Semileptonic decays of the light $J^P = 1/2^+$ ground state baryon octet

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We calculate the semileptonic baryon octet–octet transition form factors using a manifestly Lorentz covariant quark model approach based on the factorization of the contribution of valence quarks and chiral effects. We perform a detailed analysis of SU(3) breaking corrections to the hyperon semileptonic decay form factors. We present complete results on decay rates and asymmetry parameters including lepton mass effects for the rates.

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I. INTRODUCTION

The analysis of the semileptonic decays of the baryon octet $B_i \rightarrow B_j e\bar{\nu}_e$ presents an opportunity to shed light on the Cabibbo–Kobayashi–Maskawa (CKM) matrix element $V_{us}$. At zero momentum transfer, the weak baryon matrix elements for the $B_i \rightarrow B_j e\bar{\nu}_e$ transitions are determined by just two constants—the vector coupling $F_{11}^{B_i B_j}$ and its axial counterpart $G_{11}^{B_i B_j}$. In the limit of exact SU(3) symmetry $F_{11}^{B_i B_j}$ and $G_{11}^{B_i B_j}$ are expressed in terms of basic parameters—the vector couplings are given in terms of well-known Clebsch–Gordan coefficients which are fixed due to the conservation of the vector current (CVC), while the axial couplings are given in terms of the familiar SU(3) octet axial–vector couplings $F$ and $D$. The Ademollo–Gatto theorem (AGT) protects the vector form factors from leading SU(3)–breaking corrections generated by the mass difference of strange and nonstrange quarks—the first nonvanishing breaking effects begin at second order in symmetry-breaking. As emphasized in Ref. [2], the vanishing of the first–order correction to the vector hyperon form factors $F_{11}^{B_i B_j}$ presents an opportunity to determine $V_{us}$ from the direct measurement of $F_{11}^{B_1 B_s}$. The axial form factor, on the other hand, contains symmetry–breaking corrections already at first order. We note that the experimental data on baryon semileptonic decays [3] are well described by the Cabibbo theory [4], which assumes SU(3) invariance of the strong interactions. However, for a precise determination of $V_{us}$ one needs to include the leading and very likely also the subleading SU(3) breaking corrections.

The theoretical analysis of SU(3) breaking corrections to hyperon semileptonic decay form factors has been performed in various approaches, including quark and soliton models, the 1/$N_c$ expansion of QCD, chiral perturbation theory (ChPT), lattice QCD, etc. (for an overview and references see [5]). In Ref. [5] we have suggested the use of a quark-based approach, which offers the possibility to consistently include chiral corrections (both SU(3)–symmetric and SU(3)–breaking) to the baryon semileptonic form factors. By matching the baryon matrix elements to the corresponding quantities derived in baryon ChPT we reproduced the chiral expansion of physical quantities (e.g. mass, magnetic moments, slopes and the axial charge of the nucleon) at the order of accuracy at which we worked. In the valence quark calculation of the baryon matrix elements we employed a simple generic ansatz for the spatial form of the quark wave function [6, 7].

In the present paper we evaluate the baryon matrix elements within a Lorentz and gauge invariant constituent quark model [8, 9]. Note that in Refs. [10, 11] we have studied the electromagnetic properties of the baryon octet and the $\Delta(1230)$–resonance in an analogous approach. In particular, we developed an approach based on a nonlinear chirally symmetric Lagrangian which involves constituent quarks and chiral fields. In a first step, this Lagrangian was used to dress the constituent quarks with a cloud of light pseudoscalar mesons and other (virtual) heavy states using the calculational technique of infrared dimensional regularization (IDR) [12]. Then, within a formal chiral expansion, we evaluated the dressed transition operators relevant for the interaction of quarks with external fields in the presence of a virtual meson cloud. In a next step, these dressed operators were used to calculate baryon matrix elements. (A simpler and more phenomenological quark model based on similar ideas regarding the dressing of constituent quarks by the meson cloud has been developed in Refs. [13].) In the present paper we improve the quantitative determination of valence quark effects by resorting to a specific relativistic quark model [8, 11] describing the internal quark dynamics. This procedure will allow us to generate predictions for all six form factors showing up in the matrix elements of the semileptonic decays of the baryon octet. With the explicit form factors together with radiative corrections, we present predictions for the corresponding decay widths and asymmetries.

The paper is structured as follows. First, in Section II, we discuss the basic notions of our approach which is directly connected to our previous work in Refs. [5, 10, 11]. That is, we derive a chiral Lagrangian motivated by baryon ChPT [10, 11], and write it in terms of quark and mesonic degrees of freedom. Using constituent quarks dressed with a cloud of light pseudoscalar mesons and other mesons heavier than the pseudoscalar mesons, we derive dressed transition operators within the chiral expansion, which are in turn used in a Lorentz and gauge invariant quark model [8] explicitly including internal quark dynamics to calculate baryon matrix elements. In Section III we derive specific expressions for the vector and axial baryon semileptonic decay constants, while in Section IV we present the numerical analysis of the axial nucleon charge and the vector and axial vector hyperon semileptonic form factors. Finally, in Section V we summarize our results.

II. APPROACH

A. Matrix elements of semileptonic decays of the baryon octet

In Refs. [5, 10, 11] we have developed a Lorentz covariant quark approach which allowed us to study light baryon properties based on the inclusion of chiral effects in a consistent fashion by matching the quark model approach to the predictions of ChPT. In particular, our results for various baryon properties (static properties and form factors
in the Euclidean region) derived in [Ref. 9, 10, 11] using this approach satisfy the low-energy theorems and identities dictated by the infrared singularities of QCD (see, e.g., the detailed discussion in Refs. [9, 10] and a brief overview in Section III C).

The main idea is to include chiral effects in the transition quark operators, which are then sandwiched between the respective baryon states. We have developed a technique which allows us to explicitly generate chiral corrections associated with the small scale $\lambda \sim m_q$, where $m_q$ is the constituent quark mass, together with effects of the internal dynamics of the valence quarks. In particular, as a first step, we dress the bare valence quark operators by a cloud of pseudoscalar mesons and states heavier than the pseudoscalar mesons in a straightforward manner by the use of an effective chirally–invariant Lagrangian (see the explicit forms in Refs. [9, 10, 11] and the relevant expressions for the calculation of semileptonic form factors below). In particular, the Lagrangian which dynamically generates the dressing of the constituent quarks by the mesonic degrees of freedom, consists of two basic pieces $L_q$ and $L_U$:

$$L_{qU} = L_q + L_U, \quad L_q = L_q^{(1)} + L_q^{(2)} + L_q^{(3)} + L_q^{(4)} + \cdots, \quad L_U = L_U^{(2)} + \cdots. \tag{1}$$

The superscript $(i)$ attached to $L_q^{(i)}$ and $L_U^{(i)}$ denotes the low energy dimension of the Lagrangian:

$$L_q^{(2)} = -\frac{F^2}{4} \langle u_\mu u^\mu + \chi_+ \rangle, \quad L_q^{(1)} = \bar{q} \left[ iD - m + \frac{1}{2} g \gamma^5 \right] q,$$

$$L_q^{(3)} = \frac{C_7}{2} \langle (u_\mu u^\mu) \bar{q} q \rangle + \frac{C_4}{8} \bar{q} i \sigma^{\mu\nu} [u_\mu, u_\nu] q + \frac{C_7}{8} m q \bar{q} \gamma^\mu F_{\mu\nu}^+ q + \cdots,$$

$$L_q^{(4)} = \frac{E_7}{2} \bar{q} \gamma^5 \chi_{\pm} q + \frac{E_7}{3} \bar{q} \sigma^{\mu\nu} (F_{\mu\nu}^+ \bar{q} q + \cdots, \tag{2e}$$

where the symbols $\langle \rangle$, $\{ \}$ and $\{ \}$ occurring in Eq. (2) denote the trace over flavor matrices, commutator, and anticommutator, respectively. In Eq. (2) we display only the terms involved in the calculation of semileptonic vector and axial vector quark coupling constants.

We use the following notation. $q, U = u^2 = \exp(\phi/F)$ are the quark and chiral fields, respectively, where $\phi$ is the octet of pseudoscalar fields and $F$ is the octet decay constant, $\sigma_{\mu\nu} = i/2 [\gamma_{\mu}, \gamma_{\nu}], u_{\mu} = \{ u^\dagger, \nabla_{\mu} u \}$. $D_\mu$ and $\nabla_{\mu}$ are the covariant derivatives acting on the quark and chiral fields, respectively, including external vector ($v_{\mu}$) and axial ($a_{\mu}$) fields, $F_{\mu\nu}^\pm = u^\dagger F_{\mu\nu}^R u \pm u F_{\mu\nu}^L u$ is the stress tensor involving $v_{\mu}$ and $a_{\mu}$, $\chi_+ = u^\dagger \chi u^\dagger \pm u \chi^\dagger u$ and $\bar{\chi}_+ \chi = \chi_+ - \langle \chi_+ \rangle/3$ with $\chi = 2BM + \cdots$, where $B$ is the quark vacuum condensate parameter and $M = \text{diag}(\bar{m}, \bar{m}, \bar{m}_s)$ is the mass matrix of current quarks (We work in the isospin symmetry limit with $\bar{m}_u = \bar{m}_d = \bar{m} = 7$ MeV). The mass of the strange quark $\bar{m}_s$ is related to the nonstrange one via $\bar{m}_s \simeq 25 \bar{m}$.

The parameters $m = 420$ MeV and $g = 0.9$ denote the constituent quark mass and axial charge in the chiral limit (i.e., they are counted as quantities of order $O(1)$ in the chiral expansion). $C_7^q, D_1^q$ and $E_7^q$ are the SU(3) quark second-, third- and fourth–order low–energy constants (LEC's). We denote the SU(3) quark LEC's by capital letters in order to distinguish them from the SU(2) LEC's $c^i_1, d^j_1$ and $e^j_1$. Also, for the quark LEC's we use the additional superscript “$q$” to differentiate them from the analogous ChPT LEC's: $C_i, D_i, E_i$ in SU(3) and $c_i, d_i, e_i$ in SU(2). For the numerical analysis we will use: $M_q = 139.57$ MeV, $M_K = 493.67$ MeV (the charged pion and kaon masses), $M_{\pi} = 547.51$ MeV and $F = (F_{\pi} + F_K)/2$ in SU(3) with $F_{\pi} = 92.4$ MeV and $F_K/F_{\pi} = 1.22$. Using the Lagrangian (2) we can calculate the semileptonic vector and axial vector quark couplings including chiral corrections following the procedure discussed in detail in Refs. [9, 10, 11]. In Appendix A we list the results for the semileptonic quark couplings $f_{1,2,3}^{su}$, $f_{1,2,3}^{du}$, $g_{1,2,3}^{su}$, and $g_{1,2,3}^{du}$. Up to order $O(p^3)$ in the three–flavor picture.

In Refs. [9, 10, 11] we illustrated the dressing technique in the case of the electromagnetic quark operator. We performed a detailed analysis of the electromagnetic properties of the baryon octet and of the $\Delta \rightarrow N\gamma$ transition. In Ref. [9] we extended this technique to the case of vector and axial vector quark operators, deriving master formulae for the calculation of the semileptonic form factors of baryons including the effects of valence quarks together with chiral corrections. Below we briefly review the derivation of these master formulae, which will be the starting point for the present paper.

First, we define the bare vector and axial vector quark transition operators constructed from quark fields of flavor $i$ and $j$ as:

$$J_{\mu, V}(q) = \int d^4xe^{-iqx}j_{\mu, V}(x), \quad j_{\mu, V}(x) = \bar{q}_j(x)\gamma_\mu q_i(x), \tag{3a}$$

$$J_{\mu, A}(q) = \int d^4xe^{-iqx}j_{\mu, A}(x), \quad j_{\mu, A}(x) = \bar{q}_j(x)\gamma_\mu\gamma_5q_i(x). \tag{3b}$$
Next, using the chiral Lagrangian derived in Ref. [3], we construct the vector/axial vector currents with quantum numbers of the bare quark currents which include mesonic degrees of freedom. These currents are then projected on the corresponding (initial and final) quark states in order to evaluate dressed vector $f_k^j(q^2)$ and axial vector $g_k^j(q^2)$ ($k = 1, 2, 3$) quark form factors which encode the chiral corrections. Finally, using the dressed quark form factors in momentum space we can determine their Fourier–transform in coordinate space.

In the one-body approximation the dressed quark operators $J_{\mu, V(A)}^\text{dress}(x)$ and their Fourier transforms $J_{\mu, V(A)}^\text{dress}(q)$ have the forms (for an extension which also includes the two–body quark–quark interactions see Ref. [3])

$$J_{\mu, V}^\text{dress}(x) = f_{ij}^j (-\partial^2 - \mu^2) [\bar{q}_j(x) \gamma_\mu q_i(x)] + \frac{g_{ij}^j (-\partial^2 - \mu^2)}{m_i} \gamma_\mu [\bar{q}_j(x) \gamma^\mu q_i(x)] - \frac{g_{ij}^j (-\partial^2 - \mu^2)}{m_i} i \partial_\mu [\bar{q}_j(x) q_i(x)],$$

$$J_{\mu, A}^\text{dress}(q) = \int d^4 x e^{-iqx} \bar{q}_j(x) \left[ \gamma_\mu F_{ij}^j(q^2) + \frac{i \sigma_{\mu \nu} q^\nu}{m_i} g_{ij}^j(q^2) + \frac{q_\mu}{m_i} g_{ij}^j(q^2) \right] q_i(x),$$

where $m_i(j)$ denotes the dressed constituent quark mass of the $i(j)$–th flavor generated by the corresponding chiral Lagrangian (for details see Ref. [10]): $f_{i, j}^{1, 2}(q^2)$ and $g_{i, j}^{1, 2}(q^2)$ denote the quark-level vector and axial vector $i \rightarrow j$ flavor changing form factors. Up to and including the third order in the chiral expansion, the tree and loop diagrams which contribute to the dressed vector $J_{\mu, V}^\text{dress}(q)$ and axial vector $J_{\mu, A}^\text{dress}(q)$ operators, respectively, are displayed in Figs.1 and 2 of Ref. [3]. In Appendix A we present our results for the semileptonic vector $f_k^{ij} = f_k^{ij}(0)$ and axial $g_k^{ij} = g_k^{ij}(0)$ couplings at the order of accuracy at which we work – up to order $\mathcal{O}(p^3)$ in the three–flavor picture including chiral corrections (both SU(3)–symmetric and SU(3)–breaking). For simplicity we restrict our approach to the isospin symmetry limit in our consideration.

In order to calculate the vector and axial vector current transitions between baryons we sandwich the dressed quark operators between the relevant baryon states. The master formulae are:

$$\langle B_j(p') | J_{\mu, V(A)}^\text{dress}(q) | B_i(p) \rangle = (2\pi)^4 \delta^4(p' - p - q) M_{\mu, V(A)}^{B_jB_i}(p, p'),$$

$$M_{\mu, V}^{B_jB_i}(p, p') = \sum_{k=1}^3 f_{ij}^k(q^2) \langle B_j(p') | V_{\mu, k}^{ij}(0) | B_i(p) \rangle,$$

$$M_{\mu, A}^{B_jB_i}(p, p') = \sum_{k=1}^3 g_{ij}^k(q^2) \langle B_j(p') | A_{\mu, k}^{ij}(0) | B_i(p) \rangle,$$

where $B_i(p)$ denotes the baryon state and $u_{B_i}(p)$ is the baryon spinor normalized according to

$$\langle B_i(p') | B_i(p) \rangle = 2E_{B_i} (2\pi)^3 \delta^3(p - p'), \quad \bar{u}_{B_i}(p) u_{B_i}(p) = 2m_{B_i}.$$
with
\[ \Gamma^V_{\mu,1} = \gamma_\mu, \quad \Gamma^V_{\mu,2} = \frac{i\sigma_{\mu\nu}q^\nu}{m_i}, \quad \Gamma^V_{\mu,3} = \frac{q_{\mu}}{m_i}, \]
\[ \Gamma^A_{\mu,1} = \gamma_\mu\gamma_5, \quad \Gamma^A_{\mu,2} = \frac{i\sigma_{\mu\nu}q^\nu}{m_i}\gamma_5, \quad \Gamma^A_{\mu,3} = \frac{q_{\mu}\gamma_5}{m_i}. \] (10)

Next we specify the expansion of the bare matrix elements \( \langle B_j(p') | V_{ij}^{\mu,k}(0) | B_i(p) \rangle \) and \( \langle B_j(p') | A_{ij}^{\mu,k}(0) | B_i(p) \rangle \) in terms of the form factors \( V_{ik}^{B_iB_j}(q^2) \) and \( A_{ik}^{B_iB_j}(q^2) \) with \( i, j = 1, 2, 3 \) encoding the effects of the internal dynamics of valence quarks:
\[ \langle B_j(p') | V_{ij}^{\mu,k}(0) | B_i(p) \rangle = \bar{u}_{B_j}(p') \left( \gamma_\mu V_{1k}^{B_jB_i}(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{m_{B_i}} V_{2k}^{B_jB_i}(q^2) + \frac{q_{\mu}}{m_{B_i}} V_{3k}^{B_jB_i}(q^2) \right) u_{B_i}(p), \] (11a)
\[ \langle B_j(p') | A_{ij}^{\mu,k}(0) | B_i(p) \rangle = \bar{u}_{B_j}(p) \left( \gamma_\mu\gamma_5 A_{1k}^{B_jB_i}(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{m_{B_i}}\gamma_5 A_{2k}^{B_jB_i}(q^2) + \frac{q_{\mu}\gamma_5}{m_{B_i}} A_{3k}^{B_jB_i}(q^2) \right) u_{B_i}(p). \] (11b)

