Single-step multipartite entangled states generation from coupled circuit cavities

Xiao-Tao Mo and Zheng-Yuan Xu
Guangdong Provincial Key Laboratory of Quantum Engineering and Materials, and School of Physics and Telecommunication Engineering, South China Normal University, Guangzhou 510006, China

(Dated: March 28, 2019)

Green-Horne-Zeilinger states are a typical type of multipartite entangled states, which plays a central role in quantum information processing. For the generation of multipartite entangled states, the single-step method is more preferable as the needed time will not increase with the increasing of the qubit number. However, this scenario has a strict requirement that all the two-qubit interaction strengths should be the same, or the generated state will be of low quality. Here, we propose a scheme for generating multipartite entangled states of superconducting qubits, from a coupled circuit cavities scenario, where we rigorously achieve the requirement via adding an extra $z$-direction ac classical field for each qubit, leading the individual qubit-cavity coupling strength to be tunable in a wide range, and thus can be tuned to the same value. Meanwhile, in order to obtain our wanted multi-qubits interaction, $x$-direction ac classical field for each qubit is also introduced. By selecting the appropriate parameters, we numerically shown that high-fidelity multi-qubit GHZ states can be generated. In addition, we also show that the coupled cavities scenario is better than a single cavity case. Therefore, our proposal represents a promising alternative for multipartite entangled states generation.

I. INTRODUCTION

Entanglement is one of the most counterintuitive consequences of quantum physics, and nowadays it plays a central role in quantum computation and quantum communication [1]. Multiparticle entangled states are entangled states of many qubits which are indispensable resource for research in large scale quantum computation [2], multiparticle quantum communication [3], quantum simulation [4] and quantum-to-classical transition [5]. Therefore, generating entangled states of an increasing number of qubits is an important benchmark for modern quantum technology [6].

Green-Horne-Zeilinger (GHZ) states [7] are a typical type of maximally entangled states, and the generation of which has been paid much attention recently [8–21]. Conventional way of GHZ states is generated in a step by step way, based on high-fidelity quantum gates. In this way, the number of entangled qubits is only increased by one at a time, and thus the needed generation time will increase with the increasing of the number of the involved qubits. In addition, this method will also result in accumulation of individual gate operation errors when the number of entangled qubits increases. Alternatively, GHZ state can be generated in a single step [22–35], via the deliberately designed collective interaction, irrespective of the number of entangled qubits.

Meanwhile, superconducting transmon qubits, a kind of superconducting Josephson-junction qubit, are one of the promising solid-state processor for quantum state manipulation [36–38]. Recently, the setup of multipartite superconducting qubits connecting to a common bus resonator has been used for single-step entangled state generation. But, with the increasing of the number of qubits, the probability to generate a GHZ state decrease dramatically. This is because that the single-step method has a strict requirement that all the two-qubit interaction strengths should be the same. However, for each qubit, its coupling strength with the bus resonator is different so that the qubit-qubit interaction does not evolve synchronously.

Here, we present a scheme to solve the above-mentioned difficulty in a two coupled cavities scenario, where each superconducting qubit is biased by a $z$-direction magnetic flux in order to tune the qubit-resonator coupling. Specifically, this modulation can effectively make the qubit-resonator couplings to be tunable via the amplitudes of the external driving fields, so that the same coupling strength requirement for many qubits can be met. Meanwhile, in order to obtain our wanted multi-qubits interaction, different from that of in Refs. [22, 23, 33, 34], $x$-direction ac classical field for each qubit is also introduced. By selecting the appropriate parameters, we analytically proved that multi-qubit GHZ states can be generated and we also numerically simulated the obtained high fidelity. In addition, for the target GHZ generation task, we also numerically show that the coupled cavities scenario is better than a single cavity case. Therefore, our proposal represents a promising alternative for multipartite entangled GHZ states generation with superconducting qubits.

