AdS Superalgebras with Brane Charges

S. Ferrara\textsuperscript{a}, M. Porrati\textsuperscript{a,b}

(a) Theory Division CERN, Ch 1211 Geneva 23, Switzerland
(b) Department of Physics, NYU, 4 Washington Pl., New York, NY 10003, USA

ABSTRACT

We consider the inclusion of brane charges in AdS\textsubscript{5} superalgebras that contain the maximal central extension of the super-Poincaré algebra on ∂AdS\textsubscript{5}. For theories with \( N \) supersymmetries on the boundary, the maximal extension is \( OSp(1/8N, R) \), which contains the group \( Sp(8N, R) \supset U(2N, 2N) \supset SU(2, 2) \times U(N) \) as extension of the conformal group. An “intermediate” extension to \( U(2N, 2N/1) \) is also discussed, as well as the inclusion of brane charges in AdS\textsubscript{7} and AdS\textsubscript{4} superalgebras. BPS conditions in the presence of brane charges are studied in some details.
1 Introduction

It is well known that the classification of superalgebras, containing the anti-de Sitter –i.e. conformal– superalgebra has a barrier at dimension $D = 7$ [1]. This result is based on the assumption that the bosonic subalgebra of the superalgebra is the “direct product” of the conformal algebra $O(D - 1, 2)$ with an internal symmetry group $G$. This classification can be viewed, essentially, as an extension of the Haag-Lopuszanski-Sohnius theorem [2] for $D > 4$.

On the other hand, in dynamical theories with extended objects, it is already known that the super-Poincaré algebras do include, in any dimension, “central” charges [3, 4, 5, 6, 7] which at first sight violate the above assumption.

It is then natural to consider the same generalization when such charges of rank $p$ are introduced in the AdS superalgebra rather than in its Poincaré counterpart. Such extension has been studied in the literature with the goal of constructing an eleven-dimensional theory in AdS space [8, 9], or a conformal theory in ten dimensions [6].

The result of these investigations is that the conformal extension of the $D = 10$ $N = 1$ Poincaré superalgebra is $OSp(1/32, R)$, in which $Sp(32, R) \supset O(10, 2)$.

In the present paper, we consider a similar extension in the context of AdS supergravity. The difference here is that AdS$_5$ superalgebras already exist. In the absence of $p$-brane charge, they correspond to the usual superconformal algebras $U(2, 2/N)$, which occur in the classification of Haag, Lopuszanski and Sohnius (HLS) [2]. We consider here the AdS$_5$ superalgebra in the presence of AdS $p$-branes, and we show that, for any $N$-extended supergravity, such algebra is $OSp(1/8N)$, with the conformal group $O(4, 2) \sim SU(2, 2)$ embedded as follows in the real symplectic group $Sp(8N, R)$ [10]:

$$Sp(8N, R) \supset U(2N, 2N) \supset SU(2, 2) \times U(N).$$ 

Extensions of conformal superalgebras in $D = 5$ have been considered also in ref. [11], where worldsheet superalgebras for $D5$-branes in an AdS$_5$ background were proposed. Ref. [11] shows, among other things, that, in the presence of $p$-brane charges, the world-sheet superconformal group of a $D3$-brane in AdS$_5$, $U(2, 2/4)$, is extended to $OSp(1/32)$.

This paper is organized as follows: in Section 2, we consider the standard superalgebras in the 5-$D$ Minkowski space, $M_5$, and in AdS$_5$, and view them as the starting blocks for further investigations. In Section 3 we consider the $OSp(1/8N)$ algebras as algebras in AdS$_5$ in the presence of AdS $p$-branes. In Section 4 we give a general, algebraic analysis of the BPS condition in AdS$_5$, and, in particular, we study the pattern of $R$-parity breaking induced by BPS $p$-branes. In Section 5 we present an extension of the analysis performed in Sections 3 and 4 to AdS$_7$. Section 6 contains a brief description of an additional “intermediate” conformal extension of
the 5-\(D\) super-Poincaré algebra, a comment on the uniqueness of such extensions, and a brief analysis of \(AdS_4\) superalgebras and their 1/2 BPS brane configurations.

2 Maximal Central Extensions of Poincaré Superalgebras

The maximal central extension of the Poincaré superalgebra with \(n\) spinorial components of the supersymmetry charges gives an algebra with \(n(n+1)/2\) bosonic central charges\(^3\) including the space-time translations. Examples of such extensions are the \(N = 1\) superalgebra in \(D = 11\) dimensions, in the presence of two- and five-brane charges, and the IIA and IIB algebras in ten dimensions, in the presence of NS and R brane charges\(^8\).

When the space-time dimension is sufficiently low, the maximal central extensions include also BPS domain walls and BPS instantons, as it becomes obvious if one regards such algebras obtained by dimensional reduction. It is relevant to this paper to recall the central extensions in dimensions \(D = 4, 5\), because they will play an important role when the analogous five-dimensional superalgebra will be considered in \(AdS_5\), and \(M_4\) will be interpreted as its boundary.

