A $\xi$-PROJECTIVELY FLAT CONNECTION ON Kenmotsu MANIFOLDS

Vahid Pirhadi

Abstract. In this paper, we introduce a semi-symmetric non-metric connection on $\eta$-Kenmotsu manifolds that changes an $\eta$-Kenmotsu manifold into an Einstein manifold. Next, we consider an especial version of this connection and show that every Kenmotsu manifold is $\xi$-projectively flat with respect to this connection. Also, we prove that if the Kenmotsu manifold $M$ is a $\xi$-concircular flat with respect to the new connection, then $M$ is necessarily of zero scalar curvature. Then, we review the sense of $\xi$-conformally flat on Kenmotsu manifolds and show that a $\xi$-conformally flat Kenmotsu manifold with respect to the new connection is an $\eta$-Einstein with respect to the Levi-Civita connection. Finally, we prove that there is no $\xi$-conharmonically flat Kenmotsu manifold with respect to this connection.

MSC(2010): 53D15; 53C07.

Keywords: Kenmotsu manifold, $\eta$-Einstein manifold, $\xi$-concircular flat manifold, $\xi$-conformally flat manifold, $\xi$-conharmonically flat manifold, $\xi$-projectively flat manifold.

1. Introduction and Background

The sense of Kenmotsu manifolds was introduced for the first time in [4] by K. Kenmotsu. He proved that a locally Kenmotsu manifold is a warped product $I \times_f N$ of an interval $I$ and a Kaehler manifold $N$ with warping function $f(t) = se^t$, where $s$ is a non-zero constant. The semi-symmetric connections was first defined by Friedman and Schouten ([5], [1]). The linear connection $\nabla$ is named a semi-symmetric connection if the following relation holds:

\[(1.1) \quad T(X, Y) = u(Y)X - u(X)Y,\]

where $T$ is the torsion tensor of $\nabla$ and $u$ is a 1-form. The semi-symmetric connections play a prominent role in Riemannian geometry. In [7], Yano showed that the existence of a semi-symmetric connection with zero curvature tensor is equivalent to being a conformally flat manifold. The Riemannian manifold $(M^n, g)$ is said to be conformally flat if at any point $p \in M$ there is a neighbourhood $U$ around $p$ and a smooth function $f$ on $U$ such that $(U, e^{2f}g)$ is flat. It is well known that Riemannian manifolds with constant sectional curvature are conformally flat. An important tool to being conformally flat is the Weyl conformal curvature tensor that is given by [8]:

\[\text{Date: Received: November 24, 2019, Accepted: March 13, 2020.}\]

*Corresponding author.
A Riemannian manifold with zero conharmonic curvature tensor is called a conharmonically flat manifold. Semi-symmetric and quarter-symmetric connections on Kenmotsu manifolds have been studied by many authors. Recently, Haseeb and Prasad defined a semi-symmetric connection on Kenmotsu manifolds and proved that an $n$-dimensional conharmonically flat manifold is equivalent to being of constant sectional curvature. In [6], Soos defined the projective curvature tensor as follow:

$P(X, Y)Z = R(X, Y)Z - \frac{1}{n-1} \{ S(Y, Z)X - S(X, Z)Y \}.$ (1.3)

He proved that a Riemannian manifold is locally projectively flat manifold if and only if its projective curvature tensor is identically vanishes. A concircular transformation on Riemannian manifold $(M^n, g)$ is a transformation that maps every geodesic circle of $M$ to a geodesic circle. The concircular curvature tensor was first defined by Yano [9] as follows:

$Z(X, Y)W = R(X, Y)W - \frac{r}{n(n-1)} \{ g(Y, W)X - g(X, W)Y \}.$ (1.4)

He showed that a Riemannian manifold which admits a concircular transformation is necessarily of constant scalar curvature [9]. Indeed, the conharmonic curvature tensor for the Riemannian manifold $(M^n, g)$ defines as follows [3]:

$C(X, Y)Z = R(X, Y)Z - \frac{1}{n-2} \{ S(Y, Z)X - S(X, Z)Y \}$

$+ g(Y, Z)QX - g(X, Z)QY \}.$ (1.5)

