The Effect of External Forced Vibrations on the Pressure Distribution of Short Journal Bearings

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Abstract. The pressure distribution of a short journal bearing in dynamic loading conditions under the effect of forced harmonic excitation was investigated. The conventional form of the Reynolds equation was analysed numerically using the central finite difference technique with appropriate initial and boundary conditions. These numerical equations were written in FORTRAN-95 to obtain relevant results. According to the results, maximum oil pressure is obtained when external forced harmonic vibration is increased, giving a 51.92 percent increase over that obtained under self-excitation only.

1. Introduction

Journal bearings have self-excited instability, which is generally called the self-excited oil whirl. The reason for this instability is imbalance-induced vibration, caused by unbalanced mass, which is known as the self-excited vibration. This is a significant factor restricting the performance and fatigue life of all rotating systems [1, 2]. In turbo-machinery, instabilities are described by the turning of the rotor bearing systems at frequencies other than the rotating frequency of the shaft. Large amplitudes of sub-synchronous vibration can show up under certain working conditions and can lead to high amplitudes and destructive vibrations [3].

External vibrations, which are another source of instability, can seriously threaten the safety and working performance of rotating machinery. The risk severity is generally dependent upon the response level of the inner machine parts with respect to the level of the externally-applied vibrations. Reddy and Srinivas [4] studied the effect of base excitation frequencies and amplitudes on the dynamics of rotors with hydrodynamic journal bearings. Wirsing [5] studied the effect of axially oscillating journal bearings on stiffness and damping coefficients. Avramov and Borysiuk, [6] suggested a general approach to analyse the vibration of a single disk rotor with journal bearings of arbitrary length.

To the authors’ knowledge there is no extant research dealing with the effects of external harmonic excitation on the pressure distribution of journal bearings experimentally, analytically, or numerically. In this work, the pressure distribution of short journal bearings under the effect of external harmonic excitation was thus determined. This paper also includes the effects of the amplitude of external force and the frequency and damping ratios on the pressure distribution in this scenario.

2. Geometry and coordinates of journal bearings

The geometry of a journal bearing as shown in figure 1 was adopted.
Figure 1. Geometry and coordinates of journal bearing

This type of bearing consists of a circular sleeve (bearing or bushing) wrapped around a rotating shaft with a diameter larger than that of the journal (shaft); these are separated by a thin layer of lubricant.

In this diagram,
- $o_b$ - bearing centre
- $o_j$ - journal center
- $c$ - bearing radial clearance,
- $e$ - eccentricity of journal
- $R, R_1$ - the radius of journal and bearing respectively
- $h_{min}$ - the minimum film thickness at $F = c - e$
- $h_{max}$ - the maximum film thickness at $G = c + e$
- $\varepsilon$ - eccentricity ratio = $e/c$
- $w$ - the load
- $\theta$ - angular coordinate measured from the line of the centres (the circumferential direction)
- $\phi$ - attitude angle

The global coordinates system used is assumed to be Cartesian (x, y, z), with the original point fixed at the bearing bush centre. The x-coordinates are in the circumferential direction, y-coordinates match the line of centre, across the oil film thickness, and the z-coordinates are in the axial direction, across the length of the bearing perpendicular to the plane (x, y).

3. Reynold's equation

When using the short journal bearing theory based on the Ocivirk approximation [7], the Reynolds equation for laminar flow in an isoviscous incompressible fluid is

$$\frac{\partial}{\partial x} \left( h^3 \frac{\partial p}{\partial x} \right) = 6 \mu u \frac{\partial h}{\partial x} + 12 \mu \frac{\partial h}{\partial \varepsilon}$$

4. Response to external harmonic excitation

Under the effects of external harmonic excitation, the oil film thickness is composed of two parts, giving $h$ as

$$h = h_\nu(\theta, t) = h_1 + h_2$$

where $h_1$ is the oil film thickness in a steady state without the effects of vibration [8]:

$$h_1(\theta) = c(1 + \varepsilon \cos \theta)$$
When the journal bearing is subjected to external vibration such a harmonic excitation, the response will take the form of the particular integral. The particular integral represents the response of the system to forced vibration, given as

\[ h_2(t) = H_2 \sin(\omega_e t - \psi) \]  \hfill (4)

where \( \omega_e, H_2, \) and \( \psi \) are the excitation frequency, amplitude of oscillation, and the phase angle of the displacement with respect to the exciting force, respectively.

