Big Bounce Genesis

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(Dated: March 25, 2014)

We report on the possibility to use dark matter mass and its interaction cross section as a smoking gun signal of the existence of a big bounce at the early stage in the evolution of our currently observed universe. A model independent study of dark matter production in the contraction and expansion phases of the bounce universe reveals a new venue for achieving the observed relic abundance in which a significantly smaller amount of dark matter—compared to the standard cosmology—is produced and survives until today, diluted only by the cosmic expansion since the radiation dominated era. Once DM mass and its interaction strength with ordinary matter are determined by experiments, this alternative route becomes a signature of the bounce universe scenario.

In recent years there has been a concordance of effort to extend the standard model of cosmology, either by embedding it in string theory or quantum gravity theory, or simply searching for alternatives of the inflation paradigm [1].

In this letter we undertake a detailed but model independent study of dark matter production in the bounce universe scenario. An unconventional production route of the dark matter is unveiled in which twice as much dark matter is produced compared to naive expectation, due to productions during the pre-bounce contraction, in addition to productions in the post-bounce expansion.

Furthermore, compared to the standard model in which “sufficient” dark matter is assumed to be produced prior to freeze-out, in the bounce scenario there exists a possibility of “insufficient” production and hence no subsequent freeze-out.

We also analyze the implications of the existence of this new branch and argue that it can be used as a discriminating signature for the bounce universe scenario.

The Bounce Universe Scenario

To facilitate a model independent analysis we divide the bounce process into five stages as illustrated in Fig. 1:

• Phase 0: Radiation dominated era before the universe enters a matter-dominated contraction.

• Phase I: The pre-bounce contraction, in which \( H < 0 \) and \( m_\chi < T < T_b \).

• The Bounce Point: This is a highly model dependent phase but it is usually assumed to be short enough that it does not upset the known physical processes of interest. For dark matter production this is indeed the case.

• Phase II: The post-bounce expansion, in which \( H > 0 \) and \( m_\chi < T < T_b \).

• Phase III: the possible freeze-out process of dark matter, in which \( H > 0 \) and \( m_\chi > T \).

\( H \) is the Hubble constant.

In this work, except the “bounce period” indicated by dash-dot line in Fig. 1, we model the universe in the pre-bounce era and post-bounce era to be radiation dominated, \( \rho \propto a^4 \), in accordance with the philosophy of the bounce scenario. The bounce period is, however, a highly model-dependent process, the detail of which depends on the underlying physics of each specific bouncing universe model and is a hotly contested subject of many a recent investigation, for example see [4–9].

For our purpose we note: all bounce models assume that the bounce period is sufficiently short, especially when compared with the process of dark matter production. Its influence on the latter, therefore, can be safely neglected. To sum up, without committing to any particular model of the “smooth bounce,” we are still able...
to subject the process of dark matter production to a quantitative analysis within the framework of “bounce universe.” Interested readers can embed their favorite models of the Big Bounce to work out the sub-leading effects.

**The Dynamics of Dark Matter Production:**

The dark matter production can be seen to follow two distinctive routes which we call “sufficient production” and “insufficient production” as depicted in Fig. 2.

![Fig. 2](image)

**FIG. 2.** $|\mathcal{M}|$ v.s. $m_\chi$: Sufficient production (with freeze-out) and insufficient production (without freeze-out) of dark matter.

The production of dark matter is modeled by

$$\chi + \chi \longleftrightarrow \phi + \phi .$$  \hspace{1cm} (1)

Without loss of generality, we consider only one species of dark matter particles, $\chi$, which can annihilate to be a pair of the light bosonic particles, $\phi$. This reaction is reversible and its thermodynamical equation takes the following form,

$$\frac{d(n_\chi a^3)}{dt} = \left( n_\chi(0)^2 n_\phi^2 \right)^{1/2} \left[ \frac{\sigma v}{n_\phi(0)^2} - \frac{n_\chi(0)^2}{(n_\phi(0))^2} \right] ,$$  \hspace{1cm} (2)

where $n_\chi$ and $n_\phi$ being the number densities of the indicated species while $n_\chi(0)$ and $n_\phi(0)$ being their respective equilibrium number densities. As usual $a$ is the scale factor of the cosmological background with $\langle \sigma v \rangle$ being the thermally averaged cross section.

