Shortcuts to adiabaticity in non-Hermitian quantum systems without rotating-wave approximation

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Abstract: The technique of shortcuts to adiabaticity (STA) has attracted broad attention due to their possible applications in quantum information processing and quantum control. However, most studies published so far have been only focused on Hermitian systems under the rotating-wave approximation (RWA). In this paper, we propose a modified shortcuts to adiabaticity technique to realize population transfer for a non-Hermitian system without RWA. We work out an exact expression for the control function and present examples consisting of two- and three-level systems with decay to show the theory. The results suggest that the shortcuts to adiabaticity technique presented here is robust for fast passages. We also find that the decay has small effect on the population transfer in the three-level system. To shed more light on the physics behind this result, we reduce the quantum three-level system to an effective two-level one with large detunings. The shortcuts to adiabaticity technique of effective two-level system is studied. Thereby the high-fidelity population transfer can be implemented in non-Hermitian systems by our method, and it works even without RWA.

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1. Introduction

Population transfer by adiabatic evolution is among the most popular methods for coherent atomic manipulation \cite{1-10}. However, due to the requirement on adiabaticity, the manipulation is very slow. Recently, by designing nonadiabatic shortcuts to speed up quantum adiabatic process, a new technique named “shortcuts to adiabaticity” (STA) \cite{11-22} opens a new door towards to fast and robust quantum state manipulations. The shortcut techniques include counterdiabatic control protocols, transitionless quantum driving (TQD) \cite{23-51}, “fast-forward” scaling \cite{52}, and inverse engineering based on Lewis-Riesenfeld (LR) invariants \cite{33, 34}. Among the methods of shortcuts to adiabaticity, transitionless quantum driving has been intensively studied. The key idea of transitionless quantum driving is to use a shortcuts to adiabaticity by adding extra fields to cancel the nonadiabatic couplings.

The shortcuts to adiabaticity technique has proved to be successful in designing control scheme for closed quantum systems. In practice, however, interactions between a quantum system and its surroundings is not avoidable, leading to coherence loss of the closed systems. Such a system can be described by a non-Hermitian Hamiltonian, and in the past few decades, it has drawn much attention \cite{35}, especially in the context of PT-symmetric systems \cite{36, 37}. There are many amazing phenomena predicted in PT-symmetric systems, for instance, a PT-symmetric Hamiltonian possesses a higher quantum speed efficiency than Hermitian Hamiltonian systems \cite{38, 39}. Recently, an approximation of the adiabatic condition for non-Hermitian systems were derived \cite{40}. For non-Hermitian systems the usual approximations and criteria are not valid in general, so results that are applicable for Hermitian systems have to be reconsidered and modified. The extension of shortcuts to adiabaticity from Hermitian systems to non-Hermitian systems has been put forward for the Landau-Zener (LZ) model \cite{41}. Therefore, we may wonder if there is a general shortcuts to adiabaticity technique for non-Hermitian PT-Hamiltonian systems? whether the adiabatic evolution proposed in Ref. \cite{40} can be accelerated for such systems?

On the other hand, the rotating wave approximation (RWA) is in common used in atomic optics and magnetic resonance \cite{42, 44}, where the counter-rotating (CR) terms are neglected. The RWA is valid in the weak coupling regime with small detuning and weak field amplitude, in this case the contribution of the counter-rotating terms to the evolution of the system is quite small. In recent experiments, efforts are made to reach a promising method for studying strong- and ultrastrong-coupling physics \cite{45-50}, where the RWA is no longer valid. Refs. \cite{51, 52} found that the RWA may lead to faulty results. Especially, recent developments in physical implementation lead to strong coupling between qubit and cavity modes \cite{53, 54}, which requires a careful consideration of the effect of counter-rotating terms. This motivate us to ask: if can we design a shortcuts to adiabaticity technique for an open system with the counter-rotating terms?

In this paper, we propose a renewed shortcuts to adiabaticity technique for non-Hermitian quantum systems beyond RWA. By using transitionless quantum driving method, we determine the exact control to speed up the adiabatic population transfer. Then, we apply it into the two- and three-level systems with decay and without RWA. We numerically calculate the population
transfer dynamics with and without counter-rotating terms. The results show that the population transfer with the counter-rotating terms are more steady. On the other hand, we also find that the decay of the excited state has small effect on the population transfer in the three-level system. This is attributed to the fact that the population transfer $|1\rangle \rightarrow |3\rangle$ is realized by the dark state. In the case of large detuning, we reduce the three-level system to an effective two-level system. The shortcuts to adiabaticity technique is then applied to the effective two-level system, which might shed more light on the present scheme.

