Does the radial–tangential macroturbulence model adequately describe the spectral line broadening of solar-type stars? *

Yoichi Takeda
National Astronomical Observatory, 2-21-1 Osawa, Mitaka, Tokyo 181-8588
takeda.yoichi@nao.ac.jp

and

Satoru Ueno
Kwasan and Hida Observatories, Kyoto University, Kurabashira, Kamitakara, Takayama, Gifu 506-1314
ueno@kwasan.kyoto-u.ac.jp

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Abstract

In incorporating the effect of atmospheric turbulence in the broadening of spectral lines, the so-called radial-tangential macroturbulence (RTM) model has been widely used in the field of solar-type stars, which was devised from an intuitive appearance of granular velocity field of the Sun. Since this model assumes that turbulent motions are restricted to only radial and tangential directions, it has a special broadening function with notably narrow width due to the projection effect, the validity of which has not yet been confirmed in practice. With an aim to check whether this RTM model adequately represents the actual solar photospheric velocity field, we carried out an extensive study on the non-thermal velocity dispersion along the line-of-sight ($V_{\text{los}}$) by analyzing spectral lines at various points of the solar disk based on locally-averaged as well as high spatial-resolution spectra, and found the following results. First, the center-to-limb run of $V_{\text{los}}$ derived from ground-based low-resolution spectra is simply monotonic with a slightly increasing tendency, which contradicts the specific trend (an appreciable peak at $\theta \sim 45^\circ$) predicted from RTM. Second, the $V_{\text{los}}$ values derived from a large number of spectra based on high-resolution space observation revealed to follow a nearly normal distribution, without any sign of peculiar distribution expected for the RTM case. These two observational facts indicate that the actual solar velocity field is not such simply dichotomous as assumed in RTM, but directionally more chaotic. We thus conclude that RTM is not an adequate model at least for solar-type stars, which would significantly overestimate the turbulent velocity dispersion by a factor of $\sim 2$. The classical Gaussian macroturbulence model should be more reasonable in this respect.

Key words: line: profiles — Stars: solar-type — Sun: granulation — Sun: photosphere — turbulence

1. Introduction

The Doppler effect of atmospheric turbulence on the broadening of spectral lines plays a significant role in stellar spectroscopy. For example, in spectroscopic determination of projected rotational velocities ($v_{\text{e}} \sin i$) of solar-type (FGK-type) stars, it is crucial to properly eliminate the line-broadening component of turbulence origin, because it is comparable to (or predominant over) the rotational broadening in such generally slow rotators decelerated due to the magnetic braking mechanism.

To make the problem easy and tractable, a very rough approximation has been adopted in traditional stellar spectroscopy, where turbulence in stellar atmospheres is divided into “micro”-turbulence and “macro”-turbulence and separately treated, where the former (micropscopic scale) is included in the Doppler with of the line-opacity profile (like thermal velocity) while the latter (macropscopic scale) acts as a global velocity distribution function (like rotational broadening function) to be convolved with the intrinsic profile. Given that the extent of the latter ($\gtrsim 2$ km s$^{-1}$) is known to be comparatively larger and more important than the former ($\lesssim 1$ km s$^{-1}$) in solar-type dwarfs, the latter “macroturbulence” is the main issue in this context.

Regarding the velocity distribution function of macroturbulence, the so-called “radial-tangential macroturbulence” (hereinafter abbreviated as RTM) model has been widely used so far, which was introduced by Gray (1975) for the first time for analyzing the line-profiles of late-type stars. That is, the appearance of solar granular velocity field (consisting of convective cells moving upward/downward and horizontal motions between the rising/falling cells) inspired Gray (1975) to postulate that the velocity vectors are directionally restricted to being either along stellar radius or tangential to the surface while the speed of gas motion in each direction follows the random Gaussian distribution with a dispersion parameter $\zeta_{\text{RT}}$. Since then, along with the efficient Fourier transform technique (e.g., Smith & Gray 1976), Gray and his coinvestigators have extensively applied this RTM model to

* Based on data collected by the Domeless Solar Telescope at Hida Observatory (Kyoto University, Japan) and those obtained by the Solar Optical Telescope on board the Hinode satellite.
line-profile analyses of F-, G-, and K-type stars in various evolutionary stages (e.g., to determine $v_e \sin i$ by separating $\zeta_{\text{RT}}$; see, Gray 1988, 2005 for more details regarding the technical descriptions and accomplished results in this field).

However, we feel some concern regarding the applicability of RTM to the case of solar-type dwarfs. Namely, according to Gray (1984; cf. section V therein), the value of $\zeta_{\text{RT}}$ (~4 km s$^{-1}$) derived from the flux spectrum of the Sun-as-a-star based on RTM is appreciably larger than the non-thermal dispersion or typical granular velocities (~2–3 km m$^{-1}$) directly estimated from spectroscopic observations of the resolved-disk Sun. What is the cause of this difference?

In order to clarify this situation, we refer to the work of Takeda (1995b; hereinafter referred to as Paper I). In paper I, an extensive profile-fitting analysis was carried out for many (~300) blend-free lines of various strengths in the solar flux spectrum by using RTM model with an aim to investigate the relation between $\zeta_{\text{RT}}$ and the mean-formation depth (log $\tau$), and the following results were derived (cf. figure 2 in Paper I):

1. $\zeta_{\text{RT}}$ progressively increases with depth from ~2.3 km s$^{-1}$ (at log $\tau$ ~ -2) to ~3.8 km s$^{-1}$ (at log $\tau$ ~ -0.5).

2. While this depth-dependence of $\zeta_{\text{RT}}$ is qualitatively consistent as compared to the tendency of solar photospheric non-thermal velocity dispersion ($V_{\text{rad}}$, $V_{\text{tan}}$; see, e.g., figures 1–3 in Gurtovenko 1975c or figure 1 in Canfield & Beckers 1976), the former is systematically higher by ~1 km s$^{-1}$ for unknown reason, which again confirmed that $\zeta_{\text{RT}}$ tends to be larger than the directly estimated velocity dispersion.

It is worth noting here that previous determinations of non-thermal velocity dispersion mentioned above were done under the assumption of anisotropic Gaussian distribution (cf. equation (4) in subsection 2.3; i.e., near-random distribution of velocity vectors), which is markedly different from the basic assumption of RTM. As a trial, we repeated the same analysis as done in Paper I (with a fixed $v_e \sin i$ of 1.9 km s$^{-1}$) but with the classical Gaussian macroturbulence (hereinafter referred to as GM) expressed by one dispersion parameter $\eta$ instead of RTM. The resulting $\eta$ values are plotted against log $\tau$ in figure 1a, where the $\zeta_{\text{RT}}$ vs. log $\tau$ relation is also shown for comparison. It is manifest from this figure that $\eta$ is systematically smaller than $\zeta_{\text{RT}}$ (the difference amounting to a factor of ~2) and more consistent with the literature results of $V_{\text{rad}}$ or $V_{\text{tan}}$.

This $\zeta_{\text{RT}}$ vs. $\eta$ discrepancy is reasonably interpreted as due to the difference between the characteristic widths of these two broadening functions. In order to demonstrate this point, the broadening functions for RTM ($M_1$) and GM ($M_2$) are graphically displayed in figures 1b and 1c, respectively. Focusing on the zero-rotation case ($v_e \sin i = 0$), we see that the half-width at half maximum (HWHM) for $M_1$ is HWHM$_1 = 0.36\zeta_{\text{RT}}$ while that for $M_2$ is HWHM$_2 = 0.83\eta$, which yields $\zeta_{\text{RT}}/\eta \sim 2.3(\sim 0.83/0.36)$ by equating these two widths as HWHM$_1 = $ HWHM$_2$. That is, since the width of RTM is narrower than that of GM (see also Fig. 17.5 of Gray 2005), the inequality of $\zeta_{\text{RT}} > \eta$ generally holds regarding the solutions of $\zeta_{\text{RT}}$ and $\eta$ required to reproduce the observed line width. This should be the reason why $\zeta_{\text{RT}}$ is larger than $\eta$ by a factor of ~2.

Given that resulting solutions of macroturbulence are so significantly dependent upon the choice of broadening function, it is necessary to seriously consider which model represents the actual velocity field more adequately. Especially, we wonder whether RTM is ever based on a reasonable assumption, because its broadening width is appreciably narrower than the extent of turbulent velocity dispersion, which stems from the extraordinary two-direction-confined characteristics (i.e., due to the projection effect; cf. figure 2). Since the peculiarity of RTM lies in its specific angle-dependence, we would be able to give an answer to this question by studying the solar photospheric velocity dispersion at various points on the disk (i.e., from different view angles).

