Thermality in de Sitter and Holography

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Assuming the existence of a dS/CFT correspondence we study the holograms of sources moving along geodesics in the bulk by calculating the one point functions they induce in the boundary theory. In analogy with a similar study of uniformly accelerated sources in AdS spacetime, we argue that comoving geodesic observers correspond to a coordinate system on the boundary in which the one point function is constant. For dS3 we show that the conformal transformations on the boundary which achieve this - when continued suitably to Lorentzian signature - induce nontrivial Bogoliubov transformations between modes, leading to a thermal spectrum. This may be regarded as a holographic signature of thermality detected by bulk geodesic observers.

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1. Introduction and summary

There are several situations where quantum fields on curved space-times lead to thermal behavior [1]. The most dramatic example is Hawking radiation from black holes [2]. A second example is the thermal spectrum observed by uniformly accelerated detectors in flat space (the Unruh effect) [3] - which is intimately related to black hole radiation. Unruh radiation has generalizations to other space-times, e.g. anti-de-Sitter (AdS) space-times [4]. Another set of examples are cosmological space-times [1], the most well-studied instance being de-Sitter (dS) spacetime. In this spacetime any geodesic observer perceives the invariant vacuum as a thermal state [5].

While recent developments in string theory have thrown valuable light on the microscopic origin of black hole radiation [6], we know very little about thermal behavior in cosmological spacetimes. This note is a attempt to throw some light on this important issue for de Sitter spacetimes.

The microscopic origin of black hole thermodynamics led to a concrete realization of the holographic principle [7] - the AdS/CFT correspondence [8]. Conversely AdS/CFT duality has provided a physical understanding of Hawking radiation and related phenomena in terms of the dual field theory. If the holographic principle is correct in this context, quantum gravity in de Sitter space-time should have a holographic dual which is some theory living on one of the spacelike boundaries $I^\pm$. Our experience with AdS/CFT duality suggests that understanding this dual theory would throw valuable light on bulk behavior in de Sitter space-times.

Unfortunately, we do not know how to obtain de Sitter space-time from string theory in a fully satisfactory manner. In fact, under some assumptions there appears to be a no-go theorem [9], though there have been several proposals in the past as well as in recent years [10]. Nevertheless it is important to figure out what would be the holographic signature of bulk phenomena assuming that such a correspondence exists. In this spirit [11] [12] [13] propose various versions of this correspondence, in direct analogy with the AdS/CFT correspondence. For other work in this direction, see [14]. General aspects of holography in de Sitter spaces are discussed in [15]. While various aspects of these proposals lead to interesting insights into the nature of the holographic theory, a holographic explanation of the thermal behavior observed by a geodesic observer is still a mystery.

In this paper we throw some light on this issue. We use an earlier work [16], which addressed a similar question in AdS spacetimes. In the latter situation, uniformly accelerated observers measure a thermal spectrum, provided the acceleration exceeds a critical
bound [4]. The purpose of [10] was to ask: how does one understand this thermality in the holographic theory? To answer this question, [10] considered an external source moving with a uniform acceleration. The source couples to one of the supergravity fields in the bulk, e.g. the dilaton \( \Phi \). According to the standard \( AdS/CFT \) correspondence the value of the field \( \Phi \) produced by this source is equal to the one point function of the operator dual to this field in the boundary CFT [17], [18]. This provides a “hologram” of the moving source. Consider such a source which moves normal to the boundary, away from it. Generically, at some given time, the hologram has a profile which is peaked at the point where the bulk trajectory intersects the boundary, dying off away from it. The width of this profile is related to the distance of the source from the boundary in accordance with the IR/UV connection. Thus the profile is time dependent, spreading as the source in the bulk moves deeper into \( AdS \) space. We now need to understand how to describe a *comoving* bulk observer holographically. This may be done by performing a conformal coordinate transformation on the boundary such that the transformed one point function is time independent. Such a coordinate transformation on the boundary would generically mix up positive and negative frequency modes of any field in the boundary CFT. In [10] it was found that such mixing occurs only when the bulk acceleration exceeds the critical value. In fact the metric of the boundary generically now becomes time-dependent, i.e. a cosmological space-time. Some of the cosmologies obtained in this way are well known. The final result is that there is a holographic relationship between acceleration radiation in the bulk and thermal behavior in cosmologies defined on the boundary.

In the following, we adopt the same strategy for geodesic trajectories in de Sitter space. However, now there are crucial differences which make the physical interpretation a bit confusing. The boundaries are now space-like, \( I^\pm \), and the dual theory is euclidean. In planar coordinates, time evolution in the bulk maps into decreasing radial distance on the boundary. Furthermore, unlike the \( AdS/CFT \) correspondence it is not yet clear whether there is an operator correspondence in \( dS/CFT \). In fact, as we shall see below, an operator correspondence is not necessarily equivalent to a correspondence between the bulk effective action and the CFT free energy in the presence of sources. Nevertheless, in this work we will assume an operator correspondence. Then one point functions of dual operators are related to the value of the bulk field on the boundary - apart from the standard factor involving a UV cutoff.

We consider a source for some scalar field \( \Phi \) of mass \( m < 1 \) (in units where the de Sitter scale is set to unity) moving along a geodesic in three dimensional de Sitter space. We
calculate the value of the field on a cutoff boundary, and hence the one point function of the
dual operator, on the boundary $I^+$. While the field $\Phi$ is a scalar, the definition of the cutoff
boundary, and hence the one point function, depends on the coordinate system used. In
planar coordinates, the one point function peaks at the point where the geodesic intersects
$I^+$ and decays as a power of the radial distance. In analogy with [10] we then ask whether
there is a coordinate transformation on the boundary which renders the transformed one
point function constant over the entire boundary. This is indeed possible since the dual
operator has nontrivial conformal dimensions. Boundary (euclidean) correlation functions
which are single valued in planar coordinates are periodic in one of the new coordinates,
and can therefore be interpreted as thermal Green’s functions of a Lorentzian signature
theory. After an analytic continuation to Lorentzian signature, this new coordinate system
is the holographic interpretation of a comoving observer along a geodesic. The coordinate
transformation, when analytically continued in this fashion, mixes positive and negative
frequency modes of fields in the boundary theory - leading to the correct temperature. An
entirely analogous story holds for holograms in global coordinates as well.

The coordinate transformation involved is once again a conformal transformation and
is in fact the restriction of the bulk transformation which takes the planar or global co-
ordinates to “static” coordinates to the boundary. In fact, the field produced on a cutoff
boundary defined in terms of a “static” coordinate system is constant. As emphasized
above, while the field $\Phi$ is a scalar, the one point function transforms nontrivially as a
conformal field - which explains the result. The final offshoot is that thermality in the
dual description appears because of a nontrivial Bogoliubov transformation involved in
the passage to the natural holographic analogs of “comoving” bulk observers.

