Estimating Instance-dependent Label-noise Transition Matrix using DNNs

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Abstract

In label-noise learning, estimating the transition matrix is a hot topic as the matrix plays an important role in building statistically consistent classifiers. Traditionally, the transition from clean labels to noisy labels (i.e., clean label transition matrix) has been widely exploited to learn a clean label classifier by employing the noisy data. Motivated by that classifiers mostly output Bayes optimal labels for prediction, in this paper, we study to directly model the transition from Bayes optimal labels to noisy labels (i.e., Bayes label transition matrix) and learn a Bayes optimal label classifier. Note that given only noisy data, it is ill-posed to estimate either the clean label transition matrix or the Bayes label transition matrix. But favorably, Bayes optimal labels are less uncertain compared with the clean labels, i.e., the class posteriors of Bayes optimal labels are one-hot vectors while those of clean labels are not. This enables two advantages to estimate the Bayes label transition matrix, i.e., (a) we could theoretically recover a set of noisy data with Bayes optimal labels under mild conditions; (b) the feasible solution space is much smaller. By exploiting the advantages, we estimate the Bayes label transition matrix by employing a deep neural network in a parameterized way, leading to better generalization and superior classification performance.

1 Introduction

The study of classification in the presence of noisy labels has been of interest for three decades [1], but becomes more and more important in weakly supervised learning [37, 17, 46, 56, 50, 49]. The main reason behind this is that datasets are becoming bigger and bigger. To improve annotation efficiency, these large-scale datasets are often collected from crowdsourcing platforms [48], online queries [5], and image engines [19], which suffer from unavoidable label noise [51]. Recent research shows that the label noise significantly degenerates the performance of deep neural networks, since deep models easily memorize the noisy labels [56, 51].

Generally, the algorithms for combating noisy labels can be categorized into statistically inconsistent algorithms and statistically consistent algorithms. The statistically inconsistent algorithms are heuristic, such as selecting possible clean examples to train the classifier [2, 51, 55, 11, 26, 33, 14], re-weighting examples to reduce the effect of noisy labels [33, 21], correcting labels [25, 15, 56, 52], or adding regularization [10, 8, 59, 39, 19, 17, 41]. These approaches empirically work well, but there is no theoretical guarantee that the learned classifiers can converge to the optimal ones learned from clean data. To address this limitation, algorithms in the second category aim to design classifier-consistent algorithms [55, 59, 15, 21, 30, 34, 28, 7, 31, 37, 54, 22, 47, 44], where classifiers learned on noisy data will asymptotically converge to the optimal classifiers defined on the clean domain.
The label transition matrix \( T(x) \) plays an important role in building statistically consistent algorithms. Traditionally, the transition matrix \( T(x) \) is defined to relate clean distribution and noisy distribution (i.e., clean label transition matrix), where \( T(x) = P(\tilde{Y} | Y, X = x) \) and \( X \) denotes the random variable of instances/features, \( \tilde{Y} \) as the variable for the noisy label, and \( Y \) as the variable for the clean label. The clean label transition matrix is widely used to learn a clean label classifier by employing the noisy data. The learned clean label classifier is expected to predict a probability distribution over a set of pre-defined classes given an input, i.e. clean class posterior probability \( P(Y | X) \). The clean class posterior probability is the distribution from which clean labels are sampled. However, Bayes optimal labels \( Y^* \), i.e. the class labels that maximize the clean class posteriors \( Y^* | X := \arg \max_{Y} P(Y | X) \), are mostly used as the predicted labels and for computing classification accuracy. Motivated by this, in this paper, we propose to directly model the transition matrix \( T^*(x) \) that relates Bayes optimal distribution and noisy distribution, i.e., \( T^*(x) = P(\tilde{Y} | Y^*, X = x) \), where \( Y^* \) denotes the variable for Bayes optimal label. Thus a Bayes optimal label classifier can be learned by exploiting the Bayes label transition matrix directly.

Studying the transition between Bayes optimal distribution and noisy distribution is considered advantageous to that of studying the transition between clean distribution and noisy distribution. The main reason is due to that the class posteriors of Bayes optimal labels are one-hot vectors while those of clean labels are not. Two advantages can be introduced by this to better estimate the instance-dependent transition matrix: (a) We can collect a set of examples with theoretically guaranteed Bayes optimal labels out of noisy data. The intrinsic reason that Bayes optimal labels can be inferred from the noisy data while clean labels cannot is that Bayes optimal labels are the labels that maximize the clean class posteriors while clean labels are sampled from the clean class posteriors. In the presence of label noise, the labels that maximize the noisy class posteriors could be identical to those that maximize the clean class posteriors (Bayes optimal labels) under mild conditions \([5]\). Therefore some instances’ Bayes optimal labels can be inferred from their noisy class posteriors while their clean labels are impossible to infer since the clean class posteriors are unobservable, as shown in Figure \([1]\). (b) The feasible solution space of the Bayes label transition matrix is much smaller than that of the clean label transition matrix. This is because that Bayes optimal labels have less uncertainty compared with the clean labels. The transition matrix defined by Bayes optimal labels and the noisy labels is therefore sparse and can be estimated more efficiently with the same amount of training data.