A Riemannian manifold with zero conharmonic curvature tensor is called a conharmonically flat manifold. Semi-symmetric and quarter-symmetric connections on Kenmotsu manifolds have been studied by many authors. Recently, Haseeb and Prasad defined a semi-symmetric connection on Kenmotsu manifolds and proved that an $n$-dimensional conharmonically flat $\eta$-Einstein Kenmotsu manifold with respect to the semi-symmetric connection is of quasi-constant curvature and has zero scalar curvature [2]. The almost contact metric manifold $(M, \varphi, \xi, \eta, g)$ is called $\eta$-Einstein manifold if its Ricci tensor satisfies:

$S(X, Y) = ag(X, Y) + b\eta(X)\eta(Y),$ (1.6)

where $a$ and $b$ are smooth functions on $M$. Similar to (1.1), the linear connection $\nabla$ on smooth manifold $M$ is named quarter-symmetric if

$\overline{T}(X, Y) = u(Y)\phi(X) - u(X)\phi(Y),$ (1.7)

where $u$ is a 1-form, $\phi$ is a $(1, 1)$ tensor field, and $\overline{T}$ is the torsion tensor of $\nabla$. Thereafter, Zhao and et al. defined a quarter-symmetric connection on Kenmotsu manifolds and proved that every $\xi$-conformally flat Kenmotsu manifold with respect to the quarter-symmetric connection is an $\eta$-Einstein. They also showed that an $n$-dimensional $\xi$-concircular Kenmotsu manifold with respect to the quarter-symmetric is of constant scalar curvature $r = -n(n-1)$.
Motivation by these works, we define a new semi-symmetric connection on Kenmotsu manifolds and get some interesting results. The paper is organised as follows: In section 2, we give some definitions and facts on Kenmotsu manifolds. In section 3, we define a new semi-symmetric connection on Kenmotsu manifolds that changes an $\eta$-Einstein Kenmotsu manifold to an Einstein manifold. In section 4, we consider an especial version of this connection and prove that if the Kenmotsu manifold $M$ is a $\xi$-concircular flat with respect to the new connection, then $M$ is necessarily of zero scalar curvature. Then, we review the sense of $\xi$-conformally flat on Kenmotsu manifolds and show that a $\xi$-conformally flat Kenmotsu manifold with respect to the new connection is an $\eta$-Einstein with respect to the Levi-Civita connection. Finally, we prove that there is no $\xi$-conharmonically flat Kenmotsu manifold with respect to this connection.

2. Preliminaries

An almost contact metric manifold is a $(2m + 1)$-dimensional smooth manifold $M$ with a $(1, 1)$ tensor field $\varphi$, a vector field $\xi$, a 1-form $\eta$, and a Riemannian metric $g$ satisfying:

\[ \varphi^2 = -I + \eta \otimes \xi, \quad \eta(\xi) = 1, \quad \varphi(\xi) = 0, \quad \eta \circ \varphi = 0, \]

\[ g(X, Y) = g(\varphi(X), \varphi(Y)) + \eta(X)\eta(Y), \]

\[ g(\varphi(X), Y) = -g(X, \varphi(Y)), \quad g(\xi, X) = \eta(X), \]

for all $X, Y \in \chi(M)$, where $\chi(M)$ is the set of all smooth vector fields on $M$. The almost contact metric manifold $(M, \varphi, \xi, \eta, g)$ is called a Kenmotsu manifold if it satisfies:

\[ (\nabla_X \varphi)Y = -\eta(Y)\varphi(X) + g(\varphi(X), Y)\xi, \]

where $\nabla$ is the Levi-Civita connection of $g$. In $n$-dimensional Kenmotsu manifolds $M$ the following relations hold [4]:

\[ \nabla_X \xi = X - \eta(X)\xi, \]

\[ (\nabla_X \eta)(Y) = g(X, Y) - \eta(X)\eta(Y). \]

\[ R(X, Y)\xi = \eta(X)Y - \eta(Y)X. \]

\[ R(\xi, X)Y = \eta(Y)X - g(X, Y)\xi. \]

\[ \eta(R(X, Y)Z) = g(X, Z)\eta(Y) - g(Y, Z)\eta(X). \]

\[ S(X, \xi) = -(n - 1)\eta(X). \]

\[ Q(\xi) = -(n - 1)\xi. \]

Also, in $\eta$-Einstein Kenmotsu manifolds, we have [4]:

\[ a + b = -(n - 1), \quad X(b) + 2b\eta(X) = 0. \]

The linear connection $\nabla$ on Riemannian manifold $(M, g)$ is called a metric connection if $\nabla g = 0$, otherwise it is called non-metric.
3. A new connection on \(\eta\)-Einstein Kenmotsu manifolds

