Abstract
Collective, explosive flow in central heavy ion collisions manifests itself in the mass dependence of $p_T$ distributions and femtoscopic length scales, measured in the soft sector ($p_T \lesssim 1\, \text{GeV}/c$). Measured $p_T$ distributions from proton-proton collisions differ significantly from those from heavy ion collisions. This has been taken as evidence that $p+p$ collisions generate little collective flow, a conclusion in line with naive expectations. We point out possible hazards of ignoring phase-space restrictions due to conservation laws when comparing high- and low-multiplicity final states. Already in two-particle correlation functions, we see clear signals of such phase-space restrictions in low-multiplicity collisions at RHIC. We discuss how these same effects, then, must appear in the single particle spectra. We argue that the effects of energy and momentum conservation actually dominate the observed systematics, and that $p+p$ collisions may be much more similar to heavy ion collisions than generally thought.

1. Introduction and Motivation
Most of the interest in the RHIC program falls naturally on collisions between the heaviest nuclei at the highest energies, where the likelihood of generating a system, per se, is believed greatest. However, it is important to understand the broader context of these measurements; the absolute necessity for extensive systematics is a generic feature of any heavy ion study \cite{1,2}. In particular, the evolution of the physics as a function of energy may indicate the existence and location of predicted critical point in the Equation of State of QCD \cite{3}; the evolution as a function of system size (e.g. comparing $p+p$ to $Au+Au$ collisions) may reveal the emergence of bulk behaviour from the underlying structure from hadronic collisions.

It is by now well-established that heavy ion collisions at RHIC energies are dominated by collective hydro-like flow. While the degree to which the flowing medium is "perfect" \cite{4} remains under study, the strongly-coupled nature of the color-deconfined system is remarkable. It allows treatment of the system as a system, with thermodynamic quantities. Further, it promises access to the underlying Equation of State of QCD, together with transport coefficients like viscosity, sometimes viewed as a complicating factor, but which are in fact is interesting in itself \cite{5}. In central collisions, the evidence for collective flow comes from the mass dependence of transverse momentum ($p_T$) distributions and the $p_T$- and mass-dependences of femtoscopic length scales. These may be compared to hydro calculations, but are often fit with simple parameterizations such as the blast-wave \cite{6} to estimate the strength of the flow.

Surprisingly, pion HBT measurements in $p+p$ collisions at RHIC show an identical flow signal as seen in $Au+Au$ collisions \cite{7}. Indeed, similar systematics appear in several hadron-hadron
measurements [8]! This appears to be at variance with blast-wave fits to $p_T$ spectra [9], which suggest a much smaller transverse flow in $p+p$ collisions. Here, we suggest that the apparent difference between spectra from $p+p$ and $Au+Au$ collisions may be understood in terms of energy and momentum conservation effects, which are naturally stronger for the smaller system. For more details on this study, see [10].

2. A fairer comparison of spectra from A+A and p+p collisions

Figure 1 shows transverse momentum spectra for pions, kaons and protons measured by the STAR Collaboration for collisions at $\sqrt{s_{NN}}=200$ GeV. The spectral shapes evolve as the multiplicity is increased from $p+p$ collisions (at the bottom) to the highest-multiplicity $Au+Au$ collisions (top). A blast-wave [6] fit to these spectra indicates a steadily increasing (decreasing) flow velocity (freezeout temperature) with increasing multiplicity, as shown by the red circles on Figure 2. However, these fits entirely neglect effects of phasespace restrictions due to energy and momentum conservation, whose significance steadily increases with decreasing multiplicity.

In the approximation that dynamics and kinematic constraints can be factorized, the measured single-particle distribution $\tilde{f}_c$ from an $N$-particle final state is related to the “parent” distribution $f$ according to [10, 11]

$$\tilde{f}_c(p_i) \propto f(p_i) \cdot \exp \left[ -\frac{1}{2(N-1)} \left( \frac{p_{ix}}{(p_{ix})^2} + \frac{p_{iy}}{(p_{iy})^2} + \frac{p_{iz}}{(p_{iz})^2} + \frac{(E_i - \langle E \rangle)^2}{(E^2 - \langle E \rangle^2)} \right) \right],$$

where $\langle p_{ij}^n \rangle$ are average quantities of energy and 3-momentum.