This paper is organized as follows. In Sec. 2 we study the shortcuts to adiabaticity applied to a decaying [55–57] two-level system without RWA. We numerically compare the population transfer with and without counter-rotating terms. In Sec. 3 we study the shortcuts to adiabaticity applied to a decaying [58, 59] three-level system without RWA. We numerically calculate the population transfer with and without counter-rotating terms. The effect of the decay on the population transfer in the three-level system is also discussed in this section. Sec. 4 is devoted to discussion and conclusion.

### 2. Transitionless driving applied to a decaying two-level atom beyond the RWA

In this section, we present a shortcuts to adiabaticity technique for an open two-level quantum system beyond RWA. Assume the two-level system interacts with a laser electric field with linear polarization in $x$ direction $\vec{E}(t) = E_0(t) \cos[\omega_L t] \hat{x}$, the Hamiltonian of the two-level system reads,

$$H_S(t) = \frac{\hbar}{2} \left[ \Omega_R(t) (|2\rangle\langle 1| + |1\rangle\langle 2|) e^{i\omega_L t} + e^{-i\omega_L t} \right] + \omega_0(t) (|2\rangle\langle 2| - |1\rangle\langle 1|) \right].$$

where $|1\rangle (|2\rangle)$ is the ground (excited) state of the two-level atom, $\omega_0$ is the transition frequency, $\Omega_R(t)$ and $\omega_L$ are the strength and the frequency of the classical field. Defining

$$H_L(t) = i\hbar U(t) U^\dagger(t)$$

$$= \frac{\hbar \omega_L}{2} (|2\rangle\langle 2| - |1\rangle\langle 1|),$$

with unitary operator

$$U(t) = e^{-\frac{\hbar}{2} \int_{0}^{t} H_L(t') dt'}$$

$$= e^{-i\omega_L t/2} (|2\rangle\langle 2| - e^{i\omega_L t/2} |1\rangle\langle 1|),$$

we can transform the Hamiltonian (1) into the interaction picture, i.e.,

$$H(t) = \underbrace{U(t) H_S(t) U(t^\dagger)}_{\Omega(t)}$$

$$= \frac{\hbar}{2} \left[ -\Delta(t) \begin{array}{cc} \Omega(t) \\ \Omega(t) \end{array} \right].$$

Here the bases are $|1\rangle = [1, 0]^T$ and $|2\rangle = [0, 1]^T$ with $T$ denoting transposition. With

$$\Omega(t) = \Omega_R(t)(1 + e^{-2i\omega_L t}),$$

and

$$\Delta(t) = \omega_0(t) - \omega_L,$$
in which \(e^{i2\omega_{LT}t}\) are the counter-rotating terms. To describe the decay of the upper level, we add an imaginary energy \(i\Gamma\) to the energy of the excited state. The Hamiltonian then becomes non-Hermitian,
\[
\hat{H}_0(t) = \frac{\hbar}{2} \begin{bmatrix} -\Delta(t) & \Omega(t) \\ \Omega(t) & \Delta(t) - i\Gamma \end{bmatrix}.
\] (7)

The dynamics of the two-level quantum system is described by the Schrödinger equation,
\[
i\hbar \frac{\partial}{\partial t} \begin{bmatrix} C_1(t) \\ C_2(t) \end{bmatrix} = \hat{H}_0(t) \begin{bmatrix} C_1(t) \\ C_2(t) \end{bmatrix},
\] (8)
where \(C_1(t)\) and \(C_2(t)\) are probability amplitudes of the two bare states \(|1\rangle\) and \(|2\rangle\), respectively.

To design a shortcut to adiabaticity for this non-Hermitian system without RWA, we have to explore the left and right eigenstates of the non-Hermitian Hamiltonian (7). Straightforward calculations lead to left eigenstates of \(\hat{H}_0(t)\) as
\[
|\varphi_+(t)\rangle = \sin\beta|1\rangle + \cos\beta e^{i\omega_{LT}t}|2\rangle,
\]
\[
|\varphi_-(t)\rangle = \cos\beta e^{-i\omega_{LT}t}|1\rangle - \sin\beta|2\rangle,
\] (9)
where the mixing angle \(\beta = \beta(t)\) is complex and defined as
\[
\beta = \frac{1}{2} \arctan \frac{2\Omega_R \cos \omega_{LT} t}{\Delta - i\Gamma}/2,
\] (10)
and the right eigenstates can also be determined,
\[
|\hat{\varphi}_+(t)\rangle = \sin \beta^*|1\rangle + \cos \beta^* e^{i\omega_{LT}t}|2\rangle,
\]
\[
|\hat{\varphi}_-(t)\rangle = \cos \beta^* e^{-i\omega_{LT}t}|1\rangle - \sin \beta^*|2\rangle,
\] (11)
where the asterisk means complex conjugate.

