Comparison of LMA and LOW Solar Solution Predictions in an SO(10) GUT Model

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Over the last few years the evidence for neutrino oscillations between the three known neutrino flavors ($\nu_e$, $\nu_\mu$, and $\nu_\tau$) has become increasingly convincing. The atmospheric neutrino flux measurements from the Super-Kamiokande (Super-K) experiment exhibit a deficit of muon neutrinos which varies with zenith angle (and hence baseline) in a way consistent with $\nu_\mu \rightarrow \nu_\tau$ oscillations \footnote{Cf.\n\textcopyright\textit{arXiv.org}}. In addition, recent combined evidence from Super-K and the SNO experiments \footnote{Cf.\n\textcopyright\textit{arXiv.org}} indicate that some electron-neutrinos from the sun are oscillating into muon and/or tau neutrinos. While the atmospheric neutrino data with $\nu_\mu \rightarrow \nu_\tau$ oscillations points to a small region of the mixing parameter space \footnote{Cf.\n\textcopyright\textit{arXiv.org}}, the solar neutrino data is consistent with at least two regions of parameter space \footnote{Cf.\n\textcopyright\textit{arXiv.org}}, corresponding to either the Large Mixing Angle (LMA) or to the LOW MSW \footnote{Cf.\n\textcopyright\textit{arXiv.org}} solution.

Neutrino oscillation data constrain Grand Unified Theories (GUTs) which provide a theory of flavor and relate lepton masses and mixings to quark masses and mixings. It is known that the presently implied neutrino mass scales can be accommodated naturally within the framework of GUTs by the seesaw mechanism \footnote{Cf.\n\textcopyright\textit{arXiv.org}}. In practice finding an explicit GUT model for the LMA solution has been found challenging. However, one example has been constructed by Barr and one us \footnote{Cf.\n\textcopyright\textit{arXiv.org}}. In this model the Dirac and Majorana neutrino mass matrices are intimately related. It has been shown by the present authors \footnote{Cf.\n\textcopyright\textit{arXiv.org}} that by varying the Majorana mass matrix parameters any point in the presently-allowed LMA region can be accommodated. In the present paper we show that the LOW region can also be realized in the model by choosing an appropriate texture for the right-handed Majorana mass matrix. In addition to the Majorana mass matrix, we also spell out the Dirac mass matrices, list the values of the associated input parameters and give results for the quark and charged lepton sectors. For each choice of the Majorana matrix we discuss the oscillation predictions. We use our results to illustrate how Neutrino Superbeams and Neutrino Factories \footnote{Cf.\n\textcopyright\textit{arXiv.org}} can further test this GUT model. Our results suggest that, independent of which is the preferred solution (LMA or LOW), Neutrino Superbeams and Factories will be necessary to identify the correct model, and that further investment in developing them should be encouraged.

Within the framework of three-flavor mixing, the flavor eigenstates $\nu_\alpha$ ($\alpha=e,\mu,\tau$) are related to the mass eigenstates $\nu_j$ ($j=1,2,3$) in vacuum by

$$\nu_\alpha = \sum_j U_{\alpha j} \nu_j, \quad U \equiv U_{\text{MNS}} \Phi_M$$

(1)

where $U$ is the unitary $3 \times 3$ Maki-Nakagawa-Sakata (MNS) mixing matrix \footnote{Cf.\n\textcopyright\textit{arXiv.org}} times a diagonal phase matrix $\Phi_M = \text{diag}(e^{i\chi_1}, e^{i\chi_2}, 1)$. The MNS matrix is conventionally parametrized by 3 mixing angles ($\theta_{23},\theta_{12},\theta_{13}$) and a CP-violating phase, $\delta_{CP}$:

$$U_{\text{MNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \xi \cr -s_{12}c_{23} - c_{12}s_{23}s_{13}\xi & c_{12}c_{23} - s_{12}s_{23}s_{13}\xi & s_{23}\xi \cr s_{12}s_{23} - c_{12}c_{23}s_{13}\xi & -c_{12}s_{23} - s_{12}c_{23}s_{13}\xi & c_{23}\xi \end{pmatrix}$$

(2)

where $c_{jk} \equiv \cos \theta_{jk}$, $s_{jk} \equiv \sin \theta_{jk}$ and $\xi = e^{i\delta_{CP}}$. The three angles can be restricted to the first quadrant, $0 \leq \theta_{ij} \leq \pi/2$, with $\delta_{CP}$ in the range $-\pi \leq \delta_{CP} \leq \pi$, though it proves advantageous to consider $\theta_{13}$ in the fourth quadrant for the LMA solutions.

