HARROD 1939*

Lawrence E. Blume and Thomas J. Sargent

Harrod’s 1939 ‘Essay in Dynamic Theory’ is celebrated as one of the foundational papers in the modern theory of economic growth. Linked eternally to Evsey Domar, he appears in the undergraduate and graduate macroeconomics curricula, and his ‘fundamental equation’ appears as the central result of the AK model in modern textbooks. Reading his Essay today, however, the reasons for his centrality are less clear. Looking forward from 1939, we see that the main stream of economic growth theory is built on neoclassical distribution theory rather than on the Keynesian principles Harrod deployed. Looking back, we see that there were many antecedent developments in growth economics, some much closer than Harrod’s to contemporary developments. So what, then, did Harrod accomplish?

1. Harrod’s Analysis

Harrod presented what he described as a ‘tentative and preliminary attempt to give an outline of a “dynamic” theory’. His purpose, he concluded, was ‘to present a tool of analysis, not to diagnose the current condition’ (Harrod, 1936, p. 33). Harrod’s equilibrium analysis was based on three assumptions:

(i) saving is proportional to national income, \( S_t = sY_t \);

(ii) investment, the demand for saving, is proportional to the growth of national income, \( I_t = g(Y_{t+1} - Y_t) \); and

(iii) saving equals investment, the demand for saving equals the supply of saving, \( S_t = I_t \).

From this, one derives the ‘fundamental equation’,

\[
\frac{Y_{t+1} - Y_t}{Y_t} = \rho_w = \frac{s}{g},
\]

in which \( \rho_w \) is the ‘warranted’ rate of growth. Put differently, national income follows the first-order difference equation \( Y_t = \left[ g/(g-s) \right] Y_{t-1} \), with \( 1 > g > s > 0 \). Domar (1946) attained the same result in a different model, now often conflated with Harrod’s, and the same equation has recently reemerged as Piketty’s Second Fundamental Law of Capitalism (Piketty, 2014, p. 166).

Harrod supplemented his formal analysis with speculations about the consequences of deviations of actual aggregate income from warranted aggregate income. Harrod said that such deviations were bound to occur because the warranted rate of growth usually differs from a ‘natural’ rate of growth that is determined by changes in productivity and the labour force. Harrod also discussed instabilities and, in passing,

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* Corresponding author: Thomas J. Sargent, Department of Economics, New York University, 19 W. 4th Street, 6FL New York, NY 10012; Email: thomas.sargent@nyu.edu.

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disappointments of investors’ expectations, that gaps between warranted and natural growth rates would ignite.

Harrod’s article was widely read and cited for 20 or 30 years after 1939 because it raised interesting issues:

(i) the dynamic properties of a fixed-co-efficient model;
(ii) the implications of the qualification that fixed-co-efficients like the saving rate are not fixed exogenously but, instead, are determined by economic forces;
(iii) alternative senses and sources of instability;
(iv) some possible interactions between a multiplier (reflecting consumption decisions) and an accelerator (reflecting real investment decisions); and
(v) economic consequences of discrepancies between ex ante (or expected) and ex post (or realised) objects.

Harrod discussed these things in ways that readers today will find difficult to comprehend and appreciate, partly because of progress that the study of economic dynamics has made since 1939, partly because Harrod chose not to use or extend some lines of work preceding 1939 that would be more familiar to today’s reader and partly because his analysis, done without benefit of a formal model, is hard to follow and the analytic categories differ from those we use today. The explicit parts of the article leading to the difference equation for warranted aggregate income are clear enough but the paper’s informal and speculative parts are obscure. The modern reader cannot be blamed for sometimes being puzzled what the article is really about and how to relate Harrod’s ideas to those prevailing today. Some of the difficulties the Essay presents are these:

(i) What is capital? Today we think of capital as a factor of production; consequently the marginal product of capital is crucial to determining interest and wage rates, the distribution of output between capital owners and workers and so forth. For Harrod, as for Keynes, capital is something quite different. ‘no distinction is drawn in this theory between capital goods and consumption goods. In measuring the increment of capital, the two are taken together; the increment consists of total production less total consumption’ (Harrod, 1936, p. 18);
(ii) What determines s and g? The parameter g describes for Harrod the demand for saving, not as we read it today, the inverse marginal or average product of capital. ‘It may be expected’, Harrod (1936, p. 17) writes, ‘to vary as income grows and in different phases of the trade cycle; it may be somewhat dependent on the rate of interest’. Similarly, ‘s is regarded as likely to vary with a change in the size of income’ (Harrod, 1936, pp. 24–5);
(iii) What is equilibrium? Harrod is clear that the ‘warranted’ rate of growth is in fact the equilibrium growth rate of a model. If the key parameters s and g in fact vary with endogenous variables, then equilibrium is not yet determined until these additional relations are appended to the model. The easiest way to fill in the gaps, of course, is to read the Essay as a fixed-coefficients model, and this has become the tradition;
(iv) Why is equilibrium unstable? The informal and speculative stability analysis is obscure in part to the modern reader. The source of the difficulty is the same as that of our question (3). Harrod’s model is not closed. A decade later,
Baumol made a first attempt to close the model so that these questions can be answered (Baumol, 1948 1949);

(v) Is this a growth model? Harrod himself did not emphasise the contribution to economic growth. The Essay was intended to develop arguments first put forth in his 1936 book *The Trade Cycle*. Although the importance of lags had been recognised since Böhm Bawerk, Harrod is concerned to distinguish his analysis from the ‘time-lag theories’, especially those of Robertson, and from the econometric approach to dynamics being developed by Frisch and Tinbergen (Besomi, 1998). Time lags lead to second-order difference equations and the possibility of oscillatory behaviour. Harrod’s one concession to a theory of growth is this: ‘supposing damping measures could be introduced, to counteract the oscillation caused by the lag, would the system be stationary or advancing? And at what rate? Dynamic theory in my sense may throw some light upon this’ (Harrod, 1936, p. 14–5). The interaction of growth and business cycles is an intriguing possibility but in that day it would have been impossible to address the ‘cross-frequency restrictions’ that Harrod’s model might impose on aggregate time series; and

(vi) What about the role of prices; distributions of income, wealth and beliefs; labour demand and supply, and so forth. He acknowledges, as noted earlier, a dependence of $g$ on the price of capital, but so much is missing.

Although Harrod’s article seems partly about economic growth, most economists today have been convinced by Solow’s and Swan’s claim that labour–capital substitutability was the pertinent assumption for a long-run analysis and their demonstration that using that assumption in place of Harrod’s fixed-co-efficient model of investment pulls the rug out from under Harrod’s distinction between a natural and warranted growth rate.