The \(N\)-extended Poincaré superalgebra in \(D = 5\), with maximal central extension, has a \(USp(2N)\) R-symmetry\(^{12}\), and it reads

\[\{Q_A^a, Q_B^b\} = (\gamma^\mu C)_{\alpha\beta} P_\mu^A \delta_B^A + (\gamma^\mu C)_{\alpha\beta} M_{\mu\nu}^A \delta_B^A + (\gamma^\mu C)_{\alpha\beta} Z^{[AB]}_{\mu
u} + (\gamma^\mu C)_{\alpha\beta} Z^{(AB)}_{\mu},\]  

(2)

where \(Z^{[AB]}_{\mu}\) and \(Z^{(AB)}_{\mu}\) are in the antisymmetric of \(USp(2N)\) \((Z^{[AB]}_{\mu} = 0)\) and \(Z^{(AB)}_{\mu}\) is in the adjoint of \(USp(2N)\).

The standard HLS\(^2\) algebra is obtained by setting \(Z^{[AB]}_{\mu} = Z^{(AB)}_{\mu} = 0\). These charges come from strings and membranes.

3 Anti-de Sitter and Conformal Superalgebras

The Anti-de Sitter superalgebra is a modification of the Poincaré superalgebra given in Eq. (2), where \(P_\mu, M_{\mu\nu}\) span the algebra of \(O(4,2)\), and generators of \(U(N)\) are included.

This is the \(AdS_5\) superalgebra \(U(2,2/N)\). This superalgebra can be formally obtained by decomposing \(USp(2N) \rightarrow SU(N) \times U(1)\) in the former algebra:

\[\{Q_A^a, Q_B^a\} = (\gamma^\mu C)_{\alpha\beta} P_\mu^A \delta_B^A + (\gamma^\mu C)_{\alpha\beta} M_{\mu\nu}^A \delta_B^A + (\gamma^\mu C)_{\alpha\beta} Z^{[AB]}_{\mu
u} + (\gamma^\mu C)_{\alpha\beta} Z^{(AB)}_{\mu} + C_{\alpha\beta} U_{B}^A,\]  

(3)

\[\{Q_A^a, Q_B^b\} = (\gamma^\mu C)_{\alpha\beta} Z^{[AB]}_{\mu} + C_{\alpha\beta} Z^{(AB)}_{\mu} + (\gamma^\mu C)_{\alpha\beta} Z^{(AB)}_{\mu}, \text{ c.c., } \mu, \nu = 0, \ldots, 4.\]  

(4)

\(^3\)More precisely, they are bosonic central charges of the supertranslation algebra, which is a subalgebra of the super-Poincaré algebra.
Setting to zero all bosonic generators except $P_{\mu}$, $M_{\mu\nu}$ and $U^A_{\mu\nu}$, and promoting them to the (non-commutative) generators of the $SU(2,2) \times U(N)$ Lie algebra, with the fermionic generators in the $(4,N) + (\bar{4},\bar{N})$ representation, the algebra becomes
\begin{align*}
\{Q^A_{\alpha}, Q_B^{\beta}\} &= (\gamma^\mu C)_{\alpha \beta} P_{\mu} \delta^A_B + (\gamma^\mu C)^{\alpha \beta} M_{\mu \nu} + C_{\alpha \beta} U^A_{\nu},
\{Q^A_{\alpha}, Q^B_{\beta}\} &= \{Q_{A \alpha}, Q_{B \beta}\} = 0.
\end{align*}
This is the standard $AdS_5$ superalgebra considered in the literature. If realized on the four-dimensional boundary, it corresponds to the HLS superconformal algebra in $D = 4$, which is the conformal extension of the Poincaré superalgebra without central charges.

Let us now consider whether an $AdS_5$ superalgebra exists with non-vanishing $Z$ generators. The $Z$ should correspond somehow to brane charges in $AdS_5$.

