Numerical investigation of the upper part load vortex rope behaviour in a Francis turbine

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Abstract. The vortex rope in the draft tube is considered as the source of the upper part load pressure pulsations, which is one of some unstable phenomena in a Francis turbine. In this study, the pressure pulsations and the vortex rope in the draft tube of a Francis turbine at upper part load operation are investigated by a three dimensional unsteady numerical method. Firstly, the main frequencies of the pressure fluctuations in three planes, which are z=0.2m, 0.4m and 0.6m, in the draft tube are obtained, which are 0.38$f_n$, 0.67$f_n$, 1.04$f_n$, 1.33$f_n$ and 1.71$f_n$ ($f_n$ is the runner rotation frequency). The simulated main frequencies show a good agreement with the results from model tests, which are 0.34$f_n$ and 1.72$f_n$. To simplify the problem in this study, the isopressure surface of p=25000Pa is considered as a vortex rope. It is found that the shape of the three vortex rope sections is almost elliptical. Then the motions of the elliptical shaped sections, which are decomposed into two components that are the motion of geometric center and alteration of the major or minor axis of the ellipse, are analyzed. Obviously, the geometric center motion represents the precession movement, and the time evolution of the major or minor axis corresponds to the self-rotation of the vortex rope, respectively. The results show that the first dominant frequency of the geometric center motion $f_1^g$ is 0.38$f_n$, and the second dominant one $f_2^g$ is 1.71$f_n$. And the first dominant frequency of the major(minor) axis alteration $f_1^s$ is 0.67$f_n$, and the second one $f_2^s$ is 1.33$f_n$. That is, the pressure fluctuation frequencies in the draft tube are combinations of the rope precession motion frequencies and self-rotation frequencies, namely $f_1^g,f_1^s,f_2^g+f_1^s,f_2^g,f_2^s$, etc. Based on the analysis, the upper part load pressure pulsations may be induced by the behavior of the elliptical vortex rope in the draft tube.

1. Introduction
With the turbine size and capacity increasing rapidly nowadays, hydraulic stability becomes a common issue that concerns both the manufacturer and owner. The upper part load pressure fluctuation, which is also called “special pressure pulsations” in China, is one of the most crucial unstable phenomena in Francis turbines \(^1\). This kind of instability will induce pressure shocks, vibrations and noises, and even threaten the plant safety. As a consequence, the issue receives more and more attention, especially the mechanism of the unstable element.

It was considered that the cause of the upper part load pressure pulsations was the resonance phenomenon, about which two opinions exist. One is the resonance between the model turbine and test rig, and the other one is the resonance of water body of the flow passage\(^1\), and the latter is the most acceptable one\(^2\). However, what is the root cause or excitation source of the resonance? Till now, lots of work have been done to understand the mechanism of the upper part load pressure pulsations.
Dörfler was the first one who experimentally investigated this issue, and gave a qualitative explanation that the exciting source was the inertial waves with high propagation speed which travelled along the surface of the rotating cavity\(^2\).

Investigations by synchronous high speed camera observations and pressure measurement were performed to explore the mechanism of the upper part load pressure fluctuations by Koutnik et al\(^4,5\). The authors found that the vortex rope in the draft did not have a circular cross-section, but an elliptical one, as shown in figure 1(left). Based on the observation, the authors pointed out that the frequencies of the upper load pressure pulsations were connected with the motions of the elliptical vortex rope, which were decomposed into the precession movement and the self rotation, as shown in figure 1(right)\(^4,5\). A theoretical model of the elliptical shaped cross section rope was also developed by the authors\(^5,6\). Elliptical shaped cross section of the cavitating vortex rope was also captured by high-speed videos in a simplified draft tube by Kirschner\(^7\).

An experimental investigation with movie recording was performed by Nicolet et al. According to the flow visualizations, he pointed out that the vortex rope was an elliptical cross section and the frequency half of the upper part load pressure fluctuations were linked to self rotating of the vortex rope, and the changing of vortex volume, which featured breathing like pattern according to pressure, was an excitation source of the unsteady phenomenon\(^8,9\).

Numerical simulations combined with a three dimensional incompressible hydrodynamic model and a one dimensional compressible hydroacoustic model were carried out by Alligné, and the author pointed out that the undesirable fluctuation of the cavitation volume, whose frequency could match with an eigen frequency of the hydraulic system, was related to the mechanism of the upper part load pressure pulsations\(^10\).

