last ring (the last ring at this step is not necessarily the last ring of the effective number of springs, it refers to the last compressed spring ring), and also is the compression load of the full spring. The compression and deformation at this step is the maximum deformation \( \text{fem} \) of the full spring.

In this kind of spring connected by coils with different stiffness, the stiffness will be increased by one step each time when the coil is pressed and the slope of the stiffness curve will be increased by one step. In the actual spring test, the test curve is a relatively smooth curve. However, the theoretical variable-parameter coil spring load-deformation characteristic curve is a stepped polyline (see in Figure 4) because of theoretical assumptions. The breakpoints can be more clearly reflected in the following theoretical calculations and graphical output (see in Chapter 3).

![Figure 4. Characteristics of load-deformation on variable parameter coil spring.](image)

### 4.2. Characteristics of stress

In order to facilitate the analysis, the most typical expression of the maximum shear stress of the k-th cycle of the variable parameter coil spring is listed first [5]:

\[
\tau_{km} = \frac{0.16k \cdot R_k}{d_k} \left( 1 + \frac{d_k}{4r_k} \right) P_{km}
\]

In the formula:
- \( k \)—Correction factor for k-th cycle;
- \( P_{km} \)—The compressive load K-th cycle, N;
- \( d_k \)—The wire diameter at the end of the k-th circle, mm;
- \( R_k \)—The radius of the coil at the end of the k-th circle, mm;
- \( d_0 \)—Initial diameter of steel wire, mm;
- \( R_0 \)—Initial radius of the coil, mm;
- \( R_k \)—The radius of the coil at the end of the k-th circle, mm;

\[
d_k = d_0 + \frac{2 \cdot \tan(\beta)}{2 \sin \alpha \cdot \tan(\phi)} (R_k - R_0), \text{mm}
\]
It can be seen from equation (3) that the maximum shear stress \( \tau_{km} \) of the k-th single turn of the coil spring with variable parameters is not only related to the compressive load \( P_{km} \) of the ring, but also proportional to the first power of the radius \( R_k \) of the coil at the end of the ring. It is inversely proportional to the cube of the wire diameter \( d_k \) at the end of the circle. Taking a deeper look, \( R_k \) and \( d_k \) are not only related to \( d_0 \), \( R_0 \), \( \alpha \), \( \beta \), \( \varphi \) and other parameters, they are also a function of serial number \( k \) and wire length \( L \). During design, changing the structural parameters can change the pressure and stress of each ring. When the value of \( \tau_{km} \) changing with \( P_{km} \) is summarized, an upward convex curve with a gradual decrease in slope as shown in Figure 5 can be obtained (in the figure: \( P_{fm} \) - compressive load of first ring; \( P_{km} \) - compressive load of k-th ring; \( P_{em} \) — compressive load of last ring; \( \tau_{fm} \) — shear stress of first ring; \( \tau_{km} \) — shear stress of k-th ring; \( \tau_{em} \) — shear stress of last ring).

Figure 5. Stress load characteristics of typical variable parameter coil spring.

The ordinary coil spring is composed of several single turns with the same structural parameters, so the compressive load \( P_{km} \) and compressive stress \( \tau_{km} \) of each ring are equal, and the stress distribution of its also completely uniform.

The variable parameter coil spring is composed of several single turns with different structural parameters, so \( P_{km} \) and \( \tau_{km} \) in Figure 5 are not equal, and the stress distribution is also uneven.

The stress seems to be as low as possible, and the distribution seems to be as uniform as possible. But for a variable-parameter coil spring, it is not possible and is unnecessary. Firstly, it is not that the lower the stress, the better; but the higher the allowable stress, the better; because only in this way can the potential of the material be fully realized. It should be noted that zero stress means that the material is not allowed to participate in the work. Giving full play to the deformation and load bearing capacity of the material is the advantage of the variable parameter coil spring. Secondly, the uniformity of the stress distribution is a reflection of the consistency of the structural parameters. Pursuing uniformity unilaterally is tantamount to giving up the variability and rationality of the variable-parameter coil spring.

The designer’s task is to use the variability of structural parameters to achieve a reasonable distribution of stress under the premise of satisfying excellent load deformation characteristics.

In Fig. 5, the ratio of stress \( \tau_{km} \) to load \( P_{km} \) in each ring represents the rate of change of stress with load. Here it is defined as the specific stress \( R_k \) of each circle, that is

\[
R_k = R_0 e^{\frac{2\pi \tan \alpha \tan \beta k}{2}}, \quad mm
\]
The mechanical concept of $R_{ki}$ is the stress per unit load. Therefore, the lower the $R_{ki}$ value, the better.

The specific stress of the ordinary coil spring is a constant, $R_{ki} = C$. But the $R_{ki}$ value of the variable parameter coil spring changes with the coil number $k$. When the wire length $L$ changes, the stress changes with the load. Whether it is gradually increased or gradually decreased, or firstly increased and then decreased, determined by the structural parameters of each circle.

The change of the $R_{ki}$ value of the variable parameter coil spring roughly follows the following rule: the root mean square value of the product of the specific stress $R_{ki}$ of each turn tends to be a constant, that is

$$
\bar{R}_{ki} = \left( \prod_{k=1}^{n} R_{ki} \right)^{1/n} \rightarrow C, MPa / kN
$$

This constant $C$ is the $R_{ki}$ value of an ordinary coil spring with the approximately same parameters such as the effective total mass $M$ of the full spring, the number of turns $n$ and the total deformation $f_g$ of the full spring.

In the premise of formula (5), the smaller the specific stress $R_{ki}$ value of the last ring is, the better. Because the compression load at this time is the maximum compression load of the full spring $P_{km}$. If the value of $R_{ki}$ is small, and the value of $\tau_{km}$ is relatively reduced. This is the performance of the rational distribution of the variable parameter coil spring stress, and also is the embodiment of reasonable materials.

