On the PeV knee of cosmic rays spectrum and TeV cutoff of electron spectrum

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The origin of the cosmic-ray knee has remained a puzzle since its discovery over 60 years. In addition, some latest experiments have revealed a spectral cutoff of the electron around 1 TeV. We find these two spectral breaks have a similar Lorentz factor ~ 10⁶, and interpret this similarity with a threshold interaction induced by a new particle X abundant in the Galaxy. The interaction process CR + X → CR + X' can take place when the effective energy is sufficient to convert it into the mass of another unknown particle X' (as a representative to all possible threshold inelastic interactions), where the mass of X' is 10⁶ higher than that of the X with respect to the above mentioned common Lorentz factor. Thus cosmic rays will lose their energy above the threshold and produce a spectral break. Under this scenario, we can reproduce the spectral break for both the nuclei and electron, and predict a flattened spectrum for electrons after the cutoff. Given that there are uncertainties of experiments in determining the actual spectra of these breaks and their components, our model allows a wide mass range of the particle X from ultra low value to around 1 eV.

PACS numbers:

Introduction.— The origin of the knee of cosmic-ray spectrum has remained a puzzle since its first discovery 67 years ago [1]. In general, there are three types of mechanism responsible for the generation of the knee. One is related to the acceleration limit of the cosmic ray sources [2][3]. Another is related to the diffusion process of the galactic cosmic rays (CRs) [4][5]. And the third type attributes it to the electron and positron pair production induced by the interaction of cosmic ray with radiation background in a typical energy of 1 eV at the source location [6][7].

On the other hand, owing to the smaller acceleration efficiency and heavy energy loss due to the synchrotron radiation and the Inverse Compton scattering process, cosmic-ray electrons are much less abundant and bear a much softer spectrum than the nuclei. The electron spectrum is so far only measured up to about 5 TeV by various experiments [10][13]. According to these measurements, electron spectrum can be described by a broken power law. The power law index is about -3.1 before ~ 1 TeV and nearly -4.1 after ~ 1 TeV [10]. A great amount of attention has been paid to the excess of electron and positron between 10 GeV to near TeV energies. The explanation includes various mechanisms from dark matter origin (one can refer to the review [14] and references therein) to astrophysics origin [15][22]. And the break is often treated by an empirical cutoff function without dedicated study in latter case. In a brief summary, from the point of view of astrophysics origin, the break electron spectrum could be attributed either to acceleration limit due to the interaction between electron with environment photon background at source [23], or the confinement of electron on sources [24][27] or energy loss during propagation [26][27].

The energy scale of the cosmic rays knee and electron cutoff are quite different, it is fairly reasonable to explain them with independent processes. But one should be cautious that both PeV cosmic rays and TeV electron have almost same Lorentz factor γ, which is about 10⁶. It is possible that such an agreement is purely a coincidence, but one can’t rule out the possibility that this is an indication that two phenomena are related to the same physics process.

Suppose that equal Lorentz factor ~ 10⁶ does have a physics meaning, the most natural explanation behind would be that there exists an unknown light mass particle X which has a mass m_e much smaller than electron mass m_e and of course much smaller than the nuclei mass m_A too. Although the pair production by TeV electrons with 1 eV photons is also a threshold interaction and could lead to a spectrum break, it is impossible for us to explain the TeV cut off of electron spectrum by this process due to the lack of the amount of the 1 eV photons according to [29]. In this case we need at least a density of 10000 cm⁻³ for the 1 eV photons to ensure more than one interaction by assuming the cross section is 1 mb and total life time is 10⁵ yr for 1 TeV electrons, but only ~ 1 cm⁻³ of photons are known in this energy in the Galaxy. We assume the X particle is abundant in the Galaxy and can interact with TeV electrons and PeV cosmic rays nuclei to generate another heavier unknown particle X'. As a matter of fact, it should also be possible that X' is a resonance, or a pair of particles or multiple particles in the most general case, as long as they have a non zero minimal mass value in whole. One simplest example is to replace the X' by a pair of electron and positron. In this case, we need only to introduce one new particle X. But dynamic modeling is beyond the scope of this work. Here, we will assume that X' is a new particle and we will study only the kinetic effect from threshold interaction. In the rest frame of TeV electron or PeV nuclei, X particle has an energy of γm_e. When this value
is large than the mass of $X'$ in the case that the mass of the X is sufficiently small so that the mass of $X'$ is not too big than electron mass, the interaction is allowed and the $X'$ particle can be produced. Otherwise, the energy is insufficient to produce the $X'$ and the interaction is forbidden. With many-time interactions, electron and nuclei may lose energy until they are below the required threshold energy. In this scenario, electrons higher than TeV energy and nuclei higher than PeV energy may lose significant amount of their energy and make the spectral break accordingly.

