Approximation of the Schwinger–Dyson and the Bethe–Salpeter Equations and Chiral Symmetry of QCD

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Abstract

The Bethe–Salpeter equation for the pion in chiral symmetric models is studied with a special care to consistency with low-energy relations. We propose a reduction of the rainbow Schwinger–Dyson and the ladder Bethe–Salpeter equations with a dressed gluon propagator. We prove that the reduction preserves the Ward–Takahashi identity for the axial-vector current and the PCAC relation.

1 Introduction

The Bethe–Salpeter(BS) equation is a popular approach to describe the pion as a relativistic bound state of a quark and an anti-quark. As the pion is a pseudo Nambu–Goldstone(NG) boson, the results should satisfy the low-energy relations based on chiral symmetry. Therefore it is important to solve the Bethe–Salpeter equation in keeping the low-energy relations. The key idea is a consistency between the Schwinger–Dyson(SD) equation for a quark and the BS equation. If one solves the exact SD equation and the exact BS equation in QCD, the Ward–Takahashi identity for the axial-vector current implies that the NG solution of the BS amplitude is given by

$$\chi(q; P_B = 0) = \frac{1}{f_\pi} \{i\gamma_5 \frac{\lambda^a}{2}, S_F(q)\}$$

in the chiral limit, and that the exact relation

$$M^2_\pi f_\pi = -2m_0 \int_q \text{tr}[\chi(q; P_B)i\gamma_5 \frac{\lambda^a}{2}]$$

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holds for a finite quark mass $m_0$, where $S_F(q)$ is the quark propagator given as the solution of the SD equation and $\chi(q; P_B)$ is the solution of the BS equation for the pion,

$$S_F(q) := \int d^4(x - y)e^{iq(x-y)}\langle 0|T\psi(x)\overline{\psi}(y)|0\rangle,$$

(3)

$$\chi(q; P_B) := e^{iP_BX} \int d^4(x - y)e^{i\overline{q}(x-y)}\langle 0|T\psi(x)\overline{\psi}(y)|P\rangle,$$

(4)

$$\overline{\chi}(q; P_B) := e^{-iP_BX} \int d^4(y - x)e^{iq(y-x)}\langle P|T\psi(y)\overline{\psi}(x)|0\rangle.$$

(5)

$|P\rangle$ denotes a pion state with normalization condition $\langle P|P'\rangle = (2\pi)^32P_{B0}\delta^3(P - P')$ and $P_{B\mu} := (\sqrt{M_\pi^2 + P^2}, P)$ denotes the on-shell momentum. The pion decay constant $f_\pi$ is defined by

$$f_\pi := \lim_{P \to P_B} \frac{1}{P^2} \int_q \text{tr}[\overline{\chi}(q; P)i\gamma_5\frac{\lambda^a}{2}P].$$

(6)

From Eqs. (1) and (2), the Gell-Mann, Oakes and Renner (GMOR) mass formula,

$$M_\pi^2f_\pi^2 \simeq -2m_0\langle \overline{\psi}\psi \rangle_0, \quad \langle \overline{\psi}\psi \rangle_0 := -\int_q \text{tr}[S_F(q)_{m_0=0}]$$

(7)

is derived for a small quark mass $m_0 \simeq 0$. These relations are derived from the chiral symmetry of QCD and give an important starting point of the chiral perturbation theory.

In practice, the SD and BS equations can be solved only under some approximations. An effective model is often constructed and it is solved with further truncations. Such approximations may not always be consistent with chiral symmetry and the low-energy relations are often violated. It is known that if one uses the rainbow approximation for the SD equation and the ladder approximation for the BS equation, the consistency between the SD and BS equations is preserved and the low-energy relations (1) and (2) still hold. The consistency is also preserved after appropriate improvements of the rainbow–ladder approximation such as employing dressed gluon propagators [1, 2, 3, 4] in the interaction kernel.

