Rotor dynamic balancing control method based on fuzzy auto-tuning single neuron PID

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Abstract: In order to improve the control efficiency and balance accuracy of rotor automatic balance, a fuzzy self-tuning single neuron PID control method is proposed in this paper. Based on the single neuron PID control method, the fuzzy control theory is introduced to adjust the output gain K of single neuron PID control to realize single neuron PID control with variable step size. In order to verify the superiority of this method, the method we designed in this paper is compared with the traditional PID control method and the single neuron PID control method by the simulation and self-built experimental platform. Experiments and simulation results show that the fuzzy self-tuning single neuron PID control method has faster response time, less overshoot amount and fewer oscillation times than the traditional PID control method and the single neuron PID control method, and has strong robustness and good stability.

Keywords: fuzzy control theory, rotor, automatic balance, single neuron

Classification: Circuits and modules for electronic instrumentation

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1 Introduction

Recent years, many researchers devote themselves to the area of rotor dynamic balancing control. Rotor dynamic balancing control is an important research content of rotating machinery vibration detection and control [1]. On the aspect of the rotor dynamic balancing control, the key problem is how to improve the balance accuracy and balance efficiency of the control strategy [2]. For this purpose, we aim to applying a fuzzy self-tuning single neuron PID control method to improve the robustness and the stability. We have done experiments and simulation of fuzzy control single neuron PID control method.

At present, the common control strategies mainly include modal balancing method [3] and influence coefficient balancing method [4]. The influence coefficient method and the improved influence coefficient method have faster equilibrium speed, but the calculation process may produce large error. Currently, many improved control strategies are applied in the field of rotating machinery control. Wei, C.S. and D. Soffker [5] introduce the strategy which included optimization using a multiobjective genetic algorithm. But it is a time-consuming tuning procedure to design the PID controllers. Cedillo, S.G.T. and P. Bonello [6] put forward an equivalent unbalance identification method for the balancing of nonlinear squeeze-film damped rotor dynamic systems, “Active unbalance control of rotor systems using online algebraic identification methods,” Asian J. Control 15 (2013) 1627 (DOI: 10.1002/asjc.744).
nonlinear squeeze-film damped rotor dynamic systems. Beltran-Carbajal, F., G. Silva-Navarro and M. Arias-Montiel [7] put forward an active control scheme based on compensation of perturbation force signals to balance rotating machinery with unknown system parameters and variable operation speed. Zhang, Y., et al. [8] propose an improved holospectrum-based balancing method with increased accuracy and a greater scope of application. In a word, the control efficiency of these methods is not high or the robustness is not good.

Based on the single plane double counterweight balance device, we propose an intelligent control method based on single neuron to solve the problem that the conventional PID control method needs to determine the control parameters in advance. At the same time, the fuzzy control technology is introduced, and the gain of the single neuron is obtained by defining the fuzzy variables and the membership function. Finally, we demonstrate the effectiveness of this method by simulation and comparing experiments on a self-built experiment platform.

2 Materials and methods

2.1 System model

We are focusing on the rotor system as the research object, and the rotor system structure model is shown in Fig. 1. Firstly, the rotor system is modeled, and the main shaft is divided into a circular disc unit, an elastic shaft section unit and a bearing unit [9].

\[ M_d \ddot{U}_{1d} + \Omega J \dot{U}_{2d} = Q_{1d} \]
\[ M_d \ddot{U}_{2d} + \Omega J \dot{U}_{1d} = Q_{2d} \]

In the Eq. (1), \( M_d \) is the disc mass matrix, \( M_d = \begin{bmatrix} m & 0 \\ 0 & J_d \end{bmatrix} \), \( J \) is gyroscopic matrix, \( J = \begin{bmatrix} 0 & 0 \\ 0 & J_p \end{bmatrix} \), \( Q_{1d} \) and \( Q_{2d} \) is corresponding generalized force. The corresponding generalized force is:

![Fig. 1. Rotor system structure model](image-url)
In the Eq. (2), $e_\xi$, $e_\eta$ are the slight eccentric of the disc.

