Time varying default barrier as an agreement rules on bond contract

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Abstract. There are some default time rules on contract agreement of a bond. The classical default time is known as Merton Model. The most important characteristic of Merton’s model is the restriction of default time to the maturity of the debt, not taking into consideration the possibility of an early default. If the firm’s value falls down to minimal level before the maturity of the debt, but it is able to recover and meet the debt’s payment at maturity, the default would be avoided in Merton’s approach. Merton model has been expanded by Hull & White [6] and Avellaneda & Zhu [1]. They introduced time-varying default barrier for modelling distance to default process. This model use time-varying variable as a barrier. In this paper, we give a valuation of a bond with time-varying default barrier agreement. We use straight forward integration for obtaining equity and liability equation. This theory is applied in Indonesian corporate bond.

Keywords: corporate bond, default time, straight forward integration

1. Introduction

Most financial institutions devote considerable resources to the measurement and management of credit risk. Regulator for many years required bank to keep capital to reflect the credit risk they are bearing. Credit risk arises from the possibility that borrowers and counterparties in derivative transactions may default. Credit risk is the distribution of financial losses due to unexpected changes in the credit quality of a counterparty in a financial agreement [5]. At its center is the probability of default, by which we mean any type of failure to honor a financial agreement. To estimate the probability of default, we need to specify a model of investor uncertainty, a model of the available information and its evolution over time, and a model definition of the default event.

There are two kinds of mathematics frameworks for pricing credit risk that have been proposed. The first model is structural models, introduced by Merton [13]. Structural models use the capital structure to find the default probability and the mean recovery rate. Merton model has expanded by Black & Cox [2], Brennan & Schwartz [4], Kim, Ramaswamy & Sundaresan (1993), Geske (1977), and others. The second model is reduced form model, introduced by Duffie & Singleton (1999). Reduced-form models use the market spread to find the default probability and the mean recovery rate. This model has been expanded by Jarrow & Turnbull [7], Jarrow, Lando, & Turnbull [8], and others.

The most important characteristics of Merton model is the restriction of default time to the maturity of the debt, not taking into consideration the possibility of an early default, no matter what happens with the firm’s value before the maturity of the debt. There were some literatures that have been
proposed to solve the problems. Black & Cox [2] introduced First Passage Time Model that gives bondholder right to reorganize a firm if its value falls below a given barrier. Reisz & Perlich [14] point out that if the firm’s asset is below the face value of the bond at maturity date, then First Passage Time Model does not mathematically appropriate. Research for some methods for Indonesian Bond Data can be found in Maruddani et al [10-12].

To avoid the problems, Hull & White [6] and Avellaneda & Zhu [1] give a new model named Time Varying Default Barrier. This paper draws this model to analyze Indonesian corporate bond. We focus on simulating and test the model using Indonesian corporate bond to assess how well the default probability of the firm.

This research paper is set out as follows: section 2 describes the literature related to Time Varying Default Barrier. Section 3 details the data and research method used to model and valuing default probability. Section 4 presents result and discussion, and section 5 the conclusion.

2. Time varying default barrier

Time Varying Default Barrier corrected the definition of bankruptcy time in Merton’s Model. This model gives bondholder right to reorganize a firm if its value falls below a given barrier. To build this model, we use some assumptions for simplifying the analytic solution. Some assumptions are used, which are:

1. Constant return and volatility
2. No transaction costs
3. No dividends
4. No riskless arbitrage
5. Security trading is continuous
6. Risk free rate is constant for all maturities
7. Short selling proceeds is permitted

The model considers some Merton model framework, a corporation financed through a single debt and single equity issue. The debt comprises a zero coupon bond with notional value $K$ maturing at time $T$. And to avoid the inconsistency mentioned above, it is defined Time Varying Default Barrier $B(t)$ for each $t$. The barrier should meet the condition $B(t) \leq K$ for all $t \leq T$. For some constant $k > 0$, we consider a deterministic function

$$
B(t) = K \exp(-k(T-t))
$$

(1)

Which can be thought of as the face value of the debt, discounted back to time $t$ at a continuously compounding rate $k$. The firms default at

$$
\tau = \inf\{t > 0: V_t < B(t)\}
$$

(2)

Observing that

$$
\{V_t < B(t)\} = \{(m-k)t + \sigma W_t < \log L - kT\}
$$

(3)

Then we have default probability

$$
p(T) = P\left[ \min_{t \leq T} ((m-k)t + \sigma W_t) < \log L - kT \right]
$$

(4)

Now we reduced the problem to calculate the distribution of the historical low of an arithmetic Brownian Motion with drift $m-k$.

