Ferromagnetic nano-structures in Valenta model

I. Zasada∗ and B. Busiakiewicz

Solid State Physics Department, University of Lodz,
ul. Pomorska 149/153, Lodz, Poland

(Dated: January 9, 2009)

Abstract

The ferromagnetic nano-structures are recently of great interest for modern investigations. A comparison of the experimental data and theoretical results shows that the use of the standard molecular field approximation is insufficient for the description of nano-structure properties. Therefore, we use the effective field approach in order to show the usefulness of the Valenta model generalized in this way. The agreement between experiment and theory is then excellent.

The magnetization profiles and the calculated Curie temperatures are presented for the systems consisting of Ni and Co layers with different configuration of the surfaces and interfaces including terraces and wires. We have shown that the position in the system as well as the kind of neighbouring layers and their mutual interactions can determine the shape of magnetization profiles. The use of the Valenta model allows us to present all dependences in the layer resolved mode.

PACS numbers:

∗Electronic address: IZASADA@wfis.uni.lodz.pl
I. INTRODUCTION

A recent development of nanotechnologies needs still more actual and more effective methods for the description of nano-structures. In this case the ultra-thin films are very convenient systems in order to describe their properties as a starting point towards the nano-objects.

In the above circumstances the description of magnetic thin films is of fundamental significance from the theoretical point of view because of their role in the interpretation for the thermodynamics applied to inhomogeneous media and for the quantum mechanics of objects with restricted dimensions.

The problem of the thin films theory is rather old, but the model introduced by Valenta fifty years ago [1, 2] seems to be still an excellent and effective approach for the characterization of the magnetization properties and the phase transition temperature. The model formally equivalent to molecular field approximation is based on broader physical background and represents an original interpretation from the physical point of view. The model in its extended form allows us to discuss not only the magnetic properties but also the lattice thermodynamic [3], order-disorder [4], electronic phenomena [5] as well as the phase transitions in the context of the stability and surface melting [6].

Recently, a new attempt at the study of different systems and phenomena within the extended Valenta model has been made.

First of all, different properties of $AB_3$ binary alloy thin films have been discussed. The long-range order parameter and the concentration distribution, mutually dependent and dependent on the film thickness and the boundary conditions at the surfaces have been analyzed [7, 8]. An interpretation of the disordering kinetics has been proposed in connection with the behaviour of disorder which appears as layer by layer process starting form the surface plane or from the sample inside in dependence on the boundary conditions. An interdependence between the surface melting and the surface disordering observed in binary alloy thin films has been considered in the context of their mutual relations [9, 10]. The presented approach allowed to extend the phase transitions diagrams to the case when the crystallinity parameter behaviour influences the local concentration profiles and the lattice order parameter describing the alloy structure. In particular, the surface-induced disorder was described when the crystal structure is preserved and, in contrast, when the surface
melting is expected for partially disordered samples. The behaviour of the magnetization in the binary alloy thin films with respect to their lattice disorder and the relative concentration of alloy components has been also discussed [11]. An interesting fact is observed when the magnetic order appears for the lattice disorder, i.e. for higher temperatures while there is no magnetization for the lattice-ordered state. The order-disorder phenomena including the distribution of chemical component has been considered as well in terms of the short-range order in binary alloy thin films whose surfaces play an essential role [12]. Among others, two effects seem to be of particularly great interest, namely: the crossover of the site occupancy in the surface layer and the shift of atoms in the surface plane between two kinds of lattice sites. Such effects influence the diffuse low-energy electron diffraction, surface melting or spin-wave resonance conditions.

Next, different levels of extensions for the original Valenta model have been proposed in connection with its applicability for magnetic thin films description. The most important one is connected with the improvement of the entropy construction in the self consistent way when the correlations are taken into account [13]. Another kind of extension is connected with introduction of the reaction field instead of the standard molecular field. The procedure for the reaction field approach (RFA) consists in adding to the molecular field a correlation dependent term determined by means of the fluctuation-dissipation theorem. The part of the effective field arising from the reaction field does not favor one orientation over another while the molecular field is directed along the spontaneous magnetization axis. The Valenta model generalized in such a way has been applied for the description of ferromagnetic films separated by the nonmagnetic spacer forming the multilayer systems [14].

In the present paper, we show that the Valenta model can be also successfully applied to different ferromagnetic nano-structures. We start with ultra-thin multilayer systems, next we consider the systems with terraces and at the end we discuss the wires properties in connection with different environment conditions.

