The Models of Permafrost for Arid and Alpine Regions

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Abstract. Stationary models of the thermal regime of permafrost taking into account water evaporation and ice sublimation within upper layer of soil are submitted. The results of Chinese researchers in mountainous Tibet and theoretical approach of Russian scientists for Northern Tien Shan are discussed. The thickness of the frozen rocks may reach an unexpectedly very large depth up to 100 m and more. Traditional stationary model of permafrost (Model Problem #0) was examined. The authors of the paper theoretically based the thermal regime of formation permafrost of cryolithozone in the highlands (two-stratum Model Problem #1). The new mechanism of formation of the thermal regime caused evaporation and sublimation processes in dispersed rocks in conditions of arid (high land) climate are suggested. Evaporation when filtering moisture in the upper soil layers is like heat sink. Then this process becomes the primary for the temperature field variations inside coarse clastic sediments. Additionally ice sublimation can increase heat dissipation in dispersed rocks. Three-stratum Model Problem #2 with a certain subsurface volumetric layer under frost desiccation was introduced. This soil layer being between the upper one of evaporation and the deep one of permafrost may generate a significant heat effect on the depth of freezing. Temperature minimum of the thermal system is formed in subsurface layer and cause heat sink into frozen soil depth. Numerical evolutions for the proposed models correspond to experimental data on the intensity of evaporation and sublimation for the surface and subsurface layers of permafrost, as well as the processes of cooling and freezing in dispersed rocks. This research will be in demand in engineering Cryolithology and Glaciology.

1. Introduction
The formation of frozen soil layers in high-mountainous and arid regions has not yet been studied in detail, allowing to obtain sufficiently reliable representations about the process, as well as predictive and reconstructive assessments. This process may be caused by another natural factors than the influence of only negative average annual air temperature as suggested at the traditional approaches in engineering Cryolithology and Glaciology during many decades [1-9].

The staff of the Institute of Glaciology and Cryolithology in Lanzhou carried out intensive geocryological investigations along the route at the building of a high-altitude highway through the Tibet Kinghai-Hizang region. The engineering and transport development of Chinese Highlands led to
an intensive study of the physical processes of the formation of mountain permafrost [10-12]. It was found that the thickness of frozen rocks reaches unexpectedly large values: 100 m and more.

The usual estimation of the thickness of permafrost rocks for a stationary case (traditional Model Problem #0) is:

\[ h_0 = -T_s \cdot \lambda / q_s. \]  

(1)

There \( T_s \) is average annual surface temperature, \( \lambda \) is thermal conductivity of the frozen ground and \( q_s \) is geothermal flux respectively.

It is difficult to expect large values of \( h_0 \) at average annual air temperatures \( T_a \) from 0 to -3°C, if also to take into account that usually the average annual surface temperature is not exceed absolute value \( T_a \) at thermal conductivity of frozen ground 1.5-2.1 W/(m-K) and geothermal flux about 0.1 W/m².

Similar facts may be cited for the regions of Mongolia. At one time the discovery of an “island-level” permafrost cause an average annual growth rate of \( T_a \), close to zero or even positive. It was unexpected and caused theoretical notions about the severe paleoclimate and relic underground permafrost for these regions [13-16].

An important contribution to the formation of the thermal regime of the cryolithozone cause by evaporation in dispersed rocks at conditions of arid climate. This high energy process is mainly characteristic for coarse detrital deposits. According to Gorbunov [5] in the Northern Tien Shan, at an average annual air temperature of 5-6°C, the seasonal freezing of talus reaches 3 m, whereas in the loamy-gravelly soils of similar magnitude is only a few tens centimeters. Well-known lenses of frozen talus which persist throughout the year are registered even in the Caucasus.

Evaporation in dispersed rocks stipulates significant impact upon formation of thermal regime in cryolithozone under conditions of arid climate. The energy-intensive process of ice sublimation is widely spread in coarse debris.

Evaporation in the upper layers of rocks plays the role of heat sink, so that the intensity of this process may have a decisive influence on the thermal regime of the underlying soils.

