Abstract. In this work, by using measure of noncompactness, some results on the existence of $n$-tuple fixed points for multivalued contraction mappings are proved. As an application, the existence of solution for a system of integral inclusions is studied.

1. Introduction

Fixed point theory is an important area in mathematics which is closely related to real world problems and has many applications in different fields of science. It has played an important role in solving problems of uniqueness and existence of the nonlinear analysis, topology and geometry. In the literature, it has applications in many sciences like engineering [1], economy [2], optimization [3], game theory [4] and medicine [5]. Brouwer [6] proved fixed point theorem on $n$-dimensional space. Banach [7] established contraction principle which supplied fixed point theorem using contraction mapping. Later, Schauder [8] proved an extension of the Brouwer’s fixed point theorem to spaces of infinite dimension under compactness condition.

For fixed point theory of single valued mappings, one can consult in [9, 10, 11]. The case of multivalued mappings compared to single valued mappings directly applies to the real world problems [12, 13]. The literature is important in view of its. Along with that Kakutani [14] extended Brouwer’s fixed point theorem to multivalued mappings. Afterward, Nadler [15] extended Banach contraction principle from single valued mappings to multivalued mappings using Hausdorff metric. Later, classes of Nadler’s fixed point theorem was extended and generalized for various multivalued mappings in [16, 17].

Measure of noncompactness has played a fundamental role in the study of single valued and multivalued mappings, especially, in metric and topological fixed point theory. It is very useful tool to guarantee the existence of fixed point. The measure of noncompactness was defined and studied by Kuratowski [18]. Darbo [19] used this measure to generalize both Schauder’s fixed point theorem and Banach’s contraction principle for condensing operators. Recently, measure of noncompactness has been used in differential equations, integral equations, nonlinear equations as given in [20, 21, 22].

Partially ordered metric spaces are very important in fixed point theory. By using two basic concepts, Guo and Lakshmikantham [23] first gave some existence

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Theorems of the coupled fixed point for both continuous and discontinuous operators, and then they offered some applications to the initial value problems of ordinary differential equations with discontinuous right-hand sides. Bhaskar and Lakshmikantham [24] introduced coupled fixed point and established in a some coupled fixed point theorems in a partially ordered metric space. By a similar idea, Berinde and Borcut [25] established a tripled fixed point for nonlinear mapping in partially ordered complete metric spaces. Ertürk and Karakaya [26] introduced the concept of $n$-tuplet fixed point and studied existence and uniqueness of fixed point of contractive type mappings in partially ordered metric space. Moreover, by using the condensing operators, Aghajani et al. [27] presented some results on the existence of coupled fixed point Karakaya et al. [28] gave some results concerning the existence of tripled fixed point via measure of noncompactness. For different models of measure of noncompactness in [29].

The existence of fixed point for various contractive mappings has been studied by many authors under different conditions. The concept of coupled fixed point for multivalued mappings was introduced by Samet and Vetro [30] and they presented coupled fixed point theorem for multivalued nonlinear contraction mappings in a partially ordered metric space. Rao et al. [31] obtained a tripled coincidence fixed point theorems for multivalued mappings in a partially ordered metric space.

In this paper, by using condensing operator, we investigate $n$-tuplet fixed point of multivalued mappings on a Banach space. Finally, we also give an application of our result to solve a system of integral inclusions.

2. Preliminaries

Throughout this paper, $E$ is a Banach space and $P(E)$ (or $2^E$) is the set of all subsets of $E$. We denote the set

$$P_k(E) = \{X \subset E, \ X \text{ is nonempty and has a property } k\}.$$

So, $P_{rcp}(E), P_{cl,bd}(E), P_{cl,cv}(E)$ will denote the classes of all relatively compact, closed-bounded and closed-convex subsets of $E$, respectively.

A mapping $T : E \to P_k(E)$ is called a multivalued mapping or set valued mapping on $E$ into $E$. A point $x \in E$ is called a fixed point of $T$ if $x \in Tx$.

