Features of determining the efficiency of the cam wave generator of the wave gear

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Abstract. The object of study is a wave gear with a cam wave generator. One of the requirements for wave gears is to increase their efficiency. The article considers a refined method for determining the efficiency of a flexible bearing, taking into account the deformation of the flexible wheel, hard wheel, outer ring of the flexible bearing, cam shaft, shaft of the flexible wheel, as well as contact deformations of teeth, FB balls, roller paths of the outer and inner rings of the flexible bearing. According to the proposed method, the influence of the resistance moment, gear wheel displacement coefficients and radial deformation of a flexible wheel on the efficiency of a flexible bearing is investigated. The obtained dependencies expand knowledge about the influence of various factors of wave transmission of the efficiency of flexible bearings.

1. Introduction

When designing gear drives of actuators, one of the requirements for them is to increase efficiency. The works by E.G. Ginzburg [1], D.P. Volkova [2], A.I. Poletuchiy [3], V.A. Finogenova et al. are dedicated to the determination of wave gear’s efficiency. As the studies of these authors have shown, the main power losses in wave transmission occur in the wave generator and toothed gearing, and an increase in efficiency can be achieved due to the correct choice of transmission parameters.

Experimental studies of various authors have shown that the efficiency of a flexible bearing is in the range from 0.8 to 0.85, and of a toothed gearing - from 0.97 to 0.99. The theoretical definition of losses in a wave generator is based on the use of a conditional coefficient of friction [4]. This approach is approximate and does not allow to reveal the influence of geometric parameters on transmission losses.

2. Problem description

The efficiency of flexible bearings is largely determined by the forces acting on the rollers. These forces depend on the radial load and its distribution between the rollers. Using a refined methodology for determining the elastic interaction of transmission elements will enable us to study the influence of various parameters on the efficiency of a flexible bearing (FB). The article uses a mathematical model that takes into account the spatial nature of the elastic interaction of gear wheels, a flexible wheel and a flexible bearing, outer and inner rings of FB with rollers.

The aim of this work is to develop an improved methodology for determining the efficiency of flexible bearings of a wave gear transmission and theoretical studies of the influence of various factors on it.
3. Mathematical model

A wave gear with a cam wave generator and a fixed rigid wheel is shown in Fig. 1. To determine the forces acting on rollers, a spatial mathematical model of wave transmission is used [5]. This model takes into account the spatial nature of the interaction of its elements: a flexible wheel (FW), a hard wheel (HW) and an outer ring of a flexible bearing (FBOR). The calculation model takes into account the interaction between: the lateral surfaces of FW and HW teeth, FW inner surface and FBOR outer surface, FBOR inner surface and rollers.

The coworking surfaces were divided by mutually perpendicular lines. Each node of the formed grids was associated with a basic function in the form of a hexagonal pyramid. The distributed forces of the interaction of the lateral surfaces of FW and HW teeth, FW inner surface and FBOR outer surface were replaced by linear combinations of basic functions.

When compiling the resolving system of equations, the following notations were introduced: the vector of nodal forces of the transmission elements’ interaction \( \mathbf{F} = \left[ \mathbf{F}^{(1)T}, \mathbf{F}^{(2)T}, \mathbf{F}^{(3)T} \right]^T \); nodal point gap vector \( \mathbf{\delta} = \left[ \mathbf{\delta}^{(1)T}, \mathbf{\delta}^{(2)T}, \mathbf{\delta}^{(3)T} \right]^T \); displacement of elements as rigid bodies \( \mathbf{a} = \left[ \mathbf{a}^{(1)T}, \mathbf{a}^{(2)T}, \mathbf{a}^{(3)T}, \mathbf{e}^T \right]^T \).

Here \( \mathbf{F}^{(1)} \), \( \mathbf{F}^{(2)} \), \( \mathbf{F}^{(3)} \) are vectors of interaction forces HW and FW, FW and FBOR, FBOR and rollers of a flexible bearing respectively; \( \mathbf{\delta}^{(1)} \), \( \mathbf{\delta}^{(2)} \), \( \mathbf{\delta}^{(3)} \) are gap vectors at nodal points between the same elements; \( \mathbf{\delta}_0^{(1)} \), \( \mathbf{\delta}_0^{(2)} \), \( \mathbf{\delta}_0^{(3)} \) are initial gap vectors (gaps in an undeformed system); \( \mathbf{a}^{(1)} = \left[ \Delta x^{(1)}, \Delta y^{(1)}, \Delta \varphi_z^{(1)} \right]^T \), \( \mathbf{a}^{(2)} = \left[ \Delta x^{(2)}, \Delta y^{(2)} \right]^T \), \( \mathbf{a}^{(3)} = \left[ \Delta x^{(3)}, \Delta y^{(3)}, \Delta \varphi_y^{(3)}, \Delta \varphi_z^{(3)} \right]^T \), \( \mathbf{e} = \left[ e_x, e_y \right]^T \) — displacement vectors of HW, FW, FBOR and cam, respectively.

