Greybody factor for an electrically charged regular-de Sitter black holes in \( d \)-dimensions

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Abstract

In this article, we study the propagation of scalar fields in the gravitational background of a spherically symmetric, electrically charged, regular-de Sitter black holes in higher dimensions. We carry out a thorough investigation of the grey body factors, by deriving analytical expressions for both minimally and non-minimally coupled scalar fields. Then we study the profile of grey body factors in terms of particle properties, namely the angular momentum quantum number and the non-minimal coupling constant, and the spacetime properties, namely the dimension, the cosmological constant and the non-linear charge parameter. We then derive the Hawking radiation spectra for a higher-dimensional electrically charged regular-de Sitter black hole. We find that the non-minimal coupling may suppress the greybody factor and the non-linear charge parameter may enhance it, but they both suppress the energy emission rate of Hawking radiation.

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I. INTRODUCTION

The regular black hole model is one of the promising candidates to understand the short distance behaviour. Such solutions are expected when Einstein’s general relativity is coupled with non-linear electrodynamics [1]. They also remove the very existence of the central singularity and replaces it with a de Sitter core. Therefore, the curvature scalars for regular black holes are finite everywhere, including at the origin, i.e., at \( r = 0 \). Since there is no suitable candidate for a quantum theory gravity, the regular black holes can be an alternative to describe the geodesically complete spacetimes. After its first inception by Bardeen [2], the research on the singularity free black hole solutions have been continued with great interest. During the last few decades, there have been a vast number of literature on regular black holes, which includes the study of the thermal phase transition [3–8], shadow properties [9–11], particle collision and particle accelerations and so on. Recently the connections between the black hole shadow, the quasinormal modes, and the greybody factor have been explored for the electrically charged regular Bardeen black holes [12]. The regular black hole solutions have also been studied in the modified theories of gravity, such as, Einstein Gauss-Bonnet gravity, the Lovelock gravity in an asymptotically flat spacetime [13] and also in dS/AdS spacetime [14, 15]. The rotating regular black hole solutions have been one of the popular test-beds for the non-Kerr family of black holes. The rotating regular black holes have also been studied in the higher dimensions [16–18].

The black hole mechanics obey the laws of thermodynamics which is for the most parts, analogous to the laws of the ordinary thermodynamic systems. The four laws of black holes mechanics were formulated in the pioneering work of Bardeen et al [19]. The most intriguing aspects that inspired Hawking was to derive the horizon temperature using a semi-classical approach [20, 21]. The Hawking radiation by black holes is a manifestation of a quantum effect in a curved spacetime, which has not been observationally detected so far. The idea of considering the solutions in extra spacetime dimensions has given a new boost in this direction, as the extra dimensions may give rise to the creation of mini black holes in high-energy particle colliders [22–24] or cosmic ray interactions [25–28] and subsequently, one can detect the associated Hawking radiation. This radiation temperature is very ideal in the sense that in the proximity of the event horizon, the black holes behave as an ideal black body. However, as we go far away from the horizon, the usual notion of black holes
as a thermal black body is lost, leaving the black holes to behave as a grey body in the far asymptotic region. This happens because of the presence of the effective potential barrier that the particles face while travelling towards infinity. This effective potential barrier is notoriously sensitive for the black hole background we are dealing with in the present study.

In recent years, a significant number of works on Hawking radiation were investigated for higher dimensional spacetimes (see [24, 29, 30] and references therein). It is also very exciting to note that the Hawking radiation is related to the quasinormal modes [31–35] and also to the black hole shadows [12, 36, 37]. To further understand the role of Hawking temperature on the greybody factor one can refer [38, 39].

In the present article, we investigate the greybody factor for higher dimensional electrically charged Bardeen-de Sitter black holes. The greybody factor in de Sitter spacetime has its own physical relevance, as it has nonvanishing values even when angular quantum number \( l = 0 \), and in the zero-frequency limit, \( \omega = 0 \), suggesting that the zero modes propagate between the region of the event horizon and the cosmological horizon [40, 41]. The case of vanishing cosmological constant does not contain such traversable modes for both minimally and non-minimally coupled scalar fields with an arbitrary value of \( l \), where \( l \) is the angular quantum number. The analytical study of the greybody factor for Schwarzschild-de Sitter [41, 42], Reissner-Nördstrom de Sitter [43], and Einstein-Gauss-Bonnet de Sitter [44] black holes have been performed to find the arbitrary partial modes of a scalar field. The study of graviton propagation and its emission rate has also been performed for higher dimensional axisymmetric Kerr black holes [45]. In our analysis, following similar procedures, we find a general expression for greybody factor for arbitrary partial modes of electrically charged Bardeen-de Sitter black holes in higher dimensions. The results are quite interesting as the cosmological constant shows a dual role in the analysis of the power spectrum.

The article is organised as follows. In section II we will present the black hole solution, which is followed by the discussion on particle emission in section III. Then we will derive an expression for the grey body factor in section IV and study its profile in section V. We will conclude the article with a discussion on our results in section VII.
II. BACKGROUND

We start with the action describing general relativity coupled to nonlinear electrodynamics in $d$-dimensions, which is,

$$S = \frac{1}{16\pi} \int d^d x \sqrt{-g} \left[ R + 2\Lambda - 4\mathcal{L}(\mathcal{F}) \right].$$

(1)

where $d$ is the spacetime dimension, $R$ is the Ricci scalar and

$$\mathcal{F} = F_{\mu\nu} F^{\mu\nu}/4, \ F_{\mu\nu} = 2\nabla_{[\mu} A_{\nu]}$$

is the electromagnetic field strength. The variation of (1), $\delta S = 0$, yields the equations for the gravitational field and for the nonlinear Maxwell field, respectively,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu}, \ \nabla_\mu (\mathcal{L}_\mathcal{F} F^{\mu\nu}) = 0, \ \nabla_\mu (*F^{\mu\nu}) = 0,$$

