Gauss-Seidel Method for Calculation of Unbalanced Load Flow

Nazaruddin¹, Mahalla², Fauzi³
¹,²,³Electric Engineering Study Program – Politeknik Negeri Lhokseumawe

Email: nazaruddin@pnl.ac.id

Abstract. Three phase load flow is a study that will provide an overview of the voltage conditions, phase angle, current, power and power losses of each bus in each phase a, b and c. A three-phase load flow study can be used in a three-phase power system control and planning operation. The purpose of this paper is to create a three-phase load flow formulation for unbalanced load conditions. Unbalanced loads on the electric power system will affect consumers, namely a decrease in the voltage received by consumers. Three-phase power flow calculations using the Gauss-Seidel Method (GSM) and computation are done using Matlab software which will be tested on the IEEE 5-bus system. The simulation results show that the unbalanced load flow converges at 32 iterations, the total power on the reference bus (slack bus) is 54.840 MW for phase a, 53.119 MW for phase b, and 64.039 MW for phase c. The IEEE 5-bus system has the largest voltage drop occurring on bus 5 which is 7.7%, 6.9% and 8.9% respectively for phases a, b and c. The biggest power losses occur at line bus 1 to bus 2 at 0.912 MW, 0.854 MW and 1.228 MW or 2.30%, 2.20% and 2.67% respectively for phases a, b and c.

1. Introduction

Three phase power flow is a study that used to determine the voltage, phase angle, line flow and losses every bus at each a, b, and c phase. Study of three-phase power flow can be used for operation and planning three-phase electric power system. Calculation of power flow that is often used using conventional methods is the Gauss-Seidel method, Newton Raphson method and the Fast-Decoupled method. These three methods are the methods used in the transmission system, where the phase is relatively balanced [2][3][4].

There is some literature for the completion of three-phase power flows, including the authors propose a multiple solution methods [5], the application of the Fast-Decoupled method by comparing Newton's and implicit bus algorithms [6] and the development of the Newton Raphson algorithm for the completion of power flow [7]. All methods proposed by researchers are applied to radial distribution networks for unbalanced load conditions.

Balanced three-phase load is a where current flow in symmetrical loads and the load is connected to a symmetrical voltage as well. In analyzing these loads are usually assumed to be supplied by symmetrical voltage. Thus, the analysis can be carried out in a single phase. So, in this case, the load is always assumed to be balanced on each phase, whereas in reality, these expenses are not balanced. In this case, the solution uses symmetrical components [1].
Analysis of a balanced three-phase system is relatively simple compared to a three-phase solution complete with network equations. Asymmetrical component transformation will separate a balanced three-phase system into three independent systems, commonly referred to as positive, negative, and zero sequence networks. For completion in the form of a single phase that only uses positive sequential components [9].

An unbalanced system can occur because of an asymmetrical network caused by the effect of non-transposed line, difference ratio taps each phase of a three-phase transformer and abnormal operating conditions due to the release of one phase. Other conditions that cause unbalance of load are the electricity consumption of each consumer is not the same.

This study is analyzed the electric power system due to unbalanced load on each bus by designing a formulation for unbalanced three-phase power flow. In a three-phase power system, power flow studies are applied to unbalanced systems. Unbalances load can occur on networks that are not transposed and the use of each phase load is not the same. Completion of three-phase power flow using the Gauss-Seidel method (GSM) and the computational process is carried out using the Matlab programming language to be tested on the IEEE 5 bus system.

2. Modeling system and algorithm

2.1. Unbalanced three-phase system model

A three-phase network between bus i and j as shown in Figure 1.

