Dirac Neutrino Masses in NCG

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Abstract

Several models in NCG with mild changes to the standard model (SM) are introduced to discuss the neutrino mass problem. We use two constraints, Poincaré duality and gauge anomaly free, to discuss the possibility of containing right-handed neutrinos in them. Our work shows that no model in this paper, with each generation containing a right-handed neutrino, can satisfy these two constraints in the same time. So, to consist with neutrino oscillation experiment results, maybe fundamental changes to the present version of NCG are usually needed to include Dirac massive neutrinos.

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I. INTRODUCTION

Recent years, several experiments suggest that neutrino oscillations might exist [1], so it is interesting to discuss the neutrino-mass problem in NCG. Since majorana mass can’t exist in the present NCG framework, we assume that the possible neutrino mass is Dirac mass and the oscillation’s origin comes from the mixture in the lepton mass matrix.

Noncommutative geometry (NCG) gives us new insights into the SM. In the past ten years, it has developed several versions [2–5]. In this paper, we only use the Connes-Lott’s new scheme (a real spectral triplet in it) [6–8], which has achieved many successes in SM [4,9]. Because three generations of right-handed neutrinos cannot exist in it [4], we must consider other models beyond SM, such as [10–13], if it is needed to discuss the neutrino mass problem. To our opinion, any new models in NCG must obey some physical and geometrical principles. In this paper we use such two constraints: gauge anomaly cancellation (physical) and Poincaré duality (geometrical) in examining several models. Some of these models appeared before, for example in [11] the author considered the right-handed neutrinos without changing the algebra structure of SM, and concluded that if the number of the right-handed neutrinos and u-type quarks are different, the model might contain massive neutrinos (we will get different conclusions, seeing the following). Other models give changes to the algebra structure of SM, but they have not considered right-handed neutrinos. We will do some change in model-building concerning the neutrino mass problem in NCG.

This paper is organized as follows: we first give a concise description of Connes-Lott’s model for SM, then we introduce the Poincaré duality and gauge anomaly cancellation in NCG. In section III, we use these two constraints to check some models to find whether they can contain right-handed neutrinos, finally a conclusion follows.
II. NCG IN STANDARD MODEL

We refer to [4] for a clear and thorough review of Connes-Lott’s version of NCG. Here we emphasize on some basic aspects of NCG which are needed for future discussions.

The fundamental element of the Connes-Lott’s model (CL) is a real even spectral triple \((A, \mathcal{H}, D)\) with a chirality operator \(\gamma\) and an antilinear isometry operator \(J\). In which \(\mathcal{H}\) is a Hilbert space expanded by fermions and their antiparticles, \(A\) is an associative involution algebra representing on \(\mathcal{H}\), \(D\) is a self-adjoint Dirac operator, \(\gamma\) and \(J\) are also expressed as operators on \(\mathcal{H}\). In fact the triple in SM is a product of two triples: one encodes spacetime, the other concerns internal space. In SM (with one generation for example):

\[
A = C^\infty(M, C) \otimes A_F
\]

\[
\mathcal{H} = L^2(M, S) \otimes \mathcal{H}_F
\]

\[
D = (\partial \otimes 1) \oplus (1 \otimes D_F)
\]

In this paper, discussing the possibility of introducing right-hand neutrino in NCG model, we only concern the internal space, which is a finite spectral triple sometime called as the finite-part \(K - cycle\). Their definitions in SM are:

\[
A_F = C \oplus H \oplus M_3(C), \ (H \text{ is the algebra of quaternion}), \ \mathcal{H}_F = \mathcal{H}_F^+ \oplus \mathcal{H}_F^-.
\]

\(\mathcal{H}_F^+\), \(D_F\), \(J\), and \(\gamma\) are defined as:

\[
\mathcal{H}_F^+ = \begin{pmatrix}
e_R \\
u_L \\
e_L \\
u_R \\
d_R \\
u_L \\
d_L
\end{pmatrix}, \ D_F = \begin{pmatrix}
0 & M & 0 & 0 \\
M^* & 0 & 0 & 0 \\
0 & 0 & 0 & \overline{M} \\
0 & 0 & \overline{M}^* & 0
\end{pmatrix}, \ J = \begin{pmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0
\end{pmatrix} \circ C, \ \gamma = \begin{pmatrix}
1 & 0 \\
0 & -1
\end{pmatrix}
\]

