Quantum secure direct communication by Einstein-Podolsky-Rosen pairs and entanglement swapping

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We present a quantum secure direct communication scheme achieved by swapping quantum entanglement. In this scheme a set of ordered Einstein-Podolsky-Rosen (EPR) pairs is used as a quantum information channel for sending secret messages directly. After insuring the safety of the quantum channel, the sender Alice encodes the secret messages directly by applying a series of local operations on her particle sequence according to their stipulation. Using three EPR pairs, three bits of secret classical information can be faithfully transmitted from Alice to remote Bob without revealing any information to a potential eavesdropper. By both Alice and Bob’s GHZ state measurement results, Bob is able to read out the encoded secret messages directly. The protocol is completely secure if perfect quantum channel is used, because there is not a transmission of the qubits carrying the secret message between Alice and Bob in the public channel.

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1. Introduction

Since the first quantum cryptography protocol using quantum mechanics to distribute keys was proposed by Bennett and Brassard in 1984 (called BB84) \cite{1}, numerous quantum cryptographic protocols have been proposed, such as Ekert 1991 protocol (Ekert91)\cite{2}, Bennett-Brassard-Mermin 1992 protocol (BBM92)\cite{3}, B92 protocol\cite{4} and others protocols\cite{5-21}.

Different from key distribution whose purpose is to establish a common random key between two parties, a secure directly communication is to communicate important messages directly without first establishing a random key to encrypt them. Recently, Shimizu and Imoto\cite{22, 23} and Beige et al.\cite{24} proposed novel quantum secure direct communication (QSDC) schemes, in which the two parties communicate important messages directly without first establishing a shared secret key to encrypt them and the message is deterministically sent through the quantum channel, but can be read only after obtaining an additional classical information for each qubit. Boström and Felbinger\cite{25} put forward a QSDC scheme, the ”ping-pong protocol”, which is insecure if it is operated in a noisy quantum channel, as indicated by Wójcik\cite{26}. Deng et al.\cite{27} suggested a two-step quantum direct communication protocol using Einstein-Podolsky-Rosen pair block. However, in all these QSDC schemes it is necessary to send the qubits with secret messages in the public channel. Therefore, Eve can attack the qubits in transmission.

By using Einstein-Podolsky-Rosen (EPR) pairs and teleportation\cite{28}, Yan and Zhang\cite{29} presented a QSDC scheme. By means of controlled quantum teleportation\cite{30} we proposed two controlled QSDC protocols\cite{31, 32}, one using Greenberger-Horne-Zeilinger (GHZ) states and teleportation, another with entangled states different from GHZ states. Because there is not a transmission of the qubits carrying the secret messages between Alice and Bob in the public channel, they are completely secure for direct secret communication as long as perfect quantum channel is used.

Entanglement swapping\cite{33} is a method that enables one to entangle two quantum systems that do not have direct interaction with one another. In the virtue of entanglement swapping, we introduce a QSDC scheme without alternative measurements. In the scheme, the secure communication between two spatially separated parties (Alice and Bob) is achieved via initially shared EPR pairs, which function as a quantum channel for faithful transmission. After insuring the safety of the quantum channel, the sender Alice encodes the secret messages directly by applying a series of local operations on her particle sequence according to their stipulation. From both Alice and Bob’s GHZ state measurement results, which is communicated via classical channel, Bob is able to read out the encoded secret messages directly. The secret messages are faithfully transmitted from a sender (Alice) to a remote receiver (Bob) without revealing any information to a potential eavesdropper.

