New gauged $N = 8$, $D = 4$ supergravities

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Abstract

New gaugings of four-dimensional $N = 8$ supergravity are constructed, including one which has a Minkowski space vacuum that preserves $N = 2$ supersymmetry and in which the gauge group is broken to $SU(3) \times U(1)^2$. Previous gaugings used the form of the ungauged action which is invariant under a rigid $SL(8, \mathbb{R})$ symmetry and promoted a 28-dimensional subgroup $(SO(8), SO(p, 8 - p)$ or the non-semi-simple contraction $CSO(p, q, 8 - p - q))$ to a local gauge group. Here, a dual form of the ungauged action is used which is invariant under $SU^*(8)$ instead of $SL(8, \mathbb{R})$ and new theories are obtained by gauging 28-dimensional subgroups of $SU^*(8)$. The gauge groups are non-semi-simple and are different real forms of the $CSO^*(2p, 8 - 2p)$ groups, denoted as $CSO^*(2p, 8 - 2p)$, and the new theories have a rigid $SU(2)$ symmetry. The five-dimensional gauged $N = 8$ supergravities are dimensionally reduced to $D = 4$. The $D = 5$, $SO(p, 6 - p)$ gauge theories reduce, after a duality transformation, to the $D = 4$, $CSO^*(6, 2)$ gauge theory while the $SO^*(6)$ gauge theory reduces to the $D = 4$, $CSO^*(6, 2)$ gauge theory. The new theories are related to the old ones via an analytic continuation. The non-semi-simple gaugings can be dualized to forms with different gauge groups.

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1. Introduction

In addition to the $N = 8$, $D = 4$ gauged supergravity of [1] with gauge group $SO(8)$, there is a class of $N = 8$, $D = 4$ gauged supergravities with non-compact gauge group [2, 3]. The ungauged Cremmer–Julia $N = 8$ supergravity [4] has an $SL(8, \mathbb{R})$ global symmetry of the action, and in each of the gaugings of [2, 3], a 28-dimensional subgroup $K$ of the $SL(8, \mathbb{R})$ global symmetry is promoted to a local symmetry, using the 28 vector fields in the $N = 8$ supergravity supermultiplet. In [5], an exhaustive classification of such gaugings was given and it was argued that the only possible gauge groups are the ones found in [1–3]. It will be
shown here that nonetheless there are further gaugings of the $N = 8$ theory that fall outside this class, in which the gauge group is not a subgroup of $SL(8, \mathbb{R})$. The key point is that there are other dual forms of the ungauged $N = 8$ theory in which the global symmetry of the action is some group $L$ that is not contained in $SL(8, \mathbb{R})$ and there is the possibility of gauging a 28-dimensional subgroup $K$ of $L$.

The ungauged $N = 8$ supergravity in $D = 4$ of [4] has a global $E_{7(7)}$ symmetry and a local $SU(8)$ symmetry. The $E_{7(7)}$ is a duality symmetry of the equations of motion and the Cremmer–Julia action [4] is invariant under a maximal $SL(8, \mathbb{R})$ subgroup of this. The bosonic sector consists of the graviton, 28 vector fields transforming as a 28 of $SL(8, \mathbb{R})$ and 70 scalars taking values in the coset $E_{7}/SU(8)$. Gauging of this theory entails promoting a 28-dimensional subgroup $K$ of $SL(8, \mathbb{R})$ to a local symmetry. In [1], the gauging with $K = SO(8)$ was constructed, and in [2, 3] gaugings were constructed with non-compact gauge groups $K = SO(p, 8 - p)$ or certain non-semi-simple gauge groups $CSO(p, q, r)$ for all non-negative integers $p, q, r$ with $p + q + r = 8$, and these are the only possible gaugings of subgroups of $SL(8, \mathbb{R})$ [5]. When $r = 0$, $CSO(p, q, 0) = SO(p, q)$, while if $q = 0$ the notation $CSO(p, 0, r) = CSO(p, r)$ was used. The $CSO(p, q, r)$ group [3] arises from a group contraction of $SO(p + r, q)$. Decomposing the generators $L = \Lambda + \Omega + \Sigma$ of $SO(p + r, q)$ into the generators $\Lambda$ of $SO(p, q)$, the generators $\Omega$ of $SO(r)$ and the remaining $r(p + q)$ generators $\Sigma$, and performing the rescaling

$$L \to \Lambda + \frac{r(1/2)}{\Omega} \Omega + \sqrt{\frac{r}{4\Omega}} \Sigma,$$

(1.1)

and taking the contraction $\xi \to 0$ gives the $CSO(p, q, r)$ algebra. In the limit, the generators of $CSO(p, q, r)$ then consist of the generators $\Lambda$ of $SO(p, q)$, $r(r - 1)/2$ Abelian generators $\Omega$ and $r(p + q)$ generators $\Sigma$ transforming as $r$ copies of the vector representation of $SO(p, q)$. The commutation relations of $CSO(p, q, r)$ take the schematic form

$$[\Lambda, \Lambda] \sim \Lambda, \quad [\Lambda, \Sigma] \sim \Sigma, \quad [\Sigma, \Sigma] \sim \Omega,$$

(1.2)

with all other commutation relations vanishing. The Abelian generators $\Omega$ are central charges, appearing only on the right-hand side of $[\Sigma, \Sigma]$, so that the algebra is a central extension of the semi-direct product $SO(p, q) \ltimes \mathbb{R}^{r(p + q)}$. There are other real forms of $CSO(p, q, r)$ in which $U(1)^{r(r-1)/2}$ is replaced by, for example, $U(1)^{r(r-1)/2 - n} \times SO(1, 1)^{n}$. The $D = 11$ origin of these theories was found in [6] and analysed in [7].

However, there are other forms of the ungauged supergravity action related to the Cremmer–Julia theory by duality transformations which have different global duality symmetry groups $G$ and for each there will be a subgroup $L \subset G$ which is a symmetry of the action. For example, the ungauged $N = 8$, $D = 5$ supergravity has a global $E_{6(6)}$ symmetry of the action [8] and dimensional reduction gives a form of the $N = 8$, $D = 4$ supergravity with a symmetry of the action that is $L = E_{6(6)} \times \mathbb{R}^{5}$, which is not contained in $SL(8, \mathbb{R})$. Duality transformations take this to the Cremmer–Julia form with $G = E_{7(7)}$ and $L = SL(8, \mathbb{R})$. A dual form in which the symmetry of the action is $SU^{s}(8)$ will play an important role here. Some other dual forms of the theory were considered in [9]. For any of these dual forms, there is the possibility of promoting a subgroup $K$ of the symmetry $L$ of the action to a local symmetry and seeking further modifications to the theory to preserve supersymmetry. If the gauge group $K$ is not a subgroup of $SL(8, \mathbb{R})$, such a gauging cannot be one of those previously found and so must be new. This point was also noted in [10], where Scherk–Schwarz reductions of $N = 8$, $D = 5$ gauged supergravity were shown to correspond to gaugings of $N = 8$, $D = 4$ supergravity in which flat subgroups of $L = E_{6} \times \mathbb{R}^{5}$ are promoted to local symmetries.

The purpose here is to exhibit such new gaugings. It will be seen that there is a dual form of the $N = 8$ action in which the subgroup $L$ of $E_{7(7)}$, which is a symmetry of the action
is $SU^*(8)$ instead of $SL(8, \mathbb{R})$, and in this formulation one can seek to gauge 28-dimensional subgroups of $SU^*(8)$. (Recall that $SU^*(2n)$ and $SO^*(2n)$ are certain non-compact forms of $SU(2n)$ and $SO(2n)$, respectively; see section 3 for definitions and discussion of these groups.) Such gauge theories will be constructed here with gauge groups $CSO^*(2p, 8 - 2p)$ for $p = 1, 2, 3$. These gauge groups are non-semi-simple contractions of $SO^*(8)$, defined in section 3) in analogy with the contractions $CSO(q, 8 - q)$ of $SO(8)$; they are not subgroups of $SL(8, \mathbb{R})$.

The existence of one of these new gaugings was anticipated in [11]. Gauged $D = 5, N = 8$ theories were constructed with gauge group $SO(6) \sim SU(4)$ in [12, 13], and with gauge groups $SO(5, 1) \sim SU^*(4), SO(4, 2) \sim SU(2, 2)$ and $SO(3, 3) \sim SL(4, \mathbb{R})$ in [12]. In [11] a further gauging was found with gauge group $SO^*(6) \sim SU(3, 1)$. The $SU(3, 1)$ gauging has a critical point with zero cosmological constant that preserves $N = 2$ supersymmetry and breaks the gauge group down to $SU(3) \times U(1)$. It also has a global $SU(2)$ symmetry. As discussed in [11], this theory can be compactified to four dimensions to give a theory which also has a Minkowski space vacuum preserving $N = 2$ supersymmetry. This theory will be constructed here and shown to be a gauged $N = 8$ supergravity. It will be seen that reduction of the $D = 5$ theories with gauge groups $SO(p, 6 - p)$ to $D = 4$ and performing a duality transformation give the $D = 4$ theories of [3] with gauge groups $CSO(p, 6 - p, 2)$, which are non-semi-simple groups with a subgroup $SU(6 - p) \times SO(2)$. The commutation relations are of the form (1.2) with $\Lambda$ generating $SO(p, 6 - p)$, a single Abelian generator $\Omega$ and 12 generators $\Sigma$ in the $(6, 2)$ representation of $SO(p, 6 - p) \times SO(2)$. There is in addition a global $SL(2, \mathbb{R})$ symmetry, which is broken to $SL(2, \mathbb{Z})$ in the quantum theory. It will also be shown that reducing the $SO^*(6)$ gauge theory gives a new theory with gauge group $CSO^*(6, 2)$, which is non-semi-simple with a subgroup $SU(3, 1) \times U(1)$, and maximal compact subgroup $SU(3) \times U(1)$ of $SU(3) \times U(1)$. This unfortunately does not contain the group $SU(3) \times SU(2) \times U(1)$ for non-semi-simple with a subgroup $SU(3, 1) \times U(1)$, and maximal compact subgroup $SU(3) \times U(1)$ of $SU(3) \times U(1)$. This unfortunately does not contain the group $SU(3) \times SU(2) \times U(1)$ for [11], but there is in addition a global $SU(2)$ symmetry, broken to a discrete subgroup in the quantum theory. The commutation relations are of the form (1.2) with $\Lambda$ generating $SO^*(6) = SU(3, 1)$, a single Abelian generator $\Omega$ and 12 generators $\Sigma$ in the $(6, 2)$ representation of $SO^*(6) \times SO(2)$.

