ABSTRACT

The present research numerically investigates the validity of the Reynolds analogy for micro-convective water flow between Stanton number ($St$) and Fanning friction factor ($f_f$), taking into account combined fluid properties variations such as temperature-dependent density, viscosity, and thermal conductivity. The Reynolds analogy is suggested to be valid when $St$ increases for thermophysical fluid properties (TFP) with a decrease in $f_f$. This analogy, therefore, helps to find the flow regime that increases heat transfer while shear stress decreases for TFP. Hence, the Reynolds analogy for TFP helps to design and improve the performance of the different devices, including micro-scale heat exchangers for electronics cooling, internal cooling passages of turbine airfoils, and many biomedical devices. Three modified non-dimensional parameters ($\Pi_{S\rho T}$, $\Pi_{S\mu T}$, and $\Pi_{SkT}$) appear from the non-dimensionalization of the governing conservation equations. Using dimensional analysis, the dependence of the friction factor on these parameters is examined.

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TFP on the fully developed flow (FDF) [2, 3]. The effects of 
μ(T) variation on Nusselt number (Nu) and friction factor (f) for laminar flow through a circular tube were analyzed 
by Kakac et al. [4]. Harms et al. [5] studied laminar FDF in a 
semicircular duct with μ(T) variations. It was observed that 
the μ(T) variations create significant distortions in both the 
velocity and temperature distributions.

The influences of TFP on the local temperature, heat 
flux, and Nu were numerically analyzed [6]. The curvature 
effect and the effect of variation in μ(T) on laminar FC and 
FDF were analyzed [7]. The dependence of Nu and f on 
μ(T) variation was analyzed under both cooling and heating 
conditions. Pure continuum-based micro-convective 
gas flow with ρ(T) variation was numerically simulated [8]. 
The physical special effects induced due to μ(T) and k(T) 
variations were investigated for the case of laminar micro-
convective water flow [9, 10]. The physical effects induced 
due to variable gas properties in micro-convective flow 
were reported [11]. The effects of μ(T) variation can not be 
overlooked for a wide range of operating conditions in the 
entry region of straight ducts [12]. Herwig and Mahulikar 
[13] examined the effects of TFP on single-phase incompressible micro-convective flow. Mahulikar et al. [14] sug-
gested the need to examine the impacts of fluid variation 
on f. The fluid friction characteristics in laminar FDF were 
studied and the Reynolds analogy was reexamined for TFP 
[15]. The effects of TFP on thermally developing flow were 
numerically studied by Liu et al. [16], in the cooling pas-
sages of micro-channel. The effects of fluid variation on 
single-phase micro-convective compressible flow were 
investigated [17]. The physical mechanisms induced due 
to TFP in laminar micro-convective FDF were examined 
[18]. Because of TFP, a significant difference in pressure 
drop from macro to micro scale was measured. Gulhane 
and Mahulikar [19] studied the hydrodynamic and ther-
merically developing flow problem and the Graetz problem due 
to fluid properties variations. Harley et al. [20] provided 
theoretical and experimental research on compressible gas 
flow in micro-channels with a large subsonic Mach num-
ber. Kumar and Mahulikar [21] explored the effects of μ(T) 
variation on laminar micro-convective FDF. The abnormal 
HT and fluid flow observations were recognized owing to 
variability in μ(T) and these observations were clarified 
using the concept of thermal and hydrodynamic undevelop-
ment of flow. Frictional flow characteristics of micro-
convective flow for TFP were investigated [22]. Recently, 
Kumar and Mahulikar [23] numerically re-examined the 
validity of the Chilton-Colburn analogy between St Pr2/3 
and f for laminar micro-convective flow with μ(T) and k(T) 
variations. Kumar and Mahulikar [24] numerically investigat-
ged the heat transfer characteristics of convective water 
flow through a micro-tube. The effects of variation in inlet 
temperature and wall heat flux on heat transfer are studied 
for variable fluid properties. The results show that the Nu 
decreases with an increase in inlet temperature for variable 
fluid properties. The deviations produced by temperature-
dependent properties on heat transfer and frictional flow 
characteristics of water flowing through a microchannel 
are numerically investigated by Kumar and Mahulikar [25]. 
The Nu displays a significant deviation from conventional 
theory due to flattening of the radial temperature profile 
due to thermal conductivity variation. The performance of 
the heat sink is optimized with the help of the entropy 
generation minimization (EGM) method [26-28].

Kumar and Mahulikar [29] and Kumar [30] analyzed the rarefaction and non-rarefaction effects on heat transfer 
characteristics of hydrodynamically and thermally develop-
ing airflow through microtubes. Keepaiboon et al. [31] 
experimental investigated boiling heat transfer character-
istics of a refrigerant in a microfluidic channel at a high 
mass flux. They proposed the new boiling heat transfer 
correlation of a refrigerant for two-phase flow at the 
microfluidic scale Gaikwad et al. [32] discussed the EGM 
in a slip-modulated electrically actuated transport through 
an asymmetrically heated microchannel. Optimum val-
ues of geometric and thermo-physical parameters were 
introduced for which a change in the thermal transport of 
heat caused by viscous dissipation and Joule heating effect 
leads to EGM in the system. Sarma et al. [33] analyzed the 
entropy generation characteristics under the influence of 
interfacial slip for a non-Newtonian microflow. The opti-
mum value of the geometric parameter such as the channel 
wall thickness and the thermophysical parameters such as 
the Peclet number (Pe) and Biot number (Bi), were deter-
mined, leading to a minimum rate of entropy generation in 
the system. Sarma et al. [34] examined the prominent role 
of the Debye–Hückel parameter, viscoelastic parameter, the 
thermal conductivity of the wall, channel wall thickness, Bi, 
Pe, and axial temperature gradient on the entropy gener-
ation rate. They established the optimum values of the above 
parameters, leading to the EGM method.

