A nAttractor Mechanism for nAdS$_2$/nCFT$_1$
Holography

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Ubiquity of AdS$_2$

- **Extremal** black holes are important: they have the smallest possible mass for given charges so they are *ground states*.

- Spherically symmetric asymptotically flat extremal black holes in $D = 4$ have *horizon geometry* AdS$_2 \times S^2$.

- In fact *all* extremal black holes include an AdS$_2$ factor (a theorem).

- This motivates interest in AdS$_2$ *quantum gravity*. 
AdS$_2$/CFT$_1$ Holography?

- AdS$_{d+1}$/CFT$_d$ correspondence is confusing for $d = 1$.

- No finite energy excitations possible in AdS$_2$:
  Their *backreaction spoil asymptotic* AdS$_2$ boundary conditions.

- Also many other (related) unpleasantries.

- So AdS$_2$/CFT$_1$ holography is not yet well understood.
nAdS$_2$/nCFT$_1$ Holography.

- Recently developed version AdS$_2$/CFT$_1$ holography: duality between *nearly* AdS$_2$ geometry and *nearly* CFT$_1$.

- Conformal symmetry is *broken spontaneously* (by boundary conditions) and *broken dynamically* (by an anomaly).

- Interesting nCFT$_1$’s realize the symmetry breaking pattern: SYK,....

- This talk: *gravitational aspects*, especially (nearly) extreme 4D black holes with spherically symmetry.
A Canonical Setting

- 4D $\mathcal{N} = 2$ ungauged SUGRA with $n_V$ vector multiplets.

- The black hole parameters are:
  
  **Mass** $M$ not an independent variable (extremality!)
  
  **Charges** $(p^I, q_I), I = 0, \ldots n_V$
  
  Asymptotic value of complex scalars $z^i_\infty, i = 1, \ldots n_V$.

- **Extreme** black holes in this setting have been studied extensively.
The Extremal Attractor

- A radial flow: the scalars $z^i$ evolve from infinity to the horizon.

- The attractor mechanism: scalar fields at the horizon are independent of the asymptotic value of the scalar fields.

- So the horizon theory is universal: independent of moduli, including the coupling constants,....

- Many other versions of the extremal attractor mechanism: rotation, asymptotically AdS, 5D, multicenter solutions, nonBPS branch, higher derivative corrections, extended black objects, more SUSY, ...

- This talk: an attractor mechanism for nearly extreme black holes. A nAttractor.
Near Extreme Black Holes

• The “near” of $n\text{AdS}_2/n\text{CFT}_1$ appears in **two ways**.

• Black holes only *nearly* extremal so scalars at the horizon depart from their extremal attractor value.

• Also: $n\text{AdS}_2/n\text{CFT}_1$ considers the entire *near* horizon region so scalars are *not constant*.

• The corresponding symmetry breakings introduce **new scale**(s).

• These *scales are fundamental* for $n\text{AdS}_2/n\text{CFT}_1$ holography.
A Scale: the Specific Heat

- The **extremal** black hole entropy is a ground state entropy

\[
S_0 = \frac{4 \pi \ell_2^2}{4 G_4} = \frac{2 \pi}{\kappa_2^2}
\]

There is *no scale, just a large dimensionless number*.

- The **nearly** extreme black hole entropy:

\[
S = S_0 + CT
\]

- The **specific heat** \( C = 2L \) is the *symmetry breaking scale*.

- Literature: is the symmetry breaking scale universal, essentially the AdS\(_2\) scale \( \ell_2 \)?

\[
C \sim \frac{\ell_2}{\kappa_2^2}
\]

This talk: “no”.
• The nAdS$_2$ region introduces many scales that are related to the black hole parameters $(p^I, q_I), z^i_\infty$.

• A nAttractor mechanism: these scales are computed by a generalization of the extremal attractor mechanism

• There is no need to construct and analyze non extremal black hole solutions.
Non-Extreme Black Holes

- General **non-extreme** black hole depends on a single *radial function* $R(r)$:

  $$ds^2_4 = -\frac{r(r - 2m)}{R^2(r)}dt^2 + \frac{R^2(r)}{r(r - 2m)}dr^2 + R^2(r)d\Omega_2^2$$

- There is a horizon at $r = 2m$.

