Research Article

Analytical Solution for Consolidation of Vertical Drains-Impervious Columns Multiple Composite Foundation under Nonuniform Distribution of Initial Pore Pressure

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Based on the axisymmetric model, the consolidation control equations based on the assumption of equal strain are established for the vertical drains-impervious columns multi-composite foundation consolidation problem, and the analytical solutions are given for the drains-impervious columns multiple composite foundation consolidation with trapezoidal, rectangular, positive triangular and inverted triangular distribution of initial pore pressure. Through the degradation study and comparison with the existing analytical solutions. Using the derived consolidation analytical solution for parameter analysis, the consolidation characteristics of multivariate composite foundation are studied. The results show that: the non-uniform distribution of initial pore pressure has a significant effect on the consolidation of combined composite foundations, the trapezoidal distribution is faster than the rectangular distribution, the positive triangular distribution is the slowest and the inverted triangular distribution is the fastest. The smaller the initial pore pressure value at the base of the soil layer and the larger the vertical permeability coefficient of the soil, the faster the consolidation. The consolidation rate of the foundation increases with the increase of the replacement rate of impervious columns and the compression modulus. Compared with the foundation of a single pile type, the consolidation rate of combined composite foundations can be significantly increased. The rate of consolidation of the foundation increases with the increase in the replacement rate and compression modulus of impervious columns.

1. Introduction

In recent years, the development of composite foundations from a single type to a combined type has occurred and great progress has been made. Depending on the permeability of the piles, composite foundations can be divided into drained pile composite foundations and impervious pile composite foundations. The use of combined vertical drains with impervious columns to treat soft ground is a cost-effective measure, and composite foundations usually solve the problem of poor foundation drainage and inadequate bearing capacity. The vertical drains have a strong drainage capacity to facilitate the drainage of water from the foundation, the impervious columns have a good bearing capacity and can effectively improve the bearing capacity of the foundation. The combination of the two can give full play to their respective advantages with a view to reducing postwork settlement, accelerating foundation consolidation and increasing load-bearing capacity.

The theory of composite foundation consolidation is an important part of the theory of composite foundations. At present, the consolidation solutions established for composite foundations are able to take into account the effects of smearing and well resistance effects [1], changes in soil permeability coefficients [2], variable loading effects [3–5] and changes in total stresses with time and depth [6–8], among other factors. Many scholars have also investigated the theory of consolidation of composite foundations...
composed of different materials. Liu et al. [9] gave a simplified method for calculating the settlement of a composite foundation consolidation used plum blossom pile layout scheme and based on an axisymmetric consolidation model with a combination of powder jetting piles and drainage board. Prakash and Krishnamoorthy [10] investigated the effect of lime and CFG pile composite foundations on the factor of safety of embankments at different time intervals during the consolidation of foundations. Ye et al. [11] considered the problem of composite foundation consolidation with soil-cement columns and prefabricated vertical drains. Yu et al. [12] based on an axisymmetric consolidation model and the consolidation theory for double-layer ground, the consolidation characteristics of combined composite foundation reinforced by penetrated impermeable columns and partially penetrated permeable columns have been studied. Yang et al. [13] studied and analyzed a multi-pile composite foundation consisting of fully penetrated long rigid piles and partially penetrated impervious or pervious short piles. Based on the assumption of equal vertical strain, derived an analytical solution for the consolidation of long rigid piles with undrained short piles in composite foundations. However, in practical engineering, the initial pore pressure induced by external loads in the foundation is often non-uniformly distributed along the depth [14, 15]. The existing research results on the theory of consolidation of multi-composite foundations of vertical drains and impervious columns are relatively few. Therefore, the study of composite foundations with combined vertical drains and impervious columns under non-uniform initial pore pressure is of both theoretical and practical value.

In view of this, this paper firstly derives the consolidation solution for the combination of vertical drain and impervious columns with the initial pore pressure distributed along a trapezoid shape, and gives an analytical solution for consolidation with the initial pore pressure distributed in a rectangle, a positive triangle and an inverted triangle. Using the analytical solution of consolidation derived in this paper, the degradation is studied, and the consolidation law of the combined vertical drains-impervious columns multi-composite foundation is analyzed and discussed by preparing the calculation program and plotting the relevant consolidation curve.

