Higgsless Electroweak Model and Contraction of Gauge Group

N.A. Gromov
Department of Mathematics, Komi Science Center UrD, RAS,
Kommunisticheskaya st. 24, Syktyvkar 167982, Russia
E-mail: gromov@dm.komisc.ru

Abstract

A modified formulation of the Electroweak Model with 3-dimensional spherical geometry in the target space is suggested. The free Lagrangian in the spherical field space along with the standard gauge field Lagrangian form the full Higgsless Lagrangian of the model, whose second order terms reproduce the same experimentally verified fields with the same masses as the Standard Electroweak Model. The vector bosons masses are automatically generated, so there is no need in special mechanism of spontaneous symmetry breaking.

The limiting case of the modified Higgsless Electroweak Model, which corresponds to the contracted gauge group \( SU(2; j) \times U(1) \) is discussed. Within framework of the limit model Z-boson, electromagnetic and electron fields are interpreted as an external ones with respect to W-bosons and neutrino fields. The W-bosons and neutrino fields do not effect on these external fields. The masses of the all particles remain the same, but the field interactions in contracted model are more simple as compared with the standard Electroweak Model.

1 Introduction

The Standard Electroweak Model based on gauge group \( SU(2) \times U(1) \) gives a good description of electroweak processes. One of the unsolved problems is the origin of electroweak symmetry breaking. In the standard formulation the scalar field (Higgs boson) performs this task via Higgs mechanism, which generates a mass terms for vector bosons. Sufficiently artificial Higgs mechanism with its imaginary bare mass is a naive relativistic analog of the phenomenological description of superconductivity \([1]\). However, it is not yet experimentally verified whether electroweak symmetry is broken by such a Higgs mechanism, or by something else. The emergence of large number Higgsless models \([2]-[7]\) was stimulated by difficulties with Higgs boson. These models are mainly based on extra dimensions of different types or larger gauge groups. A finite electroweak model without a Higgs particle which is used a regularized quantum field theory \([8],[9]\) was developed in \([7]\).

One of the important ingredient of the Standard Model is the simple group \( SU(2) \). More then fifty years in physics it is well known the notion of group contraction \([10]\), i.e. limit operation, which transforms, for example, a simple or semisimple group to a non-semisimple one. From general point of view for better understanding of a physical system it is useful to investigate its properties for limiting values of their physical parameters. In particular, for a gauge model one of the similar limiting case corresponds to a model with contracted gauge group. The gauge theories for non-semisimple groups which Lie algebras admit invariant non-degenerate metrics was considered in \([11],[12]\).

In the present paper a modified formulation of the Higgsless Electroweak Model and its limiting case for contracted gauge group \( SU(2; j) \times U(1) \) is regarded. Firstly we observe
that the quadratic form $\phi^\dagger \phi = \phi_1^* \phi_1 + \phi_2^* \phi_2 = R^2$ of the complex matter field $\phi \in \mathbb{C}_2$ is invariant with respect to gauge group transformations $SU(2) \times U(1)$ and we can restrict fields on the quadratic form without the loss of gauge invariance of the model. This quadratic form define three dimensional sphere in four dimensional Euclidean space of the real components of $\phi$, where the noneuclidean spherical geometry is realized.

Secondly we introduce the free matter field Lagrangian in this spherical field space, which along with the standard gauge field Lagrangian form the full Higgsless Lagrangian of the model. Its second order terms reproduce the same fields as the Standard Electroweak Model but without the remaining real dynamical Higgs field. The vector bosons masses are automatically generated and are given by the same formulas as in the Standard Electroweak Model, so there is no need in special mechanism of spontaneous symmetry breaking. The fermion Lagrangian of the Standard Electroweak Model are modified by replacing of the fields $\phi$ with the restricted on the quadratic form fields in such a way that its second order terms provide the electron mass and neutrino remain massless.

We recall the definition and properties of the contracted group $SU(2; j)$ in Sec. 2. In Sec. 3, we step by step modify the main points of the Electroweak Model for the gauge group $SU(2; j) \times U(1)$. We find transformation properties of gauge and matter fields under contractions. After that we obtain the Lagrangian of the contracted model from the noncontracted one by the substitution of the transformed fields. The limiting case of the modified Higgsless Electroweak Model is regarded in Sec. 4. When contraction parameter tends to zero $j \rightarrow 0$ or takes nilpotent value $j = \iota$ the field space is fibered [16] in such a way that electromagnetic, Z-boson and electron fields are in the base whereas charged W-bosons and neutrino fields are in the fiber. Within framework of the limit model the base fields can be interpreted as an external ones with respect to the fiber fields in the sense that the fiber fields do not effect on the base fields. The field interactions are simplified under contraction. Sec. 5 is devoted to the conclusions.

