Mesoscopic fluctuations of the Coulomb drag

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When two electronic layers are brought close to each other to form a bilayer system, a current flows through one of the layers (the active layer) in order to induce a voltage \( V_0 = 0 \) in the other (passive) layer [1,2]. The effect, which is called the drag, was first predicted theoretically [1,2] in the model where the carriers in the two spatially separated layers interact via long-range Coulomb interaction. Experimentally the Coulomb drag was first observed in a three-dimensional electron gas layer where the current was driven through a two-dimensional electron gas (2DEG) [3]. Subsequent experiments studied the effect in 2DEG bilayers [4], electron-hole [5], and normal metal-superconductor systems [6]. More recently, the drag was studied in the 2DEG bilayer system in high magnetic fields [7].

The quantity, which is studied theoretically, is the transconductance \( D \). To the lowest non-vanishing order in the interlayer interaction it is proportional to the drag coefficient \( D \) (\( i \) is the density conductance of the i-th layer)

\[
\frac{D}{\pi T^2} : \frac{\partial}{\partial T} \text{exp} \left( E_0 \right) = \frac{\partial}{\partial T} \text{exp} \left( E_0 \right)
\]

As a function of temperature the observed \( D \) roughly follows the quadratic law \( D \sim T^2 \), although the ratio \( D/T^2 \) deviates from the constant value \( \frac{1}{2} \).

The \( T^2 \) dependence of the Coulomb drag coefficient follows from the Fermi liquid phase space argument. To create a current in the passive layer, it is necessary to create a pair of electron-like (i.e., states with energy greater than the Fermi energy \( E_F \)) and hole-like excitations (empty states \( E < E_F \)) in a state with non-zero momentum. The energy and momentum of the pair come from an electron in the active layer, which is moving with the driving current. In each layer, the scattering states are limited to the energies of order \( T \) relative to the Fermi level, which gives two powers of \( T \) to the drag coefficient. However, the momentum is transferred equally to electrons and holes, therefore in the case of electron-hole symmetry the drag of the electrons cancels that of the holes. Thus the effect is non-zero only due to the electron-hole symmetry. Similarly, the asymmetry is necessary for the electron and hole system in the active layer to have non-zero total momentum in the first place. The asymmetry can be expressed as a derivative of the density of states and/or the diusion constant \( D \) with respect to the chemical potential. This can be obtained rigorously in the diagrammatic form [8]. For the case of disordered layers the disorder-averaged transconductance is

\[
D_0 i = \frac{e^2}{h} \frac{2}{3} \frac{(nT)^2}{(d)^2} \frac{\theta(\pi)}{\pi^2} \frac{2}{(D)^2} \frac{\ln T_0}{2T} : (1)
\]

where for simplicity we take the layers to be identical, so that they have the same chemical potential, diusion constant and the dimensionless conductance \( g = 25.8k_B T \). The logarithm is cut at the scale \( T_0 = \frac{D}{d} \) and \( T = \frac{e^2}{h} \) is the inverse Thouless length.

Such effects of the electron-hole asymmetry are well-known, for instance the thermopower in disordered electronic systems \([1,2]\) or adiabatic pumping \([3]\). As these effects are due to the electron-hole asymmetry, the average quantities are small, since each derivative with respect to the chemical potential brings one power of the Fermi energy \( E_F \) to the denominator. On the other hand, the typical energy scale of mesoscopic effects is the Thouless energy \( E_T = \frac{hD}{dL^2} \) (\( L \) is the sample size), which is much smaller than the Fermi energy. Therefore the effects mentioned above exhibit mesoscopic fluctuations, much larger than the average.

