Infrared Phases of 2d QCD

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F = dA + [A, A] , \quad D_\mu = \partial_\mu + iA_\mu^a t_R^a
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Introduction

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For \(N_f\) fermions, chiral symmetry \(h_{l/r} = \begin{cases} \text{SO}(N_f) & \text{real} \\ \text{Sp}(N_f) & \text{pseudoreal} \\ \text{U}(N_f) & \text{complex} \end{cases} \)
In the IR we have two choices of phases.

Gapped theories: In the IR they are not necessarily trivial
- No propagating degrees of freedom, multiple vacua
- Topological order (long range entanglement)

Gapless theories: Possible dof are Goldstone bosons from continuous symmetry breaking.
- Massless dof not from broken symmetry.
- Generically becomes a CFT at long distances.

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We will see that a necessary and sufficient condition for a theory to be gapped is if $R_{\ell/r}$ are sums of real reps of $G$. 
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Starting with $\bar{\psi} \Phi \psi$, which is a $\text{SO}(\text{dim } R)_1$ WZW theory we gauge $G \subset \text{SO}(\text{dim } R)$ and we have

$$\mathcal{L}_{\text{eff}} = \frac{\text{SO}(\text{dim } R)_1}{G_{I(R)}},$$

where $I(R)$ is the Dynkin index.
Dynamics

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Symmetries and anomalies of QCD are captured by the gauged WZW.
The CFT described by this theory is gapped if \( T_{SO} - T_G = 0 \).
This holds if
\[
t_{ij}t^{a}_{kl} + t^{a}_{ik}t^{a}_{lj} + t^{a}_{il}t^{a}_{jk} = 0
\] (1)

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If nonzero, $T_{SO} - T_G$ can generate massless states.

This is true if $G_{I(R)} \subset SO(\text{dim } R)_1$ conformally embeds.

Here,
\[ T_{SO} = -\frac{1}{2} : \psi \partial \psi : , \quad T_G = \frac{1}{2(I(R) + h)} : J^a J^a : , \quad J^a = : \psi t^a \psi : \]
List of Gapped Theories

1. Take any $G$ with $R = \text{Adjoint representation}$
2. $G = S(U(N) \times U(M)), \quad SO(N) \times SO(M), \quad Sp(N) \times Sp(M)$
   with $R = (\text{fund, fund})$
3. $G = U(N), \ SO(N), \ Sp(N)$ with $R = \text{rank 2 (sym or alt)}$.
4. Isolated Theories: $(F_4, \psi_{26}), \ (\text{Spin}(9), \psi_{16})$
5. Combinations of above
List of Gapped Theories cont.

| $g$                          | $R$ | $g$                          | $R$ |
|------------------------------|-----|------------------------------|-----|
| $\forall g$                 | adjoint | su(2)                      | 5   |
| so($N$)                      |      | so(9)                       | 16  |
| u($N$)                       |      | $F_4$                       | 26  |
| so($N$)                      |      | sp(4)                       | 42  |
| sp($N$)                      |      | su(8)                       | 70  |
| u($N$)                       |      | so(16)                      | 128 |
| u($N$)                       |      | so(10) + u(1)               | 16  |
| su($M$) + su($N$) + u(1)     |      | $E_6$ + u(1)                | 27  |
| so($M$) + so($N$)            | (□,□) | su(2) + su(2)               | (2,4) |
| sp($M$) + sp($N$)            | (□,□) | su(2) + sp(3)               | (2,14) |
|                             |      | su(2) + su(6)               | (2,20) |
|                             |      | su(2) + so(12)              | (2,32) |
|                             |      | su(2) + $E_7$               | (2,56) |
**Examples**

\[ G + \text{Adj} \rightsquigarrow 2^{\text{rank}(G)} \text{ vacua: } \quad \text{IR coset given by } \frac{\text{SO}(\text{dim } R)_1}{G_h} \]

\[ \text{SU}(2) + \psi_7 \rightsquigarrow \text{gapless: } \quad \text{IR coset given by } \frac{\text{SO}(7)_1}{\text{SU}(2)_{28}} \]

i.e. fermionic tricritical Ising, \( c = \frac{7}{10} \).

\[ G + N_f \psi_{\text{fund}} \rightsquigarrow \text{gapless: except } G = \text{SO}(N), \text{U}(N), \text{and } N_f = 1 \]

theory is gapped rest are gapless:

IR coset coset given by \( \frac{\text{SO}(\nu N_f N)_1}{G_{N_f}} \), \( \nu = \begin{cases} 1 & \text{orthogonal} \\ 2 & \text{unitary} \\ 4 & \text{symplectic} \end{cases} \)
There is an algebraic description of gauged WZW as GKO cosets for $G/H$. 

Branching functions

$\chi_g(q) = \sum_{h} b_{h}^{g}(q) \chi_h(q)$

If gapped: $b_{h}^{g}(q)$ does not depend on $q$ this just gives us a count of the vacua.

If gapless: $b_{h}^{g}(q) \sim b_0 + b_1 q + b_2 q^2 + \cdots$ counts states at $L_0 = n$.

From the coset, one can derive modular data, scaling dimension of operators, OPEs, etc.
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We gave a list of \((G, R)\) QCD theories that are gapped/gapless

- Low energy theory is a TQFT or a CFT and the description is via \(SO/G\) gauged WZW (which is a rational CFT).
- There are cases where we can identify the theory as a minimal model
- It is possible to access dynamical data of the QCD theory from the RCFT.
Fin.