1. INTRODUCTION

Cataclysmic variables (CVs) are mass-transferring close binaries (Patterson 1984; Warner 1995). The primary, or accreting star, is a white dwarf. The secondary is a low-mass main-sequence star that overfills its Roche lobe and transfers matter onto the primary.

The evolution of CVs is driven by two major ingredients: angular momentum loss and the response of the secondary star to the mass loss. In the standard model of CV evolution (Rappaport et al. 1983), angular momentum loss is assumed to be extremely efficient until the secondary becomes fully convective, at which point it ceases. This implies a rapid evolutionary timescale, numerous very low mass CVs, and a high average mass accretion rate; the secondary stars in CVs lose mass on a timescale shorter than the Kelvin-Helmholtz timescale and are out of thermal balance. In this “standard” model the inferred period gap is explained by a reduction in the angular momentum loss rate at the onset of full convection, which causes the secondaries to relax to a smaller radius and fall out of contact. Such a model was consistent with the stellar data at the time it was proposed.

In an earlier paper (Andronov et al. 2003, hereafter APS03) we showed that recent stellar data on angular momentum loss are inconsistent with the standard CV model. Fully convective single stars are observed to experience angular momentum loss, and the rate of angular momentum loss saturates at a much lower level than that given by a simple extrapolation of the solar rate to the periods of interest for CVs. The inferred timescale of CV evolution is much longer. In addition, the secondary evolves during the pre-CV phase. These effects raise the possibility that the secondary stars in CVs could be significantly evolved prior to the CV phase. In APS03 we explored some simple models with chemically evolved secondaries; nuclear evolution during the CV phase was neglected. In this paper we explore the effects of the evolutionary state of the secondary in more detail.

We therefore begin in § 1.1 with a discussion of our angular momentum loss rates, which motivate the exploration of evolved CV secondaries in § 1.2. The important question of the origin of the period gap is discussed in § 1.3. In § 2 we briefly describe our models and method. We begin our results by presenting models with different evolutionary states and angular momentum loss rates in § 3. We also examine the secondary mass–orbital period relation. In § 4 we discuss tests of the braking law for single stars and the mass accretion rate in CVs. Section 5 is devoted to a discussion of various classes of solutions for the origin of the period gap. We find two new possibilities that deserve further study: tidally induced mixing of evolved secondaries and nonequilibrium CV period distribution tied to the star formation rate (SFR) in the Galaxy.

1.1. Angular Momentum Loss in CVs

There are two mechanisms for angular momentum loss in CVs. The first is gravitational radiation (Landau & Lifshitz 1962), which is effective only for short orbital periods (Patterson 1984) and therefore cannot be the only mechanism responsible for angular momentum loss. The second is angular momentum carried away by a stellar wind magnetically coupled to the surface of the secondary. In tidally locked binaries (such as CVs), this momentum is removed from the total orbital momentum of the system, causing a secular decrease of the orbital period.

There has been significant progress in the modeling of angular momentum loss from a magnetized stellar wind (see Appendix A of Keppens et al. 1995). In early models of the
The less efficient saturated magnetic braking prescription implies a slower rate of period decrease for CVs than the unsaturated prescription does. This can be tied to the mass-loss rate through the response of a star to mass loss. The net effect is a lower time-averaged mass accretion rate. The typical lifetime of a CV with an initial secondary mass of order $1 \, M_\odot$ or more becomes comparable to its main-sequence lifetime. As a result, it is not clear that nuclear evolution during the CV phase can be completely neglected.

In addition, there are two major effects that could permit secondary stars to begin the CV phase with significant internal nuclear processing. The primary must exhaust its central hydrogen prior to a common-envelope (CE) phase. The secondary will also evolve, which is an important effect prior to the CE phase only for secondary stars similar in mass to the primary. However, there is also a potential time delay between the CE phase and the onset of the CV phase. If the output of the CE phase is a relatively wide binary (compared to that of CVs) with an orbital period of a few days, then the time for the system to reach contact may be significant and could result in appreciable nuclear evolution of the secondary prior to the CV phase. Furthermore, the low inferred loss rates for very low mass secondaries may favor higher mass secondaries with shorter nuclear timescales. Therefore, CVs with evolved secondaries may contribute to CV populations. Hence, it seems reasonable to study how the evolutionary state of the secondary affects the properties of CVs.

From an observational standpoint, the evidence for a considerable fraction of evolved systems comes from the spread of the mass of secondaries at a given orbital period (Smith & Dhillon 1998). This spread is naturally explained if we assume that some secondaries experienced significant nuclear evolution prior to the CV phase (e.g., APS03). Baraffe & Kolb (2000) came to the same conclusion considering the spectral type–orbital period relation in the data by Beuermann et al. (1998). Most theoretical work has assumed that secondaries are chemically unevolved.\footnote{Recently, Podsiadlowski et al. (2003) have examined this issue in the framework of the old angular momentum loss model. We discuss their results in \S\ 3.2.}

In §§ 2.4 and 2.5 we discuss two mechanisms that can change the mass-radius relationship for the secondary star in a CV but are not included in recent models. These mechanisms are related to the existence of spots on the surface of rapidly rotating stars and extra mixing associated with tidal deformation of a star.

1.3. The Origin of the Period Gap

The study of CVs has a rich history. The theoretical framework that is traditionally employed has had some success in explaining many of the global features of the observed population of CVs. However, there are some key assumptions in the standard model, most notably concerning angular momentum loss, that are seriously inconsistent with both the observed spin-down of young, low-mass stars and theoretical developments in our understanding of stellar winds. A critical reexamination of the physical effects responsible for some of the major properties of CVs is therefore clearly warranted.

For example, the presence of only a few CVs with periods between 2 and 3 hr (the period gap) is certainly a critical component of the CV phenomenon. If the CV distribution is
in equilibrium, then the most natural interpretation is that there is a physical effect that causes stars to shrink at a characteristic orbital period of 3 hr. The standard model uses a sudden drop in the efficiency of magnetic braking for fully convective stars to achieve this. Such a phase transition would produce a sharp break in the spin-down properties of single stars, which is not seen.

However, there are other physical mechanisms that could potentially cause changes in the mass-radius relationship near the fully convective boundary. As a result, it is entirely possible that the standard model is on the right track (by solving the period gap with a change in the mass-radius relationship) but that it is not using the proper underlying physical mechanism. Alternately, the period gap could arise either from the presence of more than one underlying population or from the distribution of CV periods not being in equilibrium. In our view, all of these possibilities should be explored, and a better physical picture of the evolution of CVs will be obtained. In § 5 we discuss different possible scenarios for the formation of the period gap.

