Nonleptonic two-body decays of charmed mesons

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Nonleptonic decays of charmed mesons into two pseudoscalar mesons or one pseudoscalar meson and one vector meson are studied on the basis of a generalized factorization method considering the resonance effects in the pole model for the annihilation contributions. Large strong phases between different topological diagrams are considered in this work, simply taking the phase in the coefficients \(a_i\). We find that the annihilation-type contributions calculated in the pole model are large in both of the \(PP\) and \(PV\) modes, which make our numerical results agree with the experimental data better than those previous calculations.

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I. INTRODUCTION

Nonleptonic decays of charmed mesons are very interesting as they can provide useful information on flavor mixing, CP violation, and strong interactions. They may also shed light on any new physics signal through \(D^0 - \bar{D}^0\) mixing and rare decays.\(^2\) The CLEO-c and two B factories experiments have given many new results on this subject and more are expected soon from the BES-III experiment. Besides, theoretical studies have been in progress for decades.

With the heavy quark effective theory, many QCD-inspired approaches, such as the QCD factorization approach\(^8\), the perturbative QCD approach\(^9\), and the soft-collinear effective theory\(^10\), successfully describe the hadronic B decays. However, this is not the case for the \(D\) meson decays. These approaches do not work well here, due to the mass of charm quark, of order 1.5 GeV, which is not heavy enough for a sensible heavy quark expansion, neither light enough to apply the chiral perturbation theory.

After decades of studies, the factorization approach is still one of the effective ways to deal with the two-body charmed meson decays\(^1\). However, it is well known that some difficulties exist in the naive factorization approach: the Wilson coefficients \(a_1(\mu)\) and \(a_2(\mu)\) of effective operators are renormalization scale and \(\gamma_5\)-scheme dependent; and the color-suppressed processes are not calculated well due to the smallness of \(a_2\), etc. In order to solve these problems, the so-called generalized factorization approach was proposed\(^12\). The Wilson coefficients \(a_1(\mu)\) and \(a_2(\mu)\) are not from direct calculation any more but are effective coefficients to accommodate the important nonfactorizable corrections\(^13\). In these naive or generalized factorization approaches, there is almost no strong phase. But large strong phases have been found from \(D\) decay experiments. A corresponding large relative strong phase between the factorization coefficients \(a_1\) and \(a_2\) has been discussed in\(^14,15\).

Unsatisfied by the shortcomings of the factorization approach, the model-independent diagrammatic approach, with various topological amplitudes extracted from the data, is recently applied to two-body hadronic \(D\) decays\(^16\)–\(^19\). They use \(SU(3)\) symmetry in their analysis to avoid model calculations. All the parameters are fitted from experiments to give a better agreement with the experimental data but with less predictive power. More precise predictions are limited in this approach due to the uncontrolled \(SU(3)\) breaking effect\(^15\). These analyses also show that large annihilation type contributions are needed to explain the data, which can not be calculated in the naive or generalized factorization approaches. The kind of pure annihilation type \(D\) meson decays also needs to be systematically analyzed\(^20\).

In another aspect, the hadronic picture description of nonleptonic weak decays has a longer history, because of their nonperturbative feature. Based on the idea of vector dominance, which is discussed on strange particle decays\(^21\), the pole-dominance model of two-body nonleptonic decays is proposed\(^22\). Beyond the vector dominance pole, this model also involves scalar, pseudoscalar, and axial-vector poles. For simplicity, only the lowest-lying poles are considered. This model has already been applied to charmed meson and bottom meson decays\(^22\)–\(^24\), where it is approved that this model is more or less equivalent to the factorization approach in the first order approximation.

In this work, we will use the generalized factorization approach but with a relative strong phase between the Wilson coefficients \(a_1\) and \(a_2\) to accommodate the nonfactorizable contributions. For the uncalculable annihilation type contributions, we use the pole model. In this case, we can really calculate most of the important contributions in the hadronic \(D\) decays, which are demonstrated in the model-independent diagrammatic analysis\(^19\).

The outline of this paper is as follows. In Sec.II, we give the formulas of this work, showing the generalized factorization approach and the pole-dominance model. In Sec.III, our results are given and compared to the experimental data and those of the diagrammatic approach and the calculations considering the final-state interaction of nearby
resonances effects. Summary and conclusions are followed.

II. FORMALISM

A. The factorization approach

First we begin with the weak effective Hamiltonian $H_{\text{eff}}$ for the $\Delta C = 1$ transition [24]:

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{\text{CKM}} (C_1 O_1 + C_2 O_2) + \text{h.c.},$$

where $V_{\text{CKM}}$ is the corresponding Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, $C_{1,2}$ are the Wilson coefficients. The current-current operators $O_{1,2}$ are

$$O_1 = \bar{u}_\alpha \gamma_\mu (1 - \gamma_5) q_2 \beta \cdot \bar{q}_3 \gamma^\mu (1 - \gamma_5) c_\alpha,$$

$$O_2 = \bar{u}_\alpha \gamma_\mu (1 - \gamma_5) q_2 \alpha \cdot \bar{q}_3 \gamma^\mu (1 - \gamma_5) c_\beta,$$

where $\alpha, \beta$ are color indices, and $q_{2,3}$ are $d$ or $s$ quarks.

The color-favored emission diagram corresponding to $D \rightarrow PP$ decays, with $P$ representing a pseudoscalar meson, is shown in Fig. 1 (a). Under the factorization hypothesis, the transition matrix element of hadronic two-body charmed meson decays is factorized into two parts [11]:

$$\langle P_1 P_2 | H_{\text{eff}} | D \rangle = \langle P_1 | \bar{u}_\alpha \gamma_\mu (1 - \gamma_5) q_2 \beta | 0 \rangle \langle P_2 | \bar{q}_3 \gamma^\mu (1 - \gamma_5) c | D \rangle,$$

where $a_{1,2}(\mu) = C_{2}(\mu) + C_{1}(\mu) N_c$, $a_{2}(\mu) = C_{1}(\mu) N_c$, correspond to the color-favored tree diagram ($T$) and the color-suppressed diagram ($C$) respectively in the naive factorization, with the number of colors $N_c = 3$. They are assumed to be universal and process-independent in the naive factorization approach. The current matrix elements in Eqs. (3,4) are evaluated in terms of transition form...
factors and decay constants. For $D \to PP$ decays, the form factor is defined as follows:

$$
\langle P(k)|\bar{q}_3\gamma_{\mu}(1 - \gamma_5)c|D(p)\rangle = \left[(p + k)_{\mu} - \frac{m_D^2 - m_P^2}{q^2}q_{\mu}\right] F_{1D\to P}(q^2) \\
+ \frac{m_D^2 - m_P^2}{q^2}q_{\mu}F_{0D\to P}(q^2),
$$

where $q = p - k$, and $F_1$ are the corresponding transition form factors. The decay constants $f_P$ of pseudoscalar mesons are defined as

$$
\langle P(q)|\bar{q}_1\gamma_{\mu}(1 - \gamma_5)q_2|0\rangle = if_P q_{\mu}.
$$

In terms of decay constant and transition form factors, the decay amplitude of Figs. 1(a) and (b) are then

$$
\langle P_1P_2|H_{eff}|D\rangle_T = i\frac{G_F}{\sqrt{2}} V_{CKM} a_1 f_{P_1}(m_D^2 - m_{P_1}^2) F_{0D\to P_1}(m_{P_1}^2),
$$

$$
\langle P_1P_2|H_{eff}|D\rangle_C = i\frac{G_F}{\sqrt{2}} V_{CKM} a_2 f_{P_1}(m_D^2 - m_{P_2}^2) F_{0D\to P_2}(m_{P_2}^2).
$$

The diagrams for $D \to PV$ decays, with $V$ denoting a vector meson, are shown in Figs. 1(c) and (d). If the out-emitted particle is a pseudoscalar meson, the matrix element of Fig. 1(c) is

$$
\langle PV|H_{eff}|D\rangle = \frac{G_F}{\sqrt{2}} V_{CKM} a_1 (P|\bar{q}_3\gamma_{\mu}(1 - \gamma_5)q_2|0) (V|\vec{q}_3\gamma^\mu(1 - \gamma_5)c|D)  
$$

The $D \to V$ transition form factors are usually defined as

$$
\langle V(k)|\bar{q}_3\gamma_{\mu}(1 - \gamma_5)c|D(p)\rangle = \frac{2}{m_D + m_V}\epsilon_{\mu\rho\sigma}^* p^\rho k^\sigma V(q^2)  \\
- i\left(\epsilon_{\mu} - \frac{\epsilon^*}{q^2} q_{\mu}\right)(m_D + m_V) A_{1D\to V}(q^2)  \\
+ i\left[p + k\right]_{\mu} - \frac{m_D^2 - m_V^2}{q^2}q_{\mu}\epsilon^* \cdot q \frac{m_D + m_V}{m_D + m_V} A_{2D\to V}(q^2)  \\
- \frac{2m_V\epsilon^* \cdot q}{q^2}q_{\mu} A_{0D\to V}(q^2),
$$

where $\epsilon^*$ is the polarization vector of the vector meson, and $A_1$ and $V$ are corresponding transition form factors. Utilizing the form factor definitions we get the result for Eq. (11):

$$
\langle PV|H_{eff}|D\rangle = \frac{G_F}{\sqrt{2}} V_{CKM} a_1 f_{P} m_V A_{0D\to V}(m_V^2) 2(\epsilon^* \cdot p_D).
$$

If a vector meson is out emitted, the matrix element of Fig. 1(c) is

$$
\langle PV|H_{eff}|D\rangle = \frac{G_F}{\sqrt{2}} V_{CKM} a_1 (P|\bar{q}_3\gamma_{\mu}(1 - \gamma_5)q_2|0) (V|\vec{q}_3\gamma^\mu(1 - \gamma_5)c|D)  \\
= \frac{G_F}{\sqrt{2}} V_{CKM} a_1 f_{P} m_V F_{1D\to P}(m_V^2) 2(\epsilon^* \cdot p_D),
$$

where the decay constants $f_V$ of vector mesons are defined as

$$
\langle V(q)|\bar{q}_1\gamma_{\mu}(1 - \gamma_5)q_2|0\rangle = f_V m_V \epsilon_{\mu}^*(q).
$$

For the color-suppressed diagram, Fig 1(d), we have similar formulas as Eqs. (12,13), but with the Wilson coefficient changed from $a_1$ to $a_2$.

