Phases of Braneworlds, Spinning D3-branes and
Strongly-Coupled Gauge Theories

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ABSTRACT

A spinning nonextremal D3-brane undergoes a phase transition to a naked singularity which, from the braneworld point of view, corresponds to the apparent graviton speed passing from subluminal to superluminal. We investigate this phase transition from the dual perspectives of braneworld scenarios and holography. We discuss the relevance of the thermodynamic stability domains of a spinning D3-brane to the physics of braneworld scenarios. We also describe various gravitational Lorentz violations which arise from static D3-branes.
1 Introduction

Many of the most interesting solutions of General Relativity contain spacetime singularities, such as black holes and cosmological solutions. Brane solutions of M-theory contain classically singular spacetimes, many of which are classically exact solutions. It is believed that, close to singularities, classical gravity no longer suffices. This remains persistent motivation in the search for a quantum theory of gravitation. A key question is: do spacetime singularities even exist at all within the quantum gravitational regime of M-theory (see [1] for a recent review)?

On the other hand, some singularities can actually be resolved at the classical level, such as by oxidizing to higher dimensions [4], T-dualizing to regular solution, or adding flux corresponding to wrapped branes [3, 4]. Other singularities have been deemed “unphysical” in the first place, which often implies that the gravitational force becomes repulsive near the singularity. Thus, rather than being “resolved,” such singularities are “excised” [3, 5]. One conjecture which prohibits unphysical singularities is the cosmic censorship. In its weak form [8], the cosmic censorship conjecture states that singularities are cloaked by event horizons and, in its strong form [9], that even observers falling into black holes cannot observe a singularity.

M-theory implies the existence of extra dimensions, which are hidden either via compactification or the localization of gravity on a brane [10, 11]. In this paper, we focus on the second possibility. In particular, if all observers are confined to a braneworld then it need not be necessary to prohibit a naked singularity in the bulk, so long as there are appropriate boundary conditions for the gravitons at the singularity. That is, no conserved quantities should be lost through the singularity. In fact, a singular bulk geometry might have desirable effects, such as helping to solve the cosmological constant problem [12, 13, 14].

In addition, for some braneworld scenarios with a naked singularity in the bulk, the graviton speed increases further in the bulk [15, 16, 17, 18, 19, 20, 21]. In this case, gravitons may traverse distances on the braneworld by bending in the extra dimension. Since photons remain on the braneworld, gravitons may travel at an average speed that is greater than the speed of light on the braneworld. This may offer a non-inflationary solution to the cosmological horizon problem. On the other hand, if the singularity is hidden within the event horizon of a black hole in the bulk, then the graviton speed decreases in the bulk.

Extremal black holes in an asymptotic $AdS_5$ spacetime were derived in [22], and their

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[1] An example of such a singular spacetime is the negative-mass Schwarzschild black hole. States with arbitrarily negative energy would be allowed and the vacuum, and therefore the theory, would not be stable.
nonextremal generalization in \cite{23}. Spinning branes were first constructed in \cite{24}. There is a
one-to-one correspondence between spinning branes and R-charged AdS-black holes \cite{25} and
the non-linear spherical dimensional reduction of supergravities relating various spinning
branes with AdS-black holes was given in \cite{26}. In particular, lifting five-dimensional AdS-
black holes to ten dimensions as the near-horizon region of spinning D3-branes enables us
to explore the proposed complementarity \cite{27} of holography \cite{28} and braneworld scenarios
\cite{10, 11} in terms of phase transitions. At the same time, a study of both the gravitational
and gauge sectors within this context may lead to a more complete understanding of the
nature of these phase transitions. The holographic interpretation of asymmetrically warped
spacetimes, for which the bulk singularity is hidden behind an event horizon, was discussed
in \cite{29}.

