Energy-Efficient Cell-Free Network Assisted by Hybrid RISs
Wanting Lyu, Yue Xiu, Songjie Yang, Chau Yuen, Fellow, IEEE, and Zhongpei Zhang, Member, IEEE

Abstract—In this letter, we investigate a cell-free network aided by hybrid reconfigurable intelligent surfaces (RISs), which consists of a mixture of passive and active elements that are capable of amplifying and reflecting the incident signal. To maximize the energy efficiency (EE) of the system, we formulate a joint transmit beamforming and RIS coefficient optimization problem. To deal with the fractional objective function, Dinkelbach transform, Lagrangian dual reformulation, and quadratic transform are utilized, with a block coordinate descent (BCD) based algorithm proposed to decouple the variables. In addition, successive convex approximation (SCA) method is applied to iteratively tackle the non-convexity of the sub-problems. Simulation results illustrate the effectiveness and convergence of the proposed algorithm through analyzing the EE and sum rate performance with varying parameter settings. The proposed hybrid RIS scheme can achieve 5% and 188% gain in EE compared with existing passive and active RISs. The fast convergence of the proposed algorithm is analyzed by numerical simulations as well.

Index Terms—Beamforming, cell-free, convex optimization, energy efficiency, reconfigurable intelligent surfaces.

I. INTRODUCTION

CELL-FREE network has gained great attractiveness in beyond fifth-generation mobile communications (5G) without cell boundaries [1]. It has great potential in next generation indoor and hot-spot environment, such as shopping malls, train stations, hospitals and subways. In addition, cell-free network is particularly effective in high-mobility scenarios like vehicular networks without handover cost [2]. Despite the above advantages, conventional cell-free network requires high power consumption both for transmit power and hardware power due to numerous access points (APs) deployed, resulting in low energy efficiency (EE), which is one of the challenges needed to be overcome in future networks [3].

Recently, reconfigurable intelligent surface (RIS), has become a promising technology to overcome obstacles, enhance channel capacity and improve EE [4], [5], [6], [7], [8], [9] in various scenarios such as secure communications [10], [11], satellite networks [12], aerial-terrestrial communications [13], and Terahertz communications [14]. Moreover, RIS has been integrated into cell-free networks to replace some of the APs [15], [16], [17]. However, the ideal capacity gains provided by RIS is difficult to achieve practically owing to the “multiplicative fading” effect caused by RIS, causing extremely large path loss in the cascaded channels. Considering this, a new structure of RIS, namely active RIS has been proposed by authors of [18], where active reflecting elements are configured with radio frequency (RF) chains and power amplifiers to alleviate the severe fading effect. The active RIS, however, requires higher power consumption, and introduce non-negligible self-interference and thermal noise. Considering the trade-off between the signal amplifying effect and power consumption, hybrid RIS architecture has been proposed [19], [20], [21]. In this architecture, only a few reflecting elements are activated, introducing a lower level of transmit power and effective noise than active RIS, while significantly enhancing the signal strength compared with full RIS. Hybrid RIS provides a reliable, sustainable solution to the wireless network design with an acceptable level of cost and power consumption.

To the best of our knowledge, conventional passive RISs have been used in cell-free networks to reduce the power consumption, but the data rate is limited by the severe double-fading effect. To improve the energy efficiency (EE) of cell-free networks while alleviating the fading effect, we consider hybrid RISs with a few active elements capable of amplifying the incident signal to replace a part of APs. The main contributions of this letter are:

• Motivated by the benefits of hybrid RISs, we integrate multiple hybrid RISs into a downlink cell-free network, where EE performance is considered as the metric for improving the practical value of cell-free network.
• A highly coupled non-convex problem under the minimum rate constraint is formulated by optimizing the digital beamforming and hybrid RIS coefficients design. A block coordinate descent (BCD) based iterative algorithm is proposed to solve the problem.
• Simulation results show the effectiveness of the proposed algorithm in terms of improving energy efficiency, compared with other baselines. Moreover, trade-off between EE and sum rate is analyzed and illustrated. Convergence of the proposed algorithm is verified by numerical simulations.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. Transmission Model

