Impacts of the observed $\theta_{13}$ on the running behaviors of Dirac and Majorana neutrino mixing angles and CP-violating phases

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Abstract

The recent observation of the smallest neutrino mixing angle $\theta_{13}$ in the Daya Bay and RENO experiments motivates us to examine whether $\theta_{13} \simeq 9^\circ$ at the electroweak scale can be generated from $\theta_{13} = 0^\circ$ at a superhigh-energy scale via the radiative corrections. We find that it is difficult but not impossible in the minimal supersymmetric standard model (MSSM), and a relatively large $\theta_{13}$ may have some nontrivial impacts on the running behaviors of the other two mixing angles and CP-violating phases. In particular, we demonstrate that the CP-violating phases play a crucial role in the evolution of the mixing angles by using the one-loop renormalization-group equations of the Dirac or Majorana neutrinos in the MSSM. We also take the “correlative” neutrino mixing pattern with $\theta_{12} \simeq 35.3^\circ$, $\theta_{23} = 45^\circ$ and $\theta_{13} \simeq 9.7^\circ$ at a presumable flavor symmetry scale as an example to illustrate that the three mixing angles can receive comparably small radiative corrections and thus evolve to their best-fit values at the electroweak scale if the CP-violating phases are properly adjusted.

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I. INTRODUCTION

Since 1998, a number of successful neutrino oscillation experiments have provided us with very compelling evidence that neutrinos are massive and lepton flavors are mixed [1]. The latest Daya Bay [2] and RENO [3] reactor antineutrino oscillation experiments constitute another milestone, because their results convince us that the smallest neutrino mixing angle $\theta_{13}$ is not really small: its best-fit value is about 9° [4]. In comparison, the other two mixing angles $\theta_{12}$ and $\theta_{23}$ are about 34° and 45°, respectively [4]. These three angles appear in the standard parametrization of the $3 \times 3$ lepton flavor mixing matrix $V$,

$$V = \begin{pmatrix} 
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} 
\end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\
0 & e^{i\sigma} & 0 \\
0 & 0 & 1 \end{pmatrix}, \quad (1)$$

where $c_{ij} \equiv \cos \theta_{ij}$ and $s_{ij} \equiv \sin \theta_{ij}$ (for $ij = 12, 23$ and 13). The phase parameters $\rho$ and $\sigma$ are usually referred to as the Majorana CP-violating phases. If the massive neutrinos are the Dirac particles, $\rho$ and $\sigma$ will have no physical meaning and can be rotated away by rephasing the neutrino fields. At present the three CP-violating phases remain unknown, but one of them (i.e., the Dirac phase $\delta$) may hopefully be measured in the future long-baseline neutrino oscillation experiments since $\theta_{13}$ is already known not to be very small. The strength of leptonic CP violation in neutrino oscillations is governed by the rephasing-invariant Jarlskog parameter $J = c_{12}s_{12}c_{23}s_{23}s_{13}^{2}s_{13}\sin \delta$ [5], and that is why an appreciable value of $\theta_{13}$ is a good news for us to explore leptonic CP violation in the near future.

The fact that $\theta_{13}$ is not as small as previously expected motivates us to reconsider how it can be generated at the tree level or by quantum corrections [6]. In this paper we shall follow a model-independent way to look at the impacts of a relatively large $\theta_{13}$ on the running behaviors of the other two mixing angles and CP-violating phases for both Dirac and Majorana neutrinos by using the one-loop renormalization-group equations (RGEs) in the minimal supersymmetric standard model (MSSM). Our purpose is to find out the conditions which should be satisfied at a superhigh-energy scale where a flavor symmetry model of neutrino masses can be built, in order to obtain a phenomenologically favored neutrino mixing pattern at the electroweak scale. In section II we reexamine the one-loop RGEs of neutrino masses, mixing angles and CP-violating phase(s) by assuming a nearly degenerate neutrino mass spectrum. The running behaviors of the three mixing angles in the MSSM is numerically discussed in some detail 1. We pay particular attention to the crucial role of the CP-violating phases in the RGE evolution. Section III is devoted to the analysis of a special neutrino mixing pattern — the so-called “correlative” mixing pattern with $\theta_{12} \simeq 35.3^\circ$, $\theta_{23} = 45^\circ$ and $\theta_{13} \simeq 9.7^\circ$ [11], which satisfy the sum rule $\theta_{12} + \theta_{13} + \theta_{23} = 90^\circ$, as an example to illustrate that the three mixing angles can receive comparably small radiative corrections.

1In this work we focus on the MSSM because the three neutrino mixing angles can only receive much smaller radiative corrections in the framework of the standard model [7–9]. Moreover, the evolution of fermion masses and flavor mixing parameters in the standard model may suffer from its vacuum stability problem if the mass of the Higgs boson is about 125 GeV [10].
corrections and thus evolve to their best-fit values at the electroweak scale if the CP-violating phases are properly adjusted. A brief summary of our main results and concluding remarks is given in section IV.

II. ONE-LOOP RGES FOR DIRAC AND MAJORANA NEUTRINOS

A. The Dirac case

If the massive neutrinos are the Dirac particles, their Yukawa coupling matrix $Y_\nu$ must be extremely suppressed in magnitude to reproduce the light neutrino masses of $\mathcal{O}(1)$ eV or smaller at low energy scales. In the MSSM, the running of $Y_\nu$ from the electroweak scale $\Lambda_{EW}$ to a superhigh-energy scale $\Lambda$ is governed by the one-loop RGE [12,13]

$$16\pi^2 \frac{d\omega}{dt} = 2\alpha_d \omega + \left[(Y_iY_i^\dagger)\omega + \omega(Y_iY_i^\dagger)\right],$$

(2)

where $\omega \equiv Y_\nu Y_i^\dagger$, $t \equiv \ln(\mu/\Lambda)$ with $\mu$ being an arbitrary renormalization scale between $\Lambda_{EW}$ and $\Lambda$, $Y_i$ is the charged-lepton Yukawa coupling matrix, and $\alpha_d \approx -0.6g_1^2 - 3g_2^2 + 3y_t^2$. Here $g_1$ and $g_2$ are the gauge couplings, $y_t$ stands for the top-quark Yukawa coupling. In writing out Eq. (2), those tiny terms of $\mathcal{O}(\omega^3)$ have been safely omitted.

