Tensor Glueball Photoproduction and Decay

Stephen R. Cotanch

Department of Physics, North Carolina State University, Raleigh, NC 27695 USA

Robert A. Williams

Nuclear Physics Group, Hampton University, Hampton, VA 23668 USA

and

Jefferson Lab, 12000 Jefferson Avenue, Newport News, VA 23606 USA

PACS: 12.39.Mk; 12.40.Nn; 12.40.Vv; 25.20.Lj

Keywords: Glueball widths; Glueball photoproduction; Vector and tensor meson dominance

Abstract

Using vector meson dominance (VMD), tensor glueball photoproduction cross sections, asymmetries and widths are calculated. The predicted hadronic $V V'$ decays are comparable for different vector meson ($V = \rho, \omega$ and $\phi$) channels with the $\omega \phi$ width the largest but the radiative $\omega \gamma$ and $\phi \gamma$ decays are suppressed relative to $\rho \gamma$ by over a factor of 2. This decay profile is distinct from typical meson branching rates and may be a useful glueball detection signature. Results are compared to a previous VMD scalar glueball study.

Documenting hadron states with predominantly gluonic degrees of freedom, i.e. glueballs, has been a challenging and somewhat elusive pursuit. Even though such states are consistent with quantum chromodynamics (QCD) and predicted by both lattice simulations [1,2,3,4] and gluonic models [5,6], clear experimental evidence is lacking. The purpose of this letter is to motivate additional experimental investigations by providing estimates, based upon VMD, of tensor glueball photoproduction observables and also to detail a possible decay signature for hadronic states with a significant gluonic component. The latter entails comparable $V V'$ hadronic widths, with the largest branch to $\omega \phi$ that promptly goes to $3\pi K\bar{K}$, and somewhat suppressed $\omega \gamma$ and $\phi \gamma$ radiative decays relative to $\rho \gamma$. As discussed below, this decay signature is not expected for hadrons with a predominantly quark structure. These results are essentially model-independent since they follow directly from the general principles.
of VMD, which has been found to agree with more fundamental QCD based meson radiative calculations \[7\], and the flavor independence of quark-gluonic couplings.

Consider the radiative decay \(f_2 \rightarrow V(k')\gamma(k)\) of a neutral tensor hadron with arbitrary quark, gluon structure and mass \(M_f\). Here \(k, k'\) are the momenta of the photon and vector meson with mass \(M_V\) \((k'^2 = M_V^2)\). The most general, gauge invariant \(f_2V\gamma\) vertex is \[8,9\]

\[
< \gamma(k)V(k')|f_2 > = e^\kappa e^\lambda f^{\mu\nu} A_{\kappa\lambda\mu\nu}(k,k') ,
\]

\[
A_{\kappa\lambda\mu\nu}(k,k') = 4 \frac{g_1}{M_f^2} B_{\kappa\lambda\mu\nu} + 2 \frac{g_2}{M_f^2} C_{\kappa\lambda\mu\nu} ,
\]

\[
B_{\kappa\lambda\mu\nu}(k,k') = (\epsilon_{\kappa\lambda} k \cdot k' - k'_{\kappa} k_{\lambda}) k_{\mu} k_{\nu} ,
\]

\[
C_{\kappa\lambda\mu\nu}(k,k') = 2 g_{\kappa\lambda} k_{\mu} k_{\nu} + g_{\lambda\mu} k'_{\kappa} k_{\nu} + g_{\lambda\nu} k_{\kappa} k_{\mu} - g_{\kappa\mu} k_{\lambda} k_{\nu} - g_{\kappa\nu} k_{\lambda} k_{\mu} - k \cdot k' (g_{\kappa\mu} g_{\lambda\nu} + g_{\kappa\nu} g_{\lambda\mu}) ,
\]

where \(\kappa, \lambda\) and \(f^{\mu\nu}\) are the photon, vector meson and \(f_2\) polarization vectors and tensor, respectively. Note that there are two possible coupling constants, \(g_1\) and \(g_2\), which in VMD (also tensor meson dominance) are given by \[8\]

\[
g_1 = 0, \quad g_2 = eg_{f_2V\gamma} = e \sum_{V'} \frac{g_{f_2VV'}}{f_{V'}} ,
\]

with \(g_{f_2VV'}\) the \(f_2VV'\) hadronic coupling constant, \(f_{V'}\) the \(V'\) leptonic decay constant and the sum is over all vector meson contributions consistent with isospin conservation for the \(f_2VV'\) vertex. The radiative decay widths are