The atmospheric neutrino oscillation data indicate that

$$\Delta m^2_{32} \simeq 3.0 \times 10^{-3} \text{ eV}^2,$$
$$\sin^2 2\theta_{\text{atm}} = 1.0, \ (\geq 0.89 \text{ at } 90\% \text{ c.l.}),$$

(3)

where $\Delta m^2_{ij} \equiv m_i^2 - m_j^2$ and $m_1$, $m_2$ and $m_3$ are the mass eigenstates. The atmospheric neutrino oscillation amplitude can be expressed solely in terms of the

\[\begin{align*}
\Delta m^2_{32} &\simeq 3.0 \times 10^{-3} \text{ eV}^2, \\
\sin^2 2\theta_{\text{atm}} &\equiv 1.0, \ (\geq 0.89 \text{ at } 90\% \text{ c.l.}),
\end{align*}\]
$U_{\text{MNS}}$ matrix elements and is given by $\sin^2 2\theta_{\text{atm}} = 4|U_{e3}|^2(1-|U_{e3}|^2) \approx 4|U_{e3}|^2|U_{\nu_2}|^2 = c_{13}^2 \sin^2 2\theta_{23}$. The approximation is valid because $|U_{e3}|^2$ is known to be small [10].

The solar neutrino oscillation data from Super-K indicate that, for the LMA solution, the allowed region is approximately bounded by

$$\Delta m_{21}^2 \approx (2.2 - 17) \times 10^{-5} \text{ eV}^2,$$

$$\sin^2 2\theta_{\text{sol}} \approx (0.6 - 0.9),$$

while for the LOW solution,

$$\Delta m_{21}^2 \approx (0.3 - 2) \times 10^{-7} \text{ eV}^2,$$

$$\tan^2 \theta_{12} \approx (0.6 - 1.2),$$

where the solar neutrino oscillation amplitude is

$$\sin^2 2\theta_{\text{sol}} = 4|U_{e1}|^2|1 - |U_{e1}|^2| \approx 4|U_{e1}|^2|U_{\nu_2}|^2,$$

while $\tan^2 \theta_{12} = |U_{\nu_2}|^2$. All nine quark and charged lepton masses, plus the three CKM angles and CP phase, are well-fitted with the eight input parameters

$$m_{U} \approx 113 \text{ GeV}, \quad m_{D} \approx 1 \text{ GeV},$$

$$\sigma = 1.78, \quad \epsilon = 0.145,$$

$$\delta = 0.0086, \quad \delta' = 0.0079,$$

$$\phi = 126^\circ, \quad \eta = 8 \times 10^{-6},$$

defined at the GUT scale to fit the low scale observables after evolution downward from $\Lambda_{\text{GUT}}$:

$$m_{l}(m_{t}) = 165 \text{ GeV}, \quad m_{t} = 1.777 \text{ GeV},$$

$$m_{u}(1 \text{ GeV}) = 4.5 \text{ MeV}, \quad m_{\mu} = 105.7 \text{ MeV},$$

$$V_{us} = 0.220, \quad V_{cb} = 0.0395, \quad \delta_{CP} = 64^\circ.$$  

These lead to the following predictions:

$$m_{l}(m_{t}) = 4.25 \text{ GeV}, \quad m_{c}(m_{c}) = 1.23 \text{ GeV},$$

$$m_{s}(1 \text{ GeV}) = 148 \text{ MeV}, \quad m_{d}(1 \text{ MeV}) = 7.9 \text{ MeV},$$