According to this motivation, we decided to challenge the task of verifying the validity of RTM by using the Sun as a testbench. Our approach is simple and straightforward in the sense that we carefully examine the widths of spectral lines from the disk center to the limb by making use of the profile-fitting technique (as adopted in Paper I), by which the widths of local broadening function can be efficiently determined while eliminating the effects of intrinsic and instrumental profiles. Regarding the observational data, we employed two data sets of local intensity spectra taken at a number of points on the solar disk: (i) spatially averaged spectra (over ~50") to test whether the center-to-limb variation of the macrobroadening width predicted by RTM is observed, and (ii) spectra of fine spatial resolution (with sampling step of ~0.1"–0.3") to examine the validity of the fundamental assumption on which RTM stands.

The remainder of this article is organized as follows: We first explain the definitions of RTM and GM models in section 2, which forms the fundamental basis for the following sections. Section 3 describes our method of analysis using the profile-fitting technique. In section 4, the behavior of velocity dispersion along the line-of-sight is investigated based on the low-resolution ground-based data and compared with the prediction from RTM. The analysis of high-resolution data from space observation is presented in section 5, where statistical properties of local velocity dispersion and radial velocity are discussed. The conclusions are summarized in section 6. In addition, three spe-
cial appendices are provided, where the influence of the choice of microturbulence is checked (appendix 1), solar depth-dependent non-thermal velocity dispersions are derived to compare with the literature results (appendix 2), and the behavior of macroturbulence in solar-type stars is discussed (appendix 3).

2. Definition of macroturbulence broadening function

In this section, we briefly describe the basic definitions of representative macroturbulence broadening functions, which form the basis for the contents in later sections.

2.1. Line-profile modeling with macroturbulence

In the approximation that the intrinsic specific intensity going to a direction angle $\theta$ [$I^0(v, \theta)$] is broadened by the local macroturbulence function [$\Theta(v, \theta)$], the emergent intensity profile [$I(v, \theta)$] is expressed as

$$ I(v, \theta) = I^0(v, \theta) \otimes \Theta(v, \theta), \quad (1) $$

where $\otimes$ means “convolution.”

Similarly, when the intrinsic stellar flux [$F^0(v)$] is broadened by the integrated macroscopic line-broadening function [$M(v)$] (including the combined effects of macroturbulence and rotation), we may write the finally resulting flux profile [$F(v)$] as

$$ F(v) = F^0(v) \otimes M(v), \quad (2) $$

where an implicit assumption is made that the continuum-normalized profile of $F^0$ does not vary over the stellar disk. Generally, $M(v)$ is derived by integrating $\Theta(v, \theta)$ over the disk, while appropriately Doppler-shifting (with the assumed $v_s \sin i$) as well as multiplying by the limb-darkening factor (see Gray 1988 or Gray 2005 for more details).

2.2. Radial–tangential macroturbulence

Regarding the widely used radial-tangential macroturbulence (RTM) model (Gray 1975), $\Theta(v, \theta)$ is defined as

$$ \Theta_1(v, \theta) = \frac{A_R}{\pi^{1/2} \zeta_R \cos \theta} \exp\left[-v^2/(\zeta_R \cos \theta)^2\right] + \frac{A_T}{\pi^{1/2} \zeta_T \sin \theta} \exp\left[-v^2/(\zeta_T \sin \theta)^2\right], \quad (3) $$

though $A_R = A_T$ and $\zeta_R = \zeta_T (= \zeta_{RT})$ are usually assumed to represent the macroturbulence by only one parameter ($\zeta_{RT}$). It should be remarked that this is essentially a two-component model, in the sense that fraction $A_R$ and fraction $A_T$ of the stellar surface are covered with region R and region T (respectively) and that intrinsic intensity spectrum in each region is broadened by only either (i.e., not both) of the radial or tangential turbulent flow (cf. figure 2 for a schematic description of this model). In practice, direct application of $\Theta_1$ to solar intensity spectrum is difficult, since it has an unusual profile especially near to the disk center ($\sin \theta \sim 0$) or to the limb ($\cos \theta \sim 0$). That is, as the broadening function is defined by the sum of a Gaussian profile (with a reasonable width) and a $\delta$-function-like profile (with very narrow width and very high peak), its width at half-maximum does not represent the real velocity dispersion any more (cf. figure 3a and figure 3a'). Accordingly, this model is used primarily in stellar application after the integration over the disk has been completed. Figure 1b shows the profiles of the disk-integrated macrobroadening function for this model $M_1(v; \zeta_{RT}, v_s \sin i)$, which were computed for various values of $v_s \sin i/\zeta_{RT}$ ratio by following the procedure described in Gray (1988) (with the assumption of rigid rotation and limb-darkening coefficient of $\epsilon = 0.6$).

2.3. Anisotropic Gaussian macroturbulence

Alternatively, we can consider a Gaussian macroturbulence with an anisotropic character with respect to the radial and tangential direction. This is the case where the intrinsic intensity spectrum is broadened by gas of near-random motions (in terms of both speed and direction) following the Gaussian velocity distribution of ellipsoidal anisotropy (with dispersions of $\eta_R$ and $\eta_T$ in the radial and tangential direction, respectively). Then, the local broadening function is expressed by the convolution of two Gaussians as

$$ \Theta_2(v, \theta) \propto \exp\left[-v^2/(\eta_R \cos \theta)^2\right] \otimes \exp\left[-v^2/(\eta_T \sin \theta)^2\right] \odot \exp\left[-v^2/((\eta_R \cos \theta)^2 + (\eta_T \sin \theta)^2)^{1/2}\right] \quad (4) $$

Actually, this is the traditional turbulence model which was used by solar physicists in 1960s–1970s to derive the radial and tangential components of non-thermal velocity dispersions (cf. section 1). In the special case of $\eta_R = \eta_T (= \eta)$, equation (4) reduces to the simple isotropic Gaussian function

$$ \Theta_2(v) \propto \exp\left[-(v/\eta)^2\right]. \quad (5) $$

By integration of $\Theta_2(v)$ over the disk, the integrated macrobroadening function $M_2(v; \eta, v_s \sin i)$ can be obtained. Figure 1c display the profiles of $M_2(v; \eta, v_s \sin i)$ for different $v_s \sin i/\eta$ ratios, which were numerically computed in the same manner as in $M_1$ (though $M_2$ in this case of angle-independent $\Theta_2$ can be expressed by a simple convolution of rotational broadening function and Gaussian function).

3. Profile fitting for parameter determination

Regarding the profile-fitting analysis (to be described in the following two sections), almost the same procedure as in Paper I was adopted, except that (i) specific intensity ($I$) emergent with angle $\theta$ corresponding to each observed point is relevant here (instead of angle-integrated flux) and (ii) Gaussian line-broadening function parameterized by $V_{los}$ (velocity dispersion along the line of sight) was used for the kernel function:

$$ K(v) \propto \exp\left[-(v/V_{los})^2\right]. \quad (6) $$

That is, the intensity profile $I(v, \theta)$ emergent to direction angle $\theta$ is expressed as
\[ I(v, \theta) = I^0(v, \theta) \otimes K(v) \otimes P(v), \]

where \( P(v) \) is the instrumental profile. \( I^0(v, \theta) \) is the intrinsic profile of outgoing specific intensity at the surface, which is written by the formal solution of radiative transfer as

\[ I^0(\lambda; \theta) = \int_0^\infty S_\lambda(t_\lambda) \exp(-t_\lambda/\cos \theta)d(t_\lambda/\cos \theta), \]

where \( S_\lambda \) is the source function and \( t_\lambda \) is the optical depth in the vertical direction. Regarding the calculation of \( I^0 \), we adopted Kurucz’s (1993) ATLAS9 solar photospheric model with a microturbulent velocity of \( \xi = 0.5 \) km s\(^{-1}\) (see appendix 1 regarding the effect of changing this parameter) while assuming LTE.

Following Paper I, we adopted the algorithm described in Takeda (1995a) to search for the best-fit theoretical profile, where the following three parameters were varied for this purpose: \( \log \epsilon \) (elemental abundance), \( \text{Vlos} \) (line-of-sight velocity dispersion), and \( \Delta \lambda_\perp \) (wavelength shift) [which is equivalent to the radial velocity \( v_r \) (= \( c\Delta \lambda_\perp/\lambda \); \( c \): velocity of light)].

After the solutions of these parameters have been converged, we computed the mean depth of line formation \((\log \tau)\) defined as follows: \(^3\)

\[ (\log \tau) = \frac{\int R^2_\parallel \log \tau_{5000}(\lambda = \cos \theta)d\lambda}{\int R^2_\parallel d\lambda} \]

where \( \tau_{5000} \) is the continuum optical depth at 5000 Å, \( R^2_\parallel \) is the line depth of theoretical intrinsic profile (corresponding to the resulting solution of \( \log \epsilon \)) with respect to the continuum level \( \left( R^2_\parallel = 1 - R^2_\parallel/I^0_\parallel \right) \), and integration is done over the line profile. Besides, the local equivalent width \( (w^\lambda_\parallel) \) could be evaluated as a by-product by integrating \( R^2_\parallel \) over the wavelength.

