AN EMERGENT UNIVERSE SUPPORTED BY CHIRAL COSMOLOGICAL FIELDS IN EINSTEIN–GAUSS–BONNET GRAVITY

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May 29, 2014

Abstract

We propose the application of the chiral cosmological model (CCM) for the Einstein–Gauss–Bonnet (EGB) theory of gravitation with the aim of finding new models of the Emergent Universe (EmU) scenario. We analysed the EmU supported by two chiral cosmological fields for a spatially flat universe, while we have used three chiral fields when we investigated open and closed universes. To prove the validity of the EmU scenario we fixed the scale factor and
found the exact solution by decomposition of EGB equations and solving the chiral field dynamics equation. To this end, we suggested the decomposition of the EGB equations in such a way that the first chiral field is responsible for the Einstein part of the model, while the second field, together with kinetic interaction term, is connected with the Gauss–Bonnet part of the theory. We proved that both fields are phantom ones under this decomposition, and that the model has a solution if the kinetic interaction between the fields equals a constant. We have presented the exact solution in terms of cosmic time. This was done for a spatially flat universe. In the case of open and closed universes we introduced the third chiral field (canonical for closed and phantom for open universe) which is responsible for the EGB and curvature parts. The solution of the third field equation is obtained in quadratures. Thus we have proved that the CCM is able to support EmU scenario in EGB gravity for spatially flat, open and closed universes.

1 Introduction

The discovery of the acceleration in the expanding Universe at the end of twentieth century led to intensive investigations of modified gravity and field theories [1]. Theories such as scalar-tensor gravitation (which may be considered as Einstein’s gravitational field coupled to a self-interacting scalar field in the Einstein frame), \( f(R) \) gravitation, Lovelock gravity, and its special variant of \( 5D - 6D \) gravity as Einstein-Gauss-Bonnet (EGB) gravitation, can be related to these theories. We also mention the generalization of EGB theory to \( f(R_{GB}) \) gravity which acts in \( 4D \) spacetime [2].

On the other hand new data from astrophysical observations obtained from the cosmic project WMAP, Planck and BICEP2 provide possibilities to determine restrictions on the parameters of a basic theory according to the following scheme. We may start from investigations of gravitational dynamics and finding background solutions. Then we develop cosmological perturbation theory, and solve the perturbed equations, usually in the long-wave and short-wave approximations. Afterwards it is necessary to find a power spectrum and spectral indexes which together with the tensor-to-scalar ratio may be used to confront observational digital values.

Our study belongs to the investigation of multidimensional gravity actively developed by V. N. Melnikov and his group (for a review, see [3]). In the present work we make the first step in building a model in this scenario, namely we are searching for exact solutions for the background equations in \( 5D \) EGB gravity for the Emergent Universe (EmU). Connection to the real \( 4D \) cosmology may be performed in several
ways. One way is the consideration of the results in this article as bulk-solutions in brane cosmology. The corresponding $4D$ cosmological solutions may be obtained by the superpotential method, which is related to developments in brane cosmology [4]. Another possibility to reach $4D$ FRW cosmology is to perform a compactification procedure of $5D$ FRW cosmology, and to prove that chiral fields belong to the brane. Also we may suggest the relevance of our obtained solutions to $f(R_{GB})$ gravity which is known to act in $4D$ spacetime [2].

The idea of an EmU was proposed by Ellis and Maartens [5]. This scenario avoids the initial singularity and there is no need for a quantum era. The universe in the infinite past is in an almost static state, and then evolves into an inflationary era. Mukherjee et al [6] showed that it is possible to have a spatially flat emergent universe if one allows for the existence of a perfect fluid together with some type of exotic matter.

The article of Beesham et al [7] gives the motivation for the study of the EmU scenario within the context of chiral fields in the framework of the nonlinear sigma model. Beesham et al [8] found solutions with phantom and canonical scalar fields, including a solution valid for all time. More recently, Beesham et al [9] presented new classes of exact solutions, discussed the potential and kinetic interaction of the chiral fields and calculated key cosmological parameters.

In the work [10] the authors have studied a general approach to the EmU in EGB cosmology. Their discussion was largely qualitative, focusing on the possibility of the existence of solutions and corresponding restrictions. Some restrictions for the model with canonical and phantom scalar fields coupled to a perfect fluid with a linear equation of state have been obtained. A similar analysis was done for tachyon and phantom tachyon fields. However, in [10], the authors did not really find exact solutions, to confirm the existence of the EmU scenario in FRW cosmology derived from EGB gravity. In the present article we show the possible development of EmU scenario by virtue of the exact solutions in EGB cosmology in the framework of a chiral cosmological model.

