Analysis of the $Y(4140)$ and related molecular states with QCD sum rules

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Abstract

In this article, we assume that there exist scalar $D_s^*\bar{D}_s^*$, $D_{s1}^*\bar{D}_{s1}^*$, $B_s^*\bar{B}_s^*$ and $B_s^*\bar{B}_s^*$ molecular states, and study their masses using the QCD sum rules. The numerical results indicate that the masses are about $(250 - 500)$ MeV above the corresponding $D^* - D^*$, $D_{s1}^* - D_{s1}^*$, $B^* - B^*$ and $B_s^* - B_s^*$ thresholds, the $Y(4140)$ is unlikely a scalar $D_s^*\bar{D}_s^*$ molecular state. The scalar $D_s^*\bar{D}_s^*$, $D_{s1}^*\bar{D}_{s1}^*$, $B_s^*\bar{B}_s^*$ and $B_s^*\bar{B}_s^*$ molecular states maybe not exist, while the scalar $D_s^*\bar{D}_s^*$, $D_{s1}^*\bar{D}_{s1}^*$, $B_s^*\bar{B}_s^*$ and $B_s^*\bar{B}_s^*$ molecular states maybe exist.

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1 Introduction

Recently the CDF Collaboration observed a narrow structure (which is denoted as the $Y(4140)$ now) near the $J/\psi\phi$ threshold with statistical significance in excess of 3.8 standard deviations in exclusive $B^+ \rightarrow J/\psi\phi K^+$ decays produced in $\bar{p}p$ collisions at $\sqrt{s} = 1.96$ TeV \cite{1}. The mass and width of the structure are measured to be $4143.0 \pm 2.9 \pm 1.2$ MeV and $11.7^{+8.3}_{-5.0} \pm 3.7$ MeV, respectively. The narrow structure $Y(4140)$ is very similar to the charmonium-like state $Y(3930)$ near the $J/\psi\omega$ threshold \cite{2,3}. The mass and width of the $Y(3930)$ are $3914^{+5}_{-3} \pm 2.0$ MeV and $34^{+12}_{-5} \pm 5$ MeV, respectively \cite{3}.

There have been several explanations for the nature of the narrow structure $Y(4140)$, such as a $D_s^*\bar{D}_s^*$ molecular state \cite{4,5,6,7,8,9,10}, an exotic ($J^{PC} = 1^{-+}$) hybrid charmonium \cite{5}, a $c\bar{c}ss$ tetraquark state \cite{11}, or the effect of the $J/\psi\phi$ threshold \cite{12}.

The mass is a fundamental parameter in describing a hadron, in order to identify the $Y(4140)$ as a scalar molecular state, we must prove that its mass lies in the region $(4.1 - 4.2)$ GeV. In Ref.\cite{13}, we assume that there exists a scalar $D_s^*\bar{D}_s^*$ molecular state in the $J/\psi\phi$ invariant mass distribution, and study its mass using the QCD sum rules. The numerical result indicates that the mass is about $M_Y = (4.43 \pm 0.16)$ GeV, which is inconsistent with the experimental data. The $D_s^*\bar{D}_s^*$ is probably a virtual state and not related to the meson $Y(4140)$ \cite{13}. In this article, we extend our previous work to study the $D_s^*\bar{D}_s^*$, $D_{s1}^*\bar{D}_{s1}^*$, $B_s^*\bar{B}_s^*$ and $B_s^*\bar{B}_s^*$ molecular states in a systematic way considering the $SU(3)$ symmetry and the heavy quark symmetry.

In the QCD sum rules, the operator product expansion is used to expand the time-ordered currents into a series of quark and gluon condensates which parameterize the long distance properties of the QCD vacuum. Based on the quark-hadron duality, we can obtain copious information about the hadronic parameters at the phenomenological side \cite{14,15}. 

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The article is arranged as follows: we derive the QCD sum rules for the scalar \( D^* \bar{D}^* \), \( D_s^* \bar{D}_s^* \), \( B^* \bar{B}^* \) and \( B_s^* \bar{B}_s^* \) molecular states in section 2; in section 3, numerical results and discussions; section 4 is reserved for conclusion.

2 QCD sum rules for the \( D^* \bar{D}^* \), \( D_s^* \bar{D}_s^* \), \( B^* \bar{B}^* \) and \( B_s^* \bar{B}_s^* \) molecular states

In the following, we write down the two-point correlation functions \( \Pi_{J/\eta}(p) \) in the QCD sum rules,

\[
\Pi_{J/\eta}(p) = i \int d^4x e^{ip \cdot x} \langle 0 \mid \{ J/\eta(x), J/\eta(0) \} \mid 0 \rangle, \tag{1}
\]

\[
J(x) = \bar{Q}(x) \gamma_\mu s(x) \bar{s}(x) \gamma^\mu Q(x), \tag{2}
\]

\[
\eta(x) = \bar{Q}(x) \gamma_\mu u(x) \bar{d}(x) \gamma^\mu Q(x), \tag{3}
\]

where \( Q = c, b \). We choose the scalar currents \( J(x) \) and \( \eta(x) \) to interpolate the molecular states \( D^* \bar{D}^* \), \( D_s^* \bar{D}_s^* \), \( B^* \bar{B}^* \) and \( B_s^* \bar{B}_s^* \), respectively.

We can insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators \( J(x) \) and \( \eta(x) \) into the correlation functions \( \Pi_{J/\eta}(p) \) to obtain the hadronic representation \([14, 15]\). After isolating the ground state contributions from the pole terms of the scalar molecular states \( Y \) (we use the \( Y \) to denote the scalar molecular states \( D^* \bar{D}^* \), \( D_s^* \bar{D}_s^* \), \( B^* \bar{B}^* \) and \( B_s^* \bar{B}_s^* \)), we get the following result,

\[
\Pi_{J/\eta}(p) = \frac{\lambda_Y^2}{M_Y^2 - p^2} + \cdots, \tag{4}
\]

where the pole residue (or coupling) \( \lambda_Y \) is defined by

\[
\lambda_Y = \langle 0 \mid J/\eta(0) \mid Y(p) \rangle. \tag{5}
\]

