Abstract

The $X(3872)$ resonance has been conjectured to be a $J^{PC} = 1^{++}$ charm meson-antimeson two-body molecule. Meanwhile, there is no experimental evidence for larger, few-body compounds of multiple charm meson-antimeson pairs which would resemble larger molecules or nuclei. Here we investigate the existence of such multi-meson states to the extent of what can be deduced theoretically from essentials of the interaction between uncharged $D^0$ mesons. From a molecular $X(3872)$, we predict a $4X$ octamer with $J^{PC} = 4^{++}$ and a binding energy $B_{4X} > 2.08\,\text{MeV}$, with the assumption of a $D^0\bar{D}^0$ system close to the unitary limit (compatible with the currently known mass of the $X(3872)$). If we consider heavy-quark spin symmetry explicitly, the $J^{PC} = 2^{++} D^{*0}\bar{D}^{*0}$ system is close to unitarity, too. In this case, we predict a bound ($J^{PC} = 3^{++}$) $3X$ hexamer with $B_{3X} > 2.29\,\text{MeV}$ and a more deeply bound $4X$ octamer with $B_{4X} > 11.21\,\text{MeV}$.
**Introduction:** Systems of particles with a two-body scattering length \( a \) significantly larger than the interaction range \( R (a \gg R) \) share a series of common/universal properties, which encompass a multitude of phenomena in atomic, nuclear, and particle physics \([1]\). This invariance with respect to a continuous scale transformation, however, holds strictly only in the two-body sector. In the few-body spectrum, this *continuous* scale invariance survives only partially in a *discrete* version. An example of this is the Efimov effect \([2]\), i.e. the appearance of a geometric bound-state spectrum of three-boson systems in the unitary limit \((a/R \to \infty)\). This effect was found for the first time a decade ago in experiments with caesium atoms \([3]\), and it is now known to extend to systems of non-identical particles \([4]\) as well as systems of more than three particles \([5, 6]\). In nuclear physics, the Efimov effect plays a role in the description of the triton \([7, 8]\) and \(^4\)He \([9]\), halo nuclei \([10, 14]\), and the Hoyle state \([15, 16]\).

Compared with atoms and nucleons, it is more difficult to find instances of universality in hadronic physics where the \( X(3872) \) resonance (Belle collaboration \([17]\)) might qualify as a hadronic system close to the unitary limit. The \( X \) has been conjectured to be a hadronic molecule \([18, 19]\), more precisely, a relatively shallow bound state of two hadrons because of its proximity to the \( D^*D^0 \) threshold \((\sim 0.1 \text{ MeV})\) and its narrow width. This shallowness, in particular, is a signature of universal behaviour \([20]\).

Whether universal properties can be identified in systems composed of more than two charm mesons is an open question which is even more intriguing because charm meson-antimeson interactions produce qualitatively new features that are absent in systems of identical particles. For instance, the three-body systems \( D^0D^0D^* \) and \( D^*D^0D^0 \) neither do form trimers nor do they display the Efimov effect \([21]\). Along with heavy-quark spin symmetry (HQSS) \([22, 23]\) and the associated more tightly constrained charm meson-antimeson potential enter new features. In the two-body sector, we expect from HQSS an interaction in the \( J^{PC} = 1^{++} D^*D^0 \) \( X \)-channel identical to the one in the \( J^{PC} = 2^{++} D^*D^0 \) channel, suggesting the existence of a partner molecule of the \( X \) \([24, 25]\). Like the \( X(3872) \), this partner is expected to be shallow but its survival as a bound, a virtual, or as a resonant state depends on a number of uncertainties \([25, 27]\). HQSS challenges the original expectation of an unbound \( D^0D^0D^0 \) \( J = 2 \) three-body system, and the Efimov effect becomes a realistic possibility \([28]\).