The differential equations of motion of the shaft section are given by:

\[
\begin{cases}
M_s\ddot{U}_1 + \Omega J_s\dot{U}_2 + K_s U_1 = Q_{1s} \\
M_s\ddot{U}_2 + \Omega J_s\dot{U}_1 + K_s U_2 = Q_{2s}
\end{cases}
\] (3)

In the Eq. (3), $M_s$ is the shaft segment mass matrix, $M_s = M_{sT} + M_{sB}$, $J_s$ is rotation matrix, $Q_{1s}$ and $Q_{2s}$ is corresponding generalized force.

Bearing can be simplified to a spring with stiffness and damping, ignoring the angular stiffness and coupling terms and the differential equation of motion is:

\[
\begin{cases}
M\ddot{U}_1 + \Omega J\dot{U}_2 + K_s U_1 = Q_1 \\
M\ddot{U}_2 + \Omega J\dot{U}_1 + K_s U_2 = Q_2
\end{cases}
\] (4)

In the Eq. (4), $M$ is the disc mass matrix, $\Omega J$ is rotation matrix, $K_x$ and $K_y$ is stiffness matrix. Because the interaction between the elements of the internal forces in the equation has been eliminated in the process, and the bearing seat on the stiffness of the bearing is also equivalent to the stiffness matrix, the unbalanced generalized force is left in $Q_1$ and $Q_2$. Assuming that the bearing stiffness is isotropic, the principal axis of the two direction of the motion is the same amplitude, the phase difference is 90°. The system differential equation is equivalent to:

\[
\begin{cases}
M\dddot{U}_1 + \Omega J\ddot{U}_2 + K_s U_1 = \Omega^2[\alpha_1, \alpha_2, \cdots \alpha_{2n}]^T \cos(\Omega t) \\
\quad + [b_1, b_2, \cdots b_{2n}]^T \sin(\Omega t) \\
M\dddot{U}_2 + \Omega J\ddot{U}_1 + K_s U_2 = \Omega^2[\alpha_1, \alpha_2, \cdots \alpha_{2n}]^T \sin(\Omega t) \\
\quad - [b_1, b_2, \cdots b_{2n}]^T \cos(\Omega t)
\end{cases}
\] (5)

In the Eq. (5), $\Omega = J$. If $Z = U_1 + iU_2$, the formula can be simplified:

\[
M\dddot{Z} - i\dot{G}\dddot{Z} + KZ = \Omega^2 Q e^{i\omega}
\] (6)

### 2.2 Fuzzy auto-tuning single neuron PID control method

In this paper, a fuzzy adaptive single neuron PID control system model is shown in Fig. 2. $r(k)$ is the expected vibration of rotor, $y(k)$ is the actual vibration of rotor, $e(k)$ is the difference signal. $\Delta e(k)$ is the difference rate of change, $x_i(k)$ is the input signal of neuron which is obtained from $e(k)$; $W_i(k)$ is the network weights of neurons in each input signal $x_i(k)$; $u(k)$ is the final output step of the overall controller; $k$ is the sampling time; $i = 1, 2, 3$; $K$ is the output gain of single neuron. The basic control principle is as follows: firstly, the single neuron PID controller obtain the vibration signal by the vibration sensor, and then the difference $e(k)$ is transmitted to the fuzzy controller by comparing with the preset target vibration value, the fuzzy controller get the gain $K$ of the single neuron by the decision of the
membership function, so as to obtain the control step, finally the effective control of rotor imbalance is realized.

Artificial neural network can approximate any linear or nonlinear system and realize the control of almost all conventional nonlinear and uncertain system, so it is widely used in intelligent control system. However, it is not conducive to real-time control due to the fact that the neural network structure is complex and the weight training time is long. Considering the advantages of single neuron, such as the simple structure, the simple calculation, the fast learning speed and the easy implementation, it is suitable for the nonlinear control system with high real-time performance. In this paper, the single neuron PID controller is used to replace the PID controller to improve the control performance of the controller on the system dynamic balance.

