A New Exponential Approach for Reducing the Mean Squared Errors of the Estimators of Population Mean Using Conventional and Non-Conventional Location Parameters

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A New Exponential Approach for Reducing the Mean Squared Errors of the Estimators of Population Mean Using Conventional and Non-Conventional Location Parameters

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Classes of ratio-type estimators $t$ (say) and ratio-type exponential estimators $t_e$ (say) of the population mean are proposed, and their biases and mean squared errors under large sample approximation are presented. It is the class of ratio-type exponential estimators $t_e$ provides estimators more efficient than the ratio-type estimators.

**Keywords:** study variable, auxiliary variable, bias, mean squared error.

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**Introduction**

The use of auxiliary information at the estimation stage of a survey improves the precision of the estimate(s) of the parameter(s) under investigation. The problem of estimating the population mean or total using population mean of an auxiliary variable has been extensively discussed. Out of many ratio, product and regression methods of estimation are good examples in this context. The ratio method of estimation is most effective for estimating population mean of the study variable when there is a linear relationship between study variable and auxiliary variable and they have the positive (high) correlation. However, if the correlation between study variable and auxiliary variable is negative (high) the product method of estimation can be employed.

Let $U = (U_1, U_2, \ldots, U_N)$ be the finite population of size $N$ and the variables under study and auxiliary be denoted by $y$ and $x$ respectively. Let $\left(\bar{Y}, \bar{X}\right)$ be the population means of $(y, x)$ respectively. It is desired to estimate the population mean $\bar{Y}$...
using information on population parameters such as mean ($\bar{X}$), coefficient of variation ($C_x$), coefficient of skewness ($\beta_1(x)$), kurtosis ($\beta_2(x)$), deciles, quartiles, median, midrange (MR), Walsh average (i.e. Hodges-Lehman estimator) (HL) (and tri mean (TM) etc, associated with auxiliary variable $x$ and the correlation coefficient $\rho$ between $y$ and $x$. In this context, the reader is referred to Searls (1964), Das and Tripathi (1980), Sisodia and Dwivedi (1981), Upadhyaya and Singh (1999), Singh and Tailor (2003), Singh et al (2004), Kadilar and Cingi (2004, 2006), Yan and Tian (2010), Subramani and Kumarapandian (2012a, 2012b, 2012c), Jeelani et al (2013), Ekpenyoung and Enang (2015), Subramani et al (2015) and Abid et al (2016a,b,c).

Define:

$N$ : Population size.

$n$ : Sample size.

$f = \frac{n}{N}$ : Sampling fraction.

$\theta = \frac{1 - f}{n}$.

$S_y^2 = (N - 1)^{-1} \sum_{i=1}^{N} (y_i - \bar{Y})^2$: Population Variance of the study variable $y$.

$S_x^2 = (N - 1)^{-1} \sum_{i=1}^{N} (x_i - \bar{X})^2$: Population variance of the auxiliary variable $x$.

$C_y = S_y / \bar{Y}$: Coefficient of variation of the study variable $y$.

$C_x = S_x / \bar{X}$: Coefficient of variation of the auxiliary variable $x$.

$S_{xy} = (N - 1)^{-1} \sum_{i=1}^{N} (x_i - \bar{X})(y_i - \bar{Y})$: Covariance between $y$ and $x$.

$\rho = S_{xy} / (S_x S_y)$: Correlation coefficient between $x$ and $y$.

$C = \rho C_y / C_x$,

$M_d$: Population median of $x$.

$Q_i$: $i^{th}$ population quartile ($i$=1,2,3).

$T_m = \frac{(Q_1 + Q_2 + Q_3)}{2}$ : Tri mean.
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\[ H_i = \text{median}(X_j + X_k) / 2, 1 \leq j \leq k \leq N \] Hodges-Lehmann estimator.

\[ X_{(i)}: \text{Lowest order statistic in a population of size } N, \]
\[ X_{(N)}: \text{Highest order statistic in a population of size } N, \]
\[ M_r = \frac{X_{(i)} + X_{(N)}}{2}: \text{Mid range,} \]
\[ Q_r = (Q_3 - Q_1): \text{Inter-quartile range,} \]
\[ Q_d = \frac{Q_3 - Q_1}{2}: \text{Semi-quartile range,} \]
\[ \beta_1(x) = \frac{N \sum_{i=1}^{N} (x_i - \bar{X})^3}{(N-1)(N-2)S_x^3}: \text{Coefficient of skewness of the auxiliary variable } x, \]
\[ \beta_2(x) = \left[ \frac{N(N+1) \sum_{i=1}^{N} (x_i - \bar{X})^4}{(N-1)(N-2)(N-3)S_x^4} - \frac{3(N-1)^2}{(N-2)(N-3)} \right]: \text{Coefficient of kurtosis of } x, \]
\[ \Delta = \left[ \beta_2(x) - \beta_1^2(x) + \frac{2(N-1)^2}{(N-2)(N-3)} \right], \]
\[ Q_{da} = \frac{Q_3 + Q_1}{2}: \text{Quartile average,} \]
\[ R = \frac{Y}{X}: \text{Population ratio of means,} \]
\[ G = \frac{4}{N-1} \sum_{i=1}^{N} \frac{(2i - N - 1)}{2N} X_{(i)}: \text{Gini’s Mean Difference,} \]
\[ D = \frac{2\sqrt{\pi}}{N(N-1)} \sum_{i=1}^{N} \frac{i - N + 1}{2} X_{(i)}: \text{Downton’s method,} \]
\[ S_{pw} = \frac{\sqrt{\pi}}{N^2} \sum_{i=1}^{N} (2i - N - 1) X_{(i)}: \text{Probability Weighted Moments,} \]
\[ \mu_r(x) = \left( \frac{1}{N} \right) \sum_{i=1}^{N} (x_i - \bar{X})^r: \text{r being non-negative integer.} \]
We are interested in estimating the population mean $\bar{Y}$ of the study variable $y$ (taking value $y_i$ for $i=1, 2, \ldots, N$) from a simple random sample size $n$ drawn without replacement from the population $U$. We use the notation $\bar{y}$ and $\bar{x}$ for the sample means, which are unbiased estimators of the population mean $\bar{Y}$ and $\bar{X}$, respectively. We also denote:

$$s_{yx} = (n-1)^{-1} \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$$: Sample covariance between $y$ and $x$.

$$s_x^2 = (n-1)^{-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$$: Sample variance of $x$.

$$\hat{\beta} = \frac{s_{yx}}{s_x^2}$$: Sample regression coefficient estimate of the population regression coefficient $\beta = \frac{s_{yx}}{s_x^2}$ of $y$ on $x$.

$$\hat{R} = \frac{\bar{y}}{\bar{x}}$$: Ratio of sample means.

$$\hat{Y} = [\bar{Y} + \hat{\beta}(\bar{X} - \bar{x})]$$: Regression estimator of the population mean $\bar{Y}$.

### Existing Modified Ratio Estimators and The Suggested Class of Ratio Estimators

The usual unbiased estimator for population mean $\bar{Y}$ is defined by

$$t_0 = \bar{y}.$$ (1)

whose MSE is given by

$$MSE(\bar{y}) = \frac{(1-f)}{n} S_x^2.$$ (2)

The classical ratio estimator for the population mean $\bar{Y}$ in presence of known population mean $\bar{X}$ of the auxiliary variable $x$ is defined by

$$\bar{y}_R = \frac{\bar{y}}{\bar{x}} \bar{x} \neq 0.$$ (3)

To the first degree of approximation, the bias and MSE of the ratio estimator $\bar{y}_R$ are respectively given by

$$B(\bar{y}_R) = \frac{(1-f)}{n} \cdot \frac{1}{\bar{X}} \left( RS_x^2 - \rho S_x S_y \right)$$ (4)
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and

\[ \text{MSE}(\bar{y}_r) = \frac{(1-f)}{n} \left( S_y^2 + R^2 S_x^2 - 2 R \rho S_y S_x \right). \]  \hspace{1cm} (5)

In Table 1, the modified versions of the ratio estimator reported by Kadilar and Cingi (2004) are given, Kadilar and Cingi (2006) – type estimator, Yan and Tian (2010), Subramani and Kumarapandiyan (2012a, 2012b, 2012c, 2012d), Jeelani et al. (2013) and Abid et al. (2016) along with their biases and mean squared errors (MSEs) to the first degree of approximation, as reported in Abid et al. (2016).

Note the estimators \( t_j \) (\( j = 1 \) to 65) members of the following class of estimators of the population mean \( \bar{Y} \) defined by

\[ t = \hat{\beta} \left( \frac{a \bar{X} + b}{a \bar{x} + b} \right) = \left( \frac{\hat{\beta}}{\bar{X} - \bar{x}} \right) \left( \frac{a \bar{X} + b}{a \bar{x} + b} \right), \]  \hspace{1cm} (6)

where \( \hat{\beta} \) is the sample estimate of the population regression coefficient \( \beta \) of \( y \) on \( x \), \( a(\neq 0) \) and \( b \) are real numbers (constants) or the functions of population parameters such as population total \( X(= N\bar{X}) \), population standard deviation \( \sigma_x \), variance \( \sigma^2_x \), coefficient of variation \( c_x \),

Coefficient of skewness \( \beta_1(x) \) and kurtosis \( \beta_2(x) \), correlation coefficient \( \rho \), \( \Delta \), quartiles, deciles, median, mode, midrange, Trimean and Hodgs-Lehmann (HL) estimator etc.

Some unknown members of the suggested class of ratio-type estimators \( t \) are given in Table 2

To obtain the bias and mean squared error of the proposed class of estimators ‘t’ we write

\[ \bar{y} = \bar{Y} (1 + e_0), \bar{x} = \bar{X} (1 + e_1), s_{xy} = S_{xy} (1 + e_2), s_x^2 = S_x^2 (1 + e_3) \]

such that

\[ E(e_i) = 0 \text{ for all } i = 1, 2, 3; \]

and

\[ E(e_0^2) = \frac{(1-f)}{n} C_y^2, \quad E(e_1^2) = \frac{(1-f)}{n} C_x^2, \quad E(e_0 e_1) = \frac{(1-f)}{n} C C_x, \]

\[ E(e_0 e_2) = \frac{N(N-n)}{(N-1)(N-2)} \frac{\mu_{21}}{n} \frac{1}{\bar{X} S_{xy}} = \frac{(N-n)}{n(N-2)} \frac{\mu_{21}}{\bar{X} \mu_{11}}, \]

\[ E(e_1 e_3) = \frac{N(N-n)}{(N-1)(N-2)} \frac{\mu_{21}}{n} \frac{1}{\bar{X} S_{xy}} = \frac{(N-n)}{n(N-2)} \frac{\mu_{21}}{\bar{X} \mu_{11}}, \]
| S.No. | Estimator | MSE of ( ) | Population Ratio |
|-------|-----------|------------|------------------|
| 1.    | $\hat{Y} = \frac{\hat{Y}}{\hat{X}} \cdot \bar{X}$ | $\theta \left[ R_x^2 S_r^2 + S_r^2 (1 - \rho^2) \right]$ | $R_i = \frac{\bar{Y}}{\bar{X}} = R$ |
|       |          |            | **Kadilar and Cingi (2004)** |
| 2.    | $\hat{Y} = \frac{\hat{Y}}{(X + C_1)} \cdot (\bar{X} + C_1)$ | $\theta \left[ R_x^2 S_r^2 + S_r^2 (1 - \rho^2) \right]$ | $R_i = \frac{\bar{Y}}{(X + C_1)}$ |
|       |          |            | **Kadilar and Cingi (2004)** |
| 3.    | $\hat{Y} = \frac{\hat{Y}}{(X + \beta_1(x))} \cdot (\bar{X} + \beta_1(x))$ | $\theta \left[ R_x^2 S_r^2 + S_r^2 (1 - \rho^2) \right]$ | $R_i = \frac{\bar{Y}}{(X + \beta_1(x))}$ |
|       |          |            | **Kadilar and Cingi (2004)** |
| 4.    | $\hat{Y} = \frac{\hat{Y}}{(X \beta_1(x) + C_1)} \cdot (\bar{X} \beta_1(x) + C_1)$ | $\theta \left[ R_x^2 S_r^2 + S_r^2 (1 - \rho^2) \right]$ | $R_i = \frac{\bar{Y} \beta_1(x)}{(X \beta_1(x) + C_1)}$ |
|       |          |            | **Kadilar and Cingi (2004)** |
| 5.    | $\hat{Y} = \frac{\hat{Y}}{(\bar{X} \bar{C}_r + \beta_1(x))} \cdot (\bar{X} \bar{C}_r + \beta_1(x))$ | $\theta \left[ R_x^2 S_r^2 + S_r^2 (1 - \rho^2) \right]$ | $R_i = \frac{\bar{Y} \beta_1(x)}{(\bar{X} \beta_1(x) + \bar{C}_r)}$ |
|       |          |            | **Kadilar and Cingi (2004)** |
| 6.    | $\hat{Y} = \frac{\hat{Y}}{(X + \rho)} \cdot (\bar{X} + \rho)$ | $\theta \left[ R_x^2 S_r^2 + S_r^2 (1 - \rho^2) \right]$ | $R_i = \frac{\bar{Y}}{(X + \rho)}$ |
|       |          |            | **Kadilar and Cingi (2006) –type** |
| 7.    | $\hat{Y} = \frac{\hat{Y}}{(\bar{X} \beta_1(x) + \rho)} \cdot (\bar{X} \beta_1(x) + \rho)$ | $\theta \left[ R_x^2 S_r^2 + S_r^2 (1 - \rho^2) \right]$ | $R_i = \frac{\bar{Y} \beta_1(x)}{(\bar{X} \beta_1(x) + \rho)}$ |
|       |          |            | **Kadilar and Cingi (2006) –type** |
| 8.    | $\hat{Y} = \frac{\hat{Y}}{(X \beta_1(x) + \rho)} \cdot (\bar{X} \beta_1(x) + \rho)$ | $\theta \left[ R_x^2 S_r^2 + S_r^2 (1 - \rho^2) \right]$ | $R_i = \frac{\bar{Y} \beta_1(x)}{(\bar{X} \beta_1(x) + \rho)}$ |
|       |          |            | **Kadilar and Cingi (2006) –type** |
| 9.    | $\hat{Y} = \frac{\hat{Y}}{(X \rho + \beta_1(x))} \cdot (\bar{X} \rho + \beta_1(x))$ | $\theta \left[ R_x^2 S_r^2 + S_r^2 (1 - \rho^2) \right]$ | $R_i = \frac{\bar{Y} \rho}{(\bar{X} \rho + \beta_1(x))}$ |
|       |          |            | **Kadilar and Cingi (2006) –type** |
| 10.   | $\hat{Y} = \frac{\hat{Y}}{(X \rho + \beta_1(x))} \cdot (\bar{X} \rho + \beta_1(x))$ | $\theta \left[ R_x^2 S_r^2 + S_r^2 (1 - \rho^2) \right]$ | $R_i = \frac{\bar{Y} \rho}{(\bar{X} \rho + \beta_1(x))}$ |
|       |          |            | **Kadilar and Cingi (2006) –type** |
| 11.   | $\hat{Y} = \frac{\hat{Y}}{(X + \beta_1(x))} \cdot (\bar{X} + \beta_1(x))$ | $\theta \left[ R_x^2 S_r^2 + S_r^2 (1 - \rho^2) \right]$ | $R_i = \frac{\bar{Y}}{(X + \beta_1(x))}$ |
|       |          |            | **Yan and Tian (2010)** |
| S.No. | Estimator | MSE of (\(\sigma^2\)) Population Ratio |
|-------|-----------|-----------------------------------|
| 12.   | \(\hat{Y} = \frac{\hat{\beta}_1}{(\bar{X} + \hat{M})} \) | \(R_{12} = \frac{\bar{Y} \beta_1}{(\bar{X} + \hat{M})}\) Yan and Tian (2010) |
| 13.   | \(\hat{Y} = \frac{\hat{\beta}_1}{(\bar{M} + \hat{M})} \) | \(R_{13} = \frac{\bar{Y} \beta_1}{(\bar{X} + \hat{M})}\) Subramani and Kumarapandian (2012a) |
| 14.   | \(\hat{Y} = \frac{\hat{\beta}_1}{(C\bar{X} + \hat{M})} \) | \(R_{14} = \frac{\bar{Y} \beta_1}{(C\bar{X} + \hat{M})}\) Subramani and Kumarapandian (2012a) |
| 15.   | \(\hat{Y} = \frac{\hat{\beta}_1}{(\bar{X} + \hat{M})} \) | \(R_{15} = \frac{\bar{Y} \beta_1}{(\bar{M} + \hat{M})}\) Subramani and Kumarapandian (2012b) |
| 16.   | \(\hat{Y} = \frac{\hat{\beta}_1}{(\bar{X} + \hat{M})} \) | \(R_{16} = \frac{\bar{Y} \beta_1}{(\bar{X} + \hat{M})}\) Subramani and Kumarapandian (2012c) |
| 17.   | \(\hat{Y} = \frac{\hat{\beta}_1}{(\bar{X} + \hat{D})} \) | \(R_{17} = \frac{\bar{Y} \beta_1}{(\bar{X} + \hat{D})}\) Subramani and Kumarapandian (2012d) |
| 18.   | \(\hat{Y} = \frac{\hat{\beta}_1}{(\bar{X} + \hat{D})} \) | \(R_{18} = \frac{\bar{Y} \beta_1}{(\bar{X} + \hat{D})}\) Subramani and Kumarapandian (2012d) |
| 19.   | \(\hat{Y} = \frac{\hat{\beta}_1}{(\bar{X} + \hat{D})} \) | \(R_{19} = \frac{\bar{Y} \beta_1}{(\bar{X} + \hat{D})}\) Subramani and Kumarapandian (2012d) |
| 20.   | \(\hat{Y} = \frac{\hat{\beta}_1}{(\bar{X} + \hat{D})} \) | \(R_{20} = \frac{\bar{Y} \beta_1}{(\bar{X} + \hat{D})}\) Subramani and Kumarapandian (2012d) |
| 21.   | \(\hat{Y} = \frac{\hat{\beta}_1}{(\bar{X} + \hat{D})} \) | \(R_{21} = \frac{\bar{Y} \beta_1}{(\bar{X} + \hat{D})}\) Subramani and Kumarapandian (2012d) |
| 22.   | \(\hat{Y} = \frac{\hat{\beta}_1}{(\bar{X} + \hat{D})} \) | \(R_{22} = \frac{\bar{Y} \beta_1}{(\bar{X} + \hat{D})}\) Subramani and Kumarapandian (2012d) |
Table 1. continued