2. Quantum secure direct communication protocol by swapping quantum entanglement

Suppose two distant parties, Alice and Bob, share three EPR pairs (three of the four Bell states)

\[ |\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle), |\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle). \] (1)
Without loss of generality, assume that Alice and Bob share \( |\Phi^+\rangle_{12}, |\Psi^+\rangle_{34} \) and \( |\Phi^+\rangle_{56}, \) where Alice has qubits 1, 3 and 5, and Bob possesses 2, 4 and 6. A measurement is performed on particles 1, 3 and 5 with the GHZ basis, \( |P\rangle, |Q\rangle, |R\rangle \) and \( |S\rangle \), then the total state \( |\Phi^+\rangle_{12} \otimes |\Psi^+\rangle_{34} \otimes |\Phi^+\rangle_{56} \) is projected onto \( |R^+\rangle_{135} \otimes |P^+\rangle_{246}, |R^-\rangle_{135} \otimes |P^-\rangle_{246}, |S^+\rangle_{135} \otimes |Q^+\rangle_{246}, |S^-\rangle_{135} \otimes |Q^-\rangle_{246}, |P^+\rangle_{135} \otimes |R^+\rangle_{246}, |P^-\rangle_{135} \otimes |R^-\rangle_{246}, |Q^+\rangle_{135} \otimes |S^+\rangle_{246} \) and \( |Q^-\rangle_{135} \otimes |S^-\rangle_{246} \) with equal probability of \( 1/8 \) for each. Here the GHZ basis are eight GHZ states

\[
|P^\pm\rangle = \frac{1}{\sqrt{2}}(|000\rangle \pm |111\rangle), \quad |Q^\pm\rangle = \frac{1}{\sqrt{2}}(|001\rangle \pm |110\rangle), \quad |R^\pm\rangle = \frac{1}{\sqrt{2}}(|100\rangle \pm |111\rangle), \quad |S^\pm\rangle = \frac{1}{\sqrt{2}}(|011\rangle \pm |100\rangle).
\] (2)

Previous entanglement between qubits 1 and 2, 3 and 4, and 5 and 6 are now swapped into entanglement of qubits 1, 3 and 5, and 2, 4 and 6. Although we considered entanglement swapping with the initial state \( |\Phi^+\rangle_{12} \otimes |\Psi^+\rangle_{34} \otimes |\Phi^+\rangle_{56} \), similar results can be achieved with other Bell states. For example, when Alice and Bob originally share \( |\Phi^+\rangle_{12}, |\Phi^-\rangle_{34} \) and \( |\Phi^+\rangle_{56} \), there are eight possible measurement outcomes, \( |S^-\rangle_{135} \otimes |P^+\rangle_{246}, |S^+\rangle_{135} \otimes |P^-\rangle_{246}, |P^+\rangle_{135} \otimes |R^+\rangle_{246}, |P^-\rangle_{135} \otimes |R^-\rangle_{246}, |Q^+\rangle_{135} \otimes |S^+\rangle_{246} \) and \( |Q^-\rangle_{135} \otimes |S^-\rangle_{246} \) with equal probability 1/8.

Now two spatially separated parties, Alice and Bob, wish to communicate in secret. In order to realize privacy, the first step in our scheme is to establish quantum channel (EPR pairs). Obtaining these EPR pairs could have come about in many different ways, for instance, one of Alice or Bob prepares a sequence of EPR pairs and then send half of each pair to another. Alice and Bob then choose randomly a subset of EPR pairs, and do some appropriate tests of fidelity. Passing the test certifies that they continue to hold sufficiently pure, entangled quantum states. However, if tampering has occurred, Alice and Bob throw out the EPR pairs and start over.

After insuring the security of the quantum channel (EPR pairs), Alice and Bob begin the second step of our scheme—secure direct communication. Both Alice and Bob divide all Bell states into N ordered groups \( \{\xi(1)\}_{12}, \eta(1)\}_{34}, \zeta(1)\}_{56}, \) \( \{\xi(2)\}_{12}, \eta(2)\}_{34}, \zeta(2)\}_{56}, \) \( \cdots, \{\xi(N)\}_{12}, \eta(N)\}_{34}, \zeta(N)\}_{56} \) at random, each group \( \{\xi(i)\}_{12}, \eta(i)\}_{34}, \zeta(i)\}_{56} \) \( i = 1, 2, 3, \cdots, N \) include 3 Bell states. Particles 1, 3 and 5, and particles 2, 4 and 6 of each group belong to Alice and Bob, respectively. Alice encodes information by local operations on EPR pairs. She can perform on each of her particles 3 one of the following four unitary operations

