Abstract

Quantum trajectories are used to investigate the EPR-Bohr debate in a modern sense by examining entanglement and nonlocality. We synthesize a single “entanglement molecule” from the two scattered particles of the EPR experiment. We explicitly investigate the behavior of the entanglement molecule rather than the behaviors of the two scattered particles to gain insight into the EPR-Bohr debate. We develop the entanglement molecule’s wave function in polar form and its reduced action, both of which manifest entanglement. We next apply Jacobi’s theorem to the reduced action to generate the equation of quantum motion for the entanglement molecule to produce its quantum trajectory. The resultant quantum trajectory manifests entanglement and has retrograde segments interspersed between segments of forward motion. This alternating of forward and retrograde segments generates nonlocality and, within the entanglement molecule, action at a distance. Dissection of the equation of quantum motion for the entanglement molecule, while rendering the classical behavior of the two scattered particles, also reveals an emergent “entanglon” that maintains the entanglement between the scattered particles. The characteristics of the entanglon and its relationship to nonlocality are examined.
1 Introduction

“Can quantum mechanical description of physical reality be considered complete?” was the title that Einstein, Podolsky and Rosen (EPR) [1] and Bohr [2] used in their 1935 debate regarding reality and completeness of quantum mechanics. The issues circa 1935 were “physical reality” and “completeness of the Schrödinger wave function, \( \psi \).” Subsequently, Bell [3] and the Aspect experiments [4] have shown quantum mechanics to be nonlocal. The modern issues of the EPR-Bohr debate are entanglement and nonlocality [5,6]. Many in the physics community remain skeptical about the theoretical foundation for nonlocality in quantum mechanics notwithstanding the findings of experiments more accurate than the original Aspect experiments with regard to detection and locality loopholes [6–15]. Herein, we investigate EPR phenomena with quantum trajectories with a goal of answering the locality loophole issue. Quantum trajectories are shown to render insight on nonlocality in quantum mechanics. In the course of this investigation, analysis of the quantum trajectories revealed an additional quantum entity, introduced as an “entanglon”, that can superluminally maintain entanglement between the two EPR particles.

The quantum trajectory representation of quantum mechanics is a nonlocal, phenomenological theory that is deterministic. Herein, “deterministic” means in the spirit of EPR that if without disturbing a system one can predict with certainty the value of a physical quantity (the quantum trajectory), then there exists an element of physical reality that corresponds to such a physical quantity (the quantum trajectory) [1]. Quantum trajectories with their nonlocal character are adduced as a natural representation for investigating EPR phenomena and to render insight into how entanglement induces nonlocality. The quantum Hamilton-Jacobi equation underlies the quantum trajectory representation of quantum mechanics [16,17]. The underlying Hamilton-Jacobi formulation couches the quantum trajectory representation of quantum mechanics in a configuration space, time domain rather than a Hilbert space of wave mechanics. Faraggi and Matone, using a quantum equivalence principle that connects all physical systems by a coordinate transformation, have independently derived the quantum stationary Hamilton-Jacobi equation (QSHJE) without using any axiomatic interpretations of \( \psi \) [18,19]. With Bertoldi, they have extended their work to higher dimensions and to relativistic quantum mechanics [20]. The quantum trajectory representation of quantum mechanics contains more information then the Schrödinger wave function, \( \psi \) [16,19,21,24]. All of the foregoing has posited the quantum trajectory representation as the superior method for examining fundamental issues of quantum mechanics. The quantum trajectory representation has been used to investigate the foundations of quantum mechanics free of axiomatic interpretations of Copenhagen philosophy [16,26]. With regard to the circa 1935 issue of completeness of \( \psi \), the quantum trajectory representation has already shown the existence of microstates in \( \psi \) that provides a counterexample showing that \( \psi \) is not an exhaustive description of quantum phenomena [16,19,21,24].

This investigation studies the example considered by both EPR and Bohr in their 1935 papers where two identical particles without spin are entangled and scattered from each other in opposite directions by some interaction [12]. We investigate this example in a quantum
Hamilton-Jacobi representation and develop the quantum trajectory. Rather than examining the individual quantum trajectories of the two entangled particles, we synthesize an “EPR-molecule” from the two entangled particles and subsequently examine the EPR-molecule’s quantum trajectory to gain insight on how entanglement induces nonlocality. Synthesizing an EPR-molecule renders a reduced action in an Euclidian space rather than in the configuration space described by the two entangled particles. Synthesizing an EPR-molecule is reminiscent of synthesizing a dispherical particle for an idealized quantum Young’s diffraction experiment (a simplified double slit experiment) where it was shown that the subsequent quantum trajectory for the dispherical particle transited both slits simultaneously [25]. Quantum trajectory for multi-chromatic particles have also explained wave packet spreading [26].

The terminology “EPR-molecule” is reserved for the example considered by both EPR and Bohr in their 1935 papers where they examine identical particles recoiling in opposite directions from each other after an entangling and scattering interaction [12]. This investigation examines this situation in the limit that the recoiling particles become identical. For situations where the recoiling particles are not identical, the terminology “epr-molecule” is used. For general situations of entanglement, the terminology “entanglement molecule” is used, which is a generalization of Dür’s 2001 usage to describe entanglement among qubits [27].