As mentioned in the Introduction, the Wilson coefficients $a_1$ and $a_2$ are renormalization-scale dependent in the naive factorization approach and it fails to describe the color-suppressed processes with too small $a_2 \approx -0.1$. So the generalized factorization method is proposed to include the nonfactorizable contributions $^{13}$,

$$
a_{1eff}^i = C_2(\mu) + C_1(\mu) \left(\frac{1}{N_c} + \chi_1(\mu)\right), \quad a_{2eff}^i = C_1(\mu) + C_2(\mu) \left(\frac{1}{N_c} + \chi_2(\mu)\right),
$$

where $N_c$ is the number of colors.
where the terms $\chi_i$ characterize the nonfactorizable corrections involving vertex corrections, hard spectator interactions, final-state interactions, resonance effects, etc. These $\chi_i(\mu)$ will compensate the scale- and scheme-dependence of the Wilson coefficients, so that $a_i$’s are physical now. Without confusion, we will drop the superscript "eff" in the effective Wilson coefficients for convenience in the following discussions. In the large-$N_c$ approach, the $1/N_c$ terms are discarded\cite{25}, equally with a universal nonfactorizable term $\chi_1 = \chi_2 = -1/N_c$, hence,

$$a_1 \approx C_2(m_c) = 1.274, \quad a_2 \approx C_1(m_c) = -0.529.$$ 

(16)

This implies a null relative strong phase between the two kinds of contributions. However, the experimental data tell us that there should be a large strong phase between $a_1$ and $a_2$. On the other hand, the existence of relative phases is reasonable for the importance of inelastic final-state interactions of the $D$ meson decays, in which the on-shell intermediate states contribute imaginary parts. Therefore, we consider a relative phase between the coefficients $a_1$ and $a_2$ in this work, so that

$$a_1 = |a_1|, \quad a_2 = |a_2|e^{i\delta},$$

(17)

where we set $a_1$ real for convenience.

B. Pole-dominance Model

The annihilation type diagrams are neglected as an approximation in the factorization model. However, considerable contributions come from the weak annihilation diagrams in the $D$ decays, which can be demonstrated by the difference of lifetime between $D^0$ and $D^+$. Hence, we will calculate them in a single pole-dominance model. For simplicity, only the lowest-lying poles are considered in the single-pole model. Taking $D^0 \to \pi^+K^{*-}$ as an example, the annihilation-type diagram in the pole model is shown in Fig. 2(a). $D^0$ goes into $K^0$ via the weak interaction in Eq. (11) shown in terms of quark lines in Fig. 2(b), and then decays into $\pi^+K^{*-}$ through the strong interaction. Angular momentum should be conserved at the weak vertex and all conservation laws be preserved at the strong vertex. So it is a pseudoscalar meson as a resonant state for $D \to PV$ decays. The weak matrix element is evaluated in the vacuum insertion approximation\cite{22},

$$\langle \bar{K}^0|\mathcal{H}_{e f f}|D^0\rangle = \frac{G_F}{\sqrt{2}}V_{cs}^*V_{ud}a_E^{PV}\langle \bar{K}^0|\bar{s}\gamma_\mu(1-\gamma_5)d|0\rangle \langle 0|\bar{u}\gamma^\mu(1-\gamma_5)c|D^0\rangle$$

$$= \frac{G_F}{\sqrt{2}}V_{cs}^*V_{ud}a_E^{PV}f_Kf_Dm_D^2,$$

(18)

where the subscript $E$ of the Wilson coefficient $a_E^{PV}$ denotes a $W$-exchange diagram for $D \to PV$, otherwise $a_A^{PV}$ corresponds to $W$-annihilation contributions. In fact, the effective Wilson coefficients of the $W$-annihilation diagrams and $W$-exchange diagrams have the same form as $a_1$ and $a_2$ in Eq. (15), that is

$$a_E = C_1(\mu) + C_2(\mu)\left(\frac{1}{N_c} + \chi_E(\mu)\right), \quad a_A = C_2(\mu) + C_1(\mu)\left(\frac{1}{N_c} + \chi_A(\mu)\right),$$

(19)

where $\chi_A(E)$ represents the nonfactorizable contributions in the annihilation (exchange) process. Since the nonfactorizable contributions in these kinds of diagrams are large and with relatively different strong phases, we use different symbols to avoid confusing in our approach for these collective effective Wilson coefficients as $a_E$ and $a_A$, respectively. Strong phases relative to the emission diagrams are considered in the Wilson coefficients. The effective strong coupling constant of $K^0$ to $\pi^+K^{*-}$ is defined through the Lagrangian

$$\mathcal{L}_{VVPP} = ig_{VPP}V^\mu(P\bar{\partial}_\mu P),$$

(20)

where $g_{VPP}$ is dimensionless. Inserting the propagator of the intermediate $K^0$ meson, the decay amplitude is

$$\langle \pi^+K^{*-}|\mathcal{H}_{e f f}|D^0\rangle = \frac{G_F}{\sqrt{2}}V_{cs}^*V_{ud}a_E^{PV}f_Kf_Dg_{VPP}\frac{m_D^2}{m_D^2 - m_K^2}2(\varepsilon^* \cdot p_D).$$

(21)

Similarly, in $D \to PP$ decays, it is a scalar meson as a resonant state. The effective strong coupling constant is described by

$$\mathcal{L}_{SPPP} = -g_{SPPP}m_{SPPP},$$

(22)
where $m_S$ is the mass of the scalar meson. Besides, the scalar meson decay constant of the vector current is defined as

$$\langle S(p)|q_2\gamma\mu q_1|0\rangle = f_S p_\mu. \quad (23)$$

Therefore, the corresponding matrix element is

$$\langle PP|\mathcal{H}_{eff}|D\rangle = -\frac{G_F}{\sqrt{2}} V_{CKM} a_A(a_E) f_S f_D g_{SPP} \frac{m_D^2 m_S}{m_D^2 - m_S^2}. \quad (24)$$

As a convenience of reference, the various decay formulas of individual decay modes are collected in the appendix. Note that some of the intermediate resonances are unstable particles which have large width and therefore contribute large relative phases. These phases are absorbed in the effective Wilson coefficients $a_A$ and $a_E$ for convenience.

III. NUMERICAL ANALYSIS

A. Input Parameters

In order to calculate the emission type diagrams in the factorization approach, we need to know the transition form factors and meson decay constants. The decay constants of $\pi$, $K$, $D$ and $D_s$ are taken from the particle data group (PDG) [26], others are from [27], all of which are summarized in Table I. There exist many models to parametrize the transition form factors and their $q^2$ dependence [28–46]. In this work we shall use the dipole model [28]:

$$F(q^2) = \frac{F(0)}{(1 - \alpha_1 \frac{q^2}{m_{pole}^2} + \alpha_2 \frac{q^4}{m_{pole}^4})}, \quad (25)$$

where $m_{pole}$ is the mass of the pole. The corresponding poles are $D^*$ for $F_{0,1}^{D_\pi, D_\eta, D_s K}$, $D^*_s$ for $F_{0,1}^{D_s K, D_\eta, D_{\phi}}$, $D$ for $A_0^{D\rho, D\omega, D_s K}$, and $D_s$ for $A_0^{D_{\phi}}$. The transition form factors and $\alpha_i$ parameters of $D$ to $\pi$ and $K$ are taken from the recent CLEO-c measurement [23], $D \rightarrow \eta_q$ are from [31], and others from [28], all of which are shown in Table I.

For the final states involving $\eta$ or $\eta'$, it is convenient to consider the flavor mixing of $\eta_q$ and $\eta_s$ with a mixing angle $\phi$,

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}, \quad (26)$$

where $\eta_q$ and $\eta_s$ are defined by

$$\eta_q = \frac{1}{\sqrt{2}} (u \bar{u} + d \bar{d}), \quad \eta_s = s \bar{s}. \quad (27)$$

A recent experimental measurement from the KLOE collaboration gives the mixing angle $\phi = (40.4 \pm 0.6)^\circ$ [47]. The decay constants of $\eta$ or $\eta'$ are defined by

$$\begin{align*}
\langle 0|\bar{u}\gamma_\mu \gamma_5 u|\eta(p)\rangle &= i f_\eta^\mu p_\mu, \\
\langle 0|\bar{d}\gamma_\mu \gamma_5 d|\eta(p)\rangle &= i f_\eta^\mu p_\mu, \\
\langle 0|\bar{s}\gamma_\mu \gamma_5 s|\eta(p)\rangle &= i f_\eta^\mu p_\mu, \\
\langle 0|\bar{u}\gamma_\mu \gamma_5 u|\eta'(p)\rangle &= i f_{\eta'}^\mu p_\mu, \\
\langle 0|\bar{d}\gamma_\mu \gamma_5 d|\eta'(p)\rangle &= i f_{\eta'}^\mu p_\mu, \\
\langle 0|\bar{s}\gamma_\mu \gamma_5 s|\eta'(p)\rangle &= i f_{\eta'}^\mu p_\mu.
\end{align*} \quad (28)$$

FIG. 2: Annihilation diagrams in the pole model.
We shall use the results in [49], in several methods, such as the finite-energy sum rule [49], the generalized Nambu-Jona-Lasinio model [50], and so on. Primarily the four-quark bound states or two-quark scalar states. In this work, we use constants between the intermediate state and two final states. Some of them are obtained directly from experiments.

With the ansatz in [48], we have

$$f^u_q = f^d_f = \frac{1}{\sqrt{2}} f^0_q, \quad f^u_{q'} = f^d_{f'} = \frac{1}{\sqrt{2}} f^0_{q'}.$$  \hspace{1cm} (29)

We use that $f_q = (1.07 \pm 0.02) f_\pi$ and $f_s = (1.34 \pm 0.06) f_\pi$ from [48].

The form factors of $D \to \eta_q$ in Table II denote that of $D \to \eta_{u,d,d}$, not $D \to \frac{1}{\sqrt{2}} (u + d \bar{d})$, hence,

$$F^{D \to \eta_q} = \frac{1}{\sqrt{2}} F^{D \to \eta_{u,d,d}} \cos \phi, \quad F^{D_{s} \to \eta_q} = - F^{D_{s} \to \eta_{u,d,d}} \sin \phi,$$  \hspace{1cm} (31)

$$F^{D \to \eta_{q'}} = \frac{1}{\sqrt{2}} F^{D \to \eta_{u,d,d}} \sin \phi, \quad F^{D_{s} \to \eta_{q'}} = F^{D_{s} \to \eta_{u,d,d}} \cos \phi.$$  \hspace{1cm} (32)

In order to calculate the annihilation-type diagrams in the pole model, we have to know the effective strong coupling constants between the intermediate state and two final states. Some of them are obtained directly from experiments. Some others are related to the known ones using SU(3) symmetry. Although the intermediate states are a little off shell, in the pole model they are used as on-shell resonant states. So these on-shell strong couplings are used to calculate the annihilation diagrams in an approximation.

