Mass of $Y(4140)$ in Bethe-Salpeter equation for quarks

Xiaozhao Chen,\textsuperscript{1} $^\ast$ Xiaofu Lü,\textsuperscript{2} Renbin Shi,\textsuperscript{1} and Xiurong Guo\textsuperscript{1}

\textsuperscript{1}Department of Foundational courses, Shandong University of Science and Technology, Taian, 271019, China
\textsuperscript{2}Department of Physics, Sichuan University, Chengdu, 610064, China

(Dated: December 22, 2015)

Abstract

Using the general form of the Bethe-Salpeter wave functions for the bound states consisting of two vector fields given in our previous work, we investigate the molecular state composed of $D_s^+ D_s^-$. However, for the SU(3) symmetry the component $D_s^+ D_s^-$ is coupled with the other components $D_s^0 \bar{D}_s^0$ and $D_s^+ D_s^-$. Then we interpret the internal structure of the observed $Y(4140)$ state as a mixed state of pure molecule states $D_s^0 \bar{D}_s^0$, $D_s^+ D_s^-$ and $D_s^+ s_s D_s^-$ with quantum numbers $J^P = 0^+$. In this paper, the operator product expansion is used to introduce the nonperturbative contribution from the vacuum condensates into the interaction between two heavy mesons. The calculated mass of $Y(4140)$ is consistent with the experimental value, and we conclude that it is a more reasonable scenario to explain the structure of $Y(4140)$ as a mixture of pure molecule states.

PACS numbers: 12.40.Yx, 14.40.Rt, 12.39.Ki

$^\ast$Electronic address: chen\,xzhao@sina.com; corresponding author.
I. INTRODUCTION

The narrow state \( Y(4140) \) was discovered by CDF collaboration [1] and its structure does not fit the conventional \( c\bar{c} \) charmonium interpretation. Then possible interpretations beyond quark-antiquark state have been proposed, such as hadronic molecule state [2] and tetraquark state [3]. Following the CDF result, it is suggested in Ref. [2] that the \( Y(4140) \) is a molecular state of \( D^{*+}_sD^{*-}_s \). However, there are some defects in Ref. [2]: the numerical result sensitively depends on the adjustable parameter, the heavy vector mesons \( D^{*+}_s \) and \( D^{*-}_s \) are considered as pointlike objects, and the definite spin-parity quantum numbers \( J^P \) of the \( Y(4140) \) can not be deduced in theory. More importantly, the previous work [2] dealt with this two-body system \( D^{*+}_sD^{*-}_s \) in the formalism of quantum mechanics and the potential between two heavy mesons was constructed in perturbation theory. Therefore the nonperturbative effects in quantum chromodynamics (QCD), for example, the condensates of vacuum can not be considered in their work.

In quantum field theory, the most general form of the Bethe-Salpeter (BS) wave functions for the bound states composed of two vector fields of arbitrary spin and definite parity has been given [4]. In this work, we apply the general formalism to investigate the molecular state of \( D^{*+}_sD^{*-}_s \) and consider that the effective interaction between these two heavy mesons is derived from one light vector meson exchange. In Ref. [4], we have deduced that in our approach one light pseudoscalar meson exchange has no contribution to the potential between two heavy vector mesons. Because of the SU(3) symmetry of the light vector mesons, one strange meson (\( K^* \)) exchange should be considered. From one strange meson exchange, it is necessary to consider the mixing of three pure molecule states \( D^{*0}\bar{D}^{*0} \), \( D^{*+}D^{*-} \) and \( D^{*+}_sD^{*-}_s \), which is not considered in Ref. [2]. Therefore, we assume that the \( Y(4140) \) state is a combination of three pure molecule states \( D^{*0}\bar{D}^{*0} \), \( D^{*+}D^{*-} \) and \( D^{*+}_sD^{*-}_s \).

To construct the interaction kernels between two heavy vector mesons \( D^{*0}\bar{D}^{*0} \), \( D^{*+}D^{*-} \) and \( D^{*+}_sD^{*-}_s \), we still consider that the heavy vector meson is a bound state composed of a light quark and c-quark and investigate the interaction of the light meson with the light quark in the heavy meson, which will be emphatically reconsidered in this work. As well known, the nonperturbative contribution plays an important role in the case of QCD at low energy. It is necessary to note that the more nonperturbative effects should be taken into account when we investigate the light meson interaction with quark in the heavy meson.
In this work, we introduce the operator product expansion and obtain the heavy meson BS wave function including the contribution from the condensates of vacuum. From the improved heavy meson BS wave function, we can obtain the heavy meson form factors which contain the contribution from the nonperturbative effects of QCD. Through these further form factors we can obtain the heavy meson interaction with light meson and the potentials between two heavy vector mesons without an extra parameter. Obviously, this approach is closer to QCD than our previous works [4–7]. Then numerically solving the relativistic Schrödinger-like equation with these potentials, we obtain the wave functions of the pure molecule states $D^*\bar{D}^*$, $D^*D^*$ and $D^*_s\bar{D}^*_s$, respectively. Finally, using the coupled-channel approach, we can obtain the masses and the wave functions of the mixed states and then the definite quantum numbers of the $Y(4140)$ system can be deduced.

This paper is organized in the following way. In Sec. III we show the general form of BS wave functions for the bound states consisting of two vector fields. After constructing the interaction kernel between two heavy vector mesons, we introduce the mixed state of three pure molecule states. Sec. III shows the procedure of the instantaneous approximation. Sec. IV shows how to obtain the heavy meson BS wave function and form factor which contain the contribution from the vacuum condensates. Then the interaction potentials between two heavy vector mesons and the masses of the pure molecule states are calculated. Sec. V gives the numerical result and our conclusion is presented in Section VI.

II. THE MIXING MECHANISM

If a bound state with spin $j$ and parity $\eta_{P'}$ is composed of two vector fields with masses $M_1$ and $M_2$, respectively, the BS wave function of this bound state in the momentum representation should satisfy, [4] for $\eta_{P'} = (-1)^j$,

\[
\chi_{\lambda\pi}^{j=0}(P', p') = T^{1}_{\lambda\pi}\phi_1 + T^{2}_{\lambda\pi}\phi_2,
\]

\[
\chi_{\lambda\pi}^{j\neq0}(P', p') = \eta_{\mu_1 \cdots \mu_j}^{}[p'_{\mu_1} \cdots p'_{\mu_j}] (T^{1}_{\lambda\pi}\phi_1 + T^{2}_{\lambda\pi}\phi_2) + T^{3}_{\lambda\pi}\phi_3 + T^{4}_{\lambda\pi}\phi_4,
\]

and, for $\eta_{P'} = (-1)^{j+1}$,

\[
\chi_{\lambda\pi}^{j=0}(P', p') = \epsilon_{\lambda\pi\xi\zeta} p'_{\xi} P'_{\zeta}\psi_1,
\]

\[
\chi_{\lambda\pi}^{j\neq0}(P', p') = \eta_{\mu_1 \cdots \mu_j}^{}(p'_{\mu_1} \cdots p'_{\mu_j}) \epsilon_{\lambda\pi\xi\zeta} p'_{\xi} P'_{\zeta}\psi_1 + T^{5}_{\lambda\pi}\psi_2 + T^{6}_{\lambda\pi}\psi_3 + T^{7}_{\lambda\pi}\psi_4 + T^{8}_{\lambda\pi}\psi_5,
\]
FIG. 1: Bethe-Salpeter wave function for the bound state composed of two vector fields.

where \( P' \) is the bound state momentum, \( p' \) is the relative momentum of two vector fields, \( \eta_{\mu_1 \cdots \mu_j} \) is the polarization tensor describing the spin of the bound state, the independent tensor structures \( T_{\lambda\tau}^i \) are given in Appendix A, \( \phi_i(P',p'^2) \) and \( \psi_i(P',p'^2) \) are independent scalar functions. In this work, we have \( P' = p'_1 + p'_2 \), \( p' = \eta_2 p'_1 - \eta_1 p'_2 \), \( \eta_{1,2} \) are two positive quantities such that \( \eta_{1,2} = M_{1,2}/(M_1 + M_2) \), \( p'_1 \) and \( p'_2 \) are the momenta of two vector fields, respectively. This is presented in Fig. 1.