The $AdS_5$ extension of the Poincaré superalgebra with charges given in Eqs. (3,4) is immediate. Namely, the $Z$ generators in Eqs. (3,4) complete the superalgebra $OSp(1/8N, R)$. The $SU(2,2) \times U(N)$ generators are embedded as follows in $Sp(8N, R)$:
\begin{align*}
Sp(8N, R) &\rightarrow U(2N, 2N) \rightarrow SU(2,2) \times U(N).
\end{align*}
As it is well known [9], the $Sp(8N, R)$ algebra has a three-grading with respect to the “dilation” generator, $R$ in the decomposition $SU(2,2) \rightarrow SL(2,C) \times R$. Here, $SL(2,C)$ is the Lorentz group of the boundary of $AdS_5$. Indeed,
\begin{equation}
\mathcal{L}_{Sp(8N)} = \mathcal{L}^1 + \mathcal{L}^0 + \mathcal{L}^{-1},
\end{equation}
where $\mathcal{L}^1$ contains $P_{\mu}$ and all (dimension-1) central charges of the 4-D super-Poincaré algebra. $\mathcal{L}^{-1}$ contains $K_{\mu}$ and all (dimension −1) special-conformal central charges, while
\begin{equation}
\mathcal{L}^0 = SL(4N, R) \times R
\end{equation}
is the Lie algebra which contains, among others, the generators of the Lorentz group on $\partial AdS_5$, and $U(N)$ [10]:
\begin{equation}
SL(4N, R) \rightarrow SL(2N, C) \times U(1) \rightarrow SL(2, C) \times SU(N) \times U(1).
\end{equation}
The $OSp(1/8N)$ superalgebra has a 5-grading [9, 13, 14], in which the $\mathcal{L}^{\pm 1}$ subalgebras, in the symmetric representation of $SL(4N)$, are completed with the $8N$-dimensional fundamental representation of $Sp(8N, R)$, which splits under $SL(2, C) \times SU(N) \times U(1)$ as:
\begin{equation}
8N \rightarrow (1/2, 0, N)^{1/2} + (0, 1/2, N)^{1/2} + (0, 1/2, \bar{N})^{-1/2} + (1/2, 0, \bar{N})^{-1/2}.
\end{equation}
This splitting corresponds to writing the $AdS_5$ spinor $Q^A_{\alpha}$ as $(Q^A_{\alpha}, \bar{Q}^A_{\dot{\alpha}})$, and $Q_{\alpha A}$ as $(\bar{Q}_{\dot{\alpha} A}, S_{\alpha A})$. The spinor charges $(Q^A_{\alpha}, \bar{Q}^A_{\dot{\alpha}})$, together with $\mathcal{L}^1$, form the maximal central extension of the
super-Poincaré algebra in $D = 4$, as found in ref. [7]. The $(S_{\beta A}, \bar{S}^A_{\beta})$, together with $L^{-1}$, form an isomorphic algebra, with the substitution $P_{\mu} \to K_{\mu}$ and $Z \to Z_S$. The generators in $L^0$ appear in the mixed anti-commutators $\{Q, S\}$, as it follows from the general structure of the grading:

$$\{Q_A, \bar{Q}^B_{\dot{\beta}}\} = \sigma_{\alpha}^{\mu} P_{\mu} \delta^A_B + Z^{\alpha A}_{\alpha \beta B}, \quad (Z^{A}_{\alpha \beta A} = 0), \quad \mu = 0, \ldots, 3,$$

$$\{S_{\alpha A}, \bar{S}^B_{\dot{\beta}}\} = \epsilon_{\alpha \beta} Z^{AB} + Z^A_{\alpha \beta B}, \quad \text{c.c.,}$$

$$\{S_{\alpha A}, S_{\beta B}\} = \sigma_{\alpha}^{\mu} K_{\mu} \delta^B_A + Z^B_{\alpha \beta A}, \quad (Z^{A}_{\alpha \beta A} = 0), \quad \text{c.c.,}$$

$$\{Q_A, S_{\beta B}\} = [\epsilon_{\alpha \beta} (D + iU) + M_{\alpha \beta}] \delta^A_B + \epsilon_{\alpha \beta U_{\alpha}^A + U_{\alpha \beta}^A} \quad \text{(traceless),}$$

$$\{Q_A, \bar{Q}^B_{\dot{\beta}}\} = W^{[AB]} + W_{\alpha \beta}^{(AB)}, \quad \text{c.c.,}$$

$$\mathcal{L}_{OSP(1/8N)} = \mathcal{S}^+ + \mathcal{L}^0 + SC^-.$$

The total number of generators of $Sp(8N, R)$ is $4N(8N + 1)$, which splits, in this decomposition, as

$$(\text{Sym GL}(4N))^+ + (\text{Adj GL}(4N)) + (\text{Sym GL}(4N))^-, \quad \text{(13)}$$

$$\dim \mathcal{P}^+ = \dim \mathcal{C}^- = 8N^2 + 2N, \quad \text{(14)}$$

$$\dim \mathcal{L}^0 = 16N^2. \quad \text{(15)}$$

Note that in the usual conformal extension of the non-centrally extended $S\mathcal{P}$, that we call $\mathcal{P}_0$,

$$\mathcal{L}_{U(2,2/N)} = \mathcal{S}^+_0 + \mathcal{L}^0_0 + SC^-_0,$$

where $\dim \mathcal{P}^+_0 = \dim \mathcal{C}^-_0 = 4$, $\dim \mathcal{L}^0_0 = 15 + N^2$.

## 4 BPS States in $AdS_5$ and R-Symmetry Breaking

States that preserve some of the supersymmetries of the $D = 4$ $N$-extended super-Poincaré algebra $S\mathcal{P}^+$ can be point-like or extended. These states preserve only subgroups of the R-symmetry $U(N)$, and their breaking pattern can be analyzed in pure algebraic terms. This analysis agrees with previous studies of branes in $AdS_5$ [11, 15] in all known cases, but also predicts general patterns of R-symmetry breaking. It should be emphasized that our analysis is purely in terms of Poincaré multiplets, and that we do not know whether all breaking patterns we find here are effectively realized in the AdS/CFT correspondence [16, 17, 18, 19].

The identification of brane charges with the central charges of the super-Poincaré algebra in Eq. (2) is well established in flat space. The corresponding identification of brane charges with
some bosonic generators of $OSp(1/8N)$ should hold in $AdS_5$, \cite{11,15}. This correspondence was established explicitly in \cite{11} for a brane charge appearing in the world-volume superalgebra of a $D5$-brane in $AdS_5$.

