TORQUE REVERSAL IN ACCRETION-POWERED X-RAY PULSARS

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ABSTRACT

Accretion-powered X-ray pulsars 4U 1626–67, GX 1+4, and OAO 1657–415 have recently shown puzzling torque reversals. These reversals are characterized by short timescales, on the order of days, nearly identical spin-up and spin-down rates, and very small changes in X-ray luminosity. We propose that this phenomenon is the result of sudden dynamical changes in the accretion disks triggered by a gradual variation of mass accretion rates. These sudden torque reversals may occur at a critical accretion rate \( \dot{M}_{\text{crit}} \approx 10^{15} - 10^{16} \text{ g s}^{-1} \) when the system makes a transition from (to) a primarily Keplerian flow to (from) a substantially sub-Keplerian, radial advective flow in the inner disk. For systems near spin equilibrium, the spin-up torques in the Keplerian state are slightly larger than the spin-down torques in the advective state, in agreement with observation. The abrupt reversals could be a signature of pulsar systems near spin equilibrium with the mass accretion rates modulated on a timescale of a year or longer near the critical accretion rate. It is interesting that cataclysmic variables and black hole soft X-ray transients change their X-ray emission properties at accretion rates similar to the pulsars’ critical rate. We speculate that the dynamical change in pulsar systems shares a common physical origin with white dwarf and black hole accretion disk systems.

Subject headings: accretion, accretion disks — pulsars: general — stars: magnetic fields — X-rays: stars

1. INTRODUCTION

Detailed spin evolution on long timescales has been made available for several accretion-powered X-ray pulsars such as 4U 1626–67, GX 1+4, and OAO 1657–415 (for comprehensive reviews and data, see, e.g., Chakrabarty et al. 1993; Chakrabarty 1995; and references therein). These systems have shown puzzling abrupt spin reversals. Before and after the observed torque reversals, their spin-up and spin-down torques were largely steady. It is intriguing that the spin-up and spin-down torques before and after reversals are nearly identical. These reversals are quite different from the random torque fluctuations seen in some pulsar systems believed to be fed by winds (see Nagase 1989; Anzer & Börner 1995; and references therein). The nearly steady torques plausibly indicate the existence of ordered accretion disks. The observations indicate that the mass accretion rate is gradually modulated with small amplitudes on a timescale of at least a year, which is much longer than the typical reversal timescale.

In the disk-magnetosphere interaction models of the Ghosh-Lamb type (Ghosh & Lamb 1979a, 1979b; Campbell 1992; Yi 1995; Wang 1995), the magnetic torque is a function of the mass accretion rate, \( \dot{M} \). The sign of the torque is reversed (spin-up/down) as \( \dot{M} \) varies when the disk inner edge moves past the equilibrium radius at which the torque vanishes (see Lipunov 1992). In this picture, however, the torque variation is expected to be smooth and continuous unless \( \dot{M} \) varies discontinuously. Although this behavior may be relevant for some smooth torque reversals, the observed sudden reversals appear distinct (Chakrabarty 1995). Given the lack of any plausible mechanism for discontinuous change of \( \dot{M} \), which must be tuned to occur near spin equilibrium, it is difficult for the existing magnetized disk models to provide an explanation. The observations indicate that the mass accretion rates vary little during transition (see Chakrabarty 1995), which makes the discontinuous change of \( \dot{M} \) unlikely as an explanation.

We take the neutron star moment of inertia \( I_* = 10^{46} \text{ g cm}^2 \), radius \( R_* = 10^6 \text{ cm} \), and mass \( M_* = 1.4 M_\odot \). At a cylindrical radius \( R \) from the star, the vertical component of the dipole magnetic field \( B_v(R) = B_\phi(R^{1/2}/R) \), where \( B_\phi \) is the stellar surface field strength. The spin period is \( P_* \), or angular velocity \( \Omega_* = 2\pi/P_* \).