In experiment the quantum numbers \( J^P \) of the \( Y(4140) \) are not unambiguously determined except for \( C = + \). Assuming that the \( Y(4140) \) is a S-wave molecule state whose constituents are two heavy vector mesons \( D_s^*+ \) and \( D_s^- \), one can have \( J^P = 0^+ \) or \( 2^+ \) for this system [2]. From Eqs. (1) and (2), the BS wave function of this bound state becomes, for \( J^P = 0^+ \),

\[
\chi_{\lambda\tau}^{0^+}(P',p') = T_{\lambda\tau}^1 F_1 + T_{\lambda\tau}^2 F_2,
\]

(5)

or, for \( J^P = 2^+ \),

\[
\chi_{\lambda\tau}^{2^+}(P',p') = \eta_{\mu_1\mu_2}[p'_{\mu_1} p'_{\mu_2}] (T_{\lambda\tau}^1 G_1 + T_{\lambda\tau}^2 G_2) + T_{\lambda\tau}^3 G_3 + T_{\lambda\tau}^4 G_4],
\]

(6)

which satisfies the BS equation

\[
\chi_{\lambda\tau}(P',p') = \int \frac{id^4q'}{(2\pi)^4} \Delta_{F_{\lambda\alpha}}(p'_1) V_{\alpha\theta,\beta\kappa}(p',q';p') \chi_{\theta\kappa}(P',q') \Delta_{F_{\beta\tau}}(p'_2),
\]

(7)

where \( V_{\alpha\theta,\beta\kappa} \) is the interaction kernel, \( \Delta_{F_{\lambda\alpha}}(p'_1) \) and \( \Delta_{F_{\beta\tau}}(p'_2) \) are the propagators for the spin 1 fields, \( \Delta_{F_{\lambda\alpha}}(p'_1) = (\delta_{\lambda\alpha} + \frac{p'_{\mu_1} p'_{\mu_2}}{M_1^2} \frac{1}{p'_1^2 + M_1^2 - i\epsilon}) \), \( \Delta_{F_{\beta\tau}}(p'_2) = (\delta_{\beta\tau} + \frac{p'_{\mu_3} p'_{\mu_4}}{M_2^2} \frac{1}{p'_2^2 + M_2^2 - i\epsilon}) \) and the momentum of this bound state is set as \( P' = (0,0,0,iM) \) in the rest frame. In this approach, we find that one light pseudoscalar meson exchange has no contribution to the potential between two heavy vector mesons [4]. Then we consider that the effective interaction between \( D_s^*+ \) and \( D_s^- \) is derived from one light vector meson exchange. The charmed meson \( D_s^*+ \)
is composed of a heavy quark $c$ and a light antiquark $\bar{s}$. From the SU(3) symmetry, the Lagrangian for the interaction of light vector meson with quarks should be

$$\mathcal{L}_I = ig_0 \left( \bar{u} \gamma_\mu \bar{d} \bar{s} \right) \left( \begin{array}{ccc} \rho^0 + \frac{1}{\sqrt{3}} V_8 + V_1 & \sqrt{2} \rho^+ & \sqrt{2} K^{*+} \\ \sqrt{2} \rho^- & -\rho^0 + \frac{1}{\sqrt{3}} V_8 + V_1 & \sqrt{2} K^{*0} \\ \sqrt{2} K^{*-} & \sqrt{2} K^{*0} & -\frac{2}{\sqrt{3}} V_8 + V_1 \end{array} \right) \left( \begin{array}{c} u \\ d \\ s \end{array} \right), \quad (8)$$

where the flavor-SU(3) singlet $V_1$ and octet $V_8$ states of vector mesons mix to form the physical $\omega$ and $\phi$ mesons as

$$\phi = -V_8 \cos \theta + V_1 \sin \theta, \quad \omega = V_8 \sin \theta + V_1 \cos \theta. \quad (9)$$

Because of the SU(3) symmetry, we consider that the exchanged mesons should be the singlet $V_1$ and octet $V_8$ states.

The Lagrangian expressed as Eq. (8) gives nine S-matrix elements, as shown in Fig. 2. The graphs (a), (e), (i) in Fig. 2 represent pure molecule states $D^{*0}D^*$, $D^{*+}D^{*-}$ and $D^{*+}D^{*-}_s$, respectively; and the remaining graphs represent the coupled-channel terms between two pure molecule states. Then the observed $Y(4140)$ state can not be considered as a pure molecule state of $D^{*+}D^{*-}_s$ and we interpret it as a mixed state of pure molecule states $D^{*0}D^*$, $D^{*+}D^{*-}$ and $D^{*+}D^{*-}_s$. The BS wave function of the $Y(4140)$ state is a linear combination of these three pure molecule states as

$$\chi^{Y(4140)} = \sum_\alpha a_\alpha \chi^{D^{*0}D^*_s} + \sum_\beta a_\beta \chi^{D^{*+}D^{*-}} + \sum_\gamma a_\gamma \chi^{D^{*+}D^{*-}_s}, \quad (10)$$

where $\chi^{D^{*0}D^*_s}$, $\chi^{D^{*+}D^{*-}}$, and $\chi^{D^{*+}D^{*-}_s}$ are the eigenstates of Hamiltonian without considering the coupled-channel terms, and these eigenstates have the same quantum numbers. While the pure molecule states $D^{*0}D^*$ and $D^{*+}D^{*-}$ have been investigated in Ref. [4], we investigate the pure molecule state $D^{*+}D^{*-}_s$ as follow.

Now, we construct the interaction kernel between $D^{*+}_s$ and $D^{*-}_s$ from one light vector meson ($V_1$ and $V_8$) exchange, shown as Fig. 2(i). The contribution of Fig. 2(i) is only from the terms $ig_1 \bar{s} \gamma_\mu V_1 \bar{s}$ and $ig_8 \bar{s} \gamma_\mu V_8 \bar{s}$ in Eq. (8), where $g_1$ is the singlet-quark coupling constant and $g_8$ is the octet-quark coupling constant. From Eq. (9), we obtain the relations of $g_1$ and $g_8$

$$g_\phi = -g_8 \cos \theta + g_1 \sin \theta, \quad g_\omega = g_8 \sin \theta + g_1 \cos \theta, \quad (11)$$
where the meson-quark coupling constants $g_\omega^2 = 2.42$ and $g_\phi^2 = 13.0$ were obtained within QCD sum rules approach $[8]$, and the mixing angle $\theta = 38.58^\circ$ was obtained by KLOE$[9]$. Since the SU(3) is broken, the masses of the singlet $V_1$ and octet $V_8$ states are approximatively identified with the masses of two physically observed $\omega$ and $\phi$ mesons, respectively. Then the effective quark current is $J_\mu = i\bar{s}\gamma_\mu s$ and the S-matrix element between the heavy vector mesons is

$$V^I = g_1^2 \langle V(p'_1)|J_\mu|V(q'_1)\rangle \left( \delta_{\mu\nu} + \frac{k_\mu k_\nu}{m_\omega^2} \right) \frac{1}{k^2 + m_\omega^2} \langle V(p'_2)|J_\nu|V(q'_2)\rangle + g_8^2 \langle V(p'_1)|J_\mu|V(q'_1)\rangle \left( \delta_{\mu\nu} + \frac{k_\mu k_\nu}{m_\phi^2} \right) \frac{1}{k^2 + m_\phi^2} \langle V(p'_2)|J_\nu|V(q'_2)\rangle,$$

where $\langle V|J_\mu|V\rangle$ is the vertex of the heavy vector meson interaction with the light vector meson. From the Lorentz-structure, the matrix elements of quark current can be expressed as

$$\langle V(p'_1)|J_\mu|V(q'_1)\rangle = \frac{1}{2\sqrt{E_1(p'_1)E_1(q'_1)}} \left\{ [\varepsilon^*(p'_1) \cdot \varepsilon(q'_1)]h_1^{(v)}(k^2)(p'_1 + q'_1)_\mu - h_2^{(v)}(k^2)\{[\varepsilon^*(p'_1) \cdot q'_1]\varepsilon_\mu(q'_1)
$$

$$+ [\varepsilon(q'_1) \cdot p'_1]\varepsilon'_\mu(p'_1)\} - h_3^{(v)}(k^2)\frac{1}{M_1^2}[\varepsilon^*(p'_1) \cdot q'_1][\varepsilon(q'_1) \cdot p'_1](p'_1 + q'_1)_\mu \right\},$$

(13)
where \( p_1' = (p', ip'_{10}), p_2' = (-p', ip'_{20}), q_1' = (q', i q'_{10}), q_2' = (-q', i q'_{20}), k = p_1' - q_1' = q_2' - p_2' \) is the momentum of the light meson and \( k = p' - q' \); \( h(k^2) \) and \( \bar{h}(k^2) \) are scalar functions, the four-vector \( \varepsilon(p) = (\varepsilon + \frac{(\varepsilon \cdot p) p}{M_H(E_H(p) + M_H)}, i \frac{\varepsilon \cdot p}{M_H}) \) is the polarization vector of heavy vector meson with momentum \( p \), \( E_H(p) = \sqrt{p^2 + M_H^2} \), \( (\varepsilon, 0) \) is the polarization vector in the heavy meson rest frame. In this approach, we calculate the meson-meson interaction when the exchange-meson is off the mass shell \( (k^2 \neq -m^2) \) and the heavy meson form factors \( h(k^2) \) and \( \bar{h}(k^2) \) are necessarily required. In section [IV] we will show how to obtain the form factors containing the contribution from the vacuum condensates. Taking away the external lines including the normalizations and polarization vectors \( \varepsilon^*(p_1'), \varepsilon_\theta(q_1'), \varepsilon^*(p_2'), \varepsilon_\kappa(q_2') \) in Eq. (12), we obtain the interaction kernel from one light vector meson \((V_1 \text{ and } V_8)\) exchange

