CATEGORIZING MODELS USING SELF-ORGANIZING MAPS: AN APPLICATION TO MODIFIED GRAVITY THEORIES PROBED BY COSMIC SHEAR

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Abstract

We propose to use Self-Organizing Maps (SOM) to map the impact of physical models onto observables. Using this approach, we are able to determine how theories relate to each other given their signatures. In cosmology this will be particularly useful to determine cosmological models (such as dark energy, modified gravity or inflationary models) that should be tested by the new generation of experiments. As a first example, we apply this approach to the representation of a subset of the space of modified gravity theories probed by cosmic shear. We therefore train a SOM on shear correlation functions in the $f(R)$, dilaton and symmetron models. The results indicate these three theories have similar signatures on shear for small values of their parameters but the dilaton has different signature for higher values. We also show that modified gravity (especially the dilaton model) has a different impact on cosmic shear compared to a dynamical dark energy so both need to be tested by galaxy surveys.

1. INTRODUCTION

In many fields a large variety of theoretical models are proposed in the literature and are to be tested with current or future data. For example, in cosmology 74 inflation models were systematically tested in [Martin et al. 2014] using cosmic microwave background 2013 data from the Planck satellite [Ade et al. 2014], or in astrophysics, testing various models of exoplanets’ atmosphere will be one of the goals of experiments like the JWST as reviewed by [Madhusudhan 2019; Biller & Bonnefoy 2018]. Selecting only a subset of models to be tested given an observable is in some cases necessary as obtaining robust constraints can be time consuming, requires modeling developments specific to each theory or requires large computing power (to run Monte Carlo Markov Chains for instance). In this letter we address this challenge by presenting a new approach using unsupervised non-Gaussian dimensionality reduction algorithms in order to categorize models given observables. We especially use Self-Organizing Maps (SOMs) as they can produce 2-dimensional maps where models are assigned to cells preserving topology. Such mapping indeed allows to easily determine models that have different impacts on a given observable or missing models, and the effect of measurements on this mapping can also be added straightforwardly (but is not done in the present first study). This approach complements Principal Component Analysis (PCA) as SOMs are adequate for non-Gaussian reduction and visualization. As a first step and for purpose of example, we apply this approach to the case of modified gravity theories probed by cosmic shear.

Weak gravitational lensing is indeed a powerful probe of MG as shown early on in [Song 2005] demonstrating that some MG theories (in particular DGP as developed in [Dvali et al. 2000; Deffayet 2001; Deffayet et al. 2002]) can be distinguished from a dark energy using cosmic shear, and later in [Song 2006; Tsujikawa & Tatekawa 2008; Jain & Zhang 2008]. Tomographic cosmic shear corresponds to the correlation in redshift bins of the deformation of observed galaxy shapes due to weak lensing. It directly probes the matter distribution and is sensitive to the growth of large scale structures in the Universe, which depends on the laws of gravity. Deviations from GR were thus tested using cosmic shear data from CFHTLenS [Harnois-Déraps et al. 2015; Simpson et al. 2013; Ferté et al. 2019, KiDS [Joudaki et al. 2017; Tröster et al. 2021] and DES [Abbott et al. 2019] (in the later cases galaxy-galaxy lensing and clustering data were also used). Forecasts show these constraints will improve with future imaging surveys including the Euclid satellite [Martinelli et al. 2011; Euclid Collaboration 2020], LSST [that will be produced by the Rubin Observatory, The LSST Dark Energy Science Collaboration 2018; Hojjati et al. 2016; Ferté et al. 2019] and Roman space mission [Doré et al. 2019; Eifler et al. 2020, 2021].

One of the challenges faced by these future surveys is to decide which theories of MG to test amongst the large space of models developed in the literature, [Ishak et al. 2019] for instance prioritizes theories to be tested by LSST. The goal of the present approach is to complement past studies (for instance performing non-parametric reconstruction
Parameters varied and their range for the modified gravity models used to compute the SOM training data set (in the dilaton case, $\beta_0$ is fixed to $\Omega_{\Lambda,0}/\Omega_{m,0}$).

| Model      | Parameters | Prior range                        |
|------------|------------|------------------------------------|
| f(R)       | $B_0$      | $[10^{-10}, 10^{-2}]$              |
| Dilaton    | $\xi_0, \beta_0$ | $[10^{-6}, 5 \times 10^{-4}], 2.2$ |
| Symmetron  | $\xi_+, \beta_+$ | $[10^{-6}, 10^{-4}], [0.5, 1.5]$ |

### Table 1

2. SELF-ORGANIZING MAP OF MODIFIED GRAVITY PROBED BY COSMIC SHEAR

*Self-Organizing Maps* – Self-Organizing Maps, introduced in Kohonen [1982], are a class of unsupervised neural networks which learn and reduce dimensions (usually to 2 dimensions) of a higher dimensional data set while preserving the topology of the manifold. Dimensionality reduction makes SOMs perfect tools for visualization. The training of a SOM is an iterative process where weights of the neurons within the radius of the neighborhood function are updated according to their distance to the input data, where the neuron closest to the input data is called the Best Matching Unit (BMU). At the end of the training, the neurons are weighted so that they match the distribution of the input data.