This paper is organized as follows. In the next section, we briefly discuss five-dimensional
AdS-black holes/singularities, which arise from the dimensional reduction of spinning D3-
branes in ten dimensions. The local speed of gravitational propagation is given. In section
3, we consider braneworlds from static distributions of D3-branes. Braneworlds arising
from extremal flat D3-brane distributions are briefly reviewed. Next, it is pointed out
that apparently-supraluminal gravitational propagation arises only for branes with spherical
spatial surfaces \((k = 1)\), for which we calculate exactly the apparent graviton speed. For
flat nonextremal branes, gravitational Lorentz violations result in a massive graviton on
the brane. In section 4, we study how certain phase transitions are manifest in both the
braneworld model and the dual gauge theory. We also consider the stability domains for a
spinning D3-brane and the ramifications this has to the resulting braneworld models. We
summarize our results in section 5.

2 Spinning D3-branes and the speed of gravitons

In the near-horizon decoupling limit, a \(p\)-brane spinning in transverse directions dimension-
ally reduces to a charged black hole in a domain-wall background \cite{25, 26}. The angular
momenta of the \(p\)-brane are interpreted as the black hole electric charges under a subgroup
of the R-symmetry group. For the particular case of the spinning D3-brane, the (nontrivial)
\(S^5\)-reduction of type IIB supergravity yields an AdS-black hole in five-dimensions, whose
metric is \cite{22, 23}

\[
\begin{aligned}
\text{\(ds_5^2 = -(H_1 H_2 H_3)^{-2/3} f \, dt^2 + (H_1 H_2 H_3)^{1/3}(f^{-1} \, dr^2 + g^2 r^2 dx_j^2),\)}
\end{aligned}
\] (2.1)
where
\[ f = k - \frac{\mu}{r^2} + g^2 r^2 (H_1 H_2 H_3), \quad H_i = 1 + \frac{R_i^2}{k r^2}, \] (2.2)
and \( i, j = 1, 2, 3. \)

In (five) ten dimensions, the quantities \( 1/g, R_i \) and \( \mu \) are interpreted as the (AdS radius) D3-brane charge, (AdS-black hole charge) angular momenta and (AdS-black hole mass) energy above extremality, respectively. The extremal limit \( \mu = 0 \) corresponds to a static distribution of D3-branes. \( k = 1, 0 \) or \( -1 \), corresponding to the foliating surfaces with metric \( dx_j^2 \) being \( S^3, T^3 \) or \( H^3 \). For the case \( k = 0 \), one does the rescaling \( R_i^2/k \rightarrow R_i^2 \) before sending \( k \) to zero.

The largest value of \( r \) for which \( f = 0 \) corresponds to an event horizon. For large enough \( R_i \), there is no event horizon—i.e., there is a naked singularity. The values of \( R_i \) and \( \mu \) for which this phase transition occurs can be calculated either by setting the horizon radius to zero or by setting the temperature of the system to zero. For a distribution of branes, the geometry is singular near the distribution when \( \mu = 0 \). This will be the first class of singular braneworld geometries which we will study, in the next section. For a single nonzero \( R_i \), there is an event horizon for \( \mu \neq 0 \). For a two nonzero and equal \( R_i \), the phase transition to a singular geometry takes place for \( g^2 R^4 = \mu \). The braneworld scenario from such a geometry is discussed in section 4.1. Finally, for three equal \( R_i \), the phase transition to a singular geometry is at \( g^2 R^4 = \frac{4}{27} \mu \), as discussed in section 4.2.

With a coordinate transformation, the metric (2.1) can be expressed as
\[ ds^2_5 = (H_1 H_2 H_3)^{1/3} g^2 r^2 \left( -\frac{f}{g^2 r^2 (H_1 H_2 H_3)} dt^2 + dx_i^2 + dz^2 \right), \] (2.3)
where
\[ dz = dr/(r \sqrt{f}). \] (2.4)
In general, \( r(z) \) is the amplitude of a Jacobi elliptic function. However, frequently a less complicated coordinate transformation suffices for our purposes.