An analogy with the well-known school experience is relevant here, when the evaporation of a volatile ether from a glass freezes water poured into a saucer. Another well-known fact is the ancient way of keeping cold water in the hot climate of Central Asia: seeping through the porous walls of a clay pitcher, the moisture intensively evaporates because of the low humidity of the air from the outside and cools the water inside the vessel.

2. Modeling of thermal conditions for surface and subsurface layers of rocks

According to Ershov [13] the ice sublimation during freezing of sands may reach 2.5 g/m²·hour, this value is more than 2% per year in terms of unit volume. Close evaporation figures are obtained for underground caves of high mountains [14-16]. Probably in arid regions this value can be even greater due to filtration and evaporation of water in dispersed rocks.

To obtain quantitative estimates of the role of evaporation and sublimation in the thermal regime of permafrost, we consider Model Problems #1 and #2 for the stationary one-dimensional case with coordinate \( z \) along depth. Such approach may be used because of typical scales horizontal \( L \) and vertical \( h \) dimensions of the permafrost thick \( h << L \), i.e. the dimensionless model thin layers of ground are relatively small \( \delta = h/L << 1 \) (See figures 1).

2.1. Two-stratum Model Problem #1

The first is the two-stratum Model Problem #1 (figure 1a). For simplicity, we will assume that water evaporation (ice sublimation) is uniform in the upper layer of a porous rock of thickness \( h_e \) with intensity \( Q \). The heat conductivity equations for the frozen soil have the form:

\[ \lambda_e \frac{d^2 T_e}{dz^2} = Q, \quad 0 < z < h_e, \]  

(2)
\[ \lambda_2 \frac{d^2 T_2}{dz^2} = 0, \quad h_i < z < h. \] (3)

We indicate the boundary conditions of the problem. On the free surface, you need to set the temperature (average annual):

\[ z = 0, \quad T_i = T_f. \] (4)

The conditions of continuity of temperature and heat flux on the boundary between layers 1 and 3 (figure 1a) are necessary:

\[ z = h_1, \quad T_1 = T_2, \quad \lambda_1 \frac{d^2 T_1}{dz^2} = \lambda_2 \frac{d^2 T_2}{dz^2} \] (5)

On the base of permafrost rocks, it is necessary to set the zero temperature (the boundary of the “ice-water” phase transition) and the geothermal flux:

\[ z = h_1, \quad T_2 = 0, \quad \lambda_2 \frac{d^2 T_2}{dz^2} = q_s \] (6)

The solution of stationary heat transfer problem (the Model Problem #1 - figure 1a) has a form:

\[ T(z) = \begin{cases} 
 z \cdot \left[ Q \cdot \left( \frac{z - h_i}{2} \right) + q_s / \lambda_1 \right] + T_f, & 0 < z < h_i \\
 q_s \cdot (z - h_i) / \lambda_2 + T_f, & h_i < z < h_f 
\end{cases} \] (7)

**Figure 1.** Diagram of a two- (a) and a three-stratum (b) models of permafrost rocks:
1 - evaporation and sublimation layer, 2 – frost desiccation layer, 3 – frozen ground.

The depth of base of frozen rocks (power of permafrost) is determined with second formula (7) and first condition in (6):
In formula (8) the first term is traditional estimation (1) of permafrost thickness. The second term is an addition in estimation (1) caused evaporation (sink of heat) within upper layer \( h_v \) with power of source:

\[
Q = L_v \cdot \rho_w \cdot v.
\] (9)

There \( L_v \) is heat of evaporation, \( \rho_w \) is water density and \( v \) is relative volume of evaporated water (sublimated ice) per unit time respectively.

From formulae (8) and (9) it is deduced that at sufficient intensive evaporation when

\[
Q = \frac{L_v \cdot \rho_w \cdot v \cdot h_v}{2q_g} >> 1,
\]

the existence of frozen soil is possible even at \( T_s \approx 0^\circ C \).

