Time-Dependent Scalar Fields in Modified gravities in a stationary spacetime

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Most no-hair theorems involve the assumption that the scalar field is independent of time. Recently in [Phys. Rev. D 90, 041501(R) (2014)] the existence of time-dependent scalar hair outside a stationary black hole in general relativity was ruled out. We generalize this work to modified gravities and non-minimally coupled scalar field with an additional assumption that the spacetime is axisymmetric. It is shown that in higher-order gravity such as metric $f(R)$ gravity the time-dependent scalar hair doesn’t exist. While in Palatini $f(R)$ gravity and non-minimally coupled case the time-dependent scalar hair may exist.

I. INTRODUCTION

It is well known that black holes have no hair except the parameters of mass, electric charge, and angular momentum. More precisely, the no-hair theorem claims that all black hole solutions of the Einstein-Maxwell equations of gravitation and electromagnetism in general relativity can be completely characterized by only the three parameters. Since the long-range scalar field in the standard model of particle physics is electromagnetism, the matter field considered in the original no-hair theorem is electromagnetic field only. Nevertheless, it is still worth thinking about what the result is if we take other matter fields like non-Ablelian gauge fields and scalar fields into account.

The issue of the scalar-vacuum was first considered in 1970 in Ref. [1]. The canonical scalar hair was ruled out for scalar fields with various kinds of potential [2–4]. Recently, this proof was extended to non-canonical scalar fields [5] and Galileons [6]. Besides, black holes in Brans-Dicke and scalar-tensor theories of gravity were studied in Refs. [7, 8], which showed that the isolated stationary black holes in scalar-tensor theories of gravity are no different than in general relativity. In another word, non-minimally coupled scalar hair is also ruled out for stationary and conformally flat black holes. However, there is still the case that scalar hair does exist. Coexistence of black holes and a long-range scalar field in cosmology was presented in Ref. [9]. Other scalar hair cases can be found in Refs. [10–12]. These results are based on a same assumption: the scalar field is time-independent. In Ref. [13], the authors considered Einstein gravity minimally coupled to a dilaton scalar field and obtained an exact time-dependent spherically symmetric solution, which describes gravitational collapse to a static scalar-hairy black hole. In Ref. [14] it was shown that the stationary spacetime does not ensure that the scalar field is time-independent and time-dependent real non-canonical scalar hair was ruled out in Einstein gravity. While for the complex scalar field these arguments do not apply. Indeed Kerr black holes were found to be having time-dependent massive complex scalar hair [10, 11].

In this paper, we would like to generalize the work of Ref. [15] to some modified gravities. Since the proof only needs a small subset of the Einstein equations [16], this generalization is turned out to be possible for some cases. Among numerous modified gravities, $f(R)$ gravity, which is motivated by high-energy physics, cosmology and astrophysics, has received increased attention. It is interesting to consider the generalization of Ref. [15] to $f(R)$ gravity. For metric $f(R)$ gravity the scalar curvature $R$ in the action is constructed from the metric only. And for Palatini $f(R)$ gravity the scalar curvature $R = g^\mu\nu R_{\mu\nu}$ where the Ricci curvature $R_{\mu\nu}$ is constructed from the independent connection. Besides, since the time-independent non-minimally coupled scalar hair is ruled out [17], we also investigate the case that the scalar field is time-dependent and we will find nontrivial results.

This paper is organized as follows. In Sec. II we first investigate the time-dependent scalar field in metric $f(R)$ gravity, then generalized it to other higher-order gravity and Eddington Inspired Born-Infeld (EIBI) gravity [18]. In Sec. III time-dependent scalar field in Palatini $f(R)$ gravity is investigated. In Sec. IV we investigate the time-independent non-minimally coupled scalar. Finally the conclusion is given in Sec. V.

