Prediction of ultimate burst pressure and comparison of failure criteria for thermoplastic composite pipes

P Ge\textsuperscript{1,2}, H Xia\textsuperscript{3}, Q Liu\textsuperscript{1,2}, C Shi\textsuperscript{3}, W Xiao\textsuperscript{1,2}, X Jia\textsuperscript{1,2} and Y Xu\textsuperscript{1,2}

1 Key Laboratory of Enhanced Oil Recovery in Carbonate Fractured-vuggy Reservoirs, CNPC, Urumqi, Xinjiang 830011, China.
2 SINOPEC Northwest Company of China Petroleum and Chemical Corporation, Urumqi, Xinjiang 830011, China.
3 School of Petroleum Engineering, China University of Petroleum (East China), Qingdao, Shandong 266580, China.
E-mail: S18020108@s.upc.edu.cn

Abstract. Thermoplastic composite pipes (TCPs) are becoming the ideal substitute for traditional steel pipe due to its superiorities including light weight and corrosive resistance. The cross-section of TCPs consists of an inner liner, a laminate layer, and an outer jacket. The laminate layer is made of multi-plies of helically wound continuous fibre reinforced unidirectional tape. In the present study, a three-dimensional (3D) theoretical model and a 3D finite element model were developed to analyse the stress state of a TCP under internal pressure. With a selected failure criterion for composite laminate, the ultimate burst pressure of a TCP can be predicted. By comparing the predicted burst pressure with the experimental results, several commonly used failure criteria were compared in terms of their accuracy.

1. Introduction
In response to the increasing demand for non-corrosive pipes used in the onshore and offshore oil and gas industry, thermoplastic composite pipes (TCPs) are becoming the ideal substitute for traditional carbon steel. The cross-section of TCPs consists of an inner liner, a reinforcement layer, and an outer jacket, as shown in the Figure 1. The laminate layer is made of multi-plies of helically wound continuous fibre reinforced unidirectional tape. The three layers of a TCP are bonded together to form a solid-walled construction. TCP is widely used due to its advantages including high strength, light weight, and corrosion resistance.

![Figure 1. The three-layer construction of thermoplastic composite pipes.](image_url)

Over the past decades, TCP under internal pressure with the three-layer construction has been studied by many researchers [1]. The ‘generalized plane strain’ model was used to investigate the behaviour of a pipe reinforced by two laminate plies by Kruijer et al. [2]. Bai et al. [3] assumed uniform stress distribution through the thickness of the laminate in their theoretical method of the pipe. Actually, the...
stress through the thickness is not the uniform, especially for the high pressure cases. Xia et al. [4] studied a TCP with a laminate layer consists of four plies and its stress distributions under internal pressure based on the 3D anisotropic elasticity, which could consider the effect of non-uniformity for stresses in radial direction. Moreover, for the composite cylinder with a large number of laminate plies, 3D anisotropic constitutive relationship and the thick-walled shell theory all should be used to taken into account [5]. Xing et al. [6] investigated the stress and deformation of a multiple-ply laminate with kinds of winding angles under internal pressure, external pressure and axial force by using an analytical method and a finite element (FE) software ANSYS. Different failure criteria and failure mode were also considered. Onder et al. [7] predicted the burst pressure of pressure vessels by using the Tsai-Wu failure criterion, maximum strain and stress theories. Hastie et al. [8] investigated the failure of a TCP under the combined load of pressure, axial tension and thermal gradients based on 3D anisotropic elasticity, Maximum Stress Failure Criterion and Tsai-Hill criterion. However, investigation on the failure criteria for prediction of ultimate burst pressure for TCPs has not come to an agreement. Which criterion can predict the burst pressure more reliable is expected to be verified. The failure mode for the TCPs with a large number of laminate plies may be different than that verified for a TCP with few number of laminate plies which was proposed by Xu et al. [9].

In the present study, a theoretical method based on 3D anisotropic elasticity theory and a 3D finite element analysis (FEA) model built by ABAQUS were applied to predict the stress state for a TCP under internal pressure only. From obtained stress distributions, the maximum pressure was calculated by using the proposed method with various criteria. Results obtained were compared with the experimental results and showed the fibre failure mode was much more reliable relatively in this study. Other criteria seemed to be more conservative but much safer. This study provides useful tools and guidance for the design and analysis of TCPs, and is currently under validation through more experimental data in terms of different winding angles and different number of laminate plies.

