Application of linier fuzzy multi-objective programming model in travelling salesman problem

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Abstract. Traveling salesman problem is a problem where a salesman must visit a number of cities, each of which is visited exactly once only, and has to start from and return to the origin city. The objective of this traveling salesman is to determine an optimal travel route by minimizing travel cost or travel distance. If a decision maker wants to determine an optimal route by minimizing travel cost and distance simultaneously, they need more than one destination function, and therefore traveling salesman problem can be formulated as a multi-objective problem. In a traveling salesman problem, salesman make decisions by choosing an optimal route based on an expected measurement. In fact, a number of real problems cannot be expressed as a constraint function and resources in a definite form, so it is necessary to use fuzzy logic which allows us for making decisions based on bias or incorrect data. This paper will discuss a solution to the traveling salesman problem by using a fuzzy multi-objective model, which aims to determine an optimal route of the salesman. Results obtained from data processing of five places to visit by using a Maple 18 software included minimum cost, distance, and time with satisfaction level $\alpha = 0.89$.

1. Introduction

Researchers have discussed extensively about optimization of route selection problems; one of which is traveling salesman problem. This problem is related to an investigation of the optimum and shortest route (closed) regarding situations in $n$ cities, each of which must be visited once before returning to the starting point [1]. Traveling salesman problem is a situation when a salesman has to visit all cities. Series of destination cities should shape a route in which they are visited exactly once only and then return to a place of origin. In general, the objective of traveling salesman problem is to determine an optimal travel route by minimizing travel cost or travel distance.

However, if the decision maker wants to determine an optimal route by minimizing travel cost and travel distances simultaneously, they will need more than one destination function, and therefore the traveling salesman problem can be formulated as a multi-objective problem. Accordingly, each objective function is represented in a different dimension. A conventional programming approach is unlikely to be used to solve this sort of problem.

To optimally solve the multi-objective traveling salesman problem, it needs determination of points related to a solution space of problem solving and minimum values which match to all dimensions.
1.1. Fuzzy Sets

Fuzzy set theory provides a rigorous mathematical framework in which vague conceptual phenomena can be studied properly. It can also be considered as a suitable modelling language for situations where relations, criteria and fuzzy phenomena exist [2].

**Definition 1** [3]

If \( X \) is a collection of objects denoted generally with \( x \), then the set \( \hat{A} \) in \( X \) is a set of ordered pairs.

\[
\hat{A} = \{ (x, \mu_{\hat{A}}(x)) \mid x \in X \}
\]

where,
\( \hat{A} = \) Set notation
\( X = \) Topic variable
\( x = \) Generic element \( X \)
\( \mu_{\hat{A}}(x) = \) membership function or membership class of \( x \).

In a fuzzy set, membership values lie in the range of zero to one. If \( x \) has fuzzy membership value of \( \mu_{\hat{A}}(x) = 0 \), it means that \( x \) is not a member of set \( A \), likewise if \( x \) has membership value of fuzzy \( \mu_{\hat{A}}(x) = 1 \), it means that \( x \) is a full member of set \( A \) [2].

1.2. Operation on Fuzzy Set

Because membership function is an essential component of fuzzy set, its operation is defined by membership function. The operation concept of fuzzy set was first introduced with three basic operators which were introduced, namely slice, join, and complement [4].

**Definition 2** [3]

Membership function of \( \mu_{\mathcal{C}}(x) \) from the slice of \( \tilde{C} = \hat{A} \cap \tilde{B} \) is defined as

\[
\mu_{\mathcal{C}}(x) = \min\{\mu_{\hat{A}}(x), \mu_{\tilde{B}}(x)\}, \ x \in X.
\]

**Definition 3** [3]

Membership function of \( \mu_{\mathcal{B}}(x) \) from the join \( \tilde{D} = \hat{A} \cup \tilde{B} \) is defined as

\[
\mu_{\mathcal{B}}(x) = \max\{\mu_{\hat{A}}(x), \mu_{\tilde{B}}(x)\}, \ x \in X.
\]

**Definition 4** [3]

Membership function from the complement from fuzzy set \( \hat{A}, \mu_{\mathcal{B}}(x) \) is defined as

\[
\mu_{\mathcal{B}}(x) = 1 - \mu_{\hat{A}}(x), \ x \in X.
\]

