Chiral Imprint of a Cosmic Gauge Field on Primordial Gravitational Waves

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A cosmological gauge field with isotropic stress-energy introduces parity violation into the behavior of gravitational waves. We show that a primordial spectrum of inflationary gravitational waves develops a preferred handedness, left- or right-circularly polarized, depending on the abundance and coupling of the gauge field during the radiation era. A modest abundance of the gauge field would induce parity-violating correlations of the cosmic microwave background temperature and polarization patterns that could be detected by current and future experiments.

Since the surprising discovery that parity is violated on the atomic scale by the weak nuclear force [1], searches for broken symmetries have proven to be a remarkably effective technique for uncovering new laws of physics. Here we speculate that parity violation on cosmic scales may be the sign of a dark gauge field. Our theoretical model consists of a non-Abelian (Yang-Mills) gauge field which, as we demonstrate, behaves like radiation through cosmic history, but fluctuations of the field couple to gravity in a way that distinguishes between left- and right-handed circularly polarized gravitational waves. In this paper we demonstrate the effect of this field on a primordial spectrum of gravitational waves and evaluate its impact on the cosmic microwave background (CMB).

There is an extensive literature on speculative new physics that leads to parity violation in cosmology, e.g. Refs. [2–4], and more specifically in the gravitational sector [5]. But an asymmetry is perfectly compatible with general relativity, without the need to invoke exotic interactions, as Stueckelberg first pointed out [6]. The physics of the weak interaction contains all the necessary elements.

We consider the standard cosmological model with the sole addition of a new gauge field as a toy model, with an action given by

$$ S = \int d^4x \sqrt{-g} \left( \frac{1}{2} M_p^2 R + \mathcal{L}_m - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right) $$

$$ F^{\mu\nu} = \partial_\mu A^I_\nu - \partial_\nu A^I_\mu - g_{YM} \epsilon^{IJK} A_{J\mu} A_{K\nu} $$

where Greek letters are used to represent space-time indices, lower case Latin letters are spatial indices, and upper case Latin letters $I \in \{1,2,3\}$ are reserved for the SU(2) indices. We assume a flavor-space locked configuration for the gauge field, wherein $A^I_\mu = \phi(\tau) \delta^I_\mu$, so that the directions of the internal group space are aligned with the principle spatial axes of the Robertson-Walker spacetime, $ds^2 = a^2(\tau)(-d\tau^2 + d\sigma^2)$. This approach yields an isotropic stress-energy tensor for the gauge field [7]. The scalar potential equation of motion is $\phi'' + 2g_{YM}^2 \phi^3 = 0$ where the prime indicates derivative with respect to conformal time, with a well-known solution in terms of the Jacobi elliptic sine-amplitude function [8], $g_{YM} \phi(\tau) = c_1 \sin(c_1(\tau - \tau_i)) + c_2 |\tau - \tau_i | - 1$. The constants are determined at the initial time $\tau_i$ as $c_1^2 = \frac{1}{3} g_{YM}^2 (\phi_i^2 + g_{YM}^2 \phi_i^4)$ and $c_2 = F(\theta) - 1$, an elliptic integral of the first kind, with $\csc \theta_i = 1 + \phi_i^2 / g_{YM}^2 \phi_i^4$. The homogeneous, isotropic energy density and pressure of this Yang-Mills fluid are $\rho_{YM} = 3 \rho_{YM} = 3(\phi^2 + g_{YM}^2 \phi^4) / 2\sigma^4 = 3c_1^2 / 2g_{YM}^2 \sigma^4$. The gauge field oscillates with period $\tau = \Gamma(a/2) / \sqrt{2\pi c_1}$, but its energy density and pressure scale with equa-
tion of state $w = 1/3$, like radiation. In order that
the field oscillate on a time scale to affect cosmologi-
cal physics, the coupling must be exponentially small,
$g_{YM} \sim \mathcal{O}(H_0/M_P) \sim 10^{-60}$. An origin for this small
coupling as well as the flavor-space locked configuration
could be ascribed to an inflationary epoch, although that
is beyond the scope of our present investigation. Infla-
tionary scenarios based on a similar gauge field, but re-
quiring higher order couplings to matter fields, have been
studied elsewhere [9, 10].

