Research and Optimization of Nonlinear Damping Vibration Isolator based on Parallelogram Linkage

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Abstract. A method is proposed to analyse and optimize the performance of nonlinear damping vibration isolator based on parallelogram linkage. Establish the vibration differential equation of the system, and perform Taylor expansion of the nonlinear damping coefficient, and then obtain the steady-state response of the system under harmonic excitation through methods such as perturbation method and averaging method. The displacement transfer rate curve of the steady-state response has a high degree of fitting with the curve obtained by the multi-body dynamics software, which proves the accuracy of the method. Taking the resonant peak of the steady-state response as the optimization target, and taking the high-frequency transmission rate as the constraint condition, design and optimize the rod length and damping coefficient of the mechanism. Comparing with traditional damping, the optimized nonlinear damping has a more obvious suppression effect on the resonance peak when both transmission rates of steady-state response are close at high frequencies. The analysis method in this paper is convenient to parameterize and has good programmability, which means that it provides a simple and effective new method for the parameterization and optimization design of the nonlinear damping vibration isolator based on parallelogram linkage.

1. Introduction

Vibration isolation technology has been widely used in various engineering applications, from satisfying the ride comfort of cars [1], isolating engine vibration [2] to protecting precision instruments [3], from passive vibration isolation, semi-active vibration isolation to active vibration isolation. Vibration isolation usually needs to meet: transmissibility, support performance, energy cost, etc. Active vibration isolation technology can achieve better vibration isolation performance, but requires more energy input [4]. Semi-active vibration isolation technology consumes less energy, but requires an extra controller to control the performance of the vibration isolator [5].

In general, the excitation frequency should be greater than $\sqrt{2}$ times of the natural frequency of the passive vibration isolator. Therefore, the stiffness or damping coefficient of the passive vibration isolator can be changed through various structural designs to optimize its performance. For example, Sun et al. [6] established a dynamic model for n-layer scissor-like structure, analysed the nonlinear stiffness, friction and damping characteristics of the model, and achieved good vibration isolation performance through design and optimization of structural parameter. Su et al. [7] proposed a quasi-zero stiffness vibration system with feedback control, analysed the mechanical characteristics of the system, derived the analytical formula of amplitude frequency characteristics of the system, and obtained two amplitude frequency characteristic curves with different configurations. Yao et al. [8] proposed an X-structure
vibration isolator, established the dynamic equation of the system, and solved it with the incremental harmonic balance method, and obtained the amplitude-frequency response curve of the system.

In order to further improve the performance of the passive nonlinear damping vibration isolator used in a certain type of inspection robot suspension, this paper adopts a parallelogram linkage mechanism to achieve nonlinear damping, and uses linear springs to ensure the support of the vibration isolator and avoid the unbounded response that may occur in the nonlinear stiffness system. According to the small displacement of the vibration isolator at the static equilibrium position, the nonlinear damping coefficient is derived and approximated, and the steady-state response of the system under harmonic excitation is solved. Taking the resonance peak of the steady-state response as the optimization objective and the high-frequency transmissibility of the steady-state response as the constraint condition, optimally design the rod length and horizontal damping ratio of the parallelogram linkage mechanism. This method can provide reference and guidance for the optimization and design of nonlinear damping vibration isolator based on parallelogram linkage mechanism.

2. Nonlinear damping vibration isolator structure
As shown in figure 1, the nonlinear damping vibration isolator used in inspection robot suspension consists of vertical guide mechanism, spring, and parallelogram linkage damping mechanism. The two ends of the vertical guide mechanism are respectively connected with the inspection robot body and the wheel so that the inspection robot body is restricted to only move in the vertical direction relative to the wheel. The spring plays the role of supporting the robot body and cushioning. The parallelogram linkage damping mechanism is composed of four hinged equal-length linkages and a horizontal linear damper. The upper and lower ends are respectively connected to the inspection robot body and the wheel by hinges. The two middle hinge points are connected with both ends of the horizontal damper by hinges.

![Figure 1. Diagram of the nonlinear damping vibration isolator.](image)

When all the connecting rods of the parallelogram linkage are located on the same horizontal plane, the suspension is in the static equilibrium position. When the suspension is impacted and vibrates around the static equilibrium position, the horizontal damper is stretched or compressed by the parallelogram linkage. The horizontal damping force is converted into the vertical damping force through the parallelogram linkage. The direction of damping force is opposite to the direction of suspension movement, which will dissipate vibration energy, therefore, reduce vibration and suppress resonance peaks.

