Comparative study of femtosecond filamentation properties in the classical model and the full model for different incident pulse durations

Chenrui Jing\textsuperscript{1,2}, Xiexing Qi\textsuperscript{1,2}, Zhaohui Wang\textsuperscript{3}, Baohong Ma\textsuperscript{1,2} and Chaoliang Ding\textsuperscript{1,2}

\textsuperscript{1} College of Physics and Electronic Information, Luoyang Normal University, Luoyang 471000, People’s Republic of China
\textsuperscript{2} Key Laboratory of Electromagnetic Transformation and Detection of Henan Province, Luoyang 471000, People’s Republic of China
\textsuperscript{3} Luoyang Institute of Electro-Optical Equipment, Aviation Industry Corporation of China, Luoyang 471000, People’s Republic of China

E-mail: jing1989111@sina.com

Received 24 January 2019, revised 5 April 2019
Accepted for publication 8 May 2019
Published 23 May 2019

Abstract
We investigate the influence of incident pulse duration on the properties of femtosecond filamentation by numerical simulations. The filament intensity and plasma density are compared in the classical model and the full model under different incident pulse durations. Our results demonstrate the important role that higher-order Kerr effect (HOKE) plays in femtosecond laser filamentation, and the incident pulse duration has a significant influence on the relative contribution of HOKE, which consistently confirm the conclusions proposed by Loriot et al (2011 Laser Phys. 21 1319). Our findings provide a possible way to test the validity of the classical model and the full model in filamentation process by measuring the plasma density as a function of pulse duration through relevant experimental techniques.

Keywords: filamentation, higher-order Kerr effect, self-focusing and defocusing

(Some figures may appear in colour only in the online journal)

1. Introduction

Femtosecond laser filamentation, which results from extreme nonlinear propagation of intense femtosecond laser pulses in transparent media, has been one of the hottest topics in recent years due to its potential applications, such as harmonic generation [1–4], terahertz generation [5–8], remote sensing [9–13], etc. In the mid-1990s, the physics mechanism of femtosecond laser filamentation was interpreted as the dynamical balance between plasma defocusing and Kerr self-focusing [14]. However, this interpretation has been recently challenged by the new findings from Loriot et al [15]. They found that, under the high intensity, the sign inversion of higher-order Kerr terms appears and the main defocusing mechanism may come from higher-order Kerr terms instead of the plasma. According to different defocusing mechanisms, the model of femtosecond filamentation can be classified into the classical model and the full model. In the classical model, plasma effect is considered as the dominant defocusing mechanism. Whereas, in the full model, higher-order Kerr effect (HOKE) is considered as the dominant one.

Since the establishment of the two models, the debates of the contribution of the HOKE to the process of filamentation

2040-8978/19/065503+06$33.00 © 2019 IOP Publishing Ltd Printed in the UK
have never been stopped [16–27]. In 2010, Béjot et al investigated the influence from higher-order Kerr indices on femtosecond laser filamentation [16]. Through theoretical simulation, it was found that the HOKE provides the dominant contribution to defocusing and the plasma is not required for femtosecond laser filamentation. In the further research, by investigating the conical emission from filaments, they showed that the HOKE is necessary to reproduce the observed conical emission in experiment [18]. However, by measuring the transverse distribution of conical emission rings diverging from the filament and simulating the angle-wavelength spectrum of the pulse, another group confirmed plasma as the dominant mechanism arresting the self-focusing collapse in their conditions [21].

It is known that the major difference between the classical model and the full model is whether the higher-order Kerr terms, which is mainly determined by higher-order nonlinear indices \( n_3, n_6, n_8 \), are considered in filamentation process. Therefore, investigating the properties of higher-order nonlinearities provides a possible way to test the validity of the two models. In 2014, Spott et al. calculated the electric susceptibility of atomic hydrogen. It was found that the explanation via higher-order nonlinear terms is only applicable in a certain laser intensity range [25]. Choosing Kr as the representative case, another group confirmed that higher-order nonlinearity only plays a role under very special conditions [26]. Moreover, for other noble gases, the nonlinear refractive indices \( n_4 \) are proved to be positive over the wavelengths 250–2000 nm [27], which contradicts with the HOKE hypothesis.

Based on above considerations, in this letter, we comparatively investigate the properties of femtosecond filamentation in the classical model and full model. The evolution of laser intensity and plasma density in filament with different incident pulse durations are compared in two models. Our results demonstrate the important role that HOKE plays in filamentation and the incident pulse duration has a significant influence on the relative contribution of HOKE. Moreover, from the practical point of view, our findings provide a possible way to test the validity of the classical model and the full model by measuring the plasma density as a function of pulse duration in experiment.

