Inelastic photon scattering and the magnetic moment of the $\Delta (1232)$ resonance

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Abstract

The reaction $\gamma + p \rightarrow \gamma' + p + \pi^0$ has been suggested as a means to deduce the $\Delta^+$ magnetic moment. The cross section for this process is estimated in both the constituent quark model and an effective Lagrangian procedure. The resulting total cross section is of the order 5-10 nb, which is at the limit of present experimental capabilities.

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I. INTRODUCTION

The static properties of baryons are an important testing ground for QCD based calculations in the confinement region. In particular, a recent comparison of theoretical predictions for $\Delta$ magnetic moments in different approaches has been given in [1]. However, little experimental information is available for hadrons outside of the ground state SU(3) octet. In view of the short life-time of the resonances, such information has to come from a detailed analysis of intermediate states. It was therefore suggested by Kondratyuk and Ponomarev [2] to consider radiative $\pi^+p$ scattering as a means to measure the static properties of the $\Delta$ isobar. As a result of many experimental and theoretical efforts [3], the Particle Data Group [4] now quotes a value of $\mu_{\Delta^+} = (5.6 \pm 1.9) \mu_N$ [4] for the magnetic dipole moment of the $\Delta^{++}$ resonance. The large error bar is due to large nonresonant processes, external bremsstrahlung by initial and final state particles, and a strong background due to interactions in both the initial and final states [4][5]. A much cleaner experiment would be an electromagnetic excitation of the nucleon leading to the $\Delta$ resonance with subsequent emission of a real photon followed by the decay into a nucleon and a pion. The process $\gamma + p \rightarrow \gamma' + p' + \pi^0$ would be particularly favorable, because the signal is less disturbed by the external bremsstrahlung background. Machavariani et al. [6] have recently investigated this reaction in the framework of an effective Lagrangian model. For an incident photon with lab energy $E_{\gamma}^{lab} = 386$ MeV (equivalent to a cm energy $k = 286$ MeV or a total cm energy $W = 1267$ MeV) they find differential cross sections of the order of 0.5 nb/str² MeV, with a maximum for photons emitted
with lab energy $E_{\gamma}^{\text{lab}} \approx 65$ MeV and at an angle $\Theta' \approx 90^\circ$. If we integrate these differential cross sections over all angles of the emitted pion and photon and over the final state photon energy, we obtain a total cross section of several microbarns.

Unfortunately, the A2 collaboration at MAMI working with the TAPS detector has only been able to see 3-photon events \[6\] at a rate corresponding to a cross section of tens of nanobarns, partly originating from the reaction $\gamma + p \rightarrow p + \pi^0 + \pi^0 \[7\]$. It is the aim of this contribution to point out that we only expect total cross sections for this process in the range of 5-10 nb, which is probably at the limit of the present experimental accuracy. For this purpose we have calculated both elastic (Compton) and inelastic photon scattering through the $\Delta$ resonance in the constituent quark model \[8\]. Though this model is somewhat naive and has certain deficiencies, it has the advantage that the ratio of elastic and inelastic photon scattering can be expressed analytically. As a result we expect that at least the predicted order of magnitude of the cross section will be realistic. We find some further support for these results from a numerical calculation based on an effective Lagrangian.

II. PHOTON SCATTERING IN THE CONSTITUENT QUARK MODEL

The interaction Hamiltonian between a real photon (momentum $\vec{k}$, polarization vector $\hat{\epsilon}_\lambda$) and the 3-quark system is

$$H^{\text{int}}_{\lambda} = -\frac{1}{\sqrt{2k}} \hat{\epsilon}_\lambda \cdot \vec{J}, \quad (1)$$

where $\vec{J}$ is the current operator,

$$\vec{J} = \sum_k \frac{e}{2m_k} q(k) [\vec{p}'(k) + \vec{p}(k) + i\vec{\sigma}(k) \times (\vec{p}'(k) - \vec{p}(k))], \quad (2)$$

with $m_k$ the mass, $q(k)$ the charge in units of $e$, $\vec{\sigma}(k)$ the spin operator, $\vec{p}$ and $\vec{p}'$ the initial and final cm momenta of quark $k$, respectively, and $e^2/4\pi \approx 1/137$ the fine structure constant. In the following we shall perform all calculations in the cm frame of the initial photon-nucleon system. Furthermore we shall only be interested in static properties, i.e. magnetic moments of both ground state and $\Delta$ resonance, and of magnetic transitions between these two states. Neglecting then the convection currents, the Hamiltonian of Eq. (1) can be expressed by the magnetic moment operator $\vec{\mu}$,

