Logical approach to the analysis of economic development programs

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Abstract. The present paper proposes a logical model for building an effective economic development programmer. It is built on the basis of existing programs, which are not the best in terms of their strategic implementation, but can improve some economic indicators. As part of our model, these programs will be considered as elements of a logical scheme that should implement a development program with optimal indicators.

1. Introduction
The development of an industry or region development program is a rather complex system process that does not have a single methodology. Program quality assessment has its own criteria by which we can judge whether the program in question meets the stated expectations. As a rule, it is not always possible to predict the exact result in advance for various reasons. Therefore, each proposed economic development program can be considered unreliable in terms of mathematical forecast. However, when there are several such programs, it is possible to improve the efficiency of one common based on the rest. In cybernetics, this is called the construction of reliable schemes from unreliable elements. Based on the logic-algebraic approach, a similar scheme is proposed to implement the most effective program in the conditions under consideration.

2. Materials and methods
To build a reliable program of economic development, methods of mathematical logic, predicate logic, multi-valued logic, approaches to mathematical cybernetics and discrete mathematics will be used. Multi-valued predicates will be used to qualitatively encode the characteristics that can characterize the indicators against which economic programs will be evaluated.

As a rule, a number of indicators assesses the effectiveness of the economic program; we indicate their desired values \( Y = \{y_1, y_2, \ldots, y_l\} \) - development indicators. Each of these indicators has its own resource base, that is, it depends on a number of constituent reasons, which will be indicated as \( X = \{x_1, x_2, \ldots, x_m\} \) - these are signs of the indicator on which it depends and can be improved.

By comparing each indicator with a set of its characteristics, we obtain a matrix of characteristics of the indicators dependence on the characteristics. Each of the considered economic programs as a result of implementation can change the values of development indicators. Our task is to collect such a scheme from these programs so that economic development indicators have optimal values in terms of a set of economic characteristics.
3. Mathematical formulation of the problem

Let there be a number of economic development programs. 

\[ X = \{ x_1, x_2, \ldots, x_n \}, \quad x_i \in \{0,1,\ldots,k_i-1\}, \quad k_i \in \mathbb{Z} \text{,} \]

where \( k, m \in \{2, \ldots, N\} \). \( N m \mathbb{Z} \) is the set of resources base of indicators affected by the considered economic programs; \( X_i = \{ x_i(1), x_i(2), \ldots, x_i(n) \}, \quad i = 1, \ldots, l \)

– a vector of of indicator signs \( y_i \in Y, \quad Y = \{ y_1, y_2, \ldots, y_l \} \) – many desired indicators

\( A = \{ A_1, A_2, \ldots, A_m \} \)

– a set of programs under consideration, \( A_i(X_i, y_i) \in \{0,1\}; i = 1,2,\ldots,l; \quad j = 1,2,\ldots,n \) – assessment of the quality of this program for a specific indicator for \( X_i = \{ x_i(1), x_i(2), \ldots, x_i(n) \}, \quad i = 1,2,\ldots,l \).

Let's say that

\[ a_j(y_i) = \begin{cases} 1, & A_j(X_i) = y_i, \\ 0, & A_j(X_i) \neq y_i, \end{cases} \]

\( i = 1,2,\ldots,l; \quad j = 1,2,\ldots,n \).

those

\( 1 - A_j \) reached the desired indicator according to his capabilities \( X_i \),

\( 0 \) did not reach \( A_j \) desired indicator \( y_i \) by \( X_i \).

