Distance determination by magnitude analysis of some open clusters with GAIA era and stellar luminosity function

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Received: August 17, 2021; Accepted: October 30, 2021

Abstract. In this work, we carry out by utilizing the distance equation with providing some basic descriptive statistics for both apparent and absolute magnitudes, the distances of some open clusters (i.e., Hyades, Pleiades, IC 2391, Koposov 12, Koposov 43, NGC 1348, NGC 2112, NGC 4337, SAI 24, and SAI 94) with the assumption that all members N are scattering around a mean absolute magnitude in a Gaussian distribution. Our numerical obtained results are in good agreement with previously calculated values. In the second part of the paper, we have calculated the luminosity function of Hyades open cluster by Salpeter’s normalized function Ψ(MV) due to frequency distribution function Φ(MV), on this way, we have got a very significant relationship between them with absolute magnitudes MV (i.e., the linear correlation coefficient ∼ 0.995), and the error analysis are also given.

Key words: Open clusters; Distance equation (Malmquist bias); Density distribution function.

1. Introduction

Open clusters are mostly found in the spiral arms of the Milky Way Galaxy; therefore, they are suitable traces in the studies of Galactic disk and structure (Carraro et al. 1998; Chen et al. 2003; Joshi et al. 2016). Open clusters and/or
stellar associations have been used to determine spiral arm structure, to map the rotation curve of the Galaxy, to investigate the mechanisms of star formation and its recent history, and to constrain the initial luminosity and mass function (LF and MF) in aggregates of stars. Old clusters, and in large distances, are used to define disk abundance gradients, the cluster age-metallicity relationship, point to a complex history of chemical enrichment, and mixing in the disc (Friel 1995).

Particularly, the oldest members with Galactic open clusters, serve as excellent probes of the structure and evolution of the Galactic disk. Individual clusters provide excellent tests of stellar and dynamical evolution.

One of the most important pieces of information needed in astronomy is the distances to the stars or celestial objects. If the distance (r; pc) of a star is known as well as its proper motion (μ; mas/yr) then one can calculate its tangential velocity (V_t; km/s) to the line of sight (Robinson 1985). Also, having measured the distances to the globular clusters, we can study the distribution in the Galaxy (Cassisi et al. 2001; Duncan et al. 2001). In moving clusters if the equatorial coordinates of the vertex and the distance of each member are known, then one can easily determine the velocity of the cluster and the position of its center (Elsanhoury et al. 2013). On the other hand, the determination of distances within our Galaxy allows us to calibrate the distance indicators (Shanks 1997; Borchkhadze and Kogoshvili 1999). Moreover, determining distances would also help astronomers in their quest to understand the size and age of the universe (Willick and Batra 2001; Mazumdar and Narasimha 1990), since it would provide an independent estimation of the size of the first steps on the cosmic distance ladder. Consequently, it contributes to the theories about the origin of the universe.

The purpose of our research work is to compute the distances of some open clusters based on the so-called Malmquist bias with aid of the second Gaia data release Gaia DR21 (Cantat-Gaudin and Anders 2020).

The catalog of the second Gaia database release comes to a G-band magnitude for sources brighter than 21 mag (Weiler 2018); i.e., with 9 order of magnitude fainter than Tycho-Gaia Astrometric Solution TGAS (Michalik et al. 2015). At its faint end, the Gaia DR2 astrometric precision is accurate with that of Tycho-Gaia Astrometric Solution TGAS, whereas for stars brighter than G≲15 the precision is about ten times way better than in TGAS, thus allowed to extend membership determination to fainter and more distant objects. For about 1.3 billion sources, the Gaia DR2 catalog devoted to five astrometric parameters solutions; i.e., the central coordinates (α,δ), proper motion in both sides (μ_α*,μ_δ), and the parallaxes (π; mas). Moreover, Gaia DR2 having three broad bands; G, G_BP (Blue Prism), and G_RP (Red Prism) photometric magnitudes; i.e., G (330-1050 nm), G_BP (330-680 nm), and G_RP (630-1050 nm) with precisions at the mmag level.

1https://vizier.u-strasbg.fr/viz-bin/VizieR?-source=J/A+A/633/A99
In the present paper, the data analysis for our program open clusters; i.e., Hyades, Pleiades, IC 2391, Koposov 12, Koposov 43, NGC 1348, NGC 2112, NGC 4337, SAI 24, and SAI 94 were achieved with Section 2 and performing computation of the distances with a statistical magnitude analysis. Section 3 deals with the distribution function of the Hyades cluster. Section 3 is dedicated to the conclusion of the study.

