Dynamic Decision Model for Amount of AlF₃ Addition in Industrial Aluminum Electrolysis

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Abstract: Adding aluminum fluoride to electrolyte, is the most important to regulate electrolyte molecule ratio and the electrolysis temperature in aluminum electrolysis, which directly influences the current efficiency and energy consumption. This paper presents a decision model to decide the amount of AlF₃ addition by use of the historical data including electrolysis temperature, cell working voltage, the amount of aluminum tapping and the amount of AlF₃ addition. We established dynamic regression equations and got the regression correlation coefficient by transformation of orthogonal matrices. Selecting two electrolysis cells to check the model, the standard deviation of predicted value and the actual value within two months was 1.2046,1.7768, which can meet the requirements of practical production.

Introduction:

In the process of aluminum electrolysis, the appropriate electrolyte molecular ratio and electrolysis temperature can keep a regular cell ledge[1], and reduce the horizontal current, improve the current efficiency and help to prolong the life of cell. Research has shown that, under normal production conditions, the decrease of the electrolyte temperature can improve the current efficiency. But if the electrolysis temperature is too low, the sediment would be in bottom. Therefore, how to determine the AlF₃ addition reasonably to control of aluminum electrolysis temperature and electrolyte molecule ratio is a very important in aluminum electrolysis. At present, the technicians mainly rely on their experience to decide the amount of AlF₃ addition. Some technicians control the AlF₃ addition through the expert system, but because electrolytic cells have gradually changed, the expert system cannot make adjustment according to these changes, as well as the expert’s advice produces certain conflict sometimes, which will affect the decision of the AlF₃ addition[2]. Because of the complicity of the aluminum electrolysis, it is more difficult to establish the mechanism model [3]. AlF₃ related mainly to electrolysis temperature, work voltage, aluminum content and previous the AlF₃ addition, so the model to make the decision of AlF₃ addition can be built according to the existing data and the required electrolysis temperature.

The establishment of the model for AlF₃ addition

The model structure

Aluminum electrolysis is very complex, the establishment of a mechanism model to describe the amount of AlF₃ addition is very difficult. We selected the main factors that influence the amount of

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AlF₃ addition, which is electrolysis temperature, work voltage and the aluminum tapping content. The following is the structure of the model,

\[ F(K-1) - F(K-2) = a(V(K-1) - V(K-2)) + b(L(K-1) - L(K-2)) + c(T(K) - T(K-1)) + d \]

Here: \( F(K-1), F(K-2) \) were the amount of adding fluoride of K-1 moment and K-2 moment respectively. \( V(K-1), V(K-2) \) were the the average working voltage of K-1 moment and K-2 moment respectively. \( L(K-1), L(K-2) \) were the the aluminum content of K-1 moment and K-2 moment respectively. \( T(K), T(K-1) \) were electrolysis temperature of K moment and K-1 moment respectively. \( a, b, c, d \) were the correlation coefficient, which should be updated in time.

The correlation coefficient got by transformation of orthogonal matrices.

The data, which including electrolysis temperature, the working voltage, the amount of aluminum tapping the amount of fluoride addition were sampled from the cell 216# and 217# lasted for three months. By selecting 40 group of input data, i.e: the working voltage of first days to fortieth days, the amount of aluminum tapping, the amount of fluoride addition and electrolysis temperature from the of second day to forty-first day, and obtaining the daily variation, and getting a set of data of 39 elements, denoted as \( \Delta V(39), \Delta L(39), \Delta F(39), \Delta T(39) \) respectively.

The 39 sets of data were divided into 10 groups according to the time sequence, and each of which has 30 elements, i.e: 1-30, 2-31,... 10-39, respectively, as a group, the serial number was T1, T2,..., T10. Each vector were recorded as \( Vorg(T_i), Lorg(T_i), Forg(T_i), Torg(T_i) \).

The establishment of the independent variable matrix: \( Xorg(T) = [I, Vorg(T), Lorg(T), Torg(T)] \), The I of 30x1 elements are column vectors which all is 1;

The dependent variable vector: \( yorg(T_i) = Forg(T_i) \);

A new matrix was formed: \( Aorg(T_i) = \begin{pmatrix} Xorg(T) & yorg(T) \\ xorg(T) & yorg(T) \\ yorg(T) & yorg(T) \end{pmatrix} \);

The input data are treated by adding the weight coefficients, the \( Vn, An, Fn, Tn \), for the vector \( Vorg, Lorg, Forg, Torg \) respectively.