There are many scalar mesons discovered by the experiments. The existence of the lightest scalar nonet with the mass smaller than or close to 1 GeV has been a problem for many years [20]. It is still controversial that they are primarily the four-quark bound states or two-quark scalar states. In this work, we use $K^*_0(1430), a_0(1450), f_0(1570)$, and $f_0(1500)$ as intermediate mesons in the pole model for $D \to PP$ decays. The decay constant of $K^*_0$ is calculated in several methods, such as the finite-energy sum rule [49], the generalized Nambu-Jona-Lasinio model [50], and so on. We shall use the results in [49],

$$f_{K^*_0} = (42 \pm 2) \text{MeV}.$$  \hspace{1cm} (33)

For all other scalar mesons, we take the same value of decay constants as $K^*_0$ in the flavor SU(3) limit for simplification.

The corresponding effective strong coupling constant between $K^*_0$ and the final states of $K \pi$ is evaluated by $g_{K^*_0 K \pi} = 2.7$ from the decay of $K^*_0(1430)^0 \to \pi^- K^+$. Other couplings of $g_{SPP}$ are of the same value in the SU(3) limit. For

| $f_s$ | $f_K$ | $f_\omega$ | $f_D$ | $f_{D_s}$ |
|-------|-------|-----------|-------|-----------|
| 130   | 156   | 216       | 220   | 187       |
| 215   | 207   | 258       |       |           |

TABLE I: Meson decay constants (MeV). Those of $\pi$, $K$, $D$, and $D_s$ are from PDG [20], others are from [27].

| $F^{D \to \pi}$ | $F^{D_{s} \to \pi}$ | $F^{D \to K}$ | $F^{D_{s} \to K}$ | $F^{D \to K_{s} \to \eta_q}$ | $F^{D_{s} \to K_{s} \to \eta_q}$ |
|-----------------|-------------------|---------------|-------------------|-----------------------------|-----------------------------|
| $F(0)$          | 0.67              | 0.74          | 0.74              | 0.72                        | 0.78                        |
| $\alpha_2$      | 0.21              | 0.30          | 0.33              | 0.20                        | 0.23                        |

| $A^{D_{s} \to K}$ | $A^{D \to K}$ | $A^{D \to K_{s} \to \eta_q}$ | $A^{D_{s} \to K_{s} \to \eta_q}$ |
|------------------|--------------|-----------------------------|-----------------------------|
| $F(0)$           | 0.66         | 0.76                        | 0.67                        |
| $\alpha_2$       | 0.36         | 0.17                        | 0.20                        |

TABLE II: The $D$ meson transition form factors and dipole model parameters $\alpha_{1,2}$. The parameter $\alpha_1 = \alpha_2 + 1$ if only $\alpha_2$ is shown in the table. The form factors and $\alpha_1$ parameters of $D$ to $\pi$ and $K$ are from [20], $D \to \eta_q$ from [50], and others from [28].

where

$$f^u_q = f^d_f = \frac{1}{\sqrt{2}} f^0_q, \quad f^u_{q'} = f^d_{f'} = \frac{1}{\sqrt{2}} f^0_{q'},$$

$$f^0_q = f_q \cos \phi, \quad f^s_s = - f_s \sin \phi,$$

$$f^0_{q'} = f_q \sin \phi, \quad f^s_{s'} = f_s \cos \phi.$$  \hspace{1cm} (30)
TABLE III: Branching ratios for Cabibbo-favored decays of $D \to PP$(%). The predicted branching ratios with both annihilation- and emission-type contributions (all) and with only emission-type contributions (emission) are given together with the experimental data [53], the recent results from the diagrammatic approach [19], and the calculations considering the final-state interaction (FSI) effects of nearby resonances [51] as comparison.

| Modes          | Br(FSI)  | Br(diagrammatic) | Br(emission) | Br(all)  | Br(exp) |
|----------------|----------|------------------|--------------|----------|---------|
| $D^+ \to \pi^+ K^0$ | 2.51     | 3.08±0.36        | 3.1±2.0      | 3.1±2.0  | 3.07±0.096 |
| $D^0 \to \pi^+ K^-$ | 4.03     | 3.91±0.17        | 5.8±0.7      | 3.9±1.0  | 3.89±0.077 |
| $D^0 \to \pi^0 K^0$ | 1.35     | 2.36±0.08        | 2.1±0.6      | 2.4±0.7  | 2.38±0.09 |
| $D^0 \to \bar{K}^0 \eta$ | 0.80     | 0.98±0.05        | 0.9±0.2      | 0.8±0.2  | 0.96±0.06 |
| $D^0 \to \bar{K}^0 \eta'$ | 1.51     | 1.91±0.09        | 0.3±0.2      | 1.9±0.3  | 1.90±0.11 |
| $D_S^+ \to K^+ K^0$ | 4.79     | 2.97±0.32        | 5.1±0.9      | 3.0±0.9  | 2.98±0.08 |
| $D_S^+ \to \pi^+ \eta$ | 1.33     | 1.82±0.32        | 3.8±0.4      | 1.9±0.5  | 1.84±0.15 |
| $D_S^+ \to \pi^+ \eta'$ | 5.89     | 3.82±0.36        | 2.9±0.6      | 4.6±0.6  | 3.95±0.34 |
| $D_S^+ \to \pi^+ \pi^0$ | 1.11     | 1.23±0.08        | 2.1±0.3      | 2.1±0.3  | 2.15±0.07 |

$D \to PV$ decays, the intermediate states are pseudoscalar mesons with relatively large decay constants shown in Table 1. The corresponding effective strong coupling constants are $g_{\pi \pi} = 4.2$ obtained from $\rho^0 \to \pi^0 \pi^0$ or $\rho^0 \to \pi^+ \pi^-$, $g_{K^+ K} = 4.6$ from $K^*(892)^0 \to \pi^+ K^-$, and $g_{\phi K K} = 4.5$ from $\phi \to K^+ K^-$. For decays involving $\eta$ or $\eta'$, we assume that $g_\eta = 4.2$, $g_s = 4.6$, and $g_{s s} = 4.5$, where $g_{s s}$ couple to the states with only $u$ or $d$ quarks, $g_s$ to two of the states with $s$ quark, and $g_{s s}$ to all the three mesons with $s$ quark, so that some effects of SU(3) breaking are considered.

B. $D \to PP$

For $D \to PP$ decays, the decay rate is

$$\Gamma(D \to PP) = \frac{p}{8\pi m_D} |A|^2,$$  \hspace{1cm} (34)

where $p$ is the momentum of either meson in the final state in the center-of-mass frame, $p = \sqrt{(m_D^2 - (m_{P_1} + m_{P_2})^2)(m_D^2 - (m_{P_1} - m_{P_2})^2)/2m_D}$.

As is done in the naive factorization model, the Wilson coefficients $a_i$’s are universal and process-independent, except with a relative strong phase for different topological diagrams. Because of the important nonperturbative effect of QCD in the charm system, $a_1$ and $a_2$ should deviate a lot from the naive factorization approach. In order to give the most suitable results, we input the following values by hand, which also used the hint from the fit of diagrammatic approach:

$$a_1 = 1.25 \pm 0.10,$$
$$a_2 = (0.85 \pm 0.10)e^{i(153\pm10)°},$$
$$a_A = (0.90 \pm 0.10)e^{i(160\pm10)°},$$
$$a_E = (2.4 \pm 0.1)e^{i(55\pm10)°},$$  \hspace{1cm} (35)

where $a_1$ is the coefficient of the color-favored emission tree diagrams, $a_2$ for the color-suppressed emission diagrams, $a_A$ for the W-annihilation diagrams, and $a_E$ for the W-exchange diagrams. Large relative strong phases are considered in the $a_i$, due to unneglected inelastic final-state interactions in the $D$ decays. These values of $a_1$ and $a_2$ are not far away from the large $N_c$ limit, except that we use quite large strong phases, which are required by the experimental data. As is discussed in the diagrammatic approach [10], the W-annihilation contributions with helicity-suppressed effect are much smaller than those of the W-exchange diagrams. Therefore we use a much larger coefficient of $a_E$ than the W-annihilation coefficient $a_A$. Besides, we ignore the disconnected hairpin diagrams, $SE$ and $SA$, as discussed in [10].

The predicted branching ratios with annihilation-type contributions (all) and without annihilation type contributions (emission) together with experimental measurements of charmed mesons decay into two pseudoscalar mesons are listed in Tables III, IV, V for the Cabibbo-favored decays, the singly Cabibbo-suppressed decays, and the doubly Cabibbo-suppressed decays, respectively. There are many sources of theoretical uncertainties in the calculations. Since the decay constants of pseudoscalar and vector mesons are taken from experiments with very small errors, our
TABLE IV: Same as Table III except for singly Cabibbo-suppressed decays of $D \to PP$ ($\times 10^{-3}$).

| Modes          | Br(FSI) | Br(diagrammatic) | Br(emission) | Br(all) | Br(exp) |
|----------------|---------|------------------|--------------|---------|---------|
| $D^+ \to \pi^+\pi^0$ | 1.7     | 0.88±0.10        | 1.0±0.5      | 1.0±0.5 | 1.18±0.07 |
| $D^+ \to K^+K^0$   | 8.6     | 5.46±0.53        | 11.3±1.6     | 8.4±1.6 | 6.12±0.22 |
| $D^+ \to \pi^+\eta$ | 3.6     | 1.48±0.26        | 3.1±1.0      | 1.6±1.0 | 3.54±0.21 |
| $D^+ \to \pi^+\eta'$ | 7.9     | 3.70±0.37        | 3.7±0.7      | 5.5±0.8 | 4.68±0.29 |
| $D^0 \to \pi^+\pi^-$ | 1.59    | 2.24±0.10        | 3.0±0.4      | 2.2±0.5 | 1.45±0.05 |
| $D^0 \to \pi^0\pi^0$ | 1.16    | 1.35±0.05        | 0.7±0.2      | 0.8±0.2 | 0.81±0.05 |
| $D^0 \to K^+K^-$   | 4.56    | 1.92±0.08        | 4.4±0.5      | 3.0±0.8 | 4.07±0.10 |
| $D^0 \to K^0\bar{K}^0$ | 0.93    | 0.0±0.11         | 0.3±0.1      | 0.64±0.08 |
| $D^0 \to \pi^0\eta$ | 0.58    | 0.75±0.02        | 0.7±0.2      | 1.1±0.3 | 0.68±0.07 |
| $D^0 \to \pi^0\eta'$ | 1.7     | 0.74±0.02        | 0.6±0.1      | 0.6±0.2 | 0.91±0.13 |
| $D^0 \to \eta\eta'$ | 1.0     | 1.44±0.08        | 1.3±0.4      | 1.3±0.4 | 1.67±0.18 |
| $D^0 \to \eta\eta'$ | 2.2     | 1.19±0.04        | 2.0±0.3      | 1.1±0.1 | 1.67±0.18 |
| $D^0 \to \pi^0K^+$  | 1.6     | 0.86±0.09        | 0.9±0.2      | 0.5±0.2 | 0.62±0.23 |
| $D^0 \to \pi^+K^0$  | 4.3     | 2.73±0.26        | 4.1±0.5      | 2.8±0.6 | 2.52±0.27 |
| $D^0 \to K^+\eta$   | 2.7     | 0.78±0.09        | 0.8±0.5      | 0.8±0.5 | 1.76±0.36 |
| $D^0 \to K^+\eta'$  | 5.2     | 1.07±0.17        | 0.7±0.3      | 1.4±0.4 | 1.8±0.5  |