2. The distance equation

For our program, open clusters, who’s drawn in Table 1; into which the first column presents the cluster names, second column deals with members (candidates), the Galactic positions (l, b) of these targets devoted here with columns 3 and 4, mean right ascension of members (ICRS) at epoch = 2015.5 and mean declination of members (ICRS) at epoch = 2015.5 are given here with columns 5 and 6, columns 7 and 8 gives the proper motions in both sides (µα, µδ) with errors in units of (mas/yr), and the last column presents the parallaxes (π) with its errors in units of (mas).

let us consider that; all members are on the same distance r(pc) and spread out (scatter) around a mean absolute magnitude (Mo) in a Gaussian distribution with dispersion (σ). Then the probability p(M)dM of a member of the celestial set that has an absolute magnitude in the range (M & M+dM); i.e., frequency distribution function of the absolute magnitudes of the cluster members takes the form (Mihalas and Binney 1981; Scheffler and Elsasser 1988).

\[
p(M)\,dM = \Phi(M) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(\frac{-(M-M_0)^2}{2\sigma^2}\right), \quad -\infty \leq M \leq \infty
\]  

(1)

The distance equation can be written as (Abdel-Rahman et al. 2009)

\[
r(pc) = 10^{(m_1 - M_0 - y\sigma - A - [Fe/H])},
\]  

(2)

where \((m_1; \text{mag})\) is the faintest apparent magnitude of the cluster stars, \((A)\) is the magnitude of the interstellar absorption, \([Fe/H]\) is the metallicity, and \((y)\) is the solution of the transcendental equation.

\[
\Lambda(y) = y + e^{-\frac{y^2}{2}} \left\{ \sqrt{\frac{2}{\pi}} \left[ 1 + erf \left( \frac{y}{\sqrt{2}} \right) \right] \right\}^{-1} - \alpha = 0,
\]

and

\[
\alpha = \frac{m_1 - \overline{m}}{\sigma}
\]

where \((\overline{m}; \text{mag})\) is the average value of the apparent magnitude.

The distance \(r(pc)\) of the observed members that complete down to a fixed limiting magnitude \(\overline{m}\) in the absolute range \(M, M+dM\) is given by

\[
r(pc) = 10^{1 + 0.2(\overline{m} - M)}
\]

The distance modulus is then given by:

\[
(m - M) = m_1 - M_0 - y\sigma - A - [Fe/H],
\]
Increasing $[Fe/H]$ values affect the luminosity negatively and hence the magnitudes and the distances will increase. The metallicity must be subtracted if positive and neglected when negative.

**Table 1.** The fundamental parameters of our program open clusters devoted to Gaia DR2 (Cantat-Gaudin and Anders 2020).

| Clusters | N | l $^\text{deg}$ | b $^\text{deg}$ | RA $^\text{deg}$ | Dec. $^\text{deg}$ | $\mu_\alpha$ ± $\sigma_{\mu_\alpha}$ | $\mu_\delta$ ± $\sigma_{\mu_\delta}$ | $\pi$ ± $\sigma_{\pi}$ |
|----------|---|----------------|----------------|-----------------|-----------------|----------------|----------------|----------------|
| Hyades   | 197| 180.058        | -22.349        | 66.717$^\text{(a)}$ | 15.867$^\text{(b)}$ | 66.717$^\text{(a)}$ | 15.867$^\text{(b)}$ | 20.00$^\text{(b)}$ |
| Pleiades | 1008| 166.462       | -23.614        | 56.601          | 24.114          | 20.077±0.035  | -45.503±0.038 | 7.346±0.006  |
| IC 2391  | 224| 270.386        | -6.737         | 130.292         | -52.991         | 23.316±0.041  | -1.732±0.011  | 0.396±0.005  |
| Koposov  | 43 | 179.923        | 1.747          | 88.452          | -24.644         | 20.00          | -0.726±0.025  | 0.404±0.006  |
| NGC 1348 | 105| 146.968        | -3.709         | 53.524          | 51.409          | 1.288±0.025   | -0.726±0.025  | 0.404±0.006  |
| NGC 2112 | 719| 205.886        | -12.605        | 88.452          | 0.403           | 2.713±0.009   | 4.27±0.009    | 0.877±0.003  |
| NGC 4337 | 247| 299.316        | 4.555          | 186.022         | -58.125         | -8.841±0.006  | 1.468±0.006   | 0.357±0.003  |
| SAI 24   | 130| 138.013        | 1.493          | 44.816          | 60.566          | -0.201±0.017  | 0.14±0.017    | 0.446±0.005  |
| SAI 94   | 85 | 265.432        | -2.176         | 131.171         | -46.292         | -2.254±0.17   | 4.479±0.017   | 0.204±0.008  |

