Simulation of the absolute permeability based on the capillary pressure curves using the dumbbell model

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Abstract. The paper deals with simulation of absolute permeability of granular collectors based on the capillary pressure curves using the dumbbell model. It has been established that for assessing permeability based on the capillary data, effective porosity should be used, since residual water does not participate in the displacement of fluid from the rock sample. It was proved that when using the dumbbell model instead of a simple capillary model, accuracy of absolute permeability prediction based on the capillary pressure curves increases significantly.

1. Introduction
The issues of quantitative assessment of the absolute permeability coefficient are considered by many domestic and foreign researchers [1-9]. Mathematical models proposed by the authors are dependences of absolute permeability on capacitive characteristics of reservoirs (porosity, residual water saturation, etc.).

However, to predict the performance of oil wells, it is necessary to know phase permeability for oil and water. They are determined by the distribution of pore channels by their size. The distribution is estimated based on capillary measurements of core samples from productive formations.

In this regard, the issue of permeability assessment for reservoirs based on the capillary pressure curves is important and relevant.

To assess the absolute permeability of the reservoir based on the capillary pressure curves, the size distribution function of the pore channels is used.

The basis of the capillary method of studying void space is the position that when a water-saturated sample is exposed to a certain external pressure (for example, air in a capillary meter), water is squeezed out of pores of a certain size.

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If the pore space is represented by parallel capillaries of various sizes, at the lowest pressure acting on the sample, water is squeezed out of the capillaries of the largest cross section. Then, as the capillary pressure increases, capillaries expand to the smallest ones filled with residual water. The result of the experiment is an empirical dependence of water saturation of the sample on the capillary pressure. Based
on this relationship, pore size distribution and involvement in filtration of pores of different sizes are determined.

2. Methods and materials
A mathematical model for absolute permeability of reservoirs based on capillary measurements of core samples from productive plates of the Tyumen Formation in Western Siberia was developed. We used the results of laboratory studies of core samples U2-U6 of the Tyumen Formation of the Lovinsk field.

The statistical processing proved the accuracy of the developed absolute permeability model.

3. Results and discussion
Numerous studies show that pore channels of the rock are not similar to direct non-intersecting capillaries. On the contrary, they are a chain of pores of irregular shape, interconnected by interporous contractions. A dumbbell model in which each pore channel is represented by an alternation of pores and interporous contractions was developed [10].

In the dumbbell model, each pore channel is provided with a chain of capillaries of large and small sections. At the same time, the capillaries of the large cross section (S_p) model the pores, and those of the small cross sections (S_k) model interpore channels (contractions).

In the calculations, each channel of a variable cross section is replaced by a capillary of a constant (effective) cross section. Since in the dumbbell model all the pore channels are connected in parallel, when replacing the constant cross section by capillaries, the model turns into a simple model of parallel capillaries.

In addition to the cross section of pores and interporous capillaries, the dumbbell model is characterized by the following equivalent (effective) sections:

- S_e is the equivalent capacitive section which is determined by the porosity coefficient (K_p):
  \[ S_e N_F = K_p, \]  
  where \( N_F \) is the areal density of pore channels; \( S_e \) is the equivalent electrical cross section which is determined by the porosity parameter (P_p):
  \[ S_e N_F = P_p^{-1}, \]  

- S_f is the equivalent filtration section in accordance with the Poiseuille formula, which is determined by the absolute permeability coefficient (K_pr):
  \[ S_f N_F = 8\pi K_{pr}. \]  

All equivalent sections of the dumbbell model differ from each other, but quantitative relationships between them can be obtained. With a known value of one of the effective sections (for example, the electric one), these relations allow for calculation of all equivalent sections [10].

Let us consider the filtration and electrical sections of pore channels of the dumbbell model. Dividing equations (3) and (2) term by term, we have:

\[ \frac{S_f^2}{S_e} = 8\pi K_{pr} P_p. \]

In [11], it was shown that the square of the filtration section is very close to the product of the electric section by the section of microcapillaries (interporous contractions). It follows that the left side of this equation can be replaced by the section of microcapillaries. In the dumbbell model, the cross sections of interporous contractions are smaller than the cross sections of pores, therefore they determine the permeability coefficient.

Thus, we have

\[ S_e \approx 8\pi K_{pr} P_p \text{ или } r_k^2 = 8K_{pr} P_p, \]

where \( P_p = K_p-m \), \( m \) is the cementation rate which is close to 2; \( r_k \) is the microcapillary radius (interporous contractions).