In this letter we assume a fixed cosmology with cosmological models at the level of the cosmological observables, to inform future analyses. We do so by indicating how similar the signatures of different specific modified gravity theories are on cosmic shear, using Self-Organizing Maps. In this letter we assume a fixed cosmology with $\Omega_0 = 0.31, H_0 = 67, \Omega_b = 0.04, A_s = 2.1 \times 10^{-9}$, one massive neutrino with a mass of 0.06 eV. The training set is available on demand and the plotting code is available online.

In this work, we make predictions of the theoretical predictions of the cosmic shear real-space correlation functions at angular separation $\theta$ in redshift bins $i$ and $j$, $\xi_{ij}^\pm(\theta)$ in various modified gravity (MG) theories, for different values of their parameters. We use the cosmological code CosmoSIS [Zuntz et al. 2015] to compute the theoretical predictions of cosmic shear, where the impact of modified gravity on the matter power spectrum is computed using the latest version of MGCAMB [Zucca et al. 2019] [Hojjati et al. 2011] [Zhao et al. 2009b], implemented in CosmoSIS. For the purpose of this study we assume a linear matter power spectrum. Indeed, while various approaches have been proposed to model non-linearities in modified gravity theories (see for example Mead et al. 2016; Bose et al. 2020; Thomas 2020), an alternative is to remove measurements of cosmic shear on non-linear scales for the cosmological analysis (as done in Abbott et al. 2019 for instance). In a similar way, we use the cosmic shear correlation functions in a range of $\theta$ roughly similar to the conservative scale cuts used for the constraints on deviations to GR in Abbott et al. 2019 [11]. We therefore use predictions of $\xi_+$ between 30 and 300 arcminutes and of $\xi_-$ between 250 and 300 arcminutes for all redshift combinations.

We consider modified gravity theories that are readily available in MGCAMB and have a cosmological impact, following Hojjati et al. 2016. Cosmic shear correlation functions $\xi_{ij}^\pm(\theta)$ for 10 redshift bins (indexed by $i$ and $j$) of the source galaxies (similar to what is expected for LSST or Euclid) are then computed for 100 different values of the MG parameters. These values are linearly spaced within a range that we choose to not be informed by astrophysical constraints as cosmological analyses often use such large priors. We use:

- $f(R)$ gravity parametrised by the Compton wavelength parameter $B_0$ [Song et al. 2007; Dossett et al. 2014], varying $\log_{10}(B_0)$ linearly between -10 and -2.
- the dilaton model [Damour & Polyakov 1994] [Brax et al. 2010] parametrized by the value of the force range and the coupling to matter today, respectively $\xi_0$ and $\beta_0$. We vary $\log_{10} \xi_0$ linearly between -6 and -3.3 (above this value our current pipeline gives non-physical shear correlated functions) for $\beta_0$ set to $\Omega_{\Lambda,0}/\Omega_{m,0} = 2.2$ as indicated in Hojjati et al. 2016 (for the scalar field to explain dark energy).
- the symmetron model \cite{Hinterbichler2010} parametrized by the force range $\xi_\star$, the scale factor at the time of the force activation $a_\star$ and the coupling to matter $\beta_\star$. We vary $\log_{10}(\xi_\star)$ between -6 and -2 for $\beta_\star$ set to 0.5, 1 and 1.5, while we fix $a_\star$ for simplicity, as is done in \cite{Hojjati2016} for instance. We choose to set $a_\star$ to 0.5.

Table 1 summarizes the varied parameters and their corresponding prior ranges for each of the considered MG theories. We therefore compute shear correlation functions for a total of 500 models.

3. RESULTS AND DISCUSSION

**SOM of modified gravity theories** - After training the $6 \times 6$ square grid SOM on the training set described above, we map the training set on the SOM. Fig. 1 shows a two dimensional SOM grid with cells colored by the final number of models per neuron. In this figure and the following ones, cells in white correspond to cells with no models. The cell corresponding to the highest number of models lie in the left top corner (numbered 1 in Fig. 1 with a total of 132 models in this cell), which corresponds to General Relativity or small deviations to GR. This indicates that shear correlation functions for small values of the modified gravity parameters ($B_0$, $\xi_0$ and $\xi_\star$) are indistinguishable from GR.