(a) Perryman et al. (1998) (b) Kharchenko et al. (2016)

If we now measure the average absolute magnitude for these members, we will obtain:

$$M = \int \frac{p(M)M^3dM}{\int p(M)\sqrt{3}dM} = \int \frac{exp\left\{ -\frac{(M-M_o)^2}{2\sigma^2} - 0.6\ln 10 \right\} M^3dM}{\int exp\left\{ -\frac{(M-M_o)^2}{2\sigma^2} - 0.6\ln 10 \right\} \sqrt{3}dM} = M_o - 0.6\sigma^2\ln 10 = M_o - 1.382\sigma^2$$

That is

$$M_o = M + 1.382\sigma^2$$

Therefore the last relation between $M_o$ and $\sigma$ (Bok 1937) predicts $M_o > \bar{M}$; i.e., the stars one sees at a given absolute magnitude are, on average, more luminous than the average for all the stars in each volume. This effect (i.e., 1.382 $\sigma^2$) is called the Malmquist bias, arises because, when one selects stars of fixed absolute magnitude, the volume element containing the more distant, intrinsically luminous stars is larger than that occupied by the nearer, fainter objects. The Malmquist bias too acting an important role in connection with counts of radio Galaxies, quasars, and other objects, that have been used as cosmological probes.

Our computed distances $r_c$(pc) for our program and those estimated $r_e$(pc) with different authors are drawn like in Table 2, with formats, column 1 gives the name of the cluster, columns 2 and 3 devoted to faintest and average values of the apparent magnitudes, respectively, column 4 deals with values of the
absolute magnitudes, columns 5 and 6 devoted with dispersion $\sigma$ and the $\alpha$, respectively, column 7 gives our computed distances $r_c(\text{pc})$ with its errors (i.e., $\sqrt{\sum (\text{actual value} - \text{predicted value})^2/N}$), where $N$ is the total number of stars in each cluster, and column 8 represents estimated ones $r_e(\text{pc})$ with different authors as listed in column 9.

Fig. 1 presents a comparison between distances that were computed in our work with our program abscissa ($r_c; \text{kpc}$) and others as ordinate ($r_e; \text{kpc}$) with a correlation coefficient $\sim 92\%$. Here it is mentioned slightly that no systematic difference is shown among our statistical manipulation method and others.

**Figure 1.** Plot with error bars, showing consistent between our computed distances (abscissa; $r_c$) as listed in the seventh column of Table 2 with those (ordinate; $r_e$) obtained by different authors (eighth column). The plot highly showing the correlation coefficient ($\sim 92\%$).
Table 2. Our computed distances $r_c$(pc) with its uncertainties and other estimated ones $r_e$(pc) devoted to different authors.

