Ground State Correlations and Mean Field Using the exp(S) Method.

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Abstract

This document gives a detailed account of the terms used in the computation of the ground state mean field and the ground state correlations. While the general approach to this description is given in a separate paper (nucl-th/9802029), we give here the explicit expressions used.

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1 Preliminaries.

The purpose of this document is to give a detailed account of the terms which we use in generating the 2-body ground state correlations. The terms are expectation values of operator products in the bare ground state. In order to correct for the presence of $3p3h$ and $4p4h$ correlations, the operator products are generated via a $1/\epsilon$ expansion, as outlined in reference [3]. As such, this document can be considered as an appendix to that paper.

Expressions here are given with and without angular momentum coupling (AMC). The calculations were done in angular momentum coupling and required considerable recoupling from $(ph)$ coupling to $(pp)$ coupling and vice versa. $(ph)$ coupling is indicated by means of small or greek letters ($\ell, \lambda$); whereas $(pp)$ coupling is indicated by capital letters ($L, K$). The phase-conventions of our matrix elements and amplitudes leading to their symmetries as well as the details of the computation can be found in reference [3].

The terms can be classified as either contributing to the effective two-body matrix element or contributing to the effective single particle energy. The contributions for the effective matrix element are $B^{eff}(p_1 h_1, p_2 h_2) = \langle p_1 h_1 | V^{eff, \lambda} | h_2 \bar{p}_2 \rangle$. If this matrix element is given in ($pp$) coupling, an extra recoupling is implied according to

$$\langle p_1 h_1 | V^{eff, \lambda} | h_2 \bar{p}_2 \rangle = \sum_K (-)^{K+1}(2K+1) \left\{ \begin{array}{ccc} p_1 & h_1 & \lambda \\ h_2 & p_2 & K \end{array} \right\} \langle p_1 p_2 | V^{K, eff} | h_1 h_2 \rangle$$

(1)

We are using Einstein summation convention, so that any orbit appearing twice implies summation over that orbit.

In most cases we deal with the energy ($\omega$) dependence of the two-body matrix element the following way:

$$\sum_n \frac{b_n}{\epsilon_n + \omega} \approx \left( \sum_n \frac{b_n}{\epsilon_n} \right) \left( \frac{1}{1 + \alpha \omega} \right)$$

(2)

with

$$\alpha = \left( \sum_n \frac{b_n}{\epsilon_n} \right) / \left( \sum_n \frac{b_n}{\epsilon_n} \right)$$

(3)

This expression gives the correct value at $\omega = 0$ and $\omega = \infty$, and it also gives the correct derivative with respect to $\omega$ at ($\omega = 0$). When $\omega$ dependent single-particle energies are required, we compute these energies for a set of 9 values of $\omega$ in the range from 0 to 1 GeV, and interpolate for values inbetween.

We also define the density matrix

$$d(p_1, p_3, \omega) = -\frac{1}{2} \frac{Z_{p_3 p_2, h_4 h_2} V_{p_1 p_2, h_4 h_2}}{\epsilon_{p_2 h_2} + \epsilon_{p_1 h_4} + \omega}$$

(4)

$$d(h_1, h_3, \omega) = -\frac{1}{2} \frac{Z_{p_4 p_2, h_3 h_2} V_{p_3 p_2, h_3 h_2}}{\epsilon_{p_2 h_2} + \epsilon_{p_4 h_3} + \omega}$$

(5)

or, in angular momentum coupling

$$d(p_1, p_3, \omega) = -\frac{1}{2} \sum_\ell \frac{2\ell + 1}{2p_1 + 1} Z_{p_1 h_1, p_2 h_2}^\ell \frac{\langle p_2 h_2 | V^\ell | h_1 \bar{p}_3 \rangle}{\epsilon_{p_2 h_2} + \epsilon_{p_1 h_1} + \omega}$$

(6)

$$d(h_1, h_3, \omega) = -\frac{1}{2} \sum_\ell \frac{2\ell + 1}{2h_1 + 1} Z_{p_1 h_1, p_2 h_2}^\ell \frac{\langle p_2 h_2 | V^\ell | h_1 \bar{p}_3 \rangle}{\epsilon_{p_2 h_2} + \epsilon_{p_1 h_1} + \omega}$$

(7)

2 Mean Field $ph$ Matrix elements.

We evaluate the terms

$$\langle 0 | U | ph \rangle = \langle 0 | V_{01} | ph \rangle + \langle 0 | S_{21}, V_{10} | ph \rangle + \langle 0 | S_{31}, V_{20} | ph \rangle$$

(8)

The $S_3$ term requires a $1/\epsilon$ expansion

$$-\langle 0 | \left[ S_{31}, V_{01} \right] | ph \rangle / \epsilon - \frac{1}{2} \langle 0 \left| S_{21}, \left[ S_{21}, V_{10} \right] \right| V_{20} | ph \rangle / \epsilon$$

(9)

$$-\frac{1}{\epsilon} \langle 0 \left| S_{31}, V_{00} \right| V_{20} | ph \rangle - \frac{1}{\epsilon} \langle 0 \left| S_{41}, V_{10} \right| V_{20} | ph \rangle - \frac{1}{\epsilon} \langle 0 \left| S_{51}, V_{20} \right| V_{20} | ph \rangle - \frac{1}{\epsilon} \langle 0 \left| S_{31} \right| \left[ S_{21}, V_{20} \right] \right| V_{20} | ph \rangle$$

(10)
Up to next order, we can write the terms in the last line (keeping only terms with \(S_2\)) as

\[
\begin{align*}
&+ \frac{1}{\epsilon_3} \sum_{\ell} \langle 0 \left| \left[ S_2, V_{01} \right], V_{20} \right| \phi \rangle \\
&+ \frac{1}{\epsilon_3} \sum_{\ell} \langle 0 \left| \left[ S_2, V_{10} \right], V_{20} \right| \phi \rangle \\
&+ \frac{1}{\epsilon_3} \sum_{\ell} \langle 0 \left| \left[ S_2, [S_2, V_{10}] \right], V_{20} \right| \phi \rangle \\
&+ \frac{1}{\epsilon_3} \sum_{\ell} \langle 0 \left| \left[ S_2, [S_2, [S_2, V_{20}]] \right], V_{20} \right| \phi \rangle \\
&+ \frac{1}{\epsilon_3} \sum_{\ell} \langle 0 \left| \left[ S_2, [S_2, [S_2, [S_2, V_{20}]]] \right], V_{20} \right| \phi \rangle
\end{align*}
\]

(11)

(12)

(13)

(14)

Term 1: \(\langle 0 \left| V_{01} \right| \phi \rangle\).

\[(-)^{j_0 - j_{hs}} \sqrt{\frac{2j_{hs} + 1}{2j_p + 1}} \langle p\bar{\phi}|V_{\ell=0}|h_0,h_h\rangle \]

(15)

Term (1) is modified by terms (3b) and (3e). These terms are calculated by finding a representation of natural orbits \(\{p^n, h^n\} = \{\alpha_n\}\), in which the density matrix is diagonal with a diagonal value \(v_n\), the occupation of the natural orbits. Thus these terms can be combined as

\[
\sum_{\alpha_n} (-)^{j_0 + j_{hs}} \sqrt{\frac{2j_{hs} + 1}{2j_p + 1}} \langle p\bar{\phi}|V_{\ell=0}|\alpha_n, \alpha_n\rangle v_n
\]

(16)

Term 2: \(\langle 0 \left| [S_2, V_{10}] \right| \phi \rangle\).

\[
\begin{align*}
(a) & \quad + Z_{p_1 h_1, p_2 h_2} \langle p_1 h_1|V|pp_2\rangle & \rightarrow & \frac{1}{2} \sum_{\ell} \frac{2\ell + 1}{2j_p + 1} Z_{p_1 h_1, p_2 h_2}^\ell \langle p_1 h_1|V^\ell|h_3 p_2\rangle \\
(b) & \quad - Z_{p_1 h_1, p_2 h_2} \langle p_1 h_1|V|h_3 h_3\rangle & \rightarrow & -\frac{1}{2} \sum_{\ell} \frac{2\ell + 1}{2j_p + 1} Z_{p_1 h_1, p_2 h_2}^\ell \langle h_3 h_3|V^\ell|h_2 p_2\rangle \\
(c) & \quad + Z_{p_0 p_3 h_2}^\alpha \langle p_3 h_1|V|h_3 h_3\rangle & \rightarrow & Z_{p_0 p_3 h_2}^\alpha \langle p_2|H|h_3\rangle
\end{align*}
\]

(17)

(18)

(19)

Here \(H\) represents the one-body part of the hamiltonian. As for each term there appears also a term multiplying it by \(Z^0\), the sum of these terms is small in the hartree-fock basis as the one-body part already is small.

Term 3: \(\langle 0 \left| [S_2, V_{01}], \frac{1}{H_0} V_{20} \right| \phi \rangle\).

1. Term 3a.

\[
- Z_{p_1 h_1, p_2 h_2} V_{p_3 p_3 h_3} p_1 V_{p_2 p_2 h_2} / \epsilon
\rightarrow
- \frac{2\ell + 1}{2j_p + 1} Z_{p_1 h_1, p_2 h_2} \langle p_3 h_3|V^\ell|p_1 \bar{\phi}\rangle \frac{\langle p_2 \bar{h}_2|V^\ell|h_3 \bar{p}_3\rangle}{\epsilon_{ph} + \epsilon_{p_2 h_2} + \epsilon_{p_3 h_3}}
\]

(20)

2. Term 3b.

\[
- \frac{1}{2} Z_{p_1 h_1, p_2 h_2} V_{p_3 p_3 h_3} p_1 V_{p_2 p_2 h_2} / \epsilon
\rightarrow
(-)^{j_0 - j_{ph}} \sqrt{\frac{2j_{ph} + 1}{2j_p + 1}} \langle p\bar{\phi}|V_{\ell=0}|p_1 \bar{p}_3\rangle d(p_1, p_3, \omega = \epsilon_{ph})
\]

(21)

3. Term 3c.

\[
\frac{1}{4} \sum_{K} \frac{2K + 1}{2j_p + 1} Z_{p_2 p_2 h_2} \langle p_1 p_3|V^K|h_2 p_2\rangle \frac{\langle p_1 p_3|V^K|h_1 h_2\rangle}{\epsilon_{ph} + \epsilon_{p_1 h_1} + \epsilon_{p_2 h_2}}
\]

(22)
4. Term 3d.

\[
Z_{ph_1,p_2h_2}V_{p_3h_1h_2}V_{p_2p_3h_3}/\epsilon \\
\rightarrow \ \frac{2\ell + 1}{2j_\rho + 1} Z_{p_1h_1,p_2h_2}^\ell Z_{p_2h_2,p_3h_3}^\ell \langle p_3h_3|V^\ell|h_1h_2\rangle
\]

6. Term 3f.

\[
-\frac{1}{4} Z_{p_1h_1,p_2h_2}V_{p_3h_2h_1}V_{p_1p_2h_3}/\epsilon \\
\rightarrow -\frac{1}{4} \sum K \frac{2K + 1}{2j_\rho + 1} Z_{ph_1,p_2h_2}^0 \langle ph_1|V^K|h_3h_1\rangle \frac{\langle p_1p_2|V^K|h_3h_1\rangle}{\epsilon_{ph} + \epsilon_{p_1h_3} + \epsilon_{p_2h_3}}
\]

7. Term 3g.

\[
-\frac{1}{2} Z_{p_3h_1,p_1h_1}V_{p_3p_2h_3h_1}V_{p_2p_3h_3}/\epsilon \\
\rightarrow -(-)^{j_\rho + j_\omega} \sqrt{2j_\rho + 1} Z_{ph_1,p_1h_1}^0 \sum \frac{2\ell + 1}{2j_\rho + 1} \langle p_3h_3|V^\ell|h_1h_2\rangle \frac{B_{p_3h_3p_2h_3}^\ell}{\epsilon_{ph} + \epsilon_{p_1h_3} + \epsilon_{p_2h_3}}
\]

8. Term 3h.

\[
\frac{1}{2} Z_{p_1h_1,p_2h_2}V_{h_2h_3h_1}V_{p_1p_2h_3}/\epsilon \\
\rightarrow -(-)^{j_\rho + j_\omega} \sqrt{2j_\rho + 1} Z_{ph_1,p_1h_1}^0 \sum \frac{2\ell + 1}{2j_\rho + 1} \langle p_3h_3|V^\ell|h_2h_1\rangle \frac{\langle p_1p_2|V^K|h_3h_4\rangle}{\epsilon_{ph} + \epsilon_{p_1h_3} + \epsilon_{p_2h_4}}
\]

**Term 4:** \( \langle 0 \left[ [S_2, [S_2, V_{10}]] \cdot \frac{1}{\mathbf{H}_0} \right] V_{20} \rangle \langle ph \rangle \).

