The $K/\pi$ ratio from condensed Polyakov loops

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We perform a field-theoretical computation of hadron production in large systems at the QCD confinement phase transition associated with restoration of the $Z(3)$ global symmetry. This occurs from the decay of a condensate for the Polyakov loop. From the effective potential for the Polyakov loop, its mass just below the confinement temperature $T_c$ is in between the vacuum masses of the pion and that of the kaon. Therefore, due to phase-space restrictions the number of produced kaons is roughly an order of magnitude smaller than that of produced pions, in agreement with recent results from collisions of gold ions at the BNL-RHIC. From its mass, we estimate that the Polyakov loop condensate is characterized by a (spatial) correlation scale of $1/m_\ell \simeq 1/2$ fm. For systems of deconfined matter of about that size, the free energy may not be dominated by a condensate for the Polyakov loop, and so the process of hadronization may be qualitatively different as compared to large systems. In that vein, experimental data on hadron abundance ratios, for example $K/\pi$, in high-multiplicity $pp$ events at high energies should be very interesting.

High-energy inelastic processes provide the unique opportunity to recreate perhaps the high energy density state of QCD matter that presumably prevailed during the first microseconds of the evolution of the early universe: the so-called Quark-Gluon Plasma (QGP). Simply speaking, the QGP is the state of matter for which quarks and gluons are deconfined and chiral symmetry is restored. Even if produced, the QGP is a transient state of matter which can not be observed directly. Experimentally, one observes hadrons which are produced from the decay of the deconfined QGP state. The hadron production process itself is commonly called hadronization.

One might be able to gain some insight into the hadronization process by studying the relative populations of various hadronic states emerging from a high-energy inelastic process. In fact, one of the earliest proposed signatures for a transient QGP state is the enhanced production of strangeness, particularly the relative abundance of kaons to pions (the “$K/\pi$ ratio”). More recently, it has been discussed that a large variety of hadronic multiplicities in high-energy heavy-ion collisions resemble those of a hadron gas at a temperature $T_h$ close to $T_c$, the confinement/deconfinement transition temperature. This has been interpreted such that the QGP state hadronizes statistically, in that all hadronic states are populated according to their weight in the partition function of a hadron gas. This interpretation was partly based also on the success of the statistical hadronization model to describe relative hadron abundances in high-energy inelastic $e^+e^-$ and $pp$, or $p\bar{p}$ reactions. In such reactions, one does not expect that a thermally and chemically equilibrated QGP is formed prior to hadronization. Nevertheless, the hadronic final states appear to be populated approximately according to a statistical distribution. From $e^+e^-$ to central collisions of large nuclei, the hadronization temperature obtained from fitting the relative multiplicities was found to be more or less the same, and that it is also independent of collision energy at high energies. This was interpreted as a “limiting temperature” for hadron formation in inelastic reactions, which is consistent with the prediction of a deconfinement temperature $T_c$ from lattice QCD above which hadronic states presumably can not freeze out as asymptotic states.

Here, we perform a dynamical computation of hadron production from the decay of a deconfined state with a spontaneously broken global $Z(3)$ symmetry without assuming the formation of an equilibrated hadron gas a priori. For simplicity, we restrict ourselves to the pseudoscalar meson octet, i.e. the pions, the kaons, and the eta. At the end of the paper, we also speculate about possible differences in the “statistical hadronization” in small versus large systems.

We begin by briefly reviewing the model for the QGP suggested recently by Pisarski, which we shall adopt here to compute hadron production. The basic postulate of the so-called “Polyakov loop model” is that the free energy of the deconfined state of QCD is dominated by a condensate of gauge invariant Polyakov loops

$$
\tilde{\ell} = \frac{1}{3} \text{tr} \mathcal{P} \exp \left( i g \int_0^{1/T} A_0(\vec{x}, \tau) d\tau \right).
$$

(1)

It is based on the observation from $SU(3)$ lattice gauge theory that the Polyakov loop becomes light near $T_c$. In other words, the effective potential for $\tilde{\ell}$ must be rather flat for $T = T_c$. On the lattice, the mass of $\ell$ can be measured from the two-point function with space-like separation. These two point functions in the Polyakov loop model are very different from those of ordinary perturbation theory. The condensate is then characterized by a length scale $\xi = 1/m_\ell(T)$. In order to study hadronization, we couple the
Polyakov loop to the $SU(3) \times SU(3)$ linear sigma model and make qualitative predictions for the resulting relative multiplicities of kaons and etas to pions. The results of this study have implications for understanding the flavor composition in the final state of high-energy inelastic reactions. For simplicity, we neglect the effects of nonzero baryon charge, isospin charge, electric charge, etc. in the present analysis. This should be a reasonable first approximation for particles produced in the central region of very high energy (and high multiplicity) inelastic processes. In other cases, one may extrapolate the measured hadronic multiplicities to vanishing charges [5], for example, so as to allow for a better comparison of $e^+ e^-$, $pp$, and nucleus-nucleus collisions.

