Comment on “Subleading Corrections to Parity Violating Pion Photoproduction”

In a recent preprint, [1] Zhu et al. calculated a part of the next-to-leading order (NLO) corrections to parity-violating (PV) pion photoproduction \( \gamma p \to \pi^+ n \) in heavy-baryon chiral perturbation theory (HB\(\chi PT\)). They claim they have found the contribution as large as the leading-order (LO). If correct, the process will not be a clean way to extract the longest range PV pion-nucleon coupling constant \( h_{\pi NN}^{(1)} \), as was asserted in our previous publications [2]. In this comment we show that there is no solid evidence to support Zhu et al.’s claim. Moreover, we show that the subleading parity-violating coupling \( h_V \) cannot be extracted from observables at the order of interest based on the formalism of effective field theory.

The disagreement stems from the estimation of the coefficients of the relevant PV operators in the chiral lagrangian [3-5]

\[
\mathcal{L}^{PV} = -i h_{\pi NN}^{(1)} \pi^+ p_\pi - \frac{h_V}{\sqrt 2 F_\pi} \bar \pi \gamma^\mu n D_\mu \pi^+ + \frac{h_A^{(1)} + h_A^{(2)}}{F_\pi^2} \bar \pi \gamma^\mu \gamma_5 p n + D_\mu \pi^- + i \frac{h_A^{(1)} - h_A^{(2)}}{F_\pi^2} \bar \pi \gamma_5 \gamma^\mu n D_\mu \pi^- - i \frac{e C}{m_N F_\pi} \bar \pi \sigma^{\mu\nu} F_\mu \nu n \pi^+ + \text{h.c.} + \cdots \quad (1)
\]

Here the coupling constants \( h_{\pi NN}^{(1)} \) and \( h_V \) are not defined according to the standard practice of the effective field theory. Since the operator \( \bar \pi \gamma^\mu n D_\mu \pi^+ \) has dimension five, one unit higher than the leading operator \( \pi^+ p_\pi \), the natural size of its coefficient \( h_V/\sqrt 2 F_\pi \) is \( h_{\pi NN}^{(1)}/\Lambda_X \). In other words, the natural size of \( h_V \) is \( (\sqrt 2 F_\pi/\Lambda_X) h_{\pi NN}^{(1)} \sim (\sqrt 2 A_\pi) h_{\pi NN}^{(1)} \), not \( h_{\pi NN}^{(1)} \) as implied in the comment in Ref. [1] following Eq. (13). Indeed, naive dimensional analyses performed in Refs. [3-5] have \( |h_{\pi NN}^{(1)}| \sim 5 \times 10^{-7} \) (which is very close to the DDH “best guess estimate” [4]) and \( |h_V| \sim 5 \times 10^{-8} \). If these estimations are employed, then \( h_V \) contributes only at 10% level according to Eq. (13) in [1].

Is there any evidence that the relative sizes of \( h_{\pi NN}^{(1)} \) and \( h_V \) do not follow natural power counting? To establish this, one must show that either \( h_{\pi NN}^{(1)} \) is suppressed, or \( h_V \) is enhanced, relative to their natural orders of magnitude. As is well known, the experimental data on the former is still controversial [5], and hence it is natural to assume at this point that \( h_{\pi NN}^{(1)} \) is of order \( 5 \times 10^{-7} \). It is possible, though, that \( h_{\pi NN}^{(1)} \) is much smaller than the value suggested by naive dimensional analyses as suggested by the \( ^{18}\text{F} \) experiments [1]. If so \( h_{\pi NN}^{(1)} \) may not be the dominant effect in PV \( \gamma p \to \pi^+ n \). To determine conclusively if \( h_{\pi NN}^{(1)} \) is strongly suppressed is, in fact, one of the important motivations for our papers [4].

Phenomenologically, the size of \( h_V \) is even less certain. As far as we know, the only constraint on \( h_V \) comes from the electroweak radiative corrections (through the axial current) to the single-spin asymmetry measured by SAMPLE in PV \( \vec{e}p \) and \( \vec{e}d \) scattering [8]. Since the leading-order prediction strongly disagrees with current data, a huge size radiative correction is required to reconcile the theory and experiment. If this correction is calculated in leading order in chiral perturbation theory, as it has been done in Refs. [4,5], an usually large size (as much as 100 times larger than their natural size) of the couplings \( h_A \) and/or \( h_V \) is needed to explain the data. This, however, is a strong indication that the power counting for the radiative correction itself breaks down and the data cannot be used straightforwardly to extract a reliable \( h_V \). Given that the SAMPLE result is not yet fully understood, we believe that the benefit of doubt should be given to a natural size \( h_V \sim 5 \times 10^{-8} \).

