INTRODUCTION

We investigate the thermodynamics of mesonic bound states in hot medium consisting of a nonideal quark plasma with correlations in the pion and sigma meson interaction channels. A special emphasis is put on a correct description of the bound states' dissociation in the vicinity and beyond the Mott temperature. To this end we utilize the Polyakov-loop-extended Nambu–Jona-Lasinio model at finite temperature. The model has two order parameters: the chiral condensate, determining the value of the dynamically generated quark mass \( m(T) \) related to the chiral symmetry breaking/restoration transition, and the Polyakov loop variable \( \Phi(T) \) which is an order parameter for deconfinement. Their values are obtained from a self-consistent solutions of the coupled gap equations for these parameters which correspond to the location of the minimum of the thermodynamical potential \( \Omega(m, \Phi; T) \) in the mean field approximation (MFA).

Going beyond the MFA, we evaluate the contributions from pion and sigma meson fluctuations within the Gaussian approximation to the path-integral representation of the thermodynamic potential. Above the Mott temperature one observes the spectral broadening of bound states leading to the appearance of complex mass poles in the mesonic propagators. A proper analysis leads to coupled Bethe–Salpeter equations from which meson masses and corresponding spectral widths are obtained. Next, the equation of state for the quark-meson system is obtained in form of a generalized Beth–Uhlenbeck equation [1] which describes the effects that chiral symmetry restoration and deconfinement have on the contributions from pions and sigma mesons and allows for an adequate description of bound, resonant and scattering states on equal footing. The spectral functions in the mesonic channels are evaluated from the complex-valued polarization loop integrals. An examination of the Levinson theorem [2] proves that in order to correctly describe the Mott effect, the continuum of mesonic correlations (scattering states) has to be taken into the account. As a result of this analysis, we obtain a description of pion dissociation where the effect of the vanishing bound state is exactly compensated by the occurrence of a resonance in the continuum of scattering states.

The present work improves on previous works within the NJL model [3, 4], where unphysical quark degrees of freedom appeared in the hadronic phase due to the lack of the coupling to the Polyakov-loop and the Stefan–Boltzmann limit was not obtained due to a misplaced momentum cutoff. We also improve the recent work [5] by properly discussing the role of Levinson’s theorem in the PNJL model approach to the generalized Beth–Uhlenbeck equation of state for the quark-meson plasma, see also recent developments in this direction in [6, 7]. The model presented joins both exact limits of finite-temperature QCD—the pion gas at low temperatures and the quark-gluon Stefan–Boltzmann limit at high temperatures—within a microscopic chiral quark model approach.

1. QUARKS AND LIGHT MESONS IN THE PNJL MODEL

The definition of the Nambu–Jona-Lasinio model [8–14] with Polyakov loop [15–19] is given in the
Appendix A. Here we start from the expansion of the thermodynamic potential

$$\Omega(T, \mu) = \frac{T}{V} \ln \mathcal{Z}[T, V, \mu]$$

(1)

around homogeneous mean field values, which leads to the decomposition of the auxiliary fields $\sigma'$ and $\bar{\pi}$ into their mean field and fluctuation parts

$$\sigma' = \sigma_{\text{MF}} + \sigma, \quad \bar{\pi} = \bar{\pi} \quad (\bar{\pi}_{\text{MF}} = 0).$$

(2)

Utilizing the decomposed fields results in the factorization of the partition function into the respective parts describing mean field and fluctuation contributions:

$$\mathcal{Z}_{\text{MF}}[T, V, \mu] = \exp \left\{ -\frac{V}{T^4} \frac{\sigma_{\text{MF}}^2}{4G_S} + U(\Phi, \bar{\Phi}; T) \right\}$$

$$+ \text{Tr} \ln \left[ \beta S_{\text{MF}}^r(m) \right],$$

(3)

$$\mathcal{Z}_{\text{FL}}[T, V, \mu] = \int \mathcal{D} \sigma \mathcal{D} \bar{\pi} \mathcal{D} \pi \mathcal{D} \bar{\pi} \exp \left\{ - \int dx \int d^3 \bar{x} \right\}$$

$$\times \frac{2\sigma \sigma_{\text{MF}} + \sigma^2 + \bar{\pi}^2}{4G_S} + \text{Tr} \ln \left[ 1 - S_{\text{MF}}(m) \left( \sigma + i \gamma_5 \tau_3 \bar{\pi} \right) \right].$$

(4)

