On Fundamental Algebraic Attributes of $\omega - Q$ – Fuzzy Subring, Normal Subring and Ideal

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Abstract

In this paper, the introduced the new notion of on Fundamental Algebraic attributes of $\omega - Q - FSR$ and $\omega - Q - F1$ are defined and discussed. The Homomorphism of $\omega - Q - FSR$, $\omega - Q - FNSR$ and $\omega - Q - F1$ and their inverse images have been obtained. Some related results have been discussed in their paper.

I. Introduction

The pioneering work of I. A Zadeh on fuzzy subsets of a set in\cite{1}, A. B Chakranarty et al. invented the theory of fuzzy homomorphism and algebraic structures in 1993\cite{1}. The concept of Prime fuzzy ideals in ring was established by T.K. Mukhrjee et al. in 1989\cite{2}. R.N. Dixit et al. introduced the concept of Fuzzy rings in 1992\cite{3}. In 1996, explored the notion of K-fuzzy ideals in semi rings by B. K Chang et al. A. Prasanna et al.\cite{4}, introduced the concept on Fundamental Algebraic attributes of $\chi$ – Fuzzy Subring, Normal Subring and Ideal. R. Kumar, described the new notion of fuzzy algebra in 1993\cite{5}. In 1982, Wang-Jin Liu\cite{6}, the concept of fuzzy invariant subgroups and fuzzy ideals. A. Rosenfeld\cite{7}, explored the new notation of fuzzy groups in 1971. P.K. Sharma\cite{8}, explored the $\alpha$ - Anti Fuzzy Subgroups in 2012. In addition , more recent development, $\alpha$- Fuzzy subgroups in 2013\cite{9}. D.S. Malik et al. derived from the extension of fuzzy subrings and fuzzy ideals in 1992\cite{10}. In 1992, R. Kumar\cite{11}, introduced the new notion of Certain fuzzy ideals of rings. A. Prasanna et.al\cite{12}, introduced the new concept on Elementary Algebraic characteristic of $\omega$ – Fuzzy Subring, Normal Subring and Ideal in 2020. V. Veeramani et al. derived from the Some Properties of Intuitionistic Fuzzy Normal Subrings in 2010\cite{13}. T.K. Mukhrjee et al. proposed by the concept of on fuzzy ideals of a ring in 1987\cite{14}. A. Salaraju and R. Nagarajan\cite{15}, described the notation of a structure and Construction of Q-fuzzy Groups in 2009.

The research article is arranged as follows, section II contains the elementary basic concept of definitions related to the results which are thoroughly crucial to this research. In section III, we introduce fundamental algebraic attributes of $\omega - Q$ –fuzzy subring($\omega - Q - FSR$) and ideal($\omega - Q - F1$) and section IV, describe the algebraic structures on homomorphism of $\omega - Q$ –fuzzy subrings($\omega - Q - FSR$) normal subrings($\omega - Q - FNSR$) and ideals ($\omega - Q - F1$).

II. Preliminaries

Definition: 2.1 [7]
Let R be a ring. A function $A: R \rightarrow [0,1]$ is said to be a FSR of R if
\[ (i) A(x - y) \geq \min\{\mu_A(x), \mu_A(y)\} \]
\[ (ii) A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}, \forall x, y \in R. \]

Definition: 2.2 [13]
A FSR A of a ring R is said to be a FNSR of R if
\[ A(xy) = A(yx), \forall x, y \in R. \]

Definition: 2.3 [14]
Let R be a ring. A function $A: R \rightarrow [0,1]$ is said to be a
\[ (a) \text{ Fuzzy Left Ideal of R if} \]
\[ (i) A(x - y) \geq \min\{A(x), A(y)\} \]
\[ (ii) A(xy) \geq A(y), \forall x, y \in R \]
\[ (b) \text{ Fuzzy Right Ideal of R if} \]
\[ (i) A(x - y) \geq \min\{A(x), A(y)\} \]
\[ (ii) A(xy) \geq A(x), \forall x, y \in R \]
\[ (c) \text{ Fuzzy Ideal of R if} \]

Keywords:
Fuzzy set (FS), Fuzzy subrings (FSR), Fuzzy normal subring (FNSR), $\omega - Q$ – Fuzzy subrings ($\omega - Q - FSR$), $\omega - Q$ –fuzzy normal subring ($\omega - Q - FNSR$), Fuzzy ideal(FI), $\omega - Q$ –fuzzy ideal ($\omega - Q - F1$).