These two advantages naturally motivate us to collect a set of examples with their theoretically guaranteed Bayes optimal labels out of the noisy data and train a deep neural network to approximate the Bayes label transition matrix. The collected examples, inferred Bayes optimal labels, and their noisy labels are served as data points to optimize the deep neural network. Compared with the previous transition matrix estimation methods \([31, 45, 43]\), which made assumptions and leveraged hand-crafted priors to approximate the transition matrices, we train a deep neural network to estimate the instance-dependent label transition matrix with a reduced feasible solution space, which achieves lower approximation error, better generalization, and superior classification performance.

The rest of this paper is organized as follows. In Section \(2\) we briefly review related works on noise models and learning with noisy labels. In Section \(3\) we introduce some background knowledge and the definition of Bayes label transition matrix. In Section \(4\) we explain how to estimate Bayes label transition matrices using deep neural networks. In Section \(5\) we provide empirical evaluations of our learning algorithm. In Section \(6\) we conclude our paper.

## 2 Related Work

**Noise model.** Currently, there are several typical label noise models. Specifically, the random classification noise (RCN) model assumes that clean labels flip randomly with a constant rate \([4, 27, 28]\). The class-conditional label noise (CCN) model assumes that the flip rate depends on the
latent clean class $[45]$. The instance-dependent label noise (IDN) model considers the most general case of label noise, where the flip rate depends on its instance/features $[6][43][61]$. Obviously, the IDN model is more realistic and applicable. For example, in real-world datasets, an instance whose feature contains less information or is of poor quality may be more prone to be labeled wrongly. The bounded instance dependent label noise (BIDN) $[6]$ is a reasonable extension of IDN, where the flip rates are dependent on instances but upper bounded by a value smaller than 1. However, with only noisy data, it is a non-trivial task to model such realistic noise without any assumption $[43]$. This paper focuses on the challenging BIDN problem setting.

Learning clean distributions. It is significant to reduce the side effect of noisy labels by inferring clean distributions statistically. The label transition matrix plays an important role in such an inference process, which is used to denote the probabilities that clean labels flip into noisy labels before. We first review prior efforts under the class-dependent condition $[31]$. By exploiting the class-dependent transition matrix $T$, the training loss on noisy data can be corrected. The transition matrix $T$ can be estimated in many ways, e.g., by introducing the anchor point assumption $[21]$, by exploiting clustering $[62]$, by minimizing volume of $T$ $[18]$, and by using extra clean data $[13][35]$. To make the estimation more accurately, a slack variable $[45]$ or a multiplicative dual $T$ $[52]$ can be introduced to revise the transition matrix. As for the efforts on the instance-dependent transition matrix, existing methods rely on various assumptions, e.g., the noise rate is bounded $[6]$, the noise only depends on the parts of the instance $[43]$, and additional valuable information is available $[3]$. Although above advanced methods achieve superior performance empirically, the introduction of strong assumptions limit their applications in practice. In this paper, we propose to infer Bayes optimal distribution instead of clean distribution, as Bayes optimal distribution is less uncertain and easy to be inferred under mild conditions.

Other approaches. Other methods exist with more sophisticated training frameworks or pipelines, including but not limited to robust loss functions $[59][47][22]$, sample selection $[11][40][23]$, label correction $[36][58][60]$, (implicit) regularization $[42][57][20]$, and semi-supervised learning $[16][29]$.

3 Preliminaries

We introduce the problem setting, some important definitions, and the formulation of the proposed Bayes label transition matrix in this section.

Problem setting. This paper focuses on a classification task given a training dataset with Instance Dependent Noise (IDN), which is denoted by $\hat{S} = \{(x_i, \hat{y}_i)\}_{i=1}^n$. We consider that training examples $(x_i, \hat{y}_i)$ are drawn according to random variables $(X, \hat{Y}) \sim \mathcal{D}$, where $\mathcal{D}$ is a noisy distribution. The noise rate for class $y$ is defined as $\rho_y(x) = P(\hat{Y} = y \mid Y \neq y, x)$. This paper focuses on a reasonable IDN setting that the noise rates have upper bounds $\rho_{\max}$ as in $[6]$, i.e., $\forall (x) \in \mathcal{X}$, $0 \leq \rho_y(x) \leq \rho_{\max} < 1$. Note that the problem in $[6]$ is defined on binary classification task while we extend the problem setting to multi-class classification. Our aim is to learn a robust classifier only from the noisy data, which could assign clean labels for test data.

Clean distribution. For the observed noisy training examples, all of them have corresponding clean labels, which are unobservable. The clean training examples are denoted by $S = \{(x_i, y_i)\}_{i=1}^n$, which are considered to be drawn according to random variables $(X, Y) \sim \mathcal{D}$. The term $\mathcal{D}$ denotes the underlying clean distribution.

