Non-abelian vector boson dark matter, its unified route and signatures at the LHC

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Abstract. Vector boson dark matter (DM) appears in SU(2)_N extension (N stands for neutral) of Standard Model (SM) where an additional global U(1)_P symmetry is assumed and results in a generalized lepton number defined as: \( L = P + T_3N \). Breaking of U(1)_P leads to the breaking of \( L \) to \((-1)^L\), thus stabilizing DM through modified R = \((-1)^{3B+L+2J}\). This model, already discussed in literature, offers several novel features to elaborate upon. For example, t-channel annihilation and dominant s-channel direct search, along with co-annihilation, helps the DM to evade stringent direct search bounds from LUX and XENON1T after satisfying relic density constraints. On the other hand, the exotic particles of the model can be produced at the Large Hadron Collider (LHC) yielding multilepton final states. Hadronically quiet four lepton signal with large missing energy, in specific, is shown to provide a smoking gun signature of such a framework. We study the details of \( E(6) \rightarrow SM \otimes SU(2)_N \) breaking patterns (through D-parity odd/even cases) which yield important phenomenological consequences.

Keywords: dark matter experiments, dark matter simulations

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1 Introduction

Astrophysical observations, such as rotation curve of spiral galaxies around the cluster [1, 2], inhomogeneity in cosmic microwave background radiation (CMBR) [3], or more recent observations in Bullet cluster [4] indicate towards the existence of dark matter (DM) in the universe. A particle description of DM is much sought after and lot of efforts are made (for a brief review, see for example, [5, 6]) to accommodate DM in extensions of standard model (SM). Thermal freeze-out of weakly interacting massive particles (WIMP) (see for example, [7]) provides the most popular framework of DM phenomenology. While an experimental verification of DM is still awaited, a plethora of DM models are constructed assuming the DM to be a scalar, fermion or a vector boson.

Scalar (see for example, [8–13]) and fermionic (see for example, [14–21]) DMs can be realized with minimal extensions to SM and are discussed widely. However, vector boson DMs are rather difficult to come across as it requires an extension of the SM gauge group or even more exotic frameworks. Abelian vector boson DM appears in universal extra dimension models with conserved $KK$ parity [22–24], in little Higgs framework with $T$ parity [25, 26] or in U(1) extensions of SM (for example, see [27, 28]). In this paper we have analyzed the phenomenology of a non-abelian vector boson DM, which arises in an extension of SM by $SU(2)_N$ gauge group, that has been proposed in [29–32]. $SU(2)_N$ is broken subsequently and yields
massive gauge bosons. This gauge group is dark in the sense that SU(2)$_N$ charges do not contribute to the hypercharge. Thus all three SU(2)$_N$ gauge bosons are electromagnetic charge neutral and the lightest one can in principle be a DM. The stability of the DM is ensured by an extra global symmetry U(1)$_E$, breaking of which explicitly breaks the generalized lepton number $L = P + T_{3N}$ to $(-1)^L$ leading to a discrete $R$–parity as in supersymmetric theory. One of the main motivations of such a framework is to envisage a unified theory, $E(6)$ \cite{[30]}, which subsequently breaks to $SM \otimes SU(2)_N$. In the process of $E(6) \rightarrow SM \otimes SU(2)_N$, $D$-parity is broken in presence of an SU(2)$_N$ scalar triplet which allows $g_N$ to be different from SU(2)$_L$ gauge coupling and serves as a parameter of the theory. VEV of this triplet scalar breaks the degeneracy of the dark gauge bosons and ensures a single component DM. Absence of the SU(2)$_N$ triplet (D-parity even case) leads to a constrained degenerate DM scenario, which is not viable from direct search constraints. Intermediate symmetry breaking takes care of the fact that hypercharge is determined without SU(2)$_N$ contribution, ensuring charge neutrality of the SU(2)$_N$ gauge bosons.

Non-Abelian vector boson DM has been addressed in some other contexts as well (see for example, \cite{[33–37]}). DMs in context of SO(10) Grand Unified Theory (GUT) has also been discussed (see for example \cite{[38, 39]}), but mostly for scalars and fermions.

Phenomenology of the non-abelian vector boson DM is a major motivation of this paper and this is discussed more elaborately compared to the earlier analyses of this model \cite{[30, 31]}. In particular, non observation of DM particle in direct search experiments is putting a huge constraint on the single component DM parameter space, eventually ruling out a large class of WIMP models from spin-independent direct search cross-sections. However, the model discussed here survives in a large region of relic density allowed parameter space from WMAP \cite{[40]}/PLANCK \cite{[41]} data with current direct search bounds from LUX \cite{[42]}. This is mainly due to its t-channel annihilation and dominant s-channel direct search interaction along with co-annihilation features with heavier gauge boson. This has been shown through elaborate parameter space scan, assuming SU(2)$_N$ coupling as an independent parameter (and obeying unification prescription), unlike the previous analyses.

Extended gauge group demands the presence of exotic fermions which can be produced at the Large Hadron Collider (LHC). They subsequently decay to DM providing a variety of signatures of multilepton final states that can testify such a framework in future runs of LHC. Two most promising signals of this model are discussed in this work as ‘opposite sign dilepton’ and ‘hadronically quiet four lepton’ with large missing energy. It has been explicitly shown, by analyzing three different benchmark points, that ‘hadronically quiet four lepton’ signal is a smoking-gun signature for this model, typically because of less SM background. This potentially interesting feature was also overlooked in the earlier analyses.

The paper is organized as follows: the model and the basic formalism is discussed in section 2; DM phenomenology is discussed elaborately in section 3 including relic density and direct search constraints. In section 4, we study unified framework under $E_6$ and subsequent constraints on the model parameters. Some benchmark points, identified thereafter, are studied for collider signatures in section 5. We finally conclude in section 6.

2 Dark vector multiplet and exotic particles

The theory under consideration is SU(2)$_N \otimes SU(2)_L \otimes U(1)_Y \otimes SU(3)_C$ gauge symmetric. An additional global abelian symmetry U(1)$_P$ has also been introduced. This redefines the lepton number of the particles as $L = P + T_{3N}$. Explicit breaking of U(1)$_P$ breaks $L$ to
(-1)^L and results in conservation of a discrete symmetry $R = (-1)^{3B+L+2J}$ (very similar to $R$-parity in supersymmetry). This stabilizes the lightest particle odd under $R$-parity and serves as DM. SU(2)$_N$ charges do not contribute to the hypercharge ($Y$) and thus to electromagnetic charge, defined as $Q = T_{3L} + Y$. Hence, SU(2)$_N$ gauge bosons $X_{1,2,3}$ are electromagnetic charge neutral and the lightest of them, $X_1$ (odd under $R$), aptly fits into the criterion of a DM. Once SU(2)$_N$ symmetry is broken completely, the gauge bosons acquire masses of the scale of TeV. We adopt the same particle configuration as in [30, 31] to realise its high-scale origin, DM constraints, and collider signatures through elaborate analysis. Following is the particle content of the model, where the quantum numbers are 

The quantum numbers are 

\[ \begin{pmatrix} 1 & \chi_1^0 & \chi_2^0 \\ \phi_1^0 & \phi_2^0 & \phi_3^0 \end{pmatrix} = [2, 2, -1/2, 1; 1/2], \quad (\chi_1^0 \chi_2^0) = [1, 2, 0, 1; -1/2], \]

\[ \begin{pmatrix} \phi_1^+ & \phi_2^1 \\ \phi_1^- & \phi_3^- \end{pmatrix} = [2, 1, 1/2, 1; 0], \quad \left( \begin{array}{cc} \Delta_2^0/\sqrt{2} \\ \Delta_1^0 \end{array} \right) = [1, 3, 0, 1; 1]. \]

Vertical parentheses indicate doublet under SU(2)$_L$ and the horizontal ones indicate doublet under SU(2)$_N$. For brevity, we mention one family of the particle spectrum and assume that gauge and Yukawa interactions are flavour diagonal. The fermion sector is augmented by an exotic quark $h_q$, an exotic electron $E$ and two exotic neutrinos $N, n$. The scalar sector consists of one SU(2)$_L$ doublet, one SU(2)$_N$ doublet, a bi-doublet and an SU(2)$_N$ triplet. SU(2)$_N$ gauge bosons $X_{1,2,3}$, are not assigned any global U(1)$_R$ charge as they transform under adjoint representation. Their respective $T_{3N}$ quantum numbers are $[1, 0, 0; -1]$, leading to $R$-charges for $X_{1,2}$ as $(-1)$, and $X_3$ as $(+1)$ (since $J = 1$). The odd $R$-charged dark gauge bosons are stable as they cannot decay to a pair of SM particles or to other heavier (by construction) exotic particles. Thus, in our scenario, the lightest of $X_{1,2}$ qualifies to be DM and in case of a degeneracy, both may serve as DM candidates. Scalar potential of the model and the allowed Yukawa couplings are discussed in appendix A and appendix B respectively.

### 3 Vector boson as dark matter

This section reveals the allowed parameter space of the vector boson DM through elaborate scan. For the ease of discussion, this section is divided into several subsections as follows: DM-SM interactions and annihilation cross-sections are elaborated in subsection 3.1, Boltzmann
equation and thermal freeze out in 3.2, relic density and allowed parameter space in 3.3, direct search cross-section and constraints in 3.4 and finally effects of co-annihilation is pointed out in subsection 3.5.

In our scenario, the neutral scalars that acquire vacuum expectation value (VEV) are $\chi_{0,2,3}^0, \phi_{1,2}^0$, and $\Delta_{1,3}^0$. SU(2)$_N$ and electroweak symmetries are spontaneously broken through the VEV of $\chi_{0}^0$ ($\kappa_2$) and $\phi_{1,2}^0$ ($\nu_1,2$) respectively. Due to the presence of a bi-doublet and an SU(2)$_L$ Higgs doublet, the physical Higgs field ($h$) can be written as a linear combination of CP-even neutral components ($\phi_{1,2}$) ($h = (\nu_1 \phi_1^0 + \nu_2 \phi_2^0)/v$; where $v = \sqrt{v_1^2 + v_2^2} = 246$ GeV is the electroweak symmetry breaking scale). The model behaves as a two-Higgs doublet model of type II [43, 44], where part of the bi-doublet \((\phi_1^0, \phi_2^0)\) couples to up type quarks and \((\phi_2^0)\) couples to down type quarks. Physical scalar fields and their masses after spontaneous symmetry breaking of the model has been elaborated in [31]. The scalar potential can be found in the appendix A.

VEV of the SU(2)$_N$ triplet components $\Delta_{1,3}^0$ ($\delta_{1,2}$) contribute to the SU(2)$_N$ gauge boson masses and causes the mass splitting between two lighter dark gauge bosons as [30, 31]:

\[
m^2_{X_{1,2}} = \frac{1}{2} g_N^2 [\kappa_2^2 + v_1^2 + 2(\delta_1 \mp \delta_2)^2].
\]  (3.1)

Therefore, the absence of the triplets (with $\delta_1 = \delta_2 = 0$) will make $X_{1,2}$ degenerate: $m^2_{X_1} = m^2_{X_2} = \frac{1}{2} g_N^2 [\kappa_2^2 + v_1^2]$. The other gauge boson $X_3$ (even under $R$) mixes with the usual SM neutral gauge boson $Z$ through [30, 31]:

\[
m^2_{Z,X_3} = \frac{1}{2} \left( \begin{array}{cc}
g_N \sqrt{g_1^2 + g_2^2} v_1^2 & -g_N \sqrt{g_1^2 + g_2^2} v_1^2 \\
g_N \sqrt{g_1^2 + g_2^2} v_1^2 & g_N^2 [\kappa_2^2 + v_1^2 + 4(\delta_1^2 + \delta_2^2)] \end{array} \right).
\]  (3.2)

The mass term for $X_3$ can then be assumed as: $m^2_{X_3} \approx \frac{1}{2} g_N^2 [\kappa_2^2 + v_1^2 + 4(\delta_1^2 + \delta_2^2)]$ as $Z - X_3$ mixing is very much constrained [45]. Now, it is also evident from eq. (3.1) that the maximum splitting between $X_1, X_2$ can be achieved for $\delta_1 = \delta_2 = \delta$, where

\[
m^2_{X_1} = \frac{1}{2} g_N^2 [\kappa_2^2 + v_1^2], \quad m^2_{X_2} = m^2_{X_3} = \frac{1}{2} g_N^2 [\kappa_2^2 + v_1^2 + 8\delta^2].
\]  (3.3)