In the present work, we study the EmU universe supported by two chiral fields within the context of $5D$ EGB theory. Investigation of theories of higher dimensional gravitation is dictated by novel results in high energy physics, and particularly in string theory [1]. The classical analogue of the effective string theory is represented by the low energy effective action, containing quadratic or higher powers of curvature terms. Similar terms with high power in curvature appear after the renormalization procedure of quantum field theory in the curved space background. For these theories the field equations become fourth order and ghosts appear. To avoid these difficulties Lovelock suggested a special combination of high order terms which lead to the field
equations of the second order. This means that ghost terms will not appear.

The paper is organised as follows: In section 2, we present the equations for the chiral cosmological model in EGB gravity. In section 3, we specialise to the spatially flat EmU; we present the decomposition of EGB gravity equations and solve the dynamical equations. Section 4 deals with closed and open EmU with three chiral cosmological fields; decomposition for this case and solution for the third field equation are obtained in quadratures. Finally our discussion follows.

2 The Chiral Cosmological Model in EGB Gravity

We will follow the standard prescription for EGB gravity considering the EGB action

\[ S = \frac{1}{2} \int d^5x \sqrt{-g} \left( R + \alpha_{GB} R_{GB} \right) + S_m \]  

with a matter part \( S_m \) as an action of the CCM in 5 dimensions

\[ S_m = S_{ccm} = \int d^5x \sqrt{-g} \left[ \frac{1}{2} h_{AB} (\varphi) \varphi^A \varphi^B g^{ab} - V(\varphi) \right]. \]  

We set here the 5-dimensional Einstein gravitational constant \( \kappa = 1 \). The indices \( a, b, \ldots \) take on values 0, 1, 2, 3, 5 and \( \alpha_{GB} \) is the GB coupling parameter. The notation for the CCM corresponds to those in the work [7]. The Gauss–Bonnet term is

\[ R_{GB} = R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}. \]  

The variation of the action (1) with respect to metric tensor gives the equations of EGB gravitation

\[ G_{ab} - \alpha_{GB} H_{ab} = T_{ab}, \]  

where \( G_{ab} = R_{ab} - \frac{1}{2} g_{ab} R \) is the Einstein tensor, and \( H_{ab} \) is the Lovelock tensor defined by

\[ H_{ab} = 4R_{ac}R^c_b + 4R^{cd}R_{acbd} - 2RR_{ab} - 2R^{cde}R_{bcde} + \frac{1}{2} g_{ab} R_{GB}. \]  

The chiral cosmological field equations may be derived by varying the action (2) with respect to scalar fields \( \varphi^C \)

\[ \frac{1}{\sqrt{-g}} \partial_a \left( \sqrt{-g} \varphi^a \right) - \frac{1}{2} \frac{\partial h_{BC}}{\partial \varphi^A} \varphi^C \varphi^B g^{ab} + V_A = 0, \]
where $V_A = \frac{\partial V}{\partial \varphi}$. It should be noted that in the present article all considerations are applied to a 5-dimensional spacetime.

Let us turn our attention to cosmology. We take the metric of the 5-dimensional Friedmann–Robertson–Walker (FRW) universe in the form

$$dS^2 = -dt^2 + a(t)^2 \left( \frac{dr^2}{1 - \epsilon r^2} + r^2(d\theta + \sin^2 \theta (d\varphi^2 + \sin^2 \varphi d\chi^2)) \right), \quad (7)$$

Here $\epsilon = -1, 0, +1$ for open, spatially flat and closed universes respectively.

If we take only one effective scalar field as in earlier treatments [8, 11] we may introduce a scalar field multiplett (CCM) which will have the same gravitational properties but different field dynamics. For EGB gravity, the situation is rather different as we will demonstrate later on in this article. We can state that the more general features of the CCM are exhibited in the two component model. Thus we will consider the two component CCM with the chiral (target space) metric

$$ds_{ts} = h_{11}d\phi^2 + h_{22}(\phi, \psi)d\psi^2. \quad (8)$$

The energy momentum tensor of the target space [8] in the 5D spacetime [7] takes the form

$$T_{ab} = h_{11}\phi,\phi,_{a}\phi,_{b} + h_{22}\psi,\psi,_{a}\psi,_{b} - g_{ab} \left[ \frac{1}{2} h_{11}\phi,_{c}\phi,^{c} + \frac{1}{2} h_{22}\psi,_{c}\psi,^{c} - V(\phi, \psi) \right], \quad (9)$$

where $h_{11} = constant$.

