Inverse Design of Multi-input Multi-output 2D Metastructured Devices

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Abstract—In this work, an optimization-based inverse design method is provided for multi-input multi-output (MIMO) metastructured devices. Typically, optimization-based methods use a full-wave solver in conjunction with an optimization routine to design devices. Due to the computational cost this approach is not practical for designing electrically-large aperiodic metastructured devices. To address this issue, a 2-D circuit network solver is introduced. The circuit network solver is used in conjunction with a gradient-based optimization routine that uses the adjoint variable method to solve large-scale optimization problems like those posed by metastructured devices. To validate the inverse design method, a planar beamformer and an analog signal processor for aperture field reconstruction are designed and validated with full-wave simulations.

Index Terms—Metastructures, optimization methods, antenna radiation pattern synthesis, analog processing circuits, microwave circuits

I. INTRODUCTION

The need for electromagnetic devices that can perform multiple field transformations, or exhibit multi-input multi-output (MIMO) functionality, arises in many applications such as in antenna beamforming, mode conversion, and recently for analog signal processing [1]–[3]. A promising route to the realization of these devices is through metastructured, or subwavelength textured, devices. Metastructured devices provide large degrees of freedom allowing for a single device to perform multiple field transformations. However, the design of MIMO metastructured devices requires the solution of an inverse design problem. This entails the determination of a set of unknown device characteristics, such as its geometry or material parameters, from a set of known inputs and outputs. Inverse design problems like these often lack direct solution methods and require heuristic or optimization-based methods to be solved. Heuristic methods can significantly simplify the design problem making it analytically tractable. However, they often impose limitations on the possible inputs and outputs, and have inherent errors associated with them. To avoid the limitations of heuristic methods an optimization-based approach is adopted in this work to design MIMO metastructured devices.

Previous work in the design of MIMO metastructured devices has included optimization-based design procedures that have been used to realize beamforming networks [4], [5], and analog signal processors, [2], [3]. The design procedures in [3]–[5] use full-wave solutions to Maxwell’s equations to solve the forward problem at every step of the optimization routine. This imposes a high computational cost on these design methods and places practical limitations on their ability to produce large, complex device’s. In [2], a method to reduce the computational cost of the forward problem was introduced that used a combination of full-wave solutions and the paraxial approximation to design a cascade of metasurfaces with MIMO functionality. However, only amplitude control of the transmitted field profiles was considered. The aim of this work is to provide an optimization-based procedure for designing MIMO metastructured devices with control over both the amplitude and phase of the transmitted fields.

Optimization-based inverse design of metastructured devices presents a computational challenge for two main reasons: (1) The large number of forward problems that need to be solved with different design parameters. (2) The multi-scale nature of the forward problem: subwavelength features in the unit cells and a multi-wavelength device size. The first difficulty is unavoidable so the forward problem solver must be fast. The second difficulty makes the forward problem solver slower, particularly when full-wave solutions are used. Here, these issues are addressed using a fast 2-D circuit network solver that uses reduced-order models for the unit cells of the metastructured device.

The design procedure implemented in this work poses the input-output relationship of a MIMO metastructured device as an optimization problem over the design variables. To realize the devices, the optimization problem is solved using a fast 2-D circuit network solver in conjunction with a gradient-based optimization routine that uses the adjoint variable method to efficiently calculate the gradient [2], [6], [7]. The utility of the proposed design procedure is demonstrated through the design of a planar beamformer and an analog signal processor for aperture field decomposition.

II. 2-D CIRCUIT NETWORK SOLVER

In this section, a frequency domain solver for 2-D structures supporting either transverse electric or transverse magnetic polarized fields is introduced. Numerical solutions to these
produces a staggered grid of nodal voltages that is organized into an MxN grid with unit cells defined by four-port admittance matrices, or impedance/admittance matrices, but admittance port networks can be represented by scattering matrices, wave propagation for a 2-D unit cell is a general four port network. These four used to model the unit cell coupling a natural representation (computation domain) is discretized. Since circuit theory is circuit network solver a unit cell is selected and the device and enforce desired boundary conditions. To solve for the voltages in the network, Kirchoff’s Current Law (KCL) is imposed at every node in the network. This produces a sparse linear system whose solution is the voltage at every node in the network.

