Transmission and Conversion of Energy by Coupled Soft Gears

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Dynamical aspects of coupled deformable gears are investigated to clarify the differences of mechanical properties between the machines consist of hard materials and those of soft materials. In particular, the performances of two functions, the transmission and the conversion of the energy, are compared between the hard and soft gears systems. First, the responses of the coupled gears against a constant torque working on one of gears are focused for two types of couplings; P) a pair gears are coupled, and T) three gears are coupled with forming a regular triangle. In systems with the coupling P), we obtain trivial results that the rotational energy can be transmitted to other gear only if these gears are hard enough. On the other hand, in systems with the coupling T), the transmission of the rotational energy to one of the other gears appears only if these gears are soft enough. Second, we show the responses of this system in which one of gears have contact with a high temperature heat bath and the other gears have contact with a 0 temperature heat bath. With the coupling T), the directional rotations appear in two gears with 0 temperature heat bath. Here, the direction of these rotations change depending on the noise strength.

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Recently, the practical applications of micro- or nano-scale machines are hoped by several area of sciences in particular the biology and medical science. In the environments where micro or nano scale machines work, the influences of the thermal fluctuations is so large that it cannot be neglected. In our familiar macro-scale machines, for example, a hard gear (rotator) has been known as a useful component to realize several functions. In order to get high performance of functions from the populations of hard gears, however, each pair of these gears must be always kept closely with each other to be coupled tightly. Thus, such machines seem weak for fluctuations.

On the other hand in biological systems, some kinds of functions for example the transmissions or conversions of energy, materials and signals are realized efficiently by many types of molecular machines under the large influences of thermal fluctuations. It is remarkable that, differing from our familiar parts of macro-scale machines, they are generally soft and easy to deform. From these facts, a question arises: how different are the mechanical properties and performances between hard machines as our familiar macro-scale machines and soft machines as molecular machines to realize similar functions?

In this paper, we demonstrate the simulations of the systems consisting of hard gears (rotators) and those of soft gears (rotators) under the same situations in order to compare the differences of the mechanical properties directly between the machines with hard elements and those with soft elements. In particular, we focus on the transmission and the conversion of energy realized by coupled gears in the following two situations: P) a pair gears are coupled, and T) three gears are put where the centers of them form a regular triangle (as Gear 1, Gear 2 and Gear 3 in Fig. 1(b) and (d).). The population of some types of rotators are focused to understand the dynamical and statistical properties of soft materials like the colloidal systems with electric of magnetic moment (ferrofluid), recently. Thus, the study of the coupled gears (rotators) has the large importance in wide areas of material sciences.

For the system consists of hard gears coupled tightly, the following results are naturally predicted. When a torque works on a gear and this gear rotates in a direction, the rotational energy transmits to the other gear and both gears rotate with the coupling P). On the other hand, with the coupling T), such a transmission does not appear. For example we consider the case that a torque in the clockwise direction works on Gear 1. Due to the rotation of Gear 1, the torques in the anti-clockwise direction work on Gear 2 and 3. At the same time, however, the torque in the clockwise direction also work on Gear 2 and 3 by the interaction between them. Then, by this frustration, Gear 2 and 3 cannot rotate which means it is impossible to transmit the rotational energy to them.

However, the different types of behaviors are expected if these gears are soft to deform largely. Then in the following, we study the responses of such a coupled gear system against the inputs for different softness of each gear. First of in this paper, we construct a model of the ideal coupled gears and focus on the responses of this system against a torque in a direction working on a gear. This model belongs to a specific case of coupled oscillator systems with frustrations. In this model, we obtain the opposite properties for the energy transmission between soft gears and hard gears: The soft gear system can realize the energy transmission to another gear only with the coupling T) while the hard gear system can realize the energy transmission only with the coupling P). Second, we show the responses of this system in which one of gears have contact with a high temperature heat bath and the other gears have contact with a 0 temperature heat bath. With the coupling T), the directional...
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Now, we construct a model of ideal coupled gear system as follows. First, we prepare a group of hard rods with unit length and set the edge of each rod at a rotation axis in two dimensional space. Second, we prepare hard rods between which repulsive forces work on, and put on the other edge of each rod. By these two steps, we can make a deformable gear. The coupled gear system is constructed as Fig. 1 by preparing the above mentioned gears and fixing the centers of them. In this model, the motions of particles indicate the motion of teeth of gears. In this paper, for simplicity, we consider the over-dumping limit cases as a motion of each particle.

