GENERATION OF A SEED MAGNETIC FIELD AROUND FIRST STARS: THE BIERMANN BATTERY EFFECT

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ABSTRACT

We investigate the generation processes of magnetic fields around first stars. Since first stars are expected to form anisotropic ionization fronts in the surrounding clumpy media, magnetic fields are generated by the effects of radiation force, as well as the Biermann battery effect. We calculated the amplitude of the magnetic field generated by the effects of radiation force around the first stars in a preceding paper but the Biermann battery effects were not taken into account. In this paper, we calculate the generation of magnetic fields by the Biermann battery effect as well as the effects of radiation force, utilizing radiation hydrodynamics simulations. As a result, we find that the generated magnetic field strengths are \( \sim 10^{-19} \text{ G} - 10^{-17} \text{ G} \) at \( \sim 100 \text{ pc} - 1 \text{ kpc} \) scale, an order of magnitude larger than the results of our previous study mainly as a result of the Biermann battery effect. We also find that this result is insensitive to various physical parameters including the mass of the source star and the distance between the source and the dense clump, unless we take unlikely values of these parameters.

Key words: early universe – \( \text{H}\alpha \) regions – magnetic fields – radiative transfer

Online-only material: color figures

1. INTRODUCTION

According to theoretical studies in the last decade, first stars are expected to be very massive (\( \gtrsim 100 M_\odot \)) (e.g., Bromm et al. 2002; Nakamura & Umemura 2001; Abel et al. 2002; Yoshida 2006). Recent studies, which properly address the accretion phase of first star formation, also revealed that the primary star formed in the center of the mini-halo is not as massive but still \( \gtrsim 10 M_\odot \), although a significant fraction of first stars are less massive (\( \lesssim 1 M_\odot \)) (Stacy et al. 2010; Clark et al. 2011b, 2011a; Greif et al. 2011; Smith et al. 2011). In any case, star formation episodes in the very early universe are different from that in local galaxies. One of the reasons for this difference is that the primordial gas clouds that host the first stars lack heavy elements, though they are the most efficient coolants in interstellar clouds at \( T \lesssim 1000 \text{ K} \). Because of the lack of these coolants, the temperature of the primordial gas cloud is kept around \( \sim 1000 \text{ K} \) during its collapse for \( n_{\text{HI}} < 10^{15} \text{ cm}^{-3} \), which is much hotter than the local interstellar molecular gas clouds. Consequently, the Jeans mass of the collapsing primordial gas is much larger than that of the interstellar gas, which leads to the formation of very massive stars (e.g., Omukai 2000).

Another important difference between the star formation sites in the early universe and local molecular clouds is the strengths of magnetic fields. Typical field strength \( B \sim 10 \mu \text{G} \) in the local molecular gas results in the formation of jets from protostars and regulates the gravitational collapse of cloud cores. The effects of the magnetic field on star formation in the early universe have been studied from theoretical aspects. First of all, the coupling of the magnetic field with the primordial gas was studied using a detailed chemical reaction model (Maki & Susa 2004, 2007). They found that the magnetic field is basically frozen in the primordial gas during its collapse, different from the local interstellar gas (e.g., Nakano & Umebayashi 1986a, 1986b). Under the assumption of the flux freezing condition, the dynamical importance of the magnetic field has also been investigated by several authors. If the magnetic field is stronger than \( \lesssim 10^{-9} \text{ G} \) at \( n_{\text{HI}} = 10^3 \text{ cm}^{-3} \), the field strength is amplified to \( \sim 10^3 \text{ G} \) at \( n_{\text{HI}} = 10^{21} \text{ cm}^{-3} \), which is enough to launch bipolar outflows and to suppress fragmentation of the accretion disk (Machida et al. 2006, 2008). Tan & Blackman (2004) estimated the condition for the magnetorotational instability (MRI) to be activated in the accretion disk around the protostar by comparing the ohmic dissipation timescale with the growth timescale of MRI. They found that the condition is \( B \gtrsim 10^{-2} \text{ G} \) at \( n_{\text{HI}} = 10^{15} \text{ cm}^{-3} \), which corresponds to \( B \gtrsim 10^{-10} \text{ G} \) at \( n_{\text{HI}} = 10^3 \text{ cm}^{-3} \). We also remark that the turbulent motion powered by the accreting gas can amplify the initial field strength faster than simple flux freezing, although the effects are still under debate. Thus, the magnetic field could be of importance at the final phase of the star formation process in primordial gas clouds if \( B \gtrsim 10^{-10} - 10^{-9} \text{ G} \) at \( n_{\text{HI}} = 10^3 \text{ cm}^{-3} \), i.e., at the initial phase of the collapse of primordial gas in the mini-halos.

