Improved bounds on SUSY accompanied neutrinoless double beta decay

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Abstract

Neutrinoless double beta decay \((0\nu\beta\beta)\) induced by light Majorana neutrino exchange between two decaying nucleons with squark/slepton exchange inside one and \(W\) exchange inside the other nucleon (so–called vector–scalar exchange) gives stringent limits on \(R\)–parity violating interactions. We have extended previous work by including the tensor contribution to the transition rate. We discuss the improved limits on trilinear \(R_p\)–MSSM couplings imposed by the current experimental limit on the \(0\nu\beta\beta\) decay half–life of \(\text{^{76}Ge}\).

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In supersymmetric models, the new SUSY partners differ from the SM field content in a discrete multiplicative quantum number \(R\)–parity defined as

\[ R_p = (-1)^{3B+L+2S}. \]  

Here \(B\) denotes the baryon number, \(L\) the lepton number and \(S\) the spin of a particle leading to \(R_p = +1\) for SM particles and \(R_p = -1\) for superpartners. Thus in \(R_p\) conserving models superpartners can only be produced in pairs and the LSP is stable, leading to a natural WIMP dark matter candidate. While in the minimal supersymmetric extension (MSSM) of the standard model (SM) \(R\)–parity is assumed to be conserved, there are no theoretical reasons for \(R_p\) conservation and several GUT [1] and Superstring [2] models require \(R\)–parity violation in the low energy regime. Also the reports concerning an anomaly

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at HERA in the inelastic $e^+p$ scattering at high $Q^2$ and $x$ [3] have renewed the interest in $R_P$-SUSY (see for example [4,5]). Generally, one can add the following trilinear R-parity violating terms to the superpotential [1]

$$W_{R_P} = \lambda_{ijk} L_i L_j E_k + \lambda'_{ijk} L_i Q_j D_k + \lambda''_{ijk} U_i U_j D_k,$$

(2)

where $i, j, k$ denote generation indices, $L, Q$ denote lepton and quark doublet superfields and $E, U, D$ lepton, up- and down quark singlet superfields. Terms proportional to $\lambda, \lambda'$ violate lepton number, those proportional $\lambda''$ violate baryon number. While simultaneous presence of both kinds of terms would lead to too fast proton decay and thus is forbidden, assuming the $\lambda''$ terms to be zero no constraints on $\lambda$ and $\lambda'$ terms can be derived from proton decay.

The search for neutrinoless double beta decay, converting a nucleus $(Z, A)$ into a nucleus $(Z+2, A)$ under emission of two electrons, has been proven to belong to the most powerful tools to search for lepton number violating physics beyond the SM (for a review see [6,7]). Contributions occuring through Feynman graphs involving the exchange of superpartners as well as $R_P$-couplings $\lambda'$ have been discussed in [8–12] and yield the most stringent bound on $\lambda''_{111}$. Taking into account the fact that the SUSY partners of the left and right-handed quark states can mix with each other, also diagrams appear in which the neutrino-mediated double beta decay is accompanied by SUSY exchange in the vertices (see fig. 1). These contributions allow to constrain also combinations of couplings of higher generations $\lambda'_{11j}\lambda'_{1j1}$. They have been discussed in [13] and more extensively in [11], where however only scalar-pseudoscalar currents have been taken into account, whereas the tensor contribution to the decay rate has been neglected. On the other hand, in a recent work [14] the dominance of the tensor contribution in a general framework for neutrino mediated double beta decay has been proven. In the present letter we reanalyze SUSY–accompanied double beta decay and discuss the relative importance of the different nuclear matrix elements involved.

The mixing between scalar superpartners $\tilde{f}_{L,R}$ of the left and right-handed fermions $f_{L,R}$ occurs due to non-diagonality of the mass matrix which can be written as

$$\mathcal{M}_{\tilde{f}}^2 = \begin{pmatrix} m_{\tilde{f}_L}^2 + m_{\tilde{f}}^2 - 0.42 D_Z & -m_f (A_f + \mu \tan\beta) \\ -m_f (A_f + \mu \tan\beta) & m_{\tilde{f}_R}^2 + m_{\tilde{f}}^2 - 0.08 D_Z \end{pmatrix}. $$

(3)