(2)

where $\mathcal{L}_\mathcal{F} = \partial \mathcal{L}/\partial \mathcal{F}$. The energy-momentum tensor is given by

$$T_{\mu\nu} = \frac{1}{4\pi} (\mathcal{L}_\mathcal{F} F^{\mu\nu}_2 - g_{\mu\nu} \mathcal{L}).$$

(3)

Using Einstein and Maxwell equations it is possible associate the Bardeen black hole with an electric source. As the black hole is static and spherically symmetric, the only non-zero component of Maxwell-Faraday tensor is $F_{10}$. When $\mu = 0$, we have,

$$F^{10} = \frac{q^{d-3}}{r^{d-2}} \mathcal{L}^{-1}_\mathcal{F}.$$

(4)

The metric for a class of spherically symmetric electrically charged Bardeen de Sitter black holes is given by

$$ds^2 = - h dt^2 + \frac{dr^2}{h} + r^2 d\Omega^2_{d-2},$$

(5)

where the metric function is given as $h = 1 - \frac{m(r)}{r^{d-1}} - \frac{2\Lambda r^2}{(d-1)(d-2)}$, and the mass function $m(r) = \frac{\mu r^\alpha}{(r^\beta + q^\beta)\alpha/\beta}$. The above metric function reduces to the Bardeen solution when $\alpha = d-1$ and $\beta = d-2$, while for $\alpha = \beta = d-1$, it reduces to the Hayward solution. The parameter $\mu$ is related to the mass of the black hole $M$ by $\mu = \frac{16\pi G M}{(d-2)\Omega_{d-2}}$. Here $q$ is the nonlinear parameter arising when gravity is coupled to nonlinear electrodynamics. In the limit $q \to 0$,
the metric reduces to that of the Schwarzschild-de Sitter black hole. For the electric case, from Eqs. (2) and (3), we can write the non-zero components of Einstein equations as:

\[
\frac{\alpha \, m'(r)}{r^2} = \mathcal{L} + \frac{q^{2(d-3)}}{r^{2(d-2)}} \mathcal{L}_F^{-1}.
\]  

(6)

\[
\frac{m''(r)}{r} = \mathcal{L}.
\]  

(7)

In terms of the horizon radius \( r_h \), \( \mu \) can be expressed as

\[
\mu = \left(1 + \frac{q^\beta}{r_h^\beta}\right)^{\alpha/\beta} r_h^{\alpha-2} \left(1 - \frac{2\Lambda r_h^2}{(d-1)(d-2)}\right).
\]  

(8)

The constant \( q \) has a significant effect on the stability of regular black holes. Through perturbation analysis, it was found that regular-dS black holes are unstable in certain parameter region. In our discussions, the parameters are restricted to the stable region such that the spacetime always has two horizons, the black hole horizon \( r_h \) and the cosmological horizon \( r_c \). The Lagrangian \( \mathcal{L}(r) \) and its first derivative \( \mathcal{L}_F(r) \) are obtained when one solves the equations (6) and (7) such that,

\[
\mathcal{L}(r) = \frac{\mu r^{\alpha-3}q^\beta((\alpha-1)\alpha q^\beta - (\beta + 1)\alpha r^\beta)}{(r^\beta + q^\beta)^{\frac{\alpha + 2\beta}{\beta}}},
\]

\[
\mathcal{L}_F(r) = \frac{(r^\beta + q^\beta)^{\frac{\alpha + 2\beta}{\beta}}}{\mu \alpha (2\beta + 1) r^\alpha + 2 - \beta q^{2(d-3)}}.
\]

For \( \alpha = d - 1 \) and \( \beta = d - 2 \), we have \( \mathcal{L}(r) \) and its first derivative \( \mathcal{L}_F(r) \) for Bardeen case yielding,

\[
\mathcal{L}(r) = \frac{\mu r^{d-4}q^{d-2}((d-2)(d-1)q^{d-2} - (d-1)^2 r^{d-2})}{(r^{d-2} + q^{d-2})^{\frac{3d-5}{d-2}}},
\]

\[
\mathcal{L}_F(r) = \frac{q^{d-4}(r^{d-2} + q^{d-2})^{\frac{3d-5}{d-2}}}{\mu (d-1)(2d-3)r^{2(d-1)}}.
\]

While, for \( \alpha = d - 1 \) and \( \beta = d - 1 \), we have \( \mathcal{L}(r) \) and its first derivative \( \mathcal{L}_F(r) \) for electrically charged Hayward black hole in \( d \)-dimensions such that,

\[
\mathcal{L}(r) = \frac{\mu r^{d-4}q^{d-1}((d-2)(d-1)q^{d-1} - d(d-1)r^{d-1})}{(r^{d-1} + q^{d-1})^3},
\]

\[
\mathcal{L}_F(r) = \frac{q^{d-5}(r^{d-1} + q^{d-1})^3}{\mu (d-1)(2d-1)r^{2d-1}}.
\]
We have the corresponding expressions for the Lagrangian and its first derivatives for the electrically charged Bardeen [46] and Hayward black holes respectively, in $d = 4$ dimensions. In the subsequent analysis, we focus only on the electrically charged regular Bardeen-de Sitter black holes.

