ON THE OPTICAL PULSATIONS FROM THE GEMINGA PULSAR

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ABSTRACT

We present a model for the generation mechanism of the optical pulsations recently detected from the Geminga pulsar. We argue that the optical emission is just synchrotron radiation emitted along the open magnetic field lines at altitudes of a few light cylinder radii (which requires that Geminga is an almost aligned rotator), where charged particles acquire nonzero pitch angles as a result of the cyclotron absorption of radio waves in the magnetized pair plasma. This explains self-consistently both the apparent lack of radio emission, at least at frequencies higher than about 100 MHz, and the nature of pulsed optical emission from the Geminga pulsar. Our model implies that the power of the synchrotron radiation from Geminga peaks in the infrared band, which suggests that this pulsar should also be a source of pulsed infrared emission.

Subject headings: pulsars: individual (Geminga) — radiation mechanisms: nonthermal

1. INTRODUCTION

An optical counterpart of the Geminga source was found by Bignami et al. (1987), based upon color considerations. Martin et al. (1998) obtained an optical spectrum of Geminga and concluded that its visual flux exceeds the Rayleigh-Jeans extrapolation of its thermal soft X-rays (with the neutron star surface temperature \( \approx 5 \times 10^5 \) K) by about an order of magnitude (Halpern et al. 1996). This spectrum shows a continuous power law \( f_\lambda \propto \lambda^{-0.8 \pm 0.5} \) from 3700 to 8000 \( \AA \) with a broad absorption-like feature over the 6300–6500 \( \AA \) band (Martin et al. 1998), thus indicating existence of the dominant nonthermal synchrotron component in the optical emission of the Geminga source. Martin et al. (1998) demonstrated that the spectrum (excluding 6400 \( \AA \)) can also be fitted by a composite model consisting of a nonthermal power law with an index \( 1.9 \pm 0.5 \) and a Rayleigh-Jeans blackbody (see their Fig. 5).

Recently Shearer et al. (1998), based on their deep integrated images in \( B \) band, claimed that the optical counterpart of Geminga pulses with a period \( P = 237 \) ms, reported also at X-ray (Halpern & Holt 1992) and \( \gamma \)-ray (Bertch et al. 1992) bands. They derived a magnitude of the pulsed emission \( m_B = 26.0 \pm 0.4 \) mag and found that there is a phase agreement of the optical data with \( \gamma \)-ray and hard X-ray curves. According to Shearer et al. (1998), the form of the optical light curve resembles that of \( \gamma \)-ray and hard X-rays rather than the soft X-ray signature, which is further evidence of the predominantly magnetospheric origin of Geminga’s pulsed optical emission (Halpern et al. 1996) over a thermal one (Bignami et al. 1996). The same conclusion was drawn by Shearer et al. (1999) after detailed analysis of the pulsed component, namely that “modifications to existing phenomenological models are required to take account of emission primarily near the light cylinder.”

Harlow et al. (1998) reported results of NICMOS observations of Geminga and PSR 0656 + 14. They showed that the infrared fluxes of both objects grow with increasing wavelength and claimed that, most likely, the infrared emission is generated in the pulsars’ magnetospheres.

There is an evidence that the Geminga pulsar is either radio quiet or, more likely, visible only at low radio frequencies, below about 100 MHz (Malofeev & Malov 1997; Vats et al. 1999; Kuz’mín & Losovskii 1999). An interpulse of comparable intensity to that of the main pulse, at about a half the period, was claimed in Geminga’s radio profile by Kuzmin (1998), Vats et al. (1999), and Kuz’mín & Losovskii (1999). It is worth noting here that, according to Shearer et al. (1998), the optical signal also shows two peaks with a phase separation of \( \approx 0.5 \). In Gil et al. (1998, hereafter Paper I), we proposed a model explaining why Geminga is radio quiet at high radio frequencies, based on the cyclotron resonance of radio waves with the plasma particles in the distant magnetosphere of this pulsar. More generally, we demonstrated in Paper I that in the case of an almost aligned rotator (which is, in our opinion, the case in the Geminga pulsar) the absorption of the pulsar’s almost whole radio emission by means of the cyclotron resonance cannot be avoided. Below in this paper we further explore this idea and demonstrate that the resonant damping of radio waves induces significant pitch angles of the background plasma particles. This inevitably leads to the synchrotron reemission of the damped radio energy into the infrared band, with a power-law continuation at optical frequencies. Thus, we suggest that Geminga’s pulsed optical emission is synchrotron radiation of the plasma particles gyrating about a relatively weak magnetic field of the pulsar’s remote magnetosphere.

