Critical Coupling for Dynamical Chiral-Symmetry Breaking with an Infrared Finite Gluon Propagator

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Abstract

We compute the critical coupling constant for the dynamical chiral-symmetry breaking in a model of quantum chromodynamics, solving numerically the quark self-energy using infrared finite gluon propagators found as solutions of the Schwinger-Dyson equation for the gluon, and one gluon propagator determined in numerical lattice simulations. The gluon mass scale screens the force responsible for the chiral breaking, and the transition occurs only for a larger critical coupling constant than the one obtained with the perturbative propagator. The critical coupling shows a great sensibility to the gluon mass scale variation, as well as to the functional form of the gluon propagator.

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1 Introduction

The idea that quarks obtain effective masses as a result of a dynamical breakdown of chiral symmetry (DBCS) has received a great deal of attention in the last years [1, 2]. One of the most common methods used to study the quark mass generation is to look for solutions of the Schwinger-Dyson equation for the fermionic propagator. It is known that above a certain critical coupling \( \alpha_c \equiv g^2/4\pi \) a nontrivial self-energy solution bifurcates away from the trivial one. Numerical evaluation of this critical coupling in QCD with three and four flavors gives \( \alpha_c \sim \mathcal{O}(1) \) [3, 4].

Parallel to the study of DBCS a lot of effort has also been done to obtain the nonperturbative behavior of the gluon propagator [5, 6, 7], and perhaps one of the most interesting results is the one where it is argued that the gluon may have a dynamically generated mass [5]. The study of the infrared behavior of the gluon propagator was also performed numerically on the lattice [8], and more recent numerical simulation give strong evidence for an infrared finite gluon propagator in the Landau gauge [9]. It is worth mentioning that from the phenomenological point of view, the existence of a “massive gluon” may shed light on several reactions where long distance QCD effects can interfere, and examples of the possible consequences can be found in the literature, see, for instance, Ref. [10, 11, 12].

Much work has yet to be done about the infrared behavior of the gluon propagator, but it is clear that its implications have to be tested in all possible problems. It is possible that the constraint coming from DBCS, and other phenomenological studies [10, 11, 12] will provide a map of the infrared gluon propagator. Since the bifurcation point for DBCS was studied up to now with the perturbative \( 1/k^2 \) gluon propagator, it is natural to ask what is going to happen with the infrared finite propagators that have been found through solutions of the gluonic Schwinger-Dyson equation or using Monte Carlo methods, and, moreover, to look for the consequences of different forms of non-perturbative gluon propagators. It is intuitive that the force
necessary for condensation is going to be screened if the gluon propagator is infrared finite, therefore, the actual critical coupling constant should be larger, and this is what we will investigate in this work.

We will present the Schwinger-Dyson equation of our problem, and first we will discuss the critical coupling for the linear approximated problem in the case of a bare gluon mass. This will teach us on the general behavior of the critical coupling constant as a function of the gluon mass. Secondly, we perform a numerical calculation of the full nonlinear equation, for two different gluon propagators resulting from solutions of the gluon polarization tensor, and one obtained from numerical simulation on the lattice. In the conclusions we discuss the differences in the critical coupling for each of the “massive” gluon propagators, arguing that its value definitively gives information about the infrared gluon propagator.

2 Quark propagator Schwinger-Dyson equation

The Schwinger-Dyson equation for the quark propagator in Minkowski space is

\[
S^{-1}(p) = p - \frac{4}{3} \int \frac{d^4q}{(2\pi)^4} \gamma_\mu S(q) \Gamma_\mu(p, q) g^2 D_{\mu\nu}(q - p).
\]  

(1)

where we write the gluon propagator in the form

\[
g^2 D_{\mu\nu}(q) = \frac{4\pi\alpha(-q^2/\Lambda^2)}{q^2} \left(-g_{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right).
\]  

(2)

The propagator has been written in the Landau gauge, which will be used throughout our work. In the above equations \(\Gamma_\mu(p, q)\) is the vertex function, and \(\alpha(-q^2/\Lambda^2)\) is the QCD running coupling constant, for which we know only the ultraviolet behavior, and to solve Eq.(1) we make the same ansatz of Ref. [3, 4] about its behavior for the full momentum scale

\[
\alpha(-q^2/\Lambda^2) = \frac{12\pi/(33 - 2n_f)}{\ln(\tau + \frac{-q^2}{\Lambda^2})}.
\]  

(3)
Eq. (3) goes continuously to the perturbative result, and has already been used in phenomenological applications.