**Definition 1** (see; [32]). A mapping $\mu : P_{cl,bd}(X) \to \mathbb{R}^+$ is called a measure of noncompactness if it satisfies the following conditions:

1. $(M_1) \emptyset \notin \mu^{-1}(0) \subset P_{rcp}(X)$,
2. $(M_2) \mu(\bar{A}) = \mu(A)$, where $\bar{A}$ denotes the closure of $A$,
3. $(M_3) \mu(\text{conv } A) = \mu(A)$, where $\text{conv } A$ denotes the convex hull of $A$,
4. $(M_4) \mu$ is nondecreasing,
5. $(M_5)$ If $\{A_n\}$ is a decreasing sequence of sets in $P_{cl,bd}(X)$ satisfying $\lim_{n \to \infty} \mu(A_n) = 0$, then the intersection $A_\infty = \bigcap_{n=1}^{\infty} A_n$ is nonempty.

If $(M_4)$ holds, then $A_\infty \in P_{rcp}(X)$. For this, let $\lim_{n \to \infty} \mu(A_n) = 0$. As $A_\infty \subseteq A_n$ for each $n = 0, 1, 2, \ldots$; by the monotonicity of $\mu$, we obtain

$$\mu(A_\infty) \leq \lim_{n \to \infty} \mu(A_n) = 0.$$ 

So, by $(M_1)$, we get that $A_\infty$ is nonempty and $A_\infty \in P_{rcp}(X)$. 
Theorem 1 (see: [11]). Let $X$ be a closed and convex subset of a Banach space $E$. Then every compact, continuous map $T : X \to X$ has at least one fixed point.

Theorem 2 (see: [20]). Let $X$ be a nonempty, bounded, closed and convex subset of a Banach space $E$ and let $T : X \to X$ be a continuous mapping. Suppose that there exists a constant $k \in [0, 1)$ such that
$$
\mu(T(X)) \leq k \mu(X)
$$
for any subset $X$ of $E$, then $T$ has a fixed point.

Definition 2 (see: [32]). A multivalued mapping $T : E \to P_{cl,bd}(E)$ is called D-set-Lipschitz if there exists a continuous nondecreasing function $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ such that
$$
\mu(T(X)) \leq \varphi(\mu(X))
$$
for all $X \in P_{cl,bd}(E)$ with $T(X) \in P_{cl,bd}(E)$, where $\varphi(0) = 0$. Generally, we call the function $\varphi$ to be a D-function of $T$ on $E$.

When $\varphi(r) = kr$, $k > 0$, $T$ is called a $k$-set-Lipschitz mapping and if $k < 1$, then $T$ is called a $k$-set-contraction on $E$.

If $\varphi(r) < r$ for $r > 0$, then $T$ is called a nonlinear D-set-contraction on $E$.

Lemma 1 (see: [33]). If $\varphi$ is a D-function with $\varphi(r) < r$ for $r > 0$, then
$$
\lim_{n \to \infty} \varphi^n(t) = 0
$$
for all $t \in [0, \infty)$.

Theorem 3 (see: [32]). Let $X$ be a nonempty, bounded, closed and convex subset of a Banach space $E$ and let $T : X \to P_{cl,cv}(X)$ be a closed and nonlinear D-set-contraction. Then $T$ has a fixed point.

As a consequence of Theorem 3, we obtain a fixed point theorem of Darbo (20) type for linear set-contractions.

Corollary 1 (see: [32]). Let $X$ be a bounded, closed and convex subset of a Banach space $E$ and let $T : X \to P_{cl,cv}(X)$ be a closed and $k$-set-contraction. Then $T$ has a fixed point.

Definition 3 (see: [34]). Let $X$ be a topological space, $2^X$ the family of all subsets of $X$ and $T$ be a mapping of $X$ into $2^X$ such that $Tx$ is nonempty, for all $x \in X$. Then the mapping $T$ is called upper semicontinuous if for each closed subset $C$ of $X$,
$$
T^{-1}(C) = \{ x \in X : Tx \cap C \neq \emptyset \}
$$
is closed.

Definition 4 (see: [32]). A mapping $\mu : P_k(E) \to \mathbb{R}^+$ is called nondecreasing if $A, B \in P_k(E)$ are any two sets with $A \subseteq B$, then $\mu(A) \leq \mu(B)$, where $\subseteq$ is order relation of inclusion of sets in $P_k(E)$.

Lemma 2 (see: [35]). Let $X$ be a Banach space and $F$ be a Carathéodory multivalued mapping. Let $\Phi : L^1(H;X) \to C(H;X)$ be linear continuous mapping. Then,
$$
\Phi \circ S_F : C(H;X) \to P_{cl,cv}(C(H;X))
$$
$$
u \to (\Phi \circ S_F)(\nu) := \Phi(S_F(\nu)),
$$
is a closed graph operator in $C(H;X) \times C(H;X)$. 
Lemma 3 (see; [36]).

