Parity violating asymmetry with nuclear medium effects in deep inelastic $e^-e^+$ scattering

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Recently at JLab using polarised electron beam on unpolarised deuteron target measurements have been performed for the parity violating asymmetry($A_{PV}$) and there are future plans to measure this asymmetry using various nuclear targets. In this paper, we study $A_{PV}$ in nuclear targets like $^{12}$C, $^{56}$Fe and $^{208}$Pb, in a local density approximation using spectral function which takes into account Fermi motion, binding energy correction and nucleon correlations. Furthermore, the pion and rho cloud contributions have also been taken into account. The present model has been used earlier to study medium effects in electromagnetic as well as weak interaction induced processes in the DIS region.

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I. INTRODUCTION

The quark structure of nucleon is modified when it is bound in a nucleus. There is enough evidence from the experimental and theoretical studies of Deep Inelastic Scattering(DIS) of charged leptons and neutrinos from nuclear targets, to show that the change in the quark Parton Distribution Function(PDF) in the region of large $x$ ($x$ being the Bjorken variable) dominated by the valence quarks, is due to the Fermi motion, binding energy, nucleon off mass shell and nucleon correlations effects in the nuclear medium. In the small $x$ region of Bjorken variable dominated by sea quarks, non-nucleonic degrees of freedom like pions and/or quark clusters and nuclear shadowing play an important role. However, the models which rely on pion excess to explain the DIS of leptons on nuclei are unable to explain the observed low x behavior seen in Drell-Yan processes where no significant enhancement has been experimentally seen for the nuclear targets in the relevant region of $x$. A description of nuclear effects which can consistently explain the observed effects in DIS and DY processes has been lacking.

The process of Parity Violation in Electron Scattering (PVES) has been used to probe the quark structure of nucleons. The first observation of parity violating asymmetries in the Deep Inelastic Scattering (DIS) of polarised electrons with the deuteron target at SLAC [1] confirmed the Standard Model of Electroweak Interactions in electron sector. The most recent experiments at JLab performed with the polarised electrons on deuteron targets in the DIS [2] and resonance [3, 4] regions have measured the parity violating asymmetries with a very high precision and have determined the weak electron quark couplings, $g_{\lambda A,V}^{e,q}(=2C_{1u}-C_{1d})$ and $g_{\nu A}^{e,q}(=2C_{2u}-C_{2d})$. Specifically the high precision achieved in these experiments have made it possible for the first time to determine $g_{\nu A}^{e,q}$ [2] and verify in the DIS-Resonance region [3] the phenomenon of quark-hadron duality in the weak sector of inclusive electron scattering. Earlier experiments in the low and medium energy region performed at MAINZ [5, 6], MIT-BATES [8, 9], JLab [10, 11] and SLAC [12] have made significant contributions to the study of various aspects of the quark structure of nucleon like weak charge of the proton, vector and axial vector strangeness form factors of the nucleon, neutron densities of nuclei and neutral current transition form factors of $N-\Delta$ transition. These have been summarized in many review papers, for example see Refs. [14–16].

In the high energy region specially in the DIS region, PVES experiments provide direct access to the study of weak electron and quark couplings to the Z boson, quark(antiquark) parton distributions in nucleons and their modifications in nuclei. With the high precision achieved in present experiments and future experiments planned with the 12 GeV update at JLab [17], it shall be possible to measure the parity violating asymmetry with such a precision that a comparison with the state of the art theoretical calculations would be able to explore the physics beyond the Standard Model(BSM) in electroweak processes. With the aim of studying the parity violating effects with high precision, experiments with Hydrogen, Deuterium, and other nuclear targets like Fe, Au and Pb are planned [18]. Theoretically, following the first calculations of the parity violating asymmetry in the DIS region done by Cahn and Gilman [19] in the Bjorken limit, various corrections to the asymmetry arising due to higher twist effect, finite $Q^2$...
evolution of quark(antiquark) PDF, target mass correction, charge symmetry violation and nuclear medium effects have been done by many authors [20–26]. In future experiments done with nuclear targets to study parity violating asymmetries, it will be important to understand the nuclear medium effects as emphasized by Cloet et al. [22]. In view of these theoretical and experimental developments, we have studied in this paper nuclear medium effects in parity violating asymmetry, arising due to binding energy, Fermi motion, nucleon correlation, mesonic degree of freedom of nuclei and target mass correction. We also discuss the effect due to non-isoscalarity of nucleus on PV asymmetry in nuclei like $^{56}\text{Fe}$ and $^{208}\text{Pb}$ for which nuclear medium effects have been recently found to be important in a mean field approximation using NJL Lagrangian [27–29].