4. Analysis of low spatial-resolution spectra

4.1. Expected angle-dependence of \text{Vlos}

We are now ready to test the validity of RTM based on actual spectra at various points of the solar disk. As RTM is a two-component model, which is meaningful only in the combination of radial- and tangential-flow parts, the observed spectra to be compared must be locally averaged over a sufficient number of granular cells.

Since \( \Theta_1(v, \theta) \) has an extraordinary form (cf. subsection 2.2) and can not be directly incorporated in the analysis scheme described in section 3, we proceed with the following strategy:

- Let us first assume that the RTM model exactly holds, which is characterized by two parameters \((\zeta_R \text{ and } \zeta_T); \text{ while } A_R = A_T \text{ is assumed.})
- Then, the emergent intensity profile can be simulated by convolving the intrinsic profile \( R_0(v, \theta) \) with the RTM broadening function \( \Theta_1(v, \theta; \zeta_R, \zeta_T) \) as \( R_0 \otimes \Theta_1 \).
- Let us consider here how much \text{Vlos} value would be obtained if this RTM-broadened profile is analyzed by the Gaussian-based procedure described in section 3. This can be reasonably done by equating the HWHM (half-width at half-maximum) of \( R_0(v, \theta) \otimes \Theta_1(v, \theta; \zeta_R, \zeta_T) \) with that of \( R_0(v, \theta) \otimes \exp[-(v/\text{Vlos})^2] \), by which we can express \text{Vlos} as a function of \( \theta \) for any combination of \((\zeta_R, \zeta_T)\).
- Regarding the intrinsic profile \( R_0 \), we adopted a Gaussian profile with \( \epsilon \)-folding half-width of 1.5 km s\(^{-1}\) (typical value for the thermal motion of Fe atom plus microturbulence in the solar photosphere).

As examples, we display in figure 3 the profiles of \( \Theta_1 \) and \( R_0 \otimes \Theta_1 \) computed for two cases \((\zeta_R = \zeta_T = 2 \text{ and } 4 \text{ km s}^{-1}) \) at various angles \((\theta)\). The resulting \text{Vlos} vs. \( \theta \) relations for various combinations of \((\zeta_R, \zeta_T)\) are depicted (in solid lines) in figure 4, where the curves for the (anisotropic) Gaussian macroturbulence (GM) case are also shown (in dashed lines) for comparison [\( V^2_{\text{los}} = (\eta_\text{m} \cos \theta)^2 + (\eta_\text{T} \sin \theta)^2 \) holds in this GM case according to equation (4)].

Several notable points are summarized below regarding the trends read from figure 4:

- \text{Vlos}(GM) is almost the same order as \( \sim \eta_\text{m} \) and thus its dependence upon \( \theta \) is nearly flat or monotonic.
- On the contrary, \text{Vlos}(RTM) is significantly smaller than \( \zeta \) [and \text{Vlos}(GM)] especially near to the disk center and near to the limb, resulting in a peak of \text{Vlos}(RTM) around \( \theta \sim 45^\circ \).
- This is due to the very characteristics of \( \Theta_1; \) i.e., its width tends to be considerably narrow without reflecting the turbulence dispersion (especially around \( \theta \sim 0^\circ \) and \( \theta \sim 90^\circ \)) as clearly seen in figures 3a and 3a’.
- Besides, why the inequality relation \( \text{Vlos} < \zeta \) holds can also be understood from figure 2 (for the \( \theta \sim 0^\circ \) case).
- Another important point is that this anomaly becomes particularly manifest when \( \zeta \) outweighs \( v_\text{th} \) (1.5 km s\(^{-1}\)), while less pronounced if \( \zeta \) is comparable or smaller than \( v_\text{th} \).

We will make use of these characteristics to check the RTM model in subsection 4.4.

4.2. Observational data of Hida/DST

The ground-based observations were carried out on 2015 November 3–5 (JST) by using the 60 cm Domeless Solar Telescope (DST) with the Horizontal Spectrograph at Hida Observatory of Kyoto University (Nakai & Hattori 1985). The aspect angles of the solar rotation axis (\( P: \) position angle between the geographic north pole and the solar rotational north pole; \( B_0: \) heliographic latitude of the central point of the solar disk) in this period were \( P = +24^\circ \) and \( B_0 = +4.1^\circ \). Regarding the target positions on the solar disk, we selected 32 points on the northern
meridian line of the solar disk (from the disk center to 0.97 $R_0$ with a step of 30′′ $\approx 0.03 R_0$, where $R_0$ is the apparent radius of the solar disk) as depicted in figure 5a, at which the slit was aligned in the E–W direction. Since the disk center and the nearest-limb point correspond to $\cos \theta = 1$ and $\cos \theta = 0.24 (\equiv \sqrt{1-0.97^2})$ in this arrangement ($\theta$ is the emergent angle of the ray measured with respect to the normal to the surface), the angle range of $0^\circ \leq \theta \leq 76^\circ$ is covered by our data.

In the adopted setting of the spectrograph, our observation produced a solar spectrum covering of 153′′ (spatial) and 24 Å (wavelength) on the CCD detector with $1 \times 2$ binning (1600 pixels in the dispersion direction and 600 pixels in the spatial direction). We repeated the whole set (consecutive observations on 32 points along the center-to-limb meridian) 30 times while changing the central wavelength, and finally obtained the spectra in the wavelength regions of 5190–5450 Å, 5650–5690 Å, 5830–5870 Å, and 6050–6310 Å (about $\sim 600$ Å in total). Although most of our observations were done in quiet regions on the solar disk, active regions may have affected in some of our data, since a notable spot passed through the meridian on November 4.

The data reduction was done by following the standard procedures (dark subtraction, spectrum extraction, wavelength calibration, and continuum normalization). The 1D spectrum was extracted by integrating over 200 pixels (= 51″; i.e., $\pm 100$ pixels centered on the target point) along the spatial direction. Given that typical granule size is on the order of $\sim 1''$, our spectrum corresponds to the spatial mean of each region including several tens of granular cells, by which the condition necessary for testing the RTM model is reasonably satisfied (cf. 1st paragraph of subsection 4.1). Finally, the effect of scattered light was corrected by following the procedure described in subsection 2.3 of Takeda and Ueno (2014), where the adopted value of $\alpha$ (scattered-light fraction) was $0.10 (\lambda < 5500$ Å) and 0.15 ($\lambda > 5500$ Å) according to our estimation. Given that the main scope of this study is to measure the “widths” of spectral lines, this scattered-light correction does not have any essential influence, since it is a simple multiplication of a factor to the line-depth profile (i.e., its similarity is unaffected) as shown in equations (1) and (2) of Takeda and Ueno (2014).

The S/N ratio of the resulting spectrum (directly measured from statistical fluctuation in the narrow range of line-free continuum) turned out to be sufficiently high (typically $\sim 500–1000$). The e-folding half width of the instrumental profile (assumed to be Gaussian as $\exp(v/v_{\text{ip}})^2$ in this study) was determined to be $v_{\text{ip}} \simeq 1.3$ km s$^{-1}$ by using the lamp + I$_2$ gas cell spectrum (cf. section 2 in Takeda & Ueno 2012 for details), which corresponds to FWHM ($= 2\sqrt{\ln 2} v_{\text{ip}}$) $\simeq 2.2$ km s$^{-1}$ and the spectrum resolving power of $\sim 140000 (\simeq c/\text{FWHM})$ (c: velocity of light). Note that this

Since the wavelength vs. pixel relation was derived (not legitimately by using the comparison spectra but) based on $\sim 20–50$ solar lines in the disk-center spectrum for each region, any absolute wavelength calibration is not accomplished in our data.
Comparing the observed tendency of $V_{\text{los}}$ (figures 9a–d) with the predicted trends for both RTM and GM (figure 4), we can draw a clear conclusion: None of the $V_{\text{los}}$(RTM) vs. $\theta$ relations matches the observed center-to-limb variation (gradual increase), since the characteristic peak at $\theta \sim 45^\circ$ expected for the RTM case is lacking. Let us recall that a significantly large $\zeta_{\text{RT}}$ value of $\sim 3$–4 km s$^{-1}$ was derived from the analysis of solar flux spectrum (cf. section 1), which is evidently larger than $v_{11}$. Then, a prominent peak should be observed if the condition assumed by RTM is really realized in the solar surface. Given the absence of such a key trend, we can state that RTM is not a valid model for the solar atmospheric velocity field. In contrast, the $V_{\text{los}}$(GM) vs. $\theta$ curve predicted for $(\eta_R, \eta_T) = (2 \text{ km s}^{-1}, 3 \text{ km s}^{-1})$ satisfactorily reproduces the observed relation, which may indicate that the classical GM is a more reasonable and better representation in this respect.