Bayes optimal distribution. Given instance $X$, its Bayes optimal label is denoted by $Y^*, Y^* \mid X := \arg \max_{Y} P(Y \mid X), (X, Y) \sim \mathcal{D}$. The distribution of $(X, Y^*)$ is denoted by $\mathcal{D}^*$. Note the Bayes optimal distribution $\mathcal{D}^*$ is different from the clean distribution $\mathcal{D}$ when $P(Y \mid X) \notin \{0, 1\}$. Like clean labels, Bayes optimal labels are unobservable due to the information encoded between features and labels is corrupted by label noise $[61]$. Note that it is a non-trivial task to infer $\mathcal{D}^*$ only with the noisy training dataset $\hat{S}$. Also, the noisy label $\hat{y}$, clean label $y$, and Bayes optimal label $y^*$, for the same instance $x$ may disagree with each other $[6]$.

Other definitions. The classifier is defined as $f : \mathcal{X} \rightarrow \mathcal{Y}$, where $\mathcal{X}$ and $\mathcal{Y}$ denote the instance and label spaces respectively. Let $\mathbb{1}[:]$ be the indicator function. Define the 0-1 risk of $f$ as $R(f(X), Y) \triangleq \mathbb{1}[f(X) \neq Y]$. Define the Bayes optimal classifier $f^*$ as $f^* \triangleq \arg \min_{f} R(f(X), Y^*)$. Note that there is NP-hardness of minimizing the 0-1 risk, which is neither convex nor smooth $[2]$. We can use the softmax cross entropy loss as the surrogate loss function to approximately learn the Bayes optimal
classifier \[^2\] [6]. We aim to learn a classifier \(f\) from the noisy distribution \(\hat{D}\) which also approximately minimizes \(\mathbb{E}[1(f(X), Y^*)]\).

**Bayes label transition matrix.** Traditional instance-dependent label transition matrix encodes the probabilities that clean labels flip into noisy labels given input instances. However, due to the reasons that clean labels have more uncertainty, estimating the clean label transition matrix is relatively harder. In this paper, we focus on studying the transition from *Bayes optimal labels to noisy labels*. We define the Bayes label transition matrix that bridges the Bayes optimal distribution and noisy distribution as follows,

\[
T_{i,j}^*(X) = P(\tilde{Y} = j \mid Y^* = i, X),
\]

where \(T_{i,j}^*(X)\) denotes the \((i, j)\)-th element of the matrix \(T^*(X)\), indicating the probability of a Bayes optimal label \(i\) flipped to noisy label \(j\) for input \(X\). Given the noisy class posterior probability \(P(\tilde{Y} = 1 \mid X = x), \ldots, P(\tilde{Y} = C \mid X = x)\) (which can be learned from noisy data) and the Bayes label transition matrix \(T_{i,j}^*(x) = P(\tilde{Y} = j|Y^* = i, X = x)\), the Bayes class posterior probability \(P(Y^*|X = x)\) can be inferred, i.e., \(P(Y^* \mid X = x) = (T^*(X = x)^\top)^{-1} P(\tilde{Y} \mid X = x)\).

### 4 Method

The feasible solution space of the Bayes label transition matrix is much smaller since Bayes optimal labels are deterministic. Therefore, we propose to leverage deep neural networks to estimate a Bayes label transition matrix for each input instance. We firstly collect a distilled dataset with theoretically guaranteed Bayes optimal labels out of the noisy dataset (Section 4.1). Then, we can train a deep neural network (Bayes label transition network) on the collected distilled dataset to learn the transition between Bayes optimal labels and noisy labels (Section 4.2). The learned Bayes label transition network is then fixed and used to train a classification network on the noisy dataset in a F-Correction \[^31\] fashion (Section 4.3).

#### 4.1 Collecting Distilled Examples

In this subsection, we show how to construct a distilled dataset consists of distilled examples. We formally introduce the concept of distilled examples first and then present how to collect distilled examples automatically. The collected distilled examples can be used for training the Bayes label transition network.

**Definition 1 (Distilled examples \[^6\]).** An example \((x, \tilde{y}, y^*)\) is defined to be a distilled example if \(y^*\) is identical to the one assigned by the Bayes optimal classifier under the clean data, i.e., \(y = f^*_D(x)\).

The distilled examples can be collected out of noisy examples automatically with the following guarantee,

**Theorem 1 (\[^6\]).** Denote by \(\tilde{\eta}_y(x)\) the conditional probability \(P_D(\tilde{Y} = y|X = x)\) and \(\eta_y(x) = P_D(Y = y|X = x)\). \(\forall (x) \in X\), we have

\[
\tilde{\eta}_y(x) > \frac{1 + \rho_{\text{max}}}{2} \implies \eta_y(x) > \frac{1}{2} \implies (x, \tilde{y}, Y^* = y) \text{ is distilled};
\]

where \(\rho_{\text{max}}\) is the noise rate upper bound. Theorem 1 can be proved in a similar way as did in \[^6\] (Theorem 2 therein). Note the original theorem in \[^6\] was built on binary-classification task, we extend it to the multi-class classification problem, the extension is straightforward.