When electron scatters from a bound nucleon in a nucleus and has Fermi momentum described by a momentum distribution, the Bjorken variable for the target parton acquires a Fermi momentum dependence. The quark and antiquark PDFs should therefore be convoluted with the momentum distribution of the nucleon which takes into account various nuclear medium effects in order to calculate the structure functions, entering in the expression for PV asymmetry. Moreover, there may be additional contribution due to mesonic degrees of freedom in nuclei which may contribute to these structure functions as they do in EMC effect of charged leptons and neutrinos. We calculate parity violating asymmetry. In section-3, we present the numerical results and summarise our findings in section-4.

II. PARITY VIOLATING ASYMMETRY

A. Formalism

The parity violating asymmetry $A^{PV}$ in the scattering of polarised electron from nucleon/nucleus is defined as

$$A^{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L},$$

where $\sigma_R(\sigma_L) = \frac{d^2\sigma_R(\sigma_L)}{d\Omega dE'}$ denotes the scattering cross section for the right(left) handed polarised electron. The asymmetry arises due to the interference between amplitudes due to photon($\gamma$) and Z-boson($Z^0$) exchange [20].

The differential cross-section for electron-proton scattering takes the general form as [26]

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2 E'}{Q^4 E} \left( L_\mu^\gamma W_{\mu\gamma} - \frac{G_F Q^2}{4\sqrt{2}\pi\alpha} L_\mu^Z W_{\mu\gamma}^Z \right),$$

(1)

where $E$ and $E'$ denote the energies of the incoming and outgoing electron respectively in the lab frame. $Q^2 = -q^2 = -(\ell - \ell')^2$ is the four momentum transfer square. $l_\mu$ and $l'_\mu$ denote the four-momenta of the incoming and outgoing electron respectively. The leptonic tensors in Eq. (1) are given by

$$L_\mu^\gamma = 2(l_\mu l'_\nu + l'_\mu l_\nu - l \cdot l' g_{\mu\nu} - i\lambda\epsilon_{\mu\nu\alpha\beta}l^\alpha l^\beta),$$

$$L_\mu^Z = (g_{\nu\lambda} + \lambda g_{\nu\lambda})L_\mu^\gamma,$$

(2)

where $\lambda$ denotes the sign of the initial electron helicity with $\lambda = 1$ (right handed) and $\lambda = -1$ (left handed).

The hadronic tensors $W_{\mu\nu}^\gamma$ and $W_{\mu\nu}^{\gamma Z}$ are parameterized in terms of dimensionless structure functions as:

$$W_{\mu\nu}^\gamma = \left( -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) \frac{F_1^\gamma}{M} + \left( P_\mu - \frac{P q}{q^2} q_\mu \right) \left( P_\nu - \frac{P q}{q^2} q_\nu \right) \frac{F_2^\gamma}{M Q q}.$$

$$W_{\mu\nu}^{\gamma Z} = \left( -g_{\mu\nu} + \frac{g_\mu g_\nu}{q^2} \right) \frac{F_1^{\gamma Z}}{M} + \left( P_\mu - \frac{P q}{q^2} q_\mu \right) \left( P_\nu - \frac{P q}{q^2} q_\nu \right) \frac{F_2^{\gamma Z}}{M Q q} + \frac{i\epsilon_{\mu\nu\alpha\beta} P^\alpha q^\beta}{2 M P \cdot q} F_3^{\gamma Z}. $$

(3)

(4)

In the Bjorken limit ($Q^2, \nu \to \infty, x$ fixed), the interference structure functions $F_1^{\gamma Z}$ and $F_2^{\gamma Z}$ are related by the Callan-Gross relation, $F_2^{\gamma Z} = 2x F_1^{\gamma Z}$, similar to the electromagnetic $F_{1,2}$ structure functions $F_2^\gamma(x) = 2x F_1^\gamma(x)$. 

In section-2, we briefly describe the essential expression for the asymmetry in terms of structure functions calculated in a nuclear medium along with the contribution from mesonic degrees of freedom. In section-3, we present the
\( F_1^\gamma \) is given in terms of nucleon PDFs as \(^{20}\):

\[
F_1^\gamma(x) = \frac{1}{2} \sum_q e_q^2 (q(x) + \bar{q}(x)) ,
\]

(5a)

for the pure electromagnetic case, while

\[
F_1^{\gamma Z}(x) = \sum_q e_q g_V^q (q(x) + \bar{q}(x)) ,
\]

(6a)

\[
F_3^{\gamma Z}(x) = 2 \sum_q e_q g_A^q (q(x) - \bar{q}(x)) ,
\]

(6b)

are the structure functions occurring in the weak-electromagnetic interference term. For the numerical calculations, we have used parton distribution functions of CTEQ6.6 \(^ {36}\). The vector couplings for the u and d quarks are given respectively by \( g_V^u = -1/2 + (4/3)\sin^2\theta_W \) and \( g_V^d = 1/2 - (2/3)\sin^2\theta_W \), while the quark axial-vector couplings are \( g_A^u = 1/2 \) and \( g_A^d = -1/2 \), respectively.

