Heavy Quark Distribution Functions in Heavy Baryons

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Abstract

Using the Bethe-Salpeter (B-S) equations for heavy baryons $\Lambda_Q$, $\Sigma_Q$, $\Xi_Q$ and $\Omega_Q$ ($Q = b$ or $c$), which were established in previous work, we calculate the heavy quark distribution functions in these baryons. The numerical results indicate that these distribution functions have an obvious peak at some fraction, $\alpha_0$, of the baryon’s light-cone “plus” momentum component carried by the heavy quark, and that as $m_Q$ becomes heavier this peak becomes sharper and closer to 1. The dependence of the distribution functions on various input parameters in the B-S model is also discussed. The results are seen to be qualitatively similar to an existing phenomenological model.

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I. Introduction

A significant amount of experimental data have been accumulated on lepton nucleon deep inelastic scattering processes. From these data one can extract information on the parton distribution functions in the nucleon, which describe its nonperturbative hadronic structure. In comparison with the nucleon, much less is known about the parton distribution functions in other baryons such as Λ, Λc, and Λb. This is because it is impossible to produce targets of these short lived baryons suitable for experiments. Although the parton distribution functions in these baryons cannot be studied through deep inelastic scattering processes, it is still possible to obtain information on their parton distribution functions by measuring the fragmentation of quarks to baryons or the decays of these baryons. In fact, the parton distribution functions in Λ, Σ and Δ have been calculated in the MIT bag model dressed by mesons [1]. In this paper, we will study the heavy quark distribution functions in heavy baryons.

The dynamics inside a heavy hadron is simplified by the fact that the light degrees of freedom in a heavy hadron are blind to the flavor and spin quantum numbers of the heavy quark when its mass is much bigger than the QCD scale [2]. This makes heavy flavor physics a good area for studying nonperturbative QCD interactions. In fact, with more measurements on heavy baryons becoming available [3, 4, 5, 6], theoretical study of the structure of heavy baryons is becoming increasingly important.

The parton distribution functions are scale-dependent, and their evolution in the perturbative region is described by the famous Dokshitzer- Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations [7]. However, to determine the parton distribution functions at some low energy scale (the boundary condition for DGLAP), one needs to apply either lattice QCD [8] or nonperturbative effective models. In the heavy quark limit, the light degrees of freedom in a heavy baryon have good quantum
numbers which can be used to classify heavy baryons. Based on this fact, in a previous work we took the heavy baryon to be composed of a heavy quark and a light diquark. With this picture, we established the B-S equations for the heavy baryons $\Lambda_Q$ and $\omega_Q$ (where $\omega$ represents $\Sigma$, $\Xi$, or $\Omega$), and solved these equations numerically by assuming that their kernel contains a scalar confinement term and a one-gluon-exchange term $[9]$. It is the purpose of the present paper to calculate the heavy quark distribution functions in these heavy baryons in our B-S formalism.

In $\Lambda_Q$ and $\omega_Q$ we have $0^+$ and $1^+$ diquarks, respectively. Since in the heavy quark limit the internal dynamics of a heavy baryon are described by the light degrees of freedom, we expect that the heavy quark distribution functions in $\omega_Q$ and $\omega_Q^*$ should be the same. Furthermore, the differences among the heavy quark distribution functions in $\Sigma_Q$, $\Xi_Q$, and $\Omega_Q$ should be caused by $SU(3)$ flavor breaking effects.

The remainder of this paper is organized as follows. In Section II we derive the formulas for the heavy quark distribution functions in various heavy baryons in the B-S formalism. In Section III we present numerical results for these distribution functions and discuss their dependence on the parameters of the model. We also compare our results with the distribution functions of Guo and Kroll $[10]$. Finally, we give a summary and some suggestions for future work in Section IV.

II. Formalism for the heavy quark distribution functions in $\Lambda_Q$, $\Sigma_Q$, $\Xi_Q$, and $\Omega_Q$

Based on the picture that a heavy baryon is composed of a heavy quark and a light diquark, it was shown that the B-S equation for $\Lambda_Q$ is $[9]$ 

$$ \chi_P(p) = S_F(\lambda_1 P + p) \int \frac{d^4 q}{(2\pi)^4} G(P, p, q) \chi_P(q) S_D(-\lambda_2 P + p), $$

(1)

where $\chi_P(p)$ is the B-S wave function in momentum space, $G(P, p, q)$ is the kernel, $S_F$ and $S_D$ are the propagators of the heavy quark and light scalar diquark, respectively,
with $\lambda_1 = \frac{m_Q}{m_Q + m_D}$ and $\lambda_2 = \frac{m_D}{m_Q + m_D}$ ($m_Q$ and $m_D$ are the masses of the heavy quark and the light diquark, respectively), $P$ is the momentum of $\Lambda_Q$, and $p$ is the relative momentum of the two constituents.

Similarly, the B-S equation for $\omega_Q$ takes the form

$$\chi^\mu_P(p) = S_F(\lambda_1 P + p) \int \frac{d^4q}{(2\pi)^4} G_{\mu\nu}(P, p, q) \chi^\nu_P(q) S^\mu_P(-\lambda_2 P + p),$$

where $G_{\mu\nu}(P, p, q)$ is the kernel and $S^\mu_P$ is the propagator of the $1^+$ diquark.

In the heavy quark limit

$$\chi_0^\mu_P(p) = \phi_0^\mu_P(p) u_{\Lambda_Q}(v, s),$$

where $\phi_0^\mu_P(p)$ is a scalar function, and $u_{\Lambda_Q}(v, s)$ is the Dirac spinor for $\Lambda_Q$ with helicity $s$ and velocity $v$.

For $\omega_Q$ there are three scalar functions, $A, C$ and $D$, in the B-S wave function

$$\chi^\mu_P = AB^\nu(v) + Cv^\mu p_{\nu} B^\nu(v) + D p^\mu_{\nu} p_{\nu} B^\nu(v),$$

where $B^\mu(v) = \frac{1}{\sqrt{3}} (\gamma^\mu + v^\mu) \gamma_5 u(v)$, and $p_t \equiv p - (v \cdot p)v$. $B^\mu(v)$ satisfies the constraints $\not v B^\mu(v) = B^\mu(v)$ and $v^\mu B^\mu(v) = 0$.

The B-S equations have been solved numerically in the covariant instantaneous approximation, assuming the kernels contain a scalar confinement term, $\tilde{V}_1$, and a one-gluon-exchange term, $\tilde{V}_2$, with the following form

$$\tilde{V}_1 = \frac{8\pi \kappa}{[(p_t - q_t)^2 + \mu^2]^2} - (2\pi)^3 \delta^3(p_t - q_t) \int \frac{d^3k}{(2\pi)^3} \frac{8\pi \kappa}{(k^2 + \mu^2)^2},$$

$$\tilde{V}_2 = -\frac{16\pi}{3} \alpha_s^\text{(eff)} Q_0^2 \frac{1}{[(p_t - q_t)^2 + \mu^2][((p_t - q_t)^2 + Q_0^2)^2]},$$

where $\kappa$ and $\alpha_s^\text{(eff)}$ are coupling parameters related to scalar confinement and one-gluon-exchange, respectively, where $Q_0^2$ is a parameter associated with the gluon-diquark vertex, and where the parameter $\mu$ is introduced to avoid the infra-red divergence in numerical calculations, with the limit $\mu \to 0$ being taken at the end of the calculation.
The twist-2 heavy quark distribution function in $A^+ = 0$ gauge is defined as
\[ Q(\alpha) = \sqrt{2} P^+ \int \frac{dx^-}{2\pi} e^{-i\alpha P^+ x^-} \langle B | T \bar{\psi}_Q(x^-) \gamma^+ \psi_Q(0) | B \rangle, \] (6)
where $\psi_Q$ is the field operator of the heavy quark $Q$, $P^+ = \frac{1}{\sqrt{2}} (P^0 + P^3)$, $\gamma^+ = \frac{1}{\sqrt{2}} (\gamma^0 + \gamma^3)$, $\psi_Q(x^-)$ denotes $\psi_Q(x)$ at $x^+ = x_\perp = 0$, and $|B\rangle$ represents the heavy baryon state with the normalization $\langle B, P, \lambda | B, P', \lambda' \rangle = (2\pi)^3 P_0/m_B \delta_{\lambda,\lambda'} \delta^3(P - P')$ (we have chosen the normalization convention $\bar{u}_B u_B = 1$).

