Star Cluster Formation from Turbulent Clumps. I. The Fast Formation Limit

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Abstract

We investigate the formation and early evolution of star clusters, assuming that they form from a turbulent starless clump of a given mass bounded inside a parent self-gravitating molecular cloud characterized by a particular mass surface density. As a first step, we assume instantaneous star cluster formation and gas expulsion. We draw our initial conditions from observed properties of starless clumps. We follow the early evolution of the clusters up to 20 Myr, investigating the effects of different star formation efficiencies, primordial binary fractions and eccentricities, and primordial mass segregation levels. We investigate clumps with initial masses of $M_{\text{cl}} = 3000 M_\odot$ embedded in ambient cloud environments with mass surface densities $\Sigma_{\text{cloud}} = 0.1$ and 1 g cm$^{-2}$. We show that these models of fast star cluster formation result, in the fiducial case, in clusters that expand rapidly, even considering only the bound members. Clusters formed from higher $\Sigma_{\text{cloud}}$ environments tend to expand more quickly and thus are soon larger than clusters born from lower $\Sigma_{\text{cloud}}$ conditions. To form a young cluster of a given age, stellar mass, and mass surface density, these models need to assume a parent molecular clump that is many times denser, which is unrealistic compared to observed systems. We also show that, in these models, the initial binary properties are only slightly modified by interactions, meaning that the binary properties, e.g., at 20 Myr, are very similar to those at birth. With this study, we set up the foundation for future work, where we will investigate more realistic models of star formation compared to this instantaneous, baseline case.

Key words: galaxies: star clusters: general – galaxies: star formation – methods: numerical

1. Introduction

Most stars tend to form together in clusters (e.g., Gutermuth et al. 2009), which are created from overdense gas clumps typically found in giant molecular clouds (GMCs) (e.g., McKee & Ostriker 2007). Thus, understanding how star clusters form is a direct need to understand how and where star clusters form. It is still a matter of debate whether star cluster formation depends on the properties of the GMC environment. While theoretical studies have taught us the essential physical processes that determine a star cluster’s evolution after the gas is dispersed, the transition from the dense star-forming clump to the star cluster that emerges from the gas is not yet well understood (see, e.g., Banerjee & Kroupa 2015 for a review). In particular, it is debated whether the process is slow, with the clump evolving in a quasi-equilibrium state (Tan et al. 2006; Nakamura & Li 2007), or very rapid, with star cluster formation occurring in just a crossing time of the system (Elmegreen 2000, 2007; Hartmann & Burkert 2007).

There are numerous physical processes potentially involved in such a transition from gas clump to star cluster, including fragmentation of a magnetized, turbulent, and self-gravitating medium into a population of pre-stellar cores; collapse of these cores via rotationally supported disks into single- or multi-star systems; feedback from the forming stars, especially protostellar outflows that can maintain turbulence in the clump (Nakamura & Li 2007, 2014), and, eventually, radiative feedback processes from more massive stars (e.g., Dale et al. 2015); continued infall of gas to the clump; dispersal of clump gas by feedback; and dynamical interactions among the forming and recently formed stars as they orbit in the protocluster potential (e.g., Chatterjee & Tan 2012). This is a complicated, multiscale problem, the full solution of which is beyond current computational capabilities. Thus, approximate models are necessary. By investigating how the outcome of star cluster formation depends on the adopted approximations, we can learn which processes are most important.

Our approach in this paper and subsequent papers in this series is to accurately follow the dynamics of formed stars, including binary properties, via direct $N$-body integrations and to approximate various models for how individual stars are born within the star-forming clump. Our initial conditions are based on the turbulent core/clump model of McKee & Tan (2003, hereafter MT03), which approximates clumps as singular polytropic virialized spheres that are in pressure equilibrium with their surrounding cloud medium. This surrounding cloud is also assumed to be self-gravitating, so its ambient pressure is $P \sim G \Sigma^2$, where $\Sigma$ is the cloud mass surface density—the main variable describing different environmental conditions.

In this first paper, we start with the simplest approximation for star formation, i.e., instantaneous formation of the stellar population from the initial gas clump along with simultaneous, instantaneous expulsion of the remaining gas that is not incorporated into the stars. This approximation has often been adopted by previous $N$-body studies (e.g., Bastian & Goodwin 2006; Parker et al. 2014; Pfalzner et al. 2015). However, in comparison to these previous studies, our work is distinguished by (1) adopting initial conditions that have been explicitly developed for self-gravitating gas clumps (i.e., singular polytropic spheres as approximations for turbulent, magnetized clumps) and (2) following the full evolution of binary systems.

A number of authors have studied the dynamics of binaries in star clusters using numerical models (e.g., Kroupa et al. 1999; Kroupa & Burkert 2001; Parker et al. 2009; Kouwenhoven et al. 2010; Kaczmarek et al. 2011; Parker et al. 2011) focusing on various aspects of their dynamics. In our work, we follow the evolution of binary properties due to stellar interactions and stellar
initial conditions of the models. The observed values of the mass surface density of Galactic clumps and protoclusters are in the range from ~0.03 to 1 g cm$^{-2}$.

In the setup of our initial conditions in this paper, we make several simplifying assumptions as first steps in describing the complexity of star cluster formation: (i) the parent clump is isolated (i.e., no external tidal fields); (ii) the clump is in hydrostatic and virial equilibrium with the structure of a singular polytropic sphere (MT03); (iii) stars are born with the same velocities as their parent gas, so that their velocity dispersion profile is the same as that of the initial gas; (iv) all stars form instantaneously, and the remaining gas is also expelled instantaneously at this time; (v) the star formation efficiency (SFE) is spatially constant, which means that the stars follow the same spatial distribution as the initial gas; (vi) there is no initial spatial or kinematic substructure given to the stars, except that which results from random Poisson sampling; and (vii) following an initial test model of equal-mass stars, a standard Kroupa initial mass function (IMF) for the stars (Kroupa 2001) is adopted, and various binary properties are investigated. It should be remembered that these are starting assumptions and that many of them will be relaxed in subsequent investigations. However, first, the behavior of this simplest, idealized model needs to be understood.

2. Initial Stellar Phase-space Distributions

First, we define the physical and kinematic properties of the pre-cluster clump, i.e., mass, size, density profile, and velocity dispersion profile. Stars are born from this clump and initially follow the same phase-space (position, velocity) distribution. MT03 characterize pre-cluster clumps and pre-stellar cores as singular polytropic spheres in virial and hydrostatic equilibrium. The density profile of such clumps is then

$$\rho_c(r) = \rho_{c,cl}\left(\frac{r}{R_{c,cl}}\right)^{-k_p},$$

where $\rho_{c,cl}$ is the density at the surface of the clump, i.e., at radius $R_{c,cl}$. We adopt $k_p = 1.5$ as a fiducial value, i.e., the same as that of MT03, who made their choice based on observations of clumps reported at the time. No significant difference was found in later measurements performed by Butler & Tan (2012) in IRDCs, where they found $k_p \approx 1.6$. For simplicity and consistency with the previous analysis of MT03, we thus keep $k_p = 1.5$ as our fiducial value. The density at the surface of the clump can be expressed as

$$\rho_{c,cl} = \frac{(3 - k_p)M_{c,cl}}{4\pi R_{c,cl}^4},$$

where $M_{c,cl} = M(r < R_{c,cl})$ is the total mass of the clump.