II. THE THEORETICAL SCHEME

The proposed setup for Generating GHZ state is illustrated in Fig. 1, which consists of a two coupled cavities in the circuit QED [37] scenario. Setting $\hbar = 1$ hereafter, the Hamiltonian of the two coupled cavities is

$$H_c = \omega_r a^+ a + \omega_c b^+ b + J(ab^+ + a^+ b)$$

$$= \omega_c P_+^+ P_+ - \omega - P_- P_-,$$  \hspace{1cm} (1)

where $a^\dagger (b^\dagger)$ and $a (b)$ are the creation and annihilation operators for the cavity A (B), respectively; $J$ is the coupling strength between the two cavity modes. The two localized normal modes of this coupled system are $P_{\pm} = (a \pm b)/\sqrt{2}$, and the frequencies of them are $\omega \pm = \omega_r \pm J$, with $\omega_r$ being chose to be the same for simplicity. Otherwise, the two modes and their frequency will be slightly modified.
the coupling Hamiltonian can be generally written as

$$H_{\text{int}} = \sum_{j=1}^{N} \frac{g_j}{2} a_j^+ a_j + \sum_{j=N+1}^{2N} \frac{g_j}{2} b_j^+ b_j + H.c.,$$  

(3)

where $\sigma_j^+ = |1\rangle_j \langle 1| - |0\rangle_j \langle 0|$, $\sigma_j^- = |1\rangle_j \langle 0| - |0\rangle_j \langle 1|$, with $|0\rangle$ and $|1\rangle$ being the ground and excited states of $j$th qubit, $g_j$ is the $j$th qubit-cavity coupling strength.

Meanwhile, all the qubits are simultaneously driven by the classical field along $x$ and $z$ directions as

$$H_{dz} = \sum_{j=1}^{2N} \frac{A_j}{2} \sin(\omega_j t + \varphi) \sigma_j^z,$$

$$H_{dx} = \sum_{j=1}^{2N} \frac{\Omega_j}{2} \left[ e^{-i\omega_j t} \sigma_j^+ e^{i\omega_j t} + e^{i\omega_j t} \sigma_j^- \right],$$  

(4)

where $\Omega_j$ is the Rabi frequency of the classical field along $x$ direction and $A_j$ is the amplitude of the classical field which can drive $j$th qubit along $z$ direction.

In the interaction picture with respect to $H_0 = H_c = H_{dx} + H_{dz}$, the interaction Hamiltonian $H_w = \exp \left(iT \int H_0 dt\right)H_{\text{int}} \exp \left(-iT \int H_0 dt\right)$, with $T$ being the time-ordering operator, will be

$$H_w = \sum_{j=1}^{N} \frac{g_j}{2 \sqrt{2}} (P_+ e^{i\omega_j t} + P_- e^{i\omega_j t}) \sigma_j^+ e^{-i\omega_j t} e^{i\alpha_j \cos(\omega_j t + \varphi)}$$

$$+ \sum_{j=N+1}^{2N} \frac{g_j}{2 \sqrt{2}} (P_+ e^{i\omega_j t} - P_- e^{i\omega_j t}) \sigma_j^- e^{-i\omega_j t} e^{i\alpha_j \cos(\omega_j t + \varphi)}$$

$$+ \sum_{j=1}^{2N} \frac{\Omega_j}{2} e^{-i(\omega_j t + \varphi \delta)} \sigma_j^+ e^{i\omega_j t} e^{-i\alpha_j \cos(\omega_j t + \varphi)} + H.c.$$  

(5)

where $\alpha_j = A_j / \omega_j$. Note that

$$e^{i\alpha_j \cos(\omega_j t + \varphi)} \equiv \sum_{m=-\infty}^{\infty} i^m J_m(\alpha_j) e^{i m (\omega_j t + \varphi)},$$

(6)

thus setting $\omega_{d_j} = \omega_{\delta j}$, $\varphi = \pi/2$, and

$$|\omega_- - \omega_{\delta j} + m \omega| \gg \{ |\delta|, g_j J_1(\omega_j) \}, \quad |\omega_+ - \omega_{\delta j} + m \omega| \gg \{ |\delta|, g_j J_1(\omega_j) \}$$  

(7)

with $\delta = \omega_- - \omega_{\delta j} - \omega_j$ and $(m \neq -1)$, we can neglect the terms oscillating fast, thus Eq. (5) reduces to

$$H'_w = \sum_{j=1}^{2N} \frac{J_0(\alpha_j) \Omega_j}{2} \sigma_j^+ + \frac{1}{2 \sqrt{2}} \left[ \sum_{j=1}^{N} g_j J_1(\alpha_j) e^{i\delta t} P_+ \sigma_j^- - \sum_{j=N+1}^{2N} g_j J_1(\alpha_j) e^{-i\delta t} P_- \sigma_j^- + H.c. \right].$$  

