PAPER

Exploration of the magnetic-field-induced $5s5p^3P_0-5s^21S_0$ forbidden transition in bosonic Sr atom

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Abstract

We present experimental and theoretical studies of the effect of an external magnetic field on the $5s5p^3P_0-5s^21S_0$ forbidden transition in the bosonic Sr atom. In our ultra-cold atomic system, the excitation fraction of the forbidden transition was measured under the circumstance of the different magnetic field strengths by using the normalized detection method. Based on the perturbation theory, we calculated the magnetic-field-induced $5s5p^3P_0-5s^21S_0$ transition rate as a function of the magnetic field strength. Electronic matrix elements entering the calculation of the transition rate were also used to derive the theoretical excitation fraction as a function of the magnetic field strength. A good agreement was found between the experimental measurements and the calculations. This study should be helpful in evaluating the magnetic field effects on the forbidden transition rate with higher accuracy. Moreover, it can help to understand the ultra-cold atomic interaction in the external magnetic field.

1. Introduction

The forbidden lines are commonly used for studying astrophysical and laboratory plasmas [1]. The forbidden lines are also very important in optical clocks as their narrow natural line-width can be used for laser cooling and trapping experiments [2, 3] and clock transition interrogation [4, 5]. Moreover, in the ultra-cold atomic system, one can use the forbidden line to measure the lifetime of the metastable state [6], to observe the motion-dependent nonlinear dispersion [7], and to probe the many-body interaction [8, 9]. However, it is a challenge to predict and determine the rate of the forbidden transition due to its sensitivity to the electron correlations and the relativistic effects [10, 11].

The $5s5p^3P_0-5s^21S_0$ transition in neutral Sr atom, as a typical $E1$ forbidden transition, has been extensively studied in experiments and theories for its potential applications in quantum computing [12], optical atomic clocks [13–16], and atom interferometers [17]. This forbidden transition is induced by the hyperfine interaction in the fermionic atom, and thus referred to as the hyperfine-induced transition. In contrast to fermionic Sr, bosonic Sr has a simpler level structure and higher natural abundance. The $5s5p^3P_0-5s^21S_0$ transition can be induced as well, but by the external magnetic field instead of the internal magnetic field—the hyperfine interaction. In the bosonic Sr optical clock, this clock transition rate depends on the magnetic field strength [18, 19]. However, the determined forbidden transition rate is not accurate.

In this work, we measured the excitation fraction of $5s5p^3P_0-5s^21S_0$ forbidden transition using the normalized detection method under the circumstance of the different magnetic field strengths in our $^{88}$Sr ultra-cold atomic system. Meanwhile, we carried out a calculation on the magnetic-field-induced $5s5p^3P_0-5s^21S_0$ transition rate in the framework of the multi-configuration Dirac–Hartree–Fock (MCDHF) method. Based on the calculated transition rate, we deduced the excitation fraction as a function of the magnetic field strength. A
comparison between the experimental results and theoretical calculations was made and we found that the theoretical calculations are in good agreement with our measurements.

