Quantum speed limit for the creation and decay of quantum correlations

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We derive Margolus-Levitin and Mandelstamm-Tamm type bound on the quantum speed limit time for the creation and decay of quantum correlations by an amount in a quantum system evolving under the influence of its ambient environment. The minimum distance of a non-classical state from an appropriate set of classical states is a legitimate measure of the quantumness of the state. We consider entanglement and quantum discord measures of quantum correlations, quantified using the Bures distance-based measure. To demonstrate the impact of quantum noise on this speed limit time for quantum correlations, we estimate the quantum speed limit time for the creation and decay of quantum correlations for a two-qubit system under modified OUN dephasing and collective two-qubit decoherence channels.

I. INTRODUCTION

The investigation of quantum speed limit (QSL) time for the evolution of quantum states discusses the fundamental problem of the rate at which quantum states and their characteristics change. Since the interpretation of energy-time uncertainty principle as the limit on the time of evolution in physical process, a dedicated amount of work has been done to report the significant applications of quantum speed limit in the field of quantum information processing and associated domains. Mandelstamm, Tamm (MT) and Margolus, Levitin (ML) derived bounds on the minimum time needed for a quantum system to evolve between the orthogonal states for closed systems. Similarly, in QSL time for achieving a target fidelity for driven quantum systems has been derived. The fact that the impact of the bath on a quantum system is not always detrimental in nature has sped up the research work unraveling the characteristics of open quantum systems, and naturally the investigation of speed of evolution under non-unitary dynamics has become an active research topic. Quantum speed limit for open quantum system based on different measures has been proposed. The investigation of QSL time realizes wide range of applications in the field of quantum information processing and technology as it reflects the nature of the physical process the quantum system undergoes. Thus for example, the memory effects on the dynamics of quantum systems originating from the coupling between the system and bath results in the decay-revival mechanism of quantum correlations, which is very well captured by the quantum speed limit time for certain physical processes.

The existence of quantum correlations and coherence in a system is a valuable resource for many tasks of fundamental and practical importance. The uncontested supremacy of certain protocols in quantum regime over classical counterparts in the field of quantum metrology, quantum communication, quantum thermodynamics, etc., is achieved by manipulating and controlling quantum correlations and coherence. The interwoven connection between the coherence and quantum entanglement signifies the need of harnessing their stronger interdependence and the fundamental speed limit at which a system reveals its quantumness. Recent works in this direction reveal that speed limit is an inherent feature of system’s dynamics in the Hilbert space.

The significance of quantum correlations as a resource, and technique to generate and manipulate them seeks extensive consideration in the branch of quantum communication and technology. A number of techniques have been proposed for creating entanglement in quantum systems, which is one of the themes of the present work.

In this work, we address the question how fast quantum correlations can be processed in a system under a physical process. To this end, we derive bounds on the minimum time required for the generation and decay of quantum correlations by an amount in a quantum system for non-unitary evolution. We use distance based measures of quantum entanglement and discord to derive the QSL time valid for open quantum systems. We use the Bures metric (geometric measure) measures to estimate quantum entanglement and discord. We illustrate our ideas, for the generation and decay of quantum correlations, on two models, viz., the modified Ornstein-Uhlenbeck noise (OUN), which is dephasing, and a two-level atomic dipole-dipole interaction model. We discuss the conditions under which quantum correlations are created in an initial separable atomic system. We estimate the QSL time for the change in quantum correlations in the atomic system, and analyze how the coupling strength impacts the process.

The present work is organized as follows. In Sec. II, we discuss the preliminaries required for the current work. The derivation of the minimum time required for the generation and decay of the quantum correlations under non-unitary evolution is given in Sec. III. In Sec. IV, we consider the example of a local dephasing channel and a two two-level atomic system, and investigate the QSL...
time for the creation and decay of quantum correlations. The concluding remarks are laid down in Sec. V.

