Combined analysis of the reactions $pp \rightarrow pp$, $\pi d \rightarrow \pi d$, and $\pi d \rightarrow pp$

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Abstract

Results are presented for a combined analysis of the reactions $pp \rightarrow pp$, $\pi d \rightarrow \pi d$ and $\pi d \rightarrow pp$ over the $\sqrt{s}$ interval from pion threshold to approximately 2.4 GeV. These results for $\pi d \rightarrow pp$ and $\pi d$ elastic scattering are superior to our previous analyses of these reactions. In particular, the overall phase in $\pi d \rightarrow pp$ has now been determined. Comparisons are made with previous (separate and combined) analyses of this two-nucleon system.

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I. INTRODUCTION

An understanding of the $NN$ interaction is fundamental to studies of the more general $\pi NN$ problem [1]. Below 1 GeV, in proton laboratory kinetic energy $T_p$ for the $NN$ system, the dominant channels contributing to $NN$ inelasticity are $\pi d$ and $N\Delta$ [2]. At these energies, it is useful to employ a multi-channel formalism in analyzing all existing data simultaneously. In the present work, we have used the K-matrix formalism in order to unify the analysis of several reactions ($pp \rightarrow pp$ [3], $\pi d \rightarrow \pi d$ [4], and $\pi d \rightarrow pp$ [7]) which we have, in the past, considered separately. The range of $\sqrt{s}$ was chosen to include all of our results for the pion-induced reactions ($T_\pi = 0 - 500 \text{ MeV}$).

Clearly, we are not the first to consider this problem. A joint analysis of these three reactions, in a narrow energy range near the $N\Delta$ threshold, was recently reported by Nagata et al. [8]. This work used a mix of model-based and phenomenological results to investigate possible narrow structures in these reactions. An older work by Edwards [9] used the multi-channel K-matrix formalism to study the $J^P = 2^+$ and $3^-$ states associated with dibaryon candidates.

The present analysis differs from those carried out previously in a number of important respects. We did not restrict our study to partial-waves containing interesting structures. For $pp$ elastic scattering, all waves with $J \leq 7$ were used. Partial wave with $J \leq 5$ were retained for both $\pi d$ elastic scattering and $\pi d \rightarrow pp$. In addition, the K-matrix parameters were determined solely from our fits to the available data bases for each separate reaction. No results of outside analyses or any model approaches were used as constraints. As a result, the amplitudes found in our K-matrix fits are as “unbiased” as those coming from the separate analyses [4].

In Section II, we will outline the K-matrix formalism used in this analysis. The combined
and separate analyses will be compared in Section III. Conclusions and suggestions for further study will be given in Section IV.

II. FORMALISM

In order to analyze the reaction $\pi d \to pp$ along with elastic $pp$ and $\pi d$ scattering, we have constructed a K-matrix formalism having $pp$, $\pi d$ and $N\Delta$ channels. The energy-dependence of our global fit was obtained through a coupled-channel K-matrix form in order to ensure that unitarity would not be violated. The “$N\Delta$” channel is added to account for all channels other than $pp$ and $\pi d$. The most important thresholds are illustrated schematically in Fig. 1. That this catch-all channel is indeed mainly $N\Delta$ can be seen in Fig. 2, where the total cross sections for $pp$ and $\pi d$ scattering are broken into their components.

As the elastic $pp$ partial-wave analysis is far superior to the $\pi d$ elastic and $\pi d \to pp$ analyses, we have carried out fits in which the $pp$ partial-waves were held fixed. (The partial wave decomposition the of $pp$, $\pi d$, and $N\Delta$ systems are given in Table I.) As described below, the $pp$ amplitudes were used to fix some elements of the K-matrix, while the others were determined from a fit to the combined $\pi d$ elastic and $\pi d \to pp$ data bases.