2. Experimental measurement of $^5s^5p\ ^3P_0–5s^2\ ^1S_0$ excitation fraction

2.1. Apparatus

Our experimental setup and the simplified energy levels of $^{88}$Sr atom were shown in figures 1(a) and (b), individually. Cold atoms preparation was described in detail in [20]. The $^1S_0\rightarrow^3P_1$ transitions were used for first and second stage magnetic-optically trapping which cooled the atoms down to a few $\mu$K. The 679 nm and 707 nm lasers were the re-pumping lasers. Atoms in the $^3P_{0,2}$ metastable states were pumped to the $^3S_1$ state by using the re-pumping lasers to drive the $^3P_{0,2}\rightarrow^3S_1$ transitions. Eventually, these atoms would decay to the ground state through $^3S_1\rightarrow^3P_1\rightarrow^1S_0$ as the spontaneous decay rate from $^3P_1$ state is much larger than that from $^3P_{0,2}$ states. After the two-stage cooling, the ultra-cold bosonic Sr atoms were loaded into an optical lattice which was operated at the ‘magic wavelength’ of $\lambda_m = 813$ nm [16, 21]. The 813 nm laser beam was transferred by a single-mode-polarization-maintaining fiber. The 698 nm probe laser was locked to a stable ultra-low expansion (ULE) cavity which was placed on an enclosed vibration isolation platform, and was transferred by a noise canceled fiber link. The line-width of the laser was about 1 Hz and the Allan deviation was $1 \times 10^{-15}$ at 1 s. This laser was split into two beams, one going to the femtosecond optical frequency comb (OFC, Menlo FC1500) to have its frequency measured. The second one passed through a $\lambda/2$ wave plate (HWP) and a mechanical shutter and combined with the lattice laser using a dichroic mirror (DM). The 813 nm laser and the 698 nm laser passed through a Glan–Taylor polarizer to make sure their polarizations, and focused by an achromatic lens ($f = 200$ mm) into the vacuum chamber and onto the cold atoms. The wave-vector $k$ of the two lasers are perpendicular to the gravity. The waist of the 813 nm laser was 38 $\mu$m (1/e$^2$ radius of intensity), and the 698 nm laser beam was approximately three times larger. After exciting the vacuum chamber, the lattice laser was retro-reflected by another achromatic lens and a mirror to overlapped with the incident laser beam and formed the standing wave. The retro-reflecting mirror was low-reflecting at the 698 nm wavelength. The focusing lens were mounted on two three-dimensional translation stages respectively to optimize the optical lattice and the $^1S_0\rightarrow^3P_1$ magneto-optical trap overlapping. The static magnetic field was produced by Helmholtz coils and its direction was parallel to the linear polarization of the 698 nm probe laser.

2.2. Method

About $10^5$ atoms were trapped in the one-dimensional optical lattice. The temperature of these atoms was 8.4 $\mu$K and the lifetime of the atoms trapped in the optical lattice was 500 ms. The $^5s^5p\ ^3P_0–5s^2\ ^1S_0$ forbidden transition was probed in the Lamb–Dicke regime along the lattice longitudinal axis. During the experiment, the 813 nm laser was maintained open. We used the normalized detection method [22] to measure the $^5s^5p\ ^3P_0–5s^2\ ^1S_0$ excitation fraction. Firstly, we used the 698 nm probe laser pulse ($\pi$-pulse) to pump some of these atoms, namely $N_1$, to the $^3P_0$ state. The static magnetic field was also applied. Secondly, the remaining atoms, namely $N_2$, were read out and pushed out of the optical lattice with a 461 nm detection laser pulse. Thirdly, the atoms remained in the $^3P_0$ state were transferred to the ground state by the 679 nm and 707 nm laser pulses.

Finally, the pumped atoms ($N_1$) were also read out with another 461 nm detection laser pulse. The values of $N_1$
and $N_2$ were measured by probing the 461 nm fluorescence intensity. As the lifetime of $^3P_0$ state is very long, we neglected the population decay of $^3P_0$ state atoms in the optical lattice after the probe and detection pulses we had chosen. We assumed that atoms in the $^3P_0$ state were all transferred to the ground state. The excitation fraction was deduced from these two measurements by

$$P = \frac{N_1}{N_1 + N_2}. \quad (1)$$

We used a photomultiplier tube (PMT) to measure the 461 nm fluorescence intensity for evaluating the numbers of the atoms in the ground state. In order to obtain a high signal-to-noise ratio of the fluorescence intensity, a 461 nm filter, a lens group and an aperture were used to eliminate the background noise. The measured spectrums were presented in figures 2(a)–(d) under the circumstance of the magnetic field strength $B = 1.0, 1.2, 1.7, 2.2$ mT, respectively. During the measurements, the power of the 698 nm laser was maintained constant. The spectrums were generated by repeating the detection cycle and stepping the frequency of the 698 nm laser using an acousto-optic modulator. The experimental data were fitted with Lorentzian function. From this figure, we can see that the intensity of the spectrum peak decreased and the line-width shrunk with the magnetic field strength decreasing. When $B = 1.0$ mT, the line-width is 180 Hz which is larger than the Fourier limited width (20 Hz) due to 45 ms interrogation pulse. The line-width is broadened by many factors, for example, the atomic collisions. Since in this paper the intensity of the spectrum peak is the focus of the discussion, the line-width broadening is neglected.

