Local thermal observables in spatially open FRW spaces

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Abstract

Certain local thermal observables are considered in well-known examples of spatially open FRW spaces: Milne, open de Sitter and anti de Sitter as well as Einstein static universes. Another value for fixing the ambiguity in defining the Wick square and, hence, the local temperature is motivated in the last example. Physical consequences of that choice are discussed for static and conformal vacua in those spaces.

Keywords: conformal field theory in curved spaces, local thermal observables, quantum vacua
I. INTRODUCTION

In this research I will explore, in particular, thermal characteristics of states considered in [1]. A framework employed below is based on the idea that one can construct microscopic quantities from the field products and its derivatives which are sensitive to thermal properties of a certain quantum state [2, 3]. Among of these local thermal observables are a local temperature and a thermal energy-momentum tensor. Thus, calculating them in a quantum state under consideration and in a some reference thermal one, one may decide to which extent it is legitimate to ascribe macroscopic thermal observables to it as well as whether it corresponds to a local thermal equilibrium.

Although there exists in general no global thermal states in curved spacetimes to be taken as reference ones, it is not the case in the problem under scrutiny. I shall consider observers moving along geodesics corresponding to the integral curves of conformal Killing vector fields in spatially open FRW spaces among of which are Milne, open AdS, open dS as well as open Einstein static universes. Therefore, a conformal KMS state are chosen in all of these cases (except the last one, wherein it is just a KMS state) as the reference thermal one.

In Section II, I consider the contracting and expanding Milne universes. An alternative quantization being equivalent to the standard one and relation with the conformal vacuum are studied. Thermal properties of known states are discussed in the framework briefly outlined above. In Section III I deal with the AdS spacetime, wherein an alternative quantization of the conformal scalar field on the AdS hyperboloid will be demonstrated that is unitary equivalent to the standard one [11] and the existence of which has been motivated in [1]. Along the line of Section II a discussion of the thermal properties of static and conformal vacua is presented. In Section IV I discuss thermal local observables in the cases of the open dS space as well as the open ESU and compare them with those for the closed dS space and the closed ESU. In Section V I provide final concluding remarks.

The sign convention for the metric tensor as well as the definition of the Riemann tensor are the same as in [1]. The fundamental constants are set to unity throughout this work.

II. MILNE SPACETIME

Milne universe is a subspace of Minkowski spacetime lying in the future lightcone originating from a given point O of the manifold. Its line element has the form of the open FRW universe with the scale factor exponentially growing with the conformal time, i.e.

$$ds^2 = a^2(\eta)(d\eta^2 - d\chi^2 - \sinh^2 \chi d\Omega^2),$$

where $a(\eta) = e^\eta$ is the scale factor and $d\Omega^2 = d\theta^2 + \sin^2 \theta d\varphi^2$ element of solid angle. By the same metric with a reversed direction of $\eta$ one can cover the past lightcone of the origin.
O, wherein now $a(\eta)$ approaches zero for $\eta \to +\infty$. The future and past lightcones will be called upper and lower Milne universes in the following, respectively.

### A. Minkowski modes in Milne spacetime

To simplify calculations of the Wick square below in the conformal KMS state, one has to expand the quantum field through modes which when rescaled becomes positive frequency ones with respect to the conformal Killing vector $\partial_\eta$. These modes have been found in [4] for the upper Milne wedge which specify Minkowski vacuum. One can also obtain the minkowskian modes in the lower wedge by analytically continuing them from the Rindler spacetime to that wedge. Thus, one has

$$\Phi_{\pm lm}(\eta \pm, \chi \pm, \Omega) = \pm i e^{\mp \pi \eta / 2} e^{\tau \eta \Omega} \frac{\Gamma(1 + l + ip)}{(4\pi \sinh \chi \pm)^{l + \frac{1}{2}}} P^{l-\frac{1}{2}}(\cosh \chi \pm) Y_{lm}(\Omega), \ p \in \mathbb{R}, \ (2)$$

where $\eta \pm$ and $\chi \pm$ are defined through $t = \pm e^{\mp \eta \pm} \cosh \chi \pm$ and $r = \pm e^{\mp \eta \pm} \sinh \chi \pm$, where the plus and minus indices refer to the upper and lower Milne wedges, respectively.

