Signatures of the light gluino in the top quark production

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Abstract

If a light gluino, with a mass of the order of GeV, exists in the minimal supersymmetric extension of the Standard Model, then it can contribute to the production rate of the top quark pairs at hadron colliders via $\tilde{g}\tilde{g} \to t\bar{t}$. Because the top quark is heavy, the masses of the superpartners of the left-handed and right-handed top quarks can be very different such that a parity-violating observable can be induced in the tree level production process. We discuss the phenomenology of this parity violating asymmetry at the CERN Large Hadron Collider.
I. INTRODUCTION

In despite of the success of the Standard Model (SM) in explaining and predicting experimental data, it is widely believed that new physics has to set in at some high energy scale. One of such new physics models is the the minimal supersymmetric extension of the Standard Model (MSSM). Various supersymmetry (SUSY) models, such as gravity-mediated and gauge-mediated supersymmetry breaking models [2], have been extensively considered in the literature to explain why the masses of the superparticles are not the same as those of the SM particles. In general, the masses of the superparticles are predicted to be around a few hundred GeV or at the TeV region. There are ample studies in the literature to examine the detection of these non-standard particles in the current and future experiments, including those at the CERN Large Hadron Collider (LHC), and at the future Linear Colliders.

Among the superparticles of the MSSM, some models of SUSY breaking predict the existence of a light gluino with the masses around 1 GeV or less [6]. If this scenario is true, then there is rich phenomenology predicted for the current experimental data which can be used to either confirm or constrain models. In Ref. [3], ALEPH Collaboration used the data on the cross-sections of dijet production and the angular distributions in 4-jet production to derive the ratios of the color factors $C_A/C_F$ and $T_F/C_F$. Based on the obtained values, ALEPH excluded the existence of the gluinos with the mass lighter than 6.3 GeV at 95% confidence level. The result was criticized by Farrar [5], who argued that ALEPH’s analysis underestimated the theoretical uncertainties in the knowledge of hadronization and resummation of large logarithms arising in the separation of jets from soft radiation. If these uncertainties are taken into account, the light gluino is excluded only at 1σ level.

This problem was further examined by Csikor and Fodor in Ref. [4], where they determined the color factors of underlying gauge theory by studying the behavior of the ratios $R_\gamma = \sigma(e^+e^- \rightarrow jets)/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$, $R_\tau = \Gamma(\tau^- \rightarrow \nu_\tau + jets)/\Gamma(\tau^- \rightarrow \nu_\tau e^-\bar{\nu}_e)$, $R_Z = \Gamma(Z \rightarrow hadrons)/\Gamma(Z \rightarrow \mu^+\mu^-)$ in the region of 5 GeV to $M_Z$ scale. They concluded that the $O(\alpha_s^2)$ analysis of these quantities allows to exclude the light gluino with the mass between 3 and 5 GeV at 93% confidence level, and with the mass less than 1.5 GeV at 70.8% confidence level. If their results are combined with the $\chi^2$-distribution from ALEPH analysis, the exclusion confidence level is improved to 99.97% and 99.89%, respectively. This conclusion is quite insensitive to the overall error of ALEPH’s results; for instance, the exclusion limits of the combined analysis are still above 95% if ALEPH’s systematic error is increased by a factor of 3. However, in order to extract the number of active fermions from the experimental data, both methods [3] and [4] rely on the state-of-art usage of perturbative theory. None of the separate analyses can exclude the light gluino at the confidence level \( \geq 70\% \), and combined and complicated analysis is needed to overcome the flaws of each separate method.

Another significant limitation on the possible parameter space of the models with light gluinos was recently imposed by the negative results of the search for the production of charginos with the mass less than $m_W$ at LEP2 [21]. This result disfavors the models with the masses of all gauginos vanishing at tree level at GUT scale [6], in which gluino has the mass of the order GeV, and at the electroweak scale at least one of the charginos is necessarily lighter than $W$-boson. However, the LEP2 data can not rule out the models with the other
spectra of gaugino masses, for instance, the models of gauge-mediated symmetry breaking where the gluino can be the only light gaugino \[7\]. As mentioned before, the analysis of \[3,4\] already puts strong constraints on the possibility of the light mass of the gluino, however, due to the aforementioned theoretical difficulties it seems that more study is needed.

There are a few other methods discussed in the literature to look for light gluino. If gluino is light and hadronizes before reaching the detector, it should be possible to observe its bound states, for example, \(R^0\)-mesons, created by binding of gluon and gluino \[8\]. Although the region of \(R^0\) masses is significantly restricted by KTeV measurements \[9\], \(R^0\) can still exist in the mass region \(1.4 - 2.2\) GeV \[10\].

If the squark masses are of the order of several hundreds GeV, the light mass of gluino can lead to the noticeable peaks in the dijet invariant mass and angular distributions at TEVATRON or LHC, arising due to the resonant production of massive squarks in the quark-gluino fusion \[11\]. The already existing TEVATRON data allows to exclude the light gluino models with the masses of the lighter squarks lying between 150 and 650 GeV \[11\]; it would be desirable to continue the search for the resonant peaks at TEVATRON, as well as at LHC, where the increased dijet production cross-section would allow to cover a larger region of squark masses.

In the pQCD theory, the existence of a light gluino would change the running of the strong coupling, as well as the form of the Dokshitser-Gribov-Lipatov-Altarelli-Parizi (DGLAP) equations. Therefore, to describe the existing DIS data, it is necessary to account not only for the quark and gluon distribution functions inside the initial hadron(s), but also for the gluino distribution which has different renormalization group properties. An obvious question is whether the currently available hadronic data is consistent with the existence of light gluino. The last analysis of this type was done in 1994 publications \[17,18\], which showed that the existence of the light gluino didn’t contradict DIS data available at that time. However, those analyses did not include the more recent data from H1, ZEUS and NMC experimental groups \[12–14\] covering the region of lower \(x\) and \(Q^2\). These new data can be crucial for testing the scenario of having a light gluino in the supersymmetry models, because the existence of a light gluino would imply a slower running of the parton distribution functions from the low to high \(Q^2\).