3. Magnetic-field-induced transition

3.1. Theory

3.1.1. Magnetic-field-induced transition rate

In the presence of an external magnetic field $\mathbf{B}$, the atomic Hamiltonian is [23]

$$H = H_b + H_{\text{mn}}, \quad (2)$$

where $H_b$ is the relativistic fine-structure Hamiltonian which include the Breit interaction and the main part quantum electrodynamical (QED) effects and $H_{\text{mn}}$ is the Hamiltonian for the interaction between the external magnetic field and the atom. We choose the direction of the magnetic field as the quantization axis and assume

![Figure 2. Spectrum of the magnetic-field-induced $^3S_1^0$–$^3P_0^0$ transition with the magnetic field strength $B = 1.0, 1.2, 1.7, 2.2$ mT from (a)–(d). The red curves are Lorentzian fit of the data.](image-url)
the magnetic field does not vary throughout the atomic system, the interaction Hamiltonian $H_m$ is expressed as [24]

$$H_m = (N^{(1)} + \Delta N^{(1)}) B.$$  

(3)

Here, $\Delta N^{(1)}$ is the so-called Schwinger QED correction [24]. For an $N$-electron atom, these two operators are

$$N^{(1)} = \sum_{j=1}^{N} n^{(1)}(j) = \sum_{j=1}^{N} -\frac{\sqrt{2}}{2\alpha}(\mathbf{r}_j \cdot \mathbf{C}^{(1)}(j))^1,$$

(4)

$$\Delta N^{(1)} = \sum_{j=1}^{N} \Delta n^{(1)}(j) = \sum_{j=1}^{N} \frac{g_e - 2}{2} \beta \mathbf{S}_j,$$

(5)

Here, $i$ is the imaginary unit, $r_j$ is the radial coordinate of the $j$th electron, $C_j^1(\ldots)$ is the spherical tensor operator of rank 1, $S_j$ is the relativistic spin-matrix, $g_e$ is the $g$-factor of the electron spin corrected by the QED effects, $\alpha$ is the fine-structure constant and $\beta$ and $\beta$ are the Dirac matrices.

In the presence of the external magnetic field, only the magnetic quantum number $M_J$ remains the good quantum number. The atomic states with the same magnetic quantum number and parity are mixed due to the interaction between the external magnetic field and the atom [23]. Therefore, the atomic state wave function $|M_J\rangle$ can be written as

$$|M_J\rangle = \sum_{\ell J} d_{\ell J}|\Gamma J M_J\rangle,$$

(6)

where the atomic state wave functions $|\Gamma J M_J\rangle$ are eigenstates of the Hamiltonian $H_{so}, I$ and $M_J$ are the total and magnetic quantum numbers. According to the first-order perturbation theory, the expansion coefficient $d_{\ell J}$ is given by

$$d_{\ell J} = \langle \Gamma J M_J | H_m | \Gamma' J' M_J \rangle / (E(\Gamma' J') - E(\Gamma J)),$$

(7)

where $|\Gamma' J' M_J\rangle$ is the reference atomic state.

Using the equation (6), we deduced the transition matrix element and submitted it into the formula of the electric dipole transition rate [25]. Therefore, the magnetic-field-induced transition (MIT) rate can be obtained by [23, 26]

$$A_{MIT} = \frac{2.026 13 \times 10^{18}}{\lambda^3}$$

$$\times \sum_{q} \left| \sum_{\ell J} \sum_{\Gamma' J'} d_{\ell J} d^{\ast}_{\ell J'} (-1)^{J-J'-M_J} \begin{pmatrix} J & 1 & \ell J \\ -M_J & \frac{1}{q} & M_{J'} \end{pmatrix} \langle \Gamma J | P^{(1)} || \Gamma' J' \rangle \right|^2,$$

(8)

where $\lambda$ is the wavelength in Å and $P^{(1)}$ is the electric dipole transition operator.

3.1.2. MCDHF method

According to the MCDHF method [27], the atomic state wave function (ASF) $\Psi(\Gamma J M_J)$ is a linear combination of a number of configuration state functions (CSFs) $\Phi_j(\gamma_j J M_J)$ with the same parity $P$, total angular momentum $J$ and its component along $z$ direction $M_J$.

$$\Psi(\Gamma J M_J) = \sum_{j} c_j \Phi_j(\gamma_j J M_J),$$

(9)

where $c_j$ stands for the mixing coefficient, $\gamma$ represents the other quantum numbers to uniquely define the state. The CSFs $\Phi_j(\gamma_j J M_J)$ are constructed as linear combinations of Slater determinants, each of which is a product of one-electron Dirac orbitals.

