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Hammerstein system with a stochastic input of arbitrary/unknown autocorrelation: Identification of the dynamic linear subsystem

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Abstract
For a Hammerstein system subject to a stochastic input that is spectrally coloured, this study is first in the open literature (to the present authors’ best knowledge) to estimate its linear dynamic subsystem. This estimation is achieved without any prior knowledge nor any prior/simultaneous estimation of the preceding non-linear static subsystem. This proposed estimator can handle any temporally self-correlated input despite its potentially unknown spectrum, unknown variance and unknown mean—unlike the common assumption that the input is white and zero-mean. This proposed estimator needs observations only of the Hammerstein system's overall input and consequential output, but not any observation of any intrasubsystem signal. Furthermore, this proposed estimator can handle a linear subsystem whose input and/or output are each corrupted additively by stationary (and possibly coloured) noises of unknown probability distributions, of unknown non-zero means and of unknown autocovariances. The proposed estimate is analytically proved herein as asymptotically unbiased and as pointwise consistent; and the estimate's finite-sample convergence rate is also derived analytically.

1 | INTRODUCTION

A Hammerstein system consists of two sequential subsystems: (i) a non-linear, static (memoryless) subsystem, followed by (ii) a linear, dynamic, time-invariant, asymptotically stable subsystem. Please refer to Figure 1 for a schematic showing these two subsystems and their associated signals, which will be described in great details in Section 2.

The Hammerstein system model is practical and arises in diverse engineering applications:

(i) air acoustics microphone system [1, 2]
(ii) biomedical physiology [3–8]
(iii) electrical power generator/converter [9–14]
(iv) power electronic circuitry [15–19]
(v) radio-wave communications [20]
(vi) speech coding [21]
(vii) wind turbines [22]
(viii) electrical drives [23]
(ix) laser welding [24]
(x) diesel engine's exhaust gas recirculation [25]
(xi) sticky control valves in stiction diagnosis [26]
(xii) heat exchangers [27]
(xiii) acid–base neutralization [28]
(xiv) insulation system [29]
(xv) blast furnaces [30, 31]

This study proposes a new estimator for the Hammerstein linear dynamic subsystem's (possibly Q-tap-non-causal) impulse response \( \{ h_i \} \). This estimator relies on observations of only the input and the output of the overall Hammerstein system (i.e. \( \{ U_n, Y_n \} \)), but not of any intrasystem signal nor of any intrasystem noise (e.g. \( \{ P_n, V_n, W_n, X_n, Z_n \} \)). Furthermore, this is achieved with no prior knowledge and no prior/simultaneous estimation of the leading non-linear static subsystem.
1.1 | A spectrally coloured input in system identification

Some engineering systems' input signals are necessarily coloured in their spectra: Examples include wireless telecommunication systems [33–38]. Likewise, a biomedical/chemical/mechanical/thermal system in the real world often has a physical input that would necessarily be spectrally coloured, without the luxury of a white spectrum. The excited input 'signal' often is not an electric voltage or current, whose temporal properties can be readily adjusted, but a physical process in the industrial scale that leaves little room for discretionary adjustment.

Furthermore, an input's coloured passband spectrum could facilitate the system be identified for only the specific frequency-subband over which the system operates, rather than over a wider frequency band that is practically irrelevant for the system's subsequent operation. For example, consider a wireless transmitter required to transmit over only a limited frequency band; that transmitter's and the communication channel's behaviour at that applicable frequency band would be more exactly estimated by using a spectrally coloured input with energy concentrated only over that frequency band of interest, rather than using a white input whose energy could be dispersed also over the out-of-band frequencies.

Among the many existing estimation algorithms [39–51] of an Hammerstein system's linear dynamic subsystem, all of them explicitly deal with only a spectrally white input. This study alone affronts any coloured spectrum in the input time series, that is, the input time series' autocorrelation function may be arbitrary and/or unknown to the estimator, so long if it is a finite-power. ²

1.2 | Contribution of this work

The proposed estimator can handle the following accommodating models of the Hammerstein system, its coloured input and its unobservable internal perturbation:

(i) The unknown non-linear static subsystem may be any function that is piecewise differentiable and magnitude-bounded—thus beyond the confines of any parametric class. This non-linearity may be non-invertible.

(ii) The unknown linear dynamic subsystem's impulse response may have any finite order, which may remain unknown. Furthermore, this subsystem could be non-causal, with up to Q number of anticausal taps in its impulse response.

(iii) The Hammerstein system's input signal is modelled as random, wide-sense stationarity, Gaussian-distributed with a (possibly) unknown variance, a (possibly) non-zero unknown mean and as possibly coloured with an unknown frequency spectrum.