To proceed further we also need to introduce an ansatz for the quark-gluon vertex $\Gamma^{\mu}(p, q)$, which must satisfy a Slavnov-Taylor identity that, when we neglect ghosts, reads

$$ (p - q)_\mu \Gamma^{\mu}(p, q) = S^{-1}(p) - S^{-1}(q). $$

This identity constrains the longitudinal part of the vertex, and if we write $S^{-1}(p)$ in terms of scalar functions

$$ S^{-1}(p) = A(p) \not{p} - B(p), $$

we find the solution [13]

$$ \Gamma^{\mu}(p, q) = A(p^2) \gamma^\mu + \frac{(p - q)^\mu}{(p - q)^2} \left( [A(p^2) - A(q^2)] \not{q} - [B(p^2) - B(q^2)] \right) $$

$$ + \text{transverse part}, $$

which is a much better approximation than the use of the bare vertex. Assuming that the transverse vertex part vanishes in the Landau gauge we obtain

$$ D^{\mu\nu}(p - q)\Gamma_\nu(q, p) = D^{\mu\nu}(p - q)A(q^2)\gamma_\nu, $$

and arrive at the approximate Schwinger-Dyson equation

$$ [A(p^2) - 1] \not{p} - B(p^2) = \frac{4}{3} \int \frac{d^4q}{(2\pi)^4} g^2 D^{\mu\nu}(p - q) \gamma_\mu \frac{A(q^2)}{A(q^2)} \not{q} - B(q^2) \gamma_\nu. $$

Going to Euclidean space, we will be working with the following nonlinear coupled integral equations for the quark wave-function renormalization and self-energy

$$ [A(P^2) - 1] P^2 = \frac{16\pi}{3} \int \frac{d^4Q}{(2\pi)^4} \frac{\alpha((P - Q)^2/\Lambda^2)}{\Phi[(P - Q)^2]} $$

$$ \times \left( P \cdot Q + 2 \frac{P \cdot (P - Q) Q \cdot (P - Q)}{(P - Q)^2} \right) $$

$$ \times \frac{A^2(Q^2)}{A^2(Q^2)Q^2 + B^2(Q^2)}. $$

4
where $Q^2 = -q^2$ and $P^2 = -p^2$, and we introduced a function $\Phi[(P - Q)^2]$ which, in the case of the perturbative propagator, is simply $\Phi[(P - Q)^2] = (P - Q)^2$, for the massive bare gluon it will have the form $\Phi[(P - Q)^2] = (P - Q)^2 + m_g^2$, and will be a more complex expression in the case of a dynamically generated mass.

3 The linear problem with a massive bare gluon

Eq. (9) and Eq. (10) possess the trivial solution $A(P^2) = 1$ and $B(P^2) = 0$ for small values of the coupling constant. We can also see that $B(P^2)$ depends on $B(P^2)$ at first order, whereas $A(P^2)$ has a higher order dependence on $B(P^2)$. In order to examine the possibility that a nontrivial solution, $B(P^2)$, branches away from the trivial one at a critical coupling, $\alpha_c$, we examine the so-called bifurcation equation [14]. This involves differentiating Eq. (9) and Eq. (10) functionally with respect to $B$ and then setting $B = 0$. Since the equation for $A(P^2)$ depends at least quadratically on $B(P^2)$ it will be dropped at leading order from the bifurcation problem, and we substitute $A(P^2)$ by 1 in the bifurcation equation that will come out from Eq. (10). We will deviate from the standard bifurcation theory proceeding as in Ref. [13], and instead of substituting $Q^2 + B^2(Q^2)$ by $Q^2$ in the denominator of Eq. (10), we will replace this term by $Q^2 + \delta B^2(0)$ and define the dynamical fermion mass ($m_f$) by the normalization condition

$$\delta B(0) = m_f.$$  

(11)

We finally arrive at our bifurcation equation

$$\delta B(P^2) = \frac{16\pi}{(2\pi)^3} \int dQ^2 \int d\theta \sin^2 \theta \frac{Q^2}{Q^2 + m_f^2} \times \frac{\alpha[(P - Q)^2/\Lambda^2]}{(P - Q)^2 + m_g^2} \delta B(Q^2),$$

(12)
where we have already assumed a bare massive gluon.

