BARYOGENESIS DURING REHEATING IN NATURAL INFLATION

AND COMMENTS ON SPONTANEOUS BARYOGENESIS

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ABSTRACT: We calculate the baryon asymmetry created by the decay of a pseudo Nambu-Goldstone boson whose interactions violate baryon number conservation. Our results are in disagreement with previous results in the original spontaneous baryogenesis models for the asymmetry produced by the decay of an oscillating scalar field with B number violating derivative couplings; we find that the net baryon number density is proportional to $\theta_i^3$, where $\theta_i$ is the amplitude of the PNGB-field in natural inflation at the onset of reheating. We also discuss our disagreement with the interpretation of $\dot{\theta}$ as an effective chemical potential for baryon number in spontaneous baryogenesis models. While our calculation of the asymmetry is carried out in the context of natural inflation our approach is generally valid for baryogenesis models using decaying classical fields. In the Appendices, we include a complete derivation of the number density of particles produced by the decay of a classical scalar field; this number density is proportional to the integral over momenta of the one pair production amplitude.

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Section 1: Introduction

In this paper, 1) we calculate the baryon asymmetry obtained during reheating following natural inflation using an approach that is generally valid for baryogenesis models using decaying classical fields. Our results are in disagreement with the results presented in the original spontaneous baryogenesis papers. 2) We discuss an objection to the effective chemical potential interpretation used in models of spontaneous baryogenesis.

In natural inflation the role of the inflaton is played by a pseudo Nambu-Goldstone boson, hereafter referred to as $\theta$, with a potential of the form

$$V(\theta) = \Lambda^4 (1 - \cos \theta).$$  \hspace{1cm} (1.1)

This model was proposed to “naturally” provide the flat potential required for inflation to work \[2], \[3]. Here $\theta = \Phi / f$, where $\Phi$ is a complex scalar field and $f$ is the scale at which a global symmetry is spontaneously broken; soft explicit symmetry breaking takes place at a lower scale $\Lambda$. From eq. (1.1) one can see that the height of the potential is $2\Lambda^4$ while the width is $f$. Since the scales of spontaneous and explicit symmetry breaking can “naturally” be separated by several orders of magnitude, one can obtain $\Lambda \leq 10^{-3} f$ as required for successful inflation \[4].

In ref. \[14\] an extensive study of the conditions under which the $\theta$ field can drive inflation has been obtained. After the period of inflation, the energy density of the $\theta$ field is converted to radiation during reheating through its decay to other forms of matter as it oscillates in its potential. Below we shall assume that $\theta$ is coupled only to fermions. We treat $\theta$ as a classical scalar field coupled to quantized fermion fields $Q$ and $L$ via an interaction term of the form $\bar{Q}L e^{i\theta} + \bar{L}Q e^{-i\theta}$, where $Q$ carries baryon number but $L$ does not. We show that the decay of $\theta$ gives rise to a net baryon number density $(n_b - \bar{n}_b)$ proportional to $\theta_i^3$, where $\theta_i$ is the value of the $\theta$ field at the onset of reheating.

Our result disagrees with the calculation in the original spontaneous baryogenesis papers \[15\] where it was argued that the asymmetry is proportional to $\theta_i$ to the first power, independent of the details of the baryon number violating couplings of the $\theta$ field. Specifically, in previous work, Cohen and Kaplan \[15\] considered any theory in which a scalar field is derivatively coupled to the baryon current $J^\mu$ with a term in the interaction Lagrangian of the form $\mathcal{L}_{\text{int}} \propto \partial_\mu \theta J^\mu$, and derived an expression for the baryon asymmetry produced by the decay of the scalar field as it oscillates about its minimum. The pseudo Nambu-Goldstone boson in natural inflation can serve as an example of such a scalar field. Cohen and Kaplan obtained $|\dot{n}_B| = \Gamma f^2 |\dot{\theta}|$, where $\Gamma$ is the decay rate of the $\theta$ field and $n_B$ is the net baryon number density. This gives

$$|\Delta n_B| = \Gamma f^2 |\Delta \theta|. \hspace{1cm} (1.2)$$
Below we discuss our concerns with this conclusion and present calculations for the specific case of eq. (1.1); our results disagree with eq. (1.2). We also comment on our objections to interpreting $\dot{\theta}$ as a chemical potential when $\ddot{\theta}$ is small, as was done in ref. [15]; we argue that a Lagrangian term $\dot{\theta} J^0$ does not appear in the Hamiltonian, and therefore it is incorrect to identify $\dot{\theta}$ with an effective chemical potential for baryon number.

The framework of this paper is as follows. In Section 2, we write down the Lagrangian density for the inflaton field and present the equation of motion for $\theta$ as it oscillates during the reheating phase, as derived in ref. [16]. In Section 3 we discuss our concerns with eq. (1.2) as obtained in ref. [15] (these concerns were raised in an earlier paper [16] by two of the authors [Dolgov and Freese]). We then proceed to calculate the total baryon number and antibaryon number produced during the decay of the $\theta$ field, and find a net baryon number density $(n_b - n_{\bar{b}})$ proportional to $\theta_i^3$. We also show that the energy density of the produced particles is equal to the initial energy density of the $\theta$ field as a check on our calculation. In Section 4, we discuss how constraints on parameters in natural inflation obtained in ref. [14] affect the quantitative results for baryogenesis. We also discuss our objections to the thermodynamic generation of the baryon asymmetry via an effective chemical potential interpretation in models of spontaneous baryogenesis. Finally we summarize our results. In the Appendices we provide details of the calculations outlined in the main body of the paper. In particular, in Appendices A and B, we include derivations of the number density of particles produced by the decay of a classical scalar field; the number density of particles produced is proportional to the integral over momenta of the one pair production amplitude.

Section 2: The Model

As in ref. [16] we consider a simple model involving a complex scalar field $\Phi$ and fermion fields $Q$ and $L$ with the Lagrangian density

$$\mathcal{L} = -\partial_\mu \Phi^* \partial^\mu \Phi - V(\Phi^* \Phi) + i \bar{Q} \gamma^\mu \partial_\mu Q + i \bar{L} \gamma^\mu \partial_\mu L - m_Q \bar{Q} Q - m_L \bar{L} L + (g \Phi \bar{Q} L + h.c.). \quad (2.1)$$

Note that, despite their names, $Q$ and $L$ cannot be actual quarks and leptons, since the interaction term does not conserve color. They could, however, represent heavy fermions with other interactions with the fields of the Standard Model which fix the assignments of global charges. In particular, we shall assume that the field $Q$ carries baryon number while the field $L$ does not. The U(1) symmetry that corresponds to baryon number is therefore identified as

$$\Phi \rightarrow e^{i\alpha} \Phi, \quad Q \rightarrow e^{i\alpha} Q, \quad L \rightarrow L. \quad (2.2)$$

1 We use a metric $(-1,1,1,1)$.  

3
We assume that this global symmetry is spontaneously broken at an energy scale \( f \) via a potential of the form

\[
V(|\Phi|) = \lambda (\Phi^* \Phi - f^2/2)^2.
\]

The resulting scalar field vacuum expectation value (VEV) is

\[
\langle \Phi \rangle = fe^{i\phi/f}/\sqrt{2}.
\]

Below the scale \( f \), we can neglect the radial mode of \( \Phi \) since it is so massive that it is frozen out; \( m_{\text{radial}} = \lambda^{1/2}f \). The remaining light degree of freedom is \( \phi \), the Goldstone boson of the spontaneously broken \( U(1) \). For simplicity of notation we introduce the dimensionless angular field \( \theta \equiv \phi/f \). We then obtain an effective Lagrangian density for \( \theta \), \( Q \), and \( L \) of the form

\[
\mathcal{L}_{\text{eff}} = -\frac{f^2}{2} \partial_\mu \theta \partial^\mu \theta + i\bar{Q} \gamma^\mu \partial_\mu Q + i\bar{L} \gamma^\mu \partial_\mu L - m_Q \bar{Q} Q - m_L \bar{L} L + \left( \frac{g}{\sqrt{2}} f \bar{Q} L e^{i\theta} + \text{h.c.} \right).
\]