II. TIME-DEPENDENT SCALAR FIELD IN $f(R)$ GRAVITY

The action of $f(R)$ gravity is

\[ S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} f(R) + S_M(g_{\mu\nu}, \phi), \]

where $\phi$ denotes the matter field. The variation of the action [1] with respect to the metric $g_{\mu\nu}$ leads to the equation of motion (EoM) in $f(R)$ gravity:

\[ f_R R_{\mu\nu} - \frac{1}{2} f(R) g_{\mu\nu} + [g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu] f_R = \kappa T_{\mu\nu}, \]
where \( f_R \equiv \frac{\partial f(R)}{R} \), \( \Box = \nabla^\mu \nabla_\mu \), and the energy-momentum tensor \( T_{\mu\nu} \) is given by

\[
T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}}.
\]  

(3)

In general relativity, if the null energy condition holds, the rigidity theorem ensures that stationary spacetime must be axisymmetric. In \( f(R) \) gravity, the null energy condition does not lead to \( R_{\mu\nu}l^\mu l^\nu \geq 0 \) for all timelike vector \( l^\mu \). Therefore, the null energy condition of the matter fields does not lead to the conclusion that stationary spacetime must be axisymmetric. We have to assume that the spacetime is axisymmetric and we choose coordinates \((t, r, \theta, \phi)\) so that the metric takes the form

\[
ds^2 = -e^{\nu(r,\theta)}dt^2 + 2\rho(r,\theta)dtd\phi + e^{\psi(r,\theta)}d\theta^2 + e^{A(r,\theta)}dr^2 + e^{B(r,\theta)}d\phi^2.
\]  

(4)

One can easily verify that the following components of the Ricci tensor and Christoffel symbol vanish,

\[
R_{tr} = R_{t\theta} = R_{\phi\phi} = 0,
\]  

(5)

\[
\Gamma_{tr} = \Gamma_{t\theta} = \Gamma_{\theta\theta} = 0.
\]  

(6)

The action of the K-essence is \([23–26]\) and noting that the metric is independent of time, we have

\[
\partial_t T_{rr} = \partial_t (P_X \partial_r \phi \partial_r \phi) + g_{rr} \partial_t P = g_{rr} \partial_t P = 0.
\]  

(14)

Thus, \( \partial_t P = 0 \). Combining \( \partial_t P = 0 \) with \( \partial_t P = P_\phi \partial_t \phi \), we have \( P_\phi = 0 \) and \( \partial_t P_X = \partial_\phi \partial_X P = \partial_X \partial_\phi P = 0 \). So the \( tt \) component of Eq. \([2]\) leads to

\[
\partial_t T_{tt} = \partial_t (P_X \partial_t \phi \partial_t \phi) + g_{tt} \partial_t P = P_X \partial_t (\partial_t \phi \partial_t \phi) + g_{tt} \partial_t P = P_X \partial_t (\partial_t \phi \partial_t \phi) = 0.
\]  

(15)

Similarly one can deduce that \( \partial_t (\partial_t \phi \partial_\phi \phi) = 0 \). So it is now clear that the scalar field \( \phi \) depends at most linearly upon \( t \) and \( \phi \). Moreover, since \( \phi \) should depend periodically upon \( \phi \), it is incompatible if \( \phi \) depends linearly upon \( \phi \). Hence, we finally deduce that the only possible configuration of the scalar field is

\[
\phi = at + b,
\]  

(16)

where \( a \) and \( b \) are constants. So far we have proved that in \( f(R) \) gravity, the time-dependent non-canonical scalar field in a stationary spacetime is only a linear function of \( t \). This conclusion is the same as that of Ref. \([18]\). Following the procedure of Ref. \([18]\), for asymptotically flat and (anti-)de Sitter stationary black holes, there is no time-dependent scalar hair. Here we give a brief demonstration.

\section{Boundary conditions}

Let’s first consider the asymptotic flat condition, i.e. \( g_{\mu\nu} \rightarrow g_{\mu\nu} \) as the radial coordinate \( r \rightarrow \infty \), for which \( X \rightarrow a^2/2 \), and the \( tt \) and \( rr \) components of the energy-momentum tensor tend to

\[
T_{tt} \rightarrow a^2 P_X (\frac{a^2}{2}) - P (\frac{a^2}{2}),
\]  

(17)

\[
T_{rr} \rightarrow P (\frac{a^2}{2}).
\]  

(18)

The EoMs demand \( T_{tt} = 0 \) and \( T_{rr} = 0 \), thus \( a = 0 \), which leads to a constant scalar field.

