An Inventory Management Model for Product-Service System in Dual-Channel Supply Chain

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Abstract. This paper proposes a model to study inventory management for product and service bundles under product-service system (PSS) concept. This bundle is offered to customers under the dual-channel supply chain (DCSC) structure. By PSS concept, it is expected that the products sold will increase, then the products or services that are not selling well can be bundled with goods that sell well, so as to reduce overstock or deadstock in the inventory. In terms of distribution, using the DCSC structure, sales can increase by 20% from the previous one. This research will be conducted by combining the PSS concept in the DCSC distribution structure, in terms of inventory management. Model is developed of inventory cost minimization for DCSC-PSS by considering order quantity as a decision variable. Afterwards, the model will represent two scenarios of game theory for decision making practice DCSC-PSS, Stackelberg Leadership and Vertical Nash. The initial result of the numerical experiment was DCSC's total inventory cost, Stackelberg Leadership scenario is IDR 556,642,465.40 greater than Vertical Nash, IDR 468,614,127.10. Afterwards, sensitivity analysis based on customer acceptance (ρ) and percentage of PSS (ϕ). At last, it was concluded that Vertical Nash was better than Stackelberg Leadership for decision making in inventory management

1. Instruction

The internet has been giving significant change in almost industry practices, including supply chain (SC hereinafter) structures and operations. Before the internet introduction, SC practitioners used conventional channel (offline) to deliver their products or services to customers. Once the internet got popular for public use, the operators utilized online facility as their online catalog, online point of order, or online sales outlet. In order to widen their market range, to respond to customers’ behavior change, and increase SC profitability, people start combining both online and offline channel as a new structure. Such mixture is known as dual-channel supply chain (DCSC hereinafter).

At the early phase of DCSC introduction, some products such as books, apparels, and music are sold under this structure. In research [1], DCSC’s potential to increase sales by an average of 20% and reach new markets by utilizing internet features significantly attracted the attention of researchers to contribute to the development of the DCSC. Later on electronics, apparels, tickets, foods and even production materials are also available in this structure. In addition, in recent advancement, to sell only the product itself without further consideration about complementary service related to the product is not enough. An advanced idea to sell a bundle of product and its related services is known as product service system (PSS hereinafter). Such idea has been proved to be effectively increase service level, customer satisfaction, and company profit to some extent. However, this promising PSS has not been considered and utilized to provide advanced benefits in DCSC area.
This research attempts to study inventory management for product and service bundle under PSS concept. This bundle is offered to customer under DCSC structure. By taking advantage of wider market reach under DCSC structure, the introduction of PSS instead of offering product and its related system separately, is a challenging and fruitful idea. Such combination may yield better finance performance under proper DCSC and PSS setting.

2. Literature study

Two groups of research paper were studied, namely DCSC advancement review and PSS advancement review in giving positioning of this current research idea.

Firstly, in DCSC advancement review, starting with a proposition of DCSC terminology by [1], a number of conceptual work in DCSC domain started blooming. [2] performed a case study analysis for marketing PC in Europe by using multiple channel strategy. Almost at the same time, [3] proposed a conceptual framework that ensures the successfulness of embodying conventional sales channel with internet-based one. The authors used a case study to highlight four benefits in term of cost savings, improved differentiation, enhanced trust, and market extension in applying DCSC concept.

To be compared to this idea, all of those papers were focusing on the creation of DCSC basic concept. It is obvious that in every introduction phase of a new research branch, the most prominent topics should be conceptual works instead of descriptive or normative ones. In contrast, current idea focuses on how to normatively implement pricing and PSS concept at the same time in DCSC structure.

A paper by [4] one of the prominent works related to channel structure of DCSC. The author employed inventory management to coordinate inventory amongst conventional stores and online facility. This paper proposed some substitution inventory scenarios in undertaking this integration. The result showed that this kind of integration is financially beneficial, yet there are two limits of market proportion and service level. Later on, this author gave some enhancement on his work, in [5], by introducing the concept of centralization versus decentralization in managing DCSC inventory. Quite similar DCSC inventory management work was also undertaken by [6]. Their originality was on the assumption that the system receives stochastic demands from conventional (offline) and online customers. Moreover, they assumed that customer may switch channel when a stock-out occurs. As for the result, mixed strategy of dual-channel outperforms two-single strategies.

