Why is the three-nucleon force so odd?

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By considering a class of diagrams which has been overlooked also in the most recent literature on three-body forces, we extract a new contribution to the three-nucleon interaction which specifically acts on the triplet odd states of the two nucleon subsystem. In the static approximation, this \(3N\)-force contribution is fixed by the underlying \(2N\) interaction, so in principle there are no free parameters to adjust. The \(2N\) amplitude however enters in the \(3NF\) diagram in a form which cannot be directly accessed or constrained by \(NN\) phase-shift analysis. We conclude that this new \(3N\)-force contribution provides a mechanism which implies that the presence of the third nucleon modifies the \(p\)-wave (and possibly the \(f\)-wave) components of the \(2N\) subsystem in the triplet-isotriplet channels.

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I. INTRODUCTION

The three-nucleon system represents an ideal testing ground for the nucleon-nucleon interaction \([1]\). This testing represented a challenge which required a great deal of efforts amongst various research groups active in this area in order to provide a comparison between extremely reliable calculations and precise measurements. In this field there have been four main areas of research which proved to be crucial for the progress in our understanding of the three-nucleon problem. These four areas are: 1) \(NN\) Phase-shift analysis. 2) \(2N\) potentials. 3) \(3N\) calculations. 4) Experiments on few-nucleon systems. Recent advancements on these topics can be found in Ref. \([2]\).

The combined result of all these studies and efforts has revealed that the presence of the third nucleon modifies the interaction between the remaining two. A serious difficulty must be faced at this point: namely, how can we conveniently describe the modifications of the \(NN\) force operated by the third nucleon? Most likely, we cannot avoid ambiguities in the description of the interaction modifications by the third nucleon, since the two-nucleon amplitudes enter off-the-energy-shell in the three-body quantum mechanical equations. And it is known \([3]\), that it is possible to introduce (maybe, unrealistic) modifications in the \(NN\) off-shell structure that mimics the interaction effects due to the presence of the third nucleon. The lesson we must learn from this is that it is not sufficient to generate a phenomenological \(NN\) potential that perfectly reproduces the most up to date phase-shift analysis. One must also generate a \(NN\) potential by using theoretical insight as much as possible in order to constrain the off-shell properties of the \(NN\) amplitude. If this is not the case, a \(NN\) potential which fits precisely the experimental data but with the erroneous off-shell behavior would not provide reliable results for the \(3N\) system, nor can be used to test for the presence of \(3N\) forces.

On the other hand any realistic \(2N\) potential underbinds the triton and this is the first signal that three-body nuclear forces do play a role, and that the off-shell ambiguities are not so dramatic (if the \(2N\) potential is theoretically constrained in its one-pion-exchange term and one assumes the requirements of minimal momentum dependence and/or non-locality). The off-shell ambiguities can be seen by the fact that one must adjust separately with each \(2N\) potential the parameters of the \(3N\) force in order to reproduce exactly the experimental binding energy of the triton \([4]\) and this suggests that \(2N\) forces and \(3N\) forces must be generated consistently, within the same theoretical scheme.

Aside for the problem with the triton underbinding, other signals for possible evidences of \(3NF\) effects must be sought in the \(3N\) continuum. One signal arrived recently from the study of the unpolarized differential cross section for \(nd\) elastic scattering between 60 and 200 MeV. Continuum calculations with \(2NF\) underestimate the minimum by a 30\% effect (the Sagara discrepancy). Rigorous Faddeev calculations with the inclusion of \(3NF\) completely solve the discrepancy between 60 and 140 MeV \([5]\). A coupled-channel calculation with the explicit treatment of the \(\Delta\) isobar excitation which leads to effective \(3NF\)'s in the three-nucleon subsystem reduces the Sagara discrepancy by a considerably large fraction \([6]\). Useful insights about the importance of possible \(3NF\)-diagrams should also be obtained from the study of pion production/absorption mechanisms in the \(3N\) system, since progress has been recently made in the theoretical treatment of these reactions at the \(\Delta\) resonance \([7,8]\) and in the threshold region \([9]\).

At the present stage, however, the existing \(3NF\) models achieved only a limited success in explaining the discrepancies between theory and experiments, and it is possible that the spin-isospin structure of the full three-nucleon force is not well understood, yet. Indeed, as is well known \([10]\), the comparison between theory and
experiments reveals that existing 3N forces do not provide the correct structure of the vector analyzing powers, both for the proton, $A_{y0}$, and for the deuteron case $i T_{11}$, while the deuteron tensor polarization observables are described reasonably well. This has been evidenced also for $pd$ scattering below breakup threshold [10], where variational techniques based on the Pair Correlated Hyper-spherical Harmonic method allowed to incorporate the effects of the Coulomb interaction. The situation is still the very much same in these days as has been pointed out also in the most recent Conference on Few-Body problems, held in Taipei in March 2000 (FBXVI).

