Evolution mechanisms of small-scale vortices in wall vicinity of unsteady turbulent channel flow

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Abstract. Direct numerical simulations of unsteady turbulent channel flow are performed to study evolution of small-scale vortices in the wall vicinity. In the first stage of those numerical simulations, turbulence decays in the wall vicinity after one of the walls bounding a flow becomes free-slip, and streamwise vortices there almost disappear. In the second stage, the boundary condition is again put back to non-slip wall, and vortices in near-wall region are excited again by the imposed mean shear there. In this second stage, an increase in the skin friction coefficient $c_f$ is related to an increase in the number of the vortices in the wall vicinity. Hence, the number of small-scale vortices are detected and traced to be averaged. As a result, it is found that tiny vortices newly generated in the wall vicinity are related to the intense sweep motion attributed to the larger-scale circulating motion away from the wall. However, these vortices are soon attenuated there. The persistent small-scale vortices in the wall vicinity are connected to the vortices away from the wall, and they are shifted downward by an intense inrush motion toward the wall.

1. Introduction

One of the important problems in wall turbulence is how the streamwise vortices are generated and maintained. There are several mechanisms proposed for generation of streamwise vortices in a fully developed turbulent channel flow. One is on streamwise vortices in the very vicinity of the wall, where the vortices are generated in the region of inrush motion of high-speed fluids related to the vortices further away from the wall. This mechanism of parent-offspring vortices is actually verified in several direct simulations of turbulent channel flow (e.g., Brooke and Hanratty 1993). Recent DNS studies (Iida et al., 1998, 2010) show that even in small Reynolds number, there are hieratical structure of streamwise vortices; in the wall vicinity, there are small vortices, while in the region away from the wall, larger streamwise vortices are generated. When the skin friction coefficient $c_f$ significantly reduces at the low Reynolds number, small-scale vortices in the wall vicinity almost disappear (Iida et al., 1998, 2010). Hence, it must be dynamically important that there are two kinds of vortices of different scale at the different distance from the wall.

When it comes to evolution of streamwise vortices, however, linear vortex stretching term is generally indispensable for their evolution (Kim and Lim, 2000). Hence, removing and imposing the mean shear in the wall vicinity must attenuate and generate streamwise vortices, respectively.

In this study, we performed direct numerical simulations of unsteady turbulent channel flow; after the boundary condition of one wall is changed from non-slip to free-slip condition, it is put back to non-slip condition. Streamwise vortices in the wall vicinity disappear after the free-slip
boundary condition is imposed. With imposition of non-slip condition, however, they emerge again. These tiny vortices in the wall vicinity are detected and averaged, to investigate the most probable mechanisms of their generation and evolution. As a result of this study, we found that emergence of streamwise vortices in the wall vicinity is related to the intense sweep motion (inrush motion of high-speed fluids toward the wall), which shifts vortices to the wall vicinity.

2. Numerical Conditions

Figure 1 shows the configuration and coordinate system of the unsteady channel flow, where the streamwise, wall-normal and spanwise directions are $x$, $y$ and $z$, respectively. First, one of the walls bounding the fully developed channel flow is changed into the free-slip wall, and turbulence in this wall vicinity decays (Fig. 1a). Then, the boundary condition is again returned to non-slip wall, and turbulence in this wall vicinity is excited again (Fig. 1b). Over all the period of numerical simulations, the boundary condition of the other wall (upper wall) is set to be constant.

A mean pressure gradient is, however, determined so that the friction velocity in the upper side of the wall is constant. When a flow is assumed to be statistically steady, the relation between the mean pressure $P$ and velocity $U$ is given as

$$-\frac{1}{\rho} \frac{dP}{dx} + \nu \frac{\partial^2 U}{\partial y^2} - \frac{\partial}{\partial y} \overline{uv} = 0,$$

where $\nu$ is kinetic viscosity, while $u$ and $v$ are the streamwise and wall-normal velocity fluctuations, respectively. Hence, in a Poiseuille flow bounded by non-slip walls, $dP/dx$ and $dU/dy$ at wall are related as

$$-\frac{1}{\rho} \frac{\partial P}{\partial x} \delta = \nu \left( \frac{\partial U}{\partial y} \right)_{\text{wall}} .$$

