Entropic measures of individual mobility patterns

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Abstract. Understanding human mobility from a microscopic point of view may represent a fundamental breakthrough for the development of a statistical physics for cognitive systems and it can shed light on the applicability of macroscopic statistical laws for social systems. Even if the complexity of individual behaviors prevents a true microscopic approach, the introduction of mesoscopic models allows the study of the dynamical properties for the non-stationary states of the considered system. We propose to compute various entropy measures of the individual mobility patterns obtained from GPS data that record the movements of private vehicles in the Florence district, in order to point out new features of human mobility related to the use of time and space and to define the dynamical properties of a stochastic model that could generate similar patterns. Moreover, we can relate the predictability properties of human mobility to the distribution of time passed between two successive trips. Our analysis suggests the existence of a hierarchical structure in the mobility patterns which divides the performed activities into three different categories, according to the time cost, with different information contents. We show that a Markov process defined by using the individual mobility network is not able to reproduce this hierarchy, which seems the consequence of different strategies in the activity choice. Our results could contribute to the development of governance policies for a sustainable mobility in modern cities.

Keywords: scaling in socio-economic systems, socio-economic networks, stochastic processes

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1. Introduction

Human mobility has recently become a fruitful research field in complexity science, representing a paradigmatic example of statistical systems of cognitive particles. Thanks to the commercial spreading of the Information and Communication Technologies (ICT), it has been possible to collect large data sets on individual dynamics, offering the possibility of studying this complex system at different scales [1]. As a consequence, important steps have been made toward the characterization of individual mobility patterns: approaches in this direction have been made analyzing data from dollar bills tracking [2], mobile phone use [3]–[8], subway fare card transactions [9, 10], geographic online social networks [11] and private cars’ GPS [12]–[14] data sets. In fact, the differing features observed in the various data bases suggest different statistical behaviors or explanations. In most of the papers, the individual mobility is recorded in an indirect way (tracking bills) or in relation to other activities (internet access to a social network or use of a mobile phone) and the results may be influenced by the complex features the communication habits. Indeed, the suggested power law distributions for covered distances $p(l) \propto l^{-\gamma}$ ($\gamma \in (1, 3]$) and rest times $p(t) \propto t^{-\beta}$ ($\beta \in (1, 2]$) seem not to hold in urban contexts, where the mobility is limited by the energy consumption for the physical activity of daily travel [15] and is therefore dominated by short trips [11, 13, 16]. At the beginning of every new day, people wake up knowing that a series of tasks have to be carried out. Necessarily, they must satisfy the physiological need of eating, which represents a periodical constraint in the daily routine, and sleeping, which forces the circadian rhythm and thus imposes an end to the chain of performed activities. Besides, they have to perform duties, usually work related ones, that are precisely scheduled and others, such as shopping, social and leisure activities, that can be done at any moment during their free time. Changes of plan...
entropic measures of individual mobility patterns during the day are always possible: a planned meeting may be postponed, a new activity may be picked instead of another or something can be done just because some spare time is available. The accomplishment of the daily duties is realized traveling through the locations where each particular task has to be performed, and it is clearly the primary cause of human mobility. Therefore, understanding the structure of the daily activity pattern is a crucial step in the development of a model for human mobility, thereby leading to applications in fields like urban planning, traffic management, or epidemic spreading. But it can also contribute to a formulation of a statistical mechanics theory for complex systems. This theoretical goal requires three fundamental steps:

(i) to discover generic macroscopic behaviors (i.e. statistical laws) describing average properties of human mobility;
(ii) to characterize the individuals’ microscopic dynamics and its relation with the macroscopic properties;
(iii) to study the transient states driven by external changes and the existence of critical phenomena (i.e. phase transitions) due to interactions among individuals.

The first item has been considered by several papers and various statistical laws for human mobility have been proposed \[2\]–\[4\], \[13, 14\] trying to point out the common features and the differences with the physical models. These discoveries of statistical laws imply also the definition of relevant macroscopic observables representing the state of the system. More recently, the scientific community has also addressed the second item in order to understand the cognitive aspects at the base of individual mobility (i.e. mobility strategies) and to discover the primary causes of the macroscopic behaviors \[17\]. In contrast, the last item seems to be at present beyond the possibility of the complexity science.

