Crack orientation detection in plate-like structures via Lamb wave specular reflection analysis

Xu Li¹, Nan Zhang² and Mengsheng Zhai²

¹ Henan Key Laboratory of Underwater Intelligent Equipment, 713th Research Institute of China Shipbuilding Industry Corporation, Zhengzhou 450015, China
² School of Mechanical Engineering, Xi’an Jiaotong University, Xi’an 710049, China

E-mail: zhangnanagy233@stu.xjtu.edu.cn

Abstract. The determination of crack orientation plays an important role in the non-destructive evaluation (NDE) of plate-like structures. In this paper, the interaction between Lamb waves and cracks is investigated using the time domain spectral element method (SEM) as the simulation tool. On that basis, we analyse the geometric principle of specular reflections and propose an efficient crack angle detection algorithm. During the process, four sensors take turns to excite the Lamb wave which would result in twelve crack reflection signals. Using the corresponding actuator as the centre and the reflection length as the radius, we can obtain twelve reflection circles in total. When these reflections are specular, the mirror points of the actuators with respect to the crack line are exactly the intersection points of the circles. Thus, we can obtain the crack angle by this geometric relation. Validations on different cracks are made and the result of the algorithm shows a good agreement with the real value.

1. Introduction

The Lamb waves technique proves to be useful for structure health monitoring (SHM) due to their ability to travel a long distance without much loss and the sensitivity to micro defects. Fatigue cracking has been a hot research area for Lamb wave detection so far and many works have placed the focus on damage localization as well as the detection of crack orientation [1-4].

Following the traditional pulse-echo case, Tua et al. [2] placed an actuator and a receiver that were collinear with the defect at different locations. Since the highest peak of the reflection signal appeared when the sensor line was perpendicular to the crack, they finally found the correct direction by varying the sensor line at different angle and comparing the reflection amplitudes. Lu et al. [3] studied the forward- and back-scattering of Lamb waves by different cracks and obtained the related reflection and transmission coefficients, which could help quantitatively determine the crack orientation. Yu and Leckey [4] developed a Lamb wave-focusing array algorithm where two tips of the crack appeared as two intensified points. By connecting the two tips, the specific angle could accordingly be calculated. Xu et al. [5] used a circular transducer array with Lamb wave tomography approach and successfully determined orientation of a variety of defects from the resulting images. However, the resolution of these methods which determine the orientation by comparing the amplitude of different directions would be limited since it is hard to keep the incident wave exactly perpendicular to the crack, while the effect of the imaging approaches largely depends on the implementation of algorithms and could be tedious for real use.

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In reality, when Lamb waves interact with a long crack, specular waves would be reflected by the crack face and carry most of the incident energy. Since the reflection angles are related to the orientation of the reflection plane, the specular waves have been used for the angle detection in different areas. For the non-line-of-sight (NLOS) imaging, Tsai et al. [6] proposed an algorithm to estimate the surface normal of the target with the photons and recover its invisible shape based on the local planarity assumption. In the Lamb wave based non-destructive evaluation (NDE) of composites, Muller et al. [7] calculated the edge direction of the plate using the specular boundary reflections with a circular sensor array. Considering it is more accurate and efficient to detect orientations with specular waves, it is necessary to apply it in the field of crack inspection.

In this paper, we investigate the property of specular waves with the aid of the spectral element method and propose a crack angle detection method on that basis. A linear array composed of four sensors is used whose element excited in turn while others are listening. Since each received specular waves can be seen as being transmitted from the mirror point of the actuator with respect to the crack, the crack orientation can be determined in these geometric relations. The rest of papers is organized as follows. In section 2, the spectral element method is reviewed which is taken as the research tool. Section 3 carries out a simulation to model the interaction process between the incident Lamb wave and cracks with different lengths. On that basis, section 4 proposes the crack angle detection method, and section 5 makes a validation. A conclusion is drawn in section 6.

