Are We Far from Testing General Relativity with the Transiting Extrasolar Planet HD 209458b ‘Osiris’?

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Abstract

In this paper we investigate the possibility of measuring the general relativistic gravitoelectric correction $P^{(GE)}$ to the orbital period $P$ of the transiting exoplanet HD 209458b ‘Osiris’. It turns out that the predicted magnitude of such an effect is $\sim 0.1$ s, while the most recent determinations of the orbital period of HD 209458b with the photometric transit method are accurate to $\sim 0.01$ s. It turns out that the major limiting factor is the error in the measurement of the predicted semiamplitude of the star’s radial velocity $K_M$. Indeed, it affects the determination of the mass $m$ of the planet which, in turn, induces a systematic error in the Keplerian period $P^{(0)}$ of $\sim 8$ s. However, the present situation may change if future one-two orders of magnitude improvements in the Doppler spectroscopy technique will occur.

1 Introduction

In this paper we investigate the possibility of performing a test of general relativity, in its weak-field and slow-motion approximation, for the first time in a planetary system other than our Solar System$^1$.

More precisely, it has been shown (Soffel 1989; Mashhoon et al. 2001) that the general relativistic gravitoelectric (GE) part of the gravitational field of a static, spherically symmetric mass$^2$ also induces an additional small correction $P^{(GE)}$ to the Keplerian orbital period $P^{(0)}$ of a test particle. For

$^1$See e.g. http://www.obspm.fr/encycl/ for regularly updated information about the extrasolar planets.

$^2$It is the Schwarzschild component of the space-time metric tensor which gives rise to various tested phenomena (Will 1993) like the well known secular Einstein pericenter advance, the deflection of the electromagnetic waves, the Shapiro time delay and the de Sitter geodetic precession of the spin of an orbiting gyroscope.
a circular orbit of radius $a$ around a body of mass $M$ it is, to order $O(c^{-2})$

$$P^{(GE)} \sim \frac{3\pi}{c^2} \sqrt{GMa},$$

(1)

where $G$ is the Newtonian gravitational constant and $c$ is the speed of light in vacuum. Such an effect has not yet been directly measured. The possibility of detecting it in the Sun-Mercury system, for which it amounts to $P^{(GE)} \sim 0.3 \, \text{s}$, has preliminarily been studied in (Iorio 2005).

At first sight rather surprisingly, it might be easier to detect $P^{(GE)}$ in some of the recently discovered exoplanets than in our Solar System. Indeed, many of the exoplanets whose parameters and ephemerides are best known have masses comparable to that of Jupiter and orbital periods of just a few days. This allows for a high accuracy—to a subsecond level in some cases—in measuring their periods from combined photometric transit and spectroscopic radial velocity measurements. Indeed, it is more likely to observe a transit if the orbital period is short. Moreover, the magnitude of the disturbing effects can only be observed if the main star is not too massive and the planet is not too light. Of course, such kind of measurements could not be performed in the case of Mercury due to its extremely tiny perturbing effect on the Sun and its relatively long period of almost 88 days. Only difficult and rather sparse astrometric measurements could be done.

The possibility of testing other relativistic signals which affect the orbital motion, like the gravitoelectric secular precessions of the periastron $\omega$ and of the mean anomaly $\mathcal{M}$, and the gravitomagnetic secular precessions of $\omega$ and of the longitude of the ascending node $\Omega$, must be ruled out because the ephemerides of the extrasolar planets do not currently allow to determine all the details of the orbital paths except for the orbital period.