2. PHYSICS

In this section we describe the physics of CVs relevant to our calculations. Here and below, upper case variables denote quantities in cgs units, while lower case variables express quantities in dimensionless units relative to the Sun.

The evolution of a close binary that eventually forms a CV can be described in four stages.

1. Main-sequence evolution.—The more massive (primary) star leaves the main sequence first and expands on the red giant branch. A CE system forms when unstable mass accretion onto the secondary sets in (De Kool 1990; Iben & Livio 1993).

2. CE stage of evolution (e.g., De Kool 1990).—This phase is short (≈10⁵ yr); during this stage the secondary spirals in toward the primary, and the gravitational potential energy is absorbed by an envelope that is subsequently ejected. After the envelope is ejected, the system consists of a white dwarf and a main-sequence secondary star that does not necessarily overfill its Roche lobe but might be close to it. The final separation of the main-sequence star and the white dwarf should depend on the initial mass ratio of the stars in the binary and their initial separation. However, the current understanding of this phase of evolution is not adequate to predict the outcome (De Kool 1990). The only conclusion we can make is that at least some systems after this phase are close enough (with orbital periods below ≈5 days) to eventually form a CV.

3. Post-CE and pre-CV evolution.—During this phase stars in the binary get closer because of angular momentum loss from the system until the secondary overfills its Roche lobe.

4. CV evolution.—The outcome of the first three phases defines the starting point of the CV phase. The relevant ingredients are the masses of the white dwarf and the secondary (m₁ and m₂) and the evolutionary state of the secondary, which determines the mass-radius relationship. We characterize the evolutionary state by its central hydrogen abundance (X_c).

For the models in this section we used a primary (white dwarf) mass of 0.85 M_☉, the average measured mass of CV primaries (Patterson 1984).2 We keep it constant during the evolution. The hydrogen-rich material accumulated on the surface of the white dwarf is assumed to be lost during nova outbursts (Warner 1995). The time step used in our models is considerably larger than the nova duty cycle, which ensures that we average the behavior of the system over many such cycles.

We assume that the evolution of the secondary is unaffected by the companion until it becomes a CV. We therefore generate a grid of secondaries described by their mass and central hydrogen abundance. These two parameters are initial conditions for phase 4 and uniquely describe the behavior of the system at the onset of the mass transfer and along the CV stage of evolution.

2.1. Stellar Model

We used the Yale Rotating Stellar Evolution Code (YREC) as described in APS03 for the standard model physics of our code (equation of state, nuclear reaction rates, opacities, boundary conditions, and the heavy element mixture). We examined models with solar heavy-element abundance with a range of initial masses. Once the secondaries overfilled their Roche lobes we included mass loss in the stellar interior calculations as described below.

In order to account for mass loss, we remove mass from the outer convective region given the mass accretion rate and time step by simply decreasing the mass spacing uniformly. This corresponds to the assumption that the specific entropy of the convection is unaffected by the mass loss. We then allow the model to relax and evolve. The mass accretion rate is derived to preserve conservation of angular momentum, as described in § 2.2.

2.2. Angular Momentum and Its Loss

The orbital angular momentum of the system is given by

\[ J = (1 M_☉)^{5/3} G^{2/3} m_1 m_2 m^{-1/3} \omega^{-1/3}, \]  

while the angular momentum loss rate consists of two terms:

\[ \frac{dJ}{dt} = \left( \frac{dJ}{dt} \right)_{\text{grav}} + \left( \frac{dJ}{dt} \right)_{\text{wind}}. \]

The first term is the angular momentum loss due to gravitational radiation (Landau & Lifshitz 1962),

\[ \left( \frac{dJ}{dt} \right)_{\text{grav}} = -\frac{32}{5} \frac{G^{7/2}}{c^5} a^{-7/2} m_1^2 m_2^2 \sqrt{m \ M_☉^{9/2}}, \]

and the second is the magnetic braking term \((dJ/dt)_{\text{wind}}\) from a solar-like wind coming from the secondary. Here \(m_1, m_2\), and \(m\) are the white dwarf mass, secondary mass, and total mass, respectively, \(a\) is the separation between the stars, and \(\omega\) is the orbital speed.

For magnetic braking we use two different empirical prescriptions:

1. The Rappaport et al. (1983) model, based on early stellar spin-down data, which is assumed to operate only when the model is not fully convective.

2. The more recent empirical rule obtained from the spin-down of stars in young open clusters (Sills et al. 2000),

\[ \left( \frac{dJ}{dt} \right)_{\text{wind}} = -K_w \sqrt{\frac{r}{m}} \times \begin{cases} \omega^3, & \omega \leq \omega_{\text{crit}}, \\ \omega_{\text{crit}}^2, & \omega > \omega_{\text{crit}}, \end{cases} \]

where \(\omega_{\text{crit}}\) is the critical frequency at which the generation of the magnetic field saturates. Above this frequency the angular
momentum loss becomes linear (in \( \dot{\omega} \)) instead of to the third power. The constant \( K_\omega = 2.7 \times 10^{47} \) g cm s.

In the saturated braking model, we assume that the critical angular speed (which is set by the maximum magnetic field that the star is able to generate) is a unique function of the effective temperature of the star (the upper limit on the magnetic field is set by the effective temperature alone). Because the convection zone depth is primarily a function of effective temperature, this is not an unreasonable approximation. Evolved stars would have larger radii and longer convective overturn timescales than single stars of the same mass. This implies that the actual angular momentum loss rates for evolved secondaries would be somewhat lower than the value inferred for single stars. We include this effect in the first approximation, calculating the saturation frequency as a function of the effective temperature of the star but neglecting any dependence of \( \omega_{\text{crit}} \) on the star’s surface gravity. We therefore infer the angular momentum loss for a star by interpolation of the \( \omega_{\text{crit}}-T_{\text{eff}} \) relation for young open clusters presented in Table 1 of APS03. In this way we are able to take into account the change of magnetic braking for models of secondary stars with different evolutionary states.

### 2.3. Marginal Contact

We use the Eggleton (1983) approximation for the Roche lobe radius. In the “marginal contact” assumption, it defines the radius of a spherical stellar model that overfills its Roche lobe:

\[
R_2 = R_L = a \frac{0.49\left(m_2/m_1\right)^{2/3} + \ln \left[1 + \left(m_2/m_1\right)^{1/3}\right]}{0.6\left(m_2/m_1\right)^{2/3} + \ln \left[1 + \left(m_2/m_1\right)^{1/3}\right]}.
\]

Given the equatorial radius of the model and effective temperature provided by the YREC code, we derive the mass accretion rate (as described in APS03): it is calculated by requiring that equations (1), (2), and (5) be consistent with each other. The mass-loss rate is passed to the stellar code, which subtracts the amount of mass from the convective envelope given the mass-loss rate and time step, solves for the new structure, and evolves the model.