In four-meson systems and beyond, we expect to find new universal phenomena different from the ones known to emerge in atomic and nuclear composites \([29, 30]\). We will consider, in particular, systems of \( N = 2, 3, 4 \) \( D^0D^* \) pairs with maximum spin, i.e. \( J = 2, 3, 4 \), respectively. Bound “polymers” of this kind exhibit a characteristic scaling inversely proportional to the square of the interaction range, i.e. \( B_{2N} \propto 1/R^2 \). We infer from this scaling the Thomas collapse \([31]\) of these systems along with the implied Efimov effect. However, as the collapse is a consequence of unitarity and therefore impaired by a finite interaction range. Specifically, \( D^0D^* \) pairs can decay strongly to \( D^*D^0/D^{(*)+}D^- \) via a short-range \( D \)-wave operator \([32]\) inducing a finite interaction width. Using data on the \( X \) in support of the assumption of an infinite \( D^*D^0 \) scattering length \((\text{zero binding energy of the } X \text{ molecule})\) and disregarding HQSS, we predict a bound state of four \( X \)'s: an octamer. As the tetramer and the hexamer are unbound under these circumstances, this octamer resembles a *so-called* Brunnian \([33, 34]\) state: a generalization of a Borromean structure. Finally, we predict that HQSS, i.e. a \( D^0D^* \) interaction close to the unitary limit, will stabilize the hexamer and thus induce the transition from a Brunnian to a Borromean system (a still unbound tetramer with a hexamer resembling a Borromean bound state of \( X \)'s).
Theory and calculation method: We treat the above-mentioned “polymers” as a non-relativistic few-body problem. The charm meson and antimesons comprising these “polymers” have a ground ($D/D$) and excited ($D^{*}/D^{*}$) state. Their isospin $I = 1/2$ discriminates between neutral and charged states. Because of their mass difference, we will only consider the neutral mesons which dominate the $X$ wave function tail. As we are working in the unitary limit, we have to consider only resonant two-body interactions. There is experimental evidence for resonant behaviour in the $J^{PC} = 1^{++} D^{*0} \bar{D}^{0}$ channel ($X$), and HQSS lets us expect the $2^{++} D^{*+} \bar{D}^{*-}$ channel to be resonant, too. All other combinations are assumed to be non-resonant and set to zero. Non-resonant interaction pairs would increase the total binding of the systems slightly. Thus they do not alter our conclusions.

To describe the resonant pairs, we employ a contact two-body potential regularized with a Gaussian cutoff function

$$V(r; R_c) = C(R_c) \delta^{(3)}(r; R_c), \quad (1)$$

$$\delta^{(3)}(r; R_c) = \frac{e^{-(r/R_c)^2}}{\pi^{3/2} R_c^3}, \quad (2)$$

whose cutoff-radius ($R_c$) and coupling constant $C(R_c)$ are calibrated to reproduce the location of the $X(3872)$. We consider all mesons to have an identical mass of $m = 1933.29$ MeV, i.e. twice the reduced mass of the $D^{*0} \bar{D}^{0}$ pair within the $X$ because it is the most important resonant interaction in the multi-$X$ systems (see derivation of (4)-(12) below). Corrections are deemed to be subleading and without impact on the qualitative description of the many-particle states.

Renormalized predictions, in principle, require that observables are cutoff independent. We will show below that hexamer and octamer binding energies do not conform with this demand as they Thomas-collapse if $R_c \to 0$. In few-body systems, this type of divergence can be renormalized via an additional three-boson datum [7, 8] which, as of now, is unavailable in the few-$X$ sector.

Despite this obstacle, we can obtain information about the existence of bound states and estimates of their binding energies. To this end, we choose a cutoff range near the theory’s breakdown scale which is determined by the longest omitted short-range component of the interaction. For the $X$ this missing component is the charged channel \[1\] i.e. the $D^{*+} D^{-}$ component of the $X$ wave function [36, 37]. The characteristic momentum scale of the charged channel is $M_{ch} \approx 125$ MeV. It is sensible to expect a cutoff in the vicinity of $M_{ch} R_c \sim 1$ for which the three-body counterterm vanishes, and that it remains numerically small within some interval around it. This smallness suffices to foresee that their inclusion would have no effect on the character of a state: bound will remain bound, resonance will remain resonance, etc.. Hence, a bound state found within a cutoff range around $M_{ch} R_c \sim 1$ (specifically, we chose $R_c = 1.0 - 2.0$ fm) can reliably be considered a renormalized prediction which will not change character even with the proper calibration of a collapse-preventing counterterm.