2 The Two-Body Relativistic Correction to the Orbital Period

In the first post-Newtonian approximation, the motion of two particles of rest masses $M$ and $m$, each one moving in the gravitational field of the other, can be studied in the post-Newtonian center of mass frame (Soffel 1989). In the standard isotropic coordinate\(^3\) $r$, not to be confused with the Schwarzschild radial coordinate $r'$ which, for $M >> m$, is $r' = r(1 +$}

\(^3\)It is the coordinate used in the force models of the post-Newtonian equations of motion (Estabrook 1971) adopted, e.g., for the computation of the planetary ephemerides by the Jet Propulsion Laboratory (JPL).
\( GM/2c^2r^2 \), the general relativistic equation of motion is (Soffel 1989)

\[
\ddot{r} = -\frac{\mu}{r^3} r + \frac{\mu}{c^2 r^3} \left\{ \left[ \frac{\mu}{r} (4 + 2\nu) - (1 + 3\nu) \dot{r}^2 + \frac{3\nu}{2r^2} (r \cdot \dot{r})^2 \right] r + (r \cdot \dot{r})(4 - 2\nu) \dot{r} \right\},
\]

(2)

where \( \mu \equiv G(M + m) \), \( \nu \equiv Mm/(M + m)^2 \) and the overdot represents the time derivative.

In the case of circular orbits, i.e. \( r \cdot \dot{r} = 0 \), which often occurs for many exoplanets orbiting close to their star due to tidal circularization, the post-Newtonian acceleration is entirely radial. In polar coordinates (2) reduces to

\[
r \dot{\phi}^2 = \frac{\mu}{r^2} - \frac{\mu}{c^2 r^2} \left[ \frac{\mu}{r} (4 + 2\nu) - (1 + 3\nu)(r \dot{\phi})^2 \right].
\]

(3)

The orbital period can easily be calculated from (3) for \( r = \text{const} \equiv a \). From

\[
\dot{\phi} = \sqrt{\frac{\mu}{a^3} \frac{1 - \frac{\mu}{c^2 a} (4 + 2\nu)}{1 - \frac{\mu}{c^2 a} (1 + 3\nu)}}
\]

(4)

one gets that the time \( P \) required for \( \phi \) in describing an angular interval of \( 2\pi \) with respect to a fixed direction is \( P = P^{(0)} + P^{(\text{GE})} \) with

\[
P^{(0)} = 2\pi \sqrt{\frac{a^3}{\mu}},
\]

(5)

\[
P^{(\text{GE})} = \frac{3\pi}{c^2} \sqrt{\frac{1}{a} \frac{\mu}{c^2 a}} \left( 1 - \frac{\nu}{3} \right) + \mathcal{O}(c^{-4}).
\]

(6)

Note that for \( M \gg m \), i.e. \( \mu \to M \) and \( \nu \to 0 \), (6) reduces to (1) (Mashhoon et al 2001). Moreover, (6) agrees with the expression (A2.50) obtained by Soffel (1989) in the Brumberg representation.

### 3 What Kind of Exoplanets are Suitable for Better Measuring the Relativistic Correction to the Orbital Period?

Let us evaluate the orders of magnitude involved in a typical exoplanetary scenario with \( M = M_\odot \), \( m = m_{\text{Jup}} \), \( a = 0.1 \) astronomical units (AU); the term \( (3\pi/c^2)\sqrt{\mu a}/\nu \sim 0.1 \) s, while \( \nu/3 \sim 10^{-4} \), so that it can safely be neglected in the subsequent analysis because it would be undetectable. In order to measure \( P^{(\text{GE})} \) two requirements must be fulfilled
• The planet must be as closest as possible to its star, i.e. \( a \) must be as smallest as possible. Indeed, in this case the orbital period is short and a large number of orbital revolutions can be recorded, thus increasing the observational accuracy. The observational error \( \sigma_{P(\text{obs})} \) must typically be smaller than 0.1 s = \( 1 \times 10^{-6} \) days.

• The Keplerian period \( P(0) \) must be subtracted from the observed period in order to single out the relativistic component as \( P(\text{GE}) = P(\text{obs}) - P(0) \). As a consequence, the uncertainty in \( P(0) \) due to the errors in the measured planet’s mass and semimajor axis induces a systematic bias which affects the total error \( \sigma_{P(\text{GE})} = \sigma_{P(\text{obs})} + \sigma_{P(0)}. \) It turns out

\[
\frac{P(\text{GE})}{\sigma_{P(\text{GE})}} = \frac{3}{c^2} \frac{\mu^2}{a \sigma_\mu},
\]

(7)

\[
\frac{P(\text{GE})}{\sigma_{P(\text{GE})}} = \frac{\mu}{c^2 \sigma_a}.
\]

(8)

Thus, in order to reduce the impact of \( \sigma_{P(0)} \) the mass of the system must be as largest as possible, the semimajor axis must be as smallest as possible and, of course, the errors in \( a \) and \( \mu \) must also be small.

