Secure Polar Coding for Adversarial Wiretap Channel

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Abstract

The adversarial wiretap channel (AWTC) model is a secure communication model that eavesdropper can directly read and write fractions of the transmitted bits in legitimate communication. In this paper we propose a secure polar coding scheme to provide secure and reliable communication over the AWTC model. For the adversarial reading and writing action, we present a $\rho$ equivalent channel block and apply the non-stationary polarization on it. By comparing the polarization result of $\rho$ equivalent channel block with a $\rho$ BEC block (channel block of BEC with erase probability $\rho$), we find that the polarized subsets of $\rho$ BEC block is fully contained by the polarized subsets of $\rho$ equivalent channel block by choosing the polarization parameter $\beta$ properly. Based on this observation, we construct a secure polar coding scheme on the $\rho$ BEC blocks for the AWTC model. We theoretically prove that the proposed scheme achieves the secrecy capacity of AWTC model under both reliability and strong security criterions with an infinite block length $N$. Further, by simulations, we prove that the proposed scheme can provide secure and reliable communication over AWTC model with a finite block length $N$.

Index Terms

adversarial wiretap channels, polar codes, non-stationary polarization, secrecy capacity.

I. INTRODUCTION

Wiretap channel (WTC) model, introduced by Wyner in 1975 [1], is a primitive secure communication model in which two legitimate users Alice and Bob communicates through a noise main channel while an eavesdropper Eve wiretaps through a noise wiretap channel. Later in
wiretap channel type II (WTC-II) model is introduced in which the main channel is noiseless and the eavesdropper Eve can arbitrarily choose and directly read a fixed fraction of transmitted bits. Further in [3], WTC-II model is extended to adversarial wiretap channel (AWTC) model in which the eavesdropper Eve can directly read and write the transmitted bits with a pair of fixed fractions.

According to Wyner’s secure coding theory, channel noise of the WTC model can be used to provide perfect secrecy, and the goal for secure codes construction is achieving the secrecy capacity [1]. Polar codes [4], known as the first capacity achieving codes with low complexity, has presented excellent performance in providing the perfect secrecy over WTC models. Explicit secure polar codes have already achieved the secrecy capacities of Wyner’s WTC model [5], [6] and several extended WTC models [7]–[11]. These success of polar codes on WTC models make polar codes a considerable good option for achieving the secrecy capacity of AWTC model.

However, comparing with the WTC models, the secure codes construction of AWTC model is much more complicated. Because in AWTC model, both adversarial reading and writing directly act on the transmitted bits, and the corresponding bit index sets are arbitrarily chosen and unknown to legitimate parties. It is hard to construct the secure polar codes without precise information of eavesdropper’s actions or constant channel blocks.

A. Our Contributions

In this work, we consider the secrecy capacity achieving problem of AWTC model and intent to construct a corresponding secure polar coding scheme to provide secure and reliable communication. Our contributions are summarized as follow:

- For the adversarial reading and writing operations of AWTC model, we present an equivalent model as the $\rho$ equivalent channel block (Def. 5) which is an $N$ length non-stationary channel block composed of full-noise BECs with fraction $\rho$ and noiseless BECs with fraction $1-\rho$. Then we apply the non-stationary channel polarization [14] on the $\rho$ equivalent channel block and find that it can be fully polarized by the channel polarization operation $G_N$ into perfect full-noise channels with index set $\mathcal{H}_\rho$ and perfect noiseless channels with index set $\mathcal{L}_\rho$, even if $N$ is finite.
- Next, we take a $\rho$ BEC block (Def. 8) which is an $N$ length stationary BEC block with erase probability $\rho$, and polarize it by the channel polarization operation $G_N$ into almost full-noise channels with index set $\mathcal{H}_{\epsilon_\rho}$ and almost noiseless channels with index set $\mathcal{L}_{\epsilon_\rho}$.
Then we analyze the relationship of $\rho$ equivalent channel block and $\rho$ BEC block on the channel polarization by calculating the excluding rate $R_e$ (Def. 9) which is the sum rate of $H_{\epsilon,\rho}$ not included in $H_\rho$ and $L_{\epsilon,\rho}$ not included in $L_\rho$. We carry out a simulation to test $R_e$ on different values of triple $(N, \beta, \rho)$. From the simulation results (Fig. 3), we observe that by choosing a proper $\beta = \beta^*$ with given $(N, \rho)$, have $R_e = 0$, $H_{\epsilon,\rho} \subseteq H_\rho$ and $L_{\epsilon,\rho} \subseteq L_\rho$ (Prop. 1).

- Next, for the AWTC model, we have the actual index partition $(I_e, R_e, F_e, B_e)$ according to the polarization of $\rho$ equivalent channel block which is unknown to legitimate parties, and the index partition $(I_{\epsilon, e}, R_{\epsilon, e}, F_{\epsilon, e}, B_{\epsilon, e})$ according to the polarization of $\rho$ BEC block which is constructed by legitimate parties. By comparing these two partitions (Fig. 4), we find that using $(I_{\epsilon, e}, R_{\epsilon, e}, F_{\epsilon, e}, B_{\epsilon, e})$ as the substitution of $(I, R, F, B)$ with a proper $\beta^*$ will not compromise security or reliability. Thus we construct the secure polar coding scheme for AWTC model by apply the multi-block canning structure [6] on the index subsets $(I_{\epsilon, e}, R_{\epsilon, e}, F_{\epsilon, e}, B_{\epsilon, e})$.

- At last, we theoretically analyze the performance of the proposed secure polar coding scheme and prove that the secrecy capacity of AWTC model can be achieved when $N$ goes infinity. We also run simulations to test the performance of the scheme in finite $N$ cases. The simulation results match our theoretically analysis and prove that the proposed scheme successfully provide a reliable and secure communication over AWTC model with finite block length $N$.

