Sum-Rule Results on Exclusive Decays of Heavy Mesons∗

A. Khodjamiriana,† and R. Rückla,b

a Institut für Theoretische Physik, Universität Würzburg, D-97074 Würzburg, Germany
b Max-Planck-Institut für Physik, Werner-Heisenberg-Institut, D-80805 München, Germany

Abstract

We review QCD sum rule applications to hadronic matrix elements of exclusive $B$ and $D$ decays. Results are presented for the form factors $f^+$ and $F_0$ of $B \to \pi$ and $D \to \pi$ transitions. The predictions are used to compute the width of $D^* \to D\pi$, and to extract the CKM parameter $V_{ub}$ from the measured $B \to \pi\nu\nu$ width. Furthermore, we comment on weak annihilation in $B \to \rho\gamma$, as well as on the radiative decays $D \to \rho\gamma$, $D \to K^\ast\gamma$, and $B \to \mu\nu\gamma$.

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1 Introduction

At this conference, a large amount of new experimental data on exclusive decay modes of heavy hadrons have been presented. In order to interpret these measurements in the framework of the standard model, one has to be able to calculate hadronic matrix elements of products of quark/gluon operators. Since these matrix elements involve long-distance quark-gluon dynamics, nonperturbative methods are needed. In this talk, we report on recent progress in applying QCD sum rule methods to solve this problem. The basic tools are the operator product expansion (OPE) near the light-cone and dispersion relations in combination with quark-hadron duality. The basic nonperturbative input are the light-cone wave functions of light mesons. The wave functions of $\pi$ and $K$ mesons are known with sufficient accuracy to allow for reliable predictions of heavy-to-light form factors and couplings. Recently, one has also begun to study applications requiring $\rho$-meson and photon wave functions.

After a brief outline of the calculational procedure (section 2), results are presented for the $B \rightarrow\pi$ and $D \rightarrow\pi$ form factors (section 3), and the hadronic $B^*B\pi$ and $D^*D\pi$ couplings (section 4). From that we predict the widths of $D^* \rightarrow D\pi$ and $B \rightarrow \pi l\nu$. The latter is used to extract the value of $V_{ub}$ from the CLEO measurement of $B \rightarrow \pi l\nu$ (section 5). Finally, we demonstrate the flexibility of light-cone sum rules by summarizing results on the radiative decays $B \rightarrow \rho\gamma$, $D \rightarrow \rho\gamma$, $D \rightarrow K^*\gamma$, and $B \rightarrow \mu\nu\gamma$.

2 Outline of the light-cone sum rule approach

For definiteness, let us focus on the transition matrix element $\langle L \mid \bar{q}\Gamma_a Q \mid H \rangle$ between a heavy hadron $H$ and a light $(L)$ meson. The basic object to be calculated is the correlation function

$$F_{ab}(q,p) = \int d^4xe^{ipx}\langle L(q) \mid T\{\bar{q}(x)\Gamma_a Q(x)\bar{Q}(0)\Gamma_b q'(0)\} \mid 0\rangle.$$

Here, $L$ is taken on-shell with momentum $q$ and the quark current $\bar{Q}\Gamma_b q'$ with the external momentum $(p+q)$ is chosen to carry the quantum numbers of $H$. The appropriate combinations of Dirac matrices are denoted by $\Gamma_{a,b}$.

In contrast to conventional sum rules employing the Wilson OPE in terms of local operators, here the $T$-product of currents in (1) is expanded in terms of nonlocal operators near the light-cone, i.e. at $x^2 \approx 0$. Schematically, this expansion has the form:

$$\langle L(q) \mid T\{\bar{q}(x)\Gamma_a Q(x)\bar{Q}(0)\Gamma_b q'(0)\} \mid 0\rangle \sim \sum_i C_i(x)\langle L(q) \mid O_i(x,0) \mid 0\rangle$$

with calculable coefficients $C_i(x)$. The bilocal operators $O_i(x,0)$ are constructed out of quark and gluon fields.

Furthermore, the matrix elements of $O_i(x,0)$ appearing in (2) are parametrized in terms of so-called light-cone wave functions $\phi_i^n$ characterized by particle multiplicity and twist. For the leading two-quark operator $O_{2a}(x,0) = \bar{q}(x)\Gamma_a q'(0)$ one has, schematically,

$$\langle L(q) \mid O_{2a}(x,0) \mid 0\rangle = \sum_n \int_0^1 du e^{iuq^\nu x}\phi_{2a}^n(u)(x^2)^n,$$

where a path-ordered gauge factor has been omitted for brevity. In the momentum region $(p+q)^2 \ll m^2_Q$, the light-cone expansion is dominated by the lowest-twist wave functions, and
can therefore be truncated after a few terms. Thereby, one may keep the momentum transfer \( p^2 \) timelike, in the range \( 0 < p^2 < m_Q^2 - O(1\text{GeV}^2) \). This is very important since it allows to calculate form factors in a wide kinematical region without the need of extrapolations.