\[
\mathcal{V}_{\alpha\theta,\beta\kappa}(p', q'; P') = \frac{g_1^2}{k^2 + m_{\omega}^2} + \frac{g_8^2}{k^2 + m_{\phi}^2} \mathcal{V}_{\alpha\theta,\beta\kappa}(p', q'; P')
\]

\[
= \left( \frac{g_1^2}{k^2 + m_{\omega}^2} + \frac{g_8^2}{k^2 + m_{\phi}^2} \right) \{ h_1^{(v)}(k^2) \bar{h}_1^{(v)}(k^2) (p_1' + q_1') \cdot (p_2' + q_2') \delta_{\alpha\theta} \delta_{\beta\kappa} \\
- h_1^{(v)}(k^2) \bar{h}_2^{(v)}(k^2) \delta_{\alpha\theta} [q_{2\beta}(p_1' + q_1')_{\kappa} + (p_1' + q_1')_{\beta} p_{2\kappa}] \\
- h_2^{(v)}(k^2) \bar{h}_1^{(v)}(k^2) [q_{1\alpha}(p_2' + q_2')_{\theta} + (p_2' + q_2')_{\alpha} p_{1\theta}] \delta_{\beta\kappa} \\
+ h_2^{(v)}(k^2) \bar{h}_2^{(v)}(k^2) [q_{1\alpha} q_{2\beta} \delta_{\theta\kappa} + q_{1\alpha} \delta_{\theta\beta} p_{2\kappa} + \delta_{\alpha\beta} p_{1\theta} p_{2\kappa}] \},
\]

where \( k = (k, 0) \).

Then using the method above, we can obtain the interaction kernels from one light vector meson \((\rho^\pm \text{ and } K^*)\) exchange, as shown in Fig. [2](b), (c), (d), (f), (g), (h),

\[
\mathcal{V}_{\alpha\theta,\beta\kappa}^\rho(p', q'; P') = \frac{2g_\rho^2}{k^2 + m_\rho^2} \bar{V}_{\alpha\theta,\beta\kappa}(p', q'; P'), \quad \mathcal{V}_{\alpha\theta,\beta\kappa}^{K^*}(p', q'; P') = \frac{g_{K^*}^2}{k^2 + m_{K^*}^2} \bar{V}_{\alpha\theta,\beta\kappa}(p', q'; P'),
\]

where \( \rho \) represents \( \rho^+ \text{ and } \rho^- \) mesons, \( K^* \) represents \( K^{*+}, K^{*-}, K^{*0} \text{ and } \bar{K}^{*0} \) mesons, \( g_\rho \)

and \( g_{K^*} \) are the meson-quark coupling constants obtained within QCD sum rules approach, \( g_\rho^2 = 2.42 \), \[8\] and \( g_{K^*}^2 = 1.46 \). [10]
III. THE EXTENDED BETHE-SALPETER EQUATION

In this section, we solve the BS equation expressed as Eq. (7) with the kernel (15) in instantaneous approximation and obtain the wave function of the pure molecule state $D_s^+D_s^-$. Firstly, we assume that the quantum numbers of pure molecule state $D_s^+D_s^-$ are $J^P = 0^+$. Substituting its BS wave function given by Eq. (5) and the kernel (15) into the BS equation (7), we find that the integral of one term on the right-hand side of Eq. (5) has a simple approach to solve this BS equation as follow. Ignoring the small cross terms, we can obtain two individual equations:

$$
\mathcal{F}_{\lambda\tau}^1(P', p', p'^2) = \int \frac{idq'}{(2\pi)^4} \Delta_{F\lambda\alpha}(p_1') \mathcal{V}_{\alpha\theta,\beta\kappa}(p', q'; P') \mathcal{F}_{\beta\kappa}(P' \cdot q', q'^2) \Delta_{F\beta\tau}(p_2'),
$$

(17)

$$
\mathcal{F}_{\lambda\tau}^2(P', p', p'^2) = \int \frac{idq'}{(2\pi)^4} \Delta_{F\lambda\alpha}(p_1') \mathcal{V}_{\alpha\theta,\beta\kappa}(p', q'; P') \mathcal{F}_{\beta\kappa}(P' \cdot q', q'^2) \Delta_{F\beta\tau}(p_2'),
$$

(18)

where $\mathcal{F}_{\lambda\tau}^1(P', p', p'^2) = T_{\lambda\tau}^1 \mathcal{F}_1(P', p', p'^2)$ and $\mathcal{F}_{\lambda\tau}^2(P', p', p'^2) = T_{\lambda\tau}^2 \mathcal{F}_2(P', p', p'^2)$. Solving these two equations, respectively, one can obtain two series of eigenfunctions and eigenvalues. Because the cross terms are small, we take the BS wave function to be a linear combination of two eigenstates $\mathcal{F}_{\lambda\tau}^{10}$ and $\mathcal{F}_{\lambda\tau}^{20}$ corresponding to the lowest energy in Eqs. (17) and (18), respectively. Then in the basis provided by $\mathcal{F}_{\lambda\tau}^{10}(P', p', p'^2) = T_{\lambda\tau}^1 \mathcal{F}_{10}(P', p', p'^2)$ and $\mathcal{F}_{\lambda\tau}^{20}(P', p', p'^2) = T_{\lambda\tau}^2 \mathcal{F}_{20}(P', p', p'^2)$, the BS wave function $\chi_{\lambda\tau}^{0^+}$ is considered as

$$
\chi_{\lambda\tau}^{0^+}(P', p') = c_1 \mathcal{F}_{\lambda\tau}^{10}(P', p', p'^2) + c_2 \mathcal{F}_{\lambda\tau}^{20}(P', p', p'^2).
$$

(19)

Substituting Eq. (19) into (7) and then comparing the tensor structures in the left and right sides, we obtain an eigenvalue equation

$$
c_1 \mathcal{F}_{10}(P', p', p'^2) =
$$

$$
\frac{1}{p_1'^2 + M_1^2 - ie p_2'^2 + M_2^2 - ie} \left\{ \int \frac{idq'}{(2\pi)^4} \left( \frac{g_1^2}{k^2 + m_\omega^2} + \frac{g_2^2}{k^2 + m_\phi^2} \right) \times \{ h_1^{(v)}(k^2)\bar{h}_1^{(v)}(k^2)(p'_1 + q'_1) \cdot (p'_2 + q'_2) + 2h_1^{(v)}(k^2)\bar{h}_2^{(v)}(k^2)[(q'_1 \cdot q'_2) - (p'_1 \cdot p'_2)]
$$

$$
+ 2h_2^{(v)}(k^2)\bar{h}_1^{(v)}(k^2)[(q'_1 \cdot q'_2) - (p'_1 \cdot q'_2)] \} c_1 \mathcal{F}_{10}(P' \cdot q', q'^2)
$$

$$
+ \int \frac{idq'}{(2\pi)^4} \left( \frac{g_1^2}{k^2 + m_\omega^2} + \frac{g_2^2}{k^2 + m_\phi^2} \right) \{ 2h_1^{(v)}(k^2)\bar{h}_2^{(v)}(k^2)[q_1'^2 q_2'^2 - q_1'^2 (p'_2 \cdot q'_2)]
$$

$$
+ 2h_2^{(v)}(k^2)\bar{h}_1^{(v)}(k^2)[q_1'^2 q_2'^2 - q_2'^2 (p'_1 \cdot q'_1)] \} c_2 \mathcal{F}_{20}(P' \cdot q', q'^2) \right\}
$$

(20a)
$c_2 F_{20}(P' \cdot p', p^2) =$
\[
\frac{1}{p_1^2 + M_1^2} \frac{1}{-i e p_2^2 + M_2^2} - i e \int \frac{d^4 q'}{(2\pi)^4} \frac{1}{M_1^2 p_2^2} \left( \frac{g_1^2}{k^2 + m_\omega^2} + \frac{g_8^2}{k^2 + m_\phi^2} \right) \]
\[
\times \left\{ h_1^{(v)}(k^2) \bar{h}_2^{(v)}(k^2) [(p_1' \cdot p_2') (q_1' \cdot q_2') - (p_1' \cdot q_2') (q_1 \cdot q_2')] + h_2^{(v)}(k^2) \bar{h}_1^{(v)}(k^2) \right\} \]
\[
\times \left\{ [p_1' \cdot (p_2' + q_2') q_2' \cdot (q_1 - p_1') - (M_1^2 + (p_1' \cdot q_1')) q_2' \cdot (p_2' + q_2')] c_1 F_{10}(P' \cdot q', q^2) \right. \]
\[
+ \int \frac{d^4 q'}{(2\pi)^4} \frac{1}{M_1^2 p_2^2} \left( \frac{g_1^2}{k^2 + m_\omega^2} + \frac{g_8^2}{k^2 + m_\phi^2} \right) \left\{ h_1^{(v)}(k^2) \bar{h}_1^{(v)}(k^2) \right\} \]
\[
\times \left\{ (p_1' + q_1') \cdot (p_2' + q_2') q_2'^2 [M_1^2 + (p_1' \cdot q'_1) - q_1'^2] + h_1^{(v)}(k^2) \bar{h}_1^{(v)}(k^2) [M_1^2 (q_1' \cdot q_2')(p_2' \cdot q_2')] \right. \]
\[
- M_1^2 q_2'^2 (p_2' \cdot q_1') + q_1'^2 q_2'^2 (p_1' \cdot p_2') - q_1'^2 (p_1' \cdot q_1')(p_2' \cdot q_2') + h_2^{(v)}(k^2) \bar{h}_1^{(v)}(k^2) \right\} \]
\[
\times \left\{ q_2'^2 [p_1' \cdot (p_2' + q_2') q_2' \cdot (q_1 - p_1') - (M_1^2 + (p_1' \cdot q_1')) q_2' \cdot (p_2' + q_2')] c_2 F_{20}(P' \cdot q', q^2) \right\} \}.
\]

From this eigenvalue equation, we can obtain the eigenfunctions and eigenvalues including the contribution from the cross terms.