Constructing a new matrix \( Anew \) by making: \( Vnew = Vn \times Vorg, Lnew = An \times Lorg \), \( Fnew = Fn \times Forg, Tnew = Tn \times Torg; \)

\[ Anew = \begin{pmatrix} Xnew & ynew \\ xnew & ynew \end{pmatrix} \]

This matrix can be decomposed into:

\[ Anew(T_i) = \begin{pmatrix} \lambda_1(T_i) & 0 & 0 & 0 & 0 \\ 0 & \lambda_2(T_i) & 0 & 0 & 0 \\ 0 & 0 & \lambda_3(T_i) & 0 & 0 \\ 0 & 0 & 0 & \lambda_4(T_i) & 0 \\ 0 & 0 & 0 & 0 & \lambda_5(T_i) \end{pmatrix} \]

Where \( \lambda_1(T_i), \lambda_2(T_i), \lambda_3(T_i), \lambda_4(T_i), \lambda_5(T_i) \) is eigenvalue of the matrix \( Anew(T_i) \), \( u_1(T_i), u_2(T_i), u_3(T_i), u_4(T_i), u_5(T_i) \) were standard orthogonal eigenvectors corresponded to the eigenvalue. Each eigenvalue in \( T+1 \) moments can be calculated by previous one using the model...
\( \lambda_i(t) = g_i(t) + \delta_i . \)

The eigenvectors at the moment of \( T+l \) calculated as folloeing.

For a standard orthogonal matrix \( G = (u_1, u_2, \ldots, u_j) \), where \( u_j \) is the \( j \) column of \( G \), \( G \) can be decomposed into the following form:

\[
G = (G_{12} G_{13} L G_{15})(G_{23} G_{24} G_{25}) L (G_{45}) ;
\]

(2)

Where \( G_{ij} \) (\( l \leq i < j \leq 5 \)) have the following special form:

\[
G_{ij} = G_{ij}(\theta_{ij}) = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & \cos \theta_{ij} & 0 & -\sin \theta_{ij} & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & \sin \theta_{ij} & 0 & \cos \theta_{ij} & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix}_{5 \times 5}
\]

\((-\frac{\pi}{2} \leq \theta_{ij} \leq \frac{\pi}{2})\)

This decomposition is unique\(^{[4]}\), and each \( \theta_{ij} \) is independent. Therefore, as long as we can calculate each \( \theta_{ij} \), we can get \( G \) matrix.

The following method to solve the \( \theta_{ij} \):

Making \( G_i = G \), then \( G_i = (V_{11}, V_{12}, \ldots, V_{15}) = (u_1, u_2, \ldots, u_5) \), Due to the special structure of \( G_{ij} \), we can get the first column of \( G_i \):

\[
V_{11} = u_1 = \begin{pmatrix}
\cos \theta_{12} \cos \theta_{13} L \cos \theta_{15} \\
\sin \theta_{12} \cos \theta_{13} L \cos \theta_{15} \\
\sin \theta_{13} L \cos \theta_{15} \\
\sin \theta_{14} \cos \theta_{15} \\
\sin \theta_{15}
\end{pmatrix}
\]

And

\[
\theta_{15} = \arcsin V_{11}(5),
\]

\[
\theta_{1k} = \arcsin \left( \frac{V_{11}(k)}{\cos \theta_{15} L \cos \theta_{1k+1}} \right) , k = 2, 3, 4
\]

\((-\frac{\pi}{2} \leq \theta_{1j} \leq \frac{\pi}{2} , j = 2, 3, 4, 5)\)

By the formula (2) we can get:

\[
(G_{12} G_{13} L G_{15})^L G_i = (G_{23} G_{24} G_{25}) L (G_{45})
\]

Making:

\[
G_2 = (G_{12} G_{13} L G_{15})^L G_i = (V_{21}, L V_{22})
\]

The first and second column of \( G_2 \) can be obtained as:
\[
(V_{21}, V_{22}) = \begin{pmatrix}
1 & 0 \\
0 & \cos \theta_{22} \cos \theta_{24} \cos \theta_{25} \\
0 & \sin \theta_{22} \cos \theta_{24} \cos \theta_{25} \\
0 & \sin \theta_{22} \cos \theta_{25} \\
0 & \sin \theta_{25}
\end{pmatrix}
\]

And:

\[
\theta_{25} = \arcsin V_{22}(5), \\
\theta_{24} = \arcsin \left( \frac{V_{22}(k)}{\cos \theta_{22} \cos \theta_{24} \cos \theta_{25+1}} \right), k = 3, 4 \\
\left(\frac{-\pi}{2} \leq \theta_{2j} \leq \frac{\pi}{2}, j = 3, 4, 5\right)
\]

Similarly we can get each of the remaining \( \theta_{ij} \), until finally we can get \( \theta_{45} = \arcsin V_{45}(5) \)

With the angle at the moment of \( T+l \) by the model, we can calculate the standard orthogonal matrix \( G(T+l) \) at the moment of \( T+l \), namely standard orthogonal vector group of Anew matrix at the moment of \( T+l \), thus we can determine the matrix Anew at the moment of \( T+l \).