### 2.4. Starspots

We used a model similar to the one used by Spruit & Weiss (1986). The spots are assumed to be completely black and cylindrical, extending into the interior to a depth of 10 pressure scale heights. In our models, 50% of the surface area is covered by spots. The convective energy transport is completely suppressed within a spot. Following Spruit & Weiss (1986), we assume that convective energy transport through a spherical shell of radius \( R \) covered by spots becomes

\[
L_r = 4\pi r^2 F_0 (1 - f_s),
\]

where \( F_0 \) is the local energy flux in the areas outside the spots and \( f_s \) is the fractional area covered by spots.

The results of this model are described in § 5.2.2. In our model we completely ignore the radiative energy transport through the spots, so that we get the maximum possible effect on the stellar parameters; more realistic models of starspots would have smaller effects on the radius and luminosity of the star.

### 2.5. Rotational Mixing

Rotation can induce mixing through meridional circulation and other hydrodynamic mechanisms included in single-star models. The possible mixing of elements associated with the tidal deformation of a star has not been taken into account in models of CV secondaries. To estimate the importance of such an effect, we compare the thermal timescale of the radiative core with the timescales of mixing. For a qualitative estimate of this effect, we assume that large-scale currents caused by deformation have a characteristic timescale defined by \( \tau_{\text{KH}}/d \), where

\[
\tau_{\text{KH}} \approx \frac{GM^2}{RL}
\]

is the thermal (Kelvin-Helmholtz) timescale and \( d \) is a dimensionless parameter describing the departure from spherical symmetry. In the case of pure rotation, the departure from spherical symmetry is defined as the ratio of centripetal acceleration for a point on the equator to the gravity at this point,

\[
d \approx \frac{\omega^2 R^3}{GM}.
\]

If we are dealing with stars in a close binary system, the tidal force from the companion can be significant. For a qualitative order-of-magnitude estimate of this effect, we adopt the Roche model as a loose representation of the equipotential surfaces of the star and define the departure from spherical symmetry as

\[
d \approx \frac{\left|\psi(m_1)\right|_{R_{\text{rad}}+R_{\text{nd}}}-\left|\psi(m_1)\right|_{R_{\text{rad}}}}{\left|\psi(m_2)\right|_{R_{\text{rad}}}},
\]

where \( \psi(m_1) \) and \( \psi(m_2) \) are the gravitational potentials of the white dwarf and the secondary, \( a \) is the distance between the white dwarf and the center of the secondary, and \( R_{\text{nd}} \) is the size of the radiative core of the secondary. The results can be found in § 5.2.2.

### 3. RESULTS

The overall framework of our models discussed in § 2 is similar to that in APS03. Our approach is different from that of APS03 in the use of full stellar models to calculate the radius as a function of time, instead of assuming a mass-radius relationship. This allows us to calculate models with arbitrary initial evolutionary state and abundance, as well as models in which mass loss is sufficiently large to drive secondaries out of thermal balance. Comparison of such models with observable quantities can help constrain the physics and population of CVs. These observable quantities include the following:

1. Secondary mass–orbital period relationship.—Within the framework described above, the mass-period relation might help constrain the evolutionary state and mass-radius relationship of CV secondaries. Given the insensitivity to the mass of the primary, we can use this to learn about the pre-CV phase of evolution.
2. Mass accretion rate.—As a function of the mass of the secondary or the orbital period, this might give clues about the angular momentum loss rate and the response of the secondary to mass loss (which is a function of the evolutionary and thermal state of the secondary). However, it is difficult to make an apple-to-apple comparison between the derived and observed accretion rates. The problem lies in the timescale associated with accretion. While the luminosity of CVs and therefore the instantaneous accretion rate are determined by the physics of accretion disks, model accretion rate is averaged over long periods of time ($10^7$ yr). This means that any comparison of a derived mass accretion rate with an observed one should be taken with extreme care. We chose the recently observed data by Townsley & Bildsten (2003). They derived mass accretion rates by measuring white dwarf temperatures and calculating the impact of accretion on the thermal state of the white dwarf. The mass accretion rate measured in this way is effectively the time-averaged rate over a timescale of about $10^3$ yr.

3.1. Saturated Magnetic Braking

As discussed in APS03, modern studies of the angular momentum evolution of low-mass stars require a much milder angular momentum loss rate than that typically used for CVs. The angular momentum loss rate of fast rotators saturates at some value, and above this threshold it scales linearly with $\omega$ instead of having a $\omega^3$ dependence. This has dramatic effects on the properties of CVs. As shown in the previous paper, these angular momentum loss rates are not sufficiently large to drive the mass accretion high enough to cause the secondary to depart from normal thermal equilibrium and puff up. The secondary is then at a normal thermal balance during its lifetime as a secondary in a CV.

The evolutionary tracks for our saturated models are shown in Figure 1. Solid lines represent the models starting from the zero-age main sequence (ZAMS) with different initial secondary masses. The thick solid line is a model with an initial secondary mass of 0.9 $M_\odot$; dashed lines represent models with the same initial mass but with different initial evolutionary status (central hydrogen content). For low-mass tracks there is a noticeable bump when the system becomes fully convective, even for the lower saturated angular momentum loss rates. This happens at an orbital period of about 3 hr; the system reestablishes contact at a slightly lower period (around 2.75–2.80 hr). This is related to the sudden mixing of He$^3$ when the star becomes fully convective. Originally this He$^3$ mixing was proposed to be a cause of the period gap (D’Antona & Mazzitelli 1982). Consistent with prior results (e.g., McDermott & Taam 1989), we find that the width of this feature and the timescale are certainly not wide enough and long enough to explain the CV period gap between 2 and 3 hr. We therefore conclude that a small period gap would exist even in the case in which the mass transfer rates are insufficient to drive the secondary out of thermal equilibrium, but only when most of the CV secondaries have a central helium abundance close to the ZAMS value. The evolved tracks do not follow a unique evolutionary path $P(m_2)$ in the sense in which unevolved ones do.

3.2. Disrupted Unsaturated Magnetic Braking

In this section we summarize the results of the models with magnetic braking in the form proposed by Rappaport et al. (1983). In these models the mass accretion rate is so high that the timescale for mass loss becomes shorter than the Kelvin-Helmholtz timescale. The model is out of thermal balance and becomes larger for a given mass than a ZAMS star. When the star becomes fully convective, the torque from the magnetic wind is shut down. The star then detaches and relaxes to its normal radius in a thermal timescale. When contact is reestablished, the driving mechanism of the CV is angular momentum loss by gravitational radiation alone. It is not efficient enough to drive the star out of thermal balance, and the subsequent evolution proceeds with a considerably slower pace than when magnetic braking is operating. We ran the same set of models that we did for the unsaturated prescription.

