Pre-critical Chiral Fluctuations in Nuclear Medium — precursors of chiral restoration at $\rho_B \neq 0$ — *

T. Hatsuda$^a$ and T. Kunihiro$^b$

$^a$Physics Department, Kyoto University, Kyoto 606, Japan

$^b$Faculty of Science and Technology, Ryukoku University, Seto, Otsu 520-2194, Japan

It is shown that an enhancement in the spectral function near $2m_\pi$ threshold in the $I=J=0$ channel is a good signal of the partial restoration of chiral symmetry in nuclear medium. Several experiments are discussed to detect the enhancement, which uses hadron-nucleus and photo-nucleus reactions.

1. Introduction

When chiral symmetry is partially restored in nuclear medium $^{[1,2]}$ one can expect a large fluctuation of the quark condensate $\langle \bar{q}q \rangle$, i.e., the order parameter of the chiral transition. This implies $^{[1]}$ that there arises a softening of a collective excitation in the scalar-isoscalar channel, leading to (1) partial degeneracy of the scalar-isoscalar particle (traditionally called the $\sigma$-meson) with the pion, and (2) decrease of the decay width of $\sigma$ due to the phase space suppression caused by (2) in the reaction $\sigma \rightarrow 2\pi$: The significance of the $\sigma$ meson is discussed in $^{[1]}$; see also $^{[3]}$.

Although it is not a simple task to identify the $\sigma$ meson in free space $^{[4]}$, there will be a chance to see the elusive particle more clearly in nuclei where chiral symmetry might be partially restored. Some experiments to produce the sigma meson in a nucleus was proposed in $^{[4]}$.

One should, however, notice that describing the system with the meson in terms of the meson mass and its width may become inadequate because of the strong interaction with the environmental particles. Hence the proper observable to describes the system becomes the spectral function. Recently, a calculation of the spectral function in the sigma channel has been performed with the $\sigma$-2$\pi$ coupling incorporated in the linear $\sigma$ model at finite $T$; it was shown that the enhancement of the spectral function in the $\sigma$-channel just above the two-pion threshold is the most distinct signal of the softening $^{[5]}$. In this report, we shall show that the spectral enhancement associated with the partial chiral restoration takes place also at finite baryon density close to $\rho_0 = 0.17 \text{fm}^{-3}$.

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2. Model calculation

Before entering into the explicit model-calculation, let us describe the general features of the spectral enhancement near the two-pion threshold. Consider the propagator of the $\sigma$-meson at rest in the medium:

$$D_{\sigma}^{-1}(\omega) = \omega^2 - m_\sigma^2 - \Sigma_{\sigma}(\omega; \rho),$$

where $m_\sigma$ is the mass of $\sigma$ in the tree-level, and $\Sigma_{\sigma}(\omega; \rho)$ is the loop corrections in the vacuum as well as in the medium. The corresponding spectral function is given by

$$\rho_{\sigma}(\omega) = -\pi^{-1} \text{Im} D_{\sigma}(\omega).$$

Near the two-pion threshold, $\text{Im} \Sigma_{\sigma} \propto \theta(\omega - 2m_\pi) \sqrt{1 - \frac{4m_\pi^2}{\omega^2}}$ in the one-loop order. When chiral symmetry is being restored, $m_\sigma^* \equiv \text{“effective mass” of $\sigma$ defined as a zero of the real part of the propagator $\text{Re} D_{\sigma}^{-1}(\omega = m_\sigma^*) = 0$)}$ approaches to $m_\pi$. Therefore, there exists a density $\rho_c$ at which $\text{Re} D_{\sigma}^{-1}(\omega = 2m_\pi)$ vanishes even before the complete $\sigma$-$\pi$ degeneracy takes place; namely $\text{Re} D_{\sigma}^{-1}(\omega = 2m_\pi) = [\omega^2 - m_\sigma^2 - \text{Re} \Sigma_{\sigma}]_{\omega=2m_\pi} = 0$. At this point, the spectral function can be solely represented by the imaginary part of the self-energy;

$$\rho_{\sigma}(\omega \simeq 2m_\pi) = -\frac{1}{\pi} \text{Im} \Sigma_{\sigma} \propto \frac{\theta(\omega - 2m_\pi)}{\sqrt{1 - \frac{4m_\pi^2}{\omega^2}}},$$

which shows an enhancement of the spectral function at the $2m_\pi$ threshold. We remark that this enhancement is generically correlated with the partial restoration of chiral symmetry.

To make the argument more quantitative, let us evaluate $\rho_{\sigma}(\omega)$ in a toy model, namely the SU(2) linear $\sigma$-model:

$$\mathcal{L} = \frac{1}{4} \text{tr} [\partial M \partial M^\dagger - \mu^2 M M^\dagger - \frac{2\lambda}{4!} (M M^\dagger)^2 - h(M + M^\dagger)],$$

where tr is for the flavor index and $M = \sigma + i \vec{\pi} \cdot \vec{\tau}$. Although the model has only a limited number of parameters and is not a precise low energy representation of QCD, we emphasize that it does describe the pion dynamics qualitatively well up to 1GeV as shown by Chan and Haymaker [8], where the Padé approximant is used for the scattering matrix. The coupling constants $\mu^2$, $\lambda$ and $h$ have been determined in the vacuum to reproduce $f_\pi = 93$ MeV, $m_\pi = 140$ MeV as well as the s-wave $\pi$-$\pi$ scattering phase shift in the one-loop order.