This puzzling situation about the vector analyzing powers has been carefully analyzed in two recent publications [11,12]. In Ref. [11] it was concluded that, unless 3NF of new structure could be envisaged, the $NN$ interaction in the $^3P_J$ states has to be modified in an energy dependent way (i.e. only for energies lower than 20 MeV). The study implicitly assumes that in this energy region the results from modern phase-shift analysis [13] could be possibly corrected in the triplet $p$ waves without affecting appreciably the $NN$ data. In Ref. [12] all the possibilities for solving the problem at the level of the two-body interaction (especially introducing modifications in the $^3P_J$ channels) have been attentively investigated and then ruled out, with the conclusion that the only viable solution to the puzzle of the vector analyzing powers must come from a new 3NF contribution which has not yet been taken into account. The authors of Ref. [12] suggest a 3NF of the spin-orbit type, as a possible candidate.

In this paper we discuss the dynamical mechanism which generate a new component to the three-body force. This mechanism has been obtained starting from a formalism [14] developed for the treatment of the pion dynamics in the 3N system. By projecting out the pion degrees of freedom from that formalism one obtains a 3N dynamical equation of Alt-Grassberger-Sandhas (AGS) type [15], which incorporates the spin-off of the pion dynamics beyond that already considered in the 2N interaction. It has been shown [16] that the new pionic terms of the 3N equation can be interpreted as irreducible diagrams, contributing to the construction of a 3NF. Herein, we are mainly interested in one specific 3NF mechanism, which implies an intermediate 2N-cluster formation while a pion is “in flight”. Since $2\pi$ mesonic retardation effects do play a role here, we analyzed this role, and found that they merely act as counter terms, to be subtracted because of the presence of a known cancellation effect [17,18]. This cancellation effect is correctly taken into account in the construction of all modern 3NF’s [19]. The novelty of the approach presented herein is that we sized the effect of this cancellation more precisely, by allowing the mechanism to adjust to the effects of the nuclear medium. This was possible only because we started from an explicit treatment of the pion degree of freedom, while it would have not been possible to see the effect within the more common “instantaneous” approaches to the 3N force. The nature of these dispersive effects is well known in approaches devoted to the explicit treatment of the $\Delta$ degrees of freedom [20], however a discussion of the same effects in the presence of an explicit treatment of the pion dynamics can hardly be found in the literature.

From this new 3NF contribution we have extracted in a very natural way a specific component which acts only in the (triplet) odd waves of the 2N subsystem. We provide also the first derivation of the partial-wave expansion of this piece of 3N force, which can be used in current 3N calculations. The spin-isospin structure of this 3N force implies that at low energies the presence of the third nucleon modifies the 2N subsystem in the $^3P_J$ channels, and we suggest that this might be another possible candidate for explaining the inconsistencies registered between theory and experiments at low energies, at least in those cases most sensitive to the triplet $p$-waves, such as $A_{y0}$ and $i T_{11}$.

II. IRREDUCIBLE 3N-FORCE DIAGRAMS

The diagrammatic analysis discussed in this section is based on the systematic method developed in Ref. [14] to take into account the pion dynamics into the 3N system. The method has been originally designed for the treatment of the 3N dynamics above the pion threshold and represents an appropriate, connected-kernel, generalization of the standard Faddeev-AGS three-body equations [15,21] for the explicit inclusion of one meson degree of freedom. In the $\pi-3N$ space, the equations are labelled in terms of modified Yakubovskij-type chains of partitions. In the 3N (no-pion) sector, the labelling structure leads to Faddeev-type components. Specific rules are provided on how the pion-nucleon vertex interaction couples the two sectors. By recursive application of the quasiparticle/separable-expansion method, a modified 3N AGS-quasiparticle equation is obtained where the explicit pion dynamics is built-in. This approach has been subsequently considered in Ref. [16] where an approximated, practical scheme for the solution of these equations has been designed. To the lowest order, the approximation scheme consists simply in replacing the inelastic components of the $NN$ subamplitudes with their leading terms, represented by suitable combinations of the pion-nucleon vertex interaction. By setting to zero also these leading terms for the pion inelasticities in the $NN$ subamplitudes, the $NN$ subamplitudes become totally elastic and in this limit [14] one re-obtains the standard 3N Faddeev-AGS equation with 2N interactions, plus a completely disjoint standard Yakubovskij-GS equation [22,23] for the $\pi$-3N sector. In this limit, the input
for the two separated three- and four-body equations are the fully elastic $NN$ and $\pi N$ $t$-matrices.

In Ref. [15] it is discussed how to project out the pion degree of freedom from the treatment of Ref. [14]. The result of this procedure, i.e., cooling down the pion from the theory, can be recast in a very appealing way if one uses a finite-rank expansion of the elastic $NN$ $t$-matrix. There are methods to generate these expansions, such as the Ernst-Shakin-Thaler method [24], and with these methods very reliable and accurate separable expansions have been generated and tested [25–27].