1. Let $A \subseteq C(H;X)$ be bounded. Then $\mu(A(t)) \leq \mu(A)$ for all $t \in H$, where $A(t) = \{y(t), y \in A\} \subset X$. Furthermore, if $A$ is equicontinuous on $H$, then $\mu(A(t))$ is continuous on $H$ and $\mu(A) = \sup\{\mu(A(t)), t \in H\}$.

2. If $A \subset C(H;X)$ is bounded and equicontinuous, then

$$\mu\left(\int_0^t A(s) \, ds\right) \leq \int_0^t \mu(A(s)) \, ds,$$

for all $t \in H$, where $\int_0^t A(s) \, ds = \left\{\int_0^t x(s) \, ds : x \in A\right\}$.

3. $n$-tuplet Fixed Point Theorems and Some Related Results

In this section, we investigate $n$-tuplet fixed point property of a multivalued mapping and give some applications for special cases $n = 2$, that is, coupled fixed point.

Definition 5. Let $X$ be a nonempty set and $G : X^n \to P(X)$ be a given mapping. An element $(x_1, x_2, x_3, ..., x_n) \in X^n$ is called an $n$-tuplet fixed point of $G$ if

$$x_1 \in G(x_1, x_2, x_3, ..., x_n)$$

$$x_2 \in G(x_2, x_3, ..., x_n, x_1)$$

$$\vdots$$

$$x_n \in G(x_n, x_1, x_2, ..., x_{n-1}).$$

Remark 1. If we take as special cases $n = 2$ and $n = 3$ in Definition 5, respectively, we get coupled fixed point (see; [30]) and tripled fixed point (see; [31]).

Theorem 4. (see; [37]) Let $\mu_1, \mu_2, ..., \mu_n$ be measures of noncompactness in Banach spaces $E_1, E_2, ..., E_n$ respectively. Suppose that the function $F : [0, \infty)^n \to [0, \infty)$ is convex and $F(x_1, x_2, ..., x_n) = 0$ if and only if $x_i = 0$ for $i = 1, 2, ..., n$. Then

$$\tilde{\mu}(X) = F(\mu_1(X_1), \mu_2(X_2), ..., \mu_n(X_n)),$$

defines a measure of noncompactness in $E_1 \times E_2 \times ... \times E_n$ where $X_i$ denotes the natural projection of $X$ onto $E_i$, for $i = 1, 2, ..., n$.

Remark 2. Notice that by taking

$$F(x_1, x_2, x_3, ..., x_n) = \max\{x_1, x_2, x_3, ..., x_n\},$$

or

$$F(x_1, x_2, x_3, ..., x_n) = x_1 + x_2 + x_3 + ... + x_n,$$

for any $(x_1, x_2, x_3, ..., x_n) \in [0, \infty)^n$, the conditions of Theorem 4 are satisfied. Therefore,

$$\tilde{\mu}(X) := \max(\mu(X_1), \mu(X_2), ..., \mu(X_n)),$$

or

$$\tilde{\mu}(X) := \mu(X_1) + \mu(X_2) + ... + \mu(X_n),$$

defines measures of noncompactness in the space $E^n$, where $X_i$, $i = 1, 2, ..., n$ are the natural projections of $X$ on $E_i$.

We now give an important theorem for existence of fixed point of multivalued mapping under measure of noncompactness condition.
Theorem 5. Let $X$ be a nonempty, bounded, closed and convex subset of a Banach space $E$ and let $\mu$ be an arbitrary measure of noncompactness in it. Let $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ be a nondecreasing and upper semicontinuous function such that $\varphi (r) < r$ for all $r > 0$. Suppose that $G : X_1 \times X_2 \times \cdots \times X_n \to P_{cl,cv}(X)$ is continuous multivalued operator satisfying

$$
\mu (G(X_1 \times X_2 \times \cdots \times X_n)) \leq \varphi \left( \frac{\mu (X_1) + \mu (X_2) + \cdots + \mu (X_n)}{n} \right)
$$

for all $X_1, X_2, \ldots, X_n \subset X$. Then $G$ has at least one $n$-tuple fixed point.

