Interpretations for the $X(4160)$ observed in the double charm production at B factories

Kuang-Ta Chao$^{(a,b)}$

(a) Department of Physics, Peking University, Beijing 100871, China
(b) Center for High Energy Physics, Peking University, Beijing 100871, China

Abstract

Belle Collaboration has recently observed a new state, the $X(4160)$, in the process of double charm production $e^+e^- \rightarrow J/\psi + X(4160)$ followed by $X(4160) \rightarrow D^*\bar{D}^*$. We discuss possible interpretations for the $X(4160)$ based on the NRQCD calculations and the potential model estimates for the charmonium spectrum. We first focus on the D-wave spin-singlet $2^{-+}$ charmonium $1D_2(2D)$, which is estimated to have a small production rate of about 5% of that for $e^+e^- \rightarrow J/\psi + \eta_c(1S)$, and therefore is incompatible with the observed data for $X(4160)$. We then discuss the possibility that the $X(4160)$ is the known $J_{PC} = 1^{-+}$ charmonium state $\psi(4160)$, which can be produced via two photon fragmentation, but the production rate is much smaller than observed for $e^+e^- \rightarrow J/\psi + X(4160)$. In contrast to above two possibilities, the $\eta_c(4S)$ assignment is a likely one, which is supported by the observed relatively large production rate and non-observation of $D\bar{D}$ decay of $X(4160)$, but we have to understand why $\eta_c(4S)$ has such a low mass, which deserves further studies. The P-wave excited state $\chi_{c0}(3P)$ is also an interesting candidate, if the observed broad peak around 3.8-3.9 GeV in the recoil mass of $D\bar{D}$ against $J/\psi$ in $e^+e^- \rightarrow J/\psi + D\bar{D}$ is due to the $\chi_{c0}(2P)$ state. Measurements of production angular distributions will be helpful to distinguish between $\eta_c(4S)$ and $\chi_{c0}(3P)$ assignments. Production mechanisms in nonrelativistic QCD are emphasized.

PACS numbers:13.66.Bc, 12.38.Bx, 14.40.Gx

Using a data sample of 693 fb$^{-1}$ collected around the $\Upsilon(4S)$ with the Belle detector at the KEKB $e^+e^-$ storage rings, very recently the Belle Collaboration has reported some new results for double charmonium production in $e^+e^-$ annihilation at $\sqrt{s} = 10.6$ GeV[1]. In the measured processes $e^+e^- \rightarrow J/\psi D^{(*)}\bar{D}^{(*)}$, a new resonance state, called the $X(4160)$, is observed with a significance of 5.1 $\sigma$ in $e^+e^- \rightarrow J/\psi X(4160)$ followed by $X(4160) \rightarrow D^*\bar{D}^*$. As a hadronic resonance, the X(4160) has the following mass and width[1]

$$M = 4156^{+23}_{-20} \pm 15 \text{ MeV}, \quad \Gamma = 139^{+111}_{-61} \pm 21 \text{ MeV},$$

and production cross section

$$\sigma(e^+e^- \rightarrow J/\psi X(4160))B_{D^*\bar{D}^*} = (24.7^{+12.8}_{-8.3} \pm 5.0) \text{ fb},$$

which is large and comparable to the observed cross sections for $e^+e^- \rightarrow J/\psi + \eta_c(1S)$ and $e^+e^- \rightarrow J/\psi + \chi_{c0}(1P)$. Although at present the data for the X(4160) are still preliminary.
and more data are apparently needed to identify the nature of this new state, it is worthwhile
to discuss its possible assignments, especially in view of the great potential of finding new
particles, e.g. the $\eta_c(2S)$ and the X(3940) (for recent reviews on new hadrons with heavy
quarks, see, e.g. [2, 3]) in the $e^+e^-$ annihilation processes at B factories. It is also interesting to
study the production mechanisms of those charmonium or charmonium-like states in the double
charmonium production processes in $e^+e^-$ annihilation, since the theoretical understanding for
the double charm production is very intriguing but not totally conclusive.

In the following, we discuss some possible interpretations for the X(4160), in connection
with the double charmonium production problem.

In general, in the process $e^+e^- \rightarrow J/\psi D^+\bar{D}^*$ the $D^+\bar{D}^*$ system can have charge parity either
C=+ (if $e^+e^-$ annihilated into one photon) or C=− (if $e^+e^-$ annihilated into two photons). In
the case of C=+, the X(4160) can have $J^{PC} = 0^{++}, 0^{-+}, 1^{++}, 1^{--}, 2^{++}, ...$; while in the
case of C=−, it will have $J^{PC} = 1^{--}, 2^{--}, 1^{++}, 2^{++}, ...$ Because the two photon processes are
relatively suppressed by an additional electromagnetic coupling constant $\alpha = 1/137$, at the B
factory energy $\sqrt{s} = 10.6$ GeV the one photon processes usually have larger rates, and should
therefore be considered firstly.

I. The assignment that the X(4160) is the D-wave spin-singlet charmonium state $^1D_2(2D)$ is
disfavored by the too small theoretical rate of production in $e^+e^-$ annihilation.

This 2D state has quantum numbers $J^{PC} = 2^{--}$. There are some arguments which could
be in favor of this interpretation.

