Harmonic Summing Improves Pulsar Detection Sensitivity: A Probability Analysis

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Received 2018 May 2; revised 2018 September 25; accepted 2018 September 26; published 2018 November 13

Abstract

The practical application of the harmonic summing technique in the power-spectrum analysis for searching pulsars has exhibited the technique’s effectiveness. In this paper, theoretical verification of harmonic summing considering power’s noise-signal probability distribution is given. With the top-hat and the modified von Mises pulse profile models, contours along which spectra total power is expected to exceed the 3σ detection threshold with 0.999 confidence corresponding to m = 1, 2, 4, 8, 16, or 32 harmonics summed are given with respect to the mean pulse amplitude and the pulse duty cycle. Optimized numbers of harmonics summed relative to the duty cycles are given. The routine presented builds a theoretical estimate of the minimum detectable mean flux density, i.e., sensitivity, under the power-spectrum searching method.

Key words: pulsars; general – stars: neutron

1. Introduction

Due to its sensitive response to periodicity, discrete Fourier transform has been widely used in the search for pulsars. In this technique, a dedispersed and possibly barycentered3 N-point real timeseries, T, derived from an observation is Fourier transformed into a complex spectrum series ui + ıvi with point number M(id) = N/2 + 1, where i is the unit of imaginary number, and j is the number index ranging from 0 to M(id) − 1. A power spectrum is subsequently formed as w(j) = u(j)² + v(j)².

For a Gaussian white-noise series, Tnoise, any derived u(j), v(j), and w(j) is an independent and identical (iid) random variable, respectively; the central limit theorem expects that any sample in the u(j) or v(j) is Gaussian distributed, therefore any sample in w(j) is distributed with two degrees of freedom. The average and variance of w(j) is the variance of the noise series, Tnoise, multiplied by the point number N, NVar(Tnoise) (Groth 1975; Ransom et al. 2002). Thus, when normalizing w(j) via dividing by NVar(Tnoise), any sample in the resultant w̃(j) distributes with unity average and variance. The probability for any w̃(j) sample to exceed some power, P, is P(w̃(j) > P) = e−P. Then, the probability for every w̃(j) to be smaller or equal to the power P is P(w̃(j) ≤ P) = (1 − e−P)M(id). When letting the probability be the confidence level C3σ, ~ 0.999 (the value when integrating the standard Gaussian distribution probability density from −∞ to +3), the power P(3σ) (see Equation (14) in Vaughan et al. 1994) derived is the 3σ detection threshold. Any w̃(j) sample that exceeds this power should be noticed, as the probability for this to be induced by noise is only 1 − C3σ = 0.001; a signal is much more likely to have been presented.

w(j) can be summed with each other. For sum with m = 2, in the “Lyne-Ashworth” routine, for example, one stretches the original spectrum by a factor of two by repeating each w(j) once as the next sample, and then adds the intermediate series to the original series to form the power spectrum w̃(2). In this spectrum, samples χ² distribute, and the mean and the variance of the spectrum are both 2. As the summation is implemented between two iid samples, the number of iid samples is reduced to M(id)(2) = M(id)²/2 (though length of the spectrum is still M(id)). The summation is typically implemented three more times with m = 4, 8, and 16. In the resultant w̃(m) samples, χ² distribute with the iid sample number M(id)(m) = M(id)²/m. The average and variance of w̃(m) is m. The 3σ detection threshold, P(3σ), can be derived numerically as how P(3σ) is derived with considering P(w̃(m) > P) = ∑m−1 k=0 P(k)³/m³ e−P, which is the probability for any w̃(m) sample to exceed the power, P. These summing processes have been named “harmonic summing,” because by implementing the summation, powers at the fundamental and harmonic frequencies of a supposed signal can be added. In the sum with m = 2, powers at the signal’s fundamental and second harmonic frequencies are summed at the signal’s second harmonic frequency. In the sum with m = 4, powers at the signal’s fundamental up to the fourth harmonic frequencies are summed at the signal’s fourth harmonic frequency. The results of the sum with higher m are similar. As a complex phase has been lost when forming the power spectra, harmonic summing is incoherent summation.

