Hidden-charm scalar tetra-quark mesons

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Decays of hidden-charm scalar four-quark mesons as the partners of $D^+_s(2317)$ which has successfully been assigned to the iso-triplet four-quark scalar $F^+_t \sim [c(n|\bar{s}n)l=1]$ meson are studied. Because OZI-rule allowed strong decays are kinematically limited, their radiative decays are expected to be important.

After the observation of $D^+_s(2317)$ \cite{1} and $X(3872)$ \cite{2}, tetra-quark mesons (including meson-meson molecules) are attracting general interests \cite{3}. Under this circumstance, we have proposed that (i) $D^+_s(2317)$ is an iso-triplet tetra-quark scalar meson \cite{4} and (ii) $X(3872)$ consists of two axial-vector meson states with (approximately) degenerate masses and opposite $G$-parities, which can be realized by tetra-quark system \cite{5} without violating conservation of isospin and $G$-parity. As the next step, therefore, we study hidden-charm scalar tetra-quark mesons in this short note.

Before visiting hidden-charm scalar mesons, we here review very briefly four-quark meson states. First of all, they can be classified into the following four groups \cite{4},

$$\{qq\bar{q}q\} = \{qq\bar{q}q\} + \{qq\bar{q}q\} + \{qq\bar{q}q\} + \{qq\bar{q}q\},$$

(1)

according to symmetry property of their flavor wavefunction, where parentheses and square brackets denote symmetry and anti-symmetry, respectively, under exchange of flavors between them. Each term in the right-hand side (r.h.s.) of Eq. (1) is again classified into two groups, because there are two ways to get a color-singlet tetra-quark and anti-symmetry, respectively, under exchange of flavors between them. Each term in the right-hand side (r.h.s.) of Eq. (1) seems to be well realized by the observed scalar nonoet, $\sigma(600), \sigma(980), \sigma(1600)$, and $\kappa(800)$, as predicted in Ref. \cite{6}.

The above light $[qq][q\bar{q}]$ mesons have been extended to open-charm $[cq]\{q\bar{q}\}$ mesons, and, as the result, the charm-strange $D^+_s(2317)$ has been successfully assigned to the iso-triplet $F^+_t \sim [c(n|\bar{s}n)l=1]$ with $3_c \times 3_c$ as the lowest charm-strange scalar tetra-quark meson \cite{4}. (Our notation of the open-charm scalar tetra-quark mesons is provided in Ref. \cite{4}.) With regard to the color configuration, the $3_c \times 3_c$ state would be lower than the $6_c \times 6_c$, because the forces between two quarks (and between two antiquarks) are attractive when they are of $3_c$ (and of $3_c$), while the forces are repulsive when they are of $6_c$ (and of $6_c$) \cite{6}. However, mixings of the $3_c \times 3_c$ and $6_c \times 6_c$ states in open- and hidden-charm mesons are expected to be much smaller than that of the light sector, because QCD would be rather perturbative at the energy scale of these meson masses. Putting aside the structure of color wavefunctions, the first term $\{qq\bar{q}q\}$ in the right-hand-side (r.h.s.) of Eq. (1) seems to be well realized by the observed scalar nonoet, $\sigma(600), \sigma(980), \sigma(1600)$, and $\kappa(800)$, as predicted in Ref. \cite{6}.

The above light $[qq][q\bar{q}]$ mesons have been extended to open-charm $[cq]\{q\bar{q}\}$ mesons, and, as the result, the charm-strange $D^+_s(2317)$ has been successfully assigned to the iso-triplet $F^+_t \sim [c(n|\bar{s}n)l=1]$ with $3_c \times 3_c$ as the lowest charm-strange scalar tetra-quark meson \cite{4}. (Our notation of the open-charm scalar tetra-quark mesons is provided in Ref. \cite{4}.) With regard to the color configuration, the $3_c \times 3_c$ state would be lower than the $6_c \times 6_c$, because the forces between two quarks (and between two antiquarks) are attractive when they are of $3_c$ (and of $3_c$), while the forces are repulsive when they are of $6_c$ (and of $6_c$) \cite{6}. However, mixings of the $3_c \times 3_c$ and $6_c \times 6_c$ states in open- and hidden-charm mesons are expected to be much smaller than that of the light sector, because QCD would be rather perturbative at the energy scale of open- and hidden-charm meson masses. Putting aside the structure of color wavefunctions, the first term $\{qq\bar{q}q\}$ in the right-hand-side (r.h.s.) of Eq. (1) seems to be well realized by the observed scalar nonoet, $\sigma(600), \sigma(980), \sigma(1600)$, and $\kappa(800)$, as predicted in Ref. \cite{6}.