The global symmetry is now realized in the Goldstone mode: \( \mathcal{L}_{\text{eff}} \) is invariant under

\[
Q \rightarrow e^{i\alpha} Q, \quad L \rightarrow L, \quad \theta \rightarrow \theta + \alpha.
\]

With a rotation of the form in eq. (2.5) with \( \alpha = -\theta \), the Lagrangian can alternatively be written

\[
\mathcal{L}_{\text{eff}} = -\frac{f^2}{2} \partial_\mu \theta \partial^\mu \theta + i\bar{Q} \gamma^\mu \partial_\mu Q + i\bar{L} \gamma^\mu \partial_\mu L - m_Q \bar{Q} Q - m_L \bar{L} L + \left( \frac{g}{\sqrt{2}} f \bar{Q} L e^{i\theta} + \text{h.c.} \right) + \partial_\mu \theta J^\mu,
\]

where the fermion current derives from the \( U(1) \) symmetry; here, \( J^\mu = \bar{Q} \gamma^\mu Q \).

We now assume that the symmetry (2.2) is also subject to a small explicit breaking which gives rise to a potential as in eq. (1.1) and which provides a nonzero mass for the field \( \theta \). This explicit symmetry breaking could come from Planck scale physics. Alternatively, one can imagine a scenario similar to that involving the QCD axion where, at energy scales of the order of \( \Lambda_{\text{QCD}} \), instanton effects create the fermion condensate \( \langle \bar{\psi} \psi \rangle \sim \Lambda_{\text{QCD}}^3 \), giving rise to a mass term for the axion. Note that for the natural inflation model, the required mass scales are much higher than for the QCD axion. The width of the potential must be roughly the Planck mass in order to achieve enough e-foldings of inflation, and the height of the potential must be roughly \( \Lambda^4 \sim [10^{16} \, \text{GeV}]^4 \) in order for density perturbations appropriate for structure formation to be produced (see the Discussion section at the end of the paper for more detail). Consequently the scale at which the relevant gauge group (not QCD) must become strong is roughly the GUT scale. These and other mechanisms such as those found in technicolor and schizon models for generating a potential for pseudo Nambu-Goldstone bosons are discussed in ref. [4,14].
Initially, as the $\theta$ field rolls down towards the minimum of its potential, its potential energy drives inflation. Let $\theta_i$ be the value of the $\theta$ field at the beginning of the reheating epoch, after inflationary expansion has ended. (We shall ignore spatial variations in the $\theta$ field.) During the reheating epoch the $\theta$ field oscillates about the minimum of its potential. While $\theta$ oscillates it decays to the fields $Q$ and $L$. The interactions of the fermionic fields create a thermal bath thereby reheating the universe. Note that we must take $g \ll 1$ so that fermion masses generated for the fermions from the Yukawa coupling, $m_{\psi} \sim gf$, are small enough that the fermions can in fact be produced by decays of the pseudo Nambu-Goldstone bosons. See ref. [10] in Dolgov and Freese [16] for further discussion of this point.

The equation of motion for the $\theta$ field with the back reaction of the produced fermions was rigorously derived in the one loop approximation in ref. [16]. For small deviations of $\theta$ from the equilibrium the potential can be approximated as $V(\theta) = \frac{1}{2}m_R^2f^2\theta^2$ and the equation of motion during the oscillating phase can be effectively written in the well known form

$$\ddot{\theta} + m_R^2\theta + \Gamma \dot{\theta} = 0,$$

where $m_R$ is the renormalised $\theta$ mass defined as $\lim_{w \to \infty} m_R^2 \left[ 1 + \frac{g^2}{4\pi^2} \log(2w/m_R) \right] = m^2$, where $m$ is the bare mass of the $\theta$ field, and $\Gamma \equiv g^2m_R/8\pi$. (Our expressions above differ by a factor of 2 from those in ref. [16] because a factor of $1/\sqrt{2}$ was dropped from eq. (2.5) in ref. [16].) The solution to this equation is

$$\theta(t) = \theta_ie^{-\Gamma t/2} \cos(m_Rt).$$

where we have assumed that the initial velocity of the $\theta$ field is negligible and have therefore set an arbitrary phase in the cosine to zero. The results obtained below can be easily generalized for arbitrary initial conditions. The above solution was derived assuming $m_Q = m_L = 0$. However, it can be shown that non-zero values of $m_Q$ and $m_L$ will not change the solution for $\theta$ significantly as long as $m_Q, m_L \ll m_R$, which we shall assume below.

**Section 3: Baryogenesis**

*Previous Calculations and Concerns:* In previous work, Cohen and Kaplan [15] considered any theory in which a scalar field is derivatively coupled to the baryon current with a term in the interaction Lagrangian of the form $L_{\text{int}} \propto \partial_\mu \theta J^\mu$, and derived an expression for the baryon asymmetry produced by the decay of the scalar field as it oscillates about its minimum. From eq. (2.6) one can see that our pseudo Nambu-Goldstone boson is an example of such a scalar field as it has the appropriate coupling. Cohen and Kaplan obtained $|\dot{n}_B| = \Gamma f^2|\dot{\theta}|$, where $n_B$ is the net baryon number density. This gives

$$|\Delta n_B| = \Gamma f^2|\Delta \theta|.$$  

(3.1)
In a previous paper [16] by two of the authors [Dolgov and Freese], several concerns with this interpretation were raised. We will outline two of these concerns again here, and then proceed with a direct calculation of the baryon asymmetry. Our results will disagree with eq. (3.1).

One concern is as follows: in making the identification $|\dot{n}_B| = \Gamma f^2 |\dot{\theta}|$, one is comparing an operator equation, namely, the Euler-Lagrange equation $\ddot{\theta} + m^2 \theta = \dot{n}_B/f^2$, with an equation of the form of eq. (2.7) which is obtained after vacuum averaging. In ref. [16] the average value $\langle \dot{n}_B \rangle$ was found to be not just $-\Gamma f^2 \dot{\theta}$ but a more complicated expression (eq. (3.3) in ref. [16]).

A second concern is with regard to energy conservation. The initial energy density of the field $\theta$ which creates the baryons and antibaryons is $\rho_\theta(t_i) \sim f^2 m^2 \theta_i^2$. At the end some of this energy density has been converted to baryons and antibaryons, with energy density $\rho(t_f) > n_B E_B$ where $E_B \sim m$ is the characteristic energy of the produced fermions (note that $n_B$ refers to the difference between baryon and antibaryon number densities and not to the total number density of produced particles). Clearly it must be true that $\rho(t_f) < \rho_\theta(t_i)$. If we were to use eq. (3.1) we would see that this requires $\Gamma < \Delta \theta m$. From the definition of $\Gamma$ we see that this is satisfied for small values of coupling constant $g$ as long as $\Delta \theta$ is not too small; for small values of $\Delta \theta$, this relationship can never be satisfied.

New Calculations and Results: We now proceed to calculate the net baryon number density of the particles produced during reheating. We perform an explicit calculation and find a different result from eq. (3.1). The $\theta$ field decays to either $Q \bar{L}$ pairs or $\bar{Q}L$ pairs. (The $Q$ and $L$ fields are not the mass eigenstates. Later in this section we consider effects of oscillations between $Q$ and $L$ fields.) As mentioned earlier, we treat the $\theta$ field classically, $Q$ and $L$ are quantum fields and $Q$ carries baryon number. For now we ignore any dilution of the baryon number density due to the expansion of the universe.