2. Theoretical method

In the present study, TCPs are modelled as a multi-ply cylinder which has \( N \) plies totally as shown in the Figure 2. The inner liner and outer jacket are made of thermoplastic polymer, which are denoted as the 1\(^{\text{st}}\) ply \((k=1)\) and \(N^{\text{th}}\) ply \((k=N)\). The laminate is formed by an even number of orthotropic plies and the lay-up angle is \(\alpha\) with the opposite direction every two adjacent plies. Here, the structure and loading way are axisymmetric, so stresses and strains are independent of the hoop coordinate, \(\theta\). The displacements along the radial \(r\), axial \(z\), and tangential \(\theta\) directions can be expressed as [10]:

![TCP in cylinder coordinates](image)

\[
\begin{align*}
    u_r &= u_r(r), \\
    u_\theta &= u_\theta(r, z), \\
    u_z &= u_z(z)
\end{align*}
\]

(1)

The strain-displacement relations in cylindrical coordinates are expressed as:

\[
\begin{align*}
    \varepsilon_r^{(k)} &= \frac{du_r^{(k)}}{dz}, \\
    \varepsilon_\theta^{(k)} &= \frac{du_\theta^{(k)}}{dr}, \\
    \varepsilon_z^{(k)} &= \frac{du_z^{(k)}}{dz}, \\
    \gamma_r\theta^{(k)} &= \frac{du_r^{(k)}}{dr} - \frac{du_\theta^{(k)}}{dz}, \\
    \gamma_r z^{(k)} &= 0, \\
    \gamma_\theta z^{(k)} &= \frac{du_\theta^{(k)}}{dz} = \gamma_\theta r
\end{align*}
\]

(2)

Stresses and strains of every ply in the cylindrical coordinate system are related by the constitutive equations:
In the absence of the body force, the equilibrium equations are:

\[
\frac{d\sigma_r^{(k)}}{dr} + \frac{\sigma_r^{(k)} - \sigma_\theta^{(k)}}{r} = 0
\]

\[
\frac{d\tau_\theta^{(k)}}{dr} + 2\frac{\tau_\theta^{(k)}}{r} = 0
\]

\[
\frac{d\tau_\phi^{(k)}}{dr} + \tau_\phi^{(k)} = 0
\]

From Equations (5) and (6), the results can be obtained:

\[
\epsilon_\theta^{(k)} = \frac{A^{(k)}}{r^2}, \quad \epsilon_\phi^{(k)} = \frac{B^{(k)}}{r}
\]

Combining the constitutive equations (Equations (3)), equilibrium condition (Equation (4)), strain-displacement relations (Equation (2)) and displacement fields (Equation (1)), the solution of radial displacement for each ply can be solved as:

\[
u_r^{(k)} = D^{(k)} r^{\beta_1} + E^{(k)} r^{\beta_2} + \alpha_1^{(k)} \epsilon_\theta + \alpha_2^{(k)} \gamma_\phi r^2
\]

And \(\beta_1^{(k)}, \alpha_1^{(k)}, \alpha_2^{(k)}\) can be expressed as

\[
\beta_1^{(k)} = \sqrt{\frac{C^{(k)}_{22}}{C^{(k)}_{33}}}
\]

\[
\alpha_1^{(k)} = \frac{C^{(k)}_{12} - C^{(k)}_{13}}{C^{(k)}_{33} - C^{(k)}_{22}}
\]

\[
\alpha_2^{(k)} = \frac{C^{(k)}_{26} - 2C^{(k)}_{46}}{4C^{(k)}_{33} - C^{(k)}_{22}}
\]

Where \(D^{(k)}, E^{(k)}\) are unknown constants.