2. KEPLERIAN DISK-MAGNETOSPHERE INTERACTION AND INNER REGION

In the conventional disk-magnetosphere interaction model of the Ghosh-Lamb type (Ghosh & Lamb 1979a, 1979b), the magnetic field of an accreting neutron star penetrates a geometrically thin accretion disk and exerts a magnetic torque. Except in a narrow region near the radius where the disk is disrupted, it is assumed that the accretion disk rotation is Keplerian, \( \Omega(R) = (GM_*/R^2)^{1/2} \), the radial internal pressure gradient is small, the radial drift velocity is small, and the disk thickness becomes negligible (Campbell 1992; Yi 1995; Wang 1995; and references therein). In such a model, the \( \phi \) component of the induction equation in steady state gives the azimuthal component of the field

\[
B_\phi(R) = \frac{\gamma \Omega_* - \Omega_\phi(R)}{\alpha \Omega_\phi(R)} B_\phi(R),
\]

(1)

where we have assumed that the internal viscosity and the magnetic diffusivity are due to a single turbulent process. Here we will assume that \( \gamma \), defined as the ratio of \( R \) to the vertical velocity shear length scale \( (v_\theta/(\partial v_\phi/\partial z)) \), is \( \sim 1 \) (see Campbell 1992; Yi 1995). The parameter \( \alpha \) is the usual viscosity parameter (see Frank, King, & Raine 1992), and we take \( \alpha = 0.3 \). Although we adopt specific values of \( \gamma \) and \( \alpha \), the constraints on \( B_\phi \) could always be rescaled in such a way that the exact individual values of the three parameters are not necessary (see Kenyon, Yi, & Hartmann 1996). The inner edge of the

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disk, where the disk is magnetically disrupted, is at \( R = R_c \), determined by the condition that the magnetic torque exceeds the internal torque in the disk (Campbell 1992; Yi 1995; Wang 1995), which can be expressed as

\[
\left( \frac{R}{R_i} \right)^{7/2} = \frac{2N_i}{\dot{M}(GM\_\text{e}R_i)^{1/2}} \left[ 1 - \left( \frac{R_i}{R} \right)^{3/2} \right],
\]

where \( N_i = (\gamma/\alpha)B_i^2R_i^4 \), \( B_i = B_c(R = R_i) \), and \( R_c = (GM \_\text{e}P_i^2/4\pi^2)^{1/3} \) is the Keplerian corotation radius. Integrating the magnetic torque over the disk, and allowing for the angular momentum carried by the gas that crosses the inner edge of the disk, the torque exerted on the star by the disk is

\[
N = \frac{7}{6} N_0 \frac{1 - (8/7)\left(R_c/R_0\right)^{3/2}}{1 - (R_c/R_0)^{3/2}},
\]

where \( N_0 = \dot{M}(GM\_\text{e}R_0)^{1/2} \) (Campbell 1992; Yi 1995; Wang 1995) and the equilibrium \( N = 0 \) when \( R_c/R_0 = 0.915 \). Most of the contribution to the torque in this type of model comes from field-disk interaction in a narrow region just outside of \( R_c \). Several different phenomenological descriptions of disk-magnetosphere interaction cause little practical differences to our conclusions (Wang 1995).

Figure 1 presents examples of the typical smooth torque reversal expected in the models of the Ghosh-Lamb type. It is clear that any densely sampled spin evolution would reveal a gradual and continuous variation of the torque, a generic prediction of the Ghosh-Lamb-type model. A sudden torque reversal with nearly constant \( |N| \) is hard to explain unless there exists an unknown constraining mechanism or the \( \dot{M} \) variation is discontinuous and fine tuned. It is possible that some observed smooth torque reversals (Chakrabarty 1995) could be caused by this type of reversal model. In Figure 1, GX 1+4 event could be due to such a smooth transition.

