Inflation without self-reproduction in $F(R)$ gravity

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ABSTRACT

We investigate inflation in frames of two classes of $F(R)$ gravity and check its consistency with Planck data. It is shown that $F(R)$ inflation without self-reproduction may be constructed in close analogy with the corresponding scalar example proposed by Mukhanov for the resolution the problems of multiverse, predictability and initial conditions.

Subject headings: inflation; modified gravity; multiverse
1. Introduction

Recent observational results from different experiments indicate towards the very natural choice of inflation as the very-early universe epoch. Still, specific inflationary models are not free from various problems. One problem was indicated in Ref. (Mukhanov 2014) in relation with cosmological perturbation theory (for the introduction and review, see (Mukhanov 2005, 2013; Gorbunov & Rubakov 2011)). In order to resolve the problem of multiverse, predictability and initial conditions it has been proposed the inflationary model without self-reproduction. The explicit scalar inflationary theory without self-reproduction has been formulated there (Mukhanov 2014). Note that different approach to the resolution of above problems of multiverse and initial conditions was also developed (for a review, see, (Linde 2014)).

In order to avoid the self-reproduction of the universes at the inflationary epoch, Mukhanov proposed the following conditions (Mukhanov 2014):

1. \(1 + w(N) = 1\) at \(N = 1\) (to have graceful exit),
2. \(1 + w(N) \leq 2/3\) at \(N = N_m\) (to solve initial condition problem),
3. \(1 + w(N) \ll 1\) for \(1 < N < N_m\) (inflation),
4. \(1 + w(N) > \epsilon(N)\) for \(1 < N < N_m\) (no self-reproduction).

Here \(N\) is the e-folding defined by \(a = a_0e^{-N}\) with a constant \(a_0\) and \(w(N)\) is the equation of state parameter. Furthermore, \(\epsilon(N)\) is the energy-density of the Planck unit and it is related with the energy-density of the usual unit by \(\epsilon(N) \sim \kappa^4\rho(N)\). The purpose of this letter is to construct an inflationary model of \(F(R)\) gravity satisfying the condition of no self-reproduction. See (Capozziello 2014) for the cosmology from the inflation to the dark energy in the \(F(R)\) gravity. Note that we consider \(F(R)\) frame, so our discussion is
not applied to modified gravities which have no inflationary epoch as direct solution of dynamical equations.

2. The explicit inflationary model of $F(R)$ gravity.

First, we should note that if there is no matter, we may consider the effective EoS parameter by

$$w = -1 - \frac{2\dot{H}}{3H^2} = -1 - \frac{2H'}{3H}. \quad (1)$$

Here $H$ is the Hubble rate defined by $H = \dot{a}/a$. Therefore

$$H = H_0 e^{-\frac{1}{2} \int N dN' (1 + w(N))}. \quad (2)$$

Especially when $1 + w(N) = 1$, $H \propto e^{-\frac{4}{3}N}$ and when $1 + w(N) = 2/3$, $H \propto e^{-N}$.

It is well-known that when $F(R) \propto R^n$,

$$H \propto e^{-N/\alpha}, \quad \alpha = \frac{-1 + 3n - 2n^2}{n - 2}. \quad (3)$$

Then when one regards $w$ as a constant, we find

$$n = \frac{-\alpha + 3 \pm \sqrt{\alpha^2 + 10\alpha + 1}}{4}. \quad (4)$$

Note that if $\alpha > 0$, $n$ is real. Especially when $1 + w(N) = 1$, that is, $\alpha = 2/3$,

$$n = n^f_{\pm} \equiv \frac{7 \pm \sqrt{73}}{12} = 1.29533... , \quad -0.12866... , \quad (5)$$

and when $1 + w(N) = 2/3$, that is, $\alpha = 1$,

$$n = \frac{1 \pm \sqrt{3}}{2} = 1.366026... , \quad -0.36602... . \quad (6)$$

Let define

$$n^{i}_{\pm} \equiv \frac{-\alpha + 3 \pm \sqrt{\alpha^2 + 10\alpha + 1}}{4}, \quad (7)$$
when $1 + w(N) \leq 2/3$, that is, $\alpha \geq 1$. Note that $n^i_+$ and $n^f_+$ are positive but $n^i_-$ and $n^f_-$ are negative.