Our main intention in this section is to verify the gross behavior of the critical coupling constant with the existence of an infrared finite gluon propagator, this is the reason for having selected a bare massive gluon in Eq.(12). The details of a dynamically generated gluon mass will be left for the next section. Eq.(12) is a standard Fredholm equation with a positive kernel, and, requiring \( \delta B(P^2) \) to belong to \( L^2 \), the spectrum is discrete with a smallest value \( \alpha_c \) such that we have the trivial solution \( \delta B(P^2) \equiv 0 \) for \( 0 < \alpha < \alpha_c \), and the nontrivial one if \( \alpha \geq \alpha_c \).

We can still make some simplifier approximations before estimating \( \alpha_c \), making the following substitutions

\[
\alpha((P - Q)^2/\Lambda^2) \to \theta(P^2 - Q^2)\alpha(P^2/\Lambda^2) + \theta(Q^2 - P^2)\alpha(Q^2/\Lambda^2),
\]

(13)

and

\[
\frac{1}{(P - Q)^2 + m_g^2} = \frac{1}{P^2 + m_g^2}\theta(P^2 - Q^2) + \frac{1}{Q^2 + m_g^2}\theta(Q^2 - P^2),
\]

(14)

which is known as the angle approximation, and introduces an error of about 10\% in the calculation [4].

Defining the variables \( x = P^2/m_f^2 \), \( y = Q^2/m_f^2 \), \( \ell = \Lambda^2/m_f^2 \), \( \kappa = m_g^2/m_f^2 \), and \( f(P^2) = \delta B(P^2)/m_f \), we obtain

\[
f(x) = \frac{1}{\pi} \int dy \, K(x, y) \, f(y),
\]

(15)

where

\[
K(x, y) = \frac{\alpha(x/\ell)}{x + \kappa} \frac{y}{y + 1} \theta(x - y) + \frac{\alpha(y/\ell)}{y + \kappa} \frac{y}{y + 1} \theta(y - x).
\]

(16)

The kernel \( K \) is square integrable

\[
\| K \|^2 = \int_0^\infty dx \int_0^x dy \frac{\alpha^2(x/\ell)y^2}{(x + \kappa)^2(y + 1)^2} + \int_0^\infty dx \int_x^\infty dy \frac{\alpha^2(y/\ell)y^2}{(y + \kappa)^2(y + 1)^2},
\]

(17)

therefore Eq.(13) has a nontrivial \( L^2 \) solution for \( \alpha_c \) on a point set. The smallest eigenvalue (\( \alpha_c \)) for which Eq.(13) has a nontrivial square integrable solution, is the
Table 1: Critical coupling constant ($\alpha_c$) as a function of $\ell = \Lambda^2/m_f^2$, and $\kappa = m_g^2/m_f^2$ for $n_f = 4$.

| $\ell$ | $\kappa$ | $\alpha_c$ |
|--------|-----------|-------------|
| $10^4$ | 1         | 0.6971      |
| $10^4$ | $10^2$    | 0.9440      |
| $10^4$ | $10^3$    | 1.4853      |
| $10^6$ | 1         | 0.5568      |
| $10^6$ | $10^2$    | 0.6607      |
| $10^6$ | $10^4$    | 0.9489      |
| $10^{10}$ | $10^4$ | 0.6226      |
| $10^{10}$ | $10^6$ | 0.7822      |
| $10^{10}$ | $10^8$ | 1.2278      |

Table 1 gives the critical value $\alpha_c$ as a function of $\ell$ and $\kappa$.

The values of Table 1 were obtained with $n_f = 4$ but they do not change appreciably as we change $n_f$. As could already be seen in Eq. (17), if we increase the gluon masses we can satisfy Eq. (18) only with larger critical coupling constants, and this is what we can expect from the numerical solution of the complete nonlinear problem.