Herein, the concept of a self-entangled, quantum particle [25,26] is extended to synthesize an entanglement molecule from two entangled particles. Much of the formulation for describing EPR phenomenon is common to that for self-entangled phenomenon, but the application differs physically. Herein, we apply quantum trajectories to investigate the quantum motion of entanglement molecules. For non-identical entangled particles, the consequent epr-molecule may spread and manifest nonlocality consistent with the Aspect experiments [4]. Our investigation of EPR in a quantum trajectory representation first synthesizes the epr-molecule. We next extract the generator of the quantum motion (quantum reduced action or Hamilton’s characteristic function) for the epr-molecule from its wave function. Jacobi’s theorem then renders the quantum trajectory for the epr-molecule. The resultant quantum trajectory has retrograde segments interspersed between its segments of forward motion. This alternating of forward and retrograde segments generates the nonlocality associated with entanglement. Dissection of the equation of quantum motion for the quantum trajectory reveals the classical motion for the two recoiling particles plus motion for an emergent additional entity that contains the entanglement information. This entity is designated as the “entanglon”. The entanglon is to the entangled molecule what the chemical bond is to a standard molecule. The motion for the EPR-molecule is determined from the motion for the epr-molecule in the limit that the recoiling particles become identical.

In Section 2, we develop the formulation for applying quantum trajectories to the EPR gedanken experiment. This includes synthesizing the entanglement molecule, developing its generator of quantum motion, and subsequently developing its quantum trajectory. In Section 3, we examine a particular example of an epr-molecule. We generate its theoretical equation of quantum motion. In Section 4, we examine the corresponding example for the EPR-molecule by taking the EPR-limit of our results for the epr-example. In Section 5, we exhibit the emer-
gence of the “entanglon” in the quantum trajectory representation of quantum mechanics. In Section 6, we present conclusions. Our conclusions include a discussion of how our quantum trajectory representation differs with the Copenhagen interpretation and with Bohmian mechanics.

2 Formulation

We adopt the physical setup of the original gedanken experiment considered by EPR [1] and Bohr [2] for investigation. However, we shall examine the EPR experiment in a quantum Hamilton-Jacobi representation rather than in a Schrödinger wave function ($\psi$) representation to gain insight into the relationship between entanglement and nonlocality. Let us consider two particles, $P_1$ and $P_2$, with spatial wave functions, $\psi_1(x)$ and $\psi_2(x)$, that interact through an instantaneous impulse (kick) at time $t = 0$, rather than through an interaction over the duration between $t = 0, T$ as per EPR [1], and then become entangled for $t > 0$. The positions $(x_1, x_2)$ of the two particles are co-located at the time of impulse interaction, $t = 0$ at $x_1, x_2 = 0$. The masses of $P_1$ and $P_2$ are respectively given by $m$ and $\alpha^2 m$ where $0 < \alpha \leq 1$. The factor $\alpha$ in $\psi_2$ is inserted arbitrarily as a convenient means by which we may approach EPR in the limit $\alpha \to 1$. More about this later. For mathematical simplicity, let us conjure up some interaction and an inertial reference system, reminiscent of EPR [1] and Bohr [2], that induces the two particles to recoil from each other in opposite directions after impulse with their spatial wave functions given by

$$\psi_1(x) = \exp(ikx), \quad \psi_2(x) = \alpha \exp(-ikx + i\beta); \quad t > 0 \quad (1)$$

where $k = [2(1 + \alpha^2)mE]^{1/2}/\hbar$, $E$ is energy of the epr-molecule and $-\pi < \beta \leq \pi$. The term $\beta$ represents a phase shift between the two particles. In our chosen reference system, $(\psi_1, \psi_2)$ form a set of independent solutions of the Schrödinger equation with energy $E$ [16,19], which helps facilitate the application of the quantum trajectory representation. The wave functions $\psi_1$ and $\psi_2$ have not been normalized absolutely but do have relative normalization with regard to each other as manifested by the factor $\alpha$. By Eq. (1), $\psi_1$ and $\psi_2$ are not the wave functions for identical particles unless $\alpha = 1$. EPR [1] and Bohr [2] considered identical particles. For completeness, the particular combination of impulse interaction at $t = 0$ and particle velocities $\dot{x}_1$ and $\dot{x}_2$ for $-1 \ll t < 0$ necessary to render the particular results of Eq. (1) is generally not unique.

While the particles $P_1$ and $P_2$ have causal positions $x_1$ and $x_2$ respectively, their wave functions $\psi_1$ and $\psi_2$ in the Copenhagen interpretation render the Born probability amplitude over $x$. In the quantum trajectory representation, the set $(\psi_1, \psi_2)$ of independent solutions to the Schrödinger equation in one dimension are related to the reduced action in the quantum trajectory representation of quantum mechanics through the invariance of the Schwarzian derivative of the reduced action with regard to $x$ under a Möbius transform $(\alpha \psi_1 - b \psi_2)/(c \psi_1 - d \psi_2)$, $ad - bc \neq 0$ [19].