Kinetic equation for each particle is given by

$$\dot{r}_j^i = - \sum_{(i,j) \neq (i',j')} C_{j,i}^{i,i'} \nabla_{r_j^i} V(|r_j^i - r_{j'}^{i'}|) + F_{j}^i.$$  (1)

Here, $r_j^i$ indicates a position of $j$th particle in $i$th gear in two dimensional space, and $F_j^i$ indicates the torque working on $j$th particle in $i$th gear. In this paper, we employ $V(r) = 1/r$ as the repulsive interaction potential between particles. $C_{j,i}^{i,i'}$ gives the magnitude of the repulsive force working between $j$th particle in the $i$th gear and $j'$th particle in $i'$th gear. In the following, we set $C_{j,i}^{i,i'} = 1$ for $i \neq i'$ and $C_{j,i}^{i,i'} = A$ for $i = i'$. Here, $A$ is a parameter indicating the hardness of each gear. In the followings, we consider the case that each gear has three teeth (rods). Qualitatively similar results as those observed in this paper are found if the number of teeth in each gear is more than three.

Now, the transmission and conversion of the energy by this coupled deformable gear system with the couplings P) and (T) is studied for several hardness of gears $A$. Here, the distances between the centers of gears (the gear-gear distances) are given $L$. First, we show the responses of coupled gears against a torque in a direction working on a gear.

Figure 1(a) and 1(b) show the typical temporal evolutions of the tightly coupled hard gears (a) with the coupling P) and (b) with the coupling T), where $A = 70$ and $L = 1.8$ and torque in the clockwise direction works on the Gear 1 ($F_1^1 = F, F_2^2 = F_3^3 = 0$). Here, it is noted that we can observe only the periodic motions as the dynamical motions of the system in the presented situation independent of $A$ and $L$. (No quasi-periodic or chaotic motions appear.) In Fig. 1(a), both of two gears rotate with, respectively, the opposite directions. This means the rotational energy transmits from Gear 1 to Gear 2 in the system with the coupling P). On the other hand in Fig. 1(b), only Gear 1 rotates and each tooth of Gear 2 and 3 only oscillates in a restricted region. Thus, the rotational energy is not transmitted from Gear 1 to any other gears in the system with the coupling T). These results are consistent with the predictions for coupled hard gears.

Now, we focus on the behavior of the coupled soft gears. Figure 1(c) and (d) show the typical temporal evolutions of the coupled soft gears (c) with the coupling P) and (d) with the coupling T) under the same situation as the previous cases except the hardness of each gear ($A = 7$). As seen in them, the shape of gears are extremely different from those in hard gears systems because the teeth are pushed out from the inner area of the system by the repulsive forces between particles belonging to different gears. Then, two gears look not engaged. Thus, as shown in Fig. 2(a), only Gear 1 rotates in the system with the coupling P), which means the rotational energy is not transmitted to Gear 2. On the other hand in the system with the coupling T), the rotational energy can be transmitted from Gear 1 to Gear 2 as shown in Fig. 1(d) where Gear 2 rotates in the anti-clockwise direction due to the clockwise rotation of Gear 1. In this case, each tooth in Gear 3 oscillate in a restricted area. (Because of the symmetry, the rotational energy transmits from Gear 1 to Gear 3 if the torque in the anti-clockwise direction works on Gear 1.)

As previous, we obtained the clear evidences of the differences in the mechanical properties between the hard gear systems and the soft gear systems. In the following we study the detailed properties and mechanisms of the transmissions of soft gear system with the coupling T).

Figure 2 shows the time averaged angular velocity of each gear in (a) the hard gear system with $A = 70$ and (b) the soft gear with $A = 7$ with the coupling T) as a function of torque $F$ working on Gear 1 for $L = 1.8$ and $L = 2$. Here, the time averaged angular velocity is defined as $\lim_{\tau \to \infty} (RN(t + \tau) - RN(t))/\tau$ (RN indicates the no. of rotations of the gear.), and the clockwise direction is defined as positive direction. As in Fig. 2(a), only Gear 1 rotates in the clockwise direction for $F$ larger than a critical value if each gear is hard. On the other hand, if each gear is soft, Gear 1 rotates in the clockwise direction and Gear 2 rotates in the anti-clockwise direction in certain range of $F$ in both cases with $L = 1.8$ and $L = 2$ (Fig. 2(b)). If $F$ is given much larger, only Gear 1 always rotates and rotational energy cannot transmit to any other gears independent of $A$.

