Ghost wavefunction renormalization in asymptotically safe quantum gravity

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Received 27 April 2010
Published 26 July 2010
Online at stacks.iop.org/JPhysA/43/365403

Abstract
Motivated by Weinberg’s asymptotic safety scenario, we investigate the gravitational renormalization group flow in the Einstein–Hilbert truncation supplemented by the wavefunction renormalization of the ghost fields. The latter induces non-trivial corrections to the $\beta$-functions for Newton’s constant and the cosmological constant. The resulting ghost-improved phase diagram is investigated in detail. In particular, we find a non-trivial ultraviolet fixed point, in agreement with the asymptotic safety conjecture which also survives in the presence of extra dimensions. In four dimensions the ghost anomalous dimension at the fixed point is $\eta^*_c = -1.8$, supporting spacetime being effectively two dimensional at short distances.

PACS numbers: 04.60.−m, 11.10.Hi, 11.15.Tk

(Some figures in this article are in colour only in the electronic version)

1. Introduction
Constructing a consistent and predictive quantum theory for gravity is one of the prime challenges in theoretical high energy physics today. One proposal in this direction, which recently received a lot of attention, is Weinberg’s asymptotic safety scenario [1–3], see [4–9] for reviews. This scenario adopts Wilson’s modern viewpoint on renormalization [10], assessing that gravity has a fundamental description within the framework of non-perturbatively renormalizable quantum field theories. The key ingredient underlying this idea is a conjectured non-trivial (or non-Gaussian) fixed point (NGFP) of the gravitational renormalization group (RG) flow. For RG trajectories attracted to it at high energies, the fixed point ensures that the dimensionless coupling constants remain finite, so that physical quantities are safe from unphysical UV divergences. These trajectories, also including the one describing our world [11], span the UV critical surface $S_{\text{UV}}$ of the fixed point. The precise position of ‘our RG trajectory’ within this surface is determined by free parameters,
which have to be fixed by experiment. For a finite-dimensional $S_{\text{UV}}$ this construction is then as predictive as a standard, perturbatively renormalizable, quantum field theory. Elucidating the fixed-point structure of the gravitational RG flow and understanding the properties of the corresponding UV critical surfaces is a central aspect of the asymptotic safety program to date.

An important technical tool in this program is the functional renormalization group equation (FRGE) for gravity [12]. Formulated in terms of the Wetterich equation [13], the FRGE describes the dependence of the effective average action $\Gamma_k$ on the coarse graining scale $k$:

$$\partial_t \Gamma_k = \frac{1}{2} \text{STr} \left[ (\Gamma_k^{(2)} + R_k)^{-1} \partial_t R_k \right], \quad \partial_t = \frac{k \, d}{dk}.$$  \hspace{1cm} (1.1)

Here, $\Gamma_k^{(2)}$ denotes the second functional derivative of the effective average action with respect to the fields of the theory and $\text{STr}$ is a generalized functional trace which includes a minus sign for ghosts and fermions. Furthermore, $R_k$ is a matrix-valued IR cutoff, which provides a $k$-dependent mass-term for fluctuations with momenta $p^2 < k^2$. The interplay between the full regularized propagator $(\Gamma_k^{(2)} + R_k)^{-1}$ and $\partial_t R_k$ ensures that the $\text{STr}$ receives contributions from a small $p^2$-interval around $k^2 \approx p^2$ only, rendering the trace-contribution finite.

While (1.1) constitutes a formally exact equation, it comes with the drawback that it cannot be solved exactly. A widely used approximation scheme for obtaining non-perturbative information from the FRGE consists of making an ansatz (truncation) for $\Gamma_k$ which retains a finite number of $k$-dependent coupling constants only. Projecting the resulting RG flow onto the truncation subspace allows us to read off the $\beta$-functions for the running couplings as the coefficients of the interaction monomials retained by the ansatz without resorting to the perturbation theory. For employing truncations, an important point is to establish the robustness of the emergent physical picture. Within a given ansatz, its reliability may be tested by studying the dependence of physical quantities on the shape of the unphysical IR regulator $R_k$. A second, more laborious, route consists in extending the truncation ansatz, thereby showing that all physical results, e.g. the fixed points of the RG flow, are robust under the extension.

A natural organizing principle for the operators retained in the truncation ansatz is provided by the derivative expansion of $\Gamma_k$, ordering the interaction terms by the canonical mass dimension of the corresponding coupling constants$^2$. For gravity, this has been implemented systematically, so far focusing on the gravitational sector of $\Gamma_k$ mostly. In this class the truncation ansatz is spanned by diffeomorphism invariant operators built from the physical metric $g$, e.g. $\Gamma_k = (16\pi G_k)^{-1} \int d^d x \sqrt{g}(-R + 2\Lambda_k) + \cdots$. The most studied case, the so-called Einstein–Hilbert truncation, encompasses a scale-dependent Newton’s constant $G_k$ and the cosmological constant $\Lambda_k$, and has been analyzed in a number of works [17–23]. Subsequently, this ansatz has been refined by including higher derivative $R^2$ interactions [24–26], higher order polynomials in $R$ up to $R^8$ within the framework of $f(R)$-gravity [27–29], non-local operators [28, 30], and lately also the Weyl-squared interactions [31–33] capturing the characteristic features of higher derivative gravity. All these computations have identified a NGFP of the gravitational RG flow, providing substantial evidence for the asymptotic safety scenario. Moreover, the $f(R)$- and $C^2$-results point at the dimension of the associated UV critical surface being finite, possibly even as low as three. Notably, the essential features of this picture already emerge from the structurally significantly simpler flow equations obtained within a conformally reduced gravity framework [34–36].

1 The FRGE captures the RG-dependence of the effective action. The relation between the effective action at a fixed-point $\Gamma_*$ and the corresponding fundamental action $S$ has recently been discussed in [14].
2 For reviews on the derivative expansion in the case of scalar field theory see [15, 16].
In this work we go beyond the gravitational approximation of $\Gamma_\gamma$, including quantum effects from the ghost sector. More specifically, we will augment the Einstein–Hilbert truncation by the power-counting marginal wavefunction renormalization of the ghost fields, whose operator dimension is equal to the gravitational four-derivative couplings (for a similar computation in Yang–Mills theory see [37]). Our main motivation for this ghost-improvement originates from the analogy to QCD where this coupling plays an essential role for the IR physics of the theory [38–41]. While it is clear that there is also a non-trivial interplay between the ghosts and the gravitational $\beta$-functions in gravity, explicit computations are not yet available. Here we close this gap, computing the anomalous dimension of the ghost fields $\eta_c$ and its effect on the running of Newton’s constant and the cosmological constant.

The explicit computation is based on a new perturbative heat-kernel technique developed by Anselmi and Benini [42] which allows the systematic expansion of the flow equation in the presence of background ghost fields in a curved background. This continues the exploration of quantum gravity effects in the ghost sector [44].

The rest of the paper is organized as follows. In section 2 we derive the $\beta$-functions governing the RG dependence of Newton’s constant, the cosmological constant and the wavefunction renormalization in the ghost sector, using a perturbative heat-kernel technique for non-minimal differential operators. The ghost-improved fixed-point structure and phase diagram are analyzed in section 3 and we comment on our findings in section 4. Some technical details on the heat-kernel techniques and threshold functions employed in this paper are relegated to appendix A, while the functions determining the gravitational and ghost anomalous dimensions are defined in appendix B.

2. The ghost-improved Einstein–Hilbert truncation

The main purpose of this paper is the investigation of the gravitational RG flow, including the quantum effects captured by the wavefunction renormalization $Z^\gamma_c$ in the ghost sector. Our truncation ansatz, which will be called the ‘ghost-improved Einstein–Hilbert truncation’, encompasses three scale-dependent coupling constants: Newton’s constant $G_k$, the cosmological constant $\Lambda_k$ and the power-counting marginal $Z^\gamma_c$ multiplying the ghost kinetic term. In this section, we will derive the non-perturbative $\beta$-functions capturing the RG dependence of these couplings.

2.1. The truncation ansatz

Our ansatz for the effective average action is of the general form

$$\Gamma_k[g, C, \bar{C}; \bar{g}, c, \bar{c}] = \Gamma^\text{grav}_k[g] + \Gamma^\text{gf}_k[g; \bar{g}] + \Gamma^\text{gh}_k[g, C, \bar{C}; \bar{g}, c, \bar{c}].$$  \hspace{1cm} (2.1)

Besides the physical metric $g$ and the classical ghost fields $C, \bar{C}$, it also depends on the corresponding background fields $\bar{g}$ and $c, \bar{c}$. They are related by

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \hspace{1cm} C_{\mu} = \bar{c}_{\mu} + f_{\mu}, \hspace{1cm} \tilde{C}_{\mu} = \tilde{c}_{\mu} + \tilde{f}_{\mu},$$

where $h_{\mu\nu}$ and $f_{\mu}, \tilde{f}_{\mu}$ denote the expectation value of the quantum fluctuations around the background, which are not necessarily small. In the gravitational approximation the computations are simplified by setting the background ghost fields to zero. This, however, does not allow us to keep track of the ghost kinetic term, so we work with a non-trivial ghost background in the following.

