Provably Safe Reinforcement Learning: A Theoretical and Experimental Comparison

Hanna Krasowski*, Jakob Thumm*, Marlon Müller, Lukas Schäfer, Xiao Wang, Matthias Althoff

Abstract—Ensuring safety of reinforcement learning (RL) algorithms is crucial to unlock their potential for many real-world tasks. However, vanilla RL does not guarantee safety. In recent years, several methods have been proposed to provide safety guarantees for RL by design. Yet, there is no comprehensive comparison of these provably safe RL methods. We therefore introduce a categorization of existing provably safe RL methods, present the theoretical foundations for both continuous and discrete action spaces, and benchmark the methods’ performance empirically. The methods are categorized based on how the action is adapted by the safety method: action replacement, action projection, and action masking.

Our experiments on an inverted pendulum and quadrotor stabilization task show that all provably safe methods are indeed always safe. Furthermore, their trained performance is comparable to unsafe baselines. The benchmarking suggests that different provably safe RL approaches should be selected depending on safety specifications, RL algorithms, and type of action space.

Index Terms—Safe reinforcement learning, online verification, formal methods, action replacement, action projection, action masking.

I. INTRODUCTION

Reinforcement learning (RL) contributes to many recent advancements in challenging research fields, such as robotics [1], [2], autonomous systems [3], [4], and games [5], [6]. However, to train and deploy RL agents in the real world, safety guarantees are crucial. Without them, it is unclear if the RL agent might harm humans or seriously damage the environment or itself. For vanilla RL, unsafe or dangerous actions are explored multiple times in order to achieve the highest possible reward. As a consequence, such systems cannot be trained in the real world for safety-critical tasks. Even if one can train the agent in a controlled environment, such as a simulation where the execution of unsafe actions is of no concern, there are no safety guarantees for deploying the trained agent in the real world. Therefore, safe RL emerged, which adapts the learning process such that the agent considers safety aspects next to performance during training and operation. In this paper, we discuss provably safe RL that provides provable safety guarantees at all times. Nonetheless, we initially give a short overview of recent developments in safe RL to delimit the term provably safe RL.

A) Safe RL: Garcia et al. [7] give an overview of safe RL. Provably safe RL approaches would be part of their category teacher advice that uses external knowledge to modify the exploration. However, Garcia et al. [7] also include approaches that do not provide safety guarantees in this category. Furthermore, many new contributions to safe RL have been made since this survey was published. There are recent surveys [8], [9] that partially cover safe RL. Brunke et al. [8] introduce a general terminology for safe RL with special focus on the relation to safe control. Contrary, we focus on safe RL that can always guarantee safety and evaluate these methods experimentally. Könighofer et al. [9] give a condensed overview of artificial intelligence methods that provide formal guarantees during runtime. However, they do not discuss the implications of the safety method on the RL algorithm in greater depth as we do. Subsequently, we briefly present the recent developments in safe RL and define provably safe RL afterwards. In particular, we illustrate the safe RL approaches with respect to their degree of safety and by examining their exploration coverage.

One branch of safe RL approaches considers safety by considering it in their optimization objective. Here, the agent can explore all actions and states regardless of safety. Thus, these methods are not safe during training, especially in the beginning, but converge to a safer policy without safety guarantees after sufficient time. Most advances have been made in constrained RL [10]–[12], for which the goal of the policy is to maximize the rewards while satisfying user-defined specifications. The specifications can be formulated as constraint functions [12]–[15] or as temporal logic formulas [16]–[19]. The main advantage of these methods is that no explicit model of the agent dynamics or the environment is required as the agent learns the safety aspects through experience. Thus, such safe RL methods have a high potential in non-critical settings, where unsafe actions do not cause major damage.

A second type of safe RL approaches relies on probabilistic models or synthesizes a model from sampled data. Here, the action and state space can be restricted based on probabilities. Nonetheless, unsafe actions are sometimes not detected and can therefore be taken occasionally. Several works [20]–[22] try to determine the maximal set of safe states by starting from an often user-defined conservative set and extending it with the gathered learning experience. Other methods [23]–[29] are based on formulating probabilistic models that can identify the probability of safety for an action. In general, approaches that rely on probabilistic methods are especially applicable if measurement errors, modelling errors, and disturbances cannot

The first two authors contributed equally. H. Krasowski, J. Thumm, M. Müller, L. Schäfer, X. Wang, and M. Althoff are with the School of Computation, Information and Technology, Technical University of Munich, Garching, Germany.

This paper was produced by the IEEE Publication Technology Group. They are in Piscataway, NJ.

Manuscript received Month DD, YYYY; revised Month DD, YYYY.
be bounded by sets.

b) **Provably safe RL:** Provably safe RL is especially relevant when humans could be injured by the agent, e.g., in autonomous driving, human-robot collaboration, or when damages are costly or cannot be repaired easily as in space robotics. Provably safe RL methods provide absolute guarantees with respect to given safety specifications by capitalizing on prior system knowledge. Here, only safe actions are explored such that only states that fulfill the safety specifications are reached. To achieve provable safety in practice, we often have to weaken the specifications to legal or passive safety. Hereby, inevitable safety violations caused by other agents are not considered to be the fault of the agent and are therefore not considered in this work. Examples of proving legal safety have been presented for autonomous driving [30], and in robotics [31]. Furthermore, the verification process can use an abstraction of the real system as long as it is conformant [32] to the real system, i.e., it over-approximates both aleatoric and epistemic uncertainties, and covers all relevant safety aspects. As the complexity of the abstraction is usually significantly lower than the complexity of the real system, this eases efficient verification. We focus on provably safe RL approaches that tackle safety already during learning, and limit ourselves to model-free RL algorithms that do not explicitly learn a model of the system dynamics to optimize the policy. Thus, we exclude approaches that only verify learned policies [33], [34].

Our contributions in this work are threefold. First, we introduce a comprehensive classification of provably safe RL methods and their formal description. This categorization allows us to compare and benchmark the effects of choosing a specific type of action modification on the agent’s ability to learn and maximize the reward. Second, we formulate action masking for continuous action spaces to cover all theoretically possible variants of provably safe RL in our comparison. Third, we validate the performance of the provably safe RL methods on two common control benchmarks. This comparison provides insights on the strength and weaknesses for the different provably safe RL approaches, and allows us to provide some advice on selecting the best-suited provably safe RL approach for a specific problem independent of the safety verification method used.

The remainder of this paper is structured as follows. In Sec. II we describe preliminary concepts and the presented provably safe RL approaches from a theoretical perspective. Sec. III evaluates these approaches on a two-dimensional (2D) quadrotor stabilization task. The approaches with respect to the experiments and practical considerations are discussed in Sec. IV. In Sec. V we review the related provably safe RL literature.

