On the morphology of accretion flows with small, non-zero specific angular momentum

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ABSTRACT
The morphology of adiabatic accretion flows with small non-zero specific angular momentum has been investigated in the axisymmetric and non-viscous limit. For an initial state characterized by a Bondi flow with the specific angular momentum distributed with respect to polar angle, a travelling shock wave forms which propagates more rapidly in the equatorial plane than in the plane perpendicular to it, resulting in the formation of a hot torus. In cases where the incoming flow is restricted to lie near the equatorial plane, a strong wind forms directed away from this plane with the tendency for the formation of a non-steady shock structure. As the height of the incoming flow is increased, it is found that the resultant wind weakens. The parameter regime which delineates accretion flows characterized by a travelling shock and a nearly standing shock structure is presented.

Key words: accretion, accretion discs – hydrodynamics – methods: numerical.

1 INTRODUCTION
The process of accretion is central to our understanding of many diverse astronomical objects and phenomena. The formation of young stellar objects on the Galactic scale and the underlying cause of the activity of galactic nuclei on the extragalactic scale are just a few such examples. A realistic description of accretion flows is likely to be multidimensional and time-dependent in character, and may involve the interplay between various complex physical processes such as dissipative magnetohydrodynamics and radiation. However, in some idealized cases simple geometry or physics may suffice to facilitate solution by analytical or semi-analytical means. Such problems include spherical accretion in the adiabatic or isothermal approximation, and disc accretion driven by viscous effects.

The classical case of spherical accretion on to a stationary point mass in the hydrodynamical approximation was first considered by Bondi (1952). In this study he showed the existence of several families of solutions, and described their topology. Of importance is the transonic solution which bears his name. The extension of the accretion problem to the two-dimensional case in which the accretor is in relative motion with respect to the surrounding medium was actually investigated earlier in the particle approach by Hoyle & Lyttleton (1939) and Bondi & Hoyle (1944). Later, Ruderman & Spiegel (1971), Wolfson (1977) and Bisnovati-Kogan et al. (1979) re-examined this problem semi-analytically in the hydrodynamical approximation. However, no analytical solution exists in the general case in which the hydrodynamical effects are fully considered. In fact, the recent numerical calculations reveal solutions which are complex and time-dependent. In particular, the flows are found to exhibit a wide range of structures, including shocks, jets and transient accretion discs (see, e.g., Shima et al. 1985, Taam & Fryxell 1988 and Ruffert & Arnett 1994).

Accretion flows with large specific angular momentum have also been the subject of intensive theoretical study, following the seminal analytical work of Shakura & Sunyaev (1973). In the case where matter is viscously driven, recent numerical studies have focused on the formation of a disc wind (Eggum, Coroniti & Katz 1987, 1988) and meridional flows in geometrically thin accretion discs (see Kley & Lin 1992 and Różycka, Bodenheimer & Bell 1994). In such studies viscosity and radiation transfer are essential for a proper description of the disc. On the other hand, if the specific angular momentum of the accreting matter is low, then the flow is, in general, pressure-driven and the disc is geometrically thick. Accretion in this regime was studied in
the pioneering investigations by Hawley, Smarr & Wilson (1984a,b) and Clarke, Karpik & Henriksen (1985), and recently by Molteni, Lanzafame & Chakrabarti (1994), Ryu et al. (1995) and Igumenshchev, Chen & Abramowicz (1996). The appearance of vortices and shock waves are common features in these accretion flows.

The results from such investigations, taken as an aggregate, do not present a clear picture of disc accretion in the geometrically thick disc limit. For example, Hawley et al. (1984b) studied the time-dependent, two-dimensional flows in the vicinity of a black hole in the limit that the specific angular momentum of the accreting matter lies between that of the marginally stable orbit and the marginally bound orbit. In this case, a shock front forms and travels outward in the disc. This result was confirmed in an independent study by Clarke et al. (1985) in the Newtonian approximation. Their results, however, are in direct contrast to the earlier work of Chakrabarti (1990) and Chakrabarti & Molt­teni (1993). Furthermore, both studies demonstrated that a wind emanating from the inner disc region is formed. Further complicating the description of the accretion process is the recent study by Igumenshchev et al. (1996), who investigated similar thick discs, but with viscous effects included. In this case shock formation was absent in the accretion flow, in conflict with the conclusion reached by Molteni et al. (1994).

