The recent scientific literature contains an abundance of theoretical schemes for solid state quantum computation (QC). Collectively these schemes call for a quite bewildering range of physical systems. However, experimental progress has been slow because it is hard to prevent decoherence in an environment that supports controlled interactions. Universal QC requires both manipulation of individual qubits, and the ability to dictate their interactions with one another. The former can (in many cases) be implemented by electromagnetic pulses that do not cause severe decoherence. Control of interactions, however, is often supposed to be achieved by introducing some set of control electrodes. Even in systems where this is physically possible, their presence directly adjacent to qubits may constitute a dangerous source of decoherence. Moreover their use implies that the qubits are positioned at, or near, a complex patterned surface, inevitably associated with many decoherence channels. While there do exist a variety of interesting schemes that implement interaction switching using an electromagnetic process, typically these rely on either a conveniently passive interaction \[1\] or else they require directly taking the qubit out of its relatively safe subspace and into a space where it is subject to decay \[2\].

All-optical manipulation of atomic systems has provided the basis for some of the most successful experiments demonstrating aspects of QC \[4\,5\], and here we shall adapt these ideas for use in solid state systems. Our approach is a generalization of the barrier scheme of Benjamin and Bose \[6\,7\], which supported perpetual spin coupling by introducing a strong tuning of the single-qubit Zeeman splitting. To perform such a tuning directly would require the equivalent of a qubit-by-qubit B-field modulation, a serious experimental challenge. Other comparable schemes for perpetual coupling have equally significant drawbacks, such as the need for interaction switching during initialization \[10\]. Here we are able to dispense with these issues.

We shall begin by considering the smallest structure capable of realizing a qubit-qubit gate. There are three adjacent systems in a linear arrangement, with a permanent strong interaction existing between neighbors. Each system has two stable levels \([0]\) and \([1]\), and the central system has an additional third state \([T]\). In (a) we suppose that the outer units represent qubits \(X\) and \(Y\). The central system is acting as a barrier: in the simplest case it is 'shelved' in state \([T]\), and is therefore far off-resonance from its neighbors. In practice the laser cycling mechanism depicted in Fig. 2 can form a superior barrier. In either case, when we wish to perform a gate operation we simply allow the central system into its \([0]\) and \([1]\) subspace.

![FIG. 1: (a)-(c) The simplest structure capable of demonstrating a qubit-qubit gate. There are three adjacent systems in a linear arrangement, with a permanent strong interaction existing between neighbors. Each system has two stable levels \([0]\) and \([1]\), and the central system has an additional third state \([T]\). In (a) we suppose that the outer units represent qubits \(X\) and \(Y\). The central system is acting as a barrier: in the simplest case it is 'shelved' in state \([T]\), and is therefore far off-resonance from its neighbors. In practice the laser cycling mechanism depicted in Fig. 2 can form a superior barrier. In either case, when we wish to perform a gate operation we simply allow the central system into its \([0]\) and \([1]\) subspace. Under free evolution (b) it initially becomes entangled with \(X\) and \(Y\), but 'revives' to a product state at a certain time later, allowing the barrier to be restored (c). The net effect is a unitary entangling gate on \(X\) and \(Y\). Lower figure: (d) An \(ABAB\) pattern can suffice for a scalable architecture, but if global control \(S\) is desired then a repeating set of four distinct systems \((e)\) is appropriate.]

Optical Quantum Computation with Perpetually Coupled Spins

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The possibility of using strongly and continuously interacting spins for quantum computation has recently been discussed. Here we present a simple optical scheme that achieves this goal while avoiding the drawbacks of earlier proposals. We employ a third state, accessed by a classical laser field, to create an effective barrier to information transfer. The mechanism proves to be highly efficient both for continuous and pulsed laser modes; moreover it is very robust, tolerating high decay rates for the excited states. The approach is applicable to a broad range of systems, in particular dense structures such as solid state self-assembled (e.g., molecular) devices. Importantly, there are existing structures upon which ‘first step’ experiments could be immediately performed.
tonian reads $H_{\text{comp}} = H_Z + H_{\text{int}}$, where ($\hbar = 1$)

$$H_Z = \sum_{j=1}^{3} E_j \sigma_j^Z,$$

$$H_{\text{int}} = \sum_{j=1}^{2} J_Z \sigma_j^Z \sigma_{j+1}^Z + J_{XY} (\sigma_j^X \sigma_{j+1}^X + \sigma_j^Y \sigma_{j+1}^Y).$$

The $E_j$ are (time-independent) Zeeman energies and we have chosen a general (uniaxial) anisotropic interaction, which has both the planar $XY$ interaction ($J_Z = 0$) and the isotropic Heisenberg interaction ($J_Z = J_{XY}$) as limits. The simulations presented below will employ the $XY$ limit, but we have verified that the same qualitative behavior is seen for any $J_Z$ of order $J_{XY}$.