### B. Related Works

The first secrecy capacity achieving secure polar codes for WTC model was proposed in [5] which sets up a basic way of secure polar codes construction that use the differences of polarizations between main channel block and wiretap channel block to divide the channel index into subsets with unique reliability and security properties. Further in [6], a multi-block chaining structure was constructed as a refinement of secure polar coding scheme in [5], which is known as one of the standard method for strong security polar coding construction and widely applied in the follow-up works such as [7], [10]. One remarkable advantage of this standard strong security polar coding method is the low computational complexity for both encoding and decoding process as $O(N \log N)$. 
When the AWTC model was proposed in [3], an effective explicit secrecy capacity achieving codes named as the capacity achieving AWTC code family was also proposed which contains three building blocks: algebraic manipulation detection code (AMD code), subspace evasive sets, and folded reed-solomon code (FRS code). For capacity achieving AWTC code family in [3], the encoding complexity is $O((N \log q)^2)$ where $q$ is a prime satisfying $q > Nu$ for a $u$-folded RS codes, the combined computational complexity of the FRS decoding algorithm and subspace evasive sets intersection algorithm is $\text{poly}(1/\xi)^D/\xi \log \log 1/\xi$, the AMD verification costs $O((N \log q)^2)$, thus the total complexity of the decoding is $\text{poly}(N)$. Then for our proposed secure polar coding scheme, since we only apply the standard method of strong security polar coding construction on the $\rho$ BEC blocks, both encoding and decoding computational complexities are $O(N \log N)$, which is lower than the secure codes in [3].

C. Paper Organizations

The rest of this paper is organized as follow. Section II presents the AWTC model. Section III presents our construction of secure polar coding scheme for AWTC model. Section IV presents the theoretical analysis of performance and the simulations. Finally section V concludes the paper.

II. THE ADVERSARIAL WIRETAP CHANNEL MODEL

Notations: We define the integer interval $[a, b]$ as the integer set between $[a]$ and $[b]$. For $n \in \mathbb{N}$, define $N \triangleq 2^n$. Denote $X, Y, Z, ...$ random variables (RVs) taking values in alphabets $\mathcal{X}, \mathcal{Y}, \mathcal{Z}, ...$ and the sample values of these RVs are denoted by $x, y, z, ...$ respectively. Then $p_{XY}$ denotes the joint probability of $X$ and $Y$, and $p_X, p_Y$ denotes the marginal probabilities. Especially for channel $W$, the transition probability is defined as $W_{Y|X}$ and $W$ for simplicity. Also we denote a $N$ size vector $X^{1:N} \triangleq (X^1, X^2, ..., X^N)$. When the context makes clear that we are dealing with vectors, we write $X^N$ in place of $X^{1:N}$. And for any index set $A \subseteq [1, N]$, we define $X^A \triangleq \{X^i\}_{i \in A}$. For the polar codes, we denote $G_N$ the generator matrix, $R$ the bit reverse matrix, $F = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $\otimes$ the Kronecker product, and we have $G_N = RF^\otimes n$. Denote $A[:,]$ as the average.

Now we present the definition of adversarial wiretap channel model (AWTC) [3].

Definition 1 The adversarial wiretap channel model is defined as $(\mathcal{X}, \mathcal{Y}, \mathcal{Z}, \rho_r, \rho_w, S_r, S_w)$. In the model, legitimate parties are communicating through a noiseless channel with channel input
alphabet $X$. For $N$-length transmitted codewords $X^{1:N} \in X^N$, there are two types of adversarial actions: reading and writing.

- **Reading**: Eavesdropper can arbitrarily select an index subset $S_r \subseteq [1, N]$ with fixed fraction $\rho_r = \frac{|S_r|}{N}$ and directly reads the corresponding transmitted codewords $X^{S_r}$. For the obtained bits at eavesdropper, $Z^N$ with alphabet $Z$, has

$$Z^i = \begin{cases} X^i & \text{if } i \in S_r \\ ? & \text{if } i \in S^c_r \end{cases}$$

where “?” is the dump letter.

- **Writing**: Eavesdropper can arbitrarily select an index subset $S_w \subseteq [1, N]$ with fixed fraction $\rho_w = \frac{|S_w|}{N}$ and directly writes the corresponding transmitted codewords $X^{S_w}$. For the channel output at legitimate receiver, $Y^N$ with alphabet $Y$, has

$$Y^i = \begin{cases} ? & \text{if } i \in S_w \\ X^i & \text{if } i \in S^c_w \end{cases}$$

where “?” is the dump letter.

![Adversarial Wiretap Channel Model](image)

Fig. 1. The adversarial wiretap channel model.

The communication process over AWTC is illustrated in Fig. 1. Legitimate user Alice want to send confidential message $M$ to legitimate user Bob with the existence of an active eavesdropper Eve. Alice encodes the message $M$ into channel input $X^N$ and transmits $X^N$ to Bob through a noiseless main channel. Eve arbitrarily reads $X^{S_r}$ with fixed rate $\rho_r$ and obtains the corresponding $Z^N$. Eve also arbitrarily writes $X^{S_w}$ with fixed rate $\rho_w$. Then Bob receives the modified channel output as $Y^N$ and decodes it into estimated confidential message $\hat{M}$. 
Definition 2 For any $(2^{NR}, N)$ secure codes of AWTC, the performances can be measured as follow.

- Reliability is measured by error probability $P_e = \Pr(M \neq \hat{M})$. The reliability criterion is $\lim_{N \to \infty} P_e = 0$.
- Security is measured by information leakage $L = I(Z^N; M)$. The weak security criterion is $\lim_{N \to \infty} \frac{L}{N} = 0$; the strong security criterion is $\lim_{N \to \infty} L = 0$.

The secrecy capacity of AWTC under reliability and security criterions has been characterized in [3], which is presented as follow.

Theorem 1 (Secrecy capacity [3]) The perfect secrecy capacity of the AWTC with $(\rho_r, \rho_w)$ is

$$C_s = 1 - \rho_r - \rho_w.$$  \hfill (3)

In this paper, we intend to present a polar codes based solution for the problem of secure and reliable communication over AWTC model, and try to achieve the secrecy capacity of Theo. 1.

III. Secure Polar Codes

In this section, we present a secure coding scheme for the AWTC model by polar codes.

A. BEC Based Equivalent AWTC Model

Unlike ordinary WTC model [1], adversarial actions in AWTC are not carried out through a wiretap channel but directly work on the transmitted bits in the communication channel. Since secure polar codes are built on polarized channels, without a wiretap channel, secure polar codes cannot be directly constructed on AWTC. Thus we first need to present the channel based expression for the reading and writing actions of AWTC.

Definition 3 Denote $W_{\epsilon_1} : \{0, 1\} \to \{0, 1, ?\}$ the full-noise binary erase channel (BEC) with erase probability $\epsilon = 1$ and $I(W_{\epsilon_1}) = 0$. Denote $W_{\epsilon_0} : \{0, 1\} \to \{0, 1, ?\}$ the noiseless BEC with erase probability $\epsilon = 0$ and $I(W_{\epsilon_0}) = 0$.

