Dimensional Analysis on the Variation of Fault Parameters of Rod-shaped Object by using \( \Pi \) Theorem

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Abstract. The calculation of the pressure difference remains an important challenge in engineering. Pressure difference analysis for a typical configuration is presented, which consists of two same cylinders connected with a small cylinder. ANSYS FLUENT is utilized to explore the relationship between pressure difference and several other variables based on the \( \Pi \) theorem by changing the configuration of the model and the initial velocity of the fluid. According to \( \Pi \) theorem, the \( \Delta P/v^2 \rho \) and \( 1/v \) are related to each other, which can be fit pretty well with quadratic relation from simulation results. Simultaneously, the radius of the connected cylinder has the greatest influence on the pressure difference. Within a certain range, the smaller the radius, the greater the pressure difference. These results can be guidelines for appliances with such structures in engineering.

Keywords: \( \Pi \) theorem, CFD, Cylinder segment planes, Dimensional analysis

1. Introduction
Double cylinder pipes connected by a small cylinder pipes engineering structure plays important roles in medical treatment, machinery and other fields. Recent developments in treatment of diabetes have heightened the need for more sophisticated equipment with accurate test data. Besides, many potential advantages of such structure are utilized to liquid sampling [1] and polydimethylsiloxane (PDMS) ISF transdermal extraction tools designed [2]. In addition, monitoring the impingement surface wave and seismic wave are achievable by detecting the mechanical pressure of the nearby sea via such structure [3]. The effect of pressure on structural soundness for this kind of configuration has also been studied by Westergaard [4] and Chopra [5]. From both points of view, the application of such structure strongly depends on variation in pressure, i.e., the calculation of pressure difference between sections is the key should be addressed.

Pressure differences are necessary for engineering applications, such as in agriculture, where pressure differences on leaves can affect plants [6]. Many problems of fluid mechanics, such as flow in piping systems, were first presented under ideal condition [7]. The paper mentioned above attempts to show that the tradition method has some inaccuracy. However, most studies in the pressure difference in the similar structure have only been carried out in a small number of areas and lack of quantitative analysis of fracture surface. Therefore, the ultimate goal is to produce a relation, and the relation can be used to predict the pressure difference over a range of velocities. This paper begins by building similar structure, it will then go on to analyze the simulation data by using the CFD [8].
The aim of this paper is to measure the pressure variation at the fracture surface and investigate the relationship between the initial velocity. Five group results with different initial velocity are demonstrated (initial velocity between 0.002m/s to 0.012m/s), which is obtained from ANSYS simulation. Π theorem analysis of several basic quantities to obtain the relationship. Subsequently, the data is substituted into the relational expression and polyfit is applied to obtain the relational curve. Through fitting curve, the pressure difference for similar structures can be predicted by the training models.

In this paper, the Π theorem is used to evaluate the relationship between the basic physical quantities. By controlling the variable and scanning the data, the tread of the pressure difference as a function of velocity is fitted. These results will provide some guideline for future design and analysis for applying such structure.

2. Method

In order to investigate the correlation between initial injection velocity and pressure difference, 3D models are constructed via ANSYS ([9]). ANSYS FLUENT is utilized to carry out simulations. The overall length of the model is 300mm and the two large cylinder is set to 15mm. The radius of the small cylinder is varying from 5mm to 7.5mm while the length of large cylinder to 80mm, 100mm and 120mm. The finite element size is chosen to 1mm and the refinement module is set to the fault. A streamline is placed in the center of the structure while the pressure data of the streamline is taken for research. The structure material is aluminum and the initial speed velocity is changing from 0.002m/s to 0.012m/s. Besides, the fluid flowing is set to water—H2O(l). Subsequently, the relation formula is obtained and analyzed by means of Π theorem.

3. Result

Fig.1 illustrates an example of their configuration with parameter that the radius of the large cylinder is 15mm, the radius of the small cylinder is 7.5mm, and the total length is 300mm. As shown in Fig.1(a), the centrosymmetric structure consists of three cylinders. Thus, a fracture surface occurs in the position where the radius is changing. The pressure difference of such models is defined as the pressure variation between the two sides of the tiny cylinder in the middle. As depicted in Fig.1(b), the flow direction is set to be negative along the Z-axis and the flow velocity range is 0.002m/s to 0.012m/s.

![Figure 1.](image-url)
Fig. 2 presents the velocity and pressure distribution for initial velocity which the value is 0.002m/s. It can reasonably observe from Fig. 2 (a) that the fluid will accelerate when it passes through the fracture surface for the first time and this phenomenon can be explained by Bernoulli’s Equation and Continuity Equation [10]:

$$ P + \rho gh + \frac{1}{2} \rho v^2 = C $$

Where P is the liquid pressure, \( \rho \) is the fluid density, H is the height of the liquid level, v describes the fluid velocity and C is a constant under fixed external conditions. Eq. (1) is the basic principle adopted by hydraulics before the establishment of continuum theory equation of fluid mechanics, whose essence is the conservation of mechanical energy of fluid. Therefore, the corollary of Bernoulli’s Equation indicates when the flow rate is high, the pressure is low at constant flow height. There was a significant positive correlation between velocity and pressure difference and only qualitative analysis can be relied on for complex structures, therefore, the condition can only be assumed in an ideal state. The results obtained from the preliminary analysis of the pressure difference per unit length increases as it passes through the fracture surface are shown in Fig. 2(b) and the pressure decreases the most when the speed increases the most. This corresponds to the corollary of Bernoulli’s Equation.

