The sigma meson and chiral restoration in nuclear medium

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After giving brief discussions on the chiral condensate in hot and/or dense hadronic matter and the significance of the $\sigma$ meson in QCD, we discuss the importance of experiments using nuclear targets including ones with electro-magnetic probes for obtaining clearer confirmation of the existence of the $\sigma$ meson and for exploring the possible restoration of chiral symmetry in nuclear medium.

1 Introduction

The whole content of the present talk is based on the simple fact that chiral transition is a phase transition of the QCD vacuum with the quark condensate $\langle \bar{q}q \rangle$ being the order parameter. The relevance of the present talk to this workshop on physics with electro-magnetic probes is ensured by the fact that chiral symmetry may be at least partially restored in hot and/or dense hadronic medium, hopefully including finite nuclei. In fact, a simple argument shows that the magnitude of the condensate $\langle \langle \bar{q}q \rangle \rangle_{T,\rho}$ at finite temperature $T$ and/or density $\rho$ tends to decrease.

To get convinced of this, one may first recall that the Feynman-Hellman (F-H) theorem tells us that the scalar charge of a hadron $\langle \bar{q}_i q_i \rangle_h$ can be evaluated as a derivative of the hadron mass $m_h$ with respect to the current quark mass $m_i$;

$$\langle \bar{q}_i q_i \rangle_h = \partial m_h / \partial m_i.$$

This is because the mass $m_h$ is an eigenvalue of the QCD Hamiltonian $H_{\text{QCD}}$, which in turn contains the current quark masses in the form $\sum_i m_i \bar{q}_i q_i$. As an almost trivial extension, the quark condensate in hot and dense hadronic matter is given as a derivative of the free energy $F(T, \rho)$ of the system w.r.t. the current quark mass;

$$\langle \langle \bar{q}_i q_i \rangle \rangle_{T,\rho} = \frac{\partial F}{\partial m_i},$$

where we have assumed that $F(T, \rho)$ is given as a sum of the vacuum energy $E_v$ and the usual free energy $F$ of the hadron system (without the vacuum energy); $F = E_v + F$.

If $F$ is the free energy of a hot pion gas, for example, one readily obtains

$$\delta \langle \langle \bar{q}_i q_i \rangle \rangle_T = \sum_p n_\pi(p) \langle \pi(p) | \bar{q}_i q_i | \pi(p) \rangle,$$

where $n_\pi(p)$ is the Bose-Einstein distribution function for the pion and $\langle \pi(p) | \bar{q}_i q_i | \pi(p) \rangle = \partial E_\pi^p / \partial m_i = m_\pi \langle \bar{q}_i q_i \rangle_\pi / E_\pi^p$, with $E_\pi^p = \sqrt{m_\pi^2 + p^2}$ is the scalar charge of the pion with momentum $p$. Applying F-H theorem to Gell-Mann-Oakes-Renner relation, $f_\pi^2 m_\pi^2 = -(m_u + m_d)/2 \cdot \langle \bar{u}u + \bar{d}d \rangle$, one finds that $\langle \bar{q}_i q_i \rangle_\pi = 6.25 > 0 (i = u, d)$. Thus, the magnitude of $\langle \langle \bar{q}_i q_i \rangle \rangle_T$ decreases at finite temperature; hence a partial restoration of chiral symmetry occurs at finite temperature.

In much the same way, if $F$ is the free energy of a degenerate cold nuclear matter, one has

$$\langle \langle \bar{q}_i q_i \rangle \rangle_\rho = \left( 1 - \frac{\sum_\pi \rho}{m_\pi f_\pi^2} \right) \langle \bar{q}_i q_i \rangle,$$
where \( \Sigma_{\pi N} = (m_u + m_d)/2 \cdot \langle N|\bar{u}u + \bar{d}d|N \rangle \sim (40 - 50) \text{ MeV} \) is \( \pi \)-\( N \) sigma term. Thus, the quark condensate tends to decrease in the nuclear medium: At the normal nuclear density \( \rho_0 = 0.17 \text{fm}^{-3} \), we have about 30 \% reduction of the quark condensate. \[1\]

