Measurements and simulation studies of piezoceramics for acoustic particle detection

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Abstract. Calibration sources are an indispensable tool for all detectors. In acoustic particle detection the goal of a calibration source is to mimic neutrino signatures as expected from hadronic cascades. A simple and promising method for the emulation of neutrino signals are piezo ceramics. We will present an detailed microscopic and macroscopic understanding of these piezo ceramics.

1. Microscopic description of piezos

When an electric field is applied to a piezo material, the polarized molecules are distorted. This distortion causes the material to change its dimensions. This phenomenon is known as electrostriction or inverse piezoelectric effect. In addition, a permanently polarized material such as lead zirconate titanate (PZT) produces an electric field when the material deforms as a result of an imposed mechanical force. This phenomenon is known as the piezoelectric effect. A piezoelectric material can be modeled by combining Hook’s law for anisotropic materials and Gauss’ law for electrical displacement:

\[ T_{ik} = e_{iklm}S_{lm} + \varepsilon_{ikl}E_l; \quad D_i = \varepsilon_{ilm}S_{lm} + \varepsilon_{il}E_l; \quad \partial_k T_{ik} = \rho \varphi; \quad \partial_i D_i = 0 \]  

with \( T_{ik} \), \( D_i \), \( e_{iklm} \), \( S_{lm} \), \( \varepsilon_{ikl} \), \( E_l \), \( \varepsilon_{il} \), \( \rho \) and \( u_i \) representing the stress tensor, the electrical displacement density, the elasticity tensor, the strain tensor, the piezoelectric tensor, the electric field, the permittivity tensor, the density and the displacement, respectively [2, 6].

Commonly used piezoelectric materials (e.g. piezoceramics) have a 6mm symmetry class. This means that only 11 material parameters differ from zero. The finite element program FEMLAB [3] has been used to solve equation (1). In addition a Rayleigh model is used to account for damping.

2. Simple macroscopic model: The equivalent circuit diagram

If one is only interested in one dimension a simple piezo model can be derived by a mass spring system (fig. 1) [1]. In this model a disc of thickness \( t \), density \( \rho \) and electrode area \( A \) is described

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by:

\[ m_1 u_1' = F_{spring} + F_1 \] (2)
\[ m_2 u_2' = -F_{spring} + F_2 \] (3)

Here \( m \) is the half mass of the piezo \( m = \frac{1}{2} \rho A t \), \( u_1 \) and \( u_2 \) are the displacements of the masses, \( F_{spring} \) is the spring force \( F_{spring} = -AT_{33} \) and \( F_1 \) and \( F_2 \) are external forces. In this picture the voltage, the charge and the strain are equivalent to: \( V = tE_3 \), \( Q = AD_3 \) and \( S_3 = u_1 - u_2 \).

Further we take the symmetric case with \( F_{ext} := F_1 = -F_2 \) and \( u_1 = -u_2 \). This results in the following equation system after subtracting (2) from (3) and adding damping:

\[ m_1 u_1' + 2 \delta u_1 + \frac{2Ae_{33}}{t} u_1 = \frac{Ae_{33}}{t} V + F_{ext} \] (4)
\[ Q = \frac{Ae_{33}}{t} V + \frac{2Ae_{33}}{t} u_1 \] (5)

\[ \begin{align*}
\begin{array}{c}
\text{Figure 1. Simplified model of a piezo. The displacements at the electrodes are } u_1 \text{ and } u_2 \\
\end{array}
\end{align*} \]

One can easily transform the mechanical parts of these equations to the electrical side by taking the force-voltage mathematical analogy, the so called F-V analogy (see table 1). In this analogy the electrical current is equivalent to the particle velocity e.g. the time derivative of the displacement.

\[
\begin{array}{|c|c|}
\hline
\text{mechanical law} & \text{F-V-analogy} \\
\hline
\text{Newton's law} & F = m \frac{d}{dt} u \\
\text{Hook’s law} & F = D \int_{x_0}^{x} v \, dx' \\
\text{Friction} & F = s v \\
\hline
U = L \frac{d}{dt} I \\
U = C \int_{t_0}^{t} I \, dt' \\
U = RI \\
\hline
\end{array}
\]

\textbf{Table 1.} Comparison of mechanical and electrical laws in F-V-analogy.

