Asymptotic Normalization Coefficient and Important Astrophysical Process \( ^{15}\text{N}(p, \gamma)^{16}\text{O} \)

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Abstract. In this work we report the application of the ANC method for the determination of the non-resonant radiative capture amplitude for the important astrophysical CNO cycle reaction \( ^{15}\text{N}(p, \gamma)^{16}\text{O} \), which provides a leak from the CN cycle into the CNO bi-cycle and CNO tri-cycle. It is contributed by the resonance capture to the ground state through two strong \( 1^- \) resonances and non-resonant capture to the ground state, which interferes with the resonant capture terms. To determine more accurately the contribution from the non-resonant capture we determined the proton ANCs for the ground and seven excited states of \( ^{16}\text{O} \) by measuring the angular distributions of the peripheral \( ^{15}\text{N}(^{3}\text{He},d)^{16}\text{O} \) proton transfer reaction. Using these ANCs and proton and \( \alpha \) resonance widths determined from an \( R \)-matrix fit to the data from the \( ^{15}\text{N}(p, \alpha)^{12}\text{C} \) reaction, we calculated the astrophysical \( S \) factor for the \( ^{15}\text{N}(p, \gamma)^{16}\text{O} \) reaction. The results indicate that the direct capture contribution was previously overestimated. We find the astrophysical factor to be \( S(0) = 36.0 \pm 6.0 \) keVb, which is about a factor of two lower than the presently accepted value. We conclude that for every \( 2200 \pm 300 \) cycles of the main CN cycle one CN catalyst is lost due to this reaction.

1. ANC in peripheral radiative capture and transfer reactions

It is well known that capture of a charged particle at stellar energies occurs at distances that are large compared to the nuclear radius. Direct radiative capture reaction rates depend on the normalization of the overlap function which is fixed by the appropriate ANCs. The connection between ANCs and direct proton capture rates at low energies is straightforward to obtain. The cross section for the direct capture reaction \( A(p, \gamma)B \) can be written as

\[
\sigma = \lambda | < I_{Ap}^B(r) | \hat{O}(r) | \psi_i^{(+)}(r) > |^2 ,
\]

where \( \lambda \) contains kinematical factors, \( I_{Ap}^B(r) \) is the overlap function for \( B \rightarrow A + p, \hat{O} \) is the electromagnetic transition operator and \( \psi_i^{(+)} \) is the scattering wave in the incident channel. For peripheral reactions the overlap function may be replaced by

\[
I_{Ap}^B \approx C_{Ap}^B W_{-n,l_n+1/2}(2kr) ,
\]

where
where $C_{Ap}^B$, the ANC, defines the amplitude of the tail of the radial overlap function, $W_{\eta,i+1/2}^\pm(2\kappa \tau)$ is the Whittaker function, $\eta$ the Coulomb parameter, $l_B$ orbital angular momentum for the bound state $B = (Ap)$ and $\kappa$ is the bound state wave number. Thus peripheral direct capture cross sections are directly proportional to the squares of the ANCs. In a similar way, ANCs can be used to determine reaction rates for subthreshold states [1, 2]. In addition, ANCs can be related to the external or channel part of the radiative width for resonant capture [3]. The internal part of the the radiative width, however, depends on the wave functions in the nuclear interior. If resonance parameters are known either from measurements or calculations and ANCs are known, the resonant and non-resonant components can be used together in an $R$-matrix calculation to obtain capture cross sections.

Peripheral transfer reactions provide an excellent way to determine ANCs. Consider the proton transfer reaction $A(a,b)B$ where $a = (bp)$ and $B = (Ap)$. As was previously shown [4] we can write the DWBA cross section in the form

$$\frac{d\sigma}{d\Omega} = \sum_{jBj_a} \left( \frac{C_{ApB}^B}{b_{ApB}^2} \right)^2 \sigma_{DW}^{l_Bj_Bj_a}$$

where $\sigma_{DW}^{l_Bj_Bj_a}$ is the reduced DWBA cross section and $j_i$, $l_i$ are the total and orbital angular momenta of the transferred proton in nucleus $i$. The factors $b_{bpl_a} a_{bpl_a}$ and $b_{ApB_B} a_{ApB_B}$ are the single-particle ANCs of the bound state proton wave functions in nuclei a and B which are related to the corresponding ANC of the overlap function by

$$(C_{bpl_a}^a)^2 = S_{bpl_a}^a b_{bpl_a}^2,$$  \hspace{1cm} (4)

where $S_{bpl_a}^a$ is the spectroscopic factor. If the reaction is peripheral, the ratio

