Detection and localization of multiple small damages in beam

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Abstract
Localizing small damages often requires sensors be mounted in the proximity of damage to obtain high Signal-to-Noise Ratio in system frequency response to input excitation. The proximity requirement limits the applicability of existing schemes for low-severity damage detection as an estimate of damage location may not be known a priori. In this work it is shown that spatial locality is not a fundamental impediment; multiple small damages can still be detected with high accuracy provided that the frequency range beyond the first five natural frequencies is utilized in the Frequency response functions (FRF) curvature method. The proposed method presented in this paper applies sensitivity analysis to systematically unearth frequency ranges capable of elevating damage index peak at correct damage locations. It is a baseline-free method that employs a smoothing polynomial to emulate reference curvatures for the undamaged structure. Numerical simulation of steel-beam shows that small multiple damages of severity as low as 5% can be reliably detected by including frequency range covering 5–10\textsuperscript{th} natural frequencies. The efficacy of the scheme is also experimentally validated for the same beam. It is also found that a simple noise filtration scheme such as a Gaussian moving average filter can adequately remove false peaks from the damage index profile.

Keywords
Structural damage detection, vibration-based methods, baseline-free FRF curvature method, smoothing, frequency ranges, Gaussian moving average filter

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Introduction
Structural health monitoring (SHM) utilizes vibrational characteristics of the structure for damage detection. These characteristics are described by modal parameters such as natural frequency, mode shapes, modal curvature and modal strain energy. The damage is indicated by precise measurement of the slight deviations in modal parameters values with reference to their nominal range for an ideal healthy structure. This change in modal parameters is often attributed to stiffness loss which indicates the damage along with its location and severity. Unfortunately, these parameters usually exhibit only a minor deviation which limit the application to severe damages only.\textsuperscript{1} Consequently, early stage damages, typically less severe in nature, would go undetected in several real-life scenarios. Among the most effective and reliable schemes of localizing structural damages, the FRF curvature method is known for circumventing above modal

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parameters constraints as well as addressing limitations of SHM experimental setup such as sensor noise and unmodeled disturbances. Vast majority of prior studies utilized FRF curvature method employed the knowledge of reference curvature shapes of the undamaged structure. However, such baseline information may not be available in practice, which limit the scope of these studies. In addition, sensor noise introduces further uncertainty in damage location if we obtain curvatures using only lower natural frequencies in the FRF. Such inaccuracy is far more pronounced in case of multiple-damage scenarios naturally prone to false positives indicators of damage locations. Placing sensor closer to the damage location can help mitigate the impact of noise by boosting signal to noise ratio, however, doing so would require somewhat unrealistic presumption that damage likelihood profile is known a priori. These problems call for damage detection method capable of sensing multiple and low severity damages in the presence of noisy FRF.

To overcome these limitations, a baseline-free FRF curvature method is presented which can locate single as well as multiple damages of various level severities in the beam. The method fits third-degree smoothing polynomial to noisy FRF to generate sanitized FRF providing cleaner reference curvatures. However, the main contribution of this work lies in systematic exploration of high frequency region of FRF usually ignored in prior studies. Even though such region has lower power spectrum density, its low signal-to-noise ratio can be compensated for by measuring FRF at more responsive segments of the beam. These were identified by the sensitivity analysis for four different set of frequencies. The results show that high natural frequencies can significantly attenuate false damage-index peaks for low-severity and multiple damage scenarios. Once false peaks are removed, basic filtration scheme such as Gaussian-weighted moving average filter can raise correct damage-index peak above noise floor.

**Literature review**

Earlier studies and results are founded on utilizing mode shapes and their curvatures to detect and localize damage using damage index metric computed with the help of double derivative of displacement mode shape. The approach indirectly approximates curvature of the mode shape and becomes susceptible to amplification of approximation error resulting from differentiation applied twice. Several prior studies have proposed statistical techniques to suppress these errors and elevate damage index peak. Alternatively, curvature errors can be avoided by utilizing strain mode shapes for damage detection in beams and plate-type structures. Yet, the requirement of mode shape data of the healthy structure for comparison with that of damaged structure, makes these schemes dependent on baseline information. Some researchers have used model updating methods based on iterative process or drawn a comparison with a numerical model and acquired better identification by considering more number of nodes. However, it entails very accurate model to generate meaningfully precise results. To overcome this problem, researchers investigated ways of making traditional methods baseline-free. An instantaneous baseline measurement was introduced to detect single damage in the form of removable putty, corrosion, and a cut in a square plate. The pitch-catch Lamb wave propagation was used to acquire undamaged sensor-actuator path information that served as baseline measurement. Similarly, an ultrasonic response measured at one particular point on a structure was compared with the responses at other adjacent points to detect damage without using baseline data. The method was able to detect single crack in an aluminum plate, delamination, and de-bonding in a glass fiber reinforced polymer (GFRP) by using laser scanner vibrometer. The data extraction was further improved by employing 3D laser scanner vibrometer that captured transient 3D displacements and velocities. Although laser scanner vibrometer provides high spatial data with comparatively better accuracy but the data extraction process is quite time consuming even for smaller structures. That is why it is often not feasible to be used for large structures.

