Substantiation of two-channel structure of automatic tracking system

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Abstract. Methods for linearization the equations of the «flight» of a quadcopter in one plane in the tasks of tracking of the moving objects with onboard optical devices are proposed. The two-channel structure that forming work of these control systems as a whole and providing increase in dynamic accuracy is substantiated.

1. Introduction

Currently, aircraft complexes based on small unmanned quadcopters capable of carrying the payload are wide used [1-3]. It is important for effective solution of functional tasks of these complexes to find control methods of main complex components, namely aircraft and servosystem providing complex mission accomplishment. Choice of quadcopter flight dynamics mathematical model description method [4-6] and determination efficient of flight control methods based on this mathematical model [7-8] are the significant problems of finding control algorithms of aircraft (quadcopter) flight with accordance of overall mission accomplishment criterion. At the same time, significantly less quantity of papers are devoted to task preceding aforementioned tasks of finding control algorithms with required quality, namely task of the most efficient control system structure of similar aircraft complexes. In this paper, search and substantiation of control system structure of aircraft complex with optical device mounted on quadcopter and created for implementation of automatic tracking of mobile targets are discussed.

In [9] perspective of using of multi-channel structures providing auxiliary possibilities in tracking errors decreasing task is represented. Task formulation of automatic tracking system (ATS) structure synthesis via onboard optical device (OOD) placed on quadcopter is considered in [10] (plane variant of tracking) and in [11] (spatial variant of tracking).

2. ATS structure

In [12] time-independent vertical $X$ and horizontal $Y$ control channels of bicopter (plane representation of quadcopter) flight control is suggested to use as simplified method of linearization. In this case, ATS graded structure with OOD is suggested [12] e.g. as shown in figure 1, where designations are used from [10] and also following designations are appended: $r_{BC}$ is distance between bicopter mass center and application point of thrust force vector developed by one electric motor with airscrew; $\theta_{BC}^{auto}$ is angle of bicopter rotation about $OZ_0$ axis appeared in loop of $R_{BC}$ value of bicopter position on $O_3X_3Y_3$ plane as input signal to required $\theta_{BC}$ values setting inner loop; $W_C^g$ and $W_C^v$ are transfer
functions of $X$ and $Y$ channels controllers, respectively, and also closed loop $\theta_{BC}$: $\varphi_{\nu}^{X} = \{\hat{\varphi}_{\nu}^{X}(\Delta X) \land \varphi_{\nu}^{Y}(\theta_{BC})\}$ and $\varphi_{\nu}^{X} = \{\hat{\varphi}_{\nu}^{X}(\varepsilon_{BC}) \land \varphi_{\nu}^{Y}(\varepsilon_{BC})\}$ are elements of signals for setting of bicopter power screws rotation velocity for $X$ and $Y$ channels, respectively; $\varepsilon_{BC}$ is displacement angle (sighting angle) between sight line and direction rigidly bound to one of the bicopter construction axis ($\varepsilon_{BC}$) or to bearing finder sight line (imprecise bearing finder) places on bicopter ($\varepsilon_{BC}^*$); APD is angular position transducer; BC AD is bicopter actuation device.

During vertical channel $X$ (along $O_SX_S$) and horizontal channel $Y$ (along $O_SY_S$) ATS with OOD collaboration $Y$ channel control is more dynamic than $X$ channel control. In this case, under $X$ and $Y$ channels simultaneous control dynamics exceeding is appeared, i.e. even in case of small value of bicopter body rotation angle $\theta_{BC}$ situation when one need to change bicopter velocity vector tilt angle $\theta_{BC}$ in the interval $0^\circ - 90^\circ$ is possible. Bicopter «turning over» (crash situation), i.e. fulfilment of the condition $\theta_{BC} = \pi/2$ is possible as well.

Changing of ATS with OOD operation logic allows to avoid these crash situations. Possibility of changing the $y_{BC}^S$ coordinate only by $\theta_{BC}$ coordinate control is appeared when conditions of bicopter flight control system (FCS) two-channel structure are provided not from flight start but from moment when bicopter altitude $x_{BC}'$ maintains the constant setpoint.