The new 3$N$ equation incorporating the pion dynamics has the standard AGS form [15]

$$X_{ab} = Z_{ab} + \sum_c Z_{ac} \tau_c X_{cb}, \quad (2.1)$$

where $a$, $b$, and $c$ run over the three Faddeev components of the 3$N$ system, and the only modification refers to the driving term, which can be separated into the following structure

$$Z_{ab} = Z_{ab}^{AGS} + Z_{ab}^{3N}. \quad (2.2)$$

The first contribution is, literally, the standard AGS driving term, which many groups have been calculating for years, while the second term represents the spin-off of the pion dynamics beyond that already contained in the 2$N$ interaction. Following Ref. [15], it is possible to analyze all the pion-exchange diagrams contained in $Z_{ab}^{3N}$ and it is found that they all correspond to irreducible 3$N$ diagrams, and this establishes a link between this $\pi$-3$N$ approach and those formalisms considering irreducible 3$N$ diagrams as generators of 3$N$ interactions.

The diagrams that emerged from the analysis of $Z_{ab}^{3N}$ can be classified according to their topological structures, and we can easily recognize irreducible diagrams that are well known. But we obtain also other diagrams which have been overlooked, to the best of our knowledge.

FIG. 1. Examples of well-known irreducible 3$N$F diagrams. The Fujita-Miyazawa diagram (top) and the non-polar $\pi N$ rescattering diagram (bottom).

The first class of diagram is represented by the diagrams reported in Fig. 1, which describes a pion rescattering by a third nucleon while being exchanged between the other two. As a matter of fact, such a $2\pi$-exchange structure of the 3$N$ potential has been the only one considered in all modern calculations. This has been pointed out also very recently by Friar et al. in Refs. [28,29]. On the upper part of the figure it is shown the Fujita-Miyazawa $\Delta$-mediated interaction [30], which represents the prototype of this topological structure, and accounts for an important fraction of the generic pion rescattering process. In the lower part of Fig. 1 the more general pion-rescattering process is exhibited. The “blob” represents the $\pi N$ amplitude, where one must subtract its polar part (corresponding to a nucleon propagating in the forward direction) to avoid double counting with the nucleonic multiple-scattering contributions, since these last are summed up to all orders in the dynamical $3N$ equations. The term which must be subtracted is shown in Fig. 2 and is called sometimes the (reducible) Born term. The existing 3$N$ potentials differ mainly in the model calculation of the $\pi N$ $t$-matrix, whether it is constrained by current algebra and PCAC [31], or inspired by an effective meson-baryon Lagrangian constrained by chiral symmetry and current algebra [32], or determined by effective field-theoretic methods involving light-meson dynamics [33], or by the more systematic method of $\chi$PT [34], or finally by the form envisaged by the Fujita-Miyazawa term [35]. Since such contributions (and their short-range corrections) have been the subject of very extensive studies, we really have nothing to say in addition and will skip to the next classes.

The structure of the second class of diagrams obtained by cooling down the pion from the $\pi$-3$N$ approach has been overlooked in all modern force calculations: it corresponds to the graph represented in Fig. 3. This describes a complete correlation between one of the two nucleons exchanging the meson and the third one while the pion is “in flight”. These diagrams should not be confused with those originated by pure mesonic retardation effects, analyzed and evaluated many years ago in a sequence of papers, in Refs. [36–38].

As has been shown later in Refs. [17,18], those dia-
grams contributing to the 3NF with pure mesonic retardation effects must be excluded because they cancel out against the corresponding contributions of pionic retardation effects arising from the Born term. This cancellation is correctly taken into account in the construction of modern 3NF contributions \[29,19,39\] and occurs also when considering 3NF retardation effects coming from the exchange of heavier mesons \[33,17\]. To take into account the effects of this cancellation we subtract from the diagram in Fig. 3 the 2nd (irreducible) Born diagram shown in Fig. 4.

The reason for this subtraction can be understood in the following way: The diagram in Fig. 3 considers an irreducible contribution wherein the pion propagates while a 2N subsystem clusterizes. Obviously, such an effect does happen above the pion threshold, since the reactions \(Nd \rightarrow Nd\pi\) are observed; the problem is to determine up to what extent this mechanism is relevant at lower energies, where the \(N + (NN) + \pi\) channel is asymptotically closed but may still be important as an intermediate state. The 3NF mechanism of Fig. 3 is dynamically more complete than the one shown in Fig. 4 which has been demonstrated to cancel out against the mesonic retardation corrections of the twice iterated Born term of Fig. 2 (see Refs. \[17,34\] for details). Therefore, it is the difference between the two diagrams in Fig. 3 and 4 that survives from the mesonic retardation effects and generates a new 3NF contribution. Had we replaced the full two-body t-matrix in this diagram by the input potential (this corresponds to an “instantaneous”, Born-type approximation) then the cancellation would be matched exactly and this 3NF effect would disappear. Hence, because of this incomplete cancellation, the effect must be entirely attributed to the energy dispersion of the intermediate 2N correlation, combined with a “long leg pion” diagram.