The \(\rho_{\text{max}}\) is manually set as 0.3 in all experiments. We analyze the effect of the choice of \(\rho_{\text{max}}\) in the following experiment sections and found the proposed method is not much sensitive to the choice of the \(\rho_{\text{max}}\).

According to Theorem 1, we can obtain distilled examples by collecting all noisy examples \((x, \tilde{y})\) whose \(x\) satisfies \(\tilde{\eta}_y(x) > \frac{1 + \rho_{\text{max}}}{2}\) and then assigning the label \(y\) to it as its Bayes optimal label \(y^*\). After that, we obtain an instance of distilled example \((x^{\text{distilled}}, \tilde{y}, y^*)\), the \(\tilde{y}\) indicates the noisy label while the \(y^*\) indicates the inferred Bayes optimal label. The \(y^*\) may disagree with the
With the trained Bayes label transition network, we can get which causes label noise. A recent study [43] empirically verified that the patterns that cause label noise are commonly shared. Our empirical experiments further show that the network \( T^*(x; \theta) \) generalizes well to unseen examples and thus helps achieve superior classification performance.

### 4.2 Estimating Bayes label Transition Matrices using Distilled Examples

As discussed in Section 4.1, we can collect a set of distilled examples \((x^\text{distilled}, \hat{y}, y^*)\) from noisy data. Now we proceed to train a Bayes label transition network parameterized by \( \theta \) to estimate the instance-dependent label-noise (IDN) transition matrices, which model the probability of observing a noisy label \( \hat{y} \) given input image \( x \) and its Bayes optimal label \( y^* \):

\[
T^*_{i,j}(x^\text{distilled}, \theta) = P(\hat{Y} = j | Y^* = i, x^\text{distilled}, \theta),
\]

(3)

Specifically, the Bayes label transition network takes \( x^\text{distilled} \) as input and output a Bayes label transition matrix \( T^*(x^\text{distilled}; \theta) \in \mathbb{R}^{C \times C} \), where \( C \) is the number of classes. We can use the collected Bayes labels \( y^* \) and the estimated Bayes label transition matrix \( T^*(x^\text{distilled}; \theta) \) to infer the noisy labels. The following empirical risk on the inferred noisy labels and the ground-truth noisy labels are minimized to learn the network’s parameter \( \theta \):

\[
\hat{R}_1(\theta) = -\frac{1}{n} \sum_{i=1}^{n} \tilde{y}_i \log(y^*_i \cdot T^*(x_i^\text{distilled}; \theta)),
\]

(4)

where \( \tilde{y}_i \) and \( y^*_i \) are \( \hat{y}_i \) and \( y^*_i \) in the form of one-hot vectors, \( \tilde{y}_i \in \mathbb{R}^{1 \times C} \) and \( y^*_i \in \mathbb{R}^{1 \times C} \), respectively.

### 4.3 Training Classification Network with Bayes Label Transition Matrices

Our goal is to train a classification network \( f(\cdot | w) \) that predicts Bayes class posterior probability \( P(Y^* = i | x; w) \) parameterized by \( w \). In the training stage, we cannot observe the Bayes optimal label \( Y^* \). Instead, we only have access to noisy label \( \hat{Y} \). The probability of observing a noisy label \( \hat{Y} \) given input image \( x \) is:

\[
P(\hat{Y} = j | x; w, \theta) = \sum_{i=1}^{k} P(\hat{Y} = j | Y^* = i, x; \theta) P(Y^* = i | x; w),
\]

(5)

With the trained Bayes label transition network, we can get \( T^*_{i,j}(x; \theta) = P(\hat{Y} = j | Y^* = i, x; \theta) \) for each input \( x \). We exploit F-Correction [31], which is a typical classifier-consistent algorithm, to train the classification network. To be specific, fix the learned Bayes label transition network parameter \( \theta \), we minimize the empirical risk as follows to optimize the classification network parameter \( w \):

\[
\hat{R}_2(w) = -\frac{1}{n} \sum_{i=1}^{n} \tilde{y}_i \log(f(x_i; w) \cdot T^*(x_i; \theta)),
\]

(6)

where \( f(x_i; w) \in \mathbb{R}^{1 \times C} \). The F-Correction has been proved to be a classifier-consistent algorithm, the minimizer of \( \hat{R}(w) \) under the noisy distribution is the same as the minimizer of the original cross-entropy loss under the Bayes optimal distribution [31]. If the transition matrix \( T^* \) is estimated unbiased. However, the Bayes label transition network is trained on a biased set, i.e., the set of distilled examples. The network will generalize to the non-distilled examples if they share the same pattern with the distilled examples which causes label noise. A recent study [43] empirically verified that the patterns that cause label noise are commonly shared. Our empirical experiments further show that the network \( T^*(x; \theta) \) generalizes well to unseen examples and thus helps achieve superior classification performance.
Algorithm 1 Bounded Instance-dependent Label Noise Generation.

