New bound on penguin pollution in
\[ B^0(t) \to \pi^+\pi^- \]

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Abstract

In the presence of penguin contributions, the indirect CP asymmetry in \( B^0(t) \to \pi^+\pi^- \) measures \( \sin 2\alpha_{\text{eff}} \), where \( \alpha_{\text{eff}} \) deviates from the true value of the CKM phase \( \alpha \) by an amount \( \theta \) i.e., \( \alpha_{\text{eff}} = \alpha + \theta \). Using the measured value of direct CP asymmetry in \( B^0(t) \to \pi^+\pi^- \), we derive new bound on \( |2\theta| \).

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The study of CP violation mechanism is one of the main goals of the ongoing and future B factory experiments \([1]\). In the standard model (SM) CP violation is induced by the nonzero phase appearing in the CKM mixing matrix and is often characterized by the so called unitarity triangle \([2]\). Detection of CP violation and the accurate determination of the unitarity triangle are the major goals of experimental B Physics. Decisive information about the origin of CP violation in the flavor sector can be obtained if the three angles \( \alpha, \beta \) and \( \gamma \) of the unitarity triangle can be independently measured. The sum of these three angles must be equal to 180°, if the CKM phenomena of SM is the model for CP violation. The usual way to measure
the three angles of the unitarity triangle is generally done by considering the
time dependent rate asymmetries of B meson decays [3].

The angle β is the simplest one among these three angles, which can be
determined from the time dependent rate asymmetry of the gold plated
mode $B^0_d \rightarrow J/\psi K_S$, without any uncertainty. In fact the value of $\sin(2\beta)$
has recently been reported by the Belle [4] and BaBar [5] collaborations as

$$
\sin(2\beta) = 0.99 \pm 0.14 \ (\text{stat.}) \pm 0.06 \ (\text{syst.}) \ \ \text{(Belle)}
$$

$$
\sin(2\beta) = 0.75 \pm 0.09 \ (\text{stat.}) \pm 0.04 \ (\text{syst.}) \ \ \text{(BaBar)} \quad (1)
$$

with an average

$$
\sin(2\beta) = 0.87 \pm 0.08 \quad (2)
$$

The most difficult among these angles is the angle γ. But there are several
methods exist for its determination [6, 7, 8]. Recently, it has also been pointed
out that the decay modes $B_c \rightarrow D_s D^0, B_s \rightarrow D^0 \phi$ and $\Lambda_b \rightarrow \Lambda D^0$ [9] can be
used to cleanly determine the value of γ.

For the measurement of α, the principal decay mode considered is $B^0(t) \rightarrow \pi^+ \pi^-$. However, due to the penguin pollution, a clean determination of
the angle α is not possible considering this decay mode [10]. The problem of penguin pollution can be eliminated with the help of isospin analysis [11]. In order to perform the isospin analysis, all modes of $B \rightarrow \pi \pi$ have
to be measured. However, it is difficult to measure the branching ratio of
$B^0(B^0) \rightarrow \pi^0 \pi^0$ process, which has background problem as well as a tiny
branching ratio of $O(10^{-7})$ [12]. Therefore, in practice it is hard to perform
the isospin analysis.

To be more precise, in the presence of penguin contributions, the CP
asymmetry in $B^0(t) \rightarrow \pi^+ \pi^-$ does not measure $\sin(2\alpha)$ but $\sin(2\alpha + 2\theta)$,
where $2\theta$ parameterizes the effect of penguin contributions. There are several
methods exist in the literature to constrain the penguin pollution parameter
θ. Grossman and Quinn [13] have shown that an upper bound on the error
on $\sin(2\alpha)$ due to penguin pollution can be obtained using only the measured
rate $Br(B^0 \rightarrow \pi^0 \pi^0)$ and an upper bound on the combined rate $Br(B^0 \rightarrow \pi^0 \pi^0) + Br(\bar{B}^0 \rightarrow \pi^0 \pi^0)$. Later, Charles [14] improved the bound to some
extent. In addition the paper [14] presents several bounds based on the
flavor SU(3) symmetry, together with dynamical assumptions about the size
of certain OZI-suppressed penguin contributions. The recent one is being
proposed by Gronau et al [15]. Although the bound is an improvement
over the last two bounds but it depends on the measurement on \( Br(B^0 \rightarrow \pi^0 \pi^0) \), which posses a serious challenge for the experimentalists. Therefore, the bound given by Gronau et al may not solve the purpose immediately. This in turn necessitates further scrutiny. Since with the accumulation of more and more data samples day by day the Belle and BaBar will be in a position to analyze \( B \rightarrow \pi \pi \) mode more precisely in the near future. This raises a question whether there exists any other way to constrain the penguin parameter (\( \theta \)) exploring the experimental data, as of today.