Figure 2 shows formation scheme of bicopter rotation angles $\theta_{BC}$ and $\theta_{BC}$ which allows to explain such sequence.

Formation kinematics of sighting angle in initial bicopter flight along $O_SX_S$ axis (when $\theta_{BC} = \theta_{BC}^* = 0$) is determined by fulfilment of relation $y_{BC}^S = 0$ up to fulfilment of condition $x_{BC}'$ accurate to admissible error $\Delta x_{BC}$ (bicopter does not move along $O_SY_S$ axis, or this movement is insignificant on the assumption of noncoincidence of thrust force value $F_T$ and bicopter weight value $G_{BC} = m_{BC}g$).

Then, bicopter attitude control is implemented only along $OY_S$ axis thereby still changing of boresight attitude. After appearance of target echo in visual field of imprecise bearing finder measuring only bicopter movement parameters and start of $\varepsilon_{BC}^*$ angle measurement conditions for $P_e$ and $P_X$ switches triggering are provided, and bicopter FCS structure which included to ATS with OOD is cardinally changed.

After $P_e$ switch triggering not algorithmically formed angle $\varepsilon_{BC} = \theta_{BC}^* = \theta_{BC}^*$ but angle $\varepsilon_{BC}^*$ measured by bearing finder of imprecise loop becomes input signal for closed loop controller.

**Figure 1.** ATS graded structure.  
**Figure 2.** Bicopter rotation angles formation scheme.
In the particular case, this control can be designed for decreasing of angle \( \phi_{BC} \) dynamics towards OOD sighting line (which measured by bearing finder of precise loop) with accounting of angular movement of optical device (OD) feedback actuator (FA).

After \( P_X \) switch triggering constant value for setting of airscrews rotation velocity \( \dot{\phi}_v\left(G_{BC}/2 \cos \theta_{BC}^{*}\right) \) \text{const} is formed in complex control system (CCS). This value provides maintenance of bicopter stable vertical position \( x_{BC}^{\text{stable}} = H_{BC} \). Thus, in bicopter body rotating (\( \theta_{BC} \) angle changing) one must fulfill condition:

\[
F_T = \frac{G_{BC}}{\cos \theta_{BC}^{*}}.
\] (1)

In this case, during interval of this regime operation condition \( \theta_{BC} = 90^\circ \) is always fulfilled.

We will use plane design of optical-mechanical subsystem of OD FA shown in [10].

Further, we will consider conditions of target and bicopter mutual movements as shown in figure 3 where: \( \varepsilon_{FA} \) is displacement angle between line of fire \( \phi_{BC}^{a} \) (BS) and OOD sighting line (SL_{OD}).

Target moves with variable velocity \( V_T \) horizontally (with variable velocity sign, in general) on horizontal surface of the earth \( (x_T^S = 0) \). Bearing finder of imprecise channel is mounted on bicopter body constructively in initial position towards local vertical.

Input signal of ATS with OOD is described by relations:

\[
\begin{align*}
\phi^a_T &= \arctan \frac{y^S_{BC} - y^S_T}{x^S_{BC}} = \arctan \left( \frac{y^S_{BC0} - y^S_T(0)}{x^S_{BC} + \int (y^S_{BC} - y^S_T) \, dt} \right), \\
\varepsilon_{FA}(s) &= \Phi_{\varepsilon_{FA}}(s) \phi^a_T(s)
\end{align*}
\] (2)

where \( \varepsilon_{FA}(s) \) is transfer function of OD FA.
As true input for considered CCS is changing of \(y_T^S\) value which is coordinate of target position on horizontal surface and output is SL\(_{OD}\) angular position \(\phi_{OD}^a\), possibility to introduce auxiliary channel of intermediate control of external parameter \(\phi_{OD}^d\) due to changing of \(y_{BC}^S\) value.

