Relativistic Invariance of the Phase of a Spherical Wave, relativistic Doppler Formula and Poincaré’s expansion of space.

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Abstract

Recently Einstein’s invariance of the phase of a plane wave (1905) has been described as “questionable” [Y-S Huang 2007]. Another definition of this phase, taking into account a “relativistically induced optical anisotropy” for isotropic medium in moving, has been proposed [A. Gjurchinovski 2007]. We suggest (logically) to determine this “relativistically induced effect” if the isotropic medium is the vacuum. We prove that the basic Lorentz invariant, in vacuum, is not the phase of a plane wave (ωt – kr) but the phase of a spherical wave (ωt – kr). According to Poincaré, an isotropic spherical wave is not LTed (Lorentz transformed) into an isotropic spherical wave (Einstein 1905) but LTed into an anisotropic ellipsoid wave (c = 1, relativity of simultaneity). Poincaré’s ellipsoidal wavefront (1906) is an equiphase surface. Our approach consists in deleting (k; r → kr) and not adding (ω → k; u) a scalar product [A. Gjurchinovski 2007]. The Lorenz gauge is connected with the invariance of the phase of a spherical wave and the transverse gauge with Einstein’s invariant. From Poincaré’s invariant we deduce a Doppler formula that is not the same as Einstein’s one. We call this formula “Poincaré’s formula” because it is unseparable of Poincaré’s theory of expansion of space and therefore the measurements of Hubble.

1 Invariance of the phase of a plane wave, transverse gauge and Einstein’s spherical wavefronts

Einstein admits that the Galilean invariant, the phase of a monochromatic plane wave, \( \phi = \omega t - \mathbf{k} \cdot \mathbf{r} \), is by definition a Lorentz invariant, \( \omega t' - \mathbf{k}' \cdot \mathbf{r}' \), in frames in uniform translation K and K’ [A. Einstein (1905) paragraphe 7] (\( k = \frac{2\pi}{\lambda} = 2\pi \nu = \omega, k' = \frac{2\pi}{\lambda'} = 2\pi \nu' = \omega' \)):

\[
\phi = \omega t - \mathbf{k} \cdot \mathbf{r} = 2\pi \nu(t - x \cos \theta - y \sin \theta) = \phi' = \omega t' - \mathbf{k}' \cdot \mathbf{r}' = 2\pi \nu'(t' - x' \cos \theta' - y' \sin \theta')
\]  

(1)

The invariance (1) implies that the unprimed plane wavefront, defined by the scalar product \( \mathbf{k} \cdot \mathbf{r} = c \), is not only orthogonal (\( \varphi = 90^\circ \)) to the direction of propagation \( \mathbf{k} \) in K but also that the primed plane wavefront, defined by the scalar product \( \mathbf{k}' \cdot \mathbf{r}' = c' \), is orthogonal (\( \varphi' = 90^\circ \)) to the direction of propagation \( \mathbf{k}' \) in K’. We have therefore \( \mathbf{A} = \tilde{\mathbf{A}} \sin \phi \) and \( \mathbf{A}' = \tilde{\mathbf{A}}' \sin \phi' \) where \( \tilde{\mathbf{A}} \) and \( \tilde{\mathbf{A}}' \) is respectively the amplitude in K and K’(at two space dimensions, like Einstein):

\[
\tilde{\mathbf{A}} \perp \mathbf{k} \rightarrow \tilde{\mathbf{A}}' \perp \mathbf{k}' \quad \text{or} \quad \tilde{\mathbf{A}} \mathbf{k} = \tilde{\mathbf{A}}' \mathbf{k}' = 0 = Ak \cos \varphi = A'k' \cos \varphi' \quad \Leftrightarrow \quad \varphi = \varphi' = 90^\circ
\]  

(2)