Proof. As in Remark 2, we define the measure of noncompactness $\tilde{\mu}$ by

$$
\tilde{\mu} (X) := \mu (X_1) + \mu (X_2) + \cdots + \mu (X_n).
$$

Define the mapping $\tilde{G}(X) := G (X_1 \times X_2 \times \cdots \times X_n)$. We prove that $\tilde{G}$ satisfies all the conditions of Theorem 4. Then

Clearly,

$$
\tilde{\mu} \left( \tilde{G} (X) \right) = \tilde{\mu} \left( G (X_1 \times X_2 \times \cdots \times X_n) \right)
= \tilde{\mu} \left( G(x_1, x_2, x_3, \ldots, x_n), G(x_2, x_3, \ldots, x_n, x_1), \ldots, G(x_n, x_1, x_2, \ldots, x_{n-1}) \right)
= \mu (G(x_1, x_2, x_3, \ldots, x_n)) + \mu (G(x_2, x_3, \ldots, x_n, x_1)) + \cdots + \mu (G(x_n, x_1, x_2, \ldots, x_{n-1}))
\leq \varphi \left( \frac{\mu (X_1) + \mu (X_2) + \cdots + \mu (X_n)}{n} \right) + \varphi \left( \frac{\mu (X_2) + \mu (X_3) + \cdots + \mu (X_1)}{n} \right)
+ \cdots + \varphi \left( \frac{\mu (X_n) + \mu (X_1) + \cdots + \mu (X_{n-1})}{n} \right)
= n \varphi \left( \frac{\mu (X_1) + \mu (X_2) + \cdots + \mu (X_n)}{n} \right).
$$

Now,

$$
\frac{1}{n} \tilde{\mu} \left( \tilde{G} (X) \right) \leq \varphi \left( \frac{\mu (X_1) + \mu (X_2) + \cdots + \mu (X_n)}{n} \right)
$$

and taking $\tilde{\mu}' = \frac{1}{n} \tilde{\mu}$, we get

$$
\tilde{\mu}' \left( \tilde{G} (X) \right) \leq \varphi \left( \tilde{\mu}' (X) \right).
$$

Also, $\tilde{\mu}'$ is a measure of noncompactness. Thus, by Theorem 4 we obtain that $G$ has at least one $n$-tuple fixed point. \qed

Remark 3. If we take $\mu$ measure of noncompactness in Theorem 2 as

$$
\tilde{\mu} (X) := \max (\mu (X_1), \mu (X_2), \ldots, \mu (X_n)).
$$

We can obtain the same result.

4. Application to Inclusions Systems

The multivalued fixed point theorem of this paper has some nice applications to differential and integral systems of inclusions as an example we study the solvability of a system of differential inclusions.

Consider the following differential system

$$
\begin{cases}
  x'(t) \in A(t)x(t) + G(t, x(t), y(t)), & t \in [0, b] \\
  y'(t) \in A(t)y(t) + F(t, y(t), x(t)), & t \in [0, b]
\end{cases}
$$

(4.1)
Definition 6. A family \( \{ U(t,s) : t, s \in \Delta \} \) of bounded linear operators \( U(t,s) : X \rightarrow X \) where \( (t,s) \in \Delta := \{(t,s) \in J \times J : 0 \leq s \leq t < +\infty \} \) for \( J = [0,b] \) is called an evolution system if the following properties are satisfied,

1. \( U(t,t) = I \) where \( I \) is the identity operator in \( X \) and \( U(t,s) U(s,\tau) = U(t,\tau) \) for \( 0 \leq \tau \leq s \leq t < +\infty \),
2. The mapping \( (t,s) \rightarrow U(t,s) \) is strongly continuous, that is, there exists a constant \( M > 0 \) such that

\[
\| U(t,s) \| \leq M \quad \text{for any} \quad (t,s) \in \Delta.
\]

An evolution system \( U(t,s) \) is said to be compact if \( U(t,s) \) is compact for any \( t-s > 0 \). \( U(t,s) \) is said to be equicontinuous if \( \{ U(t,s)x : x \in M \} \) is equicontinuous at \( 0 \leq s < t \leq b \) for any bounded subset \( B \subset X \). Clearly, if \( U(t,s) \) is a compact evolution system, it must be equicontinuous. The converse is not necessarily true.

