Distributed Pricing-Based User Association for Downlink Heterogeneous Cellular Networks

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Abstract—This paper considers the optimization of the user and base-station (BS) association in a wireless downlink heterogeneous cellular network under the proportional fairness criterion. We first consider the case where each BS has a single antenna and transmits at fixed power, and propose a distributed price update strategy for a pricing-based user association scheme, in which the users are assigned to the BS based on the value of a utility function minus a price. The proposed price update algorithm is based on a coordinate descent method for solving the dual of the network utility maximization problem, and it has a rigorous performance guarantee. The main advantage of the proposed algorithm as compared to the existing subgradient method for price update is that the proposed algorithm is independent of parameter choices and can be implemented asynchronously. Further, this paper considers the joint user association and BS power control problem, and proposes an iterative dual coordinate descent and the power optimization algorithm that significantly outperforms existing approaches. Finally, this paper considers the joint user association and BS beamforming problem for the case where the BSs are equipped with multiple antennas and spatially multiplex multiple users. We incorporate dual coordinate descent with the weighted minimum mean-squared error (WMMSE) algorithm, and show that it achieves nearly the same performance as a computationally more complex benchmark algorithm (which applies the WMMSE algorithm on the entire network for BS association), while avoiding excessive BS handover.

Index Terms—Base-station association, power control, beamforming, heterogeneous networks (HetNets), load balancing, proportional fairness.

I. INTRODUCTION

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ODERN wireless networks are designed based on the cellular architecture in which multiple user terminals are associated with the base-stations (BSs) to form cells. The cellular concept has further evolved to include heterogeneous networks (HetNets) where the BSs can transmit with widely different powers at disparate locations, and consequently the cells can vary considerably in size. An essential feature of HetNet is that it allows the off-loading of traffic from the macro BSs to pico or femto BSs. By splitting the conventional macro cellular structure into small cells (i.e., femto/pico cells), the HetNets allow for more aggressive reuse of frequencies as well as improved coverage and higher overall throughput for the entire network.

A main challenge in the deployment of HetNet is the appropriate setting of the transmit power levels at different tiers of macro/pico/femto BSs and the association of users to the different BSs (or equivalently the determination of coverage area for each cell). The cell association problem is further compounded when multiple antennas are deployed at the BSs with multiple users spatially multiplexed using multiple-input and multiple-output (MIMO) beamforming techniques. Conventionally, the downlink cell coverage areas are determined according to the signal-to-interference-plus-noise ratio (SINR). Each user terminal simply associates with the BS from which the received SINR is the highest—herein referred to as the max-SINR rule. A key problem with the max-SINR BS association is that it does not account for the varying data traffic pattern in the network, hence it can lead to poor load balancing. Load balancing is essential for wireless networks with small cells, because femto/pico BSs are often deployed to alleviate traffic “hot-spots” with higher-than-average user density.

This paper addresses the downlink user association problem for HetNets from an optimization perspective under the proportional fairness criterion. We follow a pricing-based strategy in which the users are associated with the BS according to the value of a utility minus a price—a strategy first adopted in [2], where a price update method based on the subgradient algorithm is proposed. The main novelty of this paper is that we advocate an alternative price update method based on a coordinate descent approach on the dual of the network utility maximization problem. The proposed algorithm has the advantage that it is free of parameter choices and that it can be implemented asynchronously across the BSs. This paper further proposes joint optimization of BS association with downlink power control and with beamforming. We show that the proposed pricing-based distributed user association can significantly improve the conventional max-SINR association. Throughout the paper, we use the terms BS association and user association interchangeably— the former emphasizes a user perspective, while the latter a BS perspective.

A. Related Work

The BS association problem has been considered extensively in the literature. While the early works in this area [3–7] mostly deal with the code-division multiple access (CDMA) system, they already reveal that the joint optimization of BS association and transmit power can significantly improve
the overall network performance. These earlier works, as well as some of the more recent ones [8–13], tend to focus on the power-based optimization objectives, e.g., minimizing the total transmit power under a predefined set of minimum SINR constraints at the user terminals. While the power minimization formulation may be appropriate for networks with fixed rate and fixed quality-of-service (QoS) requirement, modern wireless networks often maximize the objective of the overall throughput, or more generally, a network utility function across all users in the network. In this realm, [14–17] consider the maximization of the sum rate across the network, while [18, 19] consider the weighted sum rate objective for the BS association problem. More general network utility maximization formulation is considered in [21, 20–22], which use a proportional fairness objective function, while [16] considers max-min fairness in addition.

This paper considers the network utility maximization problem under the proportional fairness, i.e., log-utility objective for the downlink of a wireless cellular network. Because the BS association problem is inherently a discrete optimization problem involving the assignment of users to BSs, finding the optimal solution for such a problem is nontrivial. While conventional BS association simply uses the max-SINR rule, it is also clear from a network utility maximization or load-balancing perspective that max-SINR is far from being adequate. In this direction, [21] proposes an intuitive idea of expanding the coverage area of small cells by adding constant bias terms to the SINR values, so as to balance the load among different cells (although [21] does not analyze what the optimal bias terms should be). Other common heuristics proposed in the literature include that in [22, 20, 23], which optimize BS association through the greedy method, and [24], which randomly assigns each user terminal to the BS with the probability proportional to the estimated throughput, and [14, 15, 25], which devise their respective methods based on the relaxation heuristic. In addition to the network utility maximization formulation, [26] addresses the BS assignment problem from a game theory perspective (as the assignment problem can be thought of as a game among the BSs), where the Nash equilibrium of the game is found.

The BS association algorithm proposed in this paper is most closely related to [2], where under fixed transmit powers, a dual pricing method based on the subgradient update is proposed. This paper adopts this pricing approach, but makes further progress in identifying an alternative price update method. Other related works on BS association assuming fixed transmit powers include [16], which considers a simple model consisting of only a single pair of macro and pico BSs, and [18], which considers a special situation where user terminals may not report their channel state information (CSI) truthfully out of selfish motivation.

For the purpose of load balancing and interference management, it has also been well recognized in the existing literature that BS association and transmit power levels need to be optimized jointly. From this joint optimization perspective, an intuitive but heuristic idea is to optimize BS association and power levels in an iterative fashion, as suggested in [15, 17, 23]. The approach of [25] addresses the joint optimization problem using duality theory, but only for a relaxed version of the problem with the discrete constraints eliminated. In general, BS association and power optimization for weighted rate-sum maximization are both challenging problems, but there are some special cases where the globally optimal solution to the joint optimization problem can be found. For example, in [27] the optimal settings of BS association and power levels that maximize the sum throughput are obtained under certain restricted conditions for the case where the number of user terminals and the number of BSs are the same. Instead of searching for globally optimal solutions, this paper treats the joint BS association and power optimization problem from an iterative optimization perspective. Our main contribution here is some key observations on the role of pricing-based BS association in this heuristic approach.