Let us consider, in particular, the case $N = 4$, and let us start our analysis with point-like states (monopoles and dyons). They are associated with a scalar central charge, that we called $Z^{[AB]}$. It can always be put in the form

$$Z^{[AB]} = \begin{pmatrix} \lambda_1 \epsilon & \lambda_2 \epsilon \\ \lambda_2 & -\lambda_1 - \lambda_2 - \lambda_3 \end{pmatrix},$$

(17)

with $\epsilon$ the $2 \times 2$ antisymmetric matrix. When $\lambda_1 = \lambda_2$ we have a $1/2$ BPS state, and the R-symmetry $SU(4)$ is broken to $USp(4) \sim O(5)$. When $\lambda_1 \neq \lambda_2$ one has $1/4$ BPS states, and $SU(4) \rightarrow USp(2) \times USp(2) \sim O(3) \times O(3)$.

String BPS states are charged under the vector charge $Z^A_{\mu B}$. Let us consider a string oriented along one of the coordinate axis. The only nonzero component of $Z^A_{\mu B}$ is, say $Z^1_{1B}$ that can be brought to the standard form:

$$Z^A_{\mu B} = \begin{pmatrix} \lambda_1 & \lambda_2 & \lambda_3 \\ \lambda_2 & -\lambda_1 - \lambda_2 - \lambda_3 \\ -\lambda_1 - \lambda_2 - \lambda_3 \end{pmatrix}. \quad (18)$$

By looking at the $SU(4) \sim O(6)$ subgroup left invariant by this matrix one can easily find all possible R-symmetry breaking patterns. Specifically, $1/2$ BPS states have $\lambda_1 = \lambda_2 = -\lambda_3$; this means that the R-symmetry is broken to $SU(2) \times SU(2) \times U(1)$, or

$$O(6) \rightarrow O(4) \times O(2). \quad (19)$$

$1/4$ BPS states have $\lambda_1 = \lambda_2$, $\lambda_3 \neq \lambda_1$. The R-symmetry breaking is in this case

$$O(6) \rightarrow SU(2) \times U(1)^2. \quad (20)$$

When $\lambda_1 = \lambda_2 = \lambda_3$ the string preserves $3/8$ of the original supersymmetries, while

$$O(6) \rightarrow SU(3) \times U(1). \quad (21)$$

Finally, for generic $\lambda_i$, one finds a $1/8$ BPS state preserving only a $U(1)^3$ subgroup of the R-symmetry.

Finally, 2-branes are sources for the antisymmetric-tensor charge $Z^{(AB)}_{\mu \nu}$, belonging to the symmetric representation of $SU(4)$. By choosing a 2-brane configuration oriented along two coordinate axis, the only nonzero charge is, say, $Z^{(AB)}_{12}$. It can be brought into the standard
In this case, $1/2$ BPS states correspond to all $\lambda_i = \lambda$, and the R-symmetry breaking pattern is

\[ SU(4) \to O(4) = O(3) \times O(3). \]  

(23)

$1/4$ BPS states arise in two cases. First, when $\lambda_1 = \lambda_2$, $\lambda_3 = \lambda_4 \neq \lambda_1$. The residual R-symmetry is, in this case

\[ SU(4) \to O(2) \times O(2). \]  

(24)

In the second case, $\lambda_1 = \lambda_2$, $\lambda_3 \neq \lambda_4$ and neither $\lambda_3$ nor $\lambda_4$ equal $\lambda_1$. The R-symmetry breaks as

\[ SU(4) \to O(2). \]  

(25)

If $\lambda_1 = \lambda_2 = \lambda_3 \neq \lambda_4$, we have $3/8$ BPS states with R-symmetry

\[ SU(4) \to O(3). \]  

(26)

Finally, if all $\lambda_i$ are different, we have $1/8$ BPS states, which completely break R-symmetry.

The $1/2$ BPS states on the boundary can be thought of as “singletons” since they have multiplicity $2^N$ ($N = 4$ in our case), if regarded as $N$-extended 4-D Poincaré multiplets.

We call $s$-singleton the usual singleton associated to a massless particle on the boundary, while we call $p$-singleton a state associated to a $p$-brane on the boundary. Thus, “photons” are $s$-singletons, monopoles or dyons are 0-singletons etc. BPS states propagating in the bulk are “bound states” of $p$-singletons, since they have multiplicity $2^{2N}$.

To summarize, a $p$-singleton breaks the original $O(6)$ R-symmetry to $O(5 - p) \times O(p + 1)$, while the $s$-singleton corresponds to $p = -1$ in this formula.

## 5 Extension to $AdS_7$ Superalgebras

The analysis performed in the previous two Sections can be extended to the $AdS_7$ case, based on the conformal algebra $O^*(8) \sim O(6, 2)$. This case is related to the conjectured duality \cite{20, 16} between M-theory on $AdS_7$ and superconformal field theories in 6-D.

Here, maximal or reduced supersymmetry correspond to the $OSp(8^*/2N)$ superalgebras, with $N = 2, 1$, respectively.