Our model has static, thermodynamic properties which are consistent with lattice data for the pressure, the energy density, and the two-point function (i.e. the mass) of the Polyakov loop. With the simplest possible $Z(3)$ symmetric potential described below, one can not only fit the above observables but actually relate their behavior, for example the pressure above $T_c$ to the electric screening mass [12]. One also obtains testable predictions for the ratio of the masses of the real and imaginary parts of the Polyakov loop [12].

\[ V(\ell) = \left( -\frac{b_2}{2} |\ell|^2 - \frac{b_3}{6} (\ell^4 + (\ell^* )^3) + \frac{1}{4} (|\ell|^2)^2 \right) b_4 T^4, \quad (2) \]

which is invariant under global $Z(3)$ transformations [1,13]. The form of $V(\ell)$ is dictated by symmetry principles [1,14]. The center of $SU(3)$, $Z(3)$, is broken at high $T$ where the global minimum of $V(\ell)$, $\ell_0(T) \to 1$, and restored at low $T$ where $\ell_0(T) \to 0$.

We assume that quarks are not important for understanding the form of the effective potential for QCD. This is motivated by the results of lattice simulations for which $P/P_{\text{ideal}}$ as a function of $T/T_c$ is remarkably insensitive to the number of flavors, $N_f = 0$, 2, 3 [11]. Thus, we neglect terms linear in $\ell$, which exist in QCD with quarks. Such terms would change the weakly first-order transition (for $N_f = 0$) which is predicted by eq. (2) into a crossover. Nevertheless, the fact that the free energy is small just below $T_c$ means that their numerical effects must be small. We emphasize that the results to be described below are not driven by the order of the phase transition and would not change in any significant way by the inclusion of such small linear terms. All that matters is that the $\ell$ field becomes rather light at $T_c$, i.e. that the transition is nearly second-order.

The coefficients $b_2(T)$, $b_3$, and $b_4$ can be chosen to reproduce lattice data for the pressure and energy density of the pure glue theory for $T \geq T_c$ [14]. For example, one may choose $b_4 \approx 0.6061$, $b_3 = 2.0$, and $b_2(T) = \left( 1 - 1.11/x \right) \left( 1 + 0.265/x \right)^2 \left( 1 + 0.300/x \right)^3 - 0.487 \times 4^{(T/T_c)}$, where $x \equiv T/T_c$. For this parameterization, one obtains the pressure and energy density $e = T dp/dT - p$ as shown in Fig. 1. The above parameterization does not yet incorporate the normalization condition that $\ell_0(T) \to 1$ for $T \to \infty$. Rather, the expectation value for $\ell$ at high temperature approaches $\ell = b_3/2 + \sqrt{b_3^2 + 4b_2(\infty^2)/2}$. We thus rescale the field and the coefficients as $\ell \to \ell/r$, $b_2(T) \to b_2(T)/r^2$, $b_3 \to b_3/r$, and $b_4 \to b_4 r^4$. This ensures the proper normalization for the Polyakov loop. The rescaling does not, of course, change the pressure and the energy density.

\[ \sigma(T) \text{ from the lattice } [11]: \quad g^2/2N_c \partial^2 V/\partial \ell^2 = -g^2 b_2(T) b_4 T^4 / 2N_c = \sigma^2(T). \]

The factor in front of $\sigma^2$ arises from the normalization of the kinetic term for $\ell$, see below. Taking $N_c/g^2 = 1$, this leads to $b_2(T) \approx -0.66 \sigma^2(T)/T^4$. 

\[ \ell/T_c \]

![FIG. 1. The energy density and the pressure for SU(3) pure-gauge theory, and our fit (see text).](image)

\[ T/T_c \]