First, we may consider the following model

$$F(R) = F_i \left( \frac{R}{R_e} \right)^{n^i_+} + F_f \left( \frac{R}{R_e} \right)^{n^f_+}. \quad (8)$$

Here we do not include the Einstein-Hilbert term $R/2\kappa^2$. The above model belongs to the class of theories with positive and negative powers of curvature. Such theories have been introduced in Ref. (Nojiri & Odintsov 2003) in order to unify inflation with dark energy (for general review of the unification of inflation with dark energy in $F(R)$ cosmology, see (Nojiri & Odintsov 2006A, 2010; Capozziello & De Laurentis 2011; Capozziello & Faraoni 2010)). When $R \gg R_e$, the first term dominates because $n^i_+$ is positive and the condition $2$ could be satisfied. On the other hand, when $R \ll R_e$, the second term dominates because $n^i_-$ is negative and the condition $1$ could be satisfied. In order to avoid the anti-gravity, we should require $F'(R) > 0$, which shows that $F_i > 0$ and $F_f < 0$.

If there is a solution $R = R_0$ for the equation $\frac{d}{dR} \left( \frac{F(R)}{R^2} \right) = 0$, $R = R_0$ corresponds to the (anti-)de Sitter space-time. In case of the model $[8]$, one gets

$$R_0 = R_e \left\{ - \frac{(n^i_+ - 2) F_i}{(n^f_- - 2) F_f} \right\}^{-\frac{1}{-n^i_+ - n^f_-}}. \quad (9)$$

Note that

$$- \frac{(n^i_+ - 2) F_i}{(n^f_- - 2) F_f} > 0. \quad (10)$$

In the de Sitter space-time, if $R = 12H_0^2$, we have $H = H_0$. Around the de Sitter solution, $F(R)$ in $[8]$ can be approximated by

$$F(R) \sim R^2 \left( F_0 + F_1 (R - R_0)^2 \right). \quad (11)$$
Here

$$F_0 \equiv \frac{n_i - n_f}{2 - n_i^f} F_i \left\{ - \frac{(n_i - 2) F_i}{n_i^f - 2} F_f \right\}^{-\frac{n_i^f}{n_i^f - n_i^f}}.$$

$$F_1 \equiv (n_i^+ - 2) \left( n_i^+ - n_i^- \right) F_i \left\{ - \frac{(n_i - 2) F_i}{n_i^f - 2} F_f \right\}^{-\frac{n_i^f}{n_i^f - n_i^f}} \frac{1}{R_0^2},$$

$$= - (n_i^- - 2) \left( n_i^- - 2 \right) \frac{F_0}{R_0^2}.$$

One may consider the perturbation around the de Sitter space-time as $H = H_0 + H_1 (N)$. Then by using the following equation, which corresponds to the first FRW equation,

$$0 = -\frac{1}{2} F(R) + 3 \left( H^2 + HH' \right) F'(R) - 18 \left( 4 H^4 H' + H^2 (H')^2 + H^3 H'' \right) F''(R),$$

we obtain the equation linearized with respect to $H_1(N)$,

$$0 = 4 R_0^2 F_1 H_1(N) + 3 \left( F_0 - R_0^2 F_1 \right) H_1'(N) - \left( F_0 + R_0^2 F_1 \right) H_1''(N),$$

whose solution is given by

$$H_1(N) = C_+ e^{\lambda_+ N} + C_- e^{\lambda_- N}.$$

Here $C_\pm$ are constants and

$$\lambda_\pm = \frac{-3 \left( F_0 - R_0^2 F_1 \right) \pm \sqrt{9 F_0^2 - 2 R_0^2 F_0 F_1 + 25 R_0^4 F_1^2}}{-2 \left( F_0 + R_0^2 F_1 \right)}.$$

If we assume $|R_0^2 F_1| \ll |F_0|$, we find

$$\lambda_+ \sim 3, \quad \lambda_- \sim -\frac{4}{3} \frac{R_0^2 F_1}{F_0}.$$

The $\lambda_+$ corresponds to the rapidly growing mode and $\lambda_-$ to the slowly growing mode. If one can choose $C_+ = 0$ by a boundary condition, the period of the inflation in terms of the e-folding $N$ is given by

$$N_m \sim \frac{1}{|\lambda_-|} \sim -\frac{3 F_0}{4 R_0^2 F_1} = \left| \frac{3}{4} \left( n_i^+ - 2 \right) \left( n_i^- - 2 \right) \right|.$$
We should require a condition $N_m \sim 60 - 70$ but Eq. (19) shows that the condition cannot be satisfied and therefore the condition (3) also cannot be satisfied.