One interesting idea was presented by [7]. They proposed 2 dynamic assignment policies namely taking benefits of online demand fulfillment from traversal e-fulfillment and still in transit inventory. By applying these policies, sales increased about 8.2% over the optimal static policy. [7] consider where and how much inventory must be allocated and stored in each location for a company that meets in-store and online demand so as to minimize total costs (holding, backorder, fixed operating, transportation and holding costs). [8] consider a two-echelon dual-channel supply chain model with production and shipping setups and developing a new inventory control policy for supply chains. [9] investigated the impact of stock-out-based consumer transfer behavior on decentralized DCSC efficiency, and designed various contracts to coordinate order quantity decisions in offline channels and inventory level decisions in online channels.

In recent years attention to inventory studies and uncertainty in DCSC demand has increased. A paper by [10] conduct research on inventory decisions in manufacturers and retailers by considering service levels. They also examined the relationship between optimal decisions. [11] focuses on location-inventory problems in centralized SC by considering demand uncertainty and lead time. [13] added third-party logistics providers to supplier-retailer supply chains, assuming demand waiting times are stochastic and short, designing transportation costs and making wholesale price discount contracts to coordinate price and order quantity decisions between the three parties.

Other researchers, namely [13] learn about optimal order quantity decisions on retail channels and manufacturer level inventory decisions on online channels, trace the manufacturer's lead time decisions that are optimal in online channels, the impact of product prices and consumer transfer behavior on optimal decisions among members SC, and compare optimal decisions between decentralized and centralized scenarios. The results of the study say that customers in online channels enjoy shorter
delivery times and therefore better services in decentralized scenarios. [15] conducts inventory determination and prices for dual-channel supply chains in the case of deteriorating products.

From the above studies it can be seen that research related to inventory models in the DCSC structure has been widely carried out. However, from all works, there is no one has considered combining the concept of product service system (PSS).

Secondly, in PSS advancement review, in the past, high quality products were expected from a series of effective and efficient design, realization and distribution processes. After customers play a more important role in providing industry feedback, manufacturers begin to consider supporting services such as maintenance, upgrading, user training and etc [16]. Therefore a shift in traditional manufacturing to combine services with a focus on providing customer solutions, not just products being important.

There are various definitions of PSS that we can find. However, one important definition by [16], [17] and [18] can represent the whole. PSS is a combination of customer-oriented products and services [16], realized in an expanded value creation network, consisting of manufacturers and suppliers and service partners [17]. In addition, PSS represents integrated product and service offerings that provide the value used [18].

A work by [19] stated that several adoption of PSS became 3 objects, namely forklifts and trucks (heavy industry), the electronics industry and the household appliances industry. They emphasize this PSS on product design especially on the life cycle perspective. They consider all phases of the product life cycle, namely manufacturing, use, maintenance, and end-of-life maintenance. In addition, they also found some engineering findings about when exactly applying PSS in product design activities.

Another work by [20] describes how to choose the right strategy in implementing PSS. They use case-based reasoning to find the right PSS strategy. In providing data, they built a case database on successful PSS cases. To describe each object, twelve indexes are grouped into three categories and further analyzed. Forty-seven successful PSS cases and their related data were collected. Then the hierarchical analytic process (AHP) is used to find the relative weight of the factors related to customer selection. This weight is then used in calculating similarities using the reasoning process based on letters. Extraction results in the recommended PSS strategy. It is proven that more than 90% of cases get the right strategy from the most similar cases.

