Computation Environments (2)
Persistently Evolutionary Semantics

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Abstract
In the manuscript titled “Computation environment (1)”, we introduced a notion called computation environment as an interactive model for computation and complexity theory. In this model, Turing machines are not autonomous entities and find their meanings through the interaction between a computist and a universal processor, and thus due to evolution of the universal processor, the meanings of Turing machines could change. In this manuscript, we discuss persistently evolutionary intensions. We introduce a new semantics, called persistently evolutionary semantics, for predicate logic that the meaning of function and predicate symbols are not already predetermined, and predicate and function symbols find their meaning through the interaction of the subject with the language. In (classic) model theory, the mathematician who studies a structure is assumed as a god who lives out of the structure, and the study of the mathematician does not effect the structure. The meaning of predicate and function symbols are assumed to be independent of the mathematician who does math. The persistently evolutionary semantics could be regarded as a start of “Interactive Model Theory” as a new paradigm in model theory (similar to the paradigm of interactive computation). In interactive model theory, we suppose that a mathematical structure should consist of two parts: 1) an intelligent agent (a subject), and 2) an environment (language), and every things should find its meaning through the interaction of these two parts.

We introduce persistently evolutionary Kripke structure for propositional and predicate logic. Also, we propose a persistently evolutionary Kripke semantics for the notion of computation, where the intension of a code of a Turing machine persistently evolve. We show that in this Kripke model the subject can never know $P = NP$.

Keywords: Interactive Model Theory, Free will, Persistent Evolution.

1 Introduction
The human being interacts with its surrounded environment, and through this interaction, the environment finds its meaning for him. He percepts its environment (that he calls it the real world) through its sensors, and in order to understand it, he constructs mental symbolic forms \[4\], and formal structures in his mind. After that, he reasons about its environment through these mental constructions.

Euclidian Geometry, Ptolemy’s Almagset, Copernicus revolution, Turing computation, and etc are some few samples of theories constructed by the human being through interaction with the world.

After the human being constructs a formal theory, he observes and means its environment through it as a window to the outer world. He proposes questions about its environment in his (constructed mental) formal theory, and tries to answer them in the context of
the same theory. For example, in astronomy, after Ptolemy’s Almagest, the human being tried to find an explanation for the irregular motions of the *wandering stars* in the context of Ptolemy’s Almagest, and in geometry the human being attempted to prove *parallel postulate* using Euclid’s first four postulates.

Model theory, a branch of mathematical logic [6], studies mathematical structure in order to determine that given a theory (a set of formulas) \( \Gamma \), which other formulas are true in all structures which all formulas in \( \Gamma \) are true in them. In (classic) model theory, the mathematician is regarded as a god who lives out of a mathematical structure, and thinking activities of the mathematician does not effect the structure. In this way, the role of the human being in developing formal systems is ignored. In this paper, we introduce a new semantics for predicate logic, that the meaning of predicate and function symbols are not already predetermined, and they find their meaning through the interaction of a subject with the logical language. We name the proposed semantics “the persistently evolutionary semantics”.

The paper is organized as follows:

In Section 2, we discuss persistently evolutionary intensions. We scrutinize that whether it is possible that the intension of a word persistently evolves whereas the subject cannot be aware of it.

In section 3, we propose persistently evolutionary Kripke semantics to formalize the notion of persistently evolutionary intensions.

In section 4 and section 5, we introduce persistently evolutionary semantics for propositional and predicate logic.

In section 6, using persistently evolutionary semantics for predicate logic, we formalize the argument of section 7 of the manuscript [7].

## 2 Persistently Evolutionary Intensions

The human being, as an intelligent agent, uses languages to express and encode the intensions (concepts) that he constructs to mean his environment. Intension refers to a property that specifies the set of all possible things that a *word* (a finite string) could describe, while extension refers to the set of all actual things the word describes. Also an intensional definition of a set of objects is to intend the set by a word, and an extensional definition of a set of objects is by listing all objects. Obviously, it is impossible to give an extensional definition for an infinite set. For example, the human being intends an infinite subset of natural numbers by the word “prime”, and he can never list all prime numbers. As another example, the human being, to define the set of all Turing machines has no way except to use the intensional definition.

In theory of computation, for every Turing machine \( T \), \( L(T) \) refers to the set of all strings that the Turing machine \( T \) halts for them. We may say the Turing machine \( T \) is an intensional definition for the set \( L(T) \), or in other words, the human being intends the set \( L(T) \) by the *word* (finite string) \( T \) (note that Turing machines can be coded in finite strings).

The human being constructs concepts to mean its environment through them. It is possible that both the human being and its environment (the real world, the nature) evolve as the human being checks that whether an object is an extension of a word. This evolution
could be persistent such that the nature (or the human being) works well-defined. That is, if an output $z$ is already provided for an input $[x, y]$ ($x$ is a word, and $y$ is a thing in order to be checked whether it is an extension of $x$) then whenever in future, the same input $[x, y]$ are chosen, the output would be the same $z$. In other words, the meaning of the word $x$ may change, but in a conservative manner, that is, all the things that the human being already realized that whether they are extensions of $x$ or not, their status remains unchanged.

**Definition 2.1** Let $w$ be a word (a finite string), and $\mathcal{O}$ a domain of objects. We say the intension of the word $w$ for a subject is persistently evolutionary (or its extension is order-sensitive) whenever in the course that the subject chooses an object $o \in \mathcal{O}$ to check that whether it is an extension of the word $w$ or not, then the intension of $w$ changes, but persistently, i.e., if the agent (the subject) has checked whether an object $d$ is an extension of $w$ already, and the answer has been yes (has been no), then whenever in future the agent checks again whether the same object $d$ is an extension, the answer would be the same yes (would be the same no). What remains unchanged is the word $w$ (the syntax), but its meaning (the semantics) changes for the subject. In this way, the set of all extensions of the word $w$ is not predetermined and it depends on the order that the agent chooses objects from the domain $\mathcal{O}$ to check whether they are extensions of $w$ or not.

