Energy scan by $\phi$ mesons and threshold energy for the confinement-deconfinement phase transition

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(Dated: May 19, 2010)

We argue that the ratio of $\phi$ mesons multiplicity over cube of the mean $p_T$ is proportional to the degeneracy of the medium produced in ultra-relativistic heavy ion collisions. The ratio extracted from the existing $\phi$ meson data in the energy range $\sqrt{s}=6.3-200$ GeV, indicate that beyond a threshold energy $\sqrt{s}_{th}=15.74 \pm 8.10$ GeV, the medium crosses over from a confined phase to a deconfined phase.

PACS numbers: 25.75.-q, 25.75.Nq, 25.75.Ag

Lattice QCD predicts [1, 2] that in ultra-relativistic heavy ion collisions, a confinement-deconfinement phase transition can occur, producing a new state of matter, Quark-Gluon Plasma (QGP). QGP is a collective state where color degrees of freedom become manifest over nuclear rather than hadronic volume. Recent experiments [3, 4] have produced convincing evidences for a confinement-deconfinement phase transition in $\sqrt{s}=200$ GeV Au+Au collisions at RHIC. One naturally wonders, whether or not there is a threshold energy for the transition. One of the aims of the STAR’s energy scan programme at RHIC is to determine the threshold energy for the confinement-deconfinement transition [7, 8]. STAR proposes to study nuclear collisions at $\sqrt{s}=5, 7.7, 11.5, 17.3, 27$ and $39$ GeV. With the existing data at $\sqrt{s}=62, 130$ and $200$ GeV, STAR will scan a large energy range, $\sqrt{s}=5-200$ GeV.

For long, strangeness enhancement is considered as a signature of QGP formation [9]. In QGP environment, $gg \rightarrow s\bar{s}$ is abundant. If not annihilated before hadronisation, early produced strange and anti-strange quarks will coalesce in to form strange hadrons and compared to elementary pp collisions, strange particle production will be enhanced. However, strangeness enhancement could also be obtained in a purely hadronic scenario, mainly due to the ‘volume effect’ [10–13]. Strangeness production in small volume elementary pp collisions can be ‘canonically’ suppressed due to ‘strict’ strangeness conservation [10–13]. In bigger volume AA collisions, locally, strangeness conservation condition can be relaxed to produce strange particles. In the language of statistical mechanics, while canonical ensemble is applicable in pp collisions, grand canonical ensemble is more appropriate in heavy ion collisions. Additionally, strange particle phase space appears to be under-saturated in elementary pp or peripheral heavy ion collisions [14, 15].

As noted in [16], several unique features of $\phi$ mesons make it an ideal probe to investigate medium properties in heavy ion collisions. They are hidden strange particle and are not affected by canonical suppression. Also they are not affected by resonance decay, have both hadronic and leptonic decay channels, mass and width are not modified in a medium [17], etc. Incidentally, experimental data for $\phi$ meson production in nuclear collisions over a large energy range $\sqrt{s}=6-200$ GeV exists. NA49 collaboration measured $\phi$ meson in 20A, 30A, 40A, 80A and 158A GeV Pb+Pb collisions [17]. The centre of mass energies are, 6.3, 7.6, 8.3, 12.3 and 17.3 GeV respectively. STAR collaboration measured $\phi$ mesons in Au+Au and Cu+Cu collisions at RHIC [18–20]. In Au+Au collisions $\phi$ mesons are measured at three energies, $\sqrt{s}=62, 130$ and $200$ GeV, and in Cu+Cu collisions at $\sqrt{s}=62$ GeV and $200$ GeV. In the following we argue that the existing $\phi$ meson data do indicate a threshold energy for the confinement-deconfinement phase transition.

