Comparing D-branes and Black Holes with 0- and 6-brane Charge

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Abstract

We consider configurations of D6-branes with D0-brane charge given by recent work of Taylor and compute interaction potentials with various D-brane probes using a 1-loop open string calculation. These results are compared to a supergravity calculation using the solution given by Sheinblatt of an extremal black hole carrying 0-brane and 6-brane charge.
1. Introduction

Among their many uses, D-branes can serve as probes of spacetime at large distances as well as distances shorter than the string scale. Following the work of we have also learned that configurations of D-branes can also serve as a microscopic description of black holes at weak coupling. The fact that D-branes can be sensitive probes of the structure of black holes at least at the one-loop level has been shown in . In order to more fully understand the correspondence between D-branes and black holes, it is useful to study many examples, including those with no supersymmetry. Once in hand, a D-brane description can be useful in understanding entropy, Hawking radiation, and possibly the information problem.

In this short note we investigate the relationship between a configuration of D6-branes that carry D0-brane charge found in and a black hole solution of IIA supergravity that carries 0-brane and 6-brane charge studied in . Since the static force between a 0-brane and 6-brane is repulsive at short and long distances, the conventional wisdom has been that these objects cannot bind together. However in it was argued that 0-6 fermionic string modes could provide an attractive potential and account for the Bekenstein-Hawking entropy of the (metastable) black hole solution. Further, in a configuration (not just a single 0-brane and 6-brane) was given that carries only 0-brane and 6-brane charge and is stable to quadratic order in fluctuations. It is interesting to reconcile these two descriptions. By examining their long distance interactions we find evidence for their equivalence as was done in for the case of black p-branes. In section 2 we consider interactions of the 6-0-brane “bound state” with other D-brane probes using the 1-loop annulus diagram in open string perturbation theory. In section 3 we compute the effective action of D-brane probes in the background of the classical supergravity solution and we find that the long distance interaction potentials agree in each case.

2. Probing 6-branes in String Theory

2.1. The 6-0 Configuration

We use the basic construction given in which can be thought of as a configuration of four coincident D6-branes with a world volume $U(4)$ gauge field strength,

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1. For a review see
2. See for a review and references
\( F_{12} = F \text{ diag}(1, 1, -1, -1) \)
\( F_{34} = F \text{ diag}(1, -1, -1, 1) \).
\( F_{56} = F \text{ diag}(1, -1, 1, -1) \).  

This carries nonzero 0-brane charge (\( \int \text{Tr} \, F \wedge F \wedge F \neq 0 \)), with vanishing 2-brane (\( \int \text{Tr} \, F \wedge F = 0 \)) and 4-brane (\( \int \text{Tr} \, F = 0 \)) charge. This gauge field configuration was shown to be stable classically to quadratic order, but overall stability of this system is an open question. Since we take the world volume gauge field strength to be diagonal, we only need consider the \( U(1)^4 \) subgroup of the gauge theory. In order to compute the one-loop scattering amplitude using open string perturbation theory, we just take a composite of four D6-branes, each with a constant background \( U(1) \) field strength.

### 2.2. Probing with 0-branes

The one-loop vacuum amplitude for open strings stretched between a 0-brane in the presence of 6-brane with a constant background magnetic field strength \( F_{(2i-1)(2i)} = \tan \pi \epsilon_i \), relative velocity \( v = \tanh \pi \nu \), and impact parameter \( b \) is given by [11],

\[
A = \frac{1}{2\pi} \int \frac{dt}{t} e^{-b^2 t} Z_B \times Z_F
\]

\[
Z_B = \frac{\Theta'_1(0|it)}{\Theta_1(\nu t|it)} f_1^{-2} \Theta_4(i \epsilon_1 t|it)^{-1} \Theta_4(i \epsilon_2 t|it)^{-1} \Theta_4(i \epsilon_3 t|it)^{-1}
\]

\[
Z_F = \frac{1}{2} \left\{ \Theta_4(\nu t|it) f_4^2 \Theta_2(i \epsilon_1 t|it) \Theta_2(i \epsilon_2 t|it) \Theta_2(i \epsilon_3 t|it) - i \frac{\Theta_4(\nu t|it)}{\Theta_1(0|it)} f_4^2 \Theta_1(i \epsilon_1 t|it) \Theta_1(i \epsilon_2 t|it) \Theta_1(i \epsilon_3 t|it) - \frac{\Theta_2(\nu t|it)}{\Theta_2(0|it)} f_4^2 \Theta_3(i \epsilon_1 t|it) \Theta_3(i \epsilon_2 t|it) \Theta_3(i \epsilon_3 t|it) \right\}
\]

in units where \( 2\pi \alpha' = 1 \). To reproduce the 6-0 configuration we add four copies of this amplitude each with \( (\epsilon_1, \epsilon_2, \epsilon_3) = (\epsilon, \epsilon, \epsilon), (\epsilon, -\epsilon, -\epsilon), (-\epsilon, -\epsilon, \epsilon), \) and \( (-\epsilon, \epsilon, -\epsilon) \) respectively. In this case however, each copy gives the same result.