In terms of these structure functions \( F_i^\gamma (i = 1 - 2) \) and \( F_i^{\gamma Z} (i = 1 - 3) \), the PVDIS asymmetry can be written:

\[
A_{PV} = - \left( \frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \right) g_A^1 \left(2xyF_1^{\gamma Z} - 2[1 - 1/y + xM/E]F_2^{\gamma Z} \right) + g_V 2 - y)F_3^{\gamma Z} ,
\]

(7)

where \( y = \nu/E \) is the lepton fractional energy loss.

The PV asymmetry in Eq. (7) may also be written as:

\[
A_{PV} = \left( \frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \right) (Y_1 a_2 + Y_3 a_3) ,
\]

(8)

where \( a_2 \) is given by:

\[
a_2(x) = -2g_A^1 F_2^{\gamma Z}(x) / F_2^1(x) ,
\]

(9a)

while \( a_3 \) is given by:

\[
a_3(x) = -2 x g_V^c F_3^{\gamma Z}(x) / F_2^1(x) ,
\]

(9b)

where \( g_V^c (= -\frac{1}{2} + 2\sin^2\theta_W) \) is the vector and \( g_A^1 (= -\frac{1}{4}) \) is the axial-vector couplings of the charged lepton.

\( a_2 \) and \( a_3 \) terms may also be written in terms of quark PDFs as

\[
a_2(x) = 2 \frac{\sum_q e_q g_V^q q(x)}{\sum_q e_q^2 q(x)} ,
\]

(10)

\[
a_3(x) = -4g_V^c \frac{\sum_q e_q g_A^q q^\ast(x)}{\sum_q e_q^2 q^\ast(x)} ,
\]

(11)

where \( q_A^\ast(x_A) = q_A(x_A) + \bar{q}_A(x_A) , q_A^\ast(x_A) = q_A(x_A) - \bar{q}_A(x_A) \) and \( q = u, d, s, c \).

If we expand \( a_2 \) about the uA \( \simeq d_A \) limit and \( s_A^\ast, c_A^\ast << u_A^\ast + d_A^\ast \), then one may write \(^ {22}\)

\[
a_2(x) \simeq \frac{9}{5} - 4\sin^2\theta_W - \frac{12}{25} \frac{u_A^\ast(x) - d_A^\ast(x) - s_A^\ast(x) + c_A^\ast(x)}{u_A^\ast(x) + d_A^\ast(x)} .
\]

(12)

Similarly for \( a_3(x) \)

\[
a_3(x) \simeq \frac{9}{5} [1 - 4\sin^2\theta_W] \left[ \frac{u_A^\ast + d_A^\ast}{u_A^\ast + d_A^\ast} \frac{u_A^\ast - d_A^\ast}{u_A^\ast + d_A^\ast} - \frac{3 u_A^\ast - d_A^\ast}{5 u_A^\ast + d_A^\ast} - \frac{2}{5} \frac{s_A^\ast}{u_A^\ast + d_A^\ast} - \frac{8}{5} \frac{c_A^\ast}{u_A^\ast + d_A^\ast} \right] .
\]

(13)
At finite $Q^2$, $R^{\gamma(Z)}$ are given in terms of the ratio of the longitudinal to transverse virtual photon cross sections as:

$$R^{\gamma(Z)} \equiv \frac{\sigma_L^{\gamma(Z)}}{\sigma_T^{\gamma(Z)}} = r^2 \frac{F_2^{\gamma(Z)}}{2xF_1^{\gamma(Z)}} - 1,$$

for both the electromagnetic ($\gamma$) and interference ($\gamma Z$) contributions, with

$$r^2 = 1 + \frac{Q^2}{\nu^2} = 1 + \frac{4M^2x^2}{Q^2}.$$  

In Eq\[16\], $Y_1$ and $Y_3$ are given by [26]:

$$Y_1 = \frac{1 + (1 - y)^2 - y^2(1 - r^2/(1 + R^{\gamma Z})) - 2xyM/E}{1 + (1 - y)^2 - y^2(1 - r^2/(1 + R^\gamma)) - 2xyM/E} \left(\frac{1 + R^{\gamma Z}}{1 + R^\gamma}\right),$$

$$Y_3 = \frac{1 - (1 - y)^2}{1 + (1 - y)^2 - y^2(1 - r^2/(1 + R^\gamma)) - 2xyM/E} \left(\frac{r^2}{1 + R^\gamma}\right).$$

In the Bjorken limit, $Y_1 = 1$, $Y_3 = \frac{1 - (1 - y)^2}{1 + (1 - y)^2}$ and the kinematical ratio $r^2 \to 1$.