As shown in Appendix A with the Bogolyubov transformation method [17], the average number density $n$ of particle antiparticle pairs produced by decay of a homogeneous classical scalar field, to lowest order in perturbation theory, is given by

$$n = \frac{1}{V} \sum_{s_1, s_2} \int \text{d}p_1 \text{d}p_2 |A|^2,$$

(3.2)

where $A$ is the one pair production amplitude, subscripts 1 and 2 refer to the final particles produced and $d^3p = d^3p/[(2\pi)^3 2p^0]$. Eq. (3.2) can also be obtained using the method presented in Sec. 4-1-1 of ref. [18], as discussed in Appendix B.

Thus, to lowest order in perturbation theory, the average number density of $Q \bar{L}$ pairs
Throughout the paper, a state produced during reheating in our model is given by

\[ n(Q, \bar{L}) = \frac{1}{V} \sum_{s_Q, s_L} \int \tilde{d}p \tilde{d}q \left| \langle Q(p, s_Q), \bar{L}(q, s_L)|0 \rangle \right|^2. \]  

(3.3)

We take

\[ Q = \sum_s \int \tilde{d}k \left[ u^s_k b^s_k e^{+ik \cdot x} + v^s_k d^{s \dagger}_k e^{-ik \cdot x} \right] \]

(3.4)

and a similar expression for \( L \). Here \( \{ b^s_k, b^{s \dagger}_k \} = \{ d^s_k, d^{s \dagger}_k \} = (2\pi)^3 2k^0 \delta^3(k - k') \delta_{ss'} \). Standard algebra gives

\[
 n(Q, \bar{L}) = \frac{1}{V} \sum_{s_Q, s_L} \int \tilde{d}p \tilde{d}q \left| \langle Q(p, s_Q), \bar{L}(q, s_L)|0 \rangle \right|^2
 = \frac{g^2 f^2}{2V} \int \tilde{d}p \tilde{d}q \left| (2\pi)^3 \delta^3(p + q) \int_{-\infty}^{\infty} dt e^{i2\omega t + i\theta(t)} \right|^2 \text{Tr}[-\not{q} + m_Q](-\not{q} - m_L)
\]

(3.5)

where \( 2\omega = p^0 + q^0 \). We obtain a similar expression for \( n(L, \bar{Q}) \) with \( \theta(t) \) replaced by \(-\theta(t)\). We set the baryon number density \( n_b \) to be equal to \( n(Q, \bar{L}) \) and the antibaryon number density \( n_{\bar{b}} \) to be equal to \( n(L, \bar{Q}) \). Then we have

\[
 n_{b,\bar{b}} = \frac{g^2 f^2}{2\pi^2} \int d\omega \omega^2 \left| \int_{-\infty}^{\infty} dt e^{i2\omega t} e^{\pm i\theta(t)} \right|^2,
\]

(3.6)

where the + sign in the exponent refers to baryon number and the − sign to antibaryon number. To carry out the integral over time we expand \( e^{i\theta} \) as

\[
 1 + i\theta - \theta^2/2,
\]

(3.7)

valid for small \( \theta \), and use

\[
 \theta(t) = \begin{cases} \theta_i e^{-\Gamma t/2} \cos(m_R t) & \text{for } t \leq 0 \\ \theta_i e^{-\Gamma t/2} \cos(m_R t) & \text{for } t \geq 0 \end{cases}
\]

(3.8)

We also use a convergence factor at early times to regularize the integral. We will examine a series of possible terms to find the first nonzero contribution in perturbation theory. The lowest order term comes from using \( e^{i\theta} = 1 \) from eq. (3.7) in eq. (3.6) and gives \( \int dt e^{2i\omega t} \propto \delta(2\omega) = 0 \) since we can not have \( \omega = 0 \) for particle production. The next term in the expansion, the \( \theta \) term in (3.7), when squared gives the same contribution to \( n_b \) and

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2 Throughout the paper, a state \( \langle A(p_1, s_1), B(p_2, s_2) \rangle \) corresponds to a final state with an \( A \) particle of momentum \( p_1 \) and spin \( s_1 \) and an anti-\( B \) particle with momentum \( p_2 \) and spin \( s_2 \).
to \( n_b \). In order to obtain an asymmetry one must consider cross terms. The lowest order cross term that gives a nonzero contribution to the baryon asymmetry is

\[
n_b - n_{\bar{b}} = 2 \times \frac{g^2 f^2}{2\pi^2} \int d\omega \omega^2 \left[ \frac{\tilde{\theta}(2\omega) \left[ \tilde{\theta}^2(2\omega) \right]^*}{2i} + \text{h.c.} \right], \tag{3.9}
\]

where h.c. refers to hermitian conjugate,

\[
\tilde{\theta}(2\omega) = \int_{-\infty}^{\infty} dt e^{2i\omega t} \theta(t) \tag{3.10a}
\]

and

\[
\tilde{\theta}^2(2\omega) = \int_{-\infty}^{\infty} dt e^{2i\omega t} \theta^2(t). \tag{3.10b}
\]

The factor of 2 in eq. (3.9) arises from the fact that the cross terms in \( n_b \) and \( n_{\bar{b}} \) terms are the same up to a minus sign. One can see from the form of eq. (3.9) that we expect the asymmetry to be proportional to \( \theta^3 \). The details of this calculation are outlined in Appendix C, and the results are presented here.

We obtain

\[
n_b = \frac{1}{4} m_R f^2 \theta_i^2 + \frac{g^2}{32\pi} m_R f^2 \theta_i^3 \tag{3.11}
\]

\[
n_{\bar{b}} = \frac{1}{4} m_R f^2 \theta_i^2 - \frac{g^2}{32\pi} m_R f^2 \theta_i^3 \tag{3.12}
\]

Therefore,

\[
n_B \equiv n_b - n_{\bar{b}} = \frac{g^2}{16\pi} m_R f^2 \theta_i^3 = \frac{1}{2} \Gamma f^2 \theta_i^3 \tag{3.13}
\]

We notice that the net baryon number density is proportional to \( \theta_i^3 \). This disagrees with the calculation in ref. [15] which gives an asymmetry proportional to \( \theta_i \). We also note that the number density of pairs of particles \( n_b + n_{\bar{b}} \) is equal to \( \frac{1}{2} m_R f^2 \theta_i^2 \). Since the energy per pair of particles is \( m_R \), the energy density in the produced particles is \( \frac{1}{2} m_R^2 f^2 \theta_i^2 \), which agrees with the initial energy density of the \( \theta \) field. We have also done the calculation of

\[
\rho_{\text{final}} = \frac{1}{V} \sum_{s_Q, s_L} \int \tilde{p} \tilde{d}q \left( p^0 + q^0 \right) \left| \langle Q(p, s_Q), \bar{L}(q, s_L)|0 \rangle^2 + \langle L(q, s_L), Q(p, s_Q)|0 \rangle \right|^2 \]

and have verified that we obtain \( \frac{1}{2} m_R^2 f^2 \theta_i^2 \).

**Mass Mixing:** In many cases eq. (3.13) is not yet the complete story because of mass mixing. As we mentioned earlier the \( Q \) and \( L \) fields are not mass eigenstates. Therefore a
particle which is produced as a $Q$ may later rotate into an $L$. This effect must be taken into account. Eq. (3.13) is completely correct for the case where the fermions $Q$ and $L$ are converted immediately to regular quarks $q$ and leptons $l$ as soon as they are produced (assuming that the temperature is low enough that the $q$ and $l$ cannot convert back into $Q$ and $L$). In that case, there is no opportunity for mixing to take place, e.g., there is no opportunity for $Q$ to convert to an $L$. On the other hand, if $Q$ and $L$ do not decay immediately into stable lighter mass particles with appropriate quark quantum numbers, they may have the chance to mix into one another. One can calculate the effects of mixing in either the $Q, L$ basis or in the basis of mass eigenstates; below we will do both.

