Coherent acceleration and Landau-Zener tunneling of Bose-Einstein condensates in 1-D optical lattices

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We have loaded Bose-Einstein condensates into one-dimensional, off-resonant optical lattices and accelerated them by chirping the frequency difference between the two lattice beams. For small values of the lattice well-depth, Bloch oscillations were observed. Landau-Zener tunneling out of the lowest lattice band, leading to a breakdown of the oscillations, was also studied. In order to allow in-trap measurements of these phenomena, we dynamically compensated for the intrinsic micromotion of the atoms in our time-orbiting potential trap.

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The properties of ultra-cold atoms in periodic light-shift potentials in one, two and three dimensions have been investigated extensively in the past ten years [1]. In near-resonant and, more recently, far-detuned optical lattices, a variety of phenomena have been studied, such as the magnetic properties of atoms in optical lattices, revivals of wave-packet oscillations, and Bloch oscillations in accelerated lattices [2]. While in most of the original optical lattice experiments the atomic clouds had temperatures in the micro-Kelvin range, corresponding to a few recoil energies of the atoms, atomic samples with sub-recoil energies are now routinely produced in Bose-Einstein condensation experiments. Since the first experimental realizations in 1995, many aspects of Bose-Einstein condensed atomic clouds (BECs) have been studied [3], ranging from collective excitations to superfluid properties and quantized vortices. So far, the majority of these experiments have been carried out essentially in harmonic-oscillator potentials provided by magnetic traps or optical dipole traps. The properties of BECs in periodic potentials constitute a vast new field of research, for instance [4][12]. Several experiments in the pulsed standing wave regime [13][14] as well as studies of the tunneling of BECs out of the potential wells of a shallow optical lattice in the presence of gravity [15], and recent search for the superfluid dynamics [16] have taken the first steps in that direction. In this paper, we present the results of experiments on BECs of $^{87}$Rb atoms in accelerated optical lattices. In particular, we demonstrate coherent acceleration of BECs adiabatically loaded into optical lattices. For small values of the lattice depth we observed Bloch oscillations, which exhibited Landau-Zener breakdown when the lattice depth was further reduced and/or the acceleration increased. We loaded the condensate into optical lattices with different spatial periods, generating the periodic optical lattice either from two counterpropagating laser beams or two laser beam encasing an angle $\theta$ different from 180 degrees.

The properties of a Bose-Einstein condensate located in a periodic optical lattice with depth $U_0$ are described through the Gross-Pitaevskii equation valid for the single-particle wavefunction. Because the nonlinear term in the Gross-Pitaevskii equation reproduces the spatial periodicity of the wavefunction, the condensate ground state is periodic. In agreement with the Bloch approach, the condensate excitation spectrum exhibits a band structure and in presence of an acceleration of the optical lattice, Bloch oscillations of the condensate should occur [6]. We present experimental results for Bloch oscillations preserving the condensate wavefunction. The role of the nonlinear interaction term of the Gross-Pitaevskii equation may be described through an effective potential in a noninteracting gas model [3][14]. In the perturbative regime of ref. [6] the effective potential is $U_{\text{eff}} = U_0/(1 + 4C)$ with $C = g/E_B$ the interaction ratio between the nonlinear interaction term $g = 4\pi\hbar^2/\hbar/Md^2$ and the lattice Bloch energy $E_B = \hbar^2(2\pi)^2/Md^2$. The parameter $C$ contains the condensate density $n_0$, the s-wave scattering length $a$, the atomic mass $M$, the lattice constant $d = \pi/\sin(\theta/2)k$, with $k$ the laser wavenumber, and $\theta$ the angle between the two laser beams creating the 1-D optical lattice. From this it follows that a small angle $\theta$ should result in a large interaction term $C$. In the following, the parameters $d$, $E_B$ and $C$ always refer to the respective lattice geometries with angle $\theta$.

