Modelling Self-Interacting CDM Haloes with a
Cosmological Boltzmann Code

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ABSTRACT
We investigate the density profiles and evolution of weakly self-interacting cold dark matter haloes using a numerical code based on the collisional Boltzmann equation. This approach is alternative to N-body techniques in following the dynamical evolution of haloes in the cosmological context and taking into account particle self-interaction. The physical case with a cross section inversely proportional to the relative velocity of the colliding particles is modelled with an unprecedented resolution, spanning five orders of magnitude on the radius for each halo. The modelled haloes cover a mass range from dwarf galaxies to galaxy clusters. We find that for $\sigma_{100} \approx 10^{-24}$ cm$^2$/GeV, where $\sigma$ is the cross section per unit mass and $v_{100}$ is the collision velocity in units of 100 km/s, soft cores in good agreement with observations on galactic as well as on galaxy cluster scales are obtained. Remarkably, the observed nearly invariance of the halo central density with mass is reproduced.

Key words: cosmology:dark matter - galaxies:formation - galaxies:haloes, methods:kinetic theory

1 INTRODUCTION
The current Cosmology based on the growth of small fluctuations through gravitational instability in a universe with cold dark matter (CDM) defines a physically well-motivated theory, capable to explain most of the properties of the large-scale structures of the universe. In this scenario, where a large fraction of the matter is supposed non-dissipative, cold and collisionless, the dark haloes form hierarchically via gravitational collapse of primordial density fluctuations. The density profiles of these dark haloes have been studied systematically using N-body simulations (Navarro, Frenk & White 1996, 1997; hereafter NFW). It was found that a simple analytical density profile with one free parameter is able to describe the typical structure of the virialized CDM haloes. On its own, this free parameter, which can be the concentration, depends only on the halo mass.

The NFW density profile goes as $\rho \propto r^{-1}$ at small radii. According to recent high-resolution simulations, the inner density profile of the haloes is even steeper than $r^{-1}$ ($\rho \propto r^{-1.5}$) (Moore et al. 1999; Jing & Suto 2000). This prediction of the CDM scenario is in potential conflict with observations: the halo inner density profiles inferred from the HI rotation curves of dwarf and low surface brightness (LSB) galaxies and from a mass map constructed by strong lensing techniques for the galaxy cluster CL0024+1654 seem to be much shallower than $r^{-1}$ (Moore 1994; Flores & Primack 1995; Burkert 1995; de Blok & McGaugh 1997; Tyson et al. 1998). Recently, the rotation curves of some of the dwarf and LSB galaxies were re-obtained using high-resolution H$\alpha$ observations, resulting the inner density profiles more concentrated than those obtained from the HI observations, but on average still shallower than $r^{-1}$ and with a large scatter (Swaters et al. 2000; van den Bosch & Swaters 2000; Dal Canton & Bernstein 2000). Besides, as Firmani et al. 2000b have shown, the halo inner density profiles of most of the observed LSB galaxies could be shallower than reported if the baryon contraction is taken into account (see also Firmani & Avila-Reese 2000).

An interesting property to be confirmed with more observational studies, in particular for galaxy clusters, is that the central density of dark matter haloes is roughly independent of the halo mass with a value around 0.02 $M_{\odot}$ pc$^{-3}$ (Firmani et al. 2000a,b). Not only the predictions of the CDM scenario, but also of alternative scenarios as the warm dark matter one are unable to predict such a scale invariance of the central density of dark haloes (Avila-Reese et al. 1998; Colin et al. 2000).
In a recent burst of activity, several alternatives to the well motivated CDM scenario were proposed in order to overcome the difficulties mentioned above and others as the excess of satellites and a little angular momentum in the galaxy disc predicted by numerical simulations. These alternative theories try to suppress the excess of central mass of the galactic haloes and may be summarized into two classes: 1) a manipulation of the power spectrum with a suppression of the small scales as an intrinsic property of a dark matter: Avila-Reese et al. 1998; Moore et al. 1999; Firmani et al. 2000a,b; Peebles 2000; Colín et al. 2000. These works make different assumptions concerning thermal energy at the beginning of the mass aggregation history (MAH). 2) Assuming for dark matter particles a different nature from collisionless: self-interacting (Spergel & Steinhardt 2000); repulsive (Goodman 2000); decaying (Cen 2000); with a limiting phase-space density (Hogan & Dalcanton 2000) and annihilating (Kaplinghat et al. 2000). Intriguingly, only the assumption of a self-interacting or annihilating cold dark matter seems to be able to reproduce the observed central density scale invariance ranging from galaxy to galaxy cluster size.

