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ABSTRACT
Improvement of sound absorption and insulation using a double-layer metamaterial (DM) in the acoustic free field is proposed in the present paper. The front layer of the DM is composed of a flexible micro-perforated plate with periodic local resonators, the backing layer is a flexible plate attached with periodic local resonators too, and an air gap exists between the double plates. Good agreement is gained between the results of the theoretical prediction and finite element simulation for the DM and the original double-layer plate (DP) while considering the vibroacoustic coupling between the sound excitation and the plates. Both theoretical and simulation results verify that the local resonators can improve the sound absorption and insulation of the DM. The underlying mechanism of the DM is investigated using the acoustic impedance and displacement pattern. Filling the gap with the porous material can further improve the sound absorption and insulation of the DM. Finally, the influences of the number and additional mass ratio of local resonators on the acoustic performance of the DM are investigated, and the practical realization of the DM is verified. The present design shows great potential for practical noise reduction in the free field.

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I. INTRODUCTION
Acoustic metamaterials have opened a new perspective for acoustic wave manipulation, which has now attracted considerable attention from scientists of physics and engineering for sound absorption. Various acoustic metamaterials (AMs), such as membrane-type AMs, critically coupled AMs, and meta-porous AMs, possess perfect acoustic absorption in the low-frequency domain with a deep-subwavelength thickness. One alternative choice is using the coiled-up structures in AM design, which can significantly reduce the AM thickness. The tunability of the absorption peak of AMs has been achieved by (1) unequal-section channels and (2) different lengths of coiled-up channels. However, most of them only have a single narrow absorption band due to the resonant characteristic.

Since many realistic applications require absorption operation for multiple or wideband frequencies, the extension of the absorption of the AM from a single/narrow band to multiple/wide bands has received great attention. Zhao et al. designed a double porosity material hybridizing the porous layer and labyrinthine channel for a multi-band absorption. Yang et al. designed an AM consisting of 16 different coiled-up Fabry-Pérot (FP) channels, and a near-perfect flat absorption spectrum from 400 Hz to 3000 Hz was obtained. Zhu et al. designed an AM, which used a porous material containing four acoustic resonators, and a high absorption (>80%) was obtained in the frequency range from 180 Hz to 550 Hz. Ji et al. investigated an AM composed of four coupled unit cells, and high and near-omnidirectional absorption (average absorption coefficient ≈93%) was achieved from 50 Hz to 63 Hz with a thickness of 15.4 cm. Peng et al. designed an AM of a composite honeycomb subsurface panel with a thickness <30 mm, which can achieve 90% sound absorption from 600 Hz to 1000 Hz.

Recently, a metamaterial using a flexible micro-perforated panel attached with local resonators was designed to improve the sound absorption of the micro-perforated panel. Until now, most
of the existing perfect sound absorbers are backed by hard walls to achieve good absorption and non-transmission, which means that they cannot be employed directly in the acoustic free field as sound-absorption materials, e.g., doors and vehicle bodies. Wang et al.\(^1\) designed an AM composed of two membranes sandwiching a porous material layer. They acquired a narrowband perfect absorption at 312 Hz and broadband insulation.

Here, we design a double-layer metamaterial (DM) in the acoustic free field. The front layer of the DM is composed of a flexible micro-perforated plate attached with periodic local resonators, the backing layer is a flexible plate with periodic local resonators too, and an air gap exists between the double plates. Both theoretical and finite element methods are used to investigate the acoustic properties of the DM. The acoustic mechanism is investigated using the acoustic impedance and displacement pattern of the plates. The sound absorption and insulation of the DM filled with the porous material are further investigated. Finally, the effects of the number and the additional mass ratio of local resonators on the acoustic performance of the DM are investigated, and an actual resonator is established for practical design.

II. THE MODEL

Figure 1 shows the acoustic structure of the designed DM, where the surface metamaterial is composed of a flexible micro-perforated plate with periodic local resonators and the backing metamaterial is composed of a flexible plate with periodic local resonators too. There is an air gap between both plates of the DM. The length and width of the surface/backing metamaterial are the same. The four boundaries of the surface/backing metamaterial are all simply supported conditions. For clear difference, the local resonators on the surface/backing metamaterial are marked 1/2, respectively. The total number and arrangement of local resonators 1 and 2 are the same. Figure 2 presents the cross section of the theoretical model. Here, we assume that a plane sound wave is normally incident on the surface metamaterial. Both the reflected and transmitted sounds are scattered to the infinitely free field without reflection for simplification. The side boundaries of the air gap in the DM are rigid.

