CURRENT MAGNIFICATION AND CIRCULATING CURRENTS IN MESOSCOPIC RINGS

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Abstract

We show that several novel effects related to persistent currents can arise in open systems, which have no analogue in closed or isolated systems. We have considered a system of a metallic ring coupled to two electron reservoirs. We show that in the presence of a transport current, persistent currents can flow in a ring even in the absence of magnetic field. This is related to the current magnification effect in the ring. In the presence of magnetic field we show that the amplitude of persistent currents is sensitive to the direction of current flow from one reservoir to another. Finally we briefly discuss the persistent currents arising due to two nonclassical effects namely, Aharonov-Bohm effect and quantum tunneling.

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I. Introduction

It is well known that Aharonov-Bohm (AB) effect reveals, in solid state physics, as a characteristic dependence of physical properties of mesoscopic systems on magnetic flux. An apt example being the AB oscillations in magneto-resistance of small conducting rings[1]. In recent years a lot of interest has been generated in the persistent circulating currents induced in a mesoscopic ring by an AB flux $\phi[2]$. These are manifestations of novel quantum effects in submicron systems beyond the atomic realm. Böttiker et al proposed, a decade ago, that an ideal one dimensional metallic ring (containing elastic scatterers) of mesoscopic size (whose dimensions are less than the inelastic mean free path or phase breaking length) can support an equilibrium circulating current in response to magnetic flux[3]. The coherent wave function (even in the presence of elastic scatterers) extending over the whole circumference of the loop leads to a response with period equal to a unit of magnetic flux $\phi_0=hc/e$. There are now three experiments confirming the existence of persistent currents in mesoscopic rings[4-6]. The experiments were performed on ensemble of many mesoscopic rings[4] as well as on single (or several) rings[5,6]. Theories based on non-interacting electron picture predict magnitude of currents much less than the experimentally observed ones. Interest in this area has increased as a result of this discrepancy of up to two orders of magnitude between theory and experiment[1]. Results of the later experiment on very weakly disordered ring agree with the theory in the ballistic regime[6]. Experimental results for disordered systems (or in the diffusive regime) have not yet been satisfactorily explained.

Application of magnetic field destroys time-reversal symmetry and as a consequence, the degeneracy of states carrying current clockwise or anticlockwise is lifted. Depending on the Fermi level uncompensated current flows in either of the directions (diamagnetic or param-
modifies the periodic boundary condition into $\psi(x + L) = \psi(x)e^{(i2\pi\phi/\phi_0)}$. This condition implies that the energy levels and hence all physical observables are periodic functions of the flux with a period $\phi_0$. The persistent current is given by the flux derivative of the total free energy of the ring. For an ideal isolated ring persistent current at zero temperature exhibits periodic saw tooth behavior as a function of $\phi$. For even number of electrons $N$ the jump discontinuities occur from the value $-(2ev_f/L)$ to $(2ev_f/L)$ at $\phi = 0, \pm\phi_0, \pm2\phi_0, ..$ etc. and at $\phi = \pm\phi_0/2, \pm3\phi_0/2$, etc. for odd $N$. Here $v_f$ is the Fermi energy. The typical magnitude of the persistent currents at $T=0$ for $L$ between 1-3 $\mu$m and for Fermi wave vector $k_f$ between $10^{10}m^{-1}$ (metallic ring) and $10^8m^{-1}$ (semiconducting ring) varies between 1 and 5 nA. We would also like to emphasize that since the magnetic field tunes the boundary condition, the persistent currents can be thought to arise due to the consequence of the sensitivity of the eigenstates to the boundary conditions. Persistent currents are purely mesoscopic effects in the sense that they are strongly suppressed when the ring size exceeds the characteristic dephasing length of the electrons or the inelastic mean free path. The quantum persistent current is a sample specific property and for a given flux, apart from $N$ dependence it exhibits sensitive dependence on the microscopic configuration of disorder and hence non-self-averaging fluctuation effects. Studies have been extended to include multichannel rings, spin-orbit coupling, disorder, electron-electron interaction effects, etc[2,7-12]. The problem of persistent currents has also facilitated the study of some fundamental problems of statistical mechanics, most notably the questions concerning the role of statistical ensemble. The disordered average current has been found to be vanishingly small for moderate disorder, when grand canonical ensemble has been used, while it is of finite amplitude within the framework of canonical ensemble[2].
reservoirs, namely open systems[13-15]. In a recent experiment Mailly et al have measured the persistent currents in both closed and open rings[6]. The reservoir acts as a source and sink for electrons and is characterized by a well defined chemical potential $\mu$. There is no phase relationship between the absorbed and the emitted electrons. Thus the reservoirs act as a source of energy dissipation as well as inelastic scatterer. All the scattering processes in the leads including the loop are assumed to be elastic. Inelastic processes occur only in the reservoirs, and hence there is a complete spatial separation between elastic and inelastic processes. Weak inelastic processes do not destroy the periodic behavior of persistent currents as a function of magnetic flux $\phi$.