| S.No. | Estimator | MSE of (.) | Population Ratio |
|-------|-----------|------------|------------------|
| 23.   | \( t_{23} = \frac{\hat{Y}_t}{(\hat{X} + D) \hat{Y}_t} \) \( \theta \left[ R_{23}^2 S_i^2 + S_i^2 (1 - \rho^2) \right] \) | \( R_{23} = \frac{\theta}{(\hat{X} + D) \hat{Y}_t} \) | Subramani and Kumarapandiyan (2012d) |
| 24.   | \( t_{24} = \frac{\hat{Y}_t}{(\hat{X} + D_0) \hat{Y}_t} \) \( \theta \left[ R_{24}^2 S_i^2 + S_i^2 (1 - \rho^2) \right] \) | \( R_{24} = \frac{\theta}{(\hat{X} + D_0) \hat{Y}_t} \) | Subramani and Kumarapandiyan (2012d) |
| 25.   | \( t_{25} = \frac{\hat{Y}_t}{(\hat{X} + D) \hat{Y}_t} \) \( \theta \left[ R_{25}^2 S_i^2 + S_i^2 (1 - \rho^2) \right] \) | \( R_{25} = \frac{\theta}{(\hat{X} + D) \hat{Y}_t} \) | Subramani and Kumarapandiyan (2012d) |
| 26.   | \( t_{26} = \frac{\hat{Y}_t}{(\hat{X} + D_0) \hat{Y}_t} \) \( \theta \left[ R_{26}^2 S_i^2 + S_i^2 (1 - \rho^2) \right] \) | \( R_{26} = \frac{\theta}{(\hat{X} + D_0) \hat{Y}_t} \) | Subramani and Kumarapandiyan (2012d) |
| 27.   | \( t_{27} = \frac{\hat{Y}_t}{(\hat{X} + D) \hat{Y}_t} \) \( \theta \left[ R_{27}^2 S_i^2 + S_i^2 (1 - \rho^2) \right] \) | \( R_{27} = \frac{\theta}{(\hat{X} + D) \hat{Y}_t} \) | Jeelani et al (2013) |
| 28.   | \( t_{28} = \frac{\hat{Y}_t}{(\hat{X} + D_0) \hat{Y}_t} \) \( \theta \left[ R_{28}^2 S_i^2 + S_i^2 (1 - \rho^2) \right] \) | \( R_{28} = \frac{\theta}{(\hat{X} + D_0) \hat{Y}_t} \) | Subramani and Kumarapandiyan (2014) |
| 29.   | \( t_{29} = \frac{\hat{Y}_t}{(\hat{X} + D) \hat{Y}_t} \) \( \theta \left[ R_{29}^2 S_i^2 + S_i^2 (1 - \rho^2) \right] \) | \( R_{29} = \frac{\theta}{(\hat{X} + D) \hat{Y}_t} \) | Subramani et al (2014) |
| 30.   | \( t_{30} = \frac{\hat{Y}_t}{(\hat{X} + D_0) \hat{Y}_t} \) \( \theta \left[ R_{30}^2 S_i^2 + S_i^2 (1 - \rho^2) \right] \) | \( R_{30} = \frac{\theta}{(\hat{X} + D_0) \hat{Y}_t} \) | Subramani et al (2014) |
| 31.   | \( t_{31} = \frac{\hat{Y}_t}{(\hat{X} + D) \hat{Y}_t} \) \( \theta \left[ R_{31}^2 S_i^2 + S_i^2 (1 - \rho^2) \right] \) | \( R_{31} = \frac{\theta}{(\hat{X} + D) \hat{Y}_t} \) | Subramani et al (2014) |
| 32.   | \( t_{32} = \frac{\hat{Y}_t}{(\hat{X} + D_0) \hat{Y}_t} \) \( \theta \left[ R_{32}^2 S_i^2 + S_i^2 (1 - \rho^2) \right] \) | \( R_{32} = \frac{\theta}{(\hat{X} + D_0) \hat{Y}_t} \) | Subramani et al (2014) |
| 33.   | \( t_{33} = \frac{\hat{Y}_t}{(\hat{X} + D) \hat{Y}_t} \) \( \theta \left[ R_{33}^2 S_i^2 + S_i^2 (1 - \rho^2) \right] \) | \( R_{33} = \frac{\theta}{(\hat{X} + D) \hat{Y}_t} \) | Subramani et al (2014) |
### Table 1. continued

| S.No. | Estimator | MSE of (\(\cdot\)) | Population Ratio |
|-------|-----------|----------------------|-----------------|
| 34.   | \(t_{34} = \frac{\hat{Y}}{(\pi \beta (x) + Q)}(X \beta (x) + Q)\) | \(\theta \left[ R_C S^2 + S^2 (1 - \rho^2) \right] \) | \(R_{34} = \frac{\beta(x)F}{(\beta(x)X + Q)}\) |
|       | Subramani et al (2014) | | |
| 35.   | \(t_{35} = \frac{\hat{Y}}{(\pi \beta (x) + Q)}(X \beta (x) + Q)\) | \(\theta \left[ R_C S^2 + S^2 (1 - \rho^2) \right] \) | \(R_{35} = \frac{\beta(x)F}{(\beta(x)X + Q)}\) |
|       | Subramani et al (2014) | | |
| 36.   | \(t_{36} = \frac{\hat{Y}}{(\pi \beta (x) + Q)}(X \beta (x) + Q)\) | \(\theta \left[ R_C S^2 + S^2 (1 - \rho^2) \right] \) | \(R_{36} = \frac{\beta(x)F}{(\beta(x)X + Q)}\) |
|       | Subramani et al (2014) | | |
| 37.   | \(t_{37} = \frac{\hat{Y}}{(\pi \beta (x) + Q)}(X \beta (x) + Q)\) | \(\theta \left[ R_C S^2 + S^2 (1 - \rho^2) \right] \) | \(R_{37} = \frac{\beta(x)F}{(\beta(x)X + Q)}\) |
|       | Subramani et al (2014) | | |
| 38.   | \(t_{38} = \frac{\hat{Y}}{(\rho \pi + Q)}(\rho X + Q)\) | \(\theta \left[ R_C S^2 + S^2 (1 - \rho^2) \right] \) | \(R_{38} = \frac{\rho F}{(\rho X + Q)}\) |
|       | Subramani et al (2014) | | |
| 39.   | \(t_{39} = \frac{\hat{Y}}{(\rho \pi + Q)}(\rho X + Q)\) | \(\theta \left[ R_C S^2 + S^2 (1 - \rho^2) \right] \) | \(R_{39} = \frac{\rho F}{(\rho X + Q)}\) |
|       | Subramani et al (2014) | | |
| 40.   | \(t_{40} = \frac{\hat{Y}}{(\rho \pi + Q)}(\rho X + Q)\) | \(\theta \left[ R_C S^2 + S^2 (1 - \rho^2) \right] \) | \(R_{40} = \frac{\rho F}{(\rho X + Q)}\) |
|       | Subramani et al (2014) | | |
| 41.   | \(t_{41} = \frac{\hat{Y}}{(\rho \pi + Q)}(\rho X + Q)\) | \(\theta \left[ R_C S^2 + S^2 (1 - \rho^2) \right] \) | \(R_{41} = \frac{\rho F}{(\rho X + Q)}\) |
|       | Subramani et al (2014) | | |
| 42.   | \(t_{42} = \frac{\hat{Y}}{(\rho \pi + Q)}(\rho X + Q)\) | \(\theta \left[ R_C S^2 + S^2 (1 - \rho^2) \right] \) | \(R_{42} = \frac{\rho F}{(\rho X + Q)}\) |
|       | Subramani et al (2014) | | |
| 43.   | \(t_{43} = \frac{\hat{Y}}{(\pi + T_a)}(X + T_a)\) | \(\theta \left[ R_C S^2 + S^2 (1 - \rho^2) \right] \) | \(R_{43} = \frac{F}{(X + T_a)}\) |
|       | Abid et al (2016a) | | |
| 44.   | \(t_{44} = \frac{\hat{Y}}{(\pi C_a + T_a)}(XC_a + T_a)\) | \(\theta \left[ R_C S^2 + S^2 (1 - \rho^2) \right] \) | \(R_{44} = \frac{FC_a}{(XC_a + T_a)}\) |
|       | Abid et al (2016a) | | |

...continued
Table 1. continued

| S.No. | Estimator                                                                 | MSE of (.)                                                                 | Population Ratio       |
|-------|---------------------------------------------------------------------------|---------------------------------------------------------------------------|------------------------|
| 45.   | $\hat{t}_{45} = \frac{\hat{y}}{(X + T_n)}(X + T_n)$                      | $\theta[R^2_S + S^2_f(1 - \rho^2)]$                                      | $R_{45} = \frac{\hat{Y} \rho}{(X + T_n)}$ |
|       | Abid et al (2016a)                                                        |                                                                           |                        |
| 46.   | $\hat{t}_{46} = \frac{\hat{y}}{(X + M_r)}(X + M_r)$                      | $\theta[R^2_S + S^2_f(1 - \rho^2)]$                                      | $R_{46} = \frac{\hat{Y}}{X + M_r}$ |
|       | Abid et al (2016a)                                                        |                                                                           |                        |
| 47.   | $\hat{t}_{47} = \frac{\hat{y}}{(X + C_h + M_r)}(X + C_h + M_r)$          | $\theta[R^2_S + S^2_f(1 - \rho^2)]$                                      | $R_{47} = \frac{\hat{Y} C_h}{X + M_r}$ |
|       | Abid et al (2016a)                                                        |                                                                           |                        |
| 48.   | $\hat{t}_{48} = \frac{\hat{y}}{(X + C_h + M_r)}(X + M_r)$                | $\theta[R^2_S + S^2_f(1 - \rho^2)]$                                      | $R_{48} = \frac{\hat{Y}}{X + M_r}$ |
|       | Abid et al (2016a)                                                        |                                                                           |                        |
| 49.   | $\hat{t}_{49} = \frac{\hat{y}}{(X + H_i)}(X + H_i)$                      | $\theta[R^2_S + S^2_f(1 - \rho^2)]$                                      | $R_{49} = \frac{\hat{Y}}{X + H_i}$ |
|       | Abid et al (2016a)                                                        |                                                                           |                        |
| 50.   | $\hat{t}_{50} = \frac{\hat{y}}{(X + C_h + H_i)}(X + C_h + H_i)$          | $\theta[R^2_S + S^2_f(1 - \rho^2)]$                                      | $R_{50} = \frac{\hat{Y} C_h}{X + H_i}$ |
|       | Abid et al (2016a)                                                        |                                                                           |                        |
| 51.   | $\hat{t}_{51} = \frac{\hat{y}}{(X + H_i)}(X + H_i)$                      | $\theta[R^2_S + S^2_f(1 - \rho^2)]$                                      | $R_{51} = \frac{\hat{Y} \rho}{X + H_i}$ |
|       | Abid et al (2016a)                                                        |                                                                           |                        |
| 52.   | $\hat{t}_{52} = \frac{\hat{y}}{(X + G)}(X + G)$                         | $\theta[R^2_S + S^2_f(1 - \rho^2)]$                                      | $R_{52} = \frac{\hat{Y}}{X + G}$ |
|       | Abid et al (2016b)                                                        |                                                                           |                        |
| 53.   | $\hat{t}_{53} = \frac{\hat{y}}{(X + G)}(X + G)$                         | $\theta[R^2_S + S^2_f(1 - \rho^2)]$                                      | $R_{53} = \frac{\hat{Y} \rho}{X + G}$ |
|       | Abid et al (2016b)                                                        |                                                                           |                        |
| 54.   | $\hat{t}_{54} = \frac{\hat{y}}{(X C_h + G)}(X C_h + G)$                  | $\theta[R^2_S + S^2_f(1 - \rho^2)]$                                      | $R_{54} = \frac{\hat{Y} C_h}{X C_h + G}$ |
|       | Abid et al (2016b)                                                        |                                                                           |                        |
| 55.   | $\hat{t}_{55} = \frac{\hat{y}}{(X + D)}(X + D)$                         | $\theta[R^2_S + S^2_f(1 - \rho^2)]$                                      | $R_{55} = \frac{\hat{Y}}{X + D}$ |
|       | Abid et al (2016b)                                                        |                                                                           |                        |
Table 1. continued

| S.No. | Estimator | MSE of () | Population Ratio |
|-------|-----------|------------|------------------|
| 56.   | $t_{s6} = \hat{Y} \left( \frac{X \rho + D}{(X \rho + D)} \right)$ | $\theta \left[ R_{s6} S_i S_j (1 - \rho^2) \right]$ | $R_{s6} = \frac{\bar{Y} \rho}{(X \rho + D)}$ |
|       | Abid et al (2016b) | | |
| 57.   | $t_{s7} = \hat{Y} \left( \frac{X C_i + D}{(X C_i + D)} \right)$ | $\theta \left[ R_{s7} S_i S_j (1 - \rho^2) \right]$ | $R_{s7} = \frac{\bar{Y} C_i}{(X C_i + D)}$ |
|       | Abid et al (2016b) | | |
| 58.   | $t_{s8} = \hat{Y} \left( X + S_{sw} \right)$ | $\theta \left[ R_{s8} S_i S_j (1 - \rho^2) \right]$ | $R_{s8} = \frac{\bar{Y} \rho}{(X + S_{sw})}$ |
|       | Abid et al (2016b) | | |
| 59.   | $t_{s9} = \hat{Y} \left( \frac{X \rho + S_{sw}}{X \rho + S_{sw}} \right)$ | $\theta \left[ R_{s9} S_i S_j (1 - \rho^2) \right]$ | $R_{s9} = \frac{\bar{Y} \rho}{(X \rho + S_{sw})}$ |
|       | Abid et al (2016b) | | |
| 60.   | $t_{s0} = \hat{Y} \left( \frac{X C_i + S_{sw}}{X C_i + S_{sw}} \right)$ | $\theta \left[ R_{s0} S_i S_j (1 - \rho^2) \right]$ | $R_{s0} = \frac{\bar{Y} C_i}{(X C_i + S_{sw})}$ |
|       | Abid et al (2016b) | | |
| 61.   | $t_{s1} = \hat{Y} \left( \frac{X \rho + D_i}{X \rho + D_i} \right)$ | $\theta \left[ R_{s1} S_i S_j (1 - \rho^2) \right]$ | $R_{s1} = \frac{\bar{Y} \rho}{(X \rho + D_i)}$ |
|       | Abid et al (2016c) | | |
| 62.   | $t_{s2} = \hat{Y} \left( \frac{X \rho + D_i}{X \rho + D_i} \right)$ | $\theta \left[ R_{s2} S_i S_j (1 - \rho^2) \right]$ | $R_{s2} = \frac{\bar{Y} \rho}{(X \rho + D_i)}$ |
|       | Abid et al (2016c) | | |
| 63.   | $t_{s3} = \hat{Y} \left( \frac{X \rho + D_i}{X \rho + D_i} \right)$ | $\theta \left[ R_{s3} S_i S_j (1 - \rho^2) \right]$ | $R_{s3} = \frac{\bar{Y} \rho}{(X \rho + D_i)}$ |
|       | Abid et al (2016c) | | |
| 64.   | $t_{s4} = \hat{Y} \left( \frac{X \rho + D_i}{X \rho + D_i} \right)$ | $\theta \left[ R_{s4} S_i S_j (1 - \rho^2) \right]$ | $R_{s4} = \frac{\bar{Y} \rho}{(X \rho + D_i)}$ |
|       | Abid et al (2016c) | | |
| 65.   | $t_{s5} = \hat{Y} \left( \frac{X \rho + D_i}{X \rho + D_i} \right)$ | $\theta \left[ R_{s5} S_i S_j (1 - \rho^2) \right]$ | $R_{s5} = \frac{\bar{Y} \rho}{(X \rho + D_i)}$ |
|       | Abid et al (2016c) | | |
| 66.   | $t_{s6} = \hat{Y} \left( \frac{X \rho + D_i}{X \rho + D_i} \right)$ | $\theta \left[ R_{s6} S_i S_j (1 - \rho^2) \right]$ | $R_{s6} = \frac{\bar{Y} \rho}{(X \rho + D_i)}$ |
|       | Abid et al (2016c) | | |
### Table 1. continued

