Cosmic Microwave Background large-scale directional anomalies as seen by Planck and WMAP

L Polastri\textsuperscript{1,2}
\textsuperscript{1} Dipartimento di Fisica e Scienze della Terra, Università degli Studi di Ferrara, via Giuseppe Saragat 1, I-44122 Ferrara, Italy
\textsuperscript{2} INFN, Sezione di Ferrara, via Giuseppe Saragat 1, I-44122 Ferrara, Italy
E-mail: linda.polastri@unife.it

Abstract. It has been found that large-scale anisotropies in the Cosmic Microwave Background are anomalous with respect to the predictions of the standard model of cosmology. We focused on the low multipole alignments, assuming the ΛCDM model and we confirmed that the quadrupole/octupole and the dipole/quadrupole/octupole alignments are anomalous with a significance up to 99.9%, for both WMAP and Planck data. Trying to explain the origin of this kind of anomalies we tested the dipolar model. This alternative phenomenological model explains the CMB hemispherical power asymmetry found in the WMAP and Planck data, so it is possible that it can solve also other CMB directional anomalies. We show that the alignments are anomalous in the dipolar model too, roughly at the same level as in ΛCDM. We conclude that the dipolar model does not provide a better fit to the data than the ΛCDM.

1. Introduction
One of the pillars of the standard model of cosmology (ΛCDM) is the cosmological principle: the Universe is described with a good approximation as homogeneous and isotropic on large scales. The general appearance of the Universe must not depend on the observers position and on the direction of observation. In accordance with this principle the Cosmic Microwave Background (CMB) should be distributed in an isotropic way, i.e. it should not have preferred directions \cite{1}. The data collected by the WMAP and Planck experiments are largely consistent with the ΛCDM model, but suggest that the CMB has a preferred direction. In particular, when expanding the CMB anisotropy pattern in spherical harmonics, there is an anomalous alignment for the dipole/quadrupole/octupole (\(\ell=1, \ell=2, \ell=3\)) and the quadrupole/octupole, see Fig. 1.

Figure 1. The quadrupole (\(\ell=2\)) and the octupole (\(\ell=3\)) as measured by Planck.
In literature it has been shown [2] that the quadrupole moment of the CMB is lower than expected in ΛCDM cosmology and that the octupole is planar and aligned with the quadrupole. Three independent anomalies have been identified that involve the quadrupole and the octupole: 1) the quadrupole with itself is anomalous at the 1-in-20 level by being low; 2) the octopole with itself is anomalous at the 1-in-20 level by being very planar; 3) the alignment between the quadrupole and octupole is anomalous at the 1-in-60 level. It is convenient to use a particular representation to study the directional anomalies: the Multipole Vectors (MV) formalism [3], see all details in Section 2.1. This formalism was used to study the WMAP and Planck data [4] and confirmed the anomalous alignment between quadrupole and octupole in both data sets [5–7].

At the moment, the origin of this kind of anomalies is not clear, however, it is difficult to imagine that the directional anomalies are due to systematic or instrumental problems, because two different experiments (WMAP and Planck) provided data in agreement with each other. Therefore it is hard to believe that two distinct experiments present instrumental errors capable of altering both results in the same way. Therefore it might be that these directional anomalies have a cosmological origin.

2. CMB directional anomalies

2.1. Multipole vectors

On the unit sphere, a function can be expanded as a linear combination of spherical harmonics:

\[ f(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y^m_\ell(\theta, \phi), \]

where \( a_{\ell m} \) are the coefficients and \( Y^m_\ell(\theta, \phi) \) are the spherical harmonics.

We decided to use the multipole vectors formalism in order to have an easier approach to the problem. This is an alternative representation of the data on a sphere which allows us to express the information contained in each set of \( a_{\ell m} \) coefficients, for any integer \( m = -\ell, ..., \ell \), in unit vectors \( \hat{v}_i \) and a corresponding amplitude \( A_\ell \):

\[ a_{\ell m} \rightarrow A_\ell, \hat{v}_1, ..., \hat{v}_\ell \]

Unfortunately no closed analytical expression for Eq. 2 is available.

The multipole vectors are invariant under transformations \( a_{\ell m} \rightarrow ca_{\ell m}, A_\ell \rightarrow cA_\ell \) and they are used to construct scalar estimators that are invariant under rotation.