II. Theoretical Analysis

To clarify the differences between provably safe RL methods, we analyze their theoretical foundations and overarching concepts in this section.

a) **Markov decision process:** The RL agent learns on a Markov decision process (MDP) that is described by the tuple \((\mathcal{S}, \mathcal{A}, T, r, \gamma)\). Hereby, we assume that the set of states \(\mathcal{S}\) is fully observable with bounded precision. Partially observable MDPs can be handled using methods like particle filtering [35] and are not further discussed in this work. Both the action space \(\mathcal{A}\) and \(\mathcal{S}\) can be either continuous or discrete. \(T(s, a, s')\) is the transition function, which in the discrete case gives the probability that the transition from state \(s\) to state \(s'\) occurs by taking action \(a\). In the continuous case, \(T(s, a, s')\) denotes the probability density function of the transition. We assume that the transition function is stationary over time. For each transition, the agent receives a reward \(r : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}\) from the environment. Finally, the discount factor \(0 < \gamma < 1\) weights the relevance of future rewards.

b) **Safety of a system:** For provably safe RL, it is required that the safety of states and actions are verifiable. Otherwise no formal claims about the safety of a system can be made. Thus, we first introduce the set of provably safe states \(\mathcal{S}_\varphi\) that contains all states in which all safety constraints are fulfilled. For verifying the safety of actions, we use a safety function \(\varphi : \mathcal{S} \times \mathcal{A} \rightarrow \{0, 1\}\)

\[
\varphi(s, a) = \begin{cases} 1, & \text{if } (s, a) \text{ is verified safe} \\ 0, & \text{otherwise.} \end{cases}
\]

Commonly, \(\varphi(s, a)\) is determined by overapproximating the set of states that are reachable by taking action \(a\) in state \(s\), and then validating if the reachable set of states is a subset of \(\mathcal{S}_\varphi\), e.g., as shown in [36], [37]. It is clear that \(\varphi(s, a)\) requires some prior knowledge about the system dynamics. We can provide this knowledge with an abstract model that contains all safety relevant behaviors of the original model. These abstract safety models can get different inputs than the RL agent, and are in general less complex than the underlying MDP. In systems where such a safety model is unavailable, provably safe RL is not applicable, and only non-provably safe approaches as discussed in Sec. I can be used. Although \(\varphi(s, a)\) only verifies one action \(a\), it may take more than the next state into account, e.g., when the safety function is formulated through a model predictive control (MPC) problem that considers multiple future time steps for identifying safety [38] (see Sec. II.B), or even guarantees stability in an infinite time horizon using a region of attraction [21]. To save online computation time, \(\varphi(s, a)\) sometimes can be computed offline prior to training for low-dimensional state and action spaces, see e.g., [39].

Based on the safety function, we define a set of provably safe actions \(\mathcal{A}_\varphi(s) = \{a | \varphi(s, a) = 1\}\) for a state \(s\), which is a subset of all possible safe actions \(\mathcal{A}_\varphi(s)\), i.e., \(\mathcal{A}_\varphi(s) \subseteq \mathcal{A}_\varphi(s) \subseteq \mathcal{A}\). All provably safe RL approaches rely on the availability of provably safe actions, so we require Assumption 1

**Assumption 1:** There is at least one provably safe initial state \(s_0^\varphi \in \mathcal{S}_\varphi\) and for all states \(s\) and all exit states exists at least one safe action \(\forall s^\varphi \in \mathcal{S}_\varphi \rightarrow \mathcal{A}_\varphi(s^\varphi) \neq \emptyset\).

In practice, this requirement can be achieved using passive safety as discussed in Sec. I. With Assumption 1, it is ensured that only provably safe states \(s^\varphi \in \mathcal{S}_\varphi\) can be reached when

1. Please note that “taking no action” is commonly considered to be part of the action space, most often with the action \(a = [0, \ldots, 0]^T\).
starting from any $s_0^e$ and taking only provably safe actions thereafter.

There are multiple ways to ensure provable safety for RL systems, which we summarize in the three categories: action replacement, where the safety method replaces all unsafe actions from the agent with safe actions, action projection, which projects unsafe actions to the safe action space, and action masking, where the agent can only choose actions from the safe action space. We have chosen this categorization as it represents the three main approaches found in the literature to modify actions and thereby ensure safety for RL. Action replacement and action projection alter the action after it is outputted by the agent, so we refer to them as post-posed methods. Whereas, action masking only lets the agent choose from the safe action space; it is therefore a preemptive safety measure. Figure 1 displays the basic concept and structure of these methods. The following subsections describe the concept, mathematical formalization, required assumptions, and practical implications of the three approaches.

A. Action replacement

The first approach to ensure the safety of actions is to replace any unsafe action outputted by the agent with a safe action before its execution. The first step of action replacement is to evaluate the safety of the agent’s action $a \in A$ using $\varphi(s, a)$. If the action given by the agent’s policy $\pi(a|s)$ is not verified as safe, it is replaced with a provably safe replacement action $\tilde{a} = \psi(s)$, where $\psi : S \rightarrow A_\varphi$ is called replacement function. Following this procedure, it is guaranteed that only safe actions $a^\varphi$ with

$$a^\varphi = \begin{cases} a \sim \pi(a|s), & \text{if } \varphi(s, a) = 1 \\ \psi(s), & \text{otherwise} \end{cases}$$

are executed. The most common replacement function mentioned in the literature \cite{37,39} is to uniformly sample from $A_\varphi(s)$. However, building $A_\varphi(s)$ online often requires to check every state-action pair for safety. In time-critical and complex scenarios this can be too time-consuming, so a single safe replacement action can be used. This replacement action might stem from a backup failsafe controller \cite{36,40} or from human feedback \cite{41}. We discuss how this action replacement alters the MDP in the Appendix and additionally refer the interested reader to \cite{37}.

When replacing an unsafe action, the agent can either be trained with the agent’s unsafe action $(s, a, s', r)$, with the safe replacement action $(s, a^\varphi, s', r)$, or both. In all cases, the next state $s'$ and reward $r$ are the true state and reward received from the environment after executing the safe action $a^\varphi$. Both learning tuples have considerable intuitions behind them. When choosing the original action $(s, a, s', r)$, we update the agent according to its current policy. Hereby, we can use the default reward given by the environment $r(s, a^\varphi)$, or add a penalty to it if an unsafe action was selected $r^*(s, a, a^\varphi) = r(s, a^\varphi) + r_{\text{penalty}}(s, a, a^\varphi)$. Learning with the original action should benefit on-policy learning, where the policy is updated based on experience collected using the most recent policy. However, utilizing the tuple $(s, a, s', r)$ for training is subject to state transitions that rely heavily on the replacement function $\psi(s)$, as demonstrated in the Appendix. This can be viewed as a form of noise on the observation-action relationship, thereby hindering effective learning. Contrary, by using the replacement action tuple $(s, a^\varphi, s', r)$, we are correctly rewarding the agent for the actual performed transition. However, this requires updating the agent with an action that did not stem from the agent’s current policy $\pi(a|s)$. This is an expected behavior in off-policy learning, so it is assumed that the safe action tuple is a better fit for off-policy than for on-policy RL. We evaluate the effect of different learning tuples on the training performance in Sec. III.
where \( s_t \) and \( a_t \) are the state and action \( l \)-steps ahead of the current time step, and \( g(\cdot) \) describes the system dynamics. For the set \( \mathcal{M} \), a controller exists that keeps the agent within this set for an infinite time. If the optimization problem is solvable, then \( \tilde{a} \) is executed. If it is not solvable, the control sequence from the previous state is used as a backup plan until the safe terminal set is reached or the optimization problem is solvable again [47]. The MPC formulation allows one to easily integrate model and measurement uncertainties. However, for an environment with dynamic obstacles, the safe terminal set can be time-dependent and we are not aware of a straightforward integration that still guarantees Assumption 1.

For both CBF and MPC approaches, nominal models are used to model the known system dynamics. The part of the system dynamics that are unknown can be modelled as disturbances

\[
\dot{s} = m(s) + b(s) \tilde{a} + d, \tag{7}
\]

where safety can be ensured if \( d \) is a time signal that is essentially bounded in time \( t \geq 0 \), \( d(t) \leq \bar{d} > 0 \) [48]. In many cases, a less conservative CBF or MPC method can be designed by modelling the disturbances as state and input dependent \( d(s, \tilde{a}) \) and learning the maximal disturbance from data as presented in [48] for CBF, and [44] for MPC models.

