On the Optimal Speed Profile for Electric Vehicles

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\textbf{ABSTRACT} The main question in eco-driving is – what speed or torque profile should the vehicle follow to minimize its energy consumption over a certain distance within a desired trip time? Various techniques to obtain globally optimal energy-efficient driving profiles have been proposed in the literature, involving optimization algorithms such as dynamic programming (DP) or sequential quadratic programming. However, these methods are difficult to implement on real vehicles due to their significant computational requirements and the need for precise a-priori knowledge of the scenario ahead. Although many predictions state that electric vehicles (EVs) represent the future of mobility, the literature lacks a realistic analysis of optimal driving profiles for EVs. This paper attempts to address the gap by providing optimal solutions obtained from DP for a variety of trip times, which are compared with simple intuitive speed profiles. For a case study EV, the results show that the DP solutions involve forms of Pulse-and-Glide (PnG) at high frequency. Hence, detailed investigations are performed to: i) prove the optimality conditions of PnG for EVs; ii) show its practical use, based on realistic electric powertrain efficiency maps; iii) propose rules for lower frequency PnG operation; and iv) use PnG to track generic speed profiles.

\textbf{INDEX TERMS} Dynamic programming, eco-driving, electric vehicles, speed profile, optimization, pulse-and-gliding.

\section{I. INTRODUCTION}

How can we minimize the energy consumption of a vehicle driven from A to B with a time constraint? This is the fundamental question of eco-driving, which consists of driving techniques and strategies to minimize energy consumption. The benefits are range extension, which is especially important for electric vehicles (EVs), and emission reduction at the tailpipe and/or at the power station.

The literature discusses many eco-driving strategies, ranging from vehicle speed [1] and gearshift [2] advices, with the latter being already implemented on production passenger cars, to algorithms based on the solution of complex optimization problems [3]. These strategies are applied to internal combustion engine (ICE) vehicles, hybrid electric vehicles (HEVs), and EVs [3].

Eco-driving algorithms commonly calculate an optimal speed profile for the vehicle or driver to follow. Dynamic programming (DP) is often used as an offline tool to this purpose, e.g., see its application to an HEV in [4]. Given its significant computational requirements, DP is not adopted for an online update of the optimal speed profile. Iterative DP is used on an EV in [5], to reduce computational effort and enhance convergence with respect to DP. Ding and Jin [6] apply DP to the offline computation of the optimal speed profile on curved roads, with focus on ICE vehicles.

An alternative technique with lower computational effort is sequential quadratic programming (SQP), which is discussed and applied to EVs and HEVs in [7]. In [8], a hierarchical control strategy is proposed for ICE vehicles, splitting the speed profile into acceleration, constant speed, and braking phases. To reduce the computational effort, the SQP optimization is performed separately for each phase. Lin \textit{et al.} [9] use the Legendre pseudospectral method, and consider the location of traffic lights for an ICE vehicle implementation. Subsequently, quasi-optimal rules are derived for the separate acceleration, constant speed, and braking phases. Pontryagin’s maximum principle is applied to an ICE vehicle in [10], which uses vehicle model linearizations to calculate the optimal profiles.
Learning algorithms have been developed for vehicles traveling on fixed routes; for example, Kim et al. [11] use a learning model predictive controller (MPC) to iteratively improve the fuel economy of an ICE vehicle. Another technique suitable for fixed routes is stochastic DP, which accounts for uncertainties in historical speed data, and generates a look-up table for future use, as presented for an HEV in [12].

