Application of shielding current in bulk HTS to control magnetic field distribution

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Abstract. Superconducting shielding current is excited when external field is applied to superconductor. In case for field cooling of bulk superconductor, shielding current is an origin of strong trapped field. When external field is changed to a properly arranged bulk HTS array, various magnetic field distribution can be formed by an excited shielding current in each bulk HTS. This paper presents a simple intuitively method to design magnetic field distribution using supercurrents in bulk high-temperature superconductor (HTS) array. In this method, an ideal current path for intended field distribution is represented by shielding currents in bulk HTS array. Expected performance can be roughly estimated by using Biot-Savart law. As examples, Maxwell coil pair and helical field generator are designed. This method can be applied to design various magnet devices using bulk HTS array.

1. Introduction

Bulk high-temperature superconductors (HTS) have a large potential for various application because of its high current density. In case for field cooling of bulk superconductor, shielding current is an origin of strong trapped field. When external field is changed to a properly arranged bulk HTS array, various magnetic field distribution can be formed by an excited shielding current in each bulk HTS. This paper presents a simple intuitively method to design magnetic field distribution using supercurrents in bulk high-temperature superconductor (HTS) array. In this method, an ideal current path for intended field distribution is represented by shielding currents in bulk HTS array. Expected performance can be roughly estimated by using Biot-Savart law. As examples, Maxwell coil pair and helical field generator are designed. This method can be applied to design various magnet devices using bulk HTS array.
Figure 1. Schematic drawing of the bulk HTS SAU. When the external magnetic field is changed by the external solenoid, supercurrent is induced in each bulk HTS. Small arrows indicate the excited induced shielding supercurrents. Shielding currents of opposite direction are excited at above and below of the central axis. Transverse periodic magnetic field is generated on the central axis by the induced shielding supercurrents.

The intended periodic magnetic field distribution can be generated by superposition of the magnetic field around the shielding current in each bulk HTS. Most important feature of the operational scheme of the undulator is that the superconducting shielding currents exited in bulk HTS are highly designed to control magnetic field distribution. By generalizing this approach to other field distribution such as constant gradient field, helical field, bulk HTS array can be used to various applications and realize latent potential of bulk HTS.

2. Design procedure of bulk HTS array

In this section, a simple intuitively way to design HTS array is described without using numerical simulation tools. The approach using bulk HTS array and external field can be characterized by designing the current path which generates intended field. The simplest example of designed field is uniform magnetic field. An infinitely long solenoid or Helmholtz coil is used to generate uniform field. We can easily imagine the equivalent shape of the bulk HTS, which are infinitely stacked hollow disks and two hollow disks separated by a distance equal to the radius of the hollow disk respectively. Although the infinitely stacked hollow disks can realize better uniformity than the two hollow disks, use of the two hollow disks is adopted in the design procedure for generalization. An original hollow disk is represented by virtual current loop which consists of segmented fan shaped hollow parts as shown in figure 2.
Figure 2. An alternative expression of hollow disk. In this example, an original ring (a) is represented using segmented three parts (b).

It is noted that the direction of excited shielding supercurrents is changed by segmentation, effect from straight lines cancels, and effect from outer circular current path can be reduced by increasing size.

2.1. Producing constant-gradient field

A Maxwell coil pair; two ring coils of radius $R$ with opposite current flow are separated with distance $\sqrt{3}R$ can generate constant gradient field [7]. As shown in figure 3, this configuration can be realized by using combination of a hollow disk and a segmented current path which is introduced above. If the outer radius of the virtual current loop of the segmented parts is large enough, almost constant gradient field is generated between the rings. In order to obtain precise numerical estimation, mutual inductance among the hollow disk and segmented pars should be carefully considered and field distribution should be tuned by controlling precise shape, radius, and position of current paths, by using numerical tools such as an FEM code.

Figure 3. An example of Maxwell coil pair represented with hollow disk and segmented parts.

2.2. Producing helical field

Generation of strong helical magnetic field in short period is attractive for various applications such as spin physics and a helical undulator which generate circular polarized synchrotron radiation or photon beams carrying an orbital angular momentum [8]. The helical field can be produced using double helix as shown in figure 4.
Transverse magnetic field $B_\perp$ can be analytically expressed as follows \[9\]
\[B_\perp = \frac{\mu_0}{\lambda_0} \left( +I - I \right) \left\{ \frac{2\pi r_0}{\lambda_0} K_0 \left( \frac{2\pi r_0}{\lambda_0} \right) + K_1 \left( \frac{2\pi r_0}{\lambda_0} \right) \right\} \]
(1).
Here, $K_n$ is the modified Bessel function of the second kind. $\mu_0$, $\lambda_0$, $I$, and $r_0$, are the permeability of free space, the period of spiral, the supercurrent, and the radius of helix, respectively. $+I$ and $-I$ represent currents with different direction in double helix. As same as the segmented hollow disk, one period of spiral current path can be expressed by segmented hollow parts as shown in figure 5.

Although the segmented outer virtual helix produce the opposite helical field to that of inner virtual loop, the opposite component rapidly decreases as increasing outer radius because the Bessel function decreases very quickly.