To solve (2), the temperature dependence of $\langle \sigma v \rangle$ is needed. We consider the interactions of dark matter and the light boson to be $\mathcal{L}_{\text{int}} = \lambda \phi^2 \chi^2$, rendering $|\mathcal{M}|^2$ temperature independent. A straightforward calculation then yields, in the limit $m_\phi \rightarrow 0$,

$$\langle \sigma v \rangle = \begin{cases} x^2 \cdot \langle \bar{\sigma} v \rangle , & m_\chi \ll T \\ 4 \cdot \langle \bar{\sigma} v \rangle , & m_\chi \gg T \end{cases}$$  \hspace{1cm} (3)

where $T$ is the temperature of cosmological background with the following redefinitions of parameters:

$$x \equiv \frac{m_\chi}{T}, \quad \langle \bar{\sigma} v \rangle \equiv \frac{1}{128\pi} \frac{|\mathcal{M}|^2}{m_\chi^2} ;$$  \hspace{1cm} (4)

the latter we shall call the “reduced” scattering cross section. Besides the micro-physics of dark matter reaction, the background evolution of universe also play a crucial role in determining the relic abundance of dark matter. Next we turn our attention to the background evolution of universe in the Bounce Scenario.

**The Standard Scenario:** The standard scenario can be achieved with choosing,

$$Y_f = \frac{1}{4\Lambda^4}, \quad \Lambda \equiv \langle \bar{\sigma} v \rangle m_\chi \Sigma ,$$  \hspace{1cm} (5)

where $Y_\Sigma$ is the abundance of dark matter at the beginning of the thermal decoupling, with $\Sigma = 60.01 \times 10^{26} \text{ eV}$ being constrained by the current astrophysical observations.

We call this case “sufficient production” in which $Y_\Sigma$ takes the thermal equilibrium value at onset of thermal decoupling. Thus the relic abundance, $Y_f$, does not carry any information regarding the early universe evolution before thermal decoupling in standard cosmology. In other words $Y_\Sigma$ is generically much larger than $(4\Lambda^4)^{-1}$. It follows that the thermal relic abundance of DM is given by [10, 11],

$$Y_f = (4\Lambda^4)^{-1} \approx \langle \bar{\sigma} v \rangle^{-1} \sim m_\chi^2 g_\chi^{-4} ,$$  \hspace{1cm} (6)

where $g_\chi$ is the coupling of dark matter particle.

Both the well-known WIMP miracle and WIMP-less miracle belong to this category. In WIMP miracle, the relation, $(g_\chi, m_\chi) \sim (g_{\text{weak}}, m_{\text{weak}})$, satisfies the observed value, $\Omega_\chi \approx 0.26$, coincidently [12]. And in WIMP-less miracle inspired by SUSY model, another combination of the dark matter’s mass and coupling is predicted to give the correct $\Omega_\chi$ [13]. Neither of them shed light on the dynamics of the early universe evolution before thermal decoupling.

To extract the information of the early universe evolution before thermal decoupling from the relic abundance of dark matter, we are thus motivated to study the other possibility—the production of dark matter in early time being insufficient that the thermal equilibrium is never reached.