![Figure 2](image_url)

**Figure 2.** (a) The velocity and (b) pressure distribution of the model with the initial velocity 0.002m/s while other parameters are the same as Fig. 1.

To give a clearer trend, the velocity and pressure distribution at the center of the models are collected and exhibited in Fig. 3. As one can see from Fig. 3(a), the velocity reaches its peak at z=0.05m (first
fracture surface) while the pressure reaches its minimum as shown in Fig. 3(b). As a matter of fact, a clear negative relationship can be deducted from Fig. 3, where the pressure decreases with the velocity increment, respectively. According to Bernoulli’s Equation (Eq. (1)), \( P \) is approximately constant and velocity increases, then \( P \) must decrease. In general, a sudden change in a model can have a huge effect on fluid flow.

4. Theoretical Analysis

\( \Pi \) theorem \([14]\) is utilized to theoretical analysis the relationship between pressure difference and velocity. According to \( \Pi \) theorem in dimensional analysis, there are seven variables involved in fluid analysis.

\[
f(\rho, v, L_1, R_1, k, \mu, \Delta P) = 0 \quad n = 7
\]  

Seven physical quantities \( n \) are involved in determining the pressure difference at the fracture surface, then, the above Eq. (2) can be obtained. \( \rho \) is the liquid density, \( v \) is the fluid velocity, \( L_1 \) and \( R_1 \) present the length and radius of the large and small cylinder, \( k \) is the mark of the modulus of elasticity. Ultimately, \( \mu \) is the viscosity, \( \Delta P \) is the pressure difference which is also the main object of this study.

\[
\begin{align*}
\text{dim } v &= \frac{L}{T^{-1}} \quad a_1 = 0, b_1 = 1, c_1 = -1 \\
\text{dim } d &= L \quad a_2 = 0, b_2 = 1, c_2 = 0 \\
\text{dim } \rho &= \frac{M}{L^3} \quad a_3 = 1, b_3 = -3, c_3 = 0
\end{align*}
\]  

Three of the above seven related variables can be selected as basic quantities, thus one can list the coefficients (Eq. (3)) where \( a_1 \sim a_3, b_1 \sim b_3, c_1 \sim c_3 \) are the numbers that describe fundamental quantities in terms of length, mass and time. Subsequently, it can be derived that \( \Pi_1 = \Delta P/v^2\rho, \quad \Pi_2 = \mu/vd, \quad \Pi_3 = \frac{L_1}{R_1}, \quad \text{and } \Pi_4 = k/R_1. \)

\[
\frac{\Delta P}{v^2\rho} = f\left(\frac{v}{R_1}, \frac{L_1}{R_1}, \frac{k}{R_1}\right)
\]  

According to the condition of dimensionless number of combinations, the \( \Pi \) terms are listed and replaced the null hypothesis function to obtain the final expression (Eq.(4)). From the formula derived from the \( \Pi \) theorem, there was a significant positive correlation between \( \Delta P/v^2\rho \) and \( 1/v \). In order to explore the relationship between the pressure difference and the initial velocity, fit these two and explore their relationship. For the sake of fitting relationship between velocity and pressure difference reasonability, five models with similar shapes are built with different radii and lengths where the detail parameters are given in Table 1.

**Table 1.** Model size data. L1, L2 represent for the length of the large and the whole model respectively. R1 is the length of the radius of the small cylinder. R2 is the radius of the large cylinder.

|   | L1(mm) | R1(mm) | L2(mm) | R2(mm) |
|---|--------|--------|--------|--------|
| 1 | 100    | 5.0    | 300    | 15     |
| 2 | 100    | 5.0    | 300    | 15     |
| 3 | 80     | 7.5    | 300    | 15     |
| 4 | 80     | 7.5    | 300    | 15     |
| 5 | 120    | 5.0    | 300    | 15     |
Figure 4. The curve of fitting relationship between $\Delta P/v^2\rho$ and $1/v$

The most striking result to emerge from the Fig. 4 is that $\Delta P/v^2\rho$ and $1/v$ are Fitted with poly to form quadratic relation distribution. According to five fitting curve. In general, as the $1/v$ increases, the $\Delta P/v^2\rho$ increases simultaneously and they are not simple linear relationships. The results displayed in Fig. 4 demonstrated that if the shape of the model is controlled within a certain range, the relationship between $\Delta P/v^2\rho$ and $1/v$ can be derived.