\[ \mathbf{v} = \mathbf{Q}^{-1} \mathbf{s} \]

In (1), \( \mathbf{Q} \) characterizes all of the interactions between the unit cells in the network, \( \mathbf{v} \) is a vector containing all of the nodal voltages, and \( \mathbf{s} \) is a vector containing the source terms. To determine the structure of \( \mathbf{Q} \) and \( \mathbf{s} \), a grid with M unit cells in the x-direction and N unit cells in the z-direction is considered. There are six types of nodes that need to be accounted for: interior nodes on the \( V_x \) grid, interior nodes on the \( V_z \) grid, and the nodes on the four boundaries. The two different types of interior nodes and the boundary nodes along the input plane are shown in Fig. 2. The three other boundary nodes can be formed in a manner analogous to Fig. 2(c).

The elements of \( \mathbf{Q} \) and \( \mathbf{s} \) in (1) are found in the following manner. For a general interior node \( V_{ij} \) on the \( V_x \) grid, shown in Fig. 2(a), KCL leads to the following equation,

\[
\begin{align*}
V_{ij}^{x+y} (Y_{11}^{x} + Y_{33}^{x}) + V_{ij}^{y-1} (Y_{12}^{x} + Y_{42}^{x}) + V_{ij}^{x-1} (Y_{31}^{x} + Y_{41}^{x}) + V_{ij}^{x+y+1} (Y_{33}^{x}) \nonumber \\
+ V_{ij}^{x+y} Y_{12}^{x} + V_{ij}^{y-1} Y_{32}^{x} + V_{ij}^{x-1} Y_{42}^{x} + V_{ij}^{x+y+1} Y_{33}^{x} = 0
\end{align*}
\]

Applying KCL at a general interior node \( V_{ij}^{x+y} \) on the \( V_z \) grid, shown in Fig. 2(b), leads to the following equation,

\[
\begin{align*}
V_{ij}^{x+y} (Y_{22}^{z} + Y_{44}^{z}) + V_{ij}^{y-1} (Y_{24}^{z} + Y_{42}^{z}) + V_{ij}^{x-1} (Y_{24}^{z} + Y_{42}^{z}) + V_{ij}^{x+y+1} (Y_{44}^{z}) \nonumber \\
+ V_{ij}^{x+y} Y_{24}^{z} + V_{ij}^{y-1} Y_{42}^{z} + V_{ij}^{x-1} Y_{42}^{z} + V_{ij}^{x+y+1} Y_{44}^{z} = 0
\end{align*}
\]

Applying KCL at the four types of boundary nodes (see Fig. 1) produces the following equations: the left boundary \( V_{i1} \) (shown in Fig. 2(c)),

\[
V_{i1}^{x+y} (Y_{22}^{z} + \frac{1}{Z_{22}^{z}}) + V_{i1}^{y-1} Y_{24}^{z} + V_{i1}^{x-1} Y_{42}^{z} + V_{i1}^{x+y+1} Y_{44}^{z} = \frac{V_{i1}^{z}}{Z_{22}^{z}}
\]
forms a linear system of 2 expressions at every node in the network using (2)-(7) which is why they were not used here. Not always necessary and does increase the computational cost to capture these effects, [9], [10]. Using higher-order modes is another limiting way that it can maintain a high level of accuracy if good models of the unit cells are developed while avoiding full-wave solutions at run time. Eliminating full-wave solutions is that it can maintain a high-level of accuracy if good models of the unit cells are developed while avoiding full-wave solutions at run time. Eliminating full-wave solutions is necessary to capture the admittance matrices of its unit cells are characterized. The major advantage of solving for the device response this way is that it can maintain a high level of accuracy if good models of the unit cells are developed while avoiding full-wave solutions at run time. Eliminating full-wave solutions reduces the computational cost of solving the forward problem significantly, making the circuit network solver useful for the optimization-based inverse design of large aperiodic metastructures. The solver does come with limitations though. One limitation is the requirement that the unit cells can be represented as a four port network. This means that the problem of interest’s unit cells must have an equivalent guided wave representation, which is not always possible if there is a continuous spectrum of propagating waves. Another limiting assumption is that all of the interactions between the unit cells can be captured using a single guided mode. Meaning that mutual coupling between the unit cells and higher-order modes excited by inclusions or discontinuities are neglected. If these interactions become significant this assumption can be relaxed, and multi-modal Y-matrices or wave matrices can be used to capture these effects, [8], [10]. Using higher-order modes is not always necessary and does increase the computational cost which is why they were not used here.