Figure 5 shows the stress distribution law of a typical variable parameter coil spring, for which the stress increases with increasing load, and the specific stress decreases with increasing load. However, there are many variable-parameter coil springs with the more special stress distribution: in the low-load area, the stress increases with increasing load, and in the high-load area, the stress decreases with increasing load; The specific stress in the high load area becomes smaller. See in Figure 6 (in the figure: $P_{fm}$—pressure and load of first ring; $P_{km}$—pressure and load of $k$th ring; $P_{em}$—compressive load of last ring; $\tau_{fm}$—shear stress of first ring; $\tau_{km}$—shear stress of $k$th ring; $\tau_{em}$—shear stress of last ring).

![Figure 6. Stress load characteristics of special variable parameter coil spring.](image)

5. Evaluation index of variable parameter coil spring
When the parameters such as the effective total mass $M$, the effective number of turns $n$, the effective spring height $H_0$ and the total compression amount $f_g$ are approximately the same, the following indicators can be used as the basis for the technical design and performance evaluation of the variable parameter coil spring.

5.1. Specific load $P_M$

It is a sign of whether the potential of PM materials with specific load can be fully utilized. It is the ratio of the last ring pressure and load $P_{em}$ (the pressure and load of full spring) to the effective total mass $M$, that is

$$P_M = \frac{P_{em}}{M}, \text{ N/kg}$$

$P_M$ is the maximum load that the unit mass can bear, for which the larger the requirement, the better. It can be seen from equation (6) that, in the case when $M$ is fixed, to increase $P_M$, it is necessary to increase the pressure and load $P_{em}$. The $P_M$ value of the ordinary coil spring is low, and the variable parameter coil spring can reach the target of 5,000N/kg under the conditions of stress.

5.2. Load amplitude $P_r$

The load amplitude is a sign of the adaptability of the coil spring to load changes. It is the difference between the last ring load $P_{em}$ and the first ring load $P_{fm}$ and the ratio to $P_{fm}$, that is

$$P_r = \left(\frac{P_{em} - P_{fm}}{P_{fm}}\right) \times 100\%$$

The larger the value of $P_r$, the better. The larger the value of $P_r$, the greater the adaptability of the coil spring to load changes. The $P_r$ value of the ordinary coil spring is zero, while the $P_r$ value of the variable parameter coil spring may be greater than 1000%.

5.3. Stiffness amplitude $R_r$

The stiffness amplitude $R_r$ is a sign of load deformation characteristics. It is the ratio of the difference between the stiffness $R_e$ of the last ring pressure merge point and the stiffness $R_f$ of the first ring pressure merge point to $R_f$, that is

$$R_r = \left(\frac{R_e - R_f}{R_f}\right) \times 100\%$$

The larger the $R_r$ value, the better. The larger the $R_r$ value, the higher the average slope of the load-deformation characteristic curve from the first ring pressure to the last ring pressure, and the smaller the frequency fluctuation range. The $R_r$ value of the ordinary coil spring is zero, and the $R_r$ value of the variable parameter coil spring can reach more than 300%.

5.4. Stress amplitude $\tau_r$

The stress amplitude $\tau_r$ can explain the uniformity of the distribution of the compressive and shear stress $\tau_{km}$ in each circle. It is the ratio of the difference between the maximum compressive shear stress $\tau_{ma}$ minus the minimum compressive shear stress $\tau_{mi}$ to $\tau_{mi}$, namely

$$\tau_r = \left(\frac{\tau_{ma} - \tau_{mi}}{\tau_{mi}}\right) \times 100\%$$

The $\tau_r$ of the variable-parameter coil spring can vary from 0 to 200%. The smaller $\tau_r$ indicates the more uniform stress distribution, but at the same time it may make the load amplitude $P_r$ and the stiffness amplitude $R_r$ smaller, so a comprehensive trade-off is needed.
5.5. Specific stress at the end \( R_{ec} \)
The specific stress \( R_{ec} \) is the ratio of the compressive shear stress \( \tau_{km} \) in each ring to the \( P_{km} \) load in each compressive ring. The final ring specific stress is the ratio of the final ring compression and shear stress \( \tau_{em} \) to the final ring compression and load \( P_{em} \), that is

\[
R_{er} = \frac{\tau_{em}}{P_{em}} \text{ MPa} / kN
\]  

(10)

\( R_{ec} \) is a sign of the rationality of the stress distribution and a reflection of the level of timber used. The design requires that the maximum stress should be increased as much as possible without exceeding the allowable stress. The design hopes that it can withstand high loads and obtain low stress.

5.6. Quality factor \( M_c \)
The quality factor \( M_c \) is an index that explains the material utilization rate. It is the ratio of the mass \( M_d \) of the ineffective part of the coil spring to the mass \( M \) of the effective part, that is

\[
M_c = \frac{M_d}{M} \times 100\% 
\]  

(11)

The smaller the value of \( M_c \), the better. \( M_c \) is small, indicating that the spring seat has less dead material. In the case of the same number of seats, the two-variable spring whose wire diameter \( d \) and spring diameter \( D \) both change can minimize the value of \( M_c \).

6. Conclusions
In this article, we theoretically expounded the concept of variable-parameter coil compression springs and classified them simply. Compared with ordinary coil springs, in this paper, we point out the advantages of variable-parameter coil springs used in vehicle suspension systems; combining its variable parameters characteristics of it, we mainly analyzed its load-deformation characteristics and stress characteristics, and the evaluation index for judging the comprehensive performance of the variable parameter coil spring is given.

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