Doubly Charged Fermions Bound States, Vector Bosons Bound States

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Abstract

We consider positronium-like bound states of doubly charged fermions. We consider also $P(\text{CP})$-parity violation in positronium like system (including quarkonium and lepton antilepton bound states and mesoatoms) which take place due to $Z^0$-bosons or Higgs bosons exchange. By model independent way we consider also vector bosons bound states in the Coulomb and Higgs potential and have shown that in the presence of the Higgs potential effective potential become less singular and in this case fall down on the center is absent. Vector bosons energy levels in the magnetic field are considered. Considered magnetism of electron gas in the finite volume.

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Doubly charged fermions bound states

It is known that left-right supersymmetric models predict the existence of doubly charged fermions superpartners of doubly charged Higgs bosons in the left-right supersymmetric models (for left-right supersymmetric model see [1],[2] and references therein, for pair production of doubly charged fermions in $e^+e^-$ colliders and its subsequent decays see [2],[5] and references therein).

In this paper we consider bound states which consist of doubly (and singly) charged Higgs bosons (for triplet Higgs bosons see [3] and [4]) and its superpartners $\delta_L^{++}\delta_L^{--}$, $\delta_L^{+-}\delta_L^{-+}$, $\delta_R^{++}\delta_R^{--}$, $\delta_R^{+-}\delta_R^{-+}$,”charginium” (i.e. bound state which consist of $\tilde{\chi}^+\tilde{\chi}^-$, for review of supersymmetric theories see e.g. [6],[7] and references therein).

This bound state are positronium-like, and for its binding energy we obtain from appropriate formula for positronium levels (For positronium levels see e.g. [8]) by substitution $\alpha \to Q^2$ the following result:

$$\Delta E_{n,l} = -\frac{\alpha^2 Q^4 m_H}{4n^2} - \frac{\alpha^4 Q^8 m_H}{2n^3} \left( \frac{1}{2l + 1} - \frac{11}{32n} \right)$$

(1)

where $m_H$ is the mass of $\delta_{L,R}^{--}$-bosons or its superpartners.

As we show below however the effect of vacuum polarization which is small in case of positronium in case of heavy fermions is larger than the correction of order $\frac{v^2}{c^2}$ described by last term in (1) (For vacuum polarization see e.g. [8]).

We consider also P,CP-violation in heavy fermion bound states, in $l_i^- l_j^+$ bound states and in mesoatoms (for $\mu^+e^-$ atoms and mesoatoms see [23],[9] respectively).

We would like to stress that the width of the $\delta_{L,R}^{--}$-bosons, decays $\Gamma(\delta_{L,R}^{--} \to$
\[ l^-l^- = \frac{\hbar^2}{8\pi} m_H \ll E_{n,l} \] at sufficiently small Yukawa coupling \( h \) and system may be treated as the bound state. The width of the doubly charged fermions decays \( \delta_{L,R}^- \rightarrow l^-l^- \) is the same order (for doubly charged fermions decays see [2]). The main contribution to the width of bound state formed from triplet Higgs bosons or its superpartner comes predominantly from its own decays into leptons. The bound states with quantum numbers \( J^{PC} = 1^{--} \) may be produced at \( e^+e^- \) through virtual \( \gamma, Z^0 \)-bosons exchanges in resonance.

At \( m_H >> m_Z \) the width of the decay of bound state of doubly charged fermions has the following form:

\[
\Gamma(\delta_{L,R}^+\delta_{L,R}^- \rightarrow l^+l^-) = \frac{\alpha^5 Q^8 m_H}{6m^3} \left( (1 + Q^{-1} g_V g_V^c) + Q^{-2} (g_A g_A^c)^2 \right) \ (2)
\]

where \( m_H \) is the mass of the doubly charged fermion, \( g_V, g_A(g_V^c, g_A^c) \)-are vector and pseudovector couplings of doubly charged fermion (leptons) to \( Z^0 \)-bosons (we parametrize interaction of \( Z \)-bosons with fermions in the form: \( L = e f \bar{f} \hat{Z} (g_V + g_A \gamma_5) f \)). From decay widths of orto- and para-positronium it is easy to obtain that: the widths of the decay into 2\( \gamma \), 3\( \gamma \) is enhanced in comparison with para- and ortopositronium decay widths (into 2\( \gamma \), 3\( \gamma \)) in \( Q^{10}, Q^{12} \times \) times, respectively.

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1Bound state which consist of doubly charged Higgs bosons at \( L = 0 \) have quantum numbers \( J^{PC} = 0^{++} \) and can not be produced at \( e^+e^- \)-collisions in resonance, however in resonance may be produced P-wave bound state of doubly charged Higgs bosons. Its decay width into \( l^+l^- \) may obtained from the width of squarkonium into \( l^+l^- \), in squarkonium decay into leptons also must be taken into account the effect of photon and \( Z \)-boson exchange (for squarkoniums, its production and decays see e.g. [3], [4] and references therein). We have

\[
\Gamma(\delta_{L,R}^+\delta_{L,R}^- \rightarrow l^+l^-) = \frac{\alpha^7 Q^{12} m_H}{32 m^5} \left( (1 + Q^{-1} g_V g_V^c)^2 + Q^{-2} (g_A g_A^c)^2 \right) \]

\[
= \frac{\alpha^7 Q^{12} m_H}{32 m^5} \left( (1 + Q^{-1} g_V g_V^c)^2 + Q^{-2} (g_A g_A^c)^2 \right) \]
In resonance the Breit-Wigner cross section of the $1^{--}$ bound state production has the following form:

$$\sigma = \frac{12\pi}{4m_H^2} \frac{\Gamma(\Delta l_R^{\pm})}{\Gamma_{\text{total}}}$$  \hspace{1cm} (3)