The contraction of these algebras gives the non-maximally centrally extended supertranslation algebra in $D = 7$. 


The maximal central extension of the supertranslation algebra, on the other hand, reads
\[
\begin{align*}
\{Q^A_\alpha, Q^B_\beta\} &= (\gamma^{\mu\nu}C)_{\alpha\beta} P_\mu Q^{AB} + (\gamma^{\mu\nu}C)_{\alpha\beta} M_{\mu\nu} Q^{AB} + (\gamma^{\mu\nu}C)_{\alpha\beta} Z^0_{\mu[AB]} + \\
&\quad + (\gamma^{\mu\nu}C)_{\alpha\beta} Z^0_{\mu\nu} + C_{\alpha\beta} U^{(AB)} + (\gamma^{\mu\nu\rho\sigma}C)_{\alpha\beta} Z^{(AB)}_{\mu\nu\rho\sigma}, \\
A, B &= 1, \ldots, 4, \quad \alpha, \beta = 1, \ldots, 8, \quad \mu, .. = 0, \ldots, 6.
\end{align*}
\]
Its extension, containing the AdS$_7$ isometry algebra $O(6, 2)$, and the R-symmetry algebra $USp(2N)$, is the superalgebra $OSp(1/16N)$, with $N = 2, 1$ respectively. The $O^*(8)$ algebra appears in the decomposition $^4$
\[
Sp(16N, R) \to USp(2N) \times O^*(8).
\]

The $OSp(1/16N)$ superalgebra can be written in a manifestly $O(6, 2)$-invariant form as follows:
\[
\begin{align*}
\{Q^A_\alpha, Q^B_\beta\}^+ &= (\gamma^{\mu\nu}C)^+_{\alpha\beta} M_{\mu\nu} \Omega^{AB} + (\gamma^{\mu\nu}C)^+_{\alpha\beta} Z^0_{\mu[AB]} + (\gamma^{\mu\nu\rho\sigma}C)^+_{\alpha\beta} Z^{(AB)}_{\mu\nu\rho\sigma} + C_{\alpha\beta}^+ U^{(AB)}, \\
\alpha, \beta &= 1, \ldots, 8, \quad \mu, .. = 0, \ldots, 7.
\end{align*}
\]
The subindex + denotes the 8-D chiral projection, and $Z^{(AB)}_{\mu\nu\rho\sigma}$ is self-dual.

This superalgebra reduces to $OSp(8^*/2N)$ by setting to zero the generators $Z^0_{\mu[AB]}$ and $Z^{(AB)}_{\mu\nu\rho\sigma}$.

Notice that for $N = 2$, the embedding (28) of $O^*(8)$ in $Sp(32, R)$ is not unique. Indeed, it differs from the embedding obtained by dimensional reduction of $OSp(1/32)$, interpreted as $AdS_{11}$ superalgebra $^3$. In this case, the superalgebra reads
\[
\begin{align*}
\{Q_\alpha, Q_\beta\} &= (\gamma^{MN}C)_{\alpha\beta} L_{MN} + (\gamma^{MNPQRS}C)_{\alpha\beta} T^+_{MNPQRS}, \\
\alpha.. &= 1, \ldots, 32, \quad M.. = 0, \ldots, 11,
\end{align*}
\]
where $Q_\alpha$ is a 12-D Majorana-Weyl spinor and $T^+_{MNPQRS}$ is self-dual. By splitting the index $M = \mu, i$ ($\mu = 0, \ldots, 7, i = 1, \ldots, 4$) it is easy to see that the above superalgebra contains two subalgebras $OSp(1/16)_{L,R}$, where $Q_\alpha$ splits as $32 = 16_L + 16_R$. The bosonic part of the superalgebra breaks as
\[
Sp(32, R) \to SO(10, 2) \to SO(4) \times SO(6, 2),
\]

$^4$This embedding is obtained by writing the symplectic metric as $\Omega_{16N} = \Omega_{2N} \otimes I_8$. Then, the generators of $Sp(16N, R)$ of factorized form $A = a \otimes I_8$ or $A = I_{2N} \otimes b$, with $a, b$ $2N \times 2N$ and $8 \times 8$ matrices, respectively, form manifestly a subgroup. Here $I_n$ is the $n \times n$ identity matrix and $\Omega_n$ is the canonical $n \times n$ symplectic metric. The symplectic condition on the $Sp(16N, R)$ generators is $\Omega A^T \Omega = A$. The reality condition can be written in different forms. By choosing $A = A^*$, the subgroup of factorized matrices is $Sp(2N, R) \times SO(8)$. By choosing instead $A = \Sigma A^* \Sigma$, with $\Sigma = \Omega_{2N} \otimes I_8$, the subgroup is $USp(2N) \times O^*(8)$ — since $a$ obeys the constraints $a^\dagger = -a, \Omega_{2N} a^\dagger \Omega_{2N} = a$, that define the $USp(2N)$ algebra, while $b$ is antisymmetric, $b^\dagger = -b$, and quaternionic, $\Omega_8 b^* \Omega_8 = -b$. The latter constraints define the generators of $O(4, Q) \sim O^*(8)$.
where $SO(4) = SO(3)_L \times SO(3)_R$, and $SO(6,2) = O^*(8)_L + O^*(8)_R$. The 528 generators of $Sp(32, R)$ are in this case arranged as two copies of $Sp(16, R)$, and 256 additional generators, $Z_{\mu i}$, $Z_{\mu\nu\rho ij}$ ($\mu.. = 0, .., 7, i, j = 1, .., 4$). These new generators are elements of the coset $Sp(32, R)/Sp(16, R)_L \times Sp(16, R)_R$. They are contained in the mixed anticommutators \{16_L, 16_R\}.

