Effect of Pions in Cosmic Rays

P. Castelo Ferreira and J. Dias de Deus

CENTRA, Instituto Superior Técnico, Av. Rovisco Pais, 1049-001 Lisboa, Portugal

Abstract. The effects of pions for vacuum polarization in background magnetic fields are considered. The effects of quark condensates is also briefly addressed. Although these effects are out of the measurement accuracy of laboratory experiments they may be relevant for gamma-ray burst propagation. In particular, for emissions from the center of the galaxy, we show that the mixing between the neutral pion and photons results in a deviation of the gamma-ray spectrum from the standard power-law in the TeV range.

Keywords: non-linear optics, QED vacuum effects, Euler-Heisenberg, quarks

PACS: 12.20.Ds, 14.40.-n, 12.38.Aw, 14.65.Bt

DIAGRAMS

When background magnetic fields are present, traveling radiation interacts perturbatively with those fields. The main two processes we are dealing with in this presentation are charged fermion/boson virtual loops [1] and neutral scalar/pseudo-scalar exchange [2]. The respective diagrams and allowed intermediate particles are presented in table 1.

TABLE 1. Diagrams and respective allowed processes for photon interaction with background electromagnetic fields.

Virtual Loops

- Electron-Positron Loops ($e\bar{e}$)
- Muon-Antimuon Loops ($\mu\bar{\mu}$)
- Scalar Mesons Loops ($\pi^+\pi^-$) (plus quark condensates $\langle q\bar{q}\rangle$)

Scalar/Pseudo-scalar Exchange

- Neutral Pion (pseudo-scalar: $\phi_\pi$)
- Axion (pseudo-scalar: $\phi_a$)
- Quark Condensates (scalar: $\phi_c$)

The processes for virtual loops are described by the Euler-Heisenberg Lagrangians, while the processes for particle exchange are described by the interaction Lagrangians coupling the scalar/pseudo-scalars to the gauge connection. We address these in the next sections following [3].
EULER-HEISENBERG LAGRANGIANS

For electron-positron virtual loops \(e\bar{e}\) we have the Euler-Heisenberg Lagrangian [1]

\[
\mathcal{L}^{(2)}_{e\bar{e}} = \xi_e \left[ 4 \left( F_{\mu\nu} F^{\mu\nu} \right)^2 + 7 \left( \varepsilon_{\mu\nu\delta\rho} F_{\mu\nu} F_{\delta\rho} \right)^2 \right],
\]

\[
\xi_e = \frac{2\alpha}{45(B_c e c^2)^2} = 1.32 \times 10^{-24} \text{T}^{-2}, \quad B_c = \frac{m_e^2 c^2}{e \hbar}.
\]

Where as usual \(\alpha = 1/137\) is the fine-structure constant, \(e\) and \(m_e = 0.5 MeV\) the charge and mass of the electron, \(c\) the speed of light and \(\hbar\) the Planck constant. The remaining contributions considered in this work will be given relatively to this one. For muon virtual loops \(\mu\bar{\mu}\) we have the Lagrangian

\[
\mathcal{L}^{(2)}_{\mu\bar{\mu}} = \xi_\mu \left[ 4 \left( F_{\mu\nu} F^{\mu\nu} \right)^2 + 7 \left( \varepsilon_{\mu\nu\delta\rho} F_{\mu\nu} F_{\delta\rho} \right)^2 \right],
\]

\[
\xi_\mu = \Delta_\mu \xi_e, \quad \Delta_\mu = \xi_\mu \xi_e = \left( \frac{m_\mu}{m_e} \right)^4 = 5.43 \times 10^{-10},
\]

with \(m_\mu = 105 MeV\) the muon mass. For charged pion loops \(\pi^+\pi^-\) we have the Lagrangian

\[
\mathcal{L}^{(2)}_{\pi^+\pi^-} = \xi_\pi \left[ 7 \left( F_{\mu\nu} F^{\mu\nu} \right)^2 + 4 \left( \varepsilon_{\mu\nu\delta\rho} F_{\mu\nu} F_{\delta\rho} \right)^2 \right],
\]

\[
\xi_\pi = \Delta_\pi \xi_e, \quad \Delta_\pi = \xi_\pi \xi_e = \frac{1}{2} \left( \frac{m_e f_\pi}{m_\pi^2} \right)^4 = 2.29 \times 10^{-11},
\]

