On the invariance of the string partition function under duality

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Abstract

We consider the $N = 1$ supersymmetric $\sigma$-model and we examine the transformation properties of the partition function under target-space duality. Contrary to what one would expect, we find that it is not, in general, invariant. In fact, besides the dilaton shift emerging from the Jacobian of the duality transformation of the bosonic part, there also exist a Jacobian for the fermionic part since fermions are also transform under the duality process. The latter is just the parity of the spin structure of the word sheet and since it cannot be compensated the dual theory is not equivalent to the original one.

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Target-space duality originally appeared as a discrete symmetry of the spectrum of states of the closed string theory compactified on a torus [1]. It was later realized that the same symmetry can be extended to arbitrary backgrounds with at least one space-like abelian isometry [2]–[8]. One may gauge this isometry and by adding a Lagrange multiplier may render the gauge field strength zero. The original theory is recovered by integrating the Lagrange multiplier while by integrating over the gauge field and then fixing the gauge the dual theory is obtained. The resulting background is in general different (even topologically) from the original one and, in addition, there exists a change of the fermion fields [2]. The original and the dual theory, although they seem to be different, are equivalent as classical theories. However, the measure in the partition function is also transforms under the process. In particular, the bosonic part of the measure give rise to the dilaton shift which is necessary for conformal invariance of the dual theory (if the original one is). On the other hand, as we will see, the fermion measure transforms as well under duality which is ultimate connected to the different way that left and right-handed fermions are transformed.

Let us consider a $N = 1$ supersymmetric $\sigma$-model defined on a 2-dim space-time $\Sigma$ with metric $g^{(2)}_{\alpha\beta}$ of $(−, +)$ signature and no antisymmetric field. The action for this model, in the conformal gauge, is

$$S = \frac{1}{2\pi} \int d^2\sigma \left( \frac{1}{2} \partial_{\mu} X^I \partial^{\mu} X^J g_{IJ} + ig_{IJ} \bar{\psi}^I D \psi^J + \frac{1}{6} R_{IJKL} \bar{\psi}^I \psi^K \bar{\psi}^J \psi^L \right),$$

where the scalars $(X^I, I, J = 1, \cdots, D)$ parametrize the D-dimensional target space $M$ and the world-sheet Majorana fermions $\psi^I$ are target-space vectors. The metric on $M$ is $g_{IJ}$, the Riemann tensor $R_{IJKL}$ is with respect to the Christoffel connection $\Gamma^I_{JK}$ and we have defined $D \psi^I = \partial \psi^I + \Gamma^I_{KL} \partial X^K \psi^L$, ($\partial = \gamma^\mu \partial_{\mu}$ with $\gamma^\mu = (i\sigma^2, \sigma^1)$ and $\bar{\gamma} = \gamma^0 \gamma^1$ is the corresponding $\gamma^5$ matrix in 2 dimensions). The action (1) is invariant under the supersymmetry transformations

$$\delta X^I = \bar{\epsilon} \psi^I, \quad \delta \psi^I = -i \bar{\phi} X^I \epsilon - \Gamma^I_{KL} \bar{\epsilon} \psi^K \psi^L.$$

as well as under reparametrizations (diffeomorphisms) of $M$

$$X^{I'} = X^{I'}(X^J), \quad \psi^{I'} = \frac{\partial X^{I'}}{\partial X^J} \psi^J.$$

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which, moreover, commute with the supersymmetry transformations of eq. (3). The partition function of the theory is obtained by integrating over all \((X^I, \psi^I)\) fields and it is given by

\[
Z = \int d\mu_b d\mu_f e^{-iS} = \int [\sqrt{g} DX^I][\frac{1}{\sqrt{g}} D\psi^I] e^{-iS},
\]

