Speck of Chaos

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It has been shown that a local perturbation applied to a single site of the one-dimensional XXZ model is enough to bring this interacting integrable spin-1/2 system to the chaotic regime. Here, we show that this is not unique to this model, but happens also to the Ising model in a transverse field and to the spin-1 Lai-Sutherland chain. The larger the system is, the smaller the amplitude of the local perturbation for the onset of chaos. We use these models to show the advantages of the correlation hole as a dynamical indicator of chaos that does not require unfolding the spectrum or separating it by symmetries. We compare it with an indicator of the eigenstate thermalization hypothesis, the Gaussian distribution of off-diagonal elements of local observables.

The term quantum chaos, as used in this work, refers to properties of the spectrum and eigenstates that are similar to those found in full random matrices, such as strongly correlated eigenvalues [1–3] and eigenstates close to random vectors [4–9]. Level statistics as in random matrices are found also in some integrable models, but they are caused by finite-size effects [10, 11] or change abruptly upon tiny variations of the Hamiltonian parameters [12, 13]. Less consensual definitions of quantum chaos include the short-time exponential growth of the so-called out-of-time ordered correlator [14–19], but the same behavior appears also near critical points of integrable models [20–23], and diffusive transport [24–26], although ballistic transport has been observed also in the chaotic XXZ model with a single defect [27].

The one-dimensional (1D) clean spin-1/2 XXZ model is an interacting integrable system, whose transport behavior has been extensively studied [26, 28]. In 2004, it was shown that the model becomes chaotic if a single site has a Zeeman splitting different from that on the other sites [29] (see also [30, 31]). At first, it was thought that the transport behavior of this single-defect XXZ model was diffusive [32], but it was later found to be ballistic [27]. In spite of that, the model shows all the expected properties of chaotic many-body quantum systems. As the system size increases, level repulsion and chaotic eigenstates emerge for smaller defect amplitudes [33]. Local observables satisfy the diagonal eigenstate thermalization hypothesis (ETH) [33, 34], and the system’s long-time dynamics manifest spectral correlations [35].

This year has seen a resurgence of interest in the single-defect XXZ model. It has been employed in studies of many-body quantum chaos [36, 37], thermalization [38], and quantum transport [39]. In the present work, we show that the onset of chaos due to the local onsite perturbation is not unique to the XXZ model. This is illustrated for the spin-1/2 Ising model in a transverse field and the spin-1 Lai-Sutherland chain. The former is among the simplest quantum models that exhibit a critical point. The latter, which has SU(3) symmetry, has been investigated in the context of Haldane gapped materials [40, 41] and quantum transport [42].

What is the best way to detect quantum chaos? The analysis of level statistics is the most common approach when one has direct access to the spectrum, as in nuclear physics [2]. It requires the separation of the eigenvalues by symmetry sectors and, depending on the chaos indicator, also the unfolding of the spectrum. But other methods have been put forward as well, such as the structure of the eigenstates [4–9] and nearly maximal entanglement entropy [43–45]. In Ref. [46] (see also [39, 45, 47]), the distinction between integrable and chaotic models is done with the analysis of the distribution of the off-diagonal matrix elements of local observables in each subspace. In Ref. [37], a new chaos indicator based on the rate of deformations of the eigenstates under small perturbations bypasses the need to unfold the spectrum and to separate it by symmetries.

Identifying all symmetries of a model is not always trivial, so having a way to detect chaos despite their presence is important in studies of both chaotic and integrable models. We show that the distribution of the off-diagonal elements of local observables diagnoses chaos also when the energy levels are not separated by subspaces. However, eigenvalues, eigenstates, and matrix elements of observables are not easily accessible to experiments that focus on time evolutions, such as those with cold atoms and ion traps. Here, we promote the use of the correlation hole [35, 48–61], which is a dynamical manifestation of chaos. This indicator does not require unfolding the spectrum or separating it by symmetries [61]. We discuss how the time scale for the onset of the correlation hole in the three single-defect models – XXZ, Ising, and Lai-Sutherland chains – depends on the defect amplitude and on the system size.