Without loss of generality, we consider the binary input case of AWTC model in Def. that $X = \{0, 1\}$, $Y = \{0, 1, ?\}$ and $Z = \{0, 1, ?\}$. Comparing the reading and writing actions with the $W_{\epsilon_1}$ and $W_{\epsilon_0}$, we can observe that
For the reading action, the read bits equals to being transmitted through $W_{e_0}$ to Eve, the unread bits equals to being transmitted through $W_{e_1}$ to Eve.

For the writing action, the written bits equals to being transmitted through $W_{e_1}$ to Bob, the unwritten bits equals to being transmitted through $W_{e_0}$ to Bob.

Thus we have the following BEC based equivalent model for AWTC model.

**Definition 4** The BEC based equivalent AWTC model is defined as $(W_w^N, W_r^N) : \mathcal{X}^N \rightarrow \mathcal{Y}^N, \mathcal{Z}^N$ that

- $W_w^N : \mathcal{X}^N \rightarrow \mathcal{Y}^N$ is the writing-equivalent $N$-length main channel block. For arbitrarily chosen $S_w \subseteq [1, N]$ with fixed fraction $\rho_w$, have

$$W_{e_1}^i : W_{e_1}^i(y = ?|x) = 1 \quad \text{if} \quad i \in S_w \quad \text{(4)}$$

$$W_{e_0}^i : W_{e_0}^i(y = x|x) = 1 \quad \text{if} \quad i \notin S_w^c.$$

- $W_r^N : \mathcal{X}^N \rightarrow \mathcal{Z}^N$ is the reading-equivalent $N$-length wiretap channel block. For arbitrarily chosen $S_r \subseteq [1, N]$ with fixed fraction $\rho_r$, have

$$W_{e_0}^i : W_{e_0}^i(z = x|x) = 1 \quad \text{if} \quad i \in S_r \quad \text{(5)}$$

$$W_{e_1}^i : W_{e_1}^i(z = ?|x) = 1 \quad \text{if} \quad i \notin S_r^c.$$

Comparing the $W_w^N$ and $W_r^N$ in Def. 4, we can observe that both the equivalent channel blocks have a same formation as follow.

**Definition 5** The $\rho$ equivalent channel block is defined as $W_{\rho}^N : \mathcal{X}^N \rightarrow \mathcal{Y}^N$ that for arbitrarily chosen $S_{\rho} \subseteq [1, N]$ with fixed fraction $\rho$,

$$W_{\rho}^i = \begin{cases} W_{e_1}^i : W_{e_1}^i(y = ?|x) = 1 & \text{if} \quad i \in S_{\rho} \\ W_{e_0}^i : W_{e_0}^i(y = x|x) = 1 & \text{if} \quad i \in S_{\rho}^c. \end{cases} \quad \text{(6)}$$

The average erase probability of the channel block is $\rho$.

**Remark 1** By setting $\rho = \rho_w$, $W_{\rho}^N = W_{\rho_w}^N$ which becomes the writing-equivalent channel block; by setting $\rho = 1 - \rho_r$, $W_{\rho}^N = W_{1- \rho_r}^N$ which becomes the reading-equivalent channel block.

Therefore, by analyzing the channel polarization results of the channel operation $G_N$ on $\rho$ channel block, we can find a way to construct the secure polar codes.
B. Discussion of Polarization

Now we discuss the polarization of the $\rho$ equivalent channel block. As defined, the $\rho$ equivalent channel block is an $N$-length non-stationary channel sequence consisted of full noise BEC $W_\epsilon$ with fraction $\rho$ and noiseless BEC $W_\epsilon^0$ with fraction $1 - \rho$. The polarization theory of non-stationary channel sequence has been studied in [12]–[14] that the channel operation $G_N$ has a similar polarization effect on the non-stationary channel sequence.

**Theorem 2** (Non-stationary polarization [14, Theo. 2]) For any B-DMC $W_{1:N}$ with different transition probabilities, the generated channels $W_{1:N}^{(i)}$ form non-stationary channel transformation $G_N$ are polarized in the sense that, for any fixed $\delta \in (0, 1)$, as $N \to \infty$, the fraction of indices $i \in [1, N]$ for which $I(W_{1:N}^{(i)}) \in (1 - \delta, 1]$ goes to $\mathbb{A}[I(W_{1:N})]$ and the fraction for which $I(W_{1:N}^{(i)}) \in [0, \delta)$ goes to $1 - \mathbb{A}[I(W_{1:N})]$. Also can be write as

$$I_\infty = \begin{cases} 1 & \text{w.p. } \mathbb{A}[I(W_{1:N})] \\ 0 & \text{w.p. } 1 - \mathbb{A}[I(W_{1:N})], \end{cases}$$

(7)

where $\mathbb{A}[I(W_{1:N})]$ is the average of the initial $I(W^{(i)})$ for all the $i \in [1, N]$.

For non-stationary polarization theory, the $2 \times 2$ kernel transformation of the channel operation $G_N$ is defined as follow.

![Fig. 2. The non-stationary $2 \times 2$ kernel transformation.](image)

**Definition 6** ([14]) Define $(W^1, W^2) \mapsto (W^-, W^+)$ the non-stationary $2 \times 2$ kernel transformation, illustrated in Fig. 2 which contains a pair of channel operation ($\Box, \square$) that

$$W^- = W^1 \Box W^2 \text{ and } W^+ = W^1 \square W^2,$$

(8)

respectively as

$$W^-(f(y^1, y^2) | u^1) = \sum_{u^2 \in \mathcal{U}} \frac{1}{2} W^1(y^1 | u^1 \oplus u^2) W^2(y^2 | u^2),$$

$$W^+(f(y^1, y^2), u^1 | u^2) = \frac{1}{2} W^1(y^1 | u^1 \oplus u^2) W^2(y^2 | u^2).$$

(10)
Definition 7 (\[4\]) For any given B-DMC $W : \mathcal{X} \rightarrow \mathcal{Y}$, the Bhattacharyya parameter is defined as

$$Z(W) \triangleq \sum_{y \in \mathcal{Y}} \sqrt{W(y|0)W(y|1)}.$$  \hspace{1cm} (11)

For BEC, have $I(W) = 1 - Z(W)$.