The activation function of the single neuron network is a linear function, and the output of the single neuron is linearly related to the local field [10]. In the control field, the single neuron control expression is

\[
\Delta u(k) = W_1 e(k) + W_2 [e(k) - e(k-1)] + W_3 [e(k) - 2e(k-1) + e(k-2)]
\]  

(7)

The PID control expression is

\[
\Delta u(k) = K_I e(k) + K_P [e(k) - e(k-1)] + K_D [e(k) - 2e(k-1) + e(k-2)]
\]  

(8)

Through the above two expressions contrast, \(K_I, K_P, K_D\) are constant, \(W_1, W_2, W_3\) are the adjustable coefficient weights. So the single neuron PID controller is the PID controller of variable coefficient.

The single neuron output is

\[
\Delta u(k) = K \left( \sum_{i=1}^{3} \bar{W}_i x_i(k) \right)
\]  

(9)

The single neuron PID controller has 3 inputs as follows:

\[
\begin{align*}
x_1(k) &= e(k) \\
x_2(k) &= \Delta e(k) = e(k) - e(k-1) \\
x_3(k) &= e(k) - 2e(k-1) + e(k-2)
\end{align*}
\]  

(10)
In the Eq. (10), $x_1(k)$ is the current vibration difference which reflects the accumulation of systematic error variation. $x_2(k)$ is the change of difference which reflects the variation of error. $x_3(k)$ is the difference of one order difference which reflects the trend of error variation. Considering the weights of the single neuron controller $W_i(k)$, the improved Hebb algorithm is proposed by the paper [11],

$$
W_i(k) = W_i(k - 1) + \eta_i e(k) u(k) x_i(k)
$$

In the Eq. (11), $\eta_i$ is the learning efficiency. Finally, the control law can be obtained by:

$$
u(k) = u(k - 1) + K \left( \sum_{i=1}^{3} \bar{W}_i(k) x_i(k) \right)$$

The input signal $x_1, x_2, x_3$ were taken into the above equation. Finally, the control law can be obtained by:

$$
u(k) = u(k - 1) + K (\bar{W}_1(k) e(k) + \bar{W}_2(k) [e(k) - e(k - 1)])$$

From the above Eq. (13), we can conclude that the output gain $K$ has a great influence on the stability and convergence of the single neuron PID controller. With the increase of $K$ value, the single neuron controller output of the adjustment step increases, the adjustment time is less and the rapidity of convergence increases. But too large $K$ will lead to a larger overshoot, and even cause system oscillation. In contrast, with the decrease of $K$ value, the control step of the controller output is reduced, and the time of adjustment is increased, but the stability of the system is good. However, the $K$ value selection is too small, it will lead to decreasing the rapid response of the system and reducing the tracking ability. In order to improve the control efficiency of the controller, the fuzzy control is used to control the output gain $K$ in this paper.

Fuzzy control has good robustness, strong anti-interference ability and has a good control effect on complex objects or accurate mathematical model of controlled object which is difficult to establish, so it is suitable for rotor dynamic balance control of the nonlinear system [12]. Based on this reason, the fuzzy control is used to carry out the auto-tuning of the parameter $K$ in this paper. The fuzzy rules designed in this paper are given: when the system deviation is big, the large $K$ value is selected to obtain the fast response speed and shorten the adjustment time; When the error is small, the value of $K$ is small. This method will not only be more efficient to control the vibration, but also avoid overshoot phenomenon, so the stability of the system is improved [13].

The fuzzy controller designed in this paper has 2 inputs: The deviation between the actual amplitude and the target amplitude $e$ and the variation rate of deviation $\Delta e$. The output of the fuzzy controller is used as the output gain of the single neuron PID controller.

