Experimental Study of Transmission Tower Damage Detection Using Time Series Model

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Abstract. In order to solve damage detection problem of transmission tower, a damage index based on inter-storey stiffness and cepstral distance is presented. First, transmission tower model is established and the tower is divided into six sub-structures. Thus, we only need to identify the sub-structure location where the damaged rods are included. Then, autoregressive model is introduced, the cepstral distance between autoregressive models under damaged and undamaged states is given, and a damage index based on inter-storey stiffness and cepstral distance is proposed. Finally, the experiment of transmission tower model is conducted to evaluate the effectiveness of detecting the damaged layer of transmission tower. The analysis results of the experimental signals show that the damage index based on inter-storey stiffness and cepstral distance is an available method for damage detection of transmission tower, and the damage feature can be well extracted with the damage index.

1. Introduction

In the process of developing ultra high voltage (UHV) transmission line, safety monitoring of transmission tower has become more and more important for electricity transmission system. Structural damage identification testing based on dynamic response is an available method for safety monitoring of transmission tower. The dynamic response of acquisition structure after driving is the focus of the damage identification method with wide use [1]. One of the reasons why this method is widely used is that it is intertwined with many technologies, such as sensing technology, signal acquisition, data processing, information management, data communication and so on. Many researchers used dynamic response to detect structural damage [2].

Structural dynamic response can be analyzed by using time series model. Time series model possesses these advantages, such as simplicity of modeling and sensitivity to conditional variation, which can effectively reflect some intrinsic characteristics of structure. Time series model has been widely used in feature excation to realize structural damage detection. AR, ARX and ARMA models have been utilized to extract damage features of simple structures like shear structure [3-5]. However, transmission tower is a three-dimensional complex structure, which will bring challenges to the damage detection. Therefore, in this paper a novel damage index based on AR model and cepstral distance is proposed to identify damage location. An experiment study is also conducted to realize damage detection via practical vibration signal such as acceleration dynamic response.

2. Transmission Tower Model

The transmission tower model under investigation is a steel tower, which is shown in Fig. 1. The main frame of the transmission tower model is composed of four circular steel tubes with a certain angle to the...
horizontal platform. The interface of each layer of the structure consists of four solid circular pipe crossbars, which are welded with circular steel tubes through steel plane. The crossover braces are composed of solid square aluminum bars, which are bolted to circular steel tubes. The experimental model has a height of 2200 mm and a base width of 600 mm. The transmission tower model can be divided into 6 sub-structures, which mean that we can just identify the location of damaged sub-structure. Thus, time series models become available method for damage detection of three-dimensional structure. Structural response should be acquired to establish the time series models. The basic dynamical equation of transmission tower can be written as

\[ M \ddot{x} + C \dot{x} + Kx = F \]  

(1)

where, \( M \), \( C \) and \( K \) are structural mass matrix, damping matrix and stiffness matrix, respectively. \( F \) is driving force. \( \ddot{x}, \dot{x} \) and \( x \) are acceleration, velocity and displacement, respectively. Thus, we can obtain the acceleration response and then utilize the response to detect damaged sub-structure location via numerical simulated response signal or practical measured signal.

![Fig. 1 Transmission tower model size](image_url)

### 3. Time Series Model and Its Damage Index

**Autoregressive Model.** Autoregressive(AR) model is a simple model to simulate acceleration time domain response. A typical AR(\( p \)) model can be written as

\[ y_i = c + \sum_{i=1}^{p} \varphi_i y_{i-1} + \varepsilon_i \]  

(2)

in which \( y_i \) is the time series data, \( c \) is a constant, \( p \) is the order of autoregressive model, \( \varphi_i \) is autoregressive coefficient, \( \varepsilon_i \) is the error. In general, the error terms should meet the homoscedastic assumption.