**OBJECTIVE AND SCOPE OF INVESTIGATION**

The important step for the analysis of water cooling 
with forced convection is the use of a similar argument, the 
Reynolds analogy. The Reynolds analogy is a powerful ana-
lytical tool since it was first proposed in the late 1800s. This 
states that the f due to fluid flowing over the wall is pro-
portional to the convective heat transfer coefficient (h). It’s 
most simple form is, St = f/2 for CFP. Earlier, the Reynolds 
analogy is valid for incompressible and laminar flows. It 
has been extended to turbulent flows in different computa-
tional as well as analytical forms by many researchers. 
Mahulikar and Herwig [9] reported that the effect of μ(T) 
and k(T) variations for water is highly significant in micro-
convexion. Gulhane and Mahulikar [18] and Kumar and 
Mahulikar [35] reported that the effect of ρ(T) variation 
for water is significant in micro-convexion due to a rapid 
increment in fluid bulk mean temperature. Therefore, the
The present research is an extension of the earlier work reported by Kumar and Mahulikar [23], to include the effect of \( \rho(T) \) variation in addition to \( \mu(T) \) and \( k(T) \) variations. Hence, the first objective of the present research is to reexamine the Reynolds analogy for the case of combined \( \rho(T) \), \( \mu(T) \), and \( k(T) \) variations without entrance effect. Due to TFP, three modified non-dimensional parameters “\( \Pi_{\text{poi}} \)”, “\( \Pi_{\text{sti}} \)”, and “\( \Pi_{\text{sti}} \)” are emerged from the dimensionless form of governing conservation equations. The role of \( \Pi_{\text{poi}} \) and \( \Pi_{\text{sti}} \) in flow friction was analyzed by Kumar and Mahulikar [23]. It is also thought that in micro-convection, \( \Pi_{\text{poi}} \) correlates with \( \tau_{\text{w}} \), \( \Pi_{\text{sti}} \) correlates with \( \tau_{\text{w}} \) to find the effects of TFP on laminar liquid micro-convection. This research would also be very helpful in improving micro-convection knowledge that offers better efficiency of micro-devices.

**DESCRIPTION OF THE PROBLEM**

A circular cross-sectional micro-tube with an aspect ratio \((L/D) = 50\) is subjected to constant wall heat flux (CWHF) boundary condition (BC) as shown in figure 1. The following data is fixed for all investigated cases: Radius \( r \times 10^{-3} \text{ m} \), and inlet temperature at axis \( T_{\text{in}} = 293 \text{ K} \). The smaller diameter and higher aspect ratio are selected to analyze the effect of steeper temperature gradients on laminar micro-convection characteristics.

**Problem Formulation**

Attention is focused on the calculation of \( \text{St}, \text{Nu}, \text{Re}, \text{Pr}, \text{Pe}, \text{Po} \). The subscript ‘m’ shows the mean value of the properties evaluated at bulk mean temperature \( T_{\text{m}} \). The cross-sectionally weighted averaged axial velocity (mean velocity) \( u_{\text{m}} \) is defined as [17]:

\[
\text{Nu} = \frac{\int \rho \cdot u(r) \cdot dA}{\rho \cdot A} = \frac{\int u(r) \cdot r \cdot dr}{\int r \cdot dr}.
\]

\( \text{Pe} = \text{Re} \cdot \text{Pr}, \text{Po} = 4f \cdot \text{Re}_{\text{D}} \). The subscript ‘m’ shows the mean value of the properties evaluated at bulk mean temperature \( T_{\text{m}} \). The cross-sectionally weighted averaged axial velocity (mean velocity) \( u_{\text{m}} \) is defined as [17]:

\[
\text{St} = \left[ \frac{\text{Nu}}{16 \cdot \text{Pr}} \right] f_{f}.
\]

From this relation, it is concluded that \( \text{St} \) increases with an increase in \( f_{f} \). Therefore, the Reynolds analogy is popularly regarded as holding when \( \text{St} \) increases for CFP with an increase in \( f_{f} \). The Reynolds analogy valid region illustrates the region in which convective HT is more emphatic at the cost of augmented \( f_{f} \). As per Reynolds analogy, the flattening of velocity profile improves \( \text{St} \) and \( \text{Nu} \), it also increases \( f_{f} \). However, the flattening or sharpening of the velocity profile is affected due to TFP, which affects \( \text{Nu} \) [10]. It was supposed that the Reynolds analogy was invalid for liquids (when \( \text{Pr} \neq 1 \)) and particularly when TFP were considered [36]. Therefore, the Reynolds analogy is revisited for TFP. From the study of Gulhane and Mahulikar [18], it is found that \( h \) increases as a result of flattening the velocity profile,

![Figure 1. Schematic of 2D (with axisymmetry) circular micro-channel with the constant wall heat flux boundary condition.](image-url)
which increases St along the flow. The flattening of the velocity profile results in radially outward flow, which can not be ignored, and radial convection has a significant effect on convective HT, because Nu α (du/∂r)z. Owing to flattening of u(r, z) profile, (du/∂r)z increases, therefore Nu also increases [10]. Due to TFP, τw α μm [15]. The reduction in μm along the flow causes a reduction in f, when Reynolds analogy is valid. Hence, Reynolds analogy is now valid when St increases with a reduction in f, for TFP. It is concluded that the Reynolds analogy helps to find the flow regime in which h increases while τw decreases for TFP. By putting St = Nu/(Re·Pr) in equation (1), can be written in this form: (1/Re) = (f/16). Therefore, the Reynolds analogy is qualitatively valid for that region, where (1/Re) and f are directly dependent.