2. ESTIMATION OF PITCH ANGLES

Mikhailovskii et al. (1982) were first to suggest that the energy lost by radio waves as a result of cyclotron damping should lead to “heating” of the ambient plasma, i.e., increase of its perpendicular temperature. Lyubarskii & Petrova (1998) treated thoroughly spontaneous reemission of the absorbed energy and concluded that in short-period pulsars a significant fraction of this energy is reemitted in the far-infrared band. They found that an electron (or
The radio luminosity of Geminga, by which we mean the integrated radio power of this pulsar as it would be in the absence of pulsar damping on radio waves (damped as a result of cyclotron resonance) to the total kinetic energy of the resonant particles produced per second above a fraction of the polar cap with the area of the particle momentum along and across the local magnetic field, respectively.

Here \( \theta \) is an angle between the wavevector \( \mathbf{k} \), and the local magnetic field \( \mathbf{B} \), and

\[
\eta = \frac{L_r}{L_{p1}}, \tag{2}
\]

is the ratio of the radio luminosity to the particle luminosity, that is, the ratio of the power of radio waves the components of the particle momentum along and across the local magnetic field, respectively.

\[
\psi \approx \theta \psi^{0.5} \text{rad}, \tag{1}
\]

in the course of cyclotron absorption of radio emission. Here \( \theta \) is an angle between the wavevector \( \mathbf{k} \), and the local magnetic field \( \mathbf{B} \), and

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number of particles (determined by the Sturrock multiplication factor \( \kappa \)) among which the energy of the absorbed waves is distributed.

3. SYNCHROTRON MODEL

The cyclotron absorption in the electron-positron plasma is a collective process. On the other hand, spontaneous reemission can be considered in terms of a single particle synchrotron radiation formalism. This is because, as we estimate below (see eq. [30], and also Table 1 and Fig. 4), the average length of emitted waves is shorter than the characteristic distance between plasma particles, that is,

\[ \lambda n^{1/3} < 1, \]

where \( n \) is a number density of particles. That is why we neglect collective effects in our calculations below.

The total power of synchrotron radiation emitted by a single electron is

\[ L_0 \approx 1.6 \times 10^{-15} B^2 \gamma_p^2 \sin^2 \psi \text{[ergs s}^{-1}], \quad (10) \]

and the critical frequency of synchrotron radiation is

\[ v_c \approx 4.2 \times 10^6 B \gamma_p^2 \sin \psi \text{[Hz]} \quad (11) \]

(Ginzburg 1979). This reemission frequency \( v_c \) is much higher than the frequency of cyclotron damping \( (v_d) \approx (2\pi) \omega_B \gamma_p^{-1}(1 - \cos \theta)^{-1} \), where \( \omega_B = (eB/mc) \); see Khechinashvili et al. 2000).

The spectral density of a single electron radiation power near \( v_c \) is

\[ I_0(v_c) \approx \frac{L_0}{v_c} \approx 3.8 \times 10^{-22} B \sin \psi \text{[ergs s}^{-1} \text{Hz}^{-1}] . \quad (12) \]

Let us notice that, if a source as a whole is moving toward an observer, the power given by equation (10) should be divided by \( \sin^2 \psi \) (see eq. [5.12] on p. 75 in Ginzburg 1979, and the discussion therein). However, this is not the case in our current application, as the emitting volume represented in Figure 1 is quasi-stationary with respect to an observer on Earth. Indeed, the particles are permanently flowing in and out but the volume as the whole does not approach an observer with a relativistic velocity. That is why the multiplier \( 1/\sin^2 \psi \) should be omitted in this consideration.

The above formulae for a single-particle synchrotron radiation are valid if the condition

\[ \psi \gamma_p \gg 1, \quad (13) \]

is fulfilled. This corresponds to relativistic transverse momenta. Below we assume that the pith angles are still not very large, so that \( \sin \psi \approx \psi \). This seems like a reasonable assumption.

