Operational Modal Analysis of Frame Building Module during Road Transport

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Abstract. A real structure performance does not necessarily match the way the structure's numerical model reacts to a load applied – even though it was constructed by specialists in the field and with the use of a state-of-the-art generation of software based on the finite element method (FEM). Inconsistencies are even greater, when the construction material is not as uniform and isotropic as steel, but wood, whose mechanical properties depend on a greater number of variables, like whether the loads apply longitudinally or across the grain. In such cases, it is necessary to customise a previously developed numerical model (model calibration) to fit real conditions of the performance. More and more frequently, to carry out experimental modal analyses (EMA), some measuring equipment and software is used to this end, which allows for estimation of credible structure modal parameters, i.e. particular normal mode shapes and their equivalents of natural frequency and modal damping values, which are necessary for credible calibration of a numerical model developed in FEM. It is not in every case, however, that one can excite vibration in a structure by means of classical modal exciters or modal hammers. Being a relatively new approach in the structure modal analysis, vibrations present in immediate surroundings of a tested object are used for this purpose. An analysis of such type is called the operational modal analysis (OMA). This study seeks to estimate credible modal parameters for one of the wooden modules of a multi-storey, multi-unit residential building, constructed by UNIH HOUSE in Bielsk Podlaski in Poland, during its test road transport to its project site. The operational modal analysis of the frame building module was conducted with the use of a dedicated software LMS Test.Lab Spectral Testing. Measurements of vibration accelerations were carried out in 12 measurement points of the tested frame building module using a 32-channel and 24-bit data acquisition hardware type SCADAS Recorder from SIEMENS with 130 dB dynamic range and signal to noise ratio – minimum 106 dB, as well as a set of 10 high sensitivity triaxial piezoelectric accelerometers type TLD356B18, manufactured by PCB Piezotronics and two uniaxial, high sensitivity accelerometers type 333B50 – also manufactured by PCB Piezotronics. As a result of the tests and analyses executed on the proposed structure modal model, it was possible to estimate modal parameters of a single module of a frame multi-storey building, which may be used for calibration of the structure numerical calibration in the FEM software.

1. Introduction
In recent years, a substantial advancement in numerical modelling and computation of internal forces, strains and stresses in the modelled structure under a load applied made the design of building structures
of a variety of types and uses considerably easier, particularly when it comes to very sophisticated shapes and unusual conditions of loads. This has not only been possible as a result of a significant increase in computational capacity of the contemporarily used computers, which allows for solving problems well beyond the capacity of solving them several years ago, but also owing to some facilitation related to improvements in the functionality and intuitiveness of data input in contemporarily used software, which is based on the Finite Element Method – FEM.

Unfortunately, it often turns out that a real structure performance does not necessarily match the way the structure’s numerical model reacts to a load applied – even though, it was constructed by specialists in the field and with the use of a state-of-the-art generation of software based on the finite element method. Inconsistencies are even greater, when the construction material is not a homogeneous and isotropic material as steel, but wood, whose mechanical properties depend on a greater number of variables, like whether the loads apply longitudinally or across the grain. The consequence of this are fractures and scratches on individual structure elements, which may occur in a building structure after they are placed in a structure, or even earlier – when they are transported to the building site.

In such cases, it is necessary to customise a previously developed numerical model (model calibration) to fit real conditions of the performance. More and more frequently, to this end some measuring equipment and experimental modal analysis software is used [1], which allows for estimation of credible structure modal parameters, i.e. particular mode shapes and corresponding to them frequencies and modal damping values, which are necessary for credible calibration of a numerical model developed in FEM. It is not in every case, however, that one can excite vibrations in a structure with proper amplitudes by means of classical modal exciters or modal hammers with built-in force sensors. Being a relatively new approach in the structure modal analysis, vibrations present in immediate surroundings of a tested object or in an object itself (the so-called operational vibrations) and taking measurements with recording only responses of a tested construction to operational excitation [2] are applied for this purpose. An analysis of such a type is called the operational modal analysis (OMA) [3].

This study seeks to estimate credible modal parameters for one of the wooden modules of a multi-storey, multi-unit residential building, constructed by UNIHOUSE in Bielsk Podlaski, during its test road transport to its project site. In contrast to aviation and motorization industry [4] in which the modal analyses are quite often used in engineering practice, in construction industry apart from few cases of modal analyses of large-size objects such as bridges [5], [6], water dams [7], stadiums [8], monuments [9], massive foundation blocks, or simple plate constructions made of reinforced concrete [10] or steel [11], have not been carried out widely.

