Design of Sliding Mode Control for Overhead Crane Systems

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Abstract. Overhead cranes are large used structures for lifting and conveying weighty loads. The design and modelling of imperceptible controllers based on the uncertainty and disturbance estimator for crane system. In this work, a SMC (Sliding Mode Control) of a crane system is proposed. The asymptotically stability system is ensured by Lyapunov functions. For validation of the proposed controller, a comparative study with LQR (Linear Quadratic Regulator) results show that both the controllers give satisfactory performance, but the SMC provides better overall performance. The proposed sliding mode control approach for overhead cranes system has shown to be more effective, robust and realistic than a LQR controller, and to be able to move cargo safely to a destination even in harsh environment. Simulation results of the study prove the favourable of the proposed controller during the transport of a crane system.

1. Introduction

The overhead cranes system is exceedingly employee to transport cargo in harbours, plant and in dissimilar industries. In order to make forceful the performance of cargo transportation, the transmit of a crane lead depart to its tendency as fast and as accurately as likely, in the same time, the swings of the cargo should be preserved as little as tolerable. However, the move of the trolley of a crane system is constantly string along with swings of the load. The deceleration and the acceleration of the overhead crane system lead up to swings of the cargo, these swings can be critical and may reason destroy and episode. So, some sort of strategies must be adopted, to improve advanced controllers to control overhead cranes system [1]. Figure 1 illustrates a typical overhead crane used in industry.

Figure 1. Typical overhead crane structure.
In recent years, there has been considerable research aimed at the design of advanced and conventional control methods for overhead cranes systems [1-9]. In [1], a SMC has been suggesting to control the location of the crane system as well as to smoulder the load skew angles. Moreover, in this work, ago all of the states are accompany to design the suggestion advanced controller, a Luenberger type watcher is designed to revalue the states of the three-dimension crane system. In [2], a novel adaptive neuro SMC for reference crane as degrees’ rope length was proposed. This control mechanism pediment from collecting the sliding surfaces of three system of the pier crane to sketch out two system sliding surfaces. In [3], the dynamic model of the overhead cranes system was derived, and its characteristics were extensively evaluated in simulations.

In [4], a new off line trolley trajectory designing method for the cargo horizontal transferring duty of crane was suggestion. In [5], a nonlinear hunting control method of three-dimension crane systems which doings steady in the presence of the initial swing angle and the change of cargo weight. In [6], a controller anchor on gain scheduling feedback to motion a load on a gantry crane from point to point within one vibration cycle and without carry on large swings was presented. In [7], an adaptive-fuzzy SMC for the robust anti-sway pursuit, of overhead cranes follow up to both system uncertainty and actuator nonlinearity was subject. In [8], anchor on partial feedback linearization, a nonlinear controller was resolve and designed for the three-dimensional moving of a crane system. The control planner was constituted by linearly collecting two components that are separately gained from the nonlinear feedback of actuated and un-actuated states. In [9], the design of robust nonlinear controllers, anchor on both conventional and hierarchical sliding mode technics for pendulum crane systems was presented.

However, there has been little discussion about uncertainty and robust control for overhead crane system. So, the main purpose of this work is to investigate the robust control design system anchor on SMC for the overhead crane system with uncertainties parameters consider. Moreover, to delivers desired settling time for angular position of overhead crane and displacement of cart and compare the performance between two methods of controller will be use.

2. Mathematical Model of Crane

In this section the dynamic equations of motion are given for overhead crane system. The configuration of the system, as shown in Figure 2, can be expressed with the generalized coordinate’s vector \( (x, \dot{x}, \theta, \dot{\theta}) \).

The grab and hang load are representing as a single scale. The grab and hang load are regarded as a single mass point as the mass of the payloads is usually much larger than that of the grab. The cart and load situation in connection to a reference position, say the origin 0 are given as \((x, y)\) and \((x_p, y_p)\) respectively. The main propositions are certain to get the effective model of the crane system:

1. Neglected the mass and elastics of the wire rope.
2. The load can be consideration as point mass.
3. No disturbance with reason of wind outside the plant, duo to the overhead crane is usually operated indoors.
4. Neglected the friction force and elongation of the rope due to tension.
5. Movement of system components is extremely present in plane.
6. The swing angle of payload is limited to \(|\theta| \leq \pi/2\), and rope length is \(L > 0\)

The dynamic model of overhead crane system can be acquired using the Euler-Lagrange approach. The kinetic and potential energy are given as follows

\[
(M + m)\ddot{x} + mL\dot{\theta}\cos\theta - mL\dot{\theta}^2\sin\theta = u
\]

\[
mL\dot{x}\cos\theta + mL^2\ddot{\theta} + mgL\sin\theta = 0
\]

where, \(M\) is the cart mass (kg), \(m\) is the load mass (kg), \(L\) is the cable length (m), the control input \(u\) (N) is produced by a DC electric motor.