(iv) In between the Hammerstein system's two subsystems, there could be additive disturbance modelled as an unobservable wide-sense stationary random sequence of \( Z_m \) of possibly non-Gaussian probability distribution, a (possibly) non-zero mean that may be a priori unknown, a (possibly) unknown variance and a (possibly) coloured self-correlation (which may also be unknown).

(v) The linear subsystem's output may be disturbed additively, before the observations are made, by a wide-sense stationary noise sequence \( \{ P_n \} \) of a (possibly) non-zero mean (this may be unknown), a (possibly) unknown variance and a (possibly) coloured self-correlation (which may also be possibly unknown). ²

This work will also analytically prove the following:

(a) The proposed estimate \( \{ \hat{h}_i \} \) is asymptotically unbiased and statistically consistent, regardless of whether the linear

Though [47, 50, 51] explicitly presume the input to be spectrally white, their methods could actually handle spectrally coloured input, though perhaps unrecognized by their authors as so. However, all these four estimators of the Hammerstein linear dynamic subsystem require first the estimation of the non-linear static subsystem—a hassle avoided in the present study. Incidentally, for a block-based system's identification using a spectrally coloured input as system excitation—that has been achieved also for a Wiener system in [52, 53], but those methods are inapplicable to the present Hammerstein system. The Wiener system and the Hammerstein system, of course, differ by how they sequentially link the non-linear subsystem and the linear subsystem—a Wiener system has linear then non-linear, whereas a Hammerstein system has non-linear then linear. Such a reversed sequencing makes fundamental differences in the system's internal dynamics and thus in the system-identification strategy. At a Wiener system, the non-linear subsystem's unobservable input signal is effectively first 'coloured' by the preceding linear subsystem, whether the Wiener system's overall observable input is itself spectrally coloured. In contrast, an Hammerstein system's non-linear subsystem is excited by an observed input (whose temporal self-correlation may be estimated from the observations), but the Hammerstein system's observed output is a superposition of time-delayed echoes from the non-linear subsystem.
subsystem has an infinite impulse response (IIR) or a finite impulse response (FIR).

(b) The proposed estimate's convergence rate will be analytically derived in Section 4 and then will be verified by the finite-sample Monte Carlo simulations in Section 5.

1.3 Organization of this study

The rest of this study is organized as follows: Section 2 will define the mathematical model of the Hammerstein system's two subsystems and will define the statistics of their associated signals and noises. Section 3 will develop a new parametric estimator of the linear subsystem. Section 4 will theoretically prove this linear subsystem estimator as statistically consistent and asymptotically unbiased. This estimate's convergence rate will also be analytically derived therein. Section 5 will verify the estimator's performance by finite-sample Monte Carlo simulations. Section 6 will conclude the entire study and will point to possible directions for future research.

2 MATHEMATICAL MODEL OF THE SUBSYSTEMS AND THEIR ASSOCIATED TIME SERIES

The Hammerstein system's constituent subsystems, along with their associated signals and noises, have been represented in the block diagram in Figure 1:

(a) The overall system's input signal \( \{ U_n, n = 1, \ldots, N \} \) has

(a-i) a mean of \( \mu_u \) that can be non-zero and unknown.

(a-ii) a variance of \( \sigma_u^2 \) (that needs not be known prior), \( \forall n \).

(a-iii) a spectrum that can be coloured and unknown.

(a-iv) a Gaussian probability distribution.\(^{17}\)

(b) The static non-linear subsystem \( m(\cdot) \) is required to satisfy only the general requirements of being piecewise differentiable and magnitude bounded.\(^{18}\) The non-linear subsystem's unobserved output equals:

\[
W_n := m(U_n). 
\] (2.1)

(c) To the aforementioned non-linear subsystem's unobserved output \( \{ W_n \} \), a temporally wide-sense stationary noise \( \{ Z_n \} \) may possibly be added. This \( \{ Z_n \} \) may be Gaussian or non-Gaussian, with a possibly unknown and possibly non-zero mean of \( \mu_z \) (unlike the \( \mu_u = 0 \) assumption in \([45, 72]\)), a possibly unknown autocovariance function and statistical independence from the system's input \( \{ U_n \} \).

This \( \{ Z_n \} \) produces the corrupted output \( V_n := W_n + Z_n \), which becomes input to the linear subsystem.