The average magnetic field in the region where synchrotron emission is generated (Fig. 1) can be estimated as

\[ \langle B \rangle = \frac{1}{\mathcal{A} - \mathcal{A}_1} \int_{\mathcal{A}_1}^{\mathcal{A}_2} B(\mathcal{A})d\mathcal{A}, \quad (14) \]

where

\[ B(\mathcal{A}) \approx 9.4 \times 10^{-2.5} \frac{\mathcal{P}^{0.5}}{\mathcal{A}^{1.5}} \frac{1}{\sin \alpha} \mathcal{A}^{-3}[G]. \quad (15) \]

Substitution of \( \mathcal{A}_1 \approx 1.1 \) and \( \mathcal{A}_2 \approx 2.4 \) into equation (14) and integration yields \( B \approx 3.3 \times 10^3 \text{ G} \). This, in combination with equation (8), provides the critical frequency (eq. [11])

\[ v_c \approx 5.9 \times 10^{10} \kappa^{-0.5} \gamma_p^{1.5} \text{[Hz]} . \quad (16) \]

Note that \( v_m \approx 0.29 v_c \) is the frequency corresponding to the maximum in the single electron synchrotron spectrum.

It is natural to assume that the spectral density of total power radiated by \( N \) electrons is

\[ I(v) = NI_0(v). \quad (17) \]

Let us then estimate \( N \), i.e., the total number of emitting particles. From Figure 1, assuming the conservation of the particle flux along the dipolar fields lines, we find that in the truncated cone with the height \( h = l \cos \rho_1 = (\mathcal{A}_2 - \mathcal{A}_1) \mathcal{A}_c \cos \rho_1 \approx 1.3 \times 10^9 \text{ cm} \) there are \( N \approx F_{p1} h \approx 4 \times 10^{30} \text{ particles} \) (particles where \( F_{p1} \) is defined from eq. [6]) at any given moment of time. Then, from equations (17), (12), and (8) we obtain

\[ I(v_c) = 2.2 \times 10^{-13} \kappa^{0.5} \gamma_p^{-0.5} \text{ ergs s}^{-1} \text{ Hz}^{-1}, \quad (18) \]

which is a theoretical value of spectral density near the critical frequency \( v_c \). Let us compare it with that provided by the observations.

As it was mentioned in § 1, the optical spectrum of the Geminga pulsar can be fitted by either the flat power law with index \( \alpha = 0.8 \pm 0.5 \) (inconsistent with a Rayleigh-Jeans spectrum) or the combined power law with \( \alpha = 1.9 \pm 0.5 \) and the blackbody (Martin et al. 1998). If the latter is the case then, within our model, we should fit only a component with the spectral density of flux

\[ f(v) = 5.6 \times 10^{-29} \left( \frac{v}{10^{12}} \right)^{-1.9} \text{[ergs cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1}] \quad (19) \]

(see Fig. 5 in Martin et al. 1998). This provides an observed spectral density of power

\[ I_{\text{obs}}(v) = \zeta f(v)d^2[\text{ergs s}^{-1} \text{ Hz}^{-1}], \quad (20) \]

where \( \zeta \) is a beaming factor and \( d \approx 5 \times 10^{20} \text{ cm} \) is a distance to the Geminga pulsar. From Figure 1, using features of the dipolar geometry, we find that \( \phi \approx 1.5 \rho_1 \approx 0.8 \text{ rad} \), which leads to the beaming factor \( \zeta \approx \phi^2 \approx 0.6 \text{sr} \) (instead

### Table 1

| Parameters and Important Conditions of the Synchrotron Model |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( \alpha \)    | \( \gamma_p \)  | \( v_c \)       | \( I(v_c) \)    |
| 1.9........    | 470             | 0.043           | 1.3 \( \times \) | 4.3 \( \times \) |
| 0.8........    | 47              | 0.14            | 1.3 \( \times \) | 1.4 \( \times \) |

Note.—Parameters and important conditions of the synchrotron model calculated for \( \kappa = 20 \) in the two cases of observed spectrum fit.
of $4\pi$ for an isotropic radiation). Hence, equation (20) yields

$$I_{\text{obs}}(v) \approx 8.4 \times 10^{12} \left(\frac{v}{10^{14}}\right)^{-1.9} \text{[ergs s}^{-1} \text{Hz}^{-1}] \, .$$

(21)

Evaluating the latter equation for the critical frequency (eq. [16]) and setting it equal to our theoretical value (eq. [18]), i.e., $I_{\text{obs}}(v) = I(v_c)$, we obtain the following condition,

$$\kappa^{-0.45} \gamma_p^{0.235} \approx 5.5 \times 10^2 \, .$$

(22)

The latter restricts available sets of $\kappa$ and $\gamma_p$ (in other words, the plasma density and an average energy of its particles) for which our model provides an optical flux comparable with the observed value.