II. THE BACKGROUND OF THE VALENTA MODEL

The model introduced by Valenta in its original formulation is formally equivalent to the molecular field approximation, and for this reason, it is at present treated as an insufficient approach for the precise description of thin films properties. However, taking into
considerations the original assumptions which are based on the deep physical background, we can modify the model in various aspects with respect to the purposes of the expected interpretations.

The Valenta model \[1, 2\] consists in two assumptions:

- the discretization of sample geometry which reflects the crystallographic lattice structure and some surface irregularities connected with the nearest neighbours for each of the surfaces. The discrete structure of variables, in particular, the distances between the neighbouring planes in the film thickness direction, leads to the formulation of the equations describing the film in the form of the difference equations with properly chosen boundary conditions.

- the thermodynamics modified for inhomogeneous systems. It turns out that a sample with thin film geometry is such an inhomogeneous system from the thermodynamic point of view, so it should be considered in terms of thermodynamics modified for small clusters \[15\] or nano-structures \[16\].

A film can be treated as a sample cut in some crystallographic orientation with respect to the surface of the crystal with a given crystallographic structure characterized by the spectrum of the nearest neighbouring atoms. In this case the atoms situated at the surfaces have their neighbourhood which is different with respect to the considered site vicinity inside a sample. The geometric situation corresponds then to the different conditions in which the atoms at the surface and the atoms inside a sample are embedded. However, the film can be divided into monoatomic layers parallel to the surface planes from which a layer in two dimensions can be treated as thermodynamically homogenous subsystem. From the thermodynamic point of view a film is then interpreted as a composition of homogenous subsystems in the sense of Néel sublattices \[17\]. The Néel idea consists here in the division of the system into several groups, sublattices, which form the homogenous subsystems gathering the physically equivalent particles, i.e., equal particles accruing in the physically identical conditions. The Néel idea applied by Valenta to thin films can be interpreted that the inhomogeneous system can be divided into homogenous subsystems which are identical with monoatomic layers parallel to the surfaces, however, embedded into different neighbourhood.

The division of a thin film into homogeneous sublattices which are monoatomic layers parallel to the surface implies in terms of quantum mechanics the assumption \[1, 2\] that the total wave function of the electrons of a thin film practically differs very little from the wave function of the state in which the components of magnetic moments in the monoatomic
layers have well defined values. In these conditions, however, the model corresponds to the situation when the Hamiltonians of the subsystems do not commute with the Hamiltonian of the system. Although the commutative rules can only be approached, they are satisfied in a very simple case when the interactions between the sublattices are determined by means of the effective field.

Thus, at high temperature, the quantum mechanical construction influences the thermodynamic interpretation. The Hamiltonian of each Nel sublattice corresponds to the integral of motion for each layer. This means that the total wave function of the state is the product of wave functions with respect to an individual sublattice. From the thermodynamic point of view this fact is equivalent to the factorization of the partition functions. The total partition function is factorized with respect to the partition functions of individual monoatomic layers. The statistical operator of a film is then a product of the statistical operators of layers due to the additive character of the effective Hamiltonians. As a consequence, the entropy is a sum of terms describing homogeneous contributions of monoatomic layer entropies independent of one another in calculations. At low temperatures the construction of a Heisenberg type quantum mechanical theory leads to the solution which can be related to the spin waves propagation or, in the magnon representation, to the quasi free particles when they are embedded in the heat bath of harmonic oscillators. This level of approximation corresponds to the case when the transversal correlations between spins are neglected. Then, the Hamiltonian is reduced to the Ising type Hamiltonian whose longitudinal correlations reduce to the MFA results. In the thin films geometry the direction perpendicular to the surface plane is distinguished in a natural way by breaking the translational symmetry whose perturbation gives the conditions in which the size effects can be observed. One of the fundamental characteristics of the considered model is a discretization of variables, at least, in the film thickness direction. The geometrical properties of a film lead to their description by means of the variational procedure which should be considered in the discrete space. The variational equations are then of difference, not differential, forms.