By the spirit of the shortcut to adiabaticity [23–31], the counterdiabatic term can be given by [60]
\[
\hat{H}_{CD}(t) = \hbar \left[ |\partial_t \varphi_+(t)\rangle \langle \varphi_+(t)| - |\partial_t \varphi_-(t)\rangle \langle \varphi_-(t)| \right]
+ |\partial_t \varphi_+(t)\rangle \langle \varphi_+(t)| - |\partial_t \varphi_-(t)\rangle \langle \varphi_-(t)|
\]
\[
\hat{H}_{CD}(t) = \hbar \left[ |\partial_t \varphi_+(t)\rangle \langle \varphi_+(t)| - |\partial_t \varphi_-(t)\rangle \langle \varphi_-(t)| \right],
\] (12)
where
\[
\langle \varphi_+(t)|\partial_t \varphi_+(t)\rangle = \pm i\omega_L \cos \frac{\beta}{2},
\]
\[
\langle \varphi_-(t)|\partial_t \varphi_-(t)\rangle = (\pm i\frac{\beta}{2} - i\frac{\omega_L}{2} \sin \beta) e^{\pm i\omega_{LT}t},
\] (13)
which is equivalent to
\[
\hat{H}_{CD}(t) = \hbar \left[ \begin{array}{cc} \frac{\omega_L}{2} \sin^2 \beta & (i\frac{\beta}{2} + i\frac{\omega_L}{2} \sin 2\beta) e^{-i\omega_{LT}t} \\ (-i\frac{\beta}{2} + i\frac{\omega_L}{2} \sin 2\beta) e^{i\omega_{LT}t} & -\frac{\omega_L}{2} \sin^2 \beta \end{array} \right] - \frac{\beta}{2} \sin \beta e^{\pm i\omega_{LT}t}.
\] (14)

Here, we should stress that the counterdiabatic terms presented in Eq. (14) is beyond RWA. However, we can easily obtain the counterdiabatic term for the RWA case by ignoring the counter-rotating terms in \(H_0(t)\). Applying the rotating wave approximation to get rid of the counter-rotating terms, we end up with
\[
H_0(t) = \frac{\hbar}{2} \begin{bmatrix} -\Delta(t) & \Omega_R(t) \\ \Omega_R(t) & \Delta(t) - i\Gamma \end{bmatrix}.
\] (15)
Fig. 1. Real (red solid line) and imaginary (blue dashed line) parts of $\Omega_a$ (a). The population evolution is driven by $\Omega_a$ (b), and $[\text{Im} \Omega_a]$ (c). The parameters chosen are $\Gamma = 2\pi \times 0.5 \text{MHz}$, $\Omega_0 = 2\pi \times 5 \text{MHz}$, $\delta = 2\pi \times 300 \text{MHz}$, $t_f = 2 \times 10^{-10} \text{s}$, $t_0 = 10^{-10} \text{s}$.

By the same procedure as above, $H_{CD}$ reads

$$H_{CD}(t) = h \begin{bmatrix} 0 & \Omega_a(t) \\ -\Omega_a(t) & 0 \end{bmatrix},$$

(16)

where $\Omega_a(t) = i\dot{\theta}/2$ is the auxiliary pulse with

$$\dot{\theta} = \frac{\Omega_R[\Delta(t) - i\Gamma/2] - \Omega_R(t)[\Delta - i\Gamma/2]}{\Omega_R^2(t) + [\Delta(t) - i\Gamma/2]^2},$$

(17)

which plays the role of the Rabi frequency for a fast-driving field. In principle, $H_{CD}(t)$ can drives the dynamics along the instantaneous eigenstates of $H_0(t)$ in arbitrarily short time, but due to the practical limitations, e.g., the laser power, the performance would depend on those limitations.

The Hamiltonian (16) has off-diagonal terms with real and imaginary parts, see Fig. 1. Practically, realizing the Hamiltonian (16) is not straightforward. From the Hamiltonian (16), we can see that the off-diagonal terms are not complex conjugate to each other except that the real part of $\Omega_a$ is zero. The imaginary parts, shown by the dip in Fig. 1(a), can be realized by a complementary laser with orthogonal polarization [12], the real parts constitute a non-Hermitian contribution to the Hamiltonian. They are, however, contribute very small to the
Fig. 2. The evolution of the population for a decaying two-level system with decay rate $\Gamma = 2\pi \times 0.5 MHz$. The population transfer is implemented by the shortcuts to adiabaticity technique under RWA [Figs. 2(a), and 2(b)], and implemented by the shortcuts to adiabaticity technique beyond RWA [Fig. 2(c)]. Note that all the counter-rotating terms are neglected in Fig. 2(a), but are considered in Fig. 2(b). The population of level $|1\rangle$ (dashed red), and $|2\rangle$ (solid blue) are shown. The other parameters: $\Omega_0 = 2\pi \times 5 MHz$, $\delta = 2\pi \times 300 MHz$, $t_f = 2 \times 10^{-10} s$, $t_0 = 10^{-10} s$, $\omega_L = 2\pi \times 10 GHz$. 
dynamics. Ignoring the real part of $\Omega$, i.e., setting $\Omega \approx i\text{Im}[\Omega]$, we plot the Figs. 1(b) and 1(c). We can see that the population transfer is almost the same. So the Hamiltonian (16) can be realized approximately. When the real parts can not be ignored, the realization mentioned above is not valid anymore.