We stress that the values in Table 1, which are approximate solutions of Eq. (18), can only give a qualitative idea of the problem, because the actual solution, without the many simplifications performed until we arrived at Eq. (17), will obviously differ from the results of Table 1.
4  The critical coupling for infrared finite propagators

In this section we solve Eq. (9) and Eq. (10) numerically without further approximations. The numerical code we used is the same of Ref. [16], and the criterion to determine the critical coupling is the one of Ref. [4]. With the perturbative gluon propagator we obtain (with \( n_f = 4 \))

\[ \alpha_c = 0.854, \]  

(19)

which is compatible with the calculations of Ref. [3, 4]. We will solve the gap equations with three different propagators which we discuss in the sequence.

One of the infrared finite propagators found in the literature was determined by Cornwall [5]

\[ \Phi(Q^2) = D_c^{-1}(Q^2) = [Q^2 + m_g^2(Q^2)]bg^2 \ln\left[\frac{Q^2 + 4m_g^2(Q^2)}{\Lambda^2}\right], \]  

(20)

where \( m_g^2(Q^2) \) is the momentum-dependent dynamical gluon mass

\[ m_g^2(Q^2) = m_g^2 \left[ \ln\left(\frac{Q^2 + 4m_g^2}{\Lambda^2}\right) \right]^{-12/11} \ln\left(\frac{4m_g^2}{\Lambda^2}\right), \]  

(21)

\( g^2 \sim 1.5 - 2 \) is the strong coupling constant, and \( b = (33 - 2n_f)/48\pi^2 \) is the leading order coefficient of the \( \beta \) function of the renormalization group equation. This form for the propagator was obtained as a fit to the numerical solution of a gauge invariant set of diagrams for the gluonic Schwinger-Dyson equation.

Another infrared finite gluon propagator has been found by Stingl and collaborators [6]. Its form agrees with that derived by Zwanziger based on considerations related to the Gribov horizon [17], and is given by

\[ \Phi(Q^2) = D_s^{-1}(Q^2) = Q^2 + \mu^4/Q^2, \]  

(22)
where $\mu$ is a scale not determined in Ref. [6]. It is interesting to note that the Bernard et al. [9] lattice result for the gluon propagator can be fitted by Eq.(20) as well as Eq.(22). These propagators, apart from some multiplying constant, approach the perturbative gluon propagator in the small mass limit.

Finally, Marenzoni et al. [9] also performed a lattice study of the gluon propagator in the Landau gauge, obtaining for its infrared behavior the following fit

$$\Phi(Q^2) = D_m^{-1}(Q^2) = m_g^2 + ZQ^2(Q^2/\Lambda^2)\eta,$$

(23)

where $m_g$, $Z$ and $\eta$ are constants determined with the numerical simulation. $m_g$ is of $O(\Lambda)$, $Z \simeq 0.4$ and $\eta \simeq 0.5$, what is slightly different from the previous propagators. The results of Bernard et al. also show the behavior $(Q^2)\eta$, but with a smaller value for $\eta$.

With the above propagators we computed the dynamical fermion mass as a function of the coupling constant. As in Ref. [4] the results were fitted by a function

$$h(\alpha) = \beta(\alpha - \alpha_c)\gamma,$$

(24)

characteristic of a phase transition phenomena. We have not found large differences in the values of the critical coupling as we variated the number of fermions, therefore, the fitting will be presented for $n_f = 4$. In Fig.1 we plot $-1/\ln B(0)$ as a function of the coupling constant, for the Cornwall propagator (see Eq.(20)). The curves in Fig.1 were obtained for $m_g = 2\Lambda$ and $m_g = 2.2\Lambda$, and as expected from the example of the previous section if we increase the gluon mass the critical coupling also increases. These gluon masses are consistent with the values determined phenomenologically in the last work of Ref. [5]. The parameters of Eq.(24) and the critical coupling are given by

$$\beta = 1.0785, \quad \gamma = 0.2535, \quad \alpha_c = 0.8692, \quad (m_g = 2.0\Lambda);$$

(25)

$$\beta = 0.8424, \quad \gamma = 0.2682, \quad \alpha_c = 1.4211, \quad (m_g = 2.2\Lambda).$$