Combining chiral effects (encoded in the chiral form factors \( f_{ik}^{ij}(q^2) \) and \( g_{ik}^{ij}(q^2) \)) and valence quark effects (encoded in the form factors \( V_{ik}^{B_iB_j}(q^2) \) and \( A_{ik}^{B_iB_j}(q^2) \)) for the vector and axial vector form factors \( F_{1,2}^{B_iB_j} \) and \( G_{1,2}^{B_iB_j} \), which govern the semileptonic transitions between octet baryons, are defined as:
\[ F_{1}^{B_iB_j}(q^2) = \sum_{k=1}^{2} f_{ik}^{ij}(q^2) V_{ik}^{B_iB_j}(q^2), \quad G_{1}^{B_iB_j}(q^2) = \sum_{k=1}^{2} g_{ik}^{ij}(q^2) A_{ik}^{B_iB_j}(q^2), \] (12a)
\[ F_{2}^{B_iB_j}(q^2) = \sum_{k=1}^{2} f_{ik}^{ij}(q^2) V_{2k}^{B_iB_j}(q^2), \quad G_{2}^{B_iB_j}(q^2) = \sum_{k=1}^{2} g_{ik}^{ij}(q^2) A_{2k}^{B_iB_j}(q^2), \] (12b)
\[ F_{3}^{B_iB_j}(q^2) = \sum_{k=1}^{3} f_{ik}^{ij}(q^2) V_{3k}^{B_iB_j}(q^2), \quad G_{3}^{B_iB_j}(q^2) = \sum_{k=1}^{3} g_{ik}^{ij}(q^2) A_{3k}^{B_iB_j}(q^2). \] (12c)

Note that the operators \( V(A)_{ij}^{\mu,k}(0) \) are proportional to \( q_{\mu} \), and therefore do not generate contributions to the baryon form factors \( F_{1,2}^{B_iB_j}(q^2) \) and \( G_{1,2}^{B_iB_j}(q^2) \). Further simplifications occur when we consider the semileptonic coupling constants of baryons at maximal recoil \( q^2 = 0 \). For the couplings encoding valence quark effects we get the following constraints due to Lorentz covariance and gauge invariance:
\[ V_{12}^{B_iB_j}(0) = A_{12}^{B_iB_j}(0) = 0, \quad V_{31}^{B_iB_j}(0) = G_{12}^{B_iB_j}(0) = 0. \] (13)

It is seen that the \( V_{31}^{B_iB_j}(0) \) and \( V_{32}^{B_iB_j}(0) \) couplings start at the first order in SU(3) breaking. In the case of the couplings \( f_{ik}^{ij} = f_{ik}^{0}(0) \) and \( g_{ik}^{ij} = g_{ik}^{0}(0) \) encoding the chiral effects we have the following results (see details in Appendix A):

1) The vector coupling \( f_{ik}^{ij} \) governing the \( d \to u \) transition is trivial and equal to unity — \( f_{ik}^{0} = 1 \), because we work in the isospin symmetry limit. In the case of the \( s \to u \) transition, the corresponding vector coupling \( f_{ik}^{1u} \) contains symmetry breaking corrections of second order in SU(3) — \( \mathcal{O}(M^4_R - M^2_S)^2 \). The coupling \( f_{ik}^{1u} \) is nothing but the statement of the Ademollo–Gatto theorem (AGT) which asserts that the coupling \( f_{ik}^{1u} \) is protected from \( \text{first–order} \) symmetry breaking corrections.

2) The coupling \( f_{ik}^{1u} \) vanishes due to isospin invariance, while the coupling \( f_{ik}^{2u} \) starts at first order in SU(3) breaking — \( f_{ik}^{2u} = \mathcal{O}(M^4_R - M^2_S)^1 \).

3) The axial vector couplings \( g_{ik}^{ij} \) are either equal to zero (e.g. the coupling \( g_{ik}^{du} \) governing the \( d \to u \) transition) or vanish at the order of accuracy that we are working at (e.g. the coupling \( g_{ik}^{su} \) governing the \( s \to u \) transition).

The set of Eqs. (10)–(12) contains our main result: we separate the effects of the internal dynamics of the valence quarks contained in the matrix elements of the bare quark operators \( V(A)_{ij}^{\mu,k}(0) \) and the effects dictated by chiral dynamics which are encoded in the relativistic form factors \( f_{ik}^{ij}(q^2) \) and \( g_{ik}^{ij}(q^2) \). Due to the factorization of chiral effects and the effects of the internal dynamics of the valence quarks the calculation of the form factors \( f(q)_{ik}^{ij}(q^2) \) which encode the chiral dynamics, on one side, and the matrix elements of \( V(A)_{ij}^{\mu,k}(0) \) which encodes the effects of the valence quarks, on the other side, can be done independently. The evaluation of the matrix elements \( V(A)_{ij}^{\mu,k}(0) \) is not restricted to small momenta squared and, therefore, can shed light on baryon form factors at higher (Euclidean)
momentum squared in comparison with ChPT. In particular, as a first step, we employ a formalism motivated by the ChPT Lagrangian for the calculation of $f(q^2)$ which is formulated in terms of constituent quark degrees of freedom. The evaluation of the matrix elements of the bare quark operators can then be relegated to quark models based on specific assumptions on the internal quark dynamics, hadronization, and confinement. Note that Eqs. (12) are valid for the calculations of dressed vector and axial vector quark operators of any flavor content. In Ref. [5] we calculated the vector and axial vector coupling constants $F_i^{B,B_i}(0)$ and $G_i^{B,B_i}(0)$. Here we extend our analysis to all six coupling constants $F_i^{B,B_i}(0)$ and $G_i^{B,B_i}(0) (i = 1, 2, 3)$.

### B. Evaluation of the matrix elements of the valence quark operators

In this section we discuss the calculation of the baryonic matrix elements

$$\langle B(p')| V_{\mu,i}^{ij}(0) | B(p) \rangle \quad \text{and} \quad \langle B(p')| A_{\mu i}^{ij}(0) | B(p) \rangle$$

induced by the bare quark operators [6]. We will consistently employ the relativistic three-quark model (RQM) [8, 9] to compute the matrix elements [14]. The RQM was previously successfully applied to the study of the properties of baryons containing light and heavy quarks [8, 9]. The main advantages of this approach are: Lorentz and gauge invariance, a small number of parameters, and the modelling of effects of strong interactions at large ($\sim 1$ fm) distances. Various properties of light and heavy baryons in electromagnetic, strong and weak decays have been successfully analyzed within this RQM [8, 9] where the effects of valence quarks have been consistently taken into account. Here we extend this approach to evaluate the effects of valence quarks in the semileptonic decays of the baryon octet.

Let us begin by briefly reviewing the basic notions of the RQM approach [8, 9]. The RQM is based on an interaction Lagrangian describing the coupling between baryons and their constituent quarks. The coupling of a baryon $B(q_1q_2q_3)$ to its constituent quarks $q_1$, $q_2$ and $q_3$ is described by the Lagrangian

$$L_{int}^{ext}(x) = g_B \bar{B}(x) \int dx_1 \int dx_2 \int dx_3 \sum_i \bar{q}_i(x_1) q_i(x_2) J_B(x_1, x_2, x_3) + h.c.$$  

where $J_B(x_1, x_2, x_3)$ is a three-quark current with the quantum numbers of the relevant baryon $B$ [14, 15]. One has

$$J_B(x_1, x_2, x_3) = \sum_{i<j} \epsilon^{a_1a_2a_3} \Gamma_i q_{a_1}^1(x_1) q_{a_2}^2(x_2) C \Gamma_j q_{a_3}^3(x_3),$$

where $\Gamma_{1,2}$ are Dirac structures, $C = \gamma^0 \gamma^2$ is the charge conjugation matrix and $a_i (i = 1, 2, 3)$ are color indices. In Appendix B we list the relevant three-quark currents for the baryon octet. The choice of light baryon three-quark currents has been discussed in detail in Refs. [14, 15].

The function $F$ is related to the scalar part of the Bethe-Salpeter amplitude and characterizes the finite size of the baryon. In the following we use a specific form for the vertex function [8, 9]

$$F(x_1, x_2, x_3) = N \delta^4(x - \sum_{i=1}^3 w_i x_i) \Phi \left( \sum_{i<j} (x_i - x_j)^2 \right)$$

where $\Phi$ is the correlation function of the three constituent quarks with masses $m_1$, $m_2$, $m_3$ and $N = 9$ is a normalization factor. With the variable $w_i$ defined by $w_i = m_i/(m_1 + m_2 + m_3)$ the function $\Phi$ depends only on the relative Jacobi coordinates $(\xi_1, \xi_2)$ via $\Phi(\xi_1^2 + \xi_2^2)$, where

$$\begin{align*}
    x_1 &= x - \frac{\xi_1}{\sqrt{2}} (w_2 + w_3) + \frac{\xi_2}{\sqrt{6}} (w_2 - w_3), \\
    x_2 &= x + \frac{\xi_1}{\sqrt{2}} w_1 - \frac{\xi_2}{\sqrt{6}} (w_1 + 2w_3), \\
    x_3 &= x + \frac{\xi_1}{\sqrt{2}} w_1 + \frac{\xi_2}{\sqrt{6}} (w_1 + 2w_2),
\end{align*}$$

and $x = \sum_{i=1}^3 w_i x_i$ is the center of mass (CM) coordinate. Expressed in terms of the relative Jacobi coordinates and the center of mass coordinate, the Fourier transform of the vertex function reads [8, 9]:

$$\Phi(\xi_1^2 + \xi_2^2) = \int \frac{d^4p_1}{(2\pi)^4} \int \frac{d^4p_2}{(2\pi)^4} e^{-ip_1 \xi_1 - ip_2 \xi_2} \bar{\Phi}(-p_1^2 - p_2^2).$$
The baryon-quark coupling constants \( g_B \) are determined by the compositeness condition \([8, 9]\) (see also \([16, 17]\)) , which implies that the renormalization constant of the hadron wave function is set equal to zero:

\[
Z_B = 1 - \Sigma'_B(m_B) = 0
\]  

(20)

where \( \Sigma'_B(m_B) = g_B^2 \Pi_B(m_B) \) is the first derivative of the baryon mass operator described by the diagram in Fig.1, and \( m_B \) is the baryon mass. To clarify the physical meaning of Eq. (20) we first want to remind the reader that the renormalization constant \( Z_B^{1/2} \) can also be interpreted as the matrix element between the physical and the corresponding bare state. For \( Z_B = 0 \) it then follows that the physical state does not contain the bare one and is described as a bound state. The interaction Lagrangian Eq. (13) and the corresponding free components describe both the constituents (quarks) and the physical particles (hadrons), which are taken to be the bound states of the constituents. As a result of the interaction, the physical particle is dressed, i.e. its mass and its wave function have to be renormalized. The condition \( Z_B = 0 \) also effectively excludes the constituent degrees of freedom from the physical space and thereby guarantees that there is no double counting for the physical observable under consideration. In this picture the constituent quarks exist in virtual states only. One of the corollaries of the compositeness condition is the absence of a direct interaction of the dressed charged particle with the electromagnetic and the weak gauge boson field. Taking into account both the tree-level diagram and the diagrams with the self-energy and counter-term insertions into the external legs (that is the tree-level diagram times \((Z_B - 1)\)) one obtains a common factor \( Z_B \) which is equal to zero \([17]\).

The quantities of interest—the matrix elements \((13)\)—are described by the triangle diagram in Fig.2(a). In case of the matrix elements \( \langle B(p') | V^i_{\mu} (0) | B(p) \rangle \) and \( \langle B(p') | A^i_{\mu} (0) | B(p) \rangle \) we need to include two additional so-called “bubble” diagrams in Figs.2(b) and 2(c) which guarantee gauge invariance of the matrix elements (see details in Refs. \([8, 9]\) and \([18, 19]\)). In particular, the “bubble” diagrams are generated by the non-local coupling of the baryon to the constituent quarks and the external gauge field which arises after gauging of the non-local strong interaction Lagrangian \((15)\) containing the vertex function \((17)\). In Appendix C we present more details of how to restore gauge invariance in the non-local strong interaction Lagrangian \((15)\) through the “bubble” diagrams in Figs.2(b) and 2(c). Note that the contributions of the bubble diagrams Figs.2(b) and 2(c) to the matrix elements \( \langle B(p') | V^i_{\mu} (0) | B(p) \rangle \) and \( \langle B(p') | A^i_{\mu} (0) | B(p) \rangle \) are suppressed. In the present application the bubble diagrams contribute less than 5 % in magnitude compared to the contribution of the triangle diagram in Fig.2(a).

In the evaluation of the quark-loop diagrams we use the free fermion propagator for the constituent quark \([8, 9]\): 

\[
i S_q(x - y) = \langle 0 | T q(x) \bar{q}(y) | 0 \rangle = \int \frac{d^4 k}{(2\pi)^4} e^{-ik(x-y)} \tilde{S}_q(k)
\]

(21)

where \( \tilde{S}_q(k) = (m_q - k - i\epsilon)^{-1} \) is the usual free fermion propagator in momentum space. The appearance of unphysical imaginary parts in Feynman diagrams can be avoided by postulating the condition that the baryon mass must be less than the sum of the constituent quark masses \( M_B < \sum_i m_q \).

In the next step we have to specify the vertex function \( \Phi \), which characterizes the finite size of the baryons and the internal quark dynamics. In principle, its functional form can be calculated from the solutions of the Bethe-Salpeter equation for baryon bound states \([20]\). In Refs. \([21]\) it was found that, using various forms for the vertex function, the basic hadron observables are relatively insensitive to the specific details of the functional form of the hadron-quark vertex form factor. Using this observation as a guiding principle, we select a simple Gaussian form for the vertex function \( \Phi \) (any choice for \( \Phi \) is appropriate as long as it falls off sufficiently fast in the ultraviolet region of Euclidean space to render the Feynman diagrams ultraviolet finite). We shall employ the Gaussian form

\[
\tilde{\Phi}(k^2_{1E}, k^2_{2E}) \equiv \exp(-18/[k^2_{1E} + k^2_{2E}]/\Lambda_B^2),
\]

(22)

where \( k_{1E} \) and \( k_{2E} \) are Euclidean momenta and \( \Lambda_B \) is a size parameter which parametrizes the distribution of quarks inside a given baryon. In previous papers \([8, 9]\) we have determined a set of parameters for the light baryons

\[
m_u = m_d = 420 \text{ MeV}, \quad m_s = 570 \text{ MeV}, \quad \Lambda_B = 0.75 - 1.25 \text{ GeV}
\]

(23)

which gives very satisfactory agreement with a wide class of experimental data. Note that most of the results are not sensitive to the actual values of \( \Lambda_B \) in the above range. We present some sample results of this approach in Table 1. These are the magnetic moments of the baryon octet and the nucleon electromagnetic radii generated with \( m_u = m_d = 420 \text{ MeV}, m_s = 570 \text{ MeV} \) and \( \Lambda_B = 1.25 \text{ GeV} \). We show the contributions both of the valence quarks (3q) and of the meson cloud. In the present paper we present a corresponding analysis for the semileptonic coupling constants of the baryon octet using this same set of model parameters.
C. Connection with chiral perturbation theory

As stressed earlier, results for the baryon properties obtained using this approach satisfy the low-energy theorems and identities dictated by the infrared singularities of QCD. As a result we can relate the parameters of our approach to those of ChPT. In particular, we have analyzed the chiral expansion of the following properties of the nucleon: mass, magnetic moments, charge radii, the $\pi N$ $\sigma$-term, axial charge and $\pi NN$ coupling constant in SU(2). We have also extended our results to SU(3) including kaon and $\eta$-meson degrees of freedom.