In addition, recent theoretical studies suggested that heating by the ambipolar diffusion process in star-forming gas clouds could change the thermal evolution of the prestellar core in cases where the field strength is as strong as \( 10^{-10} \text{ G} \) at intergalactic medium (IGM) comoving densities (Schleicher et al. 2009; Sethi et al. 2008). This process also might lead to the formation of massive black holes (Sethi et al. 2010) since such heating can shut down the \( \text{H}_2 \) cooling and open the way for the atomic cooling (e.g., Omukai & Yoshii 2003). In any case, it is important to determine the magnetic field strengths in star-forming gas clouds in the early universe in order to quantify the effects of magnetic fields on primordial star formation. In spite of their potential importance, initial seed magnetic field strengths are still unknown observationally. Observations on the distortion of the cosmic microwave background spectrum (e.g., Barrow et al. 1997; Seshadri & Subramanian 2009) and measurements of the Faraday rotation in the polarized radio emission from distant quasars (e.g., Blasi et al. 1999; Vallée 2004) imply rather mild upper limits on the field strengths in the IGM, at the level of \( B \sim 10^{-9} \text{ G} \). On the other hand, various theoretical studies predict that \( B \) may be as small as \( \lesssim 10^{-18} \text{ G} \) at IGM densities. For instance, there are models that generate the magnetic field by the Biermann battery (Biermann 1950) during the structure formation (Kulsrud et al. 1997; Xu et al. 2008). A recent numerical simulation by Sur et al.
(2010) suggests that a strong magnetic field emerges during the collapse of turbulent prestellar cores of primordial gas due to the Biermann battery and turbulent dynamo action. There are also a number of models in which the fields are generated just after the big bang (e.g., Turner & Widrow 1988; Ichiki et al. 2006).

It is also suggested that reionization of the universe inevitably generates magnetic fields. Gnedin et al. (2000) and others suggested that in their cosmological simulations that the considerable Biermann battery term arises at the ionization fronts. They predict \( B \sim 10^{-19} \) G at IGM densities at \( z = 20 \); however, they did not take into account the Biermann battery effect, since they assume that the gas is isothermal and static.

In this paper, we extend our previous study (ADS10) to investigate the generation process of magnetic fields due to the ionizing radiation from first stars with more precision. We take into consideration not only the effects of radiation force but also the Biermann battery mechanism, utilizing two-dimensional radiation hydrodynamics simulations. Then we discuss whether the magnetic field strength obtained in our study could be important for the subsequent star formation process. In Section 2, we describe the basic equations and the setup of our model. We show the results of our calculations in Section 3. Sections 4 and 5 are devoted to the discussion and summary.

2. BASIC EQUATIONS AND MODEL

2.1. Equation of Magnetic Field Generation

According to ADS10, the equation of magnetic field generation is given as

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \frac{c}{en_{e}} \nabla n_{e} \times \nabla p_{e} - \frac{c}{e} \nabla \times \mathbf{f}_{\text{rad}},
\]

where \( \mathbf{B}, \mathbf{v}, \) and \( e \) denote the magnetic flux density, the fluid velocity, and the elementary charge, respectively. \( p_{e} \) and \( n_{e} \) represent the pressure and the number density of electrons. \( \mathbf{f}_{\text{rad}} \) is the radiation force acting on an electron. The first term on the right-hand side describes the advection term of magnetic flux, whereas the second term describes the Biermann battery term (Biermann 1950), which was not included in our previous study (ADS10). The third term is the radiation term which represents the momentum transfer from photons to gas particles. Note that the dissipation term due to the resistivity of the gas is omitted since it is negligible in comparison with the other terms (ADS10).

