QUASILOCAL CENTER-OF-MASS FOR TELEPARALLEL GRAVITY

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Asymptotically flat gravitating systems have 10 conserved quantities, which lack proper local densities. It has been hoped that the teleparallel equivalent of Einstein’s GR (TEGR, aka GR\text|\|^) could solve this gravitational energy-momentum localization problem. Meanwhile a new idea: quasilocal quantities, has come into favor. The earlier quasilocal investigations focused on energy-momentum. Recently we considered quasilocal angular momentum for the teleparallel theory and found that the popular expression (unlike our “covariant-symplectic” one) gives the correct result only in a certain frame. We now report that the center-of-mass moment, which has largely been neglected, gives an even stronger requirement. We found (independent of the frame gauge) that our “covariant symplectic” Hamiltonian-boundary-term quasilocal expression succeeds for all the quasilocal quantities, while the usual expression cannot give the desired center-of-mass moment. We also conclude, contrary to hopes, that the teleparallel formulation appears to have no advantage over GR with regard to localization.

1. Introduction and Overview

Associated with the symmetries of flat Minkowski spacetime are 10 conserved quantities: energy-momentum, angular momentum and the (often overlooked) center-of-mass moment. Asymptotically flat gravitating systems also have these 10 conserved quantities. The total quantities for the whole space are well defined. However, unlike the situation for all the other matter and interaction fields which constitute its source, the gravitational field itself lacks proper local densities for these quantities. This can be understood in terms of the equivalence principle: the gravitational field cannot be detected at a point. The localization of gravitational energy-momentum still remains one of the outstanding problems in classical gravity theory.

In 1961 Møller proposed a tetrad-teleparallel\(^1\) equivalent of Einstein’s GR (TEGR, aka GR\text|\|^) as a way to solve this problem; this promising approach is still being pursued\(^2\text-}\text{5}\).
More recently a new idea, quasilocal quantities\(^7\) (see Brown and York’s seminal paper\(^8\) for references to the early work), has come into favor. Our approach has been to develop a **covariant Hamiltonian formalism**. In Hamiltonian approaches quasilocal quantities are associated with the Hamiltonian boundary term. This boundary term has, at least formally, considerable freedom, which can be related to the choice of boundary conditions. Certain principles can then be used to select “good” boundary conditions. In particular we have advocated a simple, appropriate and reasonable choice to limit this freedom: we found there are only **two covariant-symplectic** choices for each dynamic field\(^9\),\(^10\),\(^11\),\(^12\),\(^13\).

Here we consider the various proposed Hamiltonian boundary terms for the quasilocal quantities of tetrad-teleparallel gravity. Earlier investigations treated only energy-momentum\(^2\),\(^4\),\(^5\),\(^3\). Then Vu\(^14\) considered angular momentum; he found that the popular expression (unlike our “covariant-symplectic” one) gives the correct result only in a certain frame. More recent TEGR angular momentum results\(^6\) are consistent with this conclusion.

The long neglected center-of-mass moment, however, offers the most restrictive requirements, not only on the allowed asymptotic behavior of the variables\(^15\),\(^16\),\(^17\) but also on the acceptable form of the expressions. We tested the various tetrad-teleparallel Hamiltonian boundary quasilocal expressions on the asymptotic eccentric Schwarzschild solution. We found (frame gauge independently) that our “covariant symplectic” Hamiltonian-boundary-term teleparallel quasilocal expression succeeds for all of the quantities, while the usual tetrad expression does not give the desired center-of-mass moment\(^18\).

It turns out that one of our “good” teleparallel expressions is equivalent to a standard “covariant-symplectic” quasilocal Riemannian GR expression\(^10\),\(^13\) (the others are asymptotically equivalent). We conclude that, for localization purposes, GR\(\parallel\) is **not** better than GR. (This assessment would be revised if a good gauge condition for teleparallel GR is established.)