The results are:
1. Nucleon mass and $\pi N$ $\sigma$-term.

\[ m_N = \tilde{m}_N - 4c_1 M^2 - \frac{3 g_A}{32 \pi^2} \frac{M^3}{F^2} + k_1 M^4 \ln \frac{M}{\tilde{m}_N} + k_2 M^4 + \mathcal{O}(M^5), \quad (24a) \]
\[ \sigma_{\pi N} = -4c_1 M^2 - \frac{9 g_A}{64 \pi^2} \frac{M^3}{F^2} + \sigma_1 M^4 \ln \frac{M}{\tilde{m}_N} + \sigma_2 M^4 + \mathcal{O}(M^5), \quad (24b) \]

where
\[ k_1 = \frac{1}{2} \sigma_1 = -\frac{3}{32 \pi^2 F^2} \frac{g_A}{\sigma} \left( \frac{g_A}{\sigma} - 8c_1 \tilde{m}_N + c_2 \tilde{m}_N + 4c_3 \tilde{m}_N \right), \]
\[ k_2 = \tilde{e}_1 - \frac{3}{128 \pi^2 F^2} \frac{g_A}{\sigma} \left( 2 \frac{g_A}{\sigma} - c_2 \tilde{m}_N \right), \]
\[ \sigma_2 = 2\tilde{e}_1 - \frac{3}{64 \pi^2 F^2} \frac{g_A}{\sigma} \left( \frac{g_A}{\sigma} - 8c_1 \tilde{m}_N + 4c_3 \tilde{m}_N \right), \]
\[ \tilde{e}_1 = e_1 - \frac{3\lambda}{2F^2} \left( \frac{g_A}{\sigma} - 8c_1 \tilde{m}_N + c_2 \tilde{m}_N + 4c_3 \tilde{m}_N \right), \]

and
\[ \lambda(\mu) = \frac{\mu^{d-4}}{(4\pi)^2} \left( \frac{1}{d-4} - \frac{1}{2} (\ln 4\pi + \Gamma(1) + 1) \right), \quad \tilde{\lambda} = \lambda(\tilde{m}_N). \quad (26) \]

2. Magnetic moments and charge radii.

\[ \mu_p = -\frac{g_A^2 M}{8\pi F^2} \tilde{m}_N + \ldots, \]
\[ \langle r_A^2 \rangle_p = -\frac{1}{16\pi^2 F^2} \ln \frac{M}{\tilde{m}_N} + \ldots, \quad (27) \]
\[ \langle r_A^2 \rangle_p = -\frac{1}{16\pi^2 F^2} \frac{g_A^2}{\mu_p} \frac{\tilde{m}_N}{M} + \ldots, \]

3. Axial charge $g_A = G_1^{\alpha\beta}(0)$, $\pi NN$ coupling constant and induced pseudoscalar form factor $g_P(q^2) = 2G_3^{\alpha\beta}(q^2)$.

\[ g_A = g_A \left( 1 + \frac{4d_{16} M^2}{g_A \tilde{m}_N} - \frac{4d_{16} M^2}{16\pi^2 F^2} \right) + \frac{M^3}{24\pi \tilde{m}_N F^2} \left( 3 + 3 \frac{g_A}{\sigma} - 4c_3 \tilde{m}_N + 8c_4 \tilde{m}_N \right) + \mathcal{O}(M^4), \quad (28a) \]
\[ g_{\pi N} = g_A \left( \frac{M}{F^2} - \frac{4c_1 M^2}{\tilde{m}_N} + 4d_{16} - 2d_{18} \right) + \left( 12 + 3 \frac{g_A}{\sigma} \right) \frac{M^3}{16\pi^2 F^2} \tilde{m}_N \]
\[ + \left( 12 + 3 \frac{g_A}{\sigma} \right) \frac{M^3}{16\pi^2 F^2} \tilde{m}_N \left( 1 + \Delta_{GT} \right), \quad (28b) \]
\[ g_P(q^2) = 4m_N \frac{g_{\pi N}}{M^2} \frac{g_A}{M^2 - q^2} - \frac{2}{3} \frac{m_A^2}{g_A} \frac{g_P(q^2)}{M^2 - q^2} + \mathcal{O}(p^2) \quad (28c) \]

where $\langle r_A^2 \rangle$ is the axial mean–square radius, $\Delta_{GT} = -2d_{18} M^2 / g_A + \mathcal{O}(M^4)$ is the correction to the Goldberger-Treiman (GT) relation, which vanishes in the chiral limit (in full equivalence with the prediction of ChPT). Note
that the correction $\Delta_{GT}$ is related to the so-called Goldberger-Treiman discrepancy \cite{27} $\Delta_{D} = 1 - (m_{N}g_{A}/F_{\pi}g_{\pi N})$ via \cite{23}: $\Delta_{GT} = \Delta_{D}/(1 - \Delta_{D})$. In Eqs. (24) - (28) we use the standard notation for the parameters of the ChPT Lagrangian: $M$ represents the pion mass to leading-order in the chiral expansion, $F_{\pi}$ is the leptonic decay constant (F is its value in the chiral limit), $\tilde{g}_{A}$ and $\tilde{m}_{N}$ are the axial charge and mass of the nucleon in the chiral limit; $l_{i}$, $c_{i}$, $d_{i}$ and $e_{i}$ are the low-energy constants (LEC’s) with an overline indicating that the corresponding LEC’s are renormalized.

In order to reproduce the above model-independent results we need to fulfill the following matching conditions between the parameters and LECs of the ChPT Lagrangian and our chiral quark-level Lagrangian (for the quark in semileptonic baryon decays.

\begin{equation}
\frac{\tilde{m}_{N}}{m} = \left(\frac{\tilde{g}_{A}}{g}\right)^{2} = R^{2}, \tag{29a}
\end{equation}

\begin{equation}
-4c_{1}M^{2} = (\tilde{m} - 4c_{1}^{2}M^{2})R^{2}, \tag{29b}
\end{equation}

\begin{equation}
8c_{1} - c_{2} - 4c_{3} - \frac{\tilde{g}_{A}^{2}}{\tilde{m}_{N}} = \left(8c_{1}^{2} - c_{2}^{2} - 4c_{3}^{2} - \frac{\tilde{g}_{A}^{2}}{\tilde{m}_{N}}\right)R^{2}, \tag{29c}
\end{equation}

\begin{equation}
\bar{c}_{1} = \frac{3}{64\pi^{2} F^{2}} \left(\frac{2 \tilde{g}_{A}}{\tilde{m}_{N}} - c_{2}\right) = \left(\bar{c}_{1}^{2} - \frac{3}{64\pi^{2} F^{2}} \left(\frac{2 \tilde{g}_{A}}{\tilde{m}_{N}} - c_{2}\right)\right)R^{2}, \tag{29d}
\end{equation}

\begin{equation}
c_{3} - 2c_{4} = c_{3}^{2} - 2c_{4}^{2} + \frac{3}{4} \tilde{m}_{N} (1 - R^{2}), \tag{29e}
\end{equation}

\begin{equation}
\tilde{d}_{16} - \frac{\tilde{g}_{A}}{64\pi^{2} F^{2}} = \left(\tilde{d}_{16}^{2} - \frac{\tilde{g}_{A}^{2}}{64\pi^{2} F^{2}}\right)R, \tag{29f}
\end{equation}

\begin{equation}
d_{18} = \tilde{d}_{18}^{2} R, \tag{29g}
\end{equation}

\begin{equation}
d_{22} = \tilde{d}_{22}^{2} R + \frac{\tilde{g}_{A}}{R}, \tag{29h}
\end{equation}

where $R = A^{n}_{11}(0)$ and $Q = (A^{n}_{11}(0))' = dA^{n}_{11}(q^{2})/dq^{2}|_{q^{2}=0}$. In addition we deduce the following constraints on the form factors encoding valence quark effects: $A^{n}_{13}(0) = R^{3}$ and $A^{n}_{13}(0) = -2m_{N}^{2}Q$.

### III. Rates and Asymmetry Parameters in Semileptonic Decays of Baryons

In this section we present detailed theoretical expressions \cite{28} - \cite{30} for the decay rates and asymmetry parameters in semileptonic baryon decays.

The decay width is given by the expression \cite{28}

\begin{equation}
\Gamma(B_{i} \to B_{j}l\nu) = \frac{G_{F}^{2}}{384\pi^{3} m_{B_{i}}^{2}} |V_{CKM}|^{2} (1 + \delta_{rad}) \int_{m_{l}^{2}}^{\Delta^{2}} ds \left(1 - m_{l}^{2}/s\right)^{2} \sqrt{(\Sigma^{2} - s)(\Delta^{2} - s)} N(s), \tag{30}
\end{equation}

where

$$N(s) = F^{2}_{1}(s)(\Delta^{2}(4s - m_{l}^{2}) + 2\Sigma^{2}\Delta(1 + 2m_{l}^{2}/s - (\Sigma^{2} + 2s)(2s + m_{l}^{2}))) + F^{2}_{2}(s)(\Delta^{2} - s)(2\Sigma^{2} + s)(2s + m_{l}^{2})/m_{B_{i}}^{2} + 3F^{2}_{3}(s)m_{l}^{2}(\Sigma^{2} - s)/m_{B_{i}}^{2},$$

$$+ 6F_{1}(s)F_{2}(s)(\Delta^{2} - s)(2s + m_{l}^{2})\Sigma/m_{B_{i}} - 6F_{1}(s)F_{3}(s)m_{l}^{2}(\Sigma^{2} - s)\Delta/m_{B_{i}},$$

$$+ G^{2}_{1}(s)(\Sigma^{2}(4s - m_{l}^{2}) + 2\Sigma^{2}\Delta^{2}(1 + 2m_{l}^{2}/s) - (\Delta^{2} + 2s)(2s + m_{l}^{2})) + G^{2}_{2}(s)(\Sigma^{2} - s)(2\Delta^{2} + s)(2s + m_{l}^{2})/m_{B_{i}}^{2} + 3G^{2}_{3}(s)m_{l}^{2}(\Delta^{2} - s)/m_{B_{i}}^{2},$$

$$- 6G_{1}(s)G_{2}(s)(\Sigma^{2} - s)(2s + m_{l}^{2})\Delta/m_{B_{i}} + 6G_{1}(s)G_{3}(s)m_{l}^{2}(\Delta^{2} - s)\Sigma/m_{B_{i}}.$$  \tag{31}
We have introduced the notation: \( s = q^2, \Sigma = m_B + m_{\bar{B}} \), \( \Delta = m_B - m_{\bar{B}}, \beta = (m_B - m_{\bar{B}})/m_B \). The factor \( \delta_{\text{rad}} \) represents the effect of radiative corrections \( 29 \) (see Table 2). \( G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2} \) is the Fermi coupling constant, and \( m_l \) is the leptonic (electron or muon) mass. For the corresponding CKM matrix elements \( V_{\text{CKM}} = V_{u\bar{d}} \) or \( V_{u\bar{s}} \) we use the central values from \( 3 \): \( V_{u\bar{d}} = 0.97377 \) and \( V_{u\bar{s}} = 0.225 \). Also we assume that the form factors are real.

Next we simplify the master formula \( 30 \), integrating over \( s \) and including terms up to \( \mathcal{O}(\beta^2) \) where \( \beta = \Delta/m_B \) is the SU(3) breaking parameter. (In this case the term proportional to \( G_3^2 \) can be omitted because it already starts at order \( \mathcal{O}(\beta^3) \).) Also, we include the momentum dependence of the leading form factors \( F_1(s) \) and \( G_1(s) \) and neglect the momentum dependence of the others. We expand the form factors \( F_1(s), G_1(s) \) to first order in \( s \):

\[
F_1(s) = F_1(0)(1 + \frac{s}{6}(r_{F_1}^2) + \mathcal{O}(s^2)), \quad G_1(s) = G_1(0)(1 + \frac{s}{6}(r_{G_1}^2) + \mathcal{O}(s^2)),
\]

where \( \langle r_{F_1}^2 \rangle \) and \( \langle r_{G_1}^2 \rangle \) are the "charge" radii of the \( F_1 \) and \( G_1 \) form factors calculated within our approach (cf. the numerical results in Sec. \( IV \)). In addition we retain finite lepton masses. These approximations are sufficient for both the \( n \rightarrow p e^- \bar{\nu}_e \) decay and for the muonic decay modes of hyperons. We also retain terms containing the form factors \( F_3 \) and \( G_3 \). Although their effects are proportional to \( m_l^2 \) they may give a measurable contribution for muonic modes (see also the discussion in Ref. \( 30, 31 \)).

At the order of accuracy to which we work the result for the decay width reads (exact formulas can be found in \( 29, 30 \)):

\[
\Gamma(B_i \rightarrow B_j \nu_l) = \frac{G_F^2}{60\pi^3} |V_{\text{CKM}}|^2 \Delta^5 (1 + \delta_{\text{rad}}) \left\{ (F_1^2 + 3G_1^2)(1 - \frac{3}{2}\beta) R_0(x) + \beta^2 \left( \frac{6}{7}F_1 R_{F_1}(x) + \frac{12}{7}G_1 R_{G_1}(x) \right) \right. \\
\left. + \frac{4}{7}F_1^2 R_{F_1}(x) + \frac{12}{7}G_1^2 R_{G_1}(x) + F_1^2 R_{F_2}(x) + \frac{6}{7}F_1F_2 R_{F_{12}}(x) + G_1 G_2 R_{G_{12}}(x) \right) + \mathcal{O}(\beta^3),
\]

where \( F_i = F_i(0), G_i = G_i(0) \) and \( x = m_l^2/\Delta \). Here the functions \( R_i(x) \) take into account the charged lepton mass \( m_l \) (see their expressions in Appendix \( D \)). In the calculation of the asymmetry parameters we restrict ourselves to the electron modes. The expressions for the electron–neutrino \( \alpha_{\nu_e} \), electron \( \alpha_e \), neutrino \( \alpha_{\nu_e} \) and emitted baryon \( \alpha_B \) asymmetries to the order of accuracy at which we are working are given in \( 29 \).

IV. NUMERICAL RESULTS

In this section we present our numerical results for the semileptonic decays of the baryon octet—coupling constants, decay widths and asymmetry parameters. First, we calculate the vector \( V_{B_i^+, B_j^+} \) and axial vector \( A_{B_i^+, B_j^+} \) couplings representing the contribution of the pure valence quarks to the semileptonic form factors of the baryons \( F_i^{B_i^+, B_j^+} \) and \( G_i^{B_i^+, B_j^+} \), i.e., when \( f_{i,j}^{1,1} = 1, g_{i,j}^{1,1} = 1 \) and \( f_{2,3}^{i,j} = g_{2,3}^{3,3} = 0 \). This limiting case corresponds to the projection of the nonrenormalized weak quark current \( j_{\mu,V-A} = \bar{q}_i \gamma_\mu(1 - \gamma_5)q_i \) between the respective baryon states. Our results for \( V_{B_i^+, B_j^+} \) and \( A_{B_i^+, B_j^+} \) are displayed in Tables 3 and 4. In Table 3, for comparison, we also present the predictions of the naive SU(6) model for the couplings \( V_{B_i^+, B_j^+} \) and \( A_{B_i^+, B_j^+} \).

Combining the contributions of the valence quarks and chiral effects we then derive the full expressions for the semileptonic couplings constants \( F_i^{B_i^+, B_j^+} \) and \( G_i^{B_i^+, B_j^+} \). The resulting forms are listed in Tables 5, 6 and 7. For convenience, we present the results for the leading (Fermi) \( F_1^{B_i^+, B_j^+} = f_1^{i,j} V_{11}^{B_i^+, B_j^+} \) and (Gamow-Teller) \( G_1^{B_i^+, B_j^+} = g_1^{i,j} A_{11}^{B_i^+, B_j^+} \) couplings in the form of a product of their SU(3) symmetric value together with a multiplicative factor \( 1 + f_{V,A}^{B_i^+, B_j^+} \) which includes the SU(3) breaking correction \( \delta_{V,A}^{B_i^+, B_j^+} \). (We remind the reader that the quark couplings \( f_{2,3}^{i,j} \) and \( g_{2,3}^{3,3} \) do not contribute to the leading baryon couplings \( f_1^{i,j} \) and \( G_1^{B_i^+, B_j^+} \).) Note that the axial vector couplings \( g_{1,4}^{d,u} \) and \( g_{1,4}^{u,d} \) defining the \( d \rightarrow u \) and \( s \rightarrow u \) flavor transitions, respectively, are expressed in terms of the unknown LEC's \( C_{q}^{u} \) and \( D_{q}^{u} \). We fix the value of these couplings to be \( g_{d,u}^{u} = 0.874 \) and \( g_{d,u}^{u} = 0.855 \) in order to reproduce the experimental data on the semileptonic decay widths as well as the ratio \( G_1/F_1 = 1.2695 \) in \( n \rightarrow p + e^- + \bar{\nu}_e \) decay.

The nucleon axial charge in the SU(3) limit (cf. Appendix \( A \)) \( g_{A}^{SU(3)} \) is given by

\[
g_{A}^{SU(3)} = 1.258
\]
while the SU(3) breaking parameters, $\delta^B_{i,j}$ and $\delta^B_{A,j}$ are found to have the form:

\[
\begin{align*}
\delta^B_{i,j} &= -0.069 \text{ (val)} + 0.070 \text{ (ch)} = 0.001, \\
\delta^\Sigma^n_{i,j} &= -0.061 \text{ (val)} + 0.070 \text{ (ch)} = 0.009, \\
\delta^\Xi^A_{i,j} &= -0.048 \text{ (val)} + 0.070 \text{ (ch)} = 0.022, \\
\delta^\Xi^\Sigma_{i,j} &= -0.028 \text{ (val)} + 0.070 \text{ (ch)} = 0.042,
\end{align*}
\]

and

\[
\begin{align*}
\delta^n_{p,A} &= 0 \text{ (val)} + 0.009 \text{ (ch)} = 0.009, \\
\delta^\Sigma^A_{p,A} &= 0.024 \text{ (val)} + 0.009 \text{ (ch)} = 0.033, \\
\delta^\Lambda^A_{p,A} &= -0.030 \text{ (val)} - 0.013 \text{ (ch)} = -0.043, \\
\delta^\Sigma^A_{n,A} &= 0.091 \text{ (val)} - 0.013 \text{ (ch)} = 0.078, \\
\delta^\Xi^A_{p,A} &= 0.066 \text{ (val)} - 0.013 \text{ (ch)} = 0.053, \\
\delta^\Xi^\Sigma^A_{p,A} &= 0.0085 \text{ (val)} - 0.013 \text{ (ch)} = -0.0045
\end{align*}
\]

where have denoted the contributions of valence quarks and chiral effects by the round brackets (val) and (ch), respectively.

