Optomechanical devices based on traveling-wave microresonators

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We theoretically study the unique applications of optomechanics based on traveling-wave microresonators, where the optomechanical coupling of degenerate modes can be enhanced selectively by optically pumping in different directions. We show that the unique features of degenerate optical modes can be applied to the entangled photon generation of clockwise and counter-clockwise optical modes, and the nonclassicality of entangled photon pair is discussed. The coherent coupling between the clockwise and counter-clockwise optical modes and two acoustic modes is also studied, in which the relative phase of the optomechanical couplings plays a key role in the optical non-reciprocity. The parity-time symmetry of acoustic modes can be observed in the slightly deformed microresonator with the interaction of forward and backward stimulated Brillouin Scattering in the triple-resonance system. In addition, the degenerate modes are in the decoherence-free subspace, which is robust against environmental noises. Based on parameters realized in recent experiments, these optomechanical devices should be readily achievable.

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I. INTRODUCTION

Optomechanical devices [1–5] have attracted considerable attentions in integrated photonic circuits [6, 8], memory [9, 10] and high precision measurements [11–13] for practical applications. There are also excellent test bed for investigating the quantum behaviors at macroscopic level [20] and are attractive for fundamental studies on physics, such as the gravitational wave detection [1–13], quantum-to-classical transitions [16, 17], and quantum information processing [18, 19]. In past few years, many remarkable progresses have been achieved both in theory and experiment, such as ground state cooling of the mechanical resonator [21], optomechanically induced transparency [22, 23], optomechanical entanglement [24] and squeezing [25], and optical frequency conversion [26–27].

Recently, more and more studies on multimode optomechanics (MOM) [1] have been carried out, for more functional devices and applications. Because they provide more degrees of freedom, people can realize new phenomena that are not possible in single mode optomechanical systems, such as the optical non-reciprocity [28–30], parity-time symmetry [31], chiral symmetry breaking [32], and topological energy transfer [33]. Different from the triple-resonance enhanced optomechanical interaction [34, 35], the MOM in this paper is referring to more than one optical signal mode or mechanical mode. Many schemes [19, 36–38] have been proposed to realize the MOM [39] based on the coupled cavities or hybrid optomechanical systems. However, in practical physical systems, it is challenging to fabricate multiple optical or mechanical resonators that have the frequencies matching each other. In additional to the frequency mismatching problem, the MOM also requires complex design for the efficient coupling between separated resonators.

In this paper, we propose to use the traveling-wave microresonators to explore the MOM, which allows many potential new phenomena and applications. The whispering-gallery microcavities [20, 26, 40–45] is one of the candidate systems, which supports degenerate clockwise (CW) and counter-clockwise (CCW) traveling-wave modes, which have already been used in optomechanically induced non-reciprocity [47, 50]. By using the degenerate oppositely propagating optical modes, we demonstrate the entangled photon generation and the controllable optical non-reciprocity. The CW and CCW acoustic waves also enable the study of the non-Hermitian dynamics of phonons, and the parity-time symmetry of acoustic modes is proposed in the slightly deformed traveling-wave microresonator. The traveling-wave microresonator provides an excellent platform for studying the optomechanics with multiple degrees of freedom, and the schemes proposed in this work are feasible for experiments. The integration of all modes in single solid state microresonator also holds the great advantage that the noise of degenerate CW and CCW traveling-
wave modes could cancel each other, where the mode pair actually forms a decoherence-free subspace, which makes optomechanical devices robust against environmental noises and holds great potential for applications in quantum information processing.

The paper is organized as follows. In Sec. II, we show the CW and CCW traveling-wave modes in traveling-wave microresonators, and develop the Hamiltonian description of the MOM. Based on this model, we propose three unique applications in this system: the entangled photon generation of CW and CCW modes in Sec. III, the non-reciprocal conversion between CW and CCW optical modes in Sec. IV, and the phononic parity-time symmetry in Sec. V. Based on parameters realized with existing technology in recent experiments, these optomechanical devices should be readily achievable. We give a discussion about the potential advantages of the traveling-wave optomechanical system in In Sec. VI, and then summarize in Sec. VII.

