On Generalized c*-continuous Functions and Generalized c*-irresolute Functions in Topological Spaces

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Abstract The aim of this paper is to introduce the notion of generalized c*-continuous functions and generalized c*-irresolute functions in topological spaces and study their basic properties.

Keywords: gc*-continuous functions and gc*-irresolute functions

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1. Introduction

In 1963, Norman Levine introduced semi-open sets in topological spaces. Also in 1970, he introduced the concept of generalized closed sets. Palaniappan and Rao introduced regular generalized closed (briefly, rg-closed) sets in 1993. In the year 1996, Andrijevic introduced and studied b-open sets. Gnanambal introduced generalized preregular closed (briefly gpr-closed) sets in 1997. N. Levine introduced the concept of semi-continuous function in 1963. In 1980, Jain introduced totally continuous functions. In 1995, T. M. Nour introduced the concept of totally semi-continuous functions as a generalization of totally continuous functions. In 2011, S.S. Benchalli and Umadevi I Neeli introduced the concept of semi-totally continuous functions in topological spaces. In this paper we introduce generalized c*-continuous functions and generalized c*-irresolute functions in topological spaces and study their basic properties.

Section 2 deals with the preliminary concepts. In section 3, generalized c*-continuous functions are introduced and study their basic properties. The generalized c*-irresolute functions in topological spaces are introduced in section 4.

2. Preliminaries

Throughout this paper X denotes a topological space on which no separation axiom is assumed. For any subset A of X, cl(A) denotes the closure of A, int(A) denotes the interior of A, pcl(A) denotes the pre-closure of A and bcl(A) denotes the b-closure of A. Further X\A denotes the complement of A in X. The following definitions are very useful in the subsequent sections.

Definition: 2.1 A subset A of a topological space X is called
i. a semi-open set [1] if A⊆int(cl(A)) and a semi-closed set if int(cl(A))⊆A.
ii. a pre-open set [2] if A⊆int(cl(A)) and a pre-closed set if cl(int(A))⊆A.
iii. a γ-open set [4] (b-open set [5]) if A⊆cl(int(A))∪int(cl(A)) and a γ-closed set (b-closed set) if int(cl(A))∩cl(int(A))⊆A.

Definition: 2.2 A subset A of a topological space X is said to be
i. a clopen set if A is both open and closed in X.
ii. a semi-clopen set if A is both semi-open and semi-closed in X.

Definition: 2.3 [6] A subset A of a topological space X is said to be a c*-open set if int(cl(A))⊆A and cl(int(A))⊆A.

Definition: 2.4 A subset A of a topological space X is called
i. a general closed set (briefly, g-closed) [7] if cl(A)⊆H whenever A⊆H and H is open in X.
ii. a regular-generalized closed set (briefly, rg-closed) [8] if cl(A)⊆H whenever A⊆H and H is regular-open in X.
iii. a generalized pre-regular closed set (briefly, gpr-closed) [9] if pcl(A)⊆H whenever A⊆H and H is regular-open in X.
iv. a regular generalized b-closed set (briefly, rgb-closed) [10] if bcl(A)⊆H whenever A⊆H and H is regular-open in X.
v. a regular weakly generalized closed set (briefly, rwg-closed) [11] if cl(int(A))⊆H whenever A⊆H and H is regular-open in X.
vi. a semi-generalized b-closed set (briefly, sgb-closed) \([12]\) if \(\text{bcl}(A) \subseteq H\) whenever \(A \subseteq H\) and \(H\) is semi-open in \(X\).

vii. a weakly closed set (briefly, w-closed) \([13]\) (equivalently, \(\tilde{g}\)-closed \([14]\)) if \(\text{cl}(A) \subseteq H\) whenever \(A \subseteq H\) and \(H\) is semi-open in \(X\).

The complements of the above mentioned closed sets are their respectively open sets.

**Definition:** 2.5 \([6]\) A subset \(A\) of a topological space \(X\) is said to be a generalized \(c^*\)-closed set (briefly, gc*-closed set) if \(\text{cl}(A) \subseteq H\) whenever \(A \subseteq H\) and \(H\) is \(c^*\)-open in \(X\). The complement of the gc*-closed set is gc*-open \([15]\].