In the power spectrum of a timeseries containing noise and signal simultaneously, the linear attribute of the Fourier transform makes both the real and imaginary parts of sample j, where the signal is the sum of the noise and signal

³ For observations typically longer than 30 minutes, the Doppler shift in pulsar pulse frequency caused by the motion of Earth appears to be significant; the barycentering process removes the shift as if the observation was carried out at the solar system barycenter, an approximate inertial reference system. Furthermore, time-domain resampling or frequency-domain correlation are often implemented to remove the Doppler shift caused by pulsar orbital motion as if the pulsar turns to be isolated and located at the binary system barycenter.

4 The normalizing power used in Vaughan et al. (1994) is two times smaller than that used here, or the powers there are two times larger.

5 In the “Lyne-Ashworth” routine for m = 4 summation, the intermediate series is not formed by simply repeating the w̃(j) samples, and so does the summation with m = 8 or 16. Please refer to the pulsar search software package SIGPROC for the source code.
components, or \( u_{\text{tot},j} = u_{\text{noise},j} + u_{\text{sig},j} \), \( v_{\text{tot},j} = v_{\text{noise},j} + v_{\text{sig},j} \). Then the total power, \( w_{\text{tot},j}^{(1)} \), and the signal power, \( w_{\text{sig},j}^{(1)} \), are \( w_{\text{tot},j}^{(1)} = u_{\text{tot},j}^2 + v_{\text{tot},j}^2 \) and \( w_{\text{sig},j}^{(1)} = u_{\text{sig},j}^2 + v_{\text{sig},j}^2 \), respectively. Because the noise and signal are summed coherently, the distribution of the normalized total power, \( \hat{w}_{\text{tot}}^{(m)} \), is not the noise’s \( \chi^2_m \) distribution shifted by the constant signal power \( \hat{w}_{\text{sig}}^{(m)} \) (Vaughan et al. 1994), but follows the two-dimensional noise-signal distribution with a probability from 0 to some power \( \hat{w}_{\text{tot}}^{(m)} \), determined by the cumulative probability distribution function \( \hat{W}_m(\hat{w}_{\text{tot}}^{(m)}; \hat{w}_{\text{sig}}^{(m)}) \) (see Equation (16) in Groth 1975 or Equation (19) in Vaughan et al. 1994). Consequently, \( 1 - \hat{W}_m(\hat{w}_{\text{sig}}^{(m)}; \hat{w}_{\text{sig}}^{(m)}) = 0.999 \) derives the \( \hat{P}_{\text{sig},m}^{(3\sigma)} \), given which total power, \( \hat{w}_{\text{tot}}^{(m)} \), is expected to exceed the detection threshold, \( \hat{P}_{\text{sig},m}^{(3\sigma)} \), with a probability of 0.999; Vaughan et al. (1994) have provided numerical routines for realizing this. As a discrete \( N \)-point sinusoid series with amplitude, \( a \), establishes signal power \( \left( \frac{N}{2} \right)^2 \) (see Equations (15) and (16) in Ransom et al. 2002), minimum detectable pulse amplitude, i.e., sensitivity, at the 3\( \sigma \) confidence level can be derived, as long as the relation between the \( \hat{P}_{\text{sig},m}^{(3\sigma)} \) and a pulse profile model is established (Vaughan et al. 1994). We discuss the cases of two profile models in the next section.