In instantaneous approximation, Eqs. (17) and (18) become the Schrödinger type equations, respectively, (see details in Appendix B)

\[
\left( \frac{b_1^2(M)}{2\mu_R} - \frac{P^2}{2\mu_R} \right) \Psi_1^{0+}(p') = \int \frac{d^3 k}{(2\pi)^3} \psi_1^{0+}(p', k) \psi_1^{0+}(p', k), \tag{21}
\]
\[
\left( \frac{b_2^2(M)}{2\mu_R} - \frac{P^2}{2\mu_R} \right) \Psi_2^{0+}(p') = \int \frac{d^3 k}{(2\pi)^3} \psi_2^{0+}(p', k) \psi_2^{0+}(p', k), \tag{22}
\]
where $\Psi_1^{0+}(p') = \int dp_1 F_1(P' \cdot p', p'^2)$, $\Psi_2^{0+}(p') = \int dp_2 F_2(P' \cdot p', p'^2)$, $\mu_R = E_1 E_2/(E_1 + E_2) = [M^4 - (M_1^2 - M_2^2)^2]/(4M^3)$, $b_2^2(M) = [M^2 - (M_1 + M_2)^2] [M^2 - (M_1 - M_2)^2] / (4M^2)$, and the potentials between $D_s^{+}$ and $D_s^{-}$ up to the second order of the $p'/M_H$ expansion are

\[
V_1^{0+}(p', k) = h_1^{(v)}(k^2) \left( \frac{g_1^2}{k^2 + m_\omega^2} + \frac{g_8^2}{k^2 + m_\phi^2} \right) \bar{h}_1^{(v)}(k^2) \left[ -1 - \frac{4p'^2 + 5k^2}{4E_1 E_2} \right], \tag{23}
\]
\[
V_2^{0+}(p', k) = h_1^{(v)}(k^2) \left( \frac{g_1^2}{k^2 + m_\omega^2} + \frac{g_8^2}{k^2 + m_\phi^2} \right) \bar{h}_1^{(v)}(k^2) \left[ -1 - \frac{2p'^2 + 2k^2}{4M_1^2} - \frac{2p'^2 + 2k^2}{4E_1 E_2} \right]. \tag{24}
\]

The eigenvalue equation (20) becomes

\[
\begin{pmatrix}
\frac{b_1^2(M)}{2\mu_R} - \lambda & H_{12} \\
H_{21} & \frac{b_2^2(M)}{2\mu_R} - \lambda
\end{pmatrix}
\begin{pmatrix}
c_1' \\
c_2'
\end{pmatrix}
= 0, \tag{25}
\]
where the matrix elements are

\[ H_{12} = H_{21} = \int d^3p' \Psi_0^{10}(p')^* \int \frac{d^3k}{(2\pi)^3} \hat{h}_1^{(v)}(k^2) \left( \frac{g_1^2}{k^2 + m_\phi^2} + \frac{g_2^2}{k^2 + m_\phi^2} \right) \hat{h}_1^{(v)}(k^2) \frac{k^2}{E_1 E_2} \Psi_0^{10}(p', k), \]

(26)

and \( \Psi_0^{10} \) and \( \Psi_0^{20} \) are the eigenfunctions corresponding to the lowest energy in Eqs. (21) and (22), respectively; \( b_{10}^2(M)/(2\mu_R) \) and \( b_{20}^2(M)/(2\mu_R) \) are the corresponding eigenvalues.

The wave function of the pure molecule state \( D_s^+D_s^- \) becomes

\[ \Psi_{D_s^+D_s^-}^{10}(p') = c_1' \Psi_0^{10}(p') + c_2' \Psi_0^{20}(p'). \]

(27)

Then we can investigate the alternative \( J^P = 2^+ \) assignment for the pure molecule state \( D_s^+D_s^- \) using the method as above.

IV. FORM FACTORS OF HEAVY VECTOR MESONS

In our previous works [4–7] the form factors of heavy vector meson \( h(k^2) \) describing its internal structure have been calculated, but we did not consider the nonperturbative effects of QCD. In this work, we firstly improve the heavy vector meson BS wave function so that it can include the nonperturbative contribution. The BS amplitude of heavy vector mesons has the form [11]

\[ \Gamma_\mu^V(p; P) = \frac{1}{N^V} \left( \gamma_\mu + \frac{P_\mu \gamma \cdot P}{M_V^2} \right) \varphi_H(p^2), \]

(28)

where \( P \) is the momentum of the heavy vector meson, \( p \) denotes the relative momentum between quark and antiquark in heavy meson, \( N^V \) is the normalization and \( \varphi_H(p^2) = \exp(-p^2/\omega_H^2) \). Because the form factors depend on the momentum of the exchanged-meson only, we set \( P = (0, 0, 0, iM_V) \) in the rest frame to calculate these scalar functions. The authors of Ref. [11] considered the SU(3) symmetry and obtained \( \omega_D = \omega_{D^*} = \omega_{D_s^*} = 1.81 \text{Gev} \). These parameters are fixed by providing fits to the observables. As in heavy-quark effective theory (HQET) [12], we consider that the heaviest quark carries all the heavy-meson momentum. Let \( D_l^* \) denote one of \( D^{*0} \), \( D^{*+} \) and \( D_{s}^{*+} \), and \( l = u, d, s \) represents the \( u, d, s \)-antiquark in the heavy vector meson \( D^{*0} \), \( D^{*+} \) and \( D_{s}^{*+} \), respectively. Then the BS wave function of \( D_l^* \) is obtained

\[ \chi = S_c(p + P) \Gamma_\mu^V(p; P) S_l(p), \]

(29)
where $S_c(p+P)$ is c-quark propagator and $S_l(p)$ is the light quark propagator in constituent quark model.

The operator product expansion (OPE) was introduced to deal with the nonperturbative effects of QCD [13]. Its physical meaning is that the short distance behaviour is determined by the Wilson coefficients and the large distance part is included in the matrix elements of the operators $O_n$ [8]. Applying the fixed-point gauge technique, the authors of Ref. [8] have obtained the massive quark propagators which include the information of condensates

$$S_c(p+P) = -\frac{i\gamma \cdot p + \gamma \cdot P - im_c}{\gamma \cdot (p+P)} + \frac{i}{4}gt^aG_{\kappa\lambda}^a\left[\frac{1}{(p+P)^2 + m_c^2}\right] \sigma_{\kappa\lambda}(\gamma \cdot p + \gamma \cdot P + im_c) + (\gamma \cdot p + \gamma \cdot P + im_c)\sigma_{\kappa\lambda}\{ \sigma_{\kappa\lambda}(\gamma \cdot p + \gamma \cdot P + im_c) + (\gamma \cdot p + \gamma \cdot P + im_c)\sigma_{\kappa\lambda}\} \times \left\{ \sigma_{\kappa\lambda}(\gamma \cdot p + \gamma \cdot P + im_c) + (\gamma \cdot p + \gamma \cdot P + im_c)\sigma_{\kappa\lambda}\right\} \quad (30)$$

and

$$S_l(p) = -\frac{i\gamma \cdot p - im_l}{\gamma \cdot p - im_l} + \frac{i}{4}gt^aG_{\kappa\lambda}^a\left[\frac{1}{(p+P)^2 + m_l^2}\right] \sigma_{\kappa\lambda}(\gamma \cdot p + im_l) + (\gamma \cdot p + im_l)\sigma_{\kappa\lambda}\{ \sigma_{\kappa\lambda}(\gamma \cdot p + im_l) + (\gamma \cdot p + im_l)\sigma_{\kappa\lambda}\} \times \left\{ \sigma_{\kappa\lambda}(\gamma \cdot p + im_l) + (\gamma \cdot p + im_l)\sigma_{\kappa\lambda}\right\} \quad (31)$$

where $m_{c,l}$ are the constituent quark masses, $\alpha_s = g^2/4\pi$ is the QCD coupling constant, $G_{\mu\nu}^a$ is the gluon field tensor, $t^a = \lambda^a/2$ and $\lambda^a$ are the Gell-Mann matrices of the group SU(3), and $\sigma_{\kappa\lambda} = \frac{i}{2}[\gamma_\kappa, \gamma_\lambda]$.

