The enhancement of mathematical analogical reasoning ability of university students through concept attainment model

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Abstract. This study aims to see the enhancement of mathematical analogical reasoning ability of the university students through concept attainment model learning based on overall and Prior Mathematical Knowledge (PMK) and interaction of both. Quasi experiments with the design of this experimental-controlled equivalent group involved 54 of second semester students at the one of State Islamic University. The instrument used is pretest-postest. Kolmogorov-Smirnov test, Levene test, t test, two-way ANOVA test were used to analyse the data. The result of this study includes: (1) The enhancement of the mathematical analogical reasoning ability of the students who gets the learning of concept attainment model is better than the enhancement of the mathematical analogical reasoning ability of the students who gets the conventional learning as a whole and based on PMK; (2) There is no interaction between the learning that is used and PMK on enhancing mathematical analogical reasoning ability.

1. Introduction
Committee on the Undergraduate Program in mathematics provided six basic recommendations for the department, programs, and courses in mathematics. One of recommendation stated that each course in mathematics should be an activity that will assist students in the development of analytical, critical reasoning, problem solving, and communication skills. The CUPM Recommendation above explained, the task of the institution in charge of educating prospective teachers who will teach mathematics, including prepare students to have mathematical analogical reasoning ability. Institutions of Higher Education Personnel (IHEP), Elementary School [1].

Teacher Education (ESTE) which is in charge of childbirth prospective teachers who will teach mathematics partially take responsibility to prepare their students to have mathematical analogical reasoning ability. Mathematical analogical reasoning ability is the ability that could be cultivated, the lecturer has a role in the development efforts of students’ mathematical analogical reasoning ability.

The low ability on mathematical reasoning caused by less effective courses, even lectures are often one direction only. One of the causes of this low mathematical ability is probably due to the limited ability of high-level mathematical thinking of the graduates (both high school and higher education), which include the aspects of reasoning, problem solving, communication and mathematical connections, whereas this ability is needed in improving quality and the competitiveness of graduates in the world of work [2].
The ability of mathematical reasoning that will be studied is the analogical reasoning ability, namely the ability to draw conclusions based on the similarity of processes or data. Analogy is the ability to see relationships, not only the relationship of things but also the relationship between ideas, and then using them to acquire other objects or ideas [3]. Meanwhile, according to different source analogy is talking about two different things, one is not the other, but two different things are compared to one another. In analogy what we sought is the similarity of two distinct things, and draws conclusions on the basis of similarity. Thus analogy can be exploited as explanatory or as the basis of reasoning [4].

The indicator of mathematical analogical reasoning in this research consist of the students can observe the pattern (of an image or a number), the students can determine the relationship between the image pattern or number, and the students can estimate or estimate the rules that form the pattern.

Learning process will go well and creatively, if lecturer give students an opportunity to find a rule (including concepts, theories, definitions, and so on) through examples that describes/represents rules that became the source, in other words, students are guided inductively to understand a common truth [5].

Study process was presented by Bruner is in line with the theory of concept attainment model [6], concept attainment model is more focused on ways to strengthen the internal human impulses in understanding science, by digging and organizing, as well as developing the language to express it. Bruner, Goodnow, and Austin reveal concept attainment model deliberately designed to help students learn the concepts that can be used to organize information, so it makes easy for students to learn concepts in more effective way [7].

Concept attainment model has several stages of learning, the stages in concept attainment models help train students in mathematical analogy reasoning. The first learning stage, namely the presentation of the data and the identification of the concept, in this stage, students are asked to compare the characteristic features in the example and non-example, students were asked to create and test the hypothesis, then the student making the definition of the concept on the essential characteristic features, here the students are trained to analogical reasoning because students asked to compare the characteristic features in the examples and non-examples, so they can find the definition of the concept on the characteristic of concept essential features.

The second stage is the stage of testing the achievement of the concept, at this stage the students were asked to identify examples of concepts and create additional instances, here the students are trained to analogical reasoning for students to think about what characteristics are represented on a concept and what kind of example that meets the criteria of the concept.

The third stage is the stage of analytical thinking strategy, at this stage the students are trained to communicate math, because students are asked to express the concept with their own words, reveals the reasons relating to create additional instances, and write down the steps to resolve the task of the concept being studied, and formulated the mathematical concept theoretically believed that students’ mathematical analogical reasoning ability become more better [6].

Concept attainment model is an inductive learning model, designed by lecturer to help students learn the concepts and train the students in practicing high-level thinking skills [8]. Concept attainment model is very relevant to the teaching of mathematics because this model can foster understanding and appreciation of students to the concepts, principles so grows the power of reason, think logically, critically, systematically and others [9].

Concept attainment model is a learning model that aims to help students understand a particular concept, it is more appropriate when the emphasize of learning is more focused on the introduction of a new concept, so as to train high-level thinking skills [10]. The formulation of the problem are the enhancement of the mathematical analogical reasoning ability of the students who gets the learning by concept attainment model is better than the enhancement of the mathematical analogical reasoning ability of the students who gets the conventional learning as a whole and based on Prior Mathematical Knowledge (PMK)? and there an interaction between the learning that used and PMK on enhancing students' mathematical analogical reasoning ability?
2. Methods
The research plan design for this experiment is illustrated as follows [11]. O are pretest and posttest, and X is learning concept attainment model.

![Figure 1. Design a plan](image)

This research was conducted at the one of Islamic State University (UIN). The population of this research was all students of Elementary school Teacher Education (PGMI) 2nd Semester. All the population selected as sample of the study, the control class and experiment class.

