Protecting the Holographic Principle: Inflation

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I. INTRODUCTION

Although inflationary models have proven to be a very successful tool for cosmology [1], nobody knows why this period started at early times. We only know how this finished, evolving in a strong non-adiabatic and out-of-equilibrium phase, called reheating. Because inflation started at times as early as $10^{-35}$ sec, it is not difficult to believe that inflation is a natural output coming from a Grand Unified Theory (GUT) holding at Planck scale. So, we have no any idea of how inflation started, because we cannot say anything about such a theory.

However, ’t Hooft [2] proposed a crucial feature for that theory: it has to be holographic. The idea of holography is the following: if we want to reconcile quantum mechanics with gravity, we have to assume that the observable degrees of freedom of the universe are projections coming from a two-dimensional surface, where the information is stored. Recently, there has been a lot of interest in the relation between holography and string theory [3].

The seminal work of Bekenstein [4] on the universal upper bound on the entropy-energy ratio for bounded systems suggested the idea of a holographic universal (HP) principle that can be applied to cosmology. In this context, Fischler and Susskind [5] proposed a holographic principle and studied its consequences for the standard model of cosmology.

During the last years, there has been a lot of work that attempts to refine the original proposal of Fischler and Susskind (FS). This search has tried to come up with a HP which does not conflict with inflation and cosmology in general. In [6] works on this line can be found. Because the FS proposal was based on adiabatic evolution and failed for closed Friedman-Robertson-Walker ‘cosmologies’, the interest was both extend the study to non-adiabatic evolution (as reheating after inflation) and try to include the closed case. The current understanding based on these studies implies that this universal principle does not constraint inflation.

In Ref. [7] the authors propose to replace the HP by the generalized second law of thermodynamics. The same idea was developed in [8], where the authors explicitly discussed the problems of the FS proposal applied to inflationary models.

The general objective of all these works is formulate the HP in cosmology. Because the HP must be considered a universal principle of a higher status than inflation, in this work I look for a solution to the problem considering a different point of view: an attempt to derive inflation from the HP. This is so because we expect the HP, to come from a (so far unknown) GUT theory, and then I will analyze if this new point of view may constraint any inflationary model. From this declaration of principles, it is clear that we do not find the current beliefs very informative because inflation and the HP are considered (almost) independent processes.

In an interesting work, Rama [9] proposed a scenario where the holographic principle a la FS could be applied to a closed universe. He showed that this is possible considering at least one exotic field matter component with density pressure ratio $w < -1/3$, and if the present value of density parameter $\Omega$ is close to one. He also suggested that this component could be realized by some form of ‘quintessence’ [10]. However, as is stressed in [8], this model is not correct because it works under adiabatic evolution with a matter content of a typical ‘inflationary’ universe ($-1 < w < -1/3$), which is a strong non-adiabatic process.

In this Letter, I look at the consequences of the FS proposal in cosmology by considering the non-adiabatic process of reheating after inflation. I discuss the case of a closed universe, which is apparently weakly favored by observations [11], with both radiation and a real scalar field $\varphi$. At early times I found that an imminent violation of the HP forces the system to saturate it through a period of exponential growth of the scale factor. This effect can be interpreted as a response to protect the HP inside the causal volume associated. Also, I discuss the possibility to explain additional exponential expansions like the one observed now. Although it is possible to obtain an accelerated phase that resembles the quintessential inflation picture [12], it is not due to a mechanism of saturation of the HP a la Fishler-Susskind. The results of a numerical integration of the evolution from inflation to the present are shown in Figure 1. The key ingredient to have an additional phase of accelerated expansion is reheating. The extremely efficient transfer of energy from the scalar field to radiation enables us to obtain a radiation-dominated phase with a vanishing small relic of $\varphi$ fields. This relic does not confront observations be-
cause the universe remains almost flat (until very small redshift) and during matter domination, it is subdomi-
nant; however, it can dominate the matter contributions for \( z < 0.5 \). The holographic principle of Fisher and Susskind is reviewed in section II. I present the model in section III and develop the formulae to treat reheating in section IV. The effect of protection of the HP is discussed in section V, and the possibilities to have another phase of accelerated expansion is discussed in section VI. The paper ends with a discussion.