### 2. Background

In *Newtonian physics* Galilean symmetry leads to the associated conserved quantities: energy, momentum, angular momentum (AM) and the (often overlooked) center-of-mass moment (COM). (The latter is associated with the lack of a preferred inertial reference frame.) Likewise in *special relativity*, Poincaré symmetry leads to conserved quantities for particles: spacetime translations are associated with energy-momentum, \(P_\mu\), and Lorentz transformations (including both boosts and rotations) are associated with 4-dimensional angular momentum: \(L^{\mu\nu} := x^\mu P^\nu - x^\nu P^\mu\), which inseparably includes the covariant partners: angular momentum, \(L^i = x^i P^j - x^j P^i\), and the center-of-mass moment,

\[
L^{0k} = x^0 P^k - x^k P^0 \equiv ct P^k - x^k E/c. \tag{1}
\]
In classical field theory, Noether’s theorem gives conserved current local densities for energy-momentum (EM) and AM/COM; the latter also includes, in general, a spin density term: \( J_{\mu\nu} = L_{\mu\nu} + S_{\mu\nu} \).

3. Gravitational Energy-Momentum and its Localization

A basic notion of energy-momentum is the Noether conserved quantity associated with the spacetime translation symmetry. This idea works well enough in flat space—aside from the usual conserved current ambiguity of adding to the density a quantity with identically vanishing divergence. The case of conserved quantities for gravitating systems, however, is quite different. It is well known that asymptotically flat spacetimes have well defined total conserved quantities, whereas localization is problematical.

Note that the source of gravity is the energy-momentum density for matter and for all other interaction fields. Via this relation gravity absolutely identifies the source energy-momentum density, removing thereby the uncertainty in the conserved Noether translational current. It seems rather ironical that the energy-momentum density for gravity itself is not so sharply defined.

Energy-momentum should be conserved. In view of the fact that it is exchanged locally between the sources and gravity, investigators sought some kind of local description of energy-momentum for the gravitational field itself. Standard arguments (e.g., Noether symmetry), however, lead only to energy-momentum densities which are reference frame dependent pseudotensors. It seems that the gravitational field (hence any gravitating system, and consequently every physical system) has no proper energy-momentum density (similarly there is no proper angular momentum/center-of-mass density). This can be understood in terms of the equivalence principle: gravity cannot be detected at a point.

4. The Tetrad-Teleparallel Approach

In 1961 Møller proposed a certain reformulation of Einstein’s GR as a means of solving this localization problem. On the one hand it can be regarded simply as GR expressed in terms of an orthonormal frame. Alternately it can be described in terms of a different framework: teleparallel geometry (aka absolute parallel, Weitzenböck geometry). It then has a different connection, a different parallel transport, which is a kind of opposite to the Riemannian one: torsion is generally non-zero while curvature vanishes (hence parallel transport is path independent, but non-trivial). Common names for this theory are TEGR (the teleparallel equivalent of GR) and GR||. A major source of interest is that the energy-momentum density for this theory is determined by an object which is tensorial (unlike the usual GR description). More precisely it is a tensor under coordinate transformations—however it does depend on the Lorentz gauge choice for the orthonormal frame. Hence to fix the localization a frame gauge condition is necessary. Although, as far as we know, no satisfactory
gauge condition has yet been recognized, this tetrad-teleparallel approach is still regarded as promising and has been under active investigation.

5. Quasilocal Energy-Momentum

With regard to the energy-momentum localization “problem”, it is now widely believed that the proper idea is not local but rather quasilocal quantities (i.e., quantities associated with a closed 2-surface).

5.1. Covariant Hamiltonian Approach

One approach to energy-momentum is via the Hamiltonian. Energy can be regarded as the value of the Hamiltonian. The traditional Hamiltonian techniques have certain virtues but they exact a price: they are not manifestly covariant. We have developed an alternate covariant Hamiltonian formalism, which naturally yields manifestly 4-covariant expressions. Here we briefly review the key features.