$$H^{\text{int}}_{\lambda} = \pm \lambda \sqrt{\frac{k}{2}} \mu_{\lambda}, \quad (3)$$

with the sign corresponding to absorption and emission respectively, and

$$\mu_{\lambda} = \sum_k \frac{e}{2m_k} q(k) \sigma_{\lambda}(k). \quad (4)$$

For simplicity we also assume that $m_k = m_N/3$, where $m_N = 938$ MeV is the mass of the proton. If we use symmetrized quark wave functions, Eq. (4) can be cast into a form operating only on the 3rd quark,
\[ \mu_\lambda = 9 \, q(3) \, \sigma_\lambda(3) \, \mu_N \, , \]  
(5)

where \( \mu_N = e/2m_N \) is the nuclear magneton.

With the usual symmetrized isospin \((\Phi)\) and spin \((\chi)\) wave functions of the 3-quark system,

\[ | N \rangle = \frac{1}{\sqrt{2}} \left( | \Phi_{MS} \chi_{MS} + \Phi_{MAX} \chi_{MA} \rangle + | \Delta = | \Phi_S \chi_S \rangle \right) \, , \]
(6)

we obtain the well-known quark model predictions for the magnetic moments of nucleon \((n, p)\) and Delta \((\Delta^{++}, \Delta^+, \Delta^0, \Delta^-)\),

\[ \mu_p = \langle p_\uparrow^1 | \mu_0 | p_\downarrow^1 \rangle = 3\mu_N \, , \]

\[ \mu_n = \langle n_\uparrow^1 | \mu_0 | n_\downarrow^1 \rangle = -2\mu_N \]

\[ \mu_\Delta^c = \langle \Delta^c_\uparrow^3 | \mu_0 | \Delta^c_\uparrow^3 \rangle = 3 \epsilon_c \mu_N \, , \]

(7)

where we have indicated the spin projection by \( \frac{1}{2} \) or \( \frac{3}{2} \), and \( \epsilon_c \) is the charge of the respective \( \Delta^c \) in units of \( e \). For further use we also evaluate the matrix elements of \( \mu_{\pm} \), in particular

\[ \langle \Delta^+ \uparrow^3 | \mu_+ | p_\uparrow^1 \rangle = 2\sqrt{3} \mu_N \, , \]

\[ \langle \Delta^+ \uparrow^1 | \mu_+ | p_\downarrow^1 \rangle = 2 \mu_N \, . \]

(8)

The familiar helicity amplitudes are given by the matrix elements of the Hamiltonian Eq. (3),

\[ A_{1/2} = -\sqrt{\frac{k}{2}} \langle \Delta^+ \uparrow^1 | \mu_+ | p_\downarrow^1 \rangle = -\sqrt{2k} \mu_N \, , \]

\[ A_{3/2} = -\sqrt{\frac{k}{2}} \langle \Delta^+ \uparrow^3 | \mu_+ | p_\uparrow^1 \rangle = -\sqrt{6k} \mu_N \, . \]

(9)

The electromagnetic width for the decay of the \( \Delta \) (1232) is

\[ \Gamma_\gamma = \frac{k^2 E}{2\pi m_\Delta} \left( | A_{1/2} |^2 + | A_{3/2} |^2 \right) = \frac{4k^3 E}{\pi m_\Delta} \mu_N^2 \, , \]

(10)

where \( k = (m_\Delta^2 - m_N^2)/2m_\Delta = 259 \text{ MeV} \) and \( E = \sqrt{m_\Delta^2 + k^2} = 973 \text{ MeV} \) at resonance, \( W = k + E = m_\Delta \).

