Two-Fluid Studies of Edge Relaxation Events in Tokamaks

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Abstract. Large-scale numerical computation is applied to fluid-based plasma modeling of edge localized modes (ELMs). The modes are magnetohydrodynamic-like instabilities that nonlinearly release bursts of particles and energy across the flow-induced transport barrier that lies near the separatrix of closed and open magnetic flux in tokamaks. Some of the first nonlinear computations of ELMs to model the separate drifts of ions and electrons, which linearly stabilize the highest wavenumber modes and avoid a nonlinear ‘ultraviolet catastrophe’ without the use of ad hoc dissipation, are presented. The two-fluid system supports high-frequency dispersive modes that are not present in magnetohydrodynamics, and a new implicit leapfrog algorithm has been implemented to run these computations. We briefly describe the algorithm and computational requirements, in addition to physical results on nonlinear structure formation.

1. Introduction
The favorable confinement achieved in present-day tokamak plasma systems largely results from a flow-induced transport barrier that forms just inside the separatrix of the equilibrium magnetic field. Magnetic field outside the separatrix intercepts the device wall, and plasma in contact with the open magnetic field has significantly lower temperature and number density than the confined plasma inside the separatrix. Free energy associated with the large pressure and current gradients in the transition region is prone to excite edge localized modes (ELMs) that release some of the stored internal energy in discrete bursts. While the ELMs provide a self-regulating mechanism for the plasma, the projected amount of released energy for certain classes of ELM events in a device the size of ITER raises concern for the longevity of structural components.

There have been significant advances in our understanding of the linear properties of ELMs [1-2], but their nonlinear evolution is a largely unexplored topic for theoretical modeling. An early nonlinear study solves compressible reduced magnetohydrodynamics (MHD) equations in a global geometry with the core-plasma region removed [3]. A more recent study applies a reduced two-fluid model to selected toroidal harmonics in a narrow region around the separatrix [4] and
finds a toroidally localized structure emerging late in the evolution. The localization represents nonlinear coupling of multiple unstable modes and is consistent (in the context of the selected harmonics) with some laboratory observations of filamentary structures. Even with this progress, there remain open theoretical questions regarding modal coupling with a full spectrum, the impact on the global profile, the recovery cycle, and how externally controllable parameters may influence the ELM activity. In the near term, we expect to be able to apply large-scale nonlinear simulation to ELM activity in isolation and with limited interaction with core MHD dynamics. In the longer term, predictive integrated modeling must incorporate ELM dynamics as one part of a self-consistent numerical description of a magnetically confined, burning plasma.

Here we describe a recent nonlinear study of ELMs, where the computational domain includes the full core region and a large edge-plasma region. The equilibria are pre-computed numerical solutions [5] to the Grad-Shafranov force-balance equation with profiles fitted to laboratory measurements of ELM-producing discharges in the DIII-D tokamak at General Atomics [6]. Linear and nonlinear computations with a broad range of toroidal components (0 ≤ n ≤ 42) are performed with the NIMROD code [7] using a relatively new algorithm for two-fluid effects [8]. The linear computations verify the stabilizing effect of drift physics for modes of large toroidal wavenumber (n), as expected from ballooning analysis. Our largest nonlinear two-fluid simulation finds nonlinear coupling generating localization into a helical structure.

2. Fluid-based modeling
The destabilizing mechanism for ELMs exists in the single-fluid MHD model, but two-fluid effects, such as Hall electric field and gyroviscosity, have a strong influence on the linear spectrum and nonlinear evolution. We therefore focus on two-fluid modeling using the system:

\[
\frac{\partial B}{\partial t} = -\nabla \times E + \kappa_{\text{divB}} \nabla \cdot B \tag{1}
\]

\[
\mu_0 J = \nabla \times B \tag{2}
\]

