Noise-driven activation in human intermittent control: a double-well potential model

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In controlling unstable systems humans switch intermittently between the passive and active behavior instead of controlling the system in a continuous manner. The notion of noise-driven control activation provides a richer alternative to the conventional threshold-based models of intermittent motor control. The present study represents the control activation as a random walk in a continuously changing double-well potential. The match between the proposed model and the previous data on human balancing of virtual stick prompts that the double-well approach can aid in explaining complex dynamics of human behavior in control processes.

I. INTRODUCTION

Experimental observations of human-controlled processes, such as standing upright and inverted pendulum balancing (Loram et al. 2011, Milton 2013), often reveal discontinuous, or intermittent control. Investigations of core properties of human control intermittency thus may have significant implications for issues of falling in elderly (Moss and Milton 2003) and human operators’ performance (Wickens and Hollands 1999).

Hypothesis of intermittent control implies repeated switching between the passive and active control phases. Control mechanisms operating in each of these phases are often hypothesized to be independent, although interacting (see e.g. Bottaro et al. 2008, Zgonnikov et al. 2014). The high-level picture of two coupled mechanisms is indeed much simplified; the actual human control systems are characterized e.g. by significant neural response delay, sensorimotor noise, and possible presence of hierarchical, multi-level control (Milton 2013). However, even within this simple two-mechanism framework the comprehensive theory of human intermittent control is still to be developed. Understanding the physics of the control activation and control execution mechanisms (operating in the passive and active phases, respectively) via formal models of these mechanisms can provide useful insights into dynamics of human control.

The current work is aimed at modeling the control activation mechanism, which plays crucial role during the passive control phase. During this phase the operator tracks the state of the controlled system and eventually decides upon interrupting its dynamics. With respect to control activation, the concept of threshold is the key notion in the modern literature on human control. Threshold-based control assumes that while the deviation of the controlled system from the desired state remains below certain threshold value, the control remains off, and when the deviation crosses this threshold, the control switches on. Simple yet powerful approach of threshold-based control proved its efficiency in modeling human control dynamics (see e.g. reviews in Asai et al. 2013, Gawthrop et al. 2011, Milton 2013). However, it has recently been challenged by the notion of noise-driven control activation.

Recent experiments on the virtual stick balancing task have revealed high variability of the control activation mechanism in humans (Zgonnikov et al. 2014). Particularly, large deviations of the controlled system in the passive phase occur much more frequently than suggested by the threshold-driven activation. One of the possible reasons for this is that control activation in humans is defined not (or not only) by the perception limits, but by some complex cognitive processes. The simple phenomenological model of noise-driven activation reported by Zgonnikov et al. (2014) can mimic the actual
human dynamics, although it does not provide any clues on the physiological or cognitive foundations of the observed behavior. An important step towards such an explanation would be to describe the control activation dynamics on the macroscopic level, in terms of some order parameter.

The present study develops the model of noise-driven control activation based on the notion of random walk in a double-well potential. We map two cognitive states of the operator, “act” and “wait”, onto two attractors of a dynamical system. The energy landscape governing the system dynamics changes continuously in response to the deviation of the controlled system from the desired state. We instantiate the proposed model using the simple example of overdamped stick balancing. We show that the model reproduces the exponential decay of action points (system deviations triggering reaction of the operator in the passive phase). In conclusion, we argue that the concept of metastable random walk in a potential well can serve as a natural, easily adaptable framework for studying mechanisms of control activation in diverse human-controlled processes.