States of a given total angular momentum and parity ($J^P$) were parameterized by a 4x4 K-matrix ($K_J$) which coupled to an appropriate $N\Delta$ channel. Spin-mixed(2x2) $pp$ states couple to unmixed $\pi d$ states, and unmixed $pp$ states couple to spin-mixed(2x2) $\pi d$ states, so the $\pi d - pp$ system is always represented by a 3x3 matrix. For example, the T-matrix ($T_J$) for $J^P = 2^+$ (unmixed $pp$ states) is given by

$$
T_2 = \begin{pmatrix}
1D_2 & 1D_2P & 1D_2F \\
1D_2P & 3P_2 & \epsilon_2 \\
1D_2F & \epsilon_2 & 3F_2
\end{pmatrix}
\begin{pmatrix}
pp \\
\pi d_- \\
\pi d_+
\end{pmatrix}
$$

(1)
whereas the T-matrix for $J^P = 2^-$ (mixed $pp$ states) is

$$
T_2 = \begin{pmatrix}
3P_2 & \epsilon_2 & 3P_2D \\
\epsilon_2 & 3F_2 & 3F_2D \\
3P_2D & 3F_2D & 3D_2
\end{pmatrix}
\begin{pmatrix}
pp_- & pp_+ & \pi_d
\end{pmatrix}
$$

The subscripts $\pm$ denote states with $L = J \pm 1$. In the above, the mixing parameters ($\epsilon$) for elastic $pp$ and $\pi d$ scattering are different. For the reaction $\pi d \rightarrow pp$, the notation $(2S_{pp}^{J+1}L_{pp}^{L} \pi)$ of Ref. [7] is used.

Adding an $N\Delta$ channel results in a 4x4 T-matrix. Dropping the $J$-subscript, we write the K-matrix as

$$
K = \begin{pmatrix}
K_{pp} & K_0 \\
\tilde{K}_0 & K_i
\end{pmatrix}
$$

where $K_{pp}$ is the elastic $pp$ scattering sub-matrix, $K_0$ and $\tilde{K}_0$ are row and column vectors, and $K_i$ is the sub-matrix of channels involving $\pi d$ and $N\Delta$ states. This K-matrix can be re-expressed as a T-matrix

$$
T = \begin{pmatrix}
T_{pp} & T_0 \\
\tilde{T}_0 & T_i
\end{pmatrix}
$$

using the relation $T=K(1-iK)^{-1}$. We then have the correspondence

$$
T_{pp} = K_{pp}(1 - iK_{pp})^{-1},
$$

where

$$
\tilde{K}_{pp} = K_{pp} + iK_0(1 - iK_i)^{-1}\tilde{K}_0
$$

In order to ensure an exact fit to the $pp$ elastic T-matrix, given by our most recent analysis of $NN$ elastic scattering to 1.6 GeV [8], we take

$$
K_{pp} = T_{pp}(1 + iT_{pp}) - iK_0(1 - iK_i)^{-1}\tilde{K}_0.
$$
The matrix elements are then expanded as polynomials in the pion energy times appropriate phase-space factors. The $\pi d$ elastic and $\pi d \rightarrow pp$ T-matrix elements are extracted from $T_0$ and $T_i$.

III. PARTIAL-WAVE AMPLITUDES

We have fitted the amplitudes for $pp \rightarrow pp$ and the existing data bases for $\pi d \rightarrow pp$, and $\pi d \rightarrow \pi d$, using the K-matrix formalism outlined in Section II. The $\pi d$ elastic and $\pi d \rightarrow \pi d$ data bases used in this analysis are described in Refs. [5] and [7], and available from the authors [4]. The overall $\chi^2$ for our combined analysis is actually superior to that found in our single-reaction analyses. This is due to the improved parameterization scheme. A comparison is given in Table II. We should emphasize that the amplitudes for $pp$ elastic scattering are the same as those given in Ref. [3]. As mentioned above, this feature was built into our K-matrix parameterization. For this reason, we have omitted plots of the $pp$ amplitudes [3].

