Diffraction of quasimonochromatic light beams on multilayer inhomogeneous photopolymer holographic diffraction structures

D I Dudnik¹, A O Semkin¹, S N Sharangovich¹

¹Tomsk State University of Control Systems and Radioelectronics, Lenina ave. 40, Tomsk, Russia, 634050

Abstract. A theoretical model of quasimonochromatic light beams diffraction by spatially inhomogeneous transmitting multilayer diffraction structures formed in a photopolymer material by a holographic method is presented in this work. This model takes into account the heterogeneity of the amplitude profile of the first harmonic of the refractive index of each layer of the multilayer structure. The model is presented using method of transfer matrixes whose components are the solutions of the coupled waves equations of partial derivates.

1. Introduction

Today optical elements based on systems of diffraction gratings and waveguide channels are widely used [1-4]. So, searching for new materials that are promising for forming similar structures in them and controlling their optical properties seems to be very relevant. Photopolymerizing materials (PPMs) are becoming more widely used.

Multilayer structures are several volume gratings separated by optically homogeneous intermediate layers [5, 6]. Such structures are characterized by special properties due to the interference of waves reconstructed from each grating and provide the ability to control the type of selective response. Multilayer holographic structures have the prospect of application as elements of spectral filters, sensors, interconnects, multiplexers / demultiplexers in optical communication lines [7, 8].

In [5, 8–11], models of plane waves diffraction on multilayer diffraction structures are presented and take into account only a certain type of homogeneity or do not take it into account at all.

The purpose of this work is to develop a theoretical model of diffraction of quasimonochromatic light beams on multilayer inhomogeneous holographic diffraction structures (MIHDS), which will take into account the inhomogeneities of amplitude profile of the refractive index harmonics for each individual layer.

2. Approaches and limitations

In this paper, we consider the process of reading MIHDS by an arbitrarily polarized monochromatic light beam, neglecting the residual absorption in PPM $E^0(r,t)$.

For this model, it is declared that the diffraction of light beams occurs on structures for which all recording processes have ended (the entire monomer has been used up), i.e. the incident beam cannot change the spatial profile of the refractive index of the structure. Also it’s considered that the aperture
of the read beam $w>>d$, where $d$ – thickness of one MIHDS layer. The readout beam’s polarization state is uniform over the aperture.

Thus, diffraction processes will be described in the geometrical optics approximation. The effects of scattering of optical radiation in the material are also assumed to be negligibly small; to perform this approximation, samples of small thickness are considered. Diffraction is considered only at the fundamental spatial harmonic of the refractive index, since the amplitudes of higher harmonics decrease exponentially with increasing number. Moreover, the above approach to the description of diffraction is similar for higher spatial harmonics with the replacement of the corresponding amplitudes and angles of diffraction.

3. Coupled-wave equations

Let an arbitrarily polarized quasimonochromatic light beam with an amplitude profile $E_0^0(\omega, r)$, wave vector $k_0^0$ and unit complex polarization vector $e_0^0$ falls on the perturbed by inclined DS area of the PPM at an angle $\theta_p$ to the $y$-axis [12]. Then, the incident quasiplane light wave at the interface ($y=0$) can be represented as a sum of linearly polarized beams with mutually orthogonal $s$ and $p$ polarization states:

$$E_0^0(r,t) = \frac{1}{2} \left[ \sum_{m=\pm, p=\pm} \int_{-\infty}^{\infty} e_m^m \cdot e_p^p \cdot \exp[i\left(\omega_0 + \omega\right)t - k_m \cdot r] \right] d\omega + c.c.,$$

where $\omega_0$ – central frequency; $e_m^m$ – unit polarization vector; $(m=s,p)$, $s$ corresponds to a wave polarized perpendicular to the diffraction plane $YZ$, $p$ corresponds to a wave polarized in the diffraction plane $YZ$; $E_m^m(\omega, r) = (e_0^s \cdot e_m^m)E_0^0(\omega, r)$ – spatial distribution of the complex amplitude of the frequency Fourier component.