While the above results pertain to $dS_3$ we believe that the picture is similar for other
$dS_d$, though we do not present explicit computations.

In this work we have used retarded Green’s functions to determine the field due to
the geodesic source which is then identified with the one point function in the boundary
theory. While the rationale for this is clear in the AdS/CFT correspondence, it is not so
in the present case. We comment, without explicit calculations, the relevance of this issue
to recent results which concern the behavior of geodesic detectors in the bulk in the one
parameter class of de Sitter invariant states.

Section 2 contains definitions of various coordinate systems used in this work. In
Section 3 we discuss an operator version of the dS/CFT correspondence. Section 4 contains
the calculation of the hologram of a geodesic source in planar coordinates. Section 5
discusses interpretations of the hologram in both planar and global coordinates and possible
extensions to $dS_d$ for $d \neq 3$. Section 6 contains conclusions and comments.
2. Coordinate systems in de Sitter

Throughout the paper, all dimensional quantities are measured in units of the de Sitter length scale.

\( d + 1 \) dimensional de-Sitter space is a hyperboloid in \( d + 2 \) dimensional flat space with signature \((-1,1,1\cdots1)\) defined by the equation

\[-(Y^0)^2 + (Y^1)^2 + \cdots (Y^{d+1})^2 = 1\]  \(2.1\)

Various coordinate systems are given by different ways of solving this equation.

Global coordinates are defined by the embedding

\[
\begin{align*}
Y^0 &= \tan T \\
Y^1 &= \sec T \cos \theta_1 \\
Y^2 &= \sec T \sin \theta_1 \cos \theta_2 \\
&\vdots \\
Y^{d+1} &= \sec T \sin \theta_1 \cdots \sin \theta_d
\end{align*}
\]

where

\[
-\frac{\pi}{2} \leq T \leq \frac{\pi}{2} \\
0 \leq \theta_i \leq \pi \quad i = (1, \cdots (d-1)) \\
0 \leq \theta_d \leq 2\pi
\]

The de-Sitter metric is then

\[ ds^2 = \sec^2 T[-dT^2 + d\Omega_d^2] \]  \(2.4\)

where \(d\Omega_d^2\) is the metric on a unit \(S^d\) whose coordinates are \(\theta_1 \cdots \theta_d\). The penrose diagram may be drawn using \((2.4)\) directly, as shown in Fig. 1. The future infinity \(I^+\) is at \(T = \frac{\pi}{2}\) while the past infinity \(I^-\) is at \(T = -\frac{\pi}{2}\). The diagram supresses the angles \(\theta_2 \cdots \theta_{d+1}\) and north pole corresponds to \(\theta_1 = 0\) while the south pole is at \(\theta_1 = \pi\).

Planar or steady-state coordinates which cover regions I and II in Fig. 1 are defined by

\[
\begin{align*}
Y^0 &= \frac{1}{2}e^\hat{t} \rho^2 + \sinh \hat{t} \\
Y^1 &= \frac{1}{2}e^\hat{t} \rho^2 - \cosh \hat{t} \\
Y^2 &= e^\hat{t} \rho \cos \theta_2 \\
Y^3 &= e^\hat{t} \rho \sin \theta_2 \cos \theta_3 \\
&\vdots \\
Y^{d+1} &= e^\hat{t} \rho \sin \theta_2 \cdots \sin \theta_d
\end{align*}
\]  \(2.5\)
Fig. 1: Penrose diagram of de Sitter space. The curved line is a constant $\hat{t}$ surface.

The angles $\theta_i$ in (2.3) are the same as in (2.2). $\rho$ is a radial coordinate in $R^d$, $0 \leq \rho \leq \infty$ which is formed by $\rho$ and the $d-1$ angles $\theta_2 \cdots \theta_d$. The metric now reads

$$ds^2 = -d\hat{t}^2 + e^{2\hat{t}}(d\rho^2 + \rho^2 d\Omega^2_{d-1})$$  \hspace{1cm} (2.6)

It is sometimes convenient to introduce cartesian coordinates on the $R^d$ which we denote by $x^i$, and also introduce a time coordinate $y$

$$y = e^{\hat{t}}$$  \hspace{1cm} (2.7)

in terms of these the metric becomes

$$ds^2 = \frac{1}{y^2}[-dy^2 + dx^i dx^j \delta_{ij}]$$  \hspace{1cm} (2.8)

Comparing (2.2) and (2.5) it is easy to see that this coordinate system covers only regions I and II. This is because (2.3) implies that $Y^0 - Y^1 = e^{\hat{t}} > 0$, while from (2.2) we get

$$Y^0 - Y^1 = 2 \sec T \cos \left[\frac{1}{2}(T + \frac{\pi}{2} - \theta_1)\right] \sin \left[\frac{1}{2}(T - \frac{\pi}{2} + \theta_1)\right]$$  \hspace{1cm} (2.9)

Since both $[\frac{1}{2}(T + \frac{\pi}{2} - \theta_1)]$ and $[\frac{1}{2}(T - \frac{\pi}{2} + \theta_1)]$ range from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$, the first two factors in (2.9) are always positive, so that the sign is determined by the sign of $[\frac{1}{2}(T - \frac{\pi}{2} + \theta_1)]$.  

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The latter is positive in regions I and II. Planar coordinates which cover regions III and IV may be defined in an analogous fashion.

A third coordinate system will be called “static” coordinates. In Region I we have

\begin{align}
Y^0 &= \sqrt{1-r^2} \sinh t \\
Y^1 &= -\sqrt{1-r^2} \cosh t \\
Y^2 &= r \cos \theta_2 & \text{Region I} \\
Y^3 &= r \sin \theta_2 \cos \theta_3 \\
\ldots \\
Y^{d+1} &= r \sin \theta_2 \cdots \sin \theta_d
\end{align}

\text{(2.10)}

where $-\infty \leq t \leq \infty$ and $0 \leq r \leq 1$. The angles $\theta_2 \cdots \theta_{d-1}$ are the same in (2.2) and (2.3). The metric is

\begin{equation}
 ds^2 = -(1-r^2)dt^2 + \frac{dr^2}{1-r^2} + r^2 d\Omega^2_{d-1} \tag{2.11}
\end{equation}

The south pole is given by $r = 0$ while the past and future horizons are given by $r = 1$. The metric is time independent in this region, which is why these are called static coordinates.