1. Term 4a.

\[
-\frac{1}{8} Z_{p_1h_1,p_2h_2}Z_{p_3h_3,p_4h_4}V_{p_2p_3p_4}V_{p_1p_2h_3h_4}/\epsilon \\
\rightarrow -\frac{1}{8} \sum K \frac{2K + 1}{2j_\rho + 1} Z_{p_1h_1,p_2h_2}^K Z_{p_3h_3,p_4h_4}^K \langle p_3p_4|V^K|h_3h_4\rangle \frac{\langle p_4p_2|V^K|h_3h_4\rangle}{\epsilon_{ph} + \epsilon_{p_1h_3} + \epsilon_{p_2h_4}}
\]

2. Term 4b.

\[
\frac{1}{4} Z_{p_1h_1,p_2h_2}Z_{p_3h_3,p_4h_4}V_{p_2p_3p_4}V_{p_1p_2h_3h_4}/\epsilon \\
\rightarrow -\frac{1}{2} \sum \frac{2\ell + 1}{2j_\rho + 1} Z_{p_1h_1,p_2h_2}^\ell \langle h_3p_2|V^\ell|h_1h_2, \omega = \epsilon_{ph} \rangle
\]

3. Term 4c.

\[
\frac{1}{4} Z_{p_1h_1,p_2h_2}Z_{p_3h_3,p_4h_4}V_{p_2p_3p_4}V_{p_1p_4h_3h_4}/\epsilon \\
\rightarrow -\frac{1}{2} \sum \frac{2\ell + 1}{2j_\rho + 1} Z_{p_1h_1,p_2h_2}^\ell \langle h_2p_2|V^\ell|h_1h_2, \omega = \epsilon_{ph} \rangle
\]
4. Term 4d.
\[-\frac{1}{2} Z_{p_1h_1p_2h_2} Z_{p_3h_4p_4h_4} V_{ph_1p_2p_2} V_{p_1p_2h_1h_4} / \epsilon \]
\[\rightarrow \quad -\frac{1}{2} \sum_\ell \ \frac{2\ell + 1}{2j_\ell + 1} Z'_{p_1h_1p_2h_2} Z'_{p_3h_4p_4h_4} \langle p_3\bar{p}_1|V|^\ell|\bar{h}_2\bar{p}_2 \rangle \frac{\langle p_3\bar{h}_1|V|^\ell|\bar{h}_2\bar{p}_2 \rangle}{\epsilon_{ph} + \epsilon_{p_1h_1} + \epsilon_{p_2h_4}} \] (31)

5. Term 4e.
\[-\frac{1}{4} Z_{p_1h_1p_2h_2} Z_{ph_2p_2h_3} Z_{h_3h_4p_3} V_{p_1p_2h_4} / \epsilon \]
\[\rightarrow \quad + \frac{1}{4} \sum_\ell \ \frac{2\ell + 1}{2j_\ell + 1} Z'_{p_1h_1p_2h_2} \langle h_2\bar{p}_2|V|^\ell|\bar{h}_3 \rangle \ d(h_3, h_1, \omega = \epsilon_{ph}) \] (32)

6. Term 4f.
\[-\frac{1}{4} Z_{ph_1p_2h_2} Z_{p_3h_3p_4h_4} V_{p_3p_4h_3} V_{p_1p_4h_4} / \epsilon \]
\[\rightarrow \quad + \frac{1}{4} \sum_\ell \ \frac{2\ell + 1}{2j_\ell + 1} Z'_{ph_1p_2h_2} \langle p_2\bar{h}_2|V|^\ell|\bar{h}_3 \rangle \ d(p_3, p_2, \omega = \epsilon_{ph}) \] (33)

7. Term 4j.
\[Z_{ph_1p_2h_2} Z_{p_3h_3p_4h_4} V_{p_3p_4h_3} V_{p_1p_4h_4} / \epsilon \]
\[\rightarrow \quad \sum_\ell \ \frac{2\ell + 1}{2j_\ell + 1} Z'_{ph_1p_2h_2} Z'_{p_3h_3p_4h_4} \langle p_2\bar{h}_2|V|^\ell|p_5\bar{p}_3 \rangle \frac{\langle p_2\bar{h}_2|V|^\ell|p_5\bar{p}_3 \rangle}{\epsilon_{ph} + \epsilon_{p_1h_1} + \epsilon_{p_4h_4}} \] (34)

8. Term 4k.
\[\frac{1}{8} Z_{ph_1p_2h_2} Z_{p_3h_3p_4h_4} V_{p_3p_4h_3} V_{p_1p_4h_4} / \epsilon \]
\[\rightarrow \quad \frac{1}{8} \sum_\ell \ \frac{2\ell + 1}{2j_\ell + 1} Z'_{ph_1p_2h_2} Z'_{p_3h_3p_4h_4} \langle h_1h_2|V|^\ell|p_3p_4 \rangle \frac{\langle h_1h_2|V|^\ell|p_3p_4 \rangle}{\epsilon_{ph} + \epsilon_{p_1h_1} + \epsilon_{p_4h_2}} \] (35)

9. Term 4l.
\[+ \frac{1}{8} Z_{ph_1p_2h_2} Z_{p_3h_3p_4h_4} V_{h_3h_2p_2} V_{p_1p_2h_4} / \epsilon \]
\[\rightarrow \quad + \frac{1}{8} \sum_\ell \ \frac{2\ell + 1}{2j_\ell + 1} Z'_{ph_1p_2h_2} Z'_{p_3h_3p_4h_4} \langle h_3h_1|V|^\ell|p_2\bar{h}_2 \rangle \frac{\langle h_3h_1|V|^\ell|p_2\bar{h}_2 \rangle}{\epsilon_{ph} + \epsilon_{p_1h_1} + \epsilon_{p_4h_4}} \] (36)

10. Term 4o.
\[-Z_{p_1h_1p_2h_2} Z_{ph_3p_4h_4} V_{h_3p_1h_3} V_{p_1p_2h_4} / \epsilon \]
\[\rightarrow \quad - \sum_\ell \ \frac{2\ell + 1}{2j_\ell + 1} Z'_{p_1h_1p_2h_2} Z'_{ph_3p_4h_4} \langle p_1\bar{h}_1|V|^\ell|h_3\bar{h}_5 \rangle \frac{\langle p_1\bar{h}_1|V|^\ell|h_3\bar{h}_5 \rangle}{\epsilon_{ph} + \epsilon_{p_1h_5} + \epsilon_{p_4h_4}} \] (37)

Term 5: \[\langle 0 \left[ \left[ \mathcal{H}_2, \mathcal{V}_{01} \right], \frac{1}{\mathcal{H}_0} \mathcal{V}_{00} \right], \frac{1}{\mathcal{H}_0} \mathcal{V}_{20} \rangle \ |ph \rangle \].

1. Term 5a.
\[\sum_\ell \ \frac{2\ell + 1}{2j_\ell + 1} Z'_{p_1h_1p_2h_2} \langle p_1\bar{h}_1|V|^\ell|h_3p_4 \rangle \langle h_3p_4|V|^\ell|h_4\bar{p}_4 \rangle \langle h_4\bar{p}_4|V|^\ell|p_1\bar{p} \rangle \]
\[\times (\epsilon_{p_2h_2} + \epsilon_{p_3h_3} + \epsilon_{ph})(\epsilon_{p_4h_4} + \epsilon_{p_2h_2} + \epsilon_{ph}) \] (38)

2. Term 5b.
\[-\sum_\ell \ \frac{2\ell + 1}{2j_\ell + 1} Z'_{ph_1h_1p_2h_2} \langle h_3p_3|V|^\ell|h_4\bar{p}_4 \rangle \langle h_3p_3|V|^\ell|h_4\bar{p}_4 \rangle \langle h_4\bar{p}_4|V|^\ell|h_3\bar{h}_5 \rangle \]
\[\times (\epsilon_{p_2h_2} + \epsilon_{p_3h_3} + \epsilon_{ph})(\epsilon_{p_4h_4} + \epsilon_{p_2h_2} + \epsilon_{ph}) \] (39)
### 3 Z Coefficients.

All contributions are identified by a Roman numeral indicating the operator from the 1/\( \epsilon \) expansion that is generating this particular term according to the following list:

\[
\begin{aligned}
\langle 0 | V_{02} | 2p2h \rangle &+ \langle 0 | [S_2, V_{00}] | 2p2h \rangle + \frac{1}{2} \langle 0 | [S_2, [S_2, V_{20}]] | 2p2h \rangle \\
&\text{ (III,III)} \\
\end{aligned}
\]

\[
\begin{aligned}
\frac{1}{2} \frac{1}{\epsilon_{3p3h}} &\langle 0 | [S_2, [S_2, V_{01}]] | 2p2h \rangle - \frac{1}{2} \frac{1}{\epsilon_{3p3h}} &\langle 0 | [S_2, [S_2, V_{10}]] | 2p2h \rangle \\
&\text{ (IV,V)} \\
\end{aligned}
\]

\[
\begin{aligned}
\frac{1}{2} \frac{1}{\epsilon_{4p4h}} &\langle 0 | [S_2, [S_2, V_{00}]] | 2p2h \rangle - \frac{1}{2} \frac{1}{\epsilon_{4p4h}} &\langle 0 | [S_2, [S_2, V_{01}]] | 2p2h \rangle \\
&\text{ (VI,VII)} \\
\end{aligned}
\]

\[
\begin{aligned}
\frac{1}{2} \frac{1}{\epsilon_{4p4h}} &\langle 0 | [S_2, V_{01}] | 2p2h \rangle + \frac{1}{2} \frac{1}{\epsilon_{4p4h}} &\langle 0 | [S_2, V_{00}] | 2p2h \rangle \\
&\text{ (VIII)} \\
\end{aligned}
\]

\[
\begin{aligned}
\frac{1}{2} \frac{1}{\epsilon_{4p4h}} &\langle 0 | [S_2, [S_2, V_{20}]] | 2p2h \rangle - \frac{1}{2} \frac{1}{\epsilon_{4p4h}} &\langle 0 | [S_2, [S_2, V_{10}]] | 2p2h \rangle \\
&\text{ (IX)} \\
\end{aligned}
\]

\[
\begin{aligned}
\frac{1}{2} \frac{1}{\epsilon_{3p3h}} &\langle 0 | [S_2, V_{01}] | 2p2h \rangle - \frac{1}{2} \frac{1}{\epsilon_{3p3h}} &\langle 0 | [S_2, V_{10}] | 2p2h \rangle \\
&\text{ (X)} \\
\end{aligned}
\]

\[
\begin{aligned}
\frac{1}{2} \frac{1}{\epsilon_{3p3h}} &\langle 0 | [S_2, V_{00}] | 2p2h \rangle - \frac{1}{2} \frac{1}{\epsilon_{3p3h}} &\langle 0 | [S_2, V_{10}] | 2p2h \rangle \\
&\text{ (XI)} \\
\end{aligned}
\]

\[
\begin{aligned}
\frac{1}{2} \frac{1}{\epsilon_{3p3h}} &\langle 0 | [S_2, [S_2, V_{01}]] | 2p2h \rangle - \frac{1}{2} \frac{1}{\epsilon_{3p3h}} &\langle 0 | [S_2, [S_2, V_{00}]] | 2p2h \rangle \\
&\text{ (XII)} \\
\end{aligned}
\]

\[
\begin{aligned}
\frac{1}{2} \frac{1}{\epsilon_{4p4h}} &\langle 0 | [S_2, [S_2, V_{01}]] | 2p2h \rangle - \frac{1}{2} \frac{1}{\epsilon_{4p4h}} &\langle 0 | [S_2, [S_2, V_{00}]] | 2p2h \rangle \\
&\text{ (XIII)} \\
\end{aligned}
\]

\[
\begin{aligned}
\frac{1}{2} \frac{1}{\epsilon_{3p3h}} &\langle 0 | [S_2, [S_2, V_{00}]] | 2p2h \rangle - \frac{1}{2} \frac{1}{\epsilon_{3p3h}} &\langle 0 | [S_2, [S_2, V_{10}]] | 2p2h \rangle \\
&\text{ (XIV)} \\
\end{aligned}
\]

\[
\begin{aligned}
\frac{1}{2} \frac{1}{\epsilon_{3p3h}} &\langle 0 | [S_2, [S_2, V_{10}]] | 2p2h \rangle - \frac{1}{2} \frac{1}{\epsilon_{3p3h}} &\langle 0 | [S_2, [S_2, V_{01}]] | 2p2h \rangle \\
&\text{ (XV)} \\
\end{aligned}
\]

\[
\begin{aligned}
\frac{1}{2} \frac{1}{\epsilon_{4p4h}} &\langle 0 | [S_2, [S_2, V_{00}]] | 2p2h \rangle - \frac{1}{2} \frac{1}{\epsilon_{4p4h}} &\langle 0 | [S_2, [S_2, V_{10}]] | 2p2h \rangle \\
&\text{ (XVI)} \\
\end{aligned}
\]

\[
\begin{aligned}
\frac{1}{2} \frac{1}{\epsilon_{4p4h}} &\langle 0 | [S_2, [S_2, V_{00}]] | 2p2h \rangle - \frac{1}{2} \frac{1}{\epsilon_{4p4h}} &\langle 0 | [S_2, [S_2, V_{10}]] | 2p2h \rangle \\
&\text{ (XVII)} \\
\end{aligned}
\]

\[
\begin{aligned}
\frac{1}{2} \frac{1}{\epsilon_{3p3h}} &\langle 0 | [S_2, [S_2, V_{10}]] | 2p2h \rangle - \frac{1}{2} \frac{1}{\epsilon_{3p3h}} &\langle 0 | [S_2, [S_2, V_{01}]] | 2p2h \rangle \\
&\text{ (XVIII)} \\
\end{aligned}
\]

\[
\begin{aligned}
\frac{1}{2} \frac{1}{\epsilon_{3p3h}} &\langle 0 | [S_2, [S_2, V_{00}]] | 2p2h \rangle - \frac{1}{2} \frac{1}{\epsilon_{3p3h}} &\langle 0 | [S_2, [S_2, V_{10}]] | 2p2h \rangle \\
&\text{ (XIX)} \\
\end{aligned}
\]

While we have attempted to be essentially complete in including the terms \((I) \rightarrow (IV)\), from the others we have selectively included only those terms that we expected to be significant. If a term is listed but not yet included in our present treatment, it will be specifically stated.