| Clusters  | $m_l$  | $\pi$  | $M_*$ | $\sigma$ | $\alpha$ | $r_c \pm \sigma_{r_c}$ | $r_e$ | Authors                              |
|-----------|--------|--------|-------|----------|----------|------------------------|-------|--------------------------------------|
| Hyades    | 11.880 | 7.851  | 4.364 | 2.081    | 1.936    | 47.00±0.20             | 47.00±0.20 | Elsanhoury and Nouh (2019)          |
|           |        |        |       |          |          |                        |       | Lodieu et al. (2019)               |
| Pleiades  | 17.957 | 14.756 | 9.087 | 2.801    | 1.144    | 135.00±1.60            | 134.00 | Elsanhoury and Nouh (2019)          |
|           |        |        |       |          |          |                        |       | Galli et al. (2017)                |
| IC 2391   | 17.980 | 14.715 | 8.802 | 2.838    | 1.151    | 145±2.50               | 147±5.50 | van Leeuwen (2007)                 |
|           |        |        |       |          |          |                        |       | Dodd (2004)                        |
| Koposov 12| 17.993 | 15.538 | 3.512 | 1.402    | 1.752    | 2479.00±8.00           | 2479.00| Elsanhoury (2021)                  |
|           |        |        |       |          |          |                        |       | Soubiran et al. (2018)            |
|           |        |        |       |          |          |                        |       | Cantat-Gaudin et al. (2018)        |
|           |        |        |       |          |          |                        |       | Sampedro et al. (2017)             |
|           |        |        |       |          |          |                        |       | Kharchenko et al. (2013)           |
|           |        |        |       |          |          |                        |       | Yadav et al. (2011)                |
|           |        |        |       |          |          |                        |       | Froebrich et al. (2008)            |
| Koposov 43| 17.725 | 15.896 | 2.279 | 1.236    | 1.481    | 4965.00±93.00          | 4965.00| Elsanhoury (2021)                  |
|           |        |        |       |          |          |                        |       | Soubiran et al. (2018)            |
|           |        |        |       |          |          |                        |       | Cantat-Gaudin et al. (2018)        |
|           |        |        |       |          |          |                        |       | Sampedro et al. (2017)             |
|           |        |        |       |          |          |                        |       | Kharchenko et al. (2013)           |
|           |        |        |       |          |          |                        |       | Froebrich et al. (2008)            |
| NGC 1348  | 17.949 | 16.072 | 4.042 | 1.542    | 1.218    | 2475.00±11.00          | 2475.00| Bjhs et al. (2021)                 |
|           |        |        |       |          |          |                        |       | Cantat-Gaudin et al. (2018)        |
| NGC 2112  | 17.994 | 15.896 | 5.600 | 1.336    | 1.572    | 1276.00±5.00           | 1276.00| Haroon et al. (2017)               |
|           |        |        |       |          |          |                        |       | Carraro et al. (2008)              |
| NGC 4337  | 17.940 | 15.518 | 3.263 | 1.241    | 1.952    | 2846.00±0.30           | 2846.00| Bjhs et al. (2020)                 |
|           |        |        |       |          |          |                        |       | Cantat-Gaudin et al. (2018)        |
| SAI 24    | 17.984 | 15.211 | 3.428 | 1.984    | 1.398    | 2374.00±8.00           | 2374.00| Elsanhoury and Amin (2018)          |
|           |        |        |       |          |          |                        |       | Kharchenko et al. (2016)           |
| SAI 94    | 17.934 | 16.569 | 3.025 | 1.244    | 1.099    | 5471.00±4.00           | 5471.00| Elsanhoury and Amin (2019)          |
|           |        |        |       |          |          |                        |       | Kharchenko et. al. (2016)          |
3. Distribution function

The stellar luminosity function is a description of the relative number of stars of different absolute luminosities. It is often used to describe the stellar content of various parts of the Galaxy or other groups of stars, but it most commonly refers to the absolute number of stars of different absolute magnitudes in the Solar neighborhood. In this form, it is usually called van Rhijn function (van Rhijn 1936).

The detailed determination of the luminosity function of the Solar neighborhood is an extremely complicated process. Difficulties arise because of the incompleteness of existing surveys of stars of all luminosities in any sample of space, and the uncertainties in the basic data (distances and magnitudes).

The effect of stellar evolution on the observed luminosity functions for main-sequence stars, $\Phi(M_V)$ in the Solar neighborhood, were first investigated by (Salpeter 1955). He reasoned that we observe at the present only those stars which were formed less than $T(M_V)$ years ago, where $T(M_V)$ is the lifetime of a star with an absolute magnitude $M_V$ on the main sequence. For stars with $M_V > 3.5$ the lifetime $T(M_V)$ is larger than the age of the Galaxy, so that we observe all the stars ever formed. For stars brighter than $M_V = 3.5$, however, the ratio of the number we see to the number ever formed will be $T(M_V)/T(Galaxy)$ on the assumption of the uniform rate of star formation. Thus if $\Psi(M_V)$ is the luminosity function of all-stars ever formed in the Solar neighborhood; i.e., initial luminosity function and time-dependent (Miller and Scalo 1979), then the observed luminosity function is given by,

$$\Phi(M_V) = \frac{\Psi(M_V)T(M_V)}{T(Galaxy)}.$$  \hspace{1cm} (4)

Where $T(Galaxy)$ is a mean lifetime of the Galaxy.