| S.No. | Estimator | MSE of () | Population Ratio |
|-------|-----------|------------|------------------|
| 67.   | $t_{67} = \hat{Y}_t \left( \frac{X \rho + D_t}{X \rho + D_t} \right)$ | $\theta \left[ R_{67}^2 S_{67}^2 + S_{67}^2 (1 - \rho^2) \right]$ | $R_{67} = \frac{\hat{Y}_t}{X \rho + D_t}$ |
|       |           |            |      | Abid et al (2016c) |
| 68.   | $t_{68} = \hat{Y}_t \left( \frac{X \rho + D_t}{X \rho + D_t} \right)$ | $\theta \left[ R_{68}^2 S_{68}^2 + S_{68}^2 (1 - \rho^2) \right]$ | $R_{68} = \frac{\hat{Y}_t}{X \rho + D_t}$ |
|       |           |            |      | Abid et al (2016c) |
| 69.   | $t_{69} = \hat{Y}_t \left( \frac{X \rho + D_t}{X \rho + D_t} \right)$ | $\theta \left[ R_{69}^2 S_{69}^2 + S_{69}^2 (1 - \rho^2) \right]$ | $R_{69} = \frac{\hat{Y}_t}{X \rho + D_t}$ |
|       |           |            |      | Abid et al (2016c) |
| 70.   | $t_{70} = \hat{Y}_t \left( \frac{X \rho + D_{t0}}{X \rho + D_{t0}} \right)$ | $\theta \left[ R_{70}^2 S_{70}^2 + S_{70}^2 (1 - \rho^2) \right]$ | $R_{70} = \frac{\hat{Y}_t}{X \rho + D_{t0}}$ |
|       |           |            |      | Abid et al (2016c) |
| 71.   | $t_{71} = \hat{Y}_t \left( \frac{X C + D_t}{X C + D_t} \right)$ | $\theta \left[ R_{71}^2 S_{71}^2 + S_{71}^2 (1 - \rho^2) \right]$ | $R_{71} = \frac{\hat{Y}_t}{X C + D_t}$ |
|       |           |            |      | Abid et al (2016c) |
| 72.   | $t_{72} = \hat{Y}_t \left( \frac{X C + D_t}{X C + D_t} \right)$ | $\theta \left[ R_{72}^2 S_{72}^2 + S_{72}^2 (1 - \rho^2) \right]$ | $R_{72} = \frac{\hat{Y}_t}{X C + D_t}$ |
|       |           |            |      | Abid et al (2016c) |
| 73.   | $t_{73} = \hat{Y}_t \left( \frac{X C + D_t}{X C + D_t} \right)$ | $\theta \left[ R_{73}^2 S_{73}^2 + S_{73}^2 (1 - \rho^2) \right]$ | $R_{73} = \frac{\hat{Y}_t}{X C + D_t}$ |
|       |           |            |      | Abid et al (2016c) |
| 74.   | $t_{74} = \hat{Y}_t \left( \frac{X C + D_t}{X C + D_t} \right)$ | $\theta \left[ R_{74}^2 S_{74}^2 + S_{74}^2 (1 - \rho^2) \right]$ | $R_{74} = \frac{\hat{Y}_t}{X C + D_t}$ |
|       |           |            |      | Abid et al (2016c) |
| 75.   | $t_{75} = \hat{Y}_t \left( \frac{X C + D_t}{X C + D_t} \right)$ | $\theta \left[ R_{75}^2 S_{75}^2 + S_{75}^2 (1 - \rho^2) \right]$ | $R_{75} = \frac{\hat{Y}_t}{X C + D_t}$ |
|       |           |            |      | Abid et al (2016c) |
| 76.   | $t_{76} = \hat{Y}_t \left( \frac{X C + D_t}{X C + D_t} \right)$ | $\theta \left[ R_{76}^2 S_{76}^2 + S_{76}^2 (1 - \rho^2) \right]$ | $R_{76} = \frac{\hat{Y}_t}{X C + D_t}$ |
|       |           |            |      | Abid et al (2016c) |
| 77.   | $t_{77} = \hat{Y}_t \left( \frac{X C + D_t}{X C + D_t} \right)$ | $\theta \left[ R_{77}^2 S_{77}^2 + S_{77}^2 (1 - \rho^2) \right]$ | $R_{77} = \frac{\hat{Y}_t}{X C + D_t}$ |
|       |           |            |      | Abid et al (2016c) |
Table 1. continued

| S.No. | Estimator | MSE of ( ) | Population Ratio |
|-------|-----------|------------|------------------|
| 78.   | \( \hat{t}_{78} = \frac{\hat{Y}}{(\bar{X}C + D)} (\bar{X}C + D) \) | \( \theta \left[ \frac{R_{i}^{2}S_{i}^{2} + S_{i}^{2}(1 - \rho^{2})}{\bar{X}C + D} \right] \) | \( R_{78} = \frac{\bar{Y}}{(\bar{X}C + D)} \) |
|       | Abid et al (2016) |
| 79.   | \( \hat{t}_{79} = \frac{\hat{Y}}{(\bar{X}C + D)} (\bar{X}C + D) \) | \( \theta \left[ \frac{R_{i}^{2}S_{i}^{2} + S_{i}^{2}(1 - \rho^{2})}{\bar{X}C + D} \right] \) | \( R_{79} = \frac{\bar{Y}}{(\bar{X}C + D)} \) |
|       | Abid et al (2016c) |
| 80.   | \( \hat{t}_{80} = \frac{\hat{Y}}{(\bar{X}C + D)} (\bar{X}C + D) \) | \( \theta \left[ \frac{R_{i}^{2}S_{i}^{2} + S_{i}^{2}(1 - \rho^{2})}{\bar{X}C + D} \right] \) | \( R_{80} = \frac{\bar{Y}}{(\bar{X}C + D)} \) |
|       | Abid et al (2016c) |

\[
E(e_1e_3) = \frac{N(N-n)}{(N-1)(N-2)} \frac{\mu_{30}}{n} \frac{1}{\bar{X}S_{x}^{2}} = \frac{(N-n)}{n(N-2)} \frac{\mu_{30}}{\bar{X} \mu_{20}},
\]

where \( \mu_{30} = E\left[ (x_i - \bar{X}) (y_i - \bar{Y})^{3} \right] \), \( C_y = \rho_{yx} \frac{C_y}{C_x}, \) \( C_y = \frac{S_y}{\bar{Y}}, \) \( C_x = \frac{S_x}{\bar{X}} \) and \( \rho_{yx} = \frac{S_{xy}}{(S_{x}S_{y})} \). \( (r,s) \) being non-negative integers.

Expressing ‘t’ defined by (6) in terms of e’s

\[
t = \bar{Y} \left[ 1 + e_0 - \left( \frac{\beta \bar{X}}{\bar{Y}} \right) e_1 (1 + e_2) (1 + e_3)^{-1} \right] (1 + \tau e_1)^{-1}.
\]

where \( \tau = \frac{(a \bar{X})}{(a \bar{X} + b)} \).

Assume \( |e_1| < 1 \) and \( |e_3| < 1 \) so that we \( (1 + e_3)^{-1} \) and \( (1 + \tau e_1)^{-1} \) are expandable. Expanding the right hand side of (7), multiplying and neglecting terms of e’s having power greater than two we have

\[
t = \bar{Y} \left[ 1 + e_0 - \tau e_1 + \tau^2 e_3 - \tau e_0 e_1 - c \left( e_1 + e_1 e_2 - e_1 e_3 - \tau e_1 \right) \right]
\]
### Table 2. Some unknown members of the class of ratio type estimators $t$.

| S.No. | Estimator                                                                                                                                                                                                 | Values of constants |
|-------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------|
| 1     | $t'_1 = \hat{f} \left( \frac{\hat{\beta}(x) \bar{X} + \rho}{\hat{\beta}(x) \bar{x} + \rho} \right)$                                                                                       | $\hat{\beta}(x)$   |
| 2     | $t'_2 = \hat{f} \left( \frac{\beta(x) \bar{X} + C_s}{\beta(x) \bar{x} + C_s} \right)$                                                                                                                   | $\beta(x)$, $C_s$  |
| 3     | $t'_3 = \hat{f} \left( \frac{\bar{X} \beta_1(x) + C_s}{\bar{x} \beta_1(x)} \right)$                                                                                                                  | $C_s$, $\beta_1(x)$|
| 4     | $t'_4 = \hat{f} \left( \frac{\bar{X} \beta_1(x) + \beta(x)}{\bar{x} \beta_1(x) + \beta(x)} \right)$                                                                                              | $\beta_1(x)$, $\beta(x)$ |
| 5     | $t'_5 = \hat{f} \left( \frac{\bar{X} \rho + \beta(x)}{\bar{x} \rho + \beta(x)} \right)$                                                                                                      | $\rho$, $\beta(x)$ |
| 6     | $t'_6 = \hat{f} \left( \frac{M_x \bar{X} + C_s}{M_x \bar{x} + C_s} \right)$                                                                                                                          | $M_x$, $C_s$       |
| 7     | $t'_7 = \hat{f} \left( \frac{M_x \bar{X} + \beta(x)}{M_x \bar{x} + \beta(x)} \right)$                                                                                                           | $M_x$, $\beta(x)$  |
| 8     | $t'_8 = \hat{f} \left( \frac{M_x \bar{X} + \beta(x)}{M_x \bar{x} + \beta(x)} \right)$                                                                                                           | $M_x$, $\beta(x)$  |
| 9     | $t'_9 = \hat{f} \left( \frac{\bar{X} \beta_1(x) + \beta(x)}{\bar{x} \beta_1(x) + \beta(x)} \right)$                                                                                           | $\beta_1(x)$, $\beta(x)$ |
| 10    | $t'_{10} = \hat{f} \left( \frac{M_x \bar{X} + \rho}{M_x \bar{x} + \rho} \right)$                                                                                                             | $M_x$, $\rho$      |
| 11    | $t'_{11} = \hat{f} \left( \frac{\bar{X} + Q_d}{\bar{x} + Q_d} \right)$                                                                                                                           | $1$, $Q_d$         |
| 12    | $t'_{12} = \hat{f} \left( \frac{\bar{X} C_s + Q_d}{\bar{x} C_s + Q_d} \right)$                                                                                                                  | $C_s$, $Q_d$       |
| 13    | $t'_{13} = \hat{f} \left( \frac{\bar{X} M_x + Q_d}{\bar{x} M_x + Q_d} \right)$                                                                                                                 | $M_x$, $Q_d$       |
| 14    | $t'_{14} = \hat{f} \left( \frac{\bar{X} M_x + Q_d}{\bar{x} M_x + Q_d} \right)$                                                                                                                 | $M_x$, $Q_d$       |
| 15    | $t'_{15} = \hat{f} \left( \frac{Q_d \bar{X} + C_s}{Q_d \bar{x} + C_s} \right)$                                                                                                                 | $Q_d$, $C_s$       |
| 16    | $t'_{16} = \hat{f} \left( \frac{Q_d \bar{X} + \beta_1(x)}{Q_d \bar{x} + \beta_1(x)} \right)$                                                                                              | $Q_d$, $\beta_1(x)$|

continued
REDUCING THE MSE OF POPULATION ESTIMATORS

**Table 2. continued**

| S.No. | Estimator | Values of constants |
|-------|-----------|---------------------|
| 17.   | \( t_{17} = \hat{\gamma} \left( \frac{Q_d \bar{X} + \beta_1(x)}{Q_d \bar{X} + \beta_1(x)} \right) \) | \( Q_d \) \( \beta_1(x) \) |
| 18.   | \( t_{18} = \hat{\gamma} \left( \frac{Q_d \bar{X} + \rho}{Q_d \bar{X} + \rho} \right) \) | \( Q_d \) \( \rho \) |
| 19.   | \( t_{19} = \hat{\gamma} \left( \frac{Q_d \bar{X} + M_d}{Q_d \bar{X} + M_d} \right) \) | \( Q_d \) \( M_d \) |
| 20.   | \( t_{20} = \hat{\gamma} \left( \frac{\beta_1(x) \bar{X} + Q_d}{\beta_1(x) \bar{X} + Q_d} \right) \) | \( \beta_1(x) \) \( Q_d \) |
|       |           | Kumarapandiyan and Subramani (2016)-type |
| 21.   | \( t_{21} = \hat{\gamma} \left( \frac{\bar{X} \beta_2(x) + T_n}{\bar{X} \beta_2(x) + T_n} \right) \) | \( \beta_2(x) \) \( T_n \) |
| 22.   | \( t_{22} = \hat{\gamma} \left( \frac{\bar{X} \beta_2(x) + T_n}{\bar{X} \beta_2(x) + T_n} \right) \) | \( \beta_2(x) \) \( T_n \) |
| 23.   | \( t_{23} = \hat{\gamma} \left( \frac{\bar{X} M_d + T_n}{\bar{X} M_d + T_n} \right) \) | \( M_d \) \( T_n \) |
| 24.   | \( t_{24} = \hat{\gamma} \left( \frac{XQ_d + T_n}{XQ_d + T_n} \right) \) | \( Q_d \) \( T_n \) |
| 25.   | \( t_{25} = \hat{\gamma} \left( \frac{XT_n + C_s}{XT_n + C_s} \right) \) | \( T_n \) \( C_s \) |
| 26.   | \( t_{26} = \hat{\gamma} \left( \frac{X T_n + \beta_2(x)}{X T_n + \beta_2(x)} \right) \) | \( T_n \) \( \beta_2(x) \) |
| 27.   | \( t_{27} = \hat{\gamma} \left( \frac{\bar{X} T_n + \rho}{\bar{X} T_n + \rho} \right) \) | \( T_n \) \( \rho \) |
| 28.   | \( t_{28} = \hat{\gamma} \left( \frac{\bar{X} T_n + \beta_1(x)}{\bar{X} T_n + \beta_1(x)} \right) \) | \( T_n \) \( \beta_1(x) \) |
| 29.   | \( t_{29} = \hat{\gamma} \left( \frac{\bar{X} T_n + M_d}{\bar{X} T_n + M_d} \right) \) | \( T_n \) \( M_d \) |
| S.No. | Estimator                                                                 | Values of constants | a | b |
|-------|---------------------------------------------------------------------------|---------------------|---|---|
| 30.   | $t_{30} = \hat{y}\left(\frac{XT_n + Q_d}{nT_n + Q_d}\right)$             |                     | $T_n$ | $Q_d$ |
| 31.   | $t_{31} = \hat{y}\left(\frac{X\beta_1(x) + M_r}{\bar{X}\beta_1(x) + M_r}\right)$ | $\beta_1(x)$       | $M_r$ |   |
| 32.   | $t_{32} = \hat{y}\left(\frac{X\beta_1(x) + M_r}{\bar{X}\beta_1(x) + M_r}\right)$ | $\beta_1(x)$       | $M_r$ |   |
| 33.   | $t_{33} = \hat{y}\left(\frac{\bar{X}M_d + M_r}{\bar{X}M_d + M_r}\right)$  |                     | $M_d$ | $M_r$ |
| 34.   | $t_{34} = \hat{y}\left(\frac{\bar{X}Q_d + M_r}{\bar{X}Q_d + M_r}\right)$  |                     | $Q_d$ | $M_r$ |
| 35.   | $t_{35} = \hat{y}\left(\frac{\bar{X}M_r + C_x}{\bar{X}M_r + C_x}\right)$  |                     | $M_r$ | $C_x$ |
| 36.   | $t_{36} = \hat{y}\left(\frac{\bar{X}M_r + \beta_1(x)}{\bar{X}M_r + \beta_1(x)}\right)$ | $M_r$ | $\beta_1(x)$ |
| 37.   | $t_{37} = \hat{y}\left(\frac{\bar{X}M_r + \rho}{\bar{X}M_r + \rho}\right)$ |                     | $M_r$ | $\rho$ |
| 38.   | $t_{38} = \hat{y}\left(\frac{\bar{X}M_r + \beta_1(x)}{\bar{X}M_r + \beta_1(x)}\right)$ | $M_r$ | $\beta_1(x)$ |
| 39.   | $t_{39} = \hat{y}\left(\frac{\bar{X}M_r + M_d}{\bar{X}M_r + M_d}\right)$  |                     | $M_r$ | $M_d$ |
| 40.   | $t_{40} = \hat{y}\left(\frac{\bar{X}M_r + Q_d}{\bar{X}M_r + Q_d}\right)$  |                     | $M_r$ | $Q_d$ |
| 41.   | $t_{41} = \hat{y}\left(\frac{XT_n + M_r}{nT_n + M_r}\right)$             |                     | $T_n$ | $M_r$ |
| 42.   | $t_{42} = \hat{y}\left(\frac{\bar{X}M_r + X_n}{\bar{X}M_r + X_n}\right)$  |                     | $M_r$ | $T_n$ |
| 43.   | $t_{43} = \hat{y}\left(\frac{\bar{X}\beta_1(x) + H_j}{\bar{X}\beta_1(x) + H_j}\right)$ | $\beta_1(x)$       | $H_j$ |   |
### Table 2. continued