(8)

Defining $H_{w0} = \sum_{j=1}^{2N} \frac{J_0(\alpha_j) \Omega_j}{2} \sigma_j^+$. In the interaction picture, the interacting Hamiltonian will be

$$H'_{\text{int}} = e^{i\delta t} P_+ \sum_{j=1}^{N} \frac{g_j}{4 \sqrt{2}} J_1(\alpha_j) (\sigma_j^+ + | - \rangle_j \langle + |) e^{-i J_0(\alpha_j) \Omega_j t} | + \rangle_j (| - \rangle_j \langle + | e^{i J_0(\alpha_j) \Omega_j t})$$

$$- e^{i\delta t} P_- \sum_{j=N+1}^{2N} \frac{g_j}{4 \sqrt{2}} J_1(\alpha_j) (\sigma_j^- + | + \rangle_j \langle - | + \rangle_j (| + \rangle_j \langle - | e^{i J_0(\alpha_j) \Omega_j t}) + H.c.$$  

(9)
Assuming that $J_0(\alpha_j)\Omega_j \gg \{\delta, J_1(\alpha_j)g_j\}$, and eliminate the oscillate with high frequencies, $H_{\text{int}}$ reduces to

$$H_{\text{int}} = \frac{e^{i\delta t}}{4\sqrt{2}} \left[ \sum_{j=1}^{N} g_j J_1(\alpha_j) P_\downarrow \sigma_j^z - \sum_{j=N+1}^{2N} g_j J_1(\alpha_j) P_\downarrow \sigma_j^x \right] + H_{\text{c.c.}} \tag{10}$$

Note that, in the case, we can control of the effective coupling strength $g_j$ by varying the externally classical field, i.e., by controlling the amplitude $\alpha_j$, so that

$$g_j J(\alpha_j)/\sqrt{2} = g \tag{11}$$

can be met. Then $H_{\text{int}}$ can be written in the form of

$$H_{\text{int}} = \frac{g}{2} P_\downarrow J_x e^{i\delta t} + H_{\text{c.c.}}, \tag{12}$$

where we have set $J_x = J_1^x - J_2^x$ with $J_1^x = \frac{1}{2} \sum_{j=1}^{N} \sigma_j^x$ and $J_2^x = \frac{1}{2} \sum_{j=N+1}^{2N} \sigma_j^x$.

The evolution operator of the effective Hamiltonian reads

$$U(\gamma) = \exp \left( i\gamma J_2^x \right) \exp \left[ iB P_\downarrow J_x \right] \exp \left( iB^* P_\uparrow J_x \right), \tag{13}$$

where

$$\gamma = \frac{g^2}{4\delta} \left[ t + \frac{1}{i\delta} (e^{-i\delta t} - 1) \right],$$

$$B = \frac{g}{2i\delta} (e^{-i\delta t} - 1). \tag{14}$$

It is obvious that $B(t)$ is a periodic function of time and vanishes at $\delta t = 2k\pi$ where $k = 1, 2, 3, ...$. At those time intervals, the evolution operator in Eq. (13) reduces to

$$U(\tau) = \exp \left[ i\gamma(\tau) J_2^x \right], \tag{15}$$

which can be directly used to generate GHZ states when $\gamma(\tau) = \pi/2$. In this case, $\delta = \sqrt{4k}/g$, $\tau = 2\pi\sqrt{k}/g$. For a fast scheme, we can set $k = 1$.

For an initial state $|\Psi(0)\rangle = |00 \cdots 0\rangle_a \otimes |00 \cdots 0\rangle_b$ for $2N$ qubits in two cavities a and b, the final state is found to be a GHZ state of

$$|\Psi(\tau)\rangle_N = U \left( \frac{\pi}{2} \right) |\Psi(0)\rangle \tag{16}$$

$$= \frac{1}{\sqrt{2}} \left[ |00 \cdots 0\rangle_a |00 \cdots 0\rangle_b - i|11 \cdots 1\rangle_a |11 \cdots 1\rangle_b \right],$$

when $N$ is even; the detailed derivation is presented in Appendix A.