In region II we have and $1 \leq r \leq \infty$

\begin{align}
Y^0 &= \sqrt{r^2-1} \sinh t \\
Y^1 &= -\sqrt{r^2-1} \cosh t \\
Y^2 &= r \cos \theta_2 & \text{Region II} \\
Y^3 &= r \sin \theta_2 \cos \theta_3 \\
\ldots \\
Y^{d+1} &= r \sin \theta_2 \cdots \sin \theta_d
\end{align}

\text{(2.12)}

The metric is again given by (2.11), but $r$ rather than $t$ is a timelike coordinate in this region. Thus the metric is not stationary any more. However we will retain the nomenclature “static coordinates” even in this region. The future infinity $\mathcal{I}^+$ is given by $r = \infty$. It is possible to introduce Kruskal coordinates which cover the entire space-time. These are denoted by $(U, V, \theta_2 \cdots \theta_d)$ where

\begin{align}
 r &= \frac{1+UV}{1-UV} \\
t &= \frac{1}{2} \log \left( \frac{U}{V} \right) & \text{Region I} \\
 r &= \frac{1+UV}{1-UV} \\
t &= \frac{1}{2} \log \left( \frac{U}{V} \right) & \text{Region II} \tag{2.13}
\end{align}
Thus in region I $U > 0, V < 0$ while in region II $U, V > 0$. In terms of these Kruskal coordinates we have in both regions I and II

\[
\begin{align*}
Y^0 &= \frac{U + V}{1 - UV} \\
Y^1 &= \frac{V - U}{1 - UV} \\
Y^2 &= \frac{1 + UV}{1 - UV} \cos \theta_2 \\
Y^3 &= \frac{1 + UV}{1 - UV} \sin \theta_2 \cos \theta_3 \\
&\ldots \\
Y^{d+1} &= \frac{1 + UV}{1 - UV} \sin \theta_2 \ldots \sin \theta_d
\end{align*}
\]

(2.14)

3. An operator $dS/CFT$ correspondence

According to the $dS/CFT$ correspondence, physics in the bulk of $dS_d$ has a holographic dual which is a conformal field theory living on the boundary of the space-time. In global coordinates the boundaries could be either $I^+$ or $I^-$, but not both \[13\]. For bulk fields which satisfy standard wave equations with two derivatives, the boundary data are the values of the field on $I^+$ and $I^-$. Equivalently, one considers the two independent solutions of the equations of motion and specifies the data in terms of these, as in \[13\]. From the point of view of a formulation of the $dS/CFT$ correspondence which relates the bulk effective action to the free energy of the CFT in the presence of sources, both the solutions have to be retained - in contrast to the $AdS/CFT$ correspondence. As a result there are two dual CFT operators for each bulk field.

In planar coordinates which cover regions I and II, and static coordinates which cover region II, the boundary is at $I^+$. However, since these coordinates do not cover the entire space-time, one has to specify the values of various bulk fields along the horizons. Equivalently one has to again consider both the independent solutions of the equations of motion.

In the $AdS/CFT$ correspondence there is an operator formulation \[17\]. We assume that there is a similar operator version of the $dS/CFT$ correspondence. We will spell out this operator correspondence in planar coordinates, though similar considerations are valid for global coordinates as well.
Consider a massive scalar field $\Phi$ of mass $m$ in $dS_{d+1}$. We will consider the case $m < d/2$ in $dS$ units. The free Klein-Gordon equation is

$$(\nabla^2 - m^2)\Phi = 0 \quad (3.1)$$

In planar coordinates given by (2.8), the two independent solutions may be easily written down

$$\Phi_k^{(1)}(\vec{x}, y) = \frac{1}{2(2\pi)^{d/2}} \frac{y^{d/2}}{y^{d/2}} H_\nu^{(1)}(|k| y) e^{i\vec{k} \cdot \vec{x}}$$

$$\Phi_k^{(2)}(\vec{x}, y) = \frac{1}{2(2\pi)^{d/2}} \frac{y^{d/2}}{y^{d/2}} H_\nu^{(2)}(|k| y) e^{i\vec{k} \cdot \vec{x}} \quad (3.2)$$

where $H_\nu^{(i)}$ are Hankel functions and

$$\nu = +\sqrt{(d/2)^2 - m^2} \quad (3.3)$$

The modes have been chosen so that they are complex conjugates of each other and normalized according to the standard Klein Gordon inner product. Therefore the mode expansion which defines the creation/annihilation operators is

$$\Phi(\vec{x}, y) = \int \frac{d^d k}{(2\pi)^d} [\Phi_k^{(1)}(\vec{x}, y) a(\vec{k}) + \text{h.c.}] \quad (3.4)$$

The operators $a(\vec{k})$ satisfy

$$[a(\vec{k}), a^\dagger(\vec{k'})] = \delta^d(\vec{k} - \vec{k'}) \quad (3.5)$$

The Fock vacuum in these coordinates is then defined by

$$a(\vec{k}) |0> = 0 \quad \text{for all } \vec{k} \quad (3.6)$$

and the states are labelled as usual by the values of the momenta $\vec{k}$. Single particle states are

$$|\vec{k}>= a^\dagger(\vec{k}) |0> \quad (3.7)$$

Consider the mode expansion (3.4) close to $I^+$, i.e. $y = y_0 \to 0$. The leading order result for the field operator is then, upto numerical factors

$$\Phi(\vec{x}, y_0) \sim \frac{y_0^{\frac{d}{2} - \nu}}{i \Gamma(1 - \nu) \sin \pi \nu} \int d^d k \ k^\nu \ [a(\vec{k}) - a^\dagger(-\vec{k})] e^{i\vec{k} \cdot \vec{x}} \quad (3.8)$$
where $k \equiv |\vec{k}|$. Thus we can define a boundary operator $\mathcal{O}_-(k)$ by

$$\mathcal{O}_-(\vec{k}) \equiv k^\nu [a(\vec{k}) - a^\dagger(-\vec{k})] \quad (3.9)$$

which is the fourier transform of some local operator $\mathcal{O}_-(\vec{x})$ on the boundary. The power of $\gamma_0$ clearly indicates that the conformal dimension of this operator is

$$\Delta_- = \frac{d}{2} - \nu \quad (3.10)$$

This is an operator form of $dS/CFT$ correspondence.

Note that roughly half of the bulk operators are related to the specific boundary operator which arises from restricting the field to the boundary. The Hankel functions have two pieces $J_\nu(|k|y)$ and $J_{-\nu}(|k|y)$. Near $I^+$ the latter dominates and the operator which comes with it is what we have identified above. There is another operator $\mathcal{O}_+(k)$, which comes with the other Bessel function $J_\nu(|k|y)$ and is independent of $\mathcal{O}_-(k)$. In position space one has

$$\text{Lim}_{\gamma_0 \to 0} \Phi(z_0, \vec{x}) \sim (\gamma_0)^{d/2-\nu} \mathcal{O}_-(\vec{x}) + \sim (\gamma_0)^{d/2+\nu} \mathcal{O}_+(\vec{x}) \quad (3.11)$$

This is the operator manifestation of the appearance of two dual operators for a single bulk field, as observed in [13].

