IMPLICATION OF $\Omega_m$ THROUGH THE MORPHOLOGICAL ANALYSIS OF WEAK LENSING FIELDS

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ABSTRACT

We apply the morphological descriptions of a two-dimensional contour map, the so-called Minkowski functionals (the area fraction, circumference, and Euler characteristics), to the convergence field $\kappa(\theta)$ of the large-scale structure reconstructed from the shear map produced by the ray-tracing simulations. Using the perturbation theory of structure formation, it has been suggested that, with respect to the threshold in the weakly nonlinear regime, the non-Gaussian features on the Minkowski functionals are induced by the three skewness parameters of $\kappa$ that are sensitive to the density parameter of matter, $\Omega_m$. We show that, in the absence of noise due to the intrinsic ellipticities of source galaxies with which the perturbation theory results can be recovered, the accuracy of the $\Omega_m$ determination is improved by $\sim 20\%$ using the Minkowski functionals compared with the conventional method of using the direct measure of skewness.

Subject headings: cosmology: theory — gravitational lensing — methods: numerical

1. INTRODUCTION

Weak gravitational lensing caused by the large-scale structure of the universe distorts the images of distant galaxies. This phenomenon is the so-called cosmic shear, which offers us the unique opportunity to measure directly the projected power spectrum of dark matter fluctuations regardless of the relation between the dynamical states of dark matter and luminous matter (Blandford et al. 1991; Miralda-Escudé 1991; Kaiser 1992). Recently, several independent measurements of cosmic shear have been made from deep “blank-field” CCD imaging surveys, which reported significant detections of shear variance (Van Waerbeke et al. 2000; Wittman et al. 2000; Bacon, Refregier, & Ellis 2000; Kaiser, Willson, & Luppino 2000; Maoli et al. 2001).

Due to the nonlinear evolution of the density fluctuation field in the large-scale structure, the cosmic shear field on a small angular scale is expected to display significant non-Gaussian features. Even in this case, for the second-moment analysis it has been shown that the numerical results from the ray-tracing simulations are in remarkably good agreement with the theoretical predictions using the fitting formula for the nonlinear matter power spectrum (Jain, Seljak, & White 2000; Hamana, Martel, & Futamase 2000b). On the other hand, the higher order statistics can provide additional cosmological information associated with the non-Gaussian features. Especially, the normalized skewness parameter of the convergence field can be a sensitive indicator of the density parameter of matter, $\Omega_m$ (Bernardeau, Van Waerbeke, & Mellier 1997). Unfortunately, however, the highly nonlinear evolution of third-order statistics cannot be described simply by the fitting formula for the nonlinear power spectrum alone. Recently, the extended method that allows us to perform the skewness calculations in the strongly nonlinear regime has been developed using “hyper-extended perturbation theory” (HEPT; Hui 1999; Scoccimarro & Frieman 1999). Nevertheless, several works using the ray-tracing simulations have revealed that the value predicted by HEPT at relevant angular scales does not agree so well with the numerical results of the skewness parameter (Jain et al. 2000; White & Hu 2000; Hamana et al. 2000a). Moreover, we would like to stress that it is difficult to find a physical meaning for the fitting formula beyond the empirical one. Therefore, it will be worth exploring a new method for effectively extracting the non-Gaussian features of the convergence field in the weakly nonlinear regime based on the perturbation theory, which relies on a more firm physical basis for structure formation.

A possible method that we propose is the use of the Minkowski functionals with respect to the level threshold; this is motivated by the fact that the functionals give the complete morphological descriptions of a considered field (Schmalzing & Buchert 1997). For a two-dimensional case, the Minkowski functionals consist of the area fraction, circumference, and Euler characteristics of the isocontour curves, where the Euler characteristics are equivalent to the genus statistics often used in cosmology (Gott, Melott, & Dickinson 1986). Recently, Matsubara & Jain (2000) applied the genus curve to the convergence field reconstructed from the ray-tracing simulations and found that the nonlinear evolution of the convergence induces a deviation from the specific curve of the genus for the Gaussian case. On the other hand, the theoretical predictions based on the perturbation theory have shown that the non-Gaussian features of the Minkowski functionals are completely characterized by the skewness parameters of the convergence field in the weakly nonlinear regime (Matsubara 2000; also see eq. [1]). These results offer the possibility of extracting the skewness parameters using the Minkowski functionals of the reconstructed convergence field. The purpose of this Letter is thus to investigate how accurately $\Omega_m$ can be determined from the skewness parameters estimated by fitting the numerical results to theoretical predictions of the Minkowski functionals.

