THE FINE-STRUCTURE LINES OF HYDROGEN IN H II REGIONS

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ABSTRACT

The 2s1/2 state of hydrogen is metastable and overpopulated in H ii regions. In addition, the 2p states may be pumped by ambient Lyα radiation. Fine-structure transitions between these states may be observable in H ii regions at 1.1 GHz (2S1/2–2p1/2) and/or 9.9 GHz (2S1/2–2p3/2), although the details of absorption versus emission are determined by the relative populations of the 2s and 2p states. The n = 2 level populations are solved with a parameterization that allows for Lyα pumping of the 2p states. The Lyα pumping rate has long been considered uncertain, as it involves solution of the difficult Lyα transfer problem. The density of Lyα photons is set by their creation rate, easily determined from the recombination rate, and their removal rate. Here we suggest that the dominant removal mechanism of Lyα radiation in H ii regions is absorption by dust. This circumvents the need to solve the Lyα transfer problem and provides an upper limit to the rate at which the 2p states are populated by Lyα photons. In virtually all cases of interest, the 2p states are predominantly populated by recombination, rather than Lyα pumping. We then solve the radiative transfer problem for the fine-structure lines in the presence of free-free radiation. In the likely absence of Lyα pumping, the 2S1/2 → 2p1/2 lines will appear in stimulated emission, and the 2S1/2 → 2p3/2 lines in absorption. Because the final 2p states are short lived, these lines are dominated by intrinsic line width (99.8 MHz). In addition, each fine-structure line is a multiplet of three blended hyperfine transitions. Searching for the 9.9 GHz lines in high emission measure H ii regions offers the best prospects for detection. The lines are predicted to be weak; in the best cases, line-to-continuum ratios of several tenths of a percent might be expected with line strengths of tens to a hundred mK with the Green Bank Telescope. Predicted line strengths, at both 1.1 and 9.9 GHz, are given for a number of H ii regions, high emission measure components, and planetary nebulae, based on somewhat uncertain emission measures, sizes, and structures. The extraordinary width of these lines and their blended structure will complicate detection.

Subject headings: H ii regions — ISM: lines and bands — line: formation — planetary nebulae: general — radio lines: ISM

1. INTRODUCTION

Transitions between fine-structure sublevels in atomic hydrogen have never been detected astronomically. With advances in radio astronomy, however, it now appears that transitions between the 2S1/2 and 2P1/2 and between the 2S1/2 and 2P3/2 levels may soon be observable in H ii regions. Figure 1 shows the fine and hyperfine structure of the n = 2 level of atomic hydrogen. The fine-structure splitting into the 2S1/2, 2P1/2, and 2P3/2 levels is caused by a combination of spin-orbit coupling, relativistic effects, and the Lamb shift. Significantly, the 2S1/2 state is metastable owing to the transition rules for angular momentum, i.e., the 2S–1S transition is forbidden. Hydrogen atoms in the interstellar medium are removed from the 2S1/2 state principally through two-photon emission to the ground state (Breit & Teller 1940) with a decay rate per atom of τ2 = 8.227 s⁻¹ (Spitzer & Greenstein 1951), resulting in optical continuum radiation. Because this rate is some 8 orders of magnitude slower than Lyman decay, it may seem reasonable to expect that the 2S1/2 state will be overpopulated relative to other excited states under photoionization equilibrium. It must be noted, however, that trapped Lyα radiation in an H ii region will pump the 2p states. In addition, in dense H ii regions collisions, primarily with ions, will transfer hydrogen atoms between the 2s and 2p states. These effects must be taken into account in order to determine the expected strengths and signs (absorption vs. stimulated emission) of the 2S1/2–2P1/2 (1.1 GHz) and 2S1/2–2P3/2 (9.9 GHz) lines.

Apparently, Wild (1952) was one of the first to investigate the astrophysical implications of these lines, suggesting that the 2S1/2–2P3/2 line might be observed in the Sun. Purcell (1952), however, noted that collisions in the dense solar atmosphere would equilibrate the 2s and 2p populations, making detection unlikely. Townes (1957) suggested that an overpopulation of the metastable 2s states may lead to detectable lines in the interstellar medium. Shklovski (1960, p. 255) raised the possibility of Lyα pumping of the 2p states but discounted its sufficiency for producing observable 2p → 2s transitions. Pottasch (1960) argued that the Lyα depth in H ii regions is essentially infinite, such that virtually all downward Lyα transitions are balanced by a Lyα absorption, resulting in an overpopulation of 2p states relative to 2s at densities below about 10⁶ cm⁻³. Field & Partridge (1961) further considered the case in which Lyα radiation is effectively destroyed by collisional conversion of 2p states to 2s (followed by two-photon decay to the ground state). The consequent overpopulation of 2p states then led to the prediction that the 9.9 GHz lines would appear in stimulated emission in H ii regions. (This
also implied that the 1.1 GHz lines would appear in absorption.) Myers & Barrett (1972) used the Haystack 37 m radio telescope with a 16 channel filter bank spanning 1 GHz in frequency to search for the 9.9 GHz lines, but did not find any lines with antenna temperatures in excess of 0.1 K. Ershov (1987) estimated the strengths of both sets of lines in Orion A and W3(OH) under the assumption that the $2p$ states are negligibly populated with respect to $2s_{1/2}$.

