Differential Renormalization of a Yukawa Model with $\gamma_5$

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Abstract

We present a two-loop computation of the beta functions and the anomalous dimensions of a $\gamma_5$-Yukawa model using differential renormalization. The calculation is carried out in coordinate space without modifying the space-time dimension and no ad-hoc prescription for $\gamma_5$ is needed. It is shown that this procedure is specially suited for theories involving $\gamma_5$, and it should be considered in analyzing chiral gauge theories.

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I. Introduction

The differential renormalization (DR) method [1] has proven to be a simple and successful way to treat ultraviolet divergences in a renormalizable quantum field theory [2],[3],[4],[5]. The method is developed in coordinate space and does not change the dimensionality of space-time. It consists of two steps. In the first one bare amplitudes, which are too singular at short distances to admit a Fourier transform, are replaced with derivatives of less singular functions. In the second one, derivatives are integrated by parts. In doing the latter, ill-defined surface integrals arise: they correspond to counterterms that guarantee finite (renormalized) Green functions and unitarity [6]. Thus, by discarding surface integrals, one simultaneously regularizes and renormalizes divergent amplitudes.

One of the motivations to develop DR was to find a consistent way of treating theories with dimension-specific objects [1]. Theories involving $\gamma_5$ are of this type and are going to be the subject of this letter.

Dimensional regularization is the most practical and better understood regularization method used in quantum field theory but it runs into difficulties with theories involving $\gamma_5$. There is a large amount of literature and controversy (see [7] for a review) over the correct prescription for $\gamma_5$ in dimensional regularization. There are basically three different ways to deal with $\gamma_5$ when dimensional regularization of the Feynman integrals is employed, each one of them giving rise to a different regularization scheme. The first one is called naive dimensional regularization [8] and uses an anticommuting $\gamma_5$ and the 4-dimensional Dirac algebra. The second one goes under the name of dimensional reduction [9], where both the Dirac algebra and the vector fields are regarded 4-dimensional and split into D-dimensional bits plus 4-D scalars. The third one is the ’t Hooft-Veltman proposal [10] and uses a non-anticommuting $\gamma_5$ in D dimensions. It has been shown that the only systematic, uniquely fixed and consistent regularization procedure is the original prescription of ’t Hooft and Veltman [11],[12],[7]. The other
two schemes present manifestly algebraic inconsistencies; they may work for some low-loop calculations, specially when there are no closed odd parity fermion loops in the theory, but the validity of the computations to all loop orders is not guaranteed. However, in the ‘t Hooft-Veltman prescription the computational simplicity of dimensional regularization is lost and (D-4)-dimensional or evanescent counterterms have to be taken into account to obtain the correct results. This scheme is rarely used due to these computational difficulties.

It is our aim to show that DR provides a convenient way to handle theories with $\gamma_5$. Since in DR the dimensionality of space-time is kept unchanged, the 4-dimensional Dirac algebra and the standard $\gamma_5$ definition are maintained. As a result, the algebraic consistency is ensured. The simplicity of the computations is also another salient feature of DR.

As an example, we will study a massless Yukawa model with lagrangian

$$L = \frac{1}{2} (\partial \phi)^2 - \frac{\lambda}{4!} \phi^4 + i \overline{\psi} \gamma \cdot \partial \psi + i e \overline{\phi} \gamma_5 \psi.$$  (1)

This model has been analysed in the literature using dimensional regularization with two of the above prescriptions, namely, the naive dimensional regularization [13], and the ‘t Hooft-Veltman prescription [14]. Here we use DR to compute the beta functions and the anomalous dimensions up to two loops thus avoiding the mentioned difficulties inherent to dimensional regularization. In sections II and III some details of the calculations are carefully explained. Section IV shows the results and section V is devoted to the discussion.

II. One-loop order

We will start by studying the model (1) at one-loop. This will give us the chance to recall the basic techniques used in DR.

We will work in Euclidean space, the Euclidean Feynman rules and some
other conventions being given in Figure 1. Figure 2 shows the 1PI divergent graphs of the model at one and two loops.