This is in sharp contrast with the situation in $AdS/CFT$ correspondence. In Poincare coordinates of $AdS_{d+1}$

$$ds^2 = \frac{1}{z^2} [-dt^2 + dz^2 + d\vec{x} \cdot d\vec{x}] \quad (3.12)$$

the mode expansion of a massive scalar field is

$$\Phi(z, x, t) \sim z^{d/2} \int_0^\infty d\alpha \int d^{d-1} k \left( \frac{\omega}{\omega} \right)^{1/2} J_\mu(\alpha z) [b(\vec{k}, \alpha) e^{-i(\omega t - \vec{k} \cdot \vec{x})} + h.c.] \quad (3.13)$$

where

$$\omega^2 = \vec{k}^2 + \alpha^2 \quad \mu = +\sqrt{(d/2)^2 + m^2} \quad (3.14)$$

The mode expansion involves only one of the independent solutions of the Klein Gordon expansion since the other solution is not normalizable. In this case, the field operator near the boundary $z = z_0 \to 0$ gets identified with a local boundary operator $Q(x, t)$ by [17], [18]

$$\text{Lim}_{z_0 \to 0} \Phi(z_0, x, t) = (z_0)^{\mu+d/2} Q(x, t) \quad (3.15)$$
where the fourier transform of $Q(x,t)$ on the boundary, $Q(k,\omega)$ is given in terms of the annihilation and creation operators in (3.13) by \[18\]

$$Q(\omega,\vec{k}) \sim (\omega^2 - \vec{k}^2)^{\mu/2}[\theta(\omega) \tilde{b}(\omega,\vec{k}) + \theta(-\omega) \tilde{b}^\dagger(-\omega,-\vec{k})] \quad (3.16)$$

where

$$\tilde{b}(\omega,\vec{k}) = (\frac{\omega}{\alpha})^{1/2}b(\alpha,\vec{k}) \quad (3.17)$$

In this case all of the annihilation and creation operators of the bulk field are necessary to construct the boundary operator. Consequently, for a given bulk field there is only one dual operator.

In the $AdS/CFT$ correspondence, the bulk and the boundary share the same time. If there is a $dS/CFT$ correspondence, the dual theory is euclidean. From the form of the metric it is clear that rescaling time is equivalent to rescaling distances on $I^+$. In the above correspondence we have used the momenta $\vec{k}$ to label states of the CFT. This means we are considering one of the coordinates on $I^+$, $x_2$ or $x_3$ as the euclidean time. A proper interpretation would however involve a continuation to lorentzian signature on the boundary.

Assuming that there is such a $dS/CFT$ correspondence, it is clear that the correlation functions of boundary operators can be written in terms of the Green’s functions of the bulk fields. Such an assumption has been made in e.g. \[19\] and \[20\]. However it is not clear how this is related to a $dS/CFT$ correspondence based on the equality of the effective action of the bulk theory and conformal field theory free energy in the presence of sources. For example, \[19\], \[20\] show that an operator correspondence leads to different results for CFT correlators for different members of the one parameter family of de Sitter invariant vacua found in \[21\]. However if these CFT correlators are calculated according to the prescription of \[13\], they are independent of the value of the parameter. This is because the Green’s functions (satisfying the inhomogeneous equation) for different values of this vacuum parameter differ from one another by solutions of the homogeneous equation and this addition does not change the field evaluated on the boundary according to the procedure of \[22\].

In the following we will assume an operator correspondence. While we have described this in the planar coordinate system, it is clear that this can be done in global coordinates using the modes derived in \[23\] and \[21\], or for any other coordinate system.
4. Holograms of Geodesics in $dS_3$

The simplest geodesic trajectory is the worldline of the south pole. In planar coordinates this is described by $\rho = 0$ while in static coordinates in region I this corresponds to $r = 0$. All other geodesics are obtained from this by isometries of de Sitter space. It is clear that the trajectory of any point of the spatial $S^d$ in global coordinates is a geodesic. Similarly any point on the $R^d$ in planar coordinates is a geodesic as well. Because of the maximal symmetry of the space it is sufficient to consider the geodesic at the south pole.

Consider a source moving along the geodesic in $dS_3$ which couples to a bulk scalar field $\Phi$ of mass $m$. We work in planar coordinates which cover regions I and II of the Penrose diagram. According to the previous section, the leading value of the one point function of the dual operator $O_-(\vec{x})$ in this state is given in terms of the value of the scalar field by

$$<O_-(\vec{x})> \sim \lim_{y_0 \to (y_0)} (y_0)^{1-\nu} \Phi(y_0, \vec{x})$$  \hspace{1cm} (4.1)

The field $\Phi(y, \vec{x})$ is produced by the source. When the source is at $(y, \vec{x}) = (y'(\lambda), 0)$ where $\lambda$ is the proper time along the geodesic, this is given by

$$\Phi(y, \vec{x}) = \int d\lambda \ G_R(y, \vec{x}; y'(\lambda), 0)$$  \hspace{1cm} (4.2)

where $G_R(y, \vec{x}; y', \vec{x}')$ is the retarded Green’s function with $y < y'$. The latter is given by

$$G_R(y, \vec{x}; y', \vec{x}') = i\theta(y' - y) < 0|\Phi(y, \vec{x}), \Phi(y', \vec{x}')]|0 >$$  \hspace{1cm} (4.3)

where $\Phi$ denotes the field operator. This may be readily calculated using the mode expansion (3.4). For $d = 3$ one has $\nu = \sqrt{1 - m^2}$ so that $\nu < 1$ and non-integral. We can then use the expressions for the Hankel functions

$$H^{(1)}_\nu(z) = \frac{1}{i \sin \pi \nu} [J_\nu(z) - e^{-i\pi \nu} J_\nu(z)]$$

$$H^{(2)}_\nu(z) = \frac{1}{i \sin \pi \nu} [e^{i\pi \nu} J_\nu(z) - J_\nu(z)]$$  \hspace{1cm} (4.4)

The result for $G_R$ is then

$$G_R(y, \vec{x}; y', \vec{x}') = \frac{2\theta(y' - y)}{\sin \pi \nu} \int \frac{d^2k}{16\pi^4} (y'y') e^{i\vec{k} \cdot (\vec{x} - \vec{x}')} [J_{-\nu}(|k|y) J_{\nu}(|k|y') - J_{-\nu}(|k|y') J_{\nu}(|k|y)]$$  \hspace{1cm} (4.5)

\footnote{We work in the leading order of the semiclassical expansion of the bulk theory.}
Along the trajectory \( \vec{x} = 0 \) the proper time interval \( d\lambda \) is related to the increment in the coordinate time \( dy' \) by

\[
d\lambda = -\frac{dy'}{y'} \tag{4.6}
\]

Combining (4.2), (4.5) and (4.6) we finally get

\[
\Phi(y, \vec{x}) \sim \int_{\infty}^{y} \frac{dy'}{y'} \int d^2k \ (yy') e^{i\vec{k} \cdot \vec{x}} [J_{-\nu}(|k|y)J_{\nu}(|k|y') - J_{-\nu}(|k|y')J_{\nu}(|k|y)] \tag{4.7}
\]

where we have ignored inessential constants. The field \( \Phi \) is of course a scalar. Therefore the expression in any other coordinate system may be obtained by simply reexpressing the right hand side of (4.7) in terms of the new coordinates. Alternatively one can start out with a mode decomposition in the new coordinates.