(4)

where \(g = \det(g_{IJ})\) is the determinant of the target-space metric. Analytic continuation to Euclidean time is necessary for a proper definition of the partition function in eq.(4). However, such a continuation can be performed after the duality transformation. We have not considered the ghost action because the ghost sector is not affected by the presence of the background metric \([10]\) and thus, it is irrelevant for our purposes. The measure \(d\mu_b d\mu_f\) has been defined by requiring that

\[
1 = \int d\mu_f e^{-||\delta\psi||^2},
1 = \int d\mu_b e^{-||\delta x||^2},
\]

(5)

for the fermionic \(d\mu_f\) and bosonic \(d\mu_b\) measure, respectively, where

\[
||\delta\psi||^2 = \frac{1}{\pi} \int d^2\sigma \sqrt{|g(2)|} g_{IJ} \delta \bar{\psi}^I \delta \psi^J,
||\delta x||^2 = \frac{1}{\pi} \int d^2\sigma \sqrt{|g(2)|} g_{IJ} \delta X^I \delta X^J.
\]

(6)

In this way the measure employed in eq.(4) is obtained. It should be noted also that it is, in addition, invariant under target-space reparametrizations (eq.(3)). Clearly, \(d\mu_b\) is invariant under eq.(3) and \(d\mu_f\) is also invariant since \(\psi^I\) are Grassmann and thus \(d\psi^I\) transforms with the inverse Jacobian (For more details see ref.[11]).

We will assume now, that \(M\) has an abelian isometry, orthogonal, for simplicity, to the surfaces of transitivity, so that the target space metric may be written as

\[
g_{IJ}(X^K) = (g_{ij}(X^k), g_{00}(X^k)),
\]

(7)

where \(X^K = (X^k, X^0 = X)\), \((i, j, k = 1, \cdots, d - 1)\) are adapted coordinates in the target space so that the isometry is generated by \(\partial/\partial X^0\) in these coordinates. The action of this model, as follows from eqs.(11) is then

\[
S = \int d^2\sigma \left( \frac{1}{2} \partial_\mu X^i \partial^\mu X^j g_{ij} + \frac{1}{2} \partial_\mu X \partial^\mu X g_{00} + \frac{1}{2} i g_{ij} \bar{\psi}^j (\partial \psi^i + \Gamma^i_{jk} \partial X^k \psi^j + \Gamma^j_{00} \partial X^i \psi^0) + \frac{1}{2} i g_{00} \bar{\psi}^0 (\partial \psi^0 + \Gamma^0_{0k} \partial X^k \psi^0 + \Gamma^0_{k0} \partial X^k \psi^0) + \frac{2}{3} R_{00ij} \bar{\psi}^0 \psi^0 \bar{\psi}^i \psi^j + \frac{1}{6} R_{ijkl} \bar{\psi}^i \psi^j \bar{\psi}^k \psi^l \right).
\]

(8)
where the fermions have accordingly been split as \( \psi^I = (\psi^i, \psi^0) \). The dual model of (9) can be constructed as usual, by employing the first order formulation [2, 5] where one replaces \( \partial_\mu X \) by the gauge field (1-form) \( A_\mu \). Then, by enforcing the constraint \( \epsilon^{\mu\nu} \partial_\mu A_\nu = 0 \), one gets that \( A_\mu \) is a pure gauge, i.e. \( A_\mu = \partial_\mu X \). Thus, we express the partition function in the first order formulation as

\[
Z = \int [\sqrt{g} D A D \bar{X}^0 D X^i] d\mu e^{-i\bar{S}_1 - iS_2},
\]

where

\[
S_1 = \int d^2\sigma \left( \frac{1}{2} \partial_\mu X^i \partial^\mu X^j g_{ij} + \frac{1}{2} i g_{ij} \bar{\psi}^j (\partial \psi^j + \Gamma^j_{kl} \partial X^k \psi^l) + \frac{1}{2} i g_{00} \bar{\psi}^0 (\partial \psi^0 + \Gamma^0_{k0} \partial X^k \psi^0) + \frac{2}{3} R_{00ij} \bar{\psi}^i \psi^j \bar{\psi}^j \psi^l + \frac{1}{6} R_{ijkl} \bar{\psi}^i \psi^j \bar{\psi}^k \psi^l \right),
\]

