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Numerical Study of Fractional Order COVID-19 Pandemic Transmission Model in Context of ABO Blood Group

By

M. Higazy¹, ², *, F.M. Allehiany³, Emad E. Mahmoud¹, ⁴

¹Department of Mathematics, College of Science, Taif University, P.O. Box 11099, Taif 21944, Saudi Arabia; m.higazy@tu.edu.sa; e.mahmoud@tu.edu.sa

²Department of Physics and Engineering Mathematics, Faculty of Electronic Engineering, Menoufia University, Menouf 32952, Egypt

³Department of Mathematical Sciences, College of Applied Sciences, Umm Al-Qura University, P.O. Box: 715, Makkah 21955, Saudi Arabia; fmlehiany@uqu.edu.sa

⁴Department of Mathematics, Faculty of Science, Sohag University, Sohag 82524, Egypt

*Corresponding authors: m.higazy@tu.edu.sa

Abstract

The worldwide association of health (WHO) has stated that COVID-19 (the novel coronavirus disease-2019) as a pandemic. Here, the common SEIR model is generalized in order to show the dynamics of COVID-19 transmission taking into account the ABO blood group of the infected people. Fractional order Caputo derivative are used in the proposed model. Our study is guided by the results that have been obtained by Chen J, Fan H, Zhang L, et al. from three unique medical clinics in Wuhan and Shenzhen, China. In this study, the feasibility region of the proposed model are calculated plus the points of equilibrium. Also, the equilibrium points stability is examined. A unique solution existence for the proposed paradigm is proved via utilizing the fixed point theory with regards to Caputo fractional derivative. Numerical experiments of the proposed paradigm is done and we show its sensitivity to the fractional order.

Keywords: ABO blood group; COVID-19; Caputo fractional derivative; SEIR model.

1 Introduction

Probably the better vital to humankind is to save the earth on which they live. Humans has developed several kinds of tools and equipment to improve their life. However, this hard improvement now and then prompts failure in the earth system. Lately, the whole world are suffering from a new coronavirus (COVID-19) that popped up in Wuhan, China [1]. Sooner, some researches have shown that COVID-19 transmission occur between humans [2]. Since epidemics produce an important risk to humans and economy resources. A convenient realization of the illness dynamics plays a critical rule in decreasing infections between humans. The decision makers need mathematical models because they are valuable tools that can help them in international health
organizations [3]. Mathematical modeling of infectious diseases has progressed dramatically over the past 30 years and continues to rise up in several fields like epidemiology [4-5].

Until now, there are just two examinations analyzing the effect of ABO blood gathering and disease of COVID-19 are those of [6] by Jiao Zhao et al. and by Chen et al. in [7]. The study in [7] has examined the connection of the ABO blood types with the susceptibility of COVID-19 framework. Their outcomes indicated that the distribution of blood types quite transformed from that in the set utilized for control. For instance, the ABO blood types within 3,694 non infected cases in Wuhan showed classification as: 32.16% for A, 24.90% for B, 9.10% for AB and 33.84% for O, whilst the 1,775 infected cases by COVID-19 from the Hospital of Jinyintan in Wuhan displayed an ABO classification as 37.75% for A, 26.42% for B, 10.03% for AB and 25.80% O. Figures 1, 2 and 3 show these results. According to these results, cases with blood type A was related to a more danger for the infection by the novel coronavirus disease-2019 in comparison with other types, whilst blood type O was related to a less danger for the infection by the novel coronavirus disease-2019.

Figure 1: ABO blood type group distribution in 3694 uninfected cases (data come from [7]).
Many mathematicians and biologists have interested in diseases dynamics (see for instance, [8–18]). Lately, fractional-order derivatives have been utilized to model several biological and physical problems. The main causes of utilizing the fractional-order models (FDMs) is linked to frameworks that have history, memory, or effects of non-locality that can be found in several biological paradigms which shows slowly the realistic double phases refusing manner of infection or diseases. It has been investigated in several scientific papers in the literature that the generalization of the mathematical integer order systems by fractional derivative models the natural manner in a very
formal technique for instance in the method suggested in [19] by Akbari et al., in [20–27] by Etemad et al., in [28–36] by Baleanu et al., in [37, 38] by and in [39] by Talaee et al. Latterly, several researchers have been issued on the concept of fractional derivative operator called “Caputo-Fabrizio operator” (see for instance, [40–49]). Different mathematical paradigms are utilized for simulating COVID-19 transition (see for instance, [50–56]). More information and applications of fractional calculus can be found in [57], [58], [59], [60].

The main object of this work is to produce a mathematical paradigm to study the effect of the ABO blood type on the infected cases of COVID-19. We generalize the common SEIR model by Caputo fractional order derivative and via dividing the infected population into four classes according to the blood type. Following the real data from Wuhan Jinyintan Hospital displayed an ABO classification, the parameters of the suggested paradigm are estimated. In this work, the feasibility region of the proposed paradigm are calculated plus the points of equilibrium. Also, the equilibrium points’ stability is examined. A unique solution existence for the proposed paradigm is demonstrated by utilizing the theorem of fixed point with regard to fractional order derivative defined by Caputo. Numerical simulation of the proposed paradigm is done and we show its sensitivity to the fractional order and to certain parameters.

The coming sections of the article are sorted out as: Section 2 represents certain essential concepts and definitions about fractional order derivative calculus. Section 3 presents the mathematical models of the proposed ABO blood group COVID-19 model. Discussion about the model stability is given in Section 4. In Section 5, a unique solution existence of the proposed paradigm is proved. Numerical experiments and system simulation is given in Section 6. Conclusion claims are written in Section 7.

2 Some essential concepts

In this place, some essential concepts and definitions of the fractional derivative and related concepts are recalled and it can be found in many references such as [61].

**Definition 2.1.** [61] Let \( S(t) \) be an integrable function and let the fractional order be \( q \in (0, 1) \). The Caputo fractional order differentiation operator is recorded in the following lines as:

\[
\frac{d^q}{dt^q} S(t) = \frac{1}{\Gamma(m-q)} \int_0^t \frac{S^{(m)}(\tau)}{(t-\tau)^{q-m+1}} d\tau \quad \text{where} \quad m = \lceil q \rceil + 1.
\]

The corresponding fractional order integral operator with order \( q \) is written in the following line as:

\[
\frac{d^{-q}}{dt^{-q}} S(t) = \frac{1}{\Gamma(q)} \int_0^t (t-\tau)^{q-1} S(\tau) d\tau.
\]

**Definition 2.2.** [40, 62] Let \( S \in H^1(\varepsilon, \varepsilon) \) where \( \varepsilon > \varepsilon \). The fractional order Caputo-Fabrizio differentiation operator is recorded as
\[ c^F \nabla^q S(t) = \frac{\phi(q)}{(1-q)} \int_0^t S(\tau) e^{\frac{-q}{1-q}(t-\tau)} d\tau. \]