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\( (i) A(x - y) \geq \min\{A(x), A(y)\} \\
(ii) A(xy) \geq \max\{A(x), A(y)\}, \forall x, y \in R \)

**Theorem 2.4** [6]

If \( A \) be a FSR of the ring \( R \) then

(i) \( A(0) \geq A(x) \)

(ii) \( A(x) = A(x), \forall x \in R \)

(iii) If \( R \) is ring with unity 1, then \( A(1) \geq A(x), \forall x \in R \).

**Definition 2.5** [1]

Let \( X \) and \( Y \) be two non-empty sets and \( f : X \rightarrow Y \) be a mapping. Let \( A \) and \( B \) be FS of \( X \) and \( Y \) respectively. Then the image of \( A \) under the map \( f \) is denoted by \( f(A) \) and is defined as

\[
\{\sup\{A(x) : x \in f^{-1}(y)\}, \forall y \in Y \} = 0; \text{ otherwise}
\]

Also the pre-image of \( B \) under \( f \) is denoted by \( f^{-1}(B) \) and defined

\[
f^{-1}(B)(x) = Bf(x), \forall x \in X.
\]

**Definition 2.6** [1]

The mapping \( f : R_1 \rightarrow R_2 \) from the ring \( R_1 \) into a ring \( R_2 \) is called a ring homomorphism if

(i) \( f(x + y) = f(x) + f(y) \)

(ii) \( f(xy) = f(x)f(y), \forall x, y \in R_1 \).

**Definition 2.7** [14]

Let \( Q \) and \( G \) be two sets and \( \mu \) be a mapping. A mapping \( \mu : G \times Q \rightarrow [0, 1] \) is called \( Q \)-FS in \( G \). For any \( Q \)-FS \( \mu \) in \( G \) and \( t \), we define the set \( U(\mu; t) = \{x \in G / \mu(x, q) \geq t, q \in Q\} \) which is named an upper cut of \( \mu \) and may be used to the characterization of \( \mu \).

**III. On Fundamental Algebraic attributes of \( \omega - Q \)-Fuzzy Subring** [FSR] and \( \omega - Q \)-Ideal of \( \omega - F \)

**Definition 3.1**

Let \( \hat{A} \) be a fuzzy set (FS) of a ring \( \tau \) and \( Q \)-fuzzy subset (FSB) of a set \( \tau \). Let \( \omega \in [0, 1] \). Then the \( Q \)-fuzzy set \( \hat{A}^{\omega} \) of \( \tau \) is called the \( \omega - Q \)-fuzzy subset (FS) of \( \tau \) with respect to FSB \( \hat{A} \) and is defined by

\[
\hat{A}^{\omega}(\theta, q) = \{\hat{A}(\theta, q) \wedge \omega\}, \quad \forall \omega \in [0, 1] \text{ and } q \in Q.
\]

**Definition 3.2**

Let \( \hat{A} \) be a FS of a ring \( \tau \) and \( Q \)-fuzzy subset (FSB) of a set \( \tau \). Then \( \hat{A} \) is called \( \omega - Q \)-Fuzzy Subring (FSR) of \( \tau \) if \( \hat{A}^{\omega} \) is FSR of \( \tau \) i.e. if the following conditions hold

(i) \( \hat{A}^{\omega}((\theta - \varphi), q) \geq \{\hat{A}^{\omega}(\theta, q) \wedge \hat{A}^{\omega}(\varphi, q)\} \)

(ii) \( \hat{A}^{\omega}(\theta \varphi, q) \geq \{\hat{A}^{\omega}(\theta, q) \wedge \hat{A}^{\omega}(\varphi, q)\}, \forall \theta, \varphi \in \tau \text{ and } q \in Q.\)

In other words, \( \hat{A} \) is \( \omega - Q \)-FSR of \( \tau \) if \( \hat{A}^{\omega} \) is FSR of \( \tau \).

**Definition 3.3**

Let \( \hat{A} \) be a FS of a ring \( \tau \) and \( Q \)-fuzzy subset (FSB) of a set \( \tau \). Let \( \omega \in [0, 1] \). Then \( \hat{A} \) is called \( \omega - Q \)-Fuzzy Left Ideal of \( \tau \) (FLI) if

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\[\begin{align*}
\mathcal{A}\cap \psi &= \{g(\bar{\mathcal{A}}(\theta, q) \land \omega) : (\theta, q) \in \omega\} \\
\mathcal{A}\cap \psi &= \{g(\bar{\mathcal{A}}(\theta, q) \land \omega) : (\theta, q) \in \omega\} \\
\Rightarrow g(\bar{\mathcal{A}}\omega) &= \{g(\bar{\mathcal{A}})\} \omega. \forall \omega \in [0,1] \text{ and } q \in Q.
\end{align*}\]

**Theorem 3.8**

Let \(\mathcal{A}\) be FSR of a ring \(\tau\), and \(Q\) - fuzzy subset (FSB) of a set \(\tau\), then \(\mathcal{A}\) is also \(\omega - Q\) - FSR of \(\tau\).