The business cycle analysis Harrod points to in the Essay, the instability of the equilibrium growth path, could not go far without an analysis of how the economy behaves off the warranted path. In any event, the lag theorists won out. Milton Friedman’s distributed lag implementation of his permanent income model of saving as an important source of business cycle dynamics displaced Harrod’s analysis. Like Harrod’s, at least in its time series distributed lag econometric implementation, Friedman’s (1957) representation of adaptive expectations as a geometric distributed lag in disposable is a fixed-coefficient model. But it would not be a fixed-coefficient model for long. Friedman informally motivated his distributed lag in income as being an operational version of Irving Fisher’s theory of intertemporal consumption choice. That set the stage for Muth (1960) to interpret the fixed-coefficient, geometric distributed lag posited by Friedman as the outcome of a statistical prediction problem, there by providing the first concrete illustration of the cross-equation restrictions that Muth’s hypothesis of rational expectations imposes. After Muth, most of us sooner or later came to realise that for both investment and consumption functions, what we had earlier treated as fixed-co-efficients were not fixed with respect to the very things whose effects we want to study, namely, government tax and expenditure policies.
2. Harrod’s Antecedents

Reading the journals of the interwar years, one is struck by the paucity of footnotes. By today’s publishing standards, Harrod did not provide a reader much help in relating his analysis to earlier and contemporary work. This is a shame because the 1920s and 1930s were decades when some of the most important ideas underlying modern dynamic economic analysis were foreshadowed or created. From reading the article, it is impossible to tell whether Harrod was unaware of some of this work or knew about it and chose not to use it.

A list of significant available contributions that Harrod neglected would include:

(i) earlier work on the properties of fixed-coefficients models, for example, the Ricardian trade model (a timeless analysis), Leontief’s input–output analysis, and the von Neumann fixed-coefficient equilibrium model of economic growth, prices, and interest rates;

(ii) Edgeworth’s (1881) analysis of ‘out of equilibrium trades’ and their potential effects on equilibrium quantities and prices;

(iii) Pigou’s (1927) analysis of the interaction of time-to-build and gaps between anticipated and realised prices and quantities as sources of equilibrium dynamics;

(iv) what turned out to be a workhorse framework for equilibrium dynamics that J.R. Hicks (Value and Capital) created by including dated commodities within a Walrasian model of general equilibrium. Hicks’s ‘temporary equilibrium’ allowed discrepancies between anticipated and realised prices. Irving Fisher’s Theory of Interest contained key ingredients that set the stage for Hicks. In effect, Hicks showed how ‘equilibrium dynamics are a special case of static equilibrium’. Hicks’s work directly set the stage for Arrow’s and Debreu’s contingent commodities: if it is fruitful to index commodities by dates, why not also index them by random events? Doing that provides a sense in which ‘an equilibrium with risky commodities is a special case of an equilibrium without them’;

(v) Frank Ramsey’s (1928) intertemporal model of the equilibrium determination of investment and consumption. When combined with the ideas of Hicks, Arrow, Debreu, Muth and Lucas, Ramsey’s framework became the foundation of the dominant approach to equilibrium growth and business cycles that continues to be refined today;

(vi) work by Slutsky, Yule and Frisch that showed how random shocks impinging on simple low-order linear difference equations creates a rich structure for distinguishing random impulses from propagation mechanisms embedded in the difference equations;

(vii) Samuelson’s (1939) contemporary work on interactions between a multiplier and accelerator; and

(viii) the analysis of stability of an equilibrium in terms of what we would now call fast and slow dynamics.

Presumably, the ‘fast dynamics’ correspond to Samuelson’s ‘out of equilibrium’ dynamics that related rates of change of prices to excess demand, while the ‘slow
dynamics’ would be those driven by the exogenous forces pushing supply and demand curves to change over time. These were organised by Samuelson into his correspondence principle as a device for disciplining comparative static analysis.

3. After Harrod

It is worthwhile to list some of the most important subsequent contributions that made progress in addressing issues that Harrod raised.

(i) Theories based on consumer optimisation aimed at providing a theory of Harrod’s saving co-efficient. We have already mentioned the permanent income theory of consumption and saving and how it developed in a way that featured a peculiar division of labour: Friedman used Irving Fisher’s theory of optimal consumption choice conditional on exogenous forecasts of a stream of future disposal incomes (i.e. Friedman studied ‘control’); while Muth studied optimal prediction. Muth and his co-authors exploited the Theil–Simon separation of prediction and control (legitimate for linear, quadratic and Gaussian problems) extensively in their 1960 book.

(ii) Theories based on firm optimisation aimed at providing a theory of Harrod’s investment co-efficient. Like Harrod, Domar (1946) also wrote in a Keynesian framework. Unlike Harrod, he was concerned directly with economic growth. For him, \( g \) was an inverse marginal product of capital. Lerner (1944) (Economics of Control) and Haavelmo (1960) (A Study on the Theory of Investment) characterised the difficulties in linking marginal product of capital to a Keynesian investment function: a neoclassical theory of the firm gives rise to a demand function for a stock of capital, not an investment function like the one Harrod and earlier Keynesian econometric models assumed. Subsequent partially effective attempts to solve the conceptual problem raised by Haavelmo were pursued by Jorgenson and Eisner; success occurred only in the mid to late 1960s with the development of the adjustment cost model of Lucas (1967), Gould (1968), and Treadway (1969), culminating in Hayashi (1982).

(iii) Malinvaud (1953) contributed what can be regarded as a multi-sector Ramsey-Cass-Koopmans model of optimal economic growth. This is the first ‘modern’ growth model. Malinvaud raised the question of when equilibrium prices would signal an efficient allocation of resources. This led to a large and productive literature that opened many doors even before a practical answer was finally provided by the so-called Cass conditions. The same issue arises in distinguishing those prices that support weak optima from those that support full optima in overlapping generations models (themselves a development of dynamic analysis that goes back to Allais and Samuelson).

(iv) The complete markets general equilibrium models of Arrow and Debreu with all trading occurring at time zero, so that traders have no need to forecast prices. In this setting, Pigou’s (and subsequently, Lucas’s) gap between anticipated and realised outcomes does not appear.
(v) Radner’s (1972) sequential equilibrium trading model put traders in a situation wherein they have to forecast prices. This model also highlighted the importance of the distinction between complete and incomplete markets that, although implicitly present in Arrow, had not heretofore been adequately appreciated. These structures open a role for gaps between anticipated and realised prices, a gap that played a central role in Lucas’s 1972 equilibrium model of the Phillips curve. Radner also raised the problem of the lack of a clear way to value firms when markets are incomplete, a problem that remains open today.

(vi) Theoretical work on rational expectations and recursive competitive equilibria. In recursive competitive equilibria, private decision makers must forecast equilibrium prices. Nevertheless, in some important settings recursive competitive equilibria and Arrow–Debreu equilibria with all trades at time 0 have equivalent allocations. In these settings, the two equilibrium concepts they are just different ways of implementing the same equilibrium allocations. This work has helped identify and clarify the role of assumptions about market completeness in making room for gaps between anticipated and realised prices to affect equilibrium allocations.

(vii) Work of Chow, Howrey and others building souped up fixed-coefficients models capable of matching aggregate economic times series. Those models combined some kind of permanent income consumption function with a distributed lag accelerator. Something like these two components seem to be what are needed to match basic dynamics of $C, I, Y$ data (e.g. Sargent’s (1989) JPE paper.) Refinements in the 1980s and 1990s of real business cycle theories, especially their log-linear manifestations, extended this tradition by offering theories of the determinants of what earlier work had treated as fixed-coefficients.

