Foreseeing Neutrino spectra in Deep Core

Daniele Fargion and Daniele D’Armiento,
Paola Di Giacomo, Paolo Paggi
Physics Department, Rome University 1, INFN 1 Roma, Ple. A. Moro 2, Italy
E-mail: daniele.fargion@roma1.infn.it

Abstract. In the present paper we evaluate the effect of neutrino flavor mixing and possible CPT violation (MINOS like) for up-going atmospheric muon neutrinos measured with the Deep Core detector of the IceCube Neutrino Observatory at South Pole. Our evaluation of rate and spectrum is based on the known Cosmic Ray flux and its consequent atmospheric neutrino spectrum derived at Super-Kamiokande, and it takes into account propagation and flavor mixing inside the Earth, interaction into the Deep Core (variable) volume, and projection along the physical Deep Core detector. Our results differ from other ones; however when the Deep Core results will be available our predictions can be compared directly with them. Interestingly, the Minos CPT violation hint might be confirmed or not.

The observed spectrum should also exhibit an anomalous minimum, due to neutrino oscillation, offering a “silent” narrow detection window for neutrinos with energies around 20 GeV, which could be very useful for a new Neutrino Astronomy.

1. Introduction
Mixing among muon and tau neutrino flavors has been observed since nearly 13 years, and it has been already considered in several recent high energy astrophysical frameworks [1],[2],[3]. Recently MINOS results [4] hint for a CPT violation in neutrino sector. In a previous paper [5] we studied the potentialities of a hypothetical long base experiment with a neutrino beam from CERN (OPERA like experiment) or Fermilab (FNL) across the Earth to the Deep Core detector of the IceCube Neutrino Observatory at South Pole, in order to clarify such a puzzle, i.e. to confirm or not the CPT violation claim. This experiment might provide a novel signature for muon neutrino and anti-neutrino mixing, while it could more generally focus on the study of the neutrino flavor mixing [5].

In the present paper, starting from a previous work [6], we address a different scenario, i.e. the impact of neutrino flavor mixing and possible CPT violation (MINOS like) directly on the signal of up-going atmospheric muon neutrinos in the Deep Core detector. The accurate evaluation of the spectrum, elaborated here in detail, is based on the known Super-Kamiokande (SK) data [7, 8], scaled to Deep Core volumes (and environment): the production of atmospheric neutrinos up to their interactions, the corresponding muon event rate and tracks projected along each channel string\(^1\) of the Deep Core detector are considered. Results are somehow surprising and not completely in agreement with other independent evaluations [9].

\(^1\) The detector is composed by an array of "strings", each holding several Digital Optical Modules (DOMs) deployed below the surface of the South Pole ice.
2. From the atmospheric cosmic ray to the event rate in Deep Core

Let us remind the neutrino energy flux defined as

\[ \phi_{\nu} (E_{\nu}) = \frac{dN_{\nu}}{dE_{\nu} \, dt \, dA \, d\Omega} \]

that we parameterized, here for the first time, fitting the well known curve from ref.[7] for muon neutrinos with the following analytical formula:

\[ \phi_{\nu} (E_{\nu}) = 10^2 \left( \frac{1}{9.5 \, E_{\nu}^{-1.12}} + \frac{1}{4} + \frac{1}{27 \, E_{\nu}} \right)^{-1} \text{m}^{-2} \text{s}^{-1} \text{sr}^{-1} \text{GeV}, \]

where the energy \( E_{\nu} \) is expressed in GeV units; the energy range of this formula is extended from \( \sim 100 \text{ MeV} \) up to \( 1 \text{ TeV} \). This averaged isotropic spectrum must be enhanced (empirically) at horizons (with mild energy dependence) because of longer pion, kaon and muon decay flights. We disregarded for Deep Core the geomagnetic cut off influence; its role at lowest energy has been taken into account at SK for azimuthal directions). To take into account the enhancement (of horizontal with respect to vertical rate) we derived (from a fit to the SK data) a correction factor to be multiplied to the previous flux, eq.2:

\[ F(E_{\nu}, \theta) = f(E_{\nu}, \theta) + \left( 1 - \frac{\int_{0}^{\pi/2} d\theta f(E_{\nu}, \theta)}{\pi/2} \right) \]

with

\[ f(E_{\nu}, \theta) = \left[ 1 + \left( \frac{E_{\nu}}{50} \right)^{\frac{7}{2}} (\sin \theta)^{0.7} \right]^{-1}, \]