The radius of a clump in virial and pressure equilibrium with its surroundings, i.e., a larger self-gravitating cloud of a given mass surface density, $\Sigma_{\text{cloud}}$, is (MT03; Tan et al. 2013, hereafter T13)

$$R_{\text{cl}} = 0.50\left(\frac{A}{k_p \phi_{p,cl} \phi_P}\right)^{1/4}\left(\frac{M_{\text{cl}}}{3000 M_\odot}\right)^{1/2} \times \left(\frac{\Sigma_{\text{cloud}}}{1 \text{ g cm}^{-2}}\right)^{-1/2} \text{ pc},$$

where $M_{\text{cl}} = M(r < R_{\text{cl}})$ is the total mass of the clump.
\[ \sigma_{cl}(r) = \frac{\phi_{P,cl} \phi_P}{A k^2 \phi_B} \left( \frac{M_{cl}}{3000 M_\odot} \right)^{1/4} \times \left( \frac{\Sigma_{cloud}}{1 \text{ g cm}^{-2}} \right)^{1/4} \text{ km s}^{-1}, \]
The initial velocity dispersion profile of the stars then follows Equation (7). The individual stellar velocities are then assigned velocities in each of the $x$, $y$, and $z$ directions independently by drawing from a Gaussian centered at zero with width $\sigma(r)$. Note that the mass-averaged velocity dispersion of the clump/cluster is (T13)

$$\sigma_d = \frac{2(3 - k_p)}{8 - 3k_p}\sigma_{d, s} \rightarrow \frac{6}{7}\sigma_{d, s}.$$  

The resulting velocity distribution has the form of a Maxwell–Boltzmann distribution with $\sigma_{d, s}$ and with a one-dimensional velocity dispersion profile as in Equation (7). The properties of the clumps, i.e., the low- and high-$\Sigma$ cases, are summarized in Table 1.

### 2.2. The Primordial Binary Population

Observational evidence shows that about half of the star systems in the field are binaries (e.g., Duquennoy & Mayor 1991; Fischer & Marcy 1992; Mason et al. 1998; Preibisch et al. 1999; Close et al. 2003; Basri & Reiners 2006; Raghavan et al. 2010). Given the densities of star-forming clumps and young star clusters, it is likely that most of these binaries were born together inside individual cores, rather than forming via subsequent dynamical interactions (e.g., Parker & Meyer 2014). However, this is a question that our simulations will be able to address.

Theoretically, a full understanding of binary formation from a collapsing gas core is likely to require a full non-ideal magnetohydrodynamic (MHD) treatment to resolve the formation of the accretion disk and, later, its potential fragmentation. The difficulty of this problem means that, essentially, the statistical properties of primordial binaries are very uncertain, so we will investigate several different choices based on observations.

For most of our simulations that include binaries, we assume a binary system fraction, $f_{\text{bin}} = 0.5$. We adopt a period distribution from the survey (Raghavan et al. 2010) using a log-normal period distribution with a mean of $P = 293.3$ yr and a standard deviation of $\sigma_{\text{pp}} = 2.28$ (with $P$ in days). We use a companion mass ratio distribution of the form $dN/dq \propto q^{0.7}$, based on observations in young star clusters (Reggiani & Meyer 2011). The eccentricity distribution remains less well constrained. While Duquennoy & Mayor (1991) found a thermal distribution, i.e., $f(e) = 2e$ for solar-type stars in the solar neighborhood, a similar, more recent study (Raghavan et al. 2010) found a flat eccentricity distribution for the same kind of stars. However, if binaries form mainly via disk fragmentation, we would expect that they are born with near-circular orbits. In order to measure how much binaries are affected by dynamical interactions in the different models, we adopt initially circular orbits for the eccentricities in our fiducial model. We also investigate cases with initially thermal and uniform distributions of eccentricities.

### 2.3. Overview of the Cluster Models

For each of the low- and high-$\Sigma$ clumps (see Table 1), we set up a stellar cluster as described above, i.e., assuming a constant SFE($r$) and a velocity dispersion profile equal to that of the parent clump. Thus, the initial crossing time (i.e., dynamical time) is defined by the properties of the parent clump to be $t_c = R_d/\sigma_d$, i.e., 0.663 and 0.118 Myr for the $\Sigma_{\text{cloud}} = 0.1$ and 1 g cm$^{-2}$ cases, respectively.

We run 20 realizations for each set of initial conditions, which are summarized in Table 2. The simulations are run for 20 Myr, varying only the random seed between simulations in the same set; this affects the initial positions and velocities of the stars, as well as the IMF sampling.

We construct these sets of simulations starting from the simplest case and moving to the one that defines our fiducial case. We start by using only single-mass particles of $m_i = 1 M_\odot$ with no primordial binaries and with an SFE of 50% in the set equal_mass. Next, we include an IMF, assuming the Kroupa (2001) distribution with a mass range from 0.01 to 100 $M_\odot$, defining the set single_imf, again with no initial binaries. We then include 50% primordial binaries with circular orbits and other properties described in Section 2.2, defining the set binaries_50. The above three simulation sets do not include SE. We define the fiducial simulation set by assuming a Kroupa IMF and an SFE of 50%, with 50% of the stars as primordial binary systems with initial circular orbits and SE included.

Next, we test other choices and parameters of the fiducial model. We test two other eccentricity distributions: a thermal eccentricity distribution, i.e., $f(e) = 2e$ in the set binaries_th, and a uniform eccentricity distribution between 0 and 1 in the set binaries_un. An extreme scenario of mass segregation is tested in the set segregated, in which stars are sorted in descending order of individual stellar mass from the center of the cluster. We also test the extreme case in which all stars are binary systems ($f_{\text{bin}} = 1$) in the set binaries_100.

We also carry out simulations with different SFEs. These simulations only differ from the fiducial set in their SFE; i.e., the average number of stars per simulation in each set increases with the SFE, since we use the same parent clump of $M_d = 3000 M_\odot$. The SFEs investigated are SFE = 10%, 30%, 80%, and 100%, and the sets are named sfe_10, sfe_30, sfe_80, and sfe_100, respectively.

### 2.4. Numerical Methods

We follow the evolution of the star clusters for 20 Myr utilizing the direct N-body integrator NBODY6++ (Wang et al. 2015), which is a GPU/MI-optimized version of the classical and widely used direct integrator NBODY6 (Aarseth 2003). NBODY6++ has implemented special regularizations to accurately follow the evolution of binaries and high-order systems in the cluster and is able to efficiently simulate star clusters with high binary fractions with no loss of accuracy. For cases with SE, we used the recipe included in NBODY6++ based on the analytical models for single and binary SE developed by Hurley et al. (2000, 2002). The code also has implemented artificial velocity kicks to emulate asymmetrical supernova ejections. The magnitudes of the kicks are drawn from a Maxwell distribution with $\sigma = 265$ km s$^{-1}$ following the observations of Hobbs et al. (2005) on pulsar proper motions.

### 3. Results

#### 3.1. Initial Dynamical State of the Clusters

Before performing any simulation from the assumptions described in Section 2, we first derive the initial dynamical state
of the clusters by characterizing their virial ratio, i.e.,

\[ Q_i = -\frac{T_i}{\Omega}, \quad (10) \]

where \( T_i \) is the total kinetic energy of the stars and \( \Omega \) is their total gravitational potential energy. A cluster with \( Q_i < 1 \) is bound, and \( Q_i = 0.5 \) is the value for a state of virial equilibrium. We assume that the gas was expelled immediately after the stars formed; thus, \( \Omega \) only depends on the stars in the cluster, i.e., \( \Omega = \Omega_\ast \). The potential of the stars is then

\[
\Omega_\ast = -\frac{G}{2} \int_0^{R_{cl}} \left[ \frac{M(r < R_{cl})}{r} \right]^2 dr - \frac{G}{2} \int_{R_{cl}}^{\infty} \left( \frac{M_r}{r} \right)^2 dr
\]

\[
= -\left[ \frac{3 - k_\rho}{5 - 2k_\rho} \right] \frac{GM_\ast^2}{R_{cl}}. \quad (11)\]

The kinetic energy of the stars is given by

\[
T_\ast = \frac{3}{2} M_\ast \sigma^2, \quad (13)\]

where \( \sigma \) is the one-dimensional mass-averaged velocity dispersion. Assuming that the stars are born from the gas following the same dispersion profile, \( \sigma \) is related to the velocity dispersion at the surface by Equation (9).

