Search for an annual modulation of dark-matter signals with a germanium spectrometer at the Sierra Grande Laboratory

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Results of a search for dark-matter induced annual modulation using 830.5 kg-days of data collected at the Sierra Grande underground laboratory with a germanium detector are presented. The analysis of the data does not show any indication of seasonal effects.

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I. INTRODUCTION

The visible part of our galaxy appears to be embedded in a dark-matter halo whose radius extends at least 50 kpc beyond the luminous radius. The halo model and its density value are still open questions, although a customarily accepted picture features an isothermal, spherically symmetric, non-rotating halo with a local density of $\rho = [0.3 \times 1.5^{+1} -1] \text{ GeV cm}^{-3}$. Recent results from microlensing data for a flattened halo would favour a higher value of $\rho$.

The MACHO and EROS microlensing observations of baryonic dark matter indicate that MACHOs (Massive Astrophysical Compact Halo Objects) could account for at most half of the dark halo, although the scarce statistics and the model dependence of its interpretation preclude establishing firm conclusions.

It can be concluded, nevertheless, that there is plenty of room for galactic non-baryonic dark matter such as light neutrinos, WIMPs (Weakly Interacting Massive Particles), or axions. The slow moving (0.001 c), heavy (GeV–TeV), and neutral WIMPs could be scattered by the nuclei in a detector producing a detectable signal. This feature has been used over the past decade to place bounds on the cross-section and masses of WIMPs by employing low-background germanium detectors, sodium-iodide spectrometers, and liquid-Xenon scintillators.

To go beyond the mere exclusion of candidates obtained so far, and actually identify the WIMP signal out of the cosmic and environmental background, one should look for genuine signatures, such as time modulation in the rate of events at the detector. A possible source of such modulation is provided by the orbital motion of the Earth around the Sun during the solar-system journey through the galactic halo, which results in a yearly variation of the relative Earth/halo velocity. The net speed of the Earth with respect to the halo oscillates between a maximum value (June) and a minimum (December), and thus the amount of energy that can be deposited by the WIMPs in the detector, as well as their detection rates, changes periodically. This kind of modulation has a period of one year, and searches for it have already been reported in the literature using semiconductor and scintillator detectors with no definitive claim of identification thus far.

The DAMA Collaboration, however, after analyzing 4549.0 kg-days of data collected with nine 9.70-kg NaI(Tl) detectors corresponding to 1185.2 kg-days in summer and 3363.8 kg-days in winter, has presented preliminary results suggesting that a yearly modulation effect might be present in their data.

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It is timely, then, that other ongoing experiments report on their findings, obtained under different conditions with different detectors and comparable activities. In this paper the analysis and results of a search for annual modulation are presented using the data collected with a 1.033-kg germanium detector at the Sierra Grande underground facility at a depth of 1,000 m.w.e. This experiment extends over a period of 1142 days (which is 48 days longer than a full three-year cycle) with a total effective running exposure of 804 days. The fact that three complete oscillations of the putative signal have been covered increases the statistical reliability of the search, because, on the one hand, a higher level of structure must be reproduced in order to account for the expected fluctuations and, on the other, the chances of an accidental fluctuation are, for the same reason, considerably reduced. In addition, over long data-taking periods semiconductor detectors are very stable to temperature and other environmental changes [11].

A description of the underground laboratory and the experimental set-up has been given in Ref. [24]. In particular the relevant experimental parameters are: an overall energy resolution of 1.2 keV at 10.37 keV and a long-term energy threshold of 4 keV. A total background spectrum corresponding to an exposure of 830.5 kg-day is shown in Fig. 1 where the background lines at 122.1, 143.5, 840.8, 1124.5, 1173.2 and 1332.5 keV corresponding to the decay of $^{57}$Co, $^{54}$Mn, $^{65}$Zn, and $^{60}$Co are clearly recognized. Calibration of the final spectrum has been performed by using radioactive sources ($^{182}$Ta and $^{207}$Bi) and low-energy peaks (8.98 and 10.37 keV) clearly identified in the spectrum.