Finally, it is possible to generate from the study of \(Z_{ab}^{3N}\) other, more complicated, diagrams. In Fig. 5 it is shown just one example. One class embraces all possible connected 3N correlations while the exchanged pion is “in flight”. On the contrary, the diagrams of Fig. 3 represents all possible disconnected 2N correlations while the pion is “in flight”.

We summarize this section with some comments. First, the analysis performed in this section was based on a new approach developed for the description of the pion dynamics within a nonrelativistic multinucleon context.

The more practical approach in Ref. \[16\] is sufficiently systematic to generate three topologically different structures of irreducible diagrams for the 3N force. The more general approach of Ref. \[14\] will generate additional classes of irreducible (and reducible) 3NF diagrams. The first diagrammatic structure that emerged is well known and is practically the only one explored in modern few-nucleon calculations. Then there are irreducible diagrams whose topological structure was not known. Most interesting is the class of diagrams considering a 2N correlation while the pion is being exchanged. These diagrams should not be confused with the mesonic retardation effects, which produce a net cancellation amongst themselves. To our knowledge, this contribution to the 3NF has never been considered before and will be analyzed in the next two sections of this paper. Finally, we have also revealed the presence of another class based on connected 3N correlations while the pion is being exchanged. From the connected-kernel approach considered in Ref. \[14\], other more complicated classes could be produced. Their basic building blocks, however, are always the 2N t-matrices and the \(\pi N\) amplitudes, and therefore it is clear that the two fundamental ingredients for the construction of the 3NF are the ones shown in Figs. 3 and 4.
III. THE “ODD” CONTRIBUTION TO THE 3N FORCE

In this section, we will first derive a 3N interaction from the irreducible 3N-force diagram shown in Fig. 3. Moreover, we will discuss how this irreducible 3NF produces a contribution acting in the triplet odd-states for the 2N subsystem.

To derive a 3N interaction from the diagram in Fig. 3, we sum all the possible diagrams corresponding to a correlation between nucleons “1” and “2” while the pion is “in flight”. There are four of such diagrams and their sum provides the contribution for an irreducible 3N force in one single Faddeev component; namely the component where nucleon “3” represents the spectator. (This scheme of diagrammatic re-summation is a very natural and automatic consequence of the formalism discussed in Ref. 4.) We denote the resulting component of the 3N force as $V_{3N}^{0}$. The complete 3N interaction obviously will result from the sum over all three Faddeev components, or equivalently from all diagrams resulting from the cyclic permutations of the nucleons in the four diagrams mentioned above. Hence the total 3NF contribution will result from $V_{3N} = V_{3N}^{0} + V_{3N}^{1} + V_{3N}^{2}$. The component $V_{3N}^{0}$ must be calculated according to the expression:

$$V_{3N}^{0} = f_{1}G_{0}^{(4)}t_{12}G_{0}^{(4)}f_{3}^{\dagger} + f_{2}G_{0}^{(4)}t_{12}G_{0}^{(4)}f_{1}^{\dagger} + f_{3}G_{0}^{(4)}t_{12}G_{0}^{(4)}f_{2}^{\dagger},$$  \hspace{1cm} (3.1)

where $f_{1}$ ($f_{1}^{\dagger}$) represents the $\pi NN$ vertex interaction for exchanged pion production (absorption) on nucleon “1”, $G_{0}^{(4)}$ denotes the intermediate propagation of the three nucleons plus the exchanged pion, $t_{12}$ represents the subtracted 2N t-matrix, describing the correlation between nucleons “1” and “2” while the pion is “in flight”. One must observe right from the start that one cannot identify this amplitude with the on-shell 2N t-matrix. Indeed this subtracted amplitude enters off-shell in the diagram, and with an energy shift. And below pion production threshold, $t_{12}$ must be real because the free Green’s function $G_{0}^{(4)}$ is not singular in this region of the real axis.