Numerically we obtain the result \( \Gamma_\gamma \approx 0.46 \text{ MeV} \) and a branching ratio \( \Gamma_\gamma/\Gamma \approx 0.38 \% \), while the experimental value is \( (0.56 \pm 0.04) \% \), i.e. the quark model underestimates the helicity amplitudes of Eq. (3) by about 20 \%. If the quark mass (or and additional quark g-factor) is adjusted to the measured magnetic moment of the proton, the helicity amplitudes of the quark model come out even 25 \% too low. The experimental value is obtained by multiplying the \( \text{rhs} \) of Eq. (8) by \( G_{N\Delta}/2\sqrt{3} \), with \( G_{N\Delta} \approx 4 \). We note that in calculations with a harmonic oscillator potential, the \( \text{rhs} \) of Eqs. (7)-(10) is usually multiplied by a factor.
\[ \exp(-k^2/6\alpha_0^2), \] with \( \alpha_0 \) the oscillator parameter. This retardation factor adds some model dependence and leads to a stronger underestimation of the electromagnetic decay width \( \Gamma_\gamma \), which would have to be corrected by choosing \( G_N \Delta \approx 4 \exp(k^2/6\alpha_0^2) \).

Using the same matrix elements we can also evaluate the differential cross section for forward Compton scattering,

\[
\frac{d\sigma^{\text{el}}}{d\Omega dk'}(\Theta = 0) = \frac{m_N^2}{16\pi^2 EW} \frac{k'}{k} \delta(k' + E' - W) |G(W)|^2 \times \frac{1}{4} \sum_{\lambda M} |\langle pM | j_{-\lambda} | \Delta, M + \lambda \rangle \langle \Delta, M + \lambda | j_{\lambda} | pM \rangle|^2, \tag{11}
\]

with the current operator \( j_\lambda(k) = \pm k\lambda \mu_\lambda \) for absorption and emission respectively (see Eqs. (1) to (3)). The required matrix elements can be obtained from Eq. (8) and the symmetry relations (valid for \( \lambda = \pm 1 \))

\[
\langle \Delta M' | \mu_\lambda | pM \rangle = \langle \Delta, -M' | \mu_{-\lambda} | p, -M \rangle = (-)^\lambda \langle pM | \mu_{-\lambda} | \Delta M' \rangle. \tag{12}
\]

The propagator in the intermediate state takes the nonrelativistic form

\[
G(W) = (m_\Delta - W - i\frac{\Gamma(W)}{2})^{-1}, \tag{13}
\]

with an energy-dependent width \( \Gamma(W) \) fixed at \( \Gamma(m_\Delta) = 120 \text{ MeV} \). For the elastic process we have \( E = E' \) and \( k' = k \), and using Eqs. (8,11,12,13) we obtain the following prediction for forward Compton scattering,

\[
\frac{d\sigma^{\text{el}}}{d\Omega}(\Theta = 0) = \sigma_{Th} \frac{5k^4}{EW[(m_\Delta - W)^2 + \frac{1}{4}\Gamma^2]}, \tag{14}
\]

where \( \sigma_{Th} = (e^2/4\pi m_N)^2 \approx 23.6 \text{ nb} \) is the Thomson cross section. In particular the predicted cross section at resonance is

\[
\frac{d\sigma^{\text{el}}}{d\Omega}(\Theta = 0, W = m_\Delta) \approx 125 \text{ nb/sr}. \tag{15}
\]

With an angular distribution for purely magnetic transitions, \( (7 + 3\cos^2\Theta)/10 \), the total Compton cross section at resonance is \( \sigma_{\text{tot}}^{\text{el}}(W = m_\Delta) \approx 1.2 \text{ \mu b} \), which has to be compared to an experimental value of about 2.8 \mu b [9]. The quark model underestimates the data by a factor of about 2, because the cross section scales with the 4th power of the helicity amplitudes.

Along exactly the same lines we shall now estimate the inelastic cross section for the reaction of interest. This requires the following changes in Eq. (11):

(a) \( \sum_{\lambda M} \rightarrow \sum_{\lambda' M'} \),

(b) \( \langle pM | j_{-\lambda} | \Delta, M + \lambda \rangle \rightarrow \langle \Delta, M + \lambda - \lambda' | j_{-\lambda'} | \Delta, M + \lambda \rangle \),