Salpeter’s work is of great importance since it gives a means of predicting the numbers of stars at any given luminosity which have been formed in the lifetime of the Galaxy. The present-day luminosity function of the open clusters within the Solar neighborhood (e.g. Hyades) predicts contains $\approx 25-30$ white dwarfs (Chin and Stothers 1971). The discrepancy between the observed and a predicted number of these objects is possibly explained by evaporation from the cluster (Weidemann et al. 1992; Eggen 1993). One possible example of an escaped white dwarf is the P98 candidate HIP 12031 (DAwe...). It is located beyond 40 pc from the cluster center and is possibly a kinematic member.

In what follows, we focused on the Hyades open cluster (600 Myr), the data were excluded (i.e., 197 stars as members) with Hipparcos data by (Perryman et al. 1998)\(^2\), which based on observations made with the ESA Hipparcos astrometry satellite. According to (Salpeter 1955) and to obtain $\Psi(M_V)$ from $\Phi(M_V)$, we need only the bolometric magnitude ($M_{bol}$) and the mass ($M/M_\odot$) of the

\(^2\)https://vizier.u-strasbg.fr/viz-bin/VizieR?-source=J/A+A/331/81
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Table 3. Luminosity function for the Hyades ($\sum N = 197$).

| $M_V$ | Observed | $\Phi(M_V)$ | $\Psi(M_V)$ |
|-------|---------|-------------|-------------|
| 3.0   | 3.5     | 1           | 0.165       |
| 3.5   | 4.0     | 4           | 0.301       |
| 4.0   | 4.5     | 6           | 0.529       |
| 4.5   | 5.0     | 5           | 0.893       |
| 5.0   | 5.5     | 10          | 1.44        |
| 5.5   | 6.0     | 15          | 2.213       |
| 6.0   | 6.5     | 11          | 3.23        |
| 6.5   | 7.0     | 14          | 4.474       |
| 7.0   | 7.5     | 15          | 5.875       |
| 7.5   | 8.0     | 21          | 7.324       |
| 8.0   | 8.5     | 16          | 8.691       |
| 8.5   | 9.0     | 19          | 9.862       |
| 9.0   | 9.5     | 18          | 10.777      |
| 9.5   | 10.0    | 12          | 11.442      |
| 10.0  | 10.5    | 10          | 11.937      |
| 10.5  | 11.0    | 10          | 12.395      |
| 11.0  | 11.5    | 6           | 12.989      |
| 11.5  | 12.0    | 4           | 13.914      |

stars at a given $M_V$.

$$\log \Psi(M_V) = \log \Phi(M_V) + 0.4(3.50 - M_{bol.}) - \log \frac{M}{M_\odot} + 0.12$$  \hspace{1cm} (5)

The distribution of luminosities for Hyades open cluster is shown in Table 3, together with the normalized $\Psi(M_V)$ and $\Phi(M_V)$ functions. The luminosity function for Hyades open cluster is shown in Fig. 2, the normalized Salpeter’s function is $\Psi(M_V)$ shown as a solid line, while $\Phi(M_V)$ is represented by the dashed line. The fit of $\Psi(M_V)$ to the Hyades cluster is better than $\Phi(M_V)$. This finding supports that the stars in clusters, as well as single stars, obey nearly the same distribution law.

From the formulations of the above section, data of Salpeter’s luminosity function $\Psi(M_V)$, and frequency distribution function $\Phi(M_V)$, we get a very significant relationship between them with an absolute magnitude $M_V$, this relation is given as

$$\Delta_{obs.} = C_1 + C_2 M_V,$$  \hspace{1cm} (6)

where

$$\Delta_{obs.} = \Phi(M_V) - \Psi(M_V).$$  \hspace{1cm} (7)
Figure 2. Histograms of the luminosity functions of the Hyades open cluster. The solid line is Salpeter’s normalized $\Psi(M_V)$. The dashed line is the normalized general luminosity function $\Phi(M_V)$.

where the coefficients and their probable errors are

- $C_1 = 3.034 \pm 0.163$
- $C_2 = -0.552 \pm 0.026$
- The probable error of the fit is $e = 0.383$
- The average squared distance between the exact solution and the least-squares solution $Q = 0.053$ (Kopal and Sharaf 1980)
- The linear correlation coefficient between $(\Delta, M_V)$ is $r = 0.955$
- The graphical representation of the raw data and the fitted data with its absolute relative errors (i.e., $\Delta = |\Delta_{\text{obs.}} - \Delta_{\text{cal.}}|$) was given in Fig. 3, where $\Delta_{\text{obs.}}$. 


and $\Delta_{\text{cal.}}$ are those obtained from Equations (7) and (8), respectively.