2. Establishment of Consolidation Control Equation

2.1. Calculation Sketch and Basic Assumptions. Figure 1 shows a computational model for a combined vertical drain-impervious columns multi-composite foundation consolidation problem. Multi-composite foundations are mostly arranged in rectangles and triangles. In this paper, both vertical drains and impervious columns are arranged in rectangles, and one unit is taken as the research object, with the vertical drain being the center of the model and the impervious column located at the outer boundary of the model. Where H is the thickness of the soft clay layer, q is the instantaneous applied external load, \( r_w \), \( r_e \) and \( r_n \) are the radius of the vertical drain, the radius of the zone of influence and the radius of the entire unit foundation respectively. \( k_h \) and \( k_v \) are the horizontal and vertical permeability coefficients of the foundation soil respectively. \( k_w \) is the vertical permeability coefficient of the vertical drain. \( u_w \) and \( u_e \) are the pore pressure within the vertical drain and soil at any point respectively. \( E_p \) and \( E_s \) are the compression modulus of the impervious column and natural soil respectively. \( r \) and \( z \) are the radial and vertical coordinates respectively, assuming that the drainage conditions of the multi-composite foundation are permeable at the top and impermeable at the bottom.

In the derivation of this paper, the following assumptions are made about the model.

1. The equal strain condition holds, i.e., there is no lateral deformation in the foundation and the vertical deformation at any point at the same depth is equal.
2. The load is applied instantaneously and the resulting initial pore pressure is non-uniformly distributed along the depth direction.
3. Ignore the radial seepage in the vertical drains.
4. Water in the soil has both radial and vertical seepage, and the seepage obeys Darcy’s law.
5. The flow around the well is continuously equal, i.e., the flow of water from the soil into the vertical drains at any depth is equal to the increment of water flowing upwards in the well, expressed as follows:

\[
2\pi r dz k_h \frac{\partial u}{\partial r} \bigg|_{r=r_w} = -\pi r^2 dz k_w \frac{\partial^2 u}{\partial z^2} \tag{1}
\]

Figure 2 shows the four cases of initial pore pressure distribution along the depth. Where \( P_T \) and \( P_B \) are the initial pore pressure values at the top and bottom of the soil layer respectively. When \( P_T = P_B = P_w \) as shown in Figure 2(a), the initial pore pressure is evenly distributed. When \( P_T = 0 \), as shown in Figure 2(b), the initial pore pressure is distributed in a positive triangle along the depth. When \( P_B = 0 \), as shown in Figure 2(c), the initial pore pressure is distributed in an inverted triangle. When \( P_T \neq P_B \neq 0 \), as shown in Figure 2(d), the initial pore pressure is distributed in a trapezoid shape.

2.2. Solving Conditions and Control Equation. The external load is supported by the vertical drains, the soil and the impervious columns, which according to the vertical equilibrium condition gives

\[
\pi r_w^2 \bar{\sigma}_w + \pi (r_e^2 - r_w^2) \bar{\sigma}_s + \pi (r_n^2 - r_e^2) \bar{\sigma}_p = \pi r_n^2 \bar{q}, \tag{2}
\]

where \( \bar{\sigma}_w \), \( \bar{\sigma}_s \) and \( \bar{\sigma}_p \) are the average stresses in the vertical drain, soil and impervious columns respectively in a multi-composite foundation.
From basic assumption (1) it follows that

\[
\frac{\sigma_{uw} - u_w}{E_w} = \frac{\sigma_s - \sigma_k}{E_s} = \frac{\sigma_p}{E_p} = \varepsilon_v, \tag{3}
\]

where \(E_w\) is the compression modulus of the vertical drain, \(\sigma_s\) is the average pore water pressure at any depth in the soil, \(\varepsilon_v\) and \(\varepsilon_z\) are the volumetric and vertical strains in the foundation, respectively, and are equal when only vertical deformation is present.