(d) The linear dynamic subsystem's impulse response \( \{ h_i, i = -Q, -Q+1, \ldots, -Q+I-1 \} \) may have a finite or an infinite number (I) of taps, with I known prior.\(^{19}\) Moreover, the impulse response may be non-causal, that is \( Q > 0 \). This impulse response is assumed as asymptotically stable, that is,

\[
\sum_{i=1}^{\infty} |h_i| < \infty. 
\] (2.2)

For the special case of the linear subsystem having IIR, the requirement needs to be stricter as exponentially bounded:\(^{17,18,19}\)

\[
|h_i| \leq e_i |x^i|, \quad \forall i, \tag{2.3}
\]

with \( 0 < e < 1 \).\(^{20}\)

(e) To the above linear subsystem's output, a random disturbance \( \{ P_n \} \) may possibly be added to yield the overall observed output,

\[
Y_n = \sum_{i=1}^{\infty} h_i V_{n-i} + P_n. 
\] (2.4)

This \( \{ P_n \} \) may have a (possibly) non-zero mean of \( \mu_p \) and a (possibly) unknown and (possibly) coloured autocovariance function. This \( \{ P_n \} \) is statistically independent from \( \{ U_n \} \) and from \( \{ Z_n \} \).

3 A NEW PARAMETRIC ESTIMATOR OF THE LINEAR SUBSYSTEM

3.1 To account for the additive noise entering between the two subsystems

The proposed estimator can accommodate a noise process of \( \{ Z_n \} \) that arises between the two subsystems, thereby additively degrading the first subsystem’s output, that is corrupting the second subsystem’s input (see Figure 1). This study permits (NOT requires) the simultaneous presence of this internal noise \( \{ Z_n \} \) and the output noise \( \{ P_n \} \), unlike \([45, 57]\), which allow only the internal noise but disallow the output noise \( \{ P_n \} \). Also, this is unlike \([41, 46–48, 50, 51, 55, 56]\), which allow only output noise \( \{ P_n \} \) but disallow the internal noise \( \{ Z_n \} \).

These two noise processes complicate the estimation task differently, but they produce qualitatively similar degradations on the Hammerstein system.

1. The internal noise \( \{ Z_n \} \) produces an \( \sum_{i=-Q}^{-1} h_i [m(U_{n-i}) + Z_{n-i}] \) that is vertically displaced from the noiseless

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\(^1\)The input \( \{ U_n \} \) needs to be Gaussian to identify the linear subsystem, only because Bussgang's theorem in Equation (3.4) requires a Gaussian distribution of \( \{ U_n \} \).

\(^2\)That is, \( \sup_{n\rightarrow\infty}|m(n)| = m_{\text{sup}} < \infty \).

\(^3\)The proposed estimator would still work even if \( I \) is not precisely known but only an upper bound is known of \( I \).

\(^4\)Incidentally, the assumption in Equation (2.3) has also been made in \([52, 53]\).
\( \sum_{i=-Q}^{I-Q-1} b_i m(U_{n-i}) \). Please refer to Figure 2a for the degenerate case of \( I = 1 \), wherein the deterministic \( m(U_n + Q) \) is plotted as a solid curve, the realizations of the random \( m(U_{n+Q}) + Z_{n+Q} \) are shown as icons, and the deterministic \( m(U_n) + \mu_n \) is plotted as the dashed curve. Note how the dashed curve for \( m(U_{n+Q}) + \mu_n \) is vertically displaced above/below of the solid curve of \( m(U_n) \).

2. The output noise \( \{P_n\} \) produces \( P_n + \sum_{i=-Q}^{I-Q-1} b_i m(U_{n-i}) \), that is vertically displaced from the noiseless \( \sum_{i=-Q}^{I-Q-1} b_i m(U_{n-i}) \). Please refer to Figure 2b for the degenerate case of \( I = 1 \), wherein the deterministic \( b_{-Q}m(U_{n+Q}) \) is plotted again as a solid curve, the random \( P_n + b_{-Q}m(U_{n+Q}) \) are shown as icons and the deterministic \( \mu_n + b_{-Q}m(U_{n+Q}) \) is plotted as the dashed curve. Note how the dashed curve for \( \mu_n + b_{-Q}m(U_{n+Q}) \) is vertically displaced above/below the solid curve of \( b_{-Q}m(U_{n+Q}) \).

More generally, the following analysis shows that the output noise \( \{Z_n\} \) generally can ‘merge’ into the Gaussian internal noise \( \{P_n\} \).

\[
\begin{align*}
\gamma_n &:= \sum_{i=-Q}^{I-Q-1} b_i [m(U_{n-i}) + Z_{n-i}] + P_n \\
&= \sum_{i=-Q}^{I-Q-1} b_i m(U_{n-i}) + P_n + \sum_{i=-Q}^{I-Q-1} b_i Z_{n-i}.
\end{align*}
\tag{3.1}
\]

Incidentally, [39, 49, 54] also investigate a Hammerstein system with both an internal noise \( \{Z_n\} \) and an output noise \( \{P_n\} \), though those disturbances are modelled in [39, 49, 54] to be zero-mean white additive noises.