For related results based on a spectrally adjusted cutoff and a flat-space projection technique, see [43].
The gravitational part $\Gamma_{k}^{\text{grav}}[g]$ is built from the physical metric $g$ and taken to be of the Einstein–Hilbert form

$$\Gamma_{k}^{\text{grav}}[g] = 2\kappa^2 Z_k^N \int d^d x \sqrt{-g} (-R + 2\Lambda_k),$$

(2.3)

where $\kappa^2 = (32\pi G_0)^{-1}$ with $G_0$ a fixed reference scale and $Z_k^N$ denotes the wavefunction renormalization for the graviton. $\Gamma_{k}^{\text{grav}}[g]$ is supplemented by the gauge-fixing term $\Gamma_{k}^{gf}[g; \bar{g}]$. Employing the harmonic gauge, the latter reads

$$\Gamma_{k}^{gf}[h; \bar{g}] = \frac{1}{2} Z_k^N \int d^d x \sqrt{-\bar{g}} \bar{g}^\mu\nu F_\mu F_\nu, \quad F_\mu = \sqrt{Z_k} \left( D^\rho h_{\mu\rho} - \frac{1}{2} D_\mu \bar{g}^{\alpha\beta} h_{\alpha\beta} \right).$$

(2.4)

The resulting Faddeev–Popov determinant is captured by the ghost term

$$\Gamma_{k}^{gh}[g, C, \bar{g}, \bar{c}, \bar{c}] = -\sqrt{2} Z_k^C \int d^d x \sqrt{-\bar{g}} \bar{C}_\mu M^{\mu\nu} C^\nu,$$

(2.5)

with

$$M^{\mu\nu} = \bar{g}^{\mu\rho} \bar{g}^{\sigma\nu} D_\rho (g_{\mu\sigma} D_\nu + g_{\nu\sigma} D_\mu) - \bar{g}^{\mu\sigma} \bar{g}^{\nu\lambda} D_\lambda (g_{\sigma\nu} D_\mu + g_{\mu\nu} D_\sigma),$$

(2.6)

containing the wavefunction renormalization of the ghosts $Z_k^C$. The gauge choice (2.4) has the main virtue, that it allows for a straightforward comparison to earlier results [17] obtained in the Einstein–Hilbert truncation without ghost-improvement, fixing $Z_k = 1$.

Before entering into the explicit computation of the $\beta$-functions arising from the ansatz (2.1), it is useful to first consider the left-hand side of the flow equation and identify the interaction monomials whose coefficients encode the running of our coupling constants. Taking the $\partial_t$-derivative of our ansatz (2.1) and setting the fluctuation fields to zero afterward yields

$$\partial_t \Gamma_k = 2\kappa^2 \int d^d x \sqrt{\bar{g}} \left[ \left( \partial_t Z_k^N \right) \bar{R} + 2 \partial_t (Z_k^N \Lambda_k) \right] - \sqrt{2} \left( \partial_t Z_k^C \right) \int d^d x \sqrt{-\bar{g}} \bar{C}_\mu M^{\mu\nu} C^\nu + \cdots.$$ 

(2.7)

Thus it suffices to extract the Einstein–Hilbert monomials and the ghost kinetic term from the right-hand side of the flow equation. Comparing the coefficients multiplying these interactions then allows us to read off the desired $\beta$-functions for $Z_k^N$, $Z_k^C$ and $\Lambda_k$.

2.2. Quadratic forms and the inverse Hessian $(\Gamma_k^{(2)} + \mathcal{R}_k)^{-1}$

Upon specifying our truncation ansatz, we proceed by computing the second variation of $\Gamma_k$ with respect to fluctuations (2.2). To facilitate the subsequent steps, it is convenient to decompose the metric fluctuations into their traceless and trace part

$$h_{\mu\nu} = \bar{h}_{\mu\nu} + \frac{1}{d} \bar{g}_{\mu\nu} h, \quad h = \bar{g}^{\mu\nu} h_{\mu\nu}, \quad \bar{g}_{\mu\nu} \bar{h}_{\mu\nu} = 0.$$ 

(2.8)

The forms quadratic in the fluctuations can be simplified further by first identifying $g = \bar{g}$, $C = c$, $\bar{C} = \bar{c}$ and specifying a particular class of backgrounds. In our case, this class has to be general enough to distinguish the interaction monomials in the Einstein–Hilbert term and the ghost kinetic term. This can be accomplished by choosing the background metric $\bar{g}$ as the one of a $d$-dimensional sphere, implying

$$\bar{R}_{\mu\nu\sigma} = \frac{1}{d(d-1)} \bar{R}(\bar{g}_{\mu\rho} \bar{g}_{\nu\sigma} - \bar{g}_{\mu\sigma} \bar{g}_{\nu\rho}), \quad \bar{R}_{\mu\nu} = \frac{1}{d} \bar{R} \bar{g}_{\mu\nu}.$$ 

(2.9)

Moreover, the background ghost field can be taken as transversal

$$\bar{D}_\mu c^\mu = 0, \quad \bar{D}_\mu \bar{c}^\mu = 0.$$ 

(2.10)
Here the two lines are equivalent up to surface terms and define the Grassmann-valued operators

\[ I_{2T} = \frac{1}{2} \left( \delta^\mu_\nu \delta^\rho_\sigma + \delta^\mu_\rho \delta^\nu_\sigma \right) - \frac{1}{4} g_{\mu\nu} g^{\rho\sigma}, \quad I_1 = \delta^\mu_\mu. \tag{2.11} \]

Given these preliminaries, we now expand ansatz (2.1) around the backgrounds (2.9) and (2.10), retaining the pieces quadratic in the fluctuations only. For \( \Gamma^\text{grav}_{\kappa}[g] + \Gamma^\text{grav}_{\kappa}[h, \tilde{g}] \) the result has already been given in [12]

\[
\Gamma^\text{quad}\{h; \tilde{g}\} = \frac{1}{2} \kappa^2 Z^N_{\kappa} \int d^d x \sqrt{\tilde{g}} \left\{ \tilde{h}_{\mu\nu} \left[ \Delta - 2 \Lambda_k + C_T \tilde{R} \right] \tilde{h}^{\mu\nu} - \frac{d - 2}{2d} h [\Delta - 2 \Lambda_k + C_S \hat{R}] h \right\} \tag{2.12}
\]

with

\[
C_T = \frac{d^2 - 3d + 4}{d(d - 1)}, \quad C_S = \frac{d - 4}{d}, \tag{2.13}
\]

and \( \Delta \equiv - \tilde{\nabla}^2 \) denoting the covariant Laplacian constructed from the background metric. Owing to the non-trivial background ghost field, the analogous computation for \( \Gamma^\text{grav}_{\kappa} \) is slightly more involved. Thus, we first give the intermediate results obtained from expanding (2.6) contracted with a dummy vector \( C^\nu \) in \( h_{\mu\nu} \):

\[
\begin{align*}
\overline{\delta}_a M^\mu_\nu C^\nu &= \left[ \tilde{D}^2 + \frac{1}{d} \tilde{R} \right] C^\mu, \\
\delta_a M^\mu_\nu C^\nu &= \tilde{g}^{\rho\sigma} \tilde{g}^{\kappa\lambda} \hat{D}_\kappa [h_{\rho\sigma} \hat{D}_\lambda + h_{\lambda\rho} \hat{D}_\sigma + (\hat{D}_\mu h_{\rho\sigma})] C^\mu \\
&\quad - \tilde{g}^{\rho\sigma} \tilde{g}^{\kappa\lambda} \hat{D}_\kappa \left[ h_{\sigma\rho} \hat{D}_\lambda + \frac{1}{2} (\hat{D}_\mu h_{\rho\sigma}) \right] C^\mu, \\
\overline{\delta}_a^2 M^\mu_\nu C^\nu &= 0.
\end{align*}
\tag{2.14}
\]

Here the bar indicates that we expand around \( g = \tilde{g} \). Remarkably, the last line is independent of the choice of background or gauge. Based on these results, we obtain the quadratic form in the ghost sector

\[
\Gamma^\text{quad,gh} = \sqrt{2} Z^N_{\kappa} \int d^d x \sqrt{\tilde{g}} \left\{ \tilde{f}_\mu \left[ \Delta - \frac{1}{d} \tilde{R} \right] f^\mu - h A^\alpha \tilde{f}_\alpha - \tilde{h}^{\mu\nu} Q_{\mu\nu}^a \tilde{f}_a - h \tilde{A}_a f^a \right. \\
\left. - \tilde{\hat{h}}_{\mu\nu} \tilde{Q}^{\mu\nu}_a f^a \right\}
\tag{2.15}
\]