2. The spectral element method (SEM)
For the simulation of the Lamb wave detection, a platform is provided by the SEM with enough efficiency and accuracy. The SEM analysis framework is just the same as the one in FEM. Since our simulation object is a time dependent problem, we need to perform the discretization both in the domain of time and space.

2.1. Spatial discretization
For spatial discretization at a time, the whole 3D physical domain $\Omega$ needs to be divided into many non-overlapped cells $\Omega_e$, where $e = 1, 2, \cdots, n_e$. Since these elements in physical domain may have irregular shapes and thus difficult to calculate, they would be subsequently mapped into a cubic reference domain $\Lambda = [-1, 1]^3$ separately which has a local coordinate system of $\xi, \eta$ and $\gamma$. Unlike the FEM, the displacement field in the spectral element would be approximated by high order polynomials at some particular points (i.e. nodes), and these nodes are unequally distributed in the intervals (see Figure 1). Take the nodes in the $\xi$ direction as an example. If we choose the Lobatto polynomials as the approximating function, these points would be accordingly called the Gauss-Lobatto-Legendre (GLL) nodes, and are decided by

$$ (1-\xi^2)P_N'(\xi) = 0 \tag{1} $$

where $P_N'$ is the first derivative of the $N^{th}$ order Legendre polynomials. Meanwhile, the displacement vector $u_e$ in the reference element can be calculated by

$$ u_e(\xi, \eta, \gamma) = \sum_{i=1}^{n_e} N_i(\xi, \eta, \gamma) \cdot u_i' $$ \tag{2}

The parameter $u_i'$ is the displacement vector at the $i^{th}$ GLL node and $N_i$ is the corresponding shape function matrix.
As to the problem of wave propagation, the SEM has a dynamic equilibrium as the FEM [8], i.e.
\[ \mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{F}. \] (3)

In the equation above, \( \mathbf{M} \) and \( \mathbf{K} \) denote the global mass and stiffness matrix respectively. \( \mathbf{C} \) is the global damping matrix that is dependent on \( \mathbf{M} \) as \( \mathbf{C} = \mu \mathbf{M} \), where \( \mu \) is the damping coefficient. \( \mathbf{F} \) and \( \mathbf{U} \) are the global force and displacement vectors that can vary as the time proceeds. Since the solution domain is divided into many subareas and transformed to the reference space, these matrices and vector will be calculated by assembling their corresponding element terms, which can be written as follows [9],

\[ \mathbf{M}' = \rho \sum_{i} \sum_{j} \sum_{k} \omega_i \omega_j \omega_k [\mathbf{N'}(\xi_i, \eta_j, \gamma_k)^\top \mathbf{N'}(\xi_i, \eta_j, \gamma_k)] \text{det}[J^{ik}_e], \] (4)

\[ \mathbf{K}' = \sum_{i} \sum_{j} \sum_{k} \omega_i \omega_j \omega_k [\mathbf{B'}(\xi_i, \eta_j, \gamma_k)^\top \mathbf{C} \mathbf{B'}(\xi_i, \eta_j, \gamma_k)] \text{det}[J^{ik}_e], \] (5)

\[ \mathbf{F}' = \sum_{i} \sum_{j} \sum_{k} \omega_i \omega_j \omega_k [\mathbf{N'}(\xi_i, \eta_j, \gamma_k)^\top \mathbf{L}(\xi_i, \eta_j, \gamma_k)] \text{det}[J^{ik}_e]. \] (6)

