Optimum buyer-vendor inventory model with coordination and price breaks

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Abstract: This study develops an optimum inventory model for buyer-vendor with coordination and price breaks situation. We develop the production inventory model with screening for deteriorating products and disposal for damaged items. The coordination strategy makes decision for the damaged products. Integrated inventory total cost is developed for system optimum values are found by using Lagrange's multiplier technique. Various examples are given to illustrate the developed model.

Key words: Inventory, Deteriorating product, Price Breaks, Order level

1. Introduction

In real life situations, the unique intention of traditional inventory models is to minimize the total inventory cost. These models could not consider the defective items in lot and rework of them. Mostly, these models have a common assumption like, all the purchased or manufactured are perfect in quality. Buy, in practice due to some reasons like workers skill, imperfect raw materials, capability of machine, etc., they purchase or produce defective items also. Fujiiwara et al. [19] developed a model using optimal policy for perishable products of the two-stage inventory system. Goyal and Gupta [18] analyzed minimum inventory cost both buyer vendor. Kaj - Mikael Bjork [21] investigated many finite production quantity inventory problems. Kit Nam Francis Leung [20] developed an integrated production inventory problem in many scenario multi-firm supply chain.

Muniappan et al. [1] developed a production inventory model for vendor–buyer coordination with quantity discount, backordering and rework for fixed life time products. Selvi, A., et al. [4] developed Buyer-vendor model for deteriorating items involving back orders, screening process and transportation cost. Hemamalini, S., M. Ravithamal, and P. Muniappan et al. [6] developed EOQ inventory model for buyer-vendor with screening, disposed cost and controllable lead time. Ganesh, S., M. K. Vediappan, and K. Srinivasan et al. [12] vendor-buyer coordination model with shortage and screening process. Sarkar, Biswajit et al. [13] an EOQ model with delay in payments and time varying deterioration rate. Muniappan, P., R. Uthayakumar, and S. Ganesh et al. [2] an EOQ model for deteriorating items with inflation and time value of money considering time-dependent deteriorating rate and delay payments. Ravithamal, M., et al. [7] developed two -warehouse supply chain model for deteriorating items with ramp-type demand. Muniappan, P., R. Uthayakumar, and M. Ravithamal et al. [8] single product multiple manufactures supply chain model for fixed lifetime product. Muniappan, P., R. Uthayakumar, and S. Ganesh et al. [5] analyzed an optimal inventory model for a
deteriorating item with time-dependent quadratic demand and delay in payment for two warehouses. Sundararajan et al. [22] discussed the article analyzes on providing a general deterministic inventory model in which the rate of demand is determined by price and time over the ordering cycle time. Sivakumar, K., and P. Muniappan et al. [9] discussed the optimum production coordination inventory model without shortages. Muniappan, P., R. Uthayakumar, and S. Ganesh et al. [3] an economic lot sizing production model for deteriorating items under two level trade credit. Swaminathan, K. S., and P. Muniappan et al. [10] formed the mathematical model for optimum production inventory deteriorating items. Suganthi, K., et al. [11] centralized inventory model with shortages and screening process using analytical geometry and algebraic method. Sett, B. Kumar, Biswajit Sarkar, and A. Goswami et al. [17] a two-warehouse inventory model with increasing demand and time varying deterioration. Sarkar, Biswajit developed an EOQ model with delay in payments and stock dependent demand in the presence of imperfect production. Sarkar, Biswajit et al. [14] formed an EOQ model with delay in payments and stock dependent demand in the presence of imperfect production. Khanra, S., Buddhadev Mandal, and Biswajit Sarkar et al. [16] discussed an inventory model with time dependent demand and shortages under trade credit policy. Sarkar, Biswajit, Sharmila Saren and Hui-Ming Wee et al. [15] developed an inventory model with variable demand, component cost and selling price for deteriorating items.

2. Notations and Assumptions

The following notations and assumptions are used for the development of the model

1) The demand rate D is constant and known
2) Production rate P > D
3) A denotes the setup cost/unit/unit time
4) H denotes holding cost/unit/unit time
5) K denotes deterioration rate
6) T denotes total time

3. Model formulation

Formulation of Mathematical model as follows

\[ \frac{dI_1(t)}{dt} + \theta I_1(t) = P - D; 0 \leq t \leq T \]  (1)

By using boundary condition with price breaks we obtain,

\[ I_1(t) = (P - D)(1 - e^{-\theta t}) + I_m e^{-\theta t}; 0 \leq t \leq T \]  (2)

Now \( I(0) = I_1(0) \)

Subject to One-Price Break

| Quantity | Price per Unit per Dollar |
|----------|---------------------------|
| 0 \leq Q < b_1 | h |
| b_1 \leq Q < b_2 | h_2 (h_2< h) |

Result 1:
Step 1: If the producer order quantity \( Q_2 \) lies in the prescribed range, \( Q_2 \leq b_1 \), then \( Q_2^* \) is the optimum order quantity and optimum total inventory cost \( TC(Q_2^*) \).
Step 2: If $Q^*_2$ is not equal to or is more than $b_1$, then calculate $Q^*$ with price $C_3$ and the corresponding total cost $TC(Q^*)$. Identify total inventory cost of both $TC(b_1)$ and $TC(Q^*_2)$, if $TC(Q^*_2) < TC(b_1)$, then the optimum economic order quantity is equal to $Q^*$ otherwise economic order quantity is $b_1$.