### 2.2. Three-stratum Model Problem #2

We demonstrate the solution of complicated Model Problem #2 where layer of frost desiccation (layer #2 with depth \( h_f \) in figure 1b) is located between evaporation layer with thickness \( h_v \) and permafrost one with \( h=h_2 \). The problem may be applied as well as for arid regions [10-18] and some condition of Mars [19].

Formulation of the Problem #2 is similar Problem #1 is (figure 1):

\[
\lambda_1 \frac{d^2 T}{dz^2} - Q = 0, \quad 0 < z < h_v,
\]

\[
\lambda_1 \frac{d^2 T}{dz^2} = 0, \quad h_v < z < h_f,
\]

\[
\lambda_2 \frac{d^2 T}{dz^2} = 0, \quad h_f < z < h_2.
\]

We suppose that layers of evaporation and frost desiccation are of equal heat conductivity \( \lambda_f \). The boundary conditions of Problem #2 are similar Problem #1: (4) – (6). The solution of the Problem #2 is presented in form:

\[
T(z) = \begin{cases} 
T_s + z \cdot \left[ Q \cdot \left( \frac{z}{2} - h_v \right) + q_g \right] / \lambda_1, & 0 < z < h_v \\
T_s + \left( q_g \cdot z - \frac{Q \cdot h_v^2}{2} \right) / \lambda_2, & h_v < z < h_f \\
T_s + q_g \cdot (z - h_f) / \lambda_2 + \left( q_g \cdot h_f - \frac{Q \cdot h_f^2}{2} \right), & h_f < z < h_2
\end{cases}
\]

From the solution the total thickness of permafrost is defined by mean of formula:

\[
h_2 = -T_s \cdot \lambda_2 / q_g + h_f + \left( \frac{Q \cdot h_f^2}{2q_g} - h_f \right) \cdot \lambda_2 .
\] (10)

Effects of water evaporation (ice sublimation) and thermal impact upon intermediate layer of frost desiccation with low heat conductivity are submitted in the Model Problem #2. The formula (10)
results that increase of the thickness \((h_f - h_v)\) causes lowering of permafrost thickness. So that evaporation (sink of heat in layer \(h_v\)) impact on parameters permafrost oppositely.

We submit the other Model Problem which is similar Problem 
#2 at the difference. On the surface \(z = 0\) the condition of heat exchange is submitted instead of soil temperature:

\[
z = 0, \quad \lambda_i \frac{dT}{dz} = R + \alpha \cdot (T_u - T) .
\]

There \(\alpha\) is turbulent heat exchange coefficient, \(R\) is radiative balance of surface, \(T_u\) is air temperature respectively.

We show some qualitative distinctions only which follow from the text with the aim of its simplification.

Firstly, the temperature of surface soil is not a priori determined. It is calculated according to the formula:

\[
T_s = \frac{T_u + (R + q_g - Q \cdot h_i)}{\alpha}.
\]

The formula is similar one cited in the monograph of Pavlov [20]. The distinction consists in method of calculation of evaporation: in [20] it is calculated on surface, at our event it is volumetric one within layer of ground.

Secondly, the depth of permafrost base is determined by the formula:

\[
h = \frac{T_u \cdot \lambda_2}{q_g} + h_f + \left( \frac{Q \cdot h_i}{2q_g} - h_f \right) \cdot \frac{\lambda_2}{\lambda_1} + \frac{Q \cdot h_i - R}{q_g \alpha} - 1 . \tag{11}
\]

There evaporation intensity \(Q\) is determined correspondingly \((9)\). Thickness of permafrost depends on three factors: impact of air temperature (first term to the right of formula \((11)\)), sublimation and evaporation of ice in upper layer of soil (second term as well as in Model Problem 
#2) and heat exchange on the surface (third term).

Estimations of thickness permafrost are resulted for traditional Model Problem 
#0 and developed ones 
#1 and 
#2 (see table 1).

We use thermal conductivities of grounds \(\lambda_1 = 1.2\) and \(\lambda_2 = 2.1\) \(\text{W/(m \cdot K)}\) (at constant capacity and density) for changing of permafrost’ stratum’s thickness: evaporating (sublimating) surface layer \(h_v = 1-12\) m and frost desiccation one \(h_f = 2-20\) m (figure 1).