Here the mean field inverse propagator is

$$S_{\text{MF}}^{-1} = \gamma^0 (i \omega_n - \mu + A_n) - \gamma_i \cdot \bar{\pi} - m - \sigma_{\text{MF}}$$

$$= \gamma^0 (i \omega_n - \mu + A_n) - \gamma_i \cdot \bar{\pi} - m.$$  

(5)

The thermodynamic potential in the mean field approximation of the PNJL model is evaluated to be given by the following expression [19]

$$\Omega_{\text{MF}} = \frac{\sigma_{\text{MF}}^2}{4G_S} + U(\Phi, \bar{\Phi}; T)$$

$$- 2N_f \int \frac{d^3 p}{(2\pi)^3} \left\{ \frac{1}{E_p} \ln N_\phi(E_p) + \ln N^r_\phi(E_p) \right\},$$  

(6)

where factors $N_\phi, N_c$ originate from performing the trace operation and are a consequence of isospin and color symmetry. The quark energy is given by $E_p = \sqrt{p^2 + m^2}, \quad E_p^* = E_p + \mu,$ and

$$N_\phi(E_p) = \left[ 1 + 3(\Phi \bar{\Phi} e^{-\beta E_p} + e^{-3\beta E_p}) \right]$$

$$N^r_\phi(E_p) = \left[ 1 + 3(\Phi \bar{\Phi} e^{-\beta E_p} + e^{-3\beta E_p}) \right].$$

(7)

In the mean field approximation of the PNJL model the values of the constituent quark mass $m$ and the Polyakov-loop variable $\Phi$, along with its complex conjugate $\bar{\Phi}$, are obtained from the condition that the thermodynamic potential should be minimized with respect to these parameters, which is augmented by the stability conditions. For $\mu = 0$, we have $\Phi = \bar{\Phi}$ and thus the minimizing conditions are given by

$$\frac{\partial \Omega_{\text{MF}}}{\partial \sigma_{\text{MF}}} = 0, \quad \frac{\partial \Omega_{\text{MF}}}{\partial \Phi} = 0.$$  

(9)

These conditions are equivalent to a set of coupled gap equations [17, 19]. For the mass gap equation we get

$$m = m_0 + 4N_fN_cG_S^2 \int d^3 p \frac{m}{(2\pi)^3} E_p \left[ 1 - f_\phi(E_p) - f^r_\phi(E_p) \right],$$

(10)

where

$$f_\phi(E_p) = [\Phi e^{-\beta(E_p + \mu)} + 2\Phi e^{-2\beta(E_p + \mu)} + e^{-3\beta(E_p + \mu)}] / N_\phi(E_p)$$

are the so-called generalized Fermi functions, characteristic for the PNJL model. One should note that if $\Phi \rightarrow 1$, the expression (11) reduces to the standard NJL model Fermi functions. For PNJL calculations we should additionally find the values of $\Phi$ from corresponding gap equation [19] at given $T$ and $\mu$.

In order to solve (10), a set of model parameters has to be determined: the cutoff parameter $\Lambda$, the current quark mass $m_0$ (in the chiral limit $m_0 = 0$) and the coupling constant $G_S$. These parameters are fitted at $T = 0$ to reproduce physical quantities: the pion mass $M_\pi = 135$ MeV, the pion decay constant $F_\pi = 92.4$ MeV and the quark condensate $\langle \bar{q}q \rangle^{1/3} = -240.772$ MeV. The used parameters [20] are shown in Table 1.

| $m_0$, MeV | $\Lambda$, MeV | $G_S \Lambda^2$ | $F_\pi$, MeV | $M_\pi$, MeV |
|------------|---------------|----------------|--------------|--------------|
| 5.495      | 602.472       | 2.318          | (92.4)       | (135)        |

Table 1. The set of model parameters reproducing observable quantities (in brackets) and $\langle \bar{q}q \rangle^{1/3} = -240.772$ MeV [20]