Models. – The Hamiltonians for the spin-1/2 XXZ model, spin-1/2 Ising model in a transverse field, and spin-1 Lai-Sutherland model [62–64] in the presence of a single defect of amplitude $d$ in the middle of the chain are respectively given by

$$H_{XXZ} = d J S^z_{L/2} + J \sum_{k=1}^{L-1} \left( S^x_k S^x_{k+1} + S^y_k S^y_{k+1} + \Delta S^z_k S^z_{k+1} \right),$$

(1)
\[
H_{ZZ} = d J S_{L/2}^z + J h x \sum_{k=1}^{L} S_k^z - J \sum_{k=1}^{L-1} S_k^z S_{k+1}^z, \tag{2}
\]

\[
H_{S1} = d J S_{L/2}^z + J \sum_{k=1}^{L-1} [S_k^z S_{k+1}^z + (S_k^x S_{k+1}^x + S_k^y S_{k+1}^y)^2]. \tag{3}
\]

Above, \( \hbar = 1 \), \( L \) is the number of sites, \( S_k^{x,y,z} \) are spin operators acting on site \( k \), \( J \) is the coupling constant that sets the energy scale, \( \Delta \) is the anisotropy of the XXZ model, and \( h_x \) is the amplitude of the transverse field in the Ising model. Note that, contrary to the case of spin \( 1/2 \), the quadratic term in Eq. (3) is necessary to guarantee integrability.

Open boundary conditions are considered to avoid translational symmetry. To avoid parity and spin reversal, we add to \( H \) (1) and \( H \) (2) small impurities at the edges of the chain, \( \epsilon_{1, L} J S_1^z \), where \( \epsilon_{1, L} \) are random numbers in \([-0.1, 0.1]\]. In the case of the spin-1 model, we add \( \epsilon_1 J S_1^z \), which connects symmetry sectors, where the total magnetization in the \( z \)-direction differs by 1. While for the spin-1/2 models, the onset of chaos requires placing the defect \( d \) out of the borders [29], for the spin-1 model, chaos emerges when the defect \( d \) is on any site, including the edges.

The parameters used are \( J = 1, \Delta = 0.48 \), and \( h_x = 0.84 \). The XXZ model conserves total spin in the \( z \)-direction, so we study the largest subspace of dimension \( D_{XXZ} = L!/(L/2)^2 \Leftrightarrow \) for the other two models: \( D_{ZZ} = 2^L \) and \( D_{S1} = 3^L \).

Level Statistics. The most used signature of quantum chaos is the distribution of spacings between nearest unfolded energy levels [65]. For chaotic systems with real and symmetric Hamiltonian matrices, as the full random matrices from the Gaussian orthogonal ensemble (GOE), the level spacing distribution follows the Wigner surmise [2, 66], \( P_{\text{WVD}}(s) = (\pi s/2)^{\frac{1}{2}} \exp(-\pi s^2/4) \), which indicates that the eigenvalues are highly correlated and repel each other. In integrable models, where the energy levels are uncorrelated and not prohibited from crossing, the level spacing distribution is usually Poissonian, \( P_{\text{P}}(s) = e^{-s} \), but exceptions include “picketfence”-kind of spectra [67–69] and systems with an excessive number of degeneracies [70].

The crossover from integrability to chaos can be studied by an indicator that quantifies how close the level spacing distribution is to \( P_{\text{WVD}}(s) \). An example is the value of \( \beta \) obtained by fitting \( P(s) \) with the Brody distribution [71] (see also [72]),

\[
P_{\beta}(s) = (\beta + 1)bs^\beta \exp(-bs^{\beta+1}), \quad b = \left[ \Gamma \left( \frac{\beta + 2}{\beta + 1} \right) \right]^{\frac{1}{\beta+1}} \tag{4}
\]

Chaotic systems give \( \beta \sim 1 \), while the Poissonian distribution leads to \( \beta \sim 0 \).

In Figs. 1 (a) and (b), we show \( \beta \) as a function of the defect amplitude for the Ising (a) and the Lai-Sutherland model (b). (An equivalent figure for the single-defect XXZ model is in Ref. [33].) One sees that the range of values of \( d \) for which \( \beta \sim 1 \) increases with system size, indicating that in the thermodynamic limit, a single defect of infinitesimal amplitude suffices for the onset of chaos.

The level spacing distribution and the ratio of consecutive levels [73, 74] detect short-range correlations only. A more complete analysis of level statistics calls for the study of long-range correlations as well, as measured, for example, with the level number variance [2]. We verified that the level number variance for the three single-defect models with \( d \sim J \) approaches the GOE result as \( L \) increases (not shown).