In this framework, using the exclusion limit of $m_{Z'} \sim O(\text{TeV})$ from heavy neutral gauge boson search [46], we can put tentative lower limits on DM mass ($m_{X_1}$) depending on the choice of $\delta_1$. For different values of SU(2)$_N$ gauge coupling ($g_N$), the lower limit on $m_{X_1}$ is shown in figure 1. For example, with $g_N^2 = 0.4$ (light green line), $m_{X_1}$ can be 400 GeV for $\delta_1 \sim 750$ GeV. The smaller is the value of $\delta_1$, the larger is the allowed $m_{X_1}$. DM mass is equal to the heavy neutral gauge boson mass ($m_{X_1} = m_{X_3} = 1$ TeV) in the limit $\delta_1 = \delta_2 = 0$, independent of the choice of $g_N$. As $X_3$ masses can be larger than 1 TeV, points above the contours are allowed while the ones below are disfavored. It is also quite apparent from eq. (3.3), presence of the triplets gives rise to a single component DM in terms of $X_1$. When the triplet is absent or the vev is zero, all three gauge bosons are degenerate, yielding a multipartite DM framework, which necessarily requires to be heavier than $\sim 1$ TeV as $m_{Z'} \gtrsim 1$ TeV.
\[ g_N^2 = 0.1 \]
\[ g_N^2 = 0.2 \]
\[ g_N^2 = 0.3 \]
\[ g_N^2 = 0.4 \]
\[ g_N^2 = 0.5 \]
\[ g_N^2 = 0.6 \]
\[ g_N^2 = 0.7 \]
\[ g_N^2 = 0.8 \]
\[ g_N^2 = 0.9 \]
\[ g_N^2 = 1. \]

\[ 200 \ 400 \ 600 \ 800 \ 1000 \]
\[ 0 \ 200 \ 400 \ 600 \ 800 \ 1000 \]

\[ m_{X_1} [GeV] \]
\[ \delta_1 [GeV] \]

**Figure 1.** Contours of triplet VEV $\delta_1$ with DM mass $m_{X_1}$ is shown for $m_{X_3} = 1$ TeV for different choices of SU(2)$_N$ coupling ($g_N$) in the maximum splitting scenario ($\delta_1 = \delta_2$). All the points below these lines are ruled out as they predict $X_3$ lighter than TeV.

### 3.1 Annihilation channels and cross-section

Relic density of DM is evaluated through thermal freeze out, governed by the number changing processes of the DM. $X_1$ being odd under $R$-charge, cannot directly couple to a pair of SM particles, but can connect to them via the exotic fermions odd under $R$. For example, one can have couplings like $\bar{d} \ h \ q \ X_1$ or $\bar{E} \ e \ X_1$. Hence, there are different channels through which $X_1$ can annihilate to SM particles which eventually help the DM to freeze out. Here, we have categorized them systematically:

- **Annihilations to fermion pairs** $\bar{d} \ d$; $e \bar{e}$; $\nu \bar{\nu}$ by exchanging $h$, $E$, $N$ respectively as shown in figure 2,

- **Annihilation to pair of Higgs** by exchanging $\phi_3$, $h$ and through four point contact term shown in figure 3,

- **Annihilation to pair of SM fermions and gauge bosons** through Higgs exchange as in figure 4.

It is also important to note that $X_2$ being heavier, can contribute to co-annihilation of $X_1$, which in turn will alter the relic density constraint of the model. We will discuss this in details later. For annihilation cross-section, we compute only the $s$-wave contribution, where $s_0 = 4m_{X_1}^2$. Therefore, we can approximate: $\langle \sigma v \rangle \sim \sigma v |s_0|$, which simplifies relic density calculations significantly. This approximation is justified as all the diagrams (except $\phi_3$ mediated one) will have non-zero $s$-wave contribution; higher order terms are therefore negligible and will not cause any visible change in the allowed parameter space of the model. Total an-
Figure 2. Annihilations (Co-annihilations with $i \neq j$, $\{i,j\} = \{1,2\}$) to SM fermion pairs by exotic quark exchange.

Figure 3. Annihilations (Co-annihilations with $i \neq j$, $\{i,j\} = \{1,2\}$) to SM Higgs pair.

Nihilation cross-section, computed from diagrams in figures 2, 3 and 4, can be written as \[47\]:

\[
(\sigma v)_{X_1 X_1 \to \text{SM}} \big|_{s_0 = 4m_{X_1}^2} = \frac{g_N^4 m_{X_1}^2}{72\pi} \left\{ \sum_{h_q} \frac{N_c}{(m_{h_q}^2 + m_{X_1}^2)^2} + \sum_{E} \frac{1}{(m_{E}^2 + m_{X_1}^2)^2} + \sum_{N} \frac{1}{(m_{N}^2 + m_{X_1}^2)^2} \right\} \\
+ \frac{g_N^2}{2\pi} \left( \frac{v_1}{v} \right)^2 \left\{ \frac{1}{6} \frac{m_f^2}{(4m_{X_1}^2 - m_{h}^2)^2 + \Gamma_{h}^2 m_{h}^2} \left( 1 - \frac{m_f^2}{m_{X_1}^2} \right)^2 \right\} \\
+ \frac{1}{4m_{X_1}^2} \frac{m_f^4}{(4m_{X_1}^2 - m_{h}^2)^2 + \Gamma_{h}^2 m_{h}^2} \left( 1 - \frac{m_f^2}{m_{X_1}^2} \right)^2 \\
+ \frac{1}{32\pi m_{X_1}^2} \sqrt{1 - \frac{m_f^2}{m_{X_1}^2}} \left\{ \frac{1}{3} \frac{g_N^4}{v^4} + \frac{3}{(4m_{X_1}^2 - m_{h}^2)^2 + \Gamma_{h}^2 m_{h}^2} \left( \frac{v_1}{v} \right)^2 \\
+ \frac{2g_N^4 m_f^2}{(4m_{X_1}^2 - m_{h}^2)^2 + \Gamma_{h}^2 m_{h}^2} \left( \frac{v_1}{v} \right)^3 \right\}. \tag{3.4}
\]

First three terms in above equation (eq. (3.4)) comes from annihilation to SM fermions through $t-$ channel graphs as shown in figure 2 [30, 31]. $N_c = 3$ is the colour factor which appears only for the colour fermion ($h_q$) exchange. Next three terms, proportional to $(v_1/v)^2$, are collective contributions from figure 3, where $X_1$ is annihilated through SM Higgs exchange to a pair of SM fermions, neutral and charged gauge bosons respectively. Last three terms...
Figure 4. Annihilations (Co-annihilations with $i \neq j$, $\{i,j\} = \{1,2\}$) to SM fermions and gauge bosons through Higgs exchange.

Figure 5. Variations of total annihilation cross-sections $\langle \sigma v \rangle \sim \sigma v|_{s_0 = 4m_{X_1}^2}$ and contributions from different channels is shown as a function of DM mass ($m_{X_1}$) for two hypothesis. Left panel: mass of the exotic fermions are assumed to be ($m_{X_1} + 100$) GeV; right panel: mass of the exotic fermions are fixed at 500 GeV. For both cases we set $g_N^2 = 0.4$ for illustration.

are contributions from figure 3, where DM annihilates to a pair of SM Higgs. All Higgs exchange diagrams are suppressed by $(v_1/v)^2$ with $v_1 \ll v$ to adjust $X_3 - Z$ mixing and were ignored in earlier analyses [30, 31].

Independent parameters which control the phenomenology of the model are: SU(2)$_N$ gauge coupling $g_N$ and the masses of the exotic non-SM particles. This yields an effective four dimensional parameter space characterizing the DM sector of the model:

$$\{g_N, m_{X_1}, m_{h_q}, m\},$$

where we simplify the situation by assuming the uncoloured exotic masses to be the same: $m_E = m_N = m_{\phi_3} = m$. This still keeps phenomenological implications of the model intact. Different contributions to total annihilation cross section as a function of DM mass is shown in figure 5. Here we have adopted two scenarios: (i) all exotic fermions are degenerate and heavier than the DM by 100 GeV, shown in the left panel and (ii) all exotic fermion masses are set to 500 GeV and shown in the right panel of figure 5.

From both the plots in figure 5, contribution from $h_q$ exchange is visibly high due to its colour interaction. Significant enhancement in the cross-section is seen at $m_{X_1} \simeq m_h/2$ due to resonance through Higgs exchange. Smaller peaks at $m_{X_1} \simeq m_W$ and $m_{X_1} \simeq m_Z$ marks the opening of $WW$ and $ZZ$ final states. The patterns of the individual as well

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This, in principle, is not equal to SM SU(2)$_L$ coupling $g_L$ (except for D-parity conserving scenario as discussed later). The freedom of choosing $g_N$ was not considered in earlier analysis [30, 31].
as total annihilation cross-section are different in left and right panels of figure 5. In the left panel, the propagators become heavier with increase in DM mass, causing depletion in cross-section. In the right panel, the propagator masses are kept fixed at 500 GeV. Thus, when $m_{X_1}$ increases and approaches the masses of the exotic fermions, final state phase space gradually increases, increasing the cross-section. Note that, in both the plots, contribution of the Higgs exchange diagram (polynomial in $v_1/v$) is small since $v_1 \ll v$ to satisfy $Z - X_3$ mixing [45]. The horizontal black dotted line denotes the annihilation cross-section required to satisfy correct relic density. Regions above denote under-closure (larger cross section) while the region below with smaller cross-section is ruled out by over closure (larger relic density).

### 3.2 Boltzmann equation and thermal freeze out

To compute thermal freeze out of the DM, we need to solve Boltzmann’s equation (BEQ) [7]:

$$\dot{n}_{X_1} + 3H n_{X_1} = -\langle \sigma v \rangle_{X_1 X_1 \rightarrow \text{SM}} \left[ (n_{X_1})^2 - (n_{X_1}^{\text{EQ}})^2 \right],$$  \hspace{1cm} (3.6)

where $n_{X_1}$ is DM number density, $H$ is the Hubble constant. $n_{X_1}^{\text{EQ}}$ is DM number density in equilibrium given by:

$$n_{X_1}^{\text{EQ}} = \int \frac{\zeta_{X_1} d^3 p}{(2\pi)^3 2E} f_{X_1}^{\text{EQ}}, \quad \text{with equilibrium density } f_{X_1}^{\text{EQ}} = \frac{1}{e^{E/k_B T} - 1}.$$  \hspace{1cm} (3.7)

$\langle \sigma v \rangle_{X_1 X_1 \rightarrow \text{SM}}$ depicts thermal averaged annihilation cross-section of the DM defined as [7, 48]:

$$\langle \sigma v \rangle_{X_1 X_1 \rightarrow \text{SM}} = \int_{\hat{s}_0}^{\infty} d\hat{s} \frac{\zeta_{X_1}^2 \hat{s} \sqrt{\hat{s} - 4m_{X_1}^2} K_1(\frac{\sqrt{\hat{s}}}{\hat{m}_{X_1}}) \langle \sigma v \rangle_{X_1 X_1 \rightarrow \text{SM}}}{16 m_{X_1}^4 T^2 K_2(\frac{m_{X_1}}{\hat{m}_{X_1}})^2},$$  \hspace{1cm} (3.8)

where $K_{1,2}$ are the modified Bessel’s functions, $\zeta_{X_1}$ is the internal degrees of freedom associated with $X_1$, which is 3 for its vectorial nature, and $\hat{s} = (p_{X_1} + p_{X_1}')^2$. We will use eq. (3.4) for computing the thermal averaged cross-section.

In order to identify the freeze-out of the DM, we will recast BEQ in terms of co-moving density ($Y$) as:

$$Y = \frac{n_{X_1}}{s},$$  \hspace{1cm} (3.9)

here, the entropy density ($s$) of the universe is given by [7]

$$s = \frac{2\pi^2}{45} g_s(T) T^3; \quad \text{with } g_s(T) = r_k g_k \left( \frac{T_k}{T} \right)^3 \theta(T - m_k).$$  \hspace{1cm} (3.10)

where the repeated index $k \in \{ \text{all particles} \}$, is summed over. Here, $T_k$ and $g_k$ are the temperature and internal DOF of $k^{th}$ particle with $r_k = 1 (7/8)$ for boson(fermion).

Now we can rewrite eq. (3.6) in terms of $Y$ and $x = \frac{m_{X_1}}{T}$ as:

$$\frac{dY}{dx} = -\frac{x \langle \sigma v \rangle s}{H(m)} \left[ \langle \sigma v \rangle_{X_1 X_1 \rightarrow \text{SM}} \left( Y^2 - Y_{\text{EQ}}^2 \right) \right],$$  \hspace{1cm} (3.11)

where

$$H(m) = 1.66 \sqrt{g_* m_{X_1}^2 / m_{\text{Pl}}^2}; \quad g_* = \sum_i \chi_i g_i \left( \frac{T_i}{T} \right)^4,$$  \hspace{1cm} (3.12)
where $i$ runs over bosons (fermions) with $\chi_i = 1$ ($7/8$) for bosons (fermions), and $m_{Pl} = \frac{1}{\sqrt{8\pi G}} = 2.43 \times 10^{18}$ GeV.