II. BISTABLE DYNAMICS OF CONTROL ACTIVATION

A. Model background

Prior to formally specifying the model, we wish to outline the main assumptions behind it. First of all, the model is designed as an alternative to the concept of threshold conventionally used in the modern studies on motor control. As has been demonstrated recently, the operators often react to the deviations that significantly exceed the typical magnitude of perception thresholds (Zgonnikov et al. 2014). We hypothesize that in controlling unstable objects many factors other than the magnitude of stimulus (i.e., deviation from the goal state) affect the range of actual system states triggering human response (we call such states “action points”, appealing to the terminology used in the car following research (Todosiev and Barbosa 1963)). For instance, if the control process lasts for a relatively long time, the mental expenses for staying perfectly aware of the tiniest deviations may be unbearable for the operator. In this case, even the deviation that otherwise can be clearly perceived may be neglected due to energy considerations. Another relevant factor is the limited ability of the operator to precisely manipulate the unstable system. Even a highly skilled operator cannot accurately compensate for small, barely detectable deviations. In order not to destabilize the system by the imprecise interruption, the operator may prefer to wait until the deviation becomes large enough. As a result, the corrective movements need not be thoroughly planned and implemented. These and other factors may cause the greater variability of the control triggering mechanism compared to the threshold mechanism. This variability is the key experimentally observed feature to be captured in the proposed model.

Second, to describe the dynamics of the control switching mechanism, we introduce the order parameter reflecting the operator’s state of mind in regards to the controlled system. Human brain is a self-organizing system (Haken 1996, Kelso 1995). Nonlinear interactions of large number of neurons and/or neuronal populations lead to emergence of complex dynamics, which, however, can often be described on the macroscopic level by just a few variables (order parameters). This approach has been widely applied for studying perceptual and cognitive dynamics (see e.g. reviews in Kelso 1995, Rolls and Deco 2010). In our case we describe dynamics of the noise-driven switching between active and passive phase as a random walk in a double-well potential which possesses two metastable attractors matching the passive and active behavior of the operator. In physics, switching between the attractors in bistable systems is driven by random forces, often labeled as “noise”. In the context of cognitive dynamics, noise of this type typically reflects stochasticity of the neuronal firing times (Rolls and Deco 2010). Mathematically such dynamics is usually captured by additive Gaussian white noise, whose intensity does not depend on the system state (see e.g. Moreno-Bote et al. 2007). However, we assume that
in the situations when the system deviation is in some sense high, the operator’s mental state is certain (that is, “act”). Similarly, when the deviation is evidently small, the probability of switching to the active phase should be relatively small. On the other hand, the system deviations allowing for both interpretations should lead to higher uncertainty in the state of the operator, and, consequently, to larger noise intensity. Correspondingly, the proposed model utilizes multiplicative noise to reflect the assumption that the level of uncertainty of the operator’s mental state is low in unambiguous situations and high in ambiguous ones.

**B. Model description**

Following Zgonnikov et al. (2014), we exemplify our theoretical constructs using possibly the simplest task of control over unstable object, namely, overdamped stick balancing. The task dynamics in terms of the stick angle $\theta$ and cart velocity $v$ are described by the non-dimensional differential equation (for details see Zgonnikov et al. 2014)

$$\dot{\theta} = \theta - v.$$  

Prior to modeling the dynamics of operator’s behavior in the active control phase, we first introduce the mechanism of control switching, which is the core part of the proposed model. We describe the aggregated mental state of the operator by the order parameter $\xi$, which switches intermittently between two states, $\xi = 0$ (passive control phase) and $\xi = 1$ (active control phase). Its dynamics is driven jointly by the deterministic and random forces. The deterministic dynamics is defined by the potential energy landscape, which possesses two attractors corresponding to $\xi = 0$ and $\xi = 1$. The relative strength of the two attractors is defined by the state of the controlled system (in our case, the stick angle $\theta$). The switching between attractors is caused by the multiplicative noise cofactor. Such dynamics can be described by the equation

$$\tau \dot{\xi} = -\frac{\partial H}{\partial \xi} + \sqrt{\epsilon H} \zeta,$$

where $\tau$ defines the time scale of switching between the passive and active phases and $H(\xi, \theta)$ is the Hamiltonian shaping the energy landscape of the system. Generally, $\tau$ is assumed to depend on energy and its partial derivatives, $\tau = \tau(H, \partial H/\partial \xi, \partial^2 H/\partial^2 \xi)$. However, in the present paper we confine our scope to the case $\tau = \text{const}$, leaving examination of the general case for future studies.

