Phase diagram of dense quark matter in QCD-like theories

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Abstract. I report the results of a series of works on the phase diagram of theories with a different number of colors and/or quarks in a different representation than in QCD. Similarities as well as differences as compared to the real world are pointed out, focusing in particular on the interplay of confinement and chiral symmetry breaking. It will be argued that recent lattice data may provide us with a clue to understand deconfinement in cold dense quark matter.

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MOTIVATION

Our knowledge about the region of the phase diagram of quantum chromodynamics (QCD) at low temperatures and high densities is still rather rudimentary. The reasons are twofold. First, standard lattice Monte Carlo techniques suffer from the formidable sign problem at high densities. Second, available experimental data from the astrophysics of compact stars are not constraining enough so far.

In this situation, it is worthwhile to investigate theories which do not describe the real world, but still may give us some hint at the nonperturbative physics of QCD at moderate densities. I will discuss a class of theories (hereafter referred to as QCD-like) with quarks in a different representation of the (possibly also different) color group with the common property that this representation is (pseudo)real. Later, I will focus on two specific examples, namely two-color QCD with fundamental quarks and (any-color) QCD with adjoint quarks [1]. However, the most striking features of these theories follow directly from the reality of the quark representation.

The very first of them, which forms the basic motivation for their study, is the fact that the QCD-like theories do not suffer from the sign problem at nonzero baryon chemical potential. This makes lattice simulations at high density possible, and can thus provide the much needed model-independent input on the equation of state of cold dense quark matter.

Second, baryons in QCD-like theories are bosons as one can combine two quarks to make a color singlet. This means that the low-density matter looks very much different than in the real world. Indeed, finite density is accomplished by a Bose–Einstein condensate (BEC) of bosonic baryons (diquarks) rather than by the Fermi sea of nucleons. On the other hand, this brings important technical advantages. One does not have to deal with three-body physics at low baryon density. Also, at high density where quark matter is expected to deconfine, Cooper pairing of quarks results in a gauge-invariant order parameter, making dense matter a quark superfluid rather than a superconductor. Due to these reasons, one has a decent chance to describe both low- and high-density matter in QCD-like theories within a single theoretical (model) framework. In the real world, this is something that nuclear astrophysics can only dream of.

TWO-COLOR QCD

In this section, I will discuss QCD with quarks in the fundamental representation of the color SU(2) group. However, most of the conclusions hold without change for quarks in any pseudoreal representation. (This class of theories was dubbed type-II in [2].) In this case, the wave function of a color-singlet diquark is antisymmetric in color. Assuming spin-zero pairing which gives the largest energy gain, the Pauli principle implies that the wave function must also be antisymmetric in flavor. The baryon is therefore composed of two quarks of different flavors.

Thanks to the (pseudo)reality of the quark representation, the quark field has the same color transformation properties as its charge conjugate. It is then advantageous to trade the right-handed component of the quark (Dirac) spinor for the charge-conjugated left-handed quark. Instead of $N_f$ flavors of Dirac fermions one then in effect deals with $2N_f$ flavors of Weyl fermions. Consequently, the global flavor symmetry of the QCD-like theory in the chiral limit is $SU(2N_f)$ rather than the usual

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1 In case of pseudoreal quarks an even number of flavors is necessary to avoid the sign problem. With an odd number of flavors, the determinant of the Dirac operator is real, but may be negative.
FIGURE 1. Phase diagram of two-color QCD with two light quark flavors. The labels distinguish various regions with qualitatively different physical behavior. In “χSB” only the chiral condensate is nonzero; this is analogous to the hadronic phase of QCD. “BEC” labels a phase in which the system behaves as a Bose–Einstein condensed gas of tightly bound diquarks. In “BCS”, the physics is dominated by a Fermi sea of quarks, slightly distorted by loose quark pairing. See the text for explanation of the lines.

SU(N)_{L} \times SU(N)_{R} \times U(1)_{B}. Note that the baryon number U(1)_{B} is embedded in the simple group SU(2N_{f}).

