1 Overview

The last few years in cosmology have been thrilling ones, as dramatic improvements in observational technology have begun to impose stringent constraints on theoretical ideas in cosmology built up over the preceding two decades. For the purposes of this article I’ll focus on the following set of overall goals:

- To obtain a physical description of the Universe, including its global dynamics and matter content.
- To measure the cosmological parameters describing the Universe, and to develop a fundamental understanding of as many of those parameters as possible.
- To understand the origin and evolution of cosmic structures.
- To understand the physical processes which took place during the extreme heat and density of the early Universe.

Over recent years, much progress has been made on all of these topics, to the extent that it is widely believed amongst cosmologists that we may stand on the threshold of the first precision cosmology, in which the parameters necessary to describe our Universe have been identified and will soon be, in most cases at least, measured to a satisfying degree of precision. Whether this optimism has any grounding in reality remains to be seen, though so far the signs are promising in that the basic picture of cosmology, centred around the Hot Big Bang, has time and again proven the best framework for interpreting the constantly improving observational situation.

In particular, the process of cosmological parameter estimation is well underway, thanks to observations of distant Type Ia supernovae, of galaxy clustering, and of the cosmic microwave background. These have established a standard cosmological model, where the Universe is dominated by dark energy, contains substantial dark matter, and with the baryons from which we
are made comprising only around 4%. Overall this model can be described by around ten parameters (e.g. see Ref. 1), and the viable region of parameter space is starting to shrink under pressure from observations. However, it is worth bearing in mind that we seek high precision determinations at least in part because they ought to shed light on fundamental physics, and there progress has been less rapid. Some parameters are likely to have no particular fundamental importance (for instance, there would probably be little fundamental significance were the Hubble constant to turn out to be $63 \text{ km s}^{-1} \text{ Mpc}^{-1}$ rather than say $72 \text{ km s}^{-1} \text{ Mpc}^{-1}$, though accurate determination of this parameter is essential if we are to pin down other parameters), but the 10% or so measured accuracy of the baryon density is to be set against the lack of even an order-of-magnitude theoretical understanding thus far.

This article does not attempt to cover the complete range of modern cosmology, but is intended as a status report on a subset of topics which I’ve chosen as being potentially of the most interest to theoretical particle physicists. The main descriptive sections concern structure formation in the Universe and the inflationary cosmology, and the final section is a mixed bag of especially topical subjects.

2 Structure Formation in the Universe

2.1 Gravitational instability

One of the most powerful tools in cosmology is the development of structures. By ‘structure’ I mean anything corresponding to inhomogeneity within the Universe, be it galaxies, variations in the gravitational potential, or anisotropies in the cosmic microwave background. The evolution of structures proves sensitive to all the main cosmological parameters, and hence is well suited to constraining them. Different types of observation naturally probe different physical regimes, for instance small verses large scales, and also different stages of the Universe’s evolution, with the microwave background probing the Universe when it was around one thousandth of its present size.

The young universe was much closer to uniformity than the present state; for instance the irregularities in the cosmic microwave background are only around one part in $10^5$, while the present matter distribution features highly overdense galaxies with voids in between. The main driving force in this evolution, at least in its initial stages, is simply gravity; any initial overdensity will exert an unbalanced gravitational force upon neighbouring material and will tend to accrete material, amplifying the original perturbations. At least until well after the cosmic microwave background radiation is released, the
perturbation evolution is well described on all scales by linear perturbation
theory, though ultimately linear theory for the density field breaks down on
short scales as virialized galaxies begin to form. On sufficiently large scales
linear theory remains adequate even today.

The Hot Big Bang model, supplemented by gravitational instability in
order to form structures, gives an excellent broad-brush description of our
Universe. However, like any theory or model in physics, its predictions depend
on some input parameters not specified by the theory. A key goal is to measure
those parameters to a satisfying degree of accuracy. For example, the detailed
process of gravitational instability depends on

- The expansion rate of the Universe (the Hubble parameter).
- The density of the material providing the gravitational attraction.
- The physical properties of the material; for example does it only experi-
  ence gravitational attraction, or are other interactions important?
- The form of the initial perturbations that get the whole structure forma-
  tion process going.

