Fast long-range state transfer in quantum dot arrays via shortcuts to adiabaticity

Yue Ban,1,2 Xi Chen,3 and Gloria Platero1

1Instituto de Ciencia de Materiales de Madrid, CSIC, Sor Juana Inés de la Cruz 3, E-28049 Madrid, Spain
2College of Materials Science and Engineering, Shanghai University, 200444 Shanghai, People’s Republic of China
3Department of physics, Shanghai University, 200444 Shanghai, People’s Republic of China

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Rapid and efficient preparation, manipulation and transfer of quantum states through an array of quantum dots (QDs) is a demanding requisite task for quantum information processing and quantum computation in solid-state physics. Conventional adiabatic protocols, as coherent transfer by adiabatic passage (CTAP) and its variations, provide slow transfer prone to decoherence, which could lower the fidelity to some extent. To achieve the robustness against decoherence, we propose a protocol of speeding up the adiabatic charge transfer in multi-QD systems, sharing the concept of “Shortcuts to Adiabaticity” (STA). We first apply the STA techniques, including the counter-diabatic driving and inverse engineering, to speed up the direct (long range) transfer between edge dots in triple QDs. Then, we extend our analysis to a multi-dot system. We show how by implementing the modified pulses, fast adiabatic-like charge transport between the outer dots can be eventually achieved without populating intermediate dots. We discuss as well the dependence of the transfer fidelity on the operation time in the presence of dephasing. The proposed protocols for accelerating adiabatic charge transfer directly between the outer dots in a QD array offers a robust mechanism for quantum information processing, by minimizing decoherence and relaxation processes.

INTRODUCTION

Fast quantum state control and transfer in quantum dot (QD) configurations with high fidelity are significant requirements for quantum computation. With the state-of-the-art technology of quantum information transfer, surface acoustic waves are able to capture electrons and transport them over long distance [1–7]. Interestingly, charge transfer between indirectly coupled systems exists in chemical reactions [8], quantum optics [9] and biology processes [10] etc.. Long-range charge transfer, i.e., direct transfer between the edge dots, mediated by quantum superpositions has been experimentally observed in a triple QD [11, 12]. Photo-assisted long range transport has been theoretically investigated [13–15] and experimentally observed [16] as well. In particular, spatial adiabatic passage [17], including coherent transfer by adiabatic Passage (CTAP) in a triple QD system [18], provides an effective scheme to transfer charge directly between the outer dots, avoiding occupation in the middle one [19]. Thus, such electronic or matter-wave analogue of Stimulated Raman Adiabatic Passage (STIRAP) technique [20, 21] can minimize the noise and systematic errors coming from the undesirable population excitation in the middle region. Moreover, this scheme has also been extended to the more complicated solid-state quantum computing architectures, such as QD chains [22, 23]. In an all-electrical controlled multi-QD system, CTAP can be realized with straddle coupling between the dots [24], so-called straddling tunneling sequence of coherent transfer by adiabatic passage (SCTAP), or with alternating coupling schemes [25].

Nowadays, quantum information processing requires high fidelity with error tolerance less than $10^{-4}$. Although insensitive to gate errors, the commonly used adiabatic protocols provide slow transfer prone to decoherence, which could reduce the fidelity to some extent. Aiming at reducing the operation time but achieving the adiabatic-like behavior, “Shortcuts
to Adiabaticity” (STA) [26] provides different ways to speed up the adiabatic passage. One example of this, counter-diabatic driving (or equivalently transitionless quantum algorithm) [27–29] considers a supplementary interaction to cancel the diabatic driving in the reference Hamiltonian. As a result, the time evolution of the wavefunctions exactly coincides with the adiabatic solution of the reference Hamiltonian. Other implementation of STA, the inverse engineering method, based on the Lewis-Riesenfeld invariant [30], allows to inversely engineer the time evolution dynamics by setting the initial and final states firstly. STA has been applied to manipulate single or two interacting spins [28, 31], and to electrically control the spin dynamics of electrons in QDs in the timescale of nanoseconds [32, 33]. Furthermore, combining STA and STIRAP, state control with high fidelity up to the non-adiabatic regime, has been achieved in different physical systems, as N-vacancy centers in diamond [34] or $^{87}$Rb ensembles [35]. Theoretically, STIRAP assisted by different implementations of STA, such as counter-diabatic driving in tunnel-coupled quantum wells [36], fast-forward in polyatomic molecules [37] or non-hermitian shortcuts in coupled optical waveguides [38] have already been proposed to control population in three-level systems. However, fast adiabatic-like state transfer in a multi-level system via combined STIRAP and STA has not yet been addressed. The extension of this combination to a multi-QD system will have important implications for qubit transfer between distant regions and thus will become a promising physical realization for large-scale quantum information processing. Furthermore, the recent experimental implementations, including scalable gate architecture for up to nine-quantum-dot array [39], efficient detection and manipulation of charge states in a quintuple QD [40] and coherent spin shuttle through a GaAs/AlGaAs quadruple-quantum-dot array [41] motivates the investigation of fast protocols for not only the improvement of the speed and stability of direct electron transfer between the edge dots, but also for the realization of long range transfer of quantum information in these systems.