**Proof:**

Let \(\theta, \varphi \in \tau\) be any element of the ring \(\tau\), and \(Q\) - fuzzy subset (FSB) of a set \(\tau\).

Now,

\[(\mathcal{A}\omega)((\theta - \varphi), q) = \mathcal{A}(\theta, \varphi) \land \mathcal{A}\omega(q) \land \omega \geq \mathcal{A}(\theta, \varphi) \land \mathcal{A}\omega(q) \land \omega.
\]

\[\Rightarrow \mathcal{A}\omega((\theta - \varphi), q) \geq \mathcal{A}(\theta, \varphi) \land \mathcal{A}\omega(q).
\]

\[(\mathcal{A}\omega)((\theta - \varphi), q) \geq \mathcal{A}(\theta, \varphi) \land \mathcal{A}\omega(q) \land \mathcal{A}\omega(q) \land \omega \geq \mathcal{A}(\theta, \varphi) \land \mathcal{A}\omega(q) \land \omega.
\]

\[\Rightarrow \mathcal{A}\omega((\theta - \varphi), q) \geq \mathcal{A}(\theta, \varphi) \land \mathcal{A}\omega(q), \forall \omega \in [0,1] \text{ and } q \in Q.
\]

Therefore, \(\mathcal{A}\) is \(\omega - Q\) - FSR of \(\tau\).

**Proposition 3.9**

The converse of above theorem (3.8) need not be a true.

**Example 3.9.1**

Let us consider the ring \((Z_{5}, +_{5}, \times_{5})\), where \(Z_{5} = \{0,1,2,3,4,5\}\).

Define the \(Q\) - fuzzy set \(\mathcal{A}\) of \(Z_{5}\) by

\[
\mathcal{A}(\theta, q) = \begin{cases} 
0.7; & \text{if } x = 0 \\
0.5; & \text{if } x = 1, 3 \\
0.2; & \text{if } x = 2, 4.
\end{cases}
\]

It is easy to verify that \(\mathcal{A}\) is not \(\omega - Q\) - FSR of \(Z_{5}\).

However, if we take \(\omega = 0.1\), then \(\mathcal{A}\omega(\theta, q) = 0.1, \forall \theta \in Z_{5}\).

Now, it can be easily proved that \(\mathcal{A}\omega\) is FSR of \(Z_{5}\) and hence \(\mathcal{A}\) is \(\omega - Q\) - FSR of \(Z_{5}\).

**Lemma 3.10**

Let \(\mathcal{A}\) be a FS of the ring \(\tau\), and \(Q\) - fuzzy subset (FSB) of a set \(\tau\). Let \(\omega \leq L\), where \(L = \mathcal{A}(\theta, q); \forall \theta \in \tau \text{ and } q \in Q\). Then \(\mathcal{A}\) is \(\omega - Q\) - FSR of \(\tau\).

**Proof:**

Let \(\mathcal{A}\) be a FS of the ring \(\tau\), \(Q\) - fuzzy subset (FSB) of a set \(\tau\) and \(\omega \leq L\).

Since \(\omega \leq L \Rightarrow L \geq \omega\)

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Let $\tilde{A}$ is FLI of a ring $\tau$, and $Q$ — fuzzy subset (FSb) of a set $\tau$ , then $\tilde{A}$ is also $\omega-Q-\mathrm{FLI}$ of $\tau$.

Proof:

In this theorem(3.12), we need only to prove that
\[
\tilde{A}^\omega(\theta \varphi, q) \geq \tilde{A}^\omega(\varphi, q), \forall \theta, \varphi \in \tau \text{ and } q \in Q.
\]
\[
\tilde{A}^\omega(\theta \varphi, q) = \{\tilde{A}(\theta \varphi, q), \omega\}
\geq \{\tilde{A}(\theta, q) \land \omega\}
= \tilde{A}^\omega(\theta, q)
\]
Implies that $\tilde{A}^\omega(\varphi, q) \geq \tilde{A}(\theta, q), \forall \theta, \varphi \in \tau \text{ and } q \in Q$.
\[
\therefore \tilde{A} \text{ is } \omega - Q - \mathrm{FLI} \text{ of } \tau.
\]

Lemma: 3.14

Let $\tilde{A}$ is FRI of a ring $\tau$, and $Q$ — fuzzy subset (FSb) of a set $\tau$ then $\tilde{A}$ is also $\omega - Q - \mathrm{FRI}$ of $\tau$.

