Adaptive and Nonlinear Kalman Filtering for GPS Navigation Processing

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1. Introduction

The Global Positioning System (GPS) is a satellite-based navigation system that provides a user with the proper equipment access to useful and accurate positioning information anywhere on the globe. The well-known Kalman filter (Gelb, 1974; Brown & Hwang, 1997; Axelrad & Brown, 1996) provides optimal (minimum mean square error) estimate of the system state vector, and has been widely applied to the fields of navigation such as GPS receiver position/velocity determination.

To obtain good estimation solutions using the EKF approach, the designers are required to have good knowledge on both dynamic process (plant dynamics, using an estimated internal model of the dynamics of the system) and measurement models, in addition to the assumption that both the process and measurement are corrupted by zero-mean white noises. A conventional Kalman filter fails ensure error convergence due to limited knowledge of the system’s dynamic model and measurement noise. If the Kalman filter is provided with information that the process behaves a certain way, whereas, in fact, it behaves a different way, the filter will continually intend to fit an incorrect process signal.

In actual navigation filter designs, there exist model uncertainties which cannot be expressed by the linear state-space model. The linear model increases modelling errors since the actual vehicle motions are non-linear process. It is very often the case that little a priori knowledge is available concerning the manoeuvring. Hence, compensation of the uncertainties is an important task in the navigation filter design. In the modelling strategy, some phenomena are disregarded and a way to take them into account is to consider a nominal model affected by uncertainty.

The adaptive algorithm has been one of the approaches to prevent divergence problem of the EKF when precise knowledge on the system models are not available. To prevent divergence problem due to modelling errors using the EKF approach, the adaptive filter algorithm has been one of the strategies considered for estimating the state vector. Many efforts have been made to improve the estimation of the covariance matrices. Mehra (1970; 1971; 1972) classified the adaptive approaches into four categories: Bayesian, maximum likelihood, correlation and covariance matching. These methods can be applied to the Kalman filtering algorithm for realizing the adaptive Kalman filtering (Mehra, 1972; Mohamed & Schwarz, 1999). One of the adaptive fading methods (Ding, et al., 2007; Jwo & Weng, 2008) is called the strong tracking Kalman filter (STKF) (Zhou & Frank, 1996), which
is essentially a nonlinear smoother algorithm that employs suboptimal multiple fading factors, in which the softening factors are involved. STKF has several merits, such as: (1) strong robustness against model uncertainties; (2) good real-time state tracking ability even when a state jump occurs, no matter whether the system has reached steady state or not. Although it has been very common that additional fictitious process noise can be added to the system model, however, the more suitable cure for non convergence caused by un-modelled states is to correct the model. For the nonlinear estimation problem, alternatives for the model-based EKF can be employed, such as the unscented Kalman filter (UKF) approach. The UKF is a nonlinear, distribution approximation method, which uses a finite number of sigma points to propagate the probability of state distribution through the nonlinear dynamics of system. The UKF exhibits superior performance when compared with classical EKF since the series approximations in the EKF algorithm can lead to poor representations of the nonlinear functions and probability distributions of interest.

Another adaptive-like approach, referred to as the interacting multiple model (IMM) algorithm (Bar-Shalom, et al., 2001; Chen & Harigae, 2001; Deshpande & Challa, 2007; Johnston & Krishnamurthy, 2001; Kim & Hong, 2004; Lee, et al., 2005; Li & Bar-Shalom, 1993; Xu & Cui, 2007.), takes into account a set of models to represent the system behaviour patterns or system model. The overall estimates is obtained by a combination of the estimates from the filters running in parallel based on the individual models that match the system modes. The IMM-based method is realized to allow the possibility of using highly dynamic models just when required, diminishing unrealistic noise considerations in non-manoeuvring situations of the system.

The fuzzy logic adaptive system (FLAS) can be employed into the filter design, where the fuzzy logic reasoning system based on the Takagi-Sugeno (T-S) model is employed, for dynamically adjusting the softening factor according to the change in vehicle dynamics. The philosophy is based on combination of the merits by UKF and strong tracking feature.

Navigation algorithm and performances evaluation based on the above various filters are to be discussed.

Fig. 1. Techniques involved for filter design in this chapter

In the GPS navigation processing, three feasible ways to avoid the divergence problem and improve the navigation accuracy are discussed: (1) adaptive approaches assisted by heuristic search techniques to fit the dynamic model process of interest as precisely as
possible; (2) utilization of an appropriate nonlinear estimation approach after deriving a better nonlinear dynamic process model; and (3) interactive multiple model approach accounting for different manoeuvring conditions. Based on the consideration, this chapter is organized as follows. In Section 2, preliminary background on GPS navigation processing using the extended Kalman filter is reviewed. The adaptive Kalman filter is discussed in Section 3. In Section 4, the unscented Kalman filter (UKF) with nonlinear dynamic modelling approach is presented. Discussion on the IMM estimator is provided in Section 5. In Section 6, Incorporation of the fuzzy logic adaptive system (FLAS) to the GPS navigation processing is presented. Three illustrative examples on GPS navigation processing are provided in Section 7 for illustrating the performance for various approaches. Conclusion is given in Section 8.

2. GPS navigation processing using the extended Kalman filter

The discrete-time extended Kalman filter deals with the case governed by the nonlinear stochastic differential equations:

\[ \mathbf{x}_{k+1} = f_k(\mathbf{x}_k) + \mathbf{w}_k \quad (1a) \]

\[ \mathbf{z}_k = h_k(\mathbf{x}_k) + \mathbf{v}_k \quad (1b) \]

where the state vector \( \mathbf{x}_k \in \mathbb{R}^n \), process noise vector \( \mathbf{w}_k \in \mathbb{R}^n \), measurement vector \( \mathbf{z}_k \in \mathbb{R}^m \), and measurement noise vector \( \mathbf{v}_k \in \mathbb{R}^m \). In Equation (1), both the vectors \( \mathbf{w}_k \) and \( \mathbf{v}_k \) are zero mean Gaussian white sequences having zero crosscorrelation with each other:

\[
\begin{align*}
\mathbf{E}[\mathbf{w}_k \mathbf{w}_i^T] &= \begin{cases} 
\mathbf{Q}_k, & i = k \\
0, & i \neq k 
\end{cases} \\
\mathbf{E}[\mathbf{v}_k \mathbf{v}_i^T] &= \begin{cases} 
\mathbf{R}_k, & i = k \\
0, & i \neq k 
\end{cases} \\
\mathbf{E}[\mathbf{w}_k \mathbf{v}_i^T] &= 0 
\end{align*}
\]

for all \( i \) and \( k \) (2)

where \( \mathbf{Q}_k \) is the process noise covariance matrix, \( \mathbf{R}_k \) is the measurement noise covariance matrix, \( \mathbf{E}[\cdot] \) represents expectation, and superscript “T” denotes matrix transpose.