In this “insufficient production” case, $Y_\Sigma$ would be much smaller than the critical value, $(4\Lambda^4)^{-1}$, and

$$Y_f = Y_\Sigma .$$  \hspace{1cm} (7)

Furthermore $Y_\Sigma$ should be proportional to its thermal averaged cross-section, $Y_\Sigma \propto \langle \bar{\sigma} v \rangle$ [14], if the dark matter is produced thermally. Then, with Eq. (7), we have

$$Y_f = Y_\Sigma \propto \langle \bar{\sigma} v \rangle .$$  \hspace{1cm} (8)
It turns out that this linear relation—obtained in the insufficient production case—opens a new window for exploring the early universe dynamics: The detail balancing of the non-equilibrium physics governing the early universe evolution is imprinted on $Y_f$ by Eq. (8).

**THERMAL PRODUCTION OF DARK MATTER**

The thermal production of dark matter particle is favorable only when the temperature of the thermal bath is comparable to its mass, $T \geq m_\chi$. In particular, the dark matter particles are produced efficiently only during the Phase I and Phase II.

**Thermal production of dark matter during the pre-bounce contraction (Phase I):** Since the cold era before Phase I, $T < m_\chi$, can be neglected as far as the production of dark matter particles is concerned, we take

$$n_\chi = 0, \quad T \sim m_\chi, \quad H < 0, \quad (9)$$

as the initial conditions at the onset of Phase I. Production proceeds efficiently according to (1) as soon as some of the light bosons $\phi$ have energy $E_\phi \sim m_\chi$.

With

$$n_\chi^{(0)}|_{T > m_\chi} = T^3 \pi^{-2}, \quad n_\phi = n_\phi^{(0)}, \quad (\sigma v)|_{T > m_\chi} = x^2 (\tilde{\sigma} v)$$

the Boltzmann equation (2), governing the $\phi - \chi$ interaction in this phase, simplifies

$$\frac{dY}{dx} = -\Lambda (1 - \pi^4 Y^2); \quad (10)$$

with the following shorthand notations:

$$Y \equiv \frac{n_\chi}{T^3}, \quad \Lambda \equiv (\tilde{\sigma} v) m_\chi \Sigma, \quad \Sigma \equiv \frac{1}{\pi^2} \frac{M_p T_0^2}{\rho_{cr}} \left( \frac{a_0}{a_{eq}} \right)^{4/3}$$

with $M_p$ and $a_{eq}$ denoting the reduced Planck mass and the scale factor at matter-radiation equality; and $\rho_{cr}$, $T_0$, and $a_0$, are today’s total energy density, temperature and scale factor, respectively.

A complete solution of (11) for this pre-bounce contraction (denoted by the subscript _),

$$Y_- = \frac{1 - e^{2\pi^2 \Lambda (x - 1)}}{\pi^2 (1 + e^{2\pi^2 \Lambda (x - 1)})} \quad (11)$$

follows, using the initial conditions (9). At the end of the phase I, $x_- = x_b \ll 1$, we can then study two limits of this complete solution,

$$Y_-|_{x=x_b} = \begin{cases} 
\pi^{-2}, & 2\pi^2 \Lambda \gg 1 \\
\Lambda, & 2\pi^2 \Lambda \ll 1 
\end{cases} \quad (12)$$

We observed that, in the large $\Lambda$ limit, the production of dark matter particles are sufficient so that the reaction (11) reaches equilibrium during this phase; whereas, in the small $\Lambda$ limit, the production of $\chi$ is insufficient and the amount of dark matter produced is proportional to the value of $\Lambda$, in particular, to $(\tilde{\sigma} v)$.

**Thermal production of dark matter during the post-bounce expansion (Phase II):** Likewise, we proceed to study post-bounce expansion phase—skipping over the short bounce period unimportant for dark matter production—imposing the following initial conditions:

$$Y_-|_{x=x_b} = Y_+|_{x=x_b}, \quad (13)$$

where the subscript _ denotes variables in the ensuing post-bounce expansion phase. With $x_b \leq x \leq 1$ and $H > 0$, eq. (2) is simplified as

$$\frac{dY}{dx} = \Lambda (1 - \pi^4 Y^2). \quad (14)$$

The complete solution is then

$$Y_+ = \frac{1 - e^{-2\pi^2 \Lambda (1 + x - x_b)}}{\pi^2 (1 + e^{-2\pi^2 \Lambda (1 + x - 2x_b)})} \quad (15)$$

with $x = 1$ at the end of dark matter production.