II. THE FISCHLER-SUSSKIND HOLOGRAPHIC PRINCIPLE

Let us assume a closed homogeneous isotropic universe with metric

\[
\text{d}s^2 = \text{d}t^2 - a^2(t)(\text{d}x^2 + \sin^2 \chi \text{d}\Omega^2),
\]

where \( \chi \) is the azimuthal angle of \( S_3 \) and \( \Omega \) is the solid angle parametrizing the two-sphere at fixed \( \chi \). The particle horizon is

\[
\chi_H = \frac{\int_t^{t_i} \frac{dt'}{a(t')}}.
\]

where \( t_i \) is a reference initial time. Because integral (2) may diverge at small \( t_i \), a natural choice is to take \( t_i = t_{pd} = 1 \) \([8]\). The angle \( \chi_H \) determines the coordinate size of the horizon, which defines a bounded area and volume. Because the entropy density \( \sigma \equiv (\rho + p)/T \) is constant, the entropy area ratio gives

\[
\frac{S}{A} = \frac{2\chi_H - \sin 2\chi_H}{4a^2(\chi_H) \sin^2(\chi_H)}.
\]

As the universe evolves, ratio (3) increases and the system reaches a stage of saturation and later, a violation of holographic principle \([5]\). For example, for a universe filled with non-relativistic matter, \( a = a_{\text{max}} \sin^2(\chi_H/2) \) so for its maximal expansion \( \chi_H = \pi \) ratio (3) becomes violated.

III. THE MODEL

I model the universe as filled with both, a single scalar field - inflaton \( \varphi \) - and a fluid of relativistic particles (or radiation) with energy density \( \rho_m \). Assuming a homogeneous field with a slowly-time-dependent equation of state \( \rho_{\varphi} = w(t)\rho_{\varphi} \), where

\[
w(t) = \frac{\dot{\varphi}^2 - 2V(\varphi)}{\dot{\varphi}^2 + 2V(\varphi)},
\]

we can write the Friedman equation as

\[
H^2 = H_0^2 \left[ \Omega_\varphi \left( \frac{a_0}{a} \right)^{3(1+w)} + \Omega_m \left( \frac{a_0}{a} \right)^4 - \Omega_k \left( \frac{a_0}{a} \right)^2 \right],
\]

where \( a_0 \) and \( H_0 \) are the scale factor and the Hubble parameter in an arbitrary reference time \( t_0 \) (often taken to be the present time). Also, we have defined the following parameters: the density parameter \( \Omega_i = \rho_{oi}/\rho_c \) for the \( i \) component with initial energy density \( \rho_{oi} \); the critical density \( \rho_c = 3H_0^2M_p^2/8\pi \); the equivalent energy density due to curvature \( \rho_k = -3M_p^2/8\pi a^2 \); and the Planck mass \( M_p = 1.2 \times 10^{19}\text{GeV} \).

Because I am considering a pressure density ratio in the range \(-1 < w < -1/3 \), this implies that at some time \( t^* \) the first term in the square brackets of (5) will dominate over the other contributions, starting a period of ‘inflationary expansion’ (if we also have \( a > 0 \)). This exponential expansion leads to a process of ‘flattening’ of the universe due to the screening of the curvature term. The authors of \([13]\) used the latter effect to obtain an apparent spatially flat FRW universe by using a closed one.

If we extend the evolution to the present (i.e., at \( t_0 \)), the evolution of the system leads to a total density parameter

\[
\Omega = \sum_i \Omega_i = \Omega_\varphi + \Omega_m = 1 + \Omega_k \approx 1,
\]

because \( \Omega_k = \rho_{0k}/\rho_c = 3M_p^2/8\pi a_0^2H_0^2 \ll 1 \) when we evaluate it today. Globally, the effect agrees with the results of \([9]\), which to solve the violation problem of the HP bound for a closed universe, introduced an ‘exotic fluid’ - as the inflaton here - as an extra matter component. However, the presence of this component leads to a period of non-adiabatic evolution \([8]\), so the equations of motion must be corrected to consider entropy production during the reheating process.

The simplest way to do that is to consider - as a first approximation - the perturbative regime of reheating which was described by adding a friction term to the inflaton equation of motion \([14]\). In this paper, I have taken into account the most efficient energy transfer possible, based on the main results of the modern reheating theory \([16]\). A more realistic and detailed approach based on this theory will be addressed elsewhere \([17]\).

IV. PARTICLE AND/OR ENTROPY PRODUCTION

In Ref. \([8]\) Kaloper and Linde concluded that the proposal of Fischler and Susskind does not confront inflation, because it eliminates the entropy produced inside the light cone during reheating. This means that their HP is valid during adiabatic expansion. To extend the analysis through the particle creation process, we have to
learn how to compute ratio (3) in a non-adiabatic stage, i.e., during reheating.