![FIG. 2. The potential for the real part of $\ell$ at three different temperatures.](image)
The potential in eq. [2], which follows from the above-mentioned fit to the lattice data, changes extremely rapidly below $T_c$ [13], see Fig. 2. This rapid change in the effective potential with temperature has important implications for the dynamical evolution of the confinement transition [14], and for the relative hadron multiplicities from the decay of the condensate for $\ell$. The hadron production temperature $T_h$ can not vary much between events because it is impossible for the expectation value for $\ell$ to stay large below $T_c$; rather, the field is in an unstable configuration and must roll down towards the minimum of the potential. (In turn, if the effective potential was slowly varying about $T_c$, one would expect a broad range of hadronization temperatures.) Also, $T_h$ must be very close to $T_c$, which in turn implies that the $K/\pi$ ratio is well below unity. As we shall see below, if we chose $T_h$ by hand to be some 10% or more below $T_c$ (disregarding the fact that from Fig. 2 this scenario appears unphysical), then phase-space for kaon production opens up and leads to essentially equal number of kaons and pions in the final state, contrary to experimental results. Thus, the question as to whether it is some feature of QCD that the hadron production temperature is so close to the confinement temperature $T_c$ [14] is answered by the behavior of the potential for the Polyakov loop which we deduce from the lattice data.

To complete the effective theory, we add a kinetic term for $\ell$ and couple it to the octet of pseudo-Goldstone excitations. The coefficient of the kinetic term for $\ell$ is set by the fluctuations of the $SU(3)$ Wilson lines in space. Perturbative corrections to that coefficient are small [18]. We assume here a Lorentz invariant form [13,16]. The coupling of $\ell$ to the $SU_R(3) \times SU_L(3)$ linear sigma model is given by

$$\mathcal{L} = \mathcal{L}_\phi + \frac{N_c}{g^2} |\partial_\mu \ell|^2 T^2 - V(\ell) - h^2 |\ell|^2 T^2 \text{Tr} \left( \Phi \Phi^\dagger \right)$$

where $\mathcal{L}_\phi$ is the Lagrangian for the linear sigma model given below and $\Phi$ is the chiral field. The coupling $h^2 \simeq 20$ between the chiral field and $\ell$ is chosen to reproduce $m_\pi(T)$ [19]. Through this interaction term, the Polyakov loop condensate drives the chiral phase transition.

The Lagrangian of the $SU(3)_R \times SU(3)_L$ linear sigma model is given by [21]

$$\mathcal{L}_\phi = \text{Tr} \left( \partial_\mu \Phi^\dagger \partial^\mu \Phi - m^2 \Phi^\dagger \Phi + \lambda_1 \left[ \text{Tr} \left( \Phi \Phi^\dagger \right) \right]^2 - \lambda_2 \text{Tr} \left( \Phi \Phi^\dagger \right)^2 + c \left| \text{Det} \left( \Phi \right) + \text{Det} \left( \Phi^\dagger \right) \right| \right) + \text{Tr} \left[ H \left( \Phi + \Phi^\dagger \right) \right].$$

$\Phi$ is a complex $3 \times 3$ matrix parameterizing the scalar and pseudoscalar meson nonets,

$$\Phi = \frac{\lambda_a}{2} \phi_a = \frac{\lambda_a}{2} (\sigma_a + i \pi_a),$$

where $\lambda_a$ are the Gell-Mann matrices with $\lambda_0 = \sqrt{\frac{4}{3}} 1$.

The $\sigma_a$ fields are members of the scalar nonet and the $\pi_a$ fields are members of the pseudoscalar nonet. Here, $\pi^\pm \equiv (\pi_1 \pm i \pi_2)/\sqrt{2}$ and $\pi^0 \equiv \pi_3$ are the charged and neutral pions, respectively. $K^\pm \equiv (\pi_4 \pm i \pi_5)/\sqrt{2}$, $K^0 \equiv (\pi_6 + i \pi_7)/\sqrt{2}$, and $\bar{K}^0 \equiv (\pi_6 - i \pi_7)/\sqrt{2}$ are the kaons. The $\eta$ and the $\eta'$ mesons are admixtures of $\pi_0$ and $\pi_8$. The classification of the scalar nonet is not important for our purposes as discussed below.

