Optimum Reconfigurable Intelligent Surface Selection for Indoor and Outdoor Communications

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Abstract

The reconfigurable intelligent surface (RIS) is a promising technology that is anticipated to enable high spectrum and energy efficiencies in future wireless communication networks. This paper investigates optimum location-based RIS selection policies in RIS-aided wireless networks to maximize the signal-to-noise ratio (SNR) for a power path-loss model in outdoor communications and an exponential path-loss model in indoor communications. The random locations of all available RISs are modeled as a Poisson point process (PPP). To quantify the network performance, the outage probabilities and average rates attained by the proposed RIS selection policies are evaluated by deriving the distance distribution of the chosen RIS node as per the selection policies for both power and exponential path-loss models. Feedback could incur heavy signaling overhead. To reduce the overhead, we also propose limited-feedback RIS selection policies by limiting the average number of RISs that feedback their location information to the source. The outage probabilities and average rates obtained by the limited-feedback RIS selection policies are derived for both path-loss models. The numerical results show notable performance gains obtained by the proposed RIS selection policies and demonstrate that the conventional relay selection policies are not suitable for RIS-aided wireless networks.

Index Terms

Poisson point process (PPP), reconfigurable intelligent surface (RIS), stochastic geometry.

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I. INTRODUCTION

A. Background and Motivation

Globally, mobile and machine-to-machine data traffic is expected to grow at a rate of about 55% per year from 2020 to 2030, reaching 5,000 Exabytes per month in 2030 [1]. While supporting 1 Terabyte per second speeds, the sixth-generation (6G) wireless networks are expected to facilitate sensing, localization, and computing in real-time by using a smart wireless environment. One of the key enablers to realizing a smart environment for 6G systems is reconfigurable intelligent surfaces (RIS), which includes many nearly passive elements having ultra-low power consumption. Each element can electronically control the phase of the reflected radio waves to concentrate energy in the desired spatial directions [2]. As such, a RIS dynamically adapts to changing wireless channel conditions to create a favorable propagation environment and increase the energy efficiency of wireless networks [3]. Moreover, a RIS greatly decreases hardware costs and power consumption. This is because the spatial feeding method of the RIS avoids the immoderate power loss due to the massive feeding networks of phased arrays [4]. A critical milestone to realize the full scale of these advantages in a network setting is to have adaptive algorithms optimizing the selection and activation of RISs to enhance smart wireless connectivity, which will be an essential feature of future wireless systems.

Due to the possible irregular deployment nature of RISs in a given geometric area, it is appropriate to assume that the RISs are randomly distributed, in contrast to a single and fixed RIS. Similar to relay networks [5], utilizing multiple RISs for single-user communication increases the overall system complexity and signaling overhead. Thus, a well-designed adaptive single RIS-selection policy is of particular importance to achieve the benefits of the deployment of multiple RISs. Further, RISs may have limited computing power to support signal processing or edge computing. Therefore, a selection policy only utilizing location information of available RISs is more practical in this context. Motivated by these facts, this paper focuses on the location-based optimum RIS selection problem in RIS-aided wireless networks that consist of multiple randomly-distributed RISs and derives performance metrics under the optimum selection policy.

B. Related Work

Most previous work on performance analysis considers different wireless networks with single and fixed RIS setup, e.g., [6]–[12] and references therein. For a given set of locations of multiple
RISs, the signal-to-noise ratio (SNR), achievable sum-rate or energy efficiency of RIS assisted networks is maximized by jointly optimizing the transmit power vector at the transmitter and the phase shift matrix with passive beamforming at all distributed RISs in [13]–[16], but without RIS selection. The RIS with the highest instantaneous end-to-end SNR is selected among multiple fixed RISs to aid the communication in [17], where the outage probability and average sum-rate are investigated.

Spatial point processes are regarded as tractable analytical tools to model the nodes’ distribution (e.g., base stations, users, or relays) that gives a good statistical fit to physical wireless network deployments [18]. However, only a few works have so far focused on spatial network models for the deployment of RISs or the distribution of users in RIS assisted networks; see [13], [19]–[23]. In [19], environmental objects are assumed to be coated as RISs where the deployment is modeled as a modified line process with random locations and orientations. In [20], they jointly design detection weight at the randomly distributed users and the passive beamforming weight at the multiple fixed RISs. [21] considers a cellular network where the midpoints of the blockages are distributed according to a Poisson point process (PPP) and a fraction of the blockages are provided with RISs. [22] provides performance analysis of a large-scale mmWave cellular network where the locations of base stations and RISs are modeled using independent PPPs. Recently, [23] studies the coverage of a RIS-aided large-scale mmWave cellular network where the buildings with RISs are distributed based on a PPP. However, none of these papers considered location-based optimum RIS selection from multiple randomly distributed RISs. Performance characterization of RIS-aided random wireless networks with such optimum RIS selection is an open problem in the literature.

C. Our Approach and Contributions

In this paper, we consider a RIS-aided wireless network where multiple RISs are randomly distributed and a RIS is optimally selected for relaying data from a transmitter (TX) to a receiver (RX). To accommodate different types of signal propagation, this paper considers the classical power-law propagation model for outdoor communications and exponential-law (which we refer to as exp-law for brevity for the rest of the paper) propagation model for indoor communications as well. We note that selecting an optimum RIS from all RISs requires feeding the information of all RIS locations back to the TX, which could incur heavy signaling overhead. To reduce the overhead, we propose limited-feedback RIS selection policies that the source only requires
feedback location information from a part of superior RISs and then selects a RIS among these RISs feeding back. Using tools from stochastic geometry, we develop a tractable theoretical framework to obtain the outage probability and average rate for the RIS-aided wireless network under the optimum RIS selection policies with all-feedback and limited-feedback cases for both path-loss models. We emphasize that developing the theoretical framework in this paper is challenging since we need to tackle with the random instantaneous SNR values at the random RIS locations coupled with the randomness in the signal propagation over fading process. Despite these challenges, we make the following novel contributions:

- Based on the nature of the SNR in RIS-aided wireless networks, we propose location-based optimum RIS selection policies that maximize the end-to-end SNR of the communication link connecting a TX and a RX via a RIS for power-law and exp-law path-loss models. These selection policies reveal that the optimum RIS given the power-law path-loss model is the RIS that has the minimal product of the distances of the TX-RIS and RIS-RX links among all randomly-distributed RISs, while the optimum RIS given the exp-law path-loss model is the RIS that has the minimal sum of these distances.

- We derive the distance distribution of the optimum RIS node for both path-loss models. These distributions are of critical importance to obtain the outage probability of RIS-aided wireless networks. The derived distributions have broader applicability where a node that has the minimal product or sum of the distances is selected. Using the derived distance distribution and the gamma approximation for fading channels, we derive theoretical expressions for the outage probability and average rate of the optimum RIS selection policies for power-law and exp-law path-loss models.

- To characterize the system performance given limited-feedback RIS selection policy, we derive the average number of RISs feeding back and confirm the number of RISs feeding back is a Poisson random variable (RV) for both path-loss models. Using the derived mean and Poisson approximation, we obtain theoretical expressions for the outage probability and average rate under the limited-feedback RIS selection policies for both path-loss models.

We verify the derived analytical results by means of extensive numerical analysis and simulations. The potential performance improvement obtained by the optimum RIS selection policy is demonstrated by comparing the performance gains obtained by the optimum policy with those of heuristic sub-optimum policies via simulations. The numerical results also demonstrate that
limited-feedback RIS selection policies achieve almost the same outage and data rate performance as the optimum RIS selection policy with all-feedback cases while significantly reducing the feedback and signaling load. Through these theoretical and numerical results, this paper provides important guidelines for selecting the optimal nodes as relays towards a reliable yet practical RIS-aided wireless network.

We note that the results in [24] focus on the node distance distribution given the min-max optimum selection criteria, and cannot be directly applied to this paper. The key technical challenges in this paper are to solve the node distance distribution given the min-product and min-sum optimum selection criteria. These are fundamentally different problems with their own particular technical challenges requiring new solution approaches when compared to those investigated in [24]. Moreover, there are also fundamental differences between RIS-aided networks and relay-aided networks in terms of their performance analysis and behaviors, i.e., see [25]. Thus, this paper provides novel theoretical results and numerical insights that have not been studied before.

D. Notation and Paper Organization

We use the following notation in this paper: We use boldface to represent a vector. \( \mathbb{N} \) denotes the set of natural numbers and \( \mathbb{R}^2 \) denotes the two-dimensional Euclidean space. \( |x| \) and \( ||x|| \) denote the absolute value of a scalar quantity \( x \) (real or complex) and the Euclidean norm of a vector quantity \( x \), respectively. \( \exp(\cdot) \) denotes the exponential function. \( \mathbb{P}(\cdot) \) denotes probability. \( \mathbb{E}_Z[\cdot] \) is the expectation over RV \( Z \). \( \mathbb{E}_\Phi[\cdot] \) is the expectation over the spatial point process \( \Phi \).

We organize the rest of the paper as follows: Section II describes the system model, assumptions, and performance metrics. Section III focuses on the power-law and exp-law path-loss models, respectively, which cover the analysis of the optimum RIS node distance distribution and the corresponding performance metrics. Section IV focuses on performance analysis of RIS selection policy with limited-feedback for both path-loss models. Section V presents detailed numerical study, justifying the derived analytical results and revealing the impact of the key system parameters on system performance. Finally, we conclude the paper in Section VI.