Since NJL-type models are non-renormalizable, it is necessary to introduce a regularization, e.g., by a cutoff $\Lambda$ in the momentum integration. Following [19], in this study, in case of thermodynamic quantities, we use the three-dimensional momentum cutoff for vacuum terms and extend the integration till infinity for finite temperature. A comprehensive study of the differences between the two regularization procedures (with and without cutoff on the quark momentum states at finite temperature) has been performed in [21].
Solutions of the gap Eq. (10) and the corresponding gap equation for the Polyakov loop variable Φ at zero chemical potential and nonzero T are presented in Fig. 1. Above the critical temperature, which is equal to 237 MeV in the chiral limit and Tc = 251 MeV for a finite current quark mass, one observes chiral symmetry restoration indicated by the rapid decrease of the constituent quark mass and the Polyakov loop variable becoming gap equation for the Polyakov loop variable Φ given explicitly by

\[ \Pi_m(q_0, \hat{q}) = -N_c N_f \sum_{x, s' = \pm 1} \int \frac{d^2 p}{(2\pi)^3} \left( 1 - \frac{f_\phi(-s' E_{p-q}) - f_\phi(s' E_p)}{q_0 + s' E_{p-q} - E_p} \right) \left( 1 - s' \left( p \cdot (p - q) + m^2 \right) \right). \]

One easily obtains the thermodynamic potential corresponding to the chosen meson part of the partition function up to the Gaussian order given by

\[ \Omega^{(2)}_M(T, \mu) = \frac{d_M T}{2 V} \text{Tr} \ln S_M^4. \]

where the degeneracy factor \( d_M \) equals 1 for sigma mesons and 3 for pions.

Performing the calculation leads to the subsequent factorization of the thermodynamic potential into parts describing the contribution from mesonic correlations corresponding to \( \sigma \) and \( \hat{\pi} \) channels of interaction

\[ \mathcal{D}^{(2)}_{\text{FL}}(T, V, \mu) = \left[ \det \left( \frac{1}{2G_S} - \Pi_m(q_0, \hat{q}) \right) \right]^{1/2} \times \left[ \det \left( \frac{1}{2G_S} - \Pi_\sigma(q_0, \hat{q}) \right) \right]^{3/2} \]

Fig. 1. Temperature dependence of the quark masses \( m(T) \) and Polyakov loop variable \( \Phi(T) \) at \( \mu = 0 \). Results for quark mass are scaled by \( m(0) = 367.5 \text{ MeV} \).
imately degenerate, \( M_e \approx M_x \), and then both masses increase linearly with temperature.

The corresponding meson pressure is shown on Fig. 2 (right panel).

2. GENERALIZED BETH–UHLENBECK APPROACH TO THE QUARK-MESON PLASMA

Deriving the thermodynamic equation of state for a quark-meson system in the Beth and Uhlenbeck form reduces to the introduction of scattering phase shifts into the formula for the thermodynamic potential. This requires that we analytically continue the propagator into the complex plane.

In a first step, we introduce meson spectral functions \( A_M^\omega (\omega, \hat{q}) \) by utilizing the integral representation of the logarithm in (16), followed by the usual expression of the propagator by means of \( A_M^\omega (\omega, \hat{q}) \) [22], i.e.,

\[
\ln S_M^\omega = - \int_0^\infty \frac{d\omega}{2g^2} \frac{1}{2g^2 - \Pi_M(\omega, \hat{q})} \approx - \int_0^\infty \frac{d\omega}{2\pi q_0 - \omega} \int_0^\infty \frac{dg}{2g^2} A_M^\omega (\omega, \hat{q}). \tag{19}
\]

Using the fact that the spectral density is given by the discontinuity of the propagator at the real axis we arrive at

\[
\frac{dg}{2g^2} A_M^\omega (\omega, \hat{q}) = -i \frac{dg}{2g^2} (S_M^\omega (\omega + i\eta, \hat{q}) - S_M^\omega (\omega - i\eta, \hat{q})) = -i \ln \frac{1 - 2G_M(\omega - i\eta, \hat{q})}{1 - 2G_M(\omega + i\eta, \hat{q})}, \tag{20}
\]

where the argument of the logarithm is by definition the scattering matrix \( S_M(\omega, \hat{q}) \) in the Jost representation [3]. This normalized complex function can also be represented by means of a scattering phase shift \( \Theta_M(\omega, \hat{q}) = \exp[2i\Phi_M(\omega, \hat{q})] \), which allows us to identify

\[
\int_0^\infty \frac{dg}{2g^2} A_M^\omega (\omega, \hat{q}) = -i \ln S_M(\omega, \hat{q}) = 2\Phi_M(\omega, \hat{q}). \tag{21}
\]