The mass-loss rate for models with unsaturated braking are shown in Figure 2. ZAMS models roughly reproduce the period gap. However, more evolved ones fail to do it. They become fully convective at periods lower than 3 hr and become “active” again (and therefore “visible”) at periods shorter than 2 hr. In addition, the period gap becomes smaller and smaller at lower $X_c$, becoming quite narrow and completely vanishing for models with an initial central hydrogen abundance below 0.2. Figure 3 shows the boundaries of the gap as a function of central hydrogen abundance.

This might be a significant problem for the “disrupted magnetic braking model” as an explanation of the period gap. The observed mass-period relation indicates that evolved secondaries constitue a considerable fraction of all CVs (see Patterson 1984; Smith & Dhillon 1998). From a theoretical view, evolved models are also very well motivated; the secondary had to evolve during the main-sequence evolution of the pre-CE binary and during the pre-CV phase. The gap would be washed out by the population of evolved systems, which have a narrower gap at lower periods or do not have it at all.

Recently, Podsiadlowski et al. (2003) have examined this issue. Consistent with what we report here, they found that with the old angular momentum loss model, evolved secondaries
would produce a narrower gap shifted to shorter periods. They then relied on population synthesis models to claim that the period gap would nonetheless persist, largely because the evolved secondaries only dominate the population at long periods. It is difficult to quantify the errors in such synthesis models, which depend on a series of ingredients that are difficult to test. Quite apart from the uncertainty in the braking law, the outcome of the CE phase has a profound impact on such synthesis models, and our knowledge of this process is limited. For this reason we believe that the best approach is to identify regimes in which the various physical effects can be separated; the spread in mass at a fixed period just above the period gap does seem to provide such a diagnostic tool.

3.3. The Secondary Mass–Orbital Period Relationship

The secondary mass–orbital period relationships for our models are shown in Figures 4 and 5. The data are taken from Smith & Dhillon (1998). As has been shown before (e.g., Baraffe & Kolb 2000; APS03), models with different evolutionary states are able to reproduce the observational spread in the mass of the secondary for a given orbital period. Here we compare the results obtained for two magnetic braking prescriptions with a full calculation of stellar structure and therefore nuclear burning along the CV phase. The results are shown in Figures 4 (saturated braking) and 5 (unsaturated braking).

The main conclusion is that both prescriptions can generate a spread in $M$ at fixed $P$ that roughly matches that seen in the data. The existence of such a spread is therefore primarily a diagnostic of a mix of evolutionary states rather than a test of the braking law. However, there is a difference in the amount of spread for periods slightly above the period gap (3–4 hr), where the saturated prescription produces a larger range than the unsaturated model does. With improved statistics, the mass distribution in this period range could be a test of the empirical braking laws.

4. UNCERTAINTIES OF THE MODEL

Our models are based on the application of angular momentum loss inferred for single stars to the case of close binaries. Two major questions arise here:

1. How precisely is the angular momentum loss rate constrained for the single stars?
2. Is an empirical angular momentum loss rate derived for the single stars applicable to the case of close binaries?

We address these two questions in this section. We demonstrate that angular momentum loss rates for single rapidly rotating stars are constrained relatively well. The answer to the second question is less clear. There is no strong decisive theoretical reasoning or observational evidence that a star in a binary would suffer angular momentum loss similar to (or different from) that of a single star rotating at the same rate.

4.1. Magnetic Braking in Single Stars Revisited

Historically, an angular momentum loss rate was derived using the average rotational velocity ($V \sin i$) of stars (or rather, the distribution of rotations) in populations of different ages. An angular momentum loss rate is assumed to have some functional dependence on mass, radius, effective temperature, and rotational rate of the star, which is motivated by theoretical considerations and then calibrated using rotational data. Besides the observational errors, there are two main ingredients that determine the precision with which the angular momentum loss rate is constrained:

1. The assumed initial conditions.
2. The ages of the stellar populations that are used.

There are other ways of testing models of magnetized stellar winds. The X-ray luminosity of a star is one of the fingerprints of magnetic activity, and therefore it can be used to constrain the properties of the stellar magnetic fields. The observed saturation of $L_X/L_{bol}$ at high rotation rates has been used as evidence for a saturation in angular momentum loss at high rotation rates (MacGregor & Brenner 1991). Recently, Ivanova & Taam (2003) used this method to demonstrate that an alternate functional form for angular momentum loss rate that rises more steeply with increasing rotation rate is consistent with X-ray data. This is a valuable test of the idea that the magnetic field cannot increase indefinitely with increased rotation rate.

Using the magnetic wind model of Mestel & Spruit (1987), they suggested a braking law in a different form. Based on X-ray luminosity data for fast rotators from Pizzolato et al. (2003), they adopted the following functional dependence:

$$\frac{dJ}{dt} \sim \begin{cases} -\omega^3, & \omega \leq \omega_{\text{crit}}, \\ -\omega^{1.3}, & \omega > \omega_{\text{crit}}. \end{cases}$$

To test this prescription they compared the prediction of their model with the rotational velocities of stars in the Pleiades and the Hyades and of the Sun. Given the assumed values of the initial rotation rate of a solar mass star (close to breakup on ZAMS) and a Pleiades age of 70 Myr, they claimed that their braking prescription provides a better fit to the data than the prescription used in APS03. Such an angular momentum loss formula would predict time-averaged mass accretion rates an order of magnitude higher than that we could expect from a saturated law but lower than the unsaturated prescription by a comparable factor.

In this section we demonstrate that it is difficult to match the suggested braking law with the value of maximum rotational velocity of solar mass stars in open clusters of different ages. We used data on the rotations of stars in four clusters (instead of the two used by Ivanova & Taam [2003]). Furthermore, we assumed a much more realistic initial rotation rate, close to the fastest rotating ZAMS stars and to the initial angular momentum of protostars as discussed by Tinker et al. (2002). In addition, we adopted a more recent estimate for the age of the Pleiades of 130 Myr (Stauffer et al. 1998; Martin et al. 1998).