Let \((M, \varphi, \xi, \eta, g)\) be an \(\eta\)-Einstein Kenmotsu manifold whose Ricci tensor is defined by (1.6). We define the semi-symmetric non-metric connection \(\nabla\) as follows:

\[
\nabla_X Y = \nabla_X Y - a\eta(X)Y - b\eta(Y)\eta(X)\xi,
\]

(3.1)

where \(\nabla\) is the Levi-Civita connection of \(g\). Consider the torsion tensor of \(\nabla\) as:

\[
T(X, Y) = \nabla_X Y - \nabla_Y X - [X, Y],
\]

(3.2)

we get

\[
\overline{T}(X, Y) = a\eta(Y)X - \eta(X)Y.
\]

(3.3)

\[
(\overline{\nabla}g)(Y, Z) = 2\eta(X)\{ag(Y, Z) + b\eta(Z)\eta(Y) \}
\]

(3.4)

that verify \(\nabla\) is a semi-symmetric non-metric connection. Now, we have the following theorem.

**Theorem 3.1.** Let \((M, \varphi, \xi, \eta, g)\) be an \(\eta\)-Einstein Kenmotsu manifold whose Ricci tensor is given by

\[
S(X, Y) = ag(X, Y) + b\eta(X)\eta(Y),
\]

(3.5)

then \(M\) is an Einstein manifold with respect to \(\nabla\).

**Proof.** Let \(\overline{R}\) be the curvature tensor with respect to \(\nabla\) which is given by:

\[
\overline{R}(X, Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z.
\]

(3.6)

By a straight calculation, we see

\[
\nabla_X \nabla_Y Z = \nabla_X \nabla_Y Z - a\eta(X)\nabla_Y Z - \frac{b}{n}\eta(X)\eta(\nabla_Y Z)\xi - X(a)\eta(Y)Z
\]

\[
- aX(\eta(Y))Z - a\eta(Y)\nabla_X Z + a^2\eta(Y)\eta(X)Z + \frac{ab}{n}\eta(Y)\eta(X)\eta(Z)\xi
\]

\[
- X\left(\frac{b}{n}\right)\eta(Y)\eta(Z)\xi - \frac{b}{n}X(\eta(Y)\eta(Z))\xi - \frac{b}{n}\eta(Y)\eta(Z)\nabla_X \xi
\]

(3.7)

\[
- \left(\frac{b}{n} + a\right)\eta(Y)\eta(Z)\eta(X)\xi.
\]

Interchanging \(X\) and \(Y\), we obtain

\[
\overline{R}(X, Y)Z = R(X, Y)Z + \{Y(a)\eta(X) - X(a)\eta(Y)\}Z - \{X(\frac{b}{n})\eta(Y)
\]

\[
- Y(\frac{b}{n})\eta(X)\}\eta(Z)\xi - \frac{b}{n}\{\eta(Y)X - \eta(X)Y\} \eta(Z)
\]

(3.8)

\[
- \frac{b}{n}\{g(Z, X)\eta(Y)\xi - g(Z, Y)\eta(X)\xi\}.
\]

Taking a contraction of the above equation yields

\[
\overline{S}(Y, Z) = S(Y, Z) + \{Y(a)\eta(Z) - Z(a)\eta(Y)\} - \{X(\frac{b}{n})\eta(Y)
\]

(3.9)

\[
- Y(\frac{b}{n})\}\eta(Z) - b\eta(Y)\eta(Z) + \frac{b}{n}g(Y, Z).
\]
Let us consider the smooth vector fields $Y$ and $Z$ as follows:

\begin{equation}
Y = \eta(Y)\xi + \nabla_Y, \quad Z = \eta(Z)\xi + \nabla_Z,
\end{equation}

where $\nabla_Y, \nabla_Z \in \text{ker}(\eta)$. By using (2.12) and the above relations, we find

\begin{equation}
Y(a)\eta(Z) - Z(a)\eta(Y) = \xi(b_n)\eta(Y) - Y(b_n) = 0.
\end{equation}

Hence, $\mathcal{S}(Y, Z)$ can be written as:

\begin{equation}
\mathcal{S}(Y, Z) = (a + b_n)g(Y, Z).
\end{equation}

From (1.6), we get

\begin{equation}
\mathcal{S}(X, Y) = (a + b_n)g(Y, Z),
\end{equation}

and this completes the proof. \hfill \square

\section{A $\xi$-projectively flat connection}

In this section, we study the notions of $\xi$-projectively flat, $\xi$-conharmonically flat, $\xi$-concircular flat, and $\xi$-conformally flat on Kenmotsu manifolds. Putting $a = 1$ and $b = -n$ in (3.1), we get an especial version of $\nabla$ (which we denote it again by $\nabla$) that is

\begin{equation}
\nabla_X Y = \nabla_X Y - Y(X)\eta(Y) + \eta(X)\eta(Y)\xi.
\end{equation}