\[
\sigma_{00} = I = |0\rangle\langle 0| + |1\rangle\langle 1|, \quad \sigma_{01} = \sigma_x = |0\rangle\langle 0| + |1\rangle\langle 1|, \quad \sigma_{10} = i\sigma_y = |0\rangle\langle 0| - |1\rangle\langle 1|, \quad \sigma_{11} = \sigma_z = |0\rangle\langle 0| - |1\rangle\langle 1|
\]

(3)

and on each of her particles 3 one of the two operations

\[
\sigma_0 = I = |0\rangle\langle 0| + |1\rangle\langle 1|, \quad \sigma_1 = \sigma_x = |0\rangle\langle 0| + |1\rangle\langle 1|.
\]

(4)

Alice and Bob assign three bits to Alice’s operations as following encoding

\[
\sigma_{ij} \otimes \sigma_k \rightarrow ijk, \quad i,j,k = 0, 1.
\]

(5)

Alice applies local operations to each pair of her particles 1 and 3 according to the secret message sequence. For instance, if the message to be transmitted is a sequence 1100101001001110, then she performs local operations sequence \( \sigma_{11} \otimes \sigma_0, \sigma_{01} \otimes \sigma_0, \sigma_{10} \otimes \sigma_1, \sigma_{00} \otimes \sigma_1, \sigma_{01} \otimes \sigma_0 \) on particles 1 and 3 of \( \{\xi(1)\}_{12}, \eta(1)\}_{34}, \zeta(1)\}_{56} \) and \( \{\xi(2)\}_{12}, \eta(2)\}_{34}, \zeta(2)\}_{56} \) \( \cdots, \{\xi(N)\}_{12}, \eta(N)\}_{34}, \zeta(N)\}_{56} \) respectively. Alice and Bob make GHZ measurements on particles 1, 3 and 5, and 2, 4 and 6, respectively. After that Alice tells Bob her measurement results. According the outcomes of Alice’s measurement, Bob can infer the information that Alice transmits to her. The specific steps of the QSDC scheme using entanglement swapping are as follows:

(1) Alice prepare EPR pairs and then sent half of each to Bob, or vice versa. They both randomly divide all Bell states into N ordered groups \( \{\xi(1)\}_{12}, \eta(1)\}_{34}, \zeta(1)\}_{56} \) and \( \{\xi(2)\}_{12}, \eta(2)\}_{34}, \zeta(2)\}_{56} \) \( \cdots, \{\xi(N)\}_{12}, \eta(N)\}_{34}, \zeta(N)\}_{56} \) and denote \( \xi(i)\}_{12}, \eta(i)\}_{34}, \zeta(i)\}_{56} \) for three Bell states of Alice’s particles 1, 3 and 5, and Bob’s particles 2, 4 and 6 in the \( i \)-th group.

(2) Alice and Bob agree on each of local operations \( \sigma_{ij} \otimes \sigma_k \) can carry three-qubit classical information and encode \( \sigma_{ij} \otimes \sigma_k \) as \( ijk \) where \( i,j,k = 0, 1 \).

(3) Alice encodes her messages on EPR pairs. Explicitly, Alice applies a local operation on each pair of her particles 1 and 3 according to the secret message sequence.