5. Analysis of high spatial-resolution spectra

5.1. Merit of studying well-resolved surface structures

In order to ascertain the consequence of subsection 4.4 from an alternative point of view, we further carried out a similar analysis but using spectra of high spatial resolution acquired by satellite observations. Unlike the case of low-resolution spatially-averaged spectra studied in section 4, we can not employ these highly-resolved data for direct comparison with predictions from the RTM model, because each spectrum reflects the gas motion of a local part smaller than the typical size of granules, to which the concept of RTM (meaningful only for spectra averaged over granular cells; cf. figure 2) is no more applicable. Instead, however, we can make use of such observational data of high spatial resolution to verify the fundamental assumption on which the RTM model is based, since the velocity distribution (amplitude, direction) within a cell can be directly studied; e.g., whether the vectors of turbulent motions are really coordinated in two orthogonal directions as assumed in RTM (cf. figure 2). This would make a decisive touchstone.

5.2. Observational data of Hinode/SOT

Regarding the spectra used for this purpose, we adopted the data obtained by the Solar Optical Telescope (SOT; Tsuneta et al. 2008) aboard the Hinode$^5$ satellite (Kosugi et al. 2007). Since the Spectro-Polarimeter (SP; Lites et al. 2013) in Hinode/SOT provides full calibrated Stokes $IQUV$ spectra of 6301–6303 Å region (comprising two Fe I lines at 6301.498 and 6302.494 Å), we could use unpolarized $I$ spectra for our purpose, which are available as Level-I data from the Hinode Data Center$^6$ or from the SolarSoft site$^7$.

Having inspected the archived data, we decided to use the spectra obtained by normal-map mode observations of three quiet regions along the southern meridian on 2008 December 17 (the start time of each mapping was 05:43:35, 09:34:05, and 10:34:05 in UT, respectively). These SP mapping observations were done by moving the (N–S aligned) slit of 0.16 width in E–W direction by $\sim 0.1$′′, and field-of-view of in the slit direction is 129′′ (corresponds to 408 pixels on the detector), resulting in sampling steps of $\sim 0.7$′′(x or E–W direction) and $\sim 0.3$′′ (y or N–S direction). Although the total region covered by each mapping was 30″ × 129″, we used only the spectra within three 20″ × 20″ square regions centered at (0″, 0″), (0″, −700″), and (0″, −975″) corresponding to the disk center ($\theta \sim 0^\circ$), the half-right-angle view point ($\theta \sim 45^\circ$), and the limb ($\theta \sim 80^\circ$), respectively. Figure 5b indicates the locations of these three regions, for which the numbers of the resulting spectra were 8191, 8266, and 8253, respectively.

5.3. Statistical properties of $V_{\text{los}}$ and $v_{\tau}$

As done in subsection 4.3, we applied the spectrum-fitting method (cf. section 3) to these spectra and successfully established the solutions of $V_{\text{los}}$ and $v_{\tau}$, where only Fe I 6302.494 line (the slightly weaker one of the available two lines; also included in table 1) was used for this analysis, and the instrumental profile was assumed to be the Gaussian function with the $e$-folding half-width of 0.71 km s$^{-1}$ corresponding to the spectrum resolving power of $R \simeq 6302.5/0.025 = 252000$ (cf. figure 7 of Lites et al. 2013). The typical signal-to-noise ratio of these SOT spectra is around 100. Some selected examples of fitted theoretical and observed spectra are displayed in figure 10. The histograms for the resulting $V_{\text{los}}$ and $v_{\tau}$, $V_{\text{los}}$ vs. $v_{\tau}$ correlation, and the continuum brightness vs. $v_{\tau}$ relation, are graphically shown figure 11.

Although several notable trends are observed in figure 11 (e.g., blue-shift tendency of brighter points at the disk-center which should be due to rising hot bubbles), we here confine ourselves only to the main purpose of this study; i.e., checking the validity of RTM.

Let us focus on the disk-center results ($\theta \sim 0^\circ$) shown in the top row of figure 12. If the condition assumed in RTM is really existent in the solar surface (cf. figure 2), observations of disk center (i.e., line-of-sight normal to the surface) with high spatial resolution should reveal almost comparable numbers of cases with turbulent-broadened profiles (R) and those with unbroadened sharp profiles (T). Then, the following trends are expected:

— The distribution function of $V_{\text{los}}$ would show an extraordinary feature (e.g., an appreciable peak at low $V_{\text{los}}$).
— Since cells of horizontal flow (T) show no radial velocity, an unusually prominent peak would exist in the distribution of $v_{\tau}$ at $v_{\tau} \sim 0$.
— As a consequence, a considerable bias at (small $V_{\text{los}}$, $v_{\tau} \sim 0$) would be observed in the $V_{\text{los}}$ vs. $v_{\tau}$ plot.

However, none of these features is observed in the disk-center results in figure 11, where we can confirm that $V_{\text{los}}$...
as well as \( v_\perp \) follow a statistically near-normal distribution without any such expected bias as mentioned above. These observational facts suggest that the velocity vectors of solar photospheric turbulence are not confined to only two (radial and tangential) directions but more chaotic with rather random orientations. Accordingly, we have reached a decision that the basic assumption of RTM does not represent the actual solar photosphere, which means that the RTM model does not correctly describe the spectral line-broadening of solar-type stars.

6. Concluding remark

We carried out an extensive spectroscopic investigation on the non-thermal velocity dispersion along the line-of-sight by analyzing spectral lines at various points of the solar disk, in order to check whether the RTM model (which has been widely used for line-profile studies of solar-type stars) adequately represents the actual solar photospheric velocity field. Applying the profile-fitting analysis to two sets of observational data: (spatially-averaged spectra from Hida/DST observations and very high spatial-resolution spectra from Hinode/SOT observations), we found the following results.

First, the center-to-limb variation of \( V_{\text{los}} \) derived from low-resolution spectra turned out simply monotonic with a slightly increasing tendency. This apparently contradicts the characteristic trend (an appreciable peak at \( \theta \sim 45^\circ \)) expected from the RTM model. Second, the distributions of \( V_{\text{los}} \) and \( v_\perp \) values derived from spectra of very high spatial resolution revealed to show a nearly normal distribution, without any sign of anomalous distribution predicted from the RTM model.

These observational facts suggest that the fundamental assumption of RTM is not compatible with the real atmospheric velocity field of the Sun, which can not be so simple (i.e., being confined only to radial and tangential directions) but should be directionally more chaotic. We thus conclude that RTM is not an adequate model at least for solar-type stars.

It is evident that RTM significantly overestimates the turbulent velocity dispersion in the solar photosphere, which should actually be \( \sim 2 \) km s\(^{-1}\) (disk center) and \( \sim 2.5 \) km s\(^{-1}\) (limb) as evidenced from the mean (or peak) value of \( V_{\text{los}} \) derived from high-resolution data indicated in figure 11 (leftmost panels). Therefore, the fact that RTM yields \( \zeta_{\text{RT}} \sim 3-4 \) km s\(^{-1}\) for the solar macroturbulence (cf. section 1) simply means that the width of RTM broadening function (\( M_1 \)) is unreasonably too narrow. We therefore stress that, when using RTM for analyzing line profiles of solar-type stars, \( \zeta_{\text{RT}} \) should be regarded as nothing but a fudge parameter without any physical meaning. If it were carelessly associated with discussion of physical processes (e.g., in estimation of the turbulent energy budget or in comparison with the sonic velocity), erroneous results would come out.

On the other hand, the classical Gaussian macroturbulence model should be more reasonable and useful in this respect. Actually, our application of GM to the analysis of solar flux spectrum resulted in \( \eta \sim 2 \) km s\(^{-1}\) (cf. figure 1b). Likewise, the GM-based conversion formula [equation (A1)] lead to \( V_{\text{rad}}^{\text{rad}} \sim 2 \) km s\(^{-1}\) (at \( \log \tau \sim -1.5 \)) and \( V_{\text{tan}}^{\text{tan}} \sim 2.5 \) km s\(^{-1}\) (at \( \log \tau \sim -2 \)) as the non-thermal dispersion in radial and tangential direction (cf. figure 13 in appendix 2), which are in fairly good agreement with the directly evaluated results based on high-resolution observations mentioned above (note that the mean formation depth for Fe I 6302.494 line is \( \log \tau \sim -1.5 \) for the disk center and \( \log \tau \sim -2 \) for the limb; cf. figure 7b).