In the following, we briefly outline the operator product expansion for the correlation functions \( \Pi_{J/\eta}(p) \) in perturbative QCD. The calculations are performed at the large spacelike momentum region \( p^2 \ll 0 \). We write down the "full" propagators \( S_{ij}(x) \) and \( C_{ij}(x) \) of a massive quark in the presence of the vacuum condensates firstly \([15]\),

\[
S_{ij}(x) = \frac{i \delta_{ij} \not{x}}{2\pi^2 x^4} - \frac{\delta_{ij} m_s}{4\pi^2 x^2} - \frac{\delta_{ij}}{12} \langle \bar{s}s \rangle + \frac{i \delta_{ij} m_s \langle \bar{s}s \rangle}{48} - \frac{\delta_{ij} x^2}{192} \langle \bar{s}g_s \sigma G s \rangle
\]

\[
+ \frac{i \delta_{ij} x^2}{1152} m_s \langle \bar{s}g_s \sigma G s \rangle \not{x} - \frac{i}{32 \pi^2 x^2} G_{\mu \nu}^{ij} (\not{x} \sigma^{\mu \nu} + \sigma^{\mu \nu} \not{x}) + \cdots, \tag{6}
\]

\[
C_{ij}(x) = \frac{i}{(2\pi)^4} \int d^4k e^{-ik \cdot x} \left\{ \frac{\delta_{ij}}{k - m_Q} - \frac{g_s G_{\alpha \beta}^{ij}}{4} \frac{\sigma_{\alpha \beta} (k + m_Q) + (k + m_Q) \sigma_{\alpha \beta}}{(k^2 - m_Q^2)^2}
\]

\[
+ \frac{\pi^2}{3} \left( \frac{\alpha_s GG}{\pi} \right) \delta_{ij} m_Q \frac{k^2 + m_Q k}{(k^2 - m_Q^2)} + \cdots \right\}, \tag{7}
\]
where \(\langle \bar{s}g_s\sigma Gs \rangle = \langle \bar{g}g_s\sigma_{\alpha\beta}G^{\alpha\beta} \rangle\) and \(\langle \frac{\alpha_s \epsilon_{GG}}{\pi} \rangle = \langle \frac{\alpha_s \epsilon_{\alpha\beta\gamma\delta}G^{\alpha\beta}}{\pi} \rangle\), then contract the quark fields in the correlation function \(\Pi_J(p)\) with Wick theorem, and obtain the result:

\[
\Pi_J(p) = i \int d^4xe^{ip\cdot x}Tr[\gamma_\mu S_{ij}(x)\gamma_\alpha C_{ji}(-x)] \tilde{Tr}[\gamma^\mu C_{mn}(x)\gamma^\alpha S_{nm}(-x)],
\]

(8)

where the \(i, j, m\) and \(n\) are color indexes. Substitute the full \(s\) and \(Q\) quark propagators into the correlation function \(\Pi_J(p)\) and complete the integral in the coordinate space, then integrate over the variables in the momentum space, we can obtain the correlation function \(\Pi_J(p)\) at the level of the quark-gluon degrees of freedom. The correlation function \(\Pi_J(p)\) is calculated in the same way, we prefer neglect the technical details.

In the QCD sum rules for the tetraquark states (irrespective of the diquark-antidiquark type and the molecule type) which have one or two heavy quarks, we always calculate the light quark parts of the correlation functions in the coordinate-space where the masses of the \(u, d, s\) quarks are taken as small quantities and treated perturbatively, and use the momentum-space expression for the heavy quark propagators [15], then transform the resulting light-quark parts to the momentum-space with \(D\)-dimensional Fourier transform [16] [17] [18]. The main uncertainties in the QCD calculations originate from the high dimensional vacuum condensates, which are known poorly compared with the low dimensional vacuum condensates, for example, the quark condensate \(\langle \bar{q}q \rangle\) and the gluon condensate \(\langle \frac{\alpha_s \epsilon_{GG}}{\pi} \rangle\).

In this article, we carry out the operator product expansion to the vacuum condensates adding up to dimension-10 and take the assumption of vacuum saturation for the high dimensional vacuum condensates, they are always factorized to lower condensates with vacuum saturation in the QCD sum rules, and factorization works well in large \(N_c\) limit. In the real world, \(N_c = 3\), there are deviations from the factorable formula, we can introduce a factor \(\kappa\) to parameterize the deviations, for example,

\[
\langle \bar{s}s \rangle^2, \langle \bar{s}s \rangle \langle \bar{g}_s\sigma Gs \rangle, \langle \bar{g}_s\sigma Gs \rangle^2 \rightarrow \kappa \langle \bar{s}s \rangle, \kappa \langle \bar{s}s \rangle \langle \bar{g}_s\sigma Gs \rangle, \kappa \langle \bar{g}_s\sigma Gs \rangle^2.
\]

(9)

In Ref. [13], we study the mass \(M_{D_s^*D_s}^2\) with variation of the parameter \(\kappa\) at the interval \(\kappa = 0 - 2\). At the range \(M^2 = (2.6 - 3.0) \text{ GeV}^2\), the value \(\kappa = 1 \pm 1\) leads to an uncertainty about 50 MeV, which is too small to smear the discrepancy between the theoretical prediction and the experimental data. If we assume the \(\kappa\) has the typical uncertainty of the QCD sum rules, say about 30%, the correction is rather mild, we can neglect the uncertainty safely and take \(\kappa = 1\), i.e. the factorization works well. In the QCD sum rules for the masses of the \(\rho\) meson and the nucleon, the value of the \(\kappa\) is always larger than 1 [19]. In calculations, we observe that larger \(\kappa\) means slower convergence in the operator product expansion, requires larger threshold parameters, and results in larger ground state masses. We can draw the conclusion tentatively that the uncertainties (i.e. \(\kappa > 1\)) in the QCD calculations enlarge the discrepancy between the theoretical prediction and the experimental data, our predictions based on the value \(\kappa = 1\) are reasonable.

The contributions from the gluon condensates are suppressed by large denominators and would not play any significant roles for the light tetraquark states [20] [21], the heavy tetraquark state [18] and the heavy molecular state [13]. In this article, we take into account them for completeness although their contributions are rather small.

Once analytical results are obtained, then we can take the quark-hadron duality and perform Borel transform with respect to the variable \(P^2 = -p^2\), finally we obtain the
The input parameters are taken to be the standard values

The following two sum rules for the interpolating current $J(x)$:

$$\lambda_Y^2 e^{-\frac{M_Y^2}{M^2}} = \int_{4(m_Q+m_s)^2}^{s_0} ds \rho(s) e^{-\frac{s}{M^2}},$$  
(10)

where

$$\rho(s) = \rho_0(s) + \rho_{\langle \bar{s}s \rangle}(s) + \left[ \rho^A_{\langle GG \rangle}(s) + \rho^B_{\langle GG \rangle}(s) \right] \left( \frac{\alpha_s GG}{\pi} \right) + \rho_{\langle \bar{s}s \rangle 2}(s),$$  
(11)

the lengthy expressions of the spectral densities $\rho_0(s)$, $\rho_{\langle \bar{s}s \rangle}(s)$, $\rho^A_{\langle GG \rangle}(s)$, $\rho^B_{\langle GG \rangle}(s)$ and $\rho_{\langle \bar{s}s \rangle 2}(s)$ are presented in the appendix. With a simple replacement,

$$m_s, \langle \bar{s}s \rangle, \langle \bar{s}g_\sigma Gs \rangle \rightarrow m_q, \langle \bar{q}q \rangle, \langle \bar{q}g_\sigma Gq \rangle,$$  
(12)

we can obtain the corresponding two sum rules for the scalar current $\eta(x)$.