Substituting with Equations (9), (13), and (12) in Equation (10), as well as replacing \( R_{cl} \) and \( \alpha_{cl} \) with their expressions in Equations (3) and (8), respectively, we obtain

\[
Q_i = \frac{3(5 - 2k_\rho)(3 - k_\rho)}{(8 - 3k_\rho)^2(k_\rho - 1)} \frac{1}{\epsilon \phi_\ast}, \quad (14)\]

\[
Q_i \rightarrow \frac{0.51}{\epsilon}, \quad (15)\]

where the arrow shows the relation using the fiducial values for the clump. Values of \( Q_i \) versus SFE are shown for different models in Figure 1.

The dynamical state of the clusters also depends on the presence of magnetic fields in the initial clump, i.e., \( \phi_\ast \). In the absence of magnetic field support (\( \phi_\ast = 1 \)), the velocities needed for virial equilibrium are higher, and stars formed from such gas will have higher values of \( Q_i \); e.g., even in the best-case scenario, with an SFE of 100\%, we have \( Q_i \approx 1.6 \) (and \( \approx 3 \) for an SFE of 50\%). However, in the fiducial case, with approximate equipartition of the energy density from large-scale magnetic fields and turbulence (\( \phi_\ast \approx 2.8 \)), an SFE of about 50\% leads to a cluster that is marginally gravitationally bound (\( Q_i \approx 1 \)). Note that this variation of \( \phi_\ast \) also corresponds to a variation in the virial parameter of the gas clump, \( \alpha_{vir} = 5(\sigma^2 R/(GM)) \), since, for the fiducial case with \( k_\rho = 1.5 \), we have \( \alpha_{vir} = 15/(4\phi_\ast) \rightarrow 1.34 \) (see Appendix A of MT03).

### 3.2. The Bound Stellar Cluster

As discussed in Section 3.1, star clusters born from turbulent clumps bounded by high-pressure ambient environments can start with relatively high velocity dispersions. Their virial ratio after gas expulsion will depend on the global SFE and the contribution of magnetic fields to the support of the parent clump, i.e., \( \phi_\ast \). There will be significant initial mass loss of the stars that are born unbound, occurring on a timescale of a few crossing times. However, the gaseous clump is assumed to have a positive power law for the velocity dispersion with radius (see Equation (9)) and so is more likely to be left with a central gravitationally bound core. In contrast, a relaxed star cluster (e.g., with a Plummer profile) has a velocity dispersion profile that decreases with radius. We thus expect differences in the early evolution of our clusters compared to those modeled with initial Plummer profiles by, e.g.,

![Figure 1. Initial virial ratio \( Q_i \) as a function of SFE from the clump. The solid line shows the relation for our fiducial value, and the dotted lines show the relation for different values of \( \phi_\ast \).](image1)

![Figure 2. Bound mass fraction, \( f_{bound} \), as a function of SFE. Solid lines and points show the results at early times (\( 2t_\ast \)); the dashed lines and open points show the results at the end of the simulations (20 Myr). The values are the mean averages for the simulation sets (each of 20 realizations) with cases shown for \( \Sigma_{cloud} = 0.1 \ g \ cm^{-2} \) (green) and \( 1 \ g \ cm^{-2} \) (red). Error bars show the values between the 25th and 75th percentiles. Dynamical evolution leads to a general decrease of \( f_{bound} \) during the first 20 Myr of these clusters.](image2)
To measure the bound mass fraction, $f_{\text{bound}}$, at each timestep of cluster evolution, we construct the bound entity based on an accurate measure of the mean velocity of the bound stars, and we select all stars with negative total energy in the frame of reference of the bound cluster. The velocity of the bound cluster is not known a priori (although it is expected to be close to the zero velocity of the reference frame); thus, this is solved in an iterative calculation. We start by selecting all stars with negative energy inside the half mass–radius of the full cluster and then iterate until the members between iterations do not change by more than two members. This method, called “snowballing,” is described in Smith et al. (2013b).

Figure 2 shows the bound fractions measured at two crossing times and after 20 Myr for clusters with different SFEs for simulations with a parent clump with $\Sigma_{\text{cloud}} = 0.1$ g cm$^{-2}$. The initially positive radial gradient of the velocity dispersion of the star clusters causes stars in the outer parts to leave first, while the central core can remain bound. The mass fraction of this remnant bound core depends on the initial global SFE. Later dynamical evolution and internal mass loss of its members due to SE leads to a slower decrease in the mass of the bound core over time.

The results shown in Figure 2 are those for the fiducial case, i.e., with $\phi_B = 2.8$. As discussed in Section 3.1, the initial virial ratio of the star clusters depends sensitively on this value: a higher value of $\phi_B$ shifts the trend shown in this figure.

Figure 3. Average Lagrangian radii evolution for different sets of simulations for star clusters born from a parent clump with $\Sigma_{\text{cloud}} = 0.1$ g cm$^{-2}$. We show the Lagrangian radii for the 10%, 20%, 30%, 40%, 50%, 60%, 70%, 80%, and 90% masses (black solid lines). Red dashed lines are the core radii defined in Aarseth (2003). Gray shaded areas represent the regions below the 50%, 95%, and 100% mass–radius of the bound cluster.
upward so that even clusters with SFE = 10% may retain a significant bound core if $\phi_B$ is sufficiently high.

### 3.3. Global Evolution

Here we explore the evolution of the structure of the clusters with time. The fiducial case has an SFE of 50% and $Q_i = 1.02$, slightly above the criterion for global boundedness. Therefore, initial expansion and some initial mass loss is expected. We show the evolution of the Lagrangian radii with time in Figures 3 and 4 for $\Sigma_{\text{cloud}} = 0.1$ and 1 g cm$^{-2}$, respectively, where the values presented are the average of the 20 simulations performed for each set. In each figure, panels (a)–(d) show the effects of gradually adding greater degrees of realism to make the fiducial model. Panels (e)–(h) show the effects of different choices of initial binary properties and degree of initial mass segregation. The figures also show the evolution of the core radius $r_c$ (red dashed lines), which is the density-averaged distance from the density center of the cluster (see Section 15.2 of Aarseth 2003).

As expected, the clusters expand with time. The expansion rate of the outer Lagrangian radii of the clusters, i.e., of the unbound stars, is determined by the initial velocity dispersion of the parent clump, which is higher at higher-mass surface densities. Thus, the star clusters starting from a clump with $\Sigma_{\text{cloud}} = 1$ g cm$^{-2}$ are soon more extended than the clusters forming from clumps with $\Sigma_{\text{cloud}} = 0.1$ g cm$^{-2}$ of the same mass and SFE; i.e., the half mass–radius at 20 Myr of the first group is $\sim 20$ pc, compared to $\sim 10$ pc for the lower-$\Sigma$ case.