where the pion constant is \(f_\pi = 93 MeV\) and we take the pion mass to be \(m_\pi = 135 MeV\). There is one further contribution one can consider. The full distribution for the pion loops is given by the integral

\[
\Pi_{(q\bar{q})} = \int_0^\infty ds I_{(q\bar{q})}, \quad I_{(q\bar{q})} = -\frac{\alpha B}{12 f_\pi^2} \frac{1}{s^2} \left[ \alpha B \cot(\alpha Bs) - \frac{1}{s} \right],
\]

represented in figure 1. The sum over the poles gives the pion loop contribution, while the the region between the first poles at \(s = 0\) and \(s = \pi/\alpha B\) gives the quark condensate contribution

\[
\xi_c = \Delta_c \xi_e, \quad \Delta_c = \frac{\xi_c}{8 \xi_e} = \frac{15 m_e^4}{128 f_\pi^2} \ln \left( \frac{\Lambda^2}{m_\pi^2} \right) \approx 1.69 \times 10^{-10},
\]

where we have taken the QCD cut-off to be \(\Lambda \approx 300 MeV\). This result was originally computed within ChPT framework in [4] and within NLJ framework in [5]. In addition we note that the quark condensates may only exist when very high densities of energy are present \(\langle E \rangle \sim 300 MeV/fm^3\) [4]. These values are only accessible in very dense plasmas, for example in neutron stars [6] or near the center of galaxy [7].
FIGURE 1. (a) The integrand (4). The sum over the poles at \( s = (n - 1)\pi /\alpha B \) \( n > 0 \) gives the pion vacuum polarization; (b) The same integrand for energies between \( m_\pi = 135 MeV \) and \( \Lambda = 300 MeV \) as marked in (a) giving the quark condensate vacuum polarization contribution.

SCALAR/PSEUDO-SCALAR EXCHANGE

The Lagrangian for the pseudo-scalar neutral pion \( \pi^0 \) exchange is given by the Adler-Bell-Jackiw anomaly \([2, 8]\)

\[
\mathcal{L}^{(2)}_{\pi^0} = \frac{1}{4} g_{\pi\gamma\gamma} \phi_{\pi^0} \epsilon^{\mu\nu\lambda\rho} F_{\mu\nu} F_{\lambda\rho} \, , \quad g_{\pi\gamma\gamma} = \frac{\alpha}{\pi f_\pi} = 2.49 \times 10^{-2} GeV^{-1} .
\]  

(6)

We note that for the theoretical suggested axion the same Lagrangian applies with the respective appropriate coupling constant \( g_{a\gamma\gamma} \) \([2]\). In addition we can take the quark condensate discussed in the previous section as an effective scalar particle \( \phi_c \) described by the following Lagrangian

\[
\mathcal{L}^{(2)}_c = \frac{1}{4} g_{c\gamma\gamma} \phi_c F_{\mu\nu} F^{\mu\nu} \, , \quad g_{c\gamma\gamma} = \Delta c \xi_e \approx 1.69 \times 10^{-10} .
\]  

(7)

The other neutral meson contributions are lower by several orders of magnitude, although their coupling constants to photons are of the same order of magnitude of the one for \( \pi^0 \), their masses are higher \([9]\).

VACUUM BIREFRINGENCE

For radiation traveling in a background magnetic field \( B_0 \), due to the radiative corrections discussed in the previous sections, it is induced a birefringent vacuum dispersion relation \([10]\)

\[
\omega_{\perp,\parallel} = k (1 - \lambda_{\perp,\parallel} B_0^2) ,
\]

\[
\lambda_{\perp} = 8 \left( 1 + \Delta \xi_\mu + \Delta \xi_c + \frac{7}{4} \Delta \xi_{\pi^0} \right) \xi_e B_0^2 ,
\]

(8)

\[
\lambda_{\parallel} = 14 \left( 1 + \Delta \xi_\mu + \Delta \xi_{\pi^0} + \frac{4}{7} \Delta \xi_{\pi^+} \right) \xi_e B_0^2 ,
\]
which induces both a polarization rotation $\Delta \theta$ and an ellipticity $\psi$

$$\Delta \theta = \frac{1}{4} (\lambda_\parallel - \lambda_\perp) \Delta z \sin(2\theta_0), \quad \psi = -\omega \Delta \theta.$$  \hspace{1cm} (9)

Are represented in figure 2 the relative magnitude of the polarization rotation induced by the several radiative corrections discussed in this work. We note that today’s laboratory experiments accuracy is not sensitive to any of these corrections.