Where \( t \geq 0 \) and \( \phi(q) \) is the Caputo-Fabrizio normalization function which depends on \( q \) such that \( \phi(0) = \phi(1) = 1 \). If \( S \in \mathcal{L}^1(-\infty, \delta) \) its fractional order Caputo-Fabrizio differentiation is given by

\[ c^F \nabla^q S(t) = \frac{q \phi(q)}{(1-q)} \int_\delta^t \left[ S(t) - S(\tau) \right] e^{\frac{-q}{1-q}(t-\tau)} d\tau. \]

In the next line, the corresponding fractional order Caputo-Fabrizio integral operator is defined as:

\[ c^F \mathcal{I}^q S(t) = \frac{2(1-q)}{(2-q)\phi(q)} + \frac{2q}{(2-q)\phi(q)} \int_0^t S(\tau)d\tau. \]

The interested readers can consult [63] for fractional order Atangana-Baleanu differentiation operator with regard to the concept of Caputo and its corresponding fractional order integral operator.

3 Mathematical model design

In the epidemiological diseases related to viruses, mathematical modelling is very useful in studying the transmission of the disease among distinct groups because it help in managing the disease. In this section of the paper, we suggest a \( SEI_A I_B I_O I_{AB} RD \) paradigm for COVID-19 epidemic utilizing fractional differential equations as a generalization of the SEIR model [64]. The worldwide association of health (WHO) classifies the infected people into two types: one class with no symptoms and the other type has symptoms and both of them can transfer the virus to others. In the proposed system, people are classified to 8 classes: susceptible individuals (\( S \)), asymptomatic infected cases (exposed) (\( E \)), symptomatic infected people having blood type A (\( I_A \)), symptomatic infected cases having blood type B (\( I_B \)), symptomatic infected people having blood type O (\( I_O \)), symptomatic infected people having blood type AB (\( I_{AB} \)), recovered cases (\( R \)) consisting of healed cases. In Figure 1, we display the dynamics of COVID-19 pandemic via the proposed paradigm.
Figure 4. The proposed COVID-19 system’s graph representation.

SEIR model can be generalized by classifying the infected class into four subclasses due to the blood groups of the infected persons and by applying the Caputo fractional derivative operator as given in the system of Equations (1).

\[
\begin{align*}
C^q\nabla S(t) &= \Lambda - (\alpha E(t) + \beta_1 I_A(t) + \beta_2 I_B(t) + \beta_3 I_O(t) + \beta_4 I_{AB}(t))S(t) - \mu S(t), \\
C^q\nabla E(t) &= (\alpha E(t) + \beta_1 I_A(t) + \beta_2 I_B(t) + \beta_3 I_O(t) + \beta_4 I_{AB}(t))S(t) - \Sigma_{i=1}^{4} \lambda_i + \mu)E(t), \\
C^q\nabla I_A(t) &= \lambda_1 E(t) - (\tau_1 + \mu + \delta_1)I_A(t), \\
C^q\nabla I_B(t) &= \lambda_2 E(t) - (\tau_2 + \mu + \delta_2)I_B(t), \\
C^q\nabla I_O(t) &= \lambda_3 E(t) - (\tau_3 + \mu + \delta_3)I_O(t), \\
C^q\nabla I_{AB}(t) &= \lambda_4 E(t) - (\tau_4 + \mu + \delta_4)I_{AB}(t), \\
C^q\nabla R(t) &= \tau_4 I_A(t) + \tau_2 I_B(t) + \tau_3 I_O(t) + \tau_4 I_{AB}(t) - \mu R(t). \\
\end{align*}
\]

The used parameters of the suggested paradigm of COVID-19 (1) are explained inside the Table 1.

In our work, the suggested system is moderated via the fractional order Caputo differential operator. The ordinary differentiation has \(s^{-1}\) as an inverse second dimension where the fractional order differentiation \(C^q\nabla\) has a dimension of \(s^{-q}\). An additional parameter \(\varpi\) is introduced which has a related dimension \(s\) and named the universal time ([65]). Via this additional parameter, in the context of physical point of view, we can get \([\varpi^{-1}C^q\nabla] \equiv d/dt \equiv s^{-1}\). Consequently, the fractional order COVID-19 system for \(t > 0\) and \(q \in (0, 1)\) is presented in the coming lines.
Suppose that the system starting constraints are $S(0) > 0$, $E(0) > 0$, $I_A(0) > 0$, $I_B(0) > 0$, $I_O(0) > 0$, $I_{AB}(0)$, $R(0) \geq 0$.

| Symbol | The meaning |
|--------|-------------|
| $\Lambda = \phi \times N$ | $N$ is the whole population number and $\phi$ is the rate of new born. |
| $\mu$ | Human natural death rate. |
| $\alpha$ | The contagion rate caused by $E$ class to $S$ class. |
| $\beta_1$ | The contagion rate caused by $I_A$ class to $S$ class. |
| $\beta_2$ | The contagion rate caused by $I_B$ class to $S$ class. |
| $\beta_3$ | The contagion rate caused by $I_O$ class to $S$ class. |
| $\beta_4$ | The contagion rate caused by $I_{AB}$ class to $S$ class. |
| $\lambda_1$ | The rate of transmission of cases from $E$ class to $I_A$ class. |
| $\lambda_2$ | The rate of transmission of cases from $E$ class to $I_B$ class. |
| $\lambda_3$ | The rate of cases transmission from $E$ class to $I_O$ class. |
| $\lambda_4$ | The rate of transmission of cases from $E$ class to $I_{AB}$ class. |
| $\tau_1$ | The recovery rate of $I_A$ class. |
| $\tau_2$ | The recovery rate of $I_B$ class. |
| $\tau_3$ | The recovery rate of $I_O$ class. |
| $\tau_4$ | The recovery rate of $I_{AB}$ class. |
| $\delta_1$ | The death rate caused by disease. |
| $\delta_2$ | The death rate within $I_A$ class by disease. |
| $\delta_3$ | The death rate within $I_B$ class by disease. |
| $\delta_4$ | The death rate within $I_{AB}$ class by disease. |

\[
\begin{align*}
\mathcal{O}^q I_A(t) &= \lambda_1 E(t) - (\tau_1 + \mu + \delta_1) I_A(t), \\
\mathcal{O}^q I_B(t) &= \lambda_2 E(t) - (\tau_2 + \mu + \delta_2) I_B(t), \\
\mathcal{O}^q I_O(t) &= \lambda_3 E(t) - (\tau_3 + \mu + \delta_3) I_O(t), \\
\mathcal{O}^q I_{AB}(t) &= \lambda_4 E(t) - (\tau_4 + \mu + \delta_4) I_{AB}(t), \\
\mathcal{O}^q R(t) &= \tau_1 I_A(t) + \tau_2 I_B(t) + \tau_3 I_O(t) + \tau_4 I_{AB}(t) - \mu R(t).
\end{align*}
\]
3.1 Positivity of the system solutions

Consider $\Omega = \{(S,E,I_A,I_B,I_O,I_{AB},R,D) \in \mathbb{R}^+ : S + E + I_A + I_B + I_O + I_{AB} + R \leq \frac{\Lambda}{\mu}\}$, here we clear that the closed set $\Omega$ is the feasible domain of the proposed paradigm.

