Measurements of Hadronic and Transition Form Factors at BESIII

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Abstract. Motivated by the need of experimental input to improve the Standard Model prediction of the anomalous magnetic moment of the muon \( a_\mu \), the BESIII Collaboration started a dedicated program to measure hadronic cross sections as well as transition form factors (TFF) with high accuracy. The large data sets acquired by the BESIII Collaboration allow to exploit initial state radiation in order to study hadron production over a wide energy range, as well as two-photon collisions to study the momentum dependence of TFFs in the spacelike regime. The current status and ongoing investigations in both endeavors are discussed.

1 Introduction

Even though the anomalous magnetic moment of the muon \( a_\mu \) is one of the most precisely known observables in particle physics, there is a long standing discrepancy between the direct measurement [1] and the Standard Model (SM) prediction [2]. Currently, the deviation is at the level of three to four standard deviations, which is of interest, as it might indicate contributions of New Physics. Thus, new measurements of \( a_\mu \) are carried out at Fermilab [3] and are planned at J-PARC [4] with the goal to improve the accuracy by a factor of four. Similar improvements are necessary for the SM prediction. The prediction is dominated by quantum electrodynamics, but also has contributions from weak and strong interactions. Due to the asymptotic freedom of the strong interaction, its contributions to \( a_\mu \) cannot be calculated perturbatively. As a consequence, the SM uncertainties are completely dominated by the uncertainties of the hadronic contributions. These can be separated into two parts, the contribution of the hadronic vacuum polarization (HVP) and the contribution of hadronic Light-by-Light scattering (HLbL). The former has the largest contribution to the absolute value, and the latter has the largest relative uncertainty. Recently, data-driven approaches based on dispersion theory are developed and used to systematically improve the SM calculations of the hadronic contributions. New and more precise measurements of related processes allow to pin down the uncertainties in a reliable way. The experimental inputs for the data-driven approaches to both hadronic contributions are measured at the BESIII experiment.

2 The BESIII Experiment

The BESIII detector is a magnetic spectrometer [5] located at the Beijing Electron Positron Collider (BEPCII) [6]. The cylindrical core of the BESIII detector consists of a helium-
based multilayer drift chamber (MDC), a plastic scintillator time-of-flight system (TOF), and a CsI(Tl) electromagnetic calorimeter (EMC), which are all enclosed in a superconducting solenoidal magnet providing a 1.0 T magnetic field. The solenoid is supported by an octagonal flux-return yoke with resistive plate counter muon identifier modules (MUC) interleaved with steel. The acceptance of charged particles and photons is 93% over 4π solid angle. The charged-particle momentum resolution at 1 GeV/c is 0.5%, and the dE/dx resolution is 6% for the electrons from Bhabha scattering. The EMC measures photon energies with a resolution of 2.5% (5%) at 1 GeV in the barrel (end cap) region. The time resolution of the TOF barrel part is 68 ps, while that of the end cap part is 110 ps. The end cap TOF system was upgraded in 2015 with multi-gap resistive plate chamber technology, providing a time resolution of 60 ps [7, 8].

BEPCII provides e+e− collisions at center-of-mass energies √s between 2.0 GeV and 4.6 GeV. The performance is optimized at √s = 3.773 GeV, the peak of the ψ(3770) resonance, where the design luminosity of 10^{33} cm^{-2}s^{-1} has been achieved. The BESIII Collaboration has acquired the world’s largest data sets at center-of-mass energies corresponding to the narrow charmonium resonances J/ψ and ψ(3686), and at, as well as above the ψ(3770), which in combination provide more than 10 fb^{-1} of data [9, 10].

### 3 Hadronic Form Factor Measurements

Using the optical theorem, the HVP can be related to hadronic cross sections measured at e+e− colliders. The contributions of the individual cross sections are evaluated in a dispersion integral over the cross section and a kernel function. Both terms are proportional to the inverse of the squared center-of-mass energy, putting emphasis on processes at √s < 1 GeV, i.e. the low multiplicity pion final states, especially e+e− → π+π−.