In the self-consistent field (SCF) procedure, the coefficients $c_j$ and the one-electron relativistic orbitals are optimized by solving the MCDHF equations, which are derived from the variational principle. The Breit interaction

$$B_{ij} = -\frac{1}{2\gamma_i} \left[ \mathbf{\alpha}_i \cdot \mathbf{\alpha}_j + \frac{(\mathbf{\alpha}_i \cdot n_j)(\mathbf{\alpha}_j \cdot n_i)}{r_{ij}^2} \right]$$

(10)

and QED effects are included in the subsequent relativistic configuration interaction (RCI) calculation, where only the mixing coefficients are variable.
3.2. Computational method
In our calculations, we used the active space method to capture the electron correlation. The $1s^22s^22p^63s^23p^63d^{10}4s^24p^65s^2$ and $1s^22s^22p^63s^23p^63d^{10}4s^24p^65s^5p$ configurations were treated as the reference configuration for the ground state ($5s^21S_0$) and the excited states ($5s5p\ ^1D_P\)$, respectively, where the $5s$ and $5p$ electrons are the valence electrons and the others the core. The configuration expansions were generated by single (S) and double (D) excitations from the reference configuration to the active set. Starting with the ground state, the occupied spectroscopic orbitals in the reference configurations were optimized in the Dirac–Hartree–Fock (DHF) approximation and kept frozen in the following computations, and others in the active set as correlation orbitals. The valence–valence (labeled as VV) and core–valence (labeled as CV) correlations between the cores with $n = 3, 4$ and the valence electrons were systematically considered in the subsequent SCF calculation procedure. The SD excitations were restricted that at most one electron may be promoted from the core–shells. The active sets were expanded as

$$\begin{align*}
n5 &= \{3s, 3p, 3d, 4s, 4p, 4d, 4f, 5s\}, \\
n6 &= n5 + \{5p, 5d, 5f, 5g, 6s\}, \\
n7 &= n6 + \{6p, 6d, 6f, 6g, 7s\}, \\
n8 &= n7 + \{7p, 7d, 7f, 7g, 8s\}, \\
n9 &= n8 + \{8p, 8d, 8f, 9s\}, \\
n10 &= n9 + \{9p, 9d, 10s\}, \\
n11 &= n10 + \{10p, 11s\}. \tag{11}\end{align*}$$

The seven layers of virtual orbitals were added to make sure the convergence of the atomic parameters under investigation. Only the added orbitals in each layer of the active set were varied. Calculations for the excited states were performed in the same way except for the first layer of the active set $n5' = \{3s, 3p, 3d, 4s, 4p, 4d, 4f, 5s, 5p\}$.

To consider the core–core (labeled as CC) correlation of the $n = 4$ core–shell, the CSFs by unrestricted SD excitations from the reference configuration to the active set of orbitals with $n = 8$ were generated. Furthermore, the higher-order electron correlations among the $n = 4$ and $n = 5$ shells were taken into account by the multi-reference (marked as MR) SD model. The final MR computational model contains the higher-order electron correlations as well as the VV, CV and CC correlations. In this step, the \{4s^24p^65p^2; 4s^24p^4d5s5p\} and \{4s^24p^65p6s6p; 4s^24p^65p6s5p; 4s^24p^4d6p\} configurations were added to the single reference configuration set. The configuration space was expanded by replacing one or two electrons in the reference configurations with the ones in the active set $n8$. Finally, the Breit interaction and the QED correlations were evaluated. These calculations were performed in the RCI computation. We used the GRASP2K package [28] to accomplish our calculations.