2. The Contours

One model is the top-hat profile model. The top-hat, or rectangular function, is described by an amplitude, \( a \), which is the difference between the higher and lower levels, and the duty cycle, \( \delta \), which is the ratio of the span of the higher level to the domain of the function. For a continuous periodic top-hat function, a Fourier coefficient of \( n \)th harmonic is \( 2a\delta \text{sinc}(m\pi\delta) \). Then, in a discrete \( N \)-point periodic top-hat series, \( n \)th harmonic establishes power \( [N a \delta \text{sinc}(m\pi\delta)]^2 \) in a power spectrum \( w_j^{(1)} \). Thus, the minimum detectable amplitude \( a \) for duty cycle \( \delta \) can be derived with

\[
\sum_{k=1}^{m} [N a \delta \text{sinc}(k\pi\delta)]^2 = P_{\text{norm}}^{(m)} \hat{P}_{\text{norm}}^{(m)} \tag{1}
\]

where \( P_{\text{norm}} \) is the normalization power. An experiment was done to realize the derivation. First, a \( 2^{23} \) point white-noise series was generated, where each sample was drawn from the standard Gaussian distribution. Second, the series was Fourier transformed with the forming power spectrum \( w_j^{(1)} \) (\( j = 0, 1, \ldots, 2^{23} + 1 \)). Third, any \( w_j^{(1)} \) was normalized with their average (Ransom et al. 2002). Fourth, the “Lyne-Ashworth” routine was implemented for harmonic summing, obtaining \( P_{\text{norm}}^{(1)} = 21.8570 \), \( P_{\text{norm}}^{(3)} = 43.5297 \), \( P_{\text{norm}}^{(5)} = 24.3985 \), \( P_{\text{norm}}^{(7)} = 45.9351 \), \( P_{\text{norm}}^{(9)} = 28.8732 \), \( P_{\text{norm}}^{(11)} = 49.7991 \), \( P_{\text{norm}}^{(13)} = 36.6707 \), \( P_{\text{norm}}^{(15)} = 55.7288 \), \( P_{\text{norm}}^{(17)} = 50.2974 \), \( P_{\text{norm}}^{(19)} = 64.5139 \), and \( P_{\text{norm}}^{(31)} = 74.4269 \), \( P_{\text{norm}}^{(32)} = 77.2078 \). Distributions of \( \hat{w}_j^{(m)} \) were found to be in agreement with the distributions in theory. Finally, detectable minimum mean amplitudes, \( \langle a \rangle = a\delta \), at the 3\( \sigma \) confidence level for \( m = 1, 2, 4, 8, 16, \) or 32 with respect to \( \delta \in [0.005, 0.92] \) were derived. Note that in the experiment, the “Lyne-Ashworth” routine was extended with \( m \) up to 32. The results are shown by the black dashed lines in the upper panel of Figure 1; the ratio \( \langle a \rangle^{(3\sigma)} / \langle a \rangle^{(3\sigma)} \) is shown in the lower panel. In

![Figure 1](attachment:image.png)

**Figure 1.** Upper panel: the contours as a function of the pulse duty cycle (x-axis) and the mean pulse amplitude (y-axis) under the top-hat pulse profile model. Along the contours, powers of signal with \( m = 1, 2, 4, 8, 16, \) or 32 harmonics summed enable total powers at the signal frequency to exceed the 3\( \sigma \) detection thresholds at confidence level 0.999. The black dashed, blue dashed, and red dotted lines, respectively, indicate the integer-frequency case, the fractional frequency case, and the fractional frequency case with interbinning interpolation implemented. Note that the contours were derived with a \( 2^{23} \) point Gaussian white-noise series. Lower panel: the contours alternatively plotted with the y-axis changed into the amplitudes, relative to the value derived without harmonic summing. The numbers at the bottom are the optimum numbers of harmonics summed. The letters at the top label are the various duty cycle intervals, whose boundaries are indicated by the vertical dashed lines.

Table 1, the optimum numbers of harmonics are summed, and the corresponding duty cycle intervals are given. Because of the equivalence of the wide pulses to narrow negative pulses,
the optimum numbers of harmonics summed and the duty cycle intervals are symmetric, relative to the 0.5 duty cycle.