In this work, we perform Monte Carlo method to calculate this interaction between CR species and the X particles. Under this scenario, our model is validated to reproduce the spectral break of both the CR nuclei and the electrons.

**Model Construction.**— In this work, the basic argument is that both the CR nuclei and the electrons bear a common Lorentz factor $\gamma$ at their spectral break at $\sim 4$ PeV for the nuclei and $\sim 1$ TeV for the electron. According to [8, 9], a relativistic particle such as photon could well served to explain the two break spectra. From kinet point of view, a non relativistic particle should also work. Actually, a non relativistic particle is more attractive as it has much less requests on energy budget (see discussion section for details). Then, we assume that $x$ is a non relative particle and treat the particle X is at rest, i.e., $v = 0$ for simplicity.

For a deeper analysis, by assuming the dominant species of CR knee region at $\sim 4$ PeV is He and the break energy of each nuclei is proportional to its mass, we can fix the Lorentz factor $\gamma$ at $10^6$ for all CR species. Suppose that X, $X'$ both have the mass much smaller than that of the electron, a common $\gamma$ can be obtained as we wish. However, remembering that the break position of the electron spectrum is around 1 TeV, which corresponds to $\gamma_{e} \sim 2 \times 10^6$, about a factor of 2 different from the simple prediction based on the knee position of cosmic rays spectrum assuming masses of X and $X'$ are much smaller than electron mass. Then the former analysis should be modified that the mass $m_{X'}$ of the particle $X'$ should take a value around the same order of the electron mass, which means a limited fraction of the effective energy will remain in the electrons after each interaction. Thus, in order to produce the $X'$ particle, a relatively higher energy is needed and will result in an elevation of the Lorentz factor $\gamma_e$ in the case of electron. However, it should be noted that the cosmic rays composition around the knee still bear a large experimental uncertainty, so the value of $\gamma_{CR}$ is still uncertain. A Lorentz factor below $2 \times 10^6$ from the all-prticle spectrum can always be applied to explain the electron spectrum under our basic assumption by adjusting the masses of X and $X'$. For a rather unlikely situation that proton component dominates the 4 PeV knee region, our model fails for simple kinetic reason. As the electron spectrum should not be break at 1 TeV but at 2 TeV instead.

In the consideration of a simple situation, we assume a constant cross section $\sigma_{e}$ for the electron and $A^2\sigma_{p}$ for the nuclei above the threshold $m_{CR} + m_{X'}$, where A is the mass number and $A^2$ is used under the consideration of the coherence of the nucleons. Again, for simplicity, we assume that the $X'$ particle has an isotropic angular distribution in the center mass system. With these assumptions, it is expected that the break spectra will be very sharp and exhibits a very high pile-up feature just around the break energy. But for a real physics model which should have a continuously arising threshold cross section, above mentioned features will be largely reduce accordingly. Suppose such a situation that the abundant X particles may be a boson and stay in a coherent state, they will interact with the CRs in a collective manner and then the cross section will be amplified by a factor $\eta$ which should be determined by the degree of their coherence.

Denote the number density of the X in the galaxy by $n_x$ which is part of an integral with the cross section in our calculation, and the mean free time of the interaction by $\tau_{A}$ for nuclei and $\tau_{e}$ for electron. Then we can evaluate $\tau_{A} = 1/n_x A^2 \eta \sigma_{p} c$, and $\tau_{e} = 1/n_x \eta \sigma_{e} c$. The life time of CRs confines the total interaction times during CRs propagation in the Galaxy, and it is estimated as $\tau_{prop}(R) \sim 2 \times 10^6 (\frac{R}{10^{16} \text{ GV}})^{-0.6} \text{ yr}$ [30] for nuclei and $10^5 \text{ yr}$ [31] for electron under the condition that we only concern the spectrum for electron around 1 TeV. Then, the average interaction number can be obtained by dividing the life time with mean free interaction time.