In contrast, in a free-slip flow, their relation becomes

$$-2 \frac{1}{\rho} \frac{\partial P}{\partial x} \delta = \nu \left( \frac{\partial U}{\partial y} \right)_{\text{wall}} .$$

Assuming the invariance of wall friction $\nu \partial U / \partial y$ in a fully developed state, a mean pressure gradient of a Poiseuille flow is two times as large as that of a free-slip flow. All the velocities, time and distance are non-dimensionalized by $\nu$ and reference velocity $u_r = \sqrt{-\delta / \rho dP/dx}$.
where $dP/dx$ is that of a Poiseuille flow, unless other non-dimensionalization is indicated. The Reynolds number defined by the reference velocity $u_r$, $\nu$ and $\delta$ is set to be 100 in the entire simulation of decaying and transition period.

In this study, the spectral method, i.e., the Fourier spectral method in the $x$ and $z$ directions and the Chebichev tau methods in the $y$ direction, is used for the spatial discretization of the simulations; the Crank-Nicolson and the Adams-Bashforth methods for the time integral. The computational region of $4\pi\delta \times 2\delta \times 2\pi\delta$ are resolved by the grids resolution of $64 \times 65 \times 64$; results are also verified by the same numerical simulation of $128 \times 129 \times 128$ grids points. As discussed later, the numerical results agree well with each other.

### 3. Results and Discussions

In these numerical simulations, non-slip boundary condition is imposed at $t_f=100$ after the free-slip condition is imposed. Hereafter, however, $t_f=100$ is redefined as $t = 0$.

The time evolution of the mean velocity over an entire channel is shown in Fig. 2a. The initial deviation of mean velocity from the fully developed state, which is mostly in good agreement with $t = 800$, is limited to the quarter of a channel; in the rest of a channel, the mean velocity profile is in good agreement with that of fully developed state. In contrast, Fig. 2b shows the time evolution of $c_f$ and bulk Reynolds number $Re_m$ just after the non-slip boundary condition is imposed. Both $c_f$ and $Re_m$ are normalized by those of fully developed channel flow; in the fully developed state, $c_f = 0.00968$ and $Re_{m0} = 2890$, respectively. As shown in Fig. 2b, there are three kinds of regimes observed in the development of $c_f$. In the first period, $c_f$ sharply decreases ($0 < t < 400$). Then, in the second period ($400 < t < 800$), it bounces back and continues to increase. Finally, in the third period ($t > 800$), it reaches an equilibrium state after a small decay. In contrast to $c_f$, the bulk Reynolds number is almost constant over the entire calculation, and hence, the increase and decrease in $c_f$ is mostly attributed to the friction velocity on the lower wall.

Figures 3a and b show the time evolution of the instantaneous vortical structures represented by the isosurfaces of second invariant of the deformation tensor $II = -\partial u_i/\partial x_j \partial u_j/\partial x_i$ as well as distributions of intense ejection and sweep events; a and b represent the top view at $t = 420$ and $t = 580$, respectively. The comparison between $t = 420$ and 580 shows that there is a definite qualitative difference in turbulence structure; at $t = 420$, the intense ejection and sweep events in the wall vicinity are very rare, while they significantly increase at $t = 580$. This is also true in the longitudinal vortices represented by black isosurfaces. During the transition, longitudinal vortices are almost always accompanied by intense sweep events on their sides, while the ejection
Figure 3. Time evolution of vortical structures in $x-z$ plane of $y = 10$. Black isosurfaces represent $II/II_{rms} = 1$ in $0 < y < 10$. Red and blue contours represent the sweep and ejection, respectively (counter interval is 0.5 in both case). (a); $t = 420$, (b); $t = 580$.

events related to vortices are relatively weak, though at the end of the transition, the strong ejection events are also accompanied by the longitudinal vortices.

Next, detecting dozens of streamwise vortices in the near-wall region, they are temporarily traced and spatially averaged. Hereafter, the streamwise vortices are averaged in the three different ways. First, all the vortices detected at $y = 10$ and 30, which have larger streamwise length than 150, are averaged in the different time. In all cases, the core of the vortex is detected as the maximum location of $II$, and all the vortices are averaged with its core overlapped; vortices with the core of $y = 10$ and 30 are defined as case 1A and 1B, respectively.