**Governing Equations**

The following 2-dimensional, steady-state, laminar, continuum-based governing conservation equations in cylindrical coordinates (with axisymmetry) incorporating ρ(T), μ(T), and k(T) variations in dimensional and nondimensional forms, as elaborated in Gulhane and Mahulikar [18] are numerically solved.

**Continuity equation**

**Dimensional form**

\[
\left( v \cdot \frac{\partial \rho}{\partial r} \right) + \rho \left[ \left( \frac{\partial u}{\partial z} \right) + \left( \frac{\partial v}{\partial r} \right) \right] + \left( \frac{\partial p}{\partial z} \right) = 0
\]

**Non dimensional form**

\[
\frac{\bar{\nu} \cdot \bar{I}_{\text{ax}} \cdot \bar{P}_{\text{dp}}}{\bar{P}_{\text{dp}}} \left( v \cdot \frac{\partial \bar{\theta}}{\partial r} + \frac{1}{2} \frac{\partial u}{\partial z} + \frac{1}{2} \frac{\partial v}{\partial r} \right) + \bar{\rho} \left( \frac{\partial \bar{\theta}}{\partial r} + \frac{\partial \bar{\theta}}{\partial z} \right) = 0
\]

**Momentum equation [Radial direction]**

**Dimensional form**

\[
\rho \left( v \cdot \frac{\partial v}{\partial r} + u \cdot \frac{\partial v}{\partial z} \right) = -\frac{\partial p}{\partial r} + \left( \frac{\mu}{r} + \frac{\partial u}{\partial z} \right) \left( \frac{\partial u}{\partial r} \right)
\]

**Nondimensional form**

\[
\bar{P} \cdot \bar{R}_{D} \left( \frac{\partial \bar{\theta}}{\partial r} + \bar{u} \cdot \frac{\partial \bar{\theta}}{\partial z} \right) = -\left( \frac{\partial \bar{\theta}}{\partial r} \right) + \left( \frac{\bar{\mu}}{r} + \frac{\partial u}{\partial z} \right) \left( \frac{\partial u}{\partial r} \right)
\]

**Energy equation**

**Dimensional form**

\[
\rho \cdot c_{p} \left( v \cdot \frac{\partial T}{\partial r} + u \cdot \frac{\partial T}{\partial z} \right) = \left( \frac{k}{r} + \frac{\partial k}{\partial z} \right) \left( \frac{\partial T}{\partial r} \right)
\]

**Nondimensional form**

\[
\bar{P} \cdot \bar{R}_{D} \left( \frac{\partial \bar{\theta}}{\partial r} + \bar{u} \cdot \frac{\partial \bar{\theta}}{\partial z} \right) = \left( \frac{1}{r} + \frac{\partial \bar{\theta}}{\partial z} \right) \left( \frac{\partial \bar{\theta}}{\partial r} \right)
\]
The non-dimensional static pressure and non-dimensional temperature are given as \( \tilde{p} = p / (\rho m \cdot u_m) \) and \( \tilde{T} = T - T_0 / (q_m^D) \) respectively. The \( \tilde{S}_{\rho T} = (\partial p / \partial T) \) is the density-temperature sensitivity, \( \tilde{S}_{\mu T} = (\partial \mu / \partial T) \) is the viscosity-temperature sensitivity and \( \tilde{S}_{kT} = (\partial k / \partial T) \) is the thermal-conductivity-temperature sensitivity. The \( R e_p \) and \( P e_D \) are \( R e \) and \( P e \) based on diameter respectively. The laminar micro-convective flow with TFP depends upon the following non-dimensional parameters: \( Br_r = \mu \cdot u_m^2 / (q_m^D) \), \( \Pi_{\rho T} = [Br_r / Br_{\rho T}] \), \( \Pi_{\mu T} = [Br_r / Br_{\mu T}] \), \( \Pi_{k T} = [Br_r / Br_{k T}] \). The \( \Pi \) parameters are the magnitude of the product of the temperature perturbation parameter \( = f(q_m^D/D/k) \) and non-dimensional property sensitivities, \( S_{\rho T} (T/\rho) \), \( S_{\mu T} (T/\mu) \) and \( S_{k T} (T/k) \). The modified non-dimensional parameter \( \Pi_{\rho T} \) shows the comparative importance of momentum transport due to \( S_{\rho T} \) over energy transport due to fluid conduction. The \( \Pi_{\mu T} \) indicates the significance of cross-flow momentum transport over energy transport due to \( S_{\mu T} \). The \( \Pi_{k T} \) gives the comparative importance of momentum transport due to \( S_{k T} \) and energy flow due to fluid conduction. Higher values of \( \Pi \) parameters show a stronger influence on micro-convection due to temperature-dependent fluid properties. The Brinkman number \( "Br_r" \) appears in the non-dimensional energy equation as the product of \( Pr \) and Eckert numbers \( (Ec) \), \( Br_r = Pr \cdot Ec = \mu \cdot u_m^2 / (k \cdot \Delta T) \), where \( \Delta T = (T_0 - T_m) \). The \( Br_r = [S_{\rho T} \cdot u_m^2 / (\rho \cdot k)] \) is the modified \( Br \) based on \( S_{\rho T} \), the \( Br_r = [S_{\mu T} \cdot u_m^2 / (\rho \cdot k)] \) is the modified \( Br \) based on \( S_{\mu T} \) and the \( Br_r = [S_{k T} \cdot u_m^2 / (\rho \cdot k)] \) is the modified \( Br \) based on \( S_{k T} \) [9]. The other non-dimensional parameters are involved in the governing equations as:

\[
\begin{align*}
\tilde{z} &= z / D \\
\tilde{r} &= r / R \\
\tilde{u} &= u / u_m \\
\tilde{v} &= v / u_m \\
\tilde{\rho} &= \rho / \rho_m \\
\tilde{\mu} &= \mu / \mu_m \\
\tilde{k} &= k / k_m \\
\end{align*}
\]