Many questions are still open, in particular concerning the applicability of a statistical approach to individual mobility. The goal of this paper is to study the dynamical features of individual mobility that can be related to entropy measures and to contribute to the understanding of their relationship with the empirical statistical results on the use of space and time in vehicular mobility. In particular, we refer to Benford’s law \( p(t) \propto t^{-\beta} \beta \simeq 1 \) that seems to well describe the distribution of the time \( t \) between two successive trips when \( t \leq 4 \text{ h} \) and a power law \( p(k) \propto k^{-\alpha} \) with \( 1 < \alpha < 2 \) for the ranking distribution of the individuals’ most visited locations \[4, 14\] and we discuss how the structure of the individual mobility patterns can be at the base of such distribution laws. Mobility pattern entropy has been used in other papers to study the predictability properties of human mobility \[5\] and to suggest a classification of individual heterogeneity in order to define mesoscopic models that reproduce the macroscopic empirical laws. These results are based on mobile phone tracking datasets and they mainly concern the time mobility patterns of individuals (the time scale is \( \simeq 1 \text{ h} \)) which are dominated by long term activities (stay at home or work) related to the circadian rhythm. Focusing on urban mobility the previous analysis could hide the relevance of asystematic (random) mobility which characterizes many short activities that each individual performs in modern cities. In this paper we take advantage of a data base on single vehicle mobility where a sampling of each trajectory is recorded using a GPS system \[18\] in the whole of Italy. Approximately 2% of the entire vehicle population is monitored for insurance reasons and time, position and covered distance are recorded every 2 km or 30 s. Moreover, a signal is also recorded each time the engine is switched on or off. Even if one has no information on the vehicle sample for
privacy reason, this data base offers a unique opportunity to study individual mobility at a fine spatial scale on large urban areas and using long time series. More precisely in the sequel we analyze the GPS data base recorded in Florence during March 2008. In order to highlight the different roles that different duration activities have in defining the individual mobility patterns, we have chosen to use entropy as the key quantity for our study and we introduce methods of analysis different from those used in [5]. Indeed, for our purposes entropy is a good observable, because it expresses the information present in a sequence of characters (given by the statistical properties and the correlations due to the personal habits) in a real number. In particular, as the entropy per character in principle does not depend on the length of the sequence, we can then easily compare patterns of different length representing the mobility of different individuals.

The paper is organized as follows:

(i) in section 2 we discuss the main features of the GPS database used to detect the individual mobility patterns;
(ii) in section 3 we define the entropy measures considered in the paper and their properties;
(iii) in sections 4 and 5 we study the time regularity of individual mobility and we analyze the entropic properties of temporal and spatial patterns, suggesting possible explanations for the underlying dynamics;
(iv) in section 6 we propose a simple Markov model on a mobility network, which simulates the entropic properties of the empirical mobility patterns.

2. Data

In Italy, over 2% of the private vehicle population is equipped with a GPS system that tracks their path on the road network for insurance reasons. A database is recorded with geographical coordinates, time, velocity and path length of these individual trajectories sampled every 2 km, or alternatively every 30 s on highways. In addition, special signals are registered when the engine is switched on and off. If the quality of the satellite signal is good, the time precision of the recorded data is practically perfect and the space precision is of the order of 10 m. A filtering procedure has been applied to exclude positioning errors due to low signal. This database allows us to obtain extremely detailed information on human mobility since one has the possibility of directly following the people movements. In fact, we can easily and precisely define a trip as the displacement between two locations where the engine has been turned off for more than a given time threshold (we choose 5 min). In this paper we use data collected during the whole month of March 2008 in the municipality area of Florence. We study the urban mobility points of 2360 individuals among \( \approx 32,000 \) users. The selected individuals meet two criteria. The first criterion is that these vehicles have been used on each of the 20 working days of the considered month: we want to exclude occasional users. The second criterion is that there should not have been any major loss of signal. In fact, the GPS device loses the satellite signal frequently at starting points of the trajectories or when vehicles are parked inside buildings. We have then excluded all individuals who have had one or more trips where the origin lies in a different location then the preceding destination. We assume that every time the engine was off for more than 5 min this stop can be associated with an activity performed near
the parking place. From the cloud of all the parking points of a particular individual the different activity locations have been identified with a gravitational clustering algorithm. In this clustering mechanism, a maximum acceptable distance of 400 m [19] between the true destination and the parking place has been assumed. Once the locations are identified, each of them is associated with an identification number that, in analogy with the studies of the entropy of texts, we call character. Finally, we have isolated two types of mobility patterns for each individual. The first is the time pattern, where the month has been divided into equal time intervals of length $\Delta t$ and at each time interval we have associated an activity (in this case, traveling is considered as an extra activity and therefore is identified by a specific character). As it is possible that many activities have been performed in the same time frame, one activity has been randomly chosen in the case of conflict regardless of the activity’s relative duration. This procedure is the same as implemented in [5], allowing us to compare our results with precedent studies. We remark that time patterns built with time interval $\Delta t$ may neglect activities shorter than $\Delta t$, which disappear from the further analysis. The second type of pattern is the jump pattern that consists of the sequence of visited locations. According to our choice, a location is introduced in the individual mobility when the engine is switched off more than a time interval $\Delta t \geq 5$ min. Therefore activities that require a time shorter than $\Delta t$ are excluded from the sequences. If, after this exclusion, two or more identical characters appear beside each other, they collapse into a unique copy of the same character, so any repetition of the same character is neglected. Clearly, these two types of pattern carry different information. In the first case the main contribution to the mobility patterns is due to the long term activities. In the second case one focuses attention on the pattern distribution of different activities.