The $3 \times 3$ matrix $H$ is chosen to reproduce the mass splitting of the strange and nonstrange quarks with $H = H_a \lambda_a/2$ and where $H_a$ are nine external fields. The determinant terms reproduce the effects of the $U(1)_A$ anomaly in the QCD vacuum [21] by preserving the $SU(3)_R \times SU(3)_L \cong SU(3)_V \times SU(3)_A$ symmetry while explicitly breaking the $U(1)_A$ symmetry. Symmetry breaking gives the $\Phi$ field a vacuum expectation value, $\langle \Phi \rangle \equiv \bar{\sigma}_0 \lambda_0/2$.

The $\bar{\sigma}_0$ and $\bar{\sigma}_8$ are admixtures of the strange and nonstrange quark–antiquark condensates. From the PCAC relations, they are given by $\bar{\sigma}_0 = (f_{\pi} + 2f_{\bar{K}})/\sqrt{3} = 130$ MeV and $\bar{\sigma}_8 = 2(f_{\pi} - f_{\bar{K}})/\sqrt{3} = -23.8$ MeV. Here, for simplicity we freeze $\bar{\sigma}_0, \bar{\sigma}_8$ at their vacuum values, i.e. we assume that dynamical relaxation of the classical condensates does not contribute much to particle production or to the tadpole self-energies of the $\pi, K, \eta$ quantum fields.

From the numerical values of $\bar{\sigma}_0, \bar{\sigma}_8$ it is clear that the energy density of the chiral condensate is too low to contribute much to the total hadron multiplicities. The free energy of the deconfined state is largely due to the potential [2] for the Polyakov loop $\ell$ which dominates hadron production at $T_c$ [13].

The parameterization of the coupling constants is given in detail in Ref. [2] and is chosen to reproduce the masses and mixing angles of the mesons at zero temperature. Here, we take $m^2 = (342$ MeV$)^2$, $\lambda_1 = 1.4$ and $\lambda_2 = 46$. The explicit symmetry breaking terms are given by $H_0 = (286$ MeV$)^3$, $H_8 = -(311$ MeV$)^3$ and $c = 4808$ MeV.

To simplify the solution of the equations of motion for $\ell$ and the mesonic excitations, we do not consider the excitations of the scalar nonet or the $\eta'$, all of which have masses either near or above 1 GeV. We perform the standard decomposition of the meson quantum fields in terms of creation and annihilation operators times adiabatic mode functions $\pi_k^0(t)$. Here, $k$ is the mode number. To make the interaction quadratic in the meson fields, we employ a mean field factorization which respects the flavor-$SU(3)$ symmetry when fluctuations are large,

$$\pi_0^2(t, \mathbf{x}) \to \langle \pi_0^2(t) \rangle.$$  

With these approximations, the interaction part becomes

$$-\mathcal{L}_{int} = \frac{1}{2} \left( \lambda_1 + \frac{1}{2} \lambda_2 \right) \left( \pi^2 + K^2 + \eta^2 \right) \langle \phi^2 \rangle + \frac{1}{2} h^2 T^2 |\ell|^2 \left( \pi^2 + K^2 + \eta^2 \right),$$

where $\ell$ is the magnetic field.
where we introduced the short-hand notation
\[ \langle \phi^2 \rangle \equiv 3\langle \pi^2 \rangle + 4\langle K^2 \rangle + \langle \eta^2 \rangle \]
(8)
for the sum of the fluctuations. The prefactor of each term reflects the internal isospin degrees of freedom. Assuming that the up and down quarks are mass-degenerate, the equations of motion for the mesonic mode functions in Minkowski space read
\[
(\partial^2/\partial t^2 + k^2)\pi^k = \left[ -m^2 - \lambda_1 \left( \langle \phi^2 \rangle + \bar{\sigma}_0^2 + \bar{\sigma}_8^2 \right) - \lambda_2 \left( \frac{\langle \phi^2 \rangle}{2} + \bar{\sigma}_0^2 + \bar{\sigma}_8^2 - \sqrt{2}\sigma_8 + \frac{\bar{\sigma}_8^2}{6} + \frac{7\bar{\sigma}_8^2}{6} \right) + c \left( \frac{\sqrt{2}\sigma_0 + \bar{\sigma}_8}{\sqrt{3}} \right) - h^2|\ell|^2T^2 \right] \pi^k,
\]
(9)
and
\[
(\partial^2/\partial t^2 + k^2)\Pi^k = \left[ -m^2 - \lambda_1 \left( \langle \phi^2 \rangle + \bar{\sigma}_0^2 + \bar{\sigma}_8^2 \right) - \lambda_2 \left( \frac{\langle \phi^2 \rangle}{2} + \bar{\sigma}_0^2 + \bar{\sigma}_8^2 - \sqrt{2}\sigma_8 + \frac{\bar{\sigma}_8^2}{6} + \frac{7\bar{\sigma}_8^2}{6} \right) + c \left( \frac{\sqrt{2}\sigma_0 + \bar{\sigma}_8}{\sqrt{3}} \right) - h^2|\ell|^2T^2 \right] \Pi^k.
\]
(10)
The equations of motion are now \(N_k\) (coupled) equations for the mode functions, where \(N_k\) is the number of modes. In the actual computations we propagated \(N_k = 1000\) modes in parallel, taking a mode-spacing of \(\Delta k = f_\pi/100\). We checked that the observables, like the sum of the occupation numbers of all field modes, are stable under reasonable variations of \(N_k\). When discussing relative abundances of \(\pi, K, \eta\) below, we always refer to the number of occupied states of all field modes. For the Polyakov loop, only the zero-mode (condensate) is considered here.