II. THE SYSTEM

In this work, we consider an MOM system as shown in Fig. 1 (a): an integrated non-magnetic dielectric microresonator served as both optical and mechanical resonator. The microresonator possesses the rotational symmetry, leading to the degeneracy of the optical or mechanical modes that are propagating in the CW and the CCW directions. In general, the Hamiltonian description of the system can be written as

\[ H = H_0 + H_i + H_d, \]

in which \( \hbar = 1 \)

\[ H_0 = \sum_{k=-N}^{N} \omega_{c,k} a_k^\dagger a_k + \sum_{l=-M}^{M} \omega_{m,l} b_l^\dagger b_l \]  

is the system Hamiltonian, which describes the optical and mechanical modes. Due to the rotational symmetry, both optical and mechanical mode fields can be represented by the traveling field \( \overrightarrow{E}_k(r, z) e^{-ik\phi} \) and \( \overrightarrow{u}_l(r, z) e^{-il\phi} \), with \( r, z, \phi \) representing the cylindrical coordinate, \( k \) and \( l \) are the azimuthal quantum numbers. Therefore, we use \( a_k \) and \( b_l \) as the annihilation operators of the \( k \)-th optical and \( l \)-th mechanical modes, and the \( \omega_{c,k} \) and \( \omega_{m,l} \) are the corresponding mode frequencies. As mentioned above, we have \( \omega_{c,k} = \omega_{c,-k} \) and \( \omega_{m,l} = \omega_{m,-l} \) due to the CW and CCW symmetry.

The mechanical motion of the dielectric microresonator can modify the optical mode frequency, through the geometry effect and the photo-elastic effect \[41, 42\], which leads to the optomechanical interaction Hamiltonian as

\[ H_I = \sum_{j,k,l} g_{j,k,l} \left( a_j + a_j^\dagger \right) \left( a_k + a_k^\dagger \right) \left( b_l + b_l^\dagger \right). \]  

The coupling strength \( g_{j,k,l} \) is determined by the overlap of the optical electric field and the mechanical displacements or strain \[41–43\]. Since the integral \( \int d\phi e^{-i(j+k+l)\phi} = \delta(j+k+l) \), the optomechanical interaction in this traveling-wave microresonator should satisfy the selection rule that \( j \pm k \pm l = 0 \). We can separate the interaction Hamiltonians into two categories:

1. Dispersive optomechanics as shown in Fig. 1 (b).

\[ H_{I,1} = \sum_k g_{k,k,0} a_k^\dagger a_k \left( b_0 + b_0^\dagger \right). \]

Here, \( g_{k,k,0} \) is the single-photon coupling rate, which corresponds to the optical cavity shift per phonon excitation. This dispersive optomechanics indicates the mechanical motion shifts the optical mode frequency. Due to the selection rule, only the mechanical mode with \( l = 0 \), i.e. the breath-type modes \[22, 47\] can give rise to such interaction.

2. Triple-resonant optomechanics as shown in Fig. 1 (c).

\[ H_{I,2} = \sum_{j,k} g_{j,j-k} \left( a_j^\dagger a_k b_{j-k} + a_j a_k^\dagger b_{j-k} \right). \]

It describes a Brillouin scattering in this triple-resonant system \[50\], where a photon in mode \( j \) split into a photon in mode \( k \) and a phonon in mode \( j-k \). Here, \( g_{j,j-k} \) is the Brillouin scattering coupling strength, which is non-zero only when the energy \( \omega_{m} = \omega_{c,j} - \omega_{c,k} \) and momentum \( l = j-k \) conservation are satisfied.
The term \( H_d \) describes the driving of the optical modes and also the optical signal input to the system. With the weak interaction coupling, we can use strong optical driving fields to enhance optomechanical coupling strength \([26, 51]\), and the optomechanical system evolves by optical signal input.

### III. DEGENERATE PHOTON PAIR GENERATION

For dispersive optomechanics, we can choose the two degenerate CCW (\(-k\)) and CW (\(k\)) optical modes that are coupled to the breath mechanical mode, thus the interaction Hamiltonian is

\[
H = g_{k,k,0} \left( a_k^\dagger a_k + a_k^\dagger a_{-k} \right) \left( b_0 + b_0^\dagger \right).
\] (6)

The system is a general bosonic three-mode system, as shown in the Fig. 2(a). Such model can also be realized in a ring type resonator coupling to a local mechanical oscillator \([1]\). Since both \(a_k\) and \(a_{-k}\) can coherently couple with the mechanical mode, it is possible to realize the interaction between \(a_k\) and \(a_{-k}\) mediated by the mechanical mode. It has been demonstrated that the red-detuned laser pump on both CW and CCW direction can induce the effective coupling between the CW and CCW signal photons. If we pump one direction with blue-detuned laser, there will generate a photon-phonon pair (anti-Stokes process), and the red detuned laser on the other direction will convert the phonon to photon (Stokes process), thus generate entangled photons propagate in opposite directions.