| S.No. | Estimator | Values of constants | a | b |
|-------|-----------|---------------------|---|---|
| 44.   | \( t_{a} = \hat{Y} \left( \frac{X \beta(x) + H}{\bar{X} \beta(x) + H} \right) \) | \( \beta(x) \) | \( H \) | \( \) |
| 45.   | \( t_{b} = \hat{Y} \left( \frac{XM_{a} + H}{\bar{X}M_{a} + H} \right) \) | \( M_{a} \) | \( H \) | \( \) |
| 46.   | \( t_{c} = \hat{Y} \left( \frac{XQ_{a} + H}{\bar{X}Q_{a} + H} \right) \) | \( Q_{a} \) | \( H \) | \( \) |
| 47.   | \( t_{d} = \hat{Y} \left( \frac{XH + C}{\bar{X}H + C} \right) \) | \( H \) | \( C \) | \( \) |
| 48.   | \( t_{e} = \hat{Y} \left( \frac{XH + \beta(x)}{\bar{X}H + \beta(x)} \right) \) | \( H \) | \( \beta(x) \) | \( \) |
| 49.   | \( t_{f} = \hat{Y} \left( \frac{XH + \rho}{\bar{X}H + \rho} \right) \) | \( H \) | \( \rho \) | \( \) |
| 50.   | \( t_{g} = \hat{Y} \left( \frac{XH + \beta(x)}{\bar{X}H + \beta(x)} \right) \) | \( H \) | \( \beta(x) \) | \( \) |
| 51.   | \( t_{h} = \hat{Y} \left( \frac{XH + M_{a}}{\bar{X}H + M_{a}} \right) \) | \( H \) | \( M_{a} \) | \( \) |
| 52.   | \( t_{i} = \hat{Y} \left( \frac{XH + Q_{a}}{\bar{X}H + Q_{a}} \right) \) | \( H \) | \( Q_{a} \) | \( \) |
| 53.   | \( t_{j} = \hat{Y} \left( \frac{XH + M_{a}}{\bar{X}H + M_{a}} \right) \) | \( H \) | \( M_{a} \) | \( \) |
| 54.   | \( t_{k} = \hat{Y} \left( \frac{XT_{a} + H}{\bar{X}T_{a} + H} \right) \) | \( T_{a} \) | \( H \) | \( \) |
| 55.   | \( t_{l} = \hat{Y} \left( \frac{XH + T_{a}}{\bar{X}H + T_{a}} \right) \) | \( H \) | \( T_{a} \) | \( \) |
| 56.   | \( t_{m} = \hat{Y} \left( \frac{XM_{a} + H}{\bar{X}M_{a} + H} \right) \) | \( M_{a} \) | \( H \) | \( \) |
| 57.   | \( t_{n} = \hat{Y} \left( \frac{X + Q_{a}}{\bar{X} + Q_{a}} \right) \) | \( 1 \) | \( Q_{a} \) | \( \) |
Table 2. continued

| S.No. | Estimator | Values of constants |
|-------|-----------|---------------------|
|       |           | a       | b       |
| 58.   | $\hat{t}_{\alpha} = \hat{y}\left( \frac{XC + Q}{Xc + Qc} \right)$ | $C_c$   | $Q_c$   |
| 62.   | $\hat{t}_{\alpha} = \hat{y}\left( \frac{XM + Q}{Xm + Qm} \right)$ | $M_m$   | $Q_m$   |
| 63.   | $\hat{t}_{\alpha} = \hat{y}\left( \frac{XQ + Q}{XQ + Q} \right)$ | $Q_a$   | $Q_a$   |
| 64.   | $\hat{t}_{\alpha} = \hat{y}\left( \frac{XQ + C}{XQ + C} \right)$ | $Q_a$   | $C_a$   |
| 65.   | $\hat{t}_{\alpha} = \hat{y}\left( \frac{XQ + \beta(x)}{XQ + \beta(x)} \right)$ | $Q_a$   | $\beta(x)$ |
| 66.   | $\hat{t}_{\alpha} = \hat{y}\left( \frac{Q X + \rho}{Q X + \rho} \right)$ | $Q_a$   | $\rho$ |
| 67.   | $\hat{t}_{\alpha} = \hat{y}\left( \frac{Q X + \beta(x)}{Q X + \beta(x)} \right)$ | $Q_a$   | $\beta(x)$ |
| 68.   | $\hat{t}_{\alpha} = \hat{y}\left( \frac{Q X + M_d}{Q X + M_d} \right)$ | $Q_a$   | $M_d$   |
| 69.   | $\hat{t}_{\alpha} = \hat{y}\left( \frac{XQ + Q}{XQ + Q} \right)$ | $Q_a$   | $Q_a$   |
| 70.   | $\hat{t}_{\alpha} = \hat{y}\left( \frac{RT + Q}{RT + Q} \right)$ | $T_n$   | $Q_a$   |
| 71.   | $\hat{t}_{\alpha} = \hat{y}\left( \frac{XQ + T_n}{XQ + T_n} \right)$ | $Q_a$   | $T_n$   |
| 72.   | $\hat{t}_{\alpha} = \hat{y}\left( \frac{XQ + M}{XQ + M} \right)$ | $Q_a$   | $M_a$   |
| 73.   | $\hat{t}_{\alpha} = \hat{y}\left( \frac{XM + Q}{XM + Q} \right)$ | $M_r$   | $Q_a$   |
| 74.   | $\hat{t}_{\alpha} = \hat{y}\left( \frac{XH + Q}{XH + Q} \right)$ | $H_l$   | $Q_a$   |

continued
### Table 2. continued

| S.No. | Estimator | Values of constants |
|-------|-----------|---------------------|
|       |           | **a** | **b** |
| 75.   | \( \hat{t}_3 = \frac{XQ_i + Q_j}{XQ_i + H_j} \) | \( Q_x \) | \( H_j \) |
| 76.   | \( \hat{t}_6 = \frac{XQ_i + Q_j}{XQ_i + Q_k} \) | \( Q_x \) | \( Q_y \) |
| 77.   | \( \hat{t}_7 = \frac{XQ_i + Q_j}{XQ_i + Q_k} \) | \( Q_x \) | \( Q_y \) |
| 78.   | \( \hat{t}_9 = \frac{X + S_x}{X + S_y} \) | 1 | \( S_x \) |
|       | Singh (2003)-type |
| 79.   | \( \hat{t}_9 = \frac{X + S_x}{X + S_y} \) | \( C_x \) | \( S_x \) |
| 80.   | \( \hat{t}_{10} = \frac{X + S_x}{X + S_y} \) | \( \beta_j(x) \) | \( S_x \) |
|       | Singh (2003)-type |
| 81.   | \( \hat{t}_{11} = \frac{X + S_x}{X + S_y} \) | \( \beta_j(x) \) | \( S_x \) |
|       | Singh (2003)-type |
| 82.   | \( \hat{t}_{10} = \frac{X + S_x}{X + S_y} \) | \( \rho \) | \( S_x \) |
| 83.   | \( \hat{t}_{10} = \frac{X + S_x}{X + S_y} \) | \( M_d \) | \( S_x \) |
| 84.   | \( \hat{t}_{10} = \frac{X + S_x}{X + S_y} \) | \( Q_d \) | \( Q_x \) |
| 85.   | \( \hat{t}_{10} = \frac{X + S_x}{X + S_y} \) | \( S_x \) | \( C_x \) |
| 86.   | \( \hat{t}_{10} = \frac{X + S_x}{X + S_y} \) | \( S_x \) | \( \beta_j(x) \) |
| 87.   | \( \hat{t}_{10} = \frac{X + S_x}{X + S_y} \) | \( S_x \) | \( \rho \) |
Table 2. continued

| S.No. | Estimator | Values of constants |
|-------|-----------|---------------------|
|       |           | a   | b   |
| 88.   | \( t_{ia} = \hat{y} \left( \frac{X_S + \beta(x)}{\bar{x}_S + \beta(x)} \right) \) | \( S \), | \( \beta(x) \) |
| 89.   | \( t_{ia} = \hat{y} \left( \frac{X_S + M_d}{\bar{x}_S + M_d} \right) \) | \( S \), | \( M_d \) |
| 90.   | \( t_{ia} = \hat{y} \left( \frac{X_S + Q_d}{\bar{x}_S + Q_d} \right) \) | \( S \), | \( Q_d \) |
| 91.   | \( t_{ia} = \hat{y} \left( \frac{\bar{x}_S + S}{\bar{x}_S + T_n} \right) \) | \( T_n \), | \( S \) |
| 92.   | \( t_{ia} = \hat{y} \left( \frac{X_S + T_n}{\bar{x}_S + T_n} \right) \) | \( S \), | \( T_n \) |
| 93.   | \( t_{ia} = \hat{y} \left( \frac{X_S + M_r}{\bar{x}_S + M_r} \right) \) | \( S \), | \( M_r \) |
| 94.   | \( t_{ia} = \hat{y} \left( \frac{X_M + S_s}{\bar{x}_M + S_s} \right) \) | \( M_r \), | \( S_s \) |
| 95.   | \( t_{ia} = \hat{y} \left( \frac{X_H + S_s}{\bar{x}_H + S_s} \right) \) | \( H_j \), | \( S_s \) |
| 96.   | \( t_{ia} = \hat{y} \left( \frac{X_S + H_j}{\bar{x}_S + H_j} \right) \) | \( S_s \), | \( H_j \) |
| 97.   | \( t_{ia} = \hat{y} \left( \frac{X_Q + S_s}{\bar{x}_Q + S_s} \right) \) | \( Q_r \), | \( S_s \) |
| 98.   | \( t_{ia} = \hat{y} \left( \frac{X_S + Q_r}{\bar{x}_S + Q_r} \right) \) | \( S_s \), | \( Q_r \) |
| 99.   | \( t_{ia} = \hat{y} \left( \frac{X_Q + S_s}{\bar{x}_Q + S_s} \right) \) | \( Q_r \), | \( S_s \) |
| 100.  | \( t_{ia} = \hat{y} \left( \frac{X_S + Q_s}{\bar{x}_S + Q_s} \right) \) | \( S_s \), | \( Q_s \) |
| 101.  | \( t_{ia} = \hat{y} \left( \frac{X + X}{\bar{x} + X} \right) \) | 1, | \( X (= Nx) \) |

continued
## Reducing the MSE of Population Estimators

| S.No. | Estimator | Values of constants | a | b |
|-------|-----------|---------------------|---|---|
| 102. | \( t_{100} = \hat{Y} \left( \frac{X_C^* + X}{X_C^*} \right) \) | \( C_s \) | \( X = N\bar{X} \) | |
| 103. | \( t_{100} = \hat{Y} \left( \frac{X\beta_i (x) + X}{\bar{X}\beta_i (x) + X} \right) \) | \( \beta_i (x) \) | \( X = N\bar{X} \) | |
| 104. | \( t_{100} = \hat{Y} \left( \frac{X\beta_j (x) + X}{\bar{X}\beta_j (x) + X} \right) \) | \( \beta_j (x) \) | \( X = N\bar{X} \) | |
| 105. | \( t_{100} = \hat{Y} \left( \frac{X\rho + X}{\bar{X}\rho + X} \right) \) | \( \rho \) | \( X = N\bar{X} \) | |
| 102. | \( t_{100} = \hat{Y} \left( \frac{X_C^* + X}{X_C^*} \right) \) | \( C_s \) | \( X = N\bar{X} \) | |
| 103. | \( t_{100} = \hat{Y} \left( \frac{X\beta_i (x) + X}{\bar{X}\beta_i (x) + X} \right) \) | \( \beta_i (x) \) | \( X = N\bar{X} \) | |
| 104. | \( t_{100} = \hat{Y} \left( \frac{X\beta_j (x) + X}{\bar{X}\beta_j (x) + X} \right) \) | \( \beta_j (x) \) | \( X = N\bar{X} \) | |
| 105. | \( t_{100} = \hat{Y} \left( \frac{X\rho + X}{\bar{X}\rho + X} \right) \) | \( \rho \) | \( X = N\bar{X} \) | |
| 106. | \( t_{100} = \hat{Y} \left( \frac{XM_d + X}{\bar{X}M_d + X} \right) \) | \( M_d \) | \( X = N\bar{X} \) | |
| 107. | \( t_{100} = \hat{Y} \left( \frac{XQ_d + X}{\bar{X}Q_d + X} \right) \) | \( Q_d \) | \( X = N\bar{X} \) | |
| 108. | \( t_{100} = \hat{Y} \left( \frac{XS + C_s}{\bar{X}S + C_s} \right) \) | \( X = N\bar{X} \) | \( C_s \) | |
| 109. | \( t_{100} = \hat{Y} \left( \frac{XS + \beta_i (x)}{\bar{X}S + \beta_i (x)} \right) \) | \( X = N\bar{X} \) | \( \beta_i (x) \) | |
| 10. | \( t_{100} = \hat{Y} \left( \frac{XS + \rho}{\bar{X}S + \rho} \right) \) | \( X = N\bar{X} \) | \( \rho \) | |
| 111. | \( t_{111} = \hat{Y} \left( \frac{XS + \beta_i (x)}{\bar{X}S + \beta_i (x)} \right) \) | \( X = N\bar{X} \) | \( \beta_i (x) \) | |
### Table 2. continued