The local speed of gravitational propagation is given by
\[ v(z) = \sqrt{\frac{f}{g^2 r^2 H_1 H_2 H_3}}. \] (2.5)
A braneworld may lie at \( z = 0 \). While Standard Model particles are restricted to live on the braneworld, gravitons may propagate in the bulk dimension \( z \). In particular, if the speed \( v(z) \) increases away from \( z = 0 \), then graviton geodesics bend into the bulk, taking the path of least-time, and thereby traverse distances on our braneworld faster than photons.
3 Braneworlds from static D3-branes

3.1 Extremal flat D3-brane distributions

As can easily be seen from (2.5), for a braneworld scenario arising from the near-horizon region of a flat \((k = 0)\) and extremal \((\mu = 0)\) D3-brane, the speed of gravitons in the bulk is the same as the speed of light on the braneworld. For completion, we will first briefly review such examples. In the extremal limit, the angular momenta \(R_i\) become distribution parameters for a static solution. The original RS2 model \([11]\) can be dimensionally-lifted to a stack of D3-branes \([30]\) (all \(R_i = 0\)). There is no mass gap between the localized massless graviton and the continuous spectrum of massive modes in the bulk.

Nonzero \(R_i\) corresponds to a distribution of D3-branes. From the holographic viewpoint, separating some of the D3-branes in the transverse space moves the dual \(\mathcal{N} = 4\) super Yang-Mills theory onto the Coulomb branch \([31, 32, 33, 34, 35]\), for which certain scalar fields have nonzero expectation values. For a braneworld scenario arising from D3-branes distributed uniformly over a disc or three-sphere \((R_1 \neq 0, R_2, R_3 = 0\) or \(R_1 = R_2 \neq 0, R_3 = 0\), respectively) \([36]\), there is a mass gap in the graviton spectrum. In the latter case, the massive graviton spectrum is discrete. The braneworld scenario arising from D3-branes distributed uniformly over a five-sphere (all \(R_i\) equal) does not appear to have been previously studied. The geometry is AdS for \(r > R_i\) and Minkowski for \(r < R_i\). The resulting braneworld scenario is similar to the previous case, in that there is a mass gap and the massive graviton spectrum is discrete. These are our first examples of braneworld scenarios which arise from singular geometries, since the curvature blows up close to the brane distributions. Although corrections due to higher-derivative terms in the action become important, the large curvature region is small, and so it is not expected that the qualitative picture is altered \([36, 37, 38]\).

3.2 Uniform disc of extremal D3-branes (with unspecified \(k\))

For D3-branes distributed uniformly over a disc, with \(k\) unspecified, (2.3) and (2.4) yield

\[
d s^2_5 = \frac{1}{\sinh^2(1 + g|z|)}[1 + \gamma^2 \sinh^2(1 + g|z|)]^{1/3} \left( \frac{-\cosh^2(1 + g|z|)}{1 + \gamma^2 \sinh^2(1 + g|z|)} dt^2 + dx_i^2 + dz^2 \right), \tag{3.1}
\]

where

\[
\gamma \equiv \frac{gR}{\sqrt{k + (gR)^2}}. \tag{3.2}
\]

We take \(dx_i^2 = g^{-2} d\Omega_3^2\), so that \(g \to 0\) for the case \(k = 0\). This case corresponds to flat D3-branes uniformly distributed over a disc, for which \(\gamma = 1\). The resulting braneworld
scenario \[36\] has been discussed briefly in the previous section. \(\gamma = 0\) corresponds to a D3-brane with \(k = 1\), and the resulting braneworld scenario exhibits a graviton speed which increases further in the bulk and leads to an apparent “superluminal” propagation of gravity \[40\].

\(\gamma\) is a useful parameter for interpolating between the two aforementioned cases, by varying \(gR\) relative to \(k\). As we will see, the graviton wave equation and spectrum is independent of \(\gamma\), so that the nature of gravitational Lorentz violations can be studied independently of the graviton spectrum and the localization of the massless graviton.

We shall now show that there is a massless graviton mode localized on the braneworld. The fluctuations of the five-dimensional graviton satisfy the equation for a minimally-coupled scalar field given by

\[
\partial_M \sqrt{-g} g^{MN} \partial_N \Phi = 0. \tag{3.3}
\]

Taking \(\Phi = \phi(z)M(t,x_i)\), the radial wave equation for the background metric \[3.1\] is

\[
- \frac{\sinh^3(1 + g|z|)}{\cosh(1 + g|z|)} \partial_z \frac{\cosh(1 + g|z|)}{\sinh^3(1 + g|z|)} \partial_z \phi = m^2 \phi, \tag{3.4}
\]

where \(m\) is defined by

\[
\Box_{(4)} M(t,x_i) = m^2 M(t,x_i), \tag{3.5}
\]

where \(\Box_{(4)}\) is the four-dimensional Laplacian.