First, the observed X(4160) has the same mass as the $\psi(4160)$ (see the Particle Physics
Booklet [4]), which is known to be the good candidate of the D-wave spin-triplet charmonium
state $^3D_1(2D)$ with $J^{PC} = 1^{--}$, and

$$M = 4153 \pm 3 \text{ MeV}, \quad \Gamma = 103 \pm 8 \text{ MeV}, \quad \Gamma_{ee} = 0.83 \pm 0.07 \text{ KeV.} \quad (3)$$

The hyperfine splitting between the center of mass of $^3D_1(2D)$ states and the $^1D_2(2D)$ is
expected to be vanishing if the short-range spin-dependent forces are due to one-gluon exchange,
and the fine-splittings between $^3D_1(2D)$ states should be a few tens MeV. Therefore the observed
mass of the X(4160), $M = 4156^{+25}_{-20} \pm 15 \text{ MeV}$, could be compatible with the D-wave
spin-singlet charmonium state $^1D_2(2D)$, and roughly speaking, is in agreement with the potential
model predictions (see, e.g. [5, 6, 7]). (Note, however, that the S-D mixing effects should
be considered due to a sizable leptonic width observed for the $\psi(4160)$.)

Second, for the $J^{PC} = 2^{++} 2D$ charmonium state, the decay to $D\bar{D}$ is forbidden, while decays to
$D^*\bar{D}^*$ and $D^*\bar{D} + c.c.$ are allowed (also including the $D^*_s\bar{D}_s$ mesons). Therefore, the decay
$X(4160) \rightarrow D^*\bar{D}^*$ could be substantial. In ref.[7] the calculated decay rate of $2^{++}(2D) \rightarrow D^*\bar{D}^*$
is nearly equal to that of $2^{--}(2D) \rightarrow D^*\bar{D} + c.c.$ (However, the sensitivity to the model and
parameters need to be further investigated.)

However, the main problem for this assignment is that the D-wave state is expected to have
a much smaller production rate than the S-wave state such as the $\eta_c$ in double charmonium
production. The double charmonium production in $e^+e^-$ annihilation at B factories has been
studied in the framework of nonrelativistic QCD (NRQCD)[8, 9, 10, 11], in which the charmo-
nium states are treated as nonrelativistic bound systems, and the production rates can be fac-
torized into the short distance part, which can be calculated in perturbative QCD, and the long
distance part, which can be related to the wavefunction of the charmonium. Experimentally,
double charmonium production processes $e^+ + e^- \rightarrow J/\psi + \eta_c(1S)(\eta_c(2S), \chi_{c0}(1P), X(3940))$
have been observed by Belle and BaBar [12, 13, 14], but the cross sections are larger than
the leading order (LO) NRQCD calculations by almost an order of magnitude[8, 9, 11] (Note
that the numerical results can be somewhat different when taking different parameters e.g.
in [8] and in [9] but the physical conclusion is the same). The next to leading order (NLO) QCD radiative corrections are found to be very significant to increase the cross section of $e^+ + e^- \rightarrow J/\psi + \eta_c(1S)$ [15]. Moreover, the relativistic corrections further increase this cross section [16] [17]. As a result, the calculated cross section of $e^+ + e^- \rightarrow J/\psi + \eta_c(1S)$ with both NLO radiative and relativistic corrections in NRQCD may reach the lower bound of the experimental values, and could resolve the problem.

Other approaches including the light-cone methods are also discussed in the literature to resolve the discrepancy between experimental data and theory for the double charmonium production [18].

It is interesting to point out that although the LO results in NRQCD for the double charmonium production cross sections in $e^+ + e^-$ annihilation are much smaller than data, the predicted relative rates seem to be consistent with the measured values. For instance, the predicted cross sections for $J/\psi + \eta_c(1S)$ and $J/\psi + \chi_c(1P)$ are comparable and much larger than that for $J/\psi + \chi_c(1P)$ and $J/\psi + \chi_c(1P)$. These ratios of LO cross sections are indeed compatible with data. In the following, we will use the calculated LO result for $e^+ + e^- \rightarrow J/\psi + D_2$ and compare it with that for $e^+ + e^- \rightarrow J/\psi + \eta_c$, to make some predictions.

To the leading order in NRQCD with QED contribution included, the cross section for $e^+ + e^- \rightarrow \gamma^* \rightarrow \psi(nS) + D_2(mD)$ process can be expressed as [10]

$$\sigma(e^+(p_1) + e^-(p_2) \rightarrow \psi(p_3) + D_2(p_4)) = \frac{5\alpha^2 |R_{S}(0)|^2 |R_{D}(0)|^2}{192m_{\psi}^2s^2} \int_{1}^{\infty} |\tilde{M}|^2 dx,$$

and

$$|\tilde{M}_{J/\psi, D_2}|^2 = \frac{4906\pi^2(s - 6m_{\psi}^2)(32\alpha_2s^2 + 96\alpha_3m_{\psi}^2 + 3\alpha_3^2)(x^2 + 1)}{243m_{\psi}^2s^6},$$

where $x = \cos \theta$, and $\theta$ is the angle between the beam axis ($p_1^\mu$) and the $J/\psi$ momentum ($p_3^\mu$).

As in [9, 10], we take following parameters: $\sqrt{s} = 10.6$ GeV, $m_c = 1.5$ GeV, $m_3 = m_4 = 2m_c$ (in the nonrelativistic limit), $\alpha_s = 0.26$, and the wave functions at the origin are taken from a potential model calculation (see e.g. the QCD (BT) model in Ref. [19]): $|R_{S}(0)|^2 = 0.810$ GeV$^3$, and $|R_{D}(0)|^2 = 0.015$ GeV$^7$, $|R_{D}(0)|^2 = 0.024$ GeV$^7$.