Note that the SU(3) breaking corrections to the vector couplings $g^B_{i,j}$ begin at second order, in accord with the Ademollo-Gatto theorem (AGT) \[1\] (see discussion in Appendix \[E\]), while corrections to the axial couplings $g^B_{A,j}$ begin at first order. In this regard, if one works to first order in symmetry breaking, our results must be expressible in terms of a model-independent representation for the axial couplings derived in terms of the SU(3) symmetric couplings $D$ and $F$ plus four SU(3)-breaking parameters $H_i$ \[1, 32\] (cf. the discussion in Ref. \[5\])—

\[
\begin{align*}
g^np_{A} &= D + F + \frac{2}{3}(H_2 - H_3), \\
g^{\Lambda p}_{A} &= -\sqrt{\frac{3}{2}}\left(F + \frac{D}{3} + \frac{1}{9}(H_1 - 2H_2 - 3H_3 - 6H_4)\right), \\
g^{\Sigma^- n}_{A} &= D - F - \frac{1}{3}(H_1 + H_3), \\
g^{\Sigma^- \Lambda}_{A} &= \sqrt{\frac{2}{3}}\left(D + \frac{1}{3}(H_1 + H_2 + 3H_4)\right), \\
g^{\Xi^- \Lambda}_{A} &= \sqrt{\frac{3}{2}}\left(F - \frac{D}{3} + \frac{1}{9}(2H_1 - H_2 - 3H_3 + 6H_4)\right), \\
g^{\Xi^- \Sigma^0}_{A} &= \sqrt{\frac{1}{2}}\left(D + F - \frac{1}{3}(H_2 - H_3)\right), \\
g^{\Xi^- \Sigma^+}_{A} &= D + F - \frac{1}{3}(H_2 - H_3).
\end{align*}
\]

Such a representation is indeed found to hold in our model with the values

\[D = 0.7505, \quad F = 0.5075\]

for the SU(3) symmetric couplings, and

\[H_1 = -0.050, \quad H_2 = 0.011, \quad H_3 = -0.006, \quad H_4 = 0.037\]

for the SU(3) breaking terms. The components of $\delta^B_{i,j}$ which are first order in symmetry breaking—$\delta^B_{A,j}(1)$—are
From Eq. (40) one obtains a sum rule which relates the corrections \( \delta_A^{np(1)} = -2\delta_A^{\Sigma(1)} \) to a linear combination of the four remaining SU(3) breaking \( \delta_A^{(1)} \) parameters together with the SU(3)-symmetric couplings \( F \) and \( D \):

\[
\delta_A^{np(1)} = -2\delta_A^{\Sigma(1)} = \frac{2(H_2 - H_3)}{3(D + F)},
\]
\[
\delta_A^{\Lambda p(1)} = \frac{H_1 - 2H_2 - 3H_3 - 6H_4}{3(D + 3F)},
\]
\[
\delta_A^{\Sigma n(1)} = \frac{H_1 + H_3}{3(D - F)},
\]
\[
\delta_A^{\Sigma \Lambda(1)} = \frac{H_1 + H_2 + 3H_4}{3D},
\]
\[
\delta_A^{\Xi \Lambda(1)} = \frac{2H_1 - H_2 - 3H_3 + 6H_4}{3(3F - D)}.
\]

From Eq. (40) one obtains a sum rule which relates the corrections \( \delta_A^{np(1)} = -2\delta_A^{\Sigma(1)} \) to a linear combination of the four remaining SU(3) breaking \( \delta_A^{(1)} \) parameters together with the SU(3)-symmetric couplings \( F \) and \( D \):

\[
\delta_A^{np(1)} = \frac{1}{3} \left( \frac{D - 3F}{D + F} \delta_A^{\Sigma(1)} + \frac{D + 3F}{D + F} \delta_A^{\Lambda p(1)} + \frac{3(D - F)}{D + F} \delta_A^{\Sigma n(1)} + \frac{4D}{D + F} \delta_A^{\Sigma \Lambda(1)} \right).
\]

The SU(3) LEC’s from the chiral Lagrangian \( [2] \) can now be determined. Three of the four couplings \( C_6^q, C_8^q, D_{16}^q \) and \( D_{18}^q \) can be fixed by use of three constraints: the value of the nucleon axial charge in the SU(3) limit \( g_A^{SU3} = D + F = 1.258 \) together with the values of the axial quark couplings \( g_1^{da} = 0.874 \) and \( g_1^{su} = 0.855 \). Keeping, e.g., \( D_{17}^q \) undetermined we can relate the remaining three LEC’s via:

\[
C_3^p = -0.319 \text{ GeV}^{-1} \quad D_{17}^q, \quad C_4^q = -0.451 \text{ GeV}^{-1} \quad D_{17}^q, \quad D_{16}^q = 0.397 \quad D_{17}^q.
\]

In turn, the couplings \( C_6^q = -1.476, E_8^q = 0.086 \text{ GeV}^{-3}, E_8^q = 0.532 \text{ GeV}^{-3} \) are fixed from the description of magnetic moments of the baryon octet, while \( E_6^q = 1.868 \text{ GeV}^{-3} \) is found from the induced pseudoscalar form factor of the nucleon. The coupling \( D_{12}^q = 0.006 \text{ GeV}^{-2} \) is determined by fitting the slope of the form factor \( G_1^p : \langle q^2 G_1^p \rangle = 0.45 \text{ fm}^2 \).

Finally, the coupling \( D_{18}^q = -0.548 \text{ GeV}^{-2} \) is fixed by the fitting the central value of the induced pseudoscalar coupling of the nucleon \( g_{\rho p} = (M_\rho / m_N)G_1^{np}(q^2 = -0.88M_\rho^2) \simeq 8.25 \) predicted by ChPT \( [24, 25] \) together with the value of the pion–nucleon coupling constant \( g_{\pi N} = 13.10 \). It should be noted that the LEC’s \( C_6^q, E_8^q, E_8^q, D_{18}^q \) and \( D_{12}^q \) are unimportant for reproducing the semileptonic decay widths because they make no contribution to the leading baryon coupling constants \( F_2^{B_iB_j} \) and \( G_1^{B_iB_j} \).

Of particular interest is the decay \( \Sigma^- \to n e^- \bar{\nu}_e \) for which we predict \( G_1/F_1 = -0.260 \) and \( (G_1 - 0.237G_2)/F_1 = -0.278 \) (see Table 6). The latter result underestimates the experimental value \(-0.327 \pm 0.007 \pm 0.019\). However, this ratio was extracted by neglecting the \( q^2 \) dependence of the form factors \( F_1 \) and \( G_1 \) in the decay \( \Sigma^- \to n e^- \bar{\nu}_e \) decay. We find (see the discussion below) that inclusion of the \( q^2 \) dependence brings about agreement with the data for both electron and muon decay widths of the decay \( \Sigma^- \to n \nu \).

In Table 7 we present our results for the nonleading baryon semileptonic couplings \( F_{23} \) and \( G_{23} \). One can see that the pseudoscalar couplings \( G_{23}^{B_iB_j} \) are dominated by the corresponding pion or kaon pole contribution. (Here the leading contribution of the pole term is shown in brackets.) We also display the induced pseudoscalar coupling constant of the nucleon \( g_\rho \), which is fixed by the LEC \( D_{18}^q \). In Table 8 we compare our results for the ratios \( F_2^{B_iB_j}/F_1^{B_iB_j} \): i) with the predictions of the simple Cabibbo model in terms of the nucleon magnetic moments and baryon octet masses, ii) with the calculations performed in the \( 1/N_c \) expansion of QCD \( [33] \), and iii) with the results found in the SU(3) chiral quark-soliton model \( \chi QSM \) \( [34] \). Because of SU(2) invariance, we exactly reproduce the result of the Cabibbo model for the ratio \( F_2^{np}/F_1^{np} \) in neutron \( \beta \)-decay, while for the other modes we find SU(3) breaking deviations. Our result for the ratio \( F_2^{np}/F_1^{np} = -0.962 \) compares well: i) with the experimental data \( (0.97 \pm 0.14) \), ii) with the results of the \( 1/N_c \) expansion of QCD \( [33] \) \((-1.02)\), iii) with the results found in the \( \chi QSM \) model \( [34] \) \((-0.96)\), and iv) with calculations done in quenched lattice QCD \( [35] \) \((-0.85 \pm 0.45)\). Also, we have quite reasonable agreement for \( F_2^{B_iB_j}/F_1^{B_iB_j} \) with the results of the \( 1/N_c \) expansion \( [33] \) and with those of the \( \chi QSM \) approach for the remaining semileptonic modes.

Finally, we would like to stress that our results for the various semileptonic couplings of the decay mode \( \Sigma^- \to n e^- \bar{\nu}_e \) are in good agreement with the predictions of the lattice approach \( [33] \). In Table 9 we give a detailed comparison with the results of Ref. \( [33] \) using our conventions for the semileptonic matrix elements.

It is useful to parametrize our predictions for the weak magnetic couplings \( F_2 \) in terms of SU(3) symmetric couplings together with first order SU(3) symmetry-breaking parameters. As stressed in Ref. \( [2] \) there is an ambiguity in
expressing the SU(3) limit that clearly indicates the relevance of the first-order correction. It means that if in analogy to Eq. (37) we introduce a set of parameters \( \{ F^{F_2}, D^{F_2}, H^{F_2}_1 \} \) then we should apply it to \( F^{B_3,B_3}_2(0) \) or to \( m_N F^{B_i,B_j}_2(m_N) \). The second choice, \( m_N F^{B_i,B_j}_2(m_N) \), is traditionally preferred (See discussion in [2]). The difference is that we in addition multiply \( F^{B_i,B_j}_2(0) \) by the nucleon mass \( m_N \) to deal with dimensionless coupling). Otherwise the SU(3) breaking corrections will be overestimated. Within our model, we determine values for these parameters:

\[
D^{F_2} = 1.237, \quad F^{F_2} = 0.563, \quad H^{F_2}_1 = -0.246, \quad H^{F_2}_2 = 0.096, \quad H^{F_2}_3 = 0.021, \quad H^{F_2}_4 = 0.030.
\]  

(43)

Also, we can check the consistency of our results with the model-independent predictions for the second-class coupling constants \( F^{B_i,B_j} = \frac{m_N}{m_{B_i}} F^{B_i,B_j}_{3}, \frac{m_N}{m_{B_i}} G^{B_i,B_j}_{2} \) to first order in SU(3) breaking, which can be parametrized in terms of three SU(3) symmetry-breaking parameters \( H^{F}_i \) (see details in [1]):

\[
\begin{align*}
F^{np} &= 0, \\
F^{np} &= \frac{1}{\sqrt{6}} \left( -H^{F}_1 + 2H^{F}_2 + 2H^{F}_3 \right), \\
F^{F^{n}n} &= -H^{F}_1, \\
F^{F^{n}n} &= -\frac{\sqrt{2}}{3} H^{F}_3, \\
F^{\Xi^{n}n} &= \frac{1}{\sqrt{6}} \left( 2H^{F}_1 - H^{F}_2 - 2H^{F}_3 \right), \\
F^{\Xi^{n}n} &= -\frac{\sqrt{2}}{2} H^{F}_2, \\
F^{\Xi^{n}n} &= -H^{F}_2.
\end{align*}
\]  

(44)

Using Eq. (44) one can derive the following sum rules for the amplitudes \( F^{B_i,B_j} \):

\[
\begin{align*}
F^{np} &= \frac{1}{\sqrt{6}} \left( F^{F^{n}n} + 2F^{\Xi^{n}n} - F^{\Xi^{n}n} \right), \\
F^{\Xi^{n}n} &= -\frac{1}{\sqrt{6}} \left( 2F^{F^{n}n} - F^{\Xi^{n}n} \right), \\
F^{\Xi^{n}n} &= -\sqrt{6} \left( F^{np} + F^{\Xi^{n}n} \right) = F^{\Xi^{n}n} + F^{\Xi^{n}n}.
\end{align*}
\]  

(45a)

(Note that the sum rule (45a) was originally derived in [1].) When we restrict our calculation to first-order SU(3) breaking terms, we indeed fulfill the sum rules (45a) and for the SU(3)-breaking parameters we obtain \( H^{F}_i \):

\[
H^{F}_1 = 0.032, \quad H^{F}_2 = -0.028, \quad H^{F}_3 = -0.011, \quad H^{F}_{G_1} = 0.047, \quad H^{F}_{G_2} = -0.035, \quad H^{F}_{G_3} = -0.009.
\]  

(46)

Next we turn to the discussion of the semileptonic decay widths. We present our results in Table 10: i) total width \( \Gamma \) including all six couplings \( F_{1,2,3} \) and \( G_{1,2,3} \), leading \( q^2 \) dependence of \( F_1 \) and \( G_1 \) form factors and radiative corrections; ii) predictions \( \Gamma(F_1, G_1) \) are the results without inclusion of the subleading semileptonic form factors \( F_{2,3} \) and \( G_{2,3} \); iii) predictions \( \Gamma(F_1(0), G_1(0)) \) are the total widths without inclusion of the subleading semileptonic form factors \( F_{2,3} \) and \( G_{2,3} \) and of the \( q^2 \) dependence in the form factors \( F_1 \) and \( G_1 \); iv) predictions \( \Gamma^0 \) are total results without radiative corrections. For comparison we present the results of a pure SU(3) fit where we include only the \( F_1 \) and \( G_1 \) coupling constants omitting the \( q^2 \) dependence of \( F_1 \) and \( G_1 \) form factors and subleading form factors \( F_{2,3} \) and \( G_{2,3} \). The values of \( F_1 \) and \( G_1 \) are given by the Cabibbo model [2] where \( G_1 \) is expressed in terms of the SU(3) couplings \( F \) and \( D \). We fix \( F \) and \( D \) via \( F = 0.470 \) and \( D = 0.800 \). One can observe that the contribution of the subleading coupling constants \( F_{2,3} \) and \( G_{2,3} \) to the semileptonic decay width of the baryon octet is negligible. On the other hand, inclusion of \( q^2 \) dependence of the leading form factors \( F_1 \) and \( G_1 \) makes a significant difference for the \( \Lambda \to p \), \( \Sigma \to n \) and \( \Xi \to \Lambda \) decay modes. As stressed above, this \( q^2 \) dependence inclusion substantially improves agreement with the data for both decays \( \Sigma^- \to n \bar{\nu}_e \bar{\nu}_\mu (l = e, \mu) \). Specifically, the \( q^2 \) dependence yields a contribution of \( 0.78 \times 10^6 \text{ s}^{-1} (12\%) \) to the decay width of \( \Sigma^- \to n \bar{\nu}_e \bar{\nu}_\mu \) transition and \( 0.61 \times 10^6 \text{ s}^{-1} (19\%) \) to the decay width of \( \Sigma^- \to n \bar{\nu}_e \bar{\nu}_\mu \) transition.

Another interesting point of discussion – the rate ratio \( R^0_{cp} = \Gamma(\Xi^0 \to \Sigma^+ e^- \bar{\nu}_e)/\Gamma(\Xi^0 \to \Sigma^+ \mu^- \bar{\nu}_\mu) \) which has recently been measured by the KTeV Collaboration \( (R^0_{cp} = 55.6^{+22.2}_{-16.7}) \) [32]. Using a much larger data sample the NA48 Collaboration has published a preliminary value of \( (R^0_{cp} = 114.1 \pm 19.4) \) [37]. Our result \( R^0_{cp} = 114.81 \) nearly coincides
with the central value of the NA48 Collaboration and is close to the theoretical prediction of Ref. \[30\]—$R_{\Sigma^0} = 118.71$. Note, that for the corresponding ratio of the $\Sigma^-$ hyperon we find $R_{\Sigma^+} = \Gamma(\Sigma^- \to \Sigma^0\mu^-\bar{\nu}_\mu)/\Gamma(\Sigma^- \to \Sigma^0\mu^-\bar{\nu}_\mu) = 77.61$.

For comparison, we present the $\chi^2$/dof for our total results (the first column of Table 10) and the SU(3) fit: $\chi^2/8$ dof = 1.4 [this paper] and $\chi^2/8$ dof = 2.4 [SU(3) fit]. (We exclude from the $\chi^2$ analysis the results for the neutron $\beta$ decay and the poorly known data for the muonic modes of the cascade hyperons $\Xi$.)

