**Abstract.** We report on our recent calculation of the inclusive direct photo- and hadroproduction of the \( J/\psi \) meson at next-to-leading order within the factorization formalism of nonrelativistic QCD. We fit the color-octet (CO) long-distance matrix elements (LDMEs), to be extracted from experiment. The relative importance of the latter can be estimated by means of velocity scaling rules; i.e., the LDMEs are predicted to scale with a definite power of the heavy-quark velocity \( v \) in the limit \( v \ll 1 \). In this way, the theoretical predictions are organized as double expansions in \( \alpha_s \) and \( v \). A crucial feature of this formalism is that it takes into account the complete structure of the \( q\bar{q} \) Fock space, which is spanned by the states \( n = 2S+1 L_{J}^{[a]} \) with definite spin \( S \), angular momentum \( L \), total angular momentum \( J \), and color multiplicity \( a = 1, 8 \). In particular, this formalism predicts the existence of color-octet (CO) processes in nature. This means that \( q\bar{q} \) pairs are produced at short distances in CO states and subsequently evolve into physical, color-singlet (CS) quarkonia by the nonperturbative emission of soft gluons. In the limit \( v \to 0 \), the traditional CS model (CSM) is recovered in the case of \( S \)-wave quarkonia. In the case of \( J/\psi \) production, the CSM prediction is based just on the \( ^1S_0^{[1]} \) CS state, while the leading relativistic corrections, of relative order \( \mathcal{O}(v^4) \), are built up by the \( ^1S_0^{[8]}, ^3S_1^{[8]} \), and \( ^3P_0^{[8]} \) \( (J = 0, 1, 2) \) CO states.

The greatest success of NRQCD was that it was able to explain the \( J/\psi \) hadroproduction yield at the Fermilab Tevatron [2], while the CSM prediction lies orders of magnitudes below the data, even if the latter is evaluated at NLO [3, 4]. The situation is similar for the transverse momentum \( (p_T) \) distribution at BNL RHIC [5]. Also in the case of \( J/\psi \) photoproduction at DESY HERA, the CSM cross section at NLO significantly falls short of the data [6, 7]. Complete NLO calculations for the CO contributions were performed for inclusive \( J/\psi \) production in two-photon collisions [8], \( e^+e^- \) annihilation [9], and direct photoproduction [7]. As for hadroproduction at NLO, before this talk was given, the CO contributions due to intermediate \( ^1S_0^{[8]} \) and \( ^3S_1^{[8]} \) states [4] were calculated as well as the complete NLO corrections to \( \Upsilon \) production, including both \( \Upsilon^{[1]} \) and \( \Upsilon^{[8]} \) contributions [10]. In order to convincingly establish the CO mechanism and the LDME universality, it had been an urgent task to complete the NLO \( J/\psi \) hadroproduction calculation by including the full CO contributions. This was actually achieved in the work [11] presented at this conference.

Our strategy for testing NRQCD factorization in \( J/\psi \) production at NLO is as follows. We first perform a common fit of the CO LDMEs to the \( p_T \) distributions measured by CDF in hadroproduction at Tevatron Run II [12] and by H1 in photoproduction at HERA1 [13] and HERA2 [14] (see Table I and Fig. 1). We then compare the \( p_T \) distributions measured by PHENIX at RHIC [15] and CMS at the LHC [16] as well as the \( W \) and \( Z \) distributions measured by H1 at HERA1 [13] and HERA2 [14] with our respective NLO predictions based on these CO LDMEs (see Fig. 2). For details on the calculation and the input parameters used, we refer the reader to Ref. [11].

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\(^1\) Speaker
TABLE 1. NLO fit results for the $J/\psi$ CO LDMEs.

| $\langle \theta J/\psi (1S_0^8) \rangle$ | $(4.76 \pm 0.71) \times 10^{-2}$ GeV$^3$ |
| $\langle \theta J/\psi (3P_0^8) \rangle$ | $(2.65 \pm 0.91) \times 10^{-3}$ GeV$^3$ |
| $\langle \theta J/\psi (3P_1^8) \rangle$ | $(-1.32 \pm 0.35) \times 10^{-2}$ GeV$^5$ |

FIT TO HERA AND TEVATRON DATA

The $p_T$ distribution of $J/\psi$ hadroproduction measured experimentally flattens at $p_T < 3$ GeV due to nonperturbative effects, a feature that cannot be faithfully described by fixed-order perturbation theory. We, therefore, exclude the CDF data points with $p_T < 3$ GeV from our fit. We have checked that our fit results depend only feebly on the precise location of this cutoff. The fit results for the CO LDMEs corresponding to our default NLO NRQCD predictions are collected in Table 1. In Figs. 1(a) and (b), the latter (solid lines) are compared with the CDF [12] and H1 [13, 14] data, respectively.