However, one cannot exclude that the observed optical spectrum of Geminga is exclusively magnetospheric in origin. In such a case we should fit the following spectral density of flux:

$$f(v) = 7.5 \times 10^{-30} \left(\frac{v}{10^{14}}\right)^{-0.8} \text{[ergs cm}^{-2} \text{s}^{-1} \text{Hz}^{-1}] \, .$$

(23)

A numerical coefficient in the latter expression for the spectrum has been chosen such in order to provide an observed flux of $f(v_B) = 1.6 \times 10^{-30}$ ergs cm$^{-2}$ s$^{-1}$ Hz$^{-1}$ at the frequency $v_B = 6.9 \times 10^{14}$ Hz (Martin et al. 1998). Repeating the same procedure as in the previous case, we obtain, that the observed spectral density of power

$$I_{\text{obs}}(v) = 1.1 \times 10^{12} \left(\frac{v}{10^{14}}\right)^{-0.8} \text{[ergs s}^{-1} \text{Hz}^{-1}] \, ,$$

(24)

calculated for the critical frequency $v_c$ (eq. [16]), matches our theoretical value $I(v_c)$ (eq. [18]), if the condition

$$\kappa^{0.1} \gamma_p^{0.7} \approx 20 \,,$$

(25)

is fulfilled.

The spectral density of power at the critical frequency $I(v_c)$ is presented in Figure 2 as the dashed and the dotted lines, corresponding to the cases of $\alpha = 1.9$ and $\alpha = 0.8$, respectively (see eqs. [18], [22], and [25]). Both lines are marked with a label “1.”

Apparently, the model of synchrotron radiation presented above should be consistent with the energy conservation. Indeed, as we demonstrate below, the consideration based on the energy conservation provides an independent test of our model, as well as restricts the set of available free parameters ($\kappa$ and $\gamma_p$). Obviously, the fraction of the synchrotron optical radiation of Geminga observed on Earth is emitted in the same direction as the low-frequency radio waves (Paper I). Using the feature of dipolar field lines that all of them intersect any given radial line with the same angle, we can find a geometrical place of all the points in the magnetosphere where the tangents to the field lines point to the same direction$^6$ as a wavevector $k_\parallel$ of the 100 MHz radio waves. As it is seen in Figure 3, these points are placed at the radial straight lines inclined to $\mu$ at an angle $\delta \approx 13^\circ$ from the side of the nearest approach to $\mu$ and $\beta + 2\delta \approx 20^\circ$ from the furthest one. The “optically bright” region is restricted in the longitudinal direction to the altitudes along these lines, where the cyclotron damping of 0.1–1 GHz radio waves occurs. Calculations show (Khechinashvili 1999$^5$) that the longitudinal size of this region $l_\parallel \approx R_{LC}$. The dimension of this region in the transverse direction can be estimated as $l_\perp \approx R_c/\gamma_p \approx R_{LC}/\gamma_p$, where $R_c$ is a curvature radius of magnetic field lines at a corresponding altitude. Such an estimation of $l_\perp$ assumes that this is a characteristic distance at which a particle moving along the curved magnetic field line keeps emitting toward the observer (i.e., an observer stays in the emission diagram of a single particle,

$^6$ $\beta = 20^\circ$ at the nearest approach of the line of sight to the magnetic axis $\mu$, and $\beta + 2\delta = 30^\circ$ at the furthest one.
within the angle range of $\sim 1/\gamma_p$). The corresponding characteristic time is $\tau_\perp \approx 1/c \times 4 \times 10^{-2} \gamma_p^2$ s, during which a particle radiates $E_\parallel = L_\parallel \tau_\perp$ energy. Obviously, the latter should be much less than the kinetic energy of an electron $E_e = \gamma_p^2 m c^2$.

