Magnetic precessions in $\varphi_0$ junction along IV-characteristics

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We show that a current sweep along current-voltage (IV) characteristic curves of a $\varphi_0$-junction may lead to both regular and chaotic magnetization dynamics with a series of specific phase trajectories. The origin of these trajectories is related to the direct coupling between the magnetic moment and the Josephson current realized in these junctions. We demonstrate that an external electromagnetic field can control the dynamics of magnetic moment within a current interval corresponding to a Shapiro step and produce topological transformation of specific precession trajectories. Our numerical results also demonstrate appearance of DC component of superconducting current in accordance with earlier theoretical prediction [Phys. Rev. Lett. 102, 017001 (2009)]. An excellent agreement of numerical and analytical results has been found describing the ferromagnetic resonance as Josephson frequency is tuned to the ferromagnetic one. We discuss experiments which can test our results.

Superconducting spintronics is one of the intensively developing fields of condensed matter physics. An important place in this field is occupied by the investigations of Josephson junctions (JJs) coupled to magnetic systems [1]. The possibility to manipulate the magnetic properties by Josephson current and its vice versa, i.e. to influence a Josephson current by magnetic moment, has attracted much recent attention [2–4]. A central role in these phenomena is played by spin-orbit interaction in a ferromagnet without inversion symmetry. Consequently, the current-phase relation (CPR) of these junctions are given by $I = I_c \sin(\varphi - \varphi_0)$, where the phase shift $\varphi_0$ is proportional to the magnetic moment perpendicular to the gradient of the asymmetric spin-orbit potential [5]. This feature of the CPR allows one to manipulate of the internal magnetic moment using the Josephson current [5,6]. Though the static properties of the S/F/S structures are well studied both theoretically and experimentally, much less is known about the magnetic dynamics of these systems [2,6]. Recently, the presence of an anomalous phase shift of $\varphi_0$ was experimentally observed in Ref. [6] in a hybrid SNS JJ fabricated using Bi$_2$Se$_3$ (which is a topological insulator with strong spin-orbit coupling) in the presence of an in-plane magnetic field. This anomalous phase shift is observed directly through CPR measurement. This constitutes a direct experimental measurement of the spin-orbit coupling strength and opens up new possibilities for phase-controlled Josephson devices made from materials with strong spin-orbit coupling.

It was demonstrated that the DC superconducting current applied to a S/F/S $\varphi_0$-junction might produce a strong orientation effect on the ferromagnetic layered magnetic moment [11]. The application of a DC voltage to the $\varphi_0$-junction would produce current oscillations and consequently magnetic precession. This precession may be monitored by the appearance of higher harmonics in the CPR as well as by the presence of a DC component of the current that increases substantially near ferromagnetic resonance [4]. It is expected that external radiation would lead to a series of novel phenomena. Out of these, the possibility of the appearance of half-integer Shapiro steps (in addition to the conventional integer steps) and the generation of an additional magnetic precession with frequency of external radiation was already discussed in Ref. [6]. However, to the best of our knowledge, an important problem related to reciprocal influence of Josephson current and magnetization at different bias current along the current-voltage (IV)-characteristics has not been investigated till now. Furthermore, the variation of the magnetic precessions in $\varphi_0$ junction along the IV-characteristics has also not been addressed.

In this Letter we present the results on magnetic precessions in $\varphi_0$ junctions in the presence of a current sweep along its IV-characteristic curve. This allows us to find specific current intervals with very simple magnetization dynamics. We show that the origin of such simple dynamics is related to the direct coupling between magnetic moment and Josephson current realized in $\varphi_0$-junction. We also demonstrate that the interaction of the Josephson current and the magnetic moment manifests several interesting features under external electromagnetic radiation. In particular, the external radiation can tune the nature of magnetic moment precession in a current interval corresponded to the Shapiro step. We also show that such external radiation can produce a topological transformation of magnetization precession trajectories. We
numerically demonstrate an appearance of dc-component of superconducting current which was predicted theoretically in Ref. [6] and discuss experiments which can test our theory.

The magnetization dynamics of our system is described by the Landau-Lifshitz-Gilbert equation

\[
\frac{dM}{dt} = -\gamma M \times H_{\text{eff}} + \frac{\alpha}{M_0} \left( M \times \frac{dM}{dt} \right)
\]

\[
H_{\text{eff}} = \frac{K}{M_0} \left[ Gr \sin \left( \varphi - r \frac{M_y}{M_0} \right) \hat{y} + \frac{M_z^2}{M_0} \right],
\]

where \( \gamma \) is the gyromagnetic ratio, \( \alpha \) is a phenomenological damping constant, \( M_0 = ||M||, G = E_J/(KV) \), \( K \) is an anisotropic constant, \( V \) is the volume of ferromagnetic layer, \( I = 4\hbar L/\hbar v_F \), \( L \) is the length of \( F \) layer, and \( h \) denotes the exchange field in the ferromagnetic layer.