By using Eq. (2), one can define conformal modes specifying the conformal Milne vacua in the wedges as follows

$$\Phi_{+ \omega lm} = \alpha_{\omega lm} \Phi_{+ \omega lm}^M + \beta_{\omega lm} \Phi_{- \omega lm}^M, \ (3a)$$
$$\Phi_{- \omega lm} = \alpha_{\omega lm} \Phi_{- \omega lm}^M + \beta_{\omega lm} \Phi_{+ \omega lm}^M, \ (3b)$$

where $\omega \in \mathbb{R}^+$ and the Bogolyubov coefficients are set as follows

$$\alpha_{\omega lm} = \frac{\exp\left(\frac{\pi \omega}{2}\right)}{(2\sinh(\pi \omega))^l}, \quad \beta_{\omega lm} = -(-1)^l \frac{\exp\left(-\frac{\pi \omega}{2}\right)}{(2\sinh(\pi \omega))^l}. \ (4)$$

Note that the modes $\Phi_{+ \omega lm}^M$ vanish in the lower wedge, while $\Phi_{- \omega lm}^M$ are strictly zero in the upper Milne wedge and both of them can be analytically continued into the Rindler space.

By exploiting Eq. (3), one can express the Minkowski modes through the upper and lower conformal Milne ones. In other words, Minkoski vacuum can be represented as an excited state under $|0^+_C \rangle \otimes |0^-_C \rangle$. The crucial ingredient of the model that allows such a representation is a presence of the conformal symmetry, because then the commutator $[\Phi(x_1), \Phi(x_2)]$ vanishes whenever $x_1$ and $x_2$ lie in the different Milne wedges as it has been noted in [5]. Thus, the field degrees of freedom in one wedge are casually independent from those lying in the other wedge.

### B. Local thermal observables

It is argued in 2, 3 that a real thermometer is modeled in the theoretical language with the help of the Wick square of the field. This construction and a concept of the local
equilibrium have been further studied in curved spacetimes [6, 7] (see also [8]). Specifically, the squared value of the local temperature $T(x)$ measured by the thermometer in a given state $\omega$ is probed by the Wick square as follows

$$T^2(x) = 12\omega(\hat{\Phi}^2(x)),$$

(5)

wherein the Wick square is defined as $\hat{\Phi}(x_1)\hat{\Phi}(x_2) \equiv \hat{\Phi}(x_1)\hat{\Phi}(x_2) - H(x_1, x_2)\hat{1}$, where $H(x_1, x_2)$ is the Hadamard parametrix canceling the leading order divergencies at the coincidence limit $x_2 \rightarrow x_1$ in the two-point function [10].

The conformal KMS state $\omega_\beta$ specified by the KMS parameter $\beta$ can be chosen as a thermal reference one. The two-point function in it equals

$$\omega_\beta(\hat{\Phi}(x_1)\hat{\Phi}(x_2)) = \frac{i}{2\pi a(\eta_1)a(\eta_2)} \int_{-\infty}^{+\infty} dk \frac{1}{1 - e^{-\beta k}} \int_{-\infty}^{+\infty} d\eta \Delta_r(\eta + \eta_1; \eta_2) e^{ik\eta},$$

(6)

where $\Delta_r(x_1, x_2)$ is the rescaled casual propagator, i.e. $[\hat{\Phi}_r(x_1), \hat{\Phi}_r(x_2)] = i\Delta_r(x_1, x_2)\hat{1}$ and $\hat{\Phi}_r(x) \equiv a(\eta)\hat{\Phi}(x)$. Exploiting the previous Subsection, one finds

$$T^2(x) = \frac{1}{a^2(\eta)} \left( \frac{1}{\beta^2} - \frac{1}{4\pi^2} \right).$$

(7)

The Hadamard parametrix in this model coincides with the Minkowski two-point function, therefore, the local temperature in the Minkowski vacuum state is strictly zero. However, the Wick square in the conformal vacuum state is negative and coincides with Eq.(7) for $\beta \rightarrow +\infty$.