II. System and Analytical Models

A. System Model

We consider a RIS-aided wireless system in \( \mathbb{R}^2 \), as illustrated in Fig. 1, where the TX and RX are located at arbitrary locations, denoted by \( x_s \in \mathbb{R}^2 \) and \( x_d \in \mathbb{R}^2 \), respectively. Potential RISs
are randomly distributed according to a spatial homogeneous Poisson point process (HPPP) $\Phi$ with intensity $\lambda > 0$. The locations of available RISs are denoted by $\phi = \{x_1, x_2, \ldots\}$, where $x_i \in \mathbb{R}^2$ is the $i$th RIS location for $i \in \mathbb{N}$ and $\phi$ is a particular realization of $\Phi = \{X_1, X_2, \ldots\}$. We consider that each RIS is implemented with $N$ number of reflecting elements which can be adjusted individually. We denote $W = \text{diag}(\exp(j\varphi_{i,1}), \ldots, \exp(j\varphi_{i,N}))$ as phase shifts of the $i$th RIS. Further, $h_{i,n} = \alpha_{i,n} \exp(-j\psi_{i,n})$ and $g_{i,n} = \beta_{i,n} \exp(-j\varphi_{i,n})$ are fading channel between the TX and the $n$th reflecting element of the $i$th RIS $X_i$ and that between the $n$th reflecting element of the $i$th RIS $X_i$ and the RX, respectively. We assume fading channels are independent and identically distributed (i.i.d.) complex Gaussian RVs with zero mean and unit variance, i.e., $h_{i,n}, g_{i,n} \sim \mathcal{CN}(0, 1)$. Hence, magnitudes of $h_{i,n}$ and $g_{i,n}$ (i.e., $\alpha_{i,n}$ and $\beta_{i,n}$) follow the Rayleigh distribution. Furthermore, we assume the $n$th reflecting element of the $i$th RIS always perform at the optimal reflection coefficients, i.e., $\varphi_{i,n} = \psi_{i,n} + \varphi_{i,n}, i \in \mathbb{N}$. Using this assumption, the corresponding instantaneous received signal at time $t$ via RIS $X_i$ is given by [25, eq. (5)]

$$
y_i(t) = \frac{\sqrt{P} Z_i}{\sqrt{G(\|x_s - X_i\|) G(\|X_i - x_d\|)}} s(t) + w(t),$$

where $P$ denotes the transmit power, $s(t)$ is a unit energy signal, $w(t)$ is additive white Gaussian noise (AWGN) having complex Gaussian distribution with mean zero and variance $N_0$, $G(\cdot)$ is a general path-loss function, and $Z_i = \sum_{n=1}^{N} \alpha_{i,n} \beta_{i,n}$. We note that $Z_i$ are i.i.d. RVs for different RISs. Thus, for compactness, we remove subscript “$i$” from $Z_i$, i.e., $Z_i = Z$, $\forall i$, in the rest of the paper, and it will be clear from the context that $Z$ is the power gain associated with the RIS.
selected to connect TX and RX.

B. RIS Selection Policies

In this subsection, we first give the instantaneous SNR of a RIS-aided wireless network and then formulate the location-based optimum RIS selection policies that maximize the instantaneous SNR for power-law and exp-law path-loss models.

1) Instantaneous SNR: We consider one RIS is selected for aiding the communication between the TX and RX according to a given RIS selection policy which is defined as follows:

Definition 1: A RIS selection policy $\mathcal{P} : \Sigma \mapsto \mathbb{R}^2$ is a mapping from the set of all countable locally finite subsets of $\mathbb{R}^2$, denoted by $\Sigma$, to $\mathbb{R}^2$, that satisfies the condition $\mathcal{P}(\phi) \in \phi$ for all $\phi \in \Sigma$.

We denote the RIS selected by $\mathcal{P}$ as $X_\mathcal{P}$. Using (1), we write the instantaneous SNR associated with RIS $X_\mathcal{P}$ according to

$$\text{SNR}_{\text{inst}} (\mathcal{P}, \Phi, Z) = \gamma Z^2 \frac{G(\|x_s - X_\mathcal{P}\|)}{G(\|X_\mathcal{P} - x_d\|)},$$

where $\gamma = \frac{P}{N_0}$ is the average SNR.

We consider a classical power-law path-loss model and an exp-law path-loss model. In the power-law path-loss model, signal power decays as $G_{\text{pow}}(d) = d^{-\eta}$, where $d$ is the link distance and $\eta > 2$ is the path-loss exponent. In the exp-law path-loss model, signal power decays over a link as $G_{\text{exp}}(d) = \exp(\alpha d^\beta)$, where $\alpha > 0$ and $\beta > 0$ are tunable parameters [26]. We note that the exp-law model is suitable for modeling short-range communication, e.g., indoor communication, which is one of the important scenarios that RISs can be applied. We next give the instantaneous SNR for the power-law and exp-law path-loss models.

Applying $G_{\text{pow}}(d) = d^{-\eta}$ to (2), we write instantaneous SNR under the power-law path-loss model as

$$\text{SNR}_{\text{inst}}^{\text{pow}} (\mathcal{P}, \Phi, Z) = \gamma Z^2 \frac{1}{\|x_s - X_\mathcal{P}\|^\eta \|X_\mathcal{P} - x_d\|},$$

where $\gamma = \frac{P}{N_0}$ is the average SNR.

For exp-law path-loss model, we consider $\beta = 1$. It suits for indoor communications when the number of obstacles scales linearly with the distance of the link $x_s$ and $X_\mathcal{P}$ and the link $X_\mathcal{P}$ and $x_d$. In this scenario, the attenuation is mainly caused by obstructing objects [26]. Applying $G_{\text{exp}}(d) = \exp(\alpha d)$ to (2), the instantaneous SNR under the exp-law path-loss model is given by

$$\text{SNR}_{\text{inst}}^{\text{exp}} (\mathcal{P}, \Phi, Z) = \gamma Z^2 \exp(\alpha \left(\|x_s - X_\mathcal{P}\| + \|X_\mathcal{P} - x_d\|\right)).$$

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2) Optimum RIS Selection: It is easy to see that we can maximize the instantaneous SNR for the power-law model in (3) by selecting the RIS that has the minimal product of the distances of the TX-RIS and RIS-RX links over the set of RIS locations in $\Phi$, while the optimum RIS that maximizes the instantaneous SNR for the exp-law model in (4) is the one that has the minimal sum of these distances. Thus, we can formulate the optimum RIS selection problem for the power-law model as follows:

$$\begin{align*}
\text{minimize} & \quad \hat{s}_{\text{pow}}(X) \\
\text{subject to} & \quad X \in \Phi,
\end{align*}$$

(5)

where $\hat{s}_{\text{pow}}(X)$ is given by $\hat{s}_{\text{pow}}(X) = \|x_s - X\| \times \|X - x_d\|$. With the aim of maximizing (4), the optimum RIS selection problem for the exp-law model can be formulated as:

$$\begin{align*}
\text{minimize} & \quad \hat{s}_{\text{exp}}(X) \\
\text{subject to} & \quad X \in \Phi,
\end{align*}$$

(6)

where $\hat{s}_{\text{exp}}(X)$ is given by $\hat{s}_{\text{exp}}(X) = \|x_s - X\| + \|X - x_d\|$. The corresponding optimum RIS selection policies for the power-law and exp-law path-loss models are formally defined as follows:

*Selection Policy 1:* The optimum RIS selection policy for the power-law path-loss model, denote by $P_{\text{pow}}^*$, is the one solving (5) for all realizations of $\Phi$ in $\Sigma$. The optimum RIS location maximizing $\text{SNR}_{\text{inst}}^{\text{pow}}(P, \Phi, Z)$ in (3), $X_{\times}^*$, is $X_{\times}^* = \arg \min_{X \in \Phi} \hat{s}_{\text{pow}}(X)$.

*Selection Policy 2:* The optimum RIS selection policy under the exp-law path-loss model, denote by $P_{\text{exp}}^*$, is the one solving (6) for all realizations of $\Phi$ in $\Sigma$. The optimum RIS location to maximize $\text{SNR}_{\text{inst}}^{\text{exp}}(P, \Phi, Z)$ in (4), $X_{\times}^*$, is $X_{\times}^* = \arg \min_{X \in \Phi} \hat{s}_{\text{exp}}(X)$.

C. Performance Metrics

In this paper, we aim to characterize the performance metrics associated with $P_{\text{pow}}^*$ and $P_{\text{exp}}^*$. To this end, we define the performance metrics of a RIS selection policy $P$ in this subsection. We will use the averaged SNR to determine outage probability and data rate over the fading process to characterize the data performance of a RIS selection policy $P$\(^1\). We denote $E_Z[\text{SNR}_{\text{inst}}(P, \Phi, Z)]$ and $E_Z[\log_2 (1 + \text{SNR}_{\text{inst}}(P, \Phi, Z))]$ as the SNR and data rate averaged over the fading process, respectively. For compactness, we define $E_Z[\text{SNR}_{\text{inst}}(P, \Phi, Z)] \triangleq \ldots$

\(^1\)These are relevant metrics when the permissible decoding delay is large enough to average over the fading process.
SNR \((P, \Phi)\) and \(E_Z [\log_2 (1 + \text{SNR}_{\text{inst}} (P, \Phi, Z))]) \triangleq R (P, \Phi)\) in the rest of the paper. We note that SNR \((P, \Phi)\) and \(R (P, \Phi)\) are still random quantities since they depend on random RIS locations. Using SNR \((P, \Phi)\), we define the outage probability as follows:

**Definition 2:** For a target SNR \(\rho\), the SNR-outage probability \(P_{\text{out}} (P)\) achieved by the selection criteria \(P\) is given by

\[
P_{\text{out}} (P) = \Pr \{\text{SNR} (P, \Phi) \leq \rho\}.
\]  

(7)

Using \(R (P, \Phi)\), we define the average rate as follows:

**Definition 3:** The average rate achieved by a RIS selection policy \(P\) is given by

\[
R_{\text{ave}} (P) = \mathbb{E}_{\Phi} [R (P, \Phi)].
\]  

(8)

In the next sections, we will evaluate \(P_{\text{out}}\) and \(R_{\text{ave}}\) for the optimum RIS selection policies \(P_{\text{pow}}^*\) and \(P_{\text{exp}}^*\). This is a challenging problem since we need to address the distribution of optimum RIS distance functions \(\hat{s}_{\text{pow}} (X^*_s)\) and \(\hat{s}_{\text{exp}} (X^*_d)\) over the random spatial point process \(\Phi\) and the averaged performance metrics over the random fading process.

### III. Optimum RIS Distance Distribution and Performance Analysis

In this section, we first obtain the optimum RIS distance distributions under the power-law and exp-law path-loss models. The obtained distribution can be generally applied in any context where a node that has the minimal product or sum of the distances of the TX-RIS and RIS-RX links among all randomly-distributed nodes is selected. We second derive the averaged performance metrics over the fading process. Using the optimum RIS distance distribution and the averaged performance metrics, we will evaluate the outage probability and average rate for the given optimum RIS selection policy.