![Three-phase network](image)

Network parameters can be determined based on the method developed by Carson (1926). A 4x4 matrix that enters its own inductance and shared inductance, [8]:

\[
\begin{bmatrix}
Z_{aa} & Z_{ab} & Z_{ac} & Z_{an} \\
Z_{ba} & Z_{bb} & Z_{bc} & Z_{bn} \\
Z_{ca} & Z_{cb} & Z_{cc} & Z_{cn} \\
Z_{na} & Z_{nb} & Z_{nc} & Z_{nn}
\end{bmatrix}
\]

(1)

For systems with neutral conductors connected to the ground, VN and Vn as shown in Figure 1, assumed to be zero. Equation (1) without including neutral influence or neutral conductors are connected to the ground, used to calculate unbalanced power flow.

\[
\begin{bmatrix}
Z_{aa} & Z_{ab} & Z_{ac} \\
Z_{ba} & Z_{bb} & Z_{bc} \\
Z_{ca} & Z_{cb} & Z_{cc}
\end{bmatrix}
\]

(2)

The relationship between bus voltage and branch current in Figure 1 can be written:
From the theory of symmetrical components, the sequence impedance of the transmission channel conductor can be determined.

\[
V^{abc} = I^{abc}Z^{abc}
\]  \hspace{1cm} (5)

\[
Z^{abc} = \frac{V^{abc}}{I^{abc}}
\]  \hspace{1cm} (6)

So, the phase impedance

\[
Z^{abc} = \frac{AV^{012}}{AI^{012}}
\]  \hspace{1cm} (7)

\[
\frac{V^{012}}{I^{012}} = A^{-1}Z^{abc}A
\]  \hspace{1cm} (8)

sequence impedance:

\[
Z^{012} = A^{-1}Z^{abc}A
\]  \hspace{1cm} (9)

Equation (9) can be written:

\[
Z^{012} = \frac{1}{3}\begin{bmatrix}
1 & 1 & 1 \\
1 & a & a^2 \\
1 & a^2 & a
\end{bmatrix}
\begin{bmatrix}
Z_{aa} & Z_{ab} & Z_{ac} \\
Z_{ba} & Z_{bb} & Z_{bc} \\
Z_{ca} & Z_{cb} & Z_{cc}
\end{bmatrix}
\begin{bmatrix}
1 & 1 & 1 \\
1 & a & a^2 \\
1 & a^2 & a
\end{bmatrix}
\]  \hspace{1cm} (10)

To change the sequence impedance back to phase impedance

\[
V^{012} = I^{012}Z^{012}
\]

Subutilize the equation \( V^{012} = A^{-1}V^{abc} \) and \( I^{012} = A^{-1}I^{abc} \) so

\[
A^{-1}V^{abc} = Z^{012}A^{-1}I^{abc}
\]

\[
Z^{abc} = AZ^{012}A^{-1}
\]  \hspace{1cm} (11)

Equation 11 can be written:

\[
Z^{abc} = \begin{bmatrix}
1 & 1 & 1 \\
1 & a & a^2 \\
1 & a^2 & a
\end{bmatrix}
\begin{bmatrix}
Z_{00} & Z_{01} & Z_{02} \\
Z_{10} & Z_{11} & Z_{12} \\
Z_{20} & Z_{21} & Z_{22}
\end{bmatrix}
\begin{bmatrix}
1 & 1 & 1 \\
1 & a & a^2 \\
1 & a^2 & a
\end{bmatrix}
\]  \hspace{1cm} (12)
2.2. Unbalanced three phase power flow equation

The current flowing between bus i and bus j in each phase in figure 1 can be calculated based on equation 1:

\[
\begin{bmatrix}
I_{a_{ij}} \\
I_{b_{ij}} \\
I_{c_{ij}}
\end{bmatrix} =
\begin{bmatrix}
Y_{aa} & Y_{ab} & Y_{ac} \\
Y_{ba} & Y_{bb} & Y_{bc} \\
Y_{ca} & Y_{cb} & Y_{cc}
\end{bmatrix}
\begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix}
\]

Suppose the transmission line is described in the form of a three-phase system as shown in Figure 2 [1]:

![Figure 2 Three phase power system bus model](image)

From Figure 2 the three-phase current equation on the bus i can be written:

\[
I_i^p = \sum_{q=a,b,c} V_i^p \left( \sum_{j=0}^{n} y_{ij}^{p,q} - \sum_{j=1}^{n} y_{ij}^{q,p} \right) V_j^q
\]