The vacuum expectation value of the energy-momentum tensor $\hat{T}_\mu^\nu$ vanishes in Minkowski state and the conformal anomaly is absent. Hence, one obtains

$$\omega_\beta(\hat{T}_\mu^\nu) = \frac{1}{480\pi^2a^4(\eta)} \left( \frac{2\pi}{\beta} \right)^4 \left( \frac{\pi^2}{\beta^2} \right) \left( \delta_\nu^\mu - \frac{4}{3}\delta_\nu^i\delta_i^\mu \right),$$

(8)

wherein $i$ runs from 1 to 3. The thermal energy-momentum tensor $\hat{E}_\mu^\nu$ in this framework is, however, only a part of the total energy-momentum tensor $T_\mu^\nu$ (denoted by $\epsilon_{\mu\nu}$ in [7]), such that

$$\omega_\beta(\hat{E}_\mu^\nu) = \frac{1}{120\pi^2a^4(\eta)} \left( 1 + \frac{4\pi^4}{\beta^4} - \frac{5\pi^2}{\beta^2} \right) \left( \delta_\nu^\mu - \frac{4}{3}\delta_\nu^i\delta_i^\mu \right).$$

(9)

Both $\omega_\beta(\hat{T}_\mu^\nu)$ and $\omega_\beta(\hat{E}_\mu^\nu)$ are traceless and vanish in Minkowski vacuum, but $\omega_\beta(\hat{T}_\mu^\nu)$ is less then zero in conformal vacuum, while $\omega_\beta(\hat{E}_\mu^\nu)$ is positive in it. Note that neither $\omega_\beta(\hat{T}_\mu^\nu)$ nor $\omega_\beta(\hat{E}_\mu^\nu)$ are proportional to $T^4(x)$.

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1 It is a standard designation of a state in the algebraic approach to the quantum field theory, see [9, 10]. I use it below for the convenience.
III. OPEN ANTI DE SITTER SPACETIME

Anti de Sitter spacetime can be imagined as a four dimensional hyperboloid embedded in a five-dimensional space $\mathbb{R}^5$ with the line element $ds^2 = \eta_{ab}dx^adx^b$, where $a$ and $b$ run from 0 to 4 and $\eta_{ab} = \text{diag}(+,-,-,-,+)$, i.e. $\eta_{ab}x^ax^b = 1$. This spacetime is pathological from the point of view that it is not globally hyperbolic. The first reason lies in that its topology is $S \times \mathbb{R}^3$, so that it possesses closed time-like curves. This feature is cured by unwrapping the circle $S$ and considering instead its universal covering $\mathbb{R}$. The second reason consists in that its spatial infinity is time-like. Therefore, one has to set a boundary condition there to fix the energy flux through it. These allow to have a well-defined quantum theory in AdS hyperboloid [11].

Among of possible parameterizations of the AdS hyperboloid, I shall consider so-called open coordinates. The line element in these coordinates becomes

$$ds^2 = a^2(\tilde{\eta})(d\tilde{\eta}^2 - d\tilde{\chi}^2 - \sinh^2 \tilde{\chi}d\tilde{\Omega}^2), \quad (10)$$

where $a(\tilde{\eta}) = 1/ \cosh \tilde{\eta}$. By the same metric one can describe geometry inside the wedges for which $\eta \in (2\pi k, \pi + 2\pi k)$, where $\eta$ is the time coordinate in the static frame [1] and $k \in \mathbb{Z}$. The geodesics of AdS correspond to comoving geodesics in open AdS, i.e. integral curves of the conformal Killing vector $\xi = \partial_\eta$.