Our effort in this paper will be an attempt in this direction to obtain some bound on penguin induced shift \( |2 \theta| \). In contrast to the earlier bounds, we show that the measurement of \( B^0(t) \rightarrow \pi^+ \pi^- \) observables can be used to put a lower limit on \( \cos 2 \theta \).

We now present a new bound in terms of the direct CP asymmetry \( a_{dir} \) of \( B^0(t) \rightarrow \pi^+ \pi^- \), which has already been measured at KEK \[10\] and SLAC \[17\]. So it may be worthwhile to put some constraint on the penguin pollution in \( B \rightarrow \pi \pi \) in terms of \( a_{dir} \), which may help us for the extraction of \( \alpha \).

We first consider the decay mode \( B^0 \rightarrow \pi^+ \pi^- \). As it is done usually, defining \( T \) as the color favored tree amplitude, and \( P \) as the penguin ampli-
tude, the transition amplitude for \( B^0 \rightarrow \pi^+ \pi^- \) and \( \bar{B}^0 \rightarrow \pi^+ \pi^- \) modes are given as

\[
A^{+-} \equiv A(B^0 \rightarrow \pi^+ \pi^-) = Te^{i\gamma} + Pe^{-i\beta} =
\]

\[
e^{i\gamma} \left[ T - Pe^{2\alpha} \right]
\]

(3)

\[
\bar{A}^{+-} \equiv A(\bar{B}^0 \rightarrow \pi^+ \pi^-) = Te^{-i\gamma} + Pe^{i\beta} =
\]

\[
e^{-i\gamma} \left[ T - Pe^{-2\alpha} \right]
\]

(4)

where we have used the Wolfenstein parameterization of CKM matrix and substituted \( \alpha = \pi - (\beta + \gamma) \).

The time dependent rate asymmetry in \( B^0(t) \rightarrow \pi^+ \pi^- \) is given as

\[
a_{CP}(t) = \frac{\Gamma(B^0(t) \rightarrow \pi^+ \pi^-) - \Gamma(\bar{B}^0(t) \rightarrow \pi^+ \pi^-)}{\Gamma(B^0(t) \rightarrow \pi^+ \pi^-) + \Gamma(\bar{B}^0(t) \rightarrow \pi^+ \pi^-)} = a_{dir} \cos \Delta mt + a_{mix-ind} \sin \Delta mt
\]

(5)

where

\[
a_{dir} \equiv \frac{1 - |\lambda_{\pi^+ \pi^-}|^2}{1 + |\lambda_{\pi^+ \pi^-}|^2}, \quad a_{mix-ind} \equiv \frac{-2\text{Im}(\lambda_{\pi^+ \pi^-})}{1 + |\lambda_{\pi^+ \pi^-}|^2},
\]

(6)
with
\[
\lambda_{\pi^+\pi^-} \equiv e^{-2i\beta} \frac{A(B^0 \to \pi^+\pi^-)}{A(\bar{B}^0 \to \pi^+\pi^-)}
\] (7)

We now define the average branching ratio \( B_{\pi^+\pi^-} \) for the decay mode \( B^0 \to \pi^+\pi^- \) and \( \bar{B}^0 \to \pi^+\pi^- \) as
\[
B_{\pi^+\pi^-} = \frac{1}{2} \left[ Br(B^0 \to \pi^+\pi^-) + Br(\bar{B}^0 \to \pi^+\pi^-) \right]
\equiv \frac{1}{2} \left[ |A^+|^2 + |\bar{A}^+|^2 \right]
\] (8)

where we express the amplitudes squared in units of two body branching ratios. Substituting the value of \( \lambda_{\pi^+\pi^-} \) from Eq. (7), we obtain the expression for direct CP asymmetry \( (a_{dir}) \) as
\[
a_{dir} = \frac{|A^{-}|^2 - |\bar{A}^{-}|^2}{|A^{-}|^2 + |\bar{A}^{-}|^2}
\] (9)

Recently Belle Collaboration [16] reported their first measurement of the CP violating parameters in \( B^0 \to \pi^+\pi^- \) decay
\[
\begin{align*}
    a_{mix-ind} &= -1.21^{+0.38}_{-0.27} \ (stat.)^{+0.16}_{-0.13} \ (syst.) \\
    a_{dir} &= 0.94^{+0.25}_{-0.31} \ (stat.) \pm 0.09 \ (syst.)
\end{align*}
\] (10)

This is in comparison to the previous BaBar result [17]
\[
\begin{align*}
    a_{mix-ind} &= -0.01 \pm 0.37 \ (stat.) \pm 0.07 \ (syst.) \\
    a_{dir} &= 0.02 \pm 0.29 \ (stat.) \pm 0.07 \ (syst.)
\end{align*}
\] (11)