If we neglect sea quark effects, and assume an isoscalar nucleus (N=Z) then one may write Cahn-Gilman limit [19] for $a_2(x)$, $a_3(x)$ and $A_{PV}(x)$

$$a_2 = \frac{9}{5} - 4\sin^2\theta_W$$

$$a_3 = \frac{9}{5} - 4\sin^2\theta_W$$

$$A_{PV} = \frac{G_FQ^2}{4\sqrt{2}\pi\alpha} \left[ \frac{1}{9} - \frac{20}{9}\sin^2\theta_W + (1 - 4\sin^2\theta_W) \frac{1 - (1 - y)^2}{1 + (1 - y)^2} \right]$$
FIG. 2: $a_2(x)$ vs $x$ in $^{56}$Fe(left) and $^{208}$Pb(right) nuclei at $Q^2 = 5\text{GeV}^2$. The solid line is the result obtained using the full model(Spectral Function+Meson cloud contribution). Double dotted dashed line is the Cahn-Gilman limit \cite{19} and dashed line is the result obtained by Cloet et al. \cite{22}.

B. Target Mass Correction

At low $Q^2$ and high values of $x$ the scattering kinematics are modified by the nucleon mass and therefore the nucleon structure functions $F_2(x, Q^2)$ are modified. In the present work, the target mass correction(TMC) has been taken from the works of Schienbein et al. \cite{37}, and the modified structure functions are given by

$$F_2^{TMC}(x, Q^2) \simeq \frac{x^2}{\xi^2} F_2(\xi) \left[ 1 + \frac{6 \mu x \xi}{\gamma} (1 - \xi)^2 \right],$$

and

$$F_3^{TMC}(x, Q^2) \simeq \frac{x}{\xi^2} F_3(\xi) \left[ 1 - \frac{\mu x \xi}{\gamma} (1 - \xi) \ln \xi \right].$$

where $\mu = \frac{M^2}{Q^2}$, $\gamma = \sqrt{1 + \frac{4 \pi^2 M^2}{Q^2}}$ and $\xi$ is the Nachtmann variable defined as $\xi = \frac{2x}{1+\gamma}$. We have used Eq\cite{18} for $F_2^{\gamma Z}$ and Eq\cite{19} for $F_3^{\gamma Z}$ to incorporate target mass correction.

C. Nuclear Medium Effects

1. Nuclear Structure

We have used local density approximation (LDA) to incorporate nuclear medium effects. This model has been successfully used earlier to describe the photon, lepton and neutrino induced reactions in the intermediate energy region for the nuclei of the present interest. Inside the nucleus when the reaction takes place on a nucleon target, several nuclear effects like Fermi motion, binding, pion and rho meson cloud contributions must be taken into account. Presently we have implemented Fermi motion and nucleon binding through the use of a nucleon spectral function. We are using a relativistic formalism for an interacting Fermi sea and the local density approximation is used to translate results from nuclear matter to finite nuclei. The use of nucleon Green’s functions in terms of their spectral functions...
offers a way to account for the Fermi motion and binding energy of the nucleon inside the nucleus. Therefore, we construct a relativistic nucleon spectral function and define everything within a field theoretical formalism which uses the nucleon propagators written in terms of this spectral function and we ensure that the baryonic number is properly normalized. All the nuclear information needed is contained in the nucleon spectral function. We start with the relativistic Dirac propagator $G_0(p_0, p)$ for a free nucleon, written in terms of the contribution from the positive and negative energy components of the nucleon described by the Dirac spinors.

For the nucleon inside the nucleus, the relativistic nucleon propagator $G(p_0, p)$ in the interacting Fermi sea is then written by making a perturbative expansion in terms of $G_0(p_0, p)$ by retaining the positive energy contributions only (the negative energy components are suppressed). This perturbative expansion is then summed in ladder approximation in a nuclear medium which can be cast in terms of the Spectral functions of holes and particles, the details of
FIG. 5: Asymmetry A vs x at $Q^2 = 5\,\text{GeV}^2$ and electron beam energy $E=6.06\,\text{GeV}$ in $^{12}\text{C}$, $^{56}\text{Fe}$ and $^{208}\text{Pb}$ nuclear targets (Lines and points have the same meaning as in Fig. 1).

which are given in Ref. [38, 39]:

$$G(p_0, \mathbf{p}) = \frac{M}{E(\mathbf{p})} \sum_r u_r(\mathbf{p}) \bar{u}_r(\mathbf{p}) \left[ \int_{-\infty}^{\mu} d\omega \frac{S_h(\omega, \mathbf{p})}{p^0 - \omega - i\eta} + \int_{\mu}^{\infty} d\omega \frac{S_p(\omega, \mathbf{p})}{p^0 - \omega + i\eta} \right], \quad (20)$$

where $S_h(\omega, \mathbf{p})$ and $S_p(\omega, \mathbf{p})$ are the hole and particle spectral functions respectively. We ensure that the wave function is properly normalized and we get the correct Baryon number for the nucleus. Furthermore, we also calculate the kinetic energy and the binding energy per nucleon and ensure that the calculated binding energy is very close to the experimentally observed ones for $^{12}\text{C}$, $^{56}\text{Fe}$ and $^{208}\text{Pb}$ nuclei.