The mass matrix in the $(Q, L)$ basis is

$$
\begin{pmatrix}
m_Q & -g f / \sqrt{2} \\
-g f / \sqrt{2} & m_L
\end{pmatrix}.
$$

(3.15)

The mass eigenstates are

$$
\psi_1 = \frac{L + \epsilon Q}{\sqrt{1 + \epsilon^2}} \quad \text{and} \quad \psi_2 = \frac{Q - \epsilon L}{\sqrt{1 + \epsilon^2}}
$$

(3.16)

with masses $m_Q - g f / (\sqrt{2} \epsilon)$ and $m_L + g f / (\sqrt{2} \epsilon)$ respectively, where $\epsilon = \sqrt{2} g f / (\Delta m + \sqrt{(\Delta m)^2 + 2 g^2 f^2})$ and $\Delta m = m_Q - m_L$. Note that $\Delta m = 0$ corresponds to $\epsilon = 1$.

In the $\psi_1, \psi_2$ basis, one can now calculate the baryon asymmetry as a sum of terms, each of which is a product of a number density of produced particle/antiparticle pairs times the (time averaged) quark content of the pair,

$$
n_B = n(\psi_1, \bar{\psi}_2) |\langle Q | \psi_1 \rangle|^2 + n(\psi_2, \bar{\psi}_1) |\langle Q | \psi_2 \rangle|^2 - n(\psi_1, \bar{\psi}_2) |\langle Q | \bar{\psi}_2 \rangle|^2 - n(\psi_2, \bar{\psi}_1) |\langle Q | \bar{\psi}_1 \rangle|^2.
$$

(3.17)

Here $n(\psi_1, \bar{\psi}_2)$ and $n(\psi_2, \bar{\psi}_1)$ are the number densities of $\psi_1$ and $\bar{\psi}_2$ pairs and $\psi_2$ and $\bar{\psi}_1$ pairs respectively; and $|\langle Q | \psi_i \rangle|^2$ is the probability that a particle which is produced as a $\psi_i$ (where $i = 1, 2$) is measured as a $Q$. Hence, for example, the first term is the product of the number density of $\psi_1 \bar{\psi}_2$ pairs produced times the quark content of $\psi_1$.

Note that we are here computing a time averaged baryon asymmetry; actually the value of the baryon asymmetry oscillates in time, as discussed in Appendix D. From eq. (3.16) we see that the probability that $\psi_{1,2}$ is measured as a $Q$ is

$$
|\langle Q | \psi_1 \rangle|^2 = |\langle Q | \bar{\psi}_1 \rangle|^2 = \frac{\epsilon^2}{1 + \epsilon^2}
$$

(3.18)

and

$$
|\langle Q | \psi_2 \rangle|^2 = |\langle Q | \bar{\psi}_2 \rangle|^2 = \frac{1}{1 + \epsilon^2}.
$$

(3.19)
As in eq. (3.2), the number densities of particle/antiparticle pairs are obtained by squaring the production amplitudes for the pairs,

\[ n_{ij} = \frac{1}{V} \sum_{s_i, s_j} \int \tilde{d}k_i \tilde{d}k_j |A_{ij}|^2, \quad (3.20) \]

where \( i \) and \( j \) are either 1 or 2. The amplitude for production of a \( \psi_i \bar{\psi}_j \) pair is

\[ A_{ij} = \langle \psi_i, \bar{\psi}_j | i \int d^4x (\frac{g}{\sqrt{2}} f e^{i\theta} QL + h.c.) |0 \rangle. \quad (3.21) \]

Using eqs. (3.18), (3.19), and (3.20), we can write eq. (3.17) as

\[ n_B = -\frac{1}{V} \sum_{s_1, s_2} \int \tilde{d}k_1 \tilde{d}k_2 \left( \frac{1 - \epsilon^2}{1 + \epsilon^2} \right) [|A_{12}|^2 - |A_{21}|^2]. \quad (3.22) \]

Using

\[ \bar{Q}L = \left( \frac{1}{1 + \epsilon^2} \right) [\bar{\psi}_2 \psi_1 - \epsilon \bar{\psi}_2 \psi_2 + \epsilon \bar{\psi}_1 \psi_1 - \epsilon^2 \bar{\psi}_1 \psi_2] \quad (3.23) \]

and its hermitian conjugate, we calculate the relevant production amplitudes:

\[ A_{12} = \langle \psi_1, \bar{\psi}_2 | i \int d^4x (\frac{g}{\sqrt{2}} f e^{i\theta} QL + \frac{g}{\sqrt{2}} f e^{-i\theta} LQ) |0 \rangle \quad (3.24a) \]

to find

\[ A_{12} = i \frac{g}{\sqrt{2}} f \left( \frac{1}{1 + \epsilon^2} \right) \langle \psi_1, \bar{\psi}_2 \rangle \int d^4x (\bar{\psi}_1 \psi_2 e^{-i\theta} - \epsilon^2 \bar{\psi}_1 \psi_2) |0 \rangle. \quad (3.24b) \]

Now the two matrix elements in eq. (3.24b) are similar to the ones we calculated in eq. (3.5), with \( \bar{Q}L \) replaced by \( \bar{\psi}_1 \psi_2 \). Hence, we have

\[ A_{12} = \left( \frac{1}{1 + \epsilon^2} \right) (A_{L\bar{Q}} - \epsilon^2 A_{QL}). \quad (3.25) \]

Similarly,

\[ A_{21} = h.c.[A_{12}] = \left( \frac{1}{1 + \epsilon^2} \right) (-\epsilon^2 A_{L\bar{Q}} + A_{QL}). \quad (3.26) \]

Thus eq. (3.22) becomes

\[ n_B = \left( \frac{1 - \epsilon^2}{1 + \epsilon^2} \right)^2 \left( \sum_{s_Q, s_L} \int \tilde{d}k_Q \tilde{d}k_L |A_{QL}|^2 - \sum_{s_L, s_Q} \int \tilde{d}k_L \tilde{d}k_Q |A_{L\bar{Q}}|^2 \right) \]

\[ = \left( \frac{1 - \epsilon^2}{1 + \epsilon^2} \right)^2 \times \text{our previous answer}. \]
Thus we find that
\[
n_B = \frac{1}{2} \Gamma f^2 \theta_i^3 \left( \frac{1 - \epsilon^2}{1 + \epsilon^2} \right)^2.
\] (3.28)

If \( m_Q = m_L, \epsilon = 1 \) and the asymmetry vanishes because in this case the net baryon number of a \((\psi_1, \bar{\psi}_2)\) pair or a \((\psi_2, \bar{\psi}_1)\) pair is 0 and thus no baryon asymmetry is produced.

Another derivation of eq. (3.28) is given in Appendix D. In the preceding paragraphs we considered particle production and mixing in the mass eigenstate \( \psi_1, \psi_2 \) basis. In Appendix D we work in the \( Q, L \) basis. We find the oscillations of the baryon asymmetry with time, and obtain the same expression as in eq. (3.28) for the time averaged baryon asymmetry.

**Thermalization:** After the \( \theta \) field has decayed into \( \psi_1 \) and \( \psi_2 \) particles, thermal equilibrium can be established if these particles have other interactions with each other and with other particles. As long as one introduces interactions such as \( \chi \bar{\psi}_1 \psi_1 \) and \( \chi \bar{\psi}_2 \psi_2 \) as a part of a realistic model, the number of \( \psi_1 - \bar{\psi}_1 \) particles and of \( \psi_2 - \bar{\psi}_2 \) particles does not change, thereby preserving the baryon asymmetry. (Interactions such as \( \chi \bar{\psi}_1 \psi_2 + h.c. \) would, however, destroy the baryon asymmetry.) The fields \( \psi_1 \) and \( \psi_2 \) will annihilate or decay to lighter particles which will thermalize. If these interactions preserve the net baryon number, then the asymmetry will survive.

