Flat direction MSSM (A-term) Inflation

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Abstract. The Minimal Supersymmetry Standard Model contains several hundreds of D- and F-flat directions that are lifted by soft susy breaking terms as well as non-renormalizable terms. In a recent paper we find that only two of these directions, \( LLe \) and \( udd \) can accommodate inflation. The model predicts more than \( 10^3 \) e-foldings with an inflationary scale of \( H_{inf} \sim 1 \) GeV, provides a tilted spectrum compatible with WMAP3, and a negligible tensor perturbation. The reheating temperature could be as low as \( T_{rh} \sim 1 \) TeV. The model is stable under radiative as well as supergravity corrections, although a significant finetuning has to be imposed on the ratio of two parameters of the model. The RGE equations allow us to relate the model parameters with slepton and gaugino masses explorable in the LHC, while the neutralino in this model could well be within reach of present dark matter searches in the laboratory.

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INTRODUCTION

Decades of research have gone by with the purpose of finding a connection between the early universe and the fundamental interactions of particle physics. There seems to be consensus nowadays within the community that the early universe is best described by a short period of inflation, which sets up the initial conditions for the subsequent hot Big Bang. In the absence of a theory of inflation, cosmologists have proposed a plethora of different models more or less based of fundamental theories of particle interactions. In spite of the tremendous growth in data provided by the recent revolution in cosmological observations, from the cosmic microwave background to the distribution of matter of large scales, we still do not know much about the true model of inflation. In particular, we don’t know what was the fundamental scale at which it took place, whether at GUT scales (of order \( 10^{16} \) GeV) or at EW scales (of order 100 GeV), while both are still compatible with present observations. On the other hand, particle physicists have searched for decades for consistent theories of (mostly supersymmetric) extensions of the Standard Model of particle interactions. With the imminent advent of the Large Hadron Collider at CERN we will finally explore the symmetry breaking sector of the electroweak theory and perhaps discover new particles associated with the longsoughtafter extensions. It is thus natural to search for theories of inflation based on the low energy supersymmetric Standard Model. These theories typically have too large masses and couplings to sustain inflation, except along certain well defined directions in the scalar potential that are D- and F-flat. However, not all of these directions remain flat; most of them get lifted by radiative and supergravity corrections, as well as by soft-susy breaking and higher order terms in the superpotential.

In a recent paper [1], we have proposed a model of inflation based on the \( udd \) and \( LLe \).
flat directions of Minimally Supersymmetric Standard Model (for a review of MSSM flat directions, see [2, 3]). In this model the inflaton is a gauge invariant combination of either squark or slepton fields. For a choice of the soft SUSY breaking parameters $A$ and the inflaton mass $m_\phi$, the potential along the flat $uud$ and $Le$ directions is such that there is a period of slow roll inflation of sufficient duration to provide the observed spectrum of CMB perturbations and an unambiguous prediction of the spectral tilt. In the inflationary part of the MSSM potential the second derivative is vanishing and the slow roll phase is driven by the third derivative of the potential.

MSSM inflation occurs at a very low scale with $H_{\text{inf}} \sim 1 - 10$ GeV and with field values much below the Planck scale $\phi_0 \sim 10^{14} - 10^{15}$ GeV. Hence it stands in strong contrast to the conventional inflation models which are based on ad hoc gauge singlet fields and often employ field values close to Planck scale (for a review, see [4]). In such models the inflaton couplings to SM physics are unknown. As a consequence much of the post-inflationary evolution, such as reheating, thermalization, generation of baryon asymmetry and cold dark matter, is not calculable from first principles. The great virtue of MSSM inflation based on flat directions is that the inflaton couplings to Standard Model particles are known and, at least in principle, measurable in laboratory experiments such as LHC or a future Linear Collider.