Our criterion for choice of inertial reference system, for which $\psi_1$ and $\psi_2$ have the wave
numbers $k$ and $-k$, generates the relationship $x_1 = -x_2/\alpha^2$ for the positions of the two particles. This is an extension of Fine’s conservation of relative position \[5\]. For $\alpha = \pm 1$, conservation of relative position holds, and the inertial reference system is the center-of-mass inertial system. Conservation of relative position will induce loss of parameter independence and outcome independence \[6\] in the EPR experiment.

EPR and Bohr assumed that, for times sufficiently long after interaction at $t = 0$, then $x_1 + x_2 \gg 1$ sufficiently to ensure “separability” of the particles $P_1$ and $P_2$. But we herein assume that the two particles remain entangled no matter how far apart they become as first confirmed by the Aspect experiments \[4\].

For entanglement in the wave function representation of quantum mechanics, we may synthesize an epr-molecule as a simple polar wave function, $\psi_{epr}$, from the entangled pair (bipolar wave function) $\psi_1$ and $\psi_2$, by \[26\] \[28\]

$$
\psi_{epr}(x) = \left[1 + \alpha^2 + 2\alpha \cos(2kx + \beta)\right]^{1/2} \exp \left[i \arctan \left(\frac{\sin(kx) - \alpha \sin(kx + \beta)}{\cos(kx) + \alpha \cos(kx + \beta)}\right)\right]
$$

where we have dropped the subscript upon particle position $x$ by the extension of conservation of relative position. The above construction is just superpositional principle at work. It converts a bipolar Ansatz to a polar Ansatz \[17\] \[19\] \[22\] \[29\] \[31\]. In the EPR limit, then $\lim_{\alpha \to 1} \psi_{epr} \to \psi_{EPR} = 2 \cos(kx), i2 \sin(kx)$ respectively for $\beta = 0, \pi$ as expected. From Eq. \[2\], $\psi_{epr}$ has the same form as a dichromatic wave function $\psi_{dichromatic}$ investigated in Reference \[26\]. But $\psi_{epr}$ and $\psi_{dichromatic}$ represent different physics as the two spectral components of $\psi_{dichromatic}$ induce self-entanglement within a dichromatic particle.

The wave function for the epr-molecule, $\psi_{epr}$, as exhibited by Eq. \[2\], does not uniquely specify the epr-components. For example, the entanglement of a running wave function, $(1 - \alpha) \exp(ikx)$ and a standing wave function, $2\alpha \exp(-i\beta/2) \cos(kx + \beta/2)$ would also render the very same $\psi_{epr}$ given by Eq. \[2\]. By the superpositional theorem, $\psi_{epr}$ remains valid for any combination of particles as long as the collective sum of their spectral components are consistent with the right side of the upper line of Eq. \[2\].

In the wave function representation, $\psi_{epr}$ as represented by Eq. \[2\], is inherently nonlocal for it is not factorable, that is \[32\] $\psi_{epr} \neq K \psi_1 \psi_2$ where $K$ is a constant. Any measurement upon the $\psi_{epr}$ for the epr-molecule concurrently measures $\psi_1$ and $\psi_2$. Likewise, in the quantum trajectory representation of quantum mechanics, entanglement implies that the reduced action (Hamilton’s characteristic function) for the epr-molecule, $W_{epr}$, is inseparable by particles, that is $W_{epr} \neq W_{particle\ 1} + W_{particle\ 2}$.

The $\psi_{epr}$ is not the wave function representing EPR landscape. The actual wave function for the EPR-molecule, $\psi_{EPR}$, for identical particles is given by
\[ \psi_{EPR} = \lim_{\alpha \to 1} (\psi_{epr}). \]

In general, we shall investigate EPR phenomena, where \( \alpha = 1 \), by

\[ \lim_{\alpha \to 1} (\text{epr-phenomenon}) \rightarrow \text{EPR-phenomenon}. \]

This avoids directly working with standing waves to establish quantum trajectories and permits us to study the behavior of quantum trajectories and other phenomena in the limit that the complex running wave function, \( \psi_{epr} \), becomes a real standing wave function, \( \psi_{EPR} \), as \( \alpha \to 1 \).

A generator of the motion for the epr-molecule is its reduced action, \( W_{epr} \). Its reduced action may be extracted from the un-normalized \( \psi_{epr} \) as microstates do not exist for \( \psi_{epr} \) \[24\]. The reduced action is given by \[26,28\]

\[ W_{epr} = \hbar \arctan \left( \frac{\sin(kx) - \alpha \sin(kx + \beta)}{\cos(kx) + \alpha \cos(kx + \beta)} \right). \] (3)

Whereas we extracted the reduced action, \( W_{epr} \), from the Schrödinger wave function herein for convenience, Faraggi and Matone have shown that in general the reduced action may be derived from their quantum equivalence principle independent of the Schrödinger formulation of quantum mechanics \[18\]. The reduced action, \( W_{epr} \), is also the solution of the QSHJE for \( E = \hbar^2 k^2/[2m(1 + \alpha^2)] \) \[26\]. Equation (3) posits a deterministic \( W_{epr} \) in Euclidean space in contrast to \( \psi_{epr} \) with its probability amplitude being posited in Hilbert space. The absolute value of \( W_{epr} \) increases monotonically with \( x \) as the arctangent function in \( W_{epr} \) as it jumps to the next Riemann sheet whenever the the underlying tangent function becomes singular.