III. MIMO INVERSE-DESIGN PROCEDURE

The fast 2-D circuit network solver introduced in the previous section solves for the output of a device given an input and all of the unit cell’s admittance parameters. However, in a MIMO inverse-design problem the unit cell’s admittance parameters are solved for given a set of desired inputs and outputs. In this section, an optimization-based inverse design procedure is described that achieves this goal. The desired MIMO functionality is realized by formulating the design objectives as an optimization problem that can be solved using the 2-D circuit network solver in conjunction with an off-the-shelf optimization routine. The design procedure is outlined and the details of how the optimization problem is formulated and solved is provided in the following subsections.

A. Optimization Problem

The design of a multi-input multi-output device begins with a set of inputs and outputs that describe its functionality. Here, these inputs and outputs are voltage distributions along the input and output planes of the device, see Fig. 1. These voltage distributions will be referred to as \( \{v_{\text{in}}^k\} \) for the inputs and \( \{v_{\text{out}}^k\} \) for the outputs where, \( k \in \{1, 2, ..., K\} \) and \( K \) is the total number of input-output pairs. Here, the term input-output pair refers to an input voltage distribution and its associated output voltage distribution, \( (v_{\text{in}}^k, v_{\text{out}}^k) \). Specifically, \( v_{\text{out}}^k \) are the observed voltages when the network is excited by \( v_{\text{in}}^k \). To realize the MIMO network described by \( \{v_{\text{in}}^k\} \) and \( \{v_{\text{out}}^k\} \) using optimization, the device’s performance for each input-output pair must be represented by a single real number. This can be done with the following cost function for the \( k \)th input-output pair,

\[
g_k(p) = \frac{1}{2} ((v_{\text{in}}^k) - v_{\text{out}}^k)^T G (v_{\text{in}}^k) - v_{\text{out}}^k)
\]

where \( p \) is a vector containing all of the design variables in the network, the vector \( v_{\text{in}}^k(p) \) contains the voltages in the network (subject to the design variables) when it is excited by \( v_{\text{in}}^k \), and the superscript \( T \) indicates the conjugate transpose. The matrix \( G \) is diagonal and positive-semidefinite. It is used to select and scale the elements of \( v_{\text{in}}^k(p) - v_{\text{out}}^k \). The performance of the device over all of the input-output pairs is determined by summing (8) over \( k \) to produce the total cost function,

\[
g(p) = \sum_{k=1}^K g_k(p)
\]

Framing the problem in this way reduces the design of the multi-input multi-output network to finding the design that minimizes the total cost, i.e. the minimizer \( p^* \), of (9). This goal is represented by the following optimization problem

\[
\arg \min_p g(p) \\
\text{subject to : } p_{\text{lb}} \preceq p \preceq p_{\text{ub}}
\]

where \( p_{\text{lb}} \) and \( p_{\text{ub}} \) are vectors containing the lower and upper bounds of the design variables, respectively.

Now that the optimization problem has been posed an appropriate optimization algorithm needs to be selected to solve it. Since the optimization problem is in general non-convex, local optimization or gradient-based algorithms are not guaranteed to find globally optimal solutions. However, global optimization algorithms do not tend to perform well in high-dimensional design spaces like the design space of a metastructured device. In high-dimensional spaces, local methods tend to outperform global methods when they use...
gradient information to navigate the design space. However, if the gradient cannot be expressed explicitly in closed form and the dimensions of the design space are large the computational cost of calculating the gradient can become prohibitive. To avoid this issue the adjoint variable method [2], [6], [7] can be used to calculate the gradient at a reduced computational cost. For these reasons, a gradient-based optimization routine utilizing the adjoint variable method is chosen to solve (10).