| S.No. | Estimator | Values of constants |
|-------|-----------|---------------------|
|       |           | a | b |
| 112.  | $t_{112} = \hat{F} \left( \frac{\sum X + M_d}{\sum X + M_d} \right)$ | $X(=N \bar{X})$ | $M_d$ |
| 113.  | $t_{113} = \hat{F} \left( \frac{\sum X + Q_d}{\sum X + Q_d} \right)$ | $X(=N \bar{X})$ | $Q_d$ |
| 114.  | $t_{114} = \hat{F} \left( \frac{\sum T_m + X}{\sum T_m + X} \right)$ | $T_m$ | $X(=N \bar{X})$ |
| 115.  | $t_{115} = \hat{F} \left( \frac{\sum X + T_m}{\sum X + T_m} \right)$ | $X(=N \bar{X})$ | $T_m$ |
| 116.  | $t_{116} = \hat{F} \left( \frac{\sum X + M_r}{\sum X + M_r} \right)$ | $X(=N \bar{X})$ | $M_r$ |
| 117.  | $t_{117} = \hat{F} \left( \frac{\sum M_r + X}{\sum M_r + X} \right)$ | $M_r$ | $X(=N \bar{X})$ |
| 118.  | $t_{118} = \hat{F} \left( \frac{\sum H_i + X}{\sum H_i + X} \right)$ | $H_i$ | $X(=N \bar{X})$ |
| 119.  | $t_{119} = \hat{F} \left( \frac{\sum X + H_i}{\sum X + H_i} \right)$ | $X(=N \bar{X})$ | $H_i$ |
| 120.  | $t_{120} = \hat{F} \left( \frac{\sum Q_r + X}{\sum Q_r + X} \right)$ | $Q_r$ | $X(=N \bar{X})$ |
| 121.  | $t_{121} = \hat{F} \left( \frac{\sum X + Q_r}{\sum X + Q_r} \right)$ | $X(=N \bar{X})$ | $Q_r$ |
| 122.  | $t_{122} = \hat{F} \left( \frac{\sum X + S_s}{\sum X + S_s} \right)$ | $X(=N \bar{X})$ | $S_s$ |
| 123.  | $t_{123} = \hat{F} \left( \frac{\sum S_s + X}{\sum S_s + X} \right)$ | $S_s$ | $X(=N \bar{X})$ |
| 124.  | $t_{124} = \hat{F} \left( \frac{\sum Q_s + X}{\sum Q_s + X} \right)$ | $Q_s$ | $X(=N \bar{X})$ |
| 125.  | $t_{125} = \hat{F} \left( \frac{\sum X + Q_s}{\sum X + Q_s} \right)$ | $X(=N \bar{X})$ | $Q_s$ |

continued
Table 2. continued

| S.No. | Estimator | Values of constants |    |
|-------|-----------|---------------------|----|
|       |           | a   | b   |
| 126.  | \( \hat{t}_{126} = \hat{\mu} \left( \frac{X + \Delta}{\bar{X} + \Delta} \right) \) | 1   | \( \Delta \) |
| 127.  | \( \hat{t}_{127} = \hat{\mu} \left( \frac{XC_s + \Delta}{\bar{X}C_s + \Delta} \right) \) | \( C_s \) | \( \Delta \) |
| 128.  | \( \hat{t}_{128} = \hat{\mu} \left( \frac{X\beta_1(x) + \Delta}{\bar{X}\beta_1(x) + \Delta} \right) \) | \( \beta_1(x) \) | \( \Delta \) |
| 129.  | \( \hat{t}_{129} = \hat{\mu} \left( \frac{X\beta(x) + \Delta}{\bar{X}\beta(x) + \Delta} \right) \) | \( \beta(x) \) | \( \Delta \) |
| 130.  | \( \hat{t}_{130} = \hat{\mu} \left( \frac{X\rho + \Delta}{\bar{X}\rho + \Delta} \right) \) | \( \rho \) | \( \Delta \) |
| 131.  | \( \hat{t}_{131} = \hat{\mu} \left( \frac{XM_d + \Delta}{\bar{X}M_d + \Delta} \right) \) | \( M_d \) | \( \Delta \) |
| 132.  | \( \hat{t}_{132} = \hat{\mu} \left( \frac{XQ_d + \Delta}{\bar{X}Q_d + \Delta} \right) \) | \( Q_d \) | \( \Delta \) |
| 133.  | \( \hat{t}_{133} = \hat{\mu} \left( \frac{X\Delta + C_s}{\bar{X}\Delta + C_s} \right) \) | \( \Delta \) | \( C_s \) |
| 134.  | \( \hat{t}_{134} = \hat{\mu} \left( \frac{X\Delta + \beta_2(x)}{\bar{X}\Delta + \beta_2(x)} \right) \) | \( \Delta \) | \( \beta_2(x) \) |
| 135.  | \( \hat{t}_{135} = \hat{\mu} \left( \frac{X\Delta + \rho}{\bar{X}\Delta + \rho} \right) \) | \( \Delta \) | \( \rho \) |
| 136.  | \( \hat{t}_{136} = \hat{\mu} \left( \frac{XQ_s + \Delta}{\bar{X}Q_s + \Delta} \right) \) | \( Q_s \) | \( \Delta \) |
| 137.  | \( \hat{t}_{137} = \hat{\mu} \left( \frac{X\Delta + Q_s}{\bar{X}\Delta + Q_s} \right) \) | \( \Delta \) | \( Q_s \) |
| 138.  | \( \hat{t}_{138} = \hat{\mu} \left( \frac{X\Delta + S_s}{\bar{X}\Delta + S_s} \right) \) | \( \Delta \) | \( S_s \) |
| 139.  | \( \hat{t}_{139} = \hat{\mu} \left( \frac{XS_s + \Delta}{\bar{X}S_s + \Delta} \right) \) | \( S_s \) | \( \Delta \) |
### Table 2. continued

| S.No. | Estimator | Values of constants |
|-------|-----------|---------------------|
|       |           | a                   | b                   |
| 140.  | $t_{40}$  | $\Delta$            | $\beta_i(x)$        |
| 141.  | $t_{41}$  | $\Delta$            | $M_d$               |
| 142.  | $t_{42}$  | $\Delta$            | $Q_e$               |
| 143.  | $t_{43}$  | $T_n$               | $\Delta$            |
| 144.  | $t_{44}$  | $\Delta$            | $T_n$               |
| 145.  | $t_{45}$  | $\Delta$            | $M_r$               |
| 146.  | $t_{46}$  | $M_r$               | $\Delta$            |
| 147.  | $t_{47}$  | $H_j$               | $\Delta$            |
| 148.  | $t_{48}$  | $\Delta$            | $H_j$               |
| 149.  | $t_{49}$  | $Q_r$               | $\Delta$            |
| 150.  | $t_{50}$  | $\Delta$            | $Q_r$               |
| 151.  | $t_{51}$  | $\Delta$            | $X$ ($= N \bar{X}$) |
| 152.  | $t_{52}$  | $X$ ($= N \bar{X}$) | $\Delta$            |
| 153.  | $t_{53}$  | 1                   | $Q_i$               |

Al-Omar et al (2009)
Table 2. continued

| S.No. | Estimator | Values of constants |
|-------|-----------|---------------------|
|       |           | a   | b   |
| 154.  | $t_{s4} = \hat{\beta} \left( \frac{X + Q}{X + Q} \right)$ | 1   | $Q_3$ |
|       | Al-Omar et al (2009) |       |       |
| 156.  | $t_{s6} = \hat{\beta} \left( \frac{X\beta(x) + Q_2}{X\beta(x) + Q_2} \right)$ | $\beta(x)$ | $Q_2$ |
|       | Kumarapandiyan and Subramani (2016)-type |       |       |
| 159.  | $t_{s9} = \hat{\beta} \left( \frac{X\beta(x) + Q_3}{X\beta(x) + Q_3} \right)$ | $\beta(x)$ | $Q_4$ |
|       | Kumarapandiyan and Subramani (2016)-type |       |       |

Or

$$(t - \bar{Y}) = \bar{Y} \left[ e_0 - \tau e_1 + \tau^2 e_1^2 - \tau e_0 e_1 - c (e_1 + e_1 e_2 - e_1 e_3 - \tau e_1^2) \right]. \quad (8)$$

Taking expectation of both sides of (2) we get the bias of ‘t’ to the the first degree of approximation as

$$B(t) = \frac{(1-f)}{n} \left[ R_j^2 S^2 \bar{Y} - \frac{N}{(N-2)} \beta \left( \frac{\mu_{21}}{\mu_{11}} \right) \left( \frac{\mu_{30}}{\mu_{20}} \right) \right]$$

$$= \frac{(1-f)}{n} (A - B) \quad (9)$$

where

$$R_j = \frac{a\bar{Y}}{a\bar{Y} + b}, \quad A = R_j^2 \left( \frac{S^2}{\bar{Y}} \right) \quad \text{and} \quad B = \frac{N}{(N-2)} \beta \left( \frac{\mu_{21}}{\mu_{11}} \right) \left( \frac{\mu_{30}}{\mu_{20}} \right).$$

The correct biases of the estimators listed in Table 1 and 2 can be obtained from (9) just by putting the suitable values of (a, b). The biases of the estimators belonging to the class of estimators ‘t’ is negligible if the sample size n is sufficiently large (i.e., $n \rightarrow N$). It should be noted that the biases of the estimators $t_1$ to $t_{45}$ listed in Table 1 reported in Subramani and Kumarapandian (2012a,b,c,d) and Abid et al (2016a,b,c) are not correct.

The correct biases of the estimators listed in Table 1 and 2 can be obtained from (9) just by putting the suitable values of (a, b). The biases of the estimators belonging to the class of estimators ‘t’ is negligible if the sample size n is sufficiently large (i.e., $n \rightarrow N$). It should be noted that the biases of the estimators $t_1$ to $t_{45}$ listed in Table 1 reported in Subramani and Kumarapandian (2012a,b,c,d) and Abid et al (2016a,b,c) are not correct.
Squaring both sides of (8) and neglecting terms of e’s having power greater than two

\[
(t - \bar{Y}) = \bar{Y}^2 \left[ e_0^2 - \tau^2 e_i^2 + C^2 e_i^2 - 2\tau e_i e_i - 2C e_i e + 2\tau C e_i^2 \right] \quad (10)
\]

Taking expectation of both sides of (10), obtain the MSE of ‘t’ to the first degree of approximation as

\[
MSE(t) = \frac{(1-f)}{n} \left[ R_j^2 S_x^2 + S_y^2 \left(1 - \rho^2 \right) \right] \quad (11)
\]

The MSE of the estimators belonging to class of estimators ‘t’ can be obtained from (11) just by putting the suitable values of (a, b).

The proposed class of estimators ‘t’ is more efficient than the usual unbiased estimator \( \bar{y} \) if

\[
MSE(t) < MSE(\bar{y})
\]
i.e. if

\[
R_j^2 S_x^2 < \beta^2 \quad . \quad (12)
\]

The members of the proposed class of estimators ‘t’ is better than the usual unbiased estimator \( \bar{y} \) as along as the condition (12) is satisfied. Further from (5) and (11)

\[
MSE(t) < MSE(\bar{y})
\]
i.e. if

\[
R_j^2 < (R - \beta)^2 \quad . \quad (13)
\]

The members of the proposed class of estimators ‘t’ is more efficient than the usual ratio estimator \( \bar{y}_R \) as long as the condition (13) is satisfied.

**Suggested Class of Ratio-Type Exponential Estimators**

Define a class of ratio-type exponential estimators for the population mean \( \bar{Y} \) as

\[
t_e = \hat{\bar{Y}} \exp \left\{ \frac{a(\bar{X} - \bar{x})}{a(\bar{X} + \bar{x}) + 2b} \right\}
\]
REDUCING THE MSE OF POPULATION ESTIMATORS

\[
\hat{\beta} = \left\{ \bar{y} + \hat{\beta} \left( \bar{x} - \bar{x} \right) \right\} \exp \left\{ \frac{a \left( \bar{x} - \bar{x} \right)}{a \left( \bar{x} + \bar{x} \right) + 2b} \right\},
\]

(14)

where \((a, b)\) are same as defined for the class of estimators \('t'\) at (1). A large number of estimators can be identified from the proposed class of estimators \(t_e\) for suitable values of \((a, b)\). Some members of the proposed class of estimators \(t_e\) corresponding to the members of the class of estimators \(t\) are listed in Table 3.

Expressing \(t_e\) in terms of \(e\)’s we have

\[
t_e = \bar{y} \left[ 1 + e_0 - Ce_1 \left( 1 + e_2 \right) \left( 1 + e_3 \right)^{-1} \right] \exp \left\{ -\frac{\tau e_1}{2} \left( 1 + \frac{\tau}{2} \right)^{-1} \right\}
\]

(15)

where,

\[
C = \left( \frac{\beta \bar{X}}{\bar{y}} \right) = \rho \frac{C_y}{C_x}.
\]

Expanding the right hand side of (15), multiplying out and neglecting terms of \(e\)’s having power greater than two we have

\[
t_e = \bar{y} \left[ 1 + e_0 - \frac{\tau e_1}{2} - C \left( e_1 + e_1 e_2 - e_1 e_3 \right) - \frac{\tau e_0 e_1}{2} + \frac{\tau}{8} \left( 3 \tau + 4C \right) e_1^2 \right]
\]

or

\[
(t_e - \bar{y}) = \bar{y} \left[ e_0 - \frac{\tau e_1}{2} - Ce_1 - C \left( e_1 e_2 - e_1 e_3 \right) - \frac{\tau e_0 e_1}{2} + \frac{\tau}{8} \left( 3 \tau + 4C \right) e_1^2 \right].
\]

(16)

Taking expectation of both sides of (16) we get the bias of \('t_e'\) to the first degree of approximation, we have

\[
B(t_e) = \frac{1 - f}{n} \left[ 3 \frac{R_j^2 S_x^2}{\bar{y}^2} - \frac{N}{(N - 2)} \hat{\beta} \left( \frac{\mu_{21}}{\mu_{11}} - \frac{\mu_{30}}{\mu_{20}} \right) \right],
\]

\[
= \frac{1 - f}{n} \left( \frac{3}{8} A - B \right),
\]

(17)

where

\[
R_j = \frac{a \bar{y}}{(a \bar{X} + b)}, \quad A \text{ and } B \text{ are same as defined earlier.}
\]
Table 3. Some members of the class of estimators $t_{e}$ corresponding to the estimators listed in Table 1.

| S.No. | Estimators                                           | MSE                          | Values of Constants | Population Ratio |
|-------|------------------------------------------------------|------------------------------|---------------------|------------------|
| 1.    | $t_{e} = \hat{f} \exp \left( \frac{X - \bar{x}}{X + \bar{x}} \right)$ | $\theta \left[ R_{e} \frac{S_{X}^{2}}{4} + S_{Y}^{2} (1 - \rho^{2}) \right]$ | $1 \ 0 \ R_i = \frac{\bar{Y}}{X} = R$ |                  |
| 2.    | $t_{e} = \hat{f} \exp \left( \frac{X - \bar{x}}{X + \bar{x} + 2C_{x}} \right)$ | $\theta \left[ R_{e} \frac{S_{X}^{2}}{4} + S_{Y}^{2} (1 - \rho^{2}) \right]$ | $1 \ C_{x} \ R_i = \frac{\bar{Y}}{X + C_{x}}$ |                  |
| 3.    | $t_{e} = \hat{f} \exp \left\{ \frac{(\bar{X} - \bar{x})}{\bar{X} + \bar{x} + 2C_{x}} \right\}$ | $\theta \left[ R_{e} \frac{S_{X}^{2}}{4} + S_{Y}^{2} (1 - \rho^{2}) \right]$ | $1 \ \beta_{2} (x) \ R_i = \frac{\bar{Y}}{X + \beta_{2} (x)}$ |                  |
| 4.    | $t_{e} = \hat{f} \exp \left\{ \frac{(\bar{X} - \bar{x})\beta_{2} (x)}{\beta_{2} (x) (X + \bar{x}) + 2C_{x}} \right\}$ | $\theta \left[ R_{e} \frac{S_{X}^{2}}{4} + S_{Y}^{2} (1 - \rho^{2}) \right]$ | $\beta_{2} (x) \ C_{x} \ R_i = \frac{\bar{Y} \beta_{2} (x)}{(X \beta_{2} (x) + C_{x})}$ |                  |
| 5.    | $t_{e} = \hat{f} \exp \left\{ \frac{C_{x} (X - \bar{x})}{C_{x} (X + \bar{x}) + 2\beta_{2} (x)} \right\}$ | $\theta \left[ R_{e} \frac{S_{X}^{2}}{4} + S_{Y}^{2} (1 - \rho^{2}) \right]$ | $C_{x} \ \beta_{2} (x) \ R_i = \frac{\bar{Y} C_{x}}{(X C_{x} + \beta_{2} (x))}$ |                  |
| 6.    | $t_{e} = \hat{f} \exp \left\{ \frac{(\bar{X} - \bar{x})}{\bar{X} + \bar{x} + 2\rho} \right\}$ | $\theta \left[ R_{e} \frac{S_{X}^{2}}{4} + S_{Y}^{2} (1 - \rho^{2}) \right]$ | $1 \ \rho \ R_i = \frac{\bar{Y}}{X + \rho}$ |                  |
| 7.    | $t_{e} = \hat{f} \exp \left\{ \frac{C_{x} (X - \bar{x})}{C_{x} (X + \bar{x}) + 2\rho} \right\}$ | $\theta \left[ R_{e} \frac{S_{X}^{2}}{4} + S_{Y}^{2} (1 - \rho^{2}) \right]$ | $C_{x} \ \rho \ R_i = \frac{\bar{Y} C_{x}}{(X C_{x} + \rho)}$ |                  |
| 8.    | $t_{e} = \hat{f} \exp \left\{ \frac{\rho (X - \bar{x})}{\rho (X + \bar{x}) + 2C_{x}} \right\}$ | $\theta \left[ R_{e} \frac{S_{X}^{2}}{4} + S_{Y}^{2} (1 - \rho^{2}) \right]$ | $\rho \ C_{x} \ R_i = \frac{\bar{Y} \rho}{(X \rho + C_{x})}$ |                  |
| 9.    | $t_{e} = \hat{f} \exp \left\{ \frac{\beta_{2} (x) (X - \bar{x})}{\beta_{2} (x) (X + \bar{x}) + 2\rho} \right\}$ | $\theta \left[ R_{e} \frac{S_{X}^{2}}{4} + S_{Y}^{2} (1 - \rho^{2}) \right]$ | $\beta_{2} (x) \ \rho \ R_i = \frac{\bar{Y} \beta_{2} (x)}{(X \beta_{2} (x) + \rho)}$ |                  |
| 10.   | $t_{e} = \hat{f} \exp \left\{ \frac{\rho (X - \bar{x})}{\rho (X + \bar{x}) + 2\beta_{2} (x)} \right\}$ | $\theta \left[ R_{e} \frac{S_{X}^{2}}{4} + S_{Y}^{2} (1 - \rho^{2}) \right]$ | $\rho \ \beta_{2} (x) \ R_i = \frac{\bar{Y} \rho}{(X \rho + \beta_{2} (x))}$ |                  |
| 11.   | $t_{e} = \hat{f} \exp \left\{ \frac{(X - \bar{x})}{(X + \bar{x}) + 2\beta_{2} (x)} \right\}$ | $\theta \left[ R_{e} \frac{S_{X}^{2}}{4} + S_{Y}^{2} (1 - \rho^{2}) \right]$ | $1 \ \beta_{2} (x) \ R_i = \frac{\bar{Y}}{(X + \beta_{2} (x))}$ |                  |
### Table 3. continued