**Q1)** Is it possible that an intension of a word would be a persistently evolutionary one?

The answer is Yes. To answer the above question, we should first clarify what the meaning (the intension) of a word is? For a subject (the human being) the meaning of a word is given by how the subject interacts via the word with the environment (the language). This interaction is nothing except choosing objects and checking whether they are extensions of the word.

Wittgenstein says the meaning of a word is identified by how it is used.

“For a large class of cases-though not for all- in which we employ the word ‘meaning’ it can be defined thus: the meaning of a word is its use in the language”\(^1\)

The use of a word happens in time, therefore we may say that the meaning of a word exists in time, and it is an *unfinished entity* similar to a choice sequence \(^1\). The meaning of a word is not predetermined, and as time passes the word finds its meaning through the interaction of the subject with the language. The meaning is a dynamic temporal (mental) construction \(^2\). In Brouwer’s intuitionism, mathematical objects are mental construction. Brouwer’s choice sequences, as a kind of mathematical objects, are dynamic temporal objects (see page 16, \(^2\)). The meaning of a word (which is identified via how it is used by the human being) has lots of common with a choice sequence. At each stage of time, the human being only experienced a finite set of things that whether they are extensions of the word or not. Also, in a choice sequence, at each stage of time only a finite segment

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\(^1\)The quote is written from Stanford Encyclopeida of Philosophy \(^4\).

\(^2\)An object is temporal exactly if it exists in time, and it is dynamic if at some moments are part added to it or removed from it \(^2\), page 16. Another philosophical framework that knows possible the evolution of a meaning is “dynamic Semantics” (see http://plato.stanford.edu/entries/dynamic-semantics/). In this framework, meaning is context change potential.
of the sequence is determined. As the human being freely chooses another thing to check its extension status for the word, the meaning of the word may persistently change (the construction of the human’s brain (or mind) may persistently change). It is similar to the act of the human being in developing a choice sequence.

The use of a word is dependent to the human being and the way that he uses (interacts via) the word in (with) the environment. We may assume that the human being has freedom to choose things in any order that he wants to check their status of being extensions of a word. The different order of choosing objects may cause that the meaning of the word evolves in different ways. But since the human being cannot go back to the past, he just lives in one way of evolution. As soon as the human being chooses an object and checks whether it is an extension of a word \( w \), (the biological construction of) his mind may persistently evolve, and the meaning of the word \( w \) persistently changes. Therefore, it seems possible that the intension of a word would be persistently evolutionary, and

the interaction of the human being with the language may make the meaning of a word persistently evolve. Assuming the free will for the human being, the behavior of the human being is not predetermined, and as a consequence the meaning of a word needs not to be predetermined.

**Q2)** Is it possible for a subject to distinguish between persistently evolutionary intensions and static ones? In other words, is it possible for a subject to determine that whether the intension of a word is static or persistently evolutionary?

The answer is No. It is not possible for the human being to recognize whether the meaning of a word, in the course of his thinking activities, persistently evolves or remains constant. Suppose \( w \) be a word. Two cases are possible

1) the intension of the word \( w \) is static and

2) the intension of the word \( w \) is persistently evolutionary.

In both cases, at each stage of time, the human being just has experienced the status of extension of a finite set of objects. The human being just has access to his pervious experiences and does not have access to the future. So at each stage of time, all information that the human being has about the word \( w \) is a finite set of objects \( \{d_1, d_2, ..., d_n\} \) that their extension status are determined. This information of the word \( w \) are the same in both cases.

The human being cannot differ between these two cases based on his obtained information. The persistent evolution is similar to being static in view of past experiences. The difference of the persistent evolution and being static is in future. But the human being does not have access to the future. As soon as, the meaning of a word evolves then it has been evolved and the subject cannot go back to the past and experience another way of evolution.

**Example 2.2** Suppose that I am in a black box with two windows: an input window and an output one. You give natural numbers as input to the black box and receive a natural numbers as output of the black box. I do the following strategy in the black box. I plan to
output 1 for each input before you give the black box 5 or 13 as inputs. If you give 5 as input (and you have not given 13 already) then after that time, I output 2 for all future inputs that have not been already given to the black box. For those natural numbers that you have already given them as input, I still output the same 1. If you give 13 as input (and you have not given 5 already) then after that time, I output 3 for all future inputs that have not been already given to the black box. For those natural numbers that you have already given them as input, I still output the same 1.

The black box of the above example behaves well defined. But it persistently evolve through interactions with the environment, and the function that the black box provides is not a predetermined function. If one does not have access to the inner structure of the black box, he could always assume that there exists a a static machine in the black box.

**Remark 2.3 Deterministic vs. Predetermination.** Being deterministic does not force to be predetermined. The human may computationally intend a function deterministically, but it is not needed that the language to be predetermined. It may be determined as time passes by the free will of the human being.

We propose a postulate about the extension of a word as follows: Suppose $w$ is a word and $\mathcal{O}$ is a class of objects. We refer to the set of extension of $w$ by $\text{E}(w) \subseteq \mathcal{O}$. Our proposed postulate which we call it "the Postulate of Persistent Evolution", PPE, says:

**PPE:** if a subject has not yet proved that $\text{E}(w)$ is finite (or in other words, if a subject has not yet listed all the element of $\text{E}(w)$ on paper) then he could not yet disprove that the meaning (intension) of $w$ does not persistently evolve.

Suppose that a subject wants to prove that

i. the meaning of a word $w$ is static and does not persistently evolve, and

ii. the set $\text{E}(w)$ is predetermined and does not depend to the order that he chooses objects from the domain $\mathcal{O}$ to check whether they are extensions of $w$ or not.

The subject at each stage of time, only knows the status of a finite number of objects in $\mathcal{O}$ that whether they are extensions of $w$ or not. Suppose $E(w)$ is infinite. Then the subject has never written all extensions of $w$ at any stage of time. He always could know it possible that the the meaning of the word $w$ may persistently change. But since this change happens persistently, he cannot recognize whether the meaning is static or not, based on the finite history that he has access to it.

