Damage assessment through changes in mode shapes due to non-proportional damping

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Abstract. Modal parameters are often used for the structural damage assessment in the dynamic field. Usually, the changes in the modal parameters between different states are assumed as measures of damage. The frequencies are easy to identify, but in some circumstances they are not sensitive to damage and moreover they are mainly a global measure. On the contrary, the mode shapes are more suited for damage localization, but they are generally hard to identify accurately. The energy dissipation, and hence the damping, increases with damage. This feature is stable and has a monotonic behaviour, therefore, damping can be confidently used as an alternative or complementary measure for damage assessment in spite of the accuracy of its identification. However, the damping by itself suffers of the same drawbacks as the frequencies. The joint use of damping and mode shapes is an effective procedure for the damage identification. In the real world the damping is of non-proportional type and the measured mode shapes are complex. It is assumed that an increase of damage causes a modification of non-proportional damping and a variation of the modal complexity. The extent of modal complexity between two different structural states can be used to identify the damage through appropriate indicators. A number of such indicators is introduced and discussed. The effectiveness and sensitivity of the damage indicators are tested on theoretical and pseudo-experimental data.

1. Introduction

Damage assessment is an important research area in structural engineering. In the past, it has received considerable attention, especially in the dynamic field, from the pioneering work [1]. This subject, although well established, still rises interest [2] due to continuous developments in signal analysis and experimental identification techniques. The basic idea arises from the notion that the modal parameters are functions of the physical properties of the structure. Therefore, changes in the physical properties cause changes in the modal parameters. The changes in the modal parameters are assumed as damage measures.

Changes in natural frequencies have been the topic of numerous research studies due to the ease of their identification [3]. These studies have revealed that frequency changes can suffer of practical limitations in several applications, especially in the case of large structure. It is highlighted that detectable frequency changes could require high damage level or global damage.
Mode shapes are more suited for damage localization than frequencies [4], but did not receive as much consideration because their determination requires the simultaneous measurement of vibrations in a high number of points together with the use of more refined identification techniques.

Damping has seldom been used for damage assessment. This is due to the existence of multiple damping mechanisms and the large scatter in estimating damping values. A review of the literature, however, suggests that damping may prove, in some cases, to be more advantageous than detection schemes based on frequencies and mode shapes alone. Among the others, use of damping for damage detection is mentioned in several applications such as: cracked elements [5], cyclic fatigue [6], composite materials [7] and structures [8] and [9]. In the last paper damage assessment is provided by proper damage indicators based on the effects of damping on the mode shapes rather than damping itself.

This paper aims to review and deepen the approach followed in [9] by testing the effectiveness and sensitivity of these indicators. The basic assumption consists of a direct relationship among the damage increase, the energy dissipation increase and the damping increase. No account is made for the actual damping mechanism and an equivalent non-proportional viscous damping is considered. Although simplified, this model allows to capture the complex nature of the mode shapes of real structures, provided the assumption of proportional damping is relaxed. In the proposed approach the complexity level of the mode shapes (i.e. the non-proportional entity) is assumed as a measure of damage and the damage is quantified by comparing different structural states. The mode shapes are not compared directly, but through appropriate synthetic measures of the modal complexity: the damage indicators.

The paper is organized in four sections. In the first section the damage modelling and its relations to modal complexity are discussed together with the damage indicators. In the second section the analysis methodologies are introduced along with a case study. In the third section sensitivity analyses of the damage indicators are carried out using theoretical data obtained through standard modal analysis. In the fourth section the most sensitive indicator is used to evaluate the damage through the analysis of pseudo-experimental data.