The bare one-loop contribution to the fermion self-energy (Fig. 2.1) is given by

\[
S_{\text{bare}}^{1}(x, y) = -\frac{(ie)^2}{(4\pi^2)^2} \frac{1}{(x-y)^2} \frac{1}{(x-y)^2} \frac{1}{\partial_x \partial_y (x-y)^4}, \tag{2}
\]

where (let us emphasize) the standard 4-dimensional Dirac algebra has been used, and \( g \equiv \frac{e^2}{16\pi^2} \). From now on, we will use the convention that derivative operators act on the first variable of the fraction, so that \( \partial_i (x-y)^{-2} = -\partial_i (y-x)^{-2} \). Clearly, the singularity of the bare amplitude (2) lies in the \( 1/(x-y)^4 \) factor, which diverges too strongly when \( x \to y \) so as to have a Fourier transform. Let us consider for a moment \( x \neq y \). Then, the equation

\[
\frac{1}{(x-y)^4} = -\frac{1}{4} \frac{\Box \ln(x-y)^2 M^2}{(x-y)^2}, \tag{3}
\]

is an identity. Now comes the key point: DR defines the renormalized amplitude as the result of using Eq. (3) (also at \( x = y \)) with the proviso that, when computing the amplitude in momentum space, the derivatives acting on the r.h.s. above are formally integrated by parts and the additional prescription that surface integrals are discarded. The constant \( M \) has dimensions of mass and plays the role of a subtraction point. More explicitly, Eq. (3) gives for the renormalized one-loop contribution to the fermion self-energy:

\[
S_1(x, y) = \frac{g}{8\pi^2} \partial \Box \frac{\ln(x-y)^2 M^2}{(x-y)^2}. \tag{4}
\]

Now we can compute the amplitude in momentum space (setting \( y = 0 \) due to translational invariance)

\[
\hat{S}_1(p) = \int d^4xe^{ipx} \left( \frac{g}{8\pi^2} \partial \Box \frac{\ln x^2 M^2}{x^2} \right). \tag{5}
\]
Following DR, we integrate by parts discarding the surface term and are left with
\[
\hat{S}_1(p) = -\frac{ig}{2} \phi \ln \frac{p^2}{4M^2\gamma^2},
\]  
(6)
where \(\gamma = 1.781072\ldots\) is the Euler constant.

The bare one-loop boson self-energy (Fig. 2.2) is
\[
\Delta_{1}^{\text{bare}}(x,y) = -\frac{(ie)^2}{(4\pi)^2} \text{tr} \left\{ \gamma_5 \phi \frac{1}{(x-y)^2} \gamma_5 \phi \frac{1}{(y-x)^2} \right\} = \frac{16g}{\pi^2} \frac{1}{(x-y)^6},
\]  
(7)
The singular factor \(1/(x-y)^6\) in (7) now produces, upon Fourier transform, a quadratic divergence. To cure it, we have to extract two derivatives more than in (3). To this end, we use (see Appendix A of ref. [1])
\[
\frac{1}{(x-y)^6} = \frac{1}{32} \Box \Box \ln(x-y)^2 M^2 \frac{(x-y)^2}{(x-y)^2},
\]  
(8)
thus obtaining the renormalized amplitude
\[
\Delta_1(x,y) = -\frac{g}{2\pi^2} \Box \Box \ln(x-y)^2 M^2 \frac{(x-y)^2}{(x-y)^2}.
\]  
(9)
We could compute now the renormalized amplitude in momentum space, as in the previous case.

We next renormalize the one-loop Yukawa vertex (Fig. 2.3). The bare amplitude is
\[
V_{1}^{\text{bare}}(x,y,z) = \frac{(ie)^3}{(4\pi)^2} \gamma_5 \phi \frac{1}{(y-x)^2} \gamma_5 \phi \frac{1}{(x-z)^2} \gamma_5 \frac{1}{(y-z)^2}
\equiv -ie \frac{g}{4\pi} \gamma_5 F(x,y,z).
\]  
(10)
To regulate this graph we notice that the function \(F\) has a singular region when \(x \sim y \sim z\). Outside this region we can integrate by parts and obtain
\[
F(x,y,z) = \gamma_i \gamma_j \frac{\partial}{\partial x^i} \left\{ \frac{1}{(x-y)^2(y-z)^2} \frac{\partial}{\partial x^j} \frac{1}{(x-z)^2} \right\} - \gamma_i \gamma_j \frac{1}{(x-y)^2(y-z)^2} \frac{\partial^2}{\partial x^i \partial x^j} \frac{1}{(x-z)^2}.
\]  
(11)
The first term on the r.h.s. has a well-defined Fourier transform, as can be seen using dimensional analysis. In turn, the second term is singular at $x \sim y \sim z$; notice that it contains the laplacian $\gamma^i \gamma^j \partial_i \partial_j$ and that