3.3. Numerical results and discussions
3.3.1. Excitation energies and rates of $^1S_0\rightarrow^1D_P1\ E1$ transition
Table 1 shows the excitation energies (in cm$^{-1}$) and the transition rates (in s$^{-1}$) of $^1S_0\rightarrow^1D_P1\ E1$ transitions with different computational models. The transition rates in the Babushkin and the Coulomb gauges, corresponding to the nonrelativistic length and velocity gauges [29], are also displayed in this table. It can be seen from this table that the excitation energies and the transition rates converged very well when the virtual orbitals increased from $n5$ to $n11$. Comparing the results obtained from the CC model with those from $n11$, we found that the core–core correlation changes the atomic parameters considerably. For example, the $^1S_0\rightarrow^1P_1$ transition energy decreases from 14538.74 cm$^{-1}$ to 13064.84 cm$^{-1}$. However, the higher-order correlations counteract the core–core effects. The effects of the Breit interaction and the QED corrections are tiny, and thus included in the uncertainty. Two different methods, based on the convergence trend of the atomic parameters and the consistency between the transition rates in two gauges [30], were used to estimate the uncertainties of the transition rates. The error bar of the $^1S_0\rightarrow^1P_1\ E1$ transition rate reaches 2%, but 4% for the rate of $^1S_0\rightarrow^1P_1\ E1$ transition.

For comparison, other theoretical and experimental values are also presented in table 1. As far as the excitation energy of the $^1S_0\rightarrow^1P_1$ transition is concerned, our result agrees with those results obtained in [31] using the configuration interaction (CI) and many-body perturbation theory (MBPT), in [32] using the MCDF method and in [33] using the configuration interaction plus core polarization (CICP) method. For the excitation energy of the $^1S_0\rightarrow^1P_1$ transition, our calculation result is 0.06% lower than that obtained in [34, 35] by the CI + MBPT method. Comparing with the experimental values from the National Institute of Standards and Technology (NIST) database [36], we found that the relative uncertainties in the excitation energies of $^1S_0\rightarrow^1D_P1$ transitions are 0.9% and 1.9%, respectively.
Table 1. Different computational models for the excitation energies \(E\) (in \(\text{cm}^{-1}\)) and the transition rates (in s\(^{-1}\)) of \(^1S_0\rightarrow^3P_1\) (B: Babushkin gauge; C: Coulomb gauge). The numbers in square brackets are the expansion in base 10.

| Model         | \(^1S_0\rightarrow^3P_1\) | \(^1S_0\rightarrow^1P_1\) |
|---------------|-------------------------|-------------------------|
|               | \(E_{11}\) | B | C | \(E_{11}\) | B | C |
| DHF           | 8455.93 | 2.24[3] | 4.32 | 23583.72 | 4.12[8] | 1.90[8] |
| \(n\)         | 15231.16 | 4.03[4] | 5.83[4] | 23086.27 | 2.44[8] | 2.30[8] |
| \(n\)         | 14745.84 | 5.05[4] | 5.06[4] | 21942.01 | 1.99[8] | 1.81[8] |
| \(n\)         | 14648.86 | 5.04[4] | 4.28[4] | 21861.95 | 1.96[8] | 1.84[8] |
| \(n\)         | 14568.70 | 5.00[4] | 4.20[4] | 21796.77 | 1.94[8] | 1.86[8] |
| CC            | 15355.01 | 4.99[4] | 4.11[4] | 21760.23 | 1.91[8] | 1.86[8] |
| MR            | 15458.28 | 4.95[4] | 4.01[4] | 21755.66 | 1.90[8] | 1.86[8] |
| Other theories | 15453.74 | 4.96[4] | 3.95[4] | 21746.22 | 1.91[8] | 1.85[8] |
| NIST [36]     | 14343.64 | 4.52[4] | 4.46[4] | 21904.02 | 1.96[8] | 2.03[8] |
| Parkinson et al [38] | 15081 | 5.55[4] | 5.69[4] | 21981 | 1.89[8] | 1.87[8] |
| Dzuba et al [41] | 2128 | 6.11[4] | 6.41[4] | 21621 | 1.92[8] | 1.95[8] |
| Liu et al [32] | 21628.84 | 1.89[8] | 1.98[8] |
| Glowicki et al [37] | 21628.84 | 1.89[8] | 1.98[8] |
| Vaeck et al [42] | 2.22[8] | 1.87[8] |
| Safronova et al [43] | 5.51[4] | 4.11[4] |
| Cheng et al [33] | 21698.47 | 1.93[8] |
| Experiments   | 21317.51 | 4.40[12] | 2.01[8] |
| Parkinson et al [38] | 21698.45 | 2.01[6] | 2.01[8] |
| Kelly et al [44] | 2.14[5] | 2.14[5] |
| Husain et al [45] | 4.99[10] | 4.99[10] |
| Kelly et al [46] | 4.55[10] | 4.55[10] |
| Drozdowski et al [40] | 4.69[11] | 4.69[11] |
| Nagel et al [39] | 1.92[1] | 1.92[1] |
| Yasuda et al [47] | 1.90[0.1] | 1.90[0.1] |