**Definition:** 2.6 A function \(f: X \rightarrow Y\) is called a generalized \(c^*\)-irresolute function if the inverse image of every semi-open subset of \(Y\) is clopen in \(X\).

Let \(X, Y\) be two topological spaces. \(f: X \rightarrow Y\) is called a generalized \(c^*\)-continuous function if \(f^{-1}(V)\) is gc*-open in \(X\) for every open set \(V\) in \(Y\).

In this section, we introduce generalized \(c^*\)-continuous functions and study its basic properties. Now, we begin with the definition of generalized \(c^*\)-continuous function.

**Definition:** 3.1 Let \(X, Y\) be two topological spaces. A function \(f: X \rightarrow Y\) is called a generalized \(c^*\)-continuous (briefly, gc*-continuous) function if \(f^{-1}(V)\) is gc*-closed in \(X\) for every gc*-closed subset of \(Y\).

**Example:** 3.2 Let \(X = \{a, b, c, d\}\) with the topology \(\tau = \{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c, d\}\}\) and \(Y = \{1, 2, 3\}\) with the topology \(\sigma = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}\}\). Define \(f: X \rightarrow Y\) by \(f(a) = 1, f(b) = 2, f(c) = 3\). Then \(f^{-1}(1) = \{a\}\) is not gc*-open in \(X\).

**Proposition:** 3.3 Let \(X, Y\) be two topological spaces. Then \(f: X \rightarrow Y\) is gc*-continuous if and only if \(f^{-1}(U)\) is gc*-closed in \(X\) for every open set \(U\) in \(Y\).

**Proof:** Suppose \(f: X \rightarrow Y\) is gc*-continuous. Let \(U\) be an open set in \(Y\). Then \(f^{-1}(U)\) is gc*-closed in \(X\). This implies, \(f^{-1}(Y\setminus U)\) is gc*-open in \(X\). Since \(f^{-1}(Y\setminus U) = X\setminus f^{-1}(U)\), we have \(X\setminus f^{-1}(U)\) is gc*-closed in \(X\). This implies, \(f^{-1}(U)\) is gc*-open in \(X\). Conversely, assume that \(f^{-1}(U)\) is gc*-open in \(X\) for every open set \(U\) in \(Y\). Let \(V\) be a closed set in \(Y\). Then \(Y\setminus V\) is open in \(Y\). Therefore, \(f^{-1}(Y\setminus V)\) is gc*-open in \(X\). That is, \(X\setminus f^{-1}(V)\) is gc*-closed in \(X\). This implies, \(f^{-1}(V)\) is gc*-closed in \(X\). Therefore, \(f\) is gc*-continuous.

**Proposition:** 3.4 Let \(X, Y\) be two topological spaces. Then every gc*-continuous function is gc*-continuous.

**Proof:** Let \(f: X \rightarrow Y\) be a gc*-continuous function. Let \(V\) be a closed set in \(Y\). Then \(f^{-1}(V)\) is a gc*-closed set in \(X\). By Proposition 4.3 \([6]\), \(f^{-1}(V)\) is gc*-closed in \(X\). Therefore, \(f\) is gc*-continuous.

The converse of the Proposition 3.4 need not be true as seen from the following example.

**Example:** 3.5 Let \(X = \{1, 2, 3, 4\}\) and \(Y = \{a, b, c, d, e\}\). Then, \(f^{-1}(1) = \{a\}, f^{-1}(2) = \{b\}, f^{-1}(3) = \{c\}, f^{-1}(4) = \{d\}, f^{-1}(5) = \{e\}\). Then \(f^{-1}(1)\) is gc*-open in \(X\). Consider the closed set \(\{a, b, c, d\}\) in \(Y\). Then \(f^{-1}(\{a, b, c, d\}) = \{1, 2\}\), which is not a closed set in \(X\). Therefore, \(f\) is not gc*-continuous.

**Proposition:** 3.6 Let \(X, Y\) be two topological spaces. Then every gc*-continuous function is gc*-continuous.

**Proof:** Let \(f: X \rightarrow Y\) be a gc*-continuous function and \(V\) be a closed set in \(Y\). Then \(f^{-1}(V)\) is a gc*-closed set in \(X\). This implies, \(f^{-1}(V)\) is gc*-closed in \(X\). Therefore, \(f\) is gc*-continuous.

