Inequalities are abundant in a society with a number of agents competing for a limited amount of resource. Statistics of such social inequalities are usually represented by the Lorenz function $L(p)$, where $p$ fraction of the population possesses $L(p)$ fraction of the total wealth, when the population is arranged in the ascending order of their wealth. Similarly, in scientometrics, such inequalities can be represented by a plot of the citation count against the respective number of papers by a scientist, again arranged in the ascending order of their citation counts. Quantitatively, these inequalities are captured by the corresponding inequality indices, namely the Kolkata $k$ and the Hirsch $h$ indices, given by the fixed points of these nonlinear (Lorenz and citation) functions. In statistical physics of criticality, the fixed points of the Renormalization Group generator functions are studied in their self-similar limit, where its (fractal) structure converges to a unique form (macroscopic in size and alone). The statistical indices in the social science, however, correspond to the fixed points where the values of the generator function (wealth or citation sizes) are commensurately abundant in fractions or numbers (of persons or papers). It has been shown already that under extreme competitions in the markets or in the universities, the $k$ index approaches a universal limiting value, as the dynamics of competition progresses. We introduce and study these indices for the inequalities of (pre-failure) avalanches, given by their nonlinear size distributions in the Fiber Bundle Models (FBM) of non-brittle materials. We show how a prior knowledge of the terminal and (almost) universal value of the $k$ index for a wide range of disorder parameter, can help in predicting an imminent catastrophic breakdown in the model. This observation has also been complemented by noting a similar (but not identical) behavior of the Hirsch index ($h$), redefined for such avalanche statistics.

**I. INTRODUCTION**

The collective dynamics of failure or breaking in any non-brittle material sample proceeds through the failures of individual elements of the material, as the external load or stress on the sample grows. The bursts of elastic energy released (experimentally detected as acoustic emissions) until the complete breakdown of the material, are widely studied (see e.g., [3]) for the universal nature of their statistics across length and energy scales. These bursts or avalanches are often also studied in models, both analytically and numerically. An avalanche is the sequence of failure events taking place in the system in going from one stable state to the next, when the external load on the system is gradually increased. For example, in the Fiber Bundle Model or FBM (see e.g., [4]), which is an ensemble of elements having different failure thresholds collectively carrying a global load, an avalanche size is defined as the total number of elements failing, immediately or due to the internal stress redistribution continued until a stable configuration is reached, after the external load is increased on a stable configuration of the model. The avalanche size could also be measured by the amount elastic energy released from these failed elements. Its distribution would then correspond more naturally to the elastic emissions. For simplicity, however, we consider here the avalanche size to be given only by the number of failed elements. For successive increases in the external load, further avalanches of different sizes occur with various frequencies. The probability distributions of the avalanche sizes, across a broad class of systems, show the common feature of having relatively larger number of smaller events and much fewer number of large ones. Usually the biggest avalanche is proportional to the system size and causes the eventual macroscopic failure of the sample.

This inequality of ranks for the avalanches is similar to what is known to exist in societies with competing agents, for example in the distribution of wealth among individuals, distribution of citations of papers by an author, and so on. There are fewer number of rich people, like fewer papers with high citations. There are commonly used indices to characterize such social inequalities. These measures do not focus on the extreme limits of the corresponding distributions (where frequencies are either very small or very high), but generally focus on its typical attribute, having high value with commensurate value of its frequency of occurrence. This is in contrast to what is usually studied in the avalanche size ($\Delta$) distribution ($D(\Delta)$) statistics viz, the self-similar fixed point limit of asymptotic avalanche size to extract the limiting singularities in $D(\Delta)$. Social inequalities are characterized by the fixed points of the respective non-

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linear inequality distributions or functions. In particular, the social indices like the Kolkata index $k$ ([4], see [5] for a review) or the Hirsch index $h$ ([6], see [7] for a review) correspond to the bare (or unnormalized) nonlinear inequality functions, like the complementary Lorenz function $c$ for $k$ or the citation function $h$ for $h$. These index values do not directly address the extreme, most efficient or vulnerable features of the social or personal behavioral statistics. For that reason, these measures can be remarkably stable with respect to the various parameters of the distribution functions.