**Cepstral Distance.** Cepstral distance[6] of AR model can be obtained through autoregressive moving-average (ARMA) model. it can be simply described as follows:

An autoregressive moving-average model is given by

\[ y_i = c + \sum_{i=1}^{q} \varphi_i y_{i-1} + \sum_{i=1}^{q} \Phi_i \varepsilon_{i-1} \]  

(3)

in which \( \varphi_i \) and \( \Phi_i \) are the model coefficients, \( p \) and \( q \) are the model orders, and \( \varepsilon_i \) is the error. The system function of ARMA model in the Z domain is given by
\[
H(z) = \frac{\sum_{i=0}^{q} \Phi_i z^{-i}}{\sum_{i=0}^{p} \varphi_i z^{-i}} = \frac{\prod_{i=0}^{q} (1-\beta_i z^{-1})}{\prod_{i=0}^{p} (1-\alpha_i z^{-1})}
\] (4)

in which \( \alpha_i \) is the pole of ARMA model and \( \beta_i \) is the zero of ARMA model. The cepstrum of transfer function \( H(z) \) is the inverse Fourier transform of the logarithm of its power spectrum \( P(z) \) [7]:

\[
\log P(z) = \log \left[ H(z) \tilde{H}(1/z) \right] = \sum_{n=0}^{\infty} C_n z^{-n}
\]

(5)
in which \( C_n \) is cepstrum parameter. Substituting equation (4) into (5), equation (5) becomes [7]

\[
\log P(z) = -\sum_{i=0}^{p} \log |z - \alpha_i|^2 + \sum_{i=1}^{q} \log |z - \beta_i|^2 + \log \sigma^2
\]

(6)

Then, the power cepstrum can be written according to the poles and zeroes as follows:

\[
C_n = \begin{cases} 
\frac{1}{|\pi|} \sum_{i=0}^{p} \alpha_i^{|n|} - \sum_{i=1}^{q} \beta_i^{|n|} & n \neq 0 \\
\log \sigma^2 & n = 0
\end{cases}
\]

(7)

We assume that the cepstrum coefficient of model \( M \) is \( C_n \) and the cepstrum coefficient of model \( M' \) is \( C'_n \). Thus, the cepstral distance can be defined as follows:

\[
d(M, M') = \left[ \sum_{n=1}^{\infty} n |C_n - C'_n|^2 \right]^{-1/2}
\]

(8)

Due to the fact that the cepstral distance \( d \) is a Euclidean distance, the ARMA models have the following property

\[
d(MM'', MM'^*) = d(M, M')
\]

(9)

The system functions of ARMA models \( M \) and \( M' \) can be expressed as \( M = B/A \) and \( M' = B'/A' \), respectively. In addition, the third ARMA model \( M'' \) is the transfer function \( M'' = 1/BB' \). Thus, the corresponding AR models \( N \) and \( N' \) with system functions can be formed as follows:

\[
N = MM'' = \frac{1}{AB'}
\]

(10)

\[
N' = M'M'' = \frac{1}{A'B}
\]

(11)

Depending on the property (9), we have

\[
d(M, M') = d(N, N')
\]

(12)

Thus, we can find that it is sufficient to just consider AR models \( N \) and \( N' \). For AR models \( N \) and \( N' \) with the order \( p \) and \( p' \), along with the poles \( \alpha \) and \( \alpha' \), the cepstral distance of AR models can be given by

\[
d(M, M') = \left[ \log \left( \frac{\prod_{i=0}^{p} (1-\alpha_i \bar{\alpha}_i) \prod_{i=0}^{q} (1-\beta_i \bar{\beta}_i)}{\prod_{i=0}^{p'} (1-\alpha'_i \bar{\alpha}'_i) \prod_{i=0}^{q'} (1-\beta'_i \bar{\beta}'_i)} \right) \right]^{1/2}
\]

(13)

**Damage Index.** For an \( n \) storey structure the normalized cepstral distance index of the \( j \) storey can be written as
\[ V_j = \frac{d_j(M,M')}{\sum_{j=1}^{n} d_j(M,M')} \] (14)