The discrete-time extended Kalman filter algorithm for the GPS navigation processing is summarized as follows:

1. Initialize state vector and state covariance matrix: \( \mathbf{\hat{x}}_0^- \) and \( \mathbf{P}_0^- \)
2. Compute Kalman gain matrix from state covariance and estimated measurement covariance:

\[ \mathbf{K}_k = \mathbf{P}_k^- \mathbf{H}_k^T [\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^T + \mathbf{R}_k]^{-1} \]

(3)

3. Multiply prediction error vector by Kalman gain matrix to get state correction vector and update state vector:

\[ \mathbf{\hat{x}}_k = \mathbf{\hat{x}}_k^- + \mathbf{K}_k [\mathbf{z}_k - \mathbf{\hat{z}}_k^-], \quad \text{where } \mathbf{\hat{z}}_k^- = \mathbf{h}_k(\mathbf{\hat{x}}_k^-) \]

(4)

4. Update error covariance

\[ \mathbf{P}_k = [\mathbf{I} - \mathbf{K}_k \mathbf{H}_k] \mathbf{P}_k^- \]

(5)
5. Predict new state vector and state covariance matrix

\[
\hat{x}_k^- = f_{k-1}(\hat{x}_{k-1}^-) \\
\]

\[
P_{k|k}^- = \Phi_k^- P_{k-1}^- \Phi_k^- + Q_k \\
\]

where the linear approximation equations for system and measurement matrices are obtained through the relations

\[
\Phi_k^- \approx \frac{\partial f_k^-}{\partial x}|_{x = \hat{x}_k^-} ; H_k^- \approx \frac{\partial h_k^-}{\partial x}|_{x = \hat{x}_k^-} \\
\]

Equations (3-5) are the measurement update equations, and Equations (6-7) are the time update equations of the algorithm from \( k \) to step \( k+1 \). These equations incorporate a measurement value into \textit{a priori} estimation to obtain an improved \textit{a posteriori} estimation. In the above equations, \( P_k^- \) is the error covariance matrix defined by \( E[(x_k - \hat{x}_k^-)(x_k - \hat{x}_k^-)^T] \).

The Kalman filter algorithm starts with an initial condition value, \( \hat{x}_0^- \) and \( P_0^- \). When new measurement \( z_k \) becomes available with the progression of time, the estimation of states and the corresponding error covariance would follow recursively ad infinity. Fig 2 shows GPS navigation using the filter approach and the flow chart for the GPS Kalman filter. Further detailed discussion can be referred to Brown and Hwang (1997), Gelb (1974), and Jwo & Cho (2007).

**Fig. 2.** GPS navigation using the filter approach and the flow chart for the GPS Kalman filter

3. The adaptive extended Kalman filter

The implementation of Kalman filter requires the \textit{a priori} knowledge of both the process and measurement models. It is widely known that poorly designed mathematical model for the
EKF may lead to the divergence. Clearly, if the plant parameters are subject to perturbations and dynamics of the system are too complex to be characterized by an explicit mathematical model, an adaptive scheme is needed. An adaptive Kalman filter can be utilized as the noise-adaptive filter to adjust the parameters. It is well known that the process model is dependent on the dynamical characteristics of the vehicle onto which the navigation system is placed. In order to overcome the defect of the conventional Kalman filtering, several approaches of adaptive Kalman filter have been proposed.

Mehra classified the adaptive approaches into four categories: Bayesian, maximum likelihood, correlation and covariance matching. The innovation sequences have been utilized by the correlation and covariance-matching techniques to estimate the noise covariances. The basic idea behind the covariance-matching approach is to make the actual value of the covariance of the residual consistent with its theoretical value. From the incoming measurement \( z_k \) and the optimal prediction \( \hat{x}_k \) obtained in the previous step, the innovation sequence is defined as: \( \nu_k = z_k - \hat{z}_k \). The innovation represents the additional information available to the filter as a consequence of the new observation \( z_k \). An innovation of zero means that the two are in complete agreement. The mean of the corresponding error of an unbiased estimator is zero.

The innovation sequences have been utilized to estimate the noise covariance matrices (Mohamed & Schwarz, 1999; Hide et al., 2003) and system fault detection (Caliskan & Hajiyev, 2000; Jwo & Cho, 2007). The innovation-based adaptive estimation (IAE) approach is essentially a type of variance estimation (scaling to the \( Q_k \) or \( R_k \) matrices) method. The basic idea behind the covariance-matching approach is to make the actual value of the covariance of the residual consistent with its theoretical value. This leads to the estimate of process noise matrix \( Q_k \) and measurement noise matrix \( R_k \). For more detailed information derivation for these equations, see Mohamed & Schwarz (1999), and Hide et al. (2003).

One of the approaches for adaptive processing is on the incorporation of fading factors. Xia et al. (1994) proposed a concept of adaptive fading Kalman filter (AFKF) and solved the state estimation problem. The AFKF is essentially a covariance scaling-based Kalman filter (scaling to the \( P \) matrix). The approach tries to estimate a scale factor to increase the predicted variance components of the state vector. In the AFKF, suboptimal fading factors are introduced into the algorithm. The idea of fading Kalman filtering is to apply a factor matrix to the predicted covariance matrix to deliberately increase the variance of the predicted state vector:

\[
P_{k+1}^- = \lambda_k \Phi_k P_k \Phi_k^T + Q_k
\]

where \( \lambda_k = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_m) \). The main difference between different adaptive fading algorithms is on the calculation of scale factor \( \lambda_k \) (Ding, et al., 2007; Jwo & Weng, 2008). One approach is to assign the scale factors as constants. When \( \lambda_i \leq 1 \) (\( i = 1, 2, \ldots, m \)), the filtering is in a steady state processing while \( \lambda_i > 1 \), the filtering may tend to be unstable. For the case \( \lambda_i = 1 \), it deteriorates to the standard Kalman filter. There are some drawbacks with constant factors, e.g., as the filtering proceeds, the precision of the filtering will decrease because the effects of old data tend to become less and less. The ideal way is to use time varying factors that are determined according to the dynamic and observation model accuracy.
To increase the tracking capability, the time-varying suboptimal scaling factor is incorporated, for on-line tuning the covariance of the predicted state, which adjusts the filter gain, and accordingly the improved version of AFKF is developed. Various formulations have been modified by multiplying the weighting factor, see Ding, et al. (2007), and Jwo & Weng (2008).