For the analysis below it is needed to introduce coordinates that cover the rest part of the AdS hyperboloid. They can be obtained by setting $\bar{\eta} = -\tilde{\chi} - i\pi/2$ and $\bar{\chi} = \tilde{\eta} + i\pi/2$. These coordinates correspond to a parametrization of the wedges with $\eta \in (-\pi/2 + 2\pi k, +\pi/2 + 2\pi k)$,

$$ds^2 = a^2(\bar{\chi})(d\bar{\eta}^2 - d\bar{\chi}^2 - \cosh^2 \bar{\chi}d\bar{\Omega}^2), \quad (11)$$

where $a(\bar{\chi}) = 1/ \sinh \bar{\chi}$. Up to the scale factor it is similar to the Rindler universe in the spherical coordinates [4]. Therefore, the line element (11) is conformally related with the Minkowski one, specifically

$$ds^2 = \frac{4\eta_{\mu\nu}dx^\mu dx^\nu}{(1 + \eta_{\lambda\rho}x^\lambda x^\rho)^2}, \quad (12)$$

where $x^\mu = (r \tanh \tilde{\eta}, r \sin \theta \cos \varphi, r \sin \theta \sin \varphi, r \cos \theta)$ and $r = e^\chi \cosh \tilde{\eta}$, such that $t^2 - r^2 < 0$. If one takes $r^2 - t^2 > 0$, then this metric can be transformed to Eq.(10). Thus, one can cover the AdS hyperboloid by domains being conformally related to the Minkowski space. Note that changing the sign inside the conformal factor, one then obtains the de Sitter line element.

The Killing vector $\zeta = z\partial_t + t\partial_z$ in these wedges is time-like for $\theta \in \{0, \pi\}$ and sets a dynamics for an observer moving with a constant four-acceleration through AdS, while

2 It is not a restriction, because the rotation group SO(3) is a subgroup of the AdS symmetry group SO(2,3), so that one can always set those values of $\theta$ without loss of generality.
\( \sigma = \partial_t \) is the conformal Killing vector. Due to the conformal symmetry, one can expand the rescaled field through the plane modes or through the boost ones, i.e. modes being eigenfunctions of the boost operator \( \zeta \) in Minkowski space. This expansion specifies the static AdS vacuum as it gives the same two-point function. On the other hand, an observer moving along integral curves of \( \zeta \) in AdS defines Unruh modes up to the conformal factor. Hence, these vacua are thermally related as it has been first found in [12] by employing the Unruh-DeWitt detector.

A. Static modes in open AdS

As in the case of Milne universe it is convenient to expand the field through the modes which when rescaled are eigenfunctions of \( \xi \) and still define static vacuum. I have argued in [1] that it must exist. I show this below in this Subsection.

For the wedge \( \eta \in (-\pi/2, +\pi/2) \) these modes are

\[
\Phi_{\text{plm}}(\tilde{\eta}, \tilde{\chi}, \tilde{\Omega}) = \frac{e^{+ip\tilde{\eta}}}{a(\tilde{\chi})} \frac{\Gamma(1 + l + ip)}{(4\pi \cosh \tilde{\eta})^{1/2}} P_{ip-1/2}^{-1/2} (i \sinh \tilde{\eta}) Y_{lm}(\tilde{\Omega}), \quad p \in \mathbb{R} \tag{13}
\]

which are normalized on the effective Cauchy surface \( \Sigma = \Sigma_1 \cup \Sigma_2 \), i.e. these modes correspond to the transparent boundary conditions [11].