As a consistency check and physical relevance of the solutions and also the source, we check the energy conditions for $\Lambda = 0$. We identify the terms $T^t_t = -\rho$, $T^r_r = p_r$ and $T^\theta_\theta = T^{\phi}_\phi = \ldots = T^{\theta_{d-3}}_{\theta_{d-3}} = T^{\phi_{d-2}}_{\phi_{d-2}} = p_t$, where $\rho$, $p_r$, and $p_t$ are, the energy density, the radial pressure, and the tangential pressure, respectively. The energy conditions are given by

\[
SEC(r) = \rho + \sum_{i=1}^{d-1} p_i = (d - 2)p_t \geq 0,
\]

\[
WEC_{1,2}(r) = NEC_{1,2}(r) = \rho + p_{r,t} \geq 0,
\]

\[
WEC_3(r) = DEC_1(r) = \rho \geq 0,
\]

\[
DEC_{2,3}(r) = \rho - p_{r,t} \geq 0.
\] (9)

We calculate the expressions for the energy density, the radial pressure, and the tangential pressure as

\[
\rho(r) = \frac{\mu(d-1)(d-2)d^{-d-2}}{(r^d - 2 + q^d - 2)^{d-1}}\]

\[
p_r(r) = \frac{\mu(d-1)(d-2)}{(r^d - 2 + q^d - 2)^{d-2}}\]

\[
p_t(r) = \frac{\mu r d - 4 q^{-2} (-d - 2)(d - 1)q - 2 + (d - 1)^2 r^{-2})}{(r^d - 2 + q^d - 2)^{d-1}}\] (10)

Therefore, the final expressions for the energy conditions are given as

\[
SEC(r) = (d - 2)\frac{\mu r d - 4 q^{-2} (-d - 2)(d - 1)q - 2 + (d - 1)^2 r^{-2})}{(r^d - 2 + q^d - 2)^{d-1}}\]

\[
WEC_1(r) = 0,
\]

\[
WEC_2(r) = \frac{\mu(d-1)(2d - 3)r^{2(d-3)}q^{-2}}{(r^d - 2 + q^d - 2)^{d-1}}\]

\[
WEC_3(r) = DEC_1(r) = \frac{\mu(d-1)(d-2)r d - 4 q^{-2}}{(r^d - 2 + q^d - 2)^{d-1}}\]

\[
DEC_2(r) = \frac{2\mu(d-1)(d-2)r d - 4 q^{-2}}{(r^d - 2 + q^d - 2)^{d-1}}\]

\[
DEC_3(r) = \frac{\mu(d-1)(d-2)r d - 4 q^{-2} (q^d - 2 d - (d - 3)r^{-2})}{(r^d - 2 + q^d - 2)^{d-1}}\] (11)
The expressions for the energy conditions (11), and also the energy density, the radial pressure and the tangential pressure (10), reduce to the corresponding expressions for the electrically charged Bardeen black holes in four dimensions [46]. We can see from the above expressions that the strong energy conditions are violated when \( r < \left( \frac{d-2}{d-1} \right)^{1/(d-2)} q \), which reduces to \( r < (2/3)^{1/2} q \) [46] for \( d = 4 \). This is expected since this region is inside the event horizon and the metric function becomes negative at this value of \( r \), and it is expected for any black hole spacetime with a regular centre. However, the condition \( p_r = p_t \) is not met for the \( d \)-dimensional electrically charged Bardeen black holes, thereby representing the anisotropic perfect fluid with \( p_r = -\rho \).

III. THE KLEIN-GORDON EQUATION

In this section, we study the emission of scalar particles that may minimally or non-minimally coupled to gravity. Then, for the scalar field coupling, we have an additional part of the action,

\[
S_\Phi = -\frac{1}{2} \int d^d x \sqrt{-g} [\xi \Phi^2 R + \partial_\mu \Phi \partial^\mu \Phi].
\]  

(12)

Here \( \xi \) is the non-minimal coupling parameter which determines the strength of the coupling, and \( \xi = 0 \) corresponds to minimal coupling. The propagation of scalar particle is governed by the following equation of motion,

\[
\nabla_\mu \nabla^\mu \Phi = \xi R \Phi.
\]  

(13)

It is customary to make an ansatz in a spherically symmetric background as,

\[
\Phi = e^{-i\omega t} \phi(r) Y_{(d-2)}^l(\Omega),
\]  

(14)

where \( Y_{(d-2)}^l(\Omega) \) are spherical harmonics on \( S^{d-2} \). Substituting this back to the equation of motion, we have the variable separable form, from which the radial equation reads as,

\[
\frac{1}{r^{d-2}} \frac{d}{dr} \left( h r^{d-2} \frac{d\phi}{dr} \right) + \left[ \frac{\omega^2}{h} - \frac{l(l + d - 3)}{r^2} - \xi R \right] \phi = 0.
\]  

(15)

Redefining the radial function \( u(r) = r^{d-2} \phi(r) \), and introducing the tortoise coordinates \( r_* \) by using the definition \( dr_* = dr/h(r) \), we have,

\[
\frac{d^2 u}{dr_*^2} + (\omega^2 - V(r_*)) u = 0,
\]  

(16)
where the effective potential is,

\[ V(r^\ast) = h\left[ \frac{l(l+d-3)}{r^2} + \xi R + \frac{d-2}{2r} h' + \frac{(d-2)(d-4)}{4r^2} h \right]. \]  

(17)

The effective potential vanishes at the two horizons, as the metric function \( h \) also vanishes there. The dependence of effective potential on the spacetime properties (\( \Lambda, q, d \)) and particle properties (\( \xi, l \)) can be studied from the above equation. While doing so, the parameter \( m \) has to be written in terms of horizon radius \( r_h \) by using the condition \( h(r_h) = 0 \), and then the value of \( r_h \) is set to 1 (\( r_h = 1 \)). We will depict these behaviours along with the grey body factor’s profile in the following sections, as it would give an intuitive knowledge of particle propagation.

IV. GREYBODY FACTOR

The analytical solution for the radial differential equation (15) over the whole spacetime region is not feasible even for the simplest choice of \( \xi = 0 \). Here we follow an approximate method proposed by Kanti et al. [47]. In this approach, the solutions are sought at specific radial regions, i.e. at the vicinity of horizons, and those solutions are patched up naively in the low energy limit. The key point to note in this matching of the solutions in the intermediate region is that, the cosmological constant should be treated with utmost care to ensure the validity of the result with more accuracy.