Since it is a local method there is no guarantee of convergence to a globally optimal solution, i.e. the solution of (10). However, this is not a problem since the globally optimal design is not required. The required design is just one that meets the design specifications so, a solution to (10) is considered any design that satisfies the design specifications.

\[ \nabla_p (g_k(p)) = -\Re \{ (v^k - v_{out}^k)^H \overline{Q} \overline{Q}_k^{-1} \overline{V}_p^k \} \]  

(11)

where \( \overline{V}_p^k \) is the following matrix,

\[ \overline{V}_p^k = \left( \frac{\partial^2 \overline{Q}_k v^k}{\partial p_1} \frac{\partial^2 \overline{Q}_k v^k}{\partial p_2} \frac{\partial^2 \overline{Q}_k v^k}{\partial p_3} \cdots \frac{\partial^2 \overline{Q}_k v^k}{\partial p_p} \right) \]  

(12)

This matrix can be solved for analytically if expressions for the derivatives of the admittance matrix (Y-matrix) elements in \( \overline{Q}_k \), the \( \overline{Q} \) matrix for the kth input-output pair, are available to determine \( \frac{\partial^2 \overline{Q}_k v^k}{\partial p_i} \). Otherwise, it can be obtained using finite-differences to approximate \( \frac{\partial^2 \overline{Q}_k v^k}{\partial p_i} \) at a low computational cost. The efficiency of calculating the gradient using (11) can be improved by observing that the product on the right hand side of (11), excluding \( \overline{V}_p^k \), forms a vector, \( \lambda^H_k \), that can be solved for independently,

\[ \lambda^H_k = (v^k - v_{out}^k)^H \overline{Q} \overline{Q}_k^{-1} \]  

(13)

Rearranging this expression yields the adjoint problem associated with (8),

\[ \overline{Q}_k \lambda_k = \overline{Q} (v^k - v_{out}^k). \]  

(14)

This allows for the adjoint variable \( \lambda_k \) to be computed at the cost of solving a forward problem of equal complexity to the original problem (10). Using (13) in (11) yields,

\[ \nabla_p (g_k(p)) = -\Re \{ \lambda^H_k \overline{V}_p^k \} \]  

(15)

Expressing (11) in this way provides a means of obtaining the gradient of the kth cost function, (8), at the computational expense of effectively two forward problem solutions. The gradient of (9) is then obtained by summing (15) over k,

\[ \nabla_p (g(p)) = -\sum_{k=1}^{K} \Re \{ \lambda^H_k \overline{V}_p^k \}. \]  

(16)

Therefore, in a problem containing P design variables the gradient can be determined with \( 2 \times K \) forward problem solutions using (16). Rather than the \( (P+1) \times K \) forward problem solutions required by finite-differences.

IV. PLANAR METASTRUCTURED BEAMFORMER

To demonstrate the effectiveness of the design procedure presented in Section III the multi-beam antenna beamformer shown in Fig. 4 is designed. The beamformer designed using this framework provides several advantages over Butler matrices and quasi-optical beamformers such as the Rotman
The beamforming region is assumed to be lossless and reciprocal and is designed to produce nine beams that are each associated with a different input port. It will operate at 10GHz and is intended for use with an aperture antenna that has a width of \( W_{ap} = 8\lambda_0 \). To produce a close approximation to a continuous aperture field the spacing between the output ports is chosen to be \( \lambda_0/10 \). This determines the discretization of the beamforming region and thus the unit cell size. The width of the beamformer is the same as the width of the aperture antenna, \( 8\lambda_0 \), and the depth is chosen to be \( 2.4\lambda_0 \). This depth was selected to provide sufficient distance to spread out the input power without utilizing cavity effects from the edges of the beamformer. Therefore, the overall dimensions of the beamformer are \( 8\lambda_0 \times 2.4\lambda_0 \). This corresponds to a network with \( M = 80 \) unit cells in the transverse direction and \( N = 24 \) in the longitudinal direction.