The parity violation is manifest in the coupling to grav-
titational waves. To explain, consider a gravitational wave
passing through the gauge field described above. The
wave will induce a quadrupolar distortion, alternately
squeezing and stretching the stress and energy of the
gauge field. However, the gauge field itself possesses a
preferred handedness via the right-handed SU(2) struc-
ture constants. Since the fields are flavor-space locked,
we perturb the metric and gauge field
$$R_{\mu\nu} = \frac{\phi'}{a^2} + \frac{2}{a^2 M_P^2} \left[ g_{YM} \phi^4 - \phi'^2 \right] h_{\mu\nu} -$$
$$- \frac{2}{a M_P^2} \left[ (k - g_{YM} \phi) g_{YM} \phi^2 t_R + \frac{a'}{a} \phi' t_R + \phi' t_R' \right],$$
$$t_R'' = \frac{2 a'}{a} t_R' + \left[ k^2 + \frac{a''}{a} - 2 k g_{YM} \phi \right] t_R = -$$
$$- \frac{2}{a M_P^2} \left[ (k + g_{YM} \phi) g_{YM} \phi^2 h_{R} - \phi' h_{R}' \right].$$ (2)

Figure 2. Top: Gravitational wave amplitude evolution as a
function of conformal time is shown for the case $g_{YM} = 0,$
$R_{YM} = 0.03$ (blue) and wavenumber $k = 10 h$/Mpc, as com-
pared to the standard case $R_{YM} = 0$ (green). The excitations
of the gauge field are shown (red) as an offset minus a constant
times $(a t_A)^2$, to illustrate their complementary behavior. The
solid (black) lines show the results of WKB solutions for the
envelopes of the oscillatory waveforms. Bottom: The case
$g_{YM} = 10^{-60}, R_{YM} = 0.03$ is shown. The waveforms capped
with solid lines are right-handed; those with dashed lines are
left-handed.

left- and right-handed gravitational waves. These initial
conditions, and the assumption that the initial tensor
fluctuations of the gauge field vanish deep in the radiation
era, allows the YM fluid to behave like radiation at early
times. The effects of the gauge field on the subsequent
evolution of the gravitational waves are illustrated in the
figures below.

The evolution of the gravitational wave amplitude is
shown for a variety of cases in Fig. 2. We begin by exam-
ining the behavior in the case $g_{YM} = 0$, corresponding to
color electrodynamics. The background solution has $\phi'$
constant, so the $h_A$ evolution equation (where the sub-
script “A” is for ambidextrous, since there is no parity
violation in this case) has a tachyonic mass that is respon-
sible for the growth of long wavelength modes. One can
show analytically that $ah_A$ doubles for modes outside the
horizon, relative to the standard case. In the top panel of
Fig. 2, the amplitude of $(ah_A)^2$ (blue) is $2^2$ times the
amplitude of $(ah)^2$ (green) going in to the first oscillation.
For modes that enter the horizon, there is a slow change
of amplitude between $h_A$ and $t_A$. A WKB analysis for
sub-horizon modes shows that $h_A^2 + t_A^2 \propto 1/a^2$ and the
exchange is oscillatory with phase $a_0 k \int d\tau'/(a(\tau'))$ where
$$k_A^2 = \sqrt{2 R_{YM} \Omega_{rad} \rho_0 H_0}$$ [14]. The gravitational wave
spectral density $\Omega_{GW}$ is shown in Fig. 3, where the ampli-
fication and periodic modulation are clearly seen. The
WKB solution predicts a peak or dip in the spectral den-
sity every five orders of magnitude in $k$ for $R_{YM} = 0.03$
(an energy density comparable to $\Delta N_e \simeq 0.2$.). Modes

$R_{YM} = 0$ (green). The excitations
of the gauge field are shown (red) as an offset minus a constant
times $(a t_A)^2$, to illustrate their complementary behavior. The
solid (black) lines show the results of WKB solutions for the
envelopes of the oscillatory waveforms. Bottom: The case
$g_{YM} = 10^{-60}, R_{YM} = 0.03$ is shown. The waveforms capped
with solid lines are right-handed; those with dashed lines are
left-handed.

$$h_R'' + \frac{2 a'}{a} h_R' + \left[ k^2 + \frac{2}{a^2 M_P^2} \left( g_{YM} \phi^4 - \phi'^2 \right) \right] h_R = -$$
$$- \frac{2}{a M_P^2} \left[ (k - g_{YM} \phi) g_{YM} \phi^2 t_R + \frac{a'}{a} \phi' t_R + \phi' t_R' \right],$$
$$t_R'' + \frac{2 a'}{a} t_R' + \left[ k^2 + \frac{a''}{a} - 2 k g_{YM} \phi \right] t_R = -$$
$$- \frac{2}{a M_P^2} \left[ (k + g_{YM} \phi) g_{YM} \phi^2 h_{R} - \phi' h_{R}' \right].$$ (2)
that spend very little time outside the horizon after inflation experience very little growth, leading to a decay in the amplitude of modulation. Since we begin our numerical calculation at $a/a_0 \sim 10^{-16}$, much later than expected in a typical inflationary scenario, this effect can be seen as the artificial decay of the spectral density at high frequency in the figure. Note that the influence of the gauge field on the spectrum is much larger than the effect of photon and neutrino free-streaming, or brief departures from a pure radiation background when particle species become non-relativistic.