Based on the JJ and magnetic system equations, we can rewrite total system of equations (to be used in our numerical studies) in normalized units

\[
\frac{d\alpha}{dt} = \frac{\omega_0}{1 + \alpha^2} \left( -m_y m_z + Gr m_z \sin((\varphi - r m_y)) \right)
\]

\[
\frac{d\beta}{dt} = \frac{\omega_0}{1 + \alpha^2} \left( m_x m_z - \alpha [m_x m_z^2 + Gr m_x m_y \sin((\varphi - r m_y)) \right)
\]

\[
\frac{d\gamma}{dt} = \frac{\omega_0}{1 + \alpha^2} \left( m_y m_z - \alpha [m_y m_z - Gr (m_y^2 + m_z^2) \sin((\varphi - r m_y))] \right)
\]

\[
\frac{d\omega}{dt} = \frac{1}{\beta_c} \left[ I-V - \sin((\varphi - r m_y)) \right], \quad \frac{d\varphi}{dt} = V
\]

where \( \beta_c = 2eIR CR^2/\hbar \) is McCumber parameter, \( m_\alpha = M_\alpha/M_0 \) for \( \alpha = x, y, z \), and \( \omega_0 = \omega_F/\omega_c \) with \( \omega_F = \gamma K/M_0 \), \( \omega_c = 2eRI_c/\hbar \) being frequency scales set by the anisotropy constant \( K \) and the external current \( I \), respectively. Here we normalize time in units of \( \omega_c^{-1} \), external current \( I \) in units of \( I_c \), and the voltage \( V \) in units of \( V_c = I_c R \). This system of equations, solved numerically using fourth order Runge–Kutta method, yields \( m_\alpha(t) \) with \( \alpha = x, y, z \), \( V(t) \) and \( \varphi(t) \) as a function of the external bias current \( I \).

The system of equations (2) is significantly simplified in absence of dissipation (\( \alpha = 0 \)). Such simplification allows us to clearly understand the magnetization dynamics in the corresponding to the different points of the IV-curve. For this purpose we calculate the temporal dependence of \( V \) and \( m_\alpha \) at each value of bias current. In parallel, we control the dynamics of the magnetic system by solving the LLG equations at some averaged values of voltage, which is qualitatively in agreement with solutions of the system (2). We present the calculated one-loop IV-curves (obtained by increasing and decreasing \( I \)) in Fig. (a). We note that the dashed line is the IV-curve of JJ without magnetic system and displays the expected hysteresis for \( \beta_c = 25 \).

As shown in Fig. (b), we find that with increase in bias current along the zero voltage state starting with \( I = 0 \), \( m_y \) grows linearly with \( I \): \( m_y = Gr I \). This behavior continues till \( m_y = 1 \). At \( I = I_c \), \( m_y \) starts to oscillate and subsequently stabilizes to one of the possible final values \( m_y = \pm 1 \). Note that in both of these cases, we observe qualitatively the same dynamics of magnetization with sweeping along IV-characteristic. As \( I \) is decreased from \( I_c \), the amplitude of \( m_y \) oscillations is grows slowly again.

One of the main results that we find from our numerics is that the magnetization dynamics has both regular and chaotic regions along the IV-curve depending on the parameters of magnetic and Josephson subsystems. Two regular regions with the simplest magnetization trajectories are demonstrated in Fig. (b,c) where a temporal dependence of \( m_y \) in the current intervals \( 0.535 \leq I \leq 0.435 \) and \( 0.4 \leq I \leq 0.34 \) is shown. We can see that the amplitude of \( m_y \) grows for both these intervals leading to regular magnetization dynamics. In the rest of the current range, the dynamics is chaotic. In what follows, we shall concentrate on the current intervals where the dynamics is regular.

Next, we note that a sweep along the IV-characteristics leads to a ferromagnetic resonance for \( \omega_J = \omega_F \), i.e., the Josephson oscillations excite the eigenmode of magnetic system (in our normalization, at \( \omega_0 = 1 \)). Earlier analytical considerations [6, 12] predicts a damped resonance with expression for \( m_y \)

\[
m_y(t) = \frac{\omega_+ - \omega_-}{r} \sin \omega t - \frac{\alpha_+ + \alpha_-}{r} \cos \omega t
\]
where $\omega_\pm = \frac{G_0^2 - 1}{T_2} \pm \frac{\alpha_\pm}{2}$ and $\alpha_\pm = \frac{G_0^2 - 1}{T_2}$ with $\Omega_\pm = (\omega \pm 1)^2 + \alpha^2 \omega^2$. In Fig. 1(d) we compare analytical result (3) with calculated by system of equations (2) for maximal value of $m_y$ at each value of bias current. We see an excellent agreement of both results.