One may use Eq. (2) to derive the explicit RGEs of the three neutrino masses and four mixing parameters. The results can be found either in Ref. [12] where the standard parametrization of $V$ is adopted, or in Ref. [13] where the more convenient Fritzsch-Xing parametrization of $V$ [14] is used. To see the appreciable running effects, here we assume that the masses of the three neutrinos are nearly degenerate. Since neglecting the small contributions of $y_e$ and $y_\mu$ in Eq. (2) is always a good approximation, we shall also do so in our calculations. Given the near mass degeneracy $m_1 \approx m_2 \approx m_3$ together with the standard parametrization of $V$ in Eq. (1), the RGEs of $m_i$ (for $i = 1, 2, 3$) turn out to be

$$\frac{dm_1}{dt} \approx \frac{m_1}{16\pi^2} \left[\alpha_D + y_\tau^2 \left(c_{12}^2 s_{23}^2 - 2c_\delta c_{12} c_{23} s_{12} s_{23} s_{13} + \mathcal{O}(s_{13}^2)\right)\right],$$

$$\frac{dm_2}{dt} \approx \frac{m_2}{16\pi^2} \left[\alpha_D + y_\tau^2 \left(c_{12}^2 s_{23}^2 + 2c_\delta c_{12} c_{23} s_{12} s_{23} s_{13} + \mathcal{O}(s_{13}^2)\right)\right],$$

$$\frac{dm_3}{dt} \approx \frac{m_3}{16\pi^2} \left[\alpha_D + y_\tau^2 c_{23}^2 + \mathcal{O}(s_{13}^2)\right],$$

(3)

where $c_\delta \equiv \cos\delta$ and $y_\tau^2 = m_\tau^2(1 + \tan^2\beta)/v^2 \approx (1 + \tan^2\beta) \times 10^{-4}$ with $v \approx 174$ GeV and $\tan\beta$ being the ratio of the vacuum expectation values of the two Higgs doublets in the MSSM. The RGEs of $\theta_{ij}$ (for $ij = 12, 23, 13$) are found to be

$$\frac{d\theta_{12}}{dt} \approx \frac{-y_\tau^2}{8\pi^2} \left[\frac{m_1^2}{\Delta m_{21}^2} \left(c_{12}^2 s_{12} s_{23}^2 - \cos2\theta_{12} c_{23} s_{23} s_{13} c_\delta\right) - \frac{m_2^2}{\Delta m_{32}^2} c_{23} s_{23} s_{13} c_\delta + \mathcal{O}(s_{13}^2)\right] ,$$

$$\frac{d\theta_{23}}{dt} \approx \frac{-y_\tau^2}{8\pi^2} \frac{m_1^2}{\Delta m_{32}^2} c_{23} s_{23} + \mathcal{O}(s_{13}^2) ,$$

$$\frac{d\theta_{13}}{dt} \approx \frac{-y_\tau^2}{8\pi^2} \frac{m_1^2}{\Delta m_{32}^2} c_{23} c_{13} s_{13} + \mathcal{O}(s_{13}^2) .$$

(4)
In addition, the RGE of the CP-violating phase $\delta$ can be written as
\[
\frac{d\delta}{dt} \approx -\frac{y_\tau^2}{8\pi^2} \left( \frac{m_1^2}{\Delta m_{21}^2} + \frac{m_1^2}{\Delta m_{32}^2} \cos 2\theta_{12} \right) \frac{c_{23}s_{23}s_{13}s_\delta}{c_{12}s_{12}} + O(s_{13}^2),
\]  
(5)

where $s_\delta \equiv \sin \delta$ is defined. Note that the full RGE of $\delta$ actually contains the terms which are inversely proportional to $s_{13}$, but they are negligible in Eq. (5) for two reasons: (a) they are not significantly enhanced just because $\theta_{13}$ is not very small, as observed in the recent Daya Bay and RENO experiments; and (b) they are significantly suppressed by the factor $\Delta m_{32}^2/\Delta m_{21}^2$ in the case of $m_1 \approx m_2 \approx m_3$ under discussion. Finally, the RGE of the Jarlskog invariant $J$ is
\[
\frac{dJ}{dt} \approx -\frac{y_\tau^2}{8\pi^2} J \left\{ \frac{m_1^2}{\Delta m_{21}^2} \left[ \cos 2\theta_{12}s_{23}^2 + 4c_{12}s_{12}c_{23}s_{23}s_{13}c_\delta \right] + \frac{m_1^2}{\Delta m_{32}^2} \left( 3c_{23}^2 - 1 \right) + O(s_{13}^2) \right\}.
\]  
(6)

Some discussions are in order.

- Eq. (4) clearly tells us that the one-loop RGE of $\theta_{12}$ is dominated by the leading term $-\frac{y_\tau^2}{8\pi^2} \frac{m_1^2}{\Delta m_{21}^2} c_{12}s_{12}s_{23}^2$. As a consequence of $\Delta m_{21}^2 \ll |\Delta m_{32}^2|$, the solar mixing angle $\theta_{12}$ is more sensitive to radiative corrections than the other two angles $\theta_{13}$ and $\theta_{23}$. Our numerical analysis shows that $\theta_{12}$ may undergo an increase of about 15° to 20° in the MSSM with $\tan \beta = 10$ (denoted as “MSSM10” hereafter for short) or an increase of about 20° to 25° in the MSSM with $\tan \beta = 50$ (denoted as “MSSM50” hereafter for short) for arbitrary values of $\delta$, if it evolves from a flavor symmetry scale $\Lambda_{FS} \sim 10^{14}$ GeV down to the electroweak scale $\Lambda_{EW} \sim 10^2$ GeV.

- Running from $\Lambda_{FS}$ down to $\Lambda_{EW}$, the values of $\theta_{23}$ and $\theta_{13}$ can either increase or decrease, depending on the sign of $\Delta m_{32}^2$. Given the MSSM10 case for example, $\theta_{23}$ changes about 1.5° while $\theta_{13}$ changes about 0.2° from $\Lambda_{FS}$ to $\Lambda_{EW}$ (or vice versa). In the MSSM50 case, $\theta_{23}$ may change about 12° while $\theta_{13}$ changes about 6° from $\Lambda_{FS}$ to $\Lambda_{EW}$ (or vice versa). Note that the radiative corrections to $\theta_{23}$ and $\theta_{13}$ are almost independent of the CP-violating phase $\delta$, as one can easily see from Eq. (4).