As a common view, CRs propagation process in the Galaxy is regarded as a major magnetic-governed diffusion process [32, 33], where the secondary interactions and energy loss processes can take place during their journey. This process has an influence on the observed nuclei spectra by a softened index $\sim 0.6$ for the nuclei. In case of electrons, they suffer from severe energy-loss processes such as Inverse Compton scattering and synchrotron radiation, which will soften the electron spectrum more significantly than the nuclei. As measured by AMS02 [34], the spectrum of $e^+ + e^-$ presents a well single power law with the index $\sim 3.17$. Thus we can evaluate the propagation effect of the electron by a softer index as well about 0.6 softer than nuclei spectrum. In summary, propagation effect tends to transform a harder power law spectrum to a softer one. And in turn, one can take care of the propagation effect by adopting a softer spectrum before studying the interaction effect. Under these considerations, we can deal with the propagation effect and the interaction with the X separately for simplicity. And in the actual calculation, we set the primary CR spectra as a single power derived from the experiments directly, which implies the propagation effect has already been taken into account.

We perform the Monte Carlo method to calculate the
interaction between the CRs and the X during their propagation. And there are three free parameters need to be adjusted, including the X’s mass \( m_x \), the interaction parameter \( \eta n_x \sigma_p \) for the proton, and the interaction parameter \( \eta n_x \sigma_e \) for the electron. Detail information of the calculation and the results are shown in the next section.

Calculations and Results.— In the model calculation for the nuclei, species from proton to Fe are injected into the X-abundant space with the spectra suggested by Ho-randel [43] which are normalized at 1 TeV/nucleus. Under the fact that there is a several-order of difference in mass between the nuclei and the particle \( X' \), we have tested that the final cosmic rays spectra show no dependence of the input mass of x particle, so we choose an example mass 1 eV for the X particle as an illustration and the parameter \( \eta n_x \sigma_p \) is tuned to be \( 4 \times 10^{-21} \text{ cm}^{-1} \) to give the best explanation to the data. The calculated spectra of individual compositions, including proton, He, CNO and Fe are shown in the left panel in Fig. 1. And the right panel in Fig. 1 displays the all-particle spectrum.

Besides the spectra break, sharp peaks locating on the threshold energy position are clear seen in Fig. 1 for each cosmic ray component. These peaks are caused by the pile-up effect which is caused by high energy nucleus which lose their energy before they arrive at the threshold energy. Considering that there is at least 3-order of magnitude mass difference between the nuclei and the particle \( X' \), the energy loss at around the threshold is approximately \( \gamma m_{2'} \). As a result, it will lead to these narrow pile-up distributions for the nuclei spectra around their thresholds. We like to point out that these peaks might be weakened and spread into a wider range with a more realistic model in which the cross section is a smoothly rising function of center mass energy rather than a simple step function. In addition, although these sharp peaks have never been observed, after considering the actual energy resolution of experiments, we should not be expected to observe these peaks anyway. If we assume the energy resolution \( \sigma \) can be described by a gaussian distribution, the observed spectra through the formula can be obtained by following integration

\[
F_{\text{obs}}(E) = \int F_{\text{true}}(E_0) \frac{1}{\sqrt{2\pi}\sigma_{\text{res}}} \exp \left[ -\frac{(E - E_0)^2}{2\sigma_{\text{res}}^2} \right] dE_0
\]

(1)

Where the parameter \( \sigma_{\text{res}} \) denotes the energy resolution with an approximate value 20\% \( E_0 \) standing for a typical experiment, and variable \( E_0 \) denote the true energy while \( E \) denote the observed energy. Then the observed spectra \( F_{\text{obs}} \) is the result of the convolution of the true spectra \( F_{\text{true}} \) with the gaussian function. Those calculated spectra are shown in Fig. 1 as well in black solid lines.

Without surprise, all peaks are washed away after taking into account the energy resolution and model calculation agree well with observation very well when energy is less than 60 PeV. The insufficient flux above 60 PeV indicates the over large cross section has been used in the calculation when energy is far from the threshold. Despite this discrepancy, keep in mind that the dominant component He around the knee is postulated as the prerequisite for the subsequent calculations, some different components corresponding to different Lorentz factors will be discussed in the last section.

In the case that the nuclei spectrum can be described in our model with a Lorentz factor \( 10^6 \), we further apply this value in the electron spectrum calculation. Before comparing the calculated spectrum with the experiments, we plot the calculated spectra with a set of the X masses from 1 eV to 0.01 eV as shown in Fig. 2. The parameter \( \eta n_x \sigma_e \) is fixed at \( 4 \times 10^{-23} \text{ cm}^{-1} \) just for an illustration purpose, leaving the parameter \( m_x \) as the only free parameter. We also plot the original spectrum (without invoking the threshold interaction) for comparison as shown in the solid black line. It can be seen that as the X mass decreases to 0.01 eV, a prominent narrow peak will appear, otherwise, no obvious peak can be seen. We can expect that when the particle X has a much lower mass than 0.01 eV, the spectrum of the electron will behave in the same manner as the nuclei’s. Similar to the cosmic rays spectrum with pair production interaction calculated by [7] and [9], electron spectrum shows a zig zag shape after the threshold interaction, while this feature also occurs at the nuclei calculation especially for the proton and the helium in Fig. 1a. This is a typical result for the threshold interactions.