Require: Clean examples \(\{(x_i, y_i)\}_{i=1}^n\);
Require: Noise rate \(\eta\);
Require: Noise rate upper bound \(\rho_{\text{max}}\);

1: Sample instance flip rates \(q_i\) from the truncated normal distribution \(N(\eta, 0.1^2, [0, \rho_{\text{max}}])\); \(//\text{mean } \eta, \text{ variance } 0.1^2, \text{ range } [0, \rho_{\text{max}}]\)

2: Independently sample \(w_1, w_2, \ldots, w_c\) from the standard normal distribution \(N(0, 1^2)\);

3: for \(i = 1, 2, \ldots, n\) do
   4: \(p = x_i \times w_{y_i}\); \(//\text{generate instance-dependent flip rates}\)
   5: \(p_{y_i} = -\infty\); \(//\text{only consider entries that are different from the true label}\)
   6: \(p = q_i \times \text{softmax}(p)\); \(//\text{make the sum of the off-diagonal entries of the } y_i\)-th row to be \(q_i\)
   7: \(p_{y_i} = 1 - q_i\); \(//\text{set the diagonal entry to be } 1-q_i\)
   8: Randomly choose a label from the label space according to the possibilities \(p\) as noisy label \(\tilde{y}_i\);
9: end for
10: return Noisy samples \(\{(x_i, \tilde{y}_i)\}_{i=1}^n\)

5 Experiments

In this section, we first introduce the experiment setup (Section 5.1) including the datasets used (Section 5.1.1), the compared methods (Section 5.1.2), and the implementation details (Section 5.1.3). Next, we conduct ablation studies in Section 5.2. Finally, we present and analyze the experimental results on synthetic and real-world noisy datasets to show the effectiveness of the proposed method (Section 5.3).

5.1 Experiment setup

The datasets we used, the baseline methods we compared with, and the implementation details are introduced in this section.

5.1.1 Datasets

We conduct the experiment on four datasets to verify the effectiveness of our proposed method, where three of them are manually corrupted, i.e., F-MNIST, CIFAR-10, and SVHN, one of them is real-world noisy datasets, i.e., Clothing1M. F-MNIST has 28 \(\times\) 28 grayscale images of 10 classes including 60,000 training images and 10,000 test images. CIFAR-10 dataset contains 50,000 color images from 10 classes for training and 10,000 color images from 10 classes for testing both with shape of 32 \(\times\) 32 \(\times\) 3. SVHN has 10 classes of images with 73,257 training images and 26,032 test images. We manually corrupt the three datasets, i.e., F-MNIST, CIFAR-10 and SVHN with bounded instance-dependent label noise according to Algorithm 1, which is modified from [43]. The noise rate upper bound \(\rho_{\text{max}}\) in Algorithm 1 is set as 0.6 for all experiments. All experiments on those datasets with synthetic instance-dependent label noise are repeated five times to guarantee reliability. The Clothing1M has 1M images with real-world noisy labels for training and 10k images with the clean label for testing. 10% of the noisy training examples of all datasets are left out as a noisy validation set for model selection.

5.1.2 Comparison Methods

We compare the proposed method with several state-of-the-art approaches: (1) CE, which trains the classification network with the standard cross-entropy loss on noise datasets. (2) GCE [59], which unites the mean absolute error loss and the cross-entropy loss to combat noisy labels. (3) APL [24], which combines two mutually reinforcing robust loss functions, we employ its combination of NCE and RCE for comparison. (4) Decoupling [26], which trains two networks on samples whose predictions from two networks are different. (5) MentorNet [14], Co-teaching [11], and Co-teaching+ [53] mainly handle noisy labels by training networks on instances with small loss values. (6) Joint [36], which jointly optimizes the network parameters and the sample labels. The hyperparameters \(\alpha\) and \(\beta\) are set to 1.2 and 0.8, respectively. (7) DMI [47], which proposes a novel information-theoretic loss function for training neural networks robust to label noise. (8) Forward [31], Reweight [21], and T-Revision [45] utilize a class-dependent transition matrix \(T\) to correct the loss function. (9) PTD [43], estimates instance-dependent transition matrix by combing
Figure 2: Illustration of the transition matrix approximation error and the hyperparameter sensitivity. Figure (a) illustrates how the distillation threshold $\rho$ affects the approximation error for the instance-dependent transition matrix. Figure (b) illustrates how the distillation threshold $\rho$ affects the test classification performance. The error bar for standard deviation in each figure has been shaded.

part-dependent transition matrices. We compare our method with the PTD-R-V, the strongest one, in all experiments. Note that we do not compare with some methods like SELF [29] and DivideMix [16], the reasons are (1) Their proposed approaches are aggregations of multiple techniques while this paper only focuses on one (estimating transition matrix); therefore the comparison is not fair. (2) This paper firstly proves that estimating an instance-dependent transition matrix using a deep neural network is possible, which provides meaningful insight for future works.