The relevant ranges of √s are accessible at BESIII only through exploiting the initial state radiation (ISR). Radiating off a photon from the initial state reduces the effective center-of-mass energy available for producing the hadronic system. The radiative process is related to the non-radiative production by the radiator function [11], which describes the probability to emit a photon carrying a certain fraction of √s at a certain angle θγ. This approach allows to measure hadronic cross sections at BESIII down to the hadronic threshold √s = 2mπ.

The angular distribution of the emitted ISR photons calls for two complementary analysis schemes. The majority of the photons is emitted at small scattering angles along the beam axis. Their direct detection is impossible. By measuring the produced hadronic system, the photon momentum can be reconstructed from energy and momentum conservation. Since the hadronic system is boosted oppositely to the ISR photon, a threshold on the mass of the produced system exists below which the particles also escape detection. At BESIII this threshold is at approximately 1 GeV. Final states above this threshold can be reconstructed with high statistics and small background contamination. For the second analysis scheme, the detection of the ISR photon is required. Here, the differential cross section of the radiative process is smaller, however, acceptance is given from the hadronic threshold. At larger hadronic masses the background contamination increases, since coincidental neutral signals are more likely to be mistaken for low energetic ISR photons.

The approach of detecting the ISR photon in the calorimeter has been successfully used to measure the pion form factor in the reaction e+e− → γISRπ+π−. Based on 2.9 fb^{-1} of data taken at √s = 3.773 GeV, pion masses between 600 MeV and 900 MeV [12] were studied. The mass range contains the pole of the ρ resonance and makes up for approximately 70% of the HVP contribution to aμ. A high precision measurement is therefore inevitable. The dominating background contribution in this analysis is due to radiative muon pair production. Since conventional means of particle identification could not sufficiently separate pions from
muons, an artificial neural network is trained and used. Careful evaluation and correction of all systematics allows for a sub-percent accuracy of the final result. The dominating uncertainties come from the normalization to the luminosity and the radiator function. In the alternative normalization to the muon yield these uncertainties cancel, however, due to the small muon yield in the data sample the result becomes statistically limited. Though comparison of the form factor with previous measurements suggests some tensions, the evaluation of the dispersion integral for $a_{\mu}^\mu$ is in good agreement with the recent combination of the KLOE results [13]. In view of more recently acquired data sets and the findings of Ref. [14] a new evaluation of the pion form factor has been started at BESIII, also taking into account additional mass ranges below and above the $\rho$ peak. The current result is included in the recent global evaluations of $a_{\mu}^\mu$ [2, 15].

Both ISR analysis schemes are used to measure the cross sections of higher pion multiplicities in $e^+e^- \to \gamma_{\text{ISR}}\pi^+\pi^-\pi^0$ and $e^+e^- \to \gamma_{\text{ISR}}\pi^+\pi^-\pi^0\pi^0$. When the photon is detected, the events are subjected to a kinematic fit of the full final state, constrained by energy and momentum conservation and the $\pi^0$ rest masses of the photon pairs. Background can be sufficiently suppressed by rejecting events in which the combination of the ISR photon candidate with any other photon in the event has an invariant mass close to the $\pi^0$ mass. When the ISR photon remains undetected, the photon momentum is reconstructed by the kinematic fit. By requiring the photon to be collinear with the beam axis, background events are rejected. The dominating background contribution is the production of the next higher pion multiplicity with neutral pions, where one of the decay photons is mistaken for an ISR photon. The final result is calculated as the error weighted mean of both analysis schemes.

The preliminary results for the $e^+e^- \to \pi^+\pi^-\pi^0$ and $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$ cross sections are compatible with the previous measurements. Especially, the agreement with the recent BaBar result on $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$ [16], where a compatible accuracy for the contribution to $a_{\mu}^\mu$ is achieved on a significantly smaller data sample, illustrates the competitiveness of ISR analyses at BESIII. The key feature is the possibility of measuring the ISR process with undetected photons starting from hadronic masses of approximately 1 GeV.