For the $J/\psi + D_2$ (1D) (see [10]) and $J/\psi + D_2$ (2D) production, we then get the angular distributions (differential cross sections) and cross sections, which are shown in Table I, where $\theta$ is the angle between the incident beam and the $J/\psi$, and the numbers with (without) square brackets mean the cross sections without QED (with QED) contributions. These cross sections are much smaller than that predicted for the $J/\psi + \eta_c(1S)$ production [9, 10], which is also listed in Table I. We see that the cross section for $J/\psi + D_2$ (2D) production is predicted to be only about 5% of that of $J/\psi + \eta_c$, in contrast to the observed production cross section for the $J/\psi + X(4160)$ shown in eq.(2), which is comparable to that of $J/\psi + \eta_c$ [12] [13] [14].

In principle, we could also detect the $^{1}D_2$ (1D) charmonium, which should lie around 3.8 GeV, in the $e^+ + e^- \rightarrow J/\psi + D_2$ (1D) process. However, the main decay modes of $^{1}D_2$ (1D) should be decays to light hadrons via intermediate gluons, since the $^{1}D_2$ (1D) is expected to lie below the $D^* \bar{D}$ threshold. Without a dominant exclusive decay channel like $D^* \bar{D}$ or $D^* \bar{D} + c.c.$, it will be even more difficult to detect this charmonium state especially when the production cross section is small.

To sum up, although the $^{1}D_2$ (2D) charmonium could be a possible assignment for the X(4160), the predicted small production rate for $e^+ + e^- \rightarrow J/\psi + D_2$ (2D) makes this assignment very unlikely. Despite of the existing uncertainties in the theoretical calculation (e.g., the
chosen parameters, and high order corrections), this conclusion should hold, since the small number of 5% for the ratio of \(J/\psi + 1\ D_2(2D)\) production cross section to that of \(J/\psi + \eta_c\) can not be enhanced to close to the observed value (about 1) by changing the parameters or including the NLO QCD corrections (Note that the NLO QCD correction to the \(J/\psi + \eta_c\) increases this production rate by a factor of about 2\[15\]).

II. The possibility that the X(4160) is the known \(J^{PC} = 1^{--}\) charmonium state \(\psi(4160)\) should be ruled out.

The \(\psi(4160)\) is in the same mass region as the newly observed X(4160), and their widths are also comparable. Moreover, the \(\psi(4160)\) can also decay to \(D^*\bar{D}\). However, the process \(e^+e^- \rightarrow J/\psi + \psi(4160)\) can only proceed through \(e^+e^-\) annihilation into two photons due to the conserved charge parities.

In fact, the two-photon process was first studied for \(e^+e^- \rightarrow 2\gamma^* \rightarrow J/\psi + J/\psi\) in ref.[20], and it was found that the production rate is comparable to or even larger than that of the one-photon process \(e^+e^- \rightarrow \gamma^* \rightarrow J/\psi + \eta_c\) in the leading order calculation[20]. Moreover, for the inclusive double charm production process \(e^+e^- \rightarrow J/\psi + c\bar{c}\), the two-photon process \(e^+e^- \rightarrow 2\gamma^* \rightarrow J/\psi + c\bar{c}\) will prevail over the one-photon process \(e^+e^- \rightarrow \gamma^* \rightarrow J/\psi + c\bar{c}\) when \(\sqrt{s}\) becomes larger than 20 GeV[21]. This is because, in these two-photon fragmentation processes the virtualities of the photons are only about \(4m^2_c\), which is much smaller than the virtuality \(s\) in the one-photon process.

However, because the \(\psi(4160)\) is expected to be a D-wave (\(^3D_1(2D)\)) dominated charmonium state (with possibly some \(^3S_1(3S)\) admixture), its coupling to the photon is suppressed by the factor \(|\sin \phi R_{3S}(0) - \cos \phi \frac{5}{2\sqrt{2}} R''_{2D}(0)|^2\), compared with \(|R_{1S}(0)|^2\) for the \(J/\psi\). Here, we have assumed that the \(\psi(4160)\) is a mixture of the \(^3D_1(2D)\) and \(^3S_1(3S)\) states with \(\phi\) being the mixing angle:

\[
|\psi(4040)\rangle = |3S_1\rangle \cos \phi + |2^3D_1\rangle \sin \phi, \\
|\psi(4160)\rangle = -|3S_1\rangle \sin \phi + |2^3D_1\rangle \cos \phi.
\]

The above expression is only a very rough approximation, since admixtures with the charmed meson pairs due to coupled channel effects and with other S-wave states are all ignored. With this simple assumption we get leptonic decay widths for the \(\psi(4040)\) and \(\psi(4160)\):

\[
\Gamma(\psi(4040) \rightarrow e^+e^-) = 4\alpha^2 e^2_c \frac{|\cos \phi R_{3S}(0) + \sin \phi \frac{5}{2\sqrt{2}} R''_{2D}(0)|^2}{(2m_e)^2},
\]
\[
\Gamma(\psi(4160) \rightarrow e^+e^-) = 4\alpha^2 e^2_c \frac{|\sin \phi R_{3S}(0) - \cos \phi \frac{5}{2\sqrt{2}} R''_{2D}(0)|^2}{(2m_e)^2}.
\]