Zhou et al. proposed a concept of strong tracking Kalman filter (STKF) and solved the state estimation problem of a class of nonlinear systems with white noise. In the so called STKF algorithm, suboptimal fading factors are introduced into the nonlinear smoother algorithm. The STKF has several important merits, including (1) strong robustness against model uncertainties; (2) good real-time state tracking capability even when a state jump occurs, no matter whether the system has reached steady state or not. Zhou & Frank (1996) proved that a filter is called the STKF only if the filter satisfies the orthogonal principle stated as follows:

**Orthogonal principle**: The sufficient condition for a filter to be called the STKF only if the time-varying filter gain matrix be selected on-line such that the state estimation mean-square error is minimized and the innovations remain orthogonal:

$$E[x_k - \hat{x}_k][x_k - \hat{x}_k]^T = \min$$

$$E[u_{k+j}u_{k+j}^T] = 0, \ k = 0,1,2..., \ j = 1,2... \quad (10)$$

Equation (10) is required for ensuring that the innovation sequence will be remained orthogonal. The time-varying suboptimal scaling factor is incorporated, for on-line tuning the covariance of the predicted state, which adjusts the filter gain, and accordingly the STKF is developed. The suboptimal scaling factor in the time-varying filter gain matrix is given by:

$$\hat{\lambda}_{i,k} = \begin{cases} \alpha_i c_k, & \alpha_i c_k \geq 1 \\ 1, & \alpha_i c_k < 1 \end{cases} \quad (11)$$

where

$$c_k = \frac{\text{tr}[N_k]}{\text{tr}[aM_k]} \quad (12)$$

and

$$N_k = V_k - \beta R_k - H_k Q_k H_k^T \quad (13a)$$

$$M_k = H_k \Phi_k P_k \Phi_k^T H_k^T \quad (14)$$

$$V_k = \begin{cases} u_0 v_0^T, & k = 0 \\ [\rho V_{k-1} + u_k v_k^T]_+, & k \geq 1 \end{cases} \quad (15)$$

Equation (13a) can be modified by multiplying an additional parameter $\gamma$, which can be a scalar of a diagonal matrix:

$$N_k = \gamma V_k - \beta R_k - H_k Q_k H_k^T \quad (13b)$$
This parameter is introduced for increasing the tracking capability through the increase of covariance matrix of the innovation.

The key parameter in the STKF is the fading factor matrix $\lambda_k$, which is dependent on three parameters, including (1) $\alpha_i$; (2) the forgetting factor ($\rho$); (3) and the softening factor ($\beta$). These parameters are usually selected empirically. $\alpha_i \geq 1, i = 1,2,\ldots,m$, which are a priori selected. If from a priori knowledge, we have the knowledge that $x$ will have a large change, then a large $\alpha_i$ should be used so as to improve the tracking capability of the STKF. On the other hand, if no a priori knowledge about the plant dynamic, it is commonly select $\alpha_1 = \alpha_2 = \cdots = \alpha_m = 1$. In such case, the STKF based on multiple fading factors deteriorates to a STKF based on a single fading factor. The range of the forgetting factor is $0 < \rho \leq 1$, for which 0.95 is commonly used. The softening factor $\beta$ is utilized to improve the smoothness of state estimation. A larger $\beta$ (with value no less than 1) leads to better estimation accuracy; while a smaller $\beta$ provides stronger tracking capability. The value is usually determined empirically through computer simulation and $\beta = 4.5$ is a commonly selected value.

4. The unscented Kalman filter

In the EKF, the state distribution is approximated by a Gaussian random variable (GRV), which is then propagated analytically through the first-order linearization of the nonlinear system. Wan and van der Merwe (2000) pointed out that this will introduce large errors in the true posterior mean and covariance of the transformed GRV and lead to sub-optimal performance and sometimes filter divergence. The UKF addresses this problem by using a deterministic sampling approach (Crassidis, 2006). The state distribution is also approximated by a GRV, but is represented using a minimal set of sample points. These sample points are carefully chosen so as to completely capture the true mean and covariance of the GRV. When the sample points are propagated through the true nonlinear system, the posterior mean and covariance can be captured accurately to the 3rd order of Taylor series expansion for any nonlinear system.

4.1 Unscented transformation

The first step in the UKF is to sample the prior state distribution, i.e., generate the sigma points through the unscented transformation (UT) (Julier et al. 2000; Julier & Uhlmann 2002; Julier 2002). The unscented transform is a method for calculating the statistics of a random variable which undergoes a nonlinear transformation. The basic premise is that to approximate a probability distribution is easier than to approximate an arbitrary nonlinear transformation. A set of weighted samples or sigma points are deterministically chosen so that they completely capture the true mean and covariance of the random variable. The samples are propagated through true nonlinear equations without linearization of the model. Suppose the mean $\bar{x}$ and covariance $P$ of vector $x$ are known, a set of deterministic vector called sigma points can then be found. The ensemble mean and covariance of the sigma points are equal to $\bar{x}$ and $P$. The nonlinear function $y = f(x)$ is applied to each deterministic vector to obtain transformed vectors. The ensemble mean and covariance of the transformed vectors will give a good estimate of the true mean and covariance of $y$, which is the key to the unscented transformation. Illustration of properties
of UKF and EKF is shown in Fig. 3. The UKF approach estimates are expected to be closer to the true values than the EKF approach.