II. DATA ANALYSIS

A. Modulation significance

The method of Freese, et al. [25] has been customarily used to look for a modulated component in data. In that method, a part of the WIMP signal is modulated with a period $T \simeq 2\pi/\omega$ ($T = 365$ days), and the experimental rate, neglecting higher order terms, is expressed as,

$$S_{\text{tot}}(t) = B + S_0 + S_m \cos(\omega t)$$

where $B$ is the background events, $S_0$ is the unmodulated part of the WIMP signal, and $t$ is measured from the time when the maximum speed relative to the halo is achieved ($\simeq$ the 2nd of June). A variable, $r$, is defined to act as an estimator of the significance of the modulation in the signal, by means of

$$r = \frac{X}{\sigma(X)} = \frac{\sum j \cos(\omega t_j) S_j}{\sqrt{2\sum j S_j}}$$

with $S_j = S_{\text{tot}}(t = t_j)$ the number of events integrated in an energy interval $\Delta E$ at the $t_j$ day of the acquisition. The sum spans the time from the beginning to the end of the experiment.

If a modulation were present in the data, $r$ would depart gradually from zero with increasing statistics, whereas for an unmodulated case, the mean value of $r$ would be zero and the variance one. If the experiment does not run continuously, however, the expected value of $r$ will not necessarily be zero, unless one uses only data recorded on time intervals properly distributed in the cosine period (for instance in opposite days). If such is not the case, one should first find the expected value of $r$ – given the down-time periods that actually occurred during the experiment – and compare this value with that obtained from the real data.

A large number of experiments ($10^4$) were simulated with down-time periods equal to those of the actual experiment and with the assumption that $S_m = 0$ and that $S_j$ is a Poisson distribution with a mean value equal to that of the corresponding real data set integrated between 12.5 and 50 keV. The frequency distribution of the modulation significance $r$ statistically obtained in such way (as well as the complementary sine projection $s$) are shown in Fig. 2 using counting rates equal to those of the experiment for that energy window.

For instance, in the region integrated between 12.5 and 50 keV, the expected value $r$ is -4.98 and that of $s$ is 4.58 with variances of 1.00 and 0.96, respectively. A fair criterion to decide whether there is a significant presence of modulation in the data is to calculate $r$ and $s$ for the real experiment and require that they lie at least $2\sigma$ away from the mean. In this situation the hypothesis of no modulation could be rejected with a confidence level, C.L. = $\int_{-2}^{+2} P(r)dr = \text{erf}((|r| + 2)/\sqrt{2}) \geq 95\%$ (where $P(r)$ is the Gaussian probability-density function). The values

$^1$Such interval is not arbitrary but the one with the highest rate beyond the Zn and Ga X-rays typically present in Ge detectors.
obtained from the data for this particular energy window were \( r = -5.79 \) and \( s = 4.04 \) which do not satisfy the criterion mentioned above.

Because of the small differential rate of the experiment it is convenient to integrate it in a large energy interval to gain sensitivity. However, as is well-known, the selected interval should not include the energy region where the spectra recorded in June and December cross each other (the so called cross-over energy where the June rates start becoming lower than December rates). Integrating over that region could hide a fluctuation that might actually be present in the data. The cross-over energy depends on the mass of the WIMP \([26-28]\) and other parameters \([28,29]\).

This experiment integrated up to a large final energy (50 keV) –consistent with the mean energy deposited by a 10 TeV WIMP– and ramped the initial energy from 12.5 to 18 keV thus ensuring sensitivity to fluctuations of WIMPs with masses up to 1 TeV \([29]\).

The modulation-significance variables as a function of the lower limit of integration, for several energy intervals in the spectra, are plotted in Fig. 3 (crosses) along with the predicted mean values of \( r \) and \( s \) (solid lines) and the 1\( \sigma \) contours (dashed). Values for the counting rates are shown in the middle of the figure and range from 6.3 counts/kg·day for the 12.5 to 50 keV energy interval down to 3.4 counts/kg·day for the 18 to 50 keV interval. The first one (6.3 counts/kg·day) corresponds to the distribution shown in Fig. 2. Clearly, in all cases the significance obtained from the data is well consistent with the absence of modulation.