Here the two lines are equivalent up to surface terms and define the Grassmann-valued operators \( A, \tilde{A}, Q, \tilde{Q} \), and their adjoints, respectively. Their explicit expressions read

\[
\begin{align*}
\tilde{A}^a &= \frac{1}{d} \left[ \tilde{D}^2 c^a + \tilde{R}^a c_\sigma + \tilde{D}_\sigma c^a \tilde{D}^\sigma + \left( 2 - \frac{d}{2} \right) D^a c_\alpha \tilde{D}_\alpha + \left( 1 - \frac{d}{2} \right) c^\alpha \tilde{D}^\alpha \tilde{D}_a \right], \\
\tilde{A}_a &= -\frac{1}{d} [\tilde{D}^a \tilde{c}_\alpha \tilde{D}_\alpha + \tilde{D}_\alpha c^a \tilde{D}^\alpha], \\
A^a &= \frac{1}{d} \left[ \tilde{D}_\sigma c^a \tilde{D}^\sigma + \tilde{D}^a c^\alpha \tilde{D}_\alpha - \left( 1 - \frac{d}{2} \right) c^\alpha \tilde{D}_a \tilde{D}^a \right], \\
\tilde{A}_a &= -\frac{1}{d} [\tilde{D}^2 \tilde{c}_a + \tilde{D}^a \tilde{c}_\alpha \tilde{D}_\alpha + \tilde{R}_{\alpha\rho} c^\alpha \tilde{D}_\rho + \tilde{D}_\rho c^a \tilde{D}^\rho]. \tag{2.16}
\end{align*}
\]
and
\[
\bar{Q}_\mu^a = \delta_\mu^a B_c^2 c^a + R_1^a \bar{c}^a c^a + \delta_\mu^a \bar{c}^a d^a \bar{D}_a \bar{D}_a \\
+ \delta_\mu^a \bar{D}_a \bar{c}_a + \bar{D}_a c_a \bar{D}_a + \delta_\mu^a \bar{D}_a \bar{c}^a + \bar{D}_a c^a \bar{D}_a - \bar{D}_\mu c_a \bar{D}_a \\
- \frac{1}{d} \delta_\mu^a \bar{D}_a c^a D_a + R_0^a c^a D_a + 2D^a c_a D_a + \bar{D}^a D^a D_a, \\
\hat{Q}_\mu^a = \hat{D}_a \bar{D}_a \bar{c}^a + \bar{D}_a c^a \bar{D}_a - \delta_\mu^a \bar{D}_a \bar{c}^a - \bar{D}_\mu c^a \bar{D}_a \\
+ \frac{1}{d} \delta_\mu^a \bar{D}_a c^a \bar{D}_a + \bar{D}_a c^a \bar{D}_a + \bar{D}_\mu c^a \bar{D}_a, \\
\hat{Q}_\mu^a = -\delta_\mu^a \bar{D}_a \bar{D}_a c^a + \bar{D}_a c^a \bar{D}_a + \delta_\mu^a \bar{D}_a \bar{c}^a - \bar{D}_\mu c^a \bar{D}_a \\
+ \frac{1}{d} \delta_\mu^a \bar{D}_a c^a \bar{D}_a + \bar{D}_a c^a \bar{D}_a + \bar{D}_\mu c^a \bar{D}_a + \bar{D}_\mu c^a \bar{D}_a, \\
\tag{2.17}
\]
where the covariant derivatives to the left of the ghost fields act on \(c\) or \(\bar{c}\) only and \((\mu) = \frac{1}{2}(\mu + \nu)\) denotes symmetrization with unit strength. The last lines in the \(Q\)-expressions ensure that the operators are traceless in the 2T-indices \(\mu \nu\).

The quadratic forms \((2.12)\) and \((2.15)\) provide the key ingredients for constructing the IR cutoff \(R_k\) and the inverse Hessian \((\Gamma_k^{(2)} + R_k)^{-1}\) in the following.

The construction of the IR cutoff follows the prescription in [12], i.e. at the level of the path integral, the gauge-fixed action is supplemented by an IR regulator of the form
\[
\Delta_k S = \int d^4 x \sqrt{g} \left\{ \frac{1}{2} \bar{h}_{\mu \alpha} R_{k,hh}^{\mu \alpha \beta} h_{\alpha \beta} + \frac{1}{2} h R_{k,hhh} h + \bar{f}^\nu R_{k,ff} f^\nu \right\}.
\tag{2.18}
\]
The matrix \(R_k\) is designed in such a way that it adds a \(k\)-dependent mass term to the Laplacians appearing in the kinetic terms:
\[
\Delta \rightarrow P_k = \Delta + R_k,
\tag{2.19}
\]
where \(R_k = k^2 R_k^{(0)}(\Delta / k^2)\) is the scalar part of the IR regulator and \(R_k^{(0)}\) a dimensionless shape function interpolating monotonically between \(R_k^{(0)}(0) = 1\) and \(\lim_{z \rightarrow \infty} R_k^{(0)}(z) = 0\).
Comparing (2.18) to the quadratic forms \((2.12)\) and \((2.15)\), this prescription fixes
\[
R_{k,hh} = 1_{2T} k^2 Z_k^N R_k, \quad R_{k,hhh} = -\frac{d - 2}{2d} \kappa^2 Z_k^N R_k, \quad R_{k,ff} = \sqrt{2} Z_k^N R_k.
\tag{2.20}
\]
Observe that \(R_k\) inherits a non-trivial \(k\)-dependence via wavefunction renormalization factors.

Notably, it is this feature which is partially responsible for the non-perturbative character of the computation.

The next step consists in constructing the Hessian \(\Gamma_k^{(2)}\) and the inverse of \(\Gamma_k^{(2)} + R_k\). Here we follow the conventions [37, 39], using a skew-symmetric metric to couple anti-commuting fields (see appendix A of [39] for details). In terms of the multiplets
\[
\Phi = \{h_{\mu \nu}, h, f^a, f_a\}, \quad \tilde{\Phi} = \{\bar{h}_{\mu \nu}, h, \bar{f}_a, f^a\},
\tag{2.21}
\]
the Hessian is given by the following matrix:
\[
[\Gamma_k^{(2)}]^{ij}(x, y) = \frac{1}{\sqrt{g(x)} \sqrt{g(y)}} \frac{\delta^2 \Gamma_k}{\delta \Phi_i(x) \delta \Phi_j(y)},
\tag{2.22}
\]
One can explicitly verify that the resulting FRGE is equivalent to [20].
where all variations act from the left. Substituting the quadratic forms (2.12) and (2.15),

\[(\Gamma^{(2)}_k + R_{k})^{ij} = \begin{bmatrix} K & \tilde{Q} \\ \tilde{Q} & M \end{bmatrix} \]  

(2.23)

assumes block form with entries

\[K = \kappa^2 Z_k^n \text{diag} \left[ (P_k - 2\Lambda_k + C_T R)1_{2T}, -\frac{d-2}{2d}(P_k - 2\Lambda_k + C_S R) \right],\]

\[M = \sqrt{2} Z_k^e \left( P_k - \frac{1}{d} \tilde{R} \right) \text{diag} [1,1],\]

(2.24)

and

\[Q = \sqrt{2} Z_k^e \left[ \tilde{Q}^{\mu\nu} \quad Q^{\mu\nu,\alpha} \right], \quad \tilde{Q} = \sqrt{2} Z_k^e \left[ -\tilde{Q}_k^{\mu\nu} \quad \tilde{A}^\alpha \right].\]

(2.25)

The inverse of (2.23) is then found via the general inversion formula for $2 \times 2$-block matrices

\[\begin{bmatrix} K & Q \\ \tilde{Q} & M \end{bmatrix}^{-1} = \begin{bmatrix} (K - QM^{-1}\tilde{Q})^{-1} & -K^{-1}Q(M - \tilde{Q}K^{-1}Q)^{-1} \\ -M^{-1}\tilde{Q}(K - QM^{-1}\tilde{Q})^{-1} & (M - \tilde{Q}K^{-1}Q)^{-1} \end{bmatrix}.\]

(2.26)

Expanding the inverse up to second order in the background-ghost fields, and taking into account the minus sign originating from the ghost sector of the supertrace, the right-hand side of the flow equation becomes

\[\partial_t \Gamma_k = \frac{1}{2} \text{Tr}_{12} \left[ (K - M^{-1}\tilde{Q})^{-1} - K^{-1}Q(M - \tilde{Q}K^{-1}Q)^{-1} \right] \partial_t R_k^{\text{grav}}\]

\[-\frac{1}{2} \text{Tr}_{gh} \left[ (M^{-1} + M^{-1}\tilde{Q}K^{-1}Q M^{-1}) \partial_t R_k^{\text{gh}} \right],\]

\[= S_{2T} + S_0 + S_1 + S_{2T} + \mathcal{G}_0 + \cdots,\]