4 Transition Form Factor Measurements

The relation of the HLbL contribution to $a_{\mu}$ to measurable quantities is not as straightforward as it is for the HVP contribution discussed in Sect. 3. Previously, hadronic models were used to evaluate $a_{\mu}^{\text{HLbL}}$ [17, 18]. Recently, dispersive approaches have been developed [19–21] in order to determine the dominating terms of HLbL model-independently. The pole contributions of the pseudoscalar mesons $\pi^0, \eta$ and $\eta'$ are considered most important, followed by pion loop contributions and heavier resonances [22]. The experimental observables needed as input to these calculations are the pseudoscalar meson TFFs at arbitrary spacelike virtualities $Q^2 = -q^2$ and the partial waves of the process $\gamma(\star)\gamma(\star) \to \pi\pi$.

At BESIII, the process of two-photon scattering is exploited to measure the relevant information. Two-photon scattering allows for the direct production of (pseudo-)scalar and tensor states at $e^+e^-$ machines. The cross section is suppressed with respect to annihilation, but it increases with the square of the logarithm of $\sqrt{s}$. More importantly, it is directly proportional to the square of the TFF. Since the TFF decreases with increasing virtuality of the photons, it is difficult to investigate the TFF for two arbitrary virtualities at the current BESIII detector. However, the investigation of the momentum dependence is feasible when one of the photons is quasi-real. This is achieved by measuring the mesons in the final state and one of the scattered leptons, while the second lepton is reconstructed from energy and momentum conservation. By requiring its scattering angle to be small, the exchange of a quasi-real photon is selected.
Using this approach, the TFF of the $\pi^0$ is investigated. Selecting events with one lepton and two photons in the final state causes a dominating background contribution by radiative Bhabha scattering. It can be effectively suppressed by a condition on the helicity angle of the photons from a pion candidate. A condition adapted from the BaBar measurement [23] of this TFF designed to suppress radiative effects in two-photon collision events further reduces background contributions due to wrongly reconstructed hadronic final states from annihilation events. Remaining background contributions are subtracted bin-by-bin in order to determine the event yield as a function of momentum transfer. Normalizing it to the reconstruction efficiency and luminosity of the data results in the differential cross section. The TFF $|F(\pi^0\gamma^*\gamma^*(−q^2,0))|^2$ is finally determined by dividing out the point-like cross section using Monte Carlo distributions. The preliminary result is shown in Fig. 1 using the representation $Q^2 \cdot |F(−q^2, 0)|$, which illustrates the asymptotic behavior of the TFF at large values of $Q^2$.

The evaluation of the contributions to the systematic uncertainties suggests a dominating contribution of the background subtraction procedure. The influence of radiative effects on the result is being evaluated based on the most recent calculations implemented in the Ekhara 3.0 event generator [24].

As can be seen from Fig. 1, the preliminary BESIII result provides information on the $\pi^0$ TFF in the range of $0.3 \leq Q^2 [\text{GeV}^2] \leq 3.1$, which is of special relevance for the calculation of $a_{\mu}^{\text{HLSN}}$ [27]. As illustrated in the left panel of the figure, the accuracy achieved at $Q^2 < 1.5 \text{ GeV}^2$ is unprecedented, while still being competitive with previous measurements [25, 26] at larger momentum transfer. The right panel of Fig. 1 confronts the preliminary result with the dispersive construction of the TFF from related time-like data [21]. Good agreement is observed, although at low values of $Q^2$, where best accuracies are claimed, there seems to be a tension.

To bring down the uncertainties of the SM prediction of $a_{\mu}$, also the contributions of $\eta$ and $\eta'$ must be under control. At BESIII the TFFs of both mesons are investigated analogously to the $\pi^0$ TFF, however several decay modes need to be taken into account. The charged decay modes $\eta \rightarrow \pi^+\pi^-\pi^0$ and $\eta' \rightarrow \pi^+\pi^-\eta$ are studied in a common analysis strategy looking for the two-photon decay of $\pi^0/\eta$. In addition, the neutral $\eta \rightarrow 3\pi^0$ mode is analyzed. Based on
the same data sample analyzed for the \( \pi^0 \) TFF, the coverage of the same range of momentum transfers is achieved with statistical accuracy competitive previous measurements, which also combined several decay modes [25, 26].