In regard to the observational accuracy, the so far discovered exoplanets whose periods have been measured with a precision close to our stringent requirements are TrES-1 (\( \sigma_{P(\text{obs})} = 8 \times 10^{-6} \) days) (Alonso et al. 2004), OGLE-TR-56 (\( \sigma_{P(\text{obs})} = 5.9 \times 10^{-6} \) days) (Konacki et al. 2003) and HD209458b (\( \sigma_{P(\text{obs})} = 1.8 \times 10^{-7} \) days) (Wittenmyer et al. 2005).

4 The HD 209458b System

The most interesting system, for our purposes, is thus HD 209458b ’Osiris’ which, of more than 150 recently discovered extrasolar planets, is the first known to transit its star (Charbonneau et al. 2000; Henry et al. 2000). The most recent determinations of its parameters and ephemerides, based on twenty-seven transit events in four bandpasses and on more than three years of high-precision radial velocities, are due to Wittenmyer et al. (2005).

Its main star, 47 pc far from us, has a mass which has been estimated by Cody & Sasselov (2002) to lie in the range \( M/M_\odot = 1.06 \pm 0.13 \). It is important to note such a range does not come from a direct measurement, i.e. \( M \) is not one of the fifteen system’s parameters simultaneously fitted
in the global data reduction of HD 209458 by means of the Eclipsing Light Curve (ELC) code (Orosz & Hauschildt 2000). It is an interval based on stellar evolution models and measurements of luminosity and temperature: no dynamical effects related in some ways to relativistic orbital motions have been used. $M$ has been held fixed in (Wittenmyer et al. 2005) and different values for its mass have been adopted.

The multi-color transit light curves allow to measure, among other things, the inclination $i$ which amounts to 86.67 deg (Wittenmyer et al. 2005).

The measurement of the radial velocity $K_M$ curve and of the orbital period $P$ allows to determine the mass function $f$ from which the planet’s mass $m$ can, thus, be deduced, for a given value of the stellar mass $M$. The planetary mass ranges from 0.590 m$_{Jup}$ ($M/M_\odot=0.93$) to 0.697 m$_{Jup}$ ($M/M_\odot=1.19$) (Wittenmyer et al. 2005). Note that, up to now, the adopted model for $f(m)$ is entirely Newtonian; see Section 5 for a modified version also including relativistic corrections.

Finally, the semimajor axis $a$ can be obtained from the relation which links it to the measured orbital period $P^{\text{obs}}$ and the system’s masses. The Kepler’s third law is used to this aim. The so obtained values in (Wittenmyer et al. 2005) range from 0.044 AU ($M/M_\odot = 0.93$) to 0.048 AU ($M/M_\odot = 1.19$).

In regard to $P^{\text{GE}}$, it turns out that, by using the adopted range for $M$ and the measured values for $a$ it ranges from 0.095 s to 0.112 s. Note that the assumption of a circular orbit for HD 209458b, adopted also by Wittenmyer et al. (2005) in their analysis, is consistent with a tidal circularization time of order $10^8$ years (Bodenheimer, Laughlin & Lin 2003) and radial velocity observations (Mazeh et al. 2000).

The orbital period is measured independently of any gravitational theory model as one of the fifteen parameters of the fit to the observations with the ELC software. Such important parameter has been determined with high accuracy by combining the ground-based observations, the transits observed with the photomultiplier tubes (PMTs) in the Fine Guidance Sensor (FGS) of the Hubble Space Telescope (HST) and the transits observed with the Space Telescope Imaging Spectrograph (STIS) over many cycles. The quoted value in (Wittenmyer et al. 2005) is

$$P^{\text{obs}}_{(\text{Wittenmyer et al.})} = 3.52474554 \pm 1.8 \times 10^{-7}\,\text{days.} \quad (9)$$

The uncertainty in $P$ is thus 0.016 s. Similar results have also been obtained by Schultz et al. (2004) who quoted

$$P^{\text{obs}}_{(\text{Schultz et al.})} = 3.52474408 \pm 2.9 \times 10^{-7}\,\text{days,} \quad (10)$$
with an uncertainty of 0.025 s. The situation is, thus, now more favorable than that described in (Iorio 2005) in which the exoplanet OGLE-TR-132b was examined.