The plan of the paper is as follows. In section 2, the $CSO(p, q, r)$ gaugings of $N = 8$ supergravity will be reviewed. In section 3, the $SO^*(2n)$ and $SU^*(2n)$ groups will be reviewed and the duality symmetry of the $N = 8$ theory will be discussed, and the form of the ungauged theory with action invariant under $SU^*(8)$ will be constructed. In section 4, the theories in which $CSO^*(2p, 2q)$ subgroups of $SU^*(8)$ are gauged are constructed for $p = 1, 2, 3$ and their symmetries are discussed. In section 5, the $N = 8, D = 5$ gauged supergravities are dimensionally reduced to $D = 4$ and the resulting $D = 4$ gaugings identified, giving a different derivation of the $CSO^*(6, 2)$ theory. In section 6, the action of dualities on these theories is considered. For the non-semi-simple gaugings, the dualization of some of the gauge fields is possible, changing the gauge group. Then U-duality transformations are considered and suitable limits of these are shown to construct the new gaugings from old ones. In section 7, the scalar potentials are analysed and some of the physical properties are discussed.

2. The $CSO(p, q, r)$ $D = 4$ gauged supergravities

The $CSO(p, q, r)$ gaugings arise from promoting a 28-dimensional subgroup $K$ of $SL(8, \mathbb{R})$ to a local symmetry. The 28 vector fields become the gauge bosons, therefore it is necessary that the subgroup $K$ is chosen so that the 28 of $SL(8, \mathbb{R})$ becomes the adjoint of $K$. Then supersymmetry requires the addition of terms depending on the coupling constant $g$ to the
action and supersymmetry transformation rules, including a scalar potential proportional to $g^2$. In [1], the gauging with $K = SO(8)$ was constructed, and in [2, 3] gaugings were constructed with non-compact gauge groups $K = SO(p, 8 - p)$ or the non-semi-simple gauge groups $CSO(p, q, r)$ for all non-negative integers $p, q, r$ with $p + q + r = 8$. The group $CSO(p, q, r)$ is the group contraction of $SO(p + r, q)$ preserving a symmetric metric with $p$ positive, $q$ negative and $r$ zero eigenvalues. Then $CSO(p, q, 0) = SO(p, q)$ and $CSO(p, q, 1) = ISO(p, q)$. The Lie algebra of $CSO(p, q, r)$ is [3]

$$[L_{ab}, L_{cd}] = L_{ad}\eta_{bc} - L_{ac}\eta_{bd} - L_{bd}\eta_{ac} + L_{bc}\eta_{ad},$$

(2.1)

where

$$\eta_{ab} = \begin{pmatrix} 1_{p \times p} & 0 & 0 \\ 0 & -1_{q \times q} & 0 \\ 0 & 0 & 0_{r \times r} \end{pmatrix},$$

(2.2)

$a, b = 1, \ldots, 8$ and $L_{ab} = -L_{ba}$. Note that despite the non-compact gauge groups, these are unitary theories, as the vector kinetic term is not the minimal term constructed with the indefinite Cartan–Killing metric, but is constructed with a positive definite scalar-dependent matrix. The $CSO(p, q, r)$ gauging and the $CSO(q, p, r)$ gauging are equivalent.

The 70 scalars parametrize the coset $E_{7(7)}/SU(8)$, while the 28 vector fields $A_\mu$ transform as a 28 of the subgroup $SL(8, \mathbb{R}) \subset E_{7(7)}$. The 28 field strengths $F$ satisfy the Bianchi identities $dF = 0$, and it is useful to define 28 dual 2-forms $G$, transforming in the 28 of $SL(8, \mathbb{R})$, so that the field equations are of the form $dG = 0$. In the linearized theory, $G \sim \ast F$, but in the full interacting theory there are field-dependent modifications to this [4]. Then the $28 + 28$ field strengths $(F, G)$ combine into a 56-vector transforming as the 56 representation of $E_{7(7)}$.

Different bases for $E_{7(7)}$ are useful for different purposes. In [1], a basis in which the $SU(8)$ subgroup of $E_{7(7)}$ is manifest was used, with the 56 of $E_{7(7)}$ decomposing into a $28 + 28$ of $SU(8)$. In this basis, the formulae for the non-compact gaugings are rather complicated, and are dramatically simplified by going to the basis in which the subgroup $SL(8, \mathbb{R}) \subset E_{7(7)}$ is manifest, with $56 \rightarrow 28 + 28$. Details of how to transform between these bases are given in [4, 2]. Let upper indices $a, b = 1, \ldots, 8$ label the 8 of $SL(8, \mathbb{R})$ and lower indices $a, b$ label the contragredient 8. Then the field strengths in the 28 are $F_{\mu a} = -F_{a \mu}$, while the dual field strength 2-forms are $G_{ab} = -G_{ba}$. As in [4], we introduce indices $i, j = 1, \ldots, 8$ in the 8 of $SU(8)$, which are raised and lowered by complex conjugation. The scalar fields can be represented by a $56 \times 56$ matrix $V$.

$$V = \begin{pmatrix} u_{ij} & v_{ijcd} \\ \overline{v}^{klab} & \overline{v}^{kl} \end{pmatrix},$$

(2.3)

transforming under a rigid $E_7$ transformation represented by a $56 \times 56$ matrix $E$ and a local $SU(8)$ transformation $U(x)$ represented by a $56 \times 56$ matrix in the $SU(8)$ subgroup of $E_7$ as

$$V \rightarrow U(x) V E^{-1}.$$ 

(2.4)

The scalar kinetic term in the ungauged theory can be written as

$$\int d^4x \sqrt{-g} \text{tr}(D_\mu V \overline{V}^{-1} D\overline{V} V^{-1}),$$

(2.5)

where $D_\mu$ is an $SU(8)$ covariant derivative involving an $SU(8)$ connection $B_\mu$. The $SU(8)$ connection appears algebraically in the action and its field equation determines $B_\mu$ in terms of the other fields.
Minimal coupling for the group $K = \text{CSO}(p, q, r)$ uses the fact that the 28 of $\text{SL}(8, \mathbb{R})$ is the adjoint of $K \subset \text{SL}(8, \mathbb{R})$ and consists of introducing the non-Abelian field strengths
\[
F^{ab}_{\mu} = \partial_{\mu} A^{ab} - \partial_{a} A^{ab}_{\mu} - 2g A^{ab}_{[a} A^{b]}_{\mu} \eta_{cd},
\]
with gauge coupling $g$ and replacing the $\text{SU}(8)$ covariant derivative $D_{\mu}$ with the $\text{SU}(8) \times K$ covariant derivative $D_{\mu}$, so that for example
\[
D_{\mu} u_{ij}^{ab} = \partial_{\mu} u_{ij}^{ab} + \mathcal{B}_{\mu}^{ij} u_{jk}^{ab} - 2g A_{\mu}^{[ij} u_{k]}^{ab} \eta_{cd}.
\]

These minimal couplings break supersymmetry, but supersymmetry can be restored by $g$-dependent modifications of the action and supersymmetry transformations. These involve scalar-dependent $\text{SU}(8)$ tensors $A^{ij}_{1}, A^{ijk}_{2}$ given in terms of the $T$-tensor
\[
T^{ijkl} = \sqrt{2} g A_{1ij} \bar{\psi}_{\mu} \gamma_{\mu} \psi_{\nu} + \frac{1}{8} g A_{2ijk} \bar{\psi}_{\nu} \gamma_{\mu} \chi_{\lambda} + \frac{1}{16} \sqrt{2} g e_{ijklmn} A^{m}_{pq} \bar{\chi}_{k} \chi_{l} + h.c.,
\]
and the fermion bilinear terms
\[
L_{g} = \sqrt{2} g A_{1ij} \bar{\psi}_{\mu} \gamma_{\mu} \psi_{\nu} + \frac{1}{8} g A_{2ijk} \bar{\psi}_{\nu} \gamma_{\mu} \chi_{\lambda} + \frac{1}{16} \sqrt{2} g e_{ijklmn} A^{m}_{pq} \bar{\chi}_{k} \chi_{l} + h.c.,
\]
while the modifications to the supersymmetry transformations are
\[
\delta_{g} \chi_{ij} = -\sqrt{2} g A_{1ij} \gamma_{\mu} \epsilon_{\nu}, \quad \delta_{g} \chi_{ijk} = -2g A_{2ijk} \epsilon_{\nu}.
\]
The conventions are as in \[1, 2\], so that fermions with upper indices are right handed and fermions with lower indices are left handed. Note that the $T$-tensor is invariant under $K \times \text{SL}(r, \mathbb{R})$ and transforms reducibly under $\text{SU}(8)$, decomposing into the tensor $A_{1}$ transforming as a 36 and the tensor $A_{2}$ transforming as a 420.