The model can be described by the state space representation using the following notation, \(x\) for the cart position; \(\dot{x}\) for the cart velocity; \(\theta\) for the load swing angle; and \(\dot{\theta}\) for the angular velocity.
Consequently, the complete overhead crane dynamic model can be represented as follows:

\[
\begin{bmatrix}
M + m & mL\cos\theta \\
ML\cos\theta & mL^2
\end{bmatrix}
\begin{bmatrix}
\ddot{x} \\
\dot{\theta}
\end{bmatrix}
= \begin{bmatrix}
ml\dot{\theta}^2\sin\theta \\
-mgL\sin\theta
\end{bmatrix} + \begin{bmatrix}
u \\
0
\end{bmatrix}
\]  

(3)

Equation (3) can be written as follows

\[
\begin{bmatrix}
K_{11} & K_{12} \\
K_{21} & K_{22}
\end{bmatrix}
\begin{bmatrix}
\ddot{x} \\
\dot{\theta}
\end{bmatrix}
= \begin{bmatrix}
f_a + u \\
f_u
\end{bmatrix}
\]  

(4)

where,

\[K_{11} = M + m, K_{12} = K_{21} = mL\cos\theta, K_{22} = mL^2, f_a = -mgL\sin\theta, f_u = mL\dot{\theta}^2\sin\theta\]

Let us define

\[K = \begin{bmatrix}
K_{11} & K_{12} \\
K_{21} & K_{22}
\end{bmatrix}\]

Then, equation (4) can be rewritten as

\[
\begin{bmatrix}
\ddot{x} \\
\dot{\theta}
\end{bmatrix}
= K^{-1}(\theta) \begin{bmatrix}
f_a + u \\
f_u
\end{bmatrix}
\]  

(5)

Consequently, equation (5) can be written as

\[
\ddot{x} = \frac{k_{22}f_a - K_{21}f_u}{K_{11}K_{22} - K_{12}K_{21}} + \frac{k_{22}f_a - K_{21}f_u}{K_{11}K_{22} - K_{12}K_{21}}
\]

\[
\dot{\theta} = \frac{-k_{11}f_a + k_{12}f_u}{K_{11}K_{22} - K_{12}K_{21}} + \frac{-k_{12}}{K_{11}K_{22} - K_{12}K_{21}} u
\]

(6)

(7)

The equations (6) and (7) can be written as state space representation as follows

\[
\ddot{x} = f_{11} + b_{11}u
\]

\[
\dot{\theta} = f_{22} + b_{22}u
\]  

(8)

(9)
where

\[ f_{11} = \frac{K_{22}f_0 - K_{21}f_0}{K_{11}K_{22} - K_{12}K_{21}}, \quad b_{11} = \frac{K_{22}}{K_{11}K_{22} - K_{12}K_{21}} \]

\[ K_{22} = \frac{-K_{11}f_0 + K_{21}f_0}{K_{11}K_{22} - K_{12}K_{21}}, \quad b_{22} = \frac{-K_{12}}{K_{11}K_{22} - K_{12}K_{21}} \]

3. Sliding Mode Control Design

In this section, in order to be realized the desired control target, we propose control scheme, consisting of dynamic control design. The methodology is based on SMC. Firstly, the system is transformed into a special structure through input transformation, containing nominal part plus some unknown term. The unknown term is computed adaptively. The control problem is to find the control signal \( u \) such that all the states of the system converge to zero. Based on Lyapunov function, we are going to derive the dynamic control law in a series of steps, as follows:

Define the sliding mode function as

\[ s = \dot{x} + c_1 \dot{x} + c_2 \dot{\theta} + c_3 \theta \quad (10) \]

where, \( c_1, c_2, \) and \( c_3 \) are control design parameters can be determined by Hurwitz

Then

\[ \dot{s} = f_1 + c_2 f_2 + (b_{11} + c_2 b_{22}) u + c_1 \dot{x} + c_3 \dot{\theta} \quad (11) \]

