Loops versus strings\textsuperscript{1}

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Abstract

Two popular attempts to understand the quantum physics of gravitation are critically assessed. The talk on which this paper is based was intended for a general particle-physics audience.

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1 General Questions on Quantum Gravity

It is not clear at all what is the problem in quantum gravity (cf. [2] or [4] for general reviews, written in the same spirit as the present one). The answers to the following questions are not known, and I believe it can do no harm to think about them before embarking in a more technical discussion.

Actually, some people proposed that gravity should not be quantized, owing to its special properties as determining the background on which all other fields propagate. There is a whole line of thought on the possibility that gravity is not a fundamental theory, and this is certainly an alternative one has to bear in mind. Indeed, even the holographic principle of G. ’t Hooft, to be discussed later, can be interpreted in this sense.

Granting that, the next question is whether it does make any sense to consider gravitons propagating in some background; that is, whether there is some useful approximation in which there is a particle physics approach to the physics of gravitons as quanta of the gravitational field. A related question is whether semiclassical gravity, i.e., the approximation in which the source of the classical Einstein equations is replaced by the expectation value of the energy momentum tensor of some quantum theory has some physical (14) validity in some limit.

At any rate, even if it is possible at all, the at first sight easy problem of graviton interactions in an otherwise flat background has withstood analysis of several generations of physicists. The reason is that the coupling constant has mass dimension $-1$, so that the structure of the perturbative counterterms involve higher and higher orders in the curvature invariants (powers of the Riemann tensor in all possible independent contractions), schematically,

$$ S = \frac{1}{2\kappa^2_R} \int R + \int R^2 + \kappa^2_R \int R^4 + \ldots $$

Nobody knows how to make sense of this approach, except in one case, to be mentioned later on.
It could be possible, sensu stricto to stop here. But if we believe that quantum gravity should give answers to such questions as to the fate of the initial cosmological singularity, its is almost unavoidable to speak of the wave function of the universe. This brings its own set of problems, such as to whether it is possible to do quantum mechanics without classical observers or whether the wave function of the Universe has a probabilistic interpretation. Paraphrasing C. Isham [25], one would not known when to qualify a probabilistic prediction on the whole Universe as a successful one.

The aim of the present paper is to discuss in some detail established results on the field. In some strong sense, the review could be finished at once, because there are none. There are, nevertheless, some interesting attempts, which look promising from certain points of view. Perhaps the two approaches that have attracted more attention have been the loop approach, on the one hand and strings on the other. We shall try to critically assess prospects in both. Interesting related papers are [21][40].

2 The issue of background independence

One of the main differences between both attacks to the quantum gravity problem is the issue of background independence, by which it is understood that no particular background should enter into the definition of the theory itself. Any other approach is purportedly at variance with diffeomorphism invariance.

Work in particle physics in the second half of last century led to some understanding of ordinary gauge theories. Can we draw some lessons from there?

Gauge theories can be formulated in the background field approach, as introduced by B. de Witt and others (cf. [49]). In this approach, the quantum field theory depends on a background field, but not on any one in particular, and the theory enjoys background gauge invariance.

Is it enough to have quantum gravity formulated in such a way? This was, incidentally,
the way G. Hooft and M. Veltman did the first complete one-loop calculation ([45]).

It can be argued that the only vacuum expectation value consistent with diffeomorphisms invariance is

\[ \langle 0 | g_{\alpha \beta} | 0 \rangle = 0 \]  

in which case the answer to the above question ought to be in the negative, because this is a singular background and curvature invariants do not make sense. It all boils down as to whether the ground state of the theory is diffeomorphisms invariant or not. There is an example, namely three-dimensional gravity in which invariant quantization can be performed [51]. In this case at least, the ensuing theory is almost topological.

In all attempts of a canonical quantization of the gravitational field, one always ends up with an (constraint) equation corresponding physically to the fact that the total hamiltonian of a parametrization invariant theory should vanish. When expressed in the Schrödinger picture, this equation is often dubbed the *Wheeler-de Witt equation*. This equation is plagued by operator ordering and all other sorts of ambiguities. It is curious to notice that in ordinary quantum field theory there also exists a Schrödinger representation, which came recently to be controlled well enough as to be able to perform lattice computations ([27]).