For comparison, also the default predictions at LO (dashed lines) as well as those of the CSM at NLO (dot-dashed lines) and LO (dotted lines) are shown. In order to visualize the size of the NLO corrections to the hard-scattering cross sections, the LO predictions are evaluated with the same LDMEs.

We observe from Fig. 1(c) that the $3P_J^8$ short-distance cross section of hadroproduction receives sizable NLO corrections that even turn it negative at $p_T \gtrsim 7$ GeV. This is, however, not problematic because a particular CO contribution represents an unphysical quantity depending on the NRQCD scale $\mu_A$ and the choices of the renormalization scheme and is entitled to become negative as long as the full cross section remains positive.

In contrast to the situation at LO, the line shapes of the $1S_0^8$ and $3P_J^8$ contributions significantly differ at NLO. Therefore we can now, in our combined HERA-Tevatron fit, independently determine $\langle \theta J/\psi (1S_0^8) \rangle$ and $\langle \theta J/\psi (3P_J^8) \rangle$. Notice that $\langle \theta J/\psi (3P_0^8) \rangle$ comes out negative, which is not problematic for the same reasons as explained above for the short distance cross sections.

PREDICTIONS FOR FURTHER DATA

We observe from Fig. 2 that our NLO NRQCD predictions nicely describe the $p_T$ distributions from PHENIX [15] (a) and CMS [16] (b) as well as the $W$ distributions from H1 [13, 14] (c), with most of the data points falling inside the yellow (light shaded) error band. The NLO NRQCD prediction of the $z$ distribution (d) agrees with the H1 data in the intermediate $z$ range, but its slope appears to be somewhat too steep at first sight. However, the contribution due to resolved photoproduction, which is not yet included here, is expected to fill the gap in the low-$z$ range, precisely where the resolved contribution is peaked. Near the high-$z$ endpoint region, the NRQCD expansion is understood to break down, and the NRQCD series could be resummed via the introduction of universal shape functions [17], possibly in the context of soft collinear effective theory [18].

FIGURE 1. NLO NRQCD predictions of $J/\psi$ hadro- and photoproduction resulting from the fit compared to the CDF [12] and H1 [13, 14] input data.

FIGURE 2. NLO NRQCD predictions of $J/\psi$ hadro- and photoproduction resulting from the fit compared to RHIC [15], CMS [16], and H1 [13, 14] data not included in the fit.
CONCLUSIONS

We have performed a complete NLO calculation of the inclusive \( J/\psi \) direct photo- and hadroproduction cross sections including the full CS and CO contributions.

We conclude from Figs. 1(a) and (b) and 2(a)–(d) that all experimental data sets considered here significantly overshoot the NLO CSM predictions, by many experimental standard deviations. Specifically, the excess amounts to 1–2 orders of magnitude in the case of hadroproduction and typically a factor of 3 in the case of photoproduction. On the other hand, these data nicely agree with the NLO NRQCD predictions, apart from well-understood deviations in the case of the \( z \) distribution of photoproduction.

In our work for the first time a multi-process fit of the CO LDMEs is performed, which come out to be consistent with NRQCD scaling rules. Our work therefore constitutes the most rigorous evidence for the existence of CO processes in nature and the LDME universality since the introduction of the NRQCD factorization formalism 15 years ago [1].

We should remark that our theoretical predictions refer to direct \( J/\psi \) production, while the CDF and CMS data include all prompt events and the H1 and PHENIX data even non-prompt ones. However, the resulting error turns out to be small against our theoretical uncertainties and has no effect on our conclusions.

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COMPARISON WITH REF. [19]

As a last point, we compare our results with those obtained by the authors of Ref. [19], another full NLO NRQCD calculation of \( J/\psi \) hadroproduction, which appeared after this talk was held. Adopting their inputs, we find agreement with their results for the \( 3\lambda_0^{[8]} \) and \( 3\lambda_1^{[8]} \) ones multiplied by \( M_0 \) and \( M_1 \), respectively. Determining the very combinations \( M_0 \) and \( M_1 \) from a three-parameter fit just to the Tevatron data with \( p_T > 7 \) GeV, we find that \( M_1 \) has attached to it an error of almost 100%. This is because \( M_0 \) and \( M_1 \) do not precisely correspond to eigenvectors of the three-dimensional covariance matrix and the linear combination of \( \langle \sigma^{J/\psi}(1\lambda_0^{[8]}) \rangle \), \( \langle \sigma^{J/\psi}(3\lambda_1^{[8]}) \rangle \), and \( \langle \sigma^{J/\psi}(3\lambda_1^{[10]}) \rangle \) corresponding to the third eigenvector carries a sizable error feeding into \( M_1 \).



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