2. Theoretical, experimental and operational modal analysis

Provide sufficient detail to allow the work to be reproduced. A modal analysis of any mechanical system, including construction structures, can be performed as theoretical, experimental or operational. In case of the theoretical modal analysis, the so-called eigenproblem for an adopted numerical model of a tested construction is solved, as a result of which theoretical model parameters of a construction in a form of set of eigenfrequencies, corresponding damping coefficients and modes are obtained. The theoretical modal analysis is usually carried out at the project stage as conducting tests on real objects is not feasible.

The experimental modal analysis (EMA) enables practical identification of real modal parameters of a tested object and is usually used at the operation stage and more rarely at the stage of construction [2]. In this type of the analysis vibrations in a tested object are excited by means of modal hammers or adequately placed modal exciters while a force exciting vibrations of an object as well as responses of a tested system in all adopted characteristic points of the system are simultaneously recorded (usually in a form of acceleration time histories, the so-called vibrograms of acceleration). In the FFT (Fast Fourier Transform) analysis, the input spectrum of a force exciting vibrations and the output spectrum of vibration accelerations are created. On such a basis, spectral transition functions FRF (Frequency Response Function) (1) are created as well as a stabilization diagram from which modal parameters of a tested object are finally estimated.
\[ FRF = \frac{\sum F(\omega)}{\sum A(\omega)} \]  

where:
\[
\sum F(\omega) \quad \text{– Fourier transform of vibration exciting force,}
\]
\[
\sum A(\omega) \quad \text{– Fourier transform of response signal.}
\]

The *operational modal analysis* (OMA), on the other hand, enables conducting test on large-size objects which could hamper or even prevent carrying out the classic experimental modal analysis due to insufficient, with regard to a size of an object, amplitudes of excited vibrations possible to be generated by means of available hammers and modal exciters. In contrast to the classic modal analysis where a tested object has to be linear and subject to description by means of total and partial differential equations, in the operational modal analysis it is also possible to identify non-linear models [2]. Obviously, a tested system should be also congruent with *Maxwell’s principle of reciprocity* and characterized with little or proportional damping. In this type of the analysis, in given nodal points of a tested object only, responses of a tested system (output) to unknown operational excitation are recorded, i.e. kinematic excitation or such resulting from action of external forces accompanying the technological process. The modal analysis is carried out by referring recorded responses of a tested system in specific nodal points to one or more of them, and subsequently by estimating modal parameters as a result of which poles of a system are identified and modes of a system are estimated. There are many methods to estimate modal parameters in the operational modal analysis (OMA). They result from the classic approach used in the experimental modal analysis (EMA) (input-output modal identification) in which both input/excitation and output/response are recognized. As far as operational sources of vibrations are concerned, only output data of an object are known. Many estimation methods used in EMA has been applied to OMA. They can be divided into [3]: nonparametric methods (inexact and faster in performing) and parametric (using much more complex algorithms and requiring more computing power, but enabling obtaining more reliable results), methods with single or a number of modes determining structural response in a given bandwidth (*SDOF* or *MDOF* methods respectively), methods determining a number of stages of calculations necessary to identify all modal parameters (*one-stage* and *two-stage* methods) as well as methods determining the domain of implementation: *time domain* methods based on the analysis of response time histories or correlation functions or *frequency domain* methods based on spectral density functions. Overall, time domain method was better conditioned than frequency domain methods. It has been changed along with improvement of a quality of numerical conditioning as a result of adoption of polynomial basis functions formulated in the z-domain [3].

It is not an intention of the authors to describe in detail or discuss advantages and disadvantages of specific model parameters estimation methods applied in OMA due to the fact the comprehensive study on the topic can be found in [12]. Also in [13], there can be found the interesting review.

Taking into consideration the fact that the modal parameters identification methods within the *frequency domain* are far more effective in determining structural modes of tested constructions (due to almost eliminated spurious mode problem) than *time domain* methods [13], a concise review on given modal parameters identification within the *frequency domain* is presented below.

Among them, the easiest one is *Peak-Picking* method, also known as *Basic Frequency Domain* (BFD) method [3] which is based on identification of the eigenfrequencies of a tested system in a form of the peaks of power spectrum density plots (PSD plots). This method requires low damping of a tested object and well-separated frequencies. Its main disadvantage lies in in the fact that it fails to ensure any damping estimations.