We also have the equilibrium co-moving number density:

$$Y^{EQ} = 0.145 \frac{g}{g_{*S}} \left( \frac{m_{X_1}}{T} \right)^{3/2} e^{-\frac{m_{X_1}}{T}}.$$  \hspace{1cm} (3.13)

Now the Boltzmann equation can be further simplified to a compact form as:

$$\frac{dy}{dx} = -\frac{m_{X_1}}{x^2} \left[ \sigma_0 \left( y^2 - y^{EQ^2} \right) \right].$$  \hspace{1cm} (3.14)

where $y = \lambda Y$ with $\lambda = (0.264 m_{Pl} \frac{g_*/(38)}{2})$, $\sigma_0 = (\sigma v)_{X_1 X_1 \rightarrow \text{SM}}$ given in eq. (3.4).

We demonstrate the patterns of DM freeze out in figure 6. Left panel shows two different combinations of $\{m_{X_1}, g_{N}^2\} = \{380 \text{ GeV}, 0.28\}$, $\{740 \text{ GeV}, 0.58\}$; right panel shows freeze out for three different values of $g_{N}^2 = \{0.2, 0.4, 0.6\}$ for same DM mass $m_{X_1} = 300$ GeV. In the left plot, due to different choices of the DM masses, the equilibrium distributions (shown by dashed lines) are different, lighter ($m_{X_1} = 380$ GeV shown by the green line) appears above the heavier one ($m_{X_1} = 740$ GeV shown by the red line). Decoupling for these two cases occur at $x = 20$ $(T \sim 19 \text{ GeV})$ and $x = 7$ $(T \sim 105 \text{ GeV})$. Heavier component still has a smaller yield due to significantly larger coupling ($g_{N}^2 = 0.58$), thanks to larger annihilation cross section. In the right panel, we illustrate the same for a fixed DM mass. Choices of the parameters here will be explained shortly.

3.3 Relic density and allowed parameter space

After freeze-out, relic density for the DM is obtained from the yield as:

$$\Omega h^2 = \frac{m_{X_1} s_0 \sqrt{g_*}}{3 H_0^2 m_{Pl} 0.26 g_{*S}} y(x_{\infty}),$$  \hspace{1cm} (3.15)

where $y(x_{\infty})$ is the solution of eq. (3.14) at large values of $x$. This can be written in terms of annihilation cross-section under $s-$ wave approximation as [7]:

$$\Omega h^2 \simeq \frac{2.4 \times 10^{-10} \text{ GeV}^{-2}}{(\sigma v)_{\chi_1 \chi_1 \rightarrow \text{SM} \rightarrow \text{SM}}},$$  \hspace{1cm} (3.16)
Figure 7. Variation in relic density is shown as a function of DM mass for two hypothesis: on left panel, masses of the exotic fermions are proposed to be \((m_{X_1} + 100)\) GeV; and on right-panel they are fixed at 500 GeV. The SU(2)\(N\) coupling \(g_N\) is also varied within the range of \(\{0.32 : 0.78\}\) for each values of \(m_{X_1}\). Correct relic density window is shown by red band. Freeze-out points of figure 6 are also indicated.

We will therefore adopt eq. (3.16) as approximate analytical solution for evaluating relic density of the DM. The variation of relic density with DM mass is shown in figure 7. On the left hand side, we vary all the exotic fermion masses as \((m_{X_1} + 100)\) GeV while on the right hand side, we keep them fixed at 500 GeV and vary DM mass up to that (similar to figure 5). In both the plots a sharp drop in relic density is observed due to resonant enhancement in the Higgs exchange diagram at \(m_{X_1} \simeq m_h/2\). Relic density varies over a wide range (blue band) for a particular DM mass due to a large span of SU(2)\(N\) gauge coupling \(g_N\): \(\{0.32−0.78\}\). The horizontal red band depicts the allowed range of relic density following WMAP in 3\(\sigma\) range\(^3\)

\[
0.09 \leq \Omega_{\text{DM}} h^2 \leq 0.12 .
\]  

(3.17)

The points which were chosen to demonstrate thermal freeze-out in figure 6, satisfy relic density, as shown in figure 7 by \((\times)\), \((+)\) and \((\triangle)\).

To find the allowed region of parameter space satisfying eq. (3.17), we vary the parameters \(\{g_N, m_{h_q}, m_N = m_E = m, m_{X_1}\}\) as discussed earlier. In \(m_{X_1} - m_{h_q}\) plane, the allowed parameter space is shown in figure 8 for different \(g_N^2\) and \(m\). Each figure is for a specific value of \(m\) and contains different colour coded regions corresponding to different values of \(g_N^2\). For each of these regions, upper and lower boundaries correspond to maximum and minimum allowed values of \(m_{h_q}\). This pattern is expected as relic density is closed from both sides. Annihilation cross-section decreases for larger \(m_{h_q}\), causing more contribution to the relic density. Thus, for a given \(g_N^2\), a cut-off in \(m_{h_q}^{\text{max}}\) corresponds to the maximum allowed relic density. Similarly, the lower cut-off in \(m_{h_q}\) corresponds to the maximum annihilation cross-section i.e, minimum relic density. From different figures in figure 8, we observe that for small values of \(m\) only a small part of \(m_{X_1}\) is allowed. This is a consequence of the constraint: \(m_{X_1} \leq m\). Now, there exists a lower limit of \(m_{X_1}\) for a given value of \(g_N\) allowed by the VEV constraints as depicted in figure 1. The hatched regions on the left hand side of each figure in figure 8 shows the parameter space disallowed by VEV. In addition, using PLANCK limit shrinks the allowed region but keeps the interpretations same.

\(^3\)PLANCK data dictates relic density in a similar range \(\Omega_{\text{DM}} h^2 = 0.1196 \pm 0.0031\) [41]. Though a little more restrictive, would not have altered the main outcome obtained in the analysis.
Figure 8. Relic density allowed parameter space in $m_{\chi_1} - m_{h_q}$ plane for four different values of $m = m_E = m_N$: 550 GeV (top left), 1050 GeV (top right), 1550 GeV (bottom left), 1950 GeV (bottom right) for different values of $g^2_N \rightarrow \{0.1 - 0.7\}$. The hatched region is excluded by the VEV constraints.

Figure 9. Relic density allowed parameter space in $m_{\chi_1} - m_{h_q}$ plane when $m = m_N = m_E$ is varied between $\{20 - 2000\}$ GeV for different $g^2_N \rightarrow \{0.1 - 0.7\}$. The hatched region is discarded by VEV limit. Benchmark points (BP1, BP2, BP3) as discussed in table 2 of the next section are also indicated in the plot.

Figure 9 summarizes the allowed parameter space in $m_{\chi_1}$ vs $m_{h_q}$ plane when $m$ is varied upto 2 TeV. Three benchmark points (BP1, BP2, BP3), identified after imposing all constraints, are indicated in this plot and listed in table 2 in subsection 4.1.
Figure 10. Relic density compatible parameter space in $m_{X_1} - g_N^2$ plane has been described by the blue region. The grey shaded region depicts the VEV disallowance and thus excluded.

Figure 11. Relevant interactions of dark matter ($X_1$) with quarks (nucleons) for direct-search experiments.

The correlation between DM mass and coupling, satisfying right relic density, is shown in figure 10 by blue shaded region. VEV exclusion limit is shown by translucent grey region on the left hand side constraining low DM mass.

3.4 Direct detection constraints
Spin-independent direct search cross-section for vector boson DM scattering off nuclei can be expressed as [49, 50]:

$$
\sigma_{DD}^{\text{SI}} = \frac{1}{\pi} \left( \frac{m_{nu}}{m_{X_1} + m_{nu}} \right)^2 \left| \frac{Z f_p + (A - Z) f_n}{A} \right|^2,
$$

where $Z$ and $(A - Z)$ are the number of protons and neutrons in Xe nucleus respectively. The mass of nucleus is given as $m_{nu} = Am_p + (A - Z)m_n$. Here, $m_p(n)$ and $f_p(n)$ are the mass and form factor for proton (neutron).

The ratios of the form factors of proton and neutron w.r.t. their respective masses are computed using the diagrams in figure 11 by incorporating the gluonic contributions along with the twist-2 operators. Here also the sub-dominant t-channel contribution due to Higgs
mediation has been taken into account which was ignored in earlier analysis [31]. The ratios therefore read:

\[
\frac{f_i}{m_i} = (I_i) \left[ -\frac{g^2_N}{4m_h^2} \left( \frac{v_1}{v} \right)^2 - \frac{g^2_N}{16} \frac{m^2_{h_q}}{(m^2_{h_q} - m^2_{X_i})^2} \right] + \frac{3}{4} (J_i) \left[ -\frac{g^2_N}{4} \frac{m^2_{X_i}}{(m^2_{h_q} - m^2_{X_i})^2} \right]
\]

\[
-(K_i) \left( 1.19 \right) \frac{g^2_N}{54m_h^2} \left( \frac{v_1}{v} \right)^2 + \frac{g^2_N}{36} \left[ (1.19) \frac{m^2_{h_q}}{6(m^2_{h_q} - m^2_{X_i})^2} + \frac{1}{3(m^2_{h_q} - m^2_{X_i})} \right]
\]

where \( i \) stands either for proton (\( p \)) or neutron (\( n \)) and their respective form factors are \( I_p(n) = 0.052 (0.061) \); \( J_p(n) = 0.222 (0.330) \); \( K_p(n) = 0.925 (0.922) \).

Variation of \( \sigma^{qf}_{DD} \) with \( m_{X_1} \) is summarized in figure 12. In the upper panel, different colour coded regions correspond to different \( g^2_N \) scanned over \( m_{h_q} \) upto 2 TeV, allowed by relic density. Again, the maximum and minimum \( m_{h_q} \) boundaries of each \( g^2_N \) region correspond to maximum and minimum relic density for a given DM mass. \( \sigma_{DD} \) has \( s \)-channel contribution from \( h_q \) as shown in figure 11, where \( m_{h_q} \) appears in the propagator and therefore, smaller \( \sigma_{DD} \) indicates larger \( m_{h_q} \). The \( t \)-channel Higgs mediation yields sub-dominant contribution to the direct search cross-section. The VEV disallowed points are shown by hatched regions which almost entirely discard low DM masses for small \( g^2_N = \{ 0.1, 0.2 \} \). In the bottom-panel, different coloured regions depict relic density allowed parameter space for fixed \( m_{h_q} \) and for \( g^2_N = 0.1 \) – 0.6. LUX and XENON1T exclusion limits as well as the future prediction from XENONnT are shown in both the figure. The main outcome of this analysis is to observe a huge region of parameter space still allowed by direct search limits. This can be attributed to the direct search cross-section of the DM guided by \( m_{h_q} \) mediation in \( s \)-channel.\(^4\)

As \( \sigma_{DD} \) goes well below XENON1T and even XENONnT limit, it is important to look into the current projection of solar, atmospheric and diffuse supernovae neutrino background. The orange thick dotted line in figure 12 indicates the presence of 3–10 neutrino events [51, 52]. The grey shaded region below indicates the neutrino floor where DM signal will have to be separated from neutrino background.

In figure 13 we show the \( m_{h_q} - m_{X_1} \) relic density allowed parameter space for \( g^2_N = 0.4 \), while highlighting the direct search exclusion bounds (LUX on left and XENON 1T on right). Direct search disallowed region corresponds to \( s \)-channel resonance \( (m_{X_1} \sim m_{h_q}) \), where the cross-section for the DM rises over the present exclusion limit.