To study the entangling interaction between CW and CCW signal photons, we assume two driving fields of the CW and CCW optical modes with frequency detuned from the cavity as \(\Delta_{d,k'} = \omega_{d,k'} - \omega_{c,k'}\), with \(k' = \pm k\), where \(\omega_{d,k'}\) is the drive laser frequency. Let the detuning \(-\Delta_{d,k}\) be \(\Delta_{d,-k}\), and apply the standard linearization treatment, the effective Hamiltonian can be written as

\[
H_{\text{lin}} = -\Delta_{d,k} a_k^\dagger a_k - \Delta_{d,-k} a_{-k}^\dagger a_{-k} + \omega_m b_0^\dagger b_0 + G_k \left( a_0^\dagger b_0 + a_k b_k^\dagger \right) + G_{-k} \left( a_{-k}^\dagger b_0^\dagger + a_{-k} b_{-k}^\dagger \right).
\] (7)

Here, the coupling strength \(G_{k(-k)} = g_{k,k,0} a_k a_{k(-k)}\) is enhanced by the driving field amplitude \(\delta_k\). The rotating wave approximation has been applied above with the assumption \(\omega_m \gg G_{k(-k)}\), \(\kappa\), \(\gamma_m\), where \(\kappa\) and \(\gamma_m\) are the cavity and mechanical damping rates, respectively.

For studying the entanglement of the photon pair, a weak signal is sent into CW optical mode as

\[
H_s = i \sqrt{\kappa n_{\text{in}}} \left( a_k^\dagger e^{-i(\omega_s - \omega_{d,k})t} - H.c. \right),
\] (8)

where \(\kappa_{\text{in}}\) is the external coupling to the cavity. The dynamics of the density matrix (\(\rho\)) of the system is governed by the Master equation \([51]\), which reads

\[
\frac{d\rho}{dt} = -i \left[ H_{\text{lin}} + H_s, \rho \right] + \kappa \left( \mathcal{L}(a_k) + \mathcal{L}(a_{-k}) \right)
+ (n_{\text{th}} + 1) \gamma_m \mathcal{L}(b_0) + n_{\text{th}} \gamma_m \mathcal{L}(b_0^\dagger),
\] (9)

where \(\mathcal{L}(A) = A \rho A^\dagger - (A^\dagger A \rho + \rho A^\dagger A)/2\) is the Lindblad superoperator for any operator \(A\), and \(n_{\text{th}}\) is the phonon thermal excitation. In our calculation, we choose the damping rate of optical mode \(\kappa/2\pi = 15\) MHz, the damping rate of mechanical mode \(\gamma_m/2\pi = 22\) kHz, and \(\kappa_{\text{in}} = \kappa/2\) from the experimental parameters \([42]\).

![FIG. 2: (Color online) (a) Schematic diagram of entangled photon generation. (b) The parameter \(I\) as a function of the detuning \(\delta_k\) for \(G_k/2\pi = 0.3\) MHz and \(n_{\text{th}} = 0, 0.1, 0.2\). (c) The parameter \(I\) as a function of the detuning \(\delta_k\) for \(n_{\text{th}} = 0.2\) and \(G_k/2\pi = 0.3, 0.4, 0.5\) MHz. (d) The parameter \(I\) as a function of the effective coupling \(G_k\) for the detuning \(\delta_k = 0\) and \(n_{\text{th}} = 0.2\). Other parameters are \(G_{-k}/2\pi = 0.1\) MHz and \(\epsilon_s, \kappa_0 = 0.1\) MHz.

To characterize the nonclassicality of the two-mode field, we introduce the parameter \([52]\)

\[
I = \frac{\langle a_k^\dagger a_k^\dagger a_{-k}^\dagger a_{-k} \rangle}{\langle a_k^\dagger a_k^\dagger a_{-k}^\dagger a_{-k} \rangle} - 1,
\] (10)

as a measure. For coherent states, \(I = 0\), while for nonclassical states we will have \(I < 0\) \([52, 53]\).

In the Fig 2, we numerically investigate the nonclassical parameter \(I\) of the system in equilibrium state for various \(n_{\text{th}}\) and \(G_k\). We should notice that the blue-detuned drive produces the photon-phonon pairs, and it would also induce instability of the system due to the amplification. Using the well-known Routh-Hurwitz stability conditions \([54]\), we derive the necessary and sufficient
stability condition for this model as $G_k^2 - G_{2-k}^2 > -\kappa \gamma / 4$. In experiment, we can adjust the drive amplitude to guarantee $G_{2-k}^2 < \kappa \gamma / 4$, therefore the system is always stable for arbitrary $G_k$.

