Strongly Coupled QCD at Finite Baryon Density

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The analytical results obtained in the infinite mass and strong coupling limits of QCD are difficult to reconcile with the predictions of the Monomer Dimer Polymer algorithm. We have reconsidered in detail the results obtained with this simulation scheme and evidences of severe convergence problems are presented for the \(SU(3)\) and \(SU(2)\) gauge group cases.

Introduction

Finite Density QCD is affected by the well known sign problem that has prevented, in most cases, any success in simulate this theory and, until now, no solution is at sight. The only exceptions in this scenario are strong coupling simulations performed with the Monomer Dimer Polymer (MDP) algorithm \[1\]. This algorithm is able to provide results not affected by early onset and in partial agreement with the Mean Field (MF) prediction. More recently another theoretical advance has been achieved solving exactly the \(\beta = 0\) QCD in the limit of infinite mass and chemical potential \[2\]. In the next section we discuss the compatibility of MDP results with the infinite mass \(SU(3)\) solution; indeed we have found difficult to conciliate the numerical and analytical predictions. This leads us to reconsider the MDP algorithm more carefully and we have found evidences of convergence problems.

We have also considered the \(SU(2)\) case where the sign problem is not present. Results obtained using the Gran Canonical Partition Function (GCPF) formalism turn out to be in very good agreement with the Hybrid Monte Carlo (HMC) calculations \[3\] while, once again, MDP results \[4\] are inaccurate in the critical region.

\(SU(3)\) case

The main feature of infinite mass QCD \[2\] is that at zero temperature \((L_t \rightarrow \infty)\) the system undergoes a first order saturation transition and the phase of the Dirac determinant is not relevant. However at non-zero temperature \((L_t \ finite)\) the system has only a smooth crossover and the phase is relevant. The MDP results are somehow puzzling if considered in the light of the infinite mass solution. In \[1\] Karsch and Mütter saw a strong first order transition for \(L_t = 4\) and \(L_s = 4,8\) with \(m \in [0.1, 0.7]\), while we know that for \(m \rightarrow \infty\) the number density is a smooth function of \(\mu\) for any finite \(L_t\). To reconcile the \(m \rightarrow \infty\) solution and MDP results we have to suppose that the transition disappears at some large bare mass \(\bar{m}\), or that the infinite mass limit of QCD is singular. These statements seem both unnatural. The former would imply the existence of a (large) physical scale where the system behaviour changes, washing out the transition. The latter is unfeasible too, because the Dirac determinant approaches its infinite mass limit continuously. In order to have data more easily comparable with the analytic predictions we have tried to use the MDP algorithm \[4\] directly in the large mass regime. The authors of the MDP code noticed \[4\] that for small masses (i.e. \(m < 0.1\)) the

\[1\] The authors thank F. Karsch for providing the MDP code.
algorithm becomes non-ergodic. When we tried MDP simulations for $m > 1.0$ we saw a similar behaviour: for no value of $\mu$ the system moved into the saturated phase. Moreover a degradation in performance (for all mass values) has been observed increasing the lattice size but, what is more surprising, this degradation seems to be related only to the value of $L_t$. We have not been able to perform simulations for $L_t > 4$.

The behaviour of MDP code prevents any direct comparison between the large mass results and MDP ones. The limited applicability of MDP algorithm raises doubts on its general validity. We have repeated the MDP simulations at $m = 0.1$ in a $4^3 \times 4$ lattice (values used in the original paper [1]), using as initial configuration either $n(\mu) = 0$ or $n(\mu) = 1$ and $O(10^6)$ Montecarlo steps for each value of $\mu$. From fig. 1 it is evident a signal of hysteresis in the data; the $\mu = 0$ start undergoes a strong saturation transition at $\mu = 0.69$ (published result [1]), while the run with saturated start jumps in the zero density phase at $\mu = 0.58$. This result should lead us to caution on the determination of the critical point and may well reconcile the MDP results with MF predictions (the MF critical point $\mu_c^{MF} = 0.61$ lies inside the hysteresis). To be confident with MDP results we should observe several flip-flops in the Montecarlo history to conclude that a clear two state signal is present. We have considered runs of up to $O(10^9)$ configurations and we did not succeed to see any flip-flop for any $\mu$ inside the hysteresis region. Starting from a zero density configuration nothing happens until we get close to $\mu = 0.69$. At this point the system has some probability to go in the saturated phase. Once the system is in the saturated phase it never goes back. The same behaviour has been observed (near $\mu = 0.58$) for the saturated start. Varying the quark mass only changes the hysteresis position unless we reach too small ($m < 0.1$) or too large ($m > 1.0$) values. From these numerical evidences we may conclude that the hysteresis behaviour of the system is independent on the statistics for any value of $\mu$. The MDP code seems to have convergence problems in the most interesting region of $\mu$, independently on the temporal lattice extent and the quark mass value.