Putting the propagators given by Eqs. (30) and (31) into (29), one can obtain the BS wave function of heavy vector meson which has the contribution from the vacuum condensates. The BS wave function expressed as Eq. (29) is a 4 x 4 matrix which can be written as a combination of 16 linearly independent matrices $1, \gamma_\mu, \sigma_{\mu\nu}, \gamma_\mu \gamma_5, \gamma_5$: [5]

$$\chi = \Psi^S + \Psi^V_\mu \gamma_\mu + \Psi^{T\mu\nu} \sigma_{\mu\nu} + \Psi^{AV}_\mu \gamma_\mu \gamma_5 + \Psi^{Pse}_\mu \gamma_5. \quad (32)$$

The corresponding wave function in instantaneous approximation can be obtained from Eq. (32): the heavy vector meson wave function is a three-vector, its components are [6, 7]

$$\Psi^V_i(p) = \int dp_0 \frac{1}{4} tr\{\gamma_i \chi\} \quad i = 1, 2, 3. \quad (33)$$

To simplify this integral, we consider that the fourth components of momenta in the second and third terms in Eqs. (30) and (31) are equal to zero. Picking up all terms proportional
to $G^2$, we obtain the wave function of $D^*_l$ which contains the nonperturbative contribution from the gluon condensates

$$\Psi_{D^*_l}(p) = \frac{2\pi i}{N}\left\{ \exp \left[ -\frac{2p^2 - M^2_{D^*_l} - m^2_c + 2M_{D^*_l}\omega_c + \Gamma^2/4 + (M_{D^*_l} - \omega_c)i\Gamma}{\omega^2_{D^*_l}} \right] \right. $$

$$\times \left[ \frac{-M_{D^*_l} + \omega_c - \omega_l - i\Gamma/2 + M_{D^*_l} - \omega_c - \omega_l + i\Gamma/2}{(M_{D^*_l} - \omega_c - \omega_l)^2 + \Gamma^2/4} \right] \left[ \frac{p^2}{3} + \omega_c M_{D^*_l} - \omega^2_c + \frac{\Gamma^2}{4} + m_cm_l + i\left( \frac{M_{D^*_l}\Gamma}{2} - \omega_c\Gamma \right) \right] $$

$$- \frac{M_{D^*_l} - \omega_c - \omega_l + i\Gamma/2 + M_{D^*_l} - \omega_c - \omega_l - i\Gamma/2}{(M_{D^*_l} - \omega_c - \omega_l)^2 + \Gamma^2/4} \right] $$

$$\times \left\{ \exp \left[ \pm \sqrt{\frac{\omega^2_{D^*_l}}{\omega^2_{D^*_l}}} \right] \right. $$

$$\times \frac{\sqrt{p^2 + m^2_c} + \sqrt{p^2 + m^2_l}}{(p^2 + m^2_c)(p^2 + m^2_l)} \left\{ \frac{1}{72} \exp \left( \frac{-p^2}{\omega_{D^*_l}} \right) \left( m_cm_l + \frac{p^2}{3} \right) \right\} \left( m_l p^2 - m_c^4 \right)$$

$$- \frac{i}{96} \exp \left[ -\frac{2p^2 - M^2_{D^*_l} - m^2_c + 2M_{D^*_l}\omega_c + \Gamma^2/4 + (M_{D^*_l} - \omega_c)i\Gamma}{\omega^2_{D^*_l}} \right] \left( m_c p^2 - m_l^4 \right) \left( \frac{p^2}{3} \right)$$

$$\left\{ \frac{m_l}{2\omega_c(p^2 + m^2_l)} \right\} \sqrt{5}p, \quad (34)$$

where $M_{D^*_l}$ is the mass of the heavy meson $D^*_l$, $p$ is the unit momentum, $\Gamma$ is the width of resonance, $\omega_{c,l} = \sqrt{p^2 + m^2_{c,l}}$ and $l = u, d, s$. To obtain this wave function, we have used the substitution

$$\langle 0|G^a_{\nu\mu}G^b_{\sigma\rho}|0 \rangle = \frac{1}{96}\delta_{ab}(g_{\nu\rho}g_{\sigma\sigma} - g_{\nu\sigma}g_{\rho\rho})\langle 0|G^a_{\mu\nu}G^a_{\mu\rho}|0 \rangle. \quad (35)$$

Now, we calculate these form factors derived from one light vector meson (also including $\rho^\pm$ and $K^*$) exchange. After exchanging one vector meson, the final particle may not be the initial one, as shown in Fig. 2. The quark current becomes $J_\mu = i\bar{l}\gamma_\mu l$, where $l' = u, d, s$ and $l$ represent the light quarks in final and initial particles, respectively. The matrix element of the quark current $J_\mu$ between the heavy meson states (H) has the form \cite{14, 15}

$$\langle H(Q)|J_\mu(0)|H(P) \rangle = \int \frac{d^3q d^3p}{(2\pi)^6} \Psi^H_Q(q) \Gamma_\mu(p, q) \Psi^H_P(p), \quad (36)$$

where $\Gamma_\mu(p, q)$ is the two-particle vertex function and $\Psi^H_Q$ is the wave function of heavy vector meson projected onto the positive energy states of quarks and boosted to the moving reference frame with momentum Q. Fig. 3 shows the vertex function $\Gamma_\mu(p, q)$ in the impulse approximation. The corresponding vertex function of the quark-meson interaction is given.
FIG. 3: The vertex function $\Gamma$ in the impulse approximation. Diagram (a) represents the light meson interaction with $l$-antiquark in $D_{l}^{*}$ and $l'$-antiquark in $D_{l'}^{*}$. Diagram (b) represents the light meson interaction with $l$-quark in $\bar{D}_{l}^{*}$ and $l'$-quark in $\bar{D}_{l'}^{*}$.

by

$$\Gamma_{\mu}^{(1)}(p, q) = \begin{cases} \bar{v}_{\nu}(q_{1}) i\gamma_{\mu} v_{l}(p_{1})(2\pi)^{3}\delta(q_{2} - p_{2}) \\ \bar{u}_{\nu}(q_{1}) i\gamma_{\mu} u_{l}(p_{1})(2\pi)^{3}\delta(q_{2} - p_{2}) \end{cases},$$

(37)

where $u_{l}(p)$ and $v_{l}(p)$ are the spinors of the quark $l$ and antiquark $\bar{l}$, respectively,

$$u_{\lambda}(p) = \sqrt{\epsilon_{l}(p) + m_{l}} \left( \frac{1}{\epsilon_{l}(p) + m_{l}} \right) \chi_{\lambda}, \quad v_{\lambda}(p) = \sqrt{\epsilon_{l}(p) + m_{l}} \left( \frac{\sigma \cdot p}{\epsilon_{l}(p) + m_{l}} \right) \chi_{\lambda},$$

(38)

with $\epsilon_{l,c}(p) = \sqrt{p^{2} + m_{l,c}^{2}}$ and $\left[14, 15\right]$

$$p_{1,2} = \epsilon_{l,c}(p) \frac{P}{M_{H}} \pm \sum_{i=1}^{3} n^{(i)}(P) p_{i}, \quad M_{H} = \epsilon_{l}(p) + \epsilon_{c}(p),$$

$$q_{1,2} = \epsilon_{l,c}(q) \frac{Q}{M_{H}} \pm \sum_{i=1}^{3} n^{(i)}(Q) q_{i}, \quad M_{H} = \epsilon_{l}(q) + \epsilon_{c}(q),$$

and $n^{(i)}$ are three four-vectors defined by

$$n^{(i)}(P) = \left\{ \delta_{ij} + \frac{P_{i} P_{j}}{M_{H}[E_{H}(P) + M_{H}]}, \quad \frac{P_{i}}{M_{H}} \right\}, \quad E_{H}(P) = \sqrt{P^{2} + M_{H}^{2}}.$$  

The first term in Eq. (37) represents the light vector meson interaction with the $l$-antiquark in $D_{l}^{*}$ and $l'$-antiquark in $D_{l'}^{*}$, while the second term is its interaction with the $l$-quark in $\bar{D}_{l}^{*}$ and $l'$-quark in $\bar{D}_{l'}^{*}$, where $\bar{D}_{l}^{*}$ denotes the anti-particle of $D_{l}^{*}$.  

13
FIG. 4: The form factor for the vertex of heavy vector meson \(D_s^*\) coupling to light vector \((V_1\) and \(V_8\)) meson. The solid line represents the form factor including the contribution from gluon condensates; and the dashed line represents the form factor without this contribution.

