Gravitational Waves Created During the EWPT

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Abstract

We study gravitational waves generated by bubble expansion created during the Cosmological Electroweak Phase Transition (EWPT). The energy radiated via gravitational waves is produced by the stress-energy tensor created by the magnetic field produced by bubble collisions during the EWPT, which occurred about 10–11 seconds after the Big Bang.

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1 Introduction

The present work is an extension of the estimate of gravitational waves created during the Cosmological Quantum Chromodynamic Phase transition (QCDPT)\cite{1} to an estimate of gravitational waves created during the Cosmological Electroweak Phase Transition (EWPT). It is also closely related to a recent publication of gravitational radiation produced by pulsar creation\cite{2}.

The QCDPT and EWPT are first order cosmological phase transitions so they have latent heat: the quark condensate for the QCDPT and the Higgs mass for the EWPT.

For the QCDPT $q(x), \bar{q}(x)$ are the quark, antiquark fields. $| >$ is the vacuum state.
\begin{equation}
< |\bar{q}(x)q(x)| > = \text{quark condensate}
\end{equation}
\begin{equation}
< |\bar{q}(x)q(x)| > = 0 \text{ in quark gluon plasma phase}
\end{equation}
\begin{equation}
\simeq -(23 \text{ GeV})^3 \text{ in hadron phase} .
\end{equation}

Therefore for the QCDPT the latent heat $\simeq -(23 \text{ GeV})^3$.

For the EWPT, with $\phi_H$ the Higgs field, with $T_c \simeq 125 \text{ GeV}$ the critical temperature,
\begin{equation}
< |\phi_H| > = 0 \text{ for } T_c \geq 125 \text{ GeV}
\end{equation}
\begin{equation}
< |\phi_H| > = 125 \text{ GeV} \text{ for } T_c \leq 125 \text{ GeV} .
\end{equation}

Therefore for the EWPT the latent heat $\simeq 125 \text{ GeV}=M_H$, the Higgs mass.

There have been several articles published on gravitational radiation produced by Cosmological Phase Transitions: the Electroweak Phase Transition (EWPT) at about 10–11 seconds after the Big Bang and the Quantum Chromodynamics Phase Transition (QCDPT) at about $10^{-4}$ seconds.

One of the first studies, which is a basis for the present work, was “Gravitational radiation from first-order phase transitions” by Kamionkowski, Kosowsky and Turner\cite{3}. More recent studies were gravitational radiation from primodial turbulence\cite{4}, gravitational radiation from cosmological phase transition magnetic fields\cite{5}, and polarization of such gravitational radiation\cite{6}.

The present work also makes use of the stress-energy tensor produced by the magnetic wall during the EWPT\cite{7}. In order to estimate the energy radiated by gravitational waves during the EWPT one needs the nucleon mass $M_n$ in units of fm$^{-1}$, $B_W$, the magnitude of the magnetic field at the bubble wall during the EWPT, and $t_{EWPT} = 10^{-11}$ seconds after the Big Bang, the time of the EWPT. The values of these parameters used in the present work are taken from estimates in Ref\cite{7}.

The only experimental detection of gravitational waves was the observation of gravitational waves from binary black hole mergers Ref\cite{8}. Gravity waves from black hole mergers had been predicted\cite{9}\cite{10} and were in agreement within experimental and theoretical errors with Ref\cite{8}.
2 Electroweak Phase Transition (EWPT)

The Standard EW Model has fields with quanta:

Fermions (quantum spin 1/2 particles) are $e^-, \nu_e$, the $\mu$ and $\tau$ leptons, the quarks $q_u, q_d$ and two other quark generations. The EW gauge bosons (quantum spin 1) are $W^+, W^-, Z^0$ and photon $\gamma$.

Since the EWPT is a first-order phase transition there is latent heat. The latent heat for the EWPT is the Higgs boson (quantum spin 0) mass, $M_H = 125$ GeV. At the LHC[11] it was found that $M_H \simeq 125$ GeV. Also during the EWPT the quarks caused Baryogenesis, with more particles than antiparticles, due to CP (Charge Congugation and Parity) violation.

The EW diagrams are shown in the figure below.

Lepton weak interaction conserves CP—No Baryogenesis

Quark weak interaction violates CP—Baryogenesis Possible

Baryogenesis requires a first order EWPT

During the EWPT bubbles form and magnetic fields were created via bubble collisions, as shown in the figure below.

From Ref[5], $B_W \simeq 4.3 \times 10^{24}$ Gauss. Therefore $B^{EPPT}_W \simeq 4.3 \times 10^7 \times B^{QCDPT}_W$. 