In this paper, we report direct and non-adiabatic transfer of one electron between outer dots in a multi-QD system, schematically shown in Fig. 1. Firstly, we begin with the fast and direct charge transfer in a triple QD based on various STA techniques, including counter-diabatic driving and inverse engineering. Different pulses are designed to achieve the fast and direct charge transfer, by reducing the excitation in the intermediate dots. Detailed comparisons are made to show the advantage of STA, which combines speed and robustness versus other protocols. More interestingly, the STA for direct charge transfer is further extended to a multi-QD system where, in contrast with the triple QD case, both counter-diabatic driving and the inverse engineering are necessary. In such a system, the non-local counter-diabatic term couples the outer dots. After making the analogy between the triple dot and an array of $2n+1$ dots, we find the effective pulses, which should be modified to control the tunneling between the first and the second dots and between the $2n^{th}$ and $2n+1^{th}$ dots. Meanwhile, we obtain their amplitudes, which are proportional to those used in a triple QD.

The fidelity is checked with different operation times in the presence of dephasing by solving the master equation in the Lindblad form. An analytically estimated fidelity is also derived from time-dependent perturbation theory. Combining the numerical and analytical results, we prove that high fidelity can be achieved by shortening the operation time, up to the order of nanosecond. Therefore, STA really provides an efficient way for fast and robust long-range charge transfer in an array of QDs.

Our protocol provides an efficient and feasible way to achieve fast direct charge transfer in long QD arrays without exciting the populations in intermediate dots, which successfully enlarges the system scale for quantum information processing. In the state-of-the-art set up, the charge detection could be easier for the proposed long QD arrays than in a triple QD array because its influence on the outer dots is much lower. In addition, the realization of shortcuts to adiabaticity in a multi-level quantum system is extremely useful not only in the manipulation of electron transfer in QD arrays but also for light propagation in N coupled waveguides and for many other physical systems.

RESULTS

Transfer in a triple QD. We start from the electron transport in a linearly arranged triple QD where the energy levels are on resonance. The Hamiltonian ($\hbar = 1$ in dimensionless units) reads

$$H_0(t) = \Omega_{12}(t)c_1^\dagger c_2 + \Omega_{23}(t)c_2^\dagger c_3 + h.c.$$  \hspace{1cm} (1)

where two adiabatic pulses of Gaussian shape are applied between adjacent dots,

$$\Omega_{12} = \Omega_0 \exp \left[ -\frac{(t - t_f/2 - \tau)^2}{\sigma^2} \right],$$  \hspace{1cm} (2)

$$\Omega_{23} = \Omega_0 \exp \left[ -\frac{(t - t_f/2 + \tau)^2}{\sigma^2} \right],$$  \hspace{1cm} (3)

similar to Pump and Stokes pulses in a $\Lambda$– type three-level atomic system. The charge state can be transferred adiabatically from $|1\rangle$ to $|3\rangle$ without populating $|2\rangle$ via the dark state

$$|\phi_0\rangle = \cos \theta |1\rangle - \sin \theta |3\rangle,$$  \hspace{1cm} (4)

which is an instantaneous eigenstate of $H_0$ with zero energy and where the mixing angle $\tan \theta = \Omega_{12}/\Omega_{23}$ [20]. The solution to the time-dependent Schrödinger equation of $H_0$ is expressed as $|\Psi(t)\rangle = \sum_j a_j(t)|j\rangle$, in the basis of the on-site states of each dot $|j\rangle$, where the population
illustrates that fast adiabatic- 
demonstrates that 
all the figures, the parameters of the pulses are \( \sigma = \tau = t_f/6 \). The time is scaled by \( 2\pi/\Omega_0 \) and \( \Omega_0 = 10\pi \) MHz.

in each dot is \( P_j = |a_j|^2 \). Fig. 2 demonstrates that the charge transfers in a triple QD adiabatically from |1\rangle to |3\rangle with the fidelity \( F > 0.9999 \) by using the pulses designing from CTAP when the adiabatic criteria, \( \Omega_0 t_f = 100\pi \gg 1 \) is fulfilled. Nevertheless, when the final time is shorter, the fidelity will be dramatically reduced, see Fig. 2 (c). Here we set the pulse intensity about several hundred MHz, \( \Omega_0 = 10\pi \) MHz, and the operation time for the CTAP protocol \( t_f = 50 \) in units of \( 2\pi/\Omega_0 \) (= 0.02\( \mu \)s), corresponding to the timescale of 1\( \mu \)s.