### 3.1 Effective \(ph\)-\(hp\) Coupled Matrix Element.

We calculate the \(2p2h\) amplitude as

\[
\begin{aligned}
Z_{p1p2,h1h2} &= -V_{p1p2,h1h2}^{tot} / \left[ \epsilon_{p1h1} + \epsilon_{p2h2} + \Delta \epsilon_{p1h1}(\omega = \epsilon_{p2h2}) + \Delta \epsilon_{p2h2}(\omega = \epsilon_{p1h1}) \right] \\
&\text{ (40)} \\
\end{aligned}
\]

or, after angular momentum coupling,

\[
\begin{aligned}
Z_{p1h1,p2h2}^\lambda &= -\frac{\langle (p_1 \tilde{h}_1) | V^{(tot)} | (p_2 \tilde{h}_2) \rangle}{\epsilon_{p1h1} + \epsilon_{p2h2} + \Delta \epsilon_{p1h1}(\omega = \epsilon_{p2h2}) + \Delta \epsilon_{p2h2}(\omega = \epsilon_{p1h1})} \\
&\text{ (41)} \\
\end{aligned}
\]

This is the only place where we actually use the \(\omega\) dependent contribution to the \(ph\) energies. The total \(ph - hp\) matrix element \(\langle (p_1 \tilde{h}_1) | V^{(tot)} | (h_2 \tilde{h}_2) \rangle\) has eight contributions which we label \(V_1-V_8\).
1. Contribution $V_1$. The first contribution is the direct matrix element from term (I)

$$
\langle (p_1 h_1) \lambda | V^{(1),\lambda} | (h_2 p_2) \lambda \rangle = \langle (p_1 h_1) \lambda | V^{\lambda} | (h_2 p_2) \lambda \rangle
$$

2. Contribution $V_2$. The second term is the G-matrix correction generated by term (II)

$$
V^{(2)}_{p_1 p_2, h_1 h_2} = \frac{1}{2} Z_{p_1 p_2, h_1 h_2} V_{p_1 p_2, h_1 h_2} + \frac{1}{2} Z_{p_1 p_2, h_3 h_4} V_{h_3 h_4, h_1 h_2}
$$

or,

$$
\langle (p_1 h_1) \lambda | V^{(2),\lambda} | (h_2 p_2) \lambda \rangle = \sum_{\lambda} (-)^{K+1} (2K + 1) \left\{ \begin{array}{c} p_1 h_2 \\ h_2 p_2 \end{array} \right\} \langle (p_1 h_1) \lambda | V^{(2),K} | (h_2 h_2) \lambda \rangle
$$

where

$$
\langle (p_1 h_1) \lambda | V^{(2),K} | (h_2 h_2) \lambda \rangle = \frac{1}{2} \left( \langle (p_1 h_1) \lambda | V^K | (p_2 h_2) \lambda \rangle + \langle (h_1 h_2) \lambda | V^K | (h_3 h_4) \lambda \rangle Z^K_{p_1 p_2, h_3 h_4} \right)
$$

and

$$
Z^K_{p_1 p_2, h_1 h_2} = (-)^{K+1} \sum_{\ell} (2\ell + 1) \left\{ \begin{array}{c} p_1 h_2 \\ h_2 p_2 \end{array} \right\} Z^K_{p_1 h_1, p_2 h_2}
$$

Note: It is understood that any contribution given in $(pp)$ coupling in the same way as this term has to be recoupled before it is added to the rest.

3. Contribution $V_3$. The third contribution is the collectivity correction also generated from term (II)

$$
V^{(3)}_{p_1 p_2, h_1 h_2} = Z_{p_1 h_1, p_2 h_2} V_{p_1 h_1, p_2 h_2} + Z_{p_1 h_1, p_2 h_2} V_{p_1 h_1, p_2 h_2} - Z_{p_1 h_1, p_2 h_2} V_{p_1 h_1, p_2 h_2}
$$

After angular momentum coupling this becomes

$$
\langle (p_1 h_1) \lambda | V^{(3),\lambda} | (h_2 p_2) \lambda \rangle = Z^\lambda_{p_1 h_1, p_2 h_2} \langle (p_2 h_2) \lambda | V^\lambda | (p_1 h_1) \lambda \rangle + Z^\lambda_{p_1 h_1, p_2 h_2} \langle (p_2 h_2) \lambda | V^\lambda | (p_1 h_1) \lambda \rangle + \sum_{\ell} (-)^{\ell+\lambda} (2\ell + 1) \left\{ \begin{array}{c} p_1 h_2 \\ h_2 p_2 \end{array} \right\} Z^\lambda_{p_1 h_1, p_2 h_2} \langle (p_2 h_2) \lambda | V^\lambda | (p_1 h_1) \lambda \rangle
$$

with

$$
\langle (p_1 h_1) \lambda | V^{\lambda} | (p_2 h_2) \lambda \rangle = \langle (p_1 h_1) \lambda | V^{\lambda} | (p_2 h_2) \lambda \rangle + \frac{1}{2} Z^\lambda_{p_1 h_1, p_2 h_2} \langle (h_4 h_4) \lambda | V^\lambda | (p_2 h_2) \lambda \rangle
$$

This last correction to the $(ph - ph)$ matrix element arises from term (III).

4. Contribution $V_4$. Additional contributions from term (III) are

$$
V^{(4)}_{p_1 p_2, h_1 h_2} = \frac{1}{2} Z_{p_1 h_1, p_2 h_2} Z_{p_1 h_1, p_2 h_2} \langle (p_3 h_3) | V^\lambda | (h_4 \bar{p}_4) \lambda \rangle
$$

In angular momentum coupling this term is included as

$$
\langle (p_1 h_1) \lambda | V^{(4),K} | (h_2 h_2) \lambda \rangle = \frac{1}{2} Z^K_{p_1 p_2, h_3 h_4} Z^K_{p_1 p_2, h_3 h_4} \langle (p_3 h_3) | V^K | (h_4 h_4) \lambda \rangle
$$

Note. At this point all terms that do not contain an energy denominator have been listed and all two-body terms are included in our treatment.

5. Contribution $V_5$. The first energy dependent contributions are generated by term (IV). It is the first term that accounts for the presence of $3p3h$-correlations. These terms are discussed in section (3.3) below together with higher order terms that can be included using effective matrix elements. We call the sum of all the contributions from section (3.3): $V^{(5)}_{p_1 p_2, h_1 h_2}$.
6. Contribution $V_6$. These contributions are generated by term (IX) as

\[
\langle (p_1 h_1) \lambda | V^{(6),\lambda} | (h_2 p_2) \lambda \rangle = -Z^\lambda_{p_1 h_1, p_2 h_2} d(p_6, p_4, \omega = \epsilon_{p_1 h_1} + \epsilon_{p_2 h_2}) B^{\lambda}_{p_4 h_4, p_5 h_5, h_6, p_2 h_2} \tag{52}
\]

\[
\langle (p_1 h_1) \lambda | V^{(6),\lambda} | (h_2 p_2) \lambda \rangle = -Z^\lambda_{p_2 h_2, p_6 h_6} d(p_6, p_4, \omega = \epsilon_{p_1 h_1} + \epsilon_{p_2 h_2}) B^{\lambda}_{p_4 h_4, p_5 h_5, h_6, p_1 h_1} \tag{53}
\]

\[
\langle (p_1 h_1) \lambda | V^{(6),\lambda} | (h_2 p_2) \lambda \rangle = -Z^\lambda_{p_4 h_4, p_5 h_5, h_6} d(h_6, p_4, \omega = \epsilon_{p_1 h_1} + \epsilon_{p_2 h_2}) B^{\lambda}_{p_4 h_4, p_5 h_5, h_6, p_1 h_1} \tag{54}
\]

\[
\langle (p_1 h_1) \lambda | V^{(6),\lambda} | (h_2 p_2) \lambda \rangle = -Z^\lambda_{p_2 h_2, p_6 h_6} d(h_6, p_4, \omega = \epsilon_{p_1 h_1} + \epsilon_{p_2 h_2}) B^{\lambda}_{p_4 h_4, p_5 h_5, h_6, p_1 h_1} \tag{55}
\]

\[
\langle (p_1 h_1) \lambda | V^{(6),\lambda} | (h_2 p_2) \lambda \rangle = -\sum_\ell (-)^{\ell+\lambda}(2\ell+1) \left\{ \begin{array}{ccc} p_1 & h_1 & \lambda \\ p_2 & h_2 & \ell \end{array} \right\} Z^\ell_{p_1 h_1, p_2 h_2} d(p_6, p_4, \omega = \epsilon_{p_1 h_1} + \epsilon_{p_2 h_2}) B^{\ell}_{p_4 h_4, p_5 h_5, h_6, p_1 h_1, p_2 h_2} \tag{56}
\]

\[
\langle (p_1 h_1) \lambda | V^{(6),\lambda} | (h_2 p_2) \lambda \rangle = -\sum_\ell (-)^{\ell+\lambda}(2\ell+1) \left\{ \begin{array}{ccc} p_1 & h_1 & \lambda \\ p_2 & h_2 & \ell \end{array} \right\} Z^\ell_{p_2 h_2, p_6 h_6} d(p_6, p_4, \omega = \epsilon_{p_1 h_1} + \epsilon_{p_2 h_2}) B^{\ell}_{p_4 h_4, p_5 h_5, h_6, p_1 h_1, p_2 h_2} \tag{57}
\]

\[
\langle (p_1 h_1) \lambda | V^{(6),\lambda} | (h_2 p_2) \lambda \rangle = -\sum_\ell (-)^{\ell+\lambda}(2\ell+1) \left\{ \begin{array}{ccc} p_1 & h_1 & \lambda \\ p_2 & h_2 & \ell \end{array} \right\} Z^\ell_{p_2 h_2, p_6 h_6} d(h_6, p_4, \omega = \epsilon_{p_1 h_1} + \epsilon_{p_2 h_2}) B^{\ell}_{p_4 h_4, p_5 h_5, h_6, p_1 h_1, p_2 h_2} \tag{58}
\]

\[
\langle (p_1 h_1) \lambda | V^{(6),\lambda} | (h_2 p_2) \lambda \rangle = -\sum_\ell (-)^{\ell+\lambda}(2\ell+1) \left\{ \begin{array}{ccc} p_1 & h_1 & \lambda \\ p_2 & h_2 & \ell \end{array} \right\} Z^\ell_{p_2 h_2, p_6 h_6} d(h_6, p_4, \omega = \epsilon_{p_1 h_1} + \epsilon_{p_2 h_2}) B^{\ell}_{p_4 h_4, p_5 h_5, h_6, p_1 h_1, p_2 h_2} \tag{59}
\]

\[
\langle (p_1 p_2) K | V^{(6),K} | (h_1 h_2) K \rangle = -\frac{1}{4} Z^K_{p_1 p_2, h_1 h_2} d(h_4, h_5, \omega = \epsilon_{p_1 h_1} + \epsilon_{p_2 h_2}) B^K_{p_3 p_4, h_4, h_5, h_6, p_3 h_3, h_4, h_2} \tag{60}
\]

\[
\langle (p_1 p_2) K | V^{(6),K} | (h_1 h_2) K \rangle = -\frac{1}{4} Z^K_{p_1 p_2, h_1 h_2} d(h_3, h_5, \omega = \epsilon_{p_1 h_1} + \epsilon_{p_2 h_2}) B^K_{p_3 p_4, h_4, h_5, h_6, p_3 h_3, h_4, h_2} \tag{61}
\]

\[
\langle (p_1 p_2) K | V^{(6),K} | (h_1 h_2) K \rangle = -\frac{1}{4} Z^K_{p_1 p_2, h_1 h_2} d(p_4, p_5, \omega = \epsilon_{p_1 h_1} + \epsilon_{p_2 h_2}) B^K_{p_3 p_4, h_4, h_5, h_6, p_3 h_3, h_4, h_2} \tag{62}
\]

\[
\langle (p_1 p_2) K | V^{(6),K} | (h_1 h_2) K \rangle = -\frac{1}{4} Z^K_{p_1 p_2, h_1 h_2} d(p_3, p_5, \omega = \epsilon_{p_1 h_1} + \epsilon_{p_2 h_2}) B^K_{p_3 p_4, h_4, h_5, h_6, p_3 h_3, h_4, h_2} \tag{63}
\]