For provably safe RL using action projection, we can use the same learning tuples as discussed in Sec. I-A. Wang [49] use the tuple \((s, a, s', r')\) with a penalty \( r' \) proportional to the projection distance \( \text{dist}(a, \tilde{a}) \). Gros et al. [45] discuss the effects on different learning algorithms when using robust MPC for projection. For Q-learning, they find that no adaption of the learning algorithm is necessary when \((s, a, s', r')\) is used. For policy gradient methods, they argue that the projection must also be included in the gradient for stable learning [45, Sec. 3].

### C. Action masking

The two previous approaches modify unsafe actions from the agent. In action masking, we do not allow the agent to output an unsafe action in the first place (preemptive method). Hereby, a mask is added to the agent so that it can only choose from actions of the safe action set. In addition to Assumption 1 action masking in practice requires an efficient function \( \eta(s) \) that determines a sufficiently large set of provably safe actions \( \mathcal{A}_s \) for a given state \( s \). The policy function \( \pi \) is informed by the function \( \eta(s) \) and the action selection is adapted such that only actions from \( \mathcal{A}_s \) can be selected:

\[
a \sim \pi(a|\eta(s), s) \in \mathcal{A}_s, \tag{8}
\]

If \( \eta(s) \) can only verify one or a few actions efficiently, the agent cannot learn properly because the agent cannot explore different actions and find the optimal one among them. Ideally, the function \( \eta(s) \) achieves \( \mathcal{A}_s = \mathcal{A}_s \).

The action masking approaches for discrete and continuous action spaces are not easily transferable into each other, and will therefore be discussed separately in this subsection. For discrete actions, in each state the safety of each action is verified using \( \varphi(s, a) \) and all verified safe actions are added to \( \mathcal{A}_s(s) \), i.e., the discrete action mask is an informed drop-out
layer added at the end of the policy network. We define the resulting safe policy \( \pi_m(a|s) \) as

\[
\pi_m(a|s) = \varphi(s, a) \frac{\pi(a|s)}{\sum_{a' \in A_{\varphi}(s)} \pi(a'|s)}.
\]

(9)

The integration of masking in a specific learning algorithm is not trivial. The effects on policy optimization methods are discussed in [50], [51]. For RL algorithms that learn the Q-function, we exemplify the effects of discrete action masking for deep Q-network (DQN) [5], which is most commonly used for Q-learning with discrete actions. During exploration with action masking, the agent samples its actions uniformly from \( A_{\varphi} \). When the agent exploits the Q-function, it chooses only the best action among \( A_{\varphi} \), i.e., \( \arg \max_a Q(s, a) \). The temporal difference error for updating the Q-function \( Q(s, a) \) is [5] Eq. 3

\[
r(s, a) + \gamma \max_{a'} Q(s', a') - Q(s, a),
\]

(10)

where the action in the next state is \( a' \in A_{\varphi} \) in contrast to the vanilla temporal difference error where the maximum Q-value for the next state is searched among actions from \( A \). Using the adapted temporal difference error in [10], the learning updates are performed only with Q-values of actions relevant in the next state instead of the full action space.

We implement continuous action masking as a transformation of the agent’s action to the provable safe action set. The action space \( A \) is transformed into \( A_{\varphi} \) by applying the transformation in [11] to the agent’s actions \( a \), where \( A_{\varphi} \) is assumed to be convex. \( \min(\cdot) \) and \( \max(\cdot) \) return a vector containing the minimal and maximal value of the given set in each dimension respectively, and all operations are evaluated element-wise.

\[
\tilde{a} = (a - \min(A)) \frac{\max(A_{\varphi}) - \min(A_{\varphi})}{\max(A) - \min(A)} + \min(A_{\varphi})
\]

(11)

For example, given a two dimensional continuous action space \( A = [0, 1] \times [-1, 2] \), then \( \min(A) = [0, -1] \). Note that this implementation approximates the safe action set with an interval set which can be conservative.

Since the action spaces for RL are defined a priori, there must always be a valid transformation from \( a \) to \( \tilde{a} \) such that \( \tilde{a} \in A_{\varphi} \) and such that the operation is deterministic and time-invariant for a specific state and action pair. In contrast to action replacement, the agent’s action is only transformed rather than replaced with a potentially non-deterministic safe action. In this way the agent can explore the full action space \( A \) since [11] ensures that the action is transformed to the safe mapping of it. If one simply clips the agent’s action to the safe action space, the policy distribution is altered so that all probabilities for actions outside the clip range are projected to the safe action on the boundary of the safe action space. Thus, the probability distribution differs from the distribution of the current policy, which can be especially a problem for on-policy RL, and actions on the bound of the provably safe action space get disproportional more likely. In contrast, the action transformation we propose in [11] conserves the original policy and is conceptually similar to action normalization, which is commonly used in RL [52] Ch. 16.8.

As for the learning tuples, we use the learning tuple \( (s, a, s', r) \) for both discrete and continuous action masking. Using the transformed action \( \tilde{a} \) for the learning tuple leads to inconsistencies. For example, the agent chooses in state \( s_1 \) the action \( a_1 \) which is transformed to \( \tilde{a}_1 \). If the agent visits state \( s_1 \) again, and chooses \( a_2 = \tilde{a}_1 \), the executed action becomes \( \tilde{a}_2 \neq a_2 \) due to the transformation.

### III. Experimental Comparison

We compare four provably safe RL implementations. Action masking, action projection with CBFs, and the two action replacement implementations sample and failsafe, where we replace unsafe actions either with a random sample from \( A_{\varphi} \), or with the action suggested by a failsafe controller. We run all experiments for discrete and continuous action spaces. For action replacement and action projection, we compare four possible learning tuples:

- **naive** - RL agent’s action \( a \) with the reward corresponding to the executed action \( (s, a, s', r) \);
- **adaptation penalty** - is naive with a penalty in case the agent’s action had to be adapted \( (s, a, s', r^*) \);
- **safe action** - safe and possibly adapted action with the corresponding reward \( (s, a^2, s', r) \);
- **both** - in case the RL agent proposes an unsafe action, both the adaptation penalty and the safe action tuples are used for learning, otherwise naive is used.

When action replacement is used with a failsafe controller, we omit safe action and both because, especially in the discrete action space, the failsafe controller might use an action that is not in the action space. When action masking is used, only safe actions can be sampled, i.e., only the naive tuple is meaningful. For on-policy learning, proximal policy optimization (PPO) [53] is used. For off-policy learning, we use the Twin Delayed Deep Deterministic policy gradient algorithm (TD3) [54] for the continuous action space and DQN [5] for the discrete. The learning algorithms are adapted from Stable Baselines3 [55]. Every configuration is evaluated on five random seeds.

#### A. Environments

We compare the provably safe RL approaches on an inverted pendulum and a 2D quadrotor stabilization task.

**a) Inverted Pendulum:** The state of the pendulum is defined as \( s = [\theta, \dot{\theta}] \), and follows the dynamics

\[
\frac{ds}{dt} = \left( g \sin(\theta) + \frac{1}{m} \dot{\theta} \right)
\]

(12)

where \( a \) is the action, \( g \) is gravity, \( m \) is the mass of the pendulum, and \( l \) its length. We discretize the dynamics with the Euler method. The input lies within \( [\alpha_{\min}, \alpha_{\max}] = [-30 \text{rad s}^{-1}, 30 \text{rad s}^{-1}] \). The desired equilibrium state is \( s^* = [0, 0] \). The observation and reward are the same as for the OpenAI Gym Pendulum-V0 environment.