We use local density approximation (LDA) where we do not have a box of constant density, but the reaction takes place at a point $\mathbf{r}$, lying inside a volume element $d^3r$ with local density $\rho_p(\mathbf{r})$ and $\rho_n(\mathbf{r})$ corresponding to the proton and neutron densities at the point $\mathbf{r}$. This leads to the spectral functions for the protons and neutrons to be the function of local Fermi momentum given by

$$k_{F_p}(\mathbf{r}) = \left[ 3\pi^2 \rho_p(\mathbf{r}) \right]^{1/3}, \quad k_{F_n}(\mathbf{r}) = \left[ 3\pi^2 \rho_n(\mathbf{r}) \right]^{1/3}, \quad (21)$$

and therefore the equivalent normalization is

$$2 \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{\mu} S_h(\omega, p, k_{F_p}(\mathbf{r})) d\omega = \rho_p(\mathbf{r}), \quad (22)$$

For a symmetric nuclear matter of density $\rho(\mathbf{r})$, there is a unique Fermi momentum given by $k_F(\mathbf{r}) = \left[ 3\pi^2 \rho(\mathbf{r})/2 \right]^{1/3}$ for which we obtain

$$4 \int \frac{d^3p}{(2\pi)^3} \int_{\mu}^{\infty} S_h(\omega, p, k_F(\mathbf{r})) d\omega = \rho(\mathbf{r}), \quad (23)$$

leading to the normalization condition given by

$$4 \int d^3r \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{\mu} S_h(\omega, \mathbf{p}, \rho(\mathbf{r})) \, d\omega = A, \quad (24)$$

where $\rho(\mathbf{r})$ is the baryon density for the nucleus which is normalized to $A$ and is taken from the electron nucleus scattering experiments.
For an isospin symmetric nucleus, we have earlier derived expressions for the electromagnetic and weak nuclear hadronic tensor $W^{\alpha\beta}_{A} \gamma$, and $W^{\alpha\beta}_{A} \gamma Z$ in terms of the corresponding nucleonic tensors $W^{\alpha\beta}_{N} \gamma$, and $W^{\alpha\beta}_{N} \gamma Z$, respectively, from which the weak and electromagnetic structure functions $F_{2A}^{\gamma}$, $F_{2A}^{W}$ and $F_{3A}^{W}$ have been obtained \cite{30,32}. Following a similar procedure we write the expressions for the pure electromagnetic and electromagnetic-weak interference nuclear hadronic tensors respectively $W^{\alpha\beta}_{A} \gamma$, and $W^{\alpha\beta}_{A} \gamma Z$ as

$$W^{\alpha\beta}_{A} \gamma, \gamma Z = 4 \int d^{3}r \int \frac{d^{3}p}{(2\pi)^{3}} \frac{M}{E(p)} \int_{-\infty}^{\mu} dp \beta_{h}(p_{0}, p, \rho(r)) W^{\alpha\beta}(p, \rho(r)) \delta ImD_{\gamma, \gamma Z}(p, q) .$$

(25)

where the nucleon structure function $W^{\alpha\beta}(p, q)$ is given by Eq.\(3\) for the $\gamma$ exchange and by Eq.\(4\) for the $\gamma Z$ term. Using Eq.(\ref{eq:25}), with the assumption that the nucleus is at rest while the nucleon in the nucleus is not at rest, we have \cite{30}

$$F_{2A}^{\gamma}(x_{A}, Q^{2}) = 4 \int d^{3}r \int \frac{d^{3}p}{(2\pi)^{3}} \int_{-\infty}^{\mu} d\omega S_{\beta}(\omega, \rho, \rho(r)) \frac{1}{\gamma^{2}} \left(1 - \frac{2\mathbf{q}^{2}}{\gamma^{2}}\right) \left(\gamma^{2} + \frac{6x_{N}(p_{0}^{2} - p_{Z}^{2})}{Q^{2}}\right) F_{2A}^{\gamma}(x_{A}, Q^{2})$$

(26)

with $p_{0}^{2} = M + \omega$, $\gamma^{2} = 1 + 4x_{N}^{2}p_{0}^{2}/Q^{2}$, $x_{N} = Q^{2}/(2p_{0} \cdot q)$ and $x_{A} = \frac{x}{A} = \frac{1}{4\pi^{2}m_{m}^{2}}$. Similarly $F_{3A}^{\gamma Z}(x_{A}, Q^{2})$ nuclear structure function is given by \cite{30}

$$F_{3A}^{\gamma Z}(x_{A}, Q^{2}) = 4 \int d^{3}r \int \frac{d^{3}p}{(2\pi)^{3}} \frac{M}{E(p)} \int_{-\infty}^{\mu} d\omega S_{\beta}(\omega, \rho, \rho(r)) \left(\frac{p_{0} - p_{Z}}{p_{0} - p_{Z}}\right) F_{3A}^{\gamma Z}(x_{A}, Q^{2})$$

(27)

At low $Q^{2}$ and high values of $x$ the scattering kinematics is modified by the nucleon mass and therefore the nucleon structure functions $F_{2N}$ and $F_{3N}$ are modified which are given by Eqs(\ref{eq:18}) and (\ref{eq:19}) respectively.

