Comparison of time frequency offset robustness of EGF prototype filter in OFDM/OQAM system

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Abstract. As a multi-carrier system, the OFDM/OQAM system is sensitive to the synchronization error of the system. As an important part of the system, the robustness of the prototype filter to synchronization error is also very important. This paper first analyzes the influence of synchronization error on the system, and then deduces the expression of the signal to interference ratio (SIR) of the demodulation symbol. After that, the ambiguity function characteristics of extended Gauss function (EGF) prototype filters with three different expansion factors are analyzed. On this basis, the simulation comparison of the time frequency offset (TFO) robustness performance of the three EGF prototype filters with different expansion factors is carried out with the measure of SIR of the demodulation symbol.

1. Introduction

As a multi-carrier modulation (MCM) system, OFDM/OQAM system has been proved to be an effective communication scheme in wireless communication [1-2]. With its high spectrum efficiency and good interference robustness performance [3], the OFDM/OQAM system is applied to power line communication (PLC) [4] and troposcatter communication [5], and it is also regarded as one of the alternatives for 5G communication [6].

However, as a multi-carrier system, the OFDM/OQAM system is sensitive to synchronization error [7]. The time offset (TO) and carrier frequency offset (CFO) of the system will have a great influence on the signal demodulation of the system. In the OFDM/OQAM system, although a prototype filter with good time frequency focusing is adopted, inter symbol interference (ISI) and inter carrier interference (ICI) will still be generated by the large time-frequency offset of the system, resulting in the degrade of the system performance [8]. Therefore, the synchronization error robustness of system prototype filter is particularly important. Although there have been a lot of research on the TO and CFO estimation methods of OFDM/OQAM systems [7-11], these methods do not fully estimate and compensate the TO and CFO of the system. Therefore, it is another solution to enhance the ability of system to resist the synchronization error from the system itself. In [12], EGF prototype filter is proposed, which can select the corresponding extension factor according to the characteristics of the wireless channel to obtain the optimal performance of the prototype filter. In [13], the authors used the channel scatter function to select the expansion factor of the EGF function, and then designs an adaptive waveform based on the EGF function for the OFDM/OQAM system.
In this paper, the expression of the SIR of the OFDM/OQAM system is derived, and then the interference robustness of the EGF prototype filter with different expansion factors is theoretically analyzed by the ambiguity function characteristics. Finally, the performance of the system is simulated under different TFO conditions.

The rest of this paper is organized as follows. The OFDM/OQAM system model is introduced in Section 2. Section 3 is derived the expression of the SIR of the OFDM/OQAM system. Then, the comparison and analysis of the ambiguity function characteristics is illustrated in Section 4, and simulation result of the SIR of demodulation symbol with different EGF prototype filters under different TO and CFO conditions are drawn in Section 5. Finally, the conclusions of this paper are expressed in Section 6.

2. OFDM/OQAM system model

The equivalent baseband continuous-time transmission signal of the OFDM/OQAM system at the transmitter can be expressed as [14]:

\[ s(t) = \sum_{m=-\infty}^{\infty} \sum_{n=0}^{M-1} a_{m,n} g_{m,n}(t) = \sum_{m=-\infty}^{\infty} \sum_{n=0}^{M-1} a_{m,n} g(t-n\tau_0) e^{j2\pi nmv_0} e^{j\pi (m+n)} \]  

(1)

Where \( M \) is the even number of subcarrier, \( a_{m,n} \) is the real-valued OQAM symbol at the time-frequency position \((m,n)\) obtain from a QAM constellation, and \( g_{m,n}(t) \) is the prototype filter, and \( \tau_0 \) is the time offset between the real and imaginary part of complex-valued symbol, and \( v_0 \) is the subcarrier spacing and meets the equation \( v_0 = 1/T = 1/2\tau_0 \).