#### A. Distance Distribution for Optimum RIS Selection

For the sake of simplicity, we define \(Y_{\text{opt}} \triangleq \hat{s}_{\text{pow}} (X^*_s)\) and \(\Lambda_{\text{opt}} \triangleq \hat{s}_{\text{exp}} (X^*_+).\) We now derive the distribution functions for \(Y_{\text{opt}}\) and \(\Lambda_{\text{opt}}\) which are key to characterize the performance of the RIS selection policies \(P_{\text{pow}}^*\) and \(P_{\text{exp}}^*\), respectively. We will consider \(x_s = (-d, 0)^\top\) and \(x_d = (d, 0)^\top\) without loss of generality due to the stationary nature of HPPPs [27]. In the following theorems, we provide the distribution of \(Y_{\text{opt}}\) and \(\Lambda_{\text{opt}}\).
Theorem 1: The CDF of $\Upsilon_{\text{opt}}$ is given by

$$F_{\Upsilon_{\text{opt}}} (\gamma) = \begin{cases} 1 - \exp \left( \frac{-2\lambda}{\pi \gamma} \left( d^4 E \left( \frac{\gamma^2}{\pi^2} \right) + (\gamma^2 - d^4) K \left( \frac{\gamma^2}{\pi^2} \right) \right) \right) & \text{if } \gamma < d^2 \\ 1 - \exp \left( -2\lambda^2 \gamma E \left( \frac{d^4}{\gamma^2} \right) \right) & \text{if } \gamma \geq d^2. \end{cases}$$

(9)

where $E(\cdot)$ is the complete elliptic integral of the second kind and $K(\cdot)$ is the complete elliptic integral of the first kind [28]. The PDF of $\Upsilon_{\text{opt}}$ is given by

$$f_{\Upsilon_{\text{opt}}} (\gamma) = \begin{cases} \frac{2}{\pi \gamma \lambda} \exp \left( \frac{-2\lambda}{\pi \gamma^2} \left( d^4 E \left( \frac{\gamma^2}{\pi^2} \right) + (\gamma^2 - d^4) K \left( \frac{\gamma^2}{\pi^2} \right) \right) \right) K \left( \frac{\gamma^2}{\pi^2} \right) & \text{if } \gamma < d^2 \\ 2\lambda \exp \left( -2\lambda^2 \gamma E \left( \frac{d^4}{\gamma^2} \right) \right) K \left( \frac{d^4}{\gamma^2} \right) & \text{if } \gamma \geq d^2. \end{cases}$$

(10)

Proof: See Appendix A.

Theorem 2: The CDF of $\Lambda_{\text{opt}}$ is given by

$$F_{\Lambda_{\text{opt}}} (\gamma) = \begin{cases} 0 & \gamma < 2d \\ 1 - \exp \left( -\frac{\lambda \pi \gamma \sqrt{\gamma^2 - 4d^2}}{4} \right) & \gamma \geq 2d \end{cases}$$

(11)

The PDF of $\Lambda_{\text{opt}}$ is given by

$$f_{\Lambda_{\text{opt}}} (\gamma) = \begin{cases} 0 & \gamma < 2d \\ \frac{\pi \lambda (\gamma^2 - 4d^2)}{2\sqrt{\gamma^2 - 4d^2}} \exp \left( -\frac{\lambda \pi \gamma \sqrt{\gamma^2 - 4d^2}}{4} \right) & \gamma \geq 2d. \end{cases}$$

(12)

Proof: See Appendix B.

B. Averaged Performance Metrics over Fading Channel

Based on (7) and (8), we recall that $P_{\text{out}} (P)$ and $R_{\text{ave}} (P)$ relate to $\text{SNR} (P, \Phi)$ and $\text{R} (P, \Phi)$, respectively. To facilitate the derivation of $P_{\text{out}}$ and $R_{\text{ave}}$ for the optimum RIS selection policies $P_{\text{pow}}^*$ and $P_{\text{exp}}^*$, we derive $\text{SNR} (P, \Phi)$ and $\text{R} (P, \Phi)$ in this subsection. We derive $\text{SNR} (P, \Phi)$ as

$$\text{SNR} (P, \Phi) = \mathbb{E}_Z \left[ \frac{\tilde{\gamma} Z^2}{G(||x_s - X_P||) G(||X_P - x_d||)} \right]$$

$$= \frac{\mathbb{E}_Z [Z^2]}{G(||x_s - X_P||) G(||X_P - x_d||)}$$

(13)

where $\mathbb{E}_Z [Z^2]$ is derived as

$$\mathbb{E}_Z [Z^2] = \sum_{n=1}^{N} \mathbb{E} [\alpha_{i,n}^2] \mathbb{E} [\beta_{i,n}^2] + \sum_{n=1}^{N} \sum_{m=1,m \neq n}^{N} \mathbb{E} [\alpha_{i,n}] \mathbb{E} [\alpha_{i,m}] \mathbb{E} [\beta_{i,n}] \mathbb{E} [\beta_{i,m}]$$

(a) \hspace{1cm} \mathbb{E} [\alpha_{i,1}^2]^2 + N (N - 1) \mathbb{E} [\alpha_{i,1}]^4 \quad \mathbb{E} [\beta_{i,1}]^4 \equiv N + N (N - 1) \mathbb{E} [\alpha_{i,1}]^4

(b) \hspace{1cm} N + N (N - 1) \frac{\pi^2}{16}$$

(14)
where equality (a) exploits the fact that $\alpha_{i,n}$ and $\beta_{i,n}$ are i.i.d. RVs. Equality (b) is due to the fact $h_{i,1} \sim \mathcal{CN}(0, 1)$ and $\mathbb{E} \left[ \alpha_{i,1}^2 \right] = 1$. Equality (c) is because $\alpha_{i,1}$ is Rayleigh distributed with the scale parameter $\sigma = \frac{1}{\sqrt{2}}$ and $\mathbb{E} \left[ \alpha_{i,1} \right] = \sigma \sqrt{\frac{\pi}{2}} = \sqrt{\frac{\pi}{4}}$. Applying (14) to (13), we have

$$\text{SNR} \left( \mathcal{P}, \Phi \right) = \frac{\bar{\gamma} N (16 + (N - 1)\pi^2)}{16G(||x_a - X_p||)G(||X_p - x_d||)}.$$

(15)

We next derive $R(\mathcal{P}, \Phi)$ as

$$R(\mathcal{P}, \Phi) = R(\mathcal{P}, Y) = \mathbb{E}_Y \left[ \log_2 \left( 1 + \bar{\gamma} Y Z^2 \right) \right] = \int_0^\infty \log_2 \left( 1 + \bar{\gamma} Y x^2 \right) f_Z(x)dx,$$

where $Y = \frac{1}{G(||x_a - X_p||)G(||X_p - x_d||)}$ and $Y$ is a single statistic summarising the overall effect of point process $\Phi$ on the data rate. Based on (16), we study the distribution of $Z$. Since $Z$ is a sum of the product of two i.i.d. Rayleigh RVs, its exact distribution is difficult to determine for $N > 1$. We thus approximate its distribution by using a gamma RV with the shape parameter given by $k = \frac{N_0^2}{16 - \pi^2}$ and the scale parameter given by $\theta = \frac{16 - \pi^2}{4\pi}$ [9], [25]. Using the PDF of RV $Z$ which is approximated with a gamma RV and the result in [9, Eq. (20)], we rewrite $R(\mathcal{P}, \Phi)$ in (16) as

$$R(\mathcal{P}, Y) = \int_0^\infty \log_2 \left( 1 + \bar{\gamma} Y x^2 \right) f_Z(x)dx$$

$$= \int_0^\infty \log_2 \left( 1 + \bar{\gamma} Y x^2 \right) \frac{x^{k-1} \exp \left( -\frac{x}{\theta} \right)}{\theta^k \Gamma(k)} dx$$

$$= \frac{1}{\log(2)} \left[ 2 \log(\theta) + \log(\bar{\gamma} Y) + 2\psi(0)(k) \right. + \frac{\mathbf{2}_{\mathbf{F}_\mathbf{3}} \left( 1, 1; 2, \frac{3}{2} - \frac{k}{2}; 2 - \frac{k}{2}; -\frac{1}{4\theta^2 \bar{\gamma} Y} \right)}{\theta^2 \bar{\gamma} Y \left( k^2 - 3k + 2 \right)}$$

$$+ \frac{\pi(\bar{\gamma} Y)^{-\frac{k}{2}}}{\theta^k \Gamma(k + 1)} \left[ \mathbf{1}_{\mathbf{F}_\mathbf{2}} \left( \frac{k}{2}; \frac{1}{2}; \frac{k}{2} + 1; -\frac{1}{4\theta^2 \bar{\gamma} Y} \right) \right.$$

$$\left. - \mathbf{1}_{\mathbf{F}_\mathbf{2}} \left( \frac{k}{2} + \frac{1}{2}; \frac{3}{2}; \frac{k}{2} + \frac{3}{2}; -\frac{1}{4\theta^2 \bar{\gamma} Y} \right) \right] \right]$$

(17)

where we use $\doteq$ to mean that the equality is valid when the gamma approximation is applied, $\log(\cdot)$ denotes natural logarithm, $\mathbf{p}_{\mathbf{F}_{\mathbf{q}}} \left( \cdot; \cdot; \cdot \right)$ denotes the generalized hypergeometric functions [29], and $\psi^{(n)}(z)$ denotes the $n$th derivative of the digamma function [29]. Our numerical results in Section V will confirm the accuracy of gamma approximation used in (17).

We note that the calculation of (17) requires high computational complexity. To ease the complexity, using Jensen’s inequality, we also derive an upper bound on $R(\mathcal{P}, \Phi)$ as

$$R(\mathcal{P}, \Phi) = \mathbb{E}_Z \left[ \log_2 \left( 1 + \text{SNR}_{\text{inst}} \left( \mathcal{P}, \Phi, Z \right) \right) \right] \leq \log_2 \left( 1 + \mathbb{E}_Z \left[ \text{SNR}_{\text{inst}} \left( \mathcal{P}, \Phi, Z \right) \right] \right).$$

(18)
By applying (15) to (18), we rewrite (18) as
\[ R(P, Y) \leq \log_2 \left( 1 + \frac{\bar{\gamma}NY(16 + (N-1)\pi^2)}{16} \right), \]  
(19)
where we write \( \tilde{R}(P, Y) = \log_2 \left( 1 + \frac{\bar{\gamma}NY(16 + (N-1)\pi^2)}{16} \right) \) in the rest of the paper.