For phenomenological studies, however, the ladder BS equation is rather involved numerically and therefore further approximations are often used. In this case the consistency between the SD and BS equations may be lost and there is no guarantee for the low-energy relations. In this paper, we propose an approximation taking only dominant terms in the SD and BS equations. We prove that the approximation preserves the low-energy relations (1) and (2), while it may change values of $M_\pi$, $f_\pi$ and $\langle \overline{\psi}\psi \rangle_0$. 

2
2 Approximation of the SD and BS Equations

In this paper we concentrate on the rainbow SD equation for the quark propagator,

$$S^{-1}_F(q) = S^{-1}_0(q) + iC_F \int_k g^2((q - k)^2)iD^{\mu\nu}(q - k)\gamma_\mu S_F(k)\gamma_\nu$$  \(8\)

and the ladder BS equation for the pion

$$S^{-1}_F(q + B)\chi(q; P_B)S^{-1}_F(q - B) = -iC_F \int_k g^2((q - k)^2)iD^{\mu\nu}(q - k)\gamma_\mu\chi(k; P_B)\gamma_\nu,$$  \(9\)

$$q_{\pm B} := q \pm \frac{P_B}{2},$$  \(10\)

where $S_0(q)$ is the bare quark propagator defined by

$$S_0(q) := \frac{i}{\not{q} - m_0}.$$  \(11\)

The normalization condition of the BS amplitude is given by

$$\lim_{P \to P_B} -\frac{P^\mu}{2P^2}i\int_q \chi_{m_{2n_2}}(q; P_B)\frac{\partial}{\partial P^\mu}(S^{-1}_{Fm_{2n_1}(q + B)}S^{-1}_{Fm_{1}m_{2}(q - B)})\chi_{n_1m_1}(q; P_B) = 1.$$  \(12\)

The above set of the SD and BS equations has been studied by many authors in Refs.\[1, 2, 3, 4, 7\] with successes in phenomenology. However, as far as we know, the full calculation of the ladder BS equation for finite $m_0$ is reported only in Ref.\[4\]. Because the BS equation requires a large computation, the studies often make approximations, such as neglecting some terms and/or truncating the Chebyshev polynomial expansion. But these approximations of the BS equation are often inconsistent with the SD equation, and therefore the low-energy relations are violated, although the numerical result in Ref.\[4\] shows that the violation may not be so large.

In general, the solution of the SD equation is written in terms of two scalar functions $A(q^2)$ and $B(q^2)$ as

$$S_F(q) = \frac{i}{A(q^2)\not{q} - B(q^2)}.$$  \(13\)
and the solution of the BS equation is written in terms of four scalar functions $\phi_S(q; P_B)$, $\phi_P(q; P_B)$, $\phi_Q(q; P_B)$ and $\phi_T(q; P_B)$ as

$$\chi_{nm}(q; P_B) = \delta_{ji} \frac{\lambda}{2} \left[ \left( \phi_S(q; P_B) + \phi_P(q; P_B) \frac{q}{P_B} + \phi_Q(q; P_B) \frac{P_B}{q} \right) + \frac{1}{2} \phi_T(q; P_B) (P_B q - q P_B) \gamma_5 \right].$$  \hspace{1cm} (14)

The following theorem addresses the consistency of the rainbow SD and the ladder BS equations with chiral symmetry:

**Theorem 1** If one solves the rainbow SD equation (8) and the ladder BS equation (9) with the normalization condition (12), the low-energy relations (1) and (2) hold.

This is known as mutual consistency between the SD and BS equations. If one uses a further approximation inconsistent between the SD and BS equations, the low-energy relations may be violated.

We here propose an approximation which is consistent with chiral symmetry:

**Theorem 2** If one makes the approximation $A(q^2) \equiv 1$ in the SD equation (8) and neglects the $\phi_P(q; P_B), \phi_Q(q; P_B), \phi_T(q; P_B)$ terms in RHS of the BS equation (9), the low-energy relations (1) and (2) still hold.