Gauge theories can be expressed in terms of gauge invariant operators, such as Wilson loops. They obey a complicated set of equations, the loop equations, which close in the large \( N \) limit as has been shown by Makeenko and Migdal ([28]). These equations can be properly regularized, e.g. in the lattice. Their explicit solution is one of the outstanding challenges in theoretical physics. Although many conjectures have been advanced in this direction, no definitive result is available.
3 Loops

The whole philosophy of this approach is canonical, i.e., an analysis of the evolution of variables defined classically through a foliation of spacetime by a family of spacelike three-surfaces \( \Sigma_t \). The standard choice in this case (cf. for example [2]) is the three-dimensional metric, \( g_{ij} \), and its canonical conjugate, related to the extrinsic curvature. Due to the fact that the system is reparametrization invariant, the total hamiltonian vanishes, and this hamiltonian constraint is usually called the Wheeler- de Witt equation.

Here, as in any canonical approach the way one chooses the canonical variables is fundamental.

Ashtekar’s clever insight started from the definition of an original set of variables ([6]) stemming from the Einstein-Hilbert lagrangian written in the form

\[
S = \int e^a \wedge e^b \wedge R_{abcd}^e \epsilon_{abcd} \quad (3)
\]

where \( e^a \) are the one-forms associated to the tetrad,

\[
e^a \equiv e^a_{\mu} dx^\mu. \quad (4)
\]

Tetrads are defined up to a local Lorentz transformation

\[
(e^a)' \equiv L^a_b(x)e^b \quad (5)
\]

The associated \( SO(1, 3) \) connection one-form \( \omega^a_{\ b} \) is usually called the spin connection. Its field strength is the curvature expressed as a two form:

\[
R^a_{\ b} \equiv d \omega^a_{\ b} + \omega^a_{\ c} \wedge \omega^c_{\ b}. \quad (6)
\]

Ashtekar’s variables are actually based on the \( SU(2) \) self-dual connection

\[
A = \omega - i \ast \omega \quad (7)
\]

\footnotetext{Boundary terms have to be considered as well. We refer to the references for details.}
Its field strength is

\[ F \equiv dA + A \wedge A \tag{8} \]

The dynamical variables are then \((A_i, E_i \equiv F^{0i})\). The main virtue of these variables is that constraints are then linearized. One of them is exactly analogous to Gauss’law:

\[ D_i E^i = 0. \tag{9} \]

There is another one related to three-dimensional diffeomorphisms invariance,

\[ tr F_{ij} E^i = 0 \tag{10} \]

and, finally, there is the Hamiltonian constraint,

\[ tr F_{ij} E^i E^j = 0 \tag{11} \]

On a purely mathematical basis, there is no doubt that Astekhar’s variables are of a great ingenuity. As a physical tool to describe the metric of space, they are not real in general. This forces a reality condition to be imposed, which is awkward. For this reason it is usually preferred to use the Barbero-Immirzi (924) formalism in which the connexion depends on a free parameter, \(\gamma\),

\[ A^i_a = \omega^i_a + \gamma K^i_a \tag{12} \]

\(\omega\) being the spin connection and \(K\) the extrinsic curvature. When \(\gamma = i\) Astekhar’s formalism is recovered; for other values of \(\gamma\) the explicit form of the constraints is more complicated. Thiemann (177) has proposed a form for the Hamiltonian constraint which seems promising, although it is not clear whether the quantum constraint algebra is isomorphic to the classical algebra (cf.35). A comprehensive reference is 46.

Some states which satisfy the Astekhar constraints are given by the loop representation, which can be introduced from the construct (depending both on the gauge field \(A\) and on a parametrized loop \(\gamma\))

\[ W(\gamma, A) \equiv tr Pe^{\hat{\gamma}, A} \tag{13} \]

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and a functional transform mapping functionals of the gauge field $\psi(A)$ into functionals of loops, $\psi(\gamma)$:

$$\psi(\gamma) \equiv \int DAW(\gamma, A)\psi(A)$$  \hspace{1cm} (14)

When one divides by diffeomorphisms, it is found that functions of knot classes (diffeomorphisms classes of smooth, non self-intersecting loops) satisfy all the constraints.

Some particular states sought to reproduce smooth spaces at coarse graining are the *Weaves*. It is not clear to what extent they also approach the conjugate variables (that is, the extrinsic curvature) as well.