2.2. Radiation Force and Photoionization Rate

The radiation force on an electron, \( \mathbf{f}_{\text{rad}} \), involves two processes. The first one is the contribution from Thomson scattering, \( \mathbf{f}_{\text{rad},T} \), \( \mathbf{f}_{\text{rad},T} \) is given by

\[
\mathbf{f}_{\text{rad},T} = \frac{\sigma_{T}}{c} \int_{0}^{\nu_{L}} F_{0,v} d\nu + \frac{\sigma_{T}}{c} \int_{\nu_{L}}^{\infty} F_{0,v} \exp[\tau_{\nu_{L}} a(\nu)]d\nu,
\]

where \( \sigma_{T} \) denotes the Thomson scattering cross section, \( F_{0,v} \) is the unabsorbed energy flux density, \( \nu_{L} \) is the Lyman-limit frequency, \( \tau_{\nu_{L}} \) denotes the optical depth at the Lyman limit regarding the photoionization, and \( a(\nu) \) is the frequency dependence of the photoionization cross section, which is normalized at the Lyman-limit frequency.

Another source of the radiation force is the momentum transfer from photons to electrons through the photoionization process. The force, expressed as \( \mathbf{f}_{\text{rad},I} \), is

\[
\mathbf{f}_{\text{rad},I} = \frac{1}{2} \frac{n_{H}}{e} \int_{\nu_{L}}^{\infty} \sigma_{e} a(\nu) F_{0,v} \exp[\tau_{\nu_{L}} a(\nu)]d\nu,
\]

where \( \sigma_{e} \) is the photoionization cross section at the Lyman limit and \( n_{H} \) represents the number density of neutral hydrogen atoms.

In order to assess the electron number density, we also solve the following photoionization rate equation for electrons:

\[
\frac{\partial n_{e}}{\partial t} + (\mathbf{v} \cdot \nabla)n_{e} = \frac{\Gamma}{n_{H}} - \alpha_{B} n_{e} \gamma_{y} n_{H} + \kappa_{\text{coll}} n_{e} \gamma_{H} n_{H},
\]

where \( \alpha_{B} \) and \( \kappa_{\text{coll}} \) denote the case B recombination rate and collisional ionization rate per unit volume, respectively. \( \gamma_{y} \) and \( \gamma_{H} \) is the number fraction of electrons, protons, and neutral hydrogen atoms, respectively. \( n_{H} \) is the number density of hydrogen nuclei and \( \mathbf{v} \) denotes the velocity of the fluid. The photoionization rate per unit volume, \( \Gamma \), is also obtained by the formal solution of the radiation transfer equation:

\[
\Gamma = n_{H} \int_{\nu_{L}}^{\infty} \sigma_{e} F_{0,v} \exp[\tau_{\nu_{L}} a(\nu)]d\nu.
\]

2.3. Hydrodynamics with Heating/Cooling

We solve the ordinary set of hydrodynamics equations:

\[
\frac{D\rho}{Dt} = -\rho \nabla \cdot \mathbf{v},
\]

\[
\frac{D\mathbf{v}}{Dt} = \frac{1}{\rho} \nabla p - \nabla \phi_{DM},
\]

\[
\frac{D\epsilon}{Dt} = G - L - \rho \nabla \cdot \mathbf{v},
\]

and the equation of state

\[
\rho = (\gamma - 1)\rho \epsilon,
\]

where \( \rho, \mathbf{v}, \) and \( \epsilon \) are the density, pressure, and the specific energy of the fluid, respectively. \( \gamma \) denotes the specific heat ratio. \( G \) and \( L \) are the radiative heating rate and cooling rate per unit volume, respectively. \( \phi_{DM} \) is the gravitational potential of the dark matter halo (see Section 2.4). The feedback from the magnetic fields to the fluid is neglected in Equation (7), since we consider the generation of very weak magnetic fields. We also omit the self-gravitational force of gas, which is unimportant as long as we consider gas with \( n_{H} \lesssim 10^{3} \) cm\(^{-3}\).

In order to perform hydrodynamics simulations, we use the Cubic Interpolated Profile (CIP) method (Yabe & Aoki 1991). The CIP scheme basically tries to solve not only the advection of physical quantities but also the derivatives of the quantities. Using this scheme, we can capture spatially sharp profiles.
of fluids that are always expected in problems including the propagation of ionization fronts. In this paper, the CIP scheme is applied to the advection terms of Equations (4) and (6)–(8).