The derivative of (3) with respect to time \(t\) gives

\[
\frac{\partial \varepsilon_z}{\partial t} = -\frac{1}{E_{\text{com}}} \frac{\partial \pi}{\partial t}. \tag{4}
\]

Where \(\pi\) is the average pore water pressure across the foundation at any depth, \(E_{\text{com}}\) is the composite modulus of elasticity, which can be expressed as

\[
E_{\text{com}} = \frac{E_w + (N_{ew}^2 - 1)E_s + (N_{nw}^2 - N_{ew}^2)E_p}{N_{nw}^2}, \tag{5}
\]

where \(N_{ew} = (r_e/r_w)\), and \(N_{nw} = (r_n/r_w)\).

The expressions for the average pore water pressure in the foundation soil at any depth and the average pore water pressure across the foundation at any depth are

\[
\pi_s = \frac{1}{\pi(r_e^2 - r_w^2)} \int_{r_w}^{r_e} 2\pi r u_s dr, \tag{6}
\]

\[
\pi = \frac{1}{\pi r_e^2} \int_{0}^{r_e} 2\pi r u_s (r) dr = \frac{N_{ew}^2 - 1}{N_{nw}^2} \pi_s. \tag{7}
\]

According to the assumption of equal strain, the basic consolidation equation for the axially symmetric case with reference to the literature Lu et al. [2] is
\[
\frac{\partial \varepsilon_v}{\partial t} = k_v \frac{1}{\gamma_w} \frac{\partial}{\partial r} \left[ \frac{\partial u_v}{\partial r} \right] - k_v \frac{\partial^2 \varepsilon_v}{\partial z^2}, \quad r_w \leq r \leq r_c. \tag{8}
\]

To simplify the calculation, a function \( w(z, t) \), which is independent of \( r \), is introduced and can be expressed as
\[
w(z, t) = \frac{k_v}{\gamma_w} \frac{\partial^2 \varepsilon_v}{\partial z^2} + \frac{\partial \varepsilon_v}{\partial z}. \tag{9}
\]
Substituting equation (9) into equation (8) gives
\[
w(z, t) = \frac{k_v}{\gamma_w} \frac{1}{r} \frac{\partial}{\partial r} \left[ r \frac{\partial u_v}{\partial r} \right], \quad r_w \leq r \leq r_c, \tag{10}
\]
where \( \gamma_w \) is the weight of the water.

Boundary conditions:
\[
r = r_w, \quad u_z = u_w = \bar{u}_w, \tag{11}
\]
\[
r = r_c, \quad \frac{\partial u_z}{\partial r} = 0, \tag{12}
\]
\[
z = 0, \quad u = 0, \tag{13}
\]
\[
z = H, \quad \frac{\partial u}{\partial z} = 0. \tag{14}
\]

Initial condition:
\[
t = 0: \quad \bar{u}(z) = u_0(z) = P_T + (P_B - P_T) \frac{z}{H}. \tag{15}
\]

Integrating (10) with respect to \( r \) and using the boundary condition (12) yields
\[
\frac{\partial u_z}{\partial r} = w(z, t) \frac{\gamma_w}{2k_v} \left( \frac{r^2}{r_w} - r \right). \tag{16}
\]

Integrating (16) with respect to \( r \) and using the boundary condition (11) yields
\[
u_z = w(z, t) \frac{\gamma_w}{2k_v} \left( \frac{r^2}{r_w} - \frac{r^2 - r_w^2}{2} \right) + u_w. \tag{17}
\]

The combination of equation (6) and equation (17) yields
\[
\bar{u}_z = w(z, t) \frac{\gamma_w r^2 F_a}{2k_v} + \bar{u}_w, \tag{18}
\]
where \( F_a = \ln N_{ew}^2 + 4N_{ew}^4 - 1/4N_{ew}^4 - 3/4N_{ew}^2/N_{ew}^2 - 1 \).

Substituting (16) into (1) can be obtained:
\[
k_w \frac{\partial^2 \bar{u}_w}{\partial z^2} = -w(z, t) \left( N_{ew}^2 - 1 \right). \tag{19}
\]

Then from equations (4), (7), (9) and (19), the following can be derived
\[
N_{ma}^2 \frac{k_w}{\gamma_w} \frac{\partial^2 \bar{u}_w}{\partial z^2} = \frac{N_{ma}^2}{F_{com}} \frac{1}{\gamma_w} \frac{\partial \bar{u}_w}{\partial t} - k_w \frac{\partial^2 \bar{u}_w}{\partial z^2}. \tag{20}
\]