A. Near the event horizon

To solve the radial equation near the black hole event horizon we make the following coordinate transformation and redefinition of cosmological constant,

\[ r \rightarrow f(r) = \frac{h}{1 - \Lambda r^2}, \quad \tilde{\Lambda} = \frac{2\Lambda}{(d-1)(d-2)}. \]  

(18)

The transformed variable \( f \) takes values from 0 to 1 as \( r \) varies from \( r_h \) to \( r \gg r_h \). In addition to this, the derivative satisfies the relation,

\[ \frac{df}{dr} = \frac{1 - f}{r} \frac{A(r)}{1 - \Lambda r^2}. \]  

(19)

where

\[ A(r) = d - 3 + (d - 1)\tilde{\Lambda} r^2 - \frac{(d-1)(1 - \tilde{\Lambda} r^2)q^{d-2}}{r^{d-2} + q^{d-2}}, \]  

(20)
which reduces to the expression for the Schwarzschild-Tangherlini-de Sitter black holes when \( q = 0 \), as \( A_{Sds} = d-3+(d-1)\Lambda r^2 \). In terms of the transformed variables, the radial equation near the event horizon takes the form,

\[
 f(1 - f) \frac{d^2 \phi}{df^2} + (1 - B_h f) \frac{d\phi}{df} + \left[ - \frac{(\omega r_h)^2}{A_h^2} + \frac{(\omega r_h)^2}{A_h^2 f} - \frac{\lambda_h (1 - \Lambda r_h^2)}{A_h^2 (1 - f)} \right] \phi = 0. \tag{21}
\]

where we used the abbreviations,

\[
 B_h = 2 - \frac{1 - \Lambda r_h^2}{A_h^2} [(d - 3)A_h + rA'(r_h)], \quad \lambda_h = l(l + d - 3) + \xi R^{(h)} r_h^2, \tag{22}
\]

where \( A_h = A(r_h) \) and \( R^{(h)} = -h'' + (d - 2) \frac{-2r^h'(d-3)(1-h)}{r^2} \bigg|_{r_h} \) is the Ricci curvature scalar on the event horizon. We have also made use of the following approximation near the event horizon \( f \sim 0 \),

\[
 \frac{(\omega r_h)^2}{A_h^2 f (1 - f)} \sim \frac{(\omega r_h)^2 (1 - f)}{A_h^2 f} = - \frac{(\omega r_h)^2}{A_h^2} + \frac{(\omega r_h)^2}{A_h^2 f}. \tag{23}
\]

The purpose of this is, as mentioned in Ref. [48], to avoid the unphysical behaviour that arises due to the poles of Gamma function. By making the field redefinition, \( \phi = f^{\alpha_1}(1 - f)^{\beta_1} W(f) \), Eq.(21) becomes a hypergeometric equation,

\[
 f(1 - f) \frac{d^2 W}{df^2} + \left[ 1 + 2\alpha_1 - (2\alpha_1 + 2\beta_1 + B_h) f \right] \frac{dW}{df} - \frac{\omega^2 r_h^2 + A_h^2 (\alpha_1 + \beta_1) (B_h + \alpha_1 + \beta_1 - 1)}{A_h^2} W = 0. \tag{24}
\]

where the coefficients are given by,

\[
 \alpha_1 = \pm \frac{\omega r_h}{A_h}, \quad \beta_1 = \frac{1}{2} \left( 2 - B_h \pm \sqrt{(2 - B_h)^2 + \frac{4\lambda_h (1 - \Lambda r_h^2)}{A_h^2}} \right). \tag{25}
\]

The solution of the above equation (24) is the standard hypergeometric function \( F(a_1, b_1, c_1, f) \) in which the parameters \( a_1, b_1, c_1 \) are given by

\[
 a_1 = \alpha_1 + \beta_1 + \frac{1}{2} \left( B_h - 1 + \sqrt{(1 - B_h)^2 - \frac{4\omega^2 r_h^2}{A_h^2}} \right),
\]

\[
 b_1 = \alpha_1 + \beta_1 + \frac{1}{2} \left( B_h - 1 - \sqrt{(1 - B_h)^2 - \frac{4\omega^2 r_h^2}{A_h^2}} \right), \tag{26}
\]

\[
 c_1 = 1 + 2\alpha_1.
\]

By considering the original and redefined fields, \( \phi(f) \) and \( W(f) \), near the event horizon, the radial function \( \phi(f) \) takes the following form,

\[
 \phi_H = A_1 f^{\alpha_1} (1 - f)^{\beta_1} F(a_1, b_1, c_1, f) + A_2 f^{-\alpha_1} (1 - f)^{\beta_1} F(1 + a_1 - c_1, 1 + b_1 - c_1, 2 - c_1, f).
\]
in which $A_{1,2}$ are the constant coefficients. In the vicinity of event horizon,

$$\phi_H \simeq A_1 f^{\alpha_1} + A_2 f^{-\alpha_1}, \quad \text{and} \quad f \propto e^{Ah/r_h}. \quad (27)$$

Now we impose the ingoing boundary condition near the event horizon and choose $\alpha_1 = -i\frac{\omega r_h}{A_h}$, so that $A_2 = 0$. Moreover, the real part $\text{Re}(c_1 - a_1 - b_1) > 0$ is due to the convergence condition of the hypergeometric function. Therefore, we have to take the negative branch of the coefficient $\beta_1$. To this end, near the event horizon, we have,

$$\phi_H = A_1 f^{\alpha_1}(1 - f)^{\beta_1} F(a_1, b_1, c_1, f). \quad (28)$$

### B. Near the cosmological horizon

We now focus on the radial region near the cosmological horizon $r_c$, where we use a similar kind of approach. Here, the metric function $h$ is approximated as [47–50]

$$h(r) = 1 - \tilde{\Lambda}r^2 - \left(\frac{r_h}{r}\right)^{d-3}(1 - \tilde{\Lambda}r_h^2) \sim \tilde{h} = 1 - \tilde{\Lambda}r^2. \quad (29)$$

The new function $\tilde{h}$ takes values from 0, at $r = r_c$, to 1 as $r \ll r_c$. The approximation that we are following gives more accurate results in the region far away from the black hole, which is to say, for a large $r_c$ or small $\tilde{\Lambda}$. Also, the approximation scheme yields more accurate results for higher values of spacetime dimension $d$.

