Semi-active Control of SDOF System with Coulomb Friction Dampers

Shuqi Guo¹, Shaopu Yang²

¹. The Department of Engineering Mechanics, Shijiazhuang Railway Institute, No.17 North 2nd-Ring East Road, Shijiazhuang, Hebei Province, 050043 China
Email:shuqig@yahoo.com.cn

². Administrative offices of Shijiazhuang Railway Institute, Shijiazhuang Railway Institute, No.17 North 2nd-Ring East Road, Shijiazhuang, Hebei Province, 050043 China.

Abstract. Semi-active control of dry friction dampers is proposed to improve the energy dissipation characteristics of passive Coulomb friction dampers. In this paper, we propose a control law for friction dampers, which maximizes energy dissipation in an instantaneous sense with semi-active control. With harmonic balance method, the analysis on semi-active control of SDOF system with Coulomb friction is presented. The good agreement of is attained between theoretical solutions and numerical simulations.

1. Introduction
Friction dampers have been proposed for use in a broad variety of applications. For example, passive friction dampers are currently used in the railway vehicle to dissipate the unwanted vibration. The control objective is to maximize energy dissipation.

In most cases, friction dampers have been studied in a passive context. Damping performance may be greatly improved by modulating the damping force instantaneously. This notion of producing a variable damping force is apparently attributable to Karnopp [1] and coworkers who proposed varying the force of a viscous damper by controlling its orifice area.

The appeal of semi-active control is that good performance can be achieved with a fraction of the input power required of active control. Semi-active control has been applied to other types of dampers, e.g. dampers made with electrorheological fluids or magnetorheological fluids [2, 3, 7]. The fluid model employed is equivalent to Coulomb plus viscous friction that is named Bingham model. In the Bingham model, the damping force produced by Coulomb friction is the controlled force. He and Ozis discussed the methods for strong nonlinear oscillation [4,5,6].

In this paper, with the harmonic balance method, analysis on semi-active control of SDOF system with Coulomb friction dampers is presented. The good agreement of is attained between theoretical solutions and numerical simulations.

2. Model of the system and control design

To whom any correspondence should be addressed.
The schematic structure of the SDOF system is shown as Fig.1, where $F_d$ is the Coulomb friction damper, $u$ is the exciting displacement or base displacement. For simplicity, let $u = u_0 \sin(\omega t)$. Then, the govern equation reads

$$m\ddot{x} + k(x - u_0 \sin \omega t) + F_d \, \text{sgn}(\dot{x} - u_0 \omega \cos \omega t) = 0$$

where $m$ is sprung mass, $k$ is sprung stiffness, $F_d$ is Coulomb friction force. The nondimensional form of Eq.(1) is Eq.(2).

$$\ddot{y} + y + A_p \, \text{sgn}(\dot{y} - \Omega \cos \Omega \tau) = \sin \Omega \tau$$

where $A_p = \frac{F_d}{ku_0}$, $\tau = \omega_n t$, $\Omega = \frac{\omega}{\omega_n}$, $\omega_n = \sqrt{\frac{k}{m}}$, $y = \frac{x}{u_0}$.

The semi-active control design is on-off control, which is shown in Fig.2. The actuator is on, when the actuator force $F_d$ has the same direction with the designed force $F_{\text{design}}$. Or the actuator is off. In this paper, the on-off control strategy is designed to always dissipate the energy the system. Then the friction damper is on when damping force $-A_p \, \text{sgn}(\dot{y} - \Omega \cos \Omega \tau)$ has the opposite direction with velocity $\dot{y}$. Or the friction damper is off, which are shown in Eq.(3).

$$\bar{A}_p = A_p \, H(\dot{y}(\dot{y} - \Omega \cos \Omega \tau))$$

where $H()$ is Heaviside function. Considering the on-off control design, Eq.(2) are made to Eq. (4) immediately.

$$\ddot{y} + y + A_p \, \text{sgn}(\dot{y} - \Omega \cos \Omega \tau)H(\dot{y}(\dot{y} - \Omega \cos \Omega \tau)) = \sin \Omega \tau$$

**Figure 1.** SDOF system with Coulomb friction

**Figure 2.** On-off control.
3. Analysis of the SDOF system with Coulomb Friction

Assume the stable response of the system Eq. (4) is

$$ y = a \sin(\Omega \tau - \phi). $$

(5)

Substitute Eq.(5) into Eq.(4), multiplied by $\sin(\Omega \tau - \phi)$, integrating it in a cycle, one obtains Eq.(6)

$$ -a\Omega^2 \pi + a\pi + A_p I_1 = \pi \cos \phi. $$

(6)

where

$$ I_1 = \int_0^\pi \sin \theta \text{sgn}(\cos(\theta - \beta)) H(\cos \theta \cos(\theta - \beta)) d\theta, $$