Differentiating Eq.(10) with respect to $\frac{1}{M^2}$, then eliminate the pole residues $\lambda_Y$, we can obtain two sum rules for the masses of the molecular states $Y$,

$$M_Y^2 = \frac{\int_{4(m_Q+m_s)^2}^{s_0} ds \frac{d}{d(-1/M^2)} \rho(s) e^{-\frac{s}{M^2}}}{\int_{4(m_Q+m_s)^2}^{s_0} ds \rho(s) e^{-\frac{s}{M^2}}},$$  
(13)

the corresponding two sum rules for the scalar current $\eta(x)$ can be obtained analogously.

## 3 Numerical results and discussions

The input parameters are taken to be the standard values $\langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{ GeV})^3$, $\langle \bar{s}s \rangle = (0.8 \pm 0.2)\langle \bar{q}q \rangle$, $\langle \bar{s}g_\sigma Gs \rangle = m_s^2 \langle \bar{s}s \rangle$, $m_s^2 = (0.8 \pm 0.2) \text{ GeV}^2$, $\langle \frac{\alpha_s GG}{\pi} \rangle = (0.33 \text{ GeV})^4$, $m_q \approx 0$, $m_s = (0.14 \pm 0.01) \text{ GeV}$, $m_c = (1.35 \pm 0.10) \text{ GeV}$ and $m_b = (4.7 \pm 0.1) \text{ GeV}$ at the energy scale $\mu = 1 \text{ GeV}$ [14, 15, 22].

In the conventional QCD sum rules [14, 15], there are two criteria (pole dominance and convergence of the operator product expansion) for choosing the Borel parameter $M^2$ and threshold parameter $s_0$. We impose the two criteria on the molecular states to choose the Borel parameter $M^2$ and threshold parameter $s_0$.

The contributions from the different terms in the operator product expansion are shown in Figs.1-4, where (and thereafter) we use the $\langle \bar{s}s \rangle$ to denote the quark condensates $\langle \bar{q}q \rangle$, $\langle \bar{s}s \rangle$ and the $\langle \bar{s}g_\sigma Gs \rangle$ to denote the mixed condensates $\langle \bar{q}g_\sigma Gq \rangle$, $\langle \bar{s}g_\sigma Gs \rangle$. From the figures, we can see that the contributions from different terms in the operator product expansion change quickly with variation of the Borel parameter at the values $M^2 \leq 2.6 \text{ GeV}^2$ and $M^2 \leq 7.0 \text{ GeV}^2$ in the hidden charm and hidden bottom channels respectively, such an unstable behavior cannot lead to stable sum rules, our numerical results confirm this conjecture, see Figs.6-7.

The dominant contributions come from the perturbative term and the $\langle \bar{s}s \rangle + \langle \bar{s}g_\sigma Gs \rangle$ term; and the interpolating currents contain more $s$ quarks have better convergent behavior. The contribution from the terms involving the gluon condensate $\langle \frac{\alpha_s GG}{\pi} \rangle$ are very small, the gluon condensate plays a minor important role. The vacuum condensates of the high dimension $\langle \bar{s}s \rangle^2 + \langle \bar{s}s \rangle \langle \bar{s}g_\sigma Gs \rangle$ serve as a criterion for choosing the Borel parameter $M^2$ and threshold parameter $s_0$. 

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Figure 1: The contributions from the different terms with variation of the Borel parameter $M^2$ in the operator product expansion for the current $\bar{c}\gamma_{\mu}ud\gamma^\mu c$. The $A$, $B$, $C$, $D$, $E$ and $F$ correspond to the contributions from the perturbative term, $\langle \bar{s}s \rangle + \langle \bar{s}g_s\sigma Gs \rangle$ term, $\langle \bar{s}G^G \rangle$ term, $\langle \bar{s}G^G \rangle + \langle \bar{s}G^G \rangle \left[ \langle \bar{s}s \rangle + \langle \bar{s}g_s\sigma Gs \rangle + \langle \bar{s}s \rangle^2 \right]$ term, $\langle \bar{s}s \rangle^2 + \langle \bar{s}s \rangle \langle \bar{s}g_s\sigma Gs \rangle$ term and $\langle \bar{s}g_s\sigma Gs \rangle^2$ term, respectively. The notations $\alpha$, $\beta$, $\gamma$, $\lambda$, $\rho$ and $\tau$ correspond to the threshold parameters $s_0 = 21$ GeV$^2$, 22 GeV$^2$, 23 GeV$^2$, 24 GeV$^2$, 25 GeV$^2$ and 26 GeV$^2$, respectively. Here we take the central values of the input parameters.
Figure 2: The contributions from the different terms with variation of the Borel parameter $M^2$ in the operator product expansion for the current $\bar{c}\gamma_\mu s \bar{s} \gamma^\mu c$. The $A$, $B$, $C$, $D$, $E$ and $F$ correspond to the contributions from the perturbative term, $\langle \bar{s}s \rangle$ term, $\langle \bar{s}s \rangle$ term, $\langle \bar{s}s \rangle + \langle \bar{s}g_s \sigma Gs \rangle$ term, $\langle \bar{s}s \rangle + \langle \bar{s}g_s \sigma Gs \rangle$ term, $\langle \bar{s}s \rangle + \langle \bar{s}g_s \sigma Gs \rangle$ term and $\langle \bar{s}g_s \sigma Gs \rangle^2$ term, respectively. The notations $\alpha$, $\beta$, $\gamma$, $\lambda$, $\rho$ and $\tau$ correspond to the threshold parameters $s_0 = 21 \text{GeV}^2$, $22 \text{GeV}^2$, $23 \text{GeV}^2$, $24 \text{GeV}^2$, $25 \text{GeV}^2$ and $26 \text{GeV}^2$, respectively. Here we take the central values of the input parameters.
Figure 3: The contributions from the different terms with variation of the Borel parameter $M^2$ in the operator product expansion for the current $b\gamma_\mu ud\gamma^\mu b$. The $A$, $B$, $C$, $D$, $E$ and $F$ correspond to the contributions from the perturbative term, $\langle ss \rangle + \langle sg_s\sigma Gs \rangle$ term, $\langle \alpha_s GG \rangle$ term, $\langle \alpha_s GG \rangle + \langle \alpha_s GG \rangle \left[ \langle ss \rangle + \langle sg_s\sigma Gs \rangle + \langle ss \rangle^2 \right]$ term, $\langle ss \rangle^2 + \langle ss \rangle \langle sg_s\sigma Gs \rangle$ term and $\langle sg_s\sigma Gs \rangle^2$ term, respectively. The notations $\alpha$, $\beta$, $\gamma$, $\lambda$, $\rho$ and $\tau$ correspond to the threshold parameters $s_0 = 132$ GeV$^2$, 134 GeV$^2$, 136 GeV$^2$, 138 GeV$^2$, 140 GeV$^2$ and 142 GeV$^2$, respectively. Here we take the central values of the input parameters.
Figure 4: The contributions from the different terms with variation of the Borel parameter $M^2$ in the operator product expansion for the current $b \gamma_\mu s \bar{s} \gamma^\mu b$. The $A$, $B$, $C$, $D$, $E$ and $F$ correspond to the contributions from the perturbative term, $\langle \bar{s}s \rangle + \langle \bar{s}g_s Gs \rangle$ term, $\langle \bar{s}g_s Gs \rangle$ term, $\langle \bar{s}g_s Gs \rangle + \langle (\bar{s}s) + \langle \bar{s}g_s Gs \rangle + \langle \bar{s}s \rangle^2 \rangle$ term, $\langle \bar{s}s \rangle^2 + \langle \bar{s}s \rangle \langle \bar{s}g_s Gs \rangle$ term and $\langle \bar{s}g_s Gs \rangle^2$ term, respectively. The notations $\alpha$, $\beta$, $\gamma$, $\lambda$, $\rho$ and $\tau$ correspond to the threshold parameters $s_0 = 132 \text{ GeV}^2$, 134 GeV$^2$, 136 GeV$^2$, 138 GeV$^2$, 140 GeV$^2$ and 142 GeV$^2$, respectively. Here we take the central values of the input parameters.
At the values $M_{\min}^2 \geq 2.6 \text{GeV}^2$ and $s_0 \geq 23 \text{GeV}^2$, the contributions from the high dimensional condensates $\langle \bar{s}s \rangle^2 + \langle \bar{s}s \rangle \langle \bar{s}g_\sigma G \rangle$ are less than 15% (4%) in the $\bar{c}\gamma_\mu u\bar{d}\gamma^\mu c$ ($\bar{c}\gamma_\mu s\bar{s}\gamma^\mu c$) channel; the contributions from the vacuum condensate of the highest dimension $\langle \bar{s}g_\sigma G \rangle^2$ are less than 3% in all the hidden charm channels, we expect the operator product expansion is convergent in the hidden charm channels. At the values $M_{\min}^2 \geq 7.0 \text{GeV}^2$ and $s_0 \geq 136 \text{GeV}^2$, the contributions from the high dimensional condensates $\langle \bar{s}s \rangle^2 + \langle \bar{s}s \rangle \langle \bar{s}g_\sigma G \rangle$ are less than 11% (5%) in the $\bar{b}\gamma_\mu u\bar{d}\gamma^\mu b$ ($\bar{b}\gamma_\mu s\bar{s}\gamma^\mu b$) channel; the contributions from the vacuum condensate of the highest dimension $\langle \bar{s}g_\sigma G \rangle^2$ are less than 7% in all the hidden bottom channels, we expect the operator product expansion is convergent in the hidden bottom channels.