The Lobatto quadrature [10] is applied to transform the original complicated triple integrals into the computationally convenient multiplications when forming these elemental matrices. Parameters \( \omega_i, \omega_j \) and \( \omega_k \) are the quadrature weights correspond to the \( i^{th}, j^{th} \) and \( k^{th} \) points in the direction of \( \xi, \eta \) and \( \gamma \) respectively. \( \mathbf{C} \) is the elasticity matrix that only depends on the Young’s modulus \( E \) and the Poisson ratio \( \nu \) for isotropic materials. \( \mathbf{L} \) and \( \mathbf{B} \) are the load vector and the strain-displacement matrix separately, while \( \text{det}[J^{ik}_e] \) is the determinant of the Jacobian matrix that maps the target area from the reference domain \( \Lambda \) back to the physical domain \( \Omega \). It is worth mentioning that by combining the Lobatto approximating polynomials and the Lobatto quadrature, the spectral element mass matrix \( \mathbf{M}' \) would have a diagonal form in the end. This will greatly increase the computational speed and reduce the complexity of the calculation compared with the conventional FEM. By these means, each part of the equilibrium in (3) can be finally formed.

2.2. Temporal discretization
After the discretization of the space, the next turns to the time. For the temporal discretization, we firstly divide the length of time history into many steps $\Delta t$. Then, the central difference method is applied by
\[
(\frac{1}{\Delta t^2} M + \frac{1}{2\Delta t} C)U_{t+\Delta t} = F - (K - \frac{2}{\Delta t^2} M)U_t - (\frac{1}{\Delta t^2} M - \frac{1}{2\Delta t} C)U_{t-\Delta t},
\]
(7)

With this formula, the displacement $U_{t+\Delta t}$ at the next time step can be calculated by the previous two steps, i.e. $U_t$ and $U_{t-\Delta t}$. Hence, we can finally obtain the displacement vector at arbitrary discrete time point by repeating the iteration. Meanwhile, when solving the equation in (7), the diagonality of the mass matrix makes it possible to calculate the $U_{t+\Delta t}$ without the inversion of $\frac{1}{\Delta t^2} M + \frac{1}{2\Delta t} C$, which again adds to the numerical efficiency. However, as a conditional stable algorithm, the central difference method requires the timestep to satisfy the Courant-Friedrichs-Lewy (CFL) criterion [12], i.e.
\[
\Delta t \leq K \frac{\Delta x}{\max(c)},
\]
(8)

where $c$ is the velocity of the propagation wave modes and $\Delta x$ is the smallest distance between two nodes. For such a second order scheme, $K$ ranges between 0.55 and 0.60 [13].

2.3. Element by element method

The equation (3) is composed of the terms that result from the multiplication of global matrices and vectors. Considering the size of the matrix will grow as the order of the Lobatto polynomials increases, it will consume large storage space if we still assemble the global matrix firstly and multiply it with the corresponding vector. Since $M$, $C$ and $K$ are all sparse matrices assembled from their element ones, we can perform the multiplication on the elemental level instead of the global level. This is the element by element (EBE) method [14] which can be written as follows,
\[
KU = (\sum_{\varepsilon} K_{\varepsilon})U = \sum_{\varepsilon} K_{\varepsilon}u' ,
\]
(9)

where the $\Sigma$ means the assemblage of the element matrices. Therefore, we can get done the whole equation without calculating the global matrix. This would further reduce the need for the storage resources and guarantee the efficiency of SEM in the simulation of high frequency guided wave.