If the transmission signal of the OFDM/OQAM system is transmitted through the AWGN channel, and CFO and TO are introduced at the same time, the receiving signal at the receiving end can be expressed as:

\[ r(t) = e^{j2\pi\Delta t} s(t) + n(t) \]  

(2)

Where \( n(t) \) is additive Gauss white noise. Suppose the time frequency lattice position of the demodulation symbol is \((m_0,n_0)\), the demodulation symbol of the time frequency lattice can be expressed as:

\[ \hat{a}_{m_0,n_0} = \left\langle r(t), g_{m_0,n_0}(t) \right\rangle \]  

(3)

Where \( n'(t) \) is the noise item after real inner product operation, which can be expressed as follow:

\[ n'(t) = \Re \left\{ \int_{R} n(t) g^*_{m_0,n_0}(t) dt \right\} \]  

(4)

Further, substituted (1) into (3), and after a series of calculations, we can rewrite (3) as follow:

\[ \hat{a}_{m_0,n_0} = \Re \left\{ \sum_{p,q} a_{p,q} e^{j2\pi(p+q)(m_0+p)\tau_0} e^{\frac{j\pi(p+q)(n_0+q)\tau_0}{2}} e^{\frac{j\pi(p+q)}{2}} A_g(-q\tau_0 - \Delta t, -pF_0 - \Delta f) \right\} + n'(t) \]  

(5)

Where \( p = m - m_0, \ q = n - n_0 \). \( A_g(x,y) \) is the self-ambiguity function of the prototype filter, which is defined as follow:

\[ A_g(x,y) = \int g(t + \tau) g^*(t - \frac{\tau}{2}) e^{j2\pi\tau t} dt = e^{j\pi\tau(x,y)} \left\langle g(t + \tau), g(t) \right\rangle \]  

(6)
In terms of the equation (5), it can be seen that the TO $\Delta t$ and CFO $\Delta f$ of the system will affect the ambiguity function of the prototype filter, and then influence the signal demodulation process of the system. Therefore, the adaptability of different prototype filters to the TO and CFO of the OFDM/OQAM system is also different.

3. SIR of demodulated signal in OFDM/OQAM system

In (5), we set $\phi_1 = 2\pi (n_0 \tau_0 \Delta f - m_0 F_0 \Delta t + 0.5 \Delta f \Delta t)$, $\phi_2 = (\pi/2) (p + q + pq)$, $\phi_3 = \pi (q \tau_0 \Delta f - p F_0 \Delta t)$ and $\phi_4 = \pi p n_0$, and the channel is assumed to be an ideal channel without noise, that is, the influence of noise is ignored. Then, we can rewrite (5) as:

$$\hat{a}_{m_0, n_0} = \Re \left\{ \sum_{p,q} a_{m_0+p, n_0+q} e^{j \phi_1} e^{j \phi_2} e^{j \phi_3} A_g (-q \tau_0 - \Delta t, -p F_0 - \Delta f) \right\}$$

(7)

Where

$$\alpha_{m_0, n_0} = e^{j \phi_3} A_g (-\Delta t, -\Delta f)$$

(8)

and

$$J_{m_0, n_0} = \sum_{p=0, q=0} a_{m_0+p, n_0+q} e^{j \phi_2} e^{j \phi_3} e^{j \phi_4} A_g (-q \tau_0 - \Delta t, -p F_0 - \Delta f)$$

(9)

According to (7), we can find that the demodulation signal is composed of two parts. The first part is the demodulated useful signal, which is composed of an attenuation factor $\alpha_{m_0, n_0}$ and the signal $a_{m_0, n_0}$ to be demodulated, and the parameters $m_0$, $n_0$, $\Delta t$ and $\Delta f$ together determine the size of the attenuation factor. The second part is the interference of the surrounding data to the demodulation signal during the demodulation process, mainly for ICI and ISI. For the attenuation of the useful signal part, after the demodulation operation of the receiver, the Zero Forcing (ZF) equalization method can be used to compensate, and the mathematical model of the TFO error analysis, as shown in Figure 1, can be obtained.

![Figure 1. The mathematical model of the TFO error analysis](image)

From Figure 1, we can see that the signals after the equalization of OQAM/OFDM system are:

$$\hat{a}^{ZF}_{m_0, n_0} = \Re \left\{ \frac{\alpha_{m_0, n_0}}{H_{m_0, n_0}} a_{m_0, n_0} + \Re \left\{ \frac{J_{m_0, n_0}}{H_{m_0, n_0}} \right\} \right\}$$

(10)
Where \( H_{m_0,n_0} = e^{i\theta} \) is the tap coefficient for ZF equalization. Suppose the variance of the transmitted signal is \( \sigma_z^2 \), and also satisfies the characteristics of independent and identical distribution. At the same time, \( e^{i\theta} = e^{i\pi m_0} = \pm 1 \), then the power of the interference signal can be expressed as:

\[
P_J = E\left\{ |J_{ZF}|^2 \right\} = \sigma_z^2 \sum_{p,q\neq 0} \Re\left\{ e^{i\theta} e^{i\phi} A_g(-q\tau_0 - \Delta t, -pF_0 - \Delta f) \right\}^2
\]