![Relative polarization rotation](image)

**FIGURE 2.** Relative polarization rotation (9) given in terms of the $\Delta \xi_i$ for electron-positron loops ($e\bar{e}$), muon-antimuon loops ($\mu\bar{\mu}$) interchange of the neutral pion ($\pi^0$), quark condensates ($\langle q\bar{q} \rangle$) and charged pion loop ($\pi^+\pi^-$).

### $\gamma$-RAY PROPAGATION

The results of the previous sections can also be applied to high energy $\gamma$-ray bursts. In order to do so consider the propagation equations for photons in background fields [11]

$$(\omega - i\partial_z + M) \begin{bmatrix} A_\parallel \\ A_\perp \\ \phi \end{bmatrix} = 0, \quad M = \begin{bmatrix} \Delta_{\gamma\gamma} + \Delta_\parallel & 0 & \Delta_\parallel^\perp \\ 0 & \Delta_{\gamma\gamma} + \Delta_\perp & \Delta_\perp^\parallel \\ \Delta_\parallel^\phi & \Delta_\perp^\phi & \Delta_\phi \end{bmatrix},$$  \hspace{1cm} (10)

with the several entries given by

$$\Delta_{\gamma\gamma} \approx -\frac{i}{2z_0} \Gamma \ln (E), \quad \Delta_\parallel \approx 4\xi_e B^2, \quad \Delta_\perp \approx 7\xi_e B^2, \quad \Delta_\parallel^\perp = \frac{1}{2} g_{\gamma\gamma} B^\parallel^\perp, \quad \Delta_\phi = m_\phi.$$  \hspace{1cm} (11)

We will address radiation from the center of the galaxy at a distance of $z_0 = 8.5 \text{kpc}$ [12, 13]. In the last expression $E$ stands for the $\gamma$-ray energy and we took the standard power law approximation valid for energies in the TeV range corresponding to $\Gamma = -\ln(dN/dE) \approx 2.25$ [13] for which the main contributions are due to photon desintegration [14]. For non-polarized radiation in gaussian magnetic field distributions in domains of average size $s$, the conversion probability of photons to pseudo-scalars is in the saturated continuum limit $z \gg s$

$$P_{\gamma\rightarrow\phi} = \frac{1}{3} \left(1 - e^{-\frac{3n_0}{2s^2}}\right), \quad P_0 \approx 0.4 \times 10^{-7} \left(\frac{g_BGE_{10}}{m_\phi^2}\right)^2.$$  \hspace{1cm} (12)
The only measurable effect from the ones discussed in this work in the TeV range is due to the photon mixing to the neutral pion. Hence, following [11], we are taking the root mean square magnetic field strength \( B_G = 1 \mu \text{Gauss} \), the radiation energy \( E_{10} \) given in units of \( 10 \text{TeV} \), for a distance \( z = z_0 = 8.5 \text{kpc} \), the domain size \( s = 0.01 \text{pc} \) and \( g = 2.49 \times 10^4 \) with \( m_\phi = m_{\pi^0} = 135 \text{MeV} \). The resulting deviation to the power law is represented in figure 3.

![Figure 3](image)

**FIGURE 3.** Dashed line represents the deviation to the power law (continuous line) due to the photon mixing to \( \pi^0 \). The data points are from the HESS col. (July/august 2003/2004) [13].

**CONCLUSIONS**

We have shown that the exchange of the neutral pion \( \pi^0 \) with photons hold a deviation from the power law spectrum for \( \gamma \)-ray from the center of the galaxy in the TeV. This result may improve our knowledge of the effects affecting \( \gamma \)-ray bursts, hence allowing a better understanding of its characteristics at the origin.