**Boundary Conditions**

The computational field is subjected to four flow and thermal boundary conditions (BCs) as follows:

1. **Inlet (z = 0):** The laminar, FDF profiles of \( u(r) \) and \( T(r) \) at the inlet-upstream for CFP are given as: \( u_r (r) = 2u_{in} (1 - r^2) \) and \( T_r (r) = T_{in} + (q_r^m - R/k) \cdot [r^2 - (r^2/4)] \) respectively [37]. The \( u_{in} \) is the inlet mean axial velocity and \( T_{in} \) is the inlet water temperature at the axis. The variations in \( \rho (T), \mu (T) \) and \( k (T) \) are turned on from inlet downstream (\( z > 0 \)). The influence of thermophysical properties on micro-convective flow without considering entrance effects is expressed by this inlet BC [10]. The present study focuses primarily on the effects of fluid thermophysical properties on micro-convective flow only. Thus, the entrance effects are ignored. However, in general, the micro-convective characteristics are determined jointly by taking into account the effects of the entrance and fluid thermophysical properties.

2. **Outlet (z = L):** \( p_{ex} = 1.01325 \times 10^5 \) Pa (atmospheric pressure) and \( v_{ex} = 0 \) since, \( \partial u / \partial z = 0 \).

3. **Axis (r = 0):** The BC of symmetry is imposed at the axis of tube; hence, \( \partial u / \partial r = (\partial p / \partial r) = (\partial T / \partial r) = 0 \).

4. **Wall (r = R):** No-slip and no normal flow BCS are imposed at the nonporous rigid wall of the tube; therefore, \( u_r = v_r = 0 \). The constant is applied at the wall, \( q_r^m = k_s (\partial T / \partial r) \).

**Computational Domain and Numerical Methodology**

The computational field is split into the graded mesh [10,000 cells = 200 (in the axial direction) × 50 (in the radial direction)] with a finer grid spacing near the inlet and the wall. The finer grid spacing near the inlet and the wall is used to capture a rapid change in fields of temperature and flow. The governing equations (2)-(9) with BCs are solved by ANSYS FLUENT software. FLUENT is based on a finite volume differencing scheme which is 2nd-order accurate. The algorithm "Semi-Implicit Method For Pressure-Linked Equations" (SIMPLE) is used to achieve a good convergence behavior. When the residuals for continuity, momentum equations are less than \( 10^{-12} \) and less than \( 10^{-15} \) for the energy equation, the solution is deemed converged. Additional information related to the accuracy of the numerical results, the convergence of solution, and validation with benchmark cases for CFP are in [10, 15, and 23].

**RESULTS AND DISCUSSION**

**Inference from Theoretical Studies based on TFP**

For water in the temperature range of 273-373 K, the variations in \( \rho (T), \mu (T), \) and \( k (T) \) are 4%, 84%, and 21% respectively [38]. The \( p (T) \) variation for pure water in the temperature range of 274-372 K is given by the Thiesen-Scheel-Diesselhorst relation as [39]:

\[
p(T) = 1000 \left[ 1 - \left( \frac{T + 288.9414}{T - 3.9863} \right)^2 \right] \frac{kg}{m^3}
\]

where \( T \) is in °C. The pressure effect on density is normally ignored in the case of water. The \( \mu (T) \) variation for single-phase water is given as [40]:

\[
\mu (T) = \mu (T_{ref}) \left( \frac{T}{T_{ref}} \right)^\alpha \exp \left[ B \left( \frac{1}{T} - \frac{1}{T_{ref}} \right) \right] \frac{kg}{m \cdot s}
\]

where \( n = 8.9, B = 4,700, \mu (T_{ref}) = 1.005 \times 10^{-3} \) kg/ms, \( T_{ref} = 293 \) K.
The $k(T)$ variation for single-phase water is calculated by least-squares error third-order polynomial fitting of data in the operating temperature range of 274-372 K as [38]:

$$k(T) = \frac{W}{m \cdot K} = \left[ -1.51721 + 0.0151476 \cdot T - 3.5035 \times 10^{-5} \cdot T^2 + 2.74269 \times 10^{-8} \cdot T^3 \right]$$

(12)

The $c_p(T)$ variation is less than 1% within a temperature range of 274-372 K, hence, $c_p$ is assumed to be constant. The micro-convective flow has the following characteristics:

(1) **Density-temperature sensitivity**

$$S_{ρT} = \left( \frac{∂ρ}{∂T} \right) = \left[ \frac{(508929.2 \cdot T + 34673158.1)}{-1000} \left( 2 \cdot (T - 3.9863) \cdot (T + 288.9414) \right) \right]$$

$$= \left[ \frac{-508929.2 \cdot (T - 3.9863) \cdot (T + 288.9414)}{(508929.2 \cdot T + 34673158.1)^2} \right]$$

(13)

The $S_{ρT} > 0$ for 0 to 4 °C, because water density increases with increasing temperature. However, after 4 °C, $S_{ρT} < 0$ is negative because water density decreases with increasing temperature.

