Open-Circuit Ultrafast Generation of Nanoscopic Toroidal Moments: The Swift Phase Generator

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Efficient and flexible schemes for a swift, field-free control of the phase in quantum devices have far-reaching impact on energy-saving operation of quantum computing, data storage, and sensing nanodevices. A novel approach for an ultrafast generation of a field-free vector potential that is tunable in duration, sign, and magnitude, allowing to impart non-invasive, spatiotemporally controlled changes to the quantum nature of nanosystems is reported. The method relies on triggering a steady-state toroidal moment in a donut-shaped nanostructure that serves as a vector-potential generator and quantum phase modulator. Irradiated by moderately intense, few cycle THz pulses with appropriately shaped polarization states, the nano donut is brought to a steady-state where a nearby object does not experience electric or magnetic fields but feels the photo-generated vector potential. Designing the time structure of the driving THz pulses allows for launching picosecond trains of vector potentials which is the key for a contact-free optimal control of quantum coherent states. This research can trigger a new class of ultrafast quantum devices operated and switched in an energy-efficient, contact and field-free manner, enabling new techniques for use in quantum information, magnetic nanostructures, and superconducting tunnel junctions as well as in toroidally ordered systems and multiferroics.

1. Introduction

Topological optical vector or vortex beams\cite{1} inspired key discoveries in analogous matter fields such as neutron\cite{2} and electron vortex beams (twisted electrons).\cite{3,4} A further interesting photonic class are toroidal fields, first introduced by Y. B. Zel’Dovich in 1958,\cite{5} followed by numerous studies on their nature and utilization for non-radiating charge-current configurations,\cite{6–8} reciprocal interactions,\cite{9} non-reciprocal refraction of light,\cite{10} laser spaser,\cite{11} dichroic effects,\cite{12} negative refraction, backward waves,\cite{13} magnetic response\cite{14} or perfect absorption,\cite{15} to name but a few.

The question clarified here is whether in quantum nanostructures toroidal electron distributions can be triggered controllably, which might have as useful applications as their optical counterpart. A generic example for a toroidal electromagnetic field is that of a polar classical current density on the surface of a torus. Similar to optical vortex and vector beams, toroidal electromagnetic magnetic fields may propagate to the far field. In the near field, metamaterials photonic toroidal moments were also studied and are found of relevance for sensing applications. In other fields, ground states with toroidal symmetry were discussed in nuclear,\cite{16} atomic,\cite{17} and molecular physics\cite{18} as well as in magnetism and ferroelectricity.\cite{19–22}

Here, we will be concerned with the ultrafast laser-induced electronic excitations in a quantum system that form a well-defined toroidal moment. Starting from an inversion and time-reversal symmetric ground state, we demonstrate how moderately intense THz pulses pump the system to a (toroidal) state with broken time and space reversal symmetry. A key element in our study is the use of a linearly polarized few-cycle pulse after irradiating the system with a focused, cylindrical vector pulse (CVB) with radial polarization.\cite{21–23} We demonstrate that the duration, the direction, the amplitude, as well as the buildup time of the toroidal moment \( T(t) \), are externally controllable on the picoseconds time scale by tuning the driving lasers parameters. Once a steady-state toroidal moment \( T \) is established, no electric or magnetic fields are present outside the structures. However, a gauge invariant vector potential \( \mathbf{A}(\mathbf{r}) \) with well-defined and controllable properties is generated and can act on phase-sensitive systems. Hence, our setup serves as an ultrafast phase generator or modulators of phase-based devices. No wiring of the system or current driving electric or magnetic fields are needed, the photo-induced toroidal moment emerges swiftly in an open circuit setup. It will sustain after the driving pulses are over because the toroidal moment is associated with an eigen mode excitation of the undriven system. Examples for possible applications are shown in Figure 1: For a Josephson junction\cite{24} consisting of two superconductors separated by an insulating barrier, the local laser-driven toroidal moment generated atop the structure drives a supercurrent across the junction. This is readily deduced from a Ginzburg–Landau formulation in which the free-energy density contains the Lifshitz-type invariant \( T \mathbf{v} \propto \sum_{i} T_{i} \delta_{i} \varphi_{i} \), meaning that \( T \) couples directly to the superfluid velocity \( \mathbf{v} \) or to spatial \( (r) \) variations in the condensate phase \( \varphi_{i} \).\cite{27–29}