We define the two-point function
\[ M_{\beta\alpha}(P, k) = \int dx^4 e^{-ikx} \langle B | T \bar{\psi}_{Q\alpha}(x) \psi_{Q\beta}(0) | B \rangle \] (7)
and then the heavy quark distribution function can be expressed as
\[ Q(\alpha) = \int \frac{d^4k}{(2\pi)^4} \sqrt{2} P^+ \delta(k^+ - \alpha P^+) \text{Tr}[\gamma^+ M(P, k)]. \] (8)

The parameter $\alpha$ in Eqs. (6,8) corresponds to the fraction of the heavy baryon’s light-cone momentum component, $P^+$, carried by the heavy quark, $Q$. When it is in the range $0 \leq \alpha \leq 1$, $Q(\alpha)$ measures the probability to find the heavy quark with the “plus” momentum fraction $\alpha$. In principle, in a heavy baryon there is a possibility to find a heavy antiquark, which is generated from the QCD vacuum. However, since we are considering heavy quarks with masses much larger than the QCD scale $\Lambda_{\text{QCD}}$, it is very difficult to produce them from the QCD vacuum. Therefore, we neglect the heavy antiquark distribution functions. In other words, the valence heavy quark distribution function is the same as the heavy quark distribution function, $Q(\alpha)$. Since there is only one heavy quark in a heavy baryon we have the following normalization condition for $Q(\alpha)$:
\[ \int d\alpha Q(\alpha) = 1. \] (9)

The two-point function $M(P, k)$ can be evaluated in our B-S framework. After some algebra we obtain for $\Lambda_Q$ in the heavy quark limit the result
\[ M^\Lambda_{\beta\alpha}(P, k) = (\bar{u}_\Lambda Q)_\alpha [\phi_0 P(k - \lambda_1 P)]^2 S_D^{-1}(P - k)(u_\Lambda Q)_\beta. \] (10)
The propagator of the light scalar diquark has the form

\[ S_D = \frac{i}{p_f^2 - W_p^2 + i\epsilon}, \]  

(11)

where \( p_f \equiv v \cdot p - \lambda_2 m_\Lambda Q \) and \( W_p \equiv \sqrt{p_f^2 + m_D^2} \).

The propagator of the heavy quark in the heavy quark limit has the form

\[ S_F = \frac{i(1 + \not\!v)}{2(p_t + E_0 + m_D + i\epsilon)}, \]  

(12)

where \( E_0 \) is the binding energy in the heavy quark limit.

We choose to work in the rest frame of \( \Lambda Q \), in which we have \( k^0 + k^3 = \alpha m_\Lambda Q \) (\( m_\Lambda Q \) is the mass of \( \Lambda Q \)) from the \( \delta(k^+ - \alpha P^+) \) constraint in Eq. \( (8) \). Hence we find

\[ S_D^{-1} = -i[-|k_\perp|^2 + 2(1 - \alpha)m_\Lambda Q k^3 + (1 - \alpha)^2 m_\Lambda Q^2 - m_D^2]. \]  

(13)

The B-S wave function, \( \phi_{0P}(p) \), can be expressed in terms of \( \tilde{\phi}_{0P}(q_t) \equiv \int \frac{d^3q}{2\pi} \phi_{0P}(p) \) as \( \tilde{\phi}_{0P}(q_t) \).

\[ \phi_{0P}(p) = \frac{i}{(p_t + E_0 + m_D + i\epsilon)(p_t^2 - W_p^2 + i\epsilon)} \int \frac{d^3q_t}{(2\pi)^3} (\tilde{V}_1 + 2p_t \tilde{V}_2) \tilde{\phi}_{0P}(q_t). \]  

(14)

The constraint \( \delta(k^+ - \alpha P^+) \) leads to \( p_t = -k^3 + (\alpha - 1)m_\Lambda Q \), and \( |p_t|^2 = |k|^2 \).

Substituting Eqs. \( (10,14) \) into Eq. \( (8) \), we integrate out \( k^0 \) with the aid of \( \delta(k^+ - \alpha P^+) \). Furthermore, the component \( k^3 \) can also be integrated out by choosing the appropriate contour. Then we arrive at the following result:

\[ Q^{\Lambda Q}(\alpha) = \frac{1}{2\sqrt{2}\pi(1 - \alpha)} \int \frac{d^3k_\perp}{(2\pi)^2} \left[ \frac{1}{E_0 + m_D + \frac{1}{2}(\alpha - 1)m_\Lambda Q + \frac{1}{2(\alpha - 1)m_\Lambda Q}(|k_\perp|^2 + m_D^2)} \right]^2 \]

\[ \left\{ \int \frac{d^3q_t}{(2\pi)^3} \left[ \tilde{V}_1(p_t - q_t) + \left( (\alpha - 1)m_\Lambda Q + \frac{1}{(\alpha - 1)m_\Lambda Q}(|k_\perp|^2 + m_D^2) \right) \right] \tilde{V}_2(p_t - q_t) \right\}^2, \]  

(15)

where \( |p_t|^2 = |k_\perp|^2 + (k_{\text{pole}}^3)^2 \) and where \( k_{\text{pole}}^3 = \frac{1}{2(\alpha - 1)m_\Lambda Q}[(\alpha - 1)^2 m_\Lambda Q^2 - |k_\perp|^2 - m_D^2] \).

Substituting \( \tilde{V}_1 \) and \( \tilde{V}_2 \) in Eq. \( (8) \) into Eq. \( (13) \) and integrating out the angular coordinates we finally obtain

\[ Q^{\Lambda Q}(\alpha) = \frac{1}{2\sqrt{2}\pi(1 - \alpha)} \int \frac{|k_\perp|d|k_\perp|}{2\pi} \left[ E_0 + m_D + \frac{1}{2}(\alpha - 1)m_\Lambda Q + \frac{1}{2(\alpha - 1)m_\Lambda Q}(|k_\perp|^2 + m_D^2) \right]^2 \]
In deriving Eq. (16) we have used the following equations to reduce the three

where

\[
\left( \int \frac{q^2 dq}{4\pi^2} \right) \left\{ \frac{16\pi\kappa}{(p_t^2 + q_t^2 + \mu^2)^2 - 4p_t^2q_t^2} [\tilde{\phi}_{0P}(q_t) - \tilde{\phi}_{0P}(p_t)] \right. \\
+ \frac{16\pi\alpha^{(\text{eff})}Q_0^2}{3(Q_0^2 - \mu^2)} \left[ (\alpha - 1)m_{\Delta Q} + \frac{1}{(\alpha - 1)m_{\Delta Q}} (|k|_2^2 + m_D^2) \right] \\
\times \left. \frac{1}{2|q_t||q_t|} \left[ \ln \left( \frac{|p_t| + |q_t|)^2 + \mu^2}{(|p_t| - |q_t|)^2 + \mu^2} \right] - \ln \left( \frac{|p_t| + |q_t|)^2 + Q_0^2}{(|p_t| - |q_t|)^2 + Q_0^2} \right] \tilde{\phi}_{0P}(q_t) \right\}^2 ,
\]

(16)

where

\[ |p_t| = \frac{1}{2(1 - \alpha)m_{\Delta Q}} \sqrt{[(\alpha - 1)^2m_{\Delta Q}^2 - |k|_2^2 - m_D^2]^2 + 4(\alpha - 1)^2|k|_2^2m_{\Delta Q}^2} \] .

