An Improved PMF-FFT Algorithm Based on Hamming Window

Lingfeng Cheng 1*, Shuyan Ni 2, Shimiao Chen 1

(1. Company of Postgraduate Management, Space Engineering University, Beijing 101407, China;
2. Department of Electronic and Optical Engineering, Space Engineering University, Beijing 101407, China)

*Corresponding author’s e-mail: 2078039965@qq.com.cn

Abstract: When the rocket uses low-orbit satellite space-based measurement and control, due to the relatively large Doppler shift in relative motion, a spread spectrum system is needed to improve anti-interference; the spread spectrum signal under high dynamic conditions often uses partial matched filtering based on FFT (PMF-FFT) algorithm, but due to the introduction of FFT, this algorithm has serious scallop loss, which affects the system’s capture performance. Through the improved method of adding a Hamming window to the system, theoretical analysis and simulation show that it effectively suppresses scallops. Loss has a certain meaning for the acquisition of spread spectrum signals.

1. Introduction

The measurement and control of the traditional rocket test mainly relies on the ground-based measurement and control station, but the coverage of the ground-based measurement and control station is limited. Once the test flight route changes, the equipment and personnel need to be re-arranged, which imposes requirements on manpower and material resources. The application of relay satellites can greatly improve the measurement and control coverage capability, but the resources are limited and it is difficult to be competent for a large number of measurement and control tasks. In recent years, the development of Internet satellites has been in the ascendancy, with large numbers and wide coverage, and has become an important means of rocket testing. In the forward link, spread spectrum signals are usually used to improve anti-interference. The rocket and the satellite are in a fast moving state at all times, and they have a relatively large relative velocity, resulting in a large Doppler frequency shift, up to MHz, and there is a large Doppler frequency between the received signal and the transmitted signal. There is a large phase difference between the pseudo-code phase and the receiving end, which makes the synchronization unsuccessful and causes the performance of the communication system to deteriorate.

Literature [1] introduced the matched filter method to the acquisition of PN codes for the first time. It focused on reducing the time required to search for a code interval, effectively reducing the integration stay time, and becoming a fast acquisition algorithm. However, it is difficult to achieve in practice due to issues such as device size [2]. With the development of microelectronics technology, the digitization of matched filters, namely digital filters (DMF), has been realized. Due to the simple algorithm, it has been widely used. But the essence of spread spectrum signal acquisition is the process of two-dimensional acquisition of carrier frequency and pseudo-code phase. Pure DMF technology is very
sensitive to Doppler frequency shift and cannot eliminate the influence of Doppler frequency shift well, so it needs to introduce FFT technology to realize the acquisition of spread spectrum signal together [3]. In [4], a method of combining FFT and partially matched filters is proposed to reduce hardware resource consumption and realize phase search while reducing frequency offset sensitivity.

However, the FFT method also brings scallop loss, thereby increasing the probability of false alarms [5-6]. In response to this situation, this paper proposes an algorithm to add a Hamming window to the system to improve the impact of scallop loss and improve the capture ability of the system.

2. PMF-FFT capture algorithm

The PMF-FFT algorithm is an acquisition algorithm for spread spectrum signals under high dynamic conditions. The pseudo code length is \(N\), the chip width is \(T_c\), the carrier frequency offset is \(f_d\), the sampling interval is \(T_s\), and the sampling factor \(K=T_c/T_s\). The number of partially matched filters is \(M\) and the length is \(X\) \((N=K*M*X)\). The biggest difference between this capture algorithm and the pure DMF algorithm is that the output of the \(M\times X\) correlators is not directly summed, but divided into \(M\) parts and then do FFT transformation, compare the \(M\) FFT transformation results with the threshold, if the threshold is exceeded, it is considered that the signal has been captured [7], and if it does not exceed the threshold, it will continue to move the pseudo-code phase to search and capture.