[5] gymnasium.farama.org/environments/classic_control/pendulum
b) 2D quadrotor: The quadrotor can only fly in the vertical x-z-plane and rotate around the y-axis with angle θ. The state of the system is defined as \( s = [x, z, \dot{x}, \dot{z}, \theta] \) and the action as \( a = [u_1, u_2] \). The system dynamics are

\[
\frac{ds}{dt} = \begin{pmatrix}
\dot{x} \\
\dot{z} \\
u_1 K \sin(\theta) + w_1 \\
-g + u_1 K \cos(\theta) + w_2 \\
d_0 \theta - d_1 \dot{\theta} + n_0 a^2
\end{pmatrix}
\] (13)

where the noise is sampled uniformly from a compact noise set \( w = [w_1, w_2] \sim W \in \mathbb{R}^2 \), and \( K, d_0, d_1, \) and \( n_0 \) are constant system parameters (see Table I). We linearize the dynamics with a first order Taylor expansion at the equilibrium point \( s^* = [0, 0, 0, 0, 0] \) and solve the differential equations for a time-discrete system. The input ranges from \( a_{\min} = [-1.5 + \frac{g}{K}, \frac{\pi}{12}] \) to \( a_{\max} = [1.5 + \frac{g}{K}, \frac{\pi}{12}] \). The reward is defined as \( r = \exp \left( -\|s - s^*\| - 0.01 \sum_{i=1}^{2} \frac{a_i}{a_{\max}[i] - a_{\min}[i]} \right) \).

B. Computation of the Safe State Set

To obtain a possibly large set of provably safe states \( S_\varphi \) and a failsafe controller for our environments, we use the scalable approach for computing robust control invariant sets presented in [56]: for every state \( s_i \in S_\varphi \), there exists a failsafe action \( \tilde{a}_i \in A \) so that \( s_{i+1} = g(s_i, \tilde{a}_i) \in S_\varphi \). Since \( S_\varphi \subseteq S \), satisfaction of our safety specifications \( s \in S \) and \( \tilde{a} \in A \) for every future point in time follows by induction. Using the algorithm in [56] provides us with an explicit representation of \( S_\varphi \) with maximized volume, which enables a fair comparison of our provably safe RL implementations.

To retrieve \( A_\varphi \) from \( S_\varphi \) at a given state \( s \), we first convert \( S_\varphi \) from a so-called generator representation used in [56] into half-space representation, i.e., \( S_\varphi = \{ s | Cs \leq d \} \) using the open-source toolbox CORA [57]. We then formulate the dynamics of the system in the form of (4) with an additional noise term \( g(s, a) = g_a(s)a + g_u(s) + w \). The noise is assumed to stem from a compact set \( w \sim W \). Finally, we reformulate the constraints as

\[
Cg_a(s)a \leq d - Cg_u(s) - Cw.
\] (14)

This results in a polytope with \( \text{dim}(d) \times 2^{\text{dim}(W)} \) halfspaces. Finally, we use pypoman\(^3\) to obtain the halfspace-representation of \( A_\varphi \).

In theory, the approach of [56] allows us to compute \( S_\varphi \) for nonlinear systems with up to 20 dimensions, such as chemical reactors. However, the conversion to halfspace representation is computationally too complex for higher dimensional systems. Therefore, we plan to develop generator-based versions of our provably safe RL methods in future work to mitigate these shortcomings.

C. Results

The differences between the provably safe RL algorithms during training is shown in Fig. 2 for discrete PPO on the 2D quadrotor task. We selected the 2D quadrotor stabilization task as it allows for a better differentiation between provably safe RL approaches than the inverted pendulum task. The difference between the three provably safe RL categories is clearest for the discrete PPO results, but we report similar trends on all other algorithms as well. The complete set of results for all algorithms and environments can be found in the Appendix.

The safety violations for the baselines in Fig. 2b show that safety can only be guaranteed when using provably safe RL. Between the baselines, TD3 converges significantly faster than all other algorithms.

The comparison between the three provably safe RL categories action replacement (sample), projection, and masking in Fig. 2c shows that replacement is the most stable and high-performing method. Action projection and masking perform similarly. To compare how active the provably safe RL methods are, we introduce the intervention rate metric. For action replacement and projection, this indicates the average rate of RL steps in which the action was altered by the safety function. For action masking, the intervention rate compares the average volume of the provably safe action set over an episode with the volume of the provably safe action set at the equilibrium point of the system, e.g., \( \frac{V_{A_\varphi, \text{episode}}}{V_{A_\varphi, \text{equi}}} \). Fig. 2b shows that action replacement converges to almost zero safety interventions, whereas projection and masking always depend on the safety method. In general, we report that a lower intervention rate often coincides with a higher reward.

We evaluate the impact of different learning tuples on the overall performance in Fig. 3. A notable result of our evaluation is that using the adapted action \( a^\varphi \), i.e., in the safe action and both tuple, is inferior to using the agent’s action when using PPO. We expect that this also transfers to other on-policy algorithms as they assume that the action used for training is sampled from the current policy. Using an action altered by the safety method breaks this paradigm. For the off-policy algorithms TD3 and DQN, shown in the Appendix, the safe action and both tuple did not improve the performance of the naive tuple. The addition of an adaption penalty reduced the intervention rate and improved the overall performance of the replacement and projection methods. Especially in the case of action replacement with a failsafe action, the adaption penalty improved the performance significantly.

IV. DISCUSSION AND CONCLUSION

The theory and experiments clearly show that provably safe RL methods are always safe, while the baselines still violate the safety results even after the reward converged. Subsequently, we discuss four statements, which result from the experiments, and conclude.

First, we want to summarize our experience of the three provably safe RL classes to give the reader an intuition when to choose which method. Replacement is the easiest method to implement for continuous action spaces. It shows very good performance and low intervention rates by the
safety method. Hereby, using a failsafe action for replacement with an adaption penalty is simple to implement and usually among the best performing methods in our experiments and is thereby recommended if available. Still, the random sampling safe actions might outperform the failsafe action. So if sampling from the safe action space is easily available, e.g., in discrete action spaces, a failsafe controller can be omitted. Projection tends to be problematic due to floating point errors and other small numerical errors, resulting in infeasible optimization problems. We therefore have to use methods like the provably safe MPC in [47], or use a failsafe controller if the optimization problem is not solvable. Together with the higher intervention rates than action replacement and the complex implementation, we would usually not recommend to use action projection. However, if one already has a CBF or MPC formulation, it might be the most suitable solution. Action masking is particularly easy to implement for discrete action spaces and shows good performance for that setting. However, action masking can be hard to implement or very restrictive for continuous action spaces, where the safe action space significantly diverges from an axis-aligned box.

Second, the naïve learning tuple usually is among the best performing tuples while no adaption of the RL algorithms is necessary. If the intervention rate should be minimized and sometimes also to improve performance, the adaption penalty tuple can be used. This adaption penalty $r_s$ is easy to integrate in the reward function, but it might require careful reward tuning for more complex environments. For the PPO case, using the safe action $\tilde{a}$ in the training tuple (i.e., configurations both and safe action) should be avoided since we observed that they are much less stable across different hyperparameters while at the same time lead to worse performance.

Third, provably safe RL converges similarly fast or faster than the baselines. However, the performance at the beginning of the training is better for provably safe RL agents. There are two factors influencing the convergence. On the one hand, the baseline agents need to explore more actions including actions that do not lead to reaching the goal. On the other hand, safe agents do not get as much information about the environment as only safe states are explored. It should be mentioned that the safety methods create a computational overhead such that convergence speed is not necessarily aligned with wall time.