We start with a first order Lagrangian for an f-form field,

$$\mathcal{L} = d\varphi \wedge p - \Lambda(\varphi, p), \quad (2)$$

(independent variation with respect to $\varphi$ and $P$ yields pairs of first order equations). From it we construct the Hamiltonian 3-form (density)

$$\mathcal{H}(N) = \mathcal{L}_N \varphi \wedge p - i_N \mathcal{L}, \quad (3)$$

which is conserved ($d \mathcal{H}(N) = 0$ “on shell”). Consequently its spatial integral yields a conserved quantity for each choice of displacement $N$. A short calculation shows that the Hamiltonian 3-form can be expressed in the form

$$\mathcal{H}(N) = N^\mu \mathcal{H}_\mu + dB(N). \quad (4)$$

For a (finite or infinite) spatial region, the Hamiltonian—the integral of this 3-form over a spacelike hypersurface—is the generator of field displacements along the vector field $N$. The total differential term (via the generalized Stokes theorem) yields a flux integral over the closed 2-surface boundary.

For all locally diffeomorphic invariant theories the spatial surface density $\mathcal{H}_\mu$ is proportional to field equations; its value vanishes “on shell”. Hence the (conserved) value of the Hamiltonian,

$$H(N) := \int_\Sigma \mathcal{H}(N) = \int_\Sigma N^\mu \mathcal{H}_\mu + \oint_{\partial \Sigma} B(N) \approx \oint_{\partial \Sigma} B(N), \quad (5)$$

is quasilocal, being determined by the boundary term $B(N)$.

In order to have a proper Hamiltonian for the desired phase space variables, the boundary term inherited from the Lagrangian can (and indeed in general must) be adjusted (this was first nicely explained for GR by Regge and Teitelboim since then the arguments have been refined). Note that the boundary term, and
hence the value of the Hamiltonian quasilocal quantities, can be adjusted without changing the field equations or the conservation property. The freedom in the Hamiltonian boundary term is linked (via the symplectic structure of the boundary term in the Hamiltonian variation) with the freedom of choice of boundary conditions.

We found that for each dynamic field the boundary term naturally inherited from the Lagrangian, $B(N) = i_N \varphi \wedge p$, has only two alternate replacements which have “covariant-symplectic” Hamiltonian boundary variation terms. They are

\begin{align}
B_\varphi(N) := i_N \varphi \wedge \Delta p - \varepsilon \Delta \varphi \wedge i_N \overset{\circ}{\varphi}, \\
B_p(N) := i_N \overset{\circ}{\varphi} \wedge \Delta p - \varepsilon \Delta \varphi \wedge i_N \overset{\circ}{p},
\end{align}

where $\varepsilon = (-1)^f$, $\overset{\circ}{\varphi}$ and $\overset{\circ}{p}$ are the values in a reference configuration, and $\Delta \varphi := \varphi - \overset{\circ}{\varphi}$, $\Delta p := p - \overset{\circ}{p}$. These boundary expressions, respectively, correspond to Dirichlet and Neumann boundary conditions. Asymptotically, at spatial infinity, both give the same values. For our considerations here we need only consider one.

6. Tetrad Gravity

By tetrad gravity we mean theories where the only dynamic variable is the frame (sometimes referred to as a tetrad or vierbein). Technically for our purposes it is most convenient to work with the coframe $\vartheta^\mu$ (i.e. the basis one-forms, dual to the vector basis $e_\alpha$). Here we shall confine our attention to orthonormal frames. (The metric is then given by $g = g_{\alpha\beta} \vartheta^\alpha \otimes \vartheta^\beta$ with the orthonormal metric components $g_{\mu\nu} = \text{diag}(-1, +1, +1, +1)$).

Any tetrad theory can be derived from a first order Lagrangian of the form

\begin{equation}
L_{\text{tet}} = d\vartheta^\mu \wedge \tau_\mu - \Lambda_{\text{tet}}(\vartheta, \tau).
\end{equation}

The orthodox choice for $\Lambda_{\text{tet}}$ is an expression quadratic in $\tau$; then $\tau$ and $d\vartheta$ are linearly related. For a certain choice, $\Lambda_{\text{tet}} = \Lambda_{\text{GRtet}}$ (the specific expression is not needed here, a neat version with an extra field is given in Eq. (21) below), this gives GR in terms of a tetrad. It should be remarked that our formalism here applies to all tetrad theories. The generic tetrad theory is a theory for a preferred orthonormal frame with no local Lorentz gauge freedom. Only one special (albeit highly interesting) case—the tetrad version of GR—has local Lorentz gauge freedom.