**Quantitative Results:** So far we have not included the effects of the expansion of the universe. For baryon number created when \( H \leq \Gamma \), we may neglect the expansion and directly use the results obtained above in eq. (3.28) with \( \theta \) replaced with the value of \( \theta \) at \( H = \Gamma \). Since the \( \theta \) field dominates the cosmic energy density, the condition \( H = \Gamma \) fixes the amplitude of \( \theta \) at that moment to be
\[
\theta_1 = \sqrt{3/4\pi} (\Gamma m_{Pl}/f m_R) \approx 0.02 g^2 m_{Pl}/f \ll 1.
\] (3.29)

In the early stages of reheating with \( \theta > \theta_1 \), expansion of the universe must be taken into account.

The decay of the \( \theta \)-field produces relativistic \( \psi_{1,2} \) and \( \bar{\psi}_{1,2} \) with energies \( \omega \approx m_R/2 \). This state is far from thermal equilibrium (the temperature of the thermalized plasma in eq. (3.30) below may be smaller than the \( \psi \) masses). The rate of thermalization depends upon the interaction strength of the fermions created in the \( \theta \) decay. It is typically higher than the decay rate because \( g \ll 1 \) to ensure reasonable fermion masses. Thermalization could occur either through annihilation of \( \psi_1 \) and \( \bar{\psi}_1 \) or \( \psi_2 \) and \( \bar{\psi}_2 \) into light particles or through their decays and subsequent elastic scattering. Assuming that these processes are fast we can roughly estimate the reheat temperature in the instantaneous
decay approximation, $\rho_{\text{rad}} = \rho_0(t = \Gamma^{-1})$, as

$$T_{\text{reh}} = (90/8\pi^3 g_*)^{1/4} \sqrt{\Gamma m_{\text{Pl}}} \approx 0.15 g_*^{1/4} \Lambda \sqrt{m_{\text{Pl}}/f}$$  \hspace{1cm} (3.30)$$

where we have taken $m_R = \Lambda^2/f$.

The entropy density after thermalization is given by $s = 4\pi^2 g_* T_{\text{reh}}^3 / 90$. It is conserved in the comoving volume if the expansion of the universe is adiabatic, in particular in the absence of first order phase transitions as the universe cools. Baryonic charge density is also assumed to be conserved inside a comoving volume during and after thermalization and so the baryon-to-entropy ratio $n_B / s$ remains constant in the course of expansion.

First we find the baryon asymmetry produced after $H \leq \Gamma$ so that expansion may be neglected (subscript 1 refers to this case). Using eqs. (3.28), (3.29) and (3.30) we find

$$\left( \frac{n_B}{s} \right)_1 \approx 10^{-4} \frac{g_5}{g_*^{1/4}} \left( \frac{m_{\text{Pl}}}{f} \right)^{3/2} \frac{f}{\Lambda} \left( \frac{1 - \epsilon^2}{1 + \epsilon^2} \right)^2.$$  \hspace{1cm} (3.31)$$

In the models studied in ref. [14] $(f/m_{\text{Pl}}) = O(1)$ and $f/\Lambda = 10^6 - 10^3$, so to get a reasonable baryon asymmetry we need a rather large coupling, $g > 10^{-2}$ (for $\epsilon \ll 1$).

In fact the asymmetry should be noticeably larger than that given by eq. (3.31). The result that we got above refers to the case when $H < \Gamma$ but the process of particle production starts much earlier when $H \approx m_R$ and the inflaton field begins to oscillate around the bottom of the potential. The net baryon number density produced while $H > \Gamma$ is again proportional to $\theta^3$, as it is associated with the interference between the $\theta$ and the $\theta^2$ terms in $| \int dt e^{2i\omega t}(1 + i\theta - \theta^2/2)|^2$ in eq. (3.6). The generation of the asymmetry is more efficient at early times ($H > \Gamma$) since the amplitude of the $\theta$-field, which goes down with the scale factor as $R^{-3/2}$, is larger. However when $H > \Gamma$ one must include the effects of the expansion of the universe on the production of the baryon asymmetry. This makes the exact calculations considerably more complicated. Still we can roughly estimate the asymmetry in the following way. The difference between the production of particles and antiparticles is most profound at early times, $\Delta t_a \sim 1/m_R$, when $\theta$ is large.

The total number of particles produced in time $\Delta t_a$ is proportional to $\Gamma \Delta t_a n_\theta$ and, as we mention above, the baryon number asymmetry must vary as $\theta^3$. Therefore, a reasonable estimate of the net baryon number density created while $H > \Gamma$ is $n_B \sim \Gamma f^2 \theta_i^3$. Between the time of peak production of baryon asymmetry at $t_a \sim 1/m_R$ and the peak entropy production at $t_b \sim 1/\Gamma$ we will take the baryon asymmetry to be diluted by a factor of $(R_a / R_b)^3 \sim (t_a / t_b)^2 \sim (\Gamma / m_R)^2$ due to the expansion of the universe, where we have taken the universe to behave as matter dominated with $R \propto t^{2/3}$ in the usual
fashion during reheating. Thus the baryon-to-entropy ratio at time $t_b$ and afterwards is $(n_B/s)_2 \sim \Gamma f^2 \theta_1^3 (\Gamma/m_R)^2 / s$. The calculation of the entropy density is exactly the same as described above eq. (3.31), while the baryonic charge density is larger than the $H < \Gamma$ case by a factor of $(\theta_i/\theta_1)^3 (\Gamma/m_R)^2 = \theta_i/\theta_1 = m_R/\Gamma = 8\pi/g^2 \gg 1$. Consequently, we get that the total baryon asymmetry of the universe is approximately equal to

$$
(n_B/s)_2 = \frac{\theta_i}{\theta_1} \left( \frac{n_B}{s} \right)_1 \approx 3 \times 10^{-3} \frac{g^3}{g_s^{1/4}} \left( \frac{m_{Pl}}{f} \right)^{3/2} \frac{f}{\Lambda} \left( \frac{1 - \epsilon^2}{1 + \epsilon^2} \right)^2.
$$

(3.32)

Here subscript 2 refers to the case where expansion has been included. Henceforth we use eq. (3.32) as our estimate of the baryon asymmetry produced.

**Section 4: Discussion**

In ref. [14], the authors obtain constraints on the parameters $\Lambda$ and $f$. The stipulation that a large fraction of the universe after inflation have inflated by at least 60 e-foldings gives $f \geq 0.06 M_{Pl}$. A stronger constraint can be obtained by requiring the formation of galaxies to take place early enough in the history of the universe; in this way one obtains $f \geq 0.3 M_{Pl}$. A constraint on $\Lambda$ is derived by using COBE data on the density fluctuation amplitude and is plotted in fig. 1 of ref. [14]; the upper bound on $\Lambda$ thus obtained ranges from $10^{13}$ GeV to $10^{16}$ GeV for $f$ between 0.3 $M_{Pl}$ and 1.2 $M_{Pl}$. If one desires the density fluctuations from inflation to be responsible for the large scale structure of our universe and hence for the COBE anisotropy, then $\Lambda$ must be equal to the above values rather than simply being bounded by these numbers.