This approximation of the BS equation does not imply to neglect $\phi_P(q; P_B), \phi_Q(q; P_B), \phi_T(q; P_B)$ in LHS of Eq.(9) or in Eq.(12). Instead they are given by

$$\phi_P(q; P_B) = \frac{B(q_B) - B(q_{+B})}{-q^2 + P_B^2/4 + B(q_B) B(q_{+B})} \phi_S(q; P_B),$$ \hspace{1cm} (15)

$$\phi_Q(q; P_B) = \frac{B(q_B) + B(q_{+B})}{2(-q^2 + P_B^2/4 + B(q_B) B(q_{+B}))} \phi_S(q; P_B),$$ \hspace{1cm} (16)

$$\phi_T(q; P_B) = \frac{-1}{-q^2 + P_B^2/4 + B(q_B) B(q_{+B})} \phi_S(q; P_B).$$ \hspace{1cm} (17)

Before the proof of this theorem, we review how to derive the SD and BS equations. We use the Cornwall–Jackiw–Tomboulis(CJT) effective action formulation. In the rainbow–ladder approximation, the CJT action is given by

$$\Gamma[S_F] = i \text{Tr} \ln[S_F] - i \text{Tr}[S^{-1}_0 S_F] + \Gamma_{\text{loop}}[S_F],$$ \hspace{1cm} (18)

$$\Gamma_{\text{loop}}[S_F] = \frac{1}{2} \int d^4 x K^{m_1 m_2 n_1 n_2} \left( i \partial_{x_1} i \partial_{x_2} \partial_{y_1} \partial_{y_2} ight) S_{F_{m_2 n_1}}(x_2, y_1) S_{F_{n_2 m_1}}(y_2, x_1) S_F,$$ \hspace{1cm} (19)
where the symbol \( \ast \) means to take \( x_1, x_2, y_1, y_2 \rightarrow x \) after all the derivatives are operated. \( \mathcal{K} \) denotes the interaction kernel defined by

\[
\mathcal{K}^{m_1 m_2, n_1 n_2}(p_1, p_2; q_1, q_2) := g^2 \left( \frac{(p_1 + p_2 - q_1 - q_2)^2}{2} \right) 
\times iD^{\mu\nu} \left( \frac{p_1 + p_2}{2} - \frac{q_1 + q_2}{2} \right) (\gamma_\mu T^a)^{m_1 m_2} (\gamma_\nu T^a)^{n_1 n_2}.
\]

(20)

The SD equation (8) is the stability condition of the CJT action

\[
\frac{\delta \Gamma[S_F]}{\delta S_{Fmn}(x, y)} = 0,
\]

(21)

and the (homogeneous) BS equation (9) and its normalization condition (12) are derived from the inhomogeneous BS equation

\[
\frac{1}{i} \frac{\delta^2 \Gamma[S_F]}{\delta S_{Fmn}(x, y) \delta S_{Fm'n'}(y', x') G^{(2)}_{C;m'm'n'n'}(y'; x'' y'')} = \delta_{m'n'} \delta_{mn'} \delta(x'' - x) \delta(y - y'')
\]

(22)

where the repeated indices are summed or integrated and \( G^{(2)}_{C;m'm'n'n'}(y'; x'' y'') \) is the two-body connected Green function defined by

\[
G^{(2)}_{C;m'm'n'n'}(y; x', x'') := \langle 0 | T \psi_n(y) \overline{\psi}_m(x) \psi_{m'}(x') \overline{\psi}_{n'}(y') | 0 \rangle
- \langle 0 | T \psi(y) \overline{\psi}_m(x) | 0 \rangle \langle 0 | T \psi_{m'}(x') \overline{\psi}_{n'}(y') | 0 \rangle.
\]

(23)

Especially, the form of the (homogeneous) BS equation is given by

\[
\frac{\delta^2 \Gamma[S_F]}{\delta S_{Fmn}(x, y) \delta S_{Fm'n'}(y', x') \chi_{m'n'}(y'; x'; P_B)} = 0.
\]

(24)