In this paper we report on the results of two-dimensional hydrodynamical simulations which delineate the parameter regime where accretion flows are characterized by a traveling shock front and a standing shock front in the inner disc. To make comparisons with previous studies more meaningful, we restrict our study to accretion flows with small non-zero specific angular momentum in the adiabatic, non-viscous hydrodynamical approximation. Our goal is to provide an understanding of the various accretion flows in terms of the vertical extent, specific angular momentum of the incoming matter (see Hawley et al. 1984a,b), and the boundary conditions imposed on the flow. In Section 2 we outline the specific approximations underlying our investigation. The initial conditions and the boundary conditions for the suite of problems is outlined in Section 3, and their numerical results are presented in Section 4. Finally, we discuss our results in the context of previous studies in Section 5.

2 FORMULATION

In this investigation it is most convenient to use spherical coordinates \((r, \theta, \phi)\). The accretion disc is assumed to be non-self-gravitating and axisymmetric with the rotation axis coincident with the polar axis \((\theta = 0)\). The flow is assumed to be adiabatic with no local radiative cooling included. The gravitational field of the black hole is described in terms of a Newtonian potential in order to compare with previous studies

\[
\Phi = -\frac{GM}{r}, \tag{2.1}
\]

where \(M\) the mass of the central object.

The governing equations describing the flow in these coordinates can be expressed in the following form. For the equation of continuity,

\[
\frac{\partial \rho}{\partial t} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial r} \left( r^2 \rho v_r \right) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left( \rho v_{r \theta} \sin \theta \right) = 0. \tag{2.2}
\]

The equations of motion take the form

\[
\frac{\partial \rho v_r}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \rho v_r^2 \right) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left( \rho v_{r \theta} v_r \sin \theta \right) = -\frac{\partial \Phi}{\partial r} + \frac{\rho (v_r^2 + v_\theta^2)}{r}, \tag{2.3}
\]

\[
\frac{\partial \rho v_{r \theta}}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \rho v_{r \theta} v_r \right) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left( \rho v_{r \theta}^2 \sin \theta \right) = -\frac{\rho v_r v_{r \theta}}{r} + \frac{\rho v_r^2 \cot \theta}{r}. \tag{2.4}
\]

The energy equation is written as

\[
\frac{\partial \rho e}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \rho e v_r \right) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left( \rho e v_r \sin \theta \right) = -p \text{ div } \mathbf{v}, \tag{2.5}
\]

with the equation of state taken to be

\[
p = (\gamma - 1) \rho e. \tag{2.7}
\]

Here \(\rho, (v_r, v_\theta, v_\phi)\), \(e\), and \(p\) are the density, the velocities, the specific internal energy and the pressure of the flow respectively. In addition, \(e = v_r r \sin \theta\) is the specific angular momentum, and \(\gamma\) is the adiabatic index, assumed to be a constant (\(\gamma = 1.5\) or 5/3).

For definiteness we have fixed \(M = 10 M_\odot\) and \(a (\text{mastro}, \pi/2) = \ell_\phi = 2R_c c\) and have used \(R_c\) and \(R_c/c\) for the units of length and time respectively. Here, \(R_c = 2GM/c^2\), the Schwarzschild radius for a black hole of mass \(M\). In these units, the velocity is expressed in terms of the speed of light, \(c\), and the energy per unit mass in terms of \(c^2\). For variations in \(M\) and \(\ell_\phi\), the results can be scaled by changing the units of length and time to \((\ell_\phi/2R_c c)^2 R_c\) and \((\ell_\phi/2R_c c)^2 R_c/c\) respectively.

The above equations are solved using the explicit second-order Eulerian hydrodynamical PPM method (Woodward & Colella 1984). In the present study, the calculational domain is located within \(r_{in} < r < r_{out}\) and \(0 \leq \theta \leq \pi/2\), where \(r_{in}\) and \(r_{out}\) define the inner and outer boundaries respectively. The numerical grid in the \(\theta\) direction is homogeneous, while in the radial direction the width of the radial zone is given by \(\Delta r = 0.05 \Delta \theta\). We assume axial symmetry with respect to the polar axis \(\theta = 0\), and mirror symmetry with respect to the equatorial plane \(\theta = \pi/2\). At the inner radial boundary, zero derivatives with respect to radius \(r\) are used for variables such as \(v_r\) and \(v_{r \theta}\), and linear extrapolation...
tions are used for \( \rho, v, \) and \( \epsilon \). At the outer boundary, the values of all the variables are fixed in time.

