Dislocation model for the TO-period anomaly

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Abstract. The dislocation-vibration model can explain both the anomaly in the period of torsional-oscillator (TO) containing solid $^4\text{He}$, i.e. the so-called nonclassical rotational inertia (NCRI), and the anomaly in the shear modulus of solid $^4\text{He}$ on the same basis. This dislocation model is extended to understand the amplitude dependence and hysteresis of the TO period by introducing an exponential distribution function for the network pinning lengths of dislocation segments. Pinning and unpinning of dislocations by $^3\text{He}$ impurity atoms depending on the history of temperature and stress play an important role. An actual distribution of the network pinning lengths is determined from the experimental data.

1. Introduction

The anomaly in the torsional-oscillator (TO) period observed originally by Kim and Chan [1], i.e. the nonclassical rotational inertia (NCRI), has been ascribed to the supersolid transition of hcp $^4\text{He}$ inside the TO. The amplitude dependence of the TO period was interpreted as the effect of critical velocity in the supersolid model. Day and Beamish [2] have observed a similar anomaly in the shear modulus of solid $^4\text{He}$, which can be well described by pinning of dislocations by $^3\text{He}$ impurities. A dislocation-vibration model [3] for the NCRI has been proposed to explain both anomalies on the same basis. According to this model the period change is proportional to $\Lambda L^2$, where $\Lambda$ is the dislocation density and $L$ is the temperature-dependent average pinning length. The model reproduces the variation of the TO-period with temperature and with $^3\text{He}$ concentration fairly well. In this paper the origin of the amplitude dependence and the hysteresis is discussed based on the dislocation-vibration model.

By modifying the dislocation model from a delta-function distribution function for the network pinning lengths to continuous distribution functions, such as an exponential one, the observed amplitude dependence and hysteresis of the TO period can be reproduced. Moreover, an actual distribution of the network pinning lengths is determined from the experimental data.

2. Dislocation model

When the torsional oscillator containing solid helium is driven at a frequency $\omega$ and amplitude $u_0$, solid helium oscillates in phase with the TO with a larger amplitude than the TO itself. The overshoot is due to the elastic nature of the solid. It causes shear stress in solid helium which in turn causes a back-action force on the TO equivalent to the moment of inertia of solid helium.

When mobile dislocations are present in solid helium with the average pinning length $L$ and the effective dislocation density $\Lambda$, the dislocation segments vibrate in phase with the TO and the displacement and stress in solid helium are enhanced by a factor, $1 + \Lambda L^2(1 - \nu)/2\pi$, where $\nu = 0.3$ is Poisson’s ratio. The apparent moment of inertia of solid helium is enhanced by the
From this dislocation-vibration model the period change of the TO is expected to be [3]

\[ \frac{\Delta p}{p} = \frac{\Lambda L^2(1 - \nu)}{2\pi}. \]  

(1)

2.1. Pinning length

Dislocations are strongly pinned by the nodes of dislocation network, and weakly pinned by \(^{3}\text{He}\) impurity atoms. The average network pinning length \(L_N\) is assumed to be independent of temperature. The impurity pinning length \(L_i\), on the other hand, depends on temperature \(T\) and \(^{3}\text{He}\) concentration \(x_3\). The effective pinning length \(L\) is equal to \(L_i(T)\) at low temperatures, and to \(L_N\) at high temperatures. An interpolation formula for the pinning length is given by

\[ L = \frac{L_NL_i(T)}{L_N + L_i(T)}. \]  

(2)

2.2. Stress-induced breakaway

Let us assume that a dislocation segment of length \(L_N\) is pinned by a \(^{3}\text{He}\) atom. When shear stress \(\sigma\) is applied on the crystal, the dislocation segment receives a force \(L_Nb\sigma\), a half of which is the force on the pinning \(^{3}\text{He}\) atom, \(f = L_Nb\sigma/2\), where \(b\) is the Burgers vector. There is a critical force \(f_c\) above which the dislocation breaks away from the impurity pinning point. The condition for break-away is \(f > f_c\). At a given stress \(\sigma\), the force is proportional to the pinning length \(L_N\). We define a critical pinning length

\[ L_c = \frac{2f_c}{b\sigma}. \]  

(3)

When the dislocation segment is longer than \(L_c\), it can break away from a pinning point under the given stress. When the dislocation segment is shorter than \(L_c\), it remains pinned under the same stress.