The resolving system of equations has the following form:

\[
\begin{pmatrix}
\mathbf{D}^{(1)} & \mathbf{D}^{(12)} & 0 & \mathbf{G}^{(1)} & \mathbf{G}^{(12)} & 0 \\
\mathbf{D}^{(21)} & \mathbf{D}^{(22)} & \mathbf{D}^{(23)} & 0 & \mathbf{G}^{(22)} & \mathbf{G}^{(23)} \\
0 & \mathbf{D}^{(32)} & \mathbf{D}^{(33)} & 0 & 0 & \mathbf{G}^{(33)} \\
\mathbf{G}^{(11)T} & 0 & 0 & \mathbf{C}^{(1)} & 0 & 0 \\
\mathbf{G}^{(12)T} & \mathbf{G}^{(22)T} & 0 & 0 & \mathbf{C}^{(22)} & 0 \\
0 & \mathbf{G}^{(23)T} & \mathbf{G}^{(33)T} & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\mathbf{F}^{(1)} \\
\mathbf{F}^{(2)} \\
\mathbf{F}^{(3)} \\
\mathbf{a}^{(1)} \\
\mathbf{a}^{(2)} \\
\mathbf{a}^{(3)}
\end{pmatrix} = \begin{pmatrix}
\mathbf{\delta}^{(1)} - \mathbf{\delta}_0^{(1)} \\
\mathbf{\delta}^{(2)} - \mathbf{\delta}_0^{(2)} \\
\mathbf{\delta}^{(3)} - \mathbf{\delta}_0^{(3)} - \mathbf{G}^{(34)T}\mathbf{e}
\end{pmatrix}
\]

(1)

where \( \mathbf{D}^{(11)}, \mathbf{D}^{(12)}, \mathbf{D}^{(21)}, \mathbf{D}^{(22)}, \mathbf{D}^{(23)}, \mathbf{D}^{(32)}, \mathbf{D}^{(33)} \) are nodal compliance matrices; \( \mathbf{G}^{(11)}, \mathbf{G}^{(12)}, \mathbf{G}^{(22)}, \mathbf{G}^{(23)}, \mathbf{G}^{(33)}, \mathbf{G}^{(34)} \) are matrices connecting the increments of gaps at nodal points.
with displacement vectors \( \mathbf{a}^{(1)}, \mathbf{a}^{(2)}, \mathbf{a}^{(3)} \), \( \mathbf{e} \); \( C^{(1)}, C^{(2)} \) are matrices of HW and FW shafts stiffness; \( \mathbf{B} \) is vector of external loads, \( \mathbf{B} = [0, 0, M_c]^T \) (\( M_c \) is resistance moment applied to HW).

System (1) has two groups of equations and inequalities. The equations of the first group (lines 1-3) were obtained by the Bubnov – Galerkin method [6], i.e. by scalar multiplication of the equations of mutual nonpenetration of surfaces of basic functions. The equations of the second group (lines 4–6) are the equations of equilibrium of bodies. The last two inequalities and the equation express the one-sided nature of the interaction of surfaces. The solution to the resolving system of equations was carried out by the method of introducing restoring forces [7].

Let us consider a method for determining the efficiency of a cam wave generator, which consists of a cam and a flexible bearing (FB) pressed onto it. A diagram of the forces acting on the ball is shown in Figure 2. The moment of rolling resistance when the ball moves along the roller path is represented as [8]

\[
T_{bj} = \frac{k}{\rho_w} P_j^{a, b},
\]

where \( P_j \) is the normal force acting on the ball; \( \rho_w \) is the reduced radius of curvature; \( k \) is the coefficient dependent on the type of bearing, oil viscosity, temperature and angular velocity of the ball.

![Diagram of forces acting on rollers.](image)

The reduced radius of curvature \( \rho_w \), included in expression (2), is determined by the formula

\[
\rho_w = \frac{r_z r_g}{r_z + r_g},
\]

where \( r_g \) is the ball radius; \( r_z \) is the gutter radius.