The thermodynamic approach is, in general, based on the free energy functional construction:

$$F = U - TS$$

which can be obtained by means of the internal energy $U$ and the entropy $S$ calculations. The internal energy $U$ is defined by the mean value of the Hamiltonian describing the
considered system while the entropy in the pair representation is then given in the standard form \[13, 18\]:

$$
S = \sum_{\nu} \sigma_{\nu} + \sum_{<\nu\nu'>} (\sigma_{\nu\nu'} - \sigma_{\nu} - \sigma_{\nu'})
$$

(2)

where

$$
\sigma_{\nu} = k_B \left[ \left( \frac{1}{2} + m_{\nu} \right) \ln \left( \frac{1}{2} + m_{\nu} \right) \\
+ \left( \frac{1}{2} - m_{\nu} \right) \ln \left( \frac{1}{2} - m_{\nu} \right) \right]
$$

(3)

We can see from (2) that the entropy has then two contributions. One of them corresponds to the single-site entropy:

$$
S_1 = N^2 \sum_{\nu} [1 - z(\nu)] \sigma_{\nu}
$$

(4)

for \(z(\nu)\) denoting the number of nearest neighbours when the central site lies in the plane \(\nu\), namely \(z(\nu) = \sum_{\nu'} z(\nu, \nu')\) where \(z(\nu, \nu')\) stands for the number of nearest neighbours in the plane \(\nu'\) when the central site is in the plane \(\nu\). \(N\) stands for the number of spins in the linear dimension of the plane (\(nN^2\) denotes the number of the spins in the system consisting of \(n\) layers).

The contribution of pair entropy can be written as:

$$
S_2 = S_2^0 + S_2^1
$$

(5)

where, the first term contains the contribution introduced in the planes \(\nu\), i.e. in the homogeneous subsystems, namely:

$$
S_2^0 = \frac{1}{2} N^2 \sum_{\nu} z(\nu, \nu) \sigma_{\nu\nu}
$$

(6)

and, the second one contains the contribution introduced by the interactions between two monolayers, i.e. the interactions between subsystems embedded into inhomogeneous bath, namely:

$$
S_2^1 = \frac{1}{2} N^2 \sum_{\nu\nu'\neq\nu} z(\nu, \nu') \sigma_{\nu\nu'\neq\nu}
$$

(7)
The entropy in each monoatomic layer, following the calculations for homogenous system, is of the form

\[ \sigma_{\nu\nu} = k_B \left[ \left( \frac{1}{4} + m_{\nu} + c_{\nu} \right) \ln \left( \frac{1}{4} + m_{\nu} + c_{\nu} \right) 
+ 2 \left( \frac{1}{4} - c_{\nu} \right) \ln \left( \frac{1}{4} - c_{\nu} \right) 
+ \left( \frac{1}{4} - m_{\nu} + c_{\nu} \right) \ln \left( \frac{1}{4} - m_{\nu} + c_{\nu} \right) \right] \] (8)

while, the entropy contribution given by the interlayer interactions leads to:

\[ \sigma_{\nu\nu', \neq \nu} = k_B \left[ \left( \frac{1}{4} + \frac{1}{2} (m_{\nu} + m_{\nu'}) + c_{\nu\nu'} \right) \times 
\times \ln \left( \frac{1}{4} + \frac{1}{2} (m_{\nu} + m_{\nu'}) + c_{\nu\nu'} \right) + 
\times \ln \left( \frac{1}{4} + \frac{1}{2} (m_{\nu} - m_{\nu'}) - c_{\nu\nu'} \right) + 
\times \ln \left( \frac{1}{4} + \frac{1}{2} (m_{\nu} - m_{\nu'}) - c_{\nu\nu'} \right) + 
\times \ln \left( \frac{1}{4} - \frac{1}{2} (m_{\nu} - m_{\nu'}) + c_{\nu\nu'} \right) + 
\times \ln \left( \frac{1}{4} - \frac{1}{2} (m_{\nu} + m_{\nu'}) + c_{\nu\nu'} \right) \right] \] (9)

When we consider the quantities \( \sigma_{\nu}, \sigma_{\nu\nu}, \) and \( \sigma_{\nu\nu', \neq \nu}, \) the factorization of the statistical operator is evident for \( \sigma_{\nu} \) and \( \sigma_{\nu\nu} \) while the factorization for \( \sigma_{\nu\nu', \neq \nu} \) is not obvious because of the interactions between the sites localized perpendicularly to the monoatomic layers. It is worth-while to notice here that the factorization of the statistical operator is equivalent to the additive character of the entropy. The simplest illustrative example confirming the above statement can be given by the assumption that \( c_{\nu\nu', \neq \nu} = m_{\nu} m_{\nu'} \) which leads to the factorized term (9) in a self-consistent way. Taking into account the above assumption we can consider the pair entropy (6) only in the planes, while the entropy contribution due to the nearest neighbouring layers interactions is reduced to the single-site entropy term.