Accordingly, application of the simple GM would be more recommended, rather than the inadequate and complex RTM. (See appendix 3, where the trend of macroturbulence in FGK-type dwarfs is discussed in view of applying the GM model.)

Finally, some comments may be due on the future prospect in this field. Regarding the modeling of turbulent velocity field in the atmosphere of the Sun or solar-type stars, we have to mention the recent remarkable progress in the simulations of 3D time-dependent surface convection (see, e.g., Nordlund, Stein, & Asplund 2009, and the references therein), which successfully reproduce the observed characteristics of spectral lines (e.g., Asplund et al. 2000; Pereira et al. 2013, for the solar case) without any ad-hoc turbulent-velocity parameters (such as micro- and macro-turbulence in the classical case) and thus by far superior to the traditional modeling. However, given the enormous computational burden of calculating such elaborate 3D models, the simple micro/macro-turbulence model is expected to remain still in wide use for practical analysis of stellar spectra. Therefore, it would be very helpful if the behaviors of classical microturbulence as well as macroturbulence can be predicted or understood based on the realistic 3D simulations. For example, while main emphasis is placed on shift and asymmetry (bisector) of spectral lines in demonstrating the predictions of 3D models in comparison with observations, less attention seems to be paid to the “width” of spectral lines. Can the trend of apparent turbulent dispersion derived in this study (i.e., tangential component being slightly larger than the radial component, an increasing tendency with depth) be reproduced by such state-of-the-art 3D hydrodynamical models? Further contributions of theoreticians in this light would be awaited.

This work was partly carried out on the Solar Data Analysis System operated by the Astronomy Data Center in cooperation with the Hinode Science Center of the National Astronomical Observatory of Japan.
Appendix 1. Influence of microturbulence on the results

Regarding the classical microturbulence,\(^8\) which is necessary to compute the theoretical intrinsic profile to be convolved with the macroscopic velocity distribution function, we adopted 0.5 km s\(^{-1}\) throughout this study in order to maintain consistency with Paper I. This is the value obtained by analyzing the profiles of 15 stronger lines (\(\xi = 0.51 \pm 0.15\) km s\(^{-1}\); cf. subsection 5.1 in Paper I), which is in fairly good agreement with Gray’s (1977) result of \(\xi = 0.5 \pm 0.1\) km s\(^{-1}\) based on the Fourier transform analysis of line profiles.

Meanwhile, the solar \(\xi\) values based on the conventional way using equivalent widths of spectral lines (i.e., by the requirement that the resulting abundances do not show any systematic dependence on the line strength) published in various literature tend to be \(\sim 1\) km s\(^{-1}\) (see, e.g., subsection 3.2 in Takeda 1994); i.e., somewhat larger than the profile-based value of 0.5 km s\(^{-1}\) we adopted. This discordance was already remarked in subsection 6.1 of Paper I and further discussed by Takeda et al. (1996; cf. subsection 4.3 therein).

However, which solar microturbulence (high-scale or low-scale) to choose would not play any essential role in deriving the macroscopic velocity dispersions (\(V_{\text{los}}\), \(\xi\), and \(\eta\) mentioned in this study, since the former (\(\sim 0.5–1\) km s\(^{-1}\)) is quantitatively insignificant as compared to the combination of the latter (\(\gtrsim 1–2\) km s\(^{-1}\)) and the thermal velocity (\(\sim 1.3\) km s\(^{-1}\) for the case of Fe) (note that each velocity dispersion contributes to the total Doppler velocity in the form of \("(root-)square-sum")\).

In order to see whether this argument is justified, we carried out extra test calculations where all the analysis were redone by using \(\xi = 1.0\) km s\(^{-1}\) instead of the fiducial value of 0.5 km s\(^{-1}\). In figure 12 are compared the resulting values of \(V_{\text{los}}^{1.0}\), \(\xi_{\text{los}}^{1.0}\), and \(\eta_{\text{los}}^{1.0}\) with those of the standard results (\(V_{\text{los}}^{0.5}\), \(\xi_{\text{los}}^{0.5}\), and \(\eta_{\text{los}}^{0.5}\)).

We can see an interesting trend regarding the resulting difference of \(V_{\text{los}}^{1.0} - V_{\text{los}}^{0.5}\), etc. That is, as long as weaker-strength regime is concerned (\(w_{\lambda}^i\) or \(W_{\lambda}^i\) is \(\lesssim 100\) mA), the macroscopic velocity dispersions tend to slightly decrease as a consequence of using larger \(\xi\), which is reasonably understandable by considering the contribution of each velocity component to the total width as mentioned above. On the other hand, for the case of stronger lines (\(\gtrsim 100–150\) mA), this tendency is inverted, and \(V_{\text{los}}^{1.0}\), \(\xi_{\text{los}}^{1.0}\), and \(\eta_{\text{los}}^{1.0}\) turn out larger than the corresponding \(\xi = 0.5\) km s\(^{-1}\) results. This phenomenon is attributed to the effect of \(\xi\) on the degree of saturation. That is, since a line may get desaturated (or saturation may be retarded) by increasing \(\xi\) as known in the traditional curve-of-growth analysis, a line for a given equivalent width (e.g., 150 mA) is strongly saturated (i.e., boxed shape with wider width at the core) for the \(\xi = 0.5\) km s\(^{-1}\) case but not so for \(\xi = 1.0\) km s\(^{-1}\). In such circumstances, the former contributes larger broadening than the latter to the total line width, by which this trend \((V_{\text{los}}^{1.0} > V_{\text{los}}^{0.5}, \cdots\) etc\) may be explained.

In any event, figure 12 shows that the changes are only \(\pm \lesssim 0.5\) km s\(^{-1}\) for most cases (\(\pm \lesssim 10–20\%\) may be a better estimation, since the amount of variation appears to be proportional to the absolute values). We may regard these differences as comparatively insignificant, though we should keep in mind that the effect of changing \(\xi\) is different for stronger saturated lines (those with \(\gtrsim 100–150\) mA forming around \(\log \tau \sim -1.5\) to \(-2\)) from that for other weaker lines.

Appendix 2. Nature of non-thermal velocity dispersion in the solar photosphere

According to the conclusion of this paper, it is a good approximation to represent the non-thermal turbulent velocity field in the solar photosphere by an anisotropic Gaussian distribution with dispersions of \(V_{\text{rad}}\) (radial direction) and \(V_{\text{tan}}\) (tangential direction). Therefore, we can safely use the classical relation (see, e.g., section 3 in Gurzovenko 1975c and the references therein):

\[V_{\text{los}}^2 = (V_{\text{rad}}^2 \cos \theta)^2 + (V_{\text{tan}}^2 \sin \theta)^2\]  \(\text{(A1)}\)

[\(\text{note that } V_{\text{rad}}^{\text{and }} V_{\text{tan}}^{\text{are equivalent to } \eta_{\text{RT}}^{\text{and }} \eta_{\text{RT}}^{\text{in \text{equation (4)}}}\).]

Let us examine the quantitative characteristics of \(V_{\text{rad}}\) and \(V_{\text{tan}}\) by using the \(V_{\text{los}}\) data derived in section 4 for many lines at various points on the solar disk.

We regard \(V_{\text{los}}\) values near to the disk center at \(1 \geq \cos\theta > 0.95\) (\(0^\circ \leq \theta < 17^\circ\)) as practically equivalent to \(V_{\text{rad}}\), which are plotted against \(\log \tau\) in figure 13a (red symbols). Although the dispersion is rather large, we could draw a mean \(V_{\text{rad}}(\tau)\) relation (with an extrapolation at \(\log \tau \lesssim -2\)) as depicted in the solid line connecting the points at \((\log \tau, V_{\text{rad}}) = (-2.5, 1.6), (-2.0, 1.8), (-1.5, 2.0), (-0.5, 2.3),\) and \((0.0, 1.9)\). Similarly, we assume those \(V_{\text{los}}\) values near to the limb at \(0.3 < \cos\theta < 0.95\) (\(73^\circ < \theta \leq 17^\circ\)) almost equivalent to \(V_{\text{tan}}\), which are shown in figure 13b (blue symbols). In this case, however, the number of points at the important region of \(-1 \lesssim \log \tau \lesssim 0\) is insufficient. Therefore, we added the data points by making use of the \(V_{\text{los}}\) values (only for class-1 lines) observed at \(0.3 < \cos\theta < 0.95\) (\(17^\circ < \theta < 73^\circ\)), which were converted to \(V_{\text{tan}}\) with the help of equation (A1) and the mean \(V_{\text{rad}}(\tau)\) relation derived above, as plotted in green filled circles in figure 13b.