X(3872) has been observed in the $\pi^+\pi^- J/\psi$ \cite{2} and $\pi^+\pi^- \pi^0 J/\psi$ \cite{2} channels with opposite $G$-parities. With regard to its spin-parity, $J^P = 1^+$ is favored by its angular analysis \cite{10}. This implies that $X(3872)$ consists of two (approximately) degenerate axial-vector meson states with different $G$-parities, as long as $G$-parity is conserved in these decays as in the well-known strong interactions. It has been observed that this can be realized by molecular states \cite{17} based on a unitarized chiral model and also by hidden-charm tetra-quark mesons corresponding to the last two terms of the r.h.s. of Eq. (1).

Extension of the above open-charm scalar four-quark mesons to hidden-charm ones is straightforward. Their flavor wavefunctions are listed in Table I. Their mass values are estimated very crudely by using a naive quark counting with $\Delta_{cs} = m_c - m_s \simeq m_n - m_D \simeq 1$ GeV and $\Delta_{sn} = m_s - m_n \simeq m_{D_s} - m_D \simeq 100$ MeV, and by taking $m_{F_t} = m_{D_s(2121)} = 2317$ MeV as the input data. Supposing that the above estimate of hidden-charm meson masses is not very far from the true ones, isospin conserving two-body decays which satisfy the OZI rule \cite{18} would be limited. One of these decays is the $\hat{\delta}^c \rightarrow \eta, \pi$, as seen in Table I. However, it is not obvious if $\hat{\delta}^c$ and $\hat{\kappa}^c$ can decay into the $\eta, \eta'$ and $\eta, \eta K$ final states, respectively, because the roughly estimated masses of $\hat{\delta}^c$ and $\hat{\kappa}^c$ are just below the corresponding thresholds. Even if these decays were kinematically allowed, their rates would be much smaller than the rate for $\hat{\delta}^c \rightarrow \eta, \pi$, because of their small phase space volume.
The amplitude for $\Gamma(\hat{A})$ estimate deviation of the overlap from its dimensionless coupling strength.

where $S$, $F$, and $M(A \rightarrow B\pi)$ are the spin of $A$, the momentum of the final particles in the rest frame of $A$, and the amplitude for the decay, respectively. To calculate the amplitude, we use the PCAC (partially conserved axial vector current) hypothesis and a hard pion approximation in the infinite momentum frame, i.e., $|p| \rightarrow \infty$ [19, 20]. (This is an innovation of the old soft-pion technique.) In this approximation, the amplitude is evaluated as

$$M(A \rightarrow B\pi) \simeq \left( \frac{m_A^2 - m_B^2}{J} \right) \langle B|A_\pi|A \rangle,$$

(3)

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Strangeness ($S$) & 1 & 0 \\
\hline
$I = 1$ & $\delta^c \sim |nc| \bar{\sigma}_{c \bar{c}}$ & \\
\hline
$I = \frac{3}{2}$ & $\delta^c \sim |nc| \bar{\sigma}_{c \bar{c}}$ & \\
\hline
$I = 0$ & $\hat{\delta}^c \sim |nc| \bar{\sigma}_{c \bar{c}}$, $\hat{\delta}^{c*} \sim |sc| \bar{\sigma}_{c \bar{c}}$ & \\
\hline
Mass (GeV) & $\sim 3.4$ & $\sim 3.3$ \\
\hline
OZI-allowed Decay & $\eta, K$ & $\eta, \pi$ \\
\hline
Threshold (GeV) & 3.48 & 3.12 \\
\hline
\end{tabular}
\caption{Hidden-charm scalar mesons, their flavor wavefunctions, their crudely estimated mass values, their assumed two-body decay channels which conserve isospin and satisfy the OZI-rule, and their threshold energies.}
\end{table}

\begin{equation}
\Gamma(A \rightarrow B\pi) = \left( \frac{1}{2J_A + 1} \right) \left( \frac{q_c}{8\pi m_A^2} \right) \sum |M(A \rightarrow B\pi)|^2,
\end{equation}