The conjugate momentum for epr-molecule is given by

\[ \partial W_{epr}/\partial x = \frac{\hbar k}{1 + \alpha^2 + 2\alpha \cos(2kx + \beta)}. \] (4)

The conjugate momentum manifests entanglement by the cosine term in the denominator on the right side of Eq. (4). We note from Eqs. (4) and (6) that the conjugate momentum is not the mechanical momentum, i.e.,

\[ \partial W_{epr}/\partial x \neq m\dot{x}. \] (5)

The equation of quantum motion for the epr-molecule is generated from \( W_{epr} \) by Jacobi’s theorem as

\[ t_{epr} - \tau = \frac{\partial W_{epr}}{\partial E} = \frac{m(x(1 - \alpha^2))}{\hbar k[1 + \alpha^2 + 2\alpha \cos(2kx + \beta)]} \] (6)
where $t$ is time and $\tau$ specifies the epoch. The quantum trajectory for the epr-molecule is in Euclidean space and renders determinism as proposed by EPR [1] for the position of the epr-molecule as a function of time that can be predicted with certainty without disturbing the system. In the foregoing, we note that our use of “certainty” is appropriate for three reasons. First, in the Copenhagen interpretation, the Heisenberg uncertainty principle uses an insufficient subset of initial values of the necessary and sufficient set of initial values that specify unique quantum motion [19,23,33]. Second, the quantum trajectories exist in Euclidean space here while the Schrödinger wave function representation is formulated in Hilbert space [21]. And third, the quantum Hamilton-Jacobi representation contains more information than the Schrödinger wave function representation and renders a unique, deterministic quantum trajectory [19,23,24,33]. Realism follows from determinism for the epr-molecule maintains a precise, theoretical quantum trajectory independent of it being measured. Nevertheless, nothing herein implies that a measurement on an epr-molecule does not physically disturb the epr-molecule in compliance with Bohr’s complementarity principle.

The use of Jacobi’s theorem to develop an equation of quantum motion, Eq. (6), is consistent with Peres’s quantum clocks where $t - \tau = \hbar(\partial \varphi / \partial E)$ where $\varphi$ is the phase of the complex wave function of the particle under consideration [34]. Equation (6) is a generalization of this for it applies to situations where the wave function is real [19,22,24].

We also note that the development of quantum trajectories differs from those of Bohmian mechanics [35]. Bohmian mechanics assumes that the conjugate momentum, $\partial W_{epr} / \partial x$, is the mechanical momentum in contradiction to Eq. (5) and subsequently integrates it to render an equation of quantum motion that differs from Eq. (6). Recently, Ghose has shown for some entangled multiparticle systems that choosing the particle distribution in Bohmian mechanics consistent with the “quantum equilibrium hypothesis” cannot be assured: a Bohmian interpretation becomes problematic for such systems [36]. Ghose did investigate in a Bohmian representation the entanglement, Eq. (2), which is studied herein.

In closing this section, we note that measurements on $\psi_{epr}$ concurrently measure $\psi_1$ and $\psi_2$ support the position of Bohr in the EPR-Bohr debates [2]. On the other hand, the very existence of quantum trajectories for the epr-molecule supports the position of EPR with regard to reality [1]. As previously discussed, the quantum Hamilton-Jacobi representation contains more information than $\psi$ which challenges the completeness of $\psi$ which also supports the position of EPR.

3 Example

Let us consider the particular example of the quantum trajectories of an epr-molecule specified by $\hbar = 1$, $m = 1$, $k = \pi/2$, $\alpha = 0.5$, $\tau = 0$, and $\beta = 0, \pi$. The resultant quantum trajectories, which are governed by Eq. (6), are exhibited on Fig. 1 where the solid line renders the quantum trajectory for $\beta = 0$ and dashed line, $\beta = \pi$. These quantum trajectories are launched from the origin, $(t, x) = (0, 0)$. Near $x = 1, 2, 3, \cdots$, the quantum trajectory for the epr-molecule with $\beta = 0$ on Fig. 1 has turning points with regard to time, $t$, where the quantum trajectories change between forward and retrograde motion [26]. The turning points cause the quantum
trajectory to alternate between forward and retrograde motion implying nonlocality and action at a distance as the quantum trajectory at various instances of time has separate, multiple locations. Furthermore, the good behavior (at least continuous first-order derivatives) implies superluminality of the quantum trajectories at the turning points for the epr-molecule where $\dot{x} \to \pm \infty$ at the extrema in $t$. This superluminality is another manifestation of nonlocality. We note that these superluminalities at the turning points are integratable as exhibited on Fig. 1.

The quantum trajectory for the epr-molecule, as exhibited by Fig. 1, is restricted to the approximate wedge given by

$$\frac{m x}{3 \hbar k} \leq t \leq \frac{3 m x}{\hbar k}.$$  

The upper boundary of the wedge, $t_u = 3 m x/(\hbar k)$, manifests maximum destructive interference between $\psi_1$ and $\psi_2$ while the lower boundary, $t_\ell = m x/(3\hbar k)$, manifests maximum reinforcement. This may be generalized with regard to $\alpha$ by

$$\frac{(1-\alpha) m x}{(1+\alpha) \hbar k} \leq t \leq \frac{(1+\alpha) m x}{(1-\alpha) \hbar k}.  \quad (7)$$

This wedge may be densely filled by varying the phase shift $\beta$ over its range $(-\pi/2, \pi/2)$ as exhibited by Fig. 2. For $\alpha \ll 1$, latent early time reversals may be suppressed [26].