7. Contribution $V_7$. The contributions $V^{(7)}_{p_1 p_2, h_1 h_2}$ are the $V^{(2)}$ quenching terms and are generated by term (VI) as

\[
\langle (p_1 p_2) K | V^{(7)}_{p_1 p_2, h_1 h_2} | (h_1 h_2) K \rangle = -\frac{1}{2} \sum_K d(p_5, p_3, \omega = \epsilon_{p_1 h_1} + \epsilon_{p_2 h_2}) Z^K_{p_5 p_4, h_1 h_2} \langle p_3 p_4 | V^K | p_1 p_2 \rangle \tag{64}
\]

\[
\langle (p_1 p_2) K | V^{(7)}_{p_1 p_2, h_1 h_2} | (h_1 h_2) K \rangle = -\frac{1}{2} \sum_K d(h_5, h_3, \omega = \epsilon_{p_1 h_1} + \epsilon_{p_2 h_2}) Z^K_{p_1 p_2, h_5 h_4} \langle h_3 h_4 | V^K | h_1 h_2 \rangle \tag{65}
\]

\[
\langle (p_1 h_1) \lambda | V^{(7),\lambda} | (h_2 p_2) \lambda \rangle = -d(h_3, h_4, \omega = \epsilon_{p_1 h_1} + \epsilon_{p_2 h_2}) Z^\lambda_{h_3 h_3, p_2 h_2} \langle p_2 h_2 | V^\lambda | p_1 h_1 \rangle \tag{66}
\]

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\[
\langle (p_1 \bar{h}_1)_{\lambda} | V^{\text{(7).}\lambda} | (h_2 \bar{p}_2)_{\lambda} \rangle = - d(p_3, p_4, \omega = \epsilon_{p_1 h_1} + \epsilon_{p_2 h_2}) Z_{p_1 h_4, p_2 h_2}^\lambda \langle p_4 \bar{h}_4 | V^\lambda | p_1 \bar{h}_1 \rangle
\] (67)

\[
\langle (p_1 \bar{h}_1)_{\lambda} | V^{\text{(7).}\lambda} | (h_2 \bar{p}_2)_{\lambda} \rangle = - d(h_3, h_4, \omega = \epsilon_{p_1 h_1} + \epsilon_{p_2 h_2}) Z_{p_4 h_3, p_1 h_1}^\lambda \langle p_4 \bar{h}_4 | V^\lambda | p_2 \bar{h}_2 \rangle
\] (68)

\[
\langle (p_1 \bar{h}_1)_{\lambda} | V^{\text{(7).}\lambda} | (h_2 \bar{p}_2)_{\lambda} \rangle = - d(p_3, p_4, \omega = \epsilon_{p_1 h_1} + \epsilon_{p_2 h_2}) Z_{p_3 h_4, p_1 h_1}^\lambda \langle p_3 \bar{h}_4 | V^\lambda | p_2 \bar{h}_2 \rangle
\] (69)

\[
\langle (p_1 \bar{h}_1)_{\lambda} | V^{\text{(7).}\lambda} | (h_2 \bar{p}_2)_{\lambda} \rangle = - \sum_\ell (-)^{\ell+\lambda} (2\ell + 1) \left\{ \frac{p_1}{h_2} \frac{h_1}{p_2} \frac{\lambda}{\ell} \right\}
\]
\[
d(h_3, h_4, \omega = \epsilon_{p_1 h_1} + \epsilon_{p_2 h_2}) Z_{p_4 h_3, p_1 h_1}^\ell \langle p_4 \bar{h}_4 | V^\ell | p_2 \bar{h}_2 \rangle
\] (70)

\[
\langle (p_1 \bar{h}_1)_{\lambda} | V^{\text{(7).}\lambda} | (h_2 \bar{p}_2)_{\lambda} \rangle = - \sum_\ell (-)^{\ell+\lambda} (2\ell + 1) \left\{ \frac{p_1}{h_2} \frac{h_1}{p_2} \frac{\lambda}{\ell} \right\}
\]
\[
d(p_3, p_4, \omega = \epsilon_{p_1 h_1} + \epsilon_{p_2 h_2}) Z_{p_3 h_4, p_1 h_1}^\ell \langle p_3 \bar{h}_4 | V^\ell | p_2 \bar{h}_2 \rangle
\] (71)

\[
\langle (p_1 \bar{h}_1)_{\lambda} | V^{\text{(7).}\lambda} | (h_2 \bar{p}_2)_{\lambda} \rangle = - \sum_\ell (-)^{\ell+\lambda} (2\ell + 1) \left\{ \frac{p_1}{h_2} \frac{h_1}{p_2} \frac{\lambda}{\ell} \right\}
\]
\[
d(h_3, h_4, \omega = \epsilon_{p_1 h_1} + \epsilon_{p_2 h_2}) Z_{p_4 h_3, p_2 h_2}^\ell \langle p_3 \bar{h}_4 | V^\ell | p_1 \bar{h}_1 \rangle
\] (72)

\[
\langle (p_1 \bar{h}_1)_{\lambda} | V^{\text{(7).}\lambda} | (h_2 \bar{p}_2)_{\lambda} \rangle = - \sum_\ell (-)^{\ell+\lambda} (2\ell + 1) \left\{ \frac{p_1}{h_2} \frac{h_1}{p_2} \frac{\lambda}{\ell} \right\}
\]
\[
d(p_3, p_4, \omega = \epsilon_{p_1 h_1} + \epsilon_{p_2 h_2}) Z_{p_3 h_4, p_2 h_2}^\ell \langle p_4 \bar{h}_4 | V^\ell | p_1 \bar{h}_1 \rangle
\] (73)

8. Contribution $V_8$. The last term arises from the fact that the non-symmetric mean field has been symmetrized and that the single particle Hamiltonian is strictly diagonal only for one value of $\omega$. We write

\[
\langle (p_1 \bar{h}_1)_{\lambda} | V^{(8),\lambda} | (h_2 \bar{p}_2)_{\lambda} \rangle = \sum_{p_3 \neq p_1} e(p_3, p_1, \omega = \epsilon_{p_2 h_2}) Z_{p_3 h_1, p_2 h_2}^\lambda + e(p_3, p_2, \omega = \epsilon_{p_1 h_1}) Z_{p_1 h_1, p_3 h_2}^\lambda
\]
\[
+ \sum_{h_1 \neq h_3} e(h_3, h_1, \omega = \epsilon_{p_2 h_2}) Z_{p_1 h_1, p_2 h_2}^\lambda + e(h_3, h_2, \omega = \epsilon_{p_1 h_1}) Z_{p_1 h_1, p_2 h_3}^\lambda
\] (74)

with

\[
e(p_3, p_1, \omega) = - \frac{1}{4} \frac{2\ell + 1}{2p_1 + 1} \sum_\ell \left( Z_{p_3 p_1, h_4 h_2}^\ell (p_3 \bar{h}_4 | V^\ell | h_2 \bar{p}_2) - Z_{p_3 p_2, h_4 h_2}^\ell (p_3 \bar{h}_4 | V^\ell | h_2 \bar{p}_2) \right)
\]
\[
- \frac{1}{2} \sum_\ell \frac{(2\ell + 1) \langle p_3 \bar{p}_1 | V^\ell | p_1 \bar{h}_2 \rangle \langle p_1 \bar{h}_2 | V^\ell | p_3 \bar{h}_2 \rangle}{\epsilon_{p_3 h_4} + \epsilon_{p_5 h_1} + \omega}
\] (75)

3.2 Mean Field $pp$ and $hh$ Matrix Elements.

The Schrödinger equation for the mean field wave functions are written in matrix form as

\[
\langle k_1 | \mathbf{H} | k_3 \rangle = \langle k_1 | \mathbf{T} | k_3 \rangle + \langle k_1 | \mathbf{U} | k_3 \rangle = \delta_{k_1, k_3} \epsilon_{k_1}
\] (76)

where $\epsilon_k$ is the single particle energy of the orbit $k$. We list here the matrix elements of the effective one-body potential, and it is understood that the diagonal elements represent the single particle energies.

The matrix of the single particle potential $\mathbf{U}$ is computed in a symmetrized form and in $m$-representation as:

\[
U_{p_1, p_3} = \sum_h V_{p_1, h_4, p_3, h_2} - \frac{1}{4} \sum_{p_2, h_4, h_2} \left( Z_{p_1 p_2, h_4 h_2} V_{p_3, h_4, h_2} + Z_{p_2 p_3, h_4 h_2} V_{p_1, h_4, h_2} \right)
\] (77)

\[
U_{h_1, h_3} = \sum_h V_{h_1, h_4, h_3} + \frac{1}{4} \sum_{p_4, p_2, h_2} \left( Z_{h_1 p_4, h_3 h_2} V_{p_2, h_4, h_2} + Z_{p_4 p_2, h_3 h_2} V_{h_1, h_4, h_2} \right)
\] (78)

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The second term in these expressions is written in AMC as

\[- \frac{1}{4} \frac{2\ell + 1}{2j_{p1} + 1} \sum (Z_{p_1p_2,h_4h_2}^\ell (p_3h_4|V^f|h_2\bar{p}_2) + Z_{p_3p_2,h_4h_2}^\ell (p_1h_4|V^f|h_2\bar{p}_2)) \]

and

\[ + \frac{1}{4} \frac{2\ell + 1}{2j_{h_1} + 1} \sum (Z_{p_1p_2,h_2h_1}^\ell (p_4\bar{h}_3|V^f|h_2\bar{p}_2) + Z_{p_4p_2,h_2h_1}^\ell (p_4\bar{h}_1|V^f|h_2\bar{p}_2)) \]

In addition to the contributions in this section we add the \( \omega \) dependent contributions due to corrections for \( Z_{3p4h} \) and \( Z_{4p4h} \). Those will be given as contributions to the \( ph \)-energies. The \( ph \)-energies are given as

\[ \epsilon_{p_1,h_1} = \epsilon_{p_1} - \epsilon_{h_1} \]

1. Term VI. The following contributions to the single particle potential:

(a)

\[ \Delta U_{p_1h_1}(p_3, p_1, \omega) = (-)^{j_{p_5} - j_{p_3}} \frac{(2j_{p_5} + 1)}{(2j_{p_3} + 1)} \langle p_1p_3|V^{f=0}|p_5p_7\rangle d(p_5, p_7, \omega) \]

(b)

\[ \Delta U_{p_1h_1}(p_3, p_1, \omega) = (-)^{j_{p_5} - j_{p_3}} \frac{(2j_{p_5} + 1)}{(2j_{p_3} + 1)} \langle p_1p_3|h_3|l_{5\bar{l}}\rangle d(h_5, h_7, \omega) \]

(c)

\[ \Delta U_{p_1h_1}(h_3, h_1, \omega) = (-)^{j_{h_5} - j_{h_3}} \frac{(2j_{h_5} + 1)}{(2j_{h_3} + 1)} \langle h_1h_3|V^{f=0}|h_5h_7\rangle d(h_5, h_7, \omega) \]

(d)

\[ \Delta U_{p_1h_1}(h_3, h_1, \omega) = (-)^{j_{h_5} - j_{h_3}} \frac{(2j_{h_5} + 1)}{(2j_{h_3} + 1)} \langle h_1h_3|V^{f=0}|h_5h_7\rangle d(h_5, h_7, \omega) \]

(e)

\[ \Delta U_{p_1h_1}(p_3, p_1, \omega) = \frac{1}{4} \sum K \frac{2K + 1}{2j_{p_1} + 1} Z_{p_1p_5,h_3h_6}^K \langle p_3p_5|V^K|p_7p_8\rangle \langle p_7p_8|V^K|h_5h_6\rangle \]