By the above assumptions, we have

\begin{equation}
\mathcal{R}(X, Y)Z = R(X, Y)Z + \{g(Z, X)\eta(Y) - g(Z, Y)\eta(X)\}\xi
\end{equation}

\begin{equation}
+ \{\eta(Y)X - \eta(X)Y\}\eta(Z),
\end{equation}

and the Ricci tensor becomes

\begin{equation}
\mathcal{S}(X, Y) = S(X, Y) - g(X, Y) + n\eta(X)\eta(Y).
\end{equation}

**Definition 4.1.** [10] Let $(M^n, \varphi, \xi, \eta, g)$ be a Kenmotsu manifold, then $M$ is called a $\xi$-projectively flat manifold with respect to $\nabla$ if

\begin{equation}
\mathcal{P}(X, Y)\xi = 0,
\end{equation}

for all $X, Y \in \mathcal{X}(M)$, where $\mathcal{P}(X, Y)Z$ is defined by

\begin{equation}
\mathcal{P}(X, Y)Z = \mathcal{R}(X, Y)Z - \frac{1}{n-1}\{\mathcal{S}(Y, Z)X - \mathcal{S}(X, Z)Y\}.
\end{equation}

**Theorem 4.2.** Let $(M^n, \varphi, \xi, \eta, g)$ be a Kenmotsu manifold, then $M$ is a $\xi$-projectively flat manifold with respect to the connection $\nabla$.

**Proof.** Substituting $Z = \xi$ in (4.8), we see

\begin{equation}
\mathcal{R}(X, Y)\xi = R(X, Y)\xi + \{\eta(Y)X - \eta(X)Y\},
\end{equation}

using $R(X, Y)\xi = \eta(X)Y - \eta(Y)X$, we conclude that

\begin{equation}
\mathcal{R}(X, Y)\xi = 0.
\end{equation}

Also, From (4.3) and $S(X, \xi) = -(n-1)\eta(X)$ we obtain

\begin{equation}
\mathcal{S}(X, \xi) = 0,
\end{equation}

which proves the theorem. \hfill \square
Definition 4.3. [10] The Kenmotsu manifold \((M^n, \varphi, \xi, \eta, g)\) is called a \(\xi\)-concircular flat manifold with respect to the semi-symmetric connection \(\nabla\), if

\[
Z(X,Y)\xi = 0,
\]

for all \(X, Y \in \chi(M)\), where \(Z(X,Y)\) defines by

\[
Z(X,Y)W = R(X,Y)W - \frac{r}{n(n-1)} \{g(Y,W)X - g(X,W)Y\}.
\]

Theorem 4.4. If \((M^n, \varphi, \xi, \eta, g)\) is a \(\xi\)-concircular flat Kenmotsu manifold with respect to the semi-symmetric connection \(\nabla\), then \(M\) is of zero scalar curvature.

Proof. Taking a contraction of (4.3), yields \(\tau = r\). Let \(\{e_1, e_2, \ldots, e_{2m}, \xi\}\) be an orthonormal basis for \((2m+1)\)-dimensional \(M \,(n = 2m + 1)\). Since \(R(X,Y)\xi = 0\), we get

\[
Z(X,Y)\xi = -\frac{r}{n(n-1)} \{\eta(Y)X - \eta(X)Y\}.
\]

Substituting \(X = \xi\) and \(Y = e_i\) to see

\[
\eta(Y)X - \eta(X)Y \neq 0,
\]

at any point \(p \in M\). So, if \(Z(X,Y)\xi = 0\), then \(M\) is necessarily of zero scalar curvature. \(\square\)

\(\xi\)-conformally flat manifolds was first introduced by Zhen and et al. [11]. The Kenmotsu manifold \((M^n, \varphi, \xi, \eta, g)\) is said a \(\xi\)-conformally flat Kenmotsu manifolds if \(\mathcal{C}'(X,Y)\xi = 0\), where \(\mathcal{C}'(X,Y)Z\) is given by

\[
\mathcal{C}'(X,Y)Z = R(X,Y)Z - \frac{1}{n-2} (\mathcal{S}(Y,Z)X - \mathcal{S}(X,Z)Y + g(Y,Z)QX - g(X,Z)Y),
\]

(4.13)

This leads us to the following theorem.