Thus, our \( T \) acts as a direct THz switch for controlling the Josephson junction which are widely discussed for phase-qubits.
quantum computing.\cite{30} Note, this scheme for driving the junction proceeds at a low energy cost with minimal Joule heating. On the relevant time scale, normal-state magnetic and electric elements surrounding the junction are not susceptible to T and there is no electric nor magnetic fields present. Hence, T penetrates to the superconductors while the driving laser acts only on the cap layer containing the T generating structure (cf. Figure 1). By the same token, due to back-coupling, a time variation in the supercurrent results in a measurable effect on the quantum well confinement. For systems with sharp discontinuity of the conduction-band edge between the inner region of the torus (for instance GaAs) and the environmental crystal matrix (e.g., AlGaAs), the quantum well confinement potential reads $V(s) = 0$ for $r_0 - \Delta r/2 \leq s \leq r_0 + \Delta r/2$, and otherwise $V(s) = V_0$, where $V_0 = 0.5$ eV ($x = 0.4$). We will deal with appropriately doped structures such that only the intra-conduction-band dynamics is relevant. The pulses frequencies and amplitudes we employ do not allow for transitions across the band gap (cf. Figure 3). The single-particle wave functions of the conduction-band carriers have the form $\Psi_m(r) = \phi_m(s, \alpha)e^{im\phi}$, where $m = 0, \pm 1, \pm 2, \ldots$, is the angular momentum quantum number with respect to the azimuthal direction (cf. Figure 3a). Transforming\cite{32} the Laplacian in the curvilinear coordinates defined in Figure 2, we find that the local wave functions $\phi_m(s, \alpha)$ fulfill

$$\begin{align*}
-\frac{h^2}{2m^*} \left( \frac{\partial^2}{\partial s^2} + \frac{2}{s} \frac{\partial}{\partial s} + \frac{1}{s^2} \frac{\partial^2}{\partial \alpha^2} - \frac{m^2}{(R + s \cos \alpha)^2} \right) - \frac{1}{s(R + s \cos \alpha)} \left( R \partial_s + \sin \alpha \partial_{\alpha} \right) + V(s) \right) \phi_m &= E_m \phi_m \\
(1)
\end{align*}$$

where $m^* = 0.067m_e$ is the effective mass and $E_m$ are the energy eigenvalues. The above equation is not solvable analytically in general. Thus, Equation (1) is solved numerically with a finite-difference scheme. In Figure 3a, the confinement-induced electronic sub-bands in the conduction band are displayed. We selected $R = 150$ nm, $r_0 = 15$ nm and $\Delta r = 5$ nm. Although the electron dynamics in polar and radial direction cannot be separated in general, for this torus, the polar quantum number $\ell$ can be regarded as a good quantum number. Numerically, we found that every state $|\psi_m\rangle$ can be characterized by a (well-defined) angular polar state. Before, we calculated the lowest radial wave function $R_0(s)$ from Equation (1) for $R \to \infty$ (which separates the radial and polar dynamics). Then, the projection is defined by $|\ell|\phi_m\rangle$ where $R_0(s)e^{i\ell\phi}$ numerically, we found that the lowest electron states characterized by the wave functions $\phi_m(s, \alpha)$ reveal projections $|\ell|\langle \ell|\phi_m\rangle|^2 \geq 0.9$. Thus in general, for ratios $r_0/\Delta r \leq 0.1$, the description via polar and azimuthal quantum numbers, respectively $\ell$ and $m$, is a reasonable approximation, allowing to present the energy dispersion as a function of the polar and azimuthal quantum numbers $\ell$ and $m$, as shown.

2. Quantitative Results, Modeling, and Analysis

As illustrated in Figure 2, the photo-induced toroidal moment is triggered upon applying to a hollow or a full donut-shaped electronic system, a combination of two THz laser pulses. A CVB pulse\cite{32} with radial polarization is followed by a spatially homogeneous, linearly polarized pulse in the setup shown in Figure 2b. There are a variety of naturally existing systems that may serve as the tubal or donut structures (for instance, BaTiO$_3$ nanotorus, hexaphenyl-benzene, C$_{120}$ torus, or curved carbon nanotubes\cite{33–35}), or one may employ nanopattering techniques for fabricating the structure shown in Figure 1. In any case, the effects discussed below are generic to the geometry of the sample depicted in Figure 2a. For clarity, we will demonstrate the effect for a semiconductor-based tubal donut (i.e., for Figure 2), as reported, for instance, in ref. [36]. As shown in the appendix, the same effects are achievable in a full donut structure.