Taking into account both the Belle and BaBar measurements, the average of \( a_{mix-ind} \) and \( a_{dir} \) is given as
\[
\begin{align*}
    a_{mix-ind} &= -0.64 \pm 0.26 \\
    a_{dir} &= 0.49 \pm 0.21
\end{align*}
\] (12)

One can easily see from Eqs. (6) and (7) that, if we neglect the penguin contribution than we have \( a_{dir} = 0 \) and \( a_{mix-ind} = \sin 2\alpha \). That means we can measure \( \sin 2\alpha \) directly from \( B^0(t) \to \pi^+\pi^- \) decay. However, due to the presence of penguin contributions the extracted value of \( \alpha_{eff} \) from \( B^0(t) \to \pi^+\pi^- \) deviates from the true \( \alpha \) value. We define \( \alpha_{eff} \) as
\[
2\alpha_{eff} = \text{Arg} \left[ e^{-2i\beta} \bar{A}^- A^{+} \right]
\] (13)
Thus it can be seen that the observables in $B^0(t) \to \pi^+\pi^-$ are the average branching ratio and the CP asymmetries:

$$B_{\pi^+\pi^-}, \ a_{\text{dir}} \ \text{and} \ a_{\text{mix-ind}} \equiv \sin(2\alpha_{\text{eff}}) \quad (14)$$

As we have already noted that the vanishing of penguin amplitude $P$ implies $2\alpha_{\text{eff}} = 2\alpha$. Thus the magnitude of penguin amplitude can be expressed as some function of $2\theta$ (say), where $2\theta = (2\alpha_{\text{eff}} - 2\alpha)$.

From Eqs. (3) and (4) we can write

$$(2i\sin\alpha)P = e^{-i\gamma}A^{+-} - e^{+i\gamma}\overline{A}^{+-} \quad (15)$$

Substituting the values of $B$ and $a_{\text{dir}}$ we can express the squared magnitude of penguin contribution as

$$|P|^2 = \frac{B_{\pi^+\pi^-}}{1 - \cos 2\alpha} \left[ 1 - \sqrt{1 - a_{\text{dir}}^2} \cos 2\theta \right] \quad (16)$$

Now we consider the three $B \to \pi\pi$ decay amplitudes as

$$A^{+-} \equiv A(B^0 \to \pi^+\pi^-) = Te^{i\gamma} + Pe^{-i\beta} = \left( T - Pe^{i\alpha} \right) e^{i\gamma}$$

$$A^{00} \equiv A(B^0 \to \pi^0\pi^0) = \frac{1}{\sqrt{2}} \left( Ce^{i\gamma} - P e^{-i\beta} \right) = \frac{1}{\sqrt{2}} \left( C + Pe^{i\alpha} \right) e^{i\gamma}$$

$$A^{+0} \equiv A(B^+ \to \pi^+\pi^0) = \frac{1}{\sqrt{2}} \left( C + T \right) e^{i\gamma} \quad (17)$$

where the CP conserving complex amplitudes $T$, $C$ and $P$ denote the ‘tree’, ‘color suppressed’ and ‘penguin’ contributions. Here we have neglected the small electroweak penguin contributions in the decay mode $B^+ \to \pi^+\pi^0$. These amplitudes obey the isospin triangle relation

$$\frac{1}{\sqrt{2}}A^{+-} + A^{00} = A^{+0} \quad (18)$$

The corresponding $\overline{A}$ amplitudes can be obtained from the $A$ amplitudes by simply changing the signs of weak phases.

Now we define the new amplitudes as

$$B^{ij} = e^{2i\gamma} \overline{A}^{ij} \quad (19)$$
It is obvious that $|B^{ij}| = |\tilde{A}^{ij}|$. Thus we can explicitly write the $B$ amplitudes as

$$B^{+-} = \left( T - P e^{-i\alpha} \right) e^{i\gamma}$$

$$B^{00} = \frac{1}{\sqrt{2}} \left( C + P e^{-i\alpha} \right) e^{i\gamma}$$

$$B^{-0} = \frac{1}{\sqrt{2}} (C + T) e^{i\gamma} \quad (20)$$

Similar to (18) the $B$ amplitudes also obey the isospin triangle relation

$$\frac{1}{\sqrt{2}} B^{+-} + B^{00} = B^{-0} \quad (21)$$

It should be noted that $A^{+0} = B^{-0}$, so that $A$ and $B$ isospin triangles have a common base. Secondly, in the absence of penguin contributions $B^{+-} = A^{+-}$. Thus the relative phase $2\theta$ between these two amplitudes is due to penguin pollution. These two isospin triangles are depicted in Figure-1. One can easily note that the distance between the points $E$ and $H$ (i.e. the difference between $A^{+-}/\sqrt{2}$ and $B^{+-}/\sqrt{2}$) is

$$|EH| = \sqrt{2}|P| \sin \alpha \quad (22)$$

We now consider the triangle EFH with the interior angles as $\angle EFH = 2\theta$ and $\angle EHF = \theta_1$. Using the sine theorem we can write

$$\sin 2\theta = \sin \theta_1 \frac{\sqrt{2}|P| \sin \alpha}{\sqrt{2} |A^{+-}|} \quad (23)$$