Theorem 4.5. Let \((M^n, \varphi, \xi, \eta, g)\) be a \(\xi\)-conformally flat Kenmotsu manifold with respect to the semi-symmetric connection \(\nabla\), then \(M\) is an \(\eta\)-Einstein manifold with respect to the Levi-Civita connection.

Proof. The equation (4.3) implies that

\[
\mathcal{Q}(X) = Q(X) - X + n\eta(X)\xi.
\]

Using (4.7), (4.8), and the above relation, we obtain

\[
\mathcal{C}'(X,Y)\xi = -\frac{1}{n-2} \{\eta(Y)Q(X) - \eta(Y)X - \eta(X)Q(Y) + \eta(X)Y\} + \frac{r}{(n-1)(n-2)} \{\eta(Y)X - \eta(X)Y\}
\]

\[
= \frac{1}{n-2} \{\eta(Y) [-Q(X) + X + \frac{r}{n-1} X] + \eta(X) \left[Q(Y) - Y - \frac{r}{n-1} Y\right]\},
\]

(4.15)
substituting $Y = \xi$ and $X = e_i$, we get
\begin{equation}
(4.16) \quad \overline{\mathcal{C}}(e_i, \xi)\xi = \frac{1}{n-2}\left\{ -Q(e_i) + e_i + \frac{r}{n-1}e_i \right\},
\end{equation}
where \(\{e_1, e_2, ..., e_{2m}, \xi\}\) is an orthonormal basis on \(M\). Thus, if \(\overline{\mathcal{C}}(e_i, \xi)\xi = 0\), then we have
\begin{equation}
(4.17) \quad Q(e_i) = \left( -\frac{r}{n-1} + 1 \right)e_i.
\end{equation}
From (2.10), we find
\begin{equation}
(4.18) \quad S(X, Y) = \left( -\frac{r}{n-1} + 1 \right)g(X, Y) - \left( -\frac{r}{n-1} + n \right)\eta(X)\eta(Y),
\end{equation}
\[\square\]
Similarly to \(\xi\)-conformally flat manifolds, a Kenmotsu manifold is called a \(\xi\)-conharmonically flat manifold if \(\overline{\mathcal{C}}(X, Y)\xi = 0\) where \(\overline{\mathcal{C}}(X, Y)Z\) defines by
\begin{equation}
(4.19) \quad \overline{\mathcal{C}}(X, Y)Z = \overline{\mathcal{R}}(X, Y)Z - \frac{1}{n-2}\{\overline{\mathcal{S}}(X, Z)X - \overline{\mathcal{S}}(X, Y)Y \}
\end{equation}
\begin{equation}
\hspace{1.5cm} + g(Y, Z)\overline{Q}X - g(X, Z)\overline{Q}Y \}.
\end{equation}
Thus, we can state the following theorem.

**Theorem 4.6.** There is no \(\xi\)-conharmonically flat Kenmotsu manifold with respect to the semi-symmetric non-metric connection \(\nabla\).

\begin{proof}
Using (4.7) and (4.8) and putting \(Z = \xi\) in the above equation, we conclude that
\begin{equation}
(4.20) \quad \overline{\mathcal{C}}(X, Y)\xi = -\frac{1}{n-2}\{\eta(Y)\overline{Q}(X) - \eta(X)\overline{Q}(Y)\}.
\end{equation}
From (4.14) and by using the orthonormal basis \(\{e_1, e_2, ..., e_{2m}, \xi\}\), we see
\begin{equation}
(4.21) \quad \overline{\mathcal{C}}(e_i, \xi)\xi = -\frac{1}{n-2}\{Q(e_i) - e_i\}.
\end{equation}
Therefore, the equation \(\overline{\mathcal{C}}(e_i, \xi)\xi = 0\) implies that \(Q(e_i) = e_i\) and this yields
\begin{equation}
(4.22) \quad S(X, Y) = g(X, Y) - n\eta(X)\eta(Y),
\end{equation}
and this is impossible because of (2.12). \[\square\]