Substituting the vertex function \(\Gamma^{(1)}_\mu\) given by Eq. (37) into the matrix element (36) and comparing the resulting expressions with the form factor decompositions (13) and (14), we obtain

\[
\begin{align*}
\bar{h}_1^{(v)}(k^2) = & h_2^{(v)}(k^2) = \bar{h}_1^{(v)}(k^2) = \bar{h}_2^{(v)}(k^2) = F_2^{\mu}(k^2), \quad h_3^{(v)}(k^2) = \bar{h}_3^{(v)}(k^2) = 0, \\
F_2^{\mu}(k^2) = & \left( g_1^2 \left( \frac{g_1^2}{k^2 + m_\omega^2} + \frac{g_8^2}{k^2 + m_\phi^2} \right) F_2^{ss}(k^2) \right) \left[ 1 + \frac{4p^2 + 5k^2}{4E_1E_2} \right], \\
V_1^{0+}(p', k) = & -F_2^{ss}(k^2) \left( \frac{g_1^2}{k^2 + m_\omega^2} + \frac{g_8^2}{k^2 + m_\phi^2} \right) F_2^{ss}(k^2) \left[ 1 + \frac{2p^2 + 2k^2}{4M_1^2} + \frac{2p^2 + 2k^2}{4E_1E_2} \right].
\end{align*}
\]

where \(\Psi_{D_i}(p)\) is the heavy vector meson wave function expressed as Eq. (34). In Fig. 4 we give the form factor \(F_2^{ss}(k^2)\) of heavy vector meson \(D_s^*\) corresponding to one light vector meson \((V_1\) and \(V_8\)) exchange and compare it with the form factor without the contribution from gluon condensates.

Finally, the potentials given by Eqs. (23) and (24) become

\[
\begin{align*}
V_1^{0+}(p', k) = & -F_2^{ss}(k^2) \left( \frac{g_1^2}{k^2 + m_\omega^2} + \frac{g_8^2}{k^2 + m_\phi^2} \right) F_2^{ss}(k^2) \left[ 1 + \frac{4p^2 + 5k^2}{4E_1E_2} \right], \\
V_2^{0+}(p', k) = & -F_2^{ss}(k^2) \left( \frac{g_1^2}{k^2 + m_\omega^2} + \frac{g_8^2}{k^2 + m_\phi^2} \right) F_2^{ss}(k^2) \left[ 1 + \frac{2p^2 + 2k^2}{4M_1^2} + \frac{2p^2 + 2k^2}{4E_1E_2} \right].
\end{align*}
\]

(40)
V. NUMERICAL RESULT

The constituent quark masses \( m_c = 1.55 \text{GeV}, m_u = m_d = 0.33 \text{GeV}, m_s = 0.5 \text{GeV} \), the meson masses \( m_\omega = 0.782 \text{GeV}, m_\rho = m_{\rho^\pm} = 0.775 \text{GeV}, m_{K^0} = m_{K^{*0}} = 0.896 \text{GeV}, m_{K^{*+}} = m_{K^{*-}} = 0.892 \text{GeV}, m_\phi = 1.019 \text{GeV}, m_{D_s^{*+}} = m_{D_s^{*-}} = 2.112 \text{GeV} \), the width of the heavy vector meson \( \Gamma_{D_s^*} = 0.0015 \text{GeV} \) \cite{16} and the gluon condensate \( \langle 0 | \bar{\alpha} G^{a}_{\mu\nu} G^{a}_{\mu\nu} | 0 \rangle = (0.36 \text{GeV})^4 \) \cite{8}. With these potentials expressed as Eq. (40), we can numerically solve the equations (21) and (22), respectively. Subsequently, the eigenvalue equation (25) can be solved. In this approach, we can investigate the molecular state \( D_s^{*+} D_s^{*-} \) with \( J^P = 2^+ \). The obtained ground-state masses of the pure molecule state \( D_s^{*+} D_s^{*-} \) with \( J^P = 0^+, 2^+ \) are presented in Table \[\text{I}\]. In Ref. [4], we have considered one light meson (\( \sigma, \rho^0, V_1, V_8 \)) exchange and obtained the masses and wave functions of pure molecule states \( D_s^{*0} \bar{D}_s^{*0} \) and \( D_s^{*+} D_s^{*-} \) collected in Table \[\text{I}\].

| Composite State | State \( J^P \) | This work |
|-----------------|-----------------|-----------|
| \( D_s^{*0} \bar{D}_s^{*0} \) | \( 0^+ \) | 3.948 |
| | \( 2^+ \) | 4.147 |
| \( D_s^{*+} D_s^{*-} \) | \( 0^+ \) | 3.953 |
| | \( 2^+ \) | 4.153 |
| \( D_s^{*0} \bar{D}_s^{*0} \) | \( 0^+ \) | 4.184 |
| | \( 2^+ \) | 4.341 |

The meson-quark coupling constants \( g_\omega, g_\rho, g_\phi, g_{K^*} \) and the parameters \( \omega_H \) in the BS amplitude of heavy vector mesons are fixed by providing fits to observables, so there is not an adjustable parameter in our approach. Simultaneously varying the constituent quark masses \( m_s, m_c \), and the full width of the heavy vector meson \( \Gamma_{D_s^*} \) within 5\%, we find that the ratio of the numerical result difference dependent on these parameters and the binding energy is at most 5\%. Thus in our approach the calculated masses of pure molecule state depend on these parameters, but not sensitively.

For the pure molecule states \( D_s^{*0} \bar{D}_s^{*0}, D_s^{*+} D_s^{*-} \) and \( D_s^{*0} \bar{D}_s^{*0}, D_s^{*+} D_s^{*-} \), we find that the resulting ground states with \( 0^+ \) lie slightly below and the \( 2^+ \) states above the threshold and then
where \( \Psi_D \) consider that the \( Y(4140) \) is a mixed state of these three ground states with \( J^P = 0^+ \). From Eq. (10), we can obtain the wave function of the mixed state in instantaneous approximation

\[
\Psi = a'_1 \Psi_{D^*0D^*0} + a'_2 \Psi_{D^+D'^*} + a'_3 \Psi_{D^*_sD^*_s},
\]

where \( \Psi_{D^*_sD^*_s} \) is the ground-state wave function of pure molecule state \( D^*_sD^*_s \) given in Eq. (27), \( \Psi_{D^0D^*0} \) and \( \Psi_{D^+D'^*} \) are the ground-state wave functions of pure molecule states \( D^0D^0 \) and \( D^+D'^* \), respectively. Then the coupled equation can be obtained

\[
\begin{pmatrix}
\frac{\delta^2(M_{D^0D^0})}{2\mu R} & A_{12} & A_{13} \\
A_{21} & \frac{\delta^2(M_{D^+D'^*})}{2\mu R} & A_{23} \\
A_{31} & A_{32} & \frac{\delta^2(M_{D^*_sD^*_s})}{2\mu R}
\end{pmatrix}
\begin{pmatrix}
a'_1 \\
a'_2 \\
a'_3
\end{pmatrix}
= E
\begin{pmatrix}
a'_1 \\
a'_2 \\
a'_3
\end{pmatrix},
\]

where \( M_{D^0D^0}, M_{D^+D'^*} \) and \( M_{D^*_sD^*_s} \) are the ground-state masses for the pure molecule states with \( J^P = 0^+ \) given in Table I. \( A_{12}, A_{13}, A_{21}, A_{23}, A_{31} \) and \( A_{32} \) are the matrix elements between two pure molecule states corresponding to the graphs (b), (c), (d), (f), (g), (h) in Fig. 2 respectively. From the interaction kernel derived from one-\( \rho^\pm \) exchange given in Eq. (16), we obtain these matrix elements at leading order in \( p'/M_H \)

\[
A_{ij} = \int d^3 p' \Psi_{D^*_iD^*_j}(p') \int \frac{d^3k}{(2\pi)^3} F_2^{ii'}(k^2) \frac{g_\rho^2}{k^2 + m_\rho^2} F_2^{ii'}(k^2) \Psi_{D^*_iD^*_j}(p', k),
\]

where we have for \( i = 1, j = 2, \rho = \rho^+, l' = d \) and \( l = u \); for \( i = 2, j = 1, \rho = \rho^-, l' = u \) and \( l = d \). And for one-\( K^* \) exchange, the matrix elements become

\[
A_{ij} = \int d^3 p' \Psi_{D^*_iD^*_j}(p') \int \frac{d^3k}{(2\pi)^3} F_2^{ii'}(k^2) \frac{g_{K^*}}{k^2 + m_{K^*}^2} F_2^{ii'}(k^2) \Psi_{D^*_iD^*_j}(p', k),
\]

where we have for \( i = 1, j = 3, K^* = K^{*+}, l' = s \) and \( l = u \); for \( i = 2, j = 3, K^* = K^{*0}, l' = s \) and \( l = d \); for \( i = 3, j = 1, K^* = K^{*-}, l' = u \) and \( l = s \); for \( i = 3, j = 2, K^* = K^{*0}, l' = d \) and \( l = s \). In Eqs. (43) and (44), the high order terms of \( p'/M_H \) are not considered for the heavy meson masses are large.