The SD and BS equations inherit the chiral property of the CJT action Eq.(18). It is easy to show that if we apply an approximation that does not violate (global) chiral symmetry directly to the CJT action, then the SD and BS equations resulting from the approximated CJT action preserve chiral symmetry. It is, however, shown that non-trivial momentum dependences in the approximation may result in changing the axial-vector Noether current and thus in modifying the low-energy relations (1) and (2). In order to keep the low-energy relations in the original forms (1) and (2), the interaction part of the CJT action \( \Gamma_{\text{loop}}[S_F] \) must be invariant under the local axial infinitesimal transformation

\[
S_F(x, y) \rightarrow S_F'(x, y) := (1 + i \gamma_5 \frac{\lambda^\alpha}{2} \theta^\alpha(x)) S_F(x, y) (1 + i \gamma_5 \frac{\lambda^\alpha}{2} \theta^\alpha(y)).
\]

(25)
Indeed, it can be shown that the rainbow–ladder approximation (19) and (20) leaves $\Gamma_{\text{loop}}[S_F]$ invariant.

The invariance of $\Gamma_{\text{loop}}[S_F]$ under the transformation Eq. (23) leads us to an equation

$$G_{C;m''n''nm}(x'' y''; y x) = \frac{\delta (\Delta t \Gamma[S_F])}{\delta S_{Fnm}(y, x)} \left\{ \frac{\delta \Gamma[S_F]}{\delta S_{Fnp}(y', x')} \{i\gamma_5 \frac{\lambda^\alpha}{2} \theta^\alpha, S_F \}_{n'm'}(y', x') \} \right\}$$

$$= G_{C;m'n'n''nm}(x'' y''; y x) \frac{\delta^2 \Gamma[S_F]}{\delta S_{Fnm}(y, x) \delta S_{Fnp}(y', x')} \{i\gamma_5 \frac{\lambda^\alpha}{2} \theta^\alpha, S_F \}_{m'n'}(x', y')$$

$$\Rightarrow G_{C;m'n'n''nm}(x'' y''; y x) \frac{\delta^2 \Gamma[S_F]}{\delta S_{Fnm}(y, x) \delta S_{Fnp}(y', x')} \{i\gamma_5 \frac{\lambda^\alpha}{2} \theta^\alpha, S_F \}_{m'n'}(x', y') + (i\gamma_5 \frac{\lambda^\alpha}{2} \theta^\alpha(x'))_{m'l}G_{C;m'n'n''nm}(x'' y''; x' y') \}$$

$$= i\{i\gamma_5 \frac{\lambda^\alpha}{2} \theta^\alpha, S_F \}_{m'n''}(x'', y''),$$

(26)

where the SD equation (21) and the inhomogeneous BS equation (22) are used for the last equality. In the momentum space Eq. (26) becomes

$$\int_q G_{C;m'n'n''nm}(p, q; P) \{i\gamma_5 \frac{\lambda^\alpha}{2}(2m_0 + P) \}_{nm}$$

$$= i\{i\gamma_5 \frac{\lambda^\alpha}{2} S_F(p - P) \}_{m'n''} + i(S_F(p + P) i\gamma_5 \frac{\lambda^\alpha}{2})_{m'n''}.$$  

(27)

Here the Fourier transformation of $G_{C;m'n'n''nm}(x'y'; y x)$ is defined by

$$G_{C;m'n'n''nm}(x'y'; y x) = \int_{pqP} e^{-iP(x'-y') + q(y-x) + P(X'-X)} G_{C;m'n'n''nm}(p, q; P).$$

(28)

