Topology Optimization - Engineering Contribution to Architectural Design

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Abstract. The idea of the topology optimization is to find within a considered design domain the distribution of material that is optimal in some sense. Material, during optimization process, is redistributed and parts that are not necessary from objective point of view are removed. The result is a solid/void structure, for which an objective function is minimized. This paper presents an application of topology optimization to multi-material structures. The design domain defined by shape of a structure is divided into sub-regions, for which different materials are assigned. During design process material is relocated, but only within selected region. The proposed idea has been inspired by architectural designs like multi-material facades of buildings. The effectiveness of topology optimization is determined by proper choice of numerical optimization algorithm. This paper utilises very efficient heuristic method called Cellular Automata. Cellular Automata are mathematical, discrete idealization of a physical systems. Engineering implementation of Cellular Automata requires decomposition of the design domain into a uniform lattice of cells. It is assumed, that the interaction between cells takes place only within the neighbouring cells. The interaction is governed by simple, local update rules, which are based on heuristics or physical laws. The numerical studies show, that this method can be attractive alternative to traditional gradient-based algorithms. The proposed approach is evaluated by selected numerical examples of multi-material bridge structures, for which various material configurations are examined. The numerical studies demonstrated a significant influence the material sub-regions location on the final topologies. The influence of assumed volume fraction on final topologies for multi-material structures is also observed and discussed. The results of numerical calculations show, that this approach produces different results as compared with classical one-material problems.

1. Introduction

Optimization as a field of science and engineering, although well recognized and widely developed remains still up-to-date challenge. Structural optimization has gained recently more attention since large computational ability become available for designers. This process is stimulated simultaneously by variety of emerging, innovative optimization methods. It is observed that traditional gradient-based mathematical programming algorithms, in many cases, are replaced by novel and efficient heuristic methods inspired by biological, chemical or physical phenomena. These methods become useful tools for structural optimization because of their versatility and easy numerical implementation. One of these modern methods, which gained significant attention among researchers and engineers is Cellular Automaton (CA). The idea of Cellular Automata has been inspired by the behaviour of biological
tissues, where behaviour of whole tissue is governed by cells that interact only with their neighbours. The modelling of complex systems by local interaction is governed by simple local rules. Cellular Automata can be adapted to optimal sizing as well as to topology optimization. The topology optimization, introduced in late eighties of the last century, become very popular in many areas of engineering science including civil engineering and architecture. The resulting structure generated by the topology optimization procedure is ensures optimal, from an objective point of view, distribution of material within considered domain. The optimized structure gains new shape and material layout, since some parts of material are relocated and others are selectively removed. Resulting, innovative structures can be an inspiration for designers and architects. The typical formulation of topology optimization is a redistribution of one or few materials within a whole design domain. It is possible also that material is functionally graded. This paper investigates a different approach. The design domain is divided into sub-regions for which different types of material are defined and through this procedure the multi-material structure is created. The challenge of the present research is to find optimal topologies, under restriction that redistribution of material can be performed only within sub-regions selected for employed materials.

2. Topology optimization

Topology optimization is a dynamically developing research area with numerous applications to many research and engineering fields ranging from aeronautical industry [1] through civil engineering [2] to architecture [3]. In recent years’ topology optimization has been more and more often used in the process of building design. Theoretical considerations as well as description of practical implementations can be found in literature. For example, wind loads are considered in [4-6], seismic hazard problem discussed in [7] treat problem from theoretical point of view whereas the support structure for a Canopy in Doha [8] may serve as the example of a real structure design. More discussion regarding shaping of tall buildings with topology optimization one can find in [3] and [9]. Broad class of applications complement bridge structures, for which a lot of numerical examples can be found in literature e.g. [10], masts [11] and tunnel reinforcements [12, 13].

The principle of topology optimization is to find a distribution of material within a design domain which is optimal in some sense. During the optimization process material is redistributed from parts, where is not necessary from objective point of view, to the parts, where structure stiffness is required to transfer applied loads to the supports. Topology optimization procedure leads finally to material/void distribution. It can be visualized by black and white regions over the design domain, as it is shown in figure 1.