Our apparatus used to achieve Bose-Einstein condensation of $^{87}$Rb is described in detail in [17]. Essentially, $5 \times 10^7$ atoms captured in a magneto-optical trap (MOT) were transferred into a triaxial time-orbiting potential trap (TOP) [18]. Subsequently, the atoms were evaporatively cooled down to the transition temperature for Bose-Einstein condensation, and after further cooling we obtained condensates of $\approx 10^4$ atoms without a discernible thermal component in a magnetic trap with frequencies around 15 – 30 Hz. In one set of experiments, the magnetic trap was then switched off and a horizontal 1-D optical lattice was switched on, while in the other case the interaction between the condensate and the lattice took place inside the magnetic trap, which was subsequently switched off to allow time-of-flight imag-
The lattice beams were created by a 50 mW diode slave-laser injected by a grating-stabilized master-laser blue-detuned by $\Delta \approx 28 - 35$ GHz from the $^{87}$Rb resonance line. After passage through an optical fibre, the laser light was split and passed through two acousto-optic modulators (AOMs) that were separately controlled by two phase-locked RF function generators operating at frequencies around 80 MHz, with a frequency difference $\delta$. The first-order output beams of the AOMs generated the optical lattice. An acceleration of the lattice was effected by applying a linear ramp to $\delta$. For the values of the detuning and laser intensity used in our experiment, the spontaneous photon scattering rate ($\approx 10 \text{s}^{-1}$) was negligible during the interaction times of a few milliseconds. In our experimental setup, we realized a counterpropagating lattice configuration and a lattice constant $d$ of 0.39 and 1.56 $\mu$m, respectively. The typical condensate density of $n_0 \approx 5 \times 10^{13}$ cm$^{-3}$ for our trap parameters in the Thomas-Fermi limit leads to $C = 0.01$ for the counterpropagating configuration and $C = 0.17$ for $\theta = 29$ deg $^{[21]}$. We therefore expect mean-field effects to be negligible in the counter-propagating lattice geometry, whereas in the geometry with a larger lattice constant the effective potential acting on the condensate should be significantly reduced. The two lattice beams with $3$ mW each were expanded to a waist of 1.8 mm, giving a theoretical maximum lattice depth (for $\Delta \approx 28$ GHz) of $U_0 \approx 4 E_B$ for the counterpropagating lattice geometry.

![Graph](image1.png)

**FIG. 1.** Rabi oscillations between two momentum states of a Bose-Einstein condensate. Shown here is the fraction $N_1/N$ of atoms in the $|p = 2\hbar k\rangle$ momentum state as a function of time. From the measured Rabi frequency $\Omega_R/2\pi \approx 3.6$ kHz we determined the lattice depth $U_0 = 2\hbar \Omega_R \approx 0.26 E_B$.

In a preliminary experiment aimed at determining the depth of the periodic optical potential, we flashed on the counterpropagating lattice with $\delta = \hbar k^2/(2\pi M) = 15.08$ kHz for $10 - 400 \mu$s. This detuning corresponds to the first Bragg resonance, causing the condensate to undergo Rabi oscillations between the momentum states $|p = 0\rangle$ and $|p = 2\hbar k\rangle$ (see Fig. 1). From the measured Rabi frequency we could then determine the lattice depth $^{[19]}$. The results of those measurements fell short by a factor of about 2 of the theoretical prediction, which was mainly due to the 10–15% uncertainty in our intensity measurements and imperfections in the alignment and polarization of the lattice beams. As shown above, mean-field effects are expected to be negligible in this geometry.

![Graph](image2.png)

**FIG. 2.** Coherent acceleration of a Bose-Einstein condensate. In (a)-(f) $U_0 = 0.29 E_B$, $a = 9.81 \text{ m} \text{s}^{-2}$, the condensate accelerated for 0.1, 0.6, 1.1, 2.1, 3.0 and 3.9 ms, respectively. In (g) the result of 2.5 ms acceleration with the same lattice depth as above, but with $a = 25 \text{ m} \text{s}^{-2}$. In this case, a fraction of the condensate undergoes Landau-Zener tunneling out of the lowest band each time a Bragg-resonance is crossed. The separations between the different spots vary because detection occurred at different time delays.