As mentioned earlier, we include the momentum dependence of the $F_1(q^2)$ and $G_1(q^2)$ form factors up to first order in $q^2$. The slopes for $F_1$ and $G_1$ form factors calculated in our approach are found to be:

\[
\langle r_{F_1}^2 \rangle = \begin{cases} 
0.66 \text{ fm}^2, & n \to p \\
0.51 \text{ fm}^2, & \Lambda \to p \\
0.59 \text{ fm}^2, & \Sigma \to n \\
0.50 \text{ fm}^2, & \Xi \to \Lambda \\
0.43 \text{ fm}^2, & \Xi \to \Sigma 
\end{cases} \quad \text{and} \quad \langle r_{G_1}^2 \rangle = \begin{cases} 
0.45 \text{ fm}^2, & n \to p \\
0.32 \text{ fm}^2, & \Lambda \to p \\
0.40 \text{ fm}^2, & \Sigma \to n \\
0.41 \text{ fm}^2, & \Xi \to \Lambda \\
0.30 \text{ fm}^2, & \Xi \to \Sigma \\
0.28 \text{ fm}^2, & \Xi \to \Sigma
\end{cases} \quad (47)
\]

These predictions for the radii of the $F_1$ and $G_1$ form factors are consistent both with data and with the results of alternative theoretical approaches. In particular, the electroproduction and the neutrino experiments which involve $d \to u$ transitions are well fitted using dipole formulas which give $\langle r_{F_1}^2 \rangle = 0.66 \text{ fm}^2$ and $\langle r_{G_1}^2 \rangle = 0.40 \text{ fm}^2$ for the slopes of the $F_1$ and $G_1$ form factors \[38\]. For the $s \to u$ modes one expects smaller radii $\langle r_{F_1}^2 \rangle = 0.50 \text{ fm}^2$ and $\langle r_{G_1}^2 \rangle = 0.30 \text{ fm}^2$, respectively (see discussion in \[29, 38\]). For example, the authors of \[39\] find slopes of $\langle r_{F_1}^2 \rangle = 0.42 \text{ fm}^2$ and $\langle r_{G_1}^2 \rangle = 0.23 \text{ fm}^2$ for the $\Xi \to \Sigma$ transition using a generalized vector dominance ansatz for the form factors. In Refs. \[34, 39\] the $F_1$ form factor radii have been calculated in the framework of ChPT and of the $\chi$QSM model. Our results are in qualitative agreement with the full covariant result of ChPT \[39\], while the $\chi$QSM approach \[34\] gives somewhat higher values for the corresponding slopes:

\[
\langle r_{F_1}^2 \rangle = \begin{cases} 
0.44 \pm 0.06 \text{ fm}^2 \ (\text{ChPT}); & 0.72 \text{ fm}^2 \ (\chi \text{QSM}), \ \Lambda \to p \\
0.51 \pm 0.05 \text{ fm}^2 \ (\text{ChPT}); & 0.60 \text{ fm}^2 \ (\chi \text{QSM}), \ \Sigma \to n \\
0.45 \pm 0.03 \text{ fm}^2 \ (\text{ChPT}); & 0.66 \text{ fm}^2 \ (\chi \text{QSM}), \ \Xi \to \Lambda \\
0.46 \pm 0.07 \text{ fm}^2 \ (\text{ChPT}); & 0.80 \text{ fm}^2 \ (\chi \text{QSM}), \ \Xi \to \Sigma
\end{cases} \quad (48)
\]

We do not include the $q^2$ dependence of the $F_1$ form factor in the $\Sigma \to \Lambda$ transition, since it vanishes on account of the assumed degeneracy of the $u$ and $d$ quark masses.

Our approach generates a very reasonable description of the baryon semileptonic data with only two parameters—the axial couplings $g_1^{su}$ and $g_1^{su}$ responsible for the $d \to u$ and $s \to u$ transitions, which are in turn expressed in terms of the parameters of the chiral Lagrangian (see Appendix A). We remind the reader that the parameters controlling the valence quark contributions to the semileptonic properties of baryons—the constituent quark masses $m_u = m_d = 420$ MeV, $m_s = 570$ MeV and the size parameter $\Lambda_B = 1.25$ GeV—have been previously fixed via the analysis of electromagnetic properties of the baryon octet \[8, 11\]. Also, the same set of parameters ($m_u = m_d, m_s, \Lambda_B$) has been successfully used in the analysis of strong, electromagnetic and weak decays of charm and bottom baryons with light baryons in the final state \[8\]. In Table 11 we present the decay rates of hyperons divided by the squared CKM matrix elements in order to remove the uncertainty related to the values of $V_{ud}$ and $V_{us}$. Finally, in Table 12 we display the predictions for the asymmetry parameters in the electron modes.

V. SUMMARY

In this paper we have analyzed the semileptonic decay properties (coupling constants, decay widths and asymmetry parameters) of the baryon octet using a manifestly Lorentz covariant quark approach including chiral and SU(3) symmetry breaking effects.

Our main results are summarized as follows:
- We have derived results for the six couplings governing the semileptonic decays of the baryon octet, revealing both chiral and SU(3) symmetry–breaking corrections;
- We presented a numerical analysis of the decay rates and asymmetry parameters in the semileptonic decays of the baryon octet.

Our results provide a generally improved representation of hyperon semileptonic decay over the conventional SU(3)-symmetric (Cabibbo) analysis. We hope that the results of this paper can be used to reliably extract a value of the CKM matrix element $V_{us}$ from semileptonic hyperon decay data along the lines of \[2\].
APPENDIX A: CHIRAL EXPANSION OF THE VECTOR AND AXIAL VECTOR QUARK COUPLINGS

In this Appendix we list the results for the semileptonic vector and axial quark couplings (both SU(3)–symmetric and SU(3)–breaking). The corresponding SU(3) chiral quark Lagrangian \( \mathcal{L}_{SU} \) is specified in Sec.II. Below we list the results for the semileptonic quark couplings \( f_{1,2,3}^{du}, g_{1,2,3}^{du} \) and \( \bar{g}_{1,2,3}^{su} \) up to order \( \mathcal{O}(p^4) \) in the three–flavor picture.

1. Vector quark couplings.

a) Couplings \( f_{1}^{su} \) and \( f_{1}^{du} \).

The vector coupling governing the \( d \rightarrow u \) transition is trivial and equal to unity — \( f_{1}^{du} = 1 \), because we work in the isospin limit. In the case of the \( s \rightarrow u \) transition, the corresponding vector coupling \( f_{1}^{su} \) contains symmetry breaking corrections of second order in SU(3) — \( \mathcal{O}(M_K - M_\pi) \) and \( \mathcal{O}((M_K - M_\pi)^2) \). Note, that the Ademollo–Gatto theorem (AGT) protects the coupling \( f_{1}^{su} \) from first–order symmetry breaking corrections. The result for the \( f_{1}^{su} \) is

\[
\delta f_{1}^{su} = \frac{3}{16} (1 + 3g^2)(H_{\pi K} + H_{\eta K}) + 3g^2(G_{\pi K} + G_{\eta K}) = 1 + \delta f_{1}^{su}. \quad (A1)
\]

Here \( \delta f_{1}^{su} = 0.07 \) is the \( SU(3) \) breaking correction. The \( \mathcal{O}(p^2) \) functions \( H_{ab} \) and \( G_{ab} \), which show up in the context of ChPT [see, e.g., Refs. 39, 40], are defined as

\[
H_{ab} = \frac{1}{(4\pi F)^2} \left( M_a^2 + M_b^2 - \frac{2M_a^2M_b^2}{M_a^2 - M_b^2} \ln M_a^2/M_b^2 \right) = \mathcal{O}((M_a^2 - M_b^2)^2), \quad (A2a)
\]

\[
G_{ab} = -\frac{1}{(4\pi F)^2} \frac{2\pi (M_a - M_b)^2}{3m} (M_a^2 + 3M_aM_b + M_b^2) = \mathcal{O}((M_a^2 - M_b^2)^2). \quad (A2b)
\]

b) Couplings \( f_{2}^{du} \) and \( f_{3}^{su} \).

The coupling \( f_{2}^{du} \) is expressed through the linear combination of diagonal couplings \( f_{2}^{u} \) and \( f_{2}^{d} \) relevant for \( u \rightarrow u \) and \( d \rightarrow d \) transitions:

\[
f_{2}^{du} = \frac{1}{2} (f_{2}^{u} - f_{2}^{d}) = f_{2}^{SU3} + \delta f_{2}^{du}, \quad (A3a)
\]

\[
f_{2}^{u} = \frac{4}{3} f_{2}^{SU3} + \delta f_{2}^{u}, \quad (A3b)
\]

\[
f_{2}^{d} = -\frac{2}{3} f_{2}^{SU3} + \delta f_{2}^{d}, \quad (A3c)
\]

where

\[
f_{2}^{SU3} = C_{6}^{q} \left( \frac{1}{2} - \frac{3g^2 M^2}{32\pi^2 F^2} \right) + 12mE_{6}^{q} M^2 - \frac{3g^2 M m}{16\pi^2 F^2} \left( \pi + \frac{M}{m} \right) + \mathcal{O}(M^3) \quad (A4)
\]

is the SU(3) symmetric term, and \( \delta f_{2}^{du} \), \( \delta f_{2}^{u} \) and \( \delta f_{2}^{d} \) are the SU(3) breaking terms. The first–order terms read:

\[
\delta f_{2}^{u} = h_{2}^{u}(M_K^2 - M_\pi^2) + \mathcal{O}((M_K^2 - M_\pi^2)^2), \quad (A5a)
\]

\[
\delta f_{2}^{d} = -2\delta f_{2}^{u} = \frac{16}{3} m(E_{2}^{u} - E_{2}^{d})(M_K^2 - M_\pi^2), \quad (A5b)
\]

\[
\delta f_{2}^{du} = \frac{1}{7} (\delta f_{2}^{u} - \delta f_{2}^{d}), \quad (A5c)
\]

\[
h_{2}^{u} = C_{6}^{q} \frac{g^2}{48\pi^2 F^2} - \frac{16}{9} m(2E_{2}^{u} + 3E_{3}^{u}) + \frac{g^2 m}{48\pi^2 F^2 M} \left( \pi + \frac{2M}{m} \right) + \mathcal{O}(M). \quad (A5d)
\]
The coupling \( f_{su}^{s} \) is given by

\[
f_{su}^{s} = f_{2}^{SU3} + \delta f_{su}^{s}
\]

where

\[
\begin{align*}
\delta f_{su}^{s} & = (M_{K}^{2} - M_{\pi}^{2}) h_{su}^{s} + \mathcal{O}((M_{K}^{2} - M_{\pi}^{2})^{2}), \\
h_{su}^{s} & = -C_{6}^{s} \frac{g^{2}}{64 \pi^{2} F^{2}} + \frac{8}{3} m_{E} \pi^{2} \left( \frac{\pi + 2M}{m} \right) + \mathcal{O}(M).
\end{align*}
\]

(A7a)
(A7b)

Here, for convenience, we define the so-called SU(3) symmetric octet mass \( \bar{M} \) of pseudoscalar mesons as \( \bar{M}^{2} = 2\bar{m}B \) with \( \bar{m} = (m_{u} + m_{d} + m_{s})/3 = (2\bar{m} + m_{s})/3 \). Also \( c_{q}^{1}, d_{1}^{1} \) and \( C_{q}^{1}, D_{1}^{q} \) are the SU(2) and SU(3) quark low-energy constants (LEC’s). The overline on top of the LEC’s denotes renormalized quantities (see definitions in Ref. [3]).

c) Couplings \( f_{3}^{su} \) and \( f_{3}^{su} \):

The coupling \( f_{3}^{su} \) vanishes due to isospin invariance, while the coupling \( f_{3}^{su} \) starts at the first order in SU(3) breaking:

\[
f_{3}^{su} = \frac{g^{2} m^{2}}{96 \pi^{2} F^{2}} \frac{M_{K}^{2} - M_{\pi}^{2}}{M^{2}} \left( 1 - \frac{3 \pi \bar{M}}{2m} - \frac{4 \bar{M}^{2}}{m^{2}} + \mathcal{O}(M^{2}) \right) + \mathcal{O}((M_{K}^{2} - M_{\pi}^{2})^{2}).
\]

(A8)

2. Axial vector quark couplings.

a) Couplings \( g_{1}^{du} \) and \( g_{1}^{su} \):

The expressions for the axial vector couplings \( g_{1}^{du} \) and \( g_{1}^{su} \) responsible for the \( d \rightarrow u \) and \( s \rightarrow u \) transitions are as follows:

\[
\begin{align*}
g_{1}^{du} & = g_{1}^{SU3} + \delta g_{1}^{du}, \\
g_{1}^{su} & = g_{1}^{SU3} + \delta g_{1}^{su},
\end{align*}
\]

(A9a)
(A9b)

where

\[
g_{1}^{SU3} = g \left( 1 - \frac{7g^{2}M^{2}}{48 \pi^{2} F^{2}} + \frac{M^{3}}{48 \pi m F^{2}} \left( 9 + \frac{23}{2} g^{2} - 8C_{3}^{q} m + 24C_{2}^{q} m \right) \right) + 6M^{2}D_{16}^{q} + \mathcal{O}(M^{4}).
\]

(A10)

is the SU(3) symmetric term, \( \delta g_{1}^{du} \) and \( \delta g_{1}^{su} \) are the SU(3) breaking terms. Let us display the first–order terms:

\[
\begin{align*}
\delta g_{1}^{du} & = h_{1}^{du} (M_{K}^{2} - M_{\pi}^{2}) + \mathcal{O}((M_{K}^{2} - M_{\pi}^{2})^{2}), \\
\delta g_{1}^{su} & = h_{1}^{su} (M_{K}^{2} - M_{\pi}^{2}) + \mathcal{O}((M_{K}^{2} - M_{\pi}^{2})^{2}), \\
h_{1}^{du} & = -2h_{1}^{su} + \frac{g}{96 \pi^{2} F^{2}} \left( 9 + 23g^{2} \right), \\
h_{1}^{su} & = \frac{g}{96 \pi^{2} F^{2}} \left( 9 + 23g^{2} \right) - \frac{g \bar{M}}{96 \pi m F^{2}} \left( 9 + \frac{11}{2} g^{2} - 16C_{3}^{q} m + 24C_{2}^{q} m \right) - \frac{2}{3} D_{16}^{q} + \mathcal{O}(M^{2}).
\end{align*}
\]

(A11a)
(A11b)
(A11c)

b) Couplings \( g_{2}^{du} \) and \( g_{2}^{su} \):

The coupling \( g_{2}^{du} \) vanishes in the isospin limit, while the coupling \( g_{2}^{su} \) is zero at order of accuracy we are working at.

c) Couplings \( g_{3}^{du} \) and \( g_{3}^{su} \):

The couplings \( g_{3}^{du} \) and \( g_{3}^{su} \) are related to the couplings \( g_{1}^{du} \) and \( g_{1}^{su} \) via:

\[
\begin{align*}
g_{3}^{du} & = 2m^{2} \frac{g_{1}^{du}}{M_{K}^{2}} - 2D_{22}^{q} - 2D_{18}^{q}, \\
g_{3}^{su} & = 2m^{2} \frac{g_{1}^{su}}{M_{K}^{2}} - 2D_{22}^{q} - 2D_{18}^{q}.
\end{align*}
\]

(A12a)
(A12b)

The SU(3) LEC’s are fixed by: \( C_{q}^{q} = -1.476, \bar{E}_{2}^{q} = 0.086 \) GeV\(^{-3} \), \( \bar{E}_{3}^{q} = 0.532 \) GeV\(^{-3} \) from the description of the baryon octet magnetic moments, \( E_{2}^{q} = 1.868 \) from the description of the induced pseudoscalar form factor of the nucleon. The coupling \( D_{22}^{q} = 0.006 \) GeV\(^{-2} \) is fixed by fitting the slope of the form factor \( G_{1}^{mp} \). \( \langle r_{G_{1}}^{2} \rangle = 0.45 \text{ fm}^{2} \). The coupling \( D_{18}^{q} = -0.548 \) GeV\(^{-2} \) is fixed by fitting the central value of the induced pseudoscalar coupling of the nucleon \( g_{p} = (M_{p}/m_{N})G_{1}^{mp}(q^{2} = -0.88M_{p}^{2}) \simeq 8.25 \) predicted by ChPT [24, 25] and the value of the pion–nucleon coupling constant \( g_{\pi N} = 13.10 \).
APPENDIX B: THREE-QUARK BARYON CURRENTS AND FIERZ IDENTITIES

In this Appendix we specify the baryonic currents used in the main text following the approach of \[14, 15\]. The three-quark currents of the baryon octet are (we restrict ourselves to the so-called vector currents obtained in the SU(3) limit and without inclusion of terms with derivatives):

\[
\begin{align*}
J_p & = \epsilon^{a_1a_2a_3}\gamma^5 d^{a_1} u^{a_2} C\gamma_\mu u^{a_3}, \\
J_n & = -\epsilon^{a_1a_2a_3}\gamma^5 u^{a_1} d^{a_2} C\gamma_\mu d^{a_3}, \\
J_{\Sigma^+} & = \epsilon^{a_1a_2a_3}\gamma^5 s^{a_1} u^{a_2} C\gamma_\mu u^{a_3}, \\
J_{\Lambda^0} & = \sqrt{2} \epsilon^{a_1a_2a_3}\gamma^5 s^{a_1} u^{a_2} C\gamma_\mu d^{a_3}, \\
J_{\Xi^-} & = -\epsilon^{a_1a_2a_3}\gamma^5 d^{a_1} s^{a_2} C\gamma_\mu s^{a_3}, \\
J_{\Xi^0} & = -\epsilon^{a_1a_2a_3}\gamma^5 u^{a_1} s^{a_2} C\gamma_\mu s^{a_3}, \\
J_{\Xi^-} & = -\epsilon^{a_1a_2a_3}\gamma^5 u^{a_1} d^{a_2} C\gamma_\mu s^{a_3}, \\
J_{\Xi^0} & = \sqrt{\frac{2}{3}} \epsilon^{a_1a_2a_3}\gamma^5 (u^{a_1} d^{a_2} C\gamma_\mu s^{a_3} - d^{a_1} u^{a_2} C\gamma_\mu s^{a_3}).
\end{align*}
\]  

where \( C = \gamma_0 \gamma_2 \) is the charge conjugation matrix.