## 2 Contracted Special Unitary group $SU(2; j)$

Let us regard two dimensional complex fibered vector space $\Phi_2(j)$ with one dimensional base $\{\phi_1\}$ and one dimensional fiber $\{\phi_2\}$ [15]. This space has two hermitian forms: first in the base $\hat{\phi}_1 \phi_1 = |\phi_1|^2$ and second in the fiber $\hat{\phi}_2 \phi_2 = |\phi_2|^2$, where bar denotes complex conjugation. Both forms can be written by one formula

$$\phi^\dagger(j) \phi(j) = |\phi_1|^2 + j^2 |\phi_2|^2,$$

(1)

where $\phi^\dagger(j) = (\hat{\phi}_1, j \hat{\phi}_2)$, parameter $j = 1, \iota$ and $\iota$ is nilpotent unit $\iota^2 = 0$. For nilpotent unit the following heuristic rules be fulfilled: 1) division of a real or complex numbers by $\iota$ is not defined, i.e. for a real or complex $a$ the expression $\frac{a}{\iota}$ is defined only for $a = 0$, 2) however identical nilpotent units can be cancelled $\iota \iota = 1$.

The special unitary group $SU(2; j)$ is defined as a transformation group of $\Phi_2(j)$ which keep invariant the hermitian form $\langle $, i.e.

$$\phi' = \begin{pmatrix} \phi_1' \\ j \phi_2' \end{pmatrix} = \begin{pmatrix} \alpha & j \beta \\ -j \beta & \alpha \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = u(j) \phi(j),$$

$$\det u(j) = |\alpha|^2 + j^2 |\beta|^2 = 1, \quad u(j) u^\dagger(j) = 1.$$  

(2)
The fundamental representation of the one-parameter subgroups of $SU(2; j)$ are easily obtained

\[ u_1(\alpha_1; j) = e^{\alpha_1 T_1(j)} = \begin{pmatrix} \cos \frac{j\alpha_1}{2} & i \sin \frac{j\alpha_1}{2} \\ i \sin \frac{j\alpha_1}{2} & \cos \frac{j\alpha_1}{2} \end{pmatrix}, \]

\[ u_2(\alpha_2; j) = e^{\alpha_2 T_2(j)} = \begin{pmatrix} \cos \frac{j\alpha_2}{2} & \sin \frac{j\alpha_2}{2} \\ -\sin \frac{j\alpha_2}{2} & \cos \frac{j\alpha_2}{2} \end{pmatrix}, \]

\[ u_3(\alpha_3; j) = e^{\alpha_3 T_3(j)} = \begin{pmatrix} e^{i\frac{j\alpha_3}{2}} & 0 \\ 0 & e^{-i\frac{j\alpha_3}{2}} \end{pmatrix}. \]

The corresponding generators

\[ T_1(j) = j \frac{i}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = j \frac{i}{2} \tau_1, \quad T_2(j) = j \frac{i}{2} \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix} = j \frac{i}{2} \tau_2, \]

\[ T_3(j) = \frac{i}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{i}{2} \tau_3, \]

with $\tau_k$ being Pauli matrices, are subject of commutation relations

\[ [T_1(j), T_2(j)] = -j^2 T_3(j), \quad [T_3(j), T_1(j)] = -T_2(j), \]

\[ [T_2(j), T_3(j)] = -T_1(j), \]

and form the Lie algebra $su(2; j)$ with the general element

\[ T(j) = \sum_{k=1}^{3} a_k T_k(j) = \frac{i}{2} \begin{pmatrix} a_3 & j(a_1 - i a_2) \\ j(a_1 + i a_2) & -a_3 \end{pmatrix} = -T^\dagger(j). \]

There are two more or less equivalent way of group contraction. We can put the contraction parameter equal to the nilpotent unit $j = \iota$ or tend it to zero $j \to 0$. Sometimes it is convenient to use the first (mathematical) approach, sometimes the second (physical) one. For example, the matrix $u(\iota)$ [2] has non-zero nilpotent non-diagonal elements for $j = \iota$, whereas for $j \to 0$ they are formally equal to zero. Nevertheless both approaches lead to the same final results.

Let us describe the contracted group $SU(2; \iota)$ in detail. For $j = \iota$ it follows from [2] that $\det u(\iota) = |\alpha|^2 = 1$, i.e. $\alpha = e^{i\varphi}$, therefore

\[ u(\iota) = \begin{pmatrix} e^{i\varphi} & i \beta \\ -i \bar{\beta} & e^{-i\varphi} \end{pmatrix}, \quad \beta = \beta_1 + i \beta_2 \in \mathbb{C}. \]

Functions of nilpotent arguments are defined by their Taylor expansion, in particular, $\cos \iota x = 1$, $\sin \iota x = \iota x$. Then one-parameter subgroups of $SU(2; \iota)$ take the form

\[ u_1(\alpha_1; \iota) = \begin{pmatrix} 1 & 0 \\ i \iota \alpha_1 & 1 \end{pmatrix}, \quad u_2(\alpha_2; \iota) = \begin{pmatrix} 1 & i \alpha_2 \\ -i \alpha_2 & 1 \end{pmatrix}. \]

The third subgroup does not changed and is given by [5]. The simple group $SU(2)$ is contracted to the non-semisimple group $SU(2; \iota)$, which is isomorphic to the real Euclid group $E(2)$. First two generators of the Lie algebra $su(2; \iota)$ are commute $[T_1(\iota), T_2(\iota)] = 0$. Hence the fundamental representation of $SU(2; \iota)$ is the same as $SU(2)$, i.e.