2. Damage modelling and indicators
A general equivalent viscous damping capable to encompass different damping mechanisms is considered. This type of damping can be proportional (real mode shapes) or non-proportional (complex mode shapes) depending whether the system matrices satisfy the conditions expressed in [10]. In the case of framed civil structures, damage is mainly due to the formation of plastic hinges. The local constitutive relations show energy dissipation because of material hysteresis. The energy loss can be predicted by an equivalent dissipation mechanism based on viscous damping. The three quantities (damage, hysteresis and equivalent damping) are related by one to one relations at least for moderate strains and ductility. In this sense, an increase of damage causes an increase of hysteresis and consequently an increase of equivalent damping. However, the hysteretic behaviour does not lead only to energy loss, but involves complex interactions between loss of stiffness and energy dissipation. In order to make clear the effects of these two contributions they are idealized and studied by means of two elementary and independent contributions. The two elementary contributions are modeled through changes in the coefficients of the stiffness \( \mathbf{K} \) or the damping \( \mathbf{C} \) system matrices (constant mass matrix \( \mathbf{M} \)). When the damage causes a reduction of stiffness then it is modeled assuming a reduced stiffness matrix \( \mathbf{K}_r \) with \( \mathbf{C} \) kept constant. On the contrary, when the damage causes an energy dissipation then it is modeled assuming an increased damping matrix \( \mathbf{C}_r \) with \( \mathbf{K} \) kept constant. In either case the final effect is an increase of the fraction of non-proportional damping compared to the initial one (undamaged structure) and hence of the extension of the modal complexity.

In the literature several indices have been proposed to quantify the extension of the modal complexity [9], [11] and [12]. The difference between the values assumed by a particular damage indicator in two different structural states quantifies the damage occurred. In this paper five of the above indices are assumed as damage indicators:
The effectiveness and sensitivity of the five damage indicators are analysed via a 3dofs spring-dashpot system representative of basic models of framed structures. The structure is a single span three floors undamaged state, is characterized by a tridiagonal stiffness matrix \( K \), a diagonal mass matrix \( M \) and a damping matrix \( C = \beta K \) (\( \beta \) is a proportionality constant).

The initial frequencies and mode shapes are: \( f_1 = 2.13 \) Hz, \( f_2 = 5.96 \) Hz, \( f_3 = 8.61 \) Hz and \( X_1 = \{0.45, 0.80, 1.00\}, X_2 = \{1.00, 0.45, -0.80\}, X_3 = \{-0.80, 1.00, -0.45\} \). \( \beta \) is equal to 0.0075 so that the damping ratios are 5%, 14%, 21% respectively for the first, second and third mode (Figure 1b).

\[
I_1 = \frac{\| \text{Im}(X_j) \|}{\| \text{Abs}(X_j) \|}; \quad I_2 = \frac{\text{Abs}(\text{Re}(X_j)^{\dagger} \text{Im}(X_j))}{\sqrt{\text{Re}(X_j)^{\dagger} \text{Re}(X_j) \text{Im}(X_j)^{\dagger} \text{Im}(X_j)}}; \quad I_3 = \frac{\sum_{j=1}^{N} \text{Abs}(\text{Im}(X_{ij}))}{N}; \quad I_4 = \frac{\text{Abs}(\phi_{i,max}) - \text{Abs}(\phi_{i,min})}{\pi}; \quad I_5 = \frac{A_i}{A_{i,max}}.
\]

The \( I_1 \) (modal imaginary ratio) indicator is taken from [9], the \( I_2 \) (modal collinearity) and \( I_3 \) (modal polygon area) indicators are taken from [11] whereas the \( I_4 \) (modal phase difference) and \( I_5 \) (modal polygon area) indicators derive from [12]. The symbols have the following meaning: \( X_j \) is the \( i \)-th complex mode shape; \( X_{ij} \) is the \( j \)-th component of \( X_i \); \( \phi_{i,max} \) and \( \phi_{i,min} \) are the maximum and minimum phase angle of the \( i \)-th mode shape; \( A_i \) is the modal polygon area of the \( i \)-th mode shape and \( A_{i,max} \) the maximum modal polygon area; \( \| \| \) is the Euclidean 2-norm operator and \( \text{Abs}(\cdot) \) is intended as the componentwise absolute value; \( N \) is the number of degrees of freedom (dofs) of the system.