For the \(^1S_0\rightarrow^1P_1\) \(E1\) transition rate, our calculated results are in excellent agreement with ones obtained with the relativistic CI method [37] and the latest CICP computational model [33]. Compared with the experimental measurements, our calculations are consistent with the result of Parkinson et al [38] using the hook method (distortion of interferometric channel-spectra in the neighborhood of absorption lines by anomalous dispersion) and the recent measurement of Nagel et al [39] using photoassociative spectroscopy of \(^{88}\)Sr\(_2\). The numbers in the round brackets are the uncertainties of their measurements. For the \(^1S_0\rightarrow^3P_1\) intercombination transition, our calculated transition rates are in good agreement with the MCDF result [32]. Moreover, the calculated transition rates in our calculations are about 0.4% larger than the measurements by observing the exponential decay of the florescence from the \(^3P_1\) excited state [40].

### 3.3.2. Magnetic-field-induced transition

For a bosonic Sr atom, the external magnetic field mixes the \(^3P_1\) and \(^1P_1\) states with \(^3P_0\), and thus opens up a one-photon \(E1\) transition channel from the \(^3P_0\) state to the ground state. Since the magnetic field strength is weak and the energy separation between the states belonging to different configurations is large, the reference \(^3P_0\) state can be approximately expressed as [23]

\[
|5s5p^3P_0, M = 0\rangle = |5s5p^3P_0, M = 0\rangle + \sum_{i=1,3} d_{i0} |5s5p^iP_1, M = 0\rangle. \tag{12}
\]

Here, the state with prime describes the dominant component of the eigenvector. Isolated from other states, the ground state is given as

\[
|5s^2^1S_0', M = 0\rangle = |5s^2^1S_0', M = 0\rangle. \tag{13}
\]
Here, the line-width of the forbidden transition depends on the magnetic field strength. To obtain an ultra-narrow line-width, the static magnetic field strength should be extremely small [18, 19]. In this experiment, the external magnetic field strength is set to be below 5 mT.

In our previous experimental work, we measured the Rabi oscillation with different Rabi frequencies. The data were fitted with the function \( P = a \times (1 - \cos(2 \pi \Omega t) \exp(-t/\tau)) \) [20, 48] where \( \tau \) is the decoherent time scale which remain constant. The Rabi frequency \( \Omega \) of the MIT is defined as,

\[
\Omega = \frac{1}{\hbar^2} \left[ d^R_{s} \langle S_0 \| P^{(1)} \| P_i \rangle + d^R_{s} \langle S_0 \| P^{(1)} \| P_i \rangle (E \cdot B) \right]
\]

(17)

where \( E \) describes the electric vector of the 698 nm laser, \( \langle S_0 \| P^{(1)} \| P_i \rangle \) and \( \langle S_0 \| P^{(1)} \| P_i \rangle \) are the reduced matrix elements for the \( ^1S_0 \rightarrow ^3P_1 \) transitions. The values of the two matrix elements and the reduced mixing coefficients were accurately determined in previous calculations. The Rabi frequency is a function of the magnetic field strength \( B \) as the intensity of 698 nm laser \( I \) is kept constant. For a two-level atom, the excitation fraction \( P \) is obtained as
In this experiment, we used the $\pi$-pulse ($t = 1/(2\Omega)$) laser to excite atoms to the excited state and the intensity of 698 nm laser is 3.6 W cm$^{-2}$. In figure 3, we present the experimental measurements and the theoretical calculations of the excitation fraction. The solid squares are the weighted average of several measured excitation fractions with the magnetic field strength $B = 1.0, 1.2, 1.7, 2.2$ mT and the error bars are the statistical uncertainty of our measurements. The red line is the calculated excitation fraction as a function of the magnetic field strength. As can be seen from this figure, our calculated results agree with the measurements when $B = 1.2, 1.7, 2.2$ mT. The deviation in the small magnetic field strength results from the low signal-to-noise ratio of the spectrum due to the fluctuation of the cold atom number in the optical lattice. As the magnetic field strength is small, the measured excitation fraction is very small. The atom number fluctuation effect is comparable to the measured excitation fraction, resulting in the low signal-to-noise ratio of the spectrum. What’s more, in our experiment, the power fluctuation of the 461 nm laser is one of the factors to induce the uncertainty. And we assume that the excitation is substantially faster than the dominant loss mechanisms, for example, the frequency and amplitude fluctuation of the 813 nm laser and the vibration of the achromatic lens and the retro-reflecting mirror.