TABLE V: Same as Table III except for doubly Cabibbo-suppressed decays of $D \to PP$ ($\times 10^{-4}$).

| Modes          | Br(diagrammatic) | Br(emission) | Br(all) | Br(exp) |
|----------------|------------------|--------------|---------|---------|
| $D^+ \to \pi^+K^0$ | 1.98±0.22       | 2.8±0.5      | 1.7±0.5 |         |
| $D^+ \to \pi^0K^+$ | 1.59±0.15       | 3.0±0.4      | 2.2±0.4 | 1.72±0.19 |
| $D^+ \to K^+\eta$   | 0.98±0.04       | 1.3±0.2      | 1.2±0.2 | 1.08±0.17 |
| $D^+ \to K^+\eta'$  | 0.91±0.17       | 0.4±0.1      | 1.0±0.1 | 1.76±0.22 |
| $D^0 \to \pi^0K^0$  | 0.67±0.02       | 0.5±0.2      | 0.6±0.2 |         |
| $D^0 \to \pi^+K^+$  | 1.12±0.05       | 2.3±0.3      | 1.6±0.4 | 1.48±0.07 |
| $D^0 \to K^0\eta$    | 0.28±0.02       | 0.23±0.05    | 0.22±0.05 |
| $D^0 \to K^0\eta'$   | 0.55±0.03       | 0.08±0.06    | 0.5±0.1 |         |
| $D^0 \to K^+K^0$     | 0.38±0.04       | 0.7±0.4      | 0.7±0.4 |         |

*Data from $^{[33]}$  
†Data from $^{[34]}$

numerical results are not very sensitive to the variations of meson decay constants. The branching ratios are truly sensitive to the coefficients of $a_1$ and $a_2$, especially to their relative strong phases. Since the systematic errors from theoretical models are usually difficult to estimate, we show uncertainties at the tables only from the parameters $a_i$ in Eq. (35). For comparison we also show the recent results from the diagrammatic approach $^{[19]}$ and those considering final-state interaction effects of nearby resonances $^{[51]}$. It is clear that our results with large annihilation-type contributions agree with experiments much better than that of Ref. $^{[51]}$. For the Cabibbo-favored channels, which are the input data for $\chi^2$ fit in the diagrammatic approach, we have comparable results with the diagrammatic approach $^{[19]}$. For other channels, we have better agreement with experiments than the diagrammatic approach. The reason is mostly due to the SU(3) breaking effects, which had been fully neglected in the diagrammatic approach.

The branching ratio of the pure annihilation process $D^+ \to \pi^+\pi^0$ is vanished in our pole model. The resonant state that annihilates to $\pi^+\pi^0$ is a scalar meson ($0^{++}$), whose isospin could be 0, 1, or 2. However, isospin-0 would be ruled out because of charged final states, and isospin-2 is forbidden for the leading order $\Delta C = 1$ weak decay. For the case of isospin-1, its $G$ parity would be odd, which conflicts to a system of two pions whose $G$ parity is even. Therefore, no resonant states can be produced and then annihilate to $\pi^+\pi^0$. In another word, no annihilation diagrams contribute to $D^+_S \to \pi^+\pi^0$ and $D^+ \to \pi^+\pi^0$. In fact, this kind of contribution is forbidden from the isospin symmetry of $\pi^+$ and $\pi^0$ as identical particles. Simply, two pions can not form an s-wave isospin 1 state, because of the Bose-Einstein statics.

The pure annihilation process $D^0 \to K^0\bar{K}^0$, with nonzero experimental data, also demonstrates the important
interactions in \([54]\), the final-state phases of the amplitudes in \([16]\), and the two-gluon anomaly effects in \([59]\).

Besides large annihilation-type contributions, \(\eta^{(')}\) in the spacelike form factors in the factorization approach in \([52]\), some effects of the inelastic final-state interaction in \([58]\), the nonfactorizable chiral loop contributions in \([57]\), and the SU(3) breaking effect in \([19, 55]\), the nonfactorizable chiral loop contributions in \([56]\), the SU(3) breaking effect in the effective Wilson coefficients in \([15]\) for this channel.

Large branching ratios with \(\eta^{(')}\) in the final states are both measured and predicted, which are larger than those with \(\eta\) in most cases, such as \(D^0\) decays into \(K^0\eta, K^0\eta'\) and \(D_s^+\) into \(\pi^+, \pi^+\eta, \pi^+\eta'\), although the phase spaces with \(\eta'\) are smaller than those with \(\eta\). For \(\eta\), the contributions from the components of \(d\bar{d}\) and \(s\bar{s}\) are destructive due to the minus sign in the mixing matrix of Eq. (26) and the positive mixing angle\(^1\); while they are constructive for \(\eta'\). Besides, large \(W\)-exchange contributions dominate most \(\eta'\) modes, especially for \(D^0 \to \eta\eta'\) and \(D^0 \to K^0\eta'\), which demonstrates large annihilation-type contributions directly again. We also refer to this issue with \(K_0^0\) (1430) as an resonance in the spacelike form factors in the factorization approach in \([52]\), some effects of the inelastic final-state interactions in \([58]\), the final-state phases of the amplitudes in \([16]\), and the two-gluon anomaly effects in \([59]\).

C. \(D \to PV\) decays

The decay rate of \(D \to PV\) decays is

\[
\Gamma(D \to PV) = \frac{P}{8\pi m_D^2} \sum_{pol.} |A|^2, \tag{36}
\]

by summing over all the polarization states of the vector mesons.

We assume that the coefficients \(a_i\) are universal and process independent for \(D \to PV\), but they are different from those of \(D \to PP\) as discussed in \([14]\) and \([19]\). Their absolute values are larger than those of \(D \to PP\) because the soft final-state interactions make more effects on \(D \to PV\) decays. In our calculations, they are used as

\[
\begin{align*}
& a_1^{PV} = 1.32 \pm 0.10, \\
& a_2^{PV} = (0.75 \pm 0.10)e^{i(160\pm10)^0}, \\
& a_3^{PV} = (0.12 \pm 0.10)e^{i(345\pm10)^0}, \\
& a_4^{PV} = (0.62 \pm 0.10)e^{i(238\pm10)^0}.
\end{align*}
\tag{37}
\]

Similarly to the \(PP\) modes, large relative strong phases due to inelastic final-state interactions are considered in the \(a_i\). Again, the contributions from \(W\)-annihilation diagrams are smaller than the \(W\)-exchange ones. Besides, the relative strong phase between \(a_1^{PV}\) and \(a_2^{PV}\) is in accordance with the results from the diagrammatic approach \([14, 19]\).

Our prediction of branching ratios of the Cabibbo-favored, the singly Cabibbo-suppressed and the doubly Cabibbo-suppressed \(D \to PV\) decays are shown in Tables \([\text{VII}, \text{VIII}]\) and \([\text{VII}]\) respectively. The results in the third column (emission) in each of these tables are the predictions of rates with only the emission-type processes; while the results in the fourth column (all) also include the annihilation-type contributions. It is obvious that the annihilation-type contributions are of the same order as the emission-type diagrams, since the intermediate states here in the pole model are pseudoscalar mesons with relatively larger decay constants than those scalar mesons of the \(D \to PP\) case. Again, for theoretical uncertainty estimation, we use only those from the parameters \(a_i\) shown in Eq. (37), as illustration. For comparison, we also list the results of the diagrammatic approach \([10]\) and the experimental date in these tables. It is easy to see that our results with the annihilation-type contributions agree with the experimental data. This means that the single-pole contribution dominates the annihilation-type contribution in most \(D \to PV\) decay channels. For example, although the \(D^0 \to K^0\phi\) channel has no emission-type contribution, with vanishing branching ratio in the factorization approach, our pole model gives the right branching ratios agreeing with the experiment. This also confirms the calculation done in the perturbative QCD approach \([20]\). Besides, some of the SU(3) flavor symmetry breaking effects are considered in this work since the decay constants, transition form factors and effective strong coupling constants are involved.

There is no resonant state contributing to the \(W\)-exchange diagram of \(D^0 \to \pi^0\rho^0\) in the pole model, because a \(\pi^0\) would violate the \(C\) parity, similarly to the case of \(D^0 \to \eta(\eta')\omega, \eta\phi\). Besides, the single-pole annihilation diagrams

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\(^1\) The theoretical and phenomenological estimates for mixing angle \(\phi\) is \(42.2^0\) and (39.3 \pm 1.0)^0\), respectively \([45]\).
TABLE VI: Branching ratios for Cabibbo-favored decays of $D \to PV(\%)$. The predicted rates with only emission-type contributions (emission) and with both annihilation- and emission-type contributions (all) are shown in the table, compared with the experimental data [26], the fitted results from the diagrammatic approach [19] in which only the (A, A1) solution is quoted, and the results considering final-state interaction (FSI) effects of nearby resonances [51].