We now use this formula to test the extreme postulate that the parent distributions— which reflect the underlying dynamics—are identical for $p+p$ and $Au+Au$ collisions at all centralities. In this case, the ratio of two measured spectra $\tilde{f}_{c,1}$ and $\tilde{f}_{c,2}$, from events with multiplicities $N_1$ and $N_2$, will be simply the ratio of their phasespace factors:

$$\frac{\tilde{f}_{c,1}(p_T)}{\tilde{f}_{c,2}(p_T)} \propto \exp \left[ \left( \frac{1}{2(N_1-1)} - \frac{1}{2(N_2-1)} \right) \cdot \left( \frac{2p_{tx}^2}{(p_{tx})^2} + \frac{E_x^2}{(E_x)^2} - \frac{2E_x(E_x)}{(E_x)^2} + \frac{(E_x)^2}{(E_x)^2} \right) \right].$$

Figure 1: Pion (left), kaon (center) and antiproton (right) $m_T$ distributions measured by the STAR Collaboration for $\sqrt{s_{NN}}=200$ GeV collisions [9]. The lowest datapoints represent minimum-bias $p+p$ collisions, while the others come from $Au+Au$ collisions of increasing multiplicity.
The data points in Figure 3 show $\pi$, K and $p$ spectra from p+p (full points) and mid-central Au+Au (open points) collisions, divided by the spectra from the most central Au+Au collisions. Curves represent Equation 2 with $\langle p_T^2 \rangle = 0.12$ GeV$^2$, $\langle E^2 \rangle = 0.43$ GeV$^2$ and $\langle E \rangle = 0.61$ GeV. According to our postulate, the only difference between the different-multiplicity collisions is, in fact, the multiplicity: $N_{\text{cent}} = 500; N_{\text{periph}} = 18; N_{\text{p+p}} = 10$.

The curves well describe the shape, magnitude, multiplicity-, and mass-dependence of the changes in the spectra. This indicates that the multiplicity evolution of spectral shapes is driven much more by phasespace restrictions due to energy and momentum distributions than by any real change in dynamics, a rather stunning suggestion. To emphasize the point, Figure 2 shows blast-wave parameters from fits to the “corrected” spectra generated by dividing the measured distributions by the phasespace factor.

3. “The system”

The evolution of the spectral shapes for $p_T \lesssim 1$ GeV/c (at higher $p_T$, other physics takes over [10]) with event multiplicity is almost perfectly described by Equation 2 when the parameters $N, \langle p_T^2 \rangle, \langle E^2 \rangle$ and $\langle E \rangle$ are adjusted. The only parameter which changes with multiplicity cut is $N$; the rest are constant. But where do these parameters come from? Are they reasonable?

While it may seem that such parameters may be extracted directly from the data, this is not so. Firstly, it is important to include “primary” particles in the consideration of phasespace constraints, and to include all particles (including any photons, neutrinos, etc). Further, the kinematic quantities $\langle p_T^2 \rangle$, etc, are averages over the (unmeasured) parent distribution, not the measured $f$. Finally, there arises the question: “what is the system for which a finite amount of energy and momentum is shared between $N$ particles?” It is likely not the entire set of particles emitted from a collision; the “system” decaying into the mid-rapidity region is not affected by beam fragmentation. Rather, the beam (and, likely, jets) steal some energy away from the smaller system of $N$ particles.
particles, which then statistically share energy. Thus, the measured quantities $\langle p_T^2 \rangle_c$ serve as a guide to their corresponding parameters in Equation 2 but need not fix them.

Further guidance comes from PYTHIA [12] simulations, which return quantities within a factor of $\sim 2$ of the ones we use. The numbers are also consistent with an estimate assuming a Maxwell-Boltzmann distribution for the underlying “system,” with temperatures $T = 0.15 \div 0.35$ GeV.

These issues are discussed in more detail in [10]. Thus, while we may use various estimates to validate our “reasonable” parameters, we cannot derive them directly from the measured spectra themselves. In principle, they can be extracted rather directly from two-particle correlation functions. At this meeting and previously [7], STAR has shown very clear phasespace-induced signals in two-pion correlation functions from p+p collisions. Work is well underway to extract “system” kinematic parameters from the two-particle correlation functions and use them to calculate phasespace effects single-particle spectra.

4. Summary and discussion

The observed evolution of the $p_T$ distributions measured at RHIC may arise due to changes in the dynamics as the system varies from p+p to central Au+Au, differences in phasespace constraints, or both. Most interpretations, based for example on blast-wave fits to the spectra, assume a dynamical origin for the spectra differences, but ignore effects of kinematics. We have shown that phasespace constraints due to energy and momentum conservation, can alone explain most of the multiplicity evolution of the spectra. Any additional change to the spectra due to dynamical effects must be very small. This claim will become much more compelling if one can extract, directly from measured two-pion correlation functions, the system kinematic parameters that we are now claiming to be “reasonable.”

We remind the reader that a purely phasespace-based explanation for the spectra evolution breaks down for $p_T > \sim 1.5$ GeV/c [10], where non-bulk physics becomes more dominant.

Since we argue that the parent spectra are, themselves, almost identical, we are not surprised to find that blast-wave parameters for p+p and Au+Au collisions are almost identical. The degree to which this implies flow in p+p collisions (or the lack of it in Au+Au collisions) remains unclear. Since any freezeout scenario should simultaneously describe spectra and femtoscopic measurements, input from two-particle correlation functions should shed more light on this intriguing question.

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