Joint influence of the cavitating vortex core and shape of runner cone, the place where vortex rope began, was considered to be related to the upper partial load unsteady phenomenon appearance\(^11\).

Arguments on the mechanism of the upper part load pressure pulsations continue. However, it is a common view that the upper part load pressure pulsations are strongly influenced by the behavior of the vortex rope in the draft tube. The purpose of this paper is to investigate the behavior of the vortex rope in the draft tube, and try to find the influence on the pressure features at upper part load operation by a numerical approach.

![Figure 1. Vortex rope and motions of the elliptical vortex rope cross section in the draft tube.](image)

2. Numerical method

2.1. Case study

The model tests were carried out on a universal Test Stand in DFEM hydraulic laboratory. Measuring taps for pressure pulsation measurement are located from the inlet of spiral casing to elbow portion of the draft tube, more details can be found in Shi\(^1\). All pressure signals are sampled simultaneously from a computer system with sampling rate of 2000Hz in time of 16.38s. The recorded signals are analyzed by using a spectrum analyzer (FFT) to get frequency domain characteristics. The peak to peak amplitude at 97% probability of pressure pulsations in draft tube cone is shown in figure 2, and the blue ones are the lines of constant pressure pulsations (\(\Delta H/H\%\)). The upper part load pressure fluctuations...
pulsations operating zone is located within the red dashed lines, as illustrated in figure 2. It can be seen that the operating zone appears in the 70%-90% of best efficiency point (BEP), and the maximum amplitude is more than 16%.

The selected operating point to investigate vortex rope in draft tube is point A ($n_{11A}/n_{11BEP}=104\%$ and $Q_{11A}/Q_{11BEP}=84\%)$ as marked in figure 2. The time and frequency histories for pressure pulsations in the draft tube, with the measuring tap located at a height of 1.0D ($D=0.35m$, is the runner diameter) below turbine central line, of point A are illustrated in figure 3. The peak to peak amplitude at 97% probability is 15.48%, as shown in the left figure. The first dominate frequency is $0.34f_n$ ($f_n$ is runner rotation frequency), which is the vortex rope precession frequency. And the second dominate frequency is $1.72f_n$, which is also observed at other locations such as the spiral case inlet and vaneless space between wicket gates and runner in the operating point, and they are all in phase. In addition, one could clearly notice vibrations of the platform and noise during model test. In other words, the resonance of water in the fluid passage happens.

**Figure 2.** Upper part load pressure pulsations operating point (Point A).

**Figure 3.** Pressure pulsations features of operating point A in draft tube.

### 2.2. Numerical details

The continuity equation and the 3D incompressible unsteady Reynolds-averaged Navier-Stokes equations with the SST $\kappa$-ω turbulence model are solved by using commercial flow solver CFX 14.0 to investigate the pressure pulsations and vortex rope in the draft tube.

The simulation domain involves the spiral case, stay ring of 24 stay vanes, a distributor made of 24 wicket gates (guide vanes), a 13-blade runner of 0.35m diameter (the specific speed is 253.2m. Kw) and the draft tube. The commercial software package ICEM is used to generate the hexahedral grid of the spiral case, the stay vanes and the draft tube, while the mesh of the guide vanes and the runner is created by the software TurboGrid. The mesh independence is validated by comparing with experiments, as shown in figure 4. It can be seen that the results are convergent when the number of mesh is greater than $5\times10^6$, with an error less than 0.5%. A mesh with about $5.22\times10^6$ nodes and $4.86\times10^6$ cells in total is chosen for the simulations, and details of the mesh is listed in table 1.
Table 1. Mesh details

|                  | Spiral case | Stay vanes | Guide vanes | Runner | Draft tube | Total |
|------------------|-------------|------------|-------------|--------|------------|-------|
| Number of nodes  | 6.39        | 0.38       | 1.41        | 1.36   | 1.32       | 4.86  |
| Number of cells  | 0.41        | 0.43       | 1.53        | 1.41   | 1.44       | 5.22  |

In the simulations, a second order central scheme is used to discretize the diffusion term, while first order upwind scheme is used to discretize the convective term. Total pressure condition is performed for the inlet condition, and opening condition is used in the outlet, and no-slip condition is used on all solid walls. The time step for the unsteady simulations is set as $1.76 \times 10^{-4}$ s, corresponding to 1.25 degree of the runner rotation.