(c) \( \delta(k' + E' - W) \rightarrow \frac{\Gamma(W')}{2\pi[(m_\Delta - W')^2 + \frac{1}{4}\Gamma^2(W')]} \).
where $W'^2 = W^2 - 2Wk'$ is the total cm energy of the $\pi N$ system. With regard to (a) we note that there occur 6 “paths” from the initial spin projection of the nucleon to the final spin projection of the $\Delta$,

$$
\begin{align*}
\frac{1}{2} & \rightarrow \frac{3}{2} \rightarrow \frac{1}{2}, \quad -\frac{1}{2} \rightarrow +\frac{1}{2} \rightarrow \frac{3}{2}, \quad -\frac{1}{2} \rightarrow +\frac{1}{2} \rightarrow -\frac{1}{2};
\end{align*}
$$

and 3 more that can be obtained by reversing all signs. In addition to the matrix elements of Eq. (8) we need the matrix elements

$$
\begin{align*}
\langle \Delta^+ + \frac{3}{2} | \mu_+ | \Delta^+ + \frac{1}{2} \rangle &= \sqrt{6} \mu_N, \\
\langle \Delta^+ + \frac{1}{2} | \mu_+ | \Delta^+, -\frac{1}{2} \rangle &= 2\sqrt{2} \mu_N.
\end{align*}
$$

(17)

All other matrix elements follow from Eq. (12). With the above changes the analogon of Eq. (11) for inelastic scattering is

$$
\frac{d\sigma^\text{inel}}{d\Omega'dk'}(\Theta = 0) = \frac{m^2_N kk' \Gamma(W')}{16\pi^2EW} \left| \frac{G(W)}{2\pi} \right|^2 \left| \frac{G(W')}{2\pi} \Gamma(W') \right|)
\times \frac{1}{4} \sum_{\lambda'\lambda M} \langle \Delta^+, M + \lambda - \lambda' | \mu_{-\lambda'} \rangle \langle \Delta^+, M + \lambda | \mu_\lambda | pM \rangle^2.
$$

(18)

By use of Eqs. (8,17), this expression can be cast into the form

$$
\frac{d\sigma^\text{inel}}{d\Omega'dk'}(\Theta = 0) = \frac{4m^2_N kk' \Gamma(W')}{\pi^2EW} \mu^4_N \left| \frac{G(W)}{2\pi} \right|^2 \left| \frac{G(W')}{2\pi} \Gamma(W') \right|.
$$

(19)

We note that in the zero-width approximation

$$
\lim_{\Gamma \rightarrow 0} \left| \frac{G(W')}{2\pi} \Gamma(W') \right| \rightarrow \delta(m_\Delta - W'),
$$

(20)

and since $k \rightarrow m_\Delta - E_N$ at resonance, $k'$ tends to zero in that limit. As a result the inelastic cross section is suppressed with regard to Compton scattering by a factor $(k'/k)^3 \approx (\frac{\Gamma}{2(m_\Delta - m_N)})^3 \approx 10^{-2}$.

If we restrict the discussion to the $\pi^0$ channel, the density of the final states will be further reduced by $\Gamma(\Delta^+ \rightarrow p\pi^0)/\Gamma \approx 2/3$. From a more detailed calculation we find a rather constant angular distribution of the form $(11 - 3 \cos^2 \Theta)/8$, and an integrated cross section

$$
\frac{d\sigma^\text{tot}}{dk'} = \sigma_{Th} \frac{20kk' \Gamma(W')}{3EW} \left[ (m_\Delta - W)^2 + \frac{1}{4} \Gamma^2(W) \right] \left[ (m_\Delta - W')^2 + \frac{1}{4} \Gamma^2(W') \right].
$$

(21)

III. RESULTS AND DISCUSSION

Comparing the total inelastic cross section to the total elastic one, Eq. (14), we obtain the simple relation
\[ R = \frac{\sigma_{\text{inel}}^{\text{tot}}}{\sigma_{\text{el}}^{\text{tot}}} = \frac{5}{6} \int_{0}^{k_{\text{max}}^\prime} dk' \left( \frac{k'}{k} \right)^3 \frac{\Gamma(W')}{2\pi[(m_\Delta - W')^2 + \frac{1}{4}\Gamma^2(W')]}. \] (22)

The upper limit of the integral is given by the threshold of \( \pi^0 \) production, \( W' = m_N + m_\pi^0 \), at which point the energy-dependent width vanishes with the 3rd power of the pion momentum. The maximum inelastic cross section is expected for \( W \approx m_\Delta + \Gamma/2 \), i.e. \( k \approx 306 \text{ MeV} \) and \( k_{\text{max}}^\prime \approx 200 \text{ MeV} \). If we increase the initial photon energy, the phase space of Eq. (19) increases, but the excitation strength decreases, because we move away from the resonance. The opposite is true if the initial photon energy is decreased, the increase in excitation strength is compensated by a shrinking of the phase space. At energies \( k \approx 300 \text{ MeV} \), the ratio of Eq. (22) has a value of about \( 6.4 \cdot 10^{-3} \) if one uses a typical energy-dependent width.

