Pionic Fluctuations of Constituent Quarks and the Neutron Charge Radius

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Abstract

Pion loop fluctuations of the constituent $u$ and $d$ quarks are shown to give only a minute contribution to the intrinsic charge radius of the neutron, under the assumption that the pion-quark coupling constant has its conventional value, with a cut--off scale of the order of 1.0 GeV. The contribution from the pion loops to the anomalous magnetic moment of the neutron represents a small ($\sim 12$-$14\%$) increase over of the static quark model value.

The bulk of the empirical value $-0.117 \pm 0.002$ fm$^2$\cite{1} for the mean square radius of the neutron is accounted for by its magnetic moment alone\cite{2}. The mean square radius of the neutron is defined as\cite{3}:

$$< r^2_n > = -6 \lim_{q^2 \to 0} \frac{d}{dq^2} G_E(q^2), \quad (1)$$

where $G_E$ is the electric form factor. In terms of the Dirac and Pauli form factors $G_E = F_1 - (q^2/4m_n^2)F_2$, where $F_1(0) = 0$ and $F_2(0) = -1.91$ for the neutron, insertion in (1) yields the expression

$$< r^2_n > = -6 \lim_{q^2 \to 0} \frac{d}{dq^2} F_1(q^2) + \frac{3}{2} \frac{F_2(0)}{m_n^2}. \quad (2)$$

where $m_n$ the neutron mass. The numerical value of the latter (magnetic moment) term is $-0.126$ fm$^2$, and thus is already very close to the the empirical value. The (very) small difference between this value, and the empirical
value, may be viewed as an intrinsic mean square charge radius of the neutron
\[ < r^2 >_{\text{int}} = < r^2 >_{\text{exp}} - < r^2 >_{\text{mag}} = +0.009 \pm 0.002 \text{fm}^2. \] (3)

This small positive value has to arise from the \( q \)-dependence of the Dirac form factor \( F_1(q^2) \).

We here show that the lowest order pionic loop fluctuations of the constituent quarks (Fig. 1) lead to only a very small contributions to the term of order \( q^2 \) in \( F_{1n}(q^2) \), which yields a contribution to \( < r^2_n >_{\text{int}} \), the magnitude of which with conventional parameter choices are even smaller than this small empirical value.

The pion coupling to the \( u \) and \( d \) constituent quarks will be assumed to have the form
\[ \mathcal{L}_{\pi qq} = i \frac{f_{\pi qq}}{m_\pi} \bar{\psi} \gamma_5 \gamma_\mu \vec{\tau} \cdot \partial_\mu \psi. \] (4)

Here the pion-quark coupling constant is determined by the \( \pi NN \) coupling constant \( f_{\pi NN} \simeq 1 \) as \( f_{\pi qq} \simeq 3/5 f_{\pi NN} \). As the pions should decouple from the constituent quarks above the chiral symmetry restoration scale \( \Lambda_\chi \sim 4 \pi f_\pi \sim 1 \text{ GeV} \), a corresponding high momentum cut-off factor will be introduced at the pion quark vertices.

The calculation of the pionic loop fluctuations to nucleon properties may be justified at the level of constituent quarks when – as in the present case – the smallness of the effective pion-quark quark coupling suggests a converging loop expansion. Corresponding attempts to calculate such contributions as pion-nucleon fluctuations fail to yield realistic results because of the much larger pion-nucleon coupling constants and large number of intermediate baryon resonances that have to be considered in the loops in the hadronic approach \[4, 5\]. At the quark level the loop contributions automatically take into account all intermediate baryon states.