2. THE RAY-TRACING SIMULATION AND THE MINKOWSKI FUNCTIONALS

We use shear and convergence fields modeled from the ray-tracing simulations through the dark matter distribution of $N$-body simulations following the previous methods by Hamana et al. (2000b) and White & Hu (2000). The original $N$-body simulations of the large-scale structure were performed with the P$^3$M code (for details, see Jing & Fang 1994 and Jing 1998). The following discussions focus on two cosmological models, summarized in Table 1, and we used three different realizations for each model. As for the power spectrum of matter fluctuations, we assume the cold dark matter (CDM) model with the transfer function given by Bardeen et al. (1986) and the shape parameter 1 Astronomical Institute, Tohoku University, Sendai 980-8578, Japan.
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\[ \Gamma = \Omega_c h. \] All the simulations employ 256^3 (\approx 17 \text{ million}) particles in a (100 h^{-1} \text{ Mpc})^3 comoving box and start at redshift \( z_c = 36 \). The gravitational softening length \( \epsilon \) is 39 h^{-1} \text{ kpc}. We use the multiple lens plane algorithm to follow the propagations of light rays through the simulated matter distributions. In this algorithm, the matter content of each box at a certain redshift is projected onto a single plane perpendicular to the line of sight. We use typically \( \sim 20 \) equally spaced lens planes in the comoving distance between source and observer. The particle positions on each plane are interpolated onto a grid of size 2048^2. In order to avoid possible correlations between different lens planes, in each plane we choose one of the three realizations at the considered redshift and then project the mass distribution along a randomly chosen one of the three coordinate axes, translates the mass distribution by a random vector, and randomly rotate it in a unit of \( \pi/2 \). We consider a set of lens planes between the source and the observer as a different realization and use 10 such realizations to estimate the cosmic variance associated with the measurements of weak lensing fields. Further details of the ray-tracing simulation are given in Hamana et al. (2000b).

The fields that we use are 3\(^\circ\) on a side. Each light ray is traced by the Born approximation and hence can be handled as a straight line that radially extends from the observer. Throughout this Letter, we assume that all source galaxies are at a redshift of \( z_s = 1 \) and that their number densities are \( n = 30 \text{ arcmin}^{-2} \). We then make the cosmic shear field, \( \gamma(\vec{r}) \), on each grid from the ray-tracing simulations and perform the smoothing on \( \gamma(\vec{r}) \) by using a top-hat filter. Using the relation between the Fourier transforms \( k(\vec{r}) \) and \( \gamma(\vec{r}) \), \( k(\vec{r}) = [(l_x^2 - l_y^2)\gamma_x(l) + 2l_xl_y\gamma_y(l)]/l^2 \), and assuming the periodic boundary condition, \( k(\vec{r}) \) is reconstructed on each grid from the cosmic shear field, where \( k(\vec{r}) \) is the convergence field. Figure 1 shows examples of the reconstructed convergence field. To compute the Minkowski functionals, we label the convergence field by the threshold value \( \nu(\vec{r}) \) that is defined by \( \nu(\vec{r}) = k(\vec{r})/a_0 \), where \( a_0 \) is the rms of \( k \) defined by \( a_0^2 = \langle \kappa^2 \rangle \).