We recall at this point that the quark model predictions for the elastic and inelastic cross sections should be scaled by \( G_{\text{total inelastic cross section}} \) to describe the experimentally observed cross section. As a result we therefore predict a total inelastic cross section

\[ \sigma_{\text{inel}}^{\text{tot}} \approx R \left( \frac{\mu_\Delta+/\mu_p}{G_{N\Delta}/2\sqrt{3}} \right)^2 \sigma_{\text{el}}^{\text{tot}} \approx 4 \left( \frac{\mu_\Delta+}{\mu_p} \right)^2 \text{nb}, \] (23)

with \( \sigma_{\text{tot}}^{\text{el}} \approx 0.9 \text{ \( \mu \)b} \) the Compton cross section at \( k \approx 300 \text{ MeV} \).

As a further check we have evaluated the cross section for the reaction \( \gamma p \to \Delta^+ \to \gamma \Delta^+ \to \gamma \pi^0 p \) in the framework of an effective Lagrangian (see e.g. [10] for details). For the \( \Delta \) current we take the simple form

\[ \langle \Delta^+(p', s') | J^\mu(0) | \Delta^+(p, s) \rangle = + \frac{e}{2m_\Delta} \bar{\Delta}_\alpha(p', s')((p' + p)'^\mu + i\sigma^\mu\nu(p' - p)\nu G_{M1}^\Delta) \Delta^\alpha(p, s), \] (24)

where \( G_{M1}^\Delta \) is the magnetic moment of the \( \Delta^+ \) in units of the “\( \Delta \) magneton”, \( e/2m_\Delta \).

In Figs. 1-2 the differential cross sections are shown following from the effective Lagrangian for the value \( G_{M1}^\Delta = 3 \).

In Fig. 1 we show the fivefold differential cross section at an incoming photon \( cm \) energy of \( k = 286 \text{ MeV} \) (as also considered in [3]), at a photon \( cm \) angle of 90° and for three in-plane pion angles (in the \( \pi^0 p \) rest frame). The energy dependence of the cross section reflects the invariant mass distribution of the \( \pi^0 p \) system and has a maximum around an outgoing photon \( cm \) energy of 80 MeV. While this energy dependence is in good agreement with the results of Ref. [3], we obtain an absolute value for the cross section that is lower by about 3 orders of magnitude. At this energy, we show in Fig. 2 the photon angular dependence, which is seen to be quite flat with a slight maximum at 90°. For comparison, the angular dependence is well approximated in the quark model calculation by its form \((22 - 6 \cos^2 \Theta)/16\), which is shown on the same figure (dotted curve, with a global rescaling factor to match the lower curve in Fig. 2). Comparing with Ref. [3], our cross sections are again lower by nearly 3 orders of magnitude. Moreover the angular distribution given in Ref. [3] has a high maximum near 90° (in the \( lab \) and approaches very small values in the forward and backward directions, quite at variance with our nearly constant angular distributions.
In Fig. 3, we compare the quark model prediction of Eq. (23) for the total $\gamma p \to \gamma \pi^0 p$ cross section (using $\mu_{\Delta^+} = \mu_p$) with the result from the effective Lagrangian calculation as function of the total cm energy $W$. The effective Lagrangian calculation is shown both for a value $G_M^{\Delta} = 3$ and $G_M^{\Delta} = 3.66$ (which corresponds with $\mu_{\Delta^+} = \mu_p$). One sees that both calculations are in rather good agreement and yield total cross sections of the order 5 nb in the considered energy range when using $\mu_{\Delta^+} = \mu_p$.

IV. CONCLUSIONS

In spite of new and improved experimental techniques, the reaction $\gamma + p \to \gamma' + \Delta^+ \to \gamma' + p + \pi^0$ has not yet been observed. Our expectations based on a simple constituent quark model show that the integrated cross section for this process should indeed be very small, namely of the order of 5 nb. These result are corroborated by an equally simple Lagrangian approach, which also permits us to calculate the five-fold differential cross sections with the result of typically 0.25 nb/GeV sr$^2$. In agreement with the quark model prediction, the angular distribution for photon emission is rather constant.

