Stability of $2\pi$ domain walls in ferromagnetic nanorings

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Abstract—The stability of $2\pi$ domain walls in ferromagnetic nanorings is investigated via calculation of the minimum energy path that separates a $2\pi$ domain wall from the vortex state of a ferromagnetic nanoring. Trapped domains are stable when they exist between certain types of transverse domain walls, i.e., walls in which the edge defects on the same side of the magnetic strip have equal sign and thus repel. Here the energy barriers between these configurations and vortex magnetization states are obtained using the string method. Due to the geometry of a ring, two types of $2\pi$ walls must be distinguished that differ by their overall topological index and exchange energy. The minimum energy path corresponds to the expulsion of a vortex. The energy barrier for annihilation of a $2\pi$ wall is compared to the activation energy for transitions between the two ring vortex states.

I. INTRODUCTION

Recent observations [1–3] in thin ferromagnetic stripes show magnetization configurations in which the magnetization makes a full $2\pi$ turn in a localized region of the stripe, while the rest of the stripe is magnetized parallel to the edges of the stripe. These structures are stable against small applied external magnetic fields which makes them potentially useful for information storage devices [4]. The same phenomenon has been observed in thin ferromagnetic annuli [1, 4], which then results in the existence of a hierarchy of equally spaced metastable states [5].

Reliable control of domain wall structures is crucial in the successful design of magnetic nanodevices [6, 7]. Here we explore the stability of $2\pi$ domain walls in ferromagnetic nanorings using the string method [8–10]. We find the energy barrier $\Delta E$ separating two metastable configurations. This barrier determines the escape rate from a metastable state through the Arrhenius law, to leading order $e^{-\Delta E/\Delta E_c}$ [11]. The string method also gives the minimum energy path and transition state.

We distinguish two types of domain walls by their winding number in the global (local) coordinate system $\omega$ (Ω). We compare the energy barriers that separates each type of wall from the ground state to the activation energy that separates the two vortex configurations [5], [12]. A current flowing along the axis of the ring produces a circumferential field. The application of this field has two effects. First, the degeneracy of the two vortex states is lifted. Second, the field determines the width of the domain wall; in its absence the $2\pi$ walls could dissociate into two independent $\pi$ transverse walls.

II. METHOD

Following previous work [5, 12], we study a permalloy ring with the following dimensions and material properties: outer radius $R_2 = 220$ nm; inner radius $R_1 = 180$ nm, thickness $t = 2$ nm, magnetization saturation $M_s = 8 \times 10^5 \text{A/m}$, and exchange length given by $l_{ex} = \sqrt{2A/\mu_0 M_s^2} = 5.6$ nm. A current flowing along the axis of the ring produces a field $H(r) = (hH_c(R_1 + R_2)/2r)\theta$. Here $h = H/H_c$ and the characteristic field strength at midradius is $\mu_0 H_c = 73.9$ mT (for $H > H_c$, the clockwise vortex state is no longer stable). The calculations were performed at $h=0.1$.

Precessional effects do not modify the location of the critical points in the energy landscape: the exponential factor in the Arrhenius formula is unaltered if we ignore them. We consider the overdamped case so that the escape trajectory follows the negative gradient of the energy. This is done by integrating only the damping term of the Landau-Lifshitz-Gilbert equation

$$\frac{dM}{dt} = -\frac{\gamma |\alpha|}{M_s}M \times (M \times H_{eff}).$$

Here $\alpha = 1$ is the damping coefficient, $\gamma$ is the gyromagnetic constant, and $H_{eff} = -\nabla_M E/\mu_0$ is the effective magnetic field. The total micromagnetic energy $E$ is the sum of the exchange $E_{ex}$, Zeeman $E_Z$ and magnetostatic terms $E_{mag}$.

The string method is necessary to calculate the minimum energy path between two stable states $(M_A, M_B)$ when there is no a-priori knowledge of the transition state. In practice, the path is discretized in $N+1$ images between $M_A$ and $M_B$ denoted as $M_i(t) \equiv M_i(r, t)$ with $i = 0, \ldots, N$. The images are updated using a two-step iteration procedure as follows: First, each image evolves using the publicly available micromagnetic code OOMMF [13] until the time reaches some interval $\Delta t$ which we have selected to be $10$ ps. This gives a sequence of configurations:

$$M'_i \equiv M'_i(r) = M_i(t) + \int_{t}^{t+\Delta t} \frac{dM_i(t')}{dt} dt'.$$

