Construction of Perturbatively Correct Light Front Hamiltonian for (2+1)-Dimensional Gauge Theory

M.Yu. Malyshev*, E.V. Prokhvatilov†, R.A. Zubov‡, V.A. Franke§
Saint Petersburg State University, St. Petersburg, Russia

Abstract

In this paper we consider (2+1)-dimensional SU(N)-symmetric gauge theory within light front perturbation theory, regularized by the method analogous to Pauli-Villars regularization. This enables us to construct correct renormalized light front Hamiltonian.

Key words: Pauli-Villars regularization, quantization on the light front, gauge field theory.

1 Introduction

The present paper is a continuation of the previously initiated investigation of the possibility to construct correct renormalized light front (LF) Hamiltonian [1–9]. Light front coordinates are defined as follows [10]:

\[ x^\pm = (x^0 \pm x^1)/\sqrt{2}, \quad x^\perp, \]

where \(x^0, x^1, x^\perp\) are Lorentz coordinates. The Hamiltonian is obtained by canonical quantization on the LF surface, i.e. \(x^+ = 0\), where the \(x^+\) plays the role of time. This quantization leads to the appearance of the singularity at zero value of the momentum \(p_-\). So one has to regularize this singularity. The regularization can be introduced as the cutoff \(|p_-| \geq \varepsilon\) in field modes. However the results of such regularized theory can turn out to be non-equivalent to results obtained with usual quantization on the surface \(x^0 = const\) in Lorentz coordinates. Indeed, such non-equivalence is found in perturbation theory framework when perturbation theory in coupling constant, generated by canonical

* M. Yu. Malyshev works also in Petersburg Nuclear Physics Institute, Gatchina (Saint Petersburg), e-mail: mimalysh@yandex.ru, m.malyshev@spbu.ru
† E-mail: e.prokhvatilov@spbu.ru
‡ E-mail: roman.zubov@hep.phys.spbu.ru
§ E-mail: v.franke@spbu.ru
Hamiltonian on the LF, and usual perturbation theory, corresponding to the quantization on the instant time surface in Lorentz coordinates, are compared \[1,11,12\]. Beside of that, the regularization \(|p_-| \geq \varepsilon\) does not allow correct description of non-perturbative vacuum effects \[13,14\]. Let us also mention the approach to the construction of effective LF Hamiltonian, proposed for quantum chromodynamics (QCD) in papers \[15\]-\[17\]. The above mentioned perturbative non-equivalence can be, in a principle, removed via introduction of special extra fields \[1,2\] playing the role analogous to the role of extra fields in Pauli-Villars (PV) regularization \[18\]. The introduction of these fields violates gauge invariance. So the corresponding ultraviolet (UV) renormalization, which can restore this symmetry in the regularization removing limit, was proposed. In this way the renormalized LF Hamiltonian, perturbatively equivalent to the Hamiltonian of the theory quantized on the instant time surface in Lorentz coordinates, can be constructed. However the coefficients of renormalization counterterms in this Hamiltonian remain unknown parameters because their values are actually defined by the sum of infinite number Feynman diagrams. Nevertheless, if one considers, instead of (3+1)-dimensional QCD, its (2+1)-dimensional analog, the number of such diagrams becomes finite (due to superrenormalizability) and one can find explicit expressions for these coefficients in terms of original parameters of the theory including the regularization parameters. Such a model for pure gauge fields was considered in paper \[9\]. However the analysis of perturbation theory diagrams was expounded rather briefly. Besides the expression for the renormalized LF Hamiltonian also was not obtained. In the present paper we give the details for the evidence of different statements in the paper \[9\] and construct the renormalized LF Hamiltonian.

2 Possible differences between perturbation theory on the LF and one on the instant time surface in Lorentz coordinates

We consider, like in \[9\], the Yang-Mills theory in (2+1)-dimensional space-time. Diagrams of covariant perturbation theory for this model can differ for the quantization on the surface of constant time in Lorentz coordinates and for the quantization on the LF, depending on the way of the regularization.

We compare such diagrams using the following basic statement proved in the paper \[1\]: denote the propagator momenta by \(Q_{\mu} = (Q_+, Q_-, Q_{\perp})\), where \(Q_\pm = (Q_0 \pm Q_1)/\sqrt{2}\) are the momenta in LF coordinates and \(Q_0, Q_1, Q_{\perp}\) are the momenta in Lorentz coordinates. Then the expression for a diagram in Lorentz coordinates can be written as the following integral over the momenta in LF coordinates (for brevity we drop the integration in \(Q_{\perp}\)) \[1\]:

\[
\prod_i \int dQ_{\pm} \int_B dQ_{\pm} f(Q_i, p_j), \tag{1}
\]

where \(f(Q_i, p_j)\) denotes the integrand which includes delta-functions expressing momentum conservation in vertices. The integration domain in \(Q_{\pm}\) is limited by the quantity \(B\) which has the order of maximal external momentum \(p_{\pm}\) (i.e. it is bounded by the domain of the size \(s|p_{\pm}^{\max}|\), where the constant \(s\) depends on the diagram structure). If we introduce in this integral also the cutoff \(|Q_{\pm}| \geq \varepsilon\) we get the result corresponding to the diagram,
calculated in LF coordinates where such cutoff is used conventionally. The limitation of the integration over \( Q_+ \) by the domain \( B \) can be related to analogous limitation in old fashioned LF perturbation theory where the momenta \( Q_+ \geq 0 \) in the intermediate states do not exceed the total momentum. Thus we obtain that the difference between results of calculation of a diagram in Lorentz and LF coordinates is given by the sum of contributions to the integral [11] from domains with \( |Q_+| \leq \varepsilon \). In the following the different terms of this sum we call configurations as in [11,17].

Let us consider (2+1)-dimensional Yang-Mills theory with the following Lagrangian density:

\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \frac{m}{2} \varepsilon^{\mu\nu\alpha} \left( A_\mu^a \partial_\nu A^a_\alpha + \frac{2}{3} g f^{abc} A^a_\mu A^b_\nu A^c_\alpha \right),
\]

where \( A^a_\mu(x) \) is the \( SU(N) \) gauge field, \( F_{\mu\nu}^a = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^a_\mu A^b_\nu, \) \( a = 1, \ldots, N^2 - 1 \) are adjoint representation indices, \( f^{abc} \) are structure constants, \( m \) is mass, \( g \) is coupling constant and \( \varepsilon^{\mu\nu\alpha} \) is the Levi-Civita symbol. We add Chern-Simons term to regularize infrared (IR) divergences [19,20].

Further we choose LF gauge \( A_\perp = 0 \). This leads to the disappearance of the term of power four in fields \( A_\mu \) in eq-n (2).