In a previous work, adopting the self-interacting CDM scenario suggested by Spergel & Steinhardt, we have proposed a thermodynamical model able to describe the gravothermal expansion of the core in the haloes (Firmani et al. 2000a); with this model, the scale invariance of the halo central densities was predicted. The NFW density profile shows a velocity dispersion raising outwards with radius (Cole & Lacey 1996; Fukushige & Makino 1997). Therefore, if particles are self-interacting, the inward heat transfer induces a thermalization process able to avoid the formation of a cuspy core. The influence of weak self-interaction on the structure of the CDM halos has been recently explored using cosmological numerical simulations (Yoshida et al. 2000; Davé et al. 2000). These works, where constant cross sections were used, have led to interesting results, that we will discuss in comparison with ours, later.

A cross section inversely proportional to the particle velocity dispersion seems to be better motivated by observations (Firmani et al. 2000a,b; Wyithe et al. 2000). In this paper we focus on detailed dynamical predictions of the density profiles of self-interacting spherical symmetric dark matter haloes ranging from galaxy to galaxy cluster mass. The cosmological model we assume is flat with vacuum density $\Omega_\Lambda = 0.7$ and expansion rate $H_0 = 65$ km s$^{-1}$ Mpc$^{-1}$.

The change in time of the gravitational field due to the mass aggregation modifies the particle orbit in agreement with its adiabatic invariants. The particle mass is distributed along its orbit according to the fraction of the period spent in any interval. For collisionless particles this approach is similar to the secondary infall (Avila-Reese et al. 1998) and allows to calculate a virialized halo density profile for mass and angular momentum aggregation histories in a specific CDM universe. For a given universe, the mass aggregation history (MAH) is obtained making use of the extended Press-Schechter formalism based on the conditional probabilities for a Gaussian random field. For a given mass, we generate a set of MAHS through Monte Carlo simulations and we calculate the average MAH. If the perihelion to aphelion ratio of the orbits (angular momentum) is opportunely selected, the NFW profile is obtained on galaxy and galaxy cluster scales.

The effects of the collisions between particles is described by the collisional term of the Boltzmann equation and induce a further evolution on the distribution function. Our scheme will provide an accurate solution only when the collision time is sufficiently larger than the orbital period of the particles. The collision cross section multiplied the relative velocity of the colliding particles is assumed to be constant in agreement with the previous discussion in Firmani et al. 2000b. This hypothesis makes particularly efficient the Monte Carlo method implemented to describe the collisions of the particles of each discrete state ($E,J$). By each trial we move a large amount of particles from one state to another according to the probability of the process. The results show an enormous space resolution obtained with a minimum computational expense, with each run taking approximately a couple of hours on a 450MHz pentium processor.

The efficiency of our method is counterbalanced by the practical difficulty to explore solutions with collision times lesser than the particle orbital periods (high cross section), as well as halo structures which deviate from the spherical symmetry. For these cases complementary studies with N-body techniques have been planned.

2 THE COSMOLOGICAL BOLTZMANN CODE

Our dynamical approach aimed to investigate the cosmological halo density profiles is based on a solution of the collisional Boltzmann equation for dark matter. The virialized dark halo is described by the distribution function $N(E, J)$, where $N(E, J)\Delta E\Delta J$ gives the number of particles within the intervals $\Delta E$ and $\Delta J$ centred at the total energy $E$ and the angular momentum $J$. The particle population is assumed to be uniform with respect to the directions of $J$. The mass aggregation supplies new particles, each one with a total energy and angular momentum. The computation of each particle orbit is based on a gravitational field derived from the mass distribution in a self-consistent way.

3 SELF-INTERACTING DARK MATTER HALO EVOLUTION

We have calculated the evolution of haloes in a $\Lambda$CDM cosmology assuming a self-interaction cross section per unit mass ($\sigma$) given by: $\sigma_v = 10^{-24}$ cm$^2$/GeV, where $v_{100}$ is the relative velocity of the colliding particles in units of 100 km s$^{-1}$. Fig. 1 shows the present-day density profiles of haloes with masses of $6 \times 10^{11}$ and $6 \times 10^{15}$ M$_\odot$ (solid lines). For comparison, the corresponding NFW profiles are also shown (dotted lines). Remarkably, the self-interacting CDM haloes appear to be thermalized in the centre resulting in a non-singular isothermal sphere, while in the outer parts, the density profile is well described by the NFW shape which is representative of the hierarchical merger history. The non-singular isothermal profile obtained with our code confirms the halo profile (dashed line) we derived in a previous work in order to describe observations (Firmani et al. 2000b). In that case, the hydrostatic equilibrium was assumed in order to pass from a NFW profile for the outer regions to an
The self-interacting cross section: \( \sigma v_{100} = 10^{-24} \text{ cm}^2 \text{ GeV}^{-1} \). Dotted lines show the NFW profile for both haloes. Long-dashed lines draw the non-singular isothermal profile obtained in Firmani et al. 2000b (see the text).

isothermal sphere in the centre. For that “naive” model the core-radius was obtained once assigned the central density.

The evolution in redshift of the density profile is shown for an halo with \( M = 6 \times 10^{11} \, M_{\odot} \).