However, as in almost all inflationary models, a fine tuning of the initial condition is needed to place the flat direction field $\phi$ to the immediate vicinity of the saddle point $\phi_0$ at the onset of inflation. In addition, there is the question of the stability of the saddle point solution and of the existence of a slow roll regime. These are issues that we wish to address in detail in the present paper. Both supergravity and radiative corrections to the flat direction inflaton potential must be considered. Hence we need to write down and solve the renormalization group (RG) equations for the MSSM flat directions of interest. RG equations are also needed to scale the model parameters, such as the inflaton mass, down to TeV scale; since the inflaton mass is related either to squark or slepton masses, it could be measured by LHC or a future Linear Collider.

Because the inflaton couplings to ordinary matter are known, inflaton decay and thermalization are processes that can be computed in an unambiguous way. Unlike in many models with a singlet inflaton, in MSSM inflation the potential relevant for decay and thermalization cannot be adjusted independently of the slow roll part of the potential.

THE MODEL

Let us begin by considering a flat direction $\phi$ with a non-renormalizable superpotential term, $W = (\lambda_n/n)(\Phi^n/M_p^{n-3})$, where $\Phi$ is the superfield which contains the flat direction. Within MSSM all the flat directions are lifted by $n = 9$ non-renormalizable operator [2]. Together with the corresponding $A$-term and the soft mass term, it gives rise to the following scalar potential for $\phi$:

$$V = \frac{1}{2} m_\phi^2 \phi^2 + A \cos(n\theta + \theta_A) \frac{\lambda_n \phi^n}{n M_p^{n-3}} + \frac{\lambda_n^2}{M_p^{2(n-3)}} \phi^{2(n-1)}.$$

(1)
where \( m_\phi \) is the soft SUSY breaking mass for \( \phi \). Here \( \phi \) and \( \theta \) denote the radial and the angular coordinates of the complex scalar field \( \Phi = \phi \exp[i\theta] \) respectively, while \( \theta_A \) is the phase of the \( A \)-term (thus \( A \) is a positive quantity with a dimension of mass). Note that the first and third terms in Eq. (11) are positive definite, while the \( A \)-term leads to a negative contribution along the directions where \( \cos(n\theta + \theta_A) < 0 \).

The maximum impact from the \( A \)-term is obtained when \( \cos(n\theta + \theta_A) = -1 \) (which occurs for \( n \) values of \( \theta \)). Along these directions \( V \) has a secondary minimum at

\[
\phi = \phi_0 \sim (m_\phi M_P^{n-3})^{1/n-2} \ll M_P \text{ (the global minimum is at } \phi = 0) \text{, provided that}
\]

\[
A^2 \geq 8(n - 1)m_\phi^2. \tag{2}
\]

At this minimum the curvature of the potential along the radial direction is \(+m_\phi^2\) (it is easy to see that the curvature is positive along the angular direction, too), and the potential reduces to: \( V \sim m_\phi^2 \phi_0^2 \sim m_\phi^2 (m_\phi M_P^{n-3})^{2/(n-2)} \). Now consider the situation where the flat direction is trapped in the false minimum \( \phi_0 \). If its potential energy, \( V \), dominates the total energy density of the Universe, a period of inflation is obtained. The Hubble expansion rate during inflation will then be \( H_{\text{inf}} \sim (m_\phi \phi_0/M_P) \sim m_\phi (m_\phi/M_P)^{1/(n-2)} \). Note that \( H_{\text{inf}} \ll m_\phi \), which implies that the potential is too steep at the false minimum and \( \phi \) cannot climb over the barrier which separates the two minima just with the help of quantum fluctuations during inflation.

The potential barrier disappears when the inequality in Eq. (2) is saturated, i.e. when \( A^2 = 8(n - 1)m_\phi^2 \). Then both the first and second derivatives of \( V \) vanish at \( \phi_0 \), i.e. \( V'(\phi_0) = 0 \), \( V''(\phi_0) = 0 \), and the potential becomes very flat along the real direction. Around \( \phi_0 \) the field is stuck in a plateau with potential energy

\[
V(\phi_0) = \frac{(n - 2)^2}{2n(n - 1)} m_\phi^2 \phi_0^2, \quad \phi_0 = \left( \frac{m_\phi M_P^{n-3}}{\sqrt{2n-2}} \right)^{1/(n-2)}. \tag{3}
\]