\begin{equation}
P = \frac{\Omega B \tau}{1 + \Omega^2 \tau / A_{\text{MIT}}} \times [d_1^R \langle |S_0||P^{(1)}||P_1 \rangle + d_2^R \langle |S_0||P^{(1)}||P_2 \rangle ] \sin(\Omega \pi t). \tag{18}
\end{equation}

Figure 3. The $5s5p \ ^3P_0-5s^2 \ ^1S_0$ excitation fraction for different magnetic field strengths. The red line is the calculated excitation fraction as a function of magnetic field strength. The solid squares are the weighted average of several measured excitation fractions with the magnetic field strength $B = 1.0, 1.2, 1.7, 2.2$ mT and the error bars are the statistical uncertainty of our measurements.

4. Conclusion

Actually, several research groups have reported the operation and characterization of the magnetic-field-induced the $nsnp \ ^3P_0 - ns^2 \ ^1S_0$ forbidden transition in bosonic atoms in the previous publications. However, the transition rate has never been determined in these works. In this work, we measured the excitation fraction of the $5s5p \ ^3P_0 - 5s^2 \ ^1S_0$ transition using the normalized detection method under the circumstance of the different magnetic field strengths in the $^{88}\text{Sr}$ ultra-cold atomic system. Meanwhile, we systematically analyzed the mechanism of this transition and accurately calculated the forbidden transition rate. Based on the calculations, we deduced the excitation fraction as a function of the magnetic field strength. The calculations are consistent with the experiments. The measurements indicated that our theoretical calculation model is reasonable. Moreover, the calculations also verified that the experimental method to determine the magnetic field strength effects on the lifetime of the metastable state we have developed is correct. This study should be helpful in more accurate evaluating the quadratic Zeeman shift and probe ac Stark shift in the $^{88}\text{Sr}$ optical clock. Based on this, we can accurately estimate the magnetic field effect on the hyperfine energy level and the higher-order Zeeman shift of the $^{87}\text{Sr}$ atom. Moreover, we can estimate the external magnetic field effects on the ultra-cold atoms collision.
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References