is the \( A_\mu \)-independent part and

\[
S_2 = \int d^2\sigma \left( \frac{1}{2} A_\mu A^\mu g_{00} + \frac{1}{2} i g_{ij} \bar{\psi}^j (\Gamma^{ij}_{00} A^0 + \frac{1}{2} i g_{00} \bar{\psi}^0 \Gamma^{0}_{0k} A^k + \frac{1}{2} \epsilon^{\mu\nu} \partial_\mu A_\nu \bar{X}^0) \right),
\]

is the \( A_\mu \)-containing one. The constraint has been incorporated by means of the Lagrange multiplier \( \bar{X}^0 \). By integrating over \( \bar{X}^0 \) one gets the original model as usual and by integrating over \( A_\mu \) the dual model is obtained. The latter integration gives

\[
Z = \int [\sqrt{\tilde{g}} D \bar{X}^0 D X^i] M d\mu e^{-i\tilde{S}},
\]

where

\[
\tilde{S} = \int d^2\sigma \left( \frac{1}{2} \partial_\mu X^i \partial^\mu X^j g_{ij} + \frac{1}{2} \partial_\mu \bar{X}^0 \partial^\mu \bar{X}^0 + \frac{1}{2} i g_{ij} \bar{\psi}^j (\partial \psi^j + \Gamma^j_{kl} \partial X^k \psi^l)
\]

\[
- \frac{1}{g_{00}} \Gamma^i_{00} \gamma^\mu \epsilon^{\nu} \partial_\nu \bar{X}^0 \psi^0
\]

\[
+ \frac{1}{2} i g_{00} \bar{\psi}^0 (\partial \psi^0 - \frac{1}{g_{00}^2} \Gamma_{00k} \gamma^\mu \epsilon^{\nu} \partial_\nu \bar{X}^0 \psi^0 + \Gamma^0_{k0} \partial X^k \psi^0)
\]

\[
+ \frac{1}{g_{00}} \bar{\psi}^0 \Gamma_{00ij} \gamma^\mu \psi^0 \bar{\psi}^j \Gamma_{00ij} \gamma^\mu \psi^0 + \frac{1}{8} \bar{\psi}^0 \Gamma_{00ij} \gamma^\mu \psi^0 \bar{\psi}^j \Gamma_{00ij} \gamma^\mu \psi^0
\]

\[
+ \frac{1}{8} R_{00ij} \bar{\psi}^i \psi^j \bar{\psi}^j \psi^l + \frac{2}{3} R_{00ij} \bar{\psi}^i \psi^j \bar{\psi}^j \psi^l + \frac{1}{6} R_{ijkl} \bar{\psi}^i \psi^j \bar{\psi}^k \psi^l \right),
\]

is the dual action and \( \tilde{g} = \frac{1}{g_{00}} g \). Note also the factor \( M \) in eq.(12) which emerges because the original model has been obtained by integrating over scalars (\( \bar{X}^0 \)), while the dual model results after integrating over 1-forms (\( A_\mu \)). The factor \( M \) appears then because the number
of scalars (0-forms) is in general different from the number of 1-forms and it is explicitly
given by
\[ M = \left( \int [\mathcal{D}X^0] e^{-i \int d^2 \sigma \partial_\mu X^0 \sqrt{g_{00}} \partial_\mu} \right)^2. \] (14)