Let us notice that in such a consideration the multiplier $1/\sin^2 \psi$ (see the discussion below eq. [12]) should be taken into account in equation (10), as the source of the synchrotron radiation (a particle) is moving toward the observer in this case. Using equation (10) with the latter correction we find that $\chi \equiv E_\parallel/E_e \sim 8 \times 10^{-4} \leq 1$, which means that only a tiny fraction of the particle’s kinetic energy is radiated away as a synchrotron emission. Assuming that this is valid for all the radiating particles, we can estimate the total radiated power of synchrotron radiation as $L = \chi L_{\parallel \perp} \approx 6.4 \times 10^{22} \kappa \gamma_p$ ergs s$^{-1}$ (where $L_{\parallel \perp}$ is found from eq. [5]). Finally, using equation (16), we obtain

$$I(\nu) \approx \frac{L}{\nu_c} \approx 1.1 \times 10^{12} \kappa^{1.5} \gamma_p^{-0.5} \text{ergs s}^{-1} \text{Hz}^{-1}. \quad (26)$$

Let us then first assume that the synchrotron spectrum is determined by $\alpha = 1.9$ (eq. [19]). Setting the value given by equation (26) equal to the appropriate observational value of power spectral density at the critical frequency $I_{\text{obs}}(\nu_c)$ (see eqs. [21] and [16]), we obtain the following condition,

$$\kappa^{0.55} \gamma_p^{2.35} = 1.1 \times 10^7. \quad (27)$$

Naturally, for all the values of $\kappa$ and $\gamma_p$ satisfying this condition, the observational and theoretical estimations (let us recall that the latter comes from the energy conservation considerations) match each other.

The same procedure but in the case of spectrum with $\alpha = 0.8$ (eq. [23]), using equations (26) and (24), yields the following condition:

$$\kappa^{1.1} \gamma_p^{0.7} = 380. \quad (28)$$

The spectral density of power calculated from equation (26), (using eqs. [27] and [28]) in order to exclude $\gamma_p$ is plotted in Figure 2 for the two cases of the observed spectrum fit. The dashed line corresponds to the spectral index $\alpha = 1.9$, and the dotted line corresponds to $\alpha = 0.8$. Both lines are marked by the label “2.” In this figure we see that each of the latter lines intersects the corresponding line (i.e., with the same $\alpha$) marked by “1” at the point where $\kappa \approx 20$. This means that the spectral density of power estimated from the energy conservation (eq. [26]) is consistent with our previous estimation (eq. [18]) only provided that the Sturrock multiplication factor is very low. On the other hand, for the “conventional” values of the magnetospheric plasma density, corresponding to $\kappa \sim 10^3$–$10^7$, the two estimations of the spectral density diverge strongly in both cases of the observed spectrum fit (see Fig. 2). Hence, the model does not work in a dense plasma. Therefore, it requires a nonuniform distribution of magnetospheric plasma, presuming existence of large domains where plasma density is reduced significantly. Magnetospheric models based on the nonstationary plasma outflow from the pulsar imply existence of such domains. The regions between the spark-associated columns of plasma (e.g., Ruderman & Sutherland 1975; Gil & Sendyk 2000) provide an example of such “empty” spaces.

Let us check the fulfillment of the condition (9) for the waves of length $\lambda_e = c/\nu_c$. The number density of the magnetospheric plasma writes

$$n_p = \kappa n_G \left(\frac{R_0}{R}\right)^3 \approx 0.64 \, P^{-3.5} \rho^{0.5} \frac{k}{\sin \alpha} \mathcal{R}^{-3} \text{[cm}^{-3}] . \quad (29)$$

Evaluating $n_p$ from equation (29) for $\mathcal{R} \approx 1.1$ and the dynamical parameters of the Geminga pulsar, and using equation (16), we can rewrite the condition (9) as

$$\lambda_e \mathcal{R}^{1/3} \approx 7 \kappa^{0.83} \gamma_p^{-1.5} < 1. \quad (30)$$

Obviously, if this condition is fulfilled for $\lambda_e$ and $\mathcal{R}$, for all $\lambda < \lambda_e$, and $\mathcal{R} > \mathcal{R}_e$, it should be fulfilled even better. For example, at the visible wavelength $\lambda_B = 4.4 \times 10^{-5}$ cm the condition (9) writes

$$\lambda_B n_p^{1/3} \approx 6.2 \times 10^{-4} \kappa^{1/3} < 1, \quad (31)$$

which is very well satisfied for $\kappa = 20$.