In order to accelerate the adiabatic transfer, we consider first a triple QD and determine the counter-diabatic Hamiltonian \( H_1 \) such that the state evolution is exactly along the \( |\phi_0\rangle \), without generating transitions among all the eigenstates of \( H_0 \). However, this complementary counter-diabatic term \( H_1 = i\Omega_0 c_1^\dagger c_3 + h.c. \) couples the first and the third dots, and it is difficult or even impossible to implement experimentally. Thus, we look for physically feasible shortcuts by introducing a unitary transformation [43],

\[
\hat{H} = U^\dagger H U - iU^\dagger \dot{U} = \hat{\Omega}_{12} c_1^\dagger c_3 + \hat{\Omega}_{23} c_2^\dagger c_3 + h.c., \tag{5}
\]

which results in the cancellation of this non-local coupling. The adoption of the unitary operator can be done in many ways [43]. Note that \( U(0) = U(t_f) = 1 \) is the necessary boundary condition in order to keep the same dynamics of the system before and after the transformation. Fig. 3 illustrates that fast adiabatic-like charged transfer can be achieved by applying the modified pulses derived from a triple QD. To realize the charge transfer in a shorter time, e.g. \( t_f = 1 \), a stronger pulse is needed. In this case, the population of the state |2\rangle is excited during the transfer, that is, the occupation of the central dot is not negligible, as the intermediate dynamics of the system is changed after the unitary transformation.

Inverse engineering is an alternative protocol for designing STA with more flexibility [45]. To design the modified pulse directly, we parameterize the solution as follows,

\[
|\Psi(t)\rangle = \cos \chi \cos \eta |1\rangle - i \sin \eta |2\rangle - \sin \chi \cos \eta |3\rangle, \tag{6}
\]

with the unknown time-dependent parameters \( \chi \) and \( \eta \). Substituting Eq. (6) back into the time-dependent Schrödinger equation for \( \hat{H} \), Eq. (5), with the wavefunction \( |\Psi\rangle \), we obtain the following auxiliary equations:

\[
\dot{\chi} = \tan \eta (\hat{\Omega}_{12} \sin \chi + \hat{\Omega}_{23} \cos \chi), \tag{7}
\]

\[
\dot{\eta} = \hat{\Omega}_{12} \cos \chi - \hat{\Omega}_{23} \sin \chi. \tag{8}
\]

Once we set the appropriate ansatz for \( \chi \) and \( \eta \), the pulses \( \Omega_{12} \) and \( \Omega_{23} \) can be derived inversely for fast charge state transfer. When the transfer becomes adiabatic, that is, \( \dot{\chi} \approx 0 \) and \( \dot{\eta} \approx 0 \), we have \( \eta \rightarrow 0 \), \( \tan \chi \rightarrow \hat{\Omega}_{12}/\hat{\Omega}_{23} \), and the whole state evolution will thus follow the dark state.

In order to consider STA, the boundary conditions, \( \chi(0) = 0, \chi(t_f) = \pi/2 \), \( \eta(0) = 0 \) and \( \eta(t_f) = 0 \) should be imposed for charge state transfer from the initial state |1\rangle to final state |3\rangle along the solution (6). More boundary
conditions $\chi(0) = \dot{\chi}(0) = \chi(t_f) = \dot{\chi}(t_f) = 0$ and $\eta(0) = \dot{\eta}(t_f) = 0$ are required for smooth pulses [45], see Eqs. (7) and (8). To interpolate the functions of $\chi$ and $\eta$, we adopt the ansatz

$$\chi = \frac{\pi t}{2t_f} - \frac{1}{3} \sin \left( \frac{2\pi t}{t_f} \right) + \frac{1}{24} \sin \left( \frac{4\pi t}{t_f} \right),$$

$$\eta = \arctan \left( \frac{\chi}{\alpha_0} \right),$$

such that the pulses $\tilde{\Omega}_{12}$ and $\tilde{\Omega}_{23}$ can be calculated from Eqs. (7) and (8). In this situation, the fast non-adiabatic manipulation inevitably excites the population in the middle dot, which is governed by

$$i\dot{a}_2 = \tilde{\Omega}_{12}a_1 + \tilde{\Omega}_{23}a_3.$$  \hspace{1cm} (11)

In order to reduce the occupation in the central dot and thus the noise during the transfer, $\eta$ should be reduced, since the population in the central dot $P_2 = \sin^2 \eta$. The parameter $\alpha_0$ provides a degree of freedom to control the amplitude of $\eta$ (see Eq. (9)). Here we just choose $\alpha_0 = 40$ as an example. For a given $t_f$, to reduce the occupation of the middle dot requires stronger effective pulses by increasing $\alpha_0$.