In this work, we apply the measures of social inequalities to the time series of avalanches of an externally stressed disordered material. The avalanches are of different size, leading to a variation of these indices with the progressive number of avalanches. The terminal values of these indices correspond to the inequality measure in the system after the catastrophic breakdown has happened i.e., there can be no further avalanche in the system. As mentioned above, due to the fact that these measures do not correspond to the extreme limits of the avalanche distributions, the terminal values of these indices are very less sensitive to different parameters of the avalanche distributions. Therefore, these social inequality measures, when applied to the case of a stressed disordered material showing avalanche dynamics, can help in characterizing the proximity of the material from catastrophic failure point by reflecting the emerging inequality in the avalanche statistics.

In what follows, we first define the various social inequality measures (the Kolkata index $k$ and the Hirsch index $h$) and outline the way in which such measures can help in predicting imminent catastrophic breakdowns in disordered materials. Then we present numerical simulations of the FBM in performing the scaling analysis of $k$ and $h$. We then use these analyses in quantifying the efficiency of these indices in estimating breakdown points. Finally we discuss the results and conclude.

II. METHOD

Here we describe the simulations of fracture of stressed disordered materials using FBM, showing avalanche dynamics. We then describe the methods of calculating the inequality indices ($k$ and $h$) from the avalanche statistics. Then we outline how it is used to predict imminent breakdown.

In the FBM, a macroscopically large number of parallel Hooke springs or fibers are clamped between two horizontal platforms; the upper one helps hanging the bundle while the load hangs from the lower one (see e.g., [1–3]). The springs or fibers are assumed to have identical spring constant, though their individual breaking strengths are assumed to be different (given by a distribution). Once the load per fiber exceeds its own breaking threshold, it fails and this extra load is shared by the surviving fibers. If the platforms are assumed rigid, there is no local deformation around a failed fiber (and no stress concentration around the ‘defect’ created by the failed fibers). The load is shared equally by all the surviving fibers. We consider here this Equal Load Sharing (ELS) scheme [1–3] for redistributing the extra load among the surviving fibers. If this extra load per fiber exceeds the threshold strength of any of the surviving fiber, the avalanche continues. The number $\Delta$ of all the fibers failing in one go (without any increase in the external load) defines the avalanche size and we study the frequency distribution $D(\Delta)$ of avalanches as the external load is increased until complete failure of the bundle.

![Figure 1](image1.png)

**FIG. 1.** The inequality measures for the avalanche sizes in an FBM (with $N = 50000$ fibers). (a) The time series of the avalanche sizes ($\Delta$) are shown for different Weibull modulus ($m$) values. The solid lines indicate the $h$ index and the $k$ index. The left hand side scales are for the avalanche sizes ($\Delta$) and $m$, while the scale for $k$ index is shown on the right hand side. While different samples continue the dynamics for different duration, indicating different values of the critical load $\sigma_c$, the terminal values of these two indices vary only weakly with $m$. (b) The rank plot of the avalanche statistics at different stages of the failure dynamics. The colors indicate the fraction of surviving fibers at the time of measurement. The 45 degree line intercept gives the $h$ index value. (c) The Lorenz curve for different stages of the failure dynamics. Again, the colors indicate the fraction of surviving fiber, hence the stage of the dynamics. The equality line is shown, from where the Lorenz curve gets deviated as the dynamics progresses. The terminal value ($k_c$) of the $k$ index is indicated.
We then extract the values of the Kolkata index \( k \) from the fixed point \( \frac{1}{2} \) of the normalized nonlinear complementary Lorenz function. In the context of economic inequality of a country, the Lorenz function (see e.g., [2]) represents the cumulative fraction \( L(p) \) of wealth possessed by the fraction \( p \) of people of the country, when the people are arranged in the ascending order of their wealth. If every person possesses equal wealth, then the Lorenz curve (or the Lorenz function \( L(p) \)) becomes the diagonal line - called the equality line - from the origin of a unit square. But this is not what is observed. Since poor people have lower wealth, the Lorenz curve is nonlinear. Starting from the origin \( (L(0) = 0) \), it remains below the equality line and monotonically increases to unity at \( p = 1 \) \((L(1) = 1)\). Now, the Kolkata index \( k \) corresponds to the fixed point (see e.g., [1, 4, 5]) of the complementary Lorenz function \( \tilde{L}(p) \equiv 1 - L(p) \): \( \tilde{L}(k) = k \). As such, it is a normalized social inequality measure and it generalizes the century old Pareto 80-20 law [3]: It gives the fraction \( (1-k) \) of people of the country, when the people are arranged in the ascending order of their wealth. If every person possesses equal wealth, then the Lorenz function (see e.g., [2]) at the fixed point \( L(0) = 0 \) of the complementary Lorenz function \( L(1) = 1 \) has minimum value equal to 0.5, when the Lorenz curve becomes the equality line or \( L(k) = k = L(k) = 1 - k \), and has maximum value equal to unity.