In this article, we take the uniform Borel parameter $M_{\min}^2$, i.e. $M_{\min}^2 \geq 2.6 \text{GeV}^2$ and $M_{\min}^2 \geq 7.0 \text{GeV}^2$ in the hidden charm and hidden bottom channels, respectively.

In Fig.5, we show the contributions from the pole terms with variation of the Borel parameter and the threshold parameter. The pole contributions are larger than 51% (55%) at the value $M_{\max}^2 \leq 3.0 \text{GeV}^2$ and $s_0 \geq 23 \text{GeV}^2$ (24 GeV$^2$) in the $\bar{c}\gamma_\mu u\bar{d}\gamma^\mu c$ ($\bar{c}\gamma_\mu s\bar{s}\gamma^\mu c$) channel, and larger than 52% (55%) at the value $M_{\max}^2 \leq 8.0 \text{GeV}^2$ and $s_0 \geq 136 \text{GeV}^2$ (138 GeV$^2$) in the $\bar{b}\gamma_\mu u\bar{d}\gamma^\mu b$ ($\bar{b}\gamma_\mu s\bar{s}\gamma^\mu b$) channel. Again we take the uniform Borel parameter $M_{\max}^2$, i.e. $M_{\max}^2 \leq 3.0 \text{GeV}^2$ and $M_{\max}^2 \leq 8.0 \text{GeV}^2$ in the hidden charm and hidden bottom channels, respectively.

In this article, the threshold parameters are taken as $s_0 = (24 \pm 1) \text{GeV}^2$, $(25 \pm 1) \text{GeV}^2$, $(138 \pm 2) \text{GeV}^2$, and $(140 \pm 2) \text{GeV}^2$ in the $\bar{c}\gamma_\mu u\bar{d}\gamma^\mu c$, $\bar{c}\gamma_\mu s\bar{s}\gamma^\mu c$, $\bar{b}\gamma_\mu u\bar{d}\gamma^\mu b$, and $\bar{b}\gamma_\mu s\bar{s}\gamma^\mu b$ channels, respectively; the Borel parameters are taken as $M^2 = (2.6 - 3.0) \text{GeV}^2$ and $(7.0 - 8.0) \text{GeV}^2$ in the hidden charm and hidden bottom channels, respectively. In those regions, the two criteria of the QCD sum rules are full satisfied [14, 15].

Taking into account all uncertainties of the input parameters, finally we obtain the values of the masses and pole residues of the scalar molecular states $Y$, which are shown in Figs.6-7 and Tables 1-2.

From Tables 1-2, we can see that the uncertainties of the masses $M_Y$ are rather small (about 4% in the hidden charm channels and 2% in the hidden bottom channels) while the uncertainties of the pole residues $\lambda_Y$ are rather large (about $18 - 22\%$). The uncertainties of the input parameters ($\langle \bar{q}q \rangle$, $\langle \bar{s}s \rangle$, $\langle \bar{s}g_\sigma G \rangle$, $\langle \bar{q}g_\sigma Gq \rangle$, $m_s$, $m_c$ and $m_b$) vary in the range ($2 - 25\%$), the uncertainties of the pole residues $\lambda_Y$ are reasonable. We obtain the squared masses $M_Y^2$ through a fraction, the uncertainties in the numerator and denominator which originate from a given input parameter (for example, $\langle \bar{s}s \rangle$, $\langle \bar{s}g_\sigma G \rangle$) cancel out with each other, and result in small net uncertainty.

At the energy scale $\mu = 1 \text{GeV}$, $\frac{\alpha_s}{\pi} \approx 0.19$ [23], if the perturbative $O(\alpha_s)$ corrections to the perturbative term are companied with large numerical factors, $1 + \xi(s, m_Q) \frac{\alpha_s}{\pi}$, the contributions may be large. For example, we can make a crude estimation by multiplying the perturbative term with a numerical factor, say $1 + \xi(s, m_c) \frac{\alpha_s}{\pi} = 2$, in the hidden charm channel, the mass $M_{D^*_sD_s}$ decreases slightly, while the pole residue $\lambda_{D^*_sD_s}$ increases remarkably. From Figs.1-4, we can see that the main contributions come from the perturbative terms, the large corrections in the numerator and denominator cancel out with each other. In fact, the $\xi(s, m_Q)$ are complicated functions of the energy $s$ and the mass $m_Q$, such a crude estimation may be underestimate the $O(\alpha_s)$ corrections, the uncertainties originate from the $O(\alpha_s)$ corrections maybe larger.