- Entropy and temperature are encoded in the radial function:

  $$S = \frac{\pi R^2(2m)}{G_4}.$$  
  $$T = \frac{m}{2\pi R^2(2m)}.$$  

- The extremal limit is $m \to 0$ **with charges and moduli fixed**.
Near-Extreme Black Holes

- The radial function $R(r)$ depends $r$ and *also on* $M$.
- The entropy depends *only on* $M$.
- Near extremality (change mass $M$ *and* position $r$):
  \[
  \Delta S = \frac{\partial S}{\partial M} \Delta M = \frac{\pi}{G_4} \left( \frac{\partial R^2}{\partial M} \Delta M + \frac{\partial R^2}{\partial r} \Delta r \right)
  \]
- Estimate 1: $\partial_M S \sim T^{-1}$ so $\Delta M \sim T^2$ and $\Delta S \sim T$.
  Estimate 2: $\Delta r \sim m \sim T$.
- So $\partial_M R^2$ is *subleading*: at linear order $\Delta S$ follows from $R^2$ at *extremality* but at a new position $r = 2m$.
- This is a *major simplification* (addressing one aspect of “near”).
The Symmetry Breaking Scale

• Therefore, the *heat capacity* follows from the radial function *of the extremal black hole*:

\[ L = \frac{1}{2} C = \frac{2\pi^2}{G_4} R^2 \left. \frac{\partial R^2}{\partial r} \right|_{\text{hor}}. \]

• The symmetry breaking scale only depends on *moving away from the horizon*.

• Moreover, the dependence is extremely simple: just a radial derivative.
The Extremal Attractor

- Consider the **horizon** attractor: the $\text{AdS}_2 \times S^2$ solution.

- For fixed charges, the $F_{\mu \nu} F^{\mu \nu}$-type terms in the Lagrangian subject the scalars $z^i$ to the **effective potential**

$$V = 2G_4^2 \left( p^I q_I \right) \left( \begin{array}{c} \nu_{IJ} \\ (\nu^{-1})^{IK} \mu_{KJ} \end{array} \right) \left( \begin{array}{c} (\nu^{-1})^{IJ} + \mu_{IK}(\nu^{-1})^{KL} \mu_{LJ} \\ \mu_{IJ}, \nu_{IJ} \text{ are functions of } z^i \text{ determined by special geometry.} \end{array} \right) \right)$$

- The scalars $z^i$ are **constant** on the $\text{AdS}_2 \times S^2$ attractor geometry.

- So the **effective potential** $V$ is extremized: $\partial_i V = 0$

- The extremum value of the potential gives: $R^4(0) = G_4^2 V_{\text{ext}}^2$
Results of Extremization

- Notation for the resulting radial function on $\text{AdS}_2 \times S^2$:
  \[
  R^4(0) = I_4(P^I, Q_I)
  \]

- The *generating* function $I_4$ is *quartic* in the charges.

- Example ($\mathcal{N} = 4$ SUGRA): $I_4(p^I, q_I) = \vec{p}^2 \vec{q}^2 - (\vec{p} \vec{q})^2$.

- The *scalar* values at the horizon are *also encoded in* $I_4$:
  \[
  \begin{pmatrix}
  X^I_{\text{hor}} \\
  F^I_{\text{hor}}
  \end{pmatrix}
  =
  \begin{pmatrix}
  p^I \\
  q_I
  \end{pmatrix}
  - i \left( -\frac{\partial q_I}{\partial p^I} \right) I_4^{1/2}(p^I, q_I)
  \]

  Symplectic section $(X^I, F_I)$ represents scalars projectively:
  \[
  z^i = \frac{X^i}{X^0}.
  \]
Moving Away from the Horizon

- The radial function **at the horizon** depends only on charges.

- It depends on **scalars at infinity** away from the horizon.

- Parametrize scalars at infinity through “charges” $p^I_\infty, q^I_\infty$:

  \[
  \begin{pmatrix}
  \frac{X^I}{F^I}\n  \end{pmatrix}
  =
  \begin{pmatrix}
  p^I_\infty
  
  q^I_\infty
  \end{pmatrix}
  - i
  \begin{pmatrix}
  -\frac{\partial q^I_\infty}{\partial p^I_\infty}
  
  \end{pmatrix}
  I_4^{1/2}(p^I_\infty, q^I_\infty)
  \]

- So: parametrize scalars **at infinity** using the charge/scalar relation determined **at the horizon**.