With the wave function transformation

\[
\phi = \left( \frac{\cosh(1 + g|z|)}{\sinh^2(1 + g|z|)} \right)^{-1/2} \psi, \tag{3.6}
\]

the wave equation \[3.4\] can be expressed in Schrödinger form,

\[
- \partial^2_z \psi + V(z) \psi = m^2 \psi, \tag{3.7}
\]

with

\[
V(z) = \frac{4 \sinh^4(1 + g|z|) + 20 \sinh^2(1 + g|z|) + 15}{\sinh^2(2(1 + g|z|))} g^2 - \alpha g \delta(z), \tag{3.8}
\]

where \(\alpha = 2(2\sinh^2(1) + 3)/\sinh(2)\). This is a volcano-type potential. That is, the coefficient of the delta-function term is negative, and the \(g^2\) term is positive at \(z = 0\) and decreases to

\[
V(z \to \infty) = g^2. \tag{3.9}
\]

The massless wavefunction solution is given by

\[
\psi = N \sqrt{\frac{\cosh(1 + g|z|)}{\sinh^3(1 + g|z|)}} \tag{3.10}
\]
Figure 1: $v(z)/v(0)$ versus $gz$ for a braneworld from a uniform disc of extremal D3-branes with unspecified $k$. $v(z)$ increases indefinitely for $\gamma = 0$ (regular line). For $\gamma > 0$, $v(z)$ asymptotes to a maximum of $1/\gamma$, which is shown for $\gamma = .01$ (dotted line) and $.1$ (dashed line).

where $N$ is the normalization constant. Since this wavefunction is square normalizable, it corresponds to a localized massless graviton state. (3.9) indicates that there is a mass gap of $M_{\text{gap}} = g$ separating the localized massless state from the massive modes which propagate in the extra dimension.

Notice that the wave equation (3.4), and therefore the graviton spectrum, is independent of $\gamma$. However, from (3.1) we see that the speed of gravitons in the bulk depends on $\gamma$:

$$v(z) = \frac{\cosh(1 + g|z|)}{\sqrt{1 + \gamma^2 \sinh^2(1 + g|z|)}}.$$  \hfill (3.11)

As shown in Figure 1, $v(z)$ increases indefinitely away from the braneworld for $\gamma = 0$. For $\gamma > 0$, $v_{\text{maximum}} = 1/\gamma$.

Since $v(z)$ increases away from the braneworld, there is an apparent violation of causality. That is, gravitational disturbances bend into the bulk and arrive at a particular location on the braneworld earlier than does the light from the same source, the latter of which are restricted to the braneworld. Analogous to Fermat’s Principle for the propagation of light, gravitational waves will take the path of least-time, given by the geodesics.

3.3 D3-brane with $k = 1$ and the apparent speed of gravity

The near-horizon region of a single D3-brane with $k = 1$ is global $AdS_5 \times S^5$. This is the simplest supergravity solution that yields a braneworld exhibiting an apparently “superlu-
Figure 2: Apparent average graviton speed $v_{\text{average}}$ versus distance $gx_{\text{total}}$ for a braneworld from an extremal D3-brane with $k = 1$.

minal” speed of gravity. For this solution, one can exactly calculate the apparent average speed of gravity between two points on the braneworld. Thus, this example serves to illustrate characteristics which carry over to the more complicated braneworlds that exhibit apparently “superluminal” speeds of gravity, which will be discussed in section 4.