In this article, we also neglect the contributions from the perturbative corrections.
Figure 5: The contributions from the pole terms with variation of the Borel parameter $M^2$. The $A$, $B$, $C$ and $D$ denote the $\bar{c}\gamma_{\mu}d\gamma^{\mu}c$, $\bar{c}\gamma_{\mu}s\gamma^{\mu}c$, $\bar{b}\gamma_{\mu}u\gamma^{\mu}b$, and $\bar{b}\gamma_{\mu}s\gamma^{\mu}b$ channels, respectively. In the hidden charm channels, the notations $\alpha$, $\beta$, $\gamma$, $\lambda$, $\rho$ and $\tau$ correspond to the threshold parameters $s_0 = 21\text{ GeV}^2$, $22\text{ GeV}^2$, $23\text{ GeV}^2$, $24\text{ GeV}^2$, $25\text{ GeV}^2$ and $26\text{ GeV}^2$ respectively; while in the hidden bottom channels they correspond to the threshold parameters $s_0 = 132\text{ GeV}^2$, $134\text{ GeV}^2$, $136\text{ GeV}^2$, $138\text{ GeV}^2$, $140\text{ GeV}^2$ and $142\text{ GeV}^2$ respectively.
\( \mathcal{O}(\alpha_s^n) \). Those perturbative corrections can be taken into account in the leading logarithmic approximations through anomalous dimension factors. After the Borel transform, the effects of those corrections are to multiply each term on the operator product expansion side by the factor,

\[
\left[ \frac{\alpha_s(M^2)}{\alpha_s(\mu^2)} \right]^{2\Gamma_{J/\eta} - \Gamma_{O_n}} \tag{14}
\]

where the \( \Gamma_{J/\eta} \) is the anomalous dimension of the scalar interpolating current \( J/\eta(x) \), the \( \Gamma_{O_n} \) is the anomalous dimension of the local operator \( O_n(0) \) in the operator product expansion,

\[
T \left\{ J/\eta(x)J/\eta(0) \right\} = C_n(x)O_n(0), \tag{15}
\]

here the \( C_n(x) \) is the corresponding Wilson coefficient.

We carry out the operator product expansion at a special energy scale, say \( \mu = 1 \) GeV, and can not smear the scale dependence by evolving the operator product expansion side to the energy scale \( M \) through Eq.(14) as the anomalous dimension of the scalar current \( J/\eta(x) \) is unknown. Furthermore, the anomalous dimensions of the high dimensional local operators have not been calculated yet, and their values are poorly known. In this article, we set the factor \( \left[ \frac{\alpha_s(M^2)}{\alpha_s(\mu^2)} \right]^{2\Gamma_{J/\eta} - \Gamma_{O_n}} \approx 1 \), such an approximation maybe result in some scale dependence and weaken the prediction ability; further studies are still needed.

The central value of the present prediction \( M_{D_s^*D_s^*} = (4.48 \pm 0.17) \) GeV is slightly larger than our previous calculation \( M_{D_s^*D_s^*} = (4.43 \pm 0.16) \) GeV \[13\]. In the present work, we take a slightly larger threshold parameter \( s_0 = (25 \pm 1) \) GeV\(^2\) rather than \( s_0 = (24 \pm 1) \) GeV\(^2\) to take into account the \( SU(3) \) breading effects and enhance the contribution from the pole term. From Table 1, we can see that the central values of the possible scalar molecular states are about \((250 - 500)\) MeV above the corresponding \( D^* - \bar{D}^* \), \( D_s^* - \bar{D}_s^* \), \( B^* - \bar{B}^* \), \( B_s^* - \bar{B}_s^* \) thresholds respectively \[23\], the \( D^*D_s^* \), \( D_s^*D_s^* \), \( B^*B^* \), \( B_s^*B_s^* \) are probably virtual states. In the constituent quark models, the energy gap between the ground state and the first radial excited state is about 500 MeV. The central values listed in Table 1 are below the corresponding thresholds of the first radial excited meson pairs. The scalar \( D^*D_s^* \), \( D_s^*D_s^* \), \( B^*B^* \), \( B_s^*B_s^* \) molecular states maybe not exist, while the scalar \( D_s^*D_s^* \), \( D_s^*D_s^* \), \( B^*B^* \), \( B_s^*B_s^* \) molecular states maybe exist.

In Refs.\[7\][13], the same current \( \bar{c}(x)\gamma_{\mu}s(x)\bar{s}(x)\gamma^\mu c(x) \) is used to interpolate the narrow structure \( Y(4140) \), however, the conclusions are quite different. The discrepancy mainly originates from the high dimensional vacuum condensates, the vacuum condensates of dimension-9,10 and the gluon involved vacuum condensates of dimension larger than 4 are neglected in Ref.\[7\]. Those condensates are counted as \( \mathcal{O}(\frac{m_c^2}{\Lambda^2}), \mathcal{O}(\frac{m_s^2}{\Lambda^2}), \mathcal{O}(\frac{m_g^6}{\Lambda^2}) \) respectively, and the corresponding contributions are greatly enhanced at small \( M^2 \), and result in rather bad convergent behavior in the operator product expansion, we have to choose larger Borel parameter \( M^2 \), one can consult the contributions from the \( \langle \bar{s}g_s\sigma Gs \rangle^2 \) term in Figs.1-F.2-F.3-F.4-F for example. If we neglect the terms concerning those high dimensional vacuum condensates and choose the input parameters (especially the value of the \( m_c \)) as Ref.\[7\], the experimental data can be reproduced. As a byproduct, we can see that the scale dependence of the QCD sum rules only weakens the prediction ability mildly.
However, we insist on taking into account the high dimensional vacuum condensates, as the interpolating current consists of a light quark-antiquark pair and a heavy quark-antiquark pair, one of the highest dimensional vacuum condensates is $\langle \bar{s}s \rangle^2 \times \langle \bar{q}q \rangle^2$.