With the same target, inverse engineering provides a more straightforward and flexible way than transitionless quantum driving to design the pulses with $t_f = 1$, see Fig. 4, in which the occupation in the middle dot is suppressed by choosing $\alpha_0 = 40$. Fig. 4 (c) further clarifies the dependence of $P_2^{\text{max}}$ at $t = t_f/2$ with $\alpha_0$.

**Charge transfer through a Multi-QD.** The mechanism of adiabatic charge transfer through a multi-QD system can be realized by SCTAP [18, 24] via the dark state in an array of odd number of QDs. The reason for that is the symmetry of the eigenvalue structure [24]. The Hamiltonian of an electron in such a system, where the energy levels are on resonance, is written in the form

$$H_0 = \Omega_1 c_1^\dagger c_2 + \sum_{1 < k < 2n} \Omega_k c_k^\dagger c_{k+1} + \Omega_{2n} c_{2n}^\dagger c_{2n+1} + h.c. \hspace{1cm} (12)$$

where a straddling scheme of internal pulses $\Omega_s$ is considered. The un-normalized dark state is expressed as [18]

$$|\phi_0\rangle = \cos \theta |1\rangle - (-1)^n \sin \theta |2n+1\rangle - X \sum_{j=1}^{n} (-1)^{j+1} |2j-1\rangle ,$$

where $\tan \theta = \Omega_1/\Omega_2$. The dots are labelled as $1, 2, ..., n, n+1, ..., 2n$, $2n+1$ where $N = 2n + 1$ is the total number of dots in the array. The hopping rates between neighboring dots $\Omega_{12}, ..., \Omega_{n,n+1}, ..., \Omega_{2n,2n+1}$, are given by

$$\Omega_1 = \Omega_{12} = \Omega_0 \exp \left[ -\frac{(t-t_f/2 + \tau)^2}{\sigma^2} \right],$$

$$\Omega_2 = \Omega_{2n,2n+1} = \Omega_0 \exp \left[ -\frac{(t-t_f/2 - \tau)^2}{\sigma^2} \right],$$

$$\Omega_s = \Omega_{k,k+1} = \Omega_{s0} \exp \left[ -\frac{(t-t_f/2)^2}{2\sigma^2} \right],$$

where $1 < k < 2n$, $\Omega_0$ and $\Omega_{s0}$ are the maximal amplitudes of the counter-intuitive pulses and straddled transitions, respectively. In the present QD system, Gaussian-shaped pulses $\Omega_1$ and $\Omega_2$ play the same role as the Pump and Stokes pulses in optical systems for $|1\rangle \rightarrow |2\rangle$ and $|2n\rangle \rightarrow |2n+1\rangle$ transitions, respectively. Meanwhile, the quantum states corresponding to the $2l$ dots, $l = 1, 2, ..., (N-1)/2$, do not participate in the dark state and therefore remain empty. If we constrain the parameter [18]

$$X = \frac{\Omega_1 \Omega_2}{\Omega_{s0} \sqrt{\Omega_1^2 + \Omega_2^2}} \ll 1,$$

i.e., $\Omega_{s0} \gg \Omega_0$, the undesirable population in the dots $3^\text{th}$, $5^\text{th}$, $2n - 1^\text{th}$ can be effectively limited, as it has been experimentally verified by Danzl et al [44] in cold atoms systems. As a result, by applying $\Omega_1$, $\Omega_2$ pulses and the large-amplitude one $\Omega_s$, population can be transferred from the dot $1^\text{st}$ to the dot $2n + 1^\text{st}$ directly.
We would like to speed up the adiabatic transfer in five coupled QDs, i.e., $2n + 1$ dots where $n = 2$. With the application of the Gaussian-shaped pulses described in Eq. (14), the charge state can be directly transferred from dot 1 to dot 5 by SCTAP with fidelity $F = \langle |\langle 5 | \Psi(t_f) \rangle | \rangle > 0.9999$, via the dark state, when $\Omega_0 t_f = 160\pi$ and other parameters $\sigma = \tau = t_f/6$ fulfill the adiabatic criteria. In addition to $E_0 = 0$ with the corresponding dark state, $|\phi_0 \rangle = (\cos \theta, 0, X, 0, \sin \theta)^T$, four eigenvalues $E_1 = -E_2, E_3 = -E_4$ are symmetric with respect to zero energy. In order to have negligible occupation in the central dot, i.e., the 3rd dot, we adopt $\Omega_0 = 10\Omega_0$ to satisfy the condition (15). Note that the probability amplitude at the fifth dot has the opposite sign as that in the right dot of a triple QD, see Eq. (4) and Eq. (13).