The specific implementation steps of fuzzy control are as follows:
1) The fuzzy definition of the system language. The input and output of the fuzzy controller are calibrated, which are specified in a certain range, and the input and output are classified by the symmetric mode [14]. The amplitude deviation is quantified as level 5, \{-2, -1, 0, +1, +2\}, the fuzzy subset of amplitude deviation is divided into 5 parts, \{NB, NS, ZO, PS, PB\}; The variation rate of deviation is quantified as level 5, \{-2, -1, 0, +1, +2\}, the fuzzy subset of variation rate of deviation is divided into 5 parts, \{NL, NS, ZO, PS, PB\}; The gain \(K\) is quantified as level 5, \{-2, -1, 0, +1, +2\}, the fuzzy subset of the gain \(K\) is divided into 5 parts, \{NB, NS, ZO, PS, PB\}; The NB, NS, ZO, PS, PB respectively represent the large negative number, the small negative number, zero, the small positive, the large positive.

2) The construction of the system membership function. In order to facilitate the calculation and improve the efficiency of the algorithm, the triangle membership function is applied in this paper. The form of the triangle membership function is as follows:

\[
\mu_{A_i}(x_i, c_i^{-1}, c_i, c_i^{+1}) = \begin{cases} 
\frac{x_i - c_i^{-1}}{c_i^{+1} - c_i^{-1}}, & c_i^{+1} \neq c_i^{-1}, x_i \in [c_i^{-1}, c_i^{+1}] \\
\frac{c_i^{+1} - x_i}{c_i^{+1} - c_i}, & c_i \neq c_i^{+1}, x_i \in [c_i^{+1}, c_i] \\
1, & x_i = c_i^{+1} \\
0, & \text{else}
\end{cases} \quad (14)
\]

In the Eq. (14), \(\mu_{A_i}(x_i, c_i^{-1}, c_i, c_i^{+1})\) is the membership function, \(i = 1, 2\) \(j = 2, 3, 4\), \(c_i^j\) is the intersection of the membership function and the x-axis.

According to the control requirements, the input amplitude deviation and deviation change rate of the middle part are relatively narrow, the various parts of positive and negative deviations are similar, and the various parts of the gain \(K\) are similar. The membership function of each control variable is shown in Fig. 3.

![Fig. 3. Membership function of each control variable](image)

3) The fuzzy decision rule is the core of the fuzzy controller. Based on the simulation and experiment, the experimental data is analyzed in the paper, and the fuzzy rules of the output of the fuzzy controller are determined finally [15]. The decision rules for the fuzzy controller are shown in Table I. The total number of rules is \(N = \prod_{i=1}^{s} p_i\), \(s = 2\), \(p_i = 5\). By the above equation can be drawn: \(N = 25\). The fuzzy rule form of two-input-single-output fuzzy model is \(R_k\):

If \(x_1\) is \(A_1^{i_1}\) and \(x_2\) is \(A_2^{i_2}\) then \(y\) is \(B_k\).
In the formula, $A_{i}^{k}$ is the fuzzy set of $x_{i}$, $B_{k}$ is the fuzzy set corresponding to the system output fuzzy value under the corresponding prerequisite. According to the previous fuzzy reasoning method, the fuzzy decision rules can be determined.

$$R_{k} = (A_{1}^{k} \times A_{2}^{k}) \times B_{k}$$  \hspace{1cm} (15)

$$R = R_{1} \cup R_{2} \cup \cdots \cup R_{n} = \bigcup_{k=1}^{n} (A_{1}^{k} \times A_{2}^{k}) \times B_{k}$$

$$R_{1} = [(NB)_{E} \times (NB)_{EC}] \times (PB)_{K}$$

$$R_{2} = [(PB)_{E} \times (NS)_{EC}] \times (PS)_{K}$$

$$R_{25} = [(PB)_{E} \times (PB)_{EC}] \times (NB)_{K}$$

The fuzzy subset of the output language variable domain is obtained:

$$U = [E \times EC] \circ R_{1} \cup [E \times EC] \circ R_{2} \cup [E \times EC] \circ R_{3} \cup \cdots \cup [E \times EC] \circ R_{m}$$

$$U = \bigcup_{i=1}^{m} [E \times EC] \circ R_{i}$$  \hspace{1cm} (16)