The YM fluid case is shown in the second set of panels of Figs. 2, 3. The effective (squared) mass term for the gauge field tensor perturbations, $-2k g_{YM} \phi$, is tachyonic for right-handed modes. The growth (suppression) of $t_R (t_L)$ is transferred to $h_R (h_L)$ as the mode enters the horizon. Once the relative amplitude is locked in at horizon entry and the fields begin to oscillate rapidly, the slow exchange of amplitude between $h_R/L$ and $t_R/L$ again comes into play. A WKB analysis for sub-horizon modes again shows that $k^2 h_R/L + t_R/L \propto 1/a^2$ and the exchange is oscillatory with similar phase if the background field is not yet oscillatory.

To evaluate the impact of this scenario on the CMB, we have implemented the scalar and tensor perturbations of the gauge field into CAMB [16]. The gauge field has the biggest impact on tensor correlations. The scalar sector also receives corrections due to the gauge field, in the form of an anisotropic scalar shear, but the impact on the CMB scalar spectrum is small. We ignore the vector perturbations which may be shown to decay [14]. For these calculations $R_{YM}$ is the ratio between the YM fluid density and the total relativistic energy density. We assume the fraction of critical density in the relativistic fluid is fixed by slightly adjusting the number of neutrino degrees of freedom upon introducing the gauge field. We otherwise assume standard ΛCDM parameters.

The CMB polarization can be decomposed into gradient $E$-modes and curl $B$-modes. In the tensor sector the gauge field introduces two main effects. First, the left- and right-handed contributions to the BB spectrum now differ, as shown in Fig. 4. Hence, the temperature and polarization anisotropy due to gravitational waves on roughly degree scales is dominated by a superposition of right-circularly polarized gravitational waves, which imprint left-helical patterns, similar to the display in Fig. 1. It is curious to see that the individual contributions deviate strongly from ΛCDM but conspire in a way that puts the combination of both close to the expected standard cosmology result. Second, because temperature $T$ and gradient polarization $E$ are both parity even but curl polarization is parity odd, the parity violation introduced by the YM fluid allows for correlations between $TB$ and $EB$ [3]. Detecting these exotic cross correlations is a smoking gun for chiral effects in the universe. Typical
predictions of our model are plotted in Fig. 5.

There are many challenges to detecting the parity-violating cross correlations, not to mention the $B$-mode signal. Galactic foregrounds, magnetic fields, weak lensing, and other systematic effects can all produce a false positive; fortunately there is no fundamental barrier that would prevent a detection that can distinguish a primordial signal. (See Ref. [17] for a recent summary.) But there are other phenomena that could produce a parity-violating signal. First of all, cosmological birefringence (CB) can lead to $TB$ and $EB$ power spectra by rotating $E$ into $B$ through a novel coupling between electromagnetism and a cosmic pseudoscalar such as quintessence [18]. A second possibility, broadly characterized as chiral gravity, posits a modification of gravity whereby an asymmetry between left- and right-circularly polarized waves is imprinted on the primordial spectrum. The third possibility, as we have shown, is essentially cosmic circular dichroism, whereby the asymmetry develops with time from an initially symmetric primordial spectrum.

Would an actual detection directly point to chiral symmetry breaking on cosmological scales? In Ref. [19] it was shown that the $TB$ and $EB$ spectra can be used to distinguish CB effects from chiral physics. As CB rotates the $E$ into a $B$ contribution the measured $B$ spectrum would resemble the $E$ one which makes this separation into $CB$ and chirality effects feasible. In turn, putting limits on the amplitude of these spectra will put constraints on chiral physics in general and our model in particular. In what follows we compute the constraints that current and future CMB experiments would put on the parameters of our model under the assumption that $TB$ and $EB$ cross-correlations are measured.

The deviations seen in the CMB spectra would clearly have an impact on the interpretation of a precision measurement of $B$ modes [20]. As can be seen from Fig. 4, the gauge field can vary the height of the $BB$ spectrum at the reionization bump near $\ell \lesssim 10$ and at the primary acoustic peak near $\ell \sim 100$ by as much as $\pm 50\%$. However, we have the greatest leverage on new physics by focusing on the exotic cross correlations. Hence, we forecast the parameter constraints $\sigma_{\mathrm{RYM}}$ and $\sigma_{\mathrm{gYM}}$ using Fisher matrix techniques, for which the Fisher matrix reads

$$
F_{ij} = \sum_l \sum_{X,Y} \frac{\partial C_{X}^{i} }{ \partial \theta_{i} } \frac{\partial C_{Y}^{j} }{ \partial \theta_{j} } \left[ \Xi^{-1} \right]_{XY} \tag{3}
$$