For the case of an asymptotically anti-de Sitter spacetime, \( g^{tt} \rightarrow 0 \) as \( r \rightarrow \infty \). In the static spherically symmetric coordinates, the anti-de Sitter metric reads

\[
ds^2 = -(1 - \frac{\Lambda}{3} r^2) dt^2 + (1 - \frac{\Lambda}{3} r^2)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2).
\]  

(19)

The \( tt \) and \( rr \) components of the energy-momentum tensor at infinity tend to

\[
T_{tt} \rightarrow P_X P(0) - \left(1 + \frac{\Lambda r^2}{3}\right) P(0),
\]  

(20)

\[
T_{tt} \rightarrow a^2 P_X (0) - \left(1 + \frac{\Lambda r^2}{3}\right) P(0),
\]  

(21)

\[
T_{rr} \rightarrow 0.
\]  

(22)
It is clear that \( T_{\text{tt}} = 0 \) and \( T_{\text{tr}} = 0 \) yield \( P(0) = 0 \) and \( a = 0 \). So there is no time-dependent scalar hair.

For the case of an asymptotically de Sitter spacetime, in the static coordinates the metric is the same as Eq. [10]. As \( \Lambda > 0 \), there is an event horizon at \( r = \sqrt{3/\Lambda} \). Thus, \( g_{\text{rr}}(r \to \sqrt{3/\Lambda}) \to -\infty \) leads to \( T_{\text{rr}}(r \to \sqrt{3/\Lambda}) \to \infty \), which is incompatible with the geometry. Hence there is no time-dependent scalar hair.

The derivation of the time-dependent scalar field in \( f(R) \) gravity can be generalized to a large class of alternative theories of gravity under the metric form \( (4) \). As an example, consider a higher-order gravity with the action

\[
S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} \left( R + \alpha R^2 + \beta R_{\mu
u} R^{\mu
u} \right) + S_M. \tag{23}
\]

The field equations read

\[
G_{\mu\nu} + 2\alpha R \left( R_{\mu\nu} - \frac{1}{4} R g_{\mu\nu} \right) + (2\alpha + \beta) (g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu) R + 2\beta R \xi (R_{\mu\nu\sigma\tau} - \frac{1}{4} R_{\mu\nu\sigma\tau} g_{\mu\nu}) + \beta \Box \left( R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) = \kappa T_{\mu\nu}. \tag{24}
\]

Since the metric is stationary and axisymmetric, the \( tr \) and \( t\theta \) components of Eq. (24) vanish and we still have Eqs. (10) and (11). The rest derivation is the same as that in \( f(R) \) gravity. It is obvious that these arguments can apply to other alternative gravities like EiBI gravity theory.

### B. Double scalar fields

We consider the case that the matter fields are consisted of two coupled non-canonical scalar fields \( \varphi_1 \) and \( \varphi_2 \), of which the most generalized action is

\[
S_M = \int d^4x \sqrt{-g} P(\varphi_1, \varphi_2, X_1, X_2). \tag{25}
\]

This action contains the case of a complex scalar field as a special case. The energy-momentum tensor is

\[
T_{\mu\nu} = P_{X_1} \partial_\mu \varphi_1 \partial_\nu \varphi_1 + P_{X_2} \partial_\mu \varphi_2 \partial_\nu \varphi_2 + P g_{\mu\nu}. \tag{26}
\]

Therefore \( T_{0i} = 0 \) does not necessarily lead to \( \partial_\varphi_1 \partial_\varphi_1 \varphi_1 = 0 \) or \( \partial_\varphi_2 \partial_\varphi_2 \varphi_2 = 0 \), and the argument given above does not work for the double scalar field case any more.

### III. TIME-DEPENDENT SCALAR FIELD IN PALATINI \( f(R) \) GRAVITY

Now we turn to the time-dependent scalar field in Palatini \( f(R) \) gravity. The action of Palatini \( f(R) \) gravity is

\[
S_{\text{Pal}} = \frac{1}{2\kappa} \int d^4x \sqrt{-f(R)} + S_M(g_{\mu\nu}, \varphi), \tag{27}
\]

where the Ricci tensor \( R_{\mu\nu} \) is constructed with the independent connection \( \Gamma^\lambda_{\mu\nu} \) and the corresponding Ricci scalar is \( R = g^{\mu\nu} R_{\mu\nu} \). Here we still assume that the spacetime is stationary and axisymmetric. Then the metric has the same form of (4) and \( \mathcal{R} \) is independent on \( t \) and \( \varphi \).