The equilibrium values of the layer dependent magnetic order parameters \( m_{\nu} \) are obtained by the variational procedure minimizing the free energy (1) with respect to the parameters \( m_{\nu} \). The obtained set of equations can be solved numerically leading to the layer and
temperature dependent magnetization; i.e. its profile across a film as well as the Curie temperature evaluation.

III. NANO-STRUCTURES IN THE VALENTA MODEL

Originally introduced for magnetic films, the Valenta model outlined above is now generalized for different systems in which their inhomogeneity is taken into account. The Valenta model assumes the infinite dimensionality in the surface plane. In this context we can speak about the nano-dimension only in the direction perpendicular to the surface and multilayers consisted of several monoatomic planes can be considered as nano-objects with restricted dimension in one direction. We can, however, introduce one or two edges in the surface plane forming terraces, wires or atomic chains, nano-structures homogeneous only in one direction which is in perfect accordance with the original Neel idea of sublattices. We consider now, the example calculations for the systems mentioned above.

A. Ferromagnetic multilayers

The magnetic properties of multilayer systems consisting of two different kinds of magnetic films separated by nonmagnetic spacer are widely studied from the experimental as well as theoretical point of view [19, 20, 21, 22, 23]. The ferromagnetism in such systems is usually discussed in terms of the temperature dependent long-range order parameter i.e. the spontaneous magnetization. From such dependence the behaviour of the Curie temperature can be derived and analyzed.

It has been theoretically calculated and experimentally shown [14, 19] that for two ferromagnetic Ni and Co films coupled by the indirect exchange interaction $J_{\text{inter}}$ (via the Cu film - Co/Cu/Ni/Cu(100) system) one obtains two different ordering temperatures and we can observe two susceptibility signals [22] one in $T_{C,Co}$ and a weaker one in $T_{C,Ni}^*$. Then, we treat the $T_{C,Ni}^*$ as a quasi-critical phase transition temperature of the system. It has been also shown [24] that such system can exhibits an inverse behaviour. By selecting the appropriate thicknesses of Co, Cu and Ni films, we can observe the lower ordering temperature of Co than the one of Ni.

We present here the numerical calculations for these two different experimental situations
performed in the frame of RFA \[14\]. In this case the internal energy \( U \) is given in the following form:

\[
U = -N^2 z_0 \sum_\nu J_{\nu\nu} m_\nu m_\nu \\
- z_1 \sum_{\nu' \in \nu} \frac{J_{\nu\nu'}}{2} m_\nu m_{\nu'1} \\
- \sum_\nu (K_\nu - \lambda) m_\nu m_\nu \\
- \gamma \sum_\nu H_\nu^z m_\nu
\]  

(10)

and the entropy \( S \) is expressed by equation (2) reduced to its single site representation. \( z_0 \) and \( z_1 \) are the numbers of nearest neighbours of a given atom in the same monoatomic layer and in the neighbouring layers and for fcc(100) structure they are, \( z_0 = 4 \) \( z_1 = 4 \), respectively; \( J_{\nu\nu'} \) represents the exchange integral responsible for the interaction between a given spin and its nearest neighbours in the same magnetic layer (\( \nu = \nu' \)) or in the neighbouring layers (\( \nu' = \nu \pm 1 \)), the index numbers the monoatomic layers of the thin film (\( \nu = 1, ..., n \)). It is convenient to denote \( J_{\nu\nu'} \) as \( J_Ni \) and \( J_Co \) for interior Ni and Co layers while for nickel layers which are in direct connection with copper we put an interface exchange coupling \( J_{Ni}^{interface} \). The Co/Cu interface is not differentiated because it is approximately compensated by the enhancement of the magnetic moment in the topmost layer facing the vacuum \[25\] and we take the same values of the exchange integral for all monoatomic layers forming the Co film. The interaction between the Co and Ni films in the Co/Cu/Ni/Cu(100) system takes place via the interlayer exchange coupling \( J_{inter} \), which depends on the nonmagnetic spacer thickness \[14\]. The anisotropy constant \( K_Ni \) including volume and the surface anisotropy term, can be distinguished as we did in the case of the exchange coupling, namely in each magnetic film we have in-plane anisotropy \( K_Ni \) and \( K_Co \) while at the interface Ni/Cu we have \( K_{Ni}^{interface} \). Additionally, in order to take into account the boundary conditions connected with the surfaces state we consider also a perpendicular anisotropy \( \kappa_S \) which is included in the parameter \( \lambda \) calculations \[14\]. The parameter \( \lambda \) appearing in equation (4) is the correlation parameter characteristic for RFA which is independent of (\( \nu \)) due to symmetry conditions \[26\] and it is assumed to be homogeneous in the sample. This parameter is entirely determined by the crystallographic structure of the considered sample and it is not the additional parameter of the theory. For its calculation one needs only the values of exchange
integrals and anisotropy constants mentioned above and of course the crystallographic data connected with the nearest neighbours distribution.