2. \(\pi\) and \(\rho\) mesons contribution to the nuclear structure function

The attractive interactions of nucleons inside the nucleus enhances the meson cloud and this increases the probability for an incident photon or $Z$ boson to interact with a pion or a rho meson instead of a nucleon. The pion and rho meson cloud contributions to the $F_{2}$ structure function have been implemented following the many body field theoretical approach of Ref. \cite{33}. In the case of $F_{3}^{\gamma Z}$ structure function there is no contribution from pion and rho meson clouds as $F_{3}^{\gamma Z}$ only gets contribution from valence quark distributions. The expression of pion structure function $F_{2A,\pi}(x)$ and $F_{3A,\pi}(x)$ in the nucleus is obtained through same formalism as done for nuclear structure function by replacing the bound nucleon propagator by pion or rho meson propagator.

The pion and rho meson cloud contributions to the $F_{2}$ structure function have been implemented following the many body field theoretical approach described in Ref. \cite{33}. The pion structure function $F_{2A,\pi}(x_{A})$ is written as:

$$F_{2A,\pi}(x) = -6 \int d^{3}r \int \frac{d^{3}p}{(2\pi)^{3}} \theta(p^{0}) \delta ImD(p) \frac{x}{x_{\pi}} 2M F_{2A,\pi}(x) \theta(x - x_{\pi}) \theta(1 - x_{\pi})$$

(28)

where $D(p)$ the pion propagator in the medium given in terms of the pion self energy $\Pi_{\pi}$:

$$D(p) = \left|p^{0} - \vec{p}^{2} - m_{\pi}^{2} - \Pi_{\pi}(p^{0}, p)\right|^{-1} .$$

(29)

where

$$\Pi_{\pi} = \frac{f^{2}/m_{\pi}^{2} F^{2}(p)\vec{p}^{2}\Pi^{*}}{1 - f^{2}/m_{\pi}^{2} V_{L}^{2} \Pi^{*}} .$$

(30)

Here, $F(p) = (\Lambda^{2} - m_{\pi}^{2})/(\Lambda^{2} + \vec{p}^{2})$ is the $\pi NN$ form factor and $\Lambda = 1$ GeV, $f = 1.01$, $V_{L}$ is the longitudinal part of the spin-isospin interaction and $\Pi^{*}$ is the irreducible pion self energy that contains the contribution of particle - hole and delta - hole excitations. In Eq.\(28\), $\delta ImD(p)$ is given by

$$\delta ImD(p) \equiv ImD(p) - \rho \frac{\partial ImD(p)}{\partial \rho} \big|_{\rho=0}$$

(31)
and
\[
\frac{x^2}{x_\pi} = \frac{-p^0 + \vec{p}^2}{M}
\] (32)

Following the same notation as in Ref. [40], the pion structure function at LO can be written in terms of pionic PDFs as
\[
F_{2\pi}(x_\pi) = \sum_q e_q^2 x_\pi [q(x_\pi) + \bar{q}(x_\pi)]
\] (33)

where \( q(x_\pi) \) and \( \bar{q}(x_\pi) \) are the quark and antiquark parton distribution functions in the pion.

Similarly, the contribution of the \( \rho \)-meson cloud to the structure function is written as [38]
\[
F_{2,\rho}(x) = -12 \int d^3r \int \frac{d^3p}{(2\pi)^3} \theta(p^0) \delta Im D_\rho(p) \frac{x}{x_\rho} 2MF_{2,\rho}(x_\rho) \theta(x_\rho - x) \theta(1 - x_\rho) \] (34)

where \( D_\rho(p) \) is the \( \rho \)-meson propagator and \( F_{2,\rho}(x_\rho) \) is the \( \rho \)-meson structure function, which we have taken equal to the pion structure function \( F_{2\pi} \) using the valence and sea pionic PDFs from reference [40]. \( \Lambda_\rho \) in \( \rho NN \) form factor \( F(p) = (\Lambda_\rho^2 - m_\rho^2)/(\Lambda_\rho^2 + \vec{p}^2) \) has also been taken as 1 GeV.

III. RESULTS AND DISCUSSION

We have used Eq.[20] for \( F_{2A}^2 \) and \( F_{2,\rho}^2 \) and Eq.[27] for \( F_{2,\rho}^Z \) to evaluate nuclear structure functions. The meson contribution due to pion and rho mesons has been evaluated using Eq.[23] for \( F_{2A}^Z(x) \) and Eq.[34] for \( F_{2,\rho}^A(x) \). The quark(antiquark) PDF parameterizations for nucleon as determined by the CTEQ collaboration [36] and the quark(antiquark) parameterization for pion(rho) mesons as given by Gluck et al. [40] have been used to calculate the meson structure functions. The Target Mass Correction(TMC) has been incorporated using Eqs.18 and 19 in the appropriate structure functions and all the results presented here are with TMC. The asymmetry \( A^{PV} \) is calculated using \( a_3(x) \) and \( a_3(x) \) from Eq.[33] and Eq.[34] respectively. For numerical calculations, we have taken \( \sin^2\theta_W = 0.2227 \) [22, 41] and the results are presented for \( ^{12}C, \ ^{56}Fe \) and \( ^{208}Pb \) nuclei.