Under suitable conditions, however, the inner parts of the accretion disk may evolve to an optically thin, low-density state, for example, inside the disk boundary layer around a white dwarf (see Paczynski 1991; Narayan & Popham 1993). When \( \dot{M} \) is sufficiently small, this transition may occur over a broad range of radii within a disk, producing a state in which radiative losses are so inefficient that the disk retains a large fraction of the heat generated from the dissipation of orbital energy (the “advective state”; see Narayan & Yi 1995). The transition to this state is not well understood and may be affected by various external factors, including X-ray irradiation.
(see Meyer & Meyer-Hofmeister 1990) and coronal heating (Meyer & Meyer-Hofmeister 1994).

Here we will assume that, whatever the details of the transition, the disk will make the jump to a low density state when it becomes possible for it to do so. What is the critical \( M_{\text{crit}} \) below which the disk becomes hot and optically thin? Observationally, weakly magnetized cataclysmic variables generally show a trend in which the X-ray to optical flux ratio decreases as \( M \) increases. Above a critical rate of \( \sim 10^{16} \text{ g s}^{-1} \), the X-ray emission becomes extremely weak. This has been attributed to the transition of the inner region to an optically thin (X-ray emitting) hot accretion disk (Patterson & Raymond 1985; Narayan & Popham 1993) at low \( M \). Interestingly, such a critical rate is largely consistent with the critical rate \( \sim 2 \times 10^9 \alpha^2 \), or \( \sim 2 \times 10^{16} \text{ g s}^{-1} \), for \( \alpha \sim 0.3 \) based on the recently discussed advection-dominated hot accretion disks (Narayan & Yi 1995). For a given accretion rate, there exists a critical radius inside of which the disk makes a transition to sub-Keplerian rotation, while the disk at radii larger than the critical radius remains Keplerian (Narayan & Yi 1995). The exact location of this critical radius is not clearly understood yet. We assume that most of the magnetic torque contribution comes from the inner region (see Wang 1995), which is inside the critical radius, e.g., \( R_{\text{crit}} \gg R' > R_c \). The relevance of the critical \( M_{\text{crit}} \) for the cataclysmic variables becomes more striking when we consider strongly magnetized neutron star systems. The inner edge of the accretion disk around a strongly magnetized neutron star lies roughly at \( R_c \sim 5 \times 10^8 (B_9/10^5 \text{ G})^{7/2} (M/10^n \text{ g s}^{-1})^{-1/2} \text{ cm} \), which is close to the typical white dwarf dwarf star. Therefore, one may ask: What would be the effects of the transition, at \( M_{\text{crit}} \sim 10^{16} \text{ g s}^{-1} \), to the hot optically thin accretion disk on the spin evolution of the pulsars?

3. DISK TRANSITION AND TORQUE REVERSAL

We propose an explanation for the torque reversal based on such a transition. The optically thin, hot accretion disk cannot be geometrically thin or Keplerian once its internal pressure \( \sim \rho c_s^2 \) becomes a significant fraction of its orbital energy. After the transition, the disk thickness \( H \sim c_s/\Omega_k \) and radial drift velocity \( v_R \sim c_s^2/\Omega_k \) increase. The rotation of the accretion disk \( \Omega \) becomes sub-Keplerian \( \Omega < \Omega_k \) (see Narayan & Yi 1995). In the case of strongly magnetized pulsars, direct X-ray observation of such a disk transition is difficult because most of the X-ray luminosity, \( L_x \sim GM_* \dot{M}/R_* \), comes from the surface of the star and the luminosity from the disk, which is truncated well above the stellar surface, is limited to \( \sim GM_* \dot{M}/R_* \ll L_x \). The sub-Keplerian disk, however, may have observable dynamical consequences. Once the sub-Keplerian rotation is forced on the magnetosphere, the sub-Keplerian corotation radius is shifted inward, and the position of the disk inner edge with respect to the new corotation radius is relocated. As a result, the magnetic torque changes, and there could be a visible change of spin-up/down torque on the disk transition timescale.

