Oscillations of three neutrinos with all $\Delta m^2 \sim 10^{-3} \text{eV}^2$

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Abstract

Oscillations of three neutrinos with all squared mass splittings around $10^{-3} \text{eV}^2$ are not firmly excluded by solar neutrino experiments. We carefully verify that they are also perfectly compatible with atmospheric neutrino experiments: due to accidental reasons the SuperKamiokande experiment is rather insensitive to ‘solar’ $\nu_e/\nu_\mu$ oscillations, even if some characteristic small effects could become visible with more statistics. This pattern of oscillations can be excluded by new solar experiments, or cleanly discovered at KamLAND.

We also perform a fit of the most recent atmospheric SK data within the usual assumption that ‘solar’ effects are negligible.

1 Introduction

In this paper we explore the possibility that the atmospheric and solar neutrino deficits can be produced by oscillations of the three known neutrinos with comparable mass splittings $\Delta m^2 \sim 10^{-3} \text{eV}^2$ and large $\nu_e/\nu_\mu$ and $\nu_\mu/\nu_\tau$ mixings. In section 2 we recall why solar neutrino experiments do not exclude this possibility. In section 3 we show that this possibility is also perfectly consistent with atmospheric neutrino experiments, even if it is sometimes said that, since SuperKamiokande (SK) sees no anomaly in the rate of $\nu_e$ events, significant oscillations of atmospheric $\nu_e$ neutrinos are excluded. This is not the case. In short the reason is the following: atmospheric neutrinos are produced by cosmic rays in the following proportion

$$(N_{\nu_e}, N_{\nu_\mu}, N_{\nu_\tau}) \propto (1, R, 0).$$

Since $R \approx 2$ the nearly maximal $\nu_\mu/\nu_\tau$ oscillation responsible of the atmospheric $\nu$ anomaly gives oscillated neutrinos with composition $\propto (1, 1, 1)$ — the only proportion not affected by further possible oscillations. Of course this argument is only approximate ($R$ is larger than 2 for $E_\nu \gtrsim 1 \text{ GeV}$ and the $\nu_\mu/\nu_\tau$ oscillation need not be exactly maximal): a numerical computation will confirm that the conclusion is correct.

From a theoretical point of view, the possibility of explaining neutrino anomalies with comparable $\Delta m^2$ has important consequences. It is easy to build models that naturally explain large mixings, but between neutrinos with comparable mass. It is also easy to build models that give hierarchical neutrinos, but with small mixings. It is more difficult to obtain large mixing angles ($\theta_{23} \sim 1$) between hierarchical neutrinos ($\Delta m^2_{23} \gg \Delta m^2_{12}$, all oscillation parameters are precisely defined later on in eq. (4)): only few mass matrices (justifiable with various symmetries) naturally give this pattern. Thus, non hierarchical neutrinos would not give very restrictive indications on flavour physics.

The paper is structured as follows. In section 2 we explain why a large solar $\Delta m^2 \sim 10^{-3} \text{eV}^2$ is not safely excluded by solar neutrino experiments. In section 3 we describe how we will fit the SK data. Since this is a delicate task, in section 4 we test our procedure performing a complete fit of the most recent SK data, in the ‘standard’ case where the solar $\Delta m^2_{12}$ is negligibly small. In section 5 we show that SK data are perfectly compatible with a $\Delta m^2_{12}$ as large as allowed by the CHOOZ bound, and we discuss how its effects can be detected at SK and at future experiments.

2 Energy independent solar oscillations?

If solar and atmospheric neutrino experiments are not affected by unknown systematic errors or by rare statistical fluctuations, if the standard solar models (SSMs) are correct, if there are only the three known
light neutrinos, then forthcoming neutrino experiments will confirm and measure more precisely that

\[
\begin{align*}
\Delta m^2_{23} &\approx 10^{-3} \text{eV}^2, \\
\sin^2 2\theta_{12} &\approx 1
\end{align*}
\]

These values give a good fit of atmospheric neutrino data and give an acceptable fit (10\% C.L. [12]) of solar neutrino data.