Although several traditional baseline-free methods successfully detected single damage, their efficacy is compromised when reused for multiple damages especially for low severities. A study was carried out by comparing residual sum of squares of each solution of vibration response and the assumed number of cracks in a beam. The study successfully localized the multiple cracks of 20% and 30% severity, however, the proposed method being reliant on iterative numerical solution, proved sensitive to initial estimate of crack location. Another way of achieving baseline-free detection was the application of polynomial-fitting to generate smoothed data. This concept was applied in the form of multi-segment function-fitting to generate actual deflection influence line (DIL) of a damaged beam. The damage was indicated by calculating the difference between actual DIL and a constructed function. In the similar manner, a smoothing polynomial called Gapped smoothing polynomial, was fitted to the curvature mode shapes to generate damage index. The same concept has been used to detect multiple cracks up to 10% severity in beams. The main aim in these methods is to get a smoothed data that can be used as a reference, representing data from healthy structure. The squared difference of the smoothed curvature and measured curvature yields a damage index,
the peak values of which indicate the damage location/s. As this method is based on curvature mode shapes, thus it also has the same limitations associated with the approximations to obtain curvatures. It works well on noise-free data, but its effectiveness varies with the noise in the measured data. Another important consideration lies in mitigating the effect of noise and disturbances.

To overcome the problem of noise and to exploit the benefits of curvature mode shapes, a noise suppression method was used to reduce the effects of noise in the damage indices generated from Gapped smoothing method (GSM). The research also highlighted the significance of resampling the data in case of less severe damage. The same approach of noise suppression was used by employing Gaussian Kernel on FRFs obtained from beam structure. Recently, Gharehbaghi et al. used digital filters such as Finite-duration impulse response (FIR) and Infinite-duration impulse response (IIR) filters to eliminate noise from the recorded acceleration time histories. These filters can successfully eliminate noise from the specific portion of the response signal which is more contaminated with noise. In another study, the curvature mode shapes were improved by using a synergy of wavelet transform (WT) and Teager energy operator (TEO). From the simulated analysis, the proposed approach was able to detect three cracks with severity of 20%, 25%, and 30% in a steel beam. These results indicated that the higher modes provided good detection as compared to lower modes at same noise level. The higher modes such as fifth and seventh modes were used to generate TEO-WT curvature mode shapes which illustrated peaks at the three crack locations along with some smaller false peaks.

The accuracy of the curvatures is based on the quality of extracted mode shapes. These fundamental mode shapes are usually extracted from FRFs at the natural frequencies of the structure and thus, the extraction process also incurs some errors which are enhanced in the curvature mode shapes. Hence, to avoid this process FRFs and their curvatures can be used directly to detect structural damage. FRF curvature method has been used to detect single damage in a beam-type structure numerically and validated experimentally on a bridge. The damage was indicated by comparing the FRFs of both undamaged and damaged structure. The FRF curvature method relies on the frequency range considered to calculate the damage index. This study found the range of first anti-resonance in the FRFs to be more effective than any wider range of frequencies. However, on the contrary the findings of another study indicated the broadband range of FRF curvature be more effective to detect single small sized damage in beam. The broadband range spanned to include 15 natural frequencies of the beam. As discussed above that the chosen frequency range is important in the FRF curvature method. In addition to the first anti-resonance range and broadband range, a high coherence range has also been introduced in FRF curvature method. High coherence indicates the quality of FRFs and hence a good quality FRF within high coherence range is expected to generate comparatively better results. The study investigated five frequency ranges including the range before first resonance, across first resonance, across second resonance, broadband range with first four natural frequencies and high coherence range. Among the three damage cases of single and multiple damage with 5%, 12.5%, and 25% severity, only high coherence range was able to detect single damage of 5% severity. While the two ranges, the range before first resonance and high coherence were able to detect multiple damage up to the severity of 12.5%. However, for accurate detection of multiple damage the sensor was placed quite closer to the damage location which was 5 mm near a damage of 12.5% severity. In real case scenarios, the location of damage is not known so this method can be used effectively, only when the measurement points are spaced equal to or smaller than the size of the damage. Moreover, it requires the baseline information to generate the FRF curvature damage indicator. A topical research employs Multiple signal classification algorithm to detect three different type of damages in a 3D five-bay truss structure. The suggested technique provides a simple way of detection and localization in which Pseudo-spectra of healthy and damaged bay are obtained and compared, however the method is dependent on baseline information.

