Theoretical model of non-linear tire slip

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Abstract. The authors carry out the analysis of research studying the phenomenon of slipping of a wheel with a pneumatic tire. It is established that the increase in the slip angle results in the slip coefficient decrease. The values of the slip coefficient for three types of tires are determined experimentally. The previously proposed analytical expression, which makes it possible to quite accurately formulate the nonlinear relationship of the slip angle with the lateral force acting on the elastic wheel, is clarified.

1. Problem formulation

A significant number of research papers are focused on the phenomenon of the wheel tires lateral slip. As a result, it was defined that the relationship between the slip angles and the lateral forces is nonlinear. The ratio of lateral force to the slip angle is called the coefficient of tire lateral slip resistance (tire slip coefficient).

Traditionally, for small slip angles, it is believed that the tire slip coefficient is a constant value. It corresponds to the linear part of the dependence of lateral force on the tire slip angle. To assess the influence of various factors on the slip coefficient value, some authors proposed a system of correction factors. However, this greatly complicates the mathematical model of the car movement with a slip.

This article proposes a theoretical model of non-linear lateral slip of the car pneumatic tire.

2. Analysis of known lateral tire slip models

The slip phenomenon of the wheel with a pneumatic tire was specified in 1925 by G. Brulier [1]. The results of the first experimental study on this phenomenon were published in the papers by G. Becker, H. Frome, H. Maruni in 1931 [2]. A rigorous evaluation of the research in the phenomenon of slipping of the wheel with a pneumatic tire was made by A.S. Litvinov [3]. A quantitative assessment of lateral slip of an elastic wheel in the form of a relationship between the lateral force $P_y$ and slip angle $\delta$ was proposed by Ya.M. Pevzner [4]. It has the form

$$P_y = K_y \cdot \delta$$

(1)

where $K_y$ is the coefficient of the lateral slip resistance of tires (slip coefficient).

Dependence (1) is used for the small values of slip angles $\delta$. A simplified form of the dependence of lateral force on the lateral slip angle is provided in Figure 1 [7]
Figure 1. Simplified form of the dependence of lateral force on the lateral slip angle.

The curve can be divided into three sections: inclined linear, curved and horizontal linear. The inclined linear section OA (Figure 1) corresponds to non-skidding slip, the curved section AB corresponds to rolling with slipping and partial skidding. The horizontal linear section BC corresponds to pure skidding.

Dependence (1) is used for the inclined linear section OA, i.e. for relatively small slip angles $\delta$. In this case:

$$K_y = K_{y0} = \frac{\partial P_y}{\partial \delta},$$

where $K_{y0}$ is the tangent of the slope of the curve $P_y(\delta)$ at the origin (Figure 1).

At the sections AB and BC, the slip coefficient $K_y$ is variable, since $\frac{\partial P_y}{\partial \delta} = Var$. The papers [3, 5] point out that the coefficient of tire lateral slip resistance depends on a large number of factors, the main of which are: wheel dimensions and design, air pressure in the tire, forces, acting on the wheel, speed, type and condition of the road surface, trajectory of the wheel center (rectilinear, curved), trajectory curvature and the speed of its measurement in time or by displacement, nature of the application of forces acting on the wheel and the rate of change of these forces. The influence of these factors does not allow using the linear theory of slipping. Its essence is that the dependence of the lateral force on the slip angle is described by the equation

$$P_y = K_y \cdot \delta = qK_{y0} \cdot \delta = q_N \cdot q_T \cdot q_{\phi} \cdot q_V \cdot q_{\infty} \cdot q_{im} \cdot q_{2} \cdot q_{TP} \cdot q_{3Y} \cdot q_{HY} \cdot K_{y0},$$

where $q$ is the total correction factor of the slip coefficient;

$q_N$ is the correction factor taking into account the redistribution of the normal reaction between the wheels;

$q_T$ is the correction factor taking into account the influence of tangents (traction and braking forces);

$q_{\phi}$ is the correction factor taking into account the change in the wheel road adherence properties;

$q_V$ is the correction factor taking into account the change in the inclination of the wheels to the supporting surface in the case of roll;

$q_{im}$ is the correction factor taking into account the influence of air pressure in the tire;

$q_{3Y}$ is the correction factor taking into account the influence of the rear steering wheels.

Analyzing the shape of the curve shown in Figure 1, the authors can conclude that this curve can be described by the following mathematical dependence [6]

$$P_y = P_{y_{max}} \cdot \frac{\delta}{K + \delta},$$

where $P_{y_{max}}$ is the maximum lateral force corresponding to the linear horizontal section BC in the graph shown in Figure 1;
$K$ is a constant coefficient for a particular tire design at given internal pressure values.

In [6], an incorrect method was applied for the analytical determination of the parameter $K$. To determine the limit of the value of $P_y$ (equation (4)), the L'Hôpital's rule is used. This technique is used when substituting $\delta = 0$ in equation (4) to obtain the $\frac{0}{0}$ or $\frac{\infty}{\infty}$ type of uncertainty.