3 INITIAL CONDITIONS

For a polytropic equation of state, \( p = K \rho^\gamma \), where \( K \) is the polytropic constant, the Bernoulli function of the flow becomes

\[
\mathcal{B} = \frac{\gamma}{\gamma - 1} K \rho^\gamma + \frac{v^2}{2} + \frac{H^2}{2r \sin \theta} + \Phi. \tag{3.1}
\]

The initial state of the flow can be constructed under additional assumptions.

3.1 Bondi-type conditions

To construct a Bondi-type condition, we first assume \( v_\theta = 0 \) and \( \ell = 0 \). Then equations (3.1) and (2.2) become, respectively,

\[
\frac{\gamma}{\gamma - 1} K \rho^\gamma + \frac{v^2}{2} - \frac{GM}{r} = \mathcal{B} = \text{constant}, \tag{3.2}
\]

and

\[
4\pi r^2 \rho v_r = -\dot{M}, \tag{3.3}
\]

where \( \dot{M} \) is the mass accretion rate. It is common to assume the sound speed at infinity as \( c_{\infty} \), which is related to the Bernoulli parameter as \( \mathcal{B} = c_{\infty}^2 / (\gamma - 1) \), where the local adiabatic sound speed is \( c_{\infty} = \gamma p_{\infty} \rho_{\infty} = K \rho^\gamma \). With \( c_{\infty} \) one can define the Bondi radius as \( R_B = GM / c_{\infty}^2 \); thus \( \mathcal{B} / c_{\infty}^2 = [1 / 2(\gamma - 1)](R_B / R_\odot) \). For the transonic solution which we choose, specification of only two of the three constants \( \mathcal{B}, M \) and \( \gamma \) is required, because the regularity condition at the sonic point provides a constraint among the constants which gives the maximal accretion rate. For \( \gamma = 5/3 \) the sonic radius is at the origin. To consider a more general case, we use \( \gamma = 1.5 \).

In the case where the specific angular momentum does not vanish, we assume that the initial condition can be described by a superposition of the specific angular momentum on the Bondi solution. Assuming that \( v_\theta = 0 \) and that the Mach number, the polytropic constant and the Bernoulli function of the flow remain unchanged, we take the radial velocity and the density to be determined according to equation (3.2). Two cases for the distribution of specific angular momentum are considered. In the first case

\[
\ell(r, \theta) = \ell_0 (1 - \cos \theta), \tag{3.4}
\]

and the other is described by

\[
\ell(r, \theta) = \ell_0 = \text{constant}. \tag{3.5}
\]

It should be noted that, in the latter case, when \( \theta \) is small, no solution may exist since \( \ell \) does not vanish. Under such a situation, \( v_r \) is set to zero, and a numerically small quantity is assigned to \( \rho \).

We have also considered cases in which the flow is restricted near the equatorial plane. In these cases, a vertical extent of the flow, \( H \), is required. For \( r \cos \theta > H \), the flow is truncated, i.e., the velocities are set to zero, and a numerically small value is assigned to \( \rho \).

3.2 Parallel-type conditions

To compare with the previous study of Ryu et al. (1995), we consider parallel-type initial conditions in which the velocity of the incoming flow is parallel to the equatorial plane,

\[
v_r = -v \sin \theta, \quad v_\theta = -v \cos \theta, \tag{3.6}
\]

where \( v = \sqrt{v_r^2 + v_\theta^2} \). The Mach number of the flow, \( M = \sqrt{v_r^2 + v_\theta^2} / c_{\infty} \), is assumed to be constant and is a free parameter to be specified for a solution. Thus, for a given polytropic constant \( K \), solutions of the flow can be solved from a single equation (3.1). Here we have assumed that \( \mathcal{B} = 0 \) and \( \ell = \ell_0 = \text{constant} \). The vertical height of the flow is \( H \). In addition, we have used \( \gamma = 5/3 \).