Lemma 1 (\[14, Lem. 2\]) For non-stationary $2 \times 2$ kernel transformation $(W_1, W^2) \mapsto (W^-, W^+)$, have

$$Z(W^+) = Z(W^1)Z(W^2),$$

$$Z(W^-) \leq Z(W^1) + Z(W^2) - Z(W^1)Z(W^2).$$ \hspace{1cm} (13)

The second equal holds when $W_1$ and $W^2$ are BECs.

Now we consider the non-stationary channel transformation of the $\rho$ equivalent channel block $W^N_{\rho}$.

Let $N = 2^n$ and $W^{(1:N)}_{\rho N}$ be the channel sequence generated from $W^N_{\rho}$ by the non-stationary channel operation $G_N$ with $n$ levels of recursively $2 \times 2$ kernel transformation $(W^1, W^2) \mapsto (W^-, W^+)$. Since $W^N_{\rho}$ is formed by $W_{e_1}$ and $W_{e_0}$ that $Z(W_{e_1}) = 1$ and $Z(W_{e_0}) = 0$, for $i \in [1, N]$, $\rho$ fraction of $Z(W^i_{\rho})$ is 1 and $1 - \rho$ fraction of $Z(W^i_{\rho})$ is 0.

Then by Lem. [2] for each recursion level of $2 \times 2$ kernel channel transformation, there are only three possible cases for the Bhattacharyya parameter values $[Z(W^1), Z(W^2)] \mapsto [Z(W^-), Z(W^+)]$ that $(1, 1) \mapsto (1, 1)$, $(1, 0) \mapsto (1, 0)$ and $(0, 0) \mapsto (0, 0)$. Hence we can observe that after $N$ levels of kernel transformation, the fraction of $Z(W^{(i)}_{\rho N}) = 1$ remains $\rho$ and $Z(W^{(i)}_{\rho N}) = 0$ remains $1 - \rho$. Thus for non-stationary channel polarization of $W^N_{\rho}$, we have the following polarized index sets of $[1, N]$:

$$\mathcal{H}_\rho = \{i \in [1, N] : Z(W^{(i)}_{\rho N}) = 1\},$$

$$\mathcal{L}_\rho = \{i \in [1, N] : Z(W^{(i)}_{\rho N}) = 0\},$$ \hspace{1cm} (15)

where $\mathcal{H}_\rho$ is the polarized full-noise index set, and $\mathcal{L}_\rho$ is the polarized noiseless index set.

Remark 2 The $\rho$ equivalent channel block $W^N_{\rho}$ can be fully polarized by non-stationary polarization operation $G_N$ that $\mathcal{H}_\rho^c = \mathcal{L}_\rho$ and $\mathcal{H}_\rho = \mathcal{L}_\rho^c$ even if $N$ is finite.

Note that from the perspective of legitimate parties, because the index set $\mathcal{S}_\rho$ it is arbitrarily chosen with fixed fraction $\rho$ by eavesdropper and different $\mathcal{S}_\rho$ can result in different $\mathcal{H}_\rho$ and $\mathcal{L}_\rho$, they cannot know the precise $\mathcal{H}_\rho$ and $\mathcal{L}_\rho$ for constructing the secure polar codes.
Definition 8 Define a ρ BEC block as $W_{\epsilon\rho}^N$ which is an $N$-length stationary sequence of channel $W_{\epsilon\rho}$, where $W_{\epsilon\rho} : \{0, 1\} \rightarrow \{0, 1, ?\}$ is the BEC with erase probability $\epsilon = \rho$.

Remark 3 Comparing the ρ BEC block $W_{\epsilon\rho}^N$ in Def. 8 with the ρ equivalent channel block $W_{\rho}^N$ in Def. 5, both channel models have a same average erase probability as ρ. From the perspective of transmission effect, $W_{\rho}^N$ is the expectation of $W_{\epsilon\rho}^N$. Thus by the Bernoulli’s law of large Numbers, when $N$ goes infinity, the $W_{\epsilon\rho}^N$ can be equivalent to the $W_{\rho}^N$. However when $N$ is finite, these two channel models are not equivalent.

Next we analyze their relationship on the channel polarization. According to the channel polarization theory, by the channel operation $G_N$, the $\rho$ BEC block $W_{\epsilon\rho}^N$ can be polarized as follow. Let $\{W_{\epsilon\rho}^{(i)}\}_{i=1}^N$ be the channel sequence generated from $W_{\epsilon\rho}^N$ by the channel polarization operation. Then for any $0 < \beta < \frac{1}{2}$, $\delta_N = 2^{-N\beta}$, the polarized index sets of $[1, N]$ are

$$\mathcal{H}_{\epsilon\rho} = \{i \in [1, N] : Z(W_{\epsilon\rho}^{(i)}) \geq 1 - \delta_N\},$$
$$\mathcal{L}_{\epsilon\rho} = \{i \in [1, N] : Z(W_{\epsilon\rho}^{(i)}) \leq \delta_N\},$$

where $\mathcal{H}_{\epsilon\rho}$ is the polarized full-noise index set, and $\mathcal{L}_{\epsilon\rho}$ is the polarized noiseless index set.

Remark 4 For polar codes, only in case of infinite $N$, the channel block can be fully polarized by any $0 < \beta < \frac{1}{2}$, which satisfies $\mathcal{H}_{\epsilon\rho} = \mathcal{L}_{\epsilon\rho}$ and $\mathcal{H}_{\rho} = \mathcal{L}_{\rho}$. However, in case of finite $N$, the polarization cannot be perfect but directly influenced by the value of $\beta$. If $\beta$ decreases, both transmission rate and upper bound of decoding bit error rate will increase. If $\beta$ increases, both code rate $|\mathcal{L}_{\epsilon\rho}|/N$ and upper bound of decoding bit error rate will decrease.

To analyze the polarization relationship between $W_{\epsilon\rho}^N$ and $W_{\rho}^N$, we use the excluding rate $R_e$ in Def. 9 to measure the sum rate of $\mathcal{H}_{\epsilon\rho}$ excluded from $\mathcal{H}_{\rho}$ and $\mathcal{L}_{\epsilon\rho}$ excluded from $\mathcal{L}_{\rho}$. If $R_e = 0$, it means $\mathcal{H}_{\epsilon\rho} \subseteq \mathcal{H}_{\rho}$ and $\mathcal{L}_{\epsilon\rho} \subseteq \mathcal{L}_{\rho}$.