In Fig.[1] we show the results for \( a_2(x) \), the term containing the contribution of quark vector coupling, in \( ^{12}C \)(top panel). These results are presented for the free nucleon case(dotted line), with nuclear structure effects(dashed line) and using our full prescription i.e including mesonic contribution(solid line). We find that there is hardly any change in \( a_2(x) \) due to nuclear medium effects for an isoscalar nucleus like \( ^{12}C \) over its free nucleon value but there is enhancement over the Cahn-Gilman limit at lower values of \( x(x<0.5) \). This is because the enhancement of the quark structure effect in \( F_{2,\rho}^Z \) and \( F_{2A}^Z \) are almost similar and do not affect \( a_2(x) \). When mesonic effects are included we find that the results are almost unaffected.

Furthermore, in Fig. 1(middle and lower panels), we show the results for \( ^{56}Fe \) and \( ^{208}Pb \) where effects of nonisoscalarity(solid line) along with the isoscalar limit(solid line with dots) have been shown separately. We see that for isoscalar condition the effects are similar to the case of \( ^{12}C \). When the effect of nonisoscalarity is taken into account, the nuclear medium effects lead to an increase in the value of \( a_2(x) \) as compared to the free nucleon value over the whole range of \( x \). This increase is mainly due to nuclear structure effects. Mesonic effects are visible only at very low values of \( x \). The enhancement of \( a_2(x) \) due to nuclear medium effects increases with the increase of nonisoscalarity(\( N-Z \)) and is smaller at lower values of \( x \) but becomes larger at higher \( x \). For example, in the case of \( ^{56}Fe \) it is 1.5% at \( x=0.3 \) and becomes 3% at \( x=0.8 \) while in the case of \( ^{208}Pb \) it is 4% at \( x=0.3 \), which becomes 9% at \( x=0.8 \). We have also shown in the all these panels the constant values of \( a_2(x) \) obtained in Cahn-Gilman limit(\( a_2 = \frac{\pi}{2} - 4\sin^2\theta_W \)) which is obtained in the case of isoscalar nuclei and neglecting all the nuclear medium effects due to nuclear structure, mesons and sea quark effects. The effect of nuclear medium is to increase \( a_2(x) \) as compared to Cahn-Gilman limit for \( x<0.5 \) for isoscalar nuclei, and for the whole range of \( x \) for nonisoscalar nuclei.

In Fig.[2] we compare our results for the nonisoscalar nuclei like \( ^{56}Fe \) and \( ^{208}Pb \) using the full model(Spectral function+Meson cloud) for \( a_2(x) \) with the results of Cloet et al. [22]. Here, we have also shown the result obtained in the Cahn-Gilman limit for reference. We see that the results obtained by us are qualitatively similar to the results obtained by Cloet et al. [22], giving an enhancement over the Cahn-Gilman limit for the whole range of \( x \). However, as compared to the results of Cloet et al. [22], we find a smaller enhancement except at very low \( x(x<0.35) \) for \( ^{56}Fe \) and \( x<0.20 \) for \( ^{208}Pb \). Thus, in the region of present experimental interest of \( x(x>0.5) \) [17], we find the nuclear medium effects to be smaller than found by Cloet et al. [22].

In Fig.[3] we show for the first time, the results for \( a_3(x) \) the term containing the contribution of quark axial-vector couplings in the nuclear medium for \( ^{12}C \)(top panel), \( ^{56}Fe \)(middle panel) and \( ^{208}Pb \)(lower panel) and
compared our results with the Cahn-Gilman limit of $a_3 = \frac{2}{3}(1 - 4\sin^2\theta_W)$. For an isoscalar nucleus, the general effect of nuclear medium is to decrease $a_3(x)$ at small $x$ ($x<0.5$) as compared to the results obtained for the free case or in the Cahn-Gilman limit. In the case of nonisoscalar nuclei like $^{56}$Fe (middle panel) and $^{208}$Pb (lower panel), we find a similar effect with slightly larger suppression at lower $x$ ($x<0.5$) and a slightly higher enhancement at large $x$ ($x>0.5$).

The nuclear medium effects on $a_3(x)$ are quantitatively smaller than $a_2(x)$, but the $x$ dependence is quite different from $a_2(x)$.