Figure 2(c) shows the phase diagrams for the transmission properties of coupled gears as functions of the gear-gear distance $L$ and the hardness of each gear $A$. As found in Fig. 2(c), the energy transmission can realize only in the limited case with large enough $A$ and small enough $L$ in the case P). On the other hand with the coupling T), the energy transmission can realize in a wide range of $L$, if each gear is appropriately soft.

Now, we focus on the force balances of the coupled gears to clarify the mechanisms of the rotational energy transmissions between soft gears. Figure 3 shows the snap shots of the time evolution of (a) hard gears $A = 70$ and (b) soft gears $A = 7$ with $L = 1.8$. In Fig. 3(a), it is
clear that the rotation direction of two teeth around the inner area of the triangle belonging to Gear 2 and 3 and the direction of repulsive force working between them are close. Thus, Gear 2 and Gear 3 cannot rotate if each gear is hard.

If each gear is soft, the relationships between the force directions and the rotational directions change drastically as in Fig. 3(b). In this case, following force relations appear against the torque in the clockwise direction working on Gear 1. I) The tooth of Gear 1 at the inner area of the triangle pushes the left tooth of Gear 3. II) The right tooth of Gear 3, which is pushed by the left tooth through a tooth between the left and right teeth, pushes the bottom tooth in Gear 2. III) The left tooth of Gear 2, which is pushed by the bottom tooth through a tooth between the left and bottom teeth, pushes the top tooth of Gear 1. IV) The direction of the force between the left tooth in Gear 2 and the top tooth in Gear 1 is not close to the opposite direction of the movement of the left tooth in Gear 2 but close to that of the top tooth in Gear 1. Here, the fact I) induces the fact II), and II) induces III). By the facts III) and IV), the left tooth in Gear 2 can move to the inner area of the triangle, and after this, the top tooth in Gear 1 moves to the inner area. Thus, the rotational energy can transmit to Gear 2 (Gear 3) from Gear 1 if the torque in the clockwise (anti-clockwise) direction works on Gear 1.

Next, we focus on the responses of the coupled soft gears with the coupling T) against the random torque working on each particle in Gear 1. Here, we consider the situation where Gear 1 has contact with a heat bath with temperature \( \theta \), and the Gear 2 and Gear 3 have contact with a heat bath with temperature 0. In this situation, as in the followings, the directional motions appear in Gear 2 and Gear 3 in the certain range of \( \theta \), where these two gears rotate in opposite directions with each other.

Figure 4(a) and 4(b) show the typical temporal evolutions of no. of rotations of one of particles in each gear for (a) \( \theta = 0.125 \) and (b) \( \theta = 0.5 \), and Fig. 4(c) shows the time averaged angular velocity of Gear 2 as a function of \( \theta \) for \( A = 7 \) and \( L = 1.8 \). In several \( \theta \), while the motion of Gear 1 looks like Brownian motion, Gear 2 and Gear 3 move in opposite directions in average with each other. Moreover, the directions of each gears rotation change depending on \( \theta \), where Gear 2 (3) rotates in the anti-clockwise (clockwise) direction for small \( \theta \) or the clockwise (anti-clockwise) direction for large \( \theta \). (For much larger \( \theta \), the rotations of Gear 2 and Gear 3 becomes 0.) Thus, this system may play the roles of not only rectifier but also the noise strength detector.

Here, we focus on the detailed motion of the coupled soft gears due to the energy conversions. When \( \theta \) is small, the magnitude of the torque working on Gear 1 is not so large in average. In such cases, as in Fig 2(b), Gear 2 or 3 can rotate with Gear 1. Then, Gear 2 (3) can rotate in the anti-clockwise (clockwise) direction when Gear 1 happens to rotate in the clockwise (anti-clockwise) direction. Thus, Gear 2 (3) rotates in the anti-clockwise (clockwise) direction in average.