Since these conclusions are quite strong, it is useful to discuss if they are also strongly founded.

The well known ‘MSW solutions’ with \(\Delta m^2 \approx 10^{-3} \text{eV}^2\) give a poor fit of the distortion of the solar \(^8\text{B}\) spectrum observed by SuperKamiokande [1] and are ruled out at 95\% C.L. [12]. Maybe this experimental result (or the estimation of its uncertainties) is wrong. Maybe the distortion is not produced by \(\nu\) oscillations, but is due to a flux of ‘hep’ neutrinos \(\sim 15\) times higher than what predicted by SSMs [1].

At the moment, it seems more safe to believe that there are at least three (instead of one) possible oscillation solutions to the solar neutrino problem. We now discuss why even this sentence is not strongly founded: it is not safely excluded that the solar neutrino anomaly can be explained by an energy independent \(\nu_e \rightarrow \nu_\alpha\) survival probability \(P_{ee} \sim 1/2\) (as can be produced by a large \(\Delta m^2 \sim 10^{-3} \text{eV}^2\)).

The deficit \(r_1 = \Phi_1^{\text{exp}}/\Phi_1^{\text{BP98}}\) of solar neutrinos measured by the three kind of solar experiments (i.e. Cl, Ga and SK), with respect to the central values \(\Phi_1^{\text{BP98}}\) of fluxes predicted by the BP98 [3] SSM, are

\[
\begin{align*}
r_{\text{Cl}} &= 0.315 \pm 0.025 \quad [0] \quad (2a) \\
r_{\text{SK}} &= 0.47 \pm 0.02 \quad [1] \quad (2b) \\
r_{\text{Ga}} &= 0.58 \pm 0.05 \quad [2] \quad (2c)
\end{align*}
\]

(the errors do not include the SSM uncertainty). The predictions of a solar-model-independent analysis, in presence of an energy-independent \(P_{ee}\), are

\[
\begin{align*}
r_{\text{Cl}} &= P_{ee}(0.03 + 0.73R_{\text{B}} + 0.24R_{\text{Be}}) \quad (3a) \\
r_{\text{SK}} &= (0.15 + 0.85P_{ee})R_{\text{B}} \quad (3b) \\
r_{\text{Ga}} &= P_{ee}(0.60 + 0.10R_{\text{Be}} + 0.31R_{\text{B}}) \quad (3c)
\end{align*}
\]

where \(R_\alpha \equiv \Phi_\alpha/\Phi_1^{\text{BP98}}\) is the ratio of total flux of type \(\alpha\) neutrinos (\(\alpha = \text{pp}, \text{pep}, \text{7Be}, \text{13N}, \text{15O}, \text{17F}, \text{8B}, \text{hep}\)) emitted by the sun, with respect to the central value of the BP98 [3] SSM. Only two free parameters \(R_{\text{B}}\) and \(R_{\text{Be}}\) appear in eqs. (3); the others have been eliminated using the fact that the total luminosity of the sun is known, and other solid informations (see [13] for more details).

From eqs. (2) and (3) we can easily see that

- Even if SSMs are not correct (i.e. \(R_{\text{B}}\) and \(R_{\text{Be}}\) are treated as free parameters of order one), the experimental data are incompatible with an energy-independent \(P_{ee}\) only in presence of a very unlikely statistical fluctuation (\(p \approx 0.2\%\)). [4, 11, 11];

- Solar data can be explained by an energy-independent \(P_{ee}\) if BP98 is correct, but one of the experiments is affected by some unknown systematic error [14]. For example, it is sufficient to double the error quoted by the chlorine experiment [14] to have \(P_{ee} = 1/2\) compatible with experimental results (in presence of a reasonably probable, \(p \sim 10\%\), statistical fluctuation).

In conclusion we believe that the evidence for a deficit of solar neutrinos is strong, while there is yet no strong evidence for an energy dependent oscillation of solar neutrinos.