Once all the $M'_i(r)$ have been obtained, the second step in the string method is a reparametrization step used to keep these images equidistant. First the complete arc $s_N$ length of the trajectory is calculated by

$$s_0 = 0, s_i = s_{i-1} + |M'_i - M'_{i-1}|.$$
The arc lengths are renormalized using $\alpha'_i = s_i/s_N$. Finally we do a simple linear interpolation for all $i$ along the trajectory so that

$$M_i(t + \Delta t) = M'_j(i) + \frac{M'_{j(i)+1} - M'_{j(i)}}{\alpha'_{j(i)+1} - \alpha'_{j(i)}} \left( \frac{i}{N} - \alpha'_j \right)$$

(4)

where $j(i)$ is the index of the string where $\alpha'_{j+1} \geq i/N \geq \alpha'_{j(i)}$. During each step we observe the magnetic energy $E_i(t) = E_{\text{ex}}(M_i(r,t)) + E_2(M_i(r,t)) + E_{\text{mag}}(M_i(r,t))$ as indicator of how far from convergence the current step is. The iteration process is stopped when there is no visible change in the function $E_i(t)$.

III. ANNihilation OF 2\pi DOMAI

We now present the results of the string method to find the minimum energy path for destruction of a $2\pi$ wall for the two types of $2\pi$ domain wall, $\Omega = \pm 1$. Fig. 1 and Fig. 2 present the string energies and configurations after relaxation of the string. The $\Omega = +1$, 2$\pi$ wall decays into the counterclockwise configuration by the expulsion of a vortex from the inner hole. On the other hand, the $\Omega = -1$, 2$\pi$ wall decays into the counterclockwise configuration by the expulsion of an antivortex. This observation shows a correspondence between a topological defect crossing the stripe and the signature of the $2\pi$ walls being annihilated.

For comparison purposes we provide magnitudes of the energy landscape of ferromagnetic nanorings obtained with the string method. The lowest energy barrier between the counterclockwise and the clockwise vortex configurations passes through a configuration denoted as the instanton saddle [12] (with $\Omega = 0$); the activation energy of this event is $3.0 \times 10^{-19}$ J. This is consistent with our previous work on nanorings [5]. For a 2$\pi$ wall with $\Omega = 1$ the decay into the ground state has an energy barrier equal to $1.9 \times 10^{-19}$ J. The annihilation of a 2$\pi$ wall with index $\Omega = -1$ has an energy barrier equal to $2.8 \times 10^{-19}$ J. This shows the energy barrier to annihilate a $2\pi$ domain wall by the expulsion of a topological defect is comparable to that of reversal between vortex states by a instanton fluctuation.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Energy barrier for annihilation of 2$\pi$ domain wall under an external field $h = 0.1$. Fig. 2 shows the configurations for images $i=0,50,100$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Segment of the ring encompassing each 2$\pi$ domain wall. Minimum energy path for the annihilation of 2$\pi$ domain walls in consideration. (Above) 2$\pi$ wall with topological index $\Omega = -1$, (below) 2$\pi$ wall with topological index $\Omega = +1$. The configurations shown correspond to the following images in the string (left) $i=0$, (center) $i=50$, (right) $i=100$.}
\end{figure}

IV. DISCUSSION

For annuli with the dimensions being considered the magnetization is constrained to lie in the plane of the ring; for the stable states the magnetization can be considered to be independent of the radial coordinate. The exchange energy $E_{\text{ex}}$ of a stable state is given by [12]:

$$E_{\text{ex}} = \frac{\mu_0 M_s^2 h_{\text{ex}}^2}{2} \ln \left( \frac{R_2}{R_1} \right) \left[ 2\pi(1 + 2\Omega) + \int_0^{2\pi} \left( \frac{\partial \phi}{\partial \theta} \right)^2 d\theta \right]$$

(5)

where $\phi(\theta)$ is the angle that the magnetization makes with the tangent of the ring at a given angle $\theta$; $\Omega$ is the “winding number” of the magnetization with respect to the local coordinate system.