For multi-cell networks with multiple antennas at the BSs, this paper also considers the joint optimization of BS association and beamforming for the scenario where multiple users can be spatially multiplexed within each cell. In this domain, [12, 28] provide algorithms for such a joint optimization problem, but only under the power minimization objective. In [14], BS association, transmit power and beamforming vectors are optimized through coordinate descent. Note that the beamforming problem by itself (assuming fixed BS association) is well studied in the literature (e.g., [29–33]). In this regard, the WMMSE algorithm [32] is of particular interest, because it can handle weighted rate sum maximization, hence the proportional fairness objective. A recent work [19] proposes a modification of the WMMSE algorithm that is capable of optimizing BS association and the beamforming vectors jointly. The WMMSE algorithm of [19] is, however, computationally complex; further it induces excessive BS handover. One of the contributions of this paper is that the pricing-based BS association can be incorporated with WMMSE beamforming design to significantly reduce the computational complexity of joint BS association and beamforming method of [19], while achieving nearly the same performance and avoiding excessive handover.

B. Main Contributions

This paper considers the optimal joint BS association with power control and with beamforming for the downlink HetNets under the proportional fairness objective. The main contributions of this paper are as follows:

1) BS Association: For a single-input and single-output (SISO) network with fixed transmit powers and with flat-fading channels, this paper proposes a distributed pricing-based user association scheme with a price update method based on coordinate descent in the dual domain. The proposed price update algorithm has faster convergence than the conventional subgradient method [2]. It is a fundamental building block for subsequent generalizations to the frequency-selective case and to the cases with power control and MIMO beamforming. Moreover, we provide a duality-gap based analysis to bound the performance error of the proposed algorithm.

2) Joint BS Association and Power Control: This paper proposes an iterative optimization approach for the joint BS
association and power control problem. We make a key observation that the choice of BS association method is crucial in joint optimization. In particular, when used in conjunction with power control, the conventional max-SINR association tends to exacerbate load imbalance, while the proposed pricing-based association alleviates load imbalance. To quantify the performance of the proposed iterative approach, we devise a benchmark algorithm based on dual optimization and by solving the nonconvex power optimization problem from multiple starting points. We show that the proposed iterative approach provides comparable performance, while being much less computationally complex.

3) Joint BS Association and Beamforming: When BSs are equipped with multiple antennas and have the ability to spatially multiplex multiple users within each cell, this paper shows that the optimization of BS association and beamforming can be decoupled without significantly affecting the overall performance. This allows us to propose a two-stage method combining the joint BS association and power control algorithm as the first stage followed by a per-cell WMMSE step in the second stage. The proposed approach is significantly less complex than the use of WMMSE algorithm for BS association over the entire network [19], while at the same time avoiding excessive BS handover.

C. Organization of This Paper

The rest of this paper is organized as follows. Section II introduces the problem formulation for the BS association problem for a SISO network. Section III analyzes a pricing based BS association approach under fixed power. The algorithm proposed in Section III is a key component in subsequent developments. Section IV considers the joint BS association and power control problem. Section V addresses the joint BS association and beamforming problem for a MIMO network. Performance evaluations are provided in Section VI. Conclusions are drawn in Section VII.

II. BS Association Problem for SISO Networks

Consider a downlink cellular network consisting of $L$ BSs with fixed transmit power levels (which may differ from BSs to BSs), and $K$ active user terminals across the geographic area covered by the network. Both the BSs and the user terminals are equipped with a single antenna each. Let $i$ be the index of user terminals, $i \in \{1, 2, \ldots, K\}$, and let $j$ be the index of BSs, $j \in \{1, 2, \ldots, L\}$. Let the total bandwidth of the system be $W$, which is shared by all BSs (i.e., frequency reuse factor is one). To simplify the problem, we assume flat-fading channels and frequency-flat PSD levels at the BSs, thus the SINR values are constants across the frequencies. Let $h_{ij} \in \mathbb{C}$ be the channel between user $i$ and BS $j$, and let $p_j$ be the transmit PSD level at BS $j$. If the user $i$ is to be associated with the BS $j$, its SINR value is then

$$\text{SINR}_{ij}(p) = \frac{|h_{ij}|^2 p_j}{\sum_{j' \neq j} |h_{ij'}|^2 |p_{j'}| + \sigma_i^2},$$

where $p = [p_1, \cdots, p_L]^T$; $\sigma_i^2$ is the PSD of the background additive white Gaussian noise (AWGN). This paper assumes that each user is associated with one BS at a time.

This paper adopts a proportionally fair network utility optimization framework of maximizing the sum log-utility across all the users in the entire network. A key step in the problem formulation is an observation made in [2], where it is shown that for a given set of users associated with a BS, round-robin among these users is the proportionally fair schedule (assuming constant and flat-fading channel and flat transmit PSD). Hence, if a total of $k_j$ users are associated with BS $j$, in order to maximize the proportional fairness objective, each of them should be allocated $1/k_j$ of the total time/frequency resource. In this case, if a user $i$ is associated with BS $j$, its rate is given by

$$R_{ij}(p, k_j) = \frac{W}{k_j} \log \left(1 + \frac{\text{SINR}_{ij}}{\Gamma}\right),$$

where $\Gamma$ is the SNR gap determined by practical coding and modulation schemes used.

Let $x_{ij}$ be a binary variable (1 or 0) denoting whether or not user $i$ is associated with BS $j$. The BS association problem is that of jointly determining $x_{ij}$ and the transmit powers $p_j$ at each BS to maximize the overall network utility, which can be written as:

$$\text{maximize}_{X, p, k} \sum_{i,j} x_{ij} \log (R_{ij}(p, k_j))$$

subject to

$$0 \leq p_j \leq P_j, \forall j$$

$$\sum_{j} x_{ij} = 1, \forall i$$

$$\sum_{i} x_{ij} = k_j, \forall j$$

$$k_j = K$$

$$x_{ij} \in \{0, 1\}, \forall i, \forall j$$

where $X = [x_{ij}]; k = [k_1, \ldots, k_L]^T$; $P_j$ is the PSD constraint of BS $j$. Constraint (5c) ensures that each user can only associate with one BS, and constraint (5c) states that all users in the network are served. Note that although $k_j$ is completely determined by $x_{ij}$, it is convenient to keep $k_j$ as an optimization variable in subsequent analysis.

III. BS Association in SISO Networks Under Fixed Power

The joint BS association and power control problem (3) is a mixed discrete optimization (over the BS association) and nonconvex optimization problem (over the powers), for which finding its global optimum is expected to be very challenging. In this section, we focus on a simplified problem setting with transmit power spectral density (PSD) levels fixed a priori. The joint optimization problem with power control is treated in the subsequent section.

A. Problem Formulation

When $p$ is fixed in (3), all SINR values are predefined by (1). We introduce parameter

$$a_{ij} = \log \left(W \log \left(1 + \frac{\text{SINR}_{ij}}{\Gamma}\right)\right).$$

(4)
Substituting \( a_{ij} \) back into (3), we simplify the BS association problem under the fixed powers as

\[
\text{maximize}_{X, k} \quad \sum_{i,j} a_{ij}x_{ij} - \sum_j k_j \log(k_j)
\]

subject to

\[
\begin{align*}
\sum_j x_{ij} &= 1, \; \forall i \\
\sum_i x_{ij} &= k_j, \; \forall j \\
\sum_j k_j &= K \\
x_{ij} &\in \{0, 1\}, \; \forall i, \forall j
\end{align*}
\]

This rest of this section presents a pricing approach, together with a novel price update method, for solving the above problem.