We now suppose that there is an additional level available in the central system [11]. This state has an allowed transition to one of the system’s qubit states; for clarity of exposition we assume the transition is to state $|0\rangle$. We assume the transition energy $\hbar \omega_{RT} \gg kT$, where $T$ is the device’s operating temperature. An on-resonance classical laser field will excite the transition via $H_{\text{laser}} = \Omega \cos(\omega_{RT} t) (|T\rangle \langle 0| + |0\rangle \langle T|)$, with $\Omega$ denoting the Rabi frequency.

In the absence of the laser field, state $|T\rangle$ cannot become populated, and we effectively have three coupled two-level systems (Fig. 1). Suppose that at some instant the left and right-hand hands each represent a qubit, $X$ and $Y$, and the central system is in state $|1\rangle$. When this initial state evolves under $H_{\text{comp}}$, we find that the three systems become mutually entangled. However, at a time $t_R = \pi \hbar (8 J_{XY}^2 + J_Z^2)^{-1/2}$ the central system ‘revives’ [2] to state $|1\rangle$. At this instant, the transformation in the computational basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ is given by (neglecting a global phase)

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & i Q s & Q c & 0 \\ 0 & Q c & i Q s & 0 \\ 0 & 0 & 0 & W \end{pmatrix}. \tag{2}$$

Here $Q = -\exp(i \phi)$, $s = \sin(\phi)$, $c = \cos(\phi)$ and $W = -\exp(-2i \phi)$, with $\phi = \frac{\pi}{2} (1 + 8 J_{XY}^2 / J_Z^2)^{-1/2}$. $U$ is an entangling two-qubit gate, which together with arbitrary single qubit operations constitute a universal gate-set [3, 13]. The difficulty is that we must also be able to passivate the device by effectively decoupling the central barrier before and after this free evolution. One might consider simply ‘shelving’ the barrier into state $|T\rangle$ - however this would require a high fidelity $\pi$ pulse and, moreover, the third state would have to be stable (i.e., have a comparable lifetime to the $|0\rangle, |1\rangle$ states). Typical real systems will not meet this condition. However, as we shall now show, a rapid cycling of the central system to the excited state $|T\rangle$ can create an excellent barrier to qubit interaction, even if $|T\rangle$ is unstable. Thus the passive state [12] of the device is to have the laser ON, and a gate is performed simply by pausing the illumination for the period $t_R$.

In order to take account of the decay of $|T\rangle$, we have employed two different approaches. The first is a stochastic quantum jump method: the total system’s density matrix evolves coherently under $H = H_{\text{comp}} + H_{\text{laser}}$, except that at random times the population of $|T\rangle$ collapses to $|0\rangle$. The second approach is the Lindbladian formulation of the system’s dynamics [14]:

$$\dot{\rho} = -i [H, \rho] + \frac{1}{2} \gamma (2 \sigma^- \rho \sigma^+ - \sigma^+ \sigma^- \rho - \rho \sigma^+ \sigma^-). \tag{3}$$

$\sigma^+$ and $\sigma^-$ are the raising and lowering operators for the decaying transition, and $\gamma$ is the decay rate. We find that the two approaches are in excellent agreement [15].

Let us first consider a single qubit-bearing system coupled to a single barrier system, i.e., a two-level system coupled to a single three-level system (Fig. 2 schematic). This is the simplest system with which one could perform first ‘proof of principle’ experiments. In the nu-

![FIG. 2: Fidelity of the decoupling process when the two-spin system depicted in the schematic is subjected to laser excitation. Time is in units of $1/J_{XY}$, and we have set $\omega_{RT} = 1000 J_{XY}$, $\omega_{RT} = 1000 J_{XY}$, and the Rabi frequency $\Omega = 40 J_{XY}$. (a) Simulation of continuous laser excitation using Eq. (1). Successive lines show the effect of progressively greater decoherence rate $\gamma$ (in units of $1/J_{XY}$), and graph (b) ‘zooms’ on the high fidelity region. For $\gamma$ values less than $\Omega$, i.e. decay weaker than the laser excitation rate, the decoupling is successful. In contrast, the extremely high $\gamma$ plots expose the interesting phenomenon of effective laser deactivation, as described in the text. In (c) we show the same numerical simulation, but now using the stochastic quantum jumps model for the decay. The agreement with (a) is seen to be very good. Plot (d) shows the effect of a pulsed laser, with other parameters as in (a). We apply a repeating pattern of brief intense laser excitation followed by a longer dark period, such that the average laser power is constant. We plot the effect when the pulse duration is chosen to effect either a $\pi$, $2\pi$ or $4\pi$ rotation on the $\{|0\rangle, |T\rangle\}$ Bloch sphere.