In Fig. 4 we show the ratio for $\frac{a_2(x)}{a_3(x)}$ vs $x$ for $^{56}$Fe and $^{208}$Pb with nuclear structure effects and meson effect. The relative contribution of $a_3(x)$ to $a_2(x)$ is about 22% at $x \approx 0.8$, which is approximately the same as in the free nucleon case and reduces to 12% at $x \approx 0.2$ as compared to 15% in the case of free nucleon. Thus in the region of present experimental interest ($x>0.5$), the relative contribution of $a_3(x)$ as compared to $a_2(x)$ is not affected much (about 2%) due to the nuclear medium effects.

In Fig. 5 we show the results for $A_{PV}$ vs $x$ using Eq. (8) for $^{12}$C (top panel), $^{56}$Fe (middle panel) and $^{208}$Pb (lower panel) nuclear targets. Here we have also explicitly shown the effect of nonisoscalarity (solid line) vs isoscalarity (solid line with dots) for $^{56}$Fe and $^{208}$Pb nuclei. We find the effect of nonisoscalarity as the largest source of nuclear medium effects, which enhances the asymmetry over the whole range of $x$. This enhancement increases with N-Z. For example, at $x=0.8$, the nuclear medium effects give an increase of 3% in the case of $^{56}$Fe, while it becomes 8% in the case of $^{208}$Pb.

We have performed these calculations for the 3-quark flavors (up, down and strange) as well as with 4-quark flavors (up, down, strange and charm) and find the results to be within 1%. The above results are presented at $Q^2 = 5 GeV^2$. However, we have also obtained $A_{PV}$ at lower values of $Q^2$ and found it to be almost independent of $x$. Furthermore, we have found the effect of TMC (not shown here) is to increase the asymmetry by about 1-1.5% in the region of $0.3 < x < 0.8$.

**IV. SUMMARY AND CONCLUSIONS**

In this paper, we have studied the nuclear medium effects on parity violating asymmetry $A_{PV}$ in the scattering of polarised electron from the nuclear targets like $^{12}$C, $^{56}$Fe and $^{208}$Pb. Besides presenting numerical results for the asymmetry, we have also presented the numerical results for the terms $a_2(x)$ and $a_3(x)$, which determine the contributions of the quark vector and axial-vector couplings to the asymmetry $A_{PV}$. While, we have compared our results for $a_2(x)$ with the presently available results in literature, the study of nuclear medium effects in $a_3(x)$ are presented for the first time. The nuclear medium effects arise due to nuclear structure effects, pionic degrees of freedom, target mass correction and sea quarks using CTEQ parameterization for nucleon quark (antiquark) PDFs and a nuclear model which uses local density approximation to describe the bound nucleons in a finite nucleus. The momentum distribution of the bound nucleons has been described by a spectral function, which uses relativistic nucleon propagator in Fermi sea and takes into account Fermi motion, binding energy and nucleon correlations. The mesonic contributions due to pions and rho mesons have been calculated in a field theoretical model.

We conclude from the results obtained in this paper that:

The nuclear medium effects on $a_2(x)$ are dominated by the nonisoscalarity of the nucleus (=N-Z) and lead to an increase in $a_2(x)$ over the entire region of $x$ as compared to the free nucleon value and the Cahn-Gilman limit of $\frac{2}{3} - 4\sin^2\theta_W (= 0.90927)$. The enhancement is smaller at lower $x$ and becomes larger at higher $x$ and it comes mainly due to the nuclear structure effects and not due to mesonic effects. This enhancement is found to be smaller than the enhancement obtained in the work of Cloet et al. [22]. In the isoscalar limit, there is no enhancement in $a_2(x)$ over the free nucleon value but they are larger than the Cahn-Gilman limit in the low $x$ region of $x<0.5$.

In the case of $a_3(x)$, the nuclear medium effects are found to give a suppression at lower $x$ ($x<0.5$) and an enhancement at higher $x$ ($x>0.5$). However, the quantitative change in $a_3(x)$ over its free nucleon value is quite small, but is appreciable as compared to the Cahn-Gilman limit of $\frac{2}{3}(1 - 4\sin^2\theta_W) (= 0.19656)$ at very low values of $x$. The relative contribution of $a_3(x)$ to $A_{PV}$ as compared to $a_2(x)$ is not much affected by the nuclear medium effects, specially in the region of $x>0.4$. Therefore, any experimental determination of $a_3(x)$ from asymmetry measurements in future will have negligible systematic errors due to nuclear medium effects.

Finally the parity violating asymmetry $A_{PV}$ in the scattering of polarised electrons from nuclear targets is found to increase over its free nucleon value due to nuclear medium effects over the entire region of $x$. This enhancement increases with the nonisoscalarity (=N-Z) of the nucleus and is almost negligible for the isoscalar nuclei. The effect of target mass correction is to further increase the asymmetry by 1 - 1.5% in the region of $0.3 < x < 0.8$. These results will be useful in analyzing the experimental data on nuclei like $^{12}$C, $^{56}$Fe and $^{208}$Pb whenever they become available in future and would help to understand the possible signals for physics beyond the standard model.
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