The pionic fluctuations illustrated by the Feynman diagrams in Fig. 1 both contribute to the \( q \)-dependent terms in the Dirac form factors of the \( u \)- and \( d \)-quarks. Denoting those \( F_{1u} \) and \( F_{1d} \) respectively, when normalized to unity at \( q^2 = 0 \), the \( SU(6) \) wave function for the nucleon yields the following expression for the intrinsic charge radius of the neutron:
\[ < r^2_n > = -4 \lim_{q^2 \to 0} \frac{d}{dq^2} \{ F_{1u}(q^2) - F_{1d}(q^2) \}. \] (6)
Here we calculate the contributions to the Dirac form factors $F_{1u}$ and $F_{1d}$ from all the pion loop amplitudes that may be represented by the Feynman diagrams in Fig. 1, where the fermion lines represent $u$ and $d$ quarks, and the pion lines represent all the pion charge states that are allowed by charge conservation. Two corresponding seagull diagrams are also generated by the point couplings

$$\mathcal{L}_{\pi qq} = ie\frac{f_{\pi qq}}{m_\pi} \bar{\psi} \gamma_5 \gamma_\mu (\vec{\phi} \times \vec{\tau})_\mu \psi,$$

which arise by minimal substitution of the e.m. vector potential $A_\mu$ in the derivative coupling (4). Inclusion of the seagull diagrams is required by current conservation.

In the absence of hadronic vertex form factors the seagull diagrams may be dropped, if in place of the derivative coupling (4) the pseudoscalar coupling:

$$\mathcal{L}_{\pi qq} = ig_{\pi qq} \bar{\psi} \gamma_5 \tau \cdot \phi \psi,$$

Figure 1: Pionic fluctuations of the constituent quarks. The fermion lines represent $u$ and $d$ quarks. Form factors are included at the vertices as explained in the text.
is employed. The result obtained with this pseudoscalar pion-quark coupling is then equivalent to that obtained with the pseudovector coupling with inclusion of the seagull terms for the diagrams considered here.

This equivalence may be retained also in the presence of hadronic vertex form factors, provided that these are included in a way that maintains current conservation. Denote the vertex form factor to be inserted at each pion-quark vertex in the amplitudes, where the e.m. field couples to the quarks (Fig. 1b) \( f(k^2) \) \( (f(-m_π^2) = 1) \). In these amplitudes the two form factors may be combined with the pion propagator to a “modified” pion propagator

\[
v(k^2) = \frac{1}{m_π^2 + k^2} f^2(k^2). \tag{9}
\]

We shall here take the form factor function \( f(k^2) \) to have the monopole form \( (\Lambda^2 - m_π^2)/(\Lambda^2 + k^2) \), where the value of the parameter \( \Lambda \) is \( \sim \Lambda_\chi \).

To maintain current conservation, and the equivalence between the pseudovector (including seagull terms) and pseudoscalar coupling models, the vertex factors have to be inserted into the amplitude that corresponds to the diagram Fig.1a, where the e.m. field couples to the pion current, so that the product of the two pion propagators is modified to \[6, 7\]:

\[
\frac{1}{m_π^2 + k_1^2} \frac{1}{m_π^2 + k_2^2} \rightarrow \frac{v(k_1^2) - v(k_2^2)}{k_2^2 - k_1^2}. \tag{10}
\]

Here \( k_1 \) and \( k_2 \) represent the 4-momenta of the pion before and after the electromagnetic coupling. Form factors inserted by this method allows the calculation to proceed on the basis of the pseudoscalar coupling (8) without further reference to seagulls.

The e.m. vertex of the internal constituent quarks, which should be assumed to have a spatially extended structure that may be described by a form factor \( F_q(q^2) \), is described by the current matrix element

\[
<p'|j_\mu(u,d)|p> = ie(\frac{2}{3}, -\frac{1}{3})\bar{u}(p')\{\gamma_\mu + (F_q(q^2) - 1)[\gamma_\mu - \frac{\gamma \cdot qq_\mu}{q^2}]\}u(p). \tag{11}
\]

This vertex, in which the first term corresponds to pointlike quarks, maintains the requirement of current conservation, as the form factor modification appears only in a purely transverse term \[6, 3\].
The pion should similarly be described as spatially extended, in view of its empirically large mean square radius. This is well described by the ρ-meson pole in the time-like region. This is taken into account through inclusion of the pion form factor $F_\pi(q^2)$ in the e.m. coupling of the pions, which is taken to have the form

$$<k'|j_\mu(\pi^\pm, 0)|k> = ie(\mp 1, 0)\{K_\mu + (F_\pi(q^2) - 1)[K_\mu - \frac{K \cdot q q_\mu}{q^2}]\}, \quad (12)$$

where $K_\mu = k_\mu - k_\mu'$. In this vertex the form factor also appears only in a transverse term. Note as the electromagnetic vertices are considered at tree level, they may contain all information of the quark structure of the pion and the self-dressings of the pion and the constituent quarks without double counting.