The converse of the Proposition 3.6 need not be true as seen from the following example.

**Example:** 3.7 Let \(X = \{1, 2, 3, 4, 5\}\) and \(Y = \{a, b, c, d\}\). Then, \(f^{-1}(1) = \{a\}, f^{-1}(2) = \{b\}, f^{-1}(3) = \{c\}, f^{-1}(4) = \{d\}, f^{-1}(5) = \{e\}\). Then \(f^{-1}(1)\) is gc*-closed in \(X\). Consider the set \(\{a\}\) in \(Y\). Then \(f^{-1}(\{a\}) = \{1\}\), which is not a gc*-closed set in \(X\). Therefore, \(f\) is not gc*-continuous.

**Proposition:** 3.8 Let \(X, Y\) be two topological spaces. Then every semi-totally continuous function is gc*-continuous.

**Proof:** Let \(f: X \rightarrow Y\) be a semi-totally continuous function and \(V\) be an open set in \(Y\). Since every open set is semi-open, we have \(V\) is semi-open in \(Y\). Therefore, by our assumption, \(f^{-1}(V)\) is gc*-open in \(X\). Then by Proposition 3.7 \([15]\), \(f^{-1}(V)\) is a gc*-open set in \(X\). Therefore, \(f: X \rightarrow Y\) is gc*-continuous.

The converse of the Proposition 3.8 need not be true as seen from the following example.

**Example:** 3.9 In Example 3.5, define \(f: X \rightarrow Y\) by \(f(1) = b, f(2) = f(3) = d, f(4) = e\). Then \(f\) is gc*-continuous. Consider the semi-open set \(\{d\}\) in \(Y\). Then \(f^{-1}(\{d\}) = \{2, 3\}\), which is not a gc*-open set in \(X\). Therefore, \(f\) is not semi-totally continuous.

**Proposition:** 3.10 Let \(X, Y\) be two topological spaces. Then every gc*-continuous function is gc*-continuous.

**Proof:** Let \(f: X \rightarrow Y\) be a gc*-continuous function and \(V\) be an open set in \(Y\). Then \(f^{-1}(V)\) is gc*-open in \(X\). Therefore, by Proposition 3.7 \([15]\), \(f^{-1}(V)\) is gc*-open in \(X\). Therefore, \(f\) is gc*-continuous.

The converse of the Proposition 3.10 need not be true as seen from the following example.

**Example:** 3.11 In Example 3.5, define \(f: X \rightarrow Y\) by \(f(1) = b, f(2) = f(3) = d, f(4) = e\). Then \(f\) is gc*-continuous. Consider the open set \(\{d\}\) in \(Y\). Then \(f^{-1}(\{d\}) = \{2, 3\}\), which is not a gc*-closed set in \(X\). Therefore, \(f\) is not a totally-continuous function.
Proposition: 3.12 Let X,Y be two topological spaces. Then every w-continuous (g-continuous) function is gc*-continuous.

Proof: Let f: X \to Y be a w-continuous function. Let V be a closed set in Y. Then f^(-1)(V) is w-closed in X. By Proposition 4.5 [6], we have f^(-1)(V) is gc* -closed in X. Therefore, f is gc*-continuous.

The converse of the Proposition 3.12 need not be true as seen from the following example.

Example: 3.13 Let X={1,2,3} and Y={a,b,c}. Then, clearly τ=\{\{1\}, \{2\}, \{1,2\}, X\} is a topology on X and σ=\{\{a\}, \{b\}, \{a,b\}, \{c\}, \{a,c\}, \{b,c\}, \{a,b,c\}, Y\} is a topology on Y. Define f : X \to Y by f(1)=a, f(2)=b, f(3)=c. Then f is gc*-continuous. Consider the closed set \{b,c\} in Y. Then f^(-1)(\{b,c\})=\{2,3\} which is not a w-closed set in X. Therefore, f is not a w-continuous function.

Proposition: 3.14 Let X, Y be two topological spaces. Then every gc*-continuous function is rg-continuous.