It was noticed in the previous paper [9] that only finite number of UV divergent diagrams are to be renormalized in this model (these diagrams are shown explicitly in [9], indices of UV divergency of them do not exceed one).

Let us consider the variant of UV regularization of this theory in which we introduce both extra fields and higher derivatives. We take into account that the chosen gauge \( A_\perp = 0 \) generates spurious singularity in the gluon propagator (namely, additional pole in \( p_- \)). To regularize this singularity we introduce one extra field more and new parameter \( \mu \) [9]. We start from the theory with only one extra field and show that in this theory we have the differences between calculations of diagrams of perturbation theory, generated by the quantization on the constant time surface, \( x^0 = 0 \), and generated by the quantization on the LF, \( x^+ = 0 \). The Lagrangian has the following form:

\[
\mathcal{L} = -\frac{1}{4} f^{a\mu\nu} \left( \frac{\Lambda^2 + 2 \partial_+ \partial_-}{\Lambda^2} \right) f^a_{1,\mu\nu} + \frac{m}{2} \varepsilon^{\mu\nu\alpha} A^a_{1,\mu} \left( \frac{\Lambda^2 + 2 \partial_+ \partial_-}{\Lambda^2} \right) \partial_\nu A^a_{1,\alpha} +
\]

\[
-\frac{1}{4} f^{a\mu\nu} \left( \frac{\mu^2 + 2 \partial_+ \partial_-}{\mu^2} \right) f^a_{2,\mu\nu} + \frac{m}{2} \varepsilon^{\mu\nu\alpha} A^a_{2,\mu} \left( \frac{\mu^2 + 2 \partial_+ \partial_-}{\mu^2} \right) \partial_\nu A^a_{2,\alpha} +
\]

\[
+ g f^{abc} A^a_\mu A^b_\nu \partial^\mu A^c_\nu,
\]

where \( f^{a\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu, j = 1, 2, \) \( A^a_{1,\mu} \) is the original field, \( A^a_{2,\mu} \) is the extra field. The interaction term contains the sum of these fields, \( A^a_\mu = A^a_{1,\mu} + A^a_{2,\mu} \). Beside of that, we require \( A^a_{1,\perp} = 0 \).

The propagator of the field \( A_{j,\mu} \) has the following form:

\[
\Delta_{1,\mu\nu}^{ab} = \frac{i \delta^{ab}}{Q^2 - m^2 + i0} \left( g_{\mu\nu} - \frac{Q_+ n_\nu + n_\mu Q_+ + i m \varepsilon_{\mu\nu\alpha} n^\alpha}{2Q_+ Q_- + i0} \right) \frac{1}{2Q_+ Q_- + i0} \frac{1}{\Lambda^2} - 1 + i0,
\]

\[
\Delta_{2,\mu\nu}^{ab} = \frac{-i \delta^{ab}}{Q^2 - m^2 + i0} \left( g_{\mu\nu} - \frac{Q_+ n_\nu + n_\mu Q_+ + i m \varepsilon_{\mu\nu\alpha} n^\alpha}{2Q_+ Q_- + i0} \right) \frac{1}{2Q_+ Q_- + i0} \frac{1}{\mu^2} - 1 + i0,
\]
where the vector $n_\mu$ has components $n_+ = 1$, $n_- = n_\perp = 0$, $n_\mu n^\mu = 0$.

The analysis of the UV divergences in the perturbation theory with this Lagrangian shows that the introduced regularization is sufficient for the removing of these divergences \[9\]. Since the interaction terms depend only on the sum of fields, $A_\mu = A_{1,\mu} + A_{2,\mu}$, the perturbation theory can be formulated with the summary propagator:

$$\Delta_{\mu\nu} = \Delta_{1,\mu\nu} + \Delta_{2,\mu\nu} = \frac{-i\delta_{\mu\nu}}{Q^2 - m^2 + i0} \left( g_{\mu\nu} - \frac{Q_\mu n_\nu + n_\mu Q_\nu + i m \varepsilon_{\mu\nu\alpha} n^\alpha}{2Q_+ Q_- + i0} \right) R,$$

$$R = \frac{2Q_+ Q_-}{\Lambda^2} \frac{1}{1 + i0} + \frac{2Q_+ Q_-}{\mu^2} \frac{1}{1 + i0} = \frac{2Q_+ Q_- (\mu^2 - \Lambda^2)}{(2Q_+ Q_- - \mu^2 + i0) (2Q_+ Q_- - \Lambda^2 + i0)}.$$  

Note that the regularization is removed in the limit $\mu \to 0, \Lambda \to \infty$ ($R \to 1$).

Let us write the expressions for the propagator at different Lorentz indices (for the analysis below we do not use the SU(N)-indices because they are not essential there):

$$\Delta_{++}(Q) = 4Q_+^2 \Delta(Q), \quad \Delta_{+\perp}(Q) = 2Q_+ (Q_\perp + im) \Delta(Q), \quad \Delta_{\perp\perp}(Q) = 2Q_+ Q_- \Delta(Q),$$

where $\Delta(Q) = -i(\Lambda^2 - \mu^2) \prod_{l=0}^2 \left( 2Q_+ Q_- - m_l^2 + i0 \right)^{-1}$,  

$m_0^2 = m^2 + Q^2_\perp$,  

$m_1 = \Lambda$,  

$m_2 = \mu$.

Let us remind that possible differences between calculations of Feynman diagrams on the LF and in Lorentz coordinates are related to the contribution to the corresponding Feynman diagrams from the integration domains $|Q_i^-| \leq \varepsilon$, which are absent in diagrams of the theory on the LF with the regularization $|Q_i^-| \geq \varepsilon$. Let us show that the configurations contributing to such difference have the form shown in Fig. 1 where the crossed lines denote the propagators which have momenta limited by the condition $|Q_i^-| \leq \varepsilon$.

\[ \text{Figure 1: the most common kind of configuration (a). The simplest configuration example (b).} \]

Here we have extracted the ”\(\varepsilon\)-blocks” which include both the set of joined crossed lines and also the not crossed lines if the momenta $Q_-$ of the latter are limited by the
quantity of order $\varepsilon$ (further we call these lines $\Pi$-lines; for example, one gets these lines if they are joined only with crossed lines and then their momenta are limited by the domain $B$ of order $\varepsilon$ by the statement in the beginning of the present section). Besides we have extracted the $\Pi$-block which contains all remaining $\Pi$-lines. All external lines are joined to this $\Pi$-block. Let us begin with a simplest example of such a configuration (Fig. 1b). The corresponding Feynman integral has the following form:

$$I(p) = -4g^2N (\Lambda^2 - \mu^2)^2 \int dk_+ \int_{-\varepsilon}^{\varepsilon} dk_- \frac{(2p_- - k_-)^2 (p_+ - k_+)(p_- - k_-)}{(\prod_{i=0}^{l} (2(p_+ - k_+)(p_- - k_-) - m_i^2 + i0))} \times \frac{k_+^2}{\prod_{l=0}^{2} (2k_+k_- - m_l^2 + i0)}. \quad (7)$$

