Is Heavy Baryon Approach Necessary?

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Abstract

It is demonstrated that using an appropriately chosen renormalization condition one can respect power counting within the relativistic baryon chiral perturbation theory without appealing to the technique of the heavy baryon approach. Explicit calculations are performed for diagrams including two-loops. It is argued that the introduction of the heavy baryon chiral perturbation theory was useful but not necessary.

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I. INTRODUCTION

Chiral symmetry is of fundamental importance in the low energy dynamics of strongly interacting particles. Using this symmetry Weinberg and Gasser and Leutwyler have developed Chiral Perturbation Theory, a systematic and feasible scheme for calculating processes of meson-meson interactions \cite{1,2}. The Chiral Perturbation Theory possesses the feature of consistent power counting which allows systematic perturbative calculations.

The nontrivial problem appeared after Gasser, Sainio and Švarc considered processes with a single baryon \cite{3}. They noticed that there is no consistent power counting when a baryon is included; higher order loops contribute to low order (in small expansion parameters) calculations. Performing calculations at any given order of the chiral expansion one needs to include contributions of diagrams with an increasing (up to infinity) number of loops.

To avoid this drawback which makes the results of perturbative calculations unreliable, Jenkins and Manohar suggested to consider an extremely nonrelativistic limit of the original relativistic field-theoretical model \cite{4}. Integrating out heavy degrees of freedom and expanding the resulting effective action in inverse powers of the baryon mass $M$, they developed Heavy Baryon Chiral Perturbation Theory (HB\textgreek{X}PT). In the framework of HB\textgreek{X}PT the power counting is restored and thus the problem of the relativistic treatment of the sector with baryon number 1, encountered in \cite{3} is circumvented. The revival of power counting is traded for explicit relativistic invariance. Nowadays HB\textgreek{X}PT is an effective method of calculation of different processes involving electro-magnetic and strong interactions (for a review and references see \cite{5} and \cite{6}).
In this paper we investigate whether the violation of power counting is an intrinsic feature of the relativistic effective theory of pion-nucleon interactions or is only an artifact of the particular method of calculation.

In this connection let us remind the reader that the problem in the relativistic approach of multi-loop diagrams contributing to low order calculations [3] was actually encountered in the $M$ scheme. This scheme puts the effective cut-off equal to the largest involved mass scale, i.e. the nucleon mass and violates the power counting.

On the other hand, for processes involving an arbitrary number of nucleons Weinberg suggested the usage of renormalization points of the order of external momenta or less [7,8]. Such schemes put the effective cutoff for loop integrals of the order of external momenta and make power counting applicable for loop diagrams. While Weinberg considered a non-relativistic effective field theory, the same idea of an appropriate choice of renormalization condition can be useful in relativistic theory as well. As was discussed in [9–11], parts of relativistic diagrams responsible for the violation of power counting can be altered by adding counter-terms. Hence they can be removed by choosing an appropriate renormalization condition.

In the present paper we reexamine the question of validity of chiral power counting in relativistic baryon chiral perturbation theory. We work in exact chiral limit and suppress the isospin. The resulting expansion in small momenta simulates essential features of chiral perturbation theory for pion-nucleon interaction (it is straightforward to show that the isospin and the small non-vanishing mass of the pseudoscalar particle just complicate calculations and do not affect our results). Calculating one and two-loop diagrams we demonstrate that by choosing an appropriate subtraction scheme one can respect power counting in the relativistic theory without appealing to the heavy baryon technique. In other words, within the suggested scheme power counting does not fail when baryons are introduced.

In ref. [9,10,12,13] it has been argued that consistent power counting can exist within the relativistic scheme. Our approach is substantially different, we consider a conventional renormalization technique [14] and in contrast with [9,10,12] we do not split loop integrals into soft and hard parts and also we do not need to include an infinite number of counter-terms while performing renormalization at any finite order.