The geodesics can be deduced from the Lagrangian corresponding to the background metric (3.1),

$$\mathcal{L} = \frac{1}{2}g_{MN}\dot{x}^M \dot{x}^N = \frac{1}{\sinh^2(1 + g|z|)} \left( - \cosh^2(1 + g|z|) \dot{t}^2 + \dot{x}_i^2 + \dot{z}^2 \right),$$

(3.12)

where the dot represents differentiation with respect to an affine parameter. The equations of motion for $t$ and $x_i$ yield

$$\dot{t} = E \tanh^2(1 + g|z|), \quad \dot{x}_i = p_i \sinh^2(1 + g|z|),$$

(3.13)

where $E$ and $p_i$ are constants of integration. Together with the requirement that the Lagrangian (3.12) vanishes for lightlike geodesics, this yields

$$\dot{z}^2 + \sinh^4(1 + g|z|) \left( p_i^2 - \frac{E^2}{\cosh^2(1 + g|z|)} \right) = 0.$$ 

(3.14)

The turning point $z_T$ of gravitons in the bulk is at $\dot{z} = 0$: $E^2 = p_i^2 \cosh^2(1 + g|z_T|)$. $\tau$ divides out of the ratio $\dot{x}/\dot{z}$, and the remaining expression can be integrated to obtain $x(z)$.

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The apparent graviton speed is exactly calculable for the solution of general $\gamma$ from the previous section but we restrict ourselves to $\gamma = 0$ for simplicity.
The distance on the brane after which a geodesic returns to the brane is given by

$$x_{\text{total}} = 2 \int_0^{z_T} \frac{dz}{\sqrt{\frac{\cosh^2(1+g|z|)}{\cosh^2(1+g|z|)} - 1}}.$$  \hspace{1cm} (3.15)

Likewise, the corresponding time interval can be expressed as

$$t_{\text{total}} = 2 \int_0^{z_T} \frac{\cosh(1 + g|z|)}{\cosh^2(1 + g|z|) \sqrt{\frac{\cosh^2(1+g|z|)}{\cosh^2(1+g|z|)} - 1}} dz.$$  \hspace{1cm} (3.16)

The apparent average speed with which gravitons propagate a given distance along the braneworld is given by

$$v_{\text{average}} = \frac{x_{\text{total}}}{t_{\text{total}}} = \frac{g x_{\text{total}}}{\pi - 2 \arctan \left( \sqrt{\frac{\sinh^2(1)}{\tan^2(\frac{g x_{\text{total}}}{2})}} \right)}.$$  \hspace{1cm} (3.17)

which is plotted in Figure 2.

### 3.4 Nonextremal D3-branes

The case of $\mu \neq 0$ and vanishing $R_i$ corresponds to a nonextremal D3-brane. From (2.4) we find that $r(z)$ can be written in terms of Jacobi polynomials. However, for our purposes it suffices to use a less complicated coordinate \footnote{For typographical simplicity, we will drop the $'$ in the remainder of this paper.}

$$r = e + \frac{e}{1 + g|z'|},$$  \hspace{1cm} (3.18)
where \( e \equiv (\mu/g^2)^{1/4} \). The event horizon is located where \( f = 0 \), at \( r_h = e \). The graviton speed is given by

\[
v(z) = \sqrt{1 - \left(1 + \frac{1}{1 + g|z|}\right)^{-4}}.
\]  

(3.19)

As can be seen from Figure 3, \( v(z) \) decreases away from \( z = 0 \). Thus, gravitons do not appear to travel faster than the speed of light. On the contrary, graviton geodesics bend towards the braneworld. A consequence of this is that the effective four-dimensional graviton is a quasi-localized massive state, whose mass increases when further from extremality. A braneworld scenario with such characteristics which is tractable with semi-analytical methods arises from a non-extremal 5-brane in ten dimensions [39, 40].

While moving away from extremality induces a mass in the gravitational bound state from the braneworld viewpoint, from the holographic viewpoint it corresponds to giving a finite temperature to the dual gauge theory.

4 Braneworlds from spinning D3-branes: phase transitions and “superluminal” gravitons

We now consider a nonextremal, spinning D3-brane. Above certain values of angular momenta, the Hawking temperature vanishes and there is no longer an event horizon hiding the singularity. There is evidence suggesting that this point in the parameter space marks the transition from the deconfined high-density phase of the dual field theory to the Coulomb phase at finite density [41]. For the case of one non-zero angular momentum, this phase transition occurs at \( \mu = 0 \). The braneworld scenario arising from this point in parameter space was briefly reviewed in section 3.1.