The damage is essentially related to the imaginary part of the mode shapes, it is hence advisable to normalize, in some sense, the mode shapes before their insertion into the formula of the damage indicators. The normalization is carried out using the procedure proposed in [11], and it is equivalent to minimize the imaginary part of the mode shapes. Geometrically, it is equivalent to rotate the complex mode shape such that it optimally aligns with the real axis in the complex plane.

The meaning of the indicators is as follows: the \( I_1 \) indicator weights the importance of the imaginary part with respect to the overall length of the complex mode shape. The \( I_2 \) indicator is based on the fact that the degree of interdependence of the real and imaginary parts of a complex mode shape is directly affected by the damping proportionality: the higher the damping proportionality, the more correlated the real and imaginary parts of the mode shapes are; if the imaginary parts of a complex mode shape are completely dependent on the real parts, the damping is proportional. The \( I_3 \) indicator measures the degree of scatter of the complex mode shape that is directly related to the amplitude of the imaginary parts. The idea behind the \( I_4 \) and \( I_5 \) indicators relies on a geometric interpretation. If the mode shape components are plotted in the complex plane, the effects of the non-proportional damping become apparent. The \( I_4 \) indicator considers the phase differences between the dofs of a mode shape as a consequence of the non-proportionality of damping. In fact, each component of a mode shape of a system endowed of proportional damping lies on a straight line, whereas those of system endowed of non-proportional damping do not; in effect, these latter exhibit an angular dispersion equivalent to the phase differences. If the individual components of a mode shape are connected by straight lines, an \( N \)-side polygon is formed. If the damping is proportional the components of a mode shape lie on a straight line and the polygon has zero area. As the non-proportionality of the damping increases, the area of this polygon also increases. The \( I_5 \) indicator measures the area of this polygon.

3. Case study and methodology

The effectiveness and sensitivity of the five damage indicators are analysed via a 3dofs spring-dashpot system representative of basic models of framed structures. The structure is a single span three floors shear-type frame (Figure 1a). Linear degrading behaviour and viscous damping are assumed. The inter-storey height is 3 m and a mass of 20 t is condensed at each floor. The columns have a constant cross-section 30 by 30 cm² and a Young modulus \( E = 3 \times 10^7 \) kN/m². The system, in its initial undamaged state, is characterized by a tridiagonal stiffness matrix \( K \), a diagonal mass matrix \( M \) and a damping matrix \( C = \beta K \) (\( \beta \) is a proportionality constant).

The initial frequencies and mode shapes are: \( f_1 = 2.13 \) Hz, \( f_2 = 5.96 \) Hz, \( f_3 = 8.61 \) Hz and \( X_1 = \{0.45, 0.80, 1.00\}, X_2 = \{1.00, 0.45, -0.80\}, X_3 = \{-0.80, 1.00, -0.45\} \). \( \beta \) is equal to 0.0075 so that the damping ratios are 5%, 14%, 21% respectively for the first, second and third mode (Figure 1b).
For simplicity, the initial reference state coincides with the undamaged structure characterized by proportional damping and real mode shapes, so that the damage indicators are zero.

According to the damage modelling of Section 2, two elementary damage cases are studied: (a) inter-storey stiffness degradation and (b) inter-storey energy dissipation (damping) increase. Further for each case two damage scenarios are studied: localized or diffused. Localized damage involves damage alternatively at the first, second or third floor, whereas diffused damage involves a simultaneous and identical damage at each floor. In the case (a) the inter-storey stiffness $k$ is reduced stepwise up to 50% of the undamaged structure, with the damping matrix fixed at the initial (undamaged) state; similarly, in the case (b) the inter-storey dashpot constant $c$ is increased stepwise up to a maximum damping ratio of 30%, with the stiffness matrix fixed at the initial (undamaged) state. In both cases a damage factor, $\lambda_k \leq 1$ or $\lambda_c \geq 1$, has been applied respectively to $k$ ($\lambda_k k$) or $c$ ($\lambda_c c$) as appropriate.