$$|V_{ub}/V_{cb}| = 0.080, \quad \sin 2\beta = 0.64.$$  

With no extra phases present, the vertex of the CKM unitary triangle occurs near the center of the presently allowed region with $\sin 2\beta \approx 0.64$, comparing favorably with recent results [11]. The Hermitian matrices $U^T U$, $D^T D$, and $N^T N$ are diagonalized with small left-handed rotations, $U_{U}$, $U_{D}$, $U_{N}$, respectively, where $U_{l}$ is diagonalized by a left-handed rotation, $U_{L}$. This accounts for the small value of $|V_{cb}| = |(U_{U}^T U_{D})_{cb}|$, while $|U_{e3}| = |(U_{U}^T U_{\nu})_{e3}|$ will turn out to be large for any reasonable right-handed Majorana mass matrix, $M_{R}$ [12].

The effective light neutrino mass matrix, $M_{\nu}$, is obtained from the seesaw mechanism once $M_{R}$ is specified. While the large atmospheric neutrino mixing $\nu_{\mu} \leftrightarrow \nu_{\tau}$ arises primarily from the structure of the charged lepton mass matrix, the structure of $M_{R}$ determines the type of $\nu_{\tau} \leftrightarrow \nu_{\mu}, \nu_{\tau}$ solar neutrino mixing.

To obtain the LMA solution requires some fine-tuning and a hierarchical structure for $M_{R}$, but this can be explained in terms of Froggatt-Nielsen diagrams [13]. Here we restrict our attention to a slightly less general form for $M_{R}$ than that considered in [12].

$$M_{R} = \begin{pmatrix} b\eta^2 & -b\eta & a\eta \\ -b\eta & \epsilon^2 & -\epsilon \\ a\eta & -\epsilon & 1 \end{pmatrix} \Lambda_{R},$$

where the parameters $\epsilon$ and $\eta$ are those introduced in Eq. (10) for the Dirac sector. This structure for $M_{R}$ can be understood as arising from one Higgs singlet which induces a $\Delta L = 2$ transition and contributes to all nine matrix elements while, by virtue of its flavor charge assignment, a second Higgs singlet breaks lepton number but modifies only the 13 and 31 elements of $M_{R}$. As shown in detail in [12], we can introduce additional CP violation by assigning a relative phase to the two lepton number breaking Higgs singlets, whereby we set

$$a = b - a' e^{i\phi'}.$$  

On the other hand, we find the LOW solution can be obtained with the simple hierarchical structure for $M_{R}$,

$$M_{R} = \begin{pmatrix} e & d & 0 \\ d & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Lambda_{R},$$

where by the flavor charge assignments, one Higgs singlet inducing a $\Delta L = 2$ transition contributes to the 12, 21 and 33 elements, while a second Higgs singlet also breaks lepton number but contributes only to the 11 matrix element. For simplicity we keep both $d$ and $e$ real, since the leptonic CP phase is inaccessible to measurement for $\Delta m_{21}^2$ values in the LOW region.

For either the LMA or LOW version, $M_{\nu}$ is then obtained by the seesaw formula [12], $M_{\nu} = N^T M_{R}^{-1} N$. With $M_{\nu}$ complex symmetric, both $M_{\nu}^T M_{\nu}$ and $M_{\nu}$ itself can be diagonalized by the same unitary transformation, $U_{\nu}$, where in the latter case we find

$$U_{\nu}^T M_{\nu} U_{\nu} = \text{diag}(m_{1}, -m_{2}, m_{3}).$$
model parameters, are, in effect, only two additional real dimensionless GUT
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With real light neutrino masses, Uν can not be arbitrarily
phase transformed and is uniquely specified up to sign
changes on its column eigenvectors [14]. Hence U_{MNS}
is found by applying phase transformations on U_{11}U_{11} to
bring U_{11}U_{11} into the parametric form of Eq. (3) whereby
the e1, e2, µ3 and τ3 elements are real and positive, the
real parts of the µ2 and τ1 elements are positive, while
the real parts of the µ1 and τ2 elements are negative. The
inverse phase transformation of that applied on the right
can then be identified with the Majorana phase matrix,
ΦM of Eq. (3). The evolution of the predicted values
between the GUT scale and the low scales can be safely
ignored [14], since tan β \simeq 5 is moderately low and the
neutrino mass spectrum is hierarchical with the opposite
CP parities present in Eq. (3).