### B. Lagrangian Dual Analysis

The problem formulation (5) is first proposed in [2], where it is shown that a dual analysis can yield considerable insight. An important idea is that the dual variables can be interpreted as the BS-specific prices, which give rise to the dual pricing approaches for BS association.

Introduce dual variables \( \mu = [\mu_1, \ldots, \mu_L]^T \) for constraint (5c), and \( \nu \) for constraint (5d). The Lagrangian function with respect to these two constraints is

\[
L(X, k, \mu, \nu) = \sum_{i,j} a_{ij}x_{ij} - \sum_j k_j \log(k_j)
\]

\[
- \sum_j \mu_j \left( \sum_i x_{ij} - k_j \right) - \nu \left( \sum_j k_j - K \right).
\]

The dual function \( g(\cdot) \) can then be written as

\[
g(\mu, \nu) = \begin{cases}
\text{maximize} & L(X, k, \mu, \nu) \\
\text{s.t.} & \sum_j x_{ij} = 1, \; i = 1, \ldots, K \\
& x_{ij} \in \{0, 1\}, \; \forall i, \forall j
\end{cases} \quad (7)
\]

The maximization of the Lagrangian has the following explicit analytic solution:

\[
x^*_i = \begin{cases}
1, & \text{if } j = j^{(i)} \\
0, & \text{if } j \neq j^{(i)}
\end{cases}
\]

where \( j^{(i)} = \arg \max_{j'} (a_{ij'} - \mu_{j'}) \)

and

\[
k_j^* = e^{\mu_j - \nu - 1}.
\]

Note that if \( j^{(i)} \) in (8) is not unique, \( x_{ij} \) can be assigned value 1 for any of the BSs with maximum \( (a_{ij} - \mu_j) \) without affecting the value of dual function.

The solution of \( x_{ij} \) in (8) is quite intuitive. The dual variable \( \mu_j \) is the price at BS \( j \), while \( a_{ij} \) is the utility of the user \( i \) if it associates with BS \( j \). Each user maximizes its utility \( a_{ij} \) minus the price among all possible BSs, while the BSs choose their prices to balance their loads.

This pricing interpretation has already been given in [2], which also proposes a subgradient algorithm for updating the prices. The present paper carries this idea one step further by observing that we can explicitly write down the Lagrangian dual optimization problem of (5). This additional observation gives rise to a better price update method.

Substituting (8) and (9) back into (7), we obtain the dual objective in closed-form as:

\[
g(\mu, \nu) = \sum_i \max_j (a_{ij} - \mu_j) + e^{\nu - \nu - 1} + \nu K. \quad (10)
\]

The Lagrangian dual problem of (5) is now the minimization of \( g(\cdot) \) over \( \mu \) and \( \nu \):

\[
\text{minimize} \quad g(\mu, \nu)
\]

The Lagrangian duality theory in optimization states that the updating of the prices can be done via the minimization of \( g(\mu, \nu) \), e.g., using the subgradient algorithm [2]. One of the main contributions of this paper is that by taking advantage of the particular form of \( g(\mu, \nu) \), the price update can alternatively be done using a coordinate descent approach in the dual domain. In the subsequent sections, we first review the subgradient method, then present the new coordinate descent method.

After the dual solution is obtained for (11), we need to recover the primal variable \( x_{ij} \) from the dual solution. This can be done through (8), but there is the possibility that a user has more than one BS with the same maximal value for \( (a_{ij} - \mu_j) \). Such ties can be resolved using heuristics. In general, we would like to keep \( k_j \) as close to \( e^{\nu - \nu - 1} \) as possible. In our simulation experience, only a very small number of users are typically involved in ties, so tie-breaking via exhaustive search is feasible.

It should be noted that because the original optimization problem (5) is discrete in nature, solving the dual is not the same as solving the original primal problem—a positive duality gap can exist. Nevertheless, the dual optimum solution often leads to good primal solutions.

### C. Subgradient Method

To solve the dual optimization problem (11), we observe first that if \( \mu \) is fixed, then \( g(\cdot) \) is a differentiable convex function of \( \nu \), so the optimal \( \nu \) can be found as

\[
\nu^{(t+1)} = \log \frac{\sum_j e^{\mu_j - \nu - 1}}{K}, \quad (12)
\]

where the time index \( t \) is included here to indicate that \( \mu \) and \( \nu \) need to be updated iteratively in a sequential order. However, \( g(\cdot) \) is not a differentiable function of \( \mu_j \), so instead of taking its derivative with respect to \( \mu_j \), the subgradient method updates \( \mu_j \)'s in each step according to

\[
\mu_j^{(t+1)} = \mu_j^{(t)} - \alpha(t) \left( e^{\mu_j^{(t)} - \nu^{(t)} - 1} - \sum_i x_{ij}^{(t)} \right), \; j = 1, \ldots, L
\]

\[
\alpha(t) = \frac{1}{t}
\]

\[
\nu^{(t+1)} = \nu^{(t)} - \alpha(t) \left( K - \sum_j e^{\mu_j^{(t)} - \nu^{(t)} - 1} \right).
\]

\[
\text{Note that if } j^{(i)} \text{ in (8) is not unique, } x_{ij} \text{ can be assigned value 1 for any of the BSs with maximum } (a_{ij} - \mu_j) \text{ without affecting the value of dual function.}
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The solution of \( x_{ij} \) in (8) is quite intuitive. The dual variable \( \mu_j \) is the price at BS \( j \), while \( a_{ij} \) is the utility of the user \( i \) if it associates with BS \( j \). Each user maximizes its utility \( a_{ij} \) minus the price among all possible BSs, while the BSs choose their prices to balance their loads.

This pricing interpretation has already been given in [2], which also proposes a subgradient algorithm for updating the prices. The present paper carries this idea one step further by observing that we can explicitly write down the Lagrangian dual optimization problem of (5). This additional observation gives rise to a better price update method.
where \( \alpha^{(t)} \) is the step size and \( x_{ij}^{(t)} \) is determined by \( \mu_j^{(t)} \) according to (8). The use of subgradient method for price update in the BS association problem is first proposed in [2].

Because the dual problem is always convex, the subgradient method is guaranteed to converge to the globally optimal solution to the dual problem (11). However, the convergence speed of the subgradient method depends heavily on the choice of step size \( \alpha^{(t)} \). Possible choices of \( \alpha^{(t)} \) include constant step size (but the constant is difficult to choose) or diminishing step sizes (which guarantee convergence but can be quite slow in practice). As a baseline for comparison, this paper adopts the self-adaptive scheme of [34] as suggested in [2]. We refer the detailed algorithm description to [34], and only mention that the scheme involves quite a few parameters, namely \( \gamma_t, \rho \geq 1, \beta < 1 \), as well as \( \delta_1 \) and \( \delta \). Still, the convergence speed is still very much parameter dependent, as seen in the simulation section later in this paper.

We remark that because all the \( \mu_j \)'s need to be updated at the same time using the same step size (in order to ensure convergence), the distributed implementation of the subgradient method requires synchronized price updates across the BSs. This is a significant drawback, as synchronization is not necessarily easy to achieve. The main advantage of the dual coordinate descent method proposed in the next section is that it is free of parameter choices and it does not require synchronization.