Another essential input to the data-driven approaches for \( a_{\mu}^{\rm HLbL} \) comes from the two-photon production of meson pairs. Most studies in the past were based on real photons [28–30]. Only recently the Belle Collaboration published the first results on the momentum dependence of the \( \pi^0 \pi^0 \) and \( K_s K_s \) production [31, 32], albeit at large momentum transfers. At BESIII the charged pion system is studied \( \gamma\gamma^* \rightarrow \pi^+ \pi^- \). The analysis at BESIII follows the strategy successfully applied to the TFFs of single pseudoscalar mesons. Three charged tracks are detected and identified as a pair of pions and an \( e^\pm \). The main background contributions stem from the two-photon production of muon pairs, which has a six-times larger cross section, and the radiative Bhabha scattering process, in which the photon couples to a \( \rho \)-meson, decaying to two pions. Background from muon production is sufficiently understood from studies at LEP and well established MC generators [33, 34] are used to train multivariate methods for \( \pi - \mu \) separation. The background contributions due to pion production in Bhabha scattering are irreducible. They are subtracted by fitting the corresponding contributions in the mass spectra. The background subtracted distributions show clear signals of the \( f_0(980) \) and \( f_2(1270) \) resonances. The differential cross section of the reaction \( \gamma\gamma^* \rightarrow \pi^+ \pi^- \) is studied for the first time at momentum transfers \( 0.2 \leq Q^2[\text{GeV}^2] \leq 2.0 \), at invariant masses between \( 2m_{\pi} \leq M_{\pi\pi} \leq 2.0 \) GeV and at full coverage of the pion helicity angle \( \cos \theta^* \). An equivalent analysis of neutral meson systems, i.e. \( \gamma\gamma^* \rightarrow \pi^0 \pi^0 \) and \( \gamma\gamma^* \rightarrow \pi^0 \eta \) is in preparation.

As a first step towards TFF measurements at arbitrary virtualities first feasibility studies have been performed, detecting all final state particles in the detector volume. The first result of this kind of measurement was recently published by the BaBar Collaboration [35], albeit at large virtualities. Feasibility studies at BESIII show that based on the combination of the largest data sets, which correspond to an integrated luminosity of more than 10 \( \text{fb}^{-1} \), the determination of the TFFs of \( \pi^0, \eta \) and \( \eta' \) is possible for momentum transfers of around \( (Q_1^2, Q_2^2) \approx (1, 1) \text{GeV}^2 \). The statistics is expected to be sufficient to validate the \( Q^2 \) dependence of the TFFs predicted by different hadronic models [27].

5 Summary and Outlook

The BESIII detector is a uniquely suited setup to provide input to the data-driven approaches for the calculation of the hadronic contributions to the anomalous magnetic moment of the muon. The acquired high statistics data sets allow to measure hadronic cross sections exclusively with the ISR method. First results are published and included in recent global reviews. Further results will follow. The ISR studies are complemented with the measurement of inclusive hadron production, namely \( R_{\text{incl}} \), in energy scans.

The investigation of two-photon collisions allow to study the TFF of pseudoscalar mesons in the most relevant range of momentum transfer. The preliminary result of the \( \pi^0 \) TFF shows unprecedented statistical accuracy and equivalent studies for \( \eta \) and \( \eta' \) are in preparation. The important two-pion final state is studied for the first time covering momentum transfers below 3 \( \text{GeV}^2 \) and invariant masses starting from the pion threshold. Current studies of the charged pion system are extended to neutral meson systems. Also higher multiplicity final states are considered to provide input to the estimate of contributions of axial and tensor states to the HLbL contribution to \( a_{\mu} \). The feasibility of the measurement of double-virtual TFF has been tested. The development of small angle tagging detectors adds new prospects to the ISR and two-photon physics program.
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