\(\mathcal{H}_F^+\) represents the particle sector (we omit the analog representation in \(\mathcal{H}_F^-\)). \(M\) is mass matrix of fermions (in multi-generation model it also contains the mixing angles such as
CKM matrix). The operator of \( C \) in the definition of \( J \) is the complex conjugation operator. Chirality \( \gamma \) is also an operator which gives 1 to right-handed particles and \(-1\) to left-handed particles. A noncommutative geometry must fulfill some axioms such as Poincaré duality, which are described in \([3,4,7]\).

III. POINCARÉ DUALITY AND ANOMALY CANCELLATION CONSTRAINTS

A. Poincaré Duality

There is a well known property for a close Riemannian manifold: Poincaré duality, which means that there exists an isomorphism between the de Rham groups \( H^p (V) \) and \( H^{n-p} (V) \) (\( n \) is the dimension of the manifold). Connes has put forth this property into any real spectral triples \([3,7]\). In order to show how it works, we demonstrate the Poincaré duality in SM as an introduction. The representation of \( \mathcal{A}_F \) on \( \mathcal{H}_F \) is:

for lepton sector: \( \pi^+_l (\lambda, q) = \begin{pmatrix} \lambda & 0 \\ 0 & q \end{pmatrix} \otimes 1_N, \pi^-_l (\lambda, q) = \begin{pmatrix} \overline{\lambda} & 0 \\ 0 & \overline{\lambda} \end{pmatrix} \otimes 1_N \)

for quark sector: \( \pi^+_q (\lambda, q) = \begin{pmatrix} \overline{\lambda} & 0 \\ 0 & \lambda \end{pmatrix} \otimes 1_3 \otimes 1_N, \pi^-_q (\lambda, q, m) = \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{pmatrix} \otimes 1_N \)

in which \( \lambda, q, m \) are elements of \( C, H, M_3(C) \) respectively. \((+, -)\) denotes particle and anti-particle sector respectively. \( N \) denotes the number of generations of fermions.

Define:

\[ Q_{ij} = (p_i, p_j) = Tr(\gamma p_i J p_j J^\dagger) \] (1)

where \( p_i \) is the minimal-rank hermitian projection of an algebra such as: \( 1_C \) for \( C \), \( 1_H (I_2) \) for \( H \), and the diagonal matrix \( e = (1,0,0) \) for \( M_3(C) \). We choose the same bases as \([4]\) in
which \( p_1 = (-1_C) \oplus e, p_2 = 1_C \oplus 1_H, p_3 = 1_C \). Poincaré duality in NCG requires this matrix has non-vanishing determinant, that is:

\[
DetQ \neq 0
\]  
(2)

In the SM the chirality and projections take the form:

\[
\begin{align*}
\gamma & \mapsto (1, -1, -1)^N \oplus (1, 1, -1, -1)^{3N} \oplus (1, -1, 1)^N \oplus (1, -1, -1)^N \\
p_1 & \mapsto (-1, 0, 0)^N \oplus (-1, 0, 0, 0)^{3N} \oplus (-1, 1, -1)^N \oplus (e, e, e)^N \\
p_2 & \mapsto (1, 1, 1)^N \oplus (1, 1, 1, 1)^{3N} \oplus (1, 1, 1)^N \oplus (0, 0, 0, 0)^{3N} \\
p_3 & \mapsto (1, 0, 0)^N \oplus (1, 1, 0, 0)^{3N} \oplus (1, 1, 1)^N \oplus (0, 0, 0, 0)^{3N}
\end{align*}
\]

From definition (1):

\[
Q = -2 \begin{pmatrix} N & 0 & 0 \\ 0 & N & 0 \\ 0 & 0 & -N \end{pmatrix}
\]

Its determinant is obviously nonzero so that the SM in NCG satisfies Poincaré duality requirement. But if we put \( N \) generations right-handed neutrinos into SM without any other changes, we will get a vanishing matrix \( Q \) (that’s why we need other models beyond SM). In the following, (2) is used as the Poincaré duality constraint in model building.