By dropping the \( \omega \) dependence this is approximated as

\[ \Delta U_{p_1h_1}(p_3, p_1) = \frac{1}{4} \sum K \frac{2K + 1}{2j_{p_1} + 1} Z_{p_1p_5,h_3h_6}^K \langle p_3p_5|V^K|p_7p_8\rangle Z_{p_7p_8,h_5h_6}^K \]

(f) Similarly we also have

\[ \Delta U_{p_1h_1}(h_3, h_1) = -\frac{1}{4} \sum K \frac{2K + 1}{2j_{h_1} + 1} Z_{h_3h_5,p_5p_6}^K \langle h_3h_5|V^K|h_7h_8\rangle Z_{p_5p_6,h_7h_8}^K \]

(g)

\[ \Delta U_{p_1h_1}(p_4, p_1, \omega) = \sum \ell \frac{2\ell + 1}{2j_{p_1} + 1} Z_{p_1h_2,p_5h_5}^\ell \langle p_5\bar{h}_5|V^f|h_4\bar{p}_4\rangle \langle p_4\bar{h}_4|V^f|p_3\bar{h}_3\rangle \]

(h)

\[ \Delta U_{p_1h_1}(h_3, h_1, \omega) = \sum \ell \frac{2\ell + 1}{2j_{h_1} + 1} Z_{h_3h_2,p_2h_2}^\ell \langle p_2\bar{h}_2|V^f|h_4\bar{p}_4\rangle \langle p_2\bar{h}_2|V^f|p_3\bar{h}_3\rangle \]
2. Term IX. We introduce the notation

\[ B_{p^1h_1,p^2h_2}^{\ell} = \langle p^1\bar{h}_1|V^{\ell}|h_2\bar{p}_2 \rangle \]  

(91)

The following contributions to the single particle potential are included

(a)

\[ \Delta U_{p^1h_1}(h_a,h_1,\omega) = \frac{1}{8} \sum_k \frac{2K + 1}{2j_{h_1} + 1} Z_{\ell p^1h_1} \frac{B_{p^1h_1,p^2h_2}^{K}}{\epsilon_{p^1h_1} + \epsilon_{p^2h_2} + \epsilon_{p^1h_1} + \omega} Z_{\ell_{p^1h_1, p^2h_2}} B_{p^1h_1,p^2h_2}^{K} \]  

(92)

(b)

\[ \Delta U_{p^1h_1}(p_a,p_1,\omega) = \frac{1}{8} \sum_k \frac{2K + 1}{2j_{p_1} + 1} Z_{\ell p^1h_1} \frac{B_{p^1h_1,p^2h_2}^{K}}{\epsilon_{p^1h_1} + \epsilon_{p^2h_2} + \epsilon_{p^1h_1} + \omega} Z_{\ell_{p^1h_1, p^2h_2}} B_{p^1h_1,p^2h_2}^{K} \]  

(93)

(c)

\[ \Delta U_{p^1h_1}(p_a,p_1,\omega) = \frac{1}{2} \sum_{\ell} \frac{2\ell + 1}{2j_{p_1} + 1} Z_{\ell p^1h_1} B_{p^1h_1,p^2h_2}^{\ell} d(p_5,p_4,\omega' = \epsilon_{p^1h_1} + \omega) \]  

(94)

(d)

\[ \Delta U_{p^1h_1}(p_a,p_1,\omega) = \frac{1}{2} \sum_{\ell} \frac{2\ell + 1}{2j_{h_1} + 1} Z_{\ell p^1h_1} B_{p^1h_1,p^2h_2}^{\ell} d(h_5,h_4,\omega' = \epsilon_{p^1h_1} + \omega) \]  

(95)

(e)

\[ \Delta U_{p^1h_1}(h_a,h_1,\omega) = \frac{1}{2} \sum_{\ell} \frac{2\ell + 1}{2j_{h_1} + 1} Z_{\ell p^1h_1} B_{p^1h_1,p^2h_2}^{\ell} d(p_5,p_4,\omega' = \epsilon_{p^1h_1} + \omega) \]  

(96)

(f)

\[ \Delta U_{p^1h_1}(h_a,h_1,\omega) = \frac{1}{2} \sum_{\ell} \frac{2\ell + 1}{2j_{h_1} + 1} Z_{\ell p^1h_1} B_{p^1h_1,p^2h_2}^{\ell} d(h_5,h_4,\omega' = \epsilon_{p^1h_1} + \omega) \]  

(97)

(g)

\[ \Delta U_{p^1h_1}(p_a,p_1,\omega) = \frac{1}{2} Z_{\ell p^1h_1} B_{p^1h_1,p^2h_2} \]  

(98)

(h)

\[ \Delta U_{p^1h_1}(h_a,h_1,\omega) = \frac{1}{2} Z_{\ell p^1h_1} B_{p^1h_1,p^2h_2} \]  

(99)

These last two terms are included by using in Eqs. (90 and 99) the effective ph-ph matrix element implied by Eq. (90).

3.3 \(\omega\)-Dependent Contributions to Single Particle Energies Arising from 3p3h.

These terms are given here as effective ph-energies which become \(\omega\) dependent. They are given as energy matrix \(U(h_3,h_1)\) or \(U(p_3,p_1)\) as required for the single particle hamiltonian. The energies are the diagonal terms, i.e. \(h_1 = h_3 = h\).

1. Terms IV,1 and IV,17.

\[ \Delta U_{p^1h_1}(h_3,h_1,\omega) = -\frac{1}{2} \sum_{\ell} \frac{2\ell + 1}{2j_{h_1} + 1} \frac{\langle h_3\bar{h}_3|V^{\ell}|h_6\bar{p}_6 \rangle \langle h_1\bar{h}_1|V^{\ell}|h_6\bar{p}_6 \rangle}{\epsilon_{p^1h_1} + \epsilon_{p^1h_1} + \omega} \]  

(100)
2. Terms IV,13a and IV,13b.

\[ \Delta U_{p_1 h_1}(p_3, p_1, \omega) = -\frac{1}{2} \sum_{\ell} \frac{2\ell + 1}{2j_{p_1} + 1} \langle p_3 \bar{p}_3 | V^\ell | p_0 \bar{h}_0 \rangle \langle p_1 \bar{p}_5 | V^\ell | p_0 \bar{h}_0 \rangle \epsilon_{p_0 h_0} + \epsilon_{p_1 h_1} + \omega \]  \hspace{1cm} (101)

Note. The previous two terms are both used with \( \omega = \epsilon_{p_2 h_2} \).

3. Term V,1

\[ \Delta U_{p_2 h_1}(h_3, h_1, \omega) = -\frac{1}{2} \sum_{K} \frac{2K + 1}{2j_{h_1} + 1} Z_{p_0 p_4, h_4 h_3}^K \langle p_6 p_4 | V^K | p_3 h_3 \rangle \epsilon_{h_3 h_3} + \epsilon_{p_6 h_3} + \omega \]  \hspace{1cm} (102)

\[ \Delta U_{p_1 h_1}(p_3, p_1, \omega) = -\frac{1}{2} \sum_{\ell} \frac{2\ell + 1}{2j_{p_1} + 1} \langle p_3 \bar{p}_3 | V^\ell | p_4 \bar{h}_4 \rangle \langle p_1 \bar{p}_5 | V^\ell | p_4 \bar{h}_4 \rangle \epsilon_{p_0 h_0} + \epsilon_{p_1 h_1} + \omega \]  \hspace{1cm} (103)

4. Term VIII,1

\[ \Delta U_{p_1 h_1}(p_3, p_1, \omega) = \frac{1}{2} \sum_{\ell} \frac{2\ell + 1}{2j_{p_1} + 1} \langle h_3 \bar{h}_3 | V^\ell | h_5 \bar{h}_5 \rangle \langle h_3 \bar{h}_3 | V^\ell | h_5 \bar{h}_5 \rangle \epsilon_{p_0 h_0} + \epsilon_{p_1 h_1} + \omega \]  \hspace{1cm} (104)

5. Term VII,1

\[ \Delta U_{p_1 h_1}(h_3, h_1, \omega) = \frac{1}{2} \sum_{\ell} \frac{2\ell + 1}{2j_{h_1} + 1} \langle h_5 \bar{h}_5 | V^\ell | h_3 \bar{h}_3 \rangle \langle h_5 \bar{h}_5 | V^\ell | h_3 \bar{h}_3 \rangle \epsilon_{p_0 h_0} + \epsilon_{p_1 h_1} + \omega \]  \hspace{1cm} (105)
6. Term X,1.

$$\Delta U_{p_1 h_1} (h_3, h_1, \omega) = \frac{1}{2} \sum_{\ell} \frac{2\ell + 1}{2j_{h_1} + 1} d(h_8, h_3, \omega) \langle h_8 h_3 | V^\ell | h_0 p_6 \rangle \langle h_1 h_3 | V^\ell | h_0 p_6 \rangle \over \epsilon_{p_6 h_0} + \epsilon_{p_1 h_5} + \omega$$

7. Term XI,1.

$$\Delta U_{p_1 h_1} (h_3, h_1, \omega) = \frac{1}{2} \sum_{\ell} \frac{2\ell + 1}{2j_{h_1} + 1} d(h_8, h_6, \omega) \langle h_3 h_6 | V^\ell | h_0 p_6 \rangle \langle h_1 h_6 | V^\ell | h_0 p_6 \rangle \over \epsilon_{p_6 h_0} + \epsilon_{p_1 h_5} + \omega$$

3.4 Effective ph-ph Matrix Element Arising from 3p3h.

Next we include those terms that can be written similarly to Eq. (48) using an effective ph-ph matrix element. These are generated by Term IV. Similar to Eq. (48), each term gives rise to four contributions of which we list only one. The others can be obtained by making the exchange $(p_1 h_1 \leftrightarrow p_2 h_2)$, with either the recoupling $h_1 \leftrightarrow h_2$ or $p_1 \leftrightarrow p_2$.

1. Terms IV,2a.

$$ \frac{1}{2} \sum_{\ell} \frac{2\ell + 1}{2j_{h_1} + 1} d(h_8, h_6, \omega) \langle h_3 h_6 | V^\ell | h_0 p_6 \rangle \langle h_1 h_6 | V^\ell | h_0 p_6 \rangle \over \epsilon_{p_6 h_0} + \epsilon_{p_1 h_5} + \omega$$

This term can be written as

$$\langle (p_1 h_1)\lambda | V^{(2)} \rangle | (h_2 p_2)\lambda \rangle = Z^{\lambda}_{p_3 h_3, p_2 h_2} A^{\ell, \lambda}_{p_1 h_1, p_3 h_3} / (1 + \epsilon_{p_2 h_2} d A^{\ell, \lambda}_{p_1 h_1, p_3 h_3} / A^{\ell, \lambda}_{p_1 h_1, p_3 h_3})$$

where $A^{\lambda}$ (dA$^{\lambda}$) are obtained from recoupling the $A^K$ (dA$^K$) according to

$$A^{\ell, \lambda}(p_1 h_1, p_3 h_3) = \sum_K (-)^{K+1}(2K+1) \left\{ \begin{array}{ccc} p_1 & h_1 & \lambda \\ p_3 & h_3 & K \end{array} \right\} A^{\ell, \lambda}(p_1 h_3, h_1 p_3)$$

with

$$A^{\ell, \lambda}(p_1 h_3, h_1 p_3) = -\frac{1}{2} (p_1 h_3 | V^K | h_3 h_6 \rangle \langle h_1 p_3 | V^K | h_5 h_6 \rangle / (\epsilon_{p_1 h_5} + \epsilon_{p_3 h_6})$$

$$d A^{\ell, \lambda}(p_1 h_3, h_1 p_3) = -\frac{1}{4} (p_1 h_3 | V^K | h_5 h_6 \rangle \langle h_1 p_3 | V^K | p_7 p_8 \rangle Z^K_{p_7 p_8, h_5 h_6} / (\epsilon_{p_1 h_5} + \epsilon_{p_3 h_6})$$

The following corrections arise from term V

$$\Delta A^{\ell, \lambda}(p_1 h_3, h_1 p_3) = -\frac{1}{4} (p_1 h_3 | V^K | h_5 h_6 \rangle \langle h_1 p_3 | V^K | p_7 p_8 \rangle Z^K_{p_7 p_8, h_5 h_6} / (\epsilon_{p_1 h_5} + \epsilon_{p_3 h_6})$$

$$\Delta d A^{\ell, \lambda}(p_1 h_3, h_1 p_3) = -\frac{1}{4} (p_1 h_3 | V^K | h_5 h_6 \rangle \langle h_1 p_3 | V^K | p_7 p_8 \rangle Z^K_{p_7 p_8, h_5 h_6} / (\epsilon_{p_1 h_5} + \epsilon_{p_3 h_6})$$
and

\[ \Delta A^{eff,K}(p_1h_3,h_1p_3) = -\frac{1}{2} \sum \ell (-)^{K+1} (2\ell + 1) \left\{ \begin{array}{ccc} p_1 & h_3 & K \\ h_6 & h_5 & \ell \end{array} \right\} \langle p_4\bar{h}_4|V^\ell|h_3h_6\rangle\langle h_1p_3|V^K|h_5h_6\rangle Z_{p_4h_4,p_1h_5}^\ell (\epsilon_{p_1h_5} + \epsilon_{p_3h_6}) \]  

(127)

\[ \Delta dA^{eff,K}(p_1h_3,h_1p_3) = -\frac{1}{2} \sum \ell (-)^{K+1} (2\ell + 1) \left\{ \begin{array}{ccc} p_1 & h_3 & K \\ h_6 & h_5 & \ell \end{array} \right\} \langle p_4\bar{h}_4|V^\ell|h_3h_6\rangle\langle h_1p_3|V^K|h_5h_6\rangle Z_{p_4h_4,p_1h_5}^\ell (\epsilon_{p_1h_5} + \epsilon_{p_3h_6})^2 \]  

(128)

and finally,

\[ \Delta A^{eff,K}(p_1h_3,h_1p_3) = \frac{1}{2} \langle p_1h_3|V^K|h_5h_6\rangle\langle h_1p_3|V^K|h_7h_6\rangle d(h_5,h_7)/(\epsilon_{p_1h_5} + \epsilon_{p_3h_6}) \]  

(129)

\[ \Delta dA^{eff,K}(p_1h_3,h_1p_3) = \frac{1}{2} \langle p_1h_3|V^K|h_5h_6\rangle\langle h_1p_3|V^K|h_7h_6\rangle d(h_5,h_7)/(\epsilon_{p_1h_5} + \epsilon_{p_3h_6})^2 \]  

(130)

Note: Eqs. (127–128) are taken into account only when \( p_4 \) and \( h_4 \) are both of the same kind (protons or neutrons).

