Status of PHOKHARA

A. GRZELIŃSKA, IFJ PAN, Kraków, TAU’08

in collaboration with

H. CZYŻ, J. H. KÜHN, and A. WAPIENIK

The radiative return

$4\pi$ revisited

▶ experimental situation: $\tau$ vs. $e^+e^-$ data
▶ improved model
▶ model predictions

Narrow Resonances $J/\psi$ and $\psi(2S)$

Conclusions
THE RADIATIVE RETURN METHOD

\[ d\sigma(e^+e^- \rightarrow \text{hadrons} + \gamma(ISR)) = \]

\[ H(Q^2, \theta\gamma) \, d\sigma(e^+e^- \rightarrow \text{hadrons})(s = Q^2) \]

- measurement of \( R(s) \) over the full range of energies, from threshold up to \( \sqrt{s} \)
- large luminosities of factories compensate \( \alpha/\pi \) from photon radiation
- radiative corrections essential (NLO, ...)

High precision measurement of the hadronic cross-section at meson-factories

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From EVA to PHOKHARA

**EVA:** $e^+e^- \to \pi^+\pi^-\gamma$
- tagged photon ($\theta_\gamma > \theta_{cut}$)
- ISR at LO + Structure Function
- FSR: point-like pions
  
  [Binner et al.]

**PHOKHARA 6.0:** $\pi^+\pi^-, \mu^+\mu^-, 4\pi, \bar{N}N, 3\pi, KK, \Lambda(\to \cdots)\bar{\Lambda}(\to \cdots)$
- ISR at NLO: virtual corrections to one photon events and two photon emission at tree level
  
  [Czyż, Kühn, 2000]

- FSR at NLO: $\pi^+\pi^-, \mu^+\mu^-, K^+K^-$
- tagged or untagged photons
- Modular structure

http://ific.uv.es/~rodrigo/phokhara/

Henryk Czyż, A.G., J. H. Kühn, E. Nowak-Kubat, G. Rodrigo, A. Wapienik

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There are altogether four different channels accessible in $e^+ e^-$ annihilation and $\tau$ decays into four pions

\[ e^+ e^- \rightarrow 2\pi^+ 2\pi^- \]
\[ e^+ e^- \rightarrow 2\pi^0 \pi^+ \pi^- \]
\[ \tau^- \rightarrow \nu 2\pi^- \pi^+ \pi^0 \] sufficient to determine all four amplitudes
\[ \tau^- \rightarrow \nu 3\pi^0 \pi^- \]
Isospin relations: $4\pi$

\[
\langle \pi^+ \pi^- \pi_1^0 \pi_2^0 | J_\mu^3 | 0 \rangle = J_\mu (p_1, p_2, p^+, p^-)
\]

\[
\langle \pi_1^+ \pi_2^+ \pi_1^- \pi_2^- | J_\mu^3 | 0 \rangle = J_\mu (p_2^+, p_2^-, p_1^+, p_1^-) + J_\mu (p_1^+, p_2^-, p_2^+, p_1^-) + J_\mu (p_2^+, p_1^-, p_1^+, p_2^-) + J_\mu (p_1^+, p_1^-, p_2^+, p_2^-)
\]

\[
\langle \pi^- \pi_1^0 \pi_2^0 \pi_3^0 | J_\mu^- | 0 \rangle = J_\mu (p_2, p_3, p^-, p_1) + J_\mu (p_1, p_3, p^-, p_2) + J_\mu (p_1, p_2, p^-, p_3)
\]

\[
\langle \pi_1^- \pi_2^- \pi^+ \pi^0 | J_\mu^- | 0 \rangle = J_\mu (p^+, p_2, p_1, p^0) + J_\mu (p^+, p_1, p_2, p^0)
\]

J. H. Kühn (1999)

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Isospin relations: \(4\pi\)

\[
\int J_{\mu}^\text{em}(J_{\nu}^\text{em})^*d\Phi_n(Q; q_1, \ldots, q_n) = \frac{(Q_\mu Q_\nu - g_{\mu\nu}Q^2)}{6\pi} R(Q^2)
\]

\[
R(Q^2) = \sigma(e^+e^- \rightarrow \text{hadrons})(Q^2)/\sigma_{\text{point}}
\]

\[
\int J_{\mu}^- J_{\nu}^-^*d\Phi_n(Q; q_1, \ldots, q_n) = \frac{(Q_\mu Q_\nu - g_{\mu\nu}Q^2)}{3\pi} R^\tau(Q^2)
\]

\[
\frac{d\Gamma_{\tau\rightarrow\nu+\text{hadrons}}}{dQ^2} = 2 \Gamma_e \frac{|V_{ud}|^2 S_{\text{EW}}}{m_\tau^2} \left(1 - \frac{Q^2}{m_\tau^2}\right)^2 \left(1 + 2\frac{Q^2}{m_\tau^2}\right) R^\tau(Q^2)
\]
Isospin relations: $4\pi$