Consider a downlink cell-free network with \( L \) APs each configured with \( N_t \) antennas serving \( K \) single-antenna users as shown in Fig. 1. To enhance the communication quality and improve the energy efficiency, \( K \) hybrid RISs are deployed in the network, each with \( N_s \) reflecting elements. For each RIS, \( N_a \ll N_s \) active reflecting elements are predetermined.
in the set \( \mathcal{A}_r \), where \( \mathcal{A}_r \subset \{1, \ldots, N_k\} \) contains the indices of active elements of RIS \( r \) and \( |\mathcal{A}_r| = N_a \). The RIS coefficient matrix of RIS \( r \) is defined as \( \Theta_r = \text{diag}\{a_{r,1}, \ldots, a_{r,N_k}\} \), where \( a_{r,n} = |a_{r,n}|e^{j\theta_{r,n}} \) denotes the coefficient of the \( n \)-th element, with the amplitude \( |a_{r,n}| \leq a_{\text{max}} \) if the element \((r, n_k)\) is in the set \( \mathcal{A}_r \), and \( |a_{r,n}| \leq 1 \) otherwise. To combine the elements in all \( R \) RISs together and for ease of exposition, define \( n = N_a(r-1)+n_k, 1 \leq n \leq N \), where \( N = RN_n \). The elements in set \( \mathcal{A} = \bigcup \mathcal{A}_r \) are the active reflecting elements in all \( R \) RISs. Then, the overall RIS matrix can be written as \( \Theta = \text{blockdiag}\{\Theta_1, \ldots, \Theta_R\} \). To distinguish the active and passive elements, rewrite \( \Theta = \Psi + \Upsilon \), where \( \Psi = \Lambda \Theta \) and \( \Upsilon = (\mathbf{I}_N - \Lambda)\Theta \) denote the coefficient matrices of active and passive elements, respectively, where the selection matrix \( \Lambda \) is an \( N \times N \) diagonal matrix with the elements corresponding to set \( \mathcal{A} \) to be 1, and others to be 0.

Assume that all APs and all RISs serve all users over the same time-frequency resource, such that the data symbol \( s \in \mathbb{C}^{K \times 1} \) satisfying \( \mathbb{E} [ss^H] = \mathbf{I}_K \) is transmitted from the APs simultaneously. A central processing unit (CPU) connected to all APs generates the digital beamformers based on perfectly known channel state information (CSI) of all the links, where the channel estimation methods can be found in [22]. The overall transmitted signal from all the APs can then be expressed as \( \mathbf{x} = \mathbf{W} \mathbf{s} \), where \( \mathbf{W} = (\mathbf{W}_1, \ldots, \mathbf{W}_L)^T \in \mathbb{C}^{LN \times K} \) denotes the transmit beamforming matrix at all the \( L \) APs. The links from all the APs to user \( k \), from APs to RISs, and from RISs to user \( k \) are denoted as \( \mathbf{d}_k^H \in \mathbb{C}^{L \times LN} \), \( \mathbf{G} \in \mathbb{C}^{RN \times LN} \), and \( \mathbf{f}_k^H \in \mathbb{C}^{RN \times N_k} \), respectively. Thus, the received signal of user \( k \) can be expressed as

\[
y_k = (\mathbf{d}_k^H + \mathbf{f}_k^H \Theta \mathbf{G}) \mathbf{w}_s + \mathbf{f}_k^H \Psi \mathbf{z} + n_k,
\]

where \( n_k \sim \mathcal{CN}(0, \sigma_0^2) \) is the additional white Gaussian noise (AWGN) at user \( k \), \( \mathbf{z} = (\mathbf{z}_1, \ldots, \mathbf{z}_R) \in \mathbb{C}^{RN \times 1} \) and \( \mathbf{z}_r \sim \mathcal{CN}(0, \sigma_0^2 \mathbf{A}_r \mathbf{I}_K) \) is the effective noise caused by the active elements of RIS \( r \), including the AWGN and self-interference due to the full-duplex mode. Hence, the received signal-to-interference-plus-noise (SINR) at user \( k \) can be expressed as

\[
\gamma_k = \frac{\left| \mathbf{d}_k^H \mathbf{w}_k \right|^2}{\sum_{j \neq k} |\mathbf{d}_j^H \mathbf{w}_j|^2 + \sum_{r=1}^{R} \sigma_{r,k}^2 + \sigma_0^2},
\]

where \( \mathbf{w}_k \) is the \( k \)-th column of the digital beamforming matrix \( \mathbf{W} \), and \( \sigma_{r,k}^2 = \left| \mathbf{f}_r^H \Psi \mathbf{z} \right|^2 \) is the effective RIS noise power.