We can now present the basic equations of the CCM [6] and EGB gravity [4] in 5D FRW spacetime [7]. This system of equations takes the form

$$H^2 + \frac{\dot{\epsilon}}{\epsilon} + \alpha_{GB} \left( H^2 + \frac{\dot{\epsilon}}{\epsilon} \right)^2 = \frac{1}{6} \left( \frac{1}{2} h_{11}\dot{\phi}^2 + \frac{1}{2} h_{22}(\phi, \psi)\dot{\psi}^2 + V(\phi, \psi) \right), \quad (10)$$

$$\left[ 1 + 2\alpha_{GB} \left( H^2 + \frac{\dot{\epsilon}}{\epsilon} \right) \right] \left( \ddot{H} - \frac{\ddot{\epsilon}}{\epsilon} \right) = -\frac{1}{3} \left( h_{11}\dot{\phi}^2 + h_{22}(\phi, \psi)\dot{\psi}^2 \right), \quad (11)$$

$$h_{11}\ddot{\phi} + 4Hh_{11}\dot{\phi} - \frac{1}{2} \frac{\partial h_{22}}{\partial \phi}\dot{\psi}^2 + \frac{\partial V}{\partial \phi} = 0, \quad h_{11} = constant, \quad (12)$$

$$h_{22}(\phi, \psi)\ddot{\psi} + h_{22}(\phi, \psi)\dot{\psi} + 4Hh_{22}(\phi, \psi)\dot{\psi} - \frac{1}{2} \frac{\partial h_{22}}{\partial \psi}\dot{\psi}^2 + \frac{\partial V}{\partial \psi} = 0. \quad (13)$$

Our investigation of these equations will be applied first to a spatially flat EmU, i.e., with the model proposed by Mukherjee et al in the work [6].

3 Spatially flat EmU with two chiral cosmological fields

The basic EGB model equations (10)-(11) will be simplified taking into account $\epsilon = 0$:
\[ H^2 + \alpha_{GB} H^4 = \frac{1}{6} \left( \frac{1}{2} h_{11} \dot{\phi}^2 + \frac{1}{2} h_{22}(\phi, \psi) \dot{\psi}^2 + V(\phi, \psi) \right), \quad (14) \]

\[ [1 + 2\alpha_{GB} H^2] \dot{H} = -\frac{1}{3} \left( h_{11} \dot{\phi}^2 + h_{22}(\phi, \psi) \dot{\psi}^2 \right). \quad (15) \]

The CCM equations (12)-(13) do not change. We extract the potential \( V \) from Eqs. (14)-(15)

\[ \frac{V}{6} = H^2 + \frac{1}{4} \dot{H} + \alpha_{GB} H^2 (H^2 + \frac{1}{2} \dot{H}). \quad (16) \]

It should be mentioned here that Eq. (15) can be obtained from a linear combination of the chiral fields Eqs. (12) \( \dot{\phi} + (13) \dot{\psi} \) and using Eq. (14).

Let us consider the case of a constant potential. In Einstein gravity a constant potential is equivalent to a cosmological constant, admitting the exact solution [11], included in inflation scenarios, and may be considered as a useful tool for numerical solutions [12].

In EGB Gravity Eqs. (14) and (15) can be reduced to the form

\[ \dot{H} \left( \alpha_{GB} H^2 + \frac{1}{2} \right) + H^2 (1 + \alpha_{GB}) = \frac{1}{6} \Lambda \quad (17) \]

which gives us an opportunity to perform the integration. The solution of (17) can be expressed exactly by the relation

\[ t - t_* = -\alpha_{GB}(1 + \alpha_{GB}) \left[ H + \frac{1 + \alpha_{GB}}{4\alpha_{GB}} \frac{1}{2} \Lambda (1 + \alpha_{GB}) \ln \left| \frac{H + \sqrt{\frac{1}{6} \Lambda (1 + \alpha_{GB})}}{H - \sqrt{\frac{1}{6} \Lambda (1 + \alpha_{GB})}} \right| \right]. \quad (18) \]