III. THEORETICAL METHOD

In our analysis, local resonator 1(2) is equivalent to dynamic mass 1(2) based on a concentrated spring–mass–damping model. The damping effect of the local resonator is considered as complex elastic constant \(k_j(1 + i\eta_j)\), where \(\eta_j\) is the damping loss factor. Then, the sound absorption, reflection, and transmission coefficients and sound transmission loss (STL) of the DM are obtained by the transfer matrix method.

A. The equivalent mass of the local resonator

Considering the lattice constant \(a_j/b_j\) of the local resonators is much smaller than the plate bending wavelength (i.e., in a sub-wavelength scale), the local resonator can be equivalent to a dynamic mass. The equivalent dynamic mass of the local resonator \(j\) \((j = 1, 2)\) is\(^3\)

\[
m_{eq,j} = \frac{m_{r,j}}{1 - \omega^2/(\omega_{r,j}(1 + i\eta_j))^2},
\]

where \(m_{r,j}, \omega_{r,j},\) and \(\eta_{r,j}\) are the corresponding static mass, resonance frequency, and damping loss factor of the local resonator \(j\), \(\omega\) is the angular frequency, and \(i = \sqrt{-1}\). The equivalent density of the surface (backing) metamaterial including the equivalent mass of local resonators is

\[
\rho_{eq,j} = \rho_j + \frac{1}{ab_j} \frac{1}{abh_j} m_{eq,j},
\]

where \(\rho_j\) and \(h_j\) are the density and thickness of the surface \((j = 1)\) or backing \((j = 2)\) plate, \(a\) and \(b\) are the length and width of the surface and backing metamaterials, and \(J_j\) is the total number of local resonators on the corresponding plate. Here, \(J_1 = J_2\). We define the ratio of the additional mass of the local resonators to the original mass of the plate as

\[
e_j = \frac{1}{\rho_j abh_j} \frac{1}{abh_j} \frac{J_j m_{eq,j}}{m_{r,j}}.
\]

As the perforation rate \(\varphi\) of the micro-perforated plate is far less than 1, i.e., \(\varphi \ll 1\), the influence of the perforations on the plate density \(\rho_j\) is negligible.

B. Transfer matrix method

The sound pressure transmission relationship between the incident, reflected, and transmitted sound of the DM is

\[
\begin{bmatrix}
\frac{p_t + p_r}{p_i} \\
\frac{1}{\rho_i} p_t
\end{bmatrix}
= \left[ \begin{bmatrix} G \end{bmatrix} \right]
\begin{bmatrix}
\frac{p_t + p_r}{p_i} \\
\frac{1}{\rho_i} p_t
\end{bmatrix}.
\]

\[\begin{bmatrix}
\begin{bmatrix}
\frac{p_t + p_r}{p_i} \\
\frac{1}{\rho_i} p_t
\end{bmatrix}
\end{bmatrix}
= \begin{bmatrix} M \end{bmatrix} \begin{bmatrix}
\begin{bmatrix}
\frac{p_t + p_r}{p_i} \\
\frac{1}{\rho_i} p_t
\end{bmatrix}
\end{bmatrix}.
\]
where $p_i, p_v$, and $p_t$ are the incident, reflected, and transmitted sound pressure, respectively, and $p_0$ and $c_0$ are the density and sound velocity of the ambient air. $[M]$, $[G]$, and $[P]$ are the transfer matrices of the surface metamaterial, air gap, and backing metamaterial, respectively. Defining

$$ [M][G][P] = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}, $$

(5)

the reflection and transmission coefficients of the acoustic energy are

$$ R = \frac{P_r}{P_i} = \frac{T_{11} - T_{22} + T_{12} \frac{1}{\rho_s} - T_{21} \frac{1}{\rho_o} c_0}{T_{11} + T_{22} + T_{12} \frac{1}{\rho_s} + T_{21} \frac{1}{\rho_o} c_0}, $$

(6)

$$ T = \frac{P_t}{P_i} = \left( \frac{2}{T_{11} + T_{22} + T_{12} \frac{1}{\rho_s} + T_{21} \frac{1}{\rho_o} c_0} \right)^2. $$

(7)

Then, the absorption coefficient is

$$ a = 1 - R - T. $$

(8)

The sound transmission loss in dB is

$$ STL = 10 \log \frac{1}{T}. $$

(9)