In the presence of a cosmological constant the hamiltonian constraint reads:

$$\epsilon_{ijk}E^{ai}E^{bj}(F^k_{ab} + \frac{\lambda}{3}\epsilon_{abc}E^{ck}) = 0$$  \hspace{1cm} (15)

A particular class of solutions of the constraint [41] are self-dual solutions of the form

$$F^i_{ab} = -\frac{\lambda}{3}\epsilon_{abc}E^{ci}$$  \hspace{1cm} (16)

Kodama [26] has shown that the Chern-Simons state

$$\psi_{CS}(A) \equiv e^{\frac{\lambda}{2\pi}S_{CS}(A)}$$  \hspace{1cm} (17)

is a solution of the hamiltonian constraint. He even suggested that the *sign* of the coarse grained, classical cosmological constant was always positive, irrespectively of the sign of the quantum parameter $\lambda$, but it is not clear whether this result is general enough. There is some concern that this state as such is not normalizable with the usual norm. It has been argued that this is only natural, because the physical relevant norm must be very different from the naïve one (cf. [40]) and indeed normalizability of the Kodama state has been suggested as a criterion for the correctness of the physical scalar product.

Loop states in general (suitable symmetrized) can be represented as spin network [37] states: colored lines (carrying some $SU(2)$ representation) meeting at nodes where intertwining $SU(2)$ operators act. A beautiful graphical representation of the group theory has
been succesfully adapted for this purpose. There is a clear relationship between this representation and the Turaev-Viro invariants Many of these ideas have been foresighted by Penrose (cf. [31]).

There is also a path integral representation, known as spin foam (cf. [8]), a topological theory of colored surfaces representing the evolution of a spin network. These are closely related to topological BF theories, and indeed, independent generalizations have been proposed. Spin foams can also be considered as an independent approach to the quantization of the gravitational field. ([10])

In addition to its specific problems, this approach shares with all canonical approaches to covariant systems the problem of time. It is not clear its definition, at least in the absence of matter. Dynamics remains somewhat mysterious; the hamiltonian constraint does not say in what sense (with respect to what) the three-dimensional dynamics evolve.

3.1 Big results

One of the main successes of the loop approach is that the area (as well as the volume) operator is discrete. This allows, assuming that a black hole has been formed (which is a process that no one knows how to represent in this setting), to explain the formula for the black hole entropy. The result is expressed in terms of the Barbero-Immirzi parameter ([38]). The physical meaning of this dependence is not well understood.

It has been pointed out [11] that there is a potential drawback in all theories in which the area (or mass) spectrum is discrete with eigenvalues $A_n$ if the level spacing between eigenvalues $\delta A_n$ is uniform because of the predicted thermal character of Hawking’s radiation. The explicit computations in the present setting, however, lead to an space between (dimensionless) eigenvalues

$$\delta A_n \sim e^{-\sqrt{A_n}},$$

which seemingly avoids this set of problems.
It has also been pointed out that not only the spin foam, but almost all other theories of gravity can be expressed as topological BF theories with constraints. While this is undoubtedly an interesting and potentially useful remark, it is important to remember that the difference between the linear sigma model (a free field theory) and the nonlinear sigma models is just a matter of constraints. This is enough to produce a mass gap and asymptotic freedom in appropriate circumstances.

4 Strings

It should be clear by now that we probably still do not know what is exactly the problem to which string theories are the answer. At any rate, the starting point is that all elementary particles are viewed as quantized excitations of a one dimensional object, the string, which can be either open (free ends) or closed (a loop). Excellent books are available, such as [19] [33].

String theories enjoy a convoluted history. Their origin can be traced to the Veneziano model of strong interactions. A crucial step was the reinterpretation by Scherk and Schwarz (39) of the massless spin two state in the closed sector (previously thought to be related to the Pomeron) as the graviton and consequently of the whole string theory as a potential theory of quantum gravity, and potential unified theories of all interactions. Now the wheel has made a complete turn, and we are perhaps back through the Maldacena conjecture (29) to a closer relationship than previously thought with ordinary gauge theories.