If the baryon asymmetry produced above is accompanied by an equal lepton asymmetry, so that $B - L = 0$, it will be wiped out by baryon number violating sphaleron processes unless the reheat temperature is below 100 GeV. The low reheat temperature condition may be a desirable feature of our model as many inflation models have difficulty creating a high reheat temperature. Furthermore, we shall require that $T_{rehe} > 10 \text{MeV}$ so that we reproduce standard nucleosynthesis. If, in addition, one requires the density fluctuations from inflation to serve as the explanation for the COBE data rather than merely being bounded by it, then $\Lambda$ is determined as a function of $f$ as described in the previous paragraph; then the combination of these constraints implies that $10^{-14} < g < 10^{-10}$ for $\Lambda$ and $f$ equal to $10^{13}$ GeV and 0.3$M_{Pl}$ respectively, and the asymmetry generated by the mechanism considered above is by far below the necessary observed value. However if $\Lambda$ is merely bounded by COBE measurements (density fluctuations must then be generated some other way than by the inflation), then $g$ can be much larger as can the baryon asymmetry. Alternatively if a nonzero $(B - L)$ is generated, for example, if the $L$ fields carry no lepton number, then it is not destroyed by the electroweak processes and the coupling constant $g$ need not be so small.
In our perturbative calculations of the number of pairs of particles produced we have assumed that the masses of the fermions are smaller than the mass $m_R$ of the theta-field and that $gf < m_{Q,L}$; otherwise the perturbative approach is not applicable. This implies that $gf < m_R = \Lambda^2/f$ or $g < (\Lambda/f)^2$. In this case, the baryon asymmetry is rather small as $\left(\frac{2n_B}{s}\right)_2 < 10^{-3}(\Lambda/f)^5(m_{Pl}/f)^{1.5} < 10^{-18}$ (in obtaining this limit we have included the simultaneous constraint on $\Lambda$ and $f$ from density fluctuation constraints in ref. [14]). If, however, $\theta$ is not the inflaton field, as in the original version of the spontaneous baryogenesis scenario [15], then the parameters $\Lambda$ and $f$ do not necessarily satisfy the above bounds and the asymmetry may be quite large, especially if $f \ll m_{Pl}$. In such a case, one would have to redo the calculation of the entropy if $\theta$ does not dominate the energy density of the universe when it decays. A period of inflation prior to the decay of the PNGB would also be required so that $\theta$ and, consequently, the baryon asymmetry have the same sign within present-day domains of sizes 10 Mpc or greater. (Existing data do not rule out a matter symmetric universe with domains of matter and antimatter on scales of 10 Mpc or more [19].)

An interesting possibility is that the mass of fermions is not below $m_R$ and the perturbative approach is not applicable. The non-perturbative calculations in this case are more complicated and will be presented elsewhere.

We would also like to point out an objection to the mechanism of creating the baryon asymmetry thermodynamically, via an effective chemical potential interpretation, as first proposed in ref. [15] and later applied to spontaneous baryogenesis models at the electroweak phase transition [20]. The approach in ref. [15] is to identify $\dot{\theta}$ in the term $\partial_\mu \theta J^\mu = -\dot{\theta} J^0 = -\dot{\theta} n_B$ in eq. (2.6) with an effective chemical potential that biases the $B$ violating interactions in the universe to create more baryons than antibaryons. If this is the case then, as is argued in ref. [15], the net baryon number density in the thermal bath is given by the expression: $n_B \sim \dot{\theta} T^2$. However this can not be true because it contradicts the equation of motion of the Goldstone field: $\partial^2 \theta = -\partial_\mu J^\mu_B/f^2$. Assuming spatial homogeneity, this equation gives $n_B \sim \dot{\theta} f^2$. (A similar contradiction is obtained using the equation of motion for a PNGB-field in an expanding universe.) This contradiction is resolved because $\dot{\theta}$ does not shift energies of baryons and antibaryons and cannot be identified with a chemical potential. While the term $\partial_\mu J^\mu$ exists in the Lagrangian density in eq. (2.6), it does not in the Hamiltonian density

$$H = \frac{\partial L_{\text{eff}}}{\partial \dot{\phi}_i} \phi_i - L_{\text{eff}}(\phi_i, \dot{\phi}_i),$$

where $\phi_i$ represents all the fields in the Lagrangian density [21]. This is similar to the interaction of a charged particle with a magnetic field, where the energy of the particle is
not changed due to the action of the field as the force is proportional to the velocity and orthogonal to it. Thus the term $-\dot{\theta} n_B$ does not appear in the Hamiltonian density and $\dot{\theta}$ cannot be interpreted as an effective chemical potential.

As mentioned above, the approach of ref. [15] has been applied to create the baryon asymmetry in spontaneous baryogenesis models at the electroweak phase transition [20]. The role of the $\theta$ field is associated with the Higgs field in electroweak baryogenesis. Since we feel that the identification of $\dot{\theta}$ as an effective chemical potential is incorrect, these models too are subject to the same criticism.

In conclusion, we have calculated the baryon asymmetry created by a pseudo Nambu-Goldstone boson with baryon number violating couplings in the context of natural inflation. We have obtained a general result for the baryon asymmetry created by the decay of an oscillating scalar field with baryon number violating couplings and demonstrated explicitly that the asymmetry is not proportional to $\theta_1$ to the first power as claimed in earlier work. We have also discussed our objection to the thermodynamical generation of the baryon asymmetry using an effective chemical potential approach in models of spontaneous baryogenesis.

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**Appendix A: Number Density of Produced Particles in Terms of One Pair Production Amplitude**

Here we use the Bogolyubov transformation method to obtain eq. (3.2). We show that in the lowest order of perturbation theory, the average number density of particle antiparticle pairs produced by decay of the initial scalar field is given by

$$n = \frac{1}{V} \sum_{s_1, s_2} \int \frac{d\vec{p}_1}{(2\pi)^3} \frac{d\vec{p}_2}{(2\pi)^3} |A|^2,$$

where $A$ is the one pair production amplitude and subscripts 1 and 2 refer to the final particle and antiparticle produced. For simplicity we will work with scalar fields here; the generalization to production of fermions is similar and has been performed in ref. [22].

We begin with a classical scalar field $\phi(t)$ coupled to a quantum complex scalar $\chi$:

$$L_{int} = g \phi(t) \chi^* \chi. \quad (A.1)$$
At early times \( t \to -\infty \), we take \( \phi(t) = 0 \) so that \( \chi \) is expanded in terms of creation and annihilation operators,

\[
\chi = \int \tilde{d}k \left[ a_k \exp(-i\omega t + i\mathbf{k} \cdot \mathbf{x}) + b_k^\dagger \exp(i\omega t - i\mathbf{k} \cdot \mathbf{x}) \right], 
\]

where \( \omega = \sqrt{k^2 + m^2} \). Here the commutators are \([a_{k_1}, a_{k_2}^\dagger] = (2\pi)^3 2k_1^0 \delta^{(3)}(\mathbf{k}_1 - \mathbf{k}_2)\) and a similar relation holds for the antiparticle creation and annihilation operators \( b_k \). Then, at later times, \( \phi(t) \neq 0 \) and eq. (A.2) is replaced by

\[
\chi = \int \tilde{d}k \left[ a_k f_k(t) \exp(i\mathbf{k} \cdot \mathbf{x}) + b_k^\dagger f_k^*(t) \exp(-i\mathbf{k} \cdot \mathbf{x}) \right], 
\]

with equation of motion

\[
\left[ \partial_t^2 + \mathbf{k}^2 + m^2 - g\phi(t) \right] f_k(t) = 0. 
\]

The subscript on \( f_k \), and on \( \alpha_k \) and \( \beta_k \) below, refers to \(|\mathbf{k}|\) and not to the momentum four vector. For continuity at early times \( f_k(t \to -\infty) = \exp(-i\omega t) \). We also assume that \( \phi(t) \to 0 \) for \( t \to \infty \). Then we have

\[
f_k(t \to +\infty) \to \alpha_k e^{-i\omega t} + \beta_k e^{i\omega t},
\]

so that \( \chi(t) \) evolves as

\[
\chi(t \to +\infty) = \int \tilde{d}k \left[ \exp(-i\omega t + i\mathbf{k} \cdot \mathbf{x})(\alpha_k a_k + \beta_k^* b_k^\dagger) \right.

\[
+ \exp(i\omega t - i\mathbf{k} \cdot \mathbf{x})(\alpha_k^* b_k^\dagger + \beta_k a_k) \right]. 
\]

One can define new creation and annihilation operators, for particles:

\[
\tilde{a}_k = \alpha_k a_k + \beta_k^* b_k^\dagger, 
\]

and for antiparticles:

\[
\tilde{b}_k = \alpha_k b_k + \beta_k^* a_k^\dagger. 
\]

Then the operator of final particle number is given by \( \tilde{N}_k = \tilde{a}_k^\dagger \tilde{a}_k^\dagger / [2k^0 V] \).