Note that LHS of Eq. (27) contains contributions only from the second term of the CJT action (18), because $\Gamma_{\text{loop}}[S_F]$ is invariant under the transformation Eq. (23). Then the low-energy relations (1) and (2) are derived from Eq. (27) and the spectral representation of $G_{C;m'n'n''nm}(p, q; P)$

$$G_{C;mnm'n'}(y x; x' y') = \int_p e^{-iP(x' - y') + i(P - P_B) X'} i \lambda_{nm}(y, x; P_B) \lambda_{m'n'}(x', y'; P_B) \frac{P^2 - M^2 + i\epsilon}{P^2 - M^2 + i\epsilon} + \cdots$$

with

$$X := \frac{x + y}{2}, \quad X' := \frac{x' + y'}{2}, \quad P = (P_0, P), \quad P_B = (\sqrt{M^2 + P^2}, P).$$

(30)

(See section 3.1 in Ref. (3).)
Now we prove the theorem 2. Consider a projection operator,

\[ \mathcal{P}_{ba,a'^{\prime}} := \frac{1}{4}[(1)_{ba}(1)_{a'^{\prime}} + (\gamma_5)_{ba}(\gamma_5)_{a'^{\prime}}] \]

which satisfies

\[ \mathcal{P}_{ba,a'^{\prime}}(S + \gamma_5 P + \gamma_{\mu} V + \gamma_{\mu} \gamma_5 A + \gamma_{\mu} \gamma_5 T)_{\nu a'^{\prime}} = (S + \gamma_5 P)_{ba}. \]

Using this operator, we modify the interaction kernel as

\[ \mathcal{K}^{m_1 m_2, n_1 n_2}(p_1, p_2; q_1, q_2) \rightarrow \mathcal{K}^{m_1 m_2, n_1 n_2}_P(p_1, p_2; q_1, q_2) \]

\[ := \delta_{i_1 i_1'} \delta_{j_1 j_1'} \delta_{j_2 j_2'} \delta_{j_3 j_3'} \delta_{k_1 k_1'} \delta_{g_1 g_1'} \mathcal{P}_{b_1 a_1} \mathcal{P}_{b_2 a_2} \mathcal{K}^{m'_1 m'_2, n'_1 n'_2} \]

in Eq.\((33)\). This modified kernel \(\mathcal{K}_P\) has the properties

\[ \mathcal{K}^{m_1 m_2, n_1 n_2}_P(p, p'; q, q') (i \frac{\lambda_5}{2})_{ln'} = (i \frac{\lambda_5}{2})_{nl} \mathcal{K}^{m_1 m_2, n_1 n_2}_P(p, p'; q, q'), \]

\[ \mathcal{K}^{m_1 m_2, n_1 n_2}_P(p, p'; q, q') (i \gamma_5 \frac{\lambda_5}{2})_{ln'} = - (i \gamma_5 \frac{\lambda_5}{2})_{nl} \mathcal{K}^{m_1 m_2, n_1 n_2}_P(p, p'; q, q'), \]

\[ \mathcal{K}^{m_1 m_2, n_1 n_2}_P(p, p'; q, q') = \mathcal{K}^{m_1 m_2, n_1 n_2}_P(p + p'; q + q'). \]

These properties imply that the interaction term, \(\Gamma_{\text{loop}}[S_F]\), is invariant under not only the global but also the local chiral transformation Eq.\((24)\). Therefore this modification preserves the low-energy relations \((1)\) and \((2)\).\(^7\)

To complete the proof, we show that this modification of the interaction kernel corresponds to the approximation stated in the theorem 2. Under the approximation \((33)\), the SD equation \((8)\) is modified to

\[ S^{-1}_F(q) = S^{-1}_0(q) + i \frac{C_F}{4} \int_k g^2((q - k)^2) D^{\mu \nu}(q - k) \text{tr}^{(D)}[S_F(k)] \gamma_\mu \gamma_\nu, \]

where \(\text{tr}^{(D)}\) denotes the trace in the Dirac space. As the second term of RHS in Eq.\((37)\) contains only the scalar component, \(A(q^2)\) is identically 1. Thus Eq.\((37)\) is equivalent to Eq.\((8)\) with the condition \(A(q^2) \equiv 1\). Similarly, the BS equation \((4)\) is modified to