The analytical expressions for the bright states are more complicated than in the triple QD case. However, under the above constraint (15) for $\Omega_1$, $\Omega_2$ and $\Omega_s$, they can be simplified as,

$$
|\phi_1 \rangle = \left( -\sin \theta, \frac{1}{2}, 0, -\frac{1}{2} \cos \theta \right)_T,
$$

$$
|\phi_2 \rangle = \left( -\sin \theta, \frac{1}{2}, 0, \frac{1}{2} \cos \theta \right)_T,
$$

$$
|\phi_3 \rangle = \left( 0, -\frac{1}{2}, \frac{1}{2}, 0 \right)_T,
$$

$$
|\phi_4 \rangle = \left( 0, \frac{1}{2}, \frac{1}{2}, 0 \right)_T. \quad (16)
$$

By detailed inspection of the Hamiltonian, we find that the counter-diabatic term, considering a triple QD protocol, for the five-QD system becomes

$$
H_1 = i\Omega_0 c_1^T c_5 + h.c. \quad (17)
$$

The additional non-local interaction term $\Omega_n = \frac{1}{3}$ couples the two edge dots. We look for a Hamiltonian $H$ with effective pulses $\Omega_1$, $\Omega_2$ and the effective straddling one $\Omega_s$, which is equivalent to $H = H_0 + H_1$, and such that the solution of the corresponding time-dependent Schrödinger equation coincides with the instantaneous eigenstate of $H_0$. As in the case of a triple QD, one can trace out the non-adjacent interaction in a complicated way. The extension to arrays longer than five QDs becomes a much more involved task.

During the transfer in the non-adiabatic regime, similarly to the middle dot in a triple QD, population in the 2nd and 4th dots of a five-QD is excited. We label $b_j$, the probability amplitude of each on-site state $|j\rangle$ for a five-QD to avoid confusion with a triple QD. To reduce the population of the 2nd and 4th dots, we make an analogy with the triple QD and the five QD by keeping $|b_1\rangle \rightarrow |a_1\rangle$, $|b_5\rangle \rightarrow |a_3\rangle$ and by using $\Omega_s$ to achieve ideally $b_3$ equal to zero. Then, we obtain finite but small occupations for the internal even dots. Their corresponding amplitude fulfills: $|b_2|^2 + |b_4|^2 \rightarrow |a_2|^2$.

Due to the symmetry of the wavefunction $b_2 \rightarrow b_4$, we find $|b_2 - b_4|^2 \rightarrow |b_2|^2 + 2|b_2||b_4| + |b_4|^2 \rightarrow 2|a_2|^2$, and obtain $b_2 - b_4 \rightarrow \sqrt{2}|a_2|, i(b_2 - b_4) \rightarrow \sqrt{2}i\dot{a}_2$. Comparing the relation obtained from the Schrödinger equation

$$
i(b_2 - b_4) = \bar{\Omega}_1 b_1 - \bar{\Omega}_2 b_5, \quad (18)
$$

with Eq. (11) for a triple QD, the effective pulses for the five-QD system are related to the ones for the triple QD designed by inverse engineering as follows: $\Omega_1 \rightarrow \sqrt{2}\Omega_{12}$ and $\Omega_2 \rightarrow \sqrt{2}\Omega_{23}$.

The more intensity of $\bar{\Omega}_s$ has, the less population excitation the state $|3\rangle$ possesses. Hence, we keep $\Omega_s = 10\Omega_0$. Applying all the strategy discussed above, we obtain the population transfer for the five-dot system with $F > 0.9999$ for $t_f = 1$ as shown in Fig. 5. Population in the intermediate dots $P_j (j = 2, 3, 4)$ remains below 1 percent.

\section*{Discussion}

\textbf{Relation between $\Omega_{\text{max}}$ and $t_f$.} To show the efficiency of our protocol, we compare the relation of the pulse peak $\Omega_{\text{max}}$ on $t_f$ performed by inverse engineering and CTAP in a triple QD closed system, respectively, when the fidelity is above 0.9999 in both cases. As shown in Fig. 6, in a triple QD, inverse engineering is more efficient than CTAP, as the maximal intensity of the designed pulses are smaller for a given $t_f$. Here we
write down the explicit expressions of the shortcut pulses,

\[ \tilde{\Omega}_{12} = \dot{\chi} \cos \chi + \dot{\chi} \cot \eta \sin \chi, \]
\[ \tilde{\Omega}_{23} = -\dot{\eta} \sin \chi + \dot{\chi} \cot \eta \cos \chi, \]
from Eqs. (7) and (8). Obviously, their intensity could become infinite, when \( \eta \to 0 \) and \( \cot \eta \to \infty \). To avoid the divergence, we introduce the parameter \( \alpha_0 = \dot{\chi} \cot \eta \) and have the maximum value at the edges, \( \Omega_{\text{max}} = \tilde{\Omega}_{12}(t_f) = \tilde{\Omega}_{23}(0) = \alpha_0 \), by taking into account the boundary conditions. Meanwhile, the maximal population excitation in the second dot is given by