| Modes | Br(FSI) | Br(diagrammatic) | Br(emission) | Br(all) | Br(exp) |
|-------|---------|------------------|--------------|---------|---------|
| $D^0 \to K^-\rho^+$ | 11.19 | 10.8±2.2 | 12.2±1.8 | 8.8±2.2 | 10.8±0.7 |
| $D^0 \to \bar{K}^0\rho^0$ | 0.88 | 1.54±1.15 | 0.7±0.5 | 1.7±0.7 | 1.32±0.12 |
| $D^0 \to \pi^0\bar{K}^0$ | 3.49 | 2.82±0.34 | 2.3±0.7 | 2.9±1.0 | 2.82±0.35 |
| $D^0 \to \pi^+K^{*-}$ | 4.69 | 5.91±0.70 | 3.8±0.7 | 3.1±1.0 | 5.68±0.68 |
| $D^0 \to \eta\bar{K}^0$ | 0.51 | 0.96±0.32 | 0.7±0.2 | 0.7±0.2 | 0.96±0.30 |
| $D^0 \to \eta'\bar{K}^0$ | 0.005 | 0.012±0.003 | 0.003±0.001 | 0.016±0.005 | <0.11 |
| $D^0 \to \bar{K}^0\omega$ | 2.16 | 2.26±1.38 | 0.6±0.5 | 2.5±0.7 | 2.22±0.12 |
| $D^0 \to \bar{K}^0\phi$ | 0.90 | 0.868±0.139 | 0 | 0.8±0.2 | 0.868±0.060 |
| $D^+ \to \pi^+\bar{K}^0$ | 0.64 | 1.83±0.49 | 1.4±1.3 | 1.4±1.3 | 1.56±0.18 |
| $D^+ \to \bar{K}^0\rho^+$ | 11.77 | 9.2±6.7 | 15.1±3.8 | 15.1±3.8 | 9.4±2.0 |
| $D^+ \to K^+\bar{K}^0$ | 3.96 | 5.6±1.9 | 4.2±1.7 | 3.90±0.23 |
| $D^+ \to \bar{K}^0K^{*+}$ | 3.37 | 1.7±0.7 | 1.0±0.6 | 5.4±1.2 |
| $D^+ \to \eta\rho^+$ | 9.49 | 8.3±1.3 | 8.3±1.3 | 8.9±0.861 |
| $D^+ \to \eta'\rho^+$ | 2.61 | 3.0±0.5 | 3.0±0.5 | 12.2±2.0 |
| $D^+ \to \pi^+\phi$ | 2.89 | 4.38±0.35 | 4.3±0.6 | 4.3±0.6 | 4.5±0.4 |
| $D^+ \to \pi^0\rho^0$ | 0.080 | 0 | 0.4±0.4 | 0.02±0.012 |
| $D^+ \to \pi^0\rho^+$ | 0.080 | 0 | 0.4±0.4 | |
| $D^+ \to \pi^+\omega$ | 0.0 | 0 | 0 | 0.23±0.06 |

can not contribute to the $D \to \rho\eta, \pi\omega$ decays because of $G$ parity violation. The isospin of resonant state for $\rho\eta$ or $\pi\omega$ is one, so the $G$ parity of the intermediate state is odd since it is a pseudoscalar meson. However, the $G$ parity of $\rho$ and $\eta$ are both even, and that of $\pi$ and $\omega$ are both odd, so the total $G$ parity of the final states is even. Therefore, no resonant states are available for the decays of $D$ mesons into $\rho\eta$ and $\pi\omega$. It is even worse for the pure annihilation process $D^+ \to \pi^+\omega$, since its decay rate is predicted to be zero in the single-pole model, but it is not small in the experiment. This has already been discussed that this channel may be dominated by the final-state rescattering via quark exchange in [19, 53], and by hidden strangeness final-state interactions in [61]. Besides, the pure annihilation mode $D^+ \to \pi^+\rho^0$ is predicted much larger in the pole model than the experiment data. The contributions from the two diagrams in this channel are constructive since the minus sign in the normalization of $\rho^0$ is compensated by the asymmetric space wave function of the two final states which are in the $P$-wave state. These two channels make such trouble that we fail to find a reasonable solution of $AP$ and $AV$ and predict the $PV$ modes with the W-annihilation contributions in the diagrammatic approach [19]. Hence, further discussions are still needed for these two channels.

$D^+_S \to \rho\eta'$ is predicted much smaller than the mode of $D^+_S \to \rho\eta$, but the experimental branching ratios of the former is larger. This is a puzzle that the phase space of the former mode is much smaller than the latter, so its branching ratio should be smaller. In fact, the experimental measurement of $D^+_S \to \rho\eta'$ [62] is already too old. It is already questioned by the PDG [26], since this branching fraction $(12.5 \pm 2.2)\%$ considerably exceeds the recent inclusive $\eta'$ fraction of $(11.7 \pm 1.8)\%$.

IV. SUMMARY

We have calculated the branching ratios for the two-body hadronic decays of charmed mesons into $PP$ and $PV$ using the generalized factorization approach for the emission-type diagrams and the pole-dominance model for the annihilation-type diagrams. Relative strong phases between different topological diagrams, which are important in the charmed decays, are considered in this work. Most of our predicted branching fractions are in accordance with the experimental data. Besides, compared to the naive and generalized factorization models ever before, the results in this work are much better since we have considered the annihilation-type diagrams and the relative strong phases between diagrams.

We find that the annihilation-type contributions in the pole model are large for both $PP$ and $PV$ modes, which is also indicated by the difference between the life time of $D^+$ and $D^0$. Comparing with the model-independent
diagrammatic approach, we reproduce their results with our specific model considering some SU(3) breaking effects. Furthermore, we get more predictions in many $D \to PV$ decay channels, which are absent in the diagrammatic approach. Most of the results have a better agreement with experimental data than previous calculations.

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### Table VII: Same as Table VI except for singly Cabibbo-suppressed decays of $D \to PV (\times 10^{-3})$

| Modes       | Br(FSI) | Br(diagrammatic) | Br(emission) | Br(all) | Br(exp) |
|-------------|---------|------------------|--------------|---------|---------|
| $D^0 \to \pi^- \rho^+$ | 1.7     | 5.3 ± 1.7        | 3.5 ± 1.6    | 8.3 ± 0.15 |
| $D^0 \to \pi^+ \rho^-$ | 6.5     | 3.9 ± 0.46       | 1.3 ± 0.5    | 3.5 ± 0.6 | 4.97 ± 0.23 |
| $D^0 \to \pi^0 \rho^0$ | 1.7     | 2.96 ± 0.98      | 1.4 ± 0.6    | 1.4 ± 0.6 | 3.73 ± 0.22 |
| $D^0 \to K^- K^{*+}$ | 4.5     | 4.25 ± 0.86      | 5.5 ± 0.8    | 4.7 ± 0.8 | 4.38 ± 0.21 |
| $D^0 \to K^+ K^{*-}$ | 2.8     | 1.99 ± 0.24      | 2.0 ± 0.3    | 1.6 ± 0.3 | 1.56 ± 0.12 |
| $D^0 \to \bar{K}^0 K^{*0}$ | 0.99  | 0.29 ± 0.22      | 0             | 0.16 ± 0.05 | <0.9 |
| $D^0 \to K^0 \bar{K}^{*0}$ | 0.99  | 0.29 ± 0.22      | 0             | 0.16 ± 0.05 | <1.8 |
| $D^+ \to \pi^+ \omega$ | 0.08    | 0.10 ± 0.18      | 0.08 ± 0.02  | 0.08 ± 0.02 | <0.26 |
| $D^+ \to \pi^0 \phi$ | 1.1     | 1.22 ± 0.08      | 1.0 ± 0.3    | 1.0 ± 0.3 | 0.76 ± 0.05 |
| $D^+ \to \eta \phi$ | 0.57    | 0.31 ± 0.10      | 0.23 ± 0.06  | 0.23 ± 0.06 | 0.14 ± 0.05 |
| $D^+ \to \eta \omega$ | 0.24    | 1.11 ± 0.86      | 0.05 ± 0.01  | 0.05 ± 0.01 |
| $D^+ \to \eta' \phi$ | 0.10    | 0.14 ± 0.02      | 0.08 ± 0.02  | 0.08 ± 0.02 |
| $D^+ \to \eta' \omega$ | 1.9     | 3.08 ± 1.42      | 1.2 ± 0.3    | 1.2 ± 0.3 | 2.21 ± 0.23 |
| $D^+ \to \eta' \phi$ | 0.001   | 0.07 ± 0.02      | 0.0001 ± 0.0001 | 0.0001 ± 0.0001 |
| $D^{+} \to \pi^{+} \rho^{0}$ | 5.9    | 6.21 ± 0.43      | 5.1 ± 1.4    | 5.1 ± 1.4 | 5.44 ± 0.26 |
| $D^{+} \to \pi^{+} \omega$ | 0.35    | 0.3 ± 0.3        | 0.3 ± 0.3    | <0.34 |
| $D^{+} \to \pi^{0} \rho^{0}$ | 2.4    | 1.6 ± 0.6        | 1.0 ± 0.6    | 2.7 ± 0.5 |
| $D^{+} \to \eta K^{*+}$ | 0.24    | 1.0 ± 0.4        | 1.0 ± 0.4    |
| $D^{+} \to \eta' K^{*+}$ | 0.24    | 0.4 ± 0.2        | 0.6 ± 0.2    |
| $D^{+} \to K^+ \omega$ | 0.72    | 1.1 ± 0.7        | 1.8 ± 0.7    | <2.4 |
| $D^{+} \to K^+ \phi$ | 0.15    | 0.3 ± 0.3        | 0.3 ± 0.3    | <0.6 |
TABLE VIII: Same as Table VI except for doubly Cabibbo-suppressed decays of \( D \rightarrow PV(\times 10^{-4}) \)

| Modes                      | \( \text{Br( diagrammatic)} \) | \( \text{Br( emission)} \) | \( \text{Br( all)} \) | \( \text{Br( exp)} \) |
|----------------------------|---------------------------------|-----------------------------|------------------------|------------------------|
| \( D^0 \rightarrow \pi^+ K^{*0} \) | 3.59±0.72                       | 3.7 ± 0.6                   | 2.7 ± 0.6               | 3.54±1.80             |
| \( D^0 \rightarrow \pi^0 K^{*0} \) | 0.54±0.18                       | 0.6 ± 0.2                   | 0.8 ± 0.3               |                        |
| \( D^0 \rightarrow K^+\rho^- \)  | 1.45±0.17                       | 1.1 ± 0.2                   | 0.9 ± 0.3               |                        |
| \( D^0 \rightarrow K^0\rho^0 \)  | 0.91±0.51                       | 0.2 ± 0.1                   | 0.5 ± 0.2               |                        |
| \( D^0 \rightarrow K^0\omega \)   | 0.58±0.40                       | 0.2 ± 0.1                   | 0.7 ± 0.2               |                        |
| \( D^0 \rightarrow K^0\phi \)    | 0.06±0.05                       | 0                          | 0.2 ± 0.06              |                        |
| \( D^0 \rightarrow \eta K^{*0} \) | 0.33±0.08                       | 0.18 ± 0.05                 | 0.17 ± 0.05             |                        |
| \( D^0 \rightarrow \eta' K^{*0} \) | 0.0040±0.0006                   | 0.001 ± 0.001               | 0.004 ± 0.001           |                        |
| \( D^+ \rightarrow \pi^+ K^{*0} \) | 3.0 ± 1.0                       | 2.2 ± 0.9                   | 3.75±0.75               |                        |
| \( D^+ \rightarrow \pi^0 K^{*0} \) | 4.7 ± 0.9                       | 4.0 ± 0.9                   |                        |                        |
| \( D^+ \rightarrow K^+\rho^0 \)  | 1.4 ± 0.4                       | 1.0 ± 0.4                   | 2.1±0.5                 |                        |
| \( D^+ \rightarrow K^0\rho^0 \)  | 0.9 ± 0.4                       | 0.5 ± 0.4                   |                        |                        |
| \( D^+ \rightarrow K^+\omega \)   | 1.4 ± 0.5                       | 1.8 ± 0.5                   |                        |                        |
| \( D^+ \rightarrow K^+\phi \)    | 0                              | 0.2 ± 0.2                   |                        |                        |
| \( D^+ \rightarrow \eta K^{*0} \) | 1.5 ± 0.2                       | 1.4 ± 0.2                   |                        |                        |
| \( D^+ \rightarrow \eta' K^{*0} \) | 0.013±0.006                     | 0.020 ± 0.07                |                        |                        |
| \( D_S^+ \rightarrow K^+ K^{*0} \) | 0.20±0.05                      | 0.2 ± 0.2                   | 0.2 ± 0.2               |                        |
| \( D_S^+ \rightarrow K^0 K^{*0} \) | 1.17±0.86                      | 2.3 ± 0.6                   | 2.3 ± 0.6               |                        |