For the above fuzzy rules, single point fuzzification is adopted, the value of an element in the corresponding fuzzy set is not zero.

$$\mu_{i}^{k}(x_{i}) = \begin{cases} 1 & \text{if } x_{i} = x_{i}^{*} \\ 0 & \text{if else} \end{cases}$$  \hspace{1cm} (17)

The max-product reasoning method is applied in the paper,

$$\mu_{B_{k}}^{*}(y_{k}) = \max_{k=1,\ldots,N} \left\{ \sup \left[ \prod_{i=1}^{n} \mu_{A_{i}}^{*}(x_{i}) \mu_{B_{i}}(y) \right] \right\}$$  \hspace{1cm} (18)

In the Eq. (18), $k_{i} \in 1, 2, 3, 4, 5$, $k = 1, 2, 3, 4 \cdots N$. If the input variable $x$ is single-point fuzzification, the membership function of fuzzy output can be obtained:

$$\mu_{B_{k}}^{*}(y_{k}) = \max_{k=1,\ldots,N} \left[ \prod_{i=1}^{n} \mu_{A_{i}}^{*}(x_{i}) \mu_{B_{i}}(y) \right]$$  \hspace{1cm} (19)

| The amplitude deviation $e$ | The variation rate of deviation $\Delta e$ |
|-----------------------------|-----------------------------------|
| NB                         | NS | ZO | PS | PB |
| NB                         | PB |    |    |    |
| NS                         | PS |    |    |    |
| ZO                         | PS | ZO | ZO |    |
| PS                         | NS |    | NS | NS |
| PB                         | NB | NS | NS |    |

4) The clear operation. The method that the output of the fuzzy set is converted to accurate values is called clarity. Currently, there are several methods of clarity, such as the maximum membership maximum value method, the maximum membership minimum value method and the centroid method. The centroid method is applied in the fuzzy system due to the fact that it is widely used currently. The specific method is as follows, and the result is the exact output of the fuzzy controller.
y = f(x) = \frac{\sum_{k=1}^{N} y^k [\mu_{B_k}(y^k)]}{\sum_{k=1}^{N} [\mu_{B_k}(y^k)]} = \sum_{k=1}^{N} y^k \left\{ \max_{k=1,\ldots,N} \left[ \prod_{i=1}^{x} \mu_{A_i}(x_i) \mu_{B_k}(y) \right] \right\}

(20)

If \( A_i = y^k, \mu_i = \max_{k=1,\ldots,N} \left[ \prod_{i=1}^{x} \mu_{A_i}(x_i) \mu_{B_k}(y) \right] \), the above equation can be simplified as

\[ y = f(x) = \frac{\sum_{i=1}^{N} A_i \mu_i}{\sum_{i=1}^{N} \mu_i} \]

(21)

In the formula, the defuzzification value is \( y \), the output Fuzzy Set is \( \mu_i \), the fuzzy value is \( A_i \).

2.3 The design of control algorithm

Based on the above two kinds of controller design, the fuzzy self-tuning PID control algorithm is proposed in the paper and the specific steps are as follows:

1) The current vibration amplitude \( u \) is obtained by the vibration sensor, and the difference between the actual amplitude and the target amplitude \( \Delta u \) is calculated by comparing with the preset target value;
2) The three input signals of single neuron \( x_i(k) \) \( (i = 1, 2, 3) \) are calculated by the formula, then the weights of the input signals \( W_i(k) \) \( (i = 1, 2, 3) \) are obtained according to the improved supervised Hebb algorithm.
3) The output gain of the single neuron \( K \) is obtained by the fuzzy controller, after the single neuron obtaining \( K \), the final output is calculated, and that is the working time of the motor \( t \).
4) The balance operation of the amplitude and phase determine whether the vibration value is lower than the threshold. If the vibration value is lower than the threshold, the control algorithm ends. If not, the process repeat until reaching the goal set in advance. The detailed algorithm process is shown in Fig. 4.