As seen from (44) the width of the doubly charged particles decay into leptons is comparable with decay width of the ordinary quarkonium into leptons:

$$\frac{\Gamma(\Delta l_R^{\pm})}{\Gamma(\bar{q}q \to l^+l^-)} \approx \frac{Q^8\alpha^3}{3Q_q^2}\approx 0.366(\text{at } Q_q = -\frac{1}{3}), 0.092(\text{at } Q_q = \frac{2}{3})$$  \hspace{1cm} (4)

and with decay width of the squarkonium (quarkonium) into photons:

$$\frac{\Gamma(\Delta l_R^{\pm})}{\Gamma(\bar{q}q \to l^+l^-)} \approx \frac{Q^{10}\alpha^3}{3Q_q^2}\approx 13.184(\text{at } Q_q = -\frac{1}{3}), 0.824(\text{at } Q_q = \frac{2}{3})$$  \hspace{1cm} (5)

Thus, in $\gamma\gamma$-collision (13) doubly charged particles bound states may be produced more copiously than quarkonium with same mass. If beam resolution is much larger than the total width of the bound state the cross section (3) in $\sim \Gamma$ times will be smaller. For example, at $\Delta(\sqrt{s}) = 10^{-3}\sqrt{s} > \Gamma(\Delta l_R^{--} \to l^-l^-)$ we obtain about $52 * Q^8$ events per year at $m_H = 500GeV$ and yearly luminosity $L = 1000 fb^{-1}$. The main decay modes are decays into $l^+l^-, \bar{\nu}\nu, q\bar{q}, 2\gamma, 3\gamma, H^{0,\gamma}, Z^{0,\gamma}$ etc.

All above analysis is also true for $\tau^+\tau^-$ (firstly decays of $\tau^+\tau^-, \mu^+\mu^-$ bound states has been considered in [10], however in this ref. has not been considered its production in resonance) bound states which also may be produced in $e^+e^-$ collisions in resonance and also in $\gamma\gamma$-collisions. $\mu^+\mu^-$ can also produced in resonance, and at $L = 1 fb^{-1}$ we obtain $3886 * 400$ events per year. For instance at $\Delta(\sqrt{s}) = 10^{-3}\sqrt{s} > \Gamma$ we obtain in accordance with above made conclusions $3886 \tau^+\tau^-$ bound states per year.
at $L = 1 fb^{-1}$. $\mu^+\mu^-$ can also produced in resonance, and at $L = 1 fb^{-1}$ we obtain 3886*400 events per year (For $\mu^+\mu^-$ atoms see [10]. Also $\mu^+\mu^-$ atoms may be produced as a result of positron collision with electrons of target. In this case depend on $n$-numbers of electrons in $cm^{-3}$ and width of the target we obtain the enhancement on many order in comparison with $\mu^+\mu^-$ atoms production in $e^+$ and $e^-$ beams collision in c.m.system.

It must be noted that levels shift from vacuum polarization in $\tau^+\tau^-$ is also essential.

In case of tau leptons bound state the main contribution in energy shift from vacuum polarization come from electron loops corrections. In case of doubly charged fermions bound states contributes besides electrons also muons and even tau lepton and pions loops (i.e. all light flavors: $a_B << \frac{1}{m}$).

For vector doubly charged bosons (see [13]) this consideration is also true (see "Vector bosons bound states" below).

**Vacuum Polarization in doubly charged fermions bound states**

The modification of Coulomb potential between fermions from corrections to photon propagator has the following form (Uling potential):

$$V(r) = \frac{-2\alpha^2Q^2}{3\pi} \frac{1}{r} \int_1^{\infty} \frac{\exp(-2mrx)}{x} F(x) dx$$

where $F(x) = \left(1 + \frac{1}{2x^2}\right)^{\frac{\sqrt{x^2-1}}{x^2}}$. The corrections to the energy levels may be calculated by perturbation theory. E.g. for ground state ($n = 1, l = 0$) energy level shift we have:

$$\Delta E_{1,0} = -\frac{\alpha^3Q^4m_H}{3\pi} \int_1^{\infty} \frac{1}{(m_exa_B + 1)^2} F(x) dx$$

For ($n = 2, l = 0, n = 2, l = 1$) energy levels shifts we have:

$$\Delta E_{2,1} = -\frac{\alpha^3Q^4m_H}{12\pi} \int_1^{\infty} \frac{(8m_e^2a_B^2x^2 + 1)}{(2m_exa_B + 1)^4} F(x) dx$$
\[ \Delta E_{2,1} = -\frac{\alpha^3 Q^4 m_H}{72\pi} \int_1^\infty \frac{1}{(2m_e x a_B + 1)^4} F(x) dx \]  

(9)

In case of heavy fermions (doubly charged fermions bound states, \(\tau^+\tau^-\),...) \(a_B << \frac{1}{m_e}\) and it is convenient instead this general formulas to use asymptotic simplified formulas obtained from Uling potential at \(m_e r << 1\). At \(m_e r << 1\) the contribution to potential from vacuum polarization is following [?]:

\[ V(r) = \frac{2\alpha^2 Q^2}{3\pi} \frac{1}{r} \left( \ln (mr) + C + \frac{5}{6} \right) \]  

(10)

where \(C = 0.577\) is Euler constant. In positronium vice versa \(a_B >> \frac{1}{m_e}\) and Uling potential has the behaviour [3]: \(\sim e^{-\frac{2m_r}{r^{3.5}}}\) which give correction of order \(\Delta E \sim -\alpha^5 m\). In our case we obtain \(\Delta E \sim -\alpha^3 m_H \ln (m_e a_B)\) and besides electron another light fermions \((a_B = \frac{2}{m_H Q^2} << \frac{1}{m_e})\) contributing in loop.