We built the estimators using the area vectors \( q \) and \( o_j \):

\[ q = \hat{q}_{21} \times \hat{q}_{22}, \]
\[ \hat{o}_1 = \hat{o}_{32} \times \hat{o}_{33}, \]
\[ \hat{o}_2 = \hat{o}_{33} \times \hat{o}_{31}, \]
\[ \hat{o}_3 = \hat{o}_{31} \times \hat{o}_{32}, \]

where \( q_{2j} \) (with \( j = 1, 2 \)) represent the two MVs associated to the quadrupole and \( o_{3i} \) (with \( i = 1, 2, 3 \)) represent the three MVs associated to the octupole.

2.2. Estimators

We provided eight different estimators belonging to two statistics “S” and “T” [3,4,8,9]. They measure “distance” from a situation of complete misalignment, in terms of \( \cos \theta \) resulting from the scalar product of the MVs. Orthogonality is associated to zero in both cases, whereas complete alignment, i.e. parallelism, is represented by the value 1. Notice that the “T” estimators...
depend on $\cos^2 \theta$, while the “S” estimators on $\cos \theta$ [10].

We considered six estimators for the quadrupole/octupole alignment:

\begin{align*}
S &= \frac{1}{3} \sum_{j=1}^{3} |\hat{q} \cdot o_j|, \quad (4a) \\
T &= 1 - \frac{1}{3} \sum_{j=1}^{3} (1 - |\hat{q} \cdot o_j|)^2, \quad (4b) \\
S_{23} &= \frac{1}{3} \sum_{j=1}^{3} |q \cdot o_j|, \quad (4c) \\
T_{23} &= 1 - \frac{1}{3} \sum_{j=1}^{3} (1 - |q \cdot o_j|)^2, \quad (4d) \\
\hat{S}_{23} &= \frac{1}{3} \sum_{j=1}^{3} |\hat{q} \cdot \hat{o}_j|, \quad (4e) \\
\hat{T}_{23} &= 1 - \frac{1}{3} \sum_{j=1}^{3} (1 - |\hat{q} \cdot \hat{o}_j|)^2. \quad (4f)
\end{align*}

and two estimators for the dipole/quadrupole/octupole alignment:

\begin{align*}
DQOS &= \frac{1}{4} (|q \cdot d| + |o_1 \cdot d| + |o_2 \cdot d| + |o_3 \cdot d|), \quad (4g) \\
DQOT &= 1 - \frac{1}{4} \left[ (1 - |q \cdot d|^2) + (1 - |o_1 \cdot d|^2) + (1 - |o_2 \cdot d|^2) + (1 - |o_3 \cdot d|^2) \right]. \quad (4h)
\end{align*}

The symbol $\hat{\cdot}$ denotes the unit vector, the vector $d$ represents the dipole direction which reads $(l, b) = (224^\circ, -22^\circ) \pm 22^\circ$ in Galactic coordinates.

2.3. Dipolar model

In order to solve the problem of hemispherical asymmetry, Gordon et al. [11] developed a particular mechanism for breaking isotropy in large-angle fluctuations of the CMB. The pillars of this model are two: 1) a field, whose spatial fluctuations are dominated by long-wavelength contributions, 2) a non-linear response to the field by the CMB temperature fluctuations.

Statistical isotropy is preserved in the full theory, but it is spontaneously broken due to long-wavelength field fluctuations that appear as a gradient locally to the observer and carry the preferred direction into a spectrum of multipoles.

The CMB may exhibit a non-linear response in two different ways: a multiplicative modulation of the anisotropy and an additive effect that is uncorrelated with the intrinsic anisotropy.

We can define for the dipolar model the temperature fluctuation field of the CMB as:

\begin{equation}
\left( \frac{\Delta T}{T} \right)_\text{mod} (\hat{n}) = (1 + A \hat{n} \cdot \hat{p}) \left( \frac{\Delta T}{T} \right)_\text{iso} (\hat{n}),
\end{equation}

where $\hat{n}$ is the observed direction, $(\Delta T/T)_\text{mod}$ is the observed and modulated CMB temperature fluctuations, $(\Delta T/T)_\text{iso}$ is the usual isotropic CMB pattern, $A$ is the amplitude of the dipole modulation and $\hat{p}$ is a given direction. It has been found that $A = 0.07 \pm 0.02$ [6], statistically significant at $\sim 3\sigma$ and the direction $\hat{p}$ is given by $(l, b) = (224^\circ, -22^\circ) \pm 22^\circ$ in Galactic coordinates, at $\sim 3.3\sigma$ [12].
3. Results