The major variation in the predictions arises from the uncertain density of Ly$\alpha$ radiation in H II regions, which in turn is determined by the processes that destroy Ly$\alpha$ radiation. In addition to the conversion of $2p$ to $2s$ states via collisions, other processes are likely to be important. Resonantly scattered Ly$\alpha$ photons undergo a diffusion in frequency and may thus acquire a significant probability of escaping the H II region due to the much lower optical depth in the line wings (Cox & Matthews 1969; Spitzer 1978). Systematic gas motions within the region (e.g., expansion) can also contribute to the migration of Ly$\alpha$ photons away from line center. The dominant removal mechanism, however, is absorption by dust within the H II region (Kaplan & Pikelner 1970; Spitzer 1978). This last fact greatly simplifies the estimation of the Ly$\alpha$ density.

In this paper, we obtain formulae for the $2s$ and $2p$ populations in an H II region under a parameterization that characterizes the rate of $2p$ production by Ly$\alpha$ absorption in terms of the rate of $2p$ production through recombination (§ 2). Existing models for dust in H II regions and planetary nebulae are then used to place tight constraints on the Ly$\alpha$ density and thus the $2p$

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**Fig. 1.**—Fine and hyperfine structure of the $n = 2$ level of hydrogen. Allowed hyperfine transitions are shown as solid vertical lines, with arrows denoting predominant transition directions if the metastable $2s_{1/2}$ states are overpopulated with respect to the $2p$ states. (See text.) Hyperfine transition frequencies are given in MHz. The quantum number $F$ corresponds to total angular momentum, i.e., orbit + electron spin + nuclear spin. Note that level separations are not shown to scale.
production rate by Lyα photons (§ 3). The radiative transfer problem for the fine-structure lines is then solved for a uniform region (§ 4). We obtain approximate predicted line strengths for various H II regions, compact components, and planetary nebulae under the justified assumption that Lyα pumping of the 2p states is negligible (§ 5). High emission measure H II regions and/or planetary nebulae are most likely to yield detectable lines.

2. POPULATION OF THE 2s AND 2p STATES

Spitzer & Greenstein (1951) estimated that between 0.30 and 0.35 of all recombinations in an H II region reach the 2s state. Including the effects of collisions and the two-photon decay rate, the rate equation for the 2s state is

\[ f\alpha n_e n_c + C_{ps} n_e n_{2p} = A_{2s} n_{2s} + C_{sp} n_e n_{2s}, \]

where \( f \) is the fraction of recombinations producing the 2s state and \( \alpha \) is the recombination coefficient. The collision coefficients, \( C_{ps} \) and \( C_{sp} \), give the appropriate rates at which atoms are transferred from 2p to 2s and from 2s to 2p, respectively. These rates are dominated by collisions with protons, but including electron collisions (and assuming that \( n_e = n_i \) and \( T = 10^4 \) K), we have \( C_{sp} = 5.31 \times 10^{-4} \text{ cm}^3 \text{s}^{-1} \) (Seaton 1955). Since \( kT \) is much greater than the energy separations between the 2s and 2p states, the reverse rate is determined by just the ratio of statistical weights, i.e.,

\[ C_{ps} = \frac{g_{2s}}{g_{2p}} C_{sp} = \frac{1}{3} C_{sp}. \]

Similar considerations apply to the 2p states. The fraction of recombinations reaching 2p is \((1 - f)\). (It is worth noting that processes such as Lyβ absorption, which ultimately produce the 2s state through 1s → 3p → 2s, have already been taken into account in the calculation of \( f \) [Spitzer & Greenstein 1951].) Radiative decay occurs through both spontaneous Lyα emission (with \( A_{21} = 6.25 \times 10^8 \text{ s}^{-1} \)), as well as stimulated emission. We must also include radiation excitation from the ground state via Lyα absorption. Including collisions that couple to the 2s states, the rate equation for 2p then is

\[
(1 - f)\alpha n_e n_c + C_{ps} n_e n_{2p} + c \int n_{\nu}^{(1s)} d\nu \int B_{12}(\nu - \nu') n_{\nu'} d\nu' = A_{21} n_{2p} + C_{ps} n_e n_{2p} + c \int n_{\nu}^{(2p)} d\nu \int B_{21}(\nu - \nu') n_{\nu'} d\nu',
\]

where \( n_{\nu}^{(1s)} \) and \( n_{\nu}^{(2p)} \) are the densities of atoms in the 1s and 2p states per radial velocity interval, measured in frequency units, and \( n_{\nu'} \) is the photon density per frequency interval (in \text{cm}^{-3} \text{Hz}^{-1}). Now,

\[
B_{21}(\nu - \nu') = \frac{c^2}{8\pi \nu_L^3} A_{21} L(\nu - \nu'),
\]

\[
B_{12}(\nu - \nu') = 3B_{21}(\nu - \nu'),
\]

where \( \nu_L \) is the Lyα frequency and \( L(\nu - \nu') \) is the Lorentz line profile. For a thermal gas,

\[
n_{\nu}^{(1s)} = \frac{n_{1s}}{\sqrt{\pi \Delta\nu_D}} e^{-\left(\nu - \nu_0\right)^2/(\Delta\nu_D)^2},
\]

where \( (\Delta\nu_D) \) is the Doppler width. For \( T = 10^4 \) K, \( (\Delta\nu_D) = 1.29 \times 10^{11} \text{ Hz} \). Clearly, thermal widths will completely dominate the natural (Lorentz) line width of Lyα, and thus we can replace \( L(\nu - \nu') \) by the Dirac delta function. We then obtain for the absorption term

\[
c \int n_{\nu}^{(1s)} d\nu \int B_{12}(\nu - \nu') n_{\nu'} d\nu' = \frac{3c^3}{8\pi \nu_L^4} A_{21} \frac{n_{1s}}{\sqrt{\pi (\Delta\nu_D)^2}} \times \int n_{\nu} e^{-\left(\nu - \nu_0\right)^2/(\Delta\nu_D)^2} d\nu.
\]

Of course, a similar calculation can be carried out for the stimulated emission term.