From a certain point of view, their dynamics is determined by a two-dimensional nonlinear sigma model, which geometrically is a theory of imbeddings of a two-dimensional surface (the world sheet of the string) to a (usually ten-dimensional) target space:

\[ x^\mu(\xi) : \Sigma_2 \rightarrow M_n \quad (19) \]

There are two types of interactions to consider. Sigma model interactions (in a given two-dimensional surface) are defined as an expansion in powers of momentum, where a new
dimensionful parameter, $\alpha' \equiv l_s^2$ sets the scale. This scale is a priori believed to be of the order of the Planck length. The first terms in the action always include a coupling to the massless backgrounds: the spacetime metric, the two-index Maxwell like field known as the Kalb-Ramond or $b$-field, and the dilaton. To be specific,

$$S = \frac{1}{l_s^2} \int_{\Sigma^2} g_{\mu\nu}(x(\xi)) \partial_\alpha x^\mu(\xi) \partial_\beta x^\nu(\xi) \gamma^{ab}(\xi) + \ldots$$

There are also string interactions, (changing the two-dimensional surface) proportional to the string coupling constant, $g_s$, whose variations are related to the logarithmic variations of the dilaton field. Open strings (which have gluons in their spectrum) always contain closed strings (which have gravitons in their spectrum) as intermediate states in higher string order ($g_s$) corrections. This interplay open/closed is one of the most fascinating aspects of the whole string theory.

It has been discovered by Friedan (cf. [16]) that in order for the quantum theory to be consistent with all classical symmetries (diffeomorphisms and conformal invariance), the beta function of the generalized couplings must vanish:

$$\beta(g_{\mu\nu}) = R_{\mu\nu} = 0$$

This result remains until now as one of the most important ones in string theory, hinting at a deep relationship between Einstein’s equations and the renormalization group.

Polyakov ([34]) introduced the so called non-critical strings which have in general a two-dimensional cosmological constant (forbidden otherwise by Weyl invariance). The dynamics of the conformal mode (often called Liouville in this context) is, however, poorly understood.

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3There are corrections coming from both dilaton and Kalb-Ramond fields. The quoted result is the first term in an expansion in derivatives, with expansion parameter $\alpha' \equiv l_s^2$. 

4.1 General setup

Fundamental strings live in D=10 spacetime dimensions, and so a Kaluza-Klein mechanism of sorts must be at work in order to explain why we only see four non-compact dimensions at low energies. Strings have in general tachyons in their spectrum, and the only way to construct seemingly consistent string theories (cf. [17]) is to project out those states, which leads to supersymmetry. This means in turn that all low energy predictions heavily depend on the supersymmetry breaking mechanisms.

String perturbation theory is probably well defined although a full proof is not available.

Several stringy symmetries are believed to be exact: T-duality, relating large and small compactification volumes, and S-duality, relating the strong coupling regime with the weak coupling one. Besides, extended configurations (D branes); topological defects in which open strings can end are known to be important [32]. They couple to Maxwell-like fields which are p-forms called Ramond-Ramond (RR) fields. These dualities [23] relate all five string theories (namely, Heterotic $E(8) \times E(8)$ Heterotic $SO(32)$, Type I, IIA and IIB) and it is conjectured that there is an unified eleven-dimensional theory, dubbed $M$-theory of which $\mathcal{N} = 1$ supergravity in $d = 11$ dimensions is the low energy limit.

4.2 Big results

Perhaps the main result is that graviton physics in flat space is well defined for the first time, and this is no minor accomplishment.

Besides, there is evidence that at least some geometric singularities are harmless in the sense that strings do not feel them. Topology change amplitudes do not vanish in string theory.

The other Big Result [12] is that one can correctly count states of extremal black holes as a function of charges. This is at the same time astonishing and disappointing. It clearly depends strongly on the objects being BPS states (that is, on supersymmetry), and the
result has not been extended to non-supersymmetric configurations. On the other hand, as we have said, it exactly reproduces the entropy as a function of a sometimes large number of charges, without any adjustable parameter.

### 4.3 The Maldacena conjecture

Maldacena [29] proposed as a conjecture that IIB string theories in a background $AdS_5 \times S_5$ with radius $l \sim l_s(g_sN)^{1/4}$ and $N$ units of RR flux is equivalent to a four dimensional ordinary gauge theory in flat four-dimensional Minkowski space, namely $\mathcal{N} = 4$ super Yang-Mills with gauge group $SU(N)$ and coupling constant $g = g_s^{1/2}$.