Proof:

In this theorem(3.14), we need only to prove that
\[
\tilde{A}^\omega(\theta \varphi, q) \geq \tilde{A}^\omega(\theta, q), \forall \theta, \varphi \in \tau \text{ and } q \in Q.
\]
\[
\tilde{A}^\omega(\theta \varphi, q) = \{\tilde{A}(\theta \varphi, q), \omega\}
\geq \{\tilde{A}(\theta, q) \land \omega\}
= \tilde{A}^\omega(\theta, q)
\]
Implies that $\tilde{A}^\omega(\theta \varphi, q) \geq \tilde{A}^\omega(\theta, q), \forall \theta, \varphi \in \tau \text{ and } q \in Q$.
\[
\therefore \tilde{A} \text{ is } \omega - Q - \mathrm{FRI} \text{ of } \tau.
\]

Theorem: 3.15

Let $\tilde{A}$ is FI of a ring $\tau$, and $Q$ — fuzzy subset (FSb) of a set $\tau$ then $\tilde{A}$ is also $\omega - Q - \mathrm{FI}$ of $\tau$.

Proof:

Follows from Lemma (3.13) and Lemma (3.14)

The IV. Algebraic Structures on Homomorphism of $\omega$ — Fuzzy Subrings, Normal Subrings and Ideals

Theorem: 4.1

Let $g: \tau_1 \rightarrow \tau_2$ be a ring homomorphism from the ring $\tau_1$ into a ring $\tau_2$, and $Q$ — fuzzy subset (Fsb) of a set $\tau$. Let $\mathfrak{B}$ be $\omega - Q - \mathrm{FSR}$ of $\tau_2$. Then $g^{-1}(\mathfrak{B})$ is $\omega - Q - \mathrm{FSR}$ of $\tau_1$.

Proof:

Let $\mathfrak{B} \omega - Q - \mathrm{FSR}$ of $\tau_2$. Let $\theta_1, \theta_2 \in \tau_1$ be any element and $Q$ — fuzzy subset (Fsb) of a set $\tau$.

Then
\[
(i) g^{-1}(\mathfrak{B}^\omega)((\theta_1 - \theta_2), q) = \mathfrak{B}^\omega(g((\theta_1 - \theta_2), q))
\geq \{\mathfrak{B}^\omega(g(\theta_1, q)) \land \mathfrak{B}^\omega(g(\theta_2, q))\}
= \{g^{-1}(\mathfrak{B}^\omega)(\theta_1, q) \land g^{-1}(\mathfrak{B}^\omega)(\theta_2, q)\}
\geq g^{-1}(\mathfrak{B}^\omega)((\theta_1 - \theta_2), q) \geq g^{-1}(\mathfrak{B}^\omega)(\theta_1, q) \land g^{-1}(\mathfrak{B}^\omega)(\theta_2, q).
\]
\[
(ii) g^{-1}(\theta_1 \theta_2, q) = \mathfrak{B}^\omega(g(\theta_1 \theta_2, q))
\geq \{\mathfrak{B}^\omega(g(\theta_1, q)) \land \mathfrak{B}^\omega(g(\theta_2, q))\}
= \{g^{-1}(\mathfrak{B}^\omega)(\theta_1, q) \land g^{-1}(\mathfrak{B}^\omega)(\theta_2, q)\}
\geq g^{-1}(\mathfrak{B}^\omega)(\theta_1 \theta_2, q) \geq g^{-1}(\mathfrak{B}^\omega)(\theta_1, q) \land g^{-1}(\mathfrak{B}^\omega)(\theta_2, q).
\]

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Let $g: \tau_1 \rightarrow \tau_2$ be a ring homomorphism from the ring $\tau_1$ into a ring $\tau_2$, and $Q$ be a fuzzy subset (FSb) of a set $\tau$. Let $\delta$ be a ring subset of $Q$. Then $g^{-1}(\delta)$ is a ring subset of $\tau_2$.