$$R_{l_Bj_Bj_a} = \frac{\sigma_{DW}^{l_Bj_Bj_a}}{b_{ApB_B}^2 b_{bpl_a}^2}$$ \hspace{1cm} (5)

is independent of the single particle ANCs $b_{bpl_a} a_{bpl_a}$ and $b_{ApB_B} a_{ApB_B}$. Thus for surface reactions where Eq. (5) holds, the DWBA cross section is best parametrized in terms of the product of the square of the ANCs of the initial and final nuclei $(C_{ApB_B}^B)^2 (C_{bpl_a}^a)^2$. If the ANC $C_{bpl_a}^a$ is known from the independent measurements the second ANC $C_{ApB_B}^B$ can be determined from the peripheral transfer reaction $A(a,b)B$.

In this paper we apply the ANC method to determine the astrophysical factor for the important astrophysical $^{15}\mathrm{N}(p,\gamma)^{16}\mathrm{O}$ reaction, which provides a path from the CN cycle to the CNO bi-cycle and CNO tri-cycle. The measured astrophysical factor for this reaction is dominated by resonant capture through two strong $J^\pi = 1^-$ resonances at $E_R = 312$ and 962 keV and direct capture to the ground state [5, 6]. This reaction can be analyzed using the $R$-matrix method in which the reaction amplitude is written as the sum of the resonant and non-resonant ones. The contribution from the nuclear interior in the $R$-matrix approach is entirely included into the resonant amplitudes, so that the non-resonant amplitude takes into account only the external contribution. That is why the normalization of the non-resonant amplitude in the $R$-matrix approach for the reaction under consideration is determined by the ANC for $^{15}\mathrm{N} + p \rightarrow ^{16}\mathrm{O}$. In [6] the normalization of the non-resonant amplitude was varied to fit the experimental data. In order to better understand the non-resonant capture part of the low-energy astrophysical factor for $^{15}\mathrm{N}(p,\gamma)^{16}\mathrm{O}$, we have measured the ANCs for proton removal from the ground state and 7 excited states of $^{16}\mathrm{O}$ using the $^{15}\mathrm{N}(\alpha, d)^{16}\mathrm{O}$ proton transfer reaction at incident energies of around 10 MeV/A. Here we report a measurement of the ANCs using this reaction. The ANCs are used to determine, within the $R$-matrix approach, the direct capture astrophysical $S$ factors to the corresponding eight bound states and calculate the total astrophysical factor.
2. ANC for $^{15}$N + $p \rightarrow ^{16}$O from the $^{15}$N($^3$He, d)$^{16}$O reaction

The experiment was performed using a momentum analyzed 25.74 MeV $^3$He beam from the U-120M cyclotron at the Nuclear Physics Institute of Prague-Rez, CAS, incident on a nitrogen gas target. The target gas chamber contained nitrogen gas enriched to 99.99% $^{15}$N. Angular distributions were obtained for the $^3$He elastic scattering and fourteen deuteron groups from the $^{15}$N($^3$He, d)$^{16}$O reaction populating states below and above the particle emission threshold of $^{16}$O, but only eight deuteron groups corresponding to proton transfer to the bound states in $^{16}$O were further analyzed. From the optical model analysis of the measured elastic scattering in the entrance channel the optical potential describing the $^3$He + $^{15}$N scattering was determined. Optical potentials for the exit channel were taken from Fulbright [7] or calculated from global formulas derived by Daehnick et al. [8].

The analysis of the transfer reaction has been done within the DWBA. By normalization of the calculated DWBA cross section to the experimental one the proton ANCs for the eight bound states of $^{16}$O were determined using the ANC for $^3$He $\rightarrow$ d + p from [9]. The determined square of the ANC for the ground state is 192.1 $\pm$ 26.0 fm$^{-1}$. The transitions to the excited bound states are not important for astrophysical reactions and here we don’t present the measured ANCs for the excited states.

3. Astrophysical factor for the $^{15}$N(p, γ)$^{16}$O radiative capture process

We have calculated the S factor for this reaction using the two-channel, two-level $R$-matrix method. The contribution from the $\alpha$ - $^{12}$C channel is also taken into account. In the case under consideration, only two levels with $J^\pi = 1^-$ produce strong energy dependence corresponding to two resonances at 312 and 962 keV. If the channel radius is large enough, then the levels calculated for the Woods-Saxon potential will be close to the single-particle shell model ones. The closest state to the subthreshold 1$^-$ level is located at $E_x = 9.585$ MeV, but its ANC (or reduced width) is so small that we were not able to observe it. Hence we neglect this level. The other level at $E_x = 7.1169$ MeV is 5.06 MeV away from the threshold and also does not affect the behavior of the S factor at low energies. The levels located at higher energies can be included into the background, which has a very smooth energy dependence. The total reaction amplitude in the $R$-matrix approach for the capture to the ground state of $^{16}$O (in the case of the interfering resonant and non-resonant terms) is

$$M(E) = i \Omega_p(E) \sum_{\nu, \lambda} A_{\nu, \lambda}(E) [\Gamma_{\nu, \gamma}(E)]^{1/2} \tilde{\Gamma}_{\lambda p}^{1/2}(E) + M^{NR}(E).$$