Consider an $n$ dimensional random variable $x$, having the mean $\hat{x}$ and covariance $P$, and suppose that it propagates through an arbitrary nonlinear function $f$. The unscented transform creates $2n+1$ sigma vectors $X$ (a capital letter) and weighted points $W$, given by

$$X_{(0)} = \hat{x}$$  

$$X_{(i)} = \hat{x} + (\sqrt{(n+\lambda)}P)^T_i, \quad i = 1,...,n$$  

$$X_{(i+n)} = \hat{x} - (\sqrt{(n+\lambda)}P)^T_i, \quad i = 1,...,n$$  

$$W_{(m)}^0 = \frac{\lambda}{(n+\lambda)}$$  

$$W_{(c)}^0 = W_{(m)}^0 + (1-\alpha^2 + \beta)$$  

$$W_{(m)}^i = W_{(c)}^i = \frac{1}{2(n+\lambda)} \quad , \quad i = 1,...,2n$$

where $(\sqrt{(n+\lambda)}P)_i$ is the $i$th row (or column) of the matrix square root. $\sqrt{(n+\lambda)}P$ can be obtained from the lower-triangular matrix of the Cholesky factorization; $\lambda = \alpha^2 (n+k) - n$ is a scaling parameter; $\alpha$ determines the spread of the sigma points around $\hat{x}$ and is usually set to a small positive (e.g., $1e-4 \leq \alpha \leq 1$); $k$ is a secondly scaling parameter (usually set as 0); $\beta$ is used to incorporate prior knowledge of the distribution of $\bar{x}$ (When $x$ is normally distributed, $\beta = 2$ is an optimal value); $W_{(m)}^i$ is the weight for the mean associated with the $i$th point; and $W_{(c)}^i$ is the weight for the covariance associated with the $i$th point.

The sigma vectors are propagated through the nonlinear function to yield a set of transformed sigma points,

$$y_i = f(X_i) \quad i = 0,...,2n$$  

The mean and covariance of $y_i$ are approximated by a weighted average mean and covariance of the transformed sigma points as follows:

$$\bar{y}_u = \frac{2n}{i=0} W_{(m)}^i y_i$$  

$$\bar{P}_u = \frac{2n}{i=0} W_{(c)}^i (y_i - \bar{y}_u)(y_i - \bar{y}_u)^T$$

As compared to the EKF’s linear approximation, the unscented transformation is accurate to the second order for any nonlinear function.
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Fig. 3. The mapping of the UKF versus that of the EKF, through the transformation of: (1) the nonlinear function \( f(\cdot) \), shown on the top portion of the figure, and (2) its Jacobian \( F \), shown at the bottom portion of the figure. The dot-line ellipse represents the true covariance; the solid-line ellipse represents the calculated covariance (Li et al. 2006)

4.2 The unscented Kalman filter

The basic premise behind the unscented Kalman filter is it is easier to approximate a Gaussian distribution than it is to approximate an arbitrary nonlinear function. Instead of linearizing using Jacobian matrices as in the EKF and achieving first-order accuracy, the UKF uses a deterministic sampling approach to capture the mean and covariance estimates with a minimal set of sample points. To look at the detailed algorithm of the UKF, firstly, the set of sigma points are created by Equations (12) and (13). A high level of operation of the unscented Kalman filter and the flow chart for the UKF approach is shown in Fig. 4. After

![Flow chart for the unscented Kalman Filter](https://www.intechopen.com)

Fig. 4. High level of operation of the unscented Kalman filter (left) and the flow chart (right) for the unscented Kalman Filter
the sigma points are generated, the time update (prediction) step and the measurement update (correction) step are involved. The samples are propagated through true nonlinear equations; the linearization is unnecessary (i.e., calculation of Jacobian is not necessary). They can capture the states up to at least second order, where as the EKF is only a first order approximation. Further detailed discussion on the UKF can be referred to Wan & Van der Merwe (2001), Julier & Uhlmann (1997), and Julier, et al. (1995). Although recently the particle filter (Gordon, 1993; 2004) has been studied and shown promising potential, especially for nonlinear, non-Gaussian problems, the content will not be covered this chapter.

4.3 A nonlinear model

Derivation of a better, nonlinear, dynamic model not only alleviates the divergence problem by resorting to modelling the system dynamic process as precisely as possible but also improves the estimation accuracy. To construct the nonlinear dynamic model, consider a vehicle moving at the velocity represented as $V_b = u_b \hat{i} + v_b \hat{j} + w_b \hat{k}$. The velocity in the fixed frame in terms of Euler angles and body velocity components has the relation

$$V = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} C_\theta C_\psi & S_\theta S_\psi C_\psi - C_\psi S_\psi & C_\theta S_\psi + S_\theta S_\psi \\ C_\theta S_\psi & S_\theta S_\psi C_\psi + C_\psi S_\psi & C_\theta S_\psi - S_\theta S_\psi \\ -S_\theta & S_\phi & C_\phi \end{bmatrix} \begin{bmatrix} u_b \\ v_b \\ w_b \end{bmatrix}$$

where the following notations are used: $S_\phi \equiv \sin(\phi)$, $C_\phi \equiv \cos(\phi)$, $S_\theta \equiv \sin(\theta)$, $C_\theta \equiv \cos(\theta)$, $S_\psi \equiv \sin(\psi)$, and $C_\psi \equiv \cos(\psi)$. Based on the idea, the dynamic process model of the GPS receiver can be represented by the nonlinear model. Suppose that, as the non-holonomic constraint, only the longitudinal movement is considered and the lateral slippage is neglected. In case the velocity in the x-component of body frame is considered, $\| V_b \| \approx \| u_b \hat{i} \| \approx V$, the model can be simplified

$$\dot{x} = V \cos \theta \cos \psi$$
$$\dot{y} = V \cos \theta \sin \psi$$
$$\dot{z} = -V \sin \theta$$

In this case, we consider the GPS navigation filter with three position states, three angles states, and two clock states, so that the state to be estimated is a $9 \times 1$ vector. The dynamic model governed by Equation (25) with additive noise can be represented by

$$\begin{bmatrix} x \\ y \\ z \\ \phi \\ \theta \\ \psi \\ V \\ b \\ d \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \\ \dot{V} \\ \dot{b} \\ \dot{d} \end{bmatrix} = \begin{bmatrix} V \cos \theta \cos \psi \\ V \cos \theta \sin \psi \\ -V \sin \theta \\ 0 \\ 0 \\ 0 \\ 0 \\ d \\ 0 \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \end{bmatrix}$$

(26)
and

\[
F = \frac{\partial f}{\partial \mathbf{x}} = \begin{bmatrix}
0 & 0 & 0 & -VS_\theta C_\psi & -VC_\theta S_\psi & C_\theta C_\psi & 0 & 0 \\
0 & 0 & 0 & -VS_\theta S_\psi & VC_\theta C_\psi & C_\theta S_\psi & 0 & 0 \\
0 & 0 & 0 & -VC_\theta & 0 & -S_\theta & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]  \tag{27}