The aim of this section is to show the results of the study of the CMB quadrupole and octupole anomalies in order to test the cosmological principle and verify the existence of directional anomalies for two different models: the ΛCDM and dipolar one. In the first part of the analysis, starting from the Planck 2013 fiducial model, we extracted the $a_{\ell m}$ coefficients, calculated the MV using the public code by [3], built the area vectors and consequently the estimators (presented in Section 2.2). We performed $10^5$ Monte Carlo (MC) simulations to obtain a theoretical trend for the ΛCDM model. We also performed the same number of simulations based on Eq. 5 for the dipolar model.

In the second part, we extracted the estimators from the available data for two experiments: WMAP and Planck. For WMAP we analysed the Internal Linear Combination (ILC) maps for the 5, 7 and 9-year data [13–15], see bottom row of Fig. 2. The ILC method combines the available data at different frequencies to mitigate the Galactic foreground emission in the final map. Regarding the Planck mission, we consider the SMICA and NILC maps of the 2013 data release [16, 17] see top row of Fig. 2. The first one is the result of a Spectral Matching Independent Component Analysis (SMICA), which reconstructs a CMB map from the linear combination in harmonic space of several input frequency maps with weights that depend on the multipole $\ell$. NILC (Needlet-ILC) instead is a method to extract the CMB by applying the ILC technique to multi-channel observations in needlet space and compute an ILC in each zone and for each scale, allowing the ILC weights to adapt naturally to the varying stretch of the other components as a function of position and multipole.

Before analysing the maps, a “boost correction” [18, 19] has been applied to the real data, because the observed dipole is affected by the motion of the satellite with respect to the CMB rest frame, see [20] for details about the boost correction.

We present our results about the estimators in Tab. 1 and in Tab. 2 where we show the corresponding probability to exceed for both ΛCDM and dipolar models [20]. The percentage of anomaly is defined as the amount of MC realizations for which the considered estimator is smaller than the observed one. The larger the percentage the more anomalous is the alignment. In Fig. 3 we show the theoretical trend for ΛCDM and dipolar models (histograms) related to the real boosted data (vertical lines). The two histograms are very close to each other and this means that the two considered models are very similar for this type of analysis. The vertical bars stand in the tail of the histograms, thus the real data confirm the anomalous alignment in
all analysed cases. Moreover, the theoretical trends of both models confirm that they are not able to explain effectively the source of the directional anomalies.

Table 1. Values of the estimators extracted from the WMAP and Planck CMB maps.

| Estimator | WMAP ILC 5 yr | WMAP ILC 7 yr | WMAP ILC 9 yr | Planck SMICA | Planck NILC |
|-----------|---------------|---------------|---------------|--------------|-------------|
| S         | 0.799         | 0.804         | 0.807         | 0.794        | 0.804       |
| T         | 0.959         | 0.962         | 0.963         | 0.956        | 0.962       |
| S23       | 0.776         | 0.783         | 0.788         | 0.718        | 0.697       |
| T23       | 0.949         | 0.953         | 0.955         | 0.919        | 0.908       |
| S23       | 0.869         | 0.877         | 0.884         | 0.859        | 0.877       |
| T23       | 0.982         | 0.984         | 0.986         | 0.979        | 0.985       |
| DQO_S     | 0.789         | 0.792         | 0.799         | 0.774        | 0.776       |
| DQO_T     | 0.940         | 0.943         | 0.946         | 0.936        | 0.944       |

Table 2. Percentage of anomaly for the quadrupole/octupole alignment and dipole/quadrupole/octupole alignment, for the WMAP and Planck data for all the analysed estimators.