\[ u_1(\alpha_1; \iota) = e^{\alpha_1 T_1(\iota)} = \begin{pmatrix} \cos \frac{\alpha_1}{2} & i \sin \frac{\alpha_1}{2} \\ i \sin \frac{\alpha_1}{2} & \cos \frac{\alpha_1}{2} \end{pmatrix}, \]

\[ u_2(\alpha_2; \iota) = e^{\alpha_2 T_2(\iota)} = \begin{pmatrix} \cos \frac{\alpha_2}{2} & \sin \frac{\alpha_2}{2} \\ -\sin \frac{\alpha_2}{2} & \cos \frac{\alpha_2}{2} \end{pmatrix}, \]

\[ u_3(\alpha_3; \iota) = e^{\alpha_3 T_3(\iota)} = \begin{pmatrix} e^{i\frac{\alpha_3}{2}} & 0 \\ 0 & e^{-i\frac{\alpha_3}{2}} \end{pmatrix}. \]
and the rest commutators are given by (7). For the general element (8) of \(su(2; \iota)\) the corresponding group element of \(SU(2; \iota)\) is as follows

\[
u(\iota) = e^{\lambda^{(\iota)}} = \begin{pmatrix} e^{\frac{a_3}{2}} & \frac{i}{a_3} \sin \frac{a_3}{2} \\ \frac{i}{a_3} \sin \frac{a_3}{2} & e^{-i \frac{a_3}{2}} \end{pmatrix}, \quad a = a_1 + ia_2 \in C. \tag{11}\]

The actions of the unitary group \(U(1)\) and the electromagnetic subgroup \(U(1)_{em}\) in the fibered space \(\Phi_2(\iota)\) are given by the same matrices as on the space \(\Phi_2\), namely

\[
u(\beta) = e^{\beta Y} = \begin{pmatrix} e^{i \frac{\beta}{2}} & 0 \\ 0 & e^{i \frac{\beta}{2}} \end{pmatrix}, \quad u_{em}(\gamma) = e^{\gamma Q} = \begin{pmatrix} e^{i \gamma} & 0 \\ 0 & 1 \end{pmatrix}, \tag{12}\]

where \(Y = \frac{i}{2}1\), \(Q = Y + T_3\).

Representations of groups \(SU(2; \iota), U(1), U(1)_{em}\) are linear ones, that is they are realised by linear operators in the fibered space \(\Phi_2(\iota)\).

### 3 Electroweak Model for \(SU(2; j) \times U(1)\) gauge group

The fibered space \(\Phi_2(j)\) can be obtained from \(\Phi_2\) by substitution \(\phi_2 \rightarrow j \phi_2\) in (11), which induces another ones for Lie algebra \(su(2)\) generators \(T_1 \rightarrow jT_1, T_2 \rightarrow jT_2, T_3 \rightarrow T_3\). As far as the gauge fields take their values in Lie algebra, we can substitute gauge fields instead of transformation of generators, namely

\[
A_1^3 \rightarrow jA_1^1, \quad A_2^2 \rightarrow jA_2^1, \quad A_3^3 \rightarrow A_3^3, \quad B_\mu \rightarrow B_\mu. \tag{13}\]

These substitutions in the Lagrangian \(L\) of the Higgsless Electroweak Model \cite{13} give rise to the Lagrangian \(L(j)\) of the contracted model with \(U(2; j) = SU(2; j) \times U(1)\) gauge group

\[
L(j) = L_A(j) + L_\phi(j), \tag{14}\]

where

\[
L_A(j) = \frac{1}{2g^2} tr(F_{\mu\nu}(j))^2 + \frac{1}{2g'^2} tr(\hat{B}_{\mu\nu})^2 =
\]

\[
= -\frac{1}{4} j^2 (F_{\mu\nu}^1)^2 + j^2 (F_{\mu\nu}^2)^2 + (F_{\mu\nu}^3)^2 - \frac{1}{4} (B_{\mu\nu})^2 \tag{15}\]

is the gauge fields Lagrangian and

\[
L_\phi(j) = \frac{1}{2} (D_\mu \phi(j))^\dagger D_\mu \phi(j) \tag{16}\]

is the free (without any potential term) matter field Lagrangian (summation on the repeating Greek indexes is always understood). Here \(D_\mu\) are the covariant derivatives

\[
D_\mu \phi(j) = \partial_\mu \phi(j) + g \left( \sum_{k=1}^{3} T_k(j) A_\mu^k \right) \phi(j) + g' Y B_\mu \phi(j), \tag{17}\]

where \(T_k(j)\) are given by (6) and \(Y = \frac{i}{2}1\) is generator of \(U(1)\). Their actions on components of \(\phi(j)\) are given by

\[
D_\mu \phi_1 = \partial_\mu \phi_1 + \frac{i}{2} (gA_\mu^3 + g' B_\mu) \phi_1 + j^2 \frac{ig}{2} (A_\mu^1 - iA_\mu^2) \phi_2, \]