The $c$-quark mass appearing in the perturbative terms (see e.g. Eq.(18)) is usually taken to be the pole mass in the QCD sum rules, while the choice of the $m_c$ in the leading-order coefficients of the higher-dimensional terms is arbitrary [25]. The $\bar{M}S$ mass $m_c(m^2_c)$ relates with the pole mass $\hat{m}$ through the relation

$$m_c(m^2_c) = \hat{m} \left[ 1 + \frac{C_F \alpha_s(m^2_c)}{\pi} + (K - 2C_F) \left( \frac{\alpha_s}{\pi} \right)^2 + \cdots \right]^{-1}, \quad (16)$$

where $K$ depends on the flavor number $n_f$. In this article, we take the approximation $m_c \approx \hat{m}$ without the $\alpha_s$ corrections for consistency. The value listed in the Particle Data Group is $m_c(m^2_c) = 1.27^{+0.07}_{-0.11}$ GeV [21], it is reasonable to take the value $m_c = m_c(1 \text{ GeV}^2) = (1.35 \pm 0.10)$ GeV in our works. In Ref.[13], we also present the result with smaller value $m_c = 1.3$ GeV, which can move down the central value about 0.06 GeV. The central value $M_Y = 4.37$ GeV is still larger than the $D_s^* D_s^*$ threshold about 150 MeV.

We can interpolate the scalar molecular states which consist of the scalar, pseudoscalar, vector, axial-vector and tensor meson pairs with the quark currents $\bar{Q}q'qQ$, $\bar{Q}i\gamma_5 q'q'\gamma_5 Q$, $\bar{Q}\gamma_\mu q'q'\gamma_\mu Q$, $\bar{Q}\gamma_\mu \gamma_5 q'q'\gamma_\mu \gamma_5 Q$ and $\bar{Q}\sigma_{\mu\nu} q'q'\sigma_{\mu\nu} Q$, respectively. Those molecule type interpolating currents relate with the diquark-antidiquark type interpolating currents through Fierz reordering in both the Dirac spinor space and the color space,

$$\begin{pmatrix}
\bar{Q}q'qQ \\
\bar{Q}i\gamma_5 q'q'\gamma_5 Q \\
\bar{Q}\gamma_\mu q'q'\gamma_\mu Q \\
\bar{Q}\gamma_\mu \gamma_5 q'q'\gamma_\mu \gamma_5 Q \\
\bar{Q}\sigma_{\mu\nu} q'q'\sigma_{\mu\nu} Q
\end{pmatrix} =
\begin{pmatrix}
-\frac{1}{8} & -\frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{16} \\
-\frac{1}{8} & \frac{1}{8} & \frac{1}{8} & -\frac{1}{8} & \frac{1}{16} \\
-\frac{1}{8} & \frac{1}{8} & -\frac{1}{8} & \frac{1}{8} & 0 \\
-\frac{1}{8} & \frac{1}{8} & \frac{1}{8} & -\frac{1}{8} & 0 \\
\frac{3}{8} & \frac{3}{8} & \frac{3}{8} & 0 & -\frac{1}{4}
\end{pmatrix}
\begin{pmatrix}
\bar{Q}\gamma_5 C \lambda^a q'q' QC\gamma_5 \lambda^a q \\
\bar{Q}C\lambda^a q'q' QC\lambda^a q \\
\bar{Q}\gamma_\mu \gamma_5 C \lambda^a q'q' QC\gamma_\mu \lambda^a q \\
\bar{Q}\gamma_\mu C \lambda^a q'q' QC\gamma_\mu \lambda^a q \\
\bar{Q}\sigma_{\mu\nu} C \lambda^a q'q' QC\sigma_{\mu\nu} \lambda^a q
\end{pmatrix}, \quad (17)$$

where $\lambda^0 = \sqrt{\frac{2}{3}} I$, the $\lambda^a$ with $a = 1, 2, \ldots, 8$ are the Gell-Mann matrices. The $\lambda^A$ with $A = 2, 5, 7$ are anti-symmetric and the $\lambda^S$ with $S = 0, 1, 3, 4, 6, 8$ are symmetric.

We usually take the diquarks as the basic constituents following Jaffe and Wilczek [26,27] to construct the tetraquark states with the diquark and antidiquark pairs. The diquarks have five Dirac tensor structures, scalar $C\gamma_5$, pseudoscalar $C$, vector $C\gamma_\mu$, axial vector $C\gamma_\mu$ and tensor $C\sigma_{\mu\nu}$, where $C$ is the charge conjunction matrix. The structures $C\gamma_\mu$ and $C\sigma_{\mu\nu}$ are symmetric, the structures $C\gamma_5$, $C$ and $C\gamma_\mu \gamma_5$ are antisymmetric. The attractive interactions of one-gluon exchange favor formation of the diquarks in color antitriplet $\bar{7}$, flavor antitriplet $\bar{\bar{7}}$ and spin singlet $1$. [28,29].

Naively, we expect the scalar tetraquark states with the structures $C\gamma_5 \lambda^A - \gamma_5 C \lambda^A$ and $C\lambda^A - C\lambda^A$ have the smallest masses. In Refs.[30,31], we study the scalar and vector hidden charm and hidden bottom tetraquark states which consist of $C\gamma_5 \lambda^A - \gamma_5 C \lambda^A$ type and $C\gamma_\mu \lambda^A - C\lambda^A$ (and $C\gamma_\mu \gamma_5 \lambda^A - \gamma_5 C \lambda^A$) type diquark pairs respectively in a systematic way; and observe that the masses of the vector tetraquark states are about $(0.6 - 0.7)$ GeV larger than the corresponding ones of the scalar tetraquark states. Furthermore, we observe that the scalar tetraquark states with the structure $C\gamma_5 \lambda^A - \gamma_5 C \lambda^A$ have much smaller masses.
molecular states in the B search for the hidden charm molecular states predicted in the present work may be observed at the LHCb, if they exist indeed. We can, J/ψω of 2 with exchanges of the intermediate mesons distributions. Those decays maybe take place through final-state re-scattering processes. QCD sum rules is still needed.

The conclusion is not robust enough, detailed analysis with the molecular states may have smaller masses than the corresponding ¯B B̄ pairs in a standard year of running at the LHCb operational luminosity of 2 × 10^32 cm^-2 sec^-1. The scalar D* D*, D* D*, B* B* and B* B* molecular states predicted in the present work may be observed at the LHCb, if they exist indeed. We can search for the hidden charm molecular states in the D̄ D̄, D* D*, D̄ D̄, D* D*, J/ψ p, J/ψ φ, J/ψ ω, ηcπ, ηcη, · · · invariant mass distributions and search for the scalar hidden bottom molecular states in the B̄ B̄, B* B*, B̄ B̄, B* B* γ, Yρ, Yφ, Yω, ηcπ, ηcη, · · · invariant mass distributions. Those decays maybe take place through final-state re-scattering precesses with exchanges of the intermediate mesons σ, π, ρ, D, D*, · · · in the t channels.