### D. Dual Coordinate Descent (DCD) Method

The main contribution of this paper is a coordinate descent [35] approach in the dual domain for solving (11). The key idea is to recognize that the dual function is expressed in a closed form in (10). First, fixing all the \( \mu_j \)'s, we see that optimal \( \nu \) can be updated by (12). Next, fixing \( \nu \) and all \( \mu_j \)'s except one of them, we see that \( g(\cdot) \) is in fact the sum of a continuous piece-wise linear function and an exponential function. So we can take its left or right derivatives and choose \( \mu_j \) to be such that the left derivative at \( \mu_j \) is less than or equal to zero, and the right derivative is greater than or equal to zero. Mathematically, define two functions \( f_1(\cdot) \) and \( f_2(\cdot) \) as:

\[
f_1(\mu_j) = |U_j|, \tag{14}
\]

where \( U_j = \{i | a_{ij} - \mu_j = \max_{j'}(a_{i j'} - \mu_{j'}) \} \), and

\[
f_2(\mu_j) = e^{\mu_j - \nu - 1}. \tag{15}
\]

It is easy to see that the left partial derivative of \( g(\cdot) \) with respect to \( \mu_j \) is exactly \( f_2(\mu_j) - f_1(\mu_j) \). Hence, fixing all other dual variables, the \( \mu_j \) that minimizes \( g(\cdot) \) is just

\[
\mu_j^{(t+1)} = \sup \left\{ \mu_j | f_2^{(t)}(\mu_j) - f_1^{(t)}(\mu_j) \leq 0 \right\}. \tag{16}
\]

This leads to the DCD method described in Algorithm 1.

The DCD method is quite intuitive. The dual variable \( \mu_j \) is the price at BS \( j \), while \( a_{ij} \) is the utility of the user \( i \) if it is associated with BS \( j \). Each user chooses to associate with the BS that maximizes its utility minus the price, while the BSs choose their prices in an iterative fashion to balance their loads. Fig. 1 illustrates the price update condition that seeks \( \mu_j^* \) to match \( f_1(\mu_j^*) \) and \( f_2(\mu_j^*) \). Here, \( f_1(\cdot) \) is a step function. The functions \( f_1(\cdot) \) and \( f_2(\cdot) \) may not intersect, but the optimal \( \mu_j^* \) can always be determined uniquely.

As mentioned earlier, a main advantage of the DCD method is that BSs do not need to synchronize their price updates. In fact, the order of price updates in Algorithm 1 can be arbitrary. Since each dual update step always produces a dual objective value that is nonincreasing, the iterative algorithm is always guaranteed to converge.

However, it should be noted that since the dual objective (10) is not a differentiable function, coordinate descent is not guaranteed to give a global optimum solution to the dual optimization problem (11), and most likely not the optimum solution to the primal problem (because a duality gap can exist). Nevertheless, the convergence point for DCD still gives fairly good solutions to the original BS association problem.

The proposed dual coordinate descent method is inspired by the development of auction algorithm [36] for the one-to-one assignment problem. The BS assignment problem in this section can be thought of as a generalization of the assignment problem solved by the auction algorithm [36] from the 1-to-1 to the \( N \)-to-1 case.

### E. Duality Gap Bound

Although the DCD method is not guaranteed to converge to the global optimum of the dual problem, and further, because of the integer constraints, there may be a non-zero optimal duality gap between the primal and the dual problems, the Lagrangian dual analysis nevertheless gives useful upper bounds on the optimum value of the original optimization problem. In particular, \( g(\mu, \nu) \) is an upper bound on \( f_o(\mathbf{X}, \mathbf{R}) \), and the gap is tightest when \( (\mu, \nu) \) are dual optimal. The following

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**Algorithm 1 Dual Coordinate Descent Method**

Initialization: Set \( \mu_j = 0, \forall j \). Set \( \nu = \log \sum_i e^{\nu_j^{t-1}} \).

repeat

for each \( j \in \{1, \cdots, L\} \) do

1) Update \( \mu_j \) according to (16).

end for

2) Update \( \nu \) according to (12).

until the dual objective value converges.

3) Set user-BS association according to (35). Resolve ties if necessary.

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**Fig. 1: Two cases of updating \( \mu_j^* \) in dual coordinate descent**

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result shows that this optimal duality gap can be expressed analytically in closed form.

**Proposition 1.** For the BS association problem \( \mathcal{P} \), the gap between the objective function \( f_0(X, \mathbf{R}) \) obtained from the dual coordinate descent algorithm and the global optimum utility is bounded by \( \sum_j k_j \log (k_j/e^{\mu_j - \nu - 1}) \), where \( k_j \) is the number of users associated with BS \( j \) and \( (\mu, \nu) \) are the values of the dual variables at convergence.

**Proof.** See Appendix A. \( \square \)

Note that whenever \( k_j = e^{\mu_j - \nu - 1} \) for a BS \( j \), as in Fig. 1(a), the user association is close to being optimal at that BS, as it does not contribute to the duality gap. When a BS is involved in ties, the duality gap is minimized when \( k_j \) is made as close to \( e^{\mu_j - \nu - 1} \) as possible.

**IV. JOINT BS ASSOCIATION AND POWER CONTROL IN SISO NETWORKS**

Thus far, we have considered the downlink BS association problem with fixed BS transmit powers. However, the setting of downlink power levels is crucial for determining cell range, especially in a HetNet where pico BSs may have a very different transmit power as compared to macro BSs. This motivates us to investigate joint BS association and downlink power optimization.

**A. Iterative DCD and Power Control**

The main algorithm proposed in this section is a simple and straightforward iteration between pricing-based BS association and power control as shown in Algorithm 2. The idea is to run the DCD algorithm under fixed power in order to achieve better load balancing, and to run a power control method under fixed user association for interference mitigation. The power optimization algorithm should also aim to maximize the overall network utility function. One possible implementation of such a power optimization is included in Appendix B, where the Newton's method is used for maximizing the log utility. As long as both the BS association and the power control steps in the iteration aim to increase the same objective function, the overall algorithm is guaranteed to converge (albeit not necessarily to the global optimum, since the problem is not convex).

Although the main idea of iterative BS association and power optimization appears straightforward, this paper makes a key observation that the use of utility-maximization based BS association algorithm is crucial here. The following simple example shows that if instead the max-SINR association rule is used iteratively with power control, the overall process may not work well at all.

Consider a two-BS scenario with the initial BS assignment as shown in Fig. 2. If we apply power control, BS \( A \) would raise its transmit power due to the fact that it serves a large number of users, while BS \( B \) would lower its power. But once BS \( A \) increases its power, according to the max-SINR rule, it would attract even more users. Thus, the overall process may exacerbate load imbalance. This is in contrast to the pricing based BS association, which would actually reduce the number of users served by BS \( A \) (due to the higher pricing term), hence avoiding the undesirable phenomenon of overloading at BS \( A \).

**B. Direct Dual Optimization for Joint BS Association and Power Control**

The iterative BS association and power control method proposed in the previous section is simple and effective. To further quantify its performance, this section pursues an alternative direct dual optimization approach for solving the joint BS association and power control problem \( \mathcal{P} \). The algorithm proposed in this section is much more computationally complex than the iterative approach of the previous section, but it serves as a benchmark for performance comparison purpose.