(13)

with \(p=a,b,c\)

Power equation:

\[
S_i^p = V_i^p I_i^{p*}
\]

(14)

\[
S_i^p = P_i^p + jQ_i^p
\]

(15)

and

\[
P_i^p + jQ_i^p = V_i^p I_i^{p*}
\]

(16)

\[
I_i^p = \frac{P_i^p - jQ_i^p}{(V_i^p)^*}
\]

(17)

substitute equation 13 to equation 17, so that it can be written
\[
\frac{P^p - jQ^p}{(V_i^p)^*} = V_i^p \sum_{q=a,b,c} \sum_{j=0}^{K} y_{ij}^{p,q} - \sum_{j=1}^{K} y_{ij}^{p,q} V_j^q, \quad j \neq i
\]
with \( p = a, b, c \) \hspace{1cm} (18)

### 2.3. Gauss-Seidel Method

The Gauss-Seidel method is carried out through a process of repetition or iteration by specifying the estimated values for the bus voltage whose value is unknown. Then calculate a new value for each bus voltage from the estimated values on other buses that have known voltage values.

The following equation gives a relationship of the value of the current flowing into the bus \( i \) a network that contains \( K \) free node.

\[
I_i^p = \sum_{q=a,b,c} V_i^p \sum_{j=0}^{n} y_{ij}^{p,q} - \sum_{j=1}^{n} y_{ij}^{p,q} V_j^q
\]

The power equation on bus \( i \) is:

\[
P_i^p + jQ_i^p = V_i^p I_i^p^*
\]

The current values can be expressed as:

\[
I_i^p = \frac{P_i^p - jQ_i^p}{(V_i^p)^*}
\]
\[
I_i^p = \frac{P_i - jQ_i}{V_i^*}
\]

Substitution of equation (23) with (25):

\[
\sum_{k=1}^{K} y_{ik} V_k = \frac{P_i - jQ_i}{V_i^*}
\]

\[
V_i^p \sum_{q=a,b,c} \sum_{j=0}^{n} y_{ij}^{p,q} - \sum_{j=1}^{n} y_{ij}^{p,q} V_j^q = \frac{P_i^p - jQ_i^p}{(V_i^p)^*}
\]

The equation for calculating the voltage of each bus:

\[
V_i^p = \frac{\frac{P_i^{p,\text{sch}} - Q_i^{p,\text{sch}}}{V_i^p^*} + \sum_{j=1}^{n} y_{ij}^{p,q} V_j^q}{\sum_{j=0}^{n} y_{ij}^{p,q}}, \quad i \neq p
\]

with \( p = a, b, c \) \hspace{1cm} (26)

The load flow solution uses bus 1 as a reference bus or swing bus so that it is numbered 1. Suppose the swing bus voltage has been set then the voltage on bus 2 that is, can be calculated based on equation (27).

To calculate losses in a line, from equation (14), complex power : \( S_{ij} \) (from bus \( i \) to \( j \)) and \( S_{ji} \) (from bus \( j \) to \( i \)).

\[
S_{ij}^p = V_i^* I_{ij}^p
\]
\[
S_{ji}^p = V_j^* I_{ji}^p
\]

Losses in lines \( i \)-\( j \) are sums of algebra from the power flow approach of equations (28) and (29)

\[
S_{Lij} = S_{ij} + S_{ji}
\]

To determine losses in lines \( i \)-\( j \) of each phase, from equation (30) can be written:
\[ S_{\text{loss}}^p_{ij} = S_{ij}^p + S_{ji}^p, \text{ with } p = a, b, c \] (31)