The other model is the modified von Mises profile model (MVMD; see Equation (20) in Ransom et al. 2002). For this model, the equivalent width, which is the division between the area under the function (the $a$), and the function’s maximum (Equation (22) in Ransom et al. 2002), is $w_k = \frac{l_k(\kappa) - e^{-\kappa}}{2 \sinh \kappa}$. In subsequent analysis regarding this model, this $w_k$ is used to define the pulse duty cycle, $\delta$. For pulse phase in pulsar rotation, $\delta = w_k$ and the $a$ is the mean pulse amplitude. For a continuous periodic MVMD function, the Fourier coefficient of the $m$th harmonic is $\frac{2a l_m(\kappa)}{l_k(\kappa) - e^{-\kappa}}$. Therefore, in a discrete $N$-point MVMD series, the $m$th harmonic establishes power $\left[ \frac{Na l_m(\kappa)}{l_k(\kappa) - e^{-\kappa}} \right]^2$ in power spectrum, $w_j^{(1)}$. When using the power to replace the $[Na \sin(m \pi \delta)]^2$ part in Equation (1) with implementing the same experiment as for the top-hat profile model, the minimum detectable mean pulse amplitude was derived. In the computation, to obtain the concentration parameter $\kappa$, corresponding to a specific $\delta$, the bisection method was used to find the root of the equation $\frac{2a l_m(\kappa)}{l_k(\kappa) - e^{-\kappa}} - \delta = 0$. Because $\kappa$ increases dramatically as $\delta$ becomes smaller, the equation could only be solved for the $\delta$ larger than 0.03. For $\delta < 0.03$, the $\kappa$ values were calculated as $\frac{\pi}{2 \kappa^2}$, as the modified Bessel function $l_m(\kappa)$ approaches $\sqrt{\frac{\pi}{2 \kappa}}$ when $\kappa \to +\infty$. In the large $\kappa$ limit, the exponentially scaled modified Bessel function is used to approximate the ratio of $l_m(\kappa)$ to $l_0(\kappa)$, i.e., $\frac{l_m(\kappa)}{l_0(\kappa)} \sim e^{-\frac{\kappa}{\sqrt{2}}}$. The derived minimum detectable mean amplitudes are shown as a black dashed line in the upper panel of Figure 2. In the lower panel, the ratio $\frac{\sin^2 m \pi \delta}{\delta}$ is shown. The optimum numbers of harmonics summed and the corresponding duty cycle intervals are given in Table 1.

The analysis above is restricted to integer frequencies, i.e., integers between 1 and $M_a^{(1)} - 1$. This refers to the case when the power spectrum happens to sample the frequency of a signal. Signals having fractional frequencies are the more general cases, in which the "scalloping effect" occurs (Ransom et al. 2002). In the power spectrum derived from a $N$-point sinusoid series with amplitude $a$, power at the nearest integer frequency with difference $\Delta \in [0.5, 0.5]$ away from the signal frequency is the multiplication between the power $\left( \frac{1}{2} Na \right)^2$ and the factor $\sin^2(\pi \Delta)$ (Ransom et al. 2002). For narrow-pulse cases, harmonic summing algorithms in principle call the

Table 1

| Harm. No. | $\delta$ int. (Top-hat) | $\delta$ int. (MVMD) |
|-----------|-------------------------|---------------------|
| 1         | 0.55 ⋯ 0.44             | 0.50 ⋯ 0.39         |
| 4         | 0.44 ⋯ 0.38             |                    |
| 2         | 0.38 ⋯ 0.24             | 0.39 ⋯ 0.22         |
| 4         | 0.24 ⋯ 0.12             | 0.22 ⋯ 0.10         |
| 8         | 0.12 ⋯ 0.061            | 0.10 ⋯ 0.050        |
| 16        | 0.061 ⋯ 0.029           | 0.050 ⋯ 0.025       |
| 32        | 0.029 ⋯ 0.0050          | 0.025 ⋯ 0.0050      |

5 Erratum of Equations (21) and (22) in Ransom et al. 2002: the FWHM of the modified von Mises distribution should be $\pi^{-1} \arccos \left[ \frac{\sin(\kappa \pi)}{\kappa} \right]$, the maximum of the distribution should be $\frac{2a \sinh \kappa}{l_k(\kappa) - e^{-\kappa}}$.
multiplied to the left-hand side of Equation (1) to take the effect into account. The derived contours on the $\delta - \alpha$ plane under the $3\sigma$ confidence level are indicated by the blue dashed lines in Figures 1 and 2 (upper panels) for the top-hat and MVMD profile models, respectively. Relative amplitudes are the same as those of the integer-frequency case, as the $\gamma$ factor is a constant.