2. Theoretical model

The propagation model of the intense femtosecond laser pulses in gaseous medium can be described by the extended nonlinear Schrodinger equation (NLSE) [14]. Assuming a linear polarized femtosecond laser pulse propagates along the \( z \) axis, the evolution of the scalar electric field envelope \( E(r, t, z) \) is governed by the following equation:

\[
\frac{\partial E}{\partial z} = \frac{i}{2k_0} \left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \right) E - \frac{k''(r) \frac{\partial^2}{\partial t^2} E}{2} + \frac{k_0}{n_0} \Delta N_{\text{Kerr}} E \\
+ \frac{k_0\omega_p^2}{2i\omega_0^3} E = \frac{\beta_{\text{K}}}{2} |E|^{2K-2} E,
\]

where \( t \) refers to the retarded time in the reference frame of the pulse; \( k_0 = \frac{2\pi}{\lambda_0} \) and \( k'' = \frac{\partial^2}{\partial t^2}|\omega_0| \) are wave number and second order dispersion coefficient. \( \omega_0 = \sqrt{\omega_0^2 - \omega_{\text{th}}^2} \) is the plasma frequency, where \( \rho, e \) and \( m_e \) denote the plasma density, electron charge and the mass of electron, respectively, and \( \beta_{\text{K}} \) is the multiphoton absorption (MPA) coefficient, where \( K \) is the minimal number of photons needed to ionize the neutral gas molecule. On the right hand side of equation (1), the first two terms are linear effects, accounting for transverse diffraction and the normal dispersion (GVD). The last three terms are nonlinear effects, which contain the Kerr effect, plasma defocusing and MPA, respectively. In the full model, the nonlinear Kerr effect can be expressed as \( \Delta N_{\text{Kerr}} = n_2 I + n_4 I^2 + n_6 I^3 + n_8 I^4 \) where \( I \) is the laser intensity and \( n_2, n_4, n_6, n_8 \) are nonlinear refractive coefficients. However, in the classical model, only the \( n_2 \) term is contained.

The propagation equation of the electric filed envelope is always coupled with plasma density evolution. In the filamentation process, the laser intensity usually maintains in a high level. The electrons mainly generate through multiphoton ionization process and the evolution of the plasma density follows the equation [14]:

\[
\frac{\partial \rho}{\partial t} = \frac{\beta_{\text{K}}}{K\hbar\omega_0} |E|^2 K \left( 1 - \frac{\rho}{\rho_{\text{th}}} \right)
\]

Due to the low pressure condition in our simulation, the effects of avalanche ionization and plasma recombination in this equation are ignored [28]. The quantity \( \rho_{\text{th}} = 1.7 \times 10^{27} \text{ m}^{-3} \) denotes the density of neutral atoms, and all the values of the parameters used in our calculations are listed in table 1.

| \( \lambda (\text{nm}) \) | 800 |
|-------------------|------|
| Pressure (bar)    | 1    |
| \( k''(\text{cm}^6 \text{m}^{-1}) \) | 0.2  |
| \( \beta_{\text{K}} (\text{cm}^{17} \text{W}^{-9}) \) | 1.27 \times 10^{-126} |
| \( n_0 \)         | 1    |
| \( n_2 (\text{m}^2 \text{W}^{-1}) \) | 1.2 \times 10^{-23} |
| \( n_4 (\text{m}^4 \text{W}^{-2}) \) | -1.5 \times 10^{-41} |
| \( n_6 (\text{m}^6 \text{W}^{-3}) \) | 2.1 \times 10^{-58} |
| \( n_8 (\text{m}^8 \text{W}^{-4}) \) | -0.8 \times 10^{-75} |

In order to solve the NSLE, split-step Fourier method [29] is adopted in the frequency domain. The path of propagation is divided into many units, thus the whole propagation process can be regarded as propagating from one unit to the forward one under the joint action of linear and nonlinear effects. In the time and space domains, the 2D + 1 computing grid for laser pulses propagation is defined by \( dt, dr, dz \), where \( dt = \frac{T}{N_t} \), \( dr = \frac{R}{N_r} \) and \( dz = \frac{Z}{N_z} \). The parameters \( T, R, Z \) refer to the calculation ranges in time, radial, propagation direction and \( N_t, N_r, N_z \) are numbers of the grid points in these directions correspondingly. Considering the factors of computation efficiency, precision and...
numerical stability, \( N_T \) and \( N_R \) are set to be 1024. \( N_T \) is 50,000 in our simulations. By decomposing the equation (1) into linear part and nonlinear part, the partial differential equation of the two parts can be solved respectively. To integrate the linear part of equation (1) along the propagation axis, the Crank–Nicholson scheme [30] is adopted and the direct integration method is applied to solve the nonlinear part of equation (1). For solving equation (2), the 4th-order Runge–Kutta method [31] is employed.