II. ONE-LOOP APPROXIMATION

We consider a field theoretical model described by the Lagrangian:

$$\mathcal{L} = -\frac{1}{2} \phi \partial_\mu \partial^\mu \phi + \bar{\psi} (i\gamma^\mu \partial_\mu - M) \psi - g \bar{\psi} \gamma^5 \gamma^\mu \psi \partial_\mu \phi + \Delta \mathcal{L}$$  \hspace{1cm} (1)

In eq. (1) $\psi$ is a fermion field with mass $M$, $\phi$ is a neutral massless pseudoscalar field, $g$ is a coupling constant and $\Delta \mathcal{L}$ includes all counter-terms necessary to remove divergences. We use dimensional regularization with $n$ being the space-time dimension.

Lagrangian (1) suggests that analogously to the meson chiral perturbation theory [1] for diagrams containing one fermion line we can have a (naive?) power counting. We assign +4 powers (of small momenta) to each loop integration, +1 power to each derivative occurring in the interaction term, -2 powers to each scalar propagator and -1 power to each fermion propagator. Thus a resulting power $N_i$ is assigned to each particular diagram $i$. We will say...
That diagram \(i\) obeys power counting if the leading term of the result of actual calculation depends on a small momentum \(k\) as \(k^N f(k)\), where \(f(k)\) is a constant or logarithmic function of \(k\).

This power counting is badly violated if the \(\overline{\text{MS}}\) scheme is used; higher order loops do not lead to higher orders in \(k\).

Below we demonstrate that the breakdown of power counting is the result of using the \(\overline{\text{MS}}\) scheme, and, by applying an appropriately chosen renormalization condition it is possible to retain the power counting within relativistic theory.

Let us consider the one-loop correction to the fermion propagator depicted in Fig. 1 (a). The corresponding expression is:

\[
\Sigma_1 = -g^2 \int \frac{d^nq}{(2\pi)^n} \gamma_5 \frac{\not{p} + \not{q} + M}{[(p + q)^2 - M^2 + i\epsilon]} \gamma_5 \not{q} \left[ q^2 + i\epsilon \right] \left[ (p + q)^2 - M^2 + i\epsilon \right] \tag{2}
\]

where \(\not{p} = p^\mu \gamma_\mu\), \(p^\mu = Mv^\mu + k^\mu\) is off mass-shell momenta of the fermion, \(v^\mu v_\mu = 1\) and \(k^\mu (<< M)\) is a small quantity. Eq. (2) can be reduced to the following form:

\[
\Sigma_1 = g^2 MJ(01) - g^2 (n - 1) J^{n+2}(11) \not{p} \tag{3}
\]

where \(J(01)\) and \(J^{n+2}(11)\) are given in the Appendix.

Substituting the values of \(J(01)\) and \(J^{n+2}(11)\) into eq. (3) we obtain:

\[
\Sigma_1 = -M \frac{ig^2}{(4\pi)^{n/2}} \left( M^2 \right)^{n/2-1} \Gamma (1 - n/2) - \not{p} \frac{ig^2}{(4\pi)^{n/2}} \left( M^2 \right)^{n/2-1} \Gamma (1 - n/2)
\]

\[
- \not{p} \frac{ig^2}{(4\pi)^{n/2}} \left( M^2 \right)^{n/2-2} \frac{\Gamma (2 - n/2)}{n - 2} - \not{p} \frac{ig^2}{(4\pi)^{n/2}} \left( M^2 \right)^{n/2-1} \frac{\Gamma (3 - n/2)}{(2 - n)(3 - n)} (1 - z)^2
\]

\[
- \not{p} \frac{ig^2}{(4\pi)^{n/2}} \left( M^2 \right)^{n/2-1} \frac{\Gamma (4 - n/2)}{(2 - n)(3 - n)(4 - n)} (1 - z)^3 \times 2F_1(1, 4 - n/2; 5 - n; 1 - z)
\]

\[
+ \not{p} \frac{ig^2}{(4\pi)^{n/2}} \left( M^2 \right)^{n/2-1} \frac{\Gamma (n/2)}{(2 - n)} \Gamma (2 - n)(1 - z)^{n-1} \times 2F_1(n/2, n; n; 1 - z) \tag{4}
\]