Proof: Let f: X \to Y be a gc*-continuous function. Let V be a closed set in Y. Then f^(-1)(V) is a gc*-closed set in X. Therefore, f is gc*-continuous.

Proposition: 3.15 Let X,Y be two topological spaces. Then every gc*-continuous function is gpr-continuous.

Proof: Let f: X \to Y be a gc*-continuous function. Let V be a closed set in Y. Then f^(-1)(V) is gc*-closed in X. Therefore, f is gpr-continuous.

Proposition: 3.17 Let X, Y be two topological spaces. Then for any bijective function f: X \to Y, the following statements are equivalent.

i. f : X \to Y is gc*-continuous.

ii. f^(-1) : Y \to X is gc*-open.

Proof: (i) \Rightarrow (ii) Assume that f : X \to Y is gc*-continuous. Let U be an open set in Y. Since f is gc*-continuous, we have f^(-1)(U) is gc*-open in X. Therefore, f^(-1) is a gc*-open map.

Proposition: 3.18 Let X,Y and Z be topological spaces. If f : X \to Y and g : Y \to Z are continuous. Then g \circ f : X \to Z is gc*-continuous.

Proposition: 3.19 Let X,Y and Z be topological spaces. If f : X \to Y and g : Y \to Z are continuous. Then g \circ f : X \to Z is gc*-continuous.

Proof: Let V be a closed set in Z. Since g is continuous, we have g(V) is closed in Y. Also, since f is gc*-continuous, we have f^(-1)(g(V)) is gc*-closed in X. But f^(-1)(g(V))=g(f^(-1)(V)). Therefore, g\circ f(V) is gc*-closed in X. Hence g\circ f is gc*-continuous.

Proposition: 3.20 Let X,Y and Z be topological spaces. If f : X \to Y and g : Y \to Z are continuous. Then g\circ f : X \to Z is gc*-continuous.

Proof: Let V be a closed set in Z. Since g is continuous, we have g(V) is closed in Y. Also, since f is gc*-continuous, we have f^(-1)(g(V)) is gc*-closed in X. Therefore, g\circ f(V) is gc*-closed in Y. By Proposition 4.3 [6], (g\circ f)(V) is gc*-closed in Y. Therefore, g\circ f is gc*-continuous.
4. Generalized c*-irresolute Functions

In this section, we introduce generalized c*-irresolute functions in topological spaces. Also, we discuss about some of their basic properties.

Definition: 4.1 Let X and Y be two topological spaces. A function \( f: X \rightarrow Y \) is said to be a generalized c*-irresolute (brieﬂy, gc*-irresolute) function if \( f^{-1}(V) \) is gc*-closed in X for every gc*-closed set V in Y.

Example: 4.2 Let \( X=\{a,b,c,d,e\} \) and \( Y=\{1,2,3,4\} \). Then, clearly \( \tau=\{\emptyset,\{1\},\{3\},\{4\},\{1,3\},\{1,4\},\{1,3,4\}\} \) is a topology on X and \( \sigma=\{\emptyset,\{a\},\{c\},\{d\},\{a,c\},\{a,d\},\{a,c,d\},Y\} \) is a topology on Y. Define \( f: X \rightarrow Y \) by \( f(a)=1, f(b)=2, f(c)=3, f(d)=3, f(e)=4 \). Then \( f \) is not gc*-irresolute.

Proposition: 4.3 Let \( X,Y \) be two topological spaces. Then \( f: X \rightarrow Y \) is gc*-irresolute if and only if \( f^{-1}(U) \) is gc*-open in X for every gc*-open set U of Y.

Proof: Suppose \( f: X \rightarrow Y \) is gc*-irresolute. Let U be an gc*-open set in Y. Then \( Y\setminus U \) is a gc*-closed set in Y. This implies, \( f^{-1}(Y\setminus U) \) is a gc*-closed set in X. Since \( f^{-1}(Y\setminus U)=X\setminus f^{-1}(U) \), we have \( X\setminus f^{-1}(U) \) is a gc*-closed set in X. This implies, \( f^{-1}(U) \) is a gc*-open set in X.