Finally, the computational complexity of the three approaches highly depends on the scenario-specific implementation. For action projection, the main implementation challenge is to guarantee that the optimization problem is always feasible. If the optimization problem can be formulated as a QP problem, then the computational complexity is polynomial as shown by [58]. Contrary, the computational complexity of action replacement and action masking highly depends on the algorithm that identifies the safety of actions. For discrete action masking, the computational complexity scales linearly with the total number of actions $O(|A|)$. While for action replacement, we only need a single safe action, so in the ideal case, e.g., using a failsafe controller, the computational complexity is constant in regard to the total number of actions $O(1)$. If the action replacement approach needs to determine the entire set of safe actions, they have the same computational complexity as action masking. The computational complexity for identifying the continuous safe action space depends on the task-specific implementation. One possibility is to compute the safe action space using set-based reachability analysis, where

---

**Fig. 2.** Training curves for the 2D quadrotor benchmark. Top: Comparison of benchmark algorithms TD3, DQN, PPO continuous and discrete. Bottom: Comparison of the provably safe RL algorithms using discrete PPO. The left column depicts the reward. The right column shows the safety violations for the baselines, and the safety intervention rate for the provably safe RL algorithms.
In conclusion, we define a categorization for provably safe RL methods that comprises the literature from a learning perspective. We discuss these provably safe RL methods from a theoretical perspective and thereby define necessary assumptions. Four implementations of provably safe RL are tested and compared on a 2D quadrotor and an inverted pendulum stabilization task and provide insights on the best-suited method for each task. In future work, we would like to investigate how action projection can be adapted such that it does not suffer from numerical instabilities due to projecting on the bound of the safe action set. Furthermore, we plan to move towards a zonotope representation of the constraints instead of halfspaces to improve the computational complexity of our implementation and thereby enable a comparison on higher-dimensional environments.

V. RELATED WORK

In this section, we discuss previous work related to the provably safe RL methods presented in Sec. II.

a) Action replacement: One of the earliest provably safe RL works are [59], [60], which propose to use reachability analysis based on Hamilton-Jacobi-Isaacs (HJI) equations to guarantee the safety of a learning system. Hereby, they determine safe regions in the state space given bounded system disturbances. The safety is then guaranteed by replacing any learned action on the border of the safe set with the HJI optimal control to guide the system back inside the safe set. This was extended by [61] to a general safe RL framework based on HJI reachability analysis. The authors argue that replacing the unsafe action with the action that maximizes the distance to the unsafe set increases performance in uncertain real-world environments in comparison to action projection. However, HJI approaches suffer from the curse of dimensionality and are therefore only feasible for systems with specific characteristics [62]. Hunt et al. [37] introduce a safety monitor inside the environment that determines all discrete safe actions in every state and replaces any unsafe action from the agent with a random safe action. More specifically, they show how provably safe end-to-end learning can be accomplished using controller and model monitors. They train the RL agent using the naive learning tuple \((s, a, s', r)\), which is described in Sec. III.

Alshiekh et al. [39] combine a so-called safety shield with an agent that outputs \(n\) ranked actions. The shield checks the ranked actions, and either executes the highest-ranked safe action or chooses a random safe action if none of the \(n\) actions is safe. In contrast to [37], they update the agent with the safe action learning tuple \((s, a^s, s', r)\), but also propose that both can be used to obtain additional training information. In a similar fashion as Alshiekh et al. [39], Könighofer et al. [63] show that both probabilistic and deterministic shields increase the sample efficiency and performance for both action replacement and masking methods. He et al. [64] extend [39] and propose a learned shield that operates in a latent space and provides probabilistic safety. However, their approach cannot provide provably safety since the environment dynamics are approximated from data. Shao et al. [56] use a continuous reachability-based trajectory safeguard to evaluate the safety of actions based on reachability analysis. If the agent’s action is unsafe, \(n\) new actions are sampled randomly in the vicinity of the agent’s action, and the closest safe action to the original (unsafe) action is executed. If none of the \(n\) new actions is safe, a failsafe action is executed. The agent then trains
on the naive learning tuples \((s, a, s', r)\). Selim et al. [65] also verify the safety of actions from a continuous action space with reachability analysis. They propose an informed replacement such that the reachable sets are pushed away from the unsafe sets. They use the adaption penalty learning tuple and showcase that their approach achieves provable safety on three use cases. A further notable work is [66] that proposes a model predictive shield alongside the trained policy, which switches to a backup policy if the proposed action is unsafe. In slow-paced environments, one can also use a human in the loop to verify the actions as proposed by [41].

b) Action projection: Research on action projection is usually conducted on continuous action and state spaces. The main differentiating factor between studies in this category is the specification of the optimization problem for the projection. To begin with, the work of Pham et al. [67] guarantees safety using a differentiable constrained optimization layer, called OptLayer. Unfortunately, their approach is restricted to QP problems, so the system model and constraints have to be linear. Despite these limitations, they show the effectiveness of their approach on a collision avoidance task with a simple robotic manipulator. Cheng et al. [43] specify the safety constraints of the optimization problem via CBFs. To solve the optimization problem more efficiently, they add a neural network to the approach in Sec. II-B, which approximates the correction of the CBF controllers on this action. The action is then shifted by the approximated value prior to optimization. This improves the implementation efficiency while still guaranteeing safety, as the action is often already safe after the shift and no optimization problem needs to be solved. Their safe learning with CBFs shows faster convergence speed in comparison to vanilla RL when learning on a pendulum and a car following task. Gros et al. [45] and Wabersich et al. [38] implement the optimization problem as a robust MPC problem. Thus, safety is specified through safe sets in the state space. Wabersich et al. [38] evaluate their approach on a pendulum and a quadrotor task. Gros et al. [45] mainly discuss how the learning update has to be adapted if action projection is used for different RL algorithms. One downside of the MPC-based formulation is that dynamic constraints originating from moving objects or persons in the environment are not trivial to integrate. Li et al. [68] find a safe set through specifying safety constraints. Then, they project the action selected by the agent to the safe set. The dynamics of the system are approximated by a Gaussian process (GP), so that absolute safety is not given. However, they could guarantee absolute safety if they would assume bounded dynamics of the environment as the aforementioned approaches do.

c) Action masking: To the best of our knowledge, all existing literature considers action masking only for discrete action spaces. Studies that discuss and use action masking mainly differ in the verification of actions. The studies [69], [70] introduce monitors for the entire system and a controller based on differential dynamic logic. The monitors are then used to determine the set of safe actions from which the agent can choose. To identify the correct system can be challenging, thus an approach to automatically generate candidates is introduced as well. Mason et al. [71] identify an abstract safety MDP from the original MDP. Combining this abstract MDP with probabilistic safety constraints, they synthesize a policy that restricts the exploration. They evaluate their approach on a guarded flag collection task and on an assistance for dementia sufferer. However, their approach is not easily extendable to continuous observation spaces. In addition to the aforementioned works, there are works investigating action masking for autonomous driving tasks [50], [72], [73] and for games [51].

ACKNOWLEDGMENTS

The authors gratefully acknowledge the partial financial support of this work by the research training group ConVeY funded by the German Research Foundation under grant GRK 2428, by the project TRAITS funded by the German Federal Ministry of Education and Research, and by the Horizon 2020 EU Framework Project CONCERT under grant 101016007.