The complete solution can be analyzed in two limits, the sufficient production case and the insufficient production case:

$$Y_+|_{x=1} = \begin{cases} 
\pi^{-2}, & 4\pi^2 \Lambda \gg 1 \\
2\Lambda, & 4\pi^2 \Lambda \ll 1 
\end{cases} \quad (16)$$

Comparing (16) with (12), we observe that, in the large $\Lambda$ limit, the amount of dark matter attains its equilibrium value, $Y_+|_{x=1} = \pi^{-2}$; we shall be calling this case “sufficient production.” In the small $\Lambda$ limit, $4\pi^2 \Lambda \ll 1$, the dark matter particles continue to be produced, and the amount of dark matter is doubled at the end of the post-bounce expansion phase. Since it is still much less than the equilibrium value, we shall be calling this case “insufficient production.” In contrast to the standard big-bang paradigm in which “enough” dark matter is always assumed to be produced during reheating, the first case clarifies how the abundance of dark matter particle reaches the equilibrium value by thermal production. The second possibility is a remarkably new possibility offered by the bounce universe scenario.

**Thermal decoupling and possible freeze-out** As the temperature of universe continues to fall, at $T < m_\chi$, dark matter particles may undergo thermal decoupling while the cosmos is still in expansion, denoted as Phase III in Fig. 1. The backward reaction of (11), $\phi + \phi \rightarrow \chi + \chi$, is suppressed exponentially compared to the forward reaction, $\chi + \chi \rightarrow \phi + \phi$. In particular, using $n^{(0)}_\chi|_{T < m_\chi} = \ldots$
where the first term on the right-hand can be discarded for \( x > 1 \). Integrating (17) from \( x = 1 \) to \( x \to \infty \), we obtain the relic abundance of dark matter after freeze-out,

\[
Y_f \equiv Y|_{x \to \infty} = \frac{1}{4\Lambda^4 + \frac{1}{v^2}} ,
\]

where \( Y_* \) is the initial value of \( Y \) at the onset of thermal decoupling, \( Y_* = Y_+|_{x=1} \).

The final possible outcomes of relic abundance are summarized in Table I.

**TABLE I. Relic abundance of dark matter after freeze-out**

| Condition | \( \Omega_h \) | \( \langle \sigma v \rangle \) |
|-----------|----------------|------------------|
| \( 4\pi^2 \Lambda \gg 1 \) | \( 4\pi^2 \Lambda \ll 1 \) | \( Y_f \sim (4\pi^4)^{-1} \) |
| \( 4\pi^4 \gg Y_*^{-1} \) | \( \mathcal{A} \) | \( Y_f = 2\Lambda \) |
| \( 4\pi^4 \ll Y_*^{-1} \) | \( \mathcal{B} \) | \( Y_f = 2\Lambda \) |

A new possibility is thus opened up: dark matter production can be much lower than the equilibrium value and can evolve to the present era without damping with its density diluted only by cosmic expansion. This combined route of “insufficient production” and “no damping” is then the signature prediction of “bounce universe”!

**Observational Constraints:** We will now explore the observational consequences of this prediction of the Big Bounce. The relic abundance is evolved to present by

\[
\Omega_h = \frac{\rho_\gamma}{\rho_{\text{cr}}} = \frac{m_\chi Y_f T_0}{\rho_{\text{cr}}} \left( \frac{a_f T_f}{a_0 T_0} \right)^3 ,
\]