\[ P_{2}^{\text{max}} = \sin^2 \eta(t_0) = \frac{\chi^2(t_0)}{\chi^2(t_0) + \alpha_0^2}, \]
with \( t_0 = t_f/2 \), see also Fig. 4 (c). Therefore, one has to increase the intensity of the pulse, \( \Omega_{\text{max}} \), to suppress the population excitation in the second dot below 1 percent, by adjusting the parameter \( \alpha_0 \). Moreover, the maximum intensity and operation time satisfy the relation of time-energy uncertainty [33], as \( \chi(t_0) \geq \pi/(2t_f) \) yields to \( \tilde{\Omega}_{\text{max}}t_f \geq \pi/2 \tan \eta(t_0) \), where \( \eta(t_0) \) is constant when setting the constraint on the maximum population excitation. This rule also holds for larger arrays of QDs, if the amplitude of the curve is amplified by \( \sqrt{n} \) times for an array with \( 2n + 1 \) QDs.

**Fidelity vs decoherence.** Decoherence is a key issue for quantum state transfer and manipulation. In the following, we analyze the role of decoherence in order to check the feasibility of our strategy. In QDs systems, there are different sources of decoherence, such as hyperfine interaction or electron-phonon interaction. The time evolution of the density matrix is described by means of the Liouville-von Neumann-Lindblad equation [46] as

\[ i\dot{\rho} = [\hat{H}, \rho] - \frac{i}{2} \sum_k (L_k \rho \rho L_k^\dagger + \rho L_k^\dagger L_k - 2L_k \rho L_k^\dagger), \]
where \( L_k = \sqrt{\gamma}J_k \) are the operators in the system through which decoherence processes are introduced, and \( \gamma \) is the dephasing rate in the unit of \( 10^7 \text{s}^{-1} \). Here \( J_k \) are the angular momentum operators for spin \( f \), obeying the commutation relation: \([J_k, J_l] = iJ_m \epsilon_{klm} \) [47].

Next, we will consider a triple QD system as an example. Analogous to the master equation of a two-level system which can be described by a Bloch equation with a three-dimensional vector, the master equation of a three-level system can be recasted into a set of eight equations for the elements of the density matrix. The vector is chosen as

\[ \xi(t) = [\rho_{11} - \rho_{33}, \frac{1}{\sqrt{3}}(\rho_{11} + \rho_{33} - 2\rho_{22}), \rho_{12} + \rho_{21}, \rho_{21} + \rho_{12}, \rho_{13} + \rho_{31}, \rho_{31} - \rho_{13}, \rho_{23} + \rho_{32}, \rho_{32} - \rho_{23}]^T, \]
with the norm \( 2/\sqrt{3} \). Then the effective time-dependent Schrödinger equation can be written as

\[ \dot{\xi} = \mathcal{L}\xi, \]
where \( \mathcal{L} = \mathcal{L}_0 + \mathcal{L}_d \). The unperturbed pulse-controlled part is

\[ \mathcal{L}_0 = \tilde{\Omega}_{12}J_x + \tilde{\Omega}_{23}J_z, \]
whereas the \( 8 \times 8 \) matrix \( \mathcal{L}_d \)

\[ \mathcal{L}_d = -i \begin{pmatrix}
3\gamma & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 3\gamma & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \gamma & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 3\gamma & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \gamma & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \gamma & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 3\gamma & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma
\end{pmatrix}, \]
is the one which contributes to dephasing. Here \( J_x \) and \( J_z \) are the matrices of angular momentum operator for spin 7/2. In \( \mathcal{L}_d \), the dephasing terms with non-zero values are all diagonal, that demonstrates that decoherence directly

![FIG. 6. Relation between the pulse peak \( \Omega_{\text{max}} \) and \( t_f \) by using inverse engineering (solid, blue) and CTAP (dashed, red) for the charge transfer in a triple QD in a closed system. Other parameters are the same as those in Figs. 2 and 4.](image)