Taking the partial derivative of (18) twice with respect to \( z \) and using (7) gives
\[
\frac{N_{ma}^2}{N_{ew}^2 - 1} \frac{\partial^2 \bar{u}_w}{\partial z^2} = \frac{N_{ma}^2}{F_{com}} \frac{1}{\gamma_w} \frac{\partial \bar{u}_w}{\partial t} + k_w \frac{\partial^2 \bar{u}_w}{\partial z^2}. \tag{21}
\]

### 3. Solution of Average Pore Pressure and Average Consolidation of Composite Foundation

Referring to the literature Lu et al. [16], it is assumed that the solution of (22) takes the form of
\[
\bar{u} = \sum_{m=0}^{\infty} T_m(t) \sin \left( \frac{M}{H} z \right), \tag{23}
\]
where \( M = 2m + 1/2n, \ n = 0, 1, 2, \ldots \)

Substituting (23) into (22), the following can be obtained:
\[
T_m(t) + T_m(t) \cdot \beta_m = 0, \tag{24}
\]
where \( \beta_m = A (M/H)^2 - C/(H/M)^2 - B \).

Substituting the coefficients \( A, B \) and \( C \) in (22) into \( \beta_m \) gives
According to the theory of ordinary differential equations, the general solution of (24) can be written as

$$T_m(t) = A_m e^{-\beta_mt}. \quad (26)$$

Substituting equation (26) into equation (23) gives that

$$\pi = \sum_{m=0}^{\infty} A_m \sin\left(\frac{Mz}{H}\right) e^{-\beta_mt}. \quad (27)$$

Substituting the initial condition (15) into equation (27) and using the trigonometric orthogonality, the following can be obtained:

$$A_m = \frac{\int_0^H u_0(z) \sin \left(\frac{Mz}{H}\right) dz}{\int_0^H \sin^2 \left(\frac{Mz}{H}\right) dz} = \frac{2}{M} \left[ P_T + (-1)^m P_B - P_T \right]. \quad (28)$$

Substituting equation (28) into equation (27) gives that

$$\pi = \sum_{m=0}^{\infty} \frac{2}{M} \left[ P_T + (-1)^m P_B - P_T \right] \sin \left(\frac{Mz}{H}\right) e^{-\beta_mt}. \quad (29)$$

Based on the above derivation, the total average consolidation of a multi-composite foundation as defined by stress can be found as

$$U(t) = 1 - \frac{\int_0^H \pi dz}{\int_0^H \pi_0(z) dz} = 1 - \sum_{m=0}^{\infty} \frac{P_T + (-1)^m (P_B - P_T/M)}{M^2 (P_B + P_T)} 4 e^{-\beta_mt}. \quad (30)$$

In summary, (29) and (30) are the solutions for the consolidation of a vertical drain-impervious columns multi-composite foundation when a trapezoidal distribution of initial pore pressure is considered. Based on the four distributions of the initial pore pressure in Figure 2, further expressions for the average pore pressure and average consolidation for each particular case are given below.

**Case 1.** At $P_T = P_B = P_0$, the initial pore pressure is rectangularly distributed, see Figure 2(a).

Degenerate (22) and (23) to the instantaneously loaded initial pore pressure homogeneous vertical drain and impervious columns multi-composite foundation consolidation solution, i.e.

$$\beta_m = \left[ \frac{N_{ew}^2 / (N_{ew}^2 - 1)^2 (k_w/k_h) (1 + (1/N_{ew}^2 - 1) (k_w/k_h))}{(H^2 M^2)} + (1/N_{ew}^2 - 1) (k_w/k_h) (r_0^2 F_u/2) \right] E_{com k_h} / y_w \quad (25)$$

$$\pi = p_0 \sum_{m=0}^{\infty} \frac{2}{M} \sin \left(\frac{Mz}{H}\right) e^{-\beta mt}, \quad (31)$$

$$U(t) = 1 - \sum_{m=0}^{\infty} \frac{2}{M} e^{-\beta mt}. \quad (32)$$

**Case 2.** At $P_T = 0$, the initial pore pressure has a positive triangular distribution, see Figure 2(b).

According to (22) and (23), the following can be obtained:

$$\pi = p_B \sum_{m=0}^{\infty} (-1)^m \frac{2}{M} \sin \left(\frac{Mz}{H}\right) e^{-\beta mt}, \quad (33)$$

$$U(t) = 1 - \sum_{m=0}^{\infty} (-1)^m \frac{4}{M^2} e^{-\beta mt}. \quad (34)$$