\[
\Gamma_{f_2\rightarrow V\gamma} = \frac{2}{5} \alpha_e g_{f_2V\gamma}^2 M_{f_2} (1-x)^3 \left[ 1 + \frac{x}{2} + \frac{x^2}{6} \right] ,
\]

and \(\alpha_e = e^2/4\pi = 1/137.036, x = M_V^2/M_{f_2}^2\). Focusing upon isoscalar tensor hadrons \((I_{f_2} = 0)\) yields the radiative couplings

\[
g_{f_2\rho\gamma} = \frac{g_{f_2\rho\gamma}}{f_{\rho}}, \quad f_2 \rightarrow \rho\gamma , \tag{7}
\]

\[
g_{f_2\omega\gamma} = \frac{g_{f_2\omega\gamma}}{f_{\omega}} + \frac{g_{f_2\omega\phi}}{f_{\phi}}, \quad f_2 \rightarrow \omega\gamma , \tag{8}
\]

\[
g_{f_2\phi\gamma} = \frac{g_{f_2\phi\gamma}}{f_{\phi}} + \frac{g_{f_2\phi\omega}}{f_{\omega}}, \quad f_2 \rightarrow \phi\gamma . \tag{9}
\]

Since the \(\rho\) and \(\omega\) masses are almost equal \((M_{\rho} = 775.8\ \text{MeV}, M_{\omega} = 782.59\)
MeV), the ratio of the $\omega$ to $\rho$ channel decays is simply

$$R_{\omega/\rho} = \frac{\Gamma_{f_2 \to \omega \gamma}}{\Gamma_{f_2 \to \rho \gamma}} = \left(\frac{gf_2 \omega \gamma}{gf_2 \rho \gamma}\right)^2. \quad (10)$$

Application to tensor glueballs, i.e. $f_2 \to G_2$, and assuming flavor independence for the glueball-vector meson couplings, $g_{G_2VV} = g_{G_2V'V''}$, yields

$$R_{\omega/\rho} = \left(\frac{f_\rho}{f_\omega}\right)^2(1 + f_\omega f_\phi)^2. \quad (11)$$

Hence the ratio of decay widths is entirely governed by the leptonic decay constants whose magnitudes can be extracted from $V \to e^+e^-$ using

$$\Gamma_{V \to e^+e^-} = \frac{4\pi\alpha_e^2}{3} \frac{M_V}{f_V^2}. \quad (12)$$

The most recent measurements [10] yield $|f_\rho| = 4.965$, $|f_\omega| = 17.06$ and $|f_\phi| = 13.38$ for a relative reduction $R_{\omega/\rho} = 0.44$. The $\phi \gamma$ channel, which is also reduced by this factor, is further suppressed kinematically. As discussed below, suppression of radiative decays to isoscalar vector meson channels is not generally expected for tensor mesons since they will have different $g_{f_2VV'}$ couplings reflecting their various flavor contents. Note also that the relative phase between the $\omega$ and $\phi$ couplings has been assumed to be the same as between their respective decay constants. Depending upon phase convention (i.e. $\phi = \pm s\bar{s}$) the decay constants are often cited with opposite phases in the literature (e.g. the $SU(3)$ relation $f_\rho \sqrt{3} = -f_\omega \sin(\theta) = f_\phi \cos(\theta)$ [11] where $\theta \approx 40^\circ$ is the $\omega\phi$ mixing angle). Consistency requires the same relative sign between the couplings $g_{G_2\omega\omega}$ and $g_{G_2\omega\phi}$ since the latter, like the $\phi$ decay constant, is linear in the $\phi$ phase. Because $f_\omega$ and $f_\phi$ are comparable in magnitude, $R_{\omega/\rho}$ is very sensitive to this relative phase and would be dramatically lower, .0064, if indeed the net phase was negative. It is therefore important to more rigorously determine the relative phase of the vector meson coupling and decay constants and further study is recommended.