where the energy is in GeV units, and the horizontal angle \( \theta \) (complementary to the zenith angle \( \hat{\theta} \)) ranges from 0 (horizon) to \( \pi/2 \) (upward vertical direction). The total mass target in Deep Core for incoming neutrino is taken from the effective volume estimated (for each energy) in [9], without online veto. The ratio between all events (fully contained (FC), partially contained (PC), and upward stopping and through going, i.e. without on line veto, events) and only FC+PC, i.e. with on-line veto, events is unity up to \( \sim 100 \text{ GeV} \), reaching a ratio \( \sim 1.5 \) at 400 GeV, or \( \sim 2 \) at 1 TeV. At the highest energies the event rate is low and it is not so relevant in our neutrino oscillation study, mostly focused below 200 GeV and signals below 50 channels, with the channel number defined as the number of illuminated DOMs along the muon track. Therefore we considered here all the events without veto, with less than 50 channels, and within the Deep Core energy dependent volume (following Ref. [9]):

\[ V(E_{\nu}) = 1.17 \cdot 10^6 \cdot (5.5 \, E_{\nu}^{0.2} - 5.5) \cdot \left[ 3.2 + 0.8 \cdot \ln \left( \frac{E_{\nu}}{80} \right) \right] \text{m}^3. \]

with \( E_{\nu} \) in GeV units. Neutrino Charge Current (CC) cross section for muon production is considered (see ref. [10]) to be linear increasing with energy, in the energy range considered, while antineutrino cross section is taken about a half of the neutrino one. Electron neutrino and tau neutrinos CC and all flavor neutral current (NC) events are source of spherical showers whose role may influence (and pollute) only few channels (3-5); they will be discussed in detail elsewhere.

The present calculation includes neutrino oscillation probability \( P_{\nu_i \rightarrow \nu_j}(E_{\nu}, \theta), i = 1, 2, 3 \) as a function both of energy and arrival observation angle, given the Earth chord distance for each angle below the horizon; we take into account three neutrino flavor oscillations (see [5]), discussing the out-coming (observable) muon tracks. We also take into account the matter
Figure 1. The differential number of event counts per year in Deep Core as a function of the muon neutrino energy (3–35 GeV) and angle, assuming vacuum along the neutrino flight. The horizontal axis is the energy in GeV, the vertical one is the number of event counts per year in Deep Core, the depth axis is the zenith angle variable in radians.

influence on the mixing inside the Earth assuming at each zenith angle the corresponding average Earth density. We will show elsewhere that the exact integral in the variable Earth density do not change our present results in the 10–100 GeV range, but mostly induces changes at lower energies.

Thus we obtain the differential number of events (per unit energy, solid angle, and unit time) as a function of neutrino energy and the horizontal angle \( \theta \) (in our definition \( \theta \) ranges from 0° at horizon to 90° at the vertical upward direction):

\[
\frac{dN_\mu}{dE_\nu \, dt \, d\theta} (E_\nu, \theta) \cdot \left( \frac{\Delta t}{\text{yr}} \right) = 2\pi \frac{\phi_\nu(E_\nu)}{E_\nu^2} f(E_\nu, \theta) \sigma_{\text{CC}}(E_\nu, \theta) N_A V(E_\nu) \left( \frac{\Delta t}{\text{yr}} \right) P_{\nu_\mu \to \nu_\mu}(E_\nu, \theta). \tag{6}
\]

See Fig.1 for the energy range 3–35 GeV, and Fig.2 for the energy range 3–65 GeV. The event rate in one year of observation is obtained integrating over \( \theta \) from 37° below the horizon (equal to zenith angle \( \cos(\hat{z}) = \cos(\pi/2 - \theta) = \sin(\theta) = 0.6 \) in Deep Core zenith angle) to 90°, and integrating over an appropriate energy bin:

\[
\Delta N_\mu(E_\nu) = \frac{dN_\mu}{dt}(E_\nu) \left( \frac{\Delta t}{\text{yr}} \right) = \int_{E_i + \Delta E}^{E_i} \int_{\theta = 37^\circ}^{90^\circ} \frac{dN_\mu}{dE \, dt \, d\theta}(E_\nu, \theta) \cdot \left( \frac{\Delta t}{\text{yr}} \right) d\theta \, dE_\nu, \tag{7}
\]