Under internal pressure, \(P\), the boundary conditions at inner and outer radii are written as:

\[
\sigma_r^{(1)}(r_i) = P, \quad \sigma_r^{(N)}(r_{N+1}) = 0
\]

\[
\tau_\theta^{(i)}(r_i) = \tau_\theta^{(N)}(r_{N+1}) = 0, \quad \tau_\phi^{(1)}(r_i) = \tau_\phi^{(N)}(r_{N+1}) = 0
\]

Assuming the plies are bonded perfectly, the interface continuities are:

\[
u_r^{(k)}(r_k) = u_r^{(k+1)}(r_k), \quad u_\theta^{(k)}(r_k) = u_\theta^{(k+1)}(r_k), \quad u_\phi^{(k)}(r_k) = u_\phi^{(k+1)}(r_k)
\]

\[
\sigma_r^{(k)}(r_k) = \sigma_r^{(k+1)}(r_k), \quad \sigma_\theta^{(k)}(r_k) = \sigma_\theta^{(k+1)}(r_k), \quad \sigma_\phi^{(k)}(r_k) = \sigma_\phi^{(k+1)}(r_k)
\]

Axial force, \(F_N\), at the pipe end, is determined by integrating \(\sigma_r\) over the cross-sectional area. Torque is determined by the integrating \(\tau_\phi\) over the cross-sectional area. Considering the pipe with closed ends, axial equilibrium and zero torsion are expressed by the integrals:
\[
2\pi \sum_{k=1}^{N} \int_{0}^{r_{k}} \sigma_{z}^{(k)}(r) r dr = F_{N} = \pi r_{i}^{2} P, \\
2\pi \sum_{k=1}^{N} \int_{0}^{r_{k}} \varepsilon_{r\theta}^{(k)}(r) r^2 dr = 0. 
\] (16)

By substituting Equations (13) and (15) into (7), \( A^{(k)} = B^{(k)} = 0 \). For \( N \) plies there exist \( 2N + 2 \) unknowns, i.e. \( D^{(k)}, E^{(k)}, \varepsilon_{0}, \gamma_{0} \) (for \( k = 1, 2, \ldots, N \)), which can be calculated from boundary conditions, continuity conditions and axial/torque integrals. Then displacements, stresses and strains can be obtained.

3. FEA model

3.1. Model establishment

A 3D finite element (FE) model was developed by using ABAQUS/Standard nonlinear finite analysis tool, which is shown in Figure 3. It was meshed by 20-node quadratic brick, reduced integration element C3D20R. At one end of pipe, the reference point (RP-1) is fully fixed in the centre and coupled to the end face for all 6 degrees of freedom. At the opposite end, the reference point (RP-2) was fully coupled to the end face in all but the radial direction. Boundary condition applied on the reference point, RP-, and only the displacement in axial direction is allowed. A pressure was applied on the inner surface of the liner and a concentrated force, which represents the effect of pressure on the end surfaces, was applied on RP-2. A suitable mesh was established by performing a refinement exercise.

![Figure 3. The mesh model of TCP made in ABAQUS](image)

3.2. Verify the theoretical model

An example that consists of 12 layers of laminate plies is given to discuss stress distributions and verified the accuracy of theoretical model. The parameters of inner radius, wall thickness, and material properties of unidirectional Glass/PE composite plies and HDPE are listed in the Table 1 and Table 2, respectively. An analytical solution based on the aforementioned formulations was developed in MATLAB and compared with those obtained using the FEA simulation. In this section, a pipe with an internal pressure of 10 MPa was used.

| Table 1. The parameters of TCPs |
|---------------------------------|
| Dimensions                      | value |
| Inner diameter, (mm)            | 46    |
| Thickness of inner liner, \( t_{i} \) (mm) | 3.0   |
| Thickness of each reinforced ply, \( t_{r} \) (mm) | 0.3   |
| Thickness of outer jacket, \( t_{o} \) (mm) | 3.0   |
| Winding angle, \( \alpha \) (\(^{\circ}\)) | 55    |
Table 2. Material properties

| Material properties                      | Unidirectional Glass/PE composite tape | HDPE |
|------------------------------------------|----------------------------------------|------|
| Elastic Modulus (MPa)                    | $E_1$ 28000                            | $E_1$ 1100 |
|                                          | $E_2 = E_3$ 3200                       | $E_1$ 1100 |
| Poisson ratio                            | $\nu_{12} = \nu_{13}$ 0.0343           | $\nu$ 0.4 |
|                                          | $\nu_{23}$ 0.3                         |      |
| Shear modulus (MPa)                      | $G_{12} = G_{13}$ 2750                 | $G_{23}$ 1230 |
|                                          | $G_{23}$ 1230                          |      |