Current ideas in cosmology suggest that around 10 parameters may be suf-
icient to describe our Universe. At present, however, we don’t even know
the complete set of important parameters, far less have accurate values for
them all. The hope is that over the next few years we will both identify the
important parameters and measure them to high accuracy, in many cases at
the percent level.

2.2 Quantifying microwave background anisotropies

Although the strongest tests of cosmological models will always come from
the combination of all available data, cosmic microwave background (CMB)
anisotropies have received much attention lately (and are likely to be the sin-
gle most important tool for constraining inflation, as discussed in the next
section), and so it is worth spending some time defining the necessary termin-
ology.

We observe the temperature $T(\theta, \phi)$ coming from different directions. We
write this as a dimensionless perturbation and expand in spherical harmonics

$$\frac{T(\theta, \phi) - \bar{T}}{\bar{T}} = \sum_{\ell, m} a_{\ell m} Y_{\ell m}(\theta, \phi).$$

(1)
There is no unique prediction for the coefficients $a_{\ell m}$, but in the simplest inflationary cosmologies they are drawn from a gaussian distribution whose mean square is independent of $m$ and given by the **radiation angular power spectrum**

$$C_\ell = \langle |a_{\ell m}|^2 \rangle_{\text{ensemble}}$$  \hspace{1cm} (2)

The ensemble average represents the theorist’s ability to average over all possible observers in the Universe (or indeed over different quantum mechanical realizations), whereas an observer’s highest ambition is to estimate it by averaging over the multipoles of different $m$ as seen at our own location. The radiation angular power spectrum depends on all the cosmological parameters, and so it can be used to constrain them. To extract the full information polarization also has to be measured; this gives three additional power spectra, describing two independent modes of polarization, and the cross-correlation between the temperature anisotropies and one polarization mode (other cross-correlations vanish assuming absence of parity violation).

Computation of the power spectra requires a lot of physics: gravitational collapse, photon–electron interactions (and their polarization dependence), neutrino free-streaming etc. But as long as the perturbations are small, linear perturbation theory can be used which makes accurate calculations possible. A major step forward for the field was the public release of Seljak & Zaldarriaga’s code `cmbfast` which can compute the spectra within one percent accuracy for a given cosmological model in around one minute. An example spectrum is shown in Figure 1.

### 2.3 Recent CMB results

During 2000 and 2001 studies of microwave anisotropies took a huge leap forward with the first results from a new generation of instruments. First out with results was the Boomerang collaboration, followed closely by the Maxima collaboration; these made the first accurate mapping of the first peak in the angular power spectrum, corresponding to the first gravitational compression of the primordial fluid. The location of this peak is fixed primarily by the propagation of light to us after last-scattering, and is a sensitive probe of the geometry of the Universe. These results are consistent with a flat geometry, with only a small margin for error, and provided a convincing exclusion of a low-density open Universe with $\Omega_0 \sim 0.3$ which had up until then been regarded as a viable cosmology.

The first Boomerang and Maxima results gave tentative, but inconclusive, indication of further features to small angular scales. The situation improved...
further in mid 2001, with new results from the DASI experiment and a re-analysis of the Boomerang data including a much larger fraction of the total dataset. These results are shown in Figure 2, alongside a best-fitting theoretical model. These latest results show the first clear evidence for further oscillations in the angular power spectrum, as predicted in Figure 1. This observation is of particular qualitative significance for the inflationary cosmology, as discussed in the next section, and of quantitative significance for constraining the baryon density as described in the following subsection.

Figure 1. The radiation angular power spectrum for a particular cosmological model. The annotations name the different features.
2.4 The Standard Cosmological Model

The observations of the last few years have led to the establishment of a standard cosmological model, with ingredients as follows.

- Cosmological constant $\sim 66\%$
- Cold dark matter $\sim 30\%$
- Baryons $\sim 4\%$
- Photons and neutrinos $\sim 10^{-4}$
- Spatial flatness.
- Hubble constant around $70\text{ km s}^{-1}\text{ Mpc}^{-1}$.
- Initial conditions seeded by slow-roll inflation.

This model is in remarkable agreement with observational data.
Baryons

There are now three independent powerful ways of estimating the baryon density of the Universe. Listing the uncertainties at 95% confidence, we have

Nucleosynthesis: $\Omega_{\text{baryon}} h^2 = 0.019 \pm 0.002$.