4.1 Phase transition for two angular momenta

We now consider a nonextremal, spinning D3-brane with two angular momenta with the value \( R \). For \( g^2 R^4_{\text{transition}} = \mu \), there is no longer an event horizon, since \( f = g^2(2R^2 + r^2) \) is non-singular. In this case, (2.3) and (2.4) simplify to

\[
ds_5^2 = \left(1 + \cosh(1 + g|z|)\right)^{2/3}
\sinh(1 + g|z|)\left(-\frac{\cosh(1 + g|z|)}{(1 + \cosh(1 + g|z|))^2}dt^2 + dx_i^2 + dz^2\right),
\]

(4.1)

where

\[
r = \frac{\sqrt{2}R}{\sinh(1 + g|z|)}.
\]  

(4.2)
Figure 4: $v(z)/v(0)$ versus $gz$ for a braneworld from a nonextremal spinning D3-brane with all three $R_i$ equal. $R = R_{\text{transition}}$ corresponds to the graviton speed going to zero at some point in the bulk and then rising greater than the speed-of-light on the brane (regular line). For $R > R_{\text{transition}}$, the graviton speed reaches a non-zero minimum before rising greater than the speed-of-light on the brane, which is shown for $g^2 R^4 = .17 \mu$ (dashed line).

The fluctuations of the five-dimensional graviton satisfy the equation for a minimally-coupled scalar field given by (3.3), which leads to the wave equation (3.6). Therefore, there is a localized massless graviton state separated from the massive modes by a mass gap of $M_{\text{gap}} = g$. In the previous section we mentioned that, for a static D3-brane, going further from extremality by increasing $\mu$ serves to increase the mass of the quasi-localized graviton. If we consider that $\mu$ is then kept fixed while two angular momenta $R$ are increased from zero to the phase transition value $R_{\text{transition}}$, we find from the above that the effect has been to render the localized graviton massless once again. This is because, as $R$ is increased, the graviton speed does not decrease as much in the bulk.

### 4.2 Phase transition for three angular momenta

We will now consider the case of three equal $R_i = R$. For $R < R_{\text{transition}}$, where $g^2 R_{\text{transition}}^4 = \frac{4}{27} \mu$, there is an event horizon around the singularity. This corresponds to an AdS-Reissner-Nordström black hole from the five-dimensional viewpoint. This has been interpreted as the high density and high temperature deconfined phase of the dual N gauge theory.

$R > R_{\text{transition}}$ corresponds to a naked singularity. These backgrounds have been used in previous studies of gravitational Lorentz violations in braneworld scenarios [18, 13, 20, 21]. There is evidence that suggests this to be dual to the Coulomb phase at finite density of
From (2.4) we find that \( r(z) \) can in general be written in terms of Jacobi polynomials. However, for our purposes it suffices to use the less complicated coordinate given by (3.18). The graviton speed is then given by

\[
v(z) = \sqrt{1 - \frac{(1 + \frac{1}{1+g|z|})^2}{\left[(1 + \frac{1}{1+g|z|})^2 + (R/e)^2\right]^3}}. \tag{4.3}
\]

For \( R < R_{\text{transition}} \), \( v(z) \) decreases away from \( z = 0 \), as in the case of the static nonextremal D3-brane studied in section 3.3. At \( R = R_{\text{transition}} \), for which the bulk singularity becomes naked, \( v(z) \) goes to zero at some point in the bulk and then rises to a speed which is greater than \( v(0) \), the speed-of-light on the brane. For \( R > R_{\text{transition}} \), the graviton speed reaches a non-zero minimum before rising greater than \( v(0) \). This behavior is shown in Figure 4. As \( R \) increases further, the change in \( v(z) \) with respect to \( z \) is lessened, as shown in Figure 5. Note that the minimum of \( v(z) \) in the bulk is too small to be seen in this plot.

While causality is not actually violated from the higher-dimensional vantage point, the apparently “superluminal” gravitons imply the presence of a tachyon in the dual gauge theory. Indeed, at zero temperature \( (R = R_{\text{transition}}) \) a spurious gauge field vev produces a negative mass scalar term which destabilizes the moduli space of the theory \(^4\). On the supergravity side, this corresponds to radial motion of the spinning D3-branes.