$\pi$ & $2\pi$ at $\gamma=0$
steady at $\gamma=10$
$\pi$ & $2\pi$ at $\gamma=100$
$4\pi$ at $\gamma=0$
merical simulations depicted in Fig. 2, we first initialize the barrier system into state $|1\rangle$, and the qubit-system in superposition $|\text{init}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, so that the initial product state is $|1\rangle \otimes |\text{init}\rangle$. In the absence of a laser, the interaction $\hat{H}_{\text{int}}$ would cause the $|10\rangle$ element of the superposition to exchange with $|01\rangle$, thus entangling barrier and qubit. To measure the fidelity with which we can prevent this entanglement, we calculate $\mathcal{F}(t) \equiv \langle \text{ideal} | \hat{\rho}_Q(t) | \text{ideal} \rangle$. Here $| \text{ideal} \rangle = \frac{1}{\sqrt{2}}(|0\rangle + \exp(-i\omega_0 t)|1\rangle)$, i.e., the state for a perfectly isolated qubit, and $\hat{\rho}_Q(t)$ is the qubit reduced density matrix. Our choice of $|\text{init}\rangle$ means that if either the phase or amplitude of the ‘frozen’ qubit deviate, then this is reflected in the fidelity measure. Adopting an equal superposition for $|\text{init}\rangle$ is of course an arbitrary choice, however we have confirmed other initial qubit states do not lead to significantly inferior fidelity.

In Fig. 2(a) the uppermost plot shows the simplest case of continuous laser, zero decay ($\gamma = 0$). The fidelity remains close to unity at all times. This can be explained by transforming the system to the dominant rotating frame, from which analysis one would expect $\mathcal{F} = 1 - 4(\Omega^2/J^2)\sin(\Omega t/2)$. For values of $\gamma$ within an order of magnitude of the laser Rabi frequency, we see that fidelity follows the approximate trend $\mathcal{F} \approx \frac{1}{2}[1 + \exp(-t/\tau)]$ with $\tau \approx 20\Omega/\gamma$. Given that in real systems it should be possible to excite the state far faster than its spontaneous decay, this implies an excellent degree of decoupling. However initial experiments may need to work with very non-ideal systems and so we also show the effect of large $\gamma$ values. For $\gamma \gg \Omega$ the fidelity becomes oscillatory, and ultimately matches the case of zero decay, zero laser power. This can be understood by regarding the action of the decay as a quantum Zeno-effect [10] which inhibits the laser transition $|0\rangle \rightarrow |T\rangle$. If a very rapid decay mechanism could be switched, then one could elect to leave the laser on permanently and turn ‘on’ the decay for a time $t_d$ to create a perfect gate. This observation serves to underline the way that our scheme separates decoherence of $|T\rangle$ from decoherence of the qubit. Of course, the $X$ and $Y$ qubits will suffer the “usual” decoherence mechanisms that spins in solid state systems are subject to (except that we have done away with the need for gating electrodes, and indeed the need to be near a surface, and so certain mechanisms are removed). For our purpose the crucial criterion is that the native qubit lifetime be far longer than the characteristic timescale of our barrier mechanism, so that we can indeed neglect that lifetime in our analysis. Such systems exist, as we discuss later.

In Fig. 2(d) we depict the effect of a laser in pulsed mode. Although we have demonstrated in the above analysis that we do not require pulses to perform our decoupling, it is nevertheless interesting to investigate their effect, since a first-step experimental study may well use a laser in this mode. We show the effect of a train of $\pi$, $2\pi$ or $4\pi$ pulses. Both the $\pi$ and $2\pi$ pulse trains maintain fidelity well, being slightly inferior to a continuous laser with the same average power, while the $4\pi$ pulses have no effect. The mechanisms by which these pulses cause decopling are outlined in Fig. 3(a). Of course, a pulsed laser must necessarily contain a band of frequency components and so has a broader lineshape. In our simulations, the pulses we use are typically of duration $t_{\text{pulse}} = 0.1/J_{XY}$, which would imply a minimum laser bandwidth of order $10 J_{XY}$. Any other energy levels which can be excited by the laser must be separated from the levels we are exploiting by at least this amount if the pulsed mode is to work.

Having examined the laser-driven decoupling process for the two-unit system (Fig. 2), we confirmed that precisely the same decoupling effect occurs in the full three part qubit-barrier-qubit system [17].