Suppose Alice and Bob initially share Bell state \( |\Phi^+\rangle_{12}, |\Phi^+\rangle_{34}, |\Phi^+\rangle_{56} \) then the originally total state of them is

\[
|\Phi^+\rangle_{12} \otimes |\Phi^+\rangle_{34} \otimes |\Phi^+\rangle_{56} = \frac{1}{2\sqrt{2}}(|P^+\rangle_{135} \otimes |P^+\rangle_{246} + |P^-\rangle_{135} \otimes |P^-\rangle_{246} + |Q^+\rangle_{135} \otimes |Q^+\rangle_{246} + |Q^-\rangle_{135} \otimes |Q^-\rangle_{246} + |R^+\rangle_{135} \otimes |R^+\rangle_{246} + |R^-\rangle_{135} \otimes |R^-\rangle_{246} + |S^+\rangle_{135} \otimes |S^+\rangle_{246} + |S^-\rangle_{135} \otimes |S^-\rangle_{246} \).
\]

(6)

If Alice wishes to transmit 111 to Bob, then she performs a local operation \( \sigma_{11} \otimes \sigma_1 \) on particles 1 and 3 and the state \( \Phi^+_{12} \otimes |\Phi^+_{34} \) is turned into \( |\Phi^-\rangle_{12} \otimes \Psi^+_{34} \).
(4) Alice makes a GHZ state measurement on her particles 1, 3 and 5. Assume that her measurement outcome is $|P^+\rangle_{135}$, then she can induce Bob’s three particles 2, 4 and 6 in the state $|R^-\rangle_{246}$ by the following equation:

$$|\Phi^-(\rangle_{12} \otimes |\Psi^+\rangle_{34} \otimes |\Phi^+\rangle_{56} = \frac{1}{2\sqrt{2}}( |R^-\rangle_{135} \otimes |P^+\rangle_{246} + |R^+\rangle_{135} \otimes |P^-\rangle_{246} + |S^-\rangle_{135} \otimes |Q^+\rangle_{246} + |S^+\rangle_{135} \otimes |Q^-\rangle_{246}$$

$$+ |P^-\rangle_{135} \otimes |R^+\rangle_{246} + |P^+\rangle_{135} \otimes |R^-\rangle_{246} + |Q^-\rangle_{135} \otimes |S^+\rangle_{246} + |Q^+\rangle_{135} \otimes |S^-\rangle_{246} (\rangle_{7}$$

(5) Alice tells Bob that she has made a Bell measurement on her particles 1, 3 and 5 over a classical channel, but does not mention the result of her measurement.

(6) Bob performs a GHZ-type measurement on his particles 2, 4 and 6, and infers the outcome of Alice’s measurement.

From the calculation of entanglement swapping (Eq. (8)) and Bob’s measurement outcome $|R^-\rangle_{246}$, Bob could determine exactly that the outcome of Alice’s measurement should be $|R^-\rangle_{135}$ without Alice’s local operation.

(7) Bob asks and gets Alice’s measurement result publicly.

(8) Bob can read out Alice’s secret message by comparing his calculation result with Alice’s practical measurement outcome.

From the measurement result $|P^+\rangle_{135}$ announced by Alice and his calculation result $|R^-\rangle_{135}$, Bob can infer that Alice has applied a local operation $\sigma_{ij} \otimes \sigma_k$ on particles 1 and 3, thus he obtains Alice’s message 111. Finally, the two distant parties have realized deterministic secure direct communication.

Note: (A) The above protocol is also a quantum key distribution (QKD) scheme based on Bell states and entanglement swapping, if Alice applies her local operations $\sigma_{ij} \otimes \sigma_k$, as determined by $a$, a string of $3N$ random classical bits which she creates on her own. That is, depending on three random classical bit $a$ which she generates, Alice performs a local operation $\sigma_{ij} \otimes \sigma_k$ if $a = ijk$ $(i, j, k = 0, 1)$ on her particles 1 and 3. Alice and Bob agree upon in advance that each of the eight GHZ states can carry three bits classical information and encode $|P^+\rangle$, $|P^-\rangle$, $|Q^+\rangle$, $|Q^-\rangle$, $|R^+\rangle$, $|R^-\rangle$, $|S^+\rangle$, and $|S^-\rangle$ as 000, 001, 010, 011, 101, 100, 110 and 111, respectively. By Alice’s measurement result $|P^+\rangle_{135}$, both Alice and Bob derive $|P^+\rangle_{135} \sigma_{ij} \otimes \sigma_k$, $|P^+\rangle_{135}$ and share three certain bits 111 and three random bits 101 in private. Therefore, in our proposed protocol, Alice performs one local operation on her particles 1 and 3, Bob shares 3 certain bits and 3 random bits with Alice secretly.