If one now considers the first of these four diagrams with the Jacobi coordinates depicted as in Fig. 3, using the static approximation and assuming the process in the c.o.m. (center of mass) of the system, one obtains that this diagram corresponds to the following contribution:

$$D_{1} = \sum_{\alpha} \frac{f_{\pi NN} \sigma \cdot (q - q')}{m_{\pi}} \frac{1}{\sqrt{(2\pi)^{3}2\omega_{\alpha}}} G_{0}^{(4)} \tilde{t}_{12}(p, p'; E - \frac{q^{2}}{2\nu} - \omega_{\alpha}) G_{0}^{(4)} \frac{f_{\pi NN} \sigma \cdot (q - q')}{m_{\pi}} \frac{1}{\sqrt{(2\pi)^{3}2\omega_{\alpha}}} \tilde{t}_{12}^{\dagger}. \hspace{1cm} (3.2)$$

With the sum over $\alpha$ it is intended that all three isospin components of the pion field are summed up (in pseudospherical representation), while $\nu$ is the reduced mass of the spectator nucleon with respect to the c.o.m. of the pair.

To derive the final expression of this diagram, we will use the static approximation. Originally, the Green’s function on the left should read

$$G_{0}^{(4)} = \frac{1}{E - \frac{q^{2}}{2\mu} - \frac{q^{2}}{2\nu} - \omega_{\pi}}, \hspace{1cm} (3.3)$$

and similarly the one on the right should be

$$G_{0}^{(4)} = \frac{1}{E - \frac{q^{2}}{2\mu} - \frac{q^{2}}{2\nu} - \omega_{\pi}}. \hspace{1cm} (3.4)$$

Using the static approximation, we assume that

$$E \simeq \frac{p^{2}}{2\mu} + \frac{q^{2}}{2\nu} \simeq \frac{p'^{2}}{2\mu} + \frac{q'^{2}}{2\nu}. \hspace{1cm} (3.5)$$

In this case, both Green’s functions on the left and right of $\tilde{t}_{12}$ can be approximated by the same expression, namely

$$G_{0}^{(4)} \simeq \frac{1}{\omega_{\pi}}. \hspace{1cm} (3.6)$$

We introduce also $Q$ as the momentum transferred by the pion, $Q = q' - q$, hence $\omega_{\pi} = \sqrt{m_{\pi}^{2} + Q^{2}}$.

The subtracted t-matrix is estimated according to the expression

$$\tilde{t}_{12}(p, p'; E - \frac{q^{2}}{2\nu} - \omega_{\pi}(Q)) = t_{12}(p, p'; E - \frac{q^{2}}{2\nu} - \omega_{\pi}(Q)) - v_{12}(p, p'), \hspace{1cm} (3.7)$$

where the potential-like term $v_{12}(p, p')$ contains only OPE/OBE-type diagrams. The quantity $t_{12}$ depends on the Jacobi momenta $p$, $p'$, $q$, and $q'$ in a complicated way. The important feature, however, is that the energy of the 2N subsystem is shifted to negative values.
by the spectator kinetic energy and by the mesonic term \(\omega_s\). In case of heavier mesons \((\omega_s \gg \omega_r)\) the 2N energy becomes so negative (large in absolute value) that \(t_{12}\) is close to \(v_{12}\), and hence the two quantities almost cancel each other. As we will see further on, with the pion the results are quite different because this meson - the Goldstone boson of the underlying chiral theory - is so light.

One has to repeat the same derivation also for the other three diagrams and sum over all the contributions. As a result, the third Faddeev component of the irreducible \(3N\) force generated by the four terms given by Eq. \(3.1\), can be expressed as

\[
V^3_{3N} = \frac{f^2_{\pi NN}}{m^2_\pi} \frac{1}{(2\pi)^3} \times \left[ \frac{(\sigma_1 \cdot Q)(\sigma_3 \cdot Q)(\tau_1 \cdot \tau_3) + (\sigma_2 \cdot Q)(\sigma_3 \cdot Q)(\tau_2 \cdot \tau_3)}{\omega^2_\pi} \right]
\]

\[
\times \frac{1}{2\omega_\pi} \tilde{t}_{12}(p, p'; E - \frac{q^2}{2\nu} - m_\pi)
\]

\[
+ \frac{f^2_{\pi NN}}{m^2_\pi} \frac{1}{(2\pi)^3} \times \left[ \frac{(\sigma_1 \cdot Q)(\sigma_3 \cdot Q)(\tau_1 \cdot \tau_3) + (\sigma_2 \cdot Q)(\sigma_3 \cdot Q)(\tau_2 \cdot \tau_3)}{\omega^2_\pi} \right]
\]

\[
\times \frac{1}{2\omega_\pi} \tilde{t}_{12}(p, p'; E - \frac{q^2}{2\nu} - m_\pi)
\]

and the total contribution to the \(3N\) force will be given by summing up this contribution together with those obtained from this by cyclic permutations of the three nucleons.

This formula implies several aspects on which we would like to comment.

i) We have used the nonrelativistic reduction of the \(\pi NN\) vertex. ii) We have neglected nucleon recoil effects on the basis that the pion mass \(m_\pi\) is much smaller than the nucleon mass, \(M\). iii) We have made use of the static approximation, implying that the pion “in flight” exchange momentum but not energy with the nucleon “3”. iv) The form of the force obtained above is made symmetrical by the combined sum of the four diagrams, and below pion threshold the resulting expression is Hermitian, being \(\tilde{t}_{12}\) real.