Using the experimental values \(\Gamma_{ee}(\psi(4040)) = 0.86 \pm 0.07\) KeV and \(\Gamma_{ee}(\psi(4160)) = 0.83 \pm 0.07\) KeV, and \(|R_{3S}(0)|^2 = 0.455 \text{ GeV}^3\), \(|R''_{2D}(0)|^2 = 0.024 \text{ GeV}^7\), we get the mixing angle from the ratio of these two leptonic widths:

\[
\phi = -35^\circ, \quad \phi = +55^\circ.
\]

The mixing angle is unexpectedly large, and this is due to the observed largeness of the leptonic decay width of \(\psi(4160)\) (almost equal to that of the \(\psi(4040)\)). In fact, if we neglect the contribution from the 2D component of the \(\psi(4160)\), we would get an estimate for the mixing angle that is independent of potential model parameters: \(\phi \approx \pm 45^\circ\), which would be the maximum mixing. The large 3S-2D mixing is a puzzling problem in understanding the nature of \(\psi(4160)\). Other studies like the strong decays to \(D^*\bar{D}\) may be useful to clarify the 3S-2D mixing problem for the \(\psi(4160)\) (see, e.g. discussions in [22]).
Despite of the above uncertainty concerning the 3S-2D mixing, we may have a quite reasonable estimate of the production cross section of \(e^+e^- \rightarrow J/\psi + \psi(4160)\), as compared with that of \(e^+e^- \rightarrow J/\psi + J/\psi\). In the nonrelativistic limit, the charmonium masses are all approximately set to be \(M = 2m_c\) (i.e. all binding energies are neglected), and then we will have a simple relation

\[
\frac{\sigma(e^+e^- \rightarrow J/\psi + \psi(4160))}{\sigma(e^+e^- \rightarrow J/\psi + J/\psi)} = \frac{\Gamma_{ee}(\psi(4160))}{\Gamma_{ee}(J/\psi)} \approx 0.15,
\]

where the observed values \(\Gamma_{ee}(J/\psi) = 5.55 \pm 0.14 \pm 0.02\) and \(\Gamma_{ee}(\psi(4160)) = 0.83 \pm 0.07\) KeV \([4]\) are used. This relation is obtained by the observation that in the double vector-charmonium production via two virtual photons in \(e^+e^-\) annihilation at \(\sqrt{s} = 10.6\) GeV the photon fragmentation is dominant (see e.g. \([20, 21]\)), in which the virtual photon converts directly into the vector-charmonium, the same way as the leptonic decay of the vector-charmonium. As the most favorable mechanism with the minimal photon-virtuality, all vector charmonium states (e.g. \(J/\psi, \psi(2S)\), \(\psi(4040)\), \(\psi(4160)\), ...) are expected to be produced from the two photon fragmentation in \(e^+e^-\) annihilation at \(\sqrt{s} = 10.6\) GeV or even higher energies.

In ref. \([13]\), the following upper bound is given

\[
\sigma(e^+e^- \rightarrow J/\psi + J/\psi) \times B(J/\psi \rightarrow 2\text{ charged}) < 9.1\text{ fb},
\]

which will imply

\[
\sigma(e^+e^- \rightarrow J/\psi + \psi(4160)) \times B(\psi(4160) \rightarrow 2\text{ charged}) < 1.4\text{ fb},
\]

assuming \(B(\psi(4160) \rightarrow 2\text{ charged})\) is comparable to \(B(J/\psi \rightarrow 2\text{ charged})\).

This predicted cross section is much smaller than the experimental value given in eq.(2). Therefore, the \(\psi(4160)\) assignment for the \(X(4160)\) should be ruled out.

III. The \(X(4160)\) could be an excited \(0^+\) charmonium state: the \(\eta_c(4S)\) (less likely to be the \(\eta_c(3S)\)).

As a possible candidate of the \(0^-\) state, the \(X(4160)\) can be the \(\eta_c(4S)\) charmonium, which is expected to decay into \(D^*\bar{D}^*\) and \(D^*\bar{D} + c.c\), but not \(D\bar{D}\).

Note that Belle already found a new state, the \(X(3940)\), in the process \(e^+ + e^- \rightarrow J/\psi + X(3940)\) \([25]\), which has a dominant decay mode into \(D^*\bar{D}\) (with the fraction of \(X(3940)\) decays with more than two charged tracks in the final state into \(D^*\bar{D}\) being \((96^{+45}_{-32} \pm 22)\)%), and a quite narrow width \(\Gamma = 39 \pm 26\) MeV. This result has been further confirmed by Belle (see \([11]\)). The \(X(3940)\) is considered as a good candidate for the \(\eta_c(3S)\) (for discussions see, e.g. \([20, 2]\)). The problem is the low mass of \(X(3940)\) as the \(\eta_c(3S)\), compared with the \(\psi(3S)\) candidate \(\psi(4040)\). But this could be explained by the coupled channel effects that the coupling of \(\eta_c(3S)\) to the \(0^+\) and \(0^-\) charmed meson pair (in S-wave) will lower the mass of \(\eta_c(3S)\) \([20]\).