I consider the simplest model of reheating, which is based on the Born approximation [14]. Here, the inflaton evolves according to

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = -\Gamma\dot{\varphi},$$

where $$\Gamma$$ is the rate of particle production, and the evolution of the relativistic particles created is described by

$$\dot{\rho}_m + 4H\rho_m = \Gamma\rho_\varphi.$$  

(7)

Equations (6) and (7) explicitly show the non-adiabatic nature of the process. If, as usual, we define the energy density and pressure by

$$\rho = \frac{1}{2}\dot{\varphi}^2 + V(\varphi), \quad p = \frac{1}{2}\dot{\varphi}^2 - V(\varphi),$$

we can write the equation of state during the rapid oscillations phase of $$\varphi$$, i.e., $$V'' \gg H^2$$ as a temporal average during an oscillation by $$\langle (\rho + p)/\rho \rangle = w = (n-2)/(n+2)$$ where $$V(\varphi) = \varphi^n$$, so

$$\dot{\rho}_\varphi + 3H\rho_\varphi(1 + w) = -\Gamma\rho_\varphi.$$  

(9)

In the special case $$w = 0$$ (a quadratic potential), this equation describes the decay of a massive particle. A solution of this equation is

$$\rho_\varphi = M^4\left(\frac{a_i}{a}\right)^3 \exp[-\Gamma(t - t_i)],$$

(10)

where subscript $$i$$ indicates the epoch when the coherent oscillations around the minimum of the potential $$V(\varphi)$$ begins, and $$M^4$$ is the vacuum energy at that time.

If the produced particles are thermalized, we can use the expression for the entropy of radiation

$$S = \frac{2\pi^2}{45}gT^3a^3,$$

(11)

which combined with the equality $$\rho_m = \pi^2gT^4/30$$, valid for relativistic particles and Eq.(11), enables us to write a relation between the energy density and the entropy

$$\rho_m = 3\left(\frac{45}{2\pi^2g}\right)^{1/3} S^{4/3}a^{-4}. $$

(12)

For $$t < \Gamma^{-1}$$, the universe is dominated by $$\varphi$$ particles and, according to Eq.(5) and (10), it evolves as a matter-dominated universe $$a(t) \sim t^{2/3}$$. An approximated solution of (7) is

$$\rho_m \approx \frac{1}{10}M^2\Gamma M^2 \left(\frac{a}{a_i}\right)^{-4} \left(\frac{a}{a_i}\right)^{5/2} - 1,$$

which implies that, initially $$\rho_m$$ increases from 0 to $$M^2\Gamma M^2$$, and after that it decreases as $$a^{-3/2}$$ (see Figure 1). From (12) we find that the entropy grows as $$S \propto a^{15/8}$$. This means that $$\sigma$$ (which appears in front of (3)) is no longer constant, and has to be replaced by a function varying as $$\sim a^{15/8}$$ [15].

V. PROTECTING THE HP WITH INFLATION

Let us assume that the universe starts in a radiation-dominated era $$\rho_m \sim \rho_k \gg \rho_\varphi$$ at a time near the beginning of inflation, say $10^{-35}$ sec. As was discussed in Section II, such a universe will evolve towards an imminent violation of the HP. In fact, during this phase Eq.(5) can be written as

$$H^2 = \frac{8\pi}{3M^2}\rho_m \left(\frac{a_0}{a}\right)^4 - \frac{1}{a^2},$$

(13)

from which we obtain the solution

$$a(\chi_H) = A\sin(\chi_H),$$

(14)

where $$A \equiv \sqrt{8\pi\rho_m a_0^4/3M^2}$$ . By inserting (14) in (3), we find that the HP is violated as $$\chi_H \to \pi$$. Note that in this form, the bound (3) is violated also as $$\chi_H \to 0$$. This particular problem is solved by the arguments displayed under Eq.(2). Because $$a(\chi_H = 0) = a_0 \neq 0$$, then it is possible to write the solution (14) as $$a(\chi_H) = a_p + A\sin(\chi_H)$$, solving the HP bound (3) as $$\chi_H \to 0$$. However, the violation in the future ($$\chi_H \to \pi$$) remains.