### 3.2 To estimate the linear dynamic subsystem’s impulse response

Within the literature of Hammerstein system identification using a coloured random input: No study exists (to the best of the present authors’ knowledge) to identify the linear dynamic subsystem. This present study is the first to do so.

To facilitate the subsequent development, define:

\[
\begin{align*}
\gamma_{m,n}(\kappa) &:= \text{Cov}(m(U_n), U_{n-\kappa}), \\
\gamma_{y,n}(\kappa) &:= \text{Cov}(Y_n, U_{n-\kappa}).
\end{align*}
\]

Next, multiply the observed output in Equation (2.4) by \( \{U_{n+Q-s}, s = 0, 1, 2, \ldots\} \) and then take the expectation of both sides, thereby yielding:

\[
\gamma_{y,n}(s-Q) = \sum_{i=0}^{I-1} b_{-Q+i} \gamma_{m,n}(s-i),
\tag{3.2}
\]

where \( I \to \infty \) for an IIR linear subsystem, but \( I \) would be a natural number for the FIR case.\(^2\) Equation (3.2) may be re-expressed in a matrix form as:

\(^{2}\)If the Hammerstein system’s input was temporally uncorrelated, then Equation (3.2) would simplify to \( \gamma_{y,n}(s-Q) = b_{-Q+i} \gamma_{m,n}(s) \).

\(^{1}\)Only this degenerate case of \( I = 1 \) is plotted, because \( \sum_{i=-Q}^{I-Q-1} b_i m(U_{n-i} + Z_{n-i}) \) cannot be plotted on paper simultaneously versus the \( I + 1 \) independent parameters of \( m(U_{n-Q}), \ldots, m(U_{n-Q-J+1}) \).
if \( I \to \infty \) for an IIR linear subsystem, or if \( k \geq I \) for FIR.\(^3\)

A direct estimation of \( \gamma_{m,u}(k) \) would require a prior knowledge of the unknown non-linearity, \( m(u) \), as in Equation (2.1). To avoid that, the presently proposed estimator will instead use the Bussgang theorem [73] below to estimate \( \gamma_{m,u}(k) \).

Because \( \{ U_n \} \) is a Gaussian random process and because \( V_n = m(U_n) \) is a non-linear function of \( U_n \) but unobservable, an application of Bussgang theorem\(^2\) gives:

\[
\gamma_{m,u}(k) = b\gamma_u(k), \quad \forall k = 0, 1, 2, \ldots
\]

where \( b \equiv E[m'(u)] \) represents a constant. Replacing the \((i,j)\)th entry of \( \Gamma_k^{(m,u)} \) with \( b\gamma_u(i-j) \) of Equation (3.3), one has,

\[
h_k = [b\gamma_u(0); b\gamma_u(1); \ldots; b\gamma_u(k-1); b\gamma_u(k-2) \ldots b\gamma_u(0)]^T,
\]

Hence, the IIR may be estimated through,

\[
\hat{\gamma}_k = (\hat{\Gamma}_k)^{-1}\gamma_k,
\]

where

\[
\hat{\gamma}_k = (\hat{\gamma}_y(-Q), \ldots, \hat{\gamma}_y(k-1-Q))^T
\]

and

\[
\hat{\gamma}_y(k) = 1 \sum_{n=1}^{N-k} (Y_n - \bar{Y})(U_{n-k} - \bar{U}),
\]

\[
\forall k \in \{-Q, 1-Q, \ldots, k-1-Q\};
\]

\[
\hat{\gamma}_u(k) = 1 \sum_{n=1}^{N-k} (U_n - \bar{U})(U_{n-k} - \bar{U}),
\]

\[
\forall k \in \{0, 1, \ldots, k-1\}.
\]

Recall that \( \{ U_n, Y_n, \forall n \} \) are observations, from which \( \hat{U} := \frac{1}{N} \sum_{n=1}^{N} U_n \) and \( \hat{Y} := \frac{1}{N} \sum_{n=1}^{N} Y_n \) may be computed.

In contrast to the direct estimation of \( \gamma_{m,u}(k) \), this estimate in Equation (3.6) does not need any prior knowledge of the unknown non-linearity \( m(\cdot) \). The above estimate does not need any prior knowledge of \( \mu_n \) because both \( \hat{\gamma}_y(k) \) and \( \hat{\gamma}_u(k) \) are independent of \( \mu_n \).\(^2\)

The estimate of \( \{ \hat{b}_n \} \) in Equation (3.6) is admitted only within an indeterminable multiplicative constant, \( b \). However, this unknown constant scaling factor is unavoidable as may be seen from the following example in p. 539 of [73], or p. 1931 of [75]. Define a first overall system to have \( m(\cdot) \) followed by \( \{ \hat{b}_n \} \); and define a second system to have \( m(\cdot/b) \) followed by \( \{ b\hat{b}_n \} \). Then, both systems would produce identical outputs to any same input, if \( Z_u = 0, \forall u \).