II. PRELIMINARIES

A. Distance measure of quantum correlations

1. Measures of entanglement

In the literature, different methods are used to quantify entanglement. In principle a good measure of entanglement $E(\rho)$ has to satisfy the following conditions\textsuperscript{23}: i) $E(\rho) = 0$ for separable state $\rho$, ii) Local operation leaves the entanglement invariant, i.e., $E(\rho) = E(U_A \otimes U_B \rho U_A^\dagger \otimes U_B^\dagger)$, iii) Local operation and classical communication (LOCC) cannot increase the entanglement, i.e., $E(\text{LOCC} \rho) \leq E(\rho)$. Let us consider a set $\mathcal{S}$ of all quantum states, is divided into two subsets of entangled $\mathcal{S}_E$ states and product (separable) $\mathcal{S}_P$ states, respectively. Along with a set of classical states $\mathcal{S}_C$ it shows $\mathcal{S}_C \subseteq \mathcal{S}_P \subseteq \mathcal{S}_E \subseteq \mathcal{S}$. The distance measure of entanglement of quantum state $\rho$ is expressed as

$$E(\rho) = \min_{\sigma \in \mathcal{S}_P} \chi(\rho|\sigma),$$  

where $\chi$ can be any measure of distance between the density matrices. The Bures metric $\chi_B$ as a measure of entanglement,

$$E^B(\rho) = \chi_B(\rho|\sigma) = \frac{1}{2}d_B^2(\rho, \sigma) = \max_{\sigma \in \mathcal{S}_P}(1 - \sqrt{F_P(\rho)}),$$

where $F_P(\rho)$ is the maximum fidelity between the entangled state $\rho$ and a product state in $\mathcal{S}_P$. To calculate $E^B(\rho)$, instead of the minimization in Eq.\textsuperscript{1} we maximize $F_P(\rho)$ (Eq.\textsuperscript{2}), fidelity between $\rho$ and a separable state in $\mathcal{S}_P$. The fidelity is the squared overlap of quantum states $F(\rho, \sigma) = \left| \text{Tr}[\sqrt{\sigma \rho \sqrt{\sigma}}] \right|^2$.

2. Quantum Discord

Consider a bipartite quantum system $AB$ with Hilbert space $H = H_A \otimes H_B$, $N_A$ and $N_B$ arbitrary dimensions of subsystems $A$ and $B$ with spaces $H_A$ and $H_B$. The reduced subsystems $\rho_{A,B} = \text{Tr}_{B,A}(\rho)$. The total correlations of the composite system is given by the mutual information $I_{A:B}(\rho) = S(\rho_A) + S(\rho_B) - S(\rho)$, where $S(\rho) = -\sum_i \lambda_i \log \lambda_i$ is the von Neumann entropy. A classically equivalent definition of mutual information is defined $C_A(\rho) = S(\rho_B) - \min_k \sum \lambda_k' S(\rho_{B:k})$, where $\rho_{B:k} = \text{Tr}_A(E_k \otimes I_B \rho)/\text{Tr}(E_k \otimes I_B \rho)$ is the state of $B$ conditioned on outcome $k$ in $A$. $\{E_k\}$ describes the set of POVM elements. Quantum discord is defined as the difference between the two measures of information

$$D_A(\rho) = I(\rho) - C_A(\rho).$$

The quantum discord is zero for classical states ($D_A = D_B = 0$), otherwise is always non-negative and $D_A \neq D_B$. The quantum discord given in Eq.\textsuperscript{3} requires extensive numerical minimization. In Eq.\textsuperscript{4} a geometrical measure of quantum discord $D_A(\rho) = \min_{\sigma \in \mathcal{S}_C} ||\rho - \sigma||^2$ has been proposed, where $\mathcal{S}_C$ is a set of classical states.

$$D_A(\rho) = \min_{\sigma \in \mathcal{S}_C} ||\rho - \sigma||^2, \quad (4)$$

$$||A - B||^2 = \text{tr}(A - B)^2.$$ The geometrical discord was used to provide, among others, an operational meaning to quantum teleportation\textsuperscript{25}. The Bures metric for quantum discord\textsuperscript{26} is

$$D^B_A(\rho) = \chi_B(\rho|\sigma) = \frac{1}{2}d_B^2(\rho, \sigma) = \max_{\sigma \in \mathcal{S}_C}(1 - \sqrt{F_C(\rho)}), \quad (5)$$

where $F_C(\rho)$ is the maximum fidelity between $\rho$ and a classical state in $\mathcal{S}_C$.