\( H\alpha \) rotation curves (Swaters et al. 2000). The models were calculated for two cross sections: \( \sigma v_{100} = 10^{-24} \) and \( 2 \times 10^{-24} \text{ cm}^2 \text{ GeV}^{-1} \) (dashed and dotted lines, respectively).

The agreement of the self-interacting CDM model with observations is rather remarkable. In a range of scales from dwarfs to galaxy clusters, the haloes have (1) a nearly constant central density, and (2) a core radius roughly proportional to the maximum circular velocity. We stress that the only way to obtain halo central densities independent of their masses is assuming \( \sigma \propto 1/v \).

It is interesting to note that constraining the density for the cluster to the value of \( \rho_c = 0.02 M_{\odot} \text{ pc}^{-3} \) as inferred by the observation of CL0024+1654, we derive an estimate of the cross section: \( \sigma v_{100} \approx 10^{-24} \text{ cm}^2 / \text{GeV} \).

4 COMPARISON WITH PREVIOUS WORKS

In Fig.4 we show a grid of models for two different masses: \( 6 \times 10^{11} M_{\odot} \) and \( 6 \times 10^{15} M_{\odot} \) and wide range of cross sections. In order to scale the models with the mass for a given value of \( \sigma v_{100} \). Fig.3 shows that \( \rho_c \) is roughly scale invariant while \( R_c \) is proportional to \( V_m \).

With the aim to compare our models, derived with the assumption \( \sigma \propto v^{-1} \) to models with \( \sigma = \text{const} \), we will assume the latter velocity dispersion as representative of the collisional velocity. From Fig.2 of Yoshida and co-workers (2000), extrapolating the profile towards the center for the model S1W-a, with the same pattern of S1W-b and S1W-c, we read a central density of \( \rho_c = 2 - 3 \times 10^{16} M_{\odot} / h^2 \text{/ Mpc}^3 \) and a core radius \( R_c = 35 - 50 \text{ kpc/h} \) for a halo of 7.4 \( 10^{14} M_{\odot}/h \). We have simulated a cluster of \( M = 6 \times 10^{15} M_{\odot} \). Rescaling the mass of the Yoshida et al. (2000) halo (7.4 \( 10^{14} M_{\odot}/h \) and \( v = 1500 \text{ km/s} \)) to the cluster mass used here (6 \( 10^{15} M_{\odot} \) and \( v = 2400 \text{ km/s} \)), it is easy to obtain \( \rho_c = 0.013 - 0.008 M_{\odot}/\text{pc}^3 \) and \( R_c = 85 - 120 \text{ kpc} \). In
Figure 3. The plot shows a comparison between central densities and core-radii predicted by the model and the same quantities inferred by available observations of dwarfs (filled squares), LSB galaxies (empty squares) and the cluster CL0024+1654 (filled circle) (see Firmani et al. 2000b for details). Big empty squares represent LSBs Hα rotation curves (Swaters et al. 2000). Models are shown for $\sigma_{v100} = 10^{-24}$ (dotted lines) and $2 \times 10^{-24}$ cm$^2$ GeV$^{-1}$ (dashed lines).

Figure 4. The top panel shows the central densities of virialized haloes of $6 \times 10^{11} M_\odot$ (dotted-line) and $6 \times 10^{15} M_\odot$ (dashed-line) as a function of the strength of the self-interacting cross section. In the bottom panel core radii are plotted for the same haloes.

5 SUMMARY AND CONCLUSIONS

We have calculated the evolution of the density profiles of self-interacting CDM haloes using a numerical approach based on the collisional Boltzmann equation. This study presents two original points: (i) we were able to explore the halo density profiles covering about five orders of magnitude in the radius and for haloes from galaxy to galaxy cluster scales; (ii) we used a self-interacting cross section inversely proportional to the collision velocity of the particles. Analysing the results obtained with our code we conclude:

1. The interior of dark haloes may be deeply affected by self-interaction. A modest cross section value ($\sigma_{v100} \approx 10^{-24}$ cm$^2$ GeV$^{-1}$) is enough to produce soft cores.
2. Collisions between dark particles induce a thermalization process with the consequent appearance of a soft core. Thermal equilibrium in the core is established with a few collisions in a Hubble time ($\approx 4$).
3. Soft cores appear already at the very early dynamical evolution ($z \geq 40$); with time the central density decreases and the core size increases.
4. The observed scale invariance of the central density from galactic to galaxy cluster scales is reproduced only if
the cross section is inversely proportional to the collision velocity.
(5) The core radius predicted by the model increases proportional to the halo maximum rotation velocity in agreement with the observed trend.
(6) The observed central density of CL0024+1654 is consistent with assuming a self-interacting dark matter cross section given by: \( \sigma v_{100} \approx 10^{-24} \text{ cm}^2 \text{ GeV}^{-1} \).

The code is limited in analysing dark halo properties which depart from spherical symmetry. Further studies concerning dark halo triaxiality and the evolution of substructures have been planned using N-body techniques.

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