Next, a realistic calculation is implemented to explain the experiments. Considering the high precision electron spectrum measurement made by AMS02 [40]. We adopt the primarily injecting electron spectrum from fitting with AMS02 spectrum between 30 GeV and 300 GeV. The obtained index about \( \sim -3.17 \) will be used in the calculation. And then the effect of the energy resolution is considered when comparing our model calculation with the observation from other experiments [11, 13, 47]. As for the low energy MAGIC data, it has a large difference comparing with the AMS02. Given that we only concern the energy range near 1 TeV, the data points above 200 GeV are used to compare with model calculation. Besides, the spectrum measured by ATIC shows an obvious difference with other 4 experiments with a bump structure at energies of 300-800 GeV and the spectrum is obviously much harder, so we obtain the electron injecting spectrum by a direct fit to ATIC data below 300 GeV.
FIG. 1: a: The energy spectra of proton, He, CNO, and Fe. The red dashed lines are the MC calculation results, and the black solid line are the results by considering 20% energy resolution. The observation data are Tibet-III [35], KASCADE [36, 37], GRAPES-3 [38], RUNJOB [39], SOKOL [40], JACEE [41], EAS-TOP [42]. b: The calculated all-particle spectrum. The red dashed lines are the MC calculation results, and the black solid line are the results by considering 20% energy resolution. The observation data are Tibet-III [43], Akeno [44]. The normalized data are derived by combing all data with a rescale based on the extrapolation of the direct measurements [45].

FIG. 2: The calculated electron spectra with respect to different mass of the particle X. The black solid line is the original spectrum. The blue solid line, the red dashed line, the blue dotted line, and the violet dashed line correspond to the Monte Carlo calculation results with the X mass 1 eV, 0.5 eV, 0.1 eV, and 0.01 eV respectively.

with a single power law index $\sim -2.96$ in its calculation rather than using the one from AMS02 data.

To find out the optimal the mass and the interaction parameters of each experiment, the least-square fitting algorithm is employed. In view of the complexity of Monte Carlo computation, we apply a nonlinear optimization package NLopt [48] in this work. In addition, we note that there are differences for the observed flux between the AMS02 and other experiments. We consider these differences are owing to the different energy scales in different experiments. In this work, we take AMS02 energy scale as the standard one, and use parameter $\xi$ to describe the energy scale difference between one experiment with AMS02 (i.e., $\xi$ is the energy scale ratio between the two experiments). Thus three free parameters including the X mass $m_x$, the interaction parameter $\eta_n \sigma_e$, and the energy scale factor $\xi$ are tuned in the iteration to find the best fit results. The derived parameters are listed in Tab. I. From which, the cross section between X particle and electron is 2 order of magnitude smaller, giving the fact that $\eta_n$ should be same for both cases.

Detailed results concerning individual experiments are shown in Fig. 3. We can see that our model can fit reasonable well with each experiments. Moreover, the ATIC’s bump feature can also be produced in our calculation by taking a lower mass $\sim 0.4$ eV of the X into account. Although the statistic were poor, both HESS and VERITAS measured a high electron flux at the highest energy bin, indicating a possible flattening of the electron spectrum starting from a few TeV, and it is interesting to

| parameters | HESS | VERITAS | MAGIC | ATIC |
|------------|------|---------|-------|------|
| $m_x$ (eV) | 1.6  | 0.7     | 1.4   | 0.4  |
| $\eta_n \sigma_e$ ($\times 10^{-23}$ cm$^{-1}$) | 1.2  | 1.2     | 2.4   |      |
| $\xi$     | 1.08 | 1.01    | 1.09  | 1    |
FIG. 3: The calculation results of the electron spectra compared with the experiment data. The red dashed lines are the Monte Carlo calculation results. The black solid line are the results by considering energy resolution with 15% for HESS, VERITAS and MAGIC, and 2% for ATIC. And the violet dashed line in (a) is the result with an modified energy resolution 20% for HESS. The observation data are from AMS02 [34], HESS [13], VERITAS [10], ATIC [11], and MAGIC [12].

note that this is the expected behavior according to our model calculation. This unique feature is caused by the finite energy loss above the threshold energy. Besides, we find that calculations do not agree perfectly with the spectra for the HESS and VERITAS data at $\sim 2$ TeV. As mentioned in the nuclei spectra, this requests a more realistic cross section which should rise above the threshold slowly, or indicates a larger energy resolution than that claimed by those experiments above TeV. Comparing these results obtained from different experiments, we can see that X mass is very sensitive to the precision of electron spectrum. Spreading from 0.4 eV to 1.6 eV indicating that experimental uncertainty on X mass is order of 1 eV. Namely, the true X mass can be any number from 0 to about 1 eV.