### B. Gauge Anomaly Cancellation

Gauge anomaly free is required by the renormalizability in QFT. In ordinary quantum field theory the anomaly is proportional to \( \text{tr} [\gamma^5 \lambda_a \{ \lambda_b, \lambda_c \}] \) [14], where \( \lambda_i \) are the generators of the gauge group. In CL model, Gauge group is obtained from the unitary elements of the algebra \( A_F \). The anomaly free condition in NCG generally is [10]:

\[
Tr_p \left[ \gamma \left( \pi (x) + J\pi (x) J^\dagger \right) \right]^3 = 0
\]  
(3)
\( Tr_p \) is the trace on \( \mathcal{H}_F \) restricted to particle sector \( \mathcal{H}_F^+ \) and \( x \) is a unitary element of \( \mathcal{A}_F \).

Here we still use standard model as an example: we refer \[10\] to find details.

From the representations of \( \pi (\lambda, q, m) \), we get \( \pi (a, b, c) + J\pi (a, b, c) J^\dagger \) in particle sector:

\[
\pi (a, b, c) + J\pi (a, b, c) J^\dagger = \text{diag} \left( \begin{array}{ccc}
2b \\
a + b1_2 \\
c + (ub - b) 1_3 \\
c + (ub + b) 1_3 \\
a \otimes 1_3 + l_2 \otimes c + ub \otimes 1_6
\end{array} \right) \otimes 1_N
\]

in which \( i(a, b, c) \in \text{su}(2) \oplus \mathbb{R} \oplus \text{su}(3) \). We have used the decomposition \( u(3) = u(1) \oplus \text{su}(3) \), then we write the Lie-Algebra of unitaries of \( M_3(\mathbb{C}) \) in terms of \( c + ub \) (in SM, only one \( U(1) \) gauge boson exists, so here is only \( b \), no \( b' \), in models where there are more than one \( U(1) \) gauge bosons, the \( b' \) is needed), \( u \) is an arbitrary real number.

From equation (3) we can get three equations (we only concern \( U(1) \) gauge): \( u = -1/3 \), which is the same result as it from the so-called unimodularity condition \[4\]:

\[
Tr_p \left[ \gamma \left( \pi (x) + J\pi (x) J^\dagger \right) \right] = 0
\]

IV. RIGHT-HANDED NEUTRINO IN SEVERAL MODELS

We will begin to discuss several models beyond SM: Model 1 considers the possibility of introducing right-hand neutrino in NCG in terms of changing the fermion representation \( \mathcal{H}_F \); model 2 is originally studied by others \[10\] to find whether there is another \( U(1) \)
gauge boson, in which another $C$ algebra was put in; model 3 comes from the mathematics consideration on quantum group, where quaternion algebra was changed to $M_2(C)$; model 4 is a combination of model 2 and 3. They all give mild changes to the Connes-Lott’s version of SM, and we will introduce right-neutrinos in them. Our works show that the two constraints do not permit all generations have right-handed neutrino in those models. We also get a different conclusion in Model 1 from its previous conclusion in [11].

A. Model 1:

In [11], Rich Schelp thought out a possible way to put right-handed Dirac neutrinos into SM, which assumes there are $N_1$ massless generations of right-handed neutrinos and $(N - N_1)$ massive ones. The representations now are: (for those massless fermions, the representations on their right-handed particle are all zero)

$$\pi_{11}^+(\lambda, q) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & q \end{pmatrix} \otimes 1_{N_1}, \quad \pi_{12}^+(\lambda, q) = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & q \end{pmatrix} \otimes 1_{N-N_1}$$

The same assumption is put for $u$-type quarks except substituting $N_1 \rightarrow N_2$ (we omit the analog representation in antiparticle sector). Then we can write down those $p_i$.