III. MINIMUM TIME FOR THE CHANGE OF QUANTUM CORRELATIONS IN SYSTEM

Here, we derive the minimum time required to witness a change in quantum correlations (QC) by an amount $\mathcal{Q}(\rho) = |Q_C(\rho_0) - Q_C(\rho_1)|$ for a system evolving from initial state to final state under non-unitary dynamics. To estimate the QSL time for the change of QC, which we call $\tau_Q$, the distance between the initial and final states under evolution is calculated as a measure to quantify the change in quantumness of the state. In this work, we mainly consider entanglement and quantum discord as signatures of quantumness of the states. Bures distance based measures are made use of to estimate the quantum correlations.

A. Bures metric as a measure of quantum correlations

We define the quantum speed limit time for the creation or decay of entanglement by an amount $E^B(\rho)$ in a system for a non-unitary evolution of the form

$$\dot{\rho}_t = L_t \rho_t, \quad (6)$$

where, $L_t$, the generator of the dynamics. To begin with we rewrite Eq.\textsuperscript{2}

$$E^B(\rho) = \frac{1}{2}d_B^2(\rho, \sigma) = \max_{\sigma \in \mathcal{S}_P}(1 - \sqrt{F_P(\rho)}), \quad (7)$$

where,

$$F_P(\rho) = F(\rho, \sigma) = \left| \langle \psi_\rho | \phi_\sigma \rangle \right|^2, \quad (8)$$

is the Uhlmann fidelity\textsuperscript{27}. Here $|\psi_\rho\rangle$ is the purification of the state $\rho$ and the maximization is done over all purification $|\phi_\sigma\rangle$ of separable state $\sigma$ in the extended Hilbert
space $H_A \otimes H_B$. The states $\rho$ and $\sigma$ are the reduction of these pure states over the ancillary Hilbert space $H_B$: $\rho = \text{Tr}_B |\psi_\rho \rangle \langle \psi_\rho |$ and $\sigma = \text{Tr}_B |\phi_\sigma \rangle \langle \phi_\sigma |$.

We define the variation in entanglement observed on the system's evolution between initial state $\rho_0$ and final state $\rho_t$ as

$$E^B(\rho) = |E^B(\rho_0) - E^B(\rho_t)|,$$

equivalently

$$E^B(\rho) = \begin{cases} E^B(\rho_0) - E^B(\rho_t), & \text{if } E^B(\rho_0) \geq E^B(\rho_t) \\ E^B(\rho_t) - E^B(\rho_0), & \text{otherwise}. \end{cases}$$

(10)

Hereafter, $E^B(\rho)$, as in Eq. $10$, is called the change in entanglement occurred due to the decay or creation of entanglement in the system, unless otherwise mentioned. By substituting Eq. $7$ in the latter equation we get

$$E^B(\rho) = \begin{cases} -\sqrt{F^\rho_0} + \sqrt{F^\rho_t}, & \text{if } E^B(\rho_0) \geq E^B(\rho_t) \\ \sqrt{F^\rho_0} - \sqrt{F^\rho_t}, & \text{otherwise}. \end{cases}$$

(11)

We take the temporal rate of change of entanglement $E^B(\rho)$

$$\frac{d}{dt} E^B(\rho) \leq \frac{|dt|}{E^B(\rho)} = \frac{|F^\rho_t|}{2\sqrt{F^\rho_t}} \Rightarrow \frac{dE^B(\rho)}{dt} \leq \frac{|\langle \phi_\sigma | \rho_\psi(t) | \phi_\sigma \rangle + \langle \psi_\rho | \sigma_\phi(t) | \psi_\rho \rangle|}{2(1 - E^B(\rho_0))},$$