**Example 4.7.** Let \(M^7 = \{(x_1, x_2, ..., x_6, z) \in \mathbb{R}^7; z > 0\}\). Putting \(\eta = dz\) and let \(\{e_1, ..., e_7\}\) be an orthonormal basis which is given by
\begin{equation}
(4.23) \quad e_7 = \xi := \frac{\partial}{\partial z}, \quad e_i = e^{-z}\frac{\partial}{\partial x_i}, \quad i = 1, ..., 6.
\end{equation}
Next, suppose that the \((1, 1)\) tensor \(\varphi\) is given by:
\begin{equation}
(4.24) \quad \varphi(\xi) = 0, \quad \varphi(e_i) = e_{i+3}, \quad \varphi(e_j) = -e_{j-3},
\end{equation}
where \(i = 1, 2, 3\) and \(j = 4, 5, 6\). Then \((M^7, \varphi, \xi, \eta, g)\) is a Kenmotsu manifold which the Riemannian metric \(g\) is defined by \[2\]:
\begin{equation}
(4.25) \quad g = e^{2z} \sum_{i=1}^{6} dx^i \otimes dx^i + dz \otimes dz.
\end{equation}
Also, the curvature tensor and the Ricci tensor of $M^7$ can be written as follows [2]:

\begin{align}
R(X, Y)Z &= \{-g(Y, Z)X - g(X, Z)Y\}, \\
S(X, Y) &= -6g(X, Y).
\end{align}

From the above equations, we get

\begin{align}
\overline{R}(X, Y)Z &= \{-g(Y, Z)X - g(X, Z)Y\} + \{g(Z, X)\eta(Y) \\
&- g(Z, Y)\eta(X)\}{\xi} + \{\eta(Y)X - \eta(X)Y\}\eta(Z), \\
\overline{S}(X, Y) &= -7g(X, Y) + 7\eta(X)\eta(Y).
\end{align}

5. Conclusion

In this paper, we defined a new semi-symmetric non-metric connection on $\eta$-Kenmotsu manifolds that changes an $\eta$-Kenmotsu manifold into an Einstein manifold. Next, we proved that for $a = 1$ and $b = -n$ every Kenmotsu manifold is $\xi$-projectively flat with respect to this connection. Also, we showed that if the Kenmotsu manifold $M$ is a $\xi$-concircular flat with respect to the new connection, then $M$ is necessarily of zero scalar curvature. Thereafter, we demonstrated a $\xi$-conformally flat Kenmotsu manifold with respect to the new connection is an $\eta$-Einstein with respect to the Levi-Civita connection. Finally, we proved that there is no $\xi$-conharmonically flat Kenmotsu manifold with respect to this connection.

References

[1] A. Friedmann, and J. A. Schouten, Uber die Geometrie der halbsymmetrischen Ubertragungen. Mathematische Zeitschrift, 21(1): 211–223, 1924.
[2] A. Haseeb, and R. Prasad, (2017). Certain curvature conditions in Kenmotsu manifolds with respect to the semi-symmetric metric connection. Communications of the Korean Mathematical Society, 32(4): 1033–1045, 2017.
[3] Y. Ishii, On conharmonic transformations, Tensor (N. S.), 7: 73–80, 1957.
[4] K. Kenmotsu, A class of almost contact Riemannian manifolds. Tohoku Mathematical Journal, Second Series, 24(1): 93–103, 1972.
[5] J. A. Schouten, Ricci Calculus Springer, Berlin, 1954.
[6] G. Soos, Uber die geodatischen Abbildungen von Riemannschen Raumen auf projektiv-symmetrische Riemannische Räume. Acta Mathematica Hungarica, 9(3-4): 359–361, 1958.
[7] K. Yano, On semi-symmetric connection. Revue Roumaine de Math, Pure et Appliques, 15: 1570–1586, 1970.
[8] K. Yano, and M. Kon Structures on manifolds, world scientific publishing, 1989.
[9] K. Yano, Concircular geometry I. Concircular transformations. Proceedings of the Imperial Academy, 16(6): 195–200, 1940.
[10] P. Zhao, and et al. Certain Curvature Conditions on Kenmotsu Manifolds Admitting a Quarter-Symmetric Metric Connection. Publications de L’institut Mathematique, Nouvelle srie 1, 4(118): 169–181, 2018.
[11] G. Zhen, and et al. On $\xi$-conformally flat contact metric manifolds. Indian Journal of Pure and Applied Mathematics, 28 (6): 725–734, 1997.

(Vahid Pirhadi)

Faculty of Mathematical Sciences, Km. 6 Ravand Road, Kashan, Iran
Email address: v.pirhadi@kashanu.ac.ir