If a subject does not sense a change about a process, then he may (wrongly) presuppose that the process is static and independent of his interaction with the process. In spite of this, in the case that a process persistently evolves, the subject does not sense any change as well! We only sense a change whenever we discover that an event which has been sensed before is not going to be sensed similar to past. Persistent evolution always respects the past. As soon as a subject experiences an event, then whenever in future he examines the same event, he will experience it similar to past. Persistent evolution effects the future which has not been determined yet.
In other words, the postulate \( \text{PPE} \) says that

it is not possible for a subject to differ between static intensions and persistently
evolutionary one.

## 3 Persistently Evolutionary Semantics

In this section, to clarify the notion of persistently evolutionary intensions, we introduce a
kind of Kripke structures that we name Persistently Evolutionary Kripke structures.

Let \( P = \{ p_i \mid i \in I \} \) be a set of atomic propositional formulas for an index set \( I \subseteq \mathbb{N} \),
and \( A = \{ a_i \mid i \in I' \} \ (I' \subseteq \mathbb{N} \) is an index set) be a set that is assumed as the set of actions
of an agent \( ag \).

**Definition 3.1** A Persistently Evolutionary Kripke structure over a set of actions \( A \) and
a set of atomic formulas \( P \) is a tuple \( K = (S, \Pi = \{ \pi_j \mid j \in J \}, \sim_{ag}, V) \) where \( J \subseteq \mathbb{N} \) is an
index set, and

- \( S = A^* \times J \) is the set of all possible worlds (\( A^* \) is the set of all finite sequences of
actions in \( A \)). For each \( s = (\vec{x}, i) \in S \), we call \( i \) the meaning index of the state \( s \).

- \( \Pi \) is the set of meaning functions. Each \( \pi_i \in \Pi \ (i \in J) \), is a partial function from
\( S \times A \) to \( \mathcal{P}P \) (the set of finite subsets of \( P \) that its domain is \( (A^* \times \{i\}) \times A \). To
each state \( s = (\langle b_1, b_2, ..., b_n \rangle, i) \), and each action \( a \in A \) the function \( \pi_i \) corresponds
a finite subset of \( P \) as the meaning of the action \( a \), satisfying the following condition
(persistently evolutionary condition):

for each state \( s = (\langle b_1, b_2, ..., b_n \rangle, i) \in S \), if an action \( a \) is appeared in the finite sequence \( \langle b_1, b_2, ..., b_n \rangle \), and \( \langle b_1, b_2, ..., b_n \rangle = \langle b_1, b_2, ..., b_i \rangle \langle b_{i+1}, ..., b_n \rangle \), then \( \pi_i(s,a) = \pi_i(\langle b_1, b_2, ..., b_i \rangle, i, a) \).

- The agent \( ag \) is an operator which chooses actions from \( A \) and performs them. By
his operation, it makes the universe evolve. If \( s = (\vec{x}, i) \in S \) is the current state of
the model \( K \) and \( ag \) performs \( a \in A \), then the current world evolves to \( s' = (\vec{x}, \langle a, i \rangle) \).

Note that via evolution, the meaning index of the states does not change. We say
the agent \( ag \) lives in the meaning function \( \pi_i \), or in other words, the actual meaning function
for the agent \( ag \) is \( \pi_i \).

- \( V \) is a function form \( S \) to \( 2^P \) defined as follows: for each \( s \in S \), \( s = (\langle b_1, b_2, ..., b_n \rangle, j) \),

\[
V(s) = \pi_j((\langle \rangle, j), b_1) \cup (\bigcup_{1 \leq i \leq n} \pi_j((\langle b_1, b_2, ..., b_i \rangle, j), b_{i+1})).
\]

- \( \sim_{ag} \subseteq S \times S \) is a binary relation which satisfies the following condition:
for all two states \( s_1, s_2 \in S \), we have \( s_1 \sim_{ag} s_2 \) whenever \( s_1 = (\vec{x}, i) \) and \( s_2 = (\vec{x}, j) \) for some
\( \vec{x} = \langle x_1, x_2, ..., x_k \rangle \) and \( i, j \in J \) such that for all \( 1 \leq t \leq k \), \( \pi_i(\langle x_1, ..., x_{t-1}, i \rangle, x_t) = \pi_j(\langle x_1, ..., x_{t-1}, j \rangle, x_t) \).

The relation \( \sim_{ag} \) is an indistinguishability relation for the agent \( ag \). If \( s_1 \sim_{ag} s_2 \) then it
means that the agent \( ag \) cannot distinguish between these two states, since all experiences
that he has observed in both states are the same.
**Definition 3.2** Let $K = \langle S, \Pi = \{\pi_j \mid j \in J\}, \sim_{ag}, V \rangle$ be a persistently evolutionary Kripke Structure. We say a meaning function $\pi_i \in \Pi$ is static if it is not order-sensitive. That is, for every $n \in \mathbb{N}$, for every $a_1, a_2, \ldots, a_n, a \in A$, for every permutation $\delta: \{1 \ldots n\} \to \{1 \ldots n\}$,

$$\pi_i((a_{\delta(1)}, a_{\delta(2)}, \ldots, a_{\delta(n)}), i, a) = \pi_i((a_1, a_2, \ldots, a_n), i, a).$$

**Notation 3.3** For each state $s = (\vec{x}, i)$, we let $D(s) = \{s' \mid s' = (\vec{y}, i), \vec{x} is a prefix of \vec{y}\}$.

**Definition 3.4** Let $K = \langle S, \Pi = \{\pi_j \mid j \in J\}, \sim_{ag}, V \rangle$ be a persistently evolutionary Kripke Structure. Suppose $s = (\vec{a}, i), \vec{a} \in A^*$, and $i \in J$ be a current state that the agent $ag$ lives in. We may say that the agent $ag$ can never become conscious that whether his world is static or persistently evolutionary whenever for every $s' = (\vec{a}, i) \in D(s)$ there exists a meaning function $\pi_j \in \Pi$ which is not static, and for $s'' = (\vec{a}, j)$, we have $s' \sim_{ag} s''$.

**Definition 3.5** Let $P$ be a non-empty set of propositional variables. The language $L(P)$ is the smallest superset of $P$ such that

- if $\varphi, \psi \in L(P)$ then $-\varphi$, $(\varphi \land \psi), (\varphi \lor \psi), (\varphi \rightarrow \psi), K_{ag} \varphi, \Box \varphi, C_f \varphi \in L(P)$,

$C_f \varphi$ has to be read as “the formula $\varphi$ conflicts with the free will of the agent $ag$”, $K_{ag} \varphi$ has to be read as “the agent $ag$ knows $\varphi$”, and $\Box \varphi$ has to be read as “$\varphi$ is necessary true”.