### 3.5 Co-annihilation of \( X_1 \) with \( X_2 \)

One of the important aspects of the DM under consideration is the co-annihilation effect, which further helps it to evade direct search bounds. There are two R-charge odd gauge bosons (\( X_{1,2} \)) present in this model. With \( m_{X_2} > m_{X_1} \), the lightest one (\( X_1 \)) is stable and serves as DM candidate as discussed so far. The heavier one (\( X_2 \)) can contribute to the relic density of \( X_1 \) by co-annihilation to a pair of SM particles. The possible co-annihilation channels are already depicted in figures 2, 3, 4 (with \( i = 1, j = 2 \)). Total effective cross-

\(^4\)Recall that the \( t \)-channel Higgs mediation here is smaller due to \( Z - Z' \) mixing. This is in contrast to a large class of DM frameworks where direct search cross sections are dominated by large contributions from \( t \)-channel diagrams and therefore fall within the present exclusion bound. On top of this, relic density for this particular DM is also obtained through exotic leptonic contribution (illustrated in earlier section), which do not take part in direct search process allowing the model to further survive direct search cuts.
Figure 12. Spin independent direct search cross-section for the vector boson DM $X_1$ with respect to DM mass ($m_{X_1}$) for relic density allowed parameter space. Top: different values of $g_N^2$ is shown by different colours where $m_{h_L}$ is varied appropriately to yield correct relic density. Bottom: different $m_{h_L}$ values are shown by different coloured regions where $g_N^2$ is varied to yield correct relic density. Limits from LUX2016, XENON1T and XENONnT predictions are depicted by the dashed, solid and dot-dashed black lines respectively. The orange thick dashed line shows the background limit from solar, atmospheric and diffuse supernovae neutrinos while the grey shaded region below shows the neutrino floor. VEV constraints are shown by hatched regions.

section, including co-annihilation, can be written as:

$$\langle \sigma v \rangle_{\text{eff}} = (\sigma v)_{X_1X_1\rightarrow \text{SM SM}} + (\sigma v)_{X_1X_2\rightarrow \text{SM SM}} \left(1 + \frac{\Delta m}{m_{X_1}} \right)^{\frac{3}{2}} \exp(-\Delta m/m_{X_1}),$$ (3.20)

where $\Delta m = m_{X_2} - m_{X_1}$. Co-annihilation cross sections are significant for small $\Delta m$ ($m_{X_1} \sim m_{X_2}$), or has a Boltzmann suppression otherwise (as evident from the last term of eq. (3.20)). Following previous arguments, co-annihilation cross-sections are also computed at the threshold $s_0 = (m_{X_1} + m_{X_2})^2$ considering only the dominant s-wave contribution. $(\sigma v)_{X_1X_2\rightarrow \text{SM SM}}$ is obtained from eq. (3.4), by replacing $4m_{X_1} = (m_{X_1} + m_{X_2})^2$.

A scan of the relic density allowed parameter space of the model after incorporating co-annihilation effects is shown in figure 14. Here we have chosen a specific $g_N^2 = 0.4$ for
Figure 13. Direct search exclusion limits on relic density allowed parameter space of $m_{X_1} - m_{h_q}$ plane from LUX on the left hand side and from XENON nT on the right hand side. $g_N^2 = 0.4$ is chosen for illustration. Different allowed values of $m_N$ from relic density constraint are shown in different colours. VEV constraints are shown by hatched regions.

Figure 14. Direct search (spin independent) cross-section for relic density allowed parameter space for $X_1$ with co-annihilation taken into account. Different colour codes indicate different values of $m_{h_q}$. LUX 2016 and XENON IT exclusion limits are shown along with XENONnT sensitivity by the dashed, solid and dot-dashed black lines respectively. Orange thick dashed line shows the background limit estimated from solar, atmospheric and diffuse supernovae neutrinos while the orange shaded region below indicates the neutrino floor. VEV exclusion limit is represented by the grey region. Illustration. Different values of $m_{h_q}$ are shown by different coloured regions. VEV exclusion limit is shown in the left hand side by the grey band, which puts a limit on the low DM mass $\sim 200 \text{ GeV}$. The orange thick dotted line points to the neutrino background (3–10 neutrino events), with the shaded region below depicting the neutrino floor. Noteworthy feature of figure 14 is that the relic-density allowed parameter space lies safely much below the current direct search exclusion limits, a significant part of it going even lower than the XENONnT proposed limit. As has already been stated, this is a generic feature of DM models with co-
annihilation, where such contributions (as $X_1X_2 \rightarrow SM$) help the DM to thermally freeze out, but do not take part in direct search processes. To summarize, the model predicts that direct search of this DM is going to be difficult and direct search exclusions will not be able to rule-out a significant part of this model. This is one of the major outcomes of this analysis and was not pointed out in earlier analyses [30, 31].

4 Unified framework under $E(6)$

Here we have analyzed in detail how the gauge symmetry of our interest, $SU(2)_L \otimes SU(2)_N \otimes U(1)_Y \otimes SU(3)_C$, can be realized within an unified scenario. To understand this, we have adopted $E(6)$ as the unified group that has already been outlined in [30, 31]. We consider the following breaking patterns to achieve the low scale theory:

$$E(6) \xrightarrow{M_U} SU(3)_L \otimes SU(3)_R \otimes SU(3)_C \xrightarrow{M'_I} SU(2)_L \otimes U(1)_L \otimes SU(2)_R \otimes U(1)_R \otimes SU(3)_c \xrightarrow{M''_N} SU(2)_L \otimes U(1)_Y \otimes SU(3)_c \xrightarrow{EW SB} SU(3)_c \otimes U(1)_{EM}$$

In this section, we will first discuss the spontaneous symmetry breaking of $E(6)$ to the SM through multiple intermediate scales. We have computed the beta-function coefficients [53] for gauge coupling evolutions$^5$ for different intermediate scales. In the process, we have also used the extended survival hypothesis (ESH) [54] to allow the minimal fine tuning in the scalar potential. All contributing field representations and the beta coefficients for different scales are appended below:

- From $M'_I \rightarrow M_U$.

$E(6)$ can be spontaneously broken to $SU(3)_L \otimes SU(3)_R \otimes SU(3)_C \equiv G_{333}$ through the vacuum expectation value (VEV) of $650^'_H$ ($650^H$) scalar keeping D-parity intact (broken). The scalar and fermion fields that contribute to the running of the gauge couplings $\{g_{3L}, g_{3R}, g_{3C}\}$, associated with $SU(3)_L$, $SU(3)_R$, $SU(3)_C$ respectively, are:$^6$

$$27_F = [3, 3, 1] + [3, 1, 3] + [1, \bar{3}, 3], \quad 650^'_H \supset [8, 8, 1] + [1, 8, 1],$$

$$27_H \supset [3, 3, 1].$$

Here we would like to mention, when this breaking occurs via VEV of $650^H$, then only $[8, 8, 1]$ contributes to the beta-functions and D-parity remains conserved. This can be understood intuitively as one can see, the fields contributing to the beta-coefficients in the running of $g_{3L}$ and $g_{3R}$ are identical. Thus $g_{3L} = g_{3R}$ will be maintained as long as $G_{333}$ is unbroken. We compute the beta-coefficients as:

$$b_{3L} = 7/2, \quad b_{3R} = 7/2 (9/2), \quad b_{3c} = -5,$$

where the value of $b_{3R}$ in parenthesis denote the D-odd case.

$^5$We have considered the running of gauge couplings only, and ignored the contributions from Yukawa couplings.

$^6$We have relied on extended survival hypothesis (ESH) which tells that only those scalars are light which are participating in the symmetry breaking. Thus, even if we start with an unified $E(6)$ group, only relevant submultiplets are considered.
• From $M_I \to M'$. 

SU(3)_L \otimes SU(3)_R \otimes SU(3)_C$ is further broken to $SU(2)_L \otimes U(1)_L \otimes SU(2)_R \otimes U(1)_R \otimes SU(3)_C(\equiv G_{21213})$ through VEV of $(\bar{3}, 3, 1)$. The particles that contribute in the beta-functions are:

\begin{align*}
27_F &= [2, -1/2\sqrt{3}, 1, -1/\sqrt{3}, 1] + [2, 1/2\sqrt{3}, 1, 0, 3] + [1, 1/\sqrt{3}, 2, 1/2\sqrt{3}, 1] \\
&\quad + [1, 0, 2, -1/2\sqrt{3}, 3] + [2, -1/2\sqrt{3}, 2, 1/\sqrt{3}, 3] + [1, 0, 1, 1/\sqrt{3}, 3] \\
&\quad + [1, -1/\sqrt{3}, 1, 0, 3] + [1, 1/\sqrt{3}, 1, -1/\sqrt{3}, 1] \\
27_H &\supset [2, -1/2\sqrt{3}, 1, -1/\sqrt{3}, 1] + [1, 1/\sqrt{3}, 2, 1/2\sqrt{3}, 1] + [2, -1/2\sqrt{3}, 2, 1/2\sqrt{3}, 1] \\
650_H &\supset [1, 0, 3, 0, 1],
\end{align*}

under the symmetry group $G_{21213}$. The corresponding beta-coefficients are given by:

$b_{2L} = -5/6$, $b_{2R} = -5/6$ (1/6), $b_{LL} = b_{RR} = 115/18$, $b_{RL} = b_{LR} = 1/9$, $b_{3c} = -5$.

• From $M' \to M_I$.

$G_{21213}$ is broken to $SM \otimes SU(2)_N$ in such a way that the hypercharge generator is originated as one of the linear combination of $U(1)_L$ and $U(1)_R$ generators. The fermion and scalar fields that contribute to the evolutions of the gauge couplings, \{g_{2L}, g_{2N}, g_{1}, g_{3C}\}, are:

\begin{align*}
27_F &= [2, 1, 1/6, 3] + [1, 1, -2/3, 3] + [1, 2, 1/3, 3] + [1, 1, -1/3, 3] + [2, 2, -1/2, 1] \\
&\quad + [2, 1, 1/2, 1] + [1, 1, 1, 1] + [1, 2, 0, 1] \\
27_H &\supset [2, 2, -1/2, 1] + [2, 1, 1/2, 1] + [1, 2, 0, 1] \\
650'_H &\supset [1, 3, 0, 1],
\end{align*}

under the symmetry group $SU(2)_L \otimes SU(2)_N \otimes U(1)_Y \otimes SU(3)_C$. Corresponding beta-coefficients are:

$b_{2L} = -5/6$, $b_{2R} = 1/6$, $b_{1Y} = 21/2$, $b_{3c} = -5$,

respectively.

The sub-multiplet $650'_H \supset [1, 8, 1] \supset [1, 3, 0, 1]$ is considered for D-parity breaking case only. This SU(2)$_N$ triplet $\Delta \equiv [1, 3, 0, 1]$ plays a crucial role in this analysis. The VEV of this triplet breaks the mass degeneracy of the SU(2)$_N$ gauge bosons, leading to one component dark matter scenario. Otherwise, for D-parity conserving case where this triplet is absent, this framework will have two degenerate dark matter candidates.

• From $M_{EW} \to M'_I$.

In the next step, $[1, 2, 0, 1] \in 27_H$, singlet under SU(2)$_L \otimes U(1)_Y \otimes SU(3)_c$, acquires VEV and causes spontaneous braking of SU(2)$_N$ leading to SM gauge symmetry. The fermion and scalar fields which participate in the evolution of gauge couplings are (under SU(2)$_L, U(1)_Y, SU(3)_c$):

\begin{align*}
27_H &\supset [2, -1/2, 1] + [2, 1/2, 1] \\
27_F &\supset [2, 1/6, 3] + [1, -2/3, 3] + [1, 1/3, 3] + [2, -1/2, 1] + [1, 1, 1],
\end{align*}
and the corresponding beta-coefficients are:

\[ b_{2L} = -3, \quad b_{1Y} = 21/5, \quad b_{3c} = -7, \]

associated with \( \{g_{2L}, g_{1Y}, g_{3c}\} \) respectively.\(^7\) To summarize, the scalar and fermion representations, which participate in the symmetry breaking at different stages, are listed in table 1.

In the breaking pattern of \( E(6) \to \text{SU}(2)_N \otimes \text{SM} \to \text{SM} \), we have considered both D-parity conserving and non-conserving cases. For D-parity broken case, we will have one extra \( \text{SU}(2)_N \) triplet at the TeV scale, and its respective parent multiplet will be present at high scales. We have introduced three intermediate symmetry groups in between \( E(6) \) and SM. In principle, these intermediate scales are different. But we have noted, as almost all the fermions (27-dimensional) stick till \( \text{SU}(2)_N \) breaking scale, beta coefficients for \( U(1)_L \) and \( U(1)_R \) are quite large (≈ 115/18). This results in rapid growth of the respective gauge couplings, making them non-perturbative. Thus, to avoid such catastrophe, we have assumed that \( \mathcal{G}_{3_{33}} \) is broken directly to \( \text{SM} \otimes \text{SU}(2)_N \), i.e. \( M_I \) coincides with \( M'_I \). Hence, in our scenario, \( M'_I \) and \( M_I \) are the free parameters. In order to estimate the freedom of choosing \( g_N \) different from \( g_{32} \) in D-parity odd scenario, we restrict \( M'_I \) to be around TeV scale for phenomenological considerations, while unification scale to be within \( [10^{19} : 10^{16}] \) GeV. The allowed range of \( g_N \) therefore turns to be \([0.60 : 0.63]\), as shown in figure 15.