The existence of new types of particle interactions can be proved if one observes the violation of the symmetries of the Standard Model, for instance, the significant violation of the discrete symmetry with respect to space reflections (\(P\)-parity) in strong interactions. Experimental search for parity-violating effects could be performed relatively easy in the processes with \(t\)-quarks in the final state, due to the possibility to trace the polarization of the tops decaying through the channel \(t \rightarrow W^+ b\). Therefore, in this work we would like to concentrate on the production of top quark pairs. For the top quark pairs produced at hadron colliders, the SM allows the production processes \(q\bar{q}, GG \rightarrow G \rightarrow t\bar{t}\) to violate \(P\)-parity in the next-to-leading orders due to the presence of \(W\) and \(Z\) bosons in loop diagrams. However, this effect is shown to be negligible \[13\]. On the other hand, for certain choices of SUSY parameters in the MSSM, it is possible to obtain a large difference between the masses of right-stop and left-stop, which in principle can lead to some noticeable asymmetries in the production of right- and left-handed top quarks. These asymmetries arise either in the next-to-leading order of the SUSY QCD process \(q\bar{q}, GG \rightarrow \tilde{G} \rightarrow t\bar{t}\), or at the tree level of
the SUSY QCD process $\tilde{g}\tilde{g} \rightarrow t\bar{t}$. The asymmetries of the first type were studied earlier in [16]. It was shown that at TEVATRON the difference in the cross-sections of right- and left-handed $t$-quark production can be of the order $2 - 3\%$ provided the right-stop is light. This conclusion holds for a wide range of gluino masses. As it will be shown below, the asymmetries of the second type can only be noticeable if gluinos are light, and the parton density of gluinos in the nucleon is comparable with that of the sea quarks.

The primary goal of this article is to present the leading order (LO) study of the second scenario, and to evaluate the impact of the small mass of gluino on the production of $t$-quarks at the CERN Large Hadron Collider (LHC). In the first part of this study we obtained the LO distributions of the partons in the nucleon with the account for the possible non-zero contents of light gluinos. For this purpose we modified the fitting program used previously to obtain CTEQ4L parton distributions [19]. Since our study is the leading order calculation, we considered it sufficient not to perform the complete NLO analysis of parton distributions, contrary to what was done in [17,18].

In the course of the study, it was a surprise for us to find that the account for the new hadronic data from H1, ZEUS and NMC groups [12–14], which was not available at the time of the previous studies [17,18], tends to increase the overall $\chi^2$ of the fit after the inclusion of a light gluino. The reason for this is that these new data cover the region of lower $x$ and $Q^2$, thus making the analysis more sensitive to the slower running of the parton distributions in the SUSY QCD theory with a light gluino. Nonetheless, we would like to be extremely cautious about this observation and refrain from any final conclusions about the consistency of the current experimental data and the SUSY QCD theory with a light gluino before more thorough next-to-leading order global analysis of hadronic data is made. Instead, we would like to concentrate on the primary goal of this paper, namely, on the calculations of the top quark production asymmetries at the LHC. For this process, the Bjorken $x$ of the initial state partons are allowed to lie in the range

$$\frac{4m_t^2}{s} = 6.25 \cdot 10^{-4} \leq x_{1,2} \leq 1,$$

where the parton distributions are less dependent on the low $x$ data. We therefore expect the results of this work to be stable with respect to the possible changes in the parton distributions, and that these changes will not introduce an uncertainty more important than those coming from the other sources (e.g. next-to-leading order corrections).

After obtaining the parton distribution functions (PDFs) in the SUSY QCD theory with a light gluino, we calculate the degree of parity violation in the $t\bar{t}$ pairs produced via the LO $\tilde{g}\tilde{g} \rightarrow t\bar{t}$. Thus, the paper consists of 3 main sections: the description of the parton distribution functions for the SUSY QCD theory with a light gluino, the calculation of the cross-sections for the process $pp(\tilde{g}\tilde{g}) \rightarrow t_{L,R}\bar{t}$, and the numeric analysis of the asymmetries in the left- and right-handed top quark production. Finally, the conclusion summarizes the obtained results.
II. PARTON DISTRIBUTIONS

We start the construction of parton distributions by assuming that the only superparticle, actively present in the nucleons at the energies of the supercolliders, is gluino, with its mass much smaller than the typical scales of $t\bar{t}$ production (less than 1.5 GeV compared to $m_t \approx 175$ GeV). For the purpose of our calculation, we incorporate the gluino sector into the PDF evolution package, used recently to build the set of CTEQ4 unpolarized parton distributions [19]. In order to simplify the modifications in the fitting program, we used the approach close to the one adopted by the authors of GRV distributions [18]. The input scale $Q_0$ for the parton distributions was chosen to be lower than in CTEQ4L and equal to the mass of gluino (assumed to be $m_{\tilde{g}} = 0.5$ GeV in this study, unless stated otherwise). At this scale, the only input distributions are of gluons and lighter ($u$, $d$, $s$) quarks, while the non-zero PDFs of gluinos and heavy quarks are radiatively generated at scales above their mass thresholds.

In the presence of light gluino, two aspects of the leading-order evolution of parton distributions are different from that in the standard QCD. First, the one-loop $\beta$-function, determining the running of the strong coupling $\alpha_s$,

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda^2)},$$

now has the form

$$\beta_0 = 11 - \frac{2}{3} n_f - 2 n_{\tilde{g}},$$

where $n_f$ and $n_{\tilde{g}}$ are the number of active quark flavors and gluinos, respectively. In our analysis, we use $\alpha_s(M_Z) = 0.118$ and $\Lambda = 7.65$ MeV for 5 flavors. The matching of $\alpha_s$ between 4 and 5, and 5 and 6 flavors takes place at $Q = 5.0$ GeV and $Q = 175$ GeV respectively, which are defined as the bottom and top quark masses.