Similarly, the scalar glueball radiative decay widths are [12]

$$\Gamma_{G_0 \to V\gamma} = \frac{1}{8} \alpha_e g_{G_0V\gamma}^2 M_{G_0}^3 \frac{M_V^3}{M_0^2} (1 - x)^3, \quad (13)$$

where $g_{G_0V\gamma}$ is also given in VMD by Eqs. (7, 8, 9) with $f_2$ replaced by $G_0$, $M_{G_0}$ is the scalar glueball mass, $M_0$ is a reference mass fixed at 1 GeV and $x = M_V^2/M_0^2$. Again VMD predicts the same suppression factor for radiative decays to isoscalar vector meson channels.
The $G \to \gamma \gamma$ decays for $G_0$ and $G_2$ can also be obtained from VMD

$$
\Gamma_{G_0 \to \gamma \gamma} = \frac{\pi}{4} \alpha_e^2 g_{G_0 V V}^2 \left[ \left( \frac{1}{f_\rho} \right)^2 + \left( \frac{1}{f_\omega} + \frac{1}{f_\phi} \right)^2 \right]^2 \frac{M_{G_0}^3}{M_0^2},
$$

(14)

$$
\Gamma_{G_2 \to \gamma \gamma} = \frac{4\pi}{5} \alpha_e^2 g_{G_2 V V}^2 \left[ \left( \frac{1}{f_\rho} \right)^2 + \left( \frac{1}{f_\omega} + \frac{1}{f_\phi} \right)^2 \right]^2 M_{G_2}.
$$

(15)

To compute the radiative widths the hadronic couplings $g_{G V V}$ must be specified. For the scalar glueball, Ref. [12] uses the value $g_{G_0 V V} = 3.43$, but there is an error in Eq.(37) of that paper which should instead read, $g_{G_0 V V} = 3.43$, yielding the slightly larger value $g_{G_0 V V} = 4.65$. The corrected coupling is now closer to 4.23 which was obtained by an independent scalar glueball mixing analysis [13]. The tensor glueball coupling can be estimated by assuming the hadronic decays $G_2 \to V V'$ for $V = \rho, \omega$ and $\phi$ dominate and saturate the entire tensor glueball width, i.e.

$$
\Gamma_{G_2} \approx \sum_{V V'} \Gamma_{G_2 \to V V'} = 3\Gamma_{G_2 \to \rho \rho} + \Gamma_{G_2 \to \omega \omega} + \Gamma_{G_2 \to \phi \phi} + \Gamma_{G_2 \to \omega \phi}.
$$

(16)

This of course represents more of an upper bound for the coupling but for experimental planning it should provide sufficient photoproduction cross section estimates. Using Eq. (1) with the photon replaced by a second vector meson, $\gamma(k) \to V(k)$, the tensor glueball hadronic widths are

$$
\Gamma_{G_2 \to V V'} = \frac{9 g_{G_2 V V}^2}{60\pi} M_{G_2}(1 - 2x_+ + x_-^2)^{1/2}[6 - 9x_+ + 9x_-^2

- (8 - x_+ - x_-^2)x_-^2],
$$

(17)

where $S = 1/2$ if $V = V'$ and 1 otherwise, $x_+ = x \pm x'$, $x = M_V^2/M_{G_2}^2$ and $x' = M_{V'}^2/M_{G_2}^2$. For identical mesons, $V = V'$

$$
\Gamma_{G_2 \to V V} = \frac{9 g_{G_2 V V}^2}{20\pi} M_{G_2}(1 - 4x)^{1/2}[1 - 3x + 6x^2] .
$$

(18)

Taking $f_2(2010)$ (mass 2011 MeV, total width 202 MeV) and $f_2(2300)$ (mass 2297 MeV, total width 149 MeV) as tensor glueball candidates, Eqs. (16, 17, 18) yield, $g_{G_2 V V} = 1.60$, for $f_2(2010)$ and a similar value, $g_{G_2 V V} = 1.14$, for $f_2(2300)$. Adopting the average, 1.37, for the tensor coupling, the estimated glueball hadronic and radiative decays are summarized in Tables 1 and 2, respectively.

From the Tables it is clear that VMD and flavor independence predict roughly comparable hadronic $VV'$ widths. It is interesting that the largest branch...
Table 1
Tensor glueball hadronic decays in MeV

| $VV' \rightarrow$ | $\rho^0 \rho^0$ | $\omega$ | $\phi \phi$ | $\omega \phi$ |
|-------------------|----------------|---------|-------------|-------------|
| $\Gamma_{G_2(2010)} \rightarrow V V'$ | 26.2 | 25.8 | 10.3 | 33.0 |
| $\Gamma_{G_2(2300)} \rightarrow V V'$ | 37.2 | 36.8 | 20.3 | 44.7 |