| S.No. | Estimators | MSE | Values of Constants | Population Ratio |
|-------|------------|-----|---------------------|-------------------|
| 12.   | $t_{12e} = \hat{\beta} \exp \left( \frac{\beta (x) (\bar{x} - \bar{X})}{\beta (x) (\bar{x} + \bar{X}) + 2 \beta (x)} \right)$ | $\theta \left[ R_{12}^2 S_2^2 + S_2^2 (1 - \rho^2) \right]$ | $\beta (x) \beta_2 (x) \beta (x)$ | $R_{12} = \frac{\bar{Y} \beta (x) (X \beta (x) + \beta (x))}{\bar{X} (X \beta (x) + \beta (x))}$ |
| 13.   | $t_{13e} = \hat{\beta} \exp \left( \frac{(\bar{x} - \bar{X})}{(\bar{x} + \bar{X}) + 2 M_d} \right)$ | $\theta \left[ R_{13}^2 S_2^2 + S_2^2 (1 - \rho^2) \right]$ | $\beta (x) M_d$ | $R_{13} = \frac{\bar{Y}}{(X + M_d)}$ |
| 14.   | $t_{14e} = \hat{\beta} \exp \left( \frac{C_1 (\bar{x} - \bar{X})}{C_1 (\bar{x} + \bar{X}) + 2 M_d} \right)$ | $\theta \left[ R_{14}^2 S_2^2 + S_2^2 (1 - \rho^2) \right]$ | $C_1 M_d$ | $R_{14} = \frac{\bar{Y} C_1 (X \beta (x) + \beta (x))}{\bar{X} C_1 (X + M_d)}$ |
| 15.   | $t_{15e} = \hat{\beta} \exp \left( \frac{\beta (x) (\bar{x} - \bar{X})}{\beta (x) (\bar{x} + \bar{X}) + 2 M_d} \right)$ | $\theta \left[ R_{15}^2 S_2^2 + S_2^2 (1 - \rho^2) \right]$ | $\beta (x) M_d$ | $R_{15} = \frac{\bar{Y} \beta (x)}{(X \beta (x) + \beta (x))}$ |
| 16.   | $t_{16e} = \hat{\beta} \exp \left( \frac{\beta_2 (x) (\bar{x} - \bar{X})}{\beta_2 (x) (\bar{x} + \bar{X}) + 2 M_d} \right)$ | $\theta \left[ R_{16}^2 S_2^2 + S_2^2 (1 - \rho^2) \right]$ | $\beta_2 (x) M_d$ | $R_{16} = \frac{\bar{Y} \beta_2 (x) (X \beta (x) + \beta (x))}{(X \beta (x) + \beta (x))}$ |
| 17.   | $t_{17e} = \hat{\beta} \exp \left( \frac{(\bar{x} - \bar{X})}{(\bar{x} + \bar{X}) + 2 D_1} \right)$ | $\theta \left[ R_{17}^2 S_2^2 + S_2^2 (1 - \rho^2) \right]$ | $D_1$ | $R_{17} = \frac{\bar{Y}}{(X + D_1)}$ |
| 18.   | $t_{18e} = \hat{\beta} \exp \left( \frac{(\bar{x} - \bar{X})}{(\bar{x} + \bar{X}) + 2 D_2} \right)$ | $\theta \left[ R_{18}^2 S_2^2 + S_2^2 (1 - \rho^2) \right]$ | $D_2$ | $R_{18} = \frac{\bar{Y}}{(X + D_2)}$ |
| 19.   | $t_{19e} = \hat{\beta} \exp \left( \frac{(\bar{x} - \bar{X})}{(\bar{x} + \bar{X}) + 2 D_3} \right)$ | $\theta \left[ R_{19}^2 S_2^2 + S_2^2 (1 - \rho^2) \right]$ | $D_3$ | $R_{19} = \frac{\bar{Y}}{(X + D_3)}$ |
| 20.   | $t_{20e} = \hat{\beta} \exp \left( \frac{(\bar{x} - \bar{X})}{(\bar{x} + \bar{X}) + 2 D_4} \right)$ | $\theta \left[ R_{20}^2 S_2^2 + S_2^2 (1 - \rho^2) \right]$ | $D_4$ | $R_{20} = \frac{\bar{Y}}{(X + D_4)}$ |
| 21.   | $t_{21e} = \hat{\beta} \exp \left( \frac{(\bar{x} - \bar{X})}{(\bar{x} + \bar{X}) + 2 D_5} \right)$ | $\theta \left[ R_{21}^2 S_2^2 + S_2^2 (1 - \rho^2) \right]$ | $D_5$ | $R_{21} = \frac{\bar{Y}}{(X + D_5)}$ |
| 22.   | $t_{22e} = \hat{\beta} \exp \left( \frac{(\bar{x} - \bar{X})}{(\bar{x} + \bar{X}) + 2 D_6} \right)$ | $\theta \left[ R_{22}^2 S_2^2 + S_2^2 (1 - \rho^2) \right]$ | $D_6$ | $R_{22} = \frac{\bar{Y}}{(X + D_6)}$ |
| 23.   | $t_{23e} = \hat{\beta} \exp \left( \frac{(\bar{x} - \bar{X})}{(\bar{x} + \bar{X}) + 2 D_7} \right)$ | $\theta \left[ R_{23}^2 S_2^2 + S_2^2 (1 - \rho^2) \right]$ | $D_7$ | $R_{23} = \frac{\bar{Y}}{(X + D_7)}$ |
**Table 3. continued**

| S.No. | Estimators | MSE | Values of Constants | Population Ratio |
|-------|------------|-----|---------------------|-----------------|
| 24.   | $t_{24} = \hat{Y} \exp \left( \frac{(\bar{X} - \bar{X})}{(X + \bar{X} + 2D_a) \theta \left[ R^2_{\alpha} \frac{S^2_{\alpha}}{4} + S^2_{\alpha} (1 - \rho^2) \right]} \right)$ | $\beta \left[ R^2_{\alpha} \frac{S^2_{\alpha}}{4} + S^2_{\alpha} (1 - \rho^2) \right]$ | $D_a$ | $R_{24} = \frac{y}{(X + D_a)}$ |
| 25.   | $t_{25} = \hat{Y} \exp \left( \frac{(\bar{X} - \bar{X})}{(X + \bar{X} + 2D_a) \theta \left[ R^2_{\alpha} \frac{S^2_{\alpha}}{4} + S^2_{\alpha} (1 - \rho^2) \right]} \right)$ | $\beta \left[ R^2_{\alpha} \frac{S^2_{\alpha}}{4} + S^2_{\alpha} (1 - \rho^2) \right]$ | $D_a$ | $R_{25} = \frac{y}{(X + D_a)}$ |
| 26.   | $t_{26} = \hat{Y} \exp \left( \frac{(\bar{X} - \bar{X})}{(X + \bar{X} + 2D_{10}) \theta \left[ R^2_{\alpha} \frac{S^2_{\alpha}}{4} + S^2_{\alpha} (1 - \rho^2) \right]} \right)$ | $\beta \left[ R^2_{\alpha} \frac{S^2_{\alpha}}{4} + S^2_{\alpha} (1 - \rho^2) \right]$ | $D_{10}$ | $R_{26} = \frac{y}{(X + D_{10})}$ |
| 27.   | $t_{27} = \hat{Y} \exp \left( \frac{\beta(x)(\bar{X} - \bar{X})}{\beta(x)(X + \bar{X} + 2Q) \theta \left[ R^2_{\alpha} \frac{S^2_{\alpha}}{4} + S^2_{\alpha} (1 - \rho^2) \right]} \right)$ | $\beta(x) Q_a$ | $R_{27} = \frac{\hat{Y} \beta(x)}{(X \beta(x) + Q)}$ |
| 28.   | $t_{28} = \hat{Y} \exp \left( \frac{\rho(\bar{X} - \bar{X})}{\rho(X + \bar{X} + 2M_x) \theta \left[ R^2_{\alpha} \frac{S^2_{\alpha}}{4} + S^2_{\alpha} (1 - \rho^2) \right]} \right)$ | $\rho M_a$ | $R_{28} = \frac{\rho \hat{Y}}{(\rho X + M)}$ |
| 29.   | $t_{29} = \hat{Y} \exp \left( \frac{\beta(x)(\bar{X} - \bar{X})}{\beta(x)(X + \bar{X} + 2Q) \theta \left[ R^2_{\alpha} \frac{S^2_{\alpha}}{4} + S^2_{\alpha} (1 - \rho^2) \right]} \right)$ | $\beta(x) Q_1$ | $R_{29} = \frac{\hat{Y} \beta(x)}{(X \beta(x) + Q)}$ |
| 30.   | $t_{30} = \hat{Y} \exp \left( \frac{\beta(x)(\bar{X} - \bar{X})}{\beta(x)(X + \bar{X} + 2Q) \theta \left[ R^2_{\alpha} \frac{S^2_{\alpha}}{4} + S^2_{\alpha} (1 - \rho^2) \right]} \right)$ | $\beta(x) Q_3$ | $R_{30} = \frac{\hat{Y} \beta(x)}{(X \beta(x) + Q)}$ |
| 31.   | $t_{31} = \hat{Y} \exp \left( \frac{\beta(x)(\bar{X} - \bar{X})}{\beta(x)(X + \bar{X} + 2Q) \theta \left[ R^2_{\alpha} \frac{S^2_{\alpha}}{4} + S^2_{\alpha} (1 - \rho^2) \right]} \right)$ | $\beta(x) Q_r$ | $R_{31} = \frac{\hat{Y} \beta(x)}{(X \beta(x) + Q)}$ |
| 32.   | $t_{32} = \hat{Y} \exp \left( \frac{\beta(x)(\bar{X} - \bar{X})}{\beta(x)(X + \bar{X} + 2Q) \theta \left[ R^2_{\alpha} \frac{S^2_{\alpha}}{4} + S^2_{\alpha} (1 - \rho^2) \right]} \right)$ | $\beta(x) Q_a$ | $R_{32} = \frac{\hat{Y} \beta(x)}{(X \beta(x) + Q)}$ |
| 33.   | $t_{33} = \hat{Y} \exp \left( \frac{\beta(x)(\bar{X} - \bar{X})}{\beta(x)(X + \bar{X} + 2Q) \theta \left[ R^2_{\alpha} \frac{S^2_{\alpha}}{4} + S^2_{\alpha} (1 - \rho^2) \right]} \right)$ | $\beta(x) Q_a$ | $R_{33} = \frac{\hat{Y} \beta(x)}{(X \beta(x) + Q)}$ |
| 34.   | $t_{34} = \hat{Y} \exp \left( \frac{\beta(x)(\bar{X} - \bar{X})}{\beta(x)(X + \bar{X} + 2Q) \theta \left[ R^2_{\alpha} \frac{S^2_{\alpha}}{4} + S^2_{\alpha} (1 - \rho^2) \right]} \right)$ | $\beta(x) Q_1$ | $R_{34} = \frac{\hat{Y} \beta(x)}{(X \beta(x) + Q)}$ |
| 35.   | $t_{35} = \hat{Y} \exp \left( \frac{\beta(x)(\bar{X} - \bar{X})}{\beta(x)(X + \bar{X} + 2Q) \theta \left[ R^2_{\alpha} \frac{S^2_{\alpha}}{4} + S^2_{\alpha} (1 - \rho^2) \right]} \right)$ | $\beta(x) Q_3$ | $R_{35} = \frac{\hat{Y} \beta(x)}{(X \beta(x) + Q)}$ |

*continued*
### Table 3. continued

| S.No. | Estimators | MSE | Values of Constants | Population Ratio |
|-------|------------|-----|---------------------|------------------|
| 36.   | \( \bar{y}_{36} = \hat{Y} \exp \left( \frac{\beta(x)(\bar{X} - \pi)}{\beta(x)(\bar{X} + \pi) + 2Q} \right) \) | \( \theta \left[ R_3 S_3^2 + S_r^2(1 - \rho^2) \right] \) | \( R_{36} = \frac{\bar{Y} \beta(x)}{(\bar{X} \beta(x) + Q)} \) |
| 37.   | \( \bar{y}_{37} = \hat{Y} \exp \left( \frac{\beta(x)(\bar{X} - \pi)}{\beta(x)(\bar{X} + \pi) + 2Q} \right) \) | \( \theta \left[ R_3 S_3^2 + S_r^2(1 - \rho^2) \right] \) | \( R_{37} = \frac{\bar{Y} \beta(x)}{(\bar{X} \beta(x) + Q)} \) |
| 38.   | \( \bar{y}_{38} = \hat{Y} \exp \left( \frac{\rho(\bar{X} - \pi)}{\rho(\bar{X} + \pi) + 2Q} \right) \) | \( \theta \left[ R_3 S_3^2 + S_r^2(1 - \rho^2) \right] \) | \( R_{38} = \frac{\bar{Y} \rho}{(\rho \bar{X} + Q)} \) |
| 39.   | \( \bar{y}_{39} = \hat{Y} \exp \left( \frac{\rho(\bar{X} - \pi)}{\rho(\bar{X} + \pi) + 2Q} \right) \) | \( \theta \left[ R_3 S_3^2 + S_r^2(1 - \rho^2) \right] \) | \( R_{39} = \frac{\bar{Y} \rho}{(\rho \bar{X} + Q)} \) |
| 40.   | \( \bar{y}_{40} = \hat{Y} \exp \left( \frac{\rho(\bar{X} - \pi)}{\rho(\bar{X} + \pi) + 2Q} \right) \) | \( \theta \left[ R_3 S_3^2 + S_r^2(1 - \rho^2) \right] \) | \( R_{40} = \frac{\bar{Y} \rho}{(\rho \bar{X} + Q)} \) |
| 41.   | \( \bar{y}_{41} = \hat{Y} \exp \left( \frac{\rho(\bar{X} - \pi)}{\rho(\bar{X} + \pi) + 2Q} \right) \) | \( \theta \left[ R_3 S_3^2 + S_r^2(1 - \rho^2) \right] \) | \( R_{41} = \frac{\bar{Y} \rho}{(\rho \bar{X} + Q)} \) |
| 42.   | \( \bar{y}_{42} = \hat{Y} \exp \left( \frac{\rho(\bar{X} - \pi)}{\rho(\bar{X} + \pi) + 2Q} \right) \) | \( \theta \left[ R_3 S_3^2 + S_r^2(1 - \rho^2) \right] \) | \( R_{42} = \frac{\bar{Y} \rho}{(\rho \bar{X} + Q)} \) |
| 43.   | \( \bar{y}_{43} = \hat{Y} \exp \left( \frac{\rho(\bar{X} - \pi)}{\rho(\bar{X} + \pi) + 2T_m} \right) \) | \( \theta \left[ R_3 S_3^2 + S_r^2(1 - \rho^2) \right] \) | \( R_{43} = \frac{\bar{Y}}{(\bar{X} + T_m)} \) |
| 44.   | \( \bar{y}_{44} = \hat{Y} \exp \left( \frac{\rho(\bar{X} - \pi)}{\rho(\bar{X} + \pi) + 2T_m} \right) \) | \( \theta \left[ R_3 S_3^2 + S_r^2(1 - \rho^2) \right] \) | \( R_{44} = \frac{\bar{Y}}{(\bar{X} \rho + T_m)} \) |
| 45.   | \( \bar{y}_{45} = \hat{Y} \exp \left( \frac{\rho(\bar{X} - \pi)}{\rho(\bar{X} + \pi) + 2T_m} \right) \) | \( \theta \left[ R_3 S_3^2 + S_r^2(1 - \rho^2) \right] \) | \( R_{45} = \frac{\bar{Y}}{(\bar{X} + T_m)} \) |
| 46.   | \( \bar{y}_{46} = \hat{Y} \exp \left( \frac{\rho(\bar{X} - \pi)}{\rho(\bar{X} + \pi) + 2M_r} \right) \) | \( \theta \left[ R_3 S_3^2 + S_r^2(1 - \rho^2) \right] \) | \( R_{46} = \frac{\bar{Y}}{(\bar{X} + M_r)} \) |
| 47.   | \( \bar{y}_{47} = \hat{Y} \exp \left( \frac{\rho(\bar{X} - \pi)}{\rho(\bar{X} + \pi) + 2M_r} \right) \) | \( \theta \left[ R_3 S_3^2 + S_r^2(1 - \rho^2) \right] \) | \( R_{47} = \frac{\bar{Y}}{(\bar{X} + M_r)} \) |
Table 3. continued