Conclusions And Discussions.— The origin of CRs knee at around 4 PeV has still remained a puzzle since its first discovery. In the postulation that the He domains the knee which corresponds to a Lorentz factor $\gamma \sim 10^6$, we compare it with the latest measurement of the electron spectrum which has a cutoff at around 1 TeV corresponding to a Lorentz factor $\gamma \sim 2 \times 10^6$. 
and find these two similar value may indicate a latent common physical mechanism. Then we assume the CR and link it to the existence of a new particle X abundant in the Galaxy. The interaction channel \( CR + X \rightarrow CR + X' \) open when the total energy in central-mass frame is higher than the threshold \( m_{CR} + m_{X'} \), where \( m_{X'} = 10^6 m_e \) with respect to the common Lorentz factor \( 10^6 \). In addition, we postulate the X particles stay in the coherent state, thus their cross section with the CRs may be amplified. By implementing these assumptions in the Monte Carlo calculation and taking into account the effect of the energy resolution, we can reproduce the spectral break for both the nuclei and electrons and can well explain the experiment measurements. As a result, we can derive the cross section between X particle and electron is 2 order of magnitude smaller than the nuclei. A unique zig zag shape around the threshold energy has been found in our model calculation both for the nuclei and the electrons. Under this scenario, we predict that the spectrum will become flattened after the cutoff energy.

Concerning the assumption of the non-relativistic status of the X particles, they are very likely to be concentrated in the Galaxy under the attractive gravity force. From the kinetic point of view, relativistic X particle is also possible. In this case, the Xs may have two origins, including the cosmology origin and the astrophysics origin. For former case, the X particle has to be uniformly distributed in the whole universe, which means that many order of magnitude higher energy density will be required than the case of non relative X particle. For the astrophysics origin of the abundant X particle, one has to deal with a situation that a great amount of unknown radiation are escaping from galaxies. Quantitative study needs to invoke dynamic models which is beyond the scope of this paper.

Although our simple model calculation gives a good explanation to the nuclei spectra below 60 PeV, the higher energy does not agree with the observed all-particle spectrum. The abrupt cutoff is caused by the over large cross section used in the calculation for heavier nuclei, and a series sharp peaks also indicates this over-simplified assumption. In addition, the calculated spectrum for the electron verify this opinion as well. A more realistic energy-dependent cross section that arising after the threshold and decreases at higher energies is probably necessary to solve the problem. In order to insure the validity of our model calculation, we should postulate a much weaker coupling process for the elastic scattering than that for the inelastic scattering, which means a much smaller coupling between the light X particle and matter particles than that between heavy X' particle with matter particles.

In the prerequisite that He plays the major role in the knee's spectral break, a set of values of the mass of particle X between 0.4 eV to 1.6 eV can describe the experimental data well. However, keep in mind that the actual composition at knee is still unknown, and disagreement between different experiments is huge \([49–53]\). So this mass range doesn’t represent the actual X mass is in order of 1 eV. If heavier nuclei domain the knee spectrum, which indicates a lower \( \gamma \), a heavier X' thus a heavier X will be required to keep the cutoff spectrum of the electron at \( \sim 1 \) TeV. As an examples, in a possible case that the knee is composed of proton and helium such that Lorentz factor is \( \sim 2 \times 10^6 \), an arbitrary low mass of the particle X is suitable if the electron cutoff locates at 1 TeV. For a very unlikely case that the proton occupies the knee energy, our model will fail for simple kinetic reason, because a spectral break of electron have to occurs at least around 2 TeV according to our model. In a word, due to the uncertainty of the measured spectra for both nuclei and the electron, we can’t give a final determination of the X mass, but an ultra-wide range of the X mass is possible. So the precise measurement of the nuclei spectra and composition around the knee, and the explicit cutoff position of the electron spectrum are important to study the possible new physics. We look forward to the promising experiments DAMPE and LHAASO to test our model.

Acknowledgements.— The authors thank Xiaojun Bi, Shouhua Zhu, Chun Liu and Pengfei Yin for their helpful comments and suggestions. This work is supported by the Natural Sciences Foundation of China (11135010).

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