III. NUMERICAL SIMULATIONS

In order to obtain the effective Hamiltonian, several approximations have been made here. According to the postulated conditions in Eq. (7), we need to choose a suitable value of the frequency $\omega_j$ of $z$-direction magnetic. $\omega_j$ is set to around a fixed value, which should be adjusted with respect to the corresponding qubit frequency $\omega_{qj}$, in order to ensure $\delta$ to be equal for different qubits. For simplicity, we set $\omega_j/2\pi = 2\pi \times 600$ MHz in our numerical simulation. Meanwhile, suitable driving amplitudes $A_j$ of the $z$-direction classical fields can be chosen to make sure that the effective qubit-cavity coupling strengths to be the same, as shown in Eq. (11), in spite of the fact that the original qubit-cavity coupling strengths $g_j$s are not identical. Here, $A_j$s are used to tune $g_j$ to a same value $g/2\pi = 15$ (10) MHz in the two (four) qubits case. In addition, since $\alpha_j$ becomes a certain value, we also choose the amplitudes of each $x$-direction magnetic flux $\Omega_j$ to meet the condition of $J_0(\alpha_j)\Omega_j = \Omega$. For our simulation, the used master equation is

$$\frac{d\rho(t)}{dt} = -i[H_w, \rho(t)] + \kappa L(a) + \kappa L(b)$$

$$+ \sum_{j=1}^{2N} \left[ \beta L(\sigma_j^-) + \gamma L(\sigma_j^+) \right] \tag{17}$$

where $L(B) = B \rho(t)B^\dagger - B^\dagger B \rho(t)/2 - \rho(t)B^\dagger B/2$ is the Lindblad operator with $B \in \{a, b, \sigma_j^-, \sigma_j^+\}$. Here, the decay and dephasing rates for all the qubits and the decay for both cavities are all set to be equal as $\kappa = \beta = \gamma = 2\pi \times 4$ kHz.

As shown in Fig. 2, the fidelities of two qubits entangled state generation are plotted with respect to time, for both the one and two cavities cases and the other parameters are list in table I. Obviously in the Fig.2, when the $\delta\tau = 2\pi$, the fidelity of red solid line reaches up to 99.02% which is larger than the same data of the blue dashed line. This proves that the performance of generating GHZ state by a $z$-direction biasing magnetic flux in the two-cavities case is better than that.
of the one cavity case. Note that, the quantity of inter-cavity coupling $J$ exists only in the two-cavities case, and thus in the one cavity group we compensate this to the qubits’ frequency, so that the $z$-direction driving frequency $\omega_j/2\pi$ is still equal to 600MHz, be consistent with that of the two-cavity case. Besides the number of cavities, decoherence and the oscillating terms are non-negligible factors, which decrease the fidelity of the generated GHZ state. In order to make all qubit-cavity coupling strength $g_j$ to be a same value, each qubit is driven by the classical field along $z$ directions which bring in the oscillating terms that decreases the fidelity about 0.5%. Theoretically, with the increasing of the frequency $\omega_j$, the affect of the oscillating terms would decrease.

As shown in Fig. 3, the fidelities of four qubits entangled state generation are plotted with respective to time, for both the one and two cavities cases and the other parameters are list in table II. When the $\delta\tau = 2\pi$, the value of fidelity is 96.1% and 90.5% for the two- and one-cavity cases, respectively. The same conclusion we can get is that the two-cavity case shows better performance. Therefore, keeping all qubits apart to each cavity will beneficial to the increase of fidelity. This is because that the cross talk effect among the oscillating terms will be separated into to cavities and thus some of them have been suppressed.

### IV. CONCLUSION

In summary, we have put forward a project to generate GHZ state in two coupled cavities scenario. In this method, each qubit is driven simultaneously by the classical fields. Obviously, the role of the $x$-direction classical field is manipulating the qubit state. Therefore, we mainly adjust the qubit-cavity strength $g_j$ by controlling the amplitude and frequency of the $z$-direction classical field. Because of deviation of superconducting manufacturing technique or the other environmental factor, we need a way to make all qubit-cavity coupling strength $g_j$ to be a same value, which can be achieved here by selecting the appropriate parameters. In addition, we also show that the coupled cavities scenario is better than a single cavity case. Therefore, our proposal represents a promising alternative for multipartite entangled states generation with superconducting qubits.

### ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China (Grant No. 11874156) and the National Key R & D Program of China (Grant No. 2016YFA0301803).
Appendix A: Derivation details for Eq. (16)

In this Appendix, we present some derivation details in the main text. We note that

\[ |\Psi(\tau)\rangle_N = e^{i(\pi/2)^2} |\frac{N}{2}, -\frac{N}{2}\rangle_{x,a} \left| \frac{N}{2}, -\frac{N}{2}\right\rangle_{z,b} \]

\[ = e^{i(\pi/2)^2} \sum_{M_1, M_2} C_{M_1} \left| \frac{N}{2}, M_1\right\rangle_{x,a} C_{M_2} \left| \frac{N}{2}, M_2\right\rangle_{z,b} \]

\[ = \sum_{M_1, M_2} e^{i\pi(M_1-M_2)^2} C_{M_1} \left| \frac{N}{2}, M_1\right\rangle_{x,a} C_{M_2} \left| \frac{N}{2}, M_2\right\rangle_{z,b}. \]  

(A1)

\[ \exp \left[ \frac{\pi}{i} (M_1 - M_2)^2 \right] = \begin{cases} i, & M_1 - M_2 \text{ is odd;} \\ 1, & M_1 - M_2 \text{ is even.} \end{cases} \]

\[ = \frac{1}{\sqrt{2}} \left[ e^{i\pi/2} + (-1)^{(M_1 - M_2)} e^{-i\pi/2} \right] \]

\[ = \frac{1}{\sqrt{2}} \left[ e^{i\pi/2} + (-1)^{2N-M_1+M_1-M_2} e^{-i\pi/2} \right] \]

\[ = \frac{1}{\sqrt{2}} \left[ e^{i\pi/2} + (-1)^{(N/2-M_1)(-1)^{(N/2-M_1)} e^{-i\pi/2}} \right] \]

we get

\[ |\Psi(\tau)\rangle_N = \frac{1}{\sqrt{2}} \sum_{M_1, M_2} \left[ e^{i\pi/2} + (-1)^{N}(-1)^{(N/2-M_1)}(-1)^{(N/2-M_2)} e^{-i\pi/2} \right] C_{M_1} \left| \frac{N}{2}, M_1\right\rangle_{x,a} C_{M_2} \left| \frac{N}{2}, M_2\right\rangle_{z,b} \]

\[ = \frac{1}{\sqrt{2}} \left[ e^{i\pi/2} \sum_{M_1} C_{M_1} \left| \frac{N}{2}, M_1\right\rangle_{x,a} \sum_{M_2} C_{M_2} \left| \frac{N}{2}, M_2\right\rangle_{z,b} \right] \]

\[ + (-1)^N e^{-i\pi/2} \left( \sum_{M_1} (-1)^{(N/2-M_1)} C_{M_1} \left| \frac{N}{2}, M_1\right\rangle_{x,a} \sum_{M_2} (-1)^{(N/2-M_2)} C_{M_2} \left| \frac{N}{2}, M_2\right\rangle_{z,b} \right) \]

\[ = \frac{1}{\sqrt{2}} \left[ e^{i\pi/2} \left| \frac{N}{2}, -\frac{N}{2}\right\rangle_{x,a} \left| \frac{N}{2}, -\frac{N}{2}\right\rangle_{z,b} \right] + (-1)^N e^{-i\pi/2} \left| \frac{N}{2}, \frac{N}{2}\rangle_{x,a} \left| \frac{N}{2}, \frac{N}{2}\rangle_{z,b} \right] \]

\[ = \frac{1}{\sqrt{2}} \left[ e^{i\pi/2} |00\cdots0\rangle_a |00\cdots0\rangle_b + (-1)^N e^{-i\pi/2} |11\cdots1\rangle_a |11\cdots1\rangle_b \right]. \]  

(A3)

When \( N = 1 \), the final state is

\[ |\Psi(\tau)\rangle_1 = \frac{1}{\sqrt{2}} (|0\rangle_a |0\rangle_b + i|1\rangle_a |1\rangle_b) . \]  