Consider the pionic fluctuations of u and d quarks illustrated in Figs. 1a and 1b. The only processes, in which the electromagnetic field couples to the pions, are the fluctuations $u \rightarrow \pi^+ d \rightarrow u$ and $d \rightarrow \pi^- u \rightarrow d$. The flavor factors of these two processes are the same, and therefore their contributions to the u and d quark current matrix elements have the same magnitude, but opposite sign.

For $u$ quarks the fluctuations with electromagnetic coupling to the internal quark lines are $u \rightarrow \pi^+ d \rightarrow u$ and $u \rightarrow \pi^0 u \rightarrow u$. In the first the coupling is proportional to the $d$ quark charge $-e/3$, while the flavor factor from the quark charge changing vertices is $(\sqrt{2})^2$. In the second the coupling is proportional to the $u$ quark charge $2e/3$, while the flavor factor from the quark charge conserving vertices is 1. Multiplication of the charge and flavor factors shows that these two contributions cancel exactly in the case of the $u$ quark. In the case of the $d$ quark the product of charge and flavor factors for the corresponding fluctuations $d \rightarrow \pi^- u \rightarrow d$ and $d \rightarrow \pi^0 d \rightarrow d$ are on the other hand 4/3 and $-1/3$ respectively, which add up to 1. Consequently fluctuations with electromagnetic coupling to the intermediate quark only contribute to the current matrix element of the $d$ quark.

The evaluation of the net loop contributions to the form factor combination $F_{1u} - F_{1d}$ required for the neutron mean square radius (6) is particularly convenient, as the combined current matrix elements satisfy the continuity equation, without electromagnetic coupling to the external quark legs. In the absence
of form factors one obtains the result:

\[
< r_n^2 >_{\text{loops}} = \frac{g_{\pi qq}^2}{8\pi^2} \left\{ -\int_0^1 dx x^3 \left\{ \frac{1}{H(m_\pi)} - \frac{m_q^2(1-x)^2}{H(m_\pi)^2} \right\} + \int_0^1 dx (1-x)^3 \left\{ \frac{2}{H(m_\pi)} + \frac{m_q^2(1-x)^2}{H(m_\pi)^2} \right\} \right\}.
\]

(13)

Here the function \( H(m) \) is defined as

\[
H(m) = m_q^2(1-x)^2 + m^2 x.
\]

(14)

The first integral on the rhs of (13) arises from fluctuations with e.m. coupling to the pions (Fig.1a) and the second from terms with e.m. coupling to the quark lines (Fig.1b). The latter is finite in the chiral limit, while the former is divergent in that limit. That divergence also appears in other chiral pion field theoretical models, e.g. in the Skyrme model [9].

The monopole hadronic form factors at the pion-quark vertices introduced so as to maintain current conservation as described above are included by means of the following substitutions in the expression (13):

\[
\frac{1}{H(m_\pi)} \rightarrow \frac{1}{H(m_\pi)} - \frac{1}{H(\Lambda)} - \frac{x}{H(\Lambda)^2} \Lambda^2 - m_\pi^2,
\]

(15a)

\[
\left( \frac{1}{H(m_\pi)^2} \right)^2 \rightarrow \frac{1}{H(m_\pi)^2} - \frac{1}{H(\Lambda)^2} - \frac{2x}{H(\Lambda)^3} \Lambda^2 - m_\pi^2.
\]

(15b)

The pionic loop contribution to the mean square neutron radius (13) is very small and sensitive to the constituent mass value. With a quark mass value of \( m_q = 300 \text{ MeV} \) the numerical values are -0.00082 fm\(^2\) and 0.0018 fm\(^2\) for \( \Lambda = 800 \text{ MeV} \) and 1.0 GeV respectively. With \( m = 340 \text{ MeV} \) the corresponding numerical values are -0.0048 fm\(^2\) with \( \Lambda = 800 \text{ MeV} \) and -0.0023 fm\(^2\) with \( \Lambda = 1 \text{ GeV} \). The value is positive for \( \Lambda > 1.2 \text{ GeV} \), the asymptotic value for \( \Lambda \rightarrow \infty \) being 0.013fm\(^2\) with the latter constituent mass value. This latter value is close to the empirical range. The smallness of the pion loop contribution to the mean square neutron radius is a consequence of strong cancellations between the two terms in (13).