To study the features of the pressure fluctuations and vortex rope in the draft tube, 21 monitoring points are set on three planes, which are $z=0.2$m, 0.4m and 0.6m, as shown in figure 5. We label the three planes $z=0.2$m, 0.4m and 0.6m as ‘Up’, ‘Md’ and ‘Lw’, respectively. The monitoring points are named as ‘plane-r-$0$’, where $r$ represents the radial direction (indexes are 0, 1 and 2) and $0$ the circumferential direction (indexes are 0, 1, 2 and 3). For instance, the black point marked in figure 5 is ‘Up12’, and the point in the centre is named as ‘plane-00’.

3. Results and discussions
The total transient simulation time is 2s, which corresponds to 31.56 runner revolutions, and the results of the latest 5 revolutions are used to investigate pressure fluctuations and the vortex rope in the draft tube.

3.1. Amplitude features of the pressure fluctuations in the draft tube
3.2. Frequency features of the pressure pulsations in the draft tube

Pressure frequency spectra in the three planes are acquired by using Fast Fourier Transform (FFT), as shown in figure 7. The main frequency components are almost the same in the three planes, with the first dominate one $0.38f_n$, second one $0.67f_n$, third one $1.04f_n$, etc. Obviously, $0.38f_n$ is the vortex rope precession frequency. The main frequency components comparison between simulation result and experiment is listed in table 2, from which we can see that simulation result matches experimental result in an acceptable range.

**Table 2. Comparison of frequency components between model test and simulation**

|                  | Main frequency components ($f_n$) |
|------------------|-----------------------------------|
| Upper plane      | 0.38 0.67 1.04 1.33 -             |
| Middle plane     | 0.38 0.67 1.04 1.33 -             |
| Lower plane      | 0.38 0.67 1.04 1.33 1.71           |
| Model test*     | 0.34 - - - 1.72                    |

* the frequency spectrum is shown in Fig 3.

3.3. Vortex rope features
Figure 8. Helical vortex rope in the draft tube at four instants.

Figure 9. The shape of vortex rope cross section and its motions. (a) The shape of the vortex rope cross section in the three planes, (b) Sketch of motions of the elliptical shaped vortex rope section.

In order to understand the cause of the upper part load pressure pulsations, the behaviour of vortex rope in the draft tube is studied. To simplify the problem here, the helical vortex is represented by the isopressure surface corresponding to $p=-25,000$ Pa, as visualized in figure 8 at four instants. The cross sections of the vortex rope in the three planes are demonstrated in figure 9(a). One can see that the shape of the rope cross sections is almost elliptical, which is consistent with Koutnik’s results\cite{4,5}.

It was considered that the frequency of the upper part load pressure pulsations is concerned with the motions of elliptical shaped cross section. Thus, as Koutnik et al did, the motions of the ellipse are decomposed into precession movement and self rotation in the paper. Figure 9(b) shows the sketch of the ellipse motions, where $\omega_p$ and $\omega_s$ represent the angular speeds of the precession movement and self rotation, respectively. Three parameters $x_c$, $y_c$, and $R_c$, which indicate the geometric center of the ellipse, are introduced to study the precession movement, and they are defined as follows.

$$x_c = \frac{\sum_i^N x_i}{N}, \quad y_c = \frac{\sum_i^N y_i}{N}, \quad R_c = \sqrt{x_c^2 + y_c^2}$$

Where $N$ is the number of the ellipse data. What’s more, another three parameters, which are $L_a$, $L_b$ and $\bar{R}$, are employed to investigate the variation of the ellipse shape, which can be used to imply the self rotation in a sense. Here $L_a$ or $L_b$ represents the major or minor axis of the ellipse. To simplify the issue, $L_a$ is determined by $x_{\min}$ and $x_{\max}$, as shown in figure 9(b). $L_b$ and $\bar{R}$ are calculated as follows, where $S$ is the ellipse area, and $\bar{R}$ is the equivalent radius of the ellipse.
\[ L_0 = S / \pi L_a, \overline{R} = \sqrt{S / \pi} \]  

\( L_0 \) is about the sum of the first dominate frequencies of the rope motions of the self rotation and precession movement, respectively.

\begin{align*}
L_0 = S / \pi L_a, \overline{R} = \sqrt{S / \pi}
\end{align*}  

\[ \overline{R} = \sqrt{S / \pi} \]

\( \overline{R} \) is the rope precession movement.