Here, we make the change of variable, $r \rightarrow \tilde{h}(r)$, so that the radial equation near the cosmological horizon takes the form,

$$\tilde{h}(1 - \tilde{h}) \frac{d^2 \phi}{d\tilde{h}^2} + \left(1 - \frac{d + 1}{2} \tilde{h}\right) \frac{d\phi}{d\tilde{h}} + \left[\frac{(\omega r_c)^2}{4\tilde{h}} - \frac{l(l + d - 3)}{4(1 - \tilde{h})} - \frac{\xi R^{(c)} r_c^2}{4}\right] \phi = 0, \quad (30)$$

in which $R^{(c)} = -\tilde{h}'' + (d - 2)\frac{-2\tilde{h}'' + (d - 3)(1 - \tilde{h})}{2} \bigg|_{r_c}$ is the Ricci curvature scalar at $r_c$. With a field redefinition of $\phi(\tilde{h}) = \tilde{h}^{\alpha_2}(1 - \tilde{h})^{\beta_2} X(\tilde{h})$, the above equation becomes a hypergeometric equation,

$$(1 - \tilde{h})\tilde{h} \frac{d^2 X}{d\tilde{h}^2} + \left[1 + 2\alpha_2 - (2\alpha_2 + 2\beta_2 + \frac{d + 1}{2}) \tilde{h}\right] \frac{dX}{d\tilde{h}} - \frac{2(\alpha_2 + \beta_2)(\alpha_2 + \beta_2 + d - 1) + \xi R^{(c)} r_c^2}{4} X = 0, \quad (31)$$

where,

$$\alpha_2 = \pm i\frac{\omega r_c}{2}, \quad \beta_2 = -\frac{d + l - 3}{2} \quad \text{or} \quad \frac{l}{2}. \quad (32)$$
As before, the solution to this hypergeometric equation (30) gives the hypergeometric function,

\[ \phi_C = B_1 \tilde{h}^{\alpha_2} (1 - \tilde{h})^{\beta_2} F(a_2, b_2, c_2, \tilde{h}) + B_2 \tilde{h}^{-\alpha_2} (1 - \tilde{h})^{\beta_2} F(1 + a_2 - c_2, 1 + b_2 - c_2, 2 - c_2, \tilde{h}), \tag{33} \]

with the parameters

\[ a_2 = \alpha_2 + \beta_2 + \frac{d - 1 + \sqrt{(d - 1)^2 - 4 \xi R(c) r_c^2}}{4}, \tag{34} \]

\[ b_2 = \alpha_2 + \beta_2 + \frac{d - 1 - \sqrt{(d - 1)^2 - 4 \xi R(c) r_c^2}}{4}, \]

\[ c_2 = 1 + 2 \alpha_2. \]

In the above, \( B_{1,2} \) are constant coefficients. The criteria of convergence of the hypergeometric function has the requisite that \( \text{Re}(c_2 - a_2 - b_2) > 0 \), which forces us to take \( \beta_2 = -\frac{d - l - 3}{2} \).

The metric function, and hence the effective potential vanishes at the cosmological horizon \( r_c \). Therefore, we expect the solution, which is the superposition of plane waves,

\[ \phi_C = B_1 e^{-i \omega r_*} + B_2 e^{i \omega r_*}, \tag{35} \]

where \( r_* = \frac{1}{2} r_c \ln \frac{r/r_c + 1}{r/r_c - 1} \) is the tortoise coordinate near \( r_c \). The negative exponent part stands for an ingoing wave, whereas the positive one for outgoing wave. The sign of \( \alpha_2 \) just interchanges for the ingoing and outgoing waves. We take \( \alpha_2 = \frac{i \omega r_c}{2} \) here. Unlike the region near the event horizon, here both the ingoing and outgoing waves do exist. The amplitudes of these waves gives the grey body factor for the scalar fields emitted by the black hole, as,

\[ |\gamma_{\omega}|^2 = 1 - \left| \frac{B_2}{B_1} \right|^2. \tag{36} \]

C. Matching the solutions in the intermediate region

The solution is complete only if we ensure that the asymptotic solutions \( \phi_H \) and \( \phi_C \) can be matched for an arbitrary value of radial coordinate, which lies in the region between two horizons.

1. Black hole horizon

To begin with, we consider the solution near the event horizon. As \( r \gg r_h \) in the intermediate region, the function \( f \to 1 \), which allows us to shift the argument of the
hypergeometric function from $f$ to $1 - f$,

$$F(a, b, c; f) = \frac{\Gamma(c)\Gamma(c - a - b)}{\Gamma(c - a)\Gamma(c - b)} F(a, b, a + b - c + 1; 1 - f)$$

$$+ (1 - f)^{c - a - b} \frac{\Gamma(c)\Gamma(a + b - c)}{\Gamma(a)\Gamma(b)} F(c - a, c - b, c - a - b + 1; 1 - f).$$

With the small value approximation of cosmological constant $\Lambda r_h^2 \ll 1$, we have $A_h \simeq d - 3$, in the region where $r \gg r_h$. This is valid only if $\Lambda r_h^2 \simeq r^2/r_c^2 \ll 1$. For the region where $r \gg r_h$, from (18) we have

$$h \rightarrow 1 - \tilde{\Lambda} r^2 + \mathcal{O}\left(\frac{r_h^{d-3}}{r^{d-3}}\right).$$

Under this condition, the Ricci scalar $R^{(h)} \rightarrow \frac{2d\Lambda}{d-2}$. Therefore, for small values of $\xi$, the term $\xi R^{(h)} r_h^2 \rightarrow \xi \frac{2d\Lambda r_h^2}{d-2} \ll 1$ and hence it can be neglected. Hence, we have $B_h \simeq 1, \beta_1 \simeq -\frac{l}{d-3}$.