Having performed an analytic continuation into the wedges \( \eta \in (0, +\pi) \) and \( \eta \in (-\pi, 0) \) according to \( \bar{\eta}_+ = -\tilde{\chi} - \frac{i\pi}{2}, \bar{\chi}_+ = \tilde{\eta} + \frac{i\pi}{2} \) and \( \bar{\eta}_- = \tilde{\chi} - \frac{i\pi}{2}, \bar{\chi}_- = -\tilde{\eta} - \frac{i\pi}{2} \), respectively, one obtains

\[
\Phi_{\text{plm}}^{\pm}(\bar{\eta}_\pm, \bar{\chi}_\pm, \bar{\Omega}) = (\mp i)^{1/2} e^{\mp p\bar{\eta}_\pm} \frac{\Gamma(1 + l + ip)}{(4\pi \sinh \bar{\chi}_\pm)^{1/2}} P_{ip-1/2}^{-1/2} (cosh \bar{\chi}_\pm) Y_{lm}(\Omega). \tag{14}
\]

Having these modes, it is straightforward to obtain the relation between conformal and static vacua in AdS [1]. Indeed, the conformal modes are

\[
\Phi_{\omega lm}^{C+} = \alpha_{\omega lm} \Phi_{\omega lm}^{S+} + \beta_{\omega lm} \Phi_{-\omega lm}^{S+}, \tag{15a}
\]

\[
\Phi_{\omega lm}^{C-} = \alpha_{\omega lm} \Phi_{\omega lm}^{S+} + \beta_{\omega lm} \Phi_{\omega lm}^{S+}, \tag{15b}
\]

where \( \omega \in \mathbb{R}^+ \) and

\[
\alpha_{\omega lm} = \frac{\exp \left( \frac{\pi \omega}{2} \right)}{\left( 2 \sinh(\pi \omega) \right)^{1/2}}, \quad \beta_{\omega lm} = -(-1)^{1/2} \frac{\exp \left( -\frac{\pi \omega}{2} \right)}{\left( 2 \sinh(\pi \omega) \right)^{1/2}} \tag{16}
\]

One can show that the modes \( \Phi_{\omega lm}^{C+} \) vanish in wedges where \( \eta \in (-\pi + 2\pi k, 2\pi k) \), while the modes \( \Phi_{\omega lm}^{C-} \) vanish in wedges where \( \eta \in (2\pi k, +\pi + 2\pi k), k \in \mathbb{Z} \).

The modes Eq.(13) or Eq.(14) define static vacuum. Indeed, the two-point function is

\[
\omega_S(\hat{\Phi}(x_1)\hat{\Phi}(x_2)) = \sum_{lm}^{+\infty} \int_{-\infty}^{+\infty} dp \Phi_{\text{plm}}^{S}(x_1)\Phi_{\text{plm}}^{S}(x_2) = \frac{1}{8\pi^2} \frac{\cos \chi_1 \cos \chi_2}{\cos(\Delta \eta - i\varepsilon) - \cos(\zeta)},
\]
where it has been already rewritten in static AdS coordinates, \( \cos \zeta = \cos(\chi_1 - \chi_2) + \sin \chi_1 \sin \chi_2 (\cos \Theta - 1) \) and \( \cos \Theta = \cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2 \cos (\phi_1 - \phi_2) \). Comparing it with the two-point function for the closed Einstein static universe derived in [15] conformally mapped to the static AdS, one concludes the modes Eq.(14) are unitary equivalent to the static ones for the transparent boundary conditions [11].

B. Local thermal observables

In analogous manner to Subsec. II B, one finds the squared value of the local temperature ascribed to the conformal KMS state with the KMS parameter \( \beta \):

\[
T^2(x) = \frac{1}{a^2(\bar{\eta})} \left( \frac{1}{\beta^2} - \frac{1}{4\pi^2} \right) + \frac{R}{24\pi^2} - 12\alpha_0 R, \tag{17}
\]

where \( \alpha_0 \) is due to ambiguity in defining the Wick square [13] and \( R = -6(a''/a - 1)/a^2 \) Ricci scalar equaling to +12 in AdS space. If one sets \( \alpha_0 = 1/288\pi^2 \), then the Wick square is a conformally invariant field [14] (see also [8]). Another motivation for this choice of \( \alpha_0 \) is given in [7, 8]. This value of \( \alpha_0 \) is taken for granted in this Subsection.