$$\Delta_{\text{cal.}} = \Psi(M_V) - \Phi(M_V).$$

(8)

Some statistical analyses of these errors are given as follows:

- Mean (average value) = 0.675
- Median (central value) = 0.330
- Median absolute deviation = median of $|\Delta_i - \text{median}| = 0.213$
- Root mean square = $\sqrt{\frac{1}{N} \sum_{n=1}^{N}(\Delta_i)^2} = 1.147$
- Variance = $\frac{1}{N} \sum_{n=1}^{N}(\Delta_i - \bar{\Delta})^2 = 0.899$
- Mean absolute deviation = $\frac{1}{N} \sum_{n=1}^{N}|\Delta_i - \bar{\Delta}| = 0.583$

Figure 3. The graphical representation between the $\Delta_{\text{obs.}}$ vs. $M_V$ (solid line), with its relative errors (dashed line).
4. Conclusion

Generally, this paper is divided into two folds:

– Utilizing the distance equation, we have computed the distances $r_c$ (pc) for some open clusters. Malmquist started by assuming that the luminosity function is scattered around a mean absolute magnitude $M_o$ in a Gaussian distribution with dispersion $\sigma$. Although the distances calculated here are statistically devoted with magnitude analysis, they are in good agreement with other published ones ($\sim 92\%$). We must mention that $r_c$ (pc) obtained mostly photometrically which is affected by so many factors including evolutionary factors. On the other hand, and depending on statistical distribution functions, although based on averages, can give smearing out of some defects that are inherited in other distances.

– We have computed the observed luminosity function $\Phi(M_V)$ and Salpeter’s normalized $\Psi(M_V)$ for the Hyades open cluster ($\sum N = 197$) with regarding the bolometric magnitude $M_{bol}$. It is found that the Saltpeter luminosity function is fitted better to the observed frequency distribution of Hyades.

Acknowledgements. The authors deeply thank the anonymous referees for his/her constructive comments that significantly improved the quality of the paper.

This work presents results from the European Space Agency (ESA) space mission Gaia. Gaia data are being processed by the Gaia Data Processing and Analysis Consortium (DPAC). Funding for the DPAC is provided by national institutions, in particular the institutions participating in the Gaia Multi-Lateral Agreement (MLA). The Gaia mission website is https://www.cosmos.esa.int/gaia. The Gaia archive website is https://archives.esac.esa.int/gaia. This research has made use of the VizieR catalog access tool, CDS, Strasbourg, France (DOI: 10.26093/cds/vizier).

References

Abdel-Rahman, H. I., Issa, I. A., Sharaf, M. A., et al., Statistical Gaussian Distribution Function as a Distance Indicator to Stellar Groups. 2009, *Journal of Korean Astronomical Society*, 42, 71, DOI: 10.5303/JKAS.2009.42.4.071

Bisht, D., Elsanhoury, W. H., Zhu, Q., et al., An Investigation of Poorly Studied Open Cluster NGC 4337 Using Multicolor Photometric and Gaia DR2 Astrometric Data. 2020, *Astronomical Journal*, 160, 119, DOI: 10.3847/1538-3881/ab9fdd

Bisht, D., Zhu, Q., Elsanhoury, W. H., et al., Detailed analysis of the poorly studied northern open cluster NGC 1348 using multi-color photometry and GAIA EDR3 astrometry. 2021, *Publications of the ASJ*, 73, 677, DOI: 10.1093/pasj/psab029

Bok, B. J. 1937, *The Distribution of the Stars in Space*
Borchkhadze, T. M. & Kogoshvili, N. G., Concerning the errors arising through the use of Tully-Fisher relation for estimation of the Virgo cluster distance. 1999, *Astrophysics*, **42**, 25, DOI: 10.1007/BF02700910

Cantat-Gaudin, T. & Anders, F., Clusters and mirages: cataloguing stellar aggregates in the Milky Way. 2020, *Astronomy and Astrophysics*, **633**, A99, DOI: 10.1051/0004-6361/201936691

Cantat-Gaudin, T., Jordi, C., Vallenari, A., et al., A Gaia DR2 view of the open cluster population in the Milky Way. 2018, *Astronomy and Astrophysics*, **618**, A93, DOI: 10.1051/0004-6361/201833476

Carraro, G., Ng, Y. K., & Portinari, L., On the Galactic disc age-metallicity relation. 1998, *Monthly Notices of the RAS*, **296**, 1045, DOI: 10.1046/j.1365-8711.1998.01460.x

