Spatial optical solitons supported by mutual focusing

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We study composite spatial optical solitons supported by two-wave mutual focusing induced by cross-phase modulation in Kerr-like nonlinear media. We find the families of both single- and two-hump solitons and discuss their properties and stability. We also reveal remarkable similarities between recently predicted holographic solitons in photorefractive media and parametric solitons in quadratic nonlinear crystals.

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One of the standard physical mechanisms supporting the existence of spatial optical solitons is the beam self-focusing due to nonlinearity-induced change of the medium refractive index (see, e.g., Refs. and references therein). In such a case, an optical beam induces an effective waveguide and becomes self-guided in a nonlinear optical medium.

A rather different physical mechanism is responsible for the so-called mutual beam focusing which can be observed, for example, in photorefractive two-wave mixing. In this case, a single beam diffracts because its self-induced focusing is weak or negligible. However, the same beam can demonstrate focusing in the presence of the other beam, due to the mutual cross-phase-modulation interaction in a Kerr-like medium with the third-order nonlinear susceptibility. As was predicted recently, the effect of mutual focusing can be responsible for the formation of a new kind of optical spatial soliton, the so-called holographic soliton. Such a soliton is created in the absence of the self-action effect, solely due to the beam cross-coupling through the induced Bragg reflection. Because the holographic soliton is described by two modes in the frames of the coupled-mode theory, it can be regarded as a special case of vector solitons, well studied in nonlinear optics.

The concept of a vector soliton was originally associated with nonlinear interaction of two polarizations in a birefringent Kerr medium. In this case, both self- and cross-phase modulation of the beams produce a bound state of two orthogonally polarized field components in the form of a self-trapped two-component vector soliton. The existence of spatial vector solitons can be understood through the concept of induced waveguiding. Indeed, when one of the soliton components creates an effective waveguide in a nonlinear medium via the self-modulation effect, the second component can be localized as a guided mode of this waveguide, which creates a composite state in the form of a vector soliton even beyond the bifurcation point. This concept allows to introduce, in a simple and straightforward way, both single- and multi-hump vector solitons, once it is accepted that the soliton-induced waveguide can support higher-order modes.

In the case of vector solitons due to mutual focusing the physics of beam coupling cannot be grasped by the simple waveguiding arguments. In particular, it is not intuitively clear what would be the properties of such solitons when the powers of the two constituents are significantly dissimilar. Since the holographic solitons have been found only in the special case when two optical beams have equal amplitudes (and the corresponding mathematical problem reduces to a scalar equation), we wonder if the mutual self-focusing may generate stable composite solitons in a Kerr-like medium in a broad range of experimental parameters, and if they can be compared with other types of solitons studied earlier.

First of all, we note that mutual focusing is also known to support parametric spatial solitons in quadratic nonlinear media. In general, such three-wave parametric solitons consist of two coupled low-frequency beams and the sum-frequency component. In this case, the phase-matching energy-dependent interaction between the beams is a key physical mechanism for supporting the parametric solitons. One of the waves can be generated inside the crystal, however at least two waves at the input are required to generate a three-wave parametric soliton. This feature suggests that there may be a strong similarity between different types of solitons supported by mutual focusing.

In this Letter we analyze the composite spatial solitons supported by mutual beam focusing in a Kerr-like nonlinear medium in the absence of the self-action effects. We predict the existence of continuous families of single- and two-hump composite solitons, and discuss their stability and interaction.

In order to describe the mutual focusing of two optical beams in a nonlinear medium with the third-order susceptibility, we consider the model for holographic solitons recently introduced by Cohen et al.

\begin{align}
    i \frac{\partial u}{\partial z} + \frac{\partial^2 u}{\partial x^2} + \frac{|v|^2 u}{1 + |u|^2 + |v|^2} &= 0, \\
    i \frac{\partial v}{\partial z} + \frac{\partial^2 v}{\partial x^2} + \frac{|u|^2 v}{1 + |u|^2 + |v|^2} &= 0,
\end{align}

where \( u(x, z) \) and \( v(x, z) \) are the normalized envelopes of the two interacting waves in a waveguide geometry. The waves are coupled only through the effective nonlinear cross-phase modulation terms and the saturation effect
istic examples of the two-wave soliton profiles are shown for which light is localized in the central region, functions of the propagation constant $\alpha$, and find localized solutions everywhere inside the existence region of symmetric single-hump solitons.