\(^7\) \( g_1 \) is GUT normalized U(1)\(_Y\) coupling and normalization factor 3/5 is used to fit within this framework.

| Particles | \( E(6) \) | \( (\text{SU}(3))^3 \) | \( \mathcal{G}_{21213} \) | \( \text{SM} \otimes \text{SU}(2)_N \) | SM |
|-----------|-------------|----------------|-----------------|-----------------|-----|
| Scalar    | 650\(_H\)  | [8, 8, 1]      | [1, 0, 3, 0, 1] | [1, 3, 0, 1]    |     |
|           | [1, 8, 1]  |                |                 |                 |     |
|           | 27\(_H\)   | [3, 3, 1]      | [2, \(-1/2\sqrt{3}, 1, -1/\sqrt{3}, 1\)] | [2, 2, \(-1/2, 1\)] | [2, \(-1/2, 1\)] |
|           |             | [1, \sqrt{3}, 2, 1/2\sqrt{3}, 1] | [2, 1, 1/2, 1] | [1, 2, 0, 1]    |
|           | 27\(_F\)   | [3, 3, 1]      | [2, \(-1/2\sqrt{3}, 1, -1/\sqrt{3}, 1\)] | [2, 1/6, 3]     | [2, \(-1/2, 1\)] |
|           | [3, 1, 3]  | [2, 1/2\sqrt{3}, 1, 0, 3] | [1, 1, \(-2/3\)] | [1, 1, \(-2/3\)] |
|           | [1, 3, 3]  | [1, \sqrt{3}, 2, 1/2\sqrt{3}, 1] | [2, 1/3, 3]     | [1, 1/3, 3]    |
|           |             | [1, \sqrt{3}, 1, 0, 3] | [2, \(-1/2, 1\)] | [1, 1, 1]      |
|           |             | [1, \sqrt{3}, 1, -1/\sqrt{3}, 1] | [1, 2, 0, 1]    | [1, 1]        |

Table 1. The representations of scalar and fermion fields that contribute in the renormalization group evolutions of gauge couplings under different intermediate symmetries.

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\(^7\) \( g_1 \) is GUT normalized U(1)\(_Y\) coupling and normalization factor 3/5 is used to fit within this framework.
Figure 15. The unification within D-parity odd scenario has been encapsulated. The range of SU(2)_N gauge coupling, g_N, and one of the intermediate scales (M_I) have been explored with allowed unification scale (M_U), i.e., within \(10^{18} - 10^{19}\) GeV. Here, M_I is considered to be degenerate with M_f, and M'_I is chosen to be TeV scale.

For D-parity even case, g_N is no more a free parameter and equals to g_{2L}. Here also, we find consistent solutions for unification with M_I ranging between \([10^{19} : 10^{16}]\) GeV. The D-parity even case will lead to degenerate mass (m_{X_1} = m_{X_2}), as the triplet VEV is absent. Thus, we will have a degenerate two component dark matter scenario which is itself an interesting possibility. Relic density allowed parameter space of degenerate DM scenario is shown in direct search plane in association with LUX, XENON1T bounds (figure 16). It essentially indicates that DM mass has to be less than a TeV in order to satisfy them. However, the DM has to be as heavy as at least a TeV to satisfy constraints from Z - Z' mixing. Hence, the situation is phenomenologically nonviable and D-parity even case with degenerate DM doesn’t work for this model. We have to live with only D-parity broken scenario with a single component DM even if the triplet VEV is small. This is one of the main outcomes of this elaborate discussion on the unification which was not indicated in earlier analyses \([30, 31]\). We would also like to add here that the correlations between gauge coupling unification with the intermediate scales associated to SU(2)_N, is very specific to our chosen framework. If we fix the particle content of this model and keep the SU(2)_N breaking scale at TeV range, a range of values of g_N, M_f and M_U become consistent with the unification picture, as shown in figure 15. For the low-energy phenomenology of this model, the most important parameter of these three is g_N and thus we have concentrated on the possible range of g_N. Therefore, fixing g_N and varying M_{I,U} in the unification-allowed range neither impact nor change our findings in the phenomenological analysis. We would also like to add that a SUSY version of E(6) GUT, where all the 27 fermions and their superpartners are present at the low scale, is severely constrained by flavour data, FCNC \([55]\) etc. Au contraire, as we are working within a non-supersymmetric framework, these constraints can be evaded by considering the exotic scalars to be very heavy \(\gtrsim 15\) TeV, and suitable Yukawa coupling, without altering our conclusion.
4.1 Summary of dark matter phenomenology

Before we proceed further, we would like to summarize the outcome of DM phenomenology in the light of relic density, direct search and unification constraints. As we have argued to stick to the D-parity broken scenario, $g_N$ is not identical to SU(2)$_L$ coupling, and in principle, a free parameter. Hence DM mass ($m_{X_1}$) is a function of $g_N$ and SU(2)$_N$ symmetry breaking VEVs ($\kappa_2, \delta_{1, 2}$). In figure 17, on top of relic density allowed (blue) and VEV disallowed (grey) parameter space, we incorporate direct detection constraint provided by LUX in (red) region and unification constraint (green). For further phenomenological exploration in context of LHC, we have chosen three benchmark points (BP): BP1 ($\times$), BP2 (+) and BP3 ($\star$). The details of these benchmark points are listed in table 2. The BPs have been chosen within DM mass ranging between 500 GeV–1 TeV. Two of the three gauge couplings ($g_N$), chosen for the benchmark points (for BP1 and BP2), fall within the unification window but have negligible co-annihilation contributions due to large mass difference between $m_{X_1}$ and $m_{X_2}$. For BP3, a larger coupling is chosen, with considerable co-annihilation contribution. The choice of $m_{h_q}$ and $m$ follow a specific hierarchy: $m_{h_q} > m_{X_2} > m_N > m_{X_1}$, typically motivated from the collider aspect as we shall discuss in section 5.2.

5 Collider phenomenology at LHC

Collider signatures of this model yield a lot of interesting possibilities with a number of beyond SM particles, which interact with the SM through the SU(2)$_N$ or Yukawa interactions. In this work, we will particularly highlight two multi-leptonic final states:

- Opposite sign dilepton plus a single jet and missing energy (OSD: $\ell^+\ell^- + 1\ j + E_T$),
- Hadronically quiet four lepton and missing energy (HQ4l: $2\ell^+2\ell^- + E_T$).
Figure 17. Relic density (blue shaded), direct search (red shaded) and unification (green shaded) allowed parameter space in $g_N^2 - m_{X^-}$ plane. Three chosen benchmark points BP1, BP2 and BP3 (see table 2) are also shown.

Table 2. Three benchmark points (BP1, BP2, BP3) of the model are identified with input parameters \( \{g_N, m_{X_1}, m_{h_u}, m = m_N = m_E\} \). Relic density and direct detection cross-sections for these points are also mentioned. We have set \( m_{X_2} \) at 1000 GeV to obey maximum splitting scenario (see eq. (3.3)).

| Benchmark Points | \( g_N \) (GeV) | \( m_{X_1} \) (GeV) | \( m_{X_2} \) (GeV) | \( m_{h_u} \) (GeV) | \( m \) (GeV) | \( \Omega h^2 \) | \( \sigma_{DD} \) (cm\(^2\)) |
|------------------|----------------|----------------|----------------|----------------|-------------|----------------|----------------|
| BP1              | 0.59           | 480           | 1000           | 1200           | 600         | 0.09           | \( 10^{-50} \) |
| BP2              | 0.63           | 700           | 1000           | 1160           | 860         | 0.1            | \( 10^{-49} \) |
| BP3              | 0.70           | 900           | 1000           | 1740           | 920         | 0.09           | \( 10^{-49} \) |

Leptonic final states are interesting for study at LHC as the SM background contribution is relatively less. Unfortunately no excess has been reported from the existing data yet. In this section, we will show that the benchmark points chosen in table 2, satisfy existing data from CMS with center-of-mass-energy \( \sqrt{s} = 13 \text{ TeV} \) and predict signal events in the aforementioned channels at \( \sqrt{s} = 14 \text{ TeV} \). The excess of signal is subject to the final state event selection criteria and we will discuss corresponding SM background contributions in each of the cases accordingly.

5.1 Simulation methodology

We have implemented the model in **CalcHEP** [56] to generate the parton level events. These events are then fed into **PYTHIA v.6** [57] for showering and hadronization. We have used **CTEQ6L** [58] parton distribution function with renormalization (\( \mu_R \)) and factorisation (\( \mu_F \)) scales set to the sub-process center-of-mass-energy (\( \sqrt{s} \)).

All the exotic particles produced in the signal will eventually decay into SM leptons, jets and the DM \( X_1 \). This will give rise to different multilepton plus jet events, subject to
the choice of production processes, along with missing energy. To mimic the experimental environment at LHC, we use the following identification criteria:

- **Lepton** ($\ell = e, \mu$). They are identified with minimum transverse momentum ($p_T$) of 20 GeV, and with pseudorapidity $|\eta| < 2.5$ to identify them in the central region of the detector. Two leptons are treated as isolated objects if their mutual separation satisfy $\Delta R = \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2} \geq 0.2$. Lepton-jet separation must satisfy $\Delta R \geq 0.4$. $\tau$ leptons are difficult to observe in electromagnetic and muon calorimeters due to their shorter lifetime and so they are not usually classified into lepton category.

- **Jet** ($\text{jet}$). Due to SU(3) interactions, partons form hadrons after emerging out of the collision vertex. These hadrons then cluster to form jets. We have used PYCELL, the cone jet formation algorithm built within PYHTIA, to define the clustered hadrons as jets. The detector is assumed to span $|\eta| \leq 5$ and is segmented in 100-$\eta$ and 70-$\phi$ bins. The minimum transverse energy $E_T$ of each cell is taken as 0.5 GeV, while we require $E_T \geq 2$ GeV for a cell to act as a jet initiator. All the partons within $\Delta R = 0.4$ from the jet initiator cell are included in the formation of the jet, and we require $E_T \geq 20$ GeV for a group to be considered as a jet. For isolation of the jets from the unclustered objects (see below), we also impose a cut of $\Delta R > 0.4$.

- **Unclustered objects.** All the other final state particles, which are not isolated leptons and separated from jets, are considered as unclustered objects. This clearly means all the particles (electron/photon/muon) with $0 < E_T < 10$ GeV and $|\eta| < 5$ and jets with $0.5 < E_T < 20$ GeV and $|\eta| < 5$, which leave their presence in the detector, are considered as unclustered objects.

- **Missing energy** ($E_T^\text{miss}$). Though DMs produced in the decay chain of exotic particles will be missed in the detector, their transverse momentum can be estimated from the momentum imbalance in the associated visible particles (leptons and jets). This missing energy is thus defined as:

$$E_T^\text{miss} = (p_T)_{\text{vis}} - (p_T)_{\text{vis}} = \sqrt{\sum_{\ell,j} p_x^2 + \sum_{\ell,j} p_y^2}, \quad (5.1)$$

Note that the negative sign in the definition of missing energy do not carry any significance and we will always refer to the absolute value of the visible transverse momentum as missing energy. We also note that missing energy identification takes the unclustered objects, which do not fall into the lepton or jet category as defined by detector sensitivity, into account.

SM background for the corresponding signal events play a crucial role in identifying the signal significance and discovery potential of the underlying model. We generate dominant SM backgrounds using MADGRAPH [59] and shower them in PYTHIA [57]. We have appropriately used $K$ factor to match the next-to-leading-order (NLO) cross-sections for the processes contributing to SM background. The signal cross-section, which we have evaluated at tree level, may vary within 10% due to the choice of jet energy scale, parton distribution functions, smearing effects of leptons and jets, and different jet formation criteria. NLO results for the signal events, particularly involving coloured particles like $h_q$, may have larger effects than the detector simulation criteria.
Figure 18. Top panel: Feynman diagrams for producing $pp \rightarrow h_q X_1$ at the LHC. Bottom panel: production cross-section of $pp \rightarrow h_q X_1$ is plotted as a function of $m_{h_q}$ (GeV) at LHC for $\sqrt{s} = 14$ TeV, keeping the DM mass fixed at $m_{X_1} = 700$ GeV and $g_N^2 = 0.4$.

