Phase Structure of QCD Matter in a Chiral Effective Model with Quarks

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Using a unified hadron-quark effective model for the QCD equation of state, this study analyzes the phase structure of strongly interacting matter in a wide range of temperature and baryonchemical potential. At small potentials the model yields a smooth cross-over to chirally restored matter with a transition temperature and curvature in line with recent lattice QCD estimates and thermal model fits of freeze-out curves. Trajectories of constant entropy per net baryon number show a clear dependence on the particle composition in the model and on repulsive vector field interactions. Although the model might feature a critical end-point at a rather high baryonchemical potential and low temperature, probing it in heavy-ion collisions might be highly challenging due to a large thermodynamic spread of matter in the collision fireball.

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INTRODUCTION

A main objective in studying relativistic heavy-ion collisions lies in investigating the behavior of strongly-interacting matter under extreme conditions, i.e. high temperatures T and baryon densities ρB. Particularly focusing on the phase structure and on mapping the phase transitions to chirally restored and deconfined matter in the QCD phase diagram. Experimentally this can accomplished by studying observables for the phase transition at different beam energies corresponding to different excitation energies and baryon densities. While results from high-energy collisions at RHIC suggest the existence of a quark-gluon plasma (QGP) at high T and small ρB [1,2], the QCD phase structure in other regions (i.e. ρB > 0) remains largely unknown. Searching for transition signatures by scanning a range of beam energies was part of the SPS program [3], is currently performed at RHIC [4], and will also be a key objective of the CBM experiment at FAIR [5].

At ρB = 0 lattice QCD consistently shows a smooth cross-over transition with a “critical” temperature Tc ≈ 155 MeV [6,7]. At ρB > 0 lattice QCD standard lattice QCD methods fail due to the fermion sign problem and estimates from extrapolation are ambiguous. While older results suggest a critical end-point (CEP) with shift from a smooth cross-over to a first-order phase transition at finite ρB [8,10], more recent continuum extrapolated lattice estimates do not necessarily show a first order phase transition [11]. So far there are no experimental indications for a CEP to exist.

Further information on the phase structure of QCD matter is provided by effective models such as pure quark Polyakov-loop extended Nambu-Jona-Lasinio (PNJL) models [12–16] or Polyakov-quark-meson (PQM) models [17,20]. However, since in these models the Polyakov loop potentials are fixed at vanishing ρB [17], their validity for describing QCD matter decreases with higher potentials and baryon densities. To the disadvantage of these models, in the high-ρB region, baryon densities become large and baryon resonances may exhibit high multiplicities and tend to affect the phase structure significantly [21]. To circumvent these restraints, in this work the QCD phase structure is studied using a unified chiral effective model which combines hadron and quark degrees of freedom in a single partition function and provides the correct degrees of freedom in a wide range of T and ρB.

MODEL

This study of the QCD phase structure uses a chiral SU(3)-flavor σ-ω model [22,26] for describing the hadronic phase and a PNJL-type approach for quarks; see [20] for a detailed review of the model and all parameters. Particles in the model include all baryons from the octet, the decuplet and all known resonances with m ≤ 2.6 GeV [27], as well as the full set of scalar, pseudoscalar, vector, and axial vector mesons including all meson resonance states. Additionally, the three lightest quark flavors (u, d, s) are included. The fields are treated in mean field approximation and correspond to the chiral quark condensates, i.e. the scalar σ, its strange counterpart ζ, and the vector ω and φ fields. The interaction between particles and fields is described by

\[ \mathcal{L}_{\text{int}} = - \sum_i \bar{\psi}_i \left[ \gamma_0 \left( g_{\sigma \omega} \omega + g_{\phi \phi} \phi \right) + m_i^* \right] \psi_i, \]

with i running over all baryons and quarks. Scalar field couplings gσ,ζ dynamically generate the effective masses

\[ m_i^* = g_{\sigma \sigma} + g_{\mu \mu} \zeta + \delta m_i, \]

except for a small explicit mass δm (δm, = 6 MeV, δms = 105 MeV for quarks and δm, = 150 MeV for nucleons). The dynamic mass generation ensures decreasing
masses with higher $T$ and $\mu_B$ and, thus, the restoration of chiral symmetry. The vector couplings $g_{i\omega,\phi}$ generate the effective chemical potentials accordingly