If such a transition does occur at a certain critical rate, the likely upper limits on the transition timescale are the local thermal timescale \( t_{\text{th}} \sim (\alpha \Omega_k)^{-1} \) or the disk viscous-thermal timescale \( t_{\text{visc}} \sim R/(\alpha c_s) \sim (R/\Omega_k)(\alpha \Omega_k)^{-1} \sim 10^4 \text{ s} \) for \( \alpha \sim 0.3 \), \( R \sim 10^8 \text{ cm} \), and \( M \sim 10^{16} \text{ g s}^{-1} \) (see Frank et al. 1992). These timescales indicate that the local disk surface density change could occur on a timescale short enough, much less than a day, to make the transition appear almost instantaneous. The long-term gradual \( \dot{M} \) modulation determines the overall evolutionary trend and possibly affects the residuals seen in some observations (Cutler, Dennis, & Dolan 1986; Chakrabarty 1995). The sudden torque reversal does not require any short term (\( \sim 1 \) day) change of \( M \) but only that the condition \( \dot{M} \sim M_{\text{crit}} \) be satisfied around the time of reversal.

In order to model the reversal episode, we assume no dynamic vertical motion of the disk gas such as winds or outflows. The dominant effect of the transition is to reset the corotation radius and disk truncation radius \( R_c \). We take the temperature of the sub-Keplerian hot disk to be a constant fraction \( \xi \) of the local virial temperature, i.e., \( c_s^2 \sim 2R^2 \Omega_k^2 \) (see Narayan & Yi 1995). Then the ratio of the disk thickness to radius is \( H/R \sim \xi/2 \), and the radial drift velocity is \( v_R \sim -\alpha \Omega R_k \). Using the radial component of the momentum equation (see Campbell 1992), we get the sub-Keplerian rotation frequency \( \Omega/\Omega_k \sim (1 - 5\xi/2 - \alpha^2 \xi/2)^{1/2} = \mathcal{A} \), where \( \xi \rightarrow 0 \) corresponds to the usual Keplerian limit. We note that \( \Omega/\Omega_k \sim \alpha \xi < 1 \) and \( H/R \sim \xi/2 \leq 1 \). Assuming a constant \( \xi \) or \( A \), after the transition to the sub-Keplerian rotation with \( \Omega/\Omega_k = \mathcal{A} \Omega_k \), the corotation radius becomes \( R_c' = \mathcal{A}^2 R_c \). The new inner disk edge is relocated to \( R_{\text{new}} \) determined by

\[
\frac{R_{\text{new}}}{R_c} = \frac{2N_y}{N_y A} \left[ \frac{1}{1 - \left( \frac{R_c}{R_{\text{new}}} \right)^{3/2}} \right],
\]

where \( N_y = \mathcal{A} M (GM_c/R_c')^{1/2} \). The torque on the star after the transition is

\[
N_{\text{eq}}' = \frac{7}{6} \left[ 1 - \frac{8}{7} (R_c/R_c')^{3/2} \right],
\]

where \( N \rightarrow 0 \) as \( R_c/R_c' \rightarrow 0.915 \) as in the Keplerian rotation (eq. [3]). In our discussions, we take a constant \( \mathcal{A} = 0.2 \) (see Narayan & Yi 1995). The parameters \( \alpha = 0.3 \) and \( \gamma = 1 \) are assumed to be constant before and after the transition. For the Keplerian disk, the equilibrium spin \( (N = 0 \text{ in eq. [3]}) \) period is

\[
P_{\text{eq}} = \frac{[4.9 \text{ s}]}{\alpha} \left[ \frac{B_9}{10^3 \Omega_{\text{K}}} \right]^{7/6} \left[ \frac{R_*}{10^8 \text{ cm}} \right]^{1/6} \left[ \frac{M_9}{1.4 M_{\odot}} \right]^{-7/6} \left[ \frac{\dot{M}}{10^{16} \text{ g s}^{-1}} \right]^{-3/7}.
\]

For \( \Omega = A \Omega_{\text{K}} < \Omega_k \), the equilibrium spin period would become longer by a factor \( 1/A \), and the system begins to evolve toward the newly determined equilibrium after transition.