where \( N_A \) is the Avogadro number (multiplied by 10^6, because ice density \( \rho_{\text{ICE}} \approx 10^3 \text{ kg m}^{-3} \)) and \( V(E_\nu) \) is the effective interaction volume in Deep Core (expressed in m^3, like eq. (5)). The resulting event distributions are shown in figs. 3, 4, 5, and 6 for several energy resolutions (20 MeV with neutrino propagation in vacuum, and 20 Mev, 1 GeV, 2.8 GeV with neutrino propagation in matter). Then the same spectra of muon tracks has to be projected along the vertical axis of Deep Core, as shown finally in fig. 7.
Figure 2. As the previous figure but in a wider energy range (3–65 GeV), and assuming an average Earth density $\rho = 7$ during up-going neutrino flight within Earth. Most of the inner oscillating structure is lost because of the Deep Core un-ability to reveal a few GeVs muon track. The flux suppression at 20 GeV survives and is responsible of the spectrum minimum in Channel 6–9.

Figure 3. The ideal count rate that Deep Core would be able to observe with an energy resolution of 20 MeV. It is evident the difference between non oscillated case (red non oscillating curve), the mixed CPT conserved case (oscillating blue curve), and the thin-dashed CPT violated (green) alternative signal. The propagation in vacuum is assumed.
Figure 4. As the previous figure, where neutrino mixing occurs in matter. Note the little difference in spectra at very low energies.

Figure 5. As the previous figure, but with an energy resolution of 1 GeV. Most, but not all, of the low energy oscillations are lost.

The muon produced in a Charged Current interaction has nearly half the neutrino energy, and undergoes an average energy loss of \( dE_\mu/dX \simeq 200 \text{ MeV m}^{-1} \) or 1/5 GeV m\(^{-1}\). Therefore, the muon length (keeping in mind that \( E_\mu \sim E_\nu/2 \)) in ice is approximately: \( L_\mu = 5 E_\nu/2 \text{ m} \), with energy as usual expressed in GeV. We used indeed the exact muon length in ice:

\[
L_\mu = 3.9^{-1} \cdot 10^4 \ln \left( 1 + \frac{3.9}{4.0} \cdot 10^{-3} E_\nu \right) \text{ m.} \tag{8}
\]

To obtain a comparable plot of events distribution versus channel number we assumed an average projection factor, \( \langle \cos(\pi/2 - \theta) \rangle = \langle \sin \theta \rangle \) of a muon track along the DOM strings due to the
Figure 6. As the previous figure, but with an energy resolution of 2.8 GeV. The smearing of the low energy oscillations is complete.

Figure 7. The most realistic count rate per year as a function of the channel with an energy resolution of 3.88 GeV. As above the red curve marks the un-oscillated case, the blue one the CPT conservation case and the thin-dashed green line the CPT violation one.

spread of the muon track direction (in the range $0.6 < \sin \theta < 1$). We have two projections: the first one has to be $\langle \sin \theta \rangle = 0.8$ at the same plane containing the fired string; an additional projection factor 0.8 is needed because muons tracks may be often skewed with respect to the string line. The overall factor ranges (in average between 0.82 and 0.8). We assumed the medium projection factor equal to 0.72. Finally the number of events for each channel is:

$$
\Delta N_\mu(N_{ch}) = \frac{dN_\mu}{dt}(E_\nu) \left( \frac{\Delta t}{\nu T} \right) = \int_{E_i}^{E_i+\Delta E_{ch}/(\cos \theta)} \int_{\theta=90^\circ}^{\theta=37^\circ} \int_{E_{\nu}}^{E_{\nu}+\Delta E_{\nu}} \frac{dN_\mu}{dE_\nu d\theta}(E_\nu, \theta) \cdot \left( \frac{\Delta t}{\nu T} \right) \ d\theta \ dE_\nu.
$$

(9)
Figure 8. Comparison of our expected count rate (histograms) for upgoing muon tracks versus the most recent Deep Core expectations (dots) [9]. As above the red curve marks the un-oscillated case, the blue one the CPT conservation case and the thin-dashed green line the CPT violation one. The black (un–oscillated case) and red (oscillated case - CPT conserved) dots are the spectrum predictions from Ref.[9]. Our blue line should be compared to the red dots, and our red line to black dots.