To estimate this integral in the $\varepsilon \to 0$ limit we remove the dependence on $\varepsilon$ of integration limits in $k_-$ by the change $k_- \to k_-\varepsilon$ and additionally $k_+ \to k_+\varepsilon$. Then the integral takes the form:

$$-8g^2N (\Lambda^2 - \mu^2)^2 \int dk_+ \int_{-1}^{1} dk_- \frac{(2p_- - \varepsilon k_-)^2 (p_+ - k_+\varepsilon)(p_- - \varepsilon k_-)}{\prod_{l=0}^{2} (2(p_+ - k_+\varepsilon)(p_- - \varepsilon k_-) - m_l^2 + i0)} \times \frac{k_+^2}{\prod_{l=0}^{2} (2k_+k_-\varepsilon - m_l^2 + i0)} =$$

$$= -8g^2N (\Lambda^2 - \mu^2)^2 \int dk_+ \int_{-1}^{1} dk_- \frac{(2\varepsilon k_-)^2 (\varepsilon p_+ - k_+)(\varepsilon p_- - \varepsilon k_-)}{\prod_{l=0}^{2} (2(\varepsilon p_+ - k_+)(\varepsilon p_- - \varepsilon k_-) - \varepsilon m_l^2 + i0)} \times \frac{k_+^2}{\prod_{l=0}^{2} (2k_+k_- - m_l^2 + i0)}. \quad (8)$$

Since the integral over $k_+$ is convergent we get the following finite expression in the $\varepsilon \to 0$ limit:

$$-4g^2N (\Lambda^2 - \mu^2)^2 \int dk_+ \int_{-1}^{1} dk_- \frac{1}{\prod_{l=0}^{2} (2k_+k_- - m_l^2 + i0)}, \quad (9)$$

i.e. we have the difference between the calculation of such diagram on the LF and its conventional calculation in Lorentz coordinates. To estimate the dependence on $\varepsilon$ in general form we consider at first how the propagators change under the above mentioned change of integration variables. Consider the case when the propagator momentum $Q_-$ is limited, $|Q_-| \leq \varepsilon$. Let us write the $k$ instead of $Q$ (as in the Fig. 1b) so that $|k_-| \leq \varepsilon$. Let us change $k_+ \to k_+' = \varepsilon k_+$, $k_- \to k_- = k_- / \varepsilon$. Then, in accordance with the eq-n (10), we get the following for the components of the propagator:

$$\Delta(k) \to \Delta'(k') = \Delta(k') \sim O(1),$$
$$\Delta_{++}(k) \to \Delta'_{++}(k') = \Delta_{++}(k') / \varepsilon^2,$$
$$\Delta_{+-}(k) \to \Delta'_{+-}(k') = \Delta_{+-}(k') / \varepsilon,$$
$$\Delta_{-+}(k) \to \Delta'_{-+}(k') = \Delta_{-+}(k') \sim O(1). \quad (10)$$

Consider the case when the propagator momentum is $Q_\mu = p_\mu - k_\mu$ where $p_-$ is finite and $|k_-| \leq \varepsilon$. Then we have the following:

$$\Delta(p-k) \to \Delta'(p', k') = \frac{-i \varepsilon (m_1^2 - m_2^2)}{(-p_- + \varepsilon k_-)^3 \prod_{l=0}^{2} (2k_+' - 2\varepsilon p_+ + \varepsilon m_l^2 / (p_- - \varepsilon k_-) + i0 / p_-)} \sim O(\varepsilon^3),$$
\[
\Delta_{++}(p-k) \to \frac{4(\varepsilon p_{++} - k_{+}^\prime)^2\Delta'(p, k')}{\varepsilon^2} \sim O(\varepsilon),
\]
\[
\Delta_{+\perp}(p-k) \to \frac{2(\varepsilon p_{+\perp} - k_{\perp}^\prime)(p_{\perp} - k_{\perp} + im)\Delta'(p, k')}{\varepsilon} \sim O(\varepsilon^2),
\]
\[
\Delta_{\perp\perp}(p-k) \to \frac{2(\varepsilon p_{-\perp} - k_{-\perp}^\prime)(p_{-} - \varepsilon k_{-\perp}^\prime)\Delta'(p, k')}{\varepsilon} \sim O(\varepsilon^2).
\] (11)

All vertex factors can be estimated by finite quantity except of those cases when three \(\varepsilon\)-lines converge in the vertex (then they can be estimated by the quantity of order \(\varepsilon\)). Using these estimations one can easily prove that the diagram in the Fig. 1b tends to zero in \(\varepsilon \to 0\) limit at any other choice of indices. An addition of more \(\varepsilon\)-lines to the \(\varepsilon\)-block, i. e. the formation of the \(\varepsilon\)-block, does not change the estimation of the diagram in Fig. 1b. This can be shown in the following way: the \(\varepsilon\)-block consists from vertices and propagators with every vertex being proportional to \(\varepsilon\) and having one index + which leads to the appearance of the factor \(k_{+}^\prime/\varepsilon\) in the propagator adjoint to this vertex. So the dependence on \(\varepsilon\) for this vertex is cancelled. This is true for propagators having indices ++ and giving the contribution proportional to \(1/\varepsilon^2\) and for propagators having indices +\(\perp\) and giving the contribution proportional to \(1/\varepsilon\). The propagator with the indices \(\perp\perp\) gives the contribution of order \(O(1)\). The same situation takes place for the other vertices of the \(\varepsilon\)-block.

The next simple modification of the configuration in Fig. 1b has the form shown in Fig. 2a which differs from Fig. 1b by the addition of one external line. Such diagram has two \(\Pi\)-lines through which the momentum \(k\) of \(\varepsilon\)-lines goes. So, with the estimations of eq-ns (10) and (11) we obtain that the configuration tends to zero as \(\varepsilon \to 0\). The analogous situation has place when we increase the number of \(\Pi\)-lines, through which the momentum \(k\) of \(\varepsilon\)-lines goes, e. g. when we increase the number of external lines.