Generally, the cepstral distance index can just identify the change of degrees of freedom, which cannot directly find the damage site. So, a damage index based on inter-storey stiffness and cepstral distance is proposed. The index is given by

\[ C_j = \frac{V_{j+1} + V_j}{2 - V_0} \] (15)

If \( j=1 \), then \( V_{j+1}=V_0=0 \). The damage index based on inter-storey stiffness can directly detect damage site. (1)

4. Experimental Study

Experiment is utilized to validate the proposed damage detection method. The experimental model is shown in Fig. 2. Load is applied by means of a shaking table to the base floor. The system is instrumented with six accelerometers mounted at the node of each floor along the vibration direction. The acceleration response data at all floors, including the first floor have been measured and recorded. Damage can be simulated through removing the crossover bars. There are six damaged cases, which are listed in Table 1.

![Fig. 2 Transmission tower model](image-url)
Table 1 damaged cases of transmission tower model

| Cases  | Damaged   | The number of removed |
|--------|-----------|-----------------------|
| Case 1 | The 2nd layer | 2                     |
| Case 2 | The 2nd layer | 4                     |
| Case 3 | The 3rd layer | 2                     |
| Case 4 | The 3rd layer | 4                     |
| Case 5 | The 5th layer | 2                     |
| Case 6 | The 5th layer | 4                     |

In general, we can utilize Akaike information criterion to estimate the order of AR model. Here, the order \( p \) of AR model is 35. The AR models under undamaged and damaged states are utilized to establish cepstral distance. Here, the procedure of damage identification was introduced in Section 2. The measured acceleration responses under undamaged and damaged states are applied to damage detection of transmission tower. For the six damaged cases, the damage index based on time series model and cepstral distance are used to find damaged layer or sub-structure.

For Cases 1 and 2, two and four crossover bars of the second layer are removed, respectively. The damage index based on inter-storey stiffness and cepstral distance is applied to damage detection. The detection results of the experiment are depicted in Figs. 3 and 4 with the layer number plotted against the damage index value. The higher the index value is, the greater the damage probability becomes. From Figs. 3 and 4, we can see that the damage value of the second layer is obviously higher than those of the other five layers, which means the damage index based on inter-storey stiffness and cepstral distance can well find damaged site.

Fig. 3 Detection results of two damaged rods in the second layer

For Cases 3 and 4, two and four crossover bars of the third layer are removed, respectively. The damage index based on inter-storey stiffness and cepstral distance is applied to damage detection. The detection results of the experiment are depicted in Figs. 5 and 6 with the layer number plotted against the damage index value. From Figs. 5 and 6, we can observe that the damage value of the third layer is greater than those of the other five layers. Therefore, the proposed index has the potential to detect the damaged site of transmission tower.

Fig. 4 Detection results of four damaged rods in the second layer
Fig. 5 Detection results of two damaged rods in the third layer

Fig. 6 Detection results of four damaged rods in the third layer

For Cases 5 and 6, two and four crossover bars of the fifth layer are removed, respectively. The proposed damage index is also utilized to realize the damage diagnosis. The identification results of the index are shown in Figs. 7 and 8 with the layer number plotted against the damage index value. From Figs. 7 and 8, it can be observed that the values of the fifth layer are obviously higher than those of the other five layers. So, the proposed index can well identify the damaged layer of transmission tower.

Fig. 7 Detection results of two damaged rods in the fifth floor

Fig. 8 Detection results of four damaged rods in the fifth floor

5. Conclusions
In this paper, a damage index based on inter-storey stiffness and cepstral distance is presented. First, transmission tower model is established and the tower is divided into six sub-structures. Thus, we only need to identify the sub-structure location where the damaged rods are included. Then, the cepstral distance of AR models is analyzed and a damage index based on inter-storey stiffness and cepstral distance is proposed. Finally, the experiment of transmission tower model is conducted to evaluate the effectiveness of detecting the damaged layer or sub-structure location. The experimental results demonstrate that the proposed damage index is available to detect the damage of transmission tower, and the calculated values of the damaged layer are higher than those of the other undamaged layers.

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