For the $\text{CSO}(p, q, r)$ gaugings with $r > 0$, it is useful to decompose the $\text{SL}(8, \mathbb{R})$ indices $a, b = 1, \ldots, 8$ into $\text{SL}(p + q, \mathbb{R})$ indices $I, J = 1, \ldots, p + q$ and $\text{SL}(r, \mathbb{R})$ indices $\alpha, \beta = p + q + 1, \ldots, 8$, so that $a \rightarrow (I, \alpha)$ and
\[
\eta_{ab} = \begin{pmatrix}
\eta_{IJ} & 0 \\
0 & 0_{rr}
\end{pmatrix},
\]
where $\eta_{IJ} = \text{diag}(1_{p+q}, -1_{q-q})$ is the invariant metric of $\text{SO}(p, q)$. Then $A^{ab} \rightarrow (A^{IJ}, A^{I\alpha} = -A^{I\alpha}, A^{\alpha\beta})$ and
\[
F^{IJ}_{\mu} = \partial_{\mu} A^{IJ} - \partial_{I} A^{J}_{\mu} - 2g A^{KL}_{[I} A^{J]}_{\mu} \bar{\eta}_{KL},
\]
\[
F^{I\alpha}_{\mu} = \partial_{\mu} A^{I\alpha} - \partial_{I} A^{\alpha}_{\mu} - 2g A^{KL}_{[I} A^{K]I\alpha} \bar{\eta}_{KL},
\]
\[
F^{\alpha\beta}_{\mu} = \partial_{\mu} A^{\alpha\beta} - \partial_{\alpha} A^{\beta}_{\mu} - 2g A^{KL}_{[\alpha} A^{K]\beta I\alpha} \bar{\eta}_{KL}.
\]
Similarly, $u_{ij}^{ab} \rightarrow (u_{ij}^{IJ}, u_{ij}^{I\alpha}, u_{ij}^{\alpha\beta})$ etc, and the $T$-tensor becomes
\[
T^{ijkl} = \sqrt{2} g A_{1ij} \bar{\psi}_{\mu} \gamma_{\mu} \psi_{\nu} + \frac{1}{8} g A_{2ijk} \bar{\psi}_{\nu} \gamma_{\mu} \chi_{\lambda} + \frac{1}{16} \sqrt{2} g e_{ijklmn} A^{m}_{pq} \bar{\chi}_{k} \chi_{l} + h.c.,
\]
and the invariance of this under \( SO(p, q) \times SL(r, \mathbb{R}) \) is manifest. The covariant derivative can also be written in this basis, so that, for example, (2.7) becomes

\[
\mathcal{D}_\mu u_{ij}^{IJ} = \partial_\mu u_{ij}^{IJ} + B_{\mu}^k \langle u_{ij} \rangle^k - 2g A_{\mu}^{[I} u_{j]L} \eta_{KL}
\]

\[
\mathcal{D}_\mu u_{ij}^a = \partial_\mu u_{ij}^a + B_{\mu}^k \langle u_{ij} \rangle^k - 2g A_{\mu}^{[a} u_{j]L} \eta_{KL}
\]

\[
\mathcal{D}_\mu u_{ij}^{a\beta} = \partial_\mu u_{ij}^{a\beta} + B_{\mu}^k \langle u_{ij} \rangle^k - 2g A_{\mu}^{[a} u_{j]L} \eta_{KL}.
\]

(2.16)

Note that \( A^{ab} \) only appears through its curl \( dA_{ab} \) in (2.14) and (2.16).

3. Group structure and bases

The new gaugings involve the groups \( SO^*(2n) \) and \( SU^*(2n) \), and before proceeding it will be useful to recall their definitions. The group \( SO^*(2n) \) is the non-compact form of \( SO(2n) \) whose maximal compact subgroup is \( U(n) \), while \( SU^*(2n) \) is the non-compact form of \( SU(2n) \) whose maximal compact subgroup is \( USp(2n) \). Equivalently, these groups can be defined as subgroups of \( SL(2n, \mathbb{C}) \). Let \( J \) be an antisymmetric \( 2n \times 2n \) matrix satisfying \( J^2 = -1 \). The subgroup \( SU^*(2n) \) of \( SL(2n, \mathbb{C}) \) consists of those \( 2n \times 2n \) complex matrices \( U \) in \( SL(2n, \mathbb{C}) \) satisfying

\[
UJ = JU^*,
\]

where \( U^* \) is the complex conjugate of \( U \). The subgroup \( SO^*(2n) \) of \( SU^*(2n) \) consists of those matrices \( U \) also satisfying

\[
UU^t = 1,
\]

where \( U^t \) is the transpose of \( U \). Thus \( SO^*(2n) \) preserves the metric \( \delta_{ij} \) as well as the complex structure \( J \), since

\[
UJU^t = J.
\]

The subgroup of \( SL(2n, \mathbb{C}) \) of matrices satisfying (3.2) (but not necessarily (3.1)) is \( SO(2n, \mathbb{C}) \) and so \( SO^*(2n) \) is also the subgroup of \( SL(2n, \mathbb{C}) \) preserving a complex structure (3.3).

Writing \( U = e^{\lambda} \) in terms of Lie algebra generators \( \lambda_a^b (a, b = 1, \ldots, 2n) \), the generators of \( SL(2n, \mathbb{C}) \) are general traceless complex matrices \( \lambda_a^b \), \( \lambda_a^a = 0 \), while the generators of \( SU^*(2n) \) are those satisfying in addition

\[
\lambda_a^c J^{cb} = J^{ac} \lambda_c^b,
\]

(3.4)

where

\[
\lambda_a^b = (\lambda^a_b)^*.
\]

(3.5)

and the matrix \( J^{ab} \) is real and satisfies

\[
J^{ab} = J_{ab} = -J^{ba}, \quad J^{ab} J_{bc} = -\delta^a_c.
\]

(3.6)

Defining

\[
\lambda_{ab} = J_{ac} \lambda_c^b, \quad \lambda^{ab} = (\lambda_{ab})^*,
\]

(3.7)

this implies

\[
\lambda^{ab} = J^{ac} J^{bd} \lambda_{cd}.
\]

(3.8)

The generators of \( SO^*(2n) \) satisfy in addition

\[
\delta_{ac} \lambda^c_b = -\delta_{bc} \lambda^c_a.
\]

(3.9)
Defining
\[ L_{ab} = \delta_{ac} \Lambda^c_{\ b}, \quad (3.10) \]
the subgroup \( SO(2n, \mathbb{C}) \) of \( SL(2n, \mathbb{C}) \) is generated by complex antisymmetric matrices satisfying
\[ L_{ab} = -L_{ba}. \quad (3.11) \]

Then \( SO^*(2n) \) is defined as the subgroup of \( SO(2n, \mathbb{C}) \) for which the generators satisfy the reality condition
\[ L_{ab} = J^{ac} J^{bd} L_{cd}, \quad (3.12) \]
where \( L^{ab} \equiv (L_{ab})^* \). This is to be compared to the group \( SO(2n) \), which is defined as the subgroup satisfying the standard reality condition that the generators are real, \( L_{ab} \equiv (L_{ab})^* \).

Some of the lower dimensional cases are (locally) equivalent to more familiar groups:
\[ \begin{align*}
SO^*(2) & \sim SO(2), \\
SO^*(4) & \sim SO(3) \times SO(2, 1), \\
SO^*(6) & \sim SU(3, 1), \\
SO^*(8) & \sim SO(6, 2), \\
SU^*(2) & \sim SU(2), \\
SU^*(4) & \sim SO(5, 1).
\end{align*} \quad (3.13) \]

The \( N = 8 \) supermultiplet has 28 vector fields. In the Abelian theory, the 28 field strengths and their 28 duals combine into the \( 56 \) of \( E_7(\mathbb{R}) \) and the \( E_7(\mathbb{C}) \) transformations include duality transformations mixing electric and magnetic fields. In formulating the theory, one chooses 28 of the 56 field strengths \( F^A \) to be given in terms of fundamental potentials \( F^B = dA^B \) \((A, B = 1, \ldots, 28)\), and the other 28 \( G_A \) to be given by the variation of the action \( S \), \( G_A \sim \delta S/\delta F^A \), so that \( G_A = \#F^A + \cdots \) (where the dots denote terms dependent on the scalars and other fields). The vector field Bianchi identities and field equations are then
\[ dF^A = 0, \quad dG_A = 0, \quad (3.14) \]
and these 56 equations transform into each other as the \( 56 \) of \( E_7 \). Different choices of which set of 28 field strengths out of the 56 are to be fundamental, i.e., which 28 are to be given in terms of fundamental potentials, give different dual formulations of the theory, all classically equivalent.

It is often useful to choose a basis in which a subgroup of \( E_7(\mathbb{C}) \) is a manifest symmetry. In \([4]\), two bases were considered, one in which \( SU(8) \subset E_7(\mathbb{C}) \) is manifest, and the other in which \( SL(8, \mathbb{R}) \subset E_7(\mathbb{C}) \) is manifest. Under \( SL(8, \mathbb{R}) \), the \( 56 \) of \( E_7 \) decomposes into a \( 28 + 28 \). Choosing the field strengths \( F^{ab} \) in the \( 28 \) (with \( a, b = 1, \ldots, 8 \) \( SL(8, \mathbb{R}) \) indices and \( F^{ab} = -F^{ba} \)) as fundamental, \( F^{ab} = dA^{ab} \), those in the \( 28' \) are then the dual field strengths \( G_{ab} \). In this formulation, \( SL(8, \mathbb{R}) \) is a symmetry of the action while \( E_7 \) is a symmetry of the equations of motion. This is the basis best suited for gauging the \( CSO(p, q, r) \) subgroups of \( SL(8, \mathbb{R}) \). The field strengths can be combined into 28 complex combinations \( F = F + iG = F + i \ast F + \cdots \) satisfying a generalized self-duality constraint \([4]\) and these transform as a \( 28 \) under \( SU(8) \subset E_7 \). One can change to a basis in which this \( SU(8) \) is manifest (the explicit formulae relating tensors with \( SL(8, \mathbb{R}) \) indices to ones with \( SU(8) \) indices involve \( SO(8) \) gamma-matrices \([4]\)) and in this basis the \( SO(8) \) gauging is straightforward, but the expressions for the \( T \)-tensor etc in the non-compact gaugings look much more complicated than in the \( SL(8, \mathbb{R}) \) basis. Note that the 28 complex field strengths \( F \) cannot be written in terms of 28 complex potentials, and there is no formulation in which this \( SU(8) \subset E_7 \) is a rigid symmetry of the action, as it necessarily involves duality transformations. The \( SU(8) \) subgroup of the rigid \( E_7 \) symmetry should not be confused with the local \( SU(8) \) symmetry of the theory.