Here, $\Omega_p(E)$ is the phase factor in the initial channel of the reaction, $A(E)$ is the level matrix, $[\Gamma_{\nu, \gamma}(E)]^{1/2}$ is the amplitude of the radiative width of the level $\nu$ decaying to the bound state, $\tilde{\Gamma}_{\lambda p}(E) = 2 P_l(E) \gamma_{\lambda p}$ is the formal proton width of the level $\lambda$, $\gamma_{\lambda p}$ its reduced width and $P_l(E)$ is the barrier penetrability for protons; $E$ is the relative energy in the channel $p - ^{15}$N and $M^{NR}$ is the amplitude of the non-resonant radiative capture occuring in the external region. In the two-level approximation $\lambda, \nu = 1, 2$, the resonant amplitude in Eq. (6) contains four terms rather than two terms used in Ref. [6]. It constitutes one of the differences between our fit and fit in [6]. We note that in the $R$-matrix method the non-resonant capture matrix element in the internal region does not explicitly appear. This non-resonant part is given by the radial integral taken from the channel radius $r_0$ to $\infty$ and its absolute normalization is entirely determined by the ANC of the bound state. Note that the ANC and the reduced width used in the $R$-matrix approach are related by Eq. (60) of Ref. [1].

The resonant parameters in Eq. (6) and the channel radius are fitting parameters to reproduce the experimental data. We choose the channel radius in the proton channel to be $r_0 = 5.0$ fm. The proton and alpha reduced widths can be expressed in terms of the observable widths. These
widths are available in compilations [10]. But they are determined from different reactions with different uncertainties. First we have determined the proton and alpha reduced widths by fitting the measured astrophysical factor for the $^{15}\text{N}(p,\alpha)^{12}\text{C}$ reaction in the $R$-matrix approach. In this fit the channel radii for the proton and alpha channels were taken to be 5.0 and 7.0, correspondingly. The search region for proton and alpha widths was originally taken from [10] and then extended for the alpha widths for the second resonance. Since the cross section for this reaction is significantly larger than for the $^{15}\text{N}(p,\gamma)^{16}\text{O}$ reaction, it has been measured with significantly higher accuracy. This allows us to determine the proton and alpha partial widths with higher accuracy. We used two different boundary conditions. First, we fixed the second level energy at $E_2 = E_{R2} = 962$ keV, where $E_{R2}$ is the energy of the second resonance, and determined from the fit to the experimental data that the first level energy is $E_1 = 152$ keV. It means that in the R matrix approach the two levels are separated by $\approx 800$ keV. The $\chi^2$ fit per degree of freedom is $\chi^2/N = 1.27$. If we adopt the first level energy to be $E_1 = E_{R1} = 312$ keV, then the best fit is achieved for $E_2 = 1070$ keV with $\chi^2/N = 1.51$. The results of the fit are shown in Fig 1. After determining the particle reduced widths, we made the $R$-matrix fit of the $^{15}\text{N}(p,\gamma)^{16}\text{O}$ data. To achieve a better fit for this reaction, we slightly readjusted the reduced widths. Both boundary conditions gave slightly different reduced widths and similar fits to the data. The main fitting parameters in this case are the complex radiative width amplitudes, $[\Gamma_{1\gamma_1}]^{1/2}$ and $[\Gamma_{2\gamma}]^{1/2}$. The radiative width amplitude consists of the internal and external (channel) parts. The internal radiative width amplitude is real while the external one is complex since the resonant scattering wave function in the external region is described by the outgoing wave. We find that the imaginary parts of the radiative width amplitudes are important for the fit of the experimental data. The best fit is achieved if we assume that the sign of the real parts of both radiative width amplitudes is the same and opposite to the sign of their imaginary parts. At fixed particle reduced widths the radiative width amplitudes determine the absolute value of the astrophysical $S$ factor at the resonant peaks. According to [6] the $S$ factors at the resonance peaks are 400 keVb. The absolute cross section was determined in [6] by normalization of the $^{15}\text{N}(p,\alpha_1\gamma_1)^{12}\text{C}$ excitation function to the known cross section of