APPENDIX

MDP modification with action replacement

Action replacement alters the MDP on which the agent learns. Hunt et al. [67] discuss this modification for discrete action spaces and uniformly sampling from the safe action space. We generalize this discussion to use any replacement function and also include continuous action spaces. We define \( \psi(s) \) so that it randomly samples the replacement action \( \tilde{a} \) according to a replacement policy \( \pi_r(\tilde{a}|s) \) with \( \sum_{\tilde{a} \in \mathcal{A}_r(s)} \pi_r(\tilde{a}|s) = 1 \) for the discrete case, and \( \int_{\mathcal{A}_r(s)} \pi_r(\tilde{a}|s) \, d\tilde{a} = 1 \) for the continuous case, and \( \pi_r(\tilde{a}|s) \geq 0 \forall \tilde{a} \in \mathcal{A}_r(s) \). In the example of uniform sampling from \( \mathcal{A}_r(s) \), the replacement policy is \( \pi_r(\tilde{a}|s) = 1/|\mathcal{A}_r(s)| \). By replacing unsafe actions, the transition function of the MDP changes to

\[
T_\varphi(s, a, s') = \begin{cases} 
T(s, a, s'), & \text{if } \varphi(s, a) = 1 \\
T_r(s, s'), & \text{otherwise},
\end{cases}
\]

The reward function of the MDP changes accordingly to

\[
r_\varphi(s, a) = \begin{cases} 
r(s, a), & \text{if } \varphi(s, a) = 1 \\
r_r(s), & \text{otherwise},
\end{cases}
\]

In the continuous case, we get \( T_r(s, s') \) by marginalizing the transition probability density function over \( \mathcal{A}_r(s) \)

\[
T_r(s, s') = \int_{\mathcal{A}_r(s)} \pi_r(\tilde{a}|s) T(s, \tilde{a}, s') \, d\tilde{a},
\]

and \( r_r(s) \) analogously

\[
r_r(s) = \int_{\mathcal{A}_r(s)} \pi_r(\tilde{a}|s) r(s, \tilde{a}) \, d\tilde{a}.
\]

ENVIRONMENT PARAMETERS

We give an overview of all environment-specific parameters in Table I and Table II.
**TABLE I**
PENDULUM ENVIRONMENT PARAMETERS.

| Parameter     | Value       |
|---------------|-------------|
| Gravity $g$   | 9.81 m s$^{-2}$ |
| Mass $m$     | 1 kg        |
| Length $l$   | 1 m         |

**TABLE II**
2D QUADROTOR ENVIRONMENT PARAMETERS.

| Parameter     | Value       |
|---------------|-------------|
| Gravity $g$   | 9.81 m s$^{-2}$ |
| $K$           | 11/1 kg     |
| $d_0$         | 70          |
| $d_1$         | 17          |
| $\tau_0$     | 55          |
| $W$           | $[-0.1, 0.1], [-0.1, 0.1]$ |

**HYPERPARAMETERS FOR LEARNING ALGORITHMS**

We specify the hyperparameters for all learning algorithms (see Table III for PPO, Table IV for TD3, and Table V for DQN) that are different from the Stable Baselines3 [55] default values. Additionally, the code for the experiments is submitted as supplementary material to further increase the transparency.

**TABLE III**
HYPERPARAMETERS FOR PPO.

| Parameter                  | Pendulum | 2D quadrotor |
|----------------------------|----------|--------------|
| Learning rate              | 1E$^{-4}$| 5E$^{-5}$    |
| Discount factor $\gamma$  | 0.98     | 0.999        |
| Steps per update           | 2048     | 512          |
| Optimization epochs        | 20       | 30           |
| Minibatch size             | 16       | 128          |
| Max gradient clipping      | 0.9      | 0.5          |
| Entropy coefficient        | 1E$^{-3}$| 2E$^{-6}$    |
| Value function coefficient | 0.045    | 0.5          |
| Clipping range             | 0.3      | 0.1          |
| GAE $\lambda$             | 0.8      | 0.92         |
| Activation function        | ReLU     | ReLU         |
| Hidden layers              | 2        | 2            |
| Neurons per layer          | 32       | 64           |
| Training steps             | 60k      | 100k         |

**TABLE IV**
HYPERPARAMETERS FOR TD3.

| Parameter                  | Pendulum | 2D quadrotor |
|----------------------------|----------|--------------|
| Learning rate              | 3.5E$^{-3}$| 2E$^{-3}$    |
| Replay buffer size         | 1E4      | 1E5          |
| Discount factor $\gamma$  | 0.98     | 0.98         |
| Initial exploration steps  | 10E3     | 100          |
| Steps between model updates| 256      | 5            |
| Gradient steps per model update | 256 | 10          |
| Minibatch size per gradient step | 512 | 512         |
| Soft update coefficient $\tau$ | 5E$^{-3}$| 5E$^{-3}$    |
| Gaussian smoothing noise $\sigma$ | 0.2 | 0.12        |
| Activation function        | ReLU     | ReLU         |
| Hidden layers              | 2        | 2            |
| Neurons per layer          | 32       | 64           |
| Training steps             | 60k      | 100k         |