(17)

In deriving Eq. (16) we have used the following equations to reduce the three
dimensional integrations to one dimensional integrations

\[
\int \frac{d^3q_t}{(2\pi)^3} \frac{\rho(q_t^2)}{(p_t - q_t)^2 + \mu^2)^2} = \int \frac{q_t^2 dq_t}{4\pi^2} \frac{2\rho(q_t^2)}{(p_t^2 + q_t^2 + \mu^2)^2 - 4p_t^2q_t^2},
\]

(18)

and

\[
\int \frac{d^3q_t}{(2\pi)^3} \frac{\rho(q_t^2)}{(p_t - q_t)^2 + \delta^2} = \int \frac{q_t^2 dq_t}{4\pi^2} \frac{\rho(q_t^2)}{2|q_t||q_t|} \ln \left( \frac{|p_t| + |q_t|)^2 + \delta^2}{(|p_t| - |q_t|)^2 + \delta^2} \right)
\]

(19)

where \( \rho(q_t^2) \) is some arbitrary scalar function of \( q_t^2 \).

Now we turn to \( \omega_Q \). The two-point function \( M(P, k) \) in this case can be derived in a similar way and the result is

\[
M_{\alpha\beta}^{\omega_Q}(P, k) = \bar{\chi}_{\alpha}(k - \lambda_1P)S_{D\mu\nu}^{-1}(P - k)\chi_{\beta}^\nu(k - \lambda_1P).
\]

(20)

Substituting the B-S equation (2) into Eq. (20), using Eqs. (4) and (12), and working in the covariant instantaneous approximation, \( p_t = q_t \) (which ensures that the B-S equation is still covariant after this approximation), we have

\[
\text{Tr}[\gamma^+M^{\omega_Q}(P, k)] = \frac{1}{p_t + E_0 + m_D + i\epsilon} \left[ AB \gamma^+ + Cu_{\mu}p_t \cdot \vec{B} + D p^\mu p_t \cdot \vec{B} \right] \gamma^+
\]

\[
\int \frac{d^3q_t}{(2\pi)^3} \left\{ \begin{array}{l}
B_{\mu} \left[ \tilde{A}(\tilde{V}_1 + 2p_t\tilde{V}_2) - \tilde{C}(p_t \cdot q_t)^2 - p_t^2q_t^2 \tilde{V}_2 \\
+ D(p_t \cdot q_t)^2 - p_t^2q_t^2 \tilde{V}_2 \right]
+ v_{\mu}p_t \cdot B \left[ -\tilde{A}\tilde{V}_2 - \frac{p_t \cdot q_t}{p_t^2} \tilde{C}\tilde{V}_1 + D(p_t \cdot q_t)^2 \tilde{V}_2 \right] \\
+ p_{\mu}p_t \cdot B \left[ 3(p_t \cdot q_t)^2 - p_t^2q_t^2 \right] \left[ -\tilde{C}\tilde{V}_2 + D(\tilde{V}_1 + 2p_t\tilde{V}_2) \right] \end{array} \right\}.
\]

(21)
As for \( \Lambda_Q \), the B-S wave functions \( A(p_t, p_t^2) \), \( C(p_t, p_t^2) \) and \( D(p_t, p_t^2) \) are related to \( \tilde{A}(p_t^2) \), \( \tilde{C}(p_t^2) \) and \( \tilde{D}(p_t^2) \) through the following equations [4]:

\[
A(p_t, p_t^2) = \frac{-i}{(p_t + E_0 + m_D + i\epsilon)(p_t^2 - W_p^2 + i\epsilon)} \int \frac{d^3q_t}{(2\pi)^3} \left\{ \tilde{A}(q_t^2)(\tilde{V}_1 + 2p_t \tilde{V}_2) - \tilde{C}(q_t^2) \left( \frac{p_t \cdot q_t}{p_t^2} \right)^2 \tilde{V}_2 + \tilde{D}(q_t^2) \left( \frac{p_t \cdot q_t}{p_t^2} \right)^2 \left( \tilde{V}_1 + 2p_t \tilde{V}_2 \right) \right\},
\]

(22)

\[
C(p_t, p_t^2) = \frac{-i}{m_D^2(p_t + E_0 + m_D + i\epsilon)(p_t^2 - W_p^2 + i\epsilon)} \int \frac{d^3q_t}{(2\pi)^3} \left\{ -\tilde{A}(q_t^2)p_t \tilde{V}_1 + (p_t^2 + m_D^2)\tilde{V}_2 - \tilde{C}(q_t^2) \left[ \left( p_t^2 - m_D^2 \right) \frac{p_t \cdot q_t}{p_t^2} \tilde{V}_1 + p_t \left( \frac{p_t \cdot q_t}{p_t^2} \right)^2 \tilde{V}_2 \right] + \tilde{D}(q_t^2) \left( \frac{p_t \cdot q_t}{p_t^2} \right)^2 \left( p_t \tilde{V}_1 + (p_t^2 + m_D^2)\tilde{V}_2 \right) \right\},
\]

(23)

\[
D(p_t, p_t^2) = \frac{i}{m_D^2(p_t + E_0 + m_D + i\epsilon)(p_t^2 - W_p^2 + i\epsilon)} \int \frac{d^3q_t}{(2\pi)^3} \left\{ \tilde{A}(q_t^2)(\tilde{V}_1 + p_t \tilde{V}_2) + \tilde{C}(q_t^2) \left[ p_t \cdot q_t \frac{p_t \cdot q_t}{p_t^2} \tilde{V}_1 + \left( m_D^2 \right) \frac{3(p_t \cdot q_t)^2 - p_t^2 q_t^2}{2p_t^2} \tilde{V}_2 \right] + \tilde{D}(q_t^2) \left( \frac{p_t \cdot q_t}{p_t^2} \right)^2 \left( \tilde{V}_1 + p_t \tilde{V}_2 \right) \right\},
\]

(24)

and \( \tilde{A}(p_t^2) \), \( \tilde{C}(p_t^2) \) and \( \tilde{D}(p_t^2) \) obey the following three coupled integral equations

\[
\tilde{A}(p_t^2) = -\frac{1}{2W_p(E_0 + m_D - W_p)} \int \frac{d^3q_t}{(2\pi)^3} \left\{ \tilde{A}(q_t^2)(\tilde{V}_1 - 2W_p \tilde{V}_2) - \tilde{C}(q_t^2) \left( \frac{p_t \cdot q_t}{p_t^2} \right)^2 \tilde{V}_2 + \tilde{D}(q_t^2) \left( \frac{p_t \cdot q_t}{p_t^2} \right)^2 \left( \tilde{V}_1 - 2W_p \tilde{V}_2 \right) \right\},
\]