In eq. (4) \(2F_1\) is the Gauss hypergeometric function \([15]\) and we have introduced dimensionless quantity \(z \equiv p^2/M^2\).
In order to carry out the renormalization procedure it is necessary to add to eq. (4) contributions from the counter-terms. As was mentioned above all the necessary counter-terms are included in the Lagrangian and the corresponding contribution reads:

$$\delta \Sigma_1 = g^2 \delta_1 + g^2 \delta_2 \not{\!p} + g^2 \delta_3 p^2 \not{\!p} + g^2 \delta_4 p^4 \not{\!p}$$

Equation (4) fixes only the divergent parts of $\delta_i$ which have to be the same for every scheme and the choice of the particular renormalization scheme corresponds to the choice of finite parts of counter-terms. Note that $\delta_4$ is finite. We are free to add such finite counter-terms exploiting our freedom of the choice of the renormalization scheme.

We choose $\delta_4$ and the finite parts of $\delta_1$, $\delta_2$ and $\delta_3$ so as to cancel the first four terms in eq. (4). The remaining expression admits the limit $n \to 4$ and the renormalized self energy is:

$$\Sigma_R^1 = -\frac{ig^2}{32\pi^2} M^2 \not{\!p} \frac{(1-z)^3}{z} \left[ \frac{1}{z} \ln(1-z) + 1 \right]$$

Power counting states that $\Sigma_R^1$ is expected to be of a third order in $k$, and since $1-z = O(k)$, it follows that eq. (5) is in agreement with this prediction.

On the other hand it is straightforward to show that applying $\overline{MS}$ to eq. (4) and taking into account the mass and wave function renormalizations we get $\Sigma_R^1(\overline{MS}) \sim k$.

III. TWO-LOOP ANALYSIS

Two-loop corrections to the fermion propagator have more complicated structures. For illustrating purposes it is enough to consider a diagram depicted in Fig. 2 (a). The corresponding expression is:

$$\Sigma_{2(a)} = ig^4 \int \frac{d^nq_1d^nq_2 d^n\gamma_5 \gamma_5 \gamma_5 \gamma_5 \gamma_5}{(2\pi)^{2n}} J_{\mu}(1101) \not{\!p} \left[ \frac{M^2 - p^2}{2n} J(1101) - \frac{2}{p\muJ(1101)} + \left( 1 - \frac{1}{n} \right) p^2 C_2 \right]$$

Simplifying eq. (7) we obtain:

$$\Sigma_{2(a)} = ig^4 \left\{ (M^2 - p^2) \gamma^\mu J_{\mu}(1101) - 2 \not{\!p} \left[ \frac{M^2 - p^2}{2n} J(1101) - \frac{2}{p\muJ(1101)} + \left( 1 - \frac{1}{n} \right) p^2 C_2 \right] \right\}$$

+ $2MJ(01)J(01) - 4M^2 \gamma^\mu J_{\mu}(11)J(01) + 4M^2 (\not{\!p} + M) J(01)J(02)$
where \( J_\mu(1101), J(1101), C_2, J_\mu(11), J(02) \) and \( J_\mu(12) \) are given in the Appendix.