The idea of direct dual optimization is to write down the Lagrangian dual of \( \mathcal{P} \)

\[
L(X, k, \mathbf{p}, \mu, \nu) = \sum_{i,j} x_{ij} \log (R_{ij}(\mathbf{p}, k_j)) - \sum_j \mu_j \left( \sum_i x_{ij} - k_j \right) - \nu \left( \sum_j k_j - K \right),
\]

which gives

\[
g(\mu, \nu) = \max_{X, k, \mathbf{p}} \left\{ \frac{L(X, k, \mathbf{p}, \mu, \nu)}{s.t.} \right\}
\]

The above maximization problem is now over both the power \( \mathbf{p} \) and the BS association variables \( X \) and \( k \) under fixed dual variables. As before, the optimal solution for \( k_j \) can still be obtained analytically by (17), i.e., \( k_j^* = e^{\mu_j - \nu - 1} \). However, the optimization over \( X \) and \( \mathbf{p} \) is considerably more difficult because of the nonconvex and discrete nature of the problem. Here, we propose an approach of iteratively optimizing \( X \)
assuming fixed $p$ using \ref{3}, then optimizing $p$ under fixed $X$. Clearly, the solution so obtained may not be the global optimum. Thus, we choose to start from multiple initial points of $(X, p)$ in order to better approach the global optimum.

For the minimization of the dual function $g(\cdot)$, it is possible to pursue a subgradient or dual coordinate descent approach. The key is to recognize that the subgradient in \ref{13} is still valid; further the optimization of $\nu$ can still be done via \ref{12}. However, the dual coordinate descent step \ref{18} no longer applies in a straightforward fashion. Instead, to implement coordinate descent, a bisection method on $\mu_{ij}$ can be done in order to find the optimal $\mu_{ij}^*$, while holding other dual variables fixed. Bisection can be carried out based on the subderivative of $g(\cdot)$ with respect to $\mu_{ij}$, which can be calculated as $e^{\mu_{ij}} - \nu - 1 - \sum_i x_{ij}$, where $x_{ij}$ is the solution to \ref{18}. Under an ideal assumption that the true optimal solution $(X^*, k^*)$ can be found when evaluating $g(\cdot)$, we can further deduce that this dual method has the same performance bound as in Proposition 1. Finally as mentioned before, to ensure the near global optimum evaluation of $g(\cdot)$, multiple random starting points need to be tried. This gives a way to find near globally optimal solution to the overall problem.

The direct dual optimization method described above has much higher complexity than the proposed iterative BS association and power control method proposed in the previous section, but given enough starting points, it can be served as a benchmark for the proposed algorithm. The numerical simulation carried out later in the paper indicates, however, that the simpler iterative BS association and power control method proposed earlier already performs very close to the benchmark.

V. JOINT BS ASSOCIATION AND BEAMFORMING IN MIMO NETWORKS

We now further extend the BS association problem to the case where both the BSs and the users are equipped with multiple antennas, and multiple users are spatially multiplexed within each cell. The use of beamforming can significantly influence the overall effective channel gain, and consequently the optimal BS association for each user. Thus, the joint BS association and beamforming problem is highly nontrivial. Note that power control is implicitly included as part of beamforming here.

This section first reviews the state-of-the-art in this area, then proposes a novel approach of decoupling the overall problem into two subproblems where the BS association and the beamformers are optimized separately. The proposed approach has lower computational complexity; it does not require frequent BS handover; it has comparable performance to the best benchmark joint optimization algorithm in the literature.

A. Problem Formulation and Existing Approach

Consider a downlink MIMO cellular network with $M_j$ antennas at BS $j$ and $N_i$ antennas at user $i$. The channel between user $i$ and BS $j$ is denoted by matrix $H_{ij} \in \mathbb{C}^{N_i \times M_j}$. We assume one data stream per user, and up to $M_j$ users being spatially multiplexed at the same time. The channel is assumed to be flat-fading. Each BS is assumed to have a fixed total power constraint.

Because the scheduling operation, as well as transmit and receive beamformers, are designed to adapt to the channel realizations of each user, we can no longer claim that the proportionally fair scheduling would result in equal time/frequency allocation among all the users. Instead, proportionally fair scheduling over time needs to be included explicitly in the problem formulation. Toward this end, let the BS association $x_{ij}$ be fixed over time. Let $v_{ij}^{(t)} \in \mathbb{C}^{M_j}$ be the transmit vector of BS $j$ intended for user $i$ at time $t$. In order to maximize the network utility defined as the log of the long-term average rates of all users, i.e., $\sum_i \log (R_{ij}^{(t)})$, we can equivalently maximize a weighted rate sum over successive time slots:

$$
\max_{X, V^{(t)}} \sum_i \omega_i^{(t)} \sum_j x_{ij} R_{ij}^{(t)} \quad (19a)
$$

subject to

$$
\sum_i \|v_{ij}^{(t)}\|^2 \leq \bar{p}_j, \forall j \quad (19b)
$$

$$
\sum_j x_{ij} = 1, \forall i \quad (19c)
$$

$$
x_{ij} \in \{0, 1\}, \forall i, \forall j \quad (19d)
$$

where $V^{(t)} = [v_{ij}^{(t)}]$, the weight $\omega_i^{(t)}$ equals the reciprocal of each user’s long-term average rate at time $t$, and $R_{ij}^{(t)}$ is the instantaneous rate of user $i$ at time $t$ if it is associated with BS $j$ as expressed in \ref{20} at the bottom of this page (for ease of notation, time index $t$ is omitted).

Note that user scheduling within each BS is implicit in the problem formulation \ref{19}; further in \ref{19c}, $\bar{p}_j$ is the peak PSD constraint of BS $j$, and the constraint \ref{19c} enforces the rule that each user is associated with only one BS. Since $x_{ij}$ is not allowed to depend on $t$, for each optimization period with a fixed set of channels, BS handovers from time to time are not permitted.

The beamforming design problem for weighted rate-sum maximization is a difficult nonconvex problem, even when the BS association is fixed. Below, we briefly review a WMMSE approach for solving this problem for fixed BS assignment, and a generalization of the WMMSE algorithm in \ref{19} that accounts for BS association.

1) Beamforming via WMMSE with Fixed BS Association:

When user-BS association is fixed in \ref{19}, the problem reduces to a beamforming design problem with a weighted rate sum maximization objective. As proposed in \ref{33} and \ref{32}, this
beams and the number of antennas at the BS are fixed. The users are associated with the BSs based on an estimated SINR, and each user is assigned a power control strategy. The beamforming problem can be solved by solving an equivalent weighted minimum mean-square error (WMMSE) problem. We refer to [19] for the detailed description of the WMMSE algorithm.

2) WMMSE Method for BS Association: The recent work [19] further incorporates BS association into the beamforming problem by imposing a penalty term to the weighted rate-sum objective and by solving the resulting penalized WMMSE problem for each time instant \( t \). Basically, the users are penalized for being associated with more than one BS, and accordingly constraint (19) is guaranteed in the end. However, this approach does not guarantee that the user-BS association is fixed over time. Consequently, as weights \( w_i \) are updated over time, user association and user scheduling can both change. This results in rapid BS handovers, which are not desirable in practice.

