Journey Beyond the Schwarzschild Black Hole Singularity

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We present the geodesical completion of the Schwarzschild black hole in four dimensions which covers the entire space in \((u,v)\) Kruskal-Szekeres coordinates, including the spacetime behind the black and white hole singularities. The gravitational constant switches sign abruptly at the singularity, thus we interpret the other side of the singularity as a region of antigravity. The presence of such sign flips is a prediction of local (Weyl) scale invariant geodesically complete spacetimes which improve classical general relativity and string theory. We compute the geodesics for our new black hole and show that all geodesics of a test particle are complete. Hence, an ideal observer, that starts its journey in the usual space of gravity, can reach the other side of the singularity in a finite amount of proper time. As usual, an observer outside of the horizon cannot verify that such phenomena exist. However, the fact that there exist proper observers that can see this, is of fundamental significance for the construction of the correct theory and the interpretation of phenomena pertaining to black holes and cosmology close to and beyond the singularities.

The Schwarzschild black hole is a spherically symmetric solution to the vacuum Einstein equations \(R_{\mu\nu}(g) = 0\)

\[
ds^2 = - \left(1 - \frac{r_0}{r}\right) dt^2 + \left(1 - \frac{r_0}{r}\right)^{-1} dr^2 + r^2 d\Omega^2. \tag{1}
\]

Here, \(r_0 \equiv 2GM\) is the radius of the horizon, \(G_N\) is the gravitational constant, and \(M\) is the ADM mass of the black hole. Although the Ricci and scalar curvatures are zero, the curvature tensor \(R_{\mu\nu\lambda\sigma}\) blows up at the \(r = 0\) singularity. The spacetime is better described in terms of the Kruskal-Szekeres coordinates,

\[
u = \pm \left|1 - \frac{2}{r_0}\right|^{1/2} e^{(r+t)/2r_0},
\]

\[
u = \pm \left[1 - \frac{2}{r_0}\right]^{1/2} \text{Sign} \left(1 - \frac{2}{r_0}\right) e^{(r-t)/2r_0}, \tag{2}
\]

that satisfy the following properties (+ corresponds to regions I&II and – to regions III&IV in Fig.1)

when \(uv < 1\) or \(r > 0\),

\[
u = \left(1 - \frac{2}{r_0}\right) e^{t/r_0}, \quad \nu = \text{Sign} \left(1 - \frac{2}{r_0}\right) e^{t/r_0},\]

\[
r = r_0R_+(uv), \quad t = r_0 \ln \left|\frac{u}{\nu}\right|, \quad R_+(uv) \equiv 1 + \text{ProductLog}[0, \frac{uv}{e}], \tag{3}
\]

\[
ds^2 = r_0^2 \left(2 - \frac{2}{r_0 uv} \right) (-2dudv) + R_+^2 (uv) d\Omega^2.
\]

The problem occurs both in the Einstein frame and in the string frame of general relativity, as well as in string theory when it uses geodesically incomplete background geometries in the worldsheet formulation (there would be incomplete string solutions similar to particle geodesics). Usually an appeal is made to “quantum gravity”, such as string theory, to resolve the problems at singularities. However if the worldsheet formalism of strings is already geodesically incomplete we may expect that such an incomplete version of string theory would also be subject to similar problems in its classical and quantum versions. Therefore we believe that a healthier approach is to first understand and improve the geodesic completeness of gravitational and string theories at the classical level and then study the quantum effects using the improved theories. Later in this paper we will connect to the general approach \([1][2]\) to construct geodesically complete gravitational and string theories.