4. Concluding Remarks

Harrod’s article is in a Keynesian tradition of wanting something not having what were ordinarily thought of as microfoundations based on theories of choice and equilibria available in the 1930s, or at least theories of choice available to Keynes and his colleagues. The purpose was to create a New Economics (the title of Seymour Harris’s, 1947 edited collection of essays) capable of explaining observations that Keynes and his circle were convinced equilibrium classical economics could not. Most of these things (Thomas Kuhn and Edward C. Prescott would call them ‘puzzles’) involved dynamics, frictions, shocks, disappointed anticipations and apparent violations of static first-order conditions (see chapter 2 of Keynes’s General Theory). The sense in which Keynes’s General Theory was to be ‘more general’ than classical equilibrium theory was that by relaxing the restrictions imposed by those (static) first-order conditions, you could reconcile more outcomes to the theory. This approach missed the mark in two ways. First, a theory without restrictions is useless. A theory that predicts everything

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1 See Tobin’s interpretation and celebration of Keynes’s along these lines in Harris (1947).
predicts nothing, and could provide no guide for policy analysis. If the restrictions were
to be empirical, how would we be able to distinguish those relations that are stable
when policies change from those that are not (and are therefore useless for policy
analysis)? This, of course, is the essence of Marschak’s analysis of the meaning of
structural equations and, in the context of models in which private agents optimally
solve dynamic choice problems, the Lucas critique. Second, the content of the
neoclassical restrictions was not well-understood. Indeed this question was not even
seriously investigated until Sonnenschein took it up in the early 1970s, leading to the
Debreu–Mantel–Mas Colell–Sonnenschein theorem on the lack of restrictions for
equilibrium prices and the subsequent Brown and Matzkin (1996) work on restrictions
for comparative statics. The problem is not, as Keynes and his circle thought, that the
neoclassical restrictions were too severe. The framework of equilibrium theory
imposes very few restrictions per se, and the art of applied general equilibrium analysis
is to find the assumptions about the primitives, preferences and technologies, that lead
to the most useful restrictions on equilibrium outcomes.

Attempts to pursue that Keynesian vision of a more general theory attracted some
enthusiasm in the late 1960s and early 1970s but ran out of steam soon after the
publication of Barro and Grossman’s Money, Employment and Inflation, which built
on Clower’s earlier efforts to create a disequilibrium analysis. Along with the
equilibrium dynamics of Hicks and Arrow and Debreu, two broad forces eventually
subverted Keynesian economics. One was a mostly American project to put microfo-
undations underneath Keynesian economics, such as Phelps et al. (1970). The other
was the macroeconometric project begun by Tinbergen and carried forward by Klein,
Goldberger, Modigliani and others, a project that, building on possibilities that Slutsky
and Frisch had demonstrated, aimed to put a quantitative discipline on the key objects
of business cycle and growth theory.

To answer the question with which we began, Harrod’s accomplishment in the Essay
was not the establishment of a theory that provided a foundation for future
developments. His power was in asking the right questions. His struggles in the Essay
raised questions that the profession spent the next half-century addressing. The Essay is
important as a signpost on the road, not announcing a destination, but pointing in the
direction that modern economic analysis subsequently took.

Cornell University, IHS Vienna, and the Santa Fe Institute
New York University and the Hoover Institution

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2 Essentially no restrictions on prices, and only those of revealed preference on comparative statics.

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**Appendix A.** Harrod, R.F. (1939). ‘An essay in dynamic theory’, *Economic Journal*, vol. 49(193), pp. 14–33.
AN ESSAY IN DYNAMIC THEORY

1. The following pages constitute a tentative and preliminary attempt to give the outline of a "dynamic" theory. Static theory consists of a classification of terms with a view to systematic thinking, together with the extraction of such knowledge about the adjustments due to a change of circumstances as is yielded by the "laws of supply and demand." It has for some time appeared to me that it ought to be possible to develop a similar classification and system of axioms to meet the situation in which certain forces are operating steadily to increase or decrease certain magnitudes in the system. The consequent "theory" would not profess to determine the course of events in detail, but should provide a framework of concepts relevant to the study of change analogous to that provided by static theory for the study of rest.

The axiomatic basis of the theory which I propose to develop consists of three propositions—namely, (1) that the level of a community's income is the most important determinant of its supply of saving; (2) that the rate of increase of its income is an important determinant of its demand for saving, and (3) that demand is equal to supply. It thus consists in a marriage of the "acceleration principle" and the "multiplier" theory, and is a development and extension of certain arguments advanced in my Essay on the Trade Cycle.¹

2. Attempts to construct a dynamic theory have recently been proceeding upon another line—namely, by the study of time lags between certain adjustments. By the introduction of an appropriate lag the tendency of a system to oscillate can be established. In these studies there is some doubt as to the nature of the trend on which the oscillation is superimposed. Supposing

¹ Especially in Ch. 2, secs. 4–5. The "Acceleration Principle" was there designated the "Relation." There is an objection to the use of the term acceleration in this connection. The study of the condition in which demand and supply are flowing at an unaltered rate has long been known as Static Theory; this implies that the equilibrium of prices and quantities resulting therefrom is regarded as analogous to a state of rest. By analogy, therefore, a steady rate of increase of demand, which is our first matter for consideration in dynamic theory, and a major effect of which is expressed by the "Relation," should be regarded as a velocity. Acceleration would be a rate of change in this.

However, the use of the expression Acceleration Principle in the sense of my relation is rapidly accelerating in current literature, and I reluctantly bow to the force majeure of usage.
damping measures could be introduced, to counteract the oscillation caused by the lag, would the system be stationary or advancing? And at what rate? Dynamic theory in my sense may throw some light upon this.

Moreover it is possible, and this the following argument seeks to establish, that the trend of growth may itself generate forces making for oscillation. This, if so, would not impair the importance of the study of the effect of lags. But it may be that the attempt to explain the trade cycle by exclusive reference to them is an unnecessary tour de force. The study of the operation of the forces maintaining a trend of increase and the study of lags should go together.

3. The significance of what follows should not be judged solely by reference to the validity or convenience of the particular equations set forth. It involves something wider: a method of thinking, a way of approach to certain problems. It is necessary to "think dynamically." The static system of equations is set forth not only for its own beauty, but also to enable the economist to train his mind upon special problems when they arise. For instance, an economist may pose to himself the question, What would be the effect on the system of an increase of exports or of a labour-saving invention? By reference to the static equations, he then proceeds to work out the new equilibrium position supposing the new higher level of exports to be maintained in perpetuity or the labour-saving invention to be incorporated in the productive technique once for all.

But let the question be: Suppose the level of exports begins and continues to increase steadily, or suppose its rate of increase to increase, or suppose labour-saving inventions begin to be made in a steady or growing stream; then the static method will not suffice. The static theorist may hope to reduce this supposed steady increase to a succession of steps up, each having the same effect. But if the following argument is correct, the effect on the moving equilibrium of advance may often be in the opposite direction to the effect on the static equilibrium produced by each of the steps considered singly. A new method of approach—indeed, a mental revolution—is needed.

Once the mind is accustomed to thinking in terms of trends of increase, the old static formulation of problems seems stale, flat and unprofitable. This is not to deny to static theory its own appropriate sphere. It will become apparent which kind of problem belongs to each branch of study.