An effective method to overcome the scalloping effect is the Fourier interpolation. In this method, complex spectra with frequency locating at any position between adjacent two integer frequencies is formed as the weighted coherent sum of the spectra at $m$ integer frequencies around (see Equation (30) in Ransom et al. 2002). As a large $m$ leads to expensive computation, the “interbinning” case is popular. This corresponds to the $m = 2$ Fourier interpolation, but changes the coefficient from $\frac{1}{2}$ to $\frac{1}{4}$ to boost the response at half-integer frequency to be the full response (see Equation (31) in Ransom et al. 2002). The interbinning interpolation raises the efficient coefficient to $\gamma = 0.97$ on average. The derived contours are then presented as the red dotted lines in the upper panels in Figures 1 and 2.

3. Discussion

To determine the detectable minimum mean flux density or sensitivity is an essential requirement of a pulsar search program. This is a complicated problem. As described in Cordes & Chernoff (1997), sensitivity is a function of the radiometer noise, intrinsic pulse profile, pulsar period, and dispersion measure (DM), and the method used to find pulsars. The level of the radiometer noise or rms fluctuation in system temperature, $T_{sys}$, is, as manifested by the radiometer equation (see e.g., Equation (12) in O’Neil 2002), proportional to the $T_{sys}$ itself. $T_{sys}$ is a function of the source position, the telescope pointing, and the observing frequency; its sophisticated calibration procedures were described by O’Neil (2002). It has usually been found that observed timeseries exhibit the “red” power spectral features. There are both natural and artificial sources that induce the red noises. The natural sources are, for example, the emission from background and/or foreground celestial bodies (Israel & Stella 1996), and the variations of the atmospheric emission (O’Neil 2002). The artificial sources are more diverse. For example, the dependence of the temperature from the ground on the telescope azimuth, the zenith angles, the dependence of temperature from the atmosphere on the telescope zenith angle, the instability of the receiving system, and the dependence of antenna gain on telescope elevation (O’Neil 2002).

Another strong artificial source is the radio-frequency interference (RFI). The RFI is more complicated, because apart from it being telescope-dependent, it also varies from time to time. Although multiple efforts, including active surface and hardware/software filters, have been made for removing the red noises; however, they cannot be eliminated completely. By simulating pulsar signals in real observations, Lazarus et al. (2015) incorporated RFI into the analysis of sensitivity for the PALFA survey. They found at the long-period end, the predicted sensitivities were degraded by a factor of $\sim 3$ to $\sim 7$ compared with the predictions made with the Dewey et al. (1985) method. Parent et al. (2018) have further analyzed PALFA sensitivities for long-period pulsars; similar results were obtained.