5.2 Opposite sign dilepton signal

LHC being a proton-proton collider, exotic particles with colour charge ($h_q$ in our case) can be produced copiously. Opposite sign dilepton (OSD) with $1j + E_T$ arises in the model from two different production processes: (i) $pp \rightarrow h_q X_1$, through the Feynman graphs shown in the top panel of figure 18, and (ii) $pp \rightarrow h_q \bar{h}_q$, as shown by the Feynman graphs in the top panel of figure 19. Variation in production cross-sections of these processes with exotic quark mass $m_{h_q}$ is shown in the bottom panels of figure 18 and figure 19 respectively. DM mass ($m_{X_1}$) is fixed at 700 GeV and the SU(2)$_N$ gauge coupling is chosen as $g_N^2 = 0.4$ for these calculations.

Now $h_q$, once produced through SU(2)$_N$ gauge interactions, can only decay via $h_q \rightarrow X_{1,2} d$ (figure 20), depending on the availability of phase space. The branching fractions are plotted with respect to $m_{X_1}$ in the bottom panel of figure 20. Given a specific $m_{X_2}$ and $m_{h_q}$, the branching fraction to $X_1 d$ reduces and $X_2 d$ increases with $m_{X_1}$. In the limiting case, when $m_{X_1} = m_{X_2}$, the branching fractions of $h_q$ are same for both the final states. When $h_q$ decays to $dX_1$, the production of $pp \rightarrow h_q X_1$ will end in single jet plus missing energy events and $pp \rightarrow h_q \bar{h}_q$ will end with two jets plus missing energy events, which are standard signatures of many DM models including those of minimal scalar singlet extensions. SM background for such final states are also huge and hence reducing the background for a possible excess in the signal is difficult if not impossible. However, $h_q \rightarrow dX_2$ may yield some interesting possibilities if we choose the following mass hierarchy in the spectrum:

$$m_{h_q} > m_{X_2} > m = m_E = m_N > m_{X_1}.$$  \hfill(5.2)

$X_2$ can then decay to $N\bar{\nu}$, $E^+e^-$ (see figure 21). The branching fraction of $X_2$ to neutrino and charged lepton final states are equal in the limit of $m_N = m_E$, which is assumed throughout the analysis. Now, exotic leptons will decay to DM with 100% branching fraction: $N \rightarrow \nu X_1$, ...
Figure 19. Top panel: Feynman diagram for producing $pp \rightarrow h_q \bar{h}_q$ at the LHC. Bottom panel: production cross-section of $pp \rightarrow h_q \bar{h}_q$ is plotted as a function of $m_{h_q}$ (GeV) at LHC for $\sqrt{s} = 14$ TeV with $g^2_N = 0.4$.

Figure 20. Top panel: Feynman diagrams for possible decay modes of $h_q$. Bottom panel: decay branching fractions of $h_q$ as a function of $m_{X_1}$, $\text{Br}(h_q \rightarrow dX_1)$ on the left and $\text{Br}(h_q \rightarrow dX_2)$ on the right. We have chosen $m_{h_q} = 1160$ GeV and $m_{X_2} = 1000$ GeV for illustration.

$E \rightarrow eX_1$ as shown in figure 22. Hence, $pp \rightarrow h_q X_1$, $h_q \rightarrow dX_2$, $X_2 \rightarrow E^+e^-$ will lead to opposite sign same flavour dilepton, a soft jet, and missing energy in the final state. Similarly, $pp \rightarrow h_q \bar{h}_q$, $h_q \rightarrow dX_2$, $X_2 \rightarrow E^+e^-$ will also mostly yield opposite sign same flavour dilepton, two jets, and missing energy events. In the second case, many of those two jet events qualify
Figure 21. Feynman diagrams showing decays of $X_2$ to $N\bar{\nu}$, $E^+e^-$ and their conjugate final states.

Figure 22. Feynman diagrams showing decays of $E$ and $N$ to $X_1$ and SM particles.

| Benchmark Points | $m_{X_1}$ | $m_{X_2}$ | $m_{hq}$ | Br($h_q \to dX_1$) | Br($h_q \to dX_2$) |
|------------------|-----------|-----------|---------|-----------------|-----------------|
| BP1              | 480       | 1000      | 1200    | 94.8            | 5.2             |
| BP2              | 700       | 1000      | 1160    | 89.7            | 10.3            |
| BP3              | 900       | 1000      | 1740    | 57.7            | 42.3            |

Table 3. Branching ratio of $h_q \to dX_{1,2}$ for three benchmark points: BP1, BP2 and BP3.

for single jet case as the jets are soft due to their production in association with a sufficiently heavy particle $X_2$, which carries away most of the momentum in the decay chain. We will perform collider analysis only for the selected benchmark points BP1, BP2, and BP3 (table 2). Branching fractions of $h_q$ at those benchmark points are specified in table 3.

We have identified the dominant SM processes which can mimic OSD signal and consider them as backgrounds. Such processes are $t\bar{t}$+jets, $WW$+jets, $WZ$+jets, $ZZ$+jets, and also the Drell-Yan. The cross-section of the background is much larger than that of the signal and one needs to judiciously choose the cuts on the final state observables to retain the signal and reduce the background. Missing energy ($E_T$) signature of the signal is characteristically different from that of the SM background. For background, $E_T$ dominantly comes from neutrinos and energy mismatch (due to smearing and detector inefficiency). $E_T$ distributions for the signal for the BPs are plotted along with the dominant SM backgrounds in figure 23. $E_T$ peaks at a larger value for the signal and at a smaller value for the background, thus allowing us to distinguish between the model(signal) from the background. This is done by putting a sufficiently large $E_T$.

Another interesting feature emerges from the $E_T$ distribution when they are compared for the BPs as in figure 24. The peak of the distribution depends on the momentum carried away by the DM. For $pp \to h_qX_1$ (left panel of figure 24), as $m_{h_q} > m_{X_1}$, larger share of the momentum will be carried by the exotic quark $h_q$. Hence, the peak of distribution will be governed by $\Delta m = m_{h_q} - m_{X_1}$ and will shift to larger value with larger $\Delta m$. $(\Delta m)_{BP3} > (\Delta m)_{BP1} > (\Delta m)_{BP2}$ is clearly therefore reflected in the $E_T$ distributions. The same is true for the right panel of figure 24, where $E_T$ distribution is plotted for contributions
Figure 23. Missing energy distribution for ($\ell^+\ell^-+1j+E_T^n$) events at LHC from $pp \to h_qX_1$ production, for the benchmark points with dominant SM background. Top left: BP1, Top right: BP2, Bottom: BP3.

Figure 24. Missing energy ($E_T^n$) distribution in OSD events for benchmark points (BP1, BP2, BP3). On the left, we consider only $pp \to h_qX_1$ production while on the right panel $pp \to h_q\bar{h}_q$ production is considered.

only from $pp \to h_q\bar{h}_q$. This feature can help to differentiate between the benchmark points with different $\Delta m$. This can also give an idea about the mass of the exotic quark ($m_{h_q}$), given a knowledge of DM mass ($m_{X_1}$).

Cross-sections for OSD events at LHC with $\sqrt{s} = 14$ TeV and integrated luminosity $\mathcal{L} = 100$ fb$^{-1}$ are listed in table 4 in terms of actual number of events.\(^8\) On top of the basic

\[^8\] Actual number of events $N$ for luminosity $\mathcal{L}$ can be obtained for seed simulation points $N_1$ and final state events $N_2$ as $N = (\sigma \times N_2 \times \mathcal{L})/N_1$, where $\sigma$ is the corresponding production cross-section.
cuts (as discussed in 5.1), we have used following cuts to effectively separate signal from background:

- Missing Energy cut ($E_T$): $E_T > 100, 200, 300$ GeV are used. Large $E_T$ retains the signal due to large DM masses of the benchmark points and reduces background.

- Invariant mass cut: invariant mass is defined as $m_{\ell\ell}^2 = (p_{\ell^-} + p_{\ell^+})^2$ for opposite sign same flavour dileptons. We require invariant mass not to lie within the Z mass window ($|m_z - 15| \lesssim m_{\ell\ell} \lesssim |m_z + 15|$ GeV) to reduce background events from Z production.

- For jets, a moderate cut $p_T > 40$ GeV is demanded. This is particularly because of the fact that the jets produced in the signal events carry only a small fraction of the momentum and are mostly soft. The jet $p_T$ distribution for OSD events at the benchmark points are shown in figure 25. Larger $p_T$ cut kills a large fraction of signal events.

Cross-sections for the dominant SM background processes contributing to OSD events at LHC is tabulated in table 5. We have multiplied the production cross-sections generated at leading order (LO) by appropriate $K$ factors to match the NLO cross-section available in the literature. For example, for $t\bar{t}$: $K = 1.31$, $WWj$: $K = 1.30$, $WZj$: $K = 1.27$, $ZZj$: $K = 1.31$, Drell-Yan: $K = 1.2$ [59]. We estimate a limit on the final state event cross-section ($\sigma_e$) as $\sigma_e < \sigma_p/N$ where $N$ events are simulated with production cross section $\sigma_p$. A discovery significance for OSD events is shown in figure 26 in terms of luminosity (in fb$^{-1}$). The main outcome of this analysis is to see an excess in BP1 and BP2 benchmark points in OSD final states at LHC for high luminosity, while BP3 might be a little harder to explore.

### 5.2.1 Validation against observed data at LHC

We discuss here the competence of the benchmark points chosen for the analysis with the existing data at LHC. OSD events have been searched exhaustively, particularly because supersymmetry (SUSY) provides such a signal so very often. The signal selection criteria however is different from ours and is mainly guided to maximize the SUSY signal efficiency. Searches in OSD channels have been broadly divided into two different regions as described by CMS [60, 61] and ATLAS [62]: (i) on the Z-mass window or the ‘on-Z search’, where

![Figure 25](image.png)

**Figure 25.** Transverse momentum ($p_T$) of jet distribution in OSD ($\ell^+\ell^- + 1j + E_T$) events for benchmark points BP1, BP2 and BP3.
Table 4. OSD ($\ell^+\ell^-+1\,jet+\not\!E_T$) events at LHC for chosen benchmark points with $p_T^\ell > 20$, $p_T^{j} > 40$ and $|m_Z - 15| \not\leq m_{ll} \not\leq |m_Z + 15|$ at $\sqrt{s} = 14$ TeV. Number of events predicted for $\mathcal{L} = 100$ fb$^{-1}$ luminosity.

| Benchmark Points | $\sigma_{pp\to h_aX_1}$ (in pb) | $\sigma_{pp\to h_bh_q}$ (in pb) | $E_T^\ell$ (in GeV) | $E_T^{O_3D}$ (in pb) | $E_T^{O_3D}$ (in pb) | $N$ (100 fb$^{-1}$) |
|------------------|----------------|----------------|----------------|----------------|----------------|----------------|
| BP1              | 0.331          | 1.13           | > 100          | $1.77 \times 10^{-3}$ | $8.37 \times 10^{-4}$ | 260            |
|                  |                |                | > 200          | $1.53 \times 10^{-3}$ | $7.26 \times 10^{-4}$ | 225            |
|                  |                |                | > 300          | $9.70 \times 10^{-4}$ | $5.95 \times 10^{-4}$ | 156            |
| BP2              | 0.172          | 0.758          | > 100          | $1.62 \times 10^{-3}$ | $1.58 \times 10^{-3}$ | 320            |
|                  |                |                | > 200          | $1.04 \times 10^{-3}$ | $1.23 \times 10^{-3}$ | 227            |
|                  |                |                | > 300          | $5.45 \times 10^{-4}$ | $7.81 \times 10^{-4}$ | 132            |
| BP3              | 0.0199         | 0.102          | > 100          | $2.73 \times 10^{-4}$ | $1.42 \times 10^{-5}$ | 28             |
|                  |                |                | > 200          | $2.47 \times 10^{-4}$ | $1.36 \times 10^{-5}$ | 25             |
|                  |                |                | > 300          | $2.10 \times 10^{-4}$ | $1.20 \times 10^{-5}$ | 22             |

Table 5. OSD ($\ell^+\ell^-+1\,jet+\not\!E_T$) events for dominant SM background at LHC with $p_T^\ell > 20$, $p_T^{j} > 40$ and $|m_Z - 15| \not\leq m_{ll} \not\leq |m_Z + 15|$ at $\sqrt{s} = 14$ TeV and $\mathcal{L} = 100$ fb$^{-1}$ luminosity. Appropriate $K$-factors are used for different processes to match to the NLO-NLL cross-sections available in literature (see text for details).

| Process         | $\sigma_p$ (in pb) | $E_T$ (in GeV) | $\sigma^{O_3D}$ (in pb) | $N$(100 fb$^{-1}$) |
|-----------------|--------------------|----------------|-------------------------|-------------------|
| $t\bar{t} + j$  | 809.79             | > 100          | $137.66 \times 10^{-3}$ | 13766             |
|                 |                    | > 200          | < $8.09 \times 10^{-3}$ | < 1               |
|                 |                    | > 300          | < $8.09 \times 10^{-3}$ | < 1               |
| $WW + j$        | 60.58              | > 100          | 64.82                    | 6482              |
|                 |                    | > 200          | $5.45 \times 10^{-2}$    | 545               |
|                 |                    | > 300          | $1.81 \times 10^{-2}$    | 181               |
| $WZ + j$        | 0.15               | > 100          | $2.16 \times 10^{-4}$    | 21                |
|                 |                    | > 200          | $5.55 \times 10^{-5}$    | 5                 |
|                 |                    | > 300          | $2.40 \times 10^{-5}$    | 2                 |
| $ZZ + j$        | 7.63               | > 100          | $2.28 \times 10^{-4}$    | 22                |
|                 |                    | > 200          | < $7.63 \times 10^{-5}$  | < 1               |
|                 |                    | > 300          | < $7.63 \times 10^{-5}$  | < 1               |
| Drell-Yan       | 879.19             | > 100          | < $8.79 \times 10^{-3}$  | < 1               |
|                 |                    | > 200          | < $8.79 \times 10^{-3}$  | < 1               |
|                 |                    | > 300          | < $8.79 \times 10^{-3}$  | < 1               |

$|m_Z - 15| \leq m_{ll} \leq |m_Z + 15|$ GeV and (ii) out of the Z-mass window or ‘edge search’ where $m_{ll}$ does not lie within the Z-mass window. These regions are then further classified into sub-regions. On-Z search is mainly classified by number of associated jets and b-tagged jets. We only validate our benchmark points for SRA regions of CMS analysis [60], where the
Figure 26. Significance plot of OSD ($\ell^+\ell^- + 1\text{ jet} + E_T$) events at the benchmark points with $E_T > 300 \text{ GeV}$ versus luminosity (in fb$^{-1}$).