Here, $f = d, s, b, e, \mu, \tau$ and $\tilde{f}$ are their superpartners. $D_Z = M_Z^2 \cos2\beta$ with $\tan\beta = \langle H_2^0 \rangle / \langle H_1^0 \rangle$ being the ratio of vacuum expectation values of the two Higgs doublets, $m_{\tilde{f}_{L,R}}$ are soft sfermion masses, $A_f$ are soft SUSY breaking parameters describing the strength of trilinear scalar interactions, and $\mu$ is the supersymmetric Higgs(ino) mass parameter. Once sfermion mixing is included, the current eigenstates $\tilde{f}_L, \tilde{f}_R$ become superpositions of the mass eigenstates $\tilde{f}_i$ with the masses $m_{\tilde{f}_i}$ and the corresponding mixing angle $\theta_{\tilde{f}}$ is defined as

$$\sin2\theta_{\tilde{f}} = \frac{2m_{\tilde{f}_L}m_{\tilde{f}_R}}{m_{\tilde{f}_1}^2 - m_{\tilde{f}_2}^2},$$
Now it is straightforward to find the effective 4-fermion $\nu - u - d - e$ vertex induced by the sfermion exchange in the diagrams presented in fig. 1. The corresponding effective Lagrangian, after a Fiertz rearrangement, takes the form

$$L_{\text{SUSY}}^{\text{eff}}(x) = \frac{G_F}{\sqrt{2}} \left[ \frac{1}{4} \left( \eta_{(q)LR}^{n_j} - 4 \eta_{(q)LR}^{n_j} \right) \cdot U_{ni}^* \cdot \left( \bar{\nu}_i (1 + \gamma_5) e_j^c \right) \left( \bar{u}(1 + \gamma_5)d \right) - 2 \eta_{(i)LL} \cdot U_{ni} \cdot \left( \bar{\nu}_i (1 - \gamma_5) e_j^c \right) \left( \bar{u}(1 + \gamma_5)d \right) + \frac{1}{2} \eta_{(i)RR} \cdot U_{ni} \left( \bar{\nu}_i \gamma^\mu (1 + \gamma_5) e_j^c \right) \left( \bar{u} \gamma_\mu (1 - \gamma_5)d \right) + \frac{1}{8} \eta_{(i)LR} \cdot U_{ni}^* \cdot \left( \bar{\nu}_i \sigma^\mu (1 + \gamma_5) e_j^c \right) \left( \bar{u} \sigma_\mu (1 + \gamma_5)d \right) \right].$$

The $R_p$ MSSM parameters $\eta$ and neutrino mixing matrix $U_{ij}$ are defined as follows

$$\eta_{(q)LR}^{n_j} = \sum_k \frac{\lambda'_{jkL} \lambda_{nkL}}{2 \sqrt{2} G_F} \sin 2 \theta^d_k \left( \frac{1}{m_{d_1(k)}^2} - \frac{1}{m_{d_2(k)}^2} \right),$$

(6)

$$\eta_{(q)RR}^{n_j} = \sum_k \frac{\lambda'_{jkL} \lambda_{nkL}}{2 \sqrt{2} G_F} \left( \frac{\sin^2 \theta^d_k}{m_{d_1(k)}^2} + \frac{\cos^2 \theta^d_k}{m_{d_2(k)}^2} \right),$$

(7)

$$\eta_{(i)LR}^{n_j} = \sum_k \frac{\lambda'_{jk} \lambda_{nkj}}{2 \sqrt{2} G_F} \sin 2 \theta^e_k \left( \frac{1}{m_{e_1(k)}^2} - \frac{1}{m_{e_2(k)}^2} \right),$$

(8)

$$\eta_{(i)LL}^{n_j} = \sum_k \frac{\lambda'_{jk} \lambda_{nkj}}{2 \sqrt{2} G_F} \left( \frac{\cos^2 \theta^e_k}{m_{e_1(k)}^2} + \frac{\sin^2 \theta^e_k}{m_{e_2(k)}^2} \right),$$

(9)

$$\nu^0_i = \sum_j U_{ij} \nu_j.$$  

(10)