However, although the second derivative of the potential vanishes, the third does not; instead \( V'''(\phi_0) = 2(n - 2)^2 m_\phi^2/\phi_0 \). Around \( \phi = \phi_0 \) we can thus expand the potential as \( V(\phi) = V(\phi_0) + (1/3!)V'''(\phi_0)(\phi - \phi_0)^3 \). Hence, in the range \( [\phi_0 - \Delta \phi, \phi_0 + \Delta \phi] \), where \( \Delta \phi \sim H_{\text{inf}}^2/V'''(\phi_0) \sim (\phi_0^3/M_P^3) \gg H_{\text{inf}} \), the real direction has a flat potential. We can now solve the equation of motion for the \( \phi \) field in the slow-roll approximation,

\[
3H \dot{\phi} = -(V'''(\phi_0)/2)(\phi - \phi_0)^2. \]

Note that the field only feels the third derivative of the potential. Thus, if the initial conditions are such that the flat direction starts in the vicinity of \( \phi_0 \) with \( \dot{\phi} \approx 0 \), then a sufficiently large number of e-foldings of the scale factor can be generated. In fact, quantum fluctuations along the tachyonic direction \([5]\) will drive the field towards the minimum. However, quantum diffusion is stronger than the classical force, \( H_{\text{inf}}/2\pi > \dot{\phi}/H_{\text{inf}} \), for \((\phi_0 - \phi)/\phi_0 \leq (m_\phi \phi_0^2/M_P^3)^{1/2} \), but from then on, the evolution is determined by the usual slow roll. A rough estimate of the number of e-foldings is then given by

\[
N_e(\phi) = \int \frac{H_{\text{inf}}d\phi}{\dot{\phi}} \simeq \frac{\phi_0^3}{2n(n - 1)M_P^2(\phi_0 - \phi)}. \tag{4}
\]
FIGURE 1. The colored curves depict the full potential, where \( V(x) \equiv V(\phi)/(0.5 \, m_{\phi}^2 M_P^2 (m_{\phi}/M_P)^{1/2}) \),
and \( x \equiv (\lambda_n M_p/m_{\phi})^{1/4} (\phi/M_P) \). The black curve is the potential arising from the soft SUSY breaking mass term. The black dots on the colored potentials illustrate the gradual transition from minimum to the saddle point and to the maximum.

where we have assumed \( V'(\phi) \sim (\phi - \phi_0)^2 V'''(\phi_0)/2 \) (this is justified since \( V'(\phi_0) \sim 0, V'''(\phi_0) \sim 0 \)). Note that the initial displacement from \( \phi_0 \) cannot be smaller than \( H_{\text{inf}} \), due to the uncertainty from quantum fluctuations.

Inflation ends when \( \epsilon \sim 1 \), or

\[
\frac{\phi_0 - \phi}{\phi_0} \sim \left( \frac{\phi_0}{2n(n-1)M_P} \right)^{1/2}.
\]

After inflation the coherent oscillations of the flat direction excite the MS(SM) degrees of freedom and reheat the universe.

Let us now identify the possible MSSM inflaton candidates. Recall first that the highest order operators which give a non-zero A-term are those with \( n = 6 \). This happens for flat directions represented by the gauge invariant monomials \( \phi = L_i L_j e_k ; \phi = u_d i d_j \).

The flatness of the potential require that \( i \neq j \neq k \) in the former and \( i \neq j \) in the latter. For \( n = 6 \) and \( m_{\phi} \sim 1 \text{ TeV} \), as in the case of weak scale supersymmetry breaking, we find the following generic results:

a) \textit{Sub-Planckian VEVs}: In an effective field theory where the Planck scale is the cut-off, inflationary potential can be trusted only below the Planck scale, usually a challenge for the model building. In our case the flat direction VEV is sub-Planckian for the non-renormalizable operator \( n = 6 \), i.e. \( \phi_0 \sim 1 - 3 \times 10^{14} \text{ GeV} \) for \( m_{\phi} \sim 1 - 10 \text{ TeV} \), while the vacuum energy density ranges \( V \sim 10^{34} - 10^{38} \text{ (GeV)}^4 \) (we assume \( \lambda_n = 1 \); generically \( \lambda_n \leq 1 \) but its precise value depends on the nature of high energy physics);