About the condition (4), we may also regard $\epsilon \sim \kappa^2 R_e$. Eq. (16) with $C_+ = 0$ indicates

$$1 + w(N) \sim \frac{H'}{H} \sim \frac{C_- \lambda e^{\lambda - N}}{H_0}.$$  

(20)

Therefore if we choose

$$\frac{C_- \lambda}{H_0} \gg \kappa^2 R_e,$$  

(21)

the condition (4) could be satisfied.

Since the model (8) does not satisfy the condition (3), we modify the model as follows,

$$F(R) = \frac{1}{R^2} \left( F_i \left( \frac{R}{R_e} \right)^{\frac{n_i^f - 2}{\alpha}} + F_f \left( \frac{R}{R_e} \right)^{\frac{n_f^f - 2}{\alpha}} \right)^\alpha.$$  

(22)

Here $\alpha$ is a constant. The behavior when $R \gg R_e$ and $R \ll R_e$ does not change from those in the model (8) and therefore the conditions (11) and (12) can be satisfied again. The condition (4) is also satisfied with the choice (21). Therefore we only need to check the condition (3). Instead of (9), the solution describing the de Sitter space-time is given by

$$R_0 = R_e \left\{ \left( \frac{n_i^f - 2}{n_f^f - 2} \right)^{\alpha} \frac{F_i}{F_f} \right\}^{-\frac{1}{n_i^f - n_f^f}}.$$  

(23)

Around the de Sitter solution, $F(R)$ in (22) can be approximated as in (11) but instead of (12), we find

$$F_0 = \left( n_i^f - n_f^f \right)^\alpha \left( \frac{F_i}{F_f} \right)^{\frac{n_f^f - 2}{n_f^f - n_i^f}} \left( \frac{F_f}{(n_f^f - 2)^{\alpha}} \right)^{\frac{n_i^f - 2}{n_f^f - n_i^f}},$$  

$$F_1 = \frac{n_i^f + n_f^f - 4 - 4\alpha}{2\alpha R_0^2} F_0.$$  

(24)
Then instead of (19), one obtains

$$N_m \sim \frac{1}{|\lambda_-|} \sim \left| -\frac{3F_0}{4R_0^2F_1} \right| = \left| -\frac{3(n_+^l + n_-^l)}{8\alpha} \right|.$$  \hspace{1cm} (25)

By adjusting $\alpha$, we obtain $N_m \sim 60 - 70$ and the condition (3) can be satisfied.

Let us estimate the scalar index $n_s$ of the curvature perturbations and the tensor-to-scalar ratio $r$ of the density perturbation, and the running of the spectral index $\alpha_s$. The corresponding expressions for $F(R)$ gravity are given in (Bamba et al. 2014). Using the expression of $H$ in (16) with $C_+=0$, we find the following expressions for the slow-roll parameters, $\epsilon$, $\eta$, $\xi^2$ (Bamba et al. 2014),

$$\epsilon = -\lambda_- \quad \eta = -2\lambda_- \quad \xi^2 = 4\lambda_-^2.$$  \hspace{1cm} (26)

Therefore,

$$n_s - 1 = -6\epsilon + 2\eta = 2\lambda_- + \mathcal{O}(\lambda_-^2) \quad r = 16\epsilon = -16\lambda_- + \mathcal{O}(\lambda_-^2) \quad \alpha_s = 16\epsilon\eta - 24\epsilon^2 - 2\xi^2 = \mathcal{O}(\lambda_-^3).$$  \hspace{1cm} (27)

The Planck data (Ade et al. 2013A,B) suggest $n_s = 0.9603 \pm 0.0073$ (68\% CL), $r < 0.11$ (95\% CL), and $\alpha_s = -0.0134 \pm 0.0090$ (68\% CL) [the Planck and WMAP (Spergel et al. 2003; Spergel et al. 2006; Komatsu et al. 2008, 2010; Hinshaw et al. 2012)], the negative sign of which is at 1.5$\sigma$. The data $n_s \sim 0.9603$ show that $1/\lambda \sim -50$ but $r \sim 0.11$ indicates that $1/\lambda \sim -145$. This discrepancy always occurs when we consider the linearized model in (15) because this discrepancy is due to the exponential behavior in the solution (16). Therefore, if we include non-linear corrections, the above values could be improved.

