Study on Spatial Configuration of Operating Pipe String in Wellbore

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Abstract. The buckling of operation pipe string may occur under complex load in deep well. The serious buckling of the string will lead to partial wear, break or even self-locking, which is detrimental to the downhole operation. At present, most of the studies have focused on the string buckling stability and characteristics of single buckling pattern, and the spatial configuration of entire operation pipe string was seldom studied. In fact, the entire operation pipe string of deep well under complex loads may present a complex configuration consisted of non-buckling, sinusoidal buckling and helical buckling. In this study, the smooth connection condition of different deformation states of pipe string is proposed, and combining the buckling stability analysis and boundary influence characteristics, the spatial configuration of the entire operation pipe string in the borehole could be solved. A field example was given, and the variation of the spatial configuration of the entire operating string with injection pressure is analyzed in detail. The results showed that the calculation method proposed is very useful and important to determine the operation parameters in-situ, to know the working state of the downhole operation string. This works is helpful to avoid the complex situation or accidents in completion practices.

Keywords. Operation pipe string, spatial configuration, buckling, continuity condition, deep-well.

1. Introduction
The operating pipe with the upper fixed end and the lower end packer setting will generate spatial flexural deformation under high temperature and high-pressure operating environment. Due to the constraints of the shaft wall, the operating pipe with the characteristics of super length and fineness ratio may arise sinusoidal or spiral buckling in the wellbore. At present, there have been a lot of achievements in the study of unstable buckling of the operating pipe. In the early stage, Lubinski [1], Dawson and Paslay [2-3] gave the critical buckling load of the string in the inclined wellbore:

\[ F_{crs} = 2 \sqrt{\frac{EIq \sin \varphi}{r_0}} \] (1)

where \( E \) is the modulus of elasticity, Pa; \( I \) is the moment of inertia, \( m^4 \); \( q \) is the weight per unit length of the pipe, N/m; \( r_0 \) is the clearance between the pipe and the well bore, m; and \( \varphi \) is the inclination angle of the wellbore, rad.

For the curved borehole, the critical buckling load in the build-up section can be expressed as [4]
\[ F_{crs} = \frac{2EIk_0}{r_0} + 2\sqrt{\left(\frac{Elk_0}{r_0}\right)^2 + \frac{Elq \sin \varphi}{r_0}} \]  

(2)

And in the drop off interval, the critical buckling load was

\[
\begin{align*}
F_{crs} &= \frac{2EIk_0}{r_0} - 2\sqrt{\left(\frac{Elk_0}{r_0}\right)^2 + \frac{Elq \sin \varphi}{r_0}} \quad (k_0 \geq k_{test}) \\
F_{crs} &= -\frac{2EIk_0}{r_0} + 2\sqrt{\left(\frac{Elk_0}{r_0}\right)^2 + \frac{Elq \sin \varphi}{r_0}} \quad (k_0 < k_{test})
\end{align*}
\]

(3)

Where \( k_0 \) is the curvature, m\(^{-1}\), \( k_{test} = \sqrt{\frac{r_0q \sin \varphi}{El}} \).

Mitchell [5] proposed that the string will go into helical buckling when the axial compressive force exceeds 2.828 times its critical buckling load. And when the axial compressive force is less than 1.414 times of the critical buckling load, the string will change to sinusoidal buckling on unloading from a helical buckled state.

\[ F_{crh} = 2.828F_{crs} \]  

(4)

\[ F_{crh} = 1.414F_{crs} \]  

(5)

Then, in 2002, Mitchell [6-7] used assumed displacement functions to simulate the buckling state of drill string in vertical and horizontal wells without nonlinear terms. In 2005, Vaz and Mascaro [8] studied the buckling behavior of a slender rod consider self-weight with two hinged ends, and numerically solved the post-buckling configuration. In 2016, Nabil, Valery et al. [9] studied the sinusoidal buckling configuration of the super slender string in the inclined wellbore, and the gravity and friction force of the string were taken into consider. But due to the complexity of the buckling problem of the string in the actual wellbore, the current research was mostly limited to the analysis of the single buckling pattern of the downhole string. There is little research on the coexistence of different buckling states or the three-dimensional configuration of the entire operating pipe string in the wellbore, and this is not satisfied to the practical engineering.