b) \textit{Low scale inflation}: Although it is extremely hard to build an inflationary model at
low scales, for the energy density stored in the MSSM flat direction vacuum, the Hubble expansion rate comes out as low as $H_{\text{inf}} \sim 1 - 10$ GeV. It might be possible to lower the scale of inflation further to the electroweak scale;

c) Enough e-foldings: At low scales, $H_{\text{inf}} \sim \mathcal{O}(1)$ GeV, the number of e-foldings, $N_{\text{COBE}}$, required for the observationally relevant perturbations, is much less than 60 [6]. In fact the number depends on when the Universe becomes radiation dominated (note that full thermalization is not necessary as it is the relativistic equation of state which matters).

If the inflaton decays immediately after the end of inflation, which has a scale $V \sim 10^{36}$ (GeV)$^4$, we obtain $N_{\text{COBE}} \sim 47$. The relevant number of e-foldings could be greater if the scale of inflation becomes larger. For instance, if $m_\phi \sim 10$ TeV, and $V \sim 10^{38}$ (GeV)$^4$, we have $N_{\text{COBE}} \sim 50$. For the MSSM flat direction lifted by $n = 6$ non-renormalizable operators, we obtain the total number of e-foldings as $N_e \sim (\phi_0^2/m_\phi M_{\text{Pl}})^{1/2} \sim 10^3$, computed from the end of diffusion. This bout of inflation is sufficiently long to produce a patch of the Universe with no dangerous relics. Domains initially closer to $\phi_0$ will enter self-reproduction in eternal inflation.

Let us now consider adiabatic density perturbations. Despite the low inflationary scale, $H_{\text{inf}} \sim 1 - 10$ GeV, the flat direction can generate adequate density perturbations as required to explain the COBE normalization. This is due to the extreme flatness of the potential (recall that $V' = 0$), which causes the velocity of the rolling flat direction to be extremely small. Thus we find an amplitude of

$$\delta H \simeq \frac{1}{5\pi} \frac{H_{\text{inf}}^2}{\phi} \sim \frac{m_\phi M_{\text{Pl}}}{\phi_0^2} N_{\text{COBE}}^2 \sim 10^{-5},$$

(6)

for $m_\phi \sim 10^3 - 10^4$ GeV, where $\phi_0$ is given by Eq. (3). In the above expression we have used the slow roll approximation $\phi \simeq -V'''(\phi_0)(\phi_0 - \phi)^2/3H_{\text{inf}}$, and Eq. (4). Note the importance of the $n = 6$ operators lifting the flat directions $\text{LLe}$ and $\text{udd}$. Higher order operators would have allowed for larger VEVs and a large $\phi_0$, therefore leaving the amplitude of the perturbations too low.

The spectral tilt of the power spectrum is not negligible because, although $\varepsilon \sim 1/N_{\text{COBE}}^4 \ll 1$, the parameter $\eta = -2/N_{\text{COBE}}$ and thus

$$n_s = 1 + 2\eta - 6\varepsilon \simeq 1 - \frac{4}{N_{\text{COBE}}} \sim 0.92, \quad \frac{d n_s}{d \ln k} = -\frac{4}{N_{\text{COBE}}^2} \sim -0.002,$$

(7)

which agrees with the current WMAP 3-years’ data within $2\sigma$ [7], while there are essentially no tensor modes. Note that the tilt can be tuned to match the central value of the WMAP 3-years’ by chosing $\lambda_n \leq 1$.