The variations of the geometric center parameters, \( x_c, y_c \) and \( R_c \), of the three planes with the time are demonstrated in figure 10. The variations of geometric center parameters of the Upper plane and Middle plane are in regular pattern, while those in the Lower plane are irregular. Figure 11 shows the ellipse shape parameters vary with the time, and the variations in the Upper and Middle planes are in regular pattern, too. The reason of the irregularities of the parameters of the Lower plane may owe to the very unsteady nature of the vortex rope, in other words, the physical model used here is too simple. Therefore, to analyse the frequency of the parameter variations of the Lower plane is meaningless.

The frequency spectra of the motions of the elliptical cross sections in the Upper and Middle planes are listed in table 3. The first dominate frequency of the geometric motions is \( 0.38 f_n \), and it is the rope precession movement. The component of \( 0.67 f_n \) appears in the first dominate frequency of the variation of major or minor axis of the ellipse, which indicates that frequency of \( 0.67 f_n \) in the draft is induced by vortex rope self rotation. The frequencies of \( 1.33 f_n \) and \( 1.71 f_n \) are the second dominate ones of the self rotation and precession movement, respectively. The frequency of \( 1.04 f_n \), see figure 7, is about the sum of the first dominate frequencies of the rope motions.

\begin{table}[h]
\centering
\caption{First and second dominate frequencies of the motions of the elliptical cross sections}
\begin{tabular}{cccccc}
\hline
 & \( x_c \) & \( y_c \) & \( R_c \) & \( L_a \) & \( L_b \) & \( \overline{R} \) \\
\hline
\textbf{Upper plane} & & & & & & \\
First (\( f_n \)) & 0.38 & 0.38 & 0.38 & 0.67 & 0.67 & 0.38 \\
Second (\( f_n \)) & 1.71 & 1.71 & 1.33 & 1.33 & 1.33 & 0.76 \\
\textbf{Middle plane} & & & & & & \\
First (\( f_n \)) & 0.38 & 0.38 & 0.38 & 0.67 & 0.67 & 0.38 \\
Second (\( f_n \)) & 0.57 & 1.71 & 1.33 & 1.33 & 1.33 & 1.14 \\
\hline
\end{tabular}
\end{table}

\[ \text{Figure 10. The time evolution of the geometric centre parameters} \ x_c, \ y_c \ \text{and} \ R_c \ \text{of the three planes} \]

\[ \text{Figure 11. The time evolution of the ellipse shape parameters} \ L_a, \ L_b \ \text{and} \ \overline{R} \ \text{of the three planes} \]
One can notice that some diversities exist between experiment and numerical simulation in the work, namely, the first dominate frequency is $0.34f_r$ and the second one $1.72f_r$ according to experiment, while the first one is $0.38f_r$ and the frequency of $1.71f_r$ is not the second one. How to explain it? In author’s opinion, it is reasonable for the differences. The main reason is the resonance effect. As mention above, during model test, the resonance of the water in the fluid passage occurs, and the component of $1.72f_r$ is found at other locations. Because of resonance, the pressure amplitude induced by the frequency of $1.72f_r$ becomes large. On the contrary, the resonance can’t be predicted by numerical simulation just by taking the fluid into account. So the amplitude excited by the frequency of $1.71f_r$ is small, and naturally it is not the second dominate frequency. In addition, numerical methods considering more physical factors, e.g. cavitation, acoustics, structure effect, et al. could work better. However, from the point of view that the components of $0.38f_r$ and $1.71f_r$ are predicted by numerical simulation in the work, our results are in good agreement with experiment.

4. Conclusions
The upper part load pressure pulsations are of most detrimental hydraulic instabilities in Francis turbines which must be eliminated, and understanding the mechanism of the unsteady phenomenon is very important. It was considered that the cause of the upper part load pressure pulsations is connected with the vortex rope in the draft tube. Thus the vortex rope in the draft tube, which is represented by the isopressure surface of $p=-25000$Pa, is investigated by a three dimensional unsteady numerical approach. Three planes are employed to study the vortex rope. The results show that the rope cross sections are almost elliptical, which is consistent with Koutnik’s results. Then the motions of the elliptical sections, which contain precession movement and self rotation, are analysed by a simple model. The results demonstrate that the main pressure fluctuations frequencies in the draft tube are the combinations of the ellipse motions. What’s more, the frequency components of $0.38f_r$ and $1.71f_r$ predicted by the simulations are quite similar with the experimental results, which are $0.34f_r$ and $1.72f_r$. So the numerical results are reliable. Based on the analysis, the conclusion can be drawn that the cause of the upper part load pressure pulsations are strongly linked to the behaviour of the elliptical shaped cross vortex rope in the draft tube.

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