Because the \( bb_n \) (for the \( n \)th time-instant) requires the observables \( \{ Y_1, \ldots, Y_{n+1} \} \) (which extend beyond the \( n \)th time-instant), there would be estimation delay. Nonetheless, that would not change the fact that \( \hat{b}_n \) (hence \( b_n \)) may be estimated to only within an indeterminable multiplicative constant. Any non-zero mean \( \mu_z \) of the additive noise \( \{ Z_n \} \) in the linear subsystem input \( \{ V_n \} \) and thus in \( \{ Y_n \} \) would affect neither \( b \) nor \( \hat{\gamma}_y(k) \) in Equation (3.6). Any non-zero mean \( \mu_p \) of the additive noise \( \{ P_n \} \) in the linear subsystem output \( \{ Y_n \} \) would affect neither \( b \) nor \( \hat{\gamma}_y(k) \) in Equation (3.6). Furthermore, the above estimate permits the additive noises' means, \( \mu_z \) and \( \mu_p \), to be non-zero and a \textit{priori} unknown.

### 3.3 Summary of the proposed algorithm's key steps

The above-developed estimator's key algorithmic steps are summarized below:

1. Take the input/output observations \( \{ U_n, Y_n, \forall n \} \) to compute \( \hat{U} := \frac{1}{N} \sum_{n=1}^{N} U_n \) and \( \hat{Y} := \frac{1}{N} \sum_{n=1}^{N} Y_n \).
2. Substitute the input/output observations and the above-computed \( \hat{U} \) and \( \hat{Y} \) into Equation (3.7), to yield the cross-covariance \( \hat{\gamma}_{y,u}(k) \), \( \forall k \in \{-Q, 1-Q, \ldots, I-1-Q\} \).
3. Substitute the input/output observations and the above-computed \( \hat{U} \) into Equation (3.7), to yield and into Equation (3.9) to give the autocovariance \( \hat{\gamma}_u(k) \), for \( \forall k \in \{0, 1, \ldots, I-1\} \).
4. Substitute the above-computed \( \hat{\gamma}_{y,u}(k) \) and \( \hat{\gamma}_u(k) \) into Equation (3.6) to produce the estimates of the \( I \) number of

\(^2\)If the Hammerstein system's input signal was temporally uncorrelated, then \( \Gamma_k^{(m,u)} \) would simplify to \( \gamma_u(0) \Gamma_k \) in Equation (3.3).

\(^3\)Bussgang theorem: Let \( \{ X \} \) be a stationary Gaussian random process and \( \{ Y \} = m(X) \), where \( m(\cdot) \) constitutes a non-linear amplitude distortion. If \( \gamma_y(0) \) is the autocorrelation function of \( \{ X \} \) and \( \{ Y \} \) is \( \gamma_y(0) = b\gamma_u(0) \), where \( b \equiv E[m'(X)] \) is a constant that depends only on \( m(\cdot) \). That is, a time series' temporal correlation is unchanged (except by a multiplicative constant) by passing through a static non-linearity.
coefficients of the Hammerstein linear subsystem’s impulse response.

The above steps (1–3) can handle stochastically the Gaussian input that is temporally self-correlated with any (possibly unknown) autocorrelation function, with any (possibly unknown) mean or any (possibly unknown) variance.

On contrasting the above against a simpler case of the input known to be temporally uncorrelated. The above algorithmic steps (3) and (4) would simplify to steps (3) and (4) below:

(3′) Substitute the input/output observations and the above-computed \( \hat{\Omega} \) into Equation (3.9) to yield the one covariance \( \hat{\gamma}_u(0) \).

(4′) Substitute the entities in steps (2) and (3′) into Equation (3.9) to compute the linear subsystem’s estimate \( \hat{b}h \).

The key difference in computational load is the inversion of a \( k \times k \) matrix needed presently in this more widely applicable case of a self-correlated input in step (3), but only a scalar inversion in the more restricted case of an uncorrelated input in step (3′).

4 | ASYMMETRIC ANALYSIS OF THE ESTIMATOR PROPOSED IN SECTION 3 FOR THE LINEAR SUBSYSTEM

Theorem 1

(i) If the Hammerstein linear subsystem has an FIR, then \( \hat{b}h = bh + O_p(N^{-\frac{T}{2}}) \). This means that \( \hat{h} \) is a consistent estimate of \( h \).