Second, the leading order DGLAP equations now should account for the splittings $\tilde{g} \rightarrow q\bar{q}$, $\tilde{g} \rightarrow g\tilde{g}$ and $g \rightarrow g\tilde{g}$, so that the singlet equation takes the form of

$$\frac{d}{dt} \begin{pmatrix} q_S(x, Q^2) \\ G(x, Q^2) \\ \tilde{g}(x, Q^2) \end{pmatrix} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \begin{pmatrix} P_{qq}(x/y) & P_{qG}(x/y) & P_{q\tilde{g}}(x/y) \\ P_{Gq}(x/y) & P_{GG}(x/y) & P_{G\tilde{g}}(x/y) \\ P_{\tilde{g}q}(x/y) & P_{\tilde{g}G}(x/y) & P_{\tilde{g}\tilde{g}}(x/y) \end{pmatrix} \times \begin{pmatrix} q_S(y, Q^2) \\ G(y, Q^2) \\ \tilde{g}(y, Q^2) \end{pmatrix} \frac{dy}{y}.$$  

The splitting functions used in (4) can be found, e.g., in [20].

With the help of the upgraded evolution package we performed the fit of the experimental data, closely following the procedure of the construction of CTEQ4L PDF set, as described in [19]. However, for simplicity, the fit didn’t use the jet data, and the value of the strong coupling was fixed to be equal to the world-average value $\alpha_s(M_Z) = 0.118$. As a result, we obtained the set of parton distributions SUSYL, which was used throughout the rest of the paper.
III. CALCULATION OF THE MATRIX ELEMENTS

At the LHC, the $t\bar{t}$ pairs will be dominantly produced from the standard QCD processes of $q\bar{q}$ and $GG$ interactions, as shown in Figs. 1a-d. All of these diagrams preserve $P$-parity. In the SUSY QCD theory, if the mass of gluino is small ($m_{\tilde{g}} \sim 1 \text{ GeV}$), we will also expect a noticeable contribution due to the annihilation of gluinos, described by the three diagrams of Figs. 1e-g. The $s$-channel diagram of Fig. 1e is equivalent, up to a color factor, to the analogous $q\bar{q}$ diagram of Fig. 1a and doesn’t break parity; however, the parity symmetry is broken in the $t$- and $u$-channels due to the mechanism of squark mass mixing which is briefly described below.

In MSSM the left-squark and the right-squark, superpartners of the the left- and right-handed quarks, do not have definite mass but instead are a mixture of two mass eigenstates. These mass eigenstates $\tilde{q}_1$ and $\tilde{q}_2$ are related to the current eigenstates $\tilde{q}_L$ and $\tilde{q}_R$ by

$$\tilde{q}_1 = \tilde{q}_L \cos \theta_q + \tilde{q}_R \sin \theta_q, \quad \tilde{q}_2 = -\tilde{q}_L \sin \theta_q + \tilde{q}_R \cos \theta_q. \quad (5)$$

Due to this, MSSM in general allows to have nonzero asymmetries in $q\bar{q}$-pair production, defined by

$$A_q = \frac{\sigma(pp \to q_L\bar{q}) - \sigma(pp \to q_R\bar{q})}{\sigma(pp \to q_L\bar{q}) + \sigma(pp \to q_R\bar{q})}, \quad (6)$$

where $\sigma$ denotes the cross-section of the $t\bar{t}$-pair production, integrated over the relevant part of the phase space to be discussed below. The best chance to observe a non-zero $A_q$ is provided by the $t\bar{t}$ production process, where the mixing between the squarks is the largest due to the large mass of the top quark. In the following, we ignore the mass mixing for the lighter 5 quarks.

In MSSM, the squark-quark-gluino interaction Lagrangian is given by

$$L_{\tilde{g}q} = -g_s T^a_{jk} \bar{q}_k [(a_1 - b_1 \gamma_5)\tilde{q}_1 j + (a_2 - b_2 \gamma_5)\tilde{q}_2 j] \tilde{g}_a + h.c., \quad (7)$$

where $g_s$ is the strong coupling constant, $T^a$ are $SU(3)_C$ generators and $a_1$, $b_1$, $a_2$, $b_2$ are given by

$$a_1 = \frac{1}{\sqrt{2}} (\cos \theta_q - \sin \theta_q) = -b_2, \quad b_1 = -\frac{1}{\sqrt{2}} (\cos \theta_q + \sin \theta_q) = a_2. \quad (8)$$

The mixing angle $\theta_t$ and the masses $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$ can be calculated by diagonalizing the following mass matrix:

$$\begin{pmatrix} M^2_{\tilde{t}_L} & m_t m_{LR} \\ m_t m_{LR} & M^2_{\tilde{t}_R} \end{pmatrix}, \quad (9)$$
where $M_{{t_{L,R}}}^2$ and $m_{LR}$ are the parameters of the soft-breaking terms in the MSSM.