C. Performance Analysis

Based on subsections III-A and III-B, we evaluate the outage probability and average rate achieved by the optimum RIS selection policies \( P_{\text{pow}}^* \) and \( P_{\text{exp}}^* \).

1) Outage Probability: We denote \( P_{\text{out}}^{\text{pow}}(P_{\text{pow}}^*) \) and \( P_{\text{out}}^{\text{exp}}(P_{\text{exp}}^*) \) as the outage probabilities given the power-law and exp-law path-loss optimum selection policies, respectively. Using (7) and applying \( G_{\text{pow}}(d) = d^\eta \) to (15), we derive \( P_{\text{out}}^{\text{pow}}(P_{\text{pow}}^*) \) as
\[ P_{\text{out}}^{\text{pow}}(P_{\text{pow}}^*) = \Pr \{ \text{SNR}(P_{\text{pow}}^*, \Phi) \leq \rho \} = \Pr \left\{ \frac{\bar{\gamma}N(16 + (N-1)\pi^2)}{16 \Upsilon_{\text{opt}}^\eta} \leq \rho \right\} = 1 - F_{\Upsilon_{\text{opt}}} \left( \left( \frac{\bar{\gamma}N(16 + (N-1)\pi^2)}{16\rho} \right)^\frac{1}{\eta} \right), \]  
(20)
where \( F_{\Upsilon_{\text{opt}}} (\gamma) \) is the CDF of RV \( \Upsilon_{\text{opt}} \) given in (9). Applying \( G_{\text{exp}}(d) = \exp(\alpha d) \) to (15), we derive \( P_{\text{out}}^{\text{exp}}(P_{\text{exp}}^*) \) as
\[ P_{\text{out}}^{\text{exp}}(P_{\text{exp}}^*) = \Pr \{ \text{SNR}(P_{\text{exp}}^*, \Phi) \leq \rho \} = \Pr \left\{ \frac{\bar{\gamma}N(16 + (N-1)\pi^2)}{16 \exp(\alpha \Lambda_{\text{opt}})} \leq \rho \right\} = \Pr \left\{ \Lambda_{\text{opt}} > \ln \left( \frac{\bar{\gamma}N(16 + (N-1)\pi^2)}{16\rho} \right) \right\} = 1 - F_{\Lambda_{\text{opt}}} \left( \frac{1}{\alpha \ln \left( \frac{\bar{\gamma}N(16 + (N-1)\pi^2)}{16\rho} \right)} \right), \]  
(21)
where \( F_{\Lambda_{\text{opt}}} (\gamma) \) is the CDF of RV \( \Lambda_{\text{opt}} \) given in (11).

2) Average Rate: We denote \( R_{\text{ave}}^{\text{pow}}(P_{\text{pow}}^*) \) and \( R_{\text{ave}}^{\text{exp}}(P_{\text{exp}}^*) \) as the data rate obtained by the power-law and exp-law path-loss optimum selection policies, respectively. Based on the definition of \( R_{\text{ave}}(P) \) in (8), we evaluate \( R_{\text{ave}}^{\text{pow}}(P_{\text{pow}}^*) \) as
\[ R_{\text{ave}}^{\text{pow}}(P_{\text{pow}}^*) = \mathbb{E}_\Phi \left[ R(P, Y_{\text{pow}}) \right] = \int R(P, y) f_{Y_{\text{pow}}}(y) dy, \]  
(22)
where $y$ is a realization of RV $Y_{\text{pow}}$ and $Y_{\text{pow}} = \frac{1}{G_{\text{pow}}(\|x_2 - x_0\|)G_{\text{pow}}(\|x_2 - x_d\|)} = Y_{\text{opt}}^{-\eta}$. We note that $R(\mathcal{P}, Y_{\text{pow}})$ is given by (17). Using the distribution of $Y_{\text{opt}}$, we can derive the PDF of $Y_{\text{pow}} = Y_{\text{opt}}^{-\eta}$ as

$$
\begin{align*}
 f_{Y_{\text{pow}}} (y) &= \begin{cases} 
 S_1 (y) = \frac{2 \exp \left( -2y^{-\frac{\eta}{\eta}} \lambda E(x^2 y) \right) y^{-1 - \frac{1}{\eta}} \lambda K \left( d^2 y \right)}{d^2} \quad &\text{if } 0 < y \leq d^{-2\eta}, \\
 S_2 (y) = \frac{2 \exp \left( \frac{\eta}{\eta} d^2 K \left( \frac{y}{d^2} \right) \right) y^{\frac{2 - \eta}{\eta}} \lambda K \left( \frac{y}{d^2} \right)}{d^2} \quad &\text{if } y \geq d^{-2\eta}.
\end{cases}
\end{align*}
$$

(23)

There is no closed form expression for $R_{\text{ave}}^\text{pow}(\mathcal{P}_{\text{pow}}^*)$ but it can be evaluated numerically with the aid of (17), (22), and (23) by calculating the integrals given below:

$$
R_{\text{ave}}^\text{pow}(\mathcal{P}_{\text{pow}}^*) = \int_0^{d^{-2\eta}} R(\mathcal{P}, y) S_1 (y) \, dy + \int_{d^{-2\eta}}^{\infty} R(\mathcal{P}, y) S_2 (y) \, dy.
$$

(24)

As (22), we write $R_{\text{ave}}^\text{exp}(\mathcal{P}_{\text{exp}}^*)$ as

$$
R_{\text{ave}}^\text{exp}(\mathcal{P}_{\text{exp}}^*) = \int R(\mathcal{P}, y) f_{Y_{\text{exp}}} (y) \, dy,
$$

(25)

where $y$ is a realization of $Y_{\text{exp}}$ and $Y_{\text{exp}} = \exp (-\alpha \Lambda_{\text{opt}})$. $R(\mathcal{P}, Y_{\text{exp}})$ is given by (17). We then derive the PDF of $Y_{\text{exp}} = \exp (-\alpha \Lambda_{\text{opt}})$. By using variable transformation and $f_{\Lambda_{\text{opt}}} (\gamma)$ in (12), the PDF can be derived for $y \leq e^{-2\alpha d}$ as

$$
 f_{Y_{\text{exp}}} (y) = \frac{\pi \left( \frac{1}{y} \right)^{\frac{\log \left( \frac{1}{y} \right)}{\alpha^2} - 4d^2}}{2 \alpha \log \left( \frac{1}{y} \right) - 4d^2}.
$$

(26)

For $y > e^{-2\alpha d}$, the PDF is zero. With the aid of (25), (17) and (26), the average rate can be numerically calculated by evaluating the integral

$$
R_{\text{ave}}^\text{exp}(\mathcal{P}_{\text{exp}}^*) = \int_0^{e^{-2\alpha d}} R(\mathcal{P}, y) f_{Y_{\text{exp}}} (y) \, dy.
$$

(27)

Remark 1: The upper bounds on $R_{\text{ave}}^\text{pow}(\mathcal{P}_{\text{pow}}^*)$ and $R_{\text{ave}}^\text{exp}(\mathcal{P}_{\text{exp}}^*)$ via Jensen’s inequality can be obtained by replacing $R(\mathcal{P}, y)$ by $\tilde{R}(\mathcal{P}, y)$ in (24) and (27), where $\tilde{R}(\mathcal{P}, Y)$ is given in (19).

IV. LIMITED-FEEDBACK RIS SELECTION

In this section, we propose a limited-feedback RIS selection policy that selects the best RIS from a limited number of RISs feeding back. We will evaluate the average rate and outage probability attained by the proposed limited-feedback RIS selection policy for both power-law and exp-law path-loss models.
A. Limited-Feedback Strategy

In our discussion in the previous sections, we observe that a central node or source node requires location information from all RISs to pick the best one. We note that the feedback task here may not be practical. To alleviate the feedback overhead, we will consider an effective yet simple limited-feedback strategy, which is proposed as follows.

Limited-Feedback Strategy: A RIS node located \( X \in \Phi \) will send its channel quality indicator \( \hat{s}(X) \) back to the source node when \( \hat{s}(X) \leq T \), where \( T > 0 \) is a given threshold value. Here, \( \hat{s}(X) = \hat{s}_{\text{pow}}(X) \) for the power-law path-loss model, and \( \hat{s}(X) = \hat{s}_{\text{exp}}(X) \) for the exp-law path-loss model.

If no RIS feeds back its channel quality indicator, no data is transmitted by the source node. We will characterize the number of RISs feeding back given the aforementioned feedback strategy and evaluate the average rate and outage probability attained by limited-feedback RIS selection policies in the following subsections.

B. Distribution of RISs Feedback

Given the feedback strategy proposed in subsection IV-A, we denote the total number of RISs feeding back under the power-law and the exp-law RIS selection function by \( N_{\text{FB}}^{\text{pow}} \) and \( N_{\text{FB}}^{\text{exp}} \), respectively, i.e., \( N_{\text{FB}}^{\text{pow}} = \sum_{X \in \Phi} 1 \{ \hat{s}_{\text{pow}}(X) \leq T_{\text{pow}} \} \) and \( N_{\text{FB}}^{\text{exp}} = \sum_{X \in \Phi} 1 \{ \hat{s}_{\text{exp}}(X) \leq T_{\text{exp}} \} \), where \( 1 \{ \cdot \} \) is the indicator function. \( T_{\text{pow}} \) and \( T_{\text{exp}} \) are the thresholds used in the power-law and exp-law scenarios, respectively. The average number of RISs feeding back is given by \( \Xi_{\text{pow}} = E_{\Phi} [ N_{\text{FB}}^{\text{pow}} ] \) and \( \Xi_{\text{exp}} = E_{\Phi} [ N_{\text{FB}}^{\text{exp}} ] \). The sets of the RISs feeding back \( \Phi_{\text{FB}}^{\text{pow}} \) and \( \Phi_{\text{FB}}^{\text{exp}} \) are defined by \( \Phi_{\text{FB}}^{\text{pow}} = \{ X \in \Phi : \hat{s}_{\text{pow}}(X) \leq T_{\text{pow}} \} \) and \( \Phi_{\text{FB}}^{\text{exp}} = \{ X \in \Phi : \hat{s}_{\text{exp}}(X) \leq T_{\text{exp}} \} \).