The nonclassicality parameters $I$ as a function of the detuning $\delta_k = \omega_x - \omega_{c,k}$ for various thermal phonon $n_{th}$ are plotted in Fig. 2(b). As expected, the nonclassicality parameter shows negativity $I < 0$, which implies nonclassicality of the two oppositely propagating fields due to the two driving. When the detuning $\delta_k = 0$, the nonclassicality parameter reaches the smallest value. For more thermal phonon excitation $n_{th}$, more noise phonon is converted to signal mode, thus the nonclassicality reduces. This noise phonon conversion can also actually be suppressed with the increasing of the effective coupling $G_k$, as shown in Fig. 2(c) that the nonclassicality is improved for larger $G_k$. This is because when the effective coupling $G_k$ increases, the cooling of the phonon mode can be enhanced, and the thermal noise is suppressed.

As shown in Fig. 2(d), we plot the nonclassicality parameter $I$ as a function of $G_k$ at the detuning $\delta_k = 0$. The $I$ reduces with the coupling $G_k$ and approaches $-1$, which is the lower bound of the parameter $I$.

The degenerate CW and CCW signal photons travel in opposite directions, thus the two mode can be separatedly coupled to different optical waveguides [as shown in Fig. 1(a)], which enable the type-II (spatially separable) entanglement photon source on a photonic chip. Compared to the crystals used in free space parametric down conversion, the traveling-wave optomechanical approach has the advantages that narrow linewidth, long coherence time, high purity and waveguide integrated. This scheme can be used to realize quantum repeaters and nonclassical interference by using indistinguishable photon pair generated by the integrated microresonator. The potential limitation of the thermal phonon $n_{th}$ can be relaxed by coupling the mechanical mode to a third optical cavity mode, which is used to cool the mechanical mode towards the ground state. When the damping rates satisfy $\gamma_m \gg \kappa$, the steady-state entanglement can be achieved via reservoir engineering. For large mechanical noise $n_{th} \gg 0$, quantum interference can be applied to achieve robust photon entanglement in the strong coupling regime $G > \kappa$.

**IV. PHASE-CONTROLLED NON-RECIROCITY**

As shown in Fig. 1(a), the signal photon propagating in the forward and backward directions is coupling to the CCW and CW modes, therefore directional pump to the system can induce transmission difference between the forward and backward signal in the waveguide, which leads to the optical non-reciprocity. Different from the single mechanical mode induced non-reciprocity demonstrated in Ref. [47, 48], we propose a new scheme for phase-controlled non-reciprocal conversion by two mechanical mode. The underlying mechanism is that the two mechanical modes allow the paths for the coupling between the oppositely propagating signals, and effective coupling can be controlled by changing the relative phase of the two paths, which eventually leads to the one-way conversion between two modes. The schematic of the scheme is shown in Fig. 3(a), where the CW and CCW modes are dispersively coupled to two breath mechanical modes (one can be the radial expansion mode, the other one can be the out-of-plane vibration mode). In the bosonic four-mode system, the CW (CCW) optical mode is driven by lasers at two frequencies $\omega_{c,k} - \omega_{m1}$ and $\omega_{c,k} - \omega_{m2}$, with different amplitudes and phases. Under the condition that $\min (|\omega_{m1} - \omega_{m2}|, |\omega_{m1} - \omega_{m2}|) \gg \max (\kappa, \gamma_m)$, we can realize the coherent coupling between the photons and phonons, the effective interaction Hamiltonian can be written as

$$H_{in} = \sum_{j=1,2} G_{k-j}^{\dagger} b_{0j} + G_{k-j} b_{0j} + H.c.,$$

where $b_{0j}$ ($j = 1, 2$) is the annihilation operator of the mechanical mode $j$, and $G_{k-j}$ is the effective complex optomechanical coupling with a certain phase from driving fields.

By including the damping effects, the dynamics of the system obeys the quantum Langevin equations

$$\frac{dO}{dt} = -UO + \sqrt{\kappa O_{in}},$$

where $O = (a_k, a_{-k}, b_{01}, b_{02})^T$ is a vector of the operators, the input vector $O_{in} = (\epsilon_{in,k}, \epsilon_{in,-k}, 0, 0)^T$, and the coefficient matrix

$$U = \begin{pmatrix} \frac{\kappa}{\sqrt{\kappa\gamma}} & 0 & iG_{k1} & iG_{k2} \\ 0 & \frac{\kappa}{\sqrt{\kappa\gamma}} & iG_{k1} & iG_{k2} \\ iG_{k1}^* & iG_{k1}^* & \frac{\kappa}{\sqrt{\kappa\gamma}} & 0 \\ iG_{k2}^* & iG_{k2}^* & 0 & \frac{\kappa}{\sqrt{\kappa\gamma}} \end{pmatrix}.$$