Fig. 2 shows correlation functions $\xi_+^2(\theta)$ in redshift bin 2 for different cells (which are numbered from 1 to 3 in Fig. 1). The dotted and dashed lines for cells 2 and 3 show two different models associated to these two cells, while cell 1 presents negligible variations as the MG parameters are very small (e.g. the median of $B_0$ in cell 1 is $3.1 \times 10^{-9}$). We
also show for purpose of illustration the expected error bars on $\xi_{+}^{22}(\theta)$ from a survey like LSST (10 source redshift bin with 55 galaxies $\text{arcmin}^{-2}$ and a total shape noise of 0.3 over 50% of the sky), which we computed using CosmoCov [Krause & Eifler 2017; Fang et al. 2020b,a].

Other cells correspond to the various modified gravity theories used in the training, so we show in Fig. 3 a different representation of this SOM, now highlighting only the cells corresponding to each MG theory. In this case, the cells are colored according to the median of the corresponding MG parameter in each cell. The panels correspond to $f(R)$ gravity, the dilaton and symmetron models from left to right, only showing the distribution of $\xi_\star$ for $\beta_\star = 1$ in the symmetron model. The 3 theories have a large overlap in the top left part of the SOM for small values of the MG parameters. However, for larger values, the dilaton model correspond to cells different from ones corresponding to $f(R)$ and the symmetron model. This means the SOM is able to distinguish between the dilaton and other models based on tomographic cosmic shear correlation functions which indicates that its signature is different enough that it should be constrained by future analysis.

**Application: does a dynamical dark energy have the same signature as MG theories?** – Current and future galaxy surveys aim at testing if the equation of state $w$ of dark energy varies with time. The equation of state in this case often follows [Linder 2003; Chevallier & Polarski 2001] i.e., is parametrized by $w_0$ (the value of $w$ today) and $w_a$ (its dependence with time) as: $w(a) = w_0 + w_a(1 - a(t))$ where $a(t)$ is the scale factor. We want to know if constraining a dynamical dark energy using cosmic shear is sufficient to explain signatures from MG theories, using the present SOM approach. To this end, we compute the shear correlation functions for different combinations of $w_0$ and $w_a$ using the same pipeline as used in modified gravity replacing MGCamb by CAMB [Lewis et al. 2000; Howlett et al. 2012; Lewis 2014].

We thus produce a set of shear correlation functions corresponding to 96 combinations of $(w_0, w_a)$ randomly sampled from the chosen priors: $[-1.5, -0.5]$ for $w_0$, $[-1, 1]$ for $w_a$. To do so we use the apriori sampler that is readily available in CosmoSIS. We then train a SOM on a combination of this data set and the shear in modified gravity data set described above. As depicted in Fig. 4, the resulting SOM shows the signature on cosmic shear of modified gravity theories (shown in the right panel) is different from the one from a dynamical dark energy (shown in the left panel). However, the pipeline failed for some combinations of $w_0, w_a$ so we get 96 combinations instead of 100.

![Figure 3](image_url). SOM of modified gravity from Fig. 1 showing the median of the values of the parameter for each cell for $f(R)$ gravity, the dilaton (for $\beta_0 = 2.2$) and symmetron (for $\beta_\star = 1$) models from left to right.

![Figure 4](image_url). Number of models in each cell for $(w_0, w_a)$ on the left panel and MG theories on the right panel of the $6 \times 6$ Self-Organizing Map grid trained on shear correlation functions $\xi_{+}^{22}(\theta)$ computed in $(w_0, w_a)$ model and $f(R)$ gravity, dilaton and symmetron (for $\beta_\star = 0.5$, 1 and 1.5) models.
We have checked that the dilaton model in particular doesn’t overlap with a dynamical dark energy, except for parameters values close to GR. This indicates these MG theories would need to be constrained in addition to a dynamical dark energy in order to extract more information from cosmic shear.  

4. CONCLUSION

We presented a new approach to categorize models using a dimensionality reduction algorithm such as Self-Organizing Map and applied it to the case of a subset of modified gravity theories, categorizing these theories through their impact on cosmic shear. This approach is a promising way to help guide analyses in the case of a large theory space, like in cosmology. In this paper we showed for instance that the signatures left on cosmic shear from $f(R)$ gravity, the dilaton and symmetron models are similar for small deviations from GR but the dilaton can be distinguished from the other models for larger parameters values, while signatures from $f(R)$ gravity are similar to the ones left by the symmetron model for $\beta_s = 1$. Moreover we showed MG theories have a different impact on cosmic shear compared to a dynamical dark energy, indicating that MG models should indeed be explored by future surveys such as LSST or Euclid in order to further exploit cosmic shear data.

However, this first analysis has several caveats: we assume a linear matter power spectrum, consider a fixed cosmology and we use theoretical predictions of the shear correlation functions $\xi_{\pm}(\theta)$ in finely binned $\theta$. We will go beyond these assumptions in future work and develop several applications. As such, this approach can indeed be applied to other cases in cosmology such as cosmic inflation or any case of a large theory space in physics, in order to easily identify interesting directions of exploration.

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