Because the nebula is optically thick in Lyα, \( n_e \) must be considered carefully. Were escape from the nebula the primary removal mechanism, then a steady state would result in which photons are created near the line center, diffuse in frequency through resonant scattering, and are effectively removed far out in the wings. The decrease in both creation rate and diffusion rate with frequency offset results in a nearly flat distribution (\( n_\nu \)) out to the approximate frequency at which photons freely escape, beyond which it drops sharply (Capriotti 1966). We write this frequency offset as \( w(\Delta\nu) \), where \( w \) is a dimensionless parameter expressing the offset in terms of the Doppler width. We therefore assume that \( n_\nu = n_{\nu_0} / [2w(\Delta\nu_D)] \) for \( |\nu - \nu_D| > w(\Delta\nu_D) \) and \( n_\nu = 0 \) for \( |\nu - \nu_D| < w(\Delta\nu_D) \), where \( n_{\nu_0} \) is the density of Lyα photons. The 2p rate equation then becomes

\[
(1 - f)\alpha n_e n_c + C_{ps} n_e n_{2p} + \frac{3c^3}{16\pi \nu_L^4 (\Delta\nu_D)^2} \operatorname{erf}(w/w_0) n_{\nu_0} n_{1s} = A_{21} n_{2p}.
\]

Spontaneous emission completely dominates the depopulation of the 2p state, and thus the other two terms that appeared on the right-hand side of equation (3) have been dropped. Stimulated emission is negligible in comparison with spontaneous emission for any reasonable value of \( n_{\nu_0} \). Indeed, equality of spontaneous and stimulated emission would imply a radiation pressure (due to Lyα) many orders of magnitude in excess of the thermal gas pressure. In addition, the 2p → 2s collision rate is negligible for any reasonable value of \( n_e \). (We include these collisions terms insofar as they populate the 2s state, however. See eq. [1].)

As we shall see, it is quite plausible that Lyα photons are absorbed by dust before significant frequency diffusion occurs. In this case, \( n_e \) will simply reflect the thermal distribution of atoms. The resulting 2p rate equation is identical to that given above, provided that we replace \( \operatorname{erf}(w/w_0) \) with \( 2(2\pi)^{1/2} \).

At low densities, Lyα photons are created at the rate at which recombinations lead to 2p states, i.e., \((1 - f)\alpha n_e n_c\). At high densities \( (n_i > 10^8 \text{ cm}^{-3}) \), 2s states may be collisionally converted to 2p states, leading to additional Lyα photons, and thus the Lyα creation rate could be as large as \( \alpha n_e n_c \), the total recombination rate. We therefore define the Lyα lifetime as

\[
t_{\nu_0} = \frac{n_{\nu_0}}{\alpha n_e n_c},
\]

where \( 1 - f \leq r \leq 1 \). Since 2p states decay to the ground state much faster than they could be collisionally converted to 2s, all recombinations to 2p are regarded as producing a Lyα photon. Thus, \( r \) could never be smaller than \( 1 - f \).
We define

$$S = \frac{3e^3}{16\pi^2} \frac{A_{21}}{\Delta\nu_D} \frac{r}{(1-f)} \frac{\text{erf}(w)}{w} \chi n_H t_{\text{Ly}\alpha},$$

(10)

where $\chi = n_{\text{H}}/n_1$ and $n_1$ is the total hydrogen density (atomic plus ionized). The rate equations (eqs. [1] and [8]) can then be solved for the 2s and 2p populations:

$$n_{2s} = \frac{\alpha n_{p} \left[ f A_{21} + (1-f)(1+S)C_{sp} n_{p} \right]}{A_{21} \left( A_{21} + C_{sp} n_{p} \right) - C_{sp} C_{p} n_{p}^2},$$

(11)

$$n_{2p} = \frac{\alpha n_{p} \left[ (1-f)(1+S)A_{2s} + (1+S-fS)C_{sp} n_{p} \right]}{A_{21} \left( A_{21} + C_{sp} n_{p} \right) - C_{sp} C_{p} n_{p}^2}.$$

(12)

The dimensionless parameter $S$ gives the rate at which 2p states are produced by captured Ly$\alpha$ radiation in terms of the 2p creation rate from recombination (at low densities).

The ratio $n_{2p}/(3n_{2s})$ provides a determination of the relative importance of Ly$\alpha$ pumping of the 2p states and whether a fine-structure line is expected to appear in absorption or (stimulated) emission. If $n_{2p}/(3n_{2s}) > 1$, then the 9.9 GHz, 2$s_{1/2}$ to 2$p_{1/2}$ line will appear in emission and the 1.1 GHz, 2$s_{1/2}$ to 2$p_{3/2}$ line will appear in absorption. Of course, $n_{2p}/(3n_{2s}) < 1$ implies the opposite. [This assumes that the two 2p states are populated according to their relative statistical weights. To evaluate cases in which $n_{2p}/(3n_{2s})$ is of order unity, it would be necessary to separately account for the rates at which 2$p_{1/2}$ and 2$p_{3/2}$ states are created and destroyed, including the collisional rates coupling these states.] From equations (11) and (12) it follows that $n_{2p}/(3n_{2s}) > 1$ if $S > S_{\text{crit}}$, where

$$S_{\text{crit}} = \frac{3f}{(1-f)} \frac{A_{21}}{A_{2s}} = 1.14 \times 10^8.$$

(13)

(The numerical value of $S_{\text{crit}}$ corresponds to $f = 1/3$.) Because the 2p states naturally decay about $10^8$ times faster than the 2s states, population equality thus requires a pumping rate some 8 orders of magnitude faster than the approximate rate at which 2s and 2p states are formed through recombination.