3.3 Torus-type conditions

In the case of a static torus, \( v_r = 0 \) and \( v_\theta = 0 \). For a given specific angular momentum distribution, \( \ell = \text{constant} \), equation (3.1) can be solved analytically for the density distribution for a fixed \( \mathcal{B} \), which is a constant here. Specification of the solution is then determined by the polytropic constant \( K \). A general property of such an initial condition is that solutions cannot exist inside the funnel wall which, under the Newtonian potential, is determined by the condition

\[
\frac{\ell^2}{2(r \sin \theta)^2} - \frac{GM}{r} - \mathcal{B} = 0. \tag{3.7}
\]

In our case, the location of the funnel wall on the equatorial plane is simply \( 4R_\odot \) for \( \mathcal{B} = 0 \). It should be noted that, in the Newtonian approximation, the funnel wall is always closed at the inner edge of the wall independent of \( \mathcal{B} \) and \( \ell \neq 0 \). Thus accretion is forbidden for a static torus.

4 NUMERICAL RESULTS

The parameters corresponding to the numerical models are summarized in Table 1. The initial flow type, the grid size, the outer radius, the Mach number at the outer edge, \( \mathcal{M}_\text{out} \), the two constants of \( \gamma \) and \( \mathcal{B} \), and the vertical thickness of the initial incoming flow are listed. In all these models, the inner radius is fixed at \( 3R_\odot \), and the polytropic constant is \( K = 4.5 \times 10^2 \). Also note that the Mach number is defined excluding the rotational velocity. The Mach number is a calculated quantity in all models except in the parallel-type models in which it is a free parameter.

| Model | Initial Flow | \( N_r \times N_\theta \) | \( r_\text{out} \) | \( \mathcal{M}_\text{out} \) | \( \gamma \) | \( B/c^2 \) | \( H \) | \( r_\text{shock} \) |
|-------|--------------|----------------|----------------|----------------|--------|---------|-----|----------|
| 1     | Bondi \( \ell = 0 \) | 40 \times 100 | 149 | \ldots | 1.5 | \ldots | \ldots | \ldots |
| 2     | Torus \( \ell = \ell_0 \) | 40 \times 100 | 149 | \ldots | 1.5 | 10^{-5} | \ldots | \ldots |
| 3     | case 1 | 40 \times 100 | 149 | 3.46 | 1.5 | 10^{-5} | \ldots | \ldots |
| 4     | case 2 | 40 \times 100 | 149 | 3.46 | 1.5 | 10^{-5} | \ldots | \ldots |
| 5     | case 2 | 40 \times 100 | 149 | 3.46 | 1.5 | 10^{-5} | 19 | \ldots |
| 6     | case 3 | 40 \times 60 | 31 | 10 | 5/3 | 0 | 10 | \ldots |
| 7     | case 3 | 40 \times 60 | 31 | 10 | 5/3 | 0 | 8 | 21 |
| 8     | case 3 | 40 \times 60 | 31 | 10 | 5/3 | 0 | 6 | 19 |
| 9     | case 3 | 40 \times 60 | 31 | 10 | 5/3 | 0 | 5 | 7-8 |
| 10    | case 3 | 40 \times 60 | 31 | 10 | 5/3 | 0 | 4 | 5-8 |
| 11    | case 3 | 80 \times 120 | 31 | 10 | 5/3 | 0 | 4 | 5-9 |
The numerical code was tested in two cases in which analytical solutions exist relevant to the proposed study. For the case of spherical Bondi accretion (model 1), the exact solution is used as the initial condition. The flow remained spherical during the entire calculation for each of the models tested (with varying $\Omega$ and $K$). A small perturbation arises at the inner edge due to the imposed boundary condition; however, the perturbation propagated outwards and smoothed out very quickly (on a time-scale much shorter than the total evolution considered). The final state is steady, and the relative errors of the physical parameters are always less than a few per cent.

In the second test, the evolution of a static torus (model 2) was examined. The analytical solution discussed above is used as the initial condition. During the evolution, motions developed near the very surface of the torus; however, the motion in the interior of the torus is very small, with a Mach number (excluding the rotational velocity) near or less than $10^{-4}$. In this case, virtually no accretion occurred.

In the following, three distinct cases characterized by the initial conditions of the flow and the outer boundary conditions imposed are investigated. Specifically, they are described as follows.