Energy losses were: for phase transition of “ice-water” \(Q_{ph} = 3.4 \cdot 10^5\) \(\text{J/kg}\), heat of ice sublimation \(Q_{subl} = 2.3 \cdot 10^6\) \(\text{J/kg}\) and heat of water evaporation \(L_v = 5 \cdot 10^6\) \(\text{J/kg}\) at geothermal flux \(q_g = 0.05 - 0.1\) \(\text{W/m}^2\) and relative volume of evaporated water (sublimated ice) \(v = 0.1 - 2\%\) per year.

| Table 1. Estimations of permafrost’ structural parameters for the traditional Model Problem 
#0 and the developed Model Problems 
#1 and 
#2. |
|---|---|---|---|---|---|---|
| Initial Data | Numbers of Model Problems, (Calculated Formulae) |
| | 
#0, (1) | 
#1, (8) | 
#2, (11) |
| \(T_s (^\circ \text{C})\) | \(v (\% \text{year})\) | \(h_v (\text{m})\) | \(h_f (\text{m})\) | \(h_0 (\text{m})\) | \(h_1 (\text{m})\) | \(h_2 (\text{m})\) |
| -3.0 | 2.0 | 1 | 2 | 13.8 | 13.1 |
| | 2.0 | 1 | 3 | 63 | 13.8 | 12.3 |
| | 1.0 | 2 | 4 | 27.7 | 26.2 |
| | 0.5 | 4 | 6 | 55.3 | 53.8 |
| -1.0 | 0.3 | 6 | 8 | 21 | 74.3 | 72.8 |
| | 0.2 | 8 | 10 | 87.3 | 85.8 |
| 0.0 | 0.1 | 8 | 10 | 40.7 | 39.2 |
| | 0.1 | 12 | 20 | 96.0 | 90.0 |
The calculations show that evaporation and sublimation in upper layer of soil causes significant growth of the permafrost thicknesses $h_1$ or $h_2$. Existence of evaporation layer of 10-m thickness causes growth of frozen ground thickness to a few tens meters even at comparatively small relative volume of evaporated water (sublimated ice) $v$ at total amount less 2%/year within layer $h_v$.

At positive mean annual surface temperatures and occurrence of dispersed rocks in upper horizons the existence of steady state is possible: the heat sink at evaporation effectively cools lower ground.

Qualitative picture soil temperature (vs) depth is shown on the figure 2. The peculiarity of the model problems solutions is a temperature minimum in layer of evaporation (sublimation) on depth:

$$z_{\min} = h_v - \frac{q_s}{L_v \cdot \rho_w \cdot v}.$$  

![Figure 2](image)

Figure 2. Temperature distribution in the structured layers of permafrost for the traditional Model Problem #0 (line 0) and the developed Model Problems #1 and #2 (lines 1 and 2).

The Model Problem #2 may describe water evaporation (sublimation ice) within layer of frozen soil and thermal impact of drying intermediate with low heat conductivity.

The equation (10) shows that increase of thickness of layer $(h_f-h_v)$ causes lowering of permafrost thickness. So that evaporation (sublimation) at heat sink within layer $0 < z < h_v$ may affect upon parameters of permafrost oppositely.

The layer of frosty desiccation can become a “heat screen” for generation of a much smaller thickness of permafrost.

3. Conclusion
The map of Alpine road Kingrhy-Khizang of Chinese researchers shows that almost everywhere a large thick of permafrost (with upper layer consisting of debris covers) is observed. Mean temperature of frozen thick decreases along depth from -1.5°C in soft soils up to -3.5°C in firm grounds, and thick of permafrost increases up to 120 m and more accordingly.

Submitted model corresponds to experimental data of intensity of soil freezing as well grounds cooling too [13] with little studied yet averaged parameters $h_v$, $h_f$, $\lambda_1$, $\lambda_2$ etc. Nevertheless, “playing” with the parameters allows to prove the model simulations and quality of cryology interpretations.

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