Design the controller as

\[ u = -\frac{1}{b_{11} + c_2 b_{22}} \left[ f_1 + c_2 f_2 + c_1 \dot{x} + c_3 \dot{\theta} + \zeta \text{sgn}(s) \right] \quad (12) \]

where \( \zeta > 0 \)

Substituting equation (8) and (9) in equation (11), we obtain

\[ \dot{s} = -\zeta \text{sgn}(s) \quad (13) \]

Then we obtain

\[ ss' = -s \cdot \zeta \text{sgn}(s) = -\zeta |s| \leq 0 \quad (14) \]

**Hurwitz Stability Analysis**

The term \( ss' \leq 0 \) indicates that there exists \( t_s \), when \( t \geq t_s \)

we have

\[ s = \dot{x} + c_1 x + c_2 \dot{\theta} + c_3 \theta = 0 \quad (15) \]

Therefore

\[ \dot{x} = -c_1 x - c_2 \dot{\theta} - c_3 \theta \quad (16) \]

Substituting equation (16) into equation (9), we get

\[ \dot{\theta} = \frac{1}{L - c_2 \cos \theta} \left[ -g \sin \theta + \cos \theta (c_1 (-c_1 x - c_2 \dot{\theta} - c_3 \theta) + c_3 \dot{\theta}) \right] \quad (17) \]

The system equilibrium point is

\[ \theta = 0, \dot{\theta} = 0, x = 0, \dot{x} = 0 \]

In equilibrium point, let us assume

\[ \sin \theta \approx \theta, \cos \theta = 1 \]
Let us define

\[ z_1 = \theta, \ z_2 = \theta, \ z_3 = x \]

we obtain the following

\[ \dot{z}_1 = z_2 \quad (18) \]
\[ \dot{z}_2 = \frac{-gz_1 - c_1z_1 + (-c_2c_1 + c_3)z_2 - c_1^2z_3}{L - c_2} + \varepsilon_1z_1 + \varepsilon_2z_2 + \varepsilon_3z_3 \quad (19) \]
\[ \dot{z}_3 = -c_3z_1 - c_2z_2 - c_1z_3 \quad (20) \]

where, \( \varepsilon_i \) is error caused by linearization, \( i = 1, 2, 3 \), then we obtain

\[ \dot{z} = Az + \varepsilon z \quad (21) \]

where

\[ A = \begin{bmatrix} 0 & 1 & 0 \\ A_{21} & A_{22} & A_{23} \\ -c_3 & -c_2 & -c_1 \end{bmatrix}, \quad \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix} \]

Since, \( \varepsilon_i \) is a very small value, if we design the matrix \( A \) as Hurwitz, the stability of the overhead crane system \( \dot{z} = Az + \varepsilon z \) can be secured. Then if \( t \to \infty \), we have \( z \to 0 \) or \( x \to 0 \), \( \theta \to 0 \) and \( \dot{\theta} \to 0 \), moreover consider \( s \to 0 \), we have \( \dot{x} \to 0 \).

Let us assume

\[ c_2 \neq L, \ A_{21} = \frac{-g - c_1c_1}{L - c_2}, \ A_{22} = \frac{-c_2c_1 + c_3}{L - c_2}, \ 	ext{and} \ A_{23} = \frac{c_1^2}{L - c_2} \]

From \( |A - \lambda I| = 0 \)

we have

\[ \lambda^3 - (A_{22} - c_1)\lambda^2 + (-c_1A_{22} - A_{21} + c_2A_{23})\lambda - c_1A_{21} + c_3A_{23} = 0 \quad (22) \]

Compared equation (22) with equation \((\lambda + 1)(\lambda + 2)(\lambda + 3) = 1\), we can get sets of equations as follows

\[ -A_{22} + c_1 = 6 \quad (23) \]
\[ -c_1A_{22} - A_{21} + c_2A_{23} = 11 \quad (24) \]
\[ -c_1A_{21} + c_3A_{23} = 6 \quad (25) \]

The solution of the equations (23-25) are

\[ c_2 = L - g/11 \quad (26) \]
\[ c_1 = -6g/(c_2 - L) \quad (27) \]
\[ c_3 = Lc_1 + 6(c_2 - L) \quad (28) \]

### 4. Linear Quadratic Regulator (LQR) Controller

LQR is an optimal control technique that provides the superior execution with regard to some given away execution measurement. Linear quadratic optimal control takes into account the linearized equations (1), (2) of the overhead crane system. In LQR control design trouble is to planning a state
feedback controller $K$ such that the purpose role $J$ is minimized [10-11]. In this control method, a feedback gain matrix is designed such that minimizes the purpose function.

