Monopole–antimonopole: interaction, scattering and creation

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The interaction of a magnetic monopole–antimonopole pair depends on their separation as well as on a second ‘twist’ degree of freedom. This novel interaction leads to a non-trivial bound state solution known as a sphaleron and to scattering in which the monopole–antimonopoles bounce off each other and do not annihilate. The twist degree of freedom also plays a role in numerical experiments in which gauge waves collide and create monopole–antimonopole pairs. Similar gauge wavepacket scatterings in the Abelian–Higgs model lead to the production of string loops that may be relevant to superconductors. Ongoing numerical experiments to study the production of electroweak sphalerons that result in changes in the Chern–Simons number, and hence baryon number, are also described but have not yet met with success.

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1. Introduction

Magnetic monopoles have been known for over 40 years now as regular solutions in non-Abelian gauge theories [1,2]. They provide a fertile playground for theoretical ideas [3] and are also relevant to physical considerations as they are necessarily present in all grand unified models [4]. In the standard model of the electroweak interactions, only confined magnetic monopoles exist. Even so, they can lead to insight into processes such as baryon number violation [5].

In the present work, we are interested in properties of monopole–antimonopole (MM) pairs. How do monopoles interact with antimonopoles when they are at rest? How do they scatter? Can they be produced in scattering experiments? Can they play a role in particle physics?
There is a large body of work on the first two questions and there are also several excellent texts [3,6] where the reader can access results. Our recent work [7–9] provides numerical evidence for some of these works and extends them in some cases. The question of $\bar{M}M$ creation has also received attention but it is difficult to answer especially as it seems to require a description of a non-perturbative final state ($\bar{M}M$) in terms of perturbative initial states (particles). The non-perturbative state is best described in classical terms (solutions of certain differential equations) while the perturbative state is best described in terms of quanta in a quantum field theory. Thus, the process also requires a description that enables transition from quantum to classical variables. We shall largely bypass these deep questions and study the creation of $\bar{M}M$ when the initial state has a large occupation number and can be described in classical terms. After all, the initial state is up to us to prepare and we are free to set it up as we wish.

There are two applications of the work on $\bar{M}M$ creation that are more immediate. The first is that just as we can consider the creation of $\bar{M}M$, we can also consider the creation of string loops. Indeed we find initial conditions in the Abelian–Higgs model that lead to string creation. These strings are produced in loops that live for a short time and then collapse. In some regime of parameters, the Abelian–Higgs model also provides a description of superconductors, leading to the possibility of experimentally producing strings in superconductors. (This is similar to the production of string loops in He-3 by the bombardment of neutrons in an experiment that has already been done [10].) The second application of the work on $\bar{M}M$ creation is in the context of the electroweak model. Here, monopoles are confined and a monopole is always connected to an antimonopole by a Z-string. If we are able to create a confined electroweak $\bar{M}M$, it will re-annihilate just as the loops of string in the Abelian–Higgs model re-collapse. Furthermore, if the electroweak $\bar{M}M$ annihilates after some specific dynamics, the electroweak Chern–Simons number can change and lead to baryon number violation. Thus, a better understanding of the creation of monopoles may enable processes in which the baryon (and/or lepton) number changes. However, ongoing numerical work on the production of electroweak $\bar{M}M$ has not yet resulted in a change of the Chern–Simons number.

### 2. $\bar{M}M$ in SO(3) model

We consider an SO(3) gauge theory with a scalar in the adjoint representation with Lagrangian

$$\mathcal{L} = \frac{1}{2} (D^\mu \phi^a)(D_\mu \phi)^a - \frac{1}{4} W_{\mu \nu}^a W^{a \mu \nu} - \frac{\lambda}{4} (\phi^a \phi^a - \eta^2)^2,$$

where $a = 1, 2, 3$, the covariant derivative is defined as

$$(D_\mu \phi)^a = \partial_\mu \phi^a + g e^{abc} W^b_\mu \phi^c$$

and the gauge field strength is given as

$$W^{a \mu \nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g e^{abc} W^b_\mu W^c_\nu.$$  

The energy of a static field configuration is given by

$$E = \int d^3x \left[ \frac{1}{2} (D_i \phi)^a (D_i \phi)^a + \frac{1}{4} W^a_{ij} W^a_{ij} + \frac{\lambda}{4} (\phi^a \phi^a - \eta^2)^2 \right].$$