In this Letter we show that the mesoscopic fluctuations of the Coulomb drag coefficient can indeed be larger than the average Eq. (1), even if the electron systems in both layers are good metals (\( g \approx 1 \)). To characterize the fluctuations, we calculate the average square of the (random) transconductance. The result of lengthy but straightforward calculations shown below is given by

\[
D_0 i = \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right) \frac{2}{h^2} \frac{\ln T_0}{2T} \frac{2}{(D)^2} \frac{\ln T_0}{2T} : (2a)
\]

\[
D_0 i = \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right) \frac{32}{18} \frac{h^2}{3} \frac{14}{3} \frac{e^4}{h^2} \frac{E_F}{g^2} \ln d : (2b)
\]

where the numerical factor \( \approx 1 \times 10^{-1} \) is the value of the integral.
where $J_1(x)$ and $K_1(x)$ are the Bessel functions. Here, $E_T = 1$ is the dephasing time. This result is valid in the most relevant regime $L = pD/L$ and $d^3 > 1$. If $d = 1$, then the average square of the conductance is $h \frac{2i}{l} / (\frac{E_T}{g})$ with the coefficient of order unity. In what follows we restrict the experimental consequences of our results, then explain it qualitatively and nally give the rigorous calculation. The fluctuations Eq. (2) depend on temperature only through the dephasing time $l = \frac{\pi}{E_T}$ or $T = gT$ and at low enough temperatures they should inate the behavior of the conductance. Therefore the $T^2$ decrease of $\rho$ should at some temperature $T$ be almost saturated at a steady independent value. Let us estimate $T$ for the same ples used in existing experiments on using the reported parameters of the sample. Collecting the numerical factors, we write the ratio of the square of the average conductance Eq. (3) and the averaged square Eq. (4) as

$$\frac{h^2}{h^2} = \frac{gT}{E_T} \frac{1}{\ln d}$$

We take the interlayer spacing to be $d = 200A$; the screening length in GaAs is $1 = 10A$; the Thouless energy is given by $E_T = g(2L^2)$; and the dephasing time $\frac{g^2}{\hbar} \frac{1}{\ln g}$. Then we estimate $T$ as the temperature at which the ratio $\frac{h^2}{h^2} = \frac{g^2}{\hbar} \frac{1}{\ln d}$ is equal to unity

$$T = E_T 16 g^2 nL^2 1 = 0.2K$$

where the Fermi energy in the sample is $E_T' 60K$, the electron density $n = 1.1 10^4 cm^{-2}$, the size of the sample $L' 200 m$, and the conductance is calculated from the sheet resistance of the sample $R = 10 \Omega$.

The estimated $T$ is lower than the temperature range for the existing data, therefore there is no trace of the fluctuations Eq. (1) in the data. However, if one takes a dirtier sample, with the sheet resistance, for instance, $1k \Omega$, then the estimate for $T$ becomes $2K$ and the extent of the fluctuations becomes observable. To push $T$ even higher, the sample size can be also reduced.

Let us now explain Eq. (4) qualitatively. First, consider the lowest temperature regime $E_T = E_T$, so that the sample is essentially zero-dimensional (OD). The mesoscopic fluctuations of the usual conductance are universal

$$\rho = h^2\hbar^2$$

The conductance is associated with the interlayer interactions, thus possessing additional smalness. The value of such smalness can be estimated by the Golden rule arguments which is comprised by (i) phase volume $V$; (ii) matrix elements; (iii) electron-hole asymmetry (dependence of the density of states on the energy). Matrix elements in OD do not depend on energy and give smallness $1=\frac{2}{E_T}$. Therefore the phase volume is limited by temperature only, which gives the factor $T^2$. Finally, the electron-hole asymmetry $\theta$ (in (i)) is the random quantity with the typical value $1=\frac{2}{E_T}$. Putting everything together, we arrive at the estimate