The QCD sum rules is just a QCD-inspired model, we calculate the ground state mass

\[ CλA - CλB \]}

\[ 2 \]The results with the structure CλA - CλB will be presented elsewhere.
Figure 7: The pole residues of the scalar molecular states with variation of the Borel parameter $M^2$. The $A$, $B$, $C$ and $D$ denote the $\bar{c}\gamma\mu u\bar{d}\gamma\alpha c$, $\bar{c}\gamma\mu s\bar{s}\gamma\alpha c$, $\bar{b}\gamma\mu u\bar{d}\gamma\alpha b$, and $\bar{b}\gamma\mu s\bar{s}\gamma\alpha b$ channels, respectively.

| molecular states         | masses     | thresholds [21] |
|--------------------------|------------|-----------------|
| $\bar{c}\gamma\alpha u\bar{d}\gamma\alpha c$ | $4.38 \pm 0.18$ | $4.014$         |
| $\bar{c}\gamma\alpha s\bar{s}\gamma\alpha c$ | $4.48 \pm 0.17$ | $4.224$         |
| $\bar{b}\gamma\alpha u\bar{d}\gamma\alpha b$ | $11.14 \pm 0.19$ | $10.650$        |
| $\bar{b}\gamma\alpha s\bar{s}\gamma\alpha b$ | $11.24 \pm 0.18$ | $10.831$        |

Table 1: The masses (in unit of GeV) of the scalar molecular states.

| molecular states         | pole residues |
|--------------------------|---------------|
| $\bar{c}\gamma\alpha u\bar{d}\gamma\alpha c$ | $5.1 \pm 1.1$ |
| $\bar{c}\gamma\alpha s\bar{s}\gamma\alpha c$ | $6.2 \pm 1.1$ |
| $\bar{b}\gamma\alpha u\bar{d}\gamma\alpha b$ | $2.7 \pm 0.6$ |
| $\bar{b}\gamma\alpha s\bar{s}\gamma\alpha b$ | $3.2 \pm 0.7$ |

Table 2: The pole residues (in unit of $10^{-2}$ GeV$^5$ and $10^{-1}$ GeV$^5$ for the hidden charm and bottom channels respectively) of the scalar molecular states.
by imposing the two criteria (pole dominance and convergence of the operator product expansion) of the QCD sum rules. In fact, we can take smaller threshold parameter $s_0$ and larger Borel parameter $M^2$ to reproduce the experimental value of the $Y(4140)$ as a scalar $D_s^* \bar{D}_s^*$ molecular state by releasing the pole dominance condition. We usually consult the experimental data in choosing the Borel parameter $M^2$ and the threshold parameter $s_0$. The present experimental knowledge about the phenomenological hadronic spectral densities of the multiquark states (irrespective of the molecule type and the diquark-antidiquark type) is rather vague. More experimental data are still needed.

4 Conclusion

In this article, we assume that there exist the scalar $D^* \bar{D}^*$, $D_s^* \bar{D}_s^*$, $B^* \bar{B}^*$ and $B_s^* \bar{B}_s^*$ molecular states, and study their masses using the QCD sum rules. Our predictions depend heavily on the two criteria (pole dominance and convergence of the operator product expansion) of the QCD sum rules. The numerical results indicate that the masses are about $(250 - 500)$ MeV above the corresponding $D^* - \bar{D}^*$, $D_s^* - \bar{D}_s^*$, $B^* - \bar{B}^*$ and $B_s^* - \bar{B}_s^*$ thresholds, the $Y(4140)$ is unlikely a scalar $D_s^* \bar{D}_s^*$ molecular state. The scalar $D^* \bar{D}^*$, $D_s^* \bar{D}_s^*$, $B^* \bar{B}^*$, $B_s^* \bar{B}_s^*$ molecular states may not exist, while the scalar $D^* \bar{D}^*$, $D_s^* \bar{D}_s^*$, $B^* \bar{B}^*$ and $B_s^* \bar{B}_s^*$ molecular states may exist, and may be observed at the LHCb.

Appendix

The spectral densities at the level of the quark-gluon degrees of freedom:

\[
\rho_0(s) = \frac{3}{1024 \pi^6} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \alpha \beta (1 - \alpha - \beta)^3 (s - \tilde{m}_Q^2)^2 (7s^2 - 6s\tilde{m}_Q^2 + \tilde{m}_Q^4) \\
+ \frac{3}{1024 \pi^6} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \alpha \beta (1 - \alpha - \beta)^2 (s - \tilde{m}_Q^2)^3 (3s - \tilde{m}_Q^2) \\
+ \frac{3m_s m_Q}{512 \pi^6} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (\alpha + \beta)(1 - \alpha - \beta)^2 (s - \tilde{m}_Q^2)^2 (5s - 2\tilde{m}_Q^2),
\] (18)
\[
\rho_{\langle ss \rangle}(s) = \frac{3m_s\langle ss \rangle}{32\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \alpha \beta (1 - \alpha - \beta)(10s^2 - 12s\bar{m}_Q^2 + 3\bar{m}_Q^4) \\
+ \frac{3m_s\langle ss \rangle}{32\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \alpha \beta (s - \bar{m}_Q)(2s - \bar{m}_Q) \\
- \frac{m_s\langle sg_s\sigma Gs \rangle}{64\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \alpha \beta \left[ 6(2s - \bar{m}_Q^2) + s^2\delta(s - \bar{m}_Q^2) \right] \\
- \frac{3m_Q\langle ss \rangle}{32\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \alpha \beta (1 - \alpha - \beta)(s - \bar{m}_Q)(2s - \bar{m}_Q) \\
+ \frac{3m_Q\langle sg_s\sigma Gs \rangle}{128\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \alpha \beta (s - \bar{m}_Q)(3s - 2\bar{m}_Q) \\
- \frac{3m_s^2\langle ss \rangle}{8\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (s - \bar{m}_Q) \\
- \frac{m_s\langle sg_s\sigma Gs \rangle}{64\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \alpha (1 - \alpha)(3s - 2\bar{m}_Q) \\
+ \frac{3m_s^2\langle sg_s\sigma Gs \rangle}{32\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha, \tag{19}
\]