3. Interaction of oblique incident Lamb wave and the crack

A SEM model is then established to simulate the interaction of Lamb waves and the crack. The dimensions of the model are 300 mm (length) × 300 mm (width) × 2 mm (thickness), which is depicted in Figure 2(a). It is a 2024-T3 aluminium plate. The spectral finite elements employed here have the size of 0.005 mm × 0.005 mm × 0.002 mm, whose GLL points in each direction are 6, 6 and 5 respectively. For the convenience of discussion, we set up a coordinate system at the upper left corner, whose axis of z is perpendicular to the plane. A crack is placed in the upper part of the plate which is simulated by disconnecting nodes along the edges of several elements. Since our aim is to investigate the wave reflected by the crack, it is necessary to model different crack lengths for further analysis. Thus, the target length for our discussion is set to vary from 10 mm to 100 mm, which is illustrated in Figure 2(b) in detail. When performing each simulation, the corresponding crack would replace the original 120 mm crack while keeping at the same place. Considering that the defect location remains unknown when inspected in real cases, the incident wave would probably interact with it in an oblique angle. Therefore, we put the actuator in the position that is slightly right from the centre of the plate to simulate the oblique incidence.
In order to excite a single mode of A0 or S0, two identical forces are applied at the same time and in the same actuator locations both on the top and bottom sides of the plate. The directions of the forces are identical for the antisymmetric mode but opposite for the symmetric one. A five-cycle sinusoidal toneburst modulated by a Hanning window is used here as the excitation signal with a central frequency of 600 kHz. Additionally, the sampling frequency of 50 MHz is adopted here to satisfy the CFL criterion and ensure the stability of the simulation. Damping is not considered in this model and no constrain is added to the boundaries. The whole analysis of SEM is performed in the MATLAB platform, from which the result data are exported to ParaView for the further visualization.

Figure 3 shows the displacements in the z direction when the A0 mode interacts with the cracks of different length. All the snapshots are the partial enlargement drawings of the same focus area and are taken at 40 μs. Under the frequency-thickness product of 1.2 MHz·mm, the reflections only contains the A0 mode in the z direction [15].
As what can be seen in these pictures, the guided wave is firstly emitted from the actuator at the lower part of the figure. Then it is reflected by the crack body and its two tips. The former parts result in the specular waves while the latter ones cause the tip diffractions. These two reflections together form the final wave front. These similar situations have also been documented in the works [16,17] that are relevant to the wave detection of cracks. By observing the evolution process from Figure 3 (a) to (f), we can clearly find that the wave front is always composed of the diffractions from the tips no matter how long the crack is, while the range of the specular ones expands as the crack length increases. Meanwhile, most energy of the reflections are concentrated in the specular wave front which has a larger amplitude than the diffractive ones. This makes it easy to extract the information contained in the specular waves from the received signal. Moreover, by mirroring the actuator with respect to the crack line, it is of no surprise to find that the reflections (especially the specular waves) can be viewed as being transmitted from the mirror point (see Figure 4). Since the specularity has a close relation with the direction of cracks in geometry, it can be used to detect the crack’s orientation as a consequence.

4. Crack orientation detection via specular reflection analysis

4.1. Basic principle
Assume that there is an oblique straight crack with enough length lying in the direction of line \( l \). Three sensors are located at another line not far from the crack, among which the one at point \( A \) is the actuator and the other two in point \( S_1 \) and \( S_2 \) are the receivers (see Figure 5). When transmitted from point \( A \), the Lamb wave in a certain direction will be reflected by the crack at point \( R_1 \) and \( R_2 \) (denoted as reflection point) before being received by \( S_1 \) and \( S_2 \). Point \( B \) is the mirror point of \( A \) with respect to crack line \( l \), thus the line between \( A \) and \( B \) is perpendicular to \( l \) at the middle point \( M \). Benefiting from the mirroring relation, any point at the crack shares the same distance to \( A \) and \( B \), thus

\[
\| A - R_1 \| + \| R_1 - S_1 \| = \| B - R_1 \| + \| R_1 - S_2 \| ,
\]

where \( A, B, R_1, S_1 \in \mathbb{R}^2 \). Also, \( \theta_1 = \theta_2 \). According to Snell’s Law, the angle of incidence equals to the angle of reflection. Therefore, \( \alpha_i = \alpha_f \). Consequently,

\[
\theta_1 = \theta_2 = 90^\circ - \alpha_1 = 90^\circ - \alpha_2 = \theta_1 ,
\]

and

\[
\theta_1 + (\theta_2 + \alpha_1 + \alpha_2) = \theta_1 + (\theta_2 + \alpha_1 + \alpha_2) = 180^\circ .
\]

This means that point \( B, R_1 \) and \( S_1 \) are on the same straight line, thereby equation (10) evolves to

\[
\| A - R_1 \| + \| R_1 - S_1 \| = \| B - S_1 \| .
\]

From equation (13), an important conclusion can be drawn that the length of the specular reflection path from \( A \) to \( S_1 \) equals the one from the mirror point \( B \) to \( S_1 \). That is the basis of the subsequent method.