An advantage of the ratio of consecutive levels over the level spacing distribution and the level number variance is that the ratio does not require unfolding the spectrum. However, a prerequisite for all three quantities is the separation of the eigenvalues by symmetry sectors. If we mix eigenvalues from different subspaces, Poissonian statistics may emerge even when the system is chaotic [75].

Eigenstate thermalization hypothesis. Indicators of ETH based on observables can also be used to detect quantum chaos without spectrum unfolding. In chaotic systems, the infinite-time averages of local observables approach thermodynamic averages as the system size increases. This is referred to as the diagonal ETH and has been verified for the single-defect XXZ model in [33, 34] and recently in [38]. In chaotic systems, the distribution of the off-diagonal matrix elements of local operators is Gaussian [45, 46], which is called the off-diagonal ETH and has been confirmed for the single-defect XXZ model as well [39].

In Figs. 1 (c) and (d), we study the distribution of the off-
diagonal elements of the operator that breaks the integrability of the Ising (c) and the Lai-Sutherland (d) model, that is, the distribution of $\langle \psi_\beta | S^z_{L/2} | \psi_\beta \rangle$, where $| \psi_\alpha, \beta \rangle$ are the eigenstates of the Hamiltonians (2) and (3). To verify whether the distribution is Gaussian and therefore complies with ETH, we compute the ratio [45],

$$R(\omega) = \frac{\langle \psi_\alpha | S^z_{L/2} | \psi_\beta \rangle^2}{\langle \psi_\alpha | S^z_{L/2} | \psi_\beta \rangle^2},$$  \hspace{1cm} (5)

where the bar indicates the average over the off-diagonal elements for which the energy difference $\omega = | E_\beta - E_\alpha |$ lies in one of the bins of width $d\omega = 0.05$. For a Gaussian distribution [76], $R(\omega) = \pi/2$.

Figures 1 (c) and (d) show that the range of values of $\omega$ for which $R(\omega) \sim \pi/2$ increases as the system size grows. Thus, in the thermodynamic limit, $\langle \psi_\alpha | S^z_{L/2} | \psi_\beta \rangle$ should be normally distributed for any value of $\omega$. This picture does not hold for the single-defect models in the integrable limit, where distributions other than Gaussian are found.

Correlation Hole.– The results above substantiate that the single-defect models are chaotic. But which dynamical manifestation of chaos do they exhibit and how does it depend on the defect amplitude and system size? To answer these questions, we study the survival probability,

$$\langle S_p(t) \rangle = \langle \langle |\Psi(0)\rangle |\Psi(t)\rangle^2 \rangle = \sum_{\alpha, \beta = 1}^D |C^0_\alpha|^2 |C^0_\beta|^2 e^{-i(E_\alpha - E_\beta)t} \rangle,$$ \hspace{1cm} (6)

where $C^0_\alpha = \langle \psi_\alpha | \Psi(0) \rangle$ and $\langle . . \rangle$ indicates an average over initial states that have energy $E_0 = \langle \Psi(0) | H | \Psi(0) \rangle$ close to the middle of the spectrum [77]. The average is needed, because this quantity is not self-averaging at any time scale [78]. The initial states used are eigenstates of the $z$-terms in $H$ (1)-(3). The mean survival probability is related to the form factor, $K(t) = \sum_{\alpha, \beta = 1}^D (e^{-i(E_\alpha - E_\beta)t})$, but contrary to this one, the spectrum is not unfolded when analyzing $\langle S_p(t) \rangle$.

The initial decay of the survival probability is determined by the shape and bounds of the energy distribution of the initial state, which is independent of chaos [79–82]. The presence of correlated eigenvalues gets manifested later, when the dynamics resolve the discreteness of the spectrum and the mean survival probability develops a dip below its saturation point, known as correlation hole [35, 48–61]. The use of the correlation hole as an alternative to detect level repulsion was first proposed for molecules with poor line resolution [48]. The interval $t_m \leq t \leq t_H$, where the correlation hole is found, is limited by the point of its minimum, $t_m$, and by the longest time scale of the system, the so-called Heisenberg time, $t_H$, which is inversely proportional to the mean level spacing. When evolved under full random matrices, $t_m^{\text{GOE}} = (3/\pi)^{1/4} \left[59 \right]$

The onset of the correlation hole for the single-defect models is analyzed in Fig. 2. The mean survival probability for the XXZ model is shown in Figs. 2 (a) and (d), for the Ising model in Figs. 2 (b) and (e), and for the spin-1 model in Figs. 2 (c) and (f). Curves for the systems at strong chaos ($d = 0.8$) and for different systems sizes are displayed in Figs. 2 (a)-(c). They make it evident that $t_m$ grows exponentially with $L$ for the three cases. These long times are unrelated with the fact that the perturbation is local. They reflect instead the locality of the couplings, which causes the gradual and slow spread of the initial many-body states in the Hilbert space. The correlation hole also gets elongated as the system size increases, since the growth constant of the exponential behavior of $t_H$ with $L$ is larger than that for $t_m$. These features are all very similar to those observed in chaotic systems with global perturbations and local couplings [59].