According to the above discussion and on the bases of the experimental findings [19] we have chosen the following values for the exchange integrals and anisotropy constants: $J_{Ni} = 1.7 \cdot 10^{-21} J$, $J_{Ni}^{interface} = 0.5 J_{Ni}$, $J_{Co} = 3 \cdot 10^{-21} J$, $J_{inter} = -1.82 \cdot 10^{-23} J$, $K_{Ni} = 0.001 \cdot J_{Ni}$, $K_{Ni}^{interface} = 0.001 \cdot J_{Ni}^{interface}$, $K_{Co} = 0.001 \cdot J_{Co}$ and $\kappa_S = -1$.

The schematic view of the discussed multilayer system is presented in Fig. 1. First, we consider the 2ML Co/2ML Cu/4ML Ni/Cu(100). The temperature dependent layer resolved magnetization of this system is shown in Fig. 2a and we can see that for the whole range of temperatures the magnetization of Ni film is much smaller than for Co one, which is caused by the difference in the values of magnetic moments reported for these two materials [19]. If we now select a Co thickness of only 1ML, the ordering temperature of Co is below that of Ni. In Fig. 2b we present the layer-resolved temperature dependent magnetization for the 1ML Co/2ML Cu/4ML Ni/Cu(100) system. It is clearly seen that in this case we observe the quasi-critical temperature for Co layer $T^{*}_{C,Co}$ and the usual phase transition in the Ni film transition temperature $T^{*}_{C,Ni}$. We have, however, to underline that one monoatomic layer deposited on the Cu(100) substrate is determined by different physical conditions than the thin Co film containing 2ML on Cu(100). In the case of 1ML Co we cannot speak any more about magnetic moment compensation at surface and interface and we have to take into account the decrease of the magnetic moments at the interface (32exchange integral from $J_{Co} = 3 \cdot 10^{-21} J$ to $J_{Co} = 1.8 \cdot 10^{-21} J$ [23].

FIG. 1: The trilayer system Co/Cu/Ni/Cu(100) with the exchange couplings and anisotropies used in the theoretical model.
FIG. 2: The layer-dependent magnetization as a function of temperature determined for 2ML Co/2ML Cu/4ML Ni/Cu(100) system (a) and 1ML Co/2ML Cu/4ML Ni/Cu(100) system (b).

Fig. 3 shows the magnetization curves obtained as an average over all Ni and Co layers forming the Ni and Co films in the trilayer systems 2ML Co/2ML Cu/4ML Ni/Cu(100) and 1ML Co/2ML Cu/4ML Ni/Cu(100) in Fig. 3a and Fig. 3b, respectively. In both graphs we also show the average magnetization curve for the Ni film composed of 4ML and embedded in the bilayer system 2ML Cu/4ML Ni/Cu(100). In this case we have different, with respect to the trilayer system, boundary conditions characterizing the whole system surfaces state. In order to take it into account we have put a perpendicular anisotropy of different value, namely \( \kappa_S = -0.1 \). In both trilayer systems we observe the tails of remanent magnetization of Ni film for the first one and of Co film for the second one. The Ni and Co remanent magnetization changes sign at the quasi-critical temperature \( T_{C,Ni}^* \) and \( T_{C,Co}^* \), respectively, due to the antiferromagnetic coupling caused by the copper spacer of 2 ML thickness [14].

The Co film in 1ML Co/2ML Cu/4ML Ni/Cu(100) system has lower ordering temperature than the Ni and its magnetization is vanishing in the presence of ferromagnetic Ni film. Due
to the fact that the ground state moment of Co is larger than the Ni one, the element-
specific $M(T)$ curves for Co and Ni cross each other close to $T_{C,Co}^*$ (Fig. 3b). Such crossover
can be interpreted as connected with the Ni magnetization rotation from parallel (at low
temperatures) to perpendicular (at $T_{C,Co}^*$) and antiparallel (up to $T_{C,Ni}$) to the Co one.

In both considered cases the magnetization of Ni film from the trilayer system is higher
then its magnetization in bilayer system and the Curie temperature is shifted toward the
higher values by $\delta T_{C,Ni} = 40K$ in trilayer with 2ML Co and $\delta T_{C,Ni} = 34K$ in trilayer with
1ML Co. The difference between these two shifts is due to interlayer exchange coupling
$J_{\text{inter}}$ which enhances the Ni magnetization in the case of 2ML Cu/4ML Ni/Cu(100).