In the \( t \to \infty \) limit the amplitude becomes,

\[
A \to -\frac{1}{2} \int \frac{dt}{t} e^{-b^2 t} \cot vt.
\]

This is the same result as one gets from integrating out massive modes of 0 – 6 strings in the supersymmetric quantum mechanics of a D0-brane probe moving by a pure D6-brane which gives a determinant,
At short distances the 0-brane probe is not sensitive to a small background magnetic field which represents lower dimensional branes dissolved into the 6-brane.

By taking the $t \to 0$ limit we extract a long distance potential due to exchange of massless closed strings which is defined by $A = -\int_{-\infty}^{\infty} d\tau V(r^2 = b^2 + \tau^2 v^2)$,

$$V = -4 \times v \frac{\cosh 2\pi \nu - 3 \cos 2\pi \epsilon - 4 \cosh \pi \nu \sin^3 \pi \epsilon}{8 \sinh \pi \nu \cos^3 \pi \epsilon} r^{-1}$$  \hspace{1cm} (2.5)

2.3. Probing with 2-branes

Since 0-brane probes are not sensitive to differences between the constituents which form the composite (2.1), it is useful to consider other Dp-brane probes which are. First consider a D2-brane probe without any background gauge field turned on in its world volume. Using techniques of \[12,13,14,15,16\] we find the one-loop amplitude is given by,

$$A = \frac{1}{2\pi} V_2 \int \frac{dt}{t} e^{-b^2 t} Z_B \times Z_F$$

$$Z_B = \frac{\Theta'_1(0|it)}{\Theta_1(vt|it)} - \frac{i \tan \pi \epsilon}{2\pi \Theta_1(i \epsilon t|it)} f_1^{-2} \Theta_4(i \epsilon t|it)^{-1} \Theta_4(i \epsilon t|it)^{-1}$$

$$Z_F = \frac{1}{2} \left\{ \frac{\Theta_4(vt|it)}{\Theta_4(0|it)} f_4^{-2} \Theta_3(i \epsilon t|it) \Theta_2(i \epsilon t|it) \Theta_2(i \epsilon t|it) \right\}$$  \hspace{1cm} (2.6)

In this case adding the four contributions to the amplitude cancels the second term in $Z_F$ coming from the $(-1)^{FNS}$ sector. In the $t \to \infty$ short distance limit the amplitude becomes,

$$A \to 4 \times v \frac{\tan \pi \epsilon}{4\pi} \int \frac{dt}{t} e^{-b^2 t} \frac{\cosh \pi \epsilon t - \cos \pi \epsilon t}{\sin \pi \epsilon t \tanh \pi \epsilon t}. \hspace{1cm} (2.7)$$

For small $v$ there is a tachyonic instability [17] at $b^2 < \pi \epsilon$. The long distance potential in the $t \to 0$ limit is,

$$V = -4 \times v \frac{\cosh 2\pi \nu - \cos 2\pi \epsilon}{8 \sinh \pi \nu \cos^3 \pi \epsilon} (2\pi)^{-1} r^{-1}.$$  \hspace{1cm} (2.8)
2.4. Probing with 4-branes

Next we consider a D4-brane probe. The one-loop amplitude is,

\[ A = \frac{1}{2\pi} V_4 \int \frac{dt}{t} e^{-b^2 t} Z_B \times Z_F \]

\[ Z_B = \frac{1}{2\pi} \Theta'_1(0|it) \frac{\tan \pi \epsilon_1}{2\pi \Theta_1(\nu t|it)} \frac{\tan \pi \epsilon_2}{2\pi \Theta_1(\nu t|it)} f_1^{-2} \Theta_4(i \nu t|it)^{-1} \]

\[ Z_F = \frac{1}{2} \left\{ \frac{\Theta_3(\nu t|it)}{\Theta_3(0|it)} f_3^2 \Theta_3(i \nu t|it) \Theta_3(i \nu t|it) \Theta_2(i \nu t|it) \right\} \]

The calculation proceeds as in the D2-brane case. The \( t \to \infty \) limit is,

\[ A \to 4 \times \left( \frac{\tan \pi \epsilon}{4\pi} \right)^2 \frac{1}{2} \int \frac{dt}{t} e^{-b^2 t} e^{\frac{\pi \nu}{\pi \nu \cosh^2 \frac{\pi \nu}{\pi \nu}}} \frac{\cosh \pi \nu t}{\sin \pi \nu t \sinh^2 \pi \nu t}, \]

and there is a tachyonic instability when \( b^2 < \frac{\pi}{2} - \pi \epsilon \). The long distance \( (t \to 0) \) potential is,

\[ V = -4 \times v \frac{\cosh 2\pi \nu \cos 2\pi \epsilon}{8 \sinh \pi \nu \cos^3 \pi \epsilon} (2\pi)^{-2} r^{-1}. \]