Case 1. The incoming flow is supersonic and is spherically distributed with the specific angular momentum of gas varying with the polar angle as $l(r, \theta) = \ell_c(1 - \cos \theta)$.

Case 2. The incoming flow is supersonic and is spherically distributed, but the specific angular momentum of the gas is a constant.

Case 3. The incoming flow is supersonic and is parallel to the equatorial plane. In this case the thickness of the flow is restricted and the specific angular momentum of gas is a constant.

Model 3 for case 1 (see Table 1) is similar to the study first performed by Clarke et al. (1985). Qualitatively, their results that accretion occurs primarily in the polar direction and that a shock wave forms in the inner region near the inner edge which then propagates outwards are confirmed. In the subsonic regions (behind the shock) meridional internal motions develop and the density distribution achieves a nearly steady pattern. The resolution of the present calculation is higher than in Clarke et al. (1985), and hence the circulatory motion is more apparent. A typical flow pattern is illustrated in Fig. 1(a) on a small scale ($r \leq 30R_\odot$) and in Fig. 1(b) on a larger scale ($r \leq 150R_\odot$). Here, the flow vectors correspond to a quantity related to the local mass flow rate, $r^2\rho v$ (excluding the rotational velocity) and the contours correspond to the mass density. At a dimensionless time of 4830 it can be seen in Fig. 1(a) that the flow is primarily radial and directed toward the origin for $\theta < 50^\circ$, and that several circulatory cells are present near the mid-plane of the flow. From Fig. 1(b), it is evident that the shock has propagated outward to $r \sim 105$ along the mid-plane and to $r \sim 90$ in the polar direction. This difference directly reflects the presence of the centrifugal forces acting only in the direction perpendicular to the polar axis. In addition, it can also be seen that there is a small region in the mid-plane ($50 < r < 100$) in which matter is directed away from the plane. To follow the propagation of the shock wave, the Mach number ($c/v_c$) distribution (at the mid-plane) is plotted with respect to the radius at four different time intervals (see Fig. 2). The shock front is well resolved and propagates at an average dimensionless speed of 0.15, 0.075, 0.05 and 0.025 at the four intervals indicated. It is evident that the shock decelerates as a function of time. We note that although the initial conditions adopted for the flow are not identical to that in Clarke et al. (1985), the results reveal that the evolutionary pattern is very nearly the same.

The primary result that an oblique travelling shock wave forms resulting in the development of vorticity behind the shock does not change even when the angular momentum distribution is constant with respect to the polar angle (case 2). An example of the flow morphology is shown for model 4 (see Fig. 3). The flow in the mid-plane is similar to that for model 3, but the flow away from the mid-plane is significantly modified. In particular, there are two main differences in the flow morphology, resulting from the larger amount of angular momentum in the flow. Thus there is no accretion at all due to the greater contribution of the centrifugal force. That is, accretion in the planar direction is absent inside the funnel wall. In this case, a wind is found to form near the funnel wall ($r \cos \theta \sim 60$). However, due to the presence of the incoming flow, the wind is relatively weak in this case. In fact, some of the material turns back to form a vortex-like flow which moves outward as a function of time. The development of a strong wind is favoured, on the other hand, if the incoming flow is artificially truncated above a given vertical height. This tendency is illustrated in model 5, which is characterized by the same parameters as model 4 except that the flow is truncated above $H/r_{out}/8$. It can be seen that the wind develops closer to the mid-plane at $r \cos \theta \sim 10$ and extends to larger distances from the inner edge (see Fig. 4).

The radial speed of the shock differs for each case, reflecting the effect of the pressure of the incoming flow above the mid-plane of the disc. For example, the shock wave moves most slowly in model 5, where there is no incoming flow from above $H \approx 19$. The angular momentum distribution in the incident flow can also affect the speed of shock propagation, since the shock is found to travel faster in model 4 than in model 3. Here the greater centrifugal support associated with the greater angular momentum in the flow acts in the same way to assist the enhancement of the shock propagation speed.