Utilizing the above identity leads, after performing the Trace operation, to the following result

\[
\Omega_M^{(2)}(T, n) = - \frac{N_M}{2} \int_0^\infty \frac{d^3q}{(2\pi)^3} \left[ \int_0^\infty d\omega \ln \left( \omega + 2T \ln(1 - e^{-\beta\omega}) \right) \right. \\
\times \frac{d\Phi_M(\omega, \hat{q})}{d\omega} \left. \right] = - \frac{N_M}{2} \int_0^\infty \frac{d^3q}{(2\pi)^3} \left[ \int_0^\infty ds \left[ \sqrt{\hat{q}^2 + s} + 2T \ln \left( 1 - e^{-\beta\sqrt{\hat{q}^2 + s}} \right) \right] D_M(s) \right), \tag{22}
\]

where \( D_M(s) \) is a generalized mass distribution (density of states) containing all the dynamics of the system. The above expression is the so-called generalized Beth–Uhlenbeck form of the thermodynamic potential. The connection to the original Beth–Uhlenbeck expression for the second virial coefficient can be found in [3] in detail.

For an analysis of meson masses and corresponding widths, necessary to obtain a description of mesonic correlations as bound states dissolving into resonant states above the Mott temperature, one splits the polarization function \( \Pi_M(q, \hat{q}) \) according to

\[
\Pi_M(q, \hat{q}) = 4N_cN_fI_1 - 2N_cN_fP_MI_2(q_0) = I_1 - P_MI_2(q_0), \tag{23}
\]

where \( I_1 = P_MI_2(q_0) \) is defined above.
where, in the limit \( \hat{q} = 0 \) that we utilize, the integrals \( I_1 \) and \( I_2 \) are given by

\[
I_1 = \int \frac{d^3 p}{(2\pi)^3} \left[ \frac{1}{2E_p} \left( 1 - f_\alpha(E_p) - f_\alpha(E_p) \right) \right], \tag{24}
\]

\[
I_2(q_0) = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{4E_p^4} \left[ \frac{1}{2E_p-q_0} \right] \left[ 1 - f_\alpha(E_p) - f_\alpha(E_p) \right] + \frac{1}{2E_p+q_0} \right], \tag{25}
\]

with \( P_M = -q_0^2 \) for pions and \( P_M = -q_0^2 + 4m^2 \) for sigma mesons. By identifying \( q_0 = M_M - i \times \frac{1}{2} \Gamma_M \) one can perform the complex mass pole analysis leading to

\[
P_M = \frac{1}{4N_cN_fG_S} - 2I_1 \left[ I_2(q_0 = M_M - i \times \frac{1}{2} \Gamma_M) \right]^{1/2} 
\times \left( \text{Re} I_2(M_M) - i\text{Im} I_2(M_M) \right), \tag{26}
\]

which decomposes into coupled Bethe–Salpeter equations for meson mass and meson spectral width.

In the first departure beyond the pole approximation (Appendix B), it is justified above \( T_{\text{Mott}} \) to consider \( D_M(s) \) to be described by a Breit–Wigner type function

\[
A_R(s, T) = a_R \frac{M_M \Gamma_M}{(s-M_M^2)^2 + (M_M \Gamma_M)^2}, \tag{27}
\]

where \( M_M \) is the meson pole mass, \( \Gamma_M \) is the corresponding meson width and \( a_R \) is a normalization factor. Below \( T_{\text{Mott}} \), where the spectral broadening \( \Gamma_M(T) \) of the states vanishes, the above expression becomes the delta function typical for the spectral function of mesonic bound state. The meson phase shift \( \Phi_M \) corresponding to (27) should be of the form

\[
\Phi_M(s) \approx \phi_R(s) = \frac{\pi}{2} - \text{arctan} \left( \frac{4m^2 - M_M^2}{M_M \Gamma_M} \right), \tag{28}
\]

where by introducing the notation \( \phi_R(s) \) we acknowledge the fact that the above phase shift is connected with resonant properties of the mesonic correlations.

### 3. Levinson’s Theorem

**FOR QUARK-MESON THERMODYNAMICS**

In order to inspect the validity of the approach so far presented, in our analysis we consider the Levinson theorem

\[
\int_4^{\infty} ds \frac{d\Phi_M}{ds} = n \pi, \tag{29}
\]

where \( n \) denotes the number of bound states below the threshold \( 4m^2 \). Indeed, it is easy to check that the resonant phase shift \( \phi_R(s) \) alone does not fulfill (29). This implies that the scattering phase shift should be composed of at least two parts. In fact, as was demonstrated in [4], it is appropriate to decompose the scattering phase shift \( \Phi_M \) into a part corresponding to the mesonic correlation and a part describing quark-anti-quark scattering,

\[
\Phi_M = \phi_R + \phi_{sc}. \tag{30}
\]