**SU(2) case**

Let us now address the MDP convergence in the case of $SU(2)$ gauge group. The motivation is twofold: firstly we wonder if the problems present in $SU(3)$ are universal independently on the gauge group. Moreover $SU(2)$ offers us the possibility of using conventional simulation algorithms. In this case, in fact, quarks and antiquarks belong to the same (real) representation: the Dirac determinant is real and positive also for non-zero $\mu$. The main published MDP results in $SU(2)$ concern the number density and the chiral condensate in a $4^3 \times 4$ and $8^3 \times 4$ lattice at $m = 0.2$ [4]. Other results for $SU(2)$ at $\beta = 0$ and $\mu \neq 0$ are obtained using the HMC algorithm in a $4^3 \times 4$ at $m = 0.2$ [3]. In order to have results with $\mu$ varying continuously, we have used the GCPF scheme, performing simulations with $L_t = 4$ and $L_s = 4, 6, 8$, at the quark mass values $m = 0.1, 0.2, 0.4$. In fig. 2 we report the number density and the chiral condensate obtained in our simulations compared with HMC results appeared in fig. 4 of [3], error bars are reported at some values of $\mu$. It is evident that our simulation reproduces the HMC results quite accurately.
HMC suggests that sampling problems, found in $SU(3)$ with GCPF [2], are not present in $SU(2)$. Simplified models predict for $SU(2)$, at least at small temperature, a phase transition at half of the mass of the lightest baryon of the theory (degenerated with the pion at $\mu = 0$). We have then computed the pion mass in a $6^3 \times 12$ at $m = 0.1, 0.2, 0.4$. To extract the critical $\mu$ we have used the following criterium. The number density appears, increasing the volume, almost zero up to $\mu_c$, with a linear rise beyond it and flat at large $\mu$ (saturation). To identify the critical point we have computed $\partial n(\mu)/\partial \mu$ for two different volumes and defined $\mu_c$ as the position of the first crossing of the curves. In the limit $V \to \infty$ this defines correctly the $\mu$ at which the linear behaviour begins. In table we report our $\mu_c$ and one half the pion mass for different values of $m$:

| m  | $\mu_c$  | $m_\pi/2$ |
|----|--------|--------|
| 0.1| 0.340(4)| 0.3408(7) |
| 0.2| 0.485(5)| 0.4840(6) |
| 0.4| 0.693(5)| 0.6889(5) |

We can conclude from these data that our predicted $\mu_c$ moves with the quark mass in the expected way; this behaviour increases our confidence on our results. In fig. 3 we report the chiral condensate at $m=0.2$ from our data and, superimposed, the MDP results as reported in figure 2 of [4]. From fig. 3 it is evident a marked difference between MDP results and those by our simulations only for the largest lattice ($8^3 \times 4$) and again limited around $\mu_c$ as in the $SU(3)$ case. In particular the $8^3 \times 4$ results differ from ours at $\mu = 0.6$, the critical point derived in [4]. The authors of [4] have tested the independence of their results on the initial configuration only for the $4^3 \times 4$ lattice. In this case MDP results agree at a good level with ours (see $\overline{\psi}\psi$ of fig. 3). In our opinion the observed discrepancy has to be ascribed to convergence problems of the MDP algorithm, although they arise at volumes larger then in the $SU(3)$ case. Once again there are serious doubts on the accuracy that the MDP algorithm can achieve near the critical region.

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Figure 2. Number density and chiral condensate as a function of $\mu$ in a $4^3 \times 4$ at $m = 0.2$ from GCPF (line) and HMC (symbols).

Figure 3. Chiral condensate at $m = 0.2$ in $4^3 \times 4$ (dots), $6^3 \times 4$ (dashes), $8^3 \times 4$ (line) and MDP results in a $4^3 \times 4$ (diamonds) and $8^3 \times 4$ (squares).
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