Transforming the equations into the frequency domain, the intracavity field is solved as $O = (U - i\omega I)^{-1} \sqrt{\kappa O_{in}}$. Using the standard input-output theory $O_{out} = \sqrt{\kappa O} - O_{in}$, the output of the system is obtained as $O_{out} = R \sqrt{\kappa O_{in}}$, where the scattering matrix $R = \kappa (U - i\omega I)^{-1} - I$. We notice that there are two paths of the coherent conversion between CW and CCW signal photons, therefore the total phase difference $\theta$ between the path $a_k \iff b_1 \iff a_{-k}$ and the path $a_k \iff b_2 \iff a_{-k}$ plays a key role in the optical mode conversion. When the symmetry between the two paths is broken, the system becomes non-reciprocal.

To illustrate the phase-controlled non-reciprocity, we assume the effective coupling strengths are real except for $G_{k2} = |G_{k2}| e^{i\theta}$. For the convenience of expression, we define the cooperativity $C = 4|G_{k2}|^2 / \kappa \gamma$. If only one mechanical mode is coupled to the optical modes, it has been demonstrated in the experiment that the transmission is non-reciprocal only if $C_{k1} \neq C_{-k1}$, while the signal
reflection is always reciprocal \[47\]. In additional to the optomechanically induced phase shifting and absorption, the non-reciprocal conversion between two modes is also very important for information processing, and this can be realized in our scheme.

The Fig. 3(b-d) shows the conversion efficiencies between CW and CCW modes as a function of the signal frequency \(\omega\) and the pump phase differences \(\theta\). The black solid line in the Fig. 3(b) shows the reciprocal conversion when the phase \(\theta = 0\). However, when we choose the phase \(\theta = 3\pi/4\), as shown in Fig. 3(b), the nonreciprocal conversion \(|R_{k,-k}|^2 \neq |R_{-k,k}|^2\) is observed. When \(\omega \approx -\gamma_{m2}\), we find \(|R_{-k,k}|^2 \rightarrow 0\) (the red dotted line), which means that the conversion from the CW to the CCW is almost forbidden, while the opposite conversion is feasible. When \(\omega \approx \gamma_{m2}\), there is an opposite result, which is confirmed by the blue dashed line \(|R_{k,-k}|^2\) and the red dotted line \(|R_{-k,k}|^2\) as the function of the signal frequency \(\omega\). The conversional probabilities (c) and conversional ratio \(\eta\) (d) as the function of the phase \(\theta\) for the frequency \(\omega = -\gamma_{m2}\). The other parameters are \(C_{k1} = C_{-k1} = 1\), \(C_{k2} = C_{-k2} = 2.5\), \(G_{k2} = |G_{k2}| e^{i\phi}\), \(\kappa/2\pi = 15\) MHz, \(\gamma_{m1}/2\pi = 22 \times 10^{-3}\)MHz, and \(\gamma_{m2}/2\pi = 22 \times 10^{-4}\)MHz.

FIG. 3: (Color online) Optical mode conversion between CW and CCW fields. (a) Schematic diagram of phase-controlled nonreciprocity. (b) The conversion efficiency for the reciprocity \(\theta = 0\) (the black solid line \(|R_{k,-k}|^2 = |R_{-k,k}|^2\) and nonreciprocity \(\theta = 3\pi/4\) (the blue dashed line \(|R_{k,-k}|^2\) and the red dotted line \(|R_{-k,k}|^2\)) as the function of the signal frequency \(\omega\). The conversional probabilities (c) and conversional ratio \(\eta\) (d) as the function of the phase \(\theta\) for the frequency \(\omega = -\gamma_{m2}\). However, when we choose the parameters \(G_{k1} \approx -\gamma_{m2}\), as shown in Fig. 3(c), which obviously shows that the optical conversion can be periodically changed with the increasing of the phase \(\theta\). Fig. 3(d) shows the ratio \(\eta = |R_{k,-k}|^2/|R_{-k,k}|^2\) as the function of the phase \(\theta\), which can reach the nonreciprocal degree \(\eta \propto 10^2\) for the experimental parameters. It is obvious that the total phase difference \(\theta\) plays a key role in the optical mode conversion.