3. THE Ly$\alpha$ DENSITY IN H $\Pi$ REGIONS

Determination of the Ly$\alpha$ density in H $\Pi$ regions is a complicated transfer problem. It is likely, however, that the dominant mechanism for removing Ly$\alpha$ photons is quite straightforward, i.e., absorption by dust (Kaplan & Pikelner 1970; Spitzer 1978). Thus, we eschew the noncoherent radiative transfer problem and find the upper limits to $t_{\text{Ly}\alpha}$ and $S$ set by absorption. Other competing removal processes would reduce the lifetime, and therefore density, of Ly$\alpha$ photons, resulting in a lower value for $S$.

Using a silicate-graphite model for dust in H $\Pi$ regions (Aannestad 1989), with a dust-to-gas ratio of 0.009, the extinction at Ly$\alpha$ can be shown to be $N_H/(5.4 \times 10^{20} \text{ cm}^{-2})$ mag, where $N_H$ is the column density of hydrogen. The albedo for this mixture is about 0.4 at Ly$\alpha$ (Draine & Lee 1984). The lifetime of Ly$\alpha$ photons against absorption by dust can then be calculated as $t_{\text{Ly}\alpha} = (3.3 \times 10^{10} \text{ cm}^{-3} \text{ s})/n_H$. We then find that

$$S = 4.2 \times 10^7 \chi \frac{r}{(1-f)} \frac{\text{erf}(w)}{w}.$$

(14)

Since $\chi \ll 1$ throughout most of the volume of an H $\Pi$ region (Osterbrock 1989), we conclude that $S \ll S_{\text{crit}}$ and that the 2s state is overpopulated relative to the 2p states.

In the harsh environments of planetary nebulae, dust might be destroyed by shocks or hard UV radiation, or possibly separated from the ionized gas by radiation pressure (Natta & Panagia 1981; Pottasch 1987). Abundance measurements indicate, however, that various heavy elements are depleted from the gas phase in the ionized regions of NGC 7027 (Kingdon & Ferland 1997) and NGC 6445 (van Hoof et al. 2000), suggesting that dust has not been destroyed in significant quantities. In addition, planetary nebulae frequently exhibit a mid-IR spectral component characteristic of warm dust heated by the intense radiation field within the ionized region (Kwok 1980; Hoare 1990; Hoare et al. 1992). Summarizing these results, Barlow (1993) has argued that dust is a common constituent of the ionized zones of planetary nebulae, albeit with dust-to-gas ratios about an order of magnitude or more below that of the general interstellar medium.

Middlemass (1990) has modeled NGC 7027 and finds an extinction optical depth of about 0.17 at 500.7 nm. For a uniform column, the density of hydrogen in the ionized zone is about 3.5 x $10^{21} \text{ cm}^{-2}$ (Thomasson & Davies 1970) along the radius of the nebula. For the graphite dust model used by Middlemass (1990), the extinction at Ly$\alpha$ then is $N_H/(1.3 \times 10^{22} \text{ cm}^{-2})$ mag, about a factor of 20 smaller than that in a general H $\Pi$ region described above and roughly consistent with the smaller dust-to-gas ratio (≈7 x $10^{-4}$) in this object (Barlow 1993). For an albedo of 0.4, the Ly$\alpha$ lifetime is $t_{\text{Ly}\alpha} = (7.6 \times 10^{11} \text{ cm}^{-3} \text{ s})/n_H$, and thus

$$S \approx 9.8 \times 10^8 \chi \frac{r}{(1-f)} \frac{\text{erf}(w)}{w}.$$

(15)

If we assume that absorption by dust is the dominant process limiting the Ly$\alpha$ density, make the replacement $\text{erf}(w)/w \rightarrow 2(2/\pi)^{1/2}$, and let $r \approx 1$, we obtain $S = 2.3 \times 10^9 \chi$. Since it is quite unlikely that $\chi$ is as large as 0.05, as required for $S \approx S_{\text{crit}}$, we conclude also in this case that the 2p states are underpopulated relative to the 2s states.

Of course, the above estimates are upper limits to the Ly$\alpha$ lifetime, as other processes (described in §1) may contribute to the removal of these photons. Thus, values of $S$ estimated in this way are upper limits. In addition, if escape is important, then the relevant value of $\text{erf}(w)/w$ would be smaller than that substituted above.

