An analysis of undergraduate students' higher order thinking skills in Geometry

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Abstract. Higher order thinking skill (HOTS) becomes important thing and issue in teaching and learning, especially in Mathematics. By using HOTS, students could solve the complicated problems and situation that could be used in real life. So that, the students could be ready to encounter high technology developing and industry revolution 4.0. Then, students could compete with foreign students. The purpose of this study is to describe the higher order thinking skills’ students in proving geometry theorem. This study uses qualitative research and conducted to the undergraduate students. The result of this study shows that the highest HOTS score equal to 80 and the lowest HOTS score equal to 30, then the average of HOTS score is 62. The subjects with low score, only try to analyse the statement without clear reason. The subjects with medium score begin to analyse and evaluate the problem with reason although it’s not clear enough. The subjects with high score begin to create new idea in proving theorem although sometimes could explain the reason. At the end, they could draw the conclusions based on the problems.

1. Introduction
Thinking skills are the fundamental in educational process. Learning styles and higher order thinking skills (HOTS) are important aspects in teaching and learning especially at higher education institutions [1]. Thinking skills are competencies to carry out essential thinking processes in our daily lives. It is a combination of cognitive processes and the ability to solve a given task [2]. Thinking skills especially creativity is essential to support economic prosperity [3].

HOTS is the highest level in the hierarchy of cognitive processes [1]. HOTS could occur when someone gets new information, compiles, and evaluates with existing knowledge then generate this information to achieve a goal and create the solution of complex situation [1]. HOTS as the potential use of the mind to deal with new challenges because HOTS can challenge individual to interpret, analyse or manipulate information [1]. So that, developing HOT skills and cognitive abilities become the goal of science education in the world [4]. This is because meaningful knowledge resulting from significant learning requires HOTS. Teaching HOTS is an effective way of requiring clear and accurate understanding.

On the other hand, the result of the Bakry’s study showed that the differences of students’ thinking with high mathematics ability, medium mathematics ability, and low mathematics ability in solving HOTS problem. Only the student with high mathematics ability that able to create meaning, make opinion and draw conclusion. The student with medium mathematics ability only able to create meaning, make opinion. The student with low mathematics ability is not able to create meaning and draw conclusion [5].
The findings indicated that key idea was the most important factor in generating ideas among these students [3]. The difficulty in generating ideas is a main factor in influencing the student achievement. Thus, students need to study higher order thinking skills to overcome difficulties in generating ideas. Students who have higher order thinking skill, will be able to gain a deep understanding of mathematical concepts and generating it in order to explore their thought processes in solving mathematics problems [5]. That activities are expected to be applied in real life.

Taxonomy framework helps teachers realize the standards into a common language for comparison with what they personally want to achieve and presents various possibilities for consideration. The taxonomy may provide several perspectives to guide curriculum decisions [6]. In addition, teachers only focus on LOT assessment even they say want to teach and assess HOT [7]. Therefore, the need for HOTS now cannot be ignored. HOTS is important to be developed because it is hoped that Indonesian graduates can compete with graduates from other countries in various sectors. HOTS is also needed to prepare students for the challenges of the industrial revolution 4.0. Therefore, national education began to promote the improvement of HOTS.

The students in junior high school can increase the thinking ability through teaching and learning mathematics activities in classroom. This is in line with the objective of learning mathematics by the Indonesian National Education Ministry. The ministry state that the purposes of learning mathematics are (1) thinking and reasoning to draw conclusion, (2) developing creative activities that involve imagination, intuition, and discovery, originally, curiosity, conjecture and assumptions and dare to try, (3) developing problem solving skills, (4) developing the ability to integrate information and express arguments [8].

In addition, the goals of national education, namely developing critical, logical, systematic, objective, honest and disciplined thinking skills. In order to develop students higher order thinking skills, we have to provide the assessment of HOT. HOTS assessments are the set of questions or tasks based on contextual issues and situations which are non-routine or unfamiliar then demands students to analyse, evaluate, and create [7][9]. Voogt & Roblin suggested that the HOTS assessment can enhance critical thinking skills, creativity and confidence (learning self-reliance) [10]. This is in line with Widana that suggests the benefit of HOTS assessments are increasing the students’ motivation, improving learning outcomes and performing quantitative analysis [9]. Therefore, this paper focuses on describing how the students solve HOTS problems in mathematics.