2. Terms IV,15a.

\[ -\frac{1}{2} Z_{p_3h_3,p_2h_2} V_{p_3h_1p_5p_6} V_{p_5p_6h_3p_1}/\epsilon \]  

(131)

This term can be written as Eq. (121) similarly to the previous term, where the \( A^\lambda \) are the recoupled \( A^K \) according to Eq. (122) with

\[ A^{eff,K}(p_1h_3,h_1p_3) = -\frac{1}{2} \langle p_1h_3|V^K|p_5p_6\rangle\langle h_1p_3|V^K|p_5p_6\rangle/(\epsilon_{p_5h_1} + \epsilon_{p_6h_3}) \]  

(132)

\[ dA^{eff,K}(p_1h_3,h_1p_3) = -\frac{1}{2} \langle p_1h_3|V^K|p_5p_6\rangle\langle h_1p_3|V^K|p_5p_6\rangle/(\epsilon_{p_5h_1} + \epsilon_{p_6h_3})^2 \]  

(133)

with higher order corrections arising from term \( V \), as

\[ \Delta A^{eff,K}(p_1h_3,h_1p_3) = -\frac{1}{2} \langle p_1h_3|V^K|p_5p_6\rangle\langle h_1p_3|V^K|p_5p_6\rangle Z_{p_5p_6,h_3h_6}^K/(\epsilon_{p_5h_1} + \epsilon_{p_6h_3}) \]  

(134)

\[ \Delta dA^{eff,K}(p_1h_3,h_1p_3) = -\frac{1}{4} \langle p_1h_3|V^K|p_5p_6\rangle\langle h_1p_3|V^K|p_5p_6\rangle Z_{p_5p_6,h_3h_6}^K/(\epsilon_{p_5h_1} + \epsilon_{p_6h_3})^2 \]  

(135)

and

\[ \Delta A^{eff,K}(p_1h_3,h_1p_3) = -\frac{1}{2} \sum \ell (-)^{K+1} (2\ell + 1) \left\{ \begin{array}{ccc} p_1 & h_3 & K \\ p_6 & p_5 & \ell \end{array} \right\} \langle p_5\bar{p}_1|V^\ell|p_7\bar{h}_7\rangle\langle h_1p_3|V^K|p_5p_6\rangle Z_{p_5p_6,h_3h_6}^\ell (\epsilon_{p_5h_1} + \epsilon_{p_6h_3}) \]  

(136)

\[ \Delta dA^{eff,K}(p_1h_3,h_1p_3) = -\frac{1}{2} \sum \ell (-)^{K+1} (2\ell + 1) \left\{ \begin{array}{ccc} p_1 & h_3 & K \\ p_6 & p_5 & \ell \end{array} \right\} \langle p_5\bar{p}_1|V^\ell|p_7\bar{h}_7\rangle\langle h_1p_3|V^K|p_5p_6\rangle Z_{p_5p_6,h_3h_6}^\ell (\epsilon_{p_5h_1} + \epsilon_{p_6h_3})^2 \]  

(137)

and

\[ \Delta A^{eff,K}(p_1h_3,h_1p_3) = \frac{1}{2} \langle p_1h_3|V^K|p_5p_6\rangle\langle h_1p_3|V^K|p_7p_6\rangle d(p_5,p_7)/(\epsilon_{p_5h_1} + \epsilon_{p_6h_3}) \]  

(138)

\[ \Delta dA^{eff,K}(p_1h_3,h_1p_3) = \frac{1}{2} \langle p_1h_3|V^K|p_5p_6\rangle\langle h_1p_3|V^K|p_7p_6\rangle d(p_5,p_7)/(\epsilon_{p_5h_1} + \epsilon_{p_6h_3})^2 \]  

(139)

Note: Eqs. (134, 137) are taken into account only when both \( p_7 \) and \( h_7 \) are the same kind of nucleons.

3. Terms IV,12a and IV,26b.

\[ Z_{p_3h_3,p_2h_2} V_{p_3h_1p_5p_6} V_{p_5p_6h_3p_1}/\epsilon \]  

(140)
This can be written as
\[
(p_{h1})_{\Lambda}(h_{3}p_{5})_{\Lambda} \left\langle \frac{Z^{\Lambda}_{p_{h3},p_{h2}}A_{p_{h1},p_{h3}}^{\Lambda}}{1 + \epsilon_{p_{h2}}dA_{p_{h1},p_{h3}}^{\Lambda} + A_{p_{h1},p_{h3}}^{\Lambda}} \right\rangle (1 + \epsilon_{p_{h2}}dA_{p_{h1},p_{h3}}^{\Lambda} + A_{p_{h1},p_{h3}}^{\Lambda})
\]
where \(A_{p_{h1},p_{h3}}^{\Lambda} (dA)\) are the exchange terms of \(W_{\ell}^{\Lambda} (dW)\) according to
\[
A_{p_{h1},p_{h3}}^{\Lambda} = \sum_{\ell} (-)^{\ell + \lambda} (2\ell + 1) \left\{ \begin{array}{c} p_{h1} \\ h_{3} \\ p_{h3} \\ \ell \end{array} \right\} W_{\ell}^{\Lambda}(h_{3}h_{1},p_{5}p_{3})
\]
with
\[
W^{\ell}(h_{3}h_{1},p_{5}p_{3}) = -\left\langle (h_{3}h_{1})_{V}|p_{5}h_{5}\rangle_{V}|p_{3}p_{3}\rangle_{V}/(\epsilon_{p_{h5}} + \epsilon_{p_{h3}})\right\rangle
\]
\[
dW^{\ell}(h_{3}h_{1},p_{5}p_{3}) = -\left\langle (h_{3}h_{1})_{V}|p_{5}h_{5}\rangle_{V}|p_{3}p_{3}\rangle_{V}/(\epsilon_{p_{h5}} + \epsilon_{p_{h3}})^{2}\right\rangle
\]
We also have the higher order corrections
\[
\Delta W^{\ell}(h_{3}h_{1},p_{5}p_{3}) = -\left\langle (h_{3}h_{1})_{V}|p_{5}h_{5}\rangle_{V}|p_{3}p_{3}\rangle_{V}/(\epsilon_{p_{h5}} + \epsilon_{p_{h3}})\right\rangle
\]
\[
\Delta dW^{\ell}(h_{3}h_{1},p_{5}p_{3}) = -\left\langle (h_{3}h_{1})_{V}|p_{5}h_{5}\rangle_{V}|p_{3}p_{3}\rangle_{V}/(\epsilon_{p_{h5}} + \epsilon_{p_{h3}})^{2}\right\rangle
\]
and
\[
\Delta W^{\ell}(h_{3}h_{1},p_{5}p_{3}) = \left\langle (h_{3}h_{1})_{V}|p_{5}h_{5}\rangle_{V}|p_{3}p_{3}\rangle_{V}/(\epsilon_{p_{h5}} + \epsilon_{p_{h3}})\right\rangle
\]
\[
\Delta dW^{\ell}(h_{3}h_{1},p_{5}p_{3}) = \left\langle (h_{3}h_{1})_{V}|p_{5}h_{5}\rangle_{V}|p_{3}p_{3}\rangle_{V}/(\epsilon_{p_{h5}} + \epsilon_{p_{h3}})^{2}\right\rangle
\]
4. Terms IV.8b and IV.22a.

\[
Z^{\Lambda}_{p_{h3},p_{h2}}V^{p_{h3},p_{h2},h_{3},h_{1},V_{p_{h1},p_{h3}}}/\epsilon
\]
This can be written similarly to Eq. (14). Here the \(A_{\Lambda}^{\Lambda}(dA_{\Lambda}^{\Lambda})\) are the exchange terms of \(W^{\Lambda} (dW^{\Lambda})\) according to Eq. (142) where
\[
W^{\ell}(h_{3}h_{1},p_{5}p_{3}) = -\left\langle (h_{3}h_{1})_{V}|p_{5}h_{5}\rangle_{V}|p_{3}p_{3}\rangle_{V}/(\epsilon_{p_{h5}} + \epsilon_{p_{h3}})\right\rangle
\]
\[
dW^{\ell}(h_{3}h_{1},p_{5}p_{3}) = -\left\langle (h_{3}h_{1})_{V}|p_{5}h_{5}\rangle_{V}|p_{3}p_{3}\rangle_{V}/(\epsilon_{p_{h5}} + \epsilon_{p_{h3}})^{2}\right\rangle
\]
Here, the higher order corrections are as follows
\[
W^{\ell}(h_{3}h_{1},p_{5}p_{3}) = -\left\langle (h_{3}h_{1})_{V}|p_{5}h_{5}\rangle_{V}|p_{3}p_{3}\rangle_{V}/(\epsilon_{p_{h5}} + \epsilon_{p_{h3}})\right\rangle
\]
\[
dW^{\ell}(h_{3}h_{1},p_{5}p_{3}) = -\left\langle (h_{3}h_{1})_{V}|p_{5}h_{5}\rangle_{V}|p_{3}p_{3}\rangle_{V}/(\epsilon_{p_{h5}} + \epsilon_{p_{h3}})^{2}\right\rangle
\]
and
\[
W^{\ell}(h_{3}h_{1},p_{5}p_{3}) = \left\langle (h_{3}h_{1})_{V}|p_{5}h_{5}\rangle_{V}|p_{3}p_{3}\rangle_{V}/(\epsilon_{p_{h5}} + \epsilon_{p_{h3}})\right\rangle
\]
\[
dW^{\ell}(h_{3}h_{1},p_{5}p_{3}) = \left\langle (h_{3}h_{1})_{V}|p_{5}h_{5}\rangle_{V}|p_{3}p_{3}\rangle_{V}/(\epsilon_{p_{h5}} + \epsilon_{p_{h3}})^{2}\right\rangle
\]
3.5 Contributions to $V_5$.