### B. Energy Efficiency

The total power consumption of the cell-free system can be expressed as

\[
P_{\text{tot}}(\mathbf{W}, \Psi) = \sum_{l=1}^{L} P_{l,t,x}(\mathbf{W}_l) + \sum_{r=1}^{R} P_{r,t,a}(\Psi_r, \mathbf{W}) + \sum_{r=1}^{R} P_{r,R}(\mathbf{W}_r) + \sum_{k=1}^{K} P_k^U (3)
\]

circuit power of APs, RISs and UEs

Specifically, transmit power of AP \( l \) is

\[
P_{l,t,x}(\mathbf{W}_l) = \frac{\text{Tr}(\mathbf{W}_l^H \mathbf{W}_l)}{\mu^3},
\]

and transmit power of the active elements of RIS \( r \) is

\[
P_{r,t,a}(\Psi_r, \mathbf{W}_r) = \frac{1}{\mu_r} \text{Tr}(\mathbf{W}_r^H \mathbf{G}_r^H \Psi_r \mathbf{G}_r \mathbf{W}_r) + \sum_{n_r \in \mathcal{A}_r} |a_{r,n_r}|^2 \sigma_0^2,
\]

where \( \mu_A, \mu_R \in (0, 1) \) denote the amplifier efficiency factors of AP and RIS, respectively.

Thus, the energy efficiency of this system can be written as

\[
\eta = \frac{\sum_{l=1}^{L} C_{l} - R_k}{P_{\text{tot}}},
\]

where \( R_k = \log_2(1 + \gamma_k) \) is the data rate of user \( k \).

### C. Problem Formulation

To maximize the system energy efficiency, the optimization problem can be formulated as

\[
\begin{align}
\text{(P1)} \quad & \underset{\mathbf{W}, \Psi}{\text{max}} \quad \eta \\
\text{s.t.} \quad & P_{l,t,x}(\mathbf{W}_l) \leq P_{\text{max}}, \forall l \in \{1, \ldots, L\}, \quad (5a) \\
& 0 \leq \theta_n \leq 2\pi, \forall n, \quad (5c) \\
& |a_n| \leq a_{\text{max}}, \quad n \in \mathcal{A}, \quad (5d) \\
& |a_n| \leq 1, \quad n \notin \mathcal{A}, \quad (5e) \\
& R_k \geq R_{\text{th}}, \forall k, \quad (5f) \\
& P_{r,a}(\mathbf{W}_r, \Psi) \leq P_{\text{max}}, \forall r, \quad (5g)
\end{align}
\]

where (5b), (5g) are the transmit power constraints for AP \( l \) and RIS \( r \). Equation (5c), (5d) and (5e) are the phase shift and amplitude constraints for all RIS elements, respectively. Equation (5f) is to guarantee the data rate of user \( k \).

However, problem (P1) is difficult to solve due to the strongly coupled variables, non-convex fractional objective function (5a), and non-convex constraints (5f). To solve this problem, we proposed an iterative BCD algorithm in Section III.

### III. JOINT OPTIMIZATION ALGORITHM

In this section, we first transform the complex fractional objective function into equivalent concave form, and propose a BCD based algorithm to jointly optimize beamforming design \( \mathbf{W} \) at the APs and RIS coefficient \( \Theta \) at the RISs.

We first apply Dinkelbach’s method in [23]. By introducing an auxiliary variable \( \hat{y} \), the objective function (5a) can be equivalently replaced by

\[
f_1(\mathbf{W}, \Theta, \hat{y}) = \sum_{k=1}^{K} \log_2(1 + \gamma_k) - \hat{y} P_{\text{tot}}(\mathbf{W}, \Psi),
\]

because the optimal solution will be maintained after the replacement. To maximize \( f_1(\mathbf{W}, \Theta, \hat{y}) \), \( \hat{y} \) can be updated as