The nine inputs to the network are 70Ω port excitations located along the input plane, shown in Fig. 1. The inputs are spaced by 0.8\( \lambda_0 \) starting from the center line of the beamformer. This spacing aids in isolating the input ports from each other: a requirement for the simultaneous excitation of the beams. The requirement of isolation between the input ports and losslessness mandate that the radiated fields are orthogonal over a period of the radiation pattern [17]–[19]. This restricts the possible aperture fields and must be considered when selecting the desired radiation patterns of the aperture antenna. A well known set of functions that satisfy mutual orthogonality are sinc functions with appropriate angular spacings. For this reason, the aperture fields are chosen to have uniform amplitude with linear phase gradients corresponding to the following tangential wavenumbers,

\[
k_n = \frac{2\pi n}{M d}, \quad n \in \{0, \pm 1, \pm 2, ..., \pm (M - 1)\}.
\]

In (17), \( d \) is the spacing between the output ports and \( M \) is the total number of output ports. In this design the nine beams correspond to \( n = 0, \pm 1, \pm 2, \pm 3, \pm 4 \). Since \( d = \lambda_0/10 \) and \( M = 80 \), these correspond to beams at \( \theta_a = 0^\circ, \pm 7.18^\circ, \pm 14.36^\circ, \pm 22.02^\circ, \pm 30^\circ \). The output terminations are given by the input impedance of the aperture antennas ports. The input impedance for each port is 140Ω.

A. Design Specifications

B. Unit Cell Design

In order to use the design procedure presented in Section III, a suitable unit cell must be chosen and characterized.
Between the two directions to change path lengths in the x and z directions, as well as coupling should affect. The design variables should control: (1) The freedom in the response. It also provides information of six design variables should be included in the unit cell. It points to the fact that a maximum number of degrees of freedom (design variables) to allow extreme field transformations with a compact design. An arbitrary lossless and reciprocal four-port admittance matrix has a maximum of ten degrees of freedom. However, due to field averaging arguments, the degrees of freedom are reduced to effective material parameters in electrically-small structures, \[^{20},^{21}\]. The beamformer can be viewed as a lossless, reciprocal, polarization conserving medium supporting a TE wave in the x-z plane. The most general medium supporting this type of propagation is a 2-D omega medium that conserves polarization, i.e. a medium with the following material properties,

$$\overline{\mu} = \begin{pmatrix} \mu_{xx} & \mu_{xz} \\ \mu_{xz} & \mu_{zz} \end{pmatrix}, \quad \overline{\varepsilon}_{yy} = \overline{\varepsilon} = \overline{\eta} = j \begin{pmatrix} 0 & a_{xy} & 0 \\ -a_{xy} & 0 & a_{zy} \\ 0 & -a_{zy} & 0 \end{pmatrix}$$

where, \(\overline{\mu}\) and \(\overline{\varepsilon}\) are the magneto-electric and electro-magnetic tensors, respectively. This perspective provides some guidance for designing the unit cell. It points to the fact that a maximum of six design variables should be included in the unit cell. Additional design variables will only complicate the characterization of the unit cell without producing observable degrees of freedom in the response. It also provides information on what characteristics of the unit cell the design variables should affect. The design variables should control: (1) The path lengths in the x and z directions, as well as coupling between the two directions to change \(\overline{\varepsilon}_{yy}\). (2) The asymmetry in the x and z directions changes the bianisotropic response, \(\overline{\eta}\).

For these reasons the unit cell depicted in Fig. 5 is selected. The unit cell has dimensions \(dxd\) where \(d = 3\)mm. It is composed of four branches of microstrip transmission-lines meeting at a cross junction in the center. To avoid parasitic effects when interconnecting the unit cells, the transmission-line width at all of the ports is a constant value of \(w_0 = 0.25\)mm. There are six degrees of freedom in the unit cell: four lengths and two widths. Each branch contains one of the four length variables. While one of the width variables is in the x-directed branches and the other width variable is in the the z-directed branches.