As the quantum trajectory for the epr-molecule progresses out the wedge away from its launch point at the wedge’s apex at the origin $(t, x) = (0, 0)$, the durations of time spent on individual forward and retrograde segments increase. The dichromatic particle offers a precedent for understanding this motion of alternating forward and retrograde segments whose duration progressively increase as manifesting wave packet spreading [26]. Here, the analogous behavior for the epr-molecule manifests an increasing spatial displacement between its two component particles, $P_1$ and $P_2$.

There is another way to interpret the quantum trajectories exhibited in Figs. 1 and 2 where the concept of retrograde motion is replaced by invoking the use of creation and annihilation operations at the turning points [26]. At the local temporal minima, there is maximum reinforcement between $\psi_1$ and $\psi_2$, which synthesize $\psi_{epr}$, at these temporal local minima where pairs of quantum trajectories for the epr-molecule are spontaneously created. Within each pair, one quantum trajectory propagates in the $+x$ direction; the other, in the $-x$ direction. Note that these creation operations do not imply that $\psi_{epr}$ has been spectrally analyzed into $\psi_1$ and $\psi_2$ to propagate separately on the two different branches: rather the creation operations manifest spontaneous nonlocality where $\psi_{epr}$ propagates along both branches. Each branch of the pair terminates at local temporal maximum where it is annihilated along with another branch from another pair of quantum trajectories as exhibited on Fig. 1. These annihilated quantum trajectories were created at different local temporal minima and propagate in opposite directions with regard to $x$. The local temporal maxima represent points of maximum interference between $\psi_1$ and $\psi_2$ within $\psi_{epr}$.  

8
4 Quantum trajectory for EPR-molecule

The wave function for the EPR-molecule is a standing wave function. As such, its corresponding quantum trajectory is ill defined. We shall resolve its quantum trajectory by a limiting process. We still assume the conditions $\hbar = 1$, $m = 1$, $k = \pi/2$, and $\beta = 0$. For $\beta = 0$, the epr-reduced action simplifies to

$$W_{epr} = \hbar \left[ \arctan \left( \frac{1-\alpha}{1+\alpha} \tan(kx) \right) \right].$$

The EPR wave function by the upper line of Eq. (2) with $\alpha = 1$ trivially represents a standing wave function, $2 \cos(kx)$. Likewise, the limiting process, $\alpha \to (1-)$, when applied to the second line of Eq. (2), also renders

$$\lim_{\alpha \to (1-)} (\psi_{epr}) = 2 \cos(kx) = \psi_{EPR}. \quad (8)$$

Our limiting process for EPR has $\alpha$ approach 1 from below, $\alpha \to (1-)$. Concurrently, the instantaneous inertial reference frame, which is dependent upon $\alpha$, is continuously constrained throughout the limiting process to maintain the wave numbers, $k$ and $-k$ for $\psi_1$ and $\psi_2$ respectively. In the limit $\alpha \to (1-)$, both edges of the wedge exhibited in Figs. 1 and 2 become orthogonal [26]. The wedge spans the entire quadrant $t, x \geq 0$ of the $t, x$-plane. Had we chosen to take the limit of $\alpha$ approaching 1 from above, then Eq. (8) would still be valid but the wedge would have spanned the quadrant $t \geq 0, x \leq 0$ of the $t, x$-plane.

The equation of quantum motion for the EPR-molecule, which by Jacobi’s theorem, $t_{EPR} = \frac{\partial W_{EPR}}{\partial E}$, is rendered by taking the limit of $\alpha \to 1$ from below of the epr equation of quantum motion, Eq. (6). For a launch point (initial position) of $(t, x) = (0, 0)$, quantum motion for the EPR-molecule in the limit $\alpha \to (1-)$ is given by [24]

$$\lim_{\alpha \to (1-)} t_{epr} = t_{EPR} = \sum_{n=1}^{\infty} \delta[x - (2n-1)\pi/(2k)] = \sum_{n=1}^{\infty} \{ \delta[x - (2n-1)] \}, \quad x > 0, \quad \tau_{EPR} = 0 \quad (9)$$

consistent with the equation of quantum motion, Eq. (6). For $x < 0$ and the launch point still at $(t, x) = (0, 0)$, we investigate the case $1 \leq \alpha \leq \infty$ using the limiting process $\alpha \to 1$ from above. This renders

$$\lim_{\alpha \to (1+)} t_{epr} = t_{\downarrow EPR} = -\sum_{n=1}^{\infty} \delta[x - (2n-1)\pi/(2k)] = -\sum_{n=1}^{\infty} \{ \delta[x - (2n-1)] \}, \quad x < 0, \quad \tau_{\downarrow EPR} = 0 \quad (10)$$