That is true if we have “\( \tilde{\mathbf{A}} = \tilde{\mathbf{E}} \)” (Electric field), “\( \tilde{\mathbf{A}} = \tilde{\mathbf{H}} \)” (Magnetic field) but also if “\( \tilde{\mathbf{A}} = \tilde{\mathbf{A}} \)” where \( \tilde{\mathbf{A}} \) is the potential vector. With the invariance of (1) Einstein therefore implicitly adopted the transverse electromagnetic gauge for the potential vector:

\[
\text{div} \tilde{\mathbf{A}} = \text{div} \tilde{\mathbf{A}}' = 0
\]  

(3)

We showed that the Lorentz transformation (LT with \( \gamma = (1 - \beta^2)^{-\frac{1}{2}} \) and Poincaré’s units of space-time ”c = 1”)

\[
x' = \gamma(x - \beta t) \\
y' = y \\
z' = z \\
t' = \gamma(t - \beta x)
\]  

(4)

\(^1\)Einstein defines explicitly the normality in K and in K’ : ”If we call the angle \( \vartheta \) in K’ the angle between the wave-normal (the direction of the ray) and the direction of moving” [A. Einstein (1905) paragraphe 7].
modifies not only the direction of propagation \((\mathbf{k} \rightarrow \mathbf{k}')\) but also the right angle \(\varphi \rightarrow \varphi' \neq 90^\circ\) (see 8b) \[\text{[Y. Pierseaux 2007]}\]. Einstein’s invariant (1) is a Galilean invariant because the time \(t\) is constant on the wavefront in \(K\) and the Lorentz transformed (LTed) time \(t'\) is also constant on the wavefront in \(K'\). Einstein’s theory of wavefront (set of simultaneous of events) is entirely consistent with Einstein’s spherical waves in the fundamental kinematics part of his work [A. Einstein (1905)] \[\text{paragraph 3}]: "At the time \(t = \tau = 0\), when the origin of the two coordinates \((K\) and \(k)\) is common to the two systems, let a \textbf{spherical wave} be emitted therefrom, and be propagated -with the velocity \(c\) in system \(K\). If \(x, y, z\) be a point just attained by this wave, then \(x^2 + y^2 + z^2 = c^2t^2\). Transforming this equation with our equations of transformation, we obtain after a simple calculation \(\xi^2 + \eta^2 + \zeta^2 = c^2t^2\). The wave under consideration is therefore no less a \textbf{spherical wave} with velocity of propagation \(c\) when viewed in the moving system \(k\).”

According to Einstein, the spherical (isotropic) \textit{shape} of the wavefront in \(K\) is LT ed into a spherical (isotropic) shape of the wavefront in \(K'\). Given that Einstein’s plane wavefront (1) is tangent to Einstein’s spherical wavefront within each system \(K\) and \(K'\), Einstein’s theory of wavefronts (plane or spherical) is therefore inseparable, in optics and in kinematics as well, from the transverse gauge (3).

If the orthogonal plane \(\mathbf{k} \mathbf{f}\) is transformed into an orthogonal plane \(\mathbf{k}' \mathbf{f}'\) tangent to Einstein’s primed sphere, Huang’s negative frequencies are impossible because \(\mathbf{k}' \mathbf{f}' > 0\). But if \(\mathbf{k} \mathbf{f}\) is LT ed into a non-orthogonal plane \((\S)\), negative \(\omega' < 0\) Huang’s frequencies \(\mathbf{k}' \mathbf{f}' = \omega' t' < 0\) (\(t'\) is constant) become possible [Y-S Huang 2007]. This is the reason why Gjurchinovski have to introduce a "relativistically-induced optical anisotropy" [A. Gjurchinovski 2007]. The problem is that, in Einstein’s kinematics, induced by Einstein’s interpretation of invariance of light velocity with spherical wavefronts, the anisotropy does not appear. In the second part (electrodynamic and optic applications), Einstein deduces a formula for Doppler effect (5c), from the invariance (1) with LT (4),

\[
\phi = \phi' \quad \Rightarrow \quad \frac{\omega'}{\omega} = \left(\frac{\nu'}{\nu}\right)_{\text{EINSTEIN}} = \gamma(1 - \beta \cos \theta) \quad (5)
\]

in coupling with formulas for stellar aberration effect, \(\cos \theta' = \frac{\cos \theta - \beta}{1 - \beta \cos \theta} \) and \(\sin \theta' = \frac{\sin \theta}{\gamma(1 - \beta \cos \theta)}\), with the source \((\theta)\) at the infinity in \(K\) and the moving observer \((\theta')\) in \(K'\) [A. Einstein (1905)] \[\text{paragraph 7}\].