To design the beamformer, the unit cell admittance parameters need to be available as continuously differentiable functions of the lengths and widths. Here, this is achieved by constructing a reduced-order model of the unit cell from a database of full-wave simulations. The unit cells in the database were characterized in isolation using Keysight’s Momentum, and a model of Rogers RO5880 substrate neglecting dissipative losses was used. The substrate has a height of \(h = 0.787\)mm and a relative permittivity \(\varepsilon_r = 2.2\). The design variables for each unit cell in the database are chosen to form a uniformly spaced grid of the allowable lengths and widths in the design space. The allowable range of widths is \(0.2 \leq W_{1,2} \leq 0.8\)mm and the lengths vary between \(0 \leq l_i \leq l_{i\text{max}}\). Where \(l_{i\text{max}} = d/2 - w_0/2 - W\) and \(W\) corresponds to the width of the line which is connected to \(l_i\) for \(i \in \{1, 2, 3, 4\}\). The admittance parameters from the database are then spline interpolated to generate a model of the unit cell that is differentiable with respect to the variable lengths and widths.

C. Optimization and Results

Since the beamformer produces symmetric beams, the beamformer should be symmetric as well. For this reason symmetry is imposed across the center line in the forward problem solver. This results in the device having 5,760 design variables. The design variables are solved for by providing the optimization routine with the input and output terminations, the desired input and output voltage distributions, the model of the unit cell, and an initial guess of uniform lengths and widths for the design variables. The algorithm then searches the database to find a set of design variables that satisfies the design requirements. This process took \(\sim 6\) hours to produce a satisfactory design on a personal computer with an i7-9700 CPU @ 3GHz w/8 cores and 64GB RAM. The layout of the design is shown in Fig. 6.

To evaluate the performance of the beamformer, the radiation pattern of the multi-beam antenna is computed using MATLAB. The calculation assumes that the electric field across the aperture has a piece-wise uniform amplitude and phase. The equivalent magnetic currents are then determined and the total radiated electric field is calculated. The radiation patterns resulting from the circuit network solver (forward problem solver) are in excellent agreement with the desired beams, as shown in Fig. 7. The beamformer inputs have a
Fig. 6. The patterned metastructured beamformer produced by the proposed inverse-design procedure. There are nine input ports that each produce unique voltage distributions across the 80 output ports to form the desired aperture fields. These aperture fields produce beams at \( \theta_n = 0^\circ, \pm 7.18^\circ, \pm 14.48^\circ, \pm 22.02^\circ, \pm 30^\circ \). The beamformer is designed to work at 10GHz and is composed of 1920 unit cells. The width of the aperture is \( W_{ap} = 8\lambda_0 \) (24cm) and the beamformer has a depth of \( h = 2.4\lambda_0 \) (7.2cm).

Fig. 7. Radiation from an aperture antenna excited by the output voltages of the beamforming network: dashed lines correspond to the radiation pattern from the desired voltages and the solid lines correspond to the voltages calculated using the circuit network solver. The radiation pattern is computed using MATLAB by assuming a piece-wise uniform electric field across the aperture. The total radiated electric field is calculated using equivalent magnetic currents across the aperture. For clarity, only beams corresponding to positive scan angles are shown since the beams corresponding to negative scan angles are identical due to symmetry.

worst case isolation of 21.2dB and a worst case reflectance of −38dB for the broadside excitation. These results are then compared to a full-wave simulation of the beamformer performed in Keysight Momentum using the same substrate that was used in the characterization of the unit cell. The simulation, the equivalent of one forward problem solution using Momentum, took approximately 98 hours on a high-performance computing cluster with access to 15 cores and 600 GB RAM. The results of the simulation are shown in Fig. 8. Due to slight variations in the observed voltages at the beamformer ports, there are small shifts and a slight broadening of the beams. The isolation and input impedance match are slightly degraded as well. There is a worst case isolation of 16.2dB and a maximum reflectance of −15.2dB for the broadside excitation. However, the overall agreement is quite good between the full-wave and circuit network solver results.