The approach implemented by Dewey et al. (1985) examines the significance of an averaged top-hat pulse profile out of a given flux density. This is realized by first applying the radiometer equation to the top-hat pulse signal (see Equation (1) in Dewey et al. 1985, or the Appendix A1.4 in Lorimer & Kramer 2012 for the detailed derivation), then setting the entire integration time per telescope pointing as the observing integration time in the equation. The significance is indicated by the signal-to-noise ratio (S/N), which is defined as the proportion of height of the top-hat to the rms radiometer noise, and it is statistically modeled by Gaussian distributions (see Section 7.1.1.1 in Lorimer & Kramer 2012). With the integration time per pointing 2.3 minutes and the sampling time 16.8 ms, Dewey et al. (1985) set the confidence limit $\sim 7.5\sigma$ under the profile S/N. Because the detection sensitivity is partially a function of the searching method (Cordes & Chernoff 1997), the approach described is not appropriate, as the Fourier domain method was used by Dewey et al. (1985) to implement their search. The detection confidence limit should be given under the statistics in the Fourier domain rather than the statistics of pulse profile. The threshold (profile height) implied out of the profile S/N is not consistent with the threshold (spectra power) implied in the Fourier domain. The Dewey et al. (1985) method has subsequently been implemented by Johnston et al. (1992), Manchester et al. (2001), and Cordes et al. (2006) for their respective surveys, though the Fourier method was also used for searching pulsars. For the Parkes multi-beam pulsar survey, Crawford (2000) and Manchester et al. (2001) implemented a semi-analytic approach to obtain the sensitivity estimates. However, that was for including the harmonic summing into the Dewey et al. (1985) method; the inconsistency issue remains.

Vaughan et al. (1994) proposed the approach to give a sensitivity estimate for the Fourier domain searching method in the power-spectrum manner. They implemented their method with the sinusoidal pulse profile for the X-ray pulsar search. In a radio pulsar search where narrow pulses are more commonly seen, the relationship between the spectra thresholds and the top-hat and MVMD profile models has been presented in this work. The conversion of the derived minimum detectable mean amplitudes (shown in Figures 1 and 2, upper panels) into the sensitivity values in the unit of Jansky would be complicated, because a realistic conversion should include the calibration of system temperature, the response of bandpass, and the RFI etc.; these are telescope-dependent. However, for the purpose of illustrating the idea of the conversion, we present example below. In brief, the radiometer equation will be used for individual pulses since the amplitudes derived are the values in one pulse period. In the example, published system parameters of the Parkes multi-beam pulsar survey are used.

In the multi-beam survey, the sampling interval, $t_{samp}$, was configured as 0.25 ms. Then, the 35 minutes observation produced $2^{21}$ samples in the timeseries. The shortest pulsar period the survey detects is 0.50 ms. At this period, only the fundamental presents in the power spectrum (no harmonic presents). At zero DM, the effective pulse width, $W_e$, is the quadrature sum of intrinsic pulse width, $W_0$, and the sampling interval. When assuming the intrinsic pulse width to be 0.04 of the pulsar period, we have $W_e = [(0.04 \times 0.5)^2 + 0.25^2]^{1/2} \sim 0.25$ ms. The duty cycle is then $\sim 0.50$. As no harmonic presents itself at the 0.50 ms period, among the minimum mean amplitudes corresponding to this duty cycle (see Figures 1 and 2 upper panels), the $m = 1$ amplitudes should be taken to calculate the profile S/N. In the integer-frequency case, the amplitude derived with the top-hat profile model was $\sim 0.0072,$
while the amplitude derived using the MVMD profile model was \(~0.0091\). As standard deviations of the simulated white-noise series were derived as \(~1.0\) for both of the realizations for the profile models, the S/N values are then \(~0.0072\) and \(~0.0091\), respectively. For the other system parameters, the survey configured the antenna gain \(G = 0.735 \text{ K Jy}^{-1}\), the polarization number, \(n_{pol} = 2\), the central frequency, \(f_{\text{cr}} = 1, 374 \text{ MHz}\), the bandwidth, \(\Delta f_{\text{bw}} = 288 \text{ MHz}\), the digitization loss factor, \(\beta = 1.5\), and the receiver temperature, \(T_{\text{sys}} = 21 \text{ K}\) (Manchester et al. 2001). For the sky temperature, \(T_{\text{sky}}\), it is set to 427 K; this is the value of the sky position with a Galactic longitude of \(350^\circ 019\) and a Galactic latitude of \(-0^\circ 677\) measured at 408 MHz (Haslam et al. 1982).\(^8\) With the average spectral index of \(-2.5\) of the sky background (Haslam et al. 1982), we have 19 K as the position’s temperature at the central frequency. By neglecting all other contributions to \(T_{\text{sys}}\), we have \(T_{\text{sys}} = T_{\text{ext}} + T_{\text{sky}} \sim 40 \text{ K}\). Thus, sensitivity at the 0.50 ms period and zero DM was derived as \(~1.1 \text{ mJy}\) under the top-hat profile model, or as \(~1.3 \text{ mJy}\) under the MVMD profile model. In the calculation, the integration time in the radiometer equation was taken as the pulsar period. The routine described above can be used up until period 1.0 ms. From 1.0 to 2.0 ms, with the emergence of the second harmonic, the \(m = 2\) amplitudes should be taken if they are smaller than the \(m = 1\) amplitudes. From 2.0 to 4.0 ms \(m = 4\) amplitudes are preferred, from 4.0 to 8.0 ms \(m = 8\) amplitudes are preferred, from 8.0 to 16.0 ms \(m = 16\) amplitudes are preferred, and from 16.0 ms on, \(m = 32\) amplitudes are preferred. The derived sensitivities for pulsar periods from 0.50 ms to 10.0 s and DM zero are shown in Figure 3.