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (nV) = \nabla \cdot (D \nabla n) \tag{3}
\]

\[
\rho \left( \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \Pi_i \tag{4}
\]

\[
\frac{3n}{2} \left( \frac{\partial T}{\partial t} + \mathbf{V} \cdot \nabla T \right) = -nT \mathbf{V} \cdot \mathbf{V} + \mathbf{V} \cdot \left\{ n \left( \chi_{||} - \chi_{\perp} \right) \mathbf{b} \mathbf{b} + \chi_{||} \mathbf{I} \right\} \cdot \nabla T, \tag{5}
\]

where \( n \) is the electron number density, \( \rho \) is the mass density (\( m_i n \) for singly charged ions), \( T \) is temperature (multiplied by the Boltzmann constant), \( p \) is the sum of electron and ion pressures (\( 2nT \)), \( \mathbf{V} \) is the center-of-mass flow velocity, \( \mathbf{B} \) is magnetic induction, and \( \mathbf{J} \) is charge-current density. The single-temperature modeling represented by (5) assumes rapid thermal equilibration among the electron and ion species for convenience, but large thermal diffusivity (\( \chi_{||} \)) parallel to the direction (\( \hat{\mathbf{b}} \)) of the magnetic field can affect heat flow, so thermal conduction is included. The second term on the right side of equation (1) is numerical and is used to control magnetic divergence error [7]. The term on the right side of (3) is also numerical and is used to stabilize the number density representation.

The electric field (\( \mathbf{E} \)) in our two-fluid ELM computations,

\[
\mathbf{E} = \eta \mathbf{J} - \mathbf{V} \times \mathbf{B} + \frac{1}{ne} \mathbf{J} \times \mathbf{B} - \frac{T}{ne} \nabla n, \tag{6}
\]
includes the Hall effect, which allows the magnetic field to move with the electron fluid. In resistive MHD, where the last two terms on the right side of equation (6) are omitted, magnetic field is tied to the ions (apart from relatively weak diffusion). Plasma kinetics shows that electrical resistivity ($\eta$) depends on temperature as $T^{-3/2}$. The nonlinear two-fluid simulation discussed later computes its resistivity as a function of time and space from the evolving temperature distribution. The ion stress tensor includes viscous damping $\rho v \mathbf{W}$, where $v$ is the viscous diffusivity, and $\mathbf{W}$ is the traceless rate-of-strain tensor. Our two-fluid modeling also includes the Braginskii gyroviscous stress

$$
\Pi_{gv} = \frac{m_e n_T}{4eB} \left[ \hat{b} \times \mathbf{W} \cdot \left( \mathbf{I} + 3\hat{b}\hat{b} \right) - \left( \mathbf{I} + 3\hat{b}\hat{b} \right) \cdot \mathbf{W} \times \hat{b} \right]
$$

that is derived under the assumption of $\mathbf{E} \times \mathbf{B}$ drift motion being comparable to thermal speeds. This condition is not realized in tokamak plasmas, but including Braginskii gyroviscous stress is a significant step toward an accurate representation of plasma magnetization effects on flow.

The ELM computations solve equations (1-5) as initial-value problems using an implicit leapfrog method that avoids severe time-step limitations from the multi-scale nature of the system. As described in reference [8], the implicit leapfrog stagers the flow velocity temporally by 1/2 time-step from the magnetic field, number density, and temperature. The velocity equation is advanced with ideal-MHD wave effects and all viscous effects represented implicitly. The magnetic field is advanced with implicit Hall effect and resistive diffusion with other terms centered by the staggering. In addition, implicit advection (terms with $\mathbf{V} \cdot \mathbf{V}$ operators) is used throughout. Where staggering does not provide time-centering, nonlinear terms are linearized in time about the beginning of the respective advance, like the first step of Newton’s method. The resulting algebraic systems do not use outer iteration for nonlinear convergence, but the linear propagation of Hall-MHD waves is much faster than any dynamics of interest, so the system is very stiff. The spatial representation uses high-order finite elements for the poloidal cross-section of the toroidal domain, and finite Fourier series represents the toroidal direction [7]. The toroidal angle is formally an ignorable coordinate for $l=1$ poloidal mode, and finite Fourier series represents the toroidal direction [7]. The toroidal angle is formally an ignorable coordinate for linear computations, and implicit terms only couple nodes of the poloidal mesh. Nonlinear effects make the implicit operators functions of toroidal angle, so the resulting algebraic systems couple all Fourier components at all poloidal nodes in nonlinear calculations. With two-fluid terms and implicit advection, the matrices are non-Hermitian, and the preconditioned Generalized Minimal Residual method (GMRES) [9] is used. Preconditioning is accomplished with the parallel sparse solver library SuperLU_DIST [10] applied to reduced systems that drop the Fourier component coupling.