1.3. Multi-objective Linear Programming

Linear programming is an efficient technique in problem solving to obtain optimal results [5]. Linear programming can be expressed generally in scalar form which is defined in a sum as follows [6].

\[
\min z = \sum_{i=1}^{n} c_{i}x_{i}
\]

Subject to \( \sum_{i=1}^{n} a_{ij}x_{i} \leq b_{i}; \ j = 1,2,\ldots,m \)

\( x_{i} \geq 0 \).

where \( z \) is an objective function, \( x_{i} \) is a decision variable, \( m \) is the number of constraints and \( n \) is the number of decisions. Linear programming, in fact, has a single objective function, and multi-objective linear programming is a development of linear programming [7]. A vector \( X \) such as \( X^{T} = [X_{1}, \ldots, X_{n}] \), minimize objective function of \( z \) with \( n \) variables and \( m \) constraints as follow:

\[
\min z_{i} = \sum_{i=1}^{n} a_{ij}x_{j} \leq b_{i}; \ i = 1,2,\ldots,k
\]

Subject to \( \sum_{i=1}^{n} a_{ij}x_{i} \leq b_{j}; \ j = 1,2,\ldots,m \)

where, \( c_{ij}, a_{ij}, \) and \( b_{j} \) are constants.

1.4. Traveling Salesman Problem

Regarding traveling salesman problem, it is defined by two different data, i.e. the number of cities \( n \) and distance between city \( i \) and \( j \) (\( d_{ij} \)). Traveling salesman problem is divided in two types, i.e. symmetry, if \( d_{ij} = d_{ji} \), and asymmetry, if \( d_{ij} \neq d_{ji} \) [8]. Decision variable \( (x_{ij}) \) is defined as follows.

\[
x_{ij} = \begin{cases} 1, & \text{if city } j \text{ connected to city } i \, . \end{cases}
\]

\[
0, & \text{otherwise}.
\]
Mathematics model for the traveling salesman problem can be seen from this equation:

\[
\min z = \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} x_{ij}, \quad d_{ij} = \infty, \forall i = j
\]  

(8)

with these constraints,

\[
\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1, 2, ..., n
\]

(9)

\[
\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1, 2, ..., n
\]

\[
x_{ij} \in \{0,1\}.
\]

The solution shapes a tour (closed), where:

1. Coefficient of \(d_{ij}\); \(i = 1, 2, ..., n\); \(j = 1, 2, ..., n\) is distance from city \(i\) to city \(j\).
2. Decision variables that will be determined are \(x_{ij}\); \(i = 1, 2, ..., n\); \(j = 1, 2, ..., n\). If \(x_{ij} = 1\), it means the route of city \(i\) to city \(j\) will be chosen. Meanwhile, if \(x_{ij} = 0\), the route of city \(i\) to city \(j\) will not be chosen.
3. Constraint function (8) means that every place is a starting point of a tour to a destination.
4. Constraint function (9) means that every place is a destination of a tour from a starting point.

In order to make a tour (closed), it requires a solution which results in a tour instead of several sub-tours.

2. Research Model

2.1. Fuzzy Multi-objective Linear Programming

Investigation of an optimum solution in fuzzy environment will be more complicated if several objective functions are present. This will influence the analysis of fuzzy multi-criteria programming. In general, an optimization of multi-objective problem with \(k\) objective is maximized and objective function \(m\) is minimized on \(p\) constraint with \(n\) decision variables \(n\) as follows.

\[
\begin{align*}
\max Z_k(x) \\
\min Z_m(x) \\
\text{Subject to,} g_i(x) \leq b_i, \quad i = 1, 2, ..., p \\
g_i \geq 0
\end{align*}
\]

(10)

where, \(x = (x_1, x_2, ..., x_n) \in \mathbb{R}^n\) is decision variable, \(Z_k(x)\) and \(Z_m(x)\) are objective functions. When all objective functions and constraints from model (10) are fuzzy, the fuzzy model can be expressed as follows.

\[
\begin{align*}
Z_k(x) & \geq Z_k^0 \\
Z_m(x) & \leq Z_m^0 \\
\text{subject to,} & g_i(x) \leq b_i, \quad i = 1, 2, ..., p \\
g_i & \geq 0
\end{align*}
\]

(12)

where, \(Z_k^0\) and \(Z_m^0\) is an aspiration level from destination. Symbol of \(\geq\) and \(\leq\) are fuzzy version of \(\ge\) and \(\le\) which interpret ‘basically more than or equal to’ and ‘fewer than or equal to’.