**TABLE V**
HYPERPARAMETERS FOR DQN.

| Parameter                  | Pendulum | 2D quadrotor |
|----------------------------|----------|--------------|
| Learning rate              | 2E$^{-3}$| 1E$^{-4}$    |
| Replay buffer size         | 5E4      | 1E6          |
| Discount factor $\gamma$  | 0.95     | 0.99999      |
| Initial exploration steps  | 500      | 100          |
| Steps between model updates| 8        | 2            |
| Gradient steps per model update | 4   | 4            |
| Minibatch size per gradient step | 512 | 64           |
| Maximum for gradient clipping | 10   | 100          |
| Update frequency target network | 1E3 | 1E3          |
| Initial exploration probability $\epsilon$ | 1.0 | 0.137       |
| Linear interpolation steps of $\epsilon$ | 6E3 | 1E4          |
| Final exploration probability $\epsilon$ | 0.1 | 0.004       |
| Activation function        | Tanh     | Tanh         |
| Hidden layers              | 2        | 2            |
| Neurons per layer          | 32       | 64           |
| Training steps             | 60k      | 100k         |
Fig. 4. **Pendulum**: Average reward and standard deviation per training step for TD3, DQN, PPO discrete, and PPO continuous. For each configuration, five training runs with different random seeds were conducted. Each subplot contains all implemented variants. Note that for better comparability the reward for the *adaption penalty* variants is still $r$ and the adaption penalty $r^*$ is not included in the curves.
Fig. 5. 2D quadrotor: Average reward and standard deviation per training step for TD3, DQN, PPO discrete, and PPO continuous. For each configuration, five training runs with different random seeds were conducted. Each subplot contains all implemented variants. Note that for better comparability the reward for the adaption penalty variants is still $r$ and the adaption penalty $r^*$ is not included in the curves.
Fig. 6. **Pendulum**: Intervention rate for TD3, DQN, PPO discrete, and PPO continuous. Fig. (a) - (c) show the intervention rate of TD3 agents, (d) - (f) for DQN agents, (g) - (i) for discrete PPO agents, and (j) - (l) for continuous PPO agents.
Fig. 7. **2D quadrotor:** Intervention rate for TD3, DQN, PPO discrete, and PPO continuous. Fig. (a) - (c) show the intervention rate of TD3 agents, (d) - (f) for DQN agents, (g) - (i) for discrete PPO agents, and (j) - (l) for continuous PPO agents.
| Approach | Reward | | | | | | Safety Violation | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Mean | Std. Dev. | Mean | Std. Dev. | Mean | Std. Dev. | | | | | | |
| **PPO** (**continuous** | | | | | | | | | | | | |
| Projection (SafeAction) | -1.14 | 0.42 | 0.76 | 0.29 | 0.00 | 0.00 | | | | | | |
| Projection (AdaptationPenalty) | -0.07 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Projection (Both) | -0.07 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Projection (Naive) | -0.06 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Sample (SafeAction) | -0.13 | 0.16 | 0.01 | 0.02 | 0.00 | 0.00 | | | | | | |
| Sample (AdaptationPenalty) | -0.07 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Sample (Both) | -0.07 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Sample (Naive) | -0.07 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| FailSafe (AdaptationPenalty) | -0.07 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| FailSafe (Naive) | -0.07 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Baseline (Naive) | -0.06 | 0.07 | — | — | 0.00 | 0.00 | | | | | | |
| Masking (AdaptationPenalty) | -0.07 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Masking (Naive) | -0.07 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| **PPO** (**discrete** | | | | | | | | | | | | |
| Projection (SafeAction) | -0.52 | 0.55 | 0.28 | 0.42 | 0.00 | 0.00 | | | | | | |
| Projection (AdaptationPenalty) | -0.14 | 0.12 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Projection (Both) | -0.09 | 0.09 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Projection (Naive) | -0.15 | 0.27 | 0.03 | 0.10 | 0.00 | 0.00 | | | | | | |
| Sample (SafeAction) | -0.09 | 0.08 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Sample (AdaptationPenalty) | -0.13 | 0.16 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Sample (Both) | -0.10 | 0.08 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Sample (Naive) | -0.09 | 0.08 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| FailSafe (AdaptationPenalty) | -0.09 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| FailSafe (Naive) | -0.08 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Baseline (Naive) | -0.08 | 0.07 | — | — | 0.00 | 0.00 | | | | | | |
| Masking (AdaptationPenalty) | -0.07 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Masking (Naive) | -0.07 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| **TD3** | | | | | | | | | | | | |
| Projection (SafeAction) | -0.07 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Projection (AdaptationPenalty) | -0.08 | 0.08 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Projection (Both) | -0.07 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Projection (Naive) | -0.07 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Sample (SafeAction) | -0.07 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Sample (AdaptationPenalty) | -0.09 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Sample (Both) | -0.09 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Sample (Naive) | -0.07 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| FailSafe (AdaptationPenalty) | -0.09 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| FailSafe (Naive) | -0.07 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Baseline (Naive) | -0.07 | 0.07 | — | — | 0.00 | 0.00 | | | | | | |
| Masking (AdaptationPenalty) | -0.07 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Masking (Naive) | -0.07 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| **DQN** | | | | | | | | | | | | |
| Projection (SafeAction) | -0.07 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Projection (AdaptationPenalty) | -0.07 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Projection (Both) | -0.07 | 0.08 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Projection (Naive) | -0.07 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Sample (SafeAction) | -0.07 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Sample (AdaptationPenalty) | -0.09 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Sample (Both) | -0.07 | 0.08 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Sample (Naive) | -0.07 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| FailSafe (AdaptationPenalty) | -0.07 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| FailSafe (Naive) | -0.07 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Baseline (Naive) | -0.07 | 0.07 | — | — | 0.00 | 0.00 | | | | | | |
| Masking (AdaptationPenalty) | -0.07 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Masking (Naive) | -0.07 | 0.07 | 0.00 | 0.00 | 0.00 | 0.00 | | | | | | |
| Approach | Reward | Intervention Rate | Safety Violation |
|----------|-----------------|------------------|-----------------|
|          | Mean | Std. Dev. | Mean | Std. Dev. | Mean | Std. Dev. |
| **PPO (continuous)** | | | | | | |
| Projection (SafeAction) | -0.44 | 0.00 | 0.68 | 0.00 | 0.000 | 0.000 |
| Projection (AdaptionPenalty) | -0.33 | 0.07 | 0.44 | 0.05 | 0.000 | 0.000 |
| Projection (Both) | -0.39 | 0.07 | 0.57 | 0.13 | 0.000 | 0.000 |
| Projection (Naive) | -0.31 | 0.10 | 0.47 | 0.25 | 0.000 | 0.000 |
| Sample (SafeAction) | -0.43 | 0.00 | 0.86 | 0.00 | 0.000 | 0.000 |
| Sample (AdaptionPenalty) | -0.28 | 0.12 | 0.10 | 0.12 | 0.000 | 0.000 |
| Sample (Both) | -0.36 | 0.03 | 0.41 | 0.20 | 0.000 | 0.000 |
| Sample (Naive) | -0.39 | 0.06 | 0.23 | 0.13 | 0.000 | 0.000 |
| FailSafe (AdaptionPenalty) | -0.31 | 0.11 | 0.08 | 0.06 | 0.000 | 0.000 |
| FailSafe (Naive) | -0.27 | 0.11 | 0.27 | 0.29 | 0.000 | 0.000 |
| Baseline (Naive) | -0.86 | 0.01 | — | — | 0.941 | 0.009 |
| Masking (AdaptionPenalty) | -0.43 | 0.09 | 0.57 | 0.28 | 0.000 | 0.000 |
| Masking (Naive) | -0.43 | 0.09 | 0.57 | 0.28 | 0.000 | 0.000 |
| **PPO (discrete)** | | | | | | |
| Projection (SafeAction) | -0.44 | 0.01 | 0.74 | 0.21 | 0.000 | 0.000 |
| Projection (AdaptionPenalty) | -0.24 | 0.13 | 0.02 | 0.02 | 0.000 | 0.000 |
| Projection (Both) | -0.35 | 0.11 | 0.16 | 0.14 | 0.000 | 0.000 |
| Projection (Naive) | -0.34 | 0.14 | 0.46 | 0.37 | 0.000 | 0.000 |
| Sample (SafeAction) | -0.42 | 0.02 | 0.82 | 0.01 | 0.000 | 0.000 |
| Sample (AdaptionPenalty) | -0.11 | 0.10 | 0.00 | 0.00 | 0.000 | 0.000 |
| Sample (Both) | -0.38 | 0.09 | 0.32 | 0.18 | 0.000 | 0.000 |
| Sample (Naive) | -0.13 | 0.16 | 0.02 | 0.03 | 0.000 | 0.000 |
| FailSafe (AdaptionPenalty) | -0.19 | 0.15 | 0.01 | 0.03 | 0.000 | 0.000 |
| FailSafe (Naive) | -0.34 | 0.13 | 0.41 | 0.21 | 0.000 | 0.000 |
| Baseline (Naive) | -0.11 | 0.03 | — | — | 0.000 | 0.000 |
| Masking (AdaptionPenalty) | -0.25 | 0.14 | 0.28 | 0.21 | 0.000 | 0.000 |
| Masking (Naive) | -0.25 | 0.14 | 0.28 | 0.21 | 0.000 | 0.000 |
| **TD3** | | | | | | |
| Projection (SafeAction) | -0.21 | 0.03 | 0.28 | 0.10 | 0.000 | 0.000 |
| Projection (AdaptionPenalty) | -0.22 | 0.03 | 0.26 | 0.10 | 0.000 | 0.000 |
| Projection (Both) | -0.21 | 0.04 | 0.28 | 0.12 | 0.000 | 0.000 |
| Projection (Naive) | -0.25 | 0.05 | 0.26 | 0.17 | 0.000 | 0.000 |
| Sample (SafeAction) | -0.18 | 0.03 | 0.07 | 0.01 | 0.000 | 0.000 |
| Sample (AdaptionPenalty) | -0.21 | 0.04 | 0.13 | 0.05 | 0.000 | 0.000 |
| Sample (Both) | -0.21 | 0.06 | 0.06 | 0.03 | 0.000 | 0.000 |
| Sample (Naive) | -0.19 | 0.04 | 0.12 | 0.09 | 0.000 | 0.000 |
| FailSafe (AdaptionPenalty) | -0.20 | 0.03 | 0.05 | 0.02 | 0.000 | 0.000 |
| FailSafe (Naive) | -0.26 | 0.09 | 0.05 | 0.02 | 0.000 | 0.000 |
| Baseline (Naive) | -0.90 | 0.05 | — | — | 0.945 | 0.018 |
| Masking (AdaptionPenalty) | -0.16 | 0.03 | 0.03 | 0.03 | 0.000 | 0.000 |
| Masking (Naive) | -0.16 | 0.03 | 0.03 | 0.03 | 0.000 | 0.000 |
| **DQN** | | | | | | |
| Projection (SafeAction) | -0.06 | 0.02 | 0.00 | 0.01 | 0.000 | 0.000 |
| Projection (AdaptionPenalty) | -0.05 | 0.00 | 0.00 | 0.00 | 0.000 | 0.000 |
| Projection (Both) | -0.05 | 0.01 | 0.00 | 0.00 | 0.000 | 0.000 |
| Projection (Naive) | -0.09 | 0.06 | 0.12 | 0.24 | 0.000 | 0.000 |
| Sample (SafeAction) | -0.07 | 0.01 | 0.01 | 0.02 | 0.000 | 0.000 |
| Sample (AdaptionPenalty) | -0.06 | 0.03 | 0.00 | 0.00 | 0.000 | 0.000 |
| Sample (Both) | -0.07 | 0.02 | 0.00 | 0.00 | 0.000 | 0.000 |
| Sample (Naive) | -0.05 | 0.01 | 0.00 | 0.00 | 0.000 | 0.000 |
| FailSafe (AdaptionPenalty) | -0.06 | 0.02 | 0.02 | 0.04 | 0.000 | 0.000 |
| FailSafe (Naive) | -0.07 | 0.03 | 0.10 | 0.10 | 0.000 | 0.000 |
| Baseline (Naive) | -0.24 | 0.37 | — | — | 0.199 | 0.398 |
| Masking (AdaptionPenalty) | -0.15 | 0.16 | 0.14 | 0.26 | 0.000 | 0.000 |
| Masking (Naive) | -0.15 | 0.16 | 0.14 | 0.26 | 0.000 | 0.000 |
[42] A. Taylor, A. Singletary, Y. Yue, and A. Ames, “Learning for Safety-Critical Control with Control Barrier Functions,” in Proc. of the 2nd Conf. on Learning for Dynamics and Control, 2020, pp. 708–717.