3. Probing the Supergravity Black Hole Background

The supergravity solution for an extremal black hole with 0-brane and 6-brane charge given in [9] is,

\[ ds^2_{10} = -H^2 dt^2 + H^{-2} dr^2 + r^2 d\Omega^2 + dy_i dy^i \]

\[ H = 1 - \frac{g \sqrt{\alpha'}}{r} \]

\[ C^{(1)} = -\sqrt{2} g q \left[ \frac{\sqrt{\alpha'}}{r} dt + (1 - \cos \theta) d\phi \right] \]

\[ e^{2\phi} = 1 \]

In this case with a constant dilaton there is a single parameter \( q \) which is related to number of 6-branes by \( q = \frac{\sqrt{7}}{4} Q_6 \), and further the number of 0-branes is fixed by
\[ Q_0/Q_6 = V^6/(2\pi)^6 \alpha'^3. \] (3.2)

In this section we compute the effective action for various D-brane probes \[18,19,6\] by expanding the bosonic action for a Dp-brane,

\[
S_p = -T_p \int d\tau d^p \sigma e^{-\phi} \sqrt{-\det g_{\mu\nu} \partial X^\mu \partial X^\nu} + T_p \int C^{(p+1)}
\] (3.3)

around the background (3.1). In each case we choose the static gauge,

\[
X^0 = \tau
\]
\[
X^1\ldots p = \sigma^1\ldots p
\]
\[
X^{p+1}\ldots 9 = X^{p+1}\ldots 9(\tau)
\] (3.4)

and consider only velocities transverse to the branes. Expanding the term with the square root to leading order in velocities gives,

\[
-T_p V_p \int d\tau H + \frac{1}{2} T_p V_p \int d\tau H^{-1}(H^{-2} \dot{r}^2 + r^2 \dot{\Omega}^2)
\] (3.5)

As in [7] we must also make a change of variable,

\[
\frac{d\rho}{\rho} = \frac{dr}{rH}
\] (3.6)

in order to bring the velocity into the standard form \(v^2 = \dot{\rho}^2 + \rho^2 \dot{\Omega}^2\). Only the case with a 0-brane probe couples to the RR form \(C^{(1)}\) through the second term in (3.3) which gives a contribution of \[3\]

\[
-T_0 V_0 \int d\tau gg \sqrt{2\alpha'}/r
\] (3.7)

to the effective action in addition to (3.5). By expanding (3.5) and (3.7) to lowest order in \(\frac{1}{\tau}\) and using \(T_p = Q_p g^{-1}(2\pi)^{-p}(\alpha')^{-\frac{2(p+1)}{p+2}}\) we can read off the leading term in the effective potential in each case and get the result,

\[
V_{eff}^{(0)} \sim -Q_6 \sqrt{2} [1 - \sqrt{2} + \frac{3}{2} v^2] \rho^{-1}
\] (3.8)

for 0-brane probes and

\[ As in [10] we ignore the angular dependence to suppress the Lorentz force. \]
\[ V^{(p)}_{\text{eff}} \sim -Q_6 \frac{\sqrt{2}}{4} \left[1 + \frac{3}{2} v^2\right](2\pi)^{-\frac{6}{7}} \rho^{-1} \]  

(3.9)

for 2-brane and 4-brane probes.

Comparing the supergravity results (3.8), (3.9) with the string theory results (2.5), (2.8), (2.11) we find precise agreement for the static and \( v^2 \) terms if we set \( \epsilon = \frac{1}{4} \) which means \( F = 1 \). Since \( F \) is related to the relative D-brane charges through,

\[ T_0 = T_p \int_{V_6} F^3 \]  

(3.10)

using the D-brane tension formula, we get the same relation on the charges as (3.2).

4. Discussion

In this paper we have considered the interactions of D-brane probes in two very different contexts. In one case we scattered probes off a D-brane configuration carrying only 0- and 6-brane charge using open string perturbation theory, and in the other case we considered the motion of these probes in the background of a classical black hole solution. As was speculated in [8] we have found that at large distances these two objects look alike. In [9] it was argued that the supergravity solution is metastable only for large values of \( Q_6 \) and \( Q_0 \), however the probe calculation considered here seems to hold for smaller values of the charges as well.

The 6-0 D-brane configuration also has a description within the context of M(atrix) theory and it was recently considered in [20]. There it was found that M(atrix) theory gave the correct result compared with a 6-brane probe carrying a very large amount of 0-brane charge. Here, since the ratio of 0-brane and 6-brane charge is fixed by the supergravity solution with a constant dilaton, we cannot arbitrarily increase the amount of relative 0-brane charge and go to a regime where light open string modes (and hence M(atrix) theory) agree with the classical result. It would be interesting to consider the generalization of (3.1) to the case with a non-constant dilaton where this ratio is a free parameter and to study the regime where a short distance open string description is valid at long distances.

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