- The one-loop RGE of $J$ is proportional to $J$ itself, and that of $\delta$ is proportional to $\sin \delta$. Hence the evolution of $J$ or $\delta$ does not undergo a sign flip. In other words, CP violation is an intrinsic property of the lepton flavor structure: if it is present (or absent) at a given energy scale, it must be present (or absent) at any other energy scales. Evolving from $\Lambda_{FS}$ down to $\Lambda_{EW}$, $|J|$ and $|\delta|$ (for $-\pi \leq \delta \leq \pi$) will increase in the MSSM.

In the next subsection we shall see that a relatively large $\theta_{13}$ has much more interesting phenomenological consequences provided the massive neutrinos are the Majorana particles.

**B. The Majorana case**

The masses of the Majorana neutrinos are believed to be attributed to some underlying new physics at a superhigh-energy scale $\Lambda$ (e.g., via the canonical seesaw mechanism [15]).
But this kind of new physics can all point to the unique dimension-5 Weinberg operator for the neutrino masses in an effective field theory after the corresponding heavy degrees of freedom are integrated out [16]. In the MSSM, such a dimension-5 operator reads
\[
\frac{\mathcal{L}_{d=5}}{\Lambda} = \frac{1}{2} H_L^c \cdot \kappa \cdot H_2^T \ell_L + \text{h.c.},
\]
where \(\Lambda\) denotes the cutoff scale, \(\ell_L\) stands for the left-handed lepton doublet, \(H_2\) is one of the MSSM Higgs doublets, and \(\kappa\) represents the effective neutrino coupling matrix. One may obtain the effective Majorana neutrino mass matrix \(M_\nu = \kappa \nu^2 \tan^2 \beta / (1 + \tan^2 \beta)\) after spontaneous gauge symmetry breaking. The cutoff scale \(\Lambda\) actually stands for the scale of new physics, such as the mass scale of the heavy Majorana neutrinos in the canonical seesaw mechanism [15]. The evolution of \(\kappa\) from \(\Lambda\) down to the electroweak scale \(\Lambda_{\text{EW}}\) is formally independent of any details of the relevant model from which \(\kappa\) is derived. Below \(\Lambda\) the scale dependence of \(\kappa\) is described by
\[
16\pi^2 \frac{d\kappa}{dt} = \alpha_M \kappa + \left[ (Y_l Y_1^\dagger) \kappa + \kappa (Y_l Y_1^\dagger)^T \right],
\]
at the one-loop level in the MSSM [7], where \(\alpha_M \approx -1.2g_1^2 - 6g_2^2 + 6g_3^2\).

One may use Eq. (8) to derive the explicit RGEs of the three neutrino masses and six flavor mixing parameters [8,9]. Given an approximate mass degeneracy of the three neutrinos together with the standard parametrization of \(V\) in Eq. (1), the RGEs of \(m_i\) (for \(i = 1, 2, 3\)) turn out to be
\[
\frac{dm_i}{dt} \approx \frac{m_i}{16\pi^2} \left[ \alpha_M + 2y_\tau^2 \left( s_{12}^2 s_{23}^2 - 2c_\delta c_{12}^{} c_{23}^{} s_{12}^{} s_{23}^{} s_{13}^{} + \mathcal{O}(s_{13}^2) \right) \right],
\]
\[
\frac{dm_2}{dt} \approx \frac{m_2}{16\pi^2} \left[ \alpha_M + 2y_\tau^2 \left( c_{12}^2 s_{23}^2 + 2c_\delta c_{12}^{} c_{23}^{} s_{12}^{} s_{23}^{} s_{13}^{} + \mathcal{O}(s_{13}^2) \right) \right],
\]
\[
\frac{dm_3}{dt} \approx \frac{m_3}{16\pi^2} \left[ \alpha_M + 2y_\tau^2 c_{23}^2 + \mathcal{O}(s_{13}^2) \right].
\]
The RGEs of \(\theta_{ij}\) (for \(ij = 12, 23, 13\)) are found to be
\[
\frac{d\theta_{12}}{dt} \approx -\frac{y_\tau^2}{4\pi^2} \left\{ \frac{m_1^2}{\Delta m_{21}^2} s_{23} \left[ (c_{12}^{} c_{13}^{} - \cos 2\theta_{12}^{} c_{23}^{} s_{13}^{} c_\delta^{}) c_{(\rho - \sigma)} + c_{23}^{} s_{13}^{} c_\delta^{} s_{(\rho - \sigma)} \right] c_{(\rho - \sigma)} 
- \frac{m_1^2}{\Delta m_{21}^2} c_{12}^{} c_{23}^{} s_{13}^{} \left( s_{12}^2 c_\delta^{} c_\rho^{} + c_{12}^{} c_\delta^{} c_\sigma^{} \right) + \mathcal{O}(s_{13}^2) \right\},
\]
\[
\frac{d\theta_{23}}{dt} \approx -\frac{y_\tau^2}{4\pi^2} \frac{m_1^2}{\Delta m_{32}^2} c_{23}^{} \left[ s_{23} \left( s_{12}^2 c_\rho^{} + c_{12}^2 c_\sigma^{} \right) - \frac{1}{2} c_{12}^{} s_{12}^{} c_{23}^{} s_{13}^{} \left( c_{(\rho+2\sigma)} - c_{(\rho+2\sigma)} \right) + \mathcal{O}(s_{13}^2) \right],
\]
\[
\frac{d\theta_{13}}{dt} \approx \frac{y_\tau^2}{8\pi^2} \frac{m_1^2}{\Delta m_{32}^2} c_{23}^{} c_{13}^{} \left[ c_{12}^{} s_{12}^{} c_{23}^{} s_{23}^{} \left( c_{(\rho+2\sigma)} - c_{(\rho+2\sigma)} \right) 
- 2 c_{23}^{} s_{13}^{} \left( c_{12}^2 c_\delta^{} c_\rho^{} + s_{12}^2 c_\delta^{} c_\sigma^{} \right) + \mathcal{O}(s_{13}^2) \right],
\]
in which \(c_x \equiv \cos x\) and \(s_x \equiv \sin x\) (for \( x = \delta, \rho, \sigma, \rho - \sigma, \delta + \rho, \delta + \sigma, \delta + 2\rho, \delta + 2\sigma \)).
The RGE of the three CP-violating phases \(\delta, \rho\) and \(\sigma\) can be written as
\[
\frac{d\delta}{dt} \approx -\frac{y_T^2}{4\pi^2} \left\{ \frac{m_1^2}{\Delta m_{21}^2} s_{23}^2 \left[ \left( s_{23} - \frac{\cos 2\theta_{12} c_{23} s_{13} c_\delta}{c_{12} s_{12}} \right) c_{(\rho-\sigma)} + \frac{c_{23} s_{13} s_\delta}{c_{12} s_{12}} s_{(\rho-\sigma)} + O(s_{13}^2) \right] s_{(\rho-\sigma)} \right. \\
- \frac{m_1^2}{\Delta m_{32}^2} s_{13}^{-1} \left[ \frac{1}{2} c_{12} s_{12} c_{23} s_{23} \left( s_{(\delta+2\rho)} - s_{(\delta+2\rho)} \right) + \left( c_{\rho} s_\rho c_{12} + c_{\sigma} s_\sigma s_{12}^2 \right) c_{23}^2 s_{13} \\
+ \left( c_{(\delta-\rho)} s_{(\delta-\rho)} s_{12}^2 + c_{(\delta-\sigma)} s_{(\delta-\sigma)} c_{12}^2 \right) \cos 2\theta_{23} s_{13} + O(s_{13}^2) \right] \left. \right\},
\]