(25)

\[
\tilde{C}(p_t^2) = \frac{-1}{2m_D^2W_p(E_0 + m_D - W_p)} \int \frac{d^3q_t}{(2\pi)^3} \left\{ \tilde{A}(q_t^2)[W_p \tilde{V}_1 - ((E_0 + m_D)W_p + m_D^2)\tilde{V}_2] \right\} - \left( (E_0 + m_D)W_p + m_D^2 \right)\tilde{V}_2] \right\} + \tilde{D}(q_t^2) \left( \frac{p_t \cdot q_t}{p_t^2} \right)^2 \left[ -W_p \tilde{V}_1 + ((E_0 + m_D)W_p + m_D^2)\tilde{V}_2 \right] \right\},
\]

(26)
\[ \bar{D}(p_t^2) = \frac{1}{2m_D^2 W_p (E_0 + m_D - W_p)} \int \frac{d^3q_t}{(2\pi)^3} \left\{ \tilde{A}(q_t^2)(\tilde{V}_1 - W_p\tilde{V}_2) \\
+ \tilde{C}(q_t^2) \left[ -\frac{p_t \cdot q_t}{p_t^2} W_p \tilde{V}_1 + \frac{m_D^2 (3(p_t \cdot q_t)^2 - p_t^2 q_t^2)}{2p_t^4} \tilde{V}_2 \right] \right\}.
\]

(27)

Once again, we first integrate out \( k^0 \) with the help of the constraint \( \delta(k^+ - \alpha P^+) \), then we further integrate out \( k^3 \) by selecting the proper contour which contains the pole in \( k^3 \). With the aid of Eqs. (22–27), and noticing that \( \tilde{B}^{\mu} \gamma^+ B_\mu = -\frac{1}{\sqrt{2}} \) and \( p_t \cdot B \gamma^+ p_t \cdot B = \frac{1}{3\sqrt{2}} |\mathbf{k}|^2 \), we obtain:

\[ Q^{\omega Q}(\alpha) = \frac{1}{3\sqrt{2}\pi(1 - \alpha)} \int \frac{|\mathbf{k}_\perp| d|\mathbf{k}_\perp|}{2\pi} \frac{W_p}{E_0 + m_D - W_p} \left\{ -3 \tilde{A}(p_t^2)f_1(p_t^2) \right. \]
\[ + p_t^2 \tilde{C}(p_t^2)f_2(p_t^2) + p_t^2 \tilde{D}(p_t^2)f_3(p_t^2) \right\}, \]

(28)

where \( |p_t| \) is given in Eq. (17), and

\[ f_1(p_t^2) = \int \frac{d^3q_t}{(2\pi)^3} \left\{ \tilde{A}(q_t^2)(\tilde{V}_1 - 2W_p\tilde{V}_2) - \frac{1}{3} q_t^2[-\tilde{C}(q_t^2)\tilde{V}_2 + \tilde{D}(q_t^2)(\tilde{V}_1 - 2W_p\tilde{V}_2)] \right\}, \]

(29)

\[ f_2(p_t^2) = \int \frac{d^3q_t}{(2\pi)^3} \left\{ -\tilde{A}(q_t^2)\tilde{V}_2 + \tilde{C}(q_t^2)\frac{p_t \cdot q_t}{p_t^2} \tilde{V}_1 + \tilde{D}(q_t^2)\frac{(p_t \cdot q_t)^2}{p_t^4} \tilde{V}_2 \right\}, \]

(30)

\[ f_3(p_t^2) = \int \frac{d^3q_t}{(2\pi)^3} \left\{ \tilde{A}(q_t^2)(\tilde{V}_1 - 2W_p\tilde{V}_2) - \frac{(p_t \cdot q_t)^2}{p_t^4}[-\tilde{C}(q_t^2)\tilde{V}_2 + \tilde{D}(q_t^2)(\tilde{V}_1 - 2W_p\tilde{V}_2)] \right\}. \]

(31)

So far we have been working with the heavy quark limit, \( m_Q \to \infty \), of the B-S equation, but in Ref. [9] we also considered the \( 1/m_Q \) corrections to the B-S equation for \( \Lambda_Q \). To order \( 1/m_Q \), the heavy quark distribution function in \( \Lambda_Q \) is given by

\[ Q^{\Lambda_Q}(\alpha) + \Delta Q^{\Lambda_Q}(\alpha) \]

where \( \Delta Q^{\Lambda_Q}(\alpha) \) denotes the \( 1/m_Q \) corrections. To order \( 1/m_Q \), two more scalar functions appear in the B-S wave function, which can be related to \( \phi_0 p(p) \) in the
Consequently, the B-S wave function to order $1/m_Q$ is given by
\[ \chi_P(p) = \phi_0 P(p) \left[ 1 + \frac{\not{p}_t}{2m_Q} \right] u_{\Lambda_Q}(v, s). \] (32)

With Eq. (32) we can evaluate the two-point function $M_{\beta\alpha}^{\Lambda_Q}(P, k)$ to order $1/m_Q$ from Eq. (10). Note that the scalar diquark propagator in Eq. (11) remains unchanged to order $1/m_Q$. From Eq. (8), after integrating out $k^0$ and $k^3$, we have
\[
\Delta Q^{\Lambda_Q}(\alpha) = \frac{1}{2\sqrt{2\pi}(1 - \alpha)} \int \frac{|k_\perp| d|k_\perp|}{2\pi} \left[ E_0 + m_D + \frac{1}{2}(\alpha - 1)m_{\Lambda_Q} + \frac{1}{2(\alpha - 1)m_{\Lambda_Q}}(|k_\perp| + m_D^2)^2 \right.
\]
\[
\left. \int \frac{q_t^2 dq_t}{4\pi^2} \left\{ \frac{16\pi\kappa}{(p_t^2 + q_t^2 + \mu^2)^2 - 4p_t^2q_t^2} (\tilde{\phi}_0P(q_t) - \tilde{\phi}_0P(p_t)) \right. \right.
\]
\[
\left. + \frac{16\pi\alpha_s^{\text{eff}}Q_0^2}{3(Q_0^2 - \mu^2)} \left[ (\alpha - 1)m_{\Lambda_Q} + \frac{1}{(\alpha - 1)m_{\Lambda_Q}}(|k_\perp| + m_D^2) \right] \right.
\]
\[
\left. \times \frac{1}{2|p_t||q_t|} \left[ \ln \left( \frac{|p_t|}{|q_t|} \right)^2 + \mu^2 - \ln \left( \frac{|p_t|}{|q_t|} \right)^2 + \frac{Q_0^2}{Q_0^2} \tilde{\phi}_0P(q_t) \right] \right\}^2, \] (33)

where again $|p_t|$ is given in Eq. (17).

The heavy quark distribution functions have been derived at some scale $\nu_0$, which is of the order of $\Lambda_{\text{QCD}}$. The QCD running of these functions is controlled by the DGLAP equations. In our numerical calculations we will use the evolution code provided in Ref. [13] to give the heavy quark distribution functions at any higher scale. This will be done to the next-to-leading order. Since the QCD interactions are flavor independent, we can directly apply this code to the cases of heavy quarks. It should be pointed out that the scale $\nu_0$ cannot be determined in our approach. We will treat it as a free parameter and leave it to be determined by the future experimental data.

III. Numerical results

In this section we will give numerical results for the heavy quark distribution
functions based on the formulas presented in Section II. The B-S wave functions for \( \Lambda_Q \) and \( \omega_Q \) were solved numerically in our previous work by discretizing the integration region \((0, \infty)\) into \(n\) pieces (\(n\) is chosen to be sufficiently large and we use \(n\)-point Gauss quadrature rule to evaluate the integrals) and solving the eigenvalue equation with the kernel \( \tilde{V}_1 \) and \( \tilde{V}_2 \) in Eq. (3) in the covariant instantaneous approximation [3]. The normalization constants of these B-S wave functions are determined by the normalization of Isgur-Wise functions at the zero-recoil point. Substituting these numerical solutions into Eqs. (16, 28, 33), we can obtain numerical results for the heavy quark distribution functions in \( \Lambda_Q \) and \( \omega_Q \).