$$\Box \frac{1}{(x - z)^2} = -4\pi^2 \delta^{(4)}(x - z),$$

so the second term in (11) can be renormalized using (3). Finally we can write the DR renormalized graph as

$$V_1(x, y, z) = -\frac{ieg}{4\pi^4} \gamma_5 \left\{ \gamma^i \gamma^j \partial_i \left( \frac{1}{(x - y)^2} \frac{\partial}{\partial x^i} \frac{1}{(y - z)^2} \frac{\partial}{\partial x^j} \right) \right\} - \pi^2 \delta^{(4)}(x - z) \Box \ln \frac{(x - y)^2 M^2}{(x - y)^2}.$$  \hspace{2cm} (13)

The 4-point boson function has two contributions at one-loop: the bubble (Fig. 2.4) and the box (Fig. 2.5). The bare bubble is

$$\Gamma_{1a}^{\text{bare}}(x, y, z, w) = \frac{\lambda^2}{32\pi^4} \left[ \delta^{(4)}(x - y) \delta^{(4)}(z - w) \frac{1}{(x - z)^4} + \text{permutations} \right],$$

and is easily DR renormalized using (3):

$$\Gamma_{1a}(x, y, z, w) = -2h^2 \delta^{(4)}(x - y) \delta^{(4)}(z - w) \Box \ln \frac{(x - z)^2 M^2}{(x - y)^2} + \text{permutations},$$

where $h \equiv \frac{\lambda}{16\pi^2}$.

The bare box

$$\Gamma_{1b}^{\text{bare}}(x, y, z, w) =$$

$$= -\frac{(ie)^4}{(4\pi^2)^4} tr \left\{ \gamma_5 \gamma_5 \frac{1}{(x - y)^2} \gamma_5 \frac{1}{(y - w)^2} \frac{1}{(w - z)^2} \gamma_5 \frac{1}{(z - x)^2} \right\} + \text{perm.}$$

is a bit more complicated due to the presence of indices but the DR program can be straightforwardly carried out using the Dirac algebra and identifying the singular factors. In this case, we integrate by parts twice and use (3) to obtain the following DR renormalized amplitude
\[ \Gamma_{1b}(x, y, z, w) = \]
\[ = g^2 \frac{1}{\pi^4} \text{tr}\left\{ \gamma_a \gamma_b \gamma_c \gamma_d \right\} \frac{\partial}{\partial y^a} \left( \frac{1}{(y-x)^2} \frac{1}{(y-w)^2} \frac{1}{(w-z)^2} \frac{1}{(z-x)^2} \right) \]
\[ - \frac{16 g^2}{\pi^2} \delta^{(4)}(y-w) \frac{1}{(y-x)^2} \frac{\partial}{\partial z^d} \left( \frac{1}{(z-x)^2} \frac{\partial}{\partial z^d} \frac{1}{(z-w)^2} \right) \]
\[ + 16 g^2 \delta^{(4)}(y-w) \delta^{(4)}(z-w) \square \frac{\ln (x-w)^2 M^2}{(x-w)^2} + \text{perm.} \]  
\[ (17) \]

This analysis shows that all one-loop UV divergences are cured by first isolating the singularities and then by using Eqs. (3) and (8), and the integration by parts prescription. In this process an arbitrary parameter with dimensions of mass enters in a natural way. In writing the different renormalized amplitudes, one has no a priori reason to use the same \( M \) for all graphs but we will use a subtraction scheme where all the \( M' \)'s are equal. This can be done since there are no Ward identities in the theory that force us to introduce different mass scales [15]. This parameter has the physical meaning of a renormalization scale, as can be explicitly seen when the renormalization group equations are tested on the renormalized amplitudes (see Section IV).