(2) **Viscosity-temperature sensitivity**

$$S_{μT} = \left( \frac{∂μ}{∂T} \right) = \left[ \frac{-0.0099182 - 5.2377}{T} \right] \frac{kg}{m \cdot s \cdot K} < 0$$

(14)

In liquids, water has very high $S_{μT}$. The $S_{μT} < 0$ since water viscosity decreases with increasing temperature.

(3) **Thermal conductivity-temperature sensitivity**

$$S_{κT} = \left( \frac{∂κ}{∂T} \right) = \left[ \frac{0.0151476 - 7.007 \times 10^{-5}}{10^{-5} \cdot T + 8.22807 \times 10^{-8} \cdot T^2} \right] \frac{W}{m \cdot K^2} > 0$$

(15)

The $S_{κT} > 0$ as water thermal conductivity increases with increase in temperature. The profiles of $ν(r, z)$, $u(r, z)$, and $T(r, z)$ are produced and the variations of $St$, $f_r$, $Re$, and $Pr$ along the flow are examined for the case of combined $μ(T)$, $μ(T)$ and $k(T)$ variations. Table 1 gives $f_{in}, f_{max}, f_{in\max}, Re_{in}$, and $St$ along with $z_{ref}$ for $u_{min} = 0.075, 0.5, 1, 2$, and 3 m/s, for different allowable $q_w^*$.

Figure 2 shows $St$ versus $f_r$ for $u_{min} = 0.075, 1$, and 3 m/s, for different allowable $q_w^*$; for observing the validity of the Reynolds analogy. The $St$ increases along the flow and $f_r$ depends upon $τ_{w}$. Firstly the $f_r$ increases and attains its maximum value ($f_{in\max}$) at axial location $z_{ref}$, and then decreases as shown in Figure 4. From equation (1), $St$ a $f_r$ for CFP. Therefore, in general, the Reynolds analogy is supposed to be valid when $St$ increases with an increase in $f_r$ for CFP.

However, for micro-convective flow with TFP, the validity of the Reynolds analogy results in an increase in $St$ with a decrease in $f_r$. The flow regime in which $St$ increases with decreasing $f_r$ is significant for the case of $u_{min} = 0.075$ m/s. Therefore, Reynolds analogy largely valid for $u_{min} = 0.075$ m/s as shown in figure 2(a). As $u_{min}$ increases, the flow regime in which $St$ increases with decreasing $f_r$ reduces. Hence, Reynolds analogy valid region also reduces for $u_{min} = 1$ m/s as illustrated in Figure 2(b). A smaller flow regime in which $St$ increases with decreasing $f_r$ is observed in the case of $u_{min} = 3$ m/s. Therefore, the Reynolds analogy is largely invalid for $u_{min} = 3$ m/s as shown in Figure 2(c).

Figure 3 illustrates (1/Re) versus $f_r$ for the same cases as taken in figure 2, which shows reversed patterns of variations of (1/Re) versus $f_r$ and $St$ versus $f_r$. From figure 3(a), it is observed that (1/Re) and $f_r$ are directly proportional over a larger flow regime. Therefore, Reynolds analogy is largely valid for $u_{min} = 0.075$ m/s. As $u_{min}$ increases, the flow regime in which (1/Re) and $f_r$ are directly proportional reduces as clearly illustrated in figures 3(b, c). Figure 3(c) shows that the Reynolds analogy is invalid for a larger flow regime in the case of $u_{min} = 3$ m/s. Figure 4 shows $f_r$ versus $z/D$. Firstly $f_r$ increases rapidly along the flow and the maximum value of $f_r (= f_{max})$ reaches at the axial location $z_{ref}$, and after that, $f_r$ decreases along the flow as shown in figure 4. The following reasons are attributed to this, (1) The flow undevelopment happens in the locality of the inlet due to $μ(T)$ variation as $∂μ/∂(μ/$∂T$)$ > 0. (2) The water viscosity reduces with increasing temperature which decays $f_r$ along the flow as $∂μ/∂z < 0$ [18]. As $q_w^*$ increases, the axial location $z_{ref}$ moves towards the exit of micro-tube for same $u_{min}$ as given in Table 1 and also shown in Figure 4.

The variation of $Po$ along the flow is illustrated in figure 5. It is observed that $Po$ decreases with increasing $q_w^*$ and $u_{min}$. The main cause behind this is: the rate of increase in $Re$ is less than the rate of reduction in $f_r$. It is also noted that the rate of change of $Re$ increases with an increase in $q_w^*$, which is confirmed from Table 1. The deviation in $Po$ from 64 is smaller in the locality of the inlet for the case of lower $q_w^*$ as illustrated in Figure 5(a). As $q_w^*$ increases, the deviation in $Po$ from 64 also increases which is clearly shown in figures 5(a, b, c).

**Significance of the modified non-dimensional parameters, $Π_{SPr}$, $Π_{SRe}$, and $Π_{SKr}$**

Three modified non-dimensional parameters “$Π_{SPr}$”, $Π_{SRe}$ and $Π_{SKr}$ appear from the dimensionless form of
Table 1. Variation in convective flow parameter with \( u_{m,in} \) and \( q_{w}^* \)