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1 Pion effects are naively expected to enter perturbatively at subleading orders (see [35]), suggesting leading-order predictions which are indistinguishable in pionfull and pionless treatments.
**Interaction between meson pairs:** We exemplify the few-X calculations with a detailed discussion of the four-body, i.e. two-X problem. First, we treat the X as a pure $D^{*0}\bar{D}^{0}$ two-body system. This approximation disregards the shorter-range $D^{*++}\bar{D}^{-}$ component and assumes the X wave function to be

$$\Psi_X = \phi_X(r) \frac{1}{\sqrt{2}} \left[ |D^{0}\bar{D}^{*0}\rangle + |D^{*0}\bar{D}^{0}\rangle \right],$$

(3)

with the spatial two-body wave function $\phi_X(r)$. The charm meson-antimeson potential in the X channel is defined for the linear combination $D^{*0}\bar{D}^{0} + D^{0}\bar{D}^{*0}$ (the positive C-parity combination). It is practical to use a Fock representation of the potential:

$$V_X(r; R_c) = \frac{V_D(r; R_c)}{2} \left[ |D^{0}\bar{D}^{*0}\rangle \langle D^{0}\bar{D}^{*0}| + |D^{*0}\bar{D}^{0}\rangle \langle D^{*0}\bar{D}^{0}| \right],$$

(4)

$$+ \frac{V_E(r; R_c)}{2} \left[ |D^{0}\bar{D}^{*0}\rangle \langle D^{*0}\bar{D}^{0}| + |D^{*0}\bar{D}^{0}\rangle \langle D^{0}\bar{D}^{*0}| \right],$$

with a direct ($V_D$) and an exchange term ($V_E$) which combine to the potential in the X channel, $V_X = V_D + V_E$. As no negative C-parity partner of the X has been found yet, we assume $|V_D + V_E| \gg |V_D - V_E|$. Moreover, the isospin-breaking decays of the X \cite{36, 37} allow access to $V_E$, corroborating this inequality \cite{38}. Hence, we use the approximation $V_D = V_E = \frac{1}{2}V$ and

$$V_X = \frac{V}{2} \left[ |D^{0}\bar{D}^{*0}\rangle \langle D^{0}\bar{D}^{*0}| + |D^{*0}\bar{D}^{0}\rangle \langle D^{*0}\bar{D}^{0}| \right],$$

(5)

where the $(\vec{r}, R_c)$ dependence of the potential has been dropped to improve readability.

The two-X tetramer contains in principle the six possible permutations of the $|D^{0}\bar{D}^{0}D^{*0}\bar{D}^{*0}\rangle$ state that result from exchanging ground- and excited-state mesons (we assume the spins of all the $D^{*0}/\bar{D}^{*0}$ mesons/antimesons to point in the same direction). However, these permutations are further constrained by symmetries, as we require (i) positive C-parity (i.e., invariance wrt. the exchange of particles and antiparticles), and (ii) $D^{0}$ and $D^{*0}$ to obey Bose statistics which we realize with symmetric internal and spatial wave-function components as they are expected to provide the majority of the attraction (i.e., symmetric combinations of $D^{0}D^{*0}$ and $D^{*0}D^{0}$). This reduces the number of relevant states from six to two:

$$|1\rangle = \frac{1}{\sqrt{2}} \left[ |D^{0}\bar{D}^{*0}\bar{D}^{0}\bar{D}^{*0}\rangle + |D^{*0}\bar{D}^{0}\bar{D}^{*0}\bar{D}^{0}\rangle \right],$$

(6)

$$|2\rangle = \frac{1}{\sqrt{2}} \left[ |D^{0}\bar{D}^{*0}\rangle \langle D^{0}\bar{D}^{*0}| + |D^{*0}\bar{D}^{0}\rangle \langle D^{*0}\bar{D}^{0}| \right].$$

(7)

In this basis, the potential reads (insert \cite{6} and \cite{7} in \cite{5})

$$\sum_{ij} V_X(r_{ij}; R_c) \left( |1\rangle |2\rangle \right) = \left( \begin{array}{cc} 2\bar{V} & \sqrt{2}\bar{V} \\ \sqrt{2}\bar{V} & \bar{V} \end{array} \right) \left( |1\rangle |2\rangle \right),$$

(8)

\textsuperscript{2} Antisymmetric combinations – the nuclear analog are proton-proton or neutron-neutron spin-1 contributions to, e.g. $^4$He – demand an odd angular momentum with a perturbatively small effect in the leading-order framework employed in this work.
where $\bar{V}$ represents the average of the potential for all resonant pairs. Considering that $V_X$ involves particle-antiparticle interactions only, and the same ordering as in $|1\rangle$ and $|2\rangle$ (i.e., indexing particles before antiparticles):

$$\bar{V} = \frac{1}{4} [V(r_{13}) + V(r_{14}) + V(r_{23}) + V(r_{24})] .$$

The diagonalization of (8) yields

$$\sum_{ij} V_X(r_{ij}; R_c) |X_2\rangle = 3\bar{V} |X_2\rangle ,$$

as the most attractive configuration, with $|X_2\rangle = \sqrt{\frac{2}{3}}|1\rangle + \sqrt{\frac{1}{3}}|2\rangle$ being a four-meson eigenstate of $\sum V_X$. The original coupled-channel problem has thereby been recast into a single-channel form.

