Introduction to open string field theory

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Abstract. We review basic concepts of open string theory. Then we sketch briefly some aspects of string field theory and comment some applications.

1. Introduction
The quantum mechanics and special relativity are the two most fundamental frameworks in high energy physics today. When these two achievements are combined we get the quantum field theory. Using this theory and the concept of gauge symmetry we can build the standard model describing successfully all observed elementary particles and fields. However such elegant synthesis does not include the gravity. In principle, we have good reasons to expect that the higher symmetry we include in our model the more unified description we get.

If we could discover this more unified description we could obtain a complete and deep understanding of details of our universe including quantum gravity. One key step toward such unifying theory is to replace our idea of point particles by one dimensional strings. The purpose of this article is to give an introduction to some elementary concepts of string field theory.

2. Open String
A string moving in a flat Minkowski $D = 26$ dimensional space-time is described by its space-time coordinates $X^\mu(\tau, \sigma)$, where $\tau, \sigma$ are the world sheet time-like and spatial-like parameters of the string, and $\eta_{\mu\nu}$ is the space-time Minkowski metric. A good textbook to learn more about string theory can be found in [1].

2.1. Polyakov Action
The dynamic of a relativistic string is determined by the Nambu-Goto Action

$$S_{NG}[X] = - \int d\tau d\sigma \sqrt{- \left| \frac{\partial_\tau X^\mu}{\partial_\tau} \frac{\partial_\sigma X^\mu}{\partial_\sigma} \right|}.$$  \hspace{1cm} (1)

This action represent the geometrical area of the worldsheet swept by the evolution of the open string, and it is invariant by reparametrizations of the worldsheet. Since the Hamiltonian of this action is nonlinear the quantization is hard, physicists prefer to use an equivalent action proposed by Polyakov

$$S_P[X, g] = \int d\tau d\sigma \sqrt{g^{\alpha\beta}} \partial_\alpha X^\mu \partial_\beta X^\mu,$$  \hspace{1cm} (2)

were $g^{\alpha\beta}$ is the intrinsic metric.
2.2. Symmetries
The Polyakov action has space-time Poincare symmetry, two dimensional worldsheet reparametrization symmetry, and Weyl invariance. Using the last two symmetries we can gauge fix so that \( g_{\alpha\beta} = h_{\alpha\beta} = \text{diag}(-1,1) \) and obtain the action
\[
S[X] = \int d^2z \partial X^\mu \partial X_\mu
\]  
This action has infinite dimensional two dimensional conformal invariance. This means that from the point of view of the world-sheet the space-time coordinates \( X^\mu \) are free fields of a conformal field theory. This theory has infinite number of constraints associated to the vanishing of the world-sheet energy-momentum tensor \( T = \partial X^\mu \partial X_\mu \). The equation of motion is \( \partial \partial X = 0 \), and it solution is \( X(z, \bar{z}) = f(z) + g(\bar{z}) \). The boundary conditions for each freely moving endpoints are Neumann-like.

2.3. Quantization
Since the action is quadratic the quantization is simple, and we can use the radial quantization method. The spectrum is discrete with energy eigenvalues \(-1, 0, 1, 2, \ldots\). All string states are of the form \( |\Psi\rangle = \alpha_{i_1}^{\mu_1} \alpha_{i_2}^{\mu_2} \cdots \alpha_{-n_i}^{\mu_i} |0\rangle \), where \( \alpha_{i,n}^{\mu} \) are creation operators acting on the \( SL(2, R) \) vacuum \( |0\rangle \). Each elementary particles is identified with an irreducible representation of the Poincare group. In covariant quantization the string physical states must satisfy the Virasoro constraints \( (L_n - \delta_{n,0}) |\Psi\rangle = 0 \), \( n \geq 0 \).

We consider the BRST quantization because very convenient for strings. This method requires to include an anti-commuting \((b, c)\) Fadeev-Popov ghosts.
\[
S[X, b, c] = \int d^2z (\partial X^\mu \partial X_\mu + b \partial c + b \partial \bar{c})
\]  
Physical states are of the form
\[
|\Psi\rangle = \alpha_{-i_1}^{\mu_1} \alpha_{-i_2}^{\mu_2} \cdots \alpha_{-n_i}^{\mu_i} \otimes b_{-m_1} b_{-m_2} \cdots b_{-m_j} \otimes c_{-p_1} c_{-p_2} \cdots c_{-p_k} |0\rangle
\]
\( \mu_i = 0, 1, 2, \ldots; D - 1 \), \( n_i \geq 0 \), \( m_j \geq 2 \), \( p_k \geq -1 \).
The BRST operator \( Q = \int dz c(T + b \partial c) \) is anticommuting and nilpotent \( Q^2 = 0 \) only for \( D = 26 \). This operator is used to select the physical states. A physical state \( |\Psi\rangle \) must be annihilated by \( Q \) and must not be BRST trivial. This means \( Q |\Psi\rangle = 0 \) and \( |\Psi\rangle \neq |\lambda\rangle \). In other words, all physical states \( V \) belongs to the \( Q \) cohomology.

Corresponding to each physical state of the string we have a vertex operator. These vertex operators are used to compute scattering amplitudes. In general there is a standard perturbative prescription to compute this scattering amplitude involves a path integral over all configurations of the worldsheet sum over all world sheet topologies, integration over a moduli space and products of vertex operators. The scattering on-sheet tree level amplitude for four open strings in tachyon states was found by Veneziano. The generalization of this amplitude for \( n\)-point open strings in tachyon states was found by Koba-Nielsen.