3. Entropy measures

Following [5], we consider three different information entropy measures for a mobility pattern: the information entropy $S$, the temporal-uncorrelated (Shannon) entropy $S_u = -\sum_{j=1}^{N} p_j \log_2 p_j$ and the random entropy $S_r = \log_2 N$, where $p_j$ is the probability of finding the character $j$ in our sequence and $N$ is the number of distinct characters in the sequence. $S_r$ represents a maximal value of the entropy, because to represent the mobility pattern it is necessary to have at least a number of characters equal to the total number of visited activities. $S_u$ is a better measure of the entropy, as it considers also the uneven frequencies of the characters in the sequence, while still ignoring the possible compression due to the relative position of the characters, which is taken into account by the information entropy. These three values are clearly bound by the relationship $S \leq S_u \leq S_r$. The measure of the information entropy per character $S$ has been estimated using a Lempel–Ziv (LZ) algorithm estimator. This algorithm searches for repeated sequences of characters that may be exploited for the data compression of the pattern. More precisely, for a sequence of length $n$ the estimated value of entropy is

$$S = \left( \frac{\sum_{i=2}^{n} l_i}{n \log_2 n} \right)^{-1},$$

where $l_i$ is the length of the shortest string starting at position $i$ that does not appear in the part of sequence up to position $i - 1$ (inclusive). The goodness of this estimate rises
Figure 1. The downtime distribution (blue dots) for the considered users in the area of Florence during March 2008. The downtimes are computed considering vehicle stops longer than 5 min. The red line suggests an interpolation with Benford’s law \( p(t) \propto t^{-\beta} \) \( \beta = 0.95 \pm 0.04 \) for activities shorter than 4 h.

with the length \( n \) of the sequences and decreases with the broadening of the alphabet size \( N \). The information entropy is related to the concept of predictability \( \Pi \), defined as the rate of correct predictions about the value of a character in the sequence knowing all the precedent characters. The more informative is a sequence the more difficult it is to predict how it may continue: a low entropy is related to a high predictability and vice versa. An upper bound \( \Pi^m \) to the predictability of a sequence can be computed as a function of the entropy \( S \) by inversion of the formula

\[
- \Pi^m \log_2 \Pi^m - (1 - \Pi^m) \log_2 (1 - \Pi^m) + (1 - \Pi^m) \log_2 (N - 1) = S.
\]

This last equation is a consequence of Fano’s inequality [20] and the inversion is possible when \( N \) and \( \Pi^m \) are not too small.

4. **Time regularity**

We have considered 28 days, so that we have four consistent records for each day of the week. We excluded two public holidays and a Sunday when daylight saving time was introduced. Because of daylight saving time, we have had to compensate by 1 h for the last two days of the month following that Sunday. The downtime distribution plotted in the figure 1 is well interpolated by Benford’s law \( p(t) \propto t^{-\beta} \) with \( \beta \approx 1 \) [13] (red line) up to times of a few hours, and three peaks corresponding to 4 h, 8 h and 9 h (partial and full work time) can be clearly seen, with a broader one around 12 h (rest time at home)\(^3\). This fat tail distribution indicates that the use of time during mobility is dominated by

\(^3\) This value may be explained by a logarithmic time perception, or alternatively if the daily schedule is created progressively, with the activities duration limited by the free time left in the timetable.
the average regularity entropy $S_u(t_i)$ measures the information lying in the distribution of the probabilities of finding a person in a particular place in a given time frame. Every day of the week has a characteristic hour-dependency. Monday: green squares; Tuesday: red stars; Wednesday: yellow crosses; Thursday: orange diamonds; Friday: cyan circles; Saturday: magenta down triangles; Sunday: blue up triangles. With equation (2), we can link this entropic measure with the upper bound to predictability. The dot-dashed lines represent the related values of $\Pi^u(S)$, calculated for $N = 2$.

Figure 2. The average regularity entropy $S_u(t_i)$ measures the information lying in the distribution of the probabilities of finding a person in a particular place in a given time frame. Every day of the week has a characteristic hour-dependency. Monday: green squares; Tuesday: red stars; Wednesday: yellow crosses; Thursday: orange diamonds; Friday: cyan circles; Saturday: magenta down triangles; Sunday: blue up triangles. With equation (2), we can link this entropic measure with the upper bound to predictability. The dot-dashed lines represent the related values of $\Pi^u(S)$, calculated for $N = 2$. 