\[
\frac{d\rho}{dt} \approx \frac{y_T^2}{4\pi^2} \left\{ \frac{m_1^2}{\Delta m_{21}^2} s_{23}^2 c_{12}^2 \left[ \left( s_{23} - \frac{\cos 2\theta_{12} c_{23} s_{13} c_\delta}{c_{12} s_{12}} \right) c_{(\rho-\sigma)} + \frac{c_{23} s_{13} s_\delta}{c_{12} s_{12}} s_{(\rho-\sigma)} + O(s_{13}^2) \right] s_{(\rho-\sigma)} \right. \\
- \frac{m_1^2}{\Delta m_{32}^2} s_{13}^{-1} \left[ \left( c_{(\delta-\rho)} s_{(\delta-\rho)} s_{12}^2 + c_{(\delta-\sigma)} s_{(\delta-\sigma)} c_{12}^2 \right) \cos 2\theta_{23} s_{13} + O(s_{13}^2) \right] \left. \right\},
\]

\[
\frac{d\sigma}{dt} \approx \frac{y_T^2}{4\pi^2} \left\{ \frac{m_1^2}{\Delta m_{21}^2} s_{23}^2 c_{12} \left[ \left( s_{23} - \frac{\cos 2\theta_{12} c_{23} s_{13} c_\delta}{c_{12} s_{12}} \right) c_{(\rho-\sigma)} + \frac{c_{23} s_{13} s_\delta}{c_{12} s_{12}} s_{(\rho-\sigma)} + O(s_{13}^2) \right] s_{(\rho-\sigma)} \right. \\
- \frac{m_1^2}{\Delta m_{32}^2} s_{13}^{-1} \left[ \left( c_{(\delta-\rho)} s_{(\delta-\rho)} s_{12}^2 + c_{(\delta-\sigma)} s_{(\delta-\sigma)} c_{12}^2 \right) \cos 2\theta_{23} s_{13} + O(s_{13}^2) \right] \left. \right\}. \tag{11}
\]

In addition, the RGE of \( J \) is obtained as follows:

\[
\frac{d}{dt} J \approx -\frac{y_T^2}{8\pi^2} \left\{ \frac{m_1^2}{\Delta m_{21}^2} \left[ J \cos 2\theta_{12} s_{23} - \cos^2 2\theta_{12} c_{23}^2 s_{13}^2 s_{13}^2 c_\delta s_\delta \right] \\
+ \frac{m_1^2}{\Delta m_{32}^2} J \cos 2\theta_{23} + O(s_{13}^3) \right\}. \tag{12}
\]

Because of \( J \propto \sin \delta \), the running of \( J \) is also proportional to \( \sin \delta \). This situation is similar to the evolution of \( J \) in the Dirac case. But now the evolution of \( \delta \) is nonlinearly entangled with the evolution of \( \rho \) and \( \sigma \) as shown in Eq. (11), so the Majorana case is more complicated than the Dirac case.

The running behaviors of the three mixing angles and three CP-violating phases can be very different for a very small \( \theta_{13} \) and for a relative large \( \theta_{13} \). In particular, the CP-violating phases play a crucial role in the RGEs. Let us elaborate on this point in the following.

1. The running behaviors of the three mixing angles

Eq. (10) shows that the RGE running behaviors of the three neutrino mixing angles are strongly dependent on the three CP-violating phases. As for the Majorana neutrinos, the radiative corrections to the three mixing angles can be adjusted by choosing different values of the CP-violating phases \( \delta, \rho \) and \( \sigma \). To illustrate this observation, let us carry out an explicit numerical analysis. We choose \( \theta_{12} = 34^\circ, \theta_{23} = 46^\circ, \theta_{13} = 9^\circ, \delta = 90^\circ \) and \( \sigma = 30^\circ \) as the typical inputs at \( \Lambda_{\text{EW}} \) and allow \( \rho \) to vary from \( 0^\circ \) to \( 180^\circ \). Then we look at their numerical evolution to \( \Lambda_{\text{FS}} \) via the RGEs. FIGs. 1 and 2 show the possible ranges of the three mixing angles at \( \mu > \Lambda_{\text{EW}} \) (gray areas) for both normal and inverted neutrino mass hierarchies, where \( m_1 \sim 0.2 \text{ eV} \) has typically been input at \( \Lambda_{\text{EW}} \). The dashed (\( \Delta m_{23}^2 > 0 \)) and dotted-dashed (\( \Delta m_{23}^2 < 0 \)) lines in these figures represent the corresponding running
behaviors of the three mixing angles for the Dirac neutrinos (with the same input values of \(\theta_{12}, \theta_{23}, \theta_{13}\) and \(\delta\)).

The fact that the RGE of \(\theta_{12}\) is dominated by the term \(-\frac{y_{\tau}^2}{8\pi^2} \frac{m_1^2}{\Delta m_{12}^2} c_{12} s_{12} c^2_{\rho-\sigma}\) implies that the magnitude of the radiative correction to \(\theta_{12}\) depends strongly on the phase difference \((\rho - \sigma)\). Hence \(\theta_{12}\) is most sensitive to the RGE effect when \(\rho \simeq \sigma\) holds. FIG. 1 shows that in the MSSM10 case the resulting \(\theta_{12}\) at \(\Lambda_{FS}\) lies in a wide range (from 10° to 35° associated with the variation of \(\rho\)). While in the MSSM50 case the resulting \(\theta_{12}\) at \(\Lambda_{FS}\) lies in a wider range (from 7° to 55° if \(\Delta m_{23}^2 > 0\), or from 2° to 31° if \(\Delta m_{23}^2 < 0\)). If all the three CP-violating phases are freely adjusted, the allowed range of \(\theta_{12}\) at \(\Lambda_{FS}\) will become much wider (from 0.5° to 62° if \(\Delta m_{23}^2 > 0\), or from 2° to 45° if \(\Delta m_{23}^2 < 0\)).

