Control of a single-particle localization in open quantum systems

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Abstract – We investigate the possibility to control localization properties of the asymptotic state of an open quantum system with a tunable synthetic dissipation. The control mechanism relies on the matching between properties of dissipative operators, acting on neighboring sites and specified by a single control parameter, and the spatial phase structure of eigenstates of the system Hamiltonian. As a result, the latter coincide (or near coincide) with the dark states of the operators. In a disorder-free Hamiltonian with a flat band, one can either obtain a dominating localized asymptotic state or populate whole flat and/or dispersive bands, depending on the value of the control parameter. In a disordered Anderson system, the asymptotic state can be localized anywhere in the spectrum of the Hamiltonian. The dissipative control is robust with respect to an additional local dephasing.

Introduction. – Anderson localization [1] is a hallmark of modern physics [2–4], which has been observed in experiments with light, sound and matter waves [5–10]. However, the research activity in the field of open (i.e., interacting with their environments) quantum systems [11], has, to some extent, bypassed this phenomenon. Since Anderson localization is based on long-range interference effects, it is intuitive to expect that dissipation will blur the latter and thus eventually destroy the former. Another example of localization, i.e., compact states induced by the flat band topology [12–14], also involves destructive interference and therefore might be expected to be fragile with respect to any kind of dissipation.

During the last decade, it was realized that dissipative effects are not always a nuisance, especially when they can be controlled. The idea that a synthetic dissipation can be used to bring many-body systems into pure and highly entangled states [15–17], led to the creation of a field conventionally called “dissipative engineering”. However, in the context of localization, it remained unknown until very recently whether its signatures can survive in an open quantum system. Recent studies of Anderson localization in the presence of dephasing effects confirmed its destruction in the asymptotic limit, although revealed that its footprints can still be found on the way to a completely de-localized asymptotic state [18].

There were some clues that localization can survive in semi-classical and classical systems [19–22], which motivated us to search for the possibility to detect signatures of Anderson localization in open quantum systems. In ref. [23] it was demonstrated that pairwise, i.e., acting on a pair of neighboring sites only, dissipative operators (dissipators) can favor Anderson modes from a particular designated part of the spectrum. Depending on the value of a control phase parameter, dissipators sculpture the asymptotic state by selecting modes either from the lower or upper band edge of the spectrum or from the spectrum center.

In this paper we investigate the possibility to control localization properties of the asymptotic states in single-particle systems, where localization in the Hamiltonian limit is induced by flat band topology or by disorder. We demonstrate that localization signatures survive even in the presence of the most severe type of dissipation, that is a local dephasing. In other words, by introducing a synthetic dissipation into an already open (to an uncontrollable locally acting decoherence) system with Anderson or a flat-band Hamiltonian, one can observe footprints of localization in the asymptotic state.
Tunable dissipative operators. – We address the evolution of an open \( N \)-dimensional quantum system whose density operator \( \rho \) is governed by the Lindblad master equation [11,24],
\[
\dot{\rho} = \mathcal{L}(\rho) = -i[H, \rho] + \mathcal{D}(\rho).
\]
(1)
The first term on the r.h.s. captures the unitary evolution of the system given by the tight-binding lattice Hamiltonian
\[
H = \sum_j \epsilon_j b_j^\dagger b_j - \sum_{m \in \mathcal{N}(j)} b_j^\dagger b_m,
\]
(2)
where \( \epsilon_j \) are on-site energies, \( \mathcal{N}(j) \) is the set of neighbors of the \( j \)-th site, determined by the lattice topology, \( b_j \) and \( b_j^\dagger \) are the annihilation and creation operators of a boson on the \( j \)-th site. Periodic boundary conditions are imposed.

The dissipative part of the Lindblad generator \( \mathcal{L} \),
\[
\mathcal{D}(\rho) = \sum_{j=1}^S \gamma_j(t) \left[ V_j \rho V_j^\dagger - \frac{1}{2} (V_j^\dagger V_j, \rho) \right],
\]
(3)
is built from the set of \( S \) operators, \( \{V_j\}_{1, \ldots, S} \), which capture the action of the environment on the system.

The asymptotic density operator \( \rho_\infty \) is the product of the joint action of the Hamiltonian and dissipative operators. If all operators are Hermitian, \( V_j \equiv V_j^\dagger \), the asymptotic state is universal and trivial; namely, the density operator is the normalized identity \( \rho_\infty = \mathbb{1}/N \), irrespective of properties of the Hamiltonian. An example is the on-site dephasing operator,
\[
V_j^d = b_j^\dagger b_j,
\]
(4)
one of the most popular choices to study relaxation processes in open systems, single [18] and many-particle ones [25–29].