Since the point \( A, R_1 \) and \( S_1 \) are in a common triangle, it is obviously that

\[
\| A - S_1 \| < \| A - R_1 \| + \| R_1 - S_1 \| ,
\]

which indicates that the sensor at point \( S_1 \) will firstly receive the direct wave from the actuator and subsequently obtain the specular reflections. By carefully designing the excitation signal, the time-of-flight (ToF) of the direct wave and the reflections can be easily extracted from the time domain. Hence, we could determine the wave speed in current situation with the ToF of the direct wave and the distance of point \( A \) and \( S_1 \). The length of the specular reflection path \( \| B - S_1 \| \) could also be calculated thereby and the calculation process is the same for the path length \( \| B - S_2 \| \). Taking the reflection length as the radius and the corresponding receiver as the centre, we can draw two specular reflection circles \( C_1 \) and \( C_2 \) respectively. For clarity, only the arcs of the circles are drawn in the diagram and the same way is applied in the follows. The mirror point \( B \) is exactly the intersection point of those circles. Moreover, since the connection line between \( A \) and \( B \) is perpendicular to the crack line, the crack orientation can be finally determined according to the relation of their slopes.

4.2. Enhancement of the accuracy

The key step of the basic principle is to precisely locate the mirror point, which is the intersection of two reflection circles. However, this can be easily influenced by two factors. First of all, some errors could be introduced when calculating the ToFs of the direct waves and reflections, which would lead to an erroneous radius. A simple solution would be adding new sensors to form a linear sensor array. As the quantity of the resulting angles is \( n(n-1) \), where \( n \) is the number of sensors, more sensors would bring more candidate angles so that errors can be reduced by making an average. Another factor worth our attention is the tip diffractions. As shown in section 3, twelve reflections would inevitably include the diffractions in most cases. Since they cannot be seen as coming from the same mirror point of the actuator, the intersection points formed by different reflection circles would not be identical anymore. This will lead to an error if all the reflection signals are treated as the specular ones. However, as diffractions have obvious smaller amplitudes than the specular waves, to ensure the accuracy, a selection procedure must be added before the calculation.

5. Validation of the proposed algorithm
For the verification of the methods proposed in section 4, we construct the SEM model to simulate the detection processes and extract the signals received by the sensor points. An ideal case is firstly considered where the crack length is set to be 130 mm to ensure that all the reflections are specular. This model is based on the $30^\circ$ crack case and places four sensors $S_1$, $S_2$, $S_3$, and $S_4$ on a line with an interval of 50 mm. To simplify the modelling process, the crack is still set parallel to the near edge while the line of sensors is assigned with an angle oblique to it. The detailed diagram can be found in Figure 6.

In this simulation, we adopt the same excitation conditions as what have been used in section 3. Both S0 and A0 mode are tested with a central frequency of 600 kHz to select the suitable excitations. Meanwhile, the cases with no defects for both modes are also simulated to work as a baseline. By subtracting these signals, pure reflections could be extracted from the overlap of the direct waves if they cannot be separated in the defect signals. Figure 7 depicts the signals received by the other three sensor points when forces are applied at point $S_1$.