Our structure (cf. Figure 2a) is characterized by a major radius $R$ and a minor radius $r_0$. The coordinates of a charge carrier confined within the area denoted by $\Delta r$ in Figure 2a are: $x = (R + s \cos \alpha) \cos \phi$, $y = (R + s \cos \alpha) \sin \phi$, and $z = s \sin \alpha$. The angles $\phi, \alpha$ and $s$ are displayed in Figure 2a. For systems with sharp discontinuity of the conduction-band edge between the inner region of the torus (for instance GaAs) and the environmental crystal matrix (e.g., AlGaAs), the quantum well confinement potential reads $V(s) = 0$ for $r_0 - \Delta r/2 \leq s \leq r_0 + \Delta r/2$, and otherwise $V(s) = V_0$, where $V_0 = 0.5$ eV ($x = 0.4$). We will deal with appropriately doped structures such that only the intra-conduction-band dynamics is relevant. The pulses frequencies and amplitudes we employ do not allow for transitions across the band gap (cf. Figure 3). The single-particle wave functions of the conduction-band carriers have the form $\Psi_m(r) = \phi_m(s, \alpha)e^{im\phi}$, where $m = 0, \pm 1, \pm 2, \ldots$, is the angular momentum quantum number with respect to the azimuthal direction (cf. Figure 3a). Transforming\cite{32} the Laplacian in the curvilinear coordinates defined in Figure 2, we find that the local wave functions $\phi_m(s, \alpha)$ fulfill

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such that only excitations around the Fermi energy are induced from a homogeneous polar state ($m = 0$) while many $+ \ell$-states are populated implying a corresponding unidirectional current, as counter-propagating states ($+ \ell, + m$) and ($- \ell, - m$) contribute equally and hence cancel when evaluating the charge current. Applying a second (phase-delayed) linear polarized field brings about the necessary symmetry break for generating the unidirectional polar current without an azimuthal current. In the following, we inspect this process in more detail: The radially polarized CVB can be generated by a superposition of a right-hand circularly polarized ($\hat{e} = (1, i, 0)$) optical vortex with a topological charge $m_{OAM} = -1$ and a left-hand circularly polarized ($\hat{e} = (1, -i, 0)$) optical vortex with $m_{OAM} = 1$.[38] In cylindrical coordinates $\mathbf{r} = (\rho, \varphi, z)$, the vector potential of the applied optical field is

$$A_{\mathbf{r}}(\mathbf{r}, t) = \hat{e}_z A_0 e^{-\frac{\rho^2}{\omega_0^2}} e^{i \mathbf{k} \cdot \mathbf{r}} e^{-2i \tan^{-1}(z/z_0)} \times \Omega(t) \cos(\omega t).$$

The position-dependent polarization vector is $\hat{e}_z = (\cos \varphi, \sin \varphi, 0)$. The temporal envelope $\Omega(t) = \sin(\pi t / T_p)^2$ for $t \in [0, T_p]$ sets the pulse duration $T_p$. The beam waist is $w = w_0 \sqrt{1 + (z/z_0)^2}$, the curvature is $R(z) = z(1 + (z_0/z)^2)$, the Rayleigh range is $z_R = \pi w_0^2 / \lambda$, and the wave vector is $k = 2\pi / \lambda$ in the host medium. The peak amplitude is given by $A_0$ and incorporates the normalization constant $C = 2/(\sqrt{\pi} w^2)$.

Radially polarized terahertz waves are feasible experimentally for a wide range of parameters using different techniques.[39–41] Also, in the optical regime, CVBs are available, which will be needed when investigating toroidal moments in molecules such as $\text{C}_{120}$. Our simulations are performed for a CVB focused to $w_0 = 4 \mu m$. Hence, a nano-torus with a major radius of $R = 150 \text{ nm}$ is exposed to 9% of the peak amplitude of $A_{\mathbf{r}}(\mathbf{r}, t)$.
is shown. The amplitude of the CVB is chosen in a way that \( x = \frac{\hbar}{\omega} \) is clearly \( \hat{\epsilon}_z \). The combined field can be expressed by \( \hat{\epsilon}_z \) if \( \epsilon_0 = 0 \)

\[
\hat{\epsilon}_z = \frac{\hbar}{\omega} \cos(\omega t) \tag{1}
\]

where the first line represents the reversible dynamics driven by fields-induced transitions between \( \ell, m \) quantum states. The coupling of the laser pulses to the charge carriers reads

\[
\hat{H}_{\text{int}}(t) = \frac{i e \hbar}{2m} \left[ \nabla \cdot \mathbf{A}(t) + 2 \left( \mathbf{A}(t) + \mathbf{A}_0(t) \right) \cdot \nabla \right] + \frac{e^2}{2m} \left( \mathbf{A}(t) + \mathbf{A}_0(t) \right)^2 + e \Phi(t)
\]