In this nonlinear dynamic model, attitude angles are introduced. As part of the state variables, these angles can be estimated from the GPS filter in a single-antenna receiver with pseudorange observables. The process noise covariance matrix for this nonlinear model is given by the form

\[
Q_k = \begin{bmatrix}
q_{11} & q_{22} & q_{33} & q_{44} & q_{55} & q_{66} & q_{77} & q_{88} & q_{98} \\
q_{22} & q_{33} & q_{44} & q_{55} & q_{66} & q_{77} & q_{88} & q_{98} & q_{99}
\end{bmatrix}
\]  \tag{28}

in which the \( q_{ii} \) are set to be constants, \( i = 1 \ldots 7 \); \( q_{88} = S_f \Delta t + S_g \frac{\Delta t^3}{2} \); \( q_{98} = S_g \frac{\Delta t^2}{2} \); \( q_{89} = S_g \Delta t \). The linearized measurement equation based on \( n \) observables for this nonlinear model is given by:

\[
\begin{bmatrix}
\rho_1 \\
\rho_2 \\
\vdots \\
\rho_n
\end{bmatrix} = \begin{bmatrix}
\hat{\rho}_1 \\
\hat{\rho}_2 \\
\vdots \\
\hat{\rho}_n
\end{bmatrix} + \begin{bmatrix}
h_x^{(1)} & h_y^{(1)} & h_z^{(1)} & 0 & 0 & 0 & 0 & 1 & 0 \\
h_x^{(2)} & h_y^{(2)} & h_z^{(2)} & 0 & 0 & 0 & 0 & 1 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
h_x^{(3)} & h_y^{(3)} & h_z^{(3)} & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6 \\
x_7 \\
x_8 \\
x_9
\end{bmatrix} + \begin{bmatrix}
v_{\rho_1} \\
v_{\rho_2} \\
\vdots \\
v_{\rho_n}
\end{bmatrix}
\]  \tag{29}
where, as defined previously, the elements of the measurement model $H_k$ are the partial derivatives of the predicted measurements with respect to each state, which is an $(n \times 9)$ matrix.

$$
H_k = \begin{bmatrix}
    h_{x}^{(1)} & h_{y}^{(1)} & h_{z}^{(1)} & 0 & 0 & 0 & 1 & 0 \\
    h_{x}^{(2)} & h_{y}^{(2)} & h_{z}^{(2)} & 0 & 0 & 0 & 1 & 0 \\
    \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 \\
    h_{x}^{(3)} & h_{y}^{(3)} & h_{z}^{(3)} & 0 & 0 & 0 & 1 & 0 
\end{bmatrix}
$$

(30)

The expected pseudorange $h_k(\hat{x}_k)$ based on the GPS satellite position and the a priori state estimate $\hat{x}_k$ is given by

$$
h_k(\hat{x}_k) = [x_k, y_k, z_k]^T
$$

with norm

$$
\hat{r}_i = ||h_k(\hat{x}_k)|| = \sqrt{(\hat{x}_k - x_i)^2 + (\hat{y}_k - y_i)^2 + (\hat{z}_k - z_i)^2}
$$

The vector $(h_x^{(i)}, h_y^{(i)}, h_z^{(i)})$, $i = 1, \ldots, n$, denotes the line-of-sight vector from the user to the satellites:

$$
h_x^{(i)} = \frac{\hat{x}_k - x_i}{\hat{r}_i}; h_y^{(i)} = \frac{\hat{y}_k - y_i}{\hat{r}_i}; h_z^{(i)} = \frac{\hat{z}_k - z_i}{\hat{r}_i}
$$

(31)

Discussion on the linear position-velocity (PV) model can be referred to Axelrad & Brown (1996) and Brown & Hwang (1997). More detailed treatment to the nonlinear model are referred to Jwo & Lai (2008).

5. The IMM algorithm

As a structure adaptation algorithm (for tuning the $\Phi_k$ and/or $Q_k$), the IMM estimators can substantially improve navigation accuracy during vehicle manoeuvring (such as circular motion and acceleration) as well as during constant velocity straight-line motion over the conventional EKF. The IMM algorithm takes into account a set of models to represent the system behaviour patterns or system model. The approach uses model (Markov chain state) probabilities to weight the input and output of a bank of parallel Kalman filters at each time instant. The overall estimates is obtained by a combination of the estimates from the filters running in parallel based on the individual models that match the system modes. In each cycle it consists of four major steps: interaction (mixing), filtering, mode probability calculation, and combination.

According to the summary given by Lin et al. (2001), the IMM has the following properties:

1. it consists of a low bandwidth filter for the nearly uniform motion and a high bandwidth filter for the manoeuvring situations;
2. these filters interact (exchange information) with time-varying weights;
3. the final estimate is a combination of each filter’s estimate, with the weights being the mode probabilities;
4. the weights for interaction and combination are based on which model fits better the data and other factors, such as the expected transition from one mode to another.
The IMM algorithm is summarized as follows.

1. **Calculation of mixing probabilities.** The probability that mode $M^i$ was in effect at $k-1$ given that $M^j$ is in effect at $k$ conditioned on $z_{k-1}$ is given by

$$p^{ij}_{k-1} = \frac{1}{\bar{c}_j} P[j | M^i_{k-1}, z_{k-1}] P[M^i_{k-1} | z_{k-1}]$$

which are the mixing probabilities and can be written as

$$p^{ij}_{k-1} = \frac{1}{\bar{c}_j} P[j | M^i_{k-1}, z_{k-1}], i, j = 1, \ldots, r$$

where

$$P[j = P[M^i_k | M^i_{k-1}, z_{k-1}], p^{ij}_{k-1} = P[M^i_{k-1} | z_{k-1}]$$

and the normalizing constants

$$\bar{c}_j = \sum_{i=1}^{r} P[j | M^i_{k-1}, z_{k-1}]$$

2. **Mixing.** Starting with $\hat{x}^i_{k-1}$, the mixed initial condition for the filter matched to $M^j$ is computed

$$\hat{x}^0_{k-1} = \sum_{i=1}^{r} p^{ij}_{k-1} \hat{x}^i_{k-1}$$

The corresponding covariance is

$$P^0_{k-1} = \sum_{i=1}^{r} p^{ij}_{k-1} \left\{ \begin{array}{c} \hat{x}^i_{k-1} - \hat{x}^0_{k-1} \\ \hat{x}^i_{k-1} - \hat{x}^0_{k-1} \end{array} \right\} \left( \begin{array}{c} \hat{x}^i_{k-1} - \hat{x}^0_{k-1} \\ \hat{x}^i_{k-1} - \hat{x}^0_{k-1} \end{array} \right)^T$$