We propose a geodesic completion of the Schwarzschild blackhole geometry as follows. We note that there is another solution of \(R_{\mu\nu}(g) = 0\) which looks just like Eq.(1) except for replacing \(r_0\) by \(-r_0\). This solution has no horizon and corresponds to a bare singularity and is usually discarded. We propose a new interpretation of this solution. We attribute the flip of sign of \(r_0\) to be due to the flip of sign of the gravitational constant exactly at the singularity, \(G_N \to -G_N\). This flip naturally occurs in general relativity interacting with matter improved with...
local scale (Weyl) invariance \(I\) as well as in the related improved string theory \(2\). We therefore suggest, consistently with \(I[2]\), that this solution must belong to regions \(V\) and \(VI\) that are behind the black or white hole singularities, and that those are antigravity regions where gravity is repulsive rather than attractive.

We will show that the gravity regions \((I, II, III, IV)\) and the antigravity regions \((V, VI)\) are geodesically connected by exhibiting the metric \(g_{uv}\) that spans the union of all regions and by displaying the complete set of geodesics in this geometry that go through the black/white hole singularity. To begin, we use new Kruskal-Szekeres coordinates to rewrite the solution

\[
ds^2 = \left[-\left(1 + \frac{r_0}{v}\right)\right] dt^2 + \left(1 + \frac{r_0}{v}\right)^{-1}\left[dr^2 + r^2 d\Omega^2\right].
\]

The unusual overall minus sign is needed for the continuity of the metrics in Eqs.(11) at \(r = 0\). The extra sign is typical of antigravity as explained in Eq.(19). See the discussion following Eq.(19) to address any concerns about ghosts. So, in regions \(V\&VI\),

\[
\begin{align*}
uw &= \left(1 + \frac{r_0}{v}\right)^{-\frac{1}{2}} e^{(r+t)/2r_0}, \\
\nu &= \pm \left(1 + \frac{r}{v}\right)^{-\frac{1}{2}} e^{(r-t)/2r_0}, \\
v &\equiv \pm \left(1 + \frac{r}{r_0}\right)^{-\frac{1}{2}} e^{r/r_0}, \\
\nuw &\equiv t = t_0 \ln \frac{\nu}{\nuw}, \\
\R_{uv} &\equiv \left(-1 - \text{ProductLog}[\frac{-1}{\text{ewh}}]\right), \\
\frac{dr^2}{r^2} &\equiv 2\nuw\left[(1 + \R_{uv})^{-1} \R_{uv} - \nuw^2 \right].
\end{align*}
\]

The function ProductLog\([k, z]\) corresponds to branches of the Lambert function in the complex plane. For \(k = -1\) our \(\R_{uv}\) is always real and positive when \(uv > 1\).

The union of the \(\nuw \leq 1\) regions is the metric

\[
ds^2 = r_0^2 \left[-2s\R_{uv}(uv)(-2du) + s^2R^2(uv)du^2\right],
\]

\[
R(uv) = s \left(1 + \text{ProductLog}\left[-\frac{1}{2}, (-uv)^s\right]\right),
\]

\[
(1 - sR)^s e^R = uv.
\]

A plot of \(R(uv)\) and its derivative \(R'(uv)\) is given in Fig.(2), showing that \(R\) is positive for all \(uv\), vanishes at the singularity \(uv = 1\), and approaches an infinite slope \(R' \to \pm \infty\) at that point.

Generally Eq.(8) has many solutions in the complex \(R\) plane. These are expressed in terms of the many branches of the well documented function ProductLog\([k, z]\). The branch in Eq.(8) is real and positive for all real values of \(uv\). We now show that the derivative \(R'(uv)\) for all \(uv\), including \(uv = 1\), is given by

\[
sR' = \frac{e^{-R}}{-R} \left(1 - sR\right)^{-1-s} = \frac{1 - sR}{-uv R}.
\]

With this form of \(R'\) the generalized metric in Eq.(9) agrees with the metrics in Eqs.(13) for \(uv < 1\) and \(uv > 1\), and also defines the new metric at the singularity \(uv = 1\). Some care is needed to verify Eq.(9) since the derivative of \(s\) is a delta function \(s' = -2\delta(1 - uv)\).