From the above inputs, we have

$$1 - f \simeq \left(1 + q \frac{d-2}{r_h^{d-2}}\right) \frac{d-1}{d-2} \left(r_h^d r^{-d-3}\right)$$

$$\beta_1 + c_1 - a_1 - b_1 \simeq \frac{l + d - 3}{d - 3}.$$  \hspace{1cm} (39)

Now, the solution (28) in the region $r \gg r_h$ reads,

$$\phi_H \simeq \Sigma_2 r^l + \Sigma_1 r^{-l-d+3}$$

where

$$\Sigma_1 = A_1 \frac{\Gamma(c_1)\Gamma(a_1 + b_1 - c_1)}{\Gamma(a_1)\Gamma(b_1)} \left(1 + q \frac{d-2}{r_h^{d-2}}\right) \frac{(d-1)(l+d-3)}{(d-2)(d-3)} r_h^{l+d-3},$$

$$\Sigma_2 = A_1 \frac{\Gamma(c_1)\Gamma(c_1 - a_1 - b_1)}{\Gamma(c_1 - a_1)\Gamma(c_1 - b_1)} \left(1 + q \frac{d-2}{r_h^{d-2}}\right) \frac{-(d-1)}{(d-2)(d-3)} r_h^{-l}.$$  \hspace{1cm} (41)

As clearly mentioned in Ref [47], the approximations discussed above are applicable only for the expressions having the factor $(1 - f)$, not in the arguments of the gamma functions to increase the validity of the analytical results.

2. Cosmological horizon

Now we consider the other asymptotic solution, which is near the cosmological horizon. As we have done earlier, we change the argument of the hypergeometric function from $\tilde{h}$ to
$1 - \tilde{h}$ as in the intermediate region $\tilde{h} \to 1$. Here too, we make the small value approximation of the cosmological constant. In the region where $r \ll r_c$, we have,

$$1 - \tilde{h} \simeq \left( \frac{r}{r_c} \right)^2$$

(42)

and $\beta_2 \simeq -\frac{l+d-3}{2}$, $\beta_2 + c_2 - a_2 - b_2 \simeq l/2$. Following the similar procedure as earlier, we get

$$\phi_C \simeq (\Sigma_3 B_1 + \Sigma_4 B_2) r^{-(l+d-3)} + (\Sigma_5 B_1 + \Sigma_6 B_2) r^l$$

(43)

where

$$\Sigma_3 = \frac{\Gamma(c_2)\Gamma(c_2 - a_2 - b_2)}{\Gamma(c_2 - a_2)\Gamma(c_2 - b_2)} r_c^{l+d-3}, \quad \Sigma_4 = \frac{\Gamma(2 - c_2)\Gamma(c_2 - a_2 - b_2)}{\Gamma(1 - a_2)\Gamma(1 - b_2)} r_c^{l+d-3},$$

$$\Sigma_5 = \frac{\Gamma(c_2)\Gamma(a_2 + b_2 - c_2)}{\Gamma(a_2)\Gamma(b_2)} r_c^{-l}, \quad \Sigma_6 = \frac{\Gamma(2 - c_2)\Gamma(a_2 + b_2 - c_2)}{\Gamma(a_2 - c_2 + 1)\Gamma(b_2 - c_2 + 1)} r_c^{-l}.$$  

(44)

Since the two solutions (40) and (43) have the same power-law form, their smooth matching is straightforward. Matching the coefficients of the same powers of $r$ in (40) and (43), we have the constraint equations,

$$\Sigma_3 B_1 + \Sigma_4 B_2 = \Sigma_1, \quad \Sigma_5 B_1 + \Sigma_6 B_2 = \Sigma_2.$$  

(45)

Solving the above equations for $B_1$ and $B_2$ and substituting them back to the grey body factor expression, we have

$$|\gamma_\omega|^2 = 1 - \left| \Sigma_2 \Sigma_3 - \Sigma_1 \Sigma_5 \right|^2 \left/ \left| \Sigma_1 \Sigma_6 - \Sigma_2 \Sigma_4 \right| \right.^{2}.$$  

(46)

This expression has the same form as that for the Einstein gravity [47] and Einstein Gauss-Bonnet gravity [48]. However, the dependence of $\Sigma$ on spacetime and particle properties are different in all these cases. Here we have an additional parameter, the non-linear charge $q$, in addition to the cosmological constant $\Lambda$ and the non-minimal coupling $\xi$, which will definitely influence the propagation of the scalar field.

The solution we obtained for the grey body factor (46) is appropriate for small values of the cosmological constant and when the distance between the two horizons is large. Moreover, in the approximation scheme we followed, the energy of the emitted particle is not involved, which hints at the validity of the result in all energy range. However, we will see that the results are more accurate in the low energy region, and deviations from the expected values occur in the high energy region. This calls for a reevaluation of the problem with other methods, possibly a numerical investigation of Eq. (15), which we are currently working on as part of our future project.
D. Low energy limit

As the analytical expression we derived gives proper profile in the low energy limit, we will obtain the low energy limit, \( \omega \to 0 \) case, for both minimal and non-minimal coupling of scalar fields.