To find the soliton families, we solve Eq. (2) numerically and find localized solutions everywhere inside the analytically estimated existence region on the parameter plane ($\alpha_1, \alpha_2$) shown in Fig. 1(top) as a triangular domain in the inset. The characteristic dependencies of the soliton partial powers are presented in Fig. 1(top) as functions of the propagation constant $\alpha_2$. The characteristic examples of the two-wave soliton profiles are shown in Figs. 1(bottom), and they correspond to the points A, B, and C marked on the power plot. In agreement with the analytical results 2, the amplitude of the $v$ component increases for larger values of the propagation constant $\alpha_2$, and the corresponding power dependence $P_1(\alpha_2)$ is monotonic. On the other hand, the power dependence $P_2(\alpha_2)$ for the $v$ component exhibits bistability. The power increases for $\alpha_2 \to 0$ because in the low-saturation regime the amplitude of the $v$ component is
almost fixed by the value of $\alpha_1$, whereas the beam width grows as $\sqrt{\alpha_2}$. In spite of such a non-monotonic dependence of the power component, the two-wave solitons corresponding to both branches are stable. The reason is the following. In the system under consideration two partial powers are conserved independently, as the mutual interaction is incoherent. In this case, the soliton stability is defined by the generalized Vakhitov-Kolokolov criterion which, as we have verified for our model, predicts soliton stability. Thus, single-hump vector solitons supported by mutual focusing are stable.

We now search for two-hump vector solitons for which the components $u$ and $v$ have symmetric unipolar and antisymmetric bipolar profiles, respectively. We find that such solitons appear only when $\alpha_1 < \alpha_2$. This is exactly one half of the existence region for one-hump vector solitons in the inset shown in Fig. 1 (top). The characteristic examples of two-hump solitons are presented in Fig. 2 where we show the modification of the soliton profiles when one of the parameters is fixed. As the boundary of the existence domain is approached, i.e. $\alpha_2 \rightarrow \alpha_1$, the separation between two humps increases, and the two-hump solutions can be considered as a bound state of two single-hump vector solitons. However, for small $\alpha_2$ the unipolar component becomes much wider, and it carries more power than the bipolar one. In this limit the structure of two-hump vector solitons resembles that of parametric solitons in nonlinear quadratic media.

Our results suggest that there should exist a link between holographic and three-wave parametric solitons. Let us consider the properties of parametric solitons when the phase mismatch $\Delta k$ is large. Then, the generated sum-frequency wave has a small amplitude, which can be found using the cascading approximation, $E_3 \sim E_1 E_2/\Delta k$. After substituting this expression into the governing equations for the low-frequency beams, we obtain the model which is mathematically equivalent to Eqs. (11) in the low-saturation limit. Therefore, many similarities between holographic and parametric solitons are not a coincidence, but rather a manifestation of the similar physical origin of such solitons – the effect of mutual focusing.

We would like to stress that, despite some similarities, the soliton dynamics in the low- and high-saturation regimes can be substantially different, as is clearly illustrated in Figs. 3(a,b) for the soliton collisions. In both cases solitons merge, however only in the high-saturation regime strong oscillations develop, see Fig. 3(a). This happens because, in the regions with high beam intensity, the medium response is almost linear, and persistent beating between several modes can develop. On the other hand, in the low-saturation case oscillations quickly decay due to strong radiation, see Fig. 3(b).

In conclusion, we have analyzed two-component spatial solitons supported by mutual focusing of two optical beams in the absence of the self-phase modulation effects. One of the possible realization of these solitons is through the holographic focusing effect recently discussed by Cohen et al. We have demonstrated the existence of the continuous families of single- and two-hump vector solitons, and have discussed their stability. Additionally, we have emphasized a key difference between the formation of the conventional vector solitons and the two-component solitons supported by mutual focusing, but also revealed their links to the parametric quadratic solitons.

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