In order to accelerate the condensate, we adiabatically loaded it into the lattice by switching one of the lattice beams on suddenly and ramping the intensity of the other beam from 0 to its final value in 200 $\mu$s $^{[21]}$. Thereafter, the linear increase of the detuning $\delta$ provided a constant acceleration $a = \frac{\hbar}{2 \sin(\theta/2)} \frac{\delta}{dt}$ of the optical lattice, leading to a final lattice velocity $v_{\text{lat}} = \frac{\hbar}{2 \sin(\theta/2)} \delta$, where $\delta$ is the final detuning between the beams. After a few milliseconds of acceleration, the lattice beams were switched off and the condensate was imaged after another 10–15 ms of free fall. As the lattice can only transfer momentum to the condensate in units of the Bloch momentum $p_B = \hbar (2\pi/d)$, the acceleration of the condensate showed up as diffraction peaks corresponding to higher momentum classes as time increased (Fig. 2). Since for our magnetic trap parameters the initial momentum spread of the condensate (which is transferred into a spread of the lattice quasimomentum during an adiabatic switch-on) was much less than a recoil momentum of the optical lattice, the different momentum classes $|p = \pm np_B\rangle$ (where $n = 0, 1, 2, ...$) occupied by the condensate wavefunction could be resolved directly.
after the time-of-flight. The average velocity of the condensate was derived from the occupation numbers of the different momentum states. Figure 3 shows the results of the acceleration of a condensate in the counterpropagating lattice with \( U_{\text{eff}} = 0.29 E_B \) and \( a = 9.81 \text{ m s}^{-2} \). In the rest-frame of the lattice (Fig. 3 (b)), one clearly sees Bloch oscillations of the condensate velocity corresponding to a Bloch-period \( \tau_B = \frac{\hbar}{U_B a} \approx 1.2 \text{ ms} \). The shape of these oscillations agrees well with the theoretical curve calculated from the lowest energy band of the lattice. As described in [19], the acceleration process within a periodic potential can also be viewed as a succession of adiabatic rapid passages between momentum states \( \{ p = 2n \hbar k \} \) and \( \{ p = 2(n+1) \hbar k \} \). We observed a momentum transfer of up to 6\( P_B \) without a detectable reduction of the phase-space density of the condensate [22].

![Figure 3](image)

**FIG. 3.** Bloch oscillations of the condensate mean velocity \( v_m \) in an optical lattice. (a) Acceleration in the counterpropagating lattice with \( d = 0.39 \mu \text{m} \), \( U_0 \approx 0.29 E_B \) and \( a = 9.81 \text{ m s}^{-2} \). Solid line: theory. (b) Bloch oscillations in the rest frame of the lattice, along with the theoretical prediction (solid line) derived from the shape of the lowest Bloch band. (c) Acceleration in a lattice with \( d = 1.56 \mu \text{m} \), \( U_0 \approx 1.38 E_B \) and \( a = 0.94 \text{ m s}^{-2} \). In this case, the Bloch oscillations are much less pronounced. Dashed and solid lines: theory for \( U_0 = 1.38 E_B \) and \( U_{\text{eff}} \approx 0.88 E_B \).

When we increased the acceleration of the lattice or decreased the lattice depth, not all of the condensate was coherently accelerated up to the final velocity of the lattice. This can be interpreted in terms of Landau-Zener tunneling of the condensate out of the lowest band when the edge of the Brillouin zone is reached. Each time the condensate is accelerated across this edge, a fraction \( r = \exp(-a_c/a) \) with \( a_c = \frac{\pi U_{\text{eff}}}{8 \hbar P_B} \) undergoes tunneling into the first excited band (and, therefore, effectively to the continuum, as the gaps between higher bands are negligible for the shallow potentials used here). In Fig. 3 (a), the average velocity of the condensate after acceleration of the counter-propagating lattice to the Bloch velocity \( v_B = P_B / M \) is shown as a function of acceleration and lattice depth along with a theoretical prediction using the Landau-Zener tunneling probability, which gives a mean velocity \( v_m = (1-r)v_B \) of the condensate for a final velocity \( v_B \) of the lattice. When the lattice is accelerated up to a velocity \( n v_B \), a straightforward generalization of this equation yields \( v_m = v_B (1/r - 1) [1 - (1-r)^n] \) for the mean velocity of the condensate, which is in good agreement with our experimental findings (Fig. 3 (b)) [23]. In this case, a fraction \( r \) of the condensate undergoes Landau-Zener tunneling each time the Bragg resonance is crossed, with a remaining fraction \( 1-r \) of the condensate being accelerated further. Agreement with theory is also good when, instead of changing the acceleration, the lattice depth is varied at fixed acceleration (Fig. 3 (c)).