In the RWA regime, i.e., drop the terms with $e^{\pm i\omega_0 t} = 0$. It can be verified that Eq. (14) is the same as Eq. (16). Figure 2 shows the population of the ground state $|1\rangle$ (red dash lines) and the excited state $|2\rangle$ (blue solid lines) with and without RWA. In our case, we use the shortcuts to adiabaticity technology to speed up the passage, in which pulses do not need satisfy adiabaticity condition. It requires an additional laser field. Therefore we provide an example in which the adiabaticity condition fails and then apply the Hamiltonian in Eq. (16) to remedy this problem and achieve a fast full decay. Here, the coherent controls in the Hamiltonians $\tilde{H}_0(t)$ and $H_0(t)$ are chosen to be the Allen-Eberly (AE) drivings (61), i.e.,

$$\Omega_p(t) = \Omega_0\text{sech}\left[\frac{\pi(t-t_f/2)}{2\theta_0}\right].$$

$$\Delta(t) = \frac{2\delta^2\theta_0}{\pi}\tanh\left[\frac{\pi(t-t_f/2)}{2\theta_0}\right].$$

Here, we select the appropriate parameters [40, 60], in the RWA regime, we use Hamiltonian $H_0(t)$ and the counterdiabatic Hamiltonian $H_{CD}(t)$ to describe the two-level system (see Fig. 2(a)). We find that the population is transferred from the ground state into the excited state completely. However, when the RWA fails, counter-rotating terms must be taken into account, $H_{CD}(t)$ can not be used as the counterdiabatic Hamiltonian to accelerate the adiabatic dynamics governed by the reference Hamiltonian $\tilde{H}_0(t)$, see Fig. 2(b). Therefore, in the regime beyond RWA, we have to redesign the counterdiabatic terms of the shortcuts to adiabaticity according to Eq. (14). As shown in Fig. 2(c), if we apply Hamiltonian (14) into the dynamics governed by Hamiltonian $\tilde{H}_0(t)$, the population transfer efficiency can be enhanced remarkably. Our results suggest that shortcuts to adiabaticity technique well developed for Hermitian systems can be extended to non-Hermitian one. It is envisaged that our study could stimulate further studies of coherent population transfer techniques of multilevel systems for non-Hermitian systems. In the next section, we mainly discuss three-level non-Hermitian system.

3. Transitionless driving scheme applied to a decaying three-level atom beyond RWA

3.1. $H_{CD}(t)$ applied to a decaying three-level atom

In this section, we consider a three-level system of $\Lambda$-type with decay in the excited state. The two lower states are long-lived. We consider the state $|1\rangle$ is initially populated, and the other state $|3\rangle$ as the target state, and $|2\rangle$ as the excited state. Levels $|1\rangle$ and $|2\rangle$ are coupled by the Pump laser $\Omega_p(t)$, while Levels $|2\rangle$ and $|3\rangle$ are coupled by the Stokes laser $\Omega_s(t)$. We assume that there is a decay in the excited state $|2\rangle$ with rate $\Gamma$. Such a system can be described by the following time-dependent Schrödinger equation,

$$i\hbar \frac{d}{dt}c(t) = \tilde{H}_0(t)c(t),$$

where the vector $c(t) = [c_1(t), c_2(t), c_3(t)]^T$ is constructed by the three probability amplitudes of states $|1\rangle$, $|2\rangle$, and $|3\rangle$. Without the rotating wave approximation, the reference Hamiltonian of the $\Lambda$-type three-level system can be depicted as [58, 59, 62]

$$\tilde{H}_0(t) = \frac{\hbar}{2} \begin{pmatrix} 0 & \tilde{\Omega}_p(t) & 0 \\ \tilde{\Omega}_p^*(t) & 2\Delta_p - i\Gamma & \tilde{\Omega}_s(t) \\ 0 & \tilde{\Omega}_s^*(t) & 2(\Delta_p - \Delta_s) \end{pmatrix},$$

where
Fig. 3. Transition probability $P_{1\rightarrow 3}$ of the fast stimulated Raman by the shortcuts to adiabaticity under and beyond RWA. Fig. 3(a) is plotted by using the shortcuts to adiabaticity technique under RWA, but consider the CR terms. Fig. 3(b) is plotted by using the shortcuts to adiabaticity technique beyond RWA. We set the parameters: $t_f = 30\text{ns}$, $T = t_f/6$, $\tau = t_f/10$, $\Omega_0 = 2\pi \times 200\text{MHz}$, $\Delta = 2\pi \times 200\text{MHz}$, $\Gamma = 2\pi \times 100\text{MHz}$, $\omega_p = 100\text{MHz}$, $\omega_s = 80\text{MHz}$. The dashed red, dotted green, and solid blue lines describe the time-dependent population of the state $|1\rangle$, $|2\rangle$, and $|3\rangle$, respectively.