We now address the question of how our scheme could be realized in a specific system. A very promising possibility is to embody the qubit in two Zeeman sublevels of a spin active nanostructure, where $|T\rangle$ is a higher state accessed by a laser of optical frequency. Pauli blocking [2, 3] can lead to a spin-selective optical excitation of excitons in quantum dots (QDs) with an excess spin. Laser coupling to excitons in arsenide QDs can be large enough to induce optical Rabi oscillations with a period of a few picoseconds [18, 19], which is at least three orders of magnitude longer than typical decoherence times in similar structures [21, 22]. This is more than sufficient for our scheme to work with extremely high fidelity.

Spin-spin interactions in semiconductor QDs can take several forms. In certain structures, they might be due to the direct exchange mechanism, whose strength has been measured at 0.1 meV in lithographically defined dots separated by about 200 nm [22]. Alternatively, they might be mediated by conduction electrons (the RKKY interaction); suppression of Kondo resonances by this mechanism has been observed recently [23]. These experiments suggest that gates could be performed on the sub-

![FIG. 3: An explanation of the effect of fast pulses seen in Fig. 2(d). We depict the Bloch sphere for the qubit-barrier subspace $\{|10\rangle, |01\rangle\}$. The XY interaction alone would simply cause a rotation between these states - this is shown by the pole-to-pole circle on the spheres. We suppose the initial state is $|01\rangle$, then after some small time $\delta t$ we apply a rapid pulse to the barrier system. (a) A $2\pi$ pulse takes component $|10\rangle$ to $|1T\rangle$ and back, acquiring a net phase of $-1$ so that we ‘short circuit’ the Heisenberg cycle which then returns us $|01\rangle$. (b) The effect of a $\pi$ pulse plus subsequent decay: provided $\delta t \ll 1/J_{XY}$ then the effect is a beneficial Zeno-type suppression [16] of the $|01\rangle \rightarrow |10\rangle$ transition.](image-url)
nanosecond timescale, which is far shorter than measured spin relaxation times of more than 50 ns [24]. Though the lithographic dots offer a promising route to spin based QIP, we think that a first experiment testing our idea would use a pair of self assembled, Stranski-Krastanow dots, which have ideal optical properties. The excess electron spins are provided by n-doping; layers of such dots have been fabricated, with an average of one electron per dot, and spin polarization effects have been observed [26] and spin lifetimes of 20 ns have been measured. Polarized photoluminescence (PL) measurements may be used to verify the required electronic structure, and by monitoring how the positions of the lines in PL spectra move when a tuned laser is applied to the ‘blocks’ transition [30].

We could obtain a much stronger coupling between our qubits by using molecular structures. For example, the spacing between spin-active endohedral fullerenes can be as small as 1 nm. Such structures also have extremely long spin decoherence times of 50 µs [28] and exhibit the magneto-optical activity which demonstrates the presence of the required spin-dependent optical transition [30].

Single qubit operations can be performed very efficiently if the qubit units also possess a higher state (|3⟩). Further, if a laser could be targeted to specific qubits, our device could be scaled by constructing a simple repeating pattern like that of Fig. 1(d). Alternatively, one could employ the repeating ABCD arrangement of Fig 1(e). Here there are two types of qubit system and two types of barrier, where the distinction is in the ω0T transition frequencies. Such a structure supports established global control protocols [8], making it suitable for molecular-scale structures where laser targeting of individual elements is impossible.

Ultimately, for a large scale device capable of outperforming classical machines, one would look for suitable ordered two- or three-dimensional arrays [31]. Perfectly ordered 2D metallic dot arrays, with alternating dot species, have been achieved [32]. These could either act as a template for further structures, or be directly charged to create atom-like states suited to our protocol. Engineered assemblies of CdSe nanocrystal QDs might also be used; these have exhibited energy transfer on time scales of tens-of-picoseconds, while intrinsic exciton lifetimes are > 20 ns [33]. In a basis where the presence or absence of an exciton is the pseudospin, this interaction has the desired XY form. A similar interaction also exists in photosynthetic bio-molecular complexes, which can exhibit energy transfer times of 0.3 ps [33]. These structures have recently been demonstrated to be amenable to coherent quantum control [34] and could also constitute an interesting implementation scenario.

In summary, we have shown that it is possible to perform scalable all-optical QC using a solid state array of continuously and strongly interacting spins. The mechanism we employ is extremely robust and can operate with highly unstable states. Our detailed simulations have focused on the small scale structures, composed of two or three spins, that can be created with existing techniques. We have outlined ‘proof-of-principle’ experiments that could be undertaken in the immediate future.

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