In Figs. 2 (d)-(f), we fix the system size and examine how $t_m$ depends on the defect amplitude. As $d$ decreases from 0.8 toward the integrable point, the correlation hole gets postponed to later times for the XXZ and the Ising model. This is expected, since the approach to integrability reduces the correlations between the eigenvalues and the first ones to be eliminated, since the long-range correlations. It calls attention, however, that the spin-1 model does not show the same behavior. In this case, as $d$ decreases below 0.8, the correlation hole is not displaced and even its depth is hardly affected [Fig. 2 (f)]. This raises the suspicion that the border defect $\epsilon_1 S^z_1$ may suffice to bring the Lai-Sutherland chain to the chaotic regime even when $d = 0$. The reason why we did not notice this in Fig. 1 (b) may be an indication that not all symmetries of this model were identified.

Symmetries.– Poissonian level statistics may emerge in chaotic systems if eigenvalues from different subspaces are mixed. This contrasts with the correlation hole, whose appearance requires only the presence of correlated eigenvalues, not their separation by symmetry sectors. To illustrate this aspect, we show in Fig. 3 (a) the mean survival probability for a spin-1/2 model that is known to be chaotic. It has couplings between nearest- and next-nearest neighbors (NNN) and is-
This system conserves total magnetization in the $z$-direction, total spin, and it also exhibits parity and spin reversal.

In the inset of Fig. 3 (a), the level spacing distribution is obtained with the eigenvalues from the subspace with $\sum k S^z_k = 0$, but the other symmetries are disregarded, which results in a Poissonian distribution. The correlation hole, on the other hand, is evident in the main panel. The survival probability in this figure is averaged over initial states from the same subspace as in the inset.

Similar results are seen in Fig. 3 (b), which is for the spin-1 model with $d = 0$ and a very small border defect, $\varepsilon_1 = 0.05$. The inset shows a nearly Poissonian distribution, while the correlation hole is apparent in the main panel. This demonstrates that the correlation hole is a powerful tool in the identification of chaotic systems, but it should be useful also in the search for integrable models. Verifying whether a Hamiltonian remains integrable after small modifications, as done here, is a hard problem that is often avoided. In studies of integrability, the usual strategy is instead to build the Hamiltonian, using for example the quantum Yang-Baxter equations.

A natural question that arises from the discussions above is what happens to the off-diagonal ETH in the presence of symmetries [83]. It turns out that it can still detect chaos, but $R(\omega)$ is no longer $\pi/2$. Chaos is now revealed by the flatness of the curves for $R(\omega)$ at values close to integer multiples of $\pi/2$, as seen in Figs. 3 (c) for the NNN model and in Fig. 3 (d) for the spin-1 model. The specific value of $R(\omega)$ depends on the observable and the number of symmetry sectors. The following picture provides a simplified explanation.

Suppose that one has two subspaces, each with $N$ different chaotic eigenstates, and the operator associated with the symmetry sectors commutes with the observable $O$ used to compute $R(\omega)$. The distribution of the $[(N/2)(N/2 - 1)]/2$ values of $\langle \psi_\alpha O | \psi_\beta \rangle$ within each subspace is Gaussian, but by mixing the sectors, one also has $\langle (N/2)(N/2 - 1)]/2$ values $\langle \psi_\alpha O | \psi_\beta \rangle = 0$ for the cases where $| \psi_\alpha \rangle$ and $| \psi_\beta \rangle$ belong to different subspaces. As a result, $R(\omega) = \pi$, and the distribution of the off-diagonal elements is zero-inflated, similar to the ones seen in the insets of Figs. 3 (c) and (d). If instead of 2, one has $m$ subspaces, then $R(\omega) = m\pi/2$.

Conclusions.— In quantum systems with many interacting particles, integrable points are fragile, while chaos prevails. As shown here, a local perturbation applied to a single site of an integrable interacting many-body quantum system can be enough to bring it to the chaotic regime.