### 3 Fitting SK data

Reproducing what SK has really measured is a complex and delicate task. We briefly describe how we do the computation. Five basic ingredients are necessary for a fit of the SK data.

1. The experimental data: we use the most recent ones (736 days of data taking) [12].
2. The prediction for the flux of atmospheric neutrinos produced by cosmic rays [10]. We include effects due to the magnetic field of the earth and to variation of solar activity.
3. The oscillation probability for neutrinos across the earth and the atmosphere. In our case we have a generic neutrino \(3 \times 3\) mass matrix with 3 comparable \(\Delta m^2\): since we know no simple analytic approximation that takes into account all potentially relevant matter effects (the MSW [9] effect and resonances that can affect the neutrinos that cross the mantle and the core of the earth [10]) we include all matter effects with a fully numerical computation.

The disadvantage is that the transition probabilities for oscillations with long pathlength \(L \gg E_\nu/\Delta m^2\) are rapidly oscillating functions of the neutrino energy. Even for the simplest realistic model of earth density, a numerical averaging (using a sufficiently large number of \(E_\nu\)-bins) is more efficient than analytic averaging.

4. The cross section and detection efficiencies in the SK detector. A simple and safe technique has been used in [14] for fitting the old Kamiokande data. They employed the energy spectra of parent neutrinos given by the Monte Carlo simulation of the Kamiokande detector. The corresponding data for the much larger SK detector have not been published (without these...
Figure 1: $\chi^2$ of SuperKamiokande data as function of $\Delta m^2_{23}$ for $\theta_{23} = 45^\circ$, $\theta_{13} = 0$ and negligible $\Delta m^2_{12}$. In fig. (a) (b) we show the separate contribution from sub-GeV (multi-GeV) events; while all events are used in fig. (c). Continuous and dotted lines correspond to two different definitions of the $\chi^2$ (see the text).

We do not include in the fit data about ‘upward through going muons’ because they are subject to larger theoretical uncertainties and they are too energetic for being strongly affected by oscillations (they are however very interesting for excluding alternative explanations of the atmospheric $\nu$ deficit).

A $\chi^2$ function. This is a delicate point, since it requires an estimation of theoretical uncertainties in neutrino fluxes and correlation between them. For simplicity we stick to the accurate definition of and we will discuss when appropriate the effect of different definitions.

Moreover we impose the CHOOZ bound about disappearance of reactor $\bar{\nu}_e$. The CHOOZ collaboration measures the annihilation energy $E = E_{\bar{\nu}_e} + m_p - m_e + (2 - 1) m_e$ of positrons produced by inverse beta decay $\bar{\nu}_e + p \rightarrow e^+ n$ for various energy bins between 1 and 7 MeV. Since we will be interested in an oscillation pattern where the $\bar{\nu}_e$ survival probability depends on $E_{\bar{\nu}_e}$, we carefully treat the CHOOZ data, grouping them into two $E_{\bar{\nu}_e}$ bins. We correctly reproduce the ‘initial’ CHOOZ bound ($\Delta m^2_{12} < 0.9 \times 10^{-3} \text{eV}^2$ for maximal mixing at 90% C.L.). A ‘final’ analysis of the whole CHOOZ data has not yet been presented: with increased statistics, the CHOOZ bound could be improved up to $\approx 0.6 \times 10^{-3} \text{eV}^2$.

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4 Standard fit of SK data

We now begin to study the SK data assuming, as usual, that $\Delta m^2_{12}$ is too small to have relevant effects. We start with a standard fit because, beyond being interesting, allows to check our computation with other ones. We find a very satisfactory agreement with [2], where many plots of oscillations effects are presented.