Although there is much supersymmetry in the problem and the kinematics largely determine correlators, (in particular, the symmetry group $SO(2, 4) \times SO(6)$ is realized as an isometry group on the gravity side and as an $R$-symmetry group as well as conformal invariance on the gauge theory side) this is not fully so \(^4\) and the conjecture has passed many tests in the semiclassical approximation to string theory.

This is the first time that a precise holographic description of spacetime in terms of a (boundary) gauge theory is proposed and, as such it is of enormous potential interest. It has been conjectured by 't Hooft [44] and further developed by Susskind [43] that there should be much fewer degrees of freedom in quantum gravity than previously thought. The conjecture claims that it should be enough with one degree of freedom per unit Planck surface in the two-dimensional boundary of the three-dimensional volume under study. The reason for that stems from an analysis of the Bekenstein-Hawking [11][20] entropy associated to a black hole, given in terms of the two-dimensional area $A$ \(^5\) of the horizon.

\(^4\)The only correlators that are completely determined through symmetry are the two and three-point functions.

\(^5\)The area of the horizon for a Schwarzschild black hole is given by:

$$A = \frac{8\pi G^2}{c^4} M^2$$  \hspace{1cm} (22)
by
\[ S = \frac{c^3}{4G\hbar} A. \]  

This is a deep result indeed, still not fully understood.

It is true on the other hand that the Maldacena conjecture has only been checked for the time being in some corners of parameter space, namely when strings can be approximated by supergravity in the appropriate background. The way it works \[50\] is that the supergravity action corresponding to fields with prescribed boundary values is related to gauge theory correlators of certain gauge invariant operators corresponding to the particular field studied:

\[ e^{-S_{\text{sugra}}[\Phi_i]} \bigg|_{\Phi_i|_{\partial \text{AdS}} = \phi_i} = < e^{i O_i \phi_i} >_{\text{CFT}} \]  

(24)

5 Observational prospects

In the long term, advances in the field, as in any other branch of physics will be determined by experiment. The prospects here are quite dim. It has been advertised \[1\] that as a consequence of loop quantum gravity \[6\] anomalous dispersion relations of the form

\[ E^2 = (\vec{p})^2 + m^2 + E^2 \sum_{n=1} c_n \left( \frac{E}{m_P} \right)^n \]  

(26)

could explain some strange facts on the cosmic ray spectrum. Although this is an interesting suggestion (cf.\[13\]) it is not specific to loop quantum gravity; noncommutative models make similar predictions as indeed does any theory with a fundamental scale. In spite of some optimism, it is not easy to perform specific experiments which could discriminate between different quantum gravity alternatives. This should not by any means be taken as an

\[ E^2 = (\vec{p})^2 + m^2 + m^2 \sum_{n=1} c_n \left( \frac{m}{m_P} \right)^n \]  

(25)

much more difficult to observe experimentally
indication that the experiments themselves are not interesting. Nothing could be most exciting that to discover deviations from the supposed exact symmetries of Nature, and it is amazing that present observations already seemingly exclude some alternatives \cite{30}.

On the string side, perhaps some effects related to specific stringy states, such as the winding states could be experimentally verified (cf. for example some suggestions in \cite{3}). It has also been proposed that the string scale could be lowered, from the Planck scale down to the TeV regime \cite{3}. It is difficult to really pinpoint what is exactly stringy about those models, and in particular, all string predictions are difficult to disentangle from supersymmetric model predictions and rely heavily on the mechanisms of supersymmetry breaking.

6 Summary: the state of the art in quantum gravity

In the loop approach one is working with nice candidates for a quantum theory. The theories are interesting, probably related to topological field theories \cite{12} and background independence as well as diffeomorphism invariance are clearly implemented. On the other hand, it is not clear that their low energy limit is related to Einstein gravity.

Strings start from a perturbative approach more familiar to a particle physicist. However, they carry all the burden of supersymmetry and Kaluza-Klein. It has proved to be very difficult to study nontrivial non-supersymmetric dynamics.

Finally, and this applies to all approaches, the holographic ideas seem intriguing; there are many indications of a deep relationship between gravity and gauge theories.

We would like to conclude by insisting on the fact that although there is not much we know for sure on quantum effects on the gravitational field, even the few things we know are a big feat, given the difficulty to do physics without experiments.
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