1. Term IV, 20a.

$$\begin{align*}
- Z_{p_2 h_3, p_4 h_4} V_{p_1 h_1, h_3} V_{p_4 h_2 h_5} / \epsilon \\
\rightarrow \quad \langle (p_1 \bar{h}_1)_{\lambda} | V^{(5), \lambda} | (h_2 \bar{p}_2)_{\lambda} \rangle &= \sum_{\ell} \left( - \right)^{\ell+\lambda} (2\ell + 1) \left\{ \begin{array}{ccc}
h_5 & h_3 & \lambda \\
p_2 & h_2 & \ell \end{array} \right\} \\
Z^f_{p_2 h_3, p_4 h_4} \langle p_4 \bar{h}_4 | V^f | h_5 \bar{h}_2 \rangle \\
\frac{\langle p_1 \bar{h}_1 | V^\lambda | h_5 \bar{h}_3 \rangle}{\epsilon_{p_1 h_1} + \epsilon_{p_2 h_2} + \epsilon_{p_4 h_4}}
\end{align*}\quad (160)$$

We can write this as

$$\langle (p_1 \bar{h}_1)_{\lambda} | V^{(5), \lambda} | (h_2 \bar{p}_2)_{\lambda} \rangle \approx \frac{ZV^{\text{exch}, \lambda}(p_2 \bar{h}_2, h_3 \bar{h}_3)(p_1 \bar{h}_1 | V^\lambda | h_5 \bar{h}_3)}{(1 + \epsilon_{p_1 h_1} dZV^{\text{exch}, \lambda}(p_2 \bar{h}_2, h_3 \bar{h}_3)/ZV^{\text{exch}, \lambda}(p_2 h_2, h_3 \bar{h}_3))} \quad (161)$$

where the $ZV^{\text{exch}, \lambda}$ ($dZV^{\text{exch}, \lambda}$) are the recoupled $ZV^f$ ($dZV^f$) according to

$$
ZV^{\text{exch}, \lambda}(p_2 \bar{h}_2, h_3 \bar{h}_3) = \sum_{\ell} \left( - \right)^{\ell+\lambda} (2\ell + 1) \left\{ \begin{array}{ccc}
h_5 & h_3 & \lambda \\
p_2 & h_2 & \ell \end{array} \right\} ZV^f(p_2 \bar{h}_2, h_3 \bar{h}_3) \\
\quad (162)
$$

with

$$
ZV^f(p_2 \bar{h}_2, h_3 \bar{h}_3) = Z_{p_2 h_3, p_4 h_4}^f \langle p_4 \bar{h}_4 | V^f | h_5 \bar{h}_2 \rangle / (\epsilon_{p_2 h_2} + \epsilon_{p_4 h_4}) \\
dZV^f(p_2 \bar{h}_2, h_3 \bar{h}_3) = Z_{p_2 h_3, p_4 h_4}^f \langle p_4 \bar{h}_4 | V^f | h_5 \bar{h}_2 \rangle / (\epsilon_{p_2 h_2} + \epsilon_{p_4 h_4})^2 \quad (163)
$$

There are additional terms with this symmetry that can be written in the same way using effective operators. They are generated by terms V and VIII as well as XI and XIII. Term V corresponds to the following expectation value:

$$- \frac{1}{2} Z_{p_2 h_3, p_4 h_4} Z_{p_1 p_5 h_5 h_3} V_{p_6 h_4 h_3 h_8} V_{p_4 h_2 h_5} / \epsilon \quad (165)$$

This term is incorporated by replacing in [163]

$$\langle p_1 \bar{h}_1 | V^\lambda | h_5 \bar{h}_3 \rangle \rightarrow \langle p_1 \bar{h}_1 | V^{f, \lambda} | h_5 \bar{h}_3 \rangle = \langle p_1 \bar{h}_1 | V^\lambda | h_5 \bar{h}_3 \rangle + \frac{1}{2} Z_{p_1 h_1, p_6 h_3}^\lambda \langle p_5 \bar{h}_5 | V^\lambda | h_3 \bar{h}_5 \rangle \\
- \langle p_1 \bar{h}_1 | V^\lambda | h_6 \bar{h}_6 \rangle d(h_5, h_3) - \langle p_1 \bar{h}_1 | V^\lambda | h_6 \bar{h}_6 \rangle d(h_6, h_3) \quad (166)$$

Similarly, we replace in [163] and [164]

$$\langle p_4 \bar{h}_4 | V^f | h_5 \bar{h}_2 \rangle \rightarrow \langle p_4 \bar{h}_4 | V^{f, \lambda} | h_5 \bar{h}_2 \rangle = \langle p_4 \bar{h}_4 | V^f | h_5 \bar{h}_2 \rangle + Z_{p_5 h_5, p_4 h_4}^f \langle p_5 \bar{h}_5 | V^f | h_2 \bar{h}_2 \rangle \\
- \langle p_6 h_4 | V^f | h_5 \bar{h}_2 \rangle d(p_4, p_6) - \langle p_6 h_4 | V^f | h_5 \bar{h}_2 \rangle d(h_4, h_6) \quad (167)$$

2. Term IV, 16b.

$$- Z_{p_3 h_3, p_4 h_4} V_{p_1 p_5 h_5} V_{p_3 p_5 h_3 p_2} / \epsilon \quad (168)$$

gives

$$\langle (p_1 \bar{h}_1)_{\lambda} | V^{(5), \lambda} | (h_2 \bar{p}_2)_{\lambda} \rangle \approx \frac{ZV^{\text{exch}, \lambda}(p_3 \bar{h}_3, p_4 \bar{p}_4)(p_1 \bar{h}_1 | V^\lambda | p_4 \bar{p}_4)}{1 + \epsilon_{p_1 h_1} dZV^{\text{exch}, \lambda}(p_3 \bar{h}_3, p_4 \bar{p}_4)/ZV^{\text{exch}, \lambda}(p_2 h_2, p_4 \bar{p}_4)} \quad (169)$$

where the $ZV^{\text{exch}, \lambda}$ ($dZV^{\text{exch}, \lambda}$) are the recoupled $ZV^f$ ($dZV^f$) according to:

$$ZV^{\text{exch}, \lambda}(p_2 \bar{h}_2, p_5 \bar{p}_5) = \sum_{\ell} \left( - \right)^{\ell+\lambda} (2\ell + 1) \left\{ \begin{array}{ccc}
p_5 & p_4 & \lambda \\
p_2 & h_2 & \ell \end{array} \right\} ZV^f(p_4 \bar{h}_4, p_2 \bar{p}_5) \quad (170)$$

with

$$ZV^f(p_4 \bar{h}_4, p_2 \bar{p}_5) = \frac{Z_{p_2 h_2, p_4 h_4}^f \langle p_4 \bar{h}_4 | V^f | p_2 \bar{p}_5 \rangle}{\epsilon_{p_2 h_2} + \epsilon_{p_4 h_4}} \quad (171)$$

$$dZV^f(p_4 \bar{h}_4, p_2 \bar{p}_5) = \frac{Z_{p_2 h_2, p_4 h_4}^f \langle p_4 \bar{h}_4 | V^f | p_2 \bar{p}_5 \rangle}{(\epsilon_{p_2 h_2} + \epsilon_{p_4 h_4})^2} \quad (172)$$
As before, additional terms are incorporated using the substitution in (169)

\[
\langle p_1 \tilde{h}_1 | V^{\lambda} | p_4 \tilde{p}_5 \rangle \rightarrow \langle p_1 \tilde{h}_1 | V^{f_1, \lambda} | p_4 \tilde{p}_5 \rangle = \langle p_1 \tilde{h}_1 | V^{\lambda} | p_4 \tilde{p}_5 \rangle + \frac{1}{2} Z_{p_1 h_1, p_4 h_4} \langle p_6 \tilde{h}_6 | V^{\lambda} | p_3 \tilde{p}_4 \rangle - \langle p_2 \tilde{h}_2 | V^{\lambda} | p_8 \tilde{p}_8 \rangle d(p_8, p_4) - \langle p_1 \tilde{h}_1 | V^{\lambda} | p_4 \tilde{p}_5 \rangle d(p_8, p_5)
\] (173)

and also, we replace in (171) and (172)

\[
\langle p_3 \tilde{h}_3 | V^{f_2, \ell} | p_2 \tilde{p}_5 \rangle \rightarrow \langle p_3 \tilde{h}_3 | V^{f_2, \ell} | p_2 \tilde{p}_5 \rangle = \langle p_3 \tilde{h}_3 | V^{(5), \ell} | p_2 \tilde{p}_5 \rangle + Z_{p_6 h_6, p_3 h_3} \langle p_6 \tilde{h}_6 | V^{f_2} | p_5 \tilde{p}_2 \rangle - \langle p_6 \tilde{h}_3 | V^{f_2} | p_2 \tilde{p}_5 \rangle d(p_6, p_3) - \langle p_3 \tilde{h}_3 | V^{f_2} | p_2 \tilde{p}_5 \rangle d(h_3, h_6)
\] (174)

3. Terms IV, 4b and 28a.

\[
-Z_{p_1 h_1, p_4 h_4} V_{p_2 h_2 p_6} V_{p_6 h_4 h_5} / \epsilon
\] (175)

\[
-Z_{p_3 h_3, p_4 h_4} V_{p_2 p_6 h_2 p_6} V_{p_3 h_3 h_5} / \epsilon
\] (176)

These are the previous two terms with \( p_1 h_1 \) and \( p_2 h_2 \) exchanged. We incorporate them as such with all the corresponding higher order corrections included.

4. Term IV, 20b, 4a, 16a, and 28b. These terms are the exchange terms to the previous four terms with \( p_1 \) and \( p_2 \) exchanged. They are computed from the previous expressions with the additional recoupling:

\[
\langle p_1 \tilde{h}_1 | V^{(5), \lambda} | h_2 \tilde{p}_2 \rangle = \sum_{\ell} \langle h_1 h_2 | V^{K} | p_6 h_4 \rangle \left\{ \frac{p_1}{h_2} \frac{h_1}{p_2} \frac{\lambda}{\ell} \right\} \langle p_2 \tilde{h}_1 | V^{(5), \ell} | (h_2 \tilde{p}_1) \rangle (177)
\]

5. Term IV, 6a.

\[
\frac{1}{2} Z_{p_3 h_3, p_1 h_1} V_{p_3 p_6 h_3 p_2} V_{h_2 h_1 p_6} / \epsilon
\] (178)

gives

\[
\Delta B^K(p_1 p_2, h_1 h_2) \approx \langle h_1 h_2 | V^K | p_6 h_4 \rangle \frac{ZV^K(p_1 p_2, p_6 h_4)}{1 + (\epsilon_{p_2 h_2} + \epsilon_{h_1} - \epsilon_{h_4}) dZV^K(p_1 p_2, p_6 h_4)}
\] (179)

where the \( ZV^K(dZV^K) \) are the recoupled quantities from term (20) according to

\[
ZV^K(p_1 p_2, p_6 h_4) = \sum_{\ell} \langle h_1 h_2 | V^{(5), K} | p_6 h_4 \rangle \left\{ \frac{p_1}{h_4} \frac{h_2}{p_6} \frac{K}{\ell} \right\} ZV^\ell(p_2 h_4, p_6 p_1)
\] (180)

Higher order corrections are included by replacing in (179)

\[
\langle h_1 h_2 | V^K | p_6 h_4 \rangle \rightarrow \langle h_1 h_2 | V^f_{5, K} | p_6 h_4 \rangle = \langle h_1 h_2 | V^K | p_6 h_4 \rangle + \frac{1}{2} Z_{p_3 p_4, h_3 h_2} Z^K(p_3 p_4 | V^K | p_6 h_4)
\] (181)

6. Term IV, 6b.

\[
\frac{1}{2} Z_{p_3 h_3, p_1 h_1} V_{p_3 p_6 h_3 p_2} V_{h_2 h_1 p_6} / \epsilon
\] (182)

gives

\[
\Delta B^K(p_1 p_2, h_1 h_2) \approx \langle h_1 h_2 | V^K | p_6 h_4 \rangle \frac{ZV^K(p_2 p_1, p_6 h_4)}{1 + (\epsilon_{p_1 h_1} + \epsilon_{h_2} - \epsilon_{h_4}) dZV^K(p_2 p_1, p_6 h_4)}
\] (183)

For the \( ph \) coupled contribution it can be written as the transpose of term (6a).
This term is written similarly to Eq. (189) as

\[
\frac{1}{2} Z_{p_1 h_2, p_4 h_4} V_{p_1 p_2 h_6 p_3} V_{h_6 h_4 h_1 p_4} / \epsilon
\]  

(184)

gives

\[
\Delta B^K (p_1 p_2, h_1 h_2) \approx \frac{\langle p_1 p_2 | V^K | h_6 p_3 \rangle ZV^K (h_1 h_2, h_6 p_3)}{1 + (\epsilon_{p_2 h_2} + \epsilon_{p_1} - \epsilon_{p_3}) dZV^K (h_1 h_2, h_6 p_3) / ZV^K (h_1 h_2, h_6 p_3)}
\]  

(185)

where the \( ZV^K (h_1 h_2, h_6 p_3) \) are the pp-coupled \( ZV \) from term (16) according to

\[
ZV^K (h_1 h_2, h_6 p_3) = \sum_{\ell} (-)^{K+1} (2\ell + 1) \left\{ \begin{array}{c} p_3 \\ h_1 \\ h_2 \end{array} \right\} K \ell \} ZV^\ell (p_3 h_2, h_6 h_1)
\]  

(186)

Higher order corrections are included by replacing in (183)

\[
\langle p_1 p_2 | V^K | h_6 p_3 \rangle \to \langle p_1 p_2 | V^{f_3, K} | h_6 p_3 \rangle = \langle p_1 p_2 | V^K | h_6 p_3 \rangle + \frac{1}{2} Z^{K}_{p_1 p_2, h_3 h_4} \langle h_3 h_4 | V^K | h_6 p_3 \rangle
\]  