The connection of the correlation function \( F_{ab} \) with the matrix element \( \langle L \mid \bar{q} \Gamma_a Q \mid H \rangle \) of interest is obtained by writing a dispersion relation in the variable \((p + q)^2\) for \( F_{ab} \) and inserting the complete set of intermediate states \( |i\rangle \) with the quantum numbers of \( H \):

\[
F_{ab} = \sum_i \frac{\langle L \mid \bar{q} \Gamma_a Q \mid i \rangle \langle i \mid \bar{Q} \Gamma_b q' \mid 0 \rangle}{m_i^2 - (p + q)^2}.
\]

(4)

Obviously, the desired matrix element is contained in the contribution from \( |i\rangle = |H\rangle \). In order to determine it, one has to subtract the contributions of all other states \( |i\rangle \neq |H\rangle \). For the ground state \( H \) in a given channel, this is indeed possible to a reasonable approximation. To this end, one invokes quark-hadron duality to estimate the contribution of the heavier states in (4), and applies the Borel transformation in order to exponentially damp their contribution, and to diminish the sensitivity of the resulting expression for \( \langle L \mid \bar{q} \Gamma_a Q \mid H \rangle \) to the duality approximation.

One of the principal advantages of the QCD sum rule method is the universality of the nonperturbative input parameters, in the light-cone variant the nonasymptotic coefficients of the light-cone wave functions. Once these parameters have been determined from one set of measurements, one can apply the method to other observables without having to introduce new unknown parameters. Furthermore, the matrix element \( \langle H \mid \bar{Q} \Gamma_b q' \mid 0 \rangle \), multiplying the \( H \to L \) transition element in (4), and also the threshold parameter \( s_0 \), separating in \((p + q)^2\) the ground state at \((p + q)^2 = m_H^2\) from the region \((p + q)^2 > s_0\) included in the duality integral, can be estimated separately from appropriate two-point sum rules.

Another principal advantage is the possibility to estimate the accuracy of a given result within the same sum-rule framework. An analogous estimate is not possible, for example, in phenomenological quark models. Currently, one main source of uncertainties are perturbative corrections that are not yet known for sum rules of form factors and hadronic couplings. This problem will certainly be solved in near future. More difficult is it to improve our knowledge of the nonasymptotic terms in the light-cone wave functions. The lack of complete understanding leads to about 15-20\% uncertainty in the predictions presented below.

Last but not least, QCD sum rules automatically include effects due to the finiteness of the physical quark masses and can deal with heavy and light quarks. Moreover, they are not or at least less restricted to particular kinematical conditions such as zero recoil in form factors. In this respect, the sum rule method is not challenged by HQET (heavy quark effective theory) and complements lattice calculations.

### 3 The \( B \to \pi \) and \( D \to \pi \) form factors

As a first example, we consider the matrix element of the \( B \to \pi \) transition which is parametrized by two form factors:

\[
\langle \pi(q) \mid \bar{u} \gamma_\mu b \mid B(p + q) \rangle = 2 f^+(p^2) q_\mu + (f^+(p^2) + f^-(p^2)) p_\mu.
\]

(5)

In [4], the light-cone sum rule was obtained for the form factor \( f^+ \) in the region of momentum transfer \( p^2 < m_b^2 - O(\text{GeV}^2) \), and to twist 4 accuracy. Recently [3], the calculation of (4)
has been completed by deriving the sum rule for \((f^+ + f^-)\) to the same accuracy. With these results at hand, we can also predict the scalar form factor

\[
F_0(p^2) = f^+(p^2) + \frac{p^2}{m_B^2 - m^2_B} f^-(p^2).
\]

In Fig. 1, we show our results [4, 5] for \(f^+\) and \(F_0\). In order to estimate the form factor \(f^+\) at larger \(p^2\), one may use the single pole approximation involving the \(B^*\) resonance. The residue of the pole is proportional to the \(B^* B \pi\) coupling which can be extracted from the same correlation function [4] used also for \(f^+\) (see also section 4). As can be seen in Fig. 1, the pole approximation normalized in this way nicely matches the result of the direct calculation at intermediate values of \(p^2\). Also interesting to note is the consistency of our result on \(F_0\) with the Callan-Treiman relation \(\lim_{p^2 \to m_B^2} F_0(p^2) = f_B/f_\pi\) [6] although this constraint is weak because of the sizeable uncertainty in the \(B\) decay constant.

The corresponding \(D \to \pi\) form factor \(f^+\) is shown in Fig. 2. Interestingly, in this case there is almost no difference between \(F_0\) and \(f^+\).

4 The \(D^* \to D \pi\) decay width

As mentioned before, the correlation function [4] can also be used to calculate the \(B^* B \pi\) and \(D^* D \pi\) couplings [4]. The former is defined by

\[
\langle B^0(p) \pi^+(q) | B^+(p+q) \rangle = -g_{B^* B \pi} q_\mu \epsilon^\mu.
\]

In this application, one has to employ dispersion relations and sum rule methods simultaneously in the \(B\) and \(B^*\) channels. After double Borel transformation one obtains an expression for \(g_{B^* B \pi}\) the leading twist term of which depends on the pion wave functions at the momentum fraction \(u \simeq 1/2\). Numerically, we find \(g_{B^* B \pi} = 29 \pm 3\), and by an analogous calculation for \(D\) mesons, \(g_{D^* D \pi} = 12.5 \pm 1\). Since these predictions depend on light-cone wave functions at a fixed point, they are less certain than the results on form factors which involve integrals over wave functions.