Initial expansion of the bound portion of the cluster happens early, within a few crossing times, as the clusters relax to a
virialized state. The later evolution is affected by dynamical interactions between the stars (i.e., mass segregation, evolution of binaries, and dynamical ejection of stars from unstable multiple systems) and mass loss resulting from SE. The relative importance of these effects can be gauged by examining the sequence of panels (a)–(d) in Figures 3 and 4. The later-stage expansion of the bound cluster is negligible in the case of equal-mass stars. The model with an IMF but only single stars...
undergoes mass segregation that leads to noticeable expansion after about 6 Myr in the case of \( \Sigma_{\text{cloud}} = 0.1 \) g cm\(^{-2} \) and after about 1 Myr in the case of \( \Sigma_{\text{cloud}} = 1.0 \) g cm\(^{-2} \).

Note that before adding binaries and SE in our models, the evolution of the star clusters with high \( \Sigma_{\text{cloud}} \) would be exactly the same as those with low \( \Sigma_{\text{cloud}} \) after properly scaling for the initial size and crossing time (see Aarseth & Heggie 1998). However, the characteristic timescales introduced by binaries (e.g., at their typical orbital separation) and SE break this self-similarity.

When binaries are added, their presence leads to another potential source of expansion, since their binding energy couples with the internal energy of the stellar cluster, leading to a change of kinetic energy in each interaction (Heggie 1975; Hills 1975). However, little difference appears when moving from single stars to 50% binaries, even in simulations with the high-\( \Sigma \) initial condition that can suffer more interactions. As we will see in Section 3.4, the initial densities of these models are not high enough and/or do not last long enough to give binaries, on average, the chance to interact significantly with other stars.

The inclusion of SE causes the cluster to expand even more. SE starts becoming important after a few Myr, when the first massive stars lose mass by stellar winds and then explode as supernovae. The supernova explosions may cause stars to be ejected (e.g., as fast runaway stars) by either the destruction of tight binaries, velocity kicks caused by the asymmetrical explosion, or both (see the outer two lines in Figures 3 and 4).

We focus on the ejection of runaway stars in Section 3.6. For now, we see that cluster expansion is increased by this effect and by the fact that the potential well of the cluster is made shallower with the loss of mass through stellar winds and supernovae. However, the loss of runaway stars does not affect the global evolution of the cluster too much, even in the most extreme case with \( f_{\text{bin}} = 1 \). Finally, panels (e)–(h) in these figures show that variations of binary orbital properties, degree of initial mass segregation, or primordial binary fractions have relatively minor effects.

In Figure 5, we compare the evolution of several parameters of the different sets of simulations for clusters born with different initial mass surface densities: \( \Sigma_{\text{cloud}} = 0.1 \) g cm\(^{-2} \) on the left, and \( \Sigma_{\text{cloud}} = 1 \) g cm\(^{-2} \) on the right. In the first row, we show the evolution of the bound mass fraction \( f_{\text{bound}} \). Only for the purposes of this figure, in order to show the timescale on which initially unbound stars leave the cluster, we count all stars inside the 95% radius of the bound cluster as also being bound. In all panels in this figure, the values are the medians of each set of simulations with the parameters given in Table 2. Also shown in the first row is a shaded area representing the loss of mass due to SE for all stars in the fiducial simulations (including unbound stars). The second row shows the evolution of the core radii, \( r_c \), and the half-mass radii, \( r_h \). The third row shows the evolution of the effective number density, \( n_{\text{eff}} \), i.e., the number of systems (a binary is counted as one system) inside the volume defined by \( r_h \). The fourth row shows the evolution of the velocity dispersion measured inside \( r_h \), while the fifth row shows the evolution of the total binary fraction.

It takes about 1.5 \( t_{\text{cr}} \) for initially unbound stars to leave the bound cluster, leading to \( f_{\text{bound}} \) decreasing to about 0.7. The velocity profile of the clusters has a positive slope, i.e., higher speeds in the outskirts; this causes outer stars to be more likely to be unbound, with practically no chance of interacting with others. These stars leave the cluster with a velocity dispersion determined by the parent clump, i.e., \( c_{\text{cl},v} \). We refer to these as unbound stars, distinguishing them from the ejected stars that escape later due to dynamical ejections. After the first \( \sim 1.5 \) \( t_{\text{cr}} \), all initially unbound stars leave the cluster, and later evolution is determined by dynamical interactions and SE.

Simulations with equal-mass stars essentially do not lose further members. With an IMF, mass segregation does lead to some additional mass loss. When including 50% primordial binaries, mass loss at later times is moderately enhanced. Adding in SE, i.e., in the fiducial model, continues this trend, with a final value of \( f_{\text{bound}} \) ≈ 0.5. These trends are mirrored in the expansion of the clusters. Variations of binary orbital properties or primordial binary fractions are seen to have relatively minor effects.

Models with full initial mass segregation show some differences. In the case with \( \Sigma_{\text{cloud}} = 0.1 \) g cm\(^{-2} \), the number densities at the center are initially quite low, and the core of the cluster contracts significantly. After this contraction, the number density is raised in the core, which later expands quite rapidly. Even though the number densities of these clusters are never too high, the few interactions that do occur are enough to expand the cluster, and the evolution of the 50% mass–radius is determined by these interactions.

In general, the star clusters presented here expand considerably regardless of the different parameters of the simulations. The amount of expansion depends on the SFE and the initial cluster density. The top panel of Figure 6 shows the ratio of the half mass–radius at the end of the simulations, \( r_{h,1} \), and the half-radius of the bound cluster. Values are medians over the 20 simulations performed for each measurement using all stars; open circles show measurements using only the bound cluster. Values are medians over the 20 simulations performed for each set, and error bars show the region between the 25th and 75th percentiles.
mass–radius at the start, $r_{h,i}$. The differences between the models with high and low initial densities are explained mainly by the initial velocity dispersion of the parent clump. Stars that are born unbound in the denser case escape with a higher typical velocity than those in the less dense case, causing the differences in the expansion. However, when considering the bound part of the cluster, the actual size of the star clusters at 20 Myr is similar regardless of the initial density, as shown in the bottom panel of Figure 6. Differences between the sizes of the bound clusters arise when the SFEs are low. This is due to the fact that the crossing times of these bound systems are large ($t_{c} \approx 30$ Myr), and they have not yet achieved an equilibrium distribution by 20 Myr. Thus, low-SFE clusters are still in the first phase of their expansion. Regardless of the initial density,
the final size of the bound systems depends mainly on the initial SFE: a low SFE results in a more extended bound system.

### 3.4. The Effects and Evolution of Binaries

Our modeling includes a full treatment of binaries, so we are able to examine their effects and evolution in detail. A binary will be significantly perturbed by an external star (or multiples) if the potential energy of the encounter is similar to that of the initial binary, i.e.,

\[
E_{\text{bin}} = -\frac{G m_1 m_2}{2a} \sim -\frac{G (m_1 + m_2) m_2}{b},
\]

where \( b \) is the closest approach of the perturber of mass \( m_2 \), and \( m_1 \) and \( m_2 \) are the primary and secondary masses of the binary, respectively. Therefore, defining \( \mu \equiv m_1 m_2 / (m_1 + m_2) \) as the reduced mass of the binary, the closest approach needed to affect the binary properties is

\[
b \sim 2a \frac{m_2}{\mu}.
\]

We now estimate the perturbation encounter rate of a binary of a given semimajor axis \( a \) in our model star clusters. We first derive the rate assuming that stars move without significant deflection, then we include the effects of gravitational focusing. If we assume that the cluster has only single stars and binaries, the mean mass per system is \( \langle m_2 \rangle = (1 + f_{\text{bin}}) \langle m_1 \rangle \). If there are higher-order multiples, then \( \langle m_2 \rangle \) will be higher; however, these are not included as initial conditions in our models, and we will see that interactions are typically at a relatively low rate so that such multiples do not form in significant numbers during the dynamical evolution of the clusters.