In this study, on the basis of the mechanics characteristic of the operating pipe, considering the buckling mode and critical load in different type of wellbore, the transition conditions of different three-dimensional buckling configuration are proposed. And the spatial configuration of the entire operating string in wellbore are calculated.

2. The Different Buckling Mode of Pipe String in Wellbore
The buckling behavior of the compression section operating pipe in the wellbore includes sinusoidal buckling and helical buckling, as shown in figure 1.
It is assumed that the operating pipe string is close to the borehole wall while the sinusoidal or helical buckling is taken place in wellbore. Based on the theory of elastic stability, the buckling equation of the operation pipe in the curved borehole can be deduced according to the equilibrium equation of microelement, constitutive equation and geometric equation.

\[
\frac{d^4 \theta}{dx^4} - \left[ 6 \left( \frac{d \theta}{dx} \right)^2 - \frac{F_x}{EI} \right] \frac{d^2 \theta}{dx^2} + 3 \frac{M_t}{EI} \frac{d \theta}{dx} \frac{d^2 \theta}{dx^2} + \frac{q \sin \varphi}{EI r_0} \sin \theta = 0
\]

(6)

Where, \( \theta \) is the deflection angle of the section of the pipe, rad; \( M_t \) is the torque on the string, N·m; \( \varphi \) is the deviation angle, rad; \( F_x \) is the axial force on the string, N.

2.1. Sinusoidal and Helical Buckling Configuration

Equation (6) could be solved by using the perturbation method, and the solution corresponding to the sinusoidal buckling of string is [11]:

\[
\theta = \frac{8(1 - Q_1)}{12 - Q_1} \sin \eta
\]

(7)

Where, \( Q_1 = \frac{Q_0 \sin \varphi}{(1 - \varepsilon \xi \cos \varphi)^3} \), \( \eta = \frac{2}{3 \varepsilon \cos \varphi} \left[ 1 - \left(1 - \varepsilon \xi \cos \varphi \right)^{3/2} \right] \), \( Q_0 = \frac{q}{EI r_0 \omega^4} \), \( \xi = \omega x \), \( \omega = \sqrt{\frac{F_x}{2EI}} \), \( \varepsilon \) is a small quantity (that is, the axial load change caused by the weight of the string is very slow), preferably 0.01.

The solution of helical buckling is:

\[
\theta = \pm \left[ \eta - \frac{1}{5} Q_1 \sin \eta - \frac{7}{1600} \xi^2 \sin 2 \eta \right]
\]

(8)

When \( F_x \) and \( \varphi \) are given, the sinusoidal and helical shape of pipe string in the wellbore could be described by \( \theta(x) \) and \( r_0 \).
2.2. The Lateral Configuration of the Part of the Pipe String near the End

For the super long operation pipe string, the end constraint only affects the configuration of the pipe string within a certain range nearby, and for most part of the string away from the end, the influence of the end can be ignored [12]. Therefore, the analysis of string buckling always neglects the configuration of the string close to the packer end. However, for calculating the three-dimensional configuration of the entire operation pipe string, the configuration close to the ends cannot be ignored. At that time, the hypothesis that the pipe string is close to the borehole wall is no longer applicable, and the transverse deformation of the pipe string in the incomplete buckling segment near the packer end can be expressed as [12]:

\[ y = r_0 \left[ \frac{\cos(\beta L_e) - 1}{\cos(\beta L_e)} \right] + \frac{\sin \beta L_e \left[ \sin (\beta x_e) - \beta x_e \right]}{\left[ \cos(\beta L_e) - 1 \right]^2 + \sin (\beta L_e) / \sin(\beta L_e) - \beta L_e} \]