A similar study can be done by looking at the inequality of the citations of the papers by an author. For the breaking dynamics of FBM, we numerically evaluate the avalanche statistics \( D(\Delta) \), as the internal dynamics of (local) failures make progress. To extract the \( k \) index value at any time \( t \) after the start of loading the system and before the complete failure of the FBM or sample, we evaluate the Lorenz function \( L(p) \) by estimating first the fraction \( p \) of avalanches of size from 0 to \( \Delta \) from the integral

\[
p = \int_0^{\Delta} \frac{D(\delta) d\delta}{\int_0^{\infty} D(\delta) d\delta},
\]

then solving \( \Delta \) as a function of \( p \), and inserting that in the expression for cumulative avalanche size fraction

\[
L = \int_0^{\Delta} \frac{\delta D(\delta) d\delta}{\int_0^{\infty} \delta D(\delta) d\delta}.
\]

From this Lorenz function \( L(p) \) (with \( L(0) = 0 \) and \( L(1) = 1 \)), we determine the \( k \) index value by solving for the fixed point \( \tilde{L}(k) = k \) of the complementary Lorenz function \( \tilde{L}(p) \equiv 1 - L(p) \). This index value \( k \) characterizes the avalanche distribution \( D(\Delta) \) at that time \( k \) has minimum value equal to 0.5, when the Lorenz curve becomes the equality line or \( L(k) = k = \tilde{L}(k) = 1 - k \), and has maximum value equal to unity.

In the scientometrics context, the Hirsch index \( h \) corresponds to the number \( (h) \) of papers, each having commensurate number \( (h \) or more) of citations at the present or running point of the author’s carrier. Typically, if one plots the number of citations received against the number of papers, arranged in the descending order of citations, the (nonlinear) plot becomes convex towards the origin, and \( h \) index corresponds to the fixed point (intersection point of the 45 degree line) of this nonlinear function. The value of the \( h \) index reflects the author’s success (citation rate in their commensurately prolific range and not in the most successful limit (where the author is necessarily not prolific; highly appreciated or cited papers are low in number)!). The \( h \) index helps to distinguish among the authors, working on similar topics, by comparing their success rates in their commensurately prolific range. When statistically analyzed, one finds some universal scaling behavior: \( h(\sim \sqrt{N_{\text{cit}}} \) [3]) where \( N_{\text{cit}} \) and \( N_{\text{pap}} \) denote respectively the total number of papers written or the total citations received by the author.

In the context of avalanche statistics, \( D(\Delta) \), we extract the failure \( h \) index and find the scaling relation for it’s terminal value \( h_c \) as

\[
h_c = C[\sqrt{N}/\log N],
\]

where the prefactor \( C \) is a function of the Weibull moduli, characterizing the fiber strength disorder in the bundle.

The two indices defined above are monotonically increasing functions of time \( t \) in an avalanching system.
The values reach some terminal or critical limit \((k = k_c\) and \(h = h_c)\) before the catastrophic breakdown of the material. As mentioned before, these terminal values are indicative of the emerging inequalities in the dynamics and not based on the extreme events only. These values are, as we shall demonstrate in the following section, remarkably stable with respect to various threshold distributions of the FBM. Therefore, monitoring the growth of \(k\) and \(h\) values and stopping the loading process before the average terminal values \(k_c\) and \(h_c\) in the FBM, can help in loading a system and minimize the risk of overloading (hence breakdown).