When generating matrix elements it is convenient to use Fierz transformations and corresponding identities in order to interchange the quark fields. First we specify five possible spin structures \( J^{\alpha\beta,\rho\sigma} = \Gamma_1^{\alpha\beta} \otimes (C\gamma_2)^{\rho\sigma} \) defining the Fierz transformation of the baryon currents:

\[
\begin{align*}
P & = I \otimes C\gamma_5, \\
S & = \gamma_5 \otimes C, \\
A & = \gamma^\mu \otimes C\gamma_\mu \gamma_5, \\
V & = \gamma^\mu \gamma^5 \otimes C\gamma_\mu, \\
T & = \sigma^{\mu\nu} \gamma^5 \otimes C\sigma_{\mu\nu}.
\end{align*}
\]

The Fierz transformation of the structures \( J = \{P, S, A, V, T\} \) read

\[
\begin{align*}
P & = \frac{1}{4} \left( \tilde{P} + \tilde{S} - \tilde{A} + \tilde{V} + \frac{1}{2} \tilde{T} \right), \\
S & = \frac{1}{4} \left( \tilde{P} + \tilde{S} + \tilde{A} - \tilde{V} + \frac{1}{2} \tilde{T} \right), \\
A & = -\tilde{P} + \tilde{S} - \frac{1}{2} \left( \tilde{A} + \tilde{V} \right), \\
V & = \tilde{P} - \tilde{S} - \frac{1}{2} \left( \tilde{A} + \tilde{V} \right), \\
T & = 3(\tilde{P} + \tilde{S}) - \frac{1}{2} \tilde{T}.
\end{align*}
\]

Viewing the Fierz transformation in terms of a Fierz matrix \( \mathcal{F} \) one can check that \( \mathcal{F}^2 = 1 \). Using Eqs. (B3) one can derive useful identities

\[
\begin{align*}
2(P - S) - A + V & = 2(\tilde{P} - \tilde{S}) - \tilde{A} + \tilde{V}, \\
6(P + S) + T & = 6(\tilde{P} + \tilde{S}) + \tilde{T}, \\
V & = 2(P - S) - A - 2\tilde{V}, \\
T & = 6(P + S) - 2\tilde{T}.
\end{align*}
\]

The symbol \( \sim \) is used to denote Fierz-transformed matrices according to \( \tilde{J}^{\alpha\sigma,\rho\beta} = \Gamma_1^{\alpha\sigma} \otimes (C\gamma_2)^{\rho\beta} \) where \( \alpha, \beta, \rho \) and \( \sigma \) are Dirac indices.
In this section we discuss the issue of gauge invariance in the context of the calculation of the baryonic matrix elements \( \langle B(p') | V_{\mu,1}^{ij}(0) | B(p) \rangle \) and \( \langle B(p') | A_{\mu,1}^{ij}(0) | B(p) \rangle \). The nonlocal structure of the strong interaction Lagrangian leads to the breaking of local symmetries, which can be restored using minimal substitution. In our approach we use an equivalent method suggested by Mandelstam [18] based on multiplying the quark fields with path-ordered exponentials—\textit{gauge exponentials}. As a result of gauging the strong interaction Lagrangian [18] the conventional triangle diagram in Fig.2a has to be supplemented by the two additional diagrams in Figs.2b and 2c. In our previous papers we have concentrated on electromagnetic processes. For the present application we extend this procedure to the electroweak interactions. Following Terning [19] we can show that the Mandelstam method is equivalent to minimal substitution. Introducing the doublet of left fermions, \( L \), (without specifying the number of generations), the free Lagrangian (kinetic term) for \( L \) is:

\[
\mathcal{L}^L_0(x) = \bar{L}(x)i\gamma_\mu L(x) \rightarrow \int dy \bar{L}(x) \delta^4(x-y)i\gamma_\mu \left[ \mathcal{P} \exp\left( \int \frac{d^4z}{x} \Gamma^L_\mu (z) \right) L(y) \right] \\
= \int dy \bar{L}(x) \delta^4(x-y) \mathcal{P} \exp\left( \int \frac{d^4z}{x} \Gamma^L_\mu (z) \right) i \partial_\mu^{L} L(y) = \bar{L}(x)i\partial_\mu^{L} L(x) \quad (C1)
\]

where \( D^L_\mu = \partial_\mu + \Gamma^L_\mu \) and \( \Gamma^L_\mu = \frac{ig^L_\mu}{2} Y_L B_\mu \). By analogy, the Mandelstam method works for the right singlet fields \( R \)

\[
\mathcal{L}^R_0(x) = \bar{R}(x)i\gamma_\mu R(x) \rightarrow \int dy \bar{R}(x) \delta^4(x-y)i\gamma_\mu \left[ \mathcal{P} \exp\left( \int \frac{d^4z}{x} \Gamma^R_\mu (z) \right) R(y) \right] \\
= \int dy \bar{R}(x) \delta^4(x-y) \mathcal{P} \exp\left( \int \frac{d^4z}{x} \Gamma^R_\mu (z) \right) i \partial_\mu^{R} R(y) = \bar{R}(x)i\partial_\mu^{R} R(x) \quad (C2)
\]

where \( D^R_\mu = \partial_\mu + \Gamma^R_\mu \) and \( \Gamma^R_\mu = -\frac{ig^R_\mu}{2} Y_R B_\mu \). We employ the standard notation: \( W^i_\mu \) (\( i=1,2,3 \)) and \( B_\mu \) are the gauge bosons, \( g_W \) and \( g'_W \) are the corresponding coupling constants (to distinguish them from the axial charge of the quark we attach the subscript \( W \), \( Y_L \) and \( Y_R \) are the hypercharges of the left and right quarks, respectively. The set of the physical states of the gauge bosons \( (W^\pm, Z^0, A) \) is connected to the set \( (W^i, B) \) via

\[
W^\pm = \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu), Z^0_\mu = \cos \theta_w W^3_\mu - \sin \theta_w B_\mu, \quad A_\mu = \sin \theta_w W^3_\mu + \cos \theta_w B_\mu, \quad (C3)
\]

where \( \theta_w \) is the Weinberg angle which relates the electromagnetic coupling constant \( e \) and the couplings \( g_W \) and \( g'_W \) via \( e = g_w \sin \theta_w = g'_w \sin \theta_w \). The quantities \( \Gamma^L_\mu \) and \( \Gamma^R_\mu \) in terms of \( (W^\pm, Z^0, A) \) fields are given by

\[
\Gamma^L_\mu = \frac{-ig^L_\mu}{\sqrt{2}} (W^+_\mu \tau^+ + W^-_\mu \tau^-) - ie \tan \theta_w Z^0_\mu \left( \frac{\tau_3}{2 \sin^2 \theta_w} - Q \right) - ieQA_\mu, \quad (C4a)
\]

\[
\Gamma^R_\mu = \frac{ie}{2} \tan \theta_w Z^0_\mu - ieQA_\mu. \quad (C4b)
\]

In the case of the strong baryon–three–quark interaction Lagrangian it is not necessary to rewrite the Lagrangian in terms of left quark doublets and right singlets. Instead we merely substitute each quark field \( q \) by its left-handed \( q_L = (1 - \gamma_5) q/2 \) and right–handed \( q_R = (1 + \gamma_5) q/2 \) components. Then we proceed with the gauging of the theory. We only need to know the gauging for the quarks of specific flavor and handedness—\textit{e.g.}, for the left–handed \( u_L, d_L \)
and $s_L$ and the right-handed $q_R = u_R$, $d_R$ and $s_R$ quarks the gauging is

$$
u_L(y) \rightarrow \mathcal{P} \exp \left( \int \frac{dy}{x} \mu_G L \Gamma_\mu^L(z) \right) u_L(y) + \mathcal{P} \exp \left( \int \frac{dy}{x} \mu_G R \Gamma_\mu^R(z) \right) d_L(y), \quad (C5a)$$

$$d_L(y) \rightarrow \mathcal{P} \exp \left( \int \frac{dy}{x} \mu_G L \Gamma_\mu^L(z) \right) d_L(y) + \mathcal{P} \exp \left( \int \frac{dy}{x} \mu_G R \Gamma_\mu^R(z) \right) d_L(y), \quad (C5b)$$

$$s_L(y) \rightarrow \mathcal{P} \exp \left( \int \frac{dy}{x} \mu_G L \Gamma_\mu^L(z) \right) s_L(y) + \mathcal{P} \exp \left( \int \frac{dy}{x} \mu_G R \Gamma_\mu^R(z) \right) s_L(y), \quad (C5c)$$

$$q_R(y) \rightarrow \mathcal{P} \exp \left( \int \frac{dy}{x} \mu_G R \Gamma_\mu^R(z) \right) q_R(y) \quad (C5d)$$

where $(ij)$ are pairs of flavor indices. The mixed left-handed quark fields are defined as:

$$u'_L = V_{ud} u_L + V_{cd} c_L + V_{td} t_L,$$

$$d'_L = V_{ud} d_L + V_{us} s_L + V_{ub} b_L,$$

$$c'_L = V_{us} u_L + V_{cs} c_L + V_{ts} t_L,$$

$$s'_L = V_{cd} d_L + V_{cs} s_L + V_{sb} b_L. \quad (C6)$$

In the derivation of Eqs. (C5b) and (C5c) we have used the unitarity condition $\sum_k V_{ik} V_{jk}^\dagger = \delta_{ij}$ for the CKM matrix elements, which leads to the useful identities:

$$d_L = d'_L V_{ud} + s'_L V_{cd} + b'_L V_{td},$$

$$s_L = d'_L V_{us} + s'_L V_{cs} + b'_L V_{ts},$$

$$b_L = d'_L V_{ub} + s'_L V_{sb} + b'_L V_{tb}. \quad (C7)$$

In the present manuscript we restrict our considerations to semileptonic processes (i.e., processes with a single intermediate off–shell charged weak gauge boson $W^\pm$). Therefore, we expand the gauge exponentials and keep only the term linear in $W^\pm$ which gives a correction to the weak current (in addition to the standard term which comes from the gauging of the free quark Lagrangian). This is a rather important point. The use of nonlocal Lagrangians automatically requires an extension of the conventional currents dictated by the local symmetries. In addition we have an extra piece from “gauging” the strong Lagrangian which contains derivatives acting on quark fields.

For illustration we derive the weak current which governs the $n \rightarrow pW^-$ transition. The first contribution comes from “gauging” the free Lagrangian:

$$J^\mu_p(x) = \frac{g_w}{\sqrt{2}} V_{ud} \bar{u}_L(x) \gamma^\mu d_L(x) = \frac{g_w}{2\sqrt{2}} V_{ud} \bar{u}(x) O^{\mu} d(x) \quad (C8)$$

where $O^\mu = \gamma^\mu (1 - \gamma^5)$.

To derive the contribution due to “gauging” the strong interaction Lagrangian we take the three–quark currents of the proton and neutron and proceed as follows:

- We express the quark fields in terms of left– and right–handed fields. One obtains:

$$J_p = \varepsilon_{a_1 a_2 a_3} \gamma^\mu \gamma^5 \left( d^a_{L1} + d^a_{R1} \right) \left( u^a_{L1} C \gamma_\mu u^a_{R1} + u^a_{R1} C \gamma_\mu u^a_{L1} \right),$$

$$J_n = -\varepsilon_{a_1 a_2 a_3} \gamma^\mu \gamma^5 \left( u^a_{L1} + u^a_{R1} \right) \left( d^a_{L2} C \gamma_\mu d^a_{R2} + d^a_{R2} C \gamma_\mu d^a_{L2} \right).$$

- We perform the gauging using the master formulas (C5) and after some simple algebra we derive the “nonlocal” contributions to the weak current associated with the $d \rightarrow u$ flavor exchange:

$$J^\mu_d(x) = \int dy \frac{\delta L^{weak}_{BB}}{\delta W^\mu_d(x)} \quad (C9)$$
where
\[ L_{np}^{\text{weak}}(x) = \frac{g_W g_N}{\sqrt{2}} V_{ud}(x) \int dx_{123} F(x, x_1, x_2, x_3) e^{a_1 a_2 a_3} \gamma^\mu \gamma^5 d^{a_1}(x_1) d^{a_2}(x_2) C \gamma_\mu(1 + \gamma_5) u^{a_3}(x_3) I(x_2, x, W^+) \]
\[ - \frac{g_W g_N}{\sqrt{2}} V_{ud}^*(x) \int dx_{123} F(x, x_1, x_2, x_3) e^{a_1 a_2 a_3} \gamma^\mu \gamma^5 u^{a_1}(x_1) u^{a_2}(x_2) C \gamma_\mu(1 + \gamma_5) d^{a_3}(x_3) I(x_2, x, W^-) + \text{H.c.} \]

Using the Fierz transformation (see Appendix B) the Lagrangian \( L_{np}^{\text{weak}} \) can be written in a more convenient form
\[ L_{np}^{\text{weak}}(x) = - \frac{g_W g_N}{2\sqrt{2}} V_{ud}(x) \int dx_{123} F(x, x_1, x_2, x_3) e^{a_1 a_2 a_3} \gamma^\mu(1 + \gamma^5) u^{a_1}(x_1) d^{a_2}(x_2) C \gamma_\mu d^{a_3}(x_3) I(x_2, x, W^+) \]
\[ + \frac{g_W g_N}{2\sqrt{2}} V_{ud}^*(x) \int dx_{123} F(x, x_1, x_2, x_3) e^{a_1 a_2 a_3} \gamma^\mu(1 + \gamma^5) d^{a_1}(x_1) u^{a_2}(x_2) C \gamma_\mu u^{a_3}(x_3) I(x_2, x, W^-) + \text{H.c.} \] (C11)

where \( \int dx_{123} = \int dx_1 \int dx_2 \int dx_3 \) and \( I(x_2, x, W^\pm) = \int_x^z d^B p_{B}^\pm(z) \).

- We remind the reader that the function \( F(x, x_1, x_2, x_3) \) is related to the scalar part of the Bethe-Salpeter amplitude and characterizes the finite size of the baryon. We use a particular form for the vertex function defined in Eq. (17).

- The current \( J_\mu^V(x) \) generates the triangle diagram (the left diagram in Fig.1) contributing to the \( n \rightarrow pW^- \) transition, while the current \( J_\mu^A(x) \) generates the bubble diagrams (the central and right diagram in Fig.1). By analogy one can derive the currents which govern the other six modes.

- A crucial check of our gauging procedure is to check the vector and axial-vector Ward-Takahashi identities (WTI) involving matrix elements of the \( n \rightarrow pW^- \) transition. In general, for an off-shell neutron and proton with momentum \( p \) and \( p' \), respectively, and the momentum transfer \( q = p' - p \), it is convenient to write down the corresponding weak matrix elements associated with the vector and axial vector current in the form (here and in the following we omit the weak coupling \( g \) and the CKM matrices in the matrix elements):
\[ \Lambda_\mu^V(p, p') = \Lambda_\mu^{V; \pm}(p, p') + \frac{q_\mu}{q^2} \left[ \Sigma_N(p') - \Sigma_N(p) \right] \] (C12)
and
\[ \Lambda_\mu^A(p, p') = \Lambda_\mu^{A; \pm}(p, p') - \frac{q_\mu}{q^2} \left[ \gamma^5 \Sigma_N(p) + \Sigma_N(p') \gamma^5 \right] + \frac{q_\mu}{q^2} \left[ 2m_q \Lambda_P(p, p') \right]. \] (C13)

Here, \( \Lambda_\mu^{V; \pm}(p, p') \) and \( \Lambda_\mu^{A; \pm}(p, p') \) are the contributions to the vector and axial vector matrix elements orthogonal to the \( W^- \)-boson (or leptonic pair) momenta; \( \Sigma_N(p) \) is the nucleon mass operator and \( \Lambda_P(p, p') \) is the pseudoscalar nucleon vertex function.