The simplest approach is to formally take the derivative of both sides of Eq.(8) for any \(R(uv)\) and \(s(uv)\),

\[
\left[(1 - sR)^s e^R\right]' = 1.
\]

Near the singularity, for small \(R\) and well behaved \(s\), the coefficient of \(s'\) in Eq.(10) has an expansion such that the \(s'\) term becomes,

\[
\left(s'\right)' = \left(1 + \frac{3}{2} sR + O\left((sR)^2\right)\right).
\]

With our \(s(uv)\) Eq.(8), and \(s'\) \(R(uv)\) \(\to 0\), the \(s'\) term in Eq.(10) vanishes for all \(uv\), including \(uv = 1\). The remaining \(R'\) term in Eq.(10) then shows that \(R'(uv)\) is given by Eq.(9) for all \(uv\). We record here the behavior of \(R\) and \(R'\) near the singularity and far away from it

\[
uv \sim 1 : \begin{cases} R \approx (2|1 - uv|)^{1/2} \\
sR' \approx -2(1 - uv)^{-1/2} \end{cases}
\]

\[
|uv| \sim \infty : \begin{cases} R \approx \ln |uv| \ln |uv| \text{Sign}(uv) \\
sR' \approx -\frac{sR}{uv} + \frac{1}{uv} \ln |uv| \text{Sign}(uv) \end{cases}
\]

noting that these are consistent with the plots in Fig.(2).

Remarkably, the metric in Eq.(8) is a solution of the vacuum Einstein equations, \(R_{\mu\nu}(g(u,v,\Omega)) = 0\), for all \((u,v,\Omega)\). By construction, we knew that we have a solution when \(uv \neq 1\). We remark that for a metric of the form \(g(u,v,\Omega)\), which is fully specified by a single function \(R(uv)\), the Ricci tensor vanishes automatically for any \(R(uv)\). In our case, for the specific form of \(R(uv)\) given in Eqs.(8), we obtain \(R_{\mu\nu} = 0\) for all \((u,v)\) including at the black and white hole singularities at \(uv = 1\).
To study the geodesics we now consider a test particle of mass \( m \) moving in this improved black hole background. The worldline Lagrangian has the form
\[
L = \frac{1}{2}g_{\mu\nu}(x) \dot{x}^\mu \dot{x}^\nu - e^{\nu_0} \Phi e^{-\nu_0} - \frac{m^2}{2} \Omega^2,
\]
where \( e(\tau) \) is the einbein. The constraint due to \( \tau \)-reparametrization is the equation of motion with respect to \( e(\tau) \). After choosing the gauge \( e(\tau) = r_0^2 \), which corresponds to interpreting \( \tau \) as a dimensionless proper time, the constraint takes the form
\[
-2sR'(uv)\left(-2\dot{u}\dot{v} + s R^2(\dot{u}^2 - \dot{v}^2)\right) + m^2r_0^2 = 0. \tag{12}
\]
This is the \( g^{\mu\nu}p_\mu p_\nu + m^2 = 0 \) constraint for our new extended black hole metric. The canonical conjugate to the solid angle \( \Omega \) (a unit vector) is related to angular momentum \( \vec{L} \), which is conserved due to rotational symmetry in the Lagrangian or the metric \( \Omega \). Furthermore, there is also a symmetry under opposite global rescalings of \( (u, v) \rightarrow (\lambda u, \lambda^{-1} v) \). This amounts to translations of the time coordinate, \( t/r_0 \rightarrow (t/r_0 + \ln \lambda^2) \), as seen from Eqs. (3,5). Hence, there is an additional conserved quantity, that amounts to the canonical conjugate to \( t/r_0 \), which is (up to a rescaling by \( r_0 \)) a dimensionless energy parameter \( E \). Taking the conserved quantities \( (E, \vec{L}) \) into account, we rewrite the constraint in Eq. (12) as follows (see derivation below)
\[
s \left( \frac{-E^2}{uvR'} + \frac{R'(\dot{u}^2 - \dot{v}^2)}{uv} + \frac{\vec{L}^2}{R^2} \right) + m^2r_0^2 = 0. \tag{13}
\]
This equation now involves a single time-dependent degree of freedom, namely \( (uv)(\tau) \), whose solution as a function of proper time \( \tau \) will determine all geodesics. The second order equations derived from the worldline Lagrangian (geodesics) are automatically solved by the solutions of this first order differential equation because they must obey the constraint (13). Hence this determines all geodesics for all possible initial conditions \( (E, \vec{L}) \) for a test particle of small mass \( m \).