The Hamiltonian boundary term obtained from the tetrad Lagrangian is

\begin{equation}
B_{\text{tet}}(N) = i_N \vartheta^\mu \tau_\mu = N^\mu \tau_\mu.
\end{equation}

We also consider one of our “covariant-symplectic” alternatives:

\begin{equation}
B_{\vartheta}(N) := i_N \vartheta^\mu \Delta \tau_\mu + \Delta \vartheta^\mu \wedge i_N \overset{\circ}{\tau}_\mu.
\end{equation}
This is just a slight refinement, reducing to (9) if the reference values are trivial (a natural choice).

Note in particular that the boundary terms given above have the same form for all tetrad theories, independent of the specific form of $\Lambda_{tet}$. It is worth recalling here that GR, in its Riemannian formulation, has many proposed energy-momentum expressions; there is no consensus as to which is the best. In contrast, tetrad investigators1–6 basically agree on the expression (9). The general tetrad theory has an essentially unique energy-momentum flux expression, which naturally applies to the special case GRtet.

7. Metric-Compatible Gravity

We now take a more geometric approach. We start with the general class of geometries with a metric-compatible connection22,23. One of our basic variables is the orthonormal coframe $\theta^\alpha$. The other basic variable is an a priori independent metric-compatible connection one-form: $\Gamma^{\alpha\beta} = \Gamma^{[\alpha\beta]}$. These “potentials” determine the curvature 2-form:

$$ R^{\alpha\beta} := d\Gamma^{\alpha\beta} + \Gamma^{\alpha\gamma} \wedge \Gamma^{\gamma\beta}, \quad (11) $$

and the torsion 2-form:

$$ T^{\alpha} := D\theta^\alpha := d\theta^\alpha + \Gamma^{\alpha\beta} \wedge \theta^\beta. \quad (12) $$

The general metric-compatible geometric first order Lagrangian has the form

$$ \mathcal{L}_{mc} = D\theta^\mu \wedge \tau_\mu + R^{\alpha\beta} \wedge \rho_{\alpha\beta} - \Lambda_{mc}(\theta, \tau, \rho); \quad (13) $$

one of the associated “covariant-symplectic” boundary terms is

$$ B_{mc}(N) = B_\theta(N) + \Delta^{\alpha\beta} \wedge i_N \rho_{\alpha\beta} + D_\beta N^\alpha \Delta \rho_{\alpha\beta}. \quad (14) $$

The first order field equations are obtained by independent variation with respect to $\theta$, $\tau$, $\Gamma$, and $\rho$. Extra Lagrange multiplier fields, as we shall see, can be included to get the various special types of geometries.

7.1. Riemannian General Relativity

Einstein’s general relativity, in its orthodox Riemannian representation, can be obtained from the choice

$$ \Lambda_{mc} = \Lambda_{GR} := V^{\alpha\beta} \wedge (\rho_{\alpha\beta} - \eta_{\alpha\beta}). \quad (15) $$

(Here we used Trautman’s dual basis: $\eta^{\alpha\beta} := * (\theta^\alpha \wedge \theta^\beta \wedge \cdots)$, which is sometimes convenient.) The quantity $V^{\alpha\beta}$ is a Lagrange multiplier field; its variation yields $\rho_{\alpha\beta} = \eta_{\alpha\beta}$. Variation of (15), with the specification (15), with respect to $\tau_\mu$ (since $\Lambda_{GR}$ is independent of $\tau_\mu$) simply yields the Riemannian connection’s vanishing
torsion constraint: \( D\vartheta^\mu = 0 \). One consequence is that \( D\eta_{\alpha\beta} = D\vartheta^\gamma \land \eta_{\alpha\beta\gamma} = 0 \); from which it follows that the (vacuum) \( \delta\Gamma^{\alpha\beta} \) equation,
\[
\vartheta_{[\beta} \land \tau_{\alpha]} + D\eta_{\alpha\beta} = 0,
\]
yields \( \tau_\mu = 0 \). The general Hamiltonian boundary term (14) reduces then to
\[
B_{GR}(N) = \Delta\Gamma^{\alpha\beta} \land iN\eta^{\alpha\beta} + \partial_\beta N^\alpha \Delta\eta_{\alpha\beta}.
\]
This quasilocal expression was independently found from a different perspective.24