3. Numerical results and discussions

The incident laser pulse with cylindrical symmetry can be written as: \( E(r, t, 0) = A_0 \exp \left( -\frac{r^2}{\tau^2} - \frac{t^2}{\tau^2} \right) \), where \( A_0 = \sqrt{\frac{2P_0}{\pi \tau^2}} \) is the laser field amplitude, and \( \tau \) is the pulse duration.

In simulations, the beam waist is fixed at \( r_0 = 1 \) mm, the peak intensity \( I_0 \) is \( 1 \times 10^{16} \text{ W m}^{-2} \). In this case, the incident peak power \( P_{im} \sim 15.7 \text{ GW} \) is nearly two times higher than the critical power \( P_{cr} \sim 8 \text{ GW} \), which avoids multiple filaments generation. In order to make comparison between the two models, the evolution of laser intensity and the plasma density calculated in the classical model and the full model for different incident pulse durations are shown in figures 1 and 2.

Generally speaking, as can be seen in figure 1, the classical model always yields higher laser intensity in comparison with that in the full model. For different incident pulse durations, the clamped intensity is about \( 9 \times 10^{17} \text{ W m}^{-2} \) in the classical model, while in the full model, the intensity is around \( 3 \times 10^{17} \text{ W m}^{-2} \). Both values are verified to be comparable with the previous reports [14, 16, 32]. In figures 1(b) and (c), we notice that several intensity peaks appear at the longer distance, which is caused by re-focusing in filamentation process at the higher energy. With the increase of the pulse duration, the pulse energy increase, which promotes the energy exchange between the filament core and energy reservoir, thus more intensity spikes occur along with propagation [33, 34]. Besides, the collapse distance predicted in two models almost keeps constant (around \( 2.7 \) m) for different pulse durations. This is due to the fact that according to the formula \( L_C = \frac{0.3674 \lambda c}{\sqrt{(\frac{(\text{intensity})}{P_{cr}})^{0.3} - 0.852^2 + 0.0219}} \), where \( L_{DF} = \frac{\lambda \tau^2}{2} \) is the Rayleigh length [35], the collapse distance \( L_C \) is only determined by the beam waist and the incident peak power, which is independent with pulse duration. Even though the collapse distance is unchanged with the increase of pulse duration, the terminal position of filament moves backward obviously (see figure 1), which means the increase of filament length. Additionally, as is shown in figure 1, the length of filament predicted in the classical model is much shorter in comparison with the full model. This is because full model always predicts much lower plasma density in comparison with the classical model (see figure 2 in detail). As a result, the energy loss caused by MPA is suppressed, which benefits the propagation of filament to the longer distance.

![Figure 1](image-url) Evolution of laser intensity calculated by the classical model (dashed line) and the full model (solid line) for different incident pulse durations. (a) \( \tau = 60 \) fs; (b) \( \tau = 120 \) fs; (c) \( \tau = 180 \) fs.

The on-axis plasma density as a function of propagation distance with different incident pulse durations is shown in figure 2. Similar to figures 1(b), (c), several plasma density peaks also appear at the longer distance in figures 2(b) and (c). And the positions of the plasma density spikes are completely consistent with the intensity spikes in figures 1(b), (c). Taking notice of the semi-logarithmic coordinate system in figure 2, the plasma density calculated in the classical model is nearly \( 10^5 \) times higher in comparison with the full model for different incident pulse durations. This is due to the fact in the regime of
multiphoton ionization, the electron density is proportional to $I^K$. Since the major components of air are nitrogen and oxygen, the ionization potential of air is $U_i = 14.6$ eV [32], 10 photons (i.e. $K = 10$) are necessary to liberate an electron from the neutral air molecule via multiphoton ionization when the incident pulse wavelength is 800 nm. In this case, the plasma density in the classical model should be several orders of magnitude higher than that in the full model, which is coincident with our results. However, with the increase of pulse duration, the difference of plasma density calculated in the two models reduces obviously.