The steps detailed above for the tetramer can be straightforwardly applied to the hexamer and octamer. The six-body case comprises, in principle, 20 possible permutations of the $|D^0D^0D^0\bar{D}^*0\bar{D}^*0\rangle$ state, which are reduced to two states by symmetry constraints. In the eight-body case, there are 70 possible permutations of the $|D^0D^0D^0D^0\bar{D}^*0\bar{D}^*0\bar{D}^*0\bar{D}^*0\rangle$ state, which are reduced to three symmetric ones. The potential can be diagonalized, as before in the four-body case, leading to a series of eigenvalues and eigenvectors of which the most attractive configurations are

$$\sum_{ij} V_X(r; R_c) |X_3\rangle = 6\bar{V} |X_3\rangle ,$$

$$\sum_{ij} V_X(r; R_c) |X_4\rangle = 10\bar{V} |X_4\rangle .$$

Again, $\bar{V}$ represents the average of the potential experienced by the interacting pairs, while $|X_3\rangle, |X_4\rangle$ are the eigenvectors in the internal space of the interaction that correspond to the most attractive configuration.

In order to analyze the effect of HQSS on our predictions, we modify the above-derived interaction. First, we infer from HQSS a potential in the $J = 2^{++} D^*0\bar{D}^*0$ channel identical to that in the $X$ channel. Note the approximate character of this symmetry and the resulting hypothetical nature of the $2^{++}$ partner of the $X$. The HQSS extension of the two-body potential $V_X$ of (5) is

$$V_X^{HQSS} = V_X + V |D^*0\bar{D}^*0\rangle \langle D^*0\bar{D}^*0| .$$

Coupling between the $1^{++}$ and the $2^{++}$ channels is precluded in the two-body sector but, nevertheless, these transitions become possible in the few-$X$ sector, where these states appear as intermediate structures in the wave function.

We use the four-body case once again to exemplify how the additional interaction term leads to more attraction than expected earlier. In the basis (6), (7), the potential (13) now reads

$$\sum_{ij} V_X^{HQSS}(r; R_c) \left( |1\rangle \right) = \left( 2\bar{V} \sqrt{V} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right)$$

$$\begin{pmatrix} 2\bar{V} \\ 2\bar{V} \sqrt{V} \end{pmatrix} \left( |1\rangle \right) ,$$

(14)
whose diagonalization gives

\[ \sum_{ij} V_{X}^\text{HQSS}(r; R_c) |X'_2\rangle = (2 + \sqrt{2}) \bar{V} |X'_2\rangle, \]  

(15)

with the more attractive eigenvalue \((2 + \sqrt{2}) \bar{V}\) and eigenvector \(|X'_2\rangle\). For the six- and eight-body systems, the same assumptions and symmetries result in

\[ \sum_{ij} V_{X}^\text{HQSS}(r; R_c) |X'_3\rangle = \frac{1}{2} (11 + \sqrt{13}) \bar{V} |X'_3\rangle, \]  

(16)

\[ \sum_{ij} V_{X}^\text{HQSS}(r; R_c) |X'_4\rangle = \left(8 + \sqrt{22}\right) \bar{V} |X'_4\rangle, \]  

(17)

with eigenvectors \(|X'_3\rangle\) and \(|X'_4\rangle\) corresponding to configurations in which the potential is most attractive.

In both cases, with and without HQSS, the spectrum of a system composed of \(N/2\) \(X\) particles is given by the Schrödinger equation

\[ \left( -\sum_{i<j} \frac{\hbar^2}{2m} (\nabla_{ij})^2 + \eta \sum_{1 \leq i \leq N/2} V(r_{ij})(N/2+1) \leq j \leq N \right) \phi_N(r) = E \phi_N(r), \]  

(18)

with \(\phi_N(r) = \langle r_1, r_2, \cdots, r_N |(N/2)X\rangle\), \(\eta\) the eigenvalues of \([8]\) \((\eta = \{3, 6, 10\}\) respectively for \(2X, 3X,\) and \(4X\), \(r_{ij} = r_i - r_j\) with the index \(i(j)\) representing a charm meson (antimeson), where we have indexed the particles first and then the antiparticles, and with \(m\) being twice the reduced mass of the \(D_0^0D^{*0}\) system, \(i.e.\ m \approx 1933\ \text{MeV}\).

