Topological Single Electron Pumping Assisted by Majorana Fermions

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Single electron pumping based on the topological property of Majorana fermions (MFs) is proposed. The setup consists of a quantum dot and four nano topological superconductors (TSs) connected by constriction junctions, with an additional vortex located in the loop of TSs. Operation is performed by gate voltages at constriction junctions. Simulations with Bogoliubov-de Gennes equation demonstrate successfully quantum protection during switching operation.

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I. INTRODUCTION

In the electronics-based computers, computation is performed based on motions of thousands of electrons, where the accuracy of bit manipulation relies on the statistics of huge number of electrons. If one can control electrons one by one at given instants, a single electron bit may be achieved, which reduces the energy consumption to the limit. This technology is also important for forming the metrology triangle of Ohm’s law: the quantum Hall effect discovered by von Klitzing works as the conductance standard [1], and the Josephson effect provides the voltage standard [2], while realization of precise current standard remains to be a challenge [3].

Manipulation of individual electrons in condensed matters is not easy since the wave length of electrons is comparable with their separation. One way to transport single electrons is to use quantum tunneling phenomenon. One needs to puncture the time for electrons to tunnel through the barrier by introducing several quantum dots (QDs) in the circuit, since quantum tunnelings normally take place randomly in time [4, 5]. Several schemes have been proposed for single electron pumping so far, such as single electron transfer in metallic nanostructures based on the combination of Coulomb interaction between electrons and their quantum tunneling trough an insulating barrier [6], adiabatical charge pumping through a quantum dot by driving two independent parameters of the system coherently, e.g. phases of gate voltage and magnetic field [7, 8], quantum Hall effect in Corbino disk geometry combined with a time dependent flux [9].

In a topological superconductor characterized by zero-energy Majorana fermions (MFs) as quasiparticle excitations, the ground state exhibits degeneracy with respect to the parity of electron number [10–26]. This property provides a new principle for single electron pumping with unprecedented precision guaranteed by topology. In the present work we formulate a protocol for this purpose.

Our setup consists of four topological superconductors (TSs) each carrying a vortex and connected by constriction junctions; there is an additional vortex inside the loop of TSs (see Fig. 1). In order to reveal the topological property of the system, we first attach another TS to the device. We show that the edge MF of this TS can be driven in a controlled way around the loop of the four TSs by switching on and off gate voltages at constriction junctions in the designed sequence. After this process, the parity of this TS is flipped since the edge MF acquires a \( \pi \) phase due to the vortex at the center of device. Therefore, one can consider this TS as a MF qubit, and the device of four TSs as a NOT gate. We then demonstrate that the NOT gate works for single electron pumping when a QD with Coulomb blockade effect is attached when the energy of QD is adjusted appropriately. Numerical simulations based on the time-dependent Bogoliubov-de Gennes (TDBdG) equation [27] are performed, which confirms successfully the quantum protection and phase coherence during the whole process typically of several nano seconds.

FIG. 1: Schematic device setup of a NOT gate for MF qubit. Finite square samples (called bricks) of heterostructure of ferromagnetic insulator and spin-orbit coupling semiconductor are positioned on the surface of an s-wave superconductor with vortices (black dots) trapped in it. The four bricks (on the right side) form the NOT gate. The edge MF of another brick (on the left side) can be driven to circle the vortex at the center of the loop formed by the four bricks by tuning gate voltages at the point-like constriction junctions.
The remaining part of this paper is organized as follows. We discuss in Sec. II the dynamics of an edge MF in a MF-qubit driven through the NOT gate. Then we reveal the relation satisfied by the interactions in the NOT gate in Sec. III. Based on the property of NOT gate, we formulate in Sec. IV the topological single electron pumping by attaching a QD to the NOT gate, and give explicitly the energy regime for QD. In Sec. V, it is clarified that the function of NOT gate can be understood as a quantum interference between two MFs. Discussions are presented in Sec. VI with a summary given in Sec. VII.

II. NOT GATE FOR MF-QUBIT

A topological superconducting state can be achieved in a heterostructure of s-wave superconductor (s-SC), spin-orbit coupling semiconductor (SOSM), and ferromagnetic insulator (FMI) with one vortex at the sample center, where one MF appears at the vortex core and another MF at the sample edge [22, 27]. In the present device, four finite TSs are positioned on a common s-SC substrate. The core MFs are stable and do not participate in the phenomena discussed below (strictly speaking there are exponentially small contributions, which can be neglected safely), and thus will be omitted hereafter. The linear dimension of TSs should be in the regime of tens of constriction junctions, whereas \( \Gamma \) is described by the vector \((\lambda_1, \lambda_2, \lambda_3, \lambda_4) = (0, 1, 0, 1, 0)\) (see the left inset of Fig. 2). Since the qubit is isolated, there is an edge MF \( (\tilde{\gamma}_0) \) localized at the qubit. In contrary, with \( \lambda_1 = \lambda_3 = 1 \), both the unified edge of TS(1) and TS(2) and that of TS(3) and TS(4) contain two vortices, and thus there is no edge MF in the NOT gate. In the representation of \( \Omega \) the MFs \( \tilde{\gamma}_j \) for \( 1 \leq j \leq 4 \) are fused to finite energies due to the interactions.