Our studies demonstrate that the correlation hole and the distribution of off-diagonal elements of local observables can both detect chaos in the presence of symmetries. These two methods combined may assist the identification of subspaces and the search for integrable models. As an indicator of chaos, the correlation hole has the advantage of being a dynamical tool.

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[1] Fritz Haake, *Quantum Signatures of Chaos* (Springer-Verlag, Berlin, 1991).
[2] T. Guhr, A. Mueller-Gröeling, and H. A. Weidenmüller, “Random matrix theories in quantum physics: Common concepts,” Phys. Rep. 299, 189 (1998).
[3] H.-J. Stöckmann, *Quantum Chaos: an introduction* (Cambridge University Press, Cambridge, UK, 2006).
[4] Boris V. Chirikov, “An example of chaotic eigenstates in a complex atom,” Phys. Lett. A 108, 68 – 70 (1985).
[5] G. Casati, B. V. Chirikov, I. Guarneri, and F. M. Izrailev.
“Band-random-matrix model for quantum localization in conservative systems,” Phys. Rev. E 48, R1613–R1616 (1993).

[6] V. V. Flambaum, A. A. Gribakina, G. F. Gribakin, and M. G. Kozlov, “Structure of compound states in the chaotic spectrum of the ce atom: Localization properties, matrix elements, and enhancement of weak perturbations,” Phys. Rev. A 50, 267–296 (1994).

[7] V. Zelevinsky, B. A. Brown, N. Frazier, and M. Horoi, “The nuclear shell model as a testing ground for many-body quantum chaos,” Phys. Rep. 276, 85–176 (1996).

[8] L. F. Santos, F. Borgonovi, and F. M. Izrailev, “Onset of chaos and relaxation in isolated systems of interacting spins: Energy shell approach,” Phys. Rev. E 85, 036209 (2012).

[9] F. Borgonovi, F. M. Izrailev, L. F. Santos, and V. G. Zelevinsky, “Quantum chaos and thermalization in isolated systems of interacting particles,” Phys. Rep. 626, 1 (2016).

[10] S. Sorathia, F. M. Izrailev, V. G. Zelevinsky, and G. L. Celardo, “From closed to open one-dimensional anderson model: Transport versus spectral statistics,” Phys. Rev. E 86, 011142 (2012).

[11] E. Jonathan Torres-Herrera, J. A. Méndez-Bermúdez, and Lea F. Santos, “Level repulsion and dynamics in the finite one-dimensional anderson model,” Phys. Rev. E 100, 022142 (2019).

[12] L. Benet, F. Leyvraz, and T. H. Seligman, “Wigner-dyson statistics for a class of integrable models,” Phys. Rev. E 68, 045201 (2003).

[13] A. Relaño, J. Dukelsky, J. M. G. Gómez, and J. Retamosa, “Stringent numerical test of the poisson distribution for finite quantum integrable hamiltonians,” Phys. Rev. E 70, 026208 (2004).

[14] Juan Maldacena and Douglas Stanford, “Remarks on the Sachdev-Ye-Kitaev model,” Phys. Rev. D 94, 106002 (2016).

[15] Efim B. Rozenbaum, Sriram Ganeshan, and Victor Galitski, “Lyapunov exponent and out-of-time-ordered correlator’s growth rate in a chaotic system,” Phys. Rev. Lett. 118, 086801 (2017).

[16] Koji Hashimoto, Keiju Murata, and Ryosuke Yoshii, “Out-of-time-order correlators in quantum mechanics,” J. High Energy Phys. 2017, 138 (2017).

[17] Ignacio García-Mata, Marcos Saraceno, Rodolfo A. Jalabert, Augusto J. Roncaglia, and Diego A. Wisniacki, “Chaos signatures in the short and long time behavior of the out-of-time ordered correlator,” Phys. Rev. Lett. 121, 210601 (2018).

[18] Rodolfo A. Jalabert, Ignacio García-Mata, and Diego A. Wisniacki, “Semiclassical theory of out-of-time-order correlators for low-dimensional classically chaotic systems,” Phys. Rev. E 98, 062218 (2018).

[19] Jorge Chávez-Carlos, B. López-del Carpio, Miguel A. Bastarrachea-Magnani, Pavel Strážek, Sergio Lermahernández, Lea F. Santos, and Jorge G. Hirsch, “Quantum and classical Lyapunov exponents in atom-field interaction systems,” Phys. Rev. Lett. 122, 024101 (2019).