More details on evolution systems and their properties could be found in the books of Ahmed \[38\], Engel and Nagel \[39\] and Pazy \[40\].

Definition 7. We say that the couple \( (x(t), y(t)) \) \( \in C([0,b], X) \times C([0,b], X) \) is a mild solution of the evolution system (4.1) - (4.2) if it satisfies the following integral system

\[
\begin{align*}
\dot{x}(t) &= U(t,0) \varphi(x,y) + \int_0^t U(t,s) g(s) \, ds \quad \text{for } g \in S_G(x,y) \\
\dot{y}(t) &= U(t,0) \varphi(y,x) + \int_0^t U(t,s) g(s) \, ds \quad \text{for } g \in S_G(y,x),
\end{align*}
\]

for all \( t \in [0,b] \).

Theorem 6. Assume the following hypotheses

\( H1 \) \( \{ A(t) : t \in J \} \) is a family of linear operators. \( A(t) : D(A) \subset X \rightarrow X \) generates an equicontinuous evolution system \( \{ U(t,s) : (t,s) \in \Delta \} \) and

\[ |U(t,s)| \leq M. \]

\( H2 \) The multifunction \( G : J \times C([0,b] \times X \times X) \rightarrow P_{cl,cv}(X) \) is an upper Carathéodory with respect to \( x \) and \( y \) and \( \varphi : C(J;X) \rightarrow X \) is compact and

\[
\mu \left( G(t,W \times X) \right) < k\mu \left( \frac{W \times W}{2} \right) \quad \text{for any} \quad t \in J.
\]

\( H3 \) There exists a constant \( r > 0 \) such that

\[
M \left[ ||\varphi(x,y)|| + \left\{ \| g(t) \|_1 : g \in S_G(x,y), x \in A_0 \right\} \right] \leq r
\]

and

\[
M \left[ ||\varphi(y,x)|| + \left\{ \| g(t) \|_1 : g \in S_G(y,x), y \in A_0 \right\} \right] \leq r.
\]
Thus $A_0 = \{ z \in C(J; X) : \| z(t) \| \leq r \text{ for all } t \in J \}$ hold. Then the nonlocal system \((4.1) - (4.2)\) has at least one mild solution in the space $C(J; X)$.

**Proof.** To solve problem given in \((4.1) - (4.2)\), we transform it into the following fixed point problem.

Consider the multivalued operator $N : C([0, b] ; X) \rightarrow \mathcal{P}(C([0, b] ; X))$ defined by,

$$N(x, y) = \left\{ h \in C(J; X) : h(t) = U(t, 0) \varphi(x, y) + \int_0^t U(t, s) g(s) \, ds, \text{ with } g \in S_G(x, y) \right\}.$$ 

Clearly, coupled fixed points of the operator $N$ are mild solutions of system \((4.2) - (4.3)\).

Obviously, for each $y \in C([0, b] ; X)$, the set $S_G(x, y)$ is nonempty since, by $(H2)$, $G$ has a measurable selection (see $(4.1)$).

Let show that $N$ has a coupled fixed point. For that, we need to verify all the conditions of Theorem 5.

Let $A_0 = \{ z \in C([0, b] ; X) : \| z(t) \| \leq r \text{ for all } t \in [0, b] \}$. We notice that $A_0$ is closed, bounded and convex.

To show that $N(A_0 \times A_0) \subseteq A_0$, we need first to prove that the family

$$\left\{ \int_0^t U(t, s) f(s) \, ds : f \in S_F(y) \text{ and } y \in A_0 \right\}$$

is equicontinuous for $t \in J$, that is, all the functions are continuous and they have equal variation over a given neighbourhood.

In view of $(H1)$ we have that functions in the set $\{ U(t, s) : (t, s) \in \Delta \}$ are equicontinuous, i.e., for every $\varepsilon > 0$ there exists $\delta > 0$ such that $|t - \tau| < \delta$ implies $\| U(t, s) - U(\tau, s) \| < \varepsilon$ for all $U(t, s) \in \{ U(t, s) : (t, s) \in \Delta \}$.