### 1. Transfer matrix of the surface metamaterial

With the thickness of the surface metamaterial being much less than the wavelength, its transfer matrix $[M]$ can be expressed as

$$ [M] = \begin{bmatrix} 1 & Z_{m} \\ 0 & 1 \end{bmatrix}, $$

(10)

where $Z_{m}$ is the impedance of the surface metamaterial. The relationship among the average air particles’ velocity $\bar{v}_o$ in the perforation, the average pressure drop $\Delta \bar{p}_m$ between both the surfaces of the micro-perforated plate, and the average velocity $\bar{v}_{p,1}$ of the plate frame can be approximately expressed as

$$ Z_{o,R} (\bar{v}_o - \bar{v}_{p,1}) + Z_{o,I} \bar{v}_o = \Delta \bar{p}_m, $$

(11)

where $Z_{o,R}$ and $Z_{o,I}$ are the real and imaginary parts of the perforation impedance $Z_{o}$, respectively,

$$ Z_{o,R} = \frac{32 \rho_h}{d^2} \left[ \sqrt{1 + \frac{k^2}{32} + \frac{\sqrt{2}}{8} k \frac{d}{h_1}} \right], $$

(12)

$$ Z_{o,I} = \omega \rho_s h_1 \left[ 1 + \sqrt{9 + \frac{k^2}{2} + 0.85 \frac{d}{h_1}} \right], $$

(13)

where $\eta$ is the dynamic viscosity of air, $d$ is the diameter of the perforation, and $k = d / \sqrt{\rho_0 \omega / 4 \eta}$ is the perforation constant. The average velocity $\bar{v}_{p}$ of the outer micro-perforated plate surface is

$$ \bar{v}_{p} = (1 - \varphi) \bar{v}_{p,1} + \varphi \bar{v}_o, $$

(14)

where $\varphi = \pi d^2 4 a_b b_0$ is the perforation ratio, in which $a_b$ and $b_0$ are the center distances of the perforations along the $x$ and $y$ directions (see Fig. 1). $\bar{v}_{p,1}$ can be acquired by the impedance $Z_{p,1}$ of the surface metamaterial plate frame as

$$ \bar{v}_{p,1} = \frac{\Delta \bar{p}_m}{Z_{p,1}}. $$

(15)

Substituting Eq. (15) into Eq. (11), we obtain

$$ \bar{v}_o = \frac{(Z_{o,R} + Z_{o,I}) \bar{v}_{p,1}}{Z_{o}}. $$

(16)

Substituting Eq. (16) into Eq. (14), we get

$$ \bar{v}_{p} = \frac{Z_{o,R} + (1 - \varphi) Z_{o,I}}{Z_{o}} \bar{v}_{p,1} + \frac{Z_{p,1}}{Z_{o}} \bar{v}_o, $$

(17)

where $Z_{o} = \frac{Z_0}{\phi}$ is the impedance of the rigid micro-perforated plate, in which $\phi \ll 1$; Eq. (17) is approximated as

$$ \bar{v}_{p} = \frac{Z_{o,R} + Z_{p,1}}{Z_{o}} \bar{v}_{p,1}. $$

(18)

Finally, the impedance of the surface metamaterial is

$$ Z_{o} = \frac{\Delta \bar{p}_m}{\bar{v}_o} = \frac{\bar{v}_{p,1}}{Z_{o,R}} \bar{v}_{p,1} + \frac{Z_{p,1}}{Z_{o}} \bar{v}_o. $$

(19)

One can acquire $Z_{p,1}$ based on the vibroacoustic coupling between the sound excitation and the surface metamaterial. Generally, the following equilibrium equation of the surface metamaterial is satisfied:

$$ D \nabla^4 w_{p,1}(x,y,t) + \rho_{eff} h_1 \frac{\partial^2 w_{p,1}(x,y,t)}{\partial t^2} = \Delta \bar{p}_m e^{i \omega t}, $$

(20)

where $\nabla^4 = (\partial^2 / \partial x^2 + \partial^2 / \partial y^2)^2$ and $D_1 = E_1 h_1^3 / 12 (1 - \nu_1^2)$ is the bending stiffness of the micro-perforated plate, in which $E_1$ and $\nu_1$ are Young’s modulus and Poisson’s ratio, respectively. The damping is taken into account by introducing a complex Young’s modulus $E_i (1 + i \mu_i)$, where $\mu_i$ is the damping loss factor of the micro-perforated plate; $w_{p,1}$ is the displacement of the micro-perforated plate, which can be expressed as

$$ w_{p,1}(x,y,t) = w_{p,1}(x,y) e^{i \omega t} = \sum_{m=1}^{M} \sum_{n=1}^{N} A_{mn,1} X_{mn}(x) Y_{n,1}(y) e^{i \omega t}, $$