[20] Silva Pappalardi, Angelo Russomanno, Bojan žunković, Fernando Iemini, Alessandro Silva, and Rosario Fazio, “Scrambling and entanglement spreading in long-range spin chains,” Phys. Rev. B 98, 134303 (2018).

[21] Quirin Hummel, Benjamin Geiger, Juan Diego Urbina, and Klaus Richter, “Reversible quantum information spreading in many-body systems near criticality,” Phys. Rev. Lett. 123, 160401 (2019).

[22] Saúl Platowsky-Cameo, Jorge Chávez-Carlos, Miguel A. Bastarrachea-Magnani, Pavel Strážek, Sergio Lermahernández, Lea F. Santos, and Jorge G. Hirsch, “Positive quantum Lyapunov exponents in experimental systems with a regular classical limit,” Phys. Rev. E 101, 010202(R) (2020).

[23] Tianrui Xu, Thomas Scaffidi, and Xiangyu Cao, “Does scrambling equal chaos?” Phys. Rev. Lett. 124, 140602 (2020).

[24] F. Jin, R. Steinigeweg, F. Heidrich-Meisner, K. Michielsen, and H. De Raedt, “Finite-temperature charge transport in the one-dimensional hubbard model,” Phys. Rev. B 92, 205103 (2015).

[25] Thomas F. Isola, Mauro Žnidarič, “Exact localized and ballistic eigenstates in disordered chaotic spin ladders and the fermi-hubbard model,” Phys. Rev. Lett. 123, 036403 (2019).

[26] B. Bertini, F. Heidrich-Meisner, C. Karrasch, T. Prosen, R. Steinigeweg, and M. Žnidarič, “Finite-temperature transport in one-dimensional quantum lattice models,” arXiv:2003.03334.

[27] Marlon Brenes, Eduardo Mascarenhas, Marcos Rigol, and John Goold, “High-temperature coherent transport in the XXZ chain in the presence of an impurity,” Phys. Rev. B 98, 235128 (2018).

[28] X. Zotos, “Finite temperature drude weight of the one-dimensional spin-1/2 heisenberg model,” Phys. Rev. Lett. 82, 1764–1767 (1999).

[29] L. F. Santos, “Integrability of a disordered Heisenberg spin-1/2 chain,” J. Phys. A 37, 4723–4729 (2004).

[30] Lea F. Santos and Aditi Mitra, “Domain wall dynamics in integrable and chaotic spin-1/2 chains,” Phys. Rev. E 84, 016206 (2011).

[31] A. Gubin and L. F. Santos, “Quantum chaos: An introduction via chains of interacting spins 1/2,” Am. J. Phys. 80, 246–251 (2012).

[32] O. S. Barišić, P. Presloviček, A. Metavitsiadis, and X. Zotos, “Incoherent transport induced by a single static impurity in a heisenberg chain,” Phys. Rev. B 80, 125118 (2009).

[33] E. J. Torres-Herrera and Lea F. Santos, “Local quenches with global effects in interacting quantum systems,” Phys. Rev. E 89, 062110 (2014).

[34] E. J. Torres-Herrera, D. Kollmar, and L. F. Santos, “Relaxation and thermalization of isolated many-body quantum systems,” Phys. Scr. T 165, 014018 (2015).

[35] E. J. Torres-Herrera and Lea F. Santos, “Dynamical manifestations of quantum chaos: correlation hole and bulge,” Philos. Trans. Royal Soc. A 375, 20160434 (2017).

[36] Ángel L. Corps and Armando Relaño, “Distribution of the ratio of consecutive level spacings for different symmetries and degrees of chaos,” Phys. Rev. E 101, 022222 (2020).

[37] Mohit Pandey, Pieter W. Claeyss, David K. Campbell, Anatoli Polkovnikov, and Dries Sels, “Adiabatic eigenstate deformations as a sensitive probe for quantum chaos,” arXiv:2004.05043.

[38] Marlon Brenes, Tyler LeBlond, John Goold, and Marcos Rigol, “Eigenstate thermalization in a locally perturbed integrable system,” arXiv:2004.04755.

[39] Marlon Brenes, John Goold, and Marcos Rigol, “Ballistic vs diffusive low-frequency scaling in the XXZ and a locally perturbed XXZ chain,” arXiv:2005.12309.