From Eq. (9), we can derive the expressions for $m_{{t_{1,2}}}^2$ and $\theta_t$:

$$m_{{t_{1,2}}}^2 = \frac{1}{2} \left[ M_{{t_{L}}}^2 + M_{{t_{R}}}^2 \mp \sqrt{(M_{{t_{L}}}^2 - M_{{t_{R}}}^2)^2 + 4m_t^2m_{LR}^2} \right],$$

(10)

$$\tan \theta_t = \frac{m_{{t_{1}}}^2 - M_{{t_{L}}}^2}{m_t m_{LR}}.$$  

(11)

Inversely,

$$M_{{t_{R,L}}}^2 = \frac{1}{2} \left[ m_{{t_{2}}}^2 + m_{{t_{2}}}^2 \mp \sqrt{(m_{{t_{2}}}^2 - m_{{t_{1}}}^2)^2 - 4m_t^2m_{LR}^2} \right].$$  

(12)

The asymmetry $A_t$ depends on the angle of mixing $\theta_t$ in the following manner. The denominator of $A_t$ is dominated by the large contributions from the quark and gluon channels (Figs. 1a-d) and therefore shows little dependence on the masses of stops. The numerator of the asymmetry depends both on the splitting and the mixing of the squark masses. Since left/right-handed quarks couple only to left/right squarks, in the case of no mass mixing ($m_{LR} = 0$) the asymmetry is completely determined by the difference of masses $M_{{t_{L}}} - M_{{t_{R}}}$. In this limit $\theta_t \approx -\pi/2$, provided that $M_{{t_{R}}} < M_{{t_{L}}}$.

For fixed mass eigenvalues $m_{{t_{1,2}}}$, the relationship (12) for the mass parameters $M_{{t_{L,R}}}$ puts the upper bound on $m_{LR}$:

$$m_{LR} \leq \frac{m_{{t_{2}}}^2 - m_{{t_{1}}}^2}{2m_t}.$$  

(13)

For largest $m_{LR}$,

$$\theta_t = -\frac{\pi}{4}$$  

(14)

and

$$M_{{t_{L}}} = M_{{t_{R}}}. $$  

(15)

In this limit, stop mass eigenstates have the maximal mixing between the left- and right-stops, so that the asymmetry $A_t$ becomes zero. Thus, for fixed mass eigenstates, the asymmetries are expected to decrease with the growth of $m_{LR}$.

In gravity-mediated supersymmetry breaking models (mSUGRA), the masses of left- and right-stops satisfy the relations

$$M_{{t_{L}}}^2 = m_{{t_{L}}}^2 + m_t^2 + \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \cos(2\beta) \right) m_Z^2,$$

$$M_{{t_{R}}}^2 = m_{{t_{R}}}^2 + m_t^2 + \frac{2}{3} \sin^2 \theta_W \cos(2\beta) m_Z^2,$$

$$m_{LR} = -\mu \cot \beta + \lambda_t.$$  

(16)
where \( m_{\tilde{t}_L}^2, m_{\tilde{t}_R}^2 \) are the soft SUSY-breaking mass terms of left- and right-stops, \( \mu \) is the coefficient of the \( H_1-H_2 \) mixing term in the superpotential, \( \lambda_t \) is the parameter describing the strength of soft SUSY-breaking trilinear scalar interaction \( \tilde{t}_L \tilde{t}_R H_2 \), \( \tan \beta = v_u/v_d \) is the ratio of the vacuum expectation values of the two Higgs doublets. In the minimal supergravity models, the soft SUSY breaking parameters \( m_{\tilde{q}_L}^2 \) and \( m_{\tilde{q}_R}^2 \) are equal to each other, so that the mass splitting \( M_{\tilde{q}_L}^2 - M_{\tilde{q}_R}^2 \) is small, and of the same order of magnitude for all quark flavors. In this case, it is hard to expect observable asymmetries. On the other hand, in the general MSSM right- and left-squark masses \( M_{\tilde{t}_{L,R}} \) are considered to be independent parameters, in which case there is no theoretical limitations on the splitting of stop masses. In the following, the second point of view is accepted, so that \( M_{\tilde{t}_R} \) is assumed to be of the order 90-175 GeV, while \( M_{\tilde{t}_L} \) is varied between 150 and 1000 GeV.

The cross-sections entering the asymmetry (16) are calculated in a usual way by convolution of the squared and spin- and color-averaged hard scattering matrix elements \( |\mathcal{M}_{k_1k_2}|^2_{L,R} \) with the appropriate parton distributions \( f_i(x) \):

\[
\sigma(pp \to t_{L,R}\tilde{t}) = \frac{\beta}{32\pi s} \int_{-1}^1 d\cos \theta \int dx_1 dx_2 \sum_{i_1,i_2} f_{i_1}(x_1) f_{i_2}(x_2) |\mathcal{M}^{i_1i_2}(s,\tilde{t},\tilde{u})|^2_{L,R},
\]

where \( s, \tilde{t}, \tilde{u} \) are the parton Mandelstam variables, \( \beta \equiv \sqrt{1 - 4m_t^2/s} \) and the particle momenta for the partons \( q_{i_1,2} \) in the initial state are defined as \( q_{i_1}(p_1) + q_{i_2}(p_2) \rightarrow t(p_3) + \tilde{t}(p_4) \). The numerator of the asymmetry (16) is determined solely by the diagrams Figs. 1f,g (containing stops), which give the following matrix elements for the production of left-handed top:

\[
\left( \mathcal{M}_t \mathcal{M}_u^\dagger + \mathcal{M}_u \mathcal{M}_t^\dagger \right)_{L} = \sum_{i,j} \frac{a_i^2 a_j^2}{(s - m_{t_i}^2)(\tilde{u} - m_{t_j}^2)}
\times \left( 4m_t^2 \hat{s}(1 - C_i C_j) \left( (1 - C_i C_j) + (C_i - C_j) \cos \theta \right)
-2(m_t^2 - \hat{t})(m_t^2 - \hat{u})(1 - C_i^2)(1 - C_j^2) \right),
\]

\[
|\mathcal{M}_t|_{L}^2 = \sum_{i,j} \frac{a_i^2 a_j^2}{2(\hat{t} - m_{t_i}^2)(\hat{t} - m_{t_j}^2)} (m_t^2 - \hat{t})\hat{s}(1 + C_i C_j) (A + B \cos \theta),
\]

\[
|\mathcal{M}_u|_{L}^2 = \sum_{i,j} \frac{a_i^2 a_j^2}{2(\hat{u} - m_{t_i}^2)(\hat{u} - m_{t_j}^2)} (m_t^2 - \hat{u})\hat{s}(1 + C_i C_j) (A - B \cos \theta).
\]