| \( u_{m,in} \) (m/s) | \( q_{w}^* \) (W/cm²) | \( Re_{in} \) | \( Re_{ex} \) | \( f_{in} \) | \( f_{max} \) | \( f_{ex} \) | \( St_{in} \) | \( St_{ex} \) | \( \frac{\pi}{2} q_{w,max} \) |
|---------------------|-----------------|--------------|--------------|----------|----------|----------|----------|----------|----------------|
| 1.5 | 7.49 | 9.38 | 2.0799 | 2.1104 | 1.6945 | 0.0856 | 0.0977 | 0.3214 |
| 0.075 | 7.55 | 11.51 | 2.0357 | 2.0705 | 1.3745 | 0.0856 | 0.1002 | 0.3214 |
| 6 | 7.69 | 16.32 | 1.9513 | 1.9941 | 0.9636 | 0.0856 | 0.1043 | 0.3214 |
| 7.5 | 51.76 | 61.19 | 0.2857 | 0.2949 | 0.2530 | 0.0128 | 0.0135 | 1.6475 |
| 15 | 54.03 | 73.92 | 0.2580 | 0.2711 | 0.2045 | 0.0128 | 0.0139 | 1.7431 |
| 0.5 | 58.71 | 102.39 | 0.2133 | 0.2315 | 0.1432 | 0.0129 | 0.0145 | 1.8397 |
| 60 | 68.59 | 166.17 | 0.1536 | 0.1754 | 0.0868 | 0.0129 | 0.0152 | 2.1353 |
| 90 | 79.09 | 228.28 | 0.1175 | 0.1386 | 0.0644 | 0.0130 | 0.0154 | 2.3374 |
| 30 | 117.45 | 158.64 | 0.1064 | 0.1159 | 0.0908 | 0.0064 | 0.0070 | 3.6378 |
| 60 | 137.19 | 229.26 | 0.0765 | 0.0878 | 0.0601 | 0.0065 | 0.0074 | 4.1069 |
| 90 | 158.14 | 305.97 | 0.0585 | 0.0695 | 0.0445 | 0.0065 | 0.0076 | 4.5951 |
| 1 | 120 | 180.09 | 382.98 | 0.0471 | 0.0569 | 0.0360 | 0.0065 | 0.0077 | 5.1032 |
| 150 | 202.79 | 454.90 | 0.0396 | 0.0481 | 0.0311 | 0.0065 | 0.0078 | 5.4977 |
| 180 | 225.99 | 517.53 | 0.0345 | 0.0418 | 0.0283 | 0.0065 | 0.0078 | 6.0423 |
| 90 | 316.27 | 457.87 | 0.0292 | 0.0348 | 0.0282 | 0.0032 | 0.0037 | 8.9473 |
| 120 | 360.13 | 558.63 | 0.0235 | 0.0285 | 0.0229 | 0.0032 | 0.0038 | 9.9847 |
| 150 | 405.49 | 660.89 | 0.0198 | 0.0241 | 0.0194 | 0.0032 | 0.0039 | 10.8978 |
| 2 | 180 | 451.85 | 761.21 | 0.0172 | 0.0209 | 0.0172 | 0.0033 | 0.0039 | 11.8576 |
| 210 | 498.72 | 856.48 | 0.0155 | 0.0187 | 0.0157 | 0.0033 | 0.0039 | 12.8662 |
| 240 | 545.60 | 944.12 | 0.0142 | 0.0170 | 0.0147 | 0.0033 | 0.0039 | 13.9263 |
| 270 | 592.00 | 1022.11 | 0.0134 | 0.0158 | 0.0142 | 0.0033 | 0.0040 | 15.0405 |
| 90 | 474.43 | 613.13 | 0.0195 | 0.0232 | 0.0206 | 0.0022 | 0.0025 | 13.4960 |
| 120 | 540.19 | 735.21 | 0.0157 | 0.0190 | 0.0168 | 0.0022 | 0.0025 | 14.8132 |
| 150 | 608.21 | 860.91 | 0.0132 | 0.0161 | 0.0143 | 0.0022 | 0.0026 | 16.2116 |
| 3 | 180 | 677.74 | 987.12 | 0.0115 | 0.0140 | 0.0126 | 0.0022 | 0.0026 | 17.6959 |
| 210 | 748.03 | 1110.86 | 0.0103 | 0.0124 | 0.0114 | 0.0022 | 0.0026 | 19.2716 |
| 240 | 818.32 | 1229.45 | 0.0095 | 0.0113 | 0.0106 | 0.0022 | 0.0026 | 20.9442 |
| 270 | 887.92 | 1340.56 | 0.0089 | 0.0105 | 0.0100 | 0.0022 | 0.0026 | 22.7197 |

The dimensions of the nine parameters are: \([\rho] = [M^0L^{-3}], [u_u] = [L'T^{-1}], [D] = [L'], [\mu] = [M^0L^{-1}T^{-1}], [k] = [M^0L^{-1}T^{-1}],[S_{py}] = [M^0L^{-1}T^{-1}],[S_{py}] = [M^0L^{-1}T^{-1}],[S_{px}] = [M^0L^{-1}T^{-1}],[q_{w}^*] = [M^0T^{-1}].\) The \( M, L, T, \) and \( \Theta \) are the primary dimensions of mass, length, time, and temperature, respectively. There is a total of nine variables \((n = 9)\) in equation (16) and 4 primary dimensions \((m = 4)\), hence, according to the Buckingham-II theorem, there are 5 \((n-m)\) independent non-dimensional groups. Selecting \( \rho, u_u, D \) and \( k \) as the repeating variables \( (RV) \) and leaving the remaining parameters as \( \mu, S_{py}, S_{py}, S_{px} \) and \( q_{w}^* \) five non-dimensional groups are obtained as follows: \( \Pi_1 = Re_{in} = \rho u_u D / \mu \), \( \Pi_2 = Br_{w*} = \mu u_u / (q_{w}^* D) \), \( \Pi_3 = Br_{py*} = S_{py} u_u^2 \mu / (p - k) \), \( \Pi_4 = Br_{px*} = S_{px} u_u^2 \mu / (k - p) \), \( \Pi_5 = Br_{py*} = S_{py} u_u^2 \mu / (k - p) \). The \( Br_{w*} \) is Br based on \( q_{w}^* \), \( Br_{py*} \) is modified \( Br \) based on \( S_{py}^* \) and \( Br_{px*} \) is modified \( Br \) based on \( S_{px}^* \) and \( Br_{py*} \) is modified \( Br \) based on \( S_{py}^* \). The dimensionless form of the governing equations and above dimensional analysis gives a short form of equation (16) as, \( f = \phi(Re_{in}, Pi_{py*}, Pi_{px*}, Pi_{py*}) \).
emerged in continuity equation yields a strong effect on $P_0$.