Let us consider another case of configuration changing for Fig. 1b when we add one \(\varepsilon\)-line connecting \(\Pi\)- and \(\varepsilon\)-lines of the original configuration (Fig. 2b). In this case the \(\Pi\)- and \(\varepsilon\)-lines of the original configuration are divided into two parts where the momenta \(k_1\) and \(k_2\) of propagators of the divided \(\varepsilon\)-line can be chosen as loop integration variables so that they go through one of the propagator of the divided \(\Pi\)-line (see Fig. 2b). Then the estimations, corresponding to eq-ns (10) and (11), and taking into account the contribution of the new vertex adjoint to the original \(\varepsilon\)-line lead in result that the configuration goes to zero as \(\varepsilon \to 0\).
Thus the most general form of the non equal to zero configuration can be pictured in analogy with the diagram (Fig. I) in which the crossed line is changed to arbitrary \( \varepsilon \)-block.

So we come to the conclusion that in the theory under consideration we have nonzero configurations, and all of them are finite in the limit \( \varepsilon \to 0 \).

Let us show now that the method of the paper [9] where one more extra field is to be added allows to get going to zero all nonzero configurations considered above. The Lagrangian in this case has the following form:

\[
\mathcal{L} = \sum_{l=1}^{3} \left( -\frac{r_l}{4} f_{\mu}^{a\nu} \left( \frac{m_l^2 + 2 \partial_+ \partial_-}{M_l^2} \right) f_{l,\mu\nu} + r_l \frac{m_l}{2} \varepsilon^{\mu\nu\alpha} A_{l,\mu} \left( \frac{m_l^2 + 2 \partial_+ \partial_-}{M_l^2} \right) \partial_\nu A^{a}_{l,\alpha} \right) +
\]

\[+ g f^{abc} A_{\mu}^{a} A_{\nu}^{b} \partial^{\mu} A^{c\nu}, \quad (12)\]

where \( A_{l,\mu}^{a} \) is the original field, \( A_{2,\mu}^{a} \) and \( A_{3,\mu}^{a} \) are the extra fields, \( A_{l,-}^{a} = 0, A_{\mu}^{a} = A_{l,\mu}^{a} + A_{2,\mu}^{a} + A_{3,\mu}^{a} \), the parameters \( m, m_1 = \Lambda, m_2 = \mu \) are analogous to the parameters in the Lagrangian \( (3) \), \( r_l = \pm 1 \). The parameter \( m_3 \) is new parameter related to the introduction of the additional field \( A_{3,\mu}^{a} \). The quantities \( M_l^2 \) are to be expressed in terms of \( m_l \) and \( m \) so that the index of UV divergency of the summary propagator were by two units less. This summary propagator is the sum of each field propagators:

\[
\Delta_{\mu\nu}^{ab} = \frac{-i\delta^{ab}}{(k^2 - m^2 + i0)} \left( g_{\mu\nu} - \frac{k_{\mu} n_{\nu} + n_{\mu} k_{\nu} + i m \varepsilon^{\mu\nu\alpha} n^\alpha}{2 k_+ k_- + i0} \right) \left( \sum_{l=1}^{3} \frac{r_l M_l^2}{m_l^2 - 2 k_+ k_- - i0} \right). \quad (13)\]

If we reduce the sum in eq. (13) to common denominator and require the disappearance of the terms of power four and zero in \( k \) in the numerator of this sum we obtain the following conditions:

\[
\sum_{l=1}^{3} r_l M_l^2 = 0, \quad \sum_{l=1}^{3} \frac{r_l M_l^2}{m_l^2} = 0. \quad (14)\]

Further we require that the obtained expression for the summary propagator must go to the expression for the propagator of unregulated theory in the limit \( m_2^2 = \mu^2 \to 0, m_1^2 = \Lambda^2 \to \infty, m_3^2 \to \infty \), i.e.

\[
2k_+ k_- \frac{r_1 M_1^2 (m_1^2 + m_3^2) + r_2 M_2^2 (m_2^2 + m_3^2) + r_3 M_3^2 (m_1^2 + m_2^2)}{\prod_{l=1}^{3} (2k_+ k_- - m_l^2 + i0)} \to
\]

\[
\frac{r_1 M_1^2 m_3^2 + r_2 M_2^2 (m_2^2 + m_3^2) + r_3 M_3^2 m_1^2}{m_1^2 m_2^2 m_3^2} =
\]

\[= r_1 \frac{M_1^2}{m_1^2} + r_2 \left( \frac{1}{m_1^2} + \frac{1}{m_3^2} \right) M_2^2 + r_3 \frac{M_3^2}{m_3^2} \to 1. \quad (15)\]

If we take into account the second condition in eq. (14), we can satisfy the condition of eq. (15) taking, for example, \( \frac{r_2 M_2^2}{m_2^2} = -1 \). In accordance with the eq. (14) we obtain:

\[
r_1 M_1^2 = \frac{m_1^2 (m_1^2 - m_2^2)}{m_3^2 - m_1^2},
\]
\[ r_3 M_3^2 = -\frac{m_3^2(m_1^2 - m_2^2)}{m_3^2 - m_1^2}. \] (16)

Besides of that we require that only the field \( A_1 \) be remained in the regularization removing limit \( m_2^2 \to 0, m_1^2 \to \infty, m_3^2 \to \infty \). With this aim we require additionally \( \frac{m_2^2}{m_1^2} \to \infty \). We see from these facts that \( r_1 = 1, r_2 = -1, r_3 = -1 \).

To estimate the dependence of different configurations on \( \varepsilon \) let us rewrite the summary propagator in the following form:

\[ \Delta_{\mu\nu}^a(Q) = \frac{i\delta^{ab}(2Q_+ Q_{\mu\nu} - 2Q_+(Q_{\mu\nu} + Q_{\nu\mu} + i m \varepsilon_{\mu\nu\alpha} n^\alpha))}{\prod_{l=0}^3 (2Q_+ Q_- - m_l^2 + i0)} R, \] (17)

where

\[ m_0^2 = m^2 + Q_0^2, \quad R = r_1 M_1^2(m_2^2 + m_3^2) + r_2 M_2^2(m_1^2 + m_3^2) + r_3 M_3^2(m_1^2 + m_2^2). \]

The corresponding to the eq. (10) estimations for the propagator (17) keep its form, and the estimations, corresponding to the eq. (11), take the following form:

\[ \Delta'(p, k') \sim O(\varepsilon^4), \]
\[ \Delta'^{++}(p, k') \sim O(\varepsilon^2), \]
\[ \Delta'^{++}(p, k') \sim O(\varepsilon^3), \]
\[ \Delta'^{\perp\perp}(p, k') \sim O(\varepsilon^3), \] (18)

i.e. the power of \( \varepsilon \) increases comparatively to eq-ns (11). This leads to disappearance of all before considered configurations in the limit \( \varepsilon \to 0 \). Thus in the theory with the Lagrangian (12) we get the coincidence of diagram values in Lorentz coordinate instant time quantization and LF quantization.