(2.27)

with $R_k^{\text{grav}} = \text{diag}[R_{k,bb}, R_{k,jb}]$, $R_k^{\text{gh}} = \text{diag}[R_{k,jf,}, R_{k,jf}]$, and terms not contributing to the truncation indicated by the dots. As already anticipated in the last line, the full trace decomposes into operator traces on the space of traceless symmetric tensors ($2T$), scalars (0) and vectors (1). Substituting the explicit expressions for the block matrices, the traces $S_i$, which by definition, do not include $Q$ and $\tilde{Q}$ and are thus independent of the background ghost fields, become

\[S_{2T} = \frac{1}{2} \text{Tr}_{2T} \left[ \frac{1}{Z_k^e(P_k - 2\Lambda_k + C_T R)} \partial_t (Z_k^e R_k) \right],\]

\[S_0 = \frac{1}{2} \text{Tr}_0 \left[ \frac{1}{Z_k^e(P_k - 2\Lambda_k + C_S R)} \partial_t (Z_k^e R_k) \right],\]

\[S_1 = -\text{Tr}_1 \left[ \frac{1}{Z_k^e(P_k - 1/2 \tilde{R})} \partial_t (Z_k^e R_k) \right].\]

(2.28)

They give rise to the $\beta$-functions for Newton’s constant and the cosmological constant. The $\beta$-function for $Z_k^e$ is captured by the terms of second order in the background-ghost fields. Neglecting the curvature terms and making use of the cyclicity of the trace, these are found as

\[\mathcal{G}_{2T} = -\frac{Z_k^e}{\sqrt{2} \kappa^2 (Z_k^e)^2} \text{Tr}_{2T} \left[ \frac{\partial_t (Z_k^e R_k)}{P_k - 2\Lambda k} \left( Q^{\mu\nu} \frac{1}{P_k} \tilde{Q}^{\mu\nu,\alpha} - \tilde{Q}_k^{\mu\nu} \frac{1}{P_k} \tilde{Q}^{\mu\nu,\alpha} \right) \right],\]

\[\mathcal{G}_0 = \frac{2d}{d - 2} \frac{Z_k^e}{\sqrt{2} \kappa^2 (Z_k^e)^2} \text{Tr}_0 \left[ \frac{\partial_t (Z_k^e R_k)}{P_k - 2\Lambda k} \left( A^{\alpha} \frac{1}{P_k} \tilde{A}^\alpha - \tilde{A}_k^\alpha \frac{1}{P_k} \tilde{A}^\alpha \right) \right].\]
\[ G_1 = -\frac{1}{\sqrt{2}k^2 Z^2_k} \text{Tr} \left[ \frac{\partial}{\partial P_k} \left( \frac{\hat{Q}^\mu_{\alpha \beta}}{P_k} - \frac{1}{2\Lambda} \hat{Q}^\mu_{\alpha \beta} - \frac{\pi G_{\alpha \beta}}{P_k} - \frac{1}{2\Lambda} \hat{Q}^\mu_{\alpha \beta} \right) \right] \]

\[ = -\frac{2d}{d - 2} \left( \hat{A}_\mu - \frac{\Lambda k}{P_k - 2\Lambda} \hat{A}_\mu \right) \left( \hat{A}_\mu - \frac{\Lambda k}{P_k - 2\Lambda} \hat{A}_\mu \right) \right] \right]. \tag{2.29} \]

Notably, the insertions including the background ghosts, like, e.g. the \( Q \) and \( -\hat{Q} \) in \( G_1 \), always appear in pairs. Each part thereby gives exactly the same contribution to the \( G_i \), consistent with the symmetry factors of the underlying Feynman diagrams. Evaluating both parts individually provides a highly non-trivial crosscheck when computing the coefficients of the ghost kinetic term in the next subsection.

### 2.3. The ghost-improved \( \beta \)-functions

We are now in a position to construct the \( \beta \)-functions for \( Z_k^N \), \( Z_k^\epsilon \) and \( \Lambda_k \). In this context, it is useful to introduce the anomalous dimensions of Newton’s constant and the ghost wavefunction renormalization

\[ \eta_N = -\partial_i \ln \left( Z_k^N \right), \quad \eta_c = -\partial_i \ln \left( Z_k^\epsilon \right), \tag{2.30} \]

with the dimensionless couplings

\[ g_k = G_k k^{d-2} = (Z_k^N)^{-1} G_0 k^{d-2}, \quad \lambda_k = \Lambda_k k^{-2}. \tag{2.31} \]

As explained in section 2.1, the \( \beta \)-functions for Newton’s and the cosmological constant are encoded at zeroth order in the background-ghost fields. The desired interaction monomials are generated by the traces \( S \), equation (2.28), which are evaluated straightforwardly by applying the early-time heat-kernel expansion detailed in appendix A.1 followed by inserting identity (A.15). Equating the resulting coefficients with (2.7) provides the flow equations for \( \Lambda_k \) and \( G_k = \frac{1}{32\pi Z_k^N k^2} \):

\[ \partial_t \left( \frac{\Lambda_k}{8\pi G_k} \right) = \frac{k^d}{(4\pi)^{d/2}} \left( \frac{d(d + 1)}{2} \left[ \Phi^1_{d/2}(-2\lambda_k) - \frac{1}{2} \eta_N \Phi^1_{d/2}(-2\lambda_k) \right] \right. \]
\[ - \frac{1}{2} \eta_N \Phi^1_{d/2-1}(-2\lambda_k) \]
\[ - \left. \frac{d}{2} \left[ \Phi^2_{d/2}(-2\lambda_k) - \frac{1}{2} \eta_N \Phi^2_{d/2}(-2\lambda_k) \right] \right] \}

\[ = \frac{1}{16\pi G_k} \left( \frac{d(d + 1)}{12} \left[ \Phi^1_{d/2-1}(-2\lambda_k) \right] \right. \]
\[ - \frac{1}{2} \eta_N \Phi^1_{d/2-1}(-2\lambda_k) \]
\[ - \frac{d}{2} \left[ \Phi^1_{d/2-1}(-2\lambda_k) - \frac{1}{2} \eta_N \Phi^1_{d/2-1}(-2\lambda_k) \right] \]
\[ - \left. \frac{d}{3} \left[ \Phi^1_{d/2-1}(-2\lambda_k) - \frac{1}{2} \eta_N \Phi^1_{d/2-1}(-2\lambda_k) \right] \right] \}

\[ = \frac{1}{16\pi G_k} \left( \frac{d(d + 1)}{12} \left[ \Phi^1_{d/2-1}(-2\lambda_k) \right] \right. \]
\[ - \frac{1}{2} \eta_N \Phi^1_{d/2-1}(-2\lambda_k) \]
\[ - \frac{d}{2} \left[ \Phi^1_{d/2-1}(-2\lambda_k) - \frac{1}{2} \eta_N \Phi^1_{d/2-1}(-2\lambda_k) \right] \]
\[ - \left. \frac{d}{3} \left[ \Phi^1_{d/2-1}(-2\lambda_k) - \frac{1}{2} \eta_N \Phi^1_{d/2-1}(-2\lambda_k) \right] \right] \}

In the limit \( Z_k^\epsilon = 1, \eta_c = 0 \), this result agrees with earlier computations [12, 17]. The terms proportional to \( \eta_c \) are novel and capture the backreaction of the quantum effects in the ghost sector on the running of the gravitational coupling constants.