For each time frame $[t_i, t_i + \Delta t]$ with $\Delta t = 20$ min, representing a moment in the day, we have counted how many times the drivers were at their most visited location at that moment. We use this for each user to compute the regularity $R(i)$, i.e. the probability of finding the user in his most visited location at the time frame $i$, whose distribution among the users is within the $79 \pm 9\%$ confidence interval, which is consistent with the result reported in [5] based on mobile phone data. However, considering only the most visited location does not take into account the complete information contained in the visited locations signal at each time frame $i$. An entropic approach can give a more comprehensive analysis. Thus we have calculated the Shannon entropy

$$S_u(t_i) = -\sum_{j=1}^{N} p_j(t_i) \log_2 p_j(t_i)$$

for the probability distribution $p_j(i)$ to find a driver at the location $j$ at each time frame $i$. The average values of $S_u(t_i)$ across all users for the different days of the week is shown in figure 2. We remark how every day has a peak of dispersion in the afternoon when one tends to perform leisure or social activities and has its minimum value late at night (when most people are at home). The entropy value in the morning during working days corresponds to a predictability between 75% and 80% which is consistent with the predictability of
Figure 3. Left: the rescaled distributions of the time patterns entropies $S$ with different time frames $\Delta t$ are perfectly superposed, suggesting that all the dependency upon $\Delta t$ lies in the average value $\langle S \rangle$. Right: similarly, the rescaled distributions of the jump patterns entropies $S$ with different threshold $\Delta t$ are superposed and also in this case all the dependency upon the threshold lies in the average values of the entropy.

the most visited locations: this may be an indication that people mobility is related to working activities in the morning.

As might be expected, Saturday shows the greatest variability with an average entropy $\langle S_u(t_i) \rangle$ of 0.66 bit/char, $\approx 15\%$ higher than the average entropy of working days. The working days are substantially similar within the standard error ($\pm 0.01$ bit/char), if we exclude a growing tendency to spend time in unexpected places during the evenings (and on Friday afternoon). As a consequence, the average entropies slightly but steadily grow (Monday: 0.53 bit/char, Tuesday and Wednesday: 0.56 bit/char, Thursday: 0.58 bit/char, Friday: 0.61 bit/char). On Sunday, the mobility appears to have a late beginning and thus, despite high values of entropy in the afternoon, the average value of 0.57 bit/char is similar to those of working days. The relative standard deviations, representing how the value of $S_u(t_i)$ changes during the day, have consistent values across all days within the range $[0.22, 0.23]$ bit/char.

5. Pattern analysis

As our data are extremely precise in time, we can afford to create time patterns and jump patterns with short time frame values $\Delta t \geq 5$ min. This extension is important to characterize the dynamical properties of individual mobility: e.g. are the short activities chosen randomly whereas long term activities are related to individual habits according to the circadian rhythm? Might the short activities justify the applicability of a statistical physics distribution to human mobility? First we consider the dependence of the entropy distribution on the time frame threshold $\Delta t$ used to define the mobility patterns. In the figure 3, we report the distribution of the normalized entropy $S/\langle S \rangle$ using different time frames both for the time patterns and for the jump patterns. The results, regarding time frames ranging from 5 to 60 min, suggest the existence of a universal probability
distribution $p(x)$ which represents the entropy distribution of the mobility patterns according to

$$P(S; \Delta t) = \frac{1}{\langle S \rangle(\Delta t)} p\left(\frac{S}{\langle S \rangle(\Delta t)}\right).$$

(3)

Assumed that when decreasing the threshold $\Delta t$ we add new activities in the patterns and an increase of the entropy value is expected, the dynamical meaning of equation (3) is that the structure of the mobility patterns does not depend on the activity duration and we expect a self-similarity in the patterns at different time scales [21]. Since the only significant value representing the dependence of the distribution $p(S)$ on $\Delta t$ is the mean value $\langle S \rangle$, in the following we will focus on the study of the mean values of the entropy. In figure 4 we plot the dependence of average entropy values from the time frame size for the three entropy measures considered. The left panel concerns the time mobility pattern and it is remarkable to observe that the average Shannon entropy $\langle S_u \rangle$ is independent of $\Delta t$, indicating that the relative frequencies of the time patterns remain almost constant. Indeed this is a consequence of Benford’s law $p(t) \propto 1/t$ which implies that frequency of the activities of duration $t$ is $\simeq t/\Delta t$ with respect to a total length $T/\Delta t$ so that the relative frequency is $t/T$. In contrast, the information entropy decreases due to the weight of the long term activities which increases the persistence in the patterns: i.e. the probability of continuing an activity up to $t + \Delta t$ given that the activity $t$ has been carried out for a time $t$. A rough estimate of the persistence based on Benford’s law gives