In our model we have several parameters, i.e., \( \alpha_s^{(\text{eff})} \), \( \kappa \), \( Q_0^2 \), \( m_D \) and \( E_0 \). The parameter \( Q_0^2 \) is chosen to be 3.2GeV\(^2\) from the data for the electromagnetic form factor of the proton [3, 4]. The parameters \( \alpha_s^{(\text{eff})} \) and \( \kappa \) are related to each other when we solve the eigenvalue equations with fixed eigenvalues [3]. The parameter \( \kappa \) is of the order \( \Lambda_{QCD} \kappa' \) where \( \kappa' \) is around 0.2GeV\(^2\) from the potential model [5, 6]. Thus we let \( \kappa \) vary in the region between 0.02GeV\(^3\) and 0.1GeV\(^3\) [3]. The parameters \( m_D \) and \( E_0 \) are constrained by the relation \( m_D + E_0 + \frac{1}{m_Q} E_1 = m_{\Lambda_Q} - m_Q \) for \( \Lambda_Q \) (to order \( 1/m_Q \)) and \( m_D + E_0 = m_{\omega_Q} - m_Q \) for \( \omega_Q \) (\( m_Q \to \infty \)). From the heavy quark effective theory, \( m_D + E_0 \) and \( E_1 \) are independent of the flavor of the heavy quark. It was shown [3] that the value of \( E_1 \) (which is of the order \( \Lambda_{QCD} E_0 \)) influences the numerical results only slightly. In our numerical calculations we use \( m_b = 5.02\text{GeV} \) and \( m_c = 1.58\text{GeV} \) which led to consistent predictions with experiments from the B-S equation solutions in the meson case [6]. Consequently we have \( m_D + E_0 + \frac{1}{m_b} E_1 = 0.62\text{GeV} \) for \( \Lambda_Q \) (where we have neglected \( 1/m_b^2 \) corrections), and \( m_D + E_0 = 0.88\text{GeV} \) for \( \Sigma_Q \), 1.07GeV for \( \Xi_Q \), 1.12GeV for \( \Omega_Q \) in the heavy quark limit. The parameter \( m_D \) cannot be determined and hence we let it vary within some reasonable range. For \( \Lambda_Q \), we choose \( m_D \) to be in the range 0.65GeV \( \sim 0.75\text{GeV} \) [3, 17]. The axial-vector diquark mass is chosen to vary from 0.9GeV to
1GeV for $\Sigma_Q$, from 1.1GeV to 1.2GeV for $\Xi_Q$, and from 1.15GeV to 1.25GeV for $\Omega_Q$ \cite{9}. With these choices for $m_D$, the binding energy $E_0$ is negative and varies from around -30MeV to -130MeV.

With these parameters, from Eqs. (16,28,33), we obtain numerical results for the heavy quark distribution functions in $\Lambda_Q$ and $\omega_Q$. These results are shown in Figs.1 to 5. Figs.1(a), 2(a), 3(a), 4(a) and 5(a) show the dependence on $\kappa$ for a typical $m_D$, and the dependence on $m_D$ for $\kappa = 0.06\text{GeV}^3$. All these results correspond to some low energy scale which is of order several hundred MeV. They are evolved to some higher scale using the DGLAP equations. In Figs.1(b), 2(b), 3(b), 4(b) and 5(b), we show the heavy quark distribution functions at the scale $\nu^2 = 10\text{GeV}^2$, where we have taken the low energy scale $\nu_0^2$ to be 0.25GeV$^2$, as an example. From these figures we can see that for different heavy baryons with the same heavy quark flavor the shapes of the heavy quark distribution functions are rather similar. For the heavy quark distribution functions at the hadronic scale, there is an obvious peak at some “plus” momentum fraction carried by the heavy quark, $\alpha_0$, and this peak is much sharper for $b$-baryons than $c$-baryons. Furthermore, $\alpha_0$ is much closer to 1 for $b$-baryons than $c$-baryons. It can also be seen that QCD evolution makes the amplitudes of the peaks much smaller. However, the distinction between $b$-quark and $c$-quark distribution functions is still obvious at high $\nu^2$. From Figs. 1 to 5 we can also see that with the increase of $\kappa$, which represents the strength of confinement, the peaks of the heavy quark distribution functions become lower and their widths bigger. This is because the heavy quark behaves more freely when confinement is weaker.

In Ref. \cite{17} Close and Thomas calculated the $u$-quark distribution function in the nucleon in the MIT bag model, where numerical results for $\alpha_u(\alpha)$ at $\nu^2 = 5 \sim 10\text{GeV}^2$ were shown for various values of the bag radius. It was shown that $\alpha_u(\alpha)$ has a maximum value around $0.4 \sim 0.5$ at $\alpha = 0.4 \sim 0.5$. In general their
argument predicts the peak of the valence distribution (for ground state \((L = 0)\) baryons) at \((m_H - m_D)/m_H\) (where \(m_H\) is the baryon’s mass), which is in the range \(0.87 \sim 0.88\) for \(\Lambda_b\) and \(0.67 \sim 0.72\) for \(\Lambda_c\), respectively. Clearly these expectations are in excellent agreement with the peak positions at the hadronic scale, \(\nu_0^2\). This statement is also true for \(\omega_Q\). We see clearly that with the increase of quark masses, the peak positions of the quark distribution functions move closer to 1 and the peak values become bigger.

In Tables 1,2,3,4 for different heavy baryons with different parameters in our model, we list the values of \(\alpha_0\), the widths of these peaks, and the total “plus” momentum fraction, \(f\), carried by the heavy quark which is defined as

\[
 f \equiv \int d\alpha \alpha Q(\alpha). \tag{34}
\]

The width is presented by two values of \(\alpha\) at which \(Q(\alpha)\) is half of its value at the peak, \(Q(\alpha_0)\). From these tables we can see the following points: (i) The values of \(\alpha_0\), \(f\), and the widths depend not only on the light diquark mass \(m_D\) but also on \(\kappa\), and mostly the dependence on \(\kappa\) is more sensitive for \(c\)-baryons. (ii) The value of \(\alpha_0\) decreases with the increase of \(m_D\), which is reasonable. (iii) The widths of heavy quark distribution functions for \(b\)-baryons are much smaller than those for \(c\)-baryons. (iv) For \(\Lambda_Q\), \(1/m_Q\) corrections make \(\alpha_0\) and \(f\) bigger. Hence the heavy quark carries more momentum with \(1/m_Q\) corrections included. (v) Numerically, in the whole range of parameters we are considering, for \(\Lambda_b\), \(\alpha_0\) varies from 0.864 to 0.890 (0.870 to 0.890) and \(f\) varies from 0.800 to 0.857 (0.810 to 0.860) without (with) \(1/m_Q\) corrections, while for \(\Lambda_c\), \(\alpha_0\) varies from 0.670 to 0.717 (0.690 to 0.760) and \(f\) varies from 0.522 to 0.651 (0.585 to 0.672); for \(\Sigma_b\), \(\alpha_0\) varies from 0.823 to 0.846 and \(f\) varies from 0.809 to 0.834, while for \(\Sigma_c\), \(\alpha_0\) varies from 0.583 to 0.647 and \(f\) varies from 0.493 to 0.589; for \(\Xi_b\), \(\alpha_0\) varies from 0.803 to 0.820 and \(f\) varies from 0.780 to 0.809, while for \(\Xi_c\), \(\alpha_0\) varies from 0.503 to 0.580 and \(f\) varies from 0.412 to 0.520; for \(\Omega_b\), \(\alpha_0\) varies from 0.797 to 0.813 and \(f\) varies from 0.770 to 0.802,
Table 1: Values of $\alpha_0$, widths, and $f$ for $\Lambda_b$ with and without $1/m_Q$ corrections