Definition 9 The excluding rate $R_e$ is defined as

$$R_e = \frac{1}{N}(|\mathcal{H}_{\epsilon\rho} \setminus \mathcal{H}_{\rho}| + |\mathcal{L}_{\epsilon\rho} \setminus \mathcal{L}_{\rho}|),$$

where $(\mathcal{H}_{\epsilon\rho}, \mathcal{L}_{\epsilon\rho})$ are the polarized subsets of ρ BEC block $W_{\epsilon\rho}^N$, $(\mathcal{H}_{\rho}, \mathcal{L}_{\rho})$ are the polarized subsets of ρ equivalent channel block $W_{\rho}^N$.

There are four factors can influence $R_e$, which are block length $N$, polarization parameter $0 < \beta < \frac{1}{2}$, fraction $0 \leq \rho \leq 1$ and index set $S_{\rho}$. Thus we choose $N$ from $2^6$ to $2^{15}$, $\beta$ from
0.01 to 0.49 at 0.03 intervals and $\rho$ from 0.1 to 0.9 at 0.1 intervals. Then, in order to cover the randomness of index set $S_\rho$, for each value of triple $(N, \beta, \rho)$, we calculate the corresponding excluding rates for 100 arbitrarily chosen $S_\rho$ and take an average rate. The calculation results of excluding rate is illustrated in Fig. 3.

**Proposition 1** From Fig. 3, it can be observed that for the given $\rho$ and $N$ with arbitrarily chosen $S_\rho$, there exists $\beta = \beta^*$ which makes $R_e = 0$, $H_{e\rho} \subseteq H_\rho$ and $L_{e\rho} \subseteq L_\rho$. With the increasing of block length $N$, the lower bound of $\beta^*$ is decreasing.
C. Secure Polar Coding Scheme

Based on Prop. 1, now we can construct the secure polar code for AWTC model.

Remind that in AWTC model, for the adversarial writing and reading operations, the fraction pair \((\rho_w, \rho_r)\) of is publicly known and fixed, but the index pair \((S_w, S_r)\) is arbitrarily chosen by eavesdropper and unknown to legitimate parties.

Also remind that for adversarial writing operation with fraction \(\rho_w\), the equivalent channel block \(W_w^N = W_{\rho_w}^N\) and the corresponding \(\rho\) BEC block is \(W_\rho^{\rho_w}\). For adversarial reading operation with fraction \(\rho_r\), the equivalent channel block \(W_r^N = W_{1-\rho_r}^N\) and the corresponding \(\rho\) BEC block is \(W_\rho^{1-\rho_r}\).

For arbitrarily chosen \((S_w, S_r)\), the actual non-stationary polarization of equivalent channel block \(W_w^N\) and \(W_r^N\) are as follow.

\[
\begin{align*}
\mathcal{H}_w &= \{ i \in [1, N] : Z(W_w^{(i)}_{1:N}) = 1 \}, \\
\mathcal{L}_w &= \{ i \in [1, N] : Z(W_w^{(i)}_{1:N}) = 0 \}, \\
\mathcal{H}_r &= \{ i \in [1, N] : Z(W_r^{(i)}_{1:N}) = 1 \}, \\
\mathcal{L}_r &= \{ i \in [1, N] : Z(W_r^{(i)}_{1:N}) = 0 \},
\end{align*}
\]

(20)

where \(W_w^{(i)}_{1:N}\) and \(W_r^{(i)}_{1:N}\) are the generated channel by non-stationary channel operation \(G_N\) respectively from \(W_w^N\) and \(W_r^N\). With these actual polarization results, the index \([1, N]\) can be divided into following four subsets:

\[
\begin{align*}
\mathcal{I} &= \mathcal{L}_w \cap \mathcal{H}_r, \\
\mathcal{R} &= \mathcal{L}_w \cap \mathcal{H}_r^c, \\
\mathcal{F} &= \mathcal{L}_r^c \cap \mathcal{H}_r, \\
\mathcal{B} &= \mathcal{L}_r^c \cap \mathcal{H}_r^c,
\end{align*}
\]

(22)

where \(\mathcal{I}\) is secure and reliable, \(\mathcal{R}\) is insecure but reliable, \(\mathcal{F}\) is secure but unreliable, and \(\mathcal{B}\) is insecure and unreliable. Note that the index sets \((S_w, S_r)\) is arbitrarily chosen by eavesdropper, thus legitimate parties cannot know the actual polarization results \((\mathcal{H}_w, \mathcal{L}_w, \mathcal{H}_r, \mathcal{L}_r)\) or the actual division \((\mathcal{I}, \mathcal{R}, \mathcal{F}, \mathcal{B})\).

For legitimate parties, because they know the fraction pair \((\rho_w, \rho_r)\), what they can do is to build the corresponding \(\rho\) BEC blocks, as \(W_\rho^{\rho_w}\) for writing operation and \(W_\rho^{1-\rho_r}\) for reading
operation. Then according to Prop. 1 for the given \((N, \rho_w, \rho_r)\), they can choose a proper \(\beta^*\), \(\delta_N = 2^{-N\beta^*}\) to polarize the block pair \((W_{\epsilon\rho_w}^N, W_{\epsilon_1-\rho_r}^N)\) as

\[
\mathcal{H}_{\epsilon\rho_w} = \{i \in [1, N] : Z(W_{\epsilon\rho_w}^i) \geq 1 - \delta_N\},
\]
\[
\mathcal{L}_{\epsilon\rho_w} = \{i \in [1, N] : Z(W_{\epsilon\rho_w}^i) \leq \delta_N\},
\]
\[
\mathcal{H}_{\epsilon_1-\rho_r} = \{i \in [1, N] : Z(W_{\epsilon_1-\rho_r}^i) \geq 1 - \delta_N\},
\]
\[
\mathcal{L}_{\epsilon_1-\rho_r} = \{i \in [1, N] : Z(W_{\epsilon_1-\rho_r}^i) \leq \delta_N\},
\]

which satisfies \(\mathcal{H}_{\epsilon\rho_w} \subseteq \mathcal{H}_w\), \(\mathcal{L}_{\epsilon\rho_w} \subseteq \mathcal{L}_w\), \(\mathcal{H}_{\epsilon_1-\rho_r} \subseteq \mathcal{H}_r\) and \(\mathcal{L}_{\epsilon_1-\rho_r} \subseteq \mathcal{L}_r\). Further, they can divide the index \([1, N]\) as

\[
\mathcal{I}_\epsilon = \mathcal{L}_{\epsilon\rho_w} \cap \mathcal{H}_{\epsilon_1-\rho_r},
\]
\[
\mathcal{R}_\epsilon = \mathcal{L}_{\epsilon\rho_w} \cap \mathcal{H}_{\epsilon_1-\rho_r}^c,
\]
\[
\mathcal{F}_\epsilon = \mathcal{L}_{\epsilon\rho_w}^c \cap \mathcal{H}_{\epsilon_1-\rho_r},
\]
\[
\mathcal{B}_\epsilon = \mathcal{L}_{\epsilon\rho_w}^c \cap \mathcal{H}_{\epsilon_1-\rho_r}^c.
\]