The extended symmetry of course affects the physical spectrum of the theory. Put in simple terms, the invariance under the exchange of right-handed quarks and left-handed antiquarks implies that multiplets in the spectrum contain states with different baryon number. In particular, we will find mesons and baryons (diquarks) in the same multiplet. The ground state of two-color QCD exhibits, very much like real QCD, a chiral condensate which breaks the flavor symmetry as SU(2N_{f}) \rightarrow Sp(2N_{f}). There are 2N_{f}^{2} - N_{f} - 1 Nambu–Goldstone (NG) bosons. In the case N_{f} = 2, to which I will from now on restrict, this means altogether five NG bosons. Three of them form the usual isovector of pions, while the remaining two are a diquark and an antidiquark.

Even when the flavor symmetry is broken explicitly by (small) quark masses, there will still be light pseudo-NG bosons. The fact that some of them carry baryon number has far-reaching consequences: it puts the finite-density part of the phase diagram in reach of chiral perturbation theory [1]. The result of a model calculation of the phase diagram of two-color QCD with two light quark flavors [3] is shown in Fig. 1. Chiral perturbation theory successfully describes the “χPT” and “BEC” phases as well as the second-order phase transition (solid line) separating them. As the density increases, the diquarks get closely packed and the relevant degrees of freedom become the quarks themselves. The superfluid ground state is then well captured by the Bardeen–Cooper–Schrieffer (BCS) pairing of quarks around the Fermi sea. The position of the smooth (BEC–BCS) crossover between the two regimes can be indicated for example by a change of sign of the (nonrelativistic) quark chemical potential and is shown in Fig. 1 by the dash-dotted line.

In order to describe the BCS regime, one needs a model with quark degrees of freedom such as that of Nambu and Jona-Lasinio (NJL) [4, 5]. In a version of the model augmented with the Polyakov loop [3], one can simultaneously study the deconfinement phase transition. In presence of dynamical quarks, this becomes a smooth crossover, and is indicated in Fig. 1 by the dashed line (defined as a point at which the expectation value of the Polyakov loop is 0.5). We observe that the deconfinement temperature is essentially insensitive to the chemical potential. Even though this is an obvious artifact of the PNJL model, it can still be close to the actual behavior of two-color QCD, as supported by recent lattice simulations [6].

More difficult to understand seems the thermodynamic behavior of two-color matter around the second-order BEC phase transition. Lattice data for pressure, baryon density and energy density normalized to the (free quark gas) Stefan–Boltzmann limits indicate peaks in all three quantities around the BEC transition. This itself is qualitatively easy to understand already in the chiral perturbation theory. However, the peak in the energy density turns out to be an order of magnitude higher than in the other two observables. This behavior is hard to reproduce in any model with just quark or diquark degrees of freedom based on the global flavor symmetry [7].

ADJOINT QCD

In this section, I will use QCD with adjoint quarks (of two or three colors) as a specific example. However, most of what follows holds for quarks in any real representation. (This class of theories was called type-I in [2].) The important distinguishing property of this class of theories is that the center symmetry associated with the deconfinement transition is not broken by the presence of dynamical quarks. As a consequence, in the chiral limit there are still two sharp phase transitions and it is a well defined question whether these transitions coincide or not (unlike real QCD where it is somewhat moot).

As a matter of fact, it has been known for a decade that in adjoint (three-color) QCD, the chiral restoration temperature is much higher than that of deconfinement [8, 9]. (The ratio of the temperatures is about 7.8.) Unlike the case of type-II theories, here the diquark wave function is symmetric in color. This in turn changes the
OUTLOOK AND CHALLENGES

The feasibility of lattice simulations at high density makes the QCD-like theories very interesting toy models for understanding nonperturbative QCD physics in cold dense matter. At the moment, there are two main issues to be properly understood. First is the peculiar thermodynamic behavior around the BEC transition [6]. While this certainly does not describe the real world, it is important for establishing a reliable overall agreement between the results from analytic calculations and lattice simulations. Second, recent lattice data hint at the possibility of a deconfinement transition at low temperature and high chemical potential. Understanding this behavior would provide a much needed input for the construction of the equation of state of dense quark matter. It is therefore certainly worth further investigation.

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