The above simulations show that no standing shock waves are formed in the inner region of the disc. Even in the case of parallel incident flow characterized by a vertical thickness of $H$, standing shock waves may not form. In the following, we examine cases when a standing shock can be formed. Our numerical results show that the transition to this type of solution occurs at $H \approx 10$, or $H/r_{out} \approx 1/3$ (model 6). The flow pattern for model 6 is illustrated in Fig. 5 (at time 512), where it can be seen that the matter immediately behind the shock front is driven out as a wind away from the mid-plane. No standing shock is formed inside $r = 31$, although it is possible that a standing shock exists exterior to this radius. Model 7 corresponds to a boundary condition given by a parallel incident flow characterized by a vertical thickness of $H = 8R_\odot$. The flow pattern at time 4443 is shown in Fig. 6. The sonic surface at times 4443, 2759 and 1615 is represented by the heavy solid, dashed and dotted lines respectively. It can be seen that the shock front is approximately coincident with the sonic surface near $r = 21$, and that it is...
Figure 1. The flow pattern of model sequence 3 (see Table 1) at moment $t = 4830$. Panel (a) corresponds to the inner region of the flow, and panel (b) to the entire region of the flow. In (a) the density ranges from $\log \rho = -9.12$ to $-7.36$ ($A \log \rho = 0.1$), whereas in (b) the density ranges from $\log \rho = -10.52$ to $-7.56$ ($A \log \rho = 0.1$). The vector arrow is defined as $r^2 \rho v$, where $v$ is the velocity excluding $v_\phi$. The maximum amplitude of the vector corresponds to $4 \times 10^{13}$ g s$^{-1}$. Note the development of vortices in the subsonic region behind the shock and the accretion of matter near the polar direction.

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Figure 2. The spatial variation of the Mach number of the flow in the equatorial plane for model 3 for four different epochs. The solid, dotted, short-dashed and long-dashed lines correspond to 472, 1412, 2978 and 4830 respectively. Note that the shock decelerates as it propagates outward.

Figure 3. The flow pattern of model sequence 4 (see Table 1) at moment $t=3104$. The density ranges from $\log \rho = -12$ to $-8.28$ ($\Delta \log \rho = 0.1$). The maximum vector corresponds to $4 \times 10^{13}$ g s$^{-1}$. The development of vorticity in the inner regions is similar to that illustrated in Fig. 1(b) (for model 3). Here no accretion occurs as a result of the centrifugal barrier. An outflowing wind from the inner region is present.
Figure 4. The flow pattern of model sequence 5 (see Table 1) at moment $t=2766$. The density ranges from $\log \rho = -12$ to $-8.77$ ($\Delta \log \rho = 0.1$). The maximum vector corresponds to $4 \times 10^{13} \text{ g s}^{-1}$. In this case, the wind is stronger than that in Fig. 3. In addition, the shock wave moves more slowly.

Figure 5. The flow pattern of model sequence 6 (see Table 1) at moment $t=512$. The density ranges from $\log \rho = -12$ to $-7.22$ ($\Delta \log \rho = 0.1$). The vector arrow corresponds to the momentum density in the flow, with the largest vector equal to $100 \text{ g cm}^{-2} \text{ s}^{-1}$. The heavy solid lines denote a Mach number of unity.
Figure 6. The flow pattern of model sequence 7 (see Table 1) at moment $t = 4443$. The heavy solid, dashed and dotted lines are the sonic surface at moments $t = 4443$, 2759 and 1615 respectively. The shock front, which is approximately perpendicular to the equatorial plane at $r \approx 21$, is quasi-steady. Note that the vector arrow is defined as $\rho v$ with a maximum of 70 g cm$^{-2}$ s$^{-1}$. The density ranges from $\log \rho = -12$ to $-7.03$ ($\Delta \log \rho = 0.1$).

Figure 7. The flow pattern of model sequence 10 (see Table 1) at moment $t = 4161$. The heavy solid, dashed and dotted lines are the sonic surface at moments $t = 4161$, 3604 and 2041 respectively. The shock front which is approximately perpendicular to the equatorial plane at $r \approx 5$–8 is non-steady. Note that the vector arrow is defined as the momentum density with a maximum value of 500 g cm$^{-2}$ s$^{-1}$. The density ranges from $\log \rho = -12.76$ to $-6.62$ ($\Delta \log \rho = 0.1$).
Figure 8. The flow pattern of model sequence 11 (see Table 1) at moment $t = 2288$. The vector arrow is defined as the momentum density with a maximum value of 700 g cm$^{-2}$ s$^{-1}$. The density ranges from log $p = -12.0$ to $-6.64$ ($\Delta \log p = 0.1$). Note that the vortex flow in the inner region is well resolved.

fairly steady over a long time interval. The shock is approximately perpendicular to the equatorial plane and extends in the vertical direction to about $\sim 9R_G$. In comparison with model 6, matter in a larger region behind the shock is involved in the wind outflow.