Second, to clarify the evolution mechanism of tiny vortices, the newly generated streamwise vortices are selectively detected and averaged, which is defined as case 2; when the upper edges of the isosurfaces of $II$ are at $y = 10$, and their streamwise length is less than three grids points (less than $\Delta x = 30$), these isosurfaces are detected as the newly generated vortices. Moreover, time evolution of these vortices is traced by considering their advection velocity.

Finally, the vortices which have their lower edges at $y = 14$ are averaged on the three different conditions; some vortices are shifted down to reach $y = 10$, while others, though shifted downward, can not reach $y = 10$. The former and latter are defined as case 3A and 3B, respectively. In contrast, those not shifted downward at all, are defined as cases 3C. In this study, these three different groups of vortices are compared at $y = 14$, to clarify the condition for the vortex to be shifted downward. In cases 1 and 2, 10 independent numerical simulations with the different initial conditions are performed to increase the number of the detected vortices.

Time evolution of averaged streamwise vortices detected as cases 1A and 1B is presented in Fig. 5. In case 1A, the number of the vortices detected is 159 and 204 at $t = 400$ and $t = 600$, respectively, while in case 1B, the number of the vortices detected is 307 and 368 at $t = 400$ and $t = 600$, respectively. The distribution of the averaged Reynolds shear stress $\tilde{u}\tilde{v}$, which
Figure 4. Schematics of longitudinal vortices detected and averaged in cases 1, 2 and 3. (a): cases 1A and 1B vortices which cross the lines of $y = 10$ and $30$, respectively, (b): case 2 vortices with their upper edges at $y = 10$, (c): cases 3A-C vortices with their lower edges at $y = 14$. Case 3A vortices are shifted downward to reach $y = 10$. Case 3B vortices disappear before they are shifted to reach $y = 10$. Case 3C vortices are not shifted downward.

has two local maximums around the vortex, is presented as well as the velocity vectors. A local maximum with the upward motion of the vortex is an intense ejection event, while that with downward motion is an intense sweep event. By comparing their intensity, it is noted that the vortex at $y = 10$ has stronger sweep event, while that at $y = 30$ has stronger ejection. This is in good agreement with the dominance of the sweep event in the wall vicinity, and dominance of ejection event away from the wall. It is also noted that with the time, the intensity of the Reynolds shear stress increases especially in the wall vicinity, and both the ejection and sweep events are enhanced, although the sweep event is still dominant in the wall vicinity.

Next, we discuss how the vortices at $y = 10$ is generated. Figure 6 shows the averaged streamwise vortex of case 2, and its time evolution. The number of vortices detected is 301, including both clockwise ($\omega_x > 0$) and counter-clockwise ($\omega_x < 0$) vortices. Fig. 6c represents the time when the case 2 vortex is detected at $y = 10$. The vortex, defined as the isosurface of $\mathcal{H} / \mathcal{H}_{rms} > 1$, is indicated by the thin arrow in Fig. 6c, where the small circulating motion around the isosurface is clearly observed. It is noted that the tiny vortex is related to the sweep event, which has much larger scale than the vortex. In case 2, the intense ejection event is not visible at all.

Also noted is the larger scale circulating motion right above the tiny vortex. This circulating motion is more clearly observed in the upwind side of the detected vortex as indicated by the thick arrow. The sweep event related to the tiny vortex seems to be driven by this circulating motion with its core more away from the wall, which is in very good agreement with parent-offspring regeneration mechanism proposed by Brooke and Hanratty.

Time development of the vortex can be also discussed in Fig. 6, where the location of the vortex in the streamwise direction is shifted by the distance proportional to the advection velocity; the cross-streamwise plane is shifted 1 grid point ($\delta x$) by $2\nu / u_x^2$. The streamwise vortex,
which is emerged in Fig. 6c (defined as the time $T=0$), is traced back to Fig. 6a ($T=-6$) and b ($T=-2$), while its evolution is shown in Fig. 6d ($T=2$). First, in Fig. 6a, the circulation motion can not be observed at all. In contrast, in Fig. 6b, the streamwise vortex is definitely observed, though the value of $\bar{H}/\bar{H}_{rms}$ is still below 1, and hence can not be detected as a vortex in the detecting procedure. The comparison of these figures indicates that the large-scale sweep event is critically important for generation of new streamwise vortex in the wall vicinity. Moreover, it is noted from Figs. 6a to 6c that the maximum location of sweep event is shifted toward the wall when the streamwise vortex is generated, which is in good agreement with our previous study (Iida et al., 1998, 2010). Enhancement of sweep event in the wall vicinity and the shift of the maximum location of sweep event are attributed to the fluids approaching closer to wall by the strong downward motion, though not shown here.