where the detunings are defined by $\Delta_p = \omega_p - (E_2 - E_1)/\hbar$, $\Delta_s = \omega_s - (E_2 - E_3)/\hbar$ for the $\Lambda$ configuration. $\bar{\Omega}_p(t) = \Omega_p(t)(1 + e^{-2i\omega_p t})$ and $\bar{\Omega}_s(t) = \Omega_s(t)(1 + e^{-2i\omega_s t})$ are the pump and Stokes pulses (beyond RWA). The imaginary term $\Gamma$ describes the decay from $|2\rangle$. Conventional stimulated Raman adiabatic passage (STIRAP) relies on the two-photon resonance, i.e., $\Delta = \Delta_p = \Delta_s$.

The mechanism of the population transfer in STIRAP and the limitation of adiabaticity can be easily understood by introducing the so-called adiabatic states. They are connected to the bare states $|1\rangle$, $|2\rangle$ and $|3\rangle$ by the eigenstates of $\bar{H}_0(t)$,

$$|E_0(t)\rangle = \frac{1}{\Xi_0(t)}[\Omega_0(t)|1\rangle - \bar{\Omega}_s(t)|3\rangle],$$

$$|E_+(t)\rangle = \frac{1}{\Xi_1(t)}[\bar{\Omega}_p(t)|1\rangle + \epsilon_+(t)|2\rangle + \bar{\Omega}_s(t)|3\rangle],$$

$$|E_-(t)\rangle = \frac{1}{\Xi_2(t)}[\bar{\Omega}_p(t)|1\rangle + \epsilon_-(t)|2\rangle + \bar{\Omega}_s(t)|3\rangle],$$

(21)

where $\Xi_0(t) = \sqrt{[\Omega_0(t)]^2 + [\bar{\Omega}_s(t)]^2}$, $\Xi_1(t) = \sqrt{\epsilon_+^2(t) + \Xi_0^2(t)}$, and $\Xi_2(t) = \sqrt{\epsilon_-^2(t) + \Xi_0^2(t)}$. The eigenvalues of $\bar{H}_0(t)$ corresponding to the eigenstates are,

$$E_0(t) = 0, \quad E_\pm(t) = \frac{\hbar}{2}\epsilon_\pm(t).$$

(22)
with
\[ e_\pm(t) = (\Delta - i\Gamma/2) \pm \sqrt{(\Delta - i\Gamma/2)^2 + \Xi_0^2(t)}. \] (23)

In the STIRAP, the \(|E_0(t)\rangle\) eigenstate is a dark state without population on state \(|2\rangle\). The protocol allows us to produce an efficient transfer from \(|1\rangle\) to \(|3\rangle\) following the evolution of the dark state \(|E_0(t)\rangle\). Since \(\tilde{H}_0(t)\) is non-Hermitian, the instantaneous eigenstates of the Hamiltonian \(\tilde{H}_0(t)\) have to be considered, which reads
\[
|\hat{E}_0(t)\rangle = \frac{1}{\Xi_0(t)} [\Omega_p(t)|1\rangle - \Omega_p^*(t)|3\rangle],
\]
\[
|\hat{E}_+(t)\rangle = \frac{1}{\Xi_1(t)} [\Omega_p(t)|1\rangle + \hat{e}_+(t)|2\rangle + \Omega_p^*(t)|3\rangle],
\]
\[
|\hat{E}_-(t)\rangle = \frac{1}{\Xi_2(t)} [\Omega_p(t)|1\rangle + \hat{e}_-(t)|2\rangle + \Omega_p^*(t)|3\rangle],
\] (24)
with \(\hat{e}_\pm(t) = (\Delta + i\Gamma/2) \pm \sqrt{(\Delta + i\Gamma/2)^2 + \Xi_0^2(t)}\).