It is widely, though not universally, thought that the measurement of the deuterium abundance in high-redshift absorption systems gives a highly-accurate probe of the baryon density during nucleosynthesis.

Microwave background: $\Omega_{\text{baryon}} h^2 = 0.02 \pm 0.01$.

The baryon density can be inferred from the CMB spectrum, as it governs the relative heights of the first and second peaks (corresponding to compressions and rarefactions of the cosmic fluid respectively). While the Boomerang 2000 results gave a suspiciously high value for this, reanalysis in 2001 plus new results from DASI have brought the value into excellent agreement with nucleosynthesis.

Cluster baryon fraction: $\Omega_{\text{baryon}}/(\Omega_{\text{cdm}} + \Omega_{\text{baryon}}) = 0.12 \pm 0.05$.

Clusters are observed via X-ray emission from hot intracluster gas. This gas is in hydrostatic equilibrium against gravity which is principally supplied by the dark matter. For the standard cosmological model this agrees excellently with nucleosynthesis.

Cosmological constant

Famously, in 1998 two teams studying distant supernovae discovered that they were fainter than expected, and having eliminated other possible causes concluded that this was due to the expansion history of the Universe, and required a presently-accelerating cosmology\footnote{Submitted to World Scientific on October 31, 2018}. This can be brought about by a cosmological constant $\Lambda$, and if one additionally restricts to a flat geometry as motivated by the CMB, this leads to the cosmological constant density of the Standard Cosmological Model.

Now, if that was the sole evidence for a cosmological constant I wouldn’t believe it for a second. However the circumstantial evidence is extremely powerful; for instance

1. Microwave anisotropies show the Universe is flat (or close to flat), provided the initial perturbations are adiabatic.

2. Nucleosynthesis plus the cluster baryon fraction imply $\Omega_{\text{cdm}} + \Omega_{\text{baryon}} \sim 0.3$ which implies $\Omega_\Lambda \sim 0.7$. 
3. The correct galaxy power spectrum is reproduced if \((\Omega_{\text{cdm}} + \Omega_{\text{baryon}})h \simeq 0.2\) (where \(h\) is the Hubble constant in the usual units); this concurs well with direct measures of \(h\).

As a result of this and other arguments, the so-called \(\Lambda\)CDM model presently has no serious rivals.

The cosmological constant poses the twin problems of its unexpectedly small magnitude (in fundamental physics terms) and the mystery of why it should only come to dominate the Universe at the present epoch (around redshift 0.3). To address these, instead of a pure cosmological constant, one might prefer an effective one, for example a slowly-rolling potential-dominated scalar field as described in the next section for early Universe inflation. Such scenarios are known as quintessence. Current observations force such scenarios to be quite close to the pure cosmological constant, and though differences may yet be unveiled by improved experiments it appears only quite limited information will be available. It is actually becoming quite hard to construct simple quintessence models capable of matching all observations while employing plausible initial conditions.

2.5 What’s coming up?

Here is a selection (far from complete) of things to look out for in coming years which will drive further moves to precision cosmology.

| Year | Event |
|------|-------|
| Current | NASA’s Map satellite was launched in mid-2001 and is currently making an all-sky survey of the CMB (results late 2003??). |
| 2002 | Maxima and Boomerang make the first serious attempts to measure CMB polarization anisotropies. |
| 2001–2004 | Main operations phase of the Sloan Digital Sky Survey, seeking to redshift a million galaxies. |
| 2002–2005 | First systematic surveys for high-redshift galaxy clusters using X-rays and the Sunyaev–Zel’dovich effect. |
| 2007 | ESA’s Planck satellite launched, for high-resolution all-sky mapping of CMB temperature and polarization. |
| 2010?? | Launch of the LISA satellites, capable of probing a stochastic gravitational wave background (though not the inflationary one except in exceptional models). |

3 The Inflationary Cosmology
3.1 Overview and models

This section focusses on the last two of the goals listed at the start of this article. The claim is that during the very early Universe, a physical process known as **inflation** took place, which still manifests itself in our present Universe via the perturbations it left behind which later led to the development of structure in the Universe. By studying those structures, we hope to shed light on whether inflation occurred, and by what physical mechanism An extensive account of inflation appears in my textbook with David Lyth.