$$\mu_i^* = \mu_i - g_{i\omega} \omega - g_{i\phi} \phi. \quad (3)$$

Using the notation $X = \sigma^2 + \zeta^2$, the scalar meson self-interactions are introduced as

$$\mathcal{L}_{\text{scal}} = -\frac{1}{2} k_0 X + k_1 X^2 + k_2 \left( \frac{\sigma^4}{2} + \zeta^4 \right) + k_3 \sigma^2 \zeta$$

$$-k_4 \chi^4 - \frac{4}{3} \chi^4 \ln \chi \frac{4}{\chi_0} + \frac{5}{4} \chi^4 \ln \frac{\sigma^2 \zeta}{\sigma_0 \zeta_0}, \quad (4)$$

with the last two terms describing QCD trace anomaly by introducing the baryon condensate $\chi$ (dilaton field) [24].

The vector meson self-interactions are given by

$$\mathcal{L}_{\text{vec}} = \frac{1}{2} \chi \left( m_{\omega}^2 \omega^2 + m_{\phi}^2 \phi^2 \right)$$

$$+ g_4 \left( \omega^4 + \phi^4 \right) + 3 \omega^2 \phi^2 + \frac{4 \omega^3 \phi^3}{\sqrt{2}} + \frac{2 \omega \phi^3}{\sqrt{2}}. \quad (5)$$

and the explicit breaking of chiral symmetry due to non-zero current quark masses adds the following terms to the Lagrangian

$$\mathcal{L}_{\text{SB}} = -\frac{1}{\chi_0^2} \left[ m_{\pi}^2 f_{\pi} \sigma + \left( \sqrt{2} m_{k}^2 f_{k} - \frac{1}{\sqrt{2}} m_{\pi}^2 f_{\pi} \right) \zeta \right]. \quad (6)$$

The baryon couplings to the fields are fixed such as to reproduce nuclear saturation properties and vacuum masses [23, 24] and the quark couplings are chosen according to the additive quark model and to avoid free quarks from appearing in the ground state. All baryon resonance couplings are scaled to the respective nucleon couplings via $g_{B,\sigma,\zeta} = r_s g_{N,\sigma,\zeta}$ and $g_{B,\omega,\phi} = r_s g_{N,\omega,\phi}$. While the scalar coupling stays $r_s \approx 1$ to obtain a smooth cross-over at $\mu_B = 0$, the vector coupling $r_v$ is a free parameter. The repulsive effect of the vector couplings controls the particle abundances at finite $\mu_B$ (Eq. (3)) and, thus, has large impact on the resulting phase structure. In the hadron sector, reasonably large vector couplings ($r_v \approx 1$) cause the disappearance of a first-order phase transition but yield a smooth cross-over transition due to the gradual appearance of baryon resonances with higher $T$ and $\mu_B$ [21]. In contrast, all quark vector couplings have to vanish in order not to fully quench baryon number fluctuations in the transition region [23].

Quarks are introduced as in PNJL models, defining the scalar Polyakov loop field $\Phi$ by tracing the time component $A_0$ of the SU(3) color gauge field $\Phi = 1/3 \text{Tr} \left[ \exp \left( -A_0/T \right) \right]$. For static quark masses, $\Phi$ is an order parameter for deconfinement indicating the breakdown of Z(3) center symmetry. The effective Polyakov loop potential

$$U = - \left( a(T) \Phi \Phi / 2 + b (T_0/T)^3 \ln \left[ 1 - 6 \Phi \Phi \Phi \Phi / 3 (\Phi \Phi)^2 \right] \right), \quad (7)$$

with $a(T) = a_0 + a_1 (T_0/T) + a_2 (T_0/T)^2$ and the critical Polyakov temperature $T_0$ is taken from [29]. It enters the grand canonical potential and controls the transition from hadrons to quarks. It is constructed such as to reproduce quenched lattice QCD thermodynamics and known features of the deconfinement transition [29]. In the confined phase, the minimum of $U(T, \Phi, \bar{\Phi})$ is located at $\Phi = 0$ and it moves towards $\Phi \rightarrow 1$ with increasing $T$. Furthermore, $\Phi$ couples to the dilaton field

$$\chi = \chi_0 \left[ 1 - 1/4 \left( \Phi^2 + \bar{\Phi}^2 \right)^2 \right]$$

(8)

to suppress the chiral condensate in the quark phase.