**Methodology**

**Baseline-free FRF curvature method**

In general, the FRF curvature method employs the FRFs which may be in the form of displacements, velocities, or accelerations at various points on the structure for a specified frequency range. Instead of using the complex form of FRF, its imaginary part is used in this paper. These imaginary parts of FRFs – in the form of displacements – are then used to find the FRF curvatures by applying second-order central difference approximation as given in equation (1).

\[
H'(\omega)_{i,j} = \frac{H(\omega)_{i-1,j} - 2H(\omega)_{i,j} + H(\omega)_{i+1,j}}{h^2}
\]

where \(H'(\omega)_{i,j}\) and \(H(\omega)_{i,j}\) refer to the FRF curvature and imaginary part of FRF measured at a point \(i\), respectively when the excitation force is applied at point \(j\) on the beam. In equation (1), \(h\) is a constant, representing a distance between the two consecutive measurement points. Conventionally, the residual FRF
curvature $\Delta H'_{i,j}$ is obtained by taking the absolute difference between damaged and undamaged structure as,

$$\Delta H'_{i,j} = \sum_\omega |H_D'(\omega)_{i,j} - H'(\omega)_{i,j}|$$

(2)

where $H'_D(\omega)_{i,j}$ and $H'(\omega)_{i,j}$ refer to the FRF curvature of the damaged and undamaged structure, respectively. Damage index ($DI$) at a point $i$ can be derived by summing up the residual FRF curvatures obtained in equation (2).

$$DI_i = \sum \Delta H'_{i,j}$$

(3)

In this paper, instead of using data from undamaged structure, a reference is generated by fitting the FRF curvatures from the damaged structure with a cubic polynomial. The same idea has been used in several studies with modal curvatures, modal strain energy and some other parameters. The smoothed FRF curvature value $p_i$ at each point $i$, with $x_i$ being the position of the $i$th point, can be obtained as,

$$p_i = p_0 + p_1x_i + p_2x_i^2 + p_3x_i^3$$

(4)

The damage index can be obtained by finding the squared difference between the damaged and smoothed FRF curvature.

$$DI_i = (p_i - H_D(\omega)_{i,j})^2$$

(5)

The larger value of $DI$ depicts the presence of damage at that point, even a small irregularity is enough to generate a remarkable difference. However, noise in the measured FRFs may also cause the same peaks giving a false indication of damage. The other important aspect in FRF curvature method is to choose the proper frequency range. This is because the effectiveness of the method greatly depends on the selection of frequency range in the FRFs. Here, as the method is modified to be a baseline-free method, hence the effectiveness of the method must be investigated for different frequency ranges. The frequency ranges suggested in the literature$^{23-25}$ along with their description are given below and illustrated in Figure 1.

$f_0$ = frequency range immediately before the first resonance.
$f_1$ = frequency range across 1st resonance.
$f_2$ = frequency range across 2nd resonance.
$f_{WR}$ = a wide frequency range that includes the first four resonances.
$f_{HCR}$ = frequency range at high coherence (only for experimental data)

Figure 1. Four frequency ranges $f_0$, $f_1$, $f_2$ and $f_{WR}$ of the FRFs indicated by 1, 2, 3 and 4, respectively.

Sensitivity analysis for each frequency range

In addition to the regions around nodal points, there are certain regions on the beam where the response is very small. The results in the form of damage indices around these regions are often false, with peaks typically dominating the useful information. These sensitive/insensitive regions vary for the frequency range across each resonance. Therefore, a sensitivity analysis is carried out considering each frequency range. The main aim of this analysis is to find the sensitive regions along the beam length and assigning low weightage to the data from the insensitive regions. This way the useful information can be highlighted for each frequency range. The sensitivity $s_i$ at point $i$ is calculated by using the imaginary part of the FRF displacements $H_D(\omega)$ for damaged beam as,

$$s_i = \sqrt{\sum_{k = a}^b [H_D(\omega)_{i,k}]^2}$$

(6)

In equation (6), $a$ and $b$ refer to the limits of each frequency range, illustrated in Figure 1. The normalized sensitivity $s_{normi}$ at point $i$ is then computed as

$$s_{normi} = s_i/(s)_{max}$$

(7)

Figure 2 illustrates the normalized sensitivity of a cantilever beam (fixed from left end) for each frequency range. This illustration clearly shows that the sensitivity of the beam gradually decreases from 40% to 0% from middle region of the beam to the left end for $f_0$, $f_1$, and $f_{WR}$. While, for $f_2$ the sensitivity is less than 40% from 0 to 25% and 68 to 88% of beam length. This means that the damage is most likely to be detected if it occurs within these regions where the sensitivity is better. It is observed from this analysis that for most frequency ranges the beam is less sensitive ($<5\%$ for $f_0$, $f_1$, and $<10\%$ for $f_{WR}$) in the region closer to the fixed support ($0-20\%$ of beam length) and the sensitivity increases toward the free end of the beam except for $f_2$ case. This
is because for cantilever case, the lateral displacement is zero at the fixed end and it gradually increases toward the free end of the beam. The FRFs employed here are displacement/force (receptance), hence the response is also minimal around the support which results in unreliable peaks in the damage indices around this region. Thus, the damage index from equation (5) can be modified as,

$$\overline{DI}_i = \begin{cases} 0, & 0 \leq x \leq 0.2; \\ DI_i, & x > 0.2 \end{cases}$$ (8)