**Proposition 3.1.1** For the fractional system (2), the set $\Omega$ is closed positively invariant.

**Proof.** To get the differentiation of all classes fractionally, all relations in the model (2) are added. So

$$\mathcal{A}^q \nabla^q \mathbf{N}(t) = \Lambda - \mu \mathbf{N}(t) - \delta I_A - \delta I_B - \delta I_O - \delta I_{AB},$$

where $\mathbf{N}(t) = S(t) + E(t) + I_A(t) + I_B(t) + I_O(t) + I_{AB}(t) + R(t)$. Applying Laplace transformation yields (referring to results of [66], recorded as Theorem 7.2 plus Remark 7.2):

$$\mathbf{N}(t) \leq \mathbf{N}(0) E_q(-\mu \mathcal{A}^{-q} t^q) \leq \mathbf{N}(0) E_q(-\mu \mathcal{A}^{-q} t^q)$$

Where $\mathbf{N}(0)$ presents the size of the initial population. Which lead to

$$\mathbf{N}(t) \leq \mathbf{N}(0) E_q(-\mu \mathcal{A}^{-q} t^q) + \int_0^t \mathcal{A}^{-q} \tau^{-q-1} \sum_{h=0}^{\infty} \frac{(-1)^h \mathcal{A}^{-q} \tau^q}{\Gamma(hq + q)} d\tau$$

$$= \frac{\Lambda \mathcal{A}^{-q}}{\mu \mathcal{A}^{-q}} E_q(-\mu \mathcal{A}^{-q} t^q) \mathbf{N}(0) - \frac{\Lambda \mathcal{A}^{-q}}{\mu \mathcal{A}^{-q}}$$

$$= \frac{\Lambda}{\mu} + E_q(-\mu \mathcal{A}^{-q} t^q) \mathbf{N}(0) - \frac{\Lambda}{\mu}.$$ 

So if we have $\mathbf{N}(0) \leq \frac{\Lambda}{\mu}$, so for all $t > 0$ and $\mathbf{N}(0) \leq \frac{\Lambda}{\mu}$, the set $\Omega$ is closed positively invariant for the fractional system (2).

3.2 Points of equilibrium

The points of equilibrium can be calculated for the fractional order model (2) via solving the next equations simultaneously.
By solving the algebraic equations of system (2), its equilibrium points can be produced. There will be two equilibrium points: point of equilibrium without disease is obtained as $P_0 = \left( \frac{\Lambda}{\mu}, 0,0,0,0,0,0,0 \right)$ and in case of $R_0 > 1$, so the model in (2) possesses a positive point of equilibrium with cases in all classes which is symbolized here as $P_1 = (S^*, E^*, I^*_A, I^*_B, I^*_O, I^*_AB, R^*)$. Suppose that $I_A = c_1I$, $I_B = c_2I$, $I_O = c_3I$, $I_{AB} = c_4I$, $I$ is the total detected infected class and $c_1, c_2, c_3, c_4$ are constants such that $I_A + I_B + I_O + I_{AB} = I$. For simplicity, the system (2) can be represented as

\[
\begin{align*}
\sigma q^{-1} C q^2 S(t) &= \Lambda - (\alpha E(t) + BI(t))S(t) - \mu S(t), \\
\sigma q^{-1} C q^2 E(t) &= \alpha E(t) + BI(t)S(t) - (L + \mu)E(t), \\
\sigma q^{-1} C q^2 I_A(t) &= LE(t) - (T + \mu + D)I(t), \\
\sigma q^{-1} C q^2 R(t) &= TI(t) - \mu R(t).
\end{align*}
\] 

So, the endemic equilibrium point of the equivalent total framework is $P_1^* = (S^*, E^*, I^*, R^*)$ where $I_A^* + I_B^* + I_O^* + I_{AB}^* = I^*$, such that

\[
\begin{align*}
S^* &= \frac{(\mu + \gamma)(\mu + \vartheta + \Xi)}{BY + \alpha(\mu + \vartheta + \Xi)}, \\
E^* &= \frac{BY\alpha + \alpha \Lambda(\mu + \vartheta + \Xi) - \mu(\gamma + \mu)(\vartheta + \mu + \Xi)}{(\mu + \gamma)[BY + \alpha(\mu + \vartheta + \Xi)]}, \\
I^* &= \frac{\gamma[BY\alpha + \alpha \Lambda(\vartheta + \mu + \Xi) - \mu(\gamma + \mu)(\vartheta + \mu + \Xi)]}{(\mu + \gamma)[\alpha(\vartheta + \Xi + \mu) + BY](\vartheta + \mu + \Xi)}, \\
R^* &= \frac{\vartheta \gamma[\alpha \Lambda(\vartheta + \Xi + \mu) + BY\alpha - \mu(\gamma + \mu)(\vartheta + \Xi + \mu)}{(\gamma + \mu)[\alpha(\vartheta + \Xi + \mu) + BY](\vartheta + \mu + \Xi)\mu},
\end{align*}
\]

where $\gamma = \sum_{x=1}^4 c_x \gamma_x$, $\Xi = \sum_{x=1}^4 c_x \delta_x$, $\vartheta = \sum_{x=1}^4 c_x \tau_x$ and $B = \sum_{x=1}^4 c_x \beta_x$. 

\[c \nabla q^2 S(t) = 0, \]
\[c \nabla q^2 E(t) = 0, \]
\[c \nabla q^2 I_A(t) = 0, \]
\[c \nabla q^2 I_B(t) = 0, \]
\[c \nabla q^2 I_O(t) = 0, \]
\[c \nabla q^2 I_{AB}(t) = 0, \]
\[c \nabla q^2 R(t) = 0. \]
Also, $R_0$ is the basic reproduction criteria and can be estimated as in [67] via the next generation technique. To evaluate $R_0$ put the system \( (2) \) in the form:

$$\sigma^{q-1} C^q \Phi(t) = \Pi_1(\Phi(t)) - \Pi_2(\Phi(t))$$

where

$$\Pi_1(\Phi(t)) = \sigma^{1-q} \begin{bmatrix} \alpha E(t) + BI(t)S(t) \\ 0 \end{bmatrix}$$

and

$$\Pi_2(\Phi(t)) = \sigma^{1-q} \begin{bmatrix} (\gamma + \mu)E(t) \\ -\gamma E(t) + (\beta + \mu + \Xi)I(t) \end{bmatrix}$$