The magnetization behaviour depends not only on the composition but also on the thick-
ness of a system. It is clearly seen in Fig. 2 and Fig. 3. Thinner system has lower phase
transition temperature. In the case when the system is composed of two films with different
transition temperatures we can expect that if we choose the relative thicknesses of Co and
Ni films in a way that they have the same critical temperature the trilayer system will ex-
hibit only one common Curie temperature (without quasi-critical one). It is indeed observed
experimentally [24].

B. Ferromagnetic terraces

Let us now consider a simple layered structure with two terraces shown in the insets
of Fig. 4 and Fig. 5. The first terrace is semi-infinite in plane and is composed of 1
monoatomic layer while the second one is large of and is composed of 2 monoatomic layers.
The surface of the second terrace is then restricted in two dimensions and has two different
boundary conditions at the edges. One of its edges is interacting with the first terrace while
another one is exposed to vacuum. Both terraces are supported by several infinite layers
put on-top of non-magnetic material forming Ni/Cu(100) system. In this case the system
is divided in several different homogeneous subsystems and it has to be underlined that 2nd
and 4th monoatomic layers are divided into two different subsystems each. The first group
of subsystems, from 5th to 10th monoatomic layers, consists of infinite planes parallel to
the surface. The second group, from 2nd to 4th monoatomic layers, consists of semi-infinite
planes parallel to the surface. The third group, 1st and 4th monoatomic layers, consists of
semi-infinite surface planes, each of them having, however, different boundary conditions at
FIG. 3: Average Ni magnetization of a bilayer Ni film (dashed line (a) and (b)) and of Ni film (thin solid line) coupled to Co film in 2ML Co/2ML Cu/4ML Ni/Cu(100) system (a) and 1ML Co/2ML Cu/4ML Ni/Cu(100) system (b). The thick solid line ((a) and (b)) refers to the Co magnetization.

And finally the last subsystem, $s^{2^{nd}}$ monoatomic layer, forms the plane infinite in only one dimension having two different boundary conditions for each of its edges. In Fig. 4 we show the temperature dependent layer resolved magnetization for two Ni terraces ($\Delta k = 3$) supported by 7ML Ni film. Such system exhibits one phase transition temperature $T_{sC} = T_{tC} = 346K$ and different magnetization distributions in the direction perpendicular...
FIG. 4: The layer-dependent magnetization as a function of temperature determined for the system with two terraces supported by 7ML Ni in configuration shown in the inset. The second inset shows the magnetization profiles across the first terrace region (curve 1), second terrace region (curve 2) and the rest of a sample (curve 3).

to each of three surface planes (inset in Fig. 4).

Much more interesting situation appears, however, when the same two terraces are supported by only 3 ML of Ni/Cu(100). We observe then two transition temperatures lower one for the Ni support and the second one, higher, for terraced Ni structure. In Fig. 5 we present the results obtained for two widths of the second terrace, $\Delta k = 3$ and $\Delta k = 10$, respectively in Fig. 5a and Fig. 5b. In both cases the Curie temperature for the support has the same value $T_{\text{sc}} = 257\text{K}$ while the transition temperature for the terraced part of the sample depends on the second terrace width being higher for $\Delta k = 3$. Moreover, at the support Curie temperature we can notice the jump of the magnetization value for $m_{t3}$ $m_{t4}$. Above this temperature the magnetization curves $m_{t3}$ and $m_{t4}$ become identical with $m_{t2}$ and $m_{t1}$ magnetization curves, respectively (Fig. 5). We can also notice that the position of magnetization curve $m_{2}$ with respect to the rest of the curves depends on $\Delta k$. For smaller value this magnetization exhibits the dominant role having the highest values in the whole range of temperatures.
C. Ferromagnetic wires

In the frame of the Valenta model a wire can be constructed in two different ways. First of all, it can be composed with the monoatomic planes infinite in one direction with symmetric boundary conditions at both in-plane edges. The size of such wire is then determined by the number of planes and their width. The second way consists in dividing the above described wire in monoatomic chains; it means the homogeneous subsystems infinite in one direction. Fig. 6 shows the temperature dependent magnetization curves calculated for both wire types of $3 \times 3$ size. The Curie temperature is common but the magnetization distribution differs which can be clearly seen in the inset where the average magnetization for three in-plane monoatomic chains is compared with the magnetization of monoatomic planes forming the wire of the first type. We can then discuss which construction is proper. It seems that the monoatomic chains give a better resolution in magnetization calculations and its distribution is more reliable but of course the experimental results are decisive. In Fig. 7 we present the temperature dependent average magnetization calculated for three nanostructures: free-
FIG. 6: The temperature dependent magnetization determined for two types of wires: (a) constructed with monoatomic planes and (b) constructed with monoatomic chains. The inset shows the magnetization profiles for both systems (see description in the text).