To see how Eq. (13) is derived from the constraint Eq. (12), we need to clearly identify the canonical conjugate to \( t(\tau) \). For this purpose it is useful to transform to \( \tilde{t}(\tau) \), which is \( \sqrt{1 - \rho} \). After changing \( \tilde{t} \) unchanged as in Eqs. (3,5) and introduce, \( \rho = 1 - uv \), in the range \(-\infty < \rho < \infty \). Thus, the coordinate transformation and its inverse is
\[
\rho = 1 - uv, \quad \tilde{t} = \ln \left| \frac{\rho}{\sqrt{1 - \rho}} \right| \equiv \check{t}.
\]

After this change of coordinates the metric (10) becomes
\[
ds^2 = r_0^2 \left[ -s R'(\dot{t}^2 - \rho - 1 + d\rho^2(\rho - 1)^{-1}) + s R^2 d\Omega^2 \right].
\]

From the corresponding worldline Lagrangian we compute, \( E \), the canonical conjugate to \( \dot{t}(\tau) \), and express the conserved angular momentum, \( \vec{L} \), in terms of the angular velocity \( \dot{\Omega}(\tau) \)
\[
E = (\rho - 1) s R' \dot{t}, \quad \vec{L} = s R^2 \dot{t} \vec{\Omega}. \tag{14}
\]

After rewriting \( (\dot{u}, \dot{v}, \dot{\Omega}) \) in terms of \( (\dot{t}, \dot{\rho}, \vec{L}) \), the constraint (12) takes the form of Eq. (13).

Now, it is easy to get an intuitive understanding of the time development of \( (uv)(\tau) \), or equivalently \( \rho(\tau) = 1 - (uv)(\tau) \), by rewriting the constraint (13) in the form of a non-relativistic “Hamiltonian” \( \mathcal{H} \) (i.e. kinetic energy + potential energy) for one degree of freedom, subject to the condition that the corresponding “energy” level is zero, namely \( \mathcal{H} = 0 \) (the constraint), as follows
\[
\mathcal{H} \equiv \frac{1}{2} (\dot{t}^2 - \rho) + V(uv) = 0. \tag{15}
\]

This exercise identifies the potential \( V(uv) \)
\[
V(uv) \equiv \frac{u v}{2 R'(\tilde{t}^2 - s R^2 + m^2 r_0^2)} - \frac{E^2}{2 R^2}. \tag{16}
\]

Plots of \( V(uv) \) for small \( m^2 r_0^2 \) are given in Figs. (3,4).

![Fig.3](image1)

**Fig.3** - \( V(uv) \) for \( L = 0. \) Middle curve for \( m = 0. \)

![Fig.4](image2)

**Fig.4** - \( V(uv) \) for \( L \neq 0. \) Middle curve for \( m = 0. \)

The features of the plots follow from the approximate behavior near to and far from the singularity
\[
uv \simeq 1: V \simeq - \left[ \frac{uv (sL^2 + 2|1 - uv| m^2 r_0^2)}{2\sqrt{2\sqrt{1 - uv}}} + E^2 |1 - uv| \right],
\]
\[
uv \simeq \pm \infty: V \simeq \frac{uv^2}{2} \left[ -E^2 + sm^2 r_0^2 + \frac{E^2}{(\ln |uv|)^2} \right].
\]

In this potential, the dimensionless parameters \( (E^2, \vec{L}^2) \) may be considered as initial conditions for the particle.
of dimensionless mass \(m^2 r_0^2\). We omit the discussion of particles trapped in orbits around the black hole, that would occur when \(E^2 < m^2 r_0^2\), as this does not change our main points. Then, the asymptotic form of Eq. (18) for large \(|uv|\) is, \(-E^2 + p^2 + m^2 r_0^2 = 0\), indicating that \(E\) must satisfy \(E^2 > m^2 r_0^2\). We consider small values of \(|m^2 r_0^2\) since huge masses would violate the spirit that \(m\) represents a small probe for which the back reaction of the black hole can be neglected.