The oscillation parameters are precisely defined in the following way. The neutrino mixing matrix $V$ is parametrized as

$$V = R_{23}(\theta_{23}) \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & e^{i\phi} & 0 \\ 0 & 0 & 1 \end{array} \right) R_{13}(\theta_{13}) R_{12}(\theta_{12})$$

where $R_{ij}(\theta_{ij})$ represents a rotation by $\theta_{ij}$ in the $ij$ plane and $i = \{1, 2, 3\}$ are three neutrino mass eigenstates of mass $m_i$. With this parameterization $\theta_{23} \sim 45^\circ$ gives the $\nu_e/\nu_\mu$ mixing tested at SK; while $\theta_{13}$ produce the deficit of solar $\nu_e$ neutrinos ($\theta_{13}$ could be zero). We also define $\Delta m^2_{ij} = m_j^2 - m_i^2$. We can assume that $|\Delta m^2_{23}|$ is the largest splitting (see [1] for more details). With this parameterization atmospheric oscillations depend only on $\Delta m^2_{23}$, which is determined by $\theta_{23}$ and $\theta_{13}$.

In fig. 1 the continuous lines show how the $\chi^2$ depends on $\Delta m^2_{23}$ (for maximal $\theta_{23} = 45^\circ$ and zero $\theta_{13}$) fitting separately the sub-GeV and the multi-GeV events. Compared to a similar fit done by the SK collaboration, our result is less sensitive to $\Delta m^2_{23}$ events. Compared to a similar fit done by the SK collaboration, our result is less sensitive to $\Delta m^2_{23}$ events. Compared to a similar fit done by the SK collaboration, our result is less sensitive to $\Delta m^2_{23}$ events. Compared to a similar fit done by the SK collaboration, our result is less sensitive to $\Delta m^2_{23}$ events. Compared to a similar fit done by the SK collaboration, our result is less sensitive to $\Delta m^2_{23}$ events. Compared to a similar fit done by the SK collaboration, our result is less sensitive to $\Delta m^2_{23}$ events. Compared to a similar fit done by the SK collaboration, our result is less sensitive to $\Delta m^2_{23}$ events. Compared to a similar fit done by the SK collaboration, our result is less sensitive to $\Delta m^2_{23}$ events. Compared to a similar fit done by the SK collaboration, our result is less sensitive to $\Delta m^2_{23}$ events. Compared to a similar fit done by the SK collaboration, our result is less sensitive to $\Delta m^2_{23}$ events. Compared to a similar fit done by the SK collaboration, our result is less sensitive to $\Delta m^2_{23}$ events. Compared to a similar fit done by the SK collaboration, our result is less sensitive to $\Delta m^2_{23}$ events. Compared to a similar fit done by the SK collaboration, our result is less sensitive to $\Delta m^2_{23}$ events.
Figure 2: Fit of SK and Chooz data under the assumption that ‘solar’ oscillations are negligible, so that the only relevant oscillations parameters are $\Delta m^2_{23}$, $\theta_{23}$ and $\theta_{13}$. Inside the darker (lighter) areas $\chi^2 < 24$ (30) (the $\chi^2$ uses 22 experimental data; the best fit has $\chi^2_{\text{min}} = 18$). In fig. 2a the $\chi^2$ is minimized with respect to $\Delta m^2_{23}$, and in fig. 2c with respect to $\theta_{23}$.