If we accept \(X(3940)\) as the \(\eta_c(3S)\), then \(X(4160)\) should be the \(\eta_c(4S)\) if it is a \(0^-\) charmonium. In this case, the mass difference between \(\eta_c(4S)\) and \(\eta_c(3S)\) would be only 220 MeV. This mass difference is smaller than that predicted by the potential models with linear plus Coulomb potentials (see, e.g. \([5, 6, 7, 26]\)). Note that the corresponding mass difference between the \(\psi(4S)\) and \(\psi(3S)\) is about 375 MeV if the \(\psi(4S)\) is identified with the \(\psi(4415)\) and the \(\psi(3S)\) with the \(\psi(4040)\) as conventionally classified in the charmonium spectrum. An even more puzzling problem is the mass splitting between the \(\eta_c(4S)\) (if identified with \(X(4160)\)) and the \(\psi(4S)\) (if identified with \(\psi(4415)\)), which is as large as 255 MeV, compared with the mass differences between \(\eta_c(1S)\) and \(J/\psi(1S)\), \(\eta_c(2S)\) and \(\psi(2S)\), \(\eta_c(3S)\) and \(\psi(3S)\), which are only 117, 48, and 100 MeV respectively (assuming the \(X(3940)\) is identified with \(\eta_c(3S)\)). In simple potential models the mass splittings between \(0^-\)\((nS)\) and \(1^-\)\((nS)\) \((n=1,2,3,4,...)\) are
expected to be decreased as $n$ increases. Although the mass spectrum can be modified by the coupled channel effects and S-D mixing, such a big mass difference, 255 MeV, between $\eta_c(4S)$ and $\psi(4S)$ is still difficult to understand, unless the assignments for excited $1^{--}$ states are changed in some way. For instance, if the $\psi(4415)$ is not identified with the $\psi(4S)$ but with the $\psi(5S)$, as discussed in the potential model with color screening effects (see, e.g. [23, 24]), then the corresponding $\eta_c(4S)$ mass could be lowered. In this case, all higher excited states will be lowered in the mass spectra. But this is only a plausible resolution for the problem in the $\eta_c(4S)$ assignment of $X(4160)$, other approaches apparently need to be studied.

Could the $X(4160)$ be the $\eta_c(3S)$? If so, what assignment will be for the $X(3940)$. Moreover, if so, as the $\eta_c(3S)$ the mass of $X(4160)$ would be higher than that of $\psi(3S)$, which is identified with $\psi(4040)$, by 120 MeV. The positive and large mass splitting between $0^+(3S)$ and $1^{--}(3S)$ seems not acceptable in charmonium spectrum. So, $X(4160)$ can not be the $\eta_c(3S)$.

The $\eta_c(4S)$ interpretation for the $X(4160)$ is a likely one in view of the large production rates of $\eta_c(1S)$, $\eta_c(2S)$, and $\eta_c(3S)$ (if identified with the $X(3940)$) associated with $J/\psi$ in $e^+e^-$ annihilation. For the $\eta_c(4S)$ production, the angular distribution and cross section is shown in Table I. Note that the normalization can be substantially enhanced with the NLO correction (see [15]) but the angular distribution remains unchanged in this case. The form of $(1 + \cos^2 \theta)$ for the angular distribution of this assignment differs markedly from another interesting assignment, the $0^{++}$ charmonium (i.e. the $\chi_{c0}(3P)$ state), which will be discussed below.

IV. The $X(4160)$ might be an excited $0^{++}$ charmonium state: the $\chi_{c0}(3P)$ (unlikely to be the $\chi_{c0}(2P)$).

As a possible candidate of the $0^{++}$ state, the $X(4160)$ might be the $\chi_{c0}(3P)$ charmonium, which is expected to decay into $D^*\bar{D}$ and $D\bar{D}$, but not $D^*\bar{D} + c.c.$. The $D\bar{D}$ decay mode of the $X(4160)$ has not yet been seen so far. The mass of $X(4160)$ immediately indicates that it is unlikely to be the $\chi_{c0}(2P)$ state, which is predicted to lie around 3.9-4.0 GeV. The fact that the observed $Z(3930)$ can be identified with the $\chi_{c2}(2P)$ state [1] also implies that the $\chi_{c0}(2P)$ should lie well below 4160 MeV. So, $X(4160)$ can only be the $\chi_{c0}(3P)$ if it is a $0^{++}$ charmonium state. However, if in $e^+ + e^-$ annihilation both $J/\psi + \chi_{c0}(1P)$ and $J/\psi + \chi_{c0}(3P)$ are observed, why $J/\psi + \chi_{c0}(2P)$ is in the absence? In fact, according to the NRQCD calculation, the production cross section of $J/\psi + \chi_{c0}(2P)$ should be comparable to or even larger than that of $J/\psi + \chi_{c0}(1P)$ (see [10]), because the first derivative of the wavefunction at the origin for the 2P-state is usually larger than that for the 1P-state: $|R_{2P}(0)|^2 > |R_{1P}(0)|^2$ (see, e.g. [19]). To LO in NRQCD the cross sections for $e^+ + e^- \rightarrow J/\psi + \chi_{c0}(1P)$ and $e^+ + e^- \rightarrow J/\psi + \chi_{c0}(2P)$ are predicted to be 6.9 fb and 9.4 fb respectively (QED contributions are included) [10]. So, the experimental absence of $e^+ + e^- \rightarrow J/\psi + \chi_{c0}(2P)$ would be hard to understand.