Then, eye-inspecting the combined trend of these symbols, we derived the mean \(V_{\text{tan}}(\tau)\) relation as depicted in the dashed line connecting \((\log \tau, V_{\text{tan}}) = (-2.5, 2.3), (-2.0, 2.7), (-1.5, 2.9), (-1.0, 3.0), (-0.5, 2.9),\) and \((0.0, 2.7)\).

Such derived mean relations of \(V_{\text{rad}}(\tau)\) and \(V_{\text{tan}}(\tau)\) are shown together and compared with the literature results in figures 13c and 13d, from which the following characteristics are summarized:

— Roughly speaking, our results may be regarded as almost consistent with the relations derived by the previous

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\(^8\) This is by definition the microscopic turbulent velocity dispersion, the characteristic scale of which is assumed to be much smaller than the photon mean-free-path. As such, it is formally included into the Doppler width of line-opacity profile in parallel with the velocity of thermal motion.
studies in terms of the general trend and a quantitative agreement is seen.
— Especially, we could confirm that $V_{\tan}$ is systematically larger than $V_{\text{rad}}$ by $\sim 1$ km s$^{-1}$, which was already reported in various literature.
— However, our results are not necessarily compatible with the simple picture of monotonically increasing $V_{\text{rad}}$ and $V_{\tan}$ with depth suggested by many of the previous investigations. According to our mean curves, the depth-increasing tendency of $V_{\text{rad}}$ and $V_{\tan}$ manifestly seen at high layer ($\log \tau \sim -2$) reaches the ceiling around $\log \tau \sim -1 (V_{\tan})$ or $\log \tau \sim -0.5 (V_{\text{rad}})$, below which the velocity dispersion turns to slightly decrease with depth.
— Consequently, the relations we derived for $V_{\text{rad}}$ and $V_{\tan}$ show not so much a systematically large sensitivity to depth as a rather weak depth-dependence with a broad maximum around $\log \tau \sim -1$.

Appendix 3. Related topics regarding the macro-turbulence of solar-type stars

We concluded in this paper that RTM is not good but the classical GM is more preferable as the macroturbulence model for solar-type dwarfs. In connection with this consequence, we briefly mention below (1) the practical procedure for application of GM, (2) the conversion between the two systems, and (3) the empirical relation for estimating the macroturbulence.

As an example of application of Gaussian macroturbulence, we refer to the work of Takeda and Tajitsu (2009, who studied the properties of three solar twins. In their modeling, the total macroad broadening function, $f_m(v)$, was assumed to be the convolution of three Gaussian component functions $f_\alpha \propto \exp[-(v/v_\alpha)^2]$, where $\alpha$ is any of “ip” (instrumental profile), “rt” (rotation), and “mt” (macroturbulence): i.e.,

$$v_M^2 = v_{\text{ip}}^2 + v_{\text{rt}}^2 + v_{\text{mt}}^2 = v_{\text{ip}}^2 + v_{\text{rt+m}}^2, \quad (A2)$$

where $v_{\text{rt+m}}$ is the “macroturbulence+rotation.” They related these broadening parameters ($v_{\text{ip}}$, $v_{\text{rt}}$, and $v_{\text{mt}}$) to (resolving power $R$, $v_\text{esc} \sin i$, $\zeta_{\text{RT}}$) as $v_{\text{ip}} \simeq (c/R)/(2 \sqrt{\ln 2})$ ($c$ is the speed of light), $v_{\text{rt}} \simeq 0.94 v_\text{esc} \sin i$, and $v_{\text{mt}} \simeq 0.42 \zeta_{\text{RT}}$, which are based on the requirement that FWHMs of the relevant broadening profile and the Gaussian profile should be the same.9

Here, the factor (0.42) for conversion between $v_{\text{mt}}$ and $\zeta_{\text{RT}}$ was estimated from the pure radial-tangential and pure Gaussian profiles (i.e., no-rotation case of $M_1$ and $M_2$ in figures 1b and 1c), which was used to estimate $v_{\text{mt}}$ from the $\zeta_{\text{RT}}$ results obtained in Paper I. However, this factor appears to somewhat depend on the additional broadening due to rotation. For example, the results of $\zeta_{\text{RT}}$ and $\eta = v_{\text{mt}}$ derived for the case of solar flux spectrum ($v_\text{esc} \sin i = 1.9$ km s$^{-1}$) presented in figure 1a, it is slightly larger as $\eta/\zeta_{\text{RT}} \sim 0.6$. Therefore, it is not much meaningful to discuss the precise value of this ratio, for which we may only state as being around $\sim 0.5$.

Several empirical relations of $v_{\text{mac}}$ (macroturbulence) expressed in terms of atmospheric parameters (e.g., $T_{\text{eff}}$) have been proposed by several authors; e.g., Gray (1984), Valenti and Fischer (2005), Bruntt et al. (2010), and Doyle et al. (2014). Several points should be remarked regarding their practical applications:
— First, it should be clearly recognized on which macroturbulence model the relation in question is based. Among the four references mentioned above, only Bruntt et al. (2010) used GM, while RTM was employed in the other three. This must be the reason why only Bruntt et al.’s (2010) $v_{\text{mac}}$ values are appreciably lower than the others (cf. figure 4 of Doyle et al. 2014).10
— Second, if $v_{\text{mac}}$ values derived from the empirical relation are to be used for $v_\text{esc} \sin i$ determination, it is desirable to use them unchanged as they are, while following the same line-broadening model as that adopted in the original literature. Easily assuming a simple scaling relation to convert it to other system is not recommendable, which may cause some unwanted errors.
— Nevertheless, regarding the $v_{\text{mac}}$ vs. $T_{\text{eff}}$ relation derived from the lower envelope of macroturbulence plus rotational velocity distribution such as done by Valenti and Fischer (2005; cf. figure 3 therein),11 transformation by applying $v_{\text{mt}} \simeq 0.42 \zeta_{\text{RT}}$ (derived from $M_1$ and $M_2$ for the no-rotation case as mentioned above) may not be a bad approximation, since the lower envelope corresponds to $v_\text{esc} \sin i = 0$.

Bearing these points in mind, we also tried to examine how the macroturbulence ($v_{\text{mt}}$) in solar-type stars depends on $T_{\text{eff}}$ based on our own data. In figure 14 are plotted the $v_{\text{rt+m}}$ values against $T_{\text{eff}}$ (symbols), which were determined from the spectrum-fitting analysis of 6080–6089 Å region while assuming the simple Gaussian modeling for the line-broadening functions expressed by equation (A2). These results were originally derived by Takeda and Honda (2005; FGK-type dwarfs stars), Takeda et al. (2007; solar analogs), and Takeda et al. (2013; Hyades stars). The lower envelope of this $v_{\text{rt+m}}$ distribution at 6000 K $\geq T_{\text{eff}} \geq$ 5000 K can be fitted by the following analytical relation for $v_{\text{mt}}$ (solid line in the figure):

$$v_{\text{mt}} = 6.087 \times 10^{-1} - 2.352 \times 10^{-2} T_{\text{eff}} + 2.311 \times 10^{-6} T_{\text{eff}}^2, \quad (A3)$$

where $v_{\text{mt}}$ is in km s$^{-1}$ and $T_{\text{eff}}$ is in K. For comparison, the empirical relations published by Gray (1984), Valenti and Fischer (2005), Bruntt et al. (2010), and Doyle et al. (2014) are also depicted in this figure (dashed lines), where only Bruntt et al.’s (2010) $v_{\text{mac}}$ curve is shown unchanged.

9 See footnote 12 in subsection 4.2 of Takeda, Sato, and Murata (2008) for a more detailed description regarding the derivation of these conversion relations.

10 This discrepancy was already pointed out in Sect. 4.1.3 of Doyle et al. (2014), though they do not seem to be clearly aware that the macroturbulence model adopted by Bruntt et al. (2010) is Gaussian and thus different from the others.

11 It should be noted that Valenti and Fischer’s (2005) Equation (1), which is their proposed relation between $v_{\text{mac}}$ and $T_{\text{eff}}$, includes an apparent typo: The sign before the $T_{\text{eff}}$-dependent term should be ‘+’, instead of ‘−’. That is, $v_{\text{mac}} = 3.98 + (T_{\text{eff}} - 5770)/650$, where $v_{\text{mac}}$ is in km s$^{-1}$ and $T_{\text{eff}}$ is in K.
(because of the same GM as $v_{\text{nv}}$) while the $v_{\text{mac}}$ values of other three are tentatively multiplied by a factor of 0.42 (because they are based on RTM). Roughly speaking, our result is more or less favorably compared with other relations, as far as the lower temperature region (below the solar $T_{\text{eff}}$) is concerned. In particular, those of Valenti and Fischer (2005) and Bruntt et al. (2010) appear to show a reasonable consistency with our curve at $T_{\text{eff}} \lesssim 5800$ K.