V. ANALOG SIGNAL PROCESSOR

In this section, an analog signal processor is designed to demonstrate the design method’s ability to realize a variety of aperture fields. The analog signal processor samples an incident aperture field at its input ports and decomposes it into a
Fig. 9. A depiction of the analog signal processor that performs aperture field decomposition. It has K readout ports and M input ports that can interface with an aperture antenna. The upper left block of the analog signal processor’s S-matrix is $0^{K \times K}$ indicating that its input ports are impedance matched and decoupled. The block $S_{lo}^{K \times M}$ performs the inner product of the aperture field with each of the aperture basis functions, and produces the weighting coefficients at each of the readout ports. The elements of $S_{oo}^{M \times M}$ are free variables, and are neglected in the design.

The aperture basis functions used in this example are a set of discrete orthogonal polynomials with equal Euclidean norm (ensures conservation of power). Specifically, the basis functions are a regularized version of the first eleven Gram or discrete Chebyshev polynomials. The Gram polynomials are selected since they are close approximations to the minimax polynomials used for approximating functions with finite sets of polynomials. The aperture basis functions are produced using the Discrete Orthogonal Polynomial toolbox in MATLAB \[23\], and are depicted in Fig. 10. Here, eleven basis functions are used, and it is demonstrated that eleven basis functions are sufficient to approximate some non-trivial aperture fields. However, if more accuracy is needed for highly oscillatory or aperture fields with discontinuities additional basis functions can be included by making the network larger or adjusting the spacings between the readout ports.

A. Design Specifications

The analog signal processor is implemented as a planar microstrip network. It can interface with an aperture antenna that operates at 10GHz and has a width of $W_{ap} = 9.1\lambda_0 = 27.3$cm. The network has the same width as the antenna aperture and has a depth of $h = 3\lambda_0 = 9$cm. The microstrip unit cell shown in Fig. 5 is used to design the network. The unit cell size is $d = \lambda_0/10 = 3$mm, and the corresponding grid has the following dimensions, $N = 91$ and $M = 30$. Each of the eleven aperture basis functions are assigned to a 50Ω readout port. The readout ports are located along the input plane, shown in Fig. 1 and are spaced by $0.7\lambda_0$, starting from...
maximum reflectance of functions is observed. The readouts are well matched with an agreement between the desired and realized aperture basis to produce a satisfactory design. The designed network’s computer (i7-9700 CPU @ 3GHz w/8 cores with 64GB RAM) design process took approximately 10 hours on a personal terminations, and an initial set of 16,380 design variables. The

B. Optimization and Results

Field input ports are open-circuited during the design process. All of the remaining ports besides the readout and aperture have an input impedance of $140\Omega$ to the beamformer example, the aperture antenna’s input ports are located along the output plane, shown in Fig. 1. Similar field input ports, which interface with the aperture antenna, function is associated with one readout port, all of the readout the center line of the device. To ensure that each aperture basis function is associated with one readout port, all of the readout ports are required to be isolated from each other. The aperture field input ports, which interface with the aperture antenna, are located along the output plane, shown in Fig. 1. Similar to the beamformer example, the aperture antenna’s input ports have an input impedance of $140\Omega$ at broadside so, the aperture field input ports are matched to $140\Omega$ for each basis function. All of the remaining ports besides the readout and aperture field input ports are open-circuited during the design process.

Fig. 11. Comparison of the incident field profile to its approximation using the first eleven Gram polynomials with the idealized weighting coefficients and the weighting coefficients from the planar microstrip network (metastructure) using the circuit network solver. The incident aperture field is given by $V_{ap}(x) = 9.7 \cos(\pi x/W_{ap})(x - 0.009)e^{-j0.87k_0x^2}$ (19), where $x$ is the position along the aperture in meters and $k_0$ is the free space wavenumber at 10GHz. The network is tested by solving for the complex weighting coefficients produced by the network for each incident aperture field using the circuit network solver. This is done by terminating the readout ports in $50\Omega$ and the input ports in $140\Omega$, and exciting the input ports with either (18) or (19). The reconstructed fields are calculated in MATLAB using the complex weighting coefficients produced by the network and the ideal aperture basis functions, and are shown in Fig. 11 for (18) and Fig. 12 for (19). They are compared to the exact aperture fields as well, as idealized approximations of the aperture field. The weighting coefficients for the idealized approximation are computed by taking the inner product of the incident field with the first eleven Gram polynomials. The amplitude and phase of the reconstructed aperture fields show good agreement in both cases. The largest errors between the reconstructed and the exact amplitude and phase profiles occur near discontinuities or where the derivative changes sign. In these regions the reconstructed and idealized approximations of the aperture field show good agreement. Indicating that these errors are largely due to the number of basis functions used rather than issues with the design itself.

