Extracting the fragmentation functions with global analyses

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Abstract. We discuss preliminary results on the fragmentation functions obtained in a combined NLO fit to data of single-inclusive hadron production in electron-positron annihilation, proton-proton collisions, and deep-inelastic lepton-proton scattering with either pions or kaons identified in the final state.

1. Introduction

Fragmentation functions are fundamental objects which describe the collinear transition of a quark $i$ into a hadron $H$ with a fraction $z$ of its momentum, and it is usually named as $D^H_i(z)$. These fragmentation functions can only be obtained by performing global fits. In here, we describe the extraction of the fragmentation functions by using different set of data.

A field theoretical definition of the fragmentation in terms of bi-local operators was given, up to kinematical pre-factors, as

$$D^H_i(z) \approx \int dx \, e^{-iP_H x z \gamma^+} \text{Tr} \left[ \gamma^+ \langle 0 | \Psi(0) \mathcal{P} | H(P_H^+) X \rangle \langle H(P_H^+) X | \mathcal{P}' \bar{\Psi}(x) | 0 \rangle \right], \quad (1)$$

where $\mathcal{P}$ and $\mathcal{P}'$ denotes the gauge-links to render it gauge-invariant. Since the fragmentation functions are non perturbative objects, they cannot be computed from first principles and they need to be extracted by fitting the experimental data of different kind of processes. However, the scale dependence of the fragmentation functions can be obtained in pertubative QCD (pQCD) and can be determined by renormalization group equations, similar to those for parton densities (PDF).

In order to have a consistent description of the fragmentation functions, we assume momentum conservation of the fragmenting parton in the hadronization process, summarized by a sum rule stating that

$$\sum_H \int_0^1 dzzD^H_i(z, Q^2) = 1. \quad (2)$$

The main goal of the global fit is to extract the universal fragmentation functions from all available processes such that their predictions are well consistent with future experiments. Moreover, different kind of processes might help with the extraction of the fragmentation of
different flavor of quarks and they will be better constrained with more data sets. So, the processes that we are going to use for the extraction of the fragmentation are going to be described in the following.

1.1. Single-inclusive $e^+e^-$ annihilation
The cross section for the single-inclusive $e^+e^-$ annihilation (SIA) into a specific hadron $H$, $e^+e^- \rightarrow (\gamma,Z) \rightarrow H$, can be written as,

$$\frac{1}{\sigma_{tot}} \frac{d\sigma_H}{dz} = \frac{\sigma_0}{\sum_x \epsilon^2 x} \left[ 2F_1^H(z,Q^2) + F_L^H(z,Q^2) \right]$$

where $Q/2 = \sqrt{s}$ and $z_H \equiv 2p_H \cdot q/Q^2 = 2E_H/\sqrt{s}$.

$$\sigma_{tot} = \sum_q \epsilon^2 q \sigma_0 \left[ 1 + \frac{\alpha_s(Q^2)}{\pi} \right]$$

is the total cross section including its NLO $O(\alpha_s(Q^2))$ correction and $\sigma_0 = 4\pi\alpha_s^2(Q^2)/s$. The unpolarized “time-like” structure functions $F_1^H$ and $F_L^H$ are given at NLO by,

$$2F_1^H(z,Q^2) = \sum_q \epsilon^2 q \left[ D_{qH}^H + D_{qH}^q \right](z,Q^2) + \frac{\alpha_s(Q^2)}{2\pi} \left[ C_{1q}^L (D_{qH}^H + D_{qH}^q) + C_{1g}^L D_{gH}^H \right](z,Q^2)$$

$$F_L^H(z,Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \sum_q \epsilon^2 q \left[ C_{1q}^g (D_{qH}^H + D_{qH}^q) + C_{1g}^g D_{gH}^H \right](z,Q^2)$$

with $\otimes$ denoting a standard convolution and the $C_{1q,L}^{1,L}$ coefficients in the $\overline{\text{MS}}$ scheme can be found in App. A of Ref. [1].

1.2. Semi-Inclusive Deep-Inelastic Scattering
The cross section for the semi-inclusive deep-inelastic production of a hadron, $eN \rightarrow e'HX$, can be written in a factorized form in a similar way to the fully inclusive DIS case,