It is known [1] that the model has a monopole solution

$$\phi^a = \eta h(r) \hat{r}^a \quad \text{and} \quad W^a_i = \frac{(1 - k(r))}{gr} e^{aij} \hat{r}^j,$$

where $h$ and $k$ are profile functions that can be found by solving the equations of motion with the boundary conditions

$$h(0) = 0, \quad k(0) = 1, \quad h(\infty) = 1, \quad k(\infty) = 0.$$  

We will set $\eta = 1, g = 1$ in our simulations as this is equivalent to rescaling variables.
Figure 1. Higgs vectors in the xz-plane for twist = π (a) and twist = 0 (b). The Higgs zeroes are located at (0, 2) and (0, −2), shown as filled and unfilled circles. (Online version in colour.)

The $M\bar{M}$ configuration can now be written by gluing together a monopole and an antimonopole. There is some freedom in this procedure, but all we need is that the monopole and antimonopole be located with some fixed separation and that they should have a fixed relative twist. Then, the fields can be relaxed to the lowest energy configuration subject to these constraints, and/or used as initial conditions for time evolution. We choose the $M\bar{M}$ scalar field orientations to be given by

$$\hat{\phi}_1^a = (\sin \theta \cos \bar{\theta} \cos \gamma - \sin \bar{\theta} \cos \theta) \cos \left(\phi - \frac{\gamma}{2}\right) - \sin \theta \sin \gamma \sin \left(\phi - \frac{\gamma}{2}\right),$$

$$\hat{\phi}_2^a = (\sin \theta \cos \bar{\theta} \cos \gamma - \sin \bar{\theta} \cos \theta) \cos \left(\phi - \frac{\gamma}{2}\right) - \sin \theta \sin \gamma \cos \left(\phi - \frac{\gamma}{2}\right),$$

and

$$\hat{\phi}_3^a = \cos \theta \cos \bar{\theta} + \sin \theta \sin \bar{\theta} \cos \gamma.$$  

(2.7)

(2.8)

(2.9)

Note that the configuration has two free parameters. The separation of the monopole and antimonopole, $d$, is hidden in the spherical angles $(\theta, \phi)$ and $(\bar{\theta}, \bar{\phi})$ that are with respect to coordinate systems with origins at the monopole and antimonopole, respectively. The second is the twist parameter $\gamma$. The scalar field configurations for $\gamma = 0, \pi$ are illustrated in figure 1. The expression for the scalar field with the profile functions included is

$$\phi^a = h(r_m)h(\bar{r}_m)\hat{\phi}_a,$$

(2.10)

where $r_m$ and $\bar{r}_m$ are the distances to the monopole and antimonopole, respectively. For the gauge fields, we take the configuration

$$W^a_\mu = -(1 - k(r_m))(1 - k(\bar{r}_m))\epsilon^{abc}\phi^b\partial_\mu\phi^c.$$

(2.11)

(a) $M\bar{M}$ interaction

The starting configuration in equation (2.10) can now be relaxed so as to lower the field energy but with fixed values of $d$ and $\gamma$. A numerical scheme was implemented for this relaxation in [9] and the results are shown in figure 2. The energy depends on $d$ and $\gamma$. The most interesting feature is that the energy curve for $\gamma = \pi$ has a minimum. The symmetry under $\gamma \rightarrow \pi - \gamma$ then shows that the energy function has a saddle point at $\gamma = \pi$ and $d \sim 3$ (in units in which $\eta = 1$ and the vector mass and the monopole radius are 1). Thus, the SO(3) equations have a saddle point solution corresponding to a bound state of a monopole–antimonopole as was first shown to exist by Taubes in the SO(3) model in [11,12] and also discovered in the electroweak model by Manton [13].
Figure 2. (a) Total energy as a function of monopole–antimonopole separation $d$ for $\lambda = 1$ and twist varying from 0 to $\pi$. (b) Total energy as a function of monopole–antimonopole separation $d$ for twist $\gamma = \pi$ and $\lambda$ varying from 0.25 to 1.0. The sphaleron solution is at the minimum in every curve. (Online version in colour.)

Figure 3. Snapshots of a planar slice of annihilating monopole and antimonopole for $\lambda = 1$, $\gamma = 0$ and $V_z = 0.5$. The colours represent energy density. (Online version in colour.)