Using the results of the paper [2] we have shown in the paper [9] that it is possible to avoid related to parameter \( \mu \) IR divergences in the regularization removing limit if one satisfies the condition \((\ln \mu)^n/\Lambda \to 0\) in the limit \( \mu \to 0, \Lambda \to \infty \) where \( n \) is arbitrary integer number (e.g. one can take \( \mu = m^2/\Lambda \)).

Owing to superrenormalizability of considered theory in \((2+1)\)-dimensional space we need to analyze and calculate only finite number of UV divergent diagrams shown in Fig. 3.

It was proved in the paper [9] that the 1-loop diagram with two external lines (Fig. 3a),

![Figure 3a](image)

... and the 1-loop diagram with three external lines (Fig. 3b),

![Figure 3b](image)

... having the index of UV divergency equal to unity, and the 1-loop diagram with three

![Figure 3c](image)

... and the 1-loop diagram with three external lines (Fig. 3d),

![Figure 3d](image)
external lines and with the index of UV divergency equal to zero (Fig. 3î) turn out to be finite after exact analytic calculation. The remaining 2-loop diagrams with two external lines (Fig. 3b, 3c), having the index of UV divergency equal to zero, can be renormalized by the comparison with corresponding renormalized dimensionally regularized diagrams. The renormalization counterterms are found at that by the calculation of the difference of the expressions for these diagrams in the considered and dimensional regularizations. Such a procedure of the renormalization allows to restore gauge and Lorentz symmetry in the theory with the Lagrangian (12) in the regularization removing limit. In result we have to add to the Lagrangian (12) the counterterm of the form $C \ln (\Lambda/m) A^a_{\perp} A^a_{\perp}$ where the constant $C$ can be found as a result of numerical calculations of diagrams (Fig. 3, 3î). This form of the counterterm is in agreement with the remaining Lorentz invariance of the Lagrangian (12) under the following coordinate transformation: $x^+ \rightarrow \lambda x^+, \quad x^- \rightarrow \lambda^{-1} x^-$, $x^\perp \rightarrow x^\perp$. Due to this invariance the logarithmically divergent diagrams (Fig. 3b, 3c) calculated at zero external momentum (and at external Lorentz indices excluding the ”−”) are equal to zero if they have at least one external Lorentz index ”+”. Thus the only diagrams that contribute to the counterterm are those which have only external indices ”⊥”. We give in the Appendix the expression for the diagram shown in Fig. 3î.

3 The construction of the renormalized LF Hamiltonian

Consider the theory with the Lagrangian (12) and rewrite the expression for this Lagrangian in the gauge $A^a_{\perp} = 0$.

$$\mathcal{L} = \sum_{i=1}^{3} r_i \left( -\frac{\partial_+ f^a_{i,+} \partial_- f^a_{i,+}}{M_i^2} + \frac{m_i^2}{2M_i^2} (f^a_{i,+})^2 + \partial_+ A^a_{i,\perp} \frac{2\partial^2}{M_i^2} \partial_+ A^a_{i,\perp} + \frac{m_i^2}{M_i^2} \partial_+ A^a_{i,\perp} \partial_- A^a_{i,\perp} + \right.$$ 

$$+ m \left( \frac{m_i^2}{M_i^2} f^a_{i,+} A^a_{i,\perp} + \frac{2m_i^2}{M_i^2} f^a_{i,+} \partial_+ \partial_- A^a_{i,\perp} \right) - \partial_- f^a_{i,+} \left( \frac{m_i^2 + 2\partial_+ \partial_-}{M_i^2} \right) A^a_{i,\perp} \right) - g f^{abc} A^c_{\perp} A^b_{\perp} \partial_- A^a_{\perp}. \quad (19)$$

To write the action corresponding to this Lagrangian in Hamiltonian form we find at first the momentum $\Pi^a_{i,\perp}$ conjugated to the field $A^a_{i,\perp}$:

$$\Pi^a_{i,\perp} = \frac{\partial \mathcal{L}}{\partial (\partial_+ A^a_{i,\perp})} = r_i \left( 4 \frac{\partial^2}{M_i^2} \partial_+ A^a_{i,\perp} + \frac{m_i^2}{M_i^2} \partial_- A^a_{i,\perp} + \frac{2}{M_i^2} (\partial_+ - m) \partial_- f^a_{i,+} \right). \quad (20)$$

This relation allows to express the quantity $\partial_+ A^a_{i,\perp}$ in terms of variables $\Pi^a_{i,\perp}$, $A^a_{i,\perp}$ and $f^a_{i,+}$:

$$\partial_+ A^a_{i,\perp} = \frac{M_i^2}{4} \partial_-^2 \left( r_i \Pi^a_{i,\perp} - \frac{m_i^2}{M_i^2} \partial_- A^a_{i,\perp} - \frac{2}{M_i^2} (\partial_+ - m) \partial_- f^a_{i,+} \right). \quad (21)$$

In result the action can be rewritten in the following form:

$$S = \int dx^+ dx^- dx^\perp \left[ \sum_{i=1}^{3} \left( -\frac{r_i}{M_i^2} \partial_+ f^a_{i,+} \partial_- f^a_{i,+} + \Pi^a_{i,\perp} \partial_+ A^a_{i,\perp} \right) - \mathcal{H} \right], \quad (22)$$
where
\[
\mathcal{H}(\Pi^a_{I,\perp}, A^a_{I,\perp}, f^a_{I,\perp}) = \sum_{l=1}^{3} \left[ -\frac{r_{l} m_{l}^{2}}{2 M_{l}^{2}} f^{a}_{I,\perp} f^a_{I,\perp} + \frac{r_{l} m_{l}^{2}}{M_{l}^{2}} A^{a}_{I,\perp} (\partial_{\perp} - m) f^a_{I,\perp} + \frac{r_{l} M_{l}^{2}}{8} \left( r_{l} \Pi^{a}_{I,\perp} - \frac{m_{l}^{2}}{M_{l}^{2}} \partial_{\perp} A^{a}_{I,\perp} - \frac{2}{M_{l}^{2}} (\partial_{\perp} - m) \partial_{-} f^a_{I,\perp} \right) \times \partial^{-2}_{-} \left( r_{l} \Pi^{a}_{I,\perp} - \frac{m_{l}^{2}}{M_{l}^{2}} \partial_{\perp} A^{a}_{I,\perp} - \frac{2}{M_{l}^{2}} (\partial_{\perp} - m) \partial_{-} f^a_{I,\perp} \right) \right] + g f^{abc} A^{a}_{+} A^{b}_{1} \partial_{-} A^{c}_{\perp} - C \ln(\Lambda/m) A^{a}_{\perp} A^{a}_{\perp}. \tag{23}
\]