Let us note the most distinguishing feature between a confined and a deconfined state. Effective degrees of freedom of a deconfined phase is considerably larger than that of a confined phase. As an example, in Fig.1 a recent lattice simulation [2] for the temperature depen-
Can we construct an experimental observable equivalent to \( s/T^3 \), i.e. degeneracy of the medium? Variation of the observable with collision energy then can answer, whether or not threshold energy exists for the confinement-deconfinement transition. We argue that the experimental observable, \( \frac{\langle N^0 \rangle}{(p_T^3)^{\frac{3}{2}}} \), ratio of the \( \phi \) meson multiplicity \( \langle N^0 \rangle \) and cube of the \( \phi \) mesons mean transverse momentum \( \langle p_T^3 \rangle \) is effectively proportional to \( s/T^3 \), i.e. the degeneracy of the medium. The argument is based on three assumptions: (i) viscous effects are small in relativistic energy heavy ion collisions, (ii) \( \phi \) mesons multiplicity is proportional to initial entropy density and (iii) \( \phi \) mesons mean \( p_T \) is proportional to initial temperature. Let us examine the assumptions in detail.

At RHIC energy collisions, experimental data at low \( p_T < 1.5 \text{GeV} \) are consistent with ideal hydrodynamics. Assumption (i) is approximately valid in RHIC energy. However, viscous effects can be substantial at lower SPS energy. Recently Petersen and Bleicher studied elliptic flow at SPS energy. It was shown that initial conditions can have substantial effect on development of the elliptic flow. With proper initial conditions and with gradual freeze-out, elliptic flow data at SPS energy can be explained in an ideal hydrodynamic model. In ideal hydrodynamic model predictions for \( p_T \) are consistent with ideal hydrodynamics reasonably well explained the spectra. Assumption of small viscous effect in relativistic heavy ion collisions seems to be reasonable at SPS energy also. If viscous effects are small, then assumption (ii) i.e. \( \phi \) meson multiplicity is proportional to initial entropy density, is also reasonable. It is generally believed that total multiplicity is proportional to final state entropy density. If viscous effects are small, entropy is not generated and initial and final state entropy is same. Experimental data on production cross-section of various particles and anti-particles show surprisingly good agreement with thermal abundances of a hadronic resonance gas. The assumption is valid in a thermal model. However, some of the thermal models use 'strangeness under saturation' factor to explain the data. Assumption (iii), \( \phi \) mesons mean \( p_T \) is proportional to initial temperature is approximately valid in an ideal hydrodynamic model. In ideal hydrodynamic model predictions for \( p_T \) in a central (0-10%) Au+Au collisions, as a function of spatially averaged initial temperature. At the initial time \( t_i = 0.6 \text{fm} \), initial energy density is assumed to be distributed as

\[
\varepsilon(b,x,y) = \varepsilon_i[0.75N_{\text{part}}(b,x,y) + 0.25N_{\text{coll}}(b,x,y)],
\]

In Eq.1 \( b \) is the impact parameter of the collision and \( N_{\text{part}} \) and \( N_{\text{coll}} \) are the transverse profile of the average

![FIG. 2: The black circles hydrodynamic model simulations for \( \phi \) mesons mean \( p_T \) as a function of spatially averaged initial temperature \( <T_i> \). The solid line indicate that in a hydrodynamic model, \( \phi \) mesons mean \( p_T \) depend linearly on the average initial temperature.](image1)

![FIG. 3: The filled symbols are the ratio of \( \phi \) meson multiplicity over the cube of the mean \( p_T \), normalised by participant number \( (N_{\text{part}}) \) and rapidity gap \( (\Delta Y) \), in \( \text{Pb+Pb, Cu+Cu and Au+Au collisions} \), as a function of the collision energy. The solid line is a fit to the ratio by the analytical form for the step function Eq.2 the dotted line is a power law fit.](image2)
number of participants and average number of binary collisions respectively, calculated in a Glauber model. Initial fluid velocity is zero, \( v_z(x,y) = v_p(x,y) = 0 \). Hydrodynamic equations are solved with the code AZHYDRO-KOLKATA, detail of which can be found in [30–32]. We have used an equation of state, with a cross-over transition from QGP phase to hadronic phase at temperature \( T_0 =196 \text{ MeV} \) [32]. For a set of central energy density, \( \varepsilon_i \), \( \phi \) mesons mean \( p_T \)'s are calculated at the freeze-out temperature \( T_F =150 \text{ MeV} \). In ideal hydrodynamics, a linear relation between \( \langle p_T^\phi \rangle \) and average initial temperature \( \langle T_i \rangle \) is accurately observed. It is interesting to note that the relation \( \langle p_T^\phi \rangle \propto \langle T_i \rangle \) will not be valid, say for the pions or for the strange mesons \( K^+/K^- \). Pions and kaons are largely affected by resonance decay, which spoils the relation. Resonances do not contribute to \( \phi \) meson production making the relation works.