With $m = \mu - \frac{1}{2}\sigma^2$ and $L = \frac{K}{\sqrt{V_0}}$, we get

$$
p(T) = \Phi\left(\frac{\log L - mT}{\sigma\sqrt{T}}\right) + \exp(-kT)\sigma^2 \Phi\left(\frac{\log L - (m-2k)T}{\sigma\sqrt{T}}\right)
$$

(5)
3. Data and Methods

Indonesia corporate bond data is derived from publicly available databases obtained from Indonesian Bond Pricing Agency (IBPA) in 2017 on website [www.ibpa.co.id](http://www.ibpa.co.id). We use bond data issued by PT Bank Danamon Indonesia Tbk with code BDNI named Obligasi II Seri A. The profile structure of this bond is given at Table 1.

| Table 1. Profile Structure of Obligasi II Seri A Bank Danamon Indonesia Tbk. |
|-----------------------------------------------|
| Outstanding                          | Issue Term | Coupon Structure |
| 921,000,000,000                      | 5 years    | Fixed 9%         |

Total assets data of the firm is published by Indonesian Bank consists of monthly prices January 20011 until December 2016 on website [www.bi.go.id](http://www.bi.go.id). According to Wilmott (2000), investors’ main concern will be on the return on investment which refers to the percentage growth in the value of an asset. If \(X_t\) is asset value on the day, then the return from day \(i\) to day \(t+1\) is given by

\[
R_t = \ln \frac{X_t}{X_{t-1}}
\]  

If \(n\) is the number of returns in the sample, then the drift \(\mu\) can be presented by the mean of the returns distribution and volatility \(\sigma\) can be represented by the sample standard deviation.

Procedure to calculate default probability is as follow:
1. Calculate value of ln return of assets for each \(t\)
2. Test the normality distribution of ln return assets
3. Calculating default probability of bond using equation (5)

4. Results and Discussion

For deriving the probability of default, equity, and liability of the bond, we have to do some steps to fulfill the assumptions. First, we have to checked wether the natural logarithm of total assets data is normally distribution or not. Figure 1 give us the Normal Q-Q plot for the natural logarithm of total assets data. From this figure, we can see that the natural logarithm of total assets data is normally distributed. All the computation is done by R programming.

We do hypothesis test of normality distribution with Jarque Bera Test. The result of the Jarque-Bera test is given at Table 2.

![Figure 1. Normal Q-Q Plot of Natural Logarithm of BDNI Total Assets](attachment:image.png)
Then, we have to estimate some parameters. In Table 3, we give a summary for those parameters.

**Table 3. Summarize Value of the Parameters**

| Parameter                              | Value               |
|----------------------------------------|---------------------|
| Asset Value at December 2011 ($V_o$)   | Rp. 456.381.943.000.000 |
| Face Value (K)                         | Rp. 1.879.000.000.000 |
| Time to maturity ($\tau$)              | 5 years             |
| Assets volatility ($\sigma$)           | 1.9351%             |
| Interest risk free rate ($r$)          | 5.5%                |

Using R programming, we calculate of default probability for the time varying default barrier using equation (5). The result gives default probability is $3.273673E^{-5}$. We can see that the default probability of Danamon Bank bond is very small because of the outstanding of the bond is very low than the total assets value. It can be seen from Table 3, that the face value of the bond is Rp. 2,000,000,000.000 and the total assets value at the end of 2011 is Rp. 456,381,943,000.000. In normal situation, the total assets value is very sufficient for paying the principal of the bond.

5. Conclusion

The most important characteristic of classical model is the restriction of default time to the maturity of the debt, not taking into consideration the possibility of an early default, no matter what happens with the firm’s value before the maturity of the debt. If the firm’s value falls down to minimal level before the maturity of the debt, but it is able to recover and meet the debt’s payment at maturity, the default would be avoided in classical approach.

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