(21)

where $A_{mn,1}$ is the modal amplitude of the $(m,n)$ eigenmode of the surface metamaterial and $X_{mn}(x)$, $Y_{n,1}(y)$ is the corresponding shape function. For the simply supported boundary condition, the mode shape function is

$$ X_{mn}(x) = \sin \left( \frac{m \pi x}{a} \right), \quad Y_{n,1}(y) = \sin \left( \frac{n \pi y}{b} \right). $$

(22)

The velocity of the surface metamaterial can be written as

$$ v_{p,1}(x,y,t) = \frac{\partial w_{p,1}(x,y,t)}{\partial t} = \sum_{m=1}^{M} \sum_{n=1}^{N} B_{mn,1} X_{mn}(x) Y_{n,1}(y) e^{i \omega t}, $$

(23)

where $B_{mn,1} = \mu i A_{mn,1}$. Substituting Eqs. (21)–(23) into Eq. (20), $e^{i \omega t}$ can be eliminated for time-harmonic incidence; then,

$$ \sum_{m=1}^{M} \sum_{n=1}^{N} Z_{mn} B_{mn,1} X_{mn}(x) Y_{n,1}(y) = \Delta \bar{p}_m, $$

(24)
where $Z_{mn,1}$ is the $(m, n)$ modal impedance,

$$Z_{mn,1} = \frac{\rho_0 h_1}{i(\omega^2 - \omega^2_{mn,1})}.$$  

(25)

where $\omega_{mn,1}$ is the $(m, n)$ modal frequency of the surface metamaterial,

$$\omega_{mn,1} = \sqrt{\frac{D_1}{\rho_0 h_1}\left(\frac{n}{a}\right)^2 + \left(\frac{n}{b}\right)^2}.$$  

(26)

After some algebraic computing, one can acquire

$$B_{mn,1} = \frac{\Delta P_{mn,1}}{Z_{mn,1}\eta_{mn,1}},$$  

(27)

where

$$\eta_{mn,1} = \int_0^b \int_0^a X_{mn,1}(x)Y_{mn,1}(y)dxdy,$$

(28)

$$\varepsilon_{mn,1} = \int_0^b \int_0^a X_{mn,1}(x)Y_{mn,1}(y)dxdy.$$  

(29)

So, the averaged velocity of the surface metamaterial plate frame is

$$\bar{\nabla}_p,1 = \int_0^a \int_0^b \nabla_p,1|_{xy}ab = \Delta P_1 \sum_{m=1}^M \sum_{n=1}^N \frac{\varepsilon_{mn,1}}{abZ_{mn,1}\eta_{mn,1}}.$$  

(30)

Then, the impedance of the surface metamaterial plate frame is

$$Z_{p,1} = \frac{\Delta P_1}{\bar{\nabla}_p,1} = \left(\sum_{m=1}^M \sum_{n=1}^N \frac{\varepsilon_{mn,1}}{abZ_{mn,1}\eta_{mn,1}}\right)^{-1}.$$  

(31)

One can easily get the impedance $Z_m$ of the surface metamaterial according to Eq. (19) and then the transfer matrix $[M]$ according to Eq. (10).

2. Transfer matrix of the gap

The transfer matrix $[G]$ of the air gap can be expressed as

$$[G] = \begin{bmatrix} \cos(k_D) & iZ_c \sin(k_D) \\ i\sin(k_D)/Z_c & \cos(k_D) \end{bmatrix},$$  

(32)

where the characteristic wavenumber $k_c = \omega/\sqrt{\kappa/\rho}$, the characteristic impedance $Z_c = \rho/\kappa$, and $D$ is the thickness of the air gap.

3. Transfer matrix of the backing metamaterial

Similar to the surface metamaterial, the transfer matrix $[P]$ of the backing metamaterial can be expressed as

$$[P] = \begin{bmatrix} 1 & Z_p \\ 0 & 1 \end{bmatrix}.$$  

(33)

where $Z_p$ is the impedance of the backing metamaterial. The impedance derivation of the backing metamaterial is the same as that of the surface metamaterial plate frame in Sec. III B 1, see Eqs. (20)–(31).