Now, let us follow the evolution of both (3) and (5) during the transit from radiation domination to the inflationary phase. If we assume at Planck time $$t_{pl}$$ that $$\rho_m \sim \rho_k$$, then we can expect after certain time say $$t \approx 10^{-35}$$ sec, to enter an inflationary phase. In fact, because $$\rho_\varphi \sim a^{-3(1+w)}$$ with $$-1 < w < -1/3$$ decays more...
FIG. 2. This plot shows a detailed evolution of the energy densities and the holographic bound at early times. This shows how inflation appears to protect the violation of the HP.

slowly than \( \rho_k \sim a^{-2}\) and \( \rho_m \sim a^{-4}\), the former dominates. If we consider the beginning of inflation when \( \rho_\varphi \) becomes comparable to \( \rho_k \), this implies that at the time \( t \sim 10^{-37}\) sec we have

\[
\rho_\varphi \simeq 10^{-2}(10^{-1})\rho_k,
\]

for \( w = -1 \) \((-2/3)\), respectively. After \( \rho_\varphi \) becomes greater than \( \rho_k \), this component dominates the matter content in the universe and makes it inflationary in the sense that \( \rho_\varphi \gg \rho_k, \rho_m \). Here the Hubble parameter \( H \) (5) becomes nearly constant, and the coordinate size of the horizon (2) behaves like

\[
\chi_H = \int_{t_p}^{t} \frac{dt'}{a(t')} \sim H^{-1}(e^{-Ht} - e^{-Ht_p}),
\]

reaching an asymptotic constant value proportional to \( H^{-1} \). Because the scale factor grows nearly exponentially, the HP bound (3) starts to decrease, avoiding the violation in the future. A numerical integration of the system is shown in Fig. 2. In this way, the appearance of inflation saves the violation of the HP in the future. For example, in the \( w = -1 \) case, the scale factor can be written as \( a(\chi_H) \simeq \sqrt{3/8\pi} \sqrt{\sin(\pi/2 - \chi_H)} \), which behaves much like an exponential growth. The early radiation dominated phase is not necessary at all to demonstrate the role of the HP in inflating the universe. In fact, we can start the universe with a bound \( S/A \) saturated at the Planck era; this time with \( \rho_\varphi \geq \rho_k, \rho_m \).

To solve the cosmological problems, inflation must last around 60 e-folds. This means \( a_{end} = e^{60}a_{init} \). This growth makes the curvature term irrelevant for the subsequent analysis, although it was fundamental at the beginning. For example, if we assume that inflation begins when \( H^2 \sim a^{-2} \) in Eq.(5), then after the 60 e-folding we have in Eq.(5) \( a^{-2} \simeq e^{-120}H^2 \ll H^2 \), making the universe very flat.

After this 60 e-folding of inflation, particle creation starts. During this phase the formulae derived in section IV are valid, and the expression for the HP bound has to be replaced by

\[
\frac{S}{A} = \frac{\sigma \chi_H}{a^2} \times \frac{H^{-1}}{a^{1/8}},
\]

which decreases more slowly than in the previous phase. Because the curvature term has fallen more than 50 orders of magnitude from the beginning of inflation, it seems difficult to repeat a similar argument to explain the current accelerated expansion. This possibility is discussed in the next section.

VI. POSSIBLE EXTRA INFLATIONARY STAGES

After the 60 e-folding of inflation, bound 3 has fallen almost 100 orders of magnitude making impossible the existence of HP saturation for another period. However, we have seen that after reheating the universe becomes radiation-dominated, and we can consider a small relic of the inflation field, which enables us to have another accelerated period. This possibility is open if the inflation energy density does not fall more than 120 orders of magnitude during reheating.

Based on observations, we have some good evidence of acceleration for \( z < 0.5 \) and some preliminary evidence of deceleration for \( z > 0.5 \) [18]. If we assume that \( \rho_\varphi \approx \rho_m \) at \( z = 0.5 \), we can explain the current acceleration of the universe using the same field that drove inflation. Because at small redshift the curvature could be relevant, we found for \( z < 0.5 \) the field \( \varphi \) makes the universe look flat, although closed. The evolution through the matter domination to the present is very similar to what is shown in Figure 1.

VII. DISCUSSION

I have developed a consistent observational model based on the Holographic Principle which explains the role of inflation. It uses the HP as a relevant principle for inflation and leads to a scenario where it is possible to explain the current observation of accelerated expansion. In this way it reduces the amount of scalar fields needed to explain inflation and the dark energy. If reheating is efficient enough to change the relative weight of energy densities, it can also explain the coincidence problem. I have not used a particular potential form, using instead the inflaton equation of state as the relevant
object of study. After reheating, the universe becomes radiation-dominated at \( z \sim 20 \), and the inflaton energy density remains constant during all this period, making the model consistent with nucleosynthesis and making it behave similarly to the ΛCDM model. Although our model has to be tuned to get the inflation domination after \( z \sim 0.5 \), we think that its main quality – that of connect inflation and the dark energy with holographic properties of the universe – are of sufficient interest to deserve further study.

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