Running from \(\Lambda_{FS} \sim 10^{14}\) GeV down to \(\Lambda_{EW}\), the mixing angles \(\theta_{23}\) and \(\theta_{13}\) receive less significant radiative corrections. They may change one or two degrees in the MSSM10 case, as shown in FIG. 1. In the MSSM50 case shown in FIG. 2, \(\theta_{23}\) may increase or decrease in the range of 22° to 35° between the scales \(\Lambda_{EW}\) and \(\Lambda_{FS}\), whereas the change of \(\theta_{13}\) lies in the range of 4.1° to 22.5° (for \(\Delta m_{23}^2 > 0\)) or in the range of 13.0° to 30.3° (for \(\Delta m_{23}^2 < 0\)). If all the three CP-violating phases vary freely, the resulting \(\theta_{23}\) at \(\Lambda_{FS}\) lies in the range of 7.5° to 45.5° (for \(\Delta m_{23}^2 > 0\)) or in the range of 46.5° to 89° (for \(\Delta m_{23}^2 < 0\)), while the resulting \(\theta_{13}\) at \(\Lambda_{FS}\) lies in the range of 2° to 23° (for \(\Delta m_{23}^2 > 0\)) or in the range of 7.5° to 82° (for \(\Delta m_{23}^2 < 0\)). One can see that it is impossible to generate \(\theta_{13} \approx 9°\) at \(\Lambda_{EW}\) from \(\theta_{13} \approx 0°\) at \(\Lambda_{FS}\) via the radiative corrections. This observation is true even in the MSSM50 case.

We have seen that the values of the three CP-violating phases are crucial for the evolution of the three mixing angles. A very special case is \((\rho - \sigma) \simeq \pm 90°\), which leads us to

\[
\begin{align*}
\frac{d\theta_{12}}{dt} &\approx \frac{y_{\tau}^2}{4\pi^2} \frac{m_1^2}{\Delta m_{12}^2} c_{23} s_{23} s_{13} \left( s_{12} c_{(\delta+\rho)} c_{\rho} + c_{12} s_{(\delta+\rho)} s_{\rho} \right), \\
\frac{d\theta_{23}}{dt} &\approx -\frac{y_{\tau}^2}{4\pi^2} \frac{m_2^2}{\Delta m_{23}^2} c_{23} \left( s_{23} \left( s_{12} c_{\rho}^2 + c_{12} s_{\rho}^2 \right) - c_{12} s_{12} c_{23} s_{13} c_{(\delta+2\rho)} \right), \\
\frac{d\theta_{13}}{dt} &\approx \frac{y_{\tau}^2}{4\pi^2} \frac{m_3^2}{\Delta m_{32}^2} c_{23} c_{13} \left[ c_{12} s_{12} s_{23} c_{(\delta+2\rho)} - c_{23} s_{13} \left( c_{12} c_{(\delta+\rho)}^2 + s_{12} s_{(\delta+\rho)}^2 \right) \right].
\end{align*}
\tag{13}
\]

Note that the term proportional to \(m_2^2/\Delta m_{32}^2\) in the RGE of \(\theta_{12}\) in Eq. (10) is suppressed by \(\cos(\rho - \sigma) \simeq 0\) in this special case, and thus it has been omitted from Eq. (13). The three mixing angles may therefore receive comparably small radiative corrections for a modest value of \(\tan \beta\) (e.g., in the MSSM10 case). This observation was not noticed in the literature simply because \(\theta_{13}\) used to be assumed to be very small \([8,9]\). If \(\tan \beta\) is sufficiently large (e.g., in the MSSM50 case), however, the phase difference \((\rho - \sigma)\) will be able to quickly run away from its initial value \((\rho - \sigma) \sim \pm 90°\) due to the significant radiative corrections, implying that Eq. (13) is no more a good approximation of Eq. (10).

2. The radiative generation of the CP-violating phases

It is well known that one CP-violating phase can be generated from another \([17]\), simply because they are entangled in the RGEs. An especially interesting example is the Dirac phase
\( \delta \), which measures the strength of CP violation in neutrino oscillations at the electroweak scale, can be radiatively generated from the nonzero Majorana phases \( \rho \) and \( \sigma \) at a superhigh-energy scale. To illustrate, we present two numerical examples in Tables I and II to show that it is possible to radiatively generate \( \delta \) and one of the two Majorana phases from the other Majorana phase. If \( \theta_{13} \) is very small, however, the running of \( \delta \) can be significantly enhanced by the terms that are inversely proportional to \( \sin \theta_{13} \). In the MSSM10 case it has been found that even \( \delta = 90^\circ \) can be radiatively generated if \( \theta_{13} \approx 1^\circ \) is taken \cite{17}. In our numerical calculation we require \( \theta_{13} \approx 9^\circ \) at \( \Lambda_{EW} \). Thanks to the RGE running effects, we find that \(-30^\circ \leq \delta \leq 30^\circ \) at \( \Lambda_{EW} \) may result from \( \delta = 0^\circ \) at \( \Lambda_{FS} \) in the MSSM10 case. In the MSSM50 case even \( |\delta| \approx 90^\circ \) can be obtained at \( \Lambda_{EW} \), as shown in Table II.