The multiplicative noise term in Eq. (2) includes the white noise $\zeta$ of variable intensity $\sqrt{\epsilon H}$. The given dependence of the noise intensity on $H$ has been chosen for the sake of simplicity because in this case the form of the Langevin equation (2) does not depend on the stochastic process interpretation. Namely, the difference between this equation written in the Itô, Stratonovich, or Hänggi-Klimontovich forms is reduced to a minor renormalization cofactor in

**FIG. 1. Energy landscape $H(\xi)$ depending on deviation of the controlled system from the desired position (as characterized by $a(\theta)$)**
the regular drift term $\partial H/\partial \xi$ (see e.g. Mahnke et al. 2009).

The particular form of the Hamiltonian is chosen in a way that

$$\frac{\partial H}{\partial \xi} \propto \xi(\xi - 1)(\xi - a(\theta)),$$  \hspace{1cm} (3)

where $a(\theta)$ is such that $a = 1$ if $\theta = 0$ and $a \to 0$ when $\theta \to \infty$. An arbitrary function satisfying these requirements can be used; to be specific, we assume

$$a(\theta) = 1/(1 + \theta^4).$$ \hspace{1cm} (4)

As a result, the only attractor for large deviations is $\xi = 1$, while for $\theta \approx 0$ it is $\xi = 0$. Intermediate values of $\theta$ lead to bistable dynamics, so both the states become possible (Fig. 1). In addition, we impose the following constraints on $H$

$$H|_{\xi=0, \theta=0} = 0, \quad H|_{\xi=1, \theta=0} = 0,$$

$$H|_{\xi=0, \theta=1} = 1, \quad H|_{\xi=1, \theta=1} = 1.$$ \hspace{1cm} (5)

so that the random fluctuations diminish when the energy minimum $H = 0$ is achieved at the attractor states ($\xi = 0$ and $\xi = 1$) in case of unambiguous states of the controlled system ($\theta = 0$ and $\theta \gg 1$, respectively). Conditions (3–5) yield

$$H(\xi, a(\theta)) = 3\xi^4 - 4(1 + a)\xi^3 + 6a\xi^2 + 1 - a.$$ \hspace{1cm} (6)

Once the stochastic dynamics of control switching are defined by Eqs. (2), (6), the cart velocity can be easily modeled using the approach of phase space extension (Lubashevsky et al. 2003, Zgonnikov and Lubashevsky 2014). We consider the cart velocity $\upsilon$ to be a separate phase variable, so that its dynamics are governed by the equation of the form

$$\dot{\upsilon} = f(\theta, \upsilon, \xi),$$

In specifying $f(\theta, \upsilon, \xi)$, we note, first, that in the active phase ($\xi = 1$) the actions of the operator can be approximated by linear feedback $\dot{\upsilon} = \alpha \dot{\theta} - \beta \upsilon$, where $\alpha$ and $\beta$ are some constants (see Zgonnikov et al. 2014). Second, in the passive phase ($\xi = 0$), by definition, the cart velocity equals zero, which can be captured by assuming $\dot{\upsilon} \propto -\upsilon$. These considerations yield

$$\dot{\upsilon} = \xi \gamma \theta - \sigma \upsilon,$$ \hspace{1cm} (7)

where $\sigma$ and $\gamma$ are constant parameters.

Equations (2, 7) finally form the mathematical description of the model at hand.

C. Model dynamics

We simulate the model dynamics (interpreted as Ito process) using the stochastic Runge-Kutta algorithm (Roessler 2005). The model has four parameters, $\tau$, $\epsilon$, $\gamma$, and $\sigma$. The goal of this paper is to demonstrate that the proposed model in principle can capture the actual control activation dynamics. For this reason we illuminate the basic properties of the model by examining its behavior for certain physically plausible set of parameters, and do not perform a comprehensive analysis of the system dynamics depending on the parameter values (which is to be reported elsewhere). At the same time, the below results are found to be stable with respect to reasonable parameter deviations.