For the spin evolution calculation we integrate the torque equation

\[
\frac{dP_\ast}{dt} = -\frac{P_\ast^2}{T_\ast} \left[ \text{Torque} \right],
\]

where \( \text{Torque} = N \) or \( N' \) depending on the physical state of the inner disk. We take a linear increase or decrease of \( \dot{M} \) as an approximation to more complex \( \dot{M} \) variations on longer timescales. The transition occurs on a timescale \( \tau \sim P_\ast/[dP_\ast/dt] \) before and after the transition. The transition at \( M_{\text{crit}} = 10^{15} - 10^{18} \text{ g s}^{-1} \), which is determined by the fits to the observed spin evolution, is taken to be instantaneous (cf. fig. \( t_{\text{th}} \)). For each torque reversal event, we adjust \( B_\ast, \dot{M} \), and the accretion rate.
timescale, $\dot{M}/d\dot{M}/d t$. For a given initial spin period $P_*$, a fit gives a set of the above parameters.

$4 U 1626-67.—4 U 1626-67 (P_* \approx 7.7 s)$ was steadily spun-up on a timescale $\sim 10^2$ yr ($P_*/P_*^{\text{cy}} = -8.54(7) \times 10^{-15}$ s$^{-2}$) during 1979–1989. The Keplerian corotation radius $R_{\text{K}} \approx \left(GM_*/P_*^2/4\pi^2\right)^{1/3} = 6.5 \times 10^{12}$ cm. The recent BATSE detection of a sudden torque reversal to spin-down is puzzling due to its very short timescale and the nearly equal spin-up/down rates. The steady spin-down torque suggests that there remains a dynamically stable (disk) structure after the sudden reversal (for details of observations, see Chakrabarty 1995). In Figure 1a, the observed torque reversal event is reproduced by $B_*=1.2 \times 10^3 G$, $M = 4 \times 10^{15}$ g s$^{-1}$, and $d\dot{M}/dt = -5 \times 10^{12}$ g s$^{-1}$ yr$^{-1}$ which give $R_*/R_2 = 0.58$, $R'_*/R'_2 = 0.95$, and $R'_*/R_2 = 0.56$. The derived $M$ and $B_*$ are slightly lower than the previously quoted values (see Pravdo et al. 1979; Kii et al. 1986; Chakrabarty 1995). We note, however, that our estimated values ($B_*$ and $M$) can always be rescaled by changing $\alpha$ and $\gamma$ (Kenyon et al. 1996). The gradual decrease of $M$ is consistent with the observed flux decrease (Mavromatakis 1994). The fit naturally achieves the spin-down torque, which is slightly smaller than the spin-up torque. The fit requires for the transition to occur at $\dot{M}_{\text{cy}} = 3.3 \times 10^{13}$ g s$^{-1}$. The gradual decrease of the mass accretion rate on a timescale $\sim 20$ yr cannot be due to any viscous or thermal processes operating in the inner region ($\tau_{\text{G}}, \tau_{\text{t}}$).