### B. Energy-bin analysis

An alternative and common \([17,19]\) way of performing the analysis is to compute the modulation-significance variables \( r \) and \( s \) for a set of small energy bins, compatible with the detector energy resolution (rather than integrate the signal over broad energy regions), and look for their distribution. A significant departure from zero would be an indication of the presence of a modulation. In this method the question of determining the cross-over energy is avoided for all but one of the regions analyzed.

Following Ref. \([23]\), a generalization of the modulation-significance variables is used, which properly takes into account, for each energy bin, the correction for down-time periods. The new variables read:

\[
\begin{align*}
    r_0 &= \frac{\sum_j \left[ \cos(\omega t_j) - \beta \right] S_j}{\sqrt{\sum_j \left[ \cos(\omega t_j) - \beta \right]^2 S_j}} \\
    s_0 &= \frac{\sum_j \left[ \sin(\omega t_j) - \gamma \right] S_j}{\sqrt{\sum_j \left[ \sin(\omega t_j) - \gamma \right]^2 S_j}}
\end{align*}
\]

with \( \beta = \frac{1}{N} \sum_j \cos(\omega t_j) = -0.044 \) and \( \gamma = \frac{1}{N} \sum_j \sin(\omega t_j) = 0.046 \), representing the mean value of the cosine and sine oscillation for the data in consideration. \( N \) is the actual running time (804 days) out of the 1142 days of exposure. For later use it is also necessary to introduce the constant,

\[
\alpha = \langle \cos^2 \rangle = \frac{1}{N} \sum_j \cos^2(\omega t_j) = 0.531
\]

In terms of these parameters it is possible to obtain analytical expressions \([23]\) for the modulated part of the signal and its dispersion,

\[
S_m = \frac{\sum_j \left[ \cos(\omega t_j) - \beta \right] S_j}{N(\alpha - \beta^2)} \quad \text{and} \quad \sigma(S_m) = \frac{\sqrt{\sum_j \left[ \cos(\omega t_j) - \beta \right]^2 S_j}}{N(\alpha - \beta^2)}
\]

as well as for the background plus unmodulated parts (and its dispersion),

\[
b + S_0 = \frac{\sum_j \left[ \alpha - \beta \cos(\omega t_j) \right] S_j}{N(\alpha - \beta^2)} \quad \text{and} \quad \sigma(b + S_0) = \frac{\sqrt{\sum_j \left[ \alpha - \beta \cos(\omega t_j) \right]^2 S_j}}{N(\alpha - \beta^2)}
\]

For an experiment running continuously \( \alpha = 1/2 \) and \( \beta = \gamma = 0 \). Notice that in this case the dispersion (error bar) of the modulated amplitude
\[ \sigma(S_m) = \sqrt{\frac{2B}{n}} \]  

(8)

goes like the inverse square of the number of full periods in which the data were collected \((n = N/T)\) is the running time in units of \(T\), one year.) Therefore, the sensitivity of the experiment, at constant background rate, improves with the square root of the number of periods covered. Finally, similar expressions to those above may be obtained by substituting a sine for the cosine.

A set of fourteen energy bins were selected in the interval 4 to 32 keV (see Fig. 4) and the variables \( r_0 \) and \( r_s \) were calculated for each of them. They are given in Table II. Though some of the values obtained for \( r_0 \) (at 7.0, 15.3 and 21.2 keV) and for \( r_s \) (at 5.0, 6.0 and 30.1 keV) are significant, none depart from zero by more than 2-\( \sigma \) implying that this approach also produced no evidence of modulation at the 97.5% C.L. Notice that with three full oscillations under consideration a real effect should show up in both variables.