5.1.3 Implementation details

We use ResNet-18 [12] for F-MNIST, ResNet-34 networks [12] for CIFAR-10 and SVHN. We first use SGD with momentum 0.9, batch size 128, and an initial learning rate of 0.01 to warm up the network for five epochs on the noisy dataset. For Clothing1M, we use a ResNet50 pretrained on ImageNet, and the learning rate is set as 1e-3. Then, we use the warm-upped network to collect distilled examples from noisy datasets according to Section 4.1. After distilled examples collection, we train the instance-dependent transition matrix estimator network on the distilled dataset for 5 epochs. The Bayes label transition network is the same architecture as the classification network, but the last linear layer is modified according to the transition matrix shape. The optimizer of the Bayes label transition network is SGD, with a momentum of 0.9 and a learning rate of 0.01. Then, we fix the trained Bayes label transition network to train the classification network. The Bayes label transition network is used to generate a transition matrix for each input image; the transition matrix is used to correct the outputs of the classification network to bridge the Bayes posterior and the noisy posterior. The classification network is trained on the noisy dataset for 50 epochs for F-MNIST, CIFAR-10 and SVHN and for 10 epochs for Clothing1M using Adam optimizer with a learning rate of $5e^{-7}$ and weight decay of $1e^{-4}$. We also apply the transition matrix revision technique [45] to boost the performance. Note for a fair comparison, we do not use any data augmentation technique in all experiments as in [43]. All the codes are implemented in PyTorch 1.6.0 with CUDA 10.0, and run on NVIDIA Tesla V100 GPUs.

5.2 Ablation Study

The distilled dataset collection relies on the choice of distillation threshold $\rho_{\text{max}}$ (denoted as $\rho$ in the following paragraph) in Theorem 1. To further explore the effect of $\rho$, we conduct ablation studies on CIFAR-10 in this section.

In Figure 2(a), we show the instance-dependent transition matrix approximation error when employing the class-dependent transition matrix, the revised class-dependent transition matrix, and our proposed instance-dependent transition matrix estimation method. The error is measured by $\ell_1$ norm between the ground-truth transition matrix and the estimated transition matrix. For each instance, we only analyze the approximation error of a specific row because the noisy label is generated by one row of the instance-dependent transition matrix. The “Class-dependent” represents the standard class-
Table 1: Means and standard deviations (percentage) of classification accuracy on F-MNIST with different label noise levels. ‘-V’ indicates matrix revision [45].

| Method   | IDN-10%  | IDN-20%  | IDN-30%  | IDN-40%  | IDN-50%  |
|----------|----------|----------|----------|----------|----------|
| CE       | 88.65 ± 0.45 | 88.31 ± 0.37 | 85.22 ± 0.56 | 76.56 ± 2.50 | 67.42 ± 3.91 |
| GCE      | 90.86 ± 0.38 | 88.59 ± 0.26 | 86.64 ± 0.76 | 76.93 ± 1.64 | 66.69 ± 1.07 |
| APL      | 86.46 ± 0.27 | 85.32 ± 0.88 | 85.59 ± 0.85 | 74.66 ± 2.77 | 62.82 ± 0.44 |
| Decoupling | 89.83 ± 0.45 | 86.29 ± 1.13 | 86.01 ± 1.01 | 78.78 ± 0.53 | 63.73 ± 1.33 |
| MentorNet| 90.35 ± 0.64 | 87.92 ± 0.83 | 87.24 ± 0.99 | 79.01 ± 2.30 | 64.44 ± 2.97 |
| Co-teaching| 90.65 ± 0.58 | 88.77 ± 0.41 | 86.98 ± 0.67 | 78.92 ± 1.36 | 67.66 ± 2.42 |
| Co-teaching+ | 90.47 ± 0.98 | 89.15 ± 1.77 | 86.15 ± 1.04 | 79.23 ± 1.30 | 63.49 ± 2.94 |
| Joint    | 80.19 ± 0.99 | 78.46 ± 1.24 | 72.73 ± 2.44 | 65.93 ± 2.08 | 50.93 ± 3.52 |
| DMI      | 91.58 ± 0.46 | 90.33 ± 0.66 | 85.96 ± 1.52 | 77.77 ± 2.15 | 68.02 ± 1.59 |
| Forward  | 89.65 ± 0.24 | 86.11 ± 0.77 | 85.01 ± 0.43 | 78.59 ± 0.38 | 67.11 ± 1.46 |
| Reweight | 90.33 ± 0.27 | 88.81 ± 0.44 | 84.93 ± 0.42 | 76.07 ± 1.93 | 67.66 ± 1.65 |
| S2E      | 91.04 ± 0.92 | 89.93 ± 1.08 | 86.77 ± 1.15 | 76.12 ± 1.21 | 70.24 ± 2.64 |
| T-Revision| 91.36 ± 0.59 | 90.24 ± 1.01 | 85.59 ± 1.77 | 78.24 ± 1.12 | 69.04 ± 2.92 |
| PTD      | 92.03 ± 0.33 | 90.78 ± 0.64 | 87.86 ± 0.78 | 79.46 ± 1.58 | 73.38 ± 2.25 |

| Ours     | 96.06 ± 0.71 | 94.97 ± 0.33 | 91.47 ± 1.36 | 82.88 ± 2.72 | 76.35 ± 3.79 |
| Ours-V   | 96.93 ± 0.31 | 95.55 ± 0.59 | 92.24 ± 1.87 | 83.43 ± 1.72 | 76.89 ± 4.26 |