Table 2
VMD glueball electromagnetic decays in keV

| $V \rightarrow$ | $\rho$ | $\omega$ | $\phi$ | $\gamma$ |
|-----------------|-------|---------|-------|---------|
| $\Gamma_{G(1700)} \rightarrow V \gamma$ | 1950 | 844 | 453 | 15.1 |
| $\Gamma_{G_2(2010)} \rightarrow V \gamma$ | 298 | 129 | 91.6 | 1.72 |
| $\Gamma_{G_2(2300)} \rightarrow V \gamma$ | 377 | 164 | 128 | 1.96 |

is to the $\omega \phi$ channel which has a clear, novel $3\pi K \bar{K}$ prompt decay. Also noteworthy are the suppressed $\omega \gamma$ and $\phi \gamma$ decays relative to $\rho \gamma$. Hence even though the gluonic coupling has been assumed flavor blind, consistent with QCD, the glueball widths are not. As mentioned above this decay signature is not expected for mesons and several published studies find no $\omega/\rho$ suppression in meson radiative decays. Indeed Ref. [14], which also uses VMD for scalar meson radiative decays, actually predicts an enhancement for $R_{\omega/\rho}$ by an order of magnitude. Related, tensor meson decay calculations [15] to vector and pseudoscalar meson channels reveal branching ratios that are very sensitive to flavor, varying by over an order of magnitude. Further, a recent meson decay model study [16], which compliments this work by advocating radiative decays as a flavor filter to clarify glueball mixing, predicts extremely flavor dependent radiative decays of scalar mesons. That investigation incorporates decay width renormalizations from mixing (see below) with a glueball component [17] but does not include contributions from glueball decays. Similarly, Ref. [18] repeats that analysis, again not including direct glueball decays, with relativistic quark model corrections and finds the same decay pattern but all widths are reduced by 50 to 70%. Both studies detailed marked sensitivity of $f_0 \rightarrow \rho \gamma$ and $\phi \gamma$ decay widths to mixing and quark flavor.

Concerning experimental evidence for $2^{++}$ isoscalar hadrons with mass near 2 GeV, the most recent PDG report [10] list six states: $f_2(1910)$, $f_2(1950)$, $f_2(2010)$, $f_2(2150)$, $f_2(2300)$ and $f_2(2340)$. Also there is the $f_2(2220)$ which is a tensor candidate but it, along with the $f_2(1910)$ and $f_2(2150)$, is omitted from the more important PDG summary table. For the four firm tensor states there is limited decay data and no quantitative branching ratios. The specific observed decays are: $\phi \phi$ for $f_2(2010)$ and $f_2(2340)$; $\phi \phi$, $K \bar{K}$ and $\gamma \gamma$ for $f_2(2300)$; $K^*(892)\bar{K}^*(892)$, $\pi^+\pi^-$, $4\pi$, $\eta\eta$, $K \bar{K}$ and $\gamma \gamma$ for $f_2(1950)$. There is a clear need for additional, more detailed measurements.
A final comment about glueball decays is in order regarding quarkonia-glueball mixing. In addition to the investigations discussed above, there have been several other mixing studies involving scalar hadrons [13,19,20,21] but no published worked treating tensor states in the 2 GeV region which is the focus here. For all theoretical models, the isoscalar $2^{++} q\bar{q}$ states calculated in this mass region will mix with predicted nearby tensor glueballs and this will alter the unmixed decay scheme. If the mixing is weak the predicted VMD decay profile will not be appreciably modified and may be effective in identifying the existence of glueball dominated states. For strong or maximal mixing, the branching ratios will depend upon model details and the VMD predictions will be affected by hadronic couplings in the quark sector, especially their flavor dependence. In general significant mixing will distort the simple VMD glueball decay signature of suppressed $\omega\gamma$ and $\phi\gamma$ but comparable $VV'$ decay rates. An improved mixing analysis, including decay contributions from both the quark [22] and glue sectors, is in progress and will be reported in a future communication.

The glueball couplings can also be used to describe the photoproduction process, $\gamma(k, \lambda) + p(p, \sigma) \rightarrow G(q, \lambda') + p(p', \sigma')$, where the energy-momentum 4-vectors (helicities) for the photon, proton, glueball and recoil proton are $k(\lambda)$, $p(\sigma)$, $q(\lambda')$ and $p'(\sigma')$, respectively. In the helicity representation the scalar glueball photoproduction amplitude, $\langle G_0 p | \hat{T} | \gamma p \rangle$, is

$$\langle G_0 p | \hat{T} | \gamma p \rangle = \epsilon_{\mu}(\lambda) H_{\sigma'\sigma}^{\mu} \equiv \epsilon \cdot H,$$