[40] M. T. Batchelor, Xi-Wen Guan, and Norman Oelkers, “Thermal and magnetic properties of spin-1 magnetic chain compounds with large single-ion and in-plane anisotropies,” Phys. Rev. B 70, 184408 (2004).

[41] M T Batchelor, X-W Guan, N Oelkers, and A Foerster, “Thermal and magnetic properties of integrable spin-1 and spin-2 chains with applications to real compounds,” J. Stat. Mech. 2004, P10017 (2017).

[42] Maxime Dupont and Joel E. Moore, “Universal spin dynamics in infinite-temperature one-dimensional quantum magnets,” Phys. Rev. B 101, 121106 (2020).

[43] Lev Vidmar, Lucas Hackl, Eugenio Bianchi, and Marcos Rigol,
“Entanglement entropy of eigenstates of quadratic fermionic hamiltonians,” Phys. Rev. Lett. 119, 020601 (2017).

Lev Vidmar and Marcos Rigol, “Entanglement entropy of eigenstates of quantum chaotic hamiltonians,” Phys. Rev. Lett. 119, 220603 (2017).

Tyler LeBlond, Krishnanand Mallayya, Lev Vidmar, and Marcos Rigol, “Entanglement and matrix elements of observables in interacting integrable systems,” Phys. Rev. E 100, 062134 (2019).

Wouter Beugeling, Roderich Moessner, and Masudul Haque, “Off-diagonal matrix elements of local operators in many-body quantum systems,” Phys. Rev. E 91, 012144 (2015).

Ivan M. Khaymovich, Masudul Haque, and Paul A. McClarty, “Eigentestate thermalization, random matrix theory, and behemoths,” Phys. Rev. Lett. 122, 070601 (2019).

Luc Leviandier, Maurice Lombardi, Rémi Jost, and Jean Paul Pique, “Fourier transform: A tool to measure statistical level properties in very complex spectra,” Phys. Rev. Lett. 56, 2449–2452 (1986).

J. P. Pique, Y. Chen, R. W. Field, and J. L. Kinsey, “Chaos and dynamics on 0.5 ~ 300 ps time scales in vibrationally excited acetylene: Fourier transform of stimulated-emission pumping spectrum,” Phys. Rev. Lett. 58, 475–478 (1987).

T. Guhr and H.A. Weidenmüller, “Correlations in anticrossing spectra and scattering theory: analytical aspects,” Chem. Phys. 146, 21 – 38 (1990).

Joshua Wilkie and Paul Brumer, “Time-dependent manifestations of quantum chaos,” Phys. Rev. Lett. 67, 1185–1188 (1991).

U. Hartmann, H.A. Weidenmüller, and T. Guhr, “Correlations in anticrossing spectra and scattering theory: Numerical simulations,” Chem. Phys. 150, 311 – 320 (1991).

A. Delon, R. Jost, and M. Lombardi, “NO2 jet cooled visible excitation spectrum: Vibronic chaos induced by the X2 A1-A2 B2 interaction,” J. Chem. Phys. 95, 5701–5718 (1991).

M. Lombardi and T. H. Seligman, “Universal and nonuniversal statistical properties of levels and intensities for chaotic Rydberg molecules,” Phys. Rev. A 47, 3571–3586 (1993).

Y. Alhassid and R. D. Levine, “Spectral autocorrelation function in the statistical theory of energy levels,” Phys. Rev. A 46, 4650–4653 (1992).

Laurent Michaille and Jean-Paul Pique, “Influence of experimental resolution on the spectral statistics used to show quantum chaos: The case of molecular vibrational chaos,” Phys. Rev. Lett. 82, 2083–2086 (1999).

F Leyvraz, A García, H Kohler, and T H Seligman, “Fidelity under isospectral perturbations: a random matrix study,” J. Phys. A 46, 275303 (2013).

E. J. Torres-Herrera, Antonio M. García-García, and Lea F. Santos, “Generic dynamical features of quenched interacting quantum systems: Survival probability, density imbalance, and out-of-time-ordered correlator,” Phys. Rev. B 97, 060303 (2018).

Mauro Schiulaz, E. Jonathan Torres-Herrera, and Lea F. Santos, “Thouless and relaxation time scales in many-body quantum systems,” Phys. Rev. B 99, 174313 (2019).