Recall that quantum loops result in a logarithmic running of the soft supersymmetry breaking parameters $m_\phi$ and $A$. One might then worry about their impact on Eq. (2) and the success of inflation. Note however that the only requirement is that one must use the VEV-dependent values of $m_\phi(\phi)$ and $A(\phi)$ in Eq. (2) for determining $\phi_0$. We have checked that the crucial ingredient for a successful inflation, i.e. having a very flat potential such that $V'(\phi_0) = V''(\phi_0) = 0$, remains true after quantum corrections. The only difference is a small shift in the value of $\phi_0$. 


After the end of inflation, the flat direction eventually starts rolling towards its global minimum. The flat direction decays into light relativistic MS(SM) particles which obtains kinetic equilibrium rather quickly. Although the plasma heats up to a large value due to large momenta of the inflaton decay products, the process of thermalization, which requires chemical equilibrium, can be a slow process. Just to illustrate, we note that within MSSM there are other flat directions orthogonal to the inflaton. These can develop large VEVs and induce large masses to the MSSM quanta, i.e. squarks and sleptons and gauge bosons and gauginos. As a consequence, there will be a kinematical blocking for the inflaton to decay [8] which can delay thermalization. The details of thermalization would require involved calculations. However, perhaps the best guess is to assume that the flat direction mass gives the lower limit on a temperature, where all the MSSM degrees of freedom are in thermal equilibrium, $T_{rh} \sim m_\phi \sim O(1-10)$ TeV. Note that this temperature is sufficiently high for cold electroweak baryogenesis [9,10] and for both thermal and non-thermal cold dark matter production [11,12].

CONCLUSION

The existence of a saddle point in the scalar potential of the udd or LLe MSSM flat directions appears, perhaps surprisingly, to provide all the necessary ingredients for an observationally realistic model of inflation [1]. MSSM inflation takes place at a low energy scale so that it is naturally free of supergravity and super-Planckian effects. The exceptional feature of the model, which sets it apart from conventional singlet field inflation models, is the fact that here the inflaton is a gauge invariant combination of the squark or slepton fields. As a consequence, the couplings of the inflaton to the MSSM matter and gauge fields are known. This makes it possible to address the questions of reheating and gravitino production in an unambiguous way. Since udd and LLe are independently flat, therefore, if LLe is the inflaton, the udd direction can also acquire a large VEV simultaneously. This gives a large mass to gluons/gluinos which decouples them from the thermal bath, and hence suppresses thermal gravitino production. Non-thermal production of gravitinos is negligible in our model.

In the MSSM inflation model the mass of the inflaton is not a free parameter but is related to the masses of e.g. sleptons, should the LLe direction be the inflaton. We have solved the appropriate RG equation equations to relate the inflaton mass to the slepton masses at energies accessible to accelerators such as LHC and found that LHC can indeed put a constraint on the model: it may not be able to verify it, but it certainly can rule it out.

The model predictions are not modified by supergravity corrections, i.e. the observables are insensitive to the nature of Kähler potential. MSSM inflation also illustrates that it is free from any Trans-Planckian corrections. MSSM inflation retains the successes of thermal production of LSP as a dark matter and the electroweak baryogenesis within MSSM.

The existence of the saddle point requires fine-tuning [13]. As shown in Ref. [14], the parameter $\alpha$ should be small. In that paper we dealt both with the local minimum (and the constraints imposed by efficient tunneling from the local minimum onto the slow roll part of the potential) as well as with the inflection point, $V'(\phi) > 0$. We found
that a fine-tuning of the order of one part in $10^9$ is sufficient. We also found that one-loop radiative corrections induce a shift in the ratio $\alpha$ which is of the order of $10^{-2}$, making it necessary to adjust the ratio order by order up to four loops in the perturbative expansion. However, it is conceivable that the mechanism of supersymmetry breaking could remove the fine-tuning in some natural, dynamical way. For instance, $A/m$ could turn out to be a renormalization group invariant so that once the ratio is fixed e.g. by threshold corrections, it would remain fixed at all orders. This is just speculation, of course, but perhaps warranted by the simplicity and the apparent success of MSSM flat direction inflation, which is unique in being both a successful model of inflation and at the same time having a concrete and real connection to physics that can be observed in earth bound laboratories.

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