In a two-dimensional case, the Minkowski functionals are the area fraction \( v_o(\nu) \), circumference \( v_i(\nu) \), and Euler characteristics \( v_2(\nu) \) for the isocontour curve with threshold \( \nu \) that fully characterize the morphology of the field. The Euler characteristic is a purely topological quantity, which is equal to the number of isolated high-threshold regions minus the number of isolated low-threshold regions. To calculate the Minkowski functionals for the reconstructed convergence field given as pixel data, we employed the method developed by Winitzki & Kosowsky (1997).

On the other hand, under the hypothesis that the initial perturbations are Gaussian, as supported by the inflationary scenario, Matsubara (2000) recently derived the analytical formula of the Minkowski functionals based on the perturbation theory that can be applied to the weakly nonlinear convergence field.

\[ v_o(\nu) = \frac{1}{2} \text{erfc} \left( \frac{\nu}{\sqrt{2}} \right) + \frac{1}{6\sqrt{2\pi}} \frac{e^{-\nu^2/2}}{a_0s_0H_2(\nu)}, \]

\[ v_i(\nu) = \frac{1}{8\sqrt{2}} \frac{a_0}{s_0} e^{-\nu^2/2} \left\{ 1 + \frac{s_0}{6} \left[ \frac{s_1}{3} H_1(\nu) + \frac{s_1}{3} H_2(\nu) \right] \right\}, \]

\[ v_2(\nu) = \frac{1}{2\sqrt{2\pi^3}} \frac{a_0^2}{s_0} e^{-\nu^2/2} \times \left\{ H_1(\nu) + \frac{s_0}{6} \left[ \frac{s_1}{3} H_1(\nu) + \frac{s_1}{3} H_2(\nu) + \frac{s_1}{3} \right] \right\}, \]  

where \( a_0 \) is defined by \( a_0^2 = \langle \kappa^2 \rangle / \sigma_\kappa^2 \), and \( H_n(\nu) \) is the \( n \)-th order Hermite polynomial; \( s_0, s_1, \) and \( s_2 \) denote the skewness parameters defined by \( s_0 = \langle \kappa^3 \rangle / \sigma_\kappa^3 \), \( s_1 = -3/2 \langle \kappa^2 (\nabla^2 \kappa) / \sigma_\kappa^3 \rangle \), and \( s_2 = -3 \left( \langle \nabla \kappa \cdot \nabla \kappa (\nabla^2 \kappa) / \sigma_\kappa^4 \right) \), respectively, where the quantity \( s_0 \) is the skewness parameter conventionally used in the previous work on weak lensing. Equation (1) indicates that those skewness parameters can be new statistical indicators of the deviations from the specific Gaussian predictions of \( v_o(\nu) \), \( v_i(\nu) \), and \( v_2(\nu) \) with \( s_0 = s_1 = s_2 = 0 \). It should be noted that \( s_0, s_1, \) and \( s_2 \) themselves can be given as functions of the cosmological parameters for the CDM model and the smoothing scale of the top-hat filter (Bernardeau et al. 1997), which reveals that the skewness parameters are particularly sensitive to \( \Omega_m \). Therefore, we propose that comparing the theoretical predictions (eq. [1]) with their numerical (or observed) results could place constraints on the cosmological parameters. In some previous work (e.g., Matsubara & Jain 2000), the area fraction labeling the Minkowski functionals has been used instead of the threshold in order to cancel out the horizontal shift of those functionals that is due to the nonlinear evolution of the underlying density fluctuations on the high-threshold side. However, this operation merely means that the area function \( v_o(\nu) \) for the non-Gaussian field is transformed closer to its specific curve for the Gaussian case. For this reason, we do not employ the operation and simply use the threshold \( \nu \).