The diffraction geometry at the MIHDS is shown in Figure 1. $E_0^0$ – incident (reading) beam. $E_0^s$, $E_0^p$ – transmitted and diffracted beams.

![Figure 1. Diffraction pattern: at MIHDS (a); on the n-th layer of MIHDS (b).](image)

Consider diffraction at the nth layer of the MIHDS, for which the distribution of complex amplitudes of diffracting beams at the output depends on the corresponding distribution of diffracting beams at the output of previous layer.

Light field $E$ in each layer DS due to reading beam $E_0^0$ (1) diffraction on spatial harmonics can be represented as a sum of 0th and 1st diffraction orders:

$$E(r,t) = \sum_{j=0,1} E_j = \frac{1}{2} \left[ \sum_{m=\pm, p=\pm} \sum_{j=0,1} \int_{-\infty}^{\infty} e_m^m \cdot E_j^m(\omega, r) \cdot \exp[i\left(\omega_0 + \omega\right)t - k_m \cdot r] \right] d\omega + c.c.,$$

each of which is represented by two components of vector $E_j$ in the corresponding orthogonal
polarization basis defined by two orts $e^j_0$ and $e^j_1$, lying in a plane perpendicular to the axis of the beam $E_j$. Here $E_j^{m}(\omega, r)$ – slowly varying coordinate functions and are found from the equations of the first approximation of method of slowly varying amplitudes. $j=0$ corresponds to the transmitted beam, $j=1$ – corresponds to the diffracted beam with $K = j \cdot K$.

Electric field strength $E(r, t)$ in the interaction region is described by a vector wave equation following from Maxwell’s equations [12, 13]:

$$\nabla \times \nabla \times \mathbf{E}(r, t) = -\mu_0 c_0^2 \frac{\partial^2}{\partial t^2} \left[ \varepsilon(r, t) \cdot \mathbf{E}(r, t) \right],$$  \hspace{1cm} (3)

where the perturbation of the dielectric constant is represented as:

$$\varepsilon(r) = \varepsilon_0 + \Delta \varepsilon(r) = \varepsilon_0 + 0.5 \cdot n_s \left[ n_0(r) + n_e(r) \cdot e^{K \cdot r} + c.c. \right],$$  \hspace{1cm} (4)

where $\varepsilon_0 = n_a^2 \cdot \hat{I}$ – unperturbed dielectric tensor, $n_0(\nu)$ – determined by the resulting solutions for the DS recording process [14, 19, 20], $\hat{I}$ – unit vector.

According to method of slowly varying amplitudes, in the case under consideration, the Bragg diffraction of light beams on a DS in an optically inhomogeneous layer of the PPM, amplitudes $E_j^{m}(r)$ of interacting waves are determined by two systems of coupled-wave equations (CWE) in partial derivatives [12, 15]:

$$\begin{align*}
\mathbf{N}_{r0} \cdot \nabla E_0^{m}(r) &= -i C_0^{m} E_0^{m}(r) n_0(r) \exp(+i \Delta K \cdot r) \\
\mathbf{N}_{r1} \cdot \nabla E_1^{m}(r) &= -i C_1^{m} E_1^{m}(r) n_1(r) \exp(-i \Delta K \cdot r),
\end{align*}$$  \hspace{1cm} (5)

where $E_j^{m}(r)$ – amplitude profiles of beams; $\mathbf{N}_{r0,1}$ – group normals; $E_j(r) = N_{r0,1}^{m} \cdot E_j^{m}(r)$; $C_j^{m}$ – amplitude coupling coefficients; $n_0(r)$ – normalized amplitude profile of the $n$ harmonic of the refractive index of the structure; $\Delta K$ – phase mismatch vector.

The amplitude coupling coefficients included in (5) are determined as:

$$\begin{align*}
C_0^{m} &= \frac{1}{4 \epsilon} \frac{\omega}{n_0^{m}} e_0^{m} \cdot \Delta \varepsilon(r, t) \cdot e_0^{m} \\
C_1^{m} &= \frac{1}{4 \epsilon} \frac{\omega}{n_1^{m}} e_1^{m} \cdot \Delta \varepsilon(r, t) \cdot e_1^{m},
\end{align*}$$  \hspace{1cm} (6)

where $\omega$ – angular frequency of light waves; $n_j^{m}$ – refraction index; $e_j^{m}$ – unit beam polarization vectors; $\Delta \varepsilon(r, t)$ - amplitude of first harmonic of the permittivity tensor perturbation due to recording.