| S.No. | Estimators         | MSE | Values of Constants | Population Ratio |
|-------|--------------------|-----|---------------------|------------------|
| 48.   | $t_{48} = \hat{Y} \exp \left( \frac{\rho(X - \bar{X})}{\rho(X + \bar{X}) + 2M} \right)$ | $\theta \left[ R_{48}^2 \frac{S^2}{4} + S^2 \left( 1 - \rho^2 \right) \right]$ | $\rho \quad M, \quad R_{48} = \frac{\bar{Y}}{X \rho + M}$ |
| 49.   | $t_{49} = \hat{Y} \exp \left( \frac{(X - \bar{X})}{(X + \bar{X}) + 2H} \right)$ | $\theta \left[ R_{49}^2 \frac{S^2}{4} + S^2 \left( 1 - \rho^2 \right) \right]$ | $1 \quad H, \quad R_{49} = \frac{\bar{Y}}{X + H}$ |
| 50.   | $t_{50} = \hat{Y} \exp \left( \frac{C_c (X - \bar{X})}{C_c (X + \bar{X}) + 2H} \right)$ | $\theta \left[ R_{50}^2 \frac{S^2}{4} + S^2 \left( 1 - \rho^2 \right) \right]$ | $C_c \quad H, \quad R_{50} = \frac{\bar{Y}}{X \rho + H}$ |
| 51.   | $t_{51} = \hat{Y} \exp \left( \frac{\rho(X - \bar{X})}{\rho(X + \bar{X}) + 2H} \right)$ | $\theta \left[ R_{51}^2 \frac{S^2}{4} + S^2 \left( 1 - \rho^2 \right) \right]$ | $\rho \quad H, \quad R_{51} = \frac{\bar{Y}}{X + G}$ |
| 52.   | $t_{52} = \hat{Y} \exp \left( \frac{(X - \bar{X})}{(X + \bar{X}) + 2G} \right)$ | $\theta \left[ R_{52}^2 \frac{S^2}{4} + S^2 \left( 1 - \rho^2 \right) \right]$ | $1 \quad G, \quad R_{52} = \frac{\bar{Y}}{X + G}$ |
| 53.   | $t_{53} = \hat{Y} \exp \left( \frac{\rho(X - \bar{X})}{\rho(X + \bar{X}) + 2G} \right)$ | $\theta \left[ R_{53}^2 \frac{S^2}{4} + S^2 \left( 1 - \rho^2 \right) \right]$ | $\rho \quad G, \quad R_{53} = \frac{\bar{Y}}{X \rho + G}$ |
| 54.   | $t_{54} = \hat{Y} \exp \left( \frac{C_c (X - \bar{X})}{C_c (X + \bar{X}) + 2G} \right)$ | $\theta \left[ R_{54}^2 \frac{S^2}{4} + S^2 \left( 1 - \rho^2 \right) \right]$ | $C_c \quad G, \quad R_{54} = \frac{\bar{Y}}{X \rho + G}$ |
| 55.   | $t_{55} = \hat{Y} \exp \left( \frac{(X - \bar{X})}{(X + \bar{X}) + 2D} \right)$ | $\theta \left[ R_{55}^2 \frac{S^2}{4} + S^2 \left( 1 - \rho^2 \right) \right]$ | $1 \quad D, \quad R_{55} = \frac{\bar{Y}}{X + D}$ |
| 56.   | $t_{56} = \hat{Y} \exp \left( \frac{\rho(X - \bar{X})}{\rho(X + \bar{X}) + 2D} \right)$ | $\theta \left[ R_{56}^2 \frac{S^2}{4} + S^2 \left( 1 - \rho^2 \right) \right]$ | $\rho \quad D, \quad R_{56} = \frac{\bar{Y}}{X \rho + D}$ |
| 57.   | $t_{57} = \hat{Y} \exp \left( \frac{C_c (X - \bar{X})}{C_c (X + \bar{X}) + 2D} \right)$ | $\theta \left[ R_{57}^2 \frac{S^2}{4} + S^2 \left( 1 - \rho^2 \right) \right]$ | $C_c \quad D, \quad R_{57} = \frac{\bar{Y}}{X \rho + D}$ |
| 58.   | $t_{58} = \hat{Y} \exp \left( \frac{(X - \bar{X})}{(X + \bar{X}) + 2S_{pv}} \right)$ | $\theta \left[ R_{58}^2 \frac{S^2}{4} + S^2 \left( 1 - \rho^2 \right) \right]$ | $1 \quad S_{pv}, \quad R_{58} = \frac{\bar{Y}}{X + S_{pv}}$ |
| 59.   | $t_{59} = \hat{Y} \exp \left( \frac{\rho(X - \bar{X})}{\rho(X + \bar{X}) + 2S_{pv}} \right)$ | $\theta \left[ R_{59}^2 \frac{S^2}{4} + S^2 \left( 1 - \rho^2 \right) \right]$ | $\rho \quad S_{pv}, \quad R_{59} = \frac{\bar{Y}}{X \rho + S_{pv}}$ |

continued
### Table 3. continued

| S.No. | Estimators | MSE | Values of Constants | Population Ratio |
|-------|------------|-----|---------------------|------------------|
| 60.   | $t_{60} = \hat{Y} \exp \left[ -\theta \left( R_{s_0} S^2 + S^2_1 (1 - \rho^2) \right) \right] C_s S_{pw}$ | $R_{60} = \frac{\overline{Y}_C}{(X C_s + S_{pw})}$ |
| 61.   | $t_{61} = \hat{Y} \exp \left[ -\theta \left( R_{s_0} S^2 + S^2_1 (1 - \rho^2) \right) \right] \rho D_1$ | $R_{61} = \frac{\overline{Y}_\rho}{(X \rho + D_1)}$ |
| 62.   | $t_{62} = \hat{Y} \exp \left[ -\theta \left( R_{s_0} S^2 + S^2_1 (1 - \rho^2) \right) \right] \rho D_2$ | $R_{62} = \frac{\overline{Y}_\rho}{(X \rho + D_2)}$ |
| 63.   | $t_{63} = \hat{Y} \exp \left[ -\theta \left( R_{s_0} S^2 + S^2_1 (1 - \rho^2) \right) \right] \rho D_3$ | $R_{63} = \frac{\overline{Y}_\rho}{(X \rho + D_3)}$ |
| 64.   | $t_{64} = \hat{Y} \exp \left[ -\theta \left( R_{s_0} S^2 + S^2_1 (1 - \rho^2) \right) \right] \rho D_4$ | $R_{64} = \frac{\overline{Y}_\rho}{(X \rho + D_4)}$ |
| 65.   | $t_{65} = \hat{Y} \exp \left[ -\theta \left( R_{s_0} S^2 + S^2_1 (1 - \rho^2) \right) \right] \rho D_5$ | $R_{65} = \frac{\overline{Y}_\rho}{(X \rho + D_5)}$ |
| 66.   | $t_{66} = \hat{Y} \exp \left[ -\theta \left( R_{s_0} S^2 + S^2_1 (1 - \rho^2) \right) \right] \rho D_6$ | $R_{66} = \frac{\overline{Y}_\rho}{(X \rho + D_6)}$ |
| 67.   | $t_{67} = \hat{Y} \exp \left[ -\theta \left( R_{s_0} S^2 + S^2_1 (1 - \rho^2) \right) \right] \rho D_7$ | $R_{67} = \frac{\overline{Y}_\rho}{(X \rho + D_7)}$ |
| 68.   | $t_{68} = \hat{Y} \exp \left[ -\theta \left( R_{s_0} S^2 + S^2_1 (1 - \rho^2) \right) \right] \rho D_8$ | $R_{68} = \frac{\overline{Y}_\rho}{(X \rho + D_8)}$ |
| 69.   | $t_{69} = \hat{Y} \exp \left[ -\theta \left( R_{s_0} S^2 + S^2_1 (1 - \rho^2) \right) \right] \rho D_9$ | $R_{69} = \frac{\overline{Y}_\rho}{(X \rho + D_9)}$ |
| 70.   | $t_{70} = \hat{Y} \exp \left[ -\theta \left( R_{s_0} S^2 + S^2_1 (1 - \rho^2) \right) \right] \rho D_{10}$ | $R_{70} = \frac{\overline{Y}_\rho}{(X \rho + D_{10})}$ |
Table 3. continued

| S.No. | Estimators | MSE | Values of Constants | Population Ratio |
|-------|------------|-----|---------------------|------------------|
| 71.   | $t_{70} = \hat{y} \exp \left( \frac{C_s (\bar{x} - \bar{x})}{C_s (\bar{x} + \bar{x}) + 2D_i} \right)$ | $\theta \left[ R_0^2 \frac{S_i^2}{4} + S_i^2 (1 - \rho^2) \right]$ | $C_s \quad D_i \quad R_{70} = \frac{Y_{C}}{(X_{C} + D_i)}$ |
| 72.   | $t_{71} = \hat{y} \exp \left( \frac{C_s (\bar{x} - \bar{x})}{C_s (\bar{x} + \bar{x}) + 2D_i} \right)$ | $\theta \left[ R_0^2 \frac{S_i^2}{4} + S_i^2 (1 - \rho^2) \right]$ | $C_s \quad D_i \quad R_{71} = \frac{Y_{C}}{(X_{C} + D_i)}$ |
| 73.   | $t_{72} = \hat{y} \exp \left( \frac{C_s (\bar{x} - \bar{x})}{C_s (\bar{x} + \bar{x}) + 2D_i} \right)$ | $\theta \left[ R_0^2 \frac{S_i^2}{4} + S_i^2 (1 - \rho^2) \right]$ | $C_s \quad D_i \quad R_{72} = \frac{Y_{C}}{(X_{C} + D_i)}$ |
| 74.   | $t_{73} = \hat{y} \exp \left( \frac{C_s (\bar{x} - \bar{x})}{C_s (\bar{x} + \bar{x}) + 2D_i} \right)$ | $\theta \left[ R_0^2 \frac{S_i^2}{4} + S_i^2 (1 - \rho^2) \right]$ | $C_s \quad D_i \quad R_{73} = \frac{Y_{C}}{(X_{C} + D_i)}$ |
| 75.   | $t_{74} = \hat{y} \exp \left( \frac{C_s (\bar{x} - \bar{x})}{C_s (\bar{x} + \bar{x}) + 2D_i} \right)$ | $\theta \left[ R_0^2 \frac{S_i^2}{4} + S_i^2 (1 - \rho^2) \right]$ | $C_s \quad D_i \quad R_{74} = \frac{Y_{C}}{(X_{C} + D_i)}$ |
| 76.   | $t_{75} = \hat{y} \exp \left( \frac{C_s (\bar{x} - \bar{x})}{C_s (\bar{x} + \bar{x}) + 2D_i} \right)$ | $\theta \left[ R_0^2 \frac{S_i^2}{4} + S_i^2 (1 - \rho^2) \right]$ | $C_s \quad D_i \quad R_{75} = \frac{Y_{C}}{(X_{C} + D_i)}$ |
| 77.   | $t_{76} = \hat{y} \exp \left( \frac{C_s (\bar{x} - \bar{x})}{C_s (\bar{x} + \bar{x}) + 2D_i} \right)$ | $\theta \left[ R_0^2 \frac{S_i^2}{4} + S_i^2 (1 - \rho^2) \right]$ | $C_s \quad D_i \quad R_{76} = \frac{Y_{C}}{(X_{C} + D_i)}$ |
| 78.   | $t_{77} = \hat{y} \exp \left( \frac{C_s (\bar{x} - \bar{x})}{C_s (\bar{x} + \bar{x}) + 2D_i} \right)$ | $\theta \left[ R_0^2 \frac{S_i^2}{4} + S_i^2 (1 - \rho^2) \right]$ | $C_s \quad D_i \quad R_{77} = \frac{Y_{C}}{(X_{C} + D_i)}$ |
| 79.   | $t_{78} = \hat{y} \exp \left( \frac{C_s (\bar{x} - \bar{x})}{C_s (\bar{x} + \bar{x}) + 2D_i} \right)$ | $\theta \left[ R_0^2 \frac{S_i^2}{4} + S_i^2 (1 - \rho^2) \right]$ | $C_s \quad D_i \quad R_{78} = \frac{Y_{C}}{(X_{C} + D_i)}$ |
| 80.   | $t_{79} = \hat{y} \exp \left( \frac{C_s (\bar{x} - \bar{x})}{C_s (\bar{x} + \bar{x}) + 2D_i} \right)$ | $\theta \left[ R_0^2 \frac{S_i^2}{4} + S_i^2 (1 - \rho^2) \right]$ | $C_s \quad D_i \quad R_{79} = \frac{Y_{C}}{(X_{C} + D_i)}$ |
The bias of $t_e$ at (17) is negligible if the sample size $n$ is sufficiently large. The bias of the members of the proposed class of estimators can be obtained easily from (17) just by putting suitable values of the scalars $(a, b)$. Squaring both sides of (16) and neglecting terms of $e$’s having power greater than two

$$
(t_e - \bar{Y})^2 = \bar{Y}^2 \left[ e_0^2 + \frac{\tau^2 e_1^2}{4} + C^2 e_1^2 - \tau e_0 e_1 - 2 C e_i e + \tau C e_i^2 \right]
$$

(18)

Taking expectation of both sides of (18) we get the MSE of $t_e$ to the first degree of approximation as

$$
MSE(t_e) = \frac{(1-f)}{n} \left[ R_j^2 \left( \frac{S^2}{4} \right) + S_j^2 \left( 1 - \rho^2 \right) \right].
$$

(19)

The MSE of the members of the proposed class of estimators $t_e$ can be easily obtained from (19) just by putting the suitable values of $(a, b)$.

**Remark 1** Motivated by Swain (2014), define a class of ratio-type estimators for population mean $Y$ as

$$
t_s = \frac{aX + b}{a\bar{x} + b}.
$$

(20)

Thus the form of the estimators $t$ and $t_s$ taking into consideration, we define a class of ratio-cum-product-type estimators for population mean $Y$ as

$$
t_g = \frac{aX + b}{a\bar{x} + b}^g.
$$

(21)

where $g$ is a scalar taking real values. Note for $g(>0)$ the class of estimators $t_g$ generates the ratio-type estimators while for $g(<0)$ it generates product-type estimators.