(2)

where $J_A$, $q_c$, and $M(A \rightarrow B\pi)$ are the spin of $A$, the momentum of the final particles in the rest frame of $A$, and the amplitude for the decay, respectively. To calculate the amplitude, we use the PCAC (partially conserved axial vector current) hypothesis and a hard pion approximation in the infinite momentum frame, i.e., $|p| \rightarrow \infty$ [19, 20]. (This is an innovation of the old soft-pion technique.) In this approximation, the amplitude is evaluated as

$$M(A \rightarrow B\pi) \simeq \left( \frac{m_A^2 - m_B^2}{J} \right) \langle B|A_\pi|A \rangle,$$

(3)

\begin{equation}
\langle \eta_c|A_\pi-|\hat{\delta}^{c*} \rangle = \sqrt{2} \langle \eta_c|A_{\pi^0}|\hat{\delta}^{c*} \rangle = -\sqrt{2} \langle \eta_c|A_{\pi^0}|\hat{\delta}^{c*} \rangle = \sqrt{2} \langle D^+_s|A_{\pi^0}|\hat{F}^+_I \rangle.
\end{equation}

(4)

In Eq. (4), we have assumed that energy scale dependence of wavefunction overlap is mild around and above the scale of open-charm meson mass, although it was drastic between the mass scales of the light tetra-quark mesons and the open-charm ones [10, 11]. It is because the quark-gluon coupling would be (rather) perturbative in the region of the energy scale of open-charm tetra-quark meson mass and beyond, while non-perturbative at the scale of light meson masses. The last matrix element in Eq. (4) has been estimated to be $\langle \langle D^+_s|A_{\pi^0}|\hat{F}^+_I \rangle \rangle_{SU_f(4)} \sim 0.09 - 0.13$ from $\Gamma(\hat{F}^+_I \rightarrow D^+_s \pi^0)_{SU_f(4)} \simeq 5 - 10$ MeV in Refs. [10, 11]. Adopting the above value of the asymptotic matrix element as the input data, we obtain

$$\Gamma(\hat{\delta}^c \rightarrow \eta_\pi)_{SU_f(4)} \sim 5 - 10$ MeV,

(5)

where the overlap of spatial wavefunctions is still in the $SU_f(4)$ symmetry limit. However, it is not yet known how to estimate deviation of the overlap from its $SU_f(4)$ symmetry limit in the case of hidden-charm mesons, although such a deviation in the case of open-charm mesons has been estimated [10, 11] [21].

The other two-body decays of hidden-charm mesons which are kinematically allowed would be OZI-rule forbidden, as long as our estimate of their masses is not very far from the true ones. Therefore, to search for hidden-charm scalar tetra-quark scalars, their radiative decays would be important. We study them under the vector meson dominance (VMD) hypothesis in the same way as the open-charm scalar mesons which have been studied in Refs. [10, 11]. The amplitude for the radiative $S \rightarrow V\gamma$ decay is written in the form

$$M(S \rightarrow V\gamma) = F_{\mu\nu}(\gamma) G_{\mu\nu}(V) A(S \rightarrow V\gamma),$$

(6)

where $S$, $F_{\mu\nu}$ and $G_{\mu\nu}$ denote the parent scalar meson, field strengths of photon (\gamma) and vector meson (V), respectively. The amplitude for $S \rightarrow V\gamma$ decay is given by

$$A(S \rightarrow V\gamma) = \sum_{V' = \rho, \omega, \phi, \psi} \frac{X_{V'}(0)}{m_{V'}^2} A(S \rightarrow VV').$$

(7)
TABLE II: Rates for radiative decays of hidden-charm scalar mesons.

| Decay          | Pole | Rate (in MeV) |
|----------------|------|---------------|
| $\kappa^0 \rightarrow K^{*}\gamma$ | $\psi$ | $\sim 0.23$ |
| $\delta^0 \rightarrow \rho\gamma$ | $\psi$ | $\sim 0.22$ |
| $\delta^0 \rightarrow \rho \gamma$ | $\phi$ | $\sim 0.069$ |
| $\sigma^c \rightarrow \omega \gamma$ | $\psi$ | $\sim 0.22$ |
| $\sigma^c \rightarrow \psi \gamma$ | $\omega$ | $\sim 0.007$ |
| $\sigma^{sc} \rightarrow \phi \gamma$ | $\psi$ | $\sim 0.24$ |
| $\sigma^{sc} \rightarrow \psi \gamma$ | $\phi$ | $\sim 0.040$ |

under the VMD, where $X_V(0)$ is the $\gamma V$ coupling strength on the photon-mass-shell and $A(S \rightarrow VV')$ is the $SVV'$ coupling strength. The values of $X_V(0)$ have been provided as $X_\rho = 0.033 \pm 0.003$, $X_\omega = 0.011 \pm 0.001$, $X_\phi = -0.018 \pm 0.004$, $X_\psi = 0.051 \pm 0.012$, in Ref. [22]. The OZI-rule selects a possible vector meson which couples to a photon.