Namely, using (21) we can represent the total scattering phase shift \( \Phi_M \) as

\[
\Phi_M = i \ln \left[ 1 - 2G_s^2 \text{Im} I_2 \left( \frac{2G_s P_M \text{Re} I_2}{1 - 2G_s^2 I_2 + 2G_s P_M \text{Re} I_2} \right) \right], \tag{31}
\]

Then it is straightforward, using (23) and the relation between logarithm and arctan functions, to show that

\[
\Phi_M = -\text{arctan} \left[ \frac{2G_s P_M \text{Im} I_2}{1 - 2G_s^2 I_2 + 2G_s P_M \text{Re} I_2} \right], \tag{32}
\]

and, by several more consecutive steps, to obtain

\[
\Phi_M = \text{arctan} \left[ \frac{\text{Im} I_2}{\text{Re} I_2} \right] - \frac{\text{Im} I_2}{\text{Re} I_2} \frac{1 - 2G_s I_2}{2G_s^2 I_2} \frac{\text{Re} I_2}{\text{Re} I_2} \tag{33}
\]

At this point it is enough to recognize the above expression as the sum of two arctans to finally obtain

\[
\Phi_M = -\text{arctan} \left( \frac{\text{Im} I_2}{\text{Re} I_2} \right) \tag{34}
\]

\[
= -\text{arctan} \left( \frac{\text{Im} I_2}{\text{Re} I_2} \right) + \text{arctan} \left( \frac{1 - 2G_s I_1}{2G_s^2 I_2} \frac{\text{Re} I_2}{\text{Re} I_2} \right), \tag{34}
\]

\[
\text{with} \quad 1 + \frac{1 - 2G_s I_2}{2G_s^2 I_2} \frac{\text{Re} I_2}{\text{Re} I_2} = \frac{\text{Im} I_2}{\text{Re} I_2} \tag{34}
\]

\[
\text{and} \quad \text{arctan} \left( \frac{1 - 2G_s I_1}{2G_s^2 I_2} \frac{\text{Re} I_2}{\text{Re} I_2} \right) \tag{34}
\]
which proves the formula (30), where

\[
\phi_{sc} = -\arctan \left( \frac{\text{Im} I_2}{\text{Re} I_2} \right)
\]  

(35)

and

\[
\phi_R = \arctan \left( \frac{1 - 2G_S I_1}{2G_S I_2} \right) - \frac{1}{2G_S I_2} \frac{1}{\text{Re} I_2} \frac{1}{\text{Im} I_2} .
\]  

(36)

Using the conditions (26), in the above equation masses and widths could be identified in accordance with (28).

As the first part of this decomposition is independent of mesonic properties, we presume that it is connected with the scattering states’ input to the thermodynamics. On the other hand, the second part of (34) describes solely the behavior of dissolving mesons. Thus we are assured that the correct description of mesonic correlations accounts not only for bound and resonant states’ contribution, but also for the input from the scattering states.

In our analysis we will use a combined approach, where the scattering part of the phase shift \( \Phi_M \) is defined according to (35) and the resonant part is given by a delta function below \( T_{\text{Mott}} \) and by the Breit-Wigner ansatz (27) beyond it. Explicitly, we take

\[
D_M(s) = \frac{1}{\pi} \frac{d\phi_{sc}(s)}{ds} \quad \text{and} \quad \phi_{sc} = -\arctan \left( \frac{\text{Im} I_2}{\text{Re} I_2} \right)
\]

\[
\phi_R = \arctan \left( \frac{1 - 2G_S I_1}{2G_S I_2} \right) - \frac{1}{2G_S I_2} \frac{1}{\text{Re} I_2} \frac{1}{\text{Im} I_2} .
\]

We will in the following regard the states for positive real \( s \geq 4m^2 \) only.

Finally, the scattering states’ contribution to the density of states is normalized and exactly compensates the contribution from the resonance (or the bound state, resp.). This accordance with the Levinson theorem confirms the validity of the presented approach.

4. RESULTS AND DISCUSSION

The above described complex mass pole analysis leads to the results for the meson masses \( M_M \) and widths \( \Gamma_M \) as presented on Fig. 3. We observe that the sigma meson width is non-zero for all the temperatures considered, although below \( T_{\text{Mott}} \) it is not significant and therefore allows to consider sigma to be a quasi bound state. Above the Mott temperature pion and sigma masses quickly become equal and so do their spectral widths. This is the imprint of the chiral symmetry restoration where the \( \sigma \) and \( \pi \) meson, being chiral partners, become degenerate.