with $\epsilon_{\mu}(\lambda)$ the photon polarization 4-vector and $H_{\sigma'\sigma}^{\mu}$ the hadronic current obtained by application of Feynman rules to the tree level $s = (k + p)^2$, $t = (q - k)^2$ and $u = (p' - k)^2$ channel QHD diagrams. The spin-averaged scalar glueball photoproduction cross section is

$$\frac{d\sigma^{G_0}}{dT} = \frac{\pi}{4\omega_{cm}^2} \sum_{\sigma'\sigma} |\langle G_0 p | \hat{T} | \gamma p \rangle|^2 = \frac{\pi}{4\omega_{cm}^2} \sum_{\sigma'\sigma} \left[ |H_{\sigma'\sigma}^{1}|^2 + |H_{\sigma'\sigma}^{2}|^2 \right],$$

where $\omega_{cm}$ is the photon cm energy. As detailed in Ref. [12], the glueball cross section is dominated by $t$ channel exchanges for $\theta_{cm} < 65^0$. Accordingly only the $t$ channel amplitude is calculated and from Ref. [12] this is

$$H_{\sigma'\sigma}^{\mu} = \sum_{V = \rho, \omega, \phi} \frac{e g_{G_0V} g_{VNN} F_t(t) \Pi_V(t) \bar{u}(p', \sigma') [\gamma^{\mu} + i\kappa_{T}' \sigma^{\mu\alpha} k_{\alpha}'] u(p, \sigma)}{M_0},$$

with $k' = p' - p$, $t = k^2$. The hadronic form factor, $F_t(t)$, vector meson propagator, $\Pi_V(t)$, and remaining vector-nucleon couplings and transition moments are specified in Ref. [12].
Because of higher spin, the tensor glueball production amplitude is more complicated. Invoking vector and tensor dominance and Eq. (1), the photoproduction amplitude is

\[
< G_2 p | \hat{T} | \gamma p > = \frac{2 f_{\mu \nu}}{M_{G_2}^2} [2 \epsilon \cdot \hat{H} k_\mu k_\nu - k \cdot \hat{H} (\epsilon_{\mu k} + \epsilon_{k \mu}) - k \cdot k' (\epsilon_{\mu} \hat{H}_{\nu} + \epsilon_{\nu} \hat{H}_{\mu})] .
\] (20)

The hadronic current $\hat{H}$ has the same form as the scalar glueball result, $H$, except that the ratio $\frac{g_{G0}V_{G0}}{M_0}$ is replaced by $\frac{g_{G2}V_{G2}}{M_{G2}}$. Again focusing on forward angles, only $t$ channel diagrams are evaluated and since the formulation is covariant, the Gottfried-Jackson or glueball rest frame is used for mathematical convenience. The spin-averaged tensor glueball production cross section is

\[
\frac{d\sigma_{G_2}}{dt} = \frac{\pi}{4 \omega_{cm}^2} \sum_{\sigma' \sigma} \left[ a (| \hat{H}_{\sigma' \sigma}^1 |^2 + | \hat{H}_{\sigma' \sigma}^2 |^2) + b | \hat{H}_{\sigma' \sigma}^0 |^2 + (\frac{1+y}{1-y}) | \hat{H}_{\sigma' \sigma}^3 |^2 \right],
\]

where $a = 4(1-y)^2(1+y^2/6)$, $b = (1-y)^4$, $y = t/M_{G_2}^2$ and $M_{G_2} = 2.011$ GeV is the mass used in the cross section predictions presented here.

For the above specified glueball couplings, the tensor and scalar photoproduction cross sections are displayed in Figs. 1, 2 and 3. Figures 1 and 2 depict the lab energy dependence for the forward $cm$ angles $\theta_{cm} = 0^o$ and $25^o$, respectively, while Fig. 3 shows the angular distribution for 6 GeV photon lab energy. Again, since only $t$ channel amplitudes are included, results for angles greater than $60^o$ should be ignored. In contrast to the radiative widths, the cross sections are insensitive to the relative phase between the $\omega$ and $\phi$ couplings since $\rho$ exchange dominates (see Eq.(7)). While the scalar glueball cross section is somewhat larger, it is noteworthy that the magnitude of both cross sections is sufficient to expect reasonable count rates. Indeed, measurements of this process, including vector meson decays, would appear feasible for the envisioned Hall D project at Jefferson Lab.