With the above value of \(g_{D^* D \pi}\) one obtains the hadronic width \(\Gamma(D^{*+} \to D^0 \pi^+) = 32 \pm 5\) keV. This estimate is perfectly consistent with the upper limit derived from recent measurements [8, 9].

5 Extraction of \(V_{ub}\)

Furthermore, from the results on \(f^+(p^2)\) and \(F_0\) shown in Fig. 1, one can compute the widths for \(B^0 \to \pi^- l^+ \nu_l\) (\(l = e, \mu, \tau\)). We find

\[
\Gamma(B^0 \to \pi^- e^+ \nu_e) = 8.7 |V_{ub}|^2 \text{ ps}^{-1},
\]

\[
\Gamma(B^0 \to \pi^- \tau^+ \nu_\tau) = (0.7 \div 0.8) \Gamma(B^0 \to \pi^- e^+ \nu_e).
\]

Unlike \(\Gamma(B^0 \to \pi^- e^+ \nu_e)\), the width of \(B^0 \to \pi^- \tau^+ \nu_\tau\) is very sensitive to the form factor \(F_0\). The interval given in (4) reflects the uncertainty in the extrapolation of \(F_0\) from the endpoint of the curve in Fig. 1 to the value \(f_B/f_\pi = 1.0 \div 1.7\) at \(p^2 = m_B^2\) implied by current algebra [6].
Experimentally, combining the recent CLEO result for $l = e, \mu$ [11], $BR(B^0 \to \pi^- l^+ \nu_l) = (1.8 \pm 0.4 \pm 0.3 \pm 0.2) \cdot 10^{-4}$, with the world average of the $B^0$ lifetime [12], $\tau_{B^0} = 1.56 \pm 0.06$ ps, one obtains $\Gamma(B^0 \to \pi^- l^+ \nu_l) = (1.15 \pm 0.34) \cdot 10^{-4}$ ps$^{-1}$, where the errors have been added in quadrature. Comparison with [8] then yields $|V_{ub}| = 0.0036 \pm 0.0005$. The additional theoretical uncertainty is estimated to be less than 20%.

Recently, also the $B \to \rho$ form factor has been calculated with the help of a light-cone sum rule [13]. The resulting width $\Gamma(B^0 \to \rho^- l^+ \nu_l) = (14 \pm 4) |V_{ub}|^2$ ps$^{-1}$ together with (8) yields a $\rho/\pi$ ratio in agreement with the CLEO finding [11] of about 1.4.

6 Radiative decays

A further class of processes to which the method of light-cone sum rules can be applied fruitfully are exclusive radiative $B$ and $D$ decays. Recent examples include the calculations of the magnetic penguin form factor [14] in $B \to K^{*0} \gamma$ and $\rho \gamma$, and of the contribution from weak annihilation [15, 16] to $B \to \rho \gamma$. In the latter case, the light-cone expansion leads to matrix elements, respectively, hadronic wave functions associated with photon emission by a quark-antiquark pair at light-like separation. From these estimates weak annihilation is expected to contribute to $B \to \rho \gamma$ at most 10% of the penguin mediated short-distance amplitude. This effect is comparable in size to the theoretical uncertainty in the main amplitude. However, with increasing precision the long-distance effect due to weak annihilation may become non-negligible.

In radiative $D$ decays, weak annihilation plays a more pronounced role since the short-distance penguin contributions are completely negligible. Light-cone sum rules [15] predict the branching ratios $BR(D^0 \to K^{*0} \gamma) = 1.5 \cdot 10^{-4}$ and $BR(D_s \to \rho^+ \gamma) = 2.8 \cdot 10^{-5}$. The experimental observation of these modes is an interesting task for the next generation of charm experiments.

Finally, by replacing the light hadronic currents in the above applications by a leptonic current, one can treat decays like $B \to l \nu l \gamma$ in same framework. These modes are very interesting as the strong suppression for $l = e, \mu$ due to helicity conservation should be lifted by the photon emission. Indeed, explicit calculation [15] predicts a factor 10 enhancement: $\Gamma(B \to \mu \nu \mu \gamma)/\Gamma(B \to \mu \nu \mu) \simeq 11.5(180\text{MeV}/f_B)^2$.

With the few examples selected for this review we hope to have illustrated the usefulness of QCD sum rules on the light-cone.

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Figure 1: $B \to \pi$ form factors: $f^+$ (upper curve extrapolated using single-pole approximation) and $F_0$ (lower curve). The data points indicate lattice results [10].

Figure 2: The $D \to \pi$ form factor $f^+$: direct sum rule result (solid curve) in comparison to the single-pole approximation (dashed curve).