(A4)

In the Schrödinger picture, the above state can be written as

\[ \frac{1}{\sqrt{2}} \left[ (e^{-i\Omega t/2} |+\rangle_a + e^{i\Omega t/2} |\rangle_a)(e^{-i\Omega t/2} |+\rangle_b + e^{i\Omega t/2} |\rangle_b) + i(e^{-i\Omega t/2} |+\rangle_a - e^{i\Omega t/2} |\rangle_a)(e^{-i\Omega t/2} |+\rangle_b + e^{i\Omega t/2} |\rangle_b) \right] \]

where \(|\pm\rangle_a = e^{i\theta_1} |0\rangle \pm e^{-i\theta_1} |1\rangle\) and \(|\pm\rangle_b = e^{i\theta_2} |0\rangle \pm e^{-i\theta_2} |1\rangle\) with \( \Theta_n = \omega_n t/2 - \alpha_n \cos(\omega_n t + \varphi)/2 \) and \( n \in \{1, 2\} \).

[1] M.A. Nielsen and I.L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000).
[2] R. Raussendorf and H. J. Briegel, A One-Way Quantum Computer, Phys. Rev. Lett. 86, 5188 (2001).
[3] M. Hillery, V. Bužek, and A. Berthiaume, Quantum secret sharing, Phys. Rev. A 59, 1829 (1999).
[4] S. Lloyd, Universal Quantum Simulators, Science 273, 1073 (1996).
[5] A. J. Leggett, Realism and the physical world, Rep. Prog. Phys. 71, 022001 (2008).
[6] P. Zoller et al., Quantum information processing and communication, Eur. Phys. J. D 36, 203 (2005).
[7] D. M. Greenberger, M. Horne, A. Shimony, and A. Zeilinger, Bells theorem without inequalities, Am. J. Phys. 58, 1131 (1990).
[8] M. Neeley et al., Generation of three-qubit entangled states using superconducting phase qubits, Nature (London) 467, 570 (2010).
[9] C.-P. Yang, Q.-P. Su, and F. Nori, Entanglement generation and quantum information transfer between spatially-separated qubits in different cavities, New J. Phys. 15 115003 (2013).
[10] R. Barends et al., Superconducting quantum circuits at the surface code threshold for fault tolerance, Nature (London) 508, 500 (2014).
[11] S.-L. Su, X.-Q. Shao, H.-F. Wang, and S. Zhang, Scheme for entanglement generation in an atom-cavity system via dissipation, Phys. Rev. A 90, 054302 (2014).
[12] S.-L. Su, Q. Guo, H.-F. Wang, and S. Zhang, Simplified scheme for entanglement preparation with Rydberg pumping via dissipation.
pation, Phys. Rev. A 92, 022328 (2015).
[13] H. Paik et al., Experimental Demonstration of a Resonator-Induced Phase Gate in a Multiqubit Circuit-QED System, Phys. Rev. Lett. 117, 250502 (2016).
[14] C.-P. Yang, Q.-P. Su, S.-B. Zheng, and F. Nori, Entangling superconducting qubits in a multi-cavity system, New J. Phys. 18, 013025 (2016).
[15] L. Dong, Lin, Y.-F. Lin, Q.-Y. Li, H.-K. Dong, X.-M. Xi, Y.-J. Gao, Generation of three-photon polarization-entangled decoherence-free states, Ann. Phys. (N.Y.) 371, 287 (2016).
[16] M.-X. Dong, W. Zhang, Z.-B. Hou, Y.-C. Yu, S. Shi, D.-S. Ding, and B.-S. Shi, Experimental realization of narrowband four-photon Greenberger-Horne-Zeilinger state in a single cold atomic ensemble Opt. Lett. 42, 4691 (2017).
[17] X. Q. Shao, D. X. Li, Y. Q. Ji, J. H. Wu, and X. X. Yi, Ground-state blockade of Rydberg atoms and application in entanglement generation, Phys. Rev. A 96, 012328 (2017).
[18] R.-Y. Yan, Z.-B. Feng, C.-L. Zhang, M. Li, X.-J. Lu, and Y.-Q. Zhou, Fast generations of entangled states between a transmon qubit and microwave photons via shortcuts to adiabaticity, Laser Phys. Lett. 15, 115205 (2018).