(B1) Alice can also apply unitary operator

$$I = |0\rangle\langle 0| + |1\rangle\langle 1|, \quad i\sigma_y = |0\rangle\langle 1| - |1\rangle\langle 0|$$

(8)

on particles 3, and she and Bob agree beforehand as the following encoding:

$$I \rightarrow 0, \quad i\sigma_y \rightarrow 1,$$  

(9)

instead of that in the above protocol. (B2) In the above protocol, Alice can use local operations in Eq. (4) and Eq. (3) on particles 1 and 3, respectively. Moreover, Alice and Bob arrange the encoding as $\sigma_i \otimes \sigma_{jk} \rightarrow ijk$ for $i, j, k = 0, 1$. (B3) The unitary operations Eq. (4) and Eq. (3) performed by Alice in above protocol can be replaced by local operations in Eq. (5) and Eq. (6), respectively. Furthermore, Alice and Bob agree on the decoding as $I \otimes \sigma_{jk} \rightarrow 0jk, i\sigma_y \otimes \sigma_{jk} \rightarrow 1jk, j, k = 0, 1$.

(C) Particles 1, 3 and 5 play symmetric and equal role. That is, Alice can apply one unitary operator in Eq. (4) on each of one particles $m$ of her particles 1, 3 and 5, and one in Eq. (6) on each of another particles $n$ of her particles 1, 3 and 5, and Alice and Bob agree on local operations $\sigma_{ij}$ and $\sigma_k$ encoded as $ij$ and $k$, respectively; or Alice performs on each of particles $m$ of particles 1, 3 and 5 one unitary operation in Eq. (6), and on each of particles $n \neq m$ of particles 1, 3 and 5 one unitary operation in Eq. (5), and Alice and Bob encode $\sigma_{ij}$ as $ij$, and encode $I$ and $i\sigma_y$ as 0 and 1, respectively. Here $i, j, k = 0, 1$.

(D) Alice can also apply one operator in Eq. (5) on each of her particles 1, 3 and 5, and she and Bob agree beforehand as the encoding of Eq. (3).

3. Security

Our QSDC protocol is based on EPR pairs, so the proof of security is similar to those in Refs. [3, 25, 34, 35]. Once the security of the quantum channel is assured, which means that an eavesdropper Eve has not acquire particles 2, 4 and 6, i.e. Alice and Bob shares pure EPR pairs (perfect quantum channel), then no information is leaked to Eve. Hence our proposed protocol is secure, even if the shared quantum channels are public.

In summary, we present a new deterministic secure method for direct communication, where the two spatially separated parties faithfully transmit secret messages using entanglement swapping and detect eavesdroppers by the correlations of entanglement swapping results. A set of ordered Einstein-Podolsky-Rosen (EPR) pairs is used as a quantum information channel for sending secret messages directly. After insuring the safety of the quantum channel, the sender Alice encodes the secret messages directly by applying a series local operations on her particles sequence
according to their stipulation and send them to distant receiver Bob. Using three EPR pairs, three bits of secret classical information can be faithfully transmitted from Alice to remote Bob without revealing any information to a potential eavesdropper. By both Alice and Bob’s GHZ state measurement results, Bob is able to read out the encoded secret messages directly. The protocol is completely secure if perfect quantum channel is used, since there is not a transmission of the qubits carrying the secret message between Alice and Bob in the public channel. This scheme is also a quantum key distribution (QKD) scheme, in which via three EPR pairs, six bits of secret key (three certain bits and three random bits) can be shared between two spatially separated parties that play symmetric and equal roles.

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