A key result of this paper concerns the reaction of magnetic system to the superconducting current. The magnetization trajectories in the planes $m_y - m_x$, $m_z - m_x$, and $m_z - m_y$ at three values of bias current $I = 0.475, 0.450, \text{and } 0.385$ are shown in Fig. 2. They demonstrate different specific forms and some of them for distinctness are called as “fish” (j), “moon” (e), “sickle” (g) “mushroom” (h) and “apple” (e). The first current interval is characterized by complex dynamics demonstrated in Figs. (a,b,c) at $I = 0.475$. With the increasing in $I$ in the second current interval (0.535, 0.435) we observe a transformation of trajectories type “apple” to the “mushroom” in the $m_z - m_x$ plane, while the third interval (0.4, 0.34) is characterized by trajectories type “fish” and “moon”. Thus we demonstrate the possibility of controlling magnetization dynamics via external bias current.

To understand the observed temporal dependencies, we have made a detailed FFT analysis at different values of bias current which will be presented in Ref. [12] for all data shown in Fig. 1 and Fig. 2. Here we concentrate on a short description of some of them. In particular, we present results of FFT analysis of time dependencies of magnetization components and voltage for JJ with and without magnetic system at $I = 0.45$. We find that the dynamics of magnetization is determined by Josephson frequency $f = 0.07$. The existence of half harmonics in this parameter regime indicates that the excitation of magnetic dynamics happens parametrically. We also note that the effect of the magnetic oscillations on Josephson current which is manifested as a small peak in FFT of $V(t)$. Such peak is absent for JJ's without coupling to an external magnet as demonstrated in Figure 3(c).

Another important feature of the studied magnetization dynamics concerns the possibility of its control via external electromagnetic radiation. The presence of such a radiation amounts to $I \rightarrow I(t) = I + A \sin(\omega t)$ in Eq. (2), where $\omega$ is the frequency and $A$ is the amplitude of the external radiation. We find that such an external radiation can control the qualitative nature of the magnetic precession in current interval corresponding to a Shapiro step. To demonstrate this feature, we show the IV-characteristic of $\varphi_0$ junction under external radiation with frequency $\omega = 0.366$ and amplitude $A = 1$ (which demonstrates the corresponding Shapiro step at $V = 0.366$) in Fig. 4(a). The resulting magnetization precession in the $m_z - m_x$ plane at $I = 0.475, I = 0.45$ and $I = 0.385$ are presented in Fig. 4(b), Fig. 4(c) and Fig. 4(d), respectively. In sharp contrast to magnetization dynamics without radiation (see Fig. 2) demonstrating different specific precession dynamics with changing in bias current, the dynamics of magnetic precessions along the Shapiro step are very similar for all three current values.

Another central result of our work is to demonstrate that the radiation may change the topology of magnetic precession. In particular, we show the left-right transformation of “mushroom”-type precession. As shown in Fig. 4 such a change may be accomplished by changing an amplitude of radiation at a fixed DC drive current value $I = 0.45$. This transformation is related to a magnetization reversal from $-m_y$ to $+m_y$ as can be seen from change in temporal dependence of $m_y(t)$ in the presence of the external radiation as demonstrated in Fig. 4(c,d).

Next, we discuss the effect of damping on such dynamics. It was shown in Ref. [12] that damping plays an important role in the dynamics of the coupled JJ-magnet...
A = 0 ± with Ω = ω same under radiation with frequency V dependence of superconducting current on superconducting current (4). In Fig. 6 we present the analytical consideration of the effect of magnetic dynamics numerically this important result which follows from analytical consideration of the effect of magnetic dynamics on superconducting current (4). In Fig. 6 we present the voltage dependence of superconducting current $I_s(V)$ calculated by system of equations (2) at $G = 10$, $r = 0.1$ and $α = 1$ which shows a maximal $I_s$ at $V = 0.5$. In the same figure we plot the frequency dependence of $I_0$ followed from formula (4) at the same parameters. We see that the position of maximal value of $I_0$ practically coincides with the results for superconducting current $I_s(V)$. The DC contribution to the Josephson current manifests itself also as a deviation of IV-curve from the linear dependence (dashed line in Fig. 6(b)). The total IV-curve is presented in the inset to this figure. This deviation determines a dependence of $I_0$ on bias current and, as we can see, the largest deviation corresponds to the voltage $V = 0.5$.

To summarize, we point out an intriguing opportunity to observe the different type of magnetization trajectories by sweeping along the IV-characteristics of $ϕ_0$ junction due to the direct coupling between magnetic moment and Josephson current. It was demonstrated that an external electromagnetic field can control qualitative features of the dynamics of magnetic moment in a current interval which corresponds to the Shapiro step. Moreover, such radiation can also produce a topological transformation of precession trajectories. The prediction in Ref. [6] (which is verified in our numerical simulations) that the DC superconducting current in the presence of a constant voltage $V$ applied to the junction implies a dissipative regime can be easily detected experimentally. An excellent agreement between numerical and analytical results found at the ferromagnetic resonance open wide opportunities for further manipulation of system parameters and experimental verification of the magnetization dynamics the materials with strong spin-orbit coupling. This can be easily achieved by applying external radiation to the setup used in Ref. [10]. We predict that the change in topology of the magnetization dynamics would be observed in such systems as a function of amplitude of the applied radiation.

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