3. Derivation of nonlinear damping coefficient
As figure 2 shows, installing a linear damper horizontally in the parallelogram linkage can convert the horizontal linear damping into the vertical nonlinear damping by virtue of the geometric characteristics of the parallelogram linkage.
Assuming that the length of the four connecting rods of the parallelogram linkage shown in figure 2 are all \( L \), and the four hinge points are marked as 1~4 respectively. The horizontal linear damper \( c_h \) is installed between hinge point 2 and hinge point 4. When hinge point 1 and hinge point 3 coincide, it is the initial equilibrium position of the mechanism. The stiffness of the vertical linear spring is \( k \). If the displacement of the robot body is \( z_c \) and the displacement of the wheel is \( z_w \), and when the relative displacement \( \Delta z = z_c - z_w \) occurs, the relative displacement of hinge point 4 to hinge point 2 is \( h \). The geometric relationship can be obtained by equation (1) and equation (2).

\[
h = L - L \cos \alpha \quad (1)
\]

\[
\cos \alpha = \frac{\left( L^2 - \left( \frac{\Delta z}{2} \right)^2 \right)^{\frac{1}{2}}}{L} \quad (2)
\]

Derive displacement \( h \):

\[
\dot{h} = \frac{\Delta \alpha}{2(L^2 - \Delta z^2)^{\frac{1}{2}}} \Delta \dot{z} \quad (3)
\]

Analyze the force of this mechanism, as shown in figure 3.

In figure 3, \( F_{cx} \) is the vertical force of the robot body and the wheel on the parallelogram linkage. \( F_h \) is the damping force of the horizontal damper. The relationship of both forces is:

\[
F_{h2} = \frac{1}{2} F_h \cos \alpha \quad (4)
\]

\[
F_{cx2} = \frac{1}{2} F_{cx} \sin \alpha \quad (5)
\]

\[
F_{h2} = F_{cx2} \quad (6)
\]
And the damping force $F_h$ of the horizontal damper is:

$$F_h = 2h c_h$$  \hspace{1cm} (7)

So $F_{cx}$ can be written as:

$$F_{cx} = \frac{\Delta z^2 c_h}{(4L^2 - \Delta z^2)} \Delta \dot{z}$$  \hspace{1cm} (8)

According to equation (8), when the displacement $\Delta z$ is close to $2L$, the damping coefficient of the mechanism tends to infinity, and the direction of $F_{cx}$ is opposite to the direction of $\Delta \dot{z}$.

4. Solution of steady-state response

Suppose ground excitation $G = G_e \cos \omega t$, among which $G_e$ is excitation amplitude, and $\omega$ is excitation frequency, and $t$ is time. The relative displacement between the robot body and the wheel is $Y$, then nonlinear damping coefficient can be derived from equation (8):

$$C = \frac{Y^2}{(4L^2 - Y^2)} c_h \hspace{1cm} (9)$$

Establish the vibration differential equation of the suspension:

$$MY'' + KY + \frac{Y^2 Y'}{(4L^2 - Y^2)} c_h = -M \ddot{G} = M \omega^2 G_e \cos \omega t$$  \hspace{1cm} (10)

In order to simplify equation (10), perform a sixth-order Taylor expansion of nonlinear damping force at $Y=0$:

$$F_{cx} = \frac{Y^2 Y'}{(4L^2 - Y^2)} c_h = \left( \frac{Y^2}{4L^2} + \frac{Y^4}{16L^4} \right) Y c_h + R_6(x)$$  \hspace{1cm} (11)

$R_6(x)$ of equation (11) is the remainder of the Taylor expansion. The nonlinear damping coefficient obtained by the exact expression equation (9) and the approximate expression equation (11) are shown in figure 4. It shows that the nonlinear damping coefficient is an increasing function of displacement, and the approximate expressions is pretty close to the exact expression, so it is feasible to use approximate expressions in the subsequent dynamics analysis.