The spinor charges decompose as follows as representations of $SO(6,2) \times SO(4)$:

$$32 = (8_L, 2) + (8_R, 2');$$

as representations of $O^*(8)_L \times O(3)_L \times O^*(8)_R \times O(3)_R \subset Sp(16, R)_L \times Sp(16, R)_R \subset Sp(32, R)$, the spinor charges decompose instead as:

$$32 = (8_L, 2, 1, 1) + (1, 1, 8_R, 2_R).$$

If one drops the above 256 generators one finds two copies of the $OSp(1/16)$ superalgebra. By further truncation one gets $OSp(8^*/2)_L \times OSp(8^*/2)_R$. Therefore, the two embeddings only coincide when the generators are restricted to a $(1, 0)$ superconformal extension.

The superalgebra in Eq. (29), interpreted on $\partial AdS_7$, gives a conformal extension of the maximally centrally extended supertranslation algebra of type $(2, 0)$ and $(1, 0)$, respectively. For $N = 2$, this is the $OSp(1/32, R)$ algebra given in Eqs (27, 29), now written in 6-D notations:

$$\{Q^A_{\alpha}, Q^B_{\beta}\}^+ = (\gamma^\mu C)_{\alpha\beta}^+ P_{\mu}^{AB} + (\gamma^\mu C)_{\alpha\beta}^+ Z^{o[AB]}_{\mu} + (\gamma^{\mu\nu}\epsilon)_{\alpha\beta}^+ Z^{(AB)}_{\mu\nu},$$

$$\{S^A_{\alpha}, S^B_{\beta}\}^- = (\gamma^\mu C)_{\alpha\beta}^- K_{\mu}^{AB} + (\gamma^\mu C)_{\alpha\beta}^- Z^{o[AB]}_{\mu} + (\gamma^{\mu\nu}\epsilon)_{\alpha\beta}^- Z^{+(AB)}_{\mu\nu},$$

$$\{Q^A_{\alpha}, S^B_{\beta}\} = C_{\alpha\beta} U^{AB}_\cdot + (\gamma^{\mu}\epsilon)_{\alpha\beta} M^{\mu}_{AB} + C_{\alpha\beta} D^{AB}_\cdot + (\gamma^{\mu\nu}\epsilon)_{\alpha\beta} Z^{o[AB]}_{\mu\nu};$$

$$\alpha, \beta = 1, .., 4, \ \mu.. = 0, .., 5.$$  \hspace{1cm} (34)

The subindex $+$ is the chiral projector in 6-D and $Z^{+(AB)}_{\mu\nu}$ is self-dual. The BPS analysis with this boundary algebra can be done using the methods explained in Section 4. In particular, looking at the super-Poincaré algebra, we find the following patterns of R-symmetry breaking: Strings are 1/2 BPS carrying a nonzero $Z^{o[AB]}$ charge. With an appropriate choice of basis it can be cast in the form

$$Z^{o[AB]} = \begin{pmatrix} \lambda \epsilon \\ -\lambda \epsilon \end{pmatrix}.$$  \hspace{1cm} (35)

This matrix breaks $USp(4) \sim O(5) \to USp(2) \times USp(2) \sim O(4)$.

Three-branes carry a nonzero $Z^{(AB)}$ charge. It can be reduced to the following form:

$$Z^{(AB)} = \begin{pmatrix} \lambda_1 \delta \\ \lambda_2 \delta \end{pmatrix}.$$  \hspace{1cm} (36)
where $\delta$ is the $2 \times 2$ identity matrix. When $\lambda_1 = \lambda_2$, the corresponding state is 1/2 BPS, and the R-symmetry breaks as follows: $USp(4) \sim O(5) \to O(3) \times O(2)$. When $\lambda_1 \neq \lambda_2$, the state is 1/4 BPS, and the $USp(4)$ R-symmetry breaks to $O(2) \times O(2)$. Particles and membranes have the same R-symmetry breaking pattern as three-branes and strings. These results agree with the explicit analysis performed in refs. [21, 22].

### 6 Other Algebras and Uniqueness of the Extension

Let us conclude with a few additional remarks.

First, let us point out that here is a superalgebra that is intermediate between that in Eqs. (3,4) and that in Eqs. (5,6). This is the $U(2N,2N/1)$ superalgebra. It is obtained by setting to zero the right-hand side of Eq. (4), but keeping all terms in the r.h.s. of Eq. (3).