For full-wave verification of the analog signal processor’s performance, it is simulated in Keysight Momentum. The full-wave simulation took $\sim 250$ hours to complete and the results for the aperture basis functions are shown in Fig. 10. Some variations in the aperture basis functions are observed but, overall the performance matches the circuit network solver quite well. The readouts are well matched with a maximum reflectance of $-12.7\text{dB}$ for the fifth polynomial’s readout port. The worst case isolation between the readout ports is $22\text{dB}$, occurring between the readout port for the fourth polynomial and the readout port for the fifth polynomial. Two different incident aperture fields are used to test the analog signal processor’s ability to decompose an incident field. The first aperture field is given by,

$$ V_{ap}(x) = 0.5 \text{sinc}(20x)e^{-j0.09\sin(\pi/6)x} \quad \text{V} \quad (18) $$

and the second incident aperture field is given by,

$$ V_{ap}(x) = 9.7 \cos(\pi x/W_{ap})(x - 0.009)e^{-j0.87k_0x^2} \quad (19) $$

where $x$ is the position along the aperture in meters and $k_0$ is the free space wavenumber at 10GHz. The network is tested by solving for the complex weighting coefficients produced by the network for each incident aperture field using the circuit network solver. This is done by terminating the readout ports in $50\Omega$ and the input ports in $140\Omega$, and exciting the input ports with either (18) or (19). The reconstructed fields are calculated in MATLAB using the complex weighting coefficients produced by the network and the ideal aperture basis functions, and are shown in Fig. 11 for (18) and Fig. 12 for (19). They are compared to the exact aperture fields as well, as idealized approximations of the aperture field.

For full-wave verification of the analog signal processor’s performance, it is simulated in Keysight Momentum. The full-wave simulation took $\sim 250$ hours to complete and the results for the aperture basis functions are shown in Fig. 10. Some variations in the aperture basis functions are observed but, overall the performance matches the circuit network solver quite well. The readouts are well matched with a maximum reflectance of $-12.7\text{dB}$ for the fifth polynomial’s readout port. The worst case isolation between the readout ports is $22\text{dB}$, occurring between the readout port for the fourth polynomial and the readout port for the sixth polynomial. The full-wave solution for the device’s scattering matrix is then used to reconstruct the aperture fields given by (18) and (19) and the results are shown in Fig. 11 and Fig. 12 respectively. Again the results match quite well but, some errors are observed due to the errors in the aperture basis functions. The largest error is seen in the reconstructed amplitude of (12) around the position $2\lambda_0$. This is largely due to errors present in the full-wave results for the fourth aperture basis function around this position since, (19) has a significant component along this basis function.
VI. Conclusion

In this paper, an inverse-design procedure for multi-input multi-output (MIMO) metastructured devices was provided. The design procedure uses a fast 2-D circuit network solver in conjunction with a gradient-based optimization routine to produce devices with desired MIMO functions. Since the gradient must be calculated at every step of the optimization routine, and metastructures have a large number of design variables, the adjoint variable method is used to calculate the gradient. The computational efficiency gained by using the fast 2-D circuit-based solver and the adjoint variable method enables the design procedure to realize electrically large aperiodic MIMO metastructures.

The efficacy of the design procedure was then demonstrated through the design of a planar antenna beamformer and an analog signal processor for aperture field decomposition. The beamformer supports the simultaneous excitation of nine beams, contains approximately 6000 design variables, and took approximately six hours to design. The analog signal processor uses eleven orthogonal aperture basis functions to decompose incident field profiles. It contains approximately 16,400 design variables and took approximately 10 hours to design. Both of the devices were implemented in microstrip technology and their performances were verified using the Keysight method of moments solver Momentum.

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