2. Literature Review

2.1. Higher Order Thinking Skills (HOTS)

The committee identified three domains of educational or activities learning that are cognitive, affective, and psychomotor. Cognitive refers to mental skills (knowledge), affective refers to growth of feelings or emotional areas (attitude), and psychomotor refers to physical skills. The cognitive domain involves knowledge and intellectual skill development [9]. This includes the recalling or recognizing specific facts, procedural patterns, and concepts that function in the development of intellectual abilities and skills.

The Taxonomy of Educational Objectives is a scheme for classifying educational goals, objectives, and standards. Benjamin S. Bloom and team of educational psychologists developed a hierarchy of educational objectives, commonly referred to as Bloom’s Taxonomy. Taxonomy emphasizes the assessment of learning with many sample tests items provided for each category. It tries to identify six levels of the cognitive domain, from the simplest to the most complex behaviors, which contains knowledge, comprehension, application, analysis, synthesis, and evaluation [11].

Anderson and Krathwohl revised the bloom taxonomy with a two-dimensional framework become remember, understand, apply, analyze, evaluate, and create [7]. Understanding is believed to be more cognitively complex than remembering, applying belief is more cognitively complex than understanding, and so on. Then, based on the taxonomy, then it can be classified into two levels, namely Higher Order Thinking (HOT) and Lower Order Thinking (LOT). King et all define higher order thinking skills into critical, logical, reflective, metacognitive, and creative thinking [12]. Lower order thinking skills consist of remember, understand, apply. Higher order thinking skills consists of analyze, evaluate, and create [6]. Some theoreticians and researchers use different frameworks to describe higher
order skills, but all frameworks are in general agreement based on the conditions which suitable. The diagram of revised Bloom's Taxonomy is presented by Wilson in the following chart below.

![Bloom's Taxonomy Diagram](image)

Figure 1. Diagram of Bloom’s taxonomy [13]

Anderson and Krathwohl describe every category of taxonomy clearly [7]. Remember refers to retrieve relevant knowledge from long-term memory. Understand refers to determine the meaning of oral, written, and graphic communication. Apply refers to carrying out or using a procedure in a given situation. Analyse refers to break the material down into its parts and detects how the parts relate to each other and with the overall structure. Evaluate refers to makes valuation based on criteria and standards. The last create refers to brings elements together, coherent whole or make an original product.

### 3. Research Method

This research is a qualitative research which aims to analyse and describe students' higher order thinking skills. The study was conducted on 25 first semester students of the Mathematics Education study program at the University of Jember. The research was carried out on Geometry subjects, especially theorems on flat geometry. The stages of research carried out include:

- **Phase 1**: reviewing literature related to HOTS and geometry. This study aims to analyse the students’ higher order thinking skills. Next determine the indicators of HOTS that will be used
- **Phase 2**: developing HOTS test items and scoring rubrics and research instruments. Furthermore, validating the research instruments that have been developed by experts
- **Phase 3**: giving test questions to research subjects and analyse the test results
- **Phase 4**: describes the results of the analysis and student answer

Data analysis was carried out qualitatively by describing the test results based on indicators of higher order thinking skills. The research instruments used in this study were (1) HOTS test questions, (2) interview guidelines, and (3) validation sheets.

### 4. Result and Discussions

#### 4.1. Validity result

The results of the validation of the questions on the high-order thinking skills and interview guidelines for experts showed that the criteria were valid with a test item validation score of 3.5 and a validation score of 3.67. Based on the results of the validation, the questions and interview guidelines can be used in the research. However, the instrument was still revised according to the suggestions given by the validator.

#### 4.2. HOTS result

Higher order thinking skills tests were given to research subjects as many as 25 students. The test scores for higher order thinking skills are presented in Table 1 below.
Table 1. Score of HOTS’s test.

| No | Score         | Frequency |
|----|---------------|-----------|
| 1. | $80 \leq \text{score} \leq 100$ | 4         |
| 2. | $65 \leq \text{score} < 80$      | 8         |
| 3. | $50 \leq \text{score} < 60$      | 9         |
| 4. | $25 \leq \text{score} < 50$      | 4         |
| 5. | $0 \leq \text{score} < 25$       | 0         |

The lowest score is 30 and the highest score is 80, while the average student test score is 62. This shows that students' higher order thinking skills need to be developed. In addition, the fulfillment of higher order thinking indicators is described in the following diagram.

Figure 2. Diagram of indicator achievement.

Furthermore, it was further analysed and described in relation to the students' answers at each level, 1 subject was selected. Because there are four level scores that are filled with subjects, there are four subjects, hereinafter referred to as S1, S2, S3, dan S4.