To extract the hologram, i.e. the one point function of the dual operator we have to take a limit of (4.7) when \( y = y_0 \to 0 \). In this limit the dominant contribution comes from the term \( J_{-\nu}(|k|y)J_{\nu}(|k|y') \) in (4.7), which behaves as \( |k|^{-\nu}(y_0)^{-\nu}J_{\nu}(|k|y') \). Performing the angular integral in momentum space we get the result

\[
\lim_{y_0 \to 0} \Phi(y_0, \vec{x}) = (y_0)^{1-\nu} \int_{\infty}^{y_0} dy'' \int_{0}^{\infty} d|k| \ |k|^{1-\nu} J_0(|k|\rho)J_{\nu}(|k|y'') \tag{4.8}
\]

where \( \rho = |\vec{x}| \) as before. The integral over \( y' \) may be performed using

\[
\int_{\infty}^{y_0} dy' \ J_{\nu}(|k|y') = \frac{1}{|k|} [-1 + 2 \sum_{n=0}^{\infty} J_{2n+1-\nu}(|k|y_0)] \tag{4.9}
\]

Since we have \( \nu < 1 \) we get

\[
\lim_{y_0 \to 0} \Phi(y_0, \vec{x}) = (y_0)^{1-\nu} \int_{0}^{\infty} d|k| \ |k|^{-\nu} J_0(|k|\rho) \sim \left(\frac{y_0}{\rho}\right)^{1-\nu} \tag{4.10}
\]

which leads to a one point function

\[
< \mathcal{O}_-(\vec{x}) >_{\text{planar}} \sim \frac{1}{\rho^{1-\nu}} \tag{4.11}
\]

The power is appropriate for that of an operator with dimensions \((\frac{1}{2}(1-\nu), \frac{1}{2}(1-\nu))\) in the two dimensional euclidean CFT on the boundary.

### 5. Interpreting the hologram

In the bulk, an observer comoving with the geodesic will perceive the vacuum as a thermal state with a temperature \( T = 1/2\pi \) in our units. We want to see how is this reflected in the holographic dual.
5.1. Accelerated objects in AdS$_3$ (Poincare boundary)

Before we start to interpret the hologram of a geodesic source in dS$_3$ let us recall the main results of a similar calculation of holograms of accelerated objects in AdS$_3$ coupled to a bulk massless scalar field [16]. Consider the following trajectory

\[ t = \alpha z \]  

(5.1)

in a Poincare coordinate system

\[ ds^2 = \frac{1}{z^2}[-dt^2 + dz^2 + dx^2] \]  

(5.2)

This has a uniform invariant acceleration \( a \) given by

\[ a^2 = \frac{\alpha^2}{\alpha^2 - 1} \]  

(5.3)

The parameter \( \alpha \) labels the specific trajectory. A set of observers comoving with this class of objects records a Unruh temperature \( T_U = \frac{1}{2\pi} \). The local temperature measured by a particular trajectory is related to \( T_U \) by a redshift factor, leading to \( T = \frac{1}{2\pi\sqrt{\alpha^2 - 1}} \).

When such an accelerated object couples to a massless scalar field in the bulk, the one point function of the dual operator in the boundary CFT defined on a cutoff boundary at \( z = z_B \to 0 \) can be calculated along the lines of the previous section, with the result [16]

\[ <\mathcal{O}> \sim \alpha \sqrt{\alpha^2 - 1} \frac{t}{[(\alpha^2 - 1)x^2 + t^2]^{3/2}} \]  

(5.4)

As the Poincare time \( t \) increases the object moves deeper into the bulk of AdS spacetime. The hologram, given by (5.4), reflects this: the support of the one point function spreads with time.

Consider an observer on the boundary according to whom the one point function is time-independent. This would be the hologram of a bulk observer co-moving with the accelerated object. To find such observers one must therefore look for a new coordinate system in which the one point function is time independent. It turns out that this is in fact a conformal transformation on the boundary. Recalling the fact that the conformal dimensions of \( \mathcal{O} \) are \((1, 1)\) it is easy to see that the new time \( \eta \) and the new space \( \psi \) are related to the original coordinates \((t, x)\) by

\[ t \pm x = \frac{1}{\beta} e^{-\beta(\eta \mp \psi)} \]  

(5.5)
where \( \beta \) is determined by requiring that the proper time interval for any section of the trajectory is equal to the interval in terms of the new time \( \eta \),

\[
\beta = \frac{1}{\sqrt{\alpha^2 - 1}} \tag{5.6}
\]

Then the transformed one point function is

\[
<\mathcal{O}_{\eta,\psi} > \sim \alpha \sqrt{\alpha^2 - 1} \frac{\cosh \beta \psi}{[\alpha^2 \sinh^2 \beta \psi + 1]^{3/2}} \tag{5.7}
\]

The transformation (5.5) covers only one wedge of the full Minkowski space \((t,x)\) - the wedge which corresponds to the future light cone of the point \(t=x=0\). The original boundary metric becomes

\[
e^{-2\beta \eta}[-d\eta^2 + d\rho^2] \tag{5.8}
\]

This is the metric of a Milne universe. It is well known that the Minkowski vacuum appears as a thermal state in terms of particles defined according to postive frequency using the time \(\eta\), with a temperature \(T = \frac{\beta}{2\pi} \) \cite{1}, \cite{2}. This is exactly the bulk temperature. The upshot is that acceleration radiation in the bulk is interpreted as a cosmological radiation in the boundary theory.

There is in fact a good reason why (5.5) is the correct transformation. To see this we consider a different coordinate system in \(AdS_3\), which we call BTZ coordinates. The spatial coordinates are \(\rho\) with a range \(1 < \rho < \infty\) and \(\psi\) with range \(0 < \psi < \infty\) while the time coordinate \(\eta\) has a range \(-\infty < \eta < +\infty\). The metric now reads

\[
ds^2 = - (\rho^2 - 1)d\eta^2 + \frac{d\rho^2}{\rho^2 - 1} + \rho^2 d\psi^2 \tag{5.9}
\]

The boundary is now at \(\rho = \infty\), and the boundary coordinates on a cutoff boundary \(\rho = \rho_B\) are \((\eta,\psi)\). In these coordinates the accelerated trajectory (5.1) simply corresponds to

\[
\rho = \alpha \tag{5.10}
\]

It is clear that if we calculate the one point function on a cutoff boundary using these coordinates the result will be independent of the time \(\eta\).