Table 2: Means and standard deviations (percentage) of classification accuracy on CIFAR-10 with different label noise levels. ‘-V’ indicates matrix revision [45].

| Method   | IDN-10%  | IDN-20%  | IDN-30%  | IDN-40%  | IDN-50%  |
|----------|----------|----------|----------|----------|----------|
| CE       | 73.54 ± 0.14 | 71.49 ± 1.35 | 67.52 ± 1.68 | 58.63 ± 4.92 | 51.54 ± 2.70 |
| GCE      | 74.24 ± 0.89 | 72.11 ± 0.43 | 69.31 ± 0.18 | 56.86 ± 0.92 | 53.44 ± 1.28 |
| APL      | 71.12 ± 0.19 | 68.89 ± 0.27 | 65.17 ± 0.35 | 53.22 ± 2.21 | 47.31 ± 1.41 |
| Decoupling | 73.91 ± 0.37 | 74.23 ± 1.18 | 70.85 ± 1.88 | 54.73 ± 1.02 | 52.04 ± 2.09 |
| MentorNet| 74.93 ± 1.37 | 73.59 ± 1.29 | 72.32 ± 1.04 | 57.85 ± 1.88 | 52.96 ± 1.98 |
| Co-teaching+ | 75.49 ± 0.47 | 75.93 ± 0.87 | 74.86 ± 0.42 | 59.07 ± 1.03 | 55.62 ± 3.93 |
| Joint    | 74.77 ± 0.16 | 75.14 ± 0.61 | 71.92 ± 2.13 | 59.15 ± 0.87 | 53.02 ± 3.34 |
| DMI      | 75.97 ± 0.98 | 76.45 ± 0.45 | 75.93 ± 1.65 | 63.22 ± 5.37 | 55.84 ± 3.25 |
| Forward  | 72.35 ± 0.91 | 70.98 ± 0.32 | 66.53 ± 1.96 | 58.63 ± 1.25 | 52.33 ± 1.65 |
| Reweight | 73.55 ± 0.32 | 71.49 ± 0.57 | 68.76 ± 0.37 | 60.32 ± 1.03 | 52.03 ± 1.70 |
| S2E      | 75.93 ± 1.01 | 75.53 ± 0.32 | 71.21 ± 2.51 | 64.62 ± 0.68 | 56.03 ± 1.07 |
| T-Revision| 74.01 ± 0.45 | 73.42 ± 0.64 | 71.15 ± 0.43 | 59.93 ± 1.33 | 55.67 ± 2.07 |
| PTD      | 76.33 ± 0.38 | 76.05 ± 1.72 | 75.42 ± 1.33 | 65.92 ± 2.33 | 56.63 ± 1.88 |

| Ours     | 81.73 ± 0.56 | 80.26 ± 0.63 | 77.69 ± 1.37 | 71.96 ± 2.27 | 59.15 ± 3.11 |
| Ours-V   | 82.16 ± 1.01 | 80.37 ± 1.98 | 78.82 ± 1.07 | 72.93 ± 4.00 | 60.33 ± 5.29 |

dependent transition matrix learning methods [21 31], the ‘T-Revision’ indicates the class-dependent transition matrix is revised by a learnable slack variable [45]. Our proposed method estimates an instance-dependent transition matrix for each input. It can be observed that our proposed method can achieve a much lower approximation error. Figure [26] shows the classification performance of our proposed method when choosing various distillation threshold $\rho$s. When $\rho$ is not too large or too small, our method is not sensitive to the choice of $\rho$.

5.3 Comparison with the State-of-the-Arts

Results on synthetic noisy datasets. Table [12] and [3] report the classification accuracy on the datasets of F-MNIST, CIFAR-10, and SVHN, respectively.