Distribution of stress components through the thickness based on the MATLAB and FEA are shown in Figure 4. $\sigma_1, \sigma_2, \sigma_3$ and $\tau_{12}$ denote longitudinal stress, transverse stress, radial stress, and in-plane shear stress, respectively. It was found that both configurations in Theoretical results and FEA results generally matched. Among the stresses through laminate, the longitudinal stress was much higher than the others and decreased from around 85 MPa to 72 MPa in theoretical results, the axial stress and radial stress increased slightly, the shear stress decreased slowly. The tool has been set-up as a fully analytical tool allowing for faster design optimization than with FEA methods. In the following study, the analytical method is used.

Figure 4. Comparison of stress components using the two methods

4. Case study

4.1. Experiment

The TCP specimens were 0.5m long and terminated at both ends by non-reusable steel swage fittings. During the tests, the pipes were free to deform in axial direction. Short-term bursting test were carried out with a specimen at room temperature. The internal pressure was applied by a pump. When a loud bang was heard and a sharp drop of the tested pressure curve appeared, it implied that the burst of the pipe occurred. Then the pressure pump was shut down manually and the bursting point was recorded. Generally, the bursting would occur within 60 s for a 2-inch TCP. The parameters of outer radius, wall thickness are same as that listed in the Table 1 and the burst time and burst pressure are shown in the Table 3. The specimen has a winding angle of $55^\circ$.

Table 3. Results of experiment
| Test No. | Number of laminate plies | Bursting time (s) | Bursting Pressure (MPa) | Mean value (MPa) |
|---------|-------------------------|------------------|-------------------------|------------------|
| 1       | 12                      | 58               | 57.5                    |                  |
| 2       | 12                      | 42               | 59.5                    | 58.3             |
| 3       | 12                      | 35               | 58                      |                  |

4.2. Laminate failure criteria

In order to assess local stress-based material failure with the proposed theoretical method, stresses must be transformed from cylindrical coordinates to principal material coordinates shown in Figure 5, the formula for transforming is as follows [10]:

\[
\begin{bmatrix}
\sigma_1 \\ \\ \sigma_2 \\ \\ \sigma_3 \\ \\ \tau_{23} \\ \\ \tau_{31} \\ \\ \tau_{12}
\end{bmatrix}
= 
\begin{bmatrix}
m^2 & n^2 & 0 & 0 & 0 & 2mn \\ n^2 & m^2 & 0 & 0 & 0 & -2mn \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m & -n & 0 \\ 0 & 0 & 0 & n & m & 0 \\ -mn & mn & 0 & 0 & 0 & m^2-n^2
\end{bmatrix}
\begin{bmatrix}
\sigma_c \\ \\ \sigma_o \\ \\ \sigma_r \\ \\ \tau_{\theta r}
\end{bmatrix}
\]

(17)

Fibre failure mode, Tsai-Wu 3D, Tsai-Hill 3D, and Tsai-Hill 2D criteria are used to predict the failure pressure.

The expressions are displayed as following:

Fibre failure mode

\[ f = F_1\sigma_1^2 + F_{66}(\tau_{12} + \tau_{13})^2 \]  

(18)

Tsai-Wu 3D

\[ f = F_1\sigma_1 + F_2\sigma_2 + F_3\sigma_3 + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{33}\sigma_3^2 + 2F_{12}\sigma_1\sigma_2 + 2F_{23}\sigma_2\sigma_3 + 2F_{31}\sigma_1\sigma_3 + F_{44}\tau_{13}^2 + F_{35}\tau_{23}^2 + F_{66}\tau_{12}^2 \]  

(19)

Figure 5. Transformation from cylindrical coordinates to principal material coordinates