NA49 and STAR collaboration have tabulated \( \phi \) multiplicity and mean \( p_T \) in Pb+Pb and Au+Au/Cu+Cu collisions in the energy range \( \sqrt{s} =6.3-200 \text{ GeV} \) [17–20]. In Fig.4 the experimental ratio \( \langle N^\phi \rangle/\langle p_T^\phi \rangle^3 \) is shown as a function of the collision energy. In Au+Au/Cu+Cu collisions, \( \phi \) mesons are measured in a number of collisions centrality. Presently, we chose the most central ones. To account for the differences in system size, collision centrality and rapidity gap \( \Delta Y \) in NA49 and STAR data sets, \( \phi \) multiplicity is normalised by the factor 0.5\( N_{\text{part}} \Delta Y \). Uncertainty in the experimental ratio \( \langle N^\phi \rangle/\langle p_T^\phi \rangle^3 \) is large due to the large experimental error in determination of \( \phi \) meson multiplicity and mean \( p_T \). For example, in low SPS energy, uncertainty in \( \phi \) multiplicity is as large as \( \sim 20-40\% \). At RHIC energy also, \( \langle N^\phi \rangle \) is determined only within \( \sim 10-20\% \) accuracy. Mean \( p_T \) is determined more accurately, in the energy range \( \sqrt{s} =6.3-200 \text{ GeV} \), uncertainty in \( \langle p_T^\phi \rangle \) is less than 10\%. Though error bars are large, the trend of the ratio, \( R = \langle N^\phi \rangle/[0.5N_{\text{part}} \Delta Y \langle p_T^\phi \rangle^3 \rangle \) as a function of the collision energy is evident. Mimicking the temperature dependence of \( s/T^3 \), the ratio sharply rises from low SPS energy to RHIC energy. In Fig.4 the solid line is a fit to the ratio with an analytical form for the step function, \( R = \alpha [1 + \tanh (\sqrt{s} - \sqrt{s_{th}})/\Delta \sqrt{s_{th}}] \). (2)

Analytical form Eq2, well explain the data, \( \chi^2/N = 0.3 \). Fitted values are \( \alpha = 0.017 \pm 0.009 \), \( \sqrt{s_{th}} = 15.74 \pm 8.10 \text{GeV} \) and \( \Delta \sqrt{s_{th}} = 14.52 \pm 14.93 \text{GeV} \). Threshold energy can be determined only within \( \sim 50\% \) accuracy, the width of the transition is uncertain by \( \sim 100\% \). One note that presently, no data exist in the energy range 17.3-62 GeV, between the top of SPS energy and bottom of the RHIC energy. STAR energy scan programme will fill up the gap. Threshold energy and width of the transition can be determined more accurately.

One may argue that fitting the ratio \( \langle N^\phi \rangle/\langle p_T^\phi \rangle^3 \) by an analytical form for the step function is rather arbitrary, the ratio could as well be fitted by another form, without any threshold energy. As an example, in Fig.5 we have shown a fit (the dashed line) to the ratio \( R \), by a power law, \( R = A \sqrt{s} \). Power law also explains the data, but with increased \( \chi^2/N = 0.92 \). Since the step function fit as well as the power law fit give \( \chi^2/N < 1 \), from the \( \chi^2 \) analysis point of view, both the fits are equivalent and it can not be claimed that the experimental ratio \( \langle N^\phi \rangle/\langle p_T^\phi \rangle^3 \) exhibit step function like behavior. Indeed, one does wonder, whether it can be verified that the ratio \( \langle N^\phi \rangle/\langle p_T^\phi \rangle^3 \) do indeed corresponds to \( s/T^3 \), i.e.
overestimate the step function fit by 10-20%. Close similarity between lattice simulated \(s/T^3\) and experimental ratio \(\langle N^\phi \rangle /\langle p_T^\phi \rangle^3\) do support our argument that the ratio \(\langle N^\phi \rangle /\langle p_T^\phi \rangle^3\) is proportional to \(s/T^3\) or the degeneracy of the medium. To confirm that the ratio \(\langle N^\phi \rangle /\langle p_T^\phi \rangle^3\) is indeed proportional to \(s/T^3\), using the usual thermodynamic relations,