2 Poincaré’s ellipsoidal wavefront, invariance of the phase of a spherical wave and Lorenz gauge

Poincaré introduces the elongated ellipsoid in 1906: “Imagine an observer and a source involved together in the transposition. The wave surfaces emanating for the source will be \textbf{spheres}, having as centre the successive positions of the source. The distance of this centre from the present position of the source will be proportional to the time elapsed since the emission - that is to say, to the radius of the sphere. But for our observer, on account of the contraction, all these spheres will appear as \textbf{ellipsoids}. The compensation is now exact, and this is explained by Michelson’s experiments.” [H. Poincaré 1908]. So we don’t need here Poincaré’s interpretation, very special and unexpected, of Lorentz contraction (conclusion) but only the "LT of a spherical wave". Suppose that a source \(S\), at rest \(O\) in \(K\), emits a circular wavefront in \(t' = t = 0\) when \(O\) and \(O'\) coincide (we first work in two dimensions). Let us consider the relativistic invariant (light velocity "\(c = 1\)"), (4), is obviously an invariant:

\[
x^2 + y^2 = r_1^2 = t_1^2 \quad (a) \quad x'^2 + y'^2 = r'^2 = t'^2 \quad (b)
\]

The shape of (6a) is by definition a circle in \(K\) \((r = r_1 = t = t_1)\). What is the LTed shape in \(K'\) of (6b)? If the time \(t\) is fixed in \(K\) the time \(t'\) is not fixed (by LT) in \(K'\). The LTed shape of 6(b) with (4) \(t' = \gamma^{-1} t_0 - \beta x'\) is:

\[
x'^2 + y'^2 = (\gamma^{-1} t_1 - \beta x')^2 \quad \text{or} \quad (\gamma^{-1} x' + \beta t_1)^2 + y'^2 = t_1^2 \quad (7)
\]

It is the Cartesian equation of an \textbf{elongated ellipse} in \(K'\) with the observer \(O'\) at the focus \(F\). The physical meaning of Poincaré’s ellipse "\(c = 1\)" is the relativity of simultaneity: two simultaneous events in \(K\) are not simultaneous in \(K'\). In polar coordinates the equation of the ellipse \((\theta'\) is the polar angle with \(F\) as pole, \(\beta\) is eccentricity and \(\gamma t_1\) the large axis) is \((r, \theta)_{\text{cercle}(c=1)} \rightarrow (r', \theta')_{\text{ellipse}(c=1)}\):

\[
r' = \frac{t_0}{\gamma(1 + \beta \cos \theta')} = t' \quad (a) \quad \tan \varphi' = \frac{\beta \cos \theta' + 1}{\beta \sin \theta'} \quad (b)
\]
We showed that the tangent to the circle is LTed into a (non-orthogonal) tangent to the ellipse (8a) that makes the angle \( \phi' \) with the direction of propagation \( \vec{k}' \) (8b Y. Pierseaux 2007), only for longitudinal \( \theta' = 0 \) we have \( \phi' = 90^\circ \): the anisotropy in LTed frame \( K' \) is determined not only by the angle \( \theta' \) but also by the angle \( \phi' \) (2) for the tangent plane wave. At three dimensions, according to Poincaré, the spherical shape is LTed into an elongated ellipsoidal shape. Poincaré’s ellipsoid of revolution is the direct kinematical explanation of the very physical "headlight effect" (in synchrotron radiation, bremsstrahlung...): the isotropic (spherical) emission of a moving source is anisotropic (ellipsoidal) observed from a system at rest with relativistic transformation of the solid angles \( \Omega = 2\pi(1 - \cos \theta) \) and \( \Omega' = 2\pi(1 - \cos \theta') \). The reduction of the angle of aperture of the cone of emission of a moving source \( (d\Omega' = d\Omega \frac{1}{\sqrt{1 + \beta^2 \sin^2 \theta}}) \), the azimuth angle \( \psi = \phi' \) being invariant) is engraved in the ellipsoidal shape of the wavefront. So the question is: if the invariant is not the shape of a spherical wave, what is the true Lorentz invariant? The answer is: it is the phase \( \Psi \) of the spherical wave:

\[
\Psi = \omega t - kr = 2\pi \nu (t - r) \quad = \quad \Psi' = \omega' t' - k'r' = 2\pi \nu (t' - r')
\]