Turning on the connection between the qubit and TS(1), namely \( \lambda_0 = 0 \rightarrow 1 \), the wavefunction of \( \tilde{\gamma}_0 \) spreads to the unified edge of the now connected qubit, TS(1) and TS(2), since it contains three vortices. We then turn off the connection between TS1 and TS2, namely \( \lambda_1 = 1 \rightarrow 0 \). The wavefunction of edge MF collapses totally on TS2 due to the topological property. After these two switchings, the edge MF \( \tilde{\gamma}_0 \) is transported completely to TS2. Repeating this process, one can drive the edge MF \( \tilde{\gamma}_0 \) through the NOT gate in a clockwise way, and returns it back to the initial position at the qubit, with the switching sequence \((0, 1, 0, 1, 0) \rightarrow (1, 1, 0, 1, 0) \rightarrow (1, 0, 0, 1, 0) \rightarrow (1, 0, 1, 1, 0) \rightarrow (1, 0, 1, 0, 1) \rightarrow (0, 0, 1, 0, 1)\). During this process the edge MF feels the gauge field formed by the central vortex, and thus acquires a phase of \( \pi \) which makes \( \tilde{\gamma}_0 \rightarrow -\tilde{\gamma}_0 \). As the result, the electronic parity of the qubit is flipped.

In order to confirm the function of the NOT gate, we perform numerical calculations based on TDBdG equation. We first diagonalize the tight-binding BdG hamiltonian \( H_0 \) of the system in the initial stage,

\[
\tilde{H}_0 = -t_0 \sum_{i,j,\sigma} \hat{c}^\dagger_{i\sigma} \hat{c}_{j\sigma} - \mu \sum_{i,\sigma} \hat{c}^\dagger_{i\sigma} \hat{c}_{i\sigma} + \sum_{i} V_\sigma (\hat{c}^\dagger_{i\uparrow} \hat{c}_{i\downarrow} - \hat{c}^\dagger_{i\downarrow} \hat{c}_{i\uparrow}) + i t_\alpha \sum_{i,\delta} \left[ \hat{c}^\dagger_{i+\delta,\sigma} \hat{c}_{i,\sigma} \right] h.c. + \sum_{i} \left[ \Delta(i) \hat{c}^\dagger_{i\uparrow} \hat{c}_{i\downarrow} + h.c. \right],
\]

(2)

where both spin-conserved hopping \( t_0 \) and spin-flipped hopping \( t_\alpha \) are between nearest neighbors with \( a \) the grid spacing, and \( V_\sigma \) is the Zeemann energy. An edge MF state is obtained at the MF qubit with the wave function \( |\Psi(t = 0)\rangle = |\phi_{MF}\rangle \). Then we modulate dynamically the hopping parameters at the constriction junctions, which changes the MF interactions in \( \Omega \) sequentially and drives the edge MF \( \tilde{\gamma}_0 \) [27]. The evolution of the wave function is obtained by solving the TDBdG equation \( \hat{h}_0^{\Omega}\frac{d}{dt}|\Psi(t)\rangle = \hat{H}_0^{\Omega}|\Psi(t)\rangle \) based on the Chebyshev polynomials expansion [29, 30]. To monitor the evolution of the edge MF, we project the wave function onto the initial one, and evaluate the parameter \( O(t) = \langle \phi_{MF}|\Psi(t)\rangle \). As can be seen from Fig. 2, \( O(t) \) changes from positive unity at the initial stage to negative unity at the final stage, representing a sign change in the MF wave function. It is worth noticing that the conservation of the function norm as seen in Fig. 2 confirms the topological protection of the edge MF during the operating process.

III. PARITY FLIPPING IN NOT GATE

Since the parity of the whole system is conserved upon application of gate voltage as well as Cooper pair tunneling from SC substrate, the above switching operation
interactions $\Gamma$ should reverse the parities of the MF qubit and NOT gate simultaneously. In order to check the electronic parity of the NOT gate, we investigate first the signs of MF interactions $\Gamma_j$ defined in Eq. (1). As revealed in the previous work $[27]$, when $\gamma_0$ is transported to TS(2), it picks a sign $\text{sgn}(\Gamma_0 \Gamma_1)$. Therefore, the sign of edge MF $\gamma_0$ after driven through the NOT gate is given in terms of the interactions by

$$\gamma_0 \Rightarrow \text{sgn}[\Gamma_0 \Gamma_1 \Gamma_2 \Gamma_3 \Gamma_4 (-\Gamma_0)] \gamma_0.$$  \hspace{1cm} (3)

The minus sign attached to $\Gamma_0$ is due to the opposite motion of MF against the direction used to define the interaction $\Gamma_0$ in $[27]$.