4. I now propose to proceed directly to the Fundamental
Equation, constituting the marriage of the acceleration principle and the multiplier theory. This probably gives too much importance to the acceleration principle, and the necessary modification is introduced subsequently.

Let $G$ stand for the geometric rate of growth of income or output in the system, the increment being expressed as a fraction of its existing level. $G$ will vary directly with the time interval chosen—e.g., 1 per cent. per annum = 1 $\frac{1}{12}$ per cent. per month. Let $G_w$ stand for the warranted rate of growth. The warranted rate of growth is taken to be that rate of growth which, if it occurs, will leave all parties satisfied that they have produced neither more nor less than the right amount. Or, to state the matter otherwise, it will put them into a frame of mind which will cause them to give such orders as will maintain the same rate of growth. I use the unprofessional term warranted instead of equilibrium, or moving equilibrium, because, although every point on the path of output described by $G_w$ is an equilibrium point in the sense that producers, if they remain on it, will be satisfied, and be induced to keep the same rate of growth in being, the equilibrium is, for reasons to be explained, a highly unstable one.

If $x_0$ is output in period 0 and $x_1$ output in period 1, $G = \frac{x_1 - x_0}{x_0}$. Since we suppose the period to be short, $x_0$ or $x_1$ may alternatively stand in the denominator.

$x_0$ and $x_1$ are compounded of all individual outputs. I neglect questions of weighting. Even in a condition of growth, which generally speaking is steady, it is not to be supposed that all the component individuals are expanding at the same rate. Thus even in the most ideal circumstances conceivable, $G$, the actual rate of growth, would diverge from time to time from $G_w$, the warranted rate of growth, for random or seasonal causes.

Let $s$ stand for the fraction of income which individuals and corporate bodies choose to save. $s$ is total saving divided by $x_0$ or $x_1$. This may be expected to vary, with the size of income, the phase of the trade cycle, institutional changes, etc.

Let $C$ stand for the value of the capital goods required for the production of a unit increment of output. The unit of value used to measure this magnitude is the value of the unit increment of output. Thus, if it is proposed in month 1 to raise the output of shoes, so that in month 1 and all subsequent months output is one pair higher than in month 0, and the machine required to do this—neglecting all other capital that may be required—has a value 48 times the value of a pair of shoes, $C$ per month = 48.
The value of $C$ is inversely proportional to the period chosen. $C$ per annum $= 4$ in this case.\textsuperscript{1} The value of $C$ depends on the state of technology and the nature of the goods constituting the increment of output. It may be expected to vary as income grows and in different phases of the trade cycle; it may be somewhat dependent on the rate of interest.

Now, it is probably the case that in any period not the whole of the new capital is destined to look after the increment of output of consumers' goods. There may be long-range plans of capital development or a transformation of the method of producing the pre-existent level of output. These facts will be allowed for in due course. For the moment let it be assumed that all new capital goods are required for the sake of the increment of output of consumers' goods accruing.

Reserving proof for the next paragraph, we may now write the Fundamental Equation in its simplest form:\textsuperscript{2}

$$G_w = \frac{s}{C}$$

It should be noticed that the warranted rate of growth of the system appears here as an unknown term, the value of which is determined by certain "fundamental conditions"—namely, the propensity to save and the state of technology, etc. Those who define dynamic as having a cross-reference to two points of time may not regard this equation as dynamic; that particular definition of dynamic has its own interest and field of reference. I prefer to define dynamic as referring to propositions in which a rate of growth appears as an unknown variable. This equation is clearly more fundamental than those expressing lags of adjustment.

5. The proof is as follows. Let $C_p$ stand for the value of the increment of capital stock in the period divided by the increment of total output. $C_p$ is the value of the increment of capital per

\textsuperscript{1} If a month is the unit, the number of shoes added per period is 1, if a year 144. The value of $G$ per annum is 12 times as great as that of $G$ per month, since the numerator of $G$ per annum is 144 times as great and the denominator 12 times as great as the numerator and denominator respectively of $G$ per month. The number of machines added per month is 1 = 48 shoes = 48 units of increment of output. $C$ per month $= 48$. The number of machines added per year is 12 = 48 x 12 shoes. Thus the value in shoes of the annual increment of capital required to produce an annual increment of 144 shoes is 48 x 12 units. Therefore $C$ per annum $= \frac{48 \times 12}{144} = 4 = \frac{1}{12}$ of $C$ per month.

\textsuperscript{2} Since the value of $G_w$ varies directly and that of $C$ inversely with the unit period chosen, and the value of $s$ is independent of the unit, the validity of the equation is independent of the unit period chosen.

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unit increment of output actually produced. Circulating and fixed capital are lumped together.

\[ G = \frac{S}{C_p} \]  

is a truism, depending on the proposition that actual saving in a period (excess of the income in that period over consumption) is equal to the addition to the capital stock. Total saving is equal to \( sx_0 \). The addition to the capital stock is equal to \( C_p(x_1 - x_0) \). This follows from the definition of \( C_p \). And so,

\[ sx_0 = C_p(x_1 - x_0) \]

\[ \therefore \frac{s}{C_p} = \frac{x_1 - x_0}{x_0} = G \]

\( G \) is the rate of increase in total output which actually occurs; \( C_p \) is the increment in the stock of capital divided by the increment in total output which actually occurs. If the value of the increment of stock of capital per unit increment of output which actually occurs, \( C_p \), is equal to \( C \), the amount of capital per unit increment of output required by technological and other conditions (including the state of confidence, the rate of interest, etc.) then clearly the increase which actually occurs is equal to the increase which is justified by the circumstances. This means that, since \( C_p \) includes all goods (circulating and fixed capital), and is in fact production minus consumption per unit increment of output during the period, the sum of decisions to produce, to which \( G \) gives expression, are on balance justified—i.e., if \( C = C_p \), then \( G = G_w \), and (from 1(a) above)

\[ G_w = \frac{s}{C} \]

This is the fundamental equation, stated in paragraph 4, which determines the warranted rate of growth. To give numerical values to these symbols, which may be fairly representative of modern conditions: if 10 per cent. of income were saved and the capital coefficient per annum (\( C \)) were equal to 4, the warranted rate of growth would be 2\% per cent. per annum.

It may be well to emphasise at this point that no distinction is drawn in this theory between capital goods and consumption goods. In measuring the increment of capital, the two are taken together; the increment consists of total production less total consumption. Some trade-cycle theorists concern themselves with a possible lack of balance between these two categories; no doubt that has its importance. The theory here considered is more fundamental or simple; it is logically prior to the considerations regarding lack
of balance, and grasp of it is required as a preliminary to the study of them.

6. To use terminology recently employed by distinguished authorities, \( C_p \) is an \textit{ex post} quantity. I am not clear if \( C \) should be regarded as its corresponding \textit{ex ante}. \( C \) is rather that addition to capital goods in any period, which producers regard as ideally suited to the output which they are undertaking in that period. For convenience the term \textit{ex ante} when employed in this article will be used in this sense.