Figure 1. (Color online) The astrophysical $S(E)$ factor for the $^{15}\text{N}(p,\alpha)^{12}\text{C}$ reaction. The black squares and blue triangles are experimental data from [6] and [5], correspondingly. The red solid line is our $R$ matrix fit with the boundary condition $E_2 = 962$ keV, which takes into account two interfering $1^-$ resonances at $E_{R1} = 312$ and $E_{R2} = 962$ keV and non-resonant capture. The magenta dashed line is the total non-resonant capture $S$ factor given by the sum of the nonresonant capture $S$ factors to all eight bound states observed in the present ($^3\text{He},d$) experiment, and the violet dotted line is the non-resonant capture $S$ factor to the ground state.
\(\sigma = 250 \pm 35 \text{ mb at } E_R = 1134.40 \text{ keV.}\) Then from the observed relative intensity of \(\gamma\)-rays from the \(^{14}\text{N}(p, \alpha\gamma_1)^{12}\text{C}\) and \(^{15}\text{N}(p, \gamma_0)^{16}\text{O}\) reactions, the capture cross section for the latter reaction was determined. To determine accurately the uncertainty of the \(S\) factors at the resonance peaks one needs to know information, such as the errors of the measured \(\gamma\)-ray intensities and the uncertainty in the detector efficiency, which are not given in Ref. [6]. Thus we believe that a 14\% uncertainty for the \(S\) factors at the resonance peaks is too optimistic. We also assigned a 16\% uncertainty to the \(S\) factors at the resonances from the \(\approx 16\%\) uncertainties for the radiative widths for the first and second resonances given in Ref. [6]. We assigned a 16\% uncertainty to the radiative widths of both resonances in our fit. The \(\chi^2\) fit for the radiative capture is worse than for the \((p, \alpha)\) reaction because we are not able to reproduce the low-energy data from [6]. For the boundary condition \(E_2 = E_{R_2} = 962 \text{ keV}\) we get \(\chi^2/N = 2.8\) and for \(E_1 = E_{R_1} = 312 \text{ keV}\) we get \(\chi^2/N = 3.0\). We find that one of the alpha reduced widths (we cannot determine which one) has an opposite sign relative to three other particle reduced widths. In our fit we assumed that \(\gamma_{1\alpha} < 0\). The level energies used in [5] are not given, which is why we cannot compare directly our parameters and the set from [5]. The observable widths of each resonance can be expressed in terms of the reduced widths when the corresponding level energy coincides with the resonance energy. From the \(R\) matrix fit with the first level energy \(E_1 = 312 \text{ keV}\) we get the observable partial widths for the first resonance, \(\Gamma_{1p} = 1.1 \text{ keV}\) and \(\Gamma_{1a} = 90.5 \text{ keV}\). From the \(R\) matrix fit with the second level energy \(E_2 = 962 \text{ keV}\), we get the observable partial widths of the second resonance, \(\Gamma_{2p} = 99.7 \text{ keV}\) and \(\Gamma_{2a} = 50.4 \text{ keV}\). The low energy tail of our calculated \(S\) factor fitting the data from [5] goes along the lower limit of the data from [6]. Our astrophysical factor \(S(0) = 36.0 \pm 6.0 \text{ keV}\) obtained for the proton channel radius \(r_0 = 5.0 \text{ fm}\) is lower then \(S(0) = 64.0 \pm 6.0 \text{ keV}\) [6] but agrees with \(S(0) = 29.8 \pm 5.4 \text{ keV}\) [5] within uncertainties. We note that the \(R\)-matrix calculations show very little sensitivity to the variation of the channel radius \(r_0\). Decreasing (increasing) \(r_0\) increases (decreases) the integration region of the radial matrix element for the non-resonant capture, i.e. increases (decreases) the direct capture amplitude and decreases (increases) the resonant part, so that the total sum remains nearly constant. A variation of the channel radius by 33\% changes the \(S(0)\) factor by 5\%. The 17\% uncertainty of our \(S(0)\) astrophysical factor comes from the (assumed) 16\% uncertainty of the experimental data, the 10\% uncertainty of the ANCs, which results in about a 2\% uncertainty in the \(S(0)\) factor, and a 5\% uncertainty due to the dependence of the \(S\) factor on the channel radius.