Decision set of fuzzy \(D\) is defined as follows.

\[
\mu_D(x) = \min[\mu_{Z_k}(x), \mu_{Z_m}(x), \mu_{g_i}(x)]
\]

(13)

where \(\mu_{Z_k}(x)\) and \(\mu_{Z_m}(x)\) are membership functions of destination and \(\mu_{g_i}(x)\) is a membership function of constraints. Accordingly, optimum decision from solution is able to maximize the achieved aspiration level, and thus optimum solution could be given:

\[
\max_{x \in D} \min[\mu_{Z_k}(x), \mu_{Z_m}(x), \mu_{g_i}(x)] = \max_{x \in D} \mu_D(x).
\]

(14)

Implementing this appropriate membership function may result in an optimum solution [9]. For instance, \(t_k, t_m\), and \(t_i (i = 1, 2, ..., p)\) are constants subjectively chosen from tolerated disobeying from each objective function and constraint. Therefore, the membership functions for this objective are as follows.

\[
\mu_{Z_k}(x) = \begin{cases} 
0 & \text{if } Z_k(x) \leq Z_k^0 - t_k, \\
\frac{Z_k - Z_k(x)}{t_k} & \text{if } Z_k^0 - t_k \leq Z_k(x) \leq Z_k^0, \\
1 & \text{if } Z_k(x) \geq Z_k^0
\end{cases}
\]

(15)
$$\mu_{Z_m}(x) = \begin{cases} 0 & \text{If } Z_m(x) \geq Z_m^0 + t_m \\ 1 - \frac{Z_m(x) - Z_m^0}{t_m} & \text{If } Z_m^0 \leq Z_m(x) \leq Z_m^0 + t_m \\ 1 & \text{If } Z_m(x) \leq Z_m^0. \end{cases}$$

Membership functions for constraint \(i\) are as follows:

$$\mu_{g_i}(x) = \begin{cases} 0 & \text{If } g_i(x) \geq b_i + t_i \\ 1 - \frac{g_i(x) - b_i}{t_i} & \text{If } b_i \leq g_i(x) \leq b_i + t_i \\ 1 & \text{If } g_i(x) \leq b_i. \end{cases}$$

Then, when \(\alpha = \min[\mu_Z(x), \mu_{Z_m}(x), \mu_{g_i}(x)]\), where \(\alpha\) is satisfaction degree of \(x^*\) (optimum solution). Satisfaction degree in this system is ranged from 0 to 1. Maximum and minimum numbers of \(\alpha\) reflect two extreme conditions at the system, so the following model is given.

$$\max \alpha$$
with following constraints,

$$\alpha \leq 1 - \frac{Z_k^0 - Z_k(x)}{t_k}$$
(18)

$$\alpha \leq 1 - \frac{Z_m(x) - Z_m^0}{t_m}$$
(19)

$$\alpha \leq 1 - \frac{g_i(x) - b_i}{t_i}$$
(20)

$$x_{ij} \geq 0.$$  

2.2. Fuzzy Multi-objective Linear Programming on Traveling Salesman Problem

The aim of traveling salesman problem which is the most considered in determining optimum route to travel to all cities so the cost could be minimized. Situation consideration in taking decision should result in optimum solution of traveling salesman problem with minimized cost, time, and distance. Individual objective functions could be expressed for all objectives from decision maker. If \(x_{ij}\) is the link from pos \(i\) to \(j\) and

$$x_{ij} = \begin{cases} 1, \text{pos}(i) \rightarrow \text{pos}(j) \\ 0, \text{other.} \end{cases}$$
(21)

Supposed \(c_{ij}\) is a travel cost from pos \(i\) to \(j\); the total cost from a certain route is a sum of link including route. \(z_1^0\) is an aspiration level for objective function on cost minimization while \(t_1\) is tolerance, therefore the objective function is written as follows:

$$z_1: \min \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \leq z_1^0.$$  

(22)

Supposed \(d_{ij}\) is a distance from pos \(i\) to \(j\) and \(z_2^0\) is an aspiration level for objective function on cost minimization while \(t_2\) is tolerance, therefore the objective function is written as follows:

$$z_2: \min \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} x_{ij} \leq z_2^0.$$  

(23)

Supposed \(t_{ij}\) is time spent on travel from pos \(i\) to \(j\) and \(z_3^0\) adjust an aspiration level for objection function on minimization of total time while \(t_3\) is tolerance. The objective function is written as follows:

$$z_3: \min \sum_{i=1}^{n} \sum_{j=1}^{n} t_{ij} x_{ij} \leq z_3^0.$$  

(24)

Aspiration level on objective functions is determined from solving each objective function by giving obstacles on traveling salesman problem.