The expression of the data signal after down-conversion input to the receiver of each PMF:

\[
\tilde{x}(n) = \sqrt{PT_s} \sin(\pi f_d T_s) \frac{\sin(\pi f_d T_s)}{\pi f_d T_s} c(n-L)e^{-j(\Delta omTs-\varphi)}
\]  

Then the output of the \(m\)th PMF can be expressed as

\[
y(n) = \sum_{i=-(n-\Delta-1)X}^{n-1-\Delta X} x(i)c_R
\]

\[
=\sqrt{PT_s} \frac{\sin(\pi f_d T_s)}{\pi f_d T_s} \sum_{i=-(n-\Delta-1)X}^{n-1-\Delta X} c(i-L)c(n-MX)e^{-j(2\pi f_d T_s+\varphi)}
\]  

When the pseudo-code phase acquisition is successful, that is \(\varphi(i-L)(i-n+MX)=\varphi(i-L)\) \(\equiv 1\), Perform FFT transformation on the part sum of \(M\) PMF output, and finally normalize it to get

\[
G_{\text{PMF-FFT}}(k,f_d) = \sin(\pi f_d XT_s) \frac{\sin(\pi f_d XT_s)}{\pi f_d XT_s} M \sin(\pi f_d XT_s) \frac{\sin(\pi f_d XT_s)}{\pi f_d XT_s} e^{j\varphi(k,f_d)}
\]  

(3)

Where \(\varphi(k,f_d) = \pi(M-1)(f_d XT_s-k/M)-(\pi f_d(X-1)T_s\right), Equation (3) is the gain formula for any \(k\). In practical applications, when \(f_d\) is constant, we find the gain values under all \(k\) values, compare these \(M\) values, and take the largest one and compare it with the preset threshold. In formula (3), \(G_{\text{PMF-FFT}}(k,f_d) = G_{\text{PMF}}(f_d) \times G_{\text{FFT}}(k,f_d)\), \(G_{\text{PMF}}(f_d)\) is generated by the relevant accumulator, \(G_{\text{FFT}}(k,f_d)\) is caused by the incomplete compensation of the phase that exists during the fast Fourier transform, make \(\pi(f_d XT_s-k/M)\) close to zero, you can get the maximum value of \(G_{\text{FFT}}(k,f_d)\).
The length of the pseudo code is $N=1023$, $M=32$, $X=64$, $R_c=1.023\text{Mb/s}$. Two zeros are added to the last PMF. We can get the normalized gain graph of PMF-FFT and PMF.

![Diagram](image)

Figure 1 PMF-FFT algorithm implementation block diagram

(a) Pure DMF normalized gain
According to the analysis of the comparison chart in Figure 2, PMF-FFT can significantly reduce the Doppler frequency shift sensitivity, which is a great improvement for the DMF algorithm. Compared with the FFT algorithm, the complexity is lower. However, it can be seen from Figure (b) that the trough attenuation between the two peaks is large, the maximum is close to 2dB, which is not conducive to the system to capture the Doppler shift. This phenomenon is called scallop loss [8-9].

### 3. Algorithm improvement

First of all, we need to be clear that the scallop loss is caused by the compensation of the FFT algorithm. The contribution of the FFT part to the gain of the entire system is

$$G_{FFT}(k, f_d) = \frac{\sin[\pi M (f_d XT - k / M)]}{M \sin[\pi (f_d XT - k / M)]} e^{i \pi (M - 1)(f_d XT - k / M)}$$

It can be seen from Figure 3 that the amplitude of the normalized gain of the FFT part at 500Hz, 1500Hz and other trough positions is about 0.65. It can be seen that the maximum attenuation caused by the scallop effect is about 1.9dB. In order to capture the code phase smoothly, the preset threshold It cannot be higher than the minimum gain of scallop loss, but a too low threshold significantly increases the false alarm probability of capture. Therefore, the impact of scallop loss must be suppressed as much as possible.
The more commonly used methods to suppress the scallop loss caused by FFT are the zero-padded method and the window function method \[^{[10]}\]. The basis of zero-padded is to improve the spectrum by increasing the number of FFT operations, but this will significantly increase the amount of calculations, which is important for real-time performance. For a relatively high system, the consumption of hardware resources is greatly increased. The window function method is a relatively more effective and feasible method to suppress the scallop effect. By adding windows, the width of the main lobe can be increased and the frequency spectrum can be enhanced. Next, we use Hamming window to perform simulation experiments to verify the ability of the window function method to suppress the scallop effect.