Further, the WMMSE-based BS association method as proposed in [19] has high computational complexity, because the WMMSE update needs to be done between every single BS-user pair in the entire network. Also, the performance and convergence speed of the algorithm depend heavily on the parameter of the penalty term, which can only be set heuristically. Nevertheless, the method of [19] provides a useful benchmark for our proposed algorithm below.

B. Proposed Two-Stage BS Association and Beamforming

This paper formulates the joint BS association and beamforming problem in recognition of the fact that BS association typically takes place at a much larger time scale and should only adapt to the slow-fading channel characteristics, while beamforming and scheduling can take place in faster time scale. Thus, instead of jointly optimizing BS association and beamforming at each time slot, it is more sensible to decouple them in two stages. The first stage solves the BS association problem, while the second stage solves the beamforming problem assuming fixed BS association. The proposed two-stage algorithm is described below:

1) BS Association Stage: The idea is to determine BS association in the first stage based on an estimate of channel quality. For BS association purposes, we rely on a simple SISO representation of the MIMO channel and apply the joint coordinate descent and power control algorithm presented in the previous section to determine the BS association for each user.

The SISO representation for the MIMO channel is based on the fact that from a degree of freedom point of view, \( M_j \) antennas at the BS provide \( M_j \) spatial multiplex gain. Thus, we can think of a MIMO system with \( M_j \) antennas over bandwidth \( W \) as equivalently a SISO system with bandwidth \( M_j W \). More precisely, let \( |h_{ij}| \) be the average channel magnitude between BS \( j \) and user \( i \) (modeling the distance-dependent attenuation and shadowing). We estimate each user’s SINR according to (1), while accounting for the multiple \( M_j \) antennas at the BS by redefining parameter \( a_{ij} \) as

\[
a_{ij} = \log \left( M_j W \log \left( 1 + \frac{\text{SINR}_{ij}}{\Gamma} \right) \right). \tag{21}
\]

Algorithm 3 Two-Stage Joint BS Association and WMMSE Beamforming

**Initialization:** Choose \( S_j \geq M_j, \forall j \).

1) Run Algorithm 2, the joint BS association and power control (with \( a_{ij} \) calculated by (21)) until convergence. Let the result of the optimization be \((X, p)\). Associate users to BSs according to \( X \). Compute \( R_{ij} \) according to \((X, p)\) using the SISO model (3) scaled by \( M_j \).

repeat

2) Choose \( S_j \) potential users among the users associated with BS \( j \) according to \( \omega_i x_{ij} R_{ij}, \forall j \).

3) Run the WMMSE algorithm (32) for the chosen users in each cell to get the transmit beamformers and the resulting rate \( R_{ij}^{(t)} \).

4) Update the average rate for each user \( R_{ij}^{\text{avg}} \) based on \( R_{ij}^{(t)} \); set \( \omega_i = 1/R_{ij}^{\text{avg}}, \forall i \).

until \( R_{ij}^{\text{avg}} \) converges, \( \forall i \).

The joint BS association and power control algorithm can now be applied to determine the BS association. We remark here that only the BS association is of interest at this stage. The optimized power \( p_j \) serves to assist the BS association and scheduling decisions, but is further optimized in the next stage.

2) Scheduling and Beamforming Stage: After the BS association is determined, the overall problem now reduces to the beamforming vector design problem, which can be solved using the WMMSE algorithm. Our contribution in algorithm design in this stage is to point out that one can further lower the computational complexity of WMMSE by eliminate candidate users that are unlikely to be scheduled.

In the conventional WMMSE algorithm, all potential users within a cell can have their beamforming vectors updated in each step. However, because each BS \( j \) can spatially multiplex at most \( M_j \) users, to reduce the computational complexity, we may choose a subset of users who are most likely to be served to take part in the WMMSE algorithm. The simplest way to do this is to choose the users according to the estimated weighted rate \( |h_{ij}| x_{ij} R_{ij} \), where \( R_{ij} \) is calculated by the SISO model (3) scaled by \( M_j \) according to the resulting \((X, p)\) after stage one. More sophisticated scheduling can also take channel directions into account. The number of potential users chosen by the WMMSE scheduler in cell \( j \) is a parameter, called \( S_j \) in this paper, which should be greater than \( M_j \). A complete description of the two-stage method is stated in Algorithm 3.

C. Complexity Analysis

This subsection briefly analyzes the computational complexity saving of the proposed algorithm as compared to the joint BS association and beamforming algorithm of [19]. For simplicity, we assume that the number of antennas at all the BSs are the same, i.e., \( M_j = M \) for all \( j \)’s, and \( N_i = N \) for all \( i \)’s. Under fixed BS association, the conventional WMMSE algorithm has a complexity of \( O\left(K^2MN^2 + K^2M^2N + KM^3 + KN^3\right) \) per each beamforming step, where \( K \) is the number of users in the entire network.
TABLE I: Simulation Parameters

| Parameter                      | Value                                      |
|--------------------------------|--------------------------------------------|
| Channel Bandwidth              | 10 MHz                                     |
| Frequency Reuse Factor         | 1                                          |
| Duplex Mode                    | TDD                                        |
| Macro BS Max PSD               | -27 dBm/Hz                                 |
| Pico BS Max PSD                | -47 dBm/Hz                                 |
| Antenna Gain                   | 15 dBi                                     |
| SNR Gap                        | 0 dB                                       |
| Background Noise PSD           | -109 dBm/Hz                                |
| Distance-dependent Attenuation | 128.1 + 37.6 log_{10}(d), d is in km      |
| Shadowing                      | Log normal as \( N(0, \sigma^2) \), \( \sigma = 8 \) dB |

For the joint BS association and WMMSE method of [19], since the WMMSE update of each user needs to be done with respect to all \( L \) BSs in the network, parameter \( K \) in the WMMSE complexity formula for the algorithm of [19] needs to be increased by a factor of \( L \), resulting in a complexity of \( O(L^2 K^2 M N^2 + L^3 K^2 M^2 N + L K M^3 + L K N^3) \).

By contrast, in the proposed two-stage algorithm, only \( S = \sum_j S_j \) users are considered, and they are already associated with their respective BSs. Consequently, the complexity per each WMMSE iteration is reduced to \( O(S^2 M N^2 + S^2 M^2 N + S M^3 + S N^3) \). Since \( S \ll K \ll LK \), this is a significant complexity saving. In the above calculation, we ignore the complexity of the first stage, which is typically very fast. In addition, we do not account for the number of iterations in the WMMSE algorithm. However, the number of WMMSE iterations is typically smaller for the proposed algorithm than for the WMMSE algorithm of [19] since fewer users are involved. Overall, the proposed two-stage algorithm is much faster than the WMMSE algorithm of [19]. The simulation results of next section show that it performs almost as well.

VI. SIMULATION RESULTS

A. BS Association Under Fixed Powers

We first simulate the BS association algorithms with fixed powers in a downlink SISO network with a 7-cell wrap around topology, with one macro-BS and three pico-BSs per cell, and with 30 users per cell. The channel modeling parameters are as defined in Table I. The transmit PSD level is fixed at the maximum value for each BS. Fig. 3 compares the convergence behavior of the dual coordinate descent (Algorithm 1) with that of the adaptive subgradient method. Here each iteration refers to either a single update of \( \mu_j \) in the DCD method or a subgradient update of all \( \mu_j \)'s. We see that the DCD method converges to within \( 10^{-1} \) of the optimum with only two rounds of iterations per BS (i.e. 56 iterations), while the convergence of subgradient method is very sensitive to its parameters. Here, we set \( \rho = 1.2, \beta = 0.9, \) and \( \delta = 0.002 \) in the adaptive subgradient method [2], [34] and see that different settings of \( \delta \) and \( \gamma_k \) can result in very different convergence behaviors. Note that in Fig. 3 the DCD method does not converge to the optimum. This is due to the fact that it is possible for coordinate descent to get stuck in a suboptimal point. This gap is quite small in this simulation, however.