III. Two-loop order

For the sake of brevity, we will not discuss here all the graphs of the two-loop order but we will present an overview of the computations, which involve almost the same techniques used in the one-loop order, that is, isolating the singularities of bare amplitudes and using DR identities.

Two-loop bare amplitudes present two singular regions. The regularization of these amplitudes is performed from the subdivergence, treating it in the same way that it was done at one-loop, to the overall divergence. Overall
divergences require new DR identities. In our case, we only need
\[
\frac{\ln x^2 M^2}{x^4} = -\frac{1}{8} \ln^2 x^2 M^2 + \frac{2 \ln x^2 M^2}{x^2}
\]
\[
\frac{\ln x^2 M^2}{x^6} = -\frac{1}{64} \ln^2 x^2 M^2 + \frac{5 \ln x^2 M^2}{x^2}.
\] (18)

In this way two-loop 1PI are renormalized. As an example, we examine one of the diagrams which gives contribution to the fermion self-energy (Fig. 2.6), with bare amplitude
\[
S_{2a}^{\text{bare}}(x, y) = \frac{e^4}{(4\pi^2)^3} \gamma_5 \gamma_5 \int \frac{d^4u d^4v}{(y-v)^2(x-u)^2} \left\{ \frac{1}{(v-u)^2} \gamma_5 \frac{1}{(u-v)^2} \right\}
\]
\[
= -\frac{4g^2}{\pi^6} \gamma_5 \gamma_5 \int \frac{d^4u d^4v}{(y-v)^2(x-u)^2(u-v)^6}.
\] (19)

The first singular region of this amplitude is the corresponding to the sub-
divergence, when \( u \sim v \), and is cured in the same way that it was done at
the one-loop order (18). The laplacians in front of \( \ln(u-v)^2/(u-v)^2 \) are
then integrated by parts, discarding the surface terms. Then we can perform
easily the integrals over the internal points. The second singular region cor-
responds to the overall divergence, when \( x \sim y \), and is cured using (18) to
finally obtain the DR renormalized amplitude
\[
S_{2a}(x, y) = -\frac{g^2}{8\pi^2} \left( \frac{\ln^2(x-y)^2 M^2 + 3 \ln(x-y)^2 M^2}{(x-y)^2} \right).
\] (20)

Amplitudes with well located subdivergences are cured in a straightfor-
ward way, so we will not discuss them. There are other diagrams which re-
quire a supplementary effort: those that contain an overlapping divergence.
In those diagrams it is impossible to separate out the two singular regions.
Nevertheless, overlapping divergences do not represent a serious difficulty in
real-space computations, because the external points of the amplitude can be
kept separated until the regularization of the subdivergences is accomplished.
Let us take as an example of overlapping divergence the amplitude corresponding to Fig. 2.10,
\[ \Delta_{2b}^{\text{bare}}(x, y) = -\frac{e^4}{(4\pi^2)^2} tr \{ \gamma_a \gamma_b \gamma_c \gamma_d \} \times \]
\[ \int du^4 dv^4 \left\{ \frac{1}{(x-u)^2} \frac{1}{(u-y)^2} \frac{1}{(y-v)^2} \frac{1}{(v-x)^2} \frac{1}{(v-u)^2} \right\} . \quad (21) \]
In this amplitude the singular regions are \( u \sim v \sim x \) and \( u \sim v \sim y \). To renormalize it we first extract total derivatives over the external points. We notice that in two of these terms there is a laplacian acting on a fraction that allows us to obtain a delta function and perform one of the integrals over an internal point. These two terms present singular regions when \( u \sim x \) in one case, and when \( u \sim y \) in the other, and they are renormalized using (3) first, and (8) and (18) afterwards. Also the third term can easily be renormalized. After some algebra, the DR renormalized amplitude is expressed as
\[ \Delta_{2b}(x, y) = \frac{g^2}{2\pi^2} \Box \ln^2(x-y)^2 M^2 + \ln(x-y)^2 M^2 \]
\[ \frac{1}{(x-y)^2} . \quad (22) \]
The very same procedure is used to renormalize overlapping divergences of the Yukawa vertex and of the 4-point boson function: extract total derivatives and use the properties of Dirac matrices during the computation. In these diagrams, and due to the presence of indices, there are some integrals that are difficult to solve analytically, but fortunately they are finite, and they do not need to be renormalized. For our purposes, to find beta functions and anomalous dimensions, we do not need them in an explicit form.