It is clear from Eq. (18) that it is impossible to obtain \( H \) as a function on \( t \) explicitly. If we consider small \( \Lambda \) then we will obtain a collapsing universe with \( a(t) \propto \exp \left[ -\frac{(t-t_*)^2}{2\alpha_{GB}(1+\alpha_{GB})} \right] \) due to the EGB gravity influence. Taking \( \alpha_{GB} = 0 \) in (17) we obtain the solution

\[ H = \sqrt{\frac{\Lambda}{6}} \tanh \left( \sqrt{2\Lambda/3} t \right), \quad a(t) = a_* \left( \cosh \sqrt{2\Lambda/3} t \right)^{1/2}. \quad (19) \]

Observe that the solution (18) is different from the 4-dimensional result of Chervon [11]; he obtained the forms \( H = \sqrt{\frac{\Lambda}{3}} \tanh(\sqrt{3\Lambda} t), \quad a = a_* [\cosh(\sqrt{3\Lambda} t)]^{1/3} \) which are not the same as in the 5-dimensional result (18).
3.1 Decomposition of EGB system

The scale factor of the EmU we choose in the most general form [6, 8]

\[ a(t) = A \left( \beta + e^{\alpha t} \right)^m. \]  

(20)

Following the method of Beesham et al [9] we decompose the EGB and chiral fields equations in the following way:

\[ \dot{H} = -\frac{2}{3} K(\phi), \quad K(\phi) = \frac{1}{2} h_{11} \dot{\phi}^2, \]  

(21)

\[ 2 \alpha_{GB} H^2 \dot{H} = -\frac{2}{3} K(\psi), \quad K(\psi) = \frac{1}{2} h_{22}(\phi, \psi) \dot{\psi}^2, \]  

(22)

\[ H^2 + \frac{1}{4} \dot{H} = \frac{1}{6} V(\phi), \]  

(23)

\[ H^4 + \frac{1}{2} H^2 \dot{H} = \frac{1}{6 \alpha_{GB}} V(\psi). \]  

(24)

From (21) and (22) one can see that both fields should be phantom ones for the EmU, because \( \dot{H} > 0 \). Thus we can set \( h_{11} = -1 \) and transform \( h_{22} \rightarrow -h_{22} \).

Eq. (21) has the solution

\[ \phi - \phi_i = 2 \sqrt{3} m \arctan \left( \frac{e^{\alpha t/2}}{\sqrt{\beta}} \right), \]  

(25)

where \( \phi_i \) corresponds to the value at \( t = -\infty \). Using the solution (25) above we can reconstruct the potential \( V(\phi) \). The result is

\[ V(\phi) = \frac{3}{2} m \alpha^2 \sin^2 2\tilde{\phi} \left( m \tan^2 \tilde{\phi} + \frac{1}{4} \right), \]  

(26)

where \( \tilde{\phi} = \frac{\phi - \phi_i}{2 \sqrt{3} m} \).

The formulae in terms of cosmic time \( t \) for the potential and kinetic energy of \( \psi \) read

\[ V(\psi) = 6 \alpha_{GB} m^3 \alpha^4 e^{3\alpha t \frac{me^{\alpha t} + \frac{1}{2} \beta}{(\beta + e^{\alpha t})^2}}, \]  

(27)

\[ \frac{1}{2} h_{22}(\phi, \psi) \dot{\psi}^2 = 3 \alpha_{GB} m^3 \alpha^4 \beta \frac{e^{\alpha t}}{(\beta + e^{\alpha t})^2}. \]  

(28)

Note that with this result the EGB Eqs. (14) and (15) are satisfied. Our task now is to show that the dynamics of the chiral fields will not contradict the solutions of the gravitational equations.
3.2 The solution of dynamic equations

Because of the negative sign in (21) and (22), we have changed the sign of $h_{22}$ and set $h_{11} = -1$. With this, we emphasize that both chiral fields are phantom.