The number of particles in the final state of momentum \( \mathbf{k} \) is given by

\[
N_k = \langle 0 | \tilde{N}_k | 0 \rangle = |\beta_k|^2. 
\]

Thus the total number density of produced particles is

\[
n = \frac{1}{V} \frac{V}{(2\pi)^3} \int d^3k N_k = \int \frac{d^3k}{(2\pi)^3} |\beta_k|^2. 
\]
This result, obtained by the method of Bogolyubov coefficients, can be found in refs. [17,23].

Now we shall calculate $\beta_k$ in perturbation theory. Expanding $f = f_0 + f_1$, we have $f_0 = \exp(-i\omega t)$ and the equation of motion (A.4) becomes

\[(\partial_t^2 + k^2 + m^2)f_1 = g\phi(t)\exp(-i\omega t).\]  

(A.10)

Using the Green’s function method we find

\[f_1(t) = -g \int \frac{d\omega'}{2\pi} \frac{\tilde{\phi}(\omega' - \omega)}{\omega'^2 - k^2 - m^2} e^{-i\omega't},\]  

(A.11)

Taking the residue at the pole $\omega' = -\sqrt{k^2 + m^2} = -\omega$, we find the coefficient of $\exp(+i\omega t)$ to be,

\[\beta_k = ig[\tilde{\phi}(2\omega)]^*/2\omega.\]  

(A.12)

Now, for comparison, let us calculate the field theory amplitude with the interaction Lagrangian given by eq. (A.1),

\[A = \langle k_1, \bar{k}_2| i \int d^4x \ g\phi(t)\chi^*\chi|0\rangle.\]  

(A.13)

Perturbatively the matrix element is easy to calculate using eq. (A.2), and we find

\[A = ig(2\pi)^3\delta^3(\mathbf{k}_1 + \mathbf{k}_2) \int dt \ \phi(t)\exp[i(\omega_1 + \omega_2)t],\]  

(A.14)

so that

\[|A|^2 = g^2V(2\pi)^3\delta^3(\mathbf{k}_1 + \mathbf{k}_2)|\tilde{\phi}(\omega_1 + \omega_2)|^2.\]  

(A.15)

Now if we integrate over $d\mathbf{k}_1$ $d\mathbf{k}_2$, we find that

\[n = \frac{1}{V} \int d\mathbf{k}_1 \ d\mathbf{k}_2 |A|^2 = \int \frac{d^3k}{(2\pi)^3} \ g^2 |\tilde{\phi}(2\omega)|^2 \frac{1}{4\omega^2}.\]  

(A.16)

This is exactly eq. (A.9) with $\beta_k$ given by eq. (A.12). Thus we have shown that the number density of produced particles is given by the integral of the one pair production amplitude squared.

**Appendix B: Second Derivation of Number Density of Produced Particles in Terms of One Pair Production Amplitude**

Eq. (3.2) can also be obtained using the method presented in Sec. 4-1-1 of ref. [18]. (We have ignored the higher order vacuum graphs that give the exponential factor $\exp(-\bar{n})$ in eq. (4-23) of ref. [18].) We have verified that we obtain the Poisson distribution for the number of $(Q, \bar{L})$ pairs and $(\bar{Q}, L)$ pairs as in ref. [18]. Indeed the derivation of the

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Poisson distribution can be done exactly along the same lines as in ref. [18]. The only difference is that in the example considered in this book the matrix element describes the production of a single photon by an external current while in our case it gives the amplitude for production of a pair of particles. For the multiparticle production amplitude this gives rise to a different normalization, namely, in the case of the production of \(n\) photons the amplitude contains the factor \(1/\sqrt{n!}\) connected with identical photons while for the case of production of \(n\) pairs of \(QL\) (or charge conjugate) the amplitude contains \(1/n!\). In the case of photons the multiparticle amplitude squared contains the following \(n\)-dependent factors:

\[
|A_n^\gamma|^2 \sim |(n!)(1/n!)(1/\sqrt{n!})|^2 \sim 1/n!.
\]

The first factor of \(n!\) comes from \(n!\) combinations which appear when the photon production operator act on the multiphoton state \(\langle k_1, k_2, \ldots, k_n | a_k^+ \rangle\). The factor of \(1/n!\) comes from the expansion of the action \(S = \exp(i \int d^4x A^\mu J_\mu)\), and the factor of \(1/\sqrt{n!}\) comes from the normalization of the \(n\)-photon state. So the net result is proportional to \(1/n!\), which is exactly what is needed to get the Poisson distribution \(p_n = \exp(-\bar{n})\bar{n}^n/n!\). In the case of the production of \(n\)-pairs, we have the same \(1/n!\) from the expansion of the action, but now we get \(1/n!\) coming from the normalization and not \(1/\sqrt{n!}\) as before. However, the action of the product of the creation operators of \(Q\) and \(\bar{L}\), which can be symbolically written as \((a_Q^+ b_L^+)\)^n, gives now an overall factor of \(n!\) from the action of, say, \((a_Q^+)^n\), as above, and also the sum of \(n!\) equal but not interfering terms, each of them being proportional to a different delta-function of the momenta, \(\delta(p_{Qj} + p_{Lk})\). Thus in the matrix element squared we will get the same overall factor \(1/n!\) which is necessary for the Poisson distribution.

**Appendix C: Calculation of Baryon Asymmetry**

Here we calculate the lowest order nonzero contribution to the baryon asymmetry; we derive eq. (3.13) from eqs. (3.9) and (3.10). As our starting point, we have

\[
n_b - n_{\bar{b}} = \frac{g^2 f^2}{\pi^2} \int d\omega \omega^2 \left[ \frac{\tilde{\theta}(2\omega) \left[ \tilde{\theta}^2(2\omega) \right]^*}{2i} + \text{h.c.} \right], \tag{C.1}
\]

where

\[
\tilde{\theta}(2\omega) = \int_{-\infty}^{\infty} dt e^{2i\omega t} \theta(t) \tag{C.2}
\]

and

\[
\tilde{\theta}^2(2\omega) = \int_{-\infty}^{\infty} dt e^{2i\omega t} \theta^2(t). \tag{C.3}
\]

Using eq. (3.8), we find that

\[
\tilde{\theta}(2\omega) = \frac{\theta}{4i\omega} \left[ \frac{(-\Gamma/2 + im_R)}{(-\Gamma/2 + im_R) + 2i\omega} - \frac{\Gamma/2 + im_R}{(-\Gamma/2 - im_R + 2i\omega)} \right]. \tag{C.4}
\]
and
\[ \tilde{\theta}^2(2\omega)^* = -\frac{\theta_i^2}{4i\omega} \left[ \frac{(im_R + \Gamma/2)}{2im_R + 2i\omega + \Gamma} + \frac{(-im_R + \Gamma/2)}{2i\omega - 2im_R + \Gamma} + \frac{\Gamma}{2i\omega + \Gamma} \right] \] (C.5)