The relations between $\tau$ decay rates and $e^+e^-$ annihilation cross sections are:

$$R^\tau (- 0 0 0) = \frac{1}{2} R (+ + --)$$

$$R^\tau (- - + 0) = \frac{1}{2} [R (+ + --) + R (+ - 0 0)]$$
Isospin relations: $4\pi$; exp. situation

from the experimental side $e^+e^-$ cross section has been measured by:

$$e^+e^- \rightarrow 2\pi^+2\pi^-: \text{ BaBar, CMD2, SND}$$

$$e^+e^- \rightarrow 2\pi^0\pi^+\pi^-: \text{ BaBar(preliminary), CMD2, SND}$$

the $\tau$ data are from:

$$\tau^- \rightarrow \nu 3\pi^0\pi^-: \text{ ALEPH}$$

$$\tau^- \rightarrow \nu 2\pi^-\pi^+\pi^0: \text{ ALEPH, CLEO}$$
Isospin relations: $4\pi$; exp. situation

$\tau^- \to \pi^- 3\pi^0 \nu_\tau$

$v$ - the $\tau$ spectral function (normalization chosen by ALEPH)

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Isospin relations: $4\pi$; exp. situation

$\tau^- \to 2\pi^-\pi^+\pi^0\nu_\tau$

we included effects from the pion mass difference in the phase space

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The model

We updated the old $4\pi$ model from H. Czyż and J.H. Kühn Eur. Phys. J. C 18, 497 (2001) which was implemented to program PHOKHARA

- new and more accurate data
- new $\rho - \rho$ contributions
- properly modeled $\omega$ contributions
- new $\rho$ resonance $\rho(2040)$
The model

The amplitude used by H. Czyż, J.H. Kühn (2001) is schematically depicted:

\[
\begin{align*}
\rho & \rightarrow a_1 & \rho & \rightarrow \rho \\
\pi & \rightarrow \pi & \pi & \rightarrow \pi \\
\pi & \rightarrow \pi & \pi & \rightarrow \pi \\
\pi & \rightarrow \pi & \pi & \rightarrow \pi \\
\pi & \rightarrow \pi & \pi & \rightarrow \pi \\
\rho & \rightarrow \omega & \pi & \rightarrow \pi \\
\end{align*}
\]
The model

The new contributions from the omega part and $\rho$ mesons:

H. Czyž, J.H. Kühn, A. Wapienik (2008)
H. Czyž, A.G., J.H. Kühn, G. Rodrigo (2006)
The model

The SU(2) symmetric Lagrangian describing rho-pair production

$$\mathcal{L}_\rho = \frac{1}{4} \vec{F}_{\mu\nu} \cdot \vec{F}^{\mu\nu} + \frac{1}{2} (D^\mu \phi) \cdot (D_\mu \phi)$$

$$+ \frac{1}{2} m_\pi^2 \phi \cdot \phi + \frac{1}{2} m_\rho^2 \rho_\mu \cdot \rho^\mu$$

$$D_\mu \phi = \partial_\mu \phi + g \left( \vec{\rho}_\mu \times \vec{\phi} \right)$$

$$\vec{F}_{\mu\nu} = \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu - g \vec{\rho}_\mu \times \vec{\rho}_\nu$$
The fit

When we built our model we fitted its parameters to the existing data.

We fitted external masses $m_{\rho'}, m_{\rho''}, m_{\rho'''}$ and widths $\Gamma_{\rho'}, \Gamma_{\rho''}, \Gamma_{\rho'''}$ together with the couplings:

- 4 couplings in $a_1$- part
- 4 couplings in $f_0$- part
- 4 couplings in $\omega$- part
- 1 coupling in $\rho$- part

$$\chi^2 = 275, \quad n_{d.o.f} = 287$$
Comparing with $\tau$ data

$\tau^{-} \rightarrow \pi^{-}3\pi^{0}\nu_{\tau}$

the upper and lower curves represents error bars
Comparing with $\tau$ data

$\tau^- \rightarrow 2\pi^-\pi^+\pi^0\nu_\tau$

$\sqrt{Q^2}$ (GeV)

$v$

ALEPH

BaBar (+ Isospin)