As the vertical extent of the incident flow is decreased further, the position of the 'standing' shock decreases. It may be seen from Table 1 that the shock location is a sensitive function of $H$. In particular, the radial shock position decreases from $21R_G$ to $\sim 4R_G$ as $H$ is decreased by a factor of 2. The flow for a very thin incident flow characterized by $H = 4R_G$ (model 10) is shown in Fig. 7. The shock is located in the very inner region of the flow at $r_s \sim 5$ and $r \cos \theta \sim 1-2$. It can also be seen that the direction of the bulk flow in the wind region has also been affected by the height of the incident flow. Specifically, the matter is ejected closer to the 'disc' surface at $\theta > 60$ in comparison to model 7 where the wind was not restricted to lie as close to the 'disc' surface (at an angle defined by $\theta > 40$).

We point out that the shock does not reach a steady-state shape and position as the height of the incident flow is decreased. The shock wave in model 10 is found to vary in the interval $5R_G < r < 8R_G$ on a time-scale of 2000, which for a compact object of 10 $M_\odot$ is about 0.2 s. This is similar to the behaviour described by Ryu et al. (1995) for simulations characterized by incident flows of small vertical extent.

The effect of numerical resolution is studied in model 11, where the number of grid zones are doubled in each direction. The flow morphology is illustrated in Fig. 8, where the structure in the inner region exhibits a more well-defined vortex flow. However, it can be seen from Table 1 that the variability of the shock position is similar to that found in model 10, and hence the low resolution of model 10 provides a reliable description of the shock variation.

5 DISCUSSION AND CONCLUSION

We have systematically investigated the structure and evolution of accretion flows characterized by low specific angular momentum in the adiabatic and inviscid approximation. The two-dimensional hydrodynamic simulations indicate that the accretion behaviour is rich and is primarily dependent on the distribution of angular momentum and the vertical extent of the incoming flow. For a specific angular momentum distribution which significantly varies with polar angle, accretion occurs in the polar direction only. Accretion in the equatorial plane is forbidden by the centrifugal barrier in the Newtonian approximation when the central object is located inside the funnel wall (Ryu et al. 1995).

Shock waves are a common feature in these flows. For an incident flow which is approximately spherically distributed, the shock propagates outwards leaving behind a hot torus-like configuration. On the other hand, for incident flows characterized by plane-parallel geometry ($H < 10R_G$ or $H / R_{\text{out}} < 1/3$) a steady standing shock forms. For flows highly concentrated toward the equatorial plane ($H < 5R_G$) the shock exhibits non-steady behaviour.

In contrast to the incident flows described by a nearly spherical distribution, the incident flows characterized by
plane-parallel conditions result in the ejection of matter as a weak wind emanating from the inner disc region. As the flow becomes more highly concentrated toward the mid-plane, the wind increases in strength and is directed away from the disc at larger polar angles.

From this study one can conclude that the travelling shocks are transitory features which eventually terminate at large radii. The flow readjusts in this manner to the imposed initial and outer boundary conditions as it evolves to a quasi-steady hot torus configuration. Standing shock fronts, on the other hand, are not transitory features in the flow. However, the significance of axisymmetric standing shocks in the accretion process may be questioned, since their applicability rests on the requirement that the incident flow is characterized by low specific angular momentum and a restricted spatial vertical extent. These two restrictions are usually not satisfied in most astrophysical applications, since geometrically thin accretion usually corresponds to a more nearly Keplerian flow with a high specific angular momentum. Similarly, low specific angular momentum flows are primarily associated with geometrically thick flows, which do not lead to standing shock structures. Only for geometrically thin supersonic flows where there is insufficient time to maintain a hydrostatic balance in the vertical direction could such structures play a role in the accretion process, e.g., in the case of accretion resulting from tidal disruption of a star close to a massive black hole.

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