\[ z = \frac{r_0 \chi}{\beta} \left[ \frac{\cos(\beta L_e) - 1}{\cos(\beta L_e)} \right] + \frac{\sin \beta L_e \left[ 1 - \cos (\beta x_e) \right]}{\left[ \cos(\beta L_e) - 1 \right]^2 + \sin (\beta L_e) / \sin(\beta L_e) - \beta L_e} \]

where, \( \beta = \sqrt{\frac{F_{R_{e}}}{EI}} \), \( x_i = L - x \), \( L \) is the total length of pipe string; \( 0 < x_i \leq L_e \); \( L_e = 4.4934 \frac{F_{R_{e}}}{\sqrt{EI}} \) is the length of incomplete buckling section affected by end; \( \chi = 0.4225 \beta \).

\( F_{R_{e}} \) is the axial load at the packer, N.

\( \sqrt{\frac{F_{R_{e}}}{EI}} \) is the axial load at the packer, N.

For the unity of the description form, equation (9) ~ (10) can be converted to parametric equations about radial displacement \( r_e \) and rotation angle \( \theta_e \).

\[ \begin{align*}
    r_e & = \sqrt{y^2 + z^2} \\
    \theta_e & = \arccos \left( \frac{y}{r_e} \right)
\end{align*} \]

(11)

3. Calculation Method of the Spatial Configurations of Entire Operation Pipe Strings in Wellbore

In practical engineering, the spatial configuration of the entire well operating pipe string is a continuous and smooth spatial curve. The string in the upper section of the well is generally in tension and no buckling, the sinusoidal or helical buckling will occur in the section below neutral point of string when the axial compressive force exceeds the critical buckling load, and the string will occur local spatial deformation near the bottom packer. For different deformation configuration of different section, the modal function is different. In order to form a smooth continuous curve, it is necessary to satisfy the existence and continuity of the first derivative of the function of radial displacement and rotation angle at the points of junction of different pattern of the pipe string, that is, when the spatial configuration of the entire operation pipe string is formed, the non-buckling section, sinusoidal buckling section and spiral buckling section should be connected smoothly according to the continuity condition

\[ \begin{align*}
    r_i (l_i^0) & = r_{i+1} (l_{i+1}^0) \\
    \theta_i (l_i^0) & = \theta_{i+1} (l_{i+1}^0)
\end{align*} \]

(12)
The buckling characteristics of entire operation pipe string at different positions are determined by the criterion of string buckling, and the range of rotation angle can be determined by continuous conditions. The spatial configuration of the entire operation pipe string in the wellbore can be described by the standard spatial rectangular coordinate expression, which are converted from the parameter equation about radial displacement $r$ and rotation angle $\theta$

\[
\begin{align*}
  x &= x \\
  y &= r \sin \theta \\
  z &= r \cos \theta
\end{align*}
\]  

Where the $r$ and $\theta$ had been revised by equation (12)-(13).

In order to obtain accurate three-dimensional configuration of the entire operation pipe string in wellbore, it is necessary to consider the coupling between axial load and the length decrement of the entire pipe string, $\Delta L_b$, caused by buckling compression. The coupling can be solved by numerical iterative calculation method. First, the axial load of the initial state string is calculated, and the buckling state of the string is judged, then the axial force of the initial state string is corrected by calculating the additional load produced by the buckling of the string. Then, the buckling pattern of the string is judged again, the additional load is calculated and the axial force is corrected again, so the cycle is iterative until the buckling pattern is stable, that is, when the additional load tends to be a constant, the iterative cycle is stopped and the final calculation results are obtained.

The calculation formula of $\Delta L_b$ is

\[\Delta L_b = \Delta L_{\text{sin}} + \Delta L_{\text{hel}}\]  

Where, $\Delta L_{\text{sin}}$, $\Delta L_{\text{hel}}$ denote the axial deformation caused by the sinusoidal and helical buckling, respectively [12]:

\[\Delta L_{\text{sin}} = \int_0^{L_{\text{sin}}} 0.0843 \frac{F(x)}{2EI} r_0^2 dx\]  

\[\Delta L_{\text{hel}} = \int_0^{L_{\text{hel}}} 0.5026 \frac{F(x)}{2EI} r_0^2 dx\]  

Where $L_{\text{sin}}$ and $L_{\text{hel}}$ is the length of sinusoidal buckling section and helical buckling section, respectively. Therefore, the additional axial force can be calculated by Hooke's law according to the compression decrement.