III. NUMERICAL STUDIES FOR \(h\) AND \(k\) INDICES IN FBM

We consider here a FBM system consisting of \(N\) fibers \((5,000 \leq N \leq 100,000)\) having identical spring constant, but having different failure strengths \(\sigma_f\) given by the cumulative Weibull distribution

\[
F(\sigma_f) = 1 - exp\left[-(\sigma_f)^m\right].
\]  

Here \(m\) denotes the Weibull modulus and we choose the range \(1 \leq m \leq 5\) for this study. The variation in the Weibull modulus for the simulations takes care of the fact that in real samples the disorder strengths can be very different. A larger value of \(m\) makes the failure more abrupt i.e., a shorter time series of avalanche – in a limiting case leading to a brittle failure. A smaller \(m\) would eventually lead to individual, single failures. In between these two limits, there exists an avalanching quasi-brittle regime, where a temporal correlation exists in the breaking statistics which is commonly seen in experiments. In choosing the range for the Weibull parameter, therefore, we made sure that the dynamics of the model is in the quasi-brittle region, showing scale free avalanche size distribution. For \(m < 1\), the threshold distribution becomes very wide and the failure progresses through individual fiber breaking without any temporal correlation or large avalanche. For large values of \(m\), the system becomes brittle i.e., the first avalanche breaks the entire system.

The external force (stress) on the FBM increases until a fiber fails and does not increase further until the successive fibers fail due to stress reallocations. As mentioned before, the number \(\Delta\) of such failed fibers in one go (before the stress is increased further) defines the avalanche size. The external load is then increased further until the weakest surviving fiber(s) fail and causes a further avalanche. Effectively this means that the external load on the bundle increases very slowly, since the load readjustments following any fiber failure are very fast. The process then continues until the entire bundle fails. We
study the (frequency) distribution \(D(\Delta)\) of the avalanche sizes \(\Delta\) until the time \(t\) (where \(t = 0\) corresponds to the time of putting load on the bundle) and continue up to complete failure of the bundle. At each intermediate point of time \(t\), we estimate the \(k\) index value at \(t\) by evaluating first the Lorenz function \(L(p)\) following the equations (1) and (2) and then finding the fixed point \(k\) of \(\dot{L}(k) \equiv 1 - L(k) = k\). We also determine the \(h\) index values as given by the size \((\Delta = h)\) of the avalanche which matches its frequency \((D(h) = h)\) of occurrence. We average typically over 100 to 10,000 disorder configurations.

Fig. 1 shows how the values of different dynamical quantities (including those for indices \(k\) and \(h\)) change as the dynamics of breaking progresses in some representative FBM.s. The estimated values of the index \(k\) at different times \(t\) (scaled by \(N/\log N\)) before the bundle fails, are plotted in Fig. 2 until complete failure of the bundle (where the disorder of the fiber of the bundle, characterized by different Weibull moduli \((m)\), is indicated using different colors). The observed terminal values of the \(k\)-index \((k_c = 0.63 \pm 0.02)\), prior to the complete failure of the bundle seem to be weakly dependent on \(m\). Similar universality, but at a higher terminal value, was seen in the cases of citation index \(k\) of authors in the limit of extreme competitiveness [8]. Our study demonstrates that prior knowledge of this limiting value of the Kolkata index \(k\) for the growing avalanche statistics \(D(\Delta)\) would thus help predicting the imminent failure point or time (when scaled by \(N/\log N\)).

In Fig. 3 we show that the failure \(h\) index of the FBM scales as in Eq. (3) with a \(\log N\) correction factor to the scaling behavior \(h \sim \sqrt{N}\), also seen in the context of journal citations [8–9].

In demonstrating how the social index, for example, \(k\) \((h)\) could be useful in predicting the imminent failure, we load a system and continuously monitor its index value, until the value is a multiple \((q)\) of the standard deviation \(\chi_k\) \((\chi_h)\) away from the average critical (terminal) value \(k_c\) \((h_c)\) of \(k\) \((h)\). The system of course can break before that, due to sample-to-sample fluctuations of these terminal values, and in those cases we then estimated load carrying capacity \((W_{\text{max}} = N(\sigma_{\text{max}}))\) to be zero in its statistics. As can be seen in Fig. 4 for very low values of \(q\), the loading is not stopped until the average terminal value is reached, causing breakage of almost half of the samples (see Fig. 5), making the estimated safety limit of \(W_{\text{max}}\) much lower. For very high values of \(q\), the loading is stopped too soon, again making \(W_{\text{max}}\) too low. In an intermediate range of \(q\), the value of \(W_{\text{max}}\) reaches a peak. The peak value is the highest when the estimate of \(W_{\text{max}}\) is made using the knowledge of the \(k\) index terminal value, making it the most effective monitoring parameter for safe-loading. The other parameters discussed here (or any other method we know of) give lower estimate of the loading capacity \(W_{\text{max}}\) for samples characterized by Weibull distributions (at least in the range of the modulus \(m)\) considered here. We also checked this method for uniform distribution of the failure threshold in the range \((0.5 - r, 0.5 + r)\). When the system is close to the brittle limit \(r = 1/6\), the predictability using \(k\) index is less effective. However, for higher values of \(r\) (see Fig. 6), \(k\) index based loading works best, as before.