(b) $M\bar{M}$ scattering

The initial conditions for a monopole–antimonopole pair were also evolved using the classical equations of motion in [8]. Figure 3 shows snapshots of the scattering for an untwisted $M\bar{M}$, while figure 4 shows the scattering when the $M\bar{M}$ are twisted ($\gamma = \pi$). In the untwisted case, the $M\bar{M}$ annihilate, while in the twisted case they come close but not close enough to annihilate. In fact, they bounce back as seen in figure 5.

(c) $M\bar{M}$ creation

Next, we discuss the creation of $M\bar{M}$. The issue is that monopoles are described as solutions to the classical equations of motion, while particles are quanta in a quantum field theory. So $M\bar{M}$ creation requires a description of magnetic monopoles in a quantum field theory. Furthermore, a monopole is a non-perturbative solution since its magnetic charge is proportional to $1/g$, where $g$ is the gauge coupling. Perturbative calculations can only give a power law expansion in $g$ and cannot describe monopoles. Thus, any attempt at a perturbative calculation of the $M\bar{M}$ creation amplitude is sure to fail.

We take a different approach to $M\bar{M}$ creation. We do not consider $M\bar{M}$ creation from the collision of just two (or few) particles. Instead, we consider the collision of classical wavepackets of gauge particles. These classical wavepackets contain a large number of particles. With such initial conditions, it becomes possible to study $M\bar{M}$ creation because we can simply evolve the initial conditions using the classical equations of motion. However, even within this classical
framework, it is not clear what initial conditions will lead to $\bar{M}M$ creation and it requires some physical intuition, guesswork and luck to find initial conditions that successfully produce monopoles. If two wavepackets of gauge fields with sufficient energy collide, we can expect that a monopole–antimonopole pair can be created, but this is not sufficient because we also require that the monopole and antimonopole separate from each other and do not re-annihilate. On the other hand, a monopole and an antimonopole attract each other by the Coulomb force and this will tend to bring them together, causing them to annihilate. However, if the $\bar{M}M$ are twisted, as discussed in the previous sections, the twist could provide a repulsive force between the monopole and antimonopole and may help separate them. This suggests that the initial gauge wavepackets carry some parity violating structure, which is possible if they are circularly polarized.

Some more intuition may be gained from a result in magneto-hydrodynamics (MHD) that the magnetic helicity is a conserved quantity. For our purposes, the magnetic helicity can be thought of as a measure arising due to twisted or helical magnetic field lines. Furthermore, magnetic helicity likes to spread out. Therefore, if one provides initial conditions that compress magnetic

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**Figure 4.** Snapshots of a planar slice of non-annihilating monopole and antimonopole for $\lambda = 1$, $\gamma = \pi$ and $v_z = 0.5$. Except for the twist, all parameters, including snapshot times, are identical with those in figure 3. The colours represent energy density. At yet later times, the monopoles back-scatter, but are still bound and return to annihilate as discussed in the text. (Online version in colour.)

**Figure 5.** The $z$-coordinate of the monopole as a function of time for $\lambda = 1$, $v_z = 0.50$ and $\gamma / (\pi / 4) = 0, 1, 2, 3, 4$ (curves from left to right). The curves terminate once $\min(\sqrt{\phi^2 \phi^A}) \geq 0.25$ (a condition that is met after the $\bar{M}M$ have annihilated) except in the $\gamma = \pi$ case, when the $\bar{M}M$ have not annihilated even by the end of the simulation run (300 time steps with $dt = dx/2 = 0.1$). (Online version in colour.)
helicity, the system will try and resist. But the only way out is to break the MHD approximation and one way for this to happen is by the creation of magnetic monopoles. So perhaps the compression of magnetic helicity can lead to MM creation.