**Notation 3.6** Let $K$ be a Kripke model with the set of state $S$. For each subset $A \subseteq S$, $K_A$ is defined to be the same Kripke model $K$ which its set of states is restricted to the set $A$.

**Definition 3.7** In order to determine whether a formula $\varphi \in L(P)$ is true in a current world $(K, s)$, denoted by $(K, s) \models \varphi$, we look at the structure of $\varphi$: 

- $(K, s) \models p$ if and only if $p \in V(s)$
- $(K, s) \models (\varphi \lor \psi)$ if and only if $(K, s) \models \varphi$ or $(K, s) \models \psi$
- $(K, s) \models (\varphi \rightarrow \psi)$ if and only if for all $t \in D(s)$, if $(K, t) \models \varphi$ then $(K, t) \models \psi$
- $(K, s) \models (\varphi \land \psi)$ if and only if $(K, s) \models \varphi$ and $(K, s) \models \psi$
- $(K, s) \models \neg \varphi$ if and only if for all $t \in D(s)$, $(K, t) \not\models \varphi$
- $(K, s) \models \Box \varphi$ if and only if for all $t \in D(s)$, $(K, t) \models \varphi$
- $(K, s) \models K_{ag} \varphi$ if and only if for all $t \in S$, if $t \sim_{ag} s$ then $(K, t) \models \varphi$
- $(K, s) \models C_f \varphi$ if and only if there exists an infinite set $\text{Path} = \{s_0, s_1, \ldots\}$, where $s_0 = s$, and for each $i$, $s_{i+1} \in D(s_i)$ and $(K_{\text{Path}}, s_i) \not\models \varphi$

The current state of the Kripke model $K$ is not a fixed state. The current state evolves due to agent’s operation, and it is not possible for the agent to travel back in time from a state $s$ to one of its prefixes. Note that during the evolution, the meaning function does not change. That is, if the current state is $s = (\vec{x}, i)$ and due to executing an action $a$, the current state changes to be $s'$, then $s' = (\vec{x}.(a), i)$ for the same $i$. In this case, we call $\pi_i$ the actual meaning function of the universe.

The semantics of $C_f \varphi$ says that the agent $ag$ can interact with the universe and evolve it in a way that never $\varphi$ holds true. Therefore, the assumption of truth of $\varphi$ conflicts with the free will of the agent.
Definition 3.8 Let $K = \langle S, \Pi, \sim_{ag}, V \rangle$ be a persistently evolutionary Kripke structure. We say an action $a \in A$, at the state $s = (\vec{x}, i) \in S$, is a static action whenever for all $s_1, s_2 \in D(s)$, we have $\pi_i(s_1, a) = \pi_i(s_2, a)$. That is, if the agent starts from the state $s$ to perform actions, then the different orders that he may perform the actions does not make the meaning of the action ‘$a$’ change.

Remark 3.9 At each state, the agent cannot go back to past to experience his universe in different ways, thus he cannot distinguish between static actions and persistently evolutionary ones.

Example 3.10 Suppose $A = \mathbb{N}$ as a set of actions, and $P = \{p_{i,j} \mid i, j \in \mathbb{N}\}$ as a set of atomic propositions. For each finite sequence of numbers $\vec{x} = \langle x_1, x_2, ..., x_n \rangle$, and $y \in A$,

if for all $1 \leq i \leq n$, $x_i \neq 5, x_i \neq 13$, define $\pi(\vec{x}, y) = \{p_{y,1}\}$

if for some $1 \leq i \leq n$, $x_i = 5$ and for all $j < i$ $x_j \neq 13$ then

- if for some $t \leq \min(\{i \mid x_i = 5\})$, $y = x_t$ then define $\pi(\vec{x}, y) = \{p_{y,1}\}$ else define $\pi(\vec{x}, y) = \{p_{y,2}\}$.

if for some $1 \leq i \leq n$, $x_i = 13$ and for all $j < i$ $x_j \neq 5$ then

- if for some $t \leq \min(\{i \mid x_i = 13\})$, $y = x_t$ then define $\pi(\vec{x}, y) = \{p_{y,1}\}$ else define $\pi(\vec{x}, y) = \{p_{y,3}\}$.

The function $\pi$ satisfies the persistently evolutionary condition. We have $(K, \langle 1, 3 \rangle) \models p_{1,1}$. Also $(K, \langle 1, 3 \rangle) \models C_f p_{6,1}$, and $(K, \langle 1, 3 \rangle) \models C_f \neg p_{6,1}$. The value of $p_{6,1}$ is not predetermined yet and depends on the free will of the agent.

One may check that the indistinguishability relation $\sim_{ag}$ is

1) reflexive (for all $s \in S$, $s \sim_{ag} s$);

2) transitive (for all $s, t, u \in S$, if $s \sim_{ag} t$ and $t \sim_{ag} u$ then $s \sim_{ag} u$);

3) Euclidean (for all three states $s, t, u \in S$ if $s \sim_{ag} t$ and $s \sim_{ag} u$ then $t \sim_{ag} u$).

Therefore, persistently evolutionary Kripke structures are models for the standard epistemic logic $S5$ which consists of axioms $A1 - A5$ and the derivation rules $R1$ and $R2$ given below

\begin{align*}
A1: & \text{ Axioms of propositional logic} \\
A2: & (K\varphi \land K(\varphi \to \psi)) \to K\psi \\
A3: & K\varphi \to \varphi \\
A4: & K\varphi \to KK\varphi \\
A5: & \neg K\varphi \to K\neg K\varphi \\
R1: & \vdash \varphi, \vdash \varphi \to \psi \implies \vdash \psi \\
R2: & \vdash \varphi \implies K\varphi,
\end{align*}
3.1 A Kripke Model for Persistently Evolutionary Intensions

Now we describe the notion of persistently evolutionary intensions using persistently evolutionary Kripke models.