The parameters $\gamma$ and $\sigma$ characterize the system motion in the active phase. However, under some weak assumptions (essentially, $\sigma > 1$, $\gamma > \sigma$) the particular values of these parameters do not affect the system dynamics qualitatively, so in our simulations we use the values $\sigma = 3.5$, $\gamma = \sigma^2/2$ previously shown to be physically plausible (for details see Zgonnikov et al. 2014).

The parameter $\tau$ defines the time scale of the random walk $\xi$ relative to the intrinsic time scale of the controlled system; the latter is assumed to be unity for the chosen non-dimensional variables. In other words, $\tau$ characterizes the typical time needed for $\xi$ to switch from one metastable attractor to another. Consequently, we assume $\tau \ll 1$ (to be specific, $\tau = 0.2$), so that the switching occurs much faster than the transformation of the energy landscape due to changing $\theta$. The noise intensity $\epsilon$ is also assumed to be a small parameter, $\epsilon \ll 1$ (the value used for numerical simulations is $\epsilon = 0.02$).

The phase trajectory of model mimics the experimentally observed trajectories (Fig. 2). The system initially perturbed by a small deviation of $\theta$ moves along the axis $\upsilon = 0$, which represents the passive control phase ($\xi = 0$). As the angle $\theta$
increases, the energy landscape changes so that the transition to the active phase ($\xi = 1$) becomes more and more probable. This transition is induced by noise, so it occurs at probabilistically determined angle.

The simulated trajectory of the model during the active control phase represents the single corrective movement aimed at driving the stick to the vicinity of the vertical position. Such dynamics captures the basic pattern of the curved active control segments produced by human subjects. It should be also noted that the proposed model utilizes the double-well dynamics to capture not only control activation, but also the transition from the active to the passive phase as well. Although the variability of the corrective trajectories with respect to their end-points is also rather high (see Fig. 2 right frame), this transition is supposed to be determined mainly by the open-loop control mechanisms operating in the beginning of the active control phase. Correspondingly, further development of mathematical description of ballistic control is needed to capture the experimentally observed switching patterns in more detail; this would presumably aid in representing the whole spectrum of the possible active phase dynamics as well.

The distribution of the action points generated by the model decays exponentially, which captures the experimentally observed statistics (Fig. 3). Note that for small values of the stick angle the model action point statistics differs from the experimental one. One may argue this prompts that control activation occurring at small deviations may be attributed to a mechanism different from that implemented in the current model. However, we believe that the discrepancy is simply an artifact of the particular model implementation. The latter seems likely due to the fact that the model qualitatively reproduces the form of the action point distribution near the zero angle. This hypothesis is also supported by the fact that the frequency of small-magnitude action points tends to decrease with the operators’ skill, while the tail of the action point distribution remains the same (Zgonnikov et al. 2014).

Most importantly, the tail part of the numerically obtained action point distribution closely follows the experimental one. Presumably any control activation mechanism based on noisy threshold would lead to Gaussian distribution of action points (see dashed line in Fig. 3), which basically eliminates any probability of control activation at relatively large values of the system deviation. In this sense noise-driven activation as captured in the double-well model provides a much more plausible explanation of the actual human control patterns. This fact can be especially relevant in view of currently missing computational explanation of the extreme events observed in human control, such as stick falls (Cabrera and Milton 2012).

FIG. 2. Typical phase trajectory of the overdamped stick balancing exhibited by the double-well control activation model (left frame) and human subject (right frame) (see details of the experimental data in Zgonnikov et al. 2014).
FIG. 3. Probability distribution function (pdf) of action points exhibited by the model compared to the experimental data on overdamped stick balancing task. The experimental distribution is averaged over five subjects (see details of the experimental data in Zgonnikov et al. 2014).