It should also be noted that Ly$\alpha$ radiation contributes to heating the dust in the ionized region. The energy absorbed is then reradiated in the IR. In the case of NGC 7027, a mid-IR spectral component is evidently due to dust with temperature $T_d \approx 230 \text{ K}$ dominated with the ionized gas (Kwok 1980). If the heating is dominated by Ly$\alpha$ radiation, then

$$n_{\text{Ly}\alpha} \nu L_{\text{Ly}\alpha} Q_{\text{abs}}(\text{Ly}\alpha) = 4 \langle Q(a, T_d) \rangle \sigma T_d^4,$$

(16)

where $Q_{\text{abs}}(\text{Ly}\alpha)$ is the absorption efficiency at Ly$\alpha$ and $Q(a, T_d)$ is the Planck-averaged emissivity (Draine & Lee 1984), a function of grain size $a$ and temperature $T$. Large grains, being efficient emitters in the IR, will yield a larger estimate of $n_{\text{Ly}\alpha}$. Thus, we assume 1 $\mu$m graphite grains, for which $Q_{\text{abs}}(\text{Ly}\alpha) \approx 1$ and $\langle Q(1 \mu \text{m}, 230 \text{ K}) \rangle \approx 0.05$ (Draine & Lee 1984). Then,

$$n_{\text{Ly}\alpha} = 4\sigma T_d^4 \frac{(Q(1 \mu \text{m}, 230 \text{ K}))}{Q_{\text{abs}}} = 6.5 \times 10^4 \text{ cm}^{-3}.$$
We then find

\[
S = \frac{3c^3}{16\pi^2} \frac{A_{21}}{(\Delta \nu) \alpha} \frac{1}{\epsilon(w)} \left( \frac{x}{1 - x} \right) n_{s0} n_e \nonumber \\
= 2.6 \times 10^{10} \frac{\epsilon(w)}{w} \frac{x}{1 - x},
\]

where we have taken \( n_e = (1 - x)n_{H} \) (since the fraction of atoms in excited, bound states is negligible). To obtain the numerical value given above, we used the recombinination coefficient in the density-bounded case with \( T = 10^4 \) K (Osterbrock 1989). Assuming that \( \text{Ly} \alpha \) radiation is primarily removed through absorption by dust [in which case \( \epsilon(w)/w \) is replaced by \( 2(2/\pi)^{1/2} \)], then \( S < S_{\text{crit}} \) unless \( \chi \) exceeds \( 2.4 \times 10^3 \), which is unlikely (Osterbrock 1989). It should also be noted that other sources, such as continuum radiation, are likely to be important in heating the dust, thereby reducing further the implied value of \( n_{1s0} \) (and therefore \( S \)). Finally, the \( 1 \mu \text{m} \) grain size assumed here is probably an overestimate, implying that \( n_{1s0} \) is also overestimated.

4. RADIATIVE TRANSFER OF THE FINE-STRUCTURE LINES

Evidently, the \( 2s \) state is overpopulated relative to \( 2p \), and thus the fine-structure transitions will proceed from \( 2s_{1/2} \) to \( 2p_{3/2} \) via absorption and to \( 2p_{1/2} \) via stimulated emission. Although the \( 2p \) populations are probably negligible, they are included in the radiative transfer calculation. The distribution of \( 2p \) states between \( 2p_{1/2} \) and \( 2p_{3/2} \) may deviate somewhat from the statistical weights (1/3 and 2/3, respectively), in part because the separate collisional rates from \( 2s \) are not proportional to the statistical weights. Since the \( 2p \) population is most likely negligible, a detailed calculation of the distribution between \( 2p_{1/2} \) and \( 2p_{3/2} \) is not carried out here. Rather, the fractional populations of \( 2p_{1/2} \) and \( 2p_{3/2} \) are parameterized as \( \beta_{1/2} \) and \( 2 \beta_{3/2} \), respectively. If \( \beta_{1/2} = 2 \beta_{3/2} = 1 \), then these states are populated according to their statistical weights. There is the obvious constraint that \( \beta_{1/2} + 2 \beta_{3/2} = 1 \).

The fine-structure transitions are allowed electric dipole transitions, and the corresponding rates may be computed in a straightforward manner (Bethe & Salpeter 1957, p. 334), giving \( A_d = 1.597 \times 10^{-9} \text{ s}^{-1} \) \((2s_{1/2} - 2p_{1/2})\) and \( A_b = 8.78 \times 10^{-7} \text{ s}^{-1} \) \((2p_{3/2} - 2s_{1/2})\). The absorption coefficient (valid for either transition) is

\[
\kappa_{\nu} = \pm \frac{e^2}{8\pi \nu^2} \frac{g_{2s}}{g_{2p}} A_{\nu} \left( n_{2s} - \frac{\beta}{3} n_{2p} \right),
\]

where \( g \) is the degeneracy of the final state (2 for \( 2p_{1/2} \) and 4 for \( 2p_{3/2} \); \( g_{2s} = 2 \)). The minus sign corresponds to transitions to \( 2p_{1/2} \), the plus sign corresponds to transitions to \( 2p_{3/2} \), and \( \beta \) is either \( \beta_{1/2} \) or \( \beta_{3/2} \), respectively. Either final state quickly decays to the ground state via \( \text{Ly} \alpha \) with the rate \( A_{21} \). The natural line width of the \( 2s-2p \) transitions is therefore dominated by the rapid decay of the \( 2p \) state and is \( \Gamma = A_{21}/2\pi = 99.8 \text{ MHz} \). For a Lorentzian profile,

\[
A_{\nu} = \frac{A(\Gamma/2\pi)}{(\nu - \nu_T)^2 + (\Gamma/2)^2},
\]

where \( A \) is either \( A_d \) or \( A_b \) and \( \nu_T \) is the frequency of the fine-structure transition. From equations (11) and (12) we find

\[
n_{2s} - \frac{\beta}{3} n_{2p} = \frac{f \alpha n_{n_e}}{(A_{21} + C_{spni})} \left( 1 - \frac{\beta S}{S_{\text{crit}}} \right),
\]

where we have assumed that the Lyman decay rate \( A_{21} \) always dominates the collision rate \( C_{spni} n_e \). The absorption coefficient at the line center then is

\[
\kappa_{\nu}^\text{max} = \pm \frac{e^2}{8\pi \nu^2} \frac{g_{2s}}{g_{2p}} \frac{A_{\nu}}{\pi} \frac{f \alpha n_{n_e}}{(A_{21} + C_{spni})} \left( 1 - \frac{\beta S}{S_{\text{crit}}} \right).
\]