We can now examine the viable region of GUT model parameter space that is consistent with either
the LMA or LOW solar neutrino solution, and
explore the predicted relationships among the observables
sin^2 2θ_{23}, sin^2 2θ_{12}, sin^2 2θ_{13}, δ_{CP}, Δm^2_{12}, and Δm^2_{21}. We shall emphasize here the simpler cases in which there are, in effect, only two additional real dimensionless GUT model parameters, a and b in the LMA version or d and c
in the LOW version. In either version, the third parameter
Λ_R sets the scale of Δm^2_{12}. The more general CP
results obtained for the LMA solution with the presence
of a complex parameter a in Eq. (11) have been explored in
detail in [6].

The viable region of GUT model parameter space consistent
with the LMA solar solution is shown in Fig. 1. Both
parameters a and b are constrained by the data to be
close to unity, with 1.0 \lesssim a \lesssim 2.4 and 1.8 \lesssim b \lesssim 5.2.
Superimposed on the allowed region, Fig. 1(a) shows
contours of constant sin^2 2θ_{12} and contours of constant
Δm^2_{21}. Figure 1(b) similarly displays the allowed region
with contours of constant sin^2 2θ_{12} and sin^2 2θ_{13} super-
imposed. The nearly parallel nature of the contours of
Δm^2_{21} in (a) and sin^2 2θ_{13} in (b) indicates a strong
correlation between them. As the predicted Δm^2_{21}
increases, the predicted sin^2 2θ_{13} decreases. Note that if the LMA
solution is indeed the correct solution, KamLAND [14] is
expected to provide measurements of Δm^2_{21} and sin^2 2θ_{12}
to a precision of about 10% [14]. From these measure-
ments the model parameters a and b can be determined from
Fig. 1(a). Figure 1(b) can then be used to give a
prediction for sin^2 2θ_{13} with a precision also of order 10%.

FIG. 1: The viable region of GUT parameter space consistent with the present bounds on the LMA MSW solution. Contours of constant sin^2 2θ_{12} are shown together with (a) contours of constant Δm^2_{12} and (b) contours of sin^2 2θ_{13}.

TABLE I: List of four points selected in the LMA allowed parameter region to illustrate the neutrino oscillation parameter predictions of the GUT model. Here the CP phase δ_{CP} arises from φ in L alone, as no phase φ' has been introduced in M_R.

| a  | b   | Δm^2_{21} (eV^2) | Δm^2_{12} (eV^2) | tan^2 θ_{12} | sin^2 2θ_{12} | sin^2 2θ_{13} | sin^2 2θ_{23} | δ_{CP} |
|----|-----|-----------------|-----------------|-------------|--------------|---------------|---------------|--------|
| 1.0| 2.0 | 6.5 \times 10^{-5} | 3.2 \times 10^{-3} | 0.49        | 0.88         | 0.994         | 0.0008        | -4°    |
| 1.2| 2.8 | 3.3 \times 10^{-5} | 3.2 \times 10^{-3} | 0.43        | 0.84         | 0.980         | 0.0038        | -1°    |
| 1.6| 2.9 | 6.1 \times 10^{-5} | 3.2 \times 10^{-3} | 0.35        | 0.77         | 0.998         | 0.0015        | -3°    |
| 1.7| 2.7 | 10.9 \times 10^{-5} | 3.2 \times 10^{-3} | 0.32        | 0.73         | 0.996         | 0.00008       | -14°   |
| 1.7| 3.4 | 4.0 \times 10^{-5} | 3.2 \times 10^{-3} | 0.33        | 0.75         | 0.992         | 0.0033        | -2°    |
| 2.2| 3.5 | 8.8 \times 10^{-5} | 3.2 \times 10^{-3} | 0.24        | 0.63         | 0.996         | 0.0008        | -4°    |
we have selected six points in the LMA allowed parameter region to illustrate the neutrino oscillation parameter predictions of the GUT model.