The equatorial rotational velocity of a single star with solar mass and composition as a function of age for three different prescriptions is shown in Figure 6. The initial pre-main-sequence star was assumed to have a rotation period of 3 days, which would produce a ZAMS star close to the fastest observed rotators and which corresponds to the upper envelope of observed pre-main-sequence rotation rates. Starting in the pre-main-sequence, a star is allowed to spin up as it shrinks and spin down as it loses angular momentum. All magnetic braking laws were calibrated to produce the solar rotation rate at the age of the Sun. The data points are the maximum observed rotational velocities for solar mass stars in four different clusters; the Hyades (age $\sim$600 Myr), the Pleiades (age $\sim$130 Myr), $\alpha$ Per (age $\sim$60 Myr), and combined data for the young clusters IC 2391 and IC 2602 (age $\sim$30 Myr). The open data point denotes the age of the Pleiades of 70 Myr, used by Ivanova & Taam (2003), which was used until recently in many spin-down studies. However, recent brown dwarf lithium age estimates require an older age of about 120–130 Myr (Stauffer et al. 1998; Martin et al. 1998).

While the prescription in equation (6) works for the assumptions stated by Ivanova & Taam (2003), it can be seen that the suggested braking law provides a worse fit to the data when all the data are accounted for. If we decrease the saturation threshold for a solar mass star from 10 to 6 $\omega_{\text{z}}$, the prescription seems able to reproduce the angular momentum evolution for young clusters, significantly overestimating the rotation rate for stars of the age of the Hyades.

It has been shown that it is important to use all available data and proper initial conditions to constrain the spin-down properties of fast rotators. However, even if we assume that
4.2. Is Magnetic Braking Different in Single Stars and CVs?

One of the assumptions in our approach is that the empirical angular momentum loss rates derived for single stars can be applied to close binaries. We assume that the gravitational field of a close companion does not affect the dynamo or the internal structure of the star and that it does not affect the properties of the stellar wind.

Stars in close binaries show on average a higher level of magnetic activity than single stars of the same spectral type (Simon & Fekel 1987; Schrijver & Zwaan 1991). However, this would be expected because of the higher (on average) rotation rates in tidally locked binaries, which complicates the question of whether there is some additional mechanism that enhances the generated magnetic field. Basri (1987) claimed that the differences vanished when this was taken into account.

There are theoretical models of enhanced dynamo activity in tidally locked binaries (e.g., Zaqarashvili et al. 2002); however, there is no strong observational evidence for such effects. We therefore conclude that the question about the applicability of the empirical rules for single stars to the case of close binaries does not have an obvious answer. In particular, the saturation threshold could potentially be affected by the presence of a companion. However, the clear evidence for angular momentum loss in fully convective stars implies that there is no good physical basis for invoking a sharp decrease in magnetic braking as the explanation of the CV period gap.

4.3. Time-averaged Mass Accretion Rate

The main criticism of the application of the saturated magnetic braking derived for single stars to the case of CVs is that predicted mass accretion rates are much smaller than those derived from the bolometric luminosities of CVs (e.g., Ivanova & Taam 2003). While such arguments are subject to large uncertainties from our limited understanding of accretion disk physics, it is clear that an appropriately measured and rescaled mass accretion rate should be an important test for the models of evolution of CVs. The problem that is encountered is that there is no direct comparison between the observed and theoretical mass accretion rates. Even if we disregard observational uncertainties, the observed mass accretion rate usually represents the instantaneous value, while the theoretical represents the value averaged over quite long timescales (over many cycles of nuclear outbursts) of about \(10^7\) yr. For obvious reasons, if the mass accretion rate is a variable function of time, the averaged mass accretion rate should be different from the instant one. Therefore, it is important to measure the time-averaged accretion rate.

Recently, Townsley & Bildsten (2004) devised a novel way to measure time-averaged mass accretion rates in dwarf novae (DNs), determining the effective temperature of a white dwarf. Applying this method to about 30 DNs (Townsley & Bildsten 2003), they found that above the period gap, unsaturated models overestimate the time-averaged mass accretion rate, and they slightly underestimate it below the gap.

Figure 7 shows the comparison of their observations with our derived time-averaged mass accretion rates. The observed

\[\text{[Fig. 7.—Mass accretion rate as a function of period. Data were taken from}\]

Townsley & Bildsten (2003). The three panels show the derived mass accretion rate for white dwarf masses of 0.6, 0.85, and 1.1, respectively, given the obtained mass accretion rate per unit surface area. The four different lines represent models with unsaturated and saturated prescriptions with the secondary star on ZAMS or pre-evolved to central hydrogen abundance \(X_c = 0.1\). In all cases an initial secondary mass of \(0.9 M_{\odot}\) was used. [See the electronic edition of the Journal for a color version of this figure.]
mass accretion rate per unit surface was converted to a time-averaged mass accretion rate for three different white dwarf masses (0.6, 0.85, and 1.1 \(M_\odot\)). We ran models with an initial secondary mass of 0.9 \(M_\odot\), both from the ZAMS and pre-evolved to a central hydrogen abundance of \(X_c = 0.1\). We used both an unsaturated prescription and a saturated (APS03) one.

The first conclusion is that if we assume the average mass of the white dwarf to be 0.6 \(M_\odot\), the predicted mass accretion rate below the period gap would not match the data for any magnetic braking or evolutionary state of the secondary assumed. If we increase the white dwarf mass, the match with data becomes much better. This is in accord with the conclusion of Patterson (1984) that white dwarfs in CVs on average have a higher mass than single CO white dwarfs (around 0.6 \(M_\odot\)). This is most probably a manifestation of the first episode of accretion when the secondary overfills the Roche lobe for the first time and has a mass too large to accrete stably onto the white dwarf. In this case the stellar envelope expands as a result of accretion. Such accretion happens on a dynamical timescale, resulting in an accumulation of matter on the white dwarf and therefore an increase of its mass.

The second and more important conclusion is that for assumed masses of white dwarfs higher than those in the field, the data lie between the derived time-averaged mass accretion rate for both prescriptions for magnetic braking. Therefore, it is incorrect to say that the saturated magnetic braking law provides a worse fit to the data for mass accretion than the unsaturated braking law does.

Another important conclusion is that if we assume that the observed mass accretion rate is close to the actual values with correctly calculated uncertainties, then the mass-loss rates observed would be insufficient to drive the secondary star out of thermal equilibrium, and therefore this would kill the purpose of introducing the extremely efficient unsaturated braking to produce the period gap. Ivanova & Taam (2003), who recently suggested a different form of magnetic braking based on the X-ray activity of young stars, came to the same conclusion. This would also imply an intermediate torque for a given period between those predicted by saturated and unsaturated prescriptions.

The final thing to note in this section is that the measured mass accretion rate is averaged over the thermal time of the radiative envelope of the white dwarf (\(\approx 10^3\) yr), while we compare it with the theoretical estimates of mass accretion rates averaged over many cycles of nuclear nova ejections (\(\approx 10^7\) yr). If we assume that mass accretion happens in duty cycles, it would make the match between observations and saturated models much better, at the same time moving the observed values away from the derived mass-loss rates of the unsaturated prescription. So in this sense, the saturated prescription is preferable for providing a match between the theory and observations of mass accretion rates.