where the prefix $\downarrow$ in the subscripts denotes the limiting process $\alpha \to (1+)$ to generate quantum trajectories into the domain $x < 0$. The prefix $\uparrow \cup \downarrow$ denotes the union of the limiting processes $\alpha \to (1\uparrow)$. For launch point at $x = 0$, $\uparrow \cup \downarrow$EPR-molecule has positive infinite velocity for $x > 0$ and $x \neq 1, 3, 5, \cdots$ by Eq. (9) and negative infinite velocity for $x < 0$.
and \( x \neq -1, -3, -5, \ldots \) by Eq. (10) in this nonrelativistic representation. These infinite magnitudes of velocity at \( x \neq \pm 1, \pm 3, \pm 5, \ldots \) imply action at infinite distances within the EPR-molecule in this nonrelativistic examination. Also, at the trigger points of the \( \delta \)-function of Eqs. (9) and (10), \( x = \pm 1, \pm 3, \pm 5, \ldots \), the EPR-molecule has nil velocity consistent with \( \psi_{EPR} = 2 \cos(kx) \). Thus, the limiting process, \( \alpha \to 1 \), renders the expected standing wave function for \( \psi_{EPR} \) given by Eq. (8) while the limiting process also renders a consistent equation of quantum motion for \( t_{\uparrow \downarrow} \) given by Eqs. (9) and (10).

The alternative interpretation using creation and annihilation operations, which already has been discussed in Section 3, begs the question whether these operations imply high energy processes. They do not. This is shown by applying Faraggi and Matone’s effective quantum mass, \( m_{Q_{EPR}} = m(1 - \partial Q_{EPR}/\partial E) \) where \( Q \) is Bohm’s quantum potential [18], to this investigation. For the EPR-molecule, \( m_{Q_{EPR}} \) becomes

\[
\lim_{\alpha \to 1} m_{Q_{\pm EPR}} = 0, \quad x \neq \pm 1, \pm 3, \pm 5, \ldots
\]

\[
= \infty, \quad x = \pm 1, \pm 3, \pm 5, \ldots. \tag{11}
\]

Note that \( m_{Q_{\pm EPR}} \) here becomes infinite where the velocity of the EPR-molecule is nil and becomes nil where the velocity is infinite. This is consistent with conjugate momentum remaining finite [18]. Herein, neither do creation operations imply endoergic processes nor do annihilation operations imply exoergic processes.

### 5 The “entanglon”

Let us now demonstrate the emergence of the entanglon for an epr-molecule from the equation of quantum motion in the quantum trajectory representation of quantum mechanics. We shall dissect the equation of quantum motion for the synthetic epr-molecule, Eq. (6), to resolve the contributions to \( t_{epr} \) by particles \( P_1 \) and \( P_2 \) individually. These two individual contributions are insufficient by themselves to render \( t_{epr} \) for there remains a contribution due to the entanglement between the two particles. Equation (6) may be dissected as

\[
t_{epr} = \frac{mx(1 - \alpha^2)}{\hbar k[1 + \alpha^2 + 2\alpha \cos(2kx + \beta)]}
\]

\[
= \frac{mx}{\hbar k 1 + \alpha^2} - \frac{mx}{\hbar k 1 + \alpha^2 + 2\alpha \cos(2kx + \beta)} - \frac{mx}{\hbar k 1 + \alpha^2} \tag{12}
\]

where the epoch has been set as \( \tau = 0 \). The contributions to \( t_{epr} \) from particles 1 and 2 are weighted. In the EPR limit, \( \alpha \to 1 \), the contributions of particles 1 and 2 cancel each other. The remaining contribution that emerges in Eq. (12) has been allocated to an entity now
identified as the “entanglon”. Its contribution to $t_{\text{epr}}$ in Eq. (12) is identified as $t_{\text{epr}_e}$. Then, in the EPR limit, $\alpha \to (1-)$, $t_{\text{EPR}_e}$ is given by

$$t_{\text{EPR}_e} = \lim_{\alpha \to (1-)} \left( \frac{mx}{\hbar k} \frac{2\alpha \frac{1-\alpha^2}{1+\alpha^2} \cos(2kx + \beta)}{1 + \alpha^2 + 2\alpha \cos(2kx + \beta)} \right)$$

$$= \begin{cases} 
0, & x \neq 1, 3, 5, \cdots \\
\lim_{\alpha \to (1-)} \left( \frac{mx}{\hbar k} \frac{2\alpha \frac{1-\alpha^2}{1+\alpha^2}}{1 + \alpha^2 + 2\alpha \cos(2kx + \beta)} \right) \to \infty, & x = 1, 3, 5, \cdots 
\end{cases} \quad (13)$$

Hence, $t_{\text{EPR}}$ exhibits multi $\delta$-function behavior at $x = n\pi/(2k), n = 1, 3, 5, \cdots$. As the contributions to $t_{\text{EPR}}$ from particles 1 and 2 mutually cancel each other in the EPR limit $\alpha \to (1-)$ as shown by Eq. (12), we have that $t_{\text{EPR}} = t_{\text{EPR}_e}$. The $\delta$-function behavior of $t_{\text{EPR}_e}$ for the entanglon as given by Eq. (13) is consistent with the motion of the standing wave exhibited by Eq. (11) at $x = 1, 3, 5, \cdots$ for $\beta = 0$. Thus, the entanglon induces retrograde motion, which manifests nonlocality. The entanglon in the EPR limit implies action (entanglement) at infinite distances within the EPR-molecule as $t_{\text{EPR}_e} \to 0$ by Eq. (13) for $x = 1, 3, 5, \cdots$.