(12)

where $\rho_\psi(t) = |\psi_\psi(t) \rangle \langle \psi_\psi(t)|$, $\sigma_\phi(t) = |\phi_\sigma(t) \rangle \langle \phi_\sigma(t)|$ are the purification of the states $\rho$ and $\sigma$, respectively. By substituting $E^B(\rho_t)$ from Eq. $10$ we write

$$2(1 - (E^B(\rho_0) \mp E^B(\rho)) \frac{dE^B(\rho)}{dt} \leq |\langle \phi_\sigma | \rho_\psi(t) | \phi_\sigma \rangle + \langle \psi_\rho | \sigma_\phi(t) | \psi_\rho \rangle| = |\langle \phi_\sigma | L_t \rho_\psi(t) | \phi_\sigma \rangle + \langle \psi_\rho | L_t \sigma_\phi(t) | \psi_\rho \rangle|, \quad \quad \quad (13)$$

for the non-unitary evolution, where $\mp$ sign indicates the decay and creation of entanglement, respectively. We begin by providing a derivation for Margolus-Levitin bound. With $\langle \phi_\sigma | L_t \rho_\psi(t) | \phi_\sigma \rangle = \text{tr} \{ |\phi_\sigma \rangle \langle \phi_\sigma | L_t \rho_\psi(t) \}$, and $\langle \psi_\rho | L_t \sigma_\phi(t) | \psi_\rho \rangle = \text{tr} \{ |\psi_\rho \rangle \langle \psi_\rho | L_t \sigma_\phi(t) \}$, Eq. $13$ is rewritten as

$$2(1 - (E^B(\rho_0) \mp E^B(\rho)) \frac{dE^B(\rho)}{dt} \leq |\text{tr} \{ \sigma_\phi(t) L_t \rho_\psi(t) \} + \text{tr} \{ \rho_\psi(t) L_t \sigma_\phi(t) \}|.$$ 

The von-Neumann trace inequality for operators states that

$$\text{tr}[B_1 B_2] \leq \sum_{i=1}^n \mu_{1,i} \mu_{2,i},$$

where $\mu_{1,i}$ and $\mu_{2,i}$ are the singular values in the descending order for any complex matrices $B_1$ and $B_2$, respectively. For a Hermitian operator the absolute value of the eigenvalue gives the singular value. By combining Eqs. $14$ and $15$ and using triangle inequality for absolute values ($|A + B| \leq |A| + |B|$) we get

$$2(1 - (E^B(\rho_0) \mp E^B(\rho)) \frac{dE^B(\rho)}{dt} \leq \sum_{i} \mu_{1,i} q_{1,i} + \sum_{i} \mu_{2,i} q_{2,i} = \mu_{1,1} + \mu_{2,1},$$

(16)

where $\mu_{1,1}$, $\mu_{2,1}$ are the singular values of $L_t \rho_\psi(t)$ and $L_t \sigma_\phi(t)$, respectively. We have $q_{1,1} = q_{2,1} = 1$ for pure state $\sigma_\phi(t)$ and $\rho_\psi(t)$. The largest singular value of a Hermitian operator gives the operator norm $|L_t(t)|_{\text{op}} = \mu_{1,1}$ and $|L_t(t)|_{\text{op}} = \mu_{2,1}$. Equation $16$ can be rewritten using von-Neumann inequality condition as

$$2(1 - (E^B(\rho_0) \mp E^B(\rho)) \frac{dE^B(\rho)}{dt} \leq \text{tr} \{ L_t \rho_\psi(t) | \phi_\sigma \rangle \langle \phi_\sigma | \} + \text{tr} \{ L_t \sigma_\phi(t) | \psi_\rho \rangle \langle \psi_\rho | \},$$

(17)

and integrating over time we get

$$\tau \geq \tau_{QC} = \max \left\{ \frac{1}{K_{\text{op}}}, \frac{1}{K_{\text{tr}}} \right\} 2E^B(\rho) \left( 1 - \frac{2E^B(\rho_0) \mp E^B(\rho)}{2} \right),$$

(18)

where we write $K_{\text{op, tr}} = 1/\tau \int_0^\tau dt \{ |L_t \rho_\psi(t)|_{\text{op, tr}} + |L_t \sigma_\phi(t)|_{\text{op, tr}} \}$.