The equation of motion for \(\ell\) is
\[
\frac{N_c}{g^2}T^2 \frac{\partial^2 \ell}{\partial t^2} = -\frac{\delta V(\ell, \ell^*)}{\delta \ell^*} - \frac{h^2}{2}T^2\langle \phi^2 \rangle \ell.
\]
(12)
This equation follows from (f) when fluctuations of the \(\ell\) field are disregarded, as in the present qualitative analysis. Such an approximation is valid for short times, when the energy density is dominated by the zero-mode of the Polyakov loop and the contributions from fluctuations of \(\ell\) and of the chiral fields (the produced mesons) are not large.

The fluctuations of the chiral fields at any given instant are computed from their mode functions,
\[
\langle \pi^2_a(t) \rangle = \int \frac{d^3k}{(2\pi)^3} |\pi^a_k(t)|^2
\]
\[
= \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{N_k(t)}{\Omega_k(t)} + \frac{1}{2\Omega_k(t)} - \frac{1}{2\Omega_k(t = 0)} \right\}.
\]
(13)
The contribution from the vacuum fluctuations diverges. The integral is regularized using the UV cutoff \(\Lambda = N_k\Delta_k = 10f_\pi \approx 1\ GeV\), which is a sensible value for our effective theory. In practice, the dependence on this cutoff was found to be weak; if \(\Lambda\) exceeds all other mass scales, the dependence on \(\Lambda\) is only logarithmic [12,22].

In the above equation, \(N_k\) denotes the occupation number of a mode with given isospin and with momentum \(k\), subtracted for vacuum fluctuations,
\[
N_k = \frac{\Omega_k}{2} \left( |\pi^k_a| + |\pi^k_a| \right) - \frac{1}{2}, \quad (14)
\]
and \(\Omega_k = \sqrt{k^2 + m^2\pi^2 + \Pi\pi^2}\). \(\Pi\pi^2\) is the tadpole self-energy of meson field \(\pi^2\). With the approximations that lead to the interaction (f), it is obtained by taking two functional derivatives of \(\mathcal{L}_{\text{int}}\) with respect to \(\pi^2\), which is trivial since \(\mathcal{L}_{\text{int}}\) is quadratic in the dynamical fields. The expressions for \(m^2\pi^2 + \Pi\pi^2\) can simply be read off from the r.h.s. of eqs. (10,11).

For the initial condition at \(t = 0\), we consider the case that only adiabatic quantum fluctuations of the meson fields exist, but no on-shell particles, i.e.
\[
\Omega_k(0)\pi^k_a(0) = i\bar{\pi}^k_a(0) = \sqrt{\Omega_k(0)/2}.
\]
(15)

To calculate the particle production dynamically, we make some simplifications. The dynamical picture is that of a quench to a temperature \(T_h\), where the condensate for \(\ell\) decays into hadrons. (\(T_h\) is not to be confused with the decoupling temperature of the chiral field \(\Phi\) from the \(\ell\) field.) The quench approximation seems reasonable as the effective potential changes very rapidly around \(T_c\).

Dynamical simulations of the long wavelength modes on a lattice show that the \(\ell\)-condensate can be quenched to perhaps a few percent below \(T_c\) [10]. Quenching to lower temperature is hardly possible, since the \(\ell\) field eventually just “rolls down” towards the confined minimum. This is also evident from the rapidly deepening potential in Fig. 2.