Solving the equation (42), we obtain three eigenstates and the masses and channel probabilities for these eigenstates are presented in Table III. The resulting highest energy of these eigenstates is in good agreement with the experimental mass of the \( Y(4140) \) state, while the mass of \( Y(4140) \) is measured to be 4.143GeV [1]. Then we consider that this eigenstate represents the \( Y(4140) \) state and the component \( D^*_sD^*_s \) clearly dominates with a 80% probability in this mixed state. For two other eigenstates, the component \( D^*_sD^*_s \) is
TABLE II: Masses and channel probabilities of mixed states.

| Mass (GeV) | $D^{*0}D^{*0}$ | $D^{*+}D^{*-}$ | $D_s^{*+}D_s^{*-}$ |
|------------|----------------|----------------|-------------------|
| 3.945      | 45%            | 47%            | 8%                |
| 3.960      | 51%            | 48.9%          | 0.1%              |
| 4.146      | 10%            | 10%            | 80%               |

less than 10% probability and we consider that these two mixed states approximately belong to an isospin doublet corresponding the exotic state $Y(3940)$, while the mass of $Y(3940)$ is 3.943GeV in experiment [17]. The interpretation for the $Y(3940)$ state is different from the one in Ref. [4], this is because the coupled channel has not been considered in Ref. [4].

VI. CONCLUSION

For the SU(3) symmetry, the exotic state $Y(4140)$ is considered as a mixed state of three pure molecule states $D^{*0}D^*$, $D^{*+}D^{*-}$ and $D_s^{*+}D_s^{*-}$. Applying the general formalism of the BS wave functions for the bound states consisting of two vector fields, we investigate these pure molecule states. In this work, we introduce the gluon condensates into the BS wave function of the heavy meson and obtain the heavy meson form factors and the interaction between two heavy mesons including the contribution from the nonperturbative effects of QCD, which is different from our previous works. Then using the coupled-channel approach, we obtain the masses and wave functions for the mixed states of these pure molecule states with $J^P = 0^+$, which are in good agreement with experimental masses of the $Y(3940)$ and $Y(4140)$ states. Thus we can conclude that a mixing of pure molecule states should be a more credible candidate to explain the $Y(4140)$ state.

Appendix A: The tensor structures in the general form of the BS wave functions

The tensor structures in Eqs. (1), (2), (3), (4) are given below [4]

\[ T^1_{\lambda\tau} = (\eta_1\eta_2 P^\mu p^\nu - \eta_1 P^\mu p^\nu + \eta_2 P^\mu p^\nu + P^\mu p^\nu - p^\nu) g_{\lambda\tau} - (\eta_1\eta_2 P^\mu P^\nu + \eta_2 P^\mu P^\nu - \eta_1 P^\mu P^\nu - p^\mu p^\nu), \]
\[ T_{\lambda\tau}^2 = (\eta_1^2 P'^2 + 2\eta_1 P' \cdot p' + p'^2)(\eta_2^2 P'^2 - 2\eta_2 P' \cdot p' + p'^2)g_{\lambda\tau} \]
\[ + (\eta_1 \eta_2 P'^2 - \eta_1 P' \cdot p' + \eta_2 P' \cdot p' - p'^2)(\eta_1 \eta_2 P'_\lambda P'_\tau - \eta_1 P'_\lambda p'_\tau + \eta_2 P'_\lambda p'_\tau - p'_\lambda p'_\tau) \]
\[ - (\eta_1 \eta_2 P'^2 - 2\eta_1 P' \cdot p' + p'^2)(\eta_1^2 P'_\lambda P'_\tau + \eta_1 P'_\lambda p'_\tau + \eta_2 P'_\lambda p'_\tau + p'_\lambda p'_\tau) \]
\[ - (\eta_1 \eta_2 P'^2 + 2\eta_1 P' \cdot p' + p'^2)(\eta_2^2 P'_\lambda P'_\tau - \eta_2 P'_\lambda p'_\tau - \eta_2 P'_\lambda p'_\tau + p'_\lambda p'_\tau), \]
\[ T_{\lambda\tau}^3 = \frac{1}{j} p'_j \cdots p'_j g_{\mu_1:\lambda}\lambda (\eta_2^2 P'^2 + 2\eta_2 P' \cdot p' + p'^2)[(\eta_2^2 P'^2 - 2\eta_2 P' \cdot p' + p'^2)(\eta_1 P' + p')_\tau \]
\[ - (\eta_1 \eta_2 P'^2 - \eta_1 P' \cdot p' + \eta_2 P' \cdot p' - p'^2)(\eta_2 P' + p')_\lambda ] \]
\[ - p'_j \cdots p'_j [ (\eta_2^2 P'^2 + 2\eta_2 P' \cdot p' + p'^2)(\eta_2^2 P'_\lambda P'_\tau - \eta_2 P'_\lambda p'_\tau - \eta_2 P'_\lambda p'_\tau + p'_\lambda p'_\tau) \]
\[ - (\eta_1 \eta_2 P'^2 - \eta_1 P' \cdot p' + \eta_2 P' \cdot p' - p'^2)(\eta_1 \eta_2 P'_\lambda P'_\tau - \eta_1 P'_\lambda p'_\tau + \eta_2 P'_\lambda p'_\tau - p'_\lambda p'_\tau) ], \]
\[ T_{\lambda\tau}^4 = \frac{1}{j} p'_j \cdots p'_j g_{\mu_1:\lambda}\lambda (\eta_2^2 P'^2 - 2\eta_2 P' \cdot p' + p'^2)[(\eta_1 \eta_2 P'^2 - \eta_1 P' \cdot p' \]
\[ + \eta_2 P' \cdot p' - p'^2)(\eta_1 P' + p')_\lambda - (\eta_2^2 P'^2 + 2\eta_2 P' \cdot p' + p'^2)(\eta_2 P' - p')_\lambda \]
\[ - p'_j \cdots p'_j [ (\eta_1 \eta_2 P'^2 + 2\eta_1 P' \cdot p' + p'^2)(\eta_2^2 P'_\lambda P'_\tau - \eta_2 P'_\lambda p'_\tau - \eta_2 P'_\lambda p'_\tau + p'_\lambda p'_\tau) \]
\[ - (\eta_1 \eta_2 P'^2 - \eta_1 P' \cdot p' + \eta_2 P' \cdot p' - p'^2)(\eta_1 \eta_2 P'_\lambda P'_\tau - \eta_1 P'_\lambda p'_\tau + \eta_2 P'_\lambda p'_\tau - p'_\lambda p'_\tau) ], \]
\[ T_{\lambda\tau}^5 = (\eta_2 P' \cdot p' - \eta_1 P' \cdot p' - 2P'^2)p'_{(\mu_2 \cdots \mu_j \epsilon_{\mu_1})\lambda \tau \xi p'_\xi} \]
\[ + (2\eta_1 \eta_2 P' \cdot p' + \eta_2 p'^2 - \eta_1 p'^2)p'_{(\mu_2 \cdots \mu_j \epsilon_{\mu_1})\lambda \tau \xi P'_\xi} \]
\[ + p'_{(\mu_2 \cdots \mu_j \epsilon_{\mu_1})\lambda \tau \xi p'_\xi P'_\xi} + p'_{(\mu_2 \cdots \mu_j \epsilon_{\mu_1})\tau \xi \xi P'_\xi P'_\xi}, \]
\[ T_{\lambda\tau}^6 = (P' \cdot p')p'_{(\mu_2 \cdots \mu_j \epsilon_{\mu_1})\lambda \tau \xi p'_\xi} - p'^2 p'_{(\mu_2 \cdots \mu_j \epsilon_{\mu_1})\lambda \tau \xi P'_\xi} \]
\[ + p'_{(\mu_2 \cdots \mu_j \epsilon_{\mu_1})\lambda \tau \xi p'_\xi P'_\xi} - p'_{(\mu_2 \cdots \mu_j \epsilon_{\mu_1})\tau \xi \xi P'_\xi P'_\xi}, \]
\[ T_{\lambda\tau}^7 = (\eta_1 \eta_2 P'^2 - 2\eta_1 P' \cdot p' - 2P' \cdot p')p'_{(\mu_2 \cdots \mu_j \epsilon_{\mu_1})\lambda \tau \xi p'_\xi} \]
\[ + (2\eta_1 \eta_2 P'^2 + \eta_2 P' \cdot p' - \eta_1 P' \cdot p')p'_{(\mu_2 \cdots \mu_j \epsilon_{\mu_1})\lambda \tau \xi P'_\xi} \]
\[ + p'_{(\mu_2 \cdots \mu_j \epsilon_{\mu_1})\lambda \tau \xi p'_\xi P'_\xi} + p'_{(\mu_2 \cdots \mu_j \epsilon_{\mu_1})\tau \xi \xi P'_\xi P'_\xi}, \]
\[ T_{\lambda\tau}^8 = P'^2 p'_{(\mu_2 \cdots \mu_j \epsilon_{\mu_1})\lambda \tau \xi p'_\xi} - (P' \cdot p')p'_{(\mu_2 \cdots \mu_j \epsilon_{\mu_1})\lambda \tau \xi P'_\xi} \]
\[ + p'_{(\mu_2 \cdots \mu_j \epsilon_{\mu_1})\lambda \tau \xi p'_\xi P'_\xi} - p'_{(\mu_2 \cdots \mu_j \epsilon_{\mu_1})\tau \xi \xi P'_\xi P'_\xi}. \]