We impose now the boundary condition on the angle $\theta_1$. Since the triangle is closed, $\theta_1$ must lie in the range $0 < \theta_1 < 180^\circ$, which implies that $0 < \sin \theta_1 \leq 1$, the maximum value is being for $\theta_1 = 90^\circ$. Thus we can write

$$\sin 2\theta \leq \frac{2|P| \sin \alpha}{|A^{+-}|} \quad (24)$$

Squaring both sides of the above inequality and substituting the value of $4|P|^2 \sin^2 \alpha$ from Eq. (16) and $|A^{+-}|^2 = B_{\pi+\pi+}(1 + a_{dir})$, one can obtain from (24)

$$\sin^2 2\theta \leq \frac{2 \left[ 1 - \sqrt{1 - a_{dir}^2 \cos 2\theta} \right]}{1 + a_{dir}} \quad (25)$$
After simplification the above inequality can be given as

$$\left| \sqrt{1 + a_{dir}} \cos 2 \theta - \sqrt{1 - a_{dir}} \right| \geq 0 \quad (26)$$

The inequality in (26) does not provide any information regarding the relative signs between the two terms. That is we can not draw any conclusion from the above equation whether the first term is greater than or less than the second term and hence no bound on \( \cos 2 \theta \) can be obtained. Therefore, to find out the bound on \( \cos 2 \theta \) we have to consider other informations as well.

It should be noted that the equality sign holds in Eq. (25) only for \( \theta_1 = 90^\circ \). This allows us to write the inequality (25) as

$$\frac{2}{1 + a_{dir}} \left[ 1 - \sqrt{1 - a_{dir}^2} \cos 2 \theta \right] < 1 \quad , \quad (27)$$

which gives us

$$\cos 2 \theta > \frac{1}{2} \sqrt{\frac{1 - a_{dir}}{1 + a_{dir}}} \quad , \quad (28)$$

Now let us assume that \( 2 \theta < 90^\circ \), then considering the right angled triangle EIF with \( \angle EIF = 90^\circ \), we can write

$$\cos 2 \theta = \frac{|FI|}{|EF|} \leq \frac{|FH|}{|EF|} \quad , \quad (29)$$

Again the equality sign is for \( \theta_1 = 90^\circ \). Thus we obtain

$$\cos 2 \theta \leq \frac{|B^+|}{|A^-|} = \frac{|B^+| |A^+|}{|A^-|^2} \quad , \quad (30)$$

It should be noted that \( |B^+| = |\bar{A}^-| \). Now substituting \( |\bar{A}^+ A^+| = B_{\pi^+\pi^-} \sqrt{1 - a_{dir}^2} \) and \( |A^+|^2 = B_{\pi^+\pi^-} (1 + a_{dir}) \) we obtain the bound on \( \cos 2 \theta \) as

$$\cos 2 \theta \leq \sqrt{\frac{1 - a_{dir}}{1 + a_{dir}}} \quad , \quad (31)$$

The above inequality is valid only when \( 2 \theta < 90^\circ \). Combining (28) and (31) the bound on \( \cos 2 \theta \) is given (subject to above restriction) as

$$\frac{1}{2} \sqrt{\frac{1 - a_{dir}}{1 + a_{dir}}} < \cos 2 \theta \leq \frac{1 - a_{dir}}{1 + a_{dir}} \quad , \quad (32)$$
This is the new bound on $\cos 2\theta$. Here the bound on $\cos 2\theta$ is given in terms of the measurable quantity $a_{\text{dir}}$ only. It should be noted here that this bound has been derived assuming that the isospin triangles are closed and they have a common base. Thus, to the extent that isospin is violated, whether by electroweak penguin contribution or by $\pi^0 - \eta, \eta'$ mixing \cite{18}, the bound will be correspondingly weakened.

Substituting the current average value of $a_{\text{dir}}$ from Eq. (12) the bound on $\cos 2\theta$ is given as

$$0.2925 < \cos 2\theta \leq 0.585. \quad (33)$$

To summarize, in this paper we have derived a new upper bound on the penguin induced shift $|\Delta \theta|$ in $B^0 \to \pi^+\pi^-$ decay. In contrast to the earlier bounds, we have shown here that the measurement of direct CP asymmetry in $B^0(t) \to \pi^+\pi^-$ can be used to place some limit on the penguin pollution parameter.

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Figure 1: The $A$ and $B$ isospin triangles.

\[ A^{+0} = B^{-0} \]