In \(\tau^+\tau^-\) bound states only electrons give essential contribution.In doubly charged fermions bound states case contribution of muons approximately in \(\sim 3\) times smaller than electrons contribution.In case of tau-leptons bound state the muons loop contribution is negligible. Also may be essential \(\pi^+\pi^-\) loops because \(a_B << \frac{1}{m_e}\), resonance contribution (e.g. \(\rho\)-mesons contribution etc.) As pointed in [3] the hadronic contributions in vacuum polarization is negligible in positronium.

Using asymptotic behaviour of the Uling potential e.g. for \(n = 1, 2, l = 0\) and \(n = 2, l = 1\) we obtain:

\[ \Delta E_{1,0} = -\frac{\alpha^3 Q^4 m_H}{3\pi} \left( \ln \left( \frac{m_e a_B}{2} \right) + \frac{11}{6} \right) \]  

(11)

\[ \Delta E_{2,0} = -\frac{\alpha^3 Q^4 m_H}{12\pi} \left( \ln (m_e a_B) + \frac{7}{3} \right) \]  

(12)
\[
\Delta E_{2,1} = \frac{\alpha^3 Q^4 m_H}{12\pi} (\ln(m_e a_B) + \frac{8}{3})
\]

Thus the difference between \(n = 2, l = 0\) and \(n = 2, l = 1\) levels is much larger than in positronium.

In system with \(Q = 1\) beginning from tau-leptons this contribution is essentially smaller than vacuum polarization effect however it become essential and comparable with vacuum polarization effect at large charges \(Q = 2\).

Our numerical results is following: for \(\tau\)-lepton we have \(\Delta E_{1,0} \approx 10^{-3}, \Delta E_{2,1} \approx 10^{-4}\), for \(m_H \sim 500\,\text{GeV}\) we have \(\Delta E_{1,0} \approx 0.055E_{1,0}, 0.01E_{1,0}\) for \(Q = 1, 2\) respectively, for \(m_H \sim 500\,\text{GeV}\) we have \(\Delta E_{2,1} \approx 0.005E_{1,0}, 0.004E_{1,0}\) for \(Q = 1, 2\) respectively.

Our results are applicable also to bound states of heavy quarks (several hundred GeV). In heavy quarkoniums \(a_B = \frac{2}{m^2 \alpha_s} << 1/300\,\text{MeV}\) however as we see below quarkonium cannot be considered as Coulomb-like system. In this case as known vacuum polarization comes from light quarks and gluons loops. We obtain \(\Delta E_{1,0} \sim N \alpha_s^2 \ln(\frac{\mu}{m\alpha_s}) \approx 1.2E_{1,0} \sim m\alpha_s^2\) (\(N\)-number of flavours, \(\mu \sim 200\,\text{MeV}\)) which mean that vacuum polarization cannot considered as perturbation and consequently heavy quarkonium by this reason cannot be considered as Coulomb-like system. Must be solved Dirac equations numerically with potential \(-4/3\alpha_s(r)/r\), (where \(\alpha_s(r) = \frac{\alpha_s(r_0)}{1 - b \alpha_s(r_0) \ln(\frac{r}{r_0})}\)).

At small distances in potential must be included also repulsive term \(\alpha Q^2 (\frac{\hat{A}^2}{2m} \sim r^{-4} (\alpha_s(\frac{\hat{A}^2}{2m})^2 m \sim r^{-4}\) in case of QCD) which can not be considered as perturbation because appropriate integral is divergent at small \(r\).

(for Coulomb-like potential in quarkonium see e.g. [1] and ref.therein).
Vacuum polarization in quarkoniums as perturbation has been considered in [12].

It must be noted in quarkonium or squarkonium or above mentioned bound states of doubly charged fermions also give contribution to the potential from Z-bosons and Higgs bosons exchanges (see below). This corrections may be especially large for s-channel exchanges if mass of particle in the s-channel is equal to the mass of the bound state. E.g. for s-channel exchange case via scalar and vector bosons we have:

\[
\delta E = h_s^2 \frac{1}{s - m_H^2 + i\Gamma m_H} |\psi(0)|^2 - \alpha \frac{(g_V^2 + g_A^2 \sigma_1 \sigma_2)}{s - m_V^2 + i\Gamma V m_V} |\psi(0)|^2
\]  

(14)

We see that this corrections give contribution also in the width of the bound state.

**CP-Parity violation.**

We use the following parametrization of the scalar (pseudoscalar) particles interaction with fermions:

\[
L = \bar{f}(b_s + ib_p \gamma_5) f
\]  

(15)

The contribution of scalar particles into CP-violating potential take the form (for CP-violation in hydrogen see [24] and also references therein, in positronium CP-violation has been considered in [21],[?] part of decays considered below has been studied in [21]):

\[
V(r, \vec{p}) = b_s b_p \frac{1}{2m} \tilde{\Delta} \vec{n} H'(r)
\]  

(16)

where at tree levels \( H(r) = \frac{\exp(-m sr)}{r} \), \( \tilde{\Delta} = \sigma_1 - \sigma_2, \vec{n} = \frac{\vec{p}}{r} \). This formula is true only at \( v << 1 \), in general it is necessary to replace in the last formula \( i\vec{n} \to (p_f - p_i) \) where \( p_f, p_i \) are momentums of the initial and finite states.
potential $V = \vec{\Delta} \vec{n} F(r)$ we obtain \[17\]:

$$<1^{- -}|V(r)|1^{++}> = \int \psi'_p(r) \psi_S(r) F(r) \, d^3r \approx \psi'_p(0) \psi_S(0) F(r) \, d^3r$$

Also must be taken into account electric dipole moment of doubly charged fermions, tau-leptons or quarks:

$$L = A \bar{f} \sigma_{ab} \gamma_5 f F_{ab}$$

which give the following contribution in CP-violating potential:

$$V(r) = eA(\vec{\Delta} \vec{n}) \frac{1}{r^2}$$

Due to $CP$-parity violating interaction ( ) may mixed states with different $CP$-parity. For example can mixed $0^{-+}$ and $0^{++}$, $1^{+-}$ and $1^{--}$ bound states. Its lead to enhancement of some rare radiative decays.Besides may take place some decays which are forbidden by $T$-parity conservation.