If a phase transition is of 2nd order or *weak* 1st order, there exist soft modes, which are the fluctuations of the order parameter \[2\]. For the chiral transition, the fluctuation of the order parameter \( \langle (\bar{q}q)^2 \rangle \) is a scalar-isoscalar meson, which is historically called the \( \sigma \)-meson. Thus one sees that the \( \sigma \) meson becomes the soft mode of chiral transition at \( T \neq 0 \) and/or \( \rho_B \neq 0 \).\[3\] In this report, we discuss the importance of nuclear experiments with nuclear targets including ones with electro-magnetic probes for obtaining clearer confirmation of the existence of the \( \sigma \) meson and for exploring the possible restoration of chiral symmetry in nuclear medium. But, what is the \( \sigma \)?

## 2 The significance of the \( \sigma \) meson in QCD

The significance of the \( \sigma \) meson in hadron and nuclear physics may be summarized as follows\[4\]:

1. The \( \sigma \) meson is the quantum fluctuation of the order parameter \( \langle (\bar{q}q)^2 \rangle \), hence analogous to the Higgs particle in the standard model: NJL-like models\[5, 6\] and mended symmetry of Weinberg\[7\] predict that the mass \( m_{\sigma} = 400-800 \text{ MeV} \) and the width \( \Gamma \sim m_{\sigma} \).

2. Recent various analyses\[8\] have revealed the existence of a pole in the \( \pi \)-\( \pi \) \( S \)-matrix in the \( \sigma \) channel. In these analyses, significance of respecting *chiral symmetry, unitarity and crossing symmetry* to reproduce the phase shifts both in the \( \sigma \) \( (s) \)- and \( \rho \) \( (t) \)-channels with a low mass \( \sigma \) pole has been recognized\[9\].

3. Such a scalar meson has been known to be responsible for the intermediate range attraction in the nuclear force\[10\].

4. There are attempts\[11\] to show that such an collectiveness in the sigma channel can accounts for \( \Delta I = 1/2 \) enhancement in the decay \( K^0 \rightarrow 2\pi \) compared with \( K^+ \rightarrow \pi^+\pi^- \).

5. The empirical value of \( \pi \)-\( N \) sigma term \( \Sigma_{\pi N} \sim 40-50 \text{ MeV} \) implies an enhanced scalar charge of the nucleon \( \langle \bar{u}u + \bar{d}d \rangle_N \sim 8 - 10 \) provided that \( (m_u + m_d)/2 \sim 5.5 \text{MeV} \), in comparison with the naive value \( \langle \bar{u}u + \bar{d}d \rangle_N = 3 \). The collectiveness as summarized as the existence of the \( \sigma \) meson can account for such an enhancement of the scalar charge\[12\].

6. It should be noticed that the difficulties of the linear \( \sigma \) model\[13\] does not necessarily deny the linear realization of chiral symmetry where the \( \sigma \) meson appears\[14\].

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\[1\] Notice, however, that the higher order terms in the density expansion originating from the interaction energy between nucleons modify the leading order result.
3 Production of the $\sigma$-meson in nuclear medium

Is the pole observed in the $\pi$-$\pi$ phase shift really the $\sigma$ as the quantum fluctuation of the order parameter of the chiral transition? The answer could be obtained clearly if the temperature and/or density are freely changed and variations of the excitation modes, i.e., hadrons, on top of the varied vacua are traced as is usually done in condensed matter physics. The life is, unfortunately, not so easy with the QCD vacuum. Nevertheless one may notice that nuclei provide us with the baryon density and expect that nuclei might be dense enough to give rise to a partial restoration of chiral symmetry! Some years ago, the present author proposed several nuclear experiments including one using electro-magnetic probes to try to produce the $\sigma$ meson in nuclei to see a clearer evidence of the existence of the $\sigma$ meson and to explore possible restoration of chiral symmetry in nuclear medium.