Note the role of the spatially inhomogeneous electric scalar potential of the CVB, as followed from the Lorenz gauge \( \Phi(t) = -c^2 \int_{-\infty}^{t} \frac{dt'}{\epsilon_{\ell,m}} \nabla \cdot \mathbf{A}(t') \). A spatially inhomogeneous cylindrical vector beam is not divergence free. In contrast, for the homogeneous (linearly polarized) pulse, \( \nabla \cdot \mathbf{A}(t) = 0 \) applies. The second term in Equation (3) describes the irreversible dissipative relaxation dynamics caused by the coupling to acoustic phonons. The matrix elements \( R \) correspond to the Redfield tensor containing the phonon-electron matrix elements.

3. Steady-State Toroidal Moment

For clarity, we choose the two pulses in Figure 2 to have the same frequencies and magnitudes. As we are interested in a non-invasive generation of toroidal moments without damaging the sample or the heterostructure in Figure 1, we keep the pulse intensities below 1 kV cm\(^{-1}\). To ensure a swift generation of \( T \), pulse durations of two optical cycles are chosen with a photon energy of \( h\omega = 2.5 \text{ meV} \). With these laser parameters, we achieve a large number of intraband excitations obeying the propensity rules \( \Delta m = 0 \) and \( \Delta \ell \neq 0 \) and subsuming to a sizable magnitude of emergent \( T \). Figure 5 displays a snapshot of the photo-excited charge dynamics shortly after both pulses are off. The cross section of the donut sample is visible in the \( x-z \) plane revealing two circle segments, located at \( x = \pm R \), where the local azimuthal currents rotate in opposite directions meaning that the overall current is zero. In the local ring frame, the associated current density \( j = \mathbf{E} \times \mathbf{B} / c \) with \( \mathbf{j} = \mathbf{E} \times \mathbf{B} / c \), \( \mathbf{B} = (e/2m^*) \mathbf{r} \mathbf{r} \mathbf{r} \mathbf{r} | \mathbf{r} = 0 \) is a position-dependent vector pointing always in the plane defined by \( s \) and \( \alpha \) (cf. Figure 2a). These ring currents generate therefore local magnetic moments pointing in the \( \pm y \) direction (indicated by the orange area) and leading to a circulating magnetic moment \( M_0(t) = (1/2) \int dr \times j(r, t) \) with a characteristic radius \( R \). Such a solenoid current distribution on the major torus ring line is the source of the toroidal moment associated with the center of the

To evaluate physical quantities for the driven system, we utilize the density matrix method based on the single particle states that we illustrated in Figure 3. Before applying the pulses the density matrix of the sample reads \( \hat{\rho}(t) = \int_{-\infty}^{\infty} \mathbf{E}(T, t) \mathbf{B}(T, t) \) where \( \mathbf{E}(T, t) \) is the Fermi-Dirac distribution while \( E_F(t) \) is shown in Figure 3. The excitation dynamics of the system is governed by Heisenberg’s equation of motion

\[
i \hbar \frac{\partial}{\partial t} \hat{\rho}(\ell, m; \ell', m'; t) = \left[ \hat{H}_0 + \hat{H}_{\text{int}}(t), \hat{\rho}(\ell, m; \ell', m'; t) \right]
\]

\[
- \sum \mathbf{R}(\ell, m; \ell', m') \hat{\rho}(\ell, m; \ell', m'; t)
\]

\[
(3)
\]

\[
(4)
\]
Figure 5. a) Snapshot of the photo-excited (green) charge density of the torus in the x–z plane 1 ps after both pulses are completely off. The polar currents around the ring cross section (associated with the current density \( j \)) produce a circulating magnetic moment \( M \) (orange) leading to a pronounced localized toroidal moment \( T \), as needed for operating the junctions in Figure 1. The white arrows indicate the photo-generated vector potential while the color gradient characterizes its strength. Blue-violet color indicates the largest amplitude while black corresponds to a vanishing field. The pulse duration of both pulses is two optical cycles, and the color indicates the largest amplitudes while black corresponds to a vanishing junction in Figure 1. The white arrows indicate the photo-generated vector potential. b) Comparison of numerically obtained on-axis vector potential \( A_z(r = 0) \) with a fitted analytical form \( 1/(r^3) \). c) Long-distance behavior of \( A_x \) and \( A_y \). Both field components decay as \( 1/r^3 \) for \( r \to \infty \).