A work by [21] in his research divided PSS into 3 (three) main categories, namely: Product-oriented services, Use-oriented services, Results-oriented services. [22] presented a new model for evaluating the costs induced in various PSS. This model consists of two sub-models, namely a descriptive model and an explanatory model. In the descriptive model, variations in visualization in integrated product-service systems are explored. In the explanatory model, the quantification of the PSS configuration costs provided has been explained.

Other research namely [24] identify the impact of product service systems on sustainability - a structured literature review and they create tools and methods to increase the positive impact on the level of sustainability while reducing the possible negative effects. [24] describes the basic information in the game theory and game theory application for PSS, then they propose three types of PSS, as use-oriented PSS, and results-oriented PSS using the game theory framework. [25] considers a dynamic model for Use-Oriented PSS that together optimizes decisions for replacement and inventory.

There are still a number of interesting PSS works to study. But most of them are still in the conceptualization stage. The operational mechanism to make PSS closer to reality is needed. The idea of incorporating PSS features into DCSC pricing strategies is a kind of mechanism in making PSS potential work directly into industrial routines.

3. System under consideration
The conceptual model of system under discussion is given in Figure 1, which Q is the notation for order quantity. Please refer to the next section for the notation.
Figure 1. Conceptual Model

This graphical representation expresses corresponding research scope, entities involved, their relationships and decision to make. Figure 1 illustrates the conceptual model of the DCSC structure by adding the PSS concept. It is assumed that the central warehouse is an extension of the manufacturer in the distribution of products to each channel. Offline channel and online channel have their own customer demand and optimal order quantity in terms of fulfilling the customer's demand. The offline channel purchase cost is the wholesale price, \( w \). While purchase cost online channel is a unit cost, \( c \) because the assumption in this study is that online channel belongs to the central warehouse. The selling price on the offline channel, \( P_s \) is different from the selling price on the online channel, \( P_o \).

### 4. Model

In our model development, the step is started with incorporating PSS concept into commonly used offline and online demand function. Then, this function will act as the main components in constructing the proposed model.

#### 4.1. Conventional DCSC Demand and DCSC-PSS Demand Model

Referring to [27], demand function for offline and online channel, \( D_s \) and \( D_o \) respectively, can be written as follow:

\[
\begin{align}
D_s &= (1 - \rho)d_{\text{max}} - \alpha_1 P_s + \beta_1 P_o \\
D_o &= \rho d_{\text{max}} - \alpha_2 P_o + \beta_2 P_s
\end{align}
\]

where:
- \( D_s \) = annual demand in units for main product at offline channel
- \( D_o \) = annual demand in units for main product at online channel
- \( \rho \) = percentage share of customer who prefer to purchase online relative total demand
- \( d_{\text{max}} \) = maximum demand main product when price is very cheap
- \( P_s \) = price in conventional store/ offline store
- \( P_o \) = price in online store
- \( \alpha_1 \) = price sensitivity constant in offline channel
- \( \alpha_2 \) = price sensitivity constant in online channel
- \( \beta_1 \) = cross-price sensitivity constant for offline channel to substitute online channel
- \( \beta_2 \) = cross-price sensitivity constant for online channel to substitute offline channel
PSS consists of main product and additional product. The additional product can be in the form of service or other product that can support or relate to the main product. By considering PSS concept, demand product will be divided into several demands such as demand for main product, demand for additional product at offline channel and online channel respectively.

**Offline channel**

- **Main product**
  \[ D_s = (1 - \rho)d_{max} - \alpha_1 P_s + \beta_1 P_o \]  \hfill (3)

- **Additional product**
  \[ D_s' = \phi D_s \]  \hfill (4)

**Online channel**

- **Main product**
  \[ D_o = \rho d_{max} - \alpha_2 P_o + \beta_2 P_s \]  \hfill (5)

- **Additional product**
  \[ D_o' = \phi D_o \]  \hfill (6)

where:

- \( \phi \) = PSS percentage or inclusion factor of additional product for main product \((0 \leq \phi \leq 1)\)
- \( D_s \) = annual demand in units for main product at offline channel
- \( D_s' \) = annual demand in units for additional product at offline channel
- \( D_o \) = annual demand in units for main product at online channel
- \( D_o' \) = annual demand in units for additional product at online channel