We also consider non-Hermitian dissipative operators that act on pairs of neighboring sites,
\[
V_{j, n(j)}^{\text{nn}} = (b_j^\dagger + e^{i\alpha} b_{n(j)}^\dagger)(b_j - e^{-i\alpha} b_{n(j)}),
\]
(5)
where \( n(j) \in \mathcal{N}(j) \) is the index of a neighbor of the \( j \)-th site. This type of dissipation, for a one-dimensional chain, \( n(j) = j + 1 \), and in the many-body context, was introduced in refs. [15,16], where it was shown to be able to bring the system into the BEC state. In ref. [23] these dissipators were used in the context of the Anderson model, to create non-trivial asymptotic states featuring signatures of localization. Operators (5) are parametrized by a phase \( \alpha \), which makes the dissipation phase-selective. For example, when \( \alpha = 0 \), the corresponding operators try to synchronize the dynamics on the neighboring sites, by constantly recycling the anti-symmetric out-of-phase mode into the symmetric in-phase one; the effect of \( \alpha = \pi \) is the opposite.

A physical implementation of a Bose-Hubbard chain with neighboring sites coupled by such dissipators was discussed in ref. [30]. The proposed set-up consists of an array of superconductive resonators coupled by qubits; a pairwise dissipator with \( n(j) = j + 1 \) and arbitrary phase \( \alpha \) can be realized with this set-up by adjusting the position of the qubits with respect to the centers of the corresponding cavities. More specifically, the relative position of a qubit in a cavity controls the phase of a complex coupling constant \( q_j \) in the Jaynes-Cummings coupling term, \( q_j^* b_j^\dagger \sigma_j^- + q_j b_j \sigma_j^+ \), where a qubit operator \( \sigma_j^- = |g_j\rangle\langle e_j| \) [31]. To implement the dissipator with \( \alpha \neq 0 \), the coupling constant should vary as \( q_j = |q| \exp(-|i\alpha j|) \).

We address the asymptotic solution \( \rho_\infty \) of eq. (1) only. We assume that due to the absence of relevant symmetries [24,32], the density operator \( \rho(t) \) is relaxing, under the action of propagator \( P_t \), towards a unique operator (asymptotic state) \( \rho_\infty = \lim_{t \to \infty} P_t \rho_0 \) for all \( \rho_0 \). In other words, it is the unique kernel of the Lindblad generator, \( \mathcal{L}(\rho_\infty) = 0 \). To find \( \rho_\infty \) in a specified basis, we use a column-wise vectorization of the density matrix and obtain the asymptotic solution, a \( N^2 \) vector \( \rho_\infty \), as the kernel of the Liouvillian-induced \( N^2 \times N^2 \) matrix \( \mathcal{L} \). After folding the obtained vector back into the matrix form and trace-normalizing it, we end up with the asymptotic state density matrix \( \rho_\infty \) (see footnote 1). We implement both types of dissipative operators, eqs. (4), (5), simultaneously unless explicitly stated otherwise. Additionally, in the case of the Anderson Hamiltonian, we perform the averaging over \( N_r = 10^3 \) disorder realizations.

Localization on a flat band. – We first explore the possibility to shape localized asymptotic states in a flat-band lattice, where a connection between the phase properties of the dissipator and the resulting solution is more apparent. Specifically, we choose the “cross-stitch” flat-band topology [12,13], that consists of two locally cross coupled chains of size \( N \), see fig. 1(a). The disorder-free Hamiltonian, \( \epsilon_j = 0 \), possesses a horizontal (flat) band, \( E(k) = 1 \). The eigenstates of this band are localized at reciprocal sites \( (j_0, j_0 + N) \) from the upper and lower chains, with the wave function in anti-phase and zero elsewhere, \( \psi_j = -\psi_{j+N} = \delta_{j,j_0}/\sqrt{2} \), see the sketch on the right part of fig. 1(a). The dispersive band, \( E(k) = -1 - 4 \cos(k) \), \( k = 2\pi q/N, q = N/2 \ldots N/2 \), holds plane waves \( \psi_j = \psi_{j+N} = e^{ikj}/\sqrt{2N} \) which are in-phase in both chains, see the sketch on the left part of fig. 1(a).

The control can be realized, for example, by using a single non-Hermitian local dissipator, eq. (5), between the reciprocal sites \( j = N/2 \) and \( n(j) = N/2 + N \). We expect, that the compact mode localized on the central site, \( \psi_j = -\psi_{j+N} = \delta_{j,N/2}/\sqrt{2} \), becomes a dark state of the dissipator for \( \alpha = \pi \); at the same time, the in-phase

\footnote{Maximal absolute value of the elements in the r.h.s. of eq. (1) after substitution of \( \rho_\infty \) does not exceed \( 10^{-14} \).}
modes on the dispersive band are subjected to dissipation. In turn, the whole dispersive band will become a dark state for \( \alpha = 0 \), while the anti-phase compact mode at \( j = N/2 \) will nearly die out.