The difference in winding numbers ($\Delta \Omega$) of the 2$\pi$ walls considered results in an exchange energy difference $\Delta E_{\Omega}$ between these two states. Using Eq. 5 the difference can be shown to be approximately:

$$\Delta E_{\Omega} \approx 2\pi \mu_0 M_s^2 h_{\text{ex}}^2 \ln \left( \frac{R_2}{R_1} \right) \Delta \Omega = 1.298 \times 10^{-19} \text{ J.}$$

(6)
The total exchange energy difference between these $2\pi$ walls ($\Omega = \pm 1$) obtained from the micromagnetic simulation results is $1.357 \times 10^{-19}$J. Here we have ignored a contribution to this difference of terms of the form $\int \left( \frac{\partial \omega}{\partial \theta} \right)^2 d\theta$ since it is not a topological term. This shows a very good agreement between an estimate obtained from the 1D model and the full numerical simulation. The exchange energy term is the biggest contributor to the difference between the total energies of the two domain walls: the numerical values from the demagnetization and Zeeman energy are 10 times smaller. The main point is that most of the energy difference between these two types of $2\pi$ domain wall is the result of their respective topological windings. It is worth noting that this is a curvature effect as can be seen from Eq. (6): the energy difference tends to zero in the limit when the radii approaches infinity -i.e. in the limit of a straight ferromagnetic strip.

We now consider the question of how to experimentally produce these two types of $2\pi$ walls. We use the information contained in their global topological number $\omega$ and compare it to other known states of nanorings. In particular, the well-known "onion" state has $\omega = 0$. Since the onion is the remanent magnetization after saturation by an inplane uniform field, one can produce the $\Omega = -1$ wall by applying a strong field in-plane followed by a circumferential field. The two walls will approach and form a $2\pi, \Omega = -1$ wall. Changing the direction of either the in-plane field or the circumferential field will only change the final position of the $2\pi$ wall, not its topological index.

The $2\pi, \omega = 2$ domain wall cannot be produced using only uniform and circumferential fields. However, we propose the following technique to produce that configuration in nanorings: apply a strong dipolar field ($\omega = 2$) in the interior of the ring. This could be produced by a small current loop with its axis coplanar to the structure, or by bringing a magnetic tip close to the device. If this dipolar field is strong enough, two transverse walls would be produced at opposite sides of the ring; the magnetization vector at the centers of the wall will point in the opposite direction of the overall magnetization of the ring. Activating the circumferential field as the dipole strength is decreased will result in the desired configuration.

One final question is whether resistance measurements can distinguish the difference between the two types of $2\pi$ structures reported in this work. For instance, one could attempt to use anisotropic magnetoresistance effect to read the overall winding number of the configuration. An estimate of this effect can be obtained by integrating $\mathbf{J} \cdot \mathbf{M}$ along a certain segment of the ring that spans the whole $2\pi$ domain wall. Since $\mathbf{J}$ runs along $\theta$ the AMR would be proportional to $(M_B/M_s)$ which can be directly calculated from the $2\pi$ domain structure. For two electrodes located at the top and bottom of the segment shown in Fig. [2] these values are 0.042 and 0.016 for the $\Omega = +1$ and $\Omega = -1$ walls respectively. It therefore should be possible to apply a current to probe the winding number of the $2\pi$ domain.

V. CONCLUSION

We have presented results on the thermal annihilation of $2\pi$ domain walls. We differentiated between two types of $2\pi$ walls through their winding number in curved nanowires. We have observed a simple arithmetical relation between the topological index of the different configurations and the processes by which each structure decays into the ground state. The fact that the energy difference between the two states is dominated by the exchange energy allows to identify the states through their winding number. The transition path requires the motion of a singularity through the bulk: an antivortex destroys $\Omega = -1$ walls, and a vortex annihilates $\Omega = +1$ walls. Similar behavior is expected to work in linear stripes. Topological defects are known to play a role in certain types of phase transitions; here we have determined the mechanism by which they destroy $2\pi$ wall structures.

The two types of $2\pi$ domain wall correspond to distinct metastable states: the greatest contribution to the energy difference comes from the exchange energy difference. The energy can be directly associated with the topological signature of the magnetization configuration. The stability of these two states is comparable to the stability of the clockwise configuration. We suggest to use AMR noise measurements as a way to verify the presence of the two types of wall in magnetic nanowires and study their stability. Thermal fluctuations should generate both types of wall which could be recognized as two separate values of the AMR.

Further micromagnetic exploration of the $2\pi$ wall annihilation problem presented here can be done moving away from the overdamped regime by using a nonzero precessional term. The results presented here will be interesting to compare to this case. While we have presented results on a 2nm thick ring, we expect the observed transition states in thicker rings. The key parameter is the ratio of the thickness to the mean radius which should be less than approximately 0.1 [12]. We plan to explore the energy barriers and transition states as a function of thickness and ring radii.

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