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| Species | $\lambda$ (Å) | $x_{low}$ (eV) | $W_\lambda^f$ (mÅ) | Species | $\lambda$ (Å) | $x_{low}$ (eV) | $W_\lambda^f$ (mÅ) |
|---------|---------------|---------------|-----------------|---------|---------------|---------------|-----------------|
| Fe i    | 5197.929      | 4.301         | 41.6            | Fe i    | 5651.470      | 4.474         | 22.1            |
| Fe i    | 5198.711      | 2.224         | 100.0           | Fe i    | 5652.320      | 4.260         | 32.8            |
| Fe i    | 5206.801      | 4.283         | 12.1            | Fe i    | 5653.889      | 4.386         | 44.3            |
| Fe i    | 5223.187      | 3.635         | 33.3            | Fe i    | 5672.267      | 4.584         | 31.1            |
| Fe i    | 5225.525      | 0.110         | 74.7            | Fe i    | 5679.025      | 4.652         | 72.0            |
| Fe ii   | 5234.625      | 3.221         | 96.9            | Fe i    | 5680.241      | 4.186         | 16.1            |
| Fe i    | 5242.491      | 3.634         | 94.3            | Fe i    | 6065.482      | 2.608         | 125.0           |
| Fe i    | 5247.049      | 0.087         | 67.2            | Fe i    | 6082.708      | 2.223         | 38.3            |
| Fe i    | 5253.023      | 2.279         | 21.8            | Fe ii   | 6084.111      | 3.199         | 22.3            |
| Fe i    | 5279.654      | 3.301         | 8.2             | Fe i    | 6093.666      | 4.697         | 34.3            |
| Fe i    | 5285.118      | 4.434         | 32.8            | Fe i    | 6094.364      | 4.652         | 31.1            |
| Fe i    | 5288.528      | 3.695         | 65.2            | Fe i    | 6096.662      | 3.984         | 43.0            |
| Fe i    | 5300.412      | 4.593         | 8.6             | Fe i    | 6102.723      | 1.879         | 144.0           |
| Fe i    | 5301.327      | 4.386         | 4.3             | Fe i    | 6105.152      | 4.548         | 14.2            |
| Fe i    | 5308.707      | 4.256         | 9.8             | Fe i    | 6127.909      | 4.413         | 54.1            |
| Fe i    | 5320.039      | 3.642         | 23.5            | Fe i    | 6136.615      | 2.453         | 143.0           |
| Fe i    | 5321.109      | 4.434         | 47.7            | Fe i    | 6137.694      | 2.588         | 140.0           |
| Fe ii   | 5325.533      | 3.221         | 49.7            | Fe ii   | 6141.033      | 3.230         | 2.7             |
| Cr i    | 5348.312      | 1.004         | 105.0           | Fe ii   | 6149.258      | 3.889         | 39.6            |
| Fe i    | 5364.885      | 4.446         | 138.0           | Fe i    | 6151.617      | 2.176         | 53.9            |
| Fe i    | 5365.396      | 3.573         | 85.0            | Fe i    | 6165.361      | 4.143         | 49.2            |
| Fe i    | 5367.479      | 4.415         | 159.0           | Ca i    | 6169.563      | 2.526         | 120.0           |
| Fe i    | 5376.826      | 4.294         | 18.0            | Fe i    | 6173.341      | 2.223         | 74.3            |
| Fe i    | 5379.574      | 3.695         | 67.2            | Fe i    | 6180.203      | 2.727         | 59.3            |
| Fe i    | 5385.579      | 3.695         | 5.8             | Fe i    | 6187.398      | 2.832         | 5.5             |
| Fe i    | 5389.479      | 4.415         | 97.6            | Fe i    | 6187.987      | 3.943         | 55.0            |
| Fe i    | 5395.215      | 4.446         | 26.3            | Fe i    | 6191.558      | 2.433         | 136.0           |
| Fe i    | 5398.227      | 4.446         | 84.8            | Fe i    | 6199.507      | 2.559         | 6.0             |
| Fe i    | 5401.264      | 4.320         | 29.5            | Fe i    | 6200.314      | 2.608         | 78.6            |
| Fe i    | 5406.770      | 4.371         | 42.7            | Fe i    | 6213.429      | 2.223         | 83.6            |
| Fe i    | 5409.133      | 4.371         | 63.1            | Fe i    | 6220.776      | 3.882         | 23.1            |
| Fe i    | 5412.798      | 4.434         | 25.2            | Fe i    | 6221.670      | 0.859         | 2.5             |
| Fe ii   | 5414.073      | 3.221         | 34.7            | Fe i    | 6226.730      | 3.883         | 33.9            |
| Fe i    | 5415.192      | 4.386         | 199.0           | Fe i    | 6229.225      | 2.845         | 39.9            |
| Fe i    | 5417.039      | 4.415         | 39.9            | Fe i    | 6232.639      | 3.654         | 98.4            |
| Fe ii   | 5425.257      | 3.199         | 50.1            | Fe ii   | 6239.953      | 3.889         | 16.2            |
| Fe i    | 5436.297      | 4.386         | 45.2            | Fe i    | 6240.645      | 2.223         | 52.8            |
| Fe i    | 5436.587      | 2.279         | 50.1            | Fe i    | 6246.317      | 3.602         | 130.0           |
| Fe i    | 5441.354      | 4.312         | 37.0            | Fe ii   | 6247.557      | 3.892         | 61.9            |
| Fe i    | 5443.409      | 4.103         | 5.2             | Fe i    | 6252.554      | 2.404         | 128.0           |
| Fe i    | 5445.042      | 4.386         | 130.0           | Fe ii   | 6269.967      | 3.245         | 7.6             |
| Fe i    | 5650.020      | 5.099         | 41.8            | Fe i    | 6297.792      | 2.223         | 79.3            |
| Fe i    | 5659.764      | 5.085         | 47.8            | Fe i    | 6302.494      | 3.868         | 99.7            |