![FIG. 7. With the application of STA (inverse engineering) in a triple QD, (a) fidelity versus dephasing rate \( \gamma \) obtained from the full numerics (solid, blue), first order of time dependent perturbation \( F^{(1)} \) (dashed, red), second order \( F^{(2)} \) (dotted, black) and third order \( F^{(3)} \) (dot-dashed, orange), where \( t_f = 1 \). (b) Calculation of \( F \) up to forth order \( F^{(4)} \) (dashed, red). It approaches the numerical result \( F \) (solid, blue) for \( t_f = 1 \). Decreasing the operation time up to \( t_f = 0.5 \) leads to higher Fidelity \( F \) (full numerics results; dotted, black).](image)
acts on all the coordinates of the vector. The factors corresponding to $\xi_3$, $\xi_6$ and $\xi_7$ are $-i\gamma$, different from those of other coordinates of the vector. However, as $\xi_3 = \xi_6 = \xi_7 = 0$ holds during the transfer, this can simplify the following calculations in order to derive the fidelity. The fidelity for finding the state $|\psi(t_f)\rangle$ in the third dot at the final time $t_f$ is $F = \rho_{33}(t_f) = |\langle 3|\psi(t_f)\rangle|^2 = (2-3\xi_1(t_f)+\sqrt{3}\xi_2(t_f))/6$. To see the exact dependence of the fidelity on $t_f$ and the ansatz adopted by the wavefunction in the presence of the perturbation coming from decoherence, we derive the vector by time-dependent perturbation theory up to high orders, i.e. $\xi = \sum_k \xi^{(k)}$, $k = 0, 1, 2, \ldots$, which yields a corresponding expansion of the fidelity $F = \sum_k F^{(k)}$, $k = 0, 1, 2, \ldots$. For the zero order expansion without taking any perturbation in consideration, we obtain $\xi^{(0)}_1(t_f) = -1$, $\xi^{(0)}_2(t_f) = 1/\sqrt{3}$, and the ideal transfer fidelity $F^{(0)} = 1$. Expanded until the first order, the non-zero coordinate of the vector at $t_f$ is

$$-\xi_1(t_f) + \frac{1}{\sqrt{3}}\xi_2(t_f) = -\xi^{(0)}_1(t_f) + \frac{1}{\sqrt{3}}\xi^{(0)}_2(t_f)$$

$$-i\int_0^{t_f} dt_1 (-1, \frac{1}{\sqrt{3}, 0, 0, 0, 0, 0} U_0(t_f, t) L_d \xi^{(0)}(t_1),$$

where $U_0$ is the unperturbed time-evolution operator for the vector, resulting in the fidelity $F = F^{(0)} + F^{(1)} = 1 - 2\gamma t_f$. The higher orders ($n \geq 2$) can be further calculated from Dyson series, that is,

$$F^{(n)} = \frac{(-i)^n}{2} \int_0^{t_f} dt_n \int_0^{t_n} dt_{n-1} \ldots \int_0^{t_2} dt_1 \xi(t_f)^T$$

$$U(t_f - t_n) L_d(t_n) U(t_n - t_{n-1}) L_d(t_{n-1}) \ldots$$

$$U(t_2 - t_1) L_d(t_1) U(t_1) \xi(0)$$

$$= \frac{2}{3} (-3\gamma t_f)^n \frac{1}{n!}, \quad (25)$$

From the analytic calculations, we find that $F$ is only related to the dephasing rate $\gamma$ and the operation time $t_f$, whatever ansatz of the wavefunction Eq. (6) we adopt. Fig. 7 demonstrates the fidelity derived from the numerical calculations and the analytical equations by the time-dependent perturbation theory under different $t_f$. Up to the forth order, the analytical results almost coincide with the numerical ones, with $\gamma$ ranging from 0 to 0.4. It is also proved that for a given $\gamma$, shorter $t_f$ gives rise to higher $F$. Obviously, STA provides an efficient way to yield a higher fidelity under the effects of dephasing by shortening the operation time.

Furthermore, we demonstrate in Fig. 8, the fidelity $F$ in the function of $t_f$ and $\gamma$ using STA and CTAP in a triple QD. By CTAP, only when $t_f > 60$ and $\gamma < 0.01$, $\rho_{33}$ approaches to 1 and the state transfer is fulfilled, due to its vulnerability to decoherence. STA extends the regime of direct quantum state transfer to significatively smaller $t_f$. Particularly, when $t_f < 1$, fidelity remains robust against decoherence. In a large multi-QD system, the relation of $F$ to $t_f$ and $\gamma$ is quite similar.

**Feasibility of Experiments.** Although by solving the Schrödinger equation, the non-occupation of the middle dots can be theoretically checked, its direct experimental proof is much more difficult, as occupation measurement is difficult to do individually and the quantum detection charge measurement of the intermediate QD in a triple QD array would be affected by the occupation of the outer dots. However this problem can be overcome in longer arrays, which are experimentally feasible, by locating the detector at the center of the array.

For shorter operation times, stronger pulses are needed. However, the intensity of the pulses has to be limited depending of the physical setup in order to avoid strong heating. Therefore, one should find a compromise between pulse intensities and short time operation.