The distance between contiguous DOMs corresponds to a minimum muon energy of 2.8 GeV (as for a vertical muon), but because of the inclined muon track projected along the vertical string $\sin \theta \approx 0.72$, on average, to a greater energy value $2.8/0.72 = 3.88$ GeV. This projecting factor smears the event rate spectrum, and it masks most of the low energy neutrino oscillations. However the main deep (minimum) in the channel spectrum (see Fig. 7) at about 20 GeV survives the smearing and it characterizes our expected count rate.

Our results differ from an independent evaluation of the channel spectra [9], see Fig.8, especially in the lowest channel region (3-4), where we expect an excess of events. Moreover in the case of neutrino oscillations (CPT conserved case) the maximum of our spectrum is around channel 10, while it is around channel 18-20 for the spectrum from Ref.[9]. When the Deep Core telescope rate spectrum will be published our predictions can be compared with it. If at some channels (mostly 3-4-5) there will be an over production of signals, than the Minos CPT violation hint may be confirmed. It should be taken into account that NC (neutral current) events at tens GeV may spread light in spherical shape increasing and polluting the signal with thousands of events ($\approx 3000 - 4000$) in channels 2-5. However their timing and intensity structure may help in the rejection of this background. In any case channels from 6 to 8 present low atmospheric background offering a rare window for 20 GeVs neutrino astronomy. To improve the observations for this astronomy in the whole arrival directions, we suggest to double the present Deep Core array strings, allowing a simultaneous double string detection. The highest Solar Flare neutrinos
3. Conclusions
In the present paper we show step by step the procedure to estimate the Deep Core rate and spectrum based on the known Cosmic Ray rate, on the derived muon neutrino flux at SK, taking into account flavor mixing during propagation (see Appendix) and interaction in Deep Core (variable) volume, leading to lepton track signals in the Deep Core detector. Our results differ from other ones [9], see Fig 8. When the Deep Core telescope rate spectrum will be published our predictions can be compared with it, and the Minos CPT violation hint might be confirmed or not.

The minimum in the rate spectrum for atmospheric neutrinos around channel 6 offers a narrow window around twenty GeV that could be very useful for Neutrino Astronomy. By doubling the string array we may foresee a higher rate, a more complete (zenith and azimuth) atmospheric neutrino detection, and a new exciting neutrino Astronomy. The highest Solar Flare neutrinos [3] may be among the first interesting signals.

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4. Appendix: 3 flavor neutrino mixing
The evolution equation for three neutrino mixing in matter reads:

\[ i \frac{d}{dx} \Psi_{\alpha} = H \Psi_{\alpha}, \quad \text{with} \quad H = \frac{1}{2E} \left( UMU^\dagger + A \right), \quad \alpha = 1, 2, 3 \]  

(10)

which is a Schrödinger equation with a matter potential \( A \),

\[ A = \begin{pmatrix} A_{CC} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad M = \begin{pmatrix} 0 & 0 & \Delta m^2_{21} \\ 0 & \Delta m^2_{21} & 0 \\ \Delta m^2_{31} & 0 & 0 \end{pmatrix}, \quad \Psi_{\alpha} = \begin{pmatrix} \psi_{\alpha e} \\ \psi_{\alpha \mu} \\ \psi_{\alpha \tau} \end{pmatrix} \]

where \( A_{CC} = 2\sqrt{2}E G_F N_e = 0.76 \times 10^{-4} eV^2(\frac{E}{GeV})(\frac{\rho}{g/cm^3}) \) is the potential only due to \( \nu_e \) charged current interaction.

This equation is a set of 9 differential equations whose solutions for probability conversion \( P_{\alpha\beta} = \Psi_{\alpha\beta}^2 \), with \( \alpha, \beta = e, \mu, \tau \), has boundaries conditions \( P_{\alpha\beta}(0) \) equal to 1 for the starting flavor \( \mu \), 0 for the others (electron conversion into muon is negligible). This system of equations is symmetric since \( P_{\alpha\beta} = P_{\beta\alpha} \).

Among the simplest numerical solutions to show, we found a compact survival probability, for instance in vacuum for muon to muon, at a distance equal to the Earth diameter, corresponding to \( \theta = 90^\circ \), as a function of neutrino energy:

\[ P_{\mu\mu} = |0.240128 + 0.304714 e^{-2.5859/En} + 0.455158 e^{-75.96071/En}|^2 \]

The more general case taken into account in the present study, with earth matter density included, is too complex to be reported here.

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