It may be convenient to transform \(V^3_{3N}\) introducing the spin and isospin operators for the \(2N\) subsystem, \(S_{12} = \sigma_1 + \sigma_2\), and \(T_{12} = \tau_1 + \tau_2\), respectively. Then it is a matter of simple algebraic manipulations to rewrite Eq. \(3.8\) in the following form

\[
V^3_{3N} = \frac{f^2_{\pi NN}}{2m^2_\pi} \frac{1}{(2\pi)^3} \left[ \frac{(S_{12} \cdot Q)(\sigma_3 \cdot Q)(T_{12} \cdot \tau_3)}{m^2_\pi + Q^2} \right] (T_{12} \cdot \tau_3)
\]

\[
\times \tilde{t}_{12}(p, p'; E - \frac{q^2}{2\nu} - m_\pi)
\]

\[
+ \frac{f^2_{\pi NN}}{2m^2_\pi} \frac{1}{(2\pi)^3} \tilde{t}_{12}(p, p'; E - \frac{q^2}{2\nu} - m_\pi)
\]

\[
\times \frac{1}{m^2_\pi + Q^2} \frac{(S_{12} \cdot Q)(\sigma_3 \cdot Q)(T_{12} \cdot \tau_3)}{m^2_\pi + Q^2}
\]

Here, we have also approximated the two normalization factors of the pion field, i.e., the two square roots in the denominators, as

\[
\frac{1}{\sqrt{2\omega_\pi \sqrt{2\omega_\pi}}} \simeq \frac{1}{2m_\pi},
\]

and we have shifted the \(2N\) amplitudes by the pion mass \(m_\pi\), in place of \(\omega_\pi\). These approximations are really not essential, but they simplify considerably the formulas of the partial wave expansions, without really altering the physics. It is clear however that a more consistent calculation requires the employment of the exact expressions.

Both expressions Eqs. \(3.8\)-\(3.9\) are symmetrical at sight. The latter one has the advantage that it is possible to isolate from the rest the contribution given by the first two terms, which we rewrite as

\[
V^3_3 = \frac{f^2_{\pi NN}}{2m^2_\pi} \frac{1}{(2\pi)^3} \left[ \frac{(S_{12} \cdot Q)(\sigma_3 \cdot Q)(T_{12} \cdot \tau_3)}{m^2_\pi + Q^2} \right] (T_{12} \cdot \tau_3)
\]

\[
\times \tilde{t}_{12}(p, p'; E - \frac{q^2}{2\nu} - m_\pi) + h.c.
\]

The remaining part of \(V^3_{3N}\), i.e., the sum over the last four terms, mixes together the spin components of nucleon “1” with the isospin components of nucleon “2”. We will not discuss further this contribution and leave it for future studies. The last four terms in Eq. \(3.7\) is just one of the many irreducible contributions which should be added up to build the full spin-isospin structure of the \(3N\) interaction.

In the following, we will focus the attention on \(V^3_3\), which has an interesting spin-isospin structure since it depends only on the spectator and pair coordinates. Because of the presence of the spin-isospin operators \(S_{12}\) and \(T_{12}\) such term vanishes unless the \(2N\) pair is in a triplet state for both spin and isospin coordinates. And since the nucleon pair must be in an antisymmetric state
because of the generalized Pauli principle, then the allowed orbital momentum of the pair can be only odd. This means that this contribution to the irreducible 3N force acts only in triplet odd states ($3P_{3/2}$-waves, $3F_{5/2}$-waves, etc.) of the two-nucleon subsystem. Stated in other words, the third nucleon, by means of this contribution modifies selectively the triplet odd states of the 2N subsystem with respect to a free, isolated nucleon-nucleon pair. We believe that this mechanism can possibly modify those observables particularly sensitive to the triplet $p$- and $f$-waves and might therefore affect also the nucleon-deuteron vector analyzing powers.

IV. PARTIAL WAVE DECOMPOSITION

We provide the partial-wave decomposition of $V_3^\ast$. We will work in the so-called channel spin coupling since this is the most natural scheme for nucleon-deuteron scattering.

The channel-spin coupling is defined according to the following notation

$$w = (((ls)j\sigma)KL)\Gamma_z,$$  

(4.1)

where $l$, $s$, and $j$ represent respectively the orbital momentum, spin, and total spin-angular momentum of the pair (hence, $s$ represents the quantum number associated to the operator $S_{12}$). The total spin of the pair $j$ is then coupled with the intrinsic spin of the spectator $\sigma = 1/2$ to provide the so-called channel spin $K$. And finally, this is coupled to the orbital angular momentum of the third nucleon, $L$, to give the total angular momentum of the 3N system $\Gamma$ and its azimuthal component $\Gamma_z$.