The final step is the computation of \( \eta_c \). This requires extracting the background ghost kinetic term from (2.29). Unfortunately, the differential operators entering into these traces are not minimal, so the early-time heat-kernel expansion is no longer applicable and a more sophisticated method is needed. Here we follow Anselmi and Benini [42], which allows us to compute operator traces of Laplace-type operators which include arbitrary (non-Laplace type)
\( G_{2T} = -\frac{\sqrt{2}}{(4\pi)^d/2} \frac{Z_k^c}{\kappa^2(Z_k^N)^2} \left[ \frac{4d - d - 8}{4d} Q_{d/2+1}[W_1^N] - \frac{d - 2}{2d} Q_{d/2+2}[W_2^N] \right] \)
\[ \times \int d^d x \sqrt{g} \bar{c} \bar{D} \mu. \]

\( G_0 = -\frac{\sqrt{2}}{(4\pi)^d/2} \frac{Z_k^c}{\kappa^2(Z_k^N)^2} \left[ \frac{d - 4}{d(d - 2)} Q_{d/2+1}[W_1^N] - \frac{1}{d} Q_{d/2+2}[W_2^N] \right] \int d^d x \sqrt{g} \bar{c} \bar{D} \mu, \]

\( G_1 = -\frac{1}{(4\pi)^d/2} \sqrt{2} \kappa Z_k^c \left[ \frac{2d^2 - 5d - 2}{2(d - 2)} Q_{d/2+1}[W_1^N] + d Q_{d/2+2}[W_2^N] \right] \int d^d x \sqrt{g} \bar{c} \bar{D} \mu. \)

Here the \( \bar{c} \)-functions are defined in (A.4) and

\[
W_1^N = \frac{\partial_t(Z_k^N R_k)}{(p_k^2 - 2\Lambda)^2} \frac{1}{p_k^2}, \quad W_2^N = -\frac{\partial_t(Z_k^N R_k)}{(p_k^2 - 2\Lambda)^2} \frac{d}{dx} \frac{1}{p_k(x)} \bigg|_{x = \Delta},
\]

\[
W_1^c = \frac{\partial_t(Z_k^c R_k)}{p_k^2} \frac{1}{p_k^2 - 2\Lambda}, \quad W_2^c = -\frac{\partial_t(Z_k^c R_k)}{p_k^2} \frac{d}{dx} \frac{1}{p_k(x)} \bigg|_{x = \Delta}. \]

Substituting (A.16) and equating the result with the ghost kinetic term in (2.7), we finally find

\[
\partial_t Z_k^c = \frac{4Z_k^c g_k}{(4\pi)^d/2 - 1} \left\{ C_{gr} \left( \Phi_{d/2+1}^{1,1}(-2\lambda_k) - \frac{1}{2} \eta_N \Phi_{d/2+1}^{1,1}(-2\lambda_k) \right) \right. \\
+ C_{gh} \left( \Phi_{d/2+1}^{1,2}(-2\lambda_k) - \frac{1}{2} \eta_c \Phi_{d/2+1}^{1,2}(-2\lambda_k) \right) \\
+ d(\eta_N - \eta_c) \left( \Phi_{d/2+2}^{2,2}(-2\lambda_k) + \Phi_{d/2+2}^{2,2}(-2\lambda_k) \right) \right\},
\]

where

\[
C_{gr} = \frac{4d^2 - 9d - 2}{d - 2}, \quad C_{gh} = \frac{2d^2 - 5d - 2}{d - 2}. \]

Here, the \( \Phi_{d/2+1}^{1,2} \) and \( \Phi_{d/2+2}^{2,2} \)-terms originating from \( Q_{d/2+2}[W_2^c] \) drop out, due to the cancellation of the corresponding coefficients.

The \( \beta \)-functions are then obtained by solving equation (2.32) and (2.35) for \( \partial_t \lambda_k, \partial_t g_k \) and \( \eta_c \). This yields

\[
\partial_t \lambda_k = \beta_k, \quad \partial_t g_k = \beta_k = (d - 2 + \eta_N) g_k.
\]

with

\[
\beta_k = -(2 - \eta_N) \lambda_k + \frac{1}{2} g_k (4\pi)^{1-d/2} \times \left[ 2d(d + 1) \Phi_{d/2}^{1,0}(-2\lambda_k) - 8d \Phi_{d/2}^{1,0}(0) - d(d + 1) \eta_N \Phi_{d/2}^{1,0}(-2\lambda_k) + 4d \eta_c \Phi_{d/2}^{1,0}(0) \right],
\]

and the anomalous dimensions

\[
\eta_N = \frac{g B_1(\lambda) + g^2 [C_3(\lambda) C_4(\lambda) - B_1(\lambda) C_2(\lambda)]}{1 - g [B_2(\lambda) + C_2(\lambda)] + g^2 [B_2(\lambda) C_2(\lambda) - C_1(\lambda) C_3(\lambda)]},
\]

\[
\eta_c = \frac{g C_4(\lambda) + g^2 [B_1(\lambda) C_1(\lambda) - B_2(\lambda) C_4(\lambda)]}{1 - g [B_2(\lambda) + C_2(\lambda)] + g^2 [B_2(\lambda) C_2(\lambda) - C_1(\lambda) C_3(\lambda)]}.
\]
The explicit form of the functions $B_i(\lambda)$ and $C_i(\lambda)$ is given in appendix B. Equations (2.38) and (2.39) are the desired $\beta$-functions governing the RG dependence of $g_k, \lambda_k$ and $\eta_c$ and constitute the central result of this section.

Some comments are now in order. The inclusion of the wavefunction renormalization for the ghosts gives non-trivial contributions to the $\beta$-functions for $g, \lambda$. These encompass the terms proportional to $\eta_c$ in $\beta_\lambda$ and the qualitatively new $g^2$-terms in $\eta_N$. The results obtained within the standard Einstein–Hilbert truncation [7, 12] are recovered by setting $C_i = 0$. Comparing powers of $g$, the leading contributions from the ghost sector are suppressed by one power of $g$, relative to the leading Einstein–Hilbert terms. Thus in the classical regime, $g \ll 1$, the ghost improvement may be neglected. In the quantum regime close to the NGFP where $g \approx 1$, however, we expect that these corrections become important. Investigating the influence of these new terms on the gravitational RG flow is then the subject of the next section.

3. Properties of the flow equation

After deriving the $\beta$-functions (2.37) and (2.39), we now proceed by studying their properties, mostly resorting to numerical methods. In this context, it is useful to observe that $Z_c^k$ enters into $\beta_\lambda$ via $\eta_c$ only and is in turn completely determined by $g_k, \lambda_k$. Thus, substituting the explicit formula for $\eta_c$ into $\beta_\lambda$, the running of $Z_c^k$ decouples and allows us to analyze the gravitational RG flow in the two-dimensional $g, \lambda$-subsystem. Once a RG trajectory for $g_k, \lambda_k$ is found, it can be plugged back into $\eta_c$ to obtain the running of the ghost anomalous dimension. We will now exploit this decoupling and first discuss the fixed-point structure of the ghost-improved Einstein–Hilbert truncation for $d = 4$ in section 3.1, before focusing on the phase portrait and the fixed-point structure including extra-dimensions in sections 3.2 and 3.3, respectively.

3.1. Fixed points of the four-dimensional $\beta$-functions

As highlighted in the introduction, the crucial ingredient of the asymptotic safety scenario is the fixed-point structure of gravitational $\beta$-functions. Thus, we start our investigation by looking for fixed points $g^*, \lambda^*$ where $\beta_{g^*} = \beta_{\lambda^*} = 0$ simultaneously. In the vicinity of such a point, the linearized $\beta$-functions are given by $\frac{dg}{d\eta} = B_{ij}(g_j - g^*_j)$, where $B_{ij} = \frac{\partial g_j \beta_{g_i}}{\partial g_j \mid_{g=g^*}}$. $g_i = \{g, \lambda\}$. The stability coefficients $\theta_i$, defined as minus the eigenvalues of $B_{ij}$, provide an important characteristic of the fixed point. In particular, eigendirections with a positive (negative) real part $\theta$ are UV-attractive (UV-repulsive) for trajectories close to the fixed point. For the remainder of this subsection, we will set $d = 4$.

Inspection of (2.37) immediately gives the Gaussian fixed point (GFP)

$$g^* = 0, \quad \lambda^* = 0, \quad \eta^*_N = \eta^*_c = 0.$$ (3.1)

This fixed point corresponds to the free theory, and constitutes a saddle point in the $g$-$\lambda$ plane. It has one attractive and one repulsive eigendirection with stability coefficients given by the canonical mass dimensions of $G$ and $\Lambda$, respectively.

The numerical analysis of the ghost-improved $\beta$-functions also reveals a unique NGFP. Its position and properties are shown in the first two lines of table 1.

It is situated at $g^* > 0, \lambda^* > 0$ and UV-attractive in both $g, \lambda$. Substituting its position into $\eta_c$ determines the ghost anomalous dimension $\eta^*_c = -1.8$.5 For comparison, the third line

5 When including the marginal $Z_c^2$ in the set of coupling constants, $\eta^*_c$ is also the stability coefficient associated with the new (UV-irrelevant) eigendirection. Here we refrain from adopting this viewpoint, however, since the running of $\eta_c$ is completely determined by $g_k, \lambda_k$ so $Z_c^k$ is, most likely, not an essential coupling.