IV. FINITE ELEMENT SIMULATION

To validate the above theoretical method, a finite element simulation using the commercial finite element software COMSOL Multiphysics is conducted. The full multi-physical model of the DM is established, as shown in Fig. 3. Three physics modules including solid mechanics, pressure acoustics, and thermoviscous acoustics are used for the simulation. The micro-perforated plate and the backing plate adopt the solid mechanics module. The background and transmission sound fields and the air gap are modeled using the pressure acoustics module. The air inside the micro-perforations and two thin air layers on both surfaces of the micro-perforation plate are modeled by the thermoviscous acoustics module, which is able to account for the viscous and thermal dissipation inside the thermoviscous boundary layers adjacent to the perforation. The thickness of both the thin thermoviscous boundary layers is equal to that of the heat transfer boundary layer at the lowest analyzed frequency,

$$d = \sqrt{\frac{2\kappa}{\rho_0 C_p \omega}},$$  

(34)

where $\kappa$ is the air thermal conductivity coefficient and $C_p$ is the air heat capacity at constant pressure.

A plane wave radiation is added to both the end boundaries to eliminate interface acoustic reflection. The simply supported boundary condition of the double plates is realized by constraining the border displacement along the $z$ direction. In order to simplify the calculation, the local resonators on the DM are equivalent to point loads,

$$F_{ij} = \frac{\omega^2 k_c,1 m_{ij}}{k_c,1 - \omega^2 m_{ij} + i\eta_{ij}},$$  

(35)

where the subscript $j = 1, 2$ means the attributes of the surface metamaterial and backing metamaterial.
V. RESULTS AND DISCUSSIONS

A. Acoustic characteristics and mechanisms

In order to illustrate the acoustic performance of the DM, we compare the absorption/reflection/transmission coefficients of the DM and the double-layer plate (DP). Both the results of the theoretical prediction and finite element simulation are shown in Fig. 4. In our analysis, both the plates are made of aluminum, and the material parameters are listed in Table I. The geometric parameters of the DM are shown in Table II. The DP does not contain any local resonators, and the geometric dimensions are the same as those of the DM. The parameters of the spring–mass–damping model of the local resonators of the DM are listed in Table III. The parameters of local resonator 1(2) are chosen based on the (1, 1) modal frequency of the micro-perforated(backing) plate. The total number of each type of local resonators is 25, i.e., \( J_1 = J_2 = 25 \). The parameters of the air are listed in Table IV.

It can be seen from Fig. 4 that the theoretical predictions of the sound absorption [Fig. 4(a)], reflection [Fig. 4(b)], and transmission [Fig. 4(c)] coefficients coincide well with the results of the finite element simulation for the DM and DP, which demonstrates that the theoretical analysis method is accurate. The little discrepancy between the theoretical prediction and the finite element simulation, on the one hand, comes from the average calculation (average pressure drop, air particles’ velocity in the perforation, and velocity of

| Parameters | Value (mm) |
|------------|------------|
| \( a \)    | 60         |
| \( b \)    | 60         |
| \( h_1 \)  | 0.5        |
| \( d \)    | 0.5        |
| \( a_0, b_0\) | 6          |
| \( D \)    | 40         |
| \( h_2 \)  | 0.6        |
| \( a_1, b_1\) | 12        |

| Parameters | Value |
|------------|-------|
| \( \rho \) (kg m\(^{-3}\)) | 2730  |
| \( E \) (Pa) | \( 6.9 \times 10^{10} \)  |
| \( \nu \) | 0.33  |
| \( \eta \) | 0.005 |

| Parameters | Value |
|------------|-------|
| \( m_{r,1} \) (kg) | \( 19.66 \times 10^{-6} \)  |
| \( k_{r,1} \) (N m\(^{-1}\)) | 348.44  |
| \( \eta_{r,1} \) | 0.05  |
| \( m_{r,2} \) (kg) | \( 23.59 \times 10^{-6} \)  |
| \( k_{r,2} \) (N m\(^{-1}\)) | 626.28  |
| \( \eta_{r,2} \) | 0.05  |

| Parameters | Value |
|------------|-------|
| \( \rho_0 \) (kg m\(^{-3}\)) | 1.21  |
| \( c_0 \) (m s\(^{-1}\)) | 343   |
| \( \eta \) (Pa) | \( 1.81 \times 10^{-5} \)  |
| \( \kappa \) (W m\(^{-1}\) K\(^{-1}\)) | 0.024  |
| \( C_p \) (J kg K\(^{-1}\)) | 1004  |
the plate frame) and approximation treatment during the derivation of the theoretical method. On the other hand, the little discrepancy comes from the calculation precision of the finite element method.