One can instead focus on any of the maximal subgroups of \( E_7(\mathbb{C}) \), which are
\[ \begin{align*}
SU(8), & \quad SL(8, \mathbb{R}), & \quad SU(4, 4), & \quad SU^*(8), & \quad SO(6, 6) \times SL(2, \mathbb{R}), \\
SO^*(12) \times SU(2), & \quad E_{6(2)} \times U(1), & \quad E_{6(6)} \times SO(1, 1).
\end{align*} \]
The basis in which $SO(6, 6) \times SL(2, \mathbb{R})$ is manifest is useful in considering compactification from the $D = 10$ IIB theory, with the $SO(6, 6)$ the $T$-duality group and $SL(2, \mathbb{R})$ the duality symmetry of the $D = 10$ IIB supergravity [14]. The basis in which $E_{6(6)} \times SO(1, 1)$ is manifest is useful in considering compactification from $D = 5$, $N = 8$ supergravity. The ungauged $D = 5$ theory has an $E_{6(6)}$ symmetry of the action, and so compactification gives a dual form of $D = 4$, $N = 8$ supergravity which has a symmetry of the action $E_{6(6)} \times SO(1, 1)$.

Consider now the $SU^*(8)$ basis. Introducing indices $a', b', \ldots = 1, \ldots, 8$ in the fundamental representation of $SU^*(8)$, antisymmetric tensors $T^{a'b'} = -T^{b'a'}$ lie in a representation of 28 complex dimensions. However, this is reducible and one can impose the reality condition

$$T_{a'b'} \equiv (T^{a'b'})^* = J_{a'c'} J_{b'd'} T^{b'd'},$$

(3.15)

using the complex structure of $SU^*(8)$ to define the 28 representation of 28 real dimensions. From (3.12), the 28 of $SU^*(8)$ becomes the adjoint of $SO^*(8) \subset SU^*(8)$. In addition, there is a dual 28-dimensional representation, the 28', defined by antisymmetric tensors $S_{a'b'}$ satisfying a reality condition similar to (3.15) and transforming so that $S_{a'b'} T^{a'b'}$ is a singlet. Under $SU^*(8)$, the 56 of $E_{7(7)}$ decomposes as

$$56 \to 28 + 28'.$$

(3.16)

Then the 56 field strengths can be decomposed into fundamental ones $F^{a'b'} = -F^{b'a'}$ in the 28 of $SU^*(8)$, and the duals $G_{a'b'}$ in the 28'. The fundamental field strengths satisfy the reality condition

$$F_{a'b'} \equiv (F^{a'b'})^* = J_{a'c'} J_{b'd'} F^{b'd'},$$

(3.17)

and are given in terms of potentials $A_{a'b'}$ satisfying a similar reality condition, so that in the ungauged theory $F^{a'b'} = dA^{a'b'}$. The theory can then be formulated in terms of the 28 real potentials $A_{a'b'}$. It is straightforward to do this, giving a dual form of the theory in which $SU^*(8)$ is a symmetry of the action, with $SU^*(8)$ acting directly on the 28 real vector potentials in the 28. The action can be written in a way that is formally identical to the $SL(8, \mathbb{R})$-invariant Cremmer–Julia action, but with $SL(8, \mathbb{R})$ indices $a, b, \ldots$ replaced by $SU^*(8)$ indices $a', b', \ldots$, and the reality conditions on the fields changed to $SU^*(8)$-invariant ones of the type discussed above. In the following section, the gauging of 28-dimensional subgroups of this $SU^*(8)$ rigid symmetry will be considered.

4. The $CSO^*(2p, 8 - 2p)$ gaugings

The key to constructing these new gaugings is the observation that the structure of the ungauged theory in the $SL(8, \mathbb{R})$ basis is formally identical to that in the $SU^*(8)$ basis, but with indices $a, b, \ldots$ replaced by $SU^*(8)$ indices $a', b', \ldots$. Then the structure of the gaugings of subgroups of $SL(8, \mathbb{R})$ when written in the $SL(8, \mathbb{R})$ basis is identical to the structure of the gaugings of subgroups of $SU^*(8)$ when written in the $SU^*(8)$ basis. The formal equivalence means that the proofs of the $T$-tensor identities are formally the same, and hence the proof of supersymmetry for the new gaugings follows immediately. Despite appearances, this is not a trivial rewriting and the fact that the new $T$-tensors are defined with contractions in the $SU^*(8)$ basis completely changes the dependence of the potential and mass terms on the scalar fields. This is the same argument that was used in $D = 5$ in [11], where it was found that the gauging of $SO(6)$ in the $SL(6, \mathbb{R})$ basis and the gauging of $SO^*(6)$ in the $SU^*(6)$ basis were formally identical, allowing the $SO^*(6)$ gauge theory to be written down immediately.

The group $CSO(p, q)$ arises from a group contraction of $SO(p+q)$ in which the generators $L$ of $SO(p+q)$ are decomposed into $SO(p)$ generators $\Lambda, SO(q)$ generators...
The gauging of $CSO(2p, 8 − 2p)$ in the $SL(8, \mathbb{R})$ basis is as in section 2, using the 28 vector fields $A^{a\nu}$ in the 28 of $SL(8, \mathbb{R})$. The non-Abelian field strength is (2.6), the minimal couplings are as in (2.7) and then the further modifications to the action and supersymmetry transformations are given in terms of the $T$-tensor (2.8). The gauging of $CSO^∗(2p, 8 − 2p)$ in the $SU^∗(8)$ basis is formally identical, but with $SL(8, \mathbb{R})$ indices $a, b, \ldots$ replaced by $SU^∗(8)$ indices $a′, b′, \ldots$. The 28 vector field strengths $F^{a\nu}$ satisfying (3.17) are defined in terms of the 28 vector potentials $A^{a\nu}$ satisfying the reality condition

$$A^{a\nu} = (A^{a\nu})^∗ = J_{a′c} J_{b′d} A^{b′d},$$

by

$$F_{\mu\nu}^{a\nu} = \partial_{\mu} A_{\nu}^{a\nu} - \partial_{\nu} A_{\mu}^{a\nu} - 2g A_{[\mu}^{a} A_{\nu]}^{b} \eta_{c, d}^{}.$$

where $\eta_{c, d}^{}$ is the invariant metric diag$(1_{2p}, 0_{8−2p})$. In this basis, the components of the 56-bein $V$ have the same structure as in (2.3), but with primed indices $a′, b′, \ldots$ instead of unprimed ones. The covariant derivative of, for example, $u_{ij}^{m} b^c$ is

$$\mathcal{D}_\mu u_{ij}^{m} b^c = \partial_\mu A_{ij}^{a\nu} + B_{[i}^{k} u_{j]}^{m} b^c - 2g A_{(\mu}^{a} u_{ij}^{b\rho} \eta_{c, d}^{}.$$

The new $A$-tensors are given by (2.9) in terms of the new $T$-tensor

$$T_i^{jk} = \psi_{i}^{k} c^d (\eta_{m}^{m} u_{ij}^{m} b^c - \psi_{ij}^{m} b^c) + \eta_{m}^{m} u_{ij}^{m} b^c.$$

Then adding the potential (2.10) and the mass terms (2.11) and modifying the supersymmetry transformations by the terms (2.12) gives a theory which is invariant under the $N = 8$ local supersymmetry; the proof is formally equivalent to that for the $CSO(2p, 8 − 2p)$ gauging. The $CSO(2p, 8 − 2p)$ gauging has a rigid $SL(8 − 2p, \mathbb{R})$ symmetry while the $CSO^∗(2p, 8 − 2p)$ gauging has a rigid $SU^∗(8 − 2p)$ symmetry.