Proof:

It can be easily to follows from the above theorem (4.3) and Theorem (4.4)

**Theorem: 4.6**

Let $g: \tau_1 \rightarrow \tau_2$ be Surjective ring homomorphism and $\tilde{A}$ be $\omega - Q$ - FSR of $\tau_1$, and $\hat{Q}$ is $\omega - Q$ - fuzzy subset (FSb) of a set $\tau$. Then $\hat{g}(\tilde{A})$ is $\omega - Q$ -FSR of $\tau_2$.

Proof:

Let $\tilde{A}$ be $\omega - Q - Q$ -FSR of $\tau_1$.

Let $\phi_1, \phi_2 \in \tau_2$ be any element, and $Q$ be a fuzzy subset (FSb) of a set $\tau$. Then there exist some $\theta_1, \theta_2 \in \tau_1$ and $\tilde{q} \in Q$ such that $g(\theta_1) = \phi_1$ and $g(\theta_2) = \phi_2$. (Since that $\theta_1, \theta_2$ need not be unique)

(i) $g(\tilde{A})((\phi_1 - \phi_2), q) = (g(\tilde{A}))(\phi_1(\phi_2), q)$

(ii) $g(\tilde{A})((\phi_1 - \phi_2), q) \geq \tilde{A}(\phi_1 - \phi_2, q)\land \omega$.

Thus implies that $\tilde{A}(\phi_1 - \phi_2, q)\land \omega$ is FSR of $\tau_2$ and hence $g(\tilde{A})$ is $\omega - Q$ -FSR of $\tau_2$.

**Theorem: 4.7**

Let $g: \tau_1 \rightarrow \tau_2$ be Surjective ring homomorphism and $\tilde{A}$ be $\omega - FNSR$ of $\tau_1$, and $\hat{Q}$ be $\omega - Q$ - fuzzy subset (FSb) of a set $\tau$. Then $\hat{g}(\tilde{A})$ is $\omega - Q$ - FNSR of $\tau_2$.

Proof:

Let $\tilde{A}$ be $\omega - Q - Q$ -FSR of $\tau_1$.

Let $\phi_1, \phi_2 \in \tau_2$ be any element, and $Q$ be a fuzzy subset (FSb) of a set $\tau$. Then there exist some $\theta_1, \theta_2 \in \tau_1$ and $q \in Q$ such that $g(\theta_1) = \phi_1$ and $g(\theta_2) = \phi_2$. (Since that $\theta_1, \theta_2$ need not be unique)

In this view of theorem(6.6), we need only to prove that

$$g(\hat{A})^\omega(\phi_1, \phi_2, q) = g(\tilde{A}^\omega)(g(\theta_1, q)g(\theta_2, q))$$

Thus implies that $g(\tilde{A})$ is $\omega - Q$ - FNSR of $\tau_2$.

**Theorem: 4.8**

Let $g: \tau_1 \rightarrow \tau_2$ be bijective ring homomorphism and $\tilde{A}$ be $\omega - Q$ -FLI of $\tau_1$, and $\hat{Q}$ be $\omega - Q$ - fuzzy subset (FSb) of a set $\tau$. Then $\hat{g}(\tilde{A})$ is $\omega - Q$ - FLI of $\tau_2$.

Proof:

Let $\tilde{A}$ be $\omega - Q - Q$ -FLI of $\tau_1$.

Let $\phi_1, \phi_2 \in \tau_2$ be any element, and $Q$ be a fuzzy subset (FSb) of a set $\tau$. Then there exist some $\theta_1, \theta_2 \in \tau_1$ such that $g(\theta_1, q) = (\phi_1, q)$ and $g(\theta_2, q) = (\phi_2, q)$.

In this view of theorem(6.6), we need only to prove that

$$g(\tilde{A})^\omega(\phi_1, \phi_2, q) \geq \tilde{A}(\phi_1, \phi_2, q)\land \omega$$

Thus implies that $g(\tilde{A})$ is $\omega - Q$ - FLI of $\tau_2$.

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Theorem 4.9

Let \( g: \tau_1 \rightarrow \tau_2 \) be bijective ring homomorphism and \( \tilde{A} \) be \( \omega - Q \) – FRI of \( \tau_1 \), and \( Q \) – fuzzy subset (FSb) of a set \( \tau \). Then \( g(\tilde{A}) \) is \( \omega - Q \) – FRI of \( \tau_2 \).

Proof:

In this view of prof it can be obtained similar to theorem(4.8)

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Conflict of interest

All authors declare no conflict of interest in this paper.