The main difference between the case (a) and (b) is that in the second case there exists a direct relation between energy loss, damping non-proportionality and mode shapes complexity; whereas in the first case the non-proportionality arises only as an indirect effect of the stiffness loss. It is hence expected that damage related mainly to stiffness loss, case (a), is more difficult to be detected than damage related mainly to energy dissipation, case (b).

In order to work in a controlled environment the damage indicators are tested on theoretical and pseudo-experimental data. The theoretical mode shapes are derived using the standard modal analysis according to the state space method to deal with the non-proportional damping [13]. The pseudo-experimental mode shapes can be derived in principle by any identification technique. In the present context the Complex Plane Representation (CPR) method [14] is used. Briefly speaking, the CPR method is an output-only time domain technique in which the original signal is represented in the complex plane by computing its imaginary counterpart via the Hilbert transform [15]. This new representation makes it very simple to identify the phase shift of the motion between the different measurement points, and therefore it is particularly effective for the identification of complex mode shapes of general viscously damped systems. The method processes the harmonic response close to resonance and it is therefore suited with field applications based on the use of electromechanical actuators, but without the necessity of knowing the applied force, which is rarely measured in the case of civil structures.

4. Sensitivity analysis using theoretical data

4.1. Localized damage

Because of the limited room available only the results relative to damage at the columns of the lower floor are discussed. Broadly speaking, the damage indicators trend depends on the type of structure and damage scenario considered. However, the features shown below by the damage indicators are shared by the different situations that structures can actually exhibit. In this respect, the results can be considered to
have generality. In Figure 2 the damage indicators are reported against the damage factors affecting $k$ ($\lambda_k$ from 1 to 0.5; case a) or $c$ ($\lambda_c$ from 1 to 4; case b) for each of the three structural modes.

4.2. Uniform damage

Uniformly distributed damage is hardly exhibited by structures and damage is more or less always localized. However, it is interesting to analyze such a limit case. In this particular situation the non-proportionality between the damping and stiffness matrix is spatially uniform and equal to the damage factor; in this condition, the mode shapes are very close to the real ones obtained using the proportional damping (i.e. modal complexity is very small) and the damage indicators are practically zero. Thus, in the case of uniform damage the indicators are useless and it is not possible to distinguish between a healthy structure and a very damaged one. Different quantities sensitive to global damage can help to indicate whether damage is present or not. Plots of the damping ratios against the natural frequencies can help to answer the problem, Figure 3.

Figure 2. Damage indicators. Localized damage: case a (left) and b (right).

Figure 3. Damping ratio vs. natural frequency. Diffused damage: case a (left) and b (right).
4.3. Comments of the results

By the inspection of Figure 2 some conclusion can be drawn. Initially and as expected, it must be observed that the indicators are more sensitive to damage related to energy dissipation than damage related to stiffness degradation. This feature preserves regardless the mode order; however, higher order modes are generally more sensitive than lower order modes. This is partly due to the type of damping assumed and partly due to the raise of strain energy involved along with the increase of the mode order. Another note concerns the absolute values of the indicators that are contained also for important damages.

Further, as concerns the damage identifiability and quantification, the main interesting feature shared by all the indicators is their monotonical trend along with the damage increase. This allows to set univocal and unambiguous relationships between the damage level and the indicator value.

Among all, the $I_1$ indicator, that is essentially a direct measure of the modal complexity, is generally and largely the most sensitive. This is a positive result in view of the robustness of the results considered the ease of computation of $I_1$. $I_4$ is the second more sensitive indicator and $I_2$ is the less sensitive one. $I_1$ is therefore the best candidate to deal with modal complexity (damage detection) and is therefore the only one used in the following.