Related $SU_f(4)$ relation among $SVV'$ coupling strengths is given by

$$A(\kappa^{c+} \rightarrow K^{*+}\gamma) = A(\kappa^{c0} \rightarrow K^{*0}\gamma) = A(\delta^{c+} \rightarrow \rho^+\gamma) = A(\delta^{c0} \rightarrow \rho^0\gamma) = -A(\delta^{c-} \rightarrow \omega\gamma) = -A(\delta^{sc+} \rightarrow \phi\gamma) = A(\delta^{sc0} \rightarrow \rho^0\phi) \beta_1,$$

where $\beta_1$ is a parameter which provides an overlap of color and spin wavefunctions at the scale of hidden-charm meson mass. Here we take $|\beta_1| \sim 1/2$ at the scale of charmed tetra-quark meson mass $[10, 11]$, because the quark-gluon coupling is expected to be (rather) perturbative (around and) above the scale of mass of open-charm mesons. At this stage, the spatial wavefunction overlap is assumed to be in the $SU_f(4)$ symmetry limit, as in the above hadronic decays. The last coupling strength in Eq. (8) is estimated to be $[10, 11]$.

$$|A(\delta^{sc0} \rightarrow \rho^0\phi)| \simeq 0.02 \text{ (MeV)}^{-1}$$

from the measured rate $[7]$; $\Gamma(\phi \rightarrow a_0(980)\gamma)_{\text{exp}} = 0.32 \pm 0.03 \text{ keV}$.

Inserting Eq. (6) with Eq. (5) into Eq. (2), using the values of $X_V(0)$ listed above and adopting Eq. (9) as the input data, we can estimate rates for radiative decays of hidden-charm scalar mesons. The results are listed in Table II. As seen in the table, the rates for decays including $\psi$ in the final state are small because of their small phase space volumes. In particular, the $\delta^{c} \rightarrow \psi \gamma$ decay is more strongly suppressed due to the small $\gamma \omega$ coupling. Therefore, $\delta^{c}$ decays dominantly into the $\gamma \omega$ final state, and the $\delta^{c} \rightarrow \psi \gamma$ decay might not disturb the $\chi_{c0} \rightarrow \psi \gamma$ decay, even if the true mass of $\delta^{c}$ were close to that of $\chi_{c0}$. Our results are quite different from those of a typical unitarized chiral model $[22]$ with the same theoretical basis as the one $[17]$ mentioned before. It is because, in this type of chiral model, the non-loop contribution, for example, $X(3700) \rightarrow \psi \gamma \omega \rightarrow \gamma \omega$, where $\psi$ means a virtual $\psi$, has not been considered, in contrast with the present picture in which the corresponding $\sigma^{c} \rightarrow \psi \gamma \omega \rightarrow \gamma \omega$ is dominant in $\delta^{c}$ decays as discussed above.

So far we have assumed intuitively that $D^0_{s0}(2317)$ is the $I_A = 0$ component of $[cn] [s\bar{n}]_{I=1}$ with $\tilde{3}_c \times 3_c$, because the forces between two quarks (and two antiquarks) are attractive when they are of $3_c$ (and $\bar{3}_c$). Under this assumption, we have understood its narrow width and its decay property $[10, 11]$. Therefore, the low lying hidden-charm meson are expected to be of $\bar{3}_c \times 3_c$. To confirm that our intuitive assumption is feasible, two photon collision experiment would be useful. To see this, we denote the $I = 1$ hidden-charm scalar meson with $6_c \times 6_c$ as $\delta^{cs}$, and decompose $\delta^{c}$ and $\delta^{cs}$ as