\[
D_\mu \phi_2 = \partial_\mu \phi_2 + \frac{i}{2} (gA_\mu^1 + g' B_\mu) \phi_2 + j^2 \frac{ig}{2} (A_\mu^3 - iA_\mu^2) \phi_1, \]

\[
D_\mu \phi_3 = \partial_\mu \phi_3 + \frac{i}{2} (gA_\mu^2 + g' B_\mu) \phi_3 + j^2 \frac{ig}{2} (A_\mu^3 + iA_\mu^2) \phi_2. \]
The gauge fields

\[
A_\mu(x; j) = \sum_{k=1}^{3} T_k(j) A^k_\mu(x) = g \frac{i}{2} \left( \begin{array}{cc}
A^3_\mu & j(A^1_\mu + iA^2_\mu) \\
j(A^1_\mu + iA^2_\mu) & -A^3_\mu
\end{array} \right)
\]

\[
\hat{B}_\mu(x) = g' Y B_\mu(x) = g' \frac{i}{2} \left( \begin{array}{cc}
B_\mu & 0 \\
0 & B_\mu
\end{array} \right)
\]

take their values in Lie algebras \( su(2; j) \), \( u(1) \) respectively, and the stress tensors are

\[
F_{\mu \nu}(x; j) = F_{\mu \nu}^1(x; j) + [A_\mu(x; j), A_\nu(x; j)] =
\]

\[
= g \frac{i}{2} \left( \begin{array}{cc}
F^3_{\mu \nu} & j(F^1_{\mu \nu} - iF^2_{\mu \nu}) \\
j(F^1_{\mu \nu} + iF^2_{\mu \nu}) & -F^3_{\mu \nu}
\end{array} \right)
\]

\[
B_{\mu \nu} = \partial_\mu B_\nu - \partial_\nu B_\mu,
\]

or in components

\[
F^1_{\mu \nu} = F^1_{\mu \nu} + g(A^2_\mu A^3_\nu - A^3_\mu A^2_\nu), \quad F^2_{\mu \nu} = F^2_{\mu \nu} + g(A^3_\mu A^1_\nu - A^1_\mu A^3_\nu),
\]

\[
F^3_{\mu \nu} = F^3_{\mu \nu} + j^2 g(A^1_\mu A^2_\nu - A^2_\mu A^1_\nu),
\]

where \( F^k_{\mu \nu} = \partial_\mu A^k_\nu - \partial_\nu A^k_\mu \).

The Lagrangian \( L(j) \) (14) describe massless fields. In a standard approach to generate mass terms for the vector bosons the "sombrero" potential is added to the matter field Lagrangian \( L(\phi) = 1 \) (16) and after that the Higgs mechanism is used. The different way [13] is based on the fact that the quadratic form \( \phi^\dagger \phi = \rho^2 \) is invariant with respect to gauge transformations. This quadratic form define the 3-dimensional sphere \( S_3 \) of the radius \( \rho > 0 \) in the target space \( \Phi_2 \) which is \( C_2 \) or \( R_4 \) if real components are counted. In other words the radial coordinates \( R_+ \times S_3 \) are introduced in \( R_4 \). The vector boson masses are generated by the transformation of Lagrangian \( L(j) = 1 \) (14) to the coordinates on the sphere \( S_3 \) and are the same as in the standard model. Higgs boson field does not appeared if the sphere radius does not depend on the space-time coordinates \( \rho = R = const \) [13]. For \( \rho \neq const \) the real positive massless scalar field — analogy of dilaton or kind of Goldstone mode — is presented in the model [14].

The complex space \( \Phi_2(j) \) can be regarded as 4-dimensional real space \( R_4(j) \). Let us introduce the real fields

\[
\phi_1 = r(1 + i\psi_3), \quad \phi_2 = r(\psi_2 + i\psi_1).
\]

The substitution \( \phi_2 \to j\phi_2 \) induces the following substitutions

\[
\psi_1 \to j\psi_1, \quad \psi_2 \to j\psi_2, \quad \psi_3 \to \psi_3, \quad r \to r
\]

for the real fields.

For the real fields the form (11) is written as \( r^2(1 + \bar{\psi}^2(j)) = R^2 \), where \( \bar{\psi}^2(j) = j^2(\psi_1^2 + \psi_2^2) + \psi_3^2 \), therefore

\[
r = \frac{R}{\sqrt{1 + \psi^2(j)}}.
\]
Hence there are three independent real fields $\tilde{\psi}(j) = (j\psi_1, j\psi_2, \psi_3)$. These fields belong to the space $\Psi_3(j)$ with noneuclidean geometry which is realized on the 3-dimensional “sphere” of the radius $R$ in the 4-dimensional space $R_4(j)$. The fields $\tilde{\psi}(j)$ are intrinsic Beltrami coordinates on $\Psi_3(j)$. The space $\Psi_3(j = 1) \equiv S_3$ has non degenerate spherical geometry, but $\Psi_3(j = i)$ is fibered space of constant curvature with 1-dimensional base $\{\psi_3\}$ and 2-dimensional fiber $\{\psi_1, \psi_2\}$, so-called semi-spherical space $L^3$, which can be interpreted as nonrelativistic (1 + 2) kinematic with curvature or Newton kinematic $L^3$.