$$\pi(t + \Delta t/t) \propto 1 - c \ln\left(1 + \frac{\Delta t}{t}\right),$$

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where \( c \) is a suitable constant so that \( \pi(t + \Delta t/t) \to 1 \) when \( t \) increases. Therefore we expect that the entropy \( S \) tends to zero for short time frames. Indeed, most of the new characters introduced shortening \( \Delta t \) are only prolonging repetitions of the same character representing long stops. Even if we introduce new information regarding shorter activities into our analysis, the analyzed sequences will have more and longer series of iterated characters, which can be easily compressed and therefore lower the value of entropy per character.

This argument does not apply to the jump patterns where the increase of the average Shannon entropy with \( \Delta t \) is a direct indication that the number of activities in the pattern is increasing (see figure 4). We also remark that the values of \( S_u \) and \( S \) are greater in the jump patterns, as we have neglected all the repetitions that were dominating the distribution of character frequencies and are easily compressible. Moreover, the Shannon entropy measure \( S_u \) can be related to the ranking distribution of the most visited locations \( p(k) \propto k^{-\alpha} \) which has been proposed by various authors [4, 14] in mobility data. A simple computation gives

\[
S_u = -\sum_k p(k) \log_2 p(k) \simeq -\frac{\alpha-1}{\ln 2} \int_1^\infty k^{-\alpha} \ln(\alpha - 1) k^{-\alpha} \, dk = \frac{1}{\ln 2} \left( \frac{\alpha}{\alpha - 1} - \ln(\alpha - 1) \right).
\]

A value \( \alpha \simeq 1.7 \) is in accord with the GPS observations in Florence [13] and the previous crude estimate gave \( S_u \simeq 4 \) to be compared with \( S_u \simeq 3.4 \) obtained by direct calculation (see figure 4).

5.1. Time patterns

The threshold value \( \Delta t = 60 \text{ min} \) represents the same experimental conditions considered in the paper [5] for mobile phone users’ mobility. The reported results show an upper limit to the predictability of 93% which is in remarkable accord with the value of \( \Pi^m = (92\pm3)\% \) that we can derive from our measures of \( S \). The validation of this result on an independent, and rather different, data set indicates that mobility patterns obtained from both phone calls and private cars’ GPS data sets are a good representation of the human mobility with a temporal scale of one hour. But, using a time frame of this size, we are excluding a great part of the mobility, as 41% of the stops during urban mobility are shorter than one hour. As previously remarked, the information entropy tends to zero when \( \Delta t \to 0 \) (see figure 4). A numerical interpolation of the empirical data suggests that \( S \) follows a scaling law \( S(\Delta t) \propto \Delta t^\beta \), where \( \beta = 0.75 \pm 0.03 \). We will show that this scaling law is a consequence of Benford’s law for downtime distribution. We have generated synthetic sequences constituted by only by two characters (0 or 1). This choice permits us to reduce the errors of the LZ estimator, which converges more slowly to the real value of \( S \) for larger numbers of characters in the used alphabet. A random character is picked and is repeated \( r \) times, with \( r \) distributed as \( p(r) \propto r^{-1} \). Then another random character is picked and repeated, and so forth. Sequences of this kind, in which total length is equal to the total number of minutes in a month, have been generated and shorter sequences, corresponding to the different time frame \( \Delta t \), have been derived from the original sequences. The information entropy \( S_s \) of these shorter sequences has been estimated with the LZ algorithm and it turns out that the results are well interpolated by a scaling law \( S_s(\Delta t) \propto \Delta t^{\beta_s} \) where \( \beta_s = 0.73 \pm 0.03 \) (see figure 5).
Figure 5. The scaling of $\langle S \rangle$ can be explained by measuring the entropy of Monte Carlo simulated sequences that share the same Benford law distribution of the length of repeated characters. Thus, the fact that the entropy tends to zero for small time frames is only due to the progressive growth of length of the sequences of repetition of the same character that describes the same activity.

As a consequence, time patterns are not convenient to study human mobility related to short activities unless one scales the spatial resolution together with the time resolution to avoid these repeated characters. Unluckily, our data do not permit this fine analysis, and therefore we cannot extract any activity time related feature from our time patterns. However, we introduce a different approach to this issue by considering the jumps patterns.

5.2. Jump patterns

We have seen that the information entropy of the time patterns is unsuitable for describing the structure of the human mobility at short time scale since the statistical distribution of the stop duration plays the leading role in determining the value of $S$. In order to distinguish the structure of the transitions from one location to another, we have analyzed the jump patterns, where only movements are considered.