(9)

The spherical (monochromatic) wave, \( \vec{A}(r, t) = A_0 e^{i\vec{k} \cdot \vec{r} - \omega t} \), is LTed into an ellipsoidal (monochromatic) anisotropic wave, \( \vec{A}'(r, t) = A'_0 e^{i\vec{k}' \cdot \vec{r}'} \) \( (r' = t' \) variable). The LTed ellipsoidal wavefront is an equiphase surface (a non-transverse section in Minkowski's cone). First place is given by Poincaré to spherical waves and by Einstein to plane waves. Fortunately we have in optics the principle of Huygens that enable to determine any front, including the plane front, on the basis of spherical waves [Y. Pierseaux 2008]. Poincaré’s introduction of the potential four-vector \( \vec{A} \) is fundamental because if the scalar potential is zero in \( K \) \( (V = 0) \), it cannot be zero in \( K' \) \( (V' \neq 0) \) [H. Poincaré 1905]. So by the integration of potential vector \( \vec{A} \) on the spherical surface \( (d\vec{r} \cdot d\Omega = dS = r^2 d\Omega') \), and by application of the Gauss-Ostrogradski’s theorem \( (dV' = r^2 \sin \theta d\theta d\psi) \), the flux of \( \vec{A} \) is zero in \( K \), \( \text{div} \vec{A} = 0 \) \( (\vec{A} \) is in a tangent plane to the sphere \( \vec{A} \hat{k} = 0) \), but it is not zero in \( K' \):

\[
\text{div} \vec{A}' = \frac{\partial \nu'}{\partial t'} \quad (\vec{A}' \) is in a tangent plane to the ellipsoid \( K' \), \( \vec{A}'\hat{k}' \neq 0) \)

Poincaré’s ellipsoid is therefore directly connected with Lorenz gauge [Y. Pierseaux 2008]:

\[
\text{div} \vec{A} + \frac{\partial V'}{\partial t'} = \text{div} \vec{A}' + \frac{\partial V'}{\partial t'} = 0
\]

(10)

In order to cure Huang’s negative frequencies, Gjurchinovski adds a dot in \( \vec{k}', \vec{u}' = \omega' \) whilst we delete a dot in \( \vec{k}', \vec{r}' \). According to Gjurchinovski, in his Figure 3, if the medium is isotropic, the LTed vector \( \vec{u} \) is non-orthogonal to the LTed front but, if the isotropic medium is the isotropic vacuum, the LTed vector \( \vec{u} \), that is aligned onto the LTed vector wave \( \vec{k} \), is orthogonal to the LTed front. Why the electromagnetic vacuum would be the only isotropic medium that would escape to "relativistically induced optical anisotropy"? The electromagnetic vacuum is a medium exactly as the others (with a permittivity, a permeability, an impedance, 377 \( \Omega \)... and therefore we have in Lorenz gauge a non-transversal LTed wavefront. Let us note that nothing is changed at the level of electromagnetic fields because Poincaré’s longitudinal component \( \vec{A}'_\parallel \) is compensated [Y. Pierseaux & G. Rousseaux]:

\[
\vec{E}'_\parallel = -\partial_t \vec{A}'_\parallel - \nabla V' = 0
\]