Direct insertion of the solution (25) into (12), and using the formulae

\[ H = \frac{m \alpha e^{\alpha t}}{\beta + e^{\alpha t}}, \]
\[ \dot{\phi} = \frac{e^{\alpha t}}{2} \left( \frac{e^{\alpha t} (3 - e^{\alpha t})}{(\beta + e^{\alpha t})^2} \right), \]
\[ \ddot{\phi} = \sqrt{3m\beta} \frac{e^{\alpha t}}{2} \left( \frac{e^{\alpha t} (3 - e^{\alpha t})}{(\beta + e^{\alpha t})^2} \right), \]
\[ \frac{\partial V(\phi)}{\partial \phi} = \sqrt{3m\alpha^2} \left( \sin 4\tilde{\phi} \left( m \tan^2 \tilde{\phi} + \frac{1}{4} \right) + 4m \sin^2 \tilde{\phi} \tan \tilde{\phi} \right), \]

lead to conclusion that the term $\frac{1}{2} \frac{\partial h_{22}}{\partial \phi} \dot{\psi}^2$ should be equal to zero. Indeed, making reconstruction of the derivation of the $\phi$-part of the potential $\frac{\partial V(\phi)}{\partial \phi}$ in terms of cosmic time $t$ we obtain

\[ \frac{\partial V(\phi)}{\partial \phi} = \sqrt{3m\beta} \frac{e^{\alpha t}}{2} \frac{(e^{\alpha t} (8m - 1) + \beta)}{(\beta + e^{\alpha t})^2}, \]

Thus the result $\frac{1}{2} \frac{\partial h_{22}}{\partial \phi} \dot{\psi}^2 = 0$ means that $h_{22}$ is the function on $\psi$ only. With this we can set $h_{22} = 1$ without loss of generality, and look for the solution of the second field equation (13). To this end we will multiply the Eq. (13) on $\dot{\psi}$, and use the relation

\[ \dot{V}(\psi) = \lambda_1 \alpha e^{3\alpha t} \frac{(e^{\alpha t} (4m - \frac{1}{2}) + \frac{3}{2} \beta)}{(\beta + e^{\alpha t})^2}, \]

where $\lambda_1 = 6\alpha GB m^3 \alpha^4 \beta$. With this, Eq. (13) will be identically satisfied.

So our task is to find the second chiral field $\psi$. This can be done from the decomposition ansatz (28) where we taking into account $h_{22} = 1$. The solution is

\[ \psi - \psi_i = \frac{\sqrt{\lambda_1}}{\alpha} \left[ \frac{1}{\sqrt{\beta}} \arctan \left( \frac{e^{\alpha t/2}}{\sqrt{\beta}} \right) - \frac{e^{\alpha t/2}}{\beta + e^{\alpha t}} \right], \]

where $\psi_i$ corresponds to the value at $t \to -\infty$. It is clear that we could not obtain the dependance of $\psi$ on $t$ explicitly from the solution (35).

Thus the exact solution we obtained are represented by solutions for chiral cosmological fields (25) and (35); the potential $V(\phi)$ in (26) and the potential $V(\psi)$ are defined via dependence on cosmic time in (27).
4 Closed and open EmU with three chiral cosmological fields

Now we are ready to take into consideration closed and open EmU. Following the developed method we are including, in the chiral cosmological model, the third field $\chi$ which is responsible for the curvature of the FRW Universe and EGB gravity. With this aim we have to extend the decomposition with the kinetic and potential energies of $\chi$:

$$K(\chi) = \frac{1}{2} h_{33} \dot{\chi}^2 = \frac{3}{2} \frac{a^2}{a^2} \left( 1 + 2 \alpha_{GB} \left( H^2 - \dot{H} + \frac{\dot{a}}{a^2} \right) \right) \quad (36)$$

$$V(\chi) = 3 \alpha_{GB} a^2 + \frac{\epsilon}{a^2} \left( \frac{9}{2} + 3 \alpha_{GB} \left( \dot{H} + 3H^2 \right) \right) \quad (37)$$

The equation on the chiral field $\chi$ for the constant kinetic interaction $h_{33} = \text{constant}$ is

$$h_{33} \ddot{\chi} + 4Hh_{33} + \frac{\partial V(\chi)}{\partial \chi} = 0. \quad (38)$$

Multiplying Eq. (38) by $\dot{\chi}$, and inserting $\dot{\chi}$, $\ddot{\chi}$ from Eq. (36), and $\dot{V}(\chi)$ from Eq. (37), we can check that by consequence Eq. (38) is satisfied, and the very equation is satisfied too if $\dot{\chi} \neq 0$.