As an example of overlapping divergence of a vertex, we next examine the most difficult two-loop Yukawa vertex (Fig. 2.16), with bare amplitude
\[ V_{2e}^{\text{bare}}(x, y, z) = -\frac{ie^5}{(4\pi^2)^2} \gamma_5 \gamma_j \gamma_i \gamma_k \times \]
\[ \int d^4u d^4v \left\{ \partial_1 \frac{1}{(x-u)^2} \partial_2 \frac{1}{(u-y)^2} \partial_3 \frac{1}{(y-v)^2} \partial_4 \frac{1}{(v-x)^2} \frac{1}{(u-z)^2} \frac{1}{(y-v)^2} \right\} . \quad (23) \]
To proceed, we first integrate by parts over $\partial/\partial x^i$. Power counting shows that only the integrated part is too singular so as to have Fourier transform, so we focus our attention only on that part. The laplacian acting on $1/(x-u)^2$ gives a delta function, and we can then perform the integral over $u$. The final integral to be analysed is

$$
4\pi^2 \gamma_j \gamma_k \frac{1}{(x-z)^2} \frac{1}{(y-x)^2} \int d^4v \frac{1}{(x-v)^2(y-v)^2} \frac{1}{(z-v)^2} \partial_j \left( \frac{1}{(y-x)^2} \frac{1}{(z-x)^2} \right)
= 4\pi^2 \gamma_j \gamma_k \frac{1}{(x-z)^2} \frac{1}{(y-x)^2} \frac{1}{(z-x)^2} \partial_j \left( \frac{1}{(y-x)^2} \frac{1}{(z-x)^2} \right) K(z-x, y-x).
$$

(24)

Now $\partial/\partial y^j$ is integrated by parts. Once again, only the integrated part needs renormalization. This last one is separated into traceless and trace parts. The traceless combination of derivatives is finite, and the trace part is straightforwardly DR renormalized. After some algebra, the final result can be expressed as
\[ V_{2e}(x, y, z) = \frac{ieg^2}{16\pi^8}\gamma_5 \left\{ -\gamma_j\gamma_i\gamma_l\gamma_k \times \right. \]

\[
\int d^4u \frac{\partial}{\partial x^l} \left( \partial_j \left( \frac{1}{(x-u)^2} \frac{1}{(u-y)^2} \partial_k \frac{1}{(z-v)^2} \frac{1}{(u-z)^2} (y-v)^2 (x-v)^2 \right) \right)

+ 4\pi^2\gamma_j\gamma_k \frac{\partial}{\partial y^l} \left( \frac{1}{(x-z)^2} (y-x)^2 \partial_z^k K(z-x, y-x) \right)

- 4\pi^2\gamma_j\gamma_k \frac{1}{(x-z)^2} \left( \frac{\partial^2}{\partial y^l \partial z^k} - \frac{\delta_{jk}}{4} \frac{\partial^2}{\partial y \cdot \partial z} \right) K(z-x, y-x)

+ 8\pi^4 \left[ \frac{1}{16} \Box \frac{\ln(x-y)^2 M^2}{(x-y)^2} \Box \frac{\ln(x-z)^2 M^2}{(x-z)^2} \right]

+ \frac{1}{4} \frac{\partial}{\partial x^l} \left( \Box \frac{\ln(x-y)^2 M^2 \Box}{(x-y)^2} \frac{\partial}{\partial x^l} \frac{1}{(x-y)^2 (x-z)^2} \right)

+ \frac{\pi^2}{8} \delta^{(4)}(x-z) \Box \frac{\ln^2(x-y)^2 M^2 + 2 \ln(x-y)^2 M^2}{(x-y)^2} + (y \leftrightarrow z) \left\} \right. \]

Although the final expression seems to be somewhat complicated, specially for taking its Fourier transform, the computation of \( M\partial/\partial M \) on it, which is what we need to find the beta functions and the anomalous dimensions, presents no difficulty.

The remaining overlapping divergences of the Yukawa vertex and of the 4-point boson function can be treated in a similar way, and we will not explicitly discuss them here.