To derive Mandelstam-Tamm bound we rewrite Eq. $13$ as

$$2(1 - (E^B(\rho_0) \mp E^B(\rho)) \frac{dE^B(\rho)}{dt} \leq \text{tr} \{ L_t \rho_\psi(t) | \phi_\sigma \rangle \langle \phi_\sigma | \} + \text{tr} \{ L_t \sigma_\phi(t) | \psi_\rho \rangle \langle \psi_\rho | \},$$

(19)

Using Cauchy-Schwarz inequality for operators, above equation takes the form

$$2(1 - (E^B(\rho_0) \mp E^B(\rho)) \frac{dE^B(\rho)}{dt} \leq \sqrt{\text{tr} \left[ L_t \rho_\psi(t) L_t \rho_\psi(t)^\dagger \right] \text{tr} \left[ |\phi_\sigma \rangle \langle \phi_\sigma | \right]^2} + \sqrt{\text{tr} \left[ L_t \sigma_\phi(t) L_t \sigma_\phi(t)^\dagger \right] \text{tr} \left[ |\psi_\rho \rangle \langle \psi_\rho | \right]^2},$$

with $\text{tr} \{ |\phi_\sigma \rangle \langle \phi_\sigma | \} = 1 = \text{tr} \{ |\psi_\rho \rangle \langle \psi_\rho | \}$ for pure states $|\phi_\sigma \rangle$ and $|\psi_\rho \rangle$, and it gives,

$$2(1 - (E^B(\rho_0) \mp E^B(\rho)) \frac{dE^B(\rho)}{dt} \leq \sqrt{\text{tr} \left[ L_t \rho_\psi(t) L_t \rho_\psi(t)^\dagger \right]} + \sqrt{\text{tr} \left[ L_t \sigma_\phi(t) L_t \sigma_\phi(t)^\dagger \right]} = |L_t \rho_\psi|_{\text{hs}} + |L_t \sigma_\phi|_{\text{hs}},$$

(20)

where $|B|_{\text{hs}} = \sqrt{\text{tr} \{ B B^\dagger \}} = \sqrt{\sum_i \mu_i^2}$ is the Hilbert-Schmidt norm. Integrating the latter over time gives MT type bound for achieving a target quantum correlation under non-unitary dynamics.
\[ \tau \geq \tau_{QC} = \frac{1}{K_{\text{hs}}} 2E^B(\rho) \left( 1 - \frac{2E^B(\rho_0) + E^B(\rho)}{2} \right), \tag{21} \]

where \( K_{\text{hs}} = 1/\tau \int_0^\tau dt(||L_t\rho_\psi||_{\text{hs}} + ||L_t\rho_\phi||_{\text{hs}}) \).

Combining Eqs. \(18\) and \(21\) we obtain,

\[ \tau_{QC} = \max \left\{ \frac{1}{K_{\text{op}}}, \frac{1}{K_{\text{tr}}}, \frac{1}{K_{\text{hs}}} \right\} 2E^B(\rho) \left( 1 - \frac{2E^B(\rho_0) + E^B(\rho)}{2} \right), \tag{22} \]

where we have \( K_{\text{op, tr, hs}} = 1/\tau \int_0^\tau dt(||L_t\rho_\psi||_{\text{op, tr, hs}} + ||L_t\rho_\phi||_{\text{op, tr, hs}}) \). Equation \(22\) provides a unified MT-ML bound on the minimum time required to observe a change in quantum entanglement by an amount \( E^B(\rho) \) in an open quantum system for its evolution from an initial pure state to a final state.

Similarly, the minimum time for the change of quantum discord by an amount \( D_A^B(\rho) \) in a system under non-unitary dynamics is given by

\[ \tau_{QC} = \max \left\{ \frac{1}{K_{\text{op}}}, \frac{1}{K_{\text{tr}}}, \frac{1}{K_{\text{hs}}} \right\} 2D_A^B(\rho) \left( 1 - \frac{2D_A^B(\rho_0) + D_A^B(\rho)}{2} \right), \tag{23} \]

where \( D_A^B(\rho) = |D_A^B(\rho_0) - D_A^B(\rho_1)| \), is calculated using Eq. \(5\) and \(K_{\text{op, tr, hs}}\) takes the form as that in Eq. \(22\).

IV. MODELS

To demonstrate the derived speed limit time bound on the decay and creation of quantum correlations, we consider two models of decoherence channels. First channel is the modified Ornstein–Uhlenbeck noise (OUN), which is purely a dephasing channel, and the second one is a collective two-qubit decoherence model.