Let \( \text{LANGUAGE} = \{w_1, w_2, \ldots\} \) be a set of words for a subject \( IA \), and \( X = \{x_1, x_2, \ldots\} \) be an infinite set of objects that could be assumed as possible extensions of words in \( \text{LANGUAGE} \).

The subject chooses a word \( w \in \text{LANGUAGE} \) and an object \( x \in X \) to check whether \( x \) is an extension of the word \( w \) or not. Therefore, the set of actions of the Kripke model is defined to be \( A_e = \{(w_i, x_j) \mid i, j \in \mathbb{N}\} \). The set of atomic propositions is defined to be \( P_e = \{p(w_i, x_j, 0) \mid i, j \in \mathbb{N}\} \cup \{p(w_i, x_j, 1) \mid i, j \in \mathbb{N}\} \).

The agent \( IA \) chooses a word \( w_i \) and an object \( x_j \) to check whether \( x_j \) is an extension of the word \( w_i \). If at state \( s \), he chooses \( (w_i, x_j) \) then the current state evolves to \( s' = s.((w_i, x_j)) \).

We let the set of meaning functions \( \Pi_e \) to be the set of all functions \( \pi_i \) such that the following conditions:

1- For each state \( s = (\bar{x}, i) \), and action \( (w_i, x_j) \) either \( \pi_i(s, (w_i, x_j)) = p(w_i, x_j, 1) \) (we read it as “at the current state \( s \), the agent \( IA \) checked that whether \( x_j \) is an extension of the word \( w_i \) and found out the answer ‘yes’) or \( \pi_i(s, (w_i, x_j)) = p(w_i, x_j, 0) \) (we read it as “at the current state \( s \), the agent \( IA \) checked that whether \( x_j \) is an extension of the word \( w_i \) and found out the answer ‘no’).

2- Each \( \pi_i \in \Pi \) satisfies the persistently evolutionary condition.

We call the Kripke model \( K_e = (S_e, \Pi_e, \sim_{ag}, V_e) \) (introduced above) the model of persistently evolutionary intensions.

**Definition 3.11** We say the intension of a word \( w \in \text{LANGUAGE} \) is static (or its extension is not order-sensitive) at a state \( s = (\bar{x}, j) \) whenever for all \( a \in \{(w, x_i) \mid i \in \mathbb{N}\} \) and for all \( s_1 \) and \( s_2 \) in \( D(s) \), we have \( \pi_j(s_1, a) = \pi_j(s_2, a) \).

**Theorem 3.12** For each state \( s = (\bar{x}, j) \in S_e \) and each word \( w \in \text{LANGUAGE} \) there exist two states \( s_1 = (\bar{x}, t) \in S_e \) and \( s_2 = (\bar{x}, t) \in S_e \) such that \( s \sim_{ag} s_1 \) and \( s \sim_{ag} s_2 \), and the intension of \( w \) is static at \( s_1 \) and persistently evolutionary at \( s_2 \).

**Proof.** The proof is straightforward. \( \dashv \)

The above theorem says that it is not possible for the agent who lives in the persistently evolutionary Kripke model \( K_e \) to gets aware that whether the intension of a word \( w \) is static or persistently evolutionary. It is because, at each stage of time (at each state of the Kripke model \( K_e \)) the agent only observed a finite set of experiences, and as he cannot travel to the past (go back to a prefix of the current state), he cannot experience different orders of his behavior to be assured that if the actual meaning function which the universe evolves in, is order-sensitive or not.

If the agent wants to be aware of a change, then he must experience an event different from the way that he has experienced the same event already. But as the evolution happens persistently, it is impossible.
In persistently evolution, the behavior of the agent changes the future which has not yet occurred.

4 Persistently Evolutionary Semantics for Propositional Logic

Persistently evolutionary Kripke structures can be considered as models for propositional logic. Let \( P_i = \{ p_i | i \in I \} \) be a set of atomic formulas. The language \( L_l \) of propositional logic is the smallest set containing \( P_i \) satisfying the following condition:

\[
\phi, \psi \in L \Rightarrow \varphi \land \psi, \varphi \lor \psi, \neg \varphi \rightarrow \psi, K \varphi, C_f \varphi, \square \varphi \in L_l.
\]

We say \( K = \langle S, \Pi, \sim_{ag}, V \rangle \) is a Kripke structure for propositional logic whenever the set of actions is \( A = P_i \), the set of atomic formulas of the structure \( K \) is \( P = \{ p = b | p \in P_i, b \in \{0,1\} \} \), and for each \( \pi_i \in \Pi \), \( \pi_i((p_1, p_2, ..., p_n), i) = \{ p = b \} \) for some \( b \in \{0,1\} \).

Definition 4.1 For every formula \( \varphi \in L_l \), we define

\[
\begin{align*}
(K,s) \models p & \quad \text{iff} \quad p = 1 \in V(s) \\
(K,s) \models (\varphi \lor \psi) & \quad \text{iff} \quad (K,s) \models \varphi \text{ or } (K,s) \models \psi \\
(K,s) \models (\varphi \rightarrow \psi) & \quad \text{iff} \quad \text{for all } t \in D(s), \text{ if } (K,t) \models \varphi \text{ then } (K,t) \models \psi \\
(K,s) \models (\varphi \land \psi) & \quad \text{iff} \quad (K,s) \models \varphi \text{ and } (K,s) \models \psi \\
(K,s) \models \neg \varphi & \quad \text{iff} \quad \text{for all } t \in D(s), (K,t) \not\models \varphi \\
(K,s) \models K_{ag} \varphi & \quad \text{iff} \quad \text{for all } t \in S, \text{ if } t \sim_{ag} s \text{ then } (K,t) \models \varphi \\
K,s) \models C_f \varphi & \quad \text{iff} \quad \text{there exists an infinite set } \text{Path} = \{ s_0, s_1, ... \}, \text{ where } s_0 = s, \text{ and for each } i, s_{i+1} \in D(s_i) \text{ and } (K_{\text{Path}, s_i}) \not\models \varphi
\end{align*}
\]

One may check that if we omit the operator \( K, \square \), and \( C_f \) from the language \( L_l \) then the persistently evolutionary semantics is sound and complete for intuitionistic propositional logic (see Chapter 2, [8]).