**Case 3.** At $P_B = 0$, the initial pore pressure has an inverted triangular distribution, see Figure 2(c).

According to equation (29) and equation (30), the following can be obtained:

$$\pi = p_T \sum_{m=0}^{\infty} \frac{2}{M} \left[ 1 - (-1)^m \frac{1}{M} \right] \sin \left(\frac{Mz}{H}\right) e^{-\beta mt}, \quad (35)$$

$$U(t) = 1 - \sum_{m=0}^{\infty} \frac{4}{M^2} \left[ 1 - (-1)^m \frac{1}{M} \right] e^{-\beta mt}. \quad (36)$$

At this point, all the analytical solutions for multi-composite foundation consolidation of combined vertical drain-impervious columns with trapezoidal, rectangular, positive triangle and inverted triangular distribution of initial pore pressure under this model have been given.

4. Degenerations of the Obtained Solution

The degradation method with reference to the literature [1, 17] is used to analyze the degradation of the consolidation analytical solutions obtained in this paper. The above three special cases of pore pressure and consolidation degree are degenerated from (29) and (30), where (31) and (32) are the solutions for consolidation when the initial pore pressure is homogeneous and can be degenerated again, and the steps and methods of degeneration are as follows.

1. When $N_{ew} = N_{ew0}$, i.e., there are no impervious columns in the foundation. Since there are no impervious columns, the other parameters need to be degraded in the
same way, where $E_{\text{com}} = E_s$ and $N_{n_w} = n. \beta_m$ of equation (25) can be degenerated to

$$\beta_m = \frac{(k_w/k_h)(k_v/k_h)(r_c^2F_\alpha/2)((M/H)^2)(1/(n^2 - 1)) + (k_v/k_h)(1 + (1/n^2 - 1)(k_w/k_v))}{(H/M)^2 + (1/(n^2 - 1))(k_w/k_h)(r_c^2F_\alpha/2)} \frac{E_s k_h}{\gamma_w}.$$  \hspace{1cm} (35)

Equation (37) is the solution for a single vertical drain foundation, which at this point degenerates into the solution of Tang Xiaowu [18].

(2) Continuing the degeneration based on the above equation, let $n \to \infty$ and $k_h \to \infty$, then $\beta_m$ can be degenerated to

$$\beta_m = c_v \left(\frac{M}{H}\right)^2.$$ \hspace{1cm} (36)

Equation (38) is the analytical solution for one-dimensional consolidation of Terzaghi natural foundations, where $c_v$ is the vertical consolidation coefficient of the soil, $c_v = k_v E_s/\gamma_w$.

$$\Gamma_m = \frac{\left(N_{n_w}^2/(N_{n_w}^2 - 1)^2\right) + (k_w/k_h)(k_v/k_h)(r_c^2F_\alpha/2)((M/H)^2) + \left(N_{n_w}^2/(N_{n_w}^2 - 1)^2\right)(k_v/k_h)(1 + (1/N_{n_w}^2 - 1)(k_w/k_v))}{(H/M)^2 + (1/(N_{n_w}^2 - 1))(k_w/k_h)(r_c^2F_\alpha/2)} 4r_c^2Y,$$ \hspace{1cm} (38)

where $Y = (E_{\text{com}}/E_s)$.

The calculation of the solution of this paper is programmed and the effect of each parameter on the consolidation properties of the multi-composite foundation is analyzed below by plotting the consolidation curve. Figure 3 shows the consolidation curves for different distributions of initial pore pressure. The initial pore pressure distribution conditions have a significant effect on the consolidation of multi-composite foundations, with a trapezoidal distribution consolidating faster than a rectangular distribution, a positive triangular distribution consolidating the slowest, and an inverted triangular distribution consolidating the fastest. The results show that the effect of considering the non-uniform distribution of the initial pore pressure along the depth is not negligible.