**Figure 4.** The nonlinear damping coefficient obtained by the exact expression and the approximate expression ($c_h = 1$).
In order to make equation (10) dimensionless, introduce following dimensionless parameters:

\[
a = \frac{y}{L}, \quad \omega = \frac{g_e}{L}, \quad \omega_0 = \sqrt{\frac{k}{m}}, \quad \Omega = \frac{\omega}{\omega_0}, \quad \xi = \frac{c}{2ma_0}, \quad \tau = \omega_0 t
\]  

So equation (10) can be rewritten as:

\[
\ddot{a} + a + 2\xi \left(\frac{1}{4} a^2 + \frac{1}{16} a^4\right) \dot{a} = b \Omega^2 \cos \Omega \tau
\]  

It is worth noting that \(\dot{t}\) in equation (10) represents the derivation of time \(t\), \(\dot{\tau}\) in equation (13) represents the derivation of \(\tau\). The steady-state response of equation (13) is calculated by means of the average method. Let \(\Omega^2 = 1 + \varepsilon \sigma\), where \(\varepsilon\) is perturbation parameter and \(\sigma\) is tuning parameter. So equation (13) can be rewritten as:

\[
\ddot{a} + \Omega^2 a = \varepsilon f(a, \dot{a})
\]

\[
f(a, \dot{a}) = b_e \Omega^2 \cos \Omega \tau - 2\xi_e \left(\frac{1}{4} a^2 + \frac{1}{16} a^4\right) \dot{a} + \sigma a
\]

In equation (15):

\[
b_e = \frac{b}{\varepsilon}, \quad \xi_e = \frac{\xi}{\varepsilon}.
\]

Suppose the solution of equation (15) is:

\[
a = A \cos \phi
\]

\[
\dot{a} = -A \Omega \sin \phi
\]

\[
\phi = \Omega t + \theta
\]

\(A\) and \(\phi\) are the amplitude and initial phase of the steady-state response respectively, and both are slowly varying functions of time \(\tau\). After deriving the time \(\tau\) according to equation (16) and equation (17), it can get the following results:

\[
\dot{a} = \dot{A} \cos \phi - A \Omega \sin \phi - A \dot{\theta} \sin \phi
\]

\[
\ddot{a} = -\dot{A} \Omega \sin \phi - A \Omega^2 \cos \phi - A \dot{\theta} \cos \phi
\]

Plugging equation (16), equation (17) and equation (20) into equation (14) can obtain the following results:

\[
\dot{A} \cos \phi - A \dot{\theta} \sin \phi = \varepsilon f(A \cos \phi, -A \Omega \sin \phi)
\]

Combine equation (17) and equation (19) together to get:

\[
\dot{A} \cos \phi - A \dot{\theta} \sin \phi = 0
\]

Combine equation (22) and equation (21) together to get:

\[
\dot{A} = \frac{\varepsilon}{A} f(A \cos \phi, -A \Omega \sin \phi) \sin \phi
\]

\[
\dot{\theta} = \frac{\varepsilon}{A} f(A \cos \phi, -A \Omega \sin \phi) \cos \phi
\]
Since \( A \) and \( \phi \) are slowly varying functions of time \( \tau \), the average value within a period can be used to instead:

\[
\dot{A} = -\frac{\varepsilon}{2\pi\Omega} \int_0^{2\pi} f(A\cos\phi, -A\omega\sin\phi)\sin\phi = -\frac{1}{2\pi} \left( \frac{1}{8} \xi A^3\Omega + \frac{1}{64} \xi A^5\Omega + b\Omega^2\sin\theta \right) \quad (25)
\]

\[
\dot{\theta} = -\frac{\varepsilon}{2\pi A\Omega} \int_0^{2\pi} f(A\cos\phi, -A\omega\sin\phi)\cos\phi = -\frac{1}{2\pi A} (A\Omega^2 - A + b\Omega^2\cos\theta) \quad (26)
\]

At the steady-state response of the system, there is \( \dot{A} = \dot{\theta} = 0 \). Therefore, by equation (25) and equation (26), it can be further obtained that:

\[
b\Omega^2\sin\theta = -\left( \frac{1}{8} \xi A^3\Omega + \frac{1}{64} \xi A^5\Omega \right) \quad (27)
\]

\[
b\Omega^2\cos\theta = -(A\Omega^2 - A) \quad (28)
\]

Combining equation (27) and equation (28), the amplitude frequency equation and phase frequency equation of the suspension steady-state response are obtained:

\[
\left( \frac{1}{8} \xi A^3\Omega + \frac{1}{64} \xi A^5\Omega \right)^2 + (A\Omega^2 - A)^2 = b^2\Omega^4
\]

\[
\tan\theta = \frac{b\xi A^3\Omega + \xi A^5\Omega}{64(A\Omega^2 - A)} \quad (30)
\]