Appendix A: Individual formulas for various decay channels of \( D \) mesons

The different contribution formulas for Cabibbo-favored decays of \( D \rightarrow PP \) are listed as

\[
\mathcal{A}(D^+ \rightarrow \pi^+ K^{0}) = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \left( a_2 f_K (m_D^2 - m^2) F_0^{D\pi} (m_K^2) + a_1 f_K (m_D^2 - m_K^2) F_0^{DK} (m^2) \right),
\]

\[
\mathcal{A}(D^0 \rightarrow \pi^+ K^-) = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \left( a_2 f_K (m_D^2 - m_K^2) F_0^{D\pi} (m_K^2) - a_1 f_K (m^2) F_0^{DK} (m_D^2) \right),
\]

\[
\mathcal{A}(D^0 \rightarrow \pi^0 K^0) = \frac{i G_F}{2} V_{cs}^* V_{ud} \left( a_2 f_K (m_D^2 - m_D^2) F_0^{D\pi} (m_K^2) - a_2 f_K (m_D^2 - m_K^2) F_0^{DK} (m^2) \right),
\]

\[
\mathcal{A}(D^0 \rightarrow K^0 \eta) = \frac{i G_F}{2} V_{cs}^* V_{ud} \left( a_2 f_K (m_D^2 - m_K^2) F_0^{D\eta} (m^2) \cos \phi - a_1 f_K (m^2) F_0^{DK} (m_D^2 - m_K^2) \cos \phi - \sqrt{2} \sin \phi \right),
\]

\[
\mathcal{A}(D^0 \rightarrow K^0 \eta') = \frac{i G_F}{2} V_{cs}^* V_{ud} \left( a_2 f_K (m_D^2 - m_K^2) F_0^{D\eta'} (m^2) \sin \phi - a_1 f_K (m^2) F_0^{DK} (m_D^2 - m_K^2) \sin \phi + \sqrt{2} \cos \phi \right),
\]

\[
\mathcal{A}(D_S^+ \rightarrow K^+ K^0) = \frac{i G_F}{2} V_{cs}^* V_{ud} \left( a_2 f_K (m_D^2 - m_K^2) F_0^{D\eta} (m^2) \sin \phi - a_1 f_K (m^2) F_0^{DK} (m_D^2 - m_K^2) \cos \phi \right),
\]

\[
\mathcal{A}(D_S^+ \rightarrow \pi^+ \eta) = -i G_F V_{cs}^* V_{ud} \left( \frac{1}{\sqrt{2}} a_2 f_K (m_D^2 - m_K^2) F_0^{D\eta} (m^2) \sin \phi + a_1 f_K (m^2) F_0^{DK} (m_D^2 - m_K^2) \cos \phi \right),
\]

\[
\mathcal{A}(D_S^+ \rightarrow \pi^+ \eta') = i G_F V_{cs}^* V_{ud} \left( \frac{1}{\sqrt{2}} a_2 f_K (m_D^2 - m_K^2) F_0^{D\eta'} (m^2) \cos \phi - a_1 f_K (m^2) F_0^{DK} (m_D^2 - m_K^2) \sin \phi \right),
\]

where \( f_S \) and \( g_1 = 2.7 \) are, respectively, denoted as the decay constant of scalar mesons and effective strong coupling constant between the intermediate state and final states in the limit of SU(3) symmetry. Some phases from the propagators of the intermediate resonances are absorbed in the effective Wilson coefficients \( a_A \) and \( a_E \).
The formulas for singly Cabibbo-suppressed decays of $D \to PP$ are shown as

$$A(D^+ \to \pi^+\pi^0) = -i \frac{G_F}{2} V_{td}^* V_{uf} f_\pi (m_D^2 - m_\pi^2) F_0^{D\pi}(m_\pi^2)(a_1 + a_2),$$

$$A(D^+ \to K^+\eta^0) = i \frac{G_F}{\sqrt{2}} \left( a_1 V_{ts}^* V_{ud} f_K (m_D^2 - m_K^2) F_0^{DK}(m_K^2) - a_1 V_{td} V_{uf} g_1 f_D \frac{m_D^2 m_{ta}}{m_D^2 - m_{ta}^2} \right),$$

$$A(D^+ \to \pi^+\eta) = i \frac{G_F}{2} \left( \cos\phi V_{td}^* V_{ud} a_1 f_\pi (m_D^2 - m_\pi^2) F_0^{D\pi}(m_\pi^2) - 2 a_1 g_1 f_D \frac{m_D^2 m_{ta}}{m_D^2 - m_{ta}^2} \right)$$
$$+ \sqrt{2} a_2 [V_{td} V_{uf} f_D^d + V_{ta}^* V_{ud} f_\eta^0] (m_D^2 - m_\pi^2) F_0^{D\pi}(m_\pi^2),$n

$$A(D^+ \to \pi^+\eta') = i \frac{G_F}{2} \left( \sin\phi V_{td}^* V_{ud} a_1 f_\pi (m_D^2 - m_\pi^2) F_0^{D\pi}(m_\pi^2) - 2 a_1 g_1 f_D \frac{m_D^2 m_{ta}}{m_D^2 - m_{ta}^2} \right)$$
$$+ \sqrt{2} a_2 [V_{td} V_{uf} f_D^d + V_{ta}^* V_{ud} f_\eta^0] (m_D^2 - m_\pi^2) F_0^{D\pi}(m_\pi^2),$$

$$A(D^0 \to \pi^+\pi^-) = i \frac{G_F}{\sqrt{2}} V_{ts}^* V_{ud} \left( a_1 f_\pi (m_D^2 - m_\pi^2) F_0^{D\pi}(m_\pi^2) - a_1 E g_1 f_D \frac{m_D^2 m_{ta(1370)}}{m_D^2 - m_{ta(1370)}^2} \right),$$

$$A(D^0 \to \pi^0\pi^0) = -i \frac{G_F}{2} V_{ts}^* V_{ud} \left( a_2 f_\pi (m_D^2 - m_\pi^2) F_0^{D\pi}(m_\pi^2) + a_1 E g_1 f_D \frac{m_D^2 m_{ta(1370)}}{m_D^2 - m_{ta(1370)}^2} \right),$$

$$A(D^0 \to \pi^+\pi^-) = i \frac{G_F}{\sqrt{2}} V_{ts}^* V_{ud} \left( a_1 f_K (m_D^2 - m_K^2) F_0^{DK}(m_K^2) - a_1 E g_1 f_D \frac{m_D^2 m_{ta(1370)}}{m_D^2 - m_{ta(1370)}^2} \right),$$

$$A(D^0 \to \eta\pi^0) = i \frac{G_F}{2} \left( a_1 f_\pi (m_D^2 - m_\pi^2) F_0^{D\pi}(m_\pi^2) + a_1 E g_1 f_D \frac{m_D^2 m_{ta(1370)}}{m_D^2 - m_{ta(1370)}^2} \right) \cos \phi$$
$$+ \sqrt{2} a_2 E g_1 f_D \frac{m_D^2 m_{ta}}{m_D^2 - m_{ta}^2} \cos \phi,$n

$$A(D^0 \to \eta\eta') = i \frac{G_F}{2} \left( a_1 f_\pi (m_D^2 - m_\pi^2) F_0^{D\pi}(m_\pi^2) + a_1 E g_1 f_D \frac{m_D^2 m_{ta(1370)}}{m_D^2 - m_{ta(1370)}^2} \right) \cos \phi$$
$$+ \sqrt{2} a_2 E g_1 f_D \frac{m_D^2 m_{ta}}{m_D^2 - m_{ta}^2} \cos \phi,$n

$$A(D^0 \to \eta\eta') = i \frac{G_F}{\sqrt{2}} \left( a_2 f_\pi (m_D^2 - m_\pi^2) F_0^{D\pi}(m_\pi^2) + a_1 E g_1 f_D \frac{m_D^2 m_{ta(1370)}}{m_D^2 - m_{ta(1370)}^2} \right) \cos \phi$$
$$+ \sqrt{2} a_2 E g_1 f_D \frac{m_D^2 m_{ta}}{m_D^2 - m_{ta}^2} \cos \phi,$n

$$A(D^+ \to K^+\pi^0) = -i \frac{G_F}{2} \left( V_{ts}^* V_{ud} a_1 f_D (m_D^2 - m_\pi^2) F_0^{DK}(m_\pi^2) + V_{td} V_{uf} a_2 f_\pi (m_D^2 - m_K^2) F_0^{D\pi}(m_K^2) \right),$$

$$A(D^+ \to \pi^+\pi^0) = -i \frac{G_F}{\sqrt{2}} \left( V_{ts}^* V_{ud} a_1 f_D (m_D^2 - m_\pi^2) F_0^{DK}(m_\pi^2) - V_{td} V_{uf} a_2 f_\pi (m_D^2 - m_K^2) F_0^{D\pi}(m_K^2) \right),$$

$$A(D^+ \to K^+\pi^0) = -i \frac{G_F}{2} \left( V_{ts}^* V_{ud} a_1 f_D (m_D^2 - m_\pi^2) F_0^{DK}(m_\pi^2) + V_{td} V_{uf} a_2 f_\pi (m_D^2 - m_K^2) F_0^{D\pi}(m_K^2) \right) \cos \phi$$
$$- V_{ts}^* V_{ud} a_1 f_D (m_D^2 - m_\pi^2) F_0^{DK}(m_\pi^2) \sin \phi,$n

$$A(D^+ \to \pi^+\eta') = -i \frac{G_F}{\sqrt{2}} \left( V_{ts}^* V_{ud} a_1 f_D (m_D^2 - m_\pi^2) F_0^{DK}(m_\pi^2) + V_{td} V_{uf} a_2 f_\pi (m_D^2 - m_K^2) F_0^{D\pi}(m_K^2) \right) \cos \phi$$
$$- V_{ts}^* V_{ud} a_1 f_D (m_D^2 - m_\pi^2) F_0^{DK}(m_\pi^2) \sin \phi,$n