For the $CSO(p, q, r)$ gaugings considered in section 2, it was useful to decompose the $SL(8, \mathbb{R})$ indices $a, b = 1, \ldots, 8$ into $SL(p + q, \mathbb{R})$ indices $I, J = 1, \ldots, p + q$ and $SL(r, \mathbb{R})$ indices $\alpha, \beta = p + q + 1, \ldots, 8$. Similarly, for the $CSO^∗(2p, 2q)$ gauging with $p + q = 4$, it is useful to decompose the $SU^∗(8)$ indices $a′, b′ = 1, \ldots, 8$ into $SU^∗(2p)$ indices $I′, J′ = 1, \ldots, 2p$ and $SU^∗(2q)$ indices $\alpha′, \beta′ = 1, \ldots, 2q$, so that $a′ → (I′, \alpha′)$ and

$$\eta_{a′b′} = \left(\begin{array}{cc}
\delta_{I′J′} & 0 \\
0 & 0_{2q×2q}
\end{array}\right).$$

Then $A^{a\nu} → (A^{I′J′}, A^{I′a′}, A^{a′\beta})$ with field strengths and covariant derivatives given by (2.14) and (2.16), but with indices $I, \alpha$ replaced by $I′, \alpha′$ and $\eta_{k,l}$ replaced by $\delta_{k,l}$, while the $T$-tensor becomes

$$T_i^{jk} = \psi_{i}^{k} c^d (\eta_{m}^{m} u_{ij}^{m} b^c - \psi_{ij}^{m} b^c).$$
This then gives a family of gaugings with local symmetry group \( K = \text{CSO}^*(2p, 2q) \) where \( p + q = 4 \), and in addition a global symmetry \( \text{SU}^*(2q) \). The gauge groups have a subgroup

\[
\text{SO}^*(2p) \times U(1)^{q^2} \times \text{SO}(1, 1)^{q(q-1)},
\]

and the maximal compact subgroup is

\[
\tilde{K} = \text{SU}(p) \times U(1)^{q^2+1}.
\]

As usual, any solution will spontaneously break the gauge symmetry to a compact subgroup, contained in \( \tilde{K} \).

The case \( q = 0 \) is the gauging of \( \text{SO}^*(8) \). However, \( \text{SO}^*(8) = \text{SO}(6, 2) \) and is a common subgroup of both \( \text{SL}(8, \mathbb{R}) \) and of \( \text{SU}^*(8) \), so that gauging \( \text{SO}^*(8) \subset \text{SU}^*(8) \) is equivalent to the \( \text{SO}(6, 2) \) gauging of [2]. The \( \text{CSO}^*(6, 2) \) group has a maximal subgroup \( \text{SO}^*(6) \times U(1) \) where \( \text{SO}^*(6) \sim \text{SU}(3, 1) \) with maximal compact subgroup \( \tilde{K} = \text{SU}(3) \times U(1)^2 \) and the theory has a global symmetry \( \text{SU}(2) \). This will be discussed further in later sections, where it will be seen that there is an \( N = 2 \) supersymmetric Minkowski space solution with gauge symmetry broken down to the maximal compact subgroup \( \text{SU}(3) \times U(1)^2 \).

The \( \text{CSO}^*(4, 4) \) group has a maximal subgroup \( \text{SO}^*(4) \times U(1)^5 \times \text{SO}(1, 1)^2 \) with \( \text{SO}^*(4) \sim \text{SU}(2) \times \text{SU}(1, 1) \) and has maximal compact subgroup \( \tilde{K} = \text{SU}(2) \times U(1)^5 \).

The theory has a global symmetry \( \text{SU}^*(4) \sim \text{SO}(5, 1) \). There is then a family of related gaugings with the following gauge groups \( K \) with maximal subgroups \( H \times C \) and global symmetry \( U \):

| \( K \)            | \( C \)            | \( H \)          | \( U \)        |
|-------------------|-------------------|-----------------|---------------|
| \( \text{CSO}(4, 0, 4) \) | \( U(1)^8 \)      | \( \text{SO}(4) \sim \text{SU}(2) \times \text{SU}(2) \) | \( \text{SL}(4, \mathbb{R}) \sim \text{SO}(3, 3) \) |
| \( \text{CSO}(3, 1, 4) \) | \( U(1)^8 \)      | \( \text{SO}(3, 1) \sim \text{SL}(2, \mathbb{C}) \) | \( \text{SL}(4, \mathbb{R}) \sim \text{SO}(3, 3) \) |
| \( \text{CSO}(2, 2, 4) \) | \( U(1)^8 \)      | \( \text{SO}(2, 2) \sim \text{SU}(1, 1) \times \text{SU}(1, 1) \) | \( \text{SL}(4, \mathbb{R}) \sim \text{SO}(3, 3) \) |
| \( \text{CSO}(4, 4) \) | \( U(1)^3 \times \text{SO}(1, 1)^2 \) | \( \text{SO}^*(4) \sim \text{SU}(2) \times \text{SU}(1, 1) \) | \( \text{SU}^*(4) \sim \text{SO}^*(6) \) |

The \( \text{CSO}^*(2, 6) \) gauging has a subgroup \( U(1)^{10} \times \text{SO}(1, 1)^6 \) (recall \( \text{SO}^*(2) = U(1) \)), maximal compact subgroup \( \tilde{K} = U(1)^{10} \) and global symmetry \( \text{SU}^*(6) \). This is distinct from the \( \text{CSO}(2, 6) \) gauging which has a maximal compact subgroup \( U(1)^{10} \) and global symmetry \( \text{SL}(6, \mathbb{R}) \).

5. Reduction from \( D = 5 \) gauged supergravity

The ungauged \( N = 8, D = 5 \) supergravity [8] has an \( E_6 \), rigid duality symmetry and 27 Abelian vector fields transforming as a 27 of \( E_6 \), which decomposes into the \( (15^*, 1) + (6, 2) \) under the subgroup \( \text{SL}(6, \mathbb{R}) \times \text{SL}(2, \mathbb{R}) \). The gauge fields in the \( (6, 2) \) can be dualized to give massless 2-forms \( B_{mnl\alpha} \) and this is the form of the theory that was gauged in [12, 13]. As before, \( I, J = 1, \ldots, 6 \) are \( \text{SL}(6, \mathbb{R}) \) indices, \( \alpha, \beta = 1, 2 \) are \( \text{SL}(2, \mathbb{R}) \) indices while \( m, n = 0, 1, \ldots, 4 \) and \( D = 5 \) coordinate indices. In [12], gauged theories were constructed with gauge groups \( K = \text{SO}(p, 6 - p) \) for \( p = 0, 1, 2, 3 \). In the gauged theory, the subgroup \( K = \text{SO}(p, 6 - p) \) of the rigid \( \text{SL}(6, \mathbb{R}) \) symmetry is promoted to a local symmetry in which the vector fields \( A_{mI}^J \) in the \( (15^*, 1) \) of \( \text{SL}(6, \mathbb{R}) \times \text{SL}(2, \mathbb{R}) \) become the non-Abelian gauge fields transforming in the adjoint of \( K \). In the gauged theory, the 2-forms \( B_{mnl\alpha} \) become massive self-dual gauge fields satisfying a constraint of the form

\[
DB_{IJ} = gM_{IJ}^{\beta\gamma} B_{\beta\gamma},
\]

(5.1)
for some mass matrix $M$, and each massive self-dual gauge field has three degrees of freedom, the same number as a massless 2-form or vector. The action contains the terms

$$
\int \frac{1}{2g} \eta^{IJ} \epsilon^{\alpha\beta} B_{I\alpha} \wedge DB_{J\beta} + \frac{1}{2} M^{I\alpha}_{J\beta} B_{I\alpha} \wedge *B_{J\beta},
$$

(5.2)

where $\eta_{IJ}$ is the constant $SO(p, 6-p)$-invariant metric, $g$ is the gauge coupling constant and $M^{I\alpha}_{J\beta}$ is a scalar-dependent mass matrix given explicitly in [12]. $D$ is the $SO(p, 6-p)$ gauge covariant exterior derivative

$$
DB_{I\alpha} = dB_{I\alpha} - g \eta_{IJ} A^{J\beta} \wedge B_{K\alpha}.
$$

(5.3)

Varying the action (5.2) gives the constraint (5.1) with $M^{I\alpha}_{J\beta} = \epsilon_{\alpha\gamma} \eta^{IK} M^{K\gamma}_{J\beta}$. The full action of [12] has an additional coupling of the form $\int B \cdot J$ for a certain 2-form current $J$ constructed from the other fields in the theory, giving additional terms involving $J$ to (5.1) and some of the equations below, but these will be suppressed here to simplify the presentation; they do not affect the results. The action (5.2) is invariant under local $SO(p, 6-p)$ gauge transformations with $B_{I\alpha}$ transforming as a 6. As the 2-forms are massive, there is no 2-form gauge invariance, although a form of the theory with such symmetry can be constructed by introducing Stuckelberg 1-form gauge fields $a_{I\alpha}$ by the field redefinition

$$
B_{I\alpha} \rightarrow B_{I\alpha} - D_{a_{I\alpha}},
$$

(5.4)

which also introduces the gauge invariances

$$
\delta B_{I\alpha} = D\lambda_{I\alpha}, \quad \delta a_{I\alpha} = \lambda_{I\alpha},
$$

(5.5)

with 1-form parameters $\lambda_{I\alpha}$. Clearly, the 1-forms $a_{I\alpha}$ can be gauged to zero, regaining the previous formulation. The 2-forms can be thought of as obtaining their mass by eating the 1-forms. The minimal couplings break $SL(6, \mathbb{R})$ to $K$ but $SL(2, \mathbb{R})$ remains as a global symmetry.