Using the formal solution of the radiation transfer equations, the radiative heating rate \( G \) is assessed as

\[
G = n_{H_1} \int_{v_L}^{\infty} \frac{F_{0v}}{h_v} h(v - v_L) \sigma_{vl} a(v) \exp[-\tau_{vl}(v)] dv.
\]

The cooling rate, \( L \), is given as

\[
L = L_{\text{coll}} n_e n_{H_1} + L_{\text{rec}} n_e n_p + L_{\text{exc}} n_e n_{H_1} + L_{\text{ff}} n_e n_p,
\]

where \( L_{\text{coll}}, L_{\text{rec}}, L_{\text{exc}}, \) and \( L_{\text{ff}} \) are the cooling coefficients associated with collisional ionization, recombination, collisional excitation, and free–free emission. These cooling rates are taken from the compilations in Fukugita & Kawasaki (1994).

2.4. Setup

We consider a mini-halo in the IGM at redshift \( z \approx 20 \) exposed to an intense radiation flux from a nearby first star. We assume that the dark matter density profile of the mini-halo is described by the Navarro–Frenk–White profile (Navarro et al. 1997),

\[
\rho_{\text{DM}}(r) = \frac{\rho_s}{(r/r_s)^2 (1 + r/r_s)^2},
\]

where \( r \) is the radial distance from the center of the halo, \( \rho_s \) and \( r_s \) are a characteristic density and radius, which are determined by a halo mass \( M_{\text{halo}} \), collapsing at redshift \( z \) (Prada et al. 2011). We assume the gas density profile is a core-halo structure with the \( \rho \propto r^{-2} \) envelope. The core radius \( r_c \) is determined by the core density and the total gas mass \( (M_{\text{gas}} = (\Omega_{\text{b}}/\Omega_{\text{M}})M_{\text{halo}}) \).

We study various cases of core densities \( (n_0 \text{ cm}^{-3}) \), the halo mass \( M_{\text{halo}} \), and the distances between the source star and the halo center \( (D \text{ kpc}) \). We assume stationarity of the mini-halo with respect to the source star. This assumption is based upon the fact that the change in distance between the star and the halo due to the relative motion is smaller than \( D \) if we consider cosmic expansion. We remark that in case where the neighboring overdense region and the source star are contained in the same halo of \( > 10^6 \text{ M}_{\odot} \), a change of the distance due to the velocity dispersion of the halo could have a significant effect, especially for low-mass source stars.

Initially, the ambient gas is assumed to be neutral when the source star is turned on. The initial number density of the ambient gas in the intergalactic space is \( n_{\text{IGM}} = 10^{-2} \text{ cm}^{-3} \). We assume that the initial temperature is 500 K. We perform two-dimensional simulations in cylindrical coordinates assuming axial symmetry. As shown in Figure 1, the computational domain is 100 pc \( \times \) 200 pc in the \( R-z \) plane. We consider source stars of various masses, \( M_* = 500, 300, 120, 60, 25, \) and \( 9 \text{ M}_{\odot} \). The luminosities, effective temperatures, and ages of these stars are taken from the table of Schaerer (2002). The incident radiation from the source star is assumed to be perpendicular to the left edge of the computational domain.

We employ four models (A–D) of different \( M_{\text{halo}}, n_0 \), and \( D \) listed in Table 1; these are plausible values of the mini-halos in the standard ΛCDM cosmology (see Section 3.2). The number of grids we use in these simulations is basically \( 250 \times 500 \), which is confirmed to be enough to obtain physical results by the convergence study (see Section 3.4).

3. RESULTS

3.1. Typical Results

First, we consider a first star of \( 500 \text{ M}_{\odot} \) \( (t_{\text{age}} = 2 \times 10^6 \text{ yr}) \). Figure 2 shows the results for model A. The two columns correspond to the snapshots at 0.1 Myr and 2 Myr. The top row shows the color contour of the mass density of the gas. The gas density distribution is isotropic at 0.1 Myr, while it is highly disturbed by the radiation flux from the left at 2 Myr. We can also find a shock front at 2 Myr inside the core of the overdense region.

The second and third rows show the maps of the temperature and the electron density. Clearly, an ionization front is generated by the flux from the left, and it propagates into the core of the cloud. It is also worth noting that the patterns of temperature and electron density distribution are similar but not identical to each other. A slight difference between the two contour maps leads directly to the nontrivial Biermann battery term.