All these algebras can be written in a manifestly $O(4,2)$-invariant notation using the following decomposition, where $A$ and $S$ denote symmetrization and antisymmetrization, respectively ($\mu, \nu = 0, \ldots, 5$):

\begin{align}
[(4, N) \times (N, N)]_S &= [6, (N \times N)_A] + [10, (N \times N)_S], \\
(4, N) \times (N, N) &= (1, 1) + (15, 1) + (15, N^2 - 1) + (1, N^2 - 1), \\
\{Q^A, Q^B\} &= (\gamma^\mu C)^+ T^{|AB|}_\mu + (\gamma^{\mu\nu\rho} C)^+ T^{|+AB|}_{\mu\nu\rho}, \quad \text{c.c.} \\
\{Q^A, Q_B\} &= (\gamma^{\mu\nu} C) M_{\mu\nu} \delta^A_B + C T^A_B + (\gamma^{\mu\nu} C) T^{|AB|}_{\mu\nu}.
\end{align}

(37)

Here $+$ is the 6-D chiral projection and $T^{|+AB|}_{\mu\nu\rho}$ is self-dual. The space-time conformal spinors are identified here with the $(4, \bar{4})$ of $SU(2, 2)$. The $U(2N,2N/1)$ superalgebra is obtained by setting $\{Q^A, Q^B\} = 0$, while the $U(2,2/N)$ superalgebra is obtained by setting $\{Q^A, Q_B\} = 0$ and $T^{|AB|}_{\mu\nu\rho} = 0$.

We must also point out that $AdS_5$ algebras with brane charges clearly violate the Coleman-Mandula theorem [23]. This should imply that they cannot be realized as symmetries of a local world-sheet theory. In spite of this, the brane charges studied in ref. [11] are topological charges appearing in the (local) Born-Infeld action of the $D$-brane. The meaning of this result is not yet clear to us.

Let us comment now on the uniqueness of $AdS_5$ extensions of the super-Poincaré algebra with central charges. For $N$-extended supersymmetry, $OSp(1/8N)$ is the unique extension of the super-Poincaré algebra in Eq. (2) with the following properties: a) all right-hand sides of the fermionic anticommutators are nonzero, and form a simple Lie algebra; b) it contains the group $O(4,2) \times U(N)$; c) it has the same number of fermionic charges as the superconformal algebra $U(2,2/N)$. This follows from the classification of all superalgebras based on simple Lie algebras given in ref. [24]. Likewise, the “intermediate” algebra $U(2N,2N/1)$ is the unique
superconformal extension of algebra (2) where all chiral Poincaré anticommutators are set to zero. For \( N = 1 \) one can say more. In that case, indeed, it was shown in ref. [25] that only two extensions of the super-Poincaré algebra exist. One, in which all central charges are set to zero, is the usual \( U(2, 2/1) \); the other is, necessarily, \( OSp(1/8) \). Notice that in this case the “intermediate” algebra is also \( U(2, 2/1) \). In this case \( \mathcal{SP}^+ \) is just the \( N = 1 \) supertranslational algebra considered in [26, 27], while \( L_0 = R \times SO(3, 3) \),

\[
\mathcal{L}^0 = R \times SO(3, 3),
\]

since \( SL(4, R) \sim SO(3, 3) \). The standard Lorentz transformations and \( U(1) \) R-symmetry correspond to the subgroup \( SO(3, 1) \times SO(2) \subset SO(3, 3) \).

Conformal extensions exist also in lower dimensions. In \( AdS_4 \), \( OSp(N/4, R) \) can be enlarged to \( OSp(1/4N, R) \). Here the embedding of the \( O(3, 2) \sim Sp(4, R) \) isometry of \( AdS_4 \) is

\[
Sp(4N, R) \rightarrow O(N) \times Sp(4, R),
\]

Similarly to the discussion in footnote 4.

The 3-grading structure is

\[
\mathcal{L} = SP^+ + \mathcal{L}^0 + SC^-,
\]

\[
\mathcal{L}^0 = R \times SL(2N, R) \rightarrow R \times SL(2, R) \times O(N).
\]

Here \( SL(2, R) \sim O(2, 1) \) is the 3-D Lorentz group, and \( SP^+ \) is the maximal central extension of the 3-D supertranslation algebra in ref. [4].

Now, let us discuss 1/2 BPS states for \( N = 8 \). These BPS brane configurations were discussed in [28]. For this purpose, it is convenient to write the \( SP^+ \) algebra using a triality change of basis, where \( Q_{\alpha i} \) is a chiral spinor representation of \( SO(8) \):

\[
\{Q_{\alpha i}, Q_{\beta j}\} = (\gamma^\mu C)_{\alpha \beta} P_\mu \delta_{ij} + (\gamma^\mu C)_{\alpha \beta} \gamma_{ij}^{[ABCD]} Z_{\mu [ABCD]}^+ + C_{\alpha \beta} \gamma_{ij}^{[AB]} Z_{[AB]},
\]

\[
\alpha, \beta = 1, 2, \quad i, j = 1, \ldots, 8, \quad A, B = 1, \ldots, 8.
\]

\( Z_{\mu [ABCD]}^+ \) is \( O(8) \) self-dual. For zero-branes, \( Z_{[AB]} \) can be brought to a normal form by an \( SO(8) \) rotation. 1/2 BPS states correspond to 3 vanishing eigenvalues of \( Z_{[AB]} \), that break the \( O(8) \) R-symmetry as

\[
O(8) \rightarrow O(6) \times O(2).
\]