CLEO

model

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Comparing with $\tau$ data

$$\text{Br}(\tau^- \rightarrow \nu_\tau 2\pi^- \pi^+ \pi^0)$$

| Source       | Value         |
|--------------|---------------|
| PDG06        | $(4.46 \pm 0.06)\%$ |
| Model        | $(4.12 \pm 0.21)\%$ |
| BaBar (CVC)  | $(3.98 \pm 0.30)\%$ |

$$\text{Br}(\tau^- \rightarrow \nu_\tau \pi^- \omega(\pi^- \pi^+ \pi^0))$$

| Source       | Value         |
|--------------|---------------|
| PDG06        | $(1.77 \pm 0.1)\%$ |
| Model        | $(1.60 \pm 0.13)\%$ |
| BaBar (CVC)  | $(1.57 \pm 0.31)\%$ |
Comparing with $\tau$ data

$$\text{Br}(\tau^- \rightarrow \nu_\tau \pi^- 3\pi^0)$$

| Source                  | Value             |
|-------------------------|-------------------|
| PDG06                   | $(1.04 \pm 0.08)\%$ |
| model                   | $(1.06 \pm 0.09)\%$ |
| BaBar (CVC)             | $(1.02 \pm 0.05)\%$ |
Narrow Resonances

Up to now we have two narrow resonances

\[ J/\psi \text{ and } \psi(2S) \]

in the event generator PHOKHARA

They have the following masses and widths:

\[ J/\psi \rightarrow M_{J/\psi} = 3096.916 \text{ MeV}, \quad \Gamma_{J/\psi} = 93.4 \text{ keV} \]

\[ \psi(2S) \rightarrow M_{\psi(2S)} = 3686.093 \text{ MeV}, \quad \Gamma_{\psi(2S)} = 337 \text{ keV} \]
Narrow Resonances

We put narrow resonances to the following final states:

\[ \pi^+ \pi^-, \mu^+ \mu^-, KK \]

Depends on the final states one has to take into account amplitudes:

- one-photon continuum
- one-photon annihilation
- three-gluon annihilation
  - only for kaons
Form Factors

C. Bruch, A. Khodjamirian and J.H. Kühn, Eur. Phys. J. C39(2005)41
H. Czyż, A.G. and J.H. Kühn in preparation

\[ |F_{K^+}(s)|^2 \]

\[ \sqrt{s} \text{[GeV]} \]

\[ e^+e^- \rightarrow K^+K^- \]

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$|F_\pi(s)|^2$ vs $\sqrt{s}$ [GeV]

$e^+e^- \rightarrow \pi^+\pi^-$

- DM2 1989
- OLYA 1985
- CLEO-c
- J/$\psi$
- GS
- KS

Data points and curves represent the squared modulus of the form factor $F_\pi(s)$ as a function of the center-of-mass energy $\sqrt{s}$ in the center of the $\pi^+\pi^-$ channel. The figure compares experimental data with theoretical predictions and model fits.
\[ \Delta q = 14.5 \text{ MeV the detector spread} \]

\[ e^+ e^- \rightarrow J/\psi \gamma \rightarrow \pi^+ \pi^- \gamma(\gamma) \]

\[ \sqrt{s} = 10.52 \text{ GeV} \]
$e^+ e^- \rightarrow J/\psi \gamma \rightarrow \pi^+ \pi^- \gamma(\gamma)$

$\sigma_{\text{IFSNLO}} = (2.27808 \pm 0.00013) \text{ fb}$

$\sigma_{\text{ISRNLO}} = (2.32720 \pm 0.00006) \text{ fb}$

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Relative difference of the cross sections

e^+e^- \rightarrow J/\psi \gamma \rightarrow \pi^+\pi^-\gamma(\gamma)

\sqrt{s} = 10.52 \text{ GeV}
$e^+e^- \rightarrow J/\Psi \gamma \rightarrow \mu^+\mu^- \gamma(\gamma)$

$\sqrt{s} = 10.52\text{GeV}$

$\frac{d\sigma}{d\sqrt{Q^2}}$

$\sigma(\text{IFSNLO}) = (6.8527 \pm 0.0006) \text{ pb}$

$\sigma(\text{ISRNLO}) = (6.79862 \pm 0.00008) \text{ pb}$
Relative difference of the cross sections

\[ e^+e^- \rightarrow J/\psi \gamma \rightarrow \mu^+\mu^-\gamma(\gamma) \]

\[ \sqrt{s} = 10.52 \text{GeV} \]

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Summary

- $4\pi$ channels reanalysis was performed

- isospin symmetry violation not seen

- new model proposed and implemented in PHOKHARA
Summary

- implementation $J/\psi$ and $\psi(2S)$ in PHOKHARA
  - with FSR corrections included
  - required more tests