3. The running of the sum \( \delta + \rho + \sigma \)

Eq. (11) leads us to the RGE of the sum of the three CP-violating phases:

\[
\frac{d}{dt}(\delta + \rho + \sigma) \approx \frac{y_T^2}{4\pi^2} \frac{m_1^2}{\Delta m_{32}^2} \frac{1}{s_{13}} \left[ \frac{1}{2} c_{12} s_{12} c_{23} s_{23} \left( s_{\delta + 2\rho} - s_{\delta + 2\sigma} \right) \right.
\]

\[
- \left( c_{(\delta - \rho)} s_{(\delta - \rho)} s_{12}^2 + c_{(\delta - \sigma)} s_{(\delta - \sigma)} c_{12}^2 \right) \cos 2\theta_{23} s_{13}
\]

\[
+ \left( c_{\rho} s_{\rho} c_{12}^2 + c_{\sigma} s_{\sigma} s_{12}^2 \right) c_{23}^2 s_{13} + O(s_{13}^2) \right].
\]

(14)

Since the value of \( \theta_{13} \) is not small, the RGE running effect on \( (\delta + \rho + \sigma) \) is expected to be insignificant. In other words, the sum of the three CP-violating phases approximately keeps unchanged during the RGE evolution in the standard model or MSSM with a modest \( \tan \beta \). Our numerical analysis shows that \( (\delta + \rho + \sigma) \) changes less than 4\(^\circ\) when running from \( \Lambda_{FS} \) down to \( \Lambda_{EW} \) in the MSSM10 case. The stability of \( (\delta + \rho + \sigma) \) against the radiative corrections is quite impressive, unless \( \tan \beta \) is sufficiently large.

4. On the normal and inverted mass hierarchies

In all the above discussions, we have assumed that the three neutrino masses are nearly degenerate. For the purpose of completeness, here we give a brief discussion on the RGE effects by considering the hierarchical neutrino mass spectrum. The radiative corrections to the three mixing angles in both normal \( (m_1 < m_2 < m_3) \) and inverted \( (m_3 < m_1 < m_2) \) neutrino mass hierarchies are less significant than those in the case of a nearly degenerate neutrino mass spectrum. Here we focus on two special but instructive cases: i) the normal hierarchy with \( m_1 \approx 0 \) and ii) the inverted hierarchy with \( m_3 \approx 0 \). We choose \( \theta_{12} = 34^\circ, \theta_{23} = 46^\circ, \theta_{13} = 9^\circ, \delta = 90^\circ, \rho = 60^\circ \) and \( \sigma = 30^\circ \) as the typical inputs at \( \Lambda_{EW} \) and study their RGE running behaviors to \( \Lambda_{FS} \). The corresponding outputs at \( \Lambda_{FS} \) in both the MSSM10 case and the MSSM50 case are summarized in Table III. In the case of the normal hierarchy with \( m_1 \approx 0 \), the resulting values of the three mixing angles at \( \Lambda_{FS} \) are all close to their values at \( \Lambda_{EW} \), implying that the RGE running effects are insignificant. As for the inverted hierarchy with \( m_3 \approx 0 \), the evolution of \( \theta_{23} \) and \( \theta_{13} \) is also insignificant, but that of \( \theta_{12} \) is appreciable in the MSSM10 case and quite significant in the MSSM50 case.
Now let us make a brief summary. In order to obtain a phenomenologically-favored neutrino mixing pattern at the electroweak scale $\Lambda_{EW}$, we have examined the corresponding mixing pattern at a superhigh-energy scale $\Lambda_{FS}$ which might result from a certain flavor symmetry. In the MSSM10 case the values of $\theta_{23}$ and $\theta_{13}$ predicted at $\Lambda_{FS}$ are always close to their running values at $\Lambda_{EW}$, while the value of $\theta_{12}$ at $\Lambda_{FS}$ can be somewhat smaller or larger than its running value at $\Lambda_{EW}$. In the MSSM50 case the allowed ranges of the three mixing angles at $\Lambda_{FS}$ can be quite wide, as we have discussed above. However, a crucial point is that a given flavor symmetry model should be able to predict the appropriate CP-violating phases at $\Lambda_{FS}$ in order to obtain the appropriate mixing angles at $\Lambda_{EW}$ after the RGE evolution. We find that it is in general impossible to generate $\theta_{13} \approx 9^\circ$ at $\Lambda_{EW}$ from $\theta_{13} \approx 0^\circ$ at $\Lambda_{FS}$ through the radiative corrections, unless some new degrees of freedom or nontrivial running effects (such as the seesaw threshold effects [9]) are taken into account. This observation is consistent with the discussions in Ref. [18].

III. AN EXAMPLE: THE CORRELATIVE MIXING PATTERN

In this section we consider the correlative neutrino mixing pattern with $\theta_{12} \approx 35.3^\circ$, $\theta_{23} = 45^\circ$ and $\theta_{13} \approx 9.7^\circ$ [11]. The three mixing angles in this constant scenario satisfy two interesting sum rules,

$$\theta_{12} + \theta_{13} = \theta_{23} , \quad \theta_{12} + \theta_{13} + \theta_{23} = 90^\circ .$$

(15)

The latter sum rule is geometrically illustrated in FIG. 3. The corresponding lepton flavor mixing matrix is [11]

\[
V = \begin{pmatrix}
\frac{\sqrt{2} + 1}{3} & \frac{\sqrt{2} + 1}{3\sqrt{2}} & \frac{\sqrt{2} - 1}{\sqrt{6}} e^{-i\delta} \\
\frac{1}{\sqrt{6}} - \frac{\sqrt{2} - 1}{3\sqrt{2}} e^{i\delta} & \frac{1}{\sqrt{3}} - \frac{\sqrt{2} - 1}{6} e^{i\delta} & \frac{\sqrt{2} + 1}{2\sqrt{3}} \\
\frac{1}{\sqrt{6}} - \frac{\sqrt{2} - 1}{3\sqrt{2}} e^{i\delta} & \frac{1}{\sqrt{3}} - \frac{\sqrt{2} - 1}{6} e^{i\delta} & \frac{\sqrt{2} + 1}{2\sqrt{3}}
\end{pmatrix}
\begin{pmatrix}
e^{i\rho} & 0 & 0 \\
0 & e^{i\sigma} & 0 \\
0 & 0 & 1
\end{pmatrix},
\]  

(16)

which might be derived from a certain underlying flavor symmetry in a neutrino mass model at a superhigh-energy scale.

Compared with the best-fit values of the three mixing angles at $\Lambda_{EW}$ given in Ref. [4], the three mixing angles in this correlative mixing pattern at $\Lambda_{FS}$ have to receive comparably small radiative corrections during their RGE evolution. As we have mentioned in the last section, this requirement can easily be achieved in the MSSM10 case provided the condition $(\rho - \sigma) \approx \pm 90^\circ$ is satisfied for a nearly degenerate neutrino mass spectrum. Such a condition is unnecessary if the neutrino mass spectrum has a strong hierarchy.