\[ H_2 = \frac{F_0}{K \left(1 - \left(\frac{\omega_e}{\omega_n}\right)^2\right) + \left(2\xi \frac{\omega_e}{\omega_n}\right)^2} \]  \hfill (5)

\[ \psi = \tan^{-1}\left(\frac{2\xi \omega_e}{1 - \left(\frac{\omega_e}{\omega_n}\right)^2}\right) \]  \hfill (6)

where

- \( F_0 \) is the amplitude of the vibration force,
- \( \frac{\omega_e}{\omega_n} \) is the frequency ratio, and
- \( \xi \) is the damping ratio.

The natural frequency of the system, \( \omega_n \), and the stiffness, \( K \), can be found by considering the journal bearing as a simply supported beam of mass \( M \) and length \( L \), and using the double integration method [9]:

\[ \omega_n = \left(\frac{48EI}{ML^3}\right)^{1/2} \]  \hfill (7)

\[ K = \frac{48EI}{L^3} \]  \hfill (8)

where

- \( I \) is the second moment of area, \( I = \frac{\pi d^4}{64} \),
- \( d \) is the diameter of the shaft, and
- \( E \) is the modulus of elasticity of the shaft.

Equation (2), oil film thickness, in the case of harmonic excitation is

\[ h_v(\theta, t) = c(1 + \varepsilon \cos \theta) + H_2 \sin(\omega_e t - \psi) \]  \hfill (9)

5. Numerical analysis

The non-dimensional form of Reynold's equation (1) with constant viscosity is

\[ \frac{\bar{h}^3}{L} \frac{R}{R^2} \frac{\partial}{\partial \bar{z}} \left( \frac{\partial \bar{P}}{\partial \bar{z}} \right) = \frac{\partial \bar{h}}{\partial \theta} + 2 \frac{\partial \bar{h}}{\partial T} \]  \hfill (10)

where

\[ \bar{P} = \frac{P}{6\mu \omega} \left( \frac{c}{R} \right)^2 \, \bar{z} = \frac{z}{L}, \, \theta = \frac{x}{R}, \, T = \frac{tu}{R}, \, \bar{h} = \frac{h_v}{c} \]  \hfill (11)

and

\[ \bar{h} = \frac{h_v(\theta, t)}{c} = 1 + \varepsilon \cos \theta + \frac{H_2}{c} \sin(\omega_e t - \psi) \]  \hfill (12)

The governing dimensionless differential equation described above can be solved by using finite difference quotients [10] to obtain the pressure distribution in the oil film.

A grid size was chosen, of \( n \) in the circumferential direction, \( \theta \) and \( m \) in the axial direction, \( z \), as shown in figure 2. The central technique of the finite difference method was adopted.
The mesh size in the circumferential direction ($\Delta x$) and along the bearing length($\Delta z$) was defined as follows:

$$\Delta x = \frac{2\pi R}{n - 1}$$  \hspace{1cm} (13)

$$\Delta z = \frac{L - 1}{m - 1}$$  \hspace{1cm} (14)

which can be expressed in a dimensionless group as

$$\Delta \theta = \frac{\Delta x}{R}$$  \hspace{1cm} (15)

$$\Delta \bar{z} = \frac{\Delta z}{L}$$  \hspace{1cm} (16)