The constraint (15) is equivalent to a first order differential equation, \(\partial_u (uv) = \pm \sqrt{-2V(uv)}\), for the single variable \(uv\) whose solution is

\[
(\tau - \tau_0) = \pm \int_{uv_0}^{uv} \frac{dx}{\sqrt{-2V(x)}},
\]

where the ± signs are chosen according to whether the initial velocity is toward or away from the black hole. This expression can in principle be solved for \(uv\) as a function of \(\tau\), yielding the desired solution \((uv)(\tau) = F(\tau)\) where \(F(\tau)\) is fully determined. Although this appears to be complicated we note that from the plots of \(V(uv)\) alone we can easily obtain an intuitive feeling of all possible motions that \((uv)(\tau)\) can perform.

Consider the case of zero angular momentum \((L = 0, \text{Fig.}3)\). A particle that obeys the constraint in Eq. (15) has the same \(\mathcal{H}\)-energy level as the summit of the "mountain". Such a particle that comes from region \(I\) where \(uv < 0\) (approaching from the left in Fig. 3) will keep climbing the \(V(uv)\) mountain, passing into region \(II\) at the horizon at \(uv = 0\), and (in the case of \(m^2 \geq 0\) reaching the peak of the mountain where it slows down and stops momentarily at the \(uv = 1\) singularity. Note that at the peak the potential vanishes, \(V(uv = 1)|_{L=0} = 0\), and therefore \(\partial_v (uv) = 0\) to satisfy the constraint (15). In fact, at the singularity, \(\dot{u} = \dot{v} = 0\) when we examine the rest of the equations of motion that follow from the worldline Lagrangian \(L\), so the particle stops temporarily at the summit of the mountain in Fig. 3. This journey takes a finite amount of proper time \(\tau\) because the integral in Eq. (17) is finite. Clearly the summit is an unstable point, so at the subsequent moment in proper time, the particle will either slide forward to \(uv > 1\) down the mountain into region \(\mathcal{V}\) where there is antigravity, or slide back to \(uv < 1\) into region \(II\) and then \(III\) where there is gravity. It will not slide back into regions \(II\) and then \(I\) because this would cause closed timelike curves, and indeed this can be deduced from analytic investigations. Forward \(uv\) or backward \(iw\) at \(uv = 1\) are allowed solutions of the geodesic equations of motion, so both of them will happen. In either case the particle moves on to another world that is geodesically connected to the original starting point in region \(I\). This shows that particles that fall into the black hole (beyond the horizon) will inevitably end up in a new universe according to this classical analysis. Note that gravity observers in region \(III\) will interpret that the particle comes out of a white hole, while those in the antigravity region \(V\) will interpret it as coming out of a naked singularity.

A quantum analysis that treats a small probe in a static black hole (as in the present case) will reach the same conclusion and provide non-vanishing probability amplitudes for transmission to regions \(III\) and \(V\) (for tachyons as well). The computation can be performed in a WKB approximation just as in non-relativistic potential theory as presented elsewhere.

Next we consider non-zero angular momentum \((L^2 \neq 0, \text{Fig.}4)\). In this case the particle coming from regions \(I\&II\) hits an angular momentum barrier at \(uv = 1\), so classically it can only bounce back to regions \(II\&III\). However, quantum mechanically there will be a non-zero transmission probability to also tunnel into region \(V\).