Making:

\[
A_{\text{new}} = \begin{pmatrix}
A_{4 \times 4} & B_{4 \times 1} \\
C_{4 \times 1} & D_{4 \times 1}
\end{pmatrix}_{5 \times 5}
\]

Then the least squares estimate value of the correlation coefficient \( \hat{\beta} = (a, b, c, d)^T \) is \( \hat{\beta} = A^d B \).

**Decision of AlF3 addition**

**Calculation of the AlF3 addition on the same day**

The formula (1) was changed to

\[
F(K-1) = a(V(K-1) - V(K-2)) + b(L(K-L) - L(K-2)) + c(T(K) - T(K-L)) + d + F(K-2) \quad (3)
\]

The formula (3) can be used to decide the amount of AlF3 addition. In the production, the amount of AlF3 addition F(K -1) this day was calculated by using the setting temperature T(K) next day, the set voltage V(K-1) this day, the aluminum tapping amount L(K-1) this day, the temperature T(K -1) this day and some historical data (working voltage, the amount of aluminum tapping, the amount of AlF3 addition),

**Feedback correction**

In the production, the environment changes constantly, The set temperature and actual temperature have deviation. When the deviation is larger than a given value, which is determined by technician (5-10 degrees), or for greater than 5 degrees continuously, we must recalculate the model coefficients of a, b, c, d.

**The implementation of the decision**

Making a forecast to the two electrolytic cell temperature respectively. Simulation calculation was carried out by Malab programming. Weight coefficient, Vn, An,Fn ,Tn is obtained by Matlab optimization function, and experiments show that, the reasonable selection of Vn, An, Fn, Tn can improve the accuracy of the model.
The inspection of decision model

Using the following formula to calculate the standard deviation, to test the fitting degree of the model and the actual value:

\[ S = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (F_{\text{pred}} - F_{\text{real}})^2} \]  \hspace{1cm} (4)

Here, \( F_{\text{pred}}, F_{\text{real}} \) are the prediction and the actual amount of AlF3 addition; \( N \) to test the number of days. \( N=51 \).

Fig.1 is the comparison of the mode decision value with the actual value of the amount of AlF3 addition for cell 216#. In the calculation process, making \( V_n=2.0610, A_n=1.1658, F_n=0.8333, T_n=0.5810 \), By calculating, we can know that the standard deviation of the model output value and actual output value is 1.2046.

Fig.2 is the comparison of the mode decision value with the actual value of the amount of AlF3 addition for cell 217#, and making \( V_n=1.9402, A_n=1.2597, F_n=0.4913, T_n=1.0692 \). By calculating, we can know that the standard deviation of the model output value and actual output value is 1.7768.

Fig.1 is the comparison of the output value with the actual value of the amount of groove AlF3 addition model(216#)  

Fig.2 is the comparison of the output value with the actual value of the amount of groove AlF3 addition model(217#)

Conclusion

This paper established the decision model for AlF3 addition in the aluminum electrolysis by use of the electrolysis temperature, the working voltage, the aluminum tapping amount and the amount of AlF3 addition as the parameters. Through the inspection of production data, the decision model can be used as a decision making reference of the amount of AlF3 addition in the industrial aluminum electrolysis.

References

[1] Zeng Shuiping. Model Predictive Control of Superheat for Prebake Aluminum Production Cells. TMS Light Metals 2008: 347-353

[2] Liu Yexiang, Li Jie. Modern aluminum electrolysis. Bei Jing: Metallurgical Industry Press. 2008

[3] Shuiping Zeng, Lin Cui and Jinhong Li. Diagnosis System for Alumina Reduction Based on BP Neural Network, JOURNAL OF COMPUTERS, 2012,7(4):929-933

[4] Anderson, T.W., Olkin, I., UnderHill, L.G. Generation of random orthogonal matrices, SIAM Sci. Statist. Comput. 8, 625-629, 1987.