In order to realize the shortcuts to adiabaticity beyond RWA, we first write out three orthogonal projection operators corresponding to the relevant instantaneous eigenstates:

\[
\Pi_0(t) = |E_0(t)\rangle\langle \hat{E}_0(t)| = \frac{1}{\Xi_0(t)} \begin{bmatrix}
|\Omega_p(t)|^2 & 0 & -\Omega_p(t)\bar{\Omega}_s(t) \\
0 & 0 & 0 \\
-\bar{\Omega}_p(t)\Omega_s(t) & 0 & |\bar{\Omega}_p(t)|^2
\end{bmatrix},
\]
\[
\Pi_1(t) = |E_1(t)\rangle\langle \hat{E}_1(t)| = \frac{1}{\Xi_1(t)} \begin{bmatrix}
|\Omega_p(t)|^2 & \bar{\Omega}_p(t)\Omega_s(t) & \Omega_p(t)\bar{\Omega}_s(t) \\
\bar{\Omega}_p(t)\Omega_s(t) & |\Omega_s(t)|^2 \\
\Omega_p(t)\bar{\Omega}_s(t) & \Omega_s(t)\bar{\Omega}_s(t) & |\Omega_s(t)|^2
\end{bmatrix},
\]
\[
\Pi_2(t) = |E_2(t)\rangle\langle \hat{E}_2(t)| = \frac{1}{\Xi_2(t)} \begin{bmatrix}
|\Omega_p(t)|^2 & \bar{\Omega}_p(t)\Omega_s(t) & \Omega_p(t)\bar{\Omega}_s(t) \\
\bar{\Omega}_p(t)\Omega_s(t) & |\Omega_s(t)|^2 \\
\Omega_p(t)\bar{\Omega}_s(t) & \Omega_s(t)\bar{\Omega}_s(t) & |\Omega_s(t)|^2
\end{bmatrix}.
\] (25)

By the same procedure, the Hermitian \(\tilde{H}_{CD}\) Hamiltonian becomes
\[
\tilde{H}_{CD}(t) = i\hbar \sum_{j \neq k} \frac{\Pi_j(t)\partial_t \tilde{R}_0(t)\Pi_k(t)}{E_k(t) - E_j(t)}
\]
\[
= \frac{i\hbar}{\Xi_1(t)\Xi_2^2(t)} \begin{bmatrix}
|\Omega_p(t)|^2 B(t) + C(t)D(t) & \Omega_p(t)A(t) - 2i\Gamma\Omega_s(t)G(t) & \Omega_p(t)\Omega_s(t)B(t) + C(t)F(t) \\
-\bar{\Omega}_p(t)\bar{\Omega}_s(t)A^+(t) - 2i\Gamma\bar{\Omega}_s(t)G^+(t) & -\Xi_1(t)B(t) & -\bar{\Omega}_p(t)A(t) - 2i\Gamma\bar{\Omega}_s(t)G^+(t) \\
-\bar{\Omega}_p(t)\bar{\Omega}_s(t)B^+(t) - C(t)F^+(t) & \bar{\Omega}_s(t)A(t) + 2i\Gamma\bar{\Omega}_s(t)G(t) & \bar{\Omega}_s(t)^2 B(t) - C(t)D(t)
\end{bmatrix},
\] (26)
with
\[
A(t) = (2\Delta - i\Gamma)\left[\hat{\Omega}_p(t)^2\hat{\Omega}_s(t) + \hat{\Omega}_s(t)^2\hat{\Omega}_p(t)\right],
\]
\[
B(t) = \bar{\Omega}_p(t)^2\bar{\Omega}_s(t) + \bar{\Omega}_s(t)^2\bar{\Omega}_p(t) - H.C.,
\]
\[
C(t) = |\hat{e}_+(t)|^2 + |\hat{e}_-(t)|^2 + 2\Xi_0^2(t),
\]
\[
D(t) = |\Omega_p(t)|^2 \left[\hat{\Omega}_s(t)^2\Omega_p(t) - H.C.\right] + |\Omega_s(t)|^2 \left[\Omega_p(t)^2\hat{\Omega}_s(t) - H.C.\right],
\]
\[
F(t) = \bar{\Omega}_p(t)^2\bar{\Omega}_s(t)\hat{\Omega}_s(t) + \bar{\Omega}_s(t)^2\bar{\Omega}_p(t)\hat{\Omega}_p(t),
\]
\[
G(t) = \bar{\Omega}_p(t)\bar{\Omega}_s(t)\hat{\Omega}_s(t) - \bar{\Omega}_p(t)\bar{\Omega}_s(t)\hat{\Omega}_s(t).
\] (27)

Correspondently, we give the auxiliary driving for the shortcuts to adiabaticity under RWA.
The adiabaticity technique under the RWA does not work well if considering the counter-rotating terms, but beyond the RWA is not.