There is, however, a more fundamental point: \( h_V \) cannot be extracted, in principle, independent of other parameters, from observables calculated to the order of interest at present. For instance, in the leading-order chiral perturbation calculation of the isovector nucleon anapole moment [4,5], \( h_V \) appears together with \( h_A^{(2)} \) in a combination \( h_A^{(2)} - g_A h_V/2 \). Therefore, \( h_A^{(2)} \) and \( h_V \) cannot be extracted independently from the value of the anapole moment. Likewise, in the NLO result for the spin-asymmetry \( \gamma p \to \pi^+ n \), \( h_V \) and \( C \) appear in the combination \( C - \frac{g_A}{4\sqrt 2} h_V \). It is easy to show, through field redefinitions, that the above combinations are the only ones that appear in PV observables to the order of current interest. Therefore, \( h_V \) has no direct physical effects and can be set to zero by hand. (The kinematical difference between the two operators seen in ref. [4] is the result of a higher-order artifact which is partially resumed to lower order. A direct computation of the relativistic triangle diagrams confirms this.)

To prove our assertion, we use the field redefinition which is a powerful tool to eliminate the equation-of-motion operators that generally have no effect on physical observables. Introduce the following field transformation:

\[
p \to p - \frac{i h_V}{\sqrt 2 F_\pi} \pi^+ n , \\
n \to n - \frac{i h_V}{\sqrt 2 F_\pi} \pi^- p .
\]

Note that the Jacobian of this transformation is unity up to some negligible \( \mathcal{O}(h_V^2) \) corrections. The new lagrangian becomes

\[
\mathcal{L}^{PV} = -i h_{\pi NN}^{(1)} \pi^+ p_\pi + \frac{h_A^{(1)} + h_A^{(2)}}{F_\pi^2} \bar \pi \gamma^\mu \gamma_5 p n D_\mu \pi^- + \cdots
\]
\[ +i \frac{h_\pi^{(1)}}{F_\pi} \pi^\mu \gamma_5 n \pi^+ D_\mu \pi^- \]
\[ -ie \frac{C}{m_N F_\pi} \bar{\pi} \sigma^{\mu \nu} F_{\mu \nu} n \pi^+ + \text{h.c.} + \cdots , \]

where we have omitted terms having dependence on the \( \pi^0 \) field as well as those with three and more pion fields and more derivatives. The barred parameters are defined as

\[ \bar{h}_A^{(2)} = h_\pi^{(2)} A^2 \frac{h_V}{2} , \quad \bar{C} = C - \frac{\kappa_p - \kappa_n}{4\sqrt{2}} h_V . \]

So only two independent combinations of \( \bar{h}_A^{(2)} \), \( h_V \) and \( C \) appear in the effective lagrangian to the order shown. We do not know at this point whether \( h_V \) can be eliminated entirely from the effective lagrangian. However, what we show here is that an experimental determination of it is extremely difficult because of the chiral suppression associated with higher orders.

To summarize, we argue that there is no credible evidence that the relative sizes of \( h_{\pi NN}^{(1)} \) and \( h_V \) do not follow the natural power counting. In addition, through field redefinitions, we find that \( h_V \) cannot be extracted at the order of interest independent of other unknown parameters of the theory.

Jiunn-Wei Chen and Xiangdong Ji

Department of Physics, University of Maryland,
College Park, MD20742

[1] S.-L. Zhu, S.J. Puglia, B.R. Holstein and M.J. Ramsey-Musolf, hep-ph/0012253.
[2] J.W. Chen and X. Ji, hep-ph/0011230, nucl-th/0011100, to appear in Phys. Lett. B.
[3] D.B. Kaplan and M.J. Savage, Nucl. Phys. A 556, 653 (1993).
[4] S.-L. Zhu, S.J. Puglia, B.R. Holstein and M.J. Ramsey-Musolf, Phys. Rev. D 62, 033008 (2000).
[5] C.M. Maekawa, J.S. Veiga and U. van Kolck, Phys. Lett. B 488, 167 (2000).
[6] B. Desplanques, J.F. Donoghue and B.R. Holstein, Ann. Phys. (N.Y.) 124, 449 (1980).
[7] S.A. Page et al., Phys. Rev. C 35, 1119 (1987); M. Bini, T. F. Fazzini, G. Poggio, and N. Taccetti, Phys. Rev. C 38, 1195 (1988).
[8] R. Hasty, Science 290, 2117 (2000).