The results for $\pi d$ elastic scattering are also qualitatively similar, up to the limit of our single-energy analyses. In Fig. 3 we compare the main partial-waves from our single-reaction analysis [5] and combined analysis (solution C500). Significant differences begin to appear above a pion laboratory kinetic energy of 300 MeV or 2.3 GeV in $\sqrt{s}$. (The $^3D_2$ partial wave from C500 is an exception, departing from the single-reaction analysis near threshold.) The upper limit to our single-energy analyses is due to a sharp cutoff in the number of data. This is apparent in Fig. 2 of Ref. [3]. Much additional data above 300 MeV will be required before a stable solution to 500 MeV can be expected.

A comparison of results for $\pi d \rightarrow pp$ reveals the most pronounced differences. One reason for this is the overall phase which was left undetermined in Ref. [4]. There, we arbitrarily chose the $^3P_1S$ wave to be real. In the present analysis, the overall phase has been determined. In Fig. 4 we show that the $^3P_1S$ phase is very different in the combined and separate analyses. Given the large difference in overall phase, we have chosen to compare the
partial-wave amplitudes from the separate and combined analyses in terms of their moduli. This comparison is made in Fig. 5. As was the case for $\pi d$ elastic scattering, differences are most significant above approximately 2.3 GeV in $\sqrt{s}$. A similar lack of data exists above this energy.

In general we see a good agreement for the dominant amplitudes found in the separate and combined analyses. Figures 3 and 5 also display our single-energy analyses which were done in order to search for structure which may be missing from the energy-dependent fit. (Details of the single-energy analyses are given in Refs. [5,7].) A comparison of the single-energy and energy-dependent fits is given in Tables III and IV.

IV. SUMMARY AND CONCLUSIONS

We have obtained new partial-wave amplitudes for $\pi d$ elastic scattering and the reaction $\pi d \rightarrow pp$, using a K-matrix method which utilized information from our elastic $pp$ scattering analysis. In addition to producing amplitudes more tightly constrained by unitarity, we have resolved the overall phase ambiguity existing in our previous $\pi d \rightarrow pp$ analysis.

As mentioned in Section III, the combined analysis has resulted in a slightly improved fit to the $\pi d$ elastic and $\pi d \rightarrow pp$ data bases. The most noticeable differences, at the partial-wave level, appear at higher energies where the existing data are sparse. It is difficult to find cases where the fit has been dramatically improved. One exception is the set of $\pi d$ total cross section data between 300 and 500 MeV. Here the combined analysis is much more successful in reproducing the energy dependence. The combined analysis gives total cross sections which begin to rise at 500 MeV, whereas the separate analysis shows a fairly monotonic decrease from 400 to 500 MeV. The behavior seen in the combined analysis seems reasonable, as the $\pi d$ total cross sections do begin to rise just beyond the upper energy limit of our analysis. Many of the individual partial-wave amplitudes from C500 show rising imaginary parts near 500 MeV, a feature absent in the analysis of $\pi d$ elastic data alone.

The present analysis has also resulted in a unified description of the resonancelike behav-
ior previously noted in our separate analyses of $pp$ and $\pi d$ elastic scattering, and the reaction $\pi d \rightarrow pp$. This behavior has been variously described as “resonant” (due to the creation of dibaryon resonances) and “pseudo-resonant” (due to the $N\Delta$ intermediate state). We expect that our combined analysis will further constrain models based on these two mechanisms.

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Figure captions

Figure 1. Energy scale in terms of the total center-of-mass energy ($\sqrt{s}$) and the incident kinetic energies of the $pp$ ($T_p$) and $\pi d$ ($T_\pi$) initial states. The locations of relevant thresholds are also displayed.

Figure 2. Total cross sections $\sigma_{tot}$ (solid) and total elastic cross sections $\sigma_{el}$ (dashed) correspond to the C500 solution. Data for $\sigma_{tot}$ (open circles) are taken from the SAID database \[4\]. (a) Dash-dotted lines, corresponding to the C500 solution, show the total cross sections ($\sigma_{\pi d}$) for $pp \to \pi d$. The corresponding data from the SAID database \[4\] are plotted as open triangles. The remainder ($\Delta \sigma$) is given by $\sigma_{tot} - \sigma_{el} - \sigma_{\pi d}$ and plotted as a dotted line. Total cross sections for the reactions $pp \to \Delta^+ p + \Delta^{++} n$ \[4\] are plotted as dark circles. (b) Dash-dotted lines (C500) show the total cross sections ($\sigma_{pp}$) for $\pi d \to pp$. The corresponding data from the SAID database \[4\] are plotted as open triangles. The remainder ($\Delta \sigma$) is given by $\sigma_{tot} - \sigma_{el} - \sigma_{pp}$ and plotted as a dotted line.