We observe that $V_3^\ast$ has an interesting structure in the pair-spectator coordinate system. The structure is that of an OPE contribution, but in the spectator coordinates, multiplied by a full 2N interaction depending on the internal coordinates of the pair. Given this structure, it is not so difficult to perform the partial wave decomposition of $V_3^\ast$. Indeed, one can consider the spectator coordinates and separate the 3N potential into a spin-spin component and a tensor one, following the standard procedure for the OPE term (see, e.g., Refs. [10-11]),

$$\frac{(S_{12} \cdot Q)(\sigma_3 \cdot Q)}{m_s^2 + Q^2} + \frac{m_s^2}{m_s^2 + Q^2} (S_{12} \cdot \sigma_3) + \frac{Q^2}{m_s^2 + Q^2} \Sigma_{12}(\hat{Q}),$$  

(4.2)

where $(\hat{Q})$ is the angular part of the spectator momentum, and $\Sigma_{12}(\hat{Q}) = 3(S_{12} \cdot \hat{Q})(\sigma_3 \cdot \hat{Q}) - (S_{12} \cdot \sigma_3)$ is the tensor operator.

In the above equation, we have neglected the contact term, on the ground that its contribution will be unavoidably smeared out when taking into account the extended structure of the sources of the meson field. The extended nature of the sources have to be included in $V_3^\ast$ by means of phenomenological $\pi NN$ form factors. In a fully consistent calculation these formfactors should be the same as those used in the standard 2N OPE contribution. Then $V_3^\ast$ is completely fixed by the full expression of the 2N potential.

For the spin-spin part of $V_3^\ast$, we obtain the following partial-wave decomposition:

$$\langle p, q, w | V_3^\ast (\text{spin - spin}) | p', q', w' \rangle = \delta_{ss'}\delta_{\sigma\sigma_1}\delta_{LL'}\delta_{KK'} \times \frac{2}{\pi} I_L(q, q') \times (-)^{l+j+K+j'+1/2} \left\{ \begin{array}{ccc} 1 & j & j' \\ l & 1 & 1 \end{array} \right\} \left\{ \begin{array}{ccc} 1 & j & j' \\ K & 1/2 & 1/2 \end{array} \right\} \times \left[ \tilde{t}_{12}(p, p'; E - \frac{q^2}{2m_s} - m_\pi) \right]_{J''} \frac{1}{2m_\pi} + h.c. \quad (4.3)$$

For the tensor component the decomposition in partial waves is slightly more complicated, being

$$\langle p, q, w | V_3^\ast (\text{tensor}) | p', q', w' \rangle = \delta_{ss'}\delta_{\sigma\sigma_1}\delta_{TT'}\delta_{\Gamma_\gamma\Gamma_\gamma'}\hat{J}_\gamma\hat{K}_\gamma' \times \frac{2}{\pi} \sqrt{30} I_{LL'}(q, q') \left( \begin{array}{cc} L & 0 \\ 0 & L' \end{array} \right) (-)^{l+j+\Gamma+K'} \times \left\{ \begin{array}{ccc} 1 & j & j' \\ l & 1 & 1 \end{array} \right\} \left\{ \begin{array}{ccc} K & 2 & K' \\ L' & \Gamma & L \end{array} \right\} \left\{ \begin{array}{ccc} 1 & j & j' \\ K & 1/2 & 1/2 \end{array} \right\} \times \left[ \tilde{t}_{12}(p, p'; E - \frac{q^2}{2m_s} - m_\pi) \right]_{J''} \frac{1}{2m_\pi} + h.c. \quad (4.4)$$

The two quantities $I_L(q, q')$ and $I_{LL'}(q, q')$ represent well known Fourier-Bessel transforms of Yukawa-type functions,

$$I_L(q, q') = -\frac{f_{\pi NN}}{12\pi} \int_0^\infty j_L(qR) j_{L'}(q'R) \left( e^{-m_\pi R} \right) R^2 dR,$$

and

$$I_{LL'}(q, q') = -\frac{f_{\pi NN}}{12\pi} \int_0^\infty j_{L}(qR) j_{L'}(q'R) \left( e^{-m_\pi R} \right) R^2 dR \quad (4.6)$$

The resulting analytical expressions for these integrals are well known [12].

Finally, the potential matrix elements for the spin and tensor parts must be multiplied by the isotopic component, which is the same in both cases:
$\tau$ is the isotopic spin of the 2N pair, and $t$ is the total isospin of the 3N system. As ought to be expected, only for isovector pairs this matrix element is nonzero.