To illustrate, we study the RGE evolution of $V$ in Eq. (16) and present our results in Table IV. We input $\delta = -68^\circ$, $\rho = 13^\circ$ and $\sigma = 115^\circ$ at $\Lambda_{FS}$ for example, and then obtain the phenomenologically-favored results $\theta_{12} = 34.52^\circ$, $\theta_{23} = 45.98^\circ$ and $\theta_{13} = 8.83^\circ$ at $\Lambda_{EW}$.
after the radiative corrections. If the three CP-violating phases can be specified in a given flavor symmetry model at $\Lambda_{FS}$, however, the running behaviors of the three mixing angles will be more restrictive.

Given the RGEs in the framework of the standard model [7,8], the evolution of $\theta_{12}$, $\theta_{13}$ and $\theta_{23}$ in the correlative mixing pattern is insignificant but their running directions may be opposite to those in the MSSM case (depending on the CP-violating phases). If the experimental error bars of the three mixing angles turn to be sufficiently small in the future, it might be possible to see which framework is more suitable for the radiative corrections to the correlative neutrino mixing pattern.

**IV. SUMMARY**

In view of $\theta_{13} \simeq 9^\circ$ as observed in the recent Daya Bay and RENO experiments, we have reexamined the radiative corrections to the lepton flavor mixing matrix for both Dirac and Majorana neutrinos by considering both nearly degenerate and strongly hierarchical neutrino mass spectra in the framework of the MSSM. Two typical values of $\tan \beta$ (i.e., 10 and 50) have been taken in our numerical calculations. We conclude that it is difficult to generate $\theta_{13} \simeq 9^\circ$ at $\Lambda_{EW}$ from $\theta_{13} \simeq 0^\circ$ at $\Lambda_{FS}$ through the radiative corrections, unless a sufficiently large value of $\tan \beta$ is assumed or the seesaw threshold effects or some new degrees of freedom are taken into account. Therefore, we argue that it is more natural for a flavor symmetry model to predict a relatively large $\theta_{13}$ at $\Lambda_{FS}$. To illustrate this point, we have briefly discussed the correlative mixing pattern with $\theta_{12} \simeq 35.3^\circ$, $\theta_{23} = 45^\circ$ and $\theta_{13} \simeq 9.7^\circ$ as an example of this kind.

Let us remark that fixing $\theta_{13} \simeq 9^\circ$ and taking the correlative neutrino mixing pattern in our RGE analysis just serve for illustration. It is certainly possible to generate the values of $\theta_{13}$ within the present experimental limits (but not necessarily close to $9^\circ$) from $\theta_{13} \simeq 0^\circ$ at a superhigh-energy scale in the MSSM with an appropriate value of $\tan \beta$ or with the help of some new degrees of freedom. However, we stress that $\theta_{13}$ might initially be nonzero and its appreciable value might have a significant impact on the running behaviors of the other two mixing angles and CP-violating phases.

Our study clearly shows that a measurement of the Dirac CP-violating phase $\delta$ in the forthcoming long-baseline oscillation experiments and any experimental information about the Majorana CP-violating phases $\rho$ and $\sigma$ are extremely important, so as to distinguish one flavor symmetry model from another through their different sensitivities to the radiative corrections. This observation makes sense in particular after the experimental errors associated with the neutrino mixing parameters are comparable with or smaller than the magnitudes of their respective RGE running effects.

We are grateful to P. Minkowski, W. Rodejohann and H. Zhang for useful discussions. The work of S.L. is supported in part by the National Basic Research Program (973 Program) of China under Grant No. 2009CB824800, the National Natural Science Foundation of China under Grant No. 11105113, the Fujian Provincial Natural Science Foundation under Grant No. 2011J05012 and the China Postdoctoral Science Foundation funded project under Grant No. 201104340. The work of Z.Z.X. is supported in part by the National Natural Science Foundation of China under grant No. 11135009.
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FIG. 1. Running behaviors of the three mixing angles $\theta_{12}$, $\theta_{23}$ and $\theta_{13}$ from $\Lambda_{\text{FS}} \sim 10^{14}$ GeV to $\Lambda_{\text{EW}} \sim 10^2$ GeV in the MSSM10 case. We have taken $\theta_{12} = 34^\circ$, $\theta_{23} = 46^\circ$, $\theta_{13} = 9^\circ$, $\delta = 90^\circ$ and $\sigma = 30^\circ$ as the typical inputs at $\Lambda_{\text{EW}}$ for the Majorana neutrinos, and allowed $\rho$ to vary from $0^\circ$ to $180^\circ$. The gray areas show the possible ranges of the three mixing angles at $\mu > \Lambda_{\text{EW}}$. The dashed ($\Delta m_{32}^2 > 0$) and dotted-dashed ($\Delta m_{32}^2 < 0$) lines represent the corresponding running behaviors of the Dirac neutrinos with the same inputs at $\Lambda_{\text{EW}}$. 

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FIG. 2. Running behaviors of the three mixing angles $\theta_{12}$, $\theta_{23}$ and $\theta_{13}$ from $\Lambda_{\text{FS}} \sim 10^{14}$ GeV to $\Lambda_{\text{EW}} \sim 10^2$ GeV in the MSSM50 case. We have taken $\theta_{12} = 34^\circ$, $\theta_{23} = 46^\circ$, $\theta_{13} = 9^\circ$, $\delta = 90^\circ$ and $\sigma = 30^\circ$ as the typical inputs at $\Lambda_{\text{EW}}$ for the Majorana neutrinos, and allowed $\rho$ to vary from $0^\circ$ to $180^\circ$. The gray areas show the possible ranges of the three mixing angles at $\mu > \Lambda_{\text{EW}}$. The dashed ($\Delta m^2_{32} > 0$) and dotted-dashed ($\Delta m^2_{32} < 0$) lines represent the corresponding running behaviors of the Dirac neutrinos with the same inputs at $\Lambda_{\text{EW}}$. 
FIG. 3. A geometrical description of the sum rules $\theta_{12} + \theta_{13} = \theta_{23}$ and $\theta_{12} + \theta_{13} + \theta_{23} = 90^\circ$ for the correlative neutrino mixing pattern in terms of the inner angles of the right triangle $\triangle ABC$. 