\[ S^{-1}_F(q_{+B}) \chi(q; P_B) S^{-1}_F(q_{-B}) = - i \frac{C_F}{4} \int_k g^2((q - k)^2) D^{\mu \nu}(q - k) \text{tr}^{(D)}[\chi(k; P_B) \gamma_5] \gamma_\mu \gamma_5 \gamma_\nu \]

in which RHS contains only the pseudo-scalar component. This corresponds to neglecting \(\phi_P(k; P), \phi_Q(k; P)\) and \(\phi_T(k; P)\) in RHS of Eq.\((9)\). The proof is ended. \(\blacksquare\)
Theorem 2 gives a consistent way of reducing the number of degrees of the SD and BS equations and therefore enables us to save computer time. Because the deviation of $A(q^2)$ from unity is usually small and also $\phi_S(q; P_B)$ is the main term of the pion BS amplitude, this approximation will be useful in phenomenology. As this theorem guarantees that the low-energy relations (1) and (2) are satisfied, properties of chiral symmetry and its spontaneous breaking may be studied in this approximation. It is, however, noted that this approximation is regarded as modification of the model interaction as seen in Eqs.(32) and (33). As the projection (32) truncates the interaction kernel to the scalar and pseudo-scalar components, the resulting model looks very similar to the (non-localized) Nambu–Jona-Lasinio model. This indicates that the approximation may be valid for the pion, pseudo-scalar meson, but it may give large modification to the vector mesons, for instance. Thus we should consider that the approximation gives a different model, while it gives good approximation to the pseudo-scalar channel.

Further we notice the following theorem, which may sometimes be useful:

**Theorem 3** If one uses the approximation that neglects only the $\phi_T(q; P_B)$ term in RHS of the BS equation (9), the low-energy relations (1) and (2) are satisfied.

This can be proved similarly by using the projection operator

$$P'_{ba, a'd'} := \frac{1}{4} [(1)_{ba}(1)_{a'd'} + (\gamma_5)_{ba}(\gamma_5)_{a'd'} + (\gamma_\mu)_{ba}(\gamma_\mu)_{a'd'} + (\gamma_5 \gamma_\mu)_{ba}(\gamma_5 \gamma_\mu)_{a'd'}].$$

(39)

### 3 Summary and Conclusions

In this paper, we have proposed a new approximation scheme of the SD and BS equations. In general, approximations violate the low-energy properties originated from chiral symmetry. To avoid this, one must include many independent degrees in equations, although most of them are not interesting from the physical view-point and their contributions are not so large. We have proved that the low-energy relations are preserved when one takes the scalar term of the SD equation and the pseudo-scalar term of the BS equation. This approximation is consistent with the picture that the dynamical mass generation of the quark is described mainly by the scalar part in the SD equation, while the bound state structure of the pion BS (amputated)
amplitude is composed mainly of pseudo-scalar term accordingly. In Ref.[10, 11, 12, 13], the GMOR relation and its improvement were studied, where the BS equation is approximated by taking only the pseudo-scalar term although the SD solution contains both the vector and the scalar parts. This corresponds to using the projection operator

\[ P_{ba,a'b'} := \frac{1}{4} \left[ (\gamma_5)_{ba} (\gamma_5)_{a'b'} \right] \]  

instead of Eq.(31) and violates not only the local but also the global chiral invariance of \( \Gamma_{\text{loop}}[S_F] \). As a result the low-energy relations (1) and (2) may not be satisfied.

If they also approximate the SD equation by imposing \( A(q^2) \equiv 1 \), then chiral symmetry may be recovered. We have also proposed another approximation in which the tensor term of the BS equation is neglected. It is worth mentioning that the tables I and II in Ref.[4] confirms our theorem numerically.

So far we have concentrated on the effective interaction of type (20), which is based on the exchange of a dressed gluon. But it is not so difficult to apply the same approach to other types of the effective interaction. In Ref.[3], we studied an effective model in which the axial-vector current is modified. The application to such a model is obvious, for example. We hope that our approximation scheme helps analyses of the chiral effective theories of QCD.

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