The strength parameters in Fibre failure mode, Tsai-Wu 3D are given as

\[
F_1 = \frac{1}{X_y} - \frac{1}{X_x}, \quad F_2 = \frac{1}{Y_y} - \frac{1}{Y_x}, \quad F_3 = \frac{1}{Z_y} - \frac{1}{Z_x}, \quad F_{11} = \frac{1}{X_x X_y}, \quad F_{12} = \frac{1}{X_x Y_y}, \quad F_{22} = \frac{1}{Y_x Y_y}, \quad F_{33} = \frac{1}{Z_x Z_y}, \quad F_{44} = \frac{1}{S_{13} S_{23}}, \\
F_{55} = \frac{1}{S_{12} S_{23}}, \quad F_{66} = \frac{1}{S_{12} S_{13}}, \quad F_{12} = F_{21} = \frac{1}{2}\sqrt{F_{11} F_{22} F_{33}}, \quad F_{23} = F_{32} = F_{13} = \frac{1}{2}\sqrt{F_{11} F_{22} F_{33}}, \quad F_{44} = \frac{1}{2}\sqrt{F_{11} F_{22} F_{33}} \]

(20)

Tsai-Hill 3D
\[ f = F_1 \sigma_1^2 + F_2 \sigma_2^2 + F_3 \sigma_3^2 - \sigma_1 \sigma_3 (F_{11} + F_{22} - F_{33}) - \sigma_2 \sigma_3 (F_{22} + F_{33} - F_{11}) \]
\[ -\sigma_3 \sigma_1 (F_{11} + F_{33} - F_{22}) + F_{44} \tau_{23}^2 + F_{55} \tau_{13}^2 + F_{66} \tau_{12}^2 \]

Tsai-Hill 2D
\[ f = F_1 \sigma_1 + F_2 \sigma_2 + F_3 \sigma_3 + F_{11} \sigma_1^2 + F_{22} \sigma_2^2 - F_{11} \sigma_3 \sigma_1 + F_{66} \tau_{12}^2 \]

The strength parameters in Tsai-Hill 3D, Tsai-Hill 2D are given as
\[ F_{11} = \frac{1}{X_1}, F_{22} = \frac{1}{Y_2}, F_{33} = \frac{1}{Z_3}, F_{44} = \frac{1}{S_{23}^2}, F_{55} = \frac{1}{S_{13}^2}, F_{66} = \frac{1}{S_{12}^2} \]

where \( X, Y, \) and \( Z \) are strengths along directions \( 1, 2, \) and \( 3 \) respectively, subscripts ‘t’ and ‘c’ indicate tensile and compressive strengths; \( S_{13}, S_{23} \) and \( S_{12} \) are the shear strengths in the plane \( 13, 23 \) and \( 12 \) respectively.

In the following, taking the glass fibre-reinforced composite laminates in TCPs for example, the prediction of bursting pressure for various failure criteria are compared with experimental results which showed in Table 2. And the failure strength of unidirectional tapes is shown in the Table 4.

| Table 4. Failure strength of unidirectional tapes (MPa) |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| \( X_t \)      | \( X_c \)     | \( Y_t \)     | \( Y_c \)     | \( Z_t \)     | \( Z_c \)     | \( S_{12} \)   | \( S_{23} \)   | \( S_{13} \)   |
| 829            | 146           | 30            | 49            | 30            | 49            | 52             | 40             | 52             |

4.3. Results

In Figure 6 below, the theoretical predictions for the various failure criteria are presented for the burst strength of a \( \pm 55^\circ \) pipe. The result from an actual internal pressure test is also indicated by the red line. The Fibre failure mode seemed to be suitable for the prediction of burst pressure in terms of the experimental results. Tsai-Wu 3D was used to predict at a less lower level. More conservative results predicted by Tsai-Hill in 2D and 3D. It is obvious that Tsai-Hill criterion shows a drawback in predicting the failure for the composite laminate possessed the different tension and compression strength. By the contrary, Tsai-Wu 3D, which is renewed based on Tsai-Hill, showed a feasible result.

![Figure 6](image-url)

Figure 6. Comparison between theoretical prediction and experimental results for burst pressure

5. Conclusions

In the present study, a theoretical method based on 3D anisotropic elasticity theory and a 3D FEA model built by ABAQUS were applied to predict the stress state for a TCP with 12 layer of laminate plies under internal pressure. From obtained stress distributions, the ultimate burst pressure was predicted by using the proposed method with various criteria. Results obtained were compared with the experimental results and showed the Fibre failure mode was much more reliable choice in this study. Other criteria seemed to be more conservative but much safer. This study provides useful tools...
and guidance for the design and analysis of TCPs, and is currently under validation through more experimental data in terms of different winding angles and different number of laminate plies.

6. References

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