For an arbitrary recording time, the resulting normalized spatial profile of the first harmonic amplitude $n_1(r) = n_1(\nu)$ in each layer may be inhomogeneous due to recording conditions [16]. Thus, the heterogeneity should be taken into account when solving the diffraction problem. The dependence of the refractive index profile along the $y$-axis can be approximated by a function of a special form similarly to [16]:

$$n_1(\nu, c, s, t) = ch^{-1}[\varepsilon(\nu s, t)],$$  \hspace{1cm} (7)

where parameters $c, s, t$ determine, respectively, the degree of heterogeneity, asymmetry, displacement in each layer, and are set by minimizing the mean-square deviation functional of the approximating function (7) from the formed profile, modeled and presented in [14].

Figure 2 shows the most characteristic variations of the refractive index profile $n_1(\nu)$ inhomogeneities, formed during the recording of MIHDS [14]: domed (curve 1), falling (curve 2), increasing (curve 3), homogeneous (curve 4). It should be noted that function (7) allows you to approximate all possible profile types.
Figure 2. Typical variations of profile inhomogeneities in one layer of MIHDS.

4. Amplitude profiles of the diffraction field at the output of the \( n \)th layer

In accordance with [15, 16], the amplitude profiles of diffractive beams at the output of sample in near zone can be conveniently represented in aperture coordinates \( \xi_0, \xi_1 \) (Figure 1).

The coordinate transformation equations have the form [13] (index \( m \) is omitted):

\[
\xi_0 = -\eta_0 y + V_0 z, \quad \xi_1 = \eta_1 y - V_1 z,
\]

where \( V_j = N_{ij} \cdot y_0, \eta_j = N_{ij} \cdot z_0 \).

Solutions of equations (5) in each layer can be found in an analytical form by the Riemann method similarly to [17, 18] in aperture coordinates \( (\xi_0, \xi_1) \) and presented in the form of recurrence relations, with the help of which one can sequentially describe the process of spatial profiles, through distributions \( E_{0,m,n}^0(\xi) \) and \( E_{1,m,n}^1(\eta) \) at its input.

\[
E_{0,m}^0(\xi) = E_{0,m,n-1}^0 \left( -\frac{\xi}{V_1} \right) - \frac{i C_d}{2 V_0} \exp \left[ i \frac{\Delta K d_n}{2} (1 - q) \right] \cos h^{-1} \left[ c(s(1 - q) / 2 - t) \right] \times F_1(-\alpha, \alpha; 1; w) \times
\]
\[
\times E_{1,m}^{n-1} \left( \frac{\xi - \delta(1 - q)}{V_0} \right) - i \frac{C_d}{2 V_0} A \sin h \left[ \frac{c(1 + q)}{2} \right] \times F_1(1 - \alpha, 1 + \alpha; 2; w) E_{0,m,n-1}^0 \left( \frac{\xi - \delta(1 - q)}{V_0} \right) dq , (9)
\]

\[
E_{1,m}^1(\eta) = E_{1,m,n-1}^1 \left( -\frac{\eta}{V_1} \right) - \frac{i C_d}{2 V_0} \exp \left[ i \frac{\Delta K d_n}{2} (1 - q) \right] \cos h^{-1} \left[ c(s(1 - q) / 2 - t) \right] \times F_1(-\alpha, \alpha; 1; w) \times
\]
\[
\times E_{0,m}^{n-1} \left( \frac{\delta(1 - q) - \eta}{V_0} \right) - i \frac{C_d}{2 V_0} A \sin h \left[ \frac{c(1 + q)}{2} \right] \times F_1(1 - \alpha, 1 + \alpha; 2; w) E_{1,m,n-1}^1 \left( \frac{\delta(1 - q) - \eta}{V_0} \right) dq , (10)
\]