### 4.2. Cost Minimization Function

Referring to EOQ model and total inventory cost at \([26]\) with formula as below:

Total inventory cost minimization = purchase cost + order cost + holding cost

By considering PSS concept, model total inventory cost will be constructed as below:

**Offline channel**

\[
\text{TC}(Q)_{off} = (wD_s + w'D_s') + \left( \frac{\alpha_0 D_s}{Q_s} + \frac{\alpha_1 D_s'}{Q_s} \right) + \left( \frac{\beta_0 D_o}{Q_o} + \frac{\beta_0 D_o'}{Q_o} \right) + \left( \frac{\beta_1 D_o}{Q_o} + \frac{\beta_1 D_o'}{Q_o} \right) \]

\[
\text{TC}(Q)_{off} = \left[(1 - \rho)d_{max} - \alpha_1 P_s + \beta_1 P_o \right] \left[w + w'\phi + \frac{\alpha_0 + \frac{\alpha_1}{\phi} + \frac{\alpha_1}{\phi}}{\phi} + \frac{\beta_0 + \frac{\beta_0}{\phi} + \frac{\beta_0}{\phi}}{\phi} + \frac{\beta_0 + \frac{\beta_0}{\phi} + \frac{\beta_0}{\phi}}{\phi} \right] \]  \hfill (7)

**Online channel**

\[
\text{TC}(Q)_{on} = (cD_o + c'D_o') + \left( \frac{\alpha_0 D_o}{Q_o} + \frac{\alpha_0 D_o'}{Q_o} \right) + \left( \frac{\beta_0 D_o}{Q_o} + \frac{\beta_0 D_o'}{Q_o} \right) + \left( \frac{\beta_0 D_o}{Q_o} + \frac{\beta_0 D_o'}{Q_o} \right) \]

\[
\text{TC}(Q)_{on} = \left[\rho d_{max} - \alpha_2 P_o + \beta_2 P_s \right] \left[c + c'\phi + \frac{\alpha_2}{\phi} + \frac{\alpha_2}{\phi} \right] + \frac{\beta_0 + \frac{\beta_0}{\phi} + \frac{\beta_0}{\phi}}{\phi} + \frac{\beta_0 + \frac{\beta_0}{\phi} + \frac{\beta_0}{\phi}}{\phi} \]  \hfill (8)

After construct DCSC-PSS function at offline and online model as Equation (7) and (8), will be continued by making model into two different game theory scenarios, namely the Stackelberg Leadership and Vertical Nash.

**Stackelberg Leadership (Scenario 0)**

In this scenario, offline and online calculate total inventory cost separately but there is sequence between players. Decision flow is started by offline channel as follower, then followed by online channel as a leader concludes order quantity decision. Assumption for data demand maximum \((d_{max})\) is collected from previous period.
Offline channel

Total inventory cost model refers to Equation (7).

EOQ for main product

\[ Q_{o_1}^* = \sqrt{\frac{2o_o(\rho d_{max} - \alpha_2 P_o + \beta_2 P_s)}{h_o}} \]  \hspace{1cm} (9)

EOQ for additional product

\[ Q_{o_1}'^* = \sqrt{\frac{2o_o'\phi(\rho d_{max} - \alpha_2 P_o + \beta_2 P_s)}{h_o'}} \]  \hspace{1cm} (10)

where: \( Q_s = Q_{o_1}^* \) dan \( Q_s' = Q_{o_1}'^* \)

\( Q_{o_1}^* \) = optimal order quantity main product at online channel from previous period

\( Q_{o_1}'^* \) = optimal order quantity additional product at online channel from previous period

Online channel

In this scenario, assumed that demand in the central warehouse is the sum of both channels, online channel and offline channel.