The entanglon is not an “external” force carrier between particles such as the photon, graviton, etc. for the latent motions for the individual particles $P_1$ and $P_2$ of the epr-molecule remain linear with constant velocity as shown in Eq. (6). Nor does the entanglon change either wave function, $\psi_1$ or $\psi_2$. Nevertheless, the entanglon does maintain the correlation between $\psi_1$ and $\psi_2$, which it may do so superluminally. In so doing, the entanglon renders an “internal” force within the epr-molecule influencing the quantum trajectory of the epr-molecule while maintaining a coherent epr-molecule.

The entanglon also has characteristics in common with the gluon. Neither exists in isolation. When coherence within the epr-molecule is lost, then the entanglon no longer exists. There is another characteristic in common for entanglons and gluons which regards strength with range. Gluons become stronger with range. Also, as range increases, the entanglon, as well as the epr-molecule, spontaneously develops an additional pair of segments that alternate with regard to retrograde and forward motion in the quantum trajectory. These segments imply the existence of multipaths, which are inherently nonlocal, for the entanglon. The number of multipaths increase with range, which mitigates any loss of coherence between the entangled particles with range. Thus, the concept that entangled particles that are widely separated in this nonrelativistic investigation should become independent of each other due to Einstein locality is refuted.

For completeness, the forward and retrograde segments of the entanglon are reminiscent of Cramer’s transactional interpretation of quantum mechanics [37]. The transactional interpretation postulates that a quantum interaction be a standing wave synthesized from a retarded (forward-in-time) wave and an advanced (retrograde) wave.

The concept of the entanglon also supports a hierarchy of entanglement critical for an undivided universe that has been postulated in Bohmian mechanics [38].
6 Conclusions

We conclude that entangled particles may be synthesized into entanglement molecules. The quantum trajectory representation of quantum mechanics does describe causal behavior of the entanglement molecule without invoking the Born probability postulate for $\psi$. The particular quantum trajectory for an entanglement molecule may be specified by a single constant of the motion, $E$. The quantum trajectory representation including Faraggi and Matone’s quantum equivalence principle and their quantum effective mass does resolve some of the mysteries of EPR. The quantum trajectory representation renders the emergence of the entanglement which maintains coherence between widely separated, entangled entities.

Quantum trajectories in a nonrelativistic theory have shown for the EPR gedanken experiment that entanglement may be maintained superluminally. In the case of the EPR limit, entanglement is maintained instantaneously. Also, quantum trajectories in the EPR limit imply action at infinite distances in this nonrelativistic investigation. Hence, the locality loophole cannot be closed.

This opus is consistent with Copenhagen through the description of the wave function for the epr-molecule, $\psi_{epr}$ as exhibited by Eq. (2) but differs thereafter. The anticipated Copenhagen response would stipulate that a measurement upon $\psi_{epr}$ would render a probabilistic outcome for the epr-molecule. By axiomatic precept, Copenhagen has denied the very existence of the deterministic quantum trajectories, which were used herein. As noted in the Introduction, the quantum trajectory interpretation of quantum mechanics has already shown that $\psi$ is not a complete description of quantum phenomena [19,21,24].

This opus is also consistent with Bohmian mechanics [28] through the description of the epr-reduced action, $W_{epr}$ as exhibited by Eq. (3) but differs thereafter. Both representations are based upon the same quantum Hamilton-Jacobi equation and develop the same generator of quantum motion. While $W_{epr}$ is a common generator of quantum motion, quantum trajectories and Bohmian mechanics have however different equations of quantum motion. The quantum trajectory representation develops its equation of quantum motion from Jacobi’s theorem, Eq. (6). On the other hand, the Bohmian equation of quantum motion is the integration of the conjugate momentum, Eq. (1) [28].

For completeness, should a measuring process on the entangled molecule use a matched filter designed to measure some property of $\psi_1$ for example, then the measuring process will detect that property of $\psi_1$. To detect the entangled molecule, the measuring filter must be matched to the entanglement molecule as a whole.

Acknowledgements

I heartily thank Marco Matone and Alon E. Faraggi for their interesting discussions and encouragement.
References