To ensure robustness of dissipation-induced localization, we additionally introduce dephasing, by allowing the dissipators \( V^d_j \), eq. (4), to operate on every site. Recall that in the absence of \( V^{nn} \), the asymptotic density operator would be the normalized identity. We set equal rates for both types of dissipators, \( \gamma_{nn} = \gamma_d = 0.1 \).

Now we consider the dependence of the asymptotic state on the value of the parameter \( \alpha \), fig. 1(b). As expected, we observe a domination of the compact flat band mode in the central pair of sites for anti-phase local dissipation, which can be explained by dissipation-induced “pumping out” from dispersive modes. A rough estimate, \( \rho_{N/2,N/2} + \rho_{N+N/2,N+N/2} \sim 1/2 \), agrees with numerics reasonably well. The other flat band modes are not affected by this dissipative operator (they are its dark states for any value of \( \alpha \)), and share the other \( 1/2 \) between \( N-2 \) sites, \( \rho_{j,j} \sim 1/(4N) \). We note that the effect is pronounced in the much broader range, \( |\alpha| \in (\pi/2,\pi) \). In the other parameter region, \( |\alpha| \in [0, \pi/2] \), the asymptotic state looks like an almost homogeneous distribution with a noticeable drop corresponding to the suppressed central flat-band mode (the other states become populated almost equally, resulting in \( \rho_{j,j} \sim 1/(2N) \)).

The approach permits a straightforward extension by subjecting several sites to local non-Hermitian dissipation, promoting or suppressing respective compact modes. Ultimately, one has a possibility to arrange “dark bands”, the mutually exclusive excitation of the whole flat and dispersive bands, by setting pairwise dissipators at each vertical pair, and choose \( \alpha = 0 \) (dispersive regime) and \( \alpha = \pi \) (zero dispersion). The result is exact in the absence of Hermitian dissipators, \( \gamma_d = 0 \), and approximative otherwise.

Anderson localization in the presence of dissipation. – Next we consider the disorder-induced localization in the lattice described by eq. (1) with the Hamiltonian [1]

\[
H = \sum_j \epsilon_j b_j^\dagger b_j - (b_j^\dagger b_{j+1} + b_{j+1}^\dagger b_j),
\]

where \( \epsilon_k \in [-W/2, W/2] \) are random uncorrelated on-site energies, and \( W \) is the disorder strength. We recall that the eigenvalues of the Hamiltonian are restricted to a finite interval, \( E_q \in [-2 - W/2, 2 + W/2] \), while the respective

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eigenstates, $A_j^{(q)}$, are exponentially localized. The localization length is approximated by $\xi_E \approx 24(4 - E^2)/W^2$ [33], with some corrections near the band edges [34].

In ref. [23], it was demonstrated that pairwise dissipators, eq. (5), can favor specific Anderson modes. That is, the asymptotic state of the system is composed of Anderson modes, selected from around a specific point in the spectrum. For example, the in-phase dissipation, $\alpha = 0$, favors localization near the lower band edge, see fig. 2. For $\gamma_d = 0$, matrix elements of the corresponding asymptotic density operator expressed in the Anderson basis, $\rho_q,q$, can be approximated as [23]

$$\rho_q,q \propto \frac{2 - E_q}{2 + E_q}. \quad (7)$$

Instead, the anti-phase dissipation, $\alpha = \pi$, facilitates localization near the upper band edge. Moreover, by using the next-to-nearest interaction, $n(j) = j + 2$, it is possible to obtain an asymptotic state that consists of the Anderson modes from the spectrum center. In contrast to the two previous cases, the asymptotic density matrix in the direct basis does not manifest (visually) localization, as the more extended modes from the middle of the spectrum substantially overlap in space [23].

In ref. [23] we proposed that the preference by pairwise dissipators for specific Anderson modes is due to the spatial-phase properties of the latter, inherited from the seeding plane waves, the eigenstates of the Hamiltonian in the zero disorder limit [35]. Here we corroborate the conjecture.

But first we demonstrate the robustness of the phenomenon against local dephasing. We use $\alpha = 0$ as a reference case, set the rate $\gamma_{nn} = 0.1$ for the pairwise dissipators, and vary the rate of dephasing, $\gamma_d$. It turns out that not only weak dephasing, $\gamma_d \ll \gamma_{nn}$, does not noticeably change the structure of an asymptotic state but even strong dephasing, $\gamma_d \gg \gamma_{nn}$, is not able to destroy the spectral localization, see fig. 2.