All thermodynamic quantities are derived from the grand canonical potential

$$\Omega/V = -\mathcal{L}_{\text{int}} - \mathcal{L}_{\text{mes}} + \Omega_{\text{th}}/V - U_{\text{Vol}}. \quad (9)$$

with $\Omega_{\text{th}}$ including thermal contributions from mesons, baryons, and quarks ($j = u, d, s$) in the form

$$\Omega_{\text{qq}} = -T \sum_j \frac{\gamma_j}{(2\pi)^3} \int d^3k \left( \ln \left[ 1 + \Phi e^{-\frac{T}{T}(E_j^*(k)-\mu_j^*)} \right] + \ln \left[ 1 + \bar{\Phi} e^{-\frac{T}{T}(E_j^*(k)+\mu_j^*)} \right] \right), \quad (10)$$

with the spin-isospin degeneracy factor $\gamma_j$ and the single particle energy $E_j^*(k) = (k^2 + m_j^2)^{1/2}$. By minimizing $\Omega/V(T, \mu)$ with respect to the fields one obtains the self-consistent equations of motion for fields and particle densities. From these, thermodynamic variables are derived via the pressure $p = -\partial \Omega/\partial V$, the entropy density $s = \partial p/\partial T$, and the energy density $\varepsilon = Ts - pV + \sum_i \mu_i \rho_i$.

A shift in the degrees of freedom from a hadron resonance gas (HRG) at low $T$ and $\rho_B$ to a pure quark gas in the high-$T$, high-$\rho$ region is attained by including an eigenvolume $V^*_\text{ex}$ for hadrons [30, 32]. The baryon $V^*_\text{ex}^B$ is chosen according to the proton charge radius [33], mesons are assumed to exhibit half of this radius, and quarks remain point-like. At high $T$ and $\mu$, when quark multiplicities rise quickly, this formalism suppresses hadrons and establishes the shift to a pure quark phase. Thermodynamic consistency is preserved by redefining $\mu_i^*$, i.e., reducing it by the occupied volume $26$, and multiply the particle densities $\rho_i$ as well as $\varepsilon$ and $s$ with a correction factor given by the ratio of the total volume to the non-occupied sub-volume.

**RESULTS**

The chiral transition extracted from the hadron sector of the model [21] (black line in Fig. 1) can be parametrized analog to a lattice QCD estimate [34] by

$$T_c(\mu_B) = T_0 \left( 1 - 0.0193 \left( \frac{\mu_B}{T_0} \right)^2 \right)$$

(11)
with $T_0 = T_c(\mu_B = 0) = 164$ MeV. In the chiral model this curve is extracted at the point of the steepest decrease in $\sigma(T)$ at a given baryonchemical potential. The curve agrees well with different models and experimental results at different beam energies. In [42] a constant value of the interaction measure $(e - 3p)/T^4 = 7/2$ was proposed to reliably parametrize the chemical freeze-out curve. This parametrization can be reproduced in the hadronic sector of the chiral model. However, since in the chiral transition region the interaction measure increases very fast with higher $T$, other constant values of $(e - 3p)/T^4$ close to $7/2$ also yield curves close to recent freeze-out parametrizations. In general, the transition behavior is not affected by additionally taking into account the quark phase. Even in the presence of a quark phase, baryon multiplicities at the chiral transition are high and the steepest decline in $\sigma$ still takes place in the HRG. But when considering quarks, $T_c$ is shifted to slightly higher values [20]. For this reason, and due to the non-existence of direct coupling effects between $\Phi$ and the quark condensates, quarks have only minor impact on the critical temperature of the chiral transition.