\[
\rho_{\langle \bar{s}s \rangle}(s) = \frac{m_s^2\langle \bar{s}s \rangle^2}{4\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha + \frac{m_s^2\langle \bar{s}g_s\sigma Gs \rangle^2}{32\pi^2 M^6} \int_{\alpha_i}^{\alpha_f} d\alpha s^2\delta(s - \bar{m}_Q^2) \\
- \frac{m_s^2\langle \bar{s}g_s\sigma Gs \rangle}{8\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \left[ 1 + \frac{s}{M^2} \right] \delta(s - \bar{m}_Q^2) \\
- \frac{m_s m_Q\langle \bar{s}s \rangle^2}{16\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \left[ 2 + s\delta(s - \bar{m}_Q^2) \right] \\
+ \frac{5m_s m_Q\langle \bar{s}g_s\sigma Gs \rangle}{96\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \left[ 1 + \frac{s}{M^2} + \frac{s^2}{2M^4} \right] \delta(s - \bar{m}_Q^2), \tag{20}
\]
\[
\rho_{(\bar{g}G)}^A(s) = -\frac{m_Q^2}{256\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left( \frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2} \right) (1 - \alpha - \beta)^2 \left[ 2s - m_Q^2 + \frac{s^2}{6} \delta(s - m_Q^2) \right]
+ \frac{3m_s m_Q - m_Q^2}{512\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left( \frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2} \right) (1 - \alpha - \beta)^2 (3s - 2\tilde{m}_Q^2)
- \frac{m_s m_Q^3}{512\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left( \frac{1}{\alpha^3} + \frac{1}{\beta^3} \right) (\alpha + \beta)(1 - \alpha - \beta)^2 [2 + \delta(s - \tilde{m}_Q^2)]
- \frac{1}{512\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (\alpha + \beta)(1 - \alpha - \beta)^2 (10s^2 - 12s\tilde{m}_Q^2 + 3\tilde{m}_Q^4)
+ \frac{1}{256\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (\alpha + \beta)(1 - \alpha - \beta)(s - \tilde{m}_Q^2)(2s - \tilde{m}_Q^2)
- \frac{3m_s m_Q}{128\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (1 - \alpha - \beta)(s - \tilde{m}_Q^2)(3s - 2\tilde{m}_Q^2)
- \frac{m_s m_Q^3(\bar{s}s)}{96\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left( \frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2} \right) (1 - \alpha - \beta)
\left[ 1 + \frac{s}{M^2} + \frac{s^2}{2M^4} \right] \delta(s - \tilde{m}_Q^2)
- \frac{m_s m_Q^3(\bar{s}s)}{192\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left( \frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2} \right) \left[ 1 + \frac{s}{M^2} \right] \delta(s - \tilde{m}_Q^2)
+ \frac{m_s m_Q^3(\bar{s}g_sG)}{1152\pi^2 M^6} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left( \frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2} \right) \tilde{m}_Q^4 \delta(s - \tilde{m}_Q^2)
+ \frac{m_s m_Q^3(\bar{s}s)}{48\pi^2 M^2} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left( \frac{1}{\alpha^3} + \frac{1}{\beta^3} \right) \delta(s - \tilde{m}_Q^2)
+ \frac{3m_Q^3(\bar{s}s)}{192\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left( \frac{1}{\alpha^3} + \frac{1}{\beta^3} \right) (\alpha + \beta)(1 - \alpha - \beta)
\left[ 1 + \frac{s}{M^2} \right] \delta(s - \tilde{m}_Q^2)
- \frac{m_Q^3(\bar{s}g_sG)}{768\pi^2 M^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left( \frac{1}{\alpha^3} + \frac{1}{\beta^3} \right) (\alpha + \beta)\tilde{m}_Q^2 \delta(s - \tilde{m}_Q^2)
- \frac{m_Q^3(\bar{s}s)}{64\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left( \frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2} \right) (1 - \alpha - \beta) \left[ 2 + s\delta(s - \tilde{m}_Q^2) \right]
+ \frac{m_Q^3(\bar{s}g_sG)}{256\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left( \frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2} \right) \left[ 1 + \frac{s}{M^2} \right] \delta(s - \tilde{m}_Q^2)
- \frac{m_s m_Q^3(\bar{s}s)}{16\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left( \frac{1}{\alpha^2} + \frac{1}{\beta^2} \right) \delta(s - \tilde{m}_Q^2)
- \frac{m_s m_Q^3(\bar{s}s)}{64\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (\alpha + \beta) \left[ 1 + \frac{2s}{3} \delta(s - \tilde{m}_Q^2) + \frac{s^2}{6M^2} \delta(s - \tilde{m}_Q^2) \right]
+ \frac{m_Q^3(\bar{s}s)}{32\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left[ 2 + s\delta(s - \tilde{m}_Q^2) \right]
, \quad (21)
\]
\[ \rho_{(GG)}^B(s) = -\frac{m_Q^4 \langle \bar{s}s \rangle^2}{72 M^4} \int_{\alpha_i}^{\alpha_f} d\alpha \left[ \frac{1}{\alpha^3} + \frac{1}{(1-\alpha)^3} \right] \delta(s - \bar{m}_Q^2) \\
- \frac{m_s \langle s \bar{g}_s \sigma G \rangle}{192 \pi^2 M^4} \int_{\alpha_i}^{\alpha_f} d\alpha \left[ \frac{1}{\alpha^3} + \frac{1}{(1-\alpha)^3} \right] \delta(s - \bar{m}_Q^2) \\
+ \frac{m_s m_Q^2 \langle s \bar{g}_s \sigma G \rangle}{1152 \pi^2 M^4} \int_{\alpha_i}^{\alpha_f} d\alpha \left[ -\frac{1}{\alpha^2} + \frac{\alpha}{(1-\alpha)^2} \right] \bar{m}_Q^2 \delta(s - \bar{m}_Q^2) \\
- \frac{m_s m_Q^3 \langle \bar{s}s \rangle^2}{288 M^4} \int_{\alpha_i}^{\alpha_f} d\alpha \left[ \frac{1}{\alpha^3} + \frac{1}{(1-\alpha)^3} \right] \left[ 1 - \frac{s}{M^2} \right] \delta(s - \bar{m}_Q^2) \\
+ \frac{m_s^2 \langle \bar{s}s \rangle^2}{24 M^2} \int_{\alpha_i}^{\alpha_f} d\alpha \left[ \frac{1}{\alpha^2} + \frac{1}{(1-\alpha)^2} \right] \delta(s - \bar{m}_Q^2) \\
+ \frac{m_s m_Q^2 \langle s \bar{g}_s \sigma G \rangle}{64 \pi^2 M^2} \int_{\alpha_i}^{\alpha_f} d\alpha \left[ \frac{1}{\alpha^2} + \frac{1}{(1-\alpha)^2} \right] \delta(s - \bar{m}_Q^2) \\
- \frac{m_s m_Q \langle \bar{s}s \rangle^2}{96 M^4} \int_{\alpha_i}^{\alpha_f} d\alpha \left[ \frac{1-\alpha}{\alpha^2} + \frac{\alpha}{(1-\alpha)^2} \right] \bar{m}_Q^2 \delta(s - \bar{m}_Q^2) \\
+ \frac{m_s \langle \bar{s}s \rangle}{384 \pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \left[ 2 + s \delta(s - \bar{m}_Q) \right] \\
- \frac{m_Q \langle s \bar{g}_s \sigma G \rangle}{128 \pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \left[ 1 + \frac{s}{M^2} \right] \delta(s - \bar{m}_Q), \tag{22} \]

where \( \alpha_f = \frac{1+\sqrt{1-\frac{4\bar{m}_Q^2}{2}}}{2} \), \( \alpha_i = \frac{1-\sqrt{1-\frac{4\bar{m}_Q^2}{2}}}{2} \), \( \beta_i = \frac{\alpha m_Q^2}{\alpha s-\bar{m}_Q^2} \), \( \bar{m}_Q = \frac{(\alpha+\beta)m_Q^2}{\alpha \beta} \), \( \bar{m}_Q = \frac{m_Q}{\alpha(1-\alpha)} \).

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