In particular decays $0^{-+} \to 1^{--} \gamma$, $1^{--} \to 0^{-+} \gamma$, $1^{--} \to 0^{-+} \gamma$ are suppressed as $M1$ transition however as in hydrogen atom the small mixture of opposite parity states $(1^{--} - 0^{++}, 0^{++} - 0^{+-} - 0^{+-})$ mixing may enhanced this decays, because $1^{+-} \to 0^{-+} \gamma$, $0^{++} \to 1^{--} \gamma$ decays take place as $E1$ transition.

Also, Ora-Pauwell decay of orthopositronium into 3 photons (gluons) may be enhanced in the range of soft photons (gluons), because due to $CP$-parity violation $1^{--}$ is mixed with $1^{+-}$ bound state which also may decay into three photons (for positronium and bottomonium decays into 3 photons , 2 photons+gluon,3 gluons see [14] and references therein).

\[2\]The wave functions and its derivatives may be considered as constants only if $V(r)$ give essential contribution at $r << a_B$.
The differential width into 3 photons decays of the $^3S_1$ state (which is $^3S_1$ state with small mixture of $^1P_1$ state) takes the form:

$$\frac{d\Gamma(^3S'_1 \to 3\gamma)}{d\omega} \approx a_T^2 \frac{d\Gamma(^1P_1 \to 3\gamma)}{d\omega} + \frac{d\Gamma(^3S_1 \to 3\gamma)}{d\omega}$$

(20)

where $\frac{d\Gamma(^1P_1 \to 3\gamma)}{d\omega} \sim \frac{1}{\omega}$, whereas $\frac{d\Gamma(^3S \to 3\gamma)}{d\omega} \sim \omega \log \frac{m}{\omega}$, $\omega$ is photon energy, $a_T$ is the measure of the mixing between two states with different T-parity:

$$|^3S'_1 > = |^3S_1 > + a_T |^1P_1 >$$

(21)

$$a_T = \frac{<^3S_1 | V | ^1P_1 >}{E(^3S_1) - E(^1P_1)}$$

(22)

Thus at small $\omega$ we have peak instead usual Ora-Pauell formula. Analogous effect take place in quarkonium (decays into 3 photons, one photon + 2 gluons, 3 gluons). Analogously for above mentioned radiative correction for mixed $^3S_1'$ decays we have:

$$\Gamma(^3S'_1 \to ^1S_0\gamma) \approx a_T^2 \Gamma(^1P_1 \to ^1S_0\gamma) + \Gamma(^3S_1 \to ^1S_0\gamma)$$

(23)

In bottomonium the wave functions is not Coulomb like, however the behaviour of the matrix elements are expressed through $R(0), R'(0)...$ which may be expressed through decay widths of the bottomonium (e.g. $\Gamma(0^{-+} \to 2g) \sim |R_S(0)|^2, \Gamma(0^{++} \to 3g) \sim |R'(0)|^2$, see [14] and references therein).

**P-Parity violation.**

In heavy fermion bound states become large P- and CP-violation effects (because $Gm_H^2$ is large). The effect of P-violation in heavy fermion bound states is analogous to the P-violation in positronium which was calculated in [23]. Thus we can use formula (B4) of the [23] for mixing coefficient between $1^{--}$ and $1^{++}$ bound states where has been made the following
replacement (we take into account that in heavy fermions bound states s-channel contribution is suppressed as $\frac{m^2}{m_H^2}$):

$$\alpha \rightarrow \alpha Q^2, g'_V \rightarrow g_V, m_e \rightarrow m_H$$  \hspace{1cm} (24)

Using the result for $E(2^3S_1) - E(2^3P_1)$ obtained above (effect of vacuum polarization and contribution from $v^2/c^2$ corrections described by formula (1)) (B4) of the \[23\] with previous replacements we have for $2^3S_1 - 2^3P_1$ mixing:

$$a_P = \frac{-1.66 \times 10^{-2}(\frac{m_H}{500 GeV})^2 (g_V/g'_V) (1/24 m \alpha^2 Q^8 / (E(2^3S_1) - E(2^3P_1)))}{(25)}$$

Thus we see that although there is large enhancement in comparison with positronium, the effect of $P, CP$-violation in doubly charged fermions bound states case is again small for observation. Due to $P$-parity violation may mixed states with different $P$-parity. For example can mixed $1^{--}$ and $1^{++}$ bound states. It lead to enhancement of some rare radiative decays and appearance of the new channels which has been forbidden by $P$-parity conservation\[^3\].

In particular decays $0^- \rightarrow 1^- \gamma, 1^- \rightarrow 0^+ \gamma$, are suppressed as $M1$ transition however as in hydrogen atom the small mixture of opposite parity states ($1^{--} - 1^{++}$, mixing) may enhanced this decays, because $1^{--} \rightarrow 0^+ \gamma$, $0^{++} \rightarrow 1^{--} \gamma$ decays take place as $E1$ transition.