$$\frac{d\sigma_H}{dx dy dz_{2H}} = \frac{2\pi \alpha_s^2}{Q^2} \left[ (1 + (1 - y)^2/2 \sum_{q,H} \epsilon^2 q \left[ q(x,Q^2)D_{qH}^H(z_H,Q^2) + \frac{\alpha_s(Q^2)}{2\pi} \times \left[ q \otimes C_{1q}^1 \otimes D_{qH}^q + q \otimes C_{1q}^g \otimes D_{gH}^q + g \otimes C_{1g}^L \otimes D_{gH}^q \right] \right] \right] + \sum_{q,H} \epsilon^2 q \left[ q(x,Q^2)D_{qH}^H(z_H,Q^2) + \frac{\alpha_s(Q^2)}{2\pi} \times \left[ q \otimes C_{1q}^L \otimes D_{qH}^q + q \otimes C_{1q}^g \otimes D_{gH}^q + g \otimes C_{1g}^L \otimes D_{gH}^q \right] \right]$$

with $x$ and $y$ denoting the usual DIS scaling variables ($Q = sxy$), and where $z_H \equiv p_H \cdot p_N/p_N \cdot q$. The structure functions are given at NLO by,

$$2F_1^H(x,z_H,Q^2) = \sum_{q,H} \epsilon^2 q \left[ q(x,Q^2)D_{qH}^H(z_H,Q^2) + \frac{\alpha_s(Q^2)}{2\pi} \times \left[ q \otimes C_{1q}^1 \otimes D_{qH}^q + q \otimes C_{1q}^g \otimes D_{gH}^q + g \otimes C_{1g}^L \otimes D_{gH}^q \right] \right]$$

$$F_L^H(x,z_H,Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \sum_{q,H} \epsilon^2 q \left[ q \otimes C_{1q}^L \otimes D_{qH}^q + q \otimes C_{1q}^g \otimes D_{gH}^q + g \otimes C_{1g}^L \otimes D_{gH}^q \right]$$

with the NLO (\overline{\text{MS}}) coefficient functions $C_{ij}^{1,L}$ [1, 2].
1.3. Hadron-Hadron Collisions

The single-inclusive production of a hadron \( H \) at high transverse momentum \( p_T \) in hadron-hadron collisions is also amenable to QCD perturbative theory. The differential cross section can be written in factorized form as,

\[
E_H \frac{d^3 \sigma}{dp_T^3} = \sum_{a,b,c} f_a \otimes f_b \otimes d\hat{\sigma}_{ab} \otimes D^H_c, \tag{10}
\]

where the sum runs over all partonic channels \( a + b \rightarrow c + X \), with \( d\hat{\sigma}_{ab} \) the associated partonic cross section. \( d\hat{\sigma}_{ab} \) can be expanded as a power series in the strong coupling \( \alpha_s \). The \( \mathcal{O}(\alpha_s^3) \) NLO corrections are available in the literature [3].

2. Parametrization and extraction of the fragmentation functions

In the global analysis we will determine individual fragmentation functions for all partons (quarks and gluons) from data. We adopt for the fragmentations the following input distributions [4],

\[
D_i^H(z, \mu_0) = N_i z^{\alpha_i} (1 - z)^{\beta_i} [1 + \gamma_i (1 - z)^{\delta_i}] B[2 + \alpha_i, \beta_i + 1] + \gamma_i B[2 + \alpha_i, \beta_i + \delta_i + 1], \tag{11}
\]

where \( B[a,b] \) represents the Euler Beta-function and \( N_i \) is a normalization factor such that the sum rule in Eq. (5) is satisfied. We have imposed certain relations upon the individual fragmentation functions for pions and kaons. For instance, we impose isospin symmetry for the sea fragmentation functions to charged pions, i.e.

\[
D_\pi^+ = D_\pi^- \tag{12}
\]

but we allow a different normalization in the \( q + \bar{q} \) sum,

\[
D_{d+\bar{d}}^{\pi^+} = ND_{u+\bar{u}}^{\pi^+} \tag{13}
\]

For strange quarks it is assumed that

\[
D_{s+\bar{s}}^{\pi^+} = D_{s+\bar{s}}^{\pi^-} = N'D_{u+\bar{u}}^{\pi^+} \tag{14}
\]

with \( N' \) independent of \( z \).

For charged kaons we fit \( D_{u+\bar{u}}^{K^+} \) and \( D_{s+\bar{s}}^{K^+} \) independently. For the unfavored fragmentation the data are unable to discriminate between flavors and, consequently, we assume that all distributions have the same functional form:

\[
D_{s+\bar{s}}^{K^+} = D_{s+\bar{s}}^{K^-} = D_{d+\bar{d}}^{K^+} = D_{d+\bar{d}}^{K^-} \tag{15}
\]

We adopt the functional form of Eq. (14) also for the fragmentation of heavy charm and bottom quarks into charged pions and kaons but setting \( \gamma_i = 0 \). We assume that \( D_c^H = D_{c+\bar{c}}^H \) and \( D_b^H = D_{b+\bar{b}}^H \) for \( H = \pi^+, K^+ \).