5. SUMMARY, CONCLUSIONS, AND SPECULATIONS

5.1. Summary

We have used the stellar evolutionary code to calculate full models of the secondary stars in CVs in the attempt to understand how different prescriptions for magnetic braking and the evolutionary state of the secondary result in the observational properties of CV populations. The main features that we have focused on are the mass accretion rate, the mass-period relationship, and the properties of the period gap in the distribution of CVs. Our main results are as follows:

1. All models are extremely sensitive to the evolutionary state of the secondary and less sensitive to the mass of the white dwarf.

2. The models with the saturated prescription for magnetic braking dictated by the data on the spin-down of single stars do not reproduce the period gap. Models that are close to ZAMS show a shrinkage at the periods corresponding to the transition to a fully convective star, associated with the sudden mixing of He\(^+\). This feature is not broad enough (about 0.1 hr) to produce the period gap. Models with evolved secondaries do not have this feature.

3. The saturated prescription for braking is able to reproduce the size and position of the gap, but only for unevolved secondaries. It fails to produce a well-defined period gap for highly evolved systems. It is unclear to us whether CE physics is reliable enough and synthesis models are detailed enough to make predictions on the effect of evolved models on the population of CVs.

4. The data for the mass accretion rate (Townsley & Bildsten 2003) lie between the time-averaged mass accretion rates derived for saturated and unsaturated prescriptions. It is impossible to prefer one model over the other based only on this comparison with observed accretion rates. However, if we assume that there is a duty cycle of accretion, the match with saturated braking could be better.

5. The average mass of the white dwarfs in CVs is required to be higher than that of single white dwarfs to match the mass accretion rate in CVs below the period gap for any prescription for magnetic braking.

6. Both prescriptions are able to reproduce the spread in the period-mass relation if evolved secondaries are included. However, for periods between 3 and 4 hr, the scatter in mass for a given period is considerably narrower for the unsaturated prescription, and therefore better statistics with smaller uncertainties in this range could potentially be a good test of the braking law for CVs. Podsiadlowski et al. (2003) predict that unevolved stars dominate the CV population from 3 to 4.7 hr; with sufficient statistics this is testable. If Podsiadlowski et al. (2003) are correct, one would therefore expect stars just above the period gap to cluster around the unevolved mass-period relationship.

7. We gave order-of-magnitude estimates of the effects not included in the recent generation of models: mixing associated with tidal distortion of the secondary star and the effect of magnetic spots on the radius. We found that these effects might be important and therefore should be included in more sophisticated calculations.

Given the results above, it is clear that the danger of elevating the hypothesis of disrupted magnetic braking to a conclusion is quite real. Therefore, it seems important to look for other possible mechanisms for the formation of the period gap instead of the disrupted magnetic braking model. We summarize the possible scenarios below. The detailed quantitative investigation of these possibilities, however, will be the subject of some other work.

5.2. Speculations about the Origin of the Period Gap

In this section we explore the following categories of solutions:

1. A nonequilibrium period distribution (finite-age effect).
2. Changes in the mass-radius relation, possibly due to the effects of spots and tidal mixing.
3. Different populations of CVs.

5.2.1. Finite-Age Effect

The timescale for CV evolution could be long, comparable to the age of the Galaxy, if the angular momentum loss rates inferred from young single stars are applicable for CVs (see Fig. 1). An example would be a 0.9 \( M_\odot \) secondary going from an initial period of 6.5 hr to 1.5 hr in 9 Gyr.

This opens up the possibility for a period gap formation mechanism that is usually not considered: the finite-age effect. The distribution of binaries would not be steady state, and characteristics of it would be considerably dominated by the dependence of the injection rate on time.

This idea has support from recent research on star formation history in the local disk. Majewski (1993) summarizes the efforts to derive the SFR in the disk from the ages of a volume-limited sample of F-G stars (Barry 1988), the white dwarf luminosity function (Noh & Scalo 1990), and the frequency distribution of lithium abundances in red giants (Brown et al. 1989). He concludes that there is evidence for SFR fluctuations of an order of magnitude and identifies three major bursts of star formation, separated by quiescent phases:

1. A star formation epoch from 11 to 7 Gyr ago (star formation burst C).
2. A star formation epoch from 6 to 3 Gyr ago (burst B).
3. An ongoing burst of star formation, starting 2 Gyr ago (burst A).

Naturally, these variations of the SFR should have their imprint on the period distribution of CVs if their evolution is really as slow as predicted by the saturated braking law. In this case it would be logical to identify the gap as the quiescent phase in star formation (between burst A and B, or B and C). Moreover, if we tend to interpret the slight increase in the number of systems with periods of about 7 hr in the uniformly declining tail of CVs above the period gap as a feature associated with burst A, then it would be logical to conclude that the gap is formed as a result of quiescent SFR between bursts B and C. Therefore, all three spikes in the period distribution of CVs (at 2, 3, and 7 hr) could be identified as direct fingerprints of SFR bursts C, B, and A, respectively. However, it would be difficult to precisely test this idea because of the many factors contributing to the CV population.

The test for this hypothesis would be improved statistics for CVs in a single-age environment (stellar clusters) or a uniformly old population (the Galactic halo). If the period distribution were different from that observed in the solar neighborhood, it would be a clear indication that the CV population was not in equilibrium.

5.2.2. Possible Mechanisms for a Change in the Mass-Radius Relationship

There are two physical effects that we have not included in our models: (1) structural changes in CV secondaries arising from high spot coverage and (2) rotation-induced mixing in the cores of chemically evolved secondaries. These mechanisms change the mass-radius relationship and must be included in full models of stars in close binaries (as was shown in § 4); they would affect the period of the CV for a given mass of the secondary. However, these mechanisms are doubtfully responsible for the formation of the period gap, because neither of the two mechanisms shows any abrupt behavior close to 3 hr.

5.2.2.1. Starspots

There is reasonably compelling circumstantial evidence that rapidly rotating low-mass stars (above the fully convective boundary) have larger radii than predicted by standard stellar models. The radii of fully convective stars are in accord with the predictions of theoretical models. This creates the possibility that there could be a change in the mass-radius relationship near the fully convective boundary that could partially explain—or even cause—the CV period gap. In this paper we have used standard stellar models; those that begin the CV phase with significant nuclear evolution follow a different mass-radius relationship from that of unevolved models.