3. Suggested ATS structure with imprecise channel

Assume following approximate relations:

1. Setpoint of airscrews rotation velocity \(\phi_{v}^L(G_{BC}/2\cos\vartheta_{BC})_{\text{const}}\) is described by [11]

\[
\dot{\phi}_v = \left(\frac{G_{BC}}{2 \cos \vartheta_{BC}}\right)_{\text{const}} \left(\frac{G_{BC}}{2K_v^c \cos \vartheta_{BC}}\right)^{1/2},
\]

where \(K_v^c = 4\pi^2 \alpha_{\nu} D_{\nu}^4; \alpha_{\nu}\) and \(D_{\nu}\) are airscrew thrust coefficient considering blade form, cross section form and angle of attack (measured experimentally), and airscrew diameter, respectively; \(\rho\) is air density.

2. According to figure 1 \(\Delta F_{12} = F_1 - F_2\) value is described by (denoting \(\dot{\phi}_v^\vartheta\) as output signal of closed loop \(\vartheta_{BC}\) controller with \(W_{C}^\vartheta\) transfer function)

\[
\Delta F_{12} = K_v^c \Phi_{SVA}\left(\dot{\phi}_v \left(\frac{G_{BC}}{2 \cos \vartheta_{BC}}\right)_{\text{const}} + \dot{\phi}_v^\vartheta\right) - \left[\dot{\phi}_v \left(\frac{G_{BC}}{2 \cos \vartheta_{BC}}\right)_{\text{const}} - \dot{\phi}_v^\vartheta\right] = 2K_v^c \Phi_{SVA} \dot{\phi}_v^\vartheta = 2K_v^c \Phi_{SVA} W_{C}^\vartheta \varepsilon_{BC}(\varepsilon_{BC}^\star),
\]

where \(\Phi_{SVA}\) is transfer function of feedback stable velocity actuator (SVA).

3. Accounting of formula (1) the first equation of equation system from [10]

\[
\begin{align*}
V_{BC} &= \frac{1}{m_{BC}} \left[ F_T \cos \theta_{BC} \cdot \dot{\vartheta}_{BC} - G_{BC}^0 \cos \theta_{BC}\right], \\
\dot{\theta}_{BC} &= -\frac{1}{m_{BC}V_{BC}} \left[ F_T \sin \theta_{BC} \cdot \dot{\vartheta}_{BC} - G_{BC}^0 \sin \theta_{BC}\right]
\end{align*}
\]

describing the dynamics of bicopter translation movement can be written as

\[
\dot{V}_{BC} = \dot{y}_{BC}^S = \frac{1}{m_{BC}} \left( F_T \cos \frac{\pi}{2} - \vartheta_{BC}\right) = g \tan \theta_{BC}.
\]
Figure 5. Structural scheme of two-channel ATS with OOD.

Assume that value of $\vartheta_{BC}$ angle is small. Thus,

$$y_{BC}^S = \frac{\vartheta_{BC}}{s^2}. \quad (5)$$

4. According to [10] signal-flow graph of OD FA structure (ATS subsystem) can be represented as shown in figure 4 where all designations from [11] are kept.

Consider that while hardware implementation all special actions for decreasing of OD FA motor reaction torque applied to bicopter body are made. In this case, considering condition $M_R = 0$ relations for OD FA open-loop transfer function can be written as

$$W_{FA}(s) = \frac{\varphi_{OD}(s)}{e_{FA}(s)} = \frac{W_{FO}}{C_e s [T_{em}^{OD}(T_a s+1)+1]},$$

$$W_{conn}(s) = \frac{\varphi_{ODconn}(s)}{\vartheta_{BC}(s)} = \frac{W_{em}^{\prime}}{T_{em}^{OD}(T_a s+1)+1},$$

$$W_{pr}(s) = \frac{I_{OD}^{\prime}}{I_{OD}}, \quad (6)$$

where $\varphi_{OD}^{\prime} = \varphi_{OD}^{BC} + \varphi_{ODconn}$; $T_{em}^{OD} = I_{OD}R_u/C_m C_e$; $T_{em} = I_{OD}R_u/C_m C_e$; $I_{OD}^{\prime} = I_{OD} + I_{OD}; ~ I_{OD} = I_M^2 + I_{load}; ~ I_{OD} = m_{OD} r_{BC-OE} r_{OE}; m_{OD}$ is mass of moved OOD optical element; $r_{BC-OE}$ is distance between rotation center of bicopter (bicopter mass center) and control OOD optical element; $r_{OE}$ is distance between rotation center and mass center of control OOD optical element.