In these formulas

\[
C_i \equiv \frac{b_i}{a_i}, \quad i = 1, 2,
A \equiv (1 + C_i)(1 + C_j)(1 - \beta) + (1 - C_i)(1 - C_j)(1 + \beta),
B \equiv (1 + C_i)(1 + C_j)(1 - \beta) - (1 - C_i)(1 - C_j)(1 + \beta),\]
the summation \((i, j)\) goes over the two stop masses. The squared matrix element \(|\mathcal{M}_{t\bar{t}}|_L^2\) entering (17) can be written in terms of (18-20) as

\[
|\mathcal{M}_{t\bar{t}}|_L^2 = \frac{1}{256} \left( \frac{16}{3} |\mathcal{M}_t|^2 + |\mathcal{M}_u|^2 \right) + \frac{2}{3} \left( \mathcal{M}_t \mathcal{M}_t^\dagger + \mathcal{M}_u \mathcal{M}_u^\dagger \right)_L.
\]

(24)

The matrix elements for the production of right-handed top are obtained by the substitution

\[
C_{i,j} \to -C_{i,j}.
\]

(25)

If (18-20) are combined with the explicit formulas (8) for \(a_i, b_i\), it is possible to get the following expression for the difference of the matrix elements for producing the left- and right-handed top quarks in the \(t\bar{t}\) pairs:

\[
|\mathcal{M}_{t\bar{t}}|_L^2 - |\mathcal{M}_{t\bar{t}}|_R^2 = 4 \cos 2\theta_t \left\{ (X_{11} - X_{22})(\beta - \cos \theta) + (Y_{21} - Y_{12}) \cos \theta + (Z_{11} - Z_{22})(\beta + \cos \theta) \right\},
\]

(26)

where

\[
X_{ij} \equiv \frac{(m_i^2 - \hat{t})\hat{s}}{96(t - m_i^2)(t - m_j^2)},
\]

(27)

\[
Y_{ij} \equiv \frac{m_i^2 \hat{s}}{192(t - m_i^2)(\hat{u} - m_j^2)},
\]

(28)

\[
Z_{ij} \equiv \frac{(m_i^2 - \hat{u})\hat{s}}{96(\hat{u} - m_i^2)(\hat{u} - m_j^2)}.
\]

(29)

Equation (26) depends on the mass mixing angle \(\theta_t\) only through the common factor \(\cos 2\theta_t\). This proves the argument given before that for fixed \(m_{i,2}\) the asymmetry should be the largest at \(m_{LR} = 0\) and \(\theta_t = -\pi/2\).

The diagrams in Figs. 1a-e do not violate the parity and need to be included only in the denominator of the asymmetry (8). The matrix elements for the pure QCD processes (in Figs. 1a-d) are well-known, while the \(s\)-channel \(g\bar{g}\) diagram (in Fig. 1e) differs from the analogous \(q\bar{q}\) one only by a color factor:

\[
|\mathcal{M}_{\bar{g}g}|^2 = \frac{4}{9} \frac{(m_i^2 - \hat{t})^2 + (m_i^2 - \hat{u})^2 + 2m_i^2 \hat{s}}{\hat{s}^2},
\]

(30)

\[
|\mathcal{M}_{GG}|^2 = \frac{1}{16} \left( \frac{(m_i^2 - \hat{t})(m_i^2 - \hat{u})}{12\hat{s}^2} + \frac{8}{3} \frac{(m_i^2 - \hat{t})(m_i^2 - \hat{u}) - 2m_i^2(m_i^2 + \hat{t})}{(m_i^2 - \hat{t})^2} \right) \\
+ \frac{8}{3} \frac{(m_i^2 - \hat{t})(m_i^2 - \hat{u}) - 2m_i^2(m_i^2 + \hat{u})}{(m_i^2 - \hat{t})^2} - \frac{2}{3} \frac{m_i^2(\hat{s} - 4m_i^2)}{(m_i^2 - \hat{t})(m_i^2 - \hat{u})} \\
- \frac{6}{\hat{s}(m_i^2 - \hat{t})} \frac{(m_i^2 - \hat{u})(m_i^2 - \hat{t}) + m_i^2(\hat{u} - \hat{t})}{\hat{s}(m_i^2 - \hat{u})} \\
- \frac{6}{\hat{s}(m_i^2 - \hat{u})} \frac{(m_i^2 - \hat{u})(m_i^2 - \hat{t}) - m_i^2(\hat{u} - \hat{t})}{\hat{s}(m_i^2 - \hat{u})},
\]

(31)

\[
|\mathcal{M}_{\bar{g}g}|^2 = \frac{27}{32} |\mathcal{M}_{\bar{q}q}|^2.
\]

(32)
In the above, the spin and color factors in both the final and the initial states are all properly summed and averaged.