The truism stated above, 1(a), gives expression to Mr. Keynes’ proposition that saving is necessarily equal to investment—that is, to \textit{ex post} investment. Saving is not necessarily equal to \textit{ex ante} investment in this sense, since unwanted accretions or depletions of stocks may occur, or equipment may be found to have been produced in excess of, or short of, requirements.

If \textit{ex post} investment is less than \textit{ex ante} investment, this means that there has been an undesired reduction of stocks or insufficient provision of productive equipment, and there will be a stimulus to further expansion of output; conversely if \textit{ex post} investment exceeds \textit{ex ante} investment. If \textit{ex post} investment is less than \textit{ex ante} investment, saving is less than \textit{ex ante} investment. In his \textit{Treatise on Money} Mr. Keynes formulated a proposition, which has been widely felt to be enlightening, though experience has led him subsequently to condemn the definitions employed as more likely to be misconstrued than helpful. He said that if investment exceeded saving, the system would be stimulated to expand, and conversely. If for the definitions on which that proposition was based, we substitute the definition of \textit{ex ante} investment given above, it is true that if \textit{ex ante} investment exceeds saving, the system will be stimulated, and conversely. This truth may account for the feeling of satisfaction which Mr. Keynes’ proposition originally evoked and the reluctance to abandon it at his behest. In many connections we are more interested in \textit{ex ante} than in \textit{ex post} investment, the latter including as it does unwanted accretions of stocks. Mr. Keynes’ proposition of the \textit{Treatise} may still be a useful aid to thinking, if we substitute for “Investment” in it \textit{ex ante} investment as defined above.

7. Two minor points may be considered before we proceed with the main argument.

(i) It may be felt that there is something unreal in this analysis, since the increase in capital which producers will regard as right in period 1 is in the real world related not to the increase in demand from period 1 to period 2.
of total output in period 1, but to prospective increases in subsequent periods. This objection may be divided into two parts. 

(a) In view of the fact that much of the outlay of capital is connected with long-range planning, it may be held that the fundamental equation gives too much weight to the short-period effect of the acceleration principle. This objection is freely admitted and allowed for in the subsequent modification of the equation.

(b) It may further be objected that even in the sphere in which the acceleration principle holds there must be some lag between the increased provision of equipment (and stocks?) and the increased flow of output which they are designed to support. There may be some force in this. But the point is deliberately neglected in this part of the argument, along with all questions of lags. The study of these lags is of undoubtedly importance, but a division of labour in analysis is indispensable, and in this case the neglect is necessary in order to get the clearest possible view of the forces determining the trend and its influence as such. Moreover, the lag referred to in this sub-heading (b) may properly be regarded as unimportant, since, in the event of a steady advance \(G\) being maintained, the difference between \(x_1 - x_0\) and \(x_2 - x_1\) will be of the second order of small quantities. In other words, it matters not whether we regard the increment of capital as required to support the increment of total output in the same period or in the one immediately succeeding it.

8. (ii) In the demonstration given above (paras. 6 and 7) reference was made to the distinction between the \textit{ex post} and the \textit{ex ante} increase of capital goods. No reference was made to the distinction between \textit{ex post} and \textit{ex ante} saving.\(^1\) Suppose that \(G\) is not equal to \(G_p\) might not the discrepancy show itself on the other side of the equation, not in any divergence of \(C_p\) from \(C\), but in \textit{ex post} saving not being equal to \textit{ex ante} saving?

I have no very clear view as to possible causes likely to operate in a systematic way to distort \textit{ex post} from \textit{ex ante} saving, or of the probable importance of such distortions. It is said, for instance, that in a time of rising prices, fixed-income classes will not adapt their modes of life simultaneously, and so may save less than they would be disposed to do had they clearly foreseen the impending rise. \textit{Per contra} variable-income classes may not foresee their own rise of income, and so spend less than they would have been disposed to do.

\(^1\) Be it noted that \textit{ex ante} is here used of saving in a sense analogous to that defined in the expression \textit{ex ante} investment; it is the saving which savers would choose to make in any period, were they able to adapt expenditure simultaneously with the changing circumstances of the period.
This question of the possible divergence of *ex post* from *ex ante* saving must be kept entirely distinct from that of the variations in $s$ in the different phases of the trade cycle, which not only are admitted, but also play a part in the argument. $s$ may vary because the level of income or of profit is abnormally swollen or depressed.

The neglect of these possible divergences has no importance for the argument, since they will have the same effect on growth as the divergences of $C_p$ from $C$ for which they may serve as substitute. Thus if $G$ exceeds $G_e$, the right-hand side of the equation must exceed $s/C$. If the whole of this effect is found in $C_p$ it will be less than $C$, and this is a stimulus to expansion. Firms finding themselves short of stock or equipment will increase their orders. If, on the other hand, the whole of this effect is found in a divergence of *ex post* $s$ from *ex ante* $s$, *ex post* $s$ will be greater than *ex ante* $s$. Savers will find that they have saved more than they would have done had they foreseen their level of income or the level of prices correctly. Consequently they will be stimulated to expand purchases, and orders for goods will consequently be increased. Throughout the following pages the reader, whenever he finds a reference to the excess or deficiency of $C_p$ compared with $C$, may substitute, if he prefers it, a supposed deficiency or excess of *ex post* saving compared with *ex ante* saving, without affecting the course of the argument.

9. We now come to a point of major importance, constituting the difference between the dynamic equilibrium (warranted rate of growth) and the static equilibrium. Normally the latter is stable and the former unstable. This gives a *prima facie* reason for regarding the dynamic analysis as a necessary propaedeutic to trade-cycle study.

Some recent writers have been disposed to urge that the static equilibrium is not so stable as is sometimes claimed. Suppose that an increased output of a commodity, constituting a departure from equilibrium, is tried, so that its supply stands at a point at which the supply curve is above the demand curve. It is argued that, instead of a relapse at once occurring, reducing supply to the point of intersection of the supply and demand curves—this showing the stability of the old equilibrium—the upshot depends on how all parties now proceed. It is suggested that there may be a tendency to waltz round the point of intersection or, more broadly, that in the backward adjustment there may be

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1 The reader who is surprised that an excess of $G$ over $G_e$ is stimulating will find the explanation in the next paragraph.
wide repercussions disturbing the whole system. It is even held that the whole question of the stability of the static equilibrium, in the sense of the tendency of a relapse to it when a random departure occurs, is itself a dynamic problem, which cannot be looked after by the system of static equations. I have the impression that this type of criticism exaggerates the importance of this problem, and constitutes to some extent a failure to see the wood for the trees, and that on its own ground the theory of static equilibrium is well able to hold its own.

But when we look at the dynamic equilibrium, new vistas are opened. The line of output traced by the warranted rate of growth is a moving equilibrium, in the sense that it represents the one level of output at which producers will feel in the upshot that they have done the right thing, and which will induce them to continue in the same line of advance. Stock in hand and equipment available will be exactly at the level which they would wish to have them. Of course what applies to the system in general may not apply to each individual separately. But if one feels he has over-produced or over-ordered, this will be counterbalanced by an opposite experience of an equal importance in some other part of the field.