Thus
\[
\tilde{\theta}^2(2\omega)^* = \frac{\theta_i^3}{16\omega^2} \left[ \frac{(-m_R^2 - \Gamma^2/4)}{(2i\omega + \Gamma)(2i\omega + im_R - \Gamma/2)} + \frac{(m_R^2 - \Gamma^2/4 + \Gamma im_R)}{(2i\omega - 2im_R + \Gamma)(2i\omega + im_R - \Gamma/2)} \right. \\
+ \frac{\Gamma(im_R - \Gamma/2)}{(2i\omega + \Gamma)(2i\omega + im_R - \Gamma/2)} - \frac{(-m_R^2 + im_R + \Gamma^2/4)}{(2i\omega + \Gamma)(2i\omega + im_R - \Gamma/2)} \\
- \frac{m_R^2 + \Gamma^2/4}{(2i\omega - 2im_R + \Gamma)(2i\omega - im_R - \Gamma/2)} - \frac{\Gamma(im_R + \Gamma/2)}{(2i\omega + \Gamma)(2i\omega - im_R - \Gamma/2)} \].
\] (C.6)

Now we must integrate each of the terms in eq. (C.6) as indicated in eq. (C.1). The lower limit of the integral is \(m_Q + m_L \ll m_R\) and we use \(\Gamma \ll m_Q + m_L\). We find that the first term cancels with its hermitian conjugate, the third and sixth terms are 0, the second and fourth terms cancel each other, and the fifth term plus its hermitian conjugate is responsible for the final result given in eq. (3.13),
\[
n_B \equiv n_b - \bar{n}_b = \frac{g^2}{16\pi} m_R f^2 \theta_i^3 \\
= \frac{1}{2} \Gamma f^2 \theta_i^3 
\] (C.7)

### Appendix D: The Effects of Mixing in the \(Q, L\) Basis

We will consider the decay of \(\theta\) to a \(Q\bar{L}\) pair (superscript 1 for this decay channel), and the decay of \(\theta\) to a \(\bar{Q}L\) pair (superscript 2 for this decay channel). For the first decay channel, from eq. (3.16) we see that a \(Q\) produced at the time \(t = 0\) is given by
\[
\psi(0) = Q = s\psi_1 + c\psi_2, \quad (D.1a)
\]
where
\[
c = \frac{1}{\sqrt{1 + \epsilon^2}} \quad \text{and} \quad s = \frac{\epsilon}{\sqrt{1 + \epsilon^2}}. \quad (D.1b)
\]
Similarly,
\[
\bar{\chi}(0) = \bar{L} = c\bar{\psi}_1 - s\bar{\psi}_2. \quad (D.2)
\]
We will let the fields \(\psi\) and \(\chi\) evolve in time, mixing their \(Q\) and \(L\) components as they travel. The time evolution of \(\psi(t)\) can be modeled as follows:
\[
\psi(t) = (se^{-i\Delta\omega t}\psi_1 + c\psi_2)\exp(-i\omega_2 t), \quad (D.3)
\]
where $\Delta \omega = \omega_1 - \omega_2$. We now wish to ask the question: what is the $Q$ content at some time $t$ of the field $\psi$ which was initially pure $Q$? Using eq. (3.16), we can write eq. (D.3) as
\begin{equation}
\psi(t) = [(c^2 + s^2 e^{-i\Delta \omega t})Q - s c(1 - e^{-i\Delta \omega t})L] \exp(-i\omega_2 t).
\end{equation}

The quark content is given by the magnitude squared of the coefficient of the first term, so that
\begin{equation}
n_Q^{(1)}(t) = [c^4 + s^4 + 2c^2 s^2 \cos \Delta \omega t] \frac{1}{V} \sum_{s_Q, s_L} \int \tilde{dk}_Q \tilde{dk}_L |A_{QL}|^2.
\end{equation}

Similarly, from the same decay process $\theta \rightarrow Q + \bar{L}$, the $\bar{L}$ that is produced can convert to a $\bar{Q}$ so that we have
\begin{equation}
n_{\bar{Q}}^{(1)}(t) = \frac{1}{V} \sum_{s_Q, s_L} 2s^2 c^2 (1 - \cos \Delta \omega t) \int \tilde{dk}_Q \tilde{dk}_L |A_{QL}|^2.
\end{equation}

From $\theta \rightarrow L\bar{Q}$, one can obtain $Q$ at a later time from oscillations of either the $L$ or the $\bar{Q}$ and find contributions:
\begin{equation}
n_Q^{(2)}(t) = [c^4 + s^4 + 2c^2 s^2 \cos \Delta \omega t] \frac{1}{V} \sum_{s_L, s_Q} \int \tilde{dk}_L \tilde{dk}_\bar{Q} |A_{L\bar{Q}}|^2.
\end{equation}

and
\begin{equation}
n_{\bar{Q}}^{(2)}(t) = \frac{1}{V} \sum_{s_L, s_{\bar{Q}}} 2s^2 c^2 (1 - \cos \Delta \omega t) \int \tilde{dk}_L \tilde{dk}_\bar{Q} |A_{L\bar{Q}}|^2.
\end{equation}

Thus the baryon asymmetry at any time $t$ is
\begin{equation}
n_B(t) = n_Q^{(1)}(t) + n_Q^{(2)}(t) - n_{\bar{Q}}^{(1)}(t) - n_{\bar{Q}}^{(2)}(t)
= [(c^2 - s^2)^2 + 4s^2 c^2 \cos \Delta \omega t] \sum_{s_L, s_Q} (|A_{QL}|^2 - |A_{L\bar{Q}}|^2)
\end{equation}
\begin{equation}
= \left[ \left( \frac{1 - \epsilon^2}{1 + \epsilon^2} \right)^2 + 4s^2 c^2 \cos \Delta \omega t \right] \sum_{s_L, s_Q} (|A_{QL}|^2 - |A_{L\bar{Q}}|^2).
\end{equation}

One can see that the baryon asymmetry oscillates in time as a cosine about the average value. When one takes a time average, the cosine term averages to zero, and one reproduces the result in eq. (3.28),
\begin{equation}
n_B = \frac{1}{2} f^2 \theta^3 \left( \frac{1 - \epsilon^2}{1 + \epsilon^2} \right)^2.
\end{equation}

Our derivation above assumes in eqs. (D.5-D.8) that all $Q\bar{L}$ pairs and all $L\bar{Q}$ pairs were produced at the same time. If one considers that all pairs are not produced at the same time then an average over all pairs would also cancel the $\cos \Delta \omega t$ term in eqs. (D.5-D.8).
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4) Natural inflation can be realised in many realistic particle physics models in which a Nambu-Goldstone boson acquires a potential of the form in eq. (1.1). In a class of $Z_2$ symmetric models the combination of terms like $m_1 \bar{\psi} \psi$ and $m_0 (\bar{\psi} \psi e^{i \theta} + h.c.)$ can give rise to a potential as in eq. (1.1) for $\theta$ with $\Lambda^2 \sim m_0 m_1$. These “schizon” models are further described in ref. [5,6,7]. In superstring models, non-perturbative effects in the hidden sector can give rise to fermion condensates and, consequently, a potential for the model independent axion (the imaginary part of the dilaton field). In refs. [8,9], the hidden $E_8'$ sector in $E_8 \times E_8'$ heterotic string theory becomes strongly interacting, generating gaugino condensates that lead to SUSY breaking and a potential for the model independent axion. Problems with the stability of the dilaton potential in such models has prompted others to consider multiple gaugino condensation models which give a suitable potential for the axion [10,11,12]. Fermion condensates in technicolor theories can also give rise to potentials of the form above for fields coupled to the fermions. Also, a theory with an antisymmetric tensor field $B_{\mu \nu}$ (which arises, for example, in string theory) with a field strength

$$H^{\mu \nu \lambda} = \partial^\mu B^{\nu \lambda} + \partial^\lambda B^{\mu \nu} + \partial^\nu B^{\lambda \mu}$$

has an effective action which can be expressed in terms of a scalar field with a potential of the form in eq. (1.1) [13]. In a variant of these models, the tensor field can be coupled to a fundamental real scalar field $u$ with the symmetry breaking potential of the form $V(u) = (\lambda') u^2 - 6m^2/\lambda'$. This also leads to a potential as in eq. (1.1) for the scalar $\theta$.

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