Furthermore, there is limitation in the model where every city has to be visited by one of neighbour cities and vice versa, which is expressed as follows.

$$\sum_{i=1}^{n} x_{ij} = 1, \forall j$$
$$\sum_{j=1}^{n} x_{ij} = 1, \forall i.$$  

(25)

A route cannot be selected multiple times, and therefore

$$x_{ij} + x_{ji} \leq 1, \forall i, j$$
$$x_{ij} \geq 0.$$  

(26)
3. Analysis and Discussion

Object in this research is research data from a journal titled “Application of Stimulated Annealing in Solving Travelling Salesman Problem” [10]. Data is presented in Table 1. Map of stations that have to be visited by salesman is given in Figure 1.

| Route             | Cost (Rp) | Distance (km) | Time (min) |
|-------------------|-----------|---------------|------------|
| PT. X – Sei Raya  | 10,000    | 3             | 12         |
| PT. XX – Kota Baru| 12,000    | 5             | 18         |
| PT. XX – Siantan  | 12,000    | 6             | 20         |
| PT. XX - Adisucipto| 13,000   | 7             | 24         |
| PT. XX – Gajahmada| 10,000    | 3             | 10         |
| Sei Raya – Kota Baru| 14,000 | 8             | 32         |
| Sei Raya – Siantan| 16,000    | 10            | 38         |
| Sei Raya- Adisucipto| 12,000   | 6             | 25         |
| Sei Raya – Gajahmada| 12,000 | 6             | 25         |
| Kota Baru – Siantan| 18,000    | 12            | 46         |
| Kota Baru – Adisucipto| 17,000 | 12            | 42         |
| Kota Baru – Gajahmada| 12,000   | 5             | 20         |
| Siantan – Adisucipto| 18,000   | 12            | 45         |
| Siantan – Gajahmada| 14,000    | 7             | 30         |
| Adisucipto – Gajahmada| 16,000  | 10            | 35         |

**Figure 1.** Travel map of PT. XX salesman; (0) PT. XX, (1) Sei Raya, (2) Kota Baru, (3) Siantan, (4) Adisucipto, and (5) Gajahmada

3.1. Determining Aspiration Level, Tolerance, and Membership Function

Table 2 shows travel cost (c) from pos i to j in thousand rupiahs, distance (d) from pos i to j in kilometer (km) unit, and time spent on the travel (t) from pos i to j in minute unit.

| POS          | 0    | 1    | 2    | 3    | 4    | 5    |
|--------------|------|------|------|------|------|------|
| PT. XX       | (c.d.t) |      |      |      |      |      |
| Sei Raya     | (c.d.t) |      |      |      |      |      |
| Kota Baru    | (c.d.t) |      |      |      |      |      |
| Siantan      | (c.d.t) |      |      |      |      |      |
| Adisucipto   | (c.d.t) |      |      |      |      |      |
| Gajahmada    | (c.d.t) |      |      |      |      |      |
Aspiration level \( z_k^0, \ k = 1,2,3 \) is determined from solving each of objective function. In this case, solving of objective function uses obstacle function from the model of traveling salesman problem.

- Aspiration level for travel cost \( (c) \) from \( i \) to \( j \).

\[
\min z_1 = \sum_{i=0}^{5} \sum_{j=0}^{5} c_{ij} x_{ij}, \quad c_{ij} = \infty, \forall i = j
\]

with following obstacle

\[
\sum_{j=0}^{5} x_{ij} = 1, \quad i = 0, ..., 5 \quad ; \quad \sum_{i=0}^{5} x_{ij} = 1, \quad j = 0, ..., 5
\]

\[x_{ij} + x_{ji} \leq 1; \quad x_{ij} \geq 0\]

using software of Maple 18, following value is obtained \( z_1^0 = 77 \).