I begin by defining inflation. The scale factor of the Universe at a given time is measured by the scale factor \( a(t) \). In general a homogeneous and isotropic Universe has two characteristic length scales, the curvature scale and the Hubble length. The Hubble length is more important, and is given by

\[
c H^{-1} \quad \text{where} \quad H \equiv \frac{\dot{a}}{a}. \tag{3}
\]

Typically, the important thing is how the Hubble length is changing with time as compared to the expansion of the Universe, i.e. what is the behaviour of the **comoving Hubble length** \( H^{-1}/a \)?

During any standard evolution of the Universe, such as matter or radiation domination, the comoving Hubble length increases. It is then a good estimate of the size of the observable Universe. **Inflation** is defined as any epoch of the Universe’s evolution during which the comoving Hubble length is decreasing

\[
\frac{d \left( H^{-1}/a \right)}{dt} < 0 \iff \dot{a} > 0, \tag{4}
\]

and so inflation corresponds to any epoch during which the Universe has accelerated expansion. During this time, the expansion of the Universe outpaces the growth of the Hubble radius, so that physical conditions can become correlated on scales much larger than the Hubble radius, as required to solve the horizon and flatness problems.

As discussed in the last section, there is very good evidence from observations of Type Ia supernovae that the Universe is presently accelerating. This is usually attributed to the presence of a cosmological constant. This is clearly at some level good news for those interested in the possibility of inflation in the early Universe, as it indicates that inflation is possible in principle, and certainly that any purely theoretical arguments which suggest inflation is not possible should be treated with some skepticism.

If the Universe contains a fluid, or combination of fluids, with energy
density $\rho$ and pressure $p$, then
\[ \ddot{a} > 0 \iff \rho + 3p < 0, \tag{5} \]
(where the speed of light $c$ has been set to one). As we always assume a positive energy density, inflation can only take place if the Universe is dominated by a material which can have a negative pressure. Such a material is a scalar field, usually denoted $\phi$. A homogeneous scalar field has a kinetic energy and a potential energy $V(\phi)$, and has an effective energy density and pressure given by
\[ \rho = \frac{1}{2} \dot{\phi}^2 + V(\phi) \quad ; \quad p = \frac{1}{2} \dot{\phi}^2 - V(\phi). \tag{6} \]
The condition for inflation can be satisfied if the potential dominates.

A model of inflation typically amounts to choosing a form for the potential, perhaps supplemented with a mechanism for bringing inflation to an end, and perhaps may involve more than one scalar field. In an ideal world the potential would be predicted from fundamental particle physics, but unfortunately there are many proposals for possible forms. Instead, it has become customary to assume that the potential can be freely chosen, and to seek to constrain it with observations. A suitable potential needs a flat region where the potential can dominate the kinetic energy, and there should be a minimum with zero potential energy in which inflation can end. Simple examples include $V = m^2 \phi^2/2$ and $V = \lambda \phi^4$, corresponding to a massive field and to a self-interacting field respectively. Modern model building can get quite complicated — see Ref. [10] for a review.

### 3.2 Inflationary cosmology: perturbations

By far the most important aspect of inflation is that it provides a possible explanation for the origin of cosmic structures. The mechanism is fundamentally quantum mechanical; although inflation is doing its best to make the Universe homogeneous, it cannot defeat the uncertainty principle which ensures that residual inhomogeneities are left over. These are stretched to astrophysical scales by the inflationary expansion. Further, because these are determined by fundamental physics, their magnitude can be predicted independently of the initial state of the Universe before inflation. However, the magnitude does depend on the model of inflation; different potentials predict different cosmic structures.

One way to think of this is that the field experiences a quantum ‘jitter’ as it rolls down the potential. The observed temperature fluctuations in the cosmic microwave background are one part in $10^5$, which ultimately means that
the quantum effects should be suppressed compared to the classical evolution by this amount.

Inflation models generically predict two independent types of perturbation:

**Density perturbations** $\delta^2_H(k)$: These are caused by perturbations in the scalar field driving inflation, and the corresponding perturbations in the space-time metric.

**Gravitational waves** $A_T^2(k)$: These are caused by perturbations in the space-time metric alone.

They are sometimes known as scalar and tensor perturbations respectively, because of the way they transform. Density perturbations are responsible for structure formation, but gravitational waves can also affect the microwave background.