$H(.)$ Heaviside function, and

$$ \sin \beta = \frac{\sin \phi}{\sqrt{a^2 + 1 - 2a \cos \phi}}, \quad \cos \beta = \frac{a - \cos \phi}{\sqrt{a^2 + 1 - 2a \cos \phi}}. $$

(7)

When $\sin \phi \geq 0$, $0 \leq \beta \leq \pi$,

$$ I_1 = \int_{-\pi/2}^{\pi/2} \sin \theta \text{sgn}(a \cos(\theta - \beta)) d\theta + \int_{\beta - \pi/2}^{\beta + \pi/2} \sin \theta \text{sgn}(\cos(\theta - \beta)) d\theta = 2 \sin \beta. $$

And considering Eq. (7), Eq.(6) reads

$$ a\pi(1 - \Omega^2) + 2A_p \sin \beta = \pi \cos \phi. $$

(8)

When $\sin \phi < 0$, $0 > \beta > -\pi$, Eq.(6) also reads Eq.(8).

Substitute Eq.(5) into Eq.(4), multiplied by $\cos(\Omega \tau - \phi)$, integrating it in a cycle, one obtains Eq.(9)

$$ I_2 = \frac{\pi \sin \phi}{A_p}, $$

(9)

where

$$ I_2 = \int_0^\pi \cos \theta \text{sgn}(\cos(\theta - \beta)) H(\cos \theta \cos(\theta - \beta)) d\theta. $$

(10)

When $\sin \phi \geq 0$, $0 \leq \beta \leq \pi$,

$$ I_2 = \int_{-\pi/2}^{\pi/2} \cos \theta \text{sgn}(\cos(\theta - \beta)) d\theta + \int_{\beta - \pi/2}^{\beta + \pi/2} \cos \theta \text{sgn}(\cos(\theta - \beta)) d\theta = 2 + 2 \cos \beta. $$

When $\sin \phi < 0$, $0 > \beta > -\pi$, the above equation also correct. Then Eq. (9) reads Eq. (11). From Eq.(11), one obtains that $\sin \phi \geq 0$ and $0 \leq \beta \leq \pi$.

$$ \frac{\pi \sin \phi}{A_p} = 2 + 2 \cos \beta. $$

(11)

With Eq. (7), Eqs. (8) and (11) are changed to Eqs. (12) and (13) respectively.

$$ \frac{2A_p \sin \phi}{\pi \cos \phi + a\pi \Omega^2 - 1} = \sqrt{1 + a^2 - 2a \cos \phi}, $$

(12)

$$ \frac{2A_p (a - \cos \phi)}{\pi \sin \phi - 2A_p} = \sqrt{1 + a^2 - 2a \cos \phi}. $$

(13)
Eliminate the term $\sqrt{1 + a^2 - 2a \cos \phi}$ from Eqs. (12) and (13), one obtains Eq. (14).

$$2A_p \sin \phi = \pi - a^2 \pi (\Omega^2 - 1) + a \pi \cos \phi (\Omega^2 - 2).$$  \hspace{1cm} (14)

With $\sin \phi \geq 0$, one obtains $\sin \phi = \sqrt{1 - \cos^2 \phi}$. Substitute it into Eqs. (14) and make some algebra operation, one obtains

$$\left(b_1^2 + 4A_p^2\right)\cos^2 \phi + 2b_1b_2 \cos \phi + b_2^2 - 4A_p^2 = 0.$$  \hspace{1cm} (15)

where $b_1 = a\pi(\Omega^2 - 2)$ and $b_2 = \pi - a^2 \pi(\Omega^2 - 1)$. Solve $\cos \phi$ and $\sin \phi$ from Eq. (15) and Eq. (14) respectively,

$$\cos \phi = -\frac{b_1b_2 \pm 2A_p \sqrt{b_1^2 - b_2^2 - 4A_p^2}}{\left(b_1^2 + 4A_p^2\right)},$$  \hspace{1cm} (16)

$$\sin \phi = \frac{b_2 + b_1 \cos \phi}{2A_p}.$$  \hspace{1cm} (17)

From Eq. (15), one obtains Eq. (18).

$$\cos^2 \phi = -\frac{2b_1b_2 \cos \phi - b_2^2 + 4A_p^2}{b_1^2 + 4A_p^2}. $$  \hspace{1cm} (18)