All this can be presented in table 1.

| \( x_1 \) | \( x_2 \) | \( \ldots \) | \( x_m \) | \( Y \) | \( A'_1 \) | \( A'_2 \) | \( \ldots \) | \( A'_n \) |
|---|---|---|---|---|---|---|---|---|
| \( x_1(y_1) \) | \( x_2(y_1) \) | \( \ldots \) | \( x_m(y_1) \) | \( y_1 \) | \( a_1(y_1) \) | \( a_2(y_1) \) | \( \ldots \) | \( a_n(y_1) \) |
| \( x_1(y_2) \) | \( x_2(y_2) \) | \( \ldots \) | \( x_m(y_2) \) | \( y_2 \) | \( a_1(y_2) \) | \( a_2(y_2) \) | \( \ldots \) | \( a_n(y_2) \) |
| \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) |
| \( x_1(y_i) \) | \( x_2(y_i) \) | \( \ldots \) | \( x_m(y_i) \) | \( y_i \) | \( a_1(y_i) \) | \( a_2(y_i) \) | \( \ldots \) | \( a_n(y_i) \) |

\( A'_i = \{ a_1(y_1), a_2(y_2), \ldots, a_i(y_i) \}, \quad i = 1,2,\ldots,n \) – a vector, the column of values of which indicates the quality of the program under consideration for each indicator [6; 9].

Some of the indicators might not have been considered in this program. Then the following expression takes place:

\[ \exists y_i \in Y | A_i(X_i) \neq y_i, A_2(X_i) \neq y_i, \ldots, A_n(X_i) \neq y_i, \quad i = 1,2,\ldots,l; \quad j = 1,2,\ldots,n \].

It is necessary to develop a scheme for the implementation of the desired indicators based on existing programs.

\[ A_{m+1}(X_i) |_{A_{m+1}(X_i)} = y_i \text{ and } A_{m+1}(X) |_{A_{m+1}(X)} = Y. \]

Let's build a logical function reflecting the possibility of achieving the desired indicator on each set of our resources [2,4]

**Definition:** The decisive rule is:

\[ \&_{s \in 1} x_j(y_s) \rightarrow y, \quad i = 1,\ldots,l; \quad x_j(y_s) \in \{0,1,\ldots,k_i-1\}, k, m \in \{2, \ldots, N\}, N m \mathbb{Z} \]

Let there be \( n \) economic development programmes \( \{ A_1, A_2, \ldots, A_n \} \), with indicators partially attainable on a given set. For each given set of resources \( X \) we construct an assessment of the success of the program under consideration, we obtain a set of vectors \( A'_j = \{ a_1(y_1), a_2(y_2), \ldots, a_j(y_j) \}, j = 1,2,\ldots,n \) as a column \( A'_j \). Desired indicator \( y_s \), can be obtained as a result of the production rule:

\[ \&_{s \in 1} x_j(y_s) \rightarrow y, \quad x_j(y_s) \in \{0,1,\ldots,k_i-1\}, i = 1,\ldots,l; \quad s = 1,\ldots,m. \]
If the program in question has successfully worked on this indicator, then: \( A_j(X) = y_j \), \( a_j(X_i, y_i) = 1 \). In this case, there is at least one solution of the form: \( A_j(X) = y_j \), that \( \lor_{j=1}^m a_j(y_j) = 1 \). If none of the programs under consideration has reached the desired value of the indicator. \( y_j, \text{mo} \lor_{j=1}^m a_j(y_j) = 0 \).

Our entire table can be rewritten as production rules:

\[
\&_{i=1}^m x_i(y_i) \rightarrow y_i, \quad i=1, \ldots, l, x_i(y_i) \in \{0,1, \ldots, k_i-1\}, k_i, m \in [2, \ldots, N]. \quad \text{NnZ}.
\]

We wake up to choose those rules where \( \exists a_j(y_j) = 1 \) then \( \&_{i=1}^m x_i(y_i) \rightarrow y_i \), \( i=1, \ldots, l, x_i(y_i) \in \{0,1, \ldots, k_i-1\}, k_i, m \in [2, \ldots, N]. \quad \text{NnZ} \) : Choose successful solutions in each program.

The conjunction of such rules will reflect the most adequate general logical scheme for the entire set of programs under consideration:

\[
F_j(X) = \&_{i=1}^m x_i(y_i) \rightarrow y_i = \&_{i=1}^m x_i(y_i) \lor y_j.
\]

As a result, we get a disjunctive form that can be shortened to obtain the most important rules of the program under consideration. As well as those indicators on which the program \( A_j \) realized by function \( F_j \), the value of which is equal to one where we achieve the desired indicators and zero where we do not.