Figure 3. Partial-wave amplitudes of the reaction $\pi d \to \pi d$ from $T_\pi = 0$ to 500 MeV. Solid (dashed) curves give the real (imaginary) parts of amplitudes corresponding to the C500 solution. Our previous analysis (SM94) \[5\] is plotted with long dash-dotted (real part) and short dash-dotted (imaginary part) lines. The dotted curve gives the value of $\text{Im } T - T^2 - T^2_{sf}$, where $T^2_{sf}$ is the spin-flip amplitude for C500. The real (imaginary) parts of single-energy solutions are plotted as filled (open) circles. All amplitudes have been multiplied by a factor of $10^3$ and are dimensionless. Plotted are the dominant partial-wave amplitudes: (a) $^3P_0$ ($0^+$), (b) $^3S_1$ ($1^-$), (c) $^3P_2$ ($2^+$), (d) $^3D_2$ ($2^-$), (e) $^3D_3$ ($3^-$).

Figure 4. Comparison of the $^3P_1 S$ partial waves for $\pi d \to pp$ obtained in the separate and combined fits. The real (imaginary) part of solution C500 is plotted as
a solid (dashed) line. The purely real partial wave from our separate analysis (SP96) [7] is plotted as a dot-dashed line.

Figure 5. Moduli of the partial-wave amplitudes for $\pi d \rightarrow pp$ from $T_\pi = 0$ to 500 MeV. The solid and dashed curves give the amplitudes corresponding to the C500 and SP96 [7] solutions respectively. Moduli of the single-energy solutions are plotted as filled circles. All amplitudes have been multiplied by a factor of $10^3$ and are dimensionless. Only dominant partial-waves have been plotted: (a) $^1S_0P\ (0^+)$, (b) $^3P_1S\ (1^-)$, (c) $^1D_2P\ (2^+)$, (d) $^3P_2D\ (2^-)$, (e) $^3F_3D\ (3^-)$.
TABLE I. Partial wave decomposition of $pp$, $\pi d$, and $N\Delta$ systems.

| $J^P$ | $\pi d$  | $pp$   | $N\Delta$ |
|-------|---------|--------|-----------|
| 0$^+$ | $^3P_0$ | $^1S_0$ | $^5D_0$   |
| 0$^-$ | $^3P_0$ |         | $^3P_0$   |
| 1$^+$ | $^3P_1$ |         | $^3S_1$, $^3D_1$ |
| 1$^-$ | $^3S_1$, $^3D_1$ | $^3P_1$ | $^3P_1$ |
| 2$^+$ | $^3P_2$, $^3F_2$ | $^1D_2$ | $^3D_2$ |
| 2$^-$ | $^3D_2$ | $^3P_2$, $^3F_2$ | $^3P_2$, $^3F_2$ |
| 3$^+$ | $^3F_3$ |         | $^3D_3$, $^3G_3$ |
| 3$^-$ | $^3D_3$, $^3G_3$ | $^3F_3$ | $^3P_3$, $^3F_3$ |
| 4$^+$ | $^3F_4$, $^3H_4$ | $^1G_4$ | $^3G_4$ |
| 4$^-$ | $^3G_4$ | $^3F_4$, $^3H_4$ | $^3F_4$, $^3H_4$ |
TABLE II. Comparison of the combined analysis (C500) and our previous (separate) analyses. WI96 for $pp \rightarrow pp$, SM94 for $\pi d \rightarrow \pi d$, and SP96 for $\pi d \rightarrow pp$. The relevant energy ranges are: $T_\pi = 0$–500 MeV, $T_p = 288$–1290 MeV, and $\sqrt{s} = 2015$–2440 MeV.