**IV. Renormalization group constants**

After the computation of the renormalized amplitudes, it is easy to find the values of \( M\partial/\partial M \) acting on all the 1PI graphs. We present them in Fig. 3 in a pictorial form. With those values, it is possible to check that the
renormalized amplitudes satisfy renormalization group equations

\[ \left( M \frac{\partial}{\partial M} + \beta_g \frac{\partial}{\partial g} + \beta_h \frac{\partial}{\partial h} - n_\phi \gamma_\phi - n_\psi \gamma_\psi \right) \Gamma^{(n_\phi, n_\psi)} = 0. \]  

(26)

When we substitute the values of \( M \frac{\partial}{\partial M} \) in (26), we find the following values of beta functions and anomalous dimensions

\[
\begin{align*}
\beta_g &= 10g^2 + \frac{1}{6} h^2 g - 4hg^2 - \frac{57}{2} g^3 \\
\beta_h &= 3h^2 + 8hg - 48g^2 - \frac{17}{3} h^3 - 12h^2 g + 28hg^2 + 384g^3 \\
\gamma_\phi &= 2g + \frac{1}{12} h^2 - 5g^2; \quad \gamma_\psi = \frac{1}{2} g - \frac{13}{8} g^2.
\end{align*}
\]

(27)

These results coincide with the ones given in the literature, where the same computations were carried out using naive dimensional regularization [13], and the 't Hooft-Veltman scheme [14].

This model does not present closed odd parity fermion loops, so one does not expect the naive dimensional regularization to fail here. Nevertheless, one would prefer to use a procedure which does not break the algebraic consistency of the computations and such that one could always rely on its validity. These problems do not arise in [14], but there the cumbersomeness of the computations, where evanescent counterterms had to be taken into account, shows that the 't Hooft-Veltman scheme is almost impractical.

We must remark, once again, that computations using DR are simple and algebraic consistent.

V. Discussion

We have applied DR to compute the beta functions and anomalous dimensions up to two loops of the Yukawa model defined in Eq. (1).

\footnote{There is a 2 factor between the beta functions obtained in [14] and our results, due to the use of a different convention.}
DR presents some salient features. DR is just a prescription to extend distributions which are ill-defined in some points to distributions well-defined in all space-time. Bare and DR renormalized amplitudes coincide for non-singular points, so this procedure represents a minimal change in the theory.

DR is a real-space method, and computations of multiloop integrals can be done with ease. Let us recall here that the ”method of uniqueness” [16], [17], [18] which also works with coordinate amplitudes but uses dimensional regularization, has already demonstrated the powerful calculational possibilities of space-time regularization methods. DR also shares that computational simplicity. The method is specially suited to obtain renormalization group functions and renormalized amplitudes in configuration space at high loop order. The computation of these amplitudes in momentum space requires to Fourier transform and perform some difficult but well-defined integrals.

In theories in which $\gamma_5$ is present, DR preserves the algebraic consistency of the computations, since it does not alter the dimensionality of space-time, and the standard 4-dimensional Dirac algebra can be used.

It is worthwhile to mention here that DR has been used in gauge theories [3], [4], and that it reproduces correctly the chiral anomaly [3]. In gauge theories DR enforces the introduction of several mass scales, and in order to preserve the Ward identities certain correlations between them are required. The anomaly results when the Ward identities overconstraint the value of the mass parameters.

There is still no proof of the validity of DR to all orders of perturbation theory. However, it has very recently been shown [15] that there is a relationship between dimensional and differential regularization and renormalization in low loop graphs, and one expects that one could find a systematic application of these relations at higher order graphs and then yield a consistent proof of DR. One difference between the two procedures is that DR does not modify the space-time dimension, and then it allows for a natural and consistent treatment of dimension-specific theories.
DR seems to be a suited and natural method to treat theories with $\gamma_5$ and, with no doubt, deserves further consideration to evaluate chiral gauge theories.

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Figure Captions

Figure 1. Euclidean Feynman rules and other conventions used in the computations.

Figure 2. One and two loop 1PI diagrams. In parenthesis, the number of diagrams of that type.

Figure 3. The value of $M\partial/\partial M$ acting in all the considered 1PI.