Momentum Correlations for Identical Fermions

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The effects of intensity interferometry (HBT), which are mainly reflected by the measured momentum correlations, have been fully discussed since 1950’s. The analogous momentum correlations between identical fermions are however less argued. Besides the fermion statistics, the momentum correlations between identical fermions are also determined by some other aspects, such as coulomb interaction, spin interactions and resonances formation. In this paper, we discussed the factors that influence the momentum correlations between Λ’s, especially spin interaction. It is also argued that the momentum correlations between Λ’s are affected significantly by the yield ratio of Σ\textsuperscript{0}/Λ, due to the limitation of experimental measurement. The mixture of Σ\textsuperscript{0}’s and Λs would significantly weaken the measured two-particle momentum correlations of Λ’s.

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INTRODUCTION

Intensity interferometry (HBT), which was firstly proposed by Hanbury-Brown and Twiss\textsuperscript{[1]} as a method of measuring the sizes of stars, was later extended to particle physics by Goldhaber et al\textsuperscript{[2]} to extract the space-time structure of p\bar{p} annihilations by studying the angular distribution of identical pion pairs in the collisions. HBT interferometry is now regarded as a useful tool to study the space-time structures of fireballs produced in heavy-ion collisions\textsuperscript{[3, 4]}. The HBT interferometry is an effect of Bose-Einstein statistics, which leads to an enhancement of the identical boson pairs when their relative momenta is small. The effect is usually represented by the measured two-particle momentum correlations. It has been argued that some dynamical information of the expanding fireball created in heavy-ion collisions can be obtained by the correlations. For instance, the information on the anisotropic transverse flow of the fireball can be extracted by measuring the correlation functions as a function of the emission angle\textsuperscript{[5, 6, 7]}. In fact, the 2-particle momentum correlations of a many-body system are determined not only by the quantum statistics, but also the interactions between the particles produced in the source and the final-state interactions. The pure quantum statistics effects can be acquired only after all these influences have been removed carefully. For example, due to the repulsive coulomb interactions, the momentum correlations between identical charged pions are suppressed to some extent for small relative momenta, and the expected great enhancement can show up only after the coulomb interactions are removed correctly\textsuperscript{[8]}. Although the theory of HBT effects\textsuperscript{[3, 4, 5]} has been fully discussed since 1950’s, and a lot of experimental results have been acquired by relativistic heavy-ion collisions at CERN/SPS\textsuperscript{[10, 11]} and RHIC\textsuperscript{[12, 13, 14, 15, 16, 17]}, there are still a lot of open questions in this area. Both theorists and experimentists are embarrassed by the so-called “HBT puzzle”\textsuperscript{[18]} that hydrodynamic calculations, which provide perfect results of anisophic flows, yield strong disagreement with HBT radii and predict lack of energy dependence of the HBT radii for a fixed $k_T$ bin\textsuperscript{[19, 20]}. Besides “HBT” puzzle, the analogous momentum correlations between identical fermions are less argued. The conclusion can be made intuitively that the identical fermion pairs should be suppressed for small relative momentum due to Fermi-Dirac statistics. But little knowledge has been obtained by now to make a definite conclusion. Different from Boson’s correlations, the spin interactions play an important role in the two-particle momentum correlations between identical fermions. The abundance of protons produced in heavy-ion collisions guarantees the statistics for momentum correlations between them, and SPS did measure the correlations\textsuperscript{[11]}. But the complicated interactions between protons and the strong coulomb interactions are always big issues, which lead to the difficult to draw a definite conclusion. C. Greiner and B. Müller\textsuperscript{[21]} proposed the momentum correlations between Λ’s and discussed the influences by the possible existing di-Λ(H particle)\textsuperscript{[22, 23]}. WA97 measured the Λ-Λ correlations\textsuperscript{[10]} in Pb-Pb collisions at 158A GeV/c, but poor statistics refused any expected conclusions.

The relativistic heavy-ion collisions at RHIC is a factory of strangeness, where quite a lot of Λ’s are produced for central collisions. This gives us a chance to measure the momentum correlations between Λ’s with
better statistics. The most important advantage of the analysis of Λ correlations is that the coulomb interactions, if exist, can be neglected. What’s more, We hardly had any opportunity before to study the interactions between baryons with strangeness, the momentum correlations between Λ’s may give us a first glimpse of the interactions of strange baryons, however rough it might be. The investigation of the momentum correlations between Λ’s at RHIC is therefore interesting and valuable at least from this point of view.