A quite more advanced method is the Frequency Domain Decomposition (FDD) method, also called *Complex Mode Indication Function* (CMIF) or *Principal Component Analysis* (PCA) in which Singular Value Decomposition (SVD) of a matrix with the auto spectra and cross spectra between the outputs [14] is calculated. This method enables identification of closely spaced modes and different modes at the same frequency (the so-called mode-multiplicity). Nonetheless, the first generation of FDD method only enabled estimation of modal frequencies and mode shapes. Its more advanced version, the so-called
Enhanced Frequency Domain Decomposition (EFDD) additionally enables determination of a modal damping ratio [3]. Better results may be obtained in case of a third generation of FDD methods which is called Frequency-Spatial Domain Decomposition (FSDD) which uses spatial filtering (the so-called coherent averaging) to enhance the estimation of modal frequencies and damping ratios [3]. The above described methods fall into nonparametric category.

Among frequency domain parametric methods there can be distinguished rather rarely applied the Least Squares Frequency Domain (LSFD) method (used only to obtain global estimates of the mode shapes in combination with other methods providing the poles [3]), the Maximum Likelihood Frequency Domain (MLFD) method (applying a typical non-linear estimator, which has been developed to deal with noisy measurements) [13], the Least Squares Complex Frequency (LSCF) method (which was primarily “developed to provide good initial values of the parameters to the MLFD method with low computational efforts” [3]) and the Poly-Reference Least Squares Complex Frequency (p-LSCF) method – the poly-reference version of the LSCF method (also known under the commercial name the Operational PolyMAX – applied in the OMA analyses [14], [15] or the PolyMAX in case of the EMA analyses [16]), which uses the so-called right matrix-fraction model.

The p-LSCF method, just like the LSCF method facilitates obtaining very clear stabilization diagrams in wide frequency range [12], and, what is more, deals very well with the identification of closely spaced poles – which constituted a problem in the LSCF method.

Effectiveness and reliability of the p-LSCF method in the operational modal analysis have been proven inter alia in works provided by [4], [8], but still, it is not the only method which render obtaining reliable modal parameters of tested constructions possible.

As far as practical issues are concerned, for the time being good results have been also obtained in case of the Maximum Likelihood Frequency Domain method (MLFD) - an optimisation-based method which enables providing modal parameters by minimising an error norm [17] and the stochastic subspace identification method [18] (a more advanced alternative to the Peak Picking method), in which it is adopted that the so-called stochastic state space model is attained directly from measured output data or output correlations while assuming that a vibration structure excited by some unknown forces are white noise signals [14].

3. Object of the study
The object of the conducted modal analyses (figure 1) is a single wooden module of a multi-storey, multi-residential unit with number K202.6X manufactured by UNIHOUSE located in Bielsk Podlaski.

The module, with dimensions in a top view of 15.69m x 5.31m and a height of 3.18m, consists of a floor, external walls, external intermodule walls, internal division walls and ceiling. The floor is made of wooden beams with a cross-section of 60x360mm, C24 class of wood, and STEICO I-beams with a height of 360mm.

![Figure 1](image-url)
The floor is tightened with 22 mm thick OSB3 joined with beams by glue and stitches. In external walls (a module’s top walls), posts with a cross-section of 60x160 mm, C24 class of wood, and sheathing made of 12mm thick OSB3, are applied. The carrying construction of external walls (longitudinal walls) is constructed with posts with a cross-section of 50x120 m tightened with 12 mm thick OSB 3. Internal walls are built on the structure of posts (50x80 mm), C24 class of wood. Internal surface of all walls are double-sheathed with drywalls (2x12.5 mm thick). Walls and the floor are joined together by screws. The ceiling is constructed with wooden beams with a cross-section of 60x150 mm, C24 class of wood. External sheathing stiffening the ceiling is made of 12 mm thick OSB3. The internal surface of the ceiling is sheathed with drywalls (2x12.5 mm). Within the construction, standard spaces between posts and beams amounts to 600mm. Each element of the module construction is filled up with the mineral wool. The construction of K202.6X module is shown in figure 2.

Figure 2. Detailed construction of K202.6X module (with sheathing)

4. Planning and execution of measurements
In contrast to numeric analyses carried out by means a software using the Finite Elements Method, a construction model created for purposes of the modal analyses is much simpler. In this case, it is not required to apply accurate or work-consuming modelling of particular elements of the construction, details on their connections, thickness of elements and also the necessity to enter detailed data on application of construction materials. In each case of conducting the modal analysis, it is necessary to create a geometrical model of a tested object where only construction nodes will be considered in which measurements sensors are mounted. Certainly, complexity of the geometrical model is affected mostly by a quantity of available channels of a measuring instrument and a quantity of uniaxial or triaxial accelerometers that can be applied and whose parameters are adjusted to level and range of vibrations frequency of measured constructions’ responses (in most cases it is recorded in a form of a accelerogram records). In the experimental modal analyses conducted with the use of externally excited vibrations (for example by means of modal exciters), in most cases it is possible to change position of part of sensors (but not a reference sensor or sensors which remain in the same position throughout the measurement) and as a result obtain measurements in many more measuring points than a number of used accelerometers. Unfortunately, in the operational modal analysis in which only operation vibrations accompanying normal work of a tested object occur, changing positions of the sensors, even possible, is not usually applied (especially when a source of operational vibrations generates an extremely diversified vibration spectrum over time).