One can also obtain the total parton cross-sections by the integration of (18-20, 30-32) over the scattering angle $\theta$. For the $t$ and $u$-channels we define

\[
C \equiv 2(1 - \beta^2)(1 - C_iC_j)(C_i - C_j) \\
D \equiv \beta^2(1 - C_i^2)(1 - C_j^2) \\
E \equiv 2(1 - \beta^2)(1 - C_iC_j^2) - (1 - C_i^2)(1 - C_j^2),
\]

\[
v_i(\hat{s}, \beta) \equiv \frac{2m_{t_i}^2 + \hat{s} + \beta\hat{s} - 2m_{t_i}^2}{2m_{t_i}^2 + \hat{s} - \beta\hat{s} - 2m_{t_i}^2}.
\]

Then

\[
[\sigma_{\tilde{g}\tilde{g}}]_L = \sum_{i,j} \frac{g_s^4 a_i^2 a_j^2}{24576\pi} [8(1 + C_iC_j)f_1(m_{t_i}^2, m_{t_j}^2) + f_2(m_{t_i}^2, m_{t_j}^2)],
\]

\[
f_1(m_{t_i}^2, m_{t_j}^2) = \frac{1}{\beta(m_{t_i}^2 - m_{t_j}^2)} \left( -\frac{8B}{\hat{s}} \beta(m_{t_i}^2 - m_{t_j}^2) \\
+ 4(m_{t_i}^2 - m_{t_j}^2)(2Bm_{t_i}^2 + B\hat{s} + A\beta\hat{s} - 2Bm_{t_j}^2)/\hat{s}^2 \ln v_i(\hat{s}, \beta) \\
- 4(m_{t_j}^2 - m_{t_i}^2)(2Bm_{t_j}^2 + B\hat{s} + A\beta\hat{s} - 2Bm_{t_i}^2)/\hat{s}^2 \ln v_j(\hat{s}, \beta) \right),
\]

\[
f_2(m_{t_i}^2, m_{t_j}^2) = \frac{1}{\beta^2} \left\{ -\frac{8D}{\hat{s}} \beta + \left[ 2E\beta^2\hat{s}^2 + (4m_{t_i}^2 + 2\hat{s} - 4m_{t_i}^2) \times (2Dm_{t_i}^2 + D\hat{s} + C\beta\hat{s} - 2Dm_{t_j}^2) \right] \ln v_i(\hat{s}, \beta) \\
+ \left[ 2E\beta^2\hat{s}^2 + (4m_{t_j}^2 + 2\hat{s} - 4m_{t_j}^2) \times (2Dm_{t_j}^2 + D\hat{s} - C\beta\hat{s} - 2Dm_{t_i}^2) \right] \ln v_j(\hat{s}, \beta) \right\}.
\]

When $m_{t_i} = m_{t_j}$, we have

\[
f_1(m_{t_i}^2 = m_{t_j}^2) = \frac{1}{\beta} \left\{ -\frac{8B}{\hat{s}} \beta + 4(4Bm_{t_i}^2 + B\hat{s} + A\beta\hat{s} - 4Bm_{t_i}^2)/\hat{s}^2 \ln v_i(\hat{s}, \beta) \\
+ 8(m_{t_i}^2 - m_{t_i}^2)(2Bm_{t_i}^2 + B\hat{s} + A\beta\hat{s} - 2Bm_{t_i}^2) \times (\frac{1}{2m_{t_i}^2 + \hat{s} + \beta\hat{s} - 2m_{t_i}^2} - \frac{1}{2m_{t_i}^2 + \hat{s} - \beta\hat{s} - 2m_{t_i}^2})/\hat{s}^2 \right\}.
\]

Again, $[\sigma_{\tilde{g}\tilde{g}}]_R$ can be obtained by the substitution

\[
C_i \rightarrow -C_i, \quad C_j \rightarrow -C_j.
\]
The cross-sections of the other sub-processes, corresponding to (31-32), are given by

\[ \sigma_{\bar{q}q} = \frac{g_s^4}{108 \pi s} \beta (2 + \rho) \]

\[ \sigma_{GG} = \frac{g_s^4}{48 \pi s} \left( (1 + \rho + \frac{\rho^2}{16}) \ln \frac{1 + \beta}{1 - \beta} - \beta \left( \frac{7}{4} + \frac{31}{16} \rho \right) \right) \]

\[ \sigma_{\tilde{g}\tilde{g}} = \frac{g_s^4}{128 \pi s} \beta (2 + \rho) \]

where \( \rho \equiv 4m_t^2/\hat{s} \).

### IV. NUMERIC RESULTS

To estimate the largest possible asymmetries, we varied the squark mass eigenvalues \( m_{\tilde{t}_1,2} \) with \( m_{LR} \) set to be zero (see the discussion in the previous Section). No assumption was made about any model-specific relationships between the values of the mass parameters \( M_{\tilde{t}_{1,2}} \) and \( m_{LR} \) (c.f. eq. (17)).

If \( m_{LR} = 0 \), the left/right-handed quarks couple independently to the left/right-stops. Correspondingly, for \( m_{\tilde{t}_1} \neq m_{\tilde{t}_2} \), the production rates of the left- and right-handed tops will be different. The asymmetry \( A_t \) is expected to grow when the mass splitting \( m_{\tilde{t}_1} \neq m_{\tilde{t}_2} \) increases. In this work, the asymmetries were calculated for \( m_{\tilde{t}_1} = 90 \, \text{GeV} \) (which is consistent with the current LEP2 data [21]), \( m_{t} = 175 \, \text{GeV} \) and various values of \( m_{\tilde{t}_2} \). Two values of factorization scale \( \mu = m_t \) and \( 2m_t \) were used. Various sets of masses will be further denoted as \( (m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{LR}) \), with numerical values in GeV. As before, the gluino mass is assumed to be equal to 0.5 GeV.

In the SM, both \( t \) and \( \bar{t} \) decay into \( b (\bar{b}) \) and \( W^\pm \) with an almost unit probability, with a subsequent decay of the W-bosons into 2 jets or 2 leptons. In the MSSM, when both the gluino and the stop are light, the top quark can also decay via \( t \rightarrow \tilde{g} \tilde{t}_1 \), so that the branching ratio of \( t \rightarrow W^+ + b \) decreases. Assuming that all the other supersymmetric particles are heavier than the top quark, \( \theta_t = -\pi/2 \), the branching ratio for \( t \rightarrow W^+ + b \) is equal to 0.29 and 1 for \( m_{\tilde{t}_1} = 90 \, \text{GeV} \) and 175 GeV, respectively. The CDF collaboration has measured the branching ratio of \( t \rightarrow W^+ + b \) to be \( 0.87^{+0.13+0.13}_{-0.30-0.11} \) [23]. Hence, the chosen sets of the values for \( m_{\tilde{t}_1} \) and \( m_{\tilde{g}} \) are still allowed by data within 95\% c.l.