Now we analyze the actual properties of above divided subsets. Fig. 4 illustrates the comparison of the two subsets divisions \((\mathcal{I}, \mathcal{R}, \mathcal{F}, \mathcal{B})\) and \((\mathcal{I}_\epsilon, \mathcal{R}_\epsilon, \mathcal{F}_\epsilon, \mathcal{B}_\epsilon)\).

Fig. 4. Comparison of the two subsets divisions (22) and (26).

- Subset \(\mathcal{I}_\epsilon\): by choosing a proper \(\beta^*\), have \(\mathcal{L}_{\epsilon\rho_w} \subseteq \mathcal{L}_w\) and \(\mathcal{H}_{\epsilon_1-\rho_r} \subseteq \mathcal{H}_r\), thus \(\mathcal{I}_\epsilon \subseteq \mathcal{I}\), which indicates that actual polarized channels in \(\mathcal{I}_\epsilon\) are secure and reliable.
- Subset \(\mathcal{R}_\epsilon\): by choosing a proper \(\beta^*\), have \(\mathcal{R}_\epsilon \subseteq \mathcal{L}_{\epsilon\rho_w} \subseteq \mathcal{L}_w\), thus there is no intersection of \(\mathcal{R}_\epsilon\) and \((\mathcal{F}, \mathcal{B})\), which indicates that actual polarized channels in \(\mathcal{R}_\epsilon\) are reliable. As
shown in Fig. 4, the part of $\mathcal{R}_e$ included in $\mathcal{R}$ is reliable but insecure, the rest part of $\mathcal{R}_e$ is included in $\mathcal{I}$ which is secure and reliable.

- Subset $\mathcal{F}_e$: by choosing a proper $\beta^*$, have $\mathcal{F}_e \subseteq \mathcal{H}_{e1-\rho_r} \subseteq \mathcal{H}_r$, thus there is no intersection of $\mathcal{F}_e$ and $(\mathcal{R}, \mathcal{B})$, which indicates that actual polarized channels in $\mathcal{F}_e$ are secure. As shown in Fig. 4, the parts of $\mathcal{F}_e$ included in $\mathcal{F}$ is secure but unreliable, the rest part of $\mathcal{F}_e$ is included in $\mathcal{I}$ which is secure and reliable.

- Subset $\mathcal{B}_e$: by choosing a proper $\beta^*$, have $\mathcal{L}_w^c \subseteq \mathcal{L}_{e1-\rho_w}^c$ and $\mathcal{H}_r^c \subseteq \mathcal{H}_{e1-\rho_r}^c$, thus $\mathcal{B} \subseteq \mathcal{B}_e$ that $\mathcal{B}_e$ has covered all the insecure and unreliable actual polarized channels. As shown in Fig. 4, except for the entire subset $\mathcal{B}$, parts of $\mathcal{I}, \mathcal{R}$ and $\mathcal{F}$ are also included in subset $\mathcal{B}_e$.

Since the divided subsets of $\rho$ BEC blocks $(\mathcal{I}_e, \mathcal{R}_e, \mathcal{F}_e, \mathcal{B}_e)$ are fixed and known, based on the actual properties of these divided subsets, we can apply the multi-block chaining structure [6] as follow to construct a strong security polar coding scheme.

- To build the chaining structure, separate a subset $\mathcal{E}_e$ form $\mathcal{I}_e$ that satisfies $|\mathcal{E}_e| = |\mathcal{B}_e|$.
- For subset $\mathcal{I}_e \setminus \mathcal{E}_e$, since the corresponding actual polarized channels is secure and reliable, they can be used to transmit information bits.
- For subset $\mathcal{R}_e$, since the corresponding actual polarized channels is reliable but not certainly secure, it is used to transmit uniformly distributed random bits, so that the security can be guaranteed.
- For subset $\mathcal{F}_e$, since the corresponding actual polarized channels is secure but not certainly reliable, it is used to transmit publicly known frozen bits, so that the reliability can be guaranteed.
- For subset $\mathcal{E}_e$, since the corresponding actual polarized channels is secure and reliable, it is used to convey uniformly distributed random bits for the subset $\mathcal{B}_e$ of next block.
- For subset $\mathcal{B}_e$, since it includes all the insecure and unreliable actual polarized channels of subset $\mathcal{B}$, we use the chaining structure for it. In the first block, $\mathcal{B}_e$ is used to transmit random bits pre-shared by legitimate parties; in the rest blocks, $\mathcal{B}_e$ is used to transmit bit conveyed in $\mathcal{E}_e$ of previous block. Therefore both reliability and security can be guaranteed.

Finally, we propose the secure polar coding scheme for AWTC model.

- Preparing: consider the multi-block case with block number $T$, block length $N = 2^n$ and fixed fraction pair $(\rho_w, \rho_r)$;
  - build the $\rho$ BEC blocks $W_{\rho_w}^N$ for writing and $W_{e1-\rho_r}^N$ for operation;
choose a proper $\beta^*$ and polarize the block pair $(W_{\epsilon_{\rho w}}, W_{\epsilon_{1-\rho r}})$ into $(W^{(1:N)}_{\epsilon_{\rho w}N}, W^{(1:N)}_{\epsilon_{1-\rho r}N})$

- divide the index into $(I_{\epsilon}, R_{\epsilon}, F_{\epsilon}, B_{\epsilon})$ by (26) and separate $E_{\epsilon} \subset I_{\epsilon}$ that $|E_{\epsilon}| = |B_{\epsilon}|$.