It is then clear that the sign reversal of $\gamma_0$ implies $\Gamma_1 \Gamma_2 \Gamma_3 \Gamma_4 > 0$, a topological property generated by the central vortex. Hereafter, we consider explicitly the case where all interactions are positive since all possible configurations of interaction signs can be transformed to each other by gauge transformation. It is worth noting that the same sign constraint $\prod_{j=1,2N} \Gamma_j > 0$ and gauge choice are available for even number of TSs, which will be used for discussions below.

We define two regular electronic states with the four MFs, $d_1^1 = (\gamma_1 + i \gamma_2)/2$ and $d_2^1 = (\gamma_3 + i \gamma_4)/2$, as always possible even when the MFs are bounded and not free. We then rewrite the Hamiltonian $[1]$ in terms of the basis $|n_1 n_2\rangle = \{|00\rangle, |11\rangle, |10\rangle, |01\rangle\}$, with $n_i$ denoting the parity of the electronic state,

$$H_{\text{MF}} = \begin{bmatrix}
\Gamma_1' + \Gamma_2' & \Gamma_2' - \Gamma_3' & 0 & 0 \\
\Gamma_2' - \Gamma_3' & \Gamma_3' - \Gamma_4' & 0 & 0 \\
0 & 0 & \Gamma_4' - \Gamma_3' & \Gamma_4' + \Gamma_3' \\
0 & 0 & \Gamma_4' + \Gamma_3' & -\Gamma_4' - \Gamma_3'
\end{bmatrix},$$  \hspace{1cm} (4)

where $\Gamma_j' \equiv \lambda_j(t) \Gamma_j$. The four eigen energies are given by $E_{1,2} = \pm \sqrt{(\Gamma_1' + \Gamma_3')^2 + (\Gamma_2' - \Gamma_4')^2}$ for even parity, and $E_{3,4} = \pm \sqrt{(\Gamma_1' - \Gamma_3')^2 + (\Gamma_2' + \Gamma_4')^2}$ for odd parity. The switch configuration of the constriction junctions in the NOT gate changes after the operation: $\lambda_1 = \lambda_3 = 1$ and $\lambda_2 = \lambda_4 = 0$ at the initial stage, whereas the ground-state energy $E_g = -(\Gamma_1' + \Gamma_3')$ at the initial stage is achieved in the even-parity subspace, whereas $E_g = -(\Gamma_2 + \Gamma_4)$ at the final stage in the odd-parity subspace. Therefore, the electronic parity of the NOT gate is reversed after the operation of switching. It is easy to see that this parity reversal is realized because $\Gamma_1 \Gamma_2 \Gamma_3 \Gamma_4 > 0$.

![FIG. 2: (Color on line) Time evolution of the MF wave function transported through the NOT gate in terms of projections $O(t)$ to its initial wave function. The time for one switching step is $T = 4 \times 10^4 \hbar b_0$. The two insets schematically show the initial and final stages, where the solid (dotted) lines denote the on, $\lambda = 1$ (off, $\lambda = 0$) state of the constriction junctions between TSs.](image)

![FIG. 3: (a) Topological single electron pumping realized by the NOT gate and QD. A gate voltage (yellow cylinder) is used to tune the coupling between the QD and one TS of the NOT gate. (b) Working mechanism of single electron pumping (see text). The two lowest energy levels (red and blue) associated with opposite parities of the NOT gate cross each other upon the switching operation, with $\Delta E$ energy at the final stage. When $\Delta E > \epsilon_0$, the occupation energy of QD, an electron is emitted from the NOT gate to the QD to lower the energy of whole system.](image)
energy level $\epsilon_0$ in the initial stage (see Fig. 4(b)). The ground state of the total system exhibits even parity, since the NOT gate is in a state of even parity. From the parity conservation of the total system, there are two candidates for the ground state at the final stage after switching operation discussed above: vacant QD with total energy $-|\Gamma_2 - \Gamma_4|$, and/or occupied QD with total energy $-(\Gamma_2 + \Gamma_4) + \epsilon_0$, both exhibit even parity. One electron on the NOT gate is transferred to the QD upon the switching operation if $-(\Gamma_2 + \Gamma_4) + \epsilon_0 < -|\Gamma_2 - \Gamma_4|$, namely $\epsilon_0 < 2\min\{\Gamma_2, \Gamma_4\}$. Physically the inequality means that if the occupation energy of the QD is too large, electron will not jump to QD. In the same way, one can figure out that when QD is occupied at the initial stage, an electron is transferred to the NOT gate upon switching operation if $-(\Gamma_2 + \Gamma_4) < -|\Gamma_2 - \Gamma_4| + \epsilon_0$, namely $\epsilon_0 > -2\min\{\Gamma_2, \Gamma_4\}$. The physical meaning of the condition is also clear.