| $d$           | $e$       | $\Delta m_{21}^2$ (eV$^2$) | $\Delta m_{32}^2$ (eV$^2$) | $\tan^2 \theta_{12}$ | $\sin^2 2\theta_{12}$ | $\sin^2 2\theta_{23}$ | $\sin^2 2\theta_{13}$ |
|---------------|-----------|----------------------------|----------------------------|-----------------------|------------------------|------------------------|------------------------|
| $-4.2 \times 10^{-5}$ | $10.0 \times 10^{-9}$ | $1.20 \times 10^{-7}$ | $3.0 \times 10^{-3}$ | 0.56 | 0.906 | 0.911 | 0.028 |
| $-4.1 \times 10^{-5}$ | $4.0 \times 10^{-9}$ | $0.52 \times 10^{-7}$ | $3.0 \times 10^{-3}$ | 0.81 | 0.975 | 0.899 | 0.027 |
| $-3.6 \times 10^{-5}$ | $3.0 \times 10^{-9}$ | $0.64 \times 10^{-7}$ | $3.0 \times 10^{-3}$ | 0.86 | 0.980 | 0.898 | 0.030 |
| $3.6 \times 10^{-5}$ | $5.0 \times 10^{-9}$ | $0.98 \times 10^{-7}$ | $3.0 \times 10^{-3}$ | 1.00 | 0.999 | 0.914 | 0.0016 |
| $5.3 \times 10^{-5}$ | $10.0 \times 10^{-9}$ | $0.50 \times 10^{-7}$ | $3.0 \times 10^{-3}$ | 0.82 | 0.980 | 0.912 | 0.0039 |
| $5.0 \times 10^{-5}$ | $13.0 \times 10^{-9}$ | $0.85 \times 10^{-7}$ | $3.0 \times 10^{-3}$ | 0.70 | 0.966 | 0.918 | 0.0033 |

In Table II we have selected six points in the LMA allowed parameter region to illustrate the neutrino oscillation parameter predictions of the GUT model. The correlations noted above are evident. It is also striking how nearly maximal are the values for the atmospheric mixing parameter, $\sin^2 2\theta_{23}$. This apparently arises not from some additionally imposed symmetry but rather from the fine tuning between the right-handed Majorana and Dirac neutrino mass matrices, cf. Eqs. (6) and (10). However, if an additional phase is incorporated into $M_R$ for this LMA case as indicated in Eq. (11), the maximal-ity in the GUT model in question. A first measurement of $\sin^2 2\theta_{13}$ would resolve the ambiguity, and a precise measurement would test the model. For the negative $d$ version, a SuperBeam facility capable of probing down to $\sin^2 2\theta_{13} \sim 0.003$ will be able to test the model, while for the positive $d$ version the complete parameter space can only be tested with a Neutrino Factory. Table II gives the relevant mixing solutions for a set of six points. In contrast to the LMA results with small CP phases, we see that the atmospheric mixing for the LOW solution is large but not nearly so maximal.

In conclusion, we have studied predictions for a particular but representative GUT model that can accommodate both the LMA and LOW solar neutrino solutions. We find that precise measurements of $\sin^2 2\theta_{12}$, $\Delta m_{21}^2$, and $\sin^2 2\theta_{13}$ are needed to test the theory. Given the observed near maximal value of $\sin^2 2\theta_{23}$, the LMA solution, which requires some fine tuning of the $M_R$ matrix, is favored by the model. The model then predicts that the CP phase $\delta_{CP}$ is small and $\sin^2 2\theta_{13} \lesssim 0.006$. For the LOW solution which requires no fine tuning, $\sin^2 2\theta_{13}$ can be as small as this, or an order of magnitude larger, depending upon the sign of the $d$ model parameter in $M_R$. Our work suggests progress on testing GUTs can be made with Neutrino Superbeams, but ultimately a Neutrino Factory will be needed to help identify the correct
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