Also mixing of $1^{++}$ and $1^{--}$ which lead to the decays $1^{++} \rightarrow 3\gamma, 1^{--} \rightarrow 4\gamma \Gamma(1^{++} \rightarrow 3\gamma) = a^2 \Gamma(1^{--} \rightarrow 3\gamma)$

Also become possible decays:

$$3P'_1 \rightarrow 3\gamma, H^0(P^0) + \gamma$$  \hspace{1cm} (26)

\[^3\text{most of this decays has been considered in [23]}\]
Also, Ora-Pauell decay \((^3S_1 \rightarrow 3g)\) of quarkonium into 3 gluons may be enhanced in the range of soft gluons, because due to P-parity violation \(1^{-+}\) is mixed with \(1^{++}\) bound state which also may decay into three gluons (for quarkonium decays into gluons see \([14]\) and references therein) and maximal in the range of soft gluons (photons).

For \(P\)-parity violating potential in the system which consist of two different fermions we obtain the following result in the nonrelativistic approximation:

\[
V(r, \vec{p}_1, \vec{p}_2) = A + B + C + D
\]

\[
A = g_1^V g_2^A \frac{1}{2m_1} (Z'(r) i\vec{n}[\sigma_1 \sigma_2] + 2Z(r) \vec{\sigma}_2 \vec{p}_1 - i\vec{\sigma}_2 \vec{n}Z'(r))
\]

\[
B = g_1^V g_2^A \frac{1}{m_2} (Z(r) \sigma_2 \vec{p}_2 + i\vec{\sigma}_2 \vec{n}Z'(r))
\]

\[
C = g_2^V g_1^A \frac{1}{2m_2} (Z'(r) i\vec{n}[\sigma_1 \sigma_2] - 2Z(r) \vec{\sigma}_1 \vec{p}_2 - i\vec{\sigma}_1 \vec{n}Z'(r))
\]

\[
D = g_2^V g_1^A \frac{1}{m_1} (Z(r) \sigma_1 \vec{p}_1 - i\vec{\sigma}_1 \vec{n}Z'(r))
\]

where \(Z^0(r)\) is long range potential considered in \([15]\). Without leptons contribution long range potential has been considered also in \([16]-[18]\).

For fermions with different masses (e.g.\(e^{-}\mu^+, e^{-}\tau^+, \mu^-\tau^+)\) we obtain:

\[
V(r, \vec{p}_1, \vec{p}_2) = -\alpha g_V g_A (A_1 + A_2 + A_3 + A_4)
\]

where:

\[
A_1 = \frac{1}{2m_+} (2Z(r) \vec{\sigma}_+ \vec{p}_+ + i\vec{\sigma}_+ \vec{n}Z'(r))
\]
\[ A_2 = \frac{1}{2m_-}(-2Z(r)\sigma_- p_- + i\sigma_- n Z'(r)) \] (35)

\[ A_3 = \frac{1}{2m_-}(Z'(r)\bar{n}[\sigma_+ \sigma_-] + 2Z(r)\sigma_+ p_- - iZ'(r)\sigma_+ \bar{n}) \] (36)

\[ A_4 = \frac{1}{2m_+}(Z'(r)\bar{n}[\sigma_- \sigma_+] - 2Z(r)\sigma_- p_+ - iZ'(r)\sigma_- \bar{n}) \] (37)

In case of the fermion-antifermion system with same masses we have (\( p_- = -p_+ \)):

\[ V(r, p_-) = -\alpha g g_A \frac{1}{2m} (i\bar{n}S Z'(r) - iZ(r)\bar{S} p_- Z'(r)) \] (38)

Here \( S_i \) is operator of the total spin:

\[ \bar{S} = \frac{1}{2}(\sigma_1 + \sigma_2) \] (39)

The differential width into 3 photons decays of the \( ^3S_1 \) state (which is \( ^3S_1 \) state with small mixture of \(^1P_1 \) state) takes the form:

\[ \frac{d\Gamma(^3S_1 \rightarrow 3\gamma)}{d\omega} \approx a_P^2 \frac{d\Gamma(^3P_1 \rightarrow 3\gamma)}{d\omega} + \frac{d\Gamma(^3S_1 \rightarrow 3\gamma)}{d\omega} \] (40)

where \( \frac{d\Gamma(^3P_1 \rightarrow 3\gamma)}{d\omega} \sim \frac{1}{\omega} \), whereas \( \frac{d\Gamma(^3S \rightarrow 3\gamma)}{d\omega} \sim \omega \log \frac{m}{\omega} \), \( \omega \) is photon energy, \( a_P \) is the measure of the mixing:

\[ |^3S_1\rangle = |^3S_1\rangle + ia_P |^3P_1\rangle \] (41)

\[ a_P = \frac{<^3S_1|V|^1P_1>}{E(^3S_1) - E(^1P_1)} \] (42)

Thus at small \( \omega \) we have peak instead usual Ora-Pauell formula. Analogous effect take place in quarkonium (decays into 3 photons, one photon+2 gluons,
3 gluons). Analogously for above mentioned radiative correction for mixed $^3S'_1$ decays we have:

$$\Gamma(^3S'_1 \to ^1S_0 \gamma) \approx a_P^2 \Gamma(^1P_1 \to ^1S_0 \gamma) + \Gamma(^3S \to ^1S_0 \gamma) \quad (43)$$

In bottomonium the wave functions is not Coulomb like, however the behaviour of the matrix elements are expressed through $R(0), R'(0), ...$ which may be expressed through decay widths of the bottomonium (e.g. $\Gamma(0^{-+} \to 2g) \sim |R_S(0)|^2, \Gamma(1^{--} \to 3g) \sim |R'(0)|^2$, see [14] and references therein).

Our results are applicable also to bound states of heavy quarks (several hundred GeV).