A. Modified Ornstein–Uhlenbeck noise (OUN)

The dynamics of a quantum system under dephasing, a unitary process \(23\) is given by the master equation,

\[ \rho_t = \gamma(t)(s_x \rho s_x - \rho_t). \tag{24} \]

The decoherence function of OUN is

\[ p_t = e^{-\frac{\kappa}{\tau_\text{c}}(t+\frac{\lambda}{4}(e^{-\lambda t} - 1))}, \tag{25} \]

where, \(\lambda^{-1} \approx \tau_c\) defines reservoir’s finite correlation time and \(\kappa\) is the coupling strength related to qubit’s relaxation time. The decoherence rate \(\gamma(t) = -\frac{\kappa}{\tau_\text{c}}\) is calculated as

\[ \gamma(t) = \frac{\kappa(1 - e^{-\lambda t})}{4}. \tag{26} \]

It is noteworthy that calculating Bures distance gives the quantification of quantum correlations and requires the knowledge of the nearest classical states. Identifying the closest classical states makes estimating the Bures measure for quantum correlation a tricky task. Here, we use the analytical expressions of Bures distance for entanglement and discord to compute QSL time for QC. For two-qubit states, the fidelity of separability as a function of concurrence as a measure of entanglement \(32\) is given as

\[ F_p(\rho) = \max_{\sigma \in \mathcal{S}_p} F(\rho, \sigma) = \frac{1}{2}(1 + \sqrt{1 - C(\rho)^2}), \tag{27} \]

where, \(C(\rho) = \max\{0, \kappa_1 - \kappa_2 - \kappa_3 - \kappa_4\} \), \(\kappa_i^*\) are the eigenvalues of the matrix \(\sqrt{\rho} \rho \sqrt{\rho}^*\) in the descending order, with \(\rho = \sigma_y \otimes \sigma_y \rho \sigma_y \otimes \sigma_y\), \(\rho^*\) is the complex conjugate of the density matrix \(\rho\). Substituting \(F_p(\rho)\) in Eq. \(2\) we get Bures measure of entanglement

\[ E^B(\rho) = (1 - \sqrt{1 + \frac{\sqrt{1 - C(\rho)^2}}{2}}). \tag{28} \]

For maximally entangled Bell states \(E^B(\rho) = 1 - 1/\sqrt{2}\).

Similarly, we calculate QSL time for the change of quantum discord in terms of Bures distance (Eq. \(5\)). For a pure state \((\rho = |\psi\rangle \langle \psi|)\), it can be shown that \(D_A^B(\rho) = D_B^A(\rho) = E^B(\rho) = 2(1 - \sqrt{\mu_{\text{max}}})\), where \(\mu_{\text{max}}\) is the highest eigenvalue of the state \(\rho\). The equality between the Bures measure of discord and entanglement occurs from the fact that the closest product state to a pure entangled state is a pure separable state. For a two-qubit Bell diagonal state \(\rho_B = \sum_i p_i |B_i\rangle \langle B_i|\), \(|\{B_i\}\rangle\) is the maximally entangled basis set), the measure of Bures discord \(35\) is given by

FIG. 1. Quantum speed limit time in terms of Bures distance for the decay of quantum correlations by an amount \((E^B(\rho), D_A^B(\rho))\) for a maximally entangled Bell state \(|\psi^+\rangle\) evolved under OUN channel. The coupling parameter is \(\lambda = 0.1\kappa\). The actual driving time \(\tau = 1\).
\[ D_A^B(\rho) = 2\left(1 - \sqrt{F_C(\rho)}\right) = 2\left(1 - \sqrt{\frac{1 + b_{\text{max}}}{2}}\right), \quad (29) \]

where,

\[ b_{\text{max}} = \frac{1}{2} \max\left(\sqrt{(1 + c_1)^2 - (c_2 - c_3)^2} + \sqrt{(1 - c_1)^2 - (c_2 + c_4)^2}, \quad \right. \]

\[ \sqrt{(1 + c_2)^2 - (c_1 - c_3)^2} + \sqrt{(1 - c_2)^2 - (c_1 + c_4)^2}, \]

\[ \sqrt{(1 + c_3)^2 - (c_1 - c_2)^2} + \sqrt{(1 - c_3)^2 - (c_1 + c_2)^2}, \]

and \( c_i = \text{tr}(\rho_j \otimes \sigma_i) \) for \( i = 1, 3 \). For a maximally entangled Bell state \( D_A^B(\rho) = 1 - 1/\sqrt{2} \).