2.3. Distribution of the network pinning length

It is not likely that all the dislocation segments have the same network pinning length. Thus it is natural to assume that the network pinning length distributes over a wide range. An exponential distribution function is assumed for the sake of mathematical simplicity with the dislocation density \(\Lambda\), and the average of the network pinning length \(L_A\),

\[ N(L_N)dL_N = \frac{\Lambda}{L_A^2} \exp\left[-\frac{L_N}{L_A}\right] dL_N. \]  

(4)

In this case, the period change is expressed as

\[ \frac{\Delta p}{p} = \frac{(1 - \nu)}{2\pi} \int L^2 L_N N(L_N) dL_N = \frac{(1 - \nu)}{2\pi} \int L^2 L_N \frac{\Lambda}{L_A^2} \exp\left[-\frac{L_N}{L_A}\right] dL_N. \]  

(5)

3. Results

3.1. Amplitude dependence

We consider an experimental procedure in which the rim velocity (amplitude) is set at high temperature and the TO is cooled while the rim velocity is kept constant. The critical length \(L_c\) is constant throughout the cooling run. When the temperature is high enough then \(L_i >> L_N\) and dislocation segments are not pinned by \(^{3}\text{He}\) atoms. Pinning of dislocation occurs when a \(^{3}\text{He}\) atom arrives at the dislocation segment and remains on the dislocation line at lower temperature.
The arrival rate of $^3$He atoms on a typical dislocation segment of length $L_N = 5\mu$m is estimated to be 1.5 atom/s and independent of temperature because the diffusion constant of $^3$He in bulk $^4$He is independent of temperature. If the dislocation segment is longer than $L_c$, the force on the pinning atom exceeds the critical force within a period of the TO (about 1ms) and the dislocation is unpinned. Only one $^3$He atom arrives at the segment at a time and it is unpinned even when the temperature is further lowered. Thus the segments longer than $L_c$ are not pinned at all down to the lowest temperature. If the segment is shorter than $L_c$, it cannot break away from a pinning $^3$He atom. As the temperature is lowered, the number of pinning $^3$He atoms increases and the pinning length becomes shorter. The expression for the period change is divided in two parts,

$$\frac{\Delta p}{p} = \frac{(1 - \nu)}{2\pi} \int_0^{L_c} \frac{\Lambda L^2 L_N}{L^2_A} \exp \left[ -\frac{L_N}{L_A} \right] dL_N + \frac{(1 - \nu)}{2\pi} \int_{L_c}^{\infty} \frac{\Lambda L_N^3}{L^2_A} \exp \left[ -\frac{L_N}{L_A} \right] dL_N. \quad (6)$$

The first term gives a temperature-dependent contribution through the temperature dependence of $L$, while the second term gives a temperature-independent contribution.

As the initial rim velocity is increased, $L_c$ becomes smaller and the $T$-independent term becomes larger. Thus an amplitude dependence is expected. The periods at various values of $L_c$ are calculated from Eq. (6) and compared with the experimental results [4] as shown in figure 1.

**Figure 1.** Amplitude dependence of the period change calculated from Eq. (6) for various values of $L_c/L_A$. Parameters are $\Lambda = 1.9 \cdot 10^{11}$m$^{-2}$ and $L_A = 5.0 \cdot 10^{-7}$m. Experimental data [4] are plotted for comparison.

### 3.2. Hysteresis

Hysteresis of TO period was reported at first by Aoki, Graves and Kojima [5], and then by Clark, Maynard and Chan [6]. The TO-period takes various values depending on the history of temperature and stress. Aoki et al. cooled the sample down to 19mK at the rim velocity of 800 $\mu$m/s so that most of the dislocation segments were unpinned at point A in figure 2. They reduced the velocity to 0 $\mu$m/s (point B) so that all the dislocation segments were pinned and the period was decreased. When they increased the velocity again up to point C, the pinning length $L_i$(19mK) was much shorter than the critical length so that the period did not change. Namely, at point A most dislocations are unpinned, whereas at points B and C most dislocations are pinned.

**Figure 2.** Hysteresis of the period change at $f=1172.8$Hz and $T=19$mK, converted from NCRI in ref. [5] to period change.
3.3. Actual distribution function
When the temperature is sufficiently low, the first term on the right-hand side of Eq. (6) becomes negligibly small. Hence the actual distribution function for $L_N$ can be obtained by numerically differentiating the period of decreasing run from A to B in figure 2. Figure 3 shows the distribution function obtained in this way and the exponential distribution function for comparison.

![Figure 3. Distribution function for $L_N$ determined from the experimental data [5] and the exponential distribution function.](image)

4. Discussion
Clark, Maynard and Chan [6] reported that the value of NCRI at the lowest temperature was decreased successively when they warmed and cooled the TO repeatedly, each time to a higher temperature. Their results are interpreted as follows. Each time when the TO was warmed a portion of dislocation segments are unpinned. When it was cooled again, the unpinned portion remained unpinned. Thus the NCRI at the lowest temperature is a measure of how many dislocation segments are pinned.

Choi et al. [7] have reported a TO-period change in a rotating cryostat. They find that the NCRI depends on the DC velocity as well as the AC velocity and that there are crossing points from DC rotation to AC oscillation when the NCRI is measured at various DC velocities and AC velocities. If we assume that the DC rotation causes shear stress in solid helium proportional to the rotational speed, we may conclude from the present dislocation model that dislocation segments are unpinned by the DC rotation as well as AC rotation. Then, their results are explained as due to unpinning of dislocations from $^3$He impurities.

5. Conclusion
The dislocation model can explain following experimental results without introducing supersolidity: period change at low temperature, dependence on the $^3$He impurity concentration, amplitude dependence, hysteresis, and the period change in the DC rotation experiment.

References
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