If we can realize the parameters \(\gamma_{m2} \ll G \sim \kappa \ll \gamma_{m1}\), the ideal non-reciprocal optical conversion is possible. This assumption seems to be counter-intuitive because usually the damping rate of the mechanical mode is smaller than the decay rate of the cavity mode, but we can satisfy the condition when the mechanical mode is coupled to an auxiliary cavity mode \[60\]. In addition, the signal frequency \(\omega\) of ideal optical conversion can be tuned by the detuning between the driving fields and the optical modes, which has been recently demonstrated in the microwave transmission \[61\].

V. PHONONIC PARITY-TIME SYMMETRY

FIG. 4: (Color online) PT-symmetry in deformation optomechanical system. (a) Schematic diagram of PT-symmetry. The threshold coupling strength \(J_{PT}\) (b), the real \(\text{Re}(\omega)\) (c) and \(\text{Im}(\omega)\) (d) versus the \(G_i\). The black dash line in (b) shows the coupling \(J/2\pi = 16\) KHz. The red dots means that gain and loss are balanced (i.e., \(\gamma_{m} = -\gamma_{m}\)), which is the ideal PT-symmetry. The parameters are \(\omega_{m1}/2\pi = 42.3\) MHz, \(\gamma_{m}/2\pi = 4\) KHz, \(G_{-1}/2\pi = 0.14\) MHz, \(J/2\pi = 16\) KHz, and \(\kappa_{1}/2\pi = \kappa_{2}/2\pi = 3.5\) MHz.

Most of previous studies are focus on the control of multiple optical signal modes. The degenerate CW and CCW acoustic modes can also be used for interesting applications. Here, we consider a stimulated Brillouin scattering in this triple-resonant optomechanical system \[44, 50\], in which the Hamiltonian can be described as

\[
H_{\text{abs}} = \sum_{o=j,k} \omega_o \sigma_o \sigma_o^\dagger + \omega_m b_j^\dagger b_l + g_{j,k,l} \left( b_j^\dagger a_j^\dagger a_k + H.c \right),
\]

where \(a_j, a_k\) are the optical modes and \(b_l\) is the acoustic mode that satisfies the selection rule.

In a perfect rotational symmetric whispering-gallery microresonator, the CW and CCW acoustic modes are
degenerate. To study the non-Hermitian physics of the phononic mode, we consider a slightly deformed microresonator that breaks the rotational symmetry and induces an effective coupling between \( b_1 \) and \( b_{-1} \). The schematic of the system is shown in the Fig. 4(a), and the phonon coupling is described as

\[
H_{bs} = J(b_1 b_{-1} + b_1 b_{-1}), \tag{15}
\]

where \( J \) is the deformed boundary induced backscattering, i.e. the coupling between the mechanical CW and CCW modes. In practical experimental system, the \( J \) can be easily controlled to be the order of \( \gamma_m \) without degrading the high quality factor of optical and mechanical modes.

The optical wavelength is much shorter than that of acoustic modes, the smooth boundary deformation will not induce observable backscattering between the optical modes.

If we drive the optical mode \( a_j \) for the CW direction and \( a_{-j} \) for the CCW direction by two control fields, the Hamiltonian can be simplified as

\[
H_{lin} = H_0 + J(b_1^\dagger b_{-1} + b_1 b_{-1}) + \left(G_i a_j^\dagger b_l + G_{-i} a_{-j} b_{-1} + H.c\right), \tag{16}
\]

where \( G_{i(-i)} = g_{j,k} \alpha a_{j(-k)} \) is the effective optomechanical coupling by the driving field \( \alpha \).

The Hamiltonian \( H_0 = \Delta_k a_k^\dagger a_k + \Delta_{-j} a_{-j}^\dagger a_{-j} + \omega_{ml} (b_1^\dagger b_1 + b_{-1}^\dagger b_{-1}) \), in which \( \Delta_k = \omega_{c,k} - \omega_{j,k} \) and \( \Delta_{-j} = \omega_{c,j} - \omega_{-k,j} \), where \( \omega_{j,k} \) and \( \omega_{-j,-k} \) are the frequencies driving on \( a_j \) and \( a_{-k} \), respectively.