\[
p(T) = \int_0^T s(T')dT' \tag{4}
\]

\[
\varepsilon(T) = Ts - p, \tag{5}
\]

we obtained pressure and energy density for the step function fit to the ratio \(\langle N^\phi \rangle /\langle p_T^\phi \rangle^3\) and computed square speed of sound \(c_s^2 = \frac{\partial p}{\partial \varepsilon}\). In Fig.6 the dashed line is \(c_s^2\) obtained from the step function fit to the experimental ratio \(\langle N^\phi \rangle /\langle p_T^\phi \rangle^3\). The solid line is \(c_s^2\) from a parametric representation of the lattice simulations for \(s/T^3\). For \(T > 200\) MeV, \(c_s^2\) from the step function fit to the experimental ratio closely agree with \(c_s^2\) in lattice simulation. \(c_s^2\) in lattice simulation show a dip at \(T \approx 200\) MeV, \(c_s^2\) from the step function fit to the ratio show a dip at \(T \approx 180\) MeV. However, at low temperature, \(c_s^2\) is comparatively larger in lattice simulation than in the step function fit to the ratio. We note that experimental data for the ratio \(\langle N^\phi \rangle /\langle p_T^\phi \rangle^3\) do not exit below temperature \(T \approx 170\) MeV and the step function fit may not be accurate at low temperature. In the inset of Fig.6 the trace anomaly \(\frac{c_s^2}{s}\), is shown as a function of the temperature. The solid line is the trace anomaly in lattice simulation, the dashed line is that from the step function fit to \(\langle N^\phi \rangle /\langle p_T^\phi \rangle^3\). Trace anomaly from the step function is normalised by a factor \(K = 460\). Normalised trace anomaly closely agree with the lattice simulated value. We may note here that if the ratio \(\langle N^\phi \rangle /\langle p_T^\phi \rangle^3\) is fitted with a power law, \(c_s^2\) and trace anomaly from the fit do not resemble lattice simulation results. For example, square speed of sound does not show a dip or the trace anomaly does not show a peak.

To conclude, we have argued that the ratio of \(\phi\) mesons multiplicity over cube of the mean \(p_T\) corresponds to the degeneracy of the medium produced in relativistic heavy ion collisions and can signal the confinement-deconfinement phase transition. The ratio constructed from the existing data \([17,20]\) in the energy range \(\sqrt{s} = 6.3-200\) GeV, even though have large error bars, do show rapid rise from low SPS energy to high RHIC energy. Using a result of hydrodynamical analysis of \(\phi\) mesons \(p_T\) spectra \([22]\), that the average initial temperature depend logarithmically on the collision energy, the collision energy dependence of the ratio is converted in to temperature dependence. The ratio as a function of the temperature closely corresponds to lattice simulations for \(s/T^3\), the entropy density over cube of the temperature. From a step function fit to the ratio we also have extracted the threshold energy, \(\sqrt{s}_{th} = 15.74 \pm 8.10\) GeV, for the confinement-deconfinement phase transition.
[1] Karsch F, Laermann E, Petreczky P, Stickan S and Wetzorke I, 2001 Proceedings of NIC Symposium (Ed. H. Rollnik and D. Wolf, John von Neumann Institute for Computing, Jülich, NIC Series, vol.9, ISBN 3-00-009055-X, pp.173-82,2002.)

[2] M. Cheng et al., Phys. Rev. D 77, 014511 (2008) [arXiv:0710.0354 [hep-lat]].