In Fig. 3 we show the result of such a computation, using a hadron production temperature of \(T_h = 0.96T_c\) and an initial value for the expectation value of the Polyakov loop of \(\ell(t = 0) = 0.7\). The \(\ell\)-field “rolls down” towards the minimum at \(t = 0\), and thereby the energy of the deconfined state is converted into physical hadronic states.
Initially, as few hadrons have been produced, the relative number of kaons and pions fluctuates. However, it quickly settles to about $K/\pi \approx 0.15$. The relative suppression of kaon production is due to its larger mass. At $T_h = 0.96T_c$, the mass for the real part of $\ell$ (defined as the curvature of the potential about $\ell = 0$), $m_{\ell} \approx 400$ MeV, is between the vacuum masses of the pion and the kaon, hence the phase space for decay into kaons is smaller than for the decay into pions.

The nearly perfect agreement of our result on the $K/\pi$ ratio with experimental data from the CERN-SPS and BNL-RHIC accelerators is certainly a numerical coincidence. For example, variations of the coupling constants and in particular of $\alpha_s$, the coupling of $\ell$ into hadrons at $T_h$ very close to $T_c$. At such temperature then, $m_\pi < m_{\ell}(T_h) < m_K$ and so the available phase space leads to much fewer kaons than pions.

![FIG. 3. The $K^{ch}/\pi^{ch}$ (solid line) and the $\eta/\pi^{ch}$ (dashed line) ratios as functions of time.](image)

To illustrate this fact, we repeat the computation, lowering $T_h$ by hand. While this is unrealistic because it assumes that the condensate for $\ell$ retains a large expectation value even below $T_c$, it shows that as $T_h$ is lowered, the decay of the condensate for $\ell$ leaves us with essentially equal numbers of kaons and pions. This is due to the increase of $m_{\ell}$, which makes phase-space restrictions less relevant. Fig. 3 confirms this expectation. If the hadrons were produced at $T = 0.87T_c$, where $m_\ell \approx 740$ MeV is above the kaon vacuum mass, we obtain $K/\pi \approx 0.8$.

The theoretical expectation that the decay of the condensate for $\ell$ into hadrons occurs close to $T_c$, which was based on the rapid variation of the effective potential for $\ell$ from Fig. 2, is thus perfectly consistent with the constraints from experimental data on the $K/\pi$ ratio in high-energy heavy-ion scattering at the CERN-SPS and BNL-RHIC. What would perhaps be even more interesting from the point of view of learning about the presence of fundamental QCD scales in the hadronization process is to look for situations where our calculation breaks down. Fortunately, there indeed seems to be an experimentally accessible regime where this can be tested, namely high-energy, high-multiplicity $pp$ inelastic reactions.

At given energy $\sqrt{s}/A$, and at central rapidity, the $K/\pi$ ratio in heavy-ion collisions approaches that from $pp$ (or $p\bar{p}$) reactions at the same $\sqrt{s}$ as the energy increases. A natural question to ask is whether indeed they asymptotically approach the same limit. In fact, one may even compare central heavy-ion collisions at, say, BNL-RHIC energy to high-multiplicity $pp$ (or $p\bar{p}$) at much higher energies, where the charged multiplicity per participant and the energy density at central rapidity are comparable to those from heavy-ion collisions, and where one may very well produce a transient QGP state. In the absence of a length scale $1/m_{\ell}$, one would naively expect that eventually, at sufficiently high energy, the $K/\pi$ ratio from high-multiplicity $pp$ approaches that from central heavy-ion collisions at BNL-RHIC energy (also if high-multiplicity fluctuations in $pp$ are due to jets).

On the other hand, if the free energy of QCD around $T_c$ is dominated by a condensate for the Polyakov loop, hadron production is governed by a scale $m_\ell$ as explained above. Thus, hadronization from the decay of the condensate is associated with a length scale $1/m_{\ell}$. It appears quite reasonable to assume that it can only dominate hadron production in systems of size greater than $1/m_{\ell}$. (In small systems, on the other hand, fluctua-
tions dominate.) With $m_\ell$ somewhere between $m_\pi$ and $m_K$, as given by the potential for $\ell$ from Fig. 2, this condition may not hold for $pp$ reactions, regardless of how high an energy density is achieved. Note that we are not referring here to thermodynamical effects like for example canonical suppression due to exact conservation of strangeness (which becomes less important for very high energy and high-multiplicity $pp$, when there is more than one charged kaon pair per unit of rapidity). We are referring to whether or not one can probe, in principle, the deconfined phase of the non-abelian $SU(3)$ gauge theory in a system of linear dimension $1/m_\ell$ (or less), at a temperature $T \sim T_c$.