Considering the size and the weight of K202.6X module, it has been stated that obtaining proper vibrations amplitudes of the tested system with the use of modal hammers and modal exciters available in Department of Geotechnics and Structural Mechanics at the Bialystok University of Technology may be difficult with relation to a planned experiment. Regarding all damages that may occur to the module during road transport to its project site (which indicates to significant vibrations amplitudes which the module is subject to during being transported), within modal analyses it has been decided to apply operational vibrations generated and transferred to the tested construction during road transport of the said module. Due to quite a short period of time of a planned test road transport of the module and lack
of possibility to change positions of the accelerometers during driving of a truck-tractor with a platform attached, where the module is placed, the geometrical model of the tested system was limited to 12 nodes, which corresponded to the number of accelerometers with required parameters set. It was adopted that all measurement points were located on internal walls of the building module, next to the corners of the module and on one of the crosswalls. The sensors were installed at two different heights directly above the floor and just below the ceiling of the module. The nodes of the modal geometrical model of the tested object adopted for further analyses has been illustrated in figure 3.

![Figure 3. Simplified 12-node geometric model of the building module with measurement points numbered (MP – Measurement Point)](image)

Measurements of vibration accelerations (response of a tested object to operational excitation) were carried out using a 32-channel and 24-bit data acquisition hardware type SCADAS Recorder from SIEMENS with 130 dB dynamic range and signal to noise ratio - minimum 106 dB, as well as a set of 10 high sensitivity (1000 mV/g) triaxial piezoelectric accelerometers type TLD356B18, manufactured by PCB Piezotronics (measuring ranges: ±5 g pk, 0.5-3000 Hz ±5%) and two uniaxial, high sensitivity accelerometers type 333B50 - also manufactured by PCB Piezotronics (measuring ranges: ±5 g, 0.5-3000 Hz ±5%).

Triaxial piezoelectric accelerometers type TLD356B18 were installed in eight measurement points located in external corners of the module (MP 1 - MP 8) and two measurement points were situated on the lateral wall just below the ceiling (MP 9 - MP 10). Acceleration time histories were recorded in three mutually perpendicular directions: \( x \) – parallel to longitudinal axis of the module, \( y \) – transverse, \( z \) – vertical. Two uniaxial accelerometers type 333B50 were installed in the bottom part of the crosswall of the module and vibration accelerations in time domain were recorded in \( x \) direction (MP 11 - MP 12). All accelerometers utilised in the measurements (uniaxial and triaxial) were installed in proper positions within the building module by means of dedicated magnetic bases mounted to steel angles, which were first screwed to the module’s walls.

The measurement system SCADAS Recorder and one of the accelerometer type TLD356B18 are presented in figure 4.

Recording of vibration accelerations in the time domain as well as digital data processing performed later related to the operational modal analysis of the frame building module was conducted with the use of the dedicated LMS Test.Lab Spectral Testing software. This software, as far as the operational source of vibrations is concerned, provides one of the most effective algorithms for estimation of modal frequency domain parameters – the \( p \)-LSFC method (known under commercial name as Operational PolyMAX).
With regard to the construction of the tested object and the manner of generation of vibrations, after carrying out preliminary test it was assumed that the responses of the tested object (output) to unknown operational excitations would be recorded in the range of frequency up to 256 Hz (bandwidth) which corresponds to 512 Hz sample frequency.

Relevant measurements with the use of all 32 channels available were conducted during road transport of the module through streets of Bielsk Podlaski with asphalt pavement and diversified technical condition.

5. Results and discussion

The operational modal analysis performed with the use of the LMS Test.Lab Spectral Testing software which estimates modal parameters by means of $p$-LSCF method (Operational PolyMAX) requires to collect at least one recorded response of a tested object as reference. Definitely better solution is to adopt several references which would serve for calculating the necessary crosspowers. After preliminary analyses, acceleration time histories (outputs) in three different nodes of the adopted geometrical model and, simultaneously, in three different measurement directions, $5x$, $8y$ and $10z$ respectively (figure 3), were adopted as references.