| $m_D=0.65\text{GeV}$ | $\kappa(\text{GeV}^3)$ | $0.02 \ (O(1))$ | $0.02 \ (O(1/m_Q))$ | $0.10 \ (O(1))$ | $0.10 \ (O(1/m_Q))$ |
|----------------------|----------------------|------------------|------------------|------------------|------------------|
| $\alpha_0$         | 0.890               | 0.890            | 0.890            | 0.890            |
| width               | $0.797 \sim 0.923$  | $0.833 \sim 0.923$ | $0.773 \sim 0.940$ | $0.783 \sim 0.943$ |
| $f$                 | 0.857               | 0.860            | 0.822            | 0.830            |

| $m_D=0.70\text{GeV}$ | $\kappa(\text{GeV}^3)$ | $0.02 \ (O(1))$ | $0.02 \ (O(1/m_Q))$ | $0.10 \ (O(1))$ | $0.10 \ (O(1/m_Q))$ |
|----------------------|----------------------|------------------|------------------|------------------|------------------|
| $\alpha_0$         | 0.876               | 0.878            | 0.878            | 0.884            |
| width               | $0.814 \sim 0.918$  | $0.816 \sim 0.919$ | $0.762 \sim 0.935$ | $0.772 \sim 0.937$ |
| $f$                 | 0.844               | 0.848            | 0.811            | 0.820            |

| $m_D=0.75\text{GeV}$ | $\kappa(\text{GeV}^3)$ | $0.02 \ (O(1))$ | $0.02 \ (O(1/m_Q))$ | $0.10 \ (O(1))$ | $0.10 \ (O(1/m_Q))$ |
|----------------------|----------------------|------------------|------------------|------------------|------------------|
| $\alpha_0$         | 0.868               | 0.870            | 0.864            | 0.870            |
| width               | $0.789 \sim 0.912$  | $0.802 \sim 0.914$ | $0.750 \sim 0.930$ | $0.762 \sim 0.932$ |
| $f$                 | 0.829               | 0.834            | 0.800            | 0.810            |

while for $\Omega_c$, $\alpha_0$ varies from 0.533 to 0.580 and $f$ varies from 0.449 to 0.537.

As mentioned in Section II, the heavy quark distribution functions should be normalized according to Eq. (9). For some fixed set of parameters in our model, the B-S wave functions can be completely determined by solving the B-S equations numerically and fixing the normalization of the corresponding Isgur-Wise functions at the zero-recoil point. Hence by checking whether the normalization condition Eq. (9) is satisfied or not we can test whether our model works well. We have performed the integration of $Q(\alpha)$ over $\alpha$ numerically. It is found that for $\Lambda_b$ and $\Lambda_c$, in the whole range of the parameters, $\int d\alpha Q(\alpha)$ is very close to 1 (for $\Lambda_b$ and $\Lambda_c$ the deviation from 1 is of order $10^{-4}$ and $10^{-3} \sim 10^{-2}$, respectively). Furthermore, we find that $1/m_Q$ corrections make $\int d\alpha Q(\alpha)$ even closer to 1. Therefore, our model for $1/m_Q$ corrections works in the right direction. For $\omega_b$ and $\omega_c$, although $\int d\alpha Q(\alpha)$ is not as close to 1 as in the case of $\Lambda_Q$, the deviation from 1 still happens at order $10^{-2}$ at most, which is expected to be improved if $1/m_Q$ corrections are also included. Thus the momentum sum rule is satisfied in our model.
Table 2: Values of $\alpha_0$, widths, and $f$ for $\Lambda_c$ with and without $1/m_Q$ corrections

| $m_D=0.65\text{GeV}$ | \(\kappa\) (GeV$^3$) | $0.02\ (O(1))$ | $0.02\ (O(1/m_Q))$ | $0.10\ (O(1))$ | $0.10\ (O(1/m_Q))$ |
|----------------------|---------------------|------------------|------------------|------------------|------------------|
| $\alpha_0$           | 0.717               | 0.730            | 0.710            | 0.760            |
| width                | $0.580 \sim 0.807$  | $0.600 \sim 0.817$ | $0.443 \sim 0.857$ | $0.510 \sim 0.870$ |
| $f$                  | 0.651               | 0.672            | 0.569            | 0.625            |

| $m_D=0.70\text{GeV}$ | \(\kappa\) (GeV$^3$) | $0.02\ (O(1))$ | $0.02\ (O(1/m_Q))$ | $0.10\ (O(1))$ | $0.10\ (O(1/m_Q))$ |
|----------------------|---------------------|------------------|------------------|------------------|------------------|
| $\alpha_0$           | 0.670               | 0.717            | 0.697            | 0.730            |
| width                | $0.537 \sim 0.797$  | $0.563 \sim 0.807$ | $0.417 \sim 0.840$ | $0.487 \sim 0.856$ |
| $f$                  | 0.617               | 0.645            | 0.546            | 0.605            |

| $m_D=0.75\text{GeV}$ | \(\kappa\) (GeV$^3$) | $0.02\ (O(1))$ | $0.02\ (O(1/m_Q))$ | $0.10\ (O(1))$ | $0.10\ (O(1/m_Q))$ |
|----------------------|---------------------|------------------|------------------|------------------|------------------|
| $\alpha_0$           | 0.670               | 0.690            | 0.670            | 0.713            |
| width                | $0.500 \sim 0.783$  | $0.530 \sim 0.797$ | $0.387 \sim 0.823$ | $0.550 \sim 0.810$ |
| $f$                  | 0.584               | 0.618            | 0.522            | 0.585            |

Table 3: Values of $\alpha_0$, widths, and $f$ for $\Sigma_b$ and $\Sigma_c$ in the heavy quark limit

| $m_D=0.90\text{GeV}$ | \(\kappa\) (GeV$^3$) | $0.02\ (\Sigma_b)$ | $0.10\ (\Sigma_b)$ | $0.02\ (\Sigma_c)$ | $0.10\ (\Sigma_c)$ |
|----------------------|---------------------|------------------|------------------|------------------|------------------|
| $\alpha_0$           | 0.846               | 0.843            | 0.630            | 0.647            |
| width                | $0.800 \sim 0.883$  | $0.750 \sim 0.907$ | $0.517 \sim 0.720$ | $0.397 \sim 0.773$ |
| $f$                  | 0.834               | 0.834            | 0.589            | 0.528            |

| $m_D=0.95\text{GeV}$ | \(\kappa\) (GeV$^3$) | $0.02\ (\Sigma_b)$ | $0.10\ (\Sigma_b)$ | $0.02\ (\Sigma_c)$ | $0.10\ (\Sigma_c)$ |
|----------------------|---------------------|------------------|------------------|------------------|------------------|
| $\alpha_0$           | 0.840               | 0.843            | 0.610            | 0.600            |
| width                | $0.780 \sim 0.880$  | $0.733 \sim 0.903$ | $0.470 \sim 0.720$ | $0.463 \sim 0.723$ |
| $f$                  | 0.820               | 0.813            | 0.550            | 0.493            |