In the open AdS space, static vacuum is a conformal KMS state with \( \beta = 2\pi \). The squared local temperature of conformal vacuum is negative and equals Eq.(17) in the limit \( \beta \to +\infty \).

Taking into account the vacuum expectation value of \( \hat{T}^\mu_\nu \) in static vacuum in the open AdS, one derives

\[
\omega_\beta(\hat{T}^\mu_\nu) = \frac{1}{960\pi^2} \delta^\mu_\nu + \frac{1}{480\pi^2 a^4(\bar{\eta})} \left( \left( \frac{2\pi}{\beta} \right)^4 - 1 \right) \left( \delta^\mu_\nu - \frac{4}{3} \delta^\mu_i \delta^i_\nu \right), \tag{18}
\]

whereas for the thermal energy-momentum tensor \( \hat{E}^\mu_\nu \) one finds an expression to be structurally rather different from Eq.(18).

IV. DISCUSSION

An analogous result to Eq.(17) one obtains for the local temperature squared in the open de Sitter spacetime, wherein \( R = -12 \) and \( a(\bar{\eta}) = 1/\sinh \bar{\eta} \) in that equation. The total energy-momentum tensor \( \hat{T}^\mu_\nu \) in the conformal KMS state \( \omega_\beta \) is functionally given by the same Eq.(18). The conformal KMS state with \( \beta = 2\pi \) corresponds to conformal vacuum or the Chernikov-Tagirov state [15] in the closed de Sitter space.

As has been mentioned above, the Wick square is ambiguous [13]. This means one has to impose an extra condition to get rid of that. The value of \( \alpha_0 = 1/288\pi^2 \) has been motivated in [7, 8]. However, one has to set a zero value of \( \alpha_0 \) to have a zero local temperature for an observer freely moving along the time translation Killing vector in the open Einstein static
universe. Indeed, the local temperature squared can be immediately obtained from Eq. (17) by setting \( a(\bar{\eta}) = 1 \), so that if the quantum field is in static vacuum, then \( \beta \to +\infty \). Hence, \( T^2(x) = 0 \) if and only if \( \alpha_0 = 0 \), where \( R = +6 \) in the open ESU has been taken into account. The expectation values of the total and the thermal energy-momentum tensors in a KMS state defined with respect to that Killing vector coincide and are equal to

\[
\omega_{\beta}(\hat{T}_\mu^\nu) = \omega_{\beta}(\hat{E}_\mu^\nu) = \frac{\pi^2}{60\beta^4} \left( \delta_\nu^\mu - \frac{4}{3} \delta_\nu^i \delta_i^\mu \right).
\]

Note that they are both proportional to \( T^4(x) \) for vanishing \( \alpha_0 \). This result then is similar to that in Minkowski space for an inertial observer, wherein, however, the value of \( \alpha_0 \) is irrelevant, because the scalar curvature vanishes.

As has been noted above, the Chernikov-Tagirov state in the closed de Sitter space is conformal vacuum defined with respect to the conformal time \( \eta \). The local temperature squared is

\[
T^2(x) = \frac{6}{\pi^2 a^2(\eta)} \sum_{n=0}^{+\infty} \frac{n}{e^{\beta n} - 1} + \frac{R}{24\pi^2} - 12\alpha_0 R,
\]

where \( a(\eta) = 1/\sin \eta \) and \( R = -6(a''/a + 1)/a^2 \) Ricci scalar equaling to \(-12\) in dS space. This result is a slight generalisation of that obtained in [16]. The expectation value of \( \hat{T}_\mu^\nu \) in the KMS state defined with respect to \( \partial_\eta \) is given by