III. DISCUSSION

In controlling unstable systems humans often switch intermittently between the passive and active behavior instead of controlling the system in a continuous manner. The present paper argues that some intricate properties of intermittent control activation can be explained using the notion of random walk in a double-well potential. Appealing to modern understanding of human intermittent control, we propose to distinguish between the passive and active phases of operator’s behavior. We then extend the phase space of the physical system under human control by an order parameter characterizing the operator’s state of mind, which repeatedly switches between two metastable attractors corresponding to active and passive behavior. The dynamics of this order parameter is stochastic; it is defined in part by the double-well potential field (changing with the state of the controlled system), and in part by the stochastic factors. We describe the latter as state-dependent (multiplicative) noise, so that the unambiguous states of the controlled system evoke little noise compared to the uncertain ones. We demonstrate that the key characteristic of human control activation in simple stick balancing task, that is, the exponentially decaying action point distribution, is reproduced by the model. We conclude that the proposed double-well potential activation model is a physically plausible approach to modeling control activation in tasks where humans exhibit intermittent control behavior.

One of the major problems in theory of human intermittent control is understanding the mechanism of control activation, that is, transition from the passive to the active phase (Asai et al. 2013). The concept of threshold has been conventionally used to model control activation. However, a question had arisen recently as to whether threshold can fully accommodate complex, often unpredictable dynamics of human-controlled systems (Asai et al. 2013; Bottaro et al. 2008; Cabrera and Milton 2012). A more general notion, noise-driven control activation, provides a richer, intrinsically stochastic alternative to the threshold-based models of intermittent motor control (Zgonnikov et al. 2014). Still, one of the major shortcomings of the previously suggested implementation of noise-driven control switching is the lack of explicit regulations governing the activation dynamics. This in turn makes it difficult to directly extend the model suggested by Zgonnikov et al. (2014) to the processes other than overdamped stick balancing. By ascribing the two metastable attractors of a dynamical system to the possible outcomes of the control activation process, the present study overcomes this issue. As a result, the activation dynamics in different control tasks can be potentially captured using the task-specific configuration of potential landscape defining the dynamics of the control switching order parameter.

In the proposed model the dynamics of control switching is defined, first, by the regular force (i.e., the energy gradient), second, the noise term $\sqrt{\epsilon H \zeta}$, and, third, the characteristic time scale of switching $\tau$. In general case, the latter time scale may not be constant, as it is assumed in virtually all available models of perceptual switching. Specifically, $\tau$ may depend on the Hamiltonian and/or its partial derivatives; this would supposedly enable the proposed model to better capture the switching patterns found experimentally. Moreover, we speculate that in a whole class of related cognitive processes the
Hamiltonian $H$ of the macrolevel order parameter may fully define the system dynamics, as it determines the values of both the regular and stochastic forces, and the time scale of the system. This hypothesis, however, needs thorough further analysis and additional experimental evidence.

We believe that the ideas of the present work may extend beyond the specific field of motor control to more general cognitive processes. Complex, high-dimensional neural systems often exhibit low-dimensional dynamics on the macrolevel (Kelso 1995). An apt example is the phenomenon of bistable perception, which can be characterized by the double-well dynamics of switching, e.g., between two alternative interpretations of an ambiguous stimulus (Moreno-Bote et al. 2007). Evidence for double-well stochastic dynamics has also been found in other cognitive processes, for instance, perceptual categorization of speech patterns (Tuller et al. 1994) and imagined actions (van Rooij et al. 2002). Inspired by the results of the present study, we hypothesize that further quantitative investigations may provide grounds for stronger link between such processes and the concepts traditionally employed in describing dynamical systems in physics. In particular, treating the intensity of the neural noise as a state-dependent rather than constant quantity, on the one hand, is physiologically plausible (Lindner and Schimansky-Geier 2001, Silberberg et al. 2004), and, on the other hand, may potentially enhance the explanatory power of the modern cognitive models of multistable phenomena.
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