Fig. 3: Convergence behaviors of dual coordinate descent and subgradient algorithms

Fig. 4 shows the cumulative distribution function (CDF) of data rates after 56 iterations for the various BS assignment algorithms. We see that both the subgradient method and the DCD method offer substantial rate improvement to low-rate users as compared to the max-SINR BS assignment rule. For instance, the 50th-percentile rate is increased by about 33%, which is a consequence of off-loading traffic from the macro BSs to the pico BSs. The performance of the subgradient method is again parameter dependent.

Table II shows that the numerical utility \(^2\) achieved by DCD and two of the subgradient methods are almost identical, while subgradient-2 and the max-SINR method produce quite inferior results. This is consistent with the earlier convergence plot (Fig. 3) and the CDF plot (Fig. 4). In addition, the duality-gap bound calculated according to Proposition 1 for this example is about 0.45. This shows that the performance of the DCD algorithm is already very close to the global optimum. Finally, Fig. 5 displays the percentages of macro/pico users for various BS association methods. It shows that with the max-SINR BS association and subgradient-2, too many users are associated with the macro BS, while the DCD algorithm is able to achieve more balanced load by off-loading the users to pico BSs.

B. Joint BS Association and Power Control

This section considers the same network topology, but with downlink power control implemented in addition. We use Newton’s method for power control for utility maximization. Note that since the network utility maximization problem is nonconvex, only the convergence to local optimum is expected. For the implementation of the direct dual optimization approach, we choose 10 random starting points.

In Fig. 6 we observe a significant difference between max-SINR BS association and DCD-based BS association when they are implemented iteratively with power control. Further, Fig. 7 shows that the iteration between DCD and power control

\(^2\)The numerical value of the utility is computed as sum of log of user rates, where rates are in Mbps.
TABLE II: Utility values for various BS association methods after 56 iterations

| Method            | Utility |
|-------------------|---------|
| Max-SINR          | 52.86   |
| DCD               | 97.63   |
| Subgradient-1     | 97.58   |
| Subgradient-2     | 75.04   |
| Subgradient-3     | 97.66   |

TABLE III: Utility values for various joint BS association and power control methods

| Method                        | Utility |
|-------------------------------|---------|
| Iterative DCD and power control | 186.29  |
| Direct dual optimization      | 194.41  |
| Max-SINR with max power       | 52.86   |
| Max-SINR with optimised power | 193.01  |
| Iterative max-SINR and power control | 56.09  |

Fig. 4: CDF of user rates for various BS association methods after 56 iterations

Fig. 5: The percentages of macro/pico users for various BS association methods after 56 iterations

gives incremental improvement in utility, while in the max-SINR case utility actually decreases after the second iteration. These two plots validate the earlier analysis showing that the max-SINR association does not address the load balancing issue effectively and that the use of utility-maximization-based BS association is crucial when implemented with power control.

As can be seen in Fig. 6 and Table III, the direct dual optimization approach is able to provide the best performance among all the methods, but at the cost of very high complexity. In the simulation, we observe that during the updating of one single dual variable, direct dual optimization needs to call the power control algorithm approximately 1000 times, while the iterative DCD and power control method only needs to run the power control method once in each iteration.

For comparison purpose, we also implement the max-SINR BS association under the powers optimized by the duality-based approach. Now, max-SINR performs well as seen in the Fig. 6 and Table III. This shows that the problem with the max-SINR algorithm is that it is unable to induce the correct power setting, in contrast to the DCD scheme.

In Fig. 7 we show the PSD levels produced by the various methods.

Fig. 6: CDF of user rates for various joint BS association and power control methods

Fig. 7: The convergence of the iteration with power control for DCD and max-SINR

In Fig. 8, we show the PSD levels produced by the various methods.
methods. It is observed that the methods with better performance are able to suppress the overly high transmit power by the macro BSs. Further, Fig. 9 shows the percentages of users associated with the macro and pico BSs resulting from various methods. Methods with better performance tend to have higher percentages of pico users, which illustrates the benefit of off-loading traffic from macro BSs to pico BSs. Combining results from Fig. 8 and Fig. 9, we conclude that a combination of suppressing macro BS power for interference mitigation and off-loading to pico BSs for load balancing is the key to obtaining overall good system performance.

C. Joint BS Association and Beamforming

Consider again the same network topology, but for the MIMO case with 4 antennas at each of the macro and pico BSs and 2 antennas at each user. The two-stage BS association and WMMSE algorithm is compared with the max-SINR BS association under maximum power plus per-cell WMMSE, in Fig. 10 and in Table IV. The number of candidate users (the $S_j$ parameter) in the two-stage method is chosen to be 4, 6 and 8. It is observed that the two-stage method can substantially improve the max-SINR BS association: the 50th-percentile rate is almost doubled when $S = 8$. We also observe that the performance of the two-stage method improves with larger $S$, but the improvement beyond $S = 8$ is marginal for this case with 4 transmit antennas.

We also wish to compare the two-stage method with the joint BS association and WMMSE method proposed in [19]. Because this method involves implementing the WMMSE algorithm across the entire network, its complexity is very high. In fact, running such an algorithm across a 7-cell network (with 28 BSs) is already impractical. Instead, Fig. 11 compares the two algorithms in a smaller network with 3 macro BSs, 4 pico BSs, and with 105 user terminals. We observe in our simulation that the utility gains by the two-stage method and the WMMSE method of [19] are 17.82 and 23.05 respectively as compared to the max-SINR scheme. Although the WMMSE method of [19] produces overall better network utility, we observe from Fig. 11 that the majority of users do not see much performance difference between the two. In addition, we observe in the simulation that the
joint BS association and WMMSE method of [19] causes approximately 24 BS association switchings on average for each beamforming update. About 1/4 of the users are involved in BS handover in each time slot, which is not very practical. In contrast, BS association is completely fixed in the two-stage method, which is a clear advantage.