We now formulate the RIS selection policy with limited-feedback for the power-law and exp-law path-loss models as follows:

Selection Policy 3: The RIS selection policy with limited-feedback strategy under the power-law model, denote by \( \mathcal{P}_{\text{FB}}^{\text{pow}} \), is the one that solves the following optimization problem

\[
\begin{aligned}
& \text{minimize} & & \hat{s}_{\text{pow}}(X) \\
& \text{subject to} & & X \in \Phi_{\text{FB}}^{\text{pow}}
\end{aligned}
\]
Selection Policy 4: For the exp-law model, the RIS selection policy with limited-feedback strategy, which we denote by \( \mathcal{P}_{\text{exp}}^{\text{FB}} \), is the one that solves the following optimization problem

\[
\begin{align*}
\text{minimize} & \quad \mathbf{s}_{\text{exp}}(\mathbf{X}) \\
\text{subject to} & \quad \mathbf{X} \in \Phi_{\text{exp}}^{\text{FB}}
\end{align*}
\]

The distributions of \( N_{\text{pow}}^{\text{FB}} \) and \( N_{\text{exp}}^{\text{FB}} \) are a key step to quantify the performance attained by \( \mathcal{P}_{\text{pow}}^{\text{FB}} \) and \( \mathcal{P}_{\text{exp}}^{\text{FB}} \). We present the distributions of \( N_{\text{pow}}^{\text{FB}} \) and \( N_{\text{exp}}^{\text{FB}} \) in the following theorems.

**Theorem 3:** \( N_{\text{pow}}^{\text{FB}} \) is a Poisson RV with the mean \( \Xi_{\text{pow}} \) given by

\[
\Xi_{\text{pow}} = \begin{cases} 
2 \lambda \left( \frac{1}{d^2} \left( d^4 E \left( \frac{T_{\text{pow}}^2}{d^4} \right) + (T_{\text{pow}}^2 - d^4) K \left( \frac{T_{\text{pow}}^2}{d^4} \right) \right) \right) & \text{if } T_{\text{pow}} \leq d^2 \\
2 \lambda T_{\text{pow}} E \left( \frac{d^4}{T_{\text{pow}}^2} \right) & \text{if } T_{\text{pow}} > d^2
\end{cases}
\]

**Proof:** See Appendix C.

**Theorem 4:** \( N_{\text{exp}}^{\text{FB}} \) is a Poisson RV with the mean \( \Xi_{\text{exp}} \) given by

\[
\Xi_{\text{exp}} = \begin{cases} 
0 & \text{if } T_{\text{exp}} \leq 2d \\
\frac{\lambda \pi T_{\text{exp}}}{4} \sqrt{-4d^2 + T_{\text{exp}}^2} & \text{if } T_{\text{exp}} > 2d
\end{cases}
\]

**Proof:** It can be proven similarly to Theorem 3.

In Figs. IV-B, we plot the expected numbers of RISs feeding back and compare the simulated distribution of the total number of RISs feeding back with the Poisson distribution with analytical means in (30) and (31). We observe that simulated distributions match the theoretical ones perfectly, which validates the Theorems 3 and 4. As discussed in [30], the threshold value \( \Xi_{\text{pow}} \geq 5 \) enables the limited-feedback RIS selection strategy to achieve very similar outage and data rate performance as the all-feedback RIS selection strategy, while providing a massive reduction in the feedback overhead. This is because the RIS with optimum location, \( \mathbf{X}^* \), always feeds its channel quality indicators back to the source node if \( N_{\text{pow}}^{\text{FB}} \geq 1 \) and \( \Pr \{ N_{\text{pow}}^{\text{FB}} \geq 1 \} \leq 0.99 \) when \( \Xi_{\text{pow}} > 5 \) due to the exponentially decaying tail of Poisson distribution.

C. Performance Analysis

We next evaluate the outage probability and average rate attained by the class of limited-feedback RIS selection policies, parameterized by the threshold value \( T_{\text{pow}} \) and \( T_{\text{exp}} \), for the power-law and exp-law path-loss model, respectively.

The outage probability with limited feedback for the power-law and exp-law path-loss models are given in the following theorems:
Fig. 2: Average number of RISs feeding back and probability distribution of the number of RISs feeding back for $d = 1.2$ for power-law model in Figs. 3(a) and 3(b) and for exp-law model in Figs. 3(c) and 3(d).

**Theorem 5:** The outage probability $P_{\text{out}}^{\text{pow}} (P_{\text{FB}}^{\text{pow}})$ for a limited-feedback RIS selection policy $P_{\text{FB}}^{\text{pow}}$ with threshold $T_{\text{pow}}$ is equal to

$$P_{\text{out}}^{\text{pow}} (P_{\text{FB}}^{\text{pow}}) = \begin{cases} \exp (-\Xi_{\text{pow}}) & \text{if } \rho \leq \frac{\gamma N (16 + (N - 1) \pi^2)}{T_{\text{pow}} 16} \\ 1 - F_{\Upsilon_{\text{opt}} \left( \left( \frac{\gamma N (16 + (N - 1) \pi^2)}{\rho 16} \right) \frac{1}{\gamma} \right)} & \text{otherwise} \end{cases},$$

where $\Xi_{\text{pow}}$ is the average feedback load at $T_{\text{pow}}$ and given in (30).

**Proof:** To prove Theorem 5, we first write the outage event as follows:

$$\{ E_Z [\text{SNR}^{\text{pow}} (P_{\text{pow}}^*)] \leq \rho \}$$

$$= \{ \Upsilon_{\text{opt}} > T_{\text{pow}} \} \cup \left( \{ \Upsilon_{\text{opt}} \leq T_{\text{pow}} \} \cap \left\{ \frac{\gamma N (16 + (N - 1) \pi^2)}{\Upsilon_{\text{opt}} 16} \leq \rho \right\} \right).$$
Hence, $P_{\text{out}}^{\text{FB}}$ is given by

$$
P_{\text{out}}^{\text{FB}} = \Pr \{ \Upsilon_{\text{opt}} > T_{\text{pow}} \} + \Pr \left\{ \{ \Upsilon_{\text{opt}} \leq T_{\text{pow}} \} \cap \left\{ \frac{\bar{\gamma}N(16 + (N-1)\pi^2)}{\mathcal{T}_{\text{opt}}^{\text{pow}} 16} \leq \rho \right\} \right\} = \exp \left( -\Xi_{\text{pow}} \right) + \Pr \left\{ \frac{\bar{\gamma}N(16 + (N-1)\pi^2)}{\mathcal{T}_{\text{pow}}^{\text{pow}} 16} \leq \rho \right\} \quad (33)
$$

If $\rho < \frac{\bar{\gamma}N(16 + (N-1)\pi^2)}{\mathcal{T}_{\text{pow}}^{\text{pow}} 16}$, then the second summand in (33) becomes zero, and we have

$$
P_{\text{out}}^{\text{FB}, T_{\text{pow}}} = \exp \left( -\Xi_{\text{pow}} \right), \quad \text{which is the first condition in (32)}.
$$

If $\rho > \frac{\bar{\gamma}N(16 + (N-1)\pi^2)}{\mathcal{T}_{\text{pow}}^{\text{pow}} 16}$, we have

$$
P_{\text{out}}^{\text{FB}, T_{\text{pow}}} = \Pr \left\{ \frac{\bar{\gamma}N(16 + (N-1)\pi^2)}{\mathcal{T}_{\text{opt}}^{\text{pow}} 16} \leq \rho \right\} = 1 - F_{\Upsilon_{\text{opt}}} \left( \left( \frac{\bar{\gamma}N(16 + (N-1)\pi^2)}{\rho 16} \right)^{\frac{1}{\pi}} \right), \quad \text{which is the second condition in (32)}.
$$

This completes the proof of Theorem 5.

Theorem 6: The outage probability $P_{\text{out}}^{\text{exp}}$ for a limited-feedback RIS selection policy $\mathcal{P}_{\text{pow}}^{\text{FB}}$ with threshold $T_{\text{exp}}$ is given by

$$
P_{\text{out}}^{\text{exp}} (\mathcal{P}_{\text{pow}}^{\text{FB}}) = \begin{cases} 
\exp \left( -\Xi_{\text{exp}} \right) & \text{if } \rho \leq \frac{\gammaN(16 + (N-1)\pi^2)}{\exp(\alpha T_{\text{exp}}) 16} \\
1 - F_{\Upsilon_{\text{opt}}} \left( \ln \left( \frac{\bar{\gamma}N(16 + (N-1)\pi^2)}{\rho 16} \right)^{\alpha} \right) & \text{if } \frac{\gammaN(16 + (N-1)\pi^2)}{\exp(\alpha T_{\text{exp}}) 16} < \rho < \frac{\gammaN(16 + (N-1)\pi^2)}{\exp(\alpha d) 16} \\
1 & \text{if } \rho \geq \frac{\gammaN(16 + (N-1)\pi^2)}{\exp(\alpha d) 16} 
\end{cases}
$$

(34)

Proof: Theorem 6 can be proven similarly to Theorem 5.

We evaluate the average rate achieved by $\mathcal{P}_{\text{pow}}^{\text{FB}}$ and $\mathcal{P}_{\text{exp}}^{\text{pow}}$ in the following theorems:

Theorem 7: The average rate $R_{\text{ave}}^{\text{pow}} (\mathcal{P}_{\text{pow}}^{\text{FB}})$ for a limited-feedback RIS selection policy $\mathcal{P}_{\text{pow}}^{\text{FB}}$ with threshold $T_{\text{pow}}$ is given by

$$
R_{\text{ave}}^{\text{pow}} (\mathcal{P}_{\text{pow}}^{\text{FB}}) = \int_{d^{-2d}}^{d^{-2d}} \mathcal{R}(\mathcal{P}, y) S_1(y) dy + \int_{d^{-2d}}^{\infty} \mathcal{R}(\mathcal{P}, y) S_2(y) dy. 
$$

(35)

Proof: Theorem 7 can be proven by the equivalence of events $\{ N_{\text{pow}}^{\text{FB}} \geq 1 \}$ and $\{ \Upsilon_{\text{opt}} > T_{\text{pow}} \}$, and our assumption that no data will be transmitted if $\Upsilon_{\text{opt}} > T_{\text{pow}}$.