The most important difference between our fit and the result in [6] is in the contribution of the non-resonant capture to the ground state. The absolute normalization of the non-resonant capture terms is entirely determined by the corresponding ANC in the \(R\)-matrix approach. Since we have measured the ANCs, we can quite accurately determine the contribution from the non-resonant capture terms. At zero energy it contributes about 3.0\% to the total astrophysical factor. But the non-resonant capture to the ground state, which contributes about 69\% to the total non-resonant \(S\) factor, is much more important when calculating the total astrophysical factor due to its interference with the resonant capture terms. We find that our calculated non-resonant astrophysical factor for the capture to the ground state is \(S(0) = 0.86 \text{ keVb, i.e. about 9 times smaller than the } S(0) \text{ obtained in [6].} It is the main reason why our low-energy tail of the \(S\) factor goes lower than the data in [6]. Since normalization of the non-resonant capture amplitude is determined by the ANC, which has been determined with 10\% uncertainty, we do not agree with the result obtained in [6] within the hard sphere approach. Unfortunately, there is no explanation of the method used to calculate the non-resonant capture terms in [6]. We note that we also tried to fit the \(^{15}\text{N}(p, \gamma)^{16}\text{O}\) data using the expression given in [6] and failed to reproduce the low-energy tail of the \(S\) factor for the same reason. Recent low-energy direct measurements by LUNA [12] are in a very good agreement with our \(S\) factor prediction confirming the power of the indirect ANC method.
4. Summary and conclusion
To determine more accurately the contribution from the non-resonant capture to the astrophysical factor for the $^{15}\text{N}(p,\gamma)^{16}\text{O}$ CNO cycle process we determined the proton ANCs for the ground and seven excited states of $^{16}\text{O}$ by measuring the angular distributions of the peripheral $^{15}\text{N}(^{3}\text{He},d)^{16}\text{O}$ proton transfer reaction. To fit the available experimental astrophysical factors from [6, 5] we have determined the resonant proton and $\alpha$ partial widths by fitting the available experimental data for the stronger and better measured reaction $^{15}\text{N}(p,\alpha)^{12}\text{C}$. Our astrophysical factor in the energy interval 150 – 300 keV agrees well with the data from [5] but goes slightly lower than the low limit of data reported in [6]. This new astrophysical factor, $S(0) = 36.0 \pm 6.0$ keVb, obtained for $^{15}\text{N}(p,\gamma_0)^{16}\text{O}$ allows us to reevaluate the rate of leak from the CN cycle due to this reaction. In Ref. [6] it has been estimated as the ratio of the $S(0)$ factors for $^{15}\text{N}(p,\alpha)^{12}\text{C}$ and $^{15}\text{N}(p,\gamma_0)^{16}\text{O}$. The $S$ factor $S(0) = 57$ MeVb was used for $^{15}\text{N}(p,\alpha)^{12}\text{C}$. However, the later measurements [11] gave the higher value of the astrophysical factor $S(0) = 65.0 \pm 4.0$ MeVb. Our new fit for the $^{15}\text{N}(p,\alpha)^{12}\text{C}$ data gives $S(0) = 73.0 \pm 5.0$ MeVb [13].

This result overlaps, within the experimental uncertainties, with the $S$ factor recently measured via the indirect Trojan Horse method [13]. Using the data from [11] we can reevaluate the loss of catalyst in the CN cycle at 25 keV proton energy, which corresponds to the relative $p - ^{15}\text{N}$ energy $E = 23.44$ keV. We find, using our new astrophysical factor $S(E = 23.44$keV) = 38.8 \pm 6.6$ keVb for the $^{15}\text{N}(p,\gamma_0)^{16}\text{O}$ reaction and 84.1 \pm 5.9 MeVb for the $^{15}\text{N}(p,\alpha)^{12}\text{C}$ reaction, that for every 2200 \pm 300 cycles of the main CN cycle one CN catalyst is lost due to the $^{15}\text{N}(p,\gamma_0)^{16}\text{O}$ reaction, rather than 1200 \pm 100 cycles determined from data of [6] ($S(E = 23.44$keV) = 70.0 \pm 11.0$ keVb). Our result agrees with the leak rate 2600 \pm 400 cycles obtained with $S(E = 23.44$keV) = 32.0 \pm 5.8$ keVb for the $^{15}\text{N}(p,\gamma)^{16}\text{O}$ reaction in [5].

Our calculated reaction rates for $^{15}\text{N}(p,\gamma)^{16}\text{O}$ for temperatures $T_0 < 0.15$ are lower by approximately a factor of 2 than the adopted reaction rates given in the NACRE compilation [14], which were calculated using data from [6]. Extension of the LUNA measurements to the region covering the first resonance peak will allow us to update our $S$ factor fit.

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