[19] C.-P. Yang and Z.-F. Zheng, Deterministic generation of Greenberger-Horne-Zeilinger entangled states in circuit QED, Opt. Lett. 43, 5126 (2018).
[20] X. L. Wang et al., Experimental Ten-Photon Entanglement, Phys. Rev. Lett. 117, 210502 (2016).
[21] Z. Jin, S.-L. Su, A.-D. Zhu, H.-F. Wang, and S. Zhang, Engineering multipartite steady entanglement of distant atoms via dissipation, Front. Phys. 13, 134209 (2018).
[22] K. Mølmer and A. Sørensen, Multiparticle Entanglement of Hot Trapped Ions, Phys. Rev. Lett. 82, 1835 (1999).
[23] S. B. Zheng, One-Step Synthesis of Multiatom Greenberger-Horne-Zeilinger States, Phys. Rev. Lett. 87, 230404 (2001).
[24] F. Plastina, R. Fazio, and G. Massimo Palma, Macroscopic entanglement in Josephson nanocircuits, Phys. Rev. B 64, 113306 (2001).
[25] S. B. Zheng, Quantum-information processing and multiatom-entanglement engineering with a thermal cavity, Phys. Rev. A, 66, 060303 (2002).
[26] D. I. Tsomoko, S. Ashhab, and F. Nori, Fully connected network of superconducting qubits in a cavity, New J. Phys. 10, 113020 (2008).
[27] A. Galiautdinov and J. M. Martinis, Maximally entangling tripartite protocols for Josephson phase qubits, Phys. Rev. A 78, 010305(R) (2008).
[28] J. Zhang, Y.-X. Liu, C.-W. Li, T.-J. Tarn, and F. Nori, Generating stationary entangled states in superconducting qubits, Phys. Rev. A 79, 052308 (2009).
[29] C. L. Hutchison, J. M. Gambetta, A. Blais, and F. K. Wilhelm, Quantum trajectory equation for multiple qubits in circuit QED: Generating entanglement by measurement, Can. J. Phys. 87, 225 (2009).
[30] Y.-D. Wang, S. Chesi, D. Loss, and C. Bruder, One-step multi-qubit Greenberger-Horne-Zeilinger state generation in a circuit QED system, Phys. Rev. B 81, 104524 (2010).
[31] S. Aldana, Y. D. Wang, C. Bruder, Greenberger-Horne-Zeilinger generation protocol for N superconducting transmon qubits capacitively coupled to a quantum bus, Phys Rev B, 84, 134519 (2011).
[32] T. Monz, P. Schindler, J. T. Barreiro, M. Chwalla, D. Nigg, W. A. Coish, M. Harlander, W. Hansel, M. Hennrich, and R. Blatt, 14-Qubit Entanglement: Creation and Coherence, Phys. Rev. Lett. 106, 130506 (2011).
[33] Y. P. Zhong et al., Emulating Anyonic Fractional Statistical Behavior in a Superconducting Quantum Circuit, Phys. Rev. Lett. 117, 110501 (2016).
[34] C. Song, et al.,, 10-Qubit Entanglement and Parallel Logic Operations with a Superconducting Circuit, Phys. Rev. Lett. 119, 180511 (2017).
[35] Y. J. Fan, Z.-F. Zheng, Y. Zhang, D.-M. Lu, and C.-P. Yang, One-step implementation of a multi-target-qubit controlled phase gate with cat-state qubits in circuit QED, Front. Phys. 14, 21602 (2019).
[36] J. Q. You and F. Nori, Atomic physics and quantum optics using superconducting circuits, Nature (London) 474, 589 (2011).
[37] M. H. Devoret and R. J. Schoelkopf, Superconducting circuits for quantum information: An outlook, Science 339, 1169 (2013).
[38] X. Gu, A. F. Kockum, A. Miranowicz, Y.-x. Liu, and F. Nori, Microwave photonics with superconducting quantum circuits, Phys. Rep. 718-719, 1 (2017).
[39] E. Solano, G. S. Agarwal, and H. Walther, Strong-Driving-Assisted Multiparticle Entanglement in Cavity QED, Phys. Rev. Lett. 90, 027903 (2003).