After this guesswork, we write a configuration for the gauge fields that contains two circularly polarized, counter-propagating, gauge wavepackets that will be used to construct the initial conditions:

\[
\begin{align*}
\mathcal{W}_x^3 &= \partial_y f_1[(\omega f_2 - \partial_z f_2) \cos(\omega(t + (z - z_0))) - (\omega' f_3 + \partial_z f_3) \cos(\omega'(t - (z + z_0)))], \\
\mathcal{W}_y^3 &= \partial_x f_1[(\omega f_2 + \partial_z f_2) \sin(\omega(t + (z - z_0))) - (\omega' f_3 - \partial_z f_3) \sin(\omega'(t - (z + z_0)))]
\end{align*}
\]

(2.12)

and

\[
\begin{align*}
\mathcal{W}_z^3 &= \partial_x \partial_y f_1[f_2(\cos(\omega(t + (z - z_0))) - \sin(\omega(t + (z - z_0))) \\
&+ f_3(\cos(\omega'(t - (z + z_0))) - \sin(\omega'(t - (z + z_0))))],
\end{align*}
\]

where the profile functions \(f_1, f_2\) and \(f_3\) will be specified shortly. Note that this is not a solution to the field equations. It is simply a configuration that represents a gauge wavepacket that is moving in the \(-z\) direction. The configuration is complicated due to the requirement that the initial conditions need to satisfy the Gauss constraints. The chosen configuration satisfies \(\nabla \cdot \mathcal{W}^3 = 0\), and the electric field \(E^3 = -\partial_t \mathcal{W}^3\) satisfies the Gauss constraint with vanishing charge density. We will arrange for an initially vanishing charge density by setting \(\partial_t \phi^a|_{t=0} = 0\) when we choose initial conditions for the scalar field.

The profile functions are chosen to create localized packets in all directions

\[
f_1(x, y) = a \exp \left[ -\frac{(x^2 + y^2)}{2w^2} \right]
\]

(2.13)

and

\[
f_{2,3}(t \pm (z \mp z_0)) = \exp \left[ -\frac{(t \pm (z \mp z_0))^2}{2w^2} \right],
\]

(2.14)

where \(a\) is the amplitude and \(w\) is the width. The frequencies \(\omega\) and \(\omega'\) can be different in general, but here we only consider \(\omega' = \pm \omega\). The case \(\omega' = \omega\) corresponds to scattering of left- and right-handed circular polarizations, while \(\omega' = -\omega < \omega\) corresponds to scattering of left- on left-handed circular polarization waves.

Now, we can state our initial conditions for the gauge field

\[
\mathcal{W}_i^a(t = 0, x) = \mathcal{W}_i^a(t = 0, x) \quad \text{and} \quad \partial_t \mathcal{W}_i^a(t = 0, x) = \partial_t \mathcal{W}_i^a(t = 0, x).
\]

(2.15)

For the scalar field, we choose

\[
\phi^a(t = 0, x) = \frac{\eta}{\sqrt{2}} (1, 0, 1) \quad \text{and} \quad \partial_t \phi^a(t = 0, x) = 0.
\]

(2.16)

Next we need to evolve these initial conditions. The SO(3) equations of motion are written in a form inspired by Numerical Relativity that provides for numerical stability [14]. The field theory parameters in the numerical work are \(g = 0.5, \lambda = 1\) and \(\eta = 1\), and the initial condition parameters were chosen to be \(w = 0.4, z_0 = 1, a = 10, \omega = 4\) and \(\omega' = -4\).

A first sign that monopoles are produced is that the evolution produces zeroes of the Higgs field. In figure 6, the minimum value of \(|\phi|\) over the entire simulation box is plotted as a function of time. The sharp drop after some time and the persistence of the zero value indicate that monopoles have been produced and survive until the end of the evolution.

Another characteristic of magnetic monopoles is that they are topological structures and so there is a topological charge density that should be non-vanishing at the location of the monopoles. The formula for the topological charge within a surface \(S\) is given by

\[
W(S) = \frac{1}{8\pi} \oint_S \hat{d}n^i \epsilon_{ijk} \epsilon_{abc} \phi^i \phi^j \phi^k \dot{\phi}^c,
\]

(2.17)
Figure 6. Minimum value of $|\phi|$ on the lattice as a function of time showing that zeroes of the scalar field are produced after some evolution. (Online version in colour.)

Figure 7. Topological winding at late times on slices with $z = 2.9, 3.7$ and $5.7$ for simulations on a $128^3$ lattice with $dx = 0.1$ and $z = 0$ at the centre of the lattice. The total topological charges on these slices are $+2, -4$ and $+2$, respectively. (Online version in colour.)

where $\hat{\phi}^a = \phi^a / |\phi|$. Figure 7 shows three different slices of the simulation box at $z = 2.9, 3.7$ and $5.7$ at the end of the run. The plots show four positive peaks at $z = 2.9$ and $5.7$ and four negative peaks at $z = 3.7$. Thus, the run produces four monopoles and four antimonopoles.