The renormalization procedure requires that we add to eq. (8) expressions corresponding to Fig. 2 (b) and Fig. 2 (c). The expression for Fig. 2 (b) is readily obtained replacing the one-loop subdiagram of Fig. 2 (a) by counter-terms of order \( g^2 \); diagram Fig. 2 (c) corresponds to the contribution of counter-terms of order \( g^4 \). Calculating Fig. 2 (b) and adding to expression (8) we obtain:

\[
\Sigma_{2(a)} + \Sigma_{2(b)} = iM^2 g^4 \left\{ (1 - z) \gamma^\mu J_\mu(1101) - 2 \not{p} \left[ \frac{1 - z}{2n} J(1101) - \frac{2}{nM^2} p_\mu J_\mu(1101) + \left(1 - \frac{1}{n}\right) zC_2 \right] \right\}
\]

We express the counter-terms corresponding to Fig. 2 (c) as

\[
\delta \Sigma_2 = g^4 \not{p} \left( \delta_5 p + p^2 \delta_6 p + p^4 \delta_7 + p^6 \delta_8 + p^8 \delta_9 \right)
\]

As in the case of the one-loop correction, the divergent parts of \( \delta_i \) are uniquely fixed from the requirement that renormalized self energy is free of divergences. We have introduced finite terms \( \delta_8 \) and \( \delta_9 \). These terms, together with the finite parts of \( \delta_5, \delta_6 \) and \( \delta_7 \) are fixed below by the renormalization condition.

Next we express \( \delta \Sigma_2 \) in terms of \( z \) and \( M^2 \):

\[
\delta \Sigma_2 = \not{p} M^2 g^4 \left\{ \frac{\delta_5}{M^2} + \delta_6 + M^2 \delta_7 + M^4 \delta_8 + M^6 \delta_9 - (1 - z) \left[ \delta_6 + 2M^2 \delta_7 + 3M^4 \delta_8 + 4M^6 \delta_9 \right] \right\}
\]

\[
+ (1 - z)^2 \left[ M^2 \delta_7 + 3M^4 \delta_8 + 6M^6 \delta_9 \right] - (1 - z)^3 \left[ M^4 \delta_8 + 4M^6 \delta_9 \right] + (1 - z)^4 M^6 \delta_9 \right\},
\]

and we expand the analytic part of eq. (9) in \( (1 - z) \) and add \( \Sigma_{2(a)} + \Sigma_{2(b)} \) to \( \delta \Sigma_2 \). We define the renormalization scheme via the condition that \( \delta_8, \delta_9 \) and finite parts of \( \delta_5, \delta_6, \delta_7 \) are fixed so as to exactly cancel the first five terms (up to and including \( (1 - z)^4 \)) in the expansion of the analytic part. Performing all these calculations we get:

\[
\Sigma^R_2 = ig^4 \not{p} \left( \frac{M^2)^n}{(4\pi)^n} \right) \left\{ - \frac{1}{4} Li_2(1 - z) - \frac{(1 - z)^2}{4} \right\} + \frac{z(1 - z)^3}{12} - \frac{(1 - z)^4}{48 z} + \frac{29}{192}(1 - z)^4 \]

\[
+ \frac{(1 - z)^5}{12 z} + \ln(1 - z) \left[ - \frac{9}{8} (1 - z)^2 - \frac{3}{4} \ln z - \frac{3}{4} z(1 - z) - \frac{(1 - z)^3}{4 z} + \frac{(1 - z)^4}{16 z^2} \right] \}
\]

In eq. (12), \( Li_2 \) is the dilogarithm function [16].

It is straightforward to verify that the coefficient function of \( \ln(1 - z) \) as well as the analytic part of \( \Sigma^R_2 \) are of order \( (1 - z)^5 \). Therefore the two-loop correction to fermion self energy satisfies power counting.
On the other hand, applying $\overline{MS}$ scheme to $\Sigma_{2(a)}$ and taking into account the mass and wave function renormalizations we obtain $\Sigma_{2(MS)}^R \sim k$. Therefore, as was already observed in [3], we see that in the $\overline{MS}$ scheme two-loop and one-loop contributions in the fermion self energy are of the same order in $k$.

Let us summarise our approach to relativistic baryon chiral perturbation theory.