3. RESULTS

Figure 2 shows both the analytical and numerical results of the area fraction \( v_o(\nu) \) (left panels), circumference \( v_i(\nu) \) (middle panels), and Euler characteristics \( v_2(\nu) \) (right panels) per square arcminute as a function of the threshold \( \nu \) for the convergence
fields with two different smoothing scales of $\theta = 1'$ (upper panels) and $\theta = 8'$ (lower panels), respectively. In those plots, normalizations of the analytical predictions, except $v_0(\nu)$, are determined by minimizing the $\chi^2$-value for the fitting between the predictions (eq. [1]) and the numerical results. The mean values and error bars in each bin of $\nu$ are estimated from the 10 different realizations with an area of $3 \times 3$ deg$^2$, and the error corresponds to the cosmic variance associated with the measurements of the Minkowski functionals. The non-Gaussian features of the functionals for the noise-free convergence field are due to the nonlinear gravitational clustering; at negative $\nu$, it has a cutoff related to the minimum $k$ resulting from empty beams, and it has a tail at positive $\nu$ because of the collapsed halos. For the small smoothing scale of $\theta = 1'$, there are large differences between the analytical predictions and the numerical results. This is because the highly nonlinear evolution of the density field has a large effect on the convergence field. For the large smoothing scale of $\theta = 8'$, on which the convergence field is expected to be in a weakly nonlinear regime, the numerical results are broadly consistent with the analytical predictions. Note that the reason that the result of $\theta = 8'$ has larger error bars than that of $\theta = 1'$ is due to the fewer number of statistical samples.

Figure 3 shows the values of $s_{0}$, $s_{1}$, and $s_{2}$ calculated by the perturbation theory, the direct measurement of $s_{0}$ (upper left panel) from the reconstructed convergence field, and the estimations of $s_{0}$ (upper right panel), $s_{1}$ (lower left panel) and $s_{2}$ (lower right panel) obtained from the $\chi^2$ fitting between the theoretical predictions (eq. [1]) of the Minkowski functionals and their simulation results. Here we have used only the simulation data in the range of $-1.5 \leq \nu \leq 1.5$ because we expect the convergence field in this range is still in the weakly nonlinear regime and therefore can be applied to the perturbation theory predictions. In these figures, assuming that the survey of weak lensing is performed over an area of $9 \times 9$ deg$^2$, we estimated the error bars by multiplying the variance directly obtained from the 10 realizations as shown in Figure 2 by a factor of $\frac{1}{2}$. We have confirmed that the measurement of the Euler characteristics $v_0$ is also sensitive to the discreteness effect of pixel data. Therefore, to minimize the unresolved uncertainties, we determined the parameters of $s_{0}$, $s_{1}$, and $s_{2}$ in the following procedure. First, we determine $s_{0}$ from the fitting of $v_0(\nu)$ because the non-Gaussian features of $v_0(\nu)$ in the theoretical prediction (eq. [1]) depend on $s_{0}$ and $s_{1}$, where $s_{0}$ is also computed directly from the reconstructed convergence field according to the definition $s_{0} = \langle k^2 \rangle$. Similarly, by using the already determined value of $s_{0}$, we determine $s_{1}$ from the shape of $v_0$. Finally, we use the shape of the Euler characteristics $v_0(\nu)$ to determine the $s_{2}$ parameter. Note that this fitting procedure causes the large error of $s_{2}$. The upper left panel in Figure 3 shows that for all smoothing scales, the direct measurement of $s_{0}$ tends to overestimate significantly the value of $s_{0}$ calculated by the perturbation theory. This is because the direct measurement is more sensitive to the strong nonlinear rare events in the convergence distribution, such as the halos of dark matter. On the other hand, for the standard CDM (SCDM) model with $\theta = 2'$, $4'$, and $8'$, the values of $s_{0}$ obtained from our method using $v_0(\nu)$ fairly improve the estimations for $s_{0}$ predicted by the perturbation theory. For comparison, the thin lines in the upper left panel of Figure 3 show the direct measurement of $s_{0}$ in the same range of $\nu$ ($-1.5 \leq \nu \leq 1.5$) as used in our method. It is still clear that the modified direct measurement of $s_{0}$ also fails to predict its value from the perturbation theory for all the smoothing scales. Similarly, the values of $s_{1}$ obtained from our method are very similar to the values of $s_{1}$ from the perturbation theory for the smoothing scales of $\theta \simeq 2'$. However, one can see that the result of $s_{2}$ obtained from our method cannot reproduce the value of $s_{2}$ from the perturbation theory mainly because of the fitting procedure described above, and the results of $s_{0}$ and $s_{1}$ for the smallest smoothing scale of $\theta = 1'$ do not work well. For these reasons, we will not use the results of $s_{0}$ and $s_{1}$ for $\theta = 1'$ and
of $s_2$ for the determination of $\Omega_m$. On the other hand, it apparently seems that the errors of the skewness determinations for the low-density CDM model with cosmological constant ($\Lambda$CDM) are larger than those of the SCDM model. This result comes from the fact that the skewness variation $\Delta s_0 = 10$ for the flat universe models around the $\Lambda$CDM model corresponds to $|\Delta \Omega_m| = 0.05$, while $\Delta s_0 = 0.9$ around the SCDM model corresponds to the same $|\Delta \Omega_m|$. Actually, as will be shown, the relative accuracy of the $\Omega_m$ determination is not so different in both the SCDM model and the $\Lambda$CDM model.