(187)

8. Term IV, 25. This is the same as the previous term with \( h_1 \) and \( h_2 \) interchanged.

9. Term IV, 7a.

\[
-\frac{1}{2} Z_{p_3 h_3, p_4 h_4} V_{p_2 h_3 h_2 h_6} V_{p_3 p_4 p_1 h_6} / \epsilon
\]  

(188)

gives

\[
\langle (p_1 \tilde{h}_1) \lambda | V^{(5, \lambda)} | (h_2 \tilde{p}_2) \lambda \rangle = \sum_{K} \frac{1}{2} \frac{Z^K_{p_2 p_4, h_3 h_5} \langle p_3 p_4 | V^K | p_1 h_6 \rangle}{\epsilon_{p_4 h_1} + \epsilon_{p_2 h_2} + \epsilon_{p_3 h_6}} (-)^{K+1} (2K + 1) \left\{ \begin{array}{c} p_1 \\ h_3 \\ h_6 \end{array} \right\} K \lambda \} \langle p_2 \tilde{h}_2 | V^{\lambda} | h_6 \tilde{h}_3 \rangle
\]  

(189)

Higher order corrections are included by using in (180) the substitution (166)

\[
\langle p_2 \tilde{h}_2 | V^{\lambda} | h_6 \tilde{h}_3 \rangle \to \langle p_2 \tilde{h}_2 | V^{f_1, \lambda} | h_6 \tilde{h}_3 \rangle
\]  

(190)

We approximate this by

\[
\langle (p_1 \tilde{h}_1) \lambda | V^{(5, \lambda)} | (h_2 \tilde{p}_2) \lambda \rangle \approx \frac{Z^{r, \lambda}_{p_1 \tilde{h}_1, h_3 \tilde{h}_5} \langle p_2 \tilde{h}_2 | V^{\lambda} | h_5 \tilde{h}_3 \rangle}{(1 + \epsilon_{p_2 h_2} dZV^{r, \lambda}_{p_1 \tilde{h}_1, h_5 \tilde{h}_3} / ZV^{r, \lambda}_{p_1 \tilde{h}_1, h_5 \tilde{h}_3})}
\]  

(191)

Here the \( Z^{r, \lambda} \) \((dZV^{r, \lambda})\) are recoupled from pp coupling via

\[
Z^{r, \lambda}_{p_2 \tilde{h}_2, h_3 \tilde{h}_5} = (-)^{K+1} (2K + 1) \left\{ \begin{array}{c} p_1 \\ h_3 \\ h_6 \end{array} \right\} K \lambda \} ZV^K (p_1 h_6, h_1 h_3)
\]  

with

\[
ZV^K (p_1 h_6, h_1 h_3) = \frac{Z^K_{p_3 p_4, h_3 h_5} \langle p_3 p_4 | V^K | p_1 h_6 \rangle}{\epsilon_{p_4 h_1} + \epsilon_{p_3 h_6}}
\]  

(193)

\[
dZV^K (p_1 h_6, h_1 h_3) = \frac{Z^K_{p_3 p_4, h_3 h_5} \langle p_3 p_4 | V^K | p_1 h_6 \rangle}{(\epsilon_{p_4 h_1} + \epsilon_{p_3 h_6})^2}
\]  

(194)

10. Term IV, 10b.

\[
-\frac{1}{2} Z_{p_3 h_3, p_1 h_4} V_{p_7 p_2 p_3 h_2} V_{h_3 h_4 h_1 p_7} / \epsilon
\]  

(195)

This term is written similarly to Eq. (189) as

\[
\langle (p_1 \tilde{h}_1) \lambda | V^{(5, \lambda)} | (h_2 \tilde{p}_2) \lambda \rangle \approx \frac{Z^{r, \lambda}_{p_1 \tilde{h}_1, p_7 \tilde{p}_7} \langle p_2 \tilde{h}_2 | V^{\lambda} | p_3 \tilde{p}_7 \rangle}{1 + \epsilon_{p_2 h_2} dZV^{r, \lambda}_{p_1 \tilde{h}_1, p_7 \tilde{p}_7} / ZV^{r, \lambda}_{p_1 \tilde{h}_1, p_7 \tilde{p}_7}}
\]  

(196)
Here the $ZV^{r,\lambda}$ ($dZV^{r,\lambda}$) are recoupled from $pp$ coupling via
\[
ZV^{r,\lambda}(p_3\bar{h}_1, p_7\bar{p}_3) = \sum_{\kappa} \frac{1}{2} (-)^{K+1}(2K+1) \left\{ \begin{array}{ccc} p_1 & h_1 & \lambda \\ p_7 & p_3 & K \end{array} \right\} ZV^K(p_3\bar{p}_1, h_1\bar{p}_7)
\]  
(197)
where we have
\[
ZV^K(p_3\bar{p}_1, h_1\bar{p}_7) = \frac{1}{2} Z^K_{p_3\bar{p}_1,h_4\bar{h}_4} \langle h_3 h_4 | V^K | h_1 p_7 \rangle
\]  
(198)
and
\[
dZV^K(p_3\bar{p}_1, h_1\bar{p}_7) = \frac{1}{2} \frac{Z^K_{p_3\bar{p}_1,h_4\bar{h}_4} \langle h_3 h_4 | V^K | h_1 p_7 \rangle}{(\epsilon_{p_1 h_3} + \epsilon_{p_7 h_4})^2}
\]  
(199)
Higher order corrections are obtained by using in (196) the substitution (194)
\[
\langle p_2\bar{h}_2|V^\lambda|p_3\bar{p}_7 \rangle \rightarrow \langle p_2\bar{h}_2|V^{f_1,\lambda}|p_3\bar{p}_7 \rangle
\]  
(200)
11. Term IV, 21b and 10a. These are obtained from the previous two terms by interchanging $p_1h_1$ and $p_2h_2$, and they are computed that way.
12. Term IV, 21a, 7b, 24a, and 24b. These terms are the exchange terms to the previous four terms with $p_1$ and $p_2$ exchanged. They are computed from the previous expressions with the additional recoupling given by Eq. (177).
13. Term IV, 19.
\[
-Z_{p_1h_3,p_2h_4} V_{p_3h_3h_5} V_{b_4h_3h_2p_5}/\epsilon
\]  
(201)
gives
\[
\Delta B^K(p_1p_2, h_1h_2) = \frac{Z^K_{p_1p_2,h_3h_4} W^K_{h_1h_2,h_3h_4}}{1 + (\epsilon_{p_1} + \epsilon_{p_2}) dW^K_{h_1h_2,h_3h_4}/W^K_{h_1h_2,h_3h_4}}
\]  
(202)
with
\[
(d)W^K_{h_1h_2,h_3h_4} = \sum_{\ell} (-)^{K+1}(2\ell+1) \left\{ \begin{array}{ccc} h_1 & h_2 & K \\ h_3 & h_4 & \ell \end{array} \right\} (d)W^\ell_{h_1,\bar{h}_3,\bar{h}_2}
\]  
(203)
and
\[
W^\ell_{h_1,\bar{h}_3,\bar{h}_2} = -\langle h_1\bar{h}_3 | V^\ell | p_5 \bar{h}_5 \rangle \langle \bar{h}_4 h_2 | V^\ell | p_5 \bar{h}_5 \rangle / (\epsilon_{h_4} + \epsilon_{p_1} + \epsilon_{p_5 h_5})
\]  
(204)
\[
dW^\ell_{h_1,\bar{h}_3,\bar{h}_2} = -\langle h_1\bar{h}_3 | V^\ell | p_5 \bar{h}_5 \rangle \langle \bar{h}_4 h_2 | V^\ell | p_5 \bar{h}_5 \rangle / (\epsilon_{h_4} + \epsilon_{p_1} + \epsilon_{p_5 h_5})^2
\]  
(205)
14. Term IV, 3.
\[
-Z_{p_2h_3,p_1h_4} V_{p_3h_3h_5} V_{b_4h_3h_1p_5}/\epsilon
\]  
(206)
This term arises from the exchange $(p_1h_1)$ with $(p_2h_2)$. It thus results in the transpose of term (19). Combining terms (3) and (19) gives
\[
\Delta B^K(p_1p_2, h_1h_2) = Z^K_{p_1p_2,h_3h_4}
\]  
(207)
\[
\times \left[ \frac{W^K_{h_1h_2,h_3h_4}}{1 + (\epsilon_{p_1} + \epsilon_{p_2}) dW^K_{h_1h_2,h_3h_4}/W^K_{h_1h_2,h_3h_4}} + (-)^K \frac{W^K_{h_2h_1,h_3h_4}}{1 + (\epsilon_{p_1} + \epsilon_{p_2}) dW^K_{h_2h_1,h_3h_4}/W^K_{h_2h_1,h_3h_4}} \right]
\]  
(208)
Here the sum is not restricted and goes over $h_3 h_4$ as well as $h_4 h_3$. By using a restricted sum (only $h_3 h_4$, we write
\[
\Delta B^K(p_1p_2, h_1h_2) = Z^K_{p_1p_2,h_3h_4}
\]  
(209)
\[
\times \left[ \frac{W^K_{h_1h_2,h_3h_4}}{1 + (\epsilon_{p_1} + \epsilon_{p_2}) dW^K_{h_1h_2,h_3h_4}/W^K_{h_1h_2,h_3h_4}} + (-)^K \frac{W^K_{h_2h_1,h_3h_4}}{1 + (\epsilon_{p_1} + \epsilon_{p_2}) dW^K_{h_2h_1,h_3h_4}/W^K_{h_2h_1,h_3h_4}} \right]
\]  
(210)
\[
+ \frac{W^K_{h_1h_2,h_3h_4}}{1 + (\epsilon_{p_1} + \epsilon_{p_2}) dW^K_{h_1h_2,h_3h_4}/W^K_{h_1h_2,h_3h_4}} + (-)^K \frac{W^K_{h_2h_1,h_3h_4}}{1 + (\epsilon_{p_1} + \epsilon_{p_2}) dW^K_{h_2h_1,h_3h_4}/W^K_{h_2h_1,h_3h_4}} \right]
\]  
(211)
15. Term IV, 14a.

\[-Z_{p_3 p_4, h_2 h_1} V_{p_3 h_5 p_2 p_5} V_{p_4 p_5, h_5} / \epsilon\] (212)

gives

\[\Delta B^K (p_1 p_2, h_1 h_2) = \frac{Z^K_{p_3 p_4, h_1 h_2} W^K_{p_1 p_2, p_3 p_4}}{1 + (\epsilon_{h_1} + \epsilon_k) dW^K_{p_1 p_2, p_3 p_4} / W^K_{p_1 p_2, p_3 p_4}}\] (213)

This is similar to terms (3) and (19) except that

\[W^\ell_{p_1 p_2, p_3 p_4} = -\langle p_3 p_4 | V^\ell | p_5 h_5 \rangle / (\epsilon_{p_4} + \epsilon_{p_4} + \epsilon_{h_5})\] (214)

\[dW^\ell_{p_1 p_2, p_3 p_4} = -\langle p_3 p_4 | V^\ell | p_5 h_5 \rangle / (\epsilon_{p_4} + \epsilon_{p_4} + \epsilon_{h_5})^2\] (215)

16. Term IV, 14b.

\[-Z_{p_3 p_4, h_1 h_2} V_{p_3 h_5 p_2 p_5} V_{p_4 p_5, h_5} / \epsilon\] (216)

This term again arises from the exchange \((p_1 h_1)\) with \((p_2 h_2)\). It thus results in the transpose of term (14a). We combine the two terms using restricted sums as

\[\Delta B^K (p_1 p_2, h_1 h_2) = \frac{Z^K_{p_3 p_4, h_1 h_2} W^K_{p_1 p_2, p_3 p_4}}{1 + (\epsilon_{h_1} + \epsilon_{h_2}) dW^K_{p_1 p_2, p_3 p_4} / W^K_{p_1 p_2, p_3 p_4}}\] (217)

\[\times \left\{ \frac{W^K_{p_2 p_3, p_4 p_3} W^K_{p_1 p_2, p_3 p_4}}{1 + (\epsilon_{h_1} + \epsilon_{h_2}) dW^K_{p_1 p_2, p_3 p_4} / W^K_{p_1 p_2, p_3 p_4}} + (-^K) \frac{W^K_{p_3 p_4, p_3 p_4}}{1 + (\epsilon_{h_1} + \epsilon_{h_2}) dW^K_{p_1 p_2, p_3 p_4} / W^K_{p_1 p_2, p_3 p_4}} \right\}\] (218)

Note. Contributions from 3p3h have been included in full except for the last two terms (14a and 14b), which are listed for completeness.

References

[1] J. H. Heisenberg and B. Mihaila, 'Ground state correlations and mean field in \(^{16}\text{O}\)’, nucl-th/9802029 (1998).

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