First, for the DP, it can be seen from Fig. 4(a) that there are two obvious absorption dips at frequencies of 670 Hz and 820 Hz, respectively. Both absorption dips are induced by the (1, 1) modal resonances of the micro-perforated plate and the backing plate. Considering the vibroacoustic coupling between the micro-perforated plate and the backing plate, the (1, 1) modal frequency of the micro-perforated plate is 670 Hz and the (1, 1) modal frequency of the backing plate is 820 Hz. The (1, 1) modal frequencies of the two plates are well consistent with the absorption dip frequencies of the DP. As verification, Fig. 5(a) shows the displacement pattern of the DP at 670 Hz, and it can be seen that the micro-perforated plate obviously shows the (1, 1) resonance mode while the displacement of the backing plate is small and the backing plate approximates a rigid state. Furthermore, one can see that the acoustic reflection coefficient of the DP at 670 Hz [Fig. 4(b)] has a peak, while the transmission coefficient is only 0.026 [Fig. 4(c), the theoretical value used here and below], which means that the incident sound is mainly reflected on the surface of the micro-perforated plate, and only small part of the incident sound enters the perforations and induces the damping and thermoviscous friction loss of the micro-perforated plate; therefore, the absorption dip appears. Under the condition of the backing plate being almost rigid, the DP can obtain a good absorption coefficient while its input impedance matches well with the air characteristic impedance; otherwise, the absorption performance is poor. Figure 6 further shows the input normalized impedance $Z$ (solid line) of the DP. Here, $Z = Z_{in}/\rho_0 c_0$, in which $Z_{in}$ is the input impedance of the DP. It shows that the acoustic resistance of the DP reaches the minimum value 0.13 at 670 Hz and the acoustic reactance is $-1.65$; therefore, the impedance of the DP seriously mismatches with the air characteristic impedance, and the DP shows the acoustic absorption dip at 670 Hz.

Figure 5(b) shows the displacement pattern of the DP at 820 Hz. It can be seen that the backing plate shows the (1, 1) resonance state, while the micro-perforated plate is approximately rigid. At 820 Hz, the reflection coefficient of the DP is 0.35 [Fig. 4(b)] and the transmission coefficient is 0.46 [Fig. 4(c)], which indicates that the acoustic energy reflection and transmission are relatively high under the
In order to improve the sound absorption and insulation performance of the DP, we attach local resonator 1(2) to the micro-perforated (backing) plate to adjust the impedance characteristic, which forms the designed DM. The basic idea is to adjust the resonant frequency of local resonator 1(2) approaching the (1, 1) modal frequency of the micro-perforated (backing) plate, i.e., adjust the resonant frequency of local resonator 1(2) to 670 Hz (820 Hz), which can suppress the modal resonance of the micro-perforated (backing) plate; then, the impedance characteristic of the DP is adjusted to improve the sound absorption and insulation performance. As can be seen from Fig. 4(a), the sound absorption of the DM (dashed-dotted line) is better than that of the DP, especially at the absorption dip frequencies of the DP. The average absorption coefficient from 372 Hz to 1000 Hz reaches 0.74, which is better than the average absorption coefficient (0.63) of the DP within the same frequency band. Moreover, as can be seen from Figs. 4(b) and 4(c), the DM effectively reduces the sound reflection of the DP at 670 Hz and the sound transmission at 820 Hz. Figure 7 further compares the STLs of the DM and DP. It shows that the DM obviously improves the STL minimum of the DP, i.e., increasing from 2.79 dB to 8.29 dB, and the average STL of the DM from 372 Hz to 1000 Hz reaches 19.06 dB, which is also better than that (16.55 dB) of the DP. 

To further analyze the acoustic mechanisms of the acoustic absorption and insulation of the DM, Fig. 8 shows the displacement patterns of the DM at 670 Hz and 820 Hz. It can be seen that the modal vibrations of the micro-perforated plate at 670 Hz and those of the backing plate at 820 Hz are greatly suppressed due to the additional local resonators; the latter result in the backing plate being approximately rigid at 820 Hz. We observe the normalized input impedance of the DM (dashed-dotted line) in Fig. 6. Compared with the DP at 670 Hz, the DM increases its acoustic resistance and decreases the absolute value of acoustic reactance due to the additional local resonators, matching the input impedance with the air characteristic impedance; then, the sound absorption is improved. At 820 Hz, the input acoustic resistance of the DM is 0.53, the acoustic reactance approaches 0, the matching of the input impedance with the characteristic impedance of the air becomes better, and the DM has better sound absorption than the DP.