$\theta_{13} \simeq 9.7^\circ$

$\theta_{12} \simeq 35.3^\circ$

$\theta_{23} = 45^\circ$
TABLE I. Examples of the radiative generation of the CP-violating phases from $\Lambda_{FS} \sim 10^{14}$ GeV to $\Lambda_{EW} \sim 10^{2}$ GeV in the MSSM10 case.

| Parameter                  | Example I |            | Example II |            |
|----------------------------|-----------|------------|------------|------------|
| $m_1$ (eV)                 | 0.227     | 0.200      | 0.227      | 0.200      |
| $\Delta m^2_{21}$ (10^{-5} eV^2) | 20.47    | 7.59       | 21.09      | 7.59       |
| $\Delta m^2_{31}$ (10^{-3} eV^2) | 3.22     | 2.40       | 3.22       | 2.40       |
| $\theta_{12}$              | 25.1°     | 33.97°     | 23.7°      | 33.95°     |
| $\theta_{23}$              | 43.5°     | 45.99°     | 44.7°      | 45.97°     |
| $\theta_{13}$              | 7.9°      | 8.78°      | 9.0°       | 8.85°      |
| $\delta$                   | 0°        | 26.05°     | 0°         | -26.50°    |
| $\rho$                     | 55°       | 36.52°     | 0°         | 17.92°     |
| $\sigma$                   | 0°        | -5.65°     | 50°        | 55.44°     |
| $\delta + \rho + \sigma$  | 55°       | 56.92°     | 50°        | 46.86°     |

TABLE II. Examples of the radiative generation of the CP-violating phases from $\Lambda_{FS} \sim 10^{14}$ GeV to $\Lambda_{EW} \sim 10^{2}$ GeV in the MSSM50 case.

| Parameter                  | Example I |            | Example II |            |
|----------------------------|-----------|------------|------------|------------|
| $m_1$ (eV)                 | 0.245     | 0.200      | 0.245      | 0.200      |
| $\Delta m^2_{21}$ (10^{-5} eV^2) | 160.20   | 7.57       | 671.70     | 7.61       |
| $\Delta m^2_{31}$ (10^{-3} eV^2) | 16.03    | 2.40       | 10.10      | 2.39       |
| $\theta_{12}$              | 10.86°    | 34.01°     | 15.77°     | 34.03°     |
| $\theta_{23}$              | 7.50°     | 46.06°     | 44.03°     | 46.00°     |
| $\theta_{13}$              | 2.88°     | 9.04°      | 12.99°     | 9.01°      |
| $\delta$                   | 0°        | 91.48°     | 0°         | 87.93°     |
| $\rho$                     | 100°      | 9.83°      | 0°         | -62.65°    |
| $\sigma$                   | 0°        | -4.75°     | 95°        | 84.09°     |
TABLE III. Radiative corrections to the neutrino masses and flavor mixing parameters from \( \Lambda_{EW} \sim 10^2 \) GeV to \( \Lambda_{FS} \sim 10^{14} \) GeV in the MSSM with \( \tan \beta = 10 \) or 50.

| Parameter | Normal hierarchy with \( m_1 \simeq 0 \) | Inverted hierarchy with \( m_3 \simeq 0 \) |
|-----------|----------------------------------------|----------------------------------------|
|           | Input (\( \Lambda_{EW} \)) | Output (\( \Lambda_{FS} \)) | Input (\( \Lambda_{EW} \)) | Output (\( \Lambda_{FS} \)) |
| \( m_1 (eV) \) | \( 10^{-6} \) | \( 1.13 \times 10^{-6} \) | \( 1.24 \times 10^{-6} \) | \( 0.049 \) | \( 0.053 \) | \( 0.059 \) |
| \( \Delta m^2_{21} (10^{-5} \text{ eV}^2) \) | \( 7.59 \) | \( 9.80 \) | \( 12.23 \) | \( 7.59 \) | \( 9.28 \) | \( 55.10 \) |
| \( \Delta m^2_{31} (10^{-3} \text{ eV}^2) \) | \( 2.40 \) | \( 3.10 \) | \( 3.97 \) | \( -2.40 \) | \( -2.82 \) | \( -3.50 \) |
| \( \theta_{12} \) | \( 34.0^\circ \) | \( 33.98^\circ \) | \( 33.28^\circ \) | \( 34.0^\circ \) | \( 31.76^\circ \) | \( 14.06^\circ \) |
| \( \theta_{23} \) | \( 46.0^\circ \) | \( 45.96^\circ \) | \( 44.32^\circ \) | \( 46.0^\circ \) | \( 46.04^\circ \) | \( 47.75^\circ \) |
| \( \theta_{13} \) | \( 9.0^\circ \) | \( 9.00^\circ \) | \( 8.99^\circ \) | \( 9.0^\circ \) | \( 9.01^\circ \) | \( 9.26^\circ \) |
| \( \delta \) | \( 90^\circ \) | \( 90.01^\circ \) | \( 90.19^\circ \) | \( 90^\circ \) | \( 87.10^\circ \) | \( 29.18^\circ \) |
| \( \rho \) | \( 60^\circ \) | \( 60.02^\circ \) | \( 60.97^\circ \) | \( 60^\circ \) | \( 62.04^\circ \) | \( 113.93^\circ \) |
| \( \sigma \) | \( 30^\circ \) | \( 30.00^\circ \) | \( 29.95^\circ \) | \( 30^\circ \) | \( 30.86^\circ \) | \( 39.05^\circ \) |

TABLE IV. Radiative corrections to the correlative neutrino mixing pattern from \( \Lambda_{FS} \sim 10^{14} \) GeV to \( \Lambda_{EW} \sim 10^2 \) GeV in the MSSM10 case.

| Parameter | Input (\( \Lambda_{FS} \)) | Output (\( \Lambda_{EW} \)) |
|-----------|-----------------|-----------------|
| \( m_1 (eV) \) | \( 0.227 \) | \( 0.200 \) |
| \( \Delta m^2_{21} (10^{-5} \text{ eV}^2) \) | \( 15.72 \) | \( 7.59 \) |
| \( \Delta m^2_{31} (10^{-3} \text{ eV}^2) \) | \( 3.19 \) | \( 2.40 \) |
| \( \theta_{12} \) | \( 35.3^\circ \) | \( 34.52^\circ \) |
| \( \theta_{23} \) | \( 45^\circ \) | \( 45.98^\circ \) |
| \( \theta_{13} \) | \( 9.7^\circ \) | \( 8.83^\circ \) |
| \( \delta \) | \( -68^\circ \) | \( -80.88^\circ \) |
| \( \rho \) | \( 13^\circ \) | \( 19.64^\circ \) |
| \( \sigma \) | \( 115^\circ \) | \( 118.03^\circ \) |
| \( \delta + \rho + \sigma \) | \( 60^\circ \) | \( 56.79^\circ \) |