3. **Mode-matched filtering.** The estimate Equation (35) and covariance Equation (36) are used as input to the filter matched to $M^j$, which uses $z_k$ to yield $\hat{x}^j_k$ and $P^j_k$. The likelihood function corresponding to the $r$ filters

$$\Lambda^j_k = p[z_k | M^j_k, z_{k-1}]$$

are computed using the mixed initial condition Equation (35) and the associated covariance Equation (36) as

$$\Lambda^j_k = p[z_k | M^j_k, \hat{x}^j_{k-1}, P^0_k]$$

that is

$$\Lambda^j_k = N(z_k; \hat{x}^j_k, k-1; \hat{x}^j_k, k-1, \hat{x}^j_k, k-1)$$

4. **Mode probability update**
\[ \mu_k^j = \frac{1}{C} \Lambda_k^j \bar{e}_j \]  

where

\[ C = \sum_{j=1}^{r} \Lambda_k^j \bar{e}_j \]  

is the normalization constant.

5. State estimate and covariance calculation through combination. Combination of the model-conditioned estimates and covariance is done according to the mixture equations given by

\[ \hat{x}_k = \sum_{j=1}^{r} \hat{x}_k^j \mu_k^j \]  

\[ P_k = \sum_{j=1}^{r} \mu_k^j \left[ \left( \hat{x}_k^j - \hat{x}_k \right) \left( \hat{x}_k^j - \hat{x}_k \right)^T \right] \]  

The filters in the bank of parallel filters could be EKFs or UKFs to describe their individual nonlinear behaviour in each manoeuvring stage, resulting in the IMM-based extended Kalman filter (IMM-EKF) or IMM-based unscented Kalman filter (IMM-UKF) to emphasize its nonlinear consideration on system modelling. To deal with the vehicle manoeuvring in tracking and navigation applications, the two-stage estimator has been an alternative. The content will not be discussed here and the related information can be found in Alouani & Xia (1991), Blair (1993), and Keller & Darouach (1997).

6. Incorporation of the fuzzy logic adaptive system

Fuzzy logic was first developed by Zadeh in the mid-1960s for representing uncertain and imprecise knowledge. It provides an approximate but effective means of describing the behaviour of systems that are too complex, ill-defined, or not easily analyzed mathematically. A typical fuzzy system consists of three components, that is, fuzzification, fuzzy reasoning (fuzzy inference), and fuzzy defuzzification. The fuzzification process converts a crisp input value to a fuzzy value, the fuzzy inference is responsible for drawing calculations from the knowledge base, and the fuzzy defuzzification process converts the fuzzy actions into a crisp action.

The fuzzification modules: (1) transforms the error signal into a normalized fuzzy subset consisting of a subset for the range of the input values and a normalized membership function describing the degree of confidence of the input belonging to this range; (2) selects reasonable and good, ideally optimal, membership functions under certain convenient criteria meaningful to the application. The characteristics of the fuzzy adaptive system depend on the fuzzy rules and the effectiveness of the rules directly influences its performance. To obtain the best deterministic output from a fuzzy output subset, a procedure for its interpretation, known as defuzzification should be considered. The defuzzification is used to provide the deterministic values of a membership function for the output. Using fuzzy logic to infer the consequent of a set of fuzzy production rules invariably leads to fuzzy output subsets.
Fuzzy modelling is the method of describing the characteristics of a system using fuzzy inference rules. A Takagi-Sugeno (T-S) fuzzy system can be used to detect the divergence of EKF and adapt the filter. Takagi and Sugeno proposed a fuzzy modelling approach to model nonlinear systems. The T-S fuzzy system represents the conclusion by functions. The typical T-S system is shown as in Fig. 5. The output is the weighted average of the $y_k$:

$$y = \sum_{k=1}^{M} w_k \cdot y_k$$  \hspace{1cm} (30)

where the weights $w_k$ are computed as

$$w_k = \frac{\prod_{i=1}^{n} \mu_{F_k}^j (x_i)}{\sum_{j=1}^{M} \left[ \prod_{i=1}^{n} \mu_{F_j}^i (x_i) \right]}$$  \hspace{1cm} (31)

with $\sum_{i=1}^{M} w_i = 1$, and the $\mu$'s represent the membership functions.

![Fig. 5. Takagi-Sugeno (T-S) fuzzy system](https://www.intechopen.com)

The application of fuzzy logic to adaptive Kalman filtering has been becoming popular, e.g., (Sasiadek et al., 2000; Abdelnour et al., 1993; Kobayashi et al., 1995; Mostov & Soloviev, 1996). Sasiadek, Wang, and Zeremba introduced the Fuzzy Logic Adaptive System (FLAS) for adapting the process and measurement noise covariance matrices in navigation data fusion design (Sasiadek et al., 2000). Abdelnour, et al. (1993) used the exponential-weighting algorithm for detecting and correcting the divergence of the Kalman filter. Kobayashi, et al. (1995) proposed a method for generating an accurate estimate of the absolute speed of a vehicle from noisy acceleration and erroneous wheel speed information. The method employed the fuzzy logic rule-based Kalman filter to handle abrupt wheel skid and slip, and poor signal-to-noise sensor data. Mostov and Soloviev (1996) proposed the method to increase the Kalman filter order, which in turn enhances the accuracy of smoothing and thus location finding for kinematic GPS.

The process model of the KF is dependent on the dynamical characteristics of the vehicle. It is widely known that poorly designed mathematical model for the EKF may lead to the divergence. The fuzzy logic adaptive system (FLAS) can be used to adapt the gain and therefore prevent the Kalman filter from divergence. The FLAS is employed to make the necessary trade-off between accuracy and computational burden due to the increased dimension of the state vector and associated matrices. When the FLAS is employed, the
lower order state model can be used without significantly compromising accuracy. In other words, for a given accuracy, the fuzzy adaptive Kalman filter is allowed to use a lower order state model. When a designer lacks sufficient information to develop complete model or the parameters will slowly change with time, the fuzzy system can be used to adjust the performance of EKF on-line, and it will remain sensitive to parameter variations by ‘remembering’ most recent data samples.