In order to evaluate \( M \) one has to analytically continue to Euclidean space. In the case
then of constant \( g_{00} = R^2 \), \( M \) is simply \( R^{B_1 - 2B_0} \) where \( B_0, B_1 \) are the numbers of scalars
and 1-forms on the surface \( \Sigma \) in a lattice regularization. By removing the lattice we are left
only with the zero mode contribution, i.e., \( B_1 - 2B_0 = b_1 - 2b_0 = -\chi \) where \( \chi \) is the Euler
number. Thus, \( M = R^{-\chi} \) and it corresponds exactly to the dilaton shift \( \Phi - 2 \ln R \) [6]. The
non-constant \( g_{00} \) case can be worked out as well. In this case we have
\[ M = \frac{\text{det}_{0}^{-1}(\sqrt{g_{00}})}{\text{det}_{1}^{-1/2}(\sqrt{g_{00}})} = \frac{\exp \left( -\frac{1}{8} \int d^2 x \sqrt{g^{(2)}} \ln g_{00} \right)}{\exp \left( -\frac{1}{2} \int d^2 x \sqrt{g^{(2)}} \ln g_{00} \right)}, \] (15)
where \( \text{det}_{0}, \text{det}_{1} \) denote the determinants coming from integrations over zero and one-forms,
respectively. We have regularized these determinants by the inclusion of the \( e^{-\Delta_0/M^2}, e^{-\Delta_1/M^2} \)
factors where \( \Delta_0, \Delta_1 \) are Laplace-Beltrami operators for scalars and 1-forms, respectively.
This regularization corresponds exactly to the lattice regularization mentioned above. Since
\[ \Delta_0 = -\nabla^2, \]
\[ \Delta_1 = -g_{ab}^{(2)} \nabla^2 + R_{ab}^{(2)}, \] (16)
by using the short-time expansion of the heat kernel [12] for \( \nabla^2 \)
\[ Tr e^{-t\nabla^2} = \frac{1}{4\pi t} + \frac{R^{(2)}}{12\pi} + O(t), \]
we get
\[ Tr(\ln \sqrt{g_{00}} e^{-\Delta_0/M^2}) = \frac{1}{2} \int d^2 x \sqrt{g^{(2)}} \ln g_{00} \left( \frac{M^2}{4\pi} + \frac{R^{(2)}}{12\pi} + O(M^{-2}) \right), \]
\[ Tr(\ln \sqrt{g_{00}} e^{-\Delta_1/M^2}) = \frac{1}{2} \int d^2 x \sqrt{g^{(2)}} \ln g_{00} e^{-R^{(2)/M^2}} \left( \frac{M^2}{2\pi} + \frac{R^{(2)}}{6\pi} + O(M^{-2}) \right). \] (17)

Putting all together we find in the the \( M \to \infty \) limit
\[ M = \exp \left( -\frac{1}{8\pi} \int d^2 x \sqrt{g^{(2)}} \ln g_{00} R^{(2)} \right), \] (18)
which corresponds to the well known dilaton shift \[7\]. Note that there is no infinities since the dangerous $M^2$ terms in eq. (17) exactly cancel each other.

The dual action (13) above is not manifest $\mathcal{N}=1$ supersymmetric. However, it can be expressed in a manifest $\mathcal{N}=1$ form by the transformation

$$g_{00} \rightarrow \frac{1}{g_{00}},$$

$$\psi^0 \rightarrow -\frac{1}{g_{00}} \tilde{\gamma} \psi^0. \quad (19)$$

Then, (13) turns out to be

$$\tilde{S} = \int d^2\sigma \left( \frac{1}{2} \partial_\mu X^i \partial^\mu X^j \tilde{g}_{ij} + \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi \tilde{g}_{00} + \frac{1}{2} i \tilde{g}_{ij} \tilde{\psi}^i (\tilde{\partial} \tilde{\psi}^j + \tilde{\Gamma}^{ij}_{kl} \partial X^k \tilde{\psi}^l + \tilde{\Gamma}^{i}_{00} \partial \Phi \tilde{\psi}^0) + \frac{1}{2} i \tilde{g}_{00} \tilde{\psi}^0 (\tilde{\partial} \tilde{\psi}^0 + \tilde{\Gamma}^{0}_{0k} \partial \Phi \tilde{\psi}^k + \tilde{\Gamma}^{0}_{k0} \partial X^k \tilde{\psi}^0) + \frac{2}{3} \tilde{R}_{00ij} \tilde{\psi}^0 \tilde{\psi}^i \tilde{\psi}^j + \frac{1}{6} \tilde{R}_{ijkl} \tilde{\psi}^0 \tilde{\psi}^i \tilde{\psi}^j \tilde{\psi}^l \right) \quad (20)$$

where we have define

$$\tilde{g}_{ij} = g_{ij}, \quad \tilde{g}_{00} = \frac{1}{g_{00}},$$

$$\tilde{\psi}^i = \psi^i, \quad \tilde{\psi}^0 = -g_{00} \tilde{\gamma} \psi^0. \quad (21)$$