The mean rate of interactions that are able to modify the properties of a binary, \( \Gamma_b \), is proportional to the cross section defined by \( b \), i.e., \( \pi b^2 \), multiplied by the number density of perturbing systems \( n_s \) and a typical velocity in the cluster, i.e., the one-dimensional velocity dispersion, \( \sigma \). Thus, the interaction rate for binaries with a given semimajor axis \( a \) is

\[
\Gamma_b = 4\pi \left( \frac{\langle m_2 \rangle}{\mu} \right)^2 n_s \sigma.
\]

As we show in Figure 5, the effective density in our model clusters quickly falls from initial values of \( \sim 10^3 \)–\( 10^4 \) stars pc\(^{-3} \) (depending on the initial environment mass surface density) to values similar to 1 star pc\(^{-3} \) at 20 Myr, in our fiducial case. The typical velocity dispersions in the cluster, however, do not vary by much.

We can rewrite Equation (18) for a given binary of semimajor axis \( a \) and reduced mass \( \mu \) in a more convenient way as

\[
\Gamma_{b}(a, \mu) \lesssim 9.67 \times 10^{-3} \left( \frac{n_s}{10^4 \text{pc}^{-3}} \right) \left( \frac{\sigma}{2 \text{ km s}^{-1}} \right) \times \left( \frac{a}{40 \text{ au}} \right)^2 \left( \frac{\langle m_2 \rangle^2}{\mu} \right) \text{ Myr}^{-1}.
\]

The above estimate does not include the effects of gravitational focusing, which will increase the effective cross section of the encounter by the factor \( \mathcal{F} = (b_{\text{eff}}/b)^2 \), where \( b_{\text{eff}} \) is the effective impact parameter that leads to a closest approach \( b \). Treating the binary and the perturbing system as single point masses, conservation of energy and angular momentum and Equation (17) imply

\[
\mathcal{F} = 1 + \frac{G \mu}{a \sigma^2} \left( \frac{m_1 + m_2}{\langle m_2 \rangle^2} + 1 \right),
\]

where a more convenient way to express the last equation is

\[
\mathcal{F} \approx 1 + \left[ 5.55 \left( \frac{\mu}{M_\odot} \right) \left( \frac{a}{40 \text{ au}} \right)^{-1} \left( \frac{\sigma}{2 \text{ km s}^{-1}} \right)^{-2} \times \left( \frac{m_1 + m_2}{\langle m_2 \rangle} + 1 \right) \right].
\]

Then, the effective encounter rate including gravitational-focusing effects is

\[
\Gamma_{b,\text{eff}} = \Gamma_b \mathcal{F}.
\]

The interaction rates of binaries are mainly determined by the number density or perturbing systems, which vary by several orders of magnitude over the evolution of the clusters, while none of the other environmental parameters involved in Equation (22) vary by much. The other crucial parameters that determine the interaction rate of a binary are its own internal parameters—i.e., the internal binding energy \( E_{\text{bin}} \), which determines how resistant the binary is to perturbations—and the total mass of the binary, which determines the strength of the gravitational-focusing effect. These parameters vary by several orders of magnitude between members of the binary population. To give a general picture of the different available interaction rates in the simulations, we show the binary binding energy against the binary mass in Figure 7 for binaries in the set fiducial at the start (black points) and at 20 Myr (red and
green open circles). We show the values of $\Gamma_{b,\text{eff}}$ at the beginning of the simulation as contour lines, where the labels are the corresponding values in units of Myr$^{-1}$.

From Figure 7, we can define the typical binary as one with a total mass of $m_1 + m_2 \approx 0.2 \, M_\odot$ and $E_{\text{bin}} \approx 1 \, M_\odot \, \text{km}^2 \, \text{s}^{-2}$. Such a binary has an interaction rate of $\Gamma_{b,\text{eff}} \approx 0.01 \, \text{Myr}^{-1}$ in

---

**Figure 9.** Evolution of the radial location of the 10 most massive stars in example simulations drawn from the model sets that have an IMF and SFE of 50%: (a) `single_imf`, (b) `binaries_50`, (c) `fiducial`, (d) `binaries_un`, (e) `binaries_th`, (f) `segregated`, and (g) `binaries_100`. Lines show stellar distances to the density center of the cluster, and colors show the mass of the individual star. Each line represents a single star, and, in the case of a binary, there is a thinner line inside the thicker line representing the trajectory of the companion, but only if the companion is also part of the 10 most massive stars; i.e., no more than 10 lines are shown in each panel. The shaded area is the region inside the half mass-radius of the cluster measured with respect to the initial mass.
the low-\(\Sigma\) case and \(\Gamma_{b, \text{eff}} \approx 0.2 \, \text{Myr}^{-1}\) in the high-\(\Sigma\) case. These interaction rates are quite small and will fall quickly as the cluster expands. Massive binaries have higher interaction rates, as they attract other systems more efficiently; however, their binding energies are high, and their effective impact parameters become very small. Only binaries with the smallest binding energies have high enough interaction rates to be able to interchange energy effectively with the cluster. These stars are more likely to be low-mass stars.

There are several factors that determine how many interactions a binary will have during the simulations. If the binary is indeed perturbed, its binding energy will change and thus also its interaction rate. The environment may vary because of several factors—e.g., the expansion of the cluster, mass segregation, and binary fraction—and therefore it is quite complex to estimate the number of interactions a binary will have. However, we can use the results of our simulations and the initial \(\Gamma_{b, \text{eff}}\) to calculate the probability that a binary will suffer at least one important interaction during the simulation. To do so, we measure the binding energy of all binaries at \(t = 0\). At the end of the simulation, we calculate the binding energy of the binaries that have not been disrupted and compare it with their values at the start. We define “perturbed binaries” as those with a fractional change in energy of 1\%.

We also measure \(\Gamma_{b, \text{eff}}\) according to Equation (22), assuming the density and velocity values measured inside the half mass–radius of the cluster. Figure 8 shows the resulting histogram, where the value of each bin has been normalized by the total amount of (undisrupted) binaries in each bin. We constructed the histograms shown in Figure 8 by collecting all binaries from the 20 simulations performed for the set fiducial. We can see the correlation between the initial \(\Gamma_{b, \text{eff}}\) and the probability of being perturbed. There is an offset in the relation for the different initial densities. For a given initial \(\Gamma_{b, \text{eff}}\), the probability of suffering an encounter is higher in the low-\(\Sigma\) clusters. This seems counterintuitive; however, the rapid initial expansion of these clusters causes the initial \(\Gamma_{b, \text{eff}}\) to be less representative, as it does not last long (see the number density evolution in Figure 5 for \(t < 0.3 \, \text{Myr}\)).

From all the perturbed binaries, we have also highlighted the cases in which the eccentricities suffered some modification (\(\Delta e > 0.01\)), and we display this probability as the shaded areas in Figure 8. The probability of modifying the initial eccentricity appears to increase linearly with \(\Gamma_{b, \text{eff}}\) at first, but then it remains flat for higher binary interaction rates. Even stars with a high initial \(\Gamma_{b, \text{eff}}\) have only an \(\approx 30\%\) chance of modifying their initial circular orbits (\(e = 0\)) into a slightly more eccentric orbit with \(e = 0.01\). We have measured only a few rare cases in which the initial eccentricity increases by a significant factor.