What are good observables to see the softening in the sigma channel in nuclear medium? When a hadrons is put in a nucleus, the hadron may dissociate into complicated excitation to lose its identity in the medium; for example, $\sigma \leftrightarrow 2\pi$, $\sigma \leftrightarrow \text{p-h}$, $\pi^+ + \text{p-h}$, $\Delta^0 - \text{h}$ ... Then the most informative quantity is the response function or spectral function of the system. Serious calculations of the strength function were performed by Chiku and Hatsuda and Volkov et al: Their finding is an enhancement of the spectral function in the sigma channel near the $2m_\pi$ threshold. The surprise was such an enhancement had been seen by an experiment by CHAOS collaboration not at $T \neq 0$ but at $\rho_B \neq 0$ in the cross sections of $A(\pi^+, \pi^+\pi^\pm)A'$ ($A= 2 \rightarrow 208$); this experiment was actually motivated to explore possible in-medium $\pi$-$\pi$ correlations.

Then a calculation of the strength function at $\rho_B \neq 0$ was made using a linear sigma model. It was shown that chiral restoration in the nuclear medium can lead to the required enhancement near the $2m_\pi$ threshold. The result is shown in Fig.1: The upper panel shows the spectral functions in the $\sigma$ channel in nuclear medium with various densities parameterized by $\Phi(\rho)$ denoting the ratio of the quark condensate at the density $\rho$ to the vacuum value. One can see the near-$2m_\pi$ threshold enhancement of the spectral function as the chiral symmetry is restored. The lower panel shows the inverse of the Green’s function of the $\sigma$ propagator with the same $\Phi$ values; the upper curves corresponding to the smaller $\Phi$. One can see the distance between the cusps located at the $2m_\pi$-threshold and the zero decreases as the chiral symmetry is getting restored, which causes the spectral enhancement near the threshold. This was later confirmed by other groups.

![Fig.1: The spectral function in the $\sigma$ channel (upper panel) and the inverse of the $\sigma$ propagator. $\Phi$ denotes the ratio of the chiral condensates at $\rho$ and in the vacuum.](image-url)
One should notice that the state of the art calculation in the conventional reaction theory using nonlinear chiral Lagrangian but with no chiral restoration incorporated fail in reproducing the sufficient enhancement[23].

4 Discussion: The spectral enhancement in the non-linear chiral models

How can the near-threshold enhancement be described by non-linear chiral Lagrangians which are used in the conventional approaches? An answer to this question has been given by Jido et al[24]. They showed the following: (a) Although there is no explicit $\sigma$-degrees of freedom, there arises a decrease of the pion decay constant $f_{\pi}^*$ in nuclear medium. (b) This is due to a new vertex, i.e., 4$\pi$N-N vertex absent in the free space and in the previous calculations of the ($\pi, 2\pi$) reaction; see Fig.2. The vertex is responsible for the reduction of $f_{\pi}^*$ and hence for the spectral enhancement.

![Figure 1:](image1.png)

They started with a linear sigma model as given below;

$$\mathcal{L} = \frac{1}{4}\text{Tr}[\partial M \partial M^\dagger - \mu^2 MM^\dagger - \frac{2\lambda}{4!} (MM^\dagger)^2 + h(M + M^\dagger)] + \bar{\psi}(i\partial - gM_5)\psi + \cdots,$$  

where $M = \sigma + i\vec{\tau} \cdot \vec{\pi}$, $M_5 = \sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi}$, $\psi$ is the nucleon field, and Tr is for the flavor index. Making a polar decomposition $M = \sigma + i\vec{\tau} \cdot \vec{\pi} = (\langle \sigma \rangle + S)U$ with $U = e^{i\vec{\tau} \cdot \vec{\pi}}$, (4) is converted to a nonlinear chiral Lagrangian with the massive scalar field $S$;