For non-zero DM, the quadrature sum for the effective pulse width should additionally include the pulse smearing times induced by dispersion, \(\sigma_{\text{DM}}\), and scattering, \(\sigma_{\text{scat}}\). The multi-beam survey configured \(96\) frequency channels over the bandpass, \(\sigma_{\text{DM}}\), can then be represented by that at the central frequency channel. For \(\sigma_{\text{scat}}\), the values given by the NE2001 model (Cordes & Lazio 2002) were taken. The model gives \(\sigma_{\text{scat}}\) at 1000 MHz; values at the 1374 MHz were extrapolated with the spectral index \(\sim -4.4\) of the Kolmogorov spectrum for turbulence. By repeating the procedures for zero DM, sensitivities at DM 100, 300, and 1000 cm\(^{-3}\) pc were calculated as shown in Figure 3.

In the figure, the sensitivities derived via the routine originally developed by Crawford (2000) for the multi-beam survey sensitivity estimates are also shown. We see there are wide discrepancies between these values and those derived in the example. This is primarily because the estimates given in the example were drawn from the 3\(\sigma\) confidence limit, while the original estimates were drawn from the 8\(\sigma\) confidence limit. The effects of the high-pass filters with characteristic times \(\sim 2\) s and the 5 s cutoff considered in the original estimates would have further widened the discrepancies at the long-period end. The effects of the filters, cutoff, and any other factor that degrade the sensitivity predictions were not included in the example. PSR J1822–0848, with period \(\sim 2.5\) s, and PSR J1830–0052, with period \(\sim 0.3\) s, were initially discovered by the multi-beam survey. The ATNF Pulsar Catalogue\(^9\) (Manchester et al. 2005) shows both the pulsars have exhibited a mean flux density of \(\sim 0.04\) mJy. Around the periods of these pulsars, the sensitivity predictions given in this example are \(~0.01\) mJy, the 3\(\sigma\) confidence limit is comparatively low. Around the periods, the original predictions were \(~0.14\) mJy, the 8\(\sigma\) confidence limit consequently seems high.

This work is supported by (1) the National Key Projects of China, Frontier research on radio astronomy technology; (2) the

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\(^8\) The temperature value is from the data with no filtering downloaded from https://lambda.gsfc.nasa.gov/product/foreground/fg_haslam_get.cfm.

\(^9\) http://www.atnf.csiro.au/research/pulsar/psrcat/; version 1.56.
Open Project Program of the Key Laboratory of FAST, National Astronomical Observatories, Chinese Academy of Sciences; (3) the National Natural Science Foundation of China (No. 11503034, No. 11261140641, No. 11403060). M.Y. acknowledges Prof. F. Crawford for the helpful discussion and, in particular, the provision of the original program for sensitivity calculation of the multi-beam survey. M.Y. acknowledges Prof. K. J. Lee for the initial discussion. M.Y. also acknowledges the reviewer for the comments which have greatly helped in improving the manuscript.

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