Substitute Eqs. (16) and (17) into Eq. (13) and consider Eq. (18), $b_1 = a\pi(\Omega^2 - 2)$ and $b_2 = \pi - a^2 \pi(\Omega^2 - 1)$, and factor it, one obtains Eq. (19)

$$\left(a^2 - 1\right)^2\left(a^2(\Omega^2 - 1)^2 - 1\right)\left(a^2 \pi^2 - \pi^2 + 4A_p^2\right)^2\left(a^2 \pi^2(\Omega^2 - 2)^2 + 4A_p^2\right)^3 \times$$

$$\left(\pi^2 \Omega^4 \left(\Omega^2 - 1\right)^2 a^4 + \left(4A_p^2 \left(\Omega^2 + 1\right)^2 - \pi^2 \Omega^4\right) a^2 - 4A_p^2\right) = 0$$  \hspace{1cm} (19)

From Eq. (19), one obtains eleven solutions about $a$. The first five solutions are

$$a_1 = 1, a_2 = -1, a_3 = \frac{1}{1 - \Omega^2}, a_4 = \frac{1}{\Omega^2 - 1}, a_5 = \frac{\pi^2 - 4A_p^2}{\pi^2}.$$  

The sixth and seventh solutions are $\frac{a_6^2 = -4A_p^2}{\pi^2 \left(\Omega^2 - 2\right)^2}$.

From Eq. (19), the eighth to eleventh solution can solve from the following equation.

$$\pi^2 \Omega^4 \left(\Omega^2 - 1\right)^2 a^4 + \left(4A_p^2 \left(\Omega^2 + 1\right)^2 - \pi^2 \Omega^4\right) a^2 - 4A_p^2 = 0$$  \hspace{1cm} (19)

Then $a^2 = \frac{-\left(4A_p^2 \left(\Omega^2 + 1\right)^2 - \pi^2 \Omega^4\right) \pm \sqrt{\left(4A_p^2 \left(\Omega^2 + 1\right)^2 - \pi^2 \Omega^4\right)^2 + 16A_p^2 \pi^2 \Omega^4 \left(\Omega^2 - 1\right)^2}}{2\pi^2 \Omega^4 \left(\Omega^2 - 1\right)^2}$.

Apparently, there are only two meaningful solutions, i.e.
\[ a_1 = \frac{1}{\Omega^2 - 1} \]  
(19)

\[ a_2^2 = \frac{\left(4A_p^2(\Omega^2 + 1)^2 - \pi^2\Omega^4\right)^2 + 16A_p^2\pi^2\Omega^4(\Omega^2 - 1)^2}{2\pi^2\Omega^4(\Omega^2 - 1)^2} \]  
(20)

From Eq.(16), one obtains \( b_1^2 - b_2^2 + 4A_p \geq 0 \), i.e.

\[-\pi^2(a^2 - 1\left(a^2(\Omega^2 - 1)^2 - 1\right) + 4A_p^2 \geq 0\]

From the above inequality, one can evaluate \( a^2 \).

\[ a^2 \leq \frac{1}{2} + \frac{1 + \sqrt{(\Omega^2 - 2)^2\Omega^4 + 16(\Omega^2 - 1)^2\frac{A_p^2}{\pi^2}}}{2(\Omega^2 - 1)^2} \]

(21)

From the theoretical solution Eq.(20), one can plot amplitude-frequency curves with different value of \( A_p \), which are shown in Fig.3.

**Figure 3.** Amplitude-frequency curve.

Solid line: \( A_p = 0.1 \), Dotted line: \( A_p = 0.5 \), Dash line: \( A_p = 1 \), Dash-dot line: \( A_p = 2 \)

**4. Numerical simulations**

The first solution, i.e. Eq.(19), is always unstable, which can be verified by numerical simulations. The second solution, i.e. Eq.(20), have good agreement with numerical simulation, which validated by Fig. 4 and Fig. 5. Fig. 4 and Fig. 5 are the results of comparison between theoretical solution and numerical simulations with \( A_p = 0.2 \) and \( A_p = 0.3 \) respectively. The solid lines are theoretical solutions. The dot lines are numerical simulations.
5. Conclusions
Semi-active control of dry friction dampers has been proposed to improve the energy dissipation characteristics of passive dry friction dampers. In this paper, we propose control laws for friction dampers which maximize energy dissipation in an instantaneous sense with semi-active control.

With the harmonic balance method, the analysis on semi-active control of SDOF system with Coulomb friction is presented. The amplitude frequency relations are given in Fig.3. The good agreement of is attained between theoretical solutions and numerical simulations.

There are two meaning solutions. The first solution is always unstable, which is different to the case of viscous friction. The second solution is always stable.

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