At $P_0^* = \left( \frac{\Lambda}{\mu}, 0, 0 \right)$ the Jacobian matrices for $\Pi_1(\Phi(t))$ and $\Pi_2(\Phi(t))$ take the forms

$$J_{\Pi_1}(P_0) = \sigma^{1-q} \begin{bmatrix} \alpha \frac{\Lambda}{\mu} & B \frac{\Lambda}{\mu} \\ 0 & 0 \end{bmatrix}$$

and

$$J_{\Pi_2}(P_0) = \sigma^{1-q} \begin{bmatrix} (\gamma + \mu) & 0 \\ -\gamma & \beta + \mu + \Xi \end{bmatrix}.$$ 

Following [67], the basic reproduction number $R_0 = \rho(\Pi_1, \Pi_2^{-1})$, So

$$R_0 = \frac{\alpha \Lambda (\beta + \mu + \Xi) + B \Lambda \gamma}{\mu (\gamma + \mu) (\beta + \mu + \Xi)}.$$ 

This primary reproduction criteria $R_0$, is a criteria utilized to measure the transmissibility of infectious cases in epidemiologic. Since all parameters are positive, $R_0$ is directly proportional to the infection transmission rate caused by $E$ population to $S$ population ($\alpha$), the infection transmission rate caused by $I$ population to $S$ population ($B$), $\Lambda$ is the rate of new born.

4 Stability discussion

Here, the equilibrium points stability is examined. Suppose that $I_A = c_1 I$, $I_B = c_2 I$, $I_O = c_3 I$, $I_{AB} = c_4 I$, $I$ is the total detected infected class and $c_1, ..., c_4$ are constants such that $I_A + I_B + I_O + I_{AB} = I$. Then,
as in the previous section, we can let $\gamma = \sum_{x=1}^{\overline{x}} c_{x} \lambda_{x}$, $\Xi = \sum_{x=1}^{\overline{x}} c_{x} \delta_{x}$, $\vartheta = \sum_{x=1}^{\overline{x}} c_{x} \tau_{x}$ and $B = \sum_{x=1}^{\overline{x}} c_{x} \beta_{x}$. The system (2) can be rewritten as

\[
\begin{align*}
\omega^{q-1}C \nabla^{q} S(t) &= \Lambda - (\alpha E(t) + BI(t))S(t) - \mu S(t), \\
\omega^{q-1}C \nabla^{q} E(t) &= \alpha E(t) + BI(t)S(t) - (\gamma + \mu)E(t), \\
\omega^{q-1}C \nabla^{q} I_{A}(t) &= \gamma E(t) - (\vartheta + \mu + \Xi)I(t), \\
\omega^{q-1}C \nabla^{q} R(t) &= \vartheta I(t) - \mu R(t).
\end{align*}
\]

(4)

Which has the next Jacobian matrix.

\[
J = \omega^{1-q} \begin{bmatrix}
-\alpha E - BI - \mu & -\alpha S & -BS & 0 \\
\alpha E + BI & \alpha S - \gamma - \mu & BS & 0 \\
0 & \gamma & -\vartheta - \mu - \Xi & 0 \\
0 & 0 & \vartheta & -\mu
\end{bmatrix}
\]

Then at $P^*_o = (\frac{\Lambda}{\mu}, 0, 0, 0)$

\[
J(P^*_o) = \omega^{1-q} \begin{bmatrix}
-\mu & -\alpha \frac{\Lambda}{\mu} & -B \frac{\Lambda}{\mu} & 0 \\
0 & \alpha \frac{\Lambda}{\mu} - \gamma - \mu & B \frac{\Lambda}{\mu} & 0 \\
0 & \gamma & -\vartheta - \mu - \Xi & 0 \\
0 & 0 & \vartheta & -\mu
\end{bmatrix}
\]

**Proposition 4.1** The point $P^*_o = (\frac{\Lambda}{\mu}, 0, 0, 0)$ (equilibrium without disease) is unstable if the reproduction number $R_0 > 1$ and is asymptotically stable near $P^*_o$ if $R_0 < 1$.

**Proof**

At $P^*_o$, the Jacobian matrix has $-\mu$ as an eigenvalue plus the roots of the equation $\vartheta^2 + H\vartheta + \Theta = 0$.

Where

\[
H = -\frac{\alpha \Lambda - \mu (\gamma + \mu) - \mu (\vartheta + \mu + \Xi)}{\mu},
\]
and

\[ \Theta = -\frac{\alpha \Lambda(\theta + \mu + \Xi) - \mu(\bar{\gamma} + \mu)(\theta + \mu + \Xi) + YB\Lambda}{\mu}. \]

We know that all parameters are positive, then for \( R_0 < 1 \)

\[ (R_0 = \frac{\alpha \Lambda(\theta + \mu + \Xi) + B\Lambda Y}{\mu(\bar{\gamma} + \mu)(\theta + \mu + \Xi)} < 1), \]

Then

\[ \alpha \Lambda(\theta + \mu + \Xi) + B\Lambda Y < \mu(\bar{\gamma} + \mu)(\theta + \mu + \Xi) \Rightarrow H > 0. \]

And \( \alpha \Lambda - \mu(\bar{\gamma} + \mu) < \frac{-B\Lambda Y}{(\theta + \mu + \Xi)} < 0 \Rightarrow \Theta > 0. \]

Then utilizing Routh-Hurwitz stability criteria, \( P^*_0 = (\frac{\Lambda}{\mu}, 0, 0, 0) \) is asymptotically stable near \( P^*_0 \). For \( R_0 > 1, \Theta < 0 \) which means that the equation \( \theta^2 + H\theta + \Theta = 0 \) has a positive real eigenvalue which imply the instability of \( P_0 \).

At \( P^*_1 = (S^*, E^*, I^*, R^*) \), system (4) has the following Jacobian matrix:

\[
J = \begin{bmatrix}
-\alpha E - B I - \mu & -\alpha S^* & -B S^* & 0 \\
\alpha E^* + B I^* & \alpha S^* - \bar{\gamma} - \mu & B S^* & 0 \\
0 & \bar{\gamma} & -\theta - \mu - \Xi & 0 \\
0 & 0 & \theta & -\mu
\end{bmatrix}
\]

Then its characteristic polynomial takes the following form:

\[
\sigma^{1-q} (\theta + \mu)((\theta + (\bar{\gamma} + \mu + \Xi))(\theta^2 - H_1 \theta + \Theta_1),
\]

where

\[
H_1 = \alpha S^* - \bar{\gamma} - 2\mu + \frac{B S^* \bar{\gamma}}{\theta + \mu + \Xi},
\]

and

\[
\Theta_1 = (\mu + \alpha E^* + B I^*)(\bar{\gamma} + \mu) + \frac{\alpha S^* - \mu(\bar{\gamma} + \mu + \Xi) + B S^* \bar{\gamma}}{\bar{\gamma} + \mu + \Xi}.
\]
Then the eigenvalues will be $\theta_1 = -\mu$, $\theta_2 = -(\theta + \mu + \Xi)$ and the two roots of the following quadratic equation

$$(\theta^2 - H, \theta + \Theta_1) = 0.$$ 

Then $P^* = (S^*, E^*, I^*, R^*)$ is asymptotically stable in its neighborhood if $\Theta_1 > 0$, $H_1 < 0$.