The free Lagrangian (16) is transforms to the free gauge invariant matter field Lagrangian $L_{\Psi}(j)$ on $\Psi_3(j)$, which is defined with the help of the metric tensor $g_{kl}(j)$ of the space $\Psi_3(j)$

\[
g_{11} = \frac{1 + \psi_3^2 + j^2 \psi_2^2}{(1 + \psi^2(j))^2}, \quad g_{22} = \frac{1 + \psi_3^2 + j^2 \psi_1^2}{(1 + \psi^2(j))^2}, \quad g_{33} = \frac{1 + j^2 (\psi_1^2 + \psi_2^2)}{(1 + \psi^2(j))^2},
\]

\[
g_{12} = g_{21} = \frac{-j^2 \psi_1 \psi_2}{(1 + \psi^2(j))^2}, \quad g_{13} = g_{31} = \frac{-j \psi_1 \psi_3}{(1 + \psi^2(j))^2}, \quad g_{23} = g_{32} = \frac{1}{1 + \psi^2(j)}
\]

in the form

\[
L_{\psi}(j) = \frac{R^2}{2} \sum_{k,l=1}^{3} g_{kl}(j) D_\mu \psi_k(j) D_\mu \psi_l(j) = \frac{R^2 [(1 + \psi^2(j))(D_\mu \tilde{\psi}(j))^2 - (\tilde{\psi}(j), D_\mu \tilde{\psi}(j))^2]}{2(1 + \psi^2(j))^2}.
\]  

The covariant derivatives (17) are obtained from the representations of generators for the algebras $su(2), u(1)$ in the space $\Psi_3$ with the help of the substitutions of (23)

\[
T_1 \tilde{\psi}(j) = \frac{i}{2} \left( \begin{array}{c} -j(1 + j^2 \psi_1^2) \\ j(\psi_3 - j^2 \psi_1 \psi_2) \\ -j^2 (\psi_2 + \psi_1 \psi_3) \end{array} \right), \quad T_2 \tilde{\psi}(j) = \frac{i}{2} \left( \begin{array}{c} -j(\psi_3 + j^2 \psi_1 \psi_2) \\ -j(1 + j^2 \psi_2^2) \\ j^2 (\psi_1 - \psi_2 \psi_3) \end{array} \right),
\]

\[
T_3 \tilde{\psi}(j) = \frac{i}{2} \left( \begin{array}{c} j(\psi_2 + \psi_1 \psi_3) \\ j(\psi_1 + \psi_2 \psi_3) \\ 1 + \psi_3^2 \end{array} \right), \quad Y \tilde{\psi}(j) = \frac{i}{2} \left( \begin{array}{c} -j(\psi_2 + \psi_1 \psi_3) \\ j(\psi_1 - \psi_2 \psi_3) \\ -(1 + \psi_3^2) \end{array} \right)
\]

and are as follows:

\[
D_\mu \psi_1 = \partial_\mu \psi_1 - \frac{g'}{2} (\psi_2 + \psi_1 \psi_3) B_\mu + \frac{g}{2} \left[-(1 + j^2 \psi_1^2) A^1_\mu - (\psi_3 + j^2 \psi_1 \psi_2) A^2_\mu - (\psi_2 - \psi_1 \psi_3) A^3_\mu \right],
\]

\[
D_\mu \psi_2 = \partial_\mu \psi_2 + \frac{g'}{2} (\psi_1 - \psi_2 \psi_3) B_\mu + \frac{g}{2} \left[(\psi_3 - j^2 \psi_1 \psi_2) A^1_\mu - (1 + j^2 \psi_2^2) A^2_\mu + (\psi_1 + \psi_2 \psi_3) A^3_\mu \right],
\]

\[
D_\mu \psi_3 = \partial_\mu \psi_3 - \frac{g'}{2} (1 + \psi_3^2) B_\mu + \frac{g}{2} \left[-j^2 (\psi_2 + \psi_1 \psi_3) A^1_\mu + j^2 (\psi_1 - \psi_2 \psi_3) A^2_\mu + (1 + \psi_3^2) A^3_\mu \right].
\]  

6
The gauge fields Lagrangian (15) does not depend on the fields \(\phi\) and therefore remains unchanged. So the full Lagrangian (14) is given by the sum of (15) and (25).

For small fields, the second order part of the Lagrangian (25) is written as

\[
L^{(2)}_{\psi}(\jmath) = \frac{R^2}{2} \left((D_{\mu}\bar{\psi}(\jmath))^{(1)}\right)^2 = \frac{R^2}{2} \sum_{k=1}^{3} \left[(D_{\mu}\psi_k(\jmath))^{(1)}\right]^2, \tag{27}
\]

where linear terms in covariant derivatives (26) have the form

\[
(D_{\mu}\psi_1)^{(1)} = \partial_{\mu}\psi_1 - \frac{g}{2}A_\mu^1 = -\frac{g}{2}A_\mu^1, \\
(D_{\mu}\psi_2)^{(1)} = \partial_{\mu}\psi_2 - \frac{g}{2}A_\mu^2 = -\frac{g}{2}A_\mu^2, \\
(D_{\mu}\psi_3)^{(1)} = \partial_{\mu}\psi_3 + \frac{g}{2}A_\mu^3 - \frac{g'}{2}B_\mu = \frac{1}{2}\sqrt{g^2 + g'^2}Z_\mu. 
\]