In our present work we show that several novel effects related to circulating currents arise in open systems. All these effects have no analogue in closed or isolated systems. We have considered a system of metallic loop of circumference $L$ coupled to two electron reservoirs, characterized by chemical potentials $\mu_1$ and $\mu_2$ connected via ideal leads as shown in fig. (1). For the sake of simplicity we have considered 1-D free electron networks. We have introduced a $\delta$-function impurity of strength $V$ at a distance $l_3$ from the junction $J_2$, to break the spatial symmetry in the problem. The lengths of the upper and lower arms of the loop are $l_1$ and $l_2$, respectively. We have set the units $\hbar$, $m$ to unity and all the lengths are scaled with respect to the length $L$ of the circumference of the loop ($L= l_1 + l_2$). We first show that circulating currents in a ring can arise even in the absence of magnetic field, but in the presence of a transport current. This is related to current magnification in the loop and is of purely quantum mechanical origin. Next, we show that in the open systems the magnitude of persistent currents is sensitive to the direction of current flow. Finally we discuss briefly the properties of persistent currents arising simultaneously due to two non-classical effects,
II. Circulating currents in the absence of magnetic field

In this section we set the strength of the impurity to zero just for the sake of simplicity. The effect discussed here manifests itself even in the presence of impurities. To obtain transport current we must have \( \mu_1 \neq \mu_2 \) (non-equilibrium situation). The transport current will be directed from left to right or from right to left depending on whether \( \mu_1 > \mu_2 \) or \( \mu_2 > \mu_1 \). We will show that in this case a circulating current is induced in the ring by incident carriers. Existence of such currents was first discussed by Büttiker[14]. However, our analysis is qualitatively different from the earlier studies. The current injected by the reservoir into the lead around the small energy interval \( dE \) is given by \( dI_{in} = ev(dn/dE)f(E)dE \), where \( v=hk/m \) is the velocity of the carriers at the energy \( E \), \( dn/dE=1/(2\pi\hbar v) \) is the density of states in the perfect wire and \( f(E) \) is the Fermi distribution. The total current flow \( I \) in a small energy interval \( dE \) through the system is given by the current injected into the leads by reservoirs multiplied by the transmission probability \( T \). This current splits into \( I_1 \) and \( I_2 \) in the upper and lower arms such that \( I=I_1+I_2 \) (current conservation). As the upper and lower arm lengths are unequal, these two currents are different in magnitude. In our present quantum problem when one calculates the currents \( (I_1, I_2) \) in the two arms there exists two distinct possibilities. The first possibility being for a certain range of incident Fermi wave vectors the current in the two arms \( I_1 \) and \( I_2 \) are individually less than the total current \( I \), such that \( I=I_1+I_2 \). In such a situation both currents in the two arms flow in the direction of applied electric field. However, in certain energy intervals, it turns out that the current in one arm is larger than the total current \( I \) (magnification property). This implies that to conserve the total current at the junctions the current in the other arm must be negative, i.e., the current should flow against the applied external field induced by difference in the chemical potentials. In such a situation one
or persistent current in the loop. Thus the magnitude and direction of persistent current is
the same as that of the negative current. Our procedure of assigning persistent current, is
exactly the same as the procedure well known in classical LCR ac network analysis. When a
parallel resonant circuit (capacitance C connected in parallel with combination of inductance
L and resistance R) is driven by external electromotive force (generator), circulating currents
arise in the circuit at a resonant frequency[16]. This phenomenon is well known as current
magnification. It turns out that even in our quantum problem the circulating currents arise
near the antiresonances in the transmission coefficient of the loop structure coupled to leads.