![Figure 1. Initial structure with uniformly distributed material and optimized topology](image)

In topology optimization the design domain is usually discretized by finite elements, for which design variables \(d_i\) are selected. When the most common and a very efficient approach called SIMP (Solid Isotropic Material with Penalization) [14, 15] is implemented, the design variables are defined as relative densities of material. In what follows the elastic modulus \(E_i\) of each finite element can be modelled as a function of relative density \(d_i\) using power law:

\[
E_i = d_i^p E_0, \quad d_{\text{min}} \leq d_i \leq 1
\]
where $E_0$ is the elastic modulus of a solid material. The power $p$ usually chosen as 3 penalizes intermediate relative densities and drives design to a material (black) – and – void (white) structure. The majority of topology optimization results reported in the literature regard structures for which the distribution of material has been found so as to minimize their compliance $U$ under applied load subject to a total volume constraint. This problem can be formulated as the set of local minimizations posed for each element:

$$
\begin{align*}
\text{minimize} \quad & U(d_i) = d_i^p u_i^T k_i u_i \\
\text{subject to} \quad & V = \kappa V_0
\end{align*}
$$

(2)

where $u_i$ and $k_i$ are the element displacement vector and stiffness matrix, respectively. The quantity $\kappa$ is a prescribed volume fraction and $V_0$ stands for the design domain volume.

3. Topology optimization of multi-material structures

Many survey papers and books provide broad discussion on topology optimization concepts e.g. [14, 16, 17] and [18]. Simultaneously hundreds of papers present numerous results obtained for both simple models and complicated engineering structures. The vast majority of reported solutions regard structures made of one material. On the other hand, allowing for implementation of many material structures may open new possibilities for improving existing solutions. Motivation behind the present study is therefore to make an attempt to explore this area by presenting an original application of topology optimization for multi-material structures.

As the example a dual-material bridge structure presented in figure 4 has been chosen. The structure under consideration consists of two regions for which material is fixed, meaning that each region has different material. Within the approach proposed in this paper each material can be redistributed only in specified and assigned to it region. This original concept is inspired by designs of architectural structures like for example wood-concrete facades of buildings. It is worth noting that as far as topology optimization of multi-material structures is considered the approach reported in the literature is different, namely formulated problems refer to finding an optimal topology based on optimal distribution of all employed materials (see [19, 20]), what means that each material can be relocated within whole design domain.

4. Cellular Automata in topology optimization

Development in topology optimization area is stimulated by constantly growing computational effectiveness of optimization methods. The traditional gradient-based algorithms are nowadays more often replaced by various metaheuristics. Those methods have gained popularity among researchers because they are easy for numerical implementation and do not require gradient information, so that the computational cost of optimization process can be reduced. Particle Swarm Optimization, Ant Colony Optimization, Artificial Immune Systems or Cellular Automata may serve here as examples of intensively developed heuristic algorithms. In this paper the latter one has been chosen as the optimization tool, because of its simplicity, effectiveness and versatility. The first application of Cellular Automata to optimization of structures was proposed in mid-nineties of 20th century [21], but majority of papers dealing with that subject have been published during last decade (see [22, 23]). Cellular Automata (CA) are mathematical, discrete idealization of physical systems. Engineering implementation of CA requires decomposition of the design domain into a lattice of cells, states of which are updated according to local rules. These rules are based on heuristics or physical laws. The characteristic feature of CA is local interaction between a particular cell and cells forming its neighbourhood. The examples of plane (2D) neighbourhoods are presented in figure 2.
The local update rule which is applied to a design variable associated with a central cell is constructed based on information gathered among cells forming its neighbourhood. The rule can take a form of a linear combination of components, values of which are influenced by states of the neighbouring cells. First, compliance values $U_i$ are calculated for all cells while performing structural analysis. Then according to relations (3) and (4), values of both component $C_{a0}$ which is assigned to central cell and components $C_\alpha$ assigned to neighbouring cells are specified. Depending whether compliance $U_i$ of considered cell is larger or smaller than the selected threshold value $U^*$, the components $C_{a0}$ and $C_\alpha$ take positive or negative values. Finally, the values of $C_{a0}$ and $C_\alpha$ are transferred to the design variable update rule, according to (5):