The dimensional reduction to $D = 4$ of most sectors of the theory is straightforward, but the gauge and 2-form sectors have some unusual features. The reduction of the gauge fields gives vector gauge fields $A^{I\beta}_\mu$ and scalars $\sigma^{I\beta}$ in the adjoint of $K = SO(p, 6-p)$ while that of the metric gives the $D = 4$ metric, a vector field $C_\mu$ and a scalar $\phi$. The resulting theory clearly has a gauge symmetry which includes $K \times U(1)$. The 2-forms $B_{\mu\nu I\alpha}$ give 2-forms $B_{\mu\nu I\alpha}$ and vector fields $A_{\mu I\alpha}$ and the dimensional reduction of the self-duality constraint (5.1) now gives the constraint

$$
B_{I\alpha} = g M^{I\alpha}_{J\beta} * DA_{J\beta},
$$

(5.6)

so that the 2-forms $B_{\mu\nu I\alpha}$ are dual to the vector fields $A_{\mu I\alpha}$ and the theory can be formulated in terms of $A_{\mu I\alpha}$ alone. To see this in more detail, the dimensional reduction of (5.2) includes the terms

$$
\int \frac{1}{g} \eta_{IJ} \epsilon^{\alpha\beta} \left( B_{I\alpha} \wedge DA_{J\beta} - \frac{1}{2} dC \wedge A_{I\alpha} \wedge A_{J\beta} \right) + \frac{1}{2} M^{I\alpha}_{J\beta} \left[ (B_{I\alpha} - C \wedge A_{I\alpha}) \wedge * (B_{J\beta} - C \wedge A_{J\beta}) + e^{-2\phi/\sqrt{3}} A_{I\alpha} \wedge *A_{J\beta} \right] .
$$

(5.7)

The field equation for $B$ gives $B$ in terms of the other fields, and using this to eliminate $B$ from the action gives

$$
\int \frac{1}{2} \tilde{M}^{I\alpha}_{J\beta} F^{I\alpha} \wedge * F^{J\beta} - \eta_{IJ} \epsilon^{\alpha\beta} g dC \wedge A^{I\alpha} \wedge A^{J\beta} + \frac{1}{2} g^2 e^{-2\phi/\sqrt{3}} \tilde{M}^{I\alpha}_{J\beta} A^{I\alpha} \wedge * A^{J\beta} ,
$$

(5.8)
where
\[ A^{Ia} = g \eta^{IJ} g_{a\beta} A_{J\beta}, \quad F^{Ia} = D A^{Ia} = d A^{Ia} - g \eta_{JK} A^{IJ} \wedge A^{Ka}. \] (5.9)
and
\[ \tilde{M}_{IaJ\beta} = (M^{-1})_{IaJ\beta}, \quad \tilde{M}_{IaJ\beta} = \epsilon_{a\gamma} \epsilon_{b\delta} \eta_{IK} \eta_{JL} M^{K\gamma L\delta}. \] (5.10)

There is then local \( SO(p, 6 - p) \times U(1) \) gauge symmetry with gauge fields \( A^{Ia}, C \) and there remains a global \( SL(2, \mathbb{R}) \) symmetry. There are massive vector fields \( A^{Ia} \) transforming as a \( (6, 2) \) under \( SO(p, 6 - p) \times SL(2, \mathbb{R}) \) and which are singlets under \( U(1) \). They transform non-trivially under the gauge group, and their field strength is \( F = DA \); as these massive vector fields have no gauge invariance in this formulation, there is no problem with this coupling. The massive vector fields can become gauge fields by introducing a Stuckelberg scalars via the field redefinition
\[ A^{Ia} \rightarrow A^{Ia} - D \rho^{Ia}, \] (5.11)
so that the theory now has extra gauge invariances
\[ \delta A^{Ia} = D \lambda^{Ia}, \quad \delta \rho^{Ia} = \lambda^{Ia}, \] (5.12)
where \( D \) is the \( K \)-covariant derivative. The gauge fields \( A^{IJ}, A^{Ia}, C \) might then be thought of as gauge fields for the gauge group \( SO(p, 6 - p) \times \mathbb{R}_{1}^{3} \times U(1) \), but there remains the unusual coupling \( d C \wedge A \wedge A \) in (5.8). This coupling has an interesting topological interpretation [15]. The terms involving \( C \) then include
\[ \int \frac{1}{2} e^{-\phi/\sqrt{3}} d C \wedge *d C - \eta_{IJ} \epsilon_{a\beta} g d C \wedge A^{Ia} \wedge A^{J\beta}, \] (5.13)
where the kinetic term arises from the reduction of the Einstein action. As \( C \) occurs only through its field strength, it can be dualized to a new gauge field \( \tilde{C} \), so that (5.13) becomes
\[ \int \frac{1}{2} e^{-\phi/\sqrt{3}} G \wedge *G, \] (5.14)
where
\[ G = d \tilde{C} + g \eta_{IJ} \epsilon_{a\beta} A^{Ia} \wedge A^{J\beta}. \] (5.15)
Comparing with (2.14), one sees that \( F^{IJ}, F^{Ia}, F^{a\beta} = G \epsilon^{a\beta} \) have the correct form to be identified with the field strengths of the \( CSO(p, 6 - p, 2) \) gauging, and it is straightforward to check that the reduction and duality transformation indeed give the \( CSO(p, 6 - p, 2) \) gauging \( D = 4 \) supergravity. The scalars consist of the 42 scalars of the \( D = 5 \) theory, taking values in \( E_{6} / USp(8) \), and the \( 15 + 1 + 12 \) scalars \( \sigma^{IJ}, \phi, \rho^{Ia} \), giving 70 scalars in all, which is the correct number for \( N = 8, D = 4 \) supergravity. The 12 scalars \( \rho^{Ia} \) are eaten by the gauge fields \( A^{Ia} \) which become massive, breaking the gauge group to \( K \times U(1) \). As in the \( D = 5 \) theory, the gauge group is further broken to a subgroup of the maximal compact subgroup \( SO(p) \times SO(6 - p) \times U(1) \), with further vector fields becoming massive through the Higgs mechanism. The vacua correspond to critical points of the potential, and the scalar expectation values will determine which subgroup of \( SO(p) \times SO(6 - p) \times U(1) \) remains unbroken. The \( SL(2, \mathbb{R}) \) global symmetry remains, but is broken to \( SL(2, \mathbb{Z}) \) in the quantum theory.

Consider next the \( SU(3, 1) = SO^{*}(6) \) gauging of \( D = 5, N = 8 \) supergravity [11] and its reduction to \( D = 4 \). The structure of the gauging is identical in form to that of the \( SO(6) \) gauging. Instead of focusing on the \( SL(6, \mathbb{R}) \times SL(2, \mathbb{R}) \) subgroup of \( E_{6} \), the starting point in [11] was to consider the \( SU^{*}(6) \times SU(2) \) subgroup, under which the \( 27 \) decomposes as the
(16, 1) + (6, 2). Again, it is useful to introduce SU*(6) indices $I', J' = 1, \ldots, 6$ and SU(2) indices $\alpha', \beta' = 1, 2$.

The gauge fields in the (6, 2) can be dualized to give massless 2-forms $B_{mn1\alpha'}$ and this is the form of the theory that was gauged in [11]. In the SU(3, 1) gauged theory [11] the subgroup $K = SO^*(6) \sim SU(3, 1)$ of the rigid SU*(6) symmetry is promoted to a local symmetry in which the vector fields $A_{\mu}^{IJ}$ in the (16, 1) become the non-Abelian gauge fields transforming in the adjoint of SO*(6). The $B_{mn1\alpha'}$ become massive self-dual 2-forms and the SO*(6) gauged theory can be written in the SU*(6) × SU(2) basis so that it is identical in form to the SO(6) gauged theory written in the SL(6, $\mathbb{R}$) × SL(2, $\mathbb{R}$) basis, but with indices $I$, $J$ and $\alpha$, $\beta$ replaced by primed indices $I'$, $J'$ and $\alpha'$, $\beta'$. In particular, the terms in the action involving the 2-forms include

$$\int \frac{1}{2g} \eta^{I'J'} \epsilon^{\alpha'\beta'} B_{1\alpha'} \wedge DB_{J'\beta'} + \frac{1}{2} M^{IJ'} \epsilon^{\alpha'\beta'} B_{1\alpha'} \wedge *B_{J'\beta'},$$

(5.16)

and the dimensional reduction to $D = 4$ is identical in form to that considered above for the SO(6) gauging. Then dimensionally reducing the SO*(6) theory to $D = 4$ and dualizing the graviphoton $C$ as above gives precisely the CSO*(6, 2) gauged theory considered in section 4. This has an SU(2) global symmetry inherited from that of the $D = 5$ theory.

### 6. Acting with dualities

In the CSO($p, q, r$) gaugings, the $r(r - 1)/2$ gauge fields $A^{\alpha\beta}$ only appear through their exterior derivative $dA^{\alpha\beta}$, and so any set of these can be dualized to give a dual form of the theory, and in general this changes the form of the gauge group. This was seen explicitly in the last section for the CSO($p, q, 2$) gauging, in which dualizing $A^{\alpha\beta} = C_{\epsilon}^{\alpha\beta}$ to a gauge field $C$ gives the dual form with gauge group $(SO(p, 6 - p) \times \mathbb{R}^{12}) \times U(1)$ that arises from dimensional reduction of the $D = 5$, SO($p, q$) gauge theory. Similarly, dualizing the CSO*(6, 2) theory gave the $(SO^*(6) \times \mathbb{R}^{12}) \times U(1)$ gauge theory that arises from the reduction of the SO*(6) gauging in $D = 5$. Such dual forms give classically equivalent theories, even though the form of the gauge group changes.