Crosspowers were generated with frequency resolution ranging from 0.125 Hz to 0.5 Hz. Finally, estimation of the modal parameters of the tested module were carried out by means of Op. PolyMAX included in the LMS Test.Lab Spectral Testing software for crosspowers generated in resolution of 0.25 Hz. In order to obtain the most reliable modes of the tested construction, far stricter values for the tolerances of the stabilization of the modal model poles were adopted (1% of the pole vector, 1% of the frequency and 1% of the damping), in comparison with the default ones (2%, 1% and 5% respectively).

Due to complexity of the tested system (a lot of construction elements, internal division walls, many screw joints: walls-floor, walls-ceiling and stitches-wall with posts, etc.), the modal module of the tested structure was characterized by many resonant frequencies. This was the reason for reducing the range of estimation of the modal parameters of the wooden object to 20 Hz.

During validation, modes with a relatively high index of Auto-MAC (Auto Modal Assurance Criterion) which indicates the orthogonality of vectors of modes, normal for symmetric objects, were removed. This relates to the fact that in some cases the algorithm may choose different poles corresponding to the same modes. As a result, in the final modal model of the tested structure only the modes, which, according to the adopted criterion, varies between each other significantly, were taken into consideration.

The modal parameters obtained in the analyses and validation of the modal model are summarized in table 1.
Table 1. Modal parameters of the building module within the frequency range up to 20 Hz

| Mode | Frequency, Hz | Damping, % |
|------|--------------|------------|
| 1    | 0.59         | 9.97       |
| 2    | 1.52         | 25.85      |
| 3    | 2.02         | 11.75      |
| 4    | 3.40         | 4.64       |
| 5    | 5.64         | 5.35       |
| 6    | 8.37         | 5.34       |
| 7    | 9.56         | 3.25       |
| 8    | 11.84        | 1.07       |
| 9    | 12.43        | 2.72       |
| 10   | 14.14        | 0.44       |
| 11   | 15.28        | 1.14       |
| 12   | 17.52        | 4.62       |
| 13   | 18.59        | 4.05       |

Figure 5. Auto-Mac matrix between modes: view a) 2D, b) 3D

The Auto-MAC index after validation of the estimated modes were presented in a graphic form (2D and 3D) in figure 5. As shown in the illustrations below, particular modes vary between each other significantly. The maximum value of Auto-MAC for modes determined in the course of the operational modal analysis is below 46.1%.

Analysing the modal parameters from table 1, it is evident that the first three modes are characterized by relatively high level of damping. This phenomenon on the similar level is typical in damping of rigid bodies. In this case, high values of damping indicate, however, that the modal parameters do not refer to the building module itself, but to modes of the integrated system: the building module – the carriage platform, used for transportation of the module. So high levels of damping in the first three modes were certainly caused by damping obtained by wheels of the carriage platform. Next modes of the tested structure were damped to far lesser extent – from 0.44% (mode 10) to 5.35% (mode 5).

Unfortunately, the results of the operational modal analysis are usually not so good as in case of the experimental modal analysis. In this instance, it cannot be ruled out that within the analysed frequency bandwidth up to 20 Hz not all significant modes, as far as the building dynamics is concerned, were included nor state which of the determined modes are more or less important. Due to a very high level of damping, the first three modes require further analysis. Based on the conducted analyses, it cannot be stated in what extent they relate to vibrations excited by the carriage platform. This issue requires further
studies. That is why it is planned to carry on the study with the use of a different source of operational excitation or carrying out the experimental modal analysis by means of a 5 kgs large modal hammer.

6. Conclusions
As a result of the tests and the operational modal analysis carried out on the 12-node geometrical model with the use of the LMS Test.Lab Spectral Testing software which estimates the modal parameters by means of the \( p\)-LSCF method (\textit{Operational PolyMAX}) it was possible to estimate modal parameters of a single module of a frame multi-storey building. Due to significant complexity of the tested frame structure and numerous resonant frequencies of the tested module, the modal analysis was carried out with the reduced range of the frequency bandwidth to 20 Hz. Within the analysis, 13 dominant modes, with the \textit{Auto-MAC} matrix values not exceeding 46.1\%, were determined. The first three modes are characterized by a very high level of damping which is not likely to be related to damping of the modes of a single module, but rather an integrated set of the building module – the carriage platform on a wheeled chassis. The issue of such a high level of damping requires further analyses and study. It cannot be ruled out though that within the frequency bandwidth analysed in this work some other modes, significant due to the construction dynamics, occurred. For this reason, it is planned to continue the research with the use of a different source of operational excitation within the operational modal analysis or carrying out the experimental modal analysis with the use of a large modal hammer.

Regardless further study plans of the authors of this work, the estimated modal parameters within the conducted operational modal analysis may be used for calibration of the numerical model of structure in the \textit{FEM} software.

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