For F-MNIST, our method surpasses all the baseline methods by a large margin. Equipping the transition matrix revision (-V) [45] can further boost the performance of our method. For SVHN and
Table 3: Means and standard deviations (percentage) of classification accuracy on SVHN with different label noise levels. ‘-V’ indicates matrix revision.

| Method   | IDN-10%  | IDN-20%  | IDN-30%  | IDN-40%  | IDN-50%  |
|----------|----------|----------|----------|----------|----------|
| CE       | 90.39 ± 0.13 | 89.04 ± 1.32 | 85.65 ± 1.84 | 79.94 ± 2.71 | 61.01 ± 5.41 |
| GCE      | 90.82 ± 0.15 | 89.35 ± 0.94 | 86.43 ± 0.63 | 81.66 ± 1.58 | 54.77 ± 0.25 |
| APL      | 71.78 ± 0.76 | 89.48 ± 1.67 | 83.46 ± 2.17 | 77.90 ± 2.31 | 55.25 ± 3.77 |
| Decoupling | 90.55 ± 0.83 | 88.74 ± 0.77 | 85.03 ± 1.63 | 83.36 ± 2.73 | 56.76 ± 1.87 |
| MentorNet | 90.28 ± 0.52 | 89.09 ± 0.95 | 85.89 ± 0.73 | 82.63 ± 1.73 | 55.27 ± 4.14 |
| Co-teaching | 91.05 ± 0.33 | 89.56 ± 1.77 | 87.75 ± 1.37 | 84.92 ± 1.59 | 59.56 ± 2.34 |
| Co-teaching+ | 92.83 ± 0.87 | 90.73 ± 1.39 | 86.37 ± 1.66 | 75.24 ± 3.77 | 54.58 ± 3.46 |
| Joint    | 88.39 ± 0.62 | 85.37 ± 0.44 | 81.56 ± 0.43 | 78.98 ± 2.98 | 59.14 ± 3.22 |
| DMI      | 92.11 ± 0.49 | 91.63 ± 0.87 | 86.98 ± 0.36 | 81.11 ± 0.68 | 63.22 ± 3.97 |
| Forward  | 90.01 ± 0.78 | 89.77 ± 1.54 | 86.70 ± 1.44 | 80.24 ± 2.77 | 57.57 ± 1.45 |
| Reweight | 91.06 ± 0.19 | 92.01 ± 1.04 | 87.55 ± 1.71 | 83.79 ± 1.11 | 55.08 ± 1.25 |
| S2E      | 92.70 ± 0.51 | 92.02 ± 1.54 | 88.77 ± 1.77 | 83.06 ± 2.19 | 65.39 ± 2.77 |
| T-Revision | 93.07 ± 0.79 | 92.67 ± 0.88 | 88.49 ± 1.44 | 82.43 ± 1.77 | 67.64 ± 2.57 |
| PTD      | 93.77 ± 0.33 | 92.59 ± 1.07 | 89.64 ± 1.98 | 83.56 ± 2.21 | 71.57 ± 3.32 |
| Ours     | 96.05 ± 0.32 | 94.97 ± 0.58 | 93.99 ± 1.24 | 87.67 ± 1.29 | 78.13 ± 4.62 |
| Ours-V   | 96.37 ± 0.77 | 95.12 ± 0.40 | 94.69 ± 0.24 | 88.13 ± 3.23 | 78.71 ± 4.37 |

CIFAR-10, the superiority of our method is gradually revealed along with the noise rate increase, which shows that our method can handle the extremely hard situation much better. Specifically, the classification accuracy of our method is 5.83% higher than PTD (the best baseline) on CIFAR-10 in the IDN-10% case, and the performance gap is enlarged to 7.01% in the IDN-40% case. On the SVHN, the classification accuracy of our method is 2.60% higher than PTD in the IDN-10% case, 5.05% higher than PTD in the IDN-30% case, and 7.14% higher than PTD in the most challenging IDN-50% case. Overall, our proposed method exhibits a much more superior ability to handle the instance-dependent label noise problem and performs outstandingly in dealing with higher noise rate tasks.

5.3.1 Results on real-world datasets

The noise model of real-world datasets is more likely to be instance-dependent. Our proposed method also performs favorably on the challenging Clothing1M dataset (Table 4), which proves that our method is more flexible to handle such real-world noise problem.

Table 4: Classification accuracy on Clothing1M. In the experiments, only noisy samples are exploited to train and validate the deep model.

| Method   | CE   | Decoupling | MentorNet | Co-teaching | Co-teaching+ | Joint  |
|----------|------|------------|-----------|------------|-------------|--------|
|          | 68.88 | 54.53      | 56.79     | 60.15      | 65.15       | 70.88  |
| DMI      | 70.12 | 69.91      | 70.40     | 70.97      | 71.67       | 73.33  |

6 Conclusion

In this paper, we focus on training the robust classifier with the challenging instance-dependent label noise. To address the issues of existing clean label transition matrix, we propose to directly build the transition between Bayes optimal labels and noisy labels. By reducing the feasible solution of the transition matrix estimation, we prove that the instance-dependent label transition matrix that relates Bayes optimal labels and noisy labels can be directly learned using deep neural networks. The main limitation of our method comes from that the distilled examples are collected out of noisy data leading to unavoidable data distribution bias to the transition matrix estimation. Experimental results demonstrate that the proposed method is more superior in dealing with instance-dependent label noise, especially for the case of high-level noise rates.
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