The new fields

\[
W^{\pm}_\mu = \frac{1}{\sqrt{2}} \left(\hat{A}_\mu^1 \mp i\hat{A}_\mu^2\right), \quad Z_\mu = \frac{gA_\mu^3 - g'B_\mu + 2\partial_\mu\psi_3}{\sqrt{g^2 + g'^2}}, \quad A_\mu = \frac{g'A_\mu^3 + gB_\mu}{\sqrt{g^2 + g'^2}}
\]

are transformed as

\[
W^{\pm}_\mu \rightarrow jW^{\pm}_\mu, \quad Z_\mu \rightarrow Z_\mu, \quad A_\mu \rightarrow A_\mu
\]

and Lagrangian (27) is rewritten as follows

\[
L^{(2)}_{\psi} = j^2 \frac{R^2g^2}{4}W^+_{\mu}W^-_{\mu} + \frac{R^2(g^2 + g'^2)}{8}(Z_\mu)^2.
\]

The quadratic part of the full Lagrangian

\[
L_0(\jmath) = L^{(2)}_A(\jmath) + L^{(2)}_{\psi}(\jmath) =
\]

\[
= -\frac{1}{4}(F_{\mu\nu})^2 - \frac{1}{4}(Z_{\mu\nu})^2 + \frac{m_Z^2}{2}(Z_\mu)^2 + j^2 \left\{ -\frac{1}{2}W^+_{\mu\nu}W^-_{\mu\nu} + m_W^2W^+_{\mu}W^-_{\mu}\right\} \equiv \equiv L_b + j^2L_f, \tag{28}
\]

where

\[
m_W = \frac{Rg}{2}, \quad m_Z = \frac{R}{2}\sqrt{g^2 + g'^2},
\]

and \(Z_{\mu\nu} = \partial_{\mu}Z_{\nu} - \partial_{\nu}Z_{\mu}, \quad F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}, \quad W^\pm_{\mu\nu} = \partial_{\mu}W^\pm_{\nu} - \partial_{\nu}W^\pm_{\mu}\) are abelian stress tensors describes all the experimentally verified parts of the standard Electroweek Model but does not include the scalar Higgs field.

The interaction part of the full Lagrangian in the first degree of approximation is given by

\[
L^{(1)}_{int}(\jmath) = j^2 \left[L^{(3)}_A + L^{(3)}_{\psi}\right].
\]
where the third order terms of the gauge field Lagrangian (15) are

\[
L_A^{(3)} = -\frac{g}{\sqrt{g^2 + g'^2}} \left\{ i \left( \mathcal{W}_\mu^\nu \mathcal{W}_\mu^\nu - \mathcal{W}_\mu^\nu \mathcal{W}_\mu^\nu \right) (g'A_\nu + gZ_\nu) - \frac{\sqrt{2}}{g} (g'A_\nu + gZ_\nu) \left[ \mathcal{W}_\mu^\nu (\partial_\nu \psi_2 - i\partial_\nu \psi_1) + \mathcal{W}_\mu^\nu (\partial_\nu \psi_2 + i\partial_\nu \psi_1) \right] - \frac{2ig}{\sqrt{g^2 + g'^2}} (\mathcal{W}_\mu^\nu \mathcal{W}_\mu^\nu - \mathcal{W}_\mu^\nu \mathcal{W}_\mu^\nu) \partial_\nu \psi_3 - \frac{2\sqrt{2}}{\sqrt{g^2 + g'^2}} \left[ \mathcal{W}_\mu^\nu (\partial_\nu \psi_2 - i\partial_\nu \psi_1) + \mathcal{W}_\mu^\nu (\partial_\nu \psi_2 + i\partial_\nu \psi_1) \right] \partial_\nu \psi_3 + (g'\mathcal{F}_\mu^\nu + gZ_\mu^\nu) \left\{ \frac{i}{4} \left[ (\mathcal{W}_\mu^\nu)^2 - (\mathcal{W}_\mu^\nu)^2 \right] + \frac{4}{g} \partial_\mu \psi_1 \partial_\nu \psi_2 + \frac{\sqrt{2}}{g} \left[ \mathcal{W}_\mu^\nu (\partial_\nu \psi_2 - i\partial_\nu \psi_1) + \mathcal{W}_\mu^\nu (\partial_\nu \psi_2 + i\partial_\nu \psi_1) \right] \right\}\right\}
\]

and those of the matter field Lagrangian (25) are

\[
L_\psi^{(3)} = \frac{R^2 g^2}{2} \left\{ W_\mu^\nu \left[ \psi_3 (\partial_\mu \psi_2 - i\partial_\mu \psi_1) - \frac{g^2 - g'^2}{g^2 + g'^2} (\psi_2 - i\psi_1) \partial_\mu \psi_3 + \frac{g'}{\sqrt{g^2 + g'^2}} (gA_\mu - g'Z_\mu) \left( \psi_2 - i\psi_1 \right) \right] + W_\mu^\nu \left[ \psi_3 (\partial_\mu \psi_2 + i\partial_\mu \psi_1) \right] - \frac{g^2 - g'^2}{g^2 + g'^2} (\psi_2 + i\psi_1) \partial_\mu \psi_3 + \frac{g'}{\sqrt{g^2 + g'^2}} (gA_\mu - g'Z_\mu) \left( \psi_2 + i\psi_1 \right) \right] + \frac{1}{g} \sqrt{g^2 + g'^2} Z_\mu \left( \psi_1 \partial_\mu \psi_2 - \psi_2 \partial_\mu \psi_1 \right) \right\}.
\]