However, it must be noted that there is still a discrepancy of 0.126 s between the two measurements: it is as large as the relativistic effect itself. Such an uncertainty is due to the fact that, despite their very high photometric precision, neither the FGS or STIS can obtain an uninterrupted observation of the transit due to the low orbit of the HST. The data from the Canadian Microvariability and Oscillations of STars (MOST) satellite, launched in June 2003, should overcome these problems enabling to continuously observing HD 209458 for many transits with exceptionally high precision (Rucinski et al. 2003).

The fact that the predicted relativistic component of the orbital period \( P^{(GE)} \) falls into the measurability domain for HD 209458b is very important because it opens the possibility of measuring it provided that the obtainable precision is high enough. As a consequence, it may turn out that relativistic corrections should be accounted for, e.g., in modelling both the mass function and the orbital period in order to refine the measurements of \( m \) and \( a \). In Section 5 we will deal with such topics.

5 A Modified Mass Function to Order \( \mathcal{O}(c^{-2}) \)

In the barycentric frame the relation

\[
M a_M = ma_m
\]

(11)
can be assumed for the star and its planet: indeed, the post-Newtonian corrections of order \( \mathcal{O}(c^{-2}) \) to it are negligible because are proportional to (Wex 1995)

\[
\epsilon_{1PN} = \frac{1}{c^2} \frac{m}{M} \frac{1}{(1 + \frac{m}{M})^3} \left[ v^2 - \frac{G(m + M)}{r} \right],
\]

(12)
where \( v \) and \( r \) are the relative velocity and distance. Moreover, for circular orbits, as in this case, (12) vanishes. By adding \( ma_M \) to both members of (11) it is possible to straightforwardly obtain

\[
da^3 \sin^3 i = \frac{a^3 m^3 \sin^3 i}{(M + m)^3},
\]

(13)
where \( a = a_M + a_m \). By using the relation

\[
K_M = \left( \frac{2\pi a_M}{P} \right) \sin i = \left( \frac{m}{M + m} \right) \left( \frac{2\pi a}{P} \right) \sin i,
\]

(14)
where $K_M$ is the projected semi-amplitude of the star’s radial velocity, is it possible to derive the general relation

$$\frac{K_M^3 P^3}{8\pi^3} = \frac{a^3 m^3 \sin^3 i}{(m + M)^3}. \quad (15)$$

Note that the left-hand side of (15) is a phenomenologically determined quantity.

If we use the third Kepler’s law

$$P^2 = 4\pi^2 \frac{a^3}{G(M + m)} \quad (16)$$

to model the orbital period $P$ it is possible to obtain the so-called mass function $f(M,m)$ as

$$f \equiv \frac{PK_M^3}{2\pi G} = \frac{m^3 \sin^3 i}{(M + m)^2}, \quad (17)$$

For HD 209458b it amounts to $82.7 \pm 1.3 \text{ m s}^{-1}$ (Wittenmyer et al. 2005). The relation (17) is of the utmost importance because it is possible to measure the planet’s mass $m$ by keeping $M$ fixed and measuring $i, P$ and $K_M$ from the photometric transit curve and Doppler spectroscopy.