For $\tau^+\tau^-$ systems case we obtain using formula (2.5) of the [23] $a_P = -1.3 \times 10^{-6}$

**P-violation in mesoatoms and muonium**

In particular, in $\mu^-p$ atoms the effect (mixing of S- and P- states with opposite P-parity) may be enhanced in $\mu^2/m_e^2 \approx 4 \times 10^4 (\mu = \frac{m_\mu m_p}{m_\mu+m_p})$ times in comparison with hydrogen (See formulas (3.18)-(3.19) of [24] for $2S_{\frac{1}{2}} - 2P_{\frac{1}{2}}$ mixing coefficient in hydrogen). Using estimate of [24] we obtain for range of circular polarization of photon in $2S_{\frac{1}{2}}$-level decay:

$$P = 3.8 \times 10^{-1} \frac{1}{2} (1 - 4sin^2\theta_W) \times (\mu^2/m_e^2) = 0.82 \quad (44)$$

Also may be interesting $P$-violation $\mu$-mesonic ions where as well as in [24] $P$-violation may be enhanced by large charge of nuclei $Z$.

Also $P$-violation for $l_i^+l_j^-$ atoms may be obtained from formulas (3.18)-(3.19),(3.22) of [24]. In particular in case of $\mu^+e^-$-bound states the effect is the same as in hydrogen.
Analogously may be enhanced CP-violation in mesoatoms in comparison with CP-violation in hydrogen\cite{24} in $\mu^2/m_e^2 \approx 4 \times 10^4$ times.

**Vector bosons bound states**

As known\cite{26,25}, there are the two types of admissible solutions for vector bosons bound states in the Coulomb potential:

\[ l = j, \Phi = \vec{L}F(r)P^m_j, \phi = 0 \quad (45) \]

\[ l = j \pm 1, \Phi = (\vec{L}F_1(r) + i[nL]F_2(r))P^m_j, \phi = G(r)P^m_j \quad (46) \]

(see formulas (35) of ref. \cite{26}) The solution with $l = j$ lead to the Klein-Gordon equation for scalar particles, whereas the solution with $l = j \pm 1$ lead to the system of equations which described movement in the potential which have $r^{-3}$ singularity. As known such singularity lead to the fall down on the center. If however as we see below we consider besides Coulomb field generated by attractive center the Higgs field also generated by the same center (because attractive particle also obtain its mass via spontaneous symmetry breaking and consequently interact with Higgs bosons), we obtain that potential become less singular and in this case fall down on the center is absent. Also we make the conclusion about existence of bound states of vector bosons:$W^+W^-, W^-W^-, W^+W^-, W^-W^-, W^+Z^0, H^0Z^0$ etc which may be produced in $e^+e^-, e^-e^-$-collisions.

The part of lagrangian which described vector bosons including its interaction with Higgs bosons has the following form:

\[ L = -\vec{D}_a\vec{\phi}^bD^a\phi_b + \vec{D}_a\vec{\phi}^bD_b\phi^a + (\phi)^2\vec{\phi}^a\phi_a \quad (47) \]

where in accordance with notations\cite{26} $\phi_a$-is vector bosons, $\phi$-is Higgs bosons. Without Higgs bosons we obtain formula (1) of the \cite{26}. 

14
The field equations are modified due to Higgs bosons presence in comparison with [26] by the following way:

\[
(D^a D_a - k^2) \Phi^b = i e \Phi^a H_{ab} - \frac{ie}{2} D_b (k^{-2} D_b (H^{ac} F_{ac})) - D_b (\Phi^a \frac{d}{dx_a} (\log(k^2)))
\]

(48)

where \(k^2 = g^2 \phi^2(r)\). At large distances \(k^2 = m_W^2\) and we obtain equations (18) of [26]. The equation (48) may be rewritten in the following form:

\[
((W + eA_0)^2 + \Delta - k^2) \Phi = -e \phi A'_0 n_i + e \frac{d}{dx_i} (k^{-2}((W + eA_0) \bar{n} \bar{\Phi} - \Phi'A_0)) - \frac{d}{dx_i} (\bar{n} \bar{\Phi} (\log(k^2))')
\]

(49)

where

\[
((W + eA_0)^2 + \Delta - k^2) \phi = e \bar{n} \bar{\Phi} A'_0 + e (W + eA_0) (k^{-2}((W + eA_0) \bar{n} \bar{\Phi} - \phi') A'_0) - (W + eA_0) (\bar{n} \bar{\Phi} (\log(k^2))')
\]

(50)

If we consider charged vector bosons in the context of Standard Model (or its various extensions we must add also \(Z^0\)-bosons exchanges. It can be made by the following replacements in equations (49)-(50):

\[
QeA_0(r) \rightarrow QeA_0(r) + (T - 2Q \sin^2 \theta_W) g Z_0(r)
\]

(51)

where \(Z_0(r)\) is \(Z\)-boson field (radiative corrections included).

The most singular term in (48) is following:

\[
\sim \frac{ie}{2k^2} D_b (H^{ac} F_{ac}) \sim \frac{ie V^2 V'}{(m_W + C r^{-1} e^{-m_H r})^2} F_{ac} \sim \frac{e^4 r^{-3}}{(m_W + C r^{-1} e^{-m_H r})^2}
\]

(52)

If \(k^2 = m_W^2\) (i.e. constant) we obtain singular potential which lead to fall down on the center. For complete set \(^4\) of radial equations we obtain the

\(^4\) One of this equations is a result of subsidiary condition for energy level definition the three equations is enough.
following result:

\[ \Omega F_1 = \frac{2}{r^2} (F_1 + j(j + 1)F_2) - e \frac{dV}{dr} G - e \frac{d}{dr} (Q \frac{dV}{dr}) + (F_1 (\log(k^2))'), \]