5. Solution of absolute displacement transfer rate

Figure 2 shows the dimensionless absolute displacement of the robot body: \( Q = \frac{y + g}{L} = A\cos(\Omega\tau + \theta) + b\cos\Omega\tau \), so the amplitude of \( Q \) is:

\[
|Q| = (A^2 + b^2 + 2A\cos\theta)^{\frac{1}{2}} \quad (31)
\]

The relative displacement transfer rate \( T_r \) is equal to the ratio of relative displacement \( A \) to dimensionless ground excitation \( b \):

\[
T_r = \left| \frac{A}{b} \right| \quad (32)
\]

So the absolute displacement transfer rate \( T_d \) is:

\[
T_d = \left| \frac{Q}{b} \right| = ((\frac{A}{b})^2 + 2\frac{A}{b}\cos\theta + 1)^{\frac{1}{2}} = (T_r^2 + 2T_r\cos\theta + 1)^{\frac{1}{2}} \quad (33)
\]

According to equation (33), the absolute displacement transfer rate curve of the nonlinear damping system at different horizontal damping ratio \( \xi \) can be obtained, as shown in figure 5.
Figure 5 shows that the nonlinear damping mechanism can effectively suppress the resonance peak when the horizontal damping coefficient is increased, while keeping the transmission rate in the high frequency region almost unchanged. It can be seen from figure 6 that traditional linear damping will increase the transmission rate in the high-frequency region and reduce the vibration isolation rate when the resonance peak is more effectively suppressed. Therefore, by means of optimization, when the transmission rate of the nonlinear damping mechanism is close to that of the linear damping mechanism...
in the high frequency region, the nonlinear damping mechanism can suppress the resonance peak more effectively.

6. Optimization of nonlinear damping mechanism

Taking the ground excitation to rod length ratio $b$ and the horizontal damping ratio $\xi$ as variables, establish the optimal design model of the suspension system:

$$\min T_{dp} = T_d(b, \xi) \quad (\Omega = 1)$$

$$\begin{cases} T_d(b, \xi) \leq T_l(\xi) & (\Omega \geq 2) \\ \xi_{min} \leq \xi \leq \xi_{max} \\ Y_{min} \leq \xi \leq Y_{max} \end{cases}$$

The meaning of equation (34): take the maximum value of the absolute displacement transfer rate of the nonlinear damping vibration isolation mechanism as the objective function, and the optimization target is the minimum. The meaning of equation (35) is that the absolute displacement transmission rate of the nonlinear damping mechanism is lower than that of the traditional linear damping mechanism when the frequency ratio $\Omega \geq 2$, and the horizontal damping ratio $\xi$ and the ground excitation and rod length ratio $b$ need to meet a reasonable range. This is a constrained optimization design problem. Take commonly used constraint design methods to programmatically search for optimal solutions in MATLAB.

Figure 7 shows the optimized absolute displacement transfer rate curve of nonlinear damping mechanism and linear damping mechanism. It can be seen that after optimization of the nonlinear damping mechanism, the resonance peak is located at a lower frequency ratio than the linear mechanism, which expands the vibration isolation range; while the resonance peak is smaller, and the high-frequency transmission rate is slightly smaller than that of the linear mechanism. At the same time, multi-body dynamics software (ADAMS) is used to verify the absolute displacement transfer rate curve of the nonlinear damping mechanism, and the two results are highly consistent.

![Figure 7. Absolute displacement transfer rate curves of the optimized nonlinear damping mechanism ($b = 0.5$).](image-url)
7. Conclusion

- This paper deduces the nonlinear damping coefficient based on parallelogram linkage, which is simplified by Taylor expansion, and compares the nonlinear damping coefficient before and after simplification, and the difference between them is small.
- The vibration differential equation of nonlinear damping vibration isolator is established, and the steady-state response of the vibration isolator under harmonic excitation is solved by average method.
- An optimization model of nonlinear damping vibration isolator is established with the resonance peak value of the steady-state response as the optimization objective, the high-frequency transmissibility as the constraint condition, and the ratio of rod length to ground excitation and the horizontal damping ratio as the variables.
- When the transmissibility of the optimized nonlinear damping vibration isolator is close to that of the traditional linear damping system at high frequency, the resonance peak suppression effect of the nonlinear damping mechanism is more obvious.

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