torus\(^{[3]} \) pointing in the z-direction. Its value can be calculated as\(^{[9]} \)

\[
T(t) = \frac{1}{10c} \int \! dr \left[ r \, j(r, t) - 2r^2 j(r, t) \right].
\] (5)

The toroidal moment is measured in the same unit (Am\(^3\)) as the quadrupole moment \( Q \), albeit both quantities are physically different. One can show that \( Q \) for such a solenoidal current distribution disappears while \( T \) is clearly non-zero.\(^{[18]} \) Furthermore, a quadrupole moment always generates a nonzero magnetic field everywhere whereas in the case of a photo-excited nano-torus, the magnetic field is confined inside the structure (cf. Figure 5).

As discussed in connection with Figure 1, of a key importance is the vector potential \( \mathbf{A}(r, t) \) associated with \( T \) outside the torus which we evaluate as \( \mathbf{A}(r, t) = \frac{2\pi}{\mu_0} \int \! dr \frac{\mathbf{r} \cdot \mathbf{j}(r, t)}{r^2} \) (note \( \mathbf{j}(r, t) \) we evaluate as an expectation value from a full quantum propagation). Figure 5 shows how after laser excitation, the photo-generated steady-state vector potential \( \mathbf{A}(r, t) \) loops around the donut with a structure similar to that of the magnetic field of an ordinary current loop. From classical electrodynamics, the vector potential of an ultra thin torus, that is, \( \Delta r \to 0 \) and for a steady unidirectional (classical) current density \( j(r) = j_0 (s - r_0) \delta_a \) (cf. Figure 2a), where \( j_0 = I/(2\pi (R + r_0 \cos \alpha)) \) and \( I \) is the charge current one can find for \( r \gg R + r_0 \) in spherical coordinates \( r = (r, \varphi, \theta) \) and the expression \( A_r = \frac{\pi}{2} \frac{r}{4R + 2r_0} \cos \theta \), \( A_\varphi = \frac{\pi}{2} \frac{r}{4R + 2r_0} \sin \theta \), which is proportional to the volume \( V = 2\pi r_0^2 R \). The analytical form reveals the characteristic concentric loops and decay of the vector potential as \( 1/r^3 \). In Figure 5c, the components of \( \mathbf{A}(r) \) obtained from quantum calculations are contrasted with the analytical long-range behavior. The (analytical) potential on the axis \( A_x(r = 0) = \frac{\pi}{8} \frac{r^2}{R^3} \). Figure 5b confirms our results in the classical limit, namely \( A_\varphi \), is at largest at \( z = 0 \) and follows the same decay law.\(^{[14]} \)

Figure 6 illustrates the toroidal moment emergence as the circulating magnetic moment \( M_i(t) \) picks up, along with the electric field amplitudes of the applied pulses for different intensities of the CVB pulses. The rise-up time indicates how fast we can switch on the toroidal moment which is relevant for applications related to Figure 1. For an efficient current generation, the phase shift between the pulses has to be \( \pi/2 \) in which case the combined local field mimics a conventional circularly polarized local field. As expected, the transient toroidal moments reach their maxima at peak fields of the pumping pulses. After both light fields are truly off (around 3 ps), the toroidal moments dynamics is governed by the relatively slow relaxation due to acoustic phonons, but can be rectified by applying the pulses again. Once the toroidal moment has reached its steady state, the electric field associated with the rise-up time ceases and no radiation is emitted. Generally, the build-up time of the toroidal moment reflects the underlying electronic structure: it is mainly governed by the energy difference between the states with \( \ell = 1 \) and \( \ell = 0 \) (cf. Figure 3).