After $T=2$, however, $\bar{H}/\bar{H}_{rms}$ of detected streamwise vortex becomes less than unity, and can not be detected as a vortex. Our results agree well with that of Brooke and Hanratty in the sense that the tiny streamwise vortex in the wall vicinity is closely related to the sweep event, which may be generated by the circulating motion further away from the wall. Our study, however, can not further trace these vortices and make sure that they becomes the longitudinal vortex tubes, because $\bar{H}$ related to them is attenuated with time.

Next, we discuss downward shifting of vortical structures away from the wall. Figure 7 shows the case 3 vortices defined in Fig. 4 at the time when the vortices are detected at $y = 14$ (at $T = 0$). The numbers of the detected vortices are 149, 297 and 860 in cases 3A, 3B and 3C, respectively. The difference in flow structures between the cases with and without downward stretching is obvious. When the vortical structure is shifted to reach $y = 10$, which is defined as case 3A, it is accompanied by more intense sweep event than in other cases. Comparison

**Figure 5.** Time evolution of $-\bar{u}\bar{v}$, where $\bar{A}$ represents the conditional average, and velocity vectors in the cross-streamwise plane of cases 1A and 1B. (a); $t = 400$, (b); $t = 600$. The left-aside and right-hand side figures represent cases 1A and 1B, respectively. The red and blue contours represent $-\bar{u}\bar{v} > 0$ and $-\bar{u}\bar{v} < 0$, respectively. The contour interval is 0.2 in all figures.
Figure 6. Time evolution of averaged tiny vortices of the streamwise length less than 30; the time when the vortex is detected is defined as \( T = 0 \). From (a) to (d), the cross-streamwise plane is shifted 1 grid point (\( \delta x \)) by \( 2\nu/\bar{u}^2_r \). Arrows represent \( v-w \) vectors. Contour lines represent \(-\tilde{\omega}\), and contour interval is \( \Delta C = 0.1 \). Blue and red represent negative and positive values, respectively. All red contour lines are sweep events, while the ejection events are not visible. (a): \( T = -6\nu/\bar{u}^2_r \), (b): \( T = -2\nu/\bar{u}^2_r \), (c): \( T = 0 \), (d): \( T = 2\nu/\bar{u}^2_r \). The left-hand side and right-hand side figures represent the cross sections of 1 grid point upwind and downwind of middle figures, respectively. Isosurface of \( \bar{II}/\bar{II}_{rms} = 1 \) is also included, though it is only visible at \( T = 0 \) when the vortex is detected.

between cases 3B and 3C also shows that cases 3B is accompanied by more intense sweep event than case 3C which is not shifted at all. All vortices are, however, accompanied by the ejection event, though it is weaker than the sweep event. It is also marked that in case 3A, there is a definite inrush motion toward the wall as indicated by an arrow, while in others cases, downward motion is related to the spanwise flow, and hence is not directly rushed into the wall.
Figure 7. Distribution of (a); the Reynolds shear stress $-\bar{u}\bar{v}$ (contour interval $\Delta C = 0.1$), (b); streamwise velocity fluctuation ($\Delta C = 0.25$), (c); vertical velocity ($\Delta C = 0.025$), (d); streamwise vorticity ($\Delta C = 0.01$). The figures of left-hand side, middle, right-hand side represent the vortices of cases 3A, 3B, and 3C, respectively. Blue and red represent negative and positive values, respectively. In (a), isosurface of $\frac{\bar{H}}{H_{rms}} = 1$ as well as velocity vectors are also included.

Despite a definite difference in the sweep event among different cases, it is difficult to see the difference in the streamwise velocity fluctuation. This is also true in the intensity of spanwise velocity as well as streamwise vorticity. The difference in the sweep event is attributed to the vertical velocity difference, as shown in Fig. 7c, where the downward velocity ($v < 0$) takes the largest value in case 3A. In contrast, the difference in the upward velocity ($v > 0$) is negligibly small among the different cases. Hence, it is again confirmed that downward shifting of the vortex is accompanied by the stronger sweep event triggered by the stronger downward motion.