2.4. Algorithm
The algorithm of Gauss-Seidel method for three-phase load flow can be described as follows:
1) Initialize of IEEE 5-bus test system data of bus data including bus, load and generator, and line data.
2) Create the bus impedance matrix \([Z_{abc}]\)
3) Change the impedance matrix \([Z_{abc}]\) to the admittance matrix \([Y_{abc}]\)
4) Set the initial value \(V_i^{(0)} = 1.05+j0\) and \(V_j^{(0)} = 1+j0\)
5) Calculate the voltage of each bus, do the iteration process with (27)
6) Calculate \(V_i\) and \(V_j\) until it reaches convergence
7) Calculate the power on the slack bus with equation (22)
8) Calculate the current between buses using equation (21)
9) Calculate the power flow with equation (19)
10) Calculate active and reactive power losses using equation (31)

3. Test performed on a system
To test the formulation proposed in this paper is done on the IEEE 5 bus system, load data and network parameter data are shown in Table 1 and Table 2.

| Bus | Type | V (pu) | P (MW) | Q (MVAR) | P_G (MW) | Q_G (MVAR) |
|-----|------|--------|--------|----------|----------|------------|
| 1   | Slack| 1.06   | 0      | 0        | 0        | 0          |
| 2   | PQ   | 1.0    | 20     | 10       | 0        | 0          |
| 3   | PQ   | 1.0    | 45     | 15       | 0        | 0          |
| 4   | PQ   | 1.0    | 40     | 5        | 0        | 0          |
| 5   | PQ   | 1.0    | 60     | 10       | 0        | 0          |

| From | to   | R (pu) | X (pu) | 1/2B (pu) |
|------|------|--------|--------|-----------|
| 1    | 2    | 0.02   | 0.06   | 0.00      |
| 1    | 3    | 0.08   | 0.24   | 0.00      |
| 2    | 3    | 0.06   | 0.25   | 0.00      |
| 2    | 4    | 0.06   | 0.18   | 0.00      |
| 2    | 5    | 0.04   | 0.12   | 0.00      |
| 3    | 4    | 0.01   | 0.03   | 0.00      |
| 4    | 5    | 0.08   | 0.24   | 0.00      |

4. Results and Discussion
System testing carried out in the study is on the IEEE 5 bus system with a base value of 100 MVA and 150 kV, the test results are obtained by simulating the power flow for unbalanced load flow conditions with the Matlab software. Power flow simulation results for unbalanced loads show a description of the system condition in the form of parameters on each bus, which includes the voltage, power and losses of the network of each phase, namely phase a, b and c. Power flow calculation simulation using...
the Gauss-Seidel method converges in the 32nd iteration for the IEEE-5 bus system. Test results can be shown in Table 3.

| Bus Number | Phase a (Mag(pu), Angl(deg)) | Phase b (Mag(pu), Angl(deg)) | Phase c (Mag(pu), Angl(deg)) |
|------------|------------------------------|------------------------------|------------------------------|
| 1          | 0.612 (0.000)                | 0.612 (0.000)                | 0.612 (0.000)                |
| 2          | 0.587 (-3.296)               | 0.588 (-3.192)               | 0.581 (-3.882)               |
| 3          | 0.573 (-5.536)               | 0.574 (-5.348)               | 0.566 (-6.331)               |
| 4          | 0.572 (-5.892)               | 0.573 (-5.670)               | 0.565 (-6.663)               |
| 5          | 0.569 (-6.504)               | 0.570 (-6.353)               | 0.558 (-8.031)               |

The results from table 3 can be made in the form of a voltage magnitude graph for each phase for the IEEE 5 bus system can be seen in Figure 3.

Figure 3 shows the magnitude of the voltage of each bus for each phase for the IEEE 5 bus system. With the vertical axis as the voltage value in per unit (pu) and the horizontal axis is the bus number, with bus 1 as the reference bus (slack bus).

On IEEE 5 bus systems, bus 1 is a reference voltage of 0.612 pu (100%) for phases a, b and c. The minimum voltage value on bus 5 is 0.569 pu (92.3%), 0.570 pu (93.1%), and 0.558 pu (91.1%) for the a, b and c phases, so that on bus 5 experienced a voltage drop of 7.7%, 6.9% and 8.9% respectively for phases a, b and c.