Figure 7. Signals received at the sensor points of $S_2$, $S_3$, and $S_4$ respectively when forces are applied at $S_1$ to excite (a) the A0 mode and (b) S0 mode with a central frequency of 600 kHz. Pure crack reflections of the S0 mode are shown in (d) by subtracting the corresponding baseline (c) from (b).
For the original signals in Figure 7 (a) and (b), it is easy to recognize the waves directly traveling from the actuator and the receivers for both the A0 and S0 mode. However, things are different for the crack reflections. The S0 mode has a such higher speed than A0 that all the three reflections are partly covered by their direct waves (see Figure 7 (b)), which is not suitable for the detection of a small size plate. Figure 7 (d) depicts the pure crack reflections of S0 after subtracting the baseline (c). It is clear that the dispersion has distorted reflection waveforms and made them spread out as the propagation distance increases. This adds an obstacle for the extracting of ToFs. Therefore, we choose A0 as the mode for our following research.

After separating the reflections from the received signals and discarding the diffractions, the next step is to calculate the ToFs of the direct waves and specular reflections, which would be used to calculate the wave velocity and lengths of the reflection paths. Firstly, we use the cross-correlation analysis to measure the coherence between the target signal \( X(t) \) and the excitation \( Y(t) \), which is given as [18]

\[
R_{xy}(\tau) = \frac{1}{\tau} \int_{-\infty}^{\infty} X(\tau)Y(\tau - t)d\tau = \frac{1}{\tau} \int_{-\infty}^{\infty} X(\tau + t)Y(\tau)d\tau . \tag{15}
\]

Then, a Hilbert transform is performed on that basis to calculate the envelope. By locating the times where the peaks appear in the graph, we could finally obtain the ToFs of the direct waves and the reflections. Table 1 lists the detailed information when \( S_1 \) exciting the 600 kHz A0 mode. Since we can only extract the signal at the node of the spectral element, the distances between each of the sensor points have a slight deviation from the original plan. The errors of the A0 wave speed are all within 1%, which again proves the effectiveness and accuracy of the simulation. After getting the average wave speed on this plate, it is multiplied by the ToFs extracted from the reflections to obtain the corresponding path lengths. The references here are the theoretical lengths of the traveling path since all the reflections are specular with such a crack. The resulting errors are also within the limit of 1%. Thus, it is reliable to use the SEM platform for our following research.

**Table 1.** Detailed information of the three received signals when \( S_1 \) exciting the A0 mode of 600 kHz. The crack has a length of 130 mm to ensure the specular reflections.

| Sensor point | \( S_2 \) (mm) | \( S_3 \) (mm) | \( S_4 \) (mm) | Average |
|--------------|----------------|----------------|----------------|---------|
| Distance between \( S_1 \) and the point | 49.9 | 100.2 | 150.1 | - |
| Direct waves | ToF (\( \mu \)s) | 15.68 | 31.36 | 47.00 | - |
| Wave velocity (m/s) | 3183.2 | 3194.2 | 3193.3 | 3190.2 | 3176.2 |
| Error | 0.22% | 0.57% | 0.54% | 0.44% |
| Reflections | ToF (\( \mu \)s) | 27.38 | 41.52 | 56.62 | - |
| Path length (mm) | 87.35 | 132.46 | 180.63 | - |
| Reference (mm) | 86.52 | 132.48 | 180.37 | - |
| Error | 0.96% | 0.02% | 0.14% | - |

Once the lengths of the twelve traveling paths are calculated, we can use them as the radius and draw the reflection circles with the centres locating at each of the actuator points. Figure 8 depicts outcome of the 130 mm crack model. The original point is set on the sensor line that is 50 mm left from \( S_1 \). Every two of the three circles would intersect at the mirror points since all the reflections are specular. It is obvious that event under the most ideal situation, the intersection points for the same actuator do not totally overlap with each other.
As what has been analysed in section 4, the orientation detection method depends heavily on the determination of the mirror points, which further relies on the specular waves. However, considering that there is only one specular reflection point on the crack (or its extension line) for a fixed pair of incident and reflection angles, the quantity of reflection points is at most 6 for a linear array composed of 4 sensors, which depends on the crack length and its relative position with the sensor array. Therefore, it is necessary to examine the effect on different crack lengths according to the numbers of reflection points that covered by the crack.