3 Results and discussion

3.1 Simulation

In order to verify the effectiveness and superiority of the proposed control method, the rotor system is simulated and verified. The transfer function of the rotor system
is obtained by the system model in the previous part $G(s) = \frac{1}{23s + 1} e^{-3.37sx}$. In Simulink, the traditional PID control algorithm, single neuron PID algorithm (SNC) and fuzzy adaptive single neuron PID control algorithm (FSNC) are applied in the control system to verify the correctness of the method mentioned in the paper. In the traditional PID algorithm, the parameter is obtained by the trial and error method, $K_p = 0.01$, $K_i = 0.1$, $K_d = 1.75$. In the single neuron PID control algorithm, the initial weight is taken from PID, and $K = 0.02$. In the fuzzy auto-tuning single neuron PID control algorithm, the initial weight is also taken from the traditional PID, the gain $K$ is obtained by the fuzzy auto-tuning control output, the single neuron network is realized by S function. The simulation model is shown in Fig. 5. The simulation results are shown in Fig. 6.

By contrast, the overshoot of the single neuron PID control is significantly reduced compared to the traditional PID control, which is reduced from 38% to 8%, and the adjusting time is reduced from 55.3 s to 42.1 s. The overshoot of the fuzzy auto-tuning single neuron PID control is completely eliminated compared to the single neuron PID control, and the adjusting time from 42.1 s to 39.6 s. Therefore, the fuzzy auto-tuning single neuron PID control avoid overshoot and improve the control efficiency with respect to the other two ways.

In order to verify the robustness of each control method, the interference is introduced into the system when the system is running 200 s. The simulation results are shown in Fig. 7. The amplitude of the fuzzy auto-tuning single neuron PID

Fig. 5. The simulation model of the fuzzy auto-tuning single neuron PID control algorithm

Fig. 6. Simulation results of various control methods
control is only 1.5 um after the system is disturbed, while the amplitudes of traditional PID control and single neuron PID control were 2.38 um and 1.96 um. The fuzzy auto-tuning single neuron PID control quickly restore stability rapidly. Therefore the fuzzy auto-tuning single neuron PID control has good robustness.

3.2 Experimental verification

In order to verify the effectiveness and superiority of the proposed control method, the construction of the rotor system experiment platform is built as shown in Fig. 8. The vibration sensor is arranged above the bearing surface of the rotor center to obtain the change of the radial displacement of the rotor, and the balancing device is arranged at the end of the rotor to provide the unbalance compensation quality for the rotor. The controller is respectively connected with the vibration sensor and the balance device to obtain the vibration value collected by the vibration sensor, and the control signal is output to drive the balance device to compensate the unbalance.

In order to verify the balancing effect of the control method, the single neuron fuzzy auto-tuning PID control method and traditional PID control method were applied to the dynamic balance of the rotor system. The 20 experiments are carried out on the same initial conditions for the two control methods, the experimental results are shown in Fig. 9.

Comparison of the time reducing the initial vibration by 50% is shown in the Fig. 9(a). The method proposed in this paper can eliminate the initial vibration of 50% with an average of 3.9 seconds, while the traditional PID method can achieve the same effect for about 9.8 seconds; The final reductions of the vibration are
compared in the Fig. 9(b). Under the same initial conditions, the single neuron fuzzy auto-tuning PID control method can reduce the amount of vibration by 70% or so, while the traditional PID control method can only reduce about 55%. The experimental results show that the single neuron fuzzy auto-tuning PID control method is better than the PID control method.

4 Conclusion

From the result of experiments and simulation, the control efficiency is improved, and the overshoot phenomenon is avoided by applying the single neuron fuzzy auto-tuning PID control method. Therefore, the single neuron fuzzy auto-tuning PID control method has better balance accuracy and efficiency.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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