Obviously, the coefficient of lateral slip resistance of the elastic wheel can be considered as a partial derivative of the lateral force $P_y$ with respect to the slip angle $\delta$ not only for the linear part $OA$ of the characteristic (Figure 1), but also in the entire range of variation of the lateral force $P_y$ and the slip angle $\delta$

$$K_y = \frac{dP_y}{d\delta} = \frac{P_{y_{\text{max}}}}{(K + \delta)^2}$$

(5)

Analyzing expression (5), the authors can conclude that, given the increase in the slip angle $\delta$, the slip coefficient $K_y$ decreases.

From equation (4) determine the parameter $K$:

$$K = \delta \left( \frac{P_{y_{\text{max}}}}{P_y} - 1 \right).$$

(6)

After substituting expression (6) in equation (5) get:

$$K_y = \frac{P_{y_{\text{max}}}}{\delta}$$

(7)

Define $P_y$ from equation (7). To do this, transform the indicated equation (7) to the form:

$$P_y^2 - P_{y_{\text{max}}}P_y + K_y P_{y_{\text{max}}} = 0.$$  

(8)

The solution of the reduced quadratic equation (8), taking into account the root with a physical meaning, has the form:

$$P_y = 0.5 P_{y_{\text{max}}} \left( 1 - \sqrt{1 - \frac{4K_y P_{y_{\text{max}}}}{P_{y_{\text{max}}}}} \right)$$

(9)

The value of $P_{y_{\text{max}}}$ is determined by the lateral adhesion capabilities of the tire. In the absence of a tangent reaction in the spot of wheel contact with the road:

$$P_{y_{\text{max}}} = \phi R_z$$

(10)

If there is a tangent reaction in the point of wheel contact with the road:

$$P_{y_{\text{max}}} = \sqrt{\phi^2 R_z^2 - R_x^2}$$

(11)

Figure 2 shows the solid lines - graphs of the dependences of the lateral force on the slip angle for three types of tires taken as an example [3]. The same figure shows the approximating graphs [6]:
Figure 2. Dependence of the lateral force acting on the wheel on the slip angle: 1 - type P with a metal cord; 2 - type P with a textile cord; 3 - ordinary tire with a textile cord; ----- experimental data [7]; - - - approximating dependences (4) [6].

It is experimentally determined that for a P tire with a metal cord: \( P_{y,\text{max}} = 7223 \text{ N} \) and \( K = 0.2 \). For a P tire with a textile cord: \( P_{y,\text{max}} = 3006 \text{ N} \) and \( K = 0.0914 \). For an ordinary tire with a textile cord: \( P_{y,\text{max}} = 4572 \text{ N} \) and \( K = 0.261 \).

Thus, the refinement of the analytical expression (4) proposed in [6] makes it possible to get a sufficient accuracy in the formulation of the non-linear relationship of the slip angle with the lateral force acting on the elastic wheel.

Dividing and multiplying the right side of equation (9) by the slip angle \( \delta \), obtain:

\[
P_y = 0.5 P_{y,\text{max}} \left( 1 - \sqrt{1 - \frac{4K_y \delta}{P_{y,\text{max}}}} \right) = K_y' \delta
\]

(12)

where \( K_y' \) is the conditional slip coefficient for the non-linear part of the characteristic (the \( OA \) section in Figure 1.2) \( K_y' = K_{y,0} \).

\[
K_y' = 0.5 \frac{P_{y,\text{max}}}{\delta} \left( 1 - \sqrt{1 - \frac{4K_y \delta}{P_{y,\text{max}}}} \right)
\]

(13)

\( K_y \) is the coefficient of lateral slip resistance (see expression (5)).

Decomposing the expression with the root in the right side of equation (13) into a binomial series and limiting to the first two terms, get the following:

\[
\sqrt{1 - \frac{4K_y \delta}{P_{y,\text{max}}}} = \left( 1 - \frac{4K_y \delta}{P_{y,\text{max}}} \right)^{\frac{1}{2}} = 1 - \frac{2K_y \delta}{P_{y,\text{max}}} \]

(14)

After substituting (14) into (13) obtain:
\[ K_y' = K_y = \frac{dP_y}{d\delta}. \]  

(15)

This means that by determining the \( K_y(\delta) \) dependence it is possible to quite accurately obtain the tire slip angle under the influence of lateral force according to the formula:

\[ \delta = \frac{P_y}{K_y'}. \]  

(16)

over the entire range of changes for both the slip angle \( \delta \) and the lateral force \( P_y \).

On the linear part of the dependence \( P_y(\delta) \) (section OA in Figure 1) \( K_y = K_{y0} \) and expression (7) takes the form:

\[ K_y = K_{y0} = 1 - \frac{q_y}{P_{y\text{max}}} = K_{y0} - q_y \]  

(17)

where \( q_y \) is the refinement factor of the slip coefficient taking into account the slip angle, normal and tangent reaction of the road at the point of contact.

However, the approximation of the experimental curves by various mathematical curves does not allow assessing the influence of various structural and operational factors on the lateral slip of elastic tires at the design stage.

3. Conclusions

The refinement of the analytical expression proposed in [6] makes it possible to achieve a sufficient accuracy in the formulation of the non-linear relationship of the elastic tire slip angle with the lateral force acting on the elastic wheel.

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