\[
\hat{y}' = \frac{\sum_{k=1}^{K} \log_2(1 + \gamma_k)}{P_{\text{tot}}(\mathbf{W}_t, \Psi_t)}
\]

where the superscript \( t \) represents the optimum solution obtained from the last iteration.
To further tackle the sum-logarithms in (6), Lagrangian dual reformulation in [24] is utilized. Introducing a slack variable \( \hat{\epsilon} = [\hat{\epsilon}_1, \ldots, \hat{\epsilon}_K]^T \), Eq. (7) can be equivalently replaced by:

\[
f_2(W, \Theta, \hat{y}, \hat{\epsilon}) = \frac{1}{K} \log(1 + \hat{\epsilon}_k) - \sum_{k=1}^{K} \hat{\epsilon}_k + \frac{(1 + \hat{\epsilon}_k)\gamma_k}{1 + \gamma_k} - \hat{y}P_{tot}(W, \Psi). \tag{8}
\]

The optimal \( \hat{\epsilon} \) can be obtained by taking the first order partial derivative of \( f_2(W^t, \Theta^t, \hat{y}^t, \hat{\epsilon}^t) \) such that \( \frac{\partial f_2(W^t, \Theta^t, \hat{y}^t, \hat{\epsilon}^t)}{\partial \hat{\epsilon}_k} = 0 \), and then we can update \( \hat{\epsilon} \) as:

\[
\hat{\epsilon}_k = \gamma_k \tag{9}
\]

However, \( \gamma_k \) is still a non-convex fraction, which can be decoupled by introducing slack variable \( \hat{\rho}_k \), according to the quadratic transform [24], and thus the objective function can be further replaced by (10) shown at the bottom of the page, where the optimal value of \( \hat{\rho}_k \) can be determined by setting \( \frac{\partial f_3(W^t, \Theta^t, \hat{y}^t, \hat{\epsilon}^t, \hat{\rho})}{\partial \hat{\rho}_k} = 0 \) with given \( W^t, \Theta^t, \hat{y}^t, \hat{\epsilon}^t \) such that:

\[
\hat{\rho}_k = \frac{\sqrt{1 + \epsilon_k h_k^H w_k^t}}{\sum_{j=1}^{K} |h_k^H w_j^t|^2 + \sum_{r=1}^{R} \sigma^2_{r,k} + \sigma_0^2}. \tag{11}
\]

To end this, problem (P1) is equivalent to

\[
(P2) \quad \max_{W, \Theta, \hat{y}, \hat{\epsilon}, \hat{\rho}} \quad f_3(W, \Theta, \hat{y}, \hat{\epsilon}, \hat{\rho})
\]

\[
s.t. \quad (5b), (5c), (5d), (5e), (5f), (5g), \tag{12a}
\]

whose optimal solution will remain unchanged after replacing the objective function. (P2) is still non-convex due to the coupling between variables \( W \) and \( \Theta \), and the non-convex fractional constraint (5f). To decouple the variables, BCD method is applied to update the variables \( \hat{y}, \hat{\epsilon}, \hat{\rho}, W \) and \( \Theta \) alternatively, where the optimal values of variables \( \hat{y}, \hat{\epsilon}, \hat{\rho} \) have been given.

### A. Transmit Beamforming Optimization

Given fixed \( \Theta^t, \hat{y}^t, \hat{\epsilon}^t \) and \( \hat{\rho}^t \) obtained in the last iteration, (P2) can be updated as

\[
(P3) \quad \max_W \quad f_3(W|\Theta^t, \hat{y}^t, \hat{\epsilon}^t, \hat{\rho}^t)
\]

\[
s.t. \quad (5b), (5c), (5d), (5f), (5g). \tag{13a}
\]

Note that the objective function is concave with respect to \( W \). Constraints (5b) and (5g) can be transformed as

\[
P^A_{it, t_2}(W_i) = \frac{1}{\mu_i} \text{vec}(W_i)H^H\text{vec}(W_i) \tag{14}
\]

\[
P^{R}_{r,a}(W|\Psi_r(t)) = \frac{1}{\mu_R} \left( \sum_{[r,n_a] \in A_r} |a_{r,n_a}|^2 \sigma^2_{r} + \text{vec}(W)H(I \otimes \text{vec}(\Psi_r(t)H^H\Psi_r^t)\text{vec}(W)) \right) \tag{15}
\]

Note that (a), (b) are because of \( \text{Tr}(A^H B) = (\text{vec}(A))^H \text{vec}(B) \) and \( \text{Tr}(ABC) = (\text{vec}(A))^H (I \otimes \text{vec}(C)) \), respectively.