But now suppose that there is a departure from the warranted rate of growth. Suppose an excessive output, so that \( G \) exceeds \( G_w \). The consequence will be that \( C_p \), the actual increase of capital goods per unit increment of output, falls below \( C \), that which is desired. There will be, in fact, an undue depletion of stock or shortage of equipment, and the system will be stimulated to further expansion. \( G \), instead of returning to \( G_w \), will move farther from it in an upward direction, and the farther it diverges, the greater the stimulus to expansion will be. Similarly, if \( G \) falls below \( G_w \), there will be a redundance of capital goods, and a depressing influence will be exerted; this will cause a further divergence and a still stronger depressing influence; and so on. Thus in the dynamic field we have a condition opposite to that which holds in the static field. A departure from equilibrium, instead of being self-righting, will be self-aggravating. \( G_w \) represents a moving equilibrium, but a highly unstable one. Of interest this for trade-cycle analysis!

Suppose an increase in the propensity to save, which means that the values of \( s \) are increased for all levels of income. This necessarily involves, ceteris paribus, a higher rate of warranted growth. But if the actual growth was previously equal to the warranted growth, the immediate effect is to raise the warranted
rate above the actual rate. This state of affairs sets up a de-
pressing influence which will drag the actual rate progressively
farther below the warranted rate. In this as in other cases, the
movement of a dynamic determinant has an opposite effect on the
warranted path of growth to that which it has on its actual path.
How different from the order of events in static theory!

The reader may have some difficulty in the expression “stimu-
lus to expansion.” What is the significance of this, in view of
the fact that some growth is assumed as a basic condition? It
must be remembered that the value of \( G \) depends on aggregates
\( x_0 \) and \( x_1 \). These are sums of numerous quantities for which
individuals are responsible. It must be supposed that at all times
some individuals are jogging on at a steady level, others are risking
an increase of orders or output, others are willy-nilly curtailing.
\( G \) is the resultant of their separate enterprises. Some are in
any event likely to be disappointed. If \( G \) is equal to \( G_w \), it is to
be supposed that the general level of enterprise undertaken in
period 0, including in sum a certain increase over that in the
preceding period, is found to be satisfactory. Those running
short of stock balance those with surpluses. This justifies further
action on similar lines, though the individuals increasing orders
for stock in trade or planning new equipment in period 1 may not
be identical in person with those doing so in period 0. If an ex-
pansive force is in operation, more individuals, or individuals
having greater weight, will be induced by their trading position
to venture increases than did so in the preceding period. Con-
versely if a depressing force is in operation.

The dynamic theory so far stated may be summed up in two
propositions. (i) A unique warranted line of growth is determined
jointly by the propensity to save and the quantity of capital
required by technological and other considerations per unit
increment of total output. Only if producers keep to this line
will they find that on balance their production in each period has
been neither excessive nor deficient. (ii) On either side of this
line is a “field” in which centrifugal forces operate, the magnitude
of which varies directly as the distance of any point in it from the
warranted line. Departure from the warranted line sets up an
inducement to depart farther from it. The moving equilibrium
of advance is thus a highly unstable one.

The essential point here may be further explained by reference
to the expressions over-production and under-production. The
distinction between particular over-production and general over-
production is well known. In the event of particular over-
production being present, the resulting disequilibrium would be
characterized by a rate of production in excess of the warranted
rate. The economy as a whole would be in a state of disequilib-
rium, with associated pressures for reallocation of resources.

The dynamic model developed by Harrod provides a framework
for analyzing such disequilibria, emphasizing the role of
innovation and adjustment processes in economic growth.

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production, there will normally be a tendency to reduce production of the particular line, and so equilibrium will be restored. We may define general over-production as a condition in which a majority of producers, or producers representing in sum the major part of production, find they have produced or ordered too much, in the sense that they or the distributors of their goods find themselves in possession of an unwanted volume of stocks or equipment. By reference to the fundamental equation it appears that this state of things can only occur when the actual growth has been below the warranted growth—i.e., a condition of general over-production is the consequence of producers in sum producing too little. The only way in which this state of affairs could have been avoided would have been by producers in sum producing more than they did. Over-production is the consequence of production below the warranted level. Conversely, if producers find that they are continually running short of stocks and equipment, this means that they are producing above the warranted level.

But the condition of over-production, or, as we should perhaps call it, apparent over-production, will lead to a curtailment of production or orders, or a reduction in the rate of increase on balance, and consequently, so long as the fundamental conditions governing the warranted rate are unchanged, to a larger gap between actual and warranted growth, and so to an intensification of the evils which the contraction was intended to cure.

It must be noted that a rate of growth lying on either side of the warranted rate is regarded here as unwarranted. If the actual rate exceeds the warranted rate, producers on balance will not feel that they have produced or ordered too much; on the contrary, they will be running short of stocks and/or equipment. Thus they will not feel that they have produced the warranted amount plus something; on the contrary, they will feel that everything which they have produced has been warranted, and that they might warrantably have produced something more. None the less, we define their production as unwarrantably large, meaning by that that they have produced in excess of the unique amount which would leave them on balance satisfied with what they had done and prepared to go forward in the next period on similar lines.

10. The foregoing demonstration of the inherent instability of the moving equilibrium, or warranted line of advance, depends on the assumption that the values of $s$ and $C$ are independent of the value of $G$. This is formally correct. The analysis relates to a single point of time. $s$ is regarded as likely to vary with a
change in the size of income, but a change in the rate of growth at a given point of time has no effect on its size. \( C \) may also be expected to vary with the size of income, \( e.g., \) owing to the occurrence of surplus capital capacity from time to time, but the same argument for regarding it as independent of the rate of growth at a particular point of time applies.

It may be objected, however, that this method of analysis is too strict to be realistic, since the discovery that output is excessive or deficient, and the consequent emergence of a depressing or stimulating force, takes some time, and in the interval required for a reaction to be produced an appreciable change in \( s \) or \( C \) may have occurred.

Consider this with reference to an experimental increase in \( G \) above a warranted level. According to the theory of instability, any such experiment will be apparently over-justified, stocks or equipment running short in consequence of it. Is it possible that if resulting changes in the values of \( s \) or \( C \) are taken into account, this doctrine will have to be modified?

In order to justify modifying the doctrine, it would be necessary to show that, in consequence of the experimental increase, \( s \) was substantially increased or \( C \) reduced. It is unlikely that \( C \) would be reduced. The capital coefficient may often stand below the level appropriate to the technological conditions of the age, owing to the existence of surplus equipment. If this were so, the higher rate of output consequent upon the experimental increase would tend to raise \( C \). A smaller proportion of firms would come to find their capacity redundant, and a larger proportion would have to support a greater turnover by ordering extra equipment.

With saving the case is different. An expansion of activity might increase the proportion of income saved. What increase of saving is required for a modification of the instability theory?

This can be shown simply. Let \( x_e \) be an experimental increase of output above the warranted level. Let \( s_m \) stand for the fraction of the consequential income saved. The instability principle requires that

\[
C x_e > s_m x_e
\]

\[
s_m < C < \frac{s}{G_w}
\]

This condition needs interpretation. Since \( C \) and \( G_w \) do not both appear in the equation, it is necessary to define the period by which \( G_w \) is measured. This should be done by reference to the reaction time mentioned above—namely, the time required for
an undue accretion or depletion of capital goods to exert its influence upon the flow of orders. If this reaction time is six months, then $G_w$ must be measured as growth per six months.