We consider again values of $\Delta t$ ranging from 5 to 60 min, representing the time threshold under which stops are excluded from the pattern. Larger thresholds can hardly be considered as that would produce strings too short to be conveniently analyzed with the LZ algorithm. Also in this case, we focus attention on the dependence of entropy mean value on the time threshold since the distribution of the normalized entropy measures is independent of $\Delta t$ (see figure 3). From figure 4 we observe that shortening the time frame and thus introducing new characters, all the three measures of entropy rise. This is a straightforward consequence since by introducing new characters we introduce new
Figure 6. The compression ratio $\langle S \rangle / \langle S_r \rangle$ has no dependency on the threshold value. This suggests a time invariant structure of the correlations including activities shorter than 1 h.

The compression ratio is the measure of the maximum possible compression that can be performed on the string (it is equivalent to the ratio between the size of a zipped file and the size of the original file): for the considered values of $\Delta t$ in our analysis, its value lies between 52.6% and 53.6%. This fact clearly indicates that the short activities within the range 5–10 min are as compressible as the long ones within the range 50–60 min, since they have the same chance of being part of a repeated sequence that the LZ estimator uses for the compression. As a consequence, we conjecture that the mobility patterns are somehow invariant with respect of the rescaling of activity time, within the analyzed range (5–60 min). A similar observation has been considered in the paper [21]. To verify this assumption, we proceed with a different time-dependent entropic analysis. We consider a shuffling procedure for the jump patterns generated with a time frame $\Delta t_{\text{min}} = 5$ min, by randomly permuting the activities shorter than a given value $\Delta t_s$ and comparing the measured values of entropy $S$. This procedure breaks the repeated sequences that the LZ algorithm finds and uses for compressing the pattern, so that the correlations between consecutive groups of characters is lost and the value of $S$ increases. In the limiting case where all the values are shuffled, the measured value of $S$ is, in principle, equal to $S_u$. Using the shuffling procedure instead of excluding activities allows us to analyze the activity time dependency of the correlations without changing the length of the patterns.

Translated in maximum predictability through the inversion of equation (2), the values vary from 66% to 71%.
Performing a progressive shuffling of characters representing activities of which duration is under a threshold $\Delta t'$, we progressively break the correlations in the jump patterns. It can be seen that the average difference in entropy $\Delta S$ between the shuffled and un-shuffled patterns grows with the average number of permuted characters with three different slopes. The blue circles represent values of $\Delta t' \leq 115$ min, green triangles $115$ min $< \Delta t' \leq 12$ h and red diamonds $12$ h $< \Delta t'$.

For this reason, it is possible to extend the range of values $\Delta t_s$ considered in the analysis. We have computed the values of $S$ with $\Delta t_s$ spanning from 5 min to 24 h. In figure 7, we plot the average difference in entropy $\Delta S(\Delta t_s) = S(\Delta t_s) - S(\Delta t_{\text{min}})$ within the sample as a function of the average number of permuted characters $N_s$ with the same $\Delta t_s$. The greater is the slope of the curve $\Delta S(N_s)$, the faster is the variation in entropy due to the shuffling, and therefore the stronger is the breaking of the correlations due to the shuffling procedure. It is remarkable that this curve can be easily divided in three segments. The best fit with a multiple linear interpolation gives the temporal limits of these parts: the first part is limited by a time frame $\Delta t_s \leq 2$ h, the second part by $2$ h $< \Delta t_s \leq 12$ h and the last part by $\Delta t_s > 12$ h. Therefore, when we start shuffling the activities longer than 2 h with the shorter activities, the increase rate in the entropy measure changes suddenly (the derivative of the second segment is roughly one half of the derivative of the first one). Finally, when even activities longer than 12 h begin to be included in the shuffling, the entropy variation becomes negligible. These three behaviors in the information entropy suggests a possible classification of activities into three categories according to their time demand: we call them respectively short, long and very long activities. Moreover, the fact that the information introduced with shuffling is proportional to the number $N_s$ of permuted characters indicates that each class is internally homogeneous. Each character of any value $\Delta t$ within a given class brings the same amount of information when moved.