However, at this point, it is interesting to notice that Belle has observed a broad peak (but with only 3.8 $\sigma$) around 3.8-3.9 GeV in the recoil mass of $D\bar{D}$ against $J/\psi$ in the process $e^+ + e^- \rightarrow J/\psi + D + \bar{D}$ [11] (it may also be seen in the $\gamma\gamma \rightarrow D\bar{D}$ process). Is this the missing $\chi_{c0}(2P)$ state? If the bump in the 3.8-3.9 GeV region is really due to the $\chi_{c0}(2P) \rightarrow D\bar{D}$ decay, the $\chi_{c0}(3P)$ assignment for $X(4160)$ would be favored (but the $\chi_{c0}(2P)$ state should be further examined experimentally).

As discussed so far, two likely assignments for the $X(4160)$ are the $\eta_c(4S)$ and $\chi_{c0}(3P)$ charmonia. How to distinguish between them? One effectable way is to measure the angular distribution of the cross sections. The differential cross section in the case of $\chi_{c0}(3P)$ is shown in Table I (see also [10]). Compared with $(1 + \cos^2 \theta)$ in the case of $\eta_c(4S)$, the form of $(1 + 0.252\cos^2 \theta)$ for the $\chi_{c0}(3P)$ has a much weaker $\theta$ dependence, and therefore the measurements on angular distributions can be used to test the two possible assignments.
V. The X(4160) is unlikely to be the excited 2++ or 1++ charmonium state, \( \chi_{c2}(2P, 3P) \) or \( \chi_{c1}(2P, 3P) \).

Experimentally, both \( e^+ + e^- \rightarrow J/\psi + \chi_{c2}(1P) \) and \( e^+ + e^- \rightarrow J/\psi + \chi_{c1}(1P) \) have not been seen. This is consistent with the smallness of calculated cross sections for them. In fact to LO in NRQCD the cross sections for \( J/\psi + \chi_{c2}(1P) \) and \( J/\psi + \chi_{c1}(1P) \) are predicted to be 1.8 fb and 1.0 fb respectively (QED contributions are included) [10], which are much smaller than that for \( J/\psi + \chi_{c0}(1P) \) and \( J/\psi + \eta_c(1S) \). In contrast, the observed cross section for \( J/\psi + X(4160) \) is comparable to that of \( J/\psi + \chi_{c0}(1P) \) and \( J/\psi + \eta_c(1S) \). In view of both the experimental non-observation and the calculated smallness of the cross sections for \( e^+ + e^- \rightarrow J/\psi + \chi_{c2}(1P) \) and \( e^+ + e^- \rightarrow J/\psi + \chi_{c1}(1P) \) we conclude that the X(4160) is unlikely to be the excited 2++ or 1++ charmonium state, \( \chi_{c2}(2P, 3P) \) or \( \chi_{c1}(2P, 3P) \).

Note that the \( \chi_{c2}(2P, 3P) \) and \( \chi_{c1}(2P, 3P) \) interpretations for X(4160) are disfavored by the experimental absence of \( \chi_{c1,2}(1P) \) not only in the exclusive double charmonium production of \( \chi_{c1,2}(1P) \) associated with \( J/\psi \), but also in the inclusive prompt production of \( \chi_{c1,2}(1P) \) in \( e^+e^- \) annihilation [27]. In fact, recently Babar finds no evidence for prompt \( \chi_{c1,2}(1P) \) production after subtracting the contributions from prompt \( \psi(2S) \) production feed-down to \( \chi_{c1,2}(1P) \) [27]. Therefore, the assignments of X(4160) as \( \chi_{c2}(2P, 3P) \) or \( \chi_{c1}(2P, 3P) \) states are very unlikely.

VI. Non-charmonium interpretations for the X(4160): glueballs, hybrids, and charmonium-molecules.

As suggested in [28], a 0++ glueball associated with the \( J/\psi \) may be produced with a sizable rate in \( e^+e^- \) annihilation. However, as a 0++ glueball, the X(4160) would have a too large mass. Moreover, the glueball should mainly decay to light hadrons, but not \( D^*\bar{D}^* \). Nevertheless, to measure the production angular distribution parameter \( \alpha \), where the differential cross section is proportional to \( (1 + \alpha \cos^2\theta) \), will be useful to clarify the glueball interpretation (with a negative value of \( \alpha \) for the 0++ glueball).

Could the X(4160) be an exotic charmonium-hybrid? say, the 1−+ hybrid, a possible partner of the 1−− charmonium-hybrid \( c\bar{c}g \) candidate, the Y(4260) [29, 30]. However, the problem is, a hybrid does not seem to have a favorable production mechanism associated with the \( J/\psi \) in \( e^+e^- \) annihilation. Compared with the production of double charmonium states such as \( e^+ + e^- \rightarrow J/\psi + \eta_c(1S) \), the production of a \( c\bar{c}g \) hybrid associated with \( J/\psi \) requires an additional gluon production, and could therefore be relatively suppressed. But experimentally, the production rate of X(4160)\( J/\psi \) is comparable to that of \( \eta_c(1S)J/\psi \).

Whether the X(4160) can be a charmonium molecule? The well know charmonium-like state \( X(3872) \) has been suggested being a \( D^0\bar{D}^{*0} + c.c. \) molecule either as a real bound state (see, e.g. [31] and references therein) or as a virtual state (see, e.g. [32]). The most significant motivation for the molecule assignment is that the mass of \( X(3872) \) is very close to the \( D^0\bar{D}^{*0} \) threshold. However, in the case of X(4160), its mass is above the \( D^*\bar{D}^* \) threshold by about 140 MeV. This makes the X(4160) unlikely to be a \( D^*\bar{D}^* + c.c. \) molecule, since for molecules the binding energies due to meson exchanges are much less than 100 MeV.