- The **full attractor flow** has the radial function

  \[
  R^4(r) = I_4(P^I + rp^I_\infty, Q_I + rq^I_\infty)
  \]
The Symmetry Breaking Scale

• The radial derivative of $R^2$ gives the symmetry breaking scale:

$$L = \frac{\pi}{G_4} \left( p^I \frac{\partial}{\partial P^I} + q^I \frac{\partial}{\partial Q_I} \right) I_4(P^I, Q_I).$$

• The radial derivative is equivalent to a derivative in charge space.

• So the **nAttractor behavior** follows from attractor geometry.

• The derivative replaces a charge by its corresponding modulus.

• $I_4$ is quartic in the charges; $L$ is **cubic in charges** and linear in moduli.
A Flow of Many Fields

• “The” breaking scale is (essentially) the radial derivative of $R^2$.

• Other scalar fields *approach* their fixed value $z^i_{\text{hor}}$ at the horizon.

• Their radial derivatives from differentiation in charge space:

$$\frac{dz^i}{dr} = \left(p^I_\infty \frac{\partial}{\partial P^I} + q^I_\infty \frac{\partial}{\partial Q^I}\right) z^i_{\text{hor}}$$

• In general *each scalar field introduces a scale*. 
Explicit Example: The STU Model

• Eg.: \( F = \frac{X^1 X^2 X^3}{X^0} \), simplify charges so \( p^0 = 0, q_1 = q_2 = q_3 = 0 \).

• The effective potential

\[
V = \frac{1}{8 y^1 y^2 y^3} \left( q_0^2 + (p^1 y^2 y^3)^2 + (p^2 y^3 y^1)^2 + (p^3 y^1 y^2)^2 \right).
\]

\( p^i \) are M5-brane numbers, \( q_0 \) is momentum quantum number
\( y^i = -\text{Im} z^i \) are volumes of 4-cycles (in string units).

• The extremal attractor gives scalar fields \( y^i \) at the horizon as

\[
y^i_{\text{hor}} = \sqrt{\frac{q_0}{p^1 p^2 p^3}} p^i
\]

independently of their asymptotic values.

• The extremal entropy

\[
S = 4\pi V_{\text{hor}} = 2\pi \sqrt{q_0 p^1 p^2 p^3}
\]
The nAttractor Mechanism

- Present moduli \textit{at infinity} as “charges” by inverting

\[ y_i^\infty = \sqrt{\frac{q_0^\infty}{p_1^\infty p_2^\infty p_3^\infty}} p_i^\infty \]

- The \textit{symmetry breaking scale}/specific heat:

\[ L = \frac{\pi^2}{G_4} \left( p_i^\infty \frac{\partial}{\partial P_i} + q_0^\infty \frac{\partial}{\partial Q_0} \right) I_4 \]

\[ = \pi^2 q_0 p^1 p^2 p^3 R_{11} \left( \frac{1}{q_0} + \frac{1}{p^1 y_\infty y_\infty^3} + \frac{1}{p^2 y_\infty^3 y_\infty^1} + \frac{1}{p^3 y_\infty^1 y_\infty^2} \right) \]

- It \textit{depends on moduli at infinity}.

- It depends on \textit{non-trivial combinations of charges}.
The Long String Scale

- In the *dilute gas regime* the momentum charge is *small compared to background* charges (M5-branes).

- Then the symmetry breaking scale is

\[ L_{\text{long}} = 2\pi p^1 p^2 p^3 R_{11} \]

- This is the *long string scale* known from microscopic black hole models.

- Physics: low energy excitations “live” on a circle of length \( L_{\text{long}} \) rather than on a circle of radius \( R_{11} \).

- The scales for spatial dependence of the \( y^i \) is similar, but with charges permuted.
Summary

• nAdS$_2$/nCFT$_1$ holography describes the *near* horizon region of *nearly* extreme black holes.

• The *nAttractor* mechanism computes *near* horizon scalars and *near* extreme heat capacity in terms of the *extreme* attractor.

• The nAttractor introduces new parameters: the moduli, i.e. the asymptotic values of the scalars.

• It computes the *intrinsic scales* of nAdS$_2$/nCFT$_1$.

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