B. The effect of porous material in the air gap

In order to further improve the sound absorption and insulation performance of the above DM, we filled the porous material into the gap in the DM. The Johnson–Champoux–Allard (JCA) model is used to model the porous material. The effective density \( \rho_{eq} \) and bulk modulus \( K_{eq} \) of the porous material are given by

\[
\rho_{eq} = \frac{\rho_0 \alpha_\infty}{\phi} \left[ 1 + i \frac{\sigma \phi}{\omega \rho_0 \alpha_\infty} \sqrt{1 + \frac{\omega \rho_0 \eta}{2 \sigma \phi} \left( \frac{2 \alpha_\infty}{\omega \rho_0 \eta} \right)^2} \right],
\]

(36)
\[
\frac{1}{K_{eq}} = \frac{\phi}{\gamma \rho_0} \left( \gamma - (\gamma - 1) \left[ 1 - i \frac{8\eta}{\omega \rho_0 \sigma \Lambda' \sqrt{\frac{\Lambda'}{4}}} \sqrt{1 + i \frac{\omega \rho_0 \sigma \Lambda' \sqrt{\frac{\Lambda'}{4}}}{\eta}} \right] \right)^{1/2},
\]

(37)

where five parameters such as the porosity \(\phi\), the flow resistivity \(\sigma\), the tortuosity \(\alpha_{\infty}\), the viscous length \(\Lambda\), and the thermal characteristic length \(\Lambda'\) are used for describing the properties of the porous material. \(\rho_0\) is the air static pressure, \(\gamma\) is the adiabatic constant, and \(Pr\) denotes the Prandtl number. The characteristic wavenumber of the porous material is

\[
k_c = \omega \sqrt{\rho_{eq}/K_{eq}}.
\]

(38)

Then, the characteristic impedance of the porous material is

\[
Z_c = \sqrt{\rho_{eq} K_{eq}}.
\]

(39)

Figure 9 compares the acoustic properties of the DM embedded with (black solid line) and without (red dashed-dotted line) the porous material. The acoustic parameters of the porous material in the analysis are shown in Table V. Compared to the DM without the porous material, (1) it can be seen from Fig. 9(a) that the lowest frequency of the effective absorption (absorption coefficient > 0.5) of the DM filled with the porous material is decreased from 372 Hz to 313 Hz and the absorption coefficient at the absorption dip frequencies has been improved and (2) it can be seen from Fig. 9(b) that the STL minimum is increased from 8.29 dB to 10.14 dB and the average STL in the 372 Hz–1000 Hz frequency band is increased from 19.06 dB to 20.76 dB. So, we can conclude that the porous material filled in the gap of the DM can improve the sound absorption and insulation, which is induced by the viscous friction between the air and the porous material; then, the dissipation energy is converted into heat.

### C. Practical realization

For practical implementation of the proposed DM, the influence of the number of local resonators attached to the DM on the acoustic performance is first considered, as shown in Fig. 10. The number \(J_1 = J_2 = 25, 9, 1\) of additional local resonators attached to each plate is labeled cases 1, 2, and 3, respectively, where the additional mass ratio is kept as \(\varepsilon_1 = \varepsilon_2 = 10\%\).
It can be seen from Fig. 10 that decreasing the number of local resonators from 25 to 9 (cases 1–2) has little effect on the absorption when the additional mass ratio is unchanged. However, when the number of local resonators is decreased to 1 (case 3), the first (black arrow) and second (red arrow) sound absorption dips move to low frequency, while the third (blue arrow) and fourth (green arrow) sound absorption dips move toward high frequency. Nevertheless, the average sound absorption coefficient for the whole frequency band has little change (0.635, 0.635, and 0.634 in cases 1, 2, and 3, respectively).