The additional contribution to the mean square radius of the neutron from the finite radii of the pion and the quarks within the loops are

\[
< r_n^2 >_\pi = r_\pi^2 \left( \frac{g_{\pi qq}^2}{8\pi^2} \right) \int_0^1 dx x \left\{ \frac{1}{2} \left\{ \ln \frac{H(\Lambda)}{H(m_\pi)} - x \frac{\Lambda^2 - m_\pi^2}{H(\Lambda)} \right\} \right\}
\]

6
\[-2m_q^2(1-x)^2\left\{\frac{1}{H(m_\pi)} - \frac{1}{H(\Lambda)} - x\frac{\Lambda^2 - m_\pi^2}{H(\Lambda)^2}\right\}\right], \quad (16a)

\[<r_n^2> = -r_q^2\left(\frac{g_{\pi qq}}{8\pi^2}\right)\int_0^1 dx(1-x)\left\{\frac{1}{2}\left(\ln\frac{H(\Lambda)}{H(m_\pi)} - x\frac{\Lambda^2 - m_\pi^2}{H(\Lambda)}\right)\right\}

+ m_q^2(1-x)^2\left\{\frac{1}{H(m_\pi)} - \frac{1}{H(\Lambda)} - x\frac{\Lambda^2 - m_\pi^2}{H(\Lambda)^2}\right\}\right], \quad (16b)\]

respectively.

The mean square pion radius obtained from the empirical parameters in ref. \cite{10} is \( r_\pi^2 = 0.38 \text{ fm}^2 \). Realistic dynamical models for the baryon spectrum \cite{11} suggest that the (flavor averaged) mean square matter radius of the constituent \( u \) and \( d \) quarks should be about 0.13 \text{ fm}^2 if the empirical mean square radius of the proton is to be reached \cite{13}. With these values the additional contribution to the mean square radius of the neutron from the pion and (flavor averaged) constituent quark radius is very small. With \( m_q = 300 \text{ MeV} \) the value is 0.00086 \text{ fm}^2 for \( \Lambda = 800 \text{ MeV} \) and 0.0020 \text{ fm}^2 for \( \Lambda = 1 \text{ GeV} \). With \( m_q = 340 \text{ MeV} \) the corresponding values are somewhat smaller: 0.0013 \text{ fm}^2 and 0.0014 \text{ fm}^2 for \( \Lambda = 800 \text{ MeV} \) and \( \Lambda = 1 \text{ GeV} \) respectively. The smallness of these values is again a consequence of cancellations between the two contributions (16a) and (16b).

Combination of the two contributions (13) and (16) to the neutron radius then gives the following net contributions to the mean square radius of the neutron: with the quark mass 300 MeV and \( \Lambda = 800 \text{ MeV} \) the net contribution is only 0.00004 \text{ fm}^2, whereas with \( \Lambda = 1 \text{ GeV} \) it amounts to 0.004 \text{ fm}^2. In the case of \( m_q = 340 \text{ MeV} \) the net contribution is -0.005 \text{ fm}^2 for \( \Lambda = 800 \text{ MeV} \) and -0.0010 fm² for \( \Lambda = 1 \text{ GeV} \). With the smaller quark mass value the net contribution reaches 0.00 \text{ fm}^2 and thus the empirical range 0.009±0.002 \text{ fm}^2 if the cut-off parameter \( \Lambda \) is increased to 1.2 \text{ GeV}. With the larger quark mass value the empirical range is reached only by \( \Lambda = 1.5 \text{ GeV} \). The conclusion is in any case that the pionic loop fluctuations of the constituent quarks imply only a very small contribution to the intrinsic mean square radius of the neutron. The pionic loop contribution does therefore not perturb the satisfactory description of the (bulk of the) negative mean square radius of the neutron, which is implied by the empirical value of the neutron magnetic moment.