Now our task is to define the third field $\chi$ from Eq. (36). Let us note here that for an open universe ($\epsilon = -1$) the sign of $h_{33}$ should be negative, i.e., the third field $\chi$ should be phantom. For a closed universe ($\epsilon = 1$) the third field $\chi$ should be canonical. With this remark we can write the general formula for $\chi$:

$$\chi = \sqrt{3} \int dt \left( \beta + e^{\alpha t} \right)^{-m} \sqrt{1 + 2 \alpha_{GB} \left( \frac{m \alpha^2 e^{\alpha t} (me^{\alpha t} - \beta)}{(\beta + e^{\alpha t})^2} + \frac{\epsilon}{A^2(\beta + e^{\alpha t})^2m} \right)} \quad (39)$$

Thus we have obtained the solution for open and closed universes with three chiral cosmological fields. It is difficult to perform the integration for the third chiral field $\chi$ in Eq. (39). Therefore we turn our attention to the case when $\beta \approx 0$. This situation studied by Paul et al [13]. Note that we analyze the integration only for $\chi$ in Eq. (39), not for the other equation.

Taking into account $\beta \approx 0$ in Eq. (39) we obtain the general formula

$$\chi = \frac{\sqrt{3}}{A} \int dt \ e^{-2m^{\alpha t}} \sqrt{e^{2m^{\alpha t}} + 2 \alpha_{GB} \left( m^{2\alpha^2} e^{2m^{\alpha t}} + \frac{\epsilon}{A^2} \right)} \quad (40)$$

To simplify the expression (40) let us introduce a new variable and constants. Let

$$Z = e^{2m^{\alpha t}}, \quad B^2 = 1 + 2 \alpha_{GB} m^2 \alpha^2, \quad D^2 = \frac{2 \alpha_{GB}}{A^2 B^2}. \quad (41)$$
We have two possible solutions

\[ \epsilon = +1, \quad h_{33} = +1, \]
\[ \chi - \chi^* = \pm \frac{\sqrt{3}B}{A^2} \left[ -\frac{1}{2} \frac{\sqrt{Z+D^2}}{Z} + \frac{1}{4D} \ln \left( \frac{\sqrt{Z+D^2}-D}{\sqrt{Z+D^2}+D} \right) \right], \]  
\[ \epsilon = -1, \quad h_{33} = -1, \]
\[ \chi - \chi^* = \pm \frac{\sqrt{3}B}{A^2} \left[ -\frac{1}{2} \frac{\sqrt{Z-D^2}}{Z} + \frac{1}{4D} \arctan \left( \frac{\sqrt{Z-D^2}}{D} \right) \right], \quad Z \geq D^2. \]  

The inequality \( Z \geq D^2 \) means that the solution will be valid only for time

\[ t \geq \frac{\ln (2\alpha_{GB} A^{-2} \alpha^{-2})}{2mA}. \]

When \( t \to \infty \) the third chiral field tends to some constant.

Let us mention that the case \( \epsilon = +1, \quad h_{33} = -1 \) also admits a solution, while the case \( \epsilon = -1, \quad h_{33} = +1 \) may be valid only for some period of time.

5 Discussion

Our study is motivated by the great attention paid recently to modified gravity theories due to the possible "gravitational" explanation of the acceleration in the expanding Universe. Amongst modified gravity theories, the multidimensional one plays an important role both from the view of mathematical physics and from the connection to realistic 4D gravitation under compactification procedure of additional dimensions \[3\]. On the other hand an emergent universe, as well as eternal inflation and a universe with cyclic evolution have many criticisms concerning stability and singularity. In our consideration we are not concerned with these problems because of the possibility to restrict our obtained results to positive time. Consideration of negative time is only of a purely mathematical character. The methods applied to the EmU scenario are also applicable to exponential, power-law and others kinds of early inflation.

In this article we proposed application of a chiral cosmological model in EGB gravity. Considering the two-component CCM with two phantom fields in the spatially flat FRW universe we proved the existence of an exact solution and found it. In the case of closed and open universe we introduced the third field and found the exact solution in quadratures. Thus we strongly confirm the existence of the EmU scenario in EGB gravity for spatially flat, open and closed universes.
6 Acknowledgments

SVC is thankful to the University of KwaZulu-Natal, the University of Zululand and the NRF for financial support and warm hospitality during his visit in 2013 to South Africa. SDM acknowledges that this work is based upon research supported by the South African Research Chair Initiative of the Department of Science and Technology and the National Research Foundation. SVC and ASK would like to mention that the part of this work was carried out within the framework of a State Order of the Ministry of Education and Science of the Russian Federation in accordance with Project No.2014/391.

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