The power of rolling resistance forces of the \( j \)th ball along the roller paths of the outer and inner rings of flexible bearing \( N_{bj} \) and the torque is determined from the expressions

\[
N_{bj} = T_{bj} (\omega_h + \omega_c) + T_{bj} (\omega_h - \omega_c),
\]

\[
N_d = M_d \omega_h,
\]

where \( \omega_h \), \( \omega_c \), \( \omega_b \) are angular speeds of a ball, a cam and a flexible wheel; \( M_d \) is the driving moment acting on the wave generator.

When determining the efficiency of a cam wave generator \( \eta_h \), only losses caused by rolling friction of the balls along the roller paths are taken into account

\[
\eta_h = \frac{N_d - \sum_{j=1} N_{bj}}{N_d}.
\]
where \( n \) is the number of rollers.

After substituting (4) into (5), it is obtained to determine the efficiency of the wave generator

\[
\eta_h = \frac{M_d - \left(1 + 2 \frac{\omega_1}{\omega_k} - \frac{1}{\mu} \right) \sum_j \tau_j}{M_d},
\]

(6)

where \( \mu \) is wave transmission ratio.

The ratio of angular velocities included in expression (6) is determined from the condition that the ball does not slip when it rolls along the bearing rings

\[
\frac{\omega_h}{\omega_k} = \frac{\omega_1 (r - 2r_s) + \omega_2 r}{2r_s} = \frac{r(1+1/\mu) - 2r_s}{2r_s},
\]

(7)

where \( r \), \( r_s \) are the radii of roller paths of the outer ring of the bearing and the ball.

For a flexible bearing, the coefficient \( k \) is taken as equal to \( 8.3 \cdot 10^{-10} \ m^3 \).

### 4. Study results

For a theoretical study, a wave gear transmission was chosen with the following parameters: the number of FA teeth \( z_{b} = 198 \), the number of FW teeth \( z_{g} = 196 \), the displacement coefficient of FA \( x_b = 3.99 \), the displacement coefficient of FW \( x_g = 4.07 \), module \( m = 0.8 \ mm \), thickness of FW sheath \( h_0 = 3.16 \ mm \), width of the ring gear \( b_w = 30 \ mm \), the number of rollers of a flexible bearing (FB) \( n = 23 \), the outer diameter of FB \( D_B = 160 \ mm \), width of FB \( B = 18 \ mm \), the maximum deformation of FB \( w_0 = 0.9 \ mm \), tension in toothed gearing at zero load is \( 0.213 \ mm \). Torque rating \( M_n = 800 \ \text{Nm} \). The remaining dimensions of the wave transmission are adopted in accordance with GOST 30078.2-93, the dimensions of flexible bearings - according to GOST 2379-78 and the recommendations given in [9].

Figure 3 shows the dependence of a flexible bearing’s efficiency on the load. The moment of resistance \( M_C \) is applied to the flexible wheel. From the presented dependencies it is seen that with an increase in the load, the efficiency initially grows rapidly, and then the growth practically stops. With a further increase in the load, the efficiency reaches its maximum value, and then begins to decrease slightly. A good agreement between the theoretical and experimental curves confirms the reliability of the proposed technique. The experimental curve (curve 1) was obtained by V.A. Finogenov.

![Figure 3. Flexible Bearing Efficiency: 1– experiment; 2– calculation.](image)

The proposed calculation method enables us to study the effect of various parameters of determining the transmission wave on the efficiency of a flexible bearing. From the dependencies shown in Figure 4, it can be seen that an increase in the initial deformation of the flexible wheel \( w_0 \) reduces the efficiency...
of the flexible bearing. In determining these dependencies, the tension in the toothed gearing of the unloaded gear was kept constant in all calculation variants. For this, the displacement coefficient of the hard wheel was selected so that the specified tension was equal to 0.191 mm.

The resulting decrease in efficiency can be explained as follows. With an increase of $w_0$, the contact zone of the rollers with the outer and inner rings of the flexible bearing narrows and their interaction forces increase. From formulas (2) and (6) it is seen that the losses in the FB are proportional to $P^2$. Therefore, an increase in $w_0$ leads to a decrease in efficiency.

![Figure 4. Flexible bearing efficiency: 1– $w_0 = 0.9m$; 2 – $w_0 = 1.2m$](image)

5. Conclusion
1. A technique is proposed for determining the efficiency of flexible bearings of wave gears, which considers the spatial nature of the elastic interaction of its elements.
2. An increase in the initial deformation of the flexible wheel $w_0$ leads to a decrease in the efficiency of the flexible bearing.

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