- Aspiration level for distance \( (d) \) from station \( i \) to \( j \).

\[
\min z_2 = \sum_{i=0}^{5} \sum_{j=0}^{5} d_{ij} x_{ij}, \quad d_{ij} = \infty, \forall i = j
\]

with following obstacle

\[
\sum_{j=0}^{5} x_{ij} = 1, \quad i = 0, ..., 5 \quad ; \quad \sum_{i=0}^{5} x_{ij} = 1, \quad j = 0, ..., 5
\]

\[x_{ij} + x_{ji} \leq 1; \quad x_{ij} \geq 0\]

following value is obtained \( z_2^0 = 38 \).

- Aspiration level for time spent in travel \( (t) \) from \( i \) to \( j \).

\[
\min z_3 = \sum_{i=0}^{5} \sum_{j=0}^{5} t_{ij} x_{ij}, \quad t_{ij} = \infty, \forall i = j
\]

with following obstacle

\[
\sum_{j=0}^{5} x_{ij} = 1, \quad i = 0, ..., 5 \quad ; \quad \sum_{i=0}^{5} x_{ij} = 1, \quad j = 0, ..., 5
\]

\[x_{ij} + x_{ji} \leq 1; \quad x_{ij} \geq 0\]

following value is obtained \( z_3^0 = 149 \).

Additionally, tolerance value \( t_k, k = 1,2,3 \) of each objective function is determined by finding difference between coefficients of each objective on a selected route.

- Tolerance for travel cost \( (c) \) from station \( i \) to \( j \).

Based on settlement of \( z_1^0 \) aspiration level with Maple 18 software, the selected routes were \( x_{03}, x_{35}, x_{52}, x_{24}, x_{41}, x_{10} \), and therefore coefficients selected for cost objective function were \( \{c_{03}, c_{35}, c_{52}, c_{24}, c_{41}, c_{10}\} = \{12,14,12,17,12,10\} \), so that tolerance values for cost objection function were obtained as follows: \( t_1 = 17 = 10 = 7 \).

- Tolerance for distance \( (d) \) from \( i \) to \( j \).

Based on aspiration level \( z_2^0 \) with Maple 15 software, the selected routes were \( x_{02}, x_{25}, x_{53}, x_{34}, x_{41}, x_{10} \), and therefore coefficients selected for cost objective function were \( \{d_{02}, d_{25}, d_{53}, d_{34}, d_{41}, d_{10}\} = \{5,5,7,12,6,3\} \), so that tolerance values for cost objection function were obtained as follows: \( t_2 = 12 = 3 = 9 \).

- Tolerance for time spent on travel \( (t) \) from \( i \) to \( j \).

Based on settlement of \( z_3^0 \) aspiration level with Maple 15 software, the selected routes were \( x_{01}, x_{14}, x_{42}, x_{25}, x_{53}, x_{30} \), and therefore coefficients selected for cost objective function were \( \{t_{01}, t_{14}, t_{42}, t_{25}, t_{53}, t_{30}\} = \{12,25,42,20,30,20\} \), so that tolerance values for cost objection function were obtained as follows: \( t_3 = 42 = 12 = 30 \).

After determining aspiration level and tolerance, objective functions were obtained as follows.