This explains why (5.5) is the correct transformation. The point is that the transformations (5.5) are precisely the coordinate transformation between the Poincare coordinates \((t,z,x)\) and BTZ coordinates \((\eta,\rho,\psi)\) when restricted to a cutoff boundary at \(z = z_0\) or equivalently \(\rho = \frac{1}{z_0}\).
It is important to realize that while $\Phi$ is scalar under coordinate transformations, the definition of the cutoff boundary depends on the specific coordinate system used. This makes the one point function coordinate dependent - in fact it just transforms as a conformal field with the appropriate conformal dimension.

The treatment for other coordinates in $AdS$ is entirely analogous and has been discussed in detail in \[16\].

5.2. Geodesics in $dS_3$: Planar boundary

The dual theory for $dS_3$ is euclidean. It is clear from the form of the metric that in planar coordinates time evolution in the bulk maps into scale evolution on the boundary. In terms of the complex coordinate

$$z = x^2 + ix^3 = \rho e^{i\theta_2}$$

(5.11)

the scale is represented by $\rho$.

In analogy with the case of accelerated objects in $AdS_3$ we therefore ask: is there a conformal transformation

$$z \rightarrow w = w(z)$$

(5.12)

which renders the one point function (4.11) independent of $|w|$? Because of the symmetry of the problem this means that the transformed one point function is in fact a constant.

This is indeed possible. Using the fact that the operator $O_-$ has dimensions $(\frac{\Delta_-}{2}, \frac{\Delta_-}{2})$ where $\Delta_- = 1 - \nu$ it is easy to see that the transformation is

$$z = e^w$$

(5.13)

Our discussion of a possible operator correspondence in planar coordinates imply that we can label the states of this euclidean theory by the spatial momenta of the bulk theory. In other words, one of the coordinates $x^2$ or $x^3$, e.g. $x^3$ can be regarded as an euclidean time. Analytic continuation in $x^3$ then provides one definition of a quatum theory on the boundary.

Upon analytic continuation $x^3 \rightarrow ix^3$ the one point function (4.11) becomes

$$\langle O_-(\vec{x}) \rangle_{planar} \sim \frac{1}{[(x^2)^2 - (x^3)^2]^\frac{1-\nu}{2}}$$

(5.14)

This feature has been also observed in computations of the Casimir energy, see Ghezelbash et.al. in last reference in \[14\].
Now make the transformation to coordinates $(\sigma_1, \sigma_2)$

\begin{align*}
    x^2 - x^3 &= -e^{-(\sigma_2 - \sigma_3)} \\
    x^2 + x^3 &= e^{-(\sigma_2 + \sigma_3)}
\end{align*}

(5.15)

The transformed one point function is then independent of $\sigma_i$. The transformation (5.15) is precisely the transformation between Minkowski and Rindler coordinates in two dimensional flat space. As is well known this induces a nontrivial Bogoliubov transformation between modes and the Miknowski vacuum appears as a thermal state in terms of Rindler particles with the temperature given by

\[ T = \frac{1}{2\pi}. \]

(5.16)

For the special case $(AdS_3)$ we are considering, there is another way to understand this. Define

\[ w = \sigma_1 + i\sigma_2 \]

(5.17)

It is then clear that correlation functions which are single valued in $(x^2, x^3)$ would be periodic in $\sigma_2$. This means that in an alternative definition of a quantum theory on the boundary in which $\sigma_2$ is regarded as an euclidean time, these correlators would be thermal with a temperature given by (5.16). This is exactly the temperature measured by a geodesic observer in the bulk. Note this is not the usual way a field theory on a cylinder is defined. As we will see later, this argument needs a modification in higher dimensions.

Once again there is a good reason why this is the right conformal transformation. The geodesic $\vec{x} = 0$ corresponds to the point $r = 0$ in the static coordinate system in the patch I. This coordinate patch does not contain $\mathcal{I}^+$, but the coordinate $r$ can be continued to the coordinate $r$ in the static patch in region II containing $\mathcal{I}^+$. Our experience with $AdS$ leads us to expect that the one point function in these coordinates is in fact a constant.

Let us check this explicitly. Since $\Phi$ is a scalar field, all we have to do is to find the coordinate transformation relating the static and planar coordinates and express the expression for $\Phi$ at some general point, equation (4.7), in terms of static coordinates. It is important that we do this before we take any limit which takes us to a boundary since cutoff boundaries in different coordinate systems do not coincide.

The coordinate transformations can be read off from (2.5), (2.11), (2.12) and (2.14). In Kruskal coordinates we have

\begin{align*}
    \rho &= \frac{1 + UV}{2U} \\
    y &= \frac{1 - UV}{2U}
\end{align*}

(5.18)
in both regions I and II, while in terms of the \((r, t)\) coordinates we have in region I

\[
\rho = \frac{r}{\sqrt{1 - r^2}} e^{-t} \quad y = \frac{1}{\sqrt{1 - r^2}} e^{-t} \quad \text{(Region I)} \quad (5.19)
\]

while in region II

\[
\rho = \frac{r}{\sqrt{r^2 - 1}} e^{-t} \quad y = \frac{1}{\sqrt{r^2 - 1}} e^{-t} \quad \text{(Region I)} \quad (5.20)
\]

In (4.7), the point \((y, \vec{x})\) is in region II while the points labelled by \(y'\) are all in region I. Since the trajectory has \(r = r' = 0\) we have \(y' = e^{-t'}\). For a point \((r, t, \theta)\) in region II the field is

\[
\Phi(r, t, \theta_2) = \int_{-\infty}^{t+\log \tilde{r}} dt' \int dk \, d\theta \, (e^{-t'} \frac{1}{r} e^{-t}) \, e^{ik\tilde{\rho} \cos \theta} \left[ J_{-\nu}(\frac{|k|}{r} e^{-t}) J_{\nu}(|k| e^{-t'}) \right.
\]

\[
\left. - J_{-\nu}(|k| e^{-t'}) J_{\nu}(\frac{|k|}{r} e^{-t}) \right] \quad (5.21)
\]

where we have defined

\[
\tilde{r} = \sqrt{r^2 - 1} \quad \tilde{\rho} = \frac{r}{\sqrt{r^2 - 1}} e^{-t} \quad (5.22)
\]