$$\mathcal{L} = \frac{1}{2}[(\partial S)^2 - m_\pi^2 S^2] - \frac{\lambda \langle \sigma \rangle}{6} S^3 - \frac{\lambda}{4!} S^4 + \frac{(\langle \sigma \rangle + S)^2}{4} \text{Tr}[\partial U \partial U^\dagger] + \frac{(\sigma) + S}{4} h \text{Tr}[U^\dagger + U] + \mathcal{L}_{\pi N}^{(1)} - gS\bar{NN},$$  

with $\mathcal{L}_{\pi N}^{(1)} = \bar{N}(i\partial + i\gamma_5 - m_\pi^*)N$, where $(v_{\mu}, a_{\mu}) = (\xi \partial_\mu \xi^\dagger \pm \xi^\dagger \partial_\mu \xi)/2$, and $m_\pi^* = g\langle \sigma \rangle$. In the heavy-$S$ limit with $g/\lambda$ and $\langle \sigma \rangle_0 = f_\pi$ fixed, the heavy scalar field $S$ may be integrated out to give the following effective Lagrangian:

$$\mathcal{L} = \left(\frac{f_\pi^2}{4} - \frac{gf_\pi}{2m_\pi^2} \bar{NN}\right) \left(\text{Tr}[\partial U \partial U^\dagger] - \frac{h}{f_\pi} \text{Tr}[U^\dagger + U]\right).$$  

The second term in the coefficient may be replaced by $\frac{gf_\pi}{2m_\pi^2} \bar{NN}$, in nuclear medium, hence gives rise to a renormalization of $f_\pi$, which is renormalized away by a redefinition of the pion field

![Figure 2:](image2.png)
in the medium. Jido et al. [24] showed that this redefinition of the pion field in turn enhances the attraction between the pions in the $I = J = 0$ channel leading to the softening of the spectral function in the $\sigma$ channel.

The new vertex,

$$\mathcal{L}_{\text{new}} = -\frac{3g}{2\lambda f_\pi} \bar{N}N \text{Tr}[\partial U \partial U^\dagger],$$

depicted in Fig.2 has not been considered so far in the calculations of the $\pi\pi$ scattering amplitudes in nuclear matter in the non-linear chiral Lagrangian approaches [23].

5 Summary and concluding remarks

The present talk may be summarized as follows:

1. The $\sigma$ meson as the quantum fluctuation of the order parameter of the chiral transition may account for various phenomena in hadron physics which otherwise remain mysterious.

2. There have been accumulation of experimental evidence of the $\sigma$ pole in the $\pi\pi$ scattering matrix. To deduce this result, it is found essential to respect chiral symmetry, analyticity and crossing symmetry.

3. Partial restoration of chiral symmetry in hot and dense medium leads to a peculiar enhancement in the spectral function in the $\sigma$ channel near the $2m_\pi$ threshold.

4. Such an enhancement has been observed in the reaction $A(\pi^+, (\pi^+\pi^-)_{I=J=0})A'$ by CHAOS collaboration, which may be an experimental evidence of the partial restoration of chiral symmetry in heavy nuclei.

5. The spectral softening in the $\sigma$ channel is obtained both in the linear and nonlinear realization of chiral symmetry provided that the possible reduction of the quark condensate or $f_\pi$ is taken into account.

Further works are necessary to confirm that the near $2m_\pi$-threshold enhancement observed in the $(\pi^+, \pi^+\pi^-)$ reactions by CHAOS collaboration is surely due to a partial restoration of chiral symmetry in nuclear medium:

1. The strength function in the $\sigma$ channel in the wider $(\omega, q)$ region should be measured with various nuclear and electro-magnetic probes; for instance, photo- or electro-production of the $\sigma$ as well as the production by $(d, ^3\text{He})$ and $(d, t)$ reactions are interesting. We remark that formation of $\sigma$ mesic nuclei [25] by $(d, ^3\text{He})$ and $(d, t)$ reactions are proposed as was done to produce the deeply-bound pionic atoms [25].

2. To identify the $\sigma$ meson and the spectral function in that channel, detecting $2\pi^0$ and lepton pairs with $q \neq 0$ are interesting.

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