(26)
As will become clear in the following, not only the value of the gluon mass scale is important to characterize the phase transition, but the precise form of the gluon propagator will affect considerably the value of the critical coupling. In this case, as well as in the next ones, we verified that for small gluon masses we start having dynamically generated quark masses for critical couplings quite close to the value obtained with the $1/Q^2$ propagator (see Eq.(25)). After some value of the gluon mass the critical coupling deviates very fast from the value of Eq.(19). An example of this behavior is shown in Eq.(26), where the coupling constant is almost twice the value of Eq.(25), although we obtained it increasing the previous gluon mass value only by 10%! Finally, the chiral symmetry breaking for this propagator was also studied in Ref. [18] with different results. The main difference lies in the fact that in Ref. [18] it is assumed a complete cancellation between the coupling constant of the vertex function and the coupling in the denominator of Eq.(20). Therefore, the mass generation in Ref. [18] does not depend at all on the coupling constant, and it is far from clear to us that such cancellation should be performed.

Using the propagator determined by Stingl and collaborators [6], we obtain the curves shown in Fig.2 for $\mu^2 = 0.25\Lambda^2$ and $\mu^2 = 0.30\Lambda^2$, and described by Eq.(24) with

\[
\beta = 0.2482, \quad \gamma = 0.4784, \quad \alpha_c = 2.9038, \quad (\mu^2 = 0.25\Lambda^2); \quad (27)
\]

\[
\beta = 0.2946, \quad \gamma = 0.3362, \quad \alpha_c = 6.4720, \quad (\mu^2 = 0.30\Lambda^2). \quad (28)
\]

Note that the values for the critical coupling constants are quite large. We rely on these numbers based on the continuous growth of the coupling constant from a value near the one of Eq.(19) for small gluon masses, to the ones of Eq.(27) and (28) as the mass is increased. It is known for several other theories with chiral breaking for coupling constants of $O(1)$, that higher order corrections do not modify the critical behavior shown by the ladder approximation [19], and we expect the same to hold here. Comparing Fig.2 to Fig.1 we see that the dynamically generated mass is much
smaller for this propagator, than with the Cornwall one. Performing the calculation for \( \mu \approx \mathcal{O}(3.0\Lambda) \) we do not obtain a significative signal of chiral symmetry breaking, i.e. if there is a dynamical mass it is much smaller than \( \Lambda \), and do not satisfy our numerical criterion to recognize mass generation \[4\]. This result is compatible with the one of Ref. \[20\], where it was verified that the quark condensate is consistent with zero above a certain critical value of \( \mu \) for this same gluon propagator. Here we foresee a problem for the Stingl \textit{et al.} \[6\] propagator, because as shown by Cudell and Nguyen \[11\] we need \( \mu \approx \mathcal{O}(3.0\Lambda) \) to obtain a correct description of diffractive scattering with this propagator. We stress that not only the gluon mass scale is important to determine the critical coupling constant, but also the functional form of the propagator.

Fig. 3 contains the critical curve for the lattice propagator (Eq. (23)) in the case of \( m_g = 0.7\Lambda \), and with

\[
\beta = 0.4588, \quad \gamma = 0.2870, \quad \alpha_c = 3.9712, \quad (m_g = 0.7\Lambda). \tag{29}
\]

The Marenzoni \textit{et al.} propagator gives a value for the critical coupling constant which is intermediate between the other two propagators that we discussed up to now. If we increase the gluon mass we will also find a point where the symmetry is not broken anymore, however, this will occur for larger masses than the one predicted in Ref. \[9\] \((m_g \approx \Lambda)\). Comparing all the results it becomes clear that, for masses of the same order, the softer is the propagator in the infrared the larger will be the critical coupling for chiral symmetry breaking.

It is known from early studies of DBCS with the perturbative \((1/k^2)\) gluon propagator \[21\], that the chiral symmetry is broken when the product of the coupling constant \((\alpha)\) with a Casimir eigenvalue \((C_F)\), which depends on the fermion representation, is larger than a certain critical value. With the introduction of a gluon mass scale this is obviously not the case. We have found that the gluon mass scale plays an important role in this mechanism, the larger it is the stronger must be the
coupling constant to generate dynamical quark masses. In this way, there is some chance that the single-gluon-exchange approximation, as presented here, makes no sense to obtain consistent solutions of Schwinger-Dyson equations, except for very small gluon masses, or for theories with fermionic representations condensating in a channel with large eigenvalues $C_F$. For large gluon masses and fundamental representation quarks the critical coupling becomes quite large, and it is more likely that we should consider some effect due to confinement, which could be responsible for a correct treatment of DBCS.