As can be seen from Fig. 10(b), there is little change in sound insulation while decreasing the number of local resonators from 25 to 9; however, the sound insulation above 650 Hz is much enhanced when using only one local resonator. To reveal the mechanism, one can see from Fig. 5 again that the displacement has the largest amplitude at the center of the micro-perforated (backing) plate without the local resonators and little vibration exists at the four sides. The position of the local resonators with the same mass has obviously different effect on the plate vibration suppression; with more mass of the local resonators closer to the plate center, the better vibration suppression with a wider frequency band can be acquired. For one local resonator (case 3), the vibration suppression is best while the additional mass is concentrated in the plate center. The best vibration suppression of the backing plate means the best sound insulation performance around the resonance frequency (820 Hz), as shown in Fig. 10(b). Furthermore, the better vibration suppression means the separation of the absorption dips is more obvious and the first (second) and third (fourth) sound absorption dips move to opposite directions, which is induced by the vibration suppression of the micro-perforated (backing) plate.

In practical applications, the additional mass ratio of the local resonators is often required to be as small as possible. Figure 11 compares the sound absorption and insulation of the DM with different additional mass ratios \( \varepsilon_1 = \varepsilon_2 = 10\%, 5\%, \) and 2.5\%. Here, case 3 is used. As the additional mass ratio decreases, the vibration suppression effect on the plates is weakened. So, the first and second sound absorption dips shift to high frequency, while the third and fourth sound absorption dips shift to low frequency, as shown in Fig. 11(a); these variations of the absorption dips can be expected based on the above analysis. However, the average sound absorption coefficient has little change (0.634, 0.631, and 0.629 for \( \varepsilon_1 = \varepsilon_2 = 10\%, 5\%, \) and 2.5\%, respectively). From Fig. 11(b), it can be seen that the sound insulation gradually deteriorates around 820 Hz.

For practical implementation, Fig. 12 shows the schematic diagram of the micro-perforated plate with one actual resonator. Steel and soft rubber cylinders in the actual resonator serve as the mass and spring of the above local resonator. There can also be a similar design for the backing plate.

With an additional mass ratio of 5\%, the geometrical parameters of actual resonator 1(2) on the micro-perforated (backing) plate are designed according to the model resonant frequency. Both the

| TABLE VI. Material parameters of the actual resonators. |
|---------|---------|-------|------|
| \( \rho \) (kg m\( ^{-3} \)) | \( E \) (Pa) | \( v \) | \( \eta \) |
| Steel   | 7850    | \( 2 \times 10^{11} \) | 0.30  | 0  |
| Rubber  | 1100    | \( 2 \times 10^{6} \)  | 0.49  | 0.05  |

| TABLE VII. Geometric parameters of the actual resonators. |
|------------|--------|--------|--------|
|             | \( h_k \) (mm) | \( d_k \) (mm) | \( h_S \) (mm) | \( d_S \) (mm) |
| Actual resonator 1 | 10.16  | 4      | 1      | 4      |
| Actual resonator 2  | 7.66   | 4.8    | 1      | 4.8    |
actual resonators use the same materials. The material and geometric parameters of the actual resonators are listed in Tables VI and VII, respectively. Figure 13 compares the sound absorption and insulation of the DM attached with the actual resonators to those acquired from the point load model. The sound absorption and insulation coefficients of the actual model coincide well with those of the point load model, which verifies the correctness of the point load model and the practical realization of the DM. The little discrepancy between the results of both the models is due to (1) the weak coupling between the actual resonators and the air and (2) that the additional mass ratio of the actual resonators is slightly less than 5%.

VI. CONCLUSIONS

The DM composed of the flexible micro-perforated plate and homogeneous backing plate has been designed, and both parallel plates are attached with different local resonators and separated by the air gap. Both the theoretical and finite element methods are used to investigate the acoustic properties of the DM and DP in the free field. Good agreement is shown between the results of the theoretical prediction and finite element simulation for the DM and DP. The results of both methods verified that the local resonators on the DM can improve the sound absorption and insulation of the DP, especially at the model frequencies of the DP. The acoustic impedance and displacement pattern are used to reveal the details of the acoustic mechanism of the DM, which shows that the tailoring of the acoustic impedance and the model resonance of the plates by the local resonators leads to the improvement of the sound absorption and insulation.

Filling the gap with the porous material can further improve the sound absorption and insulation, the absorption coefficient of the DM above 0.5 extends from 313 Hz to 1000 Hz, and the average STL in the same frequency band is increased to 20.76 dB. Finally, the effects of reduction of the number and the additional mass ratio of the local resonators on the sound absorption and insulation of the DM are revealed. The validity of the point load model and the DM implementation are verified using the actual resonator. The present design can offer an extension to multi-layer structures for noise control applications.

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DATA AVAILABILITY

The data that support the findings of this study are available within this article.

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FIG. 13. Sound absorption (a) and STL (b) of the point load model and actual model.
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