Examples for possible approaches are given as follows. The covariance matrix of the innovation can be written as $C_{uk} = E[v_k v_k^T] = H_k P_k H_k^T + R_k$. The trace of innovation covariance matrix can be obtained through the relation: $v_k^T v_k = tr(v_k v_k^T)$. The degree of divergence (DOD) parameters for identifying the degree of change in vehicle dynamics can be determined. The innovation information at the present epoch is employed for timely reflect the change in vehicle dynamics. The DOD parameter $\xi$ can be defined as the trace of innovation covariance matrix at present epoch (i.e., the window size is one) divided by the number of satellites employed for navigation processing:

$$\xi = \frac{v_k^T v_k}{m}$$

(34)

where $v_k = [v_1 \ v_2 \ \cdots \ v_m]^T$, $m$ is the number of measurements (number of satellites). Alternatively, the averaged magnitude (absolute value) of innovation at the present epoch can also be used:

$$\zeta = \frac{1}{m} \sum_{i=1}^{m} |v_i|$$

(35)

In the FLAS, the DOD parameters are employed as the inputs for the fuzzy inference engines. By monitoring the DOD parameters, the FLAS is able to on-line tune the filter parameters according to the fuzzy rules. Fig. 6 illustrates the system architecture for GPS navigation using the FLAS-assisted filter. The block diagram of the FLAS-assisted IMM algorithm (an example on one cycle with two models) is shown in Fig. 7.

![Fig. 6. System architecture for GPS navigation using the FLAS-assisted filter](www.intechopen.com)
7. Examples

In this section, three examples are provided to illustrate the performance for various types of filter algorithms.

(A) Example 1: Incorporation of FLAS to the adaptive Kalman filter

Example 1 is taken from Jwo & Wang (2007). The simulation scenario is as follows. The simulated vehicle trajectory originates from the position of North 25.1492 degrees and East 121.7775 degrees at an altitude of 100m. This is equivalent to $T = [304239.2 \ 4911080.2 \ 2694074.3]^{T}$ m in the WGS-84 ECEF coordinate system. The location of the origin is defined as the (0,0,0) m location in the local tangent East-North-Up (ENU) frame. The three-dimensional plot of trajectory and vehicle velocity are shown in Fig. 8. The description of the vehicle motion is listed in Table 1. Assuming that the differential GPS (DGPS) mode is used most of the errors can be corrected, but the multipath and receiver measurement thermal noise cannot be eliminated. The measurement noise variances $r_{p_{i}}$ value are assumed a priori known, which is set to be $(3m)^{2}$. Let each of the white-noise spectral amplitudes that drive the random walk position states be $S_{p} = 0.1(m/sec^{2})/(rad/sec)$. Also, let the clock model spectral amplitudes be
$S_f = 0.4 \cdot 10^{-18} \text{ sec}$ and $S_g = 1.58 \cdot 10^{-18} \text{ sec}^{-1}$. These spectral amplitudes can be used to find the $Q_k$. The measurement noise covariance matrix is set as

$$R_k = \begin{bmatrix}
15 & 0 \\
15 & 15 \\
0 & 15
\end{bmatrix}$$

The parameter $\beta$ in STKF is a constant and does not change subject to the change in dynamics. When the vehicle is in high dynamic environments, a smaller softening factor ($\beta$) will be required for better tracking capability; when the vehicle is in lower dynamic environments, a larger $\beta$ will be needed for better estimation precision. Therefore, the improved versioned of STKF, which incorporates the FLAS, can be introduced for automatically adjust the value of $\beta$. For the vehicle in a very low dynamic environment, $\beta$ should be increased to a very large value, which leads $\lambda_{i,k}$ to 1 and results in the standard Kalman filter. The philosophy for defining the rules is straightforward: (1) for the case that the DOD parameter is small, our objective is to obtain results with better estimation accuracy, and a larger softening factor ($\beta$) should be applied; (2) for the case that the DOD parameter is increased, our objective is to increase the tracking capability, and a smaller softening factor should be applied. The membership functions (MFs) of input fuzzy variable DOD parameters are triangle MFs, obtained by the function:

$$\mu(x) = \begin{cases}
0 & x \leq a \\
\frac{x-a}{b-a} & a \leq x \leq b \\
\frac{c-x}{c-b} & b \leq x \leq c \\
0 & c \leq x
\end{cases}$$

| Time interval (sec) | Motion |
|---------------------|--------|
| [0-50]              | Constant velocity |
| [51-100]            | Constant acceleration |
| [101-150]           | Constant velocity |
| [151-200]           | Variable acceleration |
| [201-250]           | Constant velocity |
| [251-350]           | Circular motion, clockwise turn |
| [351-450]           | Constant velocity |

Table 1. Description of vehicle motion for Examples 1 and 2

The first-order T-S model is given. The zero-order model needs more complicated MFs and rule base and is therefore more difficult to determine. The presented FLAS is the If-Then form and consists of 3 rules. Two methods corresponding to four DOD parameters are presented.
Fig. 8. Three dimensional vehicle trajectory (left) and velocity (right) in the east, north, and vertical components

(1) Method 1 - use $\xi$ in Equation (34) as the DOD parameter
1. IF $\xi$ is zero THEN $\beta$ is $\xi + 10$
2. IF $\xi$ is small THEN $\beta$ is $\xi + 4$
3. IF $\xi$ is large THEN $\beta$ is 1

(2) Method 2 - use $\zeta$ in Equation (35) as the DOD parameter
1. IF $\zeta$ is zero THEN $\beta$ is $3\zeta + 8$
2. IF $\zeta$ is small THEN $\beta$ is $2\zeta + 4$
3. IF $\zeta$ is large THEN $\beta$ is 1

The membership functions of input fuzzy variable $\xi$ and $\zeta$ are provided in Fig. 9.

Fig. 9. Membership functions of input fuzzy variable $\xi$ (left) and $\zeta$ (right)

Figs. 10 and 11 provide the GPS navigation results for the EKF, STKF and AFSTKF approaches. It can be seen that substantial estimation accuracy improvement is obtained by using the proposed technique. In the three time intervals, 51~100, 151~200, 251~350 sec, the vehicle is manoeuvring. The mismatch of the model leads the conventional EKF to large navigation error while the FLAS timely detects the increase of DOD parameter, and then reduces softening factor so as to maintain good tracking capability. The AFSTKF has good capability to detect the change in vehicle dynamics and adjust the softening factor for preventing the divergence and remaining better navigation accuracy. The softening factors determined by the FLAS, and the corresponding fading factors are given in Fig. 12. It can be seen that when the vehicle is in high dynamic environment, $\beta$ will be tuned to a smaller value; in a low dynamic case, $\beta$ will be tuned to a very larger value. The case that $\beta$ is very
large will lead the fading factor $\lambda_{i,k}$ to 1, and the AFSTKF becomes the standard extended Kalman filter. The fact, as was predicted, can be seen in the time intervals 0-50 sec, 101-150 sec, 201-250 sec and 351-405 sec.