Given that this result was obtained, for each pattern, with a fixed string length and a fixed number of different characters, we can exclude that this result could be a consequence of some bias due to the LZ entropy estimator.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7.png}
\caption{Performing a progressive shuffling of characters representing activities of which duration is under a threshold $\Delta t'$, we progressively break the correlations in the jump patterns. It can be seen that the average difference in entropy $\Delta S$ between the shuffled and un-shuffled patterns grows with the average number of permuted characters with three different slopes. The blue circles represent values of $\Delta t' \leq 115$ min, green triangles $115$ min $< \Delta t' \leq 12$ h and red diamonds $12$ h $< \Delta t'$.}
\end{figure}
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Figure 8. Left picture: if we shuffle random characters belonging to the same class of activity ($\Delta t \leq 115$ min: blue circles, $115$ min $< \Delta t \leq 12$ h: green triangles, $12$ h $< \Delta t$: red squares) the most effective way to break the correlations between consecutive stops is to shuffle activities of intermediate length. Right: the distribution of the Shannon entropies given by the frequencies of the characters in the different classes ($\Delta t \leq 115$ min: blue dot-dashed line, $115$ min $< \Delta t \leq 12$ h: green dashed line, $12$ h $< \Delta t$: red solid line). The short and long activities provide a Gaussian-like distribution for the Shannon entropies, but the average value of the long activity is smaller denoting a lesser variability of these activities in the jump patterns. The very long activities tend to have less spread in the distribution of frequencies. The peak at $S = 0$ for the activities over 12 h indicates that frequently only one location (probably home) is visited for such a long time.

from its original position. This result is consistent with the invariance of the compression ratio changing the exclusion time threshold $\Delta t$ plotted in figure 6 for the activities in the range 5–60 min.

To avoid any consequence of the progressive shuffling with a threshold going from short to long activities, we have analyzed them separately. We have shuffled a progressive number $N_s$ of characters picked at random within each group, and we measure again the variations in the information entropy. This analysis has been performed on a set of $\approx 3800$ jump patterns where each individual has performed at least 15 activities (not necessarily different) for each class. The results, shown in figure 8 (left), show that shuffling the characters representing long activities increases the entropy faster than shuffling the characters of short activities. The very long activities have the smallest derivative, as we could have expected, as only a few locations are visited for such a long time, thus we often shuffle identical characters. Comparing figures 8 (left) and 7 we remark that if we shuffle first the short activities and then the long activities, the rate $\Delta S/N_s$ relative to the long activity part is smaller than the rate relative to the short activities, whereas if we start by shuffling the long activities, the rate is bigger. Besides, if we proceed with the shuffling of the very long activities after having shuffled the short and the long, the shuffling is ineffective, while the shuffling inside the class has a finite increasing rate for the entropy. Therefore the structure of mobility patterns involving long activities seems to contain more information than that of the short and very long activities, the last
mainly representing the circadian rhythm that introduces periodic structures in the jump patterns. This interpretation is confirmed by the Shannon entropy distributions calculated using the frequencies of characters of the three classes (see figure 8, right). The curve representing the long activities has its values concentrated under 1 bit/char, with a peak at zero, which means that we usually have only one character (probably the overnight stops at home). Comparison of distributions for the short and long activities shows that the variability in the choice of short activities is larger than that of long activities, since the $S_n$ average value of the last ones is smaller. This suggests that the different slopes in the entropy increase due to the shuffling of these classes (figure 8, left) is very significant, as the long activities have a faster rate of entropy growth with the character permutations, even if it is more probable that these permutations are ineffective because of the repetition of the same character. The previous results suggest the existence of a hierarchical structure in the mobility patterns. This hierarchy appears in the repeated sequences that the LZ algorithm recognizes for the compression. Those repeated sequences, representing individual habits, are more easily broken if we shuffle the long stops than if we shuffle the short ones. But if we shuffle first the short activities, shuffling then the long activities becomes less efficient in breaking these sequences. In our opinion, this can be explained by the assumption that a significant part of the repeated sequences are constituted by clusters of activities where a long activity plays a pivotal role between the short activities. Shuffling the long activity breaks more efficiently those sequences, whereas if we first shuffle all the short activities, then shuffling the long activities has less strong consequences. The very long (>$12 \text{ h}$) activities play the role of central nodes, which act as cornerstones of the mobility schedule. Around them repeated clusters are formed, dominated by long activities, which may correspond to daily tours [8] or part of them. We remark that this interpretation is consistent with the time usage model in which the daily schedule is created progressively, starting from the activity with the long duration and progressively using the time left in the timetable [13].

6. A simple Markov model on individual mobility networks

We finally evaluate whether the information of the observed jump patterns can be successfully modeled with discrete-time Markov processes on individual mobility networks. With each individual we associate a mobility network whose nodes are the different locations linked according to the performed trips. We have considered both from a topological point of view using the undirected adjacency matrix $a_{ij}^n$ and introducing a weight proportional to the number of performed trips for each link $w_{ij}$. Then we have computed the information entropies of the patterns generated by the Markov process, whose transition probability matrix is computed from the network adjacency matrix