where \( F_1(a, b, c; z) \) – Gauss hypergeometric function; \( w = \frac{\sin h[c(s(1 - q) / 2] \sin h[c(s(1 + q) / 2]}{\cosh cr \cos h[c(s-t)]} \right) ;
\]

\[
A = \left( cs \cdot \cosh [ct] \cosh [c(s-t)] \right)^{-1} \cdot \alpha = b^n; \quad \beta^n = \frac{d_s \cdot C_m}{\sqrt{V_0 V_1}}; \quad \delta = d_s (\eta_0 \nu_0 - \eta_1 \nu_1) / 2 V_1; \quad d_n \_ thickness of the \( n \)th layer; \quad \eta_j = \pm \sin \theta_j; \quad v_j = \cos \theta_j; \quad \theta_j \_ angles between \( N_{ij} \) and y-axis (Figure 1); \quad C^{mn}_p \_ coupling coefficients; parameters \( c, s, t \) are taken for each layer, according to the approximating function (7).

As a result, the spatial distributions of vector light fields in the 0th and 1st diffraction orders at the output of the \( n \)th layer of the MIHDS are determined by the following expressions [15, 16]:

\[
E_{0}^0(\xi) = e^{i \alpha_0} E_{0,r}^0(\xi) \exp[-i k_0^n \cdot r] + e^{i \beta_0^m} E_{0,q}^0(\xi) \exp[-i k_0^m \cdot r],
\]

\[
E_{1}^1(\eta) = e^{i \alpha_1} E_{1,r}^1(\eta) \exp[-i k_1^m \cdot r] + e^{i \beta_1^m} E_{1,q}^1(\eta) \exp[-i k_1^m \cdot r].
\]
The obtained solutions (11) completely determine both the amplitude and polarization parameters of the diffraction fields at the output of the \( n \)th layer of the MIHDS.

5. Diffraction field at the output of the MIHDS

To determine the diffraction light field at the output of the MIHDS, which consists of \( N \) DSs based on the PPM, which are separated by \( N-1 \) intermediate layers, we use the matrix method for describing the transformation of plane light waves in multilayer media.

To do this, we pass from the amplitude distributions of the frequency Fourier components of the diffracting beams (9), (10) to their angular spectra (AS):

\[
E_j(\theta) = \int_{-\infty}^{\infty} i k_j l \exp(ik_j l \theta) dl ,
\]

where \( l = \xi_0, \xi_1 \), angle \( \theta \) characterizes the direction of plane-wave components \( E_j(\theta) \) relative to wave normals.

As a result, the process of converting the frequency-angular spectra (FAS) of the interacting light fields of the 0th and 1st diffraction orders in the \( n \)th layer of the MIHDS with the thickness \( d_n \) is represented as

\[
E^n = T^n \times E'^{n-1} ,
\]

where the notation is introduced:

\[
E^n = \begin{bmatrix} E_0^n(\omega, \theta) \\ E_1^n(\omega, \theta) \end{bmatrix} ,
E'^{n-1} = \begin{bmatrix} E_0'^{n-1}(\omega, \theta) \\ E_1'^{n-1}(\omega, \theta) \end{bmatrix} ,
T^n = \begin{bmatrix} T_{00}(\omega, \theta) & T_{01}(\omega, \theta) \\ T_{10}(\omega, \theta) & T_{11}(\omega, \theta) \end{bmatrix} .
\]

Here \( T^n \) – matrix transfer function (transition matrix) of the \( n \)th layer of MIHDS; \( E_0^n(\omega, \theta) \), \( E_1^n(\omega, \theta) \) – FASs at the input and output of \( n \)th layer.