Here $\eta_{(f)LR}$ denotes the contribution vanishing in the absence of $\tilde{f}_L - \tilde{f}_R$ mixing while $\eta_{(f)LL}$ and $\eta_{(f)RR}$ in this limit correspond to the $\tilde{f}_L$ and $\tilde{f}_R$ exchange contribution in fig. 1. We use the notations $d_{(k)} = d, s, b$ and $e_{(k)} = e, \mu, \tau$. Due to the antisymmetry of the Yukawa coupling $\lambda_{njk}$ in $n_j$ it follows that $\eta_{(n)LR}^{n_j} = 0$. This is an essential difference between the slepton $\tilde{L}_L - \tilde{L}_R$ and the squark $\tilde{q}_L - \tilde{q}_R$ contributions. The latter is not imposed to vanish at any combination of indexes. Since the $\eta_{(i)LL}$ and $\eta_{(i)RR}$ contributions to the diagram fig. 1 are helicity suppressed $\propto \langle m_\nu \rangle \simeq \mathcal{O}(0.5) \text{eV}$, we can restrict ourselves to the consideration of $\eta_{(q)LR}^{n_j}$ and $\eta_{(i)LR}^{n_j}$, which contributions are momentum enhanced $\propto q \simeq p_F \simeq 100 \text{ MeV}$, where $p_F$ denotes the Fermi momentum. Here $\langle m_\nu \rangle$ and $q$ denote the effective neutrino mass and momentum entering the neutrino propagator. The first line in eq. (5) corresponds to the scalar-pseudoscalar $(S + P)$ contribution, the last line in (5) to the tensor contribution.
In the light neutrino case the $0\nu\beta\beta$ decay rate is given by
\[
T_{1/2}^{0\nu\beta\beta} = G_{01} \left\{ 4\bar{\eta}(q)M_{S+P} + \left( -\bar{\eta}(q) + \eta(q) \right) (M_{S+P} + M_T) \right\}^2 .
\] (11)
Here $G_{01}$ denotes the phase space factor defined in [15], $\eta(q) = \eta_{(q)LR}$ and effective parameters are introduced as
\[
\bar{\eta}(q) = \sum_n \Delta_n \eta(q)_{LR}.
\] (12)
For the $\bar{\eta}(q)$ summation starts from $n = 2$ and $\Delta_n$ denotes the combination of mixing matrices corresponding to heavy or sterile neutrinos
\[
\Delta_n = \sum_i U_{i,j}^* U_{i,n} U_{i,n}.
\] (13)
where the sum extends over heavy mass eigenstates $m_{\nu_i} > 10$ GeV (see [11]). In s-wave approximation for the outgoing electrons and under some assumptions according to [16,11] (the s-wave approximation is expected to affect the result less than 10 % [14]) the matrix element is
\[
M_{S+P} = \frac{F_P^{(3)}}{4Rm_e G_A} \left( M_{T'} + \frac{1}{3} M_{GT'} \right),
\] (14)
\[
M_T = \alpha_1 \left( \frac{2}{3} M_{GT'} - M_{T'} \right),
\] (15)
with (summation over nucleons $a, b$ is suppressed)
\[
M_{GT'} = \langle 0^+_f | h_R (\bar{\sigma}_a \bar{\sigma}_b) \tau^+_a \tau^+_b | 0^+_i \rangle,
\] (16)
\[
M_{T'} = \langle 0^+_f | h_{T'} \{ (\bar{\sigma}_a \bar{r}_{ab})(\bar{\sigma}_b \bar{r}_{ab}) - \frac{1}{3} (\bar{\sigma}_a \bar{\sigma}_b) \} \tau^+_a \tau^+_b | 0^+_i \rangle,
\] (17)
\[
\alpha_1 = \frac{T_1^{(3)}(0) G_V (1 - 2m_P (G_W / G_V))}{2G_A^2 R m_e}.
\] (18)
$h_R$ and $h_{T'}$ are neutrino potentials defined as
\[
h_R = \frac{2}{\pi \frac{R^2}{m_F}} \int dq dq \frac{j_0(q r_{ab}) f^2(q^2)}{\omega(\omega + E)},
\] (19)
\[
h_{T'} = \frac{2}{\pi \frac{R^2}{m_F}} \int dq dq \frac{j_0(q r_{ab}) f^2(q^2)}{\omega(\omega + E)} \{ q^2 j_0(q r_{ab}) - 3 q r_{ab} j_1(q r_{ab}) \}.
\] (20)
Here $R$ denotes the nuclear radius, $m_P$ the proton mass.

Further $\omega = \sqrt{q^2 + m_F^2}$, $q = |\bar{q}|$, $\hat{r} = \bar{r}/r$ and $j_k(q r)$ are spherical Bessel functions. $(\omega + E)^{-1}$ is the energy denominator of the perturbation theory. The form factors $F_P^{(3)}(0) = F_{1}^{(3)}(q^2)/f(q^2)$ and $T_1^{(3)}(0) = T_1^{(3)}(q^2)/f(q^2)$ with $f(q^2) = (1 + q^2/m_A^2)^{-2}$ ($m_A = 0.85$ GeV) have been calculated in the MIT bag model in [17], $G_A \approx 1.26$, $G_V \approx 1$ and the strength of the induced weak magnetism $(G_W / G_V) = \frac{\mu_{\nu} - \mu_{\mu}}{2m_p} \approx \frac{3.7}{2m_p}$ is obtained by the CVC hypothesis.