Interestingly, as shown in Fig. 7d, the strong downward motion is related to distributions
Figure 8. Time evolution of cases 3A and 3C vortical structures (isosurface of $\overline{H}/H_{\text{rms}} = 1$) and contour lines of streamwise vorticity at the same $x$–$y$ plane ($\Delta C = 0.01$). (a); $T = 0$, when the vortices are detected at $y = 14$, (b); $T = 2\nu/u_r^2$, (c); $T = 4\nu/u_r^2$. The figures of left-hand and right-hand sides represent the vortices of cases 3A and 3C, respectively, while blue and red represent negative and positive values of $\omega_1$, respectively.

of streamwise vorticity, which shows that in case 3A, the averaged vortex is accompanied by the intense streamwise vorticity of the opposite sign in its right-hand side, as indicated by other arrow. This indicates that inrush motion toward the wall is enhanced by the vorticity of opposite sign flanked by the streamwise vortex, though the mechanism for emergence of this opposite vorticity is unknown.

Figure 8 shows the averaged streamwise vortices of case 3A and 3C, and their time evolution. It is noted that the vortex of case 3A is stretched in the streamwise direction and its streamwise vorticity is intensified, while the vortex of case 3C disappears soon at $T = 2$. The difference between the case 3A and 3C at $T = 0$ is in the intensity of the streamwise vorticity of the opposite sign, which is observed in the upwind side of both vortices, indicating its importance in vortex stretching.

4. Conclusions

Direct numerical simulations of unsteady turbulent channel flow are performed. In the first stage of the numerical simulations, turbulence decays in the one side of a channel temporarily
after boundary condition is changed from no-slip wall to free-slip wall. In the second stage, the boundary condition is again put back to no-slip wall, and turbulence in near-wall region is excited again by the imposed mean shear there. The increase in the friction velocity is triggered by the enhanced sweep motion in the wall vicinity. The instantaneous flow structure in the wall vicinity also shows that in the transition period, intense sweep events are closely accompanied by the streamwise vortices there. With the time, the number of the tiny vortices in the wall vicinity increases. Hence, the generation and evolution of small-scale vortices in the wall vicinity is investigated in this second stage.

The streamwise vortices are detected by isosurfaces of $II$ which threshold value is larger than its rms value over the flow domain at the time of detection. Moreover, the streamwise vortices are averaged in the three different ways. First, all the vortices with longer streamwise length than $\Delta x = 150$ and at $y = 10$ and 30 are averaged at the specific time of the transition period. Second, to clarify the evolution mechanism of tiny vortices in the wall vicinity, the newly generated streamwise vortices are selectively detected and averaged; when the upper edge of the isosurface of $II$ is at $y = 10$, and its streamwise length is less than $\Delta x = 30$, this isosurface is detected as the newly generated vortex. Moreover, time evolution of the vortex is traced by considering its advection velocity. Finally, the vortices with the lower edge at $y = 14$ are averaged on the three different conditions, to show how vortices away from the wall is shifted downward; some vortices are shifted to $y = 10$, while others cannot reach $y = 10$, or cannot be shifted downward at all. In this study, these different groups of vortices are compared at $y = 14$.

First, by detecting all the vortices crossing the line of $y = 10$ and 30, it is found that the vortex at $y = 10$ has stronger sweep event than ejection event, while that of $y = 30$ has stronger ejection, indicating a definite difference in the intensity of ejection and sweep motions at different distances away from the wall. It is also found that in the period of the transition, both the ejection and sweep events are enhanced in the wall vicinity, although the sweep event is still dominant there.

Second, detecting dozens of newly generated streamwise vortices in the vicinity of the wall, and tracing them, it is found that these vortices are closely attributed to the sweep motion related to the larger-scale circulation motion away from the wall. However, it is also found that thus generated tiny streamwise vortices are not accompanied by the ejection event, and soon attenuated.

Finally, comparison between the vortices with and without downward shifting clearly shows that when the vortical structure is shifted to reach $y = 10$, it is accompanied by more intense sweep events in the wall vicinity. The intense sweep events are attributed to a definite inrush motion of fluids toward the wall in the sweep side of the vortex; in the other vortices, downward motion of the vortex, related to the spanwise flow, is not directly rushed into the wall. Moreover, this inrush motion is also related to the vorticity of opposite sign flanked by the streamwise vortex.

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