In practice, we solve the Schrödinger equation with the Stochastic-Variational Method (SVM \([39, 40]\)). In our implementation, this method expands the wave function in correlated Gaussian functions \((N-1) \times (N-1)\) relative Jacobi coordinates), with a non-zero interaction between the relevant pairs (meson-antimeson). We abstain from an explicit symmetrization of the spatial wave function, \(i.e.\), we do not project onto \(L = 0\) and assume that the central and parity-preserving character of the potential will produce the energetically favourable symmetric ground states in the course of the variational optimization.

**Results and conclusions:** Assuming the charm meson-antimeson interaction in the \(X\)-channel to dominate, \(i.e.\), with the average interactions \([10], [11],\) and \([12]\), we find solutions to \([18]\) of the four-body \((2X)\) and six-body \((3X)\) systems to be unbound. Adding another \(X\), we predict the eight-body \((4X)\) system bound with \(B_{4X} > 2.08\ \text{MeV}\). Including the attraction induced by HQSS, the eight-body binding energy increases to \(B_{4X}^{\text{HQSS}} > 11.21\ \text{MeV}\). Furthermore, the six-body system becomes bound with \(B_{3X} > 2.29\ \text{MeV}\). These results represent sensible lower bounds for the binding energies of the respective systems obtained at a regularization scale of about \(2\ \text{fm}\), a value deemed soft enough for an attractive three-body counterterm. Furthermore, any attraction from the non-resonant mesonic interactions (set to zero in our calculations) is expected to increase binding energies.

In figure\([1]\) we show the regulator dependence of the binding energies as a signature of the Thomas collapse. Originally this collapse is expected for (one channel) systems containing identical particles in the zero-range limit \([31]\). Here we demonstrate the occurrence of the
FIG. 1. Cutoff-radius dependence of the ground-state binding energies of few-X systems. With a resonant meson-antimeson interaction in the X channel and the \( J^{PC} = 2^{++} \) partner channel, \( 3X' \) (red square, dotted) and \( 4X' \) (blue square, dotted) clusters are bound. Solely with a resonant \( X \)-channel interaction, only the \( 4X \) (blue circle, dashed) is bound. The binding energies are proportional to \( 1/R_c^2 \) (dashed/dotted lines), and indicate a Thomas collapse of the systems. The ensuing counterterm(s) are expected to vanish within the gray-shaded area, while the total \( R_c \) range spans from the typical hadron size up to a scale set by the expected charged components of the \( X \).

collapse for a more complex system in which there is more than one channel and where a certain number of interaction pairs have been removed. A range of cutoffs over which the effect of the unenforced renormalization condition (e.g., the canonical three-body counterterm) is expected to vanish is marked in the figure (gray area).

Another effect of the reduction of resonantly interacting pairs found here is the cutoff-independent ratio between \( 4X \) and \( 3X \) energies, \( B_4/B_3 \sim 4.9 \). Compared with the ratio found in \[41\] and \[42\], \( B_4/B_3 \sim 4.6 \), we conclude that reducing the number of interacting pairs widens the gap between the energy of \( N \)- and \( (N+1) \)-boson systems. However, a single counterterm should still suffice to renormalize both systems.

In summary, we have shown how the substructure of a unitary dimer – the \( X \) – affects the spectrum of its cluster states. This spectrum differs from the one predicted for point-like bosons in the unitary limit \[42\] in an intriguing aspect. Namely, under certain assumptions about the meson-antimeson interaction, the \( X \) cluster states realize a novel generalization
of Borromean/Brunnian systems. Regardless of the enormous practical difficulties which hamper an experimental (or numerical, in the lattice) verification of our conjectures (double charm-anticharm production has only been recently achieved with the discovery of the $\Xi^{cc}_{++}$ [43] and now the fully charm tetraquark [44]), we deem the exposition of the mechanism which “delays” the formation of bound structures – the onset of binding with $4X$ and $3X$, but not necessarily with $2X$ under the assumptions we made – as a noteworthy result of the above.

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