The SO($p, q$) and CSO($p, q, r$) gauged $N = 8$ supergravities were obtained from the SO(8) gauging by acting with dualities [2, 3]. The SO(8) gauging is constructed from the ungauged theory by adding minimal couplings and the couplings (2.10) and (2.11) constructed from the $T$-tensor (2.8). These additional couplings all involve the metric $\delta_{ab}$ and explicitly break the SL(8, $\mathbb{R}$) symmetry of the ungauged action to the subgroup SO(8) preserving $\delta_{ab}$. Acting on the gauged theory with an SL(8, $\mathbb{R}$) transformation $S_a^b$ has the net effect of replacing every occurrence of $\delta_{ab}$ in the gauged theory with

$$\eta_{ab} = S_a^c S_b^d \delta_{cd}.$$  (6.1)

This is of course simply a field redefinition, giving an equivalent theory. However, every occurrence of $\delta_{ab}$ in the gauged theory is accompanied by a factor of the gauge coupling $g$. Choosing $S = \exp(-iX/2)$ where $X$ is the SO($p$) × SO($q$) invariant generator

$$X_{ab} = \begin{pmatrix} \alpha 1_{p \times p} & 0 \\ 0 & \beta 1_{q \times q} \end{pmatrix},$$

(6.2)

and

$$\alpha = -1, \quad \beta = p/q, \quad p + q = 8,$$

(6.3)
and rescaling $g \rightarrow g^{\omega t \alpha}$ has the net effect of replacing every occurrence of $\delta_{ab}$ in the gauged theory with
\[
\eta_{ab} = \begin{pmatrix} 1 & 0 \\
0 & \xi \mathbf{1}_{q \times q} \end{pmatrix},
\]
where
\[
\xi = e^{\omega - \beta' t}, \quad \eta_{ab} = \begin{pmatrix}
\xi & & & \\
& \ddots & & \\
& & \xi & \\
& & & 1
\end{pmatrix}.
\]

Then this gives a well-defined theory for all real values of the parameter $\xi$, and in particular $\xi$ can be continued to zero or to negative values. The limit $t \rightarrow \infty$, $\xi \rightarrow 0$ gives the $\text{CSO}(p, q)$ gauging, while continuing to $\xi = -1$, $t = i\pi/(\alpha - \beta)$ gives the $\text{SO}(p, q)$ gauging. The one-parameter family gives three distinct gaugings: any theory with $\xi > 0$ is equivalent (via field redefinitions) to the $\text{SO}(8)$ gauging, while any theory with $\xi < 0$ is equivalent to the $\text{SO}(p, q)$ gauging. The power of this method is that since supersymmetry is guaranteed for all $\xi > 0$, it follows by continuation that the deformed theory will also be supersymmetric for $\xi \leq 0$; see [2, 3] for further details.

Similarly, a two-parameter family of gaugings can be obtained [3] by acting with the $\text{SL}(8, \mathbb{R})$ transformations
\[
S = \exp(-tX/2 - t'X'/2),
\]
where
\[
X_{ab} = \begin{pmatrix} \alpha \mathbf{1}_{p \times p} & 0 \\
0 & \beta \mathbf{1}_{q \times q} \end{pmatrix}, \quad X'_{ab} = \begin{pmatrix} \alpha' \mathbf{1}_{p \times q} & 0 \\
0 & \beta' \mathbf{1}_{r \times r} \end{pmatrix}.
\]

\[
\alpha = \alpha' = -1, \quad \beta = p/q + r, \quad \beta' = p + q/r, \quad p + q + r = 8,
\]
and rescaling $g \rightarrow g^{\omega t \alpha \beta'}$. This has the effect of replacing every occurrence of $\delta_{ab}$ in the gauged theory with
\[
\eta_{ab} = \begin{pmatrix} 1 & 0 & 0 \\
0 & \xi \mathbf{1}_{q \times q} & 0 \\
0 & 0 & \xi \mathbf{1}_{r \times r} \end{pmatrix},
\]
where
\[
\xi = e^{\omega - \beta' t}, \quad \xi' = e^{\omega - \beta' t'},\quad \eta_{ab} = \begin{pmatrix}
\xi & & & \\
& \ddots & & \\
& & \xi & \\
& & & 1
\end{pmatrix}.
\]

This two-parameter family again divides into equivalence classes, and contains the $\text{CSO}(p, q, r)$ gauging arising when $\xi = -1$, $\xi = 0$. For $\xi > 0$, the $\xi$-family consists of the $\text{SO}(8)$, $\text{SO}(p, q + r)$ and $\text{CSO}(p, q + r)$ gaugings, when $\xi < 0$, the $\xi$-family consists of the $\text{CSO}(p + q, r)$, $\text{SO}(p, r)$ and $\text{CSO}(p, q + r)$ gaugings while for $\xi = 0$, the $\xi$-family consists of the $\text{CSO}(p + q, r)$, $\text{CSO}(p, q, r)$ and $\text{CSO}(p, q + r)$ gaugings.

The same continuation techniques can be used to generate the $\text{CSO}^*(2p, 2q)$ gaugings from the $\text{SO}^*(8)$ gauging. Starting with the $\text{SO}(6, 2) = \text{SO}^*(8)$ gauging and acting with the $\text{SU}^*(8)$ transformation $S = \exp(-tX/2)$ where $X$ is the $\text{SO}^*(2p) \times \text{SO}^*(2q)$ invariant generator
\[
X_{ab} = \begin{pmatrix} \alpha \mathbf{1}_{2p \times 2p} & 0 \\
0 & \beta \mathbf{1}_{2q \times 2q} \end{pmatrix},
\]
and
\[
\alpha = -1, \quad \beta = p/q, \quad p + q = 4.
\]
and rescaling \( g \rightarrow g e^{\alpha t} \) has the net effect of replacing \( \delta_{ab} \) in the gauged theory with
\[
\eta_{ab} = \begin{pmatrix}
1_{2p \times 2p} & 0 \\
0 & \xi 1_{2q \times 2q}
\end{pmatrix},
\]
where
\[
\xi = e^{(\alpha - \beta) t}.
\]

The limit \( t \rightarrow \infty, \xi \rightarrow 0 \) gives the \( \text{CSO}^* (2p, 2q) \) gauging, but continuing to \( \xi = -1 \) recovers one of the gaugings already considered and does not give anything new. The fact that the \( \text{CSO}^* (2p, 2q) \) gauging can be obtained as a smooth limit of a one-parameter family of consistent supersymmetric theories guarantees that the \( \text{CSO}^* (2p, 2q) \) gauged theory exists and is supersymmetric, by the arguments of [2].

An alternative way of obtaining the \( \text{CSO}^* (6, 2) \) theory is as follows. It was seen in section 5 that dimensionally reducing the \( D = 5, \text{SO}(6) \) gauged theory gives a \( D = 4 \) theory with gauge group \((\text{SO}(6) \times \mathbb{R}^{12}) \times U(1)\), which can be dualized to the \( \text{CSO}(6, 2) \) theory by dualizing the \( U(1) \) gauge field, while dimensionally reducing the \( D = 5, \text{SO}^* (6) \) gauged theory gives a \( D = 4 \) theory with gauge group \((\text{SO}^*(6) \times \mathbb{R}^{12}) \times U(1)\), which can be dualized to the \( \text{CSO}^* (6, 2) \) theory by dualizing the \( U(1) \) gauge field. Before dualizing the \( U(1) \) gauge field, the theory with gauge group \((\text{SO}(6) \times \mathbb{R}^{12}) \times U(1)\) and the one with gauge group \((\text{SO}^*(6) \times \mathbb{R}^{12}) \times U(1)\) are both deformations of the form of the \( D = 4, N = 8 \) theory with global symmetry of the action \( L = E_6 \times \mathbb{R} \), which is the dual form obtained directly by reducing the ungauged \( D = 5 \) theory. They can be thought of as gaugings of different subgroups of \( E_6 \times \mathbb{R}^* \). This means that one can act on the theory with gauge group \((\text{SO}(6) \times \mathbb{R}^{12}) \times U(1)\) using an \( E_6 \) transformation to change the couplings. In the \( \text{SL}(6, \mathbb{R}) \times \text{SL}(2, \mathbb{R}) \) basis, the generators of \( E_6 \) consist of \( \text{SL}(6, \mathbb{R}) \) and \( \text{SL}(2, \mathbb{R}) \) generators, together with generators \( \Sigma_{IJK\alpha} \) in the \((20', 2)\) of \( \text{SL}(6, \mathbb{R}) \times \text{SL}(2, \mathbb{R}) \) satisfying
\[
\Sigma_{IJK\alpha} = \Sigma_{IJK\beta} \frac{1}{6} \epsilon_{IJKLM} \epsilon_{\alpha\beta} \Sigma^{LM\beta},
\]
where \( \Sigma_{IJK\alpha} = (\Sigma_{IJK\alpha})^* \). As in [11, 12], let
\[
X_{abcd} = -\delta_{ab}^{1357} + \delta_{bc}^{2468} + \delta_{ac}^{1368} - \delta_{cd}^{2457} + \delta_{bd}^{1458} - \delta_{ad}^{2367} + \delta_{cd}^{1467} - \delta_{bc}^{2358}
\]
and decompose \( a, b = 1, \ldots, 8 \) into indices \( I, J = 1, \ldots, 6 \) and \( \alpha, \beta = 7, 8 \) to define a generator \( X_{IJK\alpha} \) of \( E_6 \). Then acting with the \( E_6 \) transformation \( \exp(itX) \), rescaling the coupling constants as above and taking the limit \( t \rightarrow i\pi/4 \) takes the theory with gauge group \((\text{SO}(6) \times \mathbb{R}^{12}) \times U(1)\) to the one with gauge group \((\text{SO}^*(6) \times \mathbb{R}^{12}) \times U(1)\).

### 7. The scalar potentials

The scalar potentials of the \( D = 4, \text{SO}(8) \) gauging was analysed in [16], and those of the non-compact \( D = 4 \) gaugings in [17]. The theories with gauge groups \( \text{SO}(4, 4) \) and \( \text{SO}(5, 3) \) have de Sitter vacua arising at local maxima of the potentials [3, 18]. The \( \text{CSO}(2, 0, 6) \) gauging has a Minkowski space solution and the potential has flat directions [3]. The structure and potentials of these models were analysed further in [17]; no other critical points are known. All of these theories have domain wall solutions that preserve half the supersymmetry [19].