Moreover, the non-convex constraint (5f) can be rewritten as

\[
|h_k^H w_k|^2 \geq \gamma_k \left( \sum_{j \neq k} |h_k^H w_j|^2 + \sum_{r=1}^{R} \sigma^2_{r,k} + \sigma_0^2 \right), \tag{16}
\]

where \( \gamma_k = 2R \mu_i - 1 \). Applying SCA method, the left hand side of the inequality can be lower bounded by its first Taylor expansion, and thus (16) can be approximated as

\[
2R \text{Re} \left( |h_k^H w_k^{(i)}|^2 |h_k^H w_k|^2 - |h_k^H w_k^{(i)}|^2 \right) \geq \gamma_k \left( \sum_{j \neq k} |h_k^H w_j|^2 + \sum_{r=1}^{R} \sigma^2_{r,k} + \sigma_0^2 \right), \tag{17}
\]

where \( \gamma_k = 2R \mu_i - 1 \). Applying SCA method, the left hand side of the inequality can be lower bounded by its first Taylor expansion, and thus (16) can be approximated as

\[
2R \text{Re} \left( |h_k^H w_k^{(i)}|^2 |h_k^H w_k|^2 - |h_k^H w_k^{(i)}|^2 \right) \geq \gamma_k \left( \sum_{j \neq k} |h_k^H w_j|^2 + \sum_{r=1}^{R} \sigma^2_{r,k} + \sigma_0^2 \right), \tag{17}
\]

where (i) denotes the value in the last iteration. Thus, (P3) can be reformulated as

\[
(P3-1) \quad \max_W \quad f_3(W|\Theta^t, \hat{y}^t, \hat{\epsilon}^t, \hat{\rho}^t)
\]

\[
s.t. \quad (14), (15), (17), \tag{18a}
\]

which is a convex problem that can be efficiently solved by standard convex solvers such as CVX [25].

### B. Hybrid RIS Coefficients Optimization

Given fixed \( W^t, \hat{y}^t, \hat{\epsilon}^t \) and \( \hat{\rho}^t \), (P2) becomes

\[
(P4) \quad \max_{\Theta} \quad f_3(\Theta, W^t, \hat{y}^t, \hat{\epsilon}^t, \hat{\rho}^t)
\]

\[
s.t. \quad (5d), (5e), (5f), (5g). \tag{19a}
\]

To make it more tractable, we first define a coefficient vector of RIS as \( \theta_r = (a_{r,1}, \ldots, a_{r,N_r})^H \). Accordingly, the active coefficient vector of RIS \( r \) can be represented as \( \Psi_r = A_r \theta_r \), where \( A_r \) denotes the active element selection matrix for RIS \( r \). Thus, the equivalent channel can be rewritten as:

\[
\tilde{h}^H = d_k^H + \frac{1}{\sum_{r=1}^{R} \theta_r^H \text{diag}(f_{r,k}^H)} \text{diag}(f_{r,k}^H) G_r = d_k^H + \theta^H \text{diag}(f_k^H) G_r, \tag{19b}
\]

and \( \theta_r = (\theta_{r,1}, \ldots, \theta_{r,R})^T \) denotes the overall RIS coefficient vector. Then, the transmit power of RIS \( r \) can be rewritten as:

\[
P_{r,a}(\theta_r|W) = \frac{1}{\mu_R} \left( \text{Tr}(\Psi_r G_r W^t H^H \Psi_r^t H^H) + \sum_{n_r \in A_r} \sigma^2_{r,n_r} \right) \tag{20a}
\]

where \( \Psi_r H, \theta_r, \sigma^2_{r} \Psi_r^t H^H \) denote the overall RIS coefficient vector. Then, the transmit power of RIS \( r \) can be rewritten as:

\[
\theta^H Q^1_{k,j} \theta + 2R \text{Re}(\theta^H Q^2_{k,j} + Q^3_{k,j}) \tag{21a}
\]

\[
\geq \gamma_k \left( \sum_{j \neq k} |h_k^H w_j|^2 + \sum_{r=1}^{R} \sigma^2_{r,k} + \sigma_0^2 \right), \tag{22a}
\]

where \( \theta^H \) denotes the optimal value of \( \theta \) in the last iteration.