These numerical experiments are proof of concept showing that a class of initial conditions can be constructed to create magnetic monopoles.

3. String creation

Just as we considered the creation of $\bar{M}M$, we can consider the creation of strings (vortices) as discussed in [15]. The problem here is that only loops of strings can be created and these will be ephemeral as they will re-collapse and annihilate. On the other hand, vortices are present in superconductors and so some of these ideas can be implemented in the laboratory.

String solutions exist in the Abelian–Higgs model that is given by the Lagrangian

$$
\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2} |D_\mu \phi|^2 - \frac{1}{4} (|\phi|^2 - \eta^2)^2,
$$

where $\phi = \phi_1 + i\phi_2$ is a complex scalar field, $D_\mu = \partial_\mu + ieA_\mu$, $A_\mu$ is the $U(1)$ gauge field with field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and $\lambda$ and $e$ are coupling constants.

The solution for a straight string along the $z$-axis is

$$
\phi = \eta f(r) e^{i\theta} \quad \text{and} \quad A_i = v(r) e^{i\theta} \frac{x_j}{e r^2} \quad (i, j = 1, 2)
$$
where we work in cylindrical coordinates \( r = \sqrt{x^2 + y^2}, \ \theta = \tan^{-1}(y/x) \), \( f(r) \) and \( v(r) \) are profile functions that vanish at the origin and go to 1 asymptotically. We will set \( \epsilon = 0.5 \) in our numerical work. The energy per unit length (also the tension) of the string is given by

\[
\mu = \pi \eta^2 F(\beta),
\]

where \( \beta \equiv 2\lambda/\varepsilon^2 \). The function \( F(\beta) \) is known numerically and is a smooth, slowly varying function with \( F(1) = 1 \), \( m_S = \sqrt{2\lambda \eta} \) equals the vector mass, \( m_V = \epsilon \eta \). For \( \beta \) not too large, the thickness of the scalar fields in the string is \( \sim m_S^{-1} \) and of the vector fields is \( \sim m_V^{-1} \). We will only consider \( \beta = 1 \).

The string is characterized by a topological winding number that is defined by

\[
n = \frac{-i}{2\pi \eta^2} \oint d\xi \phi^* \partial_i \phi = \frac{1}{2\pi} \oint \frac{d\theta}{dl},
\]

where \( \theta \) is the phase of the scalar field at a given point on the contour and \( l \) denotes the parameter along the integration curve.

We based the initial conditions for our simulations on those used for monopole–antimonopole production (see equation (2.12)) with \( W_3^\mu \) in the monopole case corresponding to \( A_\mu \) in the string case. Now, the initial conditions for the gauge fields and their time derivatives are

\[
A_i(t = 0, x) = A_i(t = 0, x),
\]

and

\[
\partial_t A_i(t = 0, x) = [\partial_t A_i(t, x)]_{t=0}.
\]

The initial conditions for the scalar field are ‘trivial’,

\[
\phi(t = 0, x) = \eta, \quad [\partial_t \phi(t, x)]_{t=0} = 0.
\]

With these initial conditions, we were able to explore the parameter space for the formation of \( U(1) \) gauge strings in two settings: (i) the prompt formation of strings from gauge fields and (ii) the formation of strings when gauge wavepackets collide. Here, we only show the results of prompt string formation in which a single wavepacket of gauge fields evolves to produce strings as in figure 8.

Unlike the case of magnetic monopoles, the string loops that are formed are short-lived as they collapse and produce radiation as is evident from figure 9. The loops may live longer if we could find initial conditions that provide them with a greater angular momentum but these too will radiate and dissipate. On the other hand, once a magnetic monopole and antimonopole pair are produced with sufficient velocity, they will move apart and survive indefinitely. Furthermore, magnetic monopoles are localized objects and so the colliding wavepackets need not be very extended. For strings, on the other hand, the wavepackets have to extend over a region that is the size of the string loop that is to be produced, and only relatively small loops can be produced. In these respects it appears that magnetic monopoles are easier to produce than strings.