$OAO 1657-415.—OAO 1657-415$ has an observed pulse period $P_* \approx 38$ s. Recent observed spin-up/down torques are $P_*/P_*^{\text{cy}} \approx -2 \times 10^{-15}$ s$^{-2}$ and $\approx 7 \times 10^{-15}$ s$^{-2}$ (Chakrabarty et al. 1993). For the torque reversal in Figure 1b, we get $B_*=10^5 G$, $M = 2.0 \times 10^{16}$ g s$^{-1}$, and $d\dot{M}/dt = -5 \times 10^{16}$ g s$^{-1}$ yr$^{-1}$. The characteristic $M$ modulation timescale is $\sim 0.3$ yr. The critical accretion rate $\dot{M}_{\text{cy}} = 1.1 \times 10^{16}$ g s$^{-1}$. $GX 1+4.—GX 1+4$ has recently shown a sudden transition from spin-down to spin-up around the spin period $P_* \approx 122$ s (Chakrabarty 1995 and references therein). $GX 1+4$ is peculiar in the sense that despite its very short spin timescale $\sim 40$ yr, the spin equilibrium has not been reached. It is likely that $M$ fluctuates or oscillates on a timescale $\sim 40$ yr near spin equilibrium. The spin-down rate in the 1980s, $P_*/P_*^{\text{cy}} \approx 3.7 \times 10^{-12}$ s$^{-2}$, is not far from the 1970s spin-up rate, $P_*/P_*^{\text{cy}} \approx 6.0 \times 10^{-12}$ s$^{-2}$. The recent 1994 torque reversal from spin-down to spin-up lasted for $\sim 100$ d. This system also showed very similar spin-down and spin-up rates. Although there is no significant spectral change in the hard X-ray emission spectra during the spin evolution, the flux appears to be increasing as spin-down torque increases (Chakrabarty 1995), which is in contradiction to the behavior expected in the Ghosh-Lamb-type model (eqs. [2] and [3]). The fit shown in Figure 1c corresponds to $B_*=3.2 \times 10^{12} G$, $M = 5.0 \times 10^{15}$ g s$^{-1}$, and $d\dot{M}/dt = 1.0 \times 10^{16}$ g s$^{-1}$ yr$^{-1}$. It is interesting to observe that the $M$ modulation timescale $\dot{M}/(dM/dt) \sim 5$ yr is not far from the detected timescale in the coherent variation of pulse frequency residual (Cutler et al. 1986; Chakrabarty 1995). The critical mass accretion rate $\dot{M}_{\text{cy}} = 6.5 \times 10^{15}$ g s$^{-1}$ lies between the two values derived above. The gradual decrease of the spin-up torque after reversal is not accurately fit in Figure 1c with the linear $M$ variation. This is not surprising given the reported unsteady behavior of the spin-down torque before the reversal (Chakrabarty 1995). Since $GX 1+4$ event is considerably more gradual than the other events, it cannot be ruled out that this event has been caused by the smooth transition of the Ghosh-Lamb type (eq. [3]) as shown in Figure 1c.

4. DISCUSSIONS

The proposed transition is most likely to occur at a critical accretion rate $\dot{M}_{\text{cy}} \sim 10^{22}–10^{23}$ g s$^{-1}$. This suggests an interesting connection between the pulsars systems and other compact accretion systems such as cataclysmic variables and black hole soft X-ray transients. We speculate that the transitions seen in these systems may be due to a common physical mechanism, i.e. disk transition to optically thin hot flow. The model indicates that the sudden torque reversal could be a signature of a pulsar system near spin-equilibrium with $M \sim \dot{M}_{\text{cy}}$.

There are some outstanding issues to look into. (1) The origin of the gradual $M$ modulation on a timescale ranging from $\sim 1$ yr to a few decades remains to be investigated (Chakrabarty 1995 and references therein). (2) It is important to quantitatively understand $\dot{M}_{\text{cy}}$ and $R_{\text{cy}}$ (see Narayan & Yi 1995). (3) Within our model, the observed smooth transitions back to spin-up (seen in OAO 1657–415 and GX 1+4) could result from the return from the advective to Keplerian flow. The characteristic timescale for such a back transition is likely to be the viscous disk formation timescale. The observed UV-delay timescale, on the order of a day, in cataclysmic variables (Livio & Pringle 1992), may be similar to the postulated reverse transition timescale. (4) It remains unexplained that in $GX 1+4$ the X-ray flux increased during the increase of the spin-down torque (Chakrabarty 1995). If X-rays come from the shocked polar accretion as $M$ decreases, the X-ray emission temperature and the apparent flux in a fixed X-ray band could decrease due to radiation drag (see, e.g., Yi & Vishniac 1994). A geometrically thick, optically thin, inner disk is more apt to scatter X-ray emission, which can make X-ray emission more conspicuous as $M$ drops.

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