We now consider the case where the current is injected from left reservoir ($\mu_1 > \mu_2$).
At temperature zero the total current flow around a small energy interval $dE$ around $E$ is
$I=(e/2\pi\hbar)T$, where $T$ is the transmission coefficient calculated at the energy $E$. It is a straight
forward exercise to set up a scattering problem and calculate the transmission coefficient ($T$)
and the currents in the upper ($I_1$) and the lower ($I_2$) arms. We closely follow our earlier method
of quantum waveguide transport on networks to calculate these quantities[15,17-19]. We have
imposed the Griffiths boundary conditions (conservation of current) and single valuedness of
the wavefunctions at the junctions. The expressions are given by

$$I = (e/2\pi\hbar)T,$$

$$T = (8(2 - \cos[2kl_1] - \cos[2kl_2] + 4\sin[kl_1]\sin[kl_2]))/\Omega,$$

$$I_1 = (e/2\pi\hbar)8(1 - \cos[2kl_2] + 2\sin[kl_1]\sin[kl_2])/\Omega,$$
\[ I_2 = (e/2\pi\hbar)^8(1 - \cos[2k_l] + 2\sin[k_l_1]\sin[k_l_2])/\Omega, \]  

\text{(4)}

where

\[ \Omega = (37 - 5\cos[2k_l] - 32\cos[k_l_1]\cos[k_l_2] - 5\cos[2k_l_2] + 5\cos[2k_l_1]\cos[2k_l_2] + 48\sin[k_l_1]\sin[k_l_2] - 4\sin[2k_l_1]\sin[2k_l_2]). \]  

\text{(5)}

In fig.(2) we have plotted the circulating currents (solid curves) in the dimensionless units \((I_c \equiv 2\pi\hbar I_c/e)\) in the small energy interval \(dE\) around the Fermi energy as a function of dimensionless wave vector \(kL\). We have chosen \(l_1/l_2=5.0/3.0\). In fig. (2) we have also plotted the transmission coefficient \(T\) for the same parameter values. We notice that the persistent current changes sign as we cross the energy or the wave vector corresponding to the first antiresonance (or transmission zero) in the transmission coefficient. It does not change sign as we cross the second antiresonance. The first antiresonance is characterized by an asymmetric zero-pole in the transmission amplitude (zero occur at a value of \(kL=(2\pi)\) and poles are given by \(kL=(6.25495-i\ 0.299976)\) and \((6.46865-i\ 1.90045))\). The proximity of the zero and pole leads to sharp variations and asymmetry in the transmission coefficient around the magnitude zero as a function of energy (around the first antiresonance). The second antiresonance is characterized by a zero along with symmetrically placed two poles and transmission coefficient is symmetric around the antiresonance. The zero is at a value \(kL=(4\pi)\) and poles are given by \(kL=(12.4105-i\ 1.07584)\) and \((12.7222-i\ 1.07584))\). Thus, we have shown that persistent currents arise near the vicinity of the antiresonances and the nature of persistent currents depend on the zero-pole structure in the transmission amplitude. In general zero-pole structure in
incommensurate ratio we mostly obtain asymmetric antiresonances. The magnitude and the width of the persistent current peaks in the vicinity of antiresonances depend on the strength of the imaginary part of the poles. If the two poles have different imaginary parts, the peak value of the persistent current will be higher for the pole with smaller imaginary part. For fixed value of Fermi energy the persistent current changes sign as we change the direction of the current flow. In equilibrium ($\mu_1 = \mu_2$) we cannot have persistent currents in the absence of magnetic field. If $\mu_1 > \mu_2$, then at zero temperature the total magnitude of persistent current is given by $I_T = \int_{\mu_1}^{\mu_2} I_c dE$. The total magnetic moment of a loop is proportional to $\oint I(l) dl$ ($l$ is taken along the loop). Owing to the current magnification property we expect that one should experimentally observe enhanced magnetic moment near the antiresonances in the transmission coefficient or the two port conductance.