5 Persistently Evolutionary Semantics for Predicate Logic

In this part, we propose a persistently evolutionary semantics for predicate logic.

A predicate language \( L_o \) contains

- a set of predicate symbols \( R \), and a natural number \( n_R \) for each \( R \in R \) as its ary,
- a set of function symbols \( F \), and a natural number \( n_f \) for each \( f \in F \),
- a set of constant symbols \( C \).

Definition 5.1 A partial \( L_o \)-structure \( N \) is given by the following data

1) a nonempty set \( N \) called the domain,
2) a partial function \( f^N : N^{n_f} \rightarrow N \), for each \( f \in F \),
3) a set \( R^N \subseteq N^{n_R} \) for each \( R \in R \),
4) a partial zero-ary function \( c^N \in N \) for each \( c \in C \). (In this way, there could be some constant symbols \( c \in C \), which are not interpreted in the structure.)

We refer to \( R^N, f^N, c^N \) as interpretations of symbols \( R, f, c \).

**Definition 5.2** \( \operatorname{TERM} \) is the smallest set containing

- variable symbols,
- constants symbols in \( C \),
- for each function symbol \( f \in \mathcal{F} \), if \( t_1, t_2, ..., t_{n_f} \in \operatorname{TERM} \) then \( f(t_1, t_2, ..., t_{n_f}) \) is a term.

The interpretation of a term \( t \), denoted by \( t^N \), is defined to be a partial function from \( N^k \) to \( N \) for some \( k \), similar to the interpretation of terms in model theory (see definition 1.1.4 of [6]). The only difference is that the interpretations are partial functions.

**Definition 5.3** \( \operatorname{FORMULA} \) is the smallest set satisfying the following conditions:

- \( \bot \in \operatorname{Formula} \),
- \( t_1, t_2 \in \operatorname{TERM} \) then \( t_1 = t_2 \in \operatorname{FORMULA} \),
- for each predicate symbol \( R \in \mathcal{R} \), if \( t_1, t_2, ..., t_{n_R} \in \operatorname{TERM} \) then \( R(t_1, t_2, ..., t_{n_R}) \in \operatorname{FORMULA} \)
- \( \varphi, \psi \in \operatorname{FORMULA} \) then \( \neg \varphi, \varphi \land \psi, \varphi \lor \psi, \varphi \rightarrow \psi, \forall y \varphi, \exists y \varphi \in \operatorname{FORMULA} \).

A persistently evolutionary Kripke structure \( K_{L_o} \) for the language \( L_o \) is defined as follows:

- a nonempty set \( O \) called the domain,
- The set of actions of the Kripke structure is
  \[
  A_O = \{ R(\bar{o}) \mid \bar{o} \in O^{n_R}, R \in \mathcal{R} \} \cup \\
  \{ f(\bar{o}) \mid \bar{o} \in O^{n_f}, f \in \mathcal{F} \} \cup \\
  \{ c \mid c \in C \}.
  \]

  The set of atomic propositions of the Kripke structure is
  \[
  P_O = \{ (R(\bar{o}) = b) \mid b \in \{0, 1\}, \bar{o} \in O^{n_R}, R \in \mathcal{R} \} \cup \\
  \{ (f(\bar{o}) = o') \mid \bar{o} \in O^{n_f}, o' \in O, f \in \mathcal{F} \} \cup \\
  \{ (c_j = o) \mid c \in C, o \in O \}.
  \]

  The set of meaning functions \( \Pi \) of the Kripke structure is the set of all functions \( \pi_i \), which satisfy persistently evolutionary condition and
  \[
  \pi(s, R(\bar{o})) = \{ (R(\bar{o}) = b) \} \text{ for some } b \in \{0, 1\}, \text{ and}
  \]
  \[
  \pi(s, f(\bar{o})) = \{ (f(\bar{o}) = o') \} \text{ for some } o' \in O.
  \]
\[ \pi(s, c) = \{(c = o)\}, \text{for some } o \in O. \]

The meaning of predicates and the value of functions are not predetermined in the Kripke structure. As soon as the agent \( ag \) chooses a predicate symbol \( R \) and a tuple \((o_1, o_2, ..., o_{n_R})\) to find the value of \( R(o_1, o_2, ..., o_{n_R}) \), the meaning function gives out an atomic proposition \((R(o_1, o_2, ..., o_{n_R}) = b), b \in \{0, 1\}\), and the current state evolves to a new state.

**Definition 5.4** Let \( s \) be a state of the persistently evolutionary Kripke model \( K_L \). The partial \( L_0 \)-structure of the state \( s \), denoted by \( N_s \), is defined as follows:

1. the domain of the structure \( N_s \) is the same domain of the Kripke model \( O \).
2. For each symbolic predicate \( R \in R \), the relation \( R^{N_s} \) is defined to be \( \{\vec{o} \mid (R(\vec{o}) = 1) \in V(s)\} \).
3. For each symbolic function \( f \in F \), the partial function \( f^{N_s} \) is defined to be \( \{(o, o') \mid (f(o) = o') \in V(s)\} \).
4. For each symbolic constant \( c \in C \), we define \( c^{N_s} = o \) if \((c = o) \in V(s)\).

We say a constant \( c \) is predetermined at a state \( s \) whenever for some \( o \in O \), \( c^{N_s} = o \). We say a predicate symbol \( R \) is predetermined for \( \vec{o} \) at a state \( s \), whenever for some \( b \in \{0, 1\} \), \((R(\vec{o}) = b) \in V(s)\). We say a function symbol \( f \) is predetermined for \( \vec{o} \) at a state \( s \), whenever for some \( o' \in O \), \( f^{N_s}(\vec{o}) = o' \). We simply can inductively define being predetermined for terms and formulas.

**Definition 5.5** Let \( K_L = \langle S, \Pi = \{\pi_i \mid i \in I\}, V \rangle \) be a persistently evolutionary Kripke structure for the language \( L_0 \). Let \( s \) be a state of this model. Also let \( \phi \) be a formula with free variables \( \vec{y} = (y_1, y_2, ..., y_n) \), and let \( \vec{\alpha} = (o_1, o_2, ..., o_n) \in O^n \). We inductively define \((K, s) \models \varphi(\vec{\alpha})\) as follows.