$$\gamma \mapsto (1, 1, -1, -1)^N \oplus (1, 1, -1, -1)^{3N} \oplus (1, 1, -1, -1)^N \oplus (1, 1, -1, -1)^{3N}$$

$$p_1 \mapsto (0, -1, 0, 0)^{N_1} \oplus (-1, -1, 0, 0)^{N-N_1} \oplus (0, -1, 0, 0)^{3N_2} \oplus (-1, -1, 0, 0)^{3(N-N_2)}$$

$$\oplus (0, -1, -1, -1)^{N_1} \oplus (-1, -1, -1, -1)^{N-N_1} \oplus (0, e, e, e)^{N_2} \oplus (e, e, e)^{N-N_2}$$

$$p_2 \mapsto (0, 1, 1, 1)^{N_1} \oplus (1, 1, 1)^{N-N_1} \oplus (0, 1, 1, 1)^{3N_2} \oplus (1, 1, 1, 1)^{3(N-N_2)}$$

$$\oplus (0, 1, 1, 1)^{N_1} \oplus (1, 1, 1)^{N-N_1} \oplus (0, 0, 0, 0)^{3N_2} \oplus (0, 0, 0, 0)^{3(N-N_2)}$$

$$p_3 \mapsto (0, 1, 0, 0)^{N_1} \oplus (1, 1, 0, 0)^{N-N_1} \oplus (0, 1, 0, 0)^{3N_2} \oplus (1, 1, 0, 0)^{3(N-N_2)}$$

$$\oplus (0, 1, 1, 1)^{N_1} \oplus (1, 1, 1)^{N-N_1} \oplus (0, 0, 0, 0)^{3N_2} \oplus (0, 0, 0, 0)^{3(N-N_2)}$$

From definition of (1):
\[ Q = -2 \begin{pmatrix} N_1 - N_2 & N - N_1 + \frac{1}{2}N_2 & N - N_1 + \frac{1}{2}N_2 \\ N - N_1 + \frac{1}{2}N_2 & N_1 & N_1 - N \\ N - N_1 + \frac{1}{2}N_2 & N_1 - N & N_1 - 2N \end{pmatrix} \]

and its determinant

\[ \text{Det}Q = 8(N_1 - N_2)N^2 \]

Poincaré duality requires \( N_1 - N_2 \neq 0 \). For the second constraint, next to calculate:

\[ \pi(x) + J\pi(x)J^\dagger = \text{diag} \left( \begin{array}{c} (0) \otimes 1_{(N-N_1)} \\ 2b \otimes 1_N \\ (a+b) \otimes 1_N \\ (-b + ub + c) \otimes 1_{(N-N_2)} \\ (b + ub + c) \otimes 1_N \\ (a + ub + c) \otimes 1_N \end{array} \right) \]

then from gauge anomaly cancellation (3), we get the following equations:

\[ N(8-2) + 3[(N - N_2)(u - 1)^3 + N(u + 1)^3 - 2Nu^3] = 0 \]  
\[ N + 3Nu = 0 \]  
\[ (N - N_2)(u - 1) + N(u + 1) - 2Nu = 0 \]

the solution is \( u = -1/3, \) and \( N_2 = 0. \) \( N_2 \) is zero means that all the \( u \) - type quarks have masses, which is different from the conclusion in [11] (in which only Poincaré duality is considered). On the other hand, \( N_1 - N_2 \neq 0, \) together with \( N_2 = 0, \) educes that if the right-handed neutrino exists in this model, the number of generations of it is less than \( N \) (in SM \( N = 3 \)). But it seems unnatural and the limited experiment results up-to-now (if correct) do not support it (those experiments need at least three massive neutrinos).