We do not expect to be able to predict the precise locations of cosmic structures from first principles (any more than one can predict the precise position of a quantum mechanical particle in a box). Rather, we need to focus on statistical measures of clustering. Simple models of inflation predict that the amplitudes of waves of a given wavenumber $k$ obey gaussian statistics, with the amplitude of each wave chosen independently and randomly from a gaussian. What it does predict is how the width of the gaussian, known as its amplitude, varies with scale; this is known as the **power spectrum**.

With current observations it is a good approximation to take the power spectra as being power laws with scale, so

$$\delta^2_H(k) = \delta^2_H(k_0) \left(\frac{k}{k_0}\right)^{n-1} \quad (7)$$

$$A_T^2(k) = A_T^2(k_0) \left(\frac{k}{k_0}\right)^{n_T} \quad (8)$$

In principle this gives four parameters — two amplitudes and two spectral indices — but in practice the spectral index of the gravitational waves is unlikely to be measured with useful accuracy, which is rather disappointing as the simplest inflation models predict a so-called consistency relation relating $n_T$ to the amplitudes of the two spectra, which would be a distinctive test of inflation. The assumption of power-laws for the spectra requires assessment both in extreme areas of parameter space and whenever observations significantly improve.
3.3 The current status of inflation

The best available constraints come from combining data from different sources; for two recent attempts see Wang et al. and Efstathiou et al. Suitable data include observations of the recent dynamics of the Universe using Type Ia supernovae, cosmic microwave anisotropy data, and galaxy correlation function data.

Currently inflation is a definite qualitative success, with striking agreement between the predictions of the simplest inflation models and observations. In particular, the locations of the microwave anisotropy power spectrum peaks are most simply interpreted as being due to an adiabatic initial perturbation spectrum in a spatially-flat Universe. The multiple peak structure strongly suggests that the perturbations already existed at a time when their corresponding scale was well outside the Hubble radius. No unambiguous evidence of nongaussianity has been seen.

Quantitatively, however, things have some way to go. At present the best that has been done is to try and constrain the parameters of the power-law approximation to the inflationary spectra. The gravitational waves have not been detected and so their amplitude has only an upper limit and their spectral index is not constrained at all. The current situation can be summarized as follows.

**Amplitude** $\delta_H$: COBE determines this (assuming no gravitational waves) to about ten percent accuracy (at one-sigma) as approximately $\delta_H = 1.9 \times 10^{-5} \Omega_0^{-0.8}$ on a scale close to the present Hubble radius (see Refs. for accurate formulae).

**Spectral index** $n$: This is thought to lie in the range $0.8 < n < 1.05$ (at 95% confidence). It would be extremely interesting were the scale-invariant case, $n = 1$, to be convincingly excluded, as that would be clear evidence of dynamical processes at work, rather than symmetries, in creating the perturbations.

**Gravitational waves** $r$: Measured in terms of the relative contribution to large-angle microwave anisotropies, the tensors are currently constrained to be no more than about 30%.

3.4 Inflation and CMB oscillations

A key property of inflationary perturbations is that they were created in the early Universe and evolved freely from then. Although a general solution to the perturbation equations has two modes, growing and decaying, only
the growing mode will remain by the time the perturbation enters the horizon. This leads directly to the prediction of an oscillatory structure in the microwave anisotropy power spectrum, as seen in Figure 1. The existence of such a structure is a robust prediction of inflation; if it is not seen then inflation cannot be the sole origin of structure.

The most significant recent development in observations pertaining to inflation is the first clear evidence for multiple peaks in the spectrum, seen by the DASI and Boomerang experiments shown in Figure 2. This is a crucial qualitative test which inflation appears to have passed, and which could have instead provided evidence against the entire inflationary paradigm for structure formation. These observations lend great support to inflation, though it must be stressed that they are not able to ‘prove’ inflation, as it may be that there are other ways to produce such an oscillatory structure.