Thus, the bosonic parts of the original and the dual theory are related by the well known duality transformation \[2\]. However, the fermion fields in the abelian direction transforms as well and it should be stressed that, in view of the $\tilde{\gamma}$ term, positive and negative chirality spinors transform differently under duality \[2\],[4]. In this way, the transformation (19) converts the dual action in a manifest $\mathcal{N}=1$ supersymmetric form. If, moreover, the measure is invariant under this transformation, one may conclude that the original and the dual pictures are descriptions of the same theory.

Ignoring the change in the bosonic measure which effectively results in the dilaton shift, we will examine the transformation of the fermionic measure

$$d\mu_f = \prod_\sigma \frac{1}{\sqrt{g_{00}(\sigma)}} d\psi^0(\sigma) \prod_\sigma \frac{1}{\det(\sqrt{g_{ij}(\sigma)})} d\psi^i(\sigma). \quad (22)$$

Since only $\psi^0$ is transformed, we will consider only the first part which, expressed in positive and negative chirality spinors $\psi^0_+, \psi^0_-$, respectively, is written as

$$\prod_\sigma \frac{1}{\sqrt{g_{00}}} d\psi^0 = \prod_\sigma \frac{1}{\sqrt{g_{00}}} d\psi^0_+ \prod_\sigma \frac{1}{\sqrt{g_{00}}} d\psi^0_. \quad (23)$$
The part of the action that contains $\psi_0^+, \psi_0^-$ is

$$S_0 = \int d^2 \sigma \left( \frac{1}{2} i g_{00} \bar{\psi}^0 (\partial \psi^0 + 2 \Gamma^0_{0k} \partial X^k \psi^0) + \frac{2}{3} R_{0i0j} \bar{\psi}^0 \psi^0 \bar{\psi} \psi^j \psi^j \right),$$

and by defining $\psi_\pm = \sqrt{g_{00}} \psi_0^\pm$ we get

$$S_0 = - \int d^2 \sigma \left( i \bar{\psi}_+ \partial_- \psi_+ + i \bar{\psi}_- \partial_+ \psi_+ + i \bar{\psi}_+ \partial_- X \Gamma_k \psi_- + i \bar{\psi}_- \partial_+ X \Gamma_k \psi_- - \frac{2}{3} R_{0i0j} \bar{\psi}_- \psi_+ \bar{\psi} \psi^j \psi^j \right),$$

where $\Gamma_k = 2 \partial_k \sqrt{g_{00}}$, $\sigma^\pm = (\sigma^0 \pm \sigma^1)$ and $\partial^\pm = \partial / \sigma^\pm$. The dual action is now of exactly the same form with $\tilde{\psi}_\pm = \mp \psi_\pm$ [2]. The measure (23) is written simply in terms of $\psi_\pm$ as

$$\prod_{\sigma} \frac{1}{\sqrt{g_{00}}} d\psi_0 = \prod_{\sigma} d\psi_+ d\psi_- ,$$

and we will examine how it transforms under

$$\psi_\pm \to \mp \psi_\pm .$$

Let us choose hermitian operators $H_+, H_-$ and let us consider the corresponding eigenvalue problems

$$H_\pm \phi_n^\pm (\sigma) = \lambda_n^\pm \phi_n^\pm (\sigma) .$$

We may expand $\psi_+, \psi_-$ in the $\phi_n^\pm$ orthonormal base as

$$\psi_\pm (\sigma) = \sum_n a_n^\pm \phi_n^\pm (\sigma) ,$$

so that eq.(26) is written as

$$\prod_{\sigma} d\psi_+ d\psi_- = \prod_n a_n^+ d\phi_n^+ .$$

It is not difficult to verify that under (27), the measure (30) transforms as

$$\prod_n a_n^+ d\phi_n^+ \to \det (-\gamma) \prod_n a_n^+ d\phi_n^+ = (-1)^{\text{dim}H_+} \prod_n a_n^+ d\phi_n^+ ,$$