Carraro, G., Villanova, S., Demarque, P., Moni Bidin, C., & McSwain, M. V., The old open cluster NGC 2112: updated estimates of fundamental parameters based on a membership analysis†. 2008, *Monthly Notices of the RAS*, **386**, 1625, DOI: 10.1111/j.1365-2966.2008.13143.x

Cassisi, S., De Santis, R., & Piersimoni, A. M., The distance to Galactic globular clusters through RR Lyrae pulsational properties. 2001, *Monthly Notices of the RAS*, **326**, 342, DOI: 10.1046/j.1365-8711.2001.04613.x

Chen, L., Hou, J. L., & Wang, J. J., On the Galactic Disk Metallicity Distribution from Open Clusters. I. New Catalogs and Abundance Gradient. 2003, *Astronomical Journal*, **125**, 1397, DOI: 10.1086/367911

Chin, C.-W. & Stothers, R., Low-Mass White Dwarfs and the Cooling Sequences in the Hyades Cluster. 1971, *Astrophysical Journal*, **163**, 555, DOI: 10.1086/150800

Dodd, R. J., Data mining in the young open cluster IC2391. 2004, *Monthly Notices of the RAS*, **355**, 959, DOI: 10.1111/j.1365-2966.2004.08378.x

Duncan, D., Chaboyer, B., Carney, B., et al., Anchoring the Population II Distance Scale: Accurate Ages for Globular Clusters and Field Halo Stars. 2001, in American Astronomical Society Meeting Abstracts, Vol. 198, *American Astronomical Society Meeting Abstracts #198*, 63.09

Eggen, O. J., Degenerate Stars in the Hyades Supercluster. 1993, *Astronomical Journal*, **106**, 642, DOI: 10.1086/116670

Elsanhoury, W. H., Photometric and kinematical analysis of Koposov 12 and Koposov 43 open clusters. 2021, *Journal of Astrophysics and Astronomy*, **42**, 90, DOI: 10.1007/s12036-021-09771-x

Elsanhoury, W. H. & Amin, M. Y., Photometric Analysis of Newly Discovered Open Clusters SAI 24 and SAI 94 Based on PPMXL Catalogue. 2019, *Serbian Astronomical Journal*, **198**, 45, DOI: 10.2298/SAJ1998045E

Elsanhoury, W. H. & Nouh, M. I., PPMXL and Gaia morphological analysis of Melotte 22 (Pleiades) and Melotte 25 (Hyades). 2019, *New Astronomy*, **72**, 19, DOI: 10.1016/j.newast.2019.04.006
Elsanhoury, W. H., Sharaf, M. A., Nouh, M. I., & Saad, A. S., The Kinematics and Velocity Ellipsoid Parameters of Open Star Clusters. 2013, *The Open Astronomy Journal*, 6, 1

Friel, E. D., The Old Open Clusters Of The Milky Way. 1995, *Annual Review of Astron and Astrophys*, 33, 381, DOI: 10.1146/annurev.aa.33.090195.002121

Froebrich, D., Meusinger, H., & Scholz, A., NTT follow-up observations of star cluster candidates from the FSR catalogue. 2008, *Monthly Notices of the RAS*, 390, 1598, DOI: 10.1111/j.1365-2966.2008.13849.x

Galli, P. A. B., Moraux, E., Bouy, H., et al., A revised moving cluster distance to the Pleiades open cluster. 2017, *Astronomy and Astrophysics*, 598, A48, DOI: 10.1051/0004-6361/201629239

Haroon, A. A., Ismaill, H. A., & Elsanhoury, W. H., Photometric and Kinematic Properties of the Nearby Open Star Cluster NGC 2112. 2017, *Astronomy and Astrophysics*, 593, A116, DOI: 10.1051/0004-6361/201628944

Joshi, Y. C., Damibis, A. K., Pandey, A. K., & Joshi, S., Study of open clusters within 1.8 kpc and understanding the Galactic structure. 2016, *Astronomy and Astrophysics*, 593, A53, DOI: 10.1051/0004-6361/201322302

Kharchenko, N. V., Piskunov, A. E., Schilbach, E., Röser, S., & Scholz, R. D., Global survey of star clusters in the Milky Way. II. The catalogue of basic parameters. 2013, *Astronomy and Astrophysics*, 558, A53, DOI: 10.1051/0004-6361/201322302