In this paper, we discussed those important factors that influence the momentum correlation between Λ’s, one of which is the spin interactions between Λ’s. It is also argued that the measured momentum correlations between Λ’s are affected significantly by the yield ratio of Σ/Λ due to the limitation of experimental measurement.

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the latter spin symmetric state. We can thus form two kinds of asymmetric total wave functions fermions with no other internal space,
\[ \psi^S(p_1, r_1; p_2, r_2) = \phi^+ \chi_A, \]
\[ \psi^T(p_1, r_1; p_2, r_2) = \phi^- \chi_S, \]
where \( \phi^+ \) and \( \phi^- \) are respectively normalized symmetric and asymmetric spatial wave functions. Therefore, the total wave function, \( \Psi_{1,2} \), can be express as a mixed state of singlet \( \psi^S \) and triplet \( \psi^T \),
\[ \Psi_{1,2} = \alpha \psi^S + \beta \psi^T, |\alpha|^2 + |\beta|^2 = 1. \] (6)
The momentum correlation function of \( \Lambda \)'s is then given by
\[ C_2^A(\vec{p}_{rel}) = 1 - (|\beta|^2 - |\alpha|^2) \exp(-p_{rel}^2 r_0^2/2), \] (7)
which depends apparently on the probability of spin singlet and triplet states. If we assume spin singlet state and each component of triplet state has the same probability, that is, \( \alpha = 1/2 \) and \( \beta = \sqrt{3}/2 \), the correlation function becomes
\[ C_2^A(\vec{p}_{rel}) = 1 - \frac{1}{2} \exp(-p_{rel}^2 r_0^2/2). \] (8)
This is exactly the formula given by Greiner and Müller [21].

Unfortunately we have in fact little (if not blank) knowledge about the spin interactions between hyperons. It is hardly to determine the value of \( \alpha \) and \( \beta \). The overall investigation on the interactions between nucleons are made mostly via \( p-p \) collisions. Deuteron, the two-particle system of \( n-p \), is the only boundary state of nucleon-nucleon. The spin of deuteron is \( S = 1 \), which indicates that the spins of neutron and proton intend to be in the same direction and form a triplet state. But we have no idea whether a \( \Lambda-\Lambda \) system prefers triplet or singlet state.

If \( \beta = \alpha = 1/\sqrt{2} \), the correlation function \( R_A \) would vanish, which means no effects of momentum correlations exhibit. When \( \beta > \alpha \), it gives \( R_A < 0 \) and Fermi-Dirac suppression would dominate. If \( \beta < \alpha \), it gives a desperate result, \( R_A > 0 \), which means that Bose-Einstein enhancement shows up!

If \( \beta = 1 \), that is, the system is in pure triplet state, \( R_A = -\exp(-p_{rel}^2 r_0^2/2) \), which represents a full Fermi-Dirac suppression for small relative momentum. On the contrary, if \( \alpha = 1 \), that is, the system is in pure singlet state, \( R_A = \exp(-p_{rel}^2 r_0^2/2) \), which represents that the correlation behaviour of \( \Lambda \)'s is totally in the same way as bosons.

THE EFFECTS OF \( \Sigma^0/\Lambda \) RATIO

The production of \( \Sigma \)'s in heavy-ion collisions gives sensitive influence on the measured \( \Lambda-\Lambda \) correlation functions. \( \Sigma^0 \) has the same value quark components as \( \Lambda \), but a little heavier. It decays to \( \Lambda \) and a low energy photon. With the mean life \( \tau \sim 10^{-20} \text{s} \), which is far longer than the life of the fireballs created by heavy-ion collisions, nearly all \( \Sigma^0 \)'s decay long after the freeze-out of the fireballs, compared with the life of the fireballs. The decay length of \( \Sigma^0 \) is of the order \( 10^4 \text{fm} \), which is small enough to prevent us distinguishing experimentally the decay vertex of \( \Sigma^0 \) between primary vertex, we can therefore hardly determine whether a \( \Lambda \), when reconstructed, is emitted directly from the fireball or from \( \Sigma^0 \) decay.