| $m_D=1.00\text{GeV}$ | \(\kappa\) (GeV$^3$) | $0.02\ (\Sigma_b)$ | $0.10\ (\Sigma_b)$ | $0.02\ (\Sigma_c)$ | $0.10\ (\Sigma_c)$ |
|----------------------|---------------------|------------------|------------------|------------------|------------------|
| $\alpha_0$           | 0.830               | 0.823            | 0.583            | 0.597            |
| width                | $0.763 \sim 0.923$  | $0.740 \sim 0.889$ | $0.430 \sim 0.707$ | $0.383 \sim 0.730$ |
| $f$                  | 0.809               | 0.814            | 0.527            | 0.523            |
Table 4: Values of $\alpha_0$, widths, and $f$ for $\Xi_b$ and $\Xi_c$ in the heavy quark limit

| $m_D=1.10\,\text{GeV}$ | $\kappa (\text{GeV}^3)$ | $\alpha_0$ | width | $f$ |
|------------------------|------------------------|------------|-------|-----|
| $\Xi_b$                | 0.02 (0.820)           | 0.10 (0.817) | 0.02 (0.553) | 0.10 (0.580) | 0.773 $\sim$ 0.857 |
| $\Xi_c$                | 0.02 (0.817)           | 0.10 (0.817) | 0.02 (0.553) | 0.10 (0.580) | 0.733 $\sim$ 0.877 |
| $m_D=1.15\,\text{GeV}$ | $\kappa (\text{GeV}^3)$ | $\alpha_0$ | width | $f$ |
| $\Xi_b$                | 0.02 (0.810)           | 0.10 (0.810) | 0.02 (0.533) | 0.10 (0.510) | 0.753 $\sim$ 0.850 |
| $\Xi_c$                | 0.02 (0.810)           | 0.10 (0.810) | 0.02 (0.533) | 0.10 (0.510) | 0.717 $\sim$ 0.875 |
| $m_D=1.20\,\text{GeV}$ | $\kappa (\text{GeV}^3)$ | $\alpha_0$ | width | $f$ |
| $\Xi_b$                | 0.02 (0.803)           | 0.10 (0.803) | 0.02 (0.513) | 0.10 (0.503) | 0.737 $\sim$ 0.853 |
| $\Xi_c$                | 0.02 (0.803)           | 0.10 (0.803) | 0.02 (0.513) | 0.10 (0.503) | 0.700 $\sim$ 0.870 |

Table 5: Values of $\alpha_0$, widths, and $f$ for $\Omega_b$ and $\Omega_c$ in the heavy quark limit

| $m_D=1.15\,\text{GeV}$ | $\kappa (\text{GeV}^3)$ | $\alpha_0$ | width | $f$ |
|------------------------|------------------------|------------|-------|-----|
| $\Omega_b$             | 0.02 (0.813)           | 0.10 (0.813) | 0.03 (0.573) | 0.10 (0.580) | 0.760 $\sim$ 0.853 |
| $\Omega_c$             | 0.02 (0.813)           | 0.10 (0.813) | 0.03 (0.573) | 0.10 (0.580) | 0.727 $\sim$ 0.873 |
| $m_D=1.20\,\text{GeV}$ | $\kappa (\text{GeV}^3)$ | $\alpha_0$ | width | $f$ |
| $\Omega_b$             | 0.02 (0.803)           | 0.10 (0.803) | 0.02 (0.553) | 0.10 (0.557) | 0.750 $\sim$ 0.894 |
| $\Omega_c$             | 0.02 (0.803)           | 0.10 (0.803) | 0.02 (0.553) | 0.10 (0.557) | 0.710 $\sim$ 0.867 |
| $m_D=1.25\,\text{GeV}$ | $\kappa (\text{GeV}^3)$ | $\alpha_0$ | width | $f$ |
| $\Omega_b$             | 0.02 (0.797)           | 0.10 (0.797) | 0.02 (0.533) | 0.10 (0.540) | 0.733 $\sim$ 0.847 |
| $\Omega_c$             | 0.02 (0.797)           | 0.10 (0.797) | 0.02 (0.533) | 0.10 (0.540) | 0.697 $\sim$ 0.863 |

| $f$ |
|-----|
| 0.773 | 0.770 | 0.474 | 0.449 |
Now we compare the present results with another phenomenological model. In Ref. [10], also based on the quark-diquark picture, \( \Lambda_Q \) is regarded as composed of a heavy quark and a scalar light diquark. The heavy baryon wave function \( \Psi_{\Lambda_Q}(\alpha, k_\perp) \) is proposed as a generalization of the Bauer-Stech-Wirbel [18] meson wave function to the quark-diquark case,

\[
\Psi_{\Lambda_Q}(x_1, k_\perp) = N_{\Lambda_Q} \alpha(1 - \alpha)^3 \exp\{-b^2[k^2_\perp + m_{\Lambda_Q}^2(\alpha - \bar{\alpha})^2]\}, \tag{35}
\]

where \( N_{\Lambda_Q} \) is the normalization constant, \( \bar{\alpha} = m_Q/m_{\Lambda_Q} \), \( k_\perp \) is the transverse momentum and the parameter \( b \) is related to the root of the average square of \( k_\perp \), \( \sqrt{\langle k^2_\perp \rangle} \). The normalization of the wave function is

\[
\int d\alpha d^2k_\perp |\Psi_{\Lambda_Q}(\alpha, k_\perp)|^2 = 1. \tag{36}
\]

The heavy quark distribution function in this model is defined in the following

\[
Q(\alpha) = \int d^2k_\perp |\Psi_{\Lambda_Q}(\alpha, k_\perp)|^2, \tag{37}
\]

which leads to

\[
Q(\alpha) = \frac{2\pi^2 N_{\Lambda_Q}^2}{b^2} \alpha^2(1 - \alpha)^6 \exp[-2b^2m_{\Lambda_Q}^2(\alpha - \bar{\alpha})^2]. \tag{38}
\]

Here \( \langle k^2_\perp \rangle \) is defined as

\[
\langle k^2_\perp \rangle = \int d\alpha d^2k_\perp k^2_\perp |\Psi_{\Lambda_Q}(\alpha, k_\perp)|^2, \tag{39}
\]

from which, using Eq. (36), we have

\[
2b^2\langle k^2_\perp \rangle = 1. \tag{40}
\]

We choose \( \sqrt{\langle k^2_\perp \rangle} \) to vary between 400MeV and 600MeV. For these two parameters we plot \( Q(\alpha) \) at the hadronic scale in Fig.6(a) and we plot \( \alpha Q(\alpha) \) at \( \nu^2 = 10\text{GeV}^2 \) in Fig.6(b). We can see from these plots that the heavy quark distribution functions in this model have a similar shape to those found in the present B-S
Table 6: Values of $\alpha_0$, widths, and $f$ for $\Lambda_b$ and $\Lambda_c$ in the model of Ref. [10]

|                | $\sqrt{\langle k^2 \rangle}$ (GeV) | 0.4 ($\Lambda_b$) | 0.6 ($\Lambda_b$) | 0.4 ($\Lambda_c$) | 0.6 ($\Lambda_c$) |
|----------------|-------------------------------------|-------------------|-------------------|-------------------|-------------------|
| $\alpha_0$     | 0.816                               | 0.764             | 0.545             | 0.460             |
| Width          | 0.766 $\sim$ 0.864                  | 0.693 $\sim$ 0.830| 0.426 $\sim$ 0.660| 0.306 $\sim$ 0.609|
| $f$            | 0.813                               | 0.758             | 0.540             | 0.456             |

model. The peak position for $Q(\alpha)$, widths of the distributions, and the total “plus” momentum fraction carried by the heavy quark $Q$, are listed in Table 6. Notice that these values are independent of the value of $m_D$ in this model. Finally, it can be seen from Table 6 that both $\alpha_0$ and $f$ in this model are smaller than those in the B-S model.