To investigate the PT-symmetric mechanical system, we use a probe field to drive the optical mode \( a_{-j} \) with the frequency \( \omega_p \). We can write the following rate equations for the coupled-resonators system

\[
\frac{dO_{in}}{dt} = -UO + \sqrt{\kappa_{in}}O_{in}, \tag{17}
\]

where \( O = (a_k, a_{-j}, b_1, b_{-1})^T \) is a vector of the operators, the input vector \( O_{in} = (0, \epsilon p, 0, 0)^T \), and the coefficient matrix

\[
U = \begin{pmatrix}
\frac{i \delta_k + \frac{\omega_p}{2}}{\sqrt{\kappa_{in}}} & iG_i & 0 & 0 \\
0 & -i\delta_j - \frac{\omega_p}{2} & iG_{-l} & 0 \\
iG_i & 0 & \frac{i \delta_l + \frac{\omega_p}{2}}{\sqrt{\kappa_{in}}} & iJ \\
0 & iG_{-l} & iJ & -i\delta_{-j} - \frac{\omega_p}{2}
\end{pmatrix}, \tag{18}
\]

where \( \delta_k = \Delta_k + \omega_p - \omega_{j,k}, \delta_{-j} = \omega_{c,j} - \omega_p, \delta_l = \omega_{ml} + \omega_p - \omega_{-k,l}, \delta_{-l} = \delta_l \).

For \( \kappa_{1,2} > > \gamma_m \), we can adiabatically eliminate the optical modes by

\[
a_k = \frac{iG_{-l} b_l + \sqrt{\kappa_{in}} \epsilon_p}{i\delta_k + \frac{\omega_p}{2}}, \tag{19}
\]

and we can obtain the effective rate equations of the acoustic modes

\[
\frac{db_l}{dt} = - \left( i\delta_l + \frac{\gamma_m}{2} + \frac{G_l^2}{i\delta_k + \frac{\omega_p}{2}} \right) b_l - iJ b_{-l}, \tag{21}
\]

\[
\frac{db_{-l}}{dt} = - \left( i\delta_{-l} + \frac{\gamma_m}{2} - \frac{G_{-l}^2}{-i\delta_{-j} + \frac{\omega_p}{2}} \right) b_{-l} - iJ b_l. \tag{22}
\]

Here we choose the driving frequency \( \omega_{j,k} = \omega_{c,k} \) and \( \omega_{j,l} = \omega_{c,j} = \omega_p \). The coupling of these two resonators creates two supermodes \( B_+ = (b_l + b_{-l})/\sqrt{2} \) and \( B_- = (b_l - b_{-l})/\sqrt{2} \) with the eigenfrequencies \( \omega_+ \) and \( \omega_- \) given as

\[
\omega_\pm = \omega_0 - \frac{i}{4} (2\gamma_m + \gamma_{j,l} - \gamma_l) \pm \frac{1}{4} \sqrt{16J^2 - (\gamma_{j,l} - \gamma_l)^2}. \tag{23}
\]

where \( \gamma_{j,l} = -\frac{4G_l^2}{\kappa_{l}}, \gamma_l = \frac{4G_l^2}{\kappa_{l}}, \) and \( \omega_0 = \delta_l + \delta_{-l} \).

Then the value of \( J \) satisfying \( 16J^2 = (\gamma_{j,l} - \gamma_l)^2 \) is the threshold coupling strength \( J_{PT} \), which is found as

\[
J_{PT} = \frac{1}{4} |\gamma_{j,l} - \gamma_l|. \tag{24}
\]

It is clear that \( J_{PT} \) depends on the driving strength of the system. For the ideal case when gain and loss are balanced (i.e., \( \gamma_{j,l} + \gamma_m = -\gamma_l - \gamma_m \)), the coupling strength \( J_{PT} \) becomes

\[
J_{PT} = \frac{\gamma_l + \gamma_m}{2}. \tag{25}
\]
The threshold coupling strength $J_{PT}$ versus the $G_1$ for the fixed $G_{\perp}/2\pi = 0.14$ MHz is plotted in the Fig. 4(b). The black dash line shows the coupling $J/2\pi = 16$ kHz and the red dot is corresponding to the ideal PT-symmetry. For $J_{PT}/J < 1$, the system is in a unbroken-symmetry phase in Fig. 4(c) and 4(d), as seen in both the non-zero difference of the real parts of the eigenfrequencies and the coalescence in their imaginary parts. As $J_{PT}/J$ approaches 1 from below, the difference in the real parts of the eigenfrequencies decreases and their imaginary parts bifurcate.