Asymptotically it agrees with accepted expressions for energy-momentum, angular momentum and the center-of-mass moment.15,16,17,20,25

7.2. Teleparallel Gravity and GR\( _\parallel \)

For teleparallel theories, in addition to a tetrad, we introduce a new (metric compatible but not symmetric) connection, \( \bar{D} \), and force it to be teleparallel. This is accomplished by simply choosing the first order Lagrangian potential to be independent of \( \rho \). (The potential \( \Lambda \) is then formally like that for a tetrad theory.)

\[
\mathcal{L}_\parallel = \bar{D}\vartheta^\mu \land \tau_\mu + \bar{R}^{\alpha\beta} \land \rho_{\alpha\beta} - \Lambda_\parallel(\vartheta, \tau).
\]

Now variation with respect to \( \rho \) yields the teleparallel condition \( \bar{R}^{\alpha\beta} = 0 \).

Since the curvature vanishes, parallel transport is path independent. Hence one can choose a frame at one point and then uniquely transport it to every other point. This preferred orthoteleparallel (OT) frame is unique up to global (i.e. rigid, constant) Lorentz transformations. (Note: teleparallel physics has a preferred frame, it does not have local Lorentz frame invariance.) In the OT frame the connection coefficients vanish, consequently the equations reduce to the tetrad form. However, the quasilocal expressions do not reduce to the tetrad form.

Comparing the Hamiltonian boundary term (14) with that of the tetrad case (9), we note extra terms involving \( \rho \), the connection’s canonically conjugate momentum field. We now show that for teleparallel theories this field generally cannot be made to vanish, unlike the connection in an OT frame. The quantity \( \rho \) must satisfy the equation obtained by variation with respect to \( \bar{\Gamma} \):
\[
\vartheta_{[\beta} \land \tau_{\alpha]} + \bar{D}\rho_{\alpha\beta} = 0.
\]
From an orthodox \( \Lambda_\parallel \), quadratic in \( \tau \), the \( \delta\tau \) equation gives a \( \tau \) linear in \( \bar{D}\vartheta \); hence for teleparallel theories, \( \tau \) is generally non-vanishing. Consequently (19) shows that \( \rho \) is generally non-vanishing. Thus, although there is always a frame in which the teleparallel connection coefficients vanish, the connection still indirectly makes an important contribution through its non-vanishing conjugate momentum field. The relation (19) also shows another surprising feature: for all teleparallel theories \( \rho \) has the gauge freedom
\[
\rho_{\alpha\beta} \rightarrow \rho_{\alpha\beta} + \bar{D}\chi_{\alpha\beta},
\]
since $D^2 \chi \simeq \bar{R} \wedge \chi = 0$.

In order to obtain $\text{GR}_{||}$, the teleparallel equivalent of GR, we may take

$$\Lambda_{\text{GR}_{||}} = V^\mu \wedge (\tau_\mu - \kappa^\alpha_\beta \wedge \eta_\alpha_\beta_\mu) - \kappa^\alpha_\gamma \wedge \kappa^\gamma_\beta \wedge \eta_\alpha_\beta,$$

where the auxiliary one-form field $\kappa$ is used like a Lagrange multiplier. Variation with respect to $\kappa$ yields

$$V^\mu \wedge \eta_\alpha_\beta_\mu + \kappa^\lambda_\alpha \wedge \eta_\lambda_\beta + \kappa^\lambda_\beta \wedge \eta_\alpha_\lambda = 0.$$  (22)

Incorporating the $\tau$ variation result, $V^\mu = \bar{D} \theta^\mu$, the first term becomes $\bar{D} \eta_\alpha_\beta$. Consequently we infer that the tensorial one-form $\kappa$ is just $\kappa^\alpha_\beta = \Gamma^\alpha_\beta - \bar{\Gamma}^\alpha_\beta$: the difference between the Levi-Civita connection coefficients (which can be thought of as a certain function of the teleparallel variables) and the teleparallel connection coefficients. Variation with respect to $V^\mu$ gives

$$\tau_\mu = \kappa^\alpha_\beta \wedge \eta_\alpha_\beta_\mu.$$  (23)

We then find a solution to (19): $\rho_\alpha_\beta = \eta_\alpha_\beta$ (we fix the $\chi_\alpha_\beta$ gauge freedom with this choice).