Table 2 summarizes the results for the $\Omega_m$ determination with a best-fit value and a 1 $\sigma$ error, which are obtained from the direct measurements of $s_0$ and from the estimations of $s_0$ and $s_2$ using the Minkowski functionals for the smoothing scales of $\theta = 2^\circ$, $4^\circ$ and $8^\circ$, respectively. Here we employed the current favored flat universe models with $\Omega_m + \Omega_{\Lambda} = 1$. The table clearly shows that our method improves the accuracy of the $\Omega_m$ determination by $\sim 20\%$ compared with that determined from the direct measure of skewness.

4. DISCUSSION

In this Letter, we addressed the issue of how accurately the density parameter $\Omega_m$ can be determined from the non-Gaussian signatures in the simulated weak lensing field based on the perturbation theory of structure formation instead of the empirical fitting formula. For this purpose, we have shown that the Minkowski functionals of convergence maps reconstructed from the cosmic shear field can be a useful new method. This is because the Minkowski functionals can effectively pick up the weakly nonlinear non-Gaussian features in the appropriate range of thresholds, in which the perturbation theory can be safely applied. In fact, our numerical results have shown that the $\Omega_m$ determination of using the Minkowski functionals produces an $\sim 20\%$ accurate best-fit value to the input value of $\Omega_m$ compared with the result of using the direct measurement of skewness. However, we still have to investigate further the possible uncertainties that are due to the limited number of numerical realizations used in this Letter by increasing the number, and this will be our future work.

In this Letter, we have not considered the effect of the intrinsic ellipticities of source galaxies on our method. Nevertheless, for practical purposes, it is critical to take into account this effect, and therefore we will need the theoretical predictions of the Minkowski functionals, including the noise effect. This study is now in progress and will be presented elsewhere. In practice, it will also be necessary to take into account the redshift distribution of source galaxies. However, previous works have quantitatively shown that, even when using a more realistic model for the redshift distribution of source galaxies as expressed by $n(z) \propto z^2 \exp\left[-(z/z_0)^2\right]$ with the mean redshift of unity, the magnitude of the cosmic shear signal is changed by only $\sim 10\%$ compared with the result of using all the sources distributed at $z_0 = 1$ (e.g., Jain et al. 2000). Therefore, we propose that changing the source distribution does not largely affect our results.

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TABLE 2

| Model          | $\Omega_m^a$ | $\Omega_m^c$ |
|----------------|--------------|--------------|
| SCDM ($\Omega_m = 1.0$)         | $0.50 \pm 0.19$ | $0.78 \pm 0.22$ |
| $\Lambda$CDM ($\Omega_m = 0.3$) | $0.24 \pm 0.05$ | $0.31 \pm 0.07$ |

$^a$ Estimated from the direct measurement of $s_0$ and from the measurements of $s_0$ and $s_2$ through the fitting of Minkowski functionals for simulation data with $\theta = 2^\circ$, $4^\circ$, and $8^\circ$. Here we employed the flat-universe models with $\Omega_m + \Omega_{\Lambda} = 1$.

$^b$ From the direct measure of $s_0$.

$^c$ From Minkowski functionals.