standing wire composed with 9 monoatomic chains in 3 x 3 arrangement; the same wire deposited on 1Ml of Ni; the same wire deposited on 1Ml of Cu. The temperature dependent magnetization of Ni bulk is shown for comparison and the inset shows the low-temperature magnetization behavior for Ni nanowires examined in three different environments. The dependence of the magnetization of Ni nanowire is qualitatively equivalent to the magnetic properties of Ni nanowires electrodeposited into self-assembled alumina arrays reported in paper [27].

IV. CONCLUDING REMARKS

The interest of various investigations and applications of ferromagnetic nano-objects in nanotechnology is a factor stimulating the development of their theoretical description in which the methodology plays an essential role. In the methodological context, the model
FIG. 7: The temperature dependent magnetization determined for three nanostructures: free-standing wire (b); the same wire deposited on 1ML of Ni (c); the same wire deposited on 1ML of Cu (a). The inset shows the low-temperature magnetization behavior for all Ni nanowires of magnetic thin films introduced by Valenta [1, 2] seems to be one of the most important methods which respect the thermodynamics of inhomogeneous systems formulated in the Nel representation.

It is worth while to mention that the Valenta model is of the Heisenberg type treatment. By this fact, it allows us to consider the spin wave description in particular, the spin wave resonance as well as the magnetization at low temperatures. In order to discuss the high temperature behaviour of magnetization, the effective molecular field is applied. Usually this approach reduces the hamiltonian to its Ising forms. However, the use of the reaction field approximation (RFA) takes also the transversal components of the spin operators into account. In this context the considerations presented in [14] and applied in the present paper bring the background for the nano-objects treatment in terms of RFA method from the point of view of their entropy formulation which should be then taken at least in the pair representation. The RFA approach is very sensitive for the description of systems with restricted dimensions due to the anisotropic character of the basic hamiltonian. The isotropic case corresponds then to the phase transition temperature equal to zero in agreement with the Mermin-Wagner theorem predictions.

Similar evaluations can be discussed in the case of correlations introduced by the constant coupling approach [28], the Bethe-Peierls-Weiss method [29], as well as the Green function
technique which is common for low and high temperature regions, leading to the magnetization profiles found in a self-consistent way. We have shown that the equivalence between the results of the Valenta model and the Green functions approach in the random phase approximation for the Ising model is evident as their property of general character. The extension of the Valenta model by the introduction of the reaction field correction corresponds to the situation in which the spin correlations are taken into account. Although this idea is common for the considerations within the Valenta model and the Green functions approach, as far as the physical interpretation is discussed, the results are now not identical due to the different description of the magnetization profiles.

Originally, the Valenta model is constructed for the homogeneous angular distribution of magnetic moments oriented in agreement with the quantization axis. We can observe, however, two orientations at least, parallel and perpendicular to the surface. Taking into account this fact, we can extend the Valenta model to the discussion concerning the angular distribution of magnetization in terms of the variational procedure derived in the discrete space. We obtain then not only two orientations of homogeneous distribution but also their inhomogeneous distribution among the sublattices.

An important generalization of the Valenta model is still expected in the characterization of the surfaces. The surface roughness and surface alloying are among the most interesting phenomena to be considered within the Valenta model. The surface roughness is a determinant of the surface texture and it appears in every scale. The spontaneous formation of defects in atomic scale on the solid surface is expected for purely entropic reasons. The study of these defects is fundamental for the surface science as well as for understanding of crystal growth, catalysis and diffusion mechanisms. The surface roughness description based on the local statistical identities leads to physically interesting properties of the surface in the context of surface phase transitions. Thus the topical use of the Valenta model is also connected with the interpretation of the STM image.

At the end, it is worth while to notice that the thermodynamic approach to the description of nano-structures within the Valenta model gives the information about the temperature dependence of diverse order parameters describing the inhomogeneous system as well as allows to distinguish the properties of any particular homogeneous subsystem which can be defined on the basis of Néel concept of sublattices. Moreover, the Valenta model construction shows the predictions for the methodological formulation of the thermodynamic of inhomogeneous
small systems.