**METHODS**

**Shortcuts for direct charge transfer in a triple QD.** In a triple QD, besides the dark state, the other instantaneous eigenstates of $H_0$, $|\phi_\alpha\rangle = |\phi_\pm\rangle$ have non-zero energies $E_\alpha = E_\pm$ and

$$|\phi_\pm\rangle = \frac{1}{\sqrt{2}}(\sin \theta|1\rangle \pm |2\rangle + \cos \theta|3\rangle). \quad (26)$$
Provided that the initial state is one of the instantaneous eigenstates, its time evolution remains if the adiabaticity criterion is fulfilled [18],

\[ |E_0 - E_\alpha| \gg |\langle \phi_0 | \phi_\alpha \rangle|, \]

This can be simplified as \( \theta \ll \sqrt{\Omega_{12}^2 + \Omega_{23}^2} \) [20, 43]. Using counter-diabatic driving, we can find a time-dependent supplementary interaction [28]

\[ H_1 = i \sum \frac{\partial}{\partial t} |\phi_\alpha \rangle \langle \phi_\alpha| = i \Omega_a c_1^\dagger c_3 + h.c., \]

with \( \Omega_a = \dot{\theta} \), in order to cancel the diabatic transition. Furthermore, an appropriate unitary transformation of \( H = H_0 + H_1 \) results in the cancellation of non-adjacent coupling. For instance, the unitary operator can be [43]

\[ U = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \cos \phi & -i \sin \phi \\ 0 & i \sin \phi & \cos \phi \end{pmatrix}. \]

Consequently, the modified pulses are derived as \( \tilde{\Omega}_{12} = \sqrt{\Omega_{12}^2 + \Omega_\alpha^2}, \tilde{\Omega}_{23} = \Omega_{23} - \phi, \phi = \arctan(\Omega_a/\Omega_{12}) \). One could also choose other unitary transformation [43] or apply an alternative shortcut with dressed-state scheme [34, 42] to cancel the additional interaction and even suppress the excitation of the intermediate state.

Alternatively, inverse engineering presents a more straightforward way to design the pulses based on the solution of the time-dependent Schrödinger equation (6). The solution is parameterized by \( \gamma \) and \( \eta \), and the modified pulses controlling the tunneling coefficients are inversely designed through Eqs. (19) and (20). Once we interpolate the functions by choosing the appropriate ansatz with the boundary conditions, we know the dynamics of the charge transfer driven by the designed pulses. Especially, to achieve the direct charge transfer, adjustable parameter offers a way to suppress the occupation in the middle dot. Furthermore, in the present case the physical implementation is different from the shortcut with dressed-state scheme, and the required intensity of pulses is much less. One more parameter can be used for further optimization [31].

The applicability of shortcuts in an array of QDs. In an array of QDs with \( 2n + 1 \) numbers, the ideal adiabatic passage via a dark state means that non-zero probability amplitudes exist in the first and the last dots without populating the even order dots. The occupation at the odd order dots are suppressed by tuning \( \Omega_{a0}/\Omega_0 \). On the other hand, the counter-diabatic term always manifests itself as the interaction between the two outer dots,

\[ H_1 = i \Omega_a c_1^\dagger c_{2n+1} + h.c. \]

This inspires us to look for \( \tilde{\Omega}_1 \) \( \tilde{\Omega}_2 \) controlling the tunneling between the 1st (2n-th) and the 2nd (2n + 1-th) dots, which could transfer the charge directly and suppress the occupation in the even order dots. Making analogy with the triple QD, we obtain, for the 2n + 1 QD system, the following expression for the Schrödinger equation

\[ i \sum_{j=1}^n (-1)^{j+1} b_{2j} = \tilde{\Omega}_1 b_1 + \tilde{\Omega}_2 (-1)^{n+1} b_{2n+1}, \]

equivalent to Eq. (11) in a triple QD. Furthermore, in order to transfer the charge directly between the outer dots, following the analogy with the triple dot we impose \( |b_1| \rightarrow |a_1|, |b_{2n+1}| \rightarrow |a_3| \) and \( |b_2|^2 + |b_4|^2 + \ldots + |b_{2n}|^2 \rightarrow |a_2|^2 \). Finally we derive the next general expression for a 2n + 1 QD system: \( |\sum_{j=1}^n (-1)^{j+1} b_{2j}| \rightarrow \sqrt{n}|a_2| \). Furthermore, the occupation of the even order dots is suppressed by considering: \( \tilde{\Omega}_1 \rightarrow \sqrt{n} \tilde{\Omega}_{12} \) and \( \tilde{\Omega}_2 \rightarrow \sqrt{n} \tilde{\Omega}_{23} \), where \( \tilde{\Omega}_{12} \) and \( \tilde{\Omega}_{23} \) are those corresponding to a triple QD.

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AUTHOR CONTRIBUTIONS

Y. B. and X. C. developed the STA and performed the numerical simulations; All authors wrote the paper and discussed the contents; G. P. supervised the whole project.

ADDITIONAL INFORMATION

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