Theorem 8: The average rate $R_{\text{ave}}^{\text{exp}} (\mathcal{P}_{\text{exp}}^{\text{FB}})$ for a given limited-feedback RIS selection policy $\mathcal{P}_{\text{exp}}^{\text{FB}}$ with threshold $T_{\text{exp}}$ is equal to

$$
R_{\text{ave}}^{\text{exp}} (\mathcal{P}_{\text{exp}}^{\text{FB}}) = \int_{\exp(-\alpha T_{\text{exp}})}^{e^{-2\alpha d}} \mathcal{R}(\mathcal{P}, y) f_{\Upsilon_{\text{exp}}} (y) dy.
$$

(36)

Proof: Theorem 8 can be proven similarly to Theorem 7.

We note that the upper bounds on $R_{\text{ave}}^{\text{pow}} (\mathcal{P}_{\text{pow}}^{\text{FB}})$ and $R_{\text{ave}}^{\text{exp}} (\mathcal{P}_{\text{exp}}^{\text{FB}})$ using Jensen’s inequality can be obtained similarly to Remark 1.
V. Numerical Results

In this section, we present simulation and numerical results to verify our derived analytical results, discuss the performance of the proposed optimum and limited-feedback RIS selection policies, and reveal the effect of system parameters on the system performance.

In our simulations, the path-loss exponent $\eta$ is taken to be 4 and the tunable parameter $\alpha$ of the exponential path-loss model is 1.037 [26]. The target SNR and the distance between TX and RX are set to $\rho = 5$ dB and $d = 1.2$ [units]. Results shown in this section are averaged over many realizations of the random locations of RISs and channel fading.

A. All-Feedback Cases

In this subsection, we focus on the proposed optimum RIS selection policies with all-feedback, i.e., the information of all RIS locations is available. To benchmark the optimum selection policy, we consider three other RIS selection schemes:

1) Min-min scheme selects the closest RIS to the TX and RX set $\{x_s, x_d\}$, i.e.,

$$\text{minimize}_{X \in \mathbb{R}^2} \min \{\|x_s - X\|, \|X - x_d\|\}, \quad X \in \Phi;$$

2) Min-max scheme selects a RIS according to

$$\text{minimize}_{X \in \mathbb{R}^2} \max \{\|x_s - X\|, \|X - x_d\|\}, \quad X \in \Phi;$$

which is the optimum scheme for a decode-and-forward relay network (see Remark 1);

3) Mid-point scheme selects the RIS that has the minimal distance to the mid-point between the TX and the RX.

In Figs. 3 and 4, we investigate the performance for power-law communications scenarios under the optimum and three other different RIS selection schemes as described above. In Fig 3, we plot the outage probability curves versus the average SNR for $N = 8$ and $N = 32$ in Fig. 3a and for $\lambda = 0.1$ and 2 [nodes/unit²] in Fig. 3b. In Fig. 3, analytical curves are obtained by using (20). The perfect agreement between analytical and simulation curves verifies the outage probability expression for the optimum RIS selection scheme in (20). In Fig. 3a, we see the outage probability decreases significantly when the number of elements increases from 8 to 32, where a RIS with $N = 8$ requires around 15 times more power than a RIS with $N = 32$ to achieve $10^{-3}$ outage probability level. For both $N = 8$ and 32, the optimum scheme has the best performance among four different schemes. Among the other three schemes, min-min is the best suboptimal one when the average SNR $\bar{\gamma}$ is less than 0 dB; otherwise, mid-point is the best
suboptimal one. For example, the optimum scheme only requires around 63% transmit power that mid-point scheme requires to achieve $10^{-3}$ outage probability level. Further, min-max always has the worst performance among these four schemes. In Fig. 3b, we observe a significant outage probability decrement when the intensity increases from $\lambda = 0.1$ to $\lambda = 2$. When $\lambda$ is very small as 0.1, all selection schemes have very similar performance. This observation indicates there is a high probability to select the same RIS by different schemes at a very low intensity and the superiority of the optimum scheme is not obvious. However, the performance gap clearly increases with $\lambda$. For example, when $\lambda$ is 2, the optimum scheme only requires around 32% transmit power that min-min scheme requires to achieve $10^{-3}$ outage probability level.

We plot the average rate versus the average SNR in Fig. 4a and the average rate versus the intensity in Fig. 4b. We only consider the optimum scheme and min-min scheme since these two schemes generally outperform the other two schemes for different sets of parameters based on Fig 3. Analytical values calculated from (24) has a perfect agreement with the simulation, which verifies the accuracy. In Fig. 4, we see that the optimum and min-min selection schemes have small rate differences for different values of $N$ at $\lambda = 0.5$. For example, the rate difference is around 0.3 [bits/sec/Hz] for $N = 16$ at SNR = 5 dB. Interestingly, the average rate increases almost linearly with SNR and the gap between the optimum scheme and min-min scheme keeps unchanged as the average SNR increases. In Fig. 4b, we see the average rate for both schemes
increases as the intensity increases, but the increase in average rate with the density is not linear as that with the average SNR. In addition, we observe that the upper bounds obtained via Jensen’s inequality is tighter as $N$ increases. The similar observation will be shown in Fig. 6.

In Figs. 5 and 6, we focus on the performance of the exp-law path-loss model. In Fig. 5, we plot the outage probability curves versus SNR $\tilde{\gamma}$, $d = 1.2$, $\alpha = 1.037$, $\rho = 5$ dB.
Fig. 6: Average Rate achieved by different RIS selection schemes for different values of $N$, $d = 1.2$, $\alpha = 1.037$.

simulations, thus validating the accuracy of (21). For both $N = 8$ and 32, the optimum scheme outperforms the other three schemes, which is similar to observations in Fig. 3. However, the other three schemes perform differently than the behavior observed in Fig. 3 for the power-law path-loss model. Specifically, the mid-point scheme always has very close performance with the optimum scheme. For example, the optimum scheme can save around 10% transmit power with respect to mid-point scheme to achieve $10^{-3}$ outage probability level. Min-min and min-max have the worst performance among these four schemes, where min-min starts to outperform min-max when SNR increases. In Fig. 5b, we see a significant outage performance increase when the intensity increases and the performance gap among the four schemes increases with the intensity, as we observed in Fig. 3b. In Fig. 5b, we also see that the performance advantage achieved by the optimum scheme over the mid-point scheme is minor when the density $\lambda = 0.1$. This means that the mid-point scheme is a good suboptimal selection scheme when the density is very small.

In Fig. 6, we plot the average rate versus the average SNR in Fig. 6a and the average rate versus intensity $\lambda$ for $N = 8, 16, 32$ in Fig. 6b. Analytical values calculated from (27) overlap with simulation, which verifies (27). Interestingly, we see that the optimum and mid-point selection schemes have very small rate differences for both $N = 16$ and 32 at $\lambda = 0.5$. For example, the rate difference is around 0.07 [bits/sec/Hz] for $N = 16$ at SNR = 5 dB. For all three cases
Fig. 7: Outage probability achieved by limited-feedback RIS selection versus the average SNR \( \bar{\gamma} \) for different values of the threshold \( T \). \( d = 1.2, N = 16, \lambda = 0.5, \) and \( \rho = 5 \) dB.

\( N = 8, 16, 32 \) in Fig. 6b, we have significant rate improvement from \( \lambda = 0.1 \) to \( \lambda = 0.5 \), and then there is a rate floor when \( \lambda \) increases further. This is due to the fact that there is a sufficient number of RISs within the neighborhood of TX and RX to support the communication. Thus, it is not worth to densify RISs in a given area beyond a certain limit.

**B. Feedback Case**

In this subsection, we focus on the network performance with limited-feedback RIS selection policies for both power-law and exp-law path-loss models. We note that \( T = \infty \) considered in this subsection corresponds to the all-feedback cases investigated in Section V-A.

In Fig. 7, we plot the outage probability achieved by limited-feedback RIS selection versus the average SNR \( \bar{\gamma} \) for the power-law model in Fig. 7a and for the exp-law model in Fig. 7b. The analytical curves in Figs. 7a and 7b are obtained by (32) and (34), respectively. These curves perfectly match with simulations, thus validating the accuracy of (32) and (34). We do not consider \( T = 1 \) in Fig. 7b since there is no RIS feeding back when \( T < 2d \) for the exp-law path-loss. For both Figs. 7a and 7b, we see that for \( T = 1, 3, 5, \) and, \( \infty \), the outage probability curves first overlap with the all-feedback cases and then keep constant as the average SNR \( \bar{\gamma} \) increases. This is because when \( \bar{\gamma} \) is small, the values of target SNR satisfy the condition \( \rho > \frac{\gamma N(16+(N-1)\pi^2)}{T_{\text{pow}}16} \) for the power-law model and the condition \( \rho > \frac{\gamma N(16+(N-1)\pi^2)}{\exp(\alpha T_{\text{exp}})16} \) for the exp-
Fig. 8: Average rate achieved by limited-feedback RIS selection for different values of the threshold $T$. $d = 1.2$ and $N = 16$.

The power-law model. In such conditions, the outage performance is the same as the all-feedback cases and does not depend on $T$ since at least one RIS feeds its location information back to the source node. When $\bar{\gamma}$ continuously increases, these conditions are not satisfied and the outage probability does only depend on the average number of RISs feeding back, without any dependence on the average SNR and fading behavior. This is because the achieved outage probability depends on whether or not there is at least one RIS feeding its location information back to the source. We also see when $T$ is larger, the outage performance is better in the flat region of outage curves. This is because when $T$ is larger, we have a higher probability of at least one RIS feeding back, thus achieving better outage performance.

We plot the average rate for limited-feedback RIS selection versus the average SNR $\bar{\gamma}$ for the power-law model in Fig. 8a and for the exp-law model in Fig. 8b. The perfect agreement between analytical curves obtained by (35) and (36) and simulations verify the accuracy of (35) and (36). For both Figs. 8a and 8b, we observe that for $\lambda = 0.1$, the performance loss from $T = \infty$ to $T = 5$ is much less than that from $T = 5$ to $T = 3$. This is due to the fact that the probability $\Pr\{N_{\text{pow}}^{\text{FB}} \geq 1\} = 1 - \exp (-\Xi_{\text{pow}})$ experience significant reduction from $T = 5$ to $T = 3$ while this probability experience minor reduction from $T = \infty$ to $T = 5$. This observation numerically demonstrates that average feedback load around 5 is enough to experience negligible optimization loss, compared to the all-feedback case. This suggests the
limited-feedback strategies significantly reduce feedback load while it almost does not sacrifice from the data rate performance.