For any point in the oil film mesh $(i, j)$, equation (10) can be written as follows:

$$\bar{h}^3_{i,j} \left( \frac{R^2}{L} \right)^2 \left( \bar{P}_{i+1,j} - 2\bar{P}_{i,j} + \bar{P}_{i-1,j} \right) = \frac{h_{i+1,j} - h_{i-1,j}}{2\Delta \bar{x}} + 2 \left( \frac{h_{i+1,j}^n - h_{i,j}^n}{\Delta T} \right)$$  \hspace{1cm} (17)

$$(\bar{P}_{i+1,j} - 2\bar{P}_{i,j} + \bar{P}_{i-1,j}) = \left[ \left( \frac{h_{i+1,j} - h_{i-1,j}}{2\Delta \bar{x}} \right) + 2 \left( \frac{h_{i+1,j}^n - h_{i,j}^n}{\Delta T} \right) \right] \left( \frac{(L/R)^2 \Delta \bar{z}^2}{h^3_{i,j}} \right)$$  \hspace{1cm} (18)

Rearranging equation (18) for $\bar{P}_{i,j}$ gives

$$\bar{P}_{i,j} = \frac{\bar{P}_{i+1,j} + \bar{P}_{i-1,j}}{2} - \left[ \left( \frac{h_{i+1,j} - h_{i-1,j}}{2\Delta \bar{x}} \right) + 2 \left( \frac{h_{i+1,j}^n - h_{i,j}^n}{\Delta T} \right) \right] \left( \frac{(L/R)^2 \Delta \bar{z}^2}{h^3_{i,j}} \right)$$  \hspace{1cm} (19)

The above equation gives the dimensionless oil film pressure in the circumference and axial directions.

### 6. Results and discussion

Figure 3 shows the oil film pressure distribution in the circumferential direction for different values of the amplitude of force causing the system to vibrate. Increase of the amplitude of the force increases
the oil film pressure. This is due to the fact that increasing the external force on the bearing system causes the gap between the bearing and journal to be decreased, causing an increase in oil film pressure.

Figure 3. Oil film pressure at different values of amplitude of forced vibration

Figure 4 shows the maximum oil film pressure during one second for different values of damping ratio at the same value of amplitude of vibration force, here equal to 20 N. It can be observed that an increase in the damping ratio causes a reduction in the oil film pressure. This is due to the fact that the damping dissipates the energy of vibration and then decreases the amplitude of vibration response.

Figure 4. Maximum oil film pressure versus time for different values of damping ratio.
Figure 5 shows the minimum oil film thickness versus the damping factor for different values of $\omega_e/\omega_n$ under the effect of external forced harmonic vibrations. It can be observed that the minimum oil film thickness increases with increases in frequency ratio, leading to decreases in the oil film pressure. It can also be observed that for higher frequency ratios, increasing the damping ratio has a greatly reduced effect on the minimum film thickness.

![Figure 5. Minimum film thickness verse damping ratio for different values of $\omega_e/\omega_n$.](image)

Figure 6 shows the distribution of oil film pressure along the circumferential direction for different ratios of $\omega_e/\omega_n$. It can be seen that the higher the frequency ratio, the lower the resultant oil film pressure. This can be attributed to the fact that the increasing the frequency ratio leads to increasing the minimum oil film thickness, causing the oil film pressure to be decreased.

![Figure 6. Oil film pressure distribution for different values of $\omega_e/\omega_n$.](image)
Figure 7 shows the oil film pressure distribution under the effect of external forced harmonic excitation in three dimensions for \( \frac{\omega}{\omega_n} = 0.5 \) and \( \xi = 0.5 \). The maximum oil film pressure obtained is 19.601 MPa at \( \theta = 174 \) deg.

![Figure 7. Oil film pressure distribution along the circumferential direction of a journal bearing in three dimensions](image)

7. Conclusions

In this work, the effect of external forced vibration on pressure distribution, including the effect of the amplitude of external force, frequency, and damping ratios, was investigated. The oil film pressure increases as the amplitude of vibration force increases, decreasing for large values of damping ratios up to 0.7. For \( \left( \frac{\omega}{\omega_n} \right) > 1 \), the oil film pressure decreases, increasing for \( \left( \frac{\omega}{\omega_n} \right) < 1 \).

Under the effect of external harmonic forced vibration only, the maximum oil film pressure was higher than seen in the self-excited case, by 51.92 % for the same specifications of both journal bearing and oil.

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