3.5 Prospects for the future

It remains possible that future observations will slap us in the face and lead to inflation being thrown out. But if not, we can expect an incremental succession of better and better observations, culminating (in terms of currently-funded projects) with the Planck satellite. Faced with observational data of exquisite quality, an initial goal will be to test whether the simplest models of inflation continue to fit the data, meaning models with a single scalar field rolling slowly in a potential $V(\phi)$ which is then to be constrained by observations. If this class of models does remain viable, we can move on to reconstruction of the inflaton potential from the data.

Planck, currently scheduled for launch in February 2007, should be highly accurate. In particular, it should be able to measure the spectral index to an accuracy better than $\pm 0.01$, and detect gravitational waves even if they are as little as 10% of the anisotropy signal. In combination with other observations, these limits could be expected to tighten significantly further, especially the tensor amplitude. Such observations would rule out almost all currently known inflationary models. Even so, there will be considerable uncertainties, so it is important not to overstate what can be achieved.

Reconstruction can only probe the small part of the potential where the field rolled while generating perturbations on observable scales. We know enough about the configuration of the Planck satellite to be able to estimate how well it should perform. Ian Grivell and I recently described a numerical technique which gives an optimal construction. Results of an example reconstruction are shown in Figure 3, where it was assumed that the true potential was $\lambda \phi^4$. The potential itself is not well determined here (the tensors are
Figure 3. Sample reconstruction of a potential, where the dashed line shows the true potential and the solid lines are thirty Monte Carlo reconstructions (real life can only provide one). The upper panel shows the potential itself which is poorly determined. However some combinations, such as \( \frac{dV/d\phi}{V^{3/2}} \), shown in the lower panel, can be determined at an accuracy of a few percent. See Ref. 16 for details.
only marginally detectable), but certain combinations, such as \( \frac{dV/d\phi}{V^{3/2}} \), are accurately constrained and would lead to high-precision constraints on inflation model parameters.

### 4 Selected topics

The previous sections overviewed the status of two areas of cosmology. In this section, I give a short account of some particular topics which have received a lot of attention lately.

#### 4.1 Does cold dark matter really work?

Of the topics in this article, perhaps the one with the greatest potential significance for elementary particle physicists is this one: Is the dark matter really cold? Until lately the opinion of the astrophysics community was united behind this assumption, based on the remarkable success with which the cold dark matter model explains many observations in structure formation. However, more recently questions have opened up as to whether or not the cold dark matter assumption gives a good fit to observations on short scales, and in particular to the structure within galaxies.

High-resolution simulations of galaxy formation indicate that the dark matter retains considerable clumpiness when the small structures which first form are absorbed into larger structures. The persistence of substructure is a success for explaining the structure of galaxy clusters, where thousands of galaxies can retain their identity upon assimilation, but fails dismally in explaining our own galaxy where only a handful of dwarf satellites are observed

Even if they were stripped of their visible baryon components, such knots of dark matter would be sufficient to destroy the observed thin disks of spiral galaxies.

Potentially related to this are two further problems:

- Dwarf galaxy cores: theory predicts that the density diverges towards the centre of halos, whereas in well-observed dwarf galaxies a uniform-density core is seen.

- Bulge constitution: enough microlensing events have been seen towards the Milky Way bulge to suggest that they explain all the dark matter in the central regions of our galaxy, leaving no room for particle dark matter.

It remains unclear whether these problems are really so robust that the cold dark matter paradigm is under serious threat. However they have been
taken sufficiently seriously as to motivate a slew of papers on alternative sark matter properties, including warm dark matter, self-interacting dark matter, annihilating dark matter or condensed dark matter. Whether any of these could provide a unified solution to the problems listed above is unclear, but needless to say all have major consequences for dark matter search strategies and particle physics phenomenology.

4.2 Do neutrinos play a role?

What role can neutrinos play in cosmology? This is a topical question as evidence mounts up in favour of a non-zero neutrino mass from solar and atmospheric neutrino experiments.

Assuming standard interactions and negligible lepton asymmetry, theory predicts

\[
\Omega_\nu = \frac{\sum_i m_{\nu_i}}{90 \, h^2 \, \text{eV}}
\]

Neutrinos could have a measurable effect on structure formation, through their free-streaming, even if \(\Omega_\nu\) were as little as 0.01, meaning \(m_\nu \sim 0.5 \, \text{eV}\) suggesting that neutrinos play only a very minor role in structure formation. However recent observations favour even smaller values. However if for some reason there were a substantial lepton asymmetry, there could be an effect at even smaller masses.