The modified damage index $\overline{DI}_i$ computed in equation (8) is then unit normalized, so that damage indices from all frequency ranges can be compared. A comparison of original and modified damage index is illustrated in Figure 3. In most frequency ranges, there is a dominant peak of damage index at the fixed end of the beam. Particularly in $f_2$, all peaks are nearly diminished at the other locations due to the dominant false peak. The modified damage index highlighted the existing peaks at the other locations of the beam by suppressing the peak around the support end. The difference is quite noticeable for $f_2$ in Figure 3(c).

**Noise suppression in FRFs**

The sensitivity analysis presented in the previous section is based on nodal points and boundary condition for each frequency range and that is even applicable to the noise-free data. However, in addition to nodal points the false peaks in the damage indices could also be due to noise in the measurement data. Hence, it is important to reduce noise in the measured response before using it to generate damage indices. Gaussian kernel and moving average filters have been used with FRFs in several research works.\textsuperscript{20,27} In this paper, a Gaussian-weighted moving average filter (GMAF) is employed by using “smoothdata” function in MATLAB. The window size of 10 is used in the moving average filter. The standard deviation is one-fifth of the window size, while mean is zero. A typical FRF with 5% noise and filtered FRF is shown in Figure 15.

An example of a damage index $DI$ for multiple damage case at locations D1 and D2, is shown in Figure 4. The damage index ($DI_{\text{original}}$) generated using the original noisy FRFs indicates many false peaks. While, the damage index ($DI_{GMAF}$) from filtered FRFs using GMAF clearly points out two damage locations, however that are accompanied by a dominant false peak on the left end around fixed support. In most cases, this false peak is dominant enough to conceal all other useful information. The modified damage index based on $f_{WR}$, obtained from equation (8) using the filtered FRFs after noise suppression ($DI_{\text{modified+GMAF}}$), noticeably indicates the two damage peaks. From this section onwards, all the results presented in the paper are modified damage indices using the filtered FRFs.

**Numerical simulations**

A simulated experiment is carried out in ANSYS on a cantilever steel beam with its length $L$, width $w$, and

![Figure 2. Sensitivity analysis of a cantilever beam based on different frequency ranges.](image)

![Figure 3. Original and modified damage indices after sensitivity analysis based on different frequency ranges.](image)

![Figure 4. Damage indices for a multiple damage case based on $f_{WR}$.](image)
height \( h \) as 1000, 20, and 50 mm, respectively. The element is Solid Quad 4 node 182 and the material properties are Young’s Modulus \( E = 200 \text{ GPa} \), Poisson’s ratio \( v = 0.3 \), and mass density \( \rho = 7850 \text{ kg/m}^3 \). The beam is supported from left end and a constant force of 10N is applied at the free end of the beam as shown in Figure 5.

Several damage scenarios consisting of single damage (SD) and multiple damage (MD) with varying severities are modeled in beam, as detailed in Table 1 and illustrated in Figure 6. The damage severity is defined as the percentage ratio of depth of the damage \( d \) to the height \( h \) of the beam. A convergence analysis is performed on an undamaged beam model for proper mesh size selection. The mesh size is varied by adjusting the element edge length. It is evident from Tables 2 and 3, that element edge length of 1 mm gives a good agreement between analytical and simulated natural frequencies, so a standard mesh size 1 mm is chosen for all the analyses. The error calculated between analytical and simulated natural frequencies varies from 0.053% to 0.85% based on first to fourth natural frequency, respectively. The first four analytical frequencies \( \omega_n \) of the cantilever beam are calculated by using equation (9) as:

### Table 1. Damage scenarios for simulated beam experiments.

| Damage scenario | Severity \((d/h) (%)\) | Location     |
|-----------------|------------------------|--------------|
| SD1             | 50                     | 800 mm       |
| SD2             | 25                     | 800 mm       |
| SD3             | 50, 25                 | 450 mm       |
| SD4             | 15                     | 800 mm       |
| SD5             | 5                      | 800 mm       |
| MD1             | 50                     | \(D_1 = 450 \text{ mm}; D_2 = 800 \text{ mm}\) |
| MD2             | 25                     | \(D_1 = 450 \text{ mm}; D_2 = 800 \text{ mm}\) |
| MD3             | 5                      | \(D_1 = 450 \text{ mm}; D_2 = 800 \text{ mm}\) |

### Table 2. Comparison of analytical and numerical natural frequencies of the undamaged beam model for varying mesh size.

