About the Frequency-Dependence of Electrical Characteristics of Quantum Devices

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Abstract  The frequency-dependence of electrical characteristics of quantum device components was researched. There were two types of nanostructures: quantum wire and junction nanostructures between two quantum wires with different cross sections. It is shown that conductivity of the first nanostructure is decreased with growth of the frequency and conductivity of the second nanostructure is increased with growth of the frequency.

Keywords  Quantum Wire, Frequency, Conductivity

1. Introduction

Interest to electrical characteristics of quantum wires is caused by new physical effects which are observed in one-dimensional conductors[1,2] and also by prospects of high-frequency applications of devices based on quantum wires[3,4].

The consecutive analysis of frequency-dependence of quantum device characteristics can be conducted in the framework of a multiphase model of charge transport[4,5]. The model was successfully applied in order to calculate the characteristics of resonant-tunneling diodes and devices based on quantum wires[6,7].

In this paper, it is shown that frequency-dependence of one-dimensional electronic gas conductivity is determined by the frequency properties of the hydrodynamic velocity of electrons.

In quantum devices, the junction between quantum wires with different cross-sections can be used as a source of nonequilibrium electrons[4,7]. Nonequilibrium effects lead to specific dependence of such junction conductivity from frequency of external signal.

2. Electric Current in Quantum Devices

The equation for the hydrodynamic velocity of electron \( \mathbf{v}(x,\lambda) \) can be written as[4]

\[
\frac{\partial \mathbf{v}(x,\lambda)}{\partial t} + (\mathbf{u}(t,r,\lambda) \cdot \nabla) \mathbf{v}(t,r,\lambda) + \mathbf{v}(t,r,\lambda) \cdot \nabla \mathbf{F}(t,r,\lambda) = -\nabla \mathbf{F}(t,r,\lambda) \times \mathbf{m}^*. 
\]

Here, the index \( \lambda \) numbers the possible electron states, \( \tau(\lambda) \) and \( \mathbf{F}(t,r,\lambda) \) - are the electron momentum relaxation time and chemical potential in \( \lambda \)-state. Other values, which are included in the equation (1) are determined as follows:

\[
\mathbf{u}(t,r,\lambda) = \mathbf{j}(t,r,\lambda)/\rho(t,r,\lambda),
\]

\[
\rho(t,r,\lambda) = \Psi^*(t,r,\lambda) \Psi(t,r,\lambda),
\]

\[
\mathbf{j}(t,r,\lambda) = -(\hbar/2m^*)(\Psi^*(t,r,\lambda) \nabla \Psi(t,r,\lambda) - \nabla \Psi^*(t,r,\lambda) \Psi(t,r,\lambda)),
\]

where \( \hbar \) is the Planck constant and electron wave functions \( \Psi(t,r,\lambda) \) satisfy Schrödinger’s equation

\[
i\hbar \partial \Psi(t,r,\lambda)/\partial t + (\hbar^2/2) \nabla \cdot \mathbf{v}(t,r,\lambda) - \mathbf{U}(t,r) \Psi(t,r,\lambda) = 0.
\]

In expression (4), \( U_{ext}(r) \) - is the built-in potential caused, for example, by the breaks of band gaps of heterostructures, \( e \) - is the electronic charge and \( \Psi(t,r) \) - is the self-consistent electric potential calculated by Poisson’s equation

\[
(\nabla \cdot \mathbf{v}(t,r)) = e(n(t,r) - N_{int}(r)).
\]

Where \( eN_{int}(r) \) - is a density of doping charge.

The equation (5) is fair at characteristic frequencies that are a lot of smaller than \( \nu_c = c/L \)

where \( c \) - is the speed of light, and \( L \) - is the typical size of structure. For modern and perspective electronic devices, the following estimation is right

\( L \approx 10^{-7} - 10^{-5} \text{ cm} \).

It means that the equation (5) is fair for frequencies smaller than \( 10^{14} \text{ Hz} \).

Electron concentration \( n(x) \) and density of electron flow \( n(t,r) \) are calculated as the sums of corresponding values in \( \lambda \)-states

\[
n(t,r) = \sum_{\lambda} n(t,r,\lambda), \quad n(t,r,\lambda) = \sum_{\lambda} n(t,r,\lambda),
\]

where

\[
n(t,r,\lambda) = \mathbf{\rho}(t,r,\lambda)f(s(t,r,\lambda)), \quad n(t,r,\lambda) = j(t,r,\lambda)f(s(t,r,\lambda)).
\]

and

\[
f(s) = (e \exp(s) + 1)^{-1},
\]
s(t, r, λ) = [E(t, r, λ) - F(t, r, λ) - m*μ(t, r, λ)]/kT.

Energy E(t, r, λ) is calculated by the formula

E(t, r, λ) = \frac{\hbar^2}{2m^*}(\nabla^2 v(t, r, λ) - \nabla \cdot j(t, r, λ)) - \frac{e}{m^*} E_0 \cos(\omega t).

The length of chemical potentials of different electrons relaxation to local chemical equilibrium is defined by the formula

L_{rel}(λ) = (\hbar \tau(λ)/m^*)^{1/2}.

Size of L_{rel} is different for different materials. It is equal to about 10 nanometers for Si, 24 nanometers for GaAs and 72 nanometers for InSb. Where structures are larger than L_{rel}, it is possible to conclude that the electronics are in a state of local chemical equilibrium.