Varying the action (27) independently with respect to the metric and connection, one can obtain the EoMs of Palatini \( f(R) \) gravity,

\[
\begin{align*}
\mathcal{R}_{(\mu\nu)} - \frac{1}{2} f(\mathcal{R}) g_{\mu\nu} &= \kappa T_{\mu\nu}, \quad \text{(28)}
\end{align*}
\]

\[
\nabla_\lambda (\sqrt{-g} f(R) g^{\mu\nu}) = 0, \tag{29}
\]

where \( \nabla_\lambda \) is defined with the independent connection \( \Gamma^\lambda_{\mu\nu} \). Let us define a conformal metric \( g_{\mu\nu} \),

\[
q_{\mu\nu} \equiv f(R) g_{\mu\nu}. \tag{30}
\]

Then, Eq. (30) implies that the independent connection \( \Gamma^\lambda_{\mu\nu} \) is the Levi-Civita connection of the conformal metric \( q_{\mu\nu} \). Under conformal transformations, the Ricci tensor \( R_{\mu\nu} \) transforms as

\[
\begin{align*}
\mathcal{R}_{\mu\nu} &= R_{\mu\nu} + 3 \frac{1}{2f(R)} (\nabla_\mu f(R)) (\nabla_\nu f(R))
- \frac{1}{f(R)} \left( \nabla_\mu \nabla_\nu + \frac{1}{2} g_{\mu\nu} \Box \right) f(R), \quad \text{(31)}
\end{align*}
\]

where \( R_{\mu\nu} \) is the Ricci tensor constructed by the spacetime metric \( g_{\mu\nu} \). Contraction with \( g^{\mu\nu} \) yields

\[
\begin{align*}
\mathcal{R} &= R + 3 \frac{1}{2f(R)} (\nabla_\mu f(R)) (\nabla_\nu f(R))
- \frac{3}{f(R)} \Box f(R). \quad \text{(32)}
\end{align*}
\]

With Eqs. (31) and (32), Eq. (28) is reduced to

\[
\begin{align*}
G_{\mu\nu} &= \frac{\kappa}{f(R)} T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \left( \mathcal{R} - \frac{1}{f(R)} \right)
+ \frac{1}{f(R)} (\nabla_\mu \nabla_\nu - g_{\mu\nu} \Box) f(R)
- \frac{3}{2f(R)} \left( \nabla_\mu f(R) (\nabla_\nu f(R) - \frac{1}{2} g_{\mu\nu} \Box f(R)) \right), \quad \text{(33)}
\end{align*}
\]

from which we can see that we still have Eqs. (10)-(11) and Eqs. (14)-(15) for Palatini \( f(R) \) gravity. Thus, the only possible configuration of the scalar field is (16).

Now we consider whether the configuration of the scalar field can be compatible to the boundary conditions. First we consider the asymptotic flat boundary condition. Note that the asymptotic flat boundary condition implies that the metric \( g_{\mu\nu} \) approaches the Minkowski metric \( \eta_{\mu\nu} \), while \( q_{\mu\nu} \) can be conformally flat. Thus Eqs. (17) and (18) no longer hold. On the other hand, \( \partial_\varphi P = 0 \) and \( \partial_\varphi \varphi = 0 \) yield \( \partial_\varphi P = 0 \), and \( P_X = P_X(\alpha^2/2) \) is a constant. Thus \( \nabla_\mu P_X = 0 \). It is easy to verify \( \Box \varphi = 0 \). Therefore, the configuration of
the scalar field of Eq. (13) is compatible with Eq. (3), the EoM of the scalar field. The asymptotic flat boundary condition no longer yields $a = 0$. Similar argument can be made in the asymptotic AdS/dS cases. Hence for all the three kinds of boundary conditions the time-dependent scalar hair may exist in Palatini $f(R)$ gravity.