For strings, the matrix \( \gamma_{ij}^{[ABCD]} Z_{\mu [ABCD]}^+ \) saturates the 1/2 BPS bound when \( Z_{\mu [ABCD]}^+ \) is a singlet under \( O(4) \times O(4) \subset O(8) \). These breaking patterns seem to agree with the explicit analysis of ref. [28].
Finally, let us comment about other symplectic extensions. An interesting embedding, corresponding to a $D5/D3$ brane system, is $OSp(8N, R) \to O^*(4) \times USp(2N)$, where $O^*(4) \sim O(2,1) \times O(3)$. This is an extension of the $OSp(4^* / 2N)$ superalgebra, and it was considered in ref. [1].

The analysis performed in this paper was made without enlarging the fermionic sector of the superalgebra. If one doubles the spinor charges, instead, other extensions such as $OSp(2/32, R)$, $OSp(1/32, R)_L \times OSp(1/32, R)_R$, or $OSp(1/64, R)$ are possible. The two latter algebras, have been proposed recently as minimal superalgebras for describing an $AdS_{11}$ version of M-theory [8, 9], or to unify space-time symmetries in extra dimensions [29].

Acknowledgements

We would like to thank R. Stora and A. Zaffaroni for useful discussions and comments. M.P. would like to thank the Scuola Normale Superiore, Pisa, Italy, for its kind hospitality during completion of this paper. M.P. is supported in part by NSF grant no. PHY-9722083. S.F. is supported in part by the EEC under TMR contract ERBFMRX-CT96-0090, ECC Science Program SCI*-CI92-0789 (INFN-Frascati), DOE grant DE-FG03-91ER40662.

References

[1] W. Nahm, Nucl. Phys. B135 (1978) 149.
[2] R. Haag, J. T. Lopuszanski and M. Sohnius, Nucl. Phys. B88 (1975) 257.
[3] J.A. de Azcarraga, J.P. Gauntlett, J.M. Izquierdo and P.K. Townsend, Phys. Rev. Lett. 63 (1989) 2443.
[4] P.K. Townsend, published in Proceedings of PASCOS/Hopkins 1995, hep-th/9507048.
[5] I. Bars, Phys. Rev. D54 (1996) 5203, hep-th/9604139.
[6] P.K. Townsend, Nucl. Phys. Proc. Suppl. 68 (1998) 11, hep-th/9708034.
[7] S. Ferrara and M. Porrati, Phys. Lett. B423 (1998) 255, hep-th/9711116.
[8] P. Horava, Phys. Rev. D59 (1999) 046004, hep-th/9712130.
[9] M. Günaydin, Nucl. Phys. B528 (1998) 432, hep-th/9803138.
[10] R. Gilmore, Lie groups, Lie algebras, and some of their applications, Wiley-Interscience, New York, NY (1974).
[11] B. Craps, J. Gomis, D. Mateos and A. Van Proeyen, hep-th/9901060.

[12] E. Cremmer, in Superspace and Supergravity, S.W. Hawking and M. Roček Eds., Cambridge U. Press, Cambridge, UK (1981).

[13] M. Günaydin, Nuovo Cim. 29A (1975) 467.

[14] I. Bars and M. Günaydin, Comm. Math. Phys. 91 (1983) 31.

[15] E. Witten, JHEP 9807 (1998) 006, hep-th/9805112; S. Gukov, M. Rangamani and E. Witten, JHEP 9812 (1998) 025, hep-th/9811048.

[16] J. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231, hep-th/9711200.

[17] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, Phys.Lett. B428 (1998) 105, hep-th/9802109.

[18] E. Witten, Adv. Theor. Math. Phys. 2 (1998) 253, hep-th/9802150.

[19] B. Zumino, J. Math. Phys. 3 (1962) 1055.

[20] P. Claus, R. Kallosh, A. Van Proeyen, Nucl. Phys. B518 (1998) 150, hep-th/9711161.

[21] J.T. Liu and R. Minasian, hep-th/9903269.

[22] C. Ahn, H. Kim and H.S. Yang, Phys. Rev. D59 (1999) 106002, hep-th/9808182.

[23] S. Coleman and J. Mandula, Phys. Rev. 159 (1967) 1251.

[24] W. Nahm, V. Rittenberg and M. Scheunert, J. Math. Phys. 17 (1976) 1626, 1640; V.G. Kac, Comm. Math. Phys. 53 (1977) 31.

[25] J.W. van Holten and A. Van Proeyen, J. Phys. A15 (1982) 3763.

[26] G. Dvali and M. Shifman, Phys. Lett. B396 (1997) 64, Erratum-ibid. B407 (1997) 452, hep-th/9612128.

[27] E. Witten, Nucl. Phys. B507 (1997) 658, hep-th/9706109.

[28] C. Ahn, H. Kim, B.-H. Lee and H.S. Yang, hep-th/9811010.

[29] I. Bars, C. Deliduman and D. Minic, hep-th/9904063.