where subscripts \( f \) and \( 0 \) denote the freeze-out and present values respectively. From current observations (\( \rho_{\text{cr}} = 3.64 \times 10^{-11} eV^4 \), \( \Omega_h = 0.26 \), \( T_0 = 2.35 \times 10^{-4} eV \), \( \left( \frac{a_f T_f}{a_0 T_0} \right)^3 \approx \frac{1}{367} \), \( a_{eq}/a_0 = 3.55 \times 10^{-4} \)) we obtain the value of thermally averaged “reduced” cross-section, scattering amplitudes and mass ranges of dark matter particles for the two possible branches, in order to reproduce the correct \( \Omega_h \) today, in Table 2. In Branch \( \mathcal{A} \), the thermal averaged cross-section is \( \langle \sigma v \rangle = 0.312 \times 10^{-31} cm^2 \), which is identical to the predictions of standard model cosmology \( \text{[15]} \). In Branch \( \mathcal{B} \), the scattering amplitude is constant and is independent of the value of \( m_\chi \). Finally these relationships derived from Big Bounce models are presented in Fig. 3.

**Discussion** In this letter we investigate dark matter production in \( T_b \gg m_\chi \), in which the majority of production occurs when \( m_\chi < T < T_b \), and the effect of bounce energy scale, \( x_b \), only appears in subleading order of relic abundance of dark matter in the case of insufficient production, \( Y_f = 2\Lambda (1 - \frac{1}{3} x_b) \) with \( x_b \ll 1 \) Eq.(15). On other hand, at intermediate temperatures, \( T_b \ll m_\chi \), the production of dark matter is highly suppressed and only insufficient production takes place; and the bounce scale, \( x_b = \frac{m_\chi}{T_b} \), appears in the leading order. Thus a straightforward integration of Eq.(16) during \( (0, T_b) \) and \( (T_b, 0) \) yields \( Y_f = \frac{1}{4\pi\Lambda} e^{-2 x_b} (1 + x_b) \) with \( x_b \gg 1 \), which indicates that the relic abundance of dark matter in this case is sensitive to the energy scale of the bounce, \( x_b \).

Imposing currently observed value of \( \Omega_h \) we obtain \( \langle \sigma v \rangle = 4.6 \times 10^{-26} eV^2 \), \( m_\chi \approx 1 \) Eq.(16). The relic abundance is enhanced by a factor, \( 2.56 \times e^{2 x_b} (1 + x_b)^{-1} \), compared with the case of \( \mathcal{B} \) in Table II. Accordingly, \( |\mathcal{M}|^2 = 18.6 \times e^{2 x_b} (1 + x_b)^{-1} \times 10^{-24} \) in this case. However, for a given combination of the coupling and mass of the dark matter particle, the energy scale of bounce should be finely tuned to give the correct \( \Omega_h \) since the factor depends on \( x_b \) exponentially. Therefore the DM production at intermediate temperatures is less natural compared to that at high temperature \( \text{[16]} \). Detailed calculations in both branches will be presented in an accompanying paper \( \text{[17]} \).

**Conclusion and Outlook** We report in this letter the possibility of using experimentally determined dark matter mass and its interaction cross section as a dis-
criminating signal for the existence of a bounce phase in the early epoch of the currently observed universe. A concrete mass and scattering cross section relation is obtained in a model independent way from the bounce universe scenario. If the experimentally determined values of DM mass and cross section do fall on this theoretical curve then the bounce universe scenario is casted into serious doubt. Even though one can consider some modification of our analysis by including detailed modelling of the bounce process, the overall shape of the DM mass–cross section curve will not be changed.

As we discover a whole new venue for dark matter production in the bounce scenario because of the existence of the phase of pre-bounce contraction, reheating is no longer called for in this setting. Therefore it is natural to expect that baryogenesis can similarly enjoy a major rethinking in the bounce universe [18]!

We would like to thank Jin U Kang and Konstantin Savvidy for many useful discussions. This research project has been supported in parts by the Jiangsu Ministry of Science and Technology under contract BK20131264. We also acknowledge 985 Grants from the Ministry of Education, and the Priority Academic Program Development for Jiangsu Higher Education Institutions (PAPD).

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