In many cases, tokamak MHD activity produces small perturbations to the large background magnetic field, density, pressure, etc., so the NIMROD implementation decomposes each physical field into a time-dependent perturbation and a steady background (or equilibrium) part. Analyzing a particular configuration is then accomplished by using the relevant profile as the steady background. For our ELM computations, we use an MHD equilibrium fitted to experimental data from the DIII-D discharge numbered 113317. Both the temperature and number density profiles drop by approximately a factor of four from just inside to outside the separatrix. To simplify modeling of the edge plasma region beyond the separatrix, the background is assigned uniform temperature and number density at the values provided by the data file for the separatrix (100 eV and 1.2×10^{19} \text{ m}^{-3}, respectively).

3. Computational results

We have studied the linear stability of the fitted equilibrium using resistive-MHD and two-fluid models. A range of toroidal harmonics is unstable, and the modes tend to be localized in the direction perpendicular to the separatrix surface. The wavelength in the azimuthal direction, i.e. within the poloidal plane and along the separatrix, decreases with increasing toroidal
wavenumber. Numerical resolution is most readily achieved with finite element basis functions of polynomial degree five or more with topologically polar meshes that are packed near the separatrix surface. For 20×120 meshes, we use artificial particle diffusivities ($D$) of 2.5-5 m$^2$/s and viscous diffusivity $v=25$ m$^2$/s. In addition, we increase the magnitude of the electrical diffusivity ($\eta/\mu_0$) function so that its value is approximately 7 m$^2$/s inside the separatrix, which is between one and two orders of magnitude larger than the physical value for the experiment. Further improvement in spatial resolution is necessary to model realistic parameters. Nonetheless, numerically resolved results at these parameters reproduce the important two-fluid stabilization for high toroidal wavenumber, as shown in figure 1. An effective electrostatic potential profile cancels ion diamagnetic drift in the computations, but gyroviscous stress from the sharp drop in pressure is strong. This drift effect completely stabilizes large-$n$ modes, but intermediate-$n$ illustrated by figure 2 are wider than the edge pressure scale and are sheared.

Figure 1. Normalized growth rates from two-fluid and MHD models for the toroidal harmonic numbers indicated. The global Alfvén time ($\tau_A$) for this equilibrium is 0.85 ms.

Figure 2. Contours of constant toroidal component of flow velocity over the poloidal plane for the $n=21$ eigenfunction from a) the MHD model and b) the two-fluid model. The modes reside along the strong equilibrium gradient just inside the separatrix surface.

The spectrum of unstable linear modes is used as the initial condition of the nonlinear two-fluid simulation. Nonlinear coupling among the unstable modes quickly excites perturbations in the linearly stable parts of the spectrum at large and small $n$-indices, as shown in figure 3. A nonlinear MHD computation with the same parameters would suffer an ‘ultraviolet catastrophe’ in that the shortest wavelength modes have the largest growth-rates. The drift effects of the two-fluid model therefore make the nonlinear evolution possible, but other normal modes of this system are dispersive at high wavenumber, increasing stiffness relative to MHD. Late in the computation, the spectrum of modes beats into a toroidally localized structure, shown in figure 4, consistent with results of the reduced-harmonic modeling described in reference [4].

The nonlinear computation ran in parallel on 344 processors, approximately 40%, of the IBM SP5 at the National Energy Research Scientific Computing (NERSC) Center. A weak parallel scaling for a similar nonlinear two-fluid problem shows that the computation achieves 76% parallel efficiency for 262 GFlops, 10% of the theoretical peak. The largest algebraic systems at each time-step couple 7.5 million complex quantities. In nonlinear MHD computations with explicit advection, operations without toroidal coupling are effective at preconditioning the Hermitian matrices. This approach to preconditioning has proven less successful in the two-fluid ELM computation, which has non-Hermitian matrices. The matrix for the magnetic advance is particularly ill conditioned due to the Hall term. Moreover, the orthogonalization step of GMRES

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makes long iteration costly for large systems. To facilitate the algebra in this computation, the
time-step is progressively reduced as nonlinear perturbations grow, until it is more than an order
of magnitude below accuracy requirements. More effective preconditioning is needed to extend
the simulated time through the ELM event, which will allow us to predict particle and heat loss.

4. Conclusions
The NIMROD computation for an unstable DIII-D equilibrium confirms the importance of two-
fluid effects for the nonlinear evolution of ELM activity. Future global-domain simulations with
equilibria unstable to core plasma modes will be able to explore the interaction of core MHD
activity with ELMs. Computationally, the present effort shows that the new two-fluid algorithm
needs more effective preconditioning for its GMRES iterations when including the Hall effect in
nonlinear simulations, and this development is underway.

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