For a maximally entangled Bell state \( |\psi^+\rangle \) under local OUN noise, using Eqs. 27 and 29 we calculate \( F_P(\rho) = F_C(\rho) = 1/2(1 + \sqrt{1 - \exp(-2\kappa(t + 1/\lambda e^{-\gamma t}))}) \), which implies the same expressions for Bures measures of entanglement and discord. This ensures that the closest separable and classical states to maximize the overlap between the correlated state are equivalent. For the maximally entangled state considered, separable state \( \sigma = \frac{1}{2}(|01\rangle\langle 01| + |10\rangle\langle 10|) \) gives the maximum fidelity. The time dependent state evolved under OUN dephasing noise maintains its Bell diagonal form, and its overlap with the same separable state \( \sigma \) provides the maximum fidelity.

In Fig. 1 ML-type bound for the speed limit time \( \tau_{\text{RE}} \) for the decay of quantum entanglement and discord for maximally entangled Bell state \( (|\psi^+\rangle) \) is the case of OUN noise is given as a function of coupling strength. We see that even though the speed limit time for the decay of quantum correlations increases in small time intervals, overall it decreases as the coupling strength increases.

### B. Collective two-qubit decoherence model

To discuss the quantum speed limit for both the creation and decay of quantum correlations, we consider a system of two two-level atoms, which have ground and excited states \(|g_i\rangle, |e_i\rangle \) (\( i = 1, 2 \)), connected to a vacuum bath by dipole transition moments \( \vec{d}_i \). Two atoms coupled to all modes of the EM field in vacuum, are located at the positions \( r_1^i \) and \( r_2^i \), respectively. The master equation for the time evolution of the atomic system coupled through the vacuum field is \[ 38 \]

\[ \frac{\partial \rho}{\partial t} = -i \sum_{i=1}^{2} \omega_i [S_i^+, \rho] - i \sum_{i \neq j} M_{i,j} [S_i^+ S_j^- + S_j^+ S_i^- - 2S_i^- S_j^+, \rho] \]

\[ - \frac{1}{2} \sum_{i,j=1}^{2} \Lambda_{i,j} (\rho S_i^+ S_j^+ + S_i^+ S_j^- \rho - 2S_i^- \rho S_j^+), \quad (30) \]

where \( S_i^+ \) (\( S_i^- \)) are the dipole raising (lowering) operators and \( S_i^0 \) the energy operator of the \( i \)th atom. \( \Lambda_{i,j} \) are the spontaneous emission rates of the atoms. \( \Lambda_{i,j} \) are the collective spontaneous emission rates arising from the coupling between the atoms through the vacuum bath and \( M_{ij} \) (\( i \neq j \)) (dipole-dipole) represent the inter-atomic coupling. The collective damping rate \( \Lambda_{i,j} \) and dipole-dipole interaction potential \( M_{ij} \) are defined

\[ \Lambda_{i,j} = \Lambda_{j,i} = \frac{3}{2} \sqrt{\Lambda_{i,j} \Lambda_{j,i}} \left\{ \left[ 1 - \left( \vec{d}_i \vec{d}_j \right)^2 \right] \sin(\mu_0 r_{ij}) \mu_0 r_{ij} \right\} \]

\[ + \left[ 1 - 3(\vec{d}_i \vec{d}_j)^2 \right] \left[ \cos(\mu_0 r_{ij}) - \sin(\mu_0 r_{ij}) \right] \left( \frac{\mu_0 r_{ij}}{\mu_0 r_{ij}} \right) \right\}, \quad (31) \]

\[ M_{ij} = \frac{3}{4} \sqrt{\Lambda_{i,j} \Lambda_{j,i}} \left\{ \left[ 1 - (\vec{d}_i \vec{d}_j)^2 \right] \cos(\mu_0 r_{ij}) \mu_0 r_{ij} \right\} \]

\[ + \left[ 1 - 3(\vec{d}_i \vec{d}_j)^2 \right] \left[ \sin(\mu_0 r_{ij}) - \cos(\mu_0 r_{ij}) \right] \left( \frac{\mu_0 r_{ij}}{\mu_0 r_{ij}} \right) \right\}, \quad (32) \]