To the first degree of approximation the bias and MSE of $t_e$ are respectively given by

$$
B(t_g) = \frac{(1-f)}{n} \left[ R_j^2 \frac{S^2}{Y} \frac{g(g+1)}{2} - \frac{\beta N}{(N-2)} \left( \frac{\mu_{11} - \mu_{30}}{\mu_{20}} \right) \right]
$$

$$
= \frac{(1-f)}{n} \left[ \frac{g(g+1)}{2} A - B \right],
$$

(22)
\[ MSE(t_s) = \left(1 - \frac{f}{n}\right) \left[g^2 R_j S_x^2 + S_y^2 (1 - \rho^2)\right], \quad (23) \]

where \(A\) and \(B\) are same as defined earlier. Putting \( g = \frac{1}{2} \) in (22) and (23) we get the bias and MSE of the Swain’s (2014)-type estimator \( t_s \) at (20) respectively as

\[ B(t_s) = \left(1 - \frac{f}{n}\right) \left[\frac{3}{8} A - B\right] \quad (24) \]

and

\[ MSE(t_s) = \left(1 - \frac{f}{n}\right) \left[\frac{R_j^2 S_x^2}{4} + S_y^2 (1 - \rho^2)\right]. \quad (25) \]

From (17), (19), (24) and (25), note the bias and MSE of the Swain’s (2014) type estimators \( t_s \) and ratio-type exponential estimator \( t_e \) defined in (14) are same up to first order of approximation.

From (11) and (23)

\[ MSE(t) - MSE(t_s) = \frac{(1 - f)}{n} R_j^2 S_x^2 + S_y^2 (1 - g^2) \]

which is positive if

\[ 1 - g^2 > 0 \]

i.e. if

\[ g^2 < 1 \]

i.e if

\[ -1 < g < 1. \quad (26) \]

The members of the class of ratio-cum-product type estimators \( t_s \) is more efficient than the corresponding members of the proposed class of ratio-type estimators \( t \) as long as the condition (26) is satisfied. From (19) and (23)

\[ MSE(t) - MSE(t_s) = \frac{(1 - f)}{n} R_j^2 S_x^2 + \left(\frac{1}{4} - g^2\right) \]

which is positive if

i.e. if

\[ \left(\frac{1}{4} - g^2\right) > 0 \quad g^2 > \frac{1}{4} \]

37
i.e. if \(-\frac{1}{2} < g < \frac{1}{2}\).  \hspace{1cm} (27)

The proposed class of ratio-cum-product type estimators \(t_g\) would always be more efficient than the corresponding members of the ratio-type exponential estimator \(t_e\) as long as the condition (27) is satisfied.

**Remark 2** It follows from (23) that either the estimator is ratio-type (i.e. \(t\) defined by (6)) or product-type defined by

\[
\hat{t}^{**} = \exp\left\{ g \left( \frac{a\bar{X} + b}{a\bar{X} + b} \right) \right\} \\
= \left[ \bar{Y} + \hat{\beta} (\bar{X} - \bar{x}) \right] \left( \frac{a\bar{X} + b}{a\bar{X} + b} \right),  \hspace{1cm} (28)
\]

the mean squared errors of ratio-type \((t)\) and product-type \((t^{**})\) to the first degree of approximation are turn out to be the same i.e.

\[
MSE(t) = MSE(t^{**}) = \frac{(1-f)}{n} \left[ R^2 \left( S_x^r + S_y^2 \left( 1 - \rho^2 \right) \right) \right].  \hspace{1cm} (29)
\]

The proposed class of estimators is always better than the ratio-type \((t)\) and product-type \((t^{**})\) estimators as long as the condition:

\[-1 < g < 1\]  \hspace{1cm} (30)

is satisfied.

**Remark 3** Define a generalized version of the ratio-cum-product-type exponential estimator \(t_e\) for the population mean \(\bar{Y}\) as

\[
t_{ge} = \hat{Y} \exp\left\{ \frac{g (\bar{X} - \bar{x})}{(X + \bar{x})} \right\} \\
= \left[ \bar{Y} + \hat{\beta} (\bar{X} - \bar{x}) \right] \exp\left\{ \frac{ga (\bar{X} - \bar{x})}{(\bar{X} + \bar{x}) + 2b} \right\},  \hspace{1cm} (31)
\]
where $\bar{x}^* = (a\bar{x} + b)$ such that $E(\bar{x}^*) = \bar{X}^* = a\bar{X} + b$; and $g$ is a scalar taking real values. For $g(>0) t_{ge}$ generates ratio-type exponential estimator, and for $g(>0)$ it generates product-type exponential estimator.

To the first degree of approximation, the bias and mean squared error of the proposed class of ratio-cum-product-type exponential estimators $t_{ge}$ are respectively given by

$$B(t_{ge}) = \frac{(1-f)}{n} \left[ g \left( \frac{g+2}{8} R_j^2 \frac{S_x^2}{\bar{Y}} - \frac{N}{N-2} \beta \left( \frac{\mu_{21}}{\mu_{11}} - \frac{\mu_{10}}{\mu_{20}} \right) \right) \right]$$

$$= \frac{(1-f)}{n} \left[ g \left( \frac{g+2}{8} A - B \right) \right] \quad (32)$$

and

$$MSE(t_{ge}) = \frac{(1-f)}{n} \left[ \left( \frac{g^2}{4} \right) R_j^2 S_x^2 + S_y^2 (1 - \rho^2) \right]. \quad (33)$$

From (23) and (33)

$$MSE(t_g) - MSE(t_{ge}) = \frac{3}{4} \frac{(1-f)}{n} g^2 R_j^2 S_x^2 \quad (34)$$

which is always positive.

It follows from (34) that the members of the proposed class of ratio-cum-product-type exponential estimators $t_{ge}$ is always better than the corresponding members of the suggested class of ratio-cum-product type estimators $t_e$.

From (19) and (33) we have

$$MSE(t_e) - MSE(t_{ge}) = \frac{(1-f)}{n} R_j^2 S_x^2 \left( 1 - g^2 \right)$$

which is positive if

$$(1 - g^2) > 0$$

i.e. if

$${-1 < g < 1}$$

i.e. if

$$|g| < 1.$$ \quad (35)
The members of the proposed class of ratio-cum product-type exponential estimators is better than the corresponding members of the suggested ratio-type exponential estimator \( \hat{t}_e \) as long as the condition (35) is satisfied. It can be also proved that the members proposed of the proposed class of ratio-cum-product-type estimators \( \hat{t}_{ge} \) is also better than the corresponding members of the product-type exponential estimator defined by

\[
\hat{t}_{e}^{**} = \hat{Y} \exp \left\{ \frac{a(\bar{x} - \bar{X})}{a(\bar{X} + \bar{X}) + 2b} \right\}
\]

\[
= \left\{ \bar{y} + \hat{\beta}(\bar{x} - \bar{X}) \right\} \exp \left\{ \frac{a(\bar{x} - \bar{X})}{a(\bar{X} + \bar{X}) + 2b} \right\}
\]

(36)

as long as the condition: \(|g| < 1\) in (35) is satisfied.

**Efficiency Comparison**

From (2) and (19)

\[
MSE(\bar{y}) - MSE(\hat{t}_e) = \frac{(1-f)}{n} S^2 \left( \beta^2 - \frac{R_j^2}{4} \right)
\]

which is positive if

\[
\beta^2 - \frac{R_j^2}{4} > 0
\]

i.e. if

\[
\frac{R_j^2}{4} < \beta^2.
\]

(37)

From (5) and (19) e

\[
MSE(\bar{y}_R) - MSE(\hat{t}_e) = \frac{(1-f)}{n} \left[ (\beta - R)^2 - \frac{R_j^2}{4} \right]
\]

which is non-negative if

\[
\left[ (\beta - R)^2 - \frac{R_j^2}{4} \right] > 0
\]
i.e. if

\[ \frac{R_j^2}{4} < (\beta - R)^2 \]  

(38)

Further from (11) and (19) we have

\[ MSE(t) - MSE(t_e) = \frac{3}{4} \left( 1 - \frac{1}{n} \right) R_j^2 S_e^2 \]  

(39)

which is always positive.

Thus from (37) – (39) it follows that the members of the proposed class of estimators \( t_e \) is:

(i) more efficient than the usual unbiased estimator \( \bar{y} \) as long as the condition (37) is satisfied.

(ii) more efficient than the usual ratio estimator \( \bar{y}_R \) as long as the condition (38) is satisfied.

(iii) is always better than the corresponding members of the \( t \)-family of estimators.

**Bias Comparison the Estimators \( t \) and \( t_e \)**

It follows from (9) and (17)

\[ |B(t_e)| < |B(t)| \]

if

\[ \left| \frac{3}{8} A - B \right| < |A - B| \]  

(40)

Since

\[ \left| \frac{3}{8} A - B \right| < \frac{3}{8} |A| + |B| \]  

(41)

and

\[ |A - B| < \sqrt{|A| + |B|} \]  

(42)
Therefore, from (40), (41) and (42)

\[ |B(t_*)| < |B(t)| \]

if

\[ \frac{3}{8}|A| + |B| < |A| + |B| \]

i.e. if \( \frac{3}{8}|A| < |A| \)

\[ \frac{5}{8}|A| > 0 \] (43)

which is always true. The members of the proposed \( t_* \) family of estimators are less biased as well as more efficient than the corresponding members of the \( t \) family. Hence, the members of the proposed class of estimators \( t_* \) is more efficient than the corresponding known members due to Kadilar and Cingi (2004), Kadilar and Cingi (2006)- type, Yan and Tian (2010), Subramani and Kumarapandian (2012a, 2012b, 2012c, 2012d), Jeelani et al (2013) and Abid et al (2016a, 2016b, 2016c) of the class of estimators \( t \).

**Empirical Study**

In support of the theoretical results, MSEs of some known estimators listed in Table 1 were computed, and corresponding estimators listed in Table 3. Natural data sets were those considered by Kadilar and Cingi (2004) and Abid et al (2012b). The findings are shown in Table 4.

Population- Source: Kadilar and Cingi (2004) and Abid et al (2016b, p.361).

\( y \): Apple production

\( x \): Number of apple trees

\( N = 106 \), \( n = 40 \), \( \bar{y} = 2212.59 \), \( \bar{x} = 27421.70 \)

\( \rho = 0.860 \), \( S_y = 11551.53 \), \( C_y = 5.22 \), \( S_x = 57460.61 \),

\( C_x = 2.10 \), \( \beta_2(x) = 34.572 \), \( \beta_1(x) = 2.122 \), \( M_d = 7297.50 \)

\( Q_d = 12156.25 \), \( G = 40201.69 \), \( S_{pr} = 35298.810 \), \( D = 35634.990 \)
Table 4. Mean Squared Errors of some known members of the class of ratio-cum-product estimators \(t\) and the corresponding members of the class of ratio-cum-product-type estimators \(t_e\)

| Known Estimators | MSE (\(t\)) | Rank | Corresponding members of \(t_e\) | MSE (\(t_e\)) | Rank |
|------------------|-------------|------|---------------------------------|-------------|------|
| \(t_1 = \frac{\hat{Y}}{\bar{X}}\) | 875480.89 | XXVI | \(t_{e_1} = \hat{f} \exp\left(\frac{\bar{X} - \bar{Y}}{\bar{X}}\right)\) | 624527.57 | XXVI |
| Kadilar and Cingi (2004) |
| \(t_2 = \frac{\hat{Y}}{(\bar{X} + \beta_1)\left(\bar{X} + C_1\right)}\) | 875429.64 | XXVI | \(t_{e_2} = \hat{f} \exp\left(\frac{\bar{X} - \beta_1}{\bar{X} + C_1}\right)\) | 624514.75 | XXVI |
| Kadilar and Cingi (2004) |
| \(t_3 = \frac{\hat{Y}}{(\bar{X} + \beta_2)\left(\bar{X} + \beta_1\right)}\) | 874638.77 | XVI | \(t_{e_3} = \hat{f} \exp\left(\frac{\bar{X} - \beta_2}{\bar{X} + \beta_1}\right)\) | 624317.04 | XVI |
| Kadilar and Cingi (2004) |
| \(t_4 = \beta_1\) | 873468.79 | XXIII | \(t_{e_4} = \hat{f} \exp\left(\frac{\bar{X} - \beta_1}{\bar{X} + \beta_1}\right)\) | 624527.20 | XXIII |
| Kadilar and Cingi (2004) |
| \(t_5 = \beta_2\) | 872451.30 | XXIX | \(t_{e_5} = \hat{f} \exp\left(\frac{\bar{X} - \beta_2}{\bar{X} + \beta_2}\right)\) | 624512.67 | XXIX |
| Kadilar and Cingi (2004) |
| \(t_6 = \beta_3\) | 871429.11 | XX | \(t_{e_6} = \hat{f} \exp\left(\frac{\bar{X} - \beta_3}{\bar{X} + \beta_3}\right)\) | 624514.62 | XX |
| Yan and Tian (2010) |

continued
Table 4. continued

| Known Estimators | MSE (t) | Rank | Corresponding members of $t_i$ | MSE (t) | Rank |
|------------------|---------|------|---------------------------------|---------|------|
| $t_{12} = \hat{\beta} \left( \tilde{X} \hat{\beta}(x) + \beta (x) \right) \left( X + \beta (x) \right)$ | 875083.64 | XVIII | $\hat{\beta} \left( \tilde{X} \hat{\beta}(x) + \beta (x) \right) \left( X + \beta (x) \right)$ | 624428.25 | XVIII |
| Yan and Tian (2010) | | | | | |
| $t_{13} = \hat{\beta} \left( \tilde{X} + M \right)$ | 749604.60 | X | $\hat{\beta} \left( \tilde{X} + M \right)$ | 593058.50 | X |
| Subramani and Kumarapandian (2012a) | | | | | |
| $t_{14} = \hat{\beta} \left( \tilde{X} + M \right) \left( X + \beta (x) \right)$ | 804446.24 | XII | $\hat{\beta} \left( \tilde{X} + M \right) \left( X + \beta (x) \right)$ | 606769.00 | XII |
| Subramani and Kumarapandian (2012a) | | | | | |
| $t_{15} = \hat{\beta} \left( \tilde{X} + M \right) \left( X + \beta (x) \right)$ | 805062.37 | XIII | $\hat{\beta} \left( \tilde{X} + M \right) \left( X + \beta (x) \right)$ | 606922.94 | XIII |
| Subramani and Kumarapandian (2012b) | | | | | |
| $t_{16} = \hat{\beta} \left( \tilde{X} + M \right) \left( X + \beta (x) \right)$ | 870388.46 | XV | $\hat{\beta} \left( \tilde{X} + M \right) \left( X + \beta (x) \right)$ | 623254.46 | XV |
| Subramani and Kumarapandian (2012b) | | | | | |
| $t_{17} = \hat{\beta} \left( \tilde{X} + \tilde{Q} \right)$ | 769827.97 | XI | $\hat{\beta} \left( \tilde{X} + \tilde{Q} \right)$ | 598114.34 | XI |
| Jeelani et al (2013) | | | | | |
| $t_{18} = \hat{\beta} \left( \tilde{X} + \tilde{G} \right)$ | 595897.22 | IV | $\hat{\beta} \left( \tilde{X} + \tilde{G} \right)$ | 554631.65 | IV |
| Abid et al (2016b) | | | | | |
| $t_{19} = \hat{\beta} \left( \tilde{X} + \tilde{D} \right)$ | 586615.71 | I | $\hat{\beta} \left( \tilde{X} + \tilde{D} \right)$ | 552311.27 | I |
| Abid et al (2016b) | | | | | |
| $t_{20} = \hat{\beta} \left( \tilde{X} + \tilde{C} + \tilde{G} \right)$ | 656912.92 | VII | $\hat{\beta} \left( \tilde{X} + \tilde{C} + \tilde{G} \right)$ | 569885.57 | VII |
| Abid et al (2016b) | | | | | |
| $t_{21} = \hat{\beta} \left( \tilde{X} + \tilde{D} \right)$ | 604155.24 | V | $\hat{\beta} \left( \tilde{X} + \tilde{D} \right)$ | 556696.15 | V |
| Abid et al (2016b) | | | | | |
| $t_{22} = \hat{\beta} \left( \tilde{X} + \tilde{D} \right)$ | 593942.29 | II | $\hat{\beta} \left( \tilde{X} + \tilde{D} \right)$ | 554142.92 | II |
| Abid et al (2016b) | | | | | |

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It is observed from Table 4 there is considerable reduction in MSEs of the proposed estimators \( (t_{1e} \text{ to } t_{16e}, t_{27e} \text{ to } t_{37e}) \) as compared to the corresponding known estimators \( (t_1 \text{ to } t_{13}, t_{27}, t_{37} \text{ to } t_{45}) \). That is the members of the proposed class of ratio-cum-product-type exponential estimators \( t_e \) is more efficient than the corresponding members of the class of ratio-cum-product-type estimators \( t \). The proposed estimators \( t_{38e} \) followed by the estimator \( t_{41e} \) have the smallest MSE among all the estimators considered in Table 4. Thus, the proposed class of ratio-cum-product-type exponential estimators \( t_e \) is justified.

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