Figure 4 reflects the influence of the initial pore pressure values at the top and bottom of the soil layer on the consolidation process. When $(P_\text{in}/P_T) = 1$ and $(P_\text{in}/P_T) = 0$ the initial pore pressure is uniformly distributed along the depth and the initial pore pressure is distributed in an inverted triangle along the depth respectively, as shown in the figure, the larger the initial pore pressure value at the bottom of the soil, the slower the consolidation of the foundation.

Figure 5 shows the effect of impervious columns replacement rate on the consolidation of a multiple composite foundation. As the radius of the foundation of the entire unit continues to decrease, this leads to a decrease in $N_{n_w}$. The smaller the impervious pile replacement rate, the slower the consolidation rate of the whole foundation, because the modulus of elasticity of the impervious pile is much larger than that of the vertical drain and the soil, the smaller the impervious pile replacement rate, the smaller the average modulus of elasticity of the whole foundation, the smaller the external load borne by the impervious pile itself, and the slower the rate of consolidation of the foundation accordingly. When $N_{n_w} = 15$, the impervious pile replacement rate is zero, i.e., there are no impervious piles in the composite foundation. It can be seen that the rate of consolidation of foundations can be significantly increased with multiple composite foundations compared to a single pile type foundation.

The effect of the vertical permeability of the soil on the consolidation of the foundation is shown in Figure 6. The effect of the soil permeability coefficient on the consolidation rate of the foundation is significant. When the radial and vertical permeability coefficients of the soil are the same, the speed of consolidation of the multi-composite foundation is the fastest, and as the vertical permeability coefficient of the soil decreases, the speed of consolidation of the foundation becomes slower, i.e., the larger the ratio of $k_h/k_v$, the slower the consolidation of the foundation.
Figure 7 gives the curve of the effect of the pile-soil compression modulus ratio on the consolidation of the foundation. The larger the pile-soil compression modulus ratio, the faster the consolidation. Engineering impervious piles can be CFG piles, cement mixer piles, or even concrete pipe piles, and the stiffness varies greatly. The increase in stiffness of impervious piles has a significant effect on the consolidation rate of composite foundations, and the consolidation rate of vertical drain-impervious piles multi-composite foundations increases significantly with the increase in the compression modulus of impervious piles.

Figure 8 shows the comparison between the consolidation solution of this paper and the existing solution under instantaneous loading. Under the same parameters, when the disturbance zone of the pile and vertical drain is not considered, the consolidation rate from fast to slow is Chen et al. [19] multi-composite foundation solution, this paper’s solution, Yu et al. [20] multi-composite foundation solution, Walker and Indraratna [21] sand drain foundation solution, and Lu et al. [17] impervious pile composite foundation solution. The reason for the faster deconsolidation of Chen et al. [19] is that the radial and vertical percolation of the drainage body is not considered, and the radial and vertical
6. Conclusion

In this paper, a model for the consolidation of multi-composite foundations with vertical drain-imperious piles is established, and the analytical solution for consolidation under consideration of non-uniform distribution of initial pore pressure along the depth, well resistance, radial and vertical seepage of the soil, etc. The consolidation characteristics of multi-composite foundations are systematically studied through parametric analysis. The main conclusions are as follows:

1. The nonuniform distribution of the initial pore pressure along the depth has a significant effect on the consolidation of multi-composite foundations, with a trapezoidal distribution of initial pore pressure resulting in faster consolidation than a rectangular distribution, a positive triangular distribution resulting in the slowest consolidation, and an inverted triangular distribution resulting in the fastest consolidation. The larger the initial pore pressure value at the base of the soil layer, the slower the consolidation.

2. When the radial and vertical permeability coefficients of the multi-composite foundation soil are the same, the foundation consolidation speed is the fastest, and increasing the vertical permeability coefficient of the soil will accelerate the consolidation speed of the multi-composite foundation.

3. The consolidation rate of the vertical drain-imperious piles multi-composite foundation increases with the increase of the replacement rate and compression modulus of the imperious pile. Increasing the stiffness of the imperious piles can accelerate the consolidation of the composite foundation.

4. The consolidation solution of this paper is compared with the existing solution, and the multi-pile composite foundation can significantly improve the consolidation rate of the foundation compared with the single pile type foundation.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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