Suppose that \( N \) \( \Lambda \)'s are reconstructed in one event, which are “all” selected to be emitted from the source. If the feed-down from multiply strange hyperons, notably \( \Xi^0 \) and \( \Xi^- \), is neglected, these \( N \) \( \Lambda \)'s mainly come from two kinds of contributions, “primordial” \( \Lambda \)'s that are really originated directly from the fireball, and “decayed” \( \Lambda \)'s that come from \( \Sigma^0 \) decays. Suppose that the yield ratio of \( \Sigma^0/\Lambda = \gamma \), typically \( \gamma \leq 1 \). The number of “primordial” \( \Lambda \)'s is then \( n_{\text{prim}} = N/(1+\gamma) \), while the number of “decayed” \( \Lambda \)'s is \( N_{\text{decay}} = \gamma N/(1+\gamma) \). Since the “decayed” \( \Lambda \)'s are nearly all produced chaotically long after freeze-out of the fireball, there should be no significant momentum correlations between them. There should also be no harm to declare that there are no momentum correlations between “decayed” \( \Lambda \)'s and “primordial” \( \Lambda \)'s.

The correlation function \( C_2 = 1 + R \) is experimentally obtained by the pairs in same events divided by pairs in mixed events. The pairs obtained from the \( N \) \( \Lambda \)'s can be separated into three parts, those between “primordial” \( \Lambda \)'s, \( N(\Lambda \Lambda) \), those between “decayed” \( \Lambda \)'s, \( N(\Sigma^0 \Sigma^0) \) and those between “primordial” and “decayed” \( \Lambda \)'s, \( N(\Lambda \Sigma^0) \),
\[ N(\Lambda \Lambda) = \frac{1}{2} \frac{N}{1+\gamma} \left( \frac{N}{1+\gamma} - 1 \right), \]
\[ N(\Sigma^0 \Sigma^0) = \frac{1}{2} \frac{\gamma N}{1+\gamma} \left( \frac{\gamma N}{1+\gamma} - 1 \right), \]
\[ N(\Lambda \Sigma^0) = \frac{\gamma N^2}{(1+\gamma)^2}. \]

Totally, \( N(\Lambda \Lambda) + N(\Sigma^0 \Sigma^0) + N(\Lambda \Sigma^0) = \frac{N}{N_{\text{pair}}} (N - 1) = N_{\text{pair}} \), where \( N_{\text{pair}} \) is the total pairs of the \( N \) \( \Lambda \)'s.

According to the discussion above, there is no contribution to the correlation function for \( N(\Sigma^0 \Sigma^0) \) and \( N(\Lambda \Sigma^0) \), that is, \( C_2 = 1 \) for these two kinds of pairs. The experimentally obtained correlation function is therefore written as
\[ C_2^{exp} = \frac{N(\Lambda \Lambda)}{N_{\text{pair}}} (1 + R_A) + \frac{N(\Sigma^0 \Sigma^0)}{N_{\text{pair}}} + \frac{N(\Lambda \Sigma^0)}{N_{\text{pair}}}, \]
\[ = 1 + \frac{N - (1 + \gamma)}{(1+\gamma)^2(N-1)} R_A, \] (9)
where \( R_A \) is the correlation function of pure \( \Lambda \)'s. For \( N \gg 1 \), \( C_2^{exp} \approx 1 + \frac{1}{(1+\gamma)^2} R_A \). If \( \gamma = 1 \), and suppose that the probability of singlet and each component of triplet
is equal,

$$C_{2}^{exp} = 1 + \frac{1}{4} R_{\Lambda} = 1 - \frac{1}{8} \exp(-p_{rel}^{2}a_{0}^{2}/2).$$ (10)

This means the the effects of pure $\Lambda$’s momentum correlations would be significantly weakened if we can not identify those $\Lambda$’s from $\Sigma^{0}$ decay. So we can hardly observe significant suppression of $\Lambda$-$\Lambda$ correlations in heavy-ion collisions.