In section 5, it was seen that the dimensional reduction of the \( N = 8, D = 5 \) theories with gauge groups \( \text{SO}(p, 6 - p) \) or \( \text{SO}^*(6) \) gives (after dualizing the graviphoton) the \( N = 8, D = 5 \) theories with gauge groups \( \text{CSO}(p, 6 - p, 2) \) or \( \text{CSO}^*(6, 2) \). If \( V_5(\chi) \) is the \( D = 5 \) scalar potential depending on 42 scalars \( \chi \) in \( E_6/U Sp(8) \), then its dimensional reduction gives a scalar potential
\[
V_d(\chi, \phi) = e^{2\phi/\sqrt{3}} V_5(\chi).
\]
where $\phi$ is the scalar coming from the reduction of the metric. The full scalar potential $V(\chi, \phi, \sigma, \rho)$ of the $D = 4$ theories also depends in principle on the scalars $\sigma^{IJ}$ or $\sigma^{IJ}$, from the reduction of the $D = 5$ gauge fields and on the Stuckelberg scalar fields $\rho^I, \rho^J$, but in fact is independent of the Stuckelberg scalars $\rho$ as the potential is necessarily invariant under the gauge invariance (5.12). Thus, the potentials have (at least) 12 flat directions and in addition are invariant under the global symmetry $SL(2, \mathbb{R})$ or $SU(2)$ as well as the local $SO(p, 6 - p) \times USp(8)$ or $SO^*(6) \times USp(8)$ symmetries. Note that the exponential dependence on $\phi$ implies that $V_4$ can only have critical points at values $\chi = \chi_0$ which are critical points of $V_5$, $\partial V_5(\chi_0) = 0$, at which the potential vanishes, $V_5(\chi_0) = 0$. However, the full potential $V$ has terms with different $\phi$ dependence. For example, dimensional reduction of the $D = 5$ vector kinetic term $N_{IJKL}(\chi) F^{IJ} F^{KL}$ for the vector fields $A^{IJ}$ gives a contribution to the scalar potential of the form

$$e^{-2\sqrt{g} \phi} N_{IJKL}(\chi) \sigma^I \sigma^J \sigma^K \sigma^L,$$

that is quartic in $\sigma$, where $N_{IJKL}(\chi) = N_{IJ|M|KL}(\chi)$ is a function of the $D = 5$ scalars $\chi$. The full $D = 4$ potential could then in principle have critical points even if the $D = 5$ potential $V_5$ or the restricted $D = 4$ potential $V_4$ do not.

The structure of the $D = 5$ potentials $V_5$ was analysed in [12, 11], and this can now be used to obtain information about the $D = 4$ potentials. For the $SO(p, 6 - p)$ gaugings it is straightforward to find the dependence on scalars in the $SL(6, \mathbb{R}) / SO(6) \times SL(2, \mathbb{R}) / SO(2)$ subspace of $E_6 / USp(8)$, parametrized by a matrix $S_I^L$ in $SL(6, \mathbb{R})$ and a matrix $S^\rho_{\alpha}$ in $SL(2, \mathbb{R})$. From [12], one finds

$$V_4 = -\frac{g^2}{32} e^{-\sqrt{\phi}} [\{\text{tr}(\eta M)\}^2 - 2 \{\text{tr}(\eta M \eta M)\}] ,$$

(7.3)

where $M_{IJ}$ is the symmetric matrix

$$M_{IJ} = S_I^K S_J^L \delta_{KL} ,$$

(7.4)

and the trace $\text{tr}$ is taken over the six $SL(6, \mathbb{R})$ indices. This is independent of $S^\rho$, as expected from $SL(2, \mathbb{R})$ invariance.

The explicit dependence of the potential $V_5$ on an $SO(p) \times SO(6 - p)$ invariant scalar $\lambda$ was found in [12]. By the Schur’s lemma argument of [16], a critical point of the potential $V_5$ restricted to such an $SO(p) \times SO(6 - p)$ invariant scalar will in fact be a critical point of the full potential. Let

$$\eta_{IJ} = \text{diag}(1, \rho, \xi_1 \xi_q) ,$$

(7.5)

where $\xi = 1$ for the $SO(6)$ gauging and $\xi = -1$ for the $SO(p, 6 - p)$ gauging. For the $SO(5)$ invariant direction, taking

$$M = \text{diag}(e^\lambda, \ldots, e^\lambda, e^{-5\lambda}) ,$$

(7.6)

gives

$$V_5 = -\frac{g^2}{32} [15 e^{2\lambda} + 10 \xi e^{-4\lambda} - e^{-10\lambda}] .$$

(7.7)

For the $SO(4) \times SO(2)$ invariant direction, taking

$$M = \text{diag}(e^{\lambda}, \ldots, e^\lambda, e^{-2\lambda}, e^{-2\lambda}) ,$$

(7.8)

gives

$$V_5 = -\frac{g^2}{4} [e^{2\lambda} + 2 \xi e^{-\lambda}] .$$

(7.9)
For the scalars in the \( SO(3) \times SO(3) \) invariant direction contained in \( SL(6, \mathbb{R}) \), taking
\[
M = \text{diag}(e^\phi, e^\phi, e^\phi, e^{-\lambda}, e^{-\lambda}, e^{-\lambda}),
\]
gives
\[
V_5 = -\frac{3g^2}{16} [\cosh (2\lambda) + 3\xi].
\]

The \( D = 5, SO(6) \) gauging has a maximally supersymmetric AdS critical point at which all scalars vanish (corresponding to \( \lambda = 0, \xi = 1 \) in each of the above cases \( 7.7 \), \( 7.9 \) and \( 7.10 \)) giving a \( D = 4 \) potential which, on setting all scalars but \( \phi \) to zero, is
\[
V_4 = -3g^2 e^{2\phi\sqrt{3}/4}. \]
The \( D = 5, SO(6) \) gauging also has a non-supersymmetric \( SO(5) \)-invariant AdS critical point with \( e^{6\lambda} = 1, \xi = 1 \) in \( 7.7 \). The \( D = 5, SO(3, 3) \) gauging has a maximally supersymmetric de Sitter critical point at which all scalars vanish (corresponding to \( \lambda = 0, \xi = -1 \) in \( 7.10 \)) giving a \( D = 4 \) potential which, on setting all scalars but \( \phi \) to zero, is
\[
V = \frac{3g^2}{8} e^{2\phi/\sqrt{3}}. \]
This is precisely the form of potential that is required for quintessence, and precisely this theory was proposed in [20] as one that could give a cosmology with quintessence. However, reinstating the scalar field \( \lambda \), the form of the potential is
\[
V_4 = \frac{3g^2}{16} e^{2\phi/\sqrt{3}} [3 - \cosh (2\lambda)].
\]
This has the desired exponential roll in the \( \phi \) direction, but is unbounded below in the \( \lambda \) direction and so it seems that the slow-roll solution used in [20] will be unstable to fluctuations in the \( \lambda \) direction.

The \( D = 5, SO^*(6) \) gauged theory and its reduction can be treated in the same way, and it is straightforward to find the dependence on scalars in the \( SU^*(6)/USp(6) \times SU(2)/SO(2) \) subspace of \( E_6/USp(8) \), parametrized by a matrix \( S_\ell' \) in \( SU^*(6) \) and a matrix \( S_{\alpha'} \) in \( SU(2) \). Remarkably, the potential is completely independent of both \( S_\ell' \) and \( S_{\alpha'} \) [11], so that the potential has a large number of flat directions. However, the potential is non-trivial in other directions, and we now turn to its dependence on one of these other directions.

The 42 scalar fields in \( E_6/USp(8) \) can be parametrized by a 27-bein \( V \) taking values in \( E_6 \). Choosing the ansatz \( V = \exp(\lambda X) \), where \( X \) is the \( E_6 \) generator constructed from the \( X_{1126} \) defined in (6.16), picks out a particular scalar field \( \lambda(x) \) in an \( SU(3) \)-invariant direction [12, 11]. The dependence on \( \lambda \) of the potential of the \( D = 5, SO(6) \) gauging was calculated in [12] and for the \( D = 5, SO^*(6) \) gauging was calculated in [11]. These then give the restricted \( D = 5 \) potentials
\[
V_5(\lambda) = \frac{3g^2}{32} [p^2 - 4\xi p - 5],
\]
where
\[
p = \cosh(4\lambda),
\]
and where the case \( \xi = 1 \) is for the \( SO(6) \) gauging and the case \( \xi = -1 \) is for the \( SO^*(6) \) gauging. The restricted \( SO(6) \) potential has the maximally supersymmetric critical point at \( p = 1, \lambda = 0 \) and a non-supersymmetric AdS critical point at \( p = 2 \), breaking the \( SO(6) \) gauge symmetry to \( SU(3) \times U(1) \) [12].

For the \( SO^*(6) \) gauging, there is a critical point at \( \lambda = 0 \) at which the potential (7.14) (with \( \xi = -1 \)) vanishes. This is then a critical point of the full potential and the \( D = 5, SO^*(6) \) gauging has a Minkowski space vacuum [11]. It was shown in [11] that this preserves
$N = 2$ supersymmetry and breaks the gauge group to $SU(3) \times U(1)$. The potential $V_3$ is independent of all the scalar fields in the $SU^*(6)/USp(6) \times SU(2)/SO(2)$ subspace of $E_6/USp(8)$ and so it has at least 16 flat directions.

As $V_3(0) = 0$, the critical point at $\lambda = 0$ is also a critical point of $V_4$, and this is in fact a critical point for the full scalar potential. The $D = 4$, $CSO^*(6, 2)$ gauged theory then has a $D = 4$ Minkowski-space solution preserving $N = 2$ supersymmetry and $SU(3) \times U(1) \times U(1)$ gauge symmetry, as well as preserving the $SU(2)$ global symmetry. There are at least 28 flat directions, corresponding to the 16 flat directions in $SU^*(6)/USp(6) \times SU(2)/SO(2)$ and the 12 Stuckelberg scalars $\rho$.

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