4. Example

RP8-3 is a deep exploration well in western China, and the tubing fracture operation parameters are listed in Table 1. The fracture pressure fluctuation curve is shown in Figure 2. The internal pressure reaches 83.00 MPa and the annulus pressure are relatively stable in 15.00 MPa. Using the iterative
calculation method and equation (1) ~ (5), the axial force and critical buckling load of tubing could be obtained, as shown in figure 3.

Table 1. Basic parameters of RP8-3 well.

| Parameter                        | Value  | Parameter                        | Value  |
|----------------------------------|--------|----------------------------------|--------|
| Depth (m)                        | 6775.00| Fluid density outside the tubing (kg/m³) | 1400.00 |
| Inner diameter of casing (mm)    | 178.19 | Fluid density in the tubing (kg/m³)  | 1100.00 |
| Outer diameter of tubing (mm)    | 88.90  | Unit weigh of tubing (N/m)         | 134.16 |
| Inner diameter of tubing (mm)    | 76.00  | Installation depth of packer (m)   | 6665.00 |

![Figure 2. Pressure curve of fracturing operation of Well RP8-3.](image)

From figure 3 we can find that, under the action of internal and external pressure difference, the tubing string buckling will occur below the depth 6260.64 m. The total length of the buckling section is 392.65 m, in which the sinusoidal buckling section is 215.47 m, the helical buckling section is 177.18 m, and the influence length of the packer is 11.71 m.

Then, using the method proposed in this study, the spatial configuration of the entire tubing string can be calculated, as shown in figure 4.

![Figure 3. Axial force and critical buckling force.](image)
Figure 4. Spatial configuration of the tubing under fracturing operation.

Figure 5 listed the different spatial configurations of tubing corresponding to the injection pressure of 60.00 MPa, 70.00 MPa, 80.00 MPa, 90.00 MPa, and 100.00 MPa, respectively. The length of different buckling pattern section of the tubing was shown in table 2. From figure 5 and table 2, it can be seen that the buckling occurred under all five injection pressures, including sinusoidal buckling and helical buckling. With the increase of injection pressure from 60.00 MPa to 100.00 MPa, the length of sinusoidal buckling section varied from 2.56 m to 215.87 m, and the length of helical buckling section increases to 610.97 m from zero. The variation of different section length with injection pressure is shown in figure 6, and we can see that a greater injection pressure would lead to a longer buckling length, especially a rapid increase of helical buckling length.

Figure 5. Spatial configuration of tubing string in wellbore under different injection pressures.
### Table 2. Length of different section of the tubing string under different injection pressures.

| Pressure/MPa | Parameter/m | Not buckling section | Sinusoidal buckling section | Helical buckling section | End influence segment |
|--------------|-------------|-----------------------|-----------------------------|--------------------------|-----------------------|
| 60.00        |             | 6647.01               | 2.56                        | 0                        | 15.43                 |
| 70.00        |             | 6602.04               | 48.22                       | 0                        | 14.74                 |
| 80.00        |             | 6323.21               | 215.87                      | 113.79                   | 12.13                 |
| 90.00        |             | 6080.35               | 206.88                      | 367.09                   | 10.68                 |
| 100.00       |             | 5828.50               | 215.87                      | 610.97                   | 9.66                  |

![Figure 6. Variation of different buckling section length respect to different injection pressures.](image)

#### 5. Conclusion
In order to accurately calculate the spatial configuration of the entire operation pipe string in a wellbore, the criteria of buckling stability in the actual wellbore and the smooth connection condition of different buckling pattern of pipe string, as well as the coupling effect of buckling deformation and axial load should be taken into account.

In fracturing operation, the injection pressure has great influence on the buckling configuration of the packer operating string. The greater the injection pressure is, the more severe the buckling is, and the faster the spiral buckling length increases.

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