The simulation results shown in tables 4 and 5 are an overview of the power flows and losses of each phase (phases a, b and c) for the IEEE 5 bus system.

| Bus from to | Line Flow | MW(a) | MVAR(a) | MW(b) | MVAR(b) | MW(c) | MVAR(c) |
|------------|-----------|-------|--------|-------|--------|-------|--------|
| 1          | 2         | 38.978| 13.706 | 37.767| 13.175 | 45.931| 17.380 |
| 1          | 3         | 15.862| 5.291  | 15.352| 5.100  | 18.108| 6.574  |
| 2          | 3         | 5.179 | 1.923  | 5.529 | 1.876  | 6.152 | 2.197  |
| 2          | 4         | 9.131 | 2.063  | 8.756 | 1.996  | 9.635 | 2.359  |
| 2          | 5         | 16.817| 3.786  | 16.627| 3.741  | 21.256| 5.160  |
| 3          | 4         | 6.971 | 0.508  | 6.313 | 0.402  | 6.292 | 0.378  |
| 4          | 5         | 1.534 | 0.268  | 1.717 | 0.291  | 3.339 | 0.626  |

| Bus Number | Power losses for the IEEE 5 bus system. |
|------------|----------------------------------------|
| 1          | 45.931                                 |
| 2          | 18.108                                 |
| 3          | 6.152                                  |
| 4          | 9.635                                  |
| 5          | 21.256                                 |
## Bus Losses

| Bus from | to | MW(a) | MVAR(a) | MW(b) | MVAR(b) | MW(c) | MVAR(c) |
|---------|----|-------|---------|-------|---------|-------|---------|
| 1       | 2  | 0.912 | 2.735   | 0.854 | 2.563   | 1.288 | 3.864   |
| 1       | 3  | 0.597 | 1.792   | 0.559 | 1.677   | 0.793 | 2.378   |
| 2       | 3  | 0.063 | 0.264   | 0.059 | 0.247   | 0.076 | 0.316   |
| 2       | 4  | 0.153 | 0.458   | 0.140 | 0.420   | 0.175 | 0.524   |
| 2       | 5  | 0.345 | 1.036   | 0.336 | 1.009   | 0.566 | 1.699   |
| 3       | 4  | 0.015 | 0.045   | 0.012 | 0.036   | 0.012 | 0.037   |
| 4       | 5  | 0.006 | 0.018   | 0.007 | 0.002   | 0.029 | 0.087   |

The results from Table 4 and Table 5 can be made in graphical form, successive graphs of power loss characteristics on the inter-bus connecting channel for the IEEE 5 bus system, as shown in Figure 4.

![Figure 4. Characteristic of power losses for the IEEE 5 bus system](image)

The highest value of power losses in the IEEE 5 bus system occurs in the connecting lines of bus 1 to bus 2, namely 0.912 MW, 0.854 MW and 1.228 MW or 2.30%, 2.20% and 2.67% respectively for a, b and c phases. The lowest power losses occur in bus 4 to bus 5 connecting lines respectively 0.006 MW, 0.007 MW, and 0.2 MW or 0.40%, 0.41% and 0.86% respectively for phase a, b and c as shown in Figure 4.

### 5. Conclusion

From the simulation results of power flow for unbalanced load for the IEEE 5 bus system, the following conclusions can be stated that the results of a power flow simulation calculation with the MathLab software converge the 32nd iteration for the IEEE 5 bus system. The highest voltage drop on the IEEE 5 bus system is on bus 5 is 7.7%, 6.9% and 8.9% respectively for phases a, b and c.

The highest power losses in the IEEE 5 bus system occur at the connecting line between bus 1 to bus 2 at 0.912 MW, 0.854 MW and 1.228 MW or 2.30%, 2.20% and 2.67% respectively for phase a, b and c. The lowest power losses are 0.006 MW, 0.007 MW, and 0.2 MW or 0.40%, 0.41% and 0.86% respectively for phases a, b and c occur on the connecting line of bus 4 to bus 5.
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