III. Persistent current dependence on the direction of the current

Here we shall discuss a phenomenon that arises only when the impurity strength $V \neq 0$. The presence of the impurity breaks the spatial symmetry of the system. We also restrict to the case of $l_1 = l_2$, to avoid the additional contribution arising due to the difference in transport current across upper and lower arms. If $\mu_1 > \mu_2$ the net current flows from left to right and vice versa if $\mu_1 < \mu_2$. For the case of $\mu_1 > \mu_2$ at zero temperature, the reservoir 1 injects a steady flux of electrons in the interval $\mu_1$ and $\mu_2$ and results in a current flowing in the right direction. These electrons moving to the right are first scattered at $J_1$, $J_2$ and then at impurity site I along with multiple scattering. If $\mu_1 < \mu_2$, (i.e., the case of current flowing in the left direction), the injected electrons from reservoir 2 are first scattered at I (the impurity site) and then at $J_2$ and $J_1$ along with multiple reflections. As there is no spatial symmetry, for these two different cases the electron wavefunction (scattering
boundary conditions across a loop. As mentioned earlier the persistent currents are sensitive to boundary conditions and hence we obtain different magnitude for the persistent currents depending on the direction of current flow[15]. We have obtained analytical expressions for the persistent currents. However, here we present our results graphically. In fig.(3) we have plotted persistent current in dimensionless units \(dj/k\) as a function of the flux \(\alpha(=2\pi\phi/\phi_0)\) for a fixed value of \(kL=(7.0)\) and \(VL=(10.0)\). The dashed and solid curves represent magnitudes of the persistent currents flowing in the loop \(dj_R/k\) and \(dj_L/k\), respectively, i.e., when the d.c current flows in the right and left directions, respectively. One can readily notice, the directional dependence of the persistent currents. In fig. (4) we have plotted the persistent currents \(dj_R/k\) and \(dj_L/k\) as a function of dimensionless impurity potential \(VL\), for a fixed value of \(kL=(7.0)\) and for \(\alpha=(0.7)\). The magnitude \(dj_L/k\) decreases monotonically to zero as \(VL \to \infty\). This is due to the fact that in this limit electrons emitted by right reservoir do not enter the loop and cannot contribute for the persistent currents. The absolute magnitude of \(dj_R/k\) saturates to a value in the same limit. This corresponds to a situation where the loop is connected to single reservoir 1 and the connection truncated at the point I(the impurity site).

At zero temperature the total contribution to the persistent current is obtained by adding all the contributions to \(dj_R/k\) and \(dj_L/k\) from levels with energies upto the chemical potentials. Thus for fixed \(|\mu_1 - \mu_2|\) we get a different persistent current depending on the direction of the current flow. In our above discussion we have restricted to a simple case of \(l_1 = l_2\). As mentioned earlier, when \(l_1 \neq l_2\), an additional contribution to the persistent current arises due to the difference in the transport current across the upper and lower arms. In such a situation the total persistent current (or associated magnetic moment) becomes asymmetric under the reversal of magnetic field. The magnitude of the transport currents in the two arms
geometry we can show that for particular value of \( \alpha (= 2\pi \phi / \phi_0) = \pi / 2, 3\pi / 2, \text{etc.} \) the total persistent current is antisymmetric with respect to the magnetic field, and for these values of magnetic field transport currents in the two arms do not exhibit the current magnification property.