5 System (2) solution: existence and uniqueness

Now, a unique solution existence of the studied paradigm (2) is to be proved in this section. The system (2) is written firstly as follows.

$$\begin{align*}
\vartheta^{q-1} C \nabla^q S(t) &= \Psi_1(t, S(t)) \\
\vartheta^{q-1} C \nabla^q E(t) &= \Psi_2(t, E(t)), \\
\vartheta^{q-1} C \nabla^q I_A(t) &= \Psi_3(t, I_A(t)), \\
\vartheta^{q-1} C \nabla^q I_B(t) &= \Psi_4(t, I_B(t)), \\
\vartheta^{q-1} C \nabla^q I_O(t) &= \Psi_5(t, I_O(t)), \\
\vartheta^{q-1} C \nabla^q I_{AB}(t) &= \Psi_6(t, I_{AB}(t)), \\
\vartheta^{q-1} C \nabla^q R(t) &= \Psi_7(t, R(t)).
\end{align*}$$

(5)

Applying fractional integral form for the two sides of equations in (5), we obtain

$$\begin{align*}
S(t) - S(0) &= \frac{\vartheta^{1-q}}{\Gamma(q)} \int_0^t \Psi_1(t, S(t))(t-\tau)d\tau, \\
E(t) - E(0) &= \frac{\vartheta^{1-q}}{\Gamma(q)} \int_0^t \Psi_2(t, E(t))(t-\tau)d\tau, \\
I_A(t) - I_A(0) &= \frac{\vartheta^{1-q}}{\Gamma(q)} \int_0^t \Psi_3(t, I_A(t))(t-\tau)d\tau, \\
I_B(t) - I_B(0) &= \frac{\vartheta^{1-q}}{\Gamma(q)} \int_0^t \Psi_4(t, I_B(t))(t-\tau)d\tau, \\
I_O(t) - I_O(0) &= \frac{\vartheta^{1-q}}{\Gamma(q)} \int_0^t \Psi_5(t, I_O(t))(t-\tau)d\tau, \\
I_{AB}(t) - I_{AB}(0) &= \frac{\vartheta^{1-q}}{\Gamma(q)} \int_0^t \Psi_6(t, I_{AB}(t))(t-\tau)d\tau, \\
R(t) - R(0) &= \frac{\vartheta^{1-q}}{\Gamma(q)} \int_0^t \Psi_7(t, R(t))(t-\tau)d\tau.
\end{align*}$$

(6)
Lipschitz condition satisfaction by the kernels $\Psi_r$, $r = 1, 2, 3, ..., 7$ is to be shown now.

**Proposition 5.1** Lipschitz condition is satisfied by the kernel $\Psi_1$, and it is contradiction if the following conditions are hold.

$$0 < (\alpha \delta_2 + \beta_1 \delta_3 + \beta_2 \delta_4 + \beta_3 \delta_5 + \beta_4 \delta_6 + \mu) < 1.$$

**Proof.**

The following is true for $S(t)$ and $S_1(t)$

$$\|\Psi_1(t, S(t)) - \Psi_1(t, S_1(t))\|$$

$$= \|(- \alpha E(t) + \beta_1 I_A(t) + \beta_2 I_B(t) + \beta_1 I_O(t) + \beta_4 I_{AB}(t)) (S(t) - S_1(t)) - \mu (S(t) - S_1(t))\|$$

$$\leq \|(- \alpha E(t) + \beta_1 I_A(t) + \beta_2 I_B(t) + \beta_1 I_O(t) + \beta_4 I_{AB}(t))\| \|S(t) - S_1(t)\| + \mu \|S(t) - S_1(t)\|$$

$$\leq (\alpha \|E(t)\| + \beta_1 \|I_A(t)\| + \beta_2 \|I_B(t)\| + \beta_1 \|I_O(t)\| + \beta_4 \|I_{AB}(t)\| + \mu) \|S(t) - S_1(t)\|$$

$$\leq (\alpha \delta_2 + \beta_1 \delta_3 + \beta_2 \delta_4 + \beta_3 \delta_5 + \beta_4 \delta_6 + \mu) \|S(t) - S_1(t)\|.$$  

Assume that $h_1 = (\alpha \delta_2 + \beta_1 \delta_3 + \beta_2 \delta_4 + \beta_3 \delta_5 + \beta_4 \delta_6 + \mu)$ where

$$\|E(t)\| \leq \delta_2, \|I_A(t)\| \leq \delta_3, \|I_B(t)\| \leq \delta_4, \|I_O(t)\| \leq \delta_5, \|I_{AB}(t)\| \leq \delta_6$$

are limited relations. Then

$$\|\Psi_1(t, S(t)) - \Psi_1(t, S_1(t))\| \leq h_1 \|S(t) - S_1(t)\|. \quad (7)$$

Consequently, the Lipschitz condition is hold for $\Psi_1$. Then $\Psi_1$ is a contraction if

$$0 < (\alpha \delta_2 + \beta_1 \delta_3 + \beta_2 \delta_4 + \beta_3 \delta_5 + \beta_4 \delta_6 + \mu) < 1.$$

Following the same reasoning, proving that Lipschitz condition is satisfied by $\Psi_r$, $r = 2, 3, ..., 7$ is obvious.
\[
\begin{align*}
\|\Psi_2(t, E(t)) - \Psi_2(t, E_i(t))\| & \leq \delta_2 \|E(t) - E_i(t)\|, \\
\|\Psi_3(t, I_A(t)) - \Psi_3(t, I_{A_l}(t))\| & \leq \delta_3 \|I_A(t) - I_{A_l}(t)\|, \\
\|\Psi_4(t, I_B(t)) - \Psi_4(t, I_{B_l}(t))\| & \leq \delta_4 \|I_B(t) - I_{B_l}(t)\|, \\
\|\Psi_5(t, I_O(t)) - \Psi_5(t, I_{O_l}(t))\| & \leq \delta_5 \|I_O(t) - I_{O_l}(t)\|, \\
\|\Psi_6(t, I_{AB}(t)) - \Psi_6(t, I_{AB_l}(t))\| & \leq \delta_6 \|I_{AB}(t) - I_{AB_l}(t)\|, \\
\|\Psi_7(t, R(t)) - \Psi_7(t, R_i(t))\| & \leq \delta_7 \|R(t) - R_i(t)\|.
\end{align*}
\]

Such that \(\|S(t)\| \leq \delta_1\), and \(\delta_2 = \alpha + \sum_{i=1}^{4} \lambda_i + \mu, \delta_3 = \tau_1 + \mu + \delta_1, \delta_4 = \tau_2 + \mu + \delta_2, \delta_5 = \tau_3 + \mu + \delta_3, \delta_6 = \tau_4 + \mu + \delta_4\) and \(\delta_7 = \mu\), are all bounded. If for all \(r = 2, 3, \ldots, 7\) we have \(0 \leq \delta_r \leq 1\) then \(\Psi_r\) are contraction for all \(r = 2, 3, \ldots, 7\).