To remove divergences from Feynman diagrams we use the forest formula of Zimmermann [14]. The forest formula is applied to individual diagrams and subtracts the overall divergence as well as the divergences corresponding to all subdiagrams. These subtractions can be implemented as counter-terms in the Lagrangian [14]. In performing actual calculations we do not necessarily need the explicit expressions for counter-terms; the subtraction scheme can be specified by pointing out the prescription for the finite parts. In this scheme the parameters of the Lagrangian are considered as finite renormalized coupling constants. In relativistic chiral perturbation theory instead of the widely used $\overline{MS}$ scheme we should apply a subtraction scheme which respects power counting. To do so we suggest the following strategy. First renormalise one-loop diagrams by expanding the analytic (in small momenta) parts in powers of a small momentum and perform covariant subtraction so as to cancel first few terms in the above mentioned expansion and respect the power counting. According to [14] the non-analytic parts readily obey power counting. For two-loop diagrams we first subtract one-loop subdiagrams and after expand analytic parts in powers of small momenta and again perform covariant subtraction so as to cancel the first few terms in the above expansion of the analytic part and respect the power counting. The non analytic parts remaining after the subdiagrams are subtracted respect power counting. For the three-loop diagrams the strategy is the same: first subtract one and two-loop subdiagrams, expand the analytic parts, etc. This iterative procedure is well defined for any number of loops. Within the suggested subtraction scheme the higher order diagrams do not contribute to lower order calculations. Consequently, coupling constants defined via low order calculations are not affected by higher order corrections. Our results remain valid when the pseudoscalar particle acquires a small mass. Note that in realistic model of baryon chiral perturbation theory one should fix finite parts of different counter-terms (specify prescriptions for subtractions) so as to respect corresponding Ward identities. As far as there are no anomalies it is always possible to satisfy this requirement within our scheme which is nothing else than the conventional renormalization with particular renormalization condition.

IV. CONCLUSION

We have demonstrated that by choosing an adequate renormalization scheme it is possible to retain the power counting in relativistic baryon chiral perturbation theory. Hence there is no necessity to invoke the heavy baryon approach. Although $\text{HB}_\chi\text{PT}$ substantially simplifies the calculations for many physical quantities, the corresponding perturbation series fails to converge in part of the low energy region [3] (this problem has been resolved by Becher and Leutwyler using the “infrared regularization” technique [12]). The original relativistic approach never encounters this problem. Hence both approaches enjoy their advantages and each has a full right to exist.
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APPENDIX A:

\[ z \equiv \frac{p^2}{M^2} \]

\[ J^{(01)} = \frac{1}{(2\pi)^n} \int \frac{d^nq}{[q^2 - M^2 + i\epsilon]} = \frac{-i}{(4\pi)^{n/2}} (M^2)^{n/2 - 1} \Gamma(1 - n/2) \quad (A1) \]

\[ J^{n+2(11)} = \frac{2}{(2\pi)^{n+1}} \int \frac{d^{n+2}q}{[q^2 + i\epsilon][(p + q)^2 - M^2 + i\epsilon]} = \frac{i}{(4\pi)^{n+1/2}} (M^2)^{n/2 - 1} \left\{ \frac{\Gamma(1 - n/2) \Gamma(n - 1)}{\Gamma(n)} \right\} \times 2F_1(1, 1 - n/2; 2 - n; 1 - z) + \Gamma(n/2) \Gamma(1 - n) (1 - z)^{n-1} 2F_1(n/2, n; n; 1 - z) \right\} \quad (A2) \]

\[ J^{(11)} = \frac{1}{(2\pi)^n} \int \frac{d^nq}{[q^2 + i\epsilon][(p + q)^2 - M^2 + i\epsilon]} = \frac{i (M^2)^{n/2 - 2}}{(4\pi)^n} \left\{ \frac{\Gamma(2 - n/2) \Gamma(n - 3)}{\Gamma(n - 2)} \right\} \times 2F_1(1, 2 - n/2; 4 - n; 1 - z) + \Gamma(n/2 - 1) \Gamma(3 - n) (1 - z)^{n-3} 2F_1(n/2 - 1, n - 2; n - 2; 1 - z) \right\} \quad (A3) \]