**B. Model 2: \( U(1) \) extension**

Another extension of standard model is discussed in paper [10], in which the algebra \( \mathcal{A}_F \) has two \( C \) algebras \( C \) and \( C' \). Now we put \( N \) generations right-handed neutrinos in
this model, leaving the weak and strong sectors unchanged as it has been done in [10]. The properties of NCG [4]: \( [\pi(a), J\pi(b)J^\dagger] = 0 \), \( [[D, \pi(a)], J\pi(b)J^\dagger] = 0 \) for \( a, b \in \mathcal{A} \), make the representation of algebra \( \mathcal{A}_F \) on antiparticle part of leptons is vectorial [4] (all belong to algebra \( C \) or \( C' \)). Without losing generality we can assume they all belong to the first \( C \) algebra. Then the algebra and its representation are: (from now on, we use the Lie-Algebra directly instead of algebra \( \mathcal{A}_F \) for convenience)

\[
\mathcal{A}_F = C \oplus C' \oplus H \oplus M_3(C);
\]

\[
\pi_R^+ (b, b') = \text{diag} ((y_b + y'_b) I_N, (y_a + y'_a) I_N, (y_a + y'_a) I_{3N}, (y_a + y'_a) I_{3N});
\]

\[
\pi_L^+ (a) = \text{diag} (aI_N, aI_{3N});
\]

\[
\pi_R^-(b, b', c) = \text{diag} (-bI_{2N}, I_{2N} \otimes (uI_3 + u'b'I_3 + c));
\]

\[
\pi_L^-(b, b', c) = \text{diag} (-bI_{2N}, I_{2N} \otimes (uI_3 + u'b'I_3 + c));
\]

in which \( y_i \) and \( y'_i \in \{-1, 0, 1\} \), and for the same \( i \), one and only one of \( \{y_i, y'_i\} \) is zero [10](it is required by the representation of Algebras). So, \( y_i y'_i = 0 \). \( u, u' \) are two real numbers. Since there are two \( C \) algebras now, we take the decomposition of \( u(3) \) as

\[
u(3) = u'(1) \oplus u(1) \oplus su(3).
\]

The minimal projection of algebra \( C' \) is \( 1'_C \), and we define \( p_4 = 1'_C \). Other \( p_i \)s (\( i = 1, 2, 3 \)) are the same as before, but right-handed neutrinos change the chirality \( \gamma \) to be the following:

\[
\gamma \mapsto (1, 1, -1, -1)^N \oplus (1, 1, -1, -1)^{3N} \oplus (1, 1, -1, -1)^N \oplus (1, 1, -1, -1)^{3N}
\]

In the same way we get the matrix:

\[
Q = \begin{pmatrix}
2N(Y_e - Y_u) & -N(2Y_e - Y_u) & -N(2Y_e - Y_u) & -N(Y'_e - Y'_u) \\
-N(2Y_e - Y_u) & 2N(Y_e - 2) & 2N(Y_e - 1) & NY'_e \\
-N(2Y_e - Y_u) & 2N(Y_e - 1) & 2NY_e & NY'_e \\
-N(Y'_e - Y'_u) & NY'_e & NY'_e & 0
\end{pmatrix}
\]

and its determinant

\[
\text{Det}Q = 4N^4 (Y'_e - Y'_u)^2
\]

in which \( Y_e = |y_e| + |y'_e|, Y_u = |y_u| + |y'_u| \), \( Y'_e = |y'_e| + |y'_e|, Y'_u = |y'_u| + |y'_u| \). Poincaré duality requires \( Y'_e \neq Y'_u \). Again, we need to calculate:
\[\pi(x) + J\pi(x)J^\dagger = diag \begin{pmatrix}
(y_v + 1)b + y'_c b' \\
(y_e + 1)b + y'_e b' \\
\ a + b \\
(y_u + u)b + (y'_u + u')b' + c \\
(y_d + u)b + (y'_d + u')b' + c \\
\ a + ub + u'b' + c
\end{pmatrix}\]