Table 1 summarizes all important physical values and conditions of the synchrotron model for both cases of a putative spectrum (i.e. with $\alpha = 0.8$ or $\alpha = 1.9$), where the mean Lorentz factor of plasma particles (eqs. [22] and [25]), the average pitch angle (eq. [8]), the critical frequency (eq. [16]), the spectral density of power (eqs. [21] and [24]), as well as condition (13) and (30) were evaluated for $\kappa = 20$. From this table it follows that our model works in both cases of the power-law fit of the spectrum, and the critical frequency of the synchrotron emission falls into infrared band (note that in the case of $\alpha = 1.9$ the critical frequency is in fact very close to the optical band). According to Table 1, both crucial conditions are well satisfied, which justifies the formalism used in our model.

We would like to stress that the calculations presented above should be regarded as rather estimative, so in fact there is a considerable range of the Sturrock multiplication factor values (near $\kappa = 20$) for which the model provides observed fluxes. At the same time, the important requirement of our model concerning the significantly low plasma density ($\kappa \leq 100$) in the region of synchrotron reemission certainly remains valid. The results of our calculations for a range of low $\kappa$-values are presented in Figure 4.

In the calculations presented above we assume that both pitch angles $\psi$ and Lorentz factors $\gamma_p$ of emitting particles remain constant while the particles pass through the “optically bright” region. The Lorentz factors obviously stay constant because $\chi \ll 1$. As for pitch angles, their stability requires that a single particle goes through at least few acts of radio absorption during the time of $\tau_\perp = t_{\text{b}}/c \approx 0.04$ s. This is equivalent to fulfillment of the following condition

$$\tau_\perp /\tau_\parallel \ll 1. \quad (32)$$

Here $\tau_\parallel = 1/\Gamma$ is a characteristic timescale of the cyclotron damping, and $\Gamma$ is the damping decrement. We found (Paper I; Khechinashvili 1995) that $\Gamma R/c \sim 100$ at the distances where the bulk of radio band (0.1–1 GHz) is typically damped. Therefore, $\Gamma \sim 2 \times 10^5$ s$^{-1}$ and $\tau_\parallel /\tau_\perp \sim 10^{-2} \ll 1$. Thus, the parameters $\gamma_p$ and $\psi$ used in our model can be treated as quasi-stationary.

We believe that the apparent spectral index $\alpha$ indicates a power-law distribution of emitting particles over energy $n(\gamma) \sim \gamma^{-q}$, where $q = 1 + 2\alpha$ (Ginzburg 1979). In particular, for $\alpha = 1.9$ we have $q = 4.8$, and for $\alpha = 0.8$ (Martin et al. 1998) $q = 2.6$. Such steep distribution functions can be
Fig. 4.—Results of the synchrotron model for the Geminga pulsar evaluated for a range of small values of the Sturrock multiplication factor $\kappa$: (a) the mean Lorentz factor, (b) the average pitch angle, (c) the critical frequency, (d) condition (13), (e) condition (30). The dashed lines correspond to the case of $\alpha = 1.9$, and the dotted lines correspond to $\alpha = 0.8$. It is seen that the synchrotron radiation peaks in the infrared band, and both conditions are fulfilled in the low-density plasma. The exception is $\kappa \gtrsim 70$ in the case of $\alpha = 0.8$, for which the condition (30) is apparently violated at the infrared wavelength $\lambda = \lambda_c$. Although, at the optical wavelength $\lambda = \lambda_0$, this condition is still well satisfied for $\kappa = 10–100$.

expected in the regions of reduced plasma density where the synchrotron optical radiation of the Geminga pulsar is believed to originate. As it was mentioned above, the spaces between the spark-associated plasma columns provide a good example of such low plasma density regions (Ruderman & Sutherland 1975; Gil & Sendyk 2000).

The net synchrotron radiation should generally be elliptically polarized. At the frequencies higher than that corresponding to the maximum in the synchrotron power spectrum ($\nu \gg \nu_m = 0.29 \nu_c$) the linear polarization is $\Pi_L = (q + 1)/(q + 7/3)$, where $q$ is a power-law index of the electron distribution function (see Pacholczyk 1977, p. 52).