Demand untuk main product

\[ D_c = D_o + D_s \]  \hspace{1cm} (11)

Demand untuk additional product

\[ D'_c = D'_o + D'_s \]  \hspace{1cm} (12)

EOQ for main product

\[ Q_s = \sqrt{\frac{2o_o\{(\rho d_{max} - \alpha_2 P_o + \beta_2 P_s) + (1-\rho)d_{max} - \alpha_1 P_s + \beta_1 P_o\}}{h_o}} \]  \hspace{1cm} (13)

EOQ for main product

\[ Q_{o_1}^* = \sqrt{\frac{2o_o\phi\{(\rho d_{max} - \alpha_2 P_o + \beta_2 P_s) + (1-\rho)d_{max} - \alpha_1 P_s + \beta_1 P_o\}}{h_o'}} \]  \hspace{1cm} (14)

Total annual inventory cost for Stackelberg Leadership (Scenario 0):

\[ TC_{scenario\ 0} = TC_{offline} + TC_{online} \]  \hspace{1cm} (15)

Vertical Nash (Scenario 1)

In this scenario, all players share their pricing data, ordering data and demand data symmetrically and then they decide order quantity simultaneously.

Offline channel

Total inventory cost model refers to Equation (8).

EOQ for main product at offline channel

\[ Q_s = \sqrt{\frac{2o_s\{(1-\rho)d_{max} - \alpha_1 P_s + \beta_1 P_o\}}{h_s}} \]  \hspace{1cm} (16)

EOQ for additional product at offline channel

\[ Q_{s}'^* = \sqrt{\frac{2o_s'\phi\{(1-\rho)d_{max} - \alpha_1 P_s + \beta_1 P_o\}}{h_s'}} \]  \hspace{1cm} (17)

EOQ for additional product at offline channel

\[ Q_{s}'^* = \sqrt{\frac{2o_s'\phi\{(1-\rho)d_{max} - \alpha_1 P_s + \beta_1 P_o\}}{h_s'}} \]  \hspace{1cm} (18)

Online channel

(19)
\[
TC(Q)_{on} = (\rho d_{max} - \alpha_2 P_o + \beta_2 P_s) \left( c + c' \phi + \frac{o_o}{Q_o} + \frac{o_o' \phi'}{Q_o'} \right) + \frac{h_o Q_o}{2} + \frac{h_o' Q_o'}{2}
\]

EOQ for main product at online channel
\[
Q_o^* = \sqrt{\frac{20_o \rho d_{max} - \alpha_2 P_o + \beta_2 P_s}{h_o}}
\]  (20)

EOQ for additional product at online channel
\[
Q_o'^* = \sqrt{\frac{2o_o' \phi (\rho d_{max} - \alpha_2 P_o + \beta_2 P_s)}{h_o'}}
\]  (21)

Total annual inventory cost for Vertical Nash (Scenario 1):
\[
T_{C_{scenario \: 1}} = T_{C_{offline}} + T_{C_{online}}
\]  (22)

5. Numerical experiment

To check proposed model, the following parameters’ value are set as given Table 1. These parameters values collected from electronics DCSC as an experiment object in Surabaya, Indonesia.

| Table 1. System Parameters |  |  |  |  |  |  |  |
|-----------------------------|---|---|---|---|---|---|---|
| Parameter | Value | Parameter | Value | Parameter | Value | Parameter | Value |
| \(c\) | IDR 300,000.00 | \(c'\) | IDR 250,000.00 | \(o_o\) | IDR 450,000.00 | \(\rho\) | 0.8 |
| \(c'\) | IDR 180,000.00 | \(o_o'\) | IDR 250,000.00 | \(\phi\) | 0.8 |
| \(w\) | IDR 350,000.00 | \(ao'\) | IDR 60,000.00 | \(\beta_1\) | 0.001 |
| \(w'\) | IDR 180,000.00 | \(h_s\) | IDR 40,000.00 | \(\beta_2\) | 0.001 |
| \(P_s\) | IDR 350,000.00 | \(h_s'\) | IDR 30,000.00 | \(\alpha_1\) | 0.001 |
| \(P_o\) | IDR 180,000.00 | \(h_o\) | IDR 18,000.00 | \(\alpha_2\) | 0.001 |
| \(o_s\) | IDR 350,000.00 | \(h_o'\) | IDR 18,000.00 | \(d_{max}\) | 1000 |