In summary, we have discussed various interpretations of the X(4160), observed by Belle in the process of double charm production \( e^+e^- \rightarrow J/\psi + X(4160) \) followed by \( X(4160) \rightarrow D^*\bar{D}^* \). The available information for this state from the data is its mass, width, a major decay mode, and its large production rate (comparable to \( \eta_c(1S) \)) associated with \( J/\psi \) in \( e^+e^- \) annihilation.

Using the leading order NRQCD calculation of the relative cross sections of double charmonium production in \( e^+e^- \) annihilation as a guide, we find that though the D-wave spin-singlet charmonium state \( ^1D_2(2D) \) (with \( J^{PC} = 2^{−+} \)) could be a candidate of X(4160), the calculated production rate is too small, only about 5% of that for \( e^+e^- \rightarrow J/\psi + \eta_c(1S) \), and therefore...
The possibility of X(4160) being the known $\psi(4160)$ produced via two-photon fragmentation in $e^+e^-$ annihilation is also discussed, but the calculated rate is much smaller (even with the 3S-2D mixing effect included) than that for $e^+e^- \rightarrow J/\psi + J/\psi$ which, however, only has a small experimental upper limit. So the $\psi(4160)$ interpretation for X(4160) should be completely ruled out.

The X(4160) is unlikely to be the excited $2^{++}$ or $1^{++}$ charmonium state, e.g., $\chi_{c2}(2P, 3P)$ or $\chi_{c1}(2P, 3P)$, because of the experimental non-observation and the calculated smallness of the cross sections for the $J/\psi + \chi_{c2}(1P)$ and $J/\psi + \chi_{c1}(1P)$ production.

The candidates of glueballs, $c\bar{c}g$ hybrids, and charmonium-molecules for the X(4160) might also be considered, but these interpretations are not very likely.

In contrast to above interpretations, the $\eta_c(4S)$ assignment for X(4160) is an interesting possibility. The production rate of $e^+e^- \rightarrow J/\psi + \eta_c(4S)$ relative to $e^+e^- \rightarrow J/\psi + \eta_c(1S)$ in NRQCD is not very small, and could be compatible with the Belle data (note that for the observed $e^+e^- \rightarrow J/\psi + \eta_c(1S)$ cross section only a lower bound of $25.6 \pm 2.8 \pm 3.4$ fb is given by Belle). And the non-observation of the $D\bar{D}$ mode of X(4160) can also be understood for this assignment. But one has to understand why $\eta_c(4S)$ has such a low mass. And if one accepts X(4160) being the $\eta_c(4S)$, then the $\psi(4415)$ can hardly be the $\psi(4S)$ as conventionally classified in charmonium spectrum.

The $\chi_{c0}(3P)$ is an even more interesting candidate for the X(4160). In particular, if the observed broad peak around 3.8-3.9 GeV in the recoil mass of $D\bar{D}$ against $J/\psi$ in $e^+e^- \rightarrow J/\psi + D + \bar{D}$ [1] is due to the $\chi_{c0}(2P)$ state, then the $\chi_{c0}(3P)$ assignment for X(4160) will be favored. However, as in the case of $\eta_c(4S)$ discussed above, one has to understand the problem of low mass values of the 3P states in the $\chi_{c0}(3P)$ assignment, compared with conventional potential model calculations. We also emphasize that measurements on the angular distributions of cross sections are useful to distinguish between the $\chi_{c0}(3P)$ and $\eta_c(4S)$ assignments for the X(4160).

In order to clarify the nature of X(4160), it will be helpful experimentally to measure the differential cross sections (the production angular distributions), which can be different in different assignments, and to measure the strong decay branching ratios into various charmed meson pairs, and to measure the quantum numbers of X(4160). Theoretically, it is certainly important to have a reliable calculation for the strong decay rates, which is not very easy considering the complexity due to the coupled channel effects, and to understand why X(4160) has a dominant decay mode into $D^*\bar{D}^*$. As for the production, as far as NRQCD is concerned, the NLO radiative corrections are only available for the $e^+e^- \rightarrow J/\psi + \eta_c$ process[15], which is confirmed by a recent independent calculation[33]. It will certainly be very useful if the NLO calculation for the P-wave and D-wave charmonium states involved in double charmonium production can be performed.

At present, we only have a limited understanding for the puzzling state X(4160). Since its finding was reported more than five months ago, there have been no theoretical papers on its interpretations. So, it is our hope that the discussion presented in this paper will stimulate more interesting discussions on this new hadronic state.

Acknowledgments. The author would like to thank C.F. Qiao, C.Z Yuan and S.L. Zhu for helpful discussions and comments. He also thanks T. Barnes and Q. Zhao for a general discussion on charmonium spectroscopy. This work was supported in part by the National Natural Science Foundation of China (No. 10675003, No 10721063), and the Research Found for Doctorial Program of Higher Education of China.
References

[1] I. Adachi et al. (Belle Collaboration), arXiv:0708.3812 [hep-ex]; P. Pakhlov (Belle Collaboration), talk given at the EPS High Energy Physics Conference, Manchester, July, 2007.

[2] E. Swanson, Phys. Rept. 429, 243 (2006).

[3] S.L. Zhu, Arxiv: 0707.2623v1 [hep-ph].