The linear system is found to be in equations (1), (2) for continuous time can be represent by state equation

$$\dot{x} = Ax + Bu$$

with, the cost function can be expressed as

$$J = \int_0^\infty (x^T Q x + u^T R u) dt$$

where, $Q$ and $R$ are the weight matrices, $Q$ is desired to be positive introduction or positive semi-definite symmetry matrix, $R$ is wanted to be positive appoint symmetry matrix. One functional method is to $Q$ and $R$ to be diagonal matrix. The estimation of the elements in $Q$ and $R$ is regarding to its subscribe to the cost function $J$.

The control law can be written as

$$u = -Kx$$

where, $K$ is determine as

$$K = R^{-1}B^TP$$

and $P$ can be determine by solution the Continuous time Algebraic Riccati Equation (CARE) as

$$A^TP + PA + Q - PB R^{-1}B^TP = 0$$

The control method for returns the feedback gain $K$ for LQR controller enable formulate by election of the design changeable matrices $Q$ and $R$, using trial and error procedure. This value of matrices, give the best pole placement control design for attitude of the crane system.

5. Result and Discussion

In this work, both simulation results are provided to check the performance of the suggestion approach. The simulation is perfect for dissimilar condition. In this simulation considered system is composed of crane system and the values of the physical data are patent in Table 1, and the initial condition is assumed for the states are $x(0) = [54.7^\circ 0.1 0]$. 

| Table 1. Physical parameters of overhead crane system. |
|-----------------------------------------------|
| Symbol | Value | Description |
|--------|-------|-------------|
| $g$    | 9.81  | Earth’s gravitational constant $(m/s^2)$ |
| $M$    | 300   | Mass of the trolley $(kg)$ |
| $m$    | 2000  | Mass of the load $(kg)$ |
| $L$    | 1.5   | Length of the steel rope $(m)$ |

To examine the validation of the proposed control methodology, the simulations for the overhead crane system were performed in MATLAB/Simulink. The revealed results of the SM and LQR controllers design for the overhead crane system can be shown in figures (3-9). The simulation results for the load swing angle, angular velocity, trolley position, trolley velocity and control input are shown in Figures 3-7. The performance of the SMC is compared with LQR controller as shown in figures (5), (6) for similar initial conditions. Fig. (7), shows the control signal, it can be seen that the control force exponentially decays as time increases. So, the closed loop for crane system is exponentially stable. The controller established on the capacity access security that the system attains the required place.
In this simulation results, the SMC gives better results than LQR controller in terms of settling time and maximum peak overshoot. The SMC successfully eliminates the oscillations occurred in the trolley motion compare with LQR controller. Moreover, the SMC reduce the maximum peak overshoot occurred in the case of LQR controller approximately to half value. Finally, SMC eliminates the steady state error occurred during simulation. The results indicate that, the SMC has faster velocity response and shorter settling time.

Finally, the rendering of the overhead crane system is tested for a validation of the proposed control scheme through several simulations with different initial conditions of the crane system, as shown in figures (8), (9) for actual trolley positions and payload angle. It can be concluded that, the proposed SMC is given better performance at different variation of initial conditions, and all the figures show that the system asymptotically approach to zero.

**Figure 3.** Swing angular for crane system.

**Figure 4.** Angular velocity for crane system.

**Figure 5.** Trolley position for crane system.

**Figure 6.** Trolley velocity for crane system.
6. Conclusion

It was presented in this study, the design of a nonlinear controller for an overhead crane based on the sliding mode control. The design procedures of sliding mode control discussed in detail. The control technique was felicitously designed for the complex process of the crane system, with each other collecting the control of payload lifting, trolley motion and payload swing abstraction. Simulation results showed that, the SMC presented better control results than the LQR controller in the terms transient response and steady state response, and needs less control effort. Further evaluation shows that the proposed control system is also robust to parameter variations. In future works, the robustness of the suggest control method should be enhanced. Accordingly, disturbance inputs should be included in the modeling and in the design of the proposed dynamic controller. And also, will consider the adaptive sliding mode control design with genetic algorithms to handle more complex models.
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