The present-day observational accuracy in measuring the period $P$ of HD 209458b suggests that general relativistic corrections might play a role in the system’s parameters determination process. Thus, we will now derive a modified expression of the right-hand side of (17) to order $O(c^{-2})$. To this aim, let us write

$$P \sim 2\pi \sqrt{\frac{a^3}{\mu} + \frac{3\pi}{c^2} \sqrt{\mu a}}. \quad (18)$$

By defining

$$\begin{align*}
x & \equiv a, \\
q & \equiv \frac{4\pi^2}{\mu}, \\
b & \equiv \frac{12\pi^2}{c^2}, \\
d & \equiv P^2,
\end{align*} \quad (19)$$

(18) can be written to order $O(c^{-2})$ as

$$qx^3 + bx^2 = d. \quad (20)$$
Let us rewrite (20) as
\[ qx + b = \frac{d}{x^2}; \tag{21} \]
since \( q, b, d, x \) are positive it admits a solution which is also unique. Let us look for a solution of the form
\[ x = x^{(0)}(1 + \epsilon), \tag{22} \]
with \( \epsilon \ll 1 \) because of order \( \mathcal{O}(c^{-2}) \) and
\[ x^{(0)} = \left( \frac{d}{q} \right)^{1/3}. \tag{23} \]
Inserting (22) into (21) yields, to order \( \mathcal{O}(c^{-2}) \)
\[ \epsilon \sim -\frac{b}{3x^{(0)}q}. \tag{24} \]
Thus,
\[ x \sim x^{(0)} - \frac{b}{3q}, \tag{25} \]
i.e.
\[ a \sim \left( \frac{\mu P^2}{4\pi^2} \right)^{1/3} - \frac{\mu}{c^2}. \tag{26} \]
For HD 209458b the relativistic correction to the semimajor axis amounts to \(-1 \times 10^{-8}\)AU.

The modified mass function can be obtained by inserting (26) into the right-hand side of (15). By using
\[ a^3 \sim \frac{\mu P^2}{4\pi^2} - \frac{3}{c^2} \left( \frac{P}{2\pi} \right)^{4/3} \mu^{5/3}, \tag{27} \]
it is straightforward to obtain
\[ \frac{K^3 M P}{2G\pi} \sim \frac{m^3 \sin^3 i}{(m + M)^2} \left[ 1 - \frac{3}{c^2} \left( \frac{2\pi\mu}{P} \right)^{2/3} \right]. \tag{28} \]
Thus one has \( f = f^{(0)} + f^{(GE)} \) with, to order \( \mathcal{O}(c^{-2}) \),
\[ f^{(0)} = \frac{m^3 \sin^3 i}{(M + m)^2}, \tag{29} \]
\[ f^{(GE)} \sim \frac{3m^3 \sin^3 i}{c^2} \left( \frac{2\pi G}{P} \right)^{2/3} \left( \frac{1}{M + m} \right)^{4/3}. \tag{30} \]
For HD 209458b $f^{(0)} = 2 \times 10^{-10} \, M_\odot$ and $f^{(GE)} = -1.5 \times 10^{-16} \, M_\odot$. The observational uncertainty in $f$ can be evaluated as

$$
\sigma_{f^{(obs)}} \leq \left( \frac{K^3_M}{2\pi G} \right) \sigma_{P^{(obs)}} + \left( \frac{3PK^2_M}{2\pi G} \right) \sigma_{K_M} = 1 \times 10^{-11} \, M_\odot.
$$

We have assumed $\sigma_{P^{(obs)}} = 0.01 \, s$ (Wittenmyer et al. 2005), and $\sigma_{K_M} = 1.3 \, m \, s^{-1}$ (Wittenmyer et al. 2005). This means that the relativistic corrections to the mass function are too small to play a role in the parameters' determination. It turns out that the major limiting factor is that due to the uncertainty in $K_M$ ($1 \times 10^{-11} \, M_\odot$); the impact of $P^{(obs)}$ is smaller amounting to $1 \times 10^{-17} \, M_\odot$.

The error in the planet’s mass $m$, which is measured from (17) and is mainly limited by $\sigma_{K_M}$ via $\sigma_{m} \sim \sigma_{f} / \sin^3 i$, also affects the measurement of $a$. The uncertainty in $a$ is

$$
\sigma_a \leq \frac{2}{3} \left[ \frac{G(M + m)}{4\pi^2 P} \right]^{1/3} \sigma_{P^{(obs)}} + \frac{1}{3} \left[ \frac{GP^2}{4\pi^2(M + m)^2} \right]^{1/3} \sigma_m = 4 \times 10^{-7} \, AU.
$$

We have used $\sigma_m = 0.034 \, m_{\text{Jup}}$ (Wittenmyer et al. 2005). The impact of $\sigma_m$ induces an error of $10^{-7} \, AU$. The uncertainty in the measured period yields an error of $10^{-8} \, AU$. Thus, the relativistic correction to $a$ is too small to be measured due to $\sigma_m$.