Figure 6 gives the variation of $P_0$ versus $\Pi_{S\rho T}$, only Reynolds’ analogy valid data has been taken for $u_m,in = 0.075$, 1, and 3

The role of $\Pi_{S\rho T}$, $\Pi_{S\mu T}$, and $\Pi_{SkT}$ in flow friction is investigated considering combined $\rho(T)$, $\mu(T)$, and $k(T)$ variations. For large $q_w$ which is mainly allowed at high $u_m,in$, $\Pi_{S\rho T}$

Figure 2. Variation of $St$ versus $f_r$: Examination of Reynolds’ analogy: (a) $u_m,in = 0.075$ m/s (Reynolds’ analogy largely valid), (b) $u_m,in = 1$ m/s, and (c) $u_m,in = 3$ m/s (Reynolds’ analogy largely invalid).

Figure 3. Variation of $(1/Re)$ versus $f_r$: (a) $u_m,in = 0.075$ m/s (Reynolds’ analogy largely valid), (b) $u_m,in = 1$ m/s, and (c) $u_m,in = 3$ m/s (Reynolds’ analogy largely invalid).
micro-convective flow. Therefore, in figure 6, the regression equations have been proposed to correlate $P_o$ with $\Pi_{S\rho T}$ at different $u_{m,in}$ in the Reynolds analogy valid region as given in Table 2.

Figure 4. Variation in $f_i$ along the flow: (a) $u_{m,in} = 0.075$ m/s, (b) $u_{m,in} = 1$ m/s, and (c) $u_{m,in} = 3$ m/s.

Figure 5. Variation in $P_o$ along the flow: (a) $u_{m,in} = 0.075$ m/s, (b) $u_{m,in} = 1$ m/s, and (c) $u_{m,in} = 3$ m/s.
As $q^*$ increases, the value of $\Pi_{S\mu T}$ also increases, which shows the effect of $\mu(T)$ variation increases on microconvective flow. Table 3 gives the regression equations which mathematically represent the correlation between $P_0$ and $\Pi_{S\mu T}$.

Figure 6. Variation of $P_0$ versus $\Pi_{S\mu T}$ (only Reynolds’ analogy valid data): (a) $u_{m,\text{in}} = 0.075$ m/s, (b) $u_{m,\text{in}} = 1$ m/s, and (c) $u_{m,\text{in}} = 3$ m/s.

Figure 7 gives the variation of $P_0$ versus $\Pi_{S\mu T}$ only Reynolds’ analogy valid data has been taken for the same cases as in figure 6. Figure 7 (a, b, c) indicates a similar pattern for different allowable, in the Reynolds’ analogy valid region. As $q^*$ increases, the value of $\Pi_{S\mu T}$ also increases, which shows the effect of $\mu(T)$ variation increases on microconvective flow. Table 3 gives the regression equations which mathematically represent the correlation between $P_0$ and $\Pi_{S\mu T}$. 

Figure 7. Variation of $P_0$ versus $\Pi_{S\mu T}$ (only Reynolds’ analogy valid data). (a) $u_{m,\text{in}} = 0.075$ m/s, (b) $u_{m,\text{in}} = 1$ m/s, and (c) $u_{m,\text{in}} = 3$ m/s.
Po and $\Pi_{\text{SkT}}$ in the Reynolds analogy valid region at $u_{\text{m,in}} = 0.075$, 1, and 3 m/s as shown in figure 7.

For large $q^*$ that is enabled at high $u_{\text{m,in}}$, $\Pi_{\text{SkT}}$ also produces a strong effect on $Po$. Figure 8 illustrates the variation of $Po$ versus $\Pi_{\text{SkT}}$ only Reynolds analogy valid data has been taken for the same cases as in figures 6, 7. From figure 8, it is observed that $\Pi_{\text{SkT}}$ increases with an increase in $q^*$ which shows the effect of $k(T)$ variation increases on micro-convective flow. Again a similar trend is observed for different allowable $q^*$ in the Reynolds analogy valid region as shown in figure 8 (a, b, c). Table 4 gives the estimated regression equations which mathematically represent the correlation between $Po$ and $\Pi_{\text{SkT}}$ for $u_{\text{m,in}} = 0.075$, 1, and 3 m/s, in the Reynolds’ analogy valid region as illustrated in figure 8.

The role of $\Pi_{\text{SkT}}$, $\Pi_{\text{S\mu T}}$, and $\Pi_{\text{S\rho T}}$ in flow friction is expressed with the help of estimated regression equations. From these equations, it is observed that as $u_{\text{m,in}}$ increases, the effect of $\rho(T)$, $\mu(T)$, and $k(T)$ variations increases on micro-convective water flow.