The particular results considered in the present paper provide an illustrative example whose generalization seems to us to have a general meaning for the discussion of the methods and their comparison at different levels of accuracy.

Acknowledgments

The authors are grateful to Professor Leszek Wojtczak, a close friend and coworker of Professor Lubo Valenta, for his stimulating discussions during the study on Valenta model.

[1] L. Valenta, Czech. J. Phys. 7, 127 (1957).
[2] L. Valenta, Czech. J. Phys. 7, 136 (1957).
[3] L. Wojtczak, S. Zajac, Bull. Acad. Polon. Sci. 16, 527 (1966).
[4] L. Valenta, A. Sukiennicki, Phys. Stat. Sol. 17, 903 (1966).
[5] L. Valenta, L. Wojtczak, Czech. J. Phys. B30, 1025 (1980).
[6] L. Valenta, Czech. J. Phys. 46, 607 (1996).
[7] F. L. Castillo Alvarado, A. Sukiennicki, L. Wojtczak, I. Zasada, Physica B 344, 477 (2004).
[8] I. Zasada, L. Wojtczak, S. Mrz, J. Alloys Compd. (2008), doi:10.1016/j.jallcom.2008.02.015.
[9] L. Wojtczak, I. Zasada, A. Sukiennicki, F. L. Castillo Alvarado, Phys. Rev. B 70, 195416 (2004).
[10] L. Wojtczak, I. Zasada, T. Rychtelska, Surf. Sci. 600, 851 (2004).
[11] A. Sukiennicki, L. Wojtczak, I. Zasada, F. L. Castillo Alvarado, JMMM 288, 137 (2005).
[12] I. Zasada, A. Sukiennicki, L. Wojtczak, F. L. Castillo Alvarado, Phys. Rev. B 74, 205402 (2006).
[13] I. Zasada, B. Busiakiewicz, L. Wojtczak, JMMM 312, 58 (2007).
[14] B. Busiakiewicz, I. Zasada, L. Wojtczak, J. Phys.: Condens. Matter 20, 095217 (2008).
[15] T. L. Hill, J. Chem. Phys. 36, 3182 (1962).
[16] H. Puszkarski, A. R. Ferchmin, Acta Physicea Superficierum 4, 55 (2001).
[17] L. Nel, Ann. de Phys. 3, 137 (1948).
[18] T. Balcerzak, Physica A317, 213 (2003).
[19] A. Scherz, C. Sorg, M. Bernien, N. Ponpandian, K. Baberschke, H. Wende, P. J. Jensen, Phys. Rev. B 72, 054447 (2005).
[20] A. Ney, F. Wilhelm, M. Farle, P. Pouloupolous, P. Srivastava, K. Baberschke, Phys. Rev. B 59, R3938 (1999).
[21] F. May, P. Srivastava, M. Farle, U. Bovensiepen, H. Wende, R. Chauvistre, K. Baberschke, J. Magn. Magn. Mater. 177-181, 1220 (1998).
[22] U. Bovensiepen, F. Wilhelm, P. Srivastava, P. Pouloupolous, M. Farle, A. Ney, K. Baberschke, Phys. Rev. Lett. 81, 2368 (1998).
[23] P. J. Jensen, K. H. Bennemann, P. Pouloupolous, M. Farle, F. Wilhelm, K. Baberschke, Phys. Rev. B 60, R14994 (1999).
[24] A. Scherz, F. Wilhelm, P. Pouloupolous, H. Wende, K. Baberschke, J. Synchrotron Rad. 8, 472 (2001).
[25] Ney A, Pouloupolous P and Baberschke K Europhys. Lett. 54, 820 (2001).
[26] M. A. Gusmao, C. Scherer, phys. stat. sol. (b) 92, 595 (1979).
[27] M. Zheng, L. Menon, H. Zeng, Y. Liu, S. Bandyopadhyay, R. D. Kirby, D. J. Sellmyer, Phys. Rev. B 62, 12282 (2000).
[28] L. Wojtczak, J. de Phys. 30, 578 (1969).
[29] J. Pearson, Phys. Rev. A 138, 213 (1965).
[30] W. Brodhorb, W. Haubeureisser, phys. stat. sol. (b) 8, K21 (1965).
[31] L. Valenta, W. Brodhorb, W. Haubeureisser, phys. stat. sol. (b) 26, 191 (1968).
[32] B. Busiakiewicz, I. Zasada, Phys. Rev. B 78, 165412 (2008).