For comparison and to correct some details in [11] we first concentrate on the $S+P$ part, i.e. $M_T = 0$ in eq. 11. Inserting the numerical value of the matrix elements $M_{GT'} = 2.95$ and $M_{T'} = 0.224$ [11] for the special case of $^{76}$Ge and the half life limit obtained from the Heidelberg–Moscow experiment, $T_{1/2}^{0\nu\beta\beta} > 1.2 \cdot 10^{25} y$, [18,7] one derives $\eta \leq 4.5 \cdot 10^{-8}$, $\bar{\eta}(0) \leq 1.1 \cdot 10^{-8}$, corresponding to
limits of

$$\lambda'_{112}\lambda'_{121} \leq 5.0 \cdot 10^{-6} \left( \frac{\Lambda_{SUSY}}{100 \text{GeV}} \right)^3$$

$$\lambda'_{113}\lambda'_{131} \leq 1.6 \cdot 10^{-7} \left( \frac{\Lambda_{SUSY}}{100 \text{GeV}} \right)^3$$

$$\Delta_n\lambda'_{311}\lambda'_{n13} \leq 9.9 \cdot 10^{-8} \left( \frac{\Lambda_{SUSY}}{100 \text{GeV}} \right)^3.$$  \hspace{1cm} (21)

Here in the last equation only the term corresponding to $\tilde{\tau}$ exchange has been kept. These limits differ from the ones given in [11] due to an erroneous factor of 2 in the definition of $\mathcal{M}_{S+P}$ in that reference as well as the improvement in the experimental bound.

Keeping the tensor part in eq. 11 the half life limit of the Heidelberg–Moscow Experiment implies:

$$\lambda'_{112}\lambda'_{121} \leq 1.1 \cdot 10^{-6} \left( \frac{\Lambda_{SUSY}}{100 \text{GeV}} \right)^3,$$

$$\lambda'_{113}\lambda'_{131} \leq 3.8 \cdot 10^{-8} \left( \frac{\Lambda_{SUSY}}{100 \text{GeV}} \right)^3$$  \hspace{1cm} (22)

for supersymmetric mass parameters of order 100 GeV. For $\Lambda_{SUSY} \sim 1$ TeV, motivated by SUSY naturalness arguments, one obtains $\lambda'_{112}\lambda'_{121} \leq 1.1 \cdot 10^{-3}$, $\lambda'_{113}\lambda'_{131} \leq 3.8 \cdot 10^{-5}$. These are by more than a factor of four more stringent than the limits obtained from the scalar pseudoscalar part considered in [11].

The uncertainty of the nuclear matrix elements involved can be estimated to be less than a factor of two. This is motivated by the fact that the $2\nu\beta\beta$ half-life of $^{76}\text{Ge}$ has been predicted correctly within a factor of 2 (the $2\nu\beta\beta$ matrix element within a factor of $\sqrt{2}$) [19]. Since the uncertainty in both parts ($S + P$ and tensor) is expected to be about the same, the improvement can still be considered as substantial.

The obtained bound should be compared with the limits obtained from tree level $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ mixing, which yield $\lambda'_{112}\lambda'_{121} \leq 1.0 \cdot 10^{-9}$ and $\lambda'_{113}\lambda'_{331} \leq 8 \cdot 10^{-8}$ respectively, for $m_{\tilde{e}} = 100$ GeV [20]. While the second generation is bounded by about three orders of magnitude more stringent from the $K$ system, for the third generation the double beta bound is most stringent. Moreover, since the masses of different exchanged particles (selectrons and squarks) enter, the limits are complementary in some sense.

In conclusion, we have performed a reanalysis of the SUSY–accompanied neutrino exchange mode of neutrinoless double beta decay. Contrary to the previous ansatz we included the tensor contribution to the decay rate, which has been shown to be the dominant contribution. This improves the limits on $\lambda'_{11j}\lambda'_{1j1}$ derived without the tensor contribution by a factor of four and thus provides the most stringent bound on $\lambda'_{113}\lambda'_{131}$.

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Fig. 1. Feynman graphs for the neutrino exchange mechanism of neutrinoless double beta decay accompanied by (a) squark and (b) slepton exchange.