This function has a number of properties [5, 11].

We transform the original table by adding a column \( A_{n+1}(X) \) and get a table of the following form table 2.

| \( x_1 \) | \( x_2 \) | \( \ldots \) | \( x_m \) | \( Y \) | \( A'_1 \) | \( A'_2 \) | \( \ldots \) | \( A'_n \) | \( A_{n+1} \) |
|---|---|---|---|---|---|---|---|---|---|
| \( x_1(y_1) \) | \( x_2(y_1) \) | \( \ldots \) | \( x_m(y_1) \) | \( y_1 \) | \( a_1(y_1) \) | \( a_2(y_1) \) | \( \ldots \) | \( a_n(y_1) \) | 1 |
| \( x_1(y_2) \) | \( x_2(y_2) \) | \( \ldots \) | \( x_m(y_2) \) | \( y_2 \) | \( a_1(y_2) \) | \( a_2(y_2) \) | \( \ldots \) | \( a_n(y_2) \) | 1 |
| \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) |
| \( x_1(y_1) \) | \( x_2(y_1) \) | \( \ldots \) | \( x_m(y_1) \) | \( y_1 \) | \( a_1(y_1) \) | \( a_2(y_1) \) | \( \ldots \) | \( a_n(y_1) \) | 1 |

4. Results

Since the economic program, on which all indicators must achieve the desired result, then all meanings \( a_{n+1}(y_i) = 1, \ i=1,2,\ldots,l \).

\( a_j(y_i) \) is a Boolean variable, that \( A_{n+1}(A'_1, A'_2, \ldots, A'_n) \) – Boolean function that takes on all tuples \((A'_1, A'_2, \ldots, A'_n)\). What can be represented like this:

\[
A_{n+1}(A'_1, A'_2, \ldots, A'_n) = \lor_{i=1}^m \&_{j=1}^m A'_{i,j}(y_i), \quad i=1,2,\ldots,l, \ j=1,2,\ldots,n.
\]

\[
A'_{i,j}(y_i) = \begin{cases} A'_j, & a_j(y_i) = 1 \\ A'_j, & a_j(y_i) = 0 \end{cases}.
\]

We will assume that \( A'_j \), this is the set of decisive rules on which the program works, \( A'_j \) – the set of solutions on which the considered program achieves the desired performance.

\[
A'_j = \&_{i=1}^m (\&_{i=1}^m x_i(y_i) \rightarrow y_i) \text{ if } a_j(y_i) = 1, \]

\[
\overline{A'_j} = \&_{i=1}^m (\&_{i=1}^m x_i(y_i) \rightarrow y_i) \text{ if } a_j(y_i) = 0.
\]

You can write it as follows:
$$A_j = \&_{i=1}^{n} \left( \vee_{i=1}^{n} x_i(y_i) \lor y_i \right) \text{ if } a_j(y_i) = 1,$$
$$\overline{A_j} = \&_{i=1}^{n} \left( \&_{i=1}^{n} x(y_i) \land \overline{y_i} \right) \text{ if } a_j(y_i) = 0.$$ 

5. Discussion

The paper proposes a scheme for constructing an economic development program to achieve the desired results for all the declared indicators. To construct this scheme, a logical analysis of the links between indicators from their resource base, an analysis of these links is used. Then the analysis of the proposed programs, and the construction of a new, most perfect on their basis [8; 10]

6. Conclusion

Various methods can be used to analyze the results of the work of economic development programs and the synthesis of more advanced programs. First of all, these are methods of deep economic analysis. Various mathematical methods and approaches are also certainly useful. The work considered data mining on indicators of economic growth and their resource base. As well as logical analysis for constructing a diagram of a perfectly working program on other less promising programs. It is hoped that mathematical logic will become a reliable tool for analyzing economic forecasts

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