VI. CONCLUSION

We considered a RIS-aided wireless network where one RIS is chosen from multiple PPP-distributed RISs to establish a communication link between the TX and RX. We considered power-law and exponential-law path-loss models to accommodate for outdoor and indoor communications, respectively. For each path-loss model, we proposed an optimum location-based RIS selection policy which aims to maximize the network SNR and derived the distance distribution of the optimum RIS node. Based on these distributions, we evaluated the outage probability and the average rate of the optimum RIS selection policies to assess the network performance for the power-law and exp-law path-loss models. We also proposed limited-feedback RIS selection policies to ease the feedback overhead for practical implementations of RIS-aided wireless networks. We derived the outage probabilities and average rates achieved by limited-feedback RIS selection policies for both path-loss models by deriving the distribution of the number of RISs feeding back. The numerical results showed the performance advantage of the proposed optimum and limited feedback RIS selection policies, demonstrated that the conventional optimum relay selection policy is not suitable for RIS-aided wireless networks, and reveal the performance gap for sub-optimum policies for power-law and exp-law path-loss models. Furthermore, the impact of system parameters, e.g., the number of reflecting elements and RIS node density, on the network performance is quantified thoroughly by means of a comprehensive numerical analysis.

APPENDIX A

PROOF OF THEOREM 1

We first present an important lemma that lays foundation for proving Theorem 1. This lemma will also be used in Appendix C for proving Theorem 3.

Lemma 1: We denote \( B_{\text{right}} (0, \tau) \) as the right half disc having non-negative first coordinates with radius \( \tau \) centering at the origin \( 0 \). We assume that \( U_r \) is a uniformly distributed random node over \( B_{\text{right}} (0, \tau) \). Let also \( \Upsilon = \hat{s}_{\text{pow}} (U_r) \). The expression for the CDF of RV \( \Upsilon \), \( F_{\Upsilon}(\gamma) \), is given by \( (A.1) \).
That is to say, for $\gamma$ is an increasing function with $d^2 < \gamma \leq \tau^2 - d^2$ when $\gamma > \tau^2 - d^2$.

We obtain the first derivative of $F(\gamma)$ as

$$F(\gamma) = \left\{ \begin{array}{ll}
\frac{2}{\pi^2} \left( d^4 E(\frac{\gamma^2}{d^2}) + (\gamma^2 - d^4) K(\frac{\gamma}{d^2}) \right) & \text{if } \gamma \leq d^2 \\
\frac{2}{\pi^2} \gamma E(\frac{\gamma}{d^2}) & \text{if } d^2 < \gamma \leq \tau^2 - d^2 \\
\frac{2}{\pi^2} \left( d^2 \sqrt{1 - \frac{(d^4 - \gamma^2 + d^2)^2}{4d^4 - 4\gamma^2}} + \gamma E(\frac{\gamma}{d^2}) \right) & \text{if } \tau^2 - d^2 < \gamma \leq d^2 + \tau^2 \\
1 & \text{if } \gamma > d^2 + \tau^2
\end{array} \right. \quad (A.1)$$

**Proof:** By using the law of cosines, $\Upsilon$ can be written as

$$\Upsilon = \sqrt{\left( \|U_t\|^2 + 2d \|U_t\| \cos \Theta + d^2 \right) \left( \|U_t\|^2 - 2d \|U_t\| \cos \Theta + d^2 \right)}$$

$$= \sqrt{d^4 + \|U_t\|^4 - 2d^2 \|U_t\|^2 \cos 2\Theta}, \quad (A.2)$$

where $\Theta$ is the angle between the non-negative $x$-axis and the line segment connecting $0$ and $U_t$. $\Theta$ is uniformly distributed over $[-\frac{\pi}{2}, \frac{\pi}{2}]$, and is not related to $\|U_t\|$ because $U_t$ is uniformly distributed. Thus, we derive the conditional CDF of $\Upsilon$ given $\{\Theta = \theta\}$ as

$$F_{\Upsilon|\Theta}(\gamma|\theta) = \Pr \{ \Upsilon^2 \leq \gamma^2 | \Theta = \theta \} = \Pr \{ d^4 + \|U_t\|^4 - 2d^2 \|U_t\|^2 \cos 2\Theta \leq \gamma^2 | \Theta = \theta \}. \quad (A.3)$$

To solve (A.3), we study the monotonicity of the function $f (\|U_t\|) = d^4 + \|U_t\|^4 - 2d^2 \|U_t\|^2 \cos 2\Theta$. We obtain the first derivative of $f (\|U_t\|)$ with respect to $\|U_t\|$ as $f' (\|U_t\|) = 4\|U_t\|^3 - 4d^2 \|U_t\| \cos 2\Theta$. Based on the expression of $f' (\|U_t\|)$, we find that $f' (\|U_t\|) > 0$ holds when the case 1) $\cos 2\Theta \leq 0$ and $\|U_t\| > 0$ or case 2) $\cos 2\Theta > 0$ and $\|U_t\| > \sqrt{d^2 \cos 2\Theta}$ is satisfied. That is to say, for $\theta \in [\frac{\pi}{4}, \frac{\pi}{2}] \cup \theta \in [-\frac{\pi}{4}, -\frac{\pi}{2}]$, $f (\|U_t\|)$ is an increasing function with $\|U_t\|$. For $\theta \in [-\frac{\pi}{4}, \frac{\pi}{4}]$, $f (\|U_t\|)$ is an increasing function when $\|U_t\| > \sqrt{d^2 \cos 2\Theta}$ and $f (\|U_t\|)$ is a decreasing function when $(\|U_t\|) < \sqrt{d^2 \cos 2\Theta}$.

We first solve (A.3) for $\theta \in [\frac{\pi}{4}, \frac{\pi}{2}] \cup [-\frac{\pi}{2}, -\frac{\pi}{4}]$. Under these conditions, only one positive root $U_1$ for $f (\|U_t\|) - \gamma^2 = 0$ exists, which is given by

$$U_1 = \sqrt{-d^2 + 2d^2 \cos 2\Theta + \frac{\sqrt{-d^4 - 2\gamma^2 + d^4 \cos 4\Theta}}{\sqrt{2}}} \quad (A.4)$$

Since $0 \leq \|U_t\| \leq \tau$ and $f (\|U_t\|)$ is an increasing function with $\|U_t\|$, thus we obtain $d^4 \leq f (\|U_t\|) \leq d^4 + \tau^4 - 2d^2 \tau^2 \cos 2\Theta$. Given $U_1$ and the range of $f (\|U_t\|)$, we solve (A.3)
for $\theta \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right] \cup \left[-\frac{\pi}{2}, -\frac{\pi}{4}\right]$ as

$$F_{T|\Theta}(\gamma|\theta, \theta \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right] \cup \left[-\frac{\pi}{2}, -\frac{\pi}{4}\right]) = \begin{cases} 
0 & \text{if } \gamma < d^2 \\
Pr \{\|U_1\| \leq U_1\} = \frac{U_1^2}{\tau^2} & \text{if } d^2 \leq \gamma \leq \sqrt{d^4 + \tau^4 - 2d^2\tau^2\cos 2\theta} \\
1 & \text{if } \gamma > \sqrt{d^4 + \tau^4 - 2d^2\tau^2\cos 2\theta}
\end{cases}$$

(5.5)

where $Pr \{\|U_1\| \leq U_1\} = \frac{U_1^2}{\tau^2}$ is based on the CDF of $\|U_1\|$, i.e., $F_{\|U_1\|}(u) = \frac{u^2}{\tau^2}$.

We next solve (A.3) for $\theta \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$. Under these conditions, two positive roots $U_1$ and $U_2$ for $f(\|U_1\|)-\gamma^2 = 0$ exist, where $U_1$ is given by (A.4) and $U_2$ is given by $U_2 = \sqrt{-d^2 + 2d^2\cos^2 \theta - \sqrt{-d^2 + 2\gamma^2 + d^4\cos 4\theta}}$. Since $f(\|U_1\|)$ is an increasing function when $\|U_1\| > \sqrt{d^2\cos 2\theta}$ and $f(\|U_1\|)$ is a decreasing function when $\|U_1\| < \sqrt{d^2\cos 2\theta}$, $f(\|U_1\|)$ obtain the minimal value $f(\sqrt{d^2\cos 2\theta}) = d^4 - d^4\cos^2 2\theta$ at $\|U_1\| = \sqrt{d^2\cos 2\theta}$. Thus, the range of $f(\|U_1\|)$ is $d^4 - d^4\cos^2 2\theta \leq f(\|U_1\|) \leq d^4 + \tau^4 - 2d^2\tau^2\cos 2\theta$. Given $U_1, U_2$, and the range of $f(\|U_1\|)$, we solve (A.3) when $\theta \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ as

$$F_{T|\Theta}(\gamma|\theta, \theta \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right] \cup \left[-\frac{\pi}{2}, -\frac{\pi}{4}\right]) = \begin{cases} 
0 & \text{if } \gamma < \sqrt{d^4 - d^4\cos^2 2\theta} \\
Pr \{U_2 \leq \|U_1\| \leq U_1\} = \frac{U_1^2 - U_2^2}{\tau^2} & \text{if } \sqrt{d^4 - d^4\cos^2 2\theta} \leq \gamma \leq d^2 \\
Pr \{\|U_1\| \leq U_1\} = \frac{U_1^2}{\tau^2} & \text{if } d^2 \leq \gamma \leq \sqrt{d^4 + \tau^4 - 2d^2\tau^2\cos 2\theta} \\
1 & \text{if } \gamma > \sqrt{d^4 + \tau^4 - 2d^2\tau^2\cos 2\theta}
\end{cases}$$

(5.6)

We will obtain $F_{T}(\gamma)$ by averaging $F_{T|\Theta}(\gamma|\theta)$ over $\Theta$. For $\theta \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right] \cup \left[-\frac{\pi}{2}, -\frac{\pi}{4}\right]$, we have

$$F_{T}(\gamma|\theta \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right] \cup \left[-\frac{\pi}{2}, -\frac{\pi}{4}\right]) = \begin{cases} 
0 & \text{if } \gamma < d^2 \\
\frac{2}{\pi} \left(\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{U_1^2}{\tau^2} d\theta + \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} U_1^2 d\theta + \int_{\frac{\pi}{4}}^{\frac{\pi}{4}} \tau^2 d\theta\right) & \text{if } d^2 \leq \gamma \leq \sqrt{d^4 + \tau^4} \\
\frac{2}{\pi} \left(\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{U_1^2 - U_2^2}{\tau^2} d\theta + \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} U_1^2 d\theta + \int_{\frac{\pi}{4}}^{\frac{\pi}{4}} \tau^2 d\theta\right) & \text{if } \sqrt{d^4 + \tau^4} \leq \gamma \leq d^2 + \tau^2 \\
\frac{2}{\pi} \left(\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{U_1^2 - U_2^2}{\tau^2} d\theta + \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} U_1^2 d\theta\right) & \text{if } \gamma \geq d^2 + \tau^2
\end{cases}$$