Finally, photon transverse asymmetry observables, $A_{\gamma \bot}$, are also predicted and are displayed in Figs. 4 and 5 for the respective $cm$ angles $0^o$ and $25^o$. The scalar glueball asymmetry is greater than the tensor and both are large and increase with energy.

In summary, both tensor and scalar glueball cross sections, asymmetries and decay observables have been predicted using VMD and flavor independence. The results indicate that photoproduction cross sections are measurable and that by detecting comparable hadronic $VV'$ decays, especially the novel $\omega\phi \rightarrow 3\pi K\bar{K}$ branch, in conjunction with suppressed $\omega\gamma$, $\phi\gamma$ transitions relative to $\rho\gamma$, it may be possible to identify states having a significant gluonic component.
Acknowledgements

This work was supported by DOE Grant No.DE-FG02-97ER41048.

References

[1] G. Bali et al., Phys. Lett. B 309 (1993) 378.
[2] J. Sexton, A. Vaccarino and D. Weingarten, Phys. Rev. Lett. 75 (1995) 4563.
[3] C. J. Morningstar and M. Peardon, Phys. Rev. D 60 (1999) 034509.
[4] B. Lucini and M. Teper, JHEP 106 (2001) 050.
[5] A. P. Szczepaniak, E. S. Swanson, C. R. Ji and S. R. Cotanch, Phys. Rev. Lett. 76 (1996) 2011.
[6] F. J. Llanes-Estrada, S. R. Cotanch, P. Bicudo, E. Ribeiro and A. P. Szczepaniak, Nucl. Phys. A 710 (2002) 45.
[7] S. R. Cotanch and P. Maris, Phys. Rev. D 68 (2003) 036006.
[8] B. Renner, Nucl. Phys. B 30 (1971) 634.
[9] Y. Oh and T.-S. H. Lee, Phys. Rev. C 69 (2004) 025201.
[10] S. Eidelman et al., Phys. Lett. B 591 (2004) 1.
[11] H. Pilkuhn et al., Nucl. Phys. B 65 (1973) 460.
[12] S. R. Cotanch and R. A. Williams, Phys. Rev. C 70 (2004) 055201.
[13] L. Burakovsky and P. R. Page, Phys. Rev. D 59 (1999) 014022.
[14] D. Black, M. Harada and J. Schechter, Phys. Rev. Lett. 88 (2002) 181603.
[15] K. Peters and E. Kempt, Phys. Lett. B 352 (1995) 467.
[16] F. E. Close, A. Donnachie and Y. S. Kalashnikova, Phys. Rev. D 67 (2003) 074031.
[17] F. E. Close and A. Kirk, Eur. Phys. J. C 21 (2001) 531.
[18] M. A. DeWitt, H. M. Choi and C. R. Ji, Phys. Rev. D 68 (2003) 054026.
[19] C. Amsler and F. E. Close, Phys. Rev. D 53 (1996) 295.
[20] W. J. Lee and D. Weingarten, Phys. Rev. D 61 (2000) 014015.
[21] F. Giacosa, T. Gutsche and A. Faessler, Phys. Rev. C 71 (2005) 025202.
[22] F. J. Llanes-Estrada and S. R. Cotanch, Nucl. Phys. A 697 (2002) 303.
Fig. 1. Predicted scalar and tensor glueball cross sections vs lab energy for $\theta_{c.m.} = 0^\circ$. 

$\gamma + p \rightarrow G(0^{++},2^{++}) + p$

$\theta_{c.m.} = 0^\circ$

$M_{0^{++}} = 1.70$ GeV

$M_{2^{++}} = 2.01$ GeV
Fig. 2. Predicted scalar and tensor glueball cross sections vs lab energy for $\theta_{\text{c.m.}} = 25^\circ$. 
Fig. 3. Predicted scalar and tensor glueball cross sections vs cm angle.
\[ A_{\perp} = \frac{(d\sigma_{\text{up}} - d\sigma_{\text{down}})}{(d\sigma_{\text{up}} + d\sigma_{\text{down}})} \]

\[ M_{0^{++}} = 1.70 \text{ GeV} \]

\[ M_{2^{++}} = 2.01 \text{ GeV} \]

\[ \gamma + p \rightarrow G(0^{++}, 2^{++}) + p \]

\[ E_{\gamma}^{\text{lab}} = 6.0 \text{ GeV} \]

Fig. 4. Predicted scalar and tensor glueball transverse asymmetry vs cm angle.
Fig. 5. Predicted scalar and tensor glueball transverse asymmetry vs lab energy.