The nucleon sector and the interaction with the nucleon of the mesons are given by,

$$\mathcal{L}_I(N, M) = -g\chi N U^5_5 N - m_0 \bar{N} U^5_5 N,$$

where we have used a polar representation $\sigma + i \vec{\pi} \cdot \vec{\gamma}_5 \equiv \chi U_5$ for convenience. The first term in (4) with the coupling constant $g$ is a standard chiral invariant coupling in the linear $\sigma$ model. Although the second term with a new parameter $m_0$ is not usually taken into account in the literatures, it is also chiral invariant and non-singular, so there is no compelling reason to dismiss it.

Under the dynamical breaking of chiral symmetry in the vacuum ($\langle \sigma \rangle_{\text{vac}} \equiv \sigma_0 \neq 0$), eq.(4) is expanded in terms of $\sigma/\sigma_0$ and $\vec{\pi}/\sigma_0$: it is found that the standard constraint $g_s = g_\rho$ is relaxed without conflicting with chiral symmetry due to the term with $m_0$; the term proportional to $m_0 \pi^2$ appears to preserve chiral symmetry.
In the following, we treat the effect of the meson-loop as well as the baryon density as a perturbation to the vacuum quantities. Therefore, our loop-expansion is valid only at relatively low densities. The full self-consistent treatment of the problem requires systematic resummation of loops similar to what was developed at finite $T$ [5].

We parametrize the chiral condensate in nuclear matter $\langle \sigma \rangle$ as $\langle \sigma \rangle \equiv \sigma_0 \Phi(\rho)$. In the linear density approximation, $\Phi(\rho) = 1 - C\rho/\rho_0$ with $C = (g_s/\sigma m^2_\sigma)\rho_0$. Instead of using $g_s$, we use $\Phi$ as a basic parameter in the following analysis. The plausible value of $\Phi(\rho = \rho_0)$ is $0.7 \sim 0.9$ [1].

3. Results and discussions

The spectral function together with $\text{Re}D^{-1}_\sigma(\omega)$ calculated with a linear sigma model are shown in Fig.1 and 2: The characteristic enhancements of the spectral function is seen just above the $2m_\pi$. It is also to be noted that even before the $\sigma$-meson mass $m^*_\sigma$ and $m_\pi$ in the medium are degenerate, i.e., the chiral-restoring point, a large enhancement of the spectral function near the $2m_\pi$ is seen.

![Figure 1](image1.png)

**Figure 1.** Spectral function for $\sigma$ and the real part of the inverse propagator for several values of $\Phi = \langle \sigma \rangle / \sigma_0$ with $m^\text{peak}_\sigma = 550$ MeV. In the lower panel, $\Phi$ decreases from bottom to top.

![Figure 2](image2.png)

**Figure 2.** Same with Fig.1 for $m^\text{peak}_\sigma = 750$ MeV

To confirm the threshold enhancement, measuring $2\pi^0$ and $2\gamma$ in experiments with hadron/photon beams off the heavy nuclear targets are useful. Measuring $\sigma \rightarrow 2\pi^0 \rightarrow 4\gamma$ is experimentally feasible [6], which is free from the $\rho$ meson background inherent in
the $\pi^+\pi^-$ measurement. Measuring of 2 $\gamma$'s from the electromagnetic decay of the $\sigma$ is interesting because of the small final state interactions, although the branching ratio is small. Nevertheless, if the enhancement is prominent, there is a chance to find the signal. When $\sigma$ has a finite three momentum, one can detect dileptons through the scalar-vector mixing in matter: $\sigma \rightarrow \gamma^* \rightarrow e^+e^-$. Recently CHAOS collaboration [7] measured the $\pi^+\pi^\pm$ invariant mass distribution $M_{\pi^+\pi^\pm}$ in the reaction $A(\pi^+, \pi^+\pi^\pm)X$ with the mass number $A$ ranging from 2 to 208: They observed that the yield for $M_{\pi^+\pi^-}^A$ near the $2m_\pi$ threshold is close to zero for $A = 2$, but increases dramatically with increasing $A$. They identified that the $\pi^+\pi^-$ pairs in this range of $M_{\pi^+\pi^-}^A$ is in the $I = J = 0$ state. The $A$ dependence of the invariant mass distribution presented in [7] near $2m_\pi$ threshold has a close resemblance to our model calculation in Fig.1 and 2, which suggests that this experiment may already provide a hint about how the partial restoration of chiral symmetry manifest itself at finite density.

We remark that $(d, ^3\text{He})$ reactions is also useful to explore the spectral enhancement because of the large incident flux. The incident kinetic energy $E$ of the deuteron in the laboratory system is estimated to be $1.1\text{GeV} < E < 10 \text{ GeV}$, to cover the spectral function in the range $2m_\pi < \omega < 750 \text{ MeV}$.

To make the calculation more realistic, one needs to incorporate the two-loop diagrams, which is expected, however, to hardly change the enhancement near the two-pi threshold discussed here.

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2 One needs also to fight with large background of photons mainly coming from $\pi^0$s.
3 See [1] for other approaches to explain the CHAOS data.