We may rewrite the action (8) or (22) in a scalar-tensor form by introducing a new scalar field $\sigma$ by

$$\sigma = -\ln(2\kappa^2F'(R)).$$  \hspace{1cm} (28)
Here, it is introduced $2\kappa^2$. We now consider a case $R \gg R_e$ and another case $R \ll R_e$.

When $R \gg R_e$, one finds
\[
\sigma \sim -\left( n_+^i - 1 \right) \ln \frac{R}{R_e} - \ln \left( 2n_+^i F_i \kappa^2 \right),
\]
and the corresponding scalar-tensor theory looks as (for general review of scalar-tensor gravity, see (Fujii & Maeda 2003, Faraoni 2004))
\[
S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left( R - \frac{3}{2} \partial_\mu \sigma \partial^\mu \sigma - V(\sigma) \right),
\]
\[
V(\sigma) \sim \frac{R(\sigma)}{F'(R(\sigma))} - \frac{F(R(\sigma))}{F'(R(\sigma))^2} = \frac{n_+^i - 1}{n_+^i} \frac{2\kappa^2 R_e e^{n_+^i - 2} \sigma}{(2n_+^i F_i \kappa^2)^{n_+^i - 1}}.
\]

On the other hand, when $R \ll R_e$, we find
\[
\sigma = -\left( n_-^f - 1 \right) \ln \frac{R}{R_e} - \ln \left( 2n_-^f F_f \kappa^2 \right),
\]
and
\[
V(\sigma) = \frac{n_-^f - 1}{n_-^f} \frac{2\kappa^2 R_e e^{n_-^f - 2} \sigma}{(2n_-^f F_f \kappa^2)^{n_-^f - 1}}.
\]

Furthermore, we also consider the case that $R \sim R_0$ and $F(R)$ in (8) or (22) can be approximated by (11).

In $F(R)$ gravity, both of the tensor mode and the scalar mode in the metric $g_{\mu\nu}$ appear as propagating modes. Rewriting $F(R)$ gravity in the scalar-tensor form, we can separate the tensor mode and the scalar mode and find the Newton law in $F(R)$ gravity by the tensor mode is identical with that in the Einstein gravity although the coupling depends on the value of the scalar in the background. The propagation of the scalar mode gives an additional correction to the Newton law.

The mass of $\sigma$ is
\[
m_{\sigma}^2 = \frac{3}{2} \frac{d^2V(\sigma)}{d\sigma^2} = \frac{3}{2} \left\{ \frac{R}{F'(R)} - \frac{4F(R)}{(F'(R))^2} + \frac{1}{F''(R)} \right\}.
\]
Then if $m_\sigma$ is not large enough, the large correction to the Newton law appears in general. For the model (8) or (22), when $R \ll R_e$, we find
\[
F(R) \sim F_f \left( \frac{R}{R_e} \right)^{n_f}.
\]
(34)

Hence, $m_\sigma^2 \sim R$. In the present universe, the order of the mass $m_\sigma$ should be that of the Hubble rate, $m_\sigma \sim H \sim 10^{-33}$ eV, which is very light and could make the correction to the Newton law very large.

In [Hu & Sawicki 2007], realistic $F(R)$ model was proposed. It has been found, however, that the model has an instability where the large curvature can be easily produced (manifestation of a possible future singularity). In the model of [Hu & Sawicki 2007], a parameter $m \sim 10^{-33}$ eV with a mass dimension is included. The parameter $m$ plays a role of the effective cosmological constant. When the curvature $R$ is large enough compared with $m^2$, $R \gg m^2$, $F(R)$ [Hu & Sawicki 2007] looks as follows:
\[
F(R) = R - c_1 m^2 + \frac{c_2 m^{2n+2}}{R^n} + O \left( R^{-2n} \right).
\]
(35)

Here $c_1$, $c_2$, and $n$ are positive dimensionless constants. Similar viable models have been proposed in [Appleby & Battye 2007; Nojiri & Odintsov 2007; Cognola et al. 2007]. Then it is possible to construct a model which behaves as (22) at the early universe but behaves as (35) at the present universe. For this purpose, we define the following function of the scalar curvature $R$,
\[
S_\pm \equiv \frac{1}{2} \left\{ 1 \pm \tanh \left( \frac{R}{R_m} - \frac{R_m}{R} \right) \right\}.
\]
(36)