The same energy bins were extracted from the data the modulated amplitude of the signal, whether present. The modulated amplitudes (in counts/keV-kg-day) as a function of the deposited energy from Table I are shown in Fig. 5. The squares correspond to \( S_m \) determined from equation (6) and the circles to substituting a sine for the square root of the counting rate so they are larger where the counting rate is larger, i.e. at low energies. Despite the fluctuations in the low-energy region, none of the calculated amplitudes lies more than 1.6-\( \sigma \) away from \( S_m = 0 \), and they are consistent with \( S_m = 0 \). For comparison and to give an idea of the required sensitivity, the signals expected from WIMPs with masses 30, 40, 50, and 200 GeV are also plotted in the figure, assuming their cross sections are those from the best exclusion derived so far [18]. The current sensitivity of the DEMOS experiment can be better appreciated by an example: a 50-GeV WIMP modulation would be detected at a 2\( \sigma \) level for a cross-section \( \gtrsim 2 \times 10^{-4} \) pb.

C. Comparing to DAMA

To compare the results presented here to those of DAMA we chose to display the modulated amplitudes extracted from the experiment, \( S_m \), as a function of the energy deposited in each detector and plotted them using the same vertical scale. Thus, Fig. 6 shows the DAMA/NaI (a) and DEMOS (b) data along with the theoretical prediction for a 60 GeV WIMP with a cross-section \( \sigma_{W,nucleon} = 1.0 \times 10^{-5} \) pb, for a halo-abundance density of 0.3 GeV/cm\(^3\). These parameters are representative of the region where the analysis of the DAMA data are interpreted in terms of a modulated signal. The prediction for DEMOS (Ge) was convoluted with the relative efficiency function of Ref. [9] whereas for DAMA we employed the published quenching factors, \( q(I)=0.09 \) and \( q(\text{Na})=0.3 \). A plot similar to our Fig. 6 was already introduced in Ref. [30] but there the normalization of the curve was done relative to the found experimental excess and is a factor eight larger than our prediction.

As stated above, the data from DEMOS [part b) of the figure] are not sensitive enough to exclude the claimed candidate. On the other hand, the same analysis applied to the data published by the DAMA collaboration does not show an improvement in sensitivity with respect to DEMOS. This leads us to conclude that the alluded candidate cannot be either excluded nor identified with current available data and that more statistics is still necessary if the tip of the neutralino region is to be probed.

Regarding the theoretical values of \( S_m \) it is worth keeping in mind that they depend on the subtraction of June minus December rates, so that slight changes in the prediction of the rates (form factors, energy-smearing or parametrizations of the rates) can lead to sizable changes in the prediction of the modulation amplitude. The inclusion of energy smearing due to detector resolution in the calculation of the rates can give rise, for the case of NaI, to a factor of two decrease in the predicted \( S_m \) [31]. Our predictions in Fig. 4 do not include this smearing.

We point out that the plots in Fig. 5 could have been displayed, alternatively, either matching the error bars of the experimental points or normalizing both plots to the peak of the predicted signal. In either case the conclusion does not vary in that the putative candidate could be there since the sensitivity of both experiments is, at the present time, not enough to either exclude nor prove it.

### III. SUMMARY AND CONCLUSIONS

Data collected during three years with a germanium spectrometer at the Sierra Grande underground laboratory have been analyzed for distinctive features of annual modulation of the signal induced by WIMP dark matter candidates. The main motivation for this analysis was the recent suggestion [22,23] that a yearly modulation signal could not be
rejected at the 90% confidence level when analyzing data obtained with a high-mass low-background scintillator detector. Two different analyses of the data were performed. First, the statistical distribution of modulation-significance variables (expected from an experiment running under the conditions of Sierra Grande) was compared with the same variables obtained from the data. Second, the data were analyzed in energy bins as an independent check of the first result and to allow for the possibility of a crossover in the expected signal. In both cases no statistically significant deviation from the null result was found, which could support the hypothesis that the data contain a modulated component. Finally, a plot was presented to be able to compare our results to those of the DAMA collaboration.