It is convenient to study the asymmetry \( A_t \) using the semileptonic modes of decay, with \( t \rightarrow b l^+ \nu_l \) \((l = e, \mu)\) and \( \bar{t} \rightarrow \bar{b} q \bar{q} \) (or vice versa), which have a branching ratio of about 0.086 and 24/81 for \( m_{\tilde{t}_1} = 90 \, \text{GeV} \) and 175 GeV, respectively. In the following we assume that it will be possible to reconstruct the kinematics of \( t \bar{t} \)-pair from the momenta of the decay products by requiring the transverse momenta \( p_T^{jets} \geq 30 \, \text{GeV} \), \( p_T^{leptons} \geq 20 \, \text{GeV} \), \( E_T \geq 20 \, \text{GeV} \), the rapidities of the jets and leptons \(|y| < 2.0\), and the jet cone separation \( \Delta R > 0.4 \) [22]. We also assume that it will be necessary to tag one \( b \)-quark with an efficiency \( C_b = 50\% \). We estimate the statistical error in the measurement of the asymmetry by

\[ \delta A_t = \frac{1}{\sqrt{LC T C_b (\sigma_{tot}^L + \sigma_{tot}^R)}} \]
where we assume the observation time \( T = 1 \) year and the luminosity \( \mathcal{L} = 100 \text{ fb}^{-1}/\text{year} \), corresponding to the second run of the LHC.

The imposed selection cuts and branching ratio significantly reduce the total cross-section of \( tt \) production, typically from around 340 pb down to 3.5 and 12 pb for \( m_{\tilde{t}_1} = 90 \text{ GeV} \) and 175 GeV, respectively. As an example, Fig. 2 shows various differential cross-sections including the \( GG, q\bar{q} \) and \( g\bar{g} \) subprocesses for the squark masses \((90, 1000, 0)\) and \( \mu = 2m_{\tilde{t}} \), obtained with the kinematic cuts and branching ratios applied. It can be readily seen that the dominant part of the \( tt \) pairs is produced due to the gluon-gluon subprocess, which contributes around 71% of the total rate. The quark-antiquark and gluino-gluino shares are 22% and 7%, respectively. The gluino contribution is comparable with the conventional QCD uncertainties in the knowledge of the total rate (about 5% to 10%), however, the presence of the light gluino will change the shape of the cross-section distributions. Thus, in principle there is a possibility to detect the light gluino by carefully fitting the event rate distributions and comparing them with the predictions of perturbative QCD.

Fig. 3 shows the sum of the differential cross-sections of left- and right-handed top quarks production \( d\sigma_L/dM_{\tilde{t}} + d\sigma_R/dM_{\tilde{t}} \), and their difference \( d\sigma_L/dM_{\tilde{t}} - d\sigma_R/dM_{\tilde{t}} \) (scaled by a factor of 100), as functions of the invariant mass of the \( tt \) pair \( M_{\tilde{t}} \). One can see that the asymmetry is most noticeable in the region of small and intermediate values of \( M_{\tilde{t}} \). This is different from the behavior of the asymmetry produced due to the presence of superpartners in the loop corrections [16]. In that case, the asymmetry becomes significant in the region of large \( M_{\tilde{t}} \), where in the case of the light gluino it can have the value of 2−3%. In this respect, we expect the minimal interference between the tree-level and loop-generated asymmetries, since the main contributions to them come from different kinematic regions.

The dependence of the cross-sections on two other kinematic parameters, the transverse momentum of the \( t \)-quark and the cosine of the scattering angle in the \( tt \) rest frame, is shown in Figs. 4 and 5. As can be seen from Fig. 5, the difference \( d\sigma_L/d\cos\theta - d\sigma_R/d\cos\theta \) changes its sign around \( \cos\theta \approx \pm 0.8 \), so that one can enhance the asymmetry by separately considering the cross-sections integrated over either large or small angles. It can also be shown that the asymmetries at small angles can be further enlarged by rejecting the events with transverse momenta larger than 100 GeV/c. For the other combinations of stop masses, \( d\sigma_L/\cos\theta - d\sigma_R/\cos\theta \) changes its sign at slightly lower \( |\cos\theta| \), approximately 0.75−0.8. We therefore present the asymmetries of the cross-sections integrated separately over the region \( |\cos\theta| \leq 0.8 \), or the region \( |\cos\theta| > 0.8 \) with \( p_T \leq 100 \text{ GeV/c} \).

Table 1 shows the values of the asymmetry \( A_t \) obtained after the integration of the rate with the aforementioned cuts in \( |\cos\theta| \) and \( p_T \). As can be seen from the Table, for various stop masses the asymmetry ranges from 0.3% to 1.1%. The behavior of the asymmetries with the growth of \( m_{\tilde{q}} \) is different at large and small angles. At \( |\cos\theta| \leq 0.8 \) the asymmetry monotonously increases with the growth of \( m_{\tilde{q}} \), while at \( |\cos\theta| > 0.8 \) the asymmetry has a maximum around \( m_{\tilde{q}} = 200 \text{ GeV} \) and then starts to decrease. At small angles \( (|\cos\theta| \leq 0.8) \) the asymmetry quickly decreases with the growth of the mass of the lighter squark and becomes practically unnoticeable for \( m_{\tilde{q}} \geq m_{\tilde{t}} \).