**Encoding:**
- $u^{I_{\epsilon} \setminus E_{\epsilon}}$ are assigned with information bits;
- $u^{E_{\epsilon} \cup R_{\epsilon}}$ are assigned with uniformly distributed random bits;
- $u^{F_{\epsilon}}$ are assigned with publicly known frozen bits;
- if $t = 1$, $u^{B_{\epsilon}}$ are assigned with pre-shared random bits;
- if $t \geq 2$, $u^{B_{\epsilon}}$ are assigned with the bits of $u^{E_{\epsilon}}$ of block $t - 1$;
- encode the $u^{N}$ into channel inputs $x^{N}$ by $x^{N} = u^{N}G_{N}$.

**Transmission:**
- Alice transmits $x^{N}$ to Bob through a noiseless communication channel;
- Eve arbitrarily chooses $S_{\epsilon}$ with fraction $\rho_{\epsilon}$ and writes $x^{S_{\epsilon}}$ into "?";
- Bob receives the modified channel outputs as $y^{N}$;
- Eve arbitrarily chooses $S_{\epsilon}$ with fraction $\rho_{\epsilon}$ and reads $x^{S_{\epsilon}}$ to obtain $z^{N}$.

**Decoding:** Bob uses the successive cancelation (SC) decoding [4] to decode $y^{N}$ into $\hat{u}^{N}$ as follow:
- if $i \in I_{\epsilon} \cup R_{\epsilon}$,
  \[ \hat{u}^{i} = \arg \max_{u \in \{0, 1\}} W^{(i)}_{\epsilon_{\rho w}N}(u|u^{1:i-1}, y^{1:N}); \]  \hspace{1cm} (27)
- if $i \in F_{\epsilon}$, $\hat{u}^{i}$ is decoded as publicly known frozen bit;
- if $i \in B_{\epsilon}$, in case of $t = 1$, $\hat{u}^{i}$ is decoded as pre-shared random bit, in case of $t \geq 2$, $\hat{u}^{i}$ is decoded as corresponding bit of $\hat{u}^{E_{\epsilon}}$ of block $t - 1$.

**IV. Performance**

In this section, we discuss the performance of the proposed secure polar coding scheme for AWTC model.

**A. Theoretical Analysis**

Consider the following Theorem for polar codes.
Theorem 3 (Decoding error rate of polar codes) \cite[Prop. 3]{5} For any B-DMC channel block $W^N$, let $A$ be an arbitrary subset of index $[1, N]$ and used as the information set for polar codes. Then the corresponding block error rate of SC decoding satisfies

$$P_e \leq \sum_{i \in A} Z(W_r^{(i)}) \tag{28}$$

Now we analyze the performance of proposed secure polar coding scheme by reliability, security and achievable secrecy rate.

Proposition 2 (Reliability) By choosing a proper $\beta^*$, reliability can be achieved by the proposed secure polar coding scheme over AWTC model.

Proof: Since the frozen bits in $F_e$ are publicly known and Bob known the pre-shared bits for the first $B_e$, then according to the multi-block chaining structure, the decoding error rate of entire $T$ blocks are determined by the SC decoding of $T$ blocks’ $I_e \cup R_e$ and $T - 1$ blocks’ $E_e$. For each codeword $X^i$, it is transmitted through actual equivalent channel $W^i_{w}$ but decoded according to BEC channel $W^i_{\rho_w}$ by legitimate parties, thus we use $\max[Z(W_r^{(i)}), Z(W_r^{(i)}_{\rho_w})]$ to analyze the reliability of polar decoding.

Then for the decoding error of $T$ blocks, have

$$P_e(T) \leq T \sum_{i \in I_e \cup R_e} \max[Z(W_r^{(i)}_{w}), Z(W_r^{(i)}_{\rho_w})] + (T - 1) \sum_{i \in E_e} \max[Z(W_r^{(i)}_{w}), Z(W_r^{(i)}_{\rho_w})] \tag{30}$$

where (a) is because $E_e \subseteq I_e \cup R_e \subseteq L_{\rho_w}$ that the corresponding $Z(W_r^{(i)}_{w}) = 0$; (b) is due to Theo. \[3\]

Therefore, when $N$ is infinite, have $\lim_{N \to \infty} P_e(T) = 0$, which indicates that reliability can be achieved. When $N$ is finite, it can be observed that if $\beta^*$ increases, the decoding error rate will decrease and a better reliability performance can be achieved.

Proposition 3 (Security) By choosing a proper $\beta^*$, strong security can be achieved by the proposed secure polar coding scheme over AWTC model.

Proof: For block $t$, let $M^t = U^{T_e} \setminus E_e$, $Z^t = Z^N$, $E^t = U^{E_e}$ and $F^t = U^{F_e}$. Then the information leakage for entire $T$ blocks is $L(T) = I(M^1:T; Z^{1:T})$. 
For multi-block chaining structure, as deduced in [6, Section IV-B], with publicly known frozen bits, have

$$L(T) \leq \sum_{t=1}^{T} I(M^t, E^t, F^t; Z^t) + I(E^0; Z^0),$$

(31)

where $I(E^0; Z^0)$ refers to the information leakage of the pre-shared bits before transmission which should be 0.

Let $a^1 < a^2 < \ldots < a^{|A|}$ be the correspondent indices of the elements $U^A$ for any subset $A$, such that $U^A \triangleq U^{a^1:a^{|A|}} = U^{a^1}, \ldots, U^{a^{|A|}}$. Since subsets $\mathcal{I}_c \cup \mathcal{F}_e$ and $\mathcal{R}_e$ match the construction of induced channel [5, Lem. 15], we have

$$I(M^t, E^t, F^t; Z^t) = I(U^{I \cup \mathcal{F}_e}; Z^N)$$

$$= \sum_{i=1}^{[I \cup \mathcal{F}_e]} I(U^{a^i}; Z^N | U^{a^{i-1}})$$

$$\overset{(a)}{=} \sum_{i=1}^{[I \cup \mathcal{F}_e]} I(U^{a^i}; U^{a^{i-1}}, Z^N)$$

$$\leq \sum_{i=1}^{[I \cup \mathcal{F}_e]} I(U^{a^i}; U^{1:a^{i-1}}, Z^N)$$

$$\overset{(b)}{=} \sum_{j \in \mathcal{I}_c \cup \mathcal{F}_e} \max \left[ I(W^{(j)}_{rN}), I(W^{(j)}_{\epsilon_1-\rho r N}) \right]$$