Thus the instability condition requires that the fraction of marginal income saved shall not be more than the fraction of total income saved multiplied by the total income and divided by the increment of warranted income per six months. Thus if the warranted growth is $2\frac{1}{2}$ per cent. per annum, or $1\frac{1}{2}$ per cent. per six months, the instability principle requires that the fraction of marginal income saved must be less than eighty times the fraction of average income saved. Supposing that the high figure of 50 per cent. is taken as the fraction of marginal income saved, the fraction of total income saved must be greater than five-eighths of 1 per cent. Thus for any normal warranted rate of growth and level of saving, the instability principle seems quite secure.

The force of this argument, however, is somewhat weakened when long-range capital outlay is taken into account. It will then appear that the attainment of a neutral or stable equilibrium of advance may not be altogether improbable in certain phases of the cycle.

11. It should be noticed that the instability theory makes the empirical verification of the acceleration principle more arduous. For it leads to the expectation that in the upward phase of the cycle the actual rate will tend to run above the warranted rate, and the accretion of capital to be less than that required by the acceleration principle; and conversely in the downward phase. Thus a finding that the volume of investment fluctuates less than is required by direct computation from the acceleration principle is consistent with the theory here set forth, in which, none the less, the acceleration principle is presented as a leading dynamic determinant.

12. It is now expedient to introduce further terms into our equation to reduce the influence of the acceleration principle. Some outlays of capital have no direct relation to the current increase of output. They may be related to a prospective long-period increase of activity, and be but slightly influenced, if at all, by the current increase of trade. Or they may be induced by new inventions calculated to cheapen production or change consumers' modes of spending their income, so that they are not related to increments of output, but are designed to revolutionise the methods for producing some portion of already existing output or to substitute one line of goods for another in the consumers' budget. There are doubtless numerous factors, including the
state of confidence and the rate of interest affecting the volume of such outlay. It may suffice for the purpose in hand to divide it into two parts.

One part, $K$, is conceived to be quite independent both of the current level of income and its current rate of growth. The other, expressed as a fraction of income, $k$, is conceived to vary with the current level of income, as distinct from its rate of growth. This seems a reasonable assumption. Long-period anticipations are bound to be influenced by the present state of prosperity or adversity: even public authorities are apt to reduce the volume of public works in a slump. Companies may relate their expenditure on long-range plans to the current state of their profit account.

Having regard to the principle that the total increase of capital is equal to the total saving in the period, our fundamental equation may be modified as follows:

$$G_m = \frac{s - k - K}{x}$$

$$\therefore \frac{s - k - K}{x_0} = \frac{x_1 - x_0}{x_0} = G$$

$$\therefore G_m = \frac{s - k - K}{x_0}$$

It must be noticed that $C$ and $C_p$ now stand not for the total increase of capital (desired and actual, respectively) per unit increment of output, but only for the net increase of capital after the capital represented by $k$ and $K$ has been subtracted.

It may be noticed that the larger the volume of outlay which will be sustained independently of the current rate of growth, the smaller is the warranted rate of growth. A larger part of savings being absorbed in such outlay, there will be a smaller part to be looked after by the acceleration principle.

13. In the following pages the expression long-range capital outlay will be used for the magnitude denoted by $xk + K$. This must not be supposed to cover all investment in durable fixed equipment; for much of that is related to, and directly governed by, the current output of consumption goods. It refers only to that part of the output of fixed equipment the production of which is not governed by the current demand for consumption goods.

If long-range capital outlay were large by comparison with that required to support the current increase in turnover of consumable

$$s x_0 = G(x_1 - x_0) + kx_0 + K$$
goods, the peculiar conditions defined in § 10 for the invalidity of the instability principle might in certain circumstances be realised. For the fraction of total income saved and devoted to the finance of the increase of current output might be very small compared with the fraction of marginal income saved. It is not, however, to be supposed that it would normally be small enough to invalidate the instability principle. For, with normal growth at 2½ per cent., saving at 10 per cent., marginal saving at 50 per cent. and the reaction time 6 months, this would mean that fifteen-sixteenths of capital would normally be devoted to long-range capital outlay and only one-sixteenth would be directly associated with the current increase of output (cf. § 10). But such a situation might well arise in certain phases of the trade cycle, especially when capital capacity was redundant and saving low. In that case a stable equilibrium of advance might for a time be achieved.

14. To complete the picture, foreign trade must be taken into account. It is reasonable to measure exports, including invisible exports and the earnings of foreign investments, in absolute terms. The value of income which may be earned in this way may be conceived to be independent both of the level of activity at home and of its growth (though in so far as the trade cycle is world-wide, its value will be de facto related to income). Let $E$ stand for this value. Imports, on the other hand, are better taken as a fraction, $i$, of the current level of income. We then have, by parity of reasoning,

$$G_w = \frac{s + i - k}{x} - \frac{K}{x} - \frac{E}{x} \quad \ldots (3) \quad 1$$

$i$ need not be equal to $\frac{E}{x}$; the difference represents an international movement of capital. The influence of the various magnitudes on the warranted rate of growth is shown by the equation.

15. The fundamental dynamic equation has been used to demonstrate the inherent tendency of the system to instability. Space forbids an application of this method of analysis to the successive phases of the trade cycle. In the course of it the values expressed by the symbols on the right-hand side of the equation undergo considerable change. As actual growth departs

1 The principle now is that saving plus income expended on imports must be equal to the increase of capital in the country plus income derived from abroad. This is deducible from the fact that income derived from the sale of home made goods to consumers at home is equal to the income devoted to their purchase. Thus:--

$$sx_0 + ix_0 = C_x(x_1 - x_0) + kx_0 + K + E$$

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upwards or downwards from the warranted level, the warranted rate itself moves,\(^1\) and may chase the actual rate in either direction. The maximum rates of advance or recession may be expected to occur at the moment when the chase is successful.

For the convenience of the reader who may be tempted to experiment with this tool, it must be observed that \(C\) is always positive. Being the total quantity of capital required in connection with increments (or decrements) of current output divided by the increment (or decrement) of that output, when the latter is negative the former is negative also, and the coefficient remains positive. \(C_p\), on the other hand, may be negative; it is not negative whenever there is a depletion of capital goods, but only when the amount of capital goods outstanding is moving in the opposite direction to the level of total output.

The formula is not well adapted to dealing with the case of zero growth. But that matter is quite simple. Zero growth is only warranted when the amount of saving is equal to the amount required for long-range capital outlay. If the amount of saving exceeds this, there will be a tendency for output to decline, and conversely.