In the future we plan to improve our calculations by including the backgrounds due to bremsstrahlung off the incoming and outgoing protons as well as radiation from nonresonant intermediate states. Though we do not expect qualitative changes by such terms, they are likely to influence the angular and energy distributions of the process. A better understanding of these backgrounds might also give a chance to analyze the data obtained with incident photons of higher energies, which would increase the counting rate by giving a larger phase space to the emitted particles.

We conclude that the electromagnetic moments of baryon resonances are among the most evasive properties of hadrons. The extremely weak signals for these moments are at the very limits of even the most advanced experimental techniques. However, such data would be invaluable for our understanding of QCD in the confinement region, and therefore a dedicated experiment is certainly desirable.

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REFERENCES

[1] T.M. Aliev, A. Özpıneci, M. Savci, [hep-ph/0002228].
[2] L. A. Kondratyuk und L. A. Ponomarov, Yad. Fiz. 7 (1968) 11. [Sov. J. Nucl. Phys. 7 (1968) 82].
[3] D. S. Beder, Nucl. Phys. B 84 (1975) 362;
   B. M. K. Nefkens et al., Phys. Rev. D 18 (1978) 3911;
   L. Heller, S. Kumano, J. C. Martinez and E.J. Moniz, Phys. Rev. C 35 (1987) 718;
   R. Wittman, Phys. Rev. C 37 (1988) 2075;
   A. Bosshard et al., Phys. Rev. D 44 (1991) 1962.
[4] Review of Particle Physics (Particle Data Group), Eur. Phys. J, C 3 (1998) 1.
[5] A. I. Machavariani, A. Faessler, and A. J. Buchmann, Nucl. Phys. A 646 (1999) 231.
[6] M. Kotulla and V. Metag, private communication.
[7] F. Härter et al., Phys. Lett. B 401 (1997) 229; F. Härter, Ph.D. thesis, University Mainz (1996).
[8] N. Isgur, G. Karl and R. Koniuk, Phys. Lett. 41 (1978) 1269; M. M. Giannini, Il Nuovo Cimento 76 A (1983) 455.
[9] D. Drechsel, M. Gorchtein, B. Pasquini, and M. Vanderhaeghen, Phys. Rev. C 61 (1999) 015204, and references quoted therein.
[10] M. Vanderhaeghen, K. Heyde, J. Ryckebusch and M. Waroquier, Nucl. Phys. A 595 (1995) 219.
\( \gamma p \rightarrow \gamma \pi^0 p : k_{cm} = 0.286 \text{ GeV}, G_{M1}^\Delta = 3 \)

\( \Theta_{cm}^{\gamma} = 90^\circ \)

\( \theta_\pi^{*} = 0^\circ \)

\( \theta_\pi^{*} = 45^\circ \)

\( \theta_\pi^{*} = 90^\circ \)

FIG. 1. Photon energy dependence of the fivefold \( \gamma p \rightarrow \gamma \Delta^+ \rightarrow \gamma \pi^0 p \) differential cross section with outgoing photon energy and angle in \( \gamma p \) cm system, and with the pion angles in the \( \pi^0 p \) rest frame.
\[ \gamma p \rightarrow \gamma \pi^0 p \quad : \quad k_{cm} = 0.286 \text{ GeV}, \quad G_{M1}^\Delta = 3 \]

\[ k'_{cm} = 0.080 \text{ GeV} \]

FIG. 2. Photon angular dependence of the fivefold \( \gamma p \rightarrow \gamma \Delta^+ \rightarrow \gamma \pi^0 p \) differential cross section. For comparison, the angular dependence \((11 - 3 \cos^2 \Theta)/8\) in the quark model calculation is shown by the dotted curve (with a global rescaling factor to match the lower curve).
\[ \gamma p \rightarrow \gamma \Delta^+ \rightarrow \gamma \pi^0 p \]

FIG. 3. Total cross section for the $\gamma p \rightarrow \gamma \Delta^+ \rightarrow \gamma \pi^0 p$ reaction as function of the total $cm$ energy $W$. The quark model calculation (with $\mu_{\Delta^+} = \mu_p$) is shown by the dashed curve. The effective Lagrangian calculation is shown both for $G_{M1}^\Delta = 3$ (dashed-dotted curve) and $G_{M1}^\Delta = 3.66$ (corresponding with $\mu_{\Delta^+} = \mu_p$, full curve).