In fig. 2 we show the results of a fit in the relevant oscillations parameters, $\Delta m^2_{23}$, $\theta_{23}$ and $\theta_{13}$. The best fit has $\chi^2_{\text{min}} = 18$ (the $\chi^2$ uses 20 experimental data from SK and 2 from Chooz). We show contour lines corresponding to the values $\chi^2 = 24$ and $\chi^2 = 30$. Using standard approximations, values of $\chi^2 - \chi^2_{\text{min}}$ can be converted into confidence levels that delimit ‘best fit regions’, and values of $\chi^2$ can be converted into confidence levels that delimit ‘exclusion regions’. Shaded areas roughly correspond to $(90 \div 99)\%$ confidence levels. In fig. 2a we minimize the $\chi^2$ with respect to $\theta_{13}$ and determine the allowed regions in the plane ($\Delta m^2_{23}, \theta_{23}$). We see that $\sin^2 2\theta_{23} > 0.8$ at 90\% C.L. Fig. 2b shows the $\chi^2$ minimized with respect to $\Delta m^2_{23}$ and determines the allowed values of the mixing angles. In fig. 2c we minimize the $\chi^2$ with respect to $\theta_{23}$ and show how the upper bound on $\theta_{13}$ depends on $\Delta m^2_{23}$. If $\Delta m^2_{23} = 3 \times 10^{-3}$ eV$^2$ the Chooz bound (dashed lines) requires a small value of $\theta_{13} \lesssim 10^\circ$. For a mass splitting below the Chooz bound, $\Delta m^2_{23} \lesssim 10^{-3}$ eV$^2$, a larger $\theta_{13}$ is not forbidden by the Chooz data, but disfavoured by SK because it generates an up/down asymmetry in the $e$-like (sub-GeV and multi-GeV) sample. However, as explained in the introduction, the sub-GeV asymmetry vanishes for $\theta_{23} = 45^\circ$, while the one in the multi-GeV sample vanishes for an appropriate value of $\theta_{23} \lesssim 45^\circ$. Since our analysis does not say that $\Delta m^2_{23} = 1 \times 10^{-3}$ eV$^2$ is disfavoured, for an appropriate value of $\theta_{23} \approx 45^\circ$, large $\theta_{13} \gtrsim 20^\circ$ are allowed.

5 Non-standard fit of SK data

Finally we assume that the ‘solar’ $\Delta m^2_{12}$ is sufficiently large to affect atmospheric neutrinos.\footnote{We do not insist on the precise correspondence since it has no particular meaning. There is no objective way of converting $\chi^2$ levels into statements like ‘oscillation parameters lie in the shaded region with 90\% probability’.} The SK observables now depend on all oscillation parameters. For simplicity we only exhibit fits where $\theta_{12} = 45^\circ$ (as suggested by the deficit of solar neutrinos, if $\theta_{13}$ is small) and $\theta_{13} = 0$, so that the CP-violating phase $\phi$ becomes irrelevant. In fig. 2 we show the result of our fit of SK atmospheric data in $\Delta m^2_{13}, \Delta m^2_{23}$ and $\theta_{23}$. Fig. 2a is done for $\Delta m^2_{12} = 0.8 \times 10^{-3}$ eV$^2$, just below the Chooz bound, and shows that a good fit is possible for appropriate values of $\Delta m^2_{13}$ and $\theta_{23}$. In fig. 2b (c) we minimize the $\chi^2$ with respect to $\Delta m^2_{13}$ ($\theta_{23}$). The main result is that values of $\Delta m^2_{12}$ as large as $10^{-3}$ eV$^2$ are compatible with the most recent SK data. Infact the upper bound on $\Delta m^2_{12} \lesssim 0.9 \times 10^{-3}$ eV$^2$ — shown in fig.s 2b,c — comes from the Chooz experiment. Even for the maximal value of $\Delta m^2_{12}$, ‘solar’ oscillations have small effect on SK observables; they produce an up/down asymmetry of $e$-like events (this observable has a dominant statistical error) accompanied by a change in the overall number of events (this observable has a dominant theoretical error). Depending on how the $\chi^2$ is defined this small effect can slightly improve or deteriorate the fit.

These small effects on $e$-like events produce the preference for values of $\theta_{23} \lesssim 50^\circ$ when $\Delta m^2_{23}$ is larger, apparent from fig.s 2a,b. We now discuss this up/down phenomenon.
asymmetry of $e$-like events in more detail because it is the most promising signal of ‘solar’ oscillations that can be observed at SK.

The qualitative features of the asymmetry are well reproduced by the rough approximation (again obtained making the simplifying assumptions used in \[ \theta \approx 45^{\circ} \])

$$
\frac{N^\nu_e}{N^\bar
nu_e} \approx 1 + \frac{R(1 + \cos 2\theta_{23}) - 2}{4} \sin^2 2\theta_{12}.
$$