$$\overset{(c)}{\leq} o(N2^{-N\beta^*}),$$

where (a) is because $U^{a^i}$ are independent from each other; (b) is because Eve can use either $W^{N}_{rN}$ or $W^{N}_{\epsilon_1-\rho r N}$ for SC decoding and her best strategy is to choose the one with higher capacity from $W^{(j)}_{rN}$ and $W^{(j)}_{\epsilon_1-\rho r N}$; (c) is because that, by choosing a proper $\beta^*$, have $(\mathcal{I}_c \cup \mathcal{F}_e) \subseteq \mathcal{H}_r$ and from [20] have $Z(W^{(j)}_{rN}) = 1$ for all $j \in \mathcal{H}_r$, thus form [22], have $I(W^{(j)}_{rN}) = 0$ for $j \in \mathcal{I}_c \cup \mathcal{F}_e$; (c) is also because [24] and [26] that for $j \in \mathcal{I}_c \cup \mathcal{F}_e$, have $I(W^{(j)}_{\epsilon_1-\rho r N}) = 1 - Z(W^{(j)}_{\epsilon_1-\rho r N}) \leq 2^{-N\beta^*}$.

Thus we have $L(T) \leq o(TN2^{-N\beta^*})$. When $N$ is infinite, have $\lim_{N \to \infty} L(T) = 0$, which indicates that strong security can be achieved. When $N$ is finite, it can be observed that if $\beta^*$ increases, the information leakage will decrease and a better security performance can be achieved.

**Proposition 4** (Secrecy rate) When $N$ is infinite, by choosing a proper $\beta^*$, the proposed secure polar coding scheme can achieve the secrecy capacity of AWTC model.
Proof: Since for entire $T$ blocks, message bits $M_1^T$ are transmitted over subset $I_e \setminus E_e$ which is proven secure and reliable with proper $\beta^*$, we have the secrecy rate as

$$R_s(T) = \frac{\sum_{\ell=1}^{T} |I_e \setminus E_e|}{TN} = \frac{|I_e \setminus E_e|}{N}. \quad (34)$$

Then when $N$ is infinite, have

$$\lim_{N \to \infty} R_s(T) = \lim_{N \to \infty} \frac{|I_e \setminus E_e|}{N} = \lim_{N \to \infty} \frac{|I_e \cup R_e| - |E_e \cup R_e|}{N} = \lim_{N \to \infty} \frac{|I_e \cup R_e| - |B_e \cup R_e|}{N} = \lim_{N \to \infty} \frac{|L_{\epsilon \rho_w}| - |H_{\epsilon 1-\rho_r}|}{N} = 1 - \rho_w - \rho_r,$$

where (a) is due to [5] Theo. 1 that

$$\lim_{N \to \infty} \frac{|L_{\epsilon \rho_w}|}{N} = I(W_{\epsilon \rho_w}) = 1 - \rho_w,$$

$$\lim_{N \to \infty} \frac{|H_{\epsilon 1-\rho_r}|}{N} = I(W_{\epsilon 1-\rho_r}) = \rho_r.$$

Thus the secrecy capacity of AWTC model can be achieved with a proper $\beta^*$ and infinite $N$. $\blacksquare$

B. Simulations

Next, we test the performance of proposed secure polar coding scheme with finite block length $N$.

First, we test the upper bound of information leakage in (33), upper bound of legitimate BER in (30) and the secrecy rate in (34). Particularly, we let $\rho_w = 0.2$, $\rho_r = 0.4$, block length $N = 2^8$ to $2^{18}$, parameter $\beta = 0.20$ to 0.32 and block number $T = 300$.

The simulation results are illustrated in Fig. 5. Fig. 5a shows the decrease of upper bound of legitimate BER with the growing of block length $N$ for all $\beta$, which matches our analysis of reliability in Prop. 2. There are some unstable point in Fig. 5a which is because the corresponding value of $\beta$ is not in the range of proper $\beta^*$ for that block length $N$. Then with the increasing of block length $N$, this unstable phenomenon is disappear, which matches Prop. 1. Fig. 5b shows the decrease of upper bound of information leakage with the growing of block length $N$ for all
Fig. 5. Simulation results of performance with finite block length $N$.

Next, we test the actual BERs for both Bob and Eve by implementing the entire secure communication process over AWTC model. Particularly, we let $\rho_w = 0.2$, $\rho_r = 0.4$, block length $N = 2^8$ to $2^{12}$, parameter $\beta = 0.22$ to $0.30$ and block number $T = 1000$. For the transmitted message, we use uniformly distributed binary random bits.

The simulation results are illustrated in Fig. 6. Fig. 6a is the BER of entire 1000 blocks for legitimate user Bob decoding the message bits, which shows that the legitimate BER decreases significantly with the increasing of block length $N$ for all the $\beta$. Fig. 6b is the BER of entire 1000
blocks for eavesdropper Eve decoding the message bits, which shows that the eavesdropper BER remains closely to 0.5 with the increasing of block length $N$ for all the $\beta$. Therefore, Fig. 6 proves that the proposed secure polar coding scheme can provide secure and reliable communication over the AWTC model with finite block length $N$.

V. CONCLUSION

In this paper, we have considered the secure coding problem of AWTC model. We have presented an channel based equivalent model of AWTC model by using the $\rho$ equivalent channel block. Then we have studied the polarization relationship of the $\rho$ equivalent channel block and the $\rho$ BEC block and found out that by choosing a proper polarization parameter $\beta = \beta^*$, the polarized subsets of $\rho$ BEC block is fully contained by the polarized subsets of $\rho$ equivalent channel block (Prop. 1). Based on this results, we have constructed a secure polar coding scheme for the AWTC model and analyzed its performance. Theoretically, we have proven that the proposed scheme achieves the secrecy capacity of AWTC model under both reliability and strong security criterions with an infinite block length $N$. Further, for the case of finite block length $N$, we have carried out simulations which proves that the proposed secure polar coding scheme can provide secure and reliable communication over AWTC model.

The containment relationship of polarized subsets for $\rho$ equivalent channel block and $\rho$ BEC block in Prop. 1 is the key element of our secure polar coding construction. In this work, we obtained this key result by observing from the simulation test, which is however not rigourous
enough, although its correctness has been proven by the communication experiments of the proposed secure polar coding scheme. Thus the theoretical proof of Prop. 1 will be our future work.

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