Figure 9 (a) depicts the traveling path of the Lamb wave for all the reflections. Under this situation, the reflection points are unequally distributed along the crack line. Ten cases with the same orientation but different positions as illustrated in Figure 9 (b). These cases have at least three specular points and will replace the original case of 130 mm when performing the simulation.

Table 2 shows the quantity of specular reflection circles that can be drawn from the corresponding sensor point. The number of resulting mirror points are also shown which, in fact, correspond to the candidates of angles that can be got in the end. The case with the most number of mirror points is J which reflects the incident wave to all the other three sensors in each excitation. As the reflection points decreases, the number of mirror points would also decline.
Table 2. Different crack cases and their corresponding resulting intersection points.

| Number of covered specular reflection points | Crack type | Number of specular reflection circles drawn from the corresponding sensor point | Number of resulting mirror points |
|---------------------------------------------|------------|-------------------------------------------------------------------------------|----------------------------------|
| 3                                           | A          | $S_1$ 3 $S_2$ 1 $S_3$ 1 $S_4$ 3                                               | 3                                |
|                                             | B          | $S_1$ 2 $S_2$ 1 $S_3$ 2 $S_4$ 2                                               | 2                                |
|                                             | C          | $S_1$ 1 $S_2$ 2 $S_3$ 1 $S_4$ 2                                               | 2                                |
|                                             | D          | $S_1$ 0 $S_2$ 2 $S_3$ 2 $S_4$ 3                                               | 3                                |
| 4                                           | E          | $S_1$ 3 $S_2$ 2 $S_3$ 2 $S_4$ 1                                               | 5                                |
|                                             | F          | $S_1$ 2 $S_2$ 2 $S_3$ 2 $S_4$ 2                                               | 4                                |
|                                             | G          | $S_1$ 1 $S_2$ 2 $S_3$ 3 $S_4$ 2                                               | 5                                |
| 5                                           | H          | $S_1$ 3 $S_2$ 3 $S_3$ 2 $S_4$ 2                                               | 8                                |
|                                             | I          | $S_1$ 2 $S_2$ 3 $S_3$ 3 $S_4$ 8                                               | 8                                |
| 6                                           | J          | $S_1$ 3 $S_2$ 3 $S_3$ 3 $S_4$ 12                                              | 12                               |

Table 3 lists the detected crack orientation of the ten crack cases. It is obvious that the outcome shows a great agreement with the real value. Most of the absolute errors are kept within 1°. Even for the cases with three specular reflection points like A, C and D, the error can also be controlled within 2°. This can be ascribed to the influence of errors which are introduced in processes of the simulation and calculation. Those cases with more reflection points (such as case F-J) would obtain more specular waves in the calculation, thus reducing the deviation brought by a single measurement.

Table 3. The outcome for each crack type.

| Crack | A    | B    | C    | D    | E    | F    | G    | H    | I    | J    |
|-------|------|------|------|------|------|------|------|------|------|------|
| Estimated orientation                   | 28.56 | 26.74 | 31.94 | 29.41 | 27.25 | 30.25 | 30.11 | 29.36 | 29.66 | 29.98 |
| Error (°)                               | 1.44  | 3.26  | 1.94  | 0.59  | 2.75  | 0.25  | 0.11  | 0.64  | 0.34  | 0.02  |

6. Conclusions
In this paper, a detection algorithm for the crack orientation is proposed based on the specularity of Lamb wave reflections. It is of great potential in structural health monitoring since the method can accurately figure out crack angles with only few sensors. Some conclusions can be obtained as follows.

1. Lamb waves are mainly composed of specular waves and tip diffractions. The specular waves can be viewed as being transmitted from the place which is the mirror point of the actuator with respect to the crack. Thus, the angle can be figured out in geometry.

2. During the analysis, the time domain spectral element method shows both its accuracy and efficiency in the configuration modelling, signals extraction and results visualization, which provides a suitable platform for the validation of method under such a high frequency.