In order to verify the universality of the system size scaling of \(k\) and \(h\) indices in FBM and also to further justify the effective monitoring process of safe-loading, we study the dynamics of the model when the threshold distribution is uniform between \((0, 1)\). In Figs. 7 and 8 the system size scaling of \(k\) and \(h\) indices are shown and they seem to obey the same scaling as was noted before for the Weibull distribution (Figs. 2 and 3). To demonstrate the
safe-loading, we choose sample-sets where the threshold distributions are uniform in \((0,r_0)\), but the value of \(r_0\) is again chosen from a uniform distribution within \((0,r_1)\). In that case, the critical load \(\sigma_c\) will vary strongly with the upper limit of the distribution, but the inequality indices will not. Fig. 8 shows the corresponding maximum loading for different values of \(r_1\). In all cases, monitoring \(k\) and \(h\) indices perform much better than monitoring \(\sigma\). Indeed, as is seen in the system size scaling of the \(h\) index, it will vary depending upon the system size (Fig. 8), while no such systematic variation exists for the \(k\) index (Fig. 7). It is therefore, the most useful monitoring parameter for safe loading studied here, as its terminal (critical) value does not have any systematic dependence on the size of the system and a rather weak dependence on the parameters of the threshold distributions, as long as the failure dynamics is not too close to brittle failure.

FIG. 7. The system size scaling of the \(k\) index for threshold distribution uniform in \((0,1)\). The terminal value of \(k\) becomes \(0.62 \pm 0.03\).

FIG. 8. The system size scaling of the \(h\) index for threshold distribution uniform in \((0,1)\).

IV. SUMMARY & DISCUSSIONS

The breaking dynamics of disordered materials under slow external loading proceeds through intermittent bursts of avalanches of a wide range of sizes. There is no characteristic size i.e., the size distribution of these avalanches are scale-free and typically follows a decaying power-law behavior in the asymptotic limit of large avalanche sizes. What intrigued researchers over the last three decades, is the emerging universality of the exponent value of the avalanche size distribution from the tectonic scale of the earthquakes (Gutenberg-Richter law) to the laboratory scale of quasi-brittle materials. Given the striking regularities in such statistics, both in experiments and in numerical model simulations, various features of avalanche size distributions have been routinely used in aiming to predict imminent catastrophic avalanches (see e.g., [10–15]).

We have studied here the failure dynamics of non-brittle materials using the Fiber Bundle Model, having fiber strengths characterized by Weibull distribution (4) and also the case of uniformly distributed fiber threshold. Specifically, we have studied here numerically the avalanche distribution \(D(\Delta)\) of the avalanches of size \(\Delta\) as the dynamics of breaking proceeds. The different values of the Weibull modulus correspond to the differences in individual samples, as is also seen in citation counts of individual authors or wealth distributions in economies of different countries.

As mentioned already, the critical behavior (characterized by the critical exponents) very near the critical or breaking point of the bundle (where the avalanche size \(\Delta\) reaches its asymptotic limit of \(O(N)\)) is very well studied (see e.g., [16–18]). Indeed, the critical behavior of the avalanche distribution \(D(\Delta) \sim \Delta^{-\gamma}\), and the universality class given by the exponent \(\gamma\) in this FBM (with equal
load sharing), have also been studied in other widely different contexts (see e.g., [14]). Although the detailed knowledge of the critical behavior for such breaking in the FBM are extremely useful in comprehending the universality class of breaking phenomena and their statistics in different contexts, because of the tiny extent of the critical region (before the complete failure point, where such critical behavior can be observed), they do not help much in the attempts to predict the breaking point or time (see however [3, 17] for overloaded FBM cases).