$$p_{i \rightarrow j}^t = \frac{a_{ij}^n}{\sum_k a_{ik}^n}, \quad (4)$$

or from the weight matrix

$$p_{i \rightarrow j}^w = \frac{w_{ij}}{\sum_k w_{ik}}. \quad (5)$$
If the first process is able to reproduce the observed information entropy, then it is the topological structure of the mobility network that carries the information to describe the individual mobility. In contrast, one has to take into account decision strategies of individuals that are represented by the weights $w_{ij}^u$. For each individual network, 10 patterns of 2000 characters have been generated to reduce both the statistical error and the systematic error due to the entropy estimator. The values of entropy found are plotted in figure 9 against the entropies measured for the empirical patterns. The results show that for low-entropy value, the Markov process with the transition matrix (5) generates patterns considerably more similar to empirical ones whereas the patterns of the Markov process based on the transition matrix (4) show a systematic deviation. However, when one considers highly informative patterns, both cases present significant differences from the empirical entropy measure. These differences can be caused by the limited length of the empirical mobility patterns, which is probably insufficient for the convergence of the LZ estimator to the real value of $S$ when the number of different characters is high. In figure 10 (left) we plot the standard error of the two Markov models as a function of the information entropy of the mobility patterns.

The numerical results confirm that the Markov process defined by the transition matrix (5) can be used to explain the observed entropy measures for individual mobility patterns in a wide range of values. Finally, using this model we have introduced a progressive rewiring in the multigraph [22] by exchanging the trips between the nodes and redefining the weights $w_{ij}^u$, in order to compare the entropy increase with the results of the shuffling on the mobility patterns (see section 5.2). The simulations plotted in figure 10 (right) show an initial linear trend with a slope (0.014 bit/char) that lies between the empirical slopes measured for short activities ($\Delta t_s < 2$ h (0.010 bit/char)) and long activities ($2$ h $< \Delta t_s < 12$ h (0.015 bit/char)) (see figure 8 left). This result verifies that the information carried by the weighted mobility network is sufficient to describe the information entropy of the dynamical process that has generated it, but it does not
take into account the hierarchial structures in the patterns according to the activities duration—which are suggested by the time-shuffling study of empirical patterns.

7. Conclusions

The analysis of entropy measures of mobility patterns from GPS signals in urban context (Florence) shows that it is possible to identify three different classes of activities: short activities $\Delta t < 2$ h, long activities $2$ h $< \Delta t < 12$ h and very long activities $12$ h $< \Delta t$. Each of these categories is homogeneous and takes part in different ways in the structure of the individual mobility patterns. The homogeneity within the short activities is confirmed by the constancy of the compression ratio observed in the jump pattern analysis. Very long activities can be identified with the overnight stop at home, together with only a few other infrequently taken alternatives. This daily return to home constitutes the base structure of the individual mobility pattern. Short and long activities are performed according to the circadian rhythm, but among them exists a hierarchical relationship, in which long activities tend to play a more central role than short activities. Long activities are most likely planned before short activities, thus playing a pivotal role for them. In fact, short activities arrange themselves around the long (or very long) activities, to form sequences that are then repeated on different days [8]. However, in contrast to what one might expect, short activities do not seem to be executed randomly. Indeed, the homogeneity in the classes indicates that even the shortest ones are eventually able to concur in a systematic mobility, indicated by the formation of repeated sequences. A recent study [7] proposes that the spatiotemporal structure of human mobility patterns can be flow-wise

Figure 10. Left: standard deviation of the generated pattern entropies from the empirical ones. The blue curve represents deviations of patterns generated with $p_{w}^{i\rightarrow j}$ while the red one represents deviations of patterns generated with $p_{t}^{i\rightarrow j}$. From this graph it can be seen that the first patterns are in good correspondence for a much wider range of values of entropies, while in the case of highly informative patterns both processes fail to emulate reality. Right: effect of progressive rewiring of the individual networks in the values of pattern entropies generated by the transition probabilities (5). For a small number of rewired links, the variation grows linearly, as seen for the empirical patterns in figure 8.
partitioned into groups of related nodes called habitats, and that those habitats tend to be more spatially cohesive than the total mobility. This suggests that the repeated sequences may reflect this habitat structure and thus be geographically related. The individuals have to optimize their daily schedule, as their mobility is limited by the mobility energy [15, 14], and therefore activities that can be performed in near locations are most likely to be made in a chain. The reason for the threshold the values 2 h and 12 h requires additional information. Probably the value of 12 h is related to the circadian rhythm, whereas the value of 2 h (or, more exactly, 115 min) may instead be a duration characteristic of the Florence area. Finally, we have shown that the information entropy measures of the mobility patterns can be reproduced by a Markov process whose transition probability matrix is computed using the empirical weights of the undirected individual mobility networks.

Further analysis on activity patterns of individuals living in different cities might lead to a better understanding and a generalization of these results.

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