| Estimator | LCDM — Dipolar | LCDM — Dipolar | LCDM — Dipolar | LCDM — Dipolar | LCDM — Dipolar |
|-----------|----------------|----------------|----------------|----------------|----------------|
| S         | 99.647 — 99.640 | 99.791 — 99.701 | 99.750 — 99.731 | 99.581 — 99.578 | 99.704 — 99.707 |
| T         | 99.824 — 99.812 | 99.856 — 99.866 | 99.873 — 99.880 | 99.775 — 99.769 | 99.856 — 99.866 |
| S23       | 99.722 — 99.724 | 99.793 — 99.791 | 99.838 — 99.830 | 98.649 — 98.606 | 97.990 — 97.951 |
| T23       | 99.863 — 99.868 | 99.892 — 99.891 | 99.905 — 99.906 | 99.217 — 99.207 | 98.861 — 98.833 |
| S23       | 98.355 — 98.308 | 98.569 — 98.539 | 98.689 — 98.676 | 98.128 — 98.089 | 98.550 — 98.523 |
| T23       | 98.654 — 98.646 | 98.839 — 98.802 | 98.901 — 98.881 | 98.420 — 98.379 | 98.839 — 98.806 |
| DQO_S     | 99.803 — 99.796 | 99.829 — 99.823 | 99.872 — 99.865 | 99.872 — 99.662 | 99.878 — 99.681 |
| DQO_T     | 99.776 — 99.779 | 99.810 — 99.808 | 99.859 — 99.851 | 99.725 — 99.728 | 99.825 — 99.823 |

4. Conclusions
We report that all the data and all the estimators that we have tested exhibit anomalous alignments for both combinations of multipoles considered, typically at the 98%-99% level, and up to 99.9% in selected cases.
The consistent pattern for the alignments observed in both WMAP and Planck data strongly disfavours an origin of the effect related to unaccounted for instrumental systematics.
We have also investigated the possibility that the phenomenological dipolar model may provide a better explanation for the existence of the observed alignments with respect to LCDM. Although the dipolar model gathered some success in explaining some anomalies, e.g. the hemispherical asymmetry, we report negative findings regarding directional anomalies: the dipolar model does not seem to be able to accommodate for the existence of anomalies significantly better than LCDM.
Figure 3. $S$ statistics for the upper row and $T$ statistics for the lower row. Green histograms for the empirical distribution of the considered estimators in $\Lambda$CDM and red for the dipolar model. Vertical lines are for the observed estimators (already boost-corrected): WMAP ILC 5 in blue, WMAP ILC 7 in pink, WMAP ILC 9 in black, Planck 2013 NILC in cyan and Planck 2013 SMICA in magenta. In each panel we show the counts in the y-axis and the estimator in the x-axis (see also [20]).

References
[1] Dodelson S 2003 Modern Cosmology (Amsterdam: Academic Press) ISBN 9780122191411
[2] de Oliveira-Costa A, Tegmark M, Zaldarriaga M and Hamilton A 2004 Phys. Rev. D69 063516
[3] Copi C J, Huterer D and Starkman G D 2004 Phys. Rev. D70 043515
[4] Copi C J, Huterer D, Schwarz D J and Starkman G D 2015 Mon. Not. Roy. Astron. Soc. 449 3458–3470
[5] Ade P A R et al. (Planck) 2014 Astron. Astrophys. 571 A16
[6] Hoftuft J, Eriksen H K, Banday A J, Gorski K M, Hansen F K and Lilje P B 2009 Astrophys. J. 699 985–989
[7] Gruppuso A and Burigana C 2009 JCAP 0908 004
[8] Gruppuso A, Burigana C and Finelli F 2007 Mon. Not. Roy. Astron. Soc. 376 907–918
[9] Abramo L R, Bernui A, Ferreira I S, Villela T and Wuensche C A 2006 Phys. Rev. D74 063506
[10] Copi C J, Huterer D, Schwarz D J and Starkman G D 2010 Adv. Astron. 2010 847541
[11] Gordon C, Hu W, Huterer D and Crawford T M 2005 Phys. Rev. D72 103002
[12] Eriksen H K, Banday A J, Gorski K M, Hansen F K and Lilje P B 2007 Astrophys. J. 660 L81–L84
[13] Hinshaw G et al. (WMAP) 2009 Astrophys. J. Suppl. 180 225–245
[14] Jarosik N et al. 2011 Astrophys. J. Suppl. 192 14
[15] Hinshaw G et al. (WMAP) 2013 Astrophys. J. Suppl. 208 19
[16] Ade P A R et al. (Planck) 2014 Astron. Astrophys. 571 A23
[17] Ade P A R et al. (Planck) 2014 Astron. Astrophys. 571 A15
[18] Kosowsky A and Kahniashvili T 2011 Phys. Rev. Lett. 106 191301
[19] Amendola L, Catena R, Masina I, Notari A, Quartin M and Quercellini C 2011 JCAP 1107 027
[20] Polastri L, Gruppuso A and Natoli P 2015 JCAP 1504 018