In addition the propagation equation (10) is also valid for light quark condensates \( (m_c \approx 20 \text{MeV}) \). Up to distances of \( z \approx 125 \text{pc} \) from the center of the galaxy the necessary energy densities for its existence are present [7]. However the perturbative saturated limit is only applicable in the GeV range considering, for example, a domain size of \( s = .01 \text{pc} \) [15] its effects are observable. In the TeV range the probability \( P_0 \) as given in (12) does not preserve unitary, hence it is not applicable. One can also add the effect of the axion [11] to the one from the \( \pi^0 \) which result should be to further reduce the spectrum over energies of \( E > 10 \text{TeV} \). Also it is expected that the effects discussed in this work are relevant near neutron stars due to the high magnetic fields present in such environments [6].

Acknowledgments – Work of PCF supported by SFRH/BPD/34566/2007.

**REFERENCES**

1. W. Heisenberg and H. Euler, Z. Physik 98 (1936) 714, physics/0605038; J. Schwinger, Phys. Rev. 82 (1951) 664.
2. L. Maiani, R. Petronzio and E. Zavattini, Phys. Lett. B175 (1986) 359; G. Raffelt and L. Stodolski, Phys. Rev. D37 (1988) 1237; E. Massó and J. Redondo, JCAP 0509 (2005) 015, hep-ph/0504202; Phys. Rev. Lett. 97 (2006) 151802, hep-ph/0606163.
3. P. Castelo Ferreira and J. Dias de Deus, Eur. J. Phys. Cxx (2008) xx, arXiv:0707.4200.
4. I. A. Shushpanov and A. V. Smilga, Phys. Lett. B402 (1997) 351, hep-ph/9703201; N. O. Agasian, I. A. Shushpanov, Phys. Lett. B472 (2000) 143, hep-ph/9911254; D. Kabat, K.-M. Lee, and E. Weinberg, Phys. Rev. D66 (2002) 014004, hep-ph/0204120; R. Gonzalez Felipe, G. M. Marques, J. E. Ribeiro, hep-th/0307290.

5. S. P. Klevanski and R. H. Lemmer, Phys. Rev. D39 (1989) 3478; P. J. A. Bicudo, J. E. F. T. Ribeiro, Phys. Rev. D42 (1990) 1611; K. G. Klimentov, Teor. Mat. Fiz. 89 (1991) 211; Theor. Math. Phys. 89 (1992) 1161; Z. Phys. C54 (1992) 323; D. Ebert, K.G. Klimentov, M.A. Vdovichenko and A.S. Vshivtsev, Phys. Rev. D61 (2000) 025005, hep-ph/9905253; R. Gonzalez Felipe, G. M. Marques, J. E. Ribeiro, Nucl. Phys. A778 (2006) 30, hep-th/0307290.

6. V. Radhakrishnan and D. J. Cooke, Astrophys. Lett. 3 (1969) 225; D. Bhattacharyya and E. P. J. van den Heuvel, Phys. Rep. 203 (1991) 1-124; Alice K. Harding, Science 251 (1991) 1033-1038; R. C. Duncan and C. Thompson, Astrophys. J. 392 (1992) 9-13; C. Kouveliotou et al., Nature 393 (1998) 235-237; Anatoly Spitkovsky, astro-ph/0310731; J. M. Lattimer and M. Prakash, Science 304 (2004) 536-542; D. J. Price and S. Rosswog, Science 312 (2006) 719-722; J.-P. De Villiers, J. F. Hawley and J. H. Krolik, astro-ph/0307260; S. Hirose, J. H. Krolik, J.-P. De Villiers and J. F. Hawley, Astrophys. J. 606 (2004) 1083-1097, astro-ph/0311500; J.-P. De Villiers, J. F. Hawley, J. H. Krolik and S. Hirose, Astrophys. J. 620 (2005) 788-888, astro-ph/0407092; J. H. Krolik, J. F. Hawley and S. Hirose, Astrophys. J. 622 (2005) 1008-1023, astro-ph/0409231.