**IV. Summary and discussion**

Based on our heavy quark and light diquark model for heavy baryons $\Lambda_Q$ and $\omega_Q$ in the B-S equation approach, we have calculated the heavy quark distribution functions in these baryons. This was done in the heavy quark limit for both $\Lambda_Q$ and $\omega_Q$. Furthermore, $1/m_Q$ corrections were also included in the case of $\Lambda_Q$. Our numerical results show that for different heavy baryons with the same heavy quark flavor the shapes of the heavy quark distribution functions are similar. At the hadronic scale, $\Lambda_{QCD}$, they have an obvious peak at some “plus” momentum fraction carried by the heavy quark, $\alpha_0$, which is much closer to 1 for $b$-baryons than $c$-baryons. In addition, the widths of these distributions are much smaller for $b$-baryons than $c$-baryons. Furthermore, we have found that the peaks of the heavy quark distribution functions become lower and their widths bigger when the confinement between the heavy quark and light diquark inside a heavy baryon is stronger. The QCD evolution of these distribution functions were also discussed by applying DGLAP equations to the next-to-leading order and the results show that
the distinction between $b$-quark and $c$-quark distribution functions is still obvious at high $\nu^2$. The evolution makes the amplitudes of the peaks much smaller, especially for the distribution functions in $b$-baryons. We also calculated the total “plus” momentum fraction carried by the heavy quark, which is much bigger for $b$-baryons than $c$-baryons. The dependence of all these numerical results on the parameters in our model was discussed in detail. We also checked the normalization condition of these distribution functions and found that it is satisfied. In addition, we compared our results with an existing phenomenological model for $\Lambda_Q$ proposed in Ref. [10]. It was found that the shapes of the heavy quark distribution functions in these two models are quite similar, although the peak positions in the B-S model are closer to 1 than those in the model of Ref. [10]. We also compared our results for the heavy quark distribution functions with the general argument of Close and Thomas based on energy and momentum conservation. It was found that the peak positions of the quark distribution functions move closer to 1 and the peak values become bigger when the quark masses increase.

Although we cannot determine the exact value of $\nu_0$, the hadronic scale at which the heavy quark distribution functions are calculated, our results do reflect nonperturbative information in the heavy baryons $\Lambda_Q$ and $\omega_Q$. We have treated $\nu_0$ as a parameter to be determined by experiment. Even though the heavy quark distribution functions cannot be measured directly from deep inelastic scattering processes, we can still expect to obtain some information about these functions from the decays of heavy baryons and fragmentation of heavy quarks to heavy baryons. In fact, the latter is directly related to experimental measurements of fragmentation processes. However, the theoretical extension to address fragmentation processes is more complicated. This is currently under investigation.
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Figure Captions

Fig.1 (a) Heavy quark distribution functions at the hadronic scale $\nu_0^2$ with the B-S equation solved for $\Lambda_Q$ in the limit $m_Q \to \infty$; (b) $\alpha Q(\alpha)$ for $\Lambda_Q$ at $\nu^2 = 10\text{GeV}^2$ in the limit $m_Q \to \infty$. The lines on the right (left) are for $\Lambda_b$ ($\Lambda_c$). The solid (dot) lines correspond to $m_D = 0.70\text{GeV}$ and $\kappa = 0.02\text{GeV}^3$ ($\kappa = 0.10\text{GeV}^3$). The dashed (dot dashed) lines correspond to $\kappa = 0.06\text{GeV}^3$ and $m_D = 0.65\text{GeV}$ ($m_D = 0.75\text{GeV}$).

Fig.2 (a) Heavy quark distribution functions at the hadronic scale $\nu_0^2$ with the B-S equation solved for $\Lambda_Q$ to order $1/m_Q$; (b) $\alpha Q(\alpha)$ for $\Lambda_Q$ at $\nu^2 = 10\text{GeV}^2$ to order $1/m_Q$. The lines on the right (left) are for $\Lambda_b$ ($\Lambda_c$). The solid (dot) lines correspond to $m_D = 0.70\text{GeV}$ and $\kappa = 0.02\text{GeV}^3$ ($\kappa = 0.10\text{GeV}^3$). The dashed (dot dashed) lines correspond to $\kappa = 0.06\text{GeV}^3$ and $m_D = 0.65\text{GeV}$ ($m_D = 0.75\text{GeV}$).

Fig.3 (a) Heavy quark distribution functions at the hadronic scale $\nu_0^2$ with the B-S equation solved for $\Sigma_Q$ in the limit $m_Q \to \infty$; (b) $\alpha Q(\alpha)$ for $\Sigma_Q$ at $\nu^2 = 10\text{GeV}^2$ in the limit $m_Q \to \infty$. The lines on the right (left) are for $\Sigma_b$ ($\Sigma_c$). The solid (dot) lines correspond to $m_D = 0.95\text{GeV}$ and $\kappa = 0.02\text{GeV}^3$ ($\kappa = 0.10\text{GeV}^3$). The dashed (dot dashed) lines correspond to $\kappa = 0.06\text{GeV}^3$ and $m_D = 0.90\text{GeV}$ ($m_D = 1.00\text{GeV}$).

Fig.4 (a) Heavy quark distribution functions at the hadronic scale $\nu_0^2$ with the B-S equation solved for $\Xi_Q$ in the limit $m_Q \to \infty$; (b) $\alpha Q(\alpha)$ for $\Xi_Q$ at $\nu^2 = 10\text{GeV}^2$ in the limit $m_Q \to \infty$. The lines on the right (left) are for $\Xi_b$ ($\Xi_c$). The solid (dot) lines correspond to $m_D = 1.15\text{GeV}$ and $\kappa = 0.02\text{GeV}^3$ ($\kappa = 0.10\text{GeV}^3$). The dashed (dot dashed) lines correspond to $\kappa = 0.06\text{GeV}^3$ and $m_D = 1.10\text{GeV}$ ($m_D = 1.20\text{GeV}$).

Fig.5 (a) Heavy quark distribution functions at the hadronic scale $\nu_0^2$ with the B-S equation solved for $\Omega_Q$ in the limit $m_Q \to \infty$; (b) $\alpha Q(\alpha)$ for $\Omega_Q$ at $\nu^2 = 10\text{GeV}^2$ in the limit $m_Q \to \infty$. The lines on the right (left) are for $\Omega_b$ ($\Omega_c$). The solid (dot) lines correspond to $m_D = 1.20\text{GeV}$ and $\kappa = 0.02\text{GeV}^3$ ($\kappa = 0.10\text{GeV}^3$). The dashed (dot dashed) lines correspond to $\kappa = 0.06\text{GeV}^3$ and $m_D = 1.15\text{GeV}$ ($m_D = 1.25\text{GeV}$).

Fig.6 (a) Heavy quark distribution functions for $\Lambda_Q$ in the model of Ref. [10]; (b) $\alpha Q(\alpha)$ for $\Lambda_Q$ at $\nu^2 = 10\text{GeV}^2$ in the model of Ref. [10]. The two lines on the right (left) are for $\Lambda_b$ ($\Lambda_c$). The solid (dashed) lines correspond to $\sqrt{\langle k_\perp \rangle^2} = 0.4\text{GeV}$ ($\sqrt{\langle k_\perp \rangle^2} = 0.6\text{GeV}$).
Fig. 1(a)

Fig. 1(b)
Fig. 2(a)

Fig. 2(b)
Fig. 3(a)

Fig. 3(b)
Fig. 4(a)

Fig. 4(b)
Fig. 5(a)

Fig. 5(b)
Fig. 6(a)

Fig. 6(b)