$$A(D^+ \to K^+\eta') = -i \frac{G_F}{\sqrt{2}} \left( V_{ts}^* V_{ud} a_1 f_D (m_D^2 - m_\pi^2) F_0^{DK}(m_\pi^2) + V_{td} V_{uf} a_2 f_\pi (m_D^2 - m_K^2) F_0^{D\pi}(m_K^2) \right) \cos \phi$$
$$- V_{ts}^* V_{ud} a_1 f_D (m_D^2 - m_\pi^2) F_0^{DK}(m_\pi^2) \sin \phi,$n

$$A(D^+ \to \pi^+\eta) = -i \frac{G_F}{2} \left( V_{ts}^* V_{ud} a_1 f_D (m_D^2 - m_\pi^2) F_0^{DK}(m_\pi^2) - V_{td} V_{uf} a_2 f_\pi (m_D^2 - m_K^2) F_0^{D\pi}(m_K^2) \right) \cos \phi$$
$$- V_{ts}^* V_{ud} a_1 f_D (m_D^2 - m_\pi^2) F_0^{DK}(m_\pi^2) \sin \phi,$n

$$A(D^+ \to \eta\eta') = -i \frac{G_F}{\sqrt{2}} \left( V_{ts}^* V_{ud} a_1 f_D (m_D^2 - m_\pi^2) F_0^{DK}(m_\pi^2) - V_{td} V_{uf} a_2 f_\pi (m_D^2 - m_K^2) F_0^{D\pi}(m_K^2) \right) \cos \phi$$
$$- V_{ts}^* V_{ud} a_1 f_D (m_D^2 - m_\pi^2) F_0^{DK}(m_\pi^2) \sin \phi,$n

$$A(D^+ \to \eta\eta') = -i \frac{G_F}{\sqrt{2}} \left( V_{ts}^* V_{ud} a_1 f_D (m_D^2 - m_\pi^2) F_0^{DK}(m_\pi^2) - V_{td} V_{uf} a_2 f_\pi (m_D^2 - m_K^2) F_0^{D\pi}(m_K^2) \right) \cos \phi$$
$$- V_{ts}^* V_{ud} a_1 f_D (m_D^2 - m_\pi^2) F_0^{DK}(m_\pi^2) \sin \phi.$$
The formulas for doubly Cabibbo-suppressed decays of $D \to PP$ are listed as

$$A(D^+ \to K^+\pi^0) = -i G_F V_{cd}^* V_{us} \left( a_{A g 1} f_s f_D \frac{m_D^2 m_{K^0_s}}{m_D^2 - m_{K^0_s}^2} + a_1 f_K (m_D^2 - m_{\pi^0}^2) F_0^{D\pi}(m_K^2) \right),$$
$$A(D^+ \to \pi^+ K^0) = -i G_F \sqrt{2} V_{cd}^* V_{us} \left( a_{A g 1} f_s f_D \frac{m_D^2 m_{K^0_s}}{m_D^2 - m_{K^0_s}^2} - a_2 f_K (m_D^2 - m_{\pi^0}^2) F_0^{D\pi}(m_K^2) \right),$$
$$A(D^+ \to K^+\eta) = G_F V_{cd}^* \left( a_{A g 1} f_s f_D \frac{m_D^2 m_{K^0_s}}{m_D^2 - m_{K^0_s}^2} (\sqrt{2} \sin \phi - \cos \phi) + a_1 f_K (m_D^2 - m_{\eta}^2) F_0^{D\eta}(m_K^2) \cos \phi \right),$$
$$A(D^+ \to K^+\eta') = G_F V_{cd}^* \left( - a_{A g 1} f_s f_D \frac{m_D^2 m_{K^0_s}}{m_D^2 - m_{K^0_s}^2} (\sin \phi + \sqrt{2} \cos \phi) + a_1 f_K (m_D^2 - m_{\eta'}^2) F_0^{D\eta'}(m_K^2) \sin \phi \right),$$
$$A(D^0 \to K^{+}\pi^{-}) = -i G_F \sqrt{2} V_{cd}^* V_{us} \left( a_{A g 1} f_s f_D \frac{m_D^2 m_{K^0_s}}{m_D^2 - m_{K^0_s}^2} - a_2 f_K (m_D^2 - m_{\pi^-}^2) F_0^{D\pi}(m_K^2) \right),$$
$$A(D^0 \to K^{0}\pi^{-}) = i G_F \sqrt{2} V_{cd}^* V_{us} \left( a_{A g 1} f_s f_D \frac{m_D^2 m_{K^0_s}}{m_D^2 - m_{K^0_s}^2} + a_2 f_K (m_D^2 - m_{\pi^-}^2) F_0^{D\pi}(m_K^2) \right),$$
$$A(D^0 \to K^0\eta) = G_F V_{cd}^* \left( a_{A g 1} f_s f_D \frac{m_D^2 m_{K^0_s}}{m_D^2 - m_{K^0_s}^2} (\sqrt{2} \sin \phi - \cos \phi) + a_2 f_K (m_D^2 - m_{\eta}^2) F_0^{D\eta}(m_K^2) \cos \phi \right),$$
$$A(D^0 \to K^0\eta') = G_F V_{cd}^* \left( - a_{A g 1} f_s f_D \frac{m_D^2 m_{K^0_s}}{m_D^2 - m_{K^0_s}^2} (\sqrt{2} \cos \phi + \sin \phi) + a_2 f_K (m_D^2 - m_{\eta'}^2) F_0^{D\eta'}(m_K^2) \sin \phi \right),$$
$$A(D^+_s \to K^0 K^{+}) = i G_F \sqrt{2} V_{cd}^* V_{us} f_K (m_{D_s}^2 - m_K^2) F_0^{D_s K} (m_K^2) (a_1 + a_2).$$

(A3)

The formulas for Cabibbo-favored decays of $D \to PV$ are shown as

$$A(D^0 \to \pi^+ K^{*-}) = \sqrt{2} G_F V_{cs}^* V_{ud} \left( a_{P V}^{P V} g_s f_k f_D \frac{m_D^2}{m_D^2 - m_K^2} + a_1^{P V} m_K \cdot f_\pi A_0^{D\pi}(m_K^2) \right) (\varepsilon \cdot p_D),$$
$$A(D^0 \to K^-\rho^+) = \sqrt{2} G_F V_{cs}^* V_{ud} \left( a_{P V}^{P V} g_s f_k f_D \frac{m_D^2}{m_D^2 - m_K^2} + a_1^{P V} m_\rho f_\rho F_1^{D\pi}(m_\rho^2) \right) (\varepsilon \cdot p_D),$$
$$A(D^0 \to \pi^0 \bar{K}^{*0}) = G_F V_{cs}^* V_{ud} \left( - a_{P V}^{P V} g_s f_k f_D \frac{m_D^2}{m_D^2 - m_K^2} + a_2^{P V} m_K \cdot f_\pi F_1^{D\pi}(m_K^2) \right) (\varepsilon \cdot p_D),$$
$$A(D^0 \to K^0\rho^0) = G_F V_{cs}^* V_{ud} \left( - a_{P V}^{P V} g_s f_k f_D \frac{m_D^2}{m_D^2 - m_K^2} + a_2^{P V} m_\rho f_\rho A_0^{D\rho}(m_\rho^2) \right) (\varepsilon \cdot p_D),$$
$$A(D^0 \to \eta\bar{K}^{*0}) = G_F V_{cs}^* V_{ud} \left( a_{P V}^{P V} f_k f_D \frac{m_D^2}{m_D^2 - m_K^2} g_s \cos \phi - \sqrt{2} g_{s*} \sin \phi \right) + a_2^{P V} m_K \cdot f_ \pi A_0^{D\pi}(m_K^2) (\varepsilon \cdot p_D),$$
$$A(D^0 \to \eta'\bar{K}^{*0}) = G_F V_{cs}^* V_{ud} \left( a_{P V}^{P V} f_k f_D \frac{m_D^2}{m_D^2 - m_K^2} g_s \sin \phi + \sqrt{2} g_{s*} \cos \phi \right) + a_2^{P V} m_K \cdot f_ \pi F_1^{D\pi}(m_K^2) (\varepsilon \cdot p_D),$$
$$A(D^0 \to \bar{K}^0\omega) = G_F V_{cs}^* V_{ud} \left( a_{P V}^{P V} f_k f_D \frac{m_D^2}{m_D^2 - m_K^2} + a_2^{P V} m_\omega f_\omega A_0^{D\omega}(m_K^2) \right) (\varepsilon \cdot p_D),$$
$$A(D^0 \to \bar{K}^0\phi) = \sqrt{2} G_F V_{cs}^* V_{ud} a_{P V}^{P V} g_{s*} f_k f_D \frac{m_D^2}{m_D^2 - m_K^2} (\varepsilon \cdot p_D),$$
$$A(D^+ \to \pi^+ \bar{K}^{*0}) = \sqrt{2} G_F V_{cs}^* V_{ud} m_K \cdot \left( a_1^{P V} f_\pi A_0^{D\pi}(m_K^2) + a_2^{P V} f_\pi F_1^{D\pi}(m_K^2) \right) (\varepsilon \cdot p_D),$$
$$A(D^+ \to K^0\rho^0) = \sqrt{2} G_F V_{cs}^* V_{ud} m_\rho \left( a_1^{P V} f_\rho A_0^{D\rho}(m_\rho^2) + a_2^{P V} f_\rho F_1^{D\rho}(m_\rho^2) \right) (\varepsilon \cdot p_D),$$
$$A(D^+_s \to K^0 K^{*0}) = \sqrt{2} G_F V_{cs}^* V_{ud} \left( a_{P V}^{P V} g_s f_k f_{D_s} \frac{m_{D_s}^2}{m_{D_s}^2 - m_{K^0}^2} + a_2^{P V} m_K \cdot f_\pi A_0^{D\pi}(m_K^2) \right) (\varepsilon \cdot p_{D_s}),$$
$$A(D^+_s \to \bar{K}^0 K^{*+}) = \sqrt{2} G_F V_{cs}^* V_{ud} \left( a_{P V}^{P V} g_s f_k f_{D_s} \frac{m_{D_s}^2}{m_{D_s}^2 - m_{K^0}^2} + a_2^{P V} m_K \cdot f_\pi A_0^{D\pi}(m_K^2) \right) (\varepsilon \cdot p_{D_s}).$$
where the effective strong coupling constants between the intermediate state and the final states for $D \to PV$ are $g_q = 4.2$, $g_s = 4.6$, and $g_{ss} = 4.5$. The formulas for singly Cabibbo-suppressed decays of $D \to PV$ are shown as