One consequence of the \( v(z) \) minimum, shown in Figure 4, is that graviton geodesics...

\(^4\)The author thanks Paolo Creminelli for pointing out this connection.
Figure 6: Stability domains for a five-dimensional AdS-black hole with three equal charges $R$. Dashed lines mark the Hawking-Page phase transition between a charged AdS black hole (b.h.) and pure AdS. Above the regular line, the AdS black hole is unstable. Above the bold line, there is a naked singularity. For braneworld scenarios, this may be the most interesting region, since here the graviton speed increases in the bulk.

Close to the braneworld will actually bend towards the braneworld. This means that there is a quasi-localized massive state, as in the case of the static nonextremal D3-brane discussed in section 3.4. As $R$ increases, the mass of the quasi-bound graviton decreases.

As the energy scale increases, gravity on the braneworld probes less of the bulk. That is, the geodesics connecting two points on the braneworld do not have time to stretch as far into the bulk when the distance between the points is decreased. Therefore, the minimum in $v(z)$ means that there is a maximum energy for which gravity propagates at apparently “superluminal” speeds. This is clearly desirable, since otherwise the gravitational loops which transmit Lorentz violations would not be suppressed at the energy scale where gravity becomes strongly interacting.

### 4.3 Stability domains

We shall now consider the thermodynamical stability domains for a spinning D3-brane $^{25, 13, 14, 15, 16, 17, 18, 19, 50, 51}$. We restrict ourselves to the case of three equal angular momenta $R$. There is evidence that the grand-canonical ensemble, rather than the canonical ensemble, is relevant when the D-brane world-volume is large. Therefore, we shall work with the former. The Hawking-Page phase transition (dashed lines in Figure 6) between a five-dimensional charged AdS black hole and pure AdS is obtained by setting the
Gibbs Euclidean action to zero, as given in [25, 49]. This gives
\[(R/e)^2 = (ge)^{2/3} - (ge)^2, \tag{4.4}\]
where \(e\) is defined by \(g^2 e^4 \equiv \mu\). From the braneworld perspective, the black hole region of parameter space corresponds to a quasi-localized massive graviton, whereas the pure AdS region corresponds to the original Randall-Sundrum model with a localized massless graviton.

Thermodynamic stability is formally equivalent to the sub-additivity of the entropy function, which for D3-branes amounts to the condition [25, 49]:
\[2 - 3(R/r_h)^2 + (R/r_h)^6 \geq 0. \tag{4.5}\]
Numerically, this yields \(R/e \approx 0.595\) (regular line in Figure 6). It has been conjectured that, above this threshold, the D3-branes split apart into fragments which move out in the radial direction carrying away some of the spin. The resulting geometry may be neither stationary nor static [44]. One can conjecture that the corresponding braneworld has a mass gap separating a quasi-localized massive graviton from a discrete graviton spectrum.

The region above \(R = R_{\text{transition}}\) (bold line in Figure 6) corresponds to a naked singularity. The singularity may ultimately be smoothed out by the true short-distance theory of gravity. Thus, without understanding the physics of the singularity, we cannot determine yet whether it significantly affects the interactions of the four-dimensional modes. Nevertheless, this may be the most interesting region from the braneworld perspective, since it seems to exhibit apparently "superluminal" gravitons.

However, it should be noted that contributions from higher-derivative gravity [52, 53], as well as logarithmic corrections in black hole thermodynamics [54], may have a significant effect on the previously-mentioned phase structure.

5 Discussion

We have considered various types of braneworlds which may arise from a D3-brane solution. Gravitons which appear to travel faster than the speed-of-light result if the D3-brane is spatially spherical. We calculated exactly the apparent graviton speed for this case. For a flat D3-brane, "superluminal" gravitons only result if the D3-brane is both nonextremal and has three nonzero spins that are above a critical value, for which case there is a naked singularity in the bulk. It has been conjectured, from the holographic viewpoint, that the naked singularity marks the transition to the finite-density Coulomb phase of the dual gauge
theory. Lastly, we discussed how the thermodynamic stability domains of the spinning D3-brane affect the physics of braneworld scenarios.

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