3. Open String Field
To understand the behavior of strings in non flat space-time backgrounds, to learn about compactification mechanisms, and symmetry breaking etc. we need to go beyond the first quantized perturbative approach. To learn about all these non perturbative phenomena in string theory, we need a second quantized approach.
3.1. Witten String Field
The vacuum of a string field is the BPZ vacuum $|\Omega\rangle$ defined by $|\Omega\rangle = c_1|0\rangle$, where $|0\rangle$ is the $SL(2, R)$ vacuum, and satisfy $a_n|\Omega\rangle = 0$, $b_n|\Omega\rangle = 0$, $c_n|\Omega\rangle = 0$, $\forall n \geq 1$, and $b_0|\Omega\rangle = 0$, $\langle \Omega|c_0|\Omega\rangle = 1$.

A string field is a functional $A[X, b, c]$ with ghost number $n_{gh} = 1$. Expanding the first lower levels we have

$$A[X(\sigma), b(\sigma), c(\sigma)] = [T(x) + A_\mu(x)\alpha^{\mu}_{-1} + B_\mu(x)\alpha^{\mu}_{-2} + C_{\mu\nu}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1} + D(x)b_{-1}c_{-1} + \cdots |\Omega\rangle + [U(x)b_{-1} + \cdots] c_0|\Omega\rangle$$

As we can see the string field contains infinite number of space-time fields of arbitrary high integer spin as coefficients. $T(x)$ is the tachyon scalar field, $A_\mu$ is the massless Yang-Mills vector potential field, $B_\mu(x)$ is a massive vector field, $C_{\mu\nu}(x)$ is a massive tensor field, $D(x)$ and $U(x)$ are auxiliary scalar fields.

The free string field action is built using the BRST $Q$ operator

$$S[A] = \frac{1}{2}\int d^{26}x(A|QA)$$

For simplicity, we omit in this action action a trace over the product of the Chan-Paton matrices associated to each open string corresponding to the gauge group $SO(32)$. This action is invariant by the gauge transformation $\delta A = QA$, and its equation of motion is $QA = 0$. In the Siegel’s gauge $b_0A = 0$ this equation of motion is equivalent to $(L_0 - 1)A = 0$. Evaluating the correlation functions in the free string action we obtain

$$S[T, A, U, \ldots] = \int d^{26}x \left[ \frac{1}{2}T(\tau + 2)T + \frac{1}{2}A_\mu A^\mu + U\partial_\mu A^\mu + U^2 + \cdots \right]$$

The gauge parameter $\Lambda$ has $n_g = 0$

$$\Lambda = [\lambda(x)b_{-1} + \nu_\mu(x)\alpha^\mu b_{-1} + \cdots |\Omega\rangle + [\omega(x)b_{-1}b_{-2} + \cdots] c_0|\Omega\rangle$$

in components the gauge transformations are $\delta T = 0$, $\delta A_\mu = \partial_\lambda$, $\delta U = \frac{1}{2}\lambda$.

In 1986 Witten generalized this action to describe interacting strings [2]

$$S[A] = \int d^{26}x \left[ \frac{1}{2}\langle A|QA \rangle + \frac{1}{3}\langle A|AA \rangle \right]$$

(5)

gauge invariant under $\delta A = QA + AA - \Lambda A$. This action is a generalization of the Chern-Simons action $S_{CS} = Tr \int d^3x e^{\mu
u\rho} \left( \frac{1}{2}A_\mu \partial_\nu A_\rho + \frac{1}{4}A_\mu A_\nu A_\rho \right)$ which is gauge invariant by $\delta A_\mu = \partial_\mu + [A_\mu, \lambda]$.

4. Applications
In a remarkable work Giddings reproduced the Veneziano amplitude using the Witten string field action [3]. He used conformal Riemann-Christoffel transformations to map the folded Witten vertex into the complex upper half plane and compute the required correlation functions. This work motivated to develop a systematic reformulation of string field theory using conformal field theory by Leclair [4]. An excellent review of conformal field theory can be found in a lecture of Ginsparg [5].

In practical computations in string field theory, all the terms in the action are expressed as n-point correlation functions with a sequence of conformal maps from the upper half plate to wedges

$$\langle A_1, \ldots A_n \rangle = \langle f^{(n)}_1 \circ A_1(0) \cdots f^{(n)}_n \circ A_n(0) \rangle,$$
\[ f_k^{(n)} = e^{\frac{2\pi i (k-1)}{n}} \left( \frac{1 + iz}{1 - iz} \right)^{2/n}, \quad n \geq 1. \]

More recently, this conformal technique helped to compute string field corrections to Yang-Mills and tachyon condensation. Finally, the quantization of the Witten string field action was completed using Batalin-Vilkovisky methods. For all these successful results we believe that string field is a very useful and interesting research area.

5. Conclusion

We explained very briefly some tools, concepts and principles used in open string and open string field theory. A lot of progress has been done using the first quantized approach of strings moving in flat space-time Minkowski background using perturbative methods. New and different kinds of symmetries have been discovered in string theory. On the other hand, open string field theory had a little slow progress in applications, but recently it has established as a solid research area.

However open string field theory has the following three problems: It contains a tachyon, it includes only bosonic string states, so it does not consider fermions. Furthermore, since the critical space-time dimension 26 is too high from a phenomenological perspective. In a related paper of this proceeding of the school we will discuss how the incorporation of supersymmetry strongly restrict the string theory in such a way that solves many of these problems.

Acknowledgments

This work had financial support from the Research Institute of the Faculty of Sciences of the National University of Engineering at Lima-Peru.

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