The Pump pulse and Stokes pulse in our numerical analysis can be described by

\[ H_{CD}(t) = \hat{h} \begin{bmatrix} 0 & A(t) & C(t) \\ -A(t) & 0 & -B(t) \\ -C(t) & B(t) & 0 \end{bmatrix}, \]  

where

\[ A(t) = \sin \theta(t) \phi(t), \] \[ B(t) = \cos \theta(t) \phi(t), \] \[ C(t) = \dot{\theta}(t), \]  

with \( \phi(t) = \frac{\Omega'(t)(\Delta - \Delta_0^2/2) - \Omega'(0)(\Delta - \Delta_0^2/2)}{2[\Omega'^2(t) + (\Delta - \Delta_0^2/2)^2]}, \) \( \dot{\theta}(t) = \frac{\Omega_p(t)\Omega_0(t) - \Omega_0(t)\Omega_p(t)}{\Omega^2(t)} \), and \( \Omega'(t) = \sqrt{\Omega_p^2 + \Omega_0^2} \), respectively.

We would like to address that when the transitions \(|1\rangle \leftrightarrow |2\rangle\) and \(|2\rangle \leftrightarrow |3\rangle\) are allowed, the transition \(|1\rangle \leftrightarrow |3\rangle\) is in general forbidden. But for artificial atom [62,67], the forbiddenness of transition is lifted.

The Pump pulse and Stokes pulse in our numerical analysis can be described by \( \Omega_p(t) = \Omega_0 \exp\left[-\frac{(t-t_f/2)^2}{2\sigma_0^2}\right] \) and \( \Omega_s(t) = \Omega_0 \exp\left[-\frac{(t+\tau-t_f/2)^2}{2\sigma_0^2}\right] \). The Gaussian pulses have the same shapes and strengths but separated by a delay of \( 2\tau \). The entire interaction time of the system evolution is \( t_f \). The constant loss rate \( \Gamma \) is a constant. The initial conditions for the populations are \( P_1(0) = 1, P_2(0) = 0 \) and \( P_3(0) = 0 \). In Fig. 3(a) and 3(b), we can find that the shortcuts to adiabaticity technique under the RWA does not work well if considering the counter-rotating effects, but beyond the RWA is not.
We use the fidelity between the final state $|\Psi(t_f)\rangle$ and the target state $|3\rangle$ to characterize the population transfer efficiency (see Fig. 4), i.e., $F = |\langle 3 |\Psi(t_f)\rangle|^2$. Figs. 4(a) and 4(c) show the fidelity beyond RWA, where we use $H_{CD}(t)$ (Eq. (26)) as the counterdiabatic Hamiltonian. Figs. 4(b) and 4(d) are also for the results beyond RWA, but $\bar{H}_{CD}$ (Eq. (26)) is chosen as the counterdiabatic Hamiltonian. The parameters used in the numerical simulation are chosen as follows: In Figs. 4(a) and 4(b), we choose $\bar{\Omega}_0 = 2\pi \times [100,800]{MHz}$ and $\Gamma = 2\pi \times [100,600]{MHz}$, respectively. In Figs. 4(c) and 4(d), the parameters are $\Delta = 2\pi \times [100,800]{MHz}$ and $\Gamma = 2\pi \times [100,600]{MHz}$, respectively. The final evolution time is 30ns. The numerical results show that the shortcuts to adiabaticity technique with the counterdiabatic Hamiltonian $H_{CD}(t)$ does not work well if the counter-rotating terms are taken into account (see Figs. 4(a) and 4(c)). The fidelity is sensitive to the decay rate and pulse parameters. The required fidelities can be reached only in narrow range of parameters. This is due to the matching-lost between the adiabatic reference Hamiltonian $\bar{H}_0(t)$ and the counterdiabatic Hamiltonian $\bar{H}_{CD}(t)$. To better the performance, we take $\bar{H}_{CD}(t)$ (Eq. (26)) as the counterdiabatic Hamiltonian to replace $H_{CD}(t)$, which are illustrated in Figs. 4(b) and 4(d). When the reference Hamiltonian and the counterdiabatic Hamiltonian are matching, the fidelity is not affected by any parameters in the coherent control field, and it is robust against the decay of the excited state.

It turned out that the previous shortcuts to adiabaticity technique under RWA does not work well if the counter-rotating terms are included. Therefore, the study of shortcuts to adiabaticity technique beyond RWA is necessary. The robustness to the population loss of the excited state was investigated and it was shown that high fidelity can be achieved even with large decay rates. The decay of the excited state has little effect on the population transfer. This is due to the fact that in the STIRAP the dark states are used, the excited state is almost not population transferred (see Fig. 4), i.e., $F(t_f) = |\langle 3 |\Psi(t_f)\rangle|^2 \\times 0.95$. The final evolution time is 30ns. The numerical results show that the shortcuts to adiabaticity technique with the counterdiabatic Hamiltonian $H_{CD}(t)$ does not work well if the counter-rotating terms are taken into account (see Figs. 4(a) and 4(c)). The fidelity is sensitive to the decay rate and pulse parameters. The required fidelities can be reached only in narrow range of parameters. This is due to the matching-lost between the adiabatic reference Hamiltonian $\bar{H}_0(t)$ and the counterdiabatic Hamiltonian $\bar{H}_{CD}(t)$. To better the performance, we take $\bar{H}_{CD}(t)$ (Eq. (26)) as the counterdiabatic Hamiltonian to replace $H_{CD}(t)$, which are illustrated in Figs. 4(b) and 4(d). When the reference Hamiltonian and the counterdiabatic Hamiltonian are matching, the fidelity is not affected by any parameters in the coherent control field, and it is robust against the decay of the excited state.