Here $R_m$ is a constant which is much larger than the scalar curvature at the present universe or the curvature on the earth. Let also $R_m$ is much smaller than the scale of the inflation, $R_m \ll R_e$, $R_0$. Then we find that when $R \gg R_m$, $S_+ \to 1$, $S_- \to 0$, very rapidly and when
\( R \ll R_m, \ S_+ \to 0, \ S_- \to 1. \) One may consider the following model by using (22):

\[
F(R) = \frac{R}{2\kappa^2} S_-(R) + \left\{ \frac{1}{R^2} \left( F_i \left( \frac{R}{R_e} \right)^{n_i^2-2} \alpha \right) + F_f \left( \frac{R}{R_e} \right)^{n_f^2-2} \alpha \right\} S_+(R). \tag{37}
\]

Then when \( R \gg R_e \), the model (22) can be reproduced. On the other hand, when \( R \ll R_e \), the Einstein gravity can be reproduced, \( F(R) \to \frac{R}{2\kappa^2} \). Note that when \( R \ll R_e \), which corresponds to the present universe, the mass (33) for the scalar mode is given by

\[
m^2_{\sigma} \sim \frac{3}{2} \frac{R^2}{R_m} \cosh^2 \left( -\frac{R_m}{R} \right), \tag{38}
\]

which is very large at the present universe or on the earth and the correction to the Newton law becomes very small. Although the model does not generate the accelerating expansion at the present universe, we can obtain the model generating the accelerating expansion by replacing \( \frac{R}{2\kappa^2} \) in the first term in (37) with the \( F(R) \) in (35). Thus, we proposed \( F(R) \) model which describes inflationary universe without self-reproduction and which behaves as General Relativity at weak curvature.

3. Discussion.

We may consider the condition (4) for the self-reproduction of general \( F(R) \) gravity. In \( F(R) \) gravity, by using the formulation of the reconstruction (Nojiri & Odintsov 2006B; Nojiri et al. 2009), one can construct the model reproducing the following Hubble rate

\[
H^2 = C f(N). \tag{39}
\]

Here \( f(N) \) is an adequate function of the e-foldings \( N \) and \( C \) is a constant. Then the condition (4) can be written as,

\[
1 + w(N) > \kappa^2 C f(N) \quad \text{for} \quad 1 < N < N_m. \tag{40}
\]
Therefore if we choose $C$ to be small enough, the condition (4) can be always satisfied. As an example, we consider the following model from Ref. (Bamba et al. 2014),

$$F(R) = C_1(6G_0 - 2R)^{3/2} \sqrt{R^{12G_0 - \frac{1}{4}}} \left[ 1 - \frac{1}{4} \left( \frac{R}{12G_0} - \frac{1}{4} \right)^2 \right]$$

$$+ C_2(6G_0 - 2R)^{3/2} L\left( \frac{1}{2}, \frac{3}{2}; \frac{R}{12G_0} - \frac{1}{4} \right),$$

which reproduces the Hubble rate,

$$(H(N))^2 = G_0 N + G_1.$$  \hfill (42)

Here $G_0(<0)$ and $G_1(>0)$ are constants. In (41), $C_1$ and $C_2$ are constants of integration, $L(u_1, u_2; y)$ is the generalized Laguerre polynomial, where $u_1$ and $u_2$ are constants and $y$ is a variable. If we choose $(N, \kappa^2 G_0, \kappa^2 G_1) = (50.0, -0.850, 95.0)$ and $(60.0, -0.950, 115)$, we obtain $(n_s, r, \alpha_s) = (0.967, 0.121, -5.42 \times 10^{-5})$ and $(0.967, 0.123, -5.55 \times 10^{-5})$, respectively, which could be consistent with the Planck data although the model (42) does not satisfy the condition (1) because there is no the exit from the inflation. Eventually, the exit should be described by another scenario, not gravitational one or by adding of extra gravitational terms. Comparing (40) and (42), we find that if we choose $G_0$ and $G_1$ small enough, the condition (1) is satisfied and the self-reproduction is prohibited.

In summary, we discussed two classes of $F(R)$ gravity which admits inflation in $F(R)$ description. On the same time, such theory may be consistent with Planck data. This shows that gravitationally-induced $F(R)$ inflation which avoids self-reproduction and resolves the problems of multiverse, predictability and initial conditions in the same sense as for scalar inflation is quite possible.

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