[4] W.M. Yao et al. (Particle Data Group), J. Physics G33, 1 (2006).

[5] E. Eichten, K. Gottfried, T. Kinoshita, K.D. Lane and T.M. Yan, Phys. Rev. D 21, 203 (1980);

[6] S. Godfrey and N. Isgur, Phys. Rev. D 32, 189 (1985).

[7] T. Barnes, S. Godfrey and E.S. Swanson, Phys. Rev. D 72, 054026 (2005).

[8] E. Braaten and J. Lee, Phys. Rev. D67, 054007 (2003); Phys. Rev. D72, 099901(E) (2005).

[9] K.Y. Liu, Z.G. He and K.T. Chao, Phys. Lett. B557, 45 (2003).

[10] K.Y. Liu, Z.G. He and K.T. Chao, Phys. Rev. D77, 014002 (2008).

[11] K. Hagiwara, E. Kou, C.-F. Qiao, Phys. Lett. B570, 39 (2003).

[12] K. Abe et al. (Belle Collaboration), Phys. Rev. Lett. 89, 142001 (2002).

[13] P. Pakhlov (Belle Collaboration), arXiv:hep-ex/0412041.

[14] B. Aubert et al. (BaBar Collaboration), Phys. Rev. D72, 031101 (2005).

[15] Y.J. Zhang, Y.J. Gao and K.T. Chao, Phys. Rev. Lett. 96, 092001(2006); see also Y.J. Zhang and K.T. Chao, Phys. Rev. Lett.98, 092003 (2007).

[16] G.T. Bodwin, D. Kang and J. Lee, Phys. Rev.D74, 014014 (2006)[hep-ph/0603186]; G.T. Bodwin, D. Kang, D. Kim, J. Lee, and C. Yu, [hep-ph/0611002]

[17] Z.G. He, Y. Fan, and K.T. Chao, Phys. Rev. D75, 074011 (2007).

[18] J.P. Ma and Z.G. Si, Phys. Rev. D70, 074007 (2004); [arXiv:hep-ph/0608221] V.V. Braguta, A.K. Likhoded, and A.V. Luchinsky, Phys. Rev. D72, 074019 (2005); [arXiv:hep-ph/0602047]; [arXiv:hep-ph/0611021] A.E. Bondar and V.L. Chernyak, Phys. Lett. B612, 215 (2005); D. Ebert and A.P. Martynenko, Phys. Rev. D74, 054008 (2006); H.-M. Choi and C.-R. Ji, arXiv: 0707.1173 [hep-ph].

[19] E.J. Eichten and C. Quigg, Phys. Rev. D52, 1726 (1995).

[20] G. T. Bodwin, J. Lee, E. Braaten, Phys. Rev. D67, 054023 (2003); Phys. Rev. Lett. 90, 162001 (2003).

[21] K.Y. Liu, Z.G. He, and K.T. Chao, Phys. Rev. D 68, 031501(R)(2003).

[22] T. Barnes, arXiv: hep-ph/0608103.

[23] Y.B. Ding, K.T. Chao and D.H. Qin, Chinese. Phys. Lett.10, 460 (1993).
[24] Y.B. Ding, K.T. Chao and D.H. Qin, Phys. Rev. D 51, 5064 (1995) [arXiv:hep-ph/9502409].

[25] K. Abe et al. (Belle Collaboration), Phys. Rev. Lett. 98, 082001 (2007).

[26] E. Eichten, K. Lane, and C. Quigg, Phys. Rev. D73, 014014 (2006); 73, 079903(E)(2006).

[27] B. Aubert et al. (BaBar Collaboration), [arXiv:0707.1633] hep-ex.

[28] S.J. Brodsky, A.S. Goldhaber, J. Lee, Phys. Rev. Lett. 91, 112001 (2003).

[29] S.L. Zhu, Phys. Lett. B625, 212 (2005).

[30] F.E. Close and P.R. Page, Phys. Lett. B628, 215 (2005).

[31] E. Braaten et al., Phys. Rev. D76, 054010 (2007) and references therein.

[32] C. Hanhart et al., Phys. Rev. D76, 034007 (2007).

[33] Bin Gong and Jian-Xiong Wang, [arXiv:0712.4220] hep-ph.
Table 1: Angular distributions and cross sections for double charmonium production in $e^+e^-$ annihilation at $\sqrt{s} = 10.6$ GeV with both QCD and QED contributions in the leading order NRQCD calculations (numbers without QED contribution are given with square brackets, see also text for the input parameters).

| Final state | Differential cross section (fb) | Cross section (fb) |
|-------------|---------------------------------|--------------------|
| $J/\psi + \eta_c(1S)$ | $2.47 \ [2.06] (1 + \cos^2 \theta)$ | 6.6 \ [5.5] |
| $J/\psi + ^1D_2(1D)$ | $0.077 \ [0.069] (1 + \cos^2 \theta)$ | 0.21 \ [0.19] |
| $J/\psi + ^1D_2(2D)$ | $0.123 \ [0.111] (1 + \cos^2 \theta)$ | 0.34 \ [0.31] |
| $J/\psi + \eta_c(4S)$ | $1.14 \ [0.95] (1 + \cos^2 \theta)$ | 3.0 \ [2.5] |
| $J/\psi + \chi_c(3P)$ | $4.7 \ [4.6] (1 + 0.252 \cos^2 \theta)$ | 10.2 \ [9.9] |