To explicitly represent the variable \( \theta \), the objective function becomes (23) shown at the bottom of the page (\( P_{hw} \) denotes the overall circuit power of the system), which is concave with respect to \( \theta \). As a result, (P4) is reformulated as

\[
(P4-1) \quad \max_{\Theta} \quad f_3(\Theta, W^t, \hat{y}^t, \hat{\epsilon}^t, \hat{\rho}^t)
\]
Algorithm 1 Proposed Algorithm for the EE Maximization Problem

1: Initialize variables \( \hat{y}(0), \hat{\epsilon}(0), \hat{\rho}(0), W(0) \) and \( \theta(0) \). Set iteration index \( i = 1 \).
2: repeat
3:   Update Dinkelbach slack variable \( \hat{y}(i) \) by (7).
4:   Update Lagrangian dual reformulation slack variable \( \hat{\epsilon}(i) \) by (9).
5:   Update quadratic transform slack variable \( \hat{\rho}(i) \) by (11).
6:   Update transmit beamforming \( W(i) \) by solving (P3-1).
7:   Update RIS coefficients \( \theta(i) \) by solving (P4-1).
8:   Update EE \( \eta \).
9: until convergence of \( \eta \).

s.t. \( (5d), (5e), (22), P_{r,a} \theta \left( W' \right) \leq P_{\text{max}} \). (24b)

Note that in each iteration, (P4-1) is a convex problem that can be easily solved by existing methods.

C. Convergence and Complexity Analysis

The overall BCD based algorithm is summarized in Algorithm 1, where in each iteration, variables \( \hat{y}, \hat{\epsilon}, \hat{\rho}, W \) and \( \theta \) are updated step by step.

1) Convergence Analysis: The overall joint transmit beamforming and hybrid RIS coefficient design optimization algorithm converges to a local optimal value through several iterations. It is because the update of each variable is an optimal solution to the corresponding sub-problem. To show this, we use \( x(i) \) to denote the optimal variable of variable \( x \) obtained from the last iteration \( i \). Then, Algorithm 1 is monotonically non-decreasing in each iteration \( i \) as

\[
\begin{align*}
& f_3(W^{(i+1)}, \theta^{(i+1)}, \hat{y}^{(i+1)}, \hat{\epsilon}^{(i+1)}, \hat{\rho}^{(i+1)}) \\
& \quad \geq f_3(W^{(i)}, \theta^{(i)}, \hat{y}^{(i)}, \hat{\epsilon}^{(i)}, \hat{\rho}^{(i)}) \\
& \quad \geq f_3(W^{(i)}, \theta^{(i)}, \hat{y}^{(i)}, \hat{\epsilon}^{(i)}, \hat{\rho}^{(i)}) \\
& \quad \geq f_3(W^{(i)}, \theta^{(i)}, \hat{y}^{(i)}, \hat{\epsilon}^{(i)}, \hat{\rho}^{(i)}) \\
& \quad \geq f_3(W^{(i)}, \theta^{(i)}, \hat{y}^{(i)}, \hat{\epsilon}^{(i)}, \hat{\rho}^{(i)})
\end{align*}
\]

(25)

Note that the convergence of SCA algorithm is given by [26], so (a) and (b) are because the updated \( W \) and \( \theta \) (or equivalently \( \theta \)) are at the Karush-Kuhn-Tucker (KKT) points of sub-problem (P3-1) and (P4-1), respectively; (c), (d), and (e) are because updating \( \hat{y}', \hat{\epsilon}', \hat{\rho}' \) maximizes the objective function, given other variables fixed. Moreover, since the optimal value of the objective \( f_3(W', \theta', \hat{y}', \hat{\epsilon}', \hat{\rho}') \) is upper bounded by the constraints, Algorithm 1 finally converges to its local optimum.