To be fully convinced of our intuitive analysis it is useful to have some analytic expressions for the geodesics. This looks complicated in 4-dimensions although there is no problem numerically. However, for the closely related 2-dimensional stringy black hole, after geodesically completing its space-time as we did above, we have explicitly constructed the full set of geodesics ofting classical analysis. This work, together with the corresponding quantum computation, will be reported in a separate publication.

In a similar way we can also discuss the geodesics whose initial conditions begin in the regions \(V\) or \(VI\).

We now give a short description of how our geodesically complete black hole spacetime in Eq. (18) fits perfectly with the Weyl symmetric re-formulation of geodesically complete gravity (SM+GR) [1] and string theory (ST) [2]. We concentrate only on the basic consequence of the Weyl symmetry, which is that dimensionful parameters are not allowed. All dimensionful constants of phenomenological significance, including the Newton constant (and therefore the string tension) emerge from Weyl-gauge fixing of some gauge degrees of freedom [1,2]. As an illustration, consider the case of the SM+GR which contains the SU(2) × U(1) Higgs doublet \(H\) and an additional singlet scalar \(\phi\) required by the Weyl-symmetric approach. \(\phi\) is compensated by the Weyl symmetry, so \(\phi\) is not a true additional degree of freedom, but participates in an important structure of the symmetry that has physical consequences. Due to the symmetry all scalars are “conformally coupled”, implying the special non-minimal coupling to the curvature

\[
\frac{1}{12} (\phi^2 - s^2) R(g), \text{with } s^2 \equiv 2H^4H.
\]

This structure is the same in low energy ST [2] but with a different interpretation of \(s\). The relative minus sign in Eq. (18) is obligatory. Weyl symmetry requires that, with the signs above, \(\phi\) has the wrong sign kinetic energy while \(H\) has the correct sign. So, \(\phi\) is a ghost but, since it can be removed by a Weyl gauge choice, this is not a problem.
If \( \phi \) were not a ghost then the curvature term would have a purely negative coefficient, \(-\frac{1}{\kappa} (\phi^2 - s^2)\), which leads to only a purely negative gravitational constant, so there are no alternatives to \( \kappa \). Therefore, the effective Planck mass \( \frac{1}{\kappa} (\phi^2 - s^2) \) (or the Newton constant) is not positive definite. At the outset of this approach in 2008 the immediate question was whether the dynamics would allow \( (\phi^2 - s^2) \) to remain always positive. It was eventually determined in 2010-2011 (references in [1] [2]) that the solutions of the field equations that do not switch sign for this quantity are non-generic and of measure zero in the phase space of initial conditions for the fields \( (\phi, s) \). So, according to the dynamics, it is untenable to insist on a limited patch of field space. By contrast, it was found that the theory becomes geodesically complete when all field configurations are included, thus solving generally the basic problem of geodesic incompleteness.

With a gauge choice, extra Weyl gauge degrees of freedom can be removed, but one can err by choosing an illegitimate gauge that corresponds to a geodesically incomplete patch. Indeed this is what happens in the “Einstein gauge” \( (E) \) and in the “string gauge” \( (s) \) [1] [2].

\[
\begin{align*}
E\text{-gauge: } & \frac{1}{12} (\phi_{E+}^2 - s_{E+}^2) = \frac{1}{16\pi G_N} \quad \text{ or } \quad \frac{d-2}{8(d-1)} (\phi_{E+}^2 - s_{E+}^2) = \frac{1}{2\kappa} e^{-2\Phi}, \quad \Phi = \text{dilaton.}