The following recursive formulas can be considered for the system (2).

\[
\begin{align*}
P_{2n} &= S_n(t) - S_{(n-1)}(t) = \frac{\sigma^{1-q}}{\Gamma(q)} \int_0^t \left(\Psi_1(\tau, S_{(n-1)}(t)) - \Psi_1(\tau, S_{(n-1)}(t))\right)(t-\tau) d\tau \\
P_{3n} &= E_n(t) - E_{(n-1)}(t) = \frac{\sigma^{1-q}}{\Gamma(q)} \int_0^t \left(\Psi_2(\tau, E_{(n-1)}(t)) - \Psi_2(\tau, E_{(n-1)}(t))\right)(t-\tau) d\tau \\
P_{4n} &= I_{An}(t) - I_{A_{(n-1)}}(t) = \frac{\sigma^{1-q}}{\Gamma(q)} \int_0^t \left(\Psi_3(\tau, I_{A_{(n-1)}}(t)) - \Psi_3(\tau, I_{A_{(n-1)}}(t))\right)(t-\tau) d\tau \\
P_{5n} &= I_{Bn}(t) - I_{B_{(n-1)}}(t) = \frac{\sigma^{1-q}}{\Gamma(q)} \int_0^t \left(\Psi_4(\tau, I_{B_{(n-1)}}(t)) - \Psi_4(\tau, I_{B_{(n-1)}}(t))\right)(t-\tau) d\tau \\
P_{6n} &= I_{On}(t) - I_{O_{(n-1)}}(t) = \frac{\sigma^{1-q}}{\Gamma(q)} \int_0^t \left(\Psi_5(\tau, I_{O_{(n-1)}}(t)) - \Psi_5(\tau, I_{O_{(n-1)}}(t))\right)(t-\tau) d\tau \\
P_{7n} &= I_{ABn}(t) - I_{AB_{(n-1)}}(t) = \frac{\sigma^{1-q}}{\Gamma(q)} \int_0^t \left(\Psi_6(\tau, I_{AB_{(n-1)}}(t)) - \Psi_6(\tau, I_{AB_{(n-1)}}(t))\right)(t-\tau) d\tau \\
P_{8n} &= I_{Obn}(t) - I_{Ob_{(n-1)}}(t) = \frac{\sigma^{1-q}}{\Gamma(q)} \int_0^t \left(\Psi_7(\tau, I_{Ob_{(n-1)}}(t)) - \Psi_7(\tau, I_{Ob_{(n-1)}}(t))\right)(t-\tau) d\tau \\
P_{9n} &= R_n(t) - R_{(n-1)}(t) = \frac{\sigma^{1-q}}{\Gamma(q)} \int_0^t \left(\Psi_8(\tau, R_{(n-1)}(t)) - \Psi_8(\tau, R_{(n-1)}(t))\right)(t-\tau) d\tau
\end{align*}
\]

Take the norm for the first relation in the above system where the initial conditions \(S_0(t) = S(0), E_0(t) = E(0), I_{A0}(t) = I_A(0), I_{B0}(t) = I_B(0), I_{O0}(t) = I_O(0), I_{AB0}(t) = I_{AB}(0)\) and \(R_0(t) = R(0)\).
\[ \|P_{1n}\| = \|S_n(t) - S_{n-1}(t)\| \]
\[ = \left\| \frac{\rho^{1-q}}{\Gamma(q)} \int_0^t (\Psi_1(t, S_{n-1}(t)) - \Psi_1(t, S_{n-1}(t))) (t - \tau) d\tau \right\| \]
\[ \leq \frac{\rho^{1-q}}{\Gamma(q)} \int_0^t (\Psi_1(t, S_{n-1}(t)) - \Psi_1(t, S_{n-1}(t))) (t - \tau) d\tau \]

Taking Lipschitz condition (7) into account, we get
\[ \|P_{1n}\| \leq \frac{\rho^{1-q}}{\Gamma(q)} h_1 \int_0^t \|P_{n-1}\| d\tau \quad (8) \]

Following the similar manner, we have
\[ \|P_{2n}\| \leq \frac{\rho^{1-q}}{\Gamma(q)} h_2 \int_0^t \|P_{2n-1}\| d\tau, \]
\[ \|P_{3n}\| \leq \frac{\rho^{1-q}}{\Gamma(q)} h_3 \int_0^t \|P_{3n-1}\| d\tau, \]
\[ \|P_{4n}\| \leq \frac{\rho^{1-q}}{\Gamma(q)} h_4 \int_0^t \|P_{4n-1}\| d\tau, \]
\[ \|P_{5n}\| \leq \frac{\rho^{1-q}}{\Gamma(q)} h_5 \int_0^t \|P_{5n-1}\| d\tau, \]
\[ \|P_{6n}\| \leq \frac{\rho^{1-q}}{\Gamma(q)} h_6 \int_0^t \|P_{6n-1}\| d\tau, \]
\[ \|P_{7n}\| \leq \frac{\rho^{1-q}}{\Gamma(q)} h_7 \int_0^t \|P_{7n-1}\| d\tau. \]

Consequently, we can record that
\[ S_n(t) = \sum_{x=1}^n P_{1x}(t), \quad E_n(t) = \sum_{x=1}^n P_{2x}(t), \quad I_{A_n}(t) = \sum_{x=1}^n P_{3x}(t), \quad I_{Bn}(t) = \sum_{x=1}^n P_{4x}(t), \quad I_{On}(t) = \sum_{x=1}^n P_{5x}(t), \]
\[ I_{A\delta n}(t) = \sum_{x=1}^n P_{6x}(t), \quad R_n(t) = \sum_{x=1}^n P_{7x}(t). \]

The existence of the solution is proved in the next proposition.