4. Generation of Pump-Probe Toroidal Pulses

Considering toroidal moment-driven operations as those sketched in Figure 1, one may wish to employ \( T(t) \) for coherent or optimal control, in which cases time-structured pulses of \( T(t) \) are needed.\(^{[35]} \) In principle, a train of \( T(t) \) is generated by...
A polarized laser pulse is now applied. The state launched by the first pulse needs to be controlled and tuning the delay time to a local circular polarization. The "helicity" can be inverted by CVB. The result—see Figure 8—is a spherical pattern which is shown in Figure 7. a) Dynamical control of the photo-excited toroidal moment $T$ by the application of pulse trains. By changing the time delay $\Delta t$ between the radially polarized CVB and the linearly polarized light field $A_1$ (cf. Figure 2b), the direction of the local circular polarization can be manipulated at wish rendering possible a change in the direction of the toroidal moment. A pulse sequence with CVB being applied first means a positive time delay $\Delta t = 0.25T_0$ and is indicated by "PS I," while if the linearly polarized pulse is applied first then $\Delta t = -0.25T_0$, and this sequence we refer to as "PS II." The individual pulses in the train have a duration of two optical cycles and a photon energy of $h\omega = 2.5$ meV with a peak amplitude of the CVB of $E_{\text{peak}} = 600$ V cm$^{-1}$. b) Fourier transform of $T(t)$ reveals the frequency spectrum of the photo-generated toroidal moment pulse.

controlling the "helicity" of the circular polarization of the local combined field of the pulses and by the time delay $\Delta t$ between the pulses (cf. Figure 2b). An important caveat however is that, our phase-coherent many-electron non-equilibrium quantum state launched by the first pulse need to be controlled and switched by another pulse to stop $T(t)$, diminish it and to launch it in the opposite direction. A priori it is not clear whether and on which time scale this is possible. Hence, full numerics is needed.

Ignoring for once the spatially inhomogeneous character of the CVB, the local combined field in the $x - z$ plane for $x = R$ is

$$A(t) \sim C_0 \left( \begin{array}{c} \Omega(t) \sin[\omega t] \\ 0 \\ \Omega(t - \Delta t) \sin[\omega(t - \Delta t)] \end{array} \right).$$

(6)

Hence, the $z$-polarized laser pulse is time-delayed relative to the CVB by $\Delta t$. Choosing now, for example, $\Delta t = 0.25T_0$ where $T_0 = 2\pi/\omega$ is one optical cycle, the combined field reads $A(t) \sim B_0(\Omega(t) \sin[\omega t], -\Omega(t - \Delta t) \cos[\omega(t - \Delta t)])^T$ which amounts clearly to a local circular polarization. The "helicity" can be inverted by tuning the delay time to $\Delta t = -0.25T_0$ meaning that the $z$-polarized laser pulse is now superimposed before the CVB. The resulting combined light field reads now $A(t) \sim B_0(\Omega(t) \sin[\omega t], -\Omega(t - \Delta t) \cos[\omega(t - \Delta t)])^T$.

By applying trains of combined laser fields with periodic variation of the time delay $\Delta t$, one can steer the direction of the photo-induced polar current loops leading to the oscillations of the toroidal moment. An example of such a pulse train scheme and its impact on the dynamical buildup of the toroidal moment $T$ is shown in Figure 7. The first pulse sequence "PS I" acts as in Figure 6, where a linear polarized light pulse $A_1$ is delayed relative to the CVB, initiating the buildup of a toroidal moment pointing in the positive $z$ direction. Following $t > 3.5$ ps, we apply twice the altered pulse sequence "PS II" where the $z$ polarized laser pulse acts before the CVB changing effectively the handedness of the local circular polarization and switching the direction of the current loops. The direct consequence is the simultaneous depletion and buildup of $T$ in the opposite direction. For times $t > 10.5$ ps, we applied the first pulse sequence twice with the result that the direction of the photo-induced current loops turns once again leading to a renewed buildup of the toroidal moment in the positive $z$-direction. Further application of these pulse sequences maintains the oscillation cycles of the photo-induced toroidal moment.