Table 1. Properties of the NGFP arising from the ghost-improved $\beta$-functions (2.37). The first two lines show the position, the universal product $g^*\lambda^*$, the ghost anomalous dimension $\eta^*_c$ and the stability coefficients of the fixed point obtained with the optimized cutoff (A.17) and exponential cutoff (A.19) with $s = 1$, respectively. For comparison, the third line displays the characteristics of the NGFP found in the standard Einstein–Hilbert truncation [29].

| Truncation | $\lambda^*$ | $g^*$ | $g^*\lambda^*$ | $\eta^*_c$ | Re$(\theta)$ | Im$(\theta)$ | Cutoff |
|------------|-------------|-------|----------------|-------------|--------------|-------------|--------|
| EH + ghost | 0.135       | 0.859 | 0.116          | -1.774      | 1.935        | 2.012       | opt    |
| EH + ghost | 0.260       | 0.355 | 0.092          | -1.846      | 2.070        | 2.439       | exp($s = 1$) |
| EH         | 0.193       | 0.707 | 0.136          | -          | 1.475        | 3.043       | opt    |

of table 1 displays the properties of the NGFP obtained within the standard Einstein–Hilbert truncation without ghost improvement. We observe that the actual numerical values of the physical product $g^*\lambda^*$ and the stability coefficients are shifted by approximately 30%. This is in the typical range for the cutoff-scheme dependence observed in [29]. Most remarkably, both the standard and the ghost-improved Einstein–Hilbert truncation give rise to the same fixed-point structure. This is highly non-trivial, as the new contributions to the gravitational $\beta$-functions are of the same order of magnitude as the other, already known, terms. We interpret this result as a striking confirmation of the gravitational fixed-point structure disclosed by the standard Einstein–Hilbert truncation [17–20, 29].

The main virtue of the ghost improvement becomes apparent while investigating the stability of the physical quantities $g^*\lambda^*$, $\theta$, $\eta^*_c$ with respect to the variation of the IR cutoff $R_k$. To illustrate this point, we resort to the exponential cutoff (A.19), and determine the properties of the NGFP for varying shape parameter $s$. Figure 1 shows the resulting cutoff-scheme dependence of the physical quantities for the standard (dashed red line) and ghost-improved (solid blue line) computation, respectively. Remarkably, the ghost improvement reduces the unphysical cutoff-scheme dependence by factors 1.4, 2.0 and 3.2 for the product $g^*\lambda^*$, Re$\theta$ and Im$\theta$, respectively. The scheme dependence of $\eta^*_c$ can only be determined in the ghost-improved computation. Here the variation is approximately 2%.

Since the cutoff-scheme dependence provides the prototypical probe for judging the reliability of a truncation, these results clearly indicate that the ghost improvement significantly improves the quality of the Einstein–Hilbert approximation. In particular, the fixed-point properties given in table 1 should be more robust as the ones obtained without ghost improvement.

3.2. The ghost-improved phase portrait

After analyzing the fixed-point structure, we determine the phase portrait resulting from the ghost-improved $\beta$-functions. We start by investigating the gravitational RG flow on the $g$-$\lambda$ plane, before studying the behavior of the anomalous dimensions along some typical sample trajectories.

The phase portrait resulting from the numerical integration of $\beta$-functions (2.37) is depicted in figure 2. We first observe that $g_k = 0$ is a fixed line, which cannot be crossed by the flow. For $g_k > 0$ the flow is dominated by the interplay of the NGFP and the GFP. In this regime, the UV behavior of the RG trajectories is controlled by the NGFP, which acts as a UV attractor. Following the RG flow from this fixed point toward the IR, the RG trajectories undergo a crossover from the NGFP to the ‘classical regime’ dominated by the GFP. Depending on whether the trajectory turns to the left (type Ia), right (type IIa) or hits the GFP (type IIa), the classical theory has a negative, positive or zero cosmological constant.
Figure 1. Stability analysis for universal quantities in the Einstein–Hilbert truncation (dashed line), previously obtained in [17], and upon including the ghost wavefunction renormalization (solid line). The ghost improvement significantly decreases the cutoff-scheme dependence.

Figure 2. The RG flow of \( g_k, \lambda_k \) obtained from the numerical integration of the ghost-improved \( \beta \)-functions (2.37) using the optimized cutoff. The solid line on the right, starting at \( \lambda = 1/2, g = 0 \), indicates a boundary of the coupling-constant space where the \( \beta \)-functions diverge. The phase portrait is in complete agreement with the one obtained from the standard Einstein–Hilbert truncation [8, 17, 29].
Figure 3. RG flow of the anomalous dimensions $\eta_N$ (upper left) and $\eta_c$ (upper right) along the sample RG trajectories of type Ia (top line), type IIa (middle line) and type IIIa (bottom line) shown in the lower left diagram. The ratio $\eta_c/\eta_N$ is shown in the lower right diagram. In the IR, this ratio approaches zero, a finite constant, or rapidly decreases after a peak for trajectories of type Ia, type IIa and type IIIa, respectively.

The trajectories with positive cosmological constants, however, cannot be continued to $k = 0$, but terminate at a finite value of $k$ while reaching the boundary of the phase space. The latter is indicated by the green line, which constitutes a singularity in the $\beta$-functions at finite $g, \lambda$. Observe that the ghost-improved phase portrait, figure 2, is in complete qualitative agreement with the standard Einstein–Hilbert truncation [8, 17, 29].

It is now illustrative to pick one sample trajectory for each of the classes discussed above and study the $k$-dependence of the anomalous dimensions $\eta_N$ and $\eta_c$ along the flow. This is shown in figure 3. In the UV, for large values of $t = \ln(k/k_0)$, the anomalous dimensions are determined by the NGFP, so that $\eta_N^* = -2$ and $\eta_c^* = -1.77$. Lowering $t$ and approaching the IR, the anomalous dimensions undergo a crossover toward the classical theory with $\eta_N \approx 0, \eta_c \approx 0$. The steep increase at the end of the type IIIa trajectory is caused by the singularity of the $\beta$-functions (green line in figure 2), and heralds the termination of the trajectory at a finite value $k$. The UV-limit of the ratio $n_c/n_N$ is governed by the NGFP and takes the value $\eta_c^*/\eta_N^* = 0.89$. Following the flow toward the IR, the ratio undergoes a crossover and asymptotes to 0, the cutoff-scheme-dependent value $54\Phi_{-1}^{3/2}/(24\Phi_{-2}^{1/2} - \Phi_{-1}^{1/2})$, or a finite value at the termination point, for trajectories of type Ia, type IIa and type IIIa, respectively.

It is worth having a closer look at the singularity causing the termination of the type IIIa trajectories. Figure 3 shows that at this point in coupling constant space, the anomalous dimensions $\eta_N$ and $\eta_c$ diverge. This can be traced back to the vanishing of the denominators in (2.39). Here, the ghost-improvement has, however, a very non-trivial effect: while the denominator arising from the standard Einstein–Hilbert truncation has a term linear in $g$, the ghost improvement adds an additional piece quadratic in $g$. This may provide an elegant mechanism for lifting this singularity by shifting the zeros of the denominator to the complex
values $g$. However, for the $B_i, C_i$ given in appendix B, this mechanism is not realized, and may require a further improvement of the truncation before becoming operational.

3.3. The NGFP in spacetimes with extra-dimensions

The $\beta$-functions (2.38) and (2.39) are continuous in the spacetime dimension. In the following, we will exploit this feature and analyze the resulting gravitational fixed-point structure for general $d$. Based on the standard Einstein–Hilbert truncation, a similar analysis has already been carried out in [17, 21, 23]. Motivated by the recent interest in asymptotically safe TeV-scale gravity models [45–48], which hinge on the existence of the NGFP in the presence of extra-dimensions, it is worthy to complement the previous results by including the ghost improvements.

Remarkably, the $d$-dimensional fixed-point structure is strikingly similar to the one obtained in four dimensions. The GFP (3.1) exists for all $d$. Furthermore, there is a unique generalization of the NGFP for all dimensions $3 \leq d \leq 25$ considered here. Its properties are summarized in figure 4. The fixed point is situated at positive $g^* > 0$, $\lambda^* > 0$ and UV attractive in both $g$ and $\lambda$. Below $d < 24$ its critical exponents are given by a complex pair $\text{Re } \theta \pm i\text{Im } \theta$. For $d \geq 24$ the imaginary part of the critical exponents vanishes and we have two real stability coefficients, which are still UV attractive. All these results are in perfect agreement with earlier findings based on the standard Einstein–Hilbert truncation. An interesting feature arises in the ratio $\eta_c^* / \eta_N^*$, with $\eta_N^* = 2 - d$, shown in the top-right diagram of figure 4. This ratio is peaked at $d = 4$, where it reaches almost unity. For both $d > 4$ and $d < 4$ the ratio decreases. It is a rather curious observation that in $d = 4$ the graviton and ghost propagators have the same anomalous dimension in the UV, highlighting this particular
4. Discussion and conclusions

In this paper we have analyzed the gravitational RG flow arising from the ghost-improved Einstein–Hilbert truncation. Besides a scale-dependent Newton’s constant and cosmological constant, this setup also takes into account the quantum effects captured by the non-trivial wavefunction renormalization in the ghost sector. Our main result is the occurrence of a unique non-GFP of the gravitational $\beta$-functions. It is located at positive Newton’s constant and cosmological constant and UV attractive in both $g$ and $\lambda$. Its properties are strikingly similar to the non-GFP found in the standard Einstein–Hilbert truncation \cite{17–23}. This finding further substantiates the asymptotic safety scenario of quantum gravity \cite{1}. The fixed point also persists for higher dimensional spacetimes, solidifying the foundations for asymptotically safe TeV-scale gravity models \cite{45–48}, possibly accessible at the LHC.