2) Computational Complexity: The computational complexity of the overall algorithm consists of the complexity for updating the variables \( \hat{y}, \hat{\epsilon}, \hat{\rho}, W \) and \( \theta \), which mainly depends on the iterative algorithm for solving \( W \) and \( \theta \). Thus, the overall computation complexity of the proposed algorithm is \( \mathcal{O}(I_0[K^{3.5} \log_2(1+\epsilon_1) + N^{3.5} \log_2(1+\epsilon_2)]) \), where \( I_0 \) is the number of iterations. Besides, \( \epsilon_1 \) and \( \epsilon_2 \) denote the accuracy of SCA algorithms.

IV. NUMERICAL RESULTS

In this section, we provide the numerical simulation results to show the effectiveness of our proposed algorithm for the hybrid-RIS aided cell-free network and compare the performance with other benchmarks.

Without loss of generality, we consider a 200 m \( \times \) 200 m square area. Specifically, \( L = 6 \) APs (each with \( N_t = 6 \) transmit antennas and \( R = 3 \) RISs are distributedly deployed in the area serving \( K = 4 \) single-antenna users simultaneously. Rician fading model is used in each link, with the Rician factor to be \( \beta = 3 \) dB. The path loss (PL) of each link is \( L = C_0(d/d_0)^{-\alpha} \), where \( C_0 = -30 \) dB is the reference PL at \( d_0 = 1 \) m, \( d \) is the length of the corresponding links, and \( \alpha = \{2.8, 2.2, 2.2\} \) are the path loss exponents of AP-UE, AP-RIS and RIS-UE links, respectively. Assume that the first \( N_a \) elements of each RIS are active, while others are passive. The equivalent noise power of active RIS elements are set as \( \sigma^2 = -76 \) dBm, and the AWGN power at the users are \( \sigma^2_0 = -80 \) dBm. The amplifier efficiencies are \( \mu^A = \mu^R = 0.8 \), and the minimum rate limit is \( R_{\text{th}} = 0 \) dB. The hardware power consumption of each AP and each UE is set as 40 dBm and 10 dBm, respectively. The circuit power of each active and passive RIS element is 30 dBm and 10 dBm, respectively. The amplifying coefficient is set as \( a^2_{\text{max}} = 20 \) dB.

To begin with, the EE performance is analyzed with increasing transmit power at the APs illustrated in Fig. 2(a). We consider each RIS are configured with \( N_a = 60 \) elements including \( N_a = 1 \) and \( N_a = 3 \) active ones. The maximum RIS transmit power is assumed as \( P_{\text{max}}^R = 10 \) dBm. We can see from the figure that the EE of the system first grows with transmit power AP increases, and then drops for each configuration. The proposed algorithm for the hybrid RIS significantly outperforms the all AP, active RIS (\( N_a = N_a \)) and random RIS coefficient cases, where all AP means to replace the RISs with APs. For fairness, the overall transmit power keeps unchanged. In addition, hybrid RIS with only one active elements can achieve higher EE than conventional passive RIS, while EE of hybrid RIS with 3 active elements is slightly lower than that of passive RIS.

We further investigate the tradeoff between the EE and sum rate with different numbers of active elements shown as

\[
\begin{align*}
& f_3(W, \theta, \hat{y}, \hat{\epsilon}, \hat{\rho}) = \sum_{k=1}^{K} 2\sqrt{1 + \epsilon_k} \text{Re} \left\{ \left( \hat{\rho}_k^H \hat{h}_k^H w_k \right) \right\} - |\hat{\rho}_k|^2 \left( \sum_{j=1}^{K} |\hat{h}_k^H w_j|^2 + \sum_{r=1}^{R} \sigma^2_{r,k} + \sigma^2_0 \right) - \hat{y} P_{\text{tot}}(W, \Psi) \\
& f_4(\theta) = \sum_{k=1}^{K} 2\sqrt{1 + \epsilon_k} \text{Re} \left\{ \hat{\rho}_k^H (d_k + \theta^H \text{diag}(f_k^H G)) w_k \right\} \\
& - |\hat{\rho}_k|^2 \left( \sum_{k=1}^{K} \theta^H Q_{k,j} \theta + 2\text{Re} \{\theta^H Q_{k,j}^2 \} + Q_{k,j}^3 + \sum_{r=1}^{R} \sigma^2_{r,k} + \sigma^2_0 \right) - \hat{y}' \left( \sum_{r=1}^{R} P_{r,a}(\theta_r | W') + \sum_{l=1}^{L} P_{l,tx}^A(W'_l) + P_{hw} \right)
\end{align*}
\]

(10)

(23)
in Fig. 2(b). With increasing sum rate, EE first grows fast, and reaches its peak when sum rate is around 65 bits/s/Hz. When sum rate continues increasing, EE drops significantly, because more power is consumed. Comparing different numbers of active elements, it can be found that hybrid RIS with only one active element outperforms passive RIS, and more active elements result in lower EE, due to more circuit power consumption on RIS.