[43] R. Cheng, G. Oroz, R. M. Murray, and J. W. Burdick, “End-to-End Safe Reinforcement Learning through Barrier Functions for Safety-Critical Continuous Control Tasks,” in Proc. of the AAAI Conf. on Artificial Intelligence, 2019, pp. 3387–3395.

[44] L. Hewing, K. P. Wabersich, M. Menner, and M. N. Zeilinger, “Learning-Based Model Predictive Control: Toward Safe Learning in Control,” Annual Review of Control, Robotics, and Autonomous Systems, vol. 3, no. 1, pp. 269–296, 2020.

[45] S. Gros, M. Zanon, and A. Bemporad, “Safe Reinforcement Learning via Projection on a Safe Set: How to Achieve Optimality?” IFAC-PapersOnLine, vol. 53, no. 2, pp. 8076–8081, 2020.

[46] P. Wieland and F. Allgöwer, “Constructive safety using control barrier functions,” IFAC Proceedings Volumes, vol. 40, no. 12, pp. 462–467, 2007.

[47] B. Schürmann, N. Kochdumper, and M. Althoff, “ReachSet Model Predictive Control for Disturbed Nonlinear Systems,” in Proc. of the IEEE Conf. on Decision and Control (CDC), 2018, pp. 3463–3470.

[48] A. J. Taylor, A. Singletary, Y. Yue, and A. D. Ames, “A Control Barrier Perspective on Episodic Learning via Projection-to-State Safety,” IEEE Control Systems Letters, vol. 5, no. 3, pp. 1019–1024, 2021.

[49] X. Wang, “Ensuring safety of learning-based motion planners using control barrier functions,” IEEE Robotics and Automation Letters, vol. 7, no. 2, pp. 4773–4780, 2022.

[50] H. Krasowski, X. Wang, and M. Althoff, “Safe Reinforcement Learning for Autonomous Lane Changing Using Set-Based Prediction,” in Proc. of the IEEE 23rd Int. Conf. on Intelligent Transportation Systems (ITSC), 2020, pp. 1–7.

[51] S. Huang and S. Ontañón, “A Closer Look at Invalid Action Masking in Policy Gradient Algorithms,” arXiv, vol. abs/2006.1, 2020.

[52] R. S. Sutton and A. G. Barto, Reinforcement Learning: An Introduction, 2nd ed. A Bradford Book, 2018.

[53] J. Schulman, F. Wolski, P. Dhariwal, A. Radford, and O. Klimov, “Proximal Policy Optimization Algorithms,” arXiv, vol. abs/1707.0, 2017.

[54] S. Fujimoto, H. Van Hoof, and D. Meger, “Addressing Function Approximation Error in Actor-Critic Methods,” in Proc. of the 35th Int. Conf. on Machine Learning (ICML), 2018, pp. 2587–2601.

[55] A. Raffin, A. Hill, A. Gleave, A. Kanervisto, M. Ernestus, and N. Dormann, “Stable-Baselines3: Reliable Reinforcement Learning Implementations,” Journal of Machine Learning Research, vol. 22, no. 268, pp. 1–8, 2021.

[56] L. Schäfer, F. Gruber, and M. Althoff, “Scalable computation of robust control invariant sets of nonlinear systems,” accepted at IEEE Transactions on Automatic Control, 2023.

[57] M. Althoff, “An Introduction to CORA 2015,” in Proc. of the Workshop on Applied Verification for Continuous and Hybrid Systems, 2015, pp. 120–151.

[58] S. A. Vavasis, “Complexity Theory: Quadratic Programming,” in Encyclopedia of Optimization, 2001, pp. 304–307.

[59] J. H. Gillula and C. J. Tomlin, “Guaranteed safe online learning of a bounded system,” in Proc. of the IEEE/RSJ Int. Conf. on Intelligent Robots and Systems (IROS), 2011, pp. 2979–2984.

[60] ——, “Guaranteed safe online learning via reachability: Tracking a ground target using a quadrotor,” in Proc. of the IEEE Int. Conf. on Robotics and Automation (ICRA), 2012, pp. 2723–2730.

[61] J. F. Fisac, A. K. Akametalu, M. N. Zeilinger, S. Kaynama, J. Gillula, and C. J. Tomlin, “A General Safety Framework for Learning-Based Control in Uncertain Robotic Systems,” IEEE Transactions on Automatic Control, vol. 64, no. 7, pp. 2737–2752, 2019.

[62] S. Herbert, J. J. Choi, S. Sanjeev, M. Gibson, K. Sreenath, and C. J. Tomlin, “Scalable learning of safety guarantees for autonomous systems using hamilton-jacobi reachability,” in Proc. of the IEEE Int. Conf. on Robotics and Automation (ICRA), 2021, pp. 5914–5920.

[63] B. Könighofer, F. Lorber, N. Jansen, and R. Bloem, “Shield Synthesis for Reinforcement Learning,” in Leveraging Applications of Formal Methods, Verification and Validation: Verification Principles, 2020, pp. 290–306.

[64] P. He, B. G. Leön, and F. Belardinelli, “Do androids dream of electric fences? safety-aware reinforcement learning with latent shielding,” in Proceedings of the Workshop on Artificial Intelligence Safety, ser. CEUR Workshop Proceedings, vol. 3087, 2022.

[65] M. Selim, A. Alauwar, S. Kousik, G. Gao, M. Pavone, and K. H. Johansson, “Safe reinforcement learning using black-box reachability analysis,” IEEE Robotics and Automation Letters, vol. 7, no. 4, pp. 10665–10672, 2022.