We underline that, according to Poincaré, the wave vector \( \vec{k}' \), the transversal electric field \( \vec{E}' \) and the transversal magnetic field \( \vec{H}' \) define a trirectangle trihedron: the non-transversality is only at the level of the potential vector \( \vec{A}' \), that, according to "the main stream" is a philosophical entity: a gauge effect would be absolutely impossible! Maybe... but the use a relativistic gauge (10) in a relativistic theory is a good idea and we are moreover able to deduce from Poincaré’s invariant (9) a Doppler formula, taking into account a fundamental anisotropy (14). In order to dot that, we have to find the LT of \( r \) into \( r' \) in (9) and to introduce the angles \( \theta \) and \( \theta' \). From (4) we have the defined positives norms \( r = \sqrt{x^2 + y^2} \) and \( r' = \sqrt{x'^2 + y'^2} \) and in polar coordinates, the relativistic law of composition of velocities is (with \( t = v, \, \frac{x}{t} = v_x, \, v_x = v \cos \theta, \, v_y = v \cos \theta, \, v'_x = v' \cos \theta', \, v'_y = v' \cos \theta' \)):

\[
\begin{align*}
\frac{v' \cos \theta'}{1 - \beta v \cos \theta} & = \frac{v \cos \theta - \beta}{1 - \beta v \cos \theta} \quad (a) \\
\frac{v' \sin \theta'}{\gamma(1 - \beta v \cos \theta)} & = \frac{v \sin \theta}{\gamma(1 - \beta v \cos \theta)} \quad (b)
\end{align*}
\]

\[
\frac{\vec{E}}{t} = \gamma(1 - \beta v \cos \theta) \quad (c)
\]

(11)

3We showed that Einstein’s cancellation of the potential \( V \) is exactly "Einstein’s deletion of aether" [Y. Pierseaux 2007]. In Poincaré’s Lorenz gauge, \( V' = A'_\parallel \neq 0 \), we have a relativistic aether like, for example, the thermodynamical cosmic background radiation (CBR), with respect to which one can only measures relative velocities.
For a light point in uniform moving (displacement under constant angle $\theta$) of a spherical front, with $\vec{v} = v = c = 1$ and $\frac{\theta}{\rho} = \theta' = c' = 1$ we have:

$$\cos \theta' = \frac{\cos \theta - \beta}{1 - \beta \cos \theta} \quad (a) \quad \sin \theta' = \frac{\sin \theta}{\gamma(1 - \beta \cos \theta)} \quad (b) \quad \frac{\nu'}{\nu} = \gamma(1 - \beta \cos \theta) = \frac{\nu'}{r} \quad (c)$$

(12)

So with (12c) and from Poincaré’s invariant (9) we deduce, exactly like Einstein, two coupled formulas for the relativistic transformation of angle and the relativistic transformation of frequency:

$$\frac{\omega'}{\omega} = \frac{\nu'}{\nu} = \frac{1}{\gamma(1 - \beta \cos \theta')} \quad (a) \quad \frac{\nu'}{r} = \frac{1}{\gamma(1 + \beta \cos \theta')} \quad (b)$$

(13)

With (13b) we find again the ellipse (8a). In order to have a true Doppler formula, the observer in K’ have to receive the signals under constant angle $\theta'$ (stellar aberration) and with constant frequency $\nu'$: at the infinity Poincaré’s spherical waves become Poincaré’s plane wave, with constant angle $\varphi'$ (8b). In the same way as Einstein’s plane wave, we have to define a infinite distance between source (in O) and the observer (in K’). So in order to compare (13a) with Einstein’s model (5) we have to change the sense (the sign) of light velocity (also in 9, see [Y. Pierseaux 2008], principle of inverse return of the light):

$$\Psi^+ = \Psi'^+ \implies \left(\frac{\omega'}{\omega}\right)_\infty = \left(\frac{\nu'}{\nu}\right)_{POINCARE} = \gamma(1 - \beta \cos \theta')$$

(14)