The expression \((22)\) shows that the canonical momentum conjugated to the field \(f^a_{I,\perp}\) generates the canonical constraint on the LF which turns out to be the constraint of the second kind. To avoid this constraint let us go to new variables with the help of Fourier transformation in \(x^{-}\) (introducing the regularization \(|p_{-}| \geq \epsilon\)):
\[
f^{a}_{I,\perp}(x) = \frac{M_{l}}{\sqrt{2\pi}} \int_{\frac{|p_{-}|}{\epsilon}}^{\infty} \frac{dp_{-}}{\sqrt{2|p_{-}|}} \left( a^{a}_{l}(p_{-}; x^{+}, x^{+}) \exp(-ip_{-} x^{-}) \right) = \frac{M_{l}}{\sqrt{2\pi}} \int_{\frac{|p_{-}|}{\epsilon}}^{\infty} \frac{dp_{-}}{\sqrt{2|p_{-}|}} \left( a^{a}_{l}(p_{-}; x^{+}, x^{+}) \exp(-ip_{-} x^{-}) + a^{a+}_{l}(p_{-}; x^{+}, x^{+}) \exp(ip_{-} x^{-}) \right). \tag{24}
\]

Then the term in the action,
\[
\sum_{l=1}^{3} \int dx^{+} dx^{-} dx^{\perp} \left( -\frac{r_{l} M_{l}^{2}}{M_{l}^{2}} \partial_{-} f^{a}_{I,\perp} \partial_{-} f^{a}_{I,\perp} \right) \tag{25}
\]
can be transformed as follows (after integration by parts in \(x^{+}\)):
\[
\sum_{l=1}^{3} \int dx^{+} dx^{\perp} \int_{\frac{|p_{-}|}{\epsilon}}^{\infty} dp_{-} \left( -ir_{l} a^{a+}_{l}(p_{-}; x^{+}, x^{+}) \partial_{+} a^{a}_{l}(p_{-}; x^{+}, x^{+}) \right). \tag{26}
\]

The obtained Hamiltonian form of the action allows to define quantum commutation relations for canonical variables:
\[
[a^{a}_{l}(p_{-}; x^{+}, x^{+}), a^{a+}_{l}(p'_{-}; x'^{+}, x'^{+})] = -r_{l} \delta_{l} \delta^{ab} \delta(p_{-} - p'_{-}) \delta(x^{+} - x'^{+}), \tag{27}
\]
\[
[A^{a}_{I,\perp}(x), \Pi^{b}_{I,\perp}(x')] = i\delta_{l} \delta^{ab} \delta(x^{+} - x'^{+}) \delta(x^{\perp} - x'^{\perp}). \tag{28}
\]

The remaining commutation relations for these fields are equal to zero. The variables \(a^{a}_{l}(p_{-}; x^{+}, x^{+})\) and \(a^{a+}_{l}(p_{-}; x^{+}, x^{+})\) can be considered as that analogous to creation and annihilation operators on the LF. Let us introduce also the Fourier transformation for the variables \(A^{a}_{I,\perp}(x)\) and \(\Pi^{b}_{I,\perp}(x)\):
\[
A^{a}_{I,\perp}(x) = M_{l} \int_{\frac{|p_{-}|}{\epsilon}}^{\infty} \frac{dp_{-}}{\sqrt{2|p_{-}|}} \left( a^{a}_{l}(p_{-}; x^{+}, x^{+}) + a^{a+}_{l}(p_{-}; x^{+}, x^{+}) \right) \exp(-ip_{-} x^{-}) = \frac{M_{l}}{m_{l}} \int_{\frac{|p_{-}|}{\epsilon}}^{\infty} \frac{dp_{-}}{\sqrt{2|p_{-}|}} \left( a^{a}_{l}(p_{-}; x^{+}, x^{+}) + a^{a+}_{l}(p_{-}; x^{+}, x^{+}) \right) \exp(-ip_{-} x^{-}). \tag{29}
\]
\begin{align*}
= \frac{M_l}{m_l} \int_{\varepsilon}^{\infty} \frac{dp_-}{\sqrt{4\pi p_-}} \left[ \left( a_{t,\perp}^a(p_-; x^+, x^+) + b_{t,\perp}^b(p_-; x^+, x^+) \right) \exp(-ip_- x^-) + \left( a_{t,\perp}^{a+}(p_-; x^+, x^+) + b_{t,\perp}^{b+}(p_-; x^+, x^+) \right) \exp(ip_- x^-) \right],
\end{align*}

\begin{align*}
\Pi_{t,\perp}^b(x) &= \frac{-im_l}{M_l \sqrt{2\pi}} \int_{(p_- > \varepsilon)} dp_- \sqrt{\frac{p_-}{2}} \left[ \left( a_{t,\perp}^b(p_-; x^+, x^+) - a_{t,\perp}^{b+}(p_-; x^+, x^+) \right) \exp(-ip_- x^-) \right] = \frac{-im_l}{M_l} \int_{\varepsilon}^{\infty} dp_- \sqrt{\frac{p_-}{4\pi}} \left[ \left( a_{t,\perp}^b(p_-; x^+, x^+) - b_{t,\perp}^{b+}(p_-; x^+, x^+) \right) \exp(-ip_- x^-) - \text{h.c.} \right],
\end{align*}

where we introduce the denotation

\[ b_{t,\perp}^a(p_-; x^+, x^+) = a_{t,\perp}^{a+}(-p_-; x^+, x^+). \]

The operators \( a_{t,\perp}^a(p_-; x^+, x^+), a_{t,\perp}^{a+}(p_-; x^+, x^+), b_{t,\perp}^b(p_-; x^+, x^+), b_{t,\perp}^{b+}(p_-; x^+, x^+) \) satisfy the following canonical commutation relations analogous to commutation relations for the creation and annihilation operators:

\begin{align*}
\left[ a_{t,\perp}^a(p_-; x^+, x^+), a_{t,\perp}^{b+}(p_-; x^+, x^+) \right] &= \delta_{ab} \delta(p_- - p_-') \delta(x^+ - x'^+),
\end{align*}

\begin{align*}
\left[ b_{t,\perp}^a(p_-; x^+, x^+), b_{t,\perp}^{b+}(p_-; x^+, x^+) \right] &= -\delta_{ab} \delta(p_- - p_-') \delta(x^+ - x'^+),
\end{align*}

and the other commutators are equal to zero.