$$
\alpha_0 = \begin{cases} 
-C_{a0} & \text{if } U_i^{(t)} < U^* \\
C_{a0} & \text{if } U_i^{(t)} \geq U^* 
\end{cases} 
$$  

$$
\alpha_k = \begin{cases} 
-C_\alpha & \text{if } U_{ik}^{(t)} < U^* \\
C_\alpha & \text{if } U_{ik}^{(t)} \geq U^* 
\end{cases} \quad k = 1...N
$$

$$
d_i^{(t+1)} = d_i^{(t)} + \Delta d_i^{(t)} \\
\Delta d_i^{(t)} = (\alpha_0 + \sum_{k=1}^{N} \alpha_k) m = \bar{\alpha} m
$$

In (5) $m$ is a move limit, $N$ refers to the number of neighbors of considered cell and $t$ is a discrete time (iteration). The above described algorithm turned out to be a very fast and efficient, what has already been reported for example in [24] and [25].

The bridge structure presented in figure 3 is considered here as the example showing effectiveness of the method. One type of material is applied, namely steel, for which a Young modulus is equal 210 GPa. Lower narrow area playing the role of a roadway is not optimized and is loaded by constant pressure 2kN/m. Lower corners of design domain are fixed. The design domain is discretized using regular grid of square finite elements. Number of elements equals 5200. Assumed volume fraction is 0.4. The value of the compliance for initial structure is equal $1.234 \times 10^{-2}$ Nm. In this paper, the initial structure consists of uniformly distributed material, for which Young modulus is multiplied by the value of established volume fraction to fulfil a total volume constraint.
The value of the compliance for resulting topology equals $1.004 \cdot 10^{-2}$ Nm. For a comparison, the topology optimization using optimality criteria method [26] has been selected. The reported value of the compliance for resulting topology in that case equals $1.077 \cdot 10^{-2}$ Nm. One can observe that the CA method allows to obtain topology with a lower value of final compliance.

5. Numerical examples of topology optimization of multi-material structures

As the numerical example of topology optimization of multi-material structure to be discussed in detail a plain elastic bridge structure presented in figure 4 has been chosen.

Two types of material are applied: concrete (drawn in grey colour), for which it was assumed, that a Young modulus is equal 30 GPa and steel (drawn in red colour), for which Young modulus is equal 210 GPa. Concrete part is located in lower region of a design domain, while steel sub-structure forms the upper part. The parts made of different materials are separated by fixed area with Young modulus 210 GPa. The fixed area which plays the role of a roadway is not optimized. The roadway is loaded by constant pressure 2 kN/m. Lower corners of design domain are fixed as well as middle points of both left and right edge. The design domain is discretized using regular grid of square finite elements. Number of elements equals 20160.

The aim of first consideration is to answer a question whether final topology depends on the value of Young modulus of applied materials? Is it different for a structure made of one material and for a structure made of few different materials? Is there a significant influence of the value of volume fraction on optimized topologies?

Firstly, the topology optimization of considered structure was performed for one material (steel) for structure as a whole for two values of volume fraction 0.5 and 0.35. Resulting topologies are shown in figures 5 and 6, respectively.
Figure 5. Final topology, one material – steel, volume fraction 0.5

Figure 6. Final topology, one material – steel, volume fraction 0.35

Then the topology optimization of considered structure was performed for one material, but this time for concrete for a whole structure (including roadway). Once again calculations were repeated for two values of volume fraction 0.5 and 0.35. Resulting topologies are shown in figures 7 and 8.