\[
\omega_{\beta}(\hat{T}_\mu^\nu) = \frac{1}{960\pi^2} \frac{1}{a^4(\eta)} \sum_{n=0}^{+\infty} \frac{n^3}{e^{\beta n} - 1} \left( \delta_\nu^\mu - \frac{4}{3} \delta_\nu^i \delta_i^\mu \right),
\]

where the vacuum expectation value of it in the Chernikov-Tagirov state has been taken into account [15], while \( \omega_{\beta}(\hat{E}_\mu^\nu) \) has structurally a different form in comparison with \( \omega_{\beta}(\hat{T}_\mu^\nu) \) given in Eq. (21).

The geodesic observer in the closed dS space moving along curves with the tangent vector \( \partial_\eta \) has to register in the standard interpretation a thermal bath with the Gibbons-Hawking temperature \( T_{GH} = 1/2\pi \) [18] what implies \( \alpha_0 = 1/192\pi^2 \). However, if \( \alpha_0 = 1/288\pi^2 \), then the local temperature is zero [7, 8].

These energy-momentum tensors in the thermal state coincide in the closed Einstein static universe. The local temperature squared can be obtained from Eq. (20) by setting \( a(\eta) = 1 \), such that \( R = -6 \) and \( \alpha_0 = 1/288\pi^2 \) [8]. Employing the vacuum expectation value of \( \hat{T}_\mu^\nu \) for the closed ESU [15, 17], one obtains

\[
\omega_{\beta}(\hat{T}_\mu^\nu) = \omega_{\beta}(\hat{E}_\mu^\nu) = \frac{1}{2\pi^2} \left( \sum_{n=0}^{+\infty} \frac{n^3}{e^{\beta n} - 1} + \frac{1}{240} \right) \left( \delta_\nu^\mu - \frac{4}{3} \delta_\nu^i \delta_i^\mu \right),
\]

which are clearly not proportional to the quartic value of the local temperature.
V. CONCLUDING REMARKS

In this research I have considered certain local thermal observables in the well-known examples of spatially open FRW spaces for a conformal linear field theory. These observables have been put forward in a series of articles [2, 3, 6–8] with a goal to define a local thermal equilibrium under the influence external fields, in particular, in curved spacetimes.

I have found that the ambiguity in defining the Wick square parametrized by $\alpha_0$ [13] has no universal value, i.e. depends on a particular situation. It has to be zero in the open ESU, otherwise the physical meaning of the local temperature $T(x)$ as defined in [2, 3] is lost.

Assuming that $\alpha_0$ has to be zero for all open FRW universes considered above, one is forced to conclude that the local temperature of Minkowski vacuum vanishes, but is real for static vacuum restricted to the open AdS and imaginary for the Chernikov-Tagirov state restricted to the open dS. Thus, the physical meaning of the local temperature as a model for a real thermometer becomes lost in the last case.

The local temperature for conformal vacua in Milne and the open dS spaces are imaginary, but is real only for a short interval of $\bar{\eta}$ close to $\bar{\eta} = 0$ in the open AdS space. However, the backreaction of the field being in conformal vacuum is infinite on horizons, so that these vacua cannot anyway be physically realized.

I have considered in [1] a spacetime that approaches open Einstein static universe at future and past time infinities and has a phase when it looks like open anti de Sitter space. During the AdS phase one might await that a freely moving observer detects a thermal condensate with a temperature $T = (2\pi a(\bar{\eta}))^{-1}$ that vanishes at $\bar{\eta} \to \pm \infty$ [1]. Qualitatively it behaves itself as the local temperature. This would not be the case, however, if there would be Milne phase, for example. This makes the concept of the local temperature as originally defined at least questionable and demands its further investigation.

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