Even though binaries with high \(\Gamma_{b, \text{eff}}\) have a greater chance to interact and exchange energy with the cluster, these systems are very rare, as can be seen by the thin black lines of Figure 8, which show the total fraction of binaries in each \(\Gamma_{b, \text{eff}}\) bin.

The variations in these effects between our considered models is small. The models with \(f_{\text{bin}} = 1\) have a value of \(\Gamma_{b}\) that is a factor of 1.33 higher than that of the simulations with \(f_{\text{bin}} = 0.5\) because of the effect on the number density of perturbing systems (\(n \propto (1 + f_{\text{bin}})^{-1}\)), and this difference becomes smaller when considering the effects of gravitational focusing. Models with initial mass segregation have central number densities about 10 times lower than those in the fiducial case, but they increase as the cluster evolves, until a point at which the few binary interactions that happen cause the cluster to expand relatively quickly. Note that most of the variables shown in Equation (19) tend to increase toward the cluster center. Especially in the case of the mass-segregated cluster, the mean mass per system is higher, which, according to Equation (19), is one of the most important parameters, since it strongly affects gravitational focusing. However, as the interactions in the center become important, the cluster expands faster; therefore, it is not possible to maintain high number densities.

If we assume that binaries are born as we have modeled them, i.e., with initially circular orbits, then to have significant modification of the orbits, such as eccentricities, requires a longer high-density phase, e.g., of several Myr. However, a longer timescale of cluster formation should not only keep the higher densities longer, it should also give more time for mass segregation, which can also boost interaction rates.

### 3.5. The Effect of Stellar Evolution

One important feature that can affect the evolution of cluster dynamics is SE, especially mass loss from winds and supernovae. En route to constructing our fiducial model set, we consider two sets with no SE, i.e., single_imf and binaries_50, the

Figure 10. Number of strong dynamical ejections (see the text) per logarithmic time interval per cluster simulation for the set fiducial compared with the set without SE, binaries_50, for the low-\(\Sigma\) clump (left) and high-\(\Sigma\) clump (right). The shaded histograms show the ejections caused by supernova explosions. The bottom panels show the evolution of the average cluster core radii.
latter only differing from the fiducial set by having SE turned off. We also show the total mass with respect to the initial as the shaded areas in the top panels of Figure 5, with a decrease by about 12% on average caused entirely by SE effects. Bound fractions at the end of the simulation in the fiducial set are \(\approx 0.55\), compared to \(\approx 0.62\) in the binaries_50 runs. Thus, we see that the decrease in the bound mass fraction can, in fact, be explained entirely by the SE mass loss, rather than a significantly increased tendency for individual stellar members to be lost from the clusters.

Another way of losing mass from the cluster is due to the sudden ejection of the members of a binary system caused by a supernova explosion. After the supernova explodes, the binding energy suddenly drops, and the system may be broken (Zwicky 1957; Blaauw 1961). The stars, both the remnant of core collapse and the secondary star of the binary, may be ejected from the cluster as runaway stars. In this case, it is expected that the models with 100% binaries (binaries_100) will experience a higher loss of members, since all supernovae occur in binary systems. However, this has only a modest effect on the bound mass fraction, as shown in Figure 5. The orbital velocity of a 10 \(M_\odot\) star in a typical binary is \(\approx 0.3\) km s\(^{-1}\) if it is in the peak of the period distribution, and it can vary from \(\approx 0.001\) to 50 km s\(^{-1}\) if we move one \(\sigma\) from the mean period. We find in our simulations that the mean escape velocity of the bound cluster varies from \(\approx 7\) to 0.6 km s\(^{-1}\) over the course of the simulation. Thus, it is not certain that a binary star will be ejected from the cluster due to binary disruption. However, we have also included velocity kicks due to asymmetric supernova explosions, with typical values of \(\approx 100\) km s\(^{-1}\). This effect is the main factor responsible for ejections of the remnants of supernova explosions. The concomitant ejection of the secondaries will depend on the binary properties at the time of the supernova explosion, which for the models presented here depends mostly on primordial binary properties.

Figure 9 shows the radial trajectories of the 10 most massive stars in example simulations drawn from the sets of investigated models. Moving from panels (a) to (b), we see the effects of primordial binaries increasing the likelihood of dynamical ejection from an unstable multiple. Then, from panels (b) to (c), we see the effects of SE, especially the ejection of stars after supernova explosions. These types of ejections are more common than fast ejections resulting from the decay of unstable multiples. Varying the initial eccentricity distribution has only minor effects compared to the fiducial model. The fully mass-segregated case leads to the most extreme concentration of the 10 most massive stars in the core of the cluster. Having 100% binaries may also lead to a greater concentration of those massive stars that are bound in the cluster to its core. However, note that there is a large degree of variation caused by stochastic sampling of the IMF in the examples shown in Figure 9.

### 3.6. Ejected Stars and Kinematic Structure

We classify the stars into three main groups: (1) unbound stars, which are born unbound from the cluster because of the initial conditions, i.e., because of the loss of gravitational potential and confining pressure due to gas expulsion; (2) bound stars, which are the stars that are still bound at the end of the simulation; and (3) ejected stars, which become unbound during cluster evolution. Among the ejected stars, we identify three different mechanisms that can lead to ejection: (A) supernova ejection, either by the kick received to the core collapse remnant and/or by the disruption of a binary that contained the supernova progenitor (see Section 3.5); (B) dynamical ejection, due to the decay of unstable triple or higher-order multiple systems or to slingshot super-elastic encounters, which leave behind a more tightly bound binary or multiple; and (C) gentle ejection, where a star finds itself unbound as a result of the global evolution of the cluster potential.

In order to obtain detailed information about the ejection events, we identify bound members in the simulations snapshot by snapshot, recording information about the stars the first time they appear unbound and comparing with the previous output time. We also record their positions and velocities at the end of the simulation.

We are especially interested in “strong” dynamical ejections that lead to relatively fast ejection velocities from the cluster, and we identify such stars as having \(\Delta T_e / \Delta t \gtrsim 2\) from the previous time output. Such stars will be easier to identify in proper-motion studies of young clusters.

Figure 10 shows the number of such strong ejection events, including supernova ejections, per cluster per logarithmic time.
interval. We compare simulations with (fiducial) and without (binaries_50) SE for the two different initial $\Sigma$ cases.

As expected, a high initial $\Sigma_{\text{cloud}}$ leads to bound clusters with smaller core radii and thus results in a larger rate of strong dynamical ejections than in the low-$\Sigma$ case. Another difference is the number of ejections after SE becomes relevant. The number of massive stars in the cluster decreases significantly after the first supernova explosions and resulting ejections. Massive stars are likely to be near the cluster center; therefore, supernova explosions lead to a drop in the central density of the cluster that has a direct effect on the subsequent number of dynamical ejections, as can be seen in the anticorrelation with core radius. Cases without SE show a constant decrease in their rate of dynamical ejection events (i.e., a flat distribution in equally logarithmically spaced time bins), while in cases with SE, the decrease becomes steeper after the first supernova.