Table 6. Results for CMS on-Z search [60] at the benchmark points for OSD events associated with 2-3 jets and $H_T > 400$. The data, signal and SM predictions are made for $\sqrt{s} = 13 \text{ TeV}$ with $\mathcal{L} = 2.3 \text{ fb}^{-1}$.

OSD signal is vetoed with 2–3 jets and zero b-jets. In this search, $H_T = \Sigma_{\text{jets}} p_T > 400 \text{ GeV}$ is imposed. The event rates of the benchmark points with such criteria are summarised in table 6, where the signal region is further divided into different $E_T$ regions. We see that all the benchmark points produce zero events at the luminosity of $2.3 \text{ fb}^{-1}$ and therefore are allowed by the observed data. This happens mainly due to the demand of number of associated jets and high $H_T$ cut which the model fails to pass through. Requiring more jets ($N_j \geq 3$) anyway produces zero events for all the benchmark points and hence we do not analyze the other signal region (SRB) of CMS analysis. The edge search or off-Z search for OSD event is divided into different invariant mass ($m_{\ell\ell}$) regions and is inclusive of number of associate jets which require a moderate $E_T > 150 \text{ GeV}$ cut for $N_j \geq 2$ and $E_T > 100 \text{ GeV}$ cut for $N_j \geq 3$. Signal events for off-Z search at the benchmark points are mentioned in table 7. Comparing with the observed data we see that all the benchmark points lie comfortably within the limit.

5.3 Hadronically quiet four lepton signal

One of the unique collider signatures that this model offers, is the hadronically quiet four lepton (HQ4l). This can arise from the production of the heavier SU(2)$_N$ gauge boson $X_2$ following the Feynman graph in left panel of figure 27. $X_2$ further decays through the exotic leptons following figure 21 and figure 22 and produces two pairs of same flavour opposite
Table 7. OSD events for CMS off-\(Z\) search [60] at the benchmark points with \(E_T > 150\,\text{GeV}\) for \(N_j \geq 2\) and \(E_T > 100\,\text{GeV}\) for \(N_j \geq 3\). The data, signal and SM background predictions are made for \(\sqrt{s} = 13\,\text{TeV}\) with \(\mathcal{L} = 2.2\,\text{fb}^{-1}\).

| \(m_\ell(\text{GeV})\) | BP1 | BP2 | BP3 | Obs. data | SM bck |
|----------------------|-----|-----|-----|-----------|------|
|                     | \(\sigma_{pp-h_qX_1}^{\text{OSD}}\) | \(\sigma_{pp-h_qX_1}^{\text{OSD}}\) | \(\sigma_{pp-h_qX_1}^{\text{OSD}}\) | \(\sigma_{pp-h_qX_1}^{\text{OSD}}\) | \(N_{\text{OSD}}\) |
| 20-70               | 0.026 | 0.25 | <1 | <0.001 | 844 | 1 |
| 70-81               | 0.003 | 0.007 | <1 | <0.001 | 0.032 | <1 |
| 81-120              | <0.003 | <0.007 | <1 | <0.001 | <0.004 | <1 |
| >120                | <0.003 | <0.007 | <1 | <0.001 | <0.004 | <1 |

Figure 27. Left: Feynman diagram for producing a pair of \(X_2\) at the LHC. Right: cross-section of \(pp \to X_2X_2\) as a function of \(m_{X_2}\) at 14 TeV LHC.

sign dilepton with large missing energy \((\ell^\pm \ell^\mp + \not{E}_T^\text{T})\). The corresponding signal can have any of the following combinations: \(e^\pm e^\mp e^\pm e^\mp\), \(e^\pm e^\pm \mu^\pm \mu^\mp\), \(\mu^\pm \mu^\mp \mu^\pm \mu^\mp\). We however choose final states inclusive of both the flavours. Hadronically quiet four lepton signature is also an artifact of the mass hierarchy as pointed out in eq. (5.2) and not necessarily a generic one for this model. The production cross-section of \(pp \to X_2X_2\) is much smaller as it is an SU(2)_L gauge interaction process. With heavier \(m_{X_2}\), the cross-section falls off sharply to a vanishingly small value as shown in the right panel of figure 27. We again note here that \(pp \to h_qh_q\) also contributes to HQ4l events, where in both legs \(h_q\) decays via \(h_q \to dX_2\) with \(X_2 \to e^+e^- X_1\) through exotic charged leptons. The signal event rates at the benchmark points are mentioned in table 8. The lepton selection criteria remains similar to OSD search, while we demand the absence of jets with \(p_T > 20\,\text{GeV}\). Although the number of signal events is small in this case, SM background is even more rare. Dominant contributions come from \(ZZ\), \(WWWW\) and \(Zt\bar{t}\) as estimated in table 9. For SM background, we have multiplied the cross-section generated at LO in \text{Madgraph} by appropriate \(K\) factors to match the NLO cross-section available in the literature (for \(WWWW\) : \(K = 1.74\), \(ZZ\) : \(K = 1.29\), \(Zt\bar{t}\) : \(K = 1.44\) [59]). Again we show that missing energy is one of the main discriminators for separating signal and background. This is shown in figure 28 for three benchmark points and for the dominant SM backgrounds.

We have also checked the efficiency of invariant mass \((m_{\ell\ell})\) cut employed in the analysis and explicitly demonstrated the \(m_{\ell\ell}\) distributions in figure 29. In the left panel, we plot \(m_{\ell\ell}\) for OSD \((\ell^+\ell^- + 1j_{\ell\ell} + \not{E}_T^\text{T})\) events for signal from BP2 and dominant SM background from \(ZZ+\text{jets}\), \(W^+W^- Z+\text{jets}\). On the right-panel we plot \(m_{\ell\ell}\) of any two same flavour opposite
Figure 28. Missing energy ($E_T$) distribution for HQ4l events at the benchmark points with dominant SM background. Top left: BP1, Top right: BP2, Bottom: BP3.

| Benchmark | $\sigma_{pp\to X_2X_2}$ (in pb) | $\sigma_{pp\to h_q\bar{h}_q}$ (in pb) | $E_{T \text{miss}}^\text{miss}$ (in pb) | $\sigma_{pp\to X_2X_2}$ (in pb) | $\sigma_{pp\to h_q\bar{h}_q}$ (in pb) | N (100 fb$^{-1}$) |
|-----------|-------------------------------|---------------------------------|----------------------------------|-------------------------------|-----------------------------------|-----------------|
| BP1       | 0.0193                        | 1.13                            | > 100 $1.88 \times 10^{-3}$      | > 200 $1.53 \times 10^{-3}$    | > 300 $1.07 \times 10^{-3}$      | 221             |
|           |                               |                                 |                                 |                               |                                   |                 |
| BP2       | 0.0264                        | 0.758                           | > 100 $2.27 \times 10^{-3}$      | > 200 $1.36 \times 10^{-3}$    | > 300 $5.43 \times 10^{-4}$      | 286             |
|           |                               |                                 |                                 |                               |                                   |                 |
| BP3       | 0.0205                        | 0.102                           | > 100 $1.73 \times 10^{-4}$      | > 200 $3.93 \times 10^{-5}$    | > 300 $1.56 \times 10^{-5}$      | 43              |

Table 8. Hadronically quiet four lepton ($\ell^+\ell^-\ell^+\ell^- + E_T$) events at LHC for chosen benchmark points with $p_T > 20$ and $|m_Z - 15| \not\leq m_{ll} \not\leq |m_Z + 15|$ for all opposite sign same flavour dilepton at $\sqrt{s} = 14$ TeV and $L = 100$ fb$^{-1}$ luminosity.

sign dilepton pair of HQ4l events for signal at BP2 with dominant SM backgrounds from $ZZ, Z\ell\ell$. For OSD, both the backgrounds $ZZ +$ jets, $W^+W^-Z +$ jets have $m_{ll}$ peaks around $Z$ mass window and reduce drastically with the invariant mass cut. For HQ4l, we can form dilepton invariant mass $m_{ll}$ depending upon same flavour or two different flavour leptons in the final state respectively. The $m_{ll}$ distribution for HQ4l events are shown for highest $m_{ll}$ obtained in the events. As can easily be seen that for $ZZ$ events, the distribution
Table 9. HQ4l ($\ell^+\ell^-\ell'^+\ell'^- + E_T^{\text{miss}}$) events for dominant SM background at LHC with $p_T^{\ell}>20$ and $|m_Z-15| \not\approx |m_{ll}| \not\approx |m_Z+15|$ for all opposite sign same flavour dilepton at $\sqrt{s}=14$ TeV and $L=100$ fb$^{-1}$ luminosity. Appropriate $K$-factors are used for different processes to match to the NLO-NLL cross-sections available in literature (see text for details).

| Process | $\sigma_{\text{production}}$ (in pb) | $E_T^{\text{miss}}$ | $\sigma_{\text{HQ4l}}$ (in pb) | $N(100 \text{ fb}^{-1})$ |
|---------|------------------------------------|----------------------|--------------------------------|--------------------------|
| ZZ      | 0.024                              | > 100                | $1.68 \times 10^{-5}$          | 1                        |
|         |                                    | > 200                | $< 1.2 \times 10^{-6}$         | < 1                      |
|         |                                    | > 300                | $< 1.2 \times 10^{-6}$         | < 1                      |
| $W^+W^-W^+W^-$ | $1.17 \times 10^{-6}$ | > 100                | $< 7.8 \times 10^{-12}$        | < 1                      |
|         |                                    | > 200                | $< 7.8 \times 10^{-12}$        | < 1                      |
|         |                                    | > 300                | $< 7.8 \times 10^{-12}$        | < 1                      |
| $Zt\bar{t}$ | 0.90                             | > 100                | $1.8 \times 10^{-4}$           | 18                       |
|         |                                    | > 200                | $4.5 \times 10^{-5}$           | 4                        |
|         |                                    | > 300                | $< 4.5 \times 10^{-5}$         | < 1                      |

Figure 29. Invariant mass ($m_{ll}$) distribution for OSD (left) and HQ4l channel (right) for BP2 compared with Z background.

peaks at Z-mass, while signal events for BP2 peaks at a larger value. We therefore, employ $|m_Z-15| \not\approx |m_{ll}| \not\approx |m_Z+15|$ cut for all the opposite sign same flavour invariant mass pairs formed in HQ4l events, which retains the signal to a large extent while the background drops significantly.