To determine the one point function in static coordinates we have to now go to the boundary at \(r = r_0 >> 1\). Then \(\tilde{r} \sim r_0\) and \(\tilde{\rho} \sim e^{-t}\). Performing (in order) the angular integration over \(\theta\), the integral over \(t'\) and finally the integral over \(|k|\) exactly as in Section 4 it is easy to see that

\[
< \mathcal{O}_- (t, \theta_2) >_{static} = \lim_{r_0 \to \infty} [(r_0)^{\nu-1} \Phi(r, t, \theta_2)] \sim \text{constant} \quad (5.23)
\]

Indeed the restriction of the coordinate transformations (5.20) to \(\mathcal{I}^+\) is precisely the conformal transformation (5.13), with the identifications

\[
w = t + i\theta_2 \quad (5.24)
\]

This explains why the conformal transformation (5.13) render the one point function constant.
5.3. Geodesics in $dS_3$ : Global boundary

The story is similar in global coordinates. Now the cutoff boundary is at $T = T_0 = \frac{\pi}{2} - \epsilon$ with $\epsilon \ll 1$. The transformation between the global coordinates and planar coordinates of regions I and II are given by directly comparing the formulae in Section 2,

$$
\begin{align*}
y &= \frac{\cos T}{\sin T - \cos \theta_1} \\
\rho &= \frac{\sin \theta}{\sin T - \cos \theta_1}
\end{align*}
$$

(5.25)

The worldline of the geodesic is now $\theta_1' = \pi$. The one point function on the boundary may be now calculated by evaluating the field in (4.7) by substituting (5.25) and finally performing the limit $\epsilon \to 0$. The final result is

$$
\langle O_{-}(\theta_1, \theta_2) \rangle_{\text{global}} = \lim_{(T_0 \to \frac{\pi}{2})}[(\cos T_0)^{\nu-1} \Phi(T, \theta_1, \theta_2)] \sim \left(\frac{1}{\sin \theta_1}\right)^{1-\nu}
$$

(5.26)

To understand this result, consider the transformation between global coordinates and static coordinates in Region II. These are

$$
\begin{align*}
r &= \sec T \sin \theta_1 \\
\tanh t &= -\cosec T \cos \theta_1
\end{align*}
$$

(5.27)

On $\mathcal{I}^+$ this becomes

$$
\tanh t = -\cos \theta_1
$$

(5.28)

or equivalently

$$
\tan \frac{\theta_1}{2} = e^t
$$

(5.29)

In terms of standard complex coordinates on $S^2$

$$
u = \tan \frac{\theta_1}{2} e^{i \theta_2}
$$

(5.30)

we therefore have the conformal transformation

$$
u = e^w
$$

(5.31)

where $w$ has been defined in (5.24). This is exactly the conformal transformation between the planar complex coordinate $z$ and $w$. Thus one might have expected that on the sphere we should have a one point function

$$
\left(\frac{1}{uu^*}\right)^{\frac{1-\nu}{2}} = \left(\frac{1}{\tan \frac{\theta_1}{2}}\right)^{1-\nu}
$$

(5.32)
which is not the same as (5.26).

However there is a Weyl anomaly here since the metric on the sphere is given in terms of \( u, \bar{u} \) by

\[
d s^2 = \frac{4u d\bar{u}}{(1 + u\bar{u})^2}
\]  

(5.33)

This means that while performing the conformal transformation we must account for this conformal factor which is not a product of a function of \( u \) and a function of \( \bar{u} \). This is exactly what has to be done to calculate two point functions of operators on the sphere. Taking this into account we should get

\[
\left( \frac{(1 + u\bar{u})^2}{u\bar{u}} \right)^{1-\nu}
\]  

(5.34)

which is precisely (5.26).

5.4. Higher dimensions

Many of the above considerations would generalize to other dimensions as well. For similar reasons, one point functions measured on the boundary defined in terms of static coordinates would be a constant. For planar and global coordinates the transformations to static coordinates are in fact exactly the ones given above for all dimensions, and so would be their restrictions to the boundary. The transformation laws for one point functions would be however different.

For \( AdS_{d+1} \) the boundary metric in static coordinates may be written as

\[
d s^2 = r_0^2 dt^2 + (1 - \mu^2) d\phi^2 + \frac{d\mu^2}{1 - \mu^2} + \mu^2 d\Omega_{d-3}^2
\]  

(5.35)

The normal way to interpret this theory would be to consider \( t \) as the euclidean time. The previous discussion suggests that there is another way to interpret the theory, viz via the analytic continuation

\[
\phi = i\eta
\]  

(5.36)

and regarding \( \eta \) as the time. Then the metric (5.35) becomes,

\[
d s^2 = dt^2 - (1 - \mu^2) d\eta^2 + \frac{d\mu^2}{1 - \mu^2} + \mu^2 d\Omega_{d-3}^2
\]  

(5.37)

This is the metric on \( dS_{d-1} \times R \). Constant \( \mu \) observers will perceive the invariant vacuum as a thermal bath. In fact one often uses the reverse of this argument to understand why there is thermality in de Sitter spacetimes from the bulk viewpoint. It is interesting that the holographic signature of thermality gets related to the question of thermality in de Sitter spacetimes again, albeit in two less dimensions.
6. Conclusions

We have offered a signature of the thermal properties of geodesic observers in the holographic theory. This is not quite an explanation. However we believe that this insight will be useful in a proper holographic understanding of cosmological spacetimes.

One important assumption in our work is that the one point function of a CFT operator is given by the value of the bulk field on the boundary, with suitable powers of the cutoff stripped off. We calculated the field on the boundary in a standard fashion using retarded Green’s functions. This is entirely analogous to treatments in the AdS/CFT correspondence. In the AdS/CFT correspondence this is almost forced upon us since the bulk and the boundary share the same “time”, and one would like to maintain causality. Things are less clear in the present case. Here the boundary is either $I^+$ or $I^-$. Our procedure gives the one point functions in the boundary on $I^+$ and a zero value on $I^-$. Presumably to get one point functions on $I^-$ one should use advanced Green’s functions in the bulk. Another possibility is to use a symmetric Green’s function. In fact the latter is suggested by an interesting result of [19]. These authors consider the one parameter class of invariant vacua in the bulk [21], or their planar coordinate analogs [20]. They show that a bulk geodesic observer would detect particles in all these vacua. However the spectrum is thermal only for a preferred value of the parameter - the one which leads to the analytic continuation of the euclidean bulk vacuum. This result could be reconciled with our results if one uses a symmetric Green’s function to obtain the field in the presence of a geodesic source, since while the retarded or advanced Green’s function does not depend on the vacuum parameter, the symmetric Green’s function does.

Our holograms are obtained from the fields produced by point sources. In a natural extension of terminology, this is a $\frac{1}{N}$ effect. In the AdS/CFT correspondence one needs nonlocal operators to probe local physics in the bulk [25]. Since we are probing the entire trajectory of a particle one should have a better description in terms of these objects and this would be a leading effect. It would be interesting to see analogs of these in the dS/CFT correspondence.