The fermion Lagrangian of the standard Electroweak Model is taken in the form (19)

\[
L_F = L_\psi^1 i\tau_\mu D_\mu L + e_\tau^1 i\tau^\nu \phi \tau^\nu \psi_\tau - h_e [e_\phi^1 (\phi^\dagger L_\psi^1) + (L_\psi^1 \phi) e_\tau],
\]

where \( L_I = \begin{pmatrix} e_I \\ \nu_{e_I} \end{pmatrix} \) is the \( SU(2) \)-doublet, \( e_r \) the \( SU(2) \)-singlet, \( h_e \) is constant and \( e_r, e_I, \nu_e \) are two component Lorentzian spinors. Here \( \tau_\mu \) are Pauli matrices, \( \tau_0 = \tau_0 = 1, \tau_k = -\tau_k \). The covariant derivatives \( D_\mu L \) are given by (17) with \( L_I \) instead of \( \phi \) and \( D_\mu e_r = (\partial_\mu + igB_\mu) e_r \). The convolution on the inner indices of \( SU(2) \)-doublet is denoted by \( (\phi^\dagger L_\psi) e_\tau \).

The matter field \( \phi \) appears in Lagrangian (30) only in mass terms. When the gauge group \( SU(2) \) is contracted to \( SU(2; j) \) and the matter field is fibered to \( \phi(j) \) the same take place with doublet \( L_I \), namely, the first component \( e_I \) does not changed, but the second component is multiplied by contraction parameter: \( \nu_{e_I} \rightarrow j \nu_{e_I} \). With the use of (22), (24) and these substitution the mass terms are rewritten in the form

\[
h_e [e_\phi^1 (\phi^\dagger L_I(j)) + (L_\psi^1(j) \phi) e_\tau] = \frac{h_e R}{\sqrt{1 + \psi^2(j)}} \left\{ e_\phi^1 e_I + e_\phi^1 e_r + e_\phi^1 e_r - e_\phi^1 e_I \right\} + i\psi_3 \left( e_\phi^1 e_r - e_\phi^1 e_I \right) + i\psi_2 \left( \nu_{e_I} e_r + e_\phi^1 \nu_{e_I} e_r - e_\phi^1 e_r \right)
\]

where the \( SU(2) \)-singlet \( e_r \) does not transforms under contraction.
4 Limiting case of Higgsless Electroweak Model

As it was mentioned the vector boson masses are automatically (without any Higgs mechanism) generated by the transformation of the free Lagrangian of the standard Electroweak Model to the Lagrangian (14), (15), (25) expressed in some coordinates on the sphere $\Psi_3(j)$. And this statement is true for both values of contraction parameter $j = 1, \iota$. When contraction parameter tends to zero $j^2 \to 0$, then the contribution of W-bosons fields to the quadratic part of the Lagrangian (28) will be small in comparison with the contribution of Z-boson and electromagnetic fields. In other words the limit Lagrangian includes only Z-boson and electromagnetic fields. Therefore charred W-bosons fields does not effect on these fields. The part $L_f$ form a new Lagrangian for W-bosons fields and their interactions with other fields. The appearance of two Lagrangians $L_b$ and $L_f$ for the limit model is in correspondence with two hermitian forms of fibered space $\Phi_2(\iota)$, which are invariant under the action of contracted gauge group $SU(2; \iota)$. Electromagnetic and Z-boson fields can be regarded as external fields with respect to the W-bosons fields.

In mathematical language the field space $\{A_\mu, Z_\mu, W_\mu^{\pm}\}$ is fibered after contraction $j = \iota$ to the base $\{A_\mu, Z_\mu\}$ and the fiber $\{W_\mu^{\pm}\}$. (In order to avoid terminological misunderstanding let us stress that we have in view locally trivial fibering, which is defined by the projection $pr : \{A_\mu, Z_\mu, W_\mu^{\pm}\} \to \{A_\mu, Z_\mu\}$ in the field space. This fibering is understood in the context of semi-Riemannian geometry [16] and has nothing to do with the principal fiber bundle.) Then $L_b$ in (28) presents Lagrangian in the base and $L_f$ is Lagrangian in the fiber. In general, properties of a fiber are depend on a points of a base and not the contrary. In this sense fields in the base are external one with respect to fields in the fiber.