For the comparison, we also give in the same Table the statistical errors \( \delta A_t \) from Eq. (10). These errors are mostly determined by \( GG \) and \( q\bar{q} \) cross-sections, so that they hardly depend on the choice of the squark masses. For most combinations of the stop masses, the
obtained values of $A_t$ can in principle be distinguished from the statistical error $\delta A_t$ at a $2\sigma$ level or better. However, what can be more important are the experimental systematic uncertainties related to the measurement of the asymmetries of the order 1%. In particular, it can be challenging to reach the necessary accuracy in the reconstruction of the kinematics of the $t\bar{t}$-pair, and the determination of the top quark polarization. Nevertheless, the predictive power of this analysis can be increased if it is combined with the search for the signature of the light gluinos in the other kinematic regions, for instance, for the loop-generated asymmetries in the production of the top-antitop pairs with large invariant masses.

V. CONCLUSION

In this work we propose a new method, based on the search of the possible violations of the discrete symmetries of the Standard Model, to test the existence of a light gluino in the MSSM. This is in contrast to many other methods presented in the literature (see the Introduction section), in which one has to assume how a light gluino hadronizes into hadron states to be compared with the experimental measurement.

We study the consequences the small mass of the gluino would have for the production of top quarks at the LHC via the tree level process $\tilde{g}\tilde{g} \rightarrow t\bar{t}$. We show that with a large mass splitting in the masses of superpartners (top-squarks) of the top quark, the gluino-gluino fusion process can generate the parity-violating asymmetry in the production of left- and right-handed $t$-quarks. Since the SM QCD theory preserves the discrete symmetry of $P$-parity, a small violation of such a symmetry may be observed from a large $t\bar{t}$ data sample at the LHC.

For $m_{\tilde{g}} \approx 0$ the largest values of the parity-violating asymmetry discussed in the previous sections is around $0.3 - 1.1\%$ for various choices of SUSY parameters. Hence, it can in principle be observed, taking into the account the high rate of the top production at the LHC. In order to measure the asymmetry with a small statistical error, the experiment should be preferably done during the second run of LHC with an integrated luminosity of $100 \text{ fb}^{-1}/\text{year}$. The rate of the top quark production does not seem to be the major obstacle for the measurement of the parity-violating asymmetry. However, it demands a good understanding of the systematic errors, better than $1\%$, to reach the precision of the measurement sufficient to test the existence of a light gluino in $t\bar{t}$ pair production.
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FIGURE CAPTIONS

Fig. 1. Leading order diagrams contributing to the production of top quarks in SUSY QCD theory.

Fig. 2. The dependence of the cross-section of $t\bar{t}$ pair production on various kinematic parameters: $t\bar{t}$ pair invariant mass $M_{t\bar{t}}$, $t$-quark transverse momentum $p_T$ and rapidity $y$. The solid line, stars, circles and dashed line correspond to the full differential cross-section and the contributions of gluon, quark and gluino subprocesses, respectively. The factorization scale $\mu = 2m_t$, the squark masses are (90,1000,0).

Fig. 3. Dependence of the sum $d\sigma_L/dM_{t\bar{t}} + d\sigma_R/dM_{t\bar{t}}$ (solid line) and the difference $d\sigma_L/dM_{t\bar{t}} - d\sigma_R/dM_{t\bar{t}}$ (dashed line, magnified by 100) of the differential cross-sections of the production of the left- and right-handed tops on the invariant mass of the $t\bar{t}$ pairs $M_{t\bar{t}}$. The factorization scale $\mu = 2m_t$, the squark masses are (90,1000,0).

Fig. 4. Dependence of the sum $d\sigma_L/dp_T + d\sigma_R/dp_T$ (solid line) and the difference $d\sigma_L/dp_T - d\sigma_R/dp_T$ (dashed line, magnified by 100) of the differential cross-sections of the production of the left- and right-handed tops on the transverse momentum of the $t$-quark $p_T$. The factorization scale $\mu = 2m_t$, the squark masses are (90,1000,0).

Fig. 5. Dependence of the sum $d\sigma_L/d \cos \theta + d\sigma_R/d \cos \theta$ (solid line) and the difference $d\sigma_L/d \cos \theta - d\sigma_R/d \cos \theta$ (dashed line, magnified by 100) of the differential cross-sections of the production of the left- and right-handed tops on the cosine of the scattering angle in the $t\bar{t}$ pair rest frame. The factorization scale $\mu = 2m_t$, the squark masses are (90,1000,0).
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\[
\mu = m_t \quad \mu = 2m_t
\]

| Masses (GeV) \[m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{LR}\] | \(\cos \theta \leq 0.8\) | \(\cos \theta > 0.8\) | \(\cos \theta \leq 0.8\) | \(\cos \theta > 0.8\) |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| \(90,150,0\)   | -0.31           | 1.14            | -0.35           | 1.20            |
| \(90,200,0\)   | -0.57           | 1.30            | -0.62           | 1.42            |
| \(90,500,0\)   | -1.31           | 1.20            | -1.39           | 1.33            |
| \(90,1000,0\)  | -1.50           | 1.05            | 1.61            | 1.20            |
| \(175,250,0\)  | -0.33           | -              | -0.36           | 1.14            |
| \(175,500,0\)  | -0.86           | -              | 0.91            | 1.14            |
| \(175,1000,0\) | -1.04           | -              | 1.11            | 1.14            |

**TABLE I.** The asymmetries \(A_t\) (in \%) predicted by SUSY QCD, and the estimated statistical errors of their measurement \(\delta A_t\) for the LHC luminosity \(\mathcal{L} = 100 \ f b^{-1}/year\). The entries with the hyphen correspond to the asymmetries which are too small to be observed.
Fig. 1
Fig. 2
Fig. 3
Fig. 4
Fig. 5