The main virtue of the ghost improvement is a significant decrease of the unphysical cutoff-scheme dependence in physical observables such as, e.g., the critical exponents of the fixed point or the universal product $g^* \lambda^*$. Since this scheme dependence constitutes a standard test for the quality of a truncation, we conclude that the ghost improvement significantly increases the robustness of the emerging physical picture. In particular, the characteristics of the fixed point displayed in table 1 should provide a better approximation of the stability coefficients emerging from the full theory, as compared to the standard Einstein–Hilbert result.

The ghost improvement hinges on the new formula (2.39) for the ghost anomalous dimension $\eta_c$, which is analytic in Newton’s constant and completely determined by $g$ and $\lambda$. At the non-GFP, we obtain $\eta^*_c = -1.77$ for the optimized and $\eta^*_c = -1.85$ for the exponential cutoff. Notably, these numbers are very close to the anomalous dimension of Newton’s constant $\eta^*_N = -2$. We believe that this is not accidental, but a consequence of spacetime becoming effectively two dimensional at short distances, where the physics is controlled by the non-GFP \cite{49, 50}. The essence of our argument is the observation that the propagator of a field with anomalous dimension $\eta$ is proportional to $p^{-2+\eta}$. Therefore, in the vicinity of the UV fixed point, both the graviton and ghost propagator behave approximately as $p^{-4}$, which translates into a logarithmic correlator in position space. In this case the spacetime seen by both fields is effectively two dimensional, suggesting $\eta^*_c = -2$ in the full theory. A similar spontaneous dimensional reduction has also been observed within the framework of causal dynamical triangulations \cite{51}, and, by now, also in a variety of other quantum gravity approaches \cite{52}. It would be very interesting to explore if this is a first hint pointing at a deeper connection between these seemingly unrelated fields.

Acknowledgments

We thank D Benedetti, A Codello, J E Daum, U Harst, P Machado, E Manrique, J Pawlowski, and M Reuter for interesting discussions and A Eichhorn and H Gies for communications on their upcoming work. The research of KG and FS is supported by the Deutsche Forschungsgemeinschaft (DFG) within the Emmy–Noether program (Grant SA/1975 1-1).

\footnote{A related argument, concluding that $d = 4$ is special, is based on the spectral dimension of spacetime computed from the running of Newton’s constant and the cosmological constant and has been put forward in \cite{49}.}

\footnote{Qualitatively, our picture is also confirmed by the very recent results \cite{43}, which study the ghost-improved Einstein–Hilbert truncation employing a spectrally adjusted cutoff. The numerical variations observed in the two computations are within the typical range expected from different cutoff schemes \cite{29}.}
Appendix A. Heat-kernel techniques

In this appendix we collect the essential formulas for evaluating the operator traces (2.28) and (2.29). We start with the traces containing only Laplacians before turning to operator insertions including the background ghost fields in appendix A.2. Their relation to the threshold-functions employed in the main text is established in appendix A.3.

A.1. Early time heat-kernel expansion

The differential operators entering into the traces (2.28) appear in the form of covariant Laplacians $\Delta = -\bar{D}^2$ only, and may thus be evaluated using standard early time heat-kernel methods for second-order differential operators (see [29] for more details). In this case the heat-kernel expansion takes the form

$$\text{Tr}[e^{-s\Delta}] = (4\pi s)^{-d/2} \int d^d x \sqrt{\bar{g}} [\text{tr}_1 a_0 + s \text{tr}_1 a_2 + \cdots] ,$$

(A.1)

with coefficients $a_0 = 1$, and $a_2 = \frac{1}{6} \bar{R} I$, and $\text{tr}_i$ denoting a trace over the internal indices

$$\text{tr}_2 T^{12} = \frac{1}{2} (d+2)(d-1) , \quad \text{tr}_1 I = d , \quad \text{tr}_0 I_0 = 1.$$  

(A.2)

Here and in the following the dots indicate interaction terms which do not contribute to the truncation.

By employing a Laplace transform, equation (A.1) can be generalized to suitable operator-valued functions $W(z)$,

$$\text{Tr}[W(\Delta)] = (4\pi)^{-d/2} \int d^d x \sqrt{\bar{g}} [Q_d/2[W] \text{tr}_1 a_0 + Q_{d/2-1}[W] \text{tr}_1 a_2 + \cdots] ,$$

(A.3)

where

$$Q_n[W] := \int_0^{\infty} ds s^{-n} \bar{W}(s) = \frac{1}{\Gamma(n)} \int_0^{\infty} dz z^{-n-1} \bar{W}(z).$$

(A.4)

Here $\bar{W}(s)$ is the Laplace transform of $W(z)$ and we used a Mellin transform to re-express the integral over $\bar{W}(s)$ in terms of the original function. Employing (A.15) to recast the functionals $Q_n$ arising from (2.28) in terms of the dimensionless threshold functions (A.13), a straightforward computation leads to the right-hand side of equation (2.32).

A.2. Applying the Anselmi–Benini perturbation method

While the traces $S_i$ can be evaluated using the standard heat-kernel techniques for Laplace operators, dealing with the $G_i$ is more involved. Here, the main technical difficulty results from the fact that their arguments contain covariant derivatives which do not organizing themselves into minimal second-order differential operators. Applying the perturbative heat-kernel technique developed by Anselmi and Benini [42], this obstruction can be circumvented by the following two-step procedure: in the first step, we use commutator relations to collect all non-minimal operators in a single operator insertion. The second step then uses the heat-kernel formula at non-coincident points to evaluate the operator traces including these ‘perturbations’.

To implement the first step, we start from the arguments in (2.29) and express the propagator sandwiched between the $A$’s (or $Q$’s) by its Laplace transform. Then, all the Laplacians reside in an exponential, that can easily be commuted with the $A$’s (or $Q$’s)

$$[\mathcal{O}, e^{-s\Delta}] = -s e^{-s\Delta} [\mathcal{O}, \Delta] + \mathcal{O}(s^2).$$

(A.5)
Undoing the Laplace transform leads to the desired form of the trace argument, including only one operator of non-Laplace type. The prefactor $s$ multiplying the commutator terms thereby translates into a derivative of the original propagator. At the level of the full trace argument, (A.5) implies

$$W(\Delta)Q^\mu\nu_a = Q^\mu\nu_a W(\Delta) - 2W'(\Delta)(D_\nu Q^\mu\nu_a)D^\sigma + \cdots,$$

and similarly for the other terms. Since our sole interest is in isolating the contribution proportional to the ghost kinetic term, all terms containing more than two covariant derivatives acting on $\bar{c}_\mu$ and $c_\mu$ or involve curvature tensors do not contribute to the truncation and may therefore be dropped.

After these manipulations, the traces $\mathcal{G}_i$ assume the generic form

$$\text{Tr}[\mathcal{O}W(\Delta)] = \int_0^\infty ds \bar{W}(s) \langle x|\mathcal{O} e^{-s\Delta}|x \rangle.$$  \hspace{1cm} (A.7)

Here all Laplacians have been collected in $W(\Delta)$ and $\mathcal{O}$ symbolizes a (matrix-valued) insertion built from two covariant derivatives acting on the background ghosts together with two (or four) covariant derivatives acting to the right. (See below for an explicit example.) The trace (A.7) is evaluated by inserting a complete set of states between the two operators

$$\langle x|\mathcal{O} e^{-s\Delta}|x \rangle = \langle x|\mathcal{O}|x'\rangle \langle x'|e^{-s\Delta}|x \rangle = \int d^d x \sqrt{g} \text{tr}[\mathcal{O}H(s, x, x')]_{x'=x}.$$  \hspace{1cm} (A.8)

Here

$$H(s, x, x') := \langle x'|e^{-s\Delta}|x \rangle = (4\pi s)^{-d/2} e^{-\frac{g_{\mu\nu}(x,x')}{16s}} \sum_{n=0}^\infty s^n A_{2n}(x, x'),$$

is the expansion of the heat-function at non-coincident points, with $A_{2n}(x, x')$ being the off-diagonal heat-kernel coefficients and $\sigma(x, x')$ denoting half the squared geodesic distance between $x$ and $x'$, satisfying

$$\sigma(x, x) = 0, \quad \frac{1}{2} \sigma^{ij} \sigma_{ij} = \sigma, \quad \sigma_{\mu\nu}(x, x) = \bar{g}_{\mu\nu}(x),$$

$$(\sigma_{(\alpha_1, \ldots, \alpha_n)}(x, x) = 0, \quad n \geq 3.$$  \hspace{1cm} (A.10)

Notably, $A_{2n}(x, x) = a_{2n}$, so all but $A_0(x, x)$ give rise to curvature terms. Furthermore, covariant derivatives acting on $A_{2n}(x, x')$ either vanish or produce higher derivative curvature terms once the coincidence limit is taken [42, 53]. Thus, while applying $\mathcal{O}$ on the function $H(s, x, x')$, the only terms contributing to the truncation are those where the covariant derivatives act pairwise on $\sigma(x, x')$.