where $\vec{\theta} = (R_{\mathrm{YM}}, g_{\mathrm{YM}})$ and $X, Y = \{TB, EB\}$. The Fisher matrix $F$ is the inverse of the covariance matrix between $R_{\mathrm{YM}}$ and $g_{\mathrm{YM}}$. The derivatives of the $C_{l}^{X}$ are obtained using CAMB. These derivatives are centered around a fiducial model: we choose $g_{\mathrm{YM}} = 10^{-56}$, $R_{\mathrm{YM}} = 0.1$ and the standard Planck $\Lambda$CDM values for the cosmological parameters [21]. The matrix $\left[ \Xi^{-1} \right]_{XY}$ is the inverse of the $TB$-$EB$ covariance matrix given by $\Xi_{X_{1}X_{2}X_{3}X_{4}} = \left( C_{X_{1}X_{2}}^{X_{3}X_{4}} + C_{X_{1}X_{4}}^{X_{2}X_{3}} \right)/(2l+1)$ where $C_{X_{1}X_{2}}^{X_{3}X_{4}} = \frac{\ell_{1}}{\ell_{1}+1} \left( W_{\ell_{1}X_{1}X_{2}} |W_{\ell_{1}}|^{-2} \right)$ and $X = \{T, E, B\}$ [19]. The instrumental parameters enter the window function $W_{\ell}^{X} \simeq \exp \left(-l^{2} \sigma_{X}^{2}/2 \right)$ due to the beam width, and the instrumental noise $w_{XX}$, where
$w_{TT}^{-1} \equiv 4\pi \sigma_T^2/N_{\text{pix}}$ and $w_{EE}^{-1} = w_{BB}^{-1} \equiv 4\pi \sigma_P^2/N_{\text{pix}}$ with the cross correlation contributions vanishing as the noise in the polarization is assumed to have no correlation to the noise in the temperature. In the window function $\sigma_b \equiv \theta_{\text{FWHM}}/\sqrt{8}\ln 2$ where the beam width is measured in radians. Similarly, the number of pixels is $N_{\text{pix}} = 4\pi \theta_{\text{FWHM}}^2$ and $\sigma_T$ and $\sigma_P$ are the temperature and polarization pixel noise. These are given by $\sigma_T^2 = (\text{NET})^2 N_{\text{pix}}/t_{\text{obs}}$ and $\sigma_P = \sqrt{2}\sigma_T$ with NET being the noise-equivalent temperature and $t_{\text{obs}}$ being the observation time. The parameters are taken from Ref. [19, 22] and summarized in Tab. I.

| Instrument | $\theta_{\text{FWHM}}$ [arcmin] | NET [µK$\sqrt{\text{s}}$] | $t_{\text{obs}}$ [y] |
|-----------|---------------------------------|---------------------------|----------------------|
| Planck    | 7.1                             | 45                        | 2                    |
| CV limited| 5                               | 0                         | 1.2                  |

Table I. Instrumental parameters for the two experiments considered in this paper. The parameters are the beamwidth $\theta_{\text{FWHM}}$, noise-equivalent temperature NET, and observation time $t_{\text{obs}}$.

The 1D marginalized confidence limits in a scenario in which the Planck satellite measures $TB$ and $EB$ cross correlations are $\sigma_{gYM} = 9.5 \times 10^{-57}$ and $\sigma_{RYM} = 0.030$. For the cosmic variance (CV) limited experiment these numbers reduce to $\sigma_{gYM} = 3.4 \times 10^{-57}$ and $\sigma_{RYM} = 8.1 \times 10^{-3}$, which would be able to make a detection of $gYM$ feasible. The 1- and 2-$\sigma$ contours are plotted in Fig. 6. For this fiducial model Planck could make a 2-$\sigma$ detection of $RYM$, but cannot exclude the ambidextrous case since $gYM = 0$ lies within its 1-$\sigma$ contour. The future looks brighter for a future satellite mission that gets closer to a CV limited experiment, which could put constraints on the chiral asymmetry; for the fiducial model, the coupling $gYM$ could be distinguished from zero at better than the two sigma level. Such a limit could be used to determine whether the gauge field is part of a dark sector that includes dark energy. If dark energy couples to the rolling gauge field, or if the gauge field is dark energy, as in a gauge-flation scenario, then the rate of cosmic acceleration may be linked to the chiral asymmetry.

Concluding, we present a simple model that breaks parity on cosmological scales. We illustrate the impact of this model on the gravitational wave spectrum and the CMB, computing the power spectra along with new $TB$ and $EB$ correlations that emerge in parity-breaking models. A detection of one of these correlations could be the sign of a flavor-space locked gauge field.

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