It's also worth noting that, even if massless, the behaviour of relic neutrinos does have to be taken into account to compute quantities such as the microwave anisotropy power spectrum. Observations of it therefore do offer an indirect confirmation that the relic neutrino population predicted by theory does in fact exist.

4.3 Are existing treatments of inflation oversimplistic?

Much of the information disseminated from the inflationary community to the broader physics and astronomy community is based around the simplest paradigm, where a single scalar field slow-rolls down a potential. While this continues to be in excellent agreement with observation, and is a powerful working hypothesis worthy of testing in its own right, much work has recently gone into studying more complicated situations. It is a continuing challenge to uncover the full phenomenology of models with more than one dynamical field. Such a situation changes many of the usual assumptions. There is no longer a unique trajectory for the inflation field, and predictions for the density perturbations may well become dependent on initial conditions. Perhaps more
importantly, the perturbations are no longer guaranteed to be adiabatic, and
isocurvature perturbations may well be non-gaussian and/or correlated with
the adiabatic component. If the single-field paradigm fails, it will be impor-
tant to understand whether there remain well-motivated inflationary models
capable of explaining the data, particularly as efforts to determine cosmolog-
ical parameters such as $h$ and $\Omega_B$ will flounder if the initial perturbations
cannot be accurately parametrized.

4.4 The braneworld

At a particle physics conference or school you can quite happily state that ‘it is
generally accepted that our Universe has more than three spatial dimensions’
without much fear of contradiction, though it is probably not necessary even
to step out of a physics building to find out how limited this general acceptance
actually is. Nevertheless, extra dimensions have been with particle theorists
consistently for a long time now, and undoubtedly they are an issue which
may be of considerable importance for early Universe cosmology.

Until recently, it had been assumed that the failure to observe the pre-
dicted extra dimensions meant that the extra ones were “curled up” to be
unobservably small. However, there is a new a new idea, the braneworld,
which proposes that at least one of these extra dimensions might be rela-
tively large, with us constrained to live on a three-dimensional brane running
through the higher-dimensional space. Gravity is able to propagate in the full
higher-dimensional space, which is known as the bulk.

This radical idea has many implications for cosmology, both in the present
and early Universe, and so far we have only scratched the surface of possible
new phenomena. Already many exciting results have been obtained; here
there is only space to consider a few pertinent questions.

a) Are there modifications to the evolution of the homogeneous Universe?
The answer appears to be yes: for example in a simple scenario (known as
Randall–Sundrum Type II) the Friedmann equation is modified at high
energies so that, after some simplifying assumptions, it reads

$$H^2 = \frac{8\pi G}{3} \left( \rho + \frac{\rho^2}{2\lambda} \right),$$

(10)

where $\lambda$ is the tension of the brane. This recovers the usual cosmology at
low energies $\rho \ll \lambda$, but otherwise we have new behaviour. This opens new
opportunities for model building, see for example Ref.

b) Are inflationary perturbations different?
Again the answer is yes — there are modifications to the formulae giving
scalar and tensor perturbations. Unfortunately the main effect of this is to introduce new degeneracies in interpreting observations, as a potential can always be found matching observations for any value of $\lambda$. The initial perturbations therefore cannot be used to test the braneworld scenario.

c) Do perturbations evolve differently after they are laid down on large scales? The answer here is less clear. It is possible that perturbation evolution is modified even at late times, e.g. perturbations in the bulk could influence the brane in a way that couldn’t be predicted from brane variables alone. Whether there is a significant effect is unclear and is likely to be model dependent.

4.5 The Ekpyrotic Universe

It has recently been proposed that the Big Bang is actually the result of the collision of two branes, dubbed the Ekpyrotic Universe. It has been claimed that this scenario can provide a resolution to the horizon and flatness problems, essentially because causality arises from the higher-dimensional theory and allows a simultaneous Big Bang everywhere on our brane, though existing implementations solve the problem by hand in the initial conditions. As I write this, it remains unclear how to successfully describe the instant of collision between the two branes (the singularity problem), and considerable controversy surrounds whether or not the scenario can also generate nearly scale-invariant adiabatic perturbations. Both aspects are required to make it a serious rival to inflation.

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