(ii) If the Hammerstein linear subsystem has an IIR that is exponentially bounded as in Equation (2.2) and if furthermore \( k = O(N^k) \) as \( N \to \infty \), then \( \hat{b}h = bh + O_p(N^{-\frac{T}{2}}) \). Here, \( h = \left[ b_1, b_2, \ldots, b_{N-1} \right]^T \) and \( \hat{h} = \left[ \hat{h}_1, 0, 0, \ldots \right]^T \).

The above steps (i) and (ii) imply that \( \hat{h} \) is an asymptotically unbiased and consistent estimate, regardless if the linear subsystem’s impulse response is finite or infinite. Theorem 1(ii) also gives a convergent rate \( O_p(N^{-\frac{T}{2}}) \), which implies \( O_p(1) \), for the linear subsystem’s impulse response estimate, if the linear subsystem is IIR.26

In the above:

(a) \( \xi_N = O_p(\delta_N) \) means that the random sequence \( \{ \xi_n, \forall n = \ldots, N - 2, N - 1, N \} \) is asymptotically bounded in probability as \( N \to \infty \), where \( \delta_N \) is a positive deterministic scalar for each \( N \).

(b) \( \xi_N = o_p(\delta_N) \) means that the random sequence \( \{ \xi_n, \forall n = \ldots, N - 2, N - 1, N \} \) asymptotically converges to zero in probability as \( N \to \infty \).

(c) \( \xi_N = O(\delta_N) \) means that the random sequence \( \{ \xi_n, \forall n = \ldots, N - 2, N - 1, N \} \) is deterministically bounded asymptotically as \( N \to \infty \).

(d) \( \xi_N = o(\delta_N) \) means that the random sequence \( \{ \xi_n, \forall n = \ldots, N - 2, N - 1, N \} \) asymptotically converges to a deterministic zero as \( N \to \infty \).

Proof of Theorem 1

(i) To evaluate the estimator’s performance if the linear subsystem having an FIR: The strict-sense stationarity of \( U_n \) gives:

\[
\hat{\gamma}_u(k) = \gamma_u(k) + O_p\left(N^{-\frac{T}{2}}\right),
\]

\[
\left\| I_j^{(a)} - I_j^{(a)-1} \right\| = O_p\left(N^{-\frac{T}{2}}\right),
\]

\[
\left\| I_j^{(a)-1} \right\| = O(1).
\]

Furthermore, using \( \left\| \hat{\gamma}_u^{(y,a)} - \gamma_u^{(y,a)} \right\| \leq \left\| \hat{\gamma}_u^{(y,a)} - \gamma_u^{(y,a)} \right\| + \left\| \gamma_u^{(y,a)} \right\| = O_p(1) \), it follows that:

\[
\left\| \hat{b}h - bh \right\| = \left\| I_j^{(a)-1} \hat{\gamma}_u^{(y,a)} - I_j^{(a)-1} \gamma_u^{(y,a)} \right\| 
\leq \left\| I_j^{(a)-1} \right\| \left\| \hat{\gamma}_u^{(y,a)} - \gamma_u^{(y,a)} \right\| 
+ \left\| I_j^{(a)-1} \right\| \left\| \gamma_u^{(y,a)} \right\| = O_p\left(N^{-\frac{T}{2}}\right).
\]

All norms above are Euclidean, with finite-size operands.27

(ii) To evaluate the performance of the linear system estimator if the linear subsystem having an IIR: Choosing \( k \) as mentioned in the Theorem 1 and using the Lemma 3 of

\[\text{With regard to the above proof for a coloured input, that proof non-trivially generalizes the spectrally white case, for which the inequality’s right side would simply be }\]

\[\left\| \hat{\gamma}_u^{(1,0)} - \gamma_u^{(1,0)} \right\| + \left\| \hat{\gamma}_u^{(0,1)} - \gamma_u^{(0,1)} \right\| \text{. The generalization of the white case's scalar difference }\hat{\gamma}_u^{(1,0)} - \gamma_u^{(1,0)} \text{ to the coloured case's matrix difference }\left\| I_j^{(a)-1} \right\| \text{ is non-trivial. Likewise, the generalization of the white case's scalar }\gamma_u^{(0,1)} \text{ to the coloured case's matrix difference }\left\| I_j^{(a)-1} \right\| \text{ is non-trivial.}\]