Example: 4.4 Let \( X=\{1,2,3\} \) and \( Y=\{a,b,c\} \). Then, clearly \( \tau=\{\emptyset,\{2\},\{3\},\{2,3\}\} \) is a topology on X and \( \sigma=\{\emptyset,\{a\},\{c\},\{a,c\}\} \) is a topology on Y. Define \( f: X \rightarrow Y \) by \( f(1)=a, f(2)=c, f(3)=b \). Then \( f \) is gc*-continuous.

Proposition: 4.5 Let \( X,Y \) be two topological spaces. Then \( f: X \rightarrow Y \) is gc*-continuous if and only if \( f^{-1}(U) \) is gc*-open in X for every gc*-closed set U in Y. Therefore, f is gc*-irresolute.

Example: 4.6 Let \( X=\{a,b,c,d\} \) with topology \( \tau=\{\emptyset,\{a\},\{d\},\{a,d\},\{a,d,e\}\} \) and \( Y=\{1,2,3,4,5\} \) with topology \( \sigma=\{\emptyset,\{1\},\{2\},\{1,2\},\{1,2,3\},\{1,2,3,4\}\} \). Define \( f: X \rightarrow Y \) by \( f(a)=2, f(b)=1, f(c)=5, f(d)=3, f(e)=4 \). Then \( f \) is gc*-irresolute.

The gc*-irresolute and w-continuous functions are independent. For example, let \( \tau=\{\emptyset,\{2\},\{3\},\{2,3\}\} \) and \( \sigma=\{\emptyset,\{a\},\{c\}\} \) is a topology on X and \( Y=\{1,2,3\} \) is a topology on Y. Define \( f: X \rightarrow Y \) by \( f(1)=a, f(2)=c, f(3)=b \). Then \( f \) is gc*-continuous. Consider the gc*-closed set \( \{2,3\} \) in Y. Then \( f^{-1}(\{2,3\})=\{1\} \), which is a gc*-closed set in \( X \).

Proposition: 4.7 Let \( X,Y \) be two topological spaces. Then every gc*-irresolute function is gc*-continuous.

Proof: Let \( f: X \rightarrow Y \) be a gc*-irresolute function and \( V \) be a closed set in Y. Then by Proposition 4.3 \[6\], \( V \) is a gc*-closed set in Y. Therefore, \( f^{-1}(V) \) is gc*-closed in X. Hence \( f \) is gc*-irresolute.

The gc*-irresolute and w-continuous functions are independent. For example, let \( \tau=\{\emptyset,\{1\},\{2\},\{1,2\}\} \) and \( \sigma=\{\emptyset,\{a\},\{b\},\{a,b\}\} \) is a topology on X and \( Y=\{1,2,3\} \) is a topology on Y. Define \( f: X \rightarrow Y \) by \( f(1)=a, f(2)=c, f(3)=b \). Then \( f \) is gc*-continuous. Consider the gc*-closed set \( \{a\} \) in Y. Then \( f^{-1}(\{a\})=\{1\} \), which is a gc*-closed set in \( X \).

Proposition: 4.8 Let \( X=\{1,2,3\} \) and \( Y=\{a,b,c\} \). Then, clearly \( \tau=\{\emptyset,\{2\},\{3\},\{2,3\}\} \) is a topology on X and \( \sigma=\{\emptyset,\{a\},\{c\}\} \) is a topology on Y. Define \( f: X \rightarrow Y \) by \( f(1)=a, f(2)=c, f(3)=b \). Then \( f \) is gc*-continuous. Consider the gc*-closed set \( \{a\} \) in Y. Then \( f^{-1}(\{a\})=\{1\} \), which is a gc*-closed set in \( X \).

Proposition: 4.9 Let \( X,Y \) and \( Z \) be topological spaces. Then \( f: X \rightarrow Y \) is a gc*-irresolute function and \( V \) be a closed set in Y. Then by Proposition 4.3 \[6\], \( V \) is a gc*-closed set in Y. Since \( f \) is gc*-irresolute, \( f^{-1}(V) \) is a gc*-closed set in X. Therefore, \( f \) is gc*-continued.