Transition matrix components \( T^n \) defined by expressions:

\[
T_{00}(\omega, \theta) = 1 - \frac{b_0^{n2}}{2} A_i \left( -i \frac{\Delta K^n d_n}{2}(1-q) \right) \sinh \left[ \frac{c s (1+q)}{2} \right] \cdot F_i(1-\alpha, 1+\alpha; 2; w) dq ,
\]

\[
T_{01}(\omega, \theta) = -i \frac{b_0^n}{2} \sqrt{v_0} \left( -i \frac{\Delta K^n d_n}{2}(1-q) \right) \cosh \left[ c s (1-q)/2 - t \right] \cdot F_i(1-\alpha, 1+\alpha; 2; w) dq ,
\]

\[
T_{10}(\omega, \theta) = -i \frac{b_1^n}{2} \sqrt{v_1} \left( -i \frac{\Delta K^n d_n}{2}(1-q) \right) \cosh \left[ c s (1-q)/2 - t \right] \cdot F_i(-\alpha, 1; w) dq ,
\]

\[
T_{11}(\omega, \theta) = 1 - \frac{b_1^{n2}}{2} A_i \left( -i \frac{\Delta K^n d_n}{2}(1-q) \right) \sinh \left[ \frac{c s (1+q)}{2} \right] \cdot F_i(1-\alpha, 1+\alpha; 2; w) dq .
\]

It should be noted that in the particular case of the interaction of plane waves in MIHDS based on DS with homogeneous profiles \( n_i(y) \), components \( T_i \) of transition matrix \( T^n \), defined by expressions (14)-(17), become known ones [7].

In MIHDS, the intermediate layer with thickness \( t_n \) (Figure 1a) gives a phase shift, and if we assume that the refractive index of the intermediate layer is equal to the refractive index of the hologram, then transition matrix \( A^n \) for such a layer is defined by:

\[
A^n = \begin{bmatrix} \exp(-i(k^2 \cdot y_0) t_n) & 0 \\ 0 & \exp(-i(k^2 \cdot y_0) t_n) \end{bmatrix} .
\]

Multiplying the transition matrices of all layers, we can obtain a relation between the input field \( E_0 \) and diffracted field \( E^N \) at the output of MIHDS with thickness \( D \):

\[
E^N = T \times E_0 ,
\]
where $T = T^N \times A^{N-1} \times T^{N-1} \times \ldots A^2 \times T^2 \times \ldots A \times T$ – matrix transfer function (transition matrix) of the entire MIHDS;

$$D = \sum_{n=1}^{N} a_n + \sum_{n=1}^{N} t_n , \quad E_0 = \begin{bmatrix} E_0(\omega, \theta) \\ 0 \end{bmatrix}, \quad E_0(\omega, \theta)$$ – of a quasiminochromatic light beam incident on the MIHDS defined by expressions (1) and (12).

### 6. Diffraction properties of MIHDS

The obtained transfer functions create the mathematical basis for calculating the selective properties of MIHDS, namely: the dependence of the diffraction efficiency on the angle of incidence and the center frequency of the reading beam. For this, in expressions (14)-(17), it is necessary to use the dependences of the phase mismatch vector module $\Delta K$ on the angle of incidence and the frequency of the reading beam, taking into account the variance of the refractive index of the PPM (19).

$$\Delta K = \Delta K(\theta) + \Delta K(\omega),$$

where $\Delta K(\theta) = (D / B)\theta$, $\Delta K(\omega) = (C - AD / B)\omega$, coefficients $A, B, C, D$ are defined in [17].

Numerical modeling was carried out for a two-layer structure with a uniform profile of the refractive index and the thickness of the intermediate layer $t_n = 10\, \text{um}$ and $t_n = 175\, \text{um}$ with the thickness of one layer of hologram $d_n = 55\, \text{um}$ (Figure 3).

![Figure 3](image-url) Selectivity of a homogeneous bilayer holographic diffraction structure at intermediate layer thicknesses $t_n = 10$ and $t_n = 175\, \text{um}$.

Figure 3 shows that for a small thickness of the intermediate layer ($t_n = 10\, \text{um}$), for a uniform appearance of the refractive index profile, the shape of the selectivity profile of the entire structure is close to that of the characteristics of a single hologram. And when this parameter is increased due to interference effects, additional local maxima and minima of light intensity are formed, while the total value of diffraction efficiency is preserved, similarly to [5].

Figure 4 presents the results of modeling the selectivity of a single hologram (Figure 4a) and a two-layer structure with an exponentially decaying refractive index profile for symmetric beam geometry during recording (Figure 4b).