In what follows we will concentrate on the discussion of the \( \pi \) meson, because it undergoes the Mott transition from the bound state to the resonant correlation in the continuum, accompanied by a jump of the scattering phase shift at threshold from \( \pi \) to 0 in accordance with the Levinson theorem.

In Fig. 4 we show the phase shift \( \Phi_M \) (lower panel) in the decomposition (30) into its resonant (upper panel) and scattering continuum (middle panel) parts, obtained from the solution of Eqs. (35) and (36).
The pion pressure resulting from utilizing the Breit–Wigner spectral function alone and together with the input from the scattering states is shown on the Fig. 5 along with the pressure of the massive pion gas. Figure 6 shows the pion pressure as compared with the quark pressure obtained from the mean field approximation of the PNJL thermodynamic potential. We notice that including the description of the scattering states results in a correct description of pion thermodynamics in the vicinity and beyond the Mott temperature.

5. CONCLUSIONS

Pion dissociation by the Mott effect in quark plasma was described within the generalized Beth–Uhlenbeck approach on the basis of a PNJL model whereby a unified description of bound, resonant and scattering states was given. As a first approximation, we utilized the Breit–Wigner ansatz for the spectral function and clarified its relation to the complex mass-pole solution of the pion Bethe–Salpeter equation. It has been demonstrated that a description of the pion Mott dissociation solely by a spectral broadening of the pion bound state when it enters the continuum of unbound states for temperatures beyond $T_{\text{Mott}}$ necessarily entails a violation of Levinson’s theorem. In order to solve this problem we have extended the approach beyond the complex mass pole approximation and solved the scattering phases in the pion channel of quark-antiquark interaction. The account for the scattering continuum in accordance with the Levinson theorem leads to a strong reduction of the pion pressure above the Mott dissociation temperature. We suggest that the behavior of the scattering phase shift in the pion channel and its temperature dependence across the Mott transition, as obtained in the present work, can be used to develop a generic ansatz for the behavior of hadronic densities of states to be used in a generalized PNJL-hadron resonance gas model [23] that embodies the Mott dissociation of hadrons.

APPENDIX

1. A NAMBU–JONA-LASINIO MODEL WITH POLYAKOV LOOP

Confinement in pure $SU(N_c)$ gauge theory can be simulated by introducing an effective potential for a complex Polyakov loop field. The PNJL Lagrangian [15–19] is

$$\mathcal{L}_{\text{PNJL}} = \mathcal{L}(\Phi, \bar{\Phi}) - U(\Phi, \bar{\Phi}; T).$$

The quark fields are coupled to the gauge field $A_{\mu}$ through the covariant derivative $D_{\mu} = \partial_{\mu} - i\gamma_{\mu}A_{\mu}$. The gauge field is $A_{\mu} = \delta_{\mu}^{\nu}A_{\nu} + i\delta_{\mu}^{\nu}A_{4}$ (the Polyakov gauge). The field $\Phi$ is determined by the trace of the Polyakov loop $L(\tilde{x})$ [17]

$$\Phi[A] = \frac{1}{N_c} \text{Tr}_c L(\tilde{x}),$$

where $L(\tilde{x}) = \mathcal{P} \exp \left[ -i \int_0^T d\tau A_{\mu}(\tilde{x}, \tau) \right]$. $\Gamma_M$ are the vertices for the scalar ($\sigma'$) and pseudoscalar ($\tilde{\pi}$) four-fermion interaction channels. The gauge sector of the Lagrangian density (38) is described by an effective potential

$$\mathcal{U}(\Phi, \bar{\Phi}; T).$$

The parameters of the effective potential (40) and (41) are summarized in Table A.
In general, the parameter $T_0$ depends on the number of active flavors and the chemical potential. In the present work we use $T_0 = 208$ MeV as has been proposed in [24].