[1] Yao K, Andersson M, Brage T, Hutton R, Jönsson P and Zou Y 2006 Phys. Rev. Lett. 97 183001
[2] Friese J et al 2011 New J. Phys. 13 125010
[3] Collopy A L, Hummon M T, Yeo M, Yan B and Ye J 2015 New J. Phys. 17 055008
[4] Tsyumennev R et al 2016 New J. Phys. 18 113002
[5] Falke Set al 2014 New J. Phys. 16 073023
[6] Yasuda M and Katori H 2004 Phys. Rev. Lett. 92 153004
[7] Westergaard P G, Christensen B T R, Trier D, Matin R, Cooper J, Holland M, Ye J and Thomsen J W 2015 Phys. Rev. Lett. 114 093002
[8] Mickelson P G, Martinez Y N, Saenz A D, Nagel S B, Chen Y C, Killian T C, Pellegrini P and Côté R 2005 Phys. Rev. Lett. 95 223002
[9] Derevianko A, Porsev S G, Kotochigova S, Tiesinga E and Julienne P S 2003 Phys. Rev. Lett. 90 063002
[10] Andersson M, Grumer J, Brage T, Zou Y and Hutton R 2016 Phys. Rev. A 93 032506
[11] Grumer J, Brage T, Andersson M, Li J, Jönsson P, Li W, Yang Y, Hutton R and Zou Y 2014 Phys. Scr. 89 114002
[12] Langer C et al 2005 Phys. Rev. Lett. 95 060502
[13] Poli N, Oates C W, Gill P and Tino G M 2013 Rev. Nuovo. Cimento. 36 555
[14] Ludlow A D, Boyd M M,Jeff, Peik E and Schmidt P O 2015 Rev. Mod. Phys. 87 637
[15] Takamoto M, Takano T and Katori H 2011 Nature 5 288
[16] Akatsuka T, Takamoto M and Katori H 2010 Phys. Rev. A 81 023402
[17] Mazzoni T, Zhang X, Del Aguilora S,Salvi L, Poli N and Tino G M 2015 Phys. Rev. A 92 033019
[18] Taichenachev A V, Yudin V I, Oates C W, Hoyt C W, Barber Z W and Hollberg L 2006 Phys. Rev. Lett. 96 083001
[19] Baillard X, Fouche M, Targat R L, Westergaard P G, Lecallier A, Croq Y L, Rovera G D, Bize S and Lemonde P 2007 Opt. Lett. 32 1812
[20] Xu Q, Liu H, Lu B, Wang Y, Yin M, Kong D, Ren J, Tian X and Chang H 2015 Chin. Opt. Lett. 13 100201
[21] Ovsianikov V D, Pal’chikov V G, Taichenachev A V, Yudin V I, Katori H and Takamoto M 2007 Phys. Rev. A 75 020501
[22] Lemonde P 2009 Eur. Phys. J. Spec. Top. 172 81
[23] Li J, Jon G, Wenxian L, Martin A, Tomas B, Roger H, Per J, Yang Y and Yaming Z 2013 Phys. Rev. A 88 013416
[24] Andersson M and Jönsson P 2008 Comput. Phys. Commun. 178 156
[25] Johnson W R 2007 Atomic Structure Theory (Berlin: Springer) p 293
[26] Grumer J, Li R W, Bernhardt D, Li J, Schippers S, Brage T, Jönsson P, Hutton R and Zou Y 2013 Phys. Rev. A 88 022513
[27] Fischer C F, Godefroid M, Brage T, Jönsson P and Gaigalas G 2016 J. Phys. B: At. Mol. Opt. Phys. 49 182004
[28] Jönsson P, Gaigalas G, Bieron J, Fischer C F and Grant I 2013 Comput. Phys. Commun. 184 2197
[29] Grant IP 174 J. Phys. B: At. Mol. Phys. 7 1458
[30] Andersson M, Liu Y, Chen C Y, Hutton R and Brage T 2008 Phys. Rev. A 78 062505
[31] Savukov I M and Johnson W R 2002 Phys. Rev. A 65 042503
[32] Liu Y, Andersson M, Brage T, Zou Y and Hutton R 2007 Phys. Rev. A 75 014502
[33] Cheng Y, Jiang J and Mitroy J 2013 Phys. Rev. A 88 022511
[34] Porsev S G, Kozlov M G, Rakhлина Y G and Derevianko A 2001 Phys. Rev. A 64 012508
[35] Porsev S G, Kozlov M G and Rakhлина Y G 2000 J. Exp. Theor. J. Exp. Theor. 72 595
[36] Sarsenettit J E and Nave G 2010 J. Phys. Chem. Ref. Data 39 033103
[37] Glowacki L 2003 J. Phys. B: At. Mol. Opt. Phys. 36 3629
[38] Parkinson W H, Reeves E M and Tomkins F S 1976 J. Phys. B: At. Mol. Phys. 9 157
[39] Negele B, Mickelson P G, Saenz A D, Martinez Y N, Chen Y C, Killian T C, Pellegrini P and Côté R 2005 Phys. Rev. Lett. 94 083004
[40] Drozdowski R, Ignacik M, Kwela J and Heldt J 1997 Z. Phys. D. 41 125
[41] Dzuba V A, Flambaum V V and Marchenko M V 2003 Phys. Rev. A 68 022506
[42] Vaeck N, Godefroid M and Hansen J E 1991 J. Phys. B: At. Mol. Opt. Phys. 24 361
[43] Safronova M S, Porsev S G, Safronova U I, Kozlov M G and Clark C W 2013 Phys. Rev. A 87 012509
[44] Kelly F M and Mathur M S 1980 Can. J. Phys. 58 1416
[45] Hussain D and Roberts G 1985 J. Chem. Soc. Faraday Trans. 81 87
[46] Kelly J F, Harris M and Gallagher A A 1988 Phys. Rev. A 37 2354
[47] Yasuda M, Kishimoto T, Takamoto M and Katori H 2006 Phys. Rev. A 73 011403
[48] Poli N, Schioppo M, Vogt S, Falke S, Stehr U, Lisdat C and Tino G M 2014 Appl. Phys. B 117 1107