It may be well to make one point with regard to a downward departure from the warranted position of sufficient importance to outlive one reaction time and bring the system within the field where the centrifugal forces have substantial strength. The downward lapse will then continue until the warranted rate, determined by the values on the right-hand side of the equation, itself moves down. This will happen when the numerator falls or the denominator rises. But in a phase of declining rate of growth the capital coefficient is not in general likely to rise. And so long as there is still some positive growth, albeit at a declining rate, the fraction of income saved is not likely to fall. Therefore, once the rate of growth is driven downwards from the warranted level, the warranted level is not itself likely to fall, or the downward movement therefore to be checked until the rate of growth becomes negative and the level of income recedes. Now, if the actual rate is standing below the warranted rate, the centrifugal force will continue to operate, driving the actual rate progressively downwards, unless or until the warranted rate itself falls to a level as low as the actual rate. But, since the actual rate is now negative, this cannot happen until the numerator of the right-

\(^1\) This idea is analogous to that propounded by Mr. D. H. Robertson that the “natural” rate of interest may be expected to vary in the different phases of the trade cycle. Cf. Economic Journal, December 1934.
hand side of the equation becomes negative—that is, until saving falls below the level required for long-range capital outlay.

16. Alongside the concept of warranted rate of growth we may introduce another, to be called the natural rate of growth. This is the maximum rate of growth allowed by the increase of population, accumulation of capital, technological improvement and the work/leisure preference schedule, supposing that there is always full employment in some sense.

There is no inherent tendency for these two rates to coincide. Indeed, there is no unique warranted rate; the value of warranted rate depends upon the phase of the trade cycle and the level of activity.

Consideration may be given to that warranted rate which would obtain in conditions of full employment; this may be regarded as the warranted rate "proper" to the economy. Prima facie it might be supposed healthier to have the "proper" warranted rate above than below the natural rate. But this is very doubtful.

The system cannot advance more quickly than the natural rate allows. If the proper warranted rate is above this, there will be a chronic tendency to depression; the depressions drag down the warranted rate below its proper level, and so keep its average value over a term of years down to the natural rate. But this reduction of the warranted rate is only achieved by having chronic unemployment.

The warranted rate is dragged down by depression; it may be twisted upwards by an inflation of prices and profit. If the proper rate is below the natural rate, the average value of the warranted rate may be sustained above its proper level over a term of years by a succession of profit booms.

Thus each state of affairs has its appropriate evils. There is much to be said for the view that it is better that the proper warranted rate should be lower rather than higher than the natural rate.

17. In order fully to grasp the dynamic principle, it is necessary to bear in mind that changes in fundamental conditions have opposite effects on the actual rate and the warranted rate. An increased amount of long-range capital outlay, an increase in the capital coefficient, an increase in the propensity to consume, and an increase in the active balance on international account, or a decline in the passive balance, are all properly thought to have a stimulating effect on the system. But they all tend, as may
readily be seen from the equation, to reduce the warranted rate. This paradox may be readily explained.

Suppose that one of these stimulants begins to operate when the actual rate is equal to the warranted rate. By depressing the warranted rate, it drags that down below the actual rate, and so automatically brings the actual rate into the field of centrifugal forces, driving it away from the warranted rate—that is, in this case, upwards. Thus the stimulant causes the system to expand.

It must not be inferred that these stimulants are only of temporary benefit. For it may be healthy for an economy to have its proper warranted rate reduced. This is likely to be so when its proper warranted rate is tending to be above the natural rate. The long-run value of the stimulant can only be assessed if it is known whether, in its absence, the proper warranted rate is running above or below the natural rate.

It is often felt that a high propensity to save should warrant a great increase in the output of wealth, and this induces an extreme aversion to accept Mr. Keynes' view that excessive saving in the modern age is hostile to prosperity. The feeling is justified to the extent that higher propensity to save does, in fact, warrant a higher rate of growth. Trouble arises if the rate of growth which it warrants is greater than that which the increase of population and the increase of technical capacity render permanently possible. And the fundamental paradox is that the more ambitious the rate warranted is, the greater the probability that the actual output will from time to time, and even persistently fall below that which the productive capacity of the population would allow.

18. Policy in this field is usually appraised by reference to its power to combat tendencies to oscillation. Our demonstration of the inherent instability of the dynamic equilibrium confirms the importance of this. But there are two points to be noticed in this connection. 1. The nature of the measures suitable for combating the tendency to oscillate may depend on whether the natural rate is above or below the proper warranted rate. 2. In addition to dealing with the tendency to oscillation when it occurs, it may be desirable to have a long-range policy designed to

1 This may be the most fundamental rational explanation of the common view that it is dangerous for an old country to be a large importer of capital. For this involves a high warranted rate of growth, and it is dangerous to have a high warranted rate when the natural rate is low. Per contra for a young country, whose natural rate is high, it is considered healthy and proper to have a large import of capital.
influence the relation between the proper warranted rate of growth and the natural rate.

If, in the absence of interference, the proper warranted rate is substantially above the natural rate, the difficulties may be too great to be dealt with by a mere anti-cycle policy. In the first place, there is the probability of a slump occurring before full employment is reached, since during the revival the warranted rate may be dangerously near the actual rate, and liable at any time to overpass it, thus generating depression. Secondly, there is an acute problem if the actual rate reaches the ceiling of full employment and is depressed to the natural rate, and therefore below the warranted rate. An attempt may then be made to drag down the warranted rate below its normal level by increasing public works \((K)\). But the difficulty of the proper warranted rate being above the natural rate will be chronic, and this means that only by keeping in being a large and growing volume of public works can the slump be prevented. In fine, the anti-cycle policy has to be converted into a permanent policy for keeping down the proper warranted rate.

19. The ideal policy would be to manipulate the proper warranted rate so that it should be equal to the natural rate. If this could be achieved—but in fact only a rough approximation would be possible—an anti-cycle policy would none the less be an indispensable supplement. For the warranted rate is bound to be disturbed by the varying incidence of inventions and fluctuations in the foreign account. An anti-cycle policy would be necessary to combat the run-away forces which come into being as soon as a substantial change occurs in the warranted rate.

20. A low rate of interest makes for a low warranted rate of increase, by encouraging high values of \(K\) and \(C\) and, possibly also, by having a depressing influence on \(s\). Since the effects of changes in the rate of interest are probably slow-working, it may be wise to use the rate of interest as a long-range weapon for reducing the warranted rate of growth, and to reserve suitable public works for use against the cycle. It is not suggested, however, that a low rate of interest has sufficient power of its own to keep down the warranted rate without the assistance of a programme of public works to be kept permanently in operation.

If permanent public works activity and a low long-term rate availed to bring the proper warranted rate into line with the natural rate, variations in the short-term rate of interest might come into their own again as an ancillary method of dealing with oscillations.
21. This essay has only touched in the most tentative way on a small fraction of the problems, theoretical and practical, of which the enunciation of a dynamic theory suggests the formulation. In the last paragraph it was implicitly hinted that our present situation is one of a relatively high proper warranted rate. The evidence for this comes from inside and outside the dynamic theory itself. According to the dynamic theory, the tendency of a system to relapse into depression before full employment is reached in the boom suggests that its proper warranted rate exceeds its natural rate. Outside evidence includes the known decline in the growth of population, which involves a decline in the natural rate. More controversial points are the tendency of a more wealthy population to save a larger fraction of its income (high value of \( s \) involves high warranted rate), and the tendency of modern progress to depress rather than elevate the value of \( C \) (low value of \( C \) involves high warranted rate).

The main object of this article, however, is to present a tool of analysis, not to diagnose present conditions.

R. F. Harrod

*Christ Church, Oxford.*