The parameters describing the fragmentation functions for pions and kaons at scale \( \mu_0 \) are determined by a standard \( \chi^2 \) minimization for \( N \) data points,

\[
\chi^2 = \sum_{i=1}^{N} \frac{(T_i - E_i)^2}{\delta E_i^2} \tag{16}
\]

where \( E_i \) is the measured value for a given observable, \( \delta E_i \) the error associated with this measurement, and \( T_i \) is the corresponding theoretical estimate for a given set of parameters. For the associated error we take, as usual, the statistical and systematical errors in quadrature in \( \delta E_i \). In order to compute the observable in a fast way, we are performing all the computation using the Mellin technique.
2.1. Mellin Technique

The integro-differential evolution equations can be straightforwardly solved analytically in Mellin n-moment space along the lines described in Ref. [5]. The Mellin moments of, for instance, the fragmentation functions $D_i^H(z, Q^2)$, are defined as,

$$D_i^H(n, Q^2) \equiv \int_0^1 dz z^{n-1} D_i^H(z, Q^2), \quad (17)$$

and can be expressed in terms of the Euler Beta functions for our ansatz at scale $\mu_0^2$. The evolved fragmentation functions in $z$-space are re-obtained by an inverse Mellin transformation given by

$$D_i^H(z, Q^2) = \frac{1}{2\pi i} \int_{C_n} dn z^{-n} D_i^H(n, Q^2), \quad (18)$$

where $C_n$ denotes an appropriately chosen contour in the complex n-plane. This method is implemented for the extraction of pion and kaon fragmentation functions.

3. Data Sets

We use charged pion and kaon production data in SIA from TPC [6] at $\sqrt{s} = 29$ GeV, SLD [7], ALEPH [8], DELPHI [9], and OPAL [10], all at $\sqrt{s} = M_Z$. We also consider the data from TASSO [11] at intermediate c.m.s. energies of $\sqrt{s} = 33$ and 44 GeV.

Due to the conceptual problem with the fragmentation functions at small $z$, the cut $z_{\min} = 0.05(0.1)$ is imposed for all pion (kaon) data sets.

ALEPH, DELPHI and TPC provide tagged results distinguishing between the sum of light $u, d, s$ quarks, charm, and bottom events.

To further constraint the fragmentation functions of different flavors, as well as to separate favored (valence) and unfavored (sea) fragmentations, we include experimental information of SIDIS. More specifically, we make use of charged pion and kaon multiplicities from the HERMES [12] and COMPASS (preliminary) [13] experiments. These data also provide an important consistency check of the flavor separation obtained from flavor tagged SIA “data”.

A wealth of data on single-inclusive hadron production from RHIC experiments [14, 15, 16, 17] have also been included in our global analysis. These encompass the $p_T$ spectrum of neutral pions at central rapidities $|\eta| \leq 0.35$ by PHENIX [14] and at two different forward rapidities $\langle \eta \rangle = 3.3$ and 3.8 by STAR [15].

BRAHMS have also published $p_T$ spectra for identified pions and kaons at two values of rapidities $\eta = 2.95$ and 3.3 [16] of which we use only the former in the fit as it has large theoretical uncertainties due to small $p_T$ values probed. In addition, there are data on $K_S^0$ production for central rapidities $|\eta| \leq 0.5$ from STAR [17]. To accommodate the $K_S^0$ data in the fit, we assume that $K_S^0 = (K^+ + K^-)/2$ with $u \rightarrow K^+$ and $d \rightarrow K^+$ fragmentation functions interchanged.

4. Results

Now, we present the preliminary results for pions and kaons fragmentation functions, and this analysis is an update and cross check of the previous DSS fit of fragmentation functions [4] where we have included a preliminary SIDIS data coming from the COMPASS experiment. We also modified the PDFs with the extraction presented by the MSTW collaboration [18].

4.1. Pion Results

The COMPASS data set have helped us to test the prediction of the fragmentation functions at $Q^2 \approx 40$ GeV with good agreement. In Figure 1 we present the SIDIS results of the global pion fits. The SIA and hadron-hadron plots have not presented a notorious difference to those shown in Ref. [4].
4.2. Kaon Results

For charged kaons, the SIDIS results have been more complicated to fit. The multiplicities are very sensitive to the PDF strange quark distribution and the net result is shown in Figure 2. As it can be seen from the plot, we have also not considered multiplicities for values of $Q^2$ larger than 25 GeV because of their large uncertainties and also because of the unusual behavior that they present. Similarly to the pion fit, the SIA and hadron-hadron plots have not changed dramatically to those presented by DSS.

There are new set of data recently released or presented in conferences, for instance Ref. [19], which will be analyzed and we expect these issues to be clarified with more experimental data.
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