The observer ($\theta'$), Einstein or Poincaré, and the source ($\theta$) are now in identical configuration for (5) and (14). We suggest to call (14) ”Poincaré’s” formula for the same reason for which Poincaré had called (4) the ”Lorentz” transformation. We underline the following contrast: Poincaré’s anisotropy (transformation of angles, headlight effect) has a physical meaning before the limit at the infinity (stellar aberration) but (13) only becomes a Doppler formula after the limit at the infinity. In (13), we have one only source in O and an infinity of observers, in moving, onto a finite ellipsoid. By inverting (13) into (14)$_\infty$ we have an infinity of sources, in moving, onto an infinite ellipsoid and one only (terrestrial) observer at the focus of this ellipsoid [Y. Pierseaux 2008 fig 2]. Moreover in non-relativistic optics we can define a Doppler formula from spherical waves, without a limit at the infinity (one considers that the source, velocity v, is ”almost” at rest during two successive fronts, velocity c). Non-relativistic hypothesis $v << c$ being unacceptable for a relativistic theory, everything happens therefore as if Poincaré’s formula should have to be applied to (very) remote objects.

3 Poincaré-Hubble’s expansion of space and the relativistic Doppler formula

Poincaré’s formula and Einstein’s formula cannot be the same because, in the former (5), the frequency is LTed as the time whilst, in the latter (14), the frequency is LTed as the inverse of the time (i.e., the lengthwave is LTed as Poincaré’s distance [Y. Pierseaux 2007]). Einstein’s four-vector of (plane) wave ($\vec{k}, \omega$) and Poincaré’s potential four-vector ($\vec{A}, V$) of spherical electromagnetic wave are antinomic. The following contrast is therefore irreducible:

$$\left(\frac{\nu'}{\nu}\right)_{EINSTEIN} = \gamma(1 - \beta \cos \theta) \quad \left(\frac{\nu'}{\nu}\right)_{POINCARE} = \gamma(1 - \beta \cos \theta')$$

(15)

The polemical question of historical priorities, between Poincaré’s supporters and Einstein’s supporters, for the theory of relativity is irrelevant because it is not the same theory. The dilation factor is in both formulas but the respective role of the angle of the source $\theta$ et $\theta'$ are reverted. Most of the experimental tests of relativistic Doppler formulas have been realized in the longitudinal configuration [H. Mandelberg & L. Witten & H. Ives & J. Stilwell] or for very high velocities, where both formulas are identical:

$$\left(\frac{\nu'}{\nu}\right)_{EINSTEIN-POINCARE} = \sqrt{\frac{1 - \beta}{1 + \beta}} = \gamma(1 - \beta)$$

(16)

Hasselkamp’s experimentation, in (almost) transversal configuration [D. & all Hasselkamp] seems to plead for Einstein’s formula, but the situation is not the same if we define the transversality with the angle of observer.
or with the angle of the source. For $\theta = 90^\circ$, the transversal effect predicted by Einstein is a contraction of duration while this one predicted by Poincaré’s formula is a dilation of duration. It is the inverse if we take $\theta' = 90^\circ$. There is no purely terrestrial experiment already realized in order to decide clearly between the two formulas (15). We underline that Einstein deduced his Doppler formula in the second part of his famous work (electrodynamics application of his kinematics) on the basis of an additional hypothesis (1). On the other hand, our deduction of Poincaré’s formula (14) is intrinsically engraved in LT (4) and therefore it becomes impossible to distinguish optics and kinematics (structure of space-time).

So only one cosmic experiment seems to plead for Poincaré’s Doppler formula, which is inseparable from his theory of the expansion of space (according to Poincaré the Lorentz contraction of units of measure induces a dilated distance, measured in “light-year”, see the quotation! and [Y. Pierseaux 2004]): this is the experiment of Hubble (1929).

On the other hand, Einstein-de Broglie’s plane wave (1) is a solution of Schrödinger equation, that is not a relativistic equation. This is the reason why quantum atomic structure should be not concerned by Poincaré’s relativistic expansion: It is impossible to make physics without ”Einstein’s quantum principle of identical units of measure” [Pierseaux Y. (2003)]. A synthesis between Einstein’s and Poincaré’s kinematics that should go into action not at the same scale, is therefore not only possible and but even necessary (i.e a synthesis between Einstein’s photons ($\mathbf{k}$, $\omega$) and Poincaré’s electromagnetic waves ($\mathbf{A}$, $V$), see note 3 and [Y. Pierseaux (2)]).

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