(5.7)

For $\theta \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$, we have

$$F_{T}(\gamma|\theta \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]) = \begin{cases} 
\frac{2}{\pi} \left(\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 0 d\theta + \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{U_1^2 - U_2^2}{\tau^2} d\theta\right) & \text{if } \gamma < d^2 \\
\frac{2}{\pi} \left(\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{U_1^2 - U_2^2}{\tau^2} d\theta\right) & \text{if } d^2 \leq \gamma \leq \sqrt{\tau^2 - d^2} \\
\frac{2}{\pi} \left(\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{U_1^2 - U_2^2}{\tau^2} d\theta\right) + \int_{\frac{\pi}{4}}^{\frac{\pi}{4}} 1 d\theta & \text{if } \tau^2 - d^2 \leq \gamma \leq \sqrt{d^4 + \tau^4} \\
\frac{2}{\pi} \left(\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 1 d\theta\right) & \text{if } \gamma \geq \sqrt{d^4 + \tau^4}
\end{cases}$$

(5.8)
Combining (A.7) and (A.8), for \( \theta \in [\frac{-\pi}{2}, \frac{\pi}{2}] \), we have

\[
F_T (\gamma | \theta \in [\frac{-\pi}{2}, \frac{\pi}{2}]) = \begin{cases} 
\frac{2}{\pi} \left( \int_0^\theta \frac{U_1^2 - U_2^2}{\tau^2} d\theta \right) & \text{if } \gamma < d^2 \\
\frac{2}{\pi} \left( \int_0^\theta \frac{U_1^2}{\tau^2} d\theta + \int_{\theta}^{\frac{\pi}{2}} \frac{U_1^2}{\tau^2} d\theta \right) & \text{if } d^2 \leq \gamma \leq \tau^2 - d^2 \\
\frac{2}{\pi} \left( \int_0^\theta \frac{U_1^2}{\tau^2} d\theta + \int_{\theta}^{\frac{\pi}{2}} \frac{U_1^2}{\tau^2} d\theta + \int_{\frac{\pi}{2}}^{\frac{\pi}{4}} \frac{U_1^2}{\tau^2} d\theta \right) & \text{if } \tau^2 - d^2 \leq \gamma \leq \sqrt{d^4 + \tau^4} \\
\frac{2}{\pi} \left( \int_0^\theta \frac{U_1^2}{\tau^2} d\theta + \int_{\theta}^{\frac{\pi}{2}} \frac{U_1^2}{\tau^2} d\theta + \int_{\frac{\pi}{2}}^{\frac{\pi}{4}} \frac{U_1^2}{\tau^2} d\theta \right) & \text{if } \gamma \geq \tau^2 + \tau^2 \\
\frac{2}{\pi} \left( \int_0^\theta \frac{U_1^2}{\tau^2} d\theta + \int_{\theta}^{\frac{\pi}{2}} \frac{U_1^2}{\tau^2} d\theta \right) & \text{if } \gamma \geq \tau^2 + \tau^2 \\
\end{cases}
\]

(A.9)

Applying [31, eq. (2.576)] given by \( \int \sqrt{a + b \cos \theta} d\theta = \frac{2}{\pi} \left( (a - b)F(c_1, \frac{1}{c_2}) + 2E(c_1, \frac{1}{c_2}) \right) \) and applying \( \int \sqrt{a + b \cos \theta} d\theta = 2\sqrt{a + bE(\frac{a}{2}, c_1)} \) to (A.9), where \( c_1 = \arcsin \sqrt{\frac{b(1 - \cos x)}{a + b}} \) and \( c_2 = \sqrt{\frac{2b}{a + b}} \), we obtain \( F_T (\gamma) \) as in (A.1) in Lemma 1. Finally, using the relation \( F_{R_{opt}}(\gamma) = 1 - \left( \lim_{r \to \infty} \exp \left( -\frac{\lambda r^2}{2} F_T(\gamma) \right) \right)^2 \) given by [32] and the result of \( F_T(\gamma) \) given in Lemma 1, we arrive at (9), which concludes the proof.

**APPENDIX B**

**PROOF OF THEOREM 2**

We assume \( U_r \) is a uniformly distributed random RIS location. We also assume that \( \Lambda = \Lambda_{\exp}(U_r) \), which is the sum of the distance between the source node and \( U_r \) and the distance between \( U_r \) the destination node. By using the law of cosines, we write \( \Lambda \) as

\[
\Lambda = \sqrt{||U_r||^2 + 2d \||U_r|| \cos \Theta + d^2} + \sqrt{||U_r||^2 - 2d \||U_r|| \cos \Theta + d^2} \quad (B.1)
\]

The conditional CDF of \( \Lambda \) given \( \{ \Theta = \theta \} \) can be expressed as

\[
F_{\Lambda|\Theta}(\gamma | \theta) = \Pr \left\{ \Lambda^2 \leq \gamma^2 | \Theta = \theta \right\} = \Pr \left\{ w(||U_r||) \leq \gamma^2 | \Theta = \theta \right\}, \quad (B.2)
\]

where

\[
w(||U_r||) = 2d^2 + 2||U_r||^2 + \sqrt{(||U_r||^2 + 2d \||U_r|| \cos \theta + d^2) \left( ||U_r||^2 - 2d \||U_r|| \cos \theta + d^2 \right)} \quad (B.3)
\]

We note that only one positive root \( W_1 \) for \( w(||U_r||) - \gamma^2 = 0 \) exist, where \( W_1 \) is given by

\[
W_1 = \frac{\sqrt{-4d^2\gamma^2 + \gamma^4}}{2\sqrt{\gamma^2 - 4d^2 \cos^2 \theta}}. \quad (B.4)
\]
Based on $w(||U_r||)$ is the increasing function with respect to $||U_r||$ and $0 \leq ||U_r|| \leq \tau$, we obtain the range of $w(||U_r||)$ is $4d^2 \leq w(||U_r||) \leq w(\tau)$. Given $W_1$ and the range of $w(||U_r||)$, we solve (B.2) as

$$F_\Lambda(\gamma) = \begin{cases} 
0 & \text{if } \gamma < 2d \\
\frac{2}{\pi\tau^2} \int_0^{\frac{\pi}{2}} W_1^2 d\theta & \text{if } 2d < \gamma \leq \sqrt{w(\tau)} \\
\frac{2}{\pi\tau^2} \int_0^{\frac{\pi}{2}} W_1^2 d\theta + \frac{2}{\pi} \int_0^{\phi} 1 d\theta & \text{if } 2\tau < \gamma \leq 2\sqrt{d^2 + \tau^2} \\
1 & \text{if } \gamma > 2\sqrt{d^2 + \tau^2}
\end{cases} \quad \text{(B.6)}$$

where $\phi_1 = \frac{1}{2} \arccos \left( \frac{d^4 + \tau^4 - (\gamma^2 - 2d^2 - 2\tau^2)^2}{2d^2\tau^2} \right)$ and $\phi_1$ is obtained by solving $\gamma^2 = w(\tau)$. Applying [29, eq. (2.562.2)], we obtain $F_\Lambda(\gamma)$ as in (B.7). Using $F_{\Lambda_{opt}}(\gamma) = \left( \lim_{\tau \to \infty} \exp \left( -\frac{\lambda\pi\tau^2}{2} F_\Lambda(\gamma) \right) \right)^2$ [32], we arrive at (11). This completes the proof.

**APPENDIX C**

**PROOF OF THEOREM 3**

We denote $B(0, \tau)$ as the disc centered at the origin 0 with radius $\tau$. We assume $\Xi_{pow}(\tau)$ as the average number of RISs located in $B(0, \tau)$ that feedback their channel quality indicators. We also assume that $U$ is a uniformly distributed random node over $B(0, \tau)$. Thus, we have

$$\Xi_{pow}(\tau) = \lambda\pi\tau^2 \Pr \left\{ S_{pow}(U) \leq T_{pow} \right\}. \quad \text{(C.1)}$$
We next obtain $\Pr \{ \hat{s}_{\text{pow}}(U) \leq T_{\text{pow}} \}$. We recall that $U_r$ is defined in Lemma 1 and $\text{vec} U_r$ is a uniformly distributed random node over right half disc $B_{\text{right}}(0, \tau)$. Similarly, we define $B_{\text{left}}(0, \tau)$ as the left half disc that centered at the origin $0$ with radius $\tau$ having negative first coordinates. We let $U_l$ is a uniformly distributed random node over left half disc $B_{\text{left}}(0, \tau)$.

Since the distribution of $U_r$ over $B_{\text{right}}$ is same as the distribution of $U_l$ over $B_{\text{left}}$, $U_r$, $U_l$, and $U_1$ are identically distributed RVs. As such, we have

$$
\Pr \{ \hat{s}_{\text{pow}}(U) \leq T_{\text{pow}} \} = \Pr \{ \hat{s}_{\text{pow}}(U_r) \leq T_{\text{pow}} \} = \Pr \{ \hat{s}_{\text{pow}}(U_l) \leq T_{\text{pow}} \}.
$$

We note that the CDF of $\Upsilon$, $F(\Upsilon)$, is given by (A.1), where $\Upsilon = \hat{s}_{\text{pow}}(U_r)$. Thus, the expression for $\Pr \{ \hat{s}_{\text{pow}}(U) \leq T_{\text{pow}} \}$ can be obtained by replacing $\gamma$ with $T_{\text{pow}}$ in (A.1). Finally, applying the expression for $\Pr \{ \hat{s}_{\text{pow}}(U) \leq T_{\text{pow}} \}$ to (C.1) and taking the limit $\Xi_{\text{pow}} = \lim_{\tau \to \Xi_{\text{pow}}(\tau)}$, we arrive at (30). This completes the proof.

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