Consider pure gauge theory with three colors, for which good lattice data exists. In $d$ Euclidean dimensions, the two-point function for the real part of the Polyakov loop behaves as

$$\langle \ell_r(r)\ell_r(0) \rangle - \langle \ell_r(0) \rangle^2 \sim \int d^d k \frac{e^{ikr}}{k^2 + m_\ell^2} \sim \left( \frac{m_\ell}{r} \right)^{(d/2 - 1)} K_{d/2-1}(m_\ell r).$$

(16)

The correlation function for the imaginary part of $\ell$ falls off more rapidly, corresponding to a larger mass for $\ell_i$. For $d = 3$, $K_{1/2}(m_\ell r) \sim 1/\sqrt{m_\ell r} \exp(m_\ell r)$, hence

$$\langle \ell_r(r)\ell_r(0) \rangle - \langle \ell_r(0) \rangle^2 \sim \frac{2}{r} e^{-m_\ell r}.$$ 

(17)

When the size of the system $r \leq 1/m_\ell$, the correlation function (17) exhibits a power-law behavior

$$\langle \ell_r(r)\ell_r(0) \rangle - \langle \ell_r(0) \rangle^2 \sim \frac{1}{r},$$

(18)

for all “accessible” distances. The $r$—dependence of the subtracted correlation function (18) is in fact that for an abelian pure gauge theory (photons), namely just the Coulomb potential. The fact that the Polyakov loop acquires a mass $m_\ell$, which is the feature of the non-abelian gauge theory, can only be probed on distance scales $r \gtrsim 1/m_\ell$. Our computation of hadron production applies to systems larger than $1/m_\ell$; when the line-line correlation function falls off exponentially and when the free energy is dominated by the condensate for $\ell$. Qualitatively, hadron production about $T_c$ in systems of size $1/m_\ell$ might differ from our model.

Data on the $K/\pi$ ratio at central rapidity from $pp$ reactions at $\sqrt{s} = 1800$ GeV at the Fermilab-Tevatron have been analysed as a function of charged multiplicity [25,26]. For the highest-multiplicity events, corresponding to $dN_{ch}/d\eta \approx 25$ the initial energy density at proper time $\tau_0 = 1$ fm/c was estimated to be 6 GeV/fm$^3$, comparable to the average value from central Au+Au collisions at BNL-RHIC energy $\sqrt{s}/A = 100-200$ GeV [22]. A first analysis [23] seemed to indicate that $K/\pi$ increases as a function of event multiplicity, from about 10% to about 14%, nearly the value from Au+Au at RHIC. However, error bars for the most inelastic events were large, and a second analysis [27] did not confirm the rising $K/\pi$ ratio but showed it to be completely flat as a function of $dN_{ch}/d\eta$. A new analysis of this and other observables with the colliding proton beams at RHIC would be most interesting.

In summary, we have discussed dynamical hadron production at the confinement transition, which is realized here as the restoration of the global $Z(3)$ symmetry for the Polyakov loop $\ell$. Specifically, the proposal to understand QCD near $T_c$ in terms of a condensate for $\ell$ [12,13] predicts that hadron production occurs very near $T_c$. This follows from the rapid change of the effective potential for the Polyakov loop about $T_c$, which we deduce from lattice data on the finite-temperature string tension, pressure, and energy density. From our effective Lagrangian, the mass of $\ell$ just below $T_c$ is greater than the vacuum mass of the pion but less than that of the kaon. Therefore, qualitatively one expects about an order of magnitude more pions than kaons to be produced, as seen in experimental data from CERN-SPS and BNL-RHIC.

The hadronization process is characterized by a length scale $1/m_\ell$, which is on the order of $1/2 - 1$ fm, just below $T_c$. Thus, in small systems hadronization may not occur from the decay of a condensate for $\ell$, regardless of the initial energy density. Differences in the $K/\pi$ ratio (and in other relative hadron multiplicities) from high-multiplicity $pp$ to heavy-ion collisions at the SPS or RHIC [8,27] could perhaps be due to a condensate scale $1/m_\ell$. If such a point of view can be established experimentally, it could provide further support for the formation of a deconfined phase in collisions of heavy ions at high energies.

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