Where:
- \(c\) = unit cost for main product
- \(c'\) = unit cost for additional product
- \(w\) = wholesale price for main product
- \(w'\) = wholesale price for additional product
- \(o_o\) = order cost per order of main product at online channel
- \(o_o'\) = order cost per order of additional product/service at online channel
- \(h_o\), \(h_s\) = holding cost per unit per year of main product at online channel, offline channel
- \(h_o'\), \(h_s'\) = holding cost per unit per year of additional product/service at online channel, offline channel

| Table 2. Comparison the results of initial solution |  |  |
| Symbol | Stackelberg Leadership | Vertical Nash |
|--------|-------------------|--------------|
| \(d_{max}\) | 1000 | 1000 |
| \(D_s\) | 190 | 190 |
| \(D_o\) | 810 | 810 |
| \(D_o'\) | 152 | 152 |
| \(D_o''\) | 648 | 648 |
| \(\phi\) | 0.8 | 0.8 |

| Symbol | Stackelberg Leadership | Vertical Nash |
|--------|-------------------|--------------|
| \(Q_s\) | 148 | 53 |
| \(Q_o\) | 173 | 156 |
| \(Q_o'\) | 127 | 36 |
| \(Q_o''\) | 149 | 134 |
| \(T_{C_{off}}\) | 104763031.40 | 101882636.50 |
| \(T_{C_{on}}\) | 451879434.00 | 366731490.60 |
### Table 3. Comparison of total cost between scenario 0 and scenario 1 to changes in factor φ

| Parameter | Scenario 0 | Scenario 1 |
|-----------|------------|------------|
| φ         | 𝑇𝐶_{DESC}  | 𝑇𝐶_{DESC}  |
| 0         | 376713502.50 | 31739660.70 |
| 0.1       | 400468286.10 | 337242628.50 |
| 0.2       | 423077983.90 | 356236893.90 |
| 0.3       | 445499287.00 | 375088236.30 |
| 0.4       | 467823069.70 | 393865956.30 |
| 0.5       | 490084531.50 | 412595677.50 |
| 0.6       | 512301724.80 | 431292175.20 |
| 0.7       | 534485373.70 | 449963224.80 |
| 0.8       | 556642465.40 | 468614127.10 |
| 0.9       | 578777853.30 | 487248564.20 |
| 1         | 600895071.00 | 505869216.70 |

For sensitivity analysis there are three parameters will be changed, such as potential demand (d\text{max}), customer acceptance (ρ) and inclusion factor of additional product for main product (ϕ) shown at Table 4.

### Table 4. Parameters used for sensitivity analysis

| Parameter | d\text{max} | ρ  | ϕ  |
|-----------|------------|----|----|
| Low       | 100        | 0.2| 0.3|
| Medium    | 500        | 0.5| 0.6|
| High      | 1000       | 0.8| 0.9|

### Table 5. Sensitivity analysis of scenario 0 with scenario 1 for low customer acceptance (ρ)

| Demand | ϕ  | ρ  | Scenario 0 𝑇𝐶_{DSC} | Scenario 1 𝑇𝐶_{DSC} |
|--------|----|----|----------------------|----------------------|
| Low    | 0.3| 0.2| 68868317.00          | 65293715.74          |
| Medium | 0.6| 0.2| 399838807.30         | 362525906.30         |
| High   | 0.9| 0.2| 899673338.10         | 811144101.50         |