From the gauge anomaly cancellation condition (3) we get:

\[(y_v + 1)^3 + (y_e + 1)^3 - 2 + 3 \left[(y_u + u)^3 + (y_d + u)^3 - 2u^3\right] = 0 \quad (10)\]

\[1 + 3u = 0 \quad (11)\]

\[(y_u + u) + (y_d + u) - 2u = 0 \quad (12)\]

\[(y_v + 1)^2y'_v + (y_e + 1)^2y'_e + 3 \left[(y_u + u)^2(y'_u + u') + (y_d + u)^2(y'_d + u') - 2u^2u'\right] = 0 \quad (13)\]

\[y'_v^3 + y'_e^3 + 3 \left[(y'_u + u')^3 + (y'_d + u')^3 - 2u'^3\right] = 0 \quad (14)\]

\[u' = 0 \quad (15)\]

\[(y'_u + u') + (y'_d + u') - 2u' = 0 \quad (16)\]

\[(y_v + 1)y'_v^2 + (y_e + 1)y'_e^2 + 3 \left[(y_u + u)(y'_u + u')^2 + (y_d + u)(y'_d + u')^2 - 2uu'^2\right] = 0 \quad (17)\]

from equations (11) and (15) we get: \(u = -1/3, u' = 0\). Which together with \(y_i y'_i = 0\), and equation (17) we can get the following equation:

\[y'_v^2 + y'_e^2 + 3u \left(y'_u^2 + y'_d^2\right) = 0 \implies Y'_e = Y'_u,\]

Obviously it conflicts with the Poincaré duality requirement. So these two constraints cannot be satisfied at the same time in this model.

\[\textbf{C. Model 3: } H \Rightarrow M_2(C)\]

There is another model with mild change to standard model described in [12], which simply changes quaternion \(H \Rightarrow M_2(C)\). Now we investigate what will happen when we input right-handed neutrinos in it.
The representation of Algebra now is:

\[
\pi^+ (\lambda, M_2) = \begin{pmatrix}
\overline{\lambda} & 0 & 0 \\
0 & \lambda & 0 \\
0 & 0 & M_2
\end{pmatrix};
\]

We use the same bases as before with a little changing: \( p_1 = -1_C \oplus e, \ p_2 = 1_C \oplus s, \ p_3 = 1_C \). ( here the minimal-rank projection of \( M_2(C) \) is not \( I_2 \), but \( s = (1, 0) \)), and the presentation of \( \gamma \) takes the same form as it in model 2.

Then the matrix \( Q \):

\[
Q = \begin{pmatrix}
0 & -2N & -2N \\
-2N & 2N & 3N \\
-2N & 3N & 4N
\end{pmatrix}
\]

The calculation shows \( \text{Det}Q = 0 \). So poincaré duality requirement alone forbids this model in NCG.

**D. Model 4: \( U(1) \) extension together with \( H \Rightarrow M_2(C) \)**

Now, an idea appears naturally that whether one can combine model 2 with model 3 to build a possible new model. We begin this work.

In the similar way as Model 3, we use the bases: \( p_1 = -1_C \oplus e, \ p_2 = 1_C \oplus s, \ p_3 = 1_C, \ p_4 = 1'_{C}. \) After carefully calculating we get:

\[
Q = \begin{pmatrix}
2N (Y_e - Y_u) & -N (2Y_e - Y_u) & -N (2Y_e - Y_u) & -N (Y_e' - Y_u') \\
-N (2Y_e - Y_u) & 2NY_e - 2N & 2NY_e - N & NY_e' \\
-N (2Y_e - Y_u) & 2N (Y_e - 1) & 2NY_e & NY_e' \\
-N (Y_e' - Y_u') & NY_e' & NY_e' & 0
\end{pmatrix}
\]

in which \( Y_e = |y_e| + |y_e'|, \ Y_e' = |y'_e| + |y'_e'|, \ Y_u = |y_u| + |y_d|, \ Y_u' = |y'_u| + |y'_d| \). The determinant is:

\[
\text{Det}Q = N^4 (Y_e' - Y_u')^2
\]
Poincaré duality condition requires: $Y_e' - Y_u' \neq 0$. As before, we next to calculate:

\[
\pi(x) + J\pi(x)J^t = \text{diag}
\begin{pmatrix}
(y_v + 1) b + y'_v b' \\
(y_e + 1) b + y'_e b' \\
a + b + xb + yb' \\
(y_u + u) b + (y'_u + u') b' + c \\
(y_d + u) b + (y'_d + u') b' + c \\
a + ub + u'b' + c + xb + yb'
\end{pmatrix}
\]

in which we use the decomposition: $u(2) = su(2) \oplus u(1) \oplus u'(1)$, $u(3) = su(3) \oplus u(1) \oplus u'(1)$, then the Lie-algebra of $M_2(C)$ is $a + xb + yb'$, the Lie-algebra of $M_3(C)$ is $c + ub + u'b'$.