It is clear that hadronically quiet four lepton channel provides a smoking gun signature of the model as the significance plot suggests in figure 30. Such a signature is not often studied and should be analyzed with existing data for a possible excess. SUSY with R-parity violation can yield four lepton final states, but doesn’t offer a large missing energy as we have here. On the other hand, SUSY with $R$-parity conservation with lighter third generation squarks may have four lepton final state and missing energy, but infected by the presence of multiple $b$ jets. Hence, four lepton final state in vector boson DM model discussed here can be disentangled from that of SUSY. The existing multilepton search criteria [61], due to large $b$-tagged jets yields zero events for signal at the benchmark points and hence allowed by the data as shown in table 10. Lastly, we note that four leptons can also arise with two soft jets and missing energy in our model from the $pp \rightarrow h_{\tilde{q}}\tilde{h}_q$ production, and its subsequent decays through heavier gauge bosons. However, the signal cross-section is smaller (for example,
Figure 30. Significance plot for hadronically quiet four lepton final state (HQ4l) events for benchmark points at LHC for $E_T > 300$ GeV with luminosity (in fb$^{-1}$).

| SR   | $E_T$ (GeV) | $H_T$ (GeV) | BP   | $\sigma^{\text{multilep}}$(fb) | $N(2.3$ fb$^{-1}$) | SM prediction | Obs. data |
|------|------------|------------|------|-------------------------------|---------------------|---------------|-----------|
| SR1  | 50–150     | 60–400     | BP1  | <0.007                        | <1                  | 19.26$^{+4.81}_{-4.80}$ | 18        |
|      |            |            | BP2  | 0.009                         | <1                  |               |           |
|      |            |            | BP3  | 0.026                         | <1                  |               |           |
| SR2  | 150–300    | 60–400     | BP1  | <0.007                        | <1                  | 1.16$^{+0.31}_{-0.29}$    | 4         |
|      |            |            | BP2  | <0.004                        | <1                  |               |           |
|      |            |            | BP3  | <0.0005                       | <1                  |               |           |
| SR3  | 50–150     | 400–600    | BP1  | <0.007                        | <1                  | 1.20$^{+0.47}_{-0.40}$ | 3         |
|      |            |            | BP2  | <0.004                        | <1                  |               |           |
|      |            |            | BP3  | <0.0005                       | <1                  |               |           |
| SR4  | 150–300    | 400–600    | BP1  | <0.007                        | <1                  | 0.29$^{+0.44}_{-0.40}$ | 0         |
|      |            |            | BP2  | <0.004                        | <1                  |               |           |
|      |            |            | BP3  | <0.0005                       | <1                  |               |           |

Table 10. Multilepton ($\geq$ 3 electrons or muons) events for CMS off-Z search [61] at the benchmark points for $\sqrt{s} = 13$ TeV with $\mathcal{L} = 2.3$ fb$^{-1}$.

BP1: $\sigma^{d+2j} = 0.17$ fb with $E_T > 300$ GeV) than the HQ4l events and the SM background is also larger. Hence, the benchmark points can easily satisfy non-observation of data in this channel and we refrain from providing a detailed analysis on that.

A few non-supersymmetric scenarios, like left-right symmetric and Type-II seesaw models [63], contain SU(2) triplet scalars with hyper charge +2 ([64, 65]). Such multiplets contain doubly charged scalars. Through the pair production of such scalars and their dominant leptonic decay modes, we can have HQ4l final states. This resembles our signal but without the large missing energy. Also, unlike our scenario here, one can construct the dilepton (same signed) invariant mass, and that distribution will peak around the mass of that doubly charged scalar. This sharp invariant mass reconstruction is not possible for our scenario and that can differentiate our model from such models containing doubly charged scalars.
6 Summary and conclusions

In this paper we have analyzed the phenomenology of a vector boson DM with SU(2)\textsubscript{N} extension of SM, in detail. The model had already been proposed in literature and is extremely relevant for current studies as it can easily evade the ever growing direct search constraints from non-observation of DM. We elaborate on relic density and direct search outcome of the DM taking care of the freedom in choosing the SU(2)\textsubscript{N} gauge coupling through elaborate parameter space scan. Several new features have emerged from this analysis including the crucial effects of co-annihilation and non-viability of a degenerate DM scenario that could emerge in absence of a scalar triplet.

We have also explicitly demonstrated the unification of the low scale model into gauge group $E(6)$ as was proposed earlier, with consistent intermediate symmetries and breaking scheme. The breaking adopted here suggests that in D-parity conserved case, SU(2)\textsubscript{N} and SU(2)\textsubscript{L} couplings are equal ($g_N = g_L$) with two degenerate vector boson DM, while for D-parity broken scenario we may have non-universality through $g_N \neq g_L$ and a single component DM emerges. The spread in the gauge coupling for D-odd case is determined by explicit calculation and the freedom is utilized for phenomenological analysis.

One of the crucial construct of the model is that the SU(2)\textsubscript{N} charges do not contribute to hypercharge and therefore the gauge bosons $X_{1,2,3}$ are neutral. The stability of the DM ($X_1$) is achieved by a modified $R = (-1)^{\text{B} + L + 2J}$ charge, through $L = T_N + P$. The phenomenology alters completely with a change in choosing the $R$ symmetry and will be discussed elsewhere. Apart from unification, the other important constraint on the model comes from small $X_3(Z')$-$Z$ mixing which demands $m_{X_3} = m_{Z'} \geq 1$ TeV. This, in turn, constrains the lower limit of the DM mass for specific $g_N$.

Relic density constraint of the model correlates coupling and mass of the DM to the exotic quark mass $m_{h_q}$ and exotic lepton mass $m$. Dominant annihilation processes are $t$-channel diagrams through exotic fermion exchange. This allows a large range of DM masses above $\sim 200$ GeV, lower than that is discarded by VEV constraints.

Direct search interaction for this model is mainly $s$-channel process and mediates only via exotic quarks. Hence, except for those regions where $m_{h_q} \sim m_{DM}$, the model is very loosely constrained by spin independent direct search bounds. Co-annihilation of DM with $X_2$ helps the DM to evade direct search constraints and the detection may go beyond the reach of XENONnT sensitivity. This is simply because co-annihilation only contributes to relic density and does not take part in direct search due to kinematical constraints. We also analyzed degenerate DM scenario in D-parity even case, where the DM mass has to be at least $m_{X_3} = m_{Z'} \geq 1$ TeV, as three of the gauge bosons (including the DM) become degenerate. But this situation is ruled out because of relic density constraints.

The exotic particles present in the model, can be produced at LHC and their subsequent decays yield multi-lepton signatures with large missing energy. Opposite sign dilepton and hadronically quiet four lepton channels are discussed in context of the present LHC data through detailed simulations. We note that the presence of only one soft jet in two lepton channel can help to distinguish this model from SM background events like $tt$ and also from other supersymmetric signals. Large missing energy cuts are shown to be effective to reduce the backgrounds and a possibility of seeing such a signal at 14 TeV emerges, albeit with large luminosity. A strong $H_T$ cut, as provided in the current LHC analysis of the data, washes away signal cross-section of this model. We therefore propose a minimal or no $H_T$ cut at all for seeing opposite sign dilepton signature of this particular scenario. However, a more
crucial signal appears in the form hadronically quiet four lepton, that has not been analyzed before. SM background for this channel is tiny, which can be further minimized by employing a high missing energy and invariant mass cut within $Z$-window. The model therefore can be tested in hadronically quiet four lepton events at future runs of LHC at 14 TeV even with a moderate integrated luminosity.

One of the important outcomes of this analysis is therefore to show that, collider search provides more sensitivity in unraveling the DM model while direct search may get delayed, even can be submerged into neutrino floor. This is in sharp contrast to many other DM scenarios. This is attributed to the $t$-channel DM annihilation and dominant $s$-channel direct search along with co-annihilation processes.

A comparative analysis on different vector boson DM frameworks will be helpful to classify and distinguish the phenomenological outcome with respect to future observations. We will discuss some of those features in our next analysis.

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A Scalar potential

The scalar potential of the model is given by:

$$V = \mu_1^2 \text{Tr} \left( \phi_{13}^\dagger \phi_{13} \right) + \mu_2^2 \left( \phi_2^\dagger \phi_2 \right) + \mu_3^2 \left( \chi \chi^\dagger \right) + \mu_4^2 \text{Tr} \left( \Delta \Delta^\dagger \right) + \left( \mu_5^2 \text{det} \Delta + \text{h.c.} \right)$$

$$+ \left( \mu_{22} \tilde{\chi} \phi_{13}^\dagger \phi_{2}^\dagger + \mu_{12} \Delta \chi^\dagger + \mu_{23} \tilde{\chi} \Delta \chi^\dagger + \text{h.c.} \right) + \frac{1}{2} \lambda_1 \left( \text{Tr} \left( \phi_{13}^\dagger \phi_{13} \right) \right)^2 + \frac{1}{2} \lambda_2 \left( \phi_2^\dagger \phi_2 \right)^2$$

$$+ \frac{1}{2} \lambda_3 \text{Tr} \left( \phi_{13}^\dagger \phi_{13} \phi_{13}^\dagger \phi_{13} \right) + \frac{1}{2} \lambda_4 \left( \chi \chi^\dagger \right)^2 + \frac{1}{2} \lambda_5 \left[ \text{Tr} \left( \Delta^\dagger \Delta \right) \right]^2$$

$$+ \frac{1}{4} \lambda_6 \text{Tr} \left( \Delta^\dagger \Delta - \Delta \Delta^\dagger \right)^2 + \tilde{\lambda}_1 \chi \phi_{13}^\dagger \phi_{13} \chi^\dagger + \tilde{\lambda}_2 \chi \phi_{13}^\dagger \phi_{13} \chi^\dagger + \tilde{\lambda}_3 \phi_{13}^\dagger \phi_{13} \phi_{13}^\dagger \phi_{13}$$

$$+ \tilde{\lambda}_4 \phi_2^\dagger \phi_{13}^\dagger \phi_{13}^\dagger \phi_2 + \tilde{\lambda}_5 \left( \phi_2^\dagger \phi_2 \right) \left( \chi^\dagger \chi \right) + \tilde{\lambda}_6 \left( \chi \chi^\dagger \right) \text{Tr} \left( \Delta^\dagger \Delta \right)$$

$$+ \tilde{\lambda}_7 \left( \Delta^\dagger \Delta - \Delta \Delta^\dagger \right) \chi^\dagger + \tilde{\lambda}_8 \left( \phi_2^\dagger \phi_2 \right) \text{Tr} \left( \Delta^\dagger \Delta \right) + \tilde{\lambda}_9 \text{Tr} \left( \phi_{13}^\dagger \phi_{13} \right) \text{Tr} \left( \Delta^\dagger \Delta \right)$$

$$+ \tilde{\lambda}_{10} \text{Tr} \left( \phi_{13} \left( \Delta^\dagger \Delta - \Delta \Delta^\dagger \right) \phi_{13}^\dagger \right),$$

(A.1)
where

$$\phi_2 = \left( \begin{array}{c} \phi_3^+ \\ \phi_3^0 \end{array} \right), \quad \tilde{\phi}_2 = \left( \begin{array}{c} \tilde{\phi}_3^0 \\ -\tilde{\phi}_3^- \end{array} \right), \quad \phi_{13} = \left( \begin{array}{c} \phi_1^0 \\ \phi_1^- \\ \phi_3^- \end{array} \right),$$

$$\tilde{\phi}_{13} = \left( \begin{array}{c} \tilde{\phi}_3^+ \\ -\tilde{\phi}_1^0 \end{array} \right), \quad \chi = (\chi_1^0 \chi_2^0), \quad \tilde{\chi} = (\tilde{\chi}_2^- - \chi_1^0).$$

(A.2)

In above potential $\mu_2^2, \mu_3$ terms explicitly break $L$ softly to $(-1)^L$.

B The Yukawa couplings

The allowed Yukawa couplings of the model [31] for quarks are:

$$\left( d\phi_1^0 - u\phi_3^- \right) d^c - \left( d\phi_3^0 - u\phi_3^- \right) h_q^c, \left( u\phi_2^0 - d\phi_2^- \right) u^c, \left( h_q^c \chi_2^0 - d_c \chi_1^0 \right) h_q.$$ (B.1)

The allowed Yukawa coupling for leptons are:

$$\left( N\phi_3^- - \nu \phi_1^- - E\phi_3^0 + e\phi_1^0 \right) e^c, \left( E\phi_2^+ - N\phi_2^0 \right) \nu^c - \left( e\phi_2^+ - \nu \phi_2^0 \right) \nu^c,$$

$$\left( EE^c - NN^c \right) \chi_2^0 - \left( eE^c - \nu N^c \right) \chi_1^0, \left( E^c\phi_1^- - N^c\phi_1^0 \right) \nu^c - \left( E^c\phi_2^+ - N^c\phi_2^0 \right) \nu^c,$$

$$n^c n^c \Delta_1^0 + (n^c \nu^c + \nu^c n^c) \Delta_2^0 / \sqrt{2} - \nu^c \nu^c \Delta_3^0.$$ (B.3)

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