Recently, there has been a significant increase in the quantity and quality of fundamental data for lower main-sequence stars. This is largely due to surveys that have discovered several eclipsing binary systems (Ribas 2003; Torres & Ribas 2002) and interferometric radius measurements (Ségransan et al. 2003). The radii of eclipsing binaries are observed to be close to theoretical expectations for the lowest mass stars \( (M < 0.3 M_\odot) \) and for higher mass stars \( (M > 0.8 M_\odot) \). However, the radii of intermediate-mass stars appear to be systematically larger than predicted by stellar interior models (Ségransan et al. 2003).

Strong observational selection effects favor the detection of tidally synchronized eclipsing binaries, which rotate significantly faster than typical low-mass field stars. There is intriguing evidence that rapidly rotating lower main-sequence stars have large starspot covering factors; these have recently been implicated as the cause of anomalies in photometric color-magnitude diagrams in young open clusters (Stauffer et al. 2003). In the presence of large covering factors, the radii of the models could be affected. This problem was explored by Spruit & Weiss (1986), and the results of their exploratory calculations are in the right sense to explain the observed trends. For stars with deep convective envelopes and radiative cores, the luminosity is insensitive to the boundary conditions, and large cool spots will tend to increase the radius while holding the luminosity nearly constant. For fully convective stars, by contrast, changes in the surface layers have a direct impact on the central temperature; spots tend to lower the mean surface effective temperature and luminosity, leaving the radii almost unaffected.

The relative changes in radius and mass of a star with a spot coverage of 50% are shown in Figure 8. Spots are assumed to be completely black. This sets an upper limit on the fractional change of stellar parameters; real spots would have some radiative energy transport through them would have less impact on a star. The change of radius is positive compared to an unspotted star, while the change in luminosity is negative. While change in radius can be significant (up to 15%) for stars with masses of 1.0–1.2 \( M_\odot \), this mechanism does not produce any abrupt transition in the interesting region of 0.2–0.4 \( M_\odot \). In addition, it is very small in this region (1%–2%).

5.2.2.2. Rotational Mixing

The second potential mechanism for changing the mass-radius relationship is rotational mixing. Our main result for chemically evolved secondaries in CVs is that they both follow a different mass-radius relationship from that of chemically unevolved stars and that they become fully convective at a different mass. Once the stars are fully convective, all of the
tracks converge on a narrow range of radii at a given mass; modest differences in the envelope helium abundance have little impact on the radius. These models, however, do not consider rotational mixing in the radiative core. Such mixing could be driven by the internal rotation of the secondary (see Pinsonneault [1997] for a discussion of some of the physical mechanisms), or it could be induced by tidal distortions from spherical symmetry. The timescale for mixing depends sensitively on the rotational period of the system; if the timescale for mixing could drop below the timescale for angular momentum loss at a critical period, this would cause a transition from a large range of radii for a given secondary mass to a small one, as the evolved stars tend toward the radii appropriate for single stars. One complicating theoretical uncertainty is the degree to which mean molecular weight gradients inhibit mixing. The appropriate timescales for three different models are shown in Figure 9.

The models in Figure 9 have a central hydrogen abundance at the onset of mass accretion of 0.7, 0.3, and 0.1, respectively. The upper curves show the thermal timescales of the radiative core for all three models as a function of rotational period. The curves in the middle represent the minimum timescale for meridional currents in the radiative interior for the stars rotating at this period.

The timescale estimates above represent only the minimum possible values; there are two important inhibiting effects. Mean molecular weight gradients can inhibit rotational mixing (Mestel 1953; Maeder & Zahn 1998) by developing a latitude-dependent $\mu$-profile. Horizontal turbulence arising from latitudinal differential rotation will tend to homogenize level surfaces, and the net impact of $\mu$-gradients will depend on the (uncertain) balance between the two. Second, even in the absence of $\mu$-gradients, horizontal turbulence will tend to decrease the efficiency of rotational mixing relative to angular momentum transport. This effect can be considerable; Pinsonneault et al. (1989) found that the diffusion coefficients for composition mixing in the Sun were $\sim30$ times smaller than those for angular momentum transport (see Chaboyer & Zahn [1992] for a theoretical exploration of this issue). In the Zahn (1992) framework, this efficiency factor is a function of position within a stellar model. Exploring these effects is beyond the scope of the current paper, and they are the subject of another paper in preparation. However, it is clear that rotational mixing can occur on an interesting timescale for evolved secondaries.

5.2.3. Different Populations of CVs

Another possibility for the origin of a period gap is that there could be different populations of CVs that have different equilibrium period distributions. A bimodal distribution could be produced if there were two distinct populations. Some of the possible sources of such effects could include the following:

1. Populations might be separated by white dwarf masses (this was first raised as a possibility by Webbink [1979] and described and compared with others by Verbunt [1984]). The two populations in this scenario are He and CO white dwarf primaries. CVs are systems that form from binaries that undergo CE evolution. They should have a considerable fraction of He white dwarfs; these form from systems in which runaway accretion from the expanding primary onto the secondary happens when the primary is on the red giant branch (see De Kool 1990 for an example). However, He white dwarfs have a considerably lower mass than CO ones (about $0.35\sim0.4M_\odot$ for the former and more than $0.6M_\odot$ for the latter). As a result, the initially lower mass He white dwarfs should accrete matter in a runaway process until the mass ratio of the primary to the secondary is high enough to allow stable accretion. This initial phase of accretion should increase the mass of the white dwarf, making subsequent CV evolution quite

Fig. 8.—Relative change in radius and luminosity of a star with an effective spot coverage of 50% as a function of mass. Spots are assumed to be completely black (see text). [See the electronic edition of the Journal for a color version of this figure.]

Fig. 9.—Timescales as a function of orbital period for three models with different secondary evolutionary states at the onset of accretion. The mass of the primary is $0.85M_\odot$ and the initial secondary mass is $0.9M_\odot$. The solid lines show the ZAMS model, dotted lines represent a model with initial $X_c = 0.3$, and dashed lines, a model with initial $X_c = 0.1$. The upper curves show the thermal timescales of a radiative core for all three models as a function of rotational period. Curves in the middle represent the minimum timescale for the classical Eddington-Sweet meridional currents at the base of the convection zone. The bottom lines represent the minimum rotational mixing timescale including tidal distortion effect. [See the electronic edition of the Journal for a color version of this figure.]
indistinguishable from that of a CV with an initially CO white dwarf. In addition, there is no obvious white dwarf mass–CV period correlation (e.g., Patterson 1984). Therefore, although different initial white dwarf masses are probably present, it is doubtful that they can explain the period gap by themselves.

2. Populations could be separated by the evolutionary state of the secondary. Systems with considerably evolved secondaries on average tend to have larger orbital periods. The larger radii of evolved systems combined with a possible change in the mass-radius relation (close to ZAMS) at the low-mass end (due to spots or mixing) could be a potential mechanism for the formation of a period gap.