In order to investigate the influence of incident pulse duration on femtosecond filamentation properties for the classical model and full model in detail, pulse duration is changed continuously from 60 to 250 fs in the followed calculations, while the other parameters are kept unchanged. In two models, the laser intensity and the plasma density around the collapse distance as a function of pulse duration are shown in figures 3 and 4, respectively. As can be seen in figure 3, with the increase of pulse duration, the filament intensity almost keeps unchanged in the full model ($\left(\frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}}}\right) \approx 2\%$), while decreases in the classical model ($\left(\frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}}}\right) \approx 7.5\%$). However, the evolution of the plasma density in filament as a function of pulse duration is quite different (see figure 4). To be specific, in the full model, the plasma density increases linearly with the increase of pulse duration, whereas, in the classical model, the plasma density increases slightly. Additionally, the discrepancy of plasma density inside the filament in two models tends to reduce when the pulse duration becomes longer.

Based on foregoing observations, we attempt to explain the results presented in figures 3 and 4 qualitatively. It is

Figure 2. Evolution of plasma density calculated by the classical model (dashed line) and the full model (solid line) for different incident pulse durations. (a) $\tau = 60$ fs; (b) $\tau = 120$ fs; (c) $\tau = 180$ fs.

Figure 3. The laser intensity at collapse distance as a function of incident pulse duration in two models. (a) Full model; (b) classical model.
known that femtosecond laser filamentation is considered as the dynamic balance between Kerr focusing and plasma defocusing in the classical model. In the full model, the nonlinear Kerr effect contains higher-order terms, i.e. $n_4 I^2$, $n_6 I^3$, $n_8 I^4$. Since the index of $n_4$, $n_8$ are negative, they will suppress self-focusing along with propagation. Under the intensity of $3 \times 10^{17}$ W m$^{-2}$, the ratio of $|\Delta n_4/\Delta n_2| = |n_4 I^2/\Delta n_2 I|$ is evaluated to be 0.375 and $|\Delta n_8/\Delta n_2| = |n_8 I^4/\Delta n_2 I|$ is 1.8, which means that the higher-order Kerr terms seem to be non-negligible in our case. As a result, in filamentation process, the concave lens effect can be considered as the collaboration between plasma defocusing and the defocusing effect originated from the negative nonlinear terms.

Since the discrepancy of plasma density calculated in two models is reduced with the increase of the pulse duration, it is reasonable to speculate that the incident pulse duration should have a significant influence on the relative contribution of HOKE. It is known that the HOKE and plasma defocusing effect can suppress beam self-focusing in filamentation process, however the mechanisms of the two effects are totally different. The HOKE which is mainly determined by laser intensity inside the filament, is an instantaneous effect. Whereas the generation of free electrons occurs in much long time scale, which means that long pulse duration will do benefit to plasma generation. Since the intensity inside the filament is almost unchanged in the full model, the plasma intensity will undoubtedly increase linearly with the increase of pulse duration. In this case, higher plasma density will promote self-defocusing, which further limit the intensity increase in the filament. Based on the above analysis, we can draw a conclusion that HOKE plays a more important role when the pulse duration is shorter, while the plasma defocusing effect will mask the HOKE when the pulse duration becomes longer.

In order to confirm the conclusions, we use ratio $\xi = \Delta n_{\text{HOKE}}/\Delta n_{\text{plasma}}$ to estimate the relative contribution of HOKE and plasma defocusing qualitatively [36]. In this equation, $\Delta n_{\text{plasma}} = 2\rho \omega_{\text{crit}}/n_0$, where $n_0$ is the linear refractive index in air and $\omega_{\text{crit}} = \frac{c n_0^2}{\rho}$ is the critical plasma density [14]. Since the intensity inside the filament is almost unchanged and the plasma density increases linearly with the increase of pulse duration in the full model, the ratio $\xi = \Delta n_{\text{HOKE}}/\Delta n_{\text{plasma}}$ will undoubtedly decrease. This means the contribution from HOKE should be reduced when the incident pulse duration becomes longer, which is consistent with our conclusions. Last, but not least, considering the applicability of equations (1) and (2), the pulse duration is not increased to several picoseconds furthermore to seek the transition point of HOKE-domain to plasma-domain in filamentation process. However, from the practical point of view, our findings provide a possible way to test the validity of the two models in filamentation process by measuring the plasma density quantitatively as a function of incident pulse duration through relevant experimental techniques.

4. Conclusion

In this paper, we theoretically investigate the evolution of filamentation properties in the classical model and the full model. It is discovered that the estimated laser intensity and the plasma density are always higher in the classical model in comparison with the full model. By continuously changing the incident pulse duration, it is found that the laser intensity as well as the plasma density at the collapse distance as a function of incident pulse duration in two models is totally different. Our findings verify that the incident pulse duration has a significant influence on the relative contribution of HOKE. Besides, from the practical point of view, these findings provide a possible way to test two models in experiment by measuring the plasma density as a function of pulse duration through experimental techniques.
Acknowledgments

This work is supported by the National Natural Science Foundation of China (Grant No. 11704174), Key Scientific Research Project of Henan Province (Grant No. 17A140023).