where $\text{dim}H_+$ is the dimension of the $H_+$ operator. Thus, in general one should expect a change of the measure if $\text{dim}H_+$ is odd and, correspondingly, a non-invariance of the theory under duality transformations. For the expansion in eq.(24), one usually employs
the eigenfunctions of the quadratic part (kinetic operator) of the fields in the action \[13\]–\[15\]. In our case, the quadratic parts of the \(\psi_\pm\) fields are simply \(\nabla_\pm = -i\partial_\pm\). If we analytically continue to Euclidean time, we get that \(\nabla_+ = \partial_z\) and \(\nabla_- = \nabla^\dagger_+ = \partial_\bar{z}\) which are no more self adjoint (\(z, \bar{z}\) are complex coordinates). This is connected to the fact that in 2-dimensional space of Euclidean signature there are no Majorana-Weyl spinors and left and right-handed spinors are complex conjugate of each other (although they can still be treated in an independent way \[16\]). However, in this case one may employ \[13\]–\[15\] the self-adjoint operators

\[
H_+ = -\nabla_z \nabla_\bar{z}, \quad H_- = -\nabla_\bar{z} \nabla_z.
\]

(32)

The expansion (29) in the eigenfunctions of the operators (32) turns out then to be

\[
\psi_+(\sigma) = \sum_n a_n \phi_n(\sigma),
\]

\[
\psi_-(\sigma) = \sum_n a_n^\dagger \phi_n^*(\sigma),
\]

(33)

so that eq.(30) is written as

\[
\prod_\sigma d\psi_+ \psi_- = \prod_n da_n da_n^\dagger.
\]

(34)

Then, we find that the measure (34) transforms as

\[
\prod_n da_n da_n^\dagger \to (-1)^{n_+} \prod_n da_n da_n^\dagger,
\]

(35)

where \(n_+\) is the number of zero modes of \(H_+\) (since the non-zero modes are paired and do not contribute as they are always even). The number now of zero modes of \(H_+\) is just the number of zero modes of the chiral Dirac operator \(\nabla_z\) (\(H_+\) is positive defined). Thus, we get that the original and the dual theory are related by \([19]\) and they are not, in general equivalent since

\[
Z[g] = (-1)^{n_+} Z[\tilde{g}].
\]

(36)

The number \(n_+\) of (definite chirality) zero modes of the Dirac operator depends on the spin structure \([1], [17]\) (it is even or odd for the even or odd spin structures, respectively) and thus, the dual theory also depend on the latter. In particular, \(n_+\) depends on the spin structure for
surfaces with genus $g \leq 2$ and in addition on the conformal class of the metric for $g \geq 3$ \cite{L7}. However, it is a topological invariant modulo 2 and thus $(-1)^{n+}$ is a topological invariant term and takes the values $(+1),(-1)$ for even and odd spin structures, respectively. In the general case with a generic metric and antisymmetric field, the analysis is more complicated but the result is the same. Even in this case left and right-handed fermions transforms in a different way under the duality process \cite{2,3}. This in turn leads to a non-trivial Jacobian for the fermionic measure in the same way that the integration over zero and one-forms produces the Jacobian for the bosonic measure. A duality invariant theory can now be written down in two ways. Either by summing over the even only spin structures, or by considering only left-moving fermions $\psi^0$. It seems that the there is no realization of the former possibility. However, there exist a well known string theory which is based exactly on the latter possibility, namely, the heterotic string and thus, heterotic string is potentially invariant under target-space duality. Finally, if there exist more than one abelian isometries and dualize an even number of them, then since each one contributes a $(-1)^{n+}$ factor, the net contribution is zero and the theory is invariant.

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