In order to perform single electron pumping starting from a switch configuration opposite to the one discussed above, one has two similar conditions on $\Gamma_1$ and $\Gamma_3$. Therefore, the necessary and sufficient condition for single electron pumping is $|\epsilon_0| < 2\min\{\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4\}$.

V. QUANTUM INTERFERENCE OF MFS

The parity flipping in the NOT gate discussed above can be understood generally as a quantum interference of MFs. We consider a loop of $2N$ TSs with a vortex at the center of the common superconducting substrate. At the initial state, the odd/even constriction junctions are on/off, with no edge MF in the system. First, we turn off $\lambda_1$, which produces two edge MFs $\hat{\gamma}_1$ and $\hat{\gamma}_2$ on TS(1) and TS(2) respectively. By turning on $\lambda_2$ and $\lambda_{2N}$, and then turning off $\lambda_3$ and $\lambda_{2N-1}$, and so on so forth, the MF $\hat{\gamma}_2$ and $\hat{\gamma}_1$ are transported to TS(2l) and TS(2l+1) respectively. Finally, we turn on $\lambda_{2l}$, which annihilates the two MFs.

After the above sequence of switchings, the odd/even constriction junctions become off/on, opposite to the initial configuration. Let us check the ground-state electronic parity for the two configurations, keeping in mind that all the coupling constants $\Gamma_j$ are positive when switched on as the topological property revealed explicitly for the NOT gate with four TSs. The ground state $|\tilde{G}\rangle$ at the initial configuration is a fully occupied state by electrons defined by $d_j^\dag = (\gamma_{2j-1} + i\gamma_{2j})/2$ for $1 \leq j \leq N$. The electronic parity of the whole system $\tilde{P} = (-1)^{\sum_{j=1}^{N}(d_j^\dag d_j)} = \prod_{j=1}^{N}(\gamma_{2j}^2 + \gamma_{2j+1}^2)$, has the eigen value $(-1)^N$.

For the final configuration, the ground state $|\tilde{G}\rangle$ is a fully occupied state by electrons defined alternatively by $d_j^\dag = (\gamma_{2j} + i\gamma_{2j+1})/2$ with $\gamma_{2N+1} \equiv \gamma_1$. Its electronic parity can be calculated as

$$\tilde{P}|\tilde{G}\rangle = -\prod_{j=1}^{N}(\gamma_{2j}^2 + \gamma_{2j+1}^2)|\tilde{G}\rangle = (-1)^{N+1}|\tilde{G}\rangle,$$

which is opposite to the one of the initial configuration.

VI. DISCUSSIONS

The mechanism of single electron pumping in the present proposal is different from previous ones in literature [6–8]. Unlike the case where electrons are driven by biased potentials between QD and electrodes [6], we switch the coupling between QD and the charge reservoir, i.e. the NOT gate, instead of the bias. Since the present single electron pumping is independent of the details of the switching process as far as it is slow enough, it is also different from the previous adiabatic pumping schemes where the details of pumping parameters during the operations have a significant effect on the direc-
tion and magnitude of the pumping current. In this sense, the present scheme can be called topological charge pumping.

In our schemes, the manipulation of edge MFs is topologically protected by edge excitation gap which sets the limitation of the working temperature. For a sample size of $150 \times 150 \text{nm}^2$, the edge excitation gap is estimated as $\sim 0.01 \Delta_0$ with $\Delta_0$ the superconduction pair potential. For $\Delta_0 = 1 \text{meV}$, the gap is $\sim 100 \text{mK}$, which is not hard to achieve experimentally in these days. It is worthy to note that there is a work by Akhmerov showing that even the occupation of higher in-gap excitations will not violate the topological protection.

VII. SUMMARY

In conclusion, we reveal that the mobility of Majorana fermion at edge of nano topological superconductor induced by a vortex can be used to implement a quantum NOT gate for Majorana qubit. The working mechanism for the NOT gate can be understood as the Aharonov-Bohm interference of two Majorana fermions. Based on this phenomenon, we formulate a scheme for topological single electron pumping. Useful applications of these devices in quantum transport and quantum computation are expected.

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