To illustrate the procedure, let us consider the following explicit example from the $A^{\mu} \bar{\Lambda}_a$ product in $\mathcal{G}_0$:

$$\mathcal{O}_0 = \left[ -\frac{1}{d} (D_\mu e^\alpha) D^\alpha \right] \left[ -\frac{1}{d} (D_\nu \bar{e}_a) D^\nu \right] = -\frac{1}{d^2} (D_\mu e^\alpha)(D_\nu \bar{e}_a) D^\alpha D^\nu + \cdots.$$  \hspace{1cm} (A.11)

Here we dropped all terms with more than two covariant derivatives acting on the background ghosts. Substituting $\mathcal{O}_0$ into (A.7) gives

$$\text{Tr}[W(\Delta)\mathcal{O}_0] = \frac{1}{2d^2(4\pi)^{d/2}} \int_0^\infty ds s^{d/2-1} \bar{W}(s) \int d^d x \sqrt{\bar{g}} (\bar{D}_\mu e^\alpha)(\bar{D}_\nu \bar{e}_a)$$

$$= \frac{1}{2d^2(4\pi)^{d/2}} Q_{d/2+1}[W] \int d^d x \sqrt{\bar{g}} e^\alpha D^\nu \bar{e}_a,$$

where we used (A.4) in the second step. Applying this algorithm systematically to all terms in (2.29) and subsequently using relation (A.16) then leads to (2.33).
A.3. Threshold functions and cutoff scheme

In order to capture the cutoff-scheme dependence and highlight the structure of the $\beta$-functions it is convenient to express the functionals $Q_n[W]$ in terms of dimensionless threshold functions. The structures appearing in the $G_i$ motivate the definitions

\[
\Phi_n^{p,q}(w) := \frac{1}{\Gamma(n)} \int_0^\infty dz z^{n-1} \frac{R^{(0)}(z) - z R^{(0)}(z)}{(z + R^{(0)}(z) + w)^{p}(z + R^{(0)}(z))^q},
\]

\[
\Phi_n^{p,q}(w) := \frac{1}{\Gamma(n)} \int_0^\infty dz z^{n-1} \frac{R^{(0)}(z)}{(z + R^{(0)}(z) + w)^{p}(z + R^{(0)}(z))^q},
\]

\[
\Phi_n^{p,q}(w) := \frac{1}{\Gamma(n)} \int_0^\infty dz z^{n-1} \frac{R^{(0)}(z)(R^{(0)}(z) - z R^{(0)}(z))}{(z + R^{(0)}(z) + w)^{p}(z + R^{(0)}(z))^q},
\]

where the prime denotes the derivative of the shape function $R^{(0)}$, defined below (2.19), with respect to its argument. Observe that the threshold functions with $p = 0$ do not depend on $w$.

These definitions naturally generalize the ones used in [12], which are recovered as special cases

\[
\Phi_n^{p,0}(w) = \Phi_n^{p}(w), \quad \Phi_n^{0,q}(w) = \Phi_n^{q}(0), \quad \Phi_n^{p,0}(w) = \Phi_n^{p}(w), \quad \Phi_n^{0,q}(w) = \Phi_n^{q}(0).
\]

These threshold functions are closely related to the $Q$-functionals (A.4). For the traces (2.28) we have

\[
Q_n \left[ \frac{\partial_t \{ Z_1^N R_t \} }{Z_1^N (P_t + w)^p} \right] = 2 k^{2(\alpha - p + 1)} \left[ \Phi_n^{p,0}(w/k^2) - \frac{1}{2} \eta, \Phi_n^{p,0}(w/k^2) \right],
\]

with the index $I = N, c$ such that the relation holds for both the gravitational and ghost wavefunction renormalization. This relation generalizes to the $Q$-functionals occurring in (2.29):

\[
Q_n \left[ f_1^N \right] = 2 Z_k^N k^{2n-4} \left[ \Phi_n^{2,1}(-2\lambda_k) - \frac{1}{2} \eta, \Phi_n^{2,1}(-2\lambda_k) \right],
\]

\[
Q_n \left[ f_1^c \right] = 2 Z_k^c k^{2n-4} \left[ \Phi_n^{2,2}(-2\lambda_k) + \tilde{\Phi}_n^{2,2}(-2\lambda_k) - \frac{1}{2} \eta, \tilde{\Phi}_n^{2,2}(-2\lambda_k) + \tilde{\Phi}_n^{2,2}(-2\lambda_k) \right].
\]

In combination with the heat-kernel formulas (appendix A.3) and (A.7), these equations allow us to find the $\beta$-functions for $Z_k^N, Z_k^c, A_k, \Lambda_k, (2.32)$ and (2.35).

We close this section with a remark on the shape functions employed in the numerical studies of section 3. Unless stated otherwise, all results are obtained with the optimized cutoff [54]:

\[
R_k^{(0),\text{opt}}(z) = (1 - z) \Theta(1 - z).
\]

In this case the integrals appearing in (A.13) can be carried out analytically:

\[
\Phi_n^{p,q}(w) = \frac{1}{\Gamma(n + 1) (1 + w)^p}, \quad \Phi_n^{p,q}(w) = \frac{1}{\Gamma(n + 2) (1 + w)^p},
\]

\[
\Phi_n^{p,q}(w) = -\frac{1}{\Gamma(n + 1) (1 + w)^p}, \quad \Phi_n^{p,q}(w) = -\frac{1}{\Gamma(n + 2) (1 + w)^p}.
\]

Here, the threshold functions degenerate such that they become independent of the index $q$.

When analyzing the cutoff-scheme dependence of our truncation, we also employ a one-parameter family of smooth exponential shape functions:

\[
R_k^{(0),\text{exp}}(z; s) = \frac{s^z}{\exp(sz) - 1}.
\]
The continuous shape parameter $s$ allows us to smoothly vary the implementation of the IR cutoff. In contrast to the optimized cutoff, the integrals in the threshold functions cannot be carried out analytically for this class of cutoffs. Thus, we have to resort to numerical integration when evaluating the threshold functions.

Appendix B. Functions determining the anomalous dimensions

In this appendix, we give the definitions of the functions $B_j(\lambda)$ and $C_j(\lambda)$, completing the construction of the anomalous dimensions for the graviton and ghost fields (2.39). In the following, all threshold functions are evaluated at the argument $w = -2\lambda$, which we then suppress for notational simplicity. The $B_j$ are exactly the same as those obtained in [12]:

$$B_1(\lambda) = \frac{1}{2}(4\pi)^{1-d/2}(d(d+1)\Phi_{d/2-1}^{1,0}-6d(d-1)\Phi_{d/2}^{2,0}-4d\Phi_{d/2-1}^{0,1} - 24\Phi_{d/2}^{0,2}),$$
$$B_2(\lambda) = -\frac{1}{6}(4\pi)^{1-d/2}(d(d+1)\Phi_{d/2-1}^{1,0} + 6d(d-1)\Phi_{d/2}^{2,0}).$$

The quantum corrections from the wavefunction renormalization of the ghosts are encoded in

$$C_1(\lambda) = (4\pi)^{1-d/2}(2C_{gr}\Phi_{d/2+1}^{2,1} - 4d(\Phi_{d/2+2}^{2,2} + \Phi_{d/2+2}^{2,1})),$$
$$C_2(\lambda) = (4\pi)^{1-d/2}(2C_{gh}\Phi_{d/2+1}^{1,2} + 4d(\Phi_{d/2+2}^{1,2} + \Phi_{d/2+2}^{0,2})),$$
$$C_3(\lambda) = \frac{1}{4}(4\pi)^{1-d/2}(2d\Phi_{d/2-1}^{0,1} + 12\Phi_{d/2}^{0,2}),$$
$$C_4(\lambda) = -(4\pi)^{1-d/2}(4C_{gr}\Phi_{d/2+1}^{2,1} + 4C_{gh}\Phi_{d/2+1}^{1,2}),$$

with the coefficients $C_{gr}$ and $C_{gh}$ defined in (2.36).

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