The bottom row shows the distribution of the generated magnetic field strength. As expected, we obtain strong magnetic fields in the neighborhood of the ionization front. In addition, strong magnetic fields are produced at the center of the core where the density is the highest. The peak field strength is as strong as \( 6 \times 10^{-18} \text{ G} \) in this case.

In Figure 3, we show the time evolution of the peak magnetic field strength in the simulated region (red curve). We also plot the peak magnetic field generated by the Biermann battery term (blue) as well as that generated by the radiation processes (green). The field strength grows almost linearly in this model, and the final strength is as large as \( \sim 6 \times 10^{-18} \text{ G} \). We also find that the radiation process is less important than the Biermann battery effect. However, it is still noteworthy that the difference between the two contributions is only a factor of \( \sim 10 \), although the nature of these processes are very different from each other.
3.2. Dependence on \( n_0, D, \text{ and } M_\ast \)

We also investigate the other models of \( n_0 \) and \( D \) listed in Table 1. The snapshots at 2 Myr for these models (B, C, and D from top to bottom) are shown in Figure 4. The left column shows the magnetic field strength, whereas the right column illustrates the number density of electrons. In model B, the generated magnetic field is of the order of \( \sim 10^{-19} \) G (top left panel), which is smaller than that in model A. Since the ionization front is not trapped by the dense core in model B (top right), the source terms of Equation (1) become very small as soon as the ionization front passes through the core. As a result, the magnetic field does not have enough time to grow. In models C and D, the core is 10 times more distant from the source star than models A and B. Therefore, the ionization front is trapped at lower density regions and becomes less sharp than that in model A. Consequently, the generated magnetic field is smaller than that in model A (middle and bottom rows).

We also plot the time evolution of the peak field strengths of models A–D in Figure 5. Models A, C, and D mostly increase monotonically, whereas model B has a clear plateau/decline. Such different behavior also comes from the fact that the ionization front immediately passes through the core in model B. In such a case, the time for the magnetic field to grow is not enough as stated above. In addition, the fluids that host the generated magnetic fields expand due to the thermal pressure of the photoionized gas. Such expansion results in the slight decline of the magnetic field strength.

The dependence on the source stellar mass \( M_\ast \) is also studied. We employ six models of \( M_\ast = 500, 300, 120, 60, 25, \) and \( 9 \, M_\odot \), while the other parameters are the same as in model A. Figure 6 shows the peak magnetic field strength as a function of \( M_\ast \). The peak field strength of each run is evaluated when it gets to the time for the death of the source star. The peak field strength basically increases as the stellar mass becomes more massive. However, the difference between the field strength of \( M_\ast = 9 \, M_\odot \) and \( 500 \, M_\odot \) is a factor of nine, which is a small difference for such a mass difference. The reason for this behavior comes from the fact that the more massive the stars are, the more ionizing photons they emit but also the
shorter lifetime they have. These two competing effects cancel each other.

In any case, the generated magnetic field strengths stay around $10^{-18} - 10^{-17}$ G if we consider stellar masses of $9 M_\odot - 500 M_\odot$, a range wide enough for first stars.

The parameters $n_0$ and $D$ we employ in this paper are reasonable values. The baryonic density of the virialized halo at $z = 20$ is $\sim 1$ cm$^{-3}$, and the sizes of the first halos are $\lesssim 100$ pc. In addition, the radii of cosmological H II regions in the numerical simulations are a few kpc (e.g., Yoshida et al. 2007). Thus, our models A–D are reasonable for the standard cosmological model.

To summarize the results of this section, the magnetic field strength generated around a first star is $10^{-19} < B < 10^{-17}$ G for $M_*= 500 M_\odot$, and a factor of a few smaller for less massive stars.