The flip side is that we know systems that contain gauge strings, while the existence of magnetic monopoles is still speculative. Gauge strings are known to exist in superconductors and, in that setting, our gauge field wavepackets correspond to photon wavepackets. This suggests that by shining light on superconductors we could produce strings within the superconductor. However, a realistic superconductor is described by a different set of equations that take into account the dependence of the model parameters on the temperature. It will be interesting to adapt our analysis to study string production in superconductors.
**4. $\tilde{M}$ in electroweak model**

The electroweak model is based on symmetry breaking

$$\frac{[SU(2) \times U(1)]}{\mathbb{Z}_2} \rightarrow U(1)_{EM}. \quad (4.1)$$

In this case, the initial symmetry group is not simply connected and it requires some care to see that there are no topological monopoles in the model. A simpler way to see the absence of topological magnetic monopoles is that the electroweak Higgs, $\Phi$, is an $SU(2)$ doublet

$$\Phi = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \quad (4.2)$$
and the minimum of the Higgs potential is given by
\[
\Phi^\dagger \Phi = \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 = \eta^2,
\] (4.3)
where \( \eta \) is the vacuum expectation value of the Higgs. Thus, the vacuum manifold is a three-sphere that does not admit incontractible two-spheres that are necessary to obtain topological monopoles.

The absence of topology in the standard model still leaves room for confined magnetic monopoles. Indeed it was shown in [16,17] that such magnetic monopoles do exist in the electroweak model. These electroweak monopoles carry magnetic charge but are attached to a string made of \( Z \) fields that connect the monopole to an antimonopole. The situation is very similar to that of a quark that carries electric charge but is confined by a quantum chromodynamics flux tube that connects it to an anti-quark of opposite electric charge.

Can we create electroweak monopole–antimonopole pairs by colliding gauge wavepackets? Even if we manage to create a monopole–antimonopole pair, they will be confined by the \( Z \)-string that will pull them together and cause them to annihilate. So electroweak monopoles can at best be created temporarily, somewhat like the string loops we discussed in §3.

There is one situation which is of interest even if electroweak monopoles are produced and that then annihilate. This is if the monopoles are produced but then annihilate after they have twisted by \( 2\pi \). A signature of such an event will be a change in the Chern–Simons number which is defined as
\[
CS = \frac{1}{32\pi^2} \epsilon^{ijk} \int d^3 x \left[ g^2 \left( W^a_{ij} W^a_{jk} - \frac{g}{3} \epsilon^{abc} W^a_i W^b_j W^c_k \right) - g^2 Y_i Y_k \right],
\] (4.4)
where \( W^a_\mu \) are the \( SU(2) \) gauge fields and \( Y_\mu \) the hypercharge gauge field. Changes in the Chern–Simons number are also indicative of baryon number violation in the electroweak model when fermions are included.

The initial conditions of §2c used to study monopole creation in the \( SO(3) \) model can be adapted to the electroweak model. Numerical experiments as of this time have not yielded a successful Chern–Simons number changing event but the search continues.

5. Conclusion

Magnetic monopoles are predicted in a wide class of particle physics models, e.g. all models of Grand Unification. Indeed, the realization that Grand Unification and standard cosmology predict an over-abundance of magnetic monopoles led to the proposal of inflationary cosmology that vastly dilutes the monopole abundance. Thus, monopoles are expected to be present in the physics that governs the universe but are not realized in our observable universe.

This peculiar circumstance is not special to magnetic monopoles. The underlying physics also admits other structures such as cars and computers but these do not occur naturally. Instead they require human intervention for their existence. Magnetic monopoles fall in this category—they may require humans to create them. Whether humans have the will to create monopoles is another question and the answer will hinge on their perceived utility. (One potential utility is to use magnetic monopoles for catalysing proton decay and to harness the released energy.)

These considerations have led us to study the interactions and dynamics of magnetic monopole–antimonopole pairs. The interaction of \( MM \) is non-trivial because of a twist that was known to exist from the 1970s [11,12]. We have described numerical work that confirms the picture and quantifies it more accurately. The twist also leads to non-trivial dynamics. The scattering of monopole–antimonopole can lead to a bounce instead of annihilation. We have used these results to intuot initial conditions that can lead to monopole creation and have successfully tested them to see the production of four monopoles and four antimonopoles. Similar studies that lead to the production of strings may be relevant to superconductors in the laboratory. When these methods are applied to the electroweak model, they can potentially generate Chern–Simons number changing events that lead to baryon number violation. However, we have not yet seen such an event in our numerical experiments and are continuing our explorations.


Data accessibility. This article has no additional data.

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