As already explained in the introduction, if $R = 2$ (as in the sub-GeV data) and $\theta_{23} = 45^{\circ}$ there is no effect. Multi-GeV neutrino events have $R \approx 3$; but are rarer and too energetic for being strongly affected by a $\Delta m^2$ below the Chooz bound. Since the cancellation makes the situation intricate, a numerical computation is necessary to determine the possible signatures of a solar $\Delta m^2$ at SK. We show in figure 3 some example of how the quantities measured by SK are affected by ‘solar’ oscillations for different values of $\Delta m^2$ if $\theta_{23} = 45^{\circ}$. The arrows on the horizontal axes denote the direction of the incoming neutrinos and correspond to the five bins of $\cos \theta_{\text{zenith}}$ used by the SK collaboration to present their results (for example $\uparrow$ refers to the up-going neutrinos that cross the core of the earth). The rate of each one of the 20 bins is normalized with respect to the no oscillation case (our predictions for unoscillated rates are in satisfactory agreement with the Monte Carlo of the SK collaboration). We see that the effect is very small for any value of $\Delta m^2$.

In figure 4 we again show the effects of ‘solar’ oscillations, but for the smallest value of $\theta_{23} = 30^{\circ}$ compatible with the SK data. There are now larger effects in the $e$ sub-GeV sample. We see that a small $\sin^2 2\theta_{23} = 3/4$ gives a poorer fit of $\mu$ events; but this fit also depends on the value of $\Delta m^2$; for this reason we do not consider useful discussing small ‘solar’ effects in the $\mu$ events. To correctly interpret fig. 4 we must remind that the overall normalization of the fluxes (i.e. the ‘1’ line in the plot) has a ~ 20% theoretical uncertainty. Moreover the ‘1’ lines for the four different data samples can be moved independently by ~ 5%. Our $\chi^2$ knows that these systematic uncertainties are highly correlated and says that (with present statistics) even the ‘solar’ effects shown in fig. 4 are “small effects”. For $\theta_{23} = 60^{\circ}$ the effects due to ‘solar’ oscillations have similar size, but opposite sign.

In all these computation we have assumed that $\Delta m^2 > 0$. Matter effects do not significantly affect the results if the $\Delta m^2$ have different signs.

### 5.1 Non zero $\theta_{13}$

So far we have assumed that $\theta_{13}, \phi = 0$. Like in the ‘standard’ scenario, a small $\theta_{13} \lesssim 20^\circ$ is allowed by the Chooz and SK data. CP violation cannot affect SK observables since integration over neutrino energy averages it to zero.

We now discuss how effects due to $\theta_{13}$ mixing can be distinguished from effects due to a ‘solar’ oscillation. In fig. 5 we show how a $\theta_{13} = \{ 0, 5^\circ, 10^\circ, 20^\circ \}$ affects the SK observables if $\Delta m^2 < \Delta m^2 = 3 \times 10^{-3} eV^2$, $\theta_{23} = \{40^\circ, 50^\circ\}$. Comparing the results of the numerical computation shown in fig. 5 with fig. 4 we notice two main differences

- $\theta_{13}$ oscillations mainly affect multi-GeV $e$-like events, while ‘solar’ oscillations can only affect $e$ sub-GeV events.
- For a given value of $\theta_{23}$, ‘solar oscillations’ and ‘$\theta_{13}$-oscillations’ produce up/down asymmetries of opposite sign. The up/down asymmetry produced by $\theta_{13}$ is different from zero and positive even if $\theta_{23} = 45^\circ$.

(In particular, a not too large $\theta_{13}$ can only produce a few % excess of sub-GeV $e$-like events). Both these
Figure 4: Effect of ‘solar’ oscillations on SK observables, normalized to the unoscillated rates, for $\theta_{12} = \theta_{23} = 45^\circ$, $\theta_{13} = 0$, $\Delta m^2_{13} = 3 \times 10^{-3}$ eV$^2$ and $\Delta m^2_{12} = \{0, 0.3, 1, 3.9\} \times 10^{-4}$ eV$^2$ (larger values have longer dashing). The arrows on the horizontal axes denote the direction of incoming neutrinos. The crosses are the experimental data (their error bars only include statistical errors).