We also have information about the velocities right after ejection ($v_i$) and at the end of the simulation ($v_\infty$). We show the corresponding distributions in Figure 11 with shaded areas for $v_i$ and solid lines for $v_\infty$. As expected, low-velocity stars show a more significant relative decrease in their velocities due to their transit out of the cluster potential. We also notice a high-velocity tail of stars appearing in the fiducial case with SE; these are the result of the dynamical ejection of massive stars, which later explode as supernovae resulting in a secondary kick for their remnants and any binary companions. Such a two-step ejection scenario has been proposed by Pfamm-Altenburg & Kroupa (2010); see also Gvaramadze et al. (2008) and has been argued to explain some O-type runaway stars and remnants with no apparent origin cluster (an alternative could be isolated massive star formation), as well as the rare observed cases of hyperfast runaways with velocities above 1000 km s$^{-1}$ (e.g., as

Figure 12. Normalized velocity vs. mass (left panels) and vs. distance (right panels) for all stars in the fiducial case for simulations with $\Sigma_{\text{cloud}} = 0.1$ g cm$^{-2}$ (top) and $\Sigma_{\text{cloud}} = 1$ g cm$^{-2}$ (bottom). Stars are separated into three groups: bound stars (black), stars born unbound (green), and ejected stars (red). Different symbols indicate whether the star is a single (filled circles) or a binary (filled stars). Velocity values are normalized by the mass average velocity dispersion of the parent clump (see Table 1). Small top and side panels show the PDFs considering all stars in the set (gray shaded area) and the fraction of the PDFs that correspond to each group of stars (lines). The escape velocity from the stellar cluster at its surface at the start of the simulation is shown by a green dashed line. In order to show some of the structure hiding in the cloud of points, contours that contain 90% of the stars in each set are shown in a lighter color, i.e., white for bound, light green for unbound, and light red for ejected.
Ejected stars have a velocity PDF that peaks between those of the unbound and bound stars. This is because the escape velocity decreases with time as the cluster expands and loses members. As discussed earlier, a cluster born in a denser state expands more quickly, and, at 20 Myr, its half mass–radius is about 10 times larger than that of the same cluster born in a lower density state. This leads to a larger relative decrease in the mean escape speed of the cluster over the 20 Myr of evolution followed by these simulations, compared to the low-$\Sigma$ case. This, in turn, causes a broadening of the ejected star velocity PDF. Otherwise, the widths of the velocity PDFs do not vary much between simulation sets.

Bound and unbound stars are more clearly distinguished in the velocity versus radial distance diagram. Bound stars have, in general, higher velocities at the center of the cluster and lower velocities at the outskirts. They thus populate different areas from the unbound and ejected stars: the higher velocities of these stars carry them further from the cluster, and the distance from the cluster is modulated also by the time when they were ejected. Supernova-induced velocity kicks also lead to the modification of a small fraction of stars in this diagram. The group of neutron stars seen in the velocity-mass diagram is another manifestation of such effects. Their velocities are a direct result of the assumed Maxwellian distribution for supernova-induced kick velocities with $\sigma = 265$ km s$^{-1}$.

Figure 13 shows the IMF of the three different classes of stars: bound, unbound, and ejected. The IMF of the unbound group (green histogram) mirrors the assumed primordial distribution; stars from all masses are initially randomly distributed and are equally likely to be born unbound. The initially bound cluster shows the same pattern; however, as the cluster evolves and stars are ejected, almost all of the massive stars are lost, resulting in the black histogram shown in Figure 13. Thus, the IMF of the ejected group (red histogram) shows a clear signature of being top-heavy—mostly a consequence of SE. Eventually, a large majority of the stars that are able to explode as supernovae are ejected from the cluster. When comparing with other simulation sets (see Figure 17), we see that, even before including SE, the IMF of the ejected stars is already top heavy, which is a result only of dynamical ejections. However, this effect is not strong enough to significantly change the shape of the bound cluster IMF.

Further variations from the set fiducial do not change these results significantly, with the exception of the segregated sets. Initial extreme mass segregation causes a very different evolution in the three groups of stars. Stars born unbound are preferentially the lowest-mass stars; therefore, the initially bound clusters have top-heavy IMFs. Later evolution causes the cluster to lose its massive stars. However, such extreme mass segregation is a very idealized model that is not expected to be a very realistic description of observed clusters.

### 3.7. Radial Structure

We now summarize the evolution of various radial distributions of stellar properties in the clusters, which, in their projected forms, are one of the most direct observables of real systems (see Figure 14 for results for the fiducial set of simulations and Figures 18 and 19 in the Appendix for the other sets). For the low- and high-$\Sigma_{\text{cloud}}$ cases, we show the radial profiles for volume density (panels (a) and (b)), projected mass surface density (panels (c) and (d)), projected one-dimensional velocity dispersion $\sigma_{\text{D}}$ (panels (e) and (f)), and projected mean stellar mass $\langle m_i \rangle$ (panels (g) and (h)).
Figure 14. Radial profiles of the fiducial simulations at 0 Myr (black), 1 Myr (red), 3 Myr (blue), 10 Myr (green), and 20 Myr (brown), with results for the low-$\Sigma_{\text{cloud}}$ shown in the left column and the high-$\Sigma_{\text{cloud}}$ in the right column. Dashed lines and circles show the mean values of the 20 realizations for the bound clusters, while solid lines show these averages for the total stellar population. Panels (a) and (b): stellar volume density radial profile as a function of spherical radial coordinates from cluster center. Panels (c) and (d): mass surface density profile as a function of projected radial coordinates from cluster center. Panels (e) and (f): one-dimensional velocity dispersion profile as a function of projected radius. Panels (g) and (h): average mass per system profile as a function of projected radius. Note that in panels (e) and (f), $\sigma_{1D}$ for all stars has been omitted because of the large variations caused by runaway stars. Note also that binary stars are treated as unresolved systems; i.e., we use their combined mass, position, and velocity to construct each profile.
These figures show the expansion of the clusters and the flattening of the initial power-law density profile, i.e., development of a constant density core. Stars born unbound separate from the cluster as it evolves and appear as an excess “halo” around the bound cluster.

As expected, the velocity dispersion profiles of the bound stars show a general trend of evolving toward smaller values as the clusters expand. This can also be seen in Figure 5 with a similar evolution for all the different simulation sets. The velocity profiles are relatively flat but with a modest tendency to decrease in the outer regions.

Panels (g) and (h) of Figure 14 show the average stellar mass per system; i.e., the masses of binary stars are combined. By this metric, we do not see significant signatures of mass segregation developing in the clusters. At later times, SE, i.e., wind mass loss and supernovae, acts to remove massive stars. Stochastic effects due to IMF sampling are still noticeable, even when averaging over 20 clusters.

The most extreme primordially segregated case we considered (set segregated) is able to maintain its mass segregation for much of the 20 Myr evolution (note that stars born unbound are mostly low-mass stars), but it becomes less prominent in the high-\(\Sigma\) case, since massive stars are ejected more efficiently by dynamical interactions. After 3 Myr, massive stars start to be lost due to SE (supernova) effects.

4. Discussion and Conclusions

We have presented a first modeling of the dynamical evolution of star clusters forming with initial conditions prescribed by the turbulent clump model of McKee & Tan (2003). These initial conditions involve idealized descriptions of star-forming protocluster clumps as singular polytropic spheres in virial equilibrium (including effects of large-scale magnetic field support) and pressure equilibrium with a surrounding cloud medium (i.e., the clump radius is set by truncation of the polytropic sphere where the local clump pressure matches the ambient cloud pressure, with the latter assumed to be due to the self-gravitating weight of the larger-scale cloud of a given mass surface density, \(\Sigma_{\text{cloud}}\)). In this first paper, we have assumed, for simplicity, that star clusters are formed instantaneously with a spatially uniform SFE. Subsequent papers will build realism into this model, in particular allowing for the effects of gradual star formation.