respectively. For identical atoms \( \Lambda_i = \Lambda_j = \Lambda = \frac{\omega_0^2}{4 \gamma \text{coh}} \). We have \( \mu_0 = \omega_0 / e, r_{ij} = |\vec{r}_i - \vec{r}_j| \) is the inter-atomic distance, \( \vec{r}_{ij} \) is the unit vector along the interatomic axis and \( \vec{d} \) the unit vector along the atomic transition dipole moments. \( \mu_0 r_{ij} \) set up a length scale into the problem and allows for a discuss of the dynamics in two regimes, a) collective \( \mu_0 r_{ij} < 1 \), b) independent \( \mu_0 r_{ij} \geq 1 \), decoherence regimes 37. The collective decoherence regime of this model is used next to investigate the speed limit of the creation and decay of the entanglement by an amount \( E^B(\rho) \) in the two two-level atomic system.

We investigate the dynamics of initially separable \( (|g_1e_2\rangle) \) and entangled Bell \( (|\psi^+\rangle) \) states. We show that \( \tau_{\text{QC}} \) decreases as the strength of inter-atomic coupling increases. Entanglement is generated in the due course of evolution of the collective atomic system for an initial separable state \( |g_1e_2\rangle \). To estimate the Bures measure of generated entanglement, the fidelity of entangled state in terms of concurrence (Eq. 27) with the closest separable state is estimated as \( F_P(\rho) = \frac{1}{2} (1 + \sqrt{1 - e^{-2\lambda t} \sin(2\Lambda \mu_0 r_{ij}) + i \sinh(\Lambda \mu_0 r_{ij})^2}) \). It’s found that the mixed separable state of the form \( \sigma = x|00\rangle\langle 00| + (1 - x)|11\rangle\langle 11| \) provides the maximum fidelity for \( x = \frac{e^{-\lambda t} F_P(\rho)}{e^{-\lambda t} F_P(\rho) + e^{-\lambda t} F_P(\rho)} \). Similarly for the dynamics of initially maximally entangled Bell state \( (|\psi^+\rangle) \), we calculate the facility of separability as \( F_P(\rho) = \frac{1}{2} (1 + \sqrt{1 - e^{-2\lambda t} \sinh(\Lambda \mu_0 r_{ij})^2}) \). In this case, closest separable state takes the form \( \sigma = x|01\rangle\langle 01| + (1 - x)|11\rangle\langle 11| \) with \( x = 4 e^{\lambda t} F_P(\rho) \). In Fig. 2 using these time dependent separable states we depict the speed limit time for the creation (\( \tau_{\text{creation}} \)) of quantum entanglement for an initial separable state and the decay (\( \tau_{\text{decay}} \)) of the initial entanglement by an amount under collective decoherence. From Fig. 2 for decay of quantum correlations, speed limit time decreases while for the creation of entanglement it is evident that QSL time rises and falls.
even though the overall envelope decreases. The QSL time for the creation and decay of quantum discord could also be estimated. The explicit analytical expression is available only for specific class of states, for instance Bell diagonal state as mentioned in the previous case. The dynamics of collective atom model does not preserve the Bell diagonal structure of states, which prevents the use of that expression for the calculation of speed limit of the dynamics of quantum discord. In general, identifying the closest classical state to estimate the Bures measure of discord for a given state $\rho$ is cumbersome. We hope to come back to this in the near future.

V. CONCLUSIONS

We derived the ML-MT type bound on the speed limit time for the creation and decay of quantum correlations by an amount in a quantum system under the influence of the surrounding environment. We used Bures distance measure to quantify entanglement and quantum discord. The minimum time required to make a change in these correlations, under the evolution, was estimated. QSL time for the change in quantum correlations in terms of operator norm gives a tighter bound for the generation and decay of entanglement and quantum discord. As illustrations of the theoretical developments, we investigated the speed limit time for the dynamics of QC for specific models. In particular, we considered the OUN dephasing channel and a dissipative two-atomic system. We showed that QSL time for quantum correlations exhibits a complex behavior as the strength of the coupling increases.

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