Appendix B: The instantaneous approximation

Next, we give the details of the instantaneous approximation. This approach has been used in Ref. [4]. Comparing the terms \((\eta_1 \eta_2 P'_\lambda P'_\tau + \eta_2 P'_\lambda p'_\tau - \eta_1 P'_\lambda P'_\tau - p'_\lambda p'_\tau)\) in both sides
of Eq. (17), we obtain

\[ F_1(P' \cdot p', p^2) = \frac{1}{p'^2 + M_1^2 - i\epsilon p^2} + \frac{1}{p'^2 + M_2^2 - i\epsilon} \int \frac{id^4q'}{(2\pi)^4} V_1(p', q'; P') F_1(P' \cdot q', q^2), \quad (B1) \]

where \( V_1(p', q'; P') \) contains all coefficients of the term \( (\eta_1\eta_2 P'_1 P'_2 + \eta_2 P'_2 p'_r - \eta_1 p'_1 P'_2 - p'_\lambda p'_\tau) \) in the right side of Eq. (17). In this paper, we set \( k = (k, 0) \) and then \( p'_{10} = q'_{10} = E_1(p'_1) = E_1(q'_1), p'_{20} = q'_{20} = E_2(p'_2) = E_2(q'_2). \) To simplify the potential, we replace the heavy meson energies \( E_1(p'_1) = E_1(q'_1) \rightarrow E_1 = (M^2 - M_1^2 + M_2^2)/(2M), E_2(p'_2) = E_2(q'_2) \rightarrow E_2 = (M^2 - M_1^2 + M_2^2)/(2M). \) The potential depends on the three-vector momentum \( V(p', q'; P') \Rightarrow V(p', q', M). \) Integrating both sides of Eq. (B1) over \( p_0' \) and multiplying by \((M + \omega_1 + \omega_2)(M^2 - (\omega_1 - \omega_2)^2), \) we obtain

\[ \left( \frac{b_2^2(M)}{2\mu_R} - \frac{p'^2}{2\mu_R} \right) \Psi_{1+}^0(p') = \int \frac{d^3k}{(2\pi)^3} V_1^{0+}(p', k) \Psi_{1+}^0(p', k) \]

and the potential between \( D_{s+} \) and \( D_{s-} \) up to the second order of the \( p'/M_H \) expansion

\[ V_1^{0+}(p', k) = h_1^{(v)}(k^2) \left( \frac{g_1^2}{k^2 + m_\omega^2} + \frac{g_2^2}{k^2 + m_\phi^2} \right) \bar{h}_1^{(v)}(k^2) \left[ -1 - \frac{4p'^2 + 5k^2}{4E_1 E_2} \right], \]

where \( \Psi_{1+}^0(p') = \int dp_0 F_1(p' \cdot p', p^2), \mu_R = E_1 E_2/(E_1 + E_2) = [M^4 - (M_1^2 - M_2^2)^2] / (4M^3), \]
\( b^2(M) = [M^2 - (M_1 + M_2)^2][M^2 - (M_1 - M_2)^2] / (4M^2), \)
\( \omega_1 = \sqrt{p'^2 + M_1^2} \) and \( \omega_2 = \sqrt{p'^2 + M_2^2}. \) And comparing the terms \( (\eta_1^2 P'_1 P'_2 + \eta_1 P'_2 p'_\tau + \eta_1 p'_1 P'_2 + p'_\lambda p'_\tau) \) in both sides of Eq. (18), we obtain

\[ p'^2 F_2(p' \cdot p', p^2) = \frac{1}{p'^2 + M_1^2 - i\epsilon p^2} + \frac{1}{p'^2 + M_2^2 - i\epsilon} \int \frac{id^4q'}{(2\pi)^4} V_2(p', q'; P') q^2 F_2(p' \cdot q', q^2). \]

Setting \( \Psi_{2+}^0(p') = \int dp_0 p'^2 F_2(p' \cdot p', p^2), \) we obtain the equation of Schrödinger type

\[ \left( \frac{b_2^2(M)}{2\mu_R} - \frac{p'^2}{2\mu_R} \right) \Psi_{2+}^0(p') = \int \frac{d^3k}{(2\pi)^3} V_2^{0+}(p', k) \Psi_{2+}^0(p', k) \]

and the potential between \( D_{s+} \) and \( D_{s-} \) up to the second order of the \( p'/M_H \) expansion

\[ V_2^{0+}(p', k) = h_1^{(v)}(k^2) \left( \frac{g_1^2}{k^2 + m_\omega^2} + \frac{g_2^2}{k^2 + m_\phi^2} \right) \bar{h}_1^{(v)}(k^2) \left[ -1 - \frac{2p'^2 + 2k^2}{4M_1^2} + \frac{2p'^2 + 2k^2}{4E_1 E_2} \right]. \]

Then the eigenfunctions in Eqs. (17) and (18) can be calculated in instantaneous approximation and we obtain the eigenfunctions \( \Psi_{10}^{0+} \) and \( \Psi_{20}^{0+} \) corresponding to the lowest energy in Eqs. (21) and (22), respectively; \( b_{10}^2(M)/(2\mu_R) \) and \( b_{20}^2(M)/(2\mu_R) \) are the corresponding eigenvalues. After integrating both sides of Eqs. (20a) and (20b) over \( p_0', \) we multiply the
resulting expressions of (20a) and (20b) from the left by the eigenfunctions $\Psi_{10}^0$ and $\Psi_{20}^0$, respectively, and then integrate both sides over the relative momentum $p'$. The eigenvalue equation (20) becomes

\[
\begin{pmatrix}
\frac{b_{10}(M)}{2\mu_R} - \lambda & H_{12} \\
H_{21} & \frac{b_{20}(M)}{2\mu_R} - \lambda
\end{pmatrix}
\begin{pmatrix}
c_1' \\
c_2'
\end{pmatrix} = 0,
\]

where the matrix elements are

\[
H_{12} = H_{21} = \int d^3p'\Psi_{10}^0(p')^* \int d^3k\tilde{h}_1^{(v)}(k^2)\left(\frac{g_1^2}{k^2 + m_\omega^2} + \frac{g_5^2}{k^2 + m_\phi^2}\right)\tilde{h}_1^{(v)}(k^2)\frac{k^2}{E_1E_2}\Psi_{20}^0(p', k).
\]

[1] T. Aaltonen et al., *Evidence for a narrow near-threshold structure in the J/ψφ mass spectrum in B⁺ → J/ψφK⁺ decays*, Phys. Rev. Lett. 102, 242002 (2009). [arXiv:0903.2229]

[2] X. Liu and S. L. Zhu, *Y(4143) is probably a molecular partner of Y(3930)*, Phys. Rev. D 80, 017502 (2009). [arXiv:0903.2529]

[3] Z. G. Wang, Y. F Tian, *Tetraquark state candidates: Y(4140), Y(4274) and X(4350)*, Int. J. Mod. Phys. A 30, 1550004 (2015). [arXiv:1502.04619]

[4] X. Chen and X. Lü, *Mass of Y(3940) in Bethe-Salpeter equation for quarks*, Eur. Phys. J. C 75:98 (2015).

[5] X. Chen, B. Wang, X. Li, X. Zeng, S. Yu and X. Lü, *Mass of X(3872) in the relativistic quark model*, Phys. Rev. D 79, 114006 (2009).

[6] X. Chen, R. Liu, R. Shi, Y. Yin, Z. Shi, A. Yang and X. Lü, *Mass of X(3872) in Bethe-Salpeter equation for quarks*, Commun. Theor. Phys. 57, 833 (2012).

[7] X. Chen, R. Liu, R. Shi and X. Lü, *Bethe-Salpeter wave functions for the bound states composed of two vector fields of arbitrary spin and their application*, Phys. Rev. D 87, 065013 (2013).

[8] L. Reinders, H. Rubinstein and S. Yazaki, *Hadron properties from QCD sum rules*, Phys. Rep. 127, 1 (1985).

[9] F. Ambrosino et al., *A global fit to determine the pseudoscalar mixing angle and the gluonium content of the η’ meson*, JHEP 0907 105 (2009). [arXiv:0906.3819]

[10] L. Reinders and H. Rubinstein, *QCD and the strange quark parameters*, Phys. Lett. B 145, 108 (1984).
[11] M. A. Ivanov, Y. L. Kalinovsky and C. D. Roberts, *Survey of heavy-meson observables*, Phys. Rev. D 60, 034018 (1999). [arXiv:nucl-th/9812063]

[12] M. Neubert, *Heavy-quark symmetry*, Phys. Rep. 245, 259 (1994).

[13] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, *QCD and resonance physics. Theoretical foundations*, Nucl. Phys. B 147, 385 (1979).

[14] R. N. Faustov, *Relativistic wavefunction and form factors of the bound system*, Ann. Phys. (N.Y.) 78, 176 (1973).

[15] R. N. Faustov, *Magnetic moment of the relativistic composite system*, Nuovo Cimento A 69, 37 (1970).

[16] K. A. Olive et al. (Particle Data Group), *Review of particle physics*, Chin. Phys. C 38, 090001 (2014).

[17] S.-K. Choi et al., *Observation of a near-threshold \( \omega J/\psi \) mass enhancement in exclusive \( B \to K\omega J/\psi \) decays*, Phys. Rev. Lett. 94, 182002 (2005). [arXiv:hep-ex/0408126]