Modeling and Simulation of Vehicle Windshield Wiper System using $H^\infty$ Loop Shaping and Robust Pole Placement Controllers

Mustefa Jibril$^1$, Messay Tadese$^2$, Eliyas Alemayehu Tadese$^3$

$^1$,$^2$School of Electrical & Computer Engineering, Dire Dawa Institute of Technology, Dire Dawa, Ethiopia
$^3$Faculty of Electrical & Computer Engineering, Jimma Institute of Technology, Jimma, Ethiopia
mustefazinet1981@gmail.com

Abstract: Vehicle windshield wiper system increases the driving safety by contributing a clear shot viewing to the driver. In this paper, modelling, designing and simulation of a vehicle windshield wiper system with robust control theory is done successfully. $H^\infty$ loop shaping and robust pole placement controllers are used to improve the wiping speed by tracking a reference speed signals. The reference speed signals used in this paper are step and sine wave signals. Comparison of the $H^\infty$ loop shaping and robust pole placement controllers based on the two reference signals is done and convincing results have been obtained. Finally the comparative results prove the effectiveness of the proposed $H^\infty$ Loop Shaping controller to improve the wiping mechanism for the given two reference signals.

Keywords: $H^\infty$ Loop Shaping, Robust Pole Placement, Windshield

1. Introduction

Driving a vehicle is complicated in harsh weather condition without using glass wiper system. Controlling the wiper speed based on the change in weather condition is one of the research area in automotive industries now a day. Recently, the wiper speed is adjusted manually controlled by the driver. Dc motors are used to drive the wiper system based on feed forward techniques. In this paper a separately excited Dc motor is designed and controlled using robust control method to improve the feed forward wiper system performance.

2. Mathematical Modelling

Consider the electromechanical car mirror wiper system shown in Figure 1.

![Electromechanical car mirror wiper system](image)

The motor shown is a servomotor, a dc motor designed in particular for use in a control system. The operation of this device is as follows: A fixed voltage is carried out to the field winding. A voltage is implemented as an input to the servo motor and the angular position $c$ of the wiper arm is the output of the device. The input voltage is implemented to the armature circuit of the dc motor. A constant voltage is carried out to the field winding. If an errors exists, the motor develops a torque to rotate the output load in the sort of way as to reduce the error to zero. For constant field current, the torque evolved by using the motor is

$$T = K_1i_a$$ (1)

Where $K_1$ is the motor torque constant and $i_a$ is the armature current.

When the armature is rotating, a voltage proportional to the fabricated from the flux and angular velocity is brought on within the armature. For a constant flux, the brought about voltage $e_b$ is without delay proportional to the angular velocity

$$e_b = K_2\frac{d\theta}{dt}$$ (2)
Where \( e_b \) is the back emf, \( K_2 \) is the back emf constant of the motor, and \( \theta \) is the angular displacement of the motor shaft.

The speed of an armature-managed dc servomotor is managed via the armature voltage \( e(t) \). The differential equation for the armature circuit is

\[
 L_a \frac{d}{dt}i_a(t) + R_a i_a(t) + K_2 \frac{d\theta}{dt} = e(t) \quad (3)
\]

Substituting Equation (2) in to Equation (3) yields:

\[
 L_a \frac{d}{dt}i_a(t) + R_a i_a(t) + K_2 \frac{d\theta}{dt} = e(t) \quad (4)
\]

Taking the Laplace transform the equation will be

\[
 sL_\omega I_a(s) + R_\omega I_a(s) + J_\theta \Theta(s) = E(s) \quad (5)
\]

The equation for torque equilibrium is

\[
 J_\theta \frac{d^2 \theta}{dt^2} + b_\theta \frac{d\theta}{dt} = T = K_i i_a(t) \quad (6)
\]

Taking the Laplace transform the equation will be

\[
 s^2 J_\theta \Theta(s) + b_\theta \Theta(s) = K_i I_a(s) \quad (7)
\]

Where \( J_0 \) is the inertia of the combination of the motor, load, and gear train referred to the motor shaft and \( b_0 \) is the viscous-friction coefficient of the aggregate of the motor, load, and gear train mentioned the motor shaft.