We get the following equations:

\[
(y_v + 1)^3 + (y_e + 1)^3 - 2(x + 1)^3 + 3 \left[(y_a + u)^3 + (y_d + u)^3 - 2(u + x)^3\right] = 0 \tag{18}
\]

\[
(x + 1) + 3(u + x) = 0 \tag{19}
\]

\[
(y_a + u) + (y_d + u) - 2(u + x) = 0 \tag{20}
\]

\[
(y_v + 1)^2 y'_v + (y_e + 1)^2 y'_e - 2(x + 1)^2 y + 3 \left[(y_a + u)^2 (y'_a + u') + (y_d + u)^2 (y'_d + u') - 2(u + x)^2 (u' + y)\right] = 0 \tag{21}
\]

\[
y'_v^3 + y'_e^3 - 2y^3 + 3 \left[(y'_a + u')^3 + (y'_d + u')^3 - 2(u' + y)^3\right] = 0 \tag{22}
\]

\[
y + 3(u' + y) = 0 \tag{23}
\]

\[
(y'_a + u') + (y'_d + u') - 2(u' + y) = 0 \tag{24}
\]

\[
(y_v + 1) y'_v^2 + (y_e + 1) y'_e^2 - 2(x + 1) y^2 + 3 \left[(y_a + u) (y'_a + u')^2 + (y_d + u) (y'_d + u')^2 - 2(u + x) (u' + y)^2\right] = 0 \tag{25}
\]

(20) and (24) tell us $2x = y_a + y_d$, $2y = y'_a + y'_d$, obviously they are integers. Together with $y_i y'_i = 0$ and (19) (23), from equation (25) we can get:

\[
16xy^2 + y'_e^2 + y'_v^2 - (4x + 1) \left(y'_a^2 + y'_d^2\right) = 0 \tag{26}
\]

which can be rewritten as:
\[
\left( y_e^2 + y'_e^2 \right) - \left( y_u^2 + y'_d^2 \right) = -16x y^2 + 4x \left( y'_u^2 + y'_d^2 \right) = Y'_e - Y'_u
\]

which is an \textbf{even} integer obviously. Since \( Y'_e, Y'_u \in \{0, 1, 2\} \), \( Y'_e - Y'_u \in \{0, \pm 1, \pm 2\} \), so
\( Y'_e - Y'_u = \pm 2 \) (\( Y'_e = Y'_u \) violates the Poincaré duality). There are only two cases:

Case 1: \( Y'_e = 2, Y'_u = 0 \)

then \( y'_u = y'_d = 0 \), so \( y = 0 \), put it in (26), we get \( Y'_e = 0 \), obviously it is inconsistent.

Case 2: \( Y'_e = 0, Y'_u = 2 \)

then \( y_u = y_d = 0 \), so \( x = 0 \), put it in (26), it obviously conflicts with the poincaré duality requirement.

So there is no proper solution in this model.

V. CONCLUSION

It is unsuccessful to give each generation a massive neutrino in every model discussed above. Maybe fundamental changes to Connes-Lott’s model are generally needed. Since majorana particles are not permitted in the present version of NCG. Of course, if the massive neutrino contains majorana particles, CL version of NCG should be replaced by a new one. Besides Connes-Lott new scheme, right-handed neutrino is also considered in other NCG versions, such as in [15], where the author discussed its effects in hypercharges’ determination, but unlike what we discussed in this paper, there is no definitely constraints to judge whether the possible existence of right-handed neutrinos is conflicted with the rigid requirements in NCG. Above all, further experimental and theoretical researches are needed to explore this issue.

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