3. Estimation of magnetic field
In case of superconducting wire, amount of coil current is a variable which is directly controllable by a power supply. Hence we can easily estimate the current distribution by using Biot-Savart law. However, in case of bulk HTS array placed in external field, shielding current is not directly controlled by external field. In order to estimate required field strength to fill hollow parts with shielding currents, a very simple situation is considered. A superconducting ring with small curvature is placed in external magnetic field as shown in figure 6. As increasing the field strength, amount of shielding
current increases and finally superconductor is filled with shielding current. At just before saturation, magnetic field at the small screened area is zero. It means that the external magnetic field strength required to fill the superconductor ring is same as the surface magnetic field strength with fully filled superconductor. As described above, the required magnetic field strength is roughly expressed as follows

\[ B_{\text{external}} = B_{\text{surface}} = \frac{\mu_0 I_{\text{bulk}}}{2\pi r_{\text{bulk}}} \]  

Here, \( \mu_0 \), \( I_{\text{bulk}} \), \( r_{\text{bulk}} \), \( B_{\text{external}} \) and \( B_{\text{surface}} \) are the permeability of free space, total amount of shielding current in the bulk ring, tube radius of the bulk ring, the applied external magnetic field, and the surface magnetic field produced by the shielding current at saturation respectively. Under this condition, magnetic field distribution can be easily calculated using Biot-Savart law.

**Figure 6.** A schematic view of shielding current in the ring superconductor.

As already mentioned in previous section, detailed calculations are required to obtain precise field distribution. In order to clear advantage of the bulk array to superconducting wire, performance of helical field generator is compared numerically. Finite-size effect should be considered for actual device, i.e. current distribution, width, and thickness of the wire and bulk are considered. Here, we adopt effective radius of helix \( r_{\text{eq}} \) and equivalent current \( I_{\text{eq}} \) for the comparison. The equivalent radius is geometrically averaged radius for finite size of coil. The equivalent current is simply defined by using width and thickness of coils or hollow bulks, as \( I_{\text{eq}} = J \times \text{width} \times \text{D} \). As an example of a double helix undulator, design parameter reported at advanced photon source in Argonne National Laboratory [10] is used. Schematic drawing for the double helix is shown in figure 7.
Figure 7. Geometrical model for double helix. Two spiral plates represent SC wires. Equivalent radius $r_{eq}$ is defined as geometrical average radius.

Inner radius, coil thickness, width of LTS wire, pitch of the helix (period of spiral), and current density of low temperature superconducting (LTS) wire are 3.15 mm, 3.84 mm, 1.0 mm, 12.0 mm, and 1.0 kA/mm$^2$, respectively. Equivalent inner radius for double helix is 4.3 mm and the total current is 3.8 kA. The expected transverse helical field is 0.23 T. As for the HTS array, inner radius, outer radius, width of current region, pitch of helix, current density of HTS bulk are set to 3.15 mm, 17 mm, 3.84 mm, 1.0 mm, 12.0 mm, and 10.0 kA/mm$^2$, respectively. The equivalent inner and outer radius of current loop for bulk HTS is 4.3 and 15 mm respectively. As the modified Bessel function in eq. (1) quickly decreases as increasing radius, when the outer radius is 15 mm, the negative transverse field strength is less than 0.6% of that of inner current path. When the operated temperature is set to around 10 K, current density of bulk HTS is about 10 kA/mm$^2$, thus the total current of 38 kA for bulk HTS array. By using this condition, strength of transverse magnetic field is calculated. The expected transverse helical field for the segmented bulk is 1.1 T for period of 12 mm and inner bore diameter of 6.3 mm. The transverse field is about 5 times larger than that of the LTS wound wire undulator. Results are listed in table 1 together with important parameters.

Table 1. Comparison of expected performance for the double helix LTS wound wire type undulator and the segmented single helix HTS bulk array type undulator

| Type                                      | Double Helix LTS Wound Wire | Segmented Single Helix HTS Bulk Array |
|-------------------------------------------|-----------------------------|--------------------------------------|
| Current Density $J_c$                     | 1.0 kA/mm$^2$               | 10.0 kA/mm$^2$                       |
| Equivalent radius of helix $r_{eq}$       | Inner 4.3 mm                | Inner 4.3 mm                         |
|                                          | Outer 15 mm                 | Outer 15 mm                          |
| Period of spiral $\lambda$               | 12 mm                       | 12 mm                                |
| Thickness of coil or disk                 | 1.0 mm                      | 1.0 mm                               |
| Transverse field $B_{\perp}$              | 0.23 T                      | 1.1 T                                |

4. Conclusion

Bulk HTS have a high latent potential for magnetic application for those high critical current density and flexibility for current path. If we can properly arrange the bulk HTSs in the external field, various interesting strong magnetic field distribution can be produced. In this paper, basic concept for designing of bulk HTS array and several examples of bulk HTS array are introduced. We hope the new strategy for magnetic field distribution control using shielding current opens new research field using bulk superconductor.
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