$$|\delta^{c}\rangle = \frac{1}{4} \left[ \chi \left[ \frac{1}{3} \tilde{3}_c \left[ \frac{1}{3} \bar{3}_c \right]_{1c} \right] \right]_{1c} = -\sqrt{\frac{1}{4}} \times \sqrt{\frac{1}{3}} \left\{ \{c\bar{c}\} \frac{1}{3} \frac{1}{3} (\bar{n}n) \frac{1}{3} \right\}_{1c} - \frac{3}{4} \times \sqrt{\frac{1}{3}} \left\{ \{c\bar{c}\} \frac{1}{3} \frac{1}{3} (\bar{n}n) \frac{3}{3} \right\}_{1c} \frac{1}{3}$$

and

$$|\delta^{cs}\rangle = \frac{1}{4} \left[ \chi \left[ \frac{1}{3} \bar{3}_c \left[ \frac{2}{3} \bar{3}_c \right]_{1c} \right] \right]_{1c} = -\sqrt{\frac{3}{4}} \times \sqrt{\frac{2}{3}} \left\{ \{c\bar{c}\} \frac{1}{3} \frac{1}{3} (\bar{n}n) \frac{1}{3} \right\}_{1c} + \frac{3}{4} \times \sqrt{\frac{2}{3}} \left\{ \{c\bar{c}\} \frac{1}{3} \frac{1}{3} (\bar{n}n) \frac{3}{3} \right\}_{1c} \frac{1}{3}$$
respectively. From the above equations, it is seen that the overlap of color and spin wavefunctions between \( \hat{\delta}^{c*} \) and \( \eta_c \pi \) states is larger by a factor \( \sqrt{6} \) than that between \( \hat{\delta}^c \) and \( \eta_c \pi \) states. Therefore, if the spatial wavefunction overlap is not very much different from each other in both cases, the rate for the \( \hat{\delta}^{c*} \rightarrow \eta_c \pi \) decay would be larger by about a factor 6 or more than that for the \( \hat{\delta}^c \rightarrow \eta_c \pi \), because it is assumed that \( m_{\hat{\delta}^{c*}} \) is larger than \( m_{\hat{\delta}^c} \). Therefore, it is expected that the cross section for the \( \gamma \gamma \rightarrow \eta_c \pi^+ \pi^- \) reaction would have a narrower (\( \lesssim 10 \) MeV) peak at \( m_{\hat{\delta}^c} \sim 3.3 \) GeV and another broader peak arising from \( \hat{\delta}^{c*} \) at a higher energy, if both of \( \hat{\delta}^c \) and \( \hat{\delta}^{c*} \) exist and \( m_{\hat{\delta}^{c*}} > m_{\hat{\delta}^c} \). If \( m_{\hat{\delta}^c} < m_{\hat{\delta}^{c*}} \), in contrast with our intuitive expectation, we need more precise information of \( m_{\hat{\delta}^{c*}} \), because the gap between \( m_{\hat{\delta}^c} \) and the \( \eta_c \pi \) threshold is not very large. If \( m_{\eta_c} + m_\pi < m_{\hat{\delta}^{c*}} < m_{\hat{\delta}^c} \), the width of \( \hat{\delta}^{c*} \) would be sensitive to \( m_{\hat{\delta}^c} \) and not necessarily broader than that of \( \hat{\delta}^c \), because the phase space volume under consideration is sensitive to it. If the mass of \( \hat{\delta}^{c*} \) is lower than \( \eta_c \pi \) threshold, the cross section would have only a peak from \( \hat{\delta}^c \) around and below \( m_{\hat{\delta}^c} \).

In summary we have studied hidden-charm scalar \([cq][\bar{c}q]\) mesons by extending the open-charm scalar \([cq][\bar{q}q]\) mesons which have been studied previously and then investigated their isospin conserving two-body decay and radiative decays. The \( \hat{\delta}^c \rightarrow \eta_c \pi \) decay will be only one two-body decay which conserves isospin and satisfies the OZI rule, if their estimated mass values are not very far from the true ones. Radiative decays of hidden-charm mesons also have been studied, because radiative channels would play an important role in search for the other hidden-charm scalar tetra-quark mesons.

To confirm our intuitive expectation that the mass of \([cn][\bar{c}n]\) with \( \bar{3}_c \times 3_c \) is lower and its width is narrower than the corresponding ones with \( 6_c \times 6_c \), it is awaited that experiments measure the cross section for the \( \gamma \gamma \rightarrow \eta_c \pi \) reaction and find two peaks: one is around \( \sim 3.3 \) GeV and narrower, while the other is beyond it and broader.

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