First we will consider the minimal coupling case \( \xi = 0 \), particularly for the lowest mode of propagation \( l = 0 \). It is not very hard to show that,

\[
\Sigma_1 \sim A_1 \frac{i\omega}{(2 - B_{h0})A_{h0}} r_r^{d-3} \left[ \left( \frac{q}{r_h} \right)^{d-2} + 1 \right]^{\frac{d-3}{d-2}} + O(\omega^2), \quad \Sigma_2 \sim A_1 + O(\omega), \quad (47)
\]

\[
\Sigma_3 \sim \frac{i\omega}{d-3} r_c^{d-2} + O(\omega^2), \quad \Sigma_4 \sim \frac{-i\omega}{d-3} r_c^{d-2} + O(\omega^2), \quad \Sigma_{5,6} \sim 1 + O(\omega). \quad (48)
\]

where,

\[
A_{h0} = (d - 3) - \frac{(d - 1)q^{d-2}}{q^{d-2} + r_h^{d-2}}, \quad (49)
\]

and

\[
B_{h0} = \frac{(-2d^2 + 3d + 7)q^{d+2}r_h^{d+2} + 2(d + 1)r_h^4 q^{2d} + (d - 3)^2 q^4 r_h^{2d}}{((d - 3)q^2 r_h^d - 2r_h^q g^d)^2}. \quad (50)
\]

Substituting these, we have the expression for the greybody factor as,

\[
|\gamma| = \frac{4A_{h0}(2 - B_{h0})(d - 3)r_c^{d+2}r_h^{d+3} \left( \frac{r_r^2(q)^d}{q^2} + 1 \right)^{\frac{d+1}{d-2}}}{\left[ A_{h0}(B_{h0} - 2)r_r^{3r_c} \left( \frac{r_r^2(q)^d}{q^2} + 1 \right)^{\frac{1}{d-2}} - (d - 3)r_c^2 r_h^2 \left( \frac{r_r^2(q)^d}{q^2} + 1 \right)^{\frac{d}{d-2}} \right]^2 + O(\omega)}. \quad (51)
\]

From the above result, it is clear that the probability of emitting low energy scalar particles is non-zero for a higher dimensional electrically charged regular black hole. Therefore, the characteristic feature of the scalar particle propagation in the de-Sitter background is intact in regular spacetime. However, we see that the non-linear charge alters the grey body factor significantly.

The explicit calculation of the limiting cases is not an easy task for the case of non minimal coupling. However, in the low energy limit, it is easy to show that,

\[
\Sigma_2\Sigma_3 = E + i\Sigma_{231} \omega + \Sigma_{232}\omega^2, \quad \Sigma_1\Sigma_5 = K + i\Sigma_{151} \omega + \Sigma_{152}\omega^2, \quad (52)
\]
\[ \Sigma_2 \Sigma_4 = E + i \Sigma_{241} \omega + \Sigma_{242} \omega^2, \quad \Sigma_1 \Sigma_6 = K + i \Sigma_{161} \omega + \Sigma_{162} \omega^2. \]  

(53)

From which we have,

\[ |\Sigma_2 \Sigma_3 - \Sigma_1 \Sigma_5|^2 \sim |\Sigma_1 \Sigma_6 - \Sigma_2 \Sigma_4|^2 = (K - E)^2 + \mathcal{O}(\omega^2). \]  

(54)

Substituting this in the expression for grey body factor, we obtain a non-zero term for the low energy scalar particle emission in the case of non-minimal coupling. The result is true for the propagation of arbitrary mode. The non-minimal coupling removes the scalar modes with non-zero low energy asymptotic grey body factor.

V. THE EFFECT OF SPACETIME AND PARTICLE PROPERTIES ON GREY BODY FACTOR

The propagation of scalar fields in the gravitational background of a charged regular de-Sitter black hole is influenced by the parameters associated with both the spacetime and particle properties. The particle properties, angular momentum number \( l \) and non-minimal coupling constant \( \xi \); and the spacetime properties, the cosmological constant \( \Lambda \) and spacetime dimension \( d \), affect in a manner similar to that of SdS spacetime. Here we have an additional spacetime feature, that is, the non-linear coupling parameter \( q \), on which we will be more focused. The dependence of the grey body factor on these parameters is studied alongside the dependence of effective potential. We study the greybody factor \( |\gamma_{\omega l}|^2 \) as a function of the dimensionless energy parameter \( \omega r_h \). To get an intuitive understanding of the particle propagation in the gravitational background, we display the effective potential below the grey body plots. Fig 1 shows the effect of particle parameters \((l, \xi)\). In the left panel first five partial waves with the angular momentum number \( l = 0, 1, 2, 3, 4 \) are shown for \( \Lambda = 0.01 \) (in units of \( r_h^{-2} \)). The solid lines represent the minimal coupling case \( \xi = 0 \) and the dashed line for non-minimal coupling \( \xi = 0.5 \). For both cases, the angular momentum number \( l \) suppresses the grey body factor, which is evident from the effective potential profile, where higher modes have higher potential barriers. Whether the scalar field is minimally coupled or not, the dominant mode is \( l = 0 \). Nevertheless, the coupling parameter \( \xi \) decreases the grey body factor for all modes, the effect is significant for the dominant mode. The corresponding effect of \( \xi \) is also seen in effective potential. For \( \xi = 0 \)
the low energy grey body factor is nonzero, as we have proved earlier, whereas, the non-minimal coupling alters this feature. In fact, in the right hand panel of Fig. 1, we have studied the dependence of particle propagation in the lowest mode, \( l = 0 \), with the coupling parameter \( \xi \). As the non-minimal coupling reduces the effective potential, the corresponding grey body factor is enhanced.

Now we focus on the effect of spacetime properties on grey body factor; however, we will discuss the role of cosmological constant separately, as it shows a bizarre characteristic. In Fig. 2 the role of \((q, d)\) are depicted for the dominant mode. The non-linear charge has an enhancing effect on the grey body factor, as it reduces the effective potential significantly. Grey body factors for different dimensions of spacetime is observed in both the presence of non-linear charge and in the limit \( q \to 0 \) cases (Fig. 2 right panel). We see that for higher dimensions, grey body is suppressed as the potential barrier increases significantly. The non-linear charge changes the saturation point of grey body factor. However, the grey body factors must approach 1 in all cases, as high energy modes can cross the potential barrier easily. We plot grey body factors in the low energy region, as the matching approach we followed in the derivation of \( \gamma_{\omega l} \) is valid in the low energy region. Moreover, the analytical approach is limited to only even dimension spacetimes.