The partition function in the path-integral representation is then given by

$$\mathcal{Z}_{\text{PNJL}}[T, V, \mu] = \int \mathcal{D} \bar{q} \mathcal{D} q \exp \left\{ - \int_0^\beta \int_0^\beta d\tau d\tilde{x} [\bar{q}(i\gamma^\mu \partial) \cdot (\bar{\sigma}_\mu - iA_\mu) - m_0 - \gamma^0 \mu q + G_s (\bar{q} \Gamma_\sigma q)^2 + G_s (\bar{q} \Gamma_\pi q)^2 - U(\Phi[A], \bar{\Phi}[A]; T) \right\},$$

where the interaction vertices are written explicitly. By means of the Hubbard–Stratonovich transformation we are able to integrate out quark degrees of freedom to arrive at the partition function written solely in terms of collective fields

$$\mathcal{Z}_{\text{PNJL}}[T, V, \mu] = \int \mathcal{D} \sigma \mathcal{D} \pi \exp \left\{ - \int_0^\beta \int_0^\beta d\tau d\tilde{x} \left( \sigma^2 + \pi^2 \over 4G_S \right) + \text{Tr} \ln[\beta S^{-1}[\sigma', \pi']] \right\},$$

where $S^{-1}[\sigma', \pi']$ is the inverse propagator given explicitly as

$$S^{-1}[\sigma', \pi'] = \gamma^0 (i\omega_n - \mu + A_0) - \tilde{\pi} \cdot \tilde{p} - m_0 - \sigma \Gamma_\sigma - \pi \Gamma_\pi.$$

The operation Tr is taken over color, flavor, Dirac and momentum indices of quark fields.

2. THE BREIT–WIGNER ANSATZ

We introduce the following pion propagator

$$S_m(p^2) = \frac{1}{1 - 2G\Pi(p^2)}. \tag{45}$$

In the first step we expand the polarization function around the mass pole

$$\Pi(p^2) = \Pi(p^2 = M^2) + (p^2 - M^2) \frac{d\Pi(p^2)}{dp^2} \bigg|_{p^2 = M^2} + \ldots \tag{46}$$

and we insert this expansion into the defined propagator

$$S_m(p^2) = \frac{1}{1 - 2G\Pi(p^2 = M^2) - 2G(p^2 - M^2) \frac{d\Pi}{dp^2} \bigg|_{p^2 = M^2}} \overset{=0}{=} \frac{1}{2G(p^2 - M^2) \frac{d\Pi}{dp^2} \bigg|_{p^2 = M^2}} = \frac{\dot{g}_{Mq\dot{q}}}{p^2 - M^2}. \tag{47}$$

where in the last step we simply defined $\dot{g}_{Mq\dot{q}}$. By means of the wave function renormalization we define a normalized propagator in the following way

$$\dot{S}_m(p^2) = \dot{g}_{Mq\dot{q}} \dot{S}_m(p^2). \tag{48}$$

Now $\dot{S}(p^2)$ is a propagator for renormalized fields $\dot{\phi} = \dot{g}_{Mq\dot{q}} \phi$. At this point we consider a complex mass pole solution

$$p^2 = \left( M^2 + i\Gamma^2 \right)^\frac{1}{2}, \tag{49}$$

where for small $\Gamma$ we get $p^2 \approx M^2 \pm i\Gamma^2$ and the corresponding propagator, in a similar way,

$$\dot{S}_m(p^2) = \frac{1}{p^2 - M^2 \mp i\Gamma^2} \overset{=0}{=} \frac{p^2 - M^2 \mp i\Gamma^2}{(p^2 - M^2)^2 + (\Gamma^2)^2}. \tag{50}$$

We introduce the spectral function by its definition to get

$$A(s = p^2) = 2i\text{Im} \dot{S}_m \pm \frac{\Gamma^2}{(s - M^2)^2 + (\Gamma^2)^2}. \tag{51}$$

ACKNOWLEDGMENTS

D.B. acknowledges support and hospitality during his stay at the University of Bielefeld where this work was completed. This work was supported in part by the Polish National Science Center (NCN) within the “Maestro” grant programme (A.W. and D.B.) under contract number 2011/02/A/ST2/00306 and by the Russian Fund for Basic Research under grants no. 11-02-01538-a (D.B.) and no. 12-01-00396, 13-01-00060 (Yu.L.K.). A.W. and A.V.F. acknowledge support from the Bogoliubov-Infeld programme for collaboration between JINR Dubna and Polish Institutions.

| Table A. Parameters of the effective potential $\mathcal{U}[A]$ |
|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $a_0$         | $a_1$         | $a_2$         | $a_3$         | $b_3$         | $b_4$         |
| 6.75          | -1.95         | 2.625         | -7.44         | 0.75          | 7.5           |