\begin{align}
A(D_S^+ \to \eta^+ \rho^-) &= -G_F V_{cd}^* V_{ud} a_{PV}^{a_0} \left( \frac{m_D^2}{m_D^2 - m_\pi^2} - \frac{2 m_D^2}{m_D^2 - m_\pi^2} \right) \sin \phi \langle \sigma \cdot p_{D_S} \rangle, \\
A(D_S^+ \to \eta^+ \rho^+) &= -G_F V_{cd}^* V_{ud} a_{PV}^{a_0} m_D f_{D_S^+} \sin \phi \langle \sigma \cdot p_{D_S} \rangle, \\
A(D_S^+ \to \eta^0 \phi) &= -G_F V_{cd}^* V_{ud} a_{PV}^{a_0} m_D F_{D_S^+} \sin \phi \langle \sigma \cdot p_{D_S} \rangle, \\
A(D_S^+ \to \eta^0 \rho^0) &= 2 G_F V_{cd}^* V_{ud} a_{PV}^{a_0} \frac{g_q f_\pi f_{D_S^+}}{m_D^2 - m_\pi^2} \sin \phi \langle \sigma \cdot p_{D_S} \rangle, \\
A(D_S^+ \to \eta^0 \omega) &= 2 G_F V_{cd}^* V_{ud} a_{PV}^{a_0} \frac{g_q f_\pi f_{D_S^+}}{m_D^2 - m_\pi^2} \sin \phi \langle \sigma \cdot p_{D_S} \rangle, \\
A(D_S^+ \to \eta^0 \omega) &= 0, \quad (A4)
\end{align}

where the effective strong coupling constants between the intermediate state and the final states for $D \to PV$ are $g_q = 4.2$, $g_s = 4.6$, and $g_{ss} = 4.5$. The formulas for singly Cabibbo-suppressed decays of $D \to PV$ are shown as
\[ A(D^+ \rightarrow \eta \rho^+) = G_F m_\rho \left( \sqrt{2} a_2^{PV} A_0^{D\rho}(m_\rho^2) [V_{cs} V_{ud} f_d + V_{cs} V_{us} f_u] + V_{cs} V_{ud} a_1^{PV} f_\rho F_1^{D\rho}(m_\rho^2) \cos \phi \right) (\epsilon^* \cdot p_D), \]
\[ A(D^+ \rightarrow \eta^' \rho^+) = G_F m_\rho \left( \sqrt{2} a_2^{PV} A_0^{D\rho}(m_\rho^2) [V_{cs} V_{ud} f_d + V_{cs} V_{us} f_u] + V_{cs} V_{ud} a_1^{PV} f_\rho F_1^{D\rho}(m_\rho^2) \sin \phi \right) (\epsilon^* \cdot p_D), \]
\[ A(D^+ \rightarrow \pi^+ \phi) = \sqrt{2} G_F V_{cs}^* V_{us} a_2^{PV} m_\phi f_\phi F_1^{D\pi}(m_\phi^2) (\epsilon^* \cdot p_D), \]
\[ A(D^+ \rightarrow \pi^+ \omega) = G_F V_{cs} V_{ud} m_\omega a_2^{PV} f_\omega F_1^{D\omega}(m_\omega^2) + a_1^{PV} f_\pi A_0^D(m_\omega^2) (\epsilon^* \cdot p_D), \]
\[ A(D_S^+ \rightarrow \pi^+ K^{\pm 0}) = \sqrt{2} G_F \left( V_{cs} V_{us} a_A^{PV} g_\omega f_\omega + V_{cs} V_{ud} a_1^{PV} m_K f_\omega A_0^{D\omega}(m_\omega^2) (\epsilon^* \cdot p_D), \right)^2 \]
\[ A(D_S^+ \rightarrow \pi^0 K^{+0}) = G_F \left( V_{cs} V_{us} a_A^{PV} g_\omega f_\omega + V_{cs} V_{ud} a_1^{PV} m_K f_\omega A_0^{D\omega}(m_\omega^2) (\epsilon^* \cdot p_D), \right)^2 \]
\[ A(D_S^+ \rightarrow K^\pm 0) = G_F \left( V_{cs} V_{us} a_A^{PV} g_\omega f_\omega + V_{cs} V_{ud} a_1^{PV} m_K f_\omega A_0^{D\omega}(m_\omega^2) (\epsilon^* \cdot p_D), \right)^2 \]
\[ A(D_S^+ \rightarrow K^0 \rho^0) = \sqrt{2} G_F \left( V_{cs} V_{us} a_A^{PV} g_\omega f_\omega + V_{cs} V_{ud} a_1^{PV} m_K f_\omega A_0^{D\omega}(m_\omega^2) (\epsilon^* \cdot p_D), \right)^2 \]
\[ A(D_S^+ \rightarrow \eta^0 K^{+0}) = G_F \left( V_{cs} V_{us} a_A^{PV} g_\omega f_\omega + V_{cs} V_{ud} a_1^{PV} m_K f_\omega A_0^{D\omega}(m_\omega^2) (\epsilon^* \cdot p_D), \right)^2 \]
\[ A(D_S^+ \rightarrow \eta^' \rho^0) = \sqrt{2} G_F \left( V_{cs} V_{us} a_A^{PV} g_\omega f_\omega + V_{cs} V_{ud} a_1^{PV} m_K f_\omega A_0^{D\omega}(m_\omega^2) (\epsilon^* \cdot p_D), \right)^2 \]
\[ A(D_S^+ \rightarrow K^0 \phi) = \sqrt{2} G_F \left( V_{cs} V_{us} a_A^{PV} g_\omega f_\omega + V_{cs} V_{ud} a_1^{PV} m_K f_\omega A_0^{D\omega}(m_\omega^2) (\epsilon^* \cdot p_D), \right)^2 \]
\[ A(D_S^+ \rightarrow \eta^0 K^0) = G_F \left( V_{cs} V_{us} a_A^{PV} g_\omega f_\omega + V_{cs} V_{ud} a_1^{PV} m_K f_\omega A_0^{D\omega}(m_\omega^2) (\epsilon^* \cdot p_D), \right)^2 \]
\[ A(D_S^+ \rightarrow \eta^' K^0) = \sqrt{2} G_F \left( V_{cs} V_{us} a_A^{PV} g_\omega f_\omega + V_{cs} V_{ud} a_1^{PV} m_K f_\omega A_0^{D\omega}(m_\omega^2) (\epsilon^* \cdot p_D), \right)^2 \]

The formulas for doubly Cabibbo-suppressed decays of \( D \rightarrow PV \) are shown as

\[ A(D^0 \rightarrow \pi^- K^{+0}) = \sqrt{2} G_F V_{cs} V_{us} \left( a_1^{PV} m_K f_\omega A_0^{D\phi}(m_\omega^2) - a_1^{PV} m_K f_\omega A_0^{D\omega}(m_\omega^2) \right) (\epsilon^* \cdot p_D), \]
\[ A(D^0 \rightarrow K^0 \phi) = \sqrt{2} G_F V_{cs} V_{us} \left( a_1^{PV} m_K f_\omega A_0^{D\phi}(m_\omega^2) - a_1^{PV} m_K f_\omega A_0^{D\omega}(m_\omega^2) \right) (\epsilon^* \cdot p_D), \]
\[ A(D^0 \rightarrow K^0 \omega) = G_F \left( a_2^{PV} m_\omega A_0^{D\omega}(m_\omega^2) - a_2^{PV} m_\omega A_0^{D\omega}(m_\omega^2) \right) (\epsilon^* \cdot p_D), \]
\[ A(D^0 \rightarrow K^0 \rho^-) = \sqrt{2} G_F V_{cs} V_{us} \left( a_1^{PV} m_K f_\omega A_0^{D\phi}(m_\omega^2) - a_1^{PV} m_K f_\omega A_0^{D\omega}(m_\omega^2) \right) (\epsilon^* \cdot p_D), \]
\[ A(D^0 \rightarrow K^0 \rho^0) = \sqrt{2} G_F V_{cs} V_{us} \left( a_1^{PV} m_K f_\omega A_0^{D\phi}(m_\omega^2) - a_1^{PV} m_K f_\omega A_0^{D\omega}(m_\omega^2) \right) (\epsilon^* \cdot p_D), \]

(A5)
\[ \mathcal{A}(D^+ \to \pi^+ K^{*0}) = \sqrt{2} G_F V_{ud}^* V_{us} \left( a_2^P g_s f_K f_D \frac{m_D^2}{m_D^2 - m_K^2} - a_1^P m_K f_K F_1^{D\pi}(m_K^2) \right) (\varepsilon \cdot p_D), \]

\[ \mathcal{A}(D^+ \to \pi^0 K^{*+}) = G_F V_{ud}^* V_{us} \left( a_2^P g_s f_K f_D \frac{m_D^2}{m_D^2 - m_K^2} - a_1^P m_K f_K F_1^{D\pi}(m_K^2) \right) (\varepsilon \cdot p_D), \]

\[ \mathcal{A}(D^+ \to K^+ \rho^0) = G_F V_{cd}^* V_{us} f_K \left( a_2^P g_s f_D \frac{m_D^2}{m_D^2 - m_K^2} + a_1^P m_\rho A_0^\rho(m_K^2) \right) (\varepsilon \cdot p_D), \]

\[ \mathcal{A}(D^+ \to K^0 \eta^+) = G_F V_{cd}^* V_{us} f_K \left( a_2^P g_s f_D \frac{m_D^2}{m_D^2 - m_K^2} + a_1^P m_\rho A_0^\rho(m_K^2) \right) (\varepsilon \cdot p_D), \]

\[ \mathcal{A}(D^+ \to K^0 \phi) = \sqrt{2} G_F V_{cd}^* V_{us} a_2^P g_s f_D \frac{m_D^2}{m_D^2 - m_K^2} (\varepsilon \cdot p_D), \]

\[ \mathcal{A}(D^+ \to \eta' K^{*0}) = \sqrt{2} G_F V_{cd}^* V_{us} \left( a_2^P f_K f_D \frac{m_D^2}{m_D^2 - m_K^2} \right) (\varepsilon \cdot p_D), \]

\[ \mathcal{A}(D^{\pm} \to K^* K^{*0}) = \sqrt{2} G_F V_{cd}^* V_{us} m_K \left( a_2^P f_K F_1^{D\pi K^*}(m_K^2) - a_1^P f_K A_0^{D\pi K^*}(m_K^2) \right) (\varepsilon \cdot p_D), \]

\[ \mathcal{A}(D^{\pm} \to K^0 K^{*+}) = \sqrt{2} G_F V_{cd}^* V_{us} m_K \left( a_2^P f_K F_1^{D\pi K^*}(m_K^2) + a_1^P f_K A_0^{D\pi K^*}(m_K^2) \right) (\varepsilon \cdot p_D). \]