Accounting of fulfillment of conditions (1-6) variant of ATS two-channel structure design can be suggested as shown in figure 5 where $\varphi_{OD}^{stable} = H_{BC}$; $W_{impr}^{BF}$ and $W_{pr}^{BF}$ are transfer functions of imprecise channel bearing finder (placed on bicopter) and precise channel bearing finder (placed behind the OOD optical elements), respectively; $W_{pr}^{BF}$ is open-loop transfer function of OD FA, i.e. precise channel of two-channel ATS with OOD.

If task of attenuation of the effect of bicopter alignment control loop to ATS with OOD precise channel, i.e. task of attenuation of angle $\vartheta_{BC}$ connection, is formulated, then it is promising to use torque motor operating in small value of coefficient CEMF $C_e$ region. In the limiting case, it is correspond to absence of connection depicted in figure 4 by dashed line. Thus, connection transfer function $W_{conn}$ can be rewritten as

$$W_{conn}(s) = \frac{I_{OD}}{I_{OD}}, \quad (7)$$
There are various methods to decreasing of disturbances of connection channel from bicopter body rotation to OOD sighting line forced rotation, i.e. it is disturbance for OD FA. e.g.,

1. in case of balanced rotating parts of OOD \((r_{BC\cdot OE} = r_{OE} = 0)\) condition \(W_{conn}(s) = 0\) is fulfilled;
2. in measuring of rotation angle \(\vartheta_{BC}\) (e.g. via gyroscopic sensor) there is the possibility to make channel compensating effect from bicopter to FA.

Figure 6 shows the structure where such compensating channel is implemented (depicted by thick line).

For full compensation of bicopter rotations \((\vartheta_{BC})\) transfer function of compensative element must be described by

\[
W_{comp}(s) = C_{e}s\left[T_{em}^{OD}(T_{a}s+1)+1\right].
\]  

(8)

Obviously, \(W_{comp}(s)\) element cannot be physically implemented to the full. As a rule, this obstacle can be overcome either using partial compensating, i.e. while using only the first derivative in compensative channel, or movement of summing point of main and compensative channels from informational actuator part to its power part.

We will use Fourier-series expansion of inverse trigonometric function:

\[
\arctan\frac{\Delta y}{H_{BC}} = \frac{\Delta y}{H_{BC}} - \frac{(\Delta y)^3}{3H_{BC}^3} + \frac{(\Delta y)^5}{5H_{BC}^5} - \ldots,
\]

from which we will choose only the first (linear) term of series. Thus, accounting of condition \(W_{conn}(s)\) simplified (linearized) variant of ATS with OOD two-channel structure can be shown as in figure 7 where \(\Phi_{\vartheta_{BC},\vartheta_{T}}\) is transfer function of formation of angle \(\vartheta_{BC}\) decreasing displacement angle \(\varepsilon_{BC}\); \(\Phi_{\varphi_{T},y_{T}}\) is closed-loop transfer function of imprecise channel of two-channel ATS.

Considering assumption that condition \(\Phi_{\vartheta_{BC},\vartheta_{T}} = 1\) in bandwidth of \(\Delta y\) control closed loop is fulfilled, transfer function \(\Phi_{\varphi_{T},y_{T}}\) is described by

\[
\Phi_{\varphi_{T},y_{T}} = \frac{s^2}{g\left(H_{BC}s+1\right)}.
\]  

(8)

Therefore, ATS with OOD imprecise channel provides formation of changing input signal \(\varphi_{T}^{o}\) forecast for precise channel taking into account compulsory use of auxiliary compensating methods in imprecise channel.
4. Conclusions
As a result of using approximation methods of linearization two-channel ATS with OOD structure is formed.
Possibility of mobile target tracking error decreasing for bicopter with auxiliary bearing finder in imprecise channel is considered.

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