The free (quadratic in fields) part of the Hamiltonian takes the following form in terms of these operators:

\begin{align*}
H &= \int dx^+ \int_{\varepsilon}^{\infty} dp_- \sum_{l=1}^{3} \frac{r_l}{2p_-} \left[ -m_l^2 a_{t,\perp}^{a+}(p_-; x^+, x^+) a_{t,\perp}^a(p_-; x^+, x^+) - \left( \partial_+ - m \right) a_{t,\perp}^{a+}(p_-; x^+, x^+) \left( \partial_+ - m \right) a_{t,\perp}^a(p_-; x^+, x^+) + m_l \left( \left( \partial_+ - m \right) a_{t,\perp}^{a+}(p_-; x^+, x^+) \left( a_{t,\perp}^a(p_-; x^+, x^+) + b_{t,\perp}^{b+}(p_-; x^+, x^+) \right) + \text{h.c.} \right) - m_l^2 \left( \frac{1 - r_l}{2} a_{t,\perp}^{a+}(p_-; x^+, x^+) + \frac{1 + r_l}{2} b_{t,\perp}^{b+}(p_-; x^+, x^+) \right) \times \left( \frac{1 - r_l}{2} a_{t,\perp}^a(p_-; x^+, x^+) + \frac{1 + r_l}{2} b_{t,\perp}^b(p_-; x^+, x^+) \right) + m_l \left( \left( \partial_+ - m \right) a_{t,\perp}^{a+}(p_-; x^+, x^+) \left( \frac{1 - r_l}{2} a_{t,\perp}^a(p_-; x^+, x^+) + \frac{1 + r_l}{2} b_{t,\perp}^b(p_-; x^+, x^+) \right) + \text{h.c.} \right) \right].
\end{align*}

The introduction of new variables analogous to the creation and annihilation operators gives rather simple dependence of the Hamiltonian on the \( m_l \). However the quadratic in fields part of the Hamiltonian has no diagonal form yet. It can be written, in a principal, in the completely diagonal form but the obtained at that expression has rather cumbersome form.
4 Conclusion

In the present paper we consider the construction of the renormalized LF Hamiltonian for SU(N)-gauge invariant Yang-Mills theory in (2+1)-dimensional space-time. In this case the perturbation theory in coupling constant is superrenormalizable and only finite number of UV divergent diagrams should be renormalized. This gives a possibility to find the counterterms renormalizing the theory explicitly in terms of original parameters. On the other side the restoration of the perturbative (to all orders in coupling constant) equivalence between the obtained LF theory and usual one, quantized on the instant time surface in Lorentz coordinates, has required the introduction of extra fields analogous to those used in the Pauli-Villars regularization. These fields generate the Fock space on the LF with indefinite metric. Nonperturbative calculations with this Hamiltonian are very complicated [8,21–24]. In the paper [25] we have considered the simplest example: the calculation of the spectrum of the anharmonic oscillator modified by the addition of extra variables analogous to extra fields of Pauli-Villars regularization. In this case one can well see how the part of the spectrum related to these extra variables become separated from the usual spectrum of the model and tends to infinite values in the limit corresponding to the removing of Pauli-Villars regularization.

As an alternative approach to the construction of the renormalized LF Hamiltonian we can mention the approach described in papers [16,17] and in paper [15] (which contains all references to the previous papers). This approach applies the method of approximate diagonalization of the Hamiltonian with the help of unitary transformation depending on the scale parameter. Such transformation is to be built using the perturbative theory in coupling constant. In this way the approximate expression for the renormalized LF Hamiltonian can be written in terms of effective ”composite particle” creation and annihilation operators. However that procedure is rather complicated and limited by only lowest perturbation theory orders.

Appendix. Calculation of diagrams contributing to the counterterm in the renormalized LF Hamiltonian.

Accordingly to eq. (17) the propagator is zero if at least one of its Lorentz indices is ”−”, i.e. $\Delta^{ab}_{\mu-} = 0$. Considering the other Lorentz indices only one can write the propagator in the form of the following matrix:

$$
\Delta^{ab}(k) = \begin{pmatrix}
\frac{\Delta_{\perp \perp}^{ab}(k)}{k^2 - m^2 + i\epsilon} & \frac{i\delta^{ab}}{k^2 - m^2 + i\epsilon}
\end{pmatrix} \begin{pmatrix}
2k_+ & k_+ - im
\end{pmatrix}.
$$

(33)

The following factor corresponds to the vertex:

$$
V^{\mu\nu\rho,abc}(p, k, q) = -gf_{abc} ((p - k)^{\rho}g^{\mu\nu} + (k - q)^{\mu}g^{\nu\rho} + (q - p)^{\nu}g^{\mu\rho}),
$$

(34)

where the metric tensor $g^{\mu\nu}$ has the following nonzero components: $g^{+-} = g^{-+} = 1$, $g^{\perp \perp} = -1$. Since propagators are connected to vertices in diagrams for Green functions it is not necessary to consider vertices with Lorentz indices ”−” (let us remark that this
would be necessary if we use Ward identities explicitly). We can form the following matrix from the vertex one of Lorentz indices of which is fixed:

\[
V_{\mu,abc}^\mu(p, k, q) = \begin{pmatrix}
V_{\mu,+,abc}(p, k, q) & V_{\mu,+,abc}(p, k, q) \\
V_{\mu,+,abc}(p, k, q) & V_{\mu,+,abc}(p, k, q)
\end{pmatrix} = -g f_{abc} \begin{pmatrix}
0 & (q - p)_- g_{\mu\perp} \\
(q - p)_- g_{\mu\perp} & (k - q)_+ g_{\mu\perp} + (q - k)_\perp
\end{pmatrix}.
\] (35)

We obtain the following expression for the diagram shown in Fig. 3a:

\[
D_{\mu_1 \mu_2, a_1 a_2}(k) = \int dq_+ dq_- dq_\perp (V_{\mu_1 \nu_1, a_1 b_1 c_1}^\mu(k, q) \Delta_{\rho_1 \nu_2}^{c_1 b_2}(k + q) V_{\mu_2 \nu_2, a_2 b_2 c_2}^{\rho_2}(k, q) \Delta_{\rho_2 \nu_1}^{c_2 b_1}(q)) =
\]

\[
= -g^2 N \delta^{a_1 a_2} \int \frac{dq_+ dq_- dq_\perp}{(q^2 - m^2 + i0)((k + q)^2 - m^2 + i0)} \times
\]

\[
\times Tr \left( \begin{pmatrix}
0 & (q_- + 2k_-) g_{\mu_1 \perp} \\
(q_- + 2k_-) g_{\mu_1 \perp} & 2q_{\mu_1} + k_{\mu_1} - (k_{\perp} + 2q_{\perp}) g_{\mu_1 \perp}
\end{pmatrix} \begin{pmatrix}
2(k_+ + q_+) & k_+ + q_- - im \\
k_+ + q_- + im & k_- - q_-
\end{pmatrix} \times
\]

\[
\times \begin{pmatrix}
0 & (q_- + 2k_-) g_{\mu_2 \perp} \\
(q_- + 2k_-) g_{\mu_2 \perp} & 2q_{\mu_2} + k_{\mu_2} - (k_{\perp} + 2q_{\perp}) g_{\mu_2 \perp}
\end{pmatrix} \begin{pmatrix}
2q_+ & q_- - im \\
q_- + im & q_-
\end{pmatrix} \right). \] (36)