As in Fig 2(b), Gear 2 or 3 cannot rotate with Gear 1 if \( F \) is much larger. Similarly, if \( \theta \) becomes large, the detailed motions of Gear 1 affects little to those of Gear 2 and Gear 3 while the random torques continue to work on Gear 2 and Gear 3 by the heat conduction from Gear 1. Then, with the certain \( \theta \), the situation where the interaction between Gear 2 and 3 is still effective may realize. In such cases, when Gear 2 happens to move in the clockwise direction, Gear 3 can also rotate in the anti-clockwise direction, or when Gear 3 happens to move in the anti-clockwise direction, Gear 2 can also rotate in the clockwise direction. In this case, the timing of the rotations of Gear 2 and 3 correlate with each other, which is consistent to as seen in Fig 4(b). Thus, Gear 2 and Gear 3 rotate in the clockwise and anti-clock directions, respectively, in average for large \( \theta \).

In this paper, the transmission and the conversion of the rotational energy by the coupled deformable gears are investigated. If each gear is soft, following dynamics are observed. A) The coupled gears can realize the transmission of rotational energy from one to the other only when three gears are coupled where they form a triangle, while the coupled hard gears can realize the transmission only when two gears are tightly coupled. B) If one of gears have contact with a high temperature heat bath and the other gears have contact with a 0 temperature heat bath with the coupling T), the directional rotations appear in two gears with 0 temperature heat bath. These results give an explicit evidence that the performances and mechanical properties of hard machines and soft machines are very different.

For each case, there are some kinds of merits and demerits. In general, the tightly coupled two hard gears are expected to realize the transmissions in wide range of the rotational energy. However, if the distance between two gears becomes a little long, these systems cannot realize the transmissions suddenly which means this system is weak for fluctuations. On the other hand, in the coupled soft gears forming a triangle, the range of the rotational energy which can be transmitted from one to others is limited. In this case, however, the such transmissions can occur almost independently of the distances between gears. This fact expects that the coupled soft gears realize such a transmission robustly against the fluctuations. This fact implies the soft gears are useful for making micro or nano scale machines. This fact may also imply that the softness of biological molecular machines are suitable to work in large influences of fluctuations.

Analytical study of the presented system, as well as the discoveries of possible dynamics and realization of functions of more varieties in models of soft machines should be important as future issues.
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[1] B. Alberts, et. al., MOLECULAR BIOLOGY OF THE CELL 4th edition (Garland Science, New York 2002)
[2] K. Kitamura, M. Tokunaga, A. H. Iwane, and T. Yanagida, Nature 397 (1999) 129.
[3] P. C. Holdsworth, M. J. P. Gingras, B. Bergersen and E. P. Chan, J. Phys. Cond. Mat. 3 (1991) 6679.
[4] J. -C. Bacri, A. Cebers and R. Perzynski, Phys. Rev. Lett. 72 (1994) 2705.
[5] C. Renner, H. Löwen and L. Barrat, H. W. Muller, Phys. Rev. E52 (1995) 5091.
[6] H. W. Muller, Phys. Rev. Lett. 82 (1999) 3907.
[7] A. Engel and P. Reimann, Phys. Rev. E70 (2004) 051107.
[8] Y. Kuramoto, Chemical Oscillations, Waves and Turbulence (Springer, Berlin, 1984).
[9] H. Daido, Phys. Rev. Lett. 68 (1992) 1073.
[10] A. Awazu, Physica D 178 (2003) 19.
[11] In cases $V(r) = (1/r)^n$ with $n > 1$, we can obtain the similar results as obtained in this paper.

FIG. 1: Typical temporal evolutions of the system consists of (a) $A = 70$ with the coupling $P$), (b) $A = 70$ with the coupling $T$), (c) $A = 7$ with the coupling $P$) and (d) $A = 7$ with the coupling $T$) ($L=1.8$). Dashed arrows indicate the direction of the torque $F$, and circles are just markers.
A. Awazu: Figure 2

FIG. 2: Time averaged angular velocity of each gear in the systems with (a) $A = 70$ and (b) $A = 7$ for $L = 1.8$ and $L = 2$. (c) Phase diagram of the system for the rotational energy transmissions.

A. Awazu: Figure 3

FIG. 3: Force Relationships between gears.
FIG. 4: Typical temporal evolutions of no. of rotations of a particle in each gear for (a) $\theta = 0.125$ and (b) $\theta = 0.5$, and (c) Time averaged angular velocity of Gear 2 and Gear 3 as a function of $\theta$ for $A = 7$ and $L = 1.8$. 

A. Awazu: Figure 4