In this analogy the mechanical part of the simplified model described so far is given by a capacitor \( C_1 = \frac{Ae_{33}^2}{t \epsilon_{3333}} \), an inductivity \( L_1 = \frac{mt^2}{2Ae_{3333}} \) and a resistor \( R_1 = \frac{\delta t^2}{Ae_{3333}} \) in series, whereas the electrical part remains a capacitance \( C_P = \frac{\epsilon_{33}A^2}{t} \) paralleled to the mechanical parts. This is known as the equivalent circuit diagram for the first resonance of a piezo. Additional resonances can be added by additional parallel circuits each with its own capacitance inductance and resistance as shown in figure 2. Adding further resonances cannot be explained in the very simple model given above. Therefore to get the values of the electrical parts in the equivalent circuit representation one has to fit to existing impedance data.
Figure 2. Equivalent circuit of a piezo. The first n resonances can be modeled.

In the equivalent circuit representation the average velocity of the displacement over the electrodes surface is proportional to the total current of all equivalent mechanical parts. The results of the finite element simulation, the equivalent circuit representation and measurements for both the impedance and displacement are plotted in figures 3 and 4, respectively. The methods to measure the impedance and the displacement are both described in [4].

Figure 3. Impedance of a PZT-5A disc of radius 12.5mm and thickness 20mm. The FEMLAB simulation (red dots) in comparison to the measurement (blue dots) and an additional fit of the equivalent circuit representation to the measurement (small blue line) is shown.

3. Simulation with coupling to water
There are several problems that arise when simulating the coupling of a piezo to water using finite element code. For low frequencies the volume of the simulated area has to be at least two times the simulated wavelength whereas at the same time the size of the finite elements has to be less than one fourth of the size of the piezo. This results in huge matrices to be inverted.
Figure 4. Displacement of a PZT-5A disc of radius 12.5mm and thickness 20mm. left: The FEMLAB simulation (solid line) and a measurement (dots) at the disc centre. right: Calculation of displacement from equivalent circuit representation using the fit parameters to the impedance measurement from figure 3 (dashed line) in comparison with the displacement averaged over the surface from the FEMLAB simulation (solid line). Figure from [4].

For high frequencies on the other hand the finite element size should be smaller than at least half the wavelength again resulting in huge matrices.

There are several methods at hand to solve these problems. One can for example add damping elements at the boundaries of the water domain. In this work we used the approach of non-reflecting boundary conditions as described in [5].

In order to suppress reflections at a boundary one has to cancel incoming waves at this boundary. For a particular wave direction with angle $\alpha$ to the boundary one must apply the following boundary condition: \( \cos(\alpha) \left( \frac{1}{c} \partial_t + \hat{n} \nabla \right) p = 0 \) with $t$, $\hat{n}$, $c$ and $p$ the time, the normal to the boundary, the sound velocity and the pressure respectively. To suppress reflections from more directions one has to multiply boundary conditions of the type discussed for one wave direction for each direction to be suppressed. In our simulation the water volume has spherical shape. In the far field the piezo acts like a point source so only first order non-reflecting boundary conditions are needed with $\alpha = 0$, as the water volume has spherical shape. This condition is equivalent to an impedance matching if simulated in the frequency domain.

At the surface of the piezo on the other hand the following boundary conditions are applied to describe the coupling to water: the pressure of the water couples back to the piezo acting as an external force. The second time derivative of the piezos displacement on the other hand acts like an accelerating acoustic source. The resonances of the impedance and the displacement are reduced by this additional mechanical load as can be seen in figure 5.

From the FEMLAB simulations one can extract the direction characteristics of an sound emitting piezo by taking the local mean pressure field (virtual detectors) at a certain distance. In figure 6 results of this simulation for $100kHz$ are plotted.
Figure 5. Impedance of a PZT-5A disc of radius 12.5mm and thickness 20mm. The FEMLAB simulation with and without water (red dots big amplitude and blue dots small amplitude) and a fit of equivalent circuit parameters to the FEMLAB simulation with water (small blue line) is plotted.

Figure 6. direction characteristics at 100kHz. left: direction characterisites of a piezo disc with radius 12.5mm and thickness 20mm at 0.35m. right: pressure field of the disc and the virtual detectors (black circles), where the local mean pressure field is taken.

4. Summary and Conclusion
We have shown that Finite Element Methods are a valuable tool to study piezoceramic sensors. Comparing results from the simulation of a piezo based on these methods with measurements yields very good agreement. Also the direction characteristics of the piezo can be extracted.
from the simulation. The measured impedance can be very well reproduced using an equivalent circuit diagram on one side and using FE methods on the other. This detailed understanding of the piezo characteristics will enable us to design both the optimal sensors for acoustic particle detection and the transducers to calibrate these sensors.

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