One particular problem arises in the case of uniform damage. In this instance any damage indicator is practically zero, due to the non-proportionality spatially uniform of the damping matrix, and cannot be used to answer the question whether damage is present or not. The representation of Figure 3 can be useful on purpose. If the damage is caused by stiffness reduction (case a) a simultaneous change of the structure frequencies (reduction) and energy dissipation (increase) takes place; whereas if it is caused by energy loss (case b) a significant addition of the energy dissipation is observed with virtually no changes of the frequencies.

5. Pseudo-experimental results

The CPR method [14] is used to identify the complex modal parameters in operating conditions. Two different excitation types are considered: distributed and point excitation. The distributed excitation consists of a forcing function whose profile coincides with mode shape of the undamped and undamaged structure. This configuration is ideal and is considered for reference. The point excitation corresponds to the current practice in civil engineering to shake the structure by means of an electromechanical actuator. For definiteness the point excitation is here assumed located at the top floor. Damage is assumed localized at the columns of the first floor. The results are presented according to the first mode (less sensitive to damage) and to the $I_1$ indicator (most sensitive indicator). The response of the structure is generated in terms of acceleration time histories with a 15% rms added white noise to account for real field conditions. The responses to both excitation types are processed via the CPR method. The identified complex mode shapes are used to compute the $I_1$ indicator.

5.1. Distributed excitation

The Figure 4 compares the theoretical and pseudo-experimental (noise free) results in the presence of distributed excitation for both damage cases (a) and (b). The comparison is carried out via the $I_1$ indicator plotted against the damage factor. In any case no differences are appreciated for the first mode. On the contrary, some deviations are observed for modes two and three. These deviations are lesser in case (a) than in case (b) where some under estimation is observed. Further, apart mode three of case (b), where the differences between theoretical and pseudo-experimental data increase with the damage factor, no such feature is detectable in the other cases. In these cases the deviation is almost constant regardless the damage level so that damage can be reliably detected.
5.2. Point excitation
As for the previous case, the Figure 5 compares the theoretical and pseudo-experimental results in the presence of point excitation for both damage cases (a) and (b). Here again, the comparison is carried out via the $I_1$ indicator plotted against the damage factor. For reference, the results of the distributed excitation are also reported. Only the results relevant to the first mode shape are shown. This is why the first mode proves to be the less sensitive in the present context and hence the one more difficult to deal with. Unlike the previous case, noisy pseudo-experimental data have been now considered. Because of that the pseudo-experimental curves are not straight lines in the Figure 5 as it happens for the theoretical data. Despite of that a good agreement is observed. Therefore experimental dynamic data collected via conventional excitation schemes of civil structures can be confidently used for damage detection purposes.

6. Conclusions
The paper is concerned with the use of the mode shapes complexity for damage assessment purposes. The proposed method is based on the observation that the damage, either due to stiffness degradation or to energy dissipation, causes an increase of the non-proportionality of the system damping matrix. In the above context, complex modes arise naturally and the amount of modal complexity is assumed
as a quantification of the structural damage. In order to get a measure of the modal complexity, several indicators derived from the literature are compared. These indicators are used to transform the identified complex mode shapes into scalar quantities readily usable in one to one relationships with the damage factor. In principle, the identification of the mode shapes can be carried out through any available signal processing procedure. Actually, the CPR method has been used because of two nice properties: it suits well with typical excitation schemes of civil structures, it proves highly robust and reliable. A simple shear-type frame structure is used as a case study. Two distinct damage cases have been considered: stiffness reduction and energy dissipation increase. In both cases all the damage indicators show a quasi-linear and monotonic behavior in the range of feasible damage. The indicators show effective for localized damage, they fail instead in the case of uniformly distributed damage because the mode shapes do not vary despite the damage level. In this case a different strategy should be followed. The best indicator happens to be the simplest one based on the ratio of the amount of the imaginary part to the whole mode vector. This indicator is used with sample examples to show its effectiveness. Theoretical and pseudo-experimental data (either noise free or noise corrupted) are compared along with the different modes excited for different excitation profile (either distributed and mode shape proportional or localized and point applied).

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