In [43] it is shown that, using a fixed target setup, highest net baryon number densities at freeze-out can be achieved in heavy-ion collisions at energies close to $E_{\text{lab}} = 30$ A GeV. Accelerators at the upcoming FAIR facility, operating between 5 and 40 GeV per nucleon, will cover this energy range. Figure 2 and 3 show the isentropic expansion paths, i.e. lines of constant entropy per net baryon number $S/A$, corresponding to these collision energies. The adiabats are depicted in the hadronic equation of state (EoS) without a quark phase and feature two values for the resonance vector couplings. In case of a rather weak coupling $r_v = 0.4$ (Fig. 2) the phase structure exhibits a first order phase transition up to $T \approx 60$ MeV, an adjacent CEP, and a smooth but rapid cross-over transition for higher temperatures (gray band). In the transition region, with increasing $T$, the rapid incline in $m_q^*$ of the baryons induces large baryon multiplicities and a sharply rising energy density and pressure along the $\mu_q$-axis (i.e. $\mu_q = \mu_B/3$) causing the adiabats to bend sharply at the chiral transition distant to the CEP.

This behavior changes for more reasonable resonance vector couplings $r_v = 0.9$ (Fig. 3) for which the changes in $\varepsilon$ and $p$ are much slower along the $\mu_q$-axis and the chiral transition takes place in a much broader $T$-range.
the EoS with larger properties to one specific point in narrow adiabat, on the other hand poses the question of this eventually allows for probing regions well outside a one collision at a specific energy. While on the one hand, but rather a large area of the phase diagram is covered inotropic adiabat corresponding to a single event and energy This means that there is no well defined and narrow isentropic adiabat in the QCD phase diagram may be even more complicated considering the sizable thermodynamic spread of fireball matter induced by initial state fluctuations as seen in dynamic models for heavy-ion collisions.

due suppressive vector field interactions. In this case, neither a first order phase transition nor a CEP exists and the chiral transition takes place in a much broader T-range. Compared to \( r_v = 0.4 \), substantial softening of the EoS with larger \( r_v \) causes the adiabats to smoothly bend at the chiral transition and to reach notably higher \( \mu_q \) while the initial \( T \) only changes on a minor scale.

The figures also depict dynamic expansion paths of the fireball at 20A GeV from ideal hydrodynamics [44] using initial densities from a geometric overlap model [45] (blue squares). Also shown are collision dynamics from the UrQMD hybrid model with fluctuating initial conditions [46] (red crosses) sampling over the central cell in 400 Au+Au collisions with a time interval of \( \Delta t = 1 \) fm between data points. In case of a soft EoS (Fig. 3), initial fluctuations cause a larger dispersion in \( T \) and \( \mu \) on an event-by-event basis. Furthermore, initial state density fluctuations create sub-regions in the fireball and cause a notable dispersion of the thermodynamic properties of the fireball matter. Due to this effect, QCD matter may spread over at least 50 MeV in \( \mu_q \) in each single event [47]. This means that there is no well defined and narrow isentropic adiabat corresponding to a single event and energy but rather a large area of the phase diagram is covered in one collision at a specific energy. While on the one hand, this eventually allows for probing regions well outside a narrow adiabat, on the other hand poses the question of how to map back final state observables to matter properties to one specific point in \( T \) and \( \mu \) on the QCD phase diagram.

Additionally including quarks (Fig. 4 using \( g_{q\omega} = 0 \)) has only minor effect on the adiabat curvature at the chiral transition. However, in the presence of a quark phase, the adiabats are slightly steeper in the chiral limit and higher initial \( T \) and \( \mu_q \) are achieved. In close analogy to the effect of the vector coupling of resonances \( r_v \) in the purely hadronic EoS, turning up the quark vector coupling \( g_{q\omega} \) from zero to finite values, causes significantly higher initial values in \( \mu_q \) and almost flat adiabat curvatures above the chiral transition in the full model including quarks.

**SUMMARY**

In summary, using an effective model with hadrons and quarks, we present a pa
terization for the chiral transition at small \( \mu_B \) agreeing well with other recent results on this topic. Isentropic adiabats in the EoS are discussed in the context of varying vector interactions: Probing the CEP and a first order phase transition at FAIR energies might be difficult due to the distance of isentropic adiabats to the first order phase transition in the model. Additionally, probing specific points in the QCD phase diagram may be even more complicated considering the sizable thermodynamic spread of fireball matter induced by initial state fluctuations as seen in dynamic models for heavy-ion collisions.

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