B field
3 Gravitational Radiation From Magnetic Fields Generated by the EWPT

The energy radiated by gravitational waves with frequency interval $d\omega$ and solid angle $d\Omega$ is

$$\frac{dE}{d\omega d\Omega} = 2G\omega^2 \Lambda_{ij,lm}(\hat{k}) T_{ij}^\gamma(k,\omega) T_{lm}(k,\omega),$$

where

$$\Lambda_{ij,lm}(\hat{k}) = \delta_{il} \delta_{jm} - \delta_{ij} \delta_{lm} / 2 + \delta_{ij} \hat{k}_l \hat{k}_m / 2 + \delta_{lm} \hat{k}_i \hat{k}_j / 2 - 2 \delta_{il} \hat{k}_j \hat{k}_m + \hat{k}_i \hat{k}_j \hat{k}_l \hat{k}_m / 2.$$  

From Eq(20) in Ref[7], which makes use of Eq(1), with $k_3 \simeq k/\sqrt{3},$

$$T_{ij}(k,\omega) = \delta_{i3} \delta_{j3} \frac{2\pi^3}{M^2} B^4_W e^{3k_3/8M^2-n^2} \delta(t-t_{\text{EWPT}}),$$

where $t_{\text{EWPT}}$ is the time of the EWPT, with

$$t_{\text{EWPT}} \simeq 10^{-11} \text{ seconds} \quad (4)$$
$$B_W \simeq 4.3 \times 10^{24} \text{ Gauss} \quad (5)$$
$$M^{-1}_N \simeq 0.2 \text{ fm} \quad (6)$$

From Eqs[123] the energy radiated by gravitational waves with frequency interval $d\omega$, eliminating the solid angle as in Ref[3], using $k_3 \simeq k/\sqrt{3}$ and [5]

$$\frac{dE}{d\omega} = \frac{8G\omega^2 \pi^7 B^4_W e^{-3k_3/4M^2-n^2} \delta(t-t_{\text{EWPT}})}{M^2} \left[ \frac{1}{2} - \frac{k_3^2}{k^2} + \frac{k_4^2}{2k^4} \right].$$

since $k_3 \simeq k/\sqrt{3}$ or $k_3/k \simeq 1/\sqrt{3}$, from Eq(5) one finds

$$\frac{dE}{d\omega} \simeq \frac{8G\omega^2 \pi^7 B^4_W e^{-k^2/18M^2_N}}{M^2}.$$  

From Refs[5,3,12]

$$k \simeq 2\pi/\lambda_o \simeq 2\pi/(10^{13} \text{ fm}).$$

Therefore, using $M^{-1}_N \simeq 0.2 \text{ fm}$

$$e^{-k^2/4M^2_N} \simeq e^0 = 1.0/,$$

from Eq(6)

$$\frac{dE}{d\omega} = \frac{8G\omega^2 \pi^7 B^4_W}{M^2}.$$ (9)

Using (with s=second)

$$G(\text{ Gauss})^4 = \frac{8.09310^{-47}}{s^2 \text{ cm}^2}$$

$$\text{cm} = 10^{13} \text{ fm},$$

and Eq(4) one obtains our final equation for the energy radiated by gravitational waves during the QCDPT

$$\frac{dE}{d\omega} = \frac{3.3610^6 \omega^2}{s^2}.$$
A review of Gravitational Wave Physics[13] discusses the generation of gravitational waves, the first detection of Gravitational waves by the LIGO detector[14], and many other aspects of Gravitational Wave Physics. An aspect of Ref[13] important for the present work is that the units for Gravitational radiation are explained.

These are used for $\frac{dE}{d\omega}$ shown in Figure 1, with $s = \text{second}$.

Figure 1: Gravitational radiation energy produced during the QCDPT as a function of frequency $\omega$. 
4 Conclusions

We have estimated the gravitational radiation energy produced by gravitational waves during the EWPT, which occurred at a time $t_{EWPT} \simeq 10^{-11}$ seconds after the Big Bang. This estimate is based on methods found in Ref[3], but the estimate is of gravitational radiation energy produced during a specific Cosmological Phase Transition, the Electroweak Phase Transition, rather than a more general study of Gravitational radiation from first-order phase transitions[3].

From Figure 1 the Gravitational radiation energy as a function of $\omega$ for the most likely values of $\omega/s$ are approximately $2 \times 10^7$ to $8 \times 10^7$ ergs/s. This is approximately $4.3 \times 10^7$ larger than the Gravitational radiation energy produced during the QCDPT[1].

Note that gravitational waves from binary black hole mergers were detected in 2016[8]. From the results shown in Figure 1, with Gravitational radiation energy $\simeq 8 \times 10^7$ ergs/s the current gravitational wave detectors, such as LIGO[14], can detect gravitational waves produced during the EWPT. We hope that such LIGO experiments are carried out soon.

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