Now we find the $\text{GR}_{||}$ quasilocal boundary expression. The teleparallel connection $\bar{\Gamma}$ is flat. We choose the reference to be flat Minkowski space; specifically we take

$$\bar{\sigma}^\alpha_\beta = \bar{\Gamma}^\alpha_\beta,$$

from which it follows that $\bar{\sigma} = 0$, consequently $\bar{\tau} = 0$. From these results we find

$$B_{\text{GR}_{||}}(N) = i_N \theta^\mu \kappa^\alpha_\beta \wedge \eta_\alpha_\beta_\mu + \Delta \bar{\Gamma}^\alpha_\beta \wedge i_N \eta_\alpha_\beta + \bar{D}^\beta \bar{N}^\alpha \Delta \eta_\alpha_\beta.$$  (25)

Our $B_{\text{GR}_{||}}$ expression asymptotically agrees with the expression found by Blagoević and Vasilic for the teleparallel total quantities at spatial infinity.

8. The Quasilocal Expressions

In this section we summarize the various quasilocal expressions in a convenient common notation: the Riemannian variables.

We first observe that, because $K = \Gamma - \bar{\Gamma}$ and the fact (24) that we choose $\Gamma$ and $\bar{\Gamma}$ to have the same reference values, the two expressions, $B_{\text{GR}_{||}}$ [25] and $B_{\text{GR}}$ [17], turn out to be equivalent. So they have the exactly the same value for all quasilocal quantities.

With this in mind the various quasilocal expressions reduce to

$$B_{\text{tet}}(N) = \Gamma^\alpha_\beta \wedge i_N \eta_\alpha_\beta,$$

$$B_\delta(N) = \Delta \Gamma^\alpha_\beta \wedge i_N \eta_\alpha_\beta,$$

$$B_{\text{GR}_{||}}(N) \equiv B_{\text{GR}}(N) = \Delta \Gamma^\alpha_\beta \wedge i_N \eta_\alpha_\beta + \bar{D}^\beta \bar{N}^\alpha \Delta \eta_\alpha_\beta.$$  (28)

Note that each of these quasilocal expressions has a “Freud type” term, linear in $N$. Essentially the same Freud type term as in $B_{\text{tet}}$ has been derived by many
investigators including Møller, Nester, Maluf and Pereira. Only the last quasilocal relation has an additional “Komar like” $DN$ term. The main point we wish to convey is that this $DN$ term is quite important for angular momentum and absolutely essential for the center-of-mass. We remark that such a term has long been recognized in GR Hamiltonian investigations as essential for the correct boundary conditions and correct total quantities at spatial infinity.

9. Evaluation

Here we discuss testing the various quasilocal expressions, especially on the asymptotic eccentric Schwarzschild geometry. Asymptotically the displacement vector field should approach a Killing vector of the asymptotic Minkowski space. Consequently it should have the Poincaré Lie algebra form

$$N^\mu = N_0^\mu + \lambda^{\mu\nu} x_\nu,$$

where $\lambda^{\mu\nu} = \lambda^{[\mu\nu]}$. (29)

The constant spacetime translation $N_0^\mu$ (leading to energy-momentum) and the spatial rotations (leading to angular momentum) were investigated for the tetrad-teleparallel theory by Vu. For GR, the center-of-mass moment was investigated by Meng. We have included their results in our tables. Here, for the tetrad-teleparallel theory, we examine the remaining quasilocal quantity, the tetrad-teleparallel COM.