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REFERENCES

1. E. Beth and G. E. Uhlenbeck, “Quantum theory of the non-ideal gas,” Physica 3, 728 (1936); 4, 915 (1937).
2. R. Dashen, S.-K. Ma, and H.J. Bernstein, “S Matrix formulation of statistical mechanics,” Phys. Rev. 187, 345–370 (1969).
3. J. Hufner, S. P. Klevansky, P. Zhuang, and H. Voss, “Thermodynamics of a quark plasma beyond the mean field: A generalized Beth–Uhlenbeck approach,” Annals Phys. 234, 225 (1994).
4. P. Zhuang, J. Hufner, and S. P. Klevansky, “Thermodynamics of quark—meson plasma in the Nambu-Jona-Lasinio model,” Nucl. Phys., Ser. A 576, 525–552 (1994).
5. S. Roessner, T. Hell, C. Ratti, and W. Weise, “The chiral and deconfinement crossover transitions: PNJL model beyond mean field,” Nucl. Phys., Ser. A 814, 118–143 (2008).
6. K. Yamazaki and T. Matsui, “Quark-hadron phase transition in the PNJL Model for interacting quarks”, Nucl. Phys., Ser. A 913, 19–50 (2013).
7. D. Blaschke, D. Zablocki, M. Buballa, and G. Roepke, Generalized Beth–Uhlenbeck approach to mesons and diquarks in hot, dense quark matter; arXiv:1305.3907 [hep-ph].
8. Y. Nambu and G. Jona-Lasinio, “Dynamical model of elementary particles based on an analogy with superconductivity,” Phys. Rev. 122, 345–358 (1961).
9. Y. Nambu and G. Jona-Lasinio, “Dynamical model of elementary particles based on an analogy with superconductivity,” Phys. Rev. 124, 246–254 (1961).
10. M. K. Volkov, “Meson Lagrangians in a superconductor quark model,” Annuals Phys. 157, 282 (1984).
11. U. Vogl and W. Weise, “The Nambu and Jona Lasinio model: its implications for hadrons and nuclei,” Prog. Part. Nucl. Phys. 27, 195–272 (1991).
12. S. P. Klevansky, “The Nambu-Jona-Lasinio model of quantum chromodynamics,” Rev. Mod. Phys. 64, 649–708 (1992).
13. T. Hatsuda and T. Kunihiro, “QCD phenomenology based on a chiral effective Lagrangian,” Phys. Rept. 247, 221–367 (1994).
14. M. Buballa, “NJL model analysis of quark matter at large density,” Phys. Rept. 407, 205–376 (2005).
15. P.N. Meisinger, T. R. Miller, and M. C. Ogilvie, “Phenomenological equations of state for the quark gluon plasma,” Phys, Rev., Ser. D 65, 034009 (2002).
16. K. Fukushima, “Chiral effective model with the Polyakov loop,” Phys. Lett., Ser. B 591, 277–284 (2004).
17. C. Ratti, M. A. Thaler, and W. Weise, “Phases of QCD: Polyakov thermodynamics and a field theoretical model,” Phys. Rept., Ser. D 73, 014019 (2006).
18. S. Roessner, C. Ratti, and W. Weise, “Polyakov loop, diquarks and the two-flavour phase diagram,” Phys. Rept., Ser. D 75, 034007 (2007).
19. H. Hansen, et al., “Mesonic correlation functions at finite temperature and density in the Nambu-Jona-Lasinio model with a Polyakov loop,” Phys. Rev., Ser. D 75, 065004 (2007).
20. H. Grigorian, “Parametrization of a nonlocal, chiral quark model in the instantaneous three-flavor case: basic formulas and tables,” Phys. Part. Nucl. Lett. 4, 223 (2007).
21. P. Costa, et al., “How parameters and regularization effect the PNJL model phase diagram and thermodynamic quantities,” Phys. Revo., Ser. D 81, 016007 (2010).
22. H. Abuki, “BCS/BEC crossover in quark matter and evolution of its static and dynamic properties: from the atomic unitary gas to color superconductivity,” Nucl. Phys., Ser. A 791, 117–164 (2007).
23. L. Turko, et al., “Mott-Hagedorn resonance gas and lattice QCD results,” Acta Phys. Polon. Supp. 5, 485 (2012).
24. B.-J. Schaefer, J. M. Pawlowski, and J. Wambach, “The phase structure of the Polyakov–Quark-Meson model,” Phys. Revo., Ser. D 76, 074023 (2007).