- if \( \phi \) is \( t_1 = t_2 \), then \((K, s) \models \phi(\vec{\alpha}) \iff t_{1}^{N_s}(\vec{\alpha}) = t_{2}^{N_s}(\vec{\alpha}) \),
- if \( \phi \) is \( R(t_1, t_2, ..., t_{n_R}) \), then \((K, s) \models \phi(\vec{\alpha}) \iff (t_{1}^{N_s}(\vec{\alpha}), t_{2}^{N_s}(\vec{\alpha}), ..., t_{n_R}^{N_s}(\vec{\alpha})) \in R^{N_s} \),
- if \( \phi \) is \( \neg \psi \), then \((K, s) \models \phi(\vec{\alpha}) \iff \forall \psi \) if \( (K, s) \models \psi(\vec{\alpha}) \) and \( s \models \psi(\vec{\alpha}) \),
- if \( \phi \) is \( \varphi \land \psi \), then \((K, s) \models \phi(\vec{\alpha}) \iff s \models \varphi(\vec{\alpha}) \) and \( s \models \psi(\vec{\alpha}) \),
- if \( \phi \) is \( \varphi \lor \psi \), then \((K, s) \models \phi(\vec{\alpha}) \iff s \models \varphi(\vec{\alpha}) \), or \( s \models \psi(\vec{\alpha}) \),
- if \( \phi \) is \( \forall x \psi(\vec{y}, x) \), then \((K, s) \models \phi(\vec{\alpha}) \iff \forall \psi \) for all \( w \in D(s) \), for all \( o' \in O \), if \( \psi(\vec{o}, o') \) is defined in the partial structure \( N_w \) (predetermined at the state \( w \)) then \( w \models \psi(\vec{o}, o') \),
- if \( \phi \) is \( \exists x \psi(\vec{y}, x) \), then \((K, s) \models \phi(\vec{\alpha}) \iff \exists \psi \) if there exists \( o' \in O \), \( s \models \psi(\vec{o}, o') \).

**Proposition 5.6** For every formula \( \varphi \in L_0 \), and every state \((K, s)\), \((K, s) \models \varphi \) then for all \( s' \in D(s) \), \((K, s') \models \varphi \).

**Proof.** It is straightforward. \( \Box \)
5.1 Free Will

One of our purpose of proposing persistently evolutionary semantics is to provide a framework to formalize the notion of free will. We discussed the notion of free will in section 4.3 of [7]. In this part, we repeat the same discussion using persistently evolutionary Kripke structures.

Let \( R \) be a one-ary predicate symbol. Let \( O = \{0, 1\}^* \) be the set of all finite strings over 0 and 1. Consider the meaning function \( \pi_j \) as follows: for \( s = (\langle R(x_1), R(x_2), \ldots, R(x_k)\rangle, j) \), and \( x \in \{0, 1\}^* \),

- if for some \( 1 \leq i \leq k \), \( x = x_i \) then \( \pi_j(s, R(x)) \) is defined to be \( \pi_j(s', R(x_i)) \) for \( s' = (\langle R(x_1), R(x_2), \ldots, R(x_{i-1})\rangle, j) \),
- if for all \( 1 \leq i \leq k \), \( x \neq x_i \), and there exists \( 1 \leq i \leq k \), such that \( x_i = x_0 \) or \( x_i = x_1 \) and for \( s' = (\langle R(x_1), R(x_2), \ldots, R(x_{i-1})\rangle, j) \), \( \pi_j(s', R(x_i)) = \{R(x_i) = 1\} \) then \( \pi_j(s, R(x)) \) is defined to be \( \{R(x) = 0\} \),
- otherwise, \( \pi_j(s, R(x)) \) is defined to be \( \{R(x) = 1\} \),

It is easy to check that the meaning function \( \pi_j \) behaves similar to the persistently evolutionary Turing machine \( PT_1 \) introduced in example 4.6 in [7]. The next theorem is a formal version of the theorem 4.9 in [7]. Let \( K' \) be the persistently evolutionary Kripke model which the set of its meaning function \( \Pi \) is \( \{\pi_j\} \).

**Theorem 5.7** Let

\[
\varphi := (\exists k \in \mathbb{N})(\forall n > k)(\exists x \in \{0, 1\}^*)(|x| = n \land R(x)).
\]

We have for the initial state \( s = (\langle \rangle, j) \),

\[
(K', s) \models \Box_C \varphi \land \Box_C \neg \varphi.
\]

**Proof.** The agent can develop the future in two ways such if the first way happens \( \varphi \) is true in the universe, but if the second happens \( \neg \varphi \) is true.

We define two ordering \( \preceq_1, \preceq_2 \) on the elements of \( \{0, 1\}^* \) as follows. Let \( x_1, x_2 \in \{0, 1\}^* \).

1. if \( |x_1| < |x_2| \) then \( x_1 \preceq_1 x_2 \),
2. if \( |x_1| = |x_2| \) then
   
   \[
   0 \preceq_1 1, \quad \text{if } x_1 \preceq_1 x_2 \text{ then } x_1a \preceq_1 x_2a, \text{ for } a \in \{0, 1\}, \quad x_10 \preceq_1 x_11.
   \]

1. if \( |x_1| + 1 < |x_2| \) then \( x_1 \preceq_2 x_2 \),
2. if \( |x_1| = |x_2| \) then
   
   \[
x_1 \preceq_2 x_2 \text{ iff } x_1 \preceq_1 x_2,
   \]
3- if $|x_1| + 1 = |x_2|$ and $|x_1|$ is even then $x_1 \preceq_2 x_2$,
4- if $|x_1| + 1 = |x_2|$ and $|x_1|$ is odd then $x_2 \preceq_2 x_1$.