| Natural frequencies | Analytical | Numerical model with different element edge length |
|---------------------|------------|--------------------------------------------------|
|                     | 15 mm      | 10 mm    | 5 mm    | 1 mm    |
| 1st                 | 16.3075    | 18.157   | 17.444  | 16.553  | 16.3161 |
| Error (%)           | 11.34%     | 6.97%    | 1.51%   | 0.053%  |
| 2nd                 | 102.198    | 113.617  | 109.29  | 103.55  | 102.062 |
| Error (%)           | 11.17%     | 6.94%    | 1.32%   | 0.13%   |
| 3rd                 | 286.153    | 317.46   | 305.64  | 289.13  | 284.929 |
| Error (%)           | 10.94%     | 6.81%    | 1.04%   | 0.428%  |
| 4th                 | 560.721    | 620.274  | 597.52  | 564.28  | 555.954 |
| Error (%)           | 10.62%     | 6.56%    | 0.635%  | 0.850%  |

### Table 3. Comparison of analytical and numerical natural frequencies of the damaged and undamaged beam.

| Natural Frequency (Hz) | Undamaged | Damage scenarios with varying damage severities |
|------------------------|-----------|-----------------------------------------------|
|                        | Analytical | Numerical | SD:5% | SD:15% | SD:25% | SD:50% | MD:5% | MD:25% | MD:50% |
| 1st                    | 16.3075    | 16.3161  | 16.32  | 16.328 | 16.335 | 16.347 | 16.314 | 15.864 |
| Error (%)              | 0.0527%    | 0.02%    | 0.07%  | 0.12%  | 0.18%  | 0.01%  | 0.46%  | 2.77%  |
| 2nd                    | 102.198    | 102.062  | 102.038| 101.928| 101.723| 100.29 | 101.938| 94.168 |
| Error (%)              | 0.33%      | 0.13%    | 0.13%  | 0.33%  | 1.37%  | 0.12%  | 1.85%  | 8.57%  |
| 3rd                    | 286.153    | 284.929  | 284.674| 283.468| 282.989| 281.219| 283.311| 262.649|
| Error (%)              | 0.42%      | 0.51%    | 0.13%  | 1.30%  | 6.53%  | 0.10%  | 1.50%  | 7.82%  |
| 4th                    | 560.721    | 555.954  | 555.218| 551.671| 545.251| 509.864| 554.755| 489.087|
| Error (%)              | 0.85%      | 0.72%    | 0.13%  | 0.97%  | 1.93%  | 8.29%  | 0.22%  | 3.08%  | 12.03% |
\[
\omega_n = (\beta_n L)^2 \left( EI/\rho AL^4 \right)^{1/2}
\]  

(9)

Where \( \beta_n L \approx (2n - 1)\pi/2 \), with \( n = 1 - 4; L, I, \) and \( A \) refer to the length, moment of inertia and cross-sectional area of the cantilever beam, respectively. The measurement data were extracted at 19 points with 55 mm spacing, with the 20th point at 10 mm spacing at the free end of the beam. The spacing is chosen in such a way that the measurement location is at least 10 mm away from the damage location.

**Results and discussion**

Since the effectiveness of the FRF curvature method relies on the chosen frequency range of FRFs, hence various frequency ranges such as \( f_0, f_1, f_2, \) and \( f_{WR} \) as mentioned earlier, are investigated for every scenario. Considering these frequency ranges, modified damage indices from the filtered FRFs are generated using a baseline-free FRF curvature method. These damage indices are shown in Figures 7 to 14 for all scenarios.

The method worked well for all frequency ranges for both 50% and 25% severe single damage cases (SD1 and SD2) as can be seen in Figures 7 and 8. Among these frequency ranges, the damage index is showing double peaks only with \( f_0 \). To check if the effectiveness of the method changes with the location of damage, damage indices are generated for a mid-span damage (SD3). From Figure 9, it is observed that except \( f_{WR} \) and \( f_2 \) none of the other two ranges provide any good results. Even \( f_2 \) show false side peaks along with one true peak for both 50% and 25% damage severity. The damage indices for all frequency ranges are better for the damage location closer to the free end of the beam as compared to the mid-span damage in a cantilever beam. Hence the effectiveness of using these frequency ranges varies with the location of the damage and none of them except \( f_{WR} \) are as effective for mid-span damage for cantilever case. However, this sensitivity to mid-span damage changes based on the boundary condition as will be described in the later section.

As the severity of the damage is further reduced (SD4), none of the above ranges provide correct damage localization as can be seen in Figure 10. To check how the detection can be improved, only \( f_{WR} \) is assessed because \( f_0, f_1, \) and \( f_2 \) cannot be altered in the FRFs. In the previous results, the range of \( f_{WR} \) includes the first four natural frequencies. It is evident from Figure 10 that results are vague at \( f_0, f_1, \) and \( f_2 \), hence higher natural frequencies are included in the range of \( f_{WR} \). Initially first ten natural frequencies were included in this range however, the results were not promising. Then the lower frequencies were excluded one by one and the range is

![Figure 6. Damage scenarios, geometry, and damage location (not to scale).](image)

![Figure 7. DI for SD1 at different frequency ranges.](image)

![Figure 8. DI for SD2 at different frequency ranges.](image)
tested as can be seen in Figure 11. It is observed that
the range from fifth to tenth natural frequency in \( f_{WR} \) gives encouraging results as the damage index peak clearly pinpoints the damage. To validate this, the damage severity is further reduced to 5% and for same range of \( f_{WR} \), the damage is clearly identified as can be seen in Figure 12.