By eliminating \( I_a(s) \) from Equations (5) and (7), we obtain

\[
 \Theta(s) = \frac{K_i}{s(L_\omega J_\theta s^3 + J_\theta R_\omega s^2 + (L_\omega b_\theta + K_2)s + b_\theta R_\omega)} \quad (8)
\]

The wiper arm speed can be evaluated by using an integrator to the wiper arm position as

\[
 W(s) = \frac{1}{s} C(s) \quad (9)
\]

Substituting Equation (10) in to Equations (9) and to Equations (8) gives us the transfer function between the applied voltage and the wiper speed as

\[
 W(s) = \frac{nK_i}{s(L_\omega J_\theta s^3 + J_\theta R_\omega s^2 + (L_\omega b_\theta + K_2)s + b_\theta R_\omega)} \quad (11)
\]

The parameters of the system is shown in Table 1 below.

| No | Parameters                          | Symbol | Values       |
|----|------------------------------------|--------|--------------|
| 1  | Inertia of the motor, load, and gear train | \( J_0 \) | \( 10^{-2} \text{ Kg m}^2 \) |
| 2  | Viscous-friction coefficient        | \( b_0 \) | \( 0.2 \text{ N ms / rad} \) |
| 3  | Back emf constant                   | \( K_2 \) | \( 0.8 \text{ V s / rad} \) |
| 4  | Motor torque constant               | \( K_1 \) | \( 0.7 \text{ N m / amp} \) |
| 5  | Motor Resistance                    | \( R_a \) | \( 1.2 \Omega \) |
| 6  | Motor Inductance                    | \( L_a \) | \( 3 \times 10^{-2} \text{ H} \) |
| 7  | Gear ratio                          | \( n \) | \( 25 \) |

Numerically the transfer function is

\[
 G(s) = \frac{17500}{3s^4 + 12s^3 + 806s^2 + 240s}
\]

The state space form will be

\[
 \begin{bmatrix}
 1 & -40 & -2686.7 & -800 & 0 \\
 1 & 0 & 0 & 0 & 0 \\
 0 & 1 & 0 & 0 & 0 \\
 0 & 0 & 1 & 0 & 0 \\
 0 & 0 & 0 & 58333 & 0 
\end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
\]

### 3. Proposed Controllers Design

#### 3.1 H infinity Loop Shaping using Glover-McFarlane method Controller Design

The block diagram of the car mirror wiper system with H infinity Loop shaping design using Glover-McFarlane method is shown in Figure 2 below.

A feedback controller, \( K_s \), is synthesized that robustly stabilizes the normalized left coprime factorization of \( G \), with a balance margin. It may be proven that the frequency response of \( K_s W_2 G W_1 \) will
be much like that of W2GW1. On the other hand, if the viable gain is simply too large, this will probably indicate an overdesigned case in appreciable of the robustness, which means that the performance of the system can also in all likelihood be progressed by the usage of a larger in computing $K_s$. The final feedback controller, $K_{\text{final}}$, is then constructed with the aid of combining the $H^\infty$ controller $K_s$, with the weighting functions $W_1$ and $W_2$ such that

$$K_{\text{final}} = W_1K_sW_2$$ (12)

We choose a pre compensator, $W_1$, and a post compensator, $W_2$ transfer functions as

$$W_1 = \frac{1}{s + 24}, \quad W_2 = \frac{1}{s + 13}$$

The $H^\infty$ controller transfer function is

$$K_s = \frac{-1.419s^5 - 109.3s^4 - 6357s^3 - 1.6e05s^2 - 1.235e06s - 3.84e05}{s^7 + 114.3s^6 + 7678s^5 + 3.05e05s^4 + 6.536e06s^3 + 6.968e07s^2 + 3.027e08s + 1.7e08}$$

Robust Pole Placement Controller Design

In a typical feedback manage system, the output, $y$, is fed back to the summing junction. It is now that the topology of the layout changes. Instead of feeding again $y$, what if we feed back all of the state variables? If each state variable is fed back to the manipulated, $u$, through a gain, $k_i$, there might be $n$ gains, $k_i$ that would be adjusted to yield the required closed-loop pole values. The feedback via the profits, $k_i$, is represented by way of the feedback vector $-K$.

The block diagram of the car mirror wiper system with robust pole placement method is shown in Figure 3.

![Figure 3 car mirror wiper system with robust pole placement method](image)

The state equations for the closed-loop system of Figure 3 can be written by inspection as

$$\dot{x} = Ax + Bu = Ax + B(-Kx) = (A - BK)x$$

$$y = Cx$$

The poles for this system is chosen as

$$P = [-1 - 2i, -1 + 2i, 4 - 3i, 4 + 3i]$$

Solving using Matlab the robust pole placement algorithm gain will be

$$K = \begin{bmatrix} -46 & -26727 & -790 & 125 \end{bmatrix}$$

4. Result and Discussion

In this section, the Simulink model design and simulation of the vehicle windshield wiper system using $H^\infty$ loop shaping and robust pole placement controllers by comparing the two proposed controllers for tracking the step and sine wave speed references.