7. M. Morris, K. Uchida and T. Do, Nature 440 (2006) 308, astro-ph/0512452.

8. S. L. Adler, C. G. Callan, D. J. Gross and R. Jackiw, Phys. Rev. D6 (1972) 2982-2988.

9. E.V. Beveren, F. Kleefeld, G. Rupp, M.D. Scadron, Mod. Phys. Lett. A17 (2002) 1673, hep-ph/0204139.

10. Z. Bialynicka-Birula and I. Bialynicka-Birula, Phys. Rev. D2 (1970) 2341; S. L. Adler, Ann. Phys. 67 (1971) 559; E. Lundström et al., Phys. Rev. Lett. 96 (2006) 083602; M. Marklund and P. K. Shukla, Rev. Mod. Phys. 78 (2006) 591; E. Zavattini and al., Phys. Rev. Lett. 96 (2006) 110406, hep-ex/0507107; J. T. Mendonça, J. Dias de Deus and P. Castelo Ferreira, Phys. Rev. Lett. 97 (2006) 100403; 97 (2006) 269901(E); S. L. Adler, J. Phys. A40 (2007) F143, hep-ph/0611267; S. Biswas, K. Melnikov, Phys. Rev. D75 (2007) 053003, hep-ph/0611345; M. Roncadelli, in XII International Workshop on Neutrino Telescopes 2007, arXiv:0706.4244; H. Gies and W. Dittrich, Phys. Lett. B431 (1998) 420, hep-ph/9804303; W. Dittrich and H. Gies, Phys. Rev. D58 (1998) 025004, hep-ph/9804375; M. Ahlers, H. Gies, J. Jaeckel and A. Ringwald, Phys. Rev. D75 (2007) 035011, hep-ph/0612098; J. T. Mendonça, Europhys. Lett. 79 (2007) 21001.

11. A. Mirizzi, G. G. Raffelt and P. D. Serpico, Phys. Rev. Lett. 95, astro-ph/0506078; Phys. Rev. D72 (2005) 023501, astro-ph/0506078; Phys. Rev. D76 (2007) 023001, arXiv:0704.3044; C. Csáki, N. Kaloper, M. Peloso and J. Terning, JCAP 0305 (2003) 005, hep-ph/0302030; A. De Angelis, O. Mansutti and M. Roncadelli, arXiv:0707.2695, arXiv:0707.4312.

12. S. Lee, Phys. Rev. D58 (1998) 043004, astro-ph/9604098; J. A. Grífsols, E. Massó and R. Toldrá, Phys. Rev. Lett. 77 (1996) 2372, astro-ph/9606028; J.-L. Han, K. Ferriere, R.N. Manchester, Astrophys. J. 610 (2004) 820, astro-ph/0404221; K. Tsuchiya and al. (CANGAROO-II), Astrophys. J. 606 (2004) L115, astro-ph/0403592; K. Kosack and al. (VERITAS), Astrophys. J. 608 (2004) L97, astro-ph/0403422; J. Albert and al. (MAGIC), Astrophys. J. 638 (2006) L101, astro-ph/0512469.

13. F. Aharonian and al. (HESS), Astron. Astrophys. 425 (2004) L13, astro-ph/0408145; Phys. Rev. Lett. 97 (2006) 221102, Erratum-ibid. 97 (2006) 249901, astro-ph/0610509.

14. A. I. Nikishov, Zh. Ekspervi. i Teor. Fiz. 41 (1961) 549; Soviet Phys.-JETP 14 (1962) 393; P. Goldreich and P. Morrison, Zh. Ekspervi. i Teor. Fiz. 45 (1963) 344; Soviet Phys.-JETP 18 (1964) 239; R. J. Gould and G. Schröder, Phys. Rev. Lett. 16 (1966) 252; Phys. Rev. 155 (1967) 1404; Phys. Rev. 155 (1967) 1408; F. W. Stecker, Nature 220 (1968) 675; Nature 224 (1969) 870; Phys. Rev. 180 (1969) 1264; Nature 226 (1969) 135; F. W. Stecker and J. Silk, Nature 221 (1969) 1229; S. L. Adler and C. Shubert, Phys. Rev. Lett. 77 (1996) 1695; A. K. Harding, M. G. Baring and P. L. Gonthier, Astrophys. J. 476 (1997) 246; M. G. Baring and A. K. Harding, Astrophys. J. 482 (1997) 372; S.L. Adler, Astrophys. J. 547 (2001) 929.

15. Christopher van Eldik et al. (HESS), contributed to 30th International Cosmic Ray Conference (ICRC 2007), Merida, Yucatan, Mexico, arXiv:0709.3729.