Consider the asymptotic spherically symmetric frame

$$\vartheta^t = (1 + \Phi) dt, \quad \vartheta^k = (1 - \Phi) dx^k,$$

with $\Phi = -\frac{M}{r}$. (30)

To test the expressions for the center-of-mass moment, displace the center:

$$\frac{1}{r} \rightarrow \frac{1}{|r-d|} = \frac{1}{r} \sum_{l=0}^{\infty} \left( \frac{d}{r} \right)^l P_l(\cos \theta) \approx \frac{1}{r} + \frac{dz}{r^3} + \ldots$$

(31)

Now evaluate the quasilocal expressions for $N^\mu$ an asymptotic boost. This choice of displacement detects the COM. Straightforward calculations, taking the asymptotic limit of the integral over a constant $r$ 2-sphere (using the obvious Minkowski

| Table 1. Quasilocal COM from different expressions |
|-----------------------------------------------|
| $$\mathcal{B}_{tet}$$ | $$\mathcal{B}_0$$ | $$\mathcal{B}_{GR||} \simeq \mathcal{B}_{GR}$$ |
|-----------------------------------------------|
| Freud term | $\frac{2}{3} M \bar{d}$ | $\frac{2}{3} M \bar{d}$ |
| Komar term | $-\frac{1}{3} M \bar{d}$ | $\frac{1}{3} M \bar{d}$ |
| Total | $\frac{2}{3} M \bar{d}$ | $\frac{2}{3} M \bar{d}$ | $M \bar{d}$ |
reference where necessary), yield the results for the various quasilocal expressions which are presented in Table 1. Note that $B_{\text{tet}}$ and $B_{\theta}$ do not give the correct COM value, whereas $B_{\text{GR}_{||}}$ and $B_{\text{GR}}$ do. (Actually it is possible to obtain the desired value from $B_{\theta}$—but not from $B_{\text{tet}}$—if one selects a certain non-trivial reference; this can be inferred from a recent COM work in GR). Table 2 summarizes the limitations of the various tetrad-teleparallel quasilocal expressions.

|             | EM         | AM         | COM       |
|-------------|------------|------------|-----------|
| $B_{\text{tet}}$ | Special frame | Special frame | Fail      |
| $B_{\theta}$   | General frame | Special frame | Special reference |
| $B_{\text{GR}_{||}}$ | General frame | General frame | General frame |

10. Summary and Conclusion

We considered an important neglected quantity: the quasilocal center-of-mass in tetrad-teleparallel gravity. (We have distinguished these two formalisms. In our terminology the former has only a tetrad field, the latter has a tetrad and a connection which has vanishing curvature.) We used the covariant Hamiltonian formalism, in which quasilocal quantities are given by the Hamiltonian boundary term, along with the covariant symplectic Hamiltonian boundary expressions. As expected, consideration of the COM not only gives the most restrictive asymptotic conditions on the variables but also gives strong constraints on the acceptable expressions.

We found that the $DN$ terms, which are absent in $B_{\text{tet}}$, the Hamiltonian boundary term of the tetrad theory, play an important role in angular momentum and an essential role in obtaining the correct center-of-mass moment. Consequently the tetrad formulation does not give the correct COM. The covariant-symplectic tetrad expression, $B_{\theta}$, can give good values with special choices of frame and reference. On the other hand the teleparallel formulation does give the correct AM and COM, quite generally, independent of the asymptotic choice of frame. We also remark that MTW Eq. (20.9), gives the necessary asymptotic form for all 10 Poincaré quasilocal quantities. We found that only the expressions $B_{\text{GR}}$ and $B_{\text{GR}_{||}}$, via the $DN$ terms, have this form.

Remarkably, one of our covariant-symplectic teleparallel Hamiltonian boundary expressions turns out to be equivalent to one of the covariant symplectic GR boundary expressions (our other boundary expressions for GR and GR$_{||}$ will differ from them quasilocally but not asymptotically). This leads us to the conclusion (which
we will revise if a good frame gauge condition, meeting all the desired criteria, is identified) that, contrary to a common hope, for localization purposes, teleparallel GR is no better (and no worse) than Einstein’s GR.

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