For the discussed pulse train application, the toroidal moment oscillates roughly with a cycle duration of 13.9 ps meaning a central frequency $\omega_T = 0.45$ THz which is confirmed by the Fourier transform of $T$ displayed in Figure 7b. Such an oscillating moment radiates with characteristics that can be inferred from the generated vector potential including retardation: $A(r, t) = (\mu_0/4\pi) \int d\mathbf{r}' j(\mathbf{r}', t')/|\mathbf{r} - \mathbf{r}'|/c$. Expanding $\mathbf{r}' = t' - \mathbf{r}'/c$ and $t' = t - |\mathbf{r} - \mathbf{r}'|/c$. One obtains the radiated "physical fields" as

$$B_{\text{rad}} = -\frac{1}{c} \mathbf{n} \times \frac{\partial A_{\text{rad}}}{\partial t}$$

$$E_{\text{rad}} = -\frac{\partial A_{\text{rad}}}{\partial t}$$

(7)

where $A_{\text{rad}}(\mathbf{r}, t) = (\mu_0/4\pi) \int d\mathbf{r}' j(\mathbf{r}', t')/|\mathbf{r} - \mathbf{r}'|$. A numerical integration reveals a spherical pattern which is shown in Figure 8 in the case of an oscillating toroidal moment generated by the charge carriers that are driven by the alternating pulse trains. In the low-frequency regime $kR \ll 1$ (which is clearly fulfilled for $\omega_T = 0.45$ THz), one can find analytically for a homogeneously distributed current density (in the ring cross section) $j(\mathbf{r}) = j_0 \delta(s - r_0) e^{i\omega t} \hat{\mathbf{z}}_\theta$, the following vector potential

$$A_{\text{rad}} \sim k^3 \sin \theta \hat{\mathbf{z}}_\theta/r$$

(8)

where $\hat{\mathbf{z}}_\theta$ is the unit vector in the $\theta$-direction in spherical coordinates $\mathbf{r} = (r, \theta, \varphi)$ (cf. Figure 2) and $k = \omega_T/c$. For $r \gg R$, the analytically and numerically obtained radiated vector potential $A_{\text{rad}}$ shows remarkable similarities, as evident from Figure 8. According to Equations (7), the emitted magnetic field behaves as $B_{\text{rad}} \sim k^3 \sin \theta \hat{\mathbf{z}}_\theta/r$, while the electric component of the
radiation is characterized by \( E_{\text{rad}} \sim k^4 \sin \theta \partial / r \). The Pointing vector \( \mathbf{S} = (1/\mu_0) E_{\text{rad}} \times B_{\text{rad}} \) associated with the radiated fields has the radial component \( \hat{e}_r \cdot \mathbf{S} \sim k^6 \sin^2 \theta / r^2 \). Here we find a direct connection with the toroidal moment \( \hat{m} \). \( \hat{m} \cdot \mathbf{S} \approx (e^2 / 4\pi c^3) |T|^2 (1 - (\hat{e}_r \cdot \mathbf{T} / |\mathbf{T}|)^2) / r^3 \), since the toroidal moment \( T \) points in the z-direction (components in \( \hat{x} \)- and \( \hat{y} \)-directions are negligible, five times smaller in magnitudes).

5. Application: Electron States in a Toroidal Vector Potential

In addition to the use of the photo-generated toroidal vector potential \( \mathbf{A}(r) \) for the applications discussed in Figure 1, we study with explicit calculations its effect on the phase coherent electron motion. To this end, let us apply \( \mathbf{A}(r) \) presented in Figure 5 to another torus \( T_2 \) placed a distance \( d_z \) parallel to the \( \mathbf{A}(r) \) generating torus (as shown in Figure 9d) and is shielded appropriately from the THz pulses. As discussed above, \( T_2 \) feels then only the generated vector potential \( \mathbf{A}(r) \) of the original torus. For clarity, we take \( T_2 \) to be ultra thin, meaning \( \Delta \tau \rightarrow 0 \) confining the electrons motion to the surface of the torus \( T_2 \). We find that \( \mathbf{A}(r) \) results in a quasi-static toroidization of \( T_2 \): Due to symmetry, in \( T_2 \), the azimuthal electron motion is unaffected since \( \mathbf{A}(r) \) is cylindrically symmetric around \( z \). Let \( \phi_{T_1} \) describe the polar angular motion around the cross section of \( T_2 \). We inspect \( |\ell (\phi_{T_1})|^2 \) that indicate the occupation probabilities of the polar states with the quantum number \( \ell \). Figure 9a shows in absence of \( \mathbf{A}(r) \) the ground state spectrum of \( T_2 \) as a function of \( \ell \). Obviously, \( \pm \ell \) states are equally populated, as should be for a current-free ground state and a toroidization of \( T_2 \) is absent. Acting with \( \mathbf{A}(r) \) on \( T_2 \), we find (when \( \mathbf{A}(r) \) is static) the spectral change depicted in Figure 9b.