Figure 7. Final topology, one material – concrete, volume fraction 0.5

Figure 8. Final topology, one material – concrete, volume fraction 0.35

The next step is optimization of a bi-material structure while the location of different materials is in accordance with figure 4. The topology generation process has been performed and the obtained final topologies are shown in figure 9 for volume fraction 0.5 and in figure 10 for volume fraction 0.35.
Figure 9. Final topology, concrete – grey, steel – red, volume fraction 0.5

Figure 10. Final topology, concrete – grey, steel – red, volume fraction 0.35

Calculations have been repeated for inverted materials: now Young modulus for lower region is equal 210 GPa and for upper one 30 GPa. The final topologies are shown in figures 11 and 12 for volume fraction 0.5 and 0.35, respectively.

Figure 11. Final topology, steel – red, concrete – grey, volume fraction 0.5

Figure 12. Final topology, steel – red, concrete – grey, volume fraction 0.35

The values of compliances for considered variants of bi-material topology optimization are summarized in table 1.
Table 1. Values of compliances for initial and final topologies

| Volume fraction | Material          | Initial compliance $[10^2 \text{ Nm}]$ | Final compliance $[10^2 \text{ Nm}]$ |
|-----------------|------------------|-----------------------------------------|--------------------------------------|
| 0.5             | Steel            | 1.576                                   | 1.247                                |
|                 | Concrete         | 13.031                                  | 8.729                                |
|                 | Concrete and steel | 4.794                               | 3.780                                |
|                 | Steel and concrete | 3.436                               | 2.583                                |
| 0.35            | Steel            | 2.251                                   | 1.701                                |
|                 | Concrete         | 15.759                                  | 11.907                               |
|                 | Concrete and steel | 6.848                               | 4.865                                |
|                 | Steel and concrete | 4.908                               | 3.992                                |

6. Discussions of results
In the first part of numerical tests, the topology optimization has been performed for structures made of only one material – either steel or concrete. Comparing results presented in figures 5 and 7 with these shown in figures 6 and 8, one can observe that final topology of a structure made of one elastic, isotropic material does not depend on the Young modulus of the material. It is necessary to underline, however, that Young modulus has the influence on final values of compliances (table 1). Next remark is, that for uniform distribution of material the final topologies generated for various volume fractions do not differ significantly from each other, see figures 5 and 6 and figures 7 and 8. As to multi-material structures, the optimization of a bridge structure, which lower part is made of concrete and upper of steel, has been discussed first. Comparing the final topologies obtained for this mixed-material structure, presented in figure 9 for volume fraction 0.5 and in figure 10 for volume fraction 0.35, one can conclude that they significantly differ from topologies generated for one material structures for the same values of volume fractions. Moreover, the topologies for a mixed-material structure obtained for various volume fractions also differ from each other. It can be observed, that for low values of volume fraction the contribution of stiffer material in final topology is much more significant (figure 9 and 10). This effect is even more visible when the stiffer material is applied in lower region of a design domain. In such a case weaker material has been completely removed for volume fraction 0.35 (figure 12). Having compared the optimal topologies and the related final compliance values, it can be summarized, that the structure tends to utilize the stronger material as much, as volume constraint allows for that. It is worth underlining, that for multi-material structures as well as for structures made of only one material the final topologies and values of final compliances do not depend on initial value of design variables and on the values of applied fixed loads. Very interesting effect however can be observed when topology optimization of structures under design dependent loads like self-weight is performed. Then the final compliances are strongly dependent on a ratio between value of applied load and a weight of a structure (density of material).

7. Conclusions
The idea of topology optimization of multi-material structures has been presented. The optimal topologies strongly depend on the ratio of the Young moduli of applied materials and on configuration of material sub-regions within the design space. It has been demonstrated, that since the assumed volume fraction has a significant influence on the final topologies proposed formulation significantly differs from typical topology optimization of one material structures. Finally, it has been also shown, that the numerical algorithm based on Cellular Automata concept can be successfully implemented to generation of optimal topologies for multi-material structures.
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