(53)

\[ \Omega F_2 = \frac{2}{r^2} F_1 - e \frac{1}{r} Q \frac{dV}{dr} - \frac{1}{r} F_1 (\log(k^2))', \]

(54)

\[ \Omega G = e \frac{dV}{dr} F_1 - e(W + eV) \frac{1}{r} Q \frac{dV}{dr} - (W + eV) F_1 (\log(k^2))', \]

(55)

\[ \frac{d\log(k^2)}{dr} F_1 + \frac{dF_1}{dr} + \frac{2}{r} F_1 - \frac{j(j + 1)}{r} F_2 + e(W + eV) G = -eQ \frac{dV}{dr}, \]

(56)

where \( \Omega \) as in \[26\] is Klein-Gordon operator:

\[ \Omega F_2 = \frac{d^2}{dr^2} + \frac{2}{r} - \frac{j(j + 1)}{r^2} + (W + eV)^2 - k^2 \]

(57)

and \( Q \) is defined as:

\[ k^2 Q = \frac{dG}{dr} - (W + eV) F_1 \]

(58)

In formulas(48)-(50) \( k^2 = (m_W + e^{\exp- \frac{m_W}{r}})^2 \).

Case \( V = 0 \) in these equations correspond to the neutral vector bosons bound states ( \( Z^0 H^0, Z^0 Z^0 \) etc.).

At small \( r \) we suppose that solutions has the following form:

\[ F_i = A_i r^s \]

(59)

\[ G = A_G r^s \]

(60)
Substituting (59)-(60) into (53)-(58) we obtain the following system of linear equations:

- \[ -4A_1 + A_2(s(s + 1) - j(j + 1) = 0 \quad (61) \]
- \[ A_1(s+1) - j(j+1) - 2 - \alpha(s-1) - 2j(j+1)A_2 - \alpha(1 + \frac{s(s - 1)}{c})A_G = 0 \quad (62) \]
- \[ (-\alpha + \frac{\alpha^3}{c^2})A_1 + (s(s + 1) - j(j + 1) - 2 - \frac{s\alpha^2}{c^2})A_G = 0 \quad (63) \]

The determinant of this system must be equal to the zero and we obtain \( s \) as a solutions of this equation. Thus, if \( c \) (which characterize the presence of Higgs bosons interaction with gauge bosons) is nonzero, the regular solutions exist.

For \( Z^0 \)-bosons case we obtain:

- \[ s = \frac{1 \pm \sqrt{1 + 4j(j + 1) + c^2 - 2}}{2} \quad (64) \]

Also if we taking into account repulsive term \( \alpha(\frac{\hat{A}}{2m})^2 \sim r^{-4} \) from Klein-Gordon equation it also prevent fall down on the center.

Bound states \( W^+W^-, W^{++}W^{--}, Z^0Z^0, Z^0H^0 \) may be produced in resonance in \( e^+e^- \)-collisions:

- \[ e^+e^- \rightarrow W^+W^-, W^{++}W^{--}, Z^0Z^0, Z^0H^0 \quad (65) \]

while \( W^-W^- \)-bound states may be produced in resonance in \( e^-e^- \)-collisions:

- \[ e^-e^- \rightarrow W^-W^- \quad (66) \]

It must be noted that bound state may be formed if \( \Gamma << W \). Thus, the solution \( l = j \) is not admissible by this reason \( (\Gamma(W^-) >> W = \frac{1}{4}m_W\alpha^2) \). However for \( l = j \pm 1 \) we expect deep levels \( (\Gamma(W^-) << W = \frac{1}{4}m_W\alpha^2) \) because we regularized at small distances singular potential. Besides in case of
$W^{++}W^{--}$ bound states the width of main mode may be $<<$ than $W = \frac{1}{2} m_W \alpha^2 p^4$ - the binding energy of $l = j W^{++}W^{--}$ bound states.

From the other hand we would like to stress that the deep level decrees mass of the vector bosons and consequently decrees width of the vector bosons because it proportional to the mass of the vector bosons, besides high velocity of the vector bosons in the bound state also make the width smaller.

**Vector bosons energy levels in the magnetic field (Reported on conference "Physics 99" in Yerevan State University 13 September 1999.)**

We choose the magnetic field as $\vec{H} = (0, 0, H)$, in this case it is convenient to choose vector potential in the following form:

$$A_x = -Hy, A_y = A_z = 0$$ (67)

Substituting $\vec{A}$ into field equation (55)-(56) we obtain the following system of equations which defined energy levels:

$$(T - \frac{eH(p_x + eHy)}{k^2} \frac{d}{dy})\Phi_1 = (-ieH + \frac{ieH(p_x + eHy)^2}{k^2})\Phi_2$$ (68)

$$(T + \frac{e^2H^2}{k^2} + \frac{eH(p_x + eHy)}{k^2} \frac{d}{dy})\Phi_2 = (ieH + \frac{ieH}{k^2} \frac{d^2}{dy^2})\Phi_1$$ (69)

Here $T = e^2 - (p_x + eHy)^2 - p_y^2 - p_z^2 - k^2$, $p_x$, $p_z$ - is constant. Components $\Phi_{0,3}$ are expressed through $\Phi_{1,2} \sim \exp (ip_x x + ip_z z)\chi_{1,2}(y)$:

$$T\Phi_3 = \frac{-ieH}{k^2}(p_z(p_x + eHy)\Phi_2 + ip_z \frac{d}{dy}\Phi_1)$$ (70)

$$T\Phi_0 = \frac{e^2H^2}{k^2}(-i(p_x + eHy)\Phi_2 - \frac{d}{dy}\Phi_1)$$ (71)

18
From comparisons of the last two formulas we see that only one of the functions $\Phi_{0,3}$ must be considered as independent.