**Proposition 5.2** Fractional order blood group classified COVID-19 (SEI_{A}I_{B}I_{O}I_{A\delta}R) transmission system (2) has a solution if there is \( \xi \) such that
\[ \frac{\sigma^{1-q}}{\Gamma(q)} x_i h_i < 1. \]

**Proof** From equations (8) and (9) plus the recursive formula, the following can be concluded.

\[ \| P_{1n}(t) \| \leq \| S(0) \| \left[ \frac{\sigma^{1-q}}{\Gamma(q)} h_1 t \right]^n, \]

\[ \| P_{2n}(t) \| \leq \| E(0) \| \left[ \frac{\sigma^{1-q}}{\Gamma(q)} h_2 t \right]^n, \]

\[ \| P_{3n}(0) \| \leq \| I_A(0) \| \left[ \frac{\sigma^{1-q}}{\Gamma(q)} h_3 t \right]^n, \]

\[ \| P_{4n}(t) \| \leq \| I_B(0) \| \left[ \frac{\sigma^{1-q}}{\Gamma(q)} h_4 t \right]^n, \]

\[ \| P_{5n}(t) \| \leq \| I_C(0) \| \left[ \frac{\sigma^{1-q}}{\Gamma(q)} h_5 t \right]^n, \]

\[ \| P_{6n}(t) \| \leq \| I_{AB}(0) \| \left[ \frac{\sigma^{1-q}}{\Gamma(q)} h_6 t \right]^n. \]

Consequently, the suggested model has a continuous solution. Then, the former functions produce solution for the system (2), and to prove that we suppose the next

\[ S(t) - S(0) = S_n(t) - C_{1n}(t), \]

\[ E(t) - E(0) = E_n(t) - C_{2n}(t), \]

\[ I_A(t) - I_A(0) = I_{An}(t) - C_{3n}(t), \]

\[ I_B(t) - I_B(0) = I_{Bn}(t) - C_{4n}(t), \]

\[ I_C(t) - I_C(0) = I_{On}(t) - C_{5n}(t), \]

\[ I_{AB}(t) - I_{AB}(0) = I_{ABn}(t) - C_{6n}(t), \]

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\[ R(t) - R(0) = R_n(t) - C_{7n}(t). \]

Then
\[
\|C_{1n}(t)\| = \left| \frac{\sigma^{1-q} t}{\Gamma(q)} \right| \int_0^t \left( \Psi_1(\tau, S(t)) - \Psi_1(\tau, S_{n-1}(t)) \right) d\tau \\
\leq \frac{\sigma^{1-q} t}{\Gamma(q)} \int_0^t \left\| \Psi_1(\tau, S(t)) - \Psi_1(\tau, S_{n-1}(t)) \right\| d\tau \\
\leq \frac{\sigma^{1-q} h_1}{\Gamma(q)} \|S(t) - S_{n-1}(t)\| t.
\]

Repeating, we get
\[
\|C_{1n}\| \leq \left[ \frac{\sigma^{1-q} t}{\Gamma(q)} \right]^{n+1} h_1^{n+1} K
\]

When \( t = x_i \)
\[
\|C_{1n}\| \leq \left[ \frac{\sigma^{1-q} x_i}{\Gamma(q)} \right]^{n+1} h_1^{n+1} K.
\]

As \( n \) approaches infinity, \( \|C_{1n}\| \) approaches zero. By the same way, \( \|C_{rn}\| \) approach zero as \( n \) approaches infinity where \( r = 2, 3, ..., 7 \), which finish the proof.  \( \blacksquare \)

The solution uniqueness is proved by supposing that there are another different solution for model as \( S^\circ(t), E^\circ(t), I_A^\circ(t), I_B^\circ(t), I_O^\circ(t), I_{Ah}^\circ(t), R^\circ(t) \), then
\[
S(t) - S^\circ(t) = \frac{\sigma^{1-q} t}{\Gamma(q)} \int_0^t \left( \Psi_1(\tau, S(t)) - \Psi_1(\tau, S^\circ(t)) \right) d\tau
\]

Taking the norm, so
\[
\|S(t) - S^\circ(t)\| = \frac{\sigma^{1-q} t}{\Gamma(q)} \left\| \Psi_1(\tau, S(t)) - \Psi_1(\tau, S^\circ(t)) \right\| d\tau
\]

Taking Lipschitz condition (7) implies that
\[
\|S(t) - S^\circ(t)\| \leq \left( \frac{\sigma^{1-q} h_1 t}{\Gamma(q)} \right) \|S(t) - S^\circ(t)\|.
\]
Then
$$\left(1 - \frac{\sigma^{1-q}}{\Gamma(q)} h \right) \| S(t) - S^\circ(t) \| \leq 0.$$ 

**Proposition 5.3** The fractional order blood group classified COVID-19 (SEI_{A,B}O_{A,B}R) transmission system (2) has a unique solution if
$$\left(1 - \frac{\sigma^{1-q}}{\Gamma(q)} h \right) > 0.$$ 

**Proof** Since condition (7) is satisfied, hence
$$\left(1 - \frac{\sigma^{1-q}}{\Gamma(q)} h \right) \| S(t) - S^\circ(t) \| \leq 0.$$ 

Which implies that $\| S(t) - S^\circ(t) \| = 0$, and as a result $S(t) = S^\circ(t)$. This is can be proved for all other system states. ■

6 Numerical experiments

In this section, using Caputo fractional derivative, the numerical solutions of the ABO blood group SEI_{A,B}O_{A,B}R COVID-19 model (2) are given at various fractional orders, namely we take $q = 0.7, 0.8, 0.9, 1$. In numerical simulations, the algorithm given by Adams-Bashforth-Moulton algorithm in [68] is used and executed within MATLAB. In addition, the model sensitivity to changes of certain parameters are to be shown. The parameters’ values are estimated and taken from the cited references. The world current rate of birth is 0.018, and the rate of death is 0.007612 ([69], [70], [64], [71]). The whole population of the world on 4/2/2020 was $N = 761010542$, as a result we have $\Lambda = \frac{\sigma \times N}{365} = 391347.06$ and $\mu = 7.612 \times 10^{10} = 2.08547 \times 10^{-5}$. Because

$\mathbf{N}(0) = S(0) + E(0) + I_A(0) + I_B(0) + I_{O}(0) + I_{AB}(0) + R(0)$, and as recorded in 4th February 2020, we have $I(0) = 24545$, $I_A(0) = 0.369 I(0)$, $I_B(0) = 0.369 I(0)$, $I_{O}(0) = 0.2192 I(0)$, $I_{AB}(0) = 0.0749 I(0)$, and $R(0) = 907 ([70])$, hence take $E(0) = 8 \times 10^4$ and $S(0) = 7610026 \times 10^4$. In addition, following the report of (WHO) given in [70] and taking into account the study in [6], the rate of mortality caused by COVID-19 can be estimated to be $\delta_1 = 3.4 \times 10^{-2}$, $\delta_2 = 3.3 \times 10^{-2}$, $\delta_3 = 3 \times 10^{-2}$ and $\delta_4 = 3.35 \times 10^{-2}$. $\alpha = 2 \times 10^{-11}$, $\beta_1 = 0.369 \times 2.2 \times 10^{-9}$, $\beta_2 = 0.3369 \times 2.2 \times 10^{-9}$, $\beta_3 = 0.2192 \times 2.2 \times 10^{-9}$ and $\beta_4 = 0.0749 \times 2.2 \times 10^{-9}$. $\lambda_1 = 0.369 \times 2.35 \times 10^{-5}$, $\lambda_2 = 0.3369 \times 2.35 \times 10^{-5}$, $\lambda_3 = 0.2192 \times 2.35 \times 10^{-5}$, $\lambda_4 = 0.0749 \times 2.35 \times 10^{-5}$, $\tau_1 = 0.03$, $\tau_2 = 0.05$, $\tau_3 = 0.06$ and $\tau_4 = 0.04$. Also, we put $\sigma = 0.99$.