**CONCLUDING REMARKS**

1. The performance of the overall system is affected by HT rates. Therefore, significant efforts have been devoted to increase the HT rates and decrease the shear stress for improvement in the performance of the system. The Reynolds analogy is valid only for that portion of the flow regime, where $St$ increases with decreasing $f$ for TFP. Therefore, the Reynolds

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**Table 2. Estimated regression equations to correlate $Po$ with $\Pi_{\text{S\rho T}}$**

| $u_{\text{m,in}}$ (m/s) | $R^2$ | Estimated regression equations |
|-------------------------|-------|-------------------------------|
| 0.075                   | 0.5412| $Po = -276.1361\cdot\Pi_{\text{S\rho T}} + 63.5879$ |
| 1                       | 0.9095| $Po = -15.3234\cdot\Pi_{\text{S\rho T}} + 53.6838$ |
| 3                       | 0.0558| $Po = 5.0029\cdot\Pi_{\text{S\rho T}} + 47.1584$ |

**Table 3. Estimated regression equations to correlate $Po$ with $\Pi_{\text{S\mu T}}$**

| $u_{\text{m,in}}$ (m/s) | $R^2$ | Estimated regression equations |
|-------------------------|-------|-------------------------------|
| 0.075                   | 0.9897| $Po = -7.3420\cdot\Pi_{\text{S\mu T}} + 64.0061$ |
| 1                       | 0.9165| $Po = -4.6980\cdot\Pi_{\text{S\mu T}} + 61.8191$ |
| 3                       | 0.3732| $Po = -2.9907\cdot\Pi_{\text{S\mu T}} + 57.1435$ |

**Table 4. Estimated regression equations to correlate $Po$ with $\Pi_{\text{SkT}}$**

| $u_{\text{m,in}}$ (m/s) | $R^2$ | Estimated regression equations |
|-------------------------|-------|-------------------------------|
| 0.075                   | 0.9984| $Po = -68.2179\cdot\Pi_{\text{SkT}} + 64.0170$ |
| 1                       | 0.9608| $Po = -57.8855\cdot\Pi_{\text{SkT}} + 62.3837$ |
| 3                       | 0.8721| $Po = -55.0776\cdot\Pi_{\text{SkT}} + 61.4054$ |
analogy helps to find the flow regime in which HT increases while τ decreases for TFP.
2. The Reynolds analogy is largely valid at low mean velocities; however, the Reynolds analogy is largely invalid at high mean velocities.
3. Direct proportionality of \(1/Re\) with \(f\) for a significant section of the flow regime, also validates the Reynolds analogy.
4. The significance of mass transport due to \(\rho(T)\) variation is given in governing equation (3) and momentum transport in axial and radial directions due to \(\mu(T)\) variation is given in governing equations (5, 7).
5. Higher values of the \(\Pi_{\rho T}, \Pi_{\mu T}\) and \(\Pi_{k T}\) parameters show a stronger effect on micro-convection due to TFP.
6. The regression equations have been proposed to correlate \(Po\) with \(\Pi_{\rho T}, \Pi_{\mu T}\) and \(\Pi_{k T}\) in the Reynolds analogy valid region.

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NOMENCLATURE

\(c_p(T)\) Temperature-dependent specific heat at constant pressure \([\text{J·kg}^{-1}·\text{K}^{-1}]\)
\(D\) Diameter of micro-tube \([\text{m}]\)
\(f\) Darcy friction factor
\(f_f\) Fanning friction factor
\(L\) Length of micro-tube \([\text{m}]\)
\(h\) Heat transfer coefficient \([\text{W}·\text{m}^{-2}·\text{K}^{-1}]\)
\(k(T)\) Temperature-dependent thermal conductivity \([\text{W}·\text{m}^{-1}·\text{K}^{-1}]\)
\(q_w\) Heat flux at the wall \([\text{W}·\text{m}^{-2}]\)
\(S_{\rho T}\) Density-temperature sensitivity \((\partial \rho / \partial T)\)
\(S_{\mu T}\) Viscosity-temperature sensitivity \((\partial \mu / \partial T)\)
\(S_{k T}\) Thermal conductivity-temperature sensitivity \((\partial k / \partial T)\)
\(S_{\rho T}\) Density-temperature sensitivity \((\partial \rho / \partial T)\)
\(T_0\) Inlet temperature \([\text{K}]\)
\(T_m\) Bulk mean temperature \([\text{K}]\)
\(T_w\) Wall temperature \([\text{K}]\)
\(u(r)\) Axial velocity profile in the radial direction \([-]\)
\((\partial u / \partial r)\) Wall velocity gradient \([\text{s}^{-1}]\)
\(\nu\) Kinematic viscosity \([\text{m}^2·\text{s}^{-1}]\)
\(\rho(T)\) Temperature-dependent density \([\text{kg}·\text{m}^{-3}]\)
\(\epsilon(T)\) Temperature-dependent specific heat at constant pressure \([\text{J}·\text{kg}^{-1}·\text{K}^{-1}]\)

Non-dimensional numbers

\(Br_{qw}\) Brinkman number based on wall heat flux \([-]\).
\(Br_{\rho T}, Br_{\mu T}, Br_{k T}\) Modified Brinkman numbers based on temperature sensitivities of \(\rho, \mu, k\) respectively.

\(Nu\) Nusselt number \((h·D)/k_m\)
\(Pe\) Peclet number \((\rho_m·u_m·c_p·D)/k_m\)
\(Po\) Poiseuille number \((f·Re)/D\)
\(Pr\) Prandtl number \((c_p·\mu_m)/k_m\)
\(Re\) Reynolds number \((\rho_m·u_m·D)/\mu_m\)
\(St\) Stanton number \((h/(\rho·u_m·c_p))\)

Subscripts

CP Constant properties
\(D\) Based on diameter
\(ex\) Value at outlet
\(in\) Value at inlet
\(m\) Mean value of properties calculated at bulk mean temperature, \(T_m\)
\(VP\) Variable properties
\(w\) Condition at wall

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.
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