Example: 4.10 Let \( X=\{1,2,3\} \) and \( Y=\{a,b,c\} \). Then, clearly \( \tau=\{\emptyset,\{1\},\{2\},\{1,2\}\} \) is a topology on X and \( \sigma=\{\emptyset,\{a\},\{b\},\{a,b\}\} \) is a topology on Y. Define \( f: X \rightarrow Y \) by \( f(1)=a, f(2)=c, f(3)=b \). Then \( f \) is gc*-continuous. Consider the gc*-closed set \( \{b\} \) in Y. Then \( f^{-1}(\{b\})=\{2\} \), which is a gc*-closed set in \( X \).

Proposition: 4.11 Let \( X,Y \) and \( Z \) be topological spaces. Then \( f: X \rightarrow Y \) and \( g: Y \rightarrow Z \) be gc*-irresolute functions, then \( g\circ f: X \rightarrow Z \) is gc*-irresolute.

Proof: Let \( V \) be a gc*-closed set in \( Z \). Then \( g^{-1}(V) \) is gc*-closed. This implies, \( f^{-1}(g^{-1}(V)) \) is gc*-closed. But \( f^{-1}(g^{-1}(V))=(g\circ f)^{-1}(V) \). Therefore \( (g\circ f)(V) \) is gc*-closed in \( X \). Hence \( g\circ f \) is gc*-irresolute.

Example: 4.12 Let \( X=\{1,2,3\} \) and \( Y=\{a,b,c\} \). Then, clearly \( \tau=\{\emptyset,\{1\},\{2\},\{1,2\}\} \) is a topology on X and \( \sigma=\{\emptyset,\{a\},\{c\}\} \) is a topology on Y. Define \( f: X \rightarrow Y \) by \( f(1)=a, f(2)=c, f(3)=b \). Then \( f \) is gc*-continuous. Consider the gc*-closed set \( \{a\} \) in Y. Then \( f^{-1}(\{a\})=\{1\} \), which is a gc*-closed set in \( X \).

Therefore \( f \) is not gc*-irresolute function.

Proposition: 4.13 Let \( X,Y \) and \( Z \) be topological spaces. Then \( f: X \rightarrow Y \) and \( g: Y \rightarrow Z \) be gc*-irresolute functions, then \( g\circ f: X \rightarrow Z \) is gc*-irresolute.

Proof: Let \( V \) be a gc*-closed set in \( Z \). Then \( g^{-1}(V) \) is gc*-closed. This implies, \( f^{-1}(g^{-1}(V))=(g\circ f)^{-1}(V) \). Therefore \( (g\circ f)(V) \) is gc*-closed in \( X \). Hence \( g\circ f \) is gc*-irresolute.

Proposition: 4.14 Let \( X,Y \) and \( Z \) be topological spaces. Then \( f: X \rightarrow Y \) and \( g: Y \rightarrow Z \) be gc*-irresolute functions, then \( g\circ f: X \rightarrow Z \) is gc*-irresolute.

Proof: Let \( V \) be a gc*-closed set in \( Z \). Then \( g^{-1}(V) \) is gc*-closed. This implies, \( f^{-1}(g^{-1}(V))=(g\circ f)^{-1}(V) \). Therefore \( (g\circ f)(V) \) is gc*-closed in \( X \). Hence \( g\circ f \) is gc*-irresolute.
Proposition: 4.12 Let $X, Y$ and $Z$ be topological spaces. If $f : X \to Y$ is $g^*$-irresolute and $g : Y \to Z$ is $g^*$-continuous, then $g \circ f : X \to Z$ is $g^*$-continuous.

**Proof:** Let $V$ be a closed set in $Z$. Since $g$ is $g^*$-continuous, we have $g^{-1}(V)$ is $g^*$-closed in $Y$. Since $f$ is $g^*$-irresolute, we have $f^{-1}(g^{-1}(V))$ is $g^*$-closed in $X$. But $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$. Therefore, $(g \circ f)^{-1}(V)$ is $g^*$-closed in $X$. Hence, $g \circ f$ is $g^*$-continuous.

5. Conclusion

In this paper we have introduced $g^*$-continuous and $g^*$-irresolute functions in topological spaces and studied some of their basic properties. Also we have studied the relationship between $g^*$-continuous functions and some of the functions already exist.

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