Representing Random Permutations as the Product of Two Involutions

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Abstract

An involution is a permutation that is its own inverse. Given a permutation $\sigma$ of $[n]$, let $N_n(\sigma)$ denote the number of ways to write $\sigma$ as a product of two involutions of $[n]$. If we endow the symmetric groups $S_n$ with uniform probability measures, then the random variables $N_n$ are asymptotically lognormal.

The proof is based upon the observation that, for most permutations $\sigma$, $N_n(\sigma)$ can be well approximated by $B_n(\sigma)$, the product of the cycle lengths of $\sigma$. Asymptotic lognormality of $N_n$ can therefore be deduced from Erdős and Turán’s theorem that $B_n$ itself is asymptotically lognormal.