Finally, we study the influence of the cosmological constant on the particle propagation. Interestingly, it depends on the value of the non-minimal coupling parameter \( \xi \). For small values of \( \xi \), cosmological constant \( \Lambda \) has an enhancing effect on grey body factor, whereas, above a particular value of \( \xi \), the effect of it is to hinder the emission of scalar fields. This competition between \( \xi \) and \( \Lambda \) is also reflected in the behaviour of effective potential. This is due to the dual role played by the \( \Lambda \) in the equations of motion. The enhancing effect is due to its feature of homogeneously distributed energy in the entire spacetime, whereas the suppressing effect is due to its role as an effective mass term through the non-minimal coupling term. These dual contributions leads to competition between \( \Lambda \) and \( \xi \). This phenomenon is also observed in SdS and Gauss-Bonnet dS spacetime.
Figure 1: Effect of particle properties on the grey body factors (upper panel) and enhancement/suppression of corresponding effective potential (lower panel) for $d = 6, \Lambda = 0.01, q = 0.5$. Left panel for $l = 0, 1, 2, 3, 4$ and $\xi = 0$ (solid lines) or $\xi = 0.5$ (dashed lines). Right panel for $l = 0$ and $\xi = 0, 0.1, 0.2, 0.3, 0.4$. 
Figure 2: Effect of spacetime properties \((q, d)\) on the grey body factors (upper panel) and enhancement/suppression of corresponding effective potential (lower panel) for \(\Lambda = 0.01, l = 0, \xi = 0.1\). Left panel for \(q = 0.5, 0.7, 0.8, 0.9, 1\) and \(d = 6\). Right panel for \(d = 6, 8, 10\) and \(q \to 0\) (solid lines) and \(q = 0.8\) (dashed lines).
Figure 3: Effect of cosmological constant $\Lambda$ on the grey body factor and competition between $\Lambda$ and $\xi$ (upper panel) and enhancement/suppression of corresponding effective potential (lower panel) for $d = 6, l = 0, q = 0.5$ and $\Lambda = 0.01, 0.1, 0.2, 0.3, 0.4$. Left panel for $\xi = 0$ and right panel for $\xi = 0.5$. 
VI. POWER SPECTRA OF HAWKING RADIATION

In this section we study the energy emission rate of the Hawking radiation. The power spectra of Hawking radiation is given by [51–53],

\[
\frac{dE}{dt d\omega} = \frac{1}{2\pi} \sum_l \frac{N_l |\gamma_{l\omega}|^2 \omega}{e^{\omega/T_{BH}} - 1},
\]

(55)

where \(N_l = \frac{(2l+d-3)(l+d-3)!}{l(d-3)!}\) is the multiplicity of the states with angular momentum \(l\), and \(T_{BH}\) is the normalised temperature of the black hole which is related to the surface gravity [39, 53],

\[
T_{BH} = \frac{1}{\sqrt{h(r_0)}} \frac{gr_h^d (d^2 - 5d - 2\Lambda r_h^2 + 6) - 2(d-2)r_h^2 q^d}{4\pi(d-2)r_h (r_h^2 q^d + qr_h^d)}.
\]

(56)

Here \(\sqrt{h(r_0)}\) is the normalisation factor with \(h(r_0)\), the value of the metric function at its global maximum \(r_0\). The global maximum is determined by the condition \(h'(r) = 0\). We study the effects of \(\xi\) and \(q\) on the power spectra, by considering only the lower modes, as the higher values of \(l\) contribute significantly less than that of lower modes and is smaller by several orders of magnitude.

First, we study the effects of \(\xi\) and \(q\) on the power spectra. The results are shown in Fig. 4. It is clear that \(\xi\), with other parameters fixed, has a suppressing effect on the Hawking radiation as in SdS case. This is in fact the sequel of what we observed in the grey body factors’ dependence on \(\xi\) in the previous section. The effect of \(q\) on power spectra is also suppressing one, which may look surprising at first sight compared to the observation in grey body factor study, where it was enhancing particle propagation. This is due to the dependence of the normalised temperature on \(q\), which results in the combined effect of diminished energy emission.

To conclude, we study the role of the cosmological constant in energy emission. The competition between \(\xi\) and \(\Lambda\) that we observed in grey body factor also has its carry-over effects in power spectra, Fig. 5. When \(\xi\) is small, the cosmological constant \(\Lambda\) enhances the Hawking radiation, whereas, for higher values of \(\xi\), the effect due to \(\Lambda\) is suppressing. The non-linear charge parameter \(q\) does not affect this interplay between \(\xi\) and \(\Lambda\).
VII. CONCLUSION AND DISCUSSION

In this article we studied the propagation of scalar particles emitted by a class of higher dimensional electrically charged regular black holes. We derived an analytic solution for the grey body factor by matching the solutions at two horizons of the de-Sitter spacetime.
The low energy limits of the solution is obtained for both minimal and non-minimal coupling of the scalar field. For the non minimal coupling case, we obtained a non-zero low energy asymptotic limit, whereas, the same limit vanishes for non-minimal coupling case. The former result is consistent with the characteristic feature of a free massless scalar field emitted in the dS black hole spacetime. We studied the profile of grey body factor along with the effective potential, to analyse the effect of spacetime and particle properties on the scalar particle propagation in the background of the gravity which is non-linearly coupled electromagnetic field. We found that the particle characteristics, namely angular momentum number and coupling parameter, suppress the grey body factor. On the other hand, the spacetime properties influences particle properties in a more complicated way. The non-linear charge parameter enhances the grey body factor, whereas, increasing spacetime dimension suppresses it, and the cosmological constant shows a dual role.

Our analysis is valid for the propagation of an arbitrary mode, which is valid effectively in the low energy regime. To fix the issue in the high energy regime, a numeric method is necessary, which we are working on as a continuation of the present work.

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