Finally, the presence of an external source in our discussion is not quite natural. The proper formulation of the problem would be to consider a wavepacket made out of bulk fields whose center follows a geodesic, as in [17]. The tails of these wavepackets then provide the one point functions necessary for the hologram. However one has to redo the analysis of thermal behavior by “comoving” set of observers in the bulk. Because of the nontrivial profile of the wavepacket one would get rather complicated transformations in the boundary theory.
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References

[1] See e.g. N.D. Birell and P.C.W. Davies, *Quantum fields in curved space* (Cambridge University Press, 1982)

[2] S. Hawking, *Nature* **248** (1974) 30; S. Hawking, *Comm. Math. Phys.* **43** (1975) 199.

[3] W. Unruh, *Phys. Rev.* **D14** (1976) 870.

[4] S. Deser and O. Levin, *Class. Quant. Gr.* **14** L163; S. Deser and O. Levin, *Phys. Rev.* **D59** (1999) 064004, hep-th/9809159; T. Jacobson, *Class. Quant. Grav.* **15** (1998) 251, gr-qc/9709043

[5] G.W. Gibbons and S. Hawking, *Phys. Rev.* **D15** (1977) 2738.

[6] A. Strominger and C. Vafa, *Phys. Lett.* **B379** (1996) 99, hep-th/9601029; C. Callan and J. Maldacena *Nucl. Phys.* **B472** (1996) 591, hep-th/9602043; A. Dhar, G. Mandal and S.R. Wadia, *Phys. Lett.* **B388** (1996) 51, hep-th/9605234; S.R. Das and S.D. Mathur, *Nucl. Phys.* **B478** (1996) 561, hep-th/9606185; J. Maldacena and A. Strominger, *Phys. Rev.* **D55** (1997) 861, hep-th/9609026.

[7] G. ’t Hooft, in “Salamfest” (1993) 0284, gr-qc/9310026; L. Susskind, *J. Math. Phys.* **36** (1995) 6377, hep-th/9409089.

[8] J. Maldacena, *Adv. Theo. Math. Phys.* **2** (1998) 231, hep-th/9711200; S. Gubser, I. Klebanov and A. Polyakov, *Phys. Lett.* **B428** (1998) 105, hep-th/9802109; E. Witten, *Adv. Theo. Math. Phys.* **2** (1998) 253, hep-th/9802150.

[9] B. de Wit, D. Smit and N.D. Hari Dass, *Nucl.Phys.* **B283** (1987) 165; J. Maldacena *Nucl.Phys.B283:165,1987 and C. Nunez, *Int.J.Mod.Phys.* **A16** (2001) 822.

[10] C. Hull, *JHEP* **9807** (1998) 021, hep-th/9806146; E. Silverstein, hep-th/0106209; C.M. Hull, *JHEP 0111* (2001) 012, hep-th/0109213; and *JHEP 0111* (2001) 061, hep-th/0110048; C. Madeiros, C. Hull and B. Spence, hep-th/0111190; G. Gibbons and C. Hull, hep-th/0111072.

[11] V. Balasubramanian, P. Horava and D. Minic, *JHEP* **0105** (2001) 043, hep-th/0103171.

[12] E. Witten, hep-th/0106109.

[13] A. Strominger, hep-th/0106113.

[14] M. Li, hep-th/0106184; D. Klemm, hep-th/0106247; S. Nojiri and S.D. Odinstov, hep-th/0106191, hep-th/0107134; A. Tolley and N. Turok, hep-th/0108119; T. Shiromizu, S. Ida and T. Torii, hep-th/0109057; B. McInnes, hep-th/0110062; A. Strominger, hep-th/0110067; V. Balasubramanian, J. de Boer and D. Minic, hep-th/0110108; B. Carniero de Cunha, hep-th/0110169; R. Cai, Y. Myung and Y. Zhang, hep-th/0110234; U. Danielsson, hep-th/0110265; S. Ogushi, hep-th/0111008; A. Petkou and G. Siopsis, hep-th/0111085; A.M. Ghezelbash and R.B.
Mann, JHEP 0201 (2002) 005, hep-th/0111217; A.M. Ghezelbash, D. Ida, R.B. Mann, T. Shiromizu, hep-th/0201004.

[15] R. Bousso, hep-th/0012052, JHEP 9906 (1999) 028 hep-th/9906022 and JHEP 0011 (2000) 038 hep-th/0010252; T. Banks, hep-th/0007146.

[16] S.R. Das and A. Zelnikov, Phys. Rev. D64 (2001) 104001 hep-th/0104198.

[17] V. Balasubramanian, P. Kraus and A. Lawrence, Phys. Rev. D59 (1999) 046003, hep-th/9805171; V. Balasubramanian, P. Kraus, A. Lawrence and S. Trivedi, Phys. Rev. D59 (1999) 104021, hep-th/9808017; T. Banks, G. Horowitz and E. Martinec, hep-th/9808; E. Keski-Vakkuri, Phys. Rev. D59 (1999) 104001, hep-th/9808037; U. Danielsson, E. Keski-Vakkuri and M. Kruczenski, JHEP 9901 (1999) 002, hep-th/9812007.

[18] S.R. Das and B. Ghosh, JHEP 0006 (2000) 043, hep-th/0005007.

[19] R. Bousso, A. Maloney and A. Strominger, hep-th/0112218.

[20] M. Spradlin and A. Volovich, hep-th/0112223.

[21] E. Mottola, Phys. Rev. D 31 (1985) 754; B. Allen, Phys. Rev. D 32 (1985) 3136.

[22] S. Giddings, Phys. Rev. Lett. 83 (1999) 2707 hep-th/9903048.

[23] N.A. Chernikov and E.A. Tagirov, Ann. Poinc. Phys. Theor. A 9 (1968) 109; E.A. Tagirov, Annals. Phys. 76 (1973) 561; R. Figari, R. Hoegh-Krohn and C. Nappi, Comm. Math. Phys. 44 (1975) 265; H. Rumpf and H.K. Urbantke, Ann. Phys. 114 (1978) 332; L. Abbott and S. Deser, Nucl. Phys. B 195 (1982) 76; A.H. Hajmi and A. Ottewill, Phys. Rev D 30 (1984) 1733; L. Ford, Phys. Rev. D 31 (1985) 710; B. Allen and A. Folacci, Phys. Rev. D 35 (1987) 3331; C.J. Burgess, Nucl. Phys. B 247 (1984) 533.

[24] T. Tanaka and M. Sasaki, Phys. Rev. D55 (1997) 6061.

[25] J. Polchinski, L. Susskind and N. Toumbas, Phys.Rev. D60 (1999) 084006, hep-th/9903223; N. Toumbas and L. Susskind, Phys.Rev. D61 (2000) 044001, hep-th/9909013.