Then, the vector and axial vector WTI are satisfied according to
\[ q^\mu \Lambda_\mu^V(p, p') = \Sigma_N(p') - \Sigma_N(p) \] (C14a)
\[ q^\mu \Lambda_\mu^A(p, p') = -\gamma^5 \Sigma_N(p) - \Sigma_N(p') \gamma^5 + 2m_q \Lambda_P(p, p'). \] (C14b)

In our derivation we have made use of the quark-level identities
\[ S_{q_2}(k + p') \gamma_\mu S_{q_1}(k) = S_{q_2}(k + p') \gamma_\mu S_{q_1}(k) + \frac{q_\mu}{q^2} [S_{q_2}(k + p') - S_{q_1}(k + p)] \]
\[ + \frac{q_\mu}{q^2} (m_{q_2} - m_{q_1}) S_{q_2}(k + p') S_{q_1}(k + p), \] (C15a)
\[ S_{q_2}(k + p') \gamma_\mu \gamma_5 S_{q_1}(k) = S_{q_2}(k + p') \gamma_\mu \gamma_5 S_{q_1}(k) - \frac{q_\mu}{q^2} [\gamma_5 S_{q_1}(k) + S_{q_2}(k + p') \gamma_5] \]
\[ + \frac{q_\mu}{q^2} (m_{q_1} + m_{q_2}) S_{q_2}(k + p') \gamma_5 S_{q_1}(k + p) \] (C15b)
which lead to the vector and axial vector WTI on the quark level:

\[ q^\mu S_{q2}(k + p')\gamma_\mu S_q(k + p) = S_{q2}(k + p) - S_{q1}(k + p) + (m_{q2} - m_{q1})S_{q2}(k + p'S_{q1}(k + p), \] (C16a)

\[ q^\mu S_{q2}(k + p')\gamma_\mu\gamma_5 S_q(k + p) = -S_{q1}(k + p')\gamma_5 - \gamma_5 S_q(k + p) + (m_{q1} + m_{q2})S_{q2}(k + p')\gamma_5 S_q(k + p). \] (C16b)

We have introduced the notation \( \Gamma_\mu = \Gamma^\nu (g_{\mu\nu} - q_\mu q_\nu/q^2) \) for the so-called Dirac matrices orthogonal to the transverse momentum \( q \). All three diagrams contribute to \( \Lambda_\mu^{|-\rangle}(p, p') \) and \( \Lambda_\mu^{|-\rangle}(p, p') \)

\[
\Lambda_\mu^{-A;\perp}(p, p') = \Lambda_\mu^{-A;\Delta}(p, p') + \Lambda_\mu^{-A;\perp}(p, p') + \Lambda_\mu^{-A;\Delta}(p, p')
\] (C17)

where

\[
\Lambda_\mu^{-A;\Delta}(p, p') = -\alpha_N \int dk_{123} \Phi(z_0) \bar{\Phi}(z_0 + z_2(q))
\times \Gamma_{1f} S_q(k_1^+\gamma^\beta\gamma^5 \text{tr}[\Gamma_{2f} S_q(k_2^+ + q)O_{1\mu} S_q(k_2^+)\gamma_\beta S_q(-k_3^+)])
\] (C18a)

\[
\Lambda_\mu^{-A;\perp}(p, p') = \alpha_N \int dk_{123} L_{2\mu} \bar{\Phi}(z_0) \int dt_2 \bar{\Phi}'(z_0 + t_2(-q))
\times \gamma^\alpha\gamma^5 S_q(k_1^+\gamma^\beta(1 + \gamma^5) \text{tr}[\gamma_\alpha S_q(k_2^+)\gamma_\beta S_q(-k_3^+)])
\] (C18b)

\[
\Lambda_\mu^{-A;\perp}(p, p') = \alpha_N \int dk_{123} L_{2\mu} \bar{\Phi}(z_0) \int dt_2 \bar{\Phi}'(z_0 + t_2(q))
\times \gamma^\alpha(1 + \gamma^5) S_q(k_1^+)\gamma^\beta S_q(k_2^+)\gamma_\beta S_q(-k_3^+)
\] (C18c)

Here \( \Gamma_{1f} \otimes \Gamma_{2f} = \gamma^\alpha\gamma^5 \otimes \gamma_\alpha - \gamma^\alpha \otimes \gamma_\alpha \gamma_5 + 2I \otimes \gamma_5 - 2\gamma_5 \otimes I \).

The expressions for \( \Sigma_N(p) \) and \( \Lambda_P(p, p') \) are given by

\[
\Sigma_N(p) = -\alpha_N \int dk_{123} \bar{\Phi}(z_0) \bar{\Phi}(z_0 + z_2(q)) \Gamma_{1f} S_q(k_1^+\gamma^5 \gamma^\alpha S_q(k_2^+)\gamma_\beta S_q(-k_3^+)) \] (C19)

and

\[
\Lambda_P(p, p') = -\alpha_N \int dk_{123} \bar{\Phi}(z_0) \bar{\Phi}(z_0 + z_2(q)) \Gamma_{1f} S_q(k_1^+\gamma^5 \gamma^\alpha S_q(k_2^+)\gamma_\beta S_q(-k_3^+)) \] (C20)

We have used the notations from our paper on magnetic moments of heavy baryons \( \mathbb{B} \):

\[
\alpha_B = 6 q_0^2, \quad k_i^+ = k_i + p \omega_i, \quad k_i^+ = k_i + p' \omega_i, \quad z_0 = -6(k_1^2 + k_2^2 + k_3^2)
\]

\[
dk_{123} = \frac{d^4k_1d^4k_2d^4k_3}{(2\pi)^{8/4}} \delta^4(k_1 + k_2 + k_3), \quad L_i = 12(k_i - \sum_{j=1}^3 k_j \omega_j),
\] (C21)

\[
z_1(q) = -12q^2(\omega_2^2 + \omega_3^2 + \omega_2^2) - L_1 q,
\]

\[
z_2(q) = -12q^2(\omega_1^2 + \omega_3^2 + \omega_1^2) - L_2 q,
\]

\[
z_3(q) = -12q^2(\omega_1^2 + \omega_1^2 + \omega_2^2) - L_3 q.
\]

By analogy one can derive the matrix elements \( \langle B(p')|V_{\mu,\perp}^{(1)}(0)|B(p) \rangle \) and \( \langle B(p')|A_{\mu,\perp}^{(2)}(0)|B(p) \rangle \) for the other six modes.


APPENDIX D: FUNCTIONS $R_i(x)$

In this Appendix we write down the functions $R_i(x = m_i/\Delta)$:

\[ R_0(x) = \sqrt{1 - x^2} \left( 1 - \frac{9}{2} x^2 - 4 x^4 \right) + \frac{15}{4} x^4 \ln \frac{1 + \sqrt{1 - x^2}}{1 - \sqrt{1 - x^2}}, \]

\[ R_{F_1}(x) = R_{F_1}^0(x) + \frac{m^2_{B_1}}{9} R_{F_1}^2(x), \]

\[ R_{F_1}^0(x) = \sqrt{1 - x^2} \left( 1 - \frac{45}{8} x^2 - \frac{37}{4} x^4 + \frac{3}{4} x^6 \right) + \frac{105}{16} x^4 \ln \frac{1 + \sqrt{1 - x^2}}{1 - \sqrt{1 - x^2}}, \]

\[ R_{F_1}^2(x) = \sqrt{1 - x^2} \left( 1 + 4 x^2 + \frac{271}{4} x^4 + 6 x^6 \right) - \frac{105}{4} x^4 (1 + \frac{x^2}{2}) \ln \frac{1 + \sqrt{1 - x^2}}{1 - \sqrt{1 - x^2}}, \]

\[ R_{G_1}(x) = R_{G_1}^0(x) + \frac{5 m^2_{B_1}}{18} (r_{G_1}^2) R_{G_1}^2(x), \]

\[ R_{G_1}^0(x) = \sqrt{1 - x^2} \left( 1 - \frac{83}{16} x^2 - \frac{173}{24} x^4 + \frac{11}{24} x^6 \right) + \frac{175}{32} x^4 \ln \frac{1 + \sqrt{1 - x^2}}{1 - \sqrt{1 - x^2}}, \]

\[ R_{G_1}^2(x) = \sqrt{1 - x^2} \left( 1 - \frac{8}{5} x^2 + \frac{319}{20} x^4 + \frac{2}{5} x^6 \right) - \frac{21}{4} x^4 (1 + \frac{x^2}{2}) \ln \frac{1 + \sqrt{1 - x^2}}{1 - \sqrt{1 - x^2}}, \]

\[ R_{F_2}(x) = \sqrt{1 - x^2} \left( 1 + \frac{9}{4} x^2 + \frac{87}{8} x^4 + 6 x^6 \right) - \frac{105}{16} x^6 \ln \frac{1 + \sqrt{1 - x^2}}{1 - \sqrt{1 - x^2}}, \]

\[ R_{F_3}(x) = x^2 R_0(x), \quad R_{G_2}(x) = (1 - x^2)^{7/2}, \]

\[ R_{F_{13}}(x) = \frac{5}{4} x^2 \sqrt{1 - x^2} \left( 1 + \frac{13}{2} x^2 \right) - \frac{15}{4} x^4 (1 + \frac{x^2}{4}) \ln \frac{1 + \sqrt{1 - x^2}}{1 - \sqrt{1 - x^2}}, \]

\[ R_{G_{12}}(x) = \sqrt{1 - x^2} \left( 1 - \frac{13}{4} x^2 + \frac{33}{8} x^4 \right) - \frac{15}{16} x^6 \ln \frac{1 + \sqrt{1 - x^2}}{1 - \sqrt{1 - x^2}}, \]

\[ R_{G_{13}}(x) = \frac{3}{2} x^2 \sqrt{1 - x^2} \left( 1 + \frac{83}{6} x^2 + \frac{8}{3} x^4 \right) - \frac{15}{2} x^4 (1 + \frac{3}{4} x^2) \ln \frac{1 + \sqrt{1 - x^2}}{1 - \sqrt{1 - x^2}}. \]

APPENDIX E: CHECK OF THE ADEMOLLO–GATTO THEOREM (AGT)

As stressed above, the Ademollo–Gatto theorem (AGT) \[1\] protects the vector form factors from leading SU(3)–breaking corrections generated by the mass difference of strange and nonstrange quarks. The first nonvanishing breaking effects start at second order in symmetry–breaking. To demonstrate that this theorem is fulfilled in our approach we consider a strangeness-changing flavor transition $B_i \rightarrow B_j e \bar{\nu}_e$. The corresponding matrix element at $q = p' - p = 0$ is written as

\[ M_{\mu, V}^{B_i \rightarrow B_j} (p, p) = \bar{u}_{B_j} (p) \gamma_\mu F_1^{B_i \rightarrow B_j} (0) u_{B_i} (p), \] (E1)

where the vector coupling constant $F_1^{B_i \rightarrow B_j} (0)$ is defined as

\[ F_1^{B_i \rightarrow B_j} (0) = f_1^{su} V_1^{B_i \rightarrow B_j}. \] (E2)

Note that we have already proved (see \[5\]) that the vector form factor $f_1^{su}$ obeys the AGT. Therefore, we merely need to demonstrate that the same is true for the form factor $V_1^{B_i \rightarrow B_j}$ encoding valence quark effects—the valence quark vector form factor. In other words, due to the factorization of chiral effects and the effects of valence quarks, both form factors—$f_1^{su}$ and $V_1^{B_i \rightarrow B_j}$—should obey the AGT. The quantity $V_1^{B_i \rightarrow B_j}$ is expressed in terms of the baryon-three-quark
coupling constants \( g_{B_i} = g_B(m_{B_i}, m_{1i}) \) and \( g_{B_j} = g_B(m_{B_j}, m_{1j}) \), the Clebsch–Gordan coefficients \( C_{V_i B_j} \) and the structure integral \( I_{B_i B_j} = I(m_{B_i}, m_{B_j}, m_{1i}, m_{1j}) \), according to the contributions from the diagrams in Fig. 2:

\[
V_{1}^{B_i B_j} = g_B g_B C_{V_i B_j} I_{B_i B_j}
\]  
(E3)

where \( m_i = m_s \) and \( m_j = m \) are the masses of strange and nonstrange quarks. In the above formulae we do not display the dependence on the spectator quark masses \( m_2 \) and \( m_3 \). Note that the coupling constant \( g_B \) is related to the structure integral \( I_{B_i B_j} \) as \( g_B^2 = 1/I_{B_i B_j} \).

Next, using the transformation of the matrix element \( M_{\mu, V}(p, p) \) under hermitian conjugation

\[
\left( M_{\mu, V}(p, p) \right)^\dagger = \bar{u}_B(p) \gamma_\mu F_{1}^{B_i B_j}(0) u_B(p) = M_{\mu, V}(p, p) = \bar{u}_B(p) \gamma_\mu F_{1}^{B_i B_j}(0) u_B(p),
\]  
(E4)

we deduce the condition \( I_{B_i B_j} = I_{B_i B_j} \) which means that the structure integral \( I(m_{B_i}, m_{B_j}, m_{1i}, m_{1j}) \) is symmetric under the transformations \( m_{B_i} \leftrightarrow m_{B_j}, m_{1i} \leftrightarrow m_{1j} \):

\[
I(m_{B_i}, m_{B_j}, m_{1i}, m_{1j}) = I(m_{B_j}, m_{B_i}, m_{1j}, m_{1i}).
\]  
(E5)

Using the latter constraint, we express the structure integral \( I_{B_i B_j} \) through the coupling constants \( g_{B_i} \) and \( g_{B_j} \), i.e. one has

\[
I_{B_i B_j} = \frac{1}{2} \left( I_{B_i B_j} + I_{B_j B_i} \right) = \frac{1}{2} \left( I_{B_i B_j} + I_{B_j B_i} + \mathcal{O}(\delta_{B_i B_j}^2, \delta_{B_j B_i}^2, \delta_{B_i B_j} \delta_{B_j B_i}) \right) = \frac{1}{2} \left( \frac{1}{g_{B_i}} + \frac{1}{g_{B_j}} + \mathcal{O}(\delta^2) \right).
\]  
(E6)

where the parameters \( \delta_{B_i B_j} = m_{B_i} - m_{B_j} = \mathcal{O}(\delta) \) and \( \delta_{ij} = m_{1i} - m_{1j} = \mathcal{O}(\delta) \) are of first order in SU(3) breaking. Using the expansion (E6) we then obtain

\[
V_{1}^{B_i B_j} = \frac{C_{V_i B_j}}{2} \left( \frac{g_{B_i}}{g_{B_j}} + \frac{g_{B_j}}{g_{B_i}} + \mathcal{O}(\delta^2) \right).
\]  
(E7)

Finally, expanding \( g_{B_i}/g_{B_j} + g_{B_j}/g_{B_i} \) in terms of the difference \( g_{B_i} - g_{B_j} \sim \mathcal{O}(\delta) \)

\[
\frac{g_{B_i}}{g_{B_j}} + \frac{g_{B_j}}{g_{B_i}} = 2 + \left( \frac{g_{B_i} - g_{B_j}}{g_{B_i}} \right)^2 + \mathcal{O}((g_{B_i} - g_{B_j})^3) = 2 + \mathcal{O}(\delta^2)
\]  
(E8)

we prove the Ademollo–Gatto theorem

\[
V_{1}^{B_i B_j} = C_{V_i B_j} (1 + \mathcal{O}(\delta^2)).
\]  
(E9)

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Fig. 1. Baryon mass operator. Bold and thin lines refer to the baryons and quarks, respectively. Quarks are labeled by the indices \( k = 1, 2, 3 \).

Fig. 2. Diagrams contributing to the matrix elements of the bare quark operators \( V_{\mu,k}^{ij}(0) \) and \( A_{\mu,k}^{ij}(0) \), \( k = 1, 2, 3 \) : triangle (a), bubble (b) and (c). Bold, thin and wiggly lines refer to the baryons, quarks and external weak field, respectively. Quarks participating in the quark flavor transition \( q_i \rightarrow q_j \) are labeled by the indices \( 1i \) and \( 1j \), while the spectator quarks – by the indices 2 and 3. Initial and final baryons are labeled by the indices \( i \) and \( j \).
Table 1. Magnetic moments of the baryon octet (in units of the nuclear magneton $\mu_N$) and nucleon electromagnetic radii (in units of fm$^2$).

| Quantity | Our results [11] | Experiment [3] |
|----------|-----------------|----------------|
|          | Valence quarks  | Meson cloud    | Total          |                     |
| $\mu_p$  | 2.530           | 0.263          | 2.793          | 2.793              |
| $\mu_n$  | $-1.530$        | $-0.383$       | $-1.913$       | $-1.913$           |
| $\mu_A$  | $-0.575$        | $-0.038$       | $-0.613$       | $-0.613 \pm 0.004$ |
| $\mu_{\Sigma^+}$ | 2.336          | 0.196          | 2.532          | 2.458 $\pm 0.010$ |
| $\mu_{\Sigma^-}$ | $-0.942$       | $-0.327$       | $-1.269$       | $-1.160 \pm 0.025$ |
| $\mu_{\Xi^0}$ | $-1.240$       | $-0.096$       | $-1.336$       | $-1.250 \pm 0.014$ |
| $\mu_{\Xi^-}$ | $-0.599$       | 0.033          | $-0.566$       | $-0.6507 \pm 0.0025$ |
| $|\mu_{\Sigma^0}|$ | 1.273           | 0.293          | 1.566          | 1.61 $\pm 0.08$    |
| $\langle r^2 \rangle_p$ | 0.700           | 0.078          | 0.778          | 0.767 $\pm 0.012$  |
| $\langle r^2 \rangle_n$ | $-0.0628$       | $-0.0542$      | $-0.117$       | $-0.1161 \pm 0.0022$ |
| $\langle r^2 \rangle_{\Sigma^+}$ | 0.637           | 0.118          | 0.755          | 0.731 $\pm 0.060$  |
| $\langle r^2 \rangle_{\Sigma^-}$ | 0.618           | 0.099          | 0.717          | 0.762 $\pm 0.019$  |

Table 2. Numerical values for the radiative corrections in % (taken from Ref. [20]).