To check if the proposed method is effective for multiple damage case too, the damage indices are generated by considering the four frequency ranges. As none of the frequency range except \( f_{WR} \) provide any useful result so the results from \( f_{WR} \) are shown here only. As can be seen in Figure 13, both damages can be clearly identified for 50% (MD1) and 25% (MD2) severity. However, only one damage is detected for the small size damage (5%, MD3). By adding higher natural frequencies in \( f_{WR} \) as for single damage case, it can be seen in Figure 14 that the second peak also becomes visible indicating the second damage. Hence, it proves that higher natural frequencies in \( f_{WR} \) can be helpful in detecting single and multiple less severe damage.

**Effect of measurement noise**

As measurement noise is inevitable in real data and it often makes a good damage detection technique ineffective. Thus, to examine whether the proposed method is reliable for real cases too, white Gaussian noise is added in the time response obtained from simulated experiments, using equation (10).

\[
nX(t) = X(t) + \left(\frac{e}{100}\sigma(X(t))R(t)\right)
\]

(10)

Here \( X(t) \) is the noise-free and \( nX(t) \) is the noisy time response, \( e \) refers to noise percentage and \( \sigma \) is standard deviation of the original time domain signal. MATLAB
command 'normrnd' is used to generate normal random data, $R(t)$. Various noise levels are investigated to analyze the behavior of the baseline-free damage detection method. The original and noisy FRFs at mid-point for SD2 with 5% noise level is shown in Figure 15. This clearly indicates that noise disturbs the FRFs valleys more than the peaks.

For further investigation, various noise levels such as $e = 0.5 - 25$ and in some cases up to $e = 45$, are added in the time domain responses, and then the baseline-free FRF curvature method considering $f_1, f_2,$ and $f_{WR}$, is used to generate damage indices. The results in Figures 16 to 21, are displayed as contour plots of the damage indices. From these results, the method considering $f_1$ detected the single damage in both SD1 and SD2 up to 5% noise level, as shown in Figure 16(a) and 17(a). The peak for SD1 is wider and shifted a bit toward left which is similar to the peak for noise-free case in Figure 7. The wider peak could be due to the severity of damage. The damage index peak moves toward right of actual damage location for noise greater than 5%. The detection is up to 2.5% and 1% noise level with $f_2$ for both SD1 and SD2, respectively as illustrated in Figures 16(b) and 17(b). A likely explanation of the low detection capability of $f_2$ in SD1 and SD2, is due to the location of damage in the insensitive region, as illustrated in Figure 2. For $f_{WR}$ by considering the first four natural frequencies, the results indicate single damage up to a noise level of 35% for both SD1 and SD2, as shown in Figures 16(c) and 17(c). There are few smaller side peaks but the peak at the damage location is dominant. When the damage location is changed as described by SD3, only $f_{WR}$ showed better results for 25% damage severity in the noise-free case. That is why only this frequency range is considered here. It can be seen in Figure 18(a) that $f_{WR}$ detected the damage up to 5% noise level, which suggested $f_{WR}$ to be comparatively less effective for mid-span damage in cantilever beam. For SD4 and SD5 as shown in Figure 18(b) and (c), the method works up to a noise level of

Figure 13. DI for MD1, MD2 and MD3 at $f_{WR}$ (1st – 4th natural frequencies).

Figure 14. DI for MD3 at $f_{WR}$ (5th – 10th natural frequencies).

Figure 15. Noisy (5%), Filtered FRF and Noise-free FRF.

Figure 16. DI after noise addition in SD1, considering: (a) $f_1$, (b) $f_2$, and (c) $f_{WR}$.

Figure 17. Damage Index (DI) using $f_1$ for noise levels up to 45% and noise-free case.
Figure 17. DI after noise addition in SD2, considering: (a) $f_1$, (b) $f_2$, and (c) $f_{WR}$.

Figure 18. DI after noise addition in: (a) SD3, considering $f_{WR}$ (1–4), (b) SD4 considering $f_{WR}$ (5–10), and (c) SD5 considering $f_{WR}$ (5–10) range of frequencies.

Figure 19. DIIs after noise addition in MD1 using $f_{WR}$ with frequency range: (a) 1–4 and (b) 4–10.

Figure 20. DIIs after noise addition in MD2 using $f_{WR}$ with frequency range: (a) 1–4 and (b) 4–10.
20% and 10%, respectively. These damage indices are generated by considering higher frequencies because the damage is less severe. There are smaller side peaks at 5% onwards for SD4 and at 10% on left for SD5, but the true peak is the dominant one.