From the results obtained, it can be seen that, in numerical simulation of selectivity of structures under study, there are local maxima, the number and width of which are determined by the thickness of the intermediate layers and volume holograms, the envelope coincides with the selectivity contour of one holographic grating, and the intensity of local minima may not reach zero.

Also, based on presented theoretical model, the selective properties of a multilayer structure consisting of three identical holograms with dome-shaped refractive index profiles (Figure 5a) and decreasing refractive index profiles on each layer (Figure 5b) were obtained.

From the results obtained, it is seen that with an increase in the number of layers, the number of local maxima increases, and the envelope still coincides with the selectivity contour of one holographic grating.
Figure 4. The calculated selectivities of the inhomogeneous single hologram (a) and the bilayer inhomogeneous holographic diffraction structure (b).

Figure 5. Selectivity of MIHDS with a decreasing (a) and domed (b) refractive index profile $n_1(y)$.

7. Conclusion
Thus, a theoretical model of the diffraction of quasimonochromatic light beams by transmitting MIHDS is presented, taking into account the spatial inhomogeneity of the amplitude profile of the refractive index first harmonic that occurs during the holographic formation of gratings in a photopolymer material. The presented analytical solutions describe the evolution of spatial profiles of light beams and their frequency-angular spectrum during diffraction on MIHDS. The obtained solutions make it possible to calculate the polarization and diffraction characteristics (diffraction efficiency and selective properties) of MIHDS, which consists of volume inhomogeneous transmitting holograms separated by intermediate layers.

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9. References

[1] Nordin G P, Johnson R V and Tanguay A R 1992 *JOSA A* 9(4) 2206-2217.
[2] Hesselman L 1994 *JOSA B* 11(9) 1800-1808.
[3] Dovolnov E A and Sharangovich S N 2006 *Russian Physics Journal* 49(11) 1189-1197.
[4] Aabbate G, Marino A and Vita F 2003 *Acta Physica Polonica A* 103 177-186.
[5] Pen E F and Rodionov M Yu 2010 *Quantum Electronics* 40(10) 919-924.
[6] Malallah R, Li H, Qi Y, Cassidy D, Muniraj I, Al-Attar N and Sheridan J T 2019 *JOSA A* 36(3) 320-334.
[7] Nordin G P and Tanguay A R 1992 *Optics Letters* 17(23) 1709-1711.
[8] Aimin Y, Liren L, Yanan Z and Jianfeng S 2008 26(1) 135-141.
[9] Yan X, Wang X and Chen Y *Applied Physics B* 125(5) 1-8.
[10] Wang S S and Magnusso R 1995 *Applied Optics* 34(14) 2414-2420.
[11] Yan X, Gao L, Yang X, Dai Y, Chen Y and Ma G 2014 *Optics Express* 22(21) 26128-26140.
[12] Semkin A O and Sharangovich S N 2013 *Bulletin of the Russian Academy of Sciences. Physics* 77(12) 1416-1419.
[13] Fedorov F I 2004 *Optics of anisotropic media* (Moscow: Editorial URSS).
[14] Dovolnov E A and Sharangovich S N 2005 *Russian Physics Journal* 48(7) 766-774.
[15] Ustyuzhanin S V and Sharangovich S N 2011 *Russian Physics Journal* 54(2) 172-179.
[16] Nozdrevatkh B F, Ustyuzhanin S V and Sharangovich S N 2010 *Physics of Wave Phenomena* 18(4) 289-293.
[17] Sharangovich S N 1995 *Radioteknika i Elektronika* 40(4) 1211-1222.
[18] Sharangovich S N 1996 *Radiotekhnika i Elektronika* 41(11) 1364-1375.
[19] Sharangovich S and Dovolnov E 2004 Proc. SPIE "Organic Optoelectronics and Photonics" 5464 411-420.
[20] Dyachenko A A and Ryabukho V P 2019 Color models of interference images of thin stratified objects in optical microscopy *Computer Optics* 43(6) 956-967 DOI: 10.18287/2412-6179-2019-43-6-956-967.