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TABLE I. Modulation-significance variables $r_0$ and $s_0$ calculated for the fourteen energy bins in which the data were divided (from 4 to 32 keV). Also shown are the modulated amplitudes and their dispersions for the cosine- (columns 4 and 5) and the sine- (columns 6 and 7) modulation analysis. The listed energies correspond to the lower end of the interval.

| E(keV) | $r_0$  | $s_0$  | $S_m$(cos) | d$S_m$(cos) | $S_m$(sin) | d$S_m$(sin) |
|-------|--------|--------|------------|-------------|------------|-------------|
| 4.0   | -0.41  | -1.05  | -0.032     | 0.080       | -0.090     | 0.088       |
| 5.0   | -1.02  | -1.66  | -0.081     | 0.079       | -0.143     | 0.087       |
| 6.0   | -0.20  | -1.35  | -0.016     | 0.078       | -0.113     | 0.084       |
| 7.0   | 1.76   | -0.27  | 0.121      | 0.069       | -0.021     | 0.076       |
| 14.2  | -0.77  | -0.92  | -0.013     | 0.017       | -0.017     | 0.018       |
| 15.3  | 1.75   | -0.16  | 0.019      | 0.011       | -0.002     | 0.012       |
| 17.7  | 0.49   | 0.01   | 0.006      | 0.013       | 0.001      | 0.001       |
| 19.4  | -0.55  | 0.86   | -0.007     | 0.012       | 0.011      | 0.013       |
| 21.2  | -1.99  | 0.76   | -0.023     | 0.012       | 0.009      | 0.012       |
| 23.0  | -0.98  | 0.68   | -0.011     | 0.011       | 0.008      | 0.011       |
| 24.7  | -0.30  | 0.83   | -0.003     | 0.010       | 0.009      | 0.011       |
| 26.5  | -0.42  | -0.37  | -0.004     | 0.010       | -0.004     | 0.010       |
| 28.3  | 0.39   | -0.36  | 0.004      | 0.009       | -0.003     | 0.010       |
| 30.1  | 1.06   | -1.66  | 0.009      | 0.009       | -0.015     | 0.009       |
FIG. 1. Total background spectrum corresponding to an exposure of 830.5 kg·day.

FIG. 2. Frequency plot of the modulation-significance variables $r$ and $s$. Each curve represents the calculated values of the variable for 10,000 simulated experiments running for 1142 days with down-time periods and background similar to the actual experiment in Sierra Grande.

FIG. 3. Predicted (solid lines) and calculated (crosses) values of the modulation-significance variables $s$ (upper) and $r$ (lower) as a function of the low-energy limit of integration, $E_i$. The dashed lines correspond to the 1-$\sigma$ contours; count rates of each data set are also shown.

FIG. 4. Selected energy bins for the analysis of section II.B. The gap between 7 and 14 keV corresponds to Zn and Ga X-rays and was not considered in the analysis.

FIG. 5. Modulated amplitude, ($S_m$), extracted from the data using Eq. (6), as a function of the deposited energy. The error bars are 1-$\sigma$. The curves correspond to the signal expected from a spin-independent WIMP with a mass of 30 (solid), 40 (dash), 50 (dot) and 200 GeV (dot-dash) and a cross section corresponding to the exclusion plot $\sigma(m_\chi)$ of ref. [18].

FIG. 6. Modulated amplitudes $S_m$ [Eq. (6) in the text] as a function of the deposited energy obtained from the data of the DAMA/NaI (a) and DEMOS (b) collaborations. In both plots the solid line corresponds to the signal expected from a 60 GeV WIMP scattering off each detector with $\sigma_{W,\text{nucleon}}=1.0 \times 10^{-5}$ pb. For the NaI case the energy resolution (which was not taken into account in the calculation) would decrease the amplitude of the curve by a factor 2 (see text). The circles in part b) of the figure correspond to the substitution of a sine for the cosine used to obtain $S_m$ and are shifted 0.5 keV to the right for clarity.
FIG. 2

\[ \langle r \rangle = -4.98 \quad \langle s \rangle = 4.58 \]