s\text{-gauge: } & \frac{1}{12} (\phi_{s-}^2 - s_{s-}^2) = -\frac{1}{16\pi G_N}, \quad \text{ or } \quad \frac{d-2}{8(d-1)} (\phi_{s-}^2 - s_{s-}^2) = \frac{1}{2\kappa} e^{-2\Phi}; \text{ in those spacetime regions gravity is repulsive (antigravity). In the corresponding worldsheet formulation of string theory the string tension also switches sign } [2]. \text{ Thus the Weyl symmetric (SM+GR) or string theory predict that, in the Einstein or string gauges, one should expect a sudden sign switch of the effective Planck mass } \frac{1}{12} (\phi^2 - s^2) \text{ at certain spacetime points that typically correspond to singularities (e.g. big bang, black holes) encountered in the Einstein or string frames. As shown in [1] [2] one may choose better Weyl gauges (e.g. “\( \gamma \)-gauge”, choose } \det (-g) \rightarrow 1, \text{ or “c-gauge”, choose } \phi \rightarrow \text{constant) that cover globally all the positive and negative patches. Then the sign switch of the effective Planck mass } \frac{1}{12} (\phi^2 - s^2) \text{ is smooth rather than abrupt. However, if one wishes to work in the Einstein or string frames, as we did in this paper, to recover the geodesically complete theory one must allow for the gravitational constant to switch sign at singularities, as in Eq. (17), and connect solutions for fields across gravity/antigravity patches. In the \( \pm \) Einstein gauges Eq. (18) becomes } \frac{(\phi_{E\pm}^2 - s_{E\pm}^2)}{12} R (g_{E\pm}) \quad \pm \frac{R (g_{E\pm})}{16\pi G_N} \frac{1}{16\pi G_N}. \quad (19)
\end{align*}
\]

where the \( \pm \) for the gravity/antigravity regions can be absorbed into a redefinition of the signature of the metric, \( g_{E\pm} = \pm g_{E\pm} \) [2]. Our new black hole in Eq. (4) is for the continuous \( g_{E\pm} \) in the union of the gravity/antigravity patches. This explains the extra minus sign in Eq. (1) and connects it to the underlying Weyl symmetric theory.

One may be worried that the sign switches of the gravitational constant or the string tension may lead to problems like unitarity or negative kinetic energy ghosts. For example, in the SM+GR action in the \( \pm \) Einstein gauge, some terms in the antigravity sector flip sign and some don’t [2] when \( g_{E\pm} \rightarrow -g_{E\pm} \) (e.g. \( F_{\mu\nu} F^{\mu\nu} \) does not, but \( R (g) \) does as in [19]). We should mention that Ref. [2] has already settled that there are no unitarity problems due to sign flips in field/string theories. As to the negative kinetic energy concerns in antigravity (as in \( -R^2 (uv) \Omega^2 \) in [3]), this apparent instability is rendered harmless by insisting that the only reasonable interpretation of the theory is by observers in the gravity sector (details and examples in an upcoming paper [3]). Such observers cannot experience the negative kinetic energy in antigravity directly, but can only detect in and out signals that interact with the antigravity region. This is no different than a closed spacetime box for which the information about its interior is scattering amplitudes for in/out states at its exterior. An analogous situation for a cosmological singularity [1][2] is treated in detail in [3]. So, there are no issues of fundamental principles.

We have demonstrated that the Schwarzschild black-hole has a geodesic completion, and that proper observers can in principle travel through the singularity. These results generalize to black holes in other dimensions as we will demonstrate in additional papers. New avenues have just opened for information to travel beyond the horizon and singularity. A similar result that was first obtained for cosmological singularities has been applied to develop a completely new perspective for the role of antigravity just before the big bang (see references in [1] [2]). Similarly, our findings must have implications for our understanding of black holes, the role of the geodesically complete spacetime in their formation and evaporation, the information loss problem, and for investigating how new physics beyond singularities in classical and quantum gravity/string theory impacts observations in our own universe. We are at the beginning of a large project.

\[\text{[1] I. Bars, P. Steinhardt, N. Turok, Phys. Rev. D89 (2014) 043515. \quad [2] I. Bars, P. Steinhardt, N. Turok, Fortsch. Phys. 62 (2014) 901. \quad [3] I. Bars, conference lectures at “String Field Theory-2015” (Sichuan Univ.), and “Convergence” (Perimeter Inst.). \quad [4] S. Gielen, N. Turok, “A perfect bounce”, arXiv: 1510.00699.}\]