We used here the following equality: \( f_{a_1 b_1 b_2} f_{a_2 b_1 b_2} = N \delta^{a_1 a_2} \). To calculate the counterterm we consider the difference between the expressions for this integral in our and in dimensional regularization, taking into account the first two terms of the Tailor series in \( k \) in the vicinity of the point \( k = 0 \). Besides we do the Euclidean rotation in the momentum space. Then one can see easily that all components of the diagram containing Lorentz index ”+” disappear. The remaining component with the indices \( \perp \perp \) can be calculated analytically and gives the finite value for it in the UV regularization removing limit. The analogous expression can be written for the diagram shown in Fig. 3d. We calculate it at zero external momenta because this diagram can diverge only logarithmically. Despite of its complicated expression this diagram allows analytical consideration after the Euclidean rotation in the momentum space. In result we obtain that this diagram is equal to zero at zero external momenta. The expressions for the remaining logarithmically divergent diagrams 3b, 3c, having Lorentz indices ”\( \perp \perp \)”, can be obtained in analogous way using the formulas (33), (35). However, these expressions turn out to be much more complicated. Besides, we were not able to consider them analytically. Only approximate numerical calculations are possible, and its results can be used, nevertheless, in nonperturbative Hamiltonian approach to this model.

Acknowledgements. Authors thank the organizers of the International Conference ”Models of Quantum Field Theory” (MQFT-2015) dedicated to the 75 years of birthday of A.N. Vasil’ev. Also they thank S.A. Paston and M.V. Kompaniets for useful discussions. The work of M.Yu. Malyshov, E.V. Prokhvatilov and R.A. Zubov was supported by the grant of SPbGU N 11.38.189.2014.

References

[1] V. A. Franke, S. A. Paston, Theor. Math. Phys., 112:3 (1997), 1117–1130, arXiv:hep-th/9901110
[2] S. A. Paston, E. V. Prokhvatilov, V. A. Franke, *Theor. Math. Phys.*, 120:3 (1999), 1164–1181, arXiv:hep-th/0002062.

[3] S. A. Paston, E. V. Prokhvatilov, V. A. Franke, *Theor. Math. Phys.*, 131:1 (2002), 516–526, arXiv:hep-th/0302016.

[4] S. A. Paston, E. V. Prokhvatilov, V. A. Franke, *Nucl. Phys. B (Proc. Suppl.)*, 108 (2002), 189–193, arXiv:hep-th/0111009.

[5] S. J. Brodsky, V. A. Franke, J. R. Hiller, G. McCartor, S. A. Paston, E. V. Prokhvatilov, *Nucl. Phys. B*, 703 (2004), 333–362, arXiv:hep-ph/0406325.

[6] S. A. Paston, E. V. Prokhvatilov, V. A. Franke, *Phys. Atom. Nucl.*, 68 (2005), 267–278, arXiv:hep-th/0501186.

[7] V. A. Franke, Yu. V. Novozhilov, S. A. Paston, E. V. Prokhvatilov, Focus on quantum field theory, chap. Quantization of Field Theory on the Light Front, 23–81, Nova science publishers, New York, 2005, arXiv:hep-th/0404031.

[8] M. Yu. Malyshev, S. A. Paston, E. V. Prokhvatilov, R. A. Zubov, *Int. J. Theor. Phys.*, 54 (2015), 169–184, arXiv:1311.4381 [hep-th].

[9] M. Yu. Malyshev, S. A. Paston, E. V. Prokhvatilov, R. A. Zubov, V. A. Franke, *Theor. Math. Phys.*, 184:3 (2015), 1314–1323, arXiv:1505.00272 [hep-th].

[10] P. A. M. Dirac, *Rev. Mod. Phys.*, 21:3 (1949), 392–398.

[11] M. Burkardt, A. Langnau, *Phys. Rev. D*, 44:4 (1991), 1187–1197.

[12] M. Burkardt, A. Langnau, *Phys. Rev. D*, 44:12 (1991), 3857–3867.

[13] E. V. Prokhvatilov, V. A. Franke, *Sov. J. Nucl. Phys.*, 49 (1989), 688.

[14] E. M. Ilgenfritz, V. A. Franke, S. A. Paston, H. J. Pirner, E. V. Prokhvatilov, *Theor. Math. Phys.*, 148:1 (2006), 948–959, arXiv:hep-th/0610020.

[15] S. D. Glazek, M. Gómez-Rocha, “Asymptotic freedom of gluons in the Fock space”, 2015, arXiv:1510.01609 [hep-th].

[16] S. D. Glazek, K. G. Wilson, *Phys. Rev. D*, 48:8 (1993), 4214–4218.

[17] S. D. Glazek, K. G. Wilson, *Phys. Rev. D*, 49:12 (1994), 5863–5872.

[18] W. Pauli, F. Villars, *Rev. Mod. Phys.*, 21:3 (1949), 434–444.

[19] S. Deser, R. Jackiw, S. Templeton, *Annals of Physics*, 281 (2000), 409–449.

[20] R. Jackiw, S. Templeton, *Phys. Rev. Lett.*, 48 (1982), 975–978.

[21] S. J. Brodsky, J. R. Hiller, G. McCartor, *Phys. Rev. D*, 64:11, 114023, arXiv:hep-ph/0107038.
[22] S. J. Brodsky, J. R. Hiller, G. McCartor, *Ann. Phys.*, **296**:2 (2002), 406–424, arXiv:hep-th/0107246.

[23] S. J. Brodsky, J. R. Hiller, G. McCartor, *Phys. Rev. D*, **60**:5, 054506, arXiv:hep-ph/9903388.

[24] S. J. Brodsky, J. R. Hiller, G. McCartor, *Phys. Rev. D*, **58**:2, 025005, arXiv:hep-th/9802120.

[25] M. Yu. Malyshev, S. A. Paston, E. V. Prokhvatilov, R. A. Zubov, V. A. Franke, “Pauli-Villars Regularization in nonperturbative Hamiltonian approach on the Light Front”, 2015, arXiv:1504.07951 [hep-th].