Let us assume that interporous contractions (microcapillaries) have different sizes and their radii are distributed according to law \( g(r) \).
Then to calculate the absolute permeability coefficient we have the following formula:

\[ K_{np} = \frac{K_n^m}{\beta} \int_0^{r_{\text{max}}} r^2 g(r) dr. \]  

(4)

Comparison of the results of laboratory measurements of air permeability in the dry sample and in the sample saturated with residual water shows that they are very similar to each other. This can be explained by the fact that residual water does not participate in filtration of fluids through the rock, i.e., only the effective part of the void space characterized by the effective porosity \( K_{pe} \) is involved in the filtration process

\[ K_{pe} = K_n (1 - K_{wo}), \]

where \( K_{wo} \) is residual water saturation.

For this reason, effective rather than open porosity should be involved in the formula linking absolute permeability with the results of capillarimetric studies.

Capillary pressure curves make it possible to obtain the size distribution of the pore channels of the effective part of the void space of the rock [8]:

\[ g(r) = \frac{dK_w}{dr}, \]

where \( K_w \) is the current coefficient of water saturation of the sample.

If we substitute this expression in formula (4) and take into account the Laplace formula, we have

\[ K_{np} = \frac{K_{pe}^2}{2} \int_{K_{wo}}^{1.0} \left( \frac{\sigma \cos \theta}{P_k} \right)^2 dK_n, \]

where \( P_k \) is capillary pressure; \( \sigma \) is surface tension; \( \cos \theta \) is the wetting angle cosine; \( K_{pe} \) is effective porosity.

The values of the surface tension and the wetting angle cosine did not change, therefore we have:

\[ K_{np} = A \cdot K_{pe}^m \int_{K_{wo}}^{1.0} dK_n \left( \frac{1}{P_k} \right)^{1.0}, \]

(5)

where \( A \) is a coefficient that takes into account constant parameters and dimensions of the formula parameters.

In the dumbbell model, the exponent \( m \) is close to 2 and characterizes the degree of rock consolidation. For the conventional capillary model, \( m \) is equal to 1.

Figures 1-4 show the results of comparison of the values of the absolute permeability coefficient, calculated based on the capillarimetric studies of core samples by formula (5) with experimental data for two values of the coefficient: \( m = 1.0 \) and \( m = 2.0 \).
Figure 1. The comparison curve for the values of absolute permeability based on experimental and calculated data for object No. 1 at \( m = 1 \) (Tyumen formation, layer \( U_{2.4} \))

\[
R = 0.9581
\]

\[
K_{np}, 10^{-3} \text{ um}^2 \\
K_{np} = 35.84 \cdot (K_{np} \cdot S) - 2.87
\]

Figure 2. The comparison curve for the values of absolute permeability based on experimental and calculated data for object No. 1 at \( m = 2 \) (Tyumen formation, layer \( U_{2.4} \))

\[
R = 0.9814
\]

\[
K_{np}, 10^{-3} \text{ um}^2 \\
K_{np} = 268.85 \cdot (K_{np}^2 \cdot S) - 0.571
\]
Figure 3. The comparison curve for the values of absolute permeability based on experimental and calculated data for object No. 2 at \( m = 1 \) (Tyumen formation, layer \( U_{2,4} \)).

\[
K_{\text{p,r}} = 20.565 (K_{\text{p,e}} \cdot S) - 1.2287 \quad (R = 0.8503)
\]

Figure 4. The comparison curve for the values of absolute permeability based on experimental and calculated data for object No. 2 at \( m = 2 \) (Tyumen formation, layer \( U_{2,4} \)).

\[
K_{\text{p,r}} = 120.12 (K_{\text{p,e}}^2 \cdot S) + 1.9094 \quad (R = 0.9414)
\]

To calculate the values of absolute permeability, we used the results of capillarymetric studies of core samples from productive layers of the Tyumen formation for two objects in Western Siberia [12–14].

These works have formulas for calculating the permeability coefficient obtained on the basis of regression analysis of capillarymetric research data. In these formulas, \( S \) is the result of numerical integration by the formula.
\[ S = \int_{K_{so}}^{1.0} \frac{dK_{f}}{P_{f}^{2}} \]

4. Conclusion

To study the size distribution of pore channels based on the capillary pressure curves, it is necessary to use a dumbbell model of void space. The capillary pressure is determined by the size of interporous contractions, and the current water saturation is determined by the capacity of the pores themselves.

When quantifying permeability based on the capillary data, effective porosity should be used, since residual water does not participate in the displacement of the fluid from the rock sample.

The analysis showed that when using the dumbbell model instead of a simple capillary model, absolute permeability prediction accuracy improves.

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