| Reaction | Separate $\chi^2$/Data | Combined $\chi^2$/Data |
|----------|------------------------|------------------------|
| $pp \rightarrow pp$ | 17380/10496 | 17380/10496 |
| $\pi d \rightarrow \pi d$ | 2745/1362 | 2418/1362 |
| $\pi d \rightarrow pp$ | 7716/4787 | 7570/4787 |
TABLE III. Comparison of single-energy (binned) and energy-dependent combined analyses of \( \pi d \) elastic scattering data. \( N_{prm} \) is the number of parameters varied in the single-energy fits. \( \chi^2_E \) is due to the energy-dependent fit (C500) taken over the same energy interval.

| \( T_\pi \) (MeV) | Range (MeV) | \( N_{prm} \) | \( \chi^2/\text{data} \) | \( \chi^2_E \) |
|-----------------|-------------|--------------|-----------------|-------------|
| 65              | 58.0 – 72.0 | 2            | 106/54          | 102         |
| 87              | 72.0 – 92.0 | 6            | 20/24           | 21          |
| 111             | 107.5 – 125.2 | 10           | 68/82           | 66          |
| 125             | 115.0 – 134.0 | 12           | 155/170         | 184         |
| 134             | 124.0 – 142.8 | 14           | 315/258         | 344         |
| 142             | 133.0 – 152.0 | 16           | 356/284         | 397         |
| 151             | 141.0 – 160.6 | 16           | 193/154         | 216         |
| 182             | 174.0 – 189.5 | 18           | 302/168         | 396         |
| 216             | 206.0 – 220.0 | 18           | 158/99          | 200         |
| 230             | 220.0 – 238.0 | 18           | 64/53           | 111         |
| 256             | 254.0 – 260.0 | 16           | 132/125         | 185         |
| 275             | 270.5 – 284.4 | 16           | 22/40           | 42          |
| 294             | 284.4 – 300.0 | 16           | 267/132         | 324         |
TABLE IV. Comparison of single-energy (binned) and energy-dependent combined analyses of $\pi d \rightarrow pp$ reaction data. $N_{prm}$ is the number of parameters varied in the single-energy fits. $\chi^2_E$ is due to the energy-dependent fit (C500) taken over the same energy interval.

| $T_\pi$ (MeV) | Range (MeV) | $N_{prm}$ | $\chi^2$/data | $\chi^2_E$ |
|---------------|-------------|-----------|----------------|-------------|
| 25            | 12.8 – 37.4 | 10        | 527/241        | 542         |
| 50            | 37.6 – 60.7 | 12        | 188/168        | 205         |
| 75            | 62.9 – 87.3 | 14        | 590/426        | 628         |
| 100           | 91.0 – 114.0| 14        | 1263/611       | 1379        |
| 125           | 113.8 – 137.1| 16      | 729/512        | 756         |
| 150           | 140.0 – 162.0| 20      | 743/630        | 792         |
| 175           | 165.0 – 187.3| 22      | 343/280        | 426         |
| 200           | 191.3 – 210.3| 20      | 120/193        | 153         |
| 225           | 217.9 – 235.9| 22      | 217/229        | 291         |
| 250           | 238.9 – 262.0| 22      | 595/483        | 685         |
| 275           | 264.9 – 285.1| 22      | 204/109        | 280         |
| 300           | 291.6 – 307.4| 24      | 198/212        | 235         |
| 325           | 318.9 – 330.0| 24      | 142/161        | 234         |
| 350           | 341.4 – 360.3| 24      | 201/185        | 233         |
| 375           | 371.4 – 375.7| 24      | 32/26          | 42          |
| 400           | 390.0 – 400.0| 24      | 19/28          | 34          |
| 425           | 417.0 – 420.0| 24      | 50/28          | 55          |
| 450           | 437.6 – 456.5| 22      | 122/48         | 231         |
| 475           | 473.8 – 487.4| 22      | 24/24          | 39          |
| 500           | 495.9 – 506.5| 22      | 49/45          | 281         |