The convergence of the proposed algorithm is verified in Fig. 2(c), from which we can see the results converges fast within around 5 iterations. Influence of different numbers of RIS elements $N_s$ is analyzed as well. By increasing $N_s$ from 20 to 60, the EE performance is improved due to the diversity gain of more RIS elements. Then the system EE drops slightly with $N_s = 80$, since a larger scale of RIS has higher circuit power consumption.

V. CONCLUSION

In this letter, we studied a hybrid RISs assisted cell-free network. The energy efficiency maximization problem was formulated by jointly optimizing the transmit beamforming and hybrid RIS coefficients design. We first transform the coupled fractional objective function into a concave form by introducing slack variables. Then, a BCD based iterative algorithm was proposed to solve the non-convex problem with SCA method. The simulation results showed that hybrid RIS can be better energy-efficient than the existing passive and active RISs with a small number of active elements. Also, the fast convergence of the proposed algorithm was verified by the numerical results. For future work, dynamic selecting the locations of active elements will be considered to further optimize the performance.

ACKNOWLEDGMENT

Any opinions, findings and conclusions or recommendations expressed in this material are those of the author(s) and do not reflect the views of the Ministry of Education, Singapore.

REFERENCES

[1] E. Nayeby, A. Ashikhmin, T. L. Marzetta, and H. Yang, “Cell-free massive MIMO systems,” in Proc. 49th Asilomar Conf. Signals Syst. Comput., 2015, pp. 695–699.
[2] M. Giordani, M. Polese, M. Mezzavilla, S. Rangan, and M. Zorzi, “Toward 6G networks: Use cases and technologies,” IEEE Commun. Mag., vol. 58, no. 3, pp. 55–61, Mar. 2020.
[3] Z. Lin et al., “SLNR-based secure energy efficient beamforming in multibeam satellite systems,” IEEE Trans. Aerosp. Electron. Syst., early access, Jul. 12, 2022. doi: 10.1109/TAES.2022.3190238.
[4] C. Huang, A. Zappone, G. C. Alexandropoulos, M. Debbah, and C. Yuen, “Reconfigurable intelligent surfaces for energy efficiency in wireless communication,” IEEE Trans. Wireless Commun., vol. 18, no. 8, pp. 4157–4170, Aug. 2019.
[5] Z. Yang et al., “Energy-efficient wireless communications with distributed reconfigurable intelligent surfaces,” IEEE Trans. Wireless Commun., vol. 21, no. 1, pp. 665–679, Jan. 2022.
[6] M. Di Renzo et al., “Smart radio environments empowered by reconfigurable intelligent surfaces,” IEEE Trans. Commun., vol. 70, no. 2, pp. 1457–1471, Feb. 2022.
[7] Y. Xu, H. Xie, Q. Wu, C. Huang, and C. Yuen, “Robust max–min energy efficiency for RIS-aided HetNets with distortion noises,” IEEE Trans. Commun., vol. 70, no. 3, pp. 2256–2268, Mar. 2021.
[8] Y. Zhang et al., “Beyond cell-free MIMO: Energy efficient reconfigurable intelligent surface aided MIMO communication,” IEEE Trans. Commun., vol. 70, no. 10, pp. 6555–6570, Oct. 2022.
[9] C. Huang et al., “Multi-hop RIS-empowered Terahertz communications: A DRL-based hybrid Beamforming design,” IEEE J. Sel. Areas Commun., vol. 39, no. 10, pp. 2505–2519, Oct. 2021.
[10] H. Niu et al., “Joint beamforming design for secure RIS-assisted IoT networks,” IEEE Internet Things J., vol. 10, no. 2, pp. 1628–1641, Jan. 2023.