Figure 5: As in fig. 4, but for a smaller $\theta_{23} = 30^\circ$. The results for $\theta_{23} = 60^\circ$ are similar, but the sign of the effects is reversed.

features can be understood comparing the approximate up/down asymmetries produced by a small $\theta_{13}$,

$$\frac{N^1_e}{N^0_e} \approx 1 + 2\theta^2_{13}(R \sin^2 \theta_{23} - 1),$$

with the corresponding approximation for ‘solar’ effects, eq. (5). We remind that $\Delta m^2_{23} \gtrsim \Delta m^2_{12}$ and that sub-GeV events have $R \approx 2$, while multi-GeV ones have $R \approx 3$. In both cases the effects cancel out if $R \approx 2$ and $\theta_{23} \approx 45^\circ$.

6 Conclusions

In conclusion a large ‘solar’ $\Delta m^2_{12} \lesssim 10^{-3}$ eV$^2$ is not safely excluded by solar neutrino experiments and

is allowed by atmospheric neutrino experiments. A ‘solar’ oscillation has little effect on SK observables and can slightly ameliorate or deteriorate the fit of SK data, depending on how the $\chi^2$ is defined. Its most clear signature at SK is an up to $\sim 15\%$ angular dependent excess (or deficit) of $e$ sub-GeV events. An indication for a $\sim 10\%$ excess of $e$-like events in the sub-GeV sample was present in the first year of data taking at SK; but the evidence has decreased in the most recent analyses with doubled statistics. However this happened in a not very nice way:

- The SK collaboration has introduced small improvements in their Monte Carlo, and obtained

†On the contrary no acceptable fit of SK data is possible for the mass pattern $\Delta m^2_{12} \sim 10^{-3}$ eV$^2$ and $\Delta m^2_{23} \sim eV^2$, sometimes invoked for reconciling the unconfirmed LSND oscillation with solar and atmospheric ones. We do not show any numerical result because this fact is sufficiently clear from the approximate analysis in *.
slightly different predictions;
- The rate of $e$ sub-GeV events in the most recent part of the sample (last 321 days of data taking) is $(18 \pm 5)\%$ lower than in the first part (first 414 days) \[29\].

Moreover, if the overall flux of atmospheric $\nu$ is somewhat lower than what current estimates indicate — as suggested by a recent (preliminary) measurement of cosmic ray fluxes \[40\] — than SK is observing a smaller deficit of $\nu_\mu$ and an excess of $\nu_e$ events. In conclusion we believe that the normalization of $\nu$ fluxes is still very uncertain and that an excess (or even a deficit) of $e$ sub-GeV events could be present. With more statistics it will be possible to use the unoscillated down-going multi-GeV events to fix the normalization, or to search for up/down asymmetries in the $e$ samples.

To conclude, a large ‘solar’ $\Delta m_{12}^2$ can be experimentally investigated in different ways:
- **SuperKamiokande**, when high-statistics will be available, could see some indication in the details of $e$ events;
- future **solar experiments** (like Borexino) could exclude this possibility;
- **KamLand** could soon observe an evident deficit of reactor $\nu_e$;
- ‘long-baseline experiments’ (like K2K and Minos) can confirm the $\nu_\mu \leftrightarrow \nu_\tau$ oscillation seen at SK. These experiments could also see the $\nu_e$ appearance signal due to a sufficiently large $\theta_{13}$. On the contrary $\nu_\tau$ appearance due only to ‘solar’ effects can be seen only if $\Delta m_{12}^2$ is very close to its maximal value, $10^{-3}$ eV$^2$. CP violating effects in $\nu_\mu \rightarrow \nu_e$ are too small to affect the experiment, except maybe in some extreme case \[22\]. All these conclusions are illustrated by fig. 5, where we show contour-plots of the relevant transition probabilities averaged over the energy spectrum of the ‘medium energy’ $\nu_\mu$ beam at Minos (see the caption for more details).
- future ‘**neutrino factories**’ (beam of $\bar{\nu}_\mu \nu_e$ produced by decay of muons) \[22\]; if $\Delta m_{12}^2$ is large enough, could study CP violation and make precision measurements of the oscillation parameters (but it could be difficult to distinguish $\theta_{13}$ effects from $\Delta m_{12}^2$ effects).