ORCID iDs

Chenrui Jing https://orcid.org/0000-0003-2788-833X

References

[1] Kanai T, Minemoto S and Sakai H 2005 Nature 435 470
[2] Lewenstein M, Balcou P, Ivanov M, L’Huillier A and Corkum P 1994 Phys. Rev. A, 49 2117
[3] Corkum P and Krausz F 2007 Nat. Phys. 3 381
[4] Chang Z, Rundquist A, Wang H, Murnane M and Kapteyn H 1997 Phys. Rev. Lett. 79 2967
[5] D’Amico C, Houard A, Franco M, Prade B, Mysyrowicz A, Couairon A and Tikhonchuk V 2007 Phys. Rev. Lett. 98 235002
[6] Tzortzakis S et al 2002 Opt. Lett. 27 1944
[7] Méchain G, Tzortzakis S, Prade B, Franco M, Mysyrowicz A and Leriche B 2003 Appl. Phys. B 77 707
[8] Cheng C, Wright E and Moloney J 2001 Phys. Rev. Lett. 87 213001
[9] Kasparian J et al 2003 Science 301 61
[10] Dogariu A, Michael J J, Scully M and Miles R 2011 Science 331 442
[11] Luo Q, Liu W and Chin S 2003 Appl. Phys. B 76 337
[12] Mitryukovskiy S, Liu Y, Ding P, Houard A and Mysyrowicz A 2014 Opt. Express 22 12750
[13] Xu H and Chin S 2011 Sensors 11 32
[14] Couairon A and Mysyrowicz A 2007 Phys. Rep. 441 47
[15] Loriot V, Hertz E, Faucher O and Lavorel B 2009 Opt. Express 17 13429
[16] Béjot P, Kasparian J, Henin S, Loriot V, Viellard T, Hertz E, Faucher O, Lavorel B and Wolf J 2010 Phys. Rev. Lett. 104 103903
[17] Polynkin P, Kolesik M, Wright E and Moloney J 2011 Phys. Rev. Lett. 106 153902
[18] Béjot P and Kasparian J 2011 Opt. Lett. 36 4812
[19] Bré C, Demircan A and Steinhäuser G 2011 Phys. Rev. Lett. 106 183902
[20] Béjot P, Hertz E, Kasparian J, Lavorel B, Wolf J and Faucher O 2011 Phys. Rev. Lett. 106 243902
[21] Kosareva O, Daigle J, Panov N, Wang T, Hosseini S, Yuan S, Roy G, Makarov V and Chin S 2011 Opt. Lett. 36 1035
[22] Ni J, Yao J, Zeng B, Chu W and Li G 2011 Phys. Rev. A 84 063846
[23] Brown J, Wright E, Moloney J and Kolesik M 2012 Opt. Lett. 37 1604
[24] Wahlstrand J, Cheng Y, Chen Y and Milchberg H 2011 Phys. Rev. Lett. 107 103901
[25] Spott A, Jaron-Becker A and Becker A 2014 Phys. Rev. A 90 013426
[26] Wang T and Kolesik M 2017 Opt. Lett. 42 4195
[27] Tarazkar M, Romanov D and Levis R 2014 Phys. Rev. A 90 062514
[28] Couairon A, Tzortzakis S and Bergé L 2002 J. Opt. Soc. Am. B 19 1117
[29] Agrawal G 1989 Nonlinear Fiber Optics (New York: Academic)
[30] Press W, Teukolsky S, Vetterling W and Flannery B 2007 Numerical Recipes (Cambridge: Cambridge University Press)
[31] Stoer J and Bulirsch R 1993 Introduction to Numerical Analysis (Berlin: Springer)
[32] Qi X, Ma C and Lin W 2016 Opt. Commun. 358 126
[33] Becker A, Aközber N, Vijayalakshmi K, Oral E, Bowden C and Chin S 2001 Appl. Phys. B 73 287
[34] Gaarde M and Couairon A 2009 Phys. Rev. Lett. 103 043901
[35] Marburger J 1975 Prog. Quantum Electron. 4 35
[36] Loriot V, Béjot P, Ettoumi W, Petit Y, Kasparian J, Henin S, Hertz E, Lavorel B, Faucher O and Wolf J 2011 Laser Phys. 21 1319