IV. Persistent currents due to evanescent modes

Let us imagine a geometry where a metallic loop is coupled to a single electron reservoir via an ideal lead. In an ideal lead the potential is assumed to be zero i.e., \( V=0 \). In the metallic loop the potential throughout the circumference is \( V \) and is positive. When injected electrons have their energies less than \( V \), these electrons can tunnel into the loop quantum mechanically and propagate inside the loop as evanescent modes and give rise to a persistent current in the presence of a magnetic field. Such currents arise simultaneously due to two nonclassical effects, namely, quantum tunneling and Aharonov Bohm effect[20]. Currents due to such evanescent modes are to be found by analytical continuation and we have found out analytical expressions for these persistent currents. In the limit \( QL >> 1 \), persistent currents in a small energy interval around \( E \) are given by \( dj = f(k, Q)e^{-QL}\sin(\phi / \phi_0) \), where \( f(k,Q) \) is a simple function of \( k \) and \( Q \). Here \( k \) is the wave vector for incident electrons i.e., \( k=\sqrt{2mE/\hbar^2} \) and \( Q=\sqrt{2m(E-V)/\hbar^2} \). As expected the persistent currents are periodic in magnetic flux with period \( \phi_0 \). Owing to the decaying nature of evanescent modes, the factor arising due to the sensitivity of the wavefunctions to the boundary conditions appears as \( e^{-QL} \). Higher harmonics in magnetic flux (say nth harmonic) also contribute to the persistent currents with a multiplication factor \( \sin(n\phi / \phi_0) \). However, these harmonics, are weighted by \( e^{-nQL} \) because for these harmonics to appear the electron has to traverse the loop \( n \) times. So, these harmonics can be neglected in the limit \( QL >> 1 \). Unlike the behavior of persistent currents
the Fermi energy as long as E < V. The total persistent current is given by sum of contributions from the electrons upto Fermi energy. Even though the current due to individual evanescent modes is small the total sum can have an observable amplitude. Especially in a real physical situation one can have a ring with extremely narrow width connected to the reservoir via an ideal wire with a much larger width. In this situation the zero point quantum potential due to the transverse confinement in the ring is much higher than the zero point energy of the ideal wire. Electrons can occupy several subbands in the connecting wire but still they have energies less than the zero point energy of the ring. All these electrons in several subband modes will propagate as evanescent modes in the ring, and in this situation a higher contribution to the total persistent current may arise.

In conclusion, we have shown that several novel effects related to persistent currents can arise in open systems which have no analogue in closed or isolated systems. All these new features can be experimentally verified. Further work in the direction of including disorder, and interaction effects is needed to put these effects on firm foundation.
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Figure captions

Fig. 1. An open metallic loop connected to two electron reservoirs.

Fig. 2. Plot of persistent current $I_c(= 2\pi \hbar I_c/e)$ versus $kL$ (solid curve) and transmission coefficient $T$ versus $kL$ (dashed curve) for $l_1/l_2=5.0/3.0$.

Fig. 3. Persistent current versus $\alpha = (2\pi \phi/\phi_0)$ for a fixed value of $kL=7.0$ and $V_L=10.0$. The dashed curve represents $dj_L/k$ and the solid curve represents $dj_R/k$.

Fig. 4. Persistent current versus impurity potential $V_L$ for a fixed value of $\alpha=0.7$ and $kL=7.0$. The dashed curve represents $dj_L/k$ and the solid curve represents $dj_R/k$. 