Now let $y_1, y_2, \ldots$ be an enumeration of element of $\{0, 1\}^*$ with respect of the ordering $\preceq_1$, and $z_1, z_2, \ldots$ be an enumeration of element of $\{0, 1\}^*$ with respect of the ordering $\preceq_2$. For each $n \in \mathbb{N}$, let $s_n = ((R(y_1), R(y_2), \ldots, R(y_n)), j)$, and $s'_n = ((R(z_1), R(z_2), \ldots, R(z_n)), j)$. Let $Path_1 = \{s_1, s_2, \ldots\}$, and $Path_2 = \{s'_1, s'_2, \ldots\}$. We are done. ⊣

Let $(K, s)$ be an arbitrary state of a persistently evolutionary Kripke model. One may easily observe that for all formula $\psi$, $(K, s) \models \Box C_f \varphi \rightarrow \Box \neg K_{ag} \psi$. It is because if $(K, s) \models C_f \psi$ then $(K, s) \not\models \psi$ and thus $(K, s) \not\models K_{ag} \psi$.

Therefore, for the formula $\varphi$ in theorem 5.7, we have $(K', s) \models \Box \neg K_{ag} \varphi \land \Box \neg K_{ag} \neg \varphi$. It says that the agent $ag$ never have evidence for $\varphi$ and never have evidence for $\neg \varphi$. Therefore the principle of “from perpetual ignorance to negation” (PIN, see Chapter 5 of [1]) is not true in persistently evolutionary Kripke models.

### 6 A Persistently Evolutionary Kripke Structure for Computation environments

In this section, we propose a persistently evolutionary Kripke structure, $K_{ce}$, for the notion of computation environments. The language of computation environment $L_{ce}$ contains

- a predicate symbol $SB$ for the successful box,
- a function symbol $TB$ for the transition box.

Let $INST_s$ and $CONF_s$ be two set introduced in the Turing computation environment (see example 3.4 of [7]). The set of actions is defined to be

$$A_{ce} = \{SB(C) \mid C \in CONF_s\} \cup \{TB(C, t) \mid C \in CONF_s, t \in INST_s\}.$$ 

The set of atomic proposition of the Kripke structure $K_{ce}$ is defined to be

$$P_{ce} = \{SB(C) = b \mid b \in \{0, 1\}, C \in CONF_s\} \cup \{TB(C, t) = C' \mid C, C' \in CONF_s, t \in INST_s\}.$$ 

The set of meaning functions $\Pi_{ce}$ is defined to be the set of all functions $\pi$ which satisfy persistently evolutionary condition, and for every $s \in S_{ce}$, $C = (q, xb_1a_{b2y}) \in CONF_s$, and $t \in INST_s$,

- if $\pi(s, SB(C)) = \{SB(C) = 1\}$ then either $C = (h, \triangle x)$ or $C = (h, x \triangle)$,
- if $C = (h, \triangle x)$ then $\pi(s, SB(C)) = \{SB(C) = 1\}$,
- $\pi(s, TB(C, t)) = \{TB(C, t) = (p, xb_1b_{2y})\}$ for $\tau = [(q, a) \rightarrow (p, c, R)]$,
- $\pi(s, TB(C, t)) = \{TB(C, t) = (p, xb_1b_{2y})\}$ for $\tau = [(q, a) \rightarrow (p, c, L)].$
if \( \tau \neq [(q, a) \to (p, c, L)] \) and \( \tau \neq [(q, a) \to (p, c, R)] \) then \( \pi(s, TB(C, t)) = \{TB(C, t) = \perp\} \).

Let \( \pi_i \) be the meaning function that behaves accord to \( SBOX_s \) and \( TBOX_s \) of the Turing computation environment. We prove

\[
(K_{ce}, (\langle \rangle, i)) \models \neg K_{ag}(P = NP).
\]

To do this, we should prove that for every finite sequence of actions \( \vec{a} \) in \( A_{ce} \), there exists a meaning function \( \pi_j \) such that \( (\vec{a}, i) \sim_{ag} (\vec{a}, j) \), and \( (K_{ce}, (\vec{a}, j)) \nmid (P = NP) \).

Suppose \( \vec{a} = \langle a_1, a_2, \ldots, a_n \rangle \), and let \( H = \{a_i \mid a_i = SB(h, x\Delta)\} \). We construct a meaning function \( \pi_j \) that considers the following boxes. For the symbol function \( TB \) the meaning function \( \pi_j \) behaves based on the transition box \( TBOX_s \). For the symbol predicate \( SB \), it behaves as follows: We persistently evolve the persistently evolutionary machine \( PT_1 \) in the way that for every \( x \in \Sigma^* \), if there exists a configuration \( C = (h, x\Delta) \) such that \( SB(C) \in H \), then the machine \( PT_1 \), after evolution, outputs 1 for \( x \) if and only if \( \pi_i((\vec{a}, i), SB(C)) = 1 \). Then we construct a successful box, denoted by \( SBOX' \), which its inner structure is similar to \( SBOX_s \) except that instead of the \( PT_1 \) machine, we replaced the above evolved \( PT_1 \) machine. Now, we let \( \pi_j \) be the meaning function that behaves accord to \( SBOX' \). Then the two followings are straightforward.

1- \( (\vec{a}, i) \sim_{ag} (\vec{a}, j) \), and
2- \( (K_{ce}, (\vec{a}, j)) \nmid (P = NP) \)

One should verify that at the state \( (\vec{a}, j) \), the formula \( P = NP \) conflicts with the free will of the agent (see the proof of theorem 5.8 of [7]), and thus we have \( (K_{ce}, (\vec{a}, j)) \nmid (P = NP) \). Therefore,

\[
(K_{ce}, (\langle \rangle, i)) \models \neg K_{ag}(P = NP).
\]

The finite sequence \( \vec{a} \) was assumed to be arbitrary. Therefore, we proved that for all finite sequence of actions \( \vec{a} \), \( (K_{ce}, (\langle \rangle, i)) \models \neg K_{ag}(P = NP) \), and it informally means that

the agent \( ag \) can never know (have evidence for) \( P = NP \).

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\[\text{Actually, since } \pi_i \text{ is the meaning function that accords with the } SBOX_s \text{ of the Turing computation environment, for all } C = (h, x\Delta) \text{ such that } SB(C) \in H \text{, we have } \pi_i((\vec{a}, i), SB(C)) = 1.\]
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