(11)

Because the prototype filter is a real valued even function and its ambiguity function is also a real value even function. Thus, (11) can be rewritten as:

\[
P_J = \sigma_z^2 \sum_{p,q\neq 0} \cos^2(\phi_2 + \phi_3) A_g^2(-q\tau_0 - \Delta t, -pF_0 - \Delta f)
\]

(12)

At the same time, the partial power of useful signals can be expressed as:

\[
P_s = E\left\{ |A_g(-\Delta t, -\Delta f) a_{m_0,n_0}|^2 \right\} = \sigma_z^2 A_g^2(-\Delta t, -\Delta f)
\]

(13)

Thus, the expression of the SIR of the demodulated signal can be deduced from the (12) and (13):

\[
SIR(\Delta t, \Delta f) = \frac{P_s}{P_J} = \frac{A_g^2(-\Delta t, -\Delta f)}{\sum_{p,q\neq 0} \cos^2(\phi_2 + \phi_3) A_g^2(-q\tau_0 - \Delta t, -pF_0 - \Delta f)}
\]

(14)

According to (14) and the above analysis, the SIR of demodulated signal is greatly related to the ambiguity function of the prototype filter. Therefore, we can analyze the TFO robustness of different prototype filters based on the signal to interference ratio of demodulated signals.

4. Performance analysis of EGF prototype filter

According to [12], we can know that EGF prototype filters exhibit different performance characteristics under the three conditions of expansion factors \( 0 < \alpha < 1 \), \( \alpha = 1 \) and \( \alpha > 1 \). When \( \alpha = 1 \), the EGF prototype filter is also called the IOTA prototype filter. It is pointed out that the ambiguity function of the prototype filter can well characterize the time frequency focusing characteristics of the prototype filter and the sensitivity to the TFO. Therefore, this paper selects three kinds of EGF prototype filters, \( \alpha = 1/2 \), \( \alpha = 1 \) and \( \alpha = 2 \), to analyze their ambiguity function characteristics. The ambiguity functions of those three prototype filters are shown in Figure 2 (a), figure 2 (b) and Figure 2 (c) respectively.

![Figure 2](image-url)
Figure 2. The ambiguity function of three EGF prototype filters

As shown in Figure 2, when $\alpha = 1/2$, the EGF prototype filter declines slowly on the time axis and declines rapidly on the frequency axis, so it should have good robustness against TO. Similarly, the EGF prototype filter declines rapidly in the direction of time axis and slows down on frequency axis for $\alpha = 2$, which means it should have good CFO robustness. On the other hand, when $\alpha = 1$, the EGF prototype filter has similar fading speed in all directions, thus, it should have the great ability to resist both TO and CFO.

5. Simulation results

This section simulates and compares the TFO robustness performance of the three prototype filters in the previous section based on the SIR of the system demodulation signal. In simulation, the number of sub carriers in the OQAM/OFDM system is 256, the sampling frequency is 10MHz, and the tap number of IOTA prototype filter is 4. At the same time, in order to facilitate the comparison of simulation results, the normalized TO $\Delta \tau$ and frequency offset $\Delta \varepsilon$ are used in the simulation.

First of all, we studied the SIR performance of different EGF prototype filters of the OFDM/QOAM system under the TO condition, the simulation results are shown in Figure 3. According to Figure 3, In the case of TO, the EGF prototype filter of $\alpha = 1/2$ has the best performance of SIR. The IOTA prototype filter is the next, and the EGF prototype filter of $\alpha = 2$ is the worst, this is consistent with the analysis of the three prototype filters in the previous section.

Figure 3. The SIR performance comparison of three EGF prototype filters with TO

Next, the SIR performance of different EGF prototype filters under the condition of CFO is studied, and the simulation results are shown in Figure 4. As we can see from the diagram, the EGF prototype filter of $\alpha = 2$ has the best performance, and $\alpha = 1/2$ has the worst performance, and the performance of the IOTA prototype filter function is still between the two prototype filters. Also, this simulation results are consistent with the analysis in the previous section.