Similar experiments were performed in the geometry leading to the larger lattice constant of 1.56 \( \mu \text{m} \). Because of the reduced Bloch velocity \( v_B \) in this geometry, the acceleration process was extremely sensitive to any initial velocity of the condensate, which in our TOP trap is intrinsically given by the micromotion [13] at the frequency of the bias field. For the trap parameters used in our experiments, the velocity amplitude of the micromotion could be of the same order of magnitude as \( v_B \) and the condensates could, therefore, have quasimomenta close to the edge of the Brillouin zone. In fact, when the standing wave was flashed on as in the Rabi-oscillation measurement described above, Bragg diffraction could be observed for zero detuning between the lattice beams, with the diffraction efficiency depending on the initial velocity of the condensate. Moreover, an initial velocity close to the band edge would have made it impossible to switch on the lattice adiabatically. In order to counteract these effects, we performed the acceleration experiments inside the magnetic trap, eliminating the velocity of the condensate relative to the lattice by phase-modulating one of the lattice beams at the same frequency and in phase with the rotating bias field of the TOP trap. In this way, in the rest frame of the lattice the micromotion was compensated. Nevertheless, a residual sloshing of the condensate with amplitudes < 3 \( \mu \text{m} \) could not be ruled out, so that the uncertainty in the initial velocity of the condensate was still around 0.5 \( \text{mm s}^{-1} \), corresponding to \( \approx 0.3 v_B \) in this geometry. In Fig. 3 (c), the results of the acceleration of the condensate with a nominal lattice depth of 1.38 \( E_B \) are shown together with the theoretical curves for \( U_0 = 1.38 E_B \) and the (assumed) effective potential \( U_{\text{eff}} \approx 0.65 U_0 \approx 0.88 E_B \). The Landau-Zener tunneling probabilities measured in this lattice geometry were compatible with the same effective potential assuming that the conversion factor between theoretical and actual lat-
tice depth was the same as in the counter-propagating geometry in which $U_{\text{eff}} \approx U_0$. In order to demonstrate unequivocally the reduction of the effective lattice potential by the interaction term, however, it would be necessary to vary $n_0$ appreciably holding all other parameters constant, which in our experimental setup was not possible without creating systematic errors due to variations in the equilibrium position of the trap (and hence the local laser intensity in the Gaussian profile) when the trap frequency was changed. As the interaction term is expected to distort the band structure of the condensate in the lattice [4], it should affect all measurable quantities (Rabi frequency, amplitude of Bloch oscillations, and tunneling probability [6]) in the same way, so that a differential measurement is necessary (as has been demonstrated in the pulsed Bragg-diffraction regime of Ref. [4]). On the theoretical side, the finite extent of the condensate leading to the occupation of only a few lattice sites and the three-dimensional nature of the condensate evolution as well as the role of the interaction term in the adiabaticity criterion for switching on the lattice will also have to be taken into account.

In summary, we have investigated the coherent acceleration of Bose-Einstein condensates adiabatically loaded into a 1-D optical lattice as well as Bloch oscillations and Landau-Zener tunneling out of the lowest Bloch band. The results obtained are in good agreement with the available theories and extend the corresponding work on ultra-cold atoms in optical lattices into the domain of Bose-Einstein condensates.

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comparable condensate density. On the other hand, the large lattice constant also means that only a few potential wells are occupied by the condensate.

[21] The adiabaticity criterion of ref. [19] refers to interband transitions due to the rate of change of the band energies of non-interacting particles. This criterion was extended in [16] to the condensate on the basis of the band energies for the effective potential. However, other criteria such as collective excitations of the condensate [6] or the perturbation of the global condensate phase (C. Williams, private communication), may impose more stringent conditions but have not been considered here.

[22] We have verified that in the process of the acceleration and tunneling, the condensed fraction is not reduced. Our investigation did not, however, test the evolution of the condensate phase, but on the basis of the Bragg scattering experiments of [13] we assume that the interaction times of our experiment should not destroy the condensate phase.

[23] These expressions are only exact if the final velocity of the lattice is an integer multiple of $v_B$, as in that case the mean velocity of the condensate is independent of the band structure.