4.1 Comparison of the proposed controllers for tracking the step speed reference

The Simulink model of the vehicle windshield wiper system using $H^\infty$ loop shaping and robust pole placement controllers for tracking the step speed reference is shown in Figure 4 below.

![Figure 4 Simulink model of the vehicle windshield wiper system using $H^\infty$ Loop Shaping and Robust Pole Placement Controllers for tracking the step speed reference](image)

The wiper system performance for the proposed controllers using a step reference (step change from 0 to $6 \text{ m/s}$) of the wiper speed simulation is shown in Figure 5 below.

![Figure 5 Simulation result for a step reference input](image)
The data of the rise time, percentage overshoot, settling time and peak value is shown in Table 1.

### Table 1 Step response data

| No | Performance Data | Robust Pole Placement | H infinity Loop Shaping |
|----|------------------|-----------------------|------------------------|
| 1  | Rise time        | 1.2 sec               | 1.22 sec               |
| 2  | Per. overshoot   | 53.4 %                | 40 %                   |
| 3  | Settling time    | 8 sec                 | 6 sec                  |
| 4  | Peak value       | 9.2 m                 | 8.4 m                  |

### 4.2 Comparison of the proposed controllers for tracking the Sine Wave speed reference

The Simulink model of the vehicle windshield wiper system using H infinity loop shaping and robust pole placement controllers for tracking the sine wave speed reference is shown in Figure 6 below.

![Simulink model](image)

Figure 6 Simulink model of the vehicle windshield wiper system using H infinity loop shaping and robust pole placement controllers for tracking the sine speed reference

The wiper system performance for the proposed controllers using a sine wave reference (wiper moving in the forward and reverse with 6 m/s) of the wiper speed simulation is shown in Figure 7 below.

![Simulation result](image)

Figure 7 Simulation result for a sine wave reference input

The simulation result shows that the vehicle windshield wiper system with H infinity loop shaping controller track the reference speed better than the vehicle windshield wiper system with robust pole placement controller.

### 5. Conclusion

The vehicle windshield wiper system is designed and simulated based on the given control signals using Matlab/Simulink and a promising result have been analyzed. The performance of the vehicle windshield wiper system is tested for wiping speed regulation using track a signal method. Comparison of the vehicle windshield wiper system with H infinity loop shaping and robust pole placement controllers is done for a step and sine wave reference speed signals and the vehicle windshield wiper system with H infinity loop shaping controller shows a good result in improving the wiping mechanism in almost the all performance measures taken. Finally the comparative results prove the effectiveness of the proposed H ∞ Loop Shaping controller to improve the wiping mechanism for the given two reference signals.

### Reference

1. Vijay S. et al. “Automatic Rain Operated Wiper and Headlight Dim and Bright Controller” International Journal of Innovative Research in Science, Engineering and Technology, Vol. 9, Issue 2, February 2020.
2. Punam W. et al. “An Automated Wiper System for Vehicles” International Journal for Research in Applied Science & Engineering Technology (IJRASET), Volume 7 Issue IV, Apr 2019.
3. Matthew B. et al. “Windshield Wipers on Connected Vehicles Produce High Accuracy Rainfall Maps” Journal of Scientific Reports, Vol. 9, 2019.
4. Varshitha P J et al. “Improvement of Auto Wiper Controller According to Rain Force” International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering, Vol. 7, Issue 5, May 2018.
5. Prajakta C. et al. “Automatic Rain Operated Wiper and Dimmer for Vehicle” International Research Journal of Engineering and Technology (IRJET), Volume: 03 Issue: 04, 2016.
6. Lubna A. et al."Design and Implementation of a Reconfigurable Automatic Rain Sensitive Windshield Wiper” International Journal of Engineering & Technology Science, Vol. 8, Issue 2, pp. 73-82, 2015.
7. Mark D. et al. “Dynamic Modelling and Experimental Validation of an Automotive Windshield Wiper System for Hardware in the Loop Simulation” Journal of Systems Science & Control Engineering, Vol. 3, Issue 1, 2015.

8. Fazle E. et al. “Intelligent Windshield for Automotive” Conference: International Conference on Computer & Information Technology (ICCIT), Vol. 17, Issue. 2, 2014.

9/21/2020