The noise model is further applied on multiple damage scenarios, MD1, MD2, and MD3. Since, multiple damage detection is challenging as compared to single damage detection, particularly under noisy conditions, so the noise levels considered for multiple damage cases are not as high as single damage cases. Initially first four natural frequencies are considered for \( f_{WR} \). The results for MD1 in Figure 19(a) indicate that both damages D1 and D2 are visible up to 0.1% and 0.3% noise level, respectively. Then higher frequencies in \( f_{WR} \) range are added, and the results are displayed in Figure 19(b). From the results, D1 can be seen up to 1% noise level, however the peak is dominant till 0.5%. Moreover, D2 is visible till 1% noise level and then the peak gradually shifts toward left of actual damage location. With smaller multiple damage in MD2, \( f_{WR} \) with first four natural frequencies gives the damage index for different noise levels as illustrated in Figure 20(a). From the figure, D1 and D2 can be seen up to the noise level of 0.3% and 0.2%, respectively. By adding higher frequencies, the detection of D1 and D2 is improved to 0.5% and 0.3% noise level, respectively as shown in Figure 20(b). For the smallest damage case in MD3 in Figure 21(a), D1 can be clearly seen up to the noise level of 0.02% while D2 is not detected. However, by including higher frequencies D2 becomes visible up to 0.05% noise level with a smaller side peak on right, as can be seen in Figure 21(b). From these results, the wide-frequency range \( f_{WR} \) comparatively performed better than the other frequency ranges. These results are shown to be further improved when this range is widened to include higher frequencies.

**Experimental results**

The experimental setup comprises two simply supported steel beams of same dimensions (1000 × 20 × 50mm), a schematic of which is shown in Figure 22. Two beams are considered so that damage can be introduced at different locations. The damage scenarios based on different severities of single and multiple damage are described in Table 4. The responses are measured at 21 points along the beam length with an accelerometer. While, the electromagnetic shaker with a random signal is used to excite the beam at point 2 (not the point shown in Figure 22). In addition to the frequency ranges investigated above, \( f_{HCR} \) a frequency range based on high coherence value is also tested on experimental data. High coherence indicates the quality of the FRFs by highlighting nonlinearity and presence of noise in a certain frequency range. The coherence function is obtained directly from the experiments or alternatively it can be calculated by using the following expression,

\[
\gamma^2(\omega) = \frac{|H(\omega)|^2 S_y(\omega)}{S_x(\omega)} , \quad [0 < \gamma^2(\omega) < 1]
\]

where, \( S_y(\omega) \) and \( S_x(\omega) \) are the power spectra of the input and response, respectively. Coherence \( \gamma^2(\omega) = 1 \) means there is no noise and nonlinearity in the FRFs. For \( f_{HCR} \), a high coherence range is selected at the frequency band where \( \gamma^2(\omega) \) approaches the highest value near 1. It can be a single or multiple range which are selected by taking an average of coherence function at each point along the length of the beam.

The beam is elastically supported from the two ends hence, simulating the free-free boundary condition.
Based on this boundary condition, a separate sensitivity analysis is conducted, and the beam was found to be insensitive around its both ends. The results of each damage scenario against each frequency range are shown in Figures 23 and 24. For S1, it can be seen in Figure 23(a) that there is a clear dominant peak at damage location for $f_0$ and $f_{WR}$. For $f_1$ and $f_{HCR}$, the peak is shifted to the left of actual damage location and that peak moved further to the left for $f_2$, but it still covers the damage location. In case of S2 as shown in Figure 23(b), the peak for $f_1$ is shifted to the right of actual damage location. All other frequency ranges detected the damage with a very small tilt toward left. From these results, most frequency ranges detected the single mid-span damage up to 25% severity without using any baseline information.

In case of multiple damage M1, the damage index for $f_0$ shows only D2 with a peak shifted to the left of actual damage location, as shown in Figure 24(a). For $f_1$, there is a very small peak to the left of D1 while dominant wider peak at D2. For $f_2$, D1 could not be detected while D2 is within the peak-width with the peak shifted to the right of actual damage location. Both D1 and D2 can be seen for $f_{WR}$ with a slight left-shift of second peak along with a smaller side peak. While for $f_{HCR}$, there is only one peak on the right of actual damage location of D2. The locations of the two damages in M2 and M3 are different from M1, and the damage severity is very small in both cases. Based on the results of M2 in Figure 24(b), D1 is detected in all cases of frequency range with some prominent side peaks in $f_1$ and $f_2$. While $f_{HCR}$ also gives one false peak on the right. In all cases D2 could not be detected at all. The results of M3 in Figure 24(c) are somewhat similar to M2. Here, the damages are very small, almost 0.5 mm in depth which indicates 2.5% severity. All frequency ranges except $f_{HCR}$, clearly show a dominant peak at the location of D1 along with many false peaks on each side. There is a very small peak at D2 for $f_{WR}$ but that is dominated by false peak on the left.