VII. CONCLUSION

This paper considers pricing-based BS association schemes for heterogeneous networks and proposes a distributed price update strategy based on a coordinate descent algorithm in the dual domain. The proposed BS association scheme can be seamlessly incorporated with power control and beamforming. In each of these cases, because BS assignment must be determined at a relatively larger time scale, we propose to implement BS association with respect to the expected average channel gains. The overall main insight of this paper is that load balancing is crucial in heterogeneous networks. Instead of assigning BSs according to SINR, a utility maximization and load balancing is crucial in heterogeneous networks. Instead of channel gains. The overall main insight of this paper is that

APPENDIX B

NEWTON’S METHOD FOR DOWNLINK POWER CONTROL

In this appendix, we describe a Newton’s method for solving the power optimization problem for maximizing the network log utility. Assuming fixed user association X (and accordingly \(k_j = \sum_i x_{ij}\) as the number of users associated with each BS. Let R be the corresponding user rates calculated by (2). We have:

\[
\begin{align*}
    f_0(X, R) &= \sum_{i,j} a_{ij}x_{ij} - \sum_j k_j \log(k_j) \\
    &= \sum_{i,j} a_{ij}x_{ij} - \sum_j k_j \log(e^{\mu_j} - \nu - 1) \\
    &- \sum_j k_j \log \left( \frac{k_j}{e^{\mu_j} - \nu - 1} \right) \\
    &= \sum_i (a_{ij} - \mu_j)x_{ij} + \sum_j k_j \\
    &+ \nu k_j - \sum_j k_j \log \left( \frac{k_j}{e^{\mu_j} - \nu - 1} \right) \\
    &= \sum_j \max(a_{ij} - \mu_j) + K + \nu K \\
    &- \sum_j k_j \log \left( \frac{k_j}{e^{\mu_j} - \nu - 1} \right) \\
    &= \sum j \max(a_{ij} - \mu_j) + \sum_j e^{\mu_j} - \nu - 1
\end{align*}
\]

+ \nu K - \sum_j k_j \log \left( \frac{k_j}{e^{\mu_j} - \nu - 1} \right) (22e)

= g(\mu, \nu) - \sum_j k_j \log \left( \frac{k_j}{e^{\mu_j} - \nu - 1} \right) (22f)

where the optimality condition on \(x_{ij}\), (8), is used in deriving (22d), and the optimality condition on \(\nu\), (12), is used in deriving (22e).

Now, let \((X^*, k^*)\) be the optimal solution to problem (5), and let \(R^*\) be the resulting user rates. By weak duality, it always holds that \(g(\mu, \nu) \geq f_0(X^*, R^*)\). Combining this result with (22d), we prove the claim

\[
f_0(X, R) \geq f_0(X^*, R^*) - \sum_j k_j \log \left( \frac{k_j}{e^{\mu_j} - \nu - 1} \right). \tag{23}
\]

APPROACH A

PROOF OF PROPOSITION 1

Let \((\mu, \nu)\) be the optimized dual variables at convergence of the DCD algorithm. Let \((X, k)\) be the primal solution recovered from the dual variable \((\mu, \nu)\) using (8) with tie-breaking if necessary, and subsequently setting \(k_j = \sum_i x_{ij}\) as the number of users associated with each BS. Let R be the corresponding user rates calculated by (2). We have:

\[
\begin{align*}
    f_0(X, R) &= \sum_{i,j} a_{ij}x_{ij} - \sum_j k_j \log(k_j) \\
    &= \sum_{i,j} a_{ij}x_{ij} - \sum_j k_j \log(e^{\mu_j} - \nu - 1) \\
    &- \sum_j k_j \log \left( \frac{k_j}{e^{\mu_j} - \nu - 1} \right) \\
    &= \sum_i (a_{ij} - \mu_j)x_{ij} + \sum_j k_j \\
    &+ \nu k_j - \sum_j k_j \log \left( \frac{k_j}{e^{\mu_j} - \nu - 1} \right) \\
    &= \sum_j \max(a_{ij} - \mu_j) + K + \nu K \\
    &- \sum_j k_j \log \left( \frac{k_j}{e^{\mu_j} - \nu - 1} \right) \\
    &= \sum j \max(a_{ij} - \mu_j) + \sum_j e^{\mu_j} - \nu - 1
\end{align*}
\]

+ \nu K - \sum_j k_j \log \left( \frac{k_j}{e^{\mu_j} - \nu - 1} \right) (22e)

= g(\mu, \nu) - \sum_j k_j \log \left( \frac{k_j}{e^{\mu_j} - \nu - 1} \right) (22f)

where the optimality condition on \(x_{ij}\), (8), is used in deriving (22d), and the optimality condition on \(\nu\), (12), is used in deriving (22e).

Now, let \((X^*, k^*)\) be the optimal solution to problem (5), and let \(R^*\) be the resulting user rates. By weak duality, it always holds that \(g(\mu, \nu) \geq f_0(X^*, R^*)\). Combining this result with (22d), we prove the claim

\[
f_0(X, R) \geq f_0(X^*, R^*) - \sum_j k_j \log \left( \frac{k_j}{e^{\mu_j} - \nu - 1} \right). \tag{23}
\]

APPENDIX B

NEWTON’S METHOD FOR DOWNLINK POWER CONTROL

In this appendix, we describe a Newton’s method for solving the power optimization problem for maximizing the network log utility. Assuming fixed user association X (and accordingly \(k_j = \sum_i x_{ij}\), the optimization problem is:

\[
\begin{align*}
    \text{maximize} & \quad \sum_{i,j} x_{ij} \log \left( R_{ij}(p, k_j) \right) \\
    \text{subject to} & \quad 0 \leq p_j \leq \mathbb{P}_j, \forall j
\end{align*}
\]

Let \(f_{\text{power}}(p)\) denote the objective function above. Introduce parameter \(r_{ij}\) as

\[
r_{ij} = \log \left( 1 + \frac{\text{SINR}_{ij}}{\Gamma} \right). \tag{25}
\]

We can write the first-order and the second-order partial derivatives of \(f_{\text{power}}(p)\) with respect to \(p_j\) as:

\[
\frac{\partial f_{\text{power}}}{\partial p_j} = \sum_i \frac{\text{SINR}_{ij}}{r_{ij}(\Gamma + \text{SINR}_{ij})} x_{ij} - \sum_i \sum_{j' \neq j} \frac{|h_{ij}|^2 \text{SINR}_{ij}^2}{|h_{ij'}|^2 r_{ij'}(\Gamma + \text{SINR}_{ij}) p_{j'}} \tag{26}
\]

and

\[
\frac{\partial^2 f_{\text{power}}}{\partial p_j^2} = -\sum_i \left( \frac{1}{r_{ij}^2} + \frac{1}{r_{ij}} \right) \frac{\text{SINR}_{ij}^2}{(\Gamma + \text{SINR}_{ij})^2} x_{ij} + \sum_i \sum_{j' \neq j} \frac{|h_{ij}||h_{ij'}|^4 (2r_{ij'} - 1) x_{ij'}}{|h_{ij'}|^4 r_{ij'}(\Gamma + \text{SINR}_{ij})^2 p_{j'}^2}. \tag{27}
\]

Following the heuristic in [37], we only use the diagonal entries of Hessian matrix in Newton’s method in order to reduce the computational complexity of inverting the Hessian. In this case, the Newton step becomes

\[
\Delta p_j = -\frac{\partial f_{\text{power}}}{\partial p_j} / \frac{\partial^2 f_{\text{power}}}{\partial p_j^2}. \quad \text{To}
\]
ensure an incremental updating direction, we further modify the Newton step as

$$\Delta p_j = \frac{\partial f_{\text{power}}}{\partial p_j} / \left| \frac{\partial^2 f_{\text{power}}}{\partial p_j^2} \right|.$$  \hfill (28)

The overall algorithm updates all $p_j$'s through

$$p_j^{(t+1)} = \left( p_j^{(t)} + \alpha_{nt} \Delta p_j \right)^{+},$$  \hfill (29)

where $\alpha_{nt}$ is the step size, which can be determined by backtracking line search [38].

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