### Table 6. Sensitivity analysis of scenario 0 with scenario 1 for medium customer acceptance (ρ)

| Demand | ϕ  | ρ  | Scenario 0 𝑇𝐶_{DSC} | Scenario 1 𝑇𝐶_{DSC} |
|--------|----|----|----------------------|----------------------|
| Low    | 0.3| 0.5| 561928222.27         | 52291654.87          |
| Medium | 0.6| 0.5| 327650977.60         | 290265916.40         |
| High   | 0.9| 0.5| 738316272.60         | 649401830.80         |
Table 7. Sensitivity analysis of scenario 0 with scenario 1 for high customer acceptance ($\rho$)

| Demand | $\phi$ | $\rho$ | Scenario 0 $TC_{DCSC}$ | Scenario 1 $TC_{DCSC}$ |
|--------|--------|--------|------------------------|------------------------|
| Low    | 0.3    | 0.8    | 43792967.14             | 38996642.95             |
| Medium | 0.6    | 0.8    | 257329205.50            | 217660313.50            |
| High   | 0.9    | 0.8    | 578777853.30            | 487248564.20            |

Table 8. Sensitivity analysis of scenario 0 with scenario 1 for low percentage PSS ($\phi$)

| Demand | $\phi$ | $\rho$ | Scenario 0 $TC_{DCSC}$ | Scenario 1 $TC_{DCSC}$ |
|--------|--------|--------|------------------------|------------------------|
| Low    | 0.3    | 0.2    | 68868317.00             | 65293715.74             |
| Medium | 0.3    | 0.5    | 285244920.20            | 252771449.00            |
| High   | 0.3    | 0.8    | 445499287.00            | 375088236.30            |

Table 9. Sensitivity analysis of scenario 0 with scenario 1 for medium percentage PSS ($\phi$)

| Demand | $\phi$ | $\rho$ | Scenario 0 $TC_{DCSC}$ | Scenario 1 $TC_{DCSC}$ |
|--------|--------|--------|------------------------|------------------------|
| Low    | 0.6    | 0.2    | 78952455.39             | 74863466.75             |
| Medium | 0.6    | 0.5    | 327650977.60            | 290265916.40            |
| High   | 0.6    | 0.8    | 512301724.80            | 431292175.20            |

Table 10. Sensitivity analysis of scenario 0 with scenario 1 for high percentage PSS ($\phi$)

| Demand | $\phi$ | $\rho$ | Scenario 0 $TC_{DCSC}$ | Scenario 1 $TC_{DCSC}$ |
|--------|--------|--------|------------------------|------------------------|
| Low    | 0.9    | 0.2    | 88923947.84             | 84314612.46             |
| Medium | 0.9    | 0.5    | 369822952.60            | 327538304.90            |
| High   | 0.9    | 0.8    | 578777853.30            | 487248564.20            |

6. Conclusion
This study has succeeded in developing an inventory model by considering the PSS concept in the DCSC structure based on the gap and the formulation of the observed problems. The higher the factor of the inclusion of additional products to the main product, does not affect the main product order quantity, otherwise it will increase the additional product order quantity. The higher customer acceptance, will affect the order quantity in each Scenario. In Scenario 0, it is found that the higher the customer acceptance, the quantity of the main product order on the offline channel increases, while the online channel remains, then the quantity of the order is an additional product, in the offline channel there is an increase in orders, while in the online channel there is no decrease or increase. In Scenario 1, it was found that the higher the customer acceptance, the quantity of the main product order on the offline channel decreased, while the online channel remained. Then the quantity of the additional product order, on the offline channel there was a decline, different from the online channel which experienced an increase in order quantity. Overall, the total inventory cost for the Vertical Nash scenario is lower than in the Stackelberg Leadership scenario. In this study, it was found that in inventory management decision making, it is better to consider scenario 0, namely Vertical Nash, than scenario 1, namely Stackelberg Leadership.
For future research, we recommended that data used is real data taken from the actual field. Developing the function of total cost inventory in the DCSC on the PSS concept by considering other aspects such as lost sales, back orders, and service levels. Consider more than one type of PSS product (there are more than one type of additional product included in the main product). Consider probabilistic or stochastic demand in inventory management in DCSC. At last, not only calculates the total inventory cost incurred but also the expected profit.

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