Figure 5 shows the time evolution of susceptible class $S(t)$ with respect to different fractional orders, namely we take $q = 0.7, 0.8, 0.9, 1$. From which we can see the effect of altering the fractional
derivative order on the evolution of the susceptible class $S(t)$. From which, As the fractional order decreases the curve become more flat and reaching the equilibrium state is delayed. The time evolution of exposed (asymptomatic) infected class $E(t)$ with respect to various fractional order is shown in Figure 6, namely we take $q \in \{1, 0.9, 0.8, 0.7\}$. It shows that lowering the fractional order slows down reaching the beak value and decreases the sharpness of the transient period. Figure 7 displays the time evolution of symptomatic infected, with blood type “A”, population $I_A(t)$ with respect to various fractional orders is displayed in Figure 8, namely we take $q \in \{1, 0.9, 0.8, 0.7\}$. The time evolution of the infected (symptomatic) with blood type “B” population $I_B(t)$ with respect to various fractional orders is displayed in Figure 9, namely we take $q \in \{1, 0.9, 0.8, 0.7\}$. The time evolution of the infected (symptomatic) with blood type “O” population $I_O(t)$ with respect to various fractional orders is displayed, namely we take $q \in \{1, 0.9, 0.8, 0.7\}$. The time evolution of the infected (symptomatic) with blood type “AB” population $I_{AB}(t)$ with respect to various fractional orders is displayed in Figure 10, namely we take $q \in \{1, 0.9, 0.8, 0.7\}$. The time evolution of the recovered population $R(t)$ with respect to various fractional order is displayed is Figure 11, namely we take $q \in \{1, 0.9, 0.8, 0.7\}$. The infected cases classified according to ABO blood group is displayed in Figure 12: (a) $I_A(t)$, $I_B(t)$, $I_O(t)$, $I_{AB}(t)$ at fractional order $q = 1$; (b) $I_A(t)$, $I_B(t)$, $I_O(t)$, $I_{AB}(t)$ at fractional order $q = 0.9$. (c) $I_A(t)$, $I_B(t)$, $I_O(t)$, $I_{AB}(t)$ at fractional order $q = 0.8$. (d) $I_A(t)$, $I_B(t)$, $I_O(t)$, $I_{AB}(t)$ at fractional order $q = 0.7$. From Figures 7, 8, 9, 10, 11, decreasing the fractional order has the same effect on the infected detected cases regardless their ABO blood type. From Figure 12, the cases with blood type “A” are greater than all other blood types cases which guarantee with the results from the real study performed in [6].
Figure 5: Time evolution of the susceptible population $S(t)$ with respect to various fractional order $q \in \{1, 0.9, 0.8, 0.7\}$.

Figure 6: Evolution of the exposed (asymptomatic) infected population $E(t)$ with respect to various fractional order $q \in \{1, 0.9, 0.8, 0.7\}$.
Figure 7: Evolution of the infected (symptomatic) with blood type “A” population $I_A(t)$ with respect to various fractional order $q \in \{1, 0.9, 0.8, 0.7\}$.

Figure 8: Evolution of the infected (symptomatic) with blood type “B” population $I_B(t)$ with respect to fractional order $q \in \{1, 0.9, 0.8, 0.7\}$.
Figure 9: Evolution of the infected (symptomatic) with blood type “O” population $I_o(t)$ with respect to various fractional order $q \in \{1, 0.9, 0.8, 0.7\}$.

Figure 10: Evolution of the infected (symptomatic) with blood type “AB” population $I_{AB}(t)$ with respect to various fractional order $q \in \{1, 0.9, 0.8, 0.7\}$.
Figure 11: Evolution of the recovered population $R(t)$ with respect to various fractional order $q \in \{1, 0.9, 0.8, 0.7\}$.

Figure 12: Infected cases $I_A(t), I_B(t), I_O(t), I_{AB}(t)$ (classified according to ABO blood group). (a) at fractional order $q = 1$; (b) at fractional order $q = 0.9$; (c) at fractional order $q = 0.8$; (d) at fractional order $q = 0.7$. 
Figure 13: The impact of altering the transmission rate ($\alpha$) caused by $E$ population to $S$ population and the transmission rate caused by $I_A$ population to $S$ population ($\beta_1$) on the number of infected cases at day 100 where $q = 1$.

In Figure 13, we show the impact of altering the transition rate ($\alpha$) caused by $E$ class to $S$ class at the same time with altering the transition rate ($\beta_1$) caused by $I_A$ class to $S$ class on the number of infected cases at day 100 with respect to $q = 1$. In Figure 14, we show the impact of altering the transition rate ($\alpha$) caused by $E$ class to class $S$ at the same time with altering the transition rate
(\(\beta_i\)) caused by \(I_A\) class to class \(S\) on the number of infected cases at day 100 with respect to fractional order \(q = 0.8\). The numerical experiments show that the other infected cases with different blood types have the same behavior with respect to their corresponding transition rates. In Figure 15, we show the evolution of the susceptible class: \(S(t, q)\). In Figure 16, we show the evolution of the infected blood type “A” population: \(I_A(t, q)\). In Figure 17, we show the evolution of the recovered class: \(R(t, q)\).

Figure 15: The dynamics of the susceptible class: \(S(t, q)\).

Figure 16: The dynamics of the infected blood type “A” population: \(I_A(t, q)\).
Figure 17: The dynamics of recovered population: $R(t, q)$.

7 Conclusion

In the presented study, the SEIR epidemic system for COVID-19 transition, utilizing the fractional derivative defined by Caputo, has been modified in order to show the dynamics of the infected cases according to their blood types. The common SEIR model has been generalized by Caputo fractional order derivative and via dividing the infected population into four classes according to the blood type. Following the real data from Wuhan Jinyintan Hospital displaying an ABO classification, the parameters of the proposed paradigm are estimated and applied to the numerical experiments. In this study, the feasibility region of the proposed model have been calculated plus the points of equilibrium. Also, the equilibrium points stability has been examined. Utilizing the fixed point theory, a unique solution existence for the proposed paradigm has been proved by with regard to fractional derivative defined by Caputo. Numerical simulation of the suggested system has been performed and the model sensitivity to the fractional order and to certain parameters has been presented. The results by the proposed model are consistent with the real data. For the future work, we suggest to study the effect of blood type on other populations.

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