The definition of Hamming window is as follows:

\[
\omega(n) = 0.54 - 0.46 \cos\left(\frac{2\pi n}{N}\right),\quad n = \frac{N}{2}, \cdots, 0, \cdots, \frac{N}{2}
\]  

(5)

Its frequency response can be expressed as

\[
W(e^{j\omega}) = 0.54U(\omega) + 0.23U(\omega - \frac{2\pi}{N}) + 0.23U(\omega + \frac{2\pi}{N})
\]  

(6)

Where \( U(\omega) = e^{j\omega/2} \sin\left(\frac{\omega N}{2}\right) / \sin(\omega/2) \), \( U(\omega) \) is the frequency response of the rectangular window, the normalized signal after the system frequency response processing of the Hamming window is expressed as

\[
G'_{FFT}(k, f_d) = [A_{FFT}(k, f_d) + 0.43A_{FFT}(k, f_d - \frac{1}{NT}) + 0.43A_{FFT}(k, f_d + \frac{1}{NT})]
\]  

(7)

Figure 4 shows that the scallop loss is effectively suppressed when the Hamming window is added. The minimum normalized amplitude at the trough is 0.82, which is about 1dB gain compared to before the window is not added.

The gain properties of FFT are similar to those of rectangular windows. The important reason for the scallop loss of FFT is that the rectangular window is narrow, which causes serious spectrum leakage. The introduction of window function can increase the width of the main lobe, reduce the spectrum leakage, and broaden the main lobe. It will make the amplitude value of the adjacent FFT intersection point higher, which effectively improves the scallop loss of the system.
4. Conclusion

The acquisition of spread spectrum signals is essentially the acquisition of code phase and carrier frequency offset. Since pure DMF and PMF are very sensitive to Doppler frequency shift, we introduced FFT into the capture, which effectively improved the Doppler frequency offset. It is a sensitive issue, but due to the spectrum leakage and fence effect of the FFT itself, it brings a large scallop loss to the system. In response to this phenomenon, an algorithm for adding a Hamming window to the system is proposed. By increasing the width of the main lobe, the scallop effect can be effectively suppressed, and a gain of about 1dB can be obtained.

References

[1] Holmes J K. Coherent spread spectrum systems[J]. New York, 1982.
[2] Lu Shanshan. Research on the acquisition technology of high dynamic spread spectrum weak signal[D]. Xi dian University, 2018.
[3] Stirling-Gallacher R A, Hulbert A P, Povey G J R. A fast acquisition technique for a direct sequence spread spectrum signal in the presence of a large Doppler shift[C]//Proceedings of ISSSTA’95 International Symposium on Spread Spectrum Techniques and Applications. IEEE, 1996, 1: 156-160.
[4] Spillard C L, Spangenberg S M, Povey G J R. A serial-parallel FFT correlator for PN code acquisition from LEO satellites[C]//1998 IEEE 5th International Symposium on Spread Spectrum Techniques and Applications-Proceedings. Spread Technology to Africa (Cat. No. 98TH8333). IEEE, 1998, 2: 446-448.
[5] Yuan Yu. Research on Fast Acquisition Algorithm of Spread Spectrum Code Based on Matched Filter and FFT [D]. Harbin Institute of Technology, 2013.
[6] Zhang Kai. Research on Fast Acquisition Method of High Dynamic Spread Spectrum Signal [D]. Xi dian University, 2014.
[7] Qiao Zishi. Design and implementation of dual-code fast synchronization spread spectrum system [D]. Chongqing University, 2017.
[8] Ma Ruofei, Ding Shipu. Research on fast acquisition technology of high Doppler spread spectrum communication[J]. Radar and Countermeasures, 2018, 38(03): 34-37.
[9] Wang Yunjing, Zhang Xiaolin, Zheng Kun. An improved PMF-FFT algorithm based on all-phase preprocessing[J]. Telemetry and Remote Control, 2018, 39(03): 15-20.

[10] Cui Yaozhong. Research on fast acquisition and tracking technology based on large frequency offset and high dynamic spread spectrum signal [D]. University of Electronic Science and Technology of China, 2019.