3. For a damage detection with a high demand of accuracy, the 600 kHz A0 mode is a good tool since it has little dispersion under this frequency- thickness product with a small wavelength. With the aid of cross correlation process, the distances between sensors and cracks can be precisely determined.

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References

[1] Mu W, Sun J, Liu G and Wang S 2019 High-Resolution Crack Localization Approach Based on Diffraction Wave. *Sensors*, 19(8), 1951

[2] Tua P S, Quek S T and Wang Q 2004 Detection of cracks in plates using piezo-actuated Lamb waves. *Smart Materials and Structures*, 13(4), 643

[3] Lu Y, Ye L, Su Z and Huang N 2007 Quantitative evaluation of crack orientation in aluminium plates based on Lamb waves. *Smart Materials and Structures*, 16(5), 1907

[4] Yu L and Leckey C A 2013 Lamb wave–based quantitative crack detection using a focusing array algorithm. *Journal of Intelligent Material Systems and Structures*, 24(9), 1138-52

[5] Xu C, Rose J L and Zhao X 2009 Detection principle of shape and orientation of corrosive defects using Lamb waves. *Journal of Robotics and Mechatronics*, 21(5), 568-73

[6] Tsai C Y, Kutulakos K N, Narasimhan S G and Sankaranarayanan A C 2017 The geometry of first-returning photons for non-line-of-sight imaging. *In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition* 7216-24

[7] Muller A, Robertson-Welsh B, Gaydecki P, Gresil M, and Soutis C 2017 Structural health monitoring using lamb wave reflections and total focusing method for image reconstruction. *Applied Composite Materials*, 24(2), 553-73

[8] He S and Ng C T 2017 Modelling and analysis of nonlinear guided waves interaction at a breathing crack using time-domain spectral finite element method. *Smart Materials and Structures*, 26(8), 085002

[9] Peng H, Meng G and Li F 2009 Modeling of wave propagation in plate structures using three-dimensional spectral element method for damage detection. *Journal of Sound and Vibration*, 320(4-5), 942-54

[10] Pozrikidis C 2005 *Introduction to finite and spectral element methods using MATLAB*. (CRC Press)

[11] Ostachowicz W, Kudela P, Krawczuk M, and Zak A 2011 *Guided waves in structures for SHM: the time-domain spectral element method*. John Wiley & Sons

[12] Chaljub E, Komatitsch D, Vilotte J P, Capdeville Y, Valette B and Festa G 2007 Spectral-element analysis in seismology. *Advances in Geophysics*, 48, 365-419

[13] Mohamed R, Demers D L and Masson P 2011 A parametric study of piezoceramic thickness effect on the generation of fundamental Lamb modes. *In Health Monitoring of Structural and Biological Systems 2011*, Vol. 7984, 79841Y

[14] Seriani G 1997 A parallel spectral element method for acoustic wave modelling. *Journal of Computational Acoustics*, 5(01), 53-69

[15] Santhanam S and Demirli R 2013 Reflection of Lamb waves obliquely incident on the free edge of a plate. *Ultrasonics*, 53(1), 271-82

[16] Jacques F, Moreau F and Ginzel E 2003 Ultrasonic backscatter sizing using phased array–developments in tip diffraction flaw sizing. *Insight-Non-Destructive Testing and Condition Monitoring*, 45(11), 724-8

[17] Rajagopal P and Lowe M J S 2006 Interaction of the fundamental shear horizontal mode with a through thickness crack in an isotropic plate. *In AIP Conference Proceedings*, Vol. 820, No. 1, 157-64

[18] Zhang Y, Wang Y, Zuo M J and Wang X 2008 Ultrasonic time-of-flight diffraction crack size identification based on cross-correlation. *In 2008 Canadian Conference on Electrical and Computer Engineering*. IEEE, 1797-800