This will be scaled according to the free-free absorption coefficient (Altenhoff et al. 1960),

\[
\kappa_{\nu} = \kappa_{\nu}^\text{max} \frac{0.212 n_{n_e}}{1 + 132 T_b^{1/3}}.
\]

The ratio of the line optical depth (at line center) \( \tau_{\nu}^\text{max} \) to the free-free continuum optical depth \( \tau_{\nu,\text{ff}}^\text{max} \) then is

\[
\frac{\tau_{\nu}^\text{max}}{\tau_{\nu,\text{ff}}^\text{max}} = \frac{K}{(A_{21} + C_{spni})} \left( 1 - \frac{\beta S}{S_{\text{crit}}} \right),
\]

where \( K = -3.0 \times 10^{-4} \text{ s}^{-1} \) for the \( 2s_{1/2} \rightarrow 2p_{1/2} \) transition and \( K = 0.41 \text{ s}^{-1} \) for the \( 2s_{1/2} \rightarrow 2p_{3/2} \) transition. We assumed that \( T = 10^4 \) K and that all ionizing photons are captured by the \( \text{H} \) \( \alpha \) region (Osterbrock 1989).

The equation of transfer is

\[
\frac{di}{dx} = (-\kappa_{\nu} - \kappa_{\nu}) I_{\nu} + J_{\nu,\text{ff}}.
\]

Note that the line produces absorption or stimulated emission only (through \( \kappa_{\nu} \)), but does not produce significant spontaneous emission. Thus, the emissivity is completely dominated by the free-free emissivity \( J_{\nu,\text{ff}} \). For an \( \text{H} \) \( \alpha \) region uniform along a ray path, the solution is

\[
T_b = T \frac{1 - e^{-(1 + R)\tau_{\nu,\text{ff}}^\text{max}}}{1 + R},
\]

where the result has been expressed in terms of brightness temperature \( T_b \). The line strength (in K) divided by the continuum brightness temperature is

\[
\frac{\Delta T_b}{T_{b,\text{cont}}} = \frac{1}{1 + R} \frac{e^{\nu - e^{\nu - \kappa_{\nu}}}}{e^{\nu} - e^{\nu - \kappa_{\nu}}} - 1.
\]

In the optically thin limit (\( \tau_{\nu,\text{ff}} \ll 1 \)), expanding (to second order) gives

\[
\Delta T_b \frac{T_{b,\text{cont}}}{T_b} = -\frac{1}{2} R \tau_{\nu,\text{ff}} \approx -\frac{1}{2} \tau_{\nu,\text{ff}}^\text{max}.
\]

This result is in agreement with that obtained by Ershov (1987) for the optically thin limit. In the optically thick limit (\( \tau_{\nu,\text{ff}} \gtrsim 1 \)),

\[
\frac{\Delta T_b}{T_{b,\text{cont}}} = -\frac{1}{1 + R}.
\]
The lines do not disappear in the optically thick limit as long as the 2s and 2p states are not in local thermodynamic equilibrium (see also Ershov 1987). The \( 10^4 \) K microwave radiation field is not sufficient to overcome the rates described in § 2, and thus it cannot establish equilibrium. For example, in the likely case where \( n_{2p}/(3n_{2s}) \ll 1 \), the 2p levels are drained by Lyman emission faster than the microwave radiation field can return these states to 2s via either absorption or stimulated emission (see also § 2).

Each fine-structure level is split into two hyperfine levels, as shown in Figure 1. The allowed transitions are also indicated. Both fine-structure lines are split into three hyperfine components. The relative intensities of the components can be calculated from the appropriate sum rules (Sobelman 1992). For the 910, 1088, and 1147 MHz lines the ratios are 1:2:1. The 9852, 9876, and 10030 MHz lines appear in the ratios 1:5:2.

### 5. PROSPECTS FOR DETECTION

The 9.9 GHz transitions are intrinsically about 3 orders of magnitude stronger than the 1.1 GHz transitions. The solution to the radiative transfer equation indicates that, in general, the line brightness temperature \( \Delta T_b \) grows as the square of the free-free optical depth in the optically thin limit (eq. [28]). One factor comes from the growth of free-free emission, which effectively forms the “background,” and the other from the proportionate growth of the line absorption. In the optically thick limit, the growth in line temperature saturates (eq. [29]). Most H II regions of interest are optically thin at 9.9 GHz and optically thick at 1.1 GHz. In general, the higher optical depth at 1.1 GHz does not significantly offset the relative weakness of these transitions. We find, therefore, that the 9.9 GHz lines offer the better prospects for detection.

Table 1 gives estimates of the line strengths for various H II regions and components, and planetary nebulae. It was assumed that \( S = 0 \), in which case the 9.9 GHz lines would appear in absorption and the 1.1 GHz lines in stimulated emission. Most of the entries in Table 1 correspond to high emission measure components in H II regions, which are listed according to the nomenclature of the original references. The published emission measure values \( E \) were used to calculate the free-free optical depth and the continuum brightness temperature (assuming in most cases \( T_c \approx 10^4 \) K). The line-to-continuum optical depth ratios \( R \) are calculated from the published electron densities (eq. [24]). Notably, our predictions for both the 1.1 and 9.9 GHz lines from Orion A (M42) are in good agreement with those of Ershov (1987).