Table 2. Shows the experimental data on the relationship between $\Delta P/v^2\rho$ and $1/v$

| fitting formula | $f(x)$ |
|-----------------|--------|
| 1               | $f(x) = -0.1x^2 + 3.9x + 4.7$ |
| 2               | $f(x) = -0.015x^2 + 0.275x + 1$ |
| 3               | $f(x) = -0.01x^2 + x + 1.28$ |
| 4               | $f(x) = 0.08x^2 + 1.4x + 4.9$ |
| 5               | $f(x) = 0.2887x^2 - 0.05x + 6.694$ |

The results obtained from the preliminary analysis of the correlation are shown in Table2. As the $1/v$ increases, a significant increase in $\Delta P/v^2\rho$ are shown and the expression forms of functions are quadratic, which can describe the relationship between them more accurately. Moreover, 1,4,5 similarity exists in groups of data, as can be seen from Table 1, their $R1$ is equal which indicates that the radius of the small cylinder has a greater effect on the pressure difference than the length of a large cylinder.

As the formula derived by $\Pi$ theorem in the previous, there are three terms which include $R1$ that exist in the formula, therefore, from Fig. 4, the change of $R1$ has the greatest impact on the fitting formula. In other words, for a model, at the same velocity, the cross-sectional area of the fracture surface will increase the pressure difference, resulting in pressure loss due to many other factors. For the error in fitting, mean square error is adopted. From model one to model five, MES is equal to 0.00056, 0.23, 0.069, 0.55 and 0.27. According to the error data, the value of MES is roughly between 0 to 0.5, which is a reasonable error range. Therefore, it can be seen that the fitting data has certain reliability.
5. Discussion

The fitting image shows an increasing trend as the initial velocity increase, however, the above fitting data can only used to a certain extent, when the velocity exceeds a critical value, turbulence forms as exhibited in Fig. 5 and there may be convection. Once the turbulence or convection is formed, it will interfere with the experimental data and lead to abnormal result. And it is not just the initial velocity that matters, others are friction between fluids, friction between fluid and pipe wall, changing the density of the fluid or temperature change. Therefore, the relationship between initial flow velocity and pressure difference can be further studied, through adding these uncontrollable variables, then, the result will be more accurate and convincing. The effects of viscosity changes can be coupled to the velocity and temperature fields. Therefore, velocity and temperature distributions vary qualitatively along microflow. Due to the change of viscosity, the velocity has obvious distortion, which changes along the flow [11], the results of this study indicate that viscosity has an effect on velocity, which in turn affects the pressure difference. So to get a more precise relationship, some uncertainty variables need be added into the simulation and analysis process.

The main research direction of this time is initial and pressure difference on fracture surface and there are other geometric influences on this model. As shown in the Table 1, the change in length of L1 is not particularly noticeable, so for the rest of the study, it is possible fix the velocity and change a geometric quantity, then solving the relationship by dimensional analysis.
Figure 6. A scatter plot of L1 and pressure difference

As exhibited in Fig. 6, the correlation between L1 and pressure difference is not obvious, and it is hard to fit into an expression. According to the forecast, the maximum pressure loss point is between 60mm and 70mm or 70mm and 80mm. In different cases, the pressure loss requirements are different, and the length of L1 can be pre-determined by specific case pair.

As for R1, a flow through a tube with a much smaller cross-section radius (effective cross section diameter less than 50%) is the main reason for observing the maximum pressure difference and causing the flow rate to increase to maximum [12]. Then, A strong relationship between pressure difference and fracture surface has been reported in the literature and this viewpoint also proves the result derived from the Π theorem, the magnitude of R1 plays a decisive role in the pressure change.

On the one hand, A further study with more focus on other geometric factors and physical factors is therefore suggested, through considering temperature, viscosity, micro-convection and other factors, the results may be more widely applicable after further fitting and it is closer to the real condition. What is surprising is that there is a method by which an approximate analytic solution can be solved. Similar results apply to domains with cylindrical outlets and disturbances in some interpreted part cylinders. In this case, a soliton flux carrier function can be constructed [13].

On the other hand, this study is based on a model, further research can be done by increasing the volume of the model, moreover, and the model is also built according to the actual situation.

6. Conclusions
In conclusion, a model composed of two large cylinders with a small cylinder inside is configured via ANSYS finite element analysis module. The initial injection velocity was set by FLUENT module and Π theorem is used to analyse the relationship between initial velocity and pressure difference. It is verified that for similar structures within a certain specification, \( \Delta P / v^2 \rho \) and \( 1/v \) have a quadratic relationship and found simultaneously that the radius of the section has the greatest influence on the pressure change. As the radius goes down, the pressure difference increases nonlinearly. These results will pave a path to analysis pressure difference more precisely for application and design based on such a structure. However, from the engineering perspective, reasonable error analysis is required to offer a reliable parameter range for practical application, e.g., in medical droppers, or in the process of monitoring seismic waves. In the future, more precise results can be obtained via finite element analysis with higher resolution and more physical quantities that affect the pressure difference.
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