The materials tested are numbers that contained in the subject of Mathematics Education II (consist of algebra, numbering patterns, sequences and arithmetic series, sequences and geometry series and also social arithmetic).

The test instrument of mathematical analogical reasoning consists of three questions that formed as description. Time Allocation for execution of the test is 2 x 50 minutes. Instruments for measuring mathematical analogical reasoning are adopted from several sources and then developed by researchers.

3. Results and Discussion
The data of pretest is collected and analyzed to determine the students’ Prior Mathematical Knowledge before this research was conducted. Then the grade is grouped by early capability categories of high, medium and low. The following are the test results pretest data equality both groups learning:

| Table 1. The normality test of normalized gain of mathematical analogical reasoning ability |
|------------------------------------------|
| Gain | Kolmogorov-Smirnov |
| Analogy Statistic | Dk | p-value |
| Experiment | .756 | 26 | .618 |
| Control | 1.267 | 26 | .081 |

From Table 1 the p-value (Asymp Sig) of the experimental class is 0.618> 0.05 = α, and the p-value (Asymp Sig) of the control class is 0.081> 0.05 = α, then the H0 is accepted. So it can be concluded that the normalized gain data of the mathematical analogical reasoning ability of students who obtained CAM and students obtained Conventional Model (CM) is normal distribution at the level of significance α = 0.05.

To test the homogeneity of the two groups of experiment and control class gain data, the Levene statistic test was used.

| Table 2. Homogeneity test of variance of normalized gain of the mathematical analagical reasoning ability |
|------------------------------------------|
| Levene Statistic | Dk1 | Dk2 | p-value |
| 2.311 | 5 | 48 | .058 |

From Table 2 obtained the value of Levene Statistic (F) of 2.311 with a significance value of 0.058. The value of significance is larger than the level of significance α = 0.05 so it can be concluded that H0
which states population variance both groups of data received. That is, the two groups of normalized gain data of the mathematical analogical reasoning ability of students having a homogeneous variance. Furthermore, since the normalized gain data group of the experimental class and the control class had a homogeneous variance and both were normally distributed, so to know the significance of the average difference of the two groups was a two-way variance (ANOVA) analysis. The summary is presented in Table 3 below.

### Table 3. The Variance Analysis of Normalized Gain of the Mathematical Analogical Reasoning Ability According to Learning Method and Category of Student Ability

| Source                      | Type III Sum of Squares | Df | Mean Square | F       | Sig.  |
|-----------------------------|-------------------------|----|-------------|---------|-------|
| Corrected Model             | 2.032*                  | 5  | 0.406       | 24.361  | 0.000 |
| Intercept                   | 20.008                  | 1  | 20.008      | 1199.486 | 0.000 |
| Learning                    | 1.856                   | 1  | 1.856       | 111.241 | 0.000 |
| early mathematical ability  | 0.164                   | 2  | 0.082       | 4.926   | 0.011 |
| Learning * early            |                         |    |             |         |       |
| mathematical ability        | 0.012                   | 2  | 0.006       | 0.355   | 0.703 |
| Error                       | 0.801                   | 48 | 0.017       |         |       |
| Total                       | 22.841                  | 54 |             |         |       |
| Corrected Total             | 2.832                   | 53 |             |         |       |

a. R Squared = 0.717 (Adjusted R Squared = 0.688)

Further testing conducted research hypothesis, there are 3 hypothesis to be tested. The 1st hypothesis is "The enhancement of the mathematical analogical reasoning ability of the students who obtained the learning of concept attainment model is better than the enhancement of the mathematical analogical reasoning ability of the students who obtained the conventional learning as a whole".

After the calculation of two-way ANOVA the results can be seen in Table 3. Obtained sig value. (1-tailed) by 0.000 < α = 0.05. Therefore, the result of the null hypothesis is rejected, it means that the enhancement of the mathematical analogical reasoning ability of the students who gets the learning of concept attainment model is better than the enhancement of the mathematical analogical reasoning ability of the students who gets the conventional learning as a whole.

The 2nd hypothesis is "The enhancement of the mathematical analogical reasoning ability of the students who gets the learning of concept attainment model is better than the enhancement of the mathematical analogical reasoning ability of the students who gets conventional learning based on the Prior Mathematical Knowledge (PMK)".

After the calculation of two-way ANOVA the results can be seen in Table 3 Obtained sig value. (1-tailed) of 0.01 < α = 0.05. Because of that, the result of the null hypothesis is rejected, it means that the enhancement of the mathematical analogical reasoning ability of the students who gets the learning of concept attainment model is better than the enhancement of the mathematical analogical reasoning ability of the students who gets conventional learning based on the Prior Mathematical Knowledge (PMK)".

The 3rd hypothesis is "There is an interaction between the learning used and Prior Mathematical Knowledge (PMK) on enhancing of students' mathematical analogical reasoning ability"

From Table 3 it can be seen the significance (sig.) for the interaction between the learning method and Prior Mathematical Knowledge (PMK) is 0.703 > α = 0.05 which means there is no interaction between the learning used and Prior Mathematical Knowledge (PMK) on enhancing of students' mathematical analogical reasoning ability.

### 4. Conclusion

The results of this study are the enhancement of mathematical analogical reasoning ability of students who gets learning by concept of attainment model is better than the enhancement of mathematical analogical reasoning ability of students who gets conventional learning as a whole and based on Prior
Mathematical Knowledge (PMK) and there is no interaction between the learning that used and the Prior Mathematical Knowledge (PMK) on enhancing of students' mathematical analogical reasoning ability. However, the learning factors and PMK each other have a significant influence on students' mathematical analogical reasoning ability.

5. References

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