KINKS IN TIME AND THEIR RELATION TO CONFINEMENT

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The time dependent formation of an electric flux tube (fundamental string) is reviewed. The main tool used for analysis is the Spacelike brane, which is a kink in time of the rolling tachyon. Both the S-brane and rolling tachyon are attempts to extend the D-brane concept to time dependent backgrounds. While S-branes are similar to Euclidean counterparts of the more familiar timelike D-branes, S-branes can smoothly change their worldvolume signature from spacelike to timelike which we interpret as the formation of a topological defect.

Here we review the results of Ref. 1 (see also references therein), in particular the derivation of actions and construction of solutions of Spacelike branes which are useful in understanding time dependent systems such as the rolling tachyon.

Recent proposals to understand the dynamics of our universe’s cosmic acceleration include searching for naturally unstable and hence time dependent backgrounds corresponding to extensions of the fruitful D-brane concept. The motivation is that the study of BPS branes has been one of the notable recent successes, providing a driving force in string theory and proving immensely useful in the study of supersymmetric and static systems. To develop our tools and intuition in time dependent backgrounds we are led to consider non-supersymmetric branes.

One such unstable background involves brane and anti-brane pairs, and their closely related non-BPS branes. Put close together, a brane and anti-brane pair will naturally interact and can annihilate, and this process can be understood as tachyon condensation. An unstable $p$-brane carries a scalar field $T$ called the tachyon field governed by a tachyon potential which is

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approximately of the form $V(T) \approx e^{-T^2}$. The tachyon field parametrizes the instability in the system. When the tachyon value is near zero, the brane system is unstable and will be driven towards the bottom of the tachyon potential at large tachyon values.

We begin our study of the dynamical evolution of the tachyon condensation process by examining the rolling tachyon solution, which is a time dependent and spatially homogenous tachyon solution

$$T = T(t = \text{time}) .$$

(1)

The tachyon values can start off near zero and then accelerate towards large tachyon values in the same way a ball rolls down a hill. At the top of its trajectory the tachyon field has value $T = 0$. This $p$ dimensional Euclidean hypersurface corresponding to the moment in time when the tachyon scalar field is everywhere zero we call a Spacelike $(p-1)$-brane. As it stands this Euclidean hypersurface does not carry much dynamics as it appears and disappears in an instant leaving no apparent trace of its existence. Despite its spacelike trajectory, it clearly does not violate causality nor does it transport energy superluminally.

To extend the S-brane lifetime we deform the S-brane worldvolume by introducing fluctuations into the rolling tachyon profile. The deformed S-brane now lives for a finite period of time and is not homogenous and as finely tuned. If we make an infinite deformation of the S-brane, it will live for an infinite length of time as shown in Figure 1.

The key point is that large fluctuations do naturally occur and they correspond to the time dependent formation of topological defects. S-branes can be thought of as initial conditions for the formation of defects.

To clarify why S-branes play this role in defect formation, we recall that for static tachyon kink configurations the zeros of the tachyon field

$$T = 0$$

(2)

are interpreted as the location of lower dimensional solitons or topological defects. In the case of string theory these solitons include branes and anti-branes. This is why in analogy, we previously identified the $T = 0$ region of a rolling tachyon or kink in time to be the location of the Spacelike brane.

The time evolution of defect formation can be followed and traces out a spacetime trajectory which is the S-brane worldvolume. In other words because both S-branes and normal defects are specified by the condition $T = 0$, slow moving and long lived Spacelike branes should be able to represent ordinary and stable timelike topological defects! (See Figure 2.)
small tachyon fluctuations make the S-brane live for a finite time

large tachyon fluctuations mean the S-brane is (infinitely ?) long lived

Figure 1. A flat S-brane can be deformed and live for more than a moment in time. Time increases as we go up the page and a horizontal plane is a moment in time.

late time remnants

T=0 Sbranes have appeared

Generic Initial Conditions

Figure 2. S-branes worldvolumes (dashed lines) allow us to follow the time evolution of initial conditions during the formation of topological defects.

To quantify this statement we have derived an effective action for Space-like branes by performing a zero mode fluctuation analysis on non-BPS branes. The S-brane action is

\[ S = \int_{\text{world volume}} \sqrt{\det(\delta_{ij} - \partial_i t \partial_j t + F_{ij})}. \]  

(3)

This action passes the following tests (Ref. 1 discusses further checks)
1) there is a scalar field, $t$, with wrong sign kinetic energy corresponding to fluctuations in time
2) gauge fields with field strength $F_{ij}$ exist on the worldvolume just like for D-branes
3) the action is real for flat Euclidean hypersurfaces of equal time
4) yet infinitely long lived solutions which smoothly change from Euclidean to Lorentzian signature are also possible. This property is based on the fact that the S-brane action is by definition defined to be spacelike relative to the open string metric whose light cone always lies within the closed string light cone, $G_{\mu\nu}^{\text{open}} \subseteq G_{\mu\nu}^{\text{closed}}$. There are therefore long lived solutions which are consistently both spacelike relative to the open string metric and also timelike relative to the closed string metric as illustrated in Figure 3.

![Figure 3. S-branes can simultaneously be spacelike relative to the open string metric and timelike relative to the closed string metric. The narrower the open string light cone becomes, the slower the S-brane is allowed to travel.](image)

5) solutions exist describing the formation of an electric flux tube

Although static descriptions of fundamental strings have been discussed, this S-brane solution describes the time dependent formation of a confined electric flux tube (see also Ref. 6). In gauge theories the bundling of gauge fields into small regions is the process of confinement, so this solution indicates that confinement can be understood as a non-perturbative but classical dynamical process. As an example, the electric S3-brane is cylindrical in shape, $\mathbb{R}^1 \times S^2$, where $r$ is the radius of the cylinder, $\chi$ goes lengthwise along the cylinder and $t$ is time. In particular, this electric S3-brane solution is

$$r = \frac{c}{t}, \quad E = 1 \tag{4}$$

where the electric field, $F_{0\chi} = E$, is constant and the critical value. The
radius of this cylinder shrinks to zero at late times while the electric field along the cylinder is constant, so this solution describes an electric flux tube which is confining into a string-like object. Using the Dirac quantization condition we found that the energy is that of a fundamental string

$$H = \frac{1}{2\pi\alpha'} \int d\chi \left( \int d^p x D \right) = \frac{1}{2\pi\alpha'} \int d\chi .$$  

(5)

Further, the S-brane coupling to RR form fields, $A$, goes to zero

$$\int_{world\ volume} A \times (t^{-1}) \overset{t \to \infty}{\longrightarrow} 0 .$$  

(6)

so the D-brane charge vanishes. The time dependent factor of $t^{-1}$ is due to the fact that the worldvolume of the S-brane shrinks to zero.

Spacelike branes are extensions of the D-brane concept to time dependent backgrounds and are useful in understanding the formation of defects. From the S-brane perspective, confinement can appear as a classical self organizing process and might lead to a hint as to the physics at the Hagedorn temperature. Spacelike branes should play a similar role in time dependent backgrounds of ordinary field theories.

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