**Note added** In the recent paper \[33\] it was observed that if $\Delta m_{23}^2 \approx 10^{-3}$ eV$^2$ the (interesting?) scenario named ‘tri-maximal mixing’ (in our language $\theta_{12} = \theta_{23} = \pi/4$ and $\sin^2 \theta_{13} = 1/3$, i.e. $\theta_{13} \approx 35^\circ$) is still compatible with SK and CHOOZ data (at least until the complete CHOOZ data will be presented), due to certain cancellations. This is also shown by fig. 6: as we have discussed, these cancellations are not a special feature of tri-maximal mixing. Quite generally SK is rather insensitive to new oscillations beyond the observed one.

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\[\theta_{23} = 40^\circ, \theta_{13} = \{0.5^\circ, 10^\circ, 20^\circ\}\]

\[\theta_{23} = 50^\circ, \theta_{13} = \{0.5^\circ, 10^\circ, 20^\circ\}\]

Figure 6: Sample of how the SK observables are affected by a non zero $\theta_{13}$ for $\Delta m_{23}^2 = 3 \times 10^{-3}$ eV$^2$ and $\theta_{23} = 40^\circ$ (left) $\theta_{23} = 50^\circ$ (right). The arrows on the horizontal axes denote the direction of the incoming neutrinos.

\[\nu \rightarrow \mu \] and an excess of $\nu_e$ events; on the contrary $\nu_\tau$ appearance due only to ‘solar’ effects can be seen only if $\Delta m_{12}^2$ is very close to its maximal value, $10^{-3}$ eV$^2$. CP violating effects in $\nu_\mu \rightarrow \nu_e$ are too small to affect the experiment, except maybe in some extreme case \[22\]. All these conclusions are illustrated by fig. 5, where we show contour-plots of the relevant transition probabilities averaged over the energy spectrum of the ‘medium energy’ $\nu_\mu$ beam at Minos (see the caption for more details).

**Note added** In the recent paper \[33\] it was observed that if $\Delta m_{23}^2 \approx 10^{-3}$ eV$^2$ the (interesting?) scenario named ‘tri-maximal mixing’ (in our language $\theta_{12} = \theta_{23} = \pi/4$ and $\sin^2 \theta_{13} = 1/3$, i.e. $\theta_{13} \approx 35^\circ$) is still compatible with SK and CHOOZ data (at least until the complete CHOOZ data will be presented), due to certain cancellations. This is also shown by fig. 6: as we have discussed, these cancellations are not a special feature of tri-maximal mixing. Quite generally SK is rather insensitive to new oscillations beyond the observed one.

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Figure 7: Contour plots of the transition probabilities $P(\nu_\mu \rightarrow e)$ (fig. [3a]), CP-conserving part of $P(\nu_\mu \rightarrow e)$ (fig. [3b]) and CP-violating part of $P(\nu_\mu \rightarrow e)$ (fig. [3c]) at MINOS (‘medium’ beam) as function of the mass splittings $(\Delta m^2_{12}, \Delta m^2_{23})$ for maximal CP-violating phase $\phi = 90^\circ$, maximal $\theta_{12} = \theta_{23} = \pi/4$ and moderately large $\theta_{13} = 20^\circ$. For this value of $\theta_{13}$, $\Delta m^2_{23} > 3 \times 10^{-3}$ is disfavoured by CHOOZ. MINOS can explore the shaded regions.

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As explained in [19] this information allows to extract the cross section and detection efficiencies of the SK detector.

[20] The energy spectra of the parent atmospheric neutrinos, corresponding to the various classes of events measured at SK (sub-GeV and multi-GeV, $e$, $\mu$ and $\tau$), as given by the most recent Monte Carlo simulation of the SuperKamiokande detector are available at the www address www.awa.tohoku.ac.jp/~etoh/atmnu/index.html.

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