The estimates in Table 1 take into account the distribution of the line strength over three hyperfine components making up each fine-structure line and the consequent line blending. At 9.9 GHz, the strongest line (9876 MHz) will be blended with the weakest line (9853 MHz). The peak temperature occurs at 9874 MHz and is 75% of that calculated using equation (26) for a single (fictitious) fine-structure line. The 10030 MHz line will be somewhat distinct, with a peak temperature equal to 32% of that predicted from equation (26), including contributions from the wings of the 9852 and 9876 MHz lines. The situation at 1.1 GHz is similar. The 1088 MHz line will appear blended with the 1147 MHz line, with a peak value of 63% at 1093 MHz. The 910 MHz line will remain distinct, with a peak value of 30%, including line wing contributions from the other two lines. The estimates in Table 1 give the line temperature at the peak of the brightest spectral feature in either the 9.9 or 1.1 GHz blended multiplet. At both frequencies, the resulting profile of a strong blended feature and a distinct weaker line, combined with Lorentzian line shapes, will provide a unique detection signature. Figure 2 shows the predicted appearance of the blended multiplet around 9.9 GHz.

The Green Bank Telescope (GBT) provides a realistic hope of detection of the 9.9 GHz lines. Having a clear aperture, it should be relatively free of standing waves in the antenna.
Thus, strong Lorentzian wings are apparent. Assumed to be negligible (see text), and thus the 9.9 GHz lines appear in absorption. The line profiles are completely dominated by natural line width, and thus strong Lorentzian wings are apparent.

Structure. Thus, it should be possible to search for very broad spectral features over bandpasses as large as 800 MHz. Nevertheless, the high emission measure H II regions that are likely to show the lines are typically quite compact. Therefore, the estimated line antenna temperatures \( \Delta T_a \) at 1.1 GHz were calculated using either the 11 arcsec beam (subscript \( G \)) or, where appropriate, the Arecibo 4\,\mathrm{arcmin} beam (subscript \( A \)).

Quite possibly the observational sensitivity will be limited by systematic effects in which the continuum antenna temperature is modulated by frequency-dependent gain variations that are not entirely removed by calibration procedures. Thus, the limiting factor may be the line-to-continuum ratio, \( \Delta T/T \), which is also tabulated for both lines.

The estimates given in Table 1 are only approximate. The underlying observations are biased in favor of component sizes to which various interferometer arrays are sensitive. In complex H II regions, structures are present on a range of scales, down to very high emission measure arcsecond-scale components (Turner & Matthews 1984). The components included in the table (typically a few tens of arcseconds in size) were selected because they contribute significantly to the total flux in the GBT beam. The more compact, arcsecond-scale components typically yield larger values of \( \Delta T/T \) (despite densities \( >10^4 \, \text{cm}^{-3} \)) and consequent collisional de-excitation), yet they tend to contribute relatively little to the total flux in the GBT beam. Conversely, extended, low emission measure components in the beam will add to the antenna temperature while contributing little line absorption. It should also be noted that the presence of structures having a wide range of densities implies that the uniform model considered in §4 is an oversimplification. Inhomogeneous structure, including clumping, in the emission regions would imply that the emission measure estimates in Table 1 represent an average over the surface of the source. In the optically thin case, i.e., most sources at 9.9 GHz, a redistribution of emission measure, and therefore optical depth, will tend to strengthen the overall estimated line strength \( \Delta T_b \) due to its \( \tau^2 \) dependence (eq. [28]). Collisional de-excitation in denser regions of the source, however, will tend to reduce the line optical depth (eq. [24]). Because \( R \) is density dependent, particularly for densities above about \( 1.5 \times 10^4 \, \text{cm}^{-3} \), the brightness temperature for a homogeneous source (eq. [26]) cannot be rescaled using some weighted optical depth; rather, detailed modeling would be required.

Three high emission measure planetary nebulae are also included in the table (IC 418, NGC 7027, and NGC 6572). These objects tend to have somewhat simpler structure than the more complex H II regions. Nevertheless, NGC 7027 has a well-documented shell structure. In this case, a number of emission-line diagnostics indicate \( n_e \approx 5 \times 10^4 \, \text{cm}^{-3} \) (Middlemass 1990). Subarcsecond radio images show that the emission originates from a shell with a peak emission measure of about \( 2.7 \times 10^6 \, \text{pc} \, \text{cm}^{-6} \) (Bryce et al. 1997). Both results are consistent with an area-filling factor of about 30%, with a characteristic emission measure of about \( 1.2 \times 10^8 \, \text{pc} \, \text{cm}^{-6} \). The higher emission measure boosts the line-to-continuum ratio, whereas the higher density acts oppositely due to collisional de-excitation. The net result in this case is a somewhat stronger estimated line with \( \Delta T_b/T_{b,\text{cont}} \approx -1.4 \times 10^{-3} \). This case is worked out in Table 1 as \{NGC 7027\} and depicted in Figure 2.