CONCLUSIONS

we investigated the momentum correlation between identical fermions, especially that between $\Lambda$’s produced in heavy-ion collisions. Different from bosons, spin interactions play an important role in the behaviour of fermion’s correlations. For $\Lambda$’s, if the spin wave function of $\Lambda$-$\Lambda$ system is symmetric, the asymmetric space wave function leads to a total suppression at small relative momentum. If the spin wave function is asymmetric, the symmetric space wave function, however, leads to a total enhancement at small relative momentum, just the same as the behaviour of identical bosons. The lack of knowledge on the spin interactions makes it complicated to draw a conclusion whether the correlation functions should suppress or enhance at small relative momentum. It is also hard to say to what extent the suppression or enhancement would be.

It is also argued that $\Sigma^{0}$/\Lambda yield ratio affects the measured momentum correlations of $\Lambda$. A high ratio would weaken the the correlation effects significantly. For $\Sigma^{0}$/\Lambda = 1, the measured effect of correlations would be reduced to only about one quarter of that of correlations between pure $\Lambda$’s. In fact, the number of $\Lambda$’s $N$ that can be reconstructed for Au-Au central collisions at RHIC[24] is quite small. In this case, the deviation of $C_{2}^{exp}$ from 1 is therefore even smaller.

The trouble is that neither the spin interactions between $\Lambda$’s nor the $\Sigma^{0}$/\Lambda yield ratio is clear to us. If the spin interactions could be clearly understood, the behaviors of pure $\Lambda$’s correlations would be determined to a great extent. The $\Sigma^{0}$/\Lambda ratio would therefore be extracted from the measured momentum correlation function. The $\Sigma^{0}$/\Lambda is important to to heavy-ion collision since it can give us information on the mechanisms of hadronization, e.g. recombination and fragmentation mechanisms[25, 26, 27]. On the other hand, if we could get a definite $\Sigma^{0}$/\Lambda yield ratio via some other approaches, the information on the interactions between $\Lambda$’s could then be obtained after the influence of $\Sigma^{0}$/\Lambda was removed.

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[1] R. Hanbury-Brown and R. Q. Twiss, Nature 178, 1046(1956)
[2] G. Goldhaber et al, Phys. Rev. 120, 300(1960)
[3] U. Heinz and B. Jacak, Ann. Rev. Nucl. Part. Sci. 49, 529(1999)
[4] S. Pratt, Phys. Rev. Lett. 53, 1219(1984)
[5] S. A. Volsoshin and W. E. Cleland, Phys. Rev. C53, 896(1996)
[6] S. A. Volsoshin and W. E. Cleland, Phys. Rev. C54, 3212(1996)
[7] U. Heinz, A. Hummel, M. A. Lisa, U. A. Wiedemann, Phys. Rev. C66 044903(2002)
[8] C. Adler et al (STAR Collaboration), Phys. Rev. Lett.87, 082301(2001)
[9] T. Csoroto, nucl-th/0505019
[10] E. Andersen et al, J. Phys. G25, 423(1999)
[11] H. Appelshauser et al., Phys. Lett. B467, 21(1999)
[12] S. Johnson, Nucl. Phys. A698, 603(2002)
[13] A. Enokizono, Nucl. Phys. A715, 595(2003)
[14] J. Adams et al (STAR Collaboration), Phys. Rev. Lett. 91, 262301(2003)
[15] J. Adams et al (STAR Collaboration), Phys. Rev. Lett. 91, 262302(2003)
[16] J. Adams et al (STAR Collaboration), Phys. Rev. Lett. 93, 012301(2004)
[17] J. Adams et al (STAR Collaboration), Phys. Rev. C71, 044906(2005)
[18] D. Magestro, J. Phys. G31, 265(2005)
[19] U. Heinz and P. Kolb, Nucl. Phys. A702 269(2002)
[20] P. Kolb and U. Heinz, nucl-th/0306084
[21] C. Greiner and B. Müller, Phys. Lett. B219, 199(1989)
[22] R. L. Jaffe, Phys. Rev. Lett. 38, 195(1977)
[23] R. L. Jaffe, Phys. Rev. Lett. 38, 1617(E)(1977)
[24] C. Adler et al (STAR Collaboration), Phys. Rev. Lett. 89, 092301(2002)
[25] V. Greco and C. Ko, J. Phys. G31, s407(2005)
[26] V. Greco and C. Ko, nucl-th/0412043
[27] R. Fries J. Phys. G31, S379(2005)