The pionic fluctuations of the constituent quarks also give a small contribution to the magnetic moment of the neutron. In units of nuclear magnetons
this contribution is

\[
F_2(0)_{\text{loops}} = -\frac{g_{\pi qq}^2}{12\pi^2} m_p m_q \int_0^1 dx (1-x)^2 (2+3x) \left\{ \frac{1}{H(m_\pi)} - \frac{1}{H(\Lambda)} - x \frac{\Lambda^2 - m_\pi^2}{H(\Lambda)^2} \right\}.
\]

(17)

Here \(m_p\) is the proton mass. The numerical value of this contribution is only \(\sim -0.23\) n.m. for \(\Lambda = 800\) MeV and \(\sim -0.26\) n.m. for \(\Lambda = 1\) GeV with \(m_q = 340\) MeV. The asymptotic value for \(\Lambda \to \infty\) is 0.37 n.m. These values represent enhancements of about \(\sim 12\)\(\sim 14\)% of the static quark model value \(-2/3(m_p/m_q)\). The latter agrees with the empirical value \(-1.91\) n.m. with \(m_q = 345\) MeV, but only if the reduction caused by the considerable “relativistic correction” that appears if the quarks are described by the Dirac current operator is neglected \[15\]. Hence there is a room for this loop correction along with exchange current contributions that are associated with the interaction between the confined quarks \[12, 13, 14\].

The pion loop contribution to the anomalous magnetic moment of the proton \(F_2^p(0)\) may also be calculated using the expression (17), provided that the bracket \((2+3x)\) in the integrand is replaced by the expression \(-(1+9x)/2\). Using the same parameter values as above for the neutron, we find the loop contribution to \(F_2^p(0)\) to fall in the range \(0.17 - 0.19\) n.m. The static quark model value for the magnetic moment of the proton is \(m_p/m_q\), and for the ratio of the neutron to proton magnetic moments \(\mu_n/\mu_p = 2/3\). This ratio, which is slightly below the empirical value 0.68, would increase by \(\sim 5\)% by inclusion of the loop contributions considered here.

The pionic loop contribution to the neutron magnetic moment was reported to be much larger than the value above in refs. \[16, 17\], without supporting formalism. The suggestion in refs. \[16, 17\] that the contribution of the pionic (and kaonic and \(\eta\)–meson) loop fluctuations of the constituent quarks to their anomalous magnetic moments explain all of the negative empirical mean square radius of the neutron is however not tenable as it neglects the other (main) quark contributions that make up the (empirical) anomalous magnetic moment of the neutron. In addition the contribution to the neutron mean square radius from the Dirac form factors of the constituent quarks to the first term of eq. (2) was neglected in refs. \[16, 17\].

The present finding is that the lowest order pionic loop fluctuations are insignificantly small for for conventional values of the coupling and cut–of scale parameters. These loop corrections are the leading ones in a \(1/m\)
expansion, being of order $(1/m)^2$. Small additional contributions that are of order $(1/m)^3$ arise from exchange currents [3, 4].

The restriction to $SU(2)$ flavor symmetry above is not essential, and the $Ks$ and $\eta s$ fluctuations of the $u$ and $d$ quarks may calculated by methods similar to those used above. The much larger masses of the $K$ and $\eta$ mesons make the numerical contributions of those fluctuations smaller than those of the pionic fluctuations however. Vector meson loop fluctuations are expected to be small, because of the much larger meson masses. Loop fluctuations in which intermediate vector mesons undergo radiative decay to pseudoscalar mesons give no contribution at all to the intrinsic charge radius of the neutron. This is direct consequence of the transversality of the transition current matrix elements, which have the generic form

$$<\pi(k')|J_\mu|V^b_\sigma(k) >= \frac{ig_{\pi\gamma}}{m_V} \epsilon_{\mu\lambda\sigma} k_\lambda k'_\gamma \delta^{ab}.$$ (18)

Here $V^b_\sigma$ denotes either the $\rho$ or the $\omega$ meson field (in the latter case the isospin index is left out).

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