3. Populations could be separated by the output of the CE phase. This is reminiscent of effect 1, with the difference that instead of having the mass of the white dwarfs divide all CVs into distinct populations, the mass of the secondary and its orbital period serve this purpose. If the systems with low initial secondary mass (0.1–0.3 \(M_\odot\)) are close to contact after the CE phase, they become CVs almost immediately. The systems with intermediate secondary masses (0.3–0.8 \(M_\odot\)) have a larger average separation than those with low-mass secondaries. This occurs because part of the initial binding energy of a binary goes to expelling the envelope; therefore, more massive secondaries do not need to move very close to a white dwarf to expel the envelope (e.g., De Kool 1990). A combination of larger separations with inefficient angular momentum loss can lead to the situation in which the characteristic timescale between the CE and CV phases exceeds the age of the Galaxy. For even more massive secondaries (0.8–1.2 \(M_\odot\)), the separation is even larger. But at the same time, the saturation frequency for magnetic braking increases exponentially with mass; therefore, systems with higher mass secondaries would evolve into contact much more rapidly. It therefore might be possible for the CV source function to have two spikes, at lower and at higher secondary masses only, with few systems born that have intermediate-mass secondaries. This could affect the period distribution in much the same fashion that the finite-age effects do.

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REFERENCES

Andronov, N., Pinsonneault, M., & Sills, A. 2003, ApJ, 582, 358 (APS03)
Baraffe, I., & Kolb, U. 2000, MNRAS, 318, 354
Barry, D. C. 1988, ApJ, 334, 436
Basri, G. 1987, ApJ, 316, 377
Beuermann, K., Baraffe, I., Kolb, U., & Weichhold, M. 1998, A&A, 339, 518
Brown, J. A., Sneden, C., Lambert, D. L., & Dutchover, E. 1989, ApJS, 71, 293
Chaboyer, B., & Zahn, J.-P. 1992, A&A, 253, 173
Charbonneau, P., & MacGregor, K. B. 1997, ApJ, 486, 502
D’Antona, F., & Mazzitelli, I. 1982, ApJ, 260, 722
De Kool, M. 1990, ApJ, 358, 189
Eggleton, P. 1983, ApJ, 268, 368
Iben, I., & Livio, M. 1993, PASP, 105, 1373
Ivanova, N., & Taam, R. E. 2003, ApJ, 599, 516
Keppens, R., MacGregor, K. B., & Charbonneau, P. 1995, A&A, 294, 469
Krishnamurthi, A., Pinsonneault, M. H., Barnes, S., & Sofia, S. 1997, ApJ, 480, 303
Landau, L. D., & Lifshitz, E. M. 1962, The Classical Theory of Fields (2nd ed.; Oxford: Pergamon)
Lanza, A. F., Rodono, M., & Rosner, R. 2000, MNRAS, 314, 398
MacGregor, K. B., & Brenner, M. 1991, ApJ, 376, 204
MacGregor, K. B., & Charbonneau, P. 1997, ApJ, 486, 484
Maeder, A., & Zahn, J.-P. 1998, A&A, 334, 1000
Majewski, S. R. 1993, ARA&A, 31, 375
Martin, E. L., Baraffe, G., Gallegos, J. E., Rebolo, R., Zapatero-Osorio, M.-R., & Bejar, V. J. S. 1998, ApJ, 499, L61
McDermott, P. N., & Taam, R. E. 1989, ApJ, 342, 1019
Mestel, L. 1953, MNRAS, 113, 716
Mestel, L., & Spruit, H. C. 1987, MNRAS, 226, 57
Montesinos, B., Thomas, J. H., Ventura, P., & Mazzitelli, I. 2001, MNRAS, 326, 877
Noh, H. R., & Scalzo, J. 1990, ApJ, 352, 605
Parker, E. N. 1993, ApJ, 408, 707
Patterson, J. 1984, ApJS, 54, 443
Pinsonneault, M. H. 1997, ARA&A, 35, 557
Pinsonneault, M. H., Kawaler, S. D., Sofia, S., & Demarque, P. 1989, ApJ, 338, 424
Pizzolato, N., Maggio, A., Micela, G., Sciortino, S., & Ventura, P. 2003, A&A, 397, 147
Podsiadlowski, P., Han, Z., & Rappaport, S. 2003, MNRAS, 340, 1214
Rappaport, S., Verbunt, F., & Joss, P. C. 1983, ApJ, 275, 713
Ribas, I. 2003, A&A, 398, 239
Schrijver, C. J., & Zwaan, C. 1991, A&A, 251, 183
Ségransan, D., Kervella, P., Forveille, T., & Queloz, D. 2003, A&A, 397, L5
Sills, A., Pinsonneault, M. H., & Terndrup, D. M. 2000, ApJ, 540, 489
Simon, T., & Fekel, F. 1987, ApJ, 316, 434
Skumanich, A. 1972, ApJ, 171, 565
Smith, D. A., & Dhillon, V. S. 1998, MNRAS, 301, 767
Solokani, S. K., Motamen, S., & Keppens, R. 1997, A&A, 325, 1039
Spruit, H. C., & Weiss, A. 1986, A&A, 166, 167
Stauffer, J. R., Hartmann, L. W., Prosser, C. F., Randich, S., Balachandran, S., Patten, B. M., Simon, T., & Giammappa, M. 1997, ApJ, 479, 776
Stauffer, J. R., Jones, B. F., Backman, D., Hartmann, L. W., Barrado y Navascués, D., Pinsonneault, M. H., Terndrup, D. M., & Muench, A. A. 2003, AJ, 126, 833
Stauffer, J. R., Schulz, G., & Kirkpatrick, J. D. 1998, ApJ, 499, L199
Tinker, J., Pinsonneault, M. H., & Terndrup, D. 2002, ApJ, 564, 877
Torres, X., & Ribas, I. 2002, ApJ, 567, 1140
Townsley, D. M., & Bildsten, L. 2003, ApJ, 596, L227
———. 2004, ApJ, 600, 390
Verbunt, F. 1984, MNRAS, 209, 227
Warner, B. 1995, Cataclysmic Variable Stars (New York: Cambridge Univ. Press)
Webbink, R. F. 1979, in IAU Colloq. 53, White Dwarfs and Variable Degenerate Stars, ed. H. M. van Horn & V. Weidemann (Rochester: Univ. Rochester), 426
Weber, E. J., & Davis, L. 1967, ApJ, 148, 217
Zahn, J.-P. 1992, A&A, 265, 115
Zaqarashvili, T., Javakhishvili G., & Belvedere, G. 2002, ApJ, 579, 810