4.3. Description of the answer to the research subject
S1 on the level with a range of values between 25 and 50. In question number 1, S1 answers the proof of the theorem is correct, while the proof should be wrong so that S1 is able to analyze and evaluate, even unable to create a correct theorem proof. In question number 2, S1 tries to analyze and create proof of the area of a triangle with a known area of the range but it is not accurate. Here's the master's job on question number 2.

Figure 3. The answer of S1 number 2

In question number 3, S1 tries to create proof of the Pythagorean theorem but has not been able to analyze and evaluate the evidence presented. Here's the master's job on question number 3.
Figure 4. The answer of S1 number 3

S2 is at a level with a value range between 50 and 65. In question number 1, S2 answers that there is a reason for proving the theorem is wrong, namely AAS postulate, which should be SAS postulate so that S2 has tried to analyze and evaluate it even though it is not yet fully, but has not been able to create a proof of the theorem. Right. In problem number 2, S2 tries to analyze, evaluate, and create proving the area of a triangle with the exact area of the span of the triangle known. In problem number 3, S2 tries to create proof of the Pythagorean theorem even though the analysis and evaluation are not completely precise. The following is the result of S2's work on question number 3.

Figure 5. The answer of S2 number 3

S3 is at a level with a range of values between 65 to 80. In question number 1, S3 answers that there is a reason for proving the theorem is wrong, namely AAS postulate, which should be SAS postulate so that S3 has tried to analyze and evaluate it even though it is not completely, but has not been able to create proof of the theorem. Right. In question number 2, S3 tries to analyze and create the proof for the area of a triangle with a known area of the triangle but has not presented the reasons for choosing a step to prove the area of the triangle. Following are the results of S3 work in question number 2.

Figure 6. The answer of S2 number 2

In problem number 3, S3 tries to create a proof for the Pythagorean theorem using the area of a square appropriately, although it has not presented the reasons for choosing the step of proving the Pythagorean theorem. Following are the results of S3 work in question number 3.
S4 is at a level with a value range between 80 and 100. In question number 1, S4 answers that there is a reason for proving the theorem is wrong, namely AAS postulate, which should be SAS postulate so that S4 has tried to analyze and evaluate it even though not completely, but has not been able to create a proof of the theorem. In problem number 2, S4 tries to analyze, evaluate, and create the proof for the area of a triangle with the exact area of the line known as well as the reasons for choosing the step of proving the area of the triangle. The following is the result of S4's work on question number 2.

**Figure 7. The answer of S3 number 3**

In problem number 3, S4 tries to create a proof for the Pythagorean theorem using trigonometric properties correctly. The following is the result of S4's work in question number 3.

**Figure 8. The answer of S4 number 2**

**Figure 9. The answer of S4 number 3**

The thing that causes errors in solving HOTS problems is that students have difficulty linking information with the strategies that will be used in solving the problem [14]. In addition, errors and difficulties occur when students have deep in encoding, transformation, process skills and comprehension [14][15]. The subjects with low score, only try to analyze sometimes without reason. So that, the subject difficult to evaluate the step in proving theorem. Sometimes the subjects don’t elaborate the whole of the proof. The subjects with medium score begin to create meaning then try to analyze and evaluate the problem with reason although less precise. The subjects could identify the fault of proving but the some of the reasons still wrong. When students have difficulty analyzing and evaluating, it shows that students have not been able to use definitions, axioms and theorems in proving a theorem. Students have difficulty in determining the direction of thinking in proving the theorem because they are not used to it. Students also have not used their reasoning optimally to direct the thought process to think systematically and directed. It causes the students were failed to draw the conclusion [5]. The subjects with high score begin to create new idea in proving theorem although sometimes could explain the reason. Then, the students could draw the conclusion based the problems [5].

The students’ answer show that they were failed to analyze and evaluate the statements in proving theorem. It is shown that the students have difficulty in process skill phase [14][15]. Apart from the
various kinds of answers and thought processes in solving HOTS problems, the students showed that they were trying to solve them. Besides that, students also expressed positive attitude [4]. It can be stated that the students were motivated to solve HOTS problems.

5. Conclusion

Based on this research, it is known that HOTS questions consist of several non-routine questions or problems which demands students to analyse, evaluate, and create but the students are not used to solving it. Students with low-level thinking skills cannot evaluate a statement because they have not been able to determine the truth value of a statement. In addition, students are still trying to create even though it is not original so that they couldn’t the extraordinary one. When students have tried to analyses, evaluate, and create a proof, sometimes they are still unable to provide the right reason. They also couldn’t provide various methods and creations. The efforts made by students in solving HOTS questions showed that they were motivated to explore their thinking skills.

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