Equation (3.31) represents an OPE potential in the spectator-pair coordinates, times a subtracted 2N $t$-matrix for the internal coordinates of the pair. This 2N amplitude is quite off-shell, because of the presence of a pion-exchange term, in addition to the standard shift due to the spectator nucleon. In the limit of a heavy meson exchange ($m_\pi \to 1$ GeV) the energy of the 2N subsystem will be large and negative, and $t_{12}$ will be dominated by its Born OBE term $v_{12}$. Hence the effect of this 3NF is suppressed because approximately $t_{12} \simeq 0$. On the contrary, as shown in Fig. 6, for the lightest meson, a 15-30% effect (at least) survives from the cancellation and this generates the “odd” contribution to the 3NF. The figure shows the comparison between the unsubtracted (solid line) and the subtracted (dashed line) $t$-matrices, for a 2N energy of -150 MeV, consistent with the calculation of the diagram shown in Fig. 3. The $t$-matrices have been calculated in the relevant triplet $p$-states, with the Bonn $B$ potential [13], which is of OBE type. Then the subtracted amplitude is given simply by Eq. (3.7). The lines show the $t_{12}$ and $t_{12}$ amplitudes as a function of the momentum $p$, while $p'$ was fixed at the value $p' = 0.89$ fm$^{-1}$. With other values of $p'$ we found the same effect. Also, in order to show that the cancellation could not produce an overall vanishing result we made a more stringent test, by assuming that when the mass of the particle “in flight” is of the order of $\Lambda \simeq 1$ GeV, then the diagram of Fig. 3 is canceled exactly against the mesonic retardation corrections of Fig. 6. In this case, the subtracted $t$-matrix entering in the pionic diagram can be evaluated according to the expression

$$t_{12}(p, p'; E - \frac{q^2}{2\nu} - \omega_\pi(Q))$$

$$\approx t_{12}(p, p'; E - \frac{q^2}{2\nu} - \omega_\pi(Q)) - t_{12}(p, p'; -\Lambda), \quad (4.8)$$

and this expression can be employed with all types of phenomenological NN potentials. We checked $t_{12}$ for an energy of the 2N subsystem around -150 MeV, for the Bonn $B$ and for the Paris potential [13], and with both interactions we found that a 10-20% effect was surviving after the subtraction, thus providing evidence that the cancellation cannot hold exactly and simultaneously in both cases of light- and heavy-meson exchanges.

However, it is also possible to modify the behavior of the 2N amplitude in this energy region, so that to obtain a 3NF contribution of larger or smaller size, without obviously altering the constraints to the 2N amplitudes from comparison with phase-shift analyses.

V. SUMMARY AND CONCLUSIONS

As discussed in the introduction, discrepancies between theoretical calculations and experimental measurements give a clear indication that 3NF’s of new structure are badly needed. The existing 3NF models do not seem to provide in full the correct spin-isospin structure of the 3N force.
In this paper, we have suggested a new mechanism which generates an irreducible $3NF$ whose structure is topologically very different from those explored up to now. Therefore, the force generated by this mechanism should be considered as an additional contribution to the full $3N$ interaction. This mechanism is generated under the hypothesis that the standard few-nucleon dynamics and the pion-exchange processes are intertwined more strongly than what has been generally assumed up to now. In particular, it is the intermediate formation of a virtual $2N$ cluster during a pion-exchange process that gives rise to this new $3NF$ term. In an instantaneous approach, such a mechanism is $100\%$ suppressed because of the presence of a well-known cancellation effect which involves the meson-retardation corrections of the reducible Born term of Fig. 2 as well as all possible irreducible $3NF$ diagrams obtained by subsuming the exchange of two pions in their variety of possible time orderings. It was known also that the same cancellation occurs when considering similar processes involving heavy-meson exchanges.

We have sized the effect of this cancellation more precisely by considering a complete $2N$ rescattering process, not just its Born term (of Fig. 2). This was possible by extrapolating the $NN$ $t$-matrix downwards to negative energy by a shift given by the spectator kinetic energy and by the relativistic energy of the meson. The result of this study reveals that a 15-30 \% contribution survives from this cancellation in case of a cluster formation while from this cancellation in case of a cluster formation while a pion is “in flight”. Instead, the cancellation is more pronounced when considering the same process while a heavy meson in “in flight”. The reason for this difference can be entirely attributed to the fact that the mass of the pion is approximately comparable to the average momenta exchanged between nucleons in nuclear process, while the mass of the heavier mesons are larger.

We have studied the spin-isospin structure of the $3NF$ generated by this new, pion-induced mechanism and we have extracted an important contribution which selectively operates in the triplet-odd waves for the $2N$ subsystem. While we acknowledge that the way to fully understand the spin-isospin structure of the $3N$ interaction is still long and difficult, we conclude that this new term may possibly contribute to this structure, especially by affecting those spin observables most sensible to the $3^P_J$ waves, such as the $Nd$ vector analyzing powers.

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