### 6 The Systematic Error in the Keplerian Period

The Keplerian period $P^{(0)}$ must be subtracted from the measured value of $P^{(obs)}$ in order to extract the relativistic correction $P^{(GE)}$, provided that the uncertainty in $P^{(0)}$ is not larger than $P^{(GE)}$ itself\(^\dagger\).

The precision with which $m$ can be determined is important for our purposes because $\sigma_m$ fixes, directly and indirectly via $\sigma_a$, the uncertainty in the knowledge of the Keplerian period

$$
\sigma_{P^{(0)}} \leq \pi \sqrt{\frac{a^3}{G(M + m)^3}} \sigma_m + 3\pi \sqrt{\frac{a}{G(M + m)}} \sigma_a = 4.145 \, s + 4.157 \, s = 8.302 \, s.
$$

Again, the error in $m$ sets the limit of the measurability of the relativistic effect which is 75 times smaller than $\sigma_{P^{(0)}}$.

\(^\dagger\)Another possible source of systematic bias is represented, in principle, by the classical correction $P^{(J_2)}$ due to the quadrupolar mass moment $J_2$ of the star (Iorio 2005). However, by assuming the fiducial value $J_2 \sim 10^{-5}$ (Miralda-Escudé 2002) it turns out that $P^{(J_2)} \sim -0.01 \, s$. 


7 Discussion and Conclusions

In this paper we have shown that the accuracy with which it is nowadays possible to determine the ephemerides of the transiting extrasolar planet HD 209458b 'Osiris' would allow, in principle, to measure the general relativistic correction to the orbital period \( P \). Indeed, the orbital period of Osiris is measured with a \( \sim 0.01 \) s precision, while the relativistic prediction for the correction \( P^{\text{GE}} \) to the Keplerian period \( P^{(0)} \) amounts to \( \sim 0.1 \) s. It is likely that near-future more accurate measurements of the orbital periods of some of the exoplanets closest to their main stars will allow to extend this possibility also to them. Indeed, one of the goals of the joint CNES-ESA COnvection ROtation and planetary Transits (COROT) mission (http://sci.esa.int/science-e/www/area/index.cfm?fareaid=39), to be launched in 2006, is the discovery of new transiting giant gaseous planets\(^5\) close to their stars, i.e. with periods ranging from a few days to three months. At present, the relativistic component of the orbital periods of the so far discovered exoplanets, other than HD 209458b, turns out to be too small by one order of magnitude to be detected.

If it was really possible to extract such a tiny slowing down of the orbital motion of the planet it would be the first test of general relativity, in its weak-field and slow-motion linearized approximation, in a planetary system other than our Solar System. Moreover, it would be the first measurement of such a consequence of the Schwarzschild gravitoelectric component of the space-time metric.

Unfortunately, in the case of HD 209458b the total error \( \sigma_{P^{\text{GE}}}^{(\text{tot})} = \sigma_{P^{(\text{obs})}} + \sigma_{P^{(0)}} \) is still larger than the investigated effect. Indeed, it turns out that the uncertainties in the planet’s mass \( m \) and semimajor axis \( a \), due to the error in the projected semiamplitude of the star’s radial velocity \( K_M \), induce a mismodelling in the Keplerian period \( \sigma_{P^{(0)}} \) of 8.302 s. One-two orders of magnitude improvements in determining \( K_M \) would be required.

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\(^5\)Also transits due to smaller telluric planets should be observed. An analogous NASA mission, called KEPLER (http://www.kepler.arc.nasa.gov/), will be orbited in 2008. Due to its higher altitude it will be able to discover, among other things, also planets with larger semimajor axes and periods up to two years.
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