The first consequence of the above assumptions is that stars follow the spatial and kinematic distributions of the parent clump, which sets the main difference between this and previous related studies in which velocity profiles of the star clusters are constructed either using isotropic velocity distributions, using the methods described by Aarseth et al. (1974) (e.g., Goodwin & Bastian 2006), or assuming that the stars are in equilibrium with their natal gas at the onset of gas expulsion (e.g., Baumgardt & Kroupa 2007). A system in equilibrium has a velocity dispersion profile that decreases with radius. The turbulent clump model, however, involves larger velocity dispersions at larger scales, so stars born on the outskirts of the clump are less likely to remain bound to the cluster. Conversely, these models contain a central, relatively tightly bound central region.

One crucial parameter that determines the amount of retained mass after the gas is expelled in this model is the contribution of magnetic fields to the support of the parent clump. Without magnetic field support, the virial ratios at the onset of gas expulsion are relatively high even for an SFE of 100%. We can infer from our results that, without magnetic fields, a 100% SFE would result in clusters that retain only \(\sim 40\%\) of the stars (see Figures 1 and 2). However, the models presented here are the worst-case scenario in terms of cluster survivability. It has been shown that gradual gas expulsion increases the amount of stellar mass retained (e.g., Mathieu 1983; Baumgardt & Kroupa 2007; Smith et al. 2013a). Another factor is the central star–to–gas mass ratio at the onset of gas expulsion (e.g., Kroupijsen et al. 2012). Our models have assumed a spatially uniform SFE, but the local SFE may be raised either by dynamical cluster relaxation before gas expulsion (Smith et al. 2011; Farias et al. 2015) or by the star formation process itself, which has been argued to be faster and globally more efficient in the densest regions of the clump (Kroupijsen 2012; Kroupijsen et al. 2012; Parmentier & Pfalzner 2013).

The fiducial clusters that we have simulated show rapid expansion, even just considering their bound members. For example, the half-mass radii show dramatic (factors of several) expansion after 1 Myr, especially in the \(\Sigma_{\text{cloud}} = 1 \text{ g cm}^{-2}\) case. Thus, for these models to explain observed young star clusters of a given age, mass, and mass surface density (see, e.g., Figure 1 of Tan et al. 2014) would require initial clumps that have mass surface densities at least 10 times greater. There is limited evidence for such dense starless clumps (Tan et al. 2014; see also Walker et al. 2016). This may indicate that some aspect of the model needs to be modified, such as the assumption of instantaneous star formation and gas expulsion.

Another feature of our work has been the full treatment of binaries, given assumed primordial binary properties. The processing of these binaries during the early phases of the dynamical evolution of star clusters can be a diagnostic of the process, i.e., by comparing their properties with observed field star and embedded cluster binary properties. We have also seen that binaries affect some aspects of the dynamical evolution of the cluster, in particular by enhancing the rate of dynamical ejection of stars. We consider that our main results on this topic so far are to show the relative importance of binaries in our model clusters depending on their input assumptions. In the clusters that we have investigated in this paper, there has been relatively little processing of the average initial binary properties, given the fairly rapid expansion of the clusters from their initial dynamical states. These results provide a baseline for comparison of future models of gradual star cluster formation.

Since we have a full treatment of binaries and SE, we are also able to make predictions for the properties of ejected stars, including via dynamical ejection from unstable triples and higher-order multiples and from supernova explosions. The models presented here are only able to reproduce high-velocity runaway stars by supernova kicks. Only the densest initial conditions lead to dynamical ejection of stars at speeds >100 km s\(^{-1}\) (but less than one per cluster), and we have not obtained dynamical ejection of any massive stars at speeds >20 km s\(^{-1}\). This probably again indicates a need for more gradual models of star formation that retain a dense cluster core for a larger number of crossing times.

In the context of our presented models that apply in the limit of fast, i.e., “instantaneous,” star cluster formation, our main conclusions are:

1. Magnetic fields that partially support the parent clump against collapse are a key factor that determines the initial velocity and bound mass of the newborn cluster. Star clusters born from clumps with no magnetic field support would need a very high SFE (>80%) to retain a significant part of the stellar mass after the gas is expelled.
2. Regardless of the different initial surface densities, the mass of the bound cluster that emerges from its natal gas is determined by the SFE. However, the expansion rate of the unbound stellar population is determined by the typical internal velocity of the parent clump. This means that clusters born in high-mass surface density environments will produce unbound populations that expand more rapidly than those produced from clusters formed in low-mass surface density environments.

3. The rapid early expansion of the bound clusters implies that at least $\sim 10 \times$ higher mass surface density starless clumps are needed to explain observed young ($\sim 1$ Myr old) clusters that have stellar mass surface densities $\gtrsim 0.1 \, \text{g cm}^{-2}$, but there is limited evidence for such clumps. This may indicate a need for more gradual models of star cluster formation.

4. The low interaction rates of binaries in our simulated clusters lead to only minor modifications of the average initial binary properties. If star cluster formation is rapid, then the present-day observed binary properties have not changed much from their primordial distributions, except for those effects induced by SE.

5. Based on velocity–position diagrams, it is possible to distinguish the bound cluster from the unbound stars, and we expect that this is made easier if gas is expelled quickly. Such diagrams also give us information on the initial properties of the parent clump, such as the initial escape velocity from the cluster.

6. High-velocity runaway stars in the clusters simulated so far are mostly due to supernova explosions that disrupt binaries. Such statistics likely indicate, again, that star cluster formation does not proceed in such a rapid manner.

It remains to be determined how the above metrics will vary for models of star clusters forming from turbulent clumps that involve gradual formation of stars and gradual gas expulsion. In addition, correlated spatial and kinematic substructures associated with turbulence and potential infalling accretion flows need to be incorporated into these models. Variations in global geometry, e.g., elongation of the clump into more filamentary configurations, also need to be explored. A wider range of initial conditions of clump properties and assumed star formation efficiencies, primordial mass segregation, and primordial binary properties needs to be investigated for this model family. By an eventual comparison of model results with observed kinematic properties of young clusters, we hope to constrain star and star cluster formation theories.

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**Appendix**

**Ancillary Results for the Full Set of Simulations**

In this Appendix we provide the results for the set of simulations with SFE = 50%, but that differ from the fiducial case in some other respects (see Table 2). Figures 15 and 16 shows the mass/position–velocity diagrams for the low and high $\Sigma$ cases, respectively, analogous to Figure 12. Figure 17 shows the corresponding IMFs for both $\Sigma$ cases constructed in the same way as in Figure 13. Figures 18 and 19 show the corresponding radial profiles comparing both $\Sigma$ cases in the same way as in Figure 14.
Figure 17. Same as Figure 13, but for simulations with $\Sigma_{\text{cloud}} = 0.1$ g cm$^{-2}$ (left set of panels) and $\Sigma_{\text{cloud}} = 1$ g cm$^{-2}$ (right set of panels). Labeled panels show the IMFs for the simulation sets: (a) single_imf, (b) binaries_50, (c) binaries_un, (d) binaries_th, (e) segregated, and (f) binaries_100.
Figure 18. Same as Figure 14, but showing radial profiles for sets equal_mass (top), single_imf (bottom left), and binaries_50 (bottom right), all with no SE.
Figure 19. Same as Figure 14, but for simulations with SE turned on. The top panels show simulations with different eccentricity distributions, binaries_un (top left) and binaries_th (top right), and the bottom panels show the most extreme cases: the set with primordial mass segregation, segregated (bottom left), and the case with 100 binaries, binaries_100 (bottom right).
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