In a related work Papavassiliou and Cornwall [22] also considered the gap equation with massive gluons coupled to the vertex equation. They found that nonsingular solutions of the vertex equation requires large gluon masses, which erase the chiral symmetry breaking! In our calculation we introduced a phenomenological coupling constant that freezes at low momentum depending on the parameter $\tau$ of Eq.(3). This parameter is determined when we find the critical coupling $\alpha_c$ for each gluon mass. Both works can be related if we choose $\tau \propto m_g^2/\Lambda^2$. With this choice when we increase the gluon mass the coupling decreases, and we may never have a coupling constant large enough to generate chiral symmetry breaking, arriving at the same inconsistency found in Ref. [22]. We stress that the determination of the critical coupling for chiral symmetry breaking, and the verification of the freezing of the coupling constant as predicted in Ref. [22], are crucial tests for the existence of a gluon mass scale, as well as can be a good indicator of the true infrared gluon propagator.

5 Conclusions

We studied the critical coupling constant for the dynamical chiral-symmetry breaking in a model of quantum chromodynamics, using infrared finite gluon propagators found as solutions of the Schwinger-Dyson equation for the gluon, as well as one
gluon propagator determined in numerical lattice simulations. We first calculated
the eigenvalue condition for the linear bifurcation equation of the quark self-energy,
finding that a bare gluon mass scale screens the force responsible for the chiral break-
ing, and the transition occurs at a larger critical coupling constant if we increase the
ratio of the gluon to fermion mass. Secondly, we solved numerically the full quark
self-energy equation for some infrared finite gluon propagators. The result confirm
our linear approximation, indicating that as we increase the gluon mass the critical
coupling constant will be larger. We also verified that the functional form of the
propagator is also important to characterize the chiral transition.

With the Cornwall propagator (Eq.(20)) and gluon masses of the order that are
expected phenomenologically, we obtain critical coupling constants not far away
from the one obtained with the $1/k^2$ propagator. For this propagator our calcula-
tion differs from the one of Ref. [18] by the reason explained in Section 4. With
the Stingl et al. propagator (Eq.(22)) the chiral transition will occur only for quite
large values of the coupling constant. If the gluon mass scale is of $O(\Lambda)$ the critical
coupling is one order of magnitude larger than the one obtained with the perturba-
tive propagator. Unfortunately, a phenomenological study of diffractive scattering
with the Stingl propagator demand gluon masses of $O(3\Lambda)$, for which there is no
symmetry breaking! This means that this propagator does not represent the actual
gluon infrared behavior, or the model of diffractive scattering of Ref. [11] must be
modified. The Marenzoni et al. propagator leads to a picture of the chiral transi-
tion that is consistent phenomenologically, but with a larger value for the critical
coupling constant. In general, the softer is the propagator in the infrared the larger
will be the critical coupling. As discussed at the end of last section, for large gluon
masses, it is possible that confinement effects cannot be discarded, due to the large
critical coupling constants involved in these cases. The value of the critical coupling
constant can be used as a tool to study the infrared behavior of the gluon prop-
agator, and associated with other phenomenological calculations (like the ones of
Ref. [10, 11, 12] may provide a map of the gluon propagator for every momenta scale.

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Figure 1: Results of the evaluation of Eq.(10) as a function of the coupling constant with the propagator of Eq.(20). We show some of the calculated points, and the curve is the result of the fitting by Eq.(24). The calculation was performed for $n_f = 4$, the upper curve is for $m_y^2 = 4.0\Lambda^2$ and the lower one is for $m_y^2 = 4.84\Lambda^2$. 
Figure 2: The same as Fig.1 for the propagator of Eq.(22), with $\mu^2 = 0.25\Lambda^2$ (upper curve) and $\mu^2 = 0.30\Lambda^2$ (lower curve).
Figure 3: The same as Fig. 2 for the propagator of Eq. (23), with $m_g^2 = 0.49 \Lambda^2$. 