The fermion Lagrangian (30) for nilpotent value of the contraction parameter $j = \iota$ is also splited on electron part in the base and neutrino part in the fiber. This means that in the limit model electron field is external one relative to neutrino field. The mass terms (31) for $j = \iota$ are

$$h_e [\bar{e}_r^l (\phi^l (\iota)L_l (\iota)) + (L^l_\iota (\iota)\phi (\iota))e_r] = \frac{h_e R}{\sqrt{1 + \psi^2_3}} \left[ e_r^1 e_r^- + e_r^- e_r^+ + i\psi_3 (e_r^- e_r^+ - e_r^1 e_r^-) \right].$$

(32)

Its second order terms $h_e R \left( e_r^1 e_r^- + e_r^- e_r^+ \right)$ provide the electron mass $m_e = h_e R$ and neutrino remain massless.

Let us note that field interactions in contracted model are more simple as compared with the standard Electroweak Model due to nullification of some terms.

5 Conclusions

The modified formulation of the Electroweak Model with the gauge group $SU(2) \times U(1)$ based on the 3-dimensional spherical geometry in the target space is suggested. This model describes all experimentally observed fields and does not include the (up to now unobserved) scalar Higgs field. The free Lagrangian in the spherical matter field space is used instead of Lagrangian with the potential of the special ”sombrero” form. The gauge field Lagrangian is the standard one. There is no need in Higgs mechanism since the vector field masses are generated automatically.
We have discussed the limiting case of the modified Higgsless Electroweak Model, which corresponds to the contracted gauge group $SU(2; j) \times U(1)$, where $j = \iota$ or $j \to 0$. The masses of the all experimentally verified particles involved in the Electroweak Model remain the same under contraction, but interactions of the fields are changed in two aspects. Firstly all field interactions become more simpler due to nullification of some terms in Lagrangian. Secondly interrelation of the fields become more complicated. All fields are divided on two classes: fields in the base (Z-boson, electromagnetic and electron) and fields in the fiber (W-bosons and neutrino). The base fields can be interpreted as external ones with respect to the fiber fields, i.e. Z-boson, electromagnetic and electron fields can interact with W-bosons and neutrino fields, but W-bosons and neutrino fields do not effect on these fields within framework of the limit model.

This work has been supported in part by the Russian Foundation for Basic Research, grant 08-01-90010-Bel-a and the program ”Fundamental problems of nonlinear dynamics” of Russian Academy of Sciences.

References

[1] D.V. Shirkov, Usp. Fiz. Nauk 179(6), 581 (2009).
[2] R. Casalbuoni, Deconstructed Higgsless models, hep-ph/0509165v1.
[3] R. S. Chivukala, E. H. Simmons, H.-J. He, M. Kurachi and M. Tanabashi, The structure of corrections to electroweak interactions in Higgsless models, hep-ph/0406073v3.
[4] A. A. Slavnov, Renormalizable electroweak model without fundamental scalar meson, hep-th/0601125v1.
[5] A. A. Slavnov, Higgs mechanism as a collective effect due to extra dimension, hep-th/0604052v1.
[6] G. Cacciapaglia, C. Csáki, G. Marandella and J. Terning, A new custodian for a realistic Higgsless model, hep-ph/0607156v1.
[7] J.W. Moffat and V.T. Toth, A finite electroweak model without a Higgs particle, arXiv:0812.1991v3 [hep-th].
[8] G.V. Efimov, Sov. J. Nucl. Phys., 2, 138 (1966).
[9] G.V. Efimov, Sov. J. Nucl. Phys., 4, 309 (1967).
[10] E. Inonu and E.P. Wigner, Proc. Nat. Acad. Sci. USA , 39, 510 (1953).
[11] C.R. Nappi and E. Witten, A WZW model based on non-semisimple group, hep-th/9310112
[12] A.A. Tseytlin, On gauge theories for non-semisimple groups, hep-th/9505129
[13] N.A. Gromov, Spherical Space permits to go without Higgs Boson, Syktyvkar 2007, Pp. 20 (Scientific reports / Komi Science Center UrD RAS, Issue 493) (in Russian); Higgsless Electroweak Theory following from the Spherical Geometry, arXiv:0705.4575v1[hep-th]; Higgsless Electroweak Model due to the Spherical Geometry, Journal of Physics: Conference Series, 2008, v. 128, 012005

[14] M.N. Chernodub, L.D. Faddeev, A.J. Niemi, Non-abelian Supercurrents and Electroweak Theory, arXiv:0804.1544v1[hep-th]; L.D. Faddeev, An alternative interpretation of the Weinberg-Salam model, arXiv:0811.3311v1[hep-th]

[15] N.A. Gromov, In book: Quantum Groups, Differential Equations and Probability Theory (Komi Science Center RAS, Syktyvkar 1994), p.3 (in Russian)

[16] N.A. Gromov, Phys. At. Nucl. 72, 794 (2009); arXiv:08100349v1 [gr-qc].

[17] R.I. Pimenov, Litov. Mat. Sb. 5, 457 (1965) (in Russian).

[18] N.A. Gromov, Contractions and Analitical Continuations of Classical Groups. Unified Approach (Komi Science Center RAS, Syktyvkar 1990) (in Russian).

[19] Rubakov V A 1999 Classical Gauge Fields (Moscow, Editorial URSS) (in Russian)