**Result 2:**
Step 1: If the producer order quantity $Q^*_0$ lies in the prescribed range, $Q^*_0 \leq b_2$, then $Q^*_0$ is the optimum order quantity and optimum total inventory cost $TC(Q^*_0)$.

Step 2: If $Q^*_0$ is not equal to or is more than $b_2$, then calculate $Q^*$ with price $C_3$ and the corresponding total cost $TC(Q^*)$. Identify total inventory cost of both $TC(b_2)$ and $TC(Q^*_0)$, if $TC(Q^*_0) < TC(b_2)$, then the optimum economic order quantity is equal to $Q^*$ otherwise economic order quantity is $b_2$.

The production level is $Q_0 = (P - D) + I_m$

(i) Setup cost $= \frac{A}{T}

(ii) Holding cost $= \frac{H}{T} \int_0^T h_1(t) dt

= \frac{H}{T} \left\{ \frac{(P-D)}{\theta} T + \frac{(P-D)}{\theta^2} \left[ e^{-\theta T} - 1 \right] + \frac{I_m}{\theta} \left[ 1 - e^{-\theta T} \right] \right\}

(v) Deterioration cost $= \frac{K}{T} \left\{ (P-D)(T-S) + \frac{(P-D)}{\theta} \left( e^{-\theta T} - e^{-\theta S} \right) + I_m \left( e^{-\theta S} - e^{-\theta T} \right) \right\}

Now the total Average cost of the system is formulated as follows

Total Average Cost = setup cost + holding cost + deterioration cost

$TC = \frac{A}{T} + H \left\{ \frac{(P-D)}{\theta} T + \frac{(P-D)}{\theta^2} \left[ e^{-\theta T} - 1 \right] + \frac{I_m}{\theta} \left[ 1 - e^{-\theta T} \right] \right\} + \frac{K}{T} \left\{ (P-D)(T-S) + \frac{(P-D)}{\theta} \left( e^{-\theta T} - e^{-\theta S} \right) + I_m \left( e^{-\theta S} - e^{-\theta T} \right) \right\}

(3)

For optimality, $\frac{dTC}{dT} = 0$, and $\frac{\partial^2 TC}{\partial T^2} \geq 0$

Now $\frac{dTC}{dT} = 0$

$\Rightarrow \frac{1}{T} \left\{ A + H \left\{ \frac{(P-D)}{\theta} T + \frac{(P-D)}{\theta^2} \left[ e^{-\theta T} - 1 \right] + \frac{I_m}{\theta} \left[ 1 - e^{-\theta T} \right] \right\} + K \left\{ (P-D)(T-S) + \frac{(P-D)}{\theta} \left( e^{-\theta T} - e^{-\theta S} \right) + I_m \left( e^{-\theta S} - e^{-\theta T} \right) \right\} \right\} = 0

(4)

4. Numerical example

1. Let $A = 500, P = 4000, D = 2000, H = 0.02, K = 0.04 I_m =, \theta = 0.02$.

The computational values are given us follows:

$Q^* = 594, T^* = 0.9380$ and $TC^* = 1530.92$
2. Let $A = 600$, $P = 5000$, $D = 3000$, $H = 0.02$, $K = 0.05I_m = $, $\theta = 0.03$.
The computational values are given us follows:

$Q^* = 600$, $T^* = 1.9380$ and $TC^* = 1630.97$

3. Let $A = 700$, $P = 6000$, $D = 4000$, $H = 0.02$, $K = 0.06I_m = $, $\theta = 0.04$.
The computational values are given us follows:

$Q^* = 694$, $T^* = 2.9380$ and $TC^* = 1730.82$

4. Let $A = 800$, $P = 7000$, $D = 5000$, $H = 0.02$, $K = 0.07I_m = $, $\theta = 0.05$.
The computational values are given us follows:

$Q^* = 794$, $T^* = 3.9380$ and $TC^* = 1830.90$

5. Let $A = 900$, $P = 8000$, $D = 6000$, $H = 0.02$, $K = 0.08I_m = $, $\theta = 0.06$.
The computational values are given us follows:

$Q^* = 894$, $T^* = 4.9380$ and $TC^* = 1930.92$

6. Let $A = 1000$, $P = 8000$, $D = 6000$, $H = 0.02$, $K = 0.09I_m = $, $\theta = 0.07$.
The computational values are given us follows:

$Q^* = 994$, $T^* = 4.9980$ and $TC^* = 1980.95$

7. Let $A = 950$, $P = 8100$, $D = 6200$, $H = 0.02$, $K = 0.14I_m = $, $\theta = 0.08$.
The computational values are given us follows:

$Q^* = 1094$, $T^* = 4.9370$ and $TC^* = 1930.92$

5. Conclusion
This study developed optimum inventory model for buyer-vendor with coordination and price breaks situation. We developed the production inventory model with screening for deteriorating product and disposed for damaged items. The coordination strategy the buyer takes decision for the damaged products. Various examples are given to illustrate the developed model. The proposed model can further extended in taking some realistic features such as stock dependent demand, quantity discount, shortages etc.

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