From the considered formulas, it follows that the current in electronic devices is created by two factors: deviations of electronic gas from the local chemical equilibrium and at nonzero values of hydrodynamic velocity of electrons. According to formula (1), the nonzero values v(t, r, λ) are caused by gradients of chemical potentials F(t, r, λ), that are deviations from chemical equilibrium of electronic gas in various spatial points. Thus, it is possible to conclude that the electronic current is the consequence of the nonequilibrium phenomena in electronic gas.

3. Frequency-Dependence of Hydrodynamic Velocity

In case when

(\partial/\partial t + (u(t, r, λ) \nabla))v(t, r, λ) = 0

the equation (1) has the solution

v(t, r, λ) = - \tau(λ) \nabla F(t, r, λ)/m^*.

Substitution of expression (16) to the formula (14) results to the Ohm’s law in the differential form and to well-known formulas for mobility and conductivity of electronic gas.

Let’s assume that the potential difference changing in time under the harmonious law with cyclic frequency \omega is applied to a spatially homogeneous sample. In this case, the gradient of chemical potential in the sample may be presented as

\nabla F = e E_0 \cos(\omega t).

where \omega is a certain constant field.

In view of spatial uniformity from (1) we shall receive

v(t, r, λ) = - [m^* (1 + \omega^2 \tau^2(λ))] E_0 \cos(\omega t) + \omega \tau(λ) \sin(\omega t).

From (18), it follows that as well as in case of classical theory, the frequency-dependence of electronic gas conductivity becomes essential at

m^* \tau(λ) = 1.

For mesoscopic structures, the momentum relaxation time is about 10^{-13} s and the factor \omega is necessary to take into account when the frequencies are more than 1 THz. Comparing expressions (13) and (16), we can see that increase of frequency results in decrease of conductivity (growth of resistance) of electronic gas.

In the development of expression (18), no assumptions about quantum dimensions of electronic gas were made. It means that formulas (18) and (19) are fair for quantum wires, which represent one-dimensional conductors.

4. Frequency Influence to Resistance of Junction between Quantum Wires

For the decision of this problem, it is necessary to consider the transport equations for different electronic phases which according to[4,5] look like

\partial u(t, r, λ)/\partial t + (\nabla u(t, r, λ)) = -h^{-1} \sum_{λ} (F(t, r, λ) -
Here, values $\Gamma_{\lambda\lambda'}$ and $\Gamma_{\lambda'\lambda}$ are the probabilities of transitions between $\lambda$ and $\lambda'$-states.

In the simplest case, when only deviations from the local chemical equilibrium between "left" and "right" electrons [4,7] are taken into account, from (20) we shall receive the system of two equations

$$\frac{\partial n_l(t,r)}{\partial t} + (\nabla, n_l(t,r)) = -\gamma\frac{1}{\hbar}(F_l(t,r) - F_r(t,r))(n_l(t,r) + n_r(t,r)),$$

$$\frac{\partial n_r(t,r)}{\partial t} + (\nabla, n_r(t,r)) = -\gamma\frac{1}{\hbar}(F_r(t,r) - F_l(t,r))(n_r(t,r) + n_l(t,r)).$$

Here: $\gamma$ is some positive dimensionless constant.

Value $t_0$ is relaxation time of electronic gas to the state of local chemical equilibrium.

From the equation (23), it follows that the source of electronic gas nonequilibrium state is the current that flows perpendicularly to boundaries of areas with different conductivities $\sigma_{l,r}$. Having made the assumptions, the equation (23) is a singularly perturbed [9]. In a stationary case, its approximate decision looks like

$$F_{-} \approx F_{0} = -\frac{\hbar(I,\nabla(\sigma_{-}/\sigma_{+}))}{2\gamma n_{+}}.$$ 

As shown in [4], the value $F_{0}$ is connected with the voltage drop in the device by the formula

$$V \approx F_{0} / e.$$ 

Thus, to within boundary effects [4], the ratio (24) defines the current-voltage characteristic (CVC) of junction between quantum wires with different thickness.

In a one-dimensional approximation, when the total electron flow is constant for CVC of junction, we shall receive the formula

$$V = J(\hbar/e^2) / 2\gamma d n_{+},$$

where $J = eI = \text{const}$

- is current density through the junction, $d$ is the effective width of the junction. Value

$$r_{J} = (\hbar/e^2) / 2\gamma d n_{+}$$

- is a specific resistance of the junction between two quantum wires with different cross-sections (dimension of $r_{J}$ - is $\Omega^{*}\text{cm}^{-2}$).
voltage increasing and a corresponding decrease in potential barriers for electrons (see Figure 5). Similarly, under conditions of negative voltage, the potential barrier for electrons increases (see Figure 6), and the current does not practically flow through the junction.

5. Conclusions

In this paper, it is shown that the frequency of an external signal \( \omega \) influences conductivity of quantum wires and devices based on them.

Conductivity of the conducting channel of a quantum wire is decreased linearly with the growth of \( \omega\).

For the junction between two conducting channels of quantum wires with different cross-sections, the inverse relationship is a representative one. Conductivity of such a junction is increased linearly with growth of the frequency, and resistance tends to zero if frequency tends to infinity. Similar frequency-dependence should be characteristic for a junction between the contact area and the conducting channel of a quantum wire.

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