We studied here numerically (for 5,000 \( \leq N \leq 100,000 \)) the avalanche distribution \( D(\Delta) \) at different times \( t \) during the failure dynamics for the bundle, under slowly but uniformly increasing load on the bundle, until complete failure. The failure index \( h \) is given by the avalanche size \( h \) of \( \Delta \) which is equal to its frequency of occurrence (\( h = D(h) \)) at any time \( t \) during the process of breaking of the bundle until its complete failure. For extracting the values of the Kolkata index \( k \) (at different times \( t \) of the dynamics) we evaluated first the Lorenz functions \( L(p) \) (giving the cumulative fraction of avalanche sizes of \( p \) fraction of avalanches when arranged from the smallest to the largest or \( N \) order avalanches), using Eqs. (1) and (3). We then look for the fixed point solution of the equation \( 1 - L(k) = k \) of the complementary Lorenz function \( \tilde{L} \equiv 1 - L \) given by the distribution \( D(\Delta) \) at that time \( t \). Fig. 1 how the values of different dynamical quantities, in particular the indices \( h \) and \( k \) change as the dynamics of breaking progresses in some representative FBMs. These estimated values of \( k \) at different times \( t \) of the bundle breaking process, before the complete failure of the bundle, are plotted in Figs. 2 and 3. The observed terminal value \( k_c = (0.62 \pm 0.03) \) of the \( k \)-index, prior to complete failure of all the studied bundles, seems to be practically independent of the nature of disorder in the bundle (e.g., Weibull modulus \( m \), or the uniformity of the distribution of fiber strengths in the bundle, or the bundle size \( N \)) though the breaking time depends strongly on \( m \) and fiber number \( N \) in the bundle). It may be noted in this connection that this average value of \( k_c \) is approximately inverse of the Golden ratio which is the precise value of \( k \)-index when the Lorenz function \( L(p) \) becomes quadratic in \( p \) (see e.g., [3]). The fixed point value \( k = \tilde{L}(k) = k = 1 - k^2 \) is then given by \( k = k_c = (\sqrt{5} - 1)/2 \approx 0.618 \). As discussed in the previous section, the advantage of monitoring the \( k \)-index value for estimating the maximum load on the bundle (clearly demonstrated in Figs. 4-6 and 9) are very encouraging. We also found (see Fig. 3) that the scaled prefactor \( (C) \) of \( h \), given by \( h/[\sqrt{N}/logN] \), approaches some fixed limiting values dependent on the Weibull modulus \( m \), at the bundle failure point or time (scaled with \( N \)).

As we mentioned in the Introduction, in social sciences the important indices try to capture the structure of the inequality distributions (e.g., wealth, citations) typically in its fixed point region, where the distribution frequency is neither very weak (as in the super rich and highly cited limit) nor very prolific (as in the poor and scarcely cited limit). As we showed here in Figs. 2 and 7 the (almost) universal terminal value of the Kolkata index \( k_c = (0.62 \pm 0.03) \) for the statistics \( D(\Delta) \) of avalanches can help in an unambiguous way in predicting the complete failure point or time of the FBM. The scaling prefactor \( (C) \) of the failure index can also help locating the macroscopic failure point of the FBM (see Figs. 3 and 8), provided the precise knowledge of \( m \) and \( N \) are available. This aspect of the terminal value \( k_c \) of \( h \)-index causes its use to be considerably limited.

In Sociophysics (see e.g., [18]) or econophysics (see e.g., [10]) it is usually argued that models and techniques of statistical physics can lead to major success in comprehending the social and economic phenomena. Our study here may be the first one to show that various statistical indices of social sciences can in turn lead to some useful predictive power for the dynamics of failures in materials: The Hirsch and Kolkata indices, given by the fixed points of the avalanche size distributions (much away from their self-similarity induced critical breaking point) seem to offer some unique and potentially useful techniques for predicting the failure of materials. We demonstrate the success of two such indices for the failure statistics of materials in almost one hundred year old Fiber Bundle Model, extensively studied both computationally and analytically in some limiting cases (see e.g., [13]); and experimentally (see e.g., [20]). Needless to mention that analyzing the experimental data for the time series of ultrasonic emissions before complete failure in materials and similar studies for some of the established theoretical models of self-organized critical dynamics in FBM (see e.g., [21]) and in earthquakes (see e.g., [22]) will be extremely important.

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