Finally, we taking into account both the influence of TO and CFO, the value of the normalized time
frequency offset is changed at the same time in the simulation, that is $\Delta \tau = \Delta \epsilon$, and the simulation results are shown in Figure 5. According to the simulation results, we can see that under the condition of both TO and CFO, the EGF prototype filter of $\alpha = 1/2$ and $\alpha = 2$ obtain a similar SIR performance, however, they are both far from the IOTA prototype filter. This is because the IOTA prototype filter has better interference robustness performance in both time and frequency domain, so it can have better performance of SIR in the presence of TO and CFO.

![Figure 4. The SIR performance comparison of three EGF prototype filters with CFO](image1)

![Figure 5. The SIR performance comparison of three EGF prototype filters with TO and CFO](image2)

From the above analysis, we can see that the SIR performance of the different prototype filters under different TFO conditions has its own merits. We can choose the corresponding prototype filter according to the characteristics of the channel in order to obtain the optimal system performance.

**6. Conclusion**

This paper first derives the SIR of the demodulated signal in the OFDM/OQAM system, and then compares the ambiguity function characteristics of the EGF filter function with different expansion factors. On this basis, the SIR performance of EGF prototype filters with different spreading factors under different TFO is simulated and compared. The simulation results show that the EGF prototype filter can be used with the corresponding expansion factor under different TFO conditions to provide a good basis for obtaining better synchronization error performance of the system.

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References

[1] Nee R V and Prasad R. 2000. *OFDM for Wireless Multimedia Communications*. OFDM for Wireless Multimedia Communications. (USA: Norwood, MA). Artech House.

[2] Wang Z and Giannakis G B. 2000. Wireless multicarrier communications. *IEEE Signal Processing Magazine*, 17(3), 29-48.

[3] Vukotić S and Vučić D. 2016, Telecommunications Forum Telfor. *Detection and classification of OFDM/QAM and OFDM/OQAM signals based on cyclostationary features*. (Serbia: Belgrade). IEEE, pp 232-235.

[4] Skrzypczak, A., Siohan P, Javadin J P. 2007, IEEE International Symposium on Power Line Communications and Its Applications. *Application of the OFDM/OQAM Modulation to Power Line Communications*. (Pisa, Italy). IEEE, vol 1635, pp, 71-76.

[5] Liu X , Chen X, Xie Z. 2016, International Conference on Network, Communication and Computing. *Application of OQAM/OFDM Modulation in Troposcatter Communication*. (Kyoto, Japan ). ACM, pp, 239-245.

[6] Aminjavaheri A, Farhang A, Rezzadzehreyhani A and Farhang-Boroujeny B. 2015. Impact of timing and frequency offsets on multicarrier waveform candidates for 5G. *Advances in Electrical Engineering*. 2014, 178-183.

[7] Baghaki A, Champagne B. 2018. Joint frequency offset, time offset, and channel estimation for OFDM/OQAM systems. *Eurasip Journal on Advances in Signal Processing*, 2018(1), 4-8.

[8] Singh P, Vasudevan K. 2017. International Conference on Signal Processing and Integrated Networks. *Preamble-based synchronization for OFDM/OQAM systems in AWGN channel*. (Noida, India ). IEEE, pp, 60-65.

[9] Fusco T, Petrella A, Tanda M. 2009. Joint Symbol Timing and CFO Estimation for OFDM/OQAM Systems in Multipath Channels. *Eurasip Journal on Advances in Signal Processing*, 2010(1), 897607.

[10] Mattera D, Tanda M. 2013. Blind Symbol Timing and CFO Estimation for OFDM/OQAM Systems. *IEEE Transactions on Wireless Communications*, 12(1):268-277.

[11] Zhao Y, Chen X, Xue L. 2016. Blind carrier frequency offset estimation for OFDM/OQAM systems based on BEM. *Systems Engineering and Electronics*, 38(6), 1435-1439.

[12] Siohan P, Roche C. 2002. Cosine-modulated filterbanks based on extended Gaussian functions. *IEEE Transactions on Signal Processing*, 48(11), 3052-3061.

[13] Hu S, Wu G, Li S. 2012. Adaptive Pulse Shaping Filter and Multiple Access Technology for OFDM/OQAM System. *Journal of Electronics and Information Technology*, 34(5), 1214-1219.

[14] Zhao Y, Chen X, Xue L, Liu J and Xie Z. 2016. Iterative Preamble-Based Time Domain Channel Estimation for OFDM/OQAM Systems. *Ieice Transactions on Communications*. E99B(10), 2221-2227.