S. Lerma-Hernández, D. Villaseñor, M. A. Bastarrachea-Magnani, E. J. Torres-Herrera, L. F. Santos, and J. G. Hirsch, “Dynamical signatures of quantum chaos and relaxation time scales in a spin-boson system,” Phys. Rev. E 100, 012218 (2019).

Javier de la Cruz, Sergio Lerma-Hernandez, and Jorge G. Hirsch, “Quantum chaos in a system with high degree of symmetries,” ArXiv:2005.06589.

G. V. Uimin, “One-dimensional problem for s = 1 with modified antiferromagnetic hamiltonian,” JETP Lett. 12, 225 (1970).

C. K. Lai, “Lattice gas with nearest-neighbor interaction in one dimension with arbitrary statistics,” J. Math. Phys. 15, 1675–1676 (1974).

Bill Sutherland, “Model for a multicomponent quantum system,” Phys. Rev. B 12, 3795–3805 (1975).

The unfolding procedure refers to the rescaling of the energies, so that the local density of states of the renormalized eigenvalues is 1.

M. L. Mehta, Random Matrices (Academic Press, Boston, USA, 1991).

M. V. Berry and M. Tabor, “Level clustering in the regular spectrum,” Proc. R. Soc. Lond. A 356, 375 – 394 (1977).

Akhilesh Pandey and Ramakrishna Ramaswamy, “Level spacings for harmonic-oscillator systems,” Phys. Rev. A 43, 4237–4243 (1991).

B. V. Chirikov and D. L. Shepelyansky, “Shnirelman peak in level spacing statistics,” Phys. Rev. Lett. 74, 518–521 (1995).

Pablo R. Zangara, Axel D. Dente, E. J. Torres-Herrera, Horacio M. Pastawski, A. Iucci, and Lea F. Santos, “Time fluctuations in isolated quantum systems of interacting particles,” Phys. Rev. E 88, 032913 (2013).

T. A. Brody, J. Flores, J. B. French, P. A. Mello, A. Pandey, and S. S. M. Wong, “Random-matrix physics: spectrum and strength fluctuations,” Rev. Mod. Phys. 53, 385 (1981).

Felix M. Izrailev, “Simple models of quantum chaos: Spectrum and eigenfunctions,” Phys. Rep. 196, 299 – 392 (1990).

Vadim Oganesyan and David A. Huse, “Localisation of interacting fermions at high temperature,” Phys. Rev. B 75, 155111 (2007).

Y. Y. Atas, E. Bogomolny, O. Giraud, and G. Roux, “Distribution of the ratio of consecutive level spacings in random matrix ensembles,” Phys. Rev. Lett. 110, 084101 (2013).

L. F. Santos, “Transport and control in one-dimensional systems,” J. Math. Phys. 50, 095211 (2009).

R. C. Geary, “The ratio of the mean deviation to the standard deviation as a test of normality,” Biometrika 27, 310 (1935).

We also average the border defects over 20 random numbers to decrease the fluctuations after saturation and we do a running average.

Mauro Schiulaz, E. Jonathan Torres-Herrera, Francisco Pérez-Bernal, and Lea F. Santos, “Self-averaging in many-body quantum systems out of equilibrium: Chaotic systems,” Phys. Rev. B 101, 174312 (2020).

E. J. Torres-Herrera and Lea F. Santos, “Quench dynamics of isolated many-body quantum systems,” Phys. Rev. A 89, 043620 (2014).

E. J. Torres-Herrera, M Vyas, and Lea F. Santos, “General features of the relaxation dynamics of interacting quantum systems,” New J. Phys. 16, 063010 (2014).

Marco Távora, E. J. Torres-Herrera, and Lea F. Santos, “Inevitable power-law behavior of isolated many-body quantum systems and how it anticipates thermalization,” Phys. Rev. A 94, 041603 (2016).

Marco Távora, E. J. Torres-Herrera, and Lea F. Santos, “Power-law decay exponents: A dynamical criterion for predicting thermalization,” Phys. Rev. A 95, 013604 (2017).

For the diagonal ETH in the presence of symmetries, see [84].

L. F. Santos and M. Rigol, “Localization and the effects of symmetries in the thermalization properties of one-dimensional quantum systems,” Phys. Rev. E 82, 031130 (2010).

For the diagonal ETH in the presence of symmetries, see [84].

L. F. Santos and M. Rigol, “Localization and the effects of symmetries in the thermalization properties of one-dimensional quantum systems,” Phys. Rev. E 82, 031130 (2010).