26This convergent rate of the proposed linear subsystem estimate subject to any arbitrarily coloured input is not inferior to but same as that in [44, 45, 47, 49, 50, 56] with simply a white input.
[32], it holds that \( \left\| \Gamma_{k}^{-1} - I_{k}^{-1} \right\| = O_p(1/k) \). Moreover, using
\[
\left\| \Gamma_{k}^{(n)} - \Gamma_{k}^{(u)} \right\| = O_p(1).
\]
\[
\left\| y_{k}^{(n)} - y_{k}^{(u)} \right\| = O_p\left( kN^{-2/3} \right),
\]
\[
\left\| (\gamma_{k}^{(n)}) - (\gamma_{k}^{(u)}) \right\| \leq O_p(1) + O(1),
\]
it holds that:
\[
\left\| h_{k} - b_{k} \right\| \approx \left\| \Gamma_{k}^{(n)} - \Gamma_{k}^{(u)} \right\| \leq \left\| \Gamma_{k}(\gamma_{k}^{(n)} - \gamma_{k}^{(n)}) \left\| \Gamma_{k}^{(n)} \right\| \right\|
\]
\[
\left\| \Gamma_{k}(\gamma_{k}^{(n)} - \gamma_{k}^{(n)}) \right\| \leq O_p\left( kN^{-2/3} \right).
\]

As the impulse response has been assumed to be exponentially bounded, the approximation error in Equation (4.3) is below \( k^{-2} \), which is \( o(1) \) if \( k \) is sufficiently large. Hence, the best convergent rate for the above Equation (4.4) is \( O_p\left( N^{-2/3} \right) \).

The proof becomes complete by selecting \( k = O\left( N^{2/3} \right) \).

5 MONTE CARLO SIMULATIONS OF THE PROPOSED ESTIMATOR OF THE LINEAR SUBSYSTEM

Monte Carlo simulations here will verify the efficacy of the earlier proposed estimator under finite-sample settings.

All simulations in this section will use the following simulation scenario:

\( \{ \xi_{n} \} \) is a temporally coloured, set to this particular MA(2) form:
\( Z_{n} = \mu + \xi_{n} + 0.5\xi_{n-1} + 0.06\xi_{n-2} \), with \( \mu = 2 \). Hence, \( \{ \xi_{n} \} \) represents a spectrally white but non-Gaussian stochastic sequence, uniformly distributed over \([-1, 1]\) at each \( n \).

The noise \( \{ Z_{n} \} \) entering between the two subsystems is temporally coloured, set to this particular MA(2) form:
\( \tilde{Z}_{n} = \mu + \tilde{\xi}_{n} + 0.5\tilde{\xi}_{n-1} + 0.06\tilde{\xi}_{n-2} \), with \( \mu = 2 \). Here, \( \{ \tilde{\xi}_{n} \} \) is non-linear and has been used in [52, 53] to model an electronic power amplifier.

(i) The Hammerstein system input \( \{ U_{n} \} \) is a second-order ‘moving average’ process (i.e. MA(2)) of \( U_{n} = \mu_{n} + e_{n} + 1.2e_{n-1} + 0.32e_{n-2} \), with \( \mu_{n} = 1 \). Here, \( \{ e_{n} \} \) represents a spectrally white stochastic sequence, having a zero mean and a unit variance, at each \( n \).

(ii) The non-linearity is set as \( m(\mu_{n}) = \text{sign}(\mu_{n}) \left( (\mu_{n}) + 1 \right) \), where sign(·) symbolizes the sign of the scalar inside the parenthesis. This dynamic non-linearity has been used in [52, 53] to model an electronic power amplifier.

(iii) The noise \( \{ Z_{n} \} \) entering between the two subsystems is temporally coloured, set to this particular MA(2) form:
\( \tilde{Z}_{n} = \mu_{n} + \tilde{\xi}_{n} + 0.5\tilde{\xi}_{n-1} + 0.06\tilde{\xi}_{n-2} \), with \( \mu_{n} = 2 \). Here, \( \{ \tilde{\xi}_{n} \} \) is non-Gaussian as in step (iii) above.

(iv) Each icon on each graph represents \( L = 1000 \) statistically independent Monte Carlo experiments, each with \( N = 400 \) snapshots, unless otherwise indicated.