\[ J^{(1101)} = \frac{1}{(2\pi)^{2n}} \int \frac{d^nq_1 d^nq_2}{[q_1^2 + i\epsilon][q_2^2 + i\epsilon][(p + q_1 + q_2)^2 - M^2 + i\epsilon]} \]

\[ = \frac{(M^2)^{n-3}}{(4\pi)^n} \left\{ \frac{\Gamma(3 - n) \Gamma^2(n/2 - 1) \Gamma(2 - n/2)}{\Gamma(n/2)} + \frac{\Gamma(4 - n) \Gamma^2(n/2 - 1) \Gamma(3 - n/2)}{\Gamma(n/2 + 1)} \right\} \]

\[ \quad + \frac{1}{12} \left[ 6 - 2\pi^2 + 15z + 12\ln(1 - z) \left( \frac{(1 - z)^2}{z} + 2(1 - z) + 2\ln z \right) \right] \right\} \quad (A4) \]

\[ J^{\mu(1101)} = \frac{1}{(2\pi)^{2n}} \int \frac{d^nq_1 d^nq_2 q_1^\mu}{[q_1^2 + i\epsilon][q_2^2 + i\epsilon][(p + q_1 + q_2)^2 - M^2 + i\epsilon]} \]

\[ = -p^{\mu} \frac{(M^2)^{n-3}}{(4\pi)^n} \left\{ \frac{\Gamma(3 - n) \Gamma(n/2 - 1) \Gamma(2 - n/2) \Gamma(n/2)}{\Gamma(n/2 + 1)} \right\} \]

\[ \right\} \quad (A5) \]
\[ + \frac{\Gamma(4-n) \Gamma(n/2-1) \Gamma(3-n/2) \Gamma(n/2)}{\Gamma(n/2+2)} z + \frac{1}{24} \left[ \frac{61}{3} - 2\pi^2 - \frac{40}{3}(1-z) - 2(1-z)^2 - 2(1-z)^3 \right. \\
\left. - 2 \frac{(1-z)^4}{z} + 12 \text{Li}_2(1-z) + \ln(1-z) \left( 12 [1-z + \ln z] + \frac{6(1-z)^2}{z} - \frac{2(1-z)^3}{z^2} \right) \right] \] (A5)

\[ J^\mu(x) = \frac{1}{(2\pi)^2} \int \frac{d^n q_1 d^n q_2 \, q_1^\mu q_2^\nu}{[q_1^2 + i\epsilon][q_2^2 + i\epsilon] \left[ (p + q_1 + q_2)^2 - M^2 + i\epsilon \right]} = C_1 g^\mu\nu + C_2 p^\mu p^\nu \] (A6)

\[ C_2 = \frac{(M^2)^{n-3}}{(4\pi)^n} \left\{ \frac{\Gamma(3-n) \Gamma(2-n/2) \Gamma^2(n/2)}{\Gamma(n/2+2)} + \frac{\Gamma(4-n) \Gamma(3-n/2) \Gamma^2(n/2)}{\Gamma(n/2+3)} z \right. \\
\left. + \frac{1}{72} \left[ \frac{241}{12} - 2\pi^2 - \frac{5}{2}(1-z) - \frac{5}{2}(1-z)^2 - \frac{3}{2}(1-z)^3 - \frac{5}{2}(1-z)^4 \right. \right. \right. \\
\left. \left. \left. + \frac{(1-z)^4 (1+z)}{z^2} \right] + 12 \text{Li}_2(1-z) + \ln(1-z) \left( 12 [1-z + \ln z] + \frac{6(1-z)^2}{z} - \frac{2(1-z)^3}{z^2} + \frac{(1-z)^4}{z^3} \right) \right\} \] (A7)
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