[3] BRAHMS Collaboration, I. Arsene et al., Nucl. Phys. A 757, 1 (2005).

[4] PHOBOS Collaboration, B. B. Back et al., Nucl. Phys. A 757, 28 (2005).

[5] PHENIX Collaboration, K. Adcox et al., Nucl. Phys. A 757 184 (2005).

[6] STAR Collaboration, J. Adams et al., Nucl. Phys. A 757 102 (2005).

[7] H. Caines [STAR Collaboration], [arXiv:0906.0305 [nucl-ex]].

[8] G. Odyniec [STAR Collaboration], J. Phys. G 35, 104164 (2008).

[9] P. Koch, B. Muller and J. Rafelski, Phys. Rept. 142, 167 (1986).

[10] J. Rafelski and M. Danos, Phys. Lett. B 97, 279 (1980).

[11] J. Cleymans, H. Oeschler and K. Redlich, Phys. Rev. C 59, 1663 (1999) [arXiv:nucl-th/9809027].

[12] S. Hamieh, K. Redlich and A. Tounsi, Phys. Lett. B 486, 61 (2000) [arXiv:hep-ph/0006024].

[13] A. Tounsi, A. Mischke and K. Redlich, Nucl. Phys. A 715, 565 (2003) [arXiv:hep-ph/0209284].

[14] F. Becattini, M. Gazdzicki, A. Keranen, J. Manninen and R. Stock, Phys. Rev. C 69, 024905 (2004) [arXiv:hep-ph/0310049].

[15] F. Becattini, J. Manninen and M. Gazdzicki, Phys. Rev. C 73, 044905 (2006) [arXiv:hep-ph/0511092].

[16] B. Mohanty and N. Xu, [arXiv:0901.0313 [nucl-ex]].

[17] C. Alt et al. [NA49 collaboration], Phys. Rev. C 78, 044907 (2008) [arXiv:0806.1937 [nucl-ex]].

[18] B. I. Abelev et al. [STAR Collaboration], Phys. Rev. Lett. 99, 112301 (2007) [arXiv:nucl-ex/0703033].

[19] B. I. Abelev et al. [STAR Collaboration], [arXiv:0809.4737 [nucl-ex]].

[20] B. I. Abelev et al. [STAR Collaboration], Phys. Lett. B 673, 183 (2009) [arXiv:0810.3979 [nucl-ex]].

[21] P. F. Kolb and U. Heinz, in Quark-Gluon Plasma 3, edited by R. C. Hwa and X.-N. Wang (World Scientific, Singapore, 2004), p. 634.

[22] P. F. Kolb, P. Huovinen, U. W. Heinz and H. Heiselberg, Phys. Lett. B 500, 232 (2001) [arXiv:hep-ph/0012137].

[23] Yu. B. Ivanov, I. N. Mishustin, V. N. Russikh and L. M. Satarov, Phys. Rev. C 80, 064904 (2009) [arXiv:0907.4140 [nucl-th]].

[24] H. Petersen and M. Bleicher, Phys. Rev. C 79, 054904 (2009) [arXiv:0901.3821 [nucl-th]].

[25] V. Roy and A. K. Chaudhuri, [arXiv:0911.4556 [nucl-th]].

[26] R. C. Hwa and K. Kajantie, Phys. Rev. D 32, 1109 (1985).

[27] P. Braun-Munzinger, D. Magestro, K. Redlich and J. Stachel, Phys. Lett. B 518, 41 (2001) [arXiv:hep-ph/0105229].

[28] J. Cleymans and K. Redlich, Phys. Rev. C 60, 054908 (1999) [arXiv:nucl-th/9903063].

[29] J. Cleymans and K. Redlich, Phys. Rev. Lett. 81, 5284 (1998) [arXiv:nucl-th/9808030].

[30] A. K. Chaudhuri, [arXiv:0801.3180 [nucl-th]].

[31] A. K. Chaudhuri, J. Phys. G 35, 104015 (2008) [arXiv:0804.3458 [hep-th]].

[32] A. K. Chaudhuri, Phys. Lett. B 681, 418 (2009) [arXiv:0909.0391 [nucl-th]].