Although the experimental results cannot be compared directly with the simulated results due to the different boundary conditions, even then the results are in agreement to the simulated results. For severe multiple damage case, only $f_{WR}$ clearly detected both damages with clear prominent peaks at the damage location. While only one damage is detected in small multiple damage cases of M2 and M3, for all frequency ranges. For free-free boundary condition, it can be seen that the mid-span damage is easily detected by most frequency ranges both for single and multiple damage cases. However, the damage closer to one end of the
beam was only detected by $f_{WR}$ (D1 for M1). The beam is insensitive closer to its ends on both sides for free-free boundary condition as compared to the cantilever beam which is insensitive around the fixed end. In the presence of noise, the mid-span damage in cantilever case (SD3) was not detected by any frequency range except $f_{WR}$ and it showed better results even for a damage severity of 5% (SD5). For multiple damage in cantilever case considering $f_{WR}$, the mid-span damage is detected in all cases which is in agreement to the experimental results, while damage closer to the free-end of the beam was difficult to detect when the severity is reduced. However, when the range is expanded to include higher frequencies the severe damage MD1 provided better results at high noise level while MD2 showed clear dominant peak for mid-span damage which was smaller for first four frequencies. Moreover, the damage closer to the free end of the beam also becomes visible which was not detected considering first four frequencies in $f_{WR}$.

It is understood that $f_0$, $f_1$, and $f_2$ cannot be changed during analysis, whereas $f_{WR}$ and $f_{HCR}$ may have alterations. There could be more than one range for which value of $\gamma^2(\omega)$ approaches the highest value. Moreover, the available frequency span for M2 and M3 was 6401 Hz, comprising of first five natural frequencies only. As proved in simulated analysis that higher natural frequencies in $f_{WR}$ can be helpful in detecting less severe single/multiple damage, it could be valid in experimental scenarios too. It is expected that if higher natural frequencies are incorporated in the analysis, the second damage can also be visible for $f_{WR}$ and maybe for $f_{HCR}$. Hence, it is concluded that $f_{WR}$ is the most appropriate frequency range in the baseline-free FRF curvature method for high to less severe single/multiple damage. It provides better results as compared to the frequency range before first resonance, across first resonance and across second resonance. Moreover, $f_{HCR}$ could also provide better results if the frequency span is increased to include higher frequencies. As there may be more than one region where the coherence value is high, so those frequency ranges can be used together to improve the results. It should be noted that in this paper the damage indices were generated without using the data from the healthy/undamaged structure. In addition to adding higher frequencies for $f_{WR}$ and $f_{HCR}$, the results can be further improved by increasing the number of measurement points along the beam length and/or by incorporating more sensors to measure the response along the beam length.

**Conclusion**

This paper presented a baseline-free FRF curvature method that is tested numerically and validated experimentally on beam-type structures using sparse measurement data. In practice the damage location is not known, hence the strength of any damage detection technique lies in localizing the damage even when the sensor is placed not close to the damage location. Using FRFs instead of modes precludes the errors associated with the modes extraction. Another advantage is that it eliminates the need for any data from a healthy structure. As the effectiveness of the FRF curvature method depends on the chosen frequency range of the FRF, hence different frequency ranges were investigated. The experimental response usually has noise in the measured data and this noise often cause false indications of damage. To tackle that, a Gaussian-weighted moving average filter is used to reduce the noise in the measured FRFs. There are certain regions along the beam length that are insensitive to damage detection and this insensitivity is associated with the boundary conditions and nodal points based on each frequency range. In this paper, a sensitivity analysis was carried out initially to indicate those insensitive regions of the beam. Based on the computed normalized sensitivity, the modified damage indices for each frequency range are generated. The simulated analysis with and without noise revealed $f_{WR}$ to be the best frequency range to detect single and multiple damage up to 5% severity. The results were improved by incorporating higher frequencies in $f_{HCR}$. These results were then validated via experiment with a different boundary condition. In the analysis of experimental data, another frequency range $f_{HCR}$ based on high coherence values, was also tested. Among all frequency ranges, only $f_{WR}$ with first four frequencies detected the multiple damage of 50% severity. From simulated results, it was proved that $f_{WR}$ worked better when the range of higher frequencies were used. In a similar manner, a large span of FRF provide more regions where the coherence value is closer to 1. Hence, $f_{HCR}$ could include more than one frequency range with high coherence value. As the experimentally extracted responses only included first five natural frequencies, hence this could not be validated. The other aspects of improving the experimental results could be, by increasing the measurement points and using more than one sensor to measure the responses. Moreover, for accurate detection and localization of less severe damage, $f_{WR}$ must include higher natural frequencies. However, the higher the natural frequencies are, the more difficult it is to obtain the experimental data with good accuracy using ordinary sensors.

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