| Decay mode | $\delta_{\text{rad}}$ | $\delta_{\nu e}^{\nu e}$ | $\delta_{\text{rad}}^e$ | $\delta_{\nu e}^{\nu e}$ | $\delta_{B}^{\text{rad}}$ |
|------------|------------------------|---------------------------|--------------------------|---------------------------|--------------------------|
| $n \to p e^- \bar{\nu}_e$ | 6.96                   | 1.98                      | 1.98                     | 2.10                      | 2.10                     |
| $\Lambda \to p e^- \bar{\nu}_e$ | 4.17                   | 1.99                      | 1.99                     | 2.10                      | 2.10                     |
| $\Xi^- \to \Sigma^- e^- \bar{\nu}_e$ | 1.85                   | 1.98                      | 1.98                     | 2.10                      | 2.10                     |
| $\Sigma^+ \to \Lambda e^+ \nu_e$ | 2.25                   | 1.99                      | 1.99                     | 2.10                      | 2.10                     |
| $\Sigma^- \to \Lambda e^- \bar{\nu}_e$ | 2.22                   | 1.99                      | 1.99                     | 2.10                      | 2.10                     |
| $\Xi^- \to \Xi^- e^- \bar{\nu}_e$ | 1.95                   | 1.98                      | 1.98                     | 2.10                      | 2.10                     |
| $\Xi^0 \to \Sigma^0 e^- \bar{\nu}_e$ | 4.36                   | 1.99                      | 1.99                     | 2.10                      | 2.10                     |
| $\Lambda \to p \mu^- \bar{\nu}_\mu$ | 6.78                   |                           |                          |                           |                          |
| $\Sigma^- \to n \mu^- \bar{\nu}_\mu$ | 1.88                   |                           |                          |                           |                          |
| $\Xi^- \to \Xi^0 \mu^- \bar{\nu}_\mu$ | 2.12                   |                           |                          |                           |                          |
| $\Xi^0 \to \Sigma^+ \mu^- \bar{\nu}_\mu$ | 6.78                   |                           |                          |                           |                          |
Table 3. Couplings $V_{11}^{B_{i}B_{j}}$ and $A_{11}^{B_{i}B_{j}}$.

| Mode     | Our results | SU(6) quark model |
|----------|-------------|-------------------|
|          | $V_{11}^{B_{i}B_{j}}$ | $A_{11}^{B_{i}B_{j}}$ | $V_{11}^{B_{i}B_{j}}$ | $A_{11}^{B_{i}B_{j}}$ |
| $n \rightarrow p$ | 1 | 1.452 | 1 | $\frac{5}{3}$ |
| $\Lambda \rightarrow p$ | $-1.146$ | $-1.039$ | $-\sqrt{\frac{3}{2}} = -1.225$ | $-\sqrt{\frac{3}{2}} = -1.225$ |
| $\Sigma^{-} \rightarrow n$ | $-0.943$ | 0.307 | $-1$ | $\frac{1}{3} = 0.333$ |
| $\Sigma^{-} \rightarrow \Lambda$ | $-0.002$ | 0.724 | 0 | $\sqrt{\frac{2}{3}} = 0.816$ |
| $\Xi^{-} \rightarrow \Lambda$ | 1.170 | 0.388 | $\sqrt{\frac{3}{2}} = 1.225$ | $\frac{1}{\sqrt{6}} = 0.408$ |
| $\Xi^{-} \rightarrow \Sigma^{0}$ | 0.689 | 1.035 | $\frac{1}{\sqrt{2}} = 0.707$ | $\frac{5}{3\sqrt{2}} = 1.179$ |
| $\Xi^{0} \rightarrow \Sigma^{+}$ | 0.975 | 1.464 | 1 | $\frac{5}{3} = 1.667$ |

Table 4. Couplings $V_{21,31}^{B_{i}B_{j}}$ and $A_{21,31}^{B_{i}B_{j}}$.

| Mode     | $V_{21}^{B_{i}B_{j}}$ | $V_{31}^{B_{i}B_{j}}$ | $A_{21}^{B_{i}B_{j}}$ | $A_{31}^{B_{i}B_{j}}$ |
|----------|---------------------|---------------------|---------------------|---------------------|
| $n \rightarrow p$ | 1.530 | 0 | 0 | 2.850 |
| $\Lambda \rightarrow p$ | $-0.840$ | $-0.093$ | $-0.042$ | $-1.431$ |
| $\Sigma^{-} \rightarrow n$ | 0.802 | $-0.288$ | $-0.047$ | 1.467 |
| $\Sigma^{-} \rightarrow \Lambda$ | 1.180 | $-0.034$ | 0.034 | 2.517 |
| $\Xi^{-} \rightarrow \Lambda$ | 0.009 | 0.231 | 0.061 | $-0.048$ |
| $\Xi^{-} \rightarrow \Sigma^{0}$ | 1.235 | 0.014 | 0.006 | 2.374 |
| $\Xi^{0} \rightarrow \Sigma^{+}$ | 1.747 | 0.019 | 0.009 | 3.357 |
Table 5. Semileptonic decay constants of baryons $F^{B_iB_j}_1$ and $G^{B_iB_j}_1$.

| Decay mode | $F^{B_iB_j}_1$ | $G^{B_iB_j}_1$ |
|------------|----------------|----------------|
| $n \rightarrow p$ | 1 | $g_A = 1.258 \left(1 + \delta_A^{np}\right) = 1.2695$ |
| $\Lambda \rightarrow p$ | $-\sqrt{\frac{3}{2}}(1 + \delta_A^{np}) = -1.226$ | $-0.928 \left(1 + \delta_A^{np}\right) = -0.888$ |
| $\Sigma^- \rightarrow n$ | $-(1 + \delta_A^{\Sigma n}) = -1.009$ | $0.243 \left(1 + \delta_A^{\Sigma n}\right) = 0.262$ |
| $\Sigma^- \rightarrow \Lambda$ | $-0.002$ | $0.613 \left(1 + \delta_A^{\Sigma \Lambda}\right) = 0.633$ |
| $\Xi^- \rightarrow \Lambda$ | $\sqrt{\frac{3}{2}}(1 + \delta_A^{\Xi \Lambda}) = 1.252$ | $0.315 \left(1 + \delta_A^{\Xi \Lambda}\right) = 0.332$ |
| $\Xi^- \rightarrow \Sigma^0$ | $\frac{1}{\sqrt{2}}(1 + \delta_A^{\Xi \Sigma}) = 0.737$ | $0.890 \left(1 + \delta_A^{\Xi \Sigma}\right) = 0.885$ |
| $\Xi^0 \rightarrow \Sigma^+$ | $1 + \delta_A^{\Xi \Sigma} = 1.042$ | $1.258 \left(1 + \delta_A^{\Xi \Sigma}\right) = 1.252$ |

Table 6. Ratios $G^{B_iB_j}_1/F^{B_iB_j}_1$.

| Decay mode | Our results | Data [3] |
|------------|-------------|----------|
| $n \rightarrow p$ | 1.2695 | $1.2695 \pm 0.0029$ |
| $\Lambda \rightarrow p$ | 0.724 | $0.718 \pm 0.015$ |
| $\Sigma^- \rightarrow n$, $G_1/F_1$ | $-0.260$ | $-0.34 \pm 0.017$ |
| $\Sigma^- \rightarrow n$, $(G_1 - 0.237G_2)/F_1$ | $-0.278$ | $-0.327 \pm 0.007 \pm 0.019$ |
| $\Xi^- \rightarrow \Lambda$ | 0.265 | $0.25 \pm 0.05$ |
| $\Xi^- \rightarrow \Sigma^0$ | 1.20 | |
| $\Xi^0 \rightarrow \Sigma^+$ | 1.20 | $1.20 \pm 0.04 \pm 0.03$ |
Table 7. Semileptonic decay constants of baryons $F_{2,3}^{B_i,B_j}$ and $G_{2,3}^{B_i,B_j}$. Here $\mu_\pi = 0.13957$ and $\mu_K = 0.493677$ are the dimensionless masses of $\pi$ and $K$ mesons.

| Decay mode | $F_{2}^{B_i,B_j}$ | $G_{2}^{B_i,B_j}$ | $F_{3}^{B_i,B_j}$ | $G_{3}^{B_i,B_j}$ |
|------------|------------------|------------------|------------------|------------------|
| $n \to p$  | 1.853            | 0.00             | 0.00             | 2.187            |
|            |                  |                  |                  | $\left(\frac{2.271}{\mu^2}\right)$ |
|            |                  |                  |                  | $g_p = 8.25$     |
| $\Lambda \to p$ | -1.226           | -0.072           | -0.067           | -2.035           |
| $\Sigma^- \to n$ | 0.971            | -0.078           | -0.055           | 0.663            |
| $\Sigma^- \to \Lambda$ | 1.206            | 0.013            | 0.016            | 1.735            |
| $\Xi^- \to \Lambda$ | 0.162            | 0.076            | 0.052            | 1.403            |
| $\Xi^- \to \Sigma^0$ | 1.770            | 0.037            | 0.035            | 3.631            |
| $\Xi^0 \to \Sigma^+$ | 2.503            | 0.052            | 0.050            | 5.137            |

Table 8. Ratios $F_{2}^{B_i,B_j}/F_{1}^{B_i,B_j}$.

| Decay mode | Cabibbo model [2] | $1/N_c$ expansion [33] | $\chi$QSM [34] | Our results |
|------------|-------------------|------------------------|----------------|------------|
| $n \to p$  | $\frac{1}{2}(\mu_p - \mu_n - 1) = 1.853$ | 1.85 | 1.57 | 1.853 |
| $\Lambda \to p$ | $\frac{m_{\Lambda}}{2m_N}(\mu_p - 1) = 1.066$ | 0.90 | 0.71 | 1 |
| $\Sigma^- \to n$ | $\frac{m_{\Sigma^-}}{m_N}(\mu_p + 2\mu_n - 1) = -1.297$ | -1.02 | -0.96 | -0.962 |
| $\Sigma^- \to \Lambda$ ($F_2$) | $\frac{-m_{\Sigma^-}}{2m_N}\sqrt{\frac{3}{2}}\mu_n = 1.490$ | 1.17 | 1.24 | 1.206 |
| $\Xi^- \to \Lambda$ | $\frac{-m_{\Xi^-}}{2m_N}(\mu_p + \mu_n - 1) = 0.085$ | 0.06 | 0.02 | 0.129 |
| $\Xi^- \to \Sigma^0$ | $\frac{m_{\Xi^-}}{2m_N}(\mu_p - \mu_n - 1) = 2.609$ | 1.85 | 2.02 | 2.402 |
| $\Xi^0 \to \Sigma^+$ | $\frac{m_{\Xi^0}}{2m_N}(\mu_p - \mu_n - 1) = 2.597$ | 1.85 | 2.402 |
Table 9. Results for the $\Sigma^{-} \rightarrow n e^{-}\bar{\nu}_{e}$ decay.

| Quantity | Lattice approach [35] | Our results |
|----------|------------------------|-------------|
| $F_{1}$  | $-0.988 \pm 0.029_{\text{lattice}} \pm 0.040_{\text{HBChPT}}$ | $-1.009$ |
| $G_{1}/F_{1}$ | $-0.287 \pm 0.052$ | $-0.260$ |
| $(G_{1} - 0.237G_{2})/F_{1}$ | $-0.37 \pm 0.08$ | $-0.278$ |
| $F_{2}/F_{1}$ | $-0.85 \pm 0.45$ | $-0.962$ |
| $F_{3}/F_{1}$ | $0.24 \pm 0.12$ | $0.055$ |
| $G_{2}/F_{1}$ | $0.35 \pm 0.15$ | $0.077$ |
| $G_{3}/F_{1}$ | $-3.42 \pm 1.85$ | $-2.180$ |

Table 10. Decay widths $\Gamma$ (in units of $10^{6}$ s$^{-1}$, for neutron decay in units of $10^{-3}$ s$^{-1}$).

| Decay mode | Our results |
|------------|-------------|
| $n \rightarrow p e^{-}\bar{\nu}_{e}$ | $1.12$ | $1.12$ | $1.12$ | $1.05$ | $1.12$ | $1.129 \pm 0.001$ |
| $\Lambda \rightarrow p e^{-}\bar{\nu}_{e}$ | $3.28$ | $3.26$ | $3.10$ | $3.15$ | $3.16$ | $3.16 \pm 0.06$ |
| $\Lambda \rightarrow p \mu^{-}\bar{\nu}_{\mu}$ | $0.57$ | $0.56$ | $0.51$ | $0.53$ | $0.52$ | $0.60 \pm 0.13$ |
| $\Sigma^{-} \rightarrow n e^{-}\bar{\nu}_{e}$ | $6.50$ | $6.50$ | $5.72$ | $6.39$ | $6.19$ | $6.88 \pm 0.24$ |
| $\Sigma^{-} \rightarrow n \mu^{-}\bar{\nu}_{\mu}$ | $3.15$ | $3.15$ | $2.54$ | $3.09$ | $2.74$ | $3.0 \pm 0.2$ |
| $\Sigma^{+} \rightarrow \Lambda e^{+}\bar{\nu}_{e}$ | $0.26$ | $0.26$ | $0.26$ | $0.25$ | $0.27$ | $0.25 \pm 0.06$ |
| $\Sigma^{-} \rightarrow \Lambda e^{-}\bar{\nu}_{e}$ | $0.43$ | $0.43$ | $0.43$ | $0.42$ | $0.45$ | $0.39 \pm 0.02$ |
| $\Xi^{-} \rightarrow \Lambda e^{-}\bar{\nu}_{e}$ | $3.35$ | $3.35$ | $3.15$ | $3.28$ | $2.80$ | $3.35 \pm 0.37$ |
| $\Xi^{-} \rightarrow \Lambda \mu^{-}\bar{\nu}_{\mu}$ | $0.96$ | $0.96$ | $0.85$ | $0.94$ | $0.76$ | $2.1^{+2.1}_{-1.3}$ |
| $\Xi^{-} \rightarrow \Sigma^{0} e^{-}\bar{\nu}_{e}$ | $0.52$ | $0.51$ | $0.50$ | $0.51$ | $0.51$ | $0.53 \pm 0.10$ |
| $\Xi^{-} \rightarrow \Sigma^{0} \mu^{-}\bar{\nu}_{\mu}$ | $0.0067$ | $0.0067$ | $0.0064$ | $0.0065$ | $0.0064$ | $< 0.05$ |
| $\Xi^{0} \rightarrow \Sigma^{+} e^{-}\bar{\nu}_{e}$ | $0.93$ | $0.93$ | $0.91$ | $0.89$ | $0.91$ | $0.93 \pm 0.14$ |
| $\Xi^{0} \rightarrow \Sigma^{+} \mu^{-}\bar{\nu}_{\mu}$ | $0.0081$ | $0.0081$ | $0.0078$ | $0.0076$ | $0.0078$ | $0.02 \pm 0.01$ |
Table 11. Predictions for $\Gamma/|V_{\text{CKM}}|^2$ (in units of $10^7$ s$^{-1}$).

| Decay mode | $\Gamma/|V_{\text{CKM}}|^2$ | Decay mode | $\Gamma/|V_{\text{CKM}}|^2$ |
|------------|-------------------------------|------------|-------------------------------|
| $\Lambda \rightarrow p e^- \bar{\nu}_e$ | 6.48                           | $\Xi^- \rightarrow \Lambda e^- \bar{\nu}_e$ | 6.62 |
| $\Lambda \rightarrow p \mu^- \bar{\nu}_\mu$ | 1.13                           | $\Xi^- \rightarrow \Lambda \mu^- \bar{\nu}_\mu$ | 1.90 |
| $\Sigma^- \rightarrow n e^- \bar{\nu}_e$ | 12.84                          | $\Xi^- \rightarrow \Sigma^0 e^- \bar{\nu}_e$ | 1.03 |
| $\Sigma^- \rightarrow n \mu^- \bar{\nu}_\mu$ | 6.22                           | $\Xi^- \rightarrow \Sigma^0 \mu^- \bar{\nu}_\mu$ | 0.013 |
| $\Sigma^+ \rightarrow \Lambda e^+ \nu_e$ | 0.027                          | $\Xi^0 \rightarrow \Sigma^+ e^- \bar{\nu}_e$ | 1.84 |
| $\Sigma^- \rightarrow \Lambda e^- \bar{\nu}_e$ | 0.045                          | $\Xi^0 \rightarrow \Sigma^+ \mu^- \bar{\nu}_\mu$ | 0.016 |

Table 12. Asymmetry parameters.

| Decay mode | $\alpha_{ee}$ | $\alpha_e$ | $\alpha_{\nu_e}$ | $\alpha_B$ |
|------------|---------------|------------|-----------------|------------|
| $n \rightarrow p e^- \bar{\nu}_e$ | -0.08        | -0.10      | 0.99            | -0.48      |
| $\Lambda \rightarrow p e^- \bar{\nu}_e$ | -0.01        | 0.02       | 0.92            | -0.60      |
| $\Sigma^- \rightarrow n e^- \bar{\nu}_e$ | 0.42         | -0.50      | -0.32           | 0.65       |
| $\Sigma^+ \rightarrow \Lambda e^+ \nu_e$ | -0.39        | -0.68      | 0.63            | 0.06       |
| $\Sigma^- \rightarrow \Lambda e^- \bar{\nu}_e$ | -0.40        | -0.69      | 0.63            | 0.07       |
| $\Xi^- \rightarrow \Lambda e^- \bar{\nu}_e$ | 0.54         | 0.23       | 0.57            | -0.54      |
| $\Xi^- \rightarrow \Sigma^0 e^- \bar{\nu}_e$ | -0.19        | -0.18      | 0.96            | -0.46      |
| $\Xi^0 \rightarrow \Sigma^+ e^- \bar{\nu}_e$ | -0.18        | -0.17      | 0.92            | -0.45      |