\[
\min z_1 = 10x_{10} + 12x_{02} + 12x_{03} + 13x_{04} + 10x_{05} + 10x_{10} + 14x_{12} + 16x_{14} + 12x_{14} + 12x_{15} + 12x_{20} + 14x_{21} + 18x_{23} + 17x_{24} + 12x_{25} + 12x_{30} + 16x_{31} + 18x_{32} + 18x_{34} + 14x_{35} + 13x_{40} + 12x_{41} + 17x_{42} + 18x_{43} + 16x_{45} + 10x_{50} + 12x_{51} + 12x_{52} + 14x_{53} + 16x_{54} \leq 77.
\]
\begin{equation}
\text{Tolerance} = t_1 = 7
\end{equation}
\begin{equation}
\text{min } z_2 = 3x_{01} + 5x_{02} + 6x_{03} + 7x_{04} + 3x_{05} + 3x_{10} + 8x_{12} + 10x_{13} + 6x_{14} + 6x_{15} + 5x_{20} + 8x_{21} + 12x_{23} + 12x_{24} + 5x_{25} + 6x_{30} + 10x_{31} + 12x_{32} + 12x_{34} + 7x_{35} + 7x_{40} + 6x_{41} + 12x_{42} + 12x_{43} + 10x_{45} + 3x_{50} + 6x_{51} + 5x_{52} + 7x_{53} + 10x_{54} \leq 38. \tag{34}
\end{equation}
\begin{equation}
\text{Tolerance} = t_2 = 9
\end{equation}
\begin{equation}
\text{min } z_3 = 12x_{01} + 18x_{02} + 20x_{03} + 24x_{04} + 10x_{05} + 12x_{10} + 32x_{12} + 38x_{13} + 25x_{14} + 25x_{15} + 18x_{20} + 32x_{21} + 46x_{23} + 42x_{24} + 20x_{25} + 20x_{30} + 38x_{31} + 46x_{32} + 45x_{34} + 30x_{35} + 24x_{40} + 25x_{41} + 42x_{42} + 45x_{43} + 35x_{45} + 10x_{50} + 25x_{51} + 20x_{52} + 30x_{53} + 35x_{54} \leq 149. \tag{35}
\end{equation}
\begin{equation}
\text{Tolerance} = t_3 = 30
\end{equation}
Below are fuzzy membership functions from each objective.
\begin{equation}
\mu(z_1) = \begin{cases} 
0 & \text{if } z_1 \geq 84 \\
1 - \frac{z_1-77}{7} & \text{if } 77 \leq z_1 \leq 84 \\
1 & \text{if } z_1 \leq 77.
\end{cases}
\end{equation}
\begin{equation}
\mu(z_2) = \begin{cases} 
0 & \text{if } z_2 \geq 47 \\
1 - \frac{z_2-38}{9} & \text{if } 38 \leq z_2 \leq 47 \\
1 & \text{if } z_2 \leq 38.
\end{cases}
\end{equation}
\begin{equation}
\mu(z_3) = \begin{cases} 
0 & \text{if } z_3 \geq 179 \\
1 - \frac{z_3-149}{30} & \text{if } 149 \leq z_3 \leq 179 \\
1 & \text{if } z_3 \leq 149.
\end{cases}
\end{equation}

3.2. Settlement of multi-objective fuzzy

Based on linear programming model of multi-objective fuzzy with maxmin approach, model settlement was carried out using Maple 18 software, solution was obtained with value of $\alpha = 0.89$, $x_{04} = 1$, $x_{12} = 1$, $x_{25} = 1$, $x_{30} = 1$, $x_{41} = 1$, $x_{53} = 1$. Value of $\alpha = 0.89$, meaning that satisfaction level on an optimal solution was 0.89 or in other words, 89% has been fulfilled.

Optimal route was obtained as follows $x_{04, x_{41, x_{12}, x_{25, x_{53}}}, x_{30}}$, meaning that selected route was $0 - 4 - 1 - 2 - 5 - 3 - 0$, and therefore poses that have to be visited by the salesman are PT.XX (0) – Adisucipto (4) – Sei Raya (1) – Kota Baru (2) – Gajahmada (5) – Siantan (3) – PT.XX (0) with
- Total travel cost: $c_{04} + c_{41} + c_{12} + c_{25} + c_{53} + c_{30} = 13 + 12 + 14 + 12 + 14 + 12 = 77$, meaning that total travel cost of the salesman is Rp. 77,000.00
- Total travel distance: $d_{04} + d_{41} + d_{12} + d_{25} + d_{53} + d_{30} = 7 + 6 + 8 + 5 + 7 + 6 = 39$, meaning that total travel distance of the salesman is 39 km.
- Total travel time: $t_{04} + t_{41} + t_{12} + t_{25} + t_{53} + t_{30} = 24 + 25 + 32 + 20 + 30 + 21 = 151$, meaning that total time spent by the salesman in his travel is 151 minute or 2 hours and 31 minutes.

4. Conclusion

The results of data processing using Maple 18 software with a satisfaction level of $\alpha = 0.89$, obtained the optimal route of $x_{04, x_{41, x_{12}, x_{25, x_{53}}}, x_{30}}$ were obtained, so that the post salesman must visit is PT.XX - Adisucipto - Sei Raya - Kota Baru - Gajahmada - Siantan - PT. XX, with a total cost of Rp. 77,000.00, a total travel distance of 39 km, and total time spent on travel of 151 minutes.

Acknowledgements

Authors would like to thanks to Rector of Universitas Padjadjaran for the full support to the research conducted through Academics Leadership Grant 2020 with Contract no. 1427/UN6.3.1/LT/2020.
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