The numerically obtained energy spectrum of \( T_2 \) in dependence on \( \ell \) evidences that now electronic states are characterized by a manifold of polar states (denoted by the dots). The strength of the occupation probabilities of these states is signaled by the size of the dots. Importantly, we identify a clear break of symmetry between the clock and anticlockwise polar motion, as evidenced by the shift (relative to \( \ell = 0 \)) of the dispersion parabola connecting the most probable \( \ell \) states (i.e., largest dots). Clearly, a unidirectionally circulating (Aharonov–Bohm) persistent polar current is present, and hence \( T_2 \) carries a toroidal moment. As \( \mathbf{A}(r) \) decays as \( 1/r^3 \), the induced toroidization in \( T_2 \) is much smaller when \( d_z \) is increased from 30 to 60 nm as shown in Figure 9c. Figure 9d summarizes the dependence on \( d_z \). Obviously, the toroidization can be enhanced at larger distances by increasing the magnitude of \( \mathbf{A}(r) \).

6. Conclusions and Outlook

Based on full-fledged quantum mechanical simulations, we demonstrated a contact-free scheme for an ultrafast laser-induced toroidal moment generation and discussed its use in phase-sensitive devices. A key element is the use of time-delayed radially polarized THz vector beam and linear polarized THz pulse. Both have picosecond durations and an electric field amplitude in the range of few hundreds V cm\(^{-1}\), avoiding thus material damage or heating effects. An appropriate tuning of the time delay between the pulses allows for controlling on the picosecond time scale the direction and the magnitude of the toroidal moment. A steady-state toroidal moment, meaning a vector potential without electric nor magnetic fields, can be achieved and maintained by pumping with pulse trains to compensate for the decay caused by relaxation due to phonons.

We discussed possible use of this scheme in superconducting tunnel junctions thanks to the fact that the generated toroidal moments break space and time inversion symmetry. This fact is also decisive for utilizing this scheme for a swift switching of ferrotoroidal domains. Furthermore, the scheme is relevant for driving dynamics in multiferroic materials possessing a toroidal moment. Appropriately shaped pulse trains generate a well-controlled oscillating toroidal moment radiating in a widely tunable frequency range and having and electric dipole character. This pattern allows for setting up radiation sources tunable from no emission to superradiant. When combined with a further electric dipole, the appropriate tuning of the frequency and the phase difference results in constructive or destructive interference between the oscillating electric dipole and the toroidal amplitudes. The combined source may not radiate due to “physical” electric or magnetic fields. In contrast, the combined radiated vector potential \( A_{\text{rad}} \) is not canceled, however, and will propagate to the far field. Such non-radiating configuration serves as a source for electromagnetic potentials.

Appendix A: Toroidal Moments of 3D Quantum Donut

Our proposed scheme works also for a full donut. This comes as no surprise as with the polar current generation an effective centrifugal potential builds up repelling the charge density to the...
donut surface and resembling our previous tubal case. To assess for this proposition we employ in the quantum dynamic simulations the confinement potential

$$V(s) = \begin{cases} V_0 & s \in M_1 \\ 0 & s \in M_2 \end{cases}$$

(A1)

which describes a circular quantum well in the local frame characterized by the coordinates $s$ and $\alpha$.

Figure A1 shows the results of the interaction of the filled torus with the laser setup proposed in the main text. The major radius is $R = 75$ nm, while the effective thickness of the donut ring is $\Delta R = 20$ nm. As one can infer from Figure A1a, the photogenerated vector potential in the center of the nano-structure shows qualitatively the same characteristics meaning a disappearing $r$-component at $t = 0$. Analogously to Figure 5a, the field lines bend around the donut rim where the solenoidal current are induced as a result of the laser excitations. In Figure A1b, the temporal buildup of the toroidal moment is shown. In the case of the filled torus, the excitation is smaller by two magnitudes in comparison to the results shown in Figure 6 at the same laser intensities, as (for the pulse frequencies) only a fraction of the carriers contributes to the toroidal moment generating current. Hence, higher pulse intensities are needed in this case.

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Conflict of Interest

The authors declare no conflict of interest.

Keywords

Josephson junctions, phase generator, topological light beams, toroidal moments

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The value of the toroidal moments is origin-specific: By moving the origin of the coordinate frame from $\tilde{r}$ to $\tilde{r}' = \tilde{r} + R_0$, the toroidal moment changes as $\tilde{T}' = \tilde{T} + R_0 \times \mathbf{M}$.