3.2. Effective two-level system for large single-photon detuning

When the detuning $\Delta(t) \gg \Omega_{p,s}(t)$, the middle state $|2\rangle$ can be eliminated adiabatically by setting $\dot{c}_2 = 0$ in Eq. (19). The three-level system can be then reduced to an effective two-level system (with levels $|1\rangle$ and $|3\rangle$) [68,70]:

$$H_{eff}(t) = \hbar \frac{\mathbf{g}}{2} \begin{bmatrix} -\Delta_{eff}(t) & \Omega_{eff}(t) \\ \Omega_{eff}^*(t) & \Delta_{eff}(t) \end{bmatrix},$$

where $\Delta_{eff}(t)$ and $\Omega_{eff}(t)$ are effective detuning and Rabi frequency, respectively,

$$\begin{align*}
\Delta_{eff}(t) &= -\frac{\Omega_{p}(t)\Omega_{p}(t)}{2\Delta - i\Gamma}, \\
\Omega_{eff}(t) &= \frac{[\Omega_{p}(t)]^2 - [\Omega_{s}(t)]^2}{4\Delta - 2i\Gamma}.
\end{align*}$$

In the large detuning limit, the three-level system reduces to an effective two-level system described by the Hamiltonian (30). Once the effective two-level Hamiltonian is obtained, we can calculate the counter-adiabatic driving according to [71]

$$H_{CD}(t) = \frac{i\hbar}{2M(t)} \begin{bmatrix} 0 & P(t) \\ -P^{*}(t) & Q(t) \end{bmatrix},$$

with

$$M(t) = \sqrt{\Omega_{eff}^2(t) + \Delta_{eff}^2(t)}.$$
Fig. 5. The population evolution of the effective two-level system by shortcuts to adiabaticity (a), and the three-level system by shortcuts to adiabaticity (b). Here the population of level $|1\rangle$ (dashed red), and $|3\rangle$ (solid blue) is presented. We set the parameters as: $t_f = 30 \text{ns}$, $T = t_f/6$, $\tau = t_f/10$, $\Omega_0 = 2\pi \times 0.16 \text{GHz}$, $\Delta = 2\pi \times 2.5 \text{GHz}$, $\Gamma = 2\pi \times 0.16 \text{GHz}$, $\omega_0 = 0.1 \text{GHz}$, $\omega_s = 0.08 \text{GHz}$. We can find that the population evolution given by the effective two-level model and by the three-level model is almost the same.

Thus the total Hamiltonian reads $H(t) = H_{\text{eff}}(t) + H_{\text{CD}}(t)$. We can directly apply the effective two-level model in the analysis of our three-level system with large single-photon detuning. The Pump pulse and Stokes pulse in our numerical analysis still can be chosen as $\Omega_p(t) = \Omega_0 \exp[-(t-\tau-t_f/2)^2]$ and $\Omega_s(t) = \Omega_0 \exp[-(t+t_f/2)^2]$. We now compare the performance of the above effective two-level system shortcuts to adiabaticity and three-level system shortcuts to adiabaticity protocols. We set the detuning $\Delta = 2\pi \times 2.5 \text{GHz}$. With $\Omega_0 = 2\pi \times 0.16 \text{GHz}$, $t_f = 30 \text{ns}$, $T = t_f/6$, and $\tau = t_f/10$, the dynamics described by the original Hamiltonian (20) is not adiabatic at all, and population can not be completely transferred from $|1\rangle$ to $|3\rangle$. Figs. 5(a) and 5(b) show the population transfer by shortcuts to adiabaticity of the effective two-level system and that of the three-level system, respectively. We can find that they are almost the same.

4. Conclusion

Employing transitionless quantum driving, we have presented a method to design a shortcut to adiabaticity for a non-Hermitian system [55][56] without RWA. We find that the previous shortcuts to adiabaticity technique with RWA does not work well if the counter-rotating terms can not be neglected, instead the present scheme leads to a high performance to finish the shortcut. Meanwhile we found that the decay of the excited state has little effect on the adiabatic population transfer in the three-level system. Our method is robust against the fluctuation of parameters, as the fidelity is still unity for a wide range of $\Gamma$, $\Omega_0$ and $\Delta$. In
the case of large detuning, we reduce the three-level system to an effective two-level system by using adiabatic elimination. The shortcuts to adiabaticity technique of effective two-level system is also studied and discussed.

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