Supplemental material for Deterministic nonlinear phase gates induced by a single qubit

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I. COMPARISON OF GATE DESIGNS

FIG. 1: (Color online) The comparison of success probability and fidelity for $\chi_3 = 0.2$ for various parameters of the Rabi interaction strength $t_1$. We assumed that allowed number of Rabi interactions are $nR = 1000$ in all cases. The input state is vacuum state. We can clearly see that 5 uses of Rabi interaction per round gives the closest operator to the ideal unitary cubic gate, which corresponds to the point $(P,F) = (1,1)$. (a) and (b) are the same figures for a different range of success probability and fidelity.

There are several ways in which the target unitary operator $\hat{U}_3 = \exp[i\chi_3 \hat{X}^3]$ can be approximated by using Rabi interactions. They vary in the number of uses of Rabi interactions $n$ before the re-initialization onto the ground state of the ancillary qubit, and the strength of each individual Rabi interaction $t_1, t_2$. The methods presented in the main text generate operators $\hat{O}_s^{(n=2)} = G_c(g|M_1(t_1 \hat{X}, t_2)|g)$, $\hat{O}_s^{(3)} = G_c(g|M_1(t_1 \hat{X}, t_2 \hat{X})|g)$, $\hat{O}_s^{(5)} = G_c(g|M_1(t_1 \hat{X}, t_2)M_1(-t_1 \hat{X}, t_2)|g)$ and $\hat{O}_s^{(10)} = G_c^2(g|M_1(t_1 \hat{X}, t_2 \hat{X})M_1(-t_1 \hat{X}, t_2 \hat{X})|^2|g)$ as approximations to $\hat{U}_3$. The last two methods correspond to $k = 1$ and $k = 2$ in the main text.

When a unitary operator $\hat{U}_3$ acts on an arbitrary state $|\psi\rangle$, the implementation success probability $\langle \psi| \hat{U}^\dagger_3 \hat{U}_3 |\psi\rangle$ is 1 regardless of the state $|\psi\rangle$ by definition. The success probability is therefore an important measure of unitarity of the implemented operators $\hat{O}_s^{(n)}$. In Fig. 1, we compared the success probability $P = \langle \psi| \hat{O}_s^{(n)} \hat{O}_s^{(n)} |\psi\rangle$ and the fidelity $F = |\langle \psi| \hat{U}_3^\dagger \hat{O}_s^{(n)} \hat{U}_3 |\psi\rangle|^2/P$ for the generated operators $\hat{O}_s^{(n)}$ for $\chi = 0.2$. The input state is vacuum state $|0\rangle$. The experimental parameters $t_1, t_2$, and the number of rounds $R$ are under the constraint $f_n(t_1, t_2) = \chi_3/R$ where the functional form of $f_n$ depends on $n$. When $R$ and $\chi_3$ are fixed, the only free parameter is one of $t_1$ or $t_2$. Therefore, curves are drawn for various Rabi interaction strength $t_1$. We can clearly see that $\hat{O}_s^{(n=5)}$ and $\hat{O}_s^{(n=10)}$ gives the closest operator to the ideal unitary cubic gate which corresponds to the point $(P,F) = (1,1)$. In addition, we also notice that $\hat{O}_s^{(n=5)}$ is better than $\hat{O}_s^{(n=10)}$, therefore implying a re-initialization is beneficial.

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