Hence, $\Pi_L \approx 0.8$ for the observed spectrum with $\alpha = 1.9$, and $\Pi_L \approx 0.7$ for the spectrum with $\alpha = 0.8$. We see that for each model spectrum the degree of expected linear polarization is very high, and one should definitely observe a polarization sweep, as a degree of the circular polarization is much lower at $\nu \gg \nu_m$ (Pacholczyk 1977).

A few other pulsars also exhibit pulsed optical emission. Among them, one can distinguish a group of very young objects (e.g., Crab and Vela). Their power-law optical spectra are flat and featureless, differing greatly from the spectra of another group of the middle-aged pulsars, such as Geminga and PSR 0656 + 14 (Kurt et al. 2000). Therefore,
it is likely that the mechanisms of the optical emission are very different in these two groups. The fluxes of the third group of relatively older pulsars (B0950 +08 and B1929 +09) are thermal-like, arising from the entire surface of the neutron star (Pavllov et al. 1996). As regards PSR 0656 +14, its spectrum in the IR-optical range can be fitted by a composite of the power law with $\alpha = 1.5$ and the blackbody (see Fig. 7 in Kurt et al. 1998), quite similar to that of Geminga. Presence of the nonthermal component in the emission of PSR 0656 +14 is independently confirmed by the pulsed fraction observed by Shearer et al. (1997). Similarity of the optical spectra of PSR 0656 +14 and Geminga may indicate that the same physical mechanisms generate their optical emission. On the other hand, PSR 0656 +14 is visible in the whole observed radio band. It exhibits an ordinary narrow pulse, contrary to the Geminga pulsar, whose erratic radio emission fills almost its entire 360° pulse window at about 100 MHz (Malofeev & Malov 1998; Kuzmin 1998; Vats et al. 1999). Therefore, one cannot assume that PSR 0656 +14 is an almost aligned rotator like Geminga, and in our opinion it is quite unlikely that the same mechanism as proposed in the present paper can provide its pulsed optical emission. We would like to stress here that it is the aligned geometry, which, in combination with the relatively high surface magnetic field, makes the Geminga pulsar so unique.

4. CONCLUSIONS

In this paper we explored consequences of the cyclotron absorption of radio waves by the pair plasma of the magnetosphere of the Geminga pulsar (Paper I). The results and conclusions can be summarized as follows:

1. We demonstrated that the relativistic charged particles should acquire significant transverse momenta and inevitably reemit the absorbed radio energy in the form of synchrotron radiation.

2. We considered both observationally derived fits of Geminga's optical spectrum (i.e., a continuous power law with the index of 0.8, and a composite spectrum consisted of a nonthermal power law with $\alpha = 1.9$ and a blackbody, see Martin et al. 1998) and found that our model provides observed optical luminosity in both cases, although only for very low values of the Sturrock multiplication factor ($\kappa \lesssim 100$).

3. Therefore, as a by result, we supported magnetospheric models presuming the nonuniform plasma outflow from the pulsar, originating in the form of spark discharges above the polar cap (e.g., Ruderman & Sutherland 1975; Gil & Sendyk 2000).

4. We found that in both cases of the observed spectrum the synchrotron radiation peaks in the infrared band, in accordance with the recent observations (Harlow et al. 1998). The predicted spectral density of power of the pulsed infrared emission from Geminga are presented in Figure 2 and Table 1.

5. The observed pulsed optical emission of the Geminga pulsar appears to be just a short-wavelength continuation of its power-law infrared spectrum.

6. In the frame of our model the shape of the optical light curve should resemble that of the low-frequency radio emission, since more absorbed radio energy should cause more intense synchrotron reemission. Indeed, as it was mentioned in §1, the optical light curve exhibits a second peak at a phase of 0.5 (Shearer et al. 1998), similar to the interpulse detected in Geminga's erratic 100 MHz radio emission (Kuzmin 1998; Vats et al. 1999; Kuzmin & Losovskii 1999).

7. Therefore, our paper is in a full agreement with a strong observational evidence of the predominantly magnetospheric origin of Geminga's pulsed IR/optical emission (Harlow et al. 1998; Shearer et al. 1998; Shearer et al. 1999).

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