### 3.3. Coherence Length

In addition, we also make mention of the coherence length of the magnetic field. Due to limited computational resources, we employ a rather small box size ($\sim 200$ pc). The resultant coherence length of the magnetic field clearly exceeds the box size in most of the cases. In addition, if the ionization structure is more or less similar to that of our previous results in ADS10, the coherence length will be as large as $\sim 1$ kpc, which is the size of the shadow.
3.4. Convergence Check

Since the equation for magnetic field generation, Equation (1), includes spatial derivatives in its source terms, we have to pay attention to the effects of the cell size on our numerical results. We check the numerical convergence of the magnetic field strengths for model A. We perform runs with $N_R \times N_z = 63 \times 125, 125 \times 250, 250 \times 500$ (canonical), $400 \times 800, 500 \times 1000$, where $N_R$ and $N_z$ are the number of grids in the $R$-axis and the $z$-axis, respectively. In Figure 7, the magnetic field probability distribution functions (PDFs) of these runs are plotted. The abscissa is the absolute value of the magnetic field strength, while the vertical axis represents the fractional grid cells that fall in the range of $[B, B + \Delta B]$, where $\Delta B = 5 \times 10^{-19}$ G. The peak field strength declines as the number of grids increases. However, the peak field strength converges at $\sim 6 \times 10^{-18}$ G and the PDFs show similar distributions for $N_R \times N_z \geq 250 \times 500$. Therefore, the numerical results of the simulations converge very well at $N_R \times N_z = 250 \times 500$, which we use in other runs.

4. DISCUSSION

In this paper, we investigate the magnetic field generated by first stars. As a result, the maximal magnetic field strength is $\lesssim 10^{-17}$ G, mainly generated by the Biermann battery mechanism.

In fact, the order of magnitude of the magnetic field generated by the Biermann battery could be assessed as

$$B \sim \frac{c}{n_e^2 e} \left( \frac{n_e}{\Delta r} \right) \left( \frac{p_e}{\Delta r} \right) \sin \theta \frac{L_{\text{age}}}{\Delta t},$$

$$\sim 5.0 \times 10^{-17} \text{G} \left( \frac{L_{\text{age}}}{2 \text{Myr}} \right) \left( \frac{\sin \theta}{0.1} \right) \left( \frac{\Delta r}{1 \text{pc}} \right)^{-2} \left( \frac{T}{10^4 \text{K}} \right),$$

where $\Delta r$ denotes the typical length scale over which $n_e$ and $p_e$ change significantly, and $\theta$ is the angle between $\nabla n_e$ and $\nabla T$. The resultant value of the above equation is roughly consistent with the results of our numerical simulations.

In this paper we explore the relative importance of the Biermann battery effect versus radiative processes. However, this is only relevant for the present setup, because the two effects depend differently on various parameters. In particular, the Biermann battery effect does not depend on the flux of the source star directly (see Equation (13)), whereas the radiation force is proportional to the flux. This means that if the magnetic field generation process occurs in the neighborhood of source objects, such as the accretion disks of the protostars/black holes, radiative processes could play a central role in the generation of the magnetic field. We will study this issue in the near future.

The field strength obtained in this paper is similar to the results of Xu et al. (2008), who investigated magnetic fields in collapsing mini-halos/prestellar cores. If we assume that the magnetic fields generated around first stars are brought into another prestellar core and evolve during the collapse of the primordial gas in a similar way to Xu et al. (2008), the magnetic field will be amplified up to $\sim 10^{-13}$ G at $10^3$ cm$^{-3}$. However, this magnetic field will not affect subsequent star formation since the magnetic field strength required for jet formation (Machida et al. 2006) and MRI activation (Tan & Blackman 2004) is $10^{-10} - 10^{-9}$ G at $10^3$ cm$^{-3}$ if we assume the simple flux freezing condition. On the other hand, recent studies suggest that the weak seed magnetic field is amplified by turbulence during the first star formation (Schleicher et al. 2010; Sur et al. 2010). In Sur et al. (2010), weak seed magnetic fields are exponentially amplified by small-scale dynamo action if they employ sufficient numerical resolutions. In this case, the magnetic fields generated by the first stars in this paper might be amplified and affect subsequent star formation. However, these studies assume a priori given turbulence and initial magnetic field. To try to settle this issue, we need cosmological MHD simulations with very high resolution.

5. SUMMARY

In summary, we have investigated the magnetic field generation process by the radiative feedback of first stars, including the effects of radiation force and the Biermann battery. As a result, we found $10^{-19} \text{G} \lesssim B \lesssim 10^{-17}$ G on the boundary of the

![Figure 7. Magnetic field probability distribution functions are plotted for various $N_z$.](figure7.png)
shadowed region, if we take reasonable parameters expected from the standard theory of cosmological structure formation. The resultant field strength with a simple assumption of flux freezing suggests that such a magnetic field is unimportant for the star formation process. However, it could be important if the magnetic field is amplified by turbulent motions of the star-forming gas cloud.