We can also investigate the time-dependent scalar field $\phi$ in scalar-tensor gravity with the action

$$S_{\text{st}} = \frac{1}{2\kappa} \int d^4x \sqrt{-g} \left[ U(\psi) R - \frac{1}{2} h(\psi) \nabla_\mu \psi \nabla^\mu \psi - V(\psi) \right] + S_{\text{M}}(g_{\mu\nu}, \phi).$$

(34)

where the action of the scalar field $\phi$ is still given by Eq. (1). Here we also assume that $g_{\mu\nu}$ is in the form of Eq. (14) and $\psi = \psi(r, \theta)$. This action is equivalent to the action (14) of metric $f(R)$ gravity if $U(\psi) = \psi$ and $h(\psi) = 0$ [28, 31], and equivalent to the action (27) of Palatini $f(R)$ gravity if $U(\psi) = \psi$ and $h(\psi) = -\frac{3}{2\kappa}$ [28, 29, 31, 32]. This case will be the same with that of time-dependent scalar field in Palatini $f(R)$ gravity: we can educe the conclusion that the scalar field $\phi$ only depend linearly on $t$, but the boundary conditions do not exclude the scalar hair, thus the scalar hair may exist.

IV. NON-MINIMALLY COUPLED SCALAR FIELD

We now give the argument for a time-dependent non-minimally coupled scalar field in a stationary spacetime. It should be noted that this is different from the case of the time-dependent scalar field in scalar-tensor. The action for the non-minimally coupled scalar field is

$$S = \int d^4x \sqrt{-g} \left[ \frac{\omega(\phi)}{\phi^2} \nabla^\mu \phi \nabla_\mu \phi - \frac{1}{2} g_{\mu\nu} \nabla^\lambda \phi \nabla_\lambda \phi + V(\phi) \right].$$

(35)

By varying with respect to $g_{\mu\nu}$ and $\phi$, one obtains the field equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{\omega(\phi)}{\phi^2} \left( \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} \nabla^\lambda \phi \nabla_\lambda \phi \right)$$

$$+ \frac{1}{\phi} \left( \nabla_\mu \nabla_\nu \phi - g_{\mu\nu} \Box \phi \right) - \frac{V(\phi)}{2\phi} g_{\mu\nu}.$$  

(36)

$$(2\omega + 3) \Box \phi = -\omega(\phi) \nabla^\lambda \phi \nabla_\lambda \phi + \phi V(\phi) - 2V.$$  

(37)

Here we still assume that the spacetime is stationary and axisymmetric. Thus we have the metric (11) and Eqs. (38)-(40). The $tr$ and $t\theta$ components of Eq. (2) imply that

$$\frac{\omega(\phi)}{\phi^2} \partial_r \phi \partial_r \phi + \frac{1}{\phi} \nabla_r \nabla_r \phi = 0,$$

$$\frac{\omega(\phi)}{\phi^2} \partial_\theta \phi \partial_\theta \phi + \frac{1}{\phi} \nabla_\theta \nabla_\theta \phi = 0.$$  

(38)

Now it is clear that $\partial_r \phi \neq 0$ no longer educes $\partial_r \phi = 0$ or $\partial_\theta \phi = 0$, thus the arguments in Sec. II no longer apply and the time-dependent non-minimally coupled scalar hair may exist.

V. CONCLUSION

In this paper, we investigated the non-canonical time-dependent scalar field in a stationary and axisymmetric spacetime in modified gravities. For a single real scalar field in metric $f(R)$ gravity, we proved that the time-dependent scalar hair does not exist for the three kinds of boundary conditions ( asymptotically flat, anti-de Sitter, and de Sitter). It was shown that the demonstration can be generalized to a large class of alternative theories of gravity like the higher-order gravity described by the action (23) and EiBI gravity. While for two coupled scalar fields, these arguments do not apply. These conclusions are the same as the time-dependent scalar field in general relativity in Ref. [13].

Though the demonstrations for a single scalar hair in general relativity and metric $f(R)$ only use a small subset of the field equations [13], the generalization to other alternative gravities may not be correct. For Palatini $f(R)$ gravity coupled with a scalar field, as the boundary conditions no longer ruled out the non-trivial configuration of the scalar field, the time-dependent scalar hair may exist outside a stationary and axisymmetric black hole. Since Palatini $f(R)$ gravity is equivalent to scalar-tensor gravity, similar argument can be applied to time-dependent scalar field in scalar-tensor gravity. For the time-independent non-minimally coupled scalar field, since the effective energy-momentum of the scalar field contains the second derivative of the scalar field, the conclusions are the same as the time-dependent scalar field in general relativity.

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