$$\rho = \frac{g^2}{\hbar} \frac{1}{\ln d} \frac{E_T^2}{h^2} (4)$$

where $h = \frac{g^2}{\hbar} (L',')$ is found similar to Eq. (3) with two important differences: (i) the fluctuations of the density of states are summed from the scale of order $T$, rather than $E_T (L')$. This suppresses the fluctuations in each layer by the factor $\frac{E_T (L')}{T}$. (ii) The matrix elements become energy dependent on the energy scale larger than $E_T$, decreasing with the transmitted energy $\frac{1}{\ln g}$. As a result, the transmitted energy is limited by $\frac{E_T (L')}{T} = 1$. Since then by $T$, so in the estimate of the phase volume we should replace $T^2$ by $T$. So we find

$$\rho = \frac{E_T (L')^2}{(E_T (L') g)^2} \frac{g^2}{T} (5)$$

FIG. 1. A. The diagram for the main in the lowest non-vanishing order in the interlayer interaction. The solid lines are the exact Green's functions of the non-interacting electron system in the presence of disorder and the wavy lines are the disorder averaged inter-layer Coulomb interaction propagators Eq. (3). B. The RPA scheme for the calculation of the interaction propagators. The dashed lines denote the disorder. The indices $i,k; l = 1,2$ indicate the layer.
Finally, to estimate the total amplitude of the transconductance fluctuations we substitute Eq. (3) into Eq. (4) to obtain

$$\frac{d}{d\tau} \left[ \frac{Z}{2} \left\{ J_{12}(\ell \ell'\ell) + J_{21}(\ell\ell'\ell) + J_{12}(\ell \ell'\ell) \right\} \right]$$

(6a)

where

$$J_{12}(\ell \ell') = \tanh \frac{\tau}{2T} \tanh \frac{\tau}{2T}$$

$$G_{12}^R(\ell \ell') G_{21}^A(\ell \ell') G_{21}^A(\ell \ell')$$

(8b)

$$G_{12}^R(\ell \ell') G_{21}^R(\ell \ell')$$

(8c)

The exact electronic Green's functions used in Eq. (3) can be written in terms of the exact wavefunctions of the system as

$$\sum_{\ell \ell'\ell} = \sum_{\ell \ell'\ell}$$

(8c)

where $\ell$ labels the exact eigenstates of the system and $\ell$ are the exact eigenvalues.

The known result for the averaged transconductance Eq. (3) is obtained by using the wavefunctions of the Green's functions of non-interacting electrons. For our purposes we only need the propagator of the interlayer interaction, which is given by (here we set the layers to be identical, so that they have the same density of states and diffusion coefficient $D$)

$$D^R(\ell \ell') = \frac{1}{2L} \frac{d^2}{dQ^2} \left[ \frac{1}{1 + (\ell \ell' + D)Q^2} \right]$$

(6)

The transconductance in the disordered two-layer system can be expressed in terms of the exact Green's functions of non-interacting, disordered electron system and the interaction propagators Eq. (6). To the lowest non-vanishing order in the interlayer interaction the transconductance is given by the diagram Fig. 1A. The left and right triangles correspond to the two layers in the system and the wavy line is the interlayer interaction propagator Eq. (6). As the electron Green's functions now depend on disorder, the $D$ is a random quantity and its moments should be averaged over disorder. Before averaging, the expression for $D$ corresponding to the diagram Fig. 1A can be written as

$$D = \frac{1}{4V} \frac{Z}{2} \frac{d!}{\theta^{d-1}} \left[ \frac{1}{2T} \frac{d}{d\tau} D_{12}^R Z_{23}^R Z_{34}^R \right]$$

(7)

where the indices indicate the spatial coordinates. Points 1;2 belong to one layer and 3;4 to the other. The triangular vertices are given by

$$J_{12}(\ell \ell') = \tanh \frac{\tau}{2T} \tanh \frac{\tau}{2T}$$

$$G_{12}^R(\ell \ell') G_{21}^A(\ell \ell') G_{21}^A(\ell \ell')$$

(8b)

$$G_{12}^R(\ell \ell') G_{21}^R(\ell \ell')$$

(8c)
The numerical coefficient can now be obtained by performing the integration without further approximations. The factor comes from the angle integration and the numerical factor in Eq. (2) from the integration over the small frequency difference.