Figure 3 plots the errors in estimating the linear subsystem's individual impulse-response tabs of \( \left\{ \frac{h_{k,b}}{b_{k}} \right\} \). Figure 3(a) verifies the proposed estimator's asymptotic unbiasedness—note how the estimation means (defined as the algebraic mean of all estimates from the \( L \) Monte Carlo trials and marked by blue * ) are very close to the true values (marked by red circles). Incidentally, the ± one-standard-deviation range is indicated by a pair of horizontal bars. Figure 3(b) verifies the proposed estimator's statistical efficiency (which has already been analytically proved in Theorem 1(i), by plotting the estimator's standard deviation \( \frac{h_{k,b}}{b_{k}} \) at each of \( i = -1, 0, 1 \) over the \( L \) Monte Carlo trials versus the number \( (N) \) of snapshots in each trial. Recall that Theorem 1(i) theoretically predicts a slope of \(-0.5\) in the convergence rate of \( \text{Var}(\tilde{b}_{i}) = O\left( \frac{1}{N} \right) \). The Monte Carlo icons give a least-squares-fit slope of \(-0.48 \) for \( \frac{h_{k,b}}{b_{k}} \) and \(-0.49 \) for \( \frac{h_{k,b}}{b_{k}} \) all within 2% of the theoretically derived \(-0.5\).

The proposed estimator's efficacy to handle coloured input, as documented above, can be contrasted against the ineffectiveness by the existing estimators of [39–46, 48, 49]. Those existing estimators were designed with white input in mind; their estimates are clearly very biased in Figure 4—thereon the bias is the difference between the estimation means (marked as ×) and the true values (marked as ◦).
The proposed estimator’s categoric improvement over these existing estimators (to handle coloured input) is further documented in Figure 5 in terms of the ‘global error’ $\text{Err}^{\text{c.b.h.}}(\cdot)$ (p. 1491 of [76]),

$$\text{Err}^{\text{c.b.h.}}(\cdot) := \frac{1}{L} \sum_{\ell=1}^{L} \left| \frac{\tilde{h}^{(\ell)} - b h}{\|b h\|} \right|,$$

where $\|\cdot\|$ represents the Euclidean norm, and $\tilde{h}^{(\ell)}$ symbolizes the $\tilde{h}$ in the $\ell$th Monte Carlo trial. Hence, this

![Figure 3](image1.png)

**Figure 3** Monte Carlo simulations verify the unbiasedness and the statistical efficiency of the proposed parametric estimator of the Hammerstein dynamic linear non-causal subsystem’s impulse response $\tilde{h}^{(\ell)}$, despite the input’s temporal correlation and non-zero mean, despite the perturbing noise entering between the two subsystems, and despite the perturbation to the output before observation.

The proposed estimator's categoric improvement over these existing estimators (to handle coloured input) is further documented in Figure 5 in terms of the ‘global error’ $\text{Err}(\tilde{b} h)$ (p. 1491 of [76]),

$$\text{Err}(\tilde{b} h) := \frac{1}{L} \sum_{\ell=1}^{L} \left| \frac{\tilde{h}^{(\ell)} - b h}{\|b h\|} \right|,$$

where $\|\cdot\|$ represents the Euclidean norm, and $\tilde{h}^{(\ell)}$ symbolizes the $\tilde{h}$ in the $\ell$th Monte Carlo trial. Hence, this

![Figure 4](image2.png)

**Figure 4** How the existing estimator from [39–46, 48, 49] is greatly biased when the input is coloured.

![Figure 5](image3.png)

**Figure 5** How the proposed estimator is more accurate than the existing estimator, under the ‘global error’ measure of $\text{Err}(\tilde{b} h)$.

$\text{Err}(\tilde{b} h)$ is one single scalar that regards the entire impulse response as a vector and that measures the error in estimating this vector. Figure 5 shows the proposed estimator’s $\text{Err}(\tilde{b} h) \rightarrow 0$ as the sample size $N$ increases from 32 to 2048, whereas the existing estimators suffer a high error floor near 0.9.

### 6 | CONCLUSION

For a Hammerstein system subjected to a spectrally coloured random input, the constituent linear dynamic subsystem is estimated here for the first time in the open literature (to the best
knowledge of the present authors), without any prior knowledge/estimation/parameterization of the non-linear static subsystem. Here the proposed estimator has been analytically proved as consistent and asymptotically unbiased. Its efficacy and derived statistics are affirmed by Monte Carlo simulations.

For the Hammerstein system's other constituent subsystem (i.e. the non-linear static subsystem), a non-parametric estimator has been developed in a companion study [77] for a similar signal/noise statistics model as indicated herein. These two subsystems' estimators are algorithmically independent from each other, in the sense that each may be computed without computing the other, or that the two may be computed simultaneously in parallel.

This work's proposed estimator can handle a coloured input of any given power spectrum, but follow-up investigations could adapt the input's spectrum to the Hammerstein system to be identified.

This work's proposed estimator processes the input/output observations in a batch. Future follow-up investigations could develop an iterative algorithm to update/modify a recent estimate by incorporating new observations without recomputing for the entire observed dataset, thereby reducing the computational load to facilitate a real-time application.

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