Then, given some $\varepsilon > 0$ let $\delta = \frac{\varepsilon}{\| g \|_\infty}$ such that $|t - \tau| < \delta$, we have

$$\left| \int_0^t U(t, s) g(s) \, ds - \int_0^\tau U(\tau, s) g(s) \, ds \right| \leq \int_0^\tau |U(t, s) - U(\tau, s)| \| g(s) \| \, ds.$$

As $\{ U(t, s) : (t, s) \in \Delta \}$ is equicontinuous, so we have

$$\left| \int_0^t U(t, s) g(s) \, ds - \int_0^\tau U(\tau, s) g(s) \, ds \right| \leq \varepsilon \| g \|_\infty |t - \tau|$$

$$< \varepsilon \| g \|_\infty \frac{\varepsilon'}{\varepsilon \| g \|_\infty} = \varepsilon'.$$

Hence we conclude that $\left\{ \int_0^t U(t, s) g(s) \, ds : g \in S_G(x, y) \text{ and } (x, y) \in A_0 \times A_0 \right\}$ is equicontinuous for $t \in J$.

Now, we show that $N(A_0 \times A_0) \subseteq A_0$. For $t \in J$, we have

$$|h(t)| = \left| U(t, 0) \varphi(x, y) + \int_0^t U(t, s) g(s) \, ds \right|$$

$$\leq |U(t, 0) \varphi(x, y)| + \int_0^t |U(t, s) g(s)| \, ds$$

$$\leq M \| \varphi(x, y) \| + M \| g \|_1$$

$$= M \| \varphi(x, y) \| + \| g \|_1 \leq r.$$

Thus $N(A_0 \times A_0) \subseteq A_0$. 

Further, it is easy to see that $N$ is convex value.

Now, let us show that $N$ has a closed graph, let $x_n \to x$, $y_n \to y$ and $h_n \to h$

such that $h_n(t) \in N(x_n, y_n)$ and we show that $h(t) \in N(x, y)$.

Now, there exists a sequence $g_n \in S_G(x_n, y_n)$ such that

$$ h_n(t) = U(t, 0) \varphi(x_n, y_n) + \int_0^t U(t, s) g_n(s) ds. $$

Consider the linear operator $\Phi : L^1([0, b]; X) \to C([0, b]; X)$ defined by

$$ \Phi f(t) = \int_0^t U(t, s) g(s) ds. $$

Clearly, $\Phi$ is linear and continuous. So by Lemma 2, we get that $\Phi \circ S_G(x, y)$ is a closed graph operator. Further, we have

$$ h_n(t) - U(t, 0) \varphi(x_n, y_n) \in \Phi \circ S_G(x, y). $$

Since $x_n \to x$, $y_n \to y$ and $h_n \to h$, therefore

$$ h(t) - U(t, 0) \varphi(x, y) \in \Phi \circ S_G(x, y). $$

That is, there exists a function $g \in S_G(x, y)$ such that

$$ h(t) = U(t, 0) \varphi(y) + \int_0^t U(t, s) g(s) ds. $$

Therefore $N$ has a closed graph, hence $N$ has closed values on $C([0, b] \times X \times X, X)$.

We know that the family $\left\{ \int_0^t U(t, s) f(s) ds, f \in S_F(W(t)) \right\}$ is equicontinuous, hence by Lemma 3, we have

$$ \mu \left( \int_0^t U(t, s) g(s) ds, g \in S_G(W(t) \times W(t)) \right) \leq \int_0^t \mu(U(t, s) g(s), g \in S_G(W(t) \times W(t))) ds $$

$$ \leq M \int_0^t \mu(g), g \in S_G(W(t) \times W(t))) ds $$

$$ \leq M t \mu(G(t, W(t))). $$

Therefore

$$ \mu(N(W \times W)) = \mu N \left( U(t, 0) \varphi(W(t) \times W(t)) + \int_0^t U(t, s) g(s) ds, g \in S_G(W(t) \times W(t)) \right) $$

$$ \leq \mu (U(t, 0) \varphi(W(t) \times W(t))) + \mu \left( \int_0^t U(t, s) g(s) ds, g \in S_G(W(t) \times W(t)) \right) $$

$$ \leq M \mu(\varphi(W(t) \times W(t))) + M t \mu(G(t, W(t))). $$

In view of $(H2)$, we get

$$ \mu(N(W \times W)) \leq M b k \mu \left( \frac{W \times W}{2} \right). $$

Therefore, for $M b k < 1$, we obtain that $N$ has at least one coupled fixed point.

Hence, the system (1.1) - (1.2) has at least one solution.
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