The product of two triangular vertices, Cooperon contribution is obtained by interchanging of vertices and in one of the triangles (the direction of arrows should be changed respectively).

In conclusion, we have described the mesoscopic fluctuations of the Coulomb drag coefficient or the transconductance. The fluctuations are characterized by the average square of the random, disorder dependent transconductance Eq. (3). Compared to the averaged transconductance Eq. (1), the fluctuations Eq. (2) are determined by the Thouless energy, rather than the Fermi energy, as the average. Therefore, there exists an intermediate temperature regime where the fluctuations are greater than the average, which results in the weak (1/T) term in the temperature dependence of the transconductance in this regime. Moreover, in this regime the is a random, sample dependent quantity, so that the sign of the measured value is also random. Since the average transconductance Eq. (1) grows as T², at higher temperatures (T > Tc) the fluctuations are small and the measured is roughly equal to the average. This was the case in the existing experiments [3] For the samples used in [3] we estimated the crossover temperature Tc = 0.2K, which was below the temperature range used in these experiments. To observe the effect of the fluctuations, one needs to take a different sample of smaller size. Then Tc can be equal to several Kelvin, and the saturation of to a value with the random sign can be observed.

Finally, we notice that the Coulomb drag coefficient may also be presented as the product of two random numbers \( p_0 ' a_1 a_2 \) where \( a_1, a_2 \) characterize each layer. If the disorder is correlated, the average \( h_0 a_1 a_2 \) appears, which leads to the results of Ref. [14]. In this respect, the results of Ref. [13] are just a particular manifestation of the mesoscopic fluctuations of \( p_0 \), discussed in this Letter.

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[1] M. B. Pogrebniak, Fiz. Tekh. Poluprovodn. 11, 637 (1977).
[2] F. M. Price, Physica (Amsterdam) 117B, 750 (1983).
[3] P. M. Solomon, P. J. Price, D. J. Frank, and D. C. La Tulipe, Phys. Rev. Lett. 63, 2508 (1989).
[4] T. J. Gramila, J. P. Eisenstein, A. H. MacDonald, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. 66, 1216 (1991).
[5] U. Simon, P. M. Solomon, and H. Shtrikman, Phys. Rev. Lett. 68, 1196 (1992).
[6] N. Giordano and J. D. Moore, Phys. Rev. B, 50, 3633 (1994).
[7] M. P. Lilly, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. 80, 1714 (1998).
[8] T. J. Gramila, J. P. Eisenstein, A. H. MacDonald, L. N. Pfeiffer, and K. W. West, Phys. Rev. B, 47, 12957 (1993).
[9] H. C. Tso, P. Vasilopoulos, and F. M. Peeters, Phys. Rev. Lett. 68, 2516 (1992).
[10] A. K. Kamenev and Y. Oreg, Phys. Rev. B, 52, 7516 (1995).
[11] A. V. Andreevich, B. L. Althuuser, A. G. Aronov, and A. Yu. Zvyuzin, Pis'ma Zh. Eksp. Teor. Fiz. 45, 237 (1987) [Sov. Phys. JETP Lett. 45, 295 (1987)].
[12] Fei Zhou, B. L. Althuuser, and B. Z. Spivak, Phys. Rev. Lett. 82, 608 (1999).
[13] M. Abramowicz and I. Stegun, Handbook of Mathematical Functions (Dover, New York, 1972).
[14] I. A. Leiner, B. L. Althuuser, and M. E. Gershenson, W. Aves in Random Media, (1999).
[15] B. L. Althuuser and A. G. Aronov, in Electron-Electron Interactions in Disordered Systems, eds. A. L. Efros, M. Pollak (North-Holland, Amsterdam, 1985).
[16] I. V. Gomyi, A. G. Yashenko, and B. V. Khveschenko, Phys. Rev. Lett. 83, 152 (1999).