At large distances we have $\Phi_{1,2} = A_i \exp (ip_xx + ip_zz) \exp \left( -\frac{1}{2}eH(y + \frac{p_x}{eH})^2 \right)$. After substitution:

$$\Phi_i = \exp (ip_xx + ip_zz) \exp \left( -\frac{1}{2}eH(y + \frac{p_x}{eH})^2 \right) \chi_i$$  \hspace{1cm} (72)

into equations (68),(69) we obtain:

$$X''_i + \left( y + \frac{p_x}{eH} \right) A_{ik} X'_k + B_{ik} X_k = 0 \hspace{1cm} (73)$$

where $X_1 = \chi_1, X_2 = \chi_2 - \frac{\omega}{2} \chi_1, A_{11} = -\omega^2 - 2eH, A_{12} = 0, A_{21} = \omega^2, A_{22} = \omega^2 - 2eH, B_{11} = \Omega_0 - \omega^2, B_{22} = -\omega^2, B_{21} = \Omega_0 + \omega^2 - k^2, B_{22} = \Omega_0 + \omega^2, \Omega_0 = \epsilon^2 - p_z^2 - k^2, \omega = \frac{eH}{k}$.

After appropriate diagonalizations we obtain two independent differential equations:

$$Z''_1 - 2\left( y + \frac{p_x}{eH} \right) Z'_1 + 2n_1 Z_1 = 0, \hspace{1cm} (74)$$
$$Z''_2 - 2\left( y + \frac{p_x}{eH} \right) Z'_2 + 2n_2 Z_2 = 0 \hspace{1cm} (75)$$

which solutions are Hermits polinoms if

$$(P^T a^{-1} (S^T BS) P)_{11} = -4n_1 \hspace{1cm} (76)$$
$$(P^T a^{-1} (S^T BS) P)_{22} = -4n_2 \hspace{1cm} (77)$$

where $n_{1,2}$- are integer positive numbers. In equations (76)(77) $S^T S = I, P^T P = I, S^T A S = a$ where in matrix $a$ only non-diagonal elements are nonzero.

This conditions (76)(77) defined energy levels $\epsilon_{1,2}(p_z^2, H, n_{1,2})$ of vector bosons in magnetic field where $\epsilon_{1,2}(p_z^2, H, n_{1,2})$-are solutions of the equations (76)(77) respectively and must be find numerically. Substituting $\Phi_{1,2}$
into differential equation (which must be solved numerically, numerical computation are in progress.) we also obtain energy levels.

At weak field limit ($eH \ll k^2$) the term $\frac{i}{2}D_b(k^{-2}D_b(H^{ac}F_{ae}))$ may be neglected and we obtain from (68)(69) the following expression for energy levels:

$$\epsilon^2 = k^2 + p_z^2 + 2eH(n + \frac{1}{2} \pm \frac{1}{2}) \quad (78)$$

From (70) we obtain:

$$\epsilon^2 = k^2 + p_z^2 + 2eH(n + \frac{1}{2}) \quad (79)$$

New functions $Z_i$ are expressed through $X_i$ by the following way:

$$Z_i = PSX_i. \quad (80)$$

Magnetism of Electron Gas in the Finite Volume (Reported on conference ”Physics 99” in Yerevan State University 13 September 1999.)

We also calculate the properties of the electron gas in the finite volume. We consider two kinds of the finite volumes: cylinder (with finite length or infinite length) and in box with sizes (axbxc). (For energy levels in magnetic field and some qualitative discussion about energy levels in magnetic field inside box has been made in [27]). Due to modification of energy levels also modified the thermodynamical and chemical potential:

$$\Omega = -kT \sum_m ln(1 \pm \exp(\frac{\mu \pm \omega}{kT})) \quad (81)$$

$$N = \sum_m \frac{1}{e^{(\frac{\omega-\mu}{kT})} + 1} \quad (82)$$
and consequently magnetization which is expressed through $\Omega$ by the following way (see e.g. [28]):

$$\vec{M} = -\frac{1}{V}(\frac{\partial \Omega}{\partial H})_{T,V,\mu}$$  \hspace{1cm} (83)

If electrons motion is restricted by cylinder (which described by boundary condition $\psi(R, z) = 0$) we have the following condition which define energy levels:

$$F(-\lambda_{1,2} + l + \frac{1}{2}, l + 1, \frac{eH}{m_e}R^2) = 0$$  \hspace{1cm} (84)

Here $F$ is hypergeometric function. Using this formula we calculate $M$ of the electron gas in the cylinder. Our numerical results for energy levels in cylinder are presented on the Fig.1 of the ref.[29]. At large $R$ or large $H$ we return to Landau levels.

The ordinary case i.e. particle movement in infinite space $R \to \infty$ correspond to the:

$$- \lambda + l + \frac{1}{2} = 0$$  \hspace{1cm} (85)

This condition lead to the Landau levels.

If magnetic field is perpendicular to the layer we obtain:

$$E_{m,k} = \sqrt{m^2 + |e|H(2m + 1 + \sigma) + (\frac{\pi k}{a})^2}$$  \hspace{1cm} (86)

where $m = 0, 1, 2, 3...; k = 1, 2, 3...$

This formula has been obtained from formula of the ref.[8] on p.148 at $p_z = \frac{\pi k}{a}$

On Fig.2,3 of the ref.[29] are presented energy levels for case of magnetic field which is parallel to the layer.

The magnetization and chemical potential versus $H$ at fixed $R$ (and width in case of layer) and electrons concentration $n = \frac{N}{V}$ is presented on
the Fig.4-Fig.11 of the ref. [29] for cases of the layer (magnetic field is parallel to the layer) and cylinder respectively.

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