To observe the phenomena of the PT-symmetry, we can calculate the intracavity field by the Eq. (17). When the system is in the steady state $\frac{d\hat{a}}{dt} = 0$, we obtain

\[
a_k = \frac{-iG_1b_j}{i\delta + \kappa_2/2},
\]

\[
a^\dagger_{-j} = \frac{iG_{-j}b_{-j} + \sqrt{\kappa_m}\epsilon_p}{i\delta + \frac{\kappa_2}{2}},
\]

\[
b_j = \frac{iJ_{-j}b_{-j}}{F_2(\delta)^{-1}},
\]

\[
b_{-j} = \frac{iG_{-j}b_j + \sqrt{\kappa_m}\epsilon_p}{i\delta + \frac{\kappa_2}{2}} / \left[ F_1(\delta) + \frac{J^2}{F_2(\delta)} \right],
\]

where $F_1(\delta) = -i\delta - 2\omega + \frac{G_{-j}^2}{\kappa + \kappa_2}$ and $F_2(\delta) = -i\delta - 2\omega - \frac{G_j^2}{\kappa + \kappa_2}$, in which the detuning $\delta = \omega_0 - \omega_{-j}$.

The spectra of the steady-state intracavity field are shown in Fig. 5. When we choose the coupling $G_1/2\pi = 0.14$ MHz, we have the threshold coupling strength $J_{PT} < J$, which is in the unbroken-PT-symmetric region. Fig. 5(a) shows the intracavity field $|a_{-j}|^2$ as the function of detuning $\delta$, and there are two resonance peaks, which are corresponding to non-zero difference of the real parts of the eigenfrequencies in the Fig. 4(b). The imaginary parts of the eigenfrequencies in Fig. 4(c) show the coalescence, which is confirmed with the same width of the two peaks. The intracavity field $|a_k|^2$ is plotted in the Fig. 5(b), which shows the similar phenomena. When $J_{PT} > J$ with the coupling $G_1/2\pi = 0.2$ MHz, we observe the broken-PT-symmetric phenomena, and there is one peak in the Fig. 5(c) and 5(d). By the deformation optomechanical system, we demonstrate that the parity-time symmetry of acoustic modes can be observed. More importantly, gain could be provided only by optical fields without nonlinear processes. Based on parameters realized with existing technology in recent experiments, these optomechanical devices should be readily achievable.

The parameters we choose above satisfy $\text{Im}(\omega - \omega_0) < 0$, which means that all modes have the loss. When $\text{Im}(\omega - \omega_0) > 0$, it is obvious that the system is unstable, and we can calculate the intracavity field with driving pulse numerically by the Eq. (17). In the linear regime, the transmission is reciprocal regardless of whether the symmetry is broken or unbroken.

VI. DISCUSSION

The traveling-wave optomechanical system can be realized in the solid state whispering gallery microresonators, with various shapes and materials \[64\]. Those high-quality materials usually support not only high quality factor (Q) optical modes, but also supports high Q mechanical modes. Especially, in polished single crystal resonators, the mechanical quality factor can be as high as $10^9$ \[65\]. Therefore, this type microresonator has great potential for the coherent optomechanical interactions.

Compared with the optical modes, the mechanical modes have much longer lifetime and can be used as quantum memory. In compare with the usual single mode optomechanics, the traveling-wave microresonators have two unique advantages, the mode degeneracy and suppression of dephasing. By employing the degenerate CW and CCW acoustic modes, the binary bosonic memory is possible \[54\]. The degenerate traveling modes have the remarkable properties that the modes bear the same noise source, and the noises cancel in most of applications. For example, if a single phonon excitation is stored as a superposition of CW and CCW directions, their phase coherence can be insensitive to the external vibration and thermal noise. Because the dominated dephasing noise of both CW and CCW are the same, the relative phase between CW and CCW mode conserves. Therefore, the natural degenerate modes in traveling-wave microresonators form a decoherence-free subspace and preserve the coherence.

In practical applications, the degeneracy of the CW and CCW modes would also save the laser source. We can use one pump laser to study the coherent interaction in two directions. Besides the CW and CCW modes studied in this work, the traveling-wave optomechanical system supports optical modes and mechanical modes in a large frequency range, with the mode frequency approximately proportional to the angular momentum \[l\]. By choosing a proper material, there will be hundreds of optical modes covering the visible, IR and mid-IR frequency range. Similarly, the acoustic wave modes also show a comb like spectrum, and have the frequency range from few MHz to GHz \[60\]. So, the applications discussed in this paper on the CW and CCW modes can be generalized to the modes with very different frequencies.

VII. CONCLUSION

In summary, we theoretically study the multimode optomechanics based on the traveling-wave microresonators, where the degenerate clockwise and counterclockwise optical and acoustical modes can be controlled selectively by optically pumping in different directions. By the unique properties of the traveling-wave optomechanical systems, we have proposed several unique applications of the system. Based on the parameters realized in recent experiments, these optomechanical devices should
be readily achievable, even on the level of single-photon with practical noises.

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