Fig. 10. Navigation errors for the STKF method (left) and the EKF method (right)

Fig. 11. East, north and up components of the navigation errors and the corresponding $1-\sigma$ bound based on the STKF method (left) and AFSTKF method (right)

Fig. 12. The softening factors (top) and fading factors (bottom)
Example 2 is taken from Jwo & Lai (2008), which has the same simulation scenario as used in Example 1. In order to improve the navigation estimation accuracy, utilization of a nonlinear model for better description on the vehicle dynamic will be more plausible to achieve better estimation accuracy. For comparison purpose, the same values of process noise covariance parameters are utilized for both the EKF and UKF: $\sigma_i = 3$, $i = 1...7$, for which

![Comparison of GPS positioning errors for EKF and UKF when the proposed nonlinear dynamic model is used](https://www.intechopen.com)

Fig. 13. Comparison of GPS positioning errors for EKF and UKF when the proposed nonlinear dynamic model is used
case $\alpha = 0.007$, $\beta = 2$, $\gamma = 0$. In the three time intervals, 51\textasciitilde100, 151\textasciitilde200, 251\textasciitilde350 s, the vehicle is conducting manoeuvring. The significant mismatch of the model leads the large errors in the conventional EKF solution, while the UKF with nonlinear model that better describes the vehicle dynamics is more able to achieve better navigation accuracy. Comparison of the positioning errors is shown in Fig. 13.

(C) Example 3: utilization of the IMM algorithm
Example 3 is taken from Jwo & Tseng (2008). Two models of the vehicle are defined: the non-manoeuvring model (constant velocity) reproduces properly straight trajectories of the vehicle; the manoeuvring model (constant acceleration) considers sharp turns and brusque accelerations. In the lower dynamic environment, the dynamic process of the GPS receiver is represented by the CV or PV (Position-Velocity) model, in which the state to be estimated is a $8 \times 1$ vector, including three position states, three velocity states, and two clock states. While considering the vehicle manoeuvring (or turning), the CA or PVA (Position-Velocity-Acceleration) model will be more suitable than the PV one. The PVA model adds three acceleration variables to the original PV model, leading to the $11 \times 1$ state vector. The following transition probability matrices of the Markov chain were used

$$P_{ij} = \begin{bmatrix} 0.99 & 0.01 \\ 0.01 & 0.99 \end{bmatrix} \quad (85)$$

The simulation scenario in this example is made as follows. The location of the origin is defined as the (0,0,0)m location in the local tangent East-North-Up (ENU) coordinate frame. The two dimensional simulated vehicle trajectory and vehicle velocity are shown in Fig. 14. Furthermore, the description of the vehicle motion is listed in Table 2. The mode probability of the IMM filter is depicted in Fig. 15. Fig. 16 provides the GPS navigation results for the standard EKF and IMM-EKF approaches. In the three time intervals, 1001-3000, 4001-5000, 5501-6500s, the vehicle is manoeuvring. The mismatch of the model leads the conventional EKF to large navigation error. Using the IMM-EKF, a reduction of approximately 8.14 m RMSE has been achieved.

Fig. 14. Simulated vehicle trajectory (left) and the corresponding vehicle velocity (right) in the east and north components.
Table 2. Description of vehicle motion for example 3

| Time interval (sec) | Motion                        |
|--------------------|-------------------------------|
| [0-1000]           | Constant velocity             |
| [1001-3000]        | Circular motion               |
| [3001-4000]        | Constant velocity             |
| [4001-5000]        | Clockwise turn                |
| [5001-5500]        | Constant velocity             |
| [5501-6500]        | Counter-clockwise turn        |
| [6501-8000]        | Constant velocity             |

Fig. 15. Model probability of the IMM-EKF

8. Conclusion

The divergence problem due to modelling errors is critical in Kalman filter applications. The conventional extended Kalman filter does not present the capability to monitor the change of parameters due to changes in vehicle dynamics. In this chapter, three feasible ways to avoid the divergence problem and to further improve the GPS navigation accuracy are discussed: (1) adaptive approaches assisted by heuristic search techniques to fit the dynamic model process of interest as precisely as possible; (2) utilization of an appropriate nonlinear estimation approach after deriving a better nonlinear dynamic process model; and (3) interactive multiple model approach accounting for different manoeuvring conditions.

In Example 1, the FLAS is incorporated into the traditional strong tracking Kalman filter (STKF) approach for determining the softening factors, resulting in the adaptive fuzzy strong tracking Kalman filter (AFSTKF). Through the use of fuzzy logic, the FLAS has been employed as a mechanism for timely detecting the dynamical changes and implementing the on-line tuning of filter parameters by monitoring the innovation information so as to maintain good tracking capability. By use of the FLAS, lower order of filter model can be utilized and, therefore, less computational effort will be sufficient without compromising estimation accuracy significantly.
An unscented Kalman filtering approach with nonlinear modelling has been presented to be superior to the EKF, due to the fact that the UKF is able to deal with the nonlinear formulation, while the linear model does not reflect the actual dynamic behaviour when the vehicle is manoeuvring. The UKF with nonlinear model ensures better description on the vehicle dynamics and will be able to achieve better navigation accuracy.

The other alternative approach for designing an adaptive Kalman filter is the interacting multiple model (IMM) algorithm. The use of an IMM method also allows exploiting the benefits of high dynamic models in the problem of vehicle navigation. An IMM-based method has been presented to be able to improve the estimation accuracy. Two models that represent different manoeuvring conditions have been conducted. Simulation experiments for GPS navigation have been carried out to discuss the accessibility and performance improvement using various approaches.

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The aim of this book is to provide an overview of recent developments in Kalman filter theory and their applications in engineering and scientific fields. The book is divided into 24 chapters and organized in five blocks corresponding to recent advances in Kalman filtering theory, applications in medical and biological sciences, tracking and positioning systems, electrical engineering and, finally, industrial processes and communication networks.

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