Kharchenko, N. V., Piskunov, A. E., Schilbach, E., Röser, S., & Scholz, R. D., Global survey of star clusters in the Milky Way. V. Integrated JHK_s magnitudes and luminosity functions. 2016, *Astronomy and Astrophysics*, 585, A101, DOI: 10.1051/0004-6361/201527292

Kopal, Z. & Sharaf, M. A., Linear Analysis of the Lightcurves of Eclipsing Variables - Part One - Analysis of the Fractional Loss of Light. 1980, *Astrophysics and Space Science*, 70, 77, DOI: 10.1007/BF00641665

Lodieu, N., Pérez-Garrido, A., Smart, R. L., & Silvotti, R., A 5D view of the α Per, Pleiades, and Praesepe clusters. 2019, *Astronomy and Astrophysics*, 628, A66, DOI: 10.1051/0004-6361/201935533

Mazumdar, A. & Narasimha, D., Distance to the Virgo cluster and estimation of the Hubble Constant. 1999, *Bulletin of the Astronomical Society of India*, 27, 267

Michalik, D., Lindegren, L., & Hobbs, D., The Tycho-Gaia astrometric solution: How to get 2.5 million parallaxes with less than one year of Gaia data. 2015, *Astronomy and Astrophysics*, 574, A115, DOI: 10.1051/0004-6361/201425310

Mihalas, D. & Binney, J. 1981, *Galactic astronomy. Structure and kinematics*

Miller, G. E. & Scalo, J. M., The Initial Mass Function and Stellar Birthrate in the Solar Neighborhood. 1979, *Astrophysical Journal, Supplement*, 41, 513, DOI: 10.1086/190629

Perryman, M. A. C., Brown, A. G. A., Lebreton, Y., et al., The Hyades: distance, structure, dynamics, and age. 1998, *Astronomy and Astrophysics*, 331, 81
Rowan-Robinson, M. 1985, *The cosmological distance ladder: distance and time in the universe*

Salpeter, E. E., The Luminosity Function and Stellar Evolution. 1955, *Astrophysical Journal*, **121**, 161, DOI: 10.1086/145971

Sampedro, L., Dias, W. S., Alfaro, E. J., Monteiro, H., & Molino, A., A multimembership catalogue for 1876 open clusters using UCAC4 data. 2017, *Monthly Notices of the RAS*, **470**, 3937, DOI: 10.1093/mnras/stx1485

Scheffler, H. & Elsasser, H., Book-Review - Physics of the Galaxy and Interstellar Matter. 1988, *Science*, **240**, 1680

Shanks, T., A test of Tully-Fisher distance estimates using Cepheids and SNIa. 1997, *Monthly Notices of the RAS*, **290**, L77, DOI: 10.1093/mnras/290.4.L77

Soubiran, C., Cantat-Gaudin, T., Romero-Gómez, M., et al., Open cluster kinematics with Gaia DR2. 2018, *Astronomy and Astrophysics*, **619**, A155, DOI: 10.1051/0004-6361/201834020

van Leeuwen, F., Validation of the new Hipparcos reduction. 2007, *Astronomy and Astrophysics*, **474**, 653, DOI: 10.1051/0004-6361:20078357

van Rhijn, P. J., The Absorption of Light in Interstellar Galactic Space and the Galactic Density Distribution. 1936, *Publications of the Kapteyn Astronomical Laboratory Groningen*, **47**, 1

Weidemann, V., Jordan, S., Iben, Icko, J., & Casertano, S., White Dwarfs in the Halo of the Hyades Cluster: The Case of the Missing White Dwarfs. 1992, *Astronomical Journal*, **104**, 1876, DOI: 10.1086/116364

Weiler, M., Revised Gaia Data Release 2 passbands. 2018, *Astronomy and Astrophysics*, **617**, A138, DOI: 10.1051/0004-6361/201833462

Willick, J. A. & Batra, P., A Determination of the Hubble Constant from Cepheid Distances and a Model of the Local Peculiar Velocity Field. 2001, *Astrophysical Journal*, **548**, 564, DOI: 10.1086/319005

Yadav, R. K. S., Glushkhova, E. V., Sariya, D. P., et al., Optical photometric study of the open clusters Koposov 12, Koposov 53 and Koposov 77. 2011, *Monthly Notices of the RAS*, **414**, 652, DOI: 10.1111/j.1365-2966.2011.18430.x