Generally, compact, high emission measure objects tend to give stronger line-to-continuum ratios, despite their higher densities (which result in collisional de-excitation). This is because H II regions tend to be loosely organized along domains of constant excitation parameter \( U \) (Habing & Israel 1979). Using size \( d \) as a free parameter, then \( E \propto U^3/d^2 \). For \( E < 3.8 \times 10^8 \, \text{pc} \, \text{cm}^{-6} \), the H II region is optically thin at 9.9 GHz, and for \( S = 0 \)

\[
\frac{\Delta T_b}{T_{b,\text{cont}}} \approx -10^{-2} \frac{E_b}{n_4 + 1.5} ,
\]

where \( E_b = E/(10^8 \, \text{pc} \, \text{cm}^{-6}) \) and \( n_4 = n/(10^4 \, \text{cm}^{-3}) \). Thus, in the low-density limit \( (n_4 \ll 1.5) \) the line-to-continuum ratio grows in direct proportion to \( E \), and therefore to \( d^{-2} \) for fixed \( U \). Of course, the most compact objects have densities in excess of \( 1.5 \times 10^4 \, \text{cm}^{-3} \), in which case the line-to-continuum ratio increases with \( d^{-1/2} \propto E^{1/4} \) for fixed \( U \), since \( n_4 \propto (U/d)^{3/2} \). For emission measures above \( \approx 4 \times 10^8 \, \text{pc} \, \text{cm}^{-6} \), an H II region becomes optically thick at 9.9 GHz, and the advantage of increased emission measure is lost. In such cases, higher densities will reduce the line-to-continuum ratio.

As discussed above, the 1.1 GHz lines are considerably weaker. Under the most favorable conditions of high free-free optical depth and low density, we find

\[
\frac{\Delta T_b}{T_{b,\text{cont}}} \approx - R \approx 3.6 \times 10^{-5} ,
\]

assuming, as discussed above, that \( S = 0 \). These conditions are not uncommon, and exceptionally high emission measures are not required to achieve high optical depth at 1.1 GHz. In the case of M42 the optical depth at 1.1 GHz is 1.2, and the beam diluted line strength would be about 4 mK, versus a continuum antenna temperature of \( \approx 400 \, \text{K} \).

6. Conclusions

The metastable \( 2s_{1/2} \) state of hydrogen is likely overpopulated in H II regions. Ly\( \alpha \) pumping of the \( 2p \) states is expected.
to be negligible due to absorption of Lyα radiation by dust. Thus, the $2s_{1/2} \rightarrow 2p_{3/2}$ transitions (9.9 GHz) are predicted to appear in absorption, and the $2s_{1/2} \rightarrow 2p_{1/2}$ transitions (1.1 GHz) in stimulated emission. Because of the short lifetime of the final $2p$ states, the width of the lines is dominated completely by intrinsic line width. In effect, then, the power is distributed over $\approx 100$ MHz of line width, resulting in very weak lines. In addition, the power is distributed over three strongly blended hyperfine lines in each multiplet.

Searching for the 9.9 GHz lines in high emission measure H ii regions offers the best prospects for detection. In the optically thin limit, the line strength varies as the square of the free-free optical depth. Predicted line-to-continuum ratios (in absorption) range up to several tenths of a percent in W58A, including the effects of line blending. With the Green Bank Telescope, the predicted peak absorption-line strength may reach $\Delta T_a \approx -170$ mK in this case, allowing for the redistribution of line strength over the three hyperfine lines. Other high-emission H ii regions are expected to show somewhat weaker 9.9 GHz lines, for example, with line-to-continuum ratios of about 0.1% and line strengths of tens of mK with the Green Bank Telescope. These predictions are uncertain, however, owing to biases inherent in estimating emission measures as well as selection effects in various interferometric surveys of compact H ii regions.

These conclusions apply to thermal sources. An important extension of this work would consider the broad-line regions of active galactic nuclei and quasars, in which a strong nonthermal microwave radiation field could influence the populations of the $2s$ and $2p$ levels, as well as provide a background for line absorption or stimulated emission.

In general, detection of the fine-structure lines of hydrogen will be challenging due to the extraordinary line width and blended structure. The observations will require meticulous baseline calibration and subtraction.

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**REFERENCES**

Osterbrock, D. E. 1989, Astrophysics of Gaseous Nebulae and Active Galactic Nuclei (Mill Valley: University Science Books)

Pottasch, S. R. 1960, ApJ, 131, 202

———. 1987, in Late Stages of Stellar Evolution, ed. S. Kwok & S. R. Pottasch (Dordrecht: Reidel), 355

Purcell, E. M. 1952, ApJ, 116, 457

Schraml, J., & Mezger, P. G. 1969, ApJ, 156, 269

Seaton, M. J. 1955, Proc. Phys. Soc. London A, 68, 457

Shklovski, I. S. 1960, Cosmic Radio Waves (Cambridge: Harvard Univ. Press)

Sobelman, I. I. 1992, Atomic Spectra and Radiative Transitions (2nd ed.; Berlin: Springer)

Spitzer, L. 1978, Physical Processes in the Interstellar Medium (New York: Wiley)

Spitzer, L., & Greenstein, J. L. 1951, ApJ, 114, 407

Thomasson, P., & Davies, J. G. 1970, MNRAS, 150, 359

Townes, C. H. 1957, in IAU Symp. 4, Radio Astronomy, ed. H. C. Van de Hulst (Cambridge: Cambridge Univ. Press), 92

Turner, B. E., & Matthews, H. E. 1984, ApJ, 277, 164

van Gorkom, J. H., Goss, W. M., Shaver, P. A., Schwartz, U. J., & Harten, R. H. 1980, A&A, 89, 150

van Hoof, P. A. M., Van de Steene, G. C., Beintema, D. A., Martin, P. G., Pottasch, S. R., & Ferland, G. J. 2000, ApJ, 532, 384

Webster, W. J., & Altenhoff, W. J. 1970, AJ, 75, 896

Wild, J. P. 1952, ApJ, 115, 206

Wynn-Williams, C. G. 1971, MNRAS, 151, 397