1. A. Einstein, B. Podolski and N. Rosen, Phys. Rev. 47, 777 (1935).
2. N. Bohr, Phys. Rev. 48, 696 (1935).
3. J. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, 1987).
4. A. Aspect, P. Grangier and G. Roger, Phys. Rev. Lett. 47, 460 (1981); 49, 91 (1982); A. Aspect, J. Dalibard and G. Roger, Phys. Rev. Lett. 49, 1804 (1982).
5. A. Fine, <http://plato.stanford.edu/archives/sum2004/entries/qt-epr/>.
6. D. Home and A. Whitaker, *Einstein’s Struggles with Quantum Theory* (Springer, New York, 2007).
7. P. G. Kwiat, K. Mattle, H. Weinfurter, A. Zeilinger, A. V. Sergienko, and Y. H. Shih, Phys. Rev. Lett. 75, 4337 (1995).
8. J. R. Togerson, D. Branning, C. H. Monken, and L. Mandel, Phys. Lett. A 204, 323 (1995).
9. G. Di Giuseppe, F. De Martini, and D. Boschi, Phys. Rev. A 56, 176 (1997).
10. D. Boschi, S. Branca, F. De Martini, and L. Hardy, Phys. Rev. Lett. 79, 2755 (1997).
11. G. Weihs, T. Jennewein, C. Simon, H. Weinfurter, and A. Zeilinger, Phys. Rev. Lett. 81, 5039 (1998).
12. D. Bouwmeester, J.-W. Pan, M. Daniell, H. Weinfurter, and A. Zeilinger, Phys. Rev. Lett. 82, 1345 (1999).
13. W. Tittel, J. Brendel, N. Gisin, and H. Zbinden, Phys. Rev. A 59, 4150 (1999).
14. M. A. Rowe, D. Klepinski, V. Meyer, C. A. Sackett, V. M. Itano, C. Monroe, and D. J. Wineland, Nature 409, 791 (2001).
15. S. Gröblacher, T. Paterek, R. Kaltenbaek, C. Brukner, M. Żukowski, M. Aspelmeyer, and A. Zeilinger, Nature 446, 1469 (2007).
16. E. R. Floyd, Phys. Rev. D 34, 3246 (1982).
17. E. R. Floyd, *Gravitation and Cosmology: From the Hubble Radius to the Planck Scale; Proceedings of Symposium in Honour of the 80th Birthday of Jean-pierre Vigier*, ed. by R. L. Amoroso et al (Kluwer Academic, 721 Dordrecht, 2002), extended version [quant-ph/0009070].
18. A. E. Faraggi and M. Matone, Phys. Rev. Lett. 78, 163 (1997) [hep-th/9606063], Phys. Lett. B 437, 369 (1998) [hep-th/9711028], B 445, 77 (1999) [hep-th/9809125], 357 (1999) [hep-th/9809126], B 450, 34 (1999) [hep-th/9705108], A 249, 180 (1998) [hep-th/9801033].
19. A. E. Faraggi and M. Matone, Int. J. Mod. Phys. A 15, 1869 (2000) [hep-th/98090127].
20. G. Bertoldi, A. E. Faraggi and M. Matone, Class. Quant. Grav. 17 3965 (2000) [hep-th/9909201]
21. R. Carroll, Can. J. Phys. 77, 319 (1999) [quant-ph/9904081], *Quantum Theory, Deformation and Integrability* (Elsevier, 2000, Amsterdam) pp. 50–56, *Uncertainty, Trajectories, and Duality*, [quant-ph/0309023]
22. E. R. Floyd, Phys. Rev. D 26, 1339 (1982).
23. E. R. Floyd, Phys. Rev. D 29, 1842 (1982).
24. E. R. Floyd, Found. Phys. Lett. 9, 489 (1996), quant-ph/9707051.
25. E. R. Floyd, Found. Phys. 37, 1403 (2007), quant-ph/0605121.
26. E. R. Floyd, Found. Phys. 37, 1386 (2007), quant-ph/0605120.
27. W. Dür, Phys. Rev. A 62, 020303(R) (2001).
28. P. R. Holland, The Quantum Theory of Motion (Cambridge University Press, Cambridge, 1993) pp. 86–87, 141–146.
29. E. R. Floyd, Phys. Rev. D 25, 1547 (1982).
30. B. Poirier, J. Chem. Phys. 121, 4501 (2004).
31. R. E. Wyatt, Quantum Dynamics with Trajectories: Introduction to Quantum Hydrodynamics (Springer, New York, 2005).
32. J. F. Clauser and M. A. Horne, Phys. Rev. D 10, 526 ((1974).
33. E. R. Floyd, Int. J. Mod. Phys. A 15, 1363 (2000), quant-ph/9907092.
34. A. Peres, Amer. J. Phys. 48, 552 (1980).
35. D. Bohm, Phys. Rev. 85, 166 (1953).
36. P. Ghose, Adv. Sc. Lett., vol 2, pp 97-99 (2009), arXiv:0905.2037v1.
37. J. G. Cramer. Rev. Mod. Phys. 58, 647-688, July (1986).
38. D. Bohm and B. J. Hiley, The Undivided Universe (London, Routledge, 1993).
Figure Captions

Fig. 1. Motion, $x(t)$, of the epr-molecule for $\tau = 0$, $A = 1$, $B = 0.5$, $k = \pi/2$ and $\beta = 0$ as a solid line and for $\beta = \pi$ as a dashed line.

Fig. 2. Motion of the epr-molecule, $x(t)$, for $\tau = 0$, $A = 1$, $B = 0.5$, $k = \pi/2$ and $\beta = 0, \pi/4, \pi/2, \cdots, 7\pi/4$ for a set of trajectories. All trajectories are displayed as solid lines.
This figure "Fig1_QTEPR.png" is available in "png" format from:

http://arxiv.org/ps/1001.4575v1
This figure "Fig2_QTEPR.PNG" is available in "PNG" format from:

http://arxiv.org/ps/1001.4575v1