Next we consider the zero-disorder limit in more detail. There the basis of the Hamiltonian is formed by the plane waves $\psi_j = e^{ik_j}/\sqrt{N}$, with the spectrum $E(k) = -2\cos(k)$, $k = 2\pi q/N, q = -N/2, \ldots, N/2$. It is

Fig. 4: (Color online) Anderson modes in Fourier space: color-coded (a) spatial harmonics $|F(k,q)|^2$ for the wave number $k$ vs. the mode number $q$ and (b) their spectral density $P(k,E)$. Asymptotic state of the open Anderson system: color-coded diagonal elements (averaged over $N_r = 10^3$ disorder realizations) of an asymptotic density matrix $\rho_{q,q}$ in the Fourier (c) and Anderson (d) basis controlled by the phase of dissipators $\alpha$. The parameters are $W = 2, \gamma_{nn} = \gamma_d = 0.1, N = 100$. 

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Fig. 5: (Color online) Anderson modes in Fourier space: color-coded (a) spatial harmonics \(|F(k,q)|^2\) for the wave number \(k\) vs. the mode number \(q\) and (b) their spectral density \(P(E,k)\). Asymptotic state of the open Anderson system: color-coded diagonal elements (averaged over \(N_r = 10^3\) disorder realizations) of an asymptotic density matrix \(\rho_{q,q}\) in the Fourier (c) and Anderson (d) basis controlled by the phase of dissipators \(\alpha\). The parameters are \(W = 10, \gamma_{nn} = \gamma_d = 0.1, N = 100\).

It is straightforward to see that for specific values of the phase parameter, \(\alpha = 2\pi q/N\), the plane wave with respective \(k = \alpha\) becomes a dark state of all \(V_{nnj}\), while all other eigenstates do not. In this case (and under assumption of zero dephasing, \(\gamma_d = 0\)), the plane wave with \(k = \alpha\) is the asymptotic state of the open system. We find that when \(\alpha\) does not coincide with one of the wave vectors or dephasing is present, the asymptotic state remains very close to the former dark state, with the plane waves with \(k \approx \alpha\) contributing most, see fig. 3.

Disorder leads to Anderson localization of all eigenstates, also taking them away from the pool of dark states of \(V_{nn}^J\), eq. (5). However, it is known that Anderson modes inherit phase properties of the original plane waves (at least in the regime of weak disorder), although their amplitudes decay exponentially in space \([35]\). Therefore, the selective effect of local dissipators persists.

We analyze the structure of Anderson modes, \(A_{k}^{(q)}\), in the plane-wave (Fourier) basis for different disorder strength. Note, that while exponential localization in the direct space assumes de-localization in the plane-wave basis, it does not exclude inhomogeneity of the distribution in there. The expansion coefficients, \(F(k,q) = \sum A_{k}^{(q)} e^{ikj}/\sqrt{N}\), have pronounced maxima along the linear dependence, \(q \propto \pm k_{\text{max}}\), see fig. 4(a).

We also calculate the spectral density of the expansion coefficients

\[
P(k,E) = \lim_{\Delta E \to 0} \frac{1}{\Delta E} \sum_{E(q) \in [E,E+\Delta E]} |F(k,q)|^2,
\]

which closely reproduces dispersion relation for the disorder-free system, fig. 4(b). It is noteworthy that these features are present even in the strong disorder regime, \(W = 10\), see fig. 5(a), (b).

The relation between the spatial structure of Anderson modes and their position in the spectrum gives a clue about the way to select the modes to form the asymptotic state. Evidently, this can be done by varying the phase parameter \(\alpha\) of pairwise dissipators. Numerical results reveal a well-shaped maximum in the diagonal elements of the asymptotic density expressed in the plane-wave basis, \(\rho_{k,k}\), such that \(k_{\text{max}} \propto \alpha\), see fig. 4(c). We also calculate the spectral density of the diagonal elements in the Anderson
basis, $\rho_{q,q}$.

\[ \rho(E) = \lim_{\Delta E \to 0} \frac{1}{\Delta E} \sum_{q : E(q) \in [E, E + \Delta E]} \rho_{q,q}. \]  

(9)

Plotted as a function of $\alpha$, it reveals the region of the Anderson spectrum whose modes contribute most to the asymptotic state, fig. 4(d). Similarly, though with less sharp results, this control recipe can be used in the case of strong disorder, $W = 10$; see fig. 5(c), (d).

Conclusions. – We demonstrated that synthetic dissipation can be used to control the localization properties of the asymptotic states of single-particle quantum systems. The control mechanism relies on the phase properties of the localized modes of the system Hamiltonian, so that the modes appear as dark (or near dark) states of synthetic dissipators. Our findings are relevant to a broad range of single-particle systems; these are other classes of flat bands, disordered flat band lattices, quasiperiodic ( Aubry-André) potentials (for both, localization in direct and momentum space). Finally, we would like to speculate about the applicability of the idea in the context of many-body localization [25–29]. However, this perspective, though very intriguing, is ambiguous at the moment.

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