Table 1. Data of adopted 86 spectral lines.
Fig. 1. (a) Results of radial–tangential macroturbulence ($\zeta_{\text{RT}}$; open circles) and Gaussian macroturbulence ($\eta$; filled circles) plotted against $\langle \log \tau \rangle$, which were determined from our analysis of Kurucz et al.'s (1984) solar flux spectrum by two different macrobroadening functions ($M_1$ and $M_2$). (b) Radial-tangential macroturbulence plus rotational broadening function $M_1(v; \zeta_{\text{RT}}, v_e \sin i)$ (for seven $v_e \sin i/\zeta_{\text{RT}}$ values from 0.0 to 3.0 with an increment of 0.5) plotted against $v/\zeta_{\text{RT}}$. (c) Gaussian macroturbulence plus rotational broadening function $M_2(v; \eta, v_e \sin i)$ (for seven $v_e \sin i/\eta$ values) plotted against $v/\eta$. 
Fig. 2. Schematic description of the concept of radial–tangential macroturbulence for the specific case of disk-center observation (the line-of-sight of the observer is perpendicular to the solar surface), which explains why the width of the broadening function is considerably smaller than the actual velocity dispersion.
Fig. 3. (a) Local distribution function of radial-tangential macroturbulence $[\Theta_1(v)]$ expressed by equation (3) corresponding to the parameters of $\zeta_R = \zeta_T = 2 \text{ km s}^{-1}$ and $A_R = A_T = 0.5$, computed for eight $\sin\theta$ values of 0.01, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, and 0.7. (Note that only the results for $0^\circ < \theta < 45^\circ$ are given here, since those for $45^\circ < \theta < 90^\circ$ can be derived by making use of the symmetric property in this case.) The curves are normalized at the line-center ($v = 0$), where the values of $\Theta_1(0)$ are 14.2, 1.55, 0.849, 0.618, 0.507, 0.445, 0.411, and 0.399, respectively. (b) Line-depth profiles $[R(v); \text{normalized at line-center}]$ simulated by convolving $\Theta_1(v)$ with a Gaussian intrinsic profile having an e-folding half-width of $v_{th} = 1.5 \text{ km s}^{-1}$. Panels (a') and (b') are almost the same as (a) and (b) but for $\zeta_R = \zeta_T = 4 \text{ km s}^{-1}$. 
Fig. 4. Run of the theoretically expected values of $V_{\text{los}}$ with $\theta$, which were computed for different combinations of radial and tangential components of the assumed turbulence models (along with the Gaussian intrinsic thermal profile with an $e$-folding half-width of 1.5 km s$^{-1}$). Solid lines correspond to the case of radial–tangential macroturbulence, while dashed lines are the cases of anisotropic Gaussian macroturbulence. For example, in the panel indicated as (R=2, T=3), the solid line shows the $\theta$-dependence of $V_{\text{los}}$ computed for ($\zeta_R = 2$ km s$^{-1}$, $\zeta_T = 3$ km s$^{-1}$), while the dashed line is for the result corresponding to ($\eta_R = 2$ km s$^{-1}$, $\eta_T = 3$ km s$^{-1}$).
Fig. 5. Graphical description of the observed points on the solar disk, at which the spectral data obtained in this study were taken. The upper figure (a) corresponds to the Hida/DST observation on 2015 November 3–5 (32 points on the northern meridian from the disk center to 0.97\(R_\odot\) with a step of 30\(''\) \(\approx\) 0.03\(R_\odot\), while spatially averaged over 51\(''\) along the E–W direction). The lower figure (b) is for the Hinode/SOT observation on 2008 December 17 (three squares of 20\(''\) \(\times\) 20\(''\) on the southern meridian at \(\theta = 0^\circ\), 45\(^\circ\), and 80\(^\circ\)). While N, S, E, and W are the directions in reference to the Sun (based on solar rotation), those in the equatorial coordinate system on the celestial sphere (defined by the rotation of Earth) are also denoted as \(N'\), \(S'\), \(E'\), and \(W'\).
Fig. 6. Spectrum-fitting examples of Hida/DST data analysis. Shown here are the selected representative cases of weak (Fe I 5308.707; left panels), medium-strength (Fe I 5321.109; center panels), and rather strong (Fe I 6302.494; right panels) lines. The panels in upper, middle, and lower row correspond to disk center (θ = 0°), intermediate point (θ = 45°), and near-limb (θ = 78°), respectively. The observed spectra are shown in red circles while the best-fit theoretical spectra are depicted in blue lines.
Fig. 7. Panels (a), (b), and (c) show the plots of $V_{\text{los}}$ (line-of-sight velocity dispersion), $\langle \log \tau \rangle$ (mean depth of line formation), and $w_{\lambda}$ (equivalent width) against $\theta$ (direction angle), respectively, which were derived from the analysis of Hida/DST data for three representative lines (the same lines as in figure 6).
Fig. 8. The upper panels show the 200 raw unnormalized Hida/DST spectra at each pixel (before being spatially averaged) for the Fe I 5308.707 (left), Fe I 5321.109 (center), and Fe I 6302.494 (right) lines. The histograms of \( V_{\text{los}} \) and \( v_r - \langle v_r \rangle \) resulting from the analysis on these raw spectra are presented in the middle and lower panels, respectively (the mean value as well as the standard deviation of the distribution are also given). The downward arrows indicate the standard solution of \( V_{\text{los}} \) derived from the spatially averaged spectra.
Fig. 9. Panels (a)–(d) show the $V_{\text{los}}$ vs. $\theta$ diagram corresponding to each of the line-strength classes (cf. table 1), which were derived from the analysis of Hida/DST data: (a) unsaturated class-1 lines (filled circles), (b) moderately saturated class-2 lines (open circles), (c) strongly saturated class-3 lines (crosses), and (d) all lines of class 1–3. The curve of growth constructed by using the data of Fe I lines presented in table 1 (on the assumption of $\xi = 0.5$ km s$^{-1}$ and $\theta_{\text{exc}} = 5040/T_{\text{exc}} = 1$) is displayed in panel (e), which demonstrates the saturation degree of each line-strength class.
Fig. 10. Examples of Fe i 6302.494 line-profile fitting based on the Hinode/SOT data. Shown here are the selected 5 representative cases of different \((v_t, V_{\text{los}})\) solutions indicated in the figure. The observed spectra are shown in red circles while the best-fit theoretical spectra are depicted in blue lines. Panels (a), (b), and (c) correspond to \(\theta = 0^\circ\), \(\theta = 45^\circ\), and \(\theta = 80^\circ\), respectively.
Fig. 11. Histograms and correlations of velocity parameters derived from the analysis of Hinode/SOT spectra. From left to right: Histogram of $V_{\text{los}}$, histogram of $v_r$, $V_{\text{los}}$ vs. $v_r$ correlation, and $I_{\text{cont}}$ (continuum intensity) vs. $v_r$ correlation. The mean value and the standard deviation are also indicated in each histogram panel. The upper, middle, and lower panels correspond to $\theta = 0^\circ$, $\theta = 45^\circ$, and $\theta = 80^\circ$, respectively.
Fig. 12. Effect of changing the microturbulence ($\xi$) from the fiducial value of 0.5 km s$^{-1}$ adopted in this study to 1.0 km s$^{-1}$ on the results of $V_{\text{los}}$ (left panels), $\zeta_{\text{RT}}$ (center panels), and $\eta$ (right panels) derived in section 1 ($\zeta_{\text{RT}}$, $\eta$) and subsection 4.3 ($V_{\text{los}}$).

Upper panels: Comparison of $\xi = 1.0$ km s$^{-1}$ and $\xi = 0.5$ km s$^{-1}$ results. Middle panels: Difference between $\xi = 1.0$ km s$^{-1}$ and $\xi = 0.5$ km s$^{-1}$ results plotted against the equivalent width ($w_{\lambda}$ in panel (d) is that derived from our intensity spectrum, while Meylan et al.’s (1993) values given in table 1 are adopted for $W_{\lambda}$ in panels (e) and (f)). Lower panels: Difference between $\xi = 1.0$ km s$^{-1}$ and $\xi = 0.5$ km s$^{-1}$ results plotted against the mean formation depth.
Fig. 13. The left two panels show the derivation of $V^{\text{rad}}$ and $V^{\text{tan}}$ (radial and tangential components of anisotropic Gaussian macroturbulence) based on the $V_{\text{los}}$ data derived from our analysis of Hida/DST spectra. (a): Results of $V_{\text{los}}$ (line-of-sight velocity dispersion) at $1 \geq \cos \theta > 0.95$ ($0^\circ \leq \theta < 17^\circ$), which we regard as $V^{\text{rad}}$, plotted against $\langle \log \tau \rangle$, where the thick solid line (drawn by eye-judgement) represents the approximate mean trend. (b): The $V_{\text{los}}$ values at $0.3 > \cos \theta$ ($73^\circ < \theta$), which we regard as $V^{\text{tan}}$, are plotted against $\langle \log \tau \rangle$. The light green symbols show the $V^{\text{tan}}$ results specially estimated from $V_{\text{los}}$ at $0.95 > \cos \theta > 0.3$ ($17^\circ < \theta < 73^\circ$) by using equation (A1) and the already derived mean $V^{\text{rad}}(\tau)$ relation. The eye-judged trend of mean $V^{\text{tan}}(\tau)$ is also shown (thick dashed line). In these two panels (a) and (b), the data derived from the line-strength class 1, 2, and 3 (defined in table 1) are expressed in filled circles, open circles, and crosses, respectively. The right panels (c) and (d) summarize the various literature results regarding the depth-dependence of $V^{\text{rad}}$ and $V^{\text{tan}}$, where the mean relations derived by ourselves are also overplotted by thick solid and thick dashed lines, respectively. Note that most of the literature data were read from Canfield and Becker’s (1976) figure 1 except for the data of Ayres (1977; open circles in panel (c)) and Kostik (1982). A77 ··· Ayres (1977); C71 ··· Canfield (1971); G75a ··· Gurtovenko (1975a); G75b ··· Gurtovenko (1975b); G75c ··· Gurtovenko (1975c); H67 ··· Holweger (1967); KG74 ··· Kondrashova and Gurtovenko (1974); K82 ··· Kostik (1982).
Fig. 14. Gaussian-approximated rotation plus macroturbulence broadening parameter ($v_{r+m}$; derived from the spectrum-fitting analysis of 6080–6089 Å region) plotted against $T_{\text{eff}}$. Filled circles · · · FGK-type stars (Takeda & Honda 2005); open triangles · · · solar-analog stars (Takeda et al. 2007); crosses · · · Hyades stars of $T_{\text{eff}} \leq 6310$ K (Takeda et al. 2013). Analytical relations of macroturbulence (reduced to the scale of Gaussian $v_{\text{mt}}$) as function of $T_{\text{eff}}$ proposed by previous studies (Gray 1984; Valenti & Fischer 2005; Bruntt et al. 2010; Doyle et al. 2014) are shown by dashed lines, where the relation $v_{\text{rot}} = 0.42 \zeta_{\text{RT}}$ was applied to convert the $\zeta_{\text{RT}}$ results (of Gray, Valenti & Fischer, and Doyle et al.) into $v_{\text{rot}}$ while Bruntt et al.’s Gaussian values were used unchanged. The analytical relation for $v_{\text{mt}}$ given by equation (A3), which fits the envelope of this $v_{r+m}$ distribution, is also depicted (solid line).