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REFERENCES

Abel, T., Bryan, G. L., & Norman, M. L. 2002, Science, 295, 93
Ando, M., Doi, K., & Susa, H. 2010, ApJ, 716, 1566
Barrow, J. D., Ferreira, P. G., & Silk, J. 1997, Phys. Rev. Lett., 78, 3610
Biermann, L. 1950, Z. Nat.forsch. A, 5, 65
Blasi, P., Burles, S., & Olinto, A. V. 1999, ApJ, 514, L79
Bromm, V., Coppi, P. S., & Larson, R. B. 2002, ApJ, 564, 23
Clark, P. C., Glover, S. C. O., Klessen, R. S., & Bromm, V. 2011a, ApJ, 727, 110
Clark, P. C., Glover, S. C. O., Smith, R. J., et al. 2011b, Science, 331, 1040
Fukugita, M., & Kawasaki, M. 1994, MNRAS, 269, 563
Gnedin, N. Y., Ferrara, A., & Zweibel, E. G. 2000, ApJ, 539, 505
Greif, T. H., Springel, V., White, S. D. M., et al. 2011, ApJ, 737, 75
Ichiki, K., Takahashi, K., Ohno, H., Hanayama, H., & Sugiyama, N. 2006, Science, 311, 827
Kulsrud, R. M., Cen, R., Ostriker, J. P., & Ryu, D. 1997, ApJ, 480, 481
Langer, M., Puget, J., & Aghanim, N. 2003, Phys. Rev. D, 67, 43505
Machida, M. N., Matsumoto, T., & Inutsuka, S.-i. 2008, ApJ, 685, 690
Machida, M. N., Omukai, K., Matsumoto, T., & Inutsuka, S. 2006, ApJ, 647, L1
Maki, H., & Susa, H. 2004, ApJ, 609, 467
Maki, H., & Susa, H. 2007, PASJ, 59, 787
Nakamura, F., & Umemura, M. 2001, ApJ, 548, 19
Nakano, T., & Umebayashi, T. 1998a, MNRAS, 291, 319
Nakano, T., & Umebayashi, T. 1998b, MNRAS, 298, 663
Navarro, J. F., Frenk, C. S., & White, S. D. M. 1997, ApJ, 490, 493
Omukai, K. 2000, ApJ, 534, 809
Omukai, K., & Yoshii, Y. 2003, ApJ, 599, 746
Prada, F., Klypin, A. A., Cuesta, A. J., Betancort-Rijo, J. E., & Primack, J. 2011, arXiv:1104.5130
Schaefer, D. 2002, A&A, 382, 28
Schleicher, D. R. G., Banerjee, R., Sur, S., et al. 2010, arXiv:1003.1135
Schleicher, D. R. G., Galli, D., Glover, S. C. O., et al. 2009, ApJ, 703, 1096
Seshadri, T. R., & Subramanian, K. 2009, Phys. Rev. Lett., 103, 081303
Sethi, S., Haiman, Z., & Pandey, K. 2010, ApJ, 721, 615
Sethi, S. K., Nath, B. B., & Subramanian, K. 2008, MNRAS, 387, 1589
Smith, R. J., Glover, S. C. O., Clark, P. C., Greif, T., & Klessen, R. S. 2011, MNRAS, 414, 3633
Stacy, A., Greif, T. H., & Bromm, V. 2010, MNRAS, 403, 45
Sur, S., Schleicher, D. R. G., Banerjee, R., Federrath, C., & Klessen, R. S. 2010, ApJ, 721, L134
Tan, J. C., & Blackman, E. G. 2004, ApJ, 603, 401
Turner, M. S., & Widrow, L. M. 1998, Phys. Rev. D, 57, 2743
Vallée, J. P. 2004, New Astron. Rev., 48, 763
Xu, H., O'Shea, B., Collins, D., et al. 2008, ApJ, 688, L57
Yabe, T., & Aoki, T. 1991, Comput. Phys. Commun., 66, 219
Yoshida, N. 2006, New Astron. Rev., 50, 19
Yoshida, N., Oh, S. P., Kitayama, T., & Hernquist, L. 2007, ApJ, 663, 687