Eco-driving is often applied to adaptive cruise control (ACC). In [13] an energy-efficient ACC is proposed for an EV, which uses route knowledge and DP to calculate an optimal speed profile at the beginning of the trip, while an MPC adapts the behavior to the local situation, e.g., to enforce safety distances. Similarly, Lim et al. [14] propose a two-stage hierarchical system for an ICE vehicle, where quadratic programming (QP) is used for both long-term optimization and local adaptation to traffic conditions, with results and computation time compared to those of SQP. In [15] the same authors apply an estimation of distribution algorithm to an HEV. Han et al. [16] propose a single-stage ACC with a local adaptation MPC for connected and automated EVs. Madhusudhanan [17] presents an ACC with an MPC that reduces unnecessary braking when approaching traffic lights on EVs, while Asadi and Vahidi [18] describe a similar algorithm for ICE vehicles. However, neither [17] nor [18] account for powertrain efficiency in the optimal control problem, while [16] uses a simplified powertrain efficiency characteristic.

Studies also separately analyze optimal driving behaviors during acceleration, constant speed, and braking phases. In [19] an analytical algorithm and DP are used to calculate the optimal acceleration profile of an EV, while Vaz et al. [20] use a genetic algorithm. However, as the acceleration maneuvers are considered completed when the desired speed is reached, the total traveled distance is different for each acceleration profile. Although the results are relevant, for optimal speed profiling the comparison should be made for a fixed trip distance.

Simulation results to find an optimal constant speed are provided by Tesla in [1]; to the same purpose, dynamometer and driving tests have been conducted in [21] and [22]. The studies in [23]–[25] derive an optimal driving technique for an ICE vehicle, i.e., the so-called pulse-and-glide (PnG), based on repeated acceleration at constant torque (pulse) and freewheeling (glide or coast) phases to maintain a constant average speed, while making the powertrain operate at its maximum efficiency in the acceleration phases. A graphical approach using the shape of the brake specific fuel consumption characteristics of an ICE shows that, under certain conditions, PnG is more efficient than cruising at constant speed. Also, these studies discuss the effect of the PnG duty cycle and frequency. In [26], the same authors extend the technique to HEVs. PnG has also been experimentally tested by Lee et al. [27] with an HEV on a dynamometer.

For the braking phase, Koehler et al. [28] explain that freewheeling to a stop is the optimal braking method, but if the required stopping distance is short, an optimization problem should be solved.

Based on these techniques, several studies have proposed advanced driver-assistance systems (ADAS) that facilitate eco-driving. For example, the solutions in [28]–[30] (with [30] implemented on production cars) provide messages via an on-board screen suggesting when to release the accelerator pedal and start freewheeling. In [31] a simple rule-based eco-driving controller limits the EV battery power, and includes an eco-indicator changing color according to the driver’s behavior.

In summary, most available studies implement optimization algorithms to calculate energy-efficient speed profiles. However, the literature lacks a realistic analysis focused on EVs. Han et al. [32] and Sciarretta and Vahidi [33] use Pontryagin’s maximum principle to determine the optimal speed profile for ICE vehicles and EVs. In the EV analysis, the electric powertrain power loss characteristics are simplified, i.e., powertrain efficiency is considered a monotonically decreasing function of torque. As a result, it is beneficial to operate the powertrain at as low as possible torque, and PnG is not useful. However, the hypothesis on the shape of the electric powertrain efficiency map may not be realistic in general.

This simulation study focuses on EVs, and attempts to address the identified gaps with the following contributions:

- An analysis of speed profiles consisting of a sequence of constant acceleration command, constant speed, and constant regenerative braking input, which are intuitive and easy for a human driver to track. The driving technique of these scenarios is referred to as constant pedal position technique (CPPT). For given average EV speeds during the trip, the resulting profiles with minimum energy consumption, i.e., using optimal CPPT (Opt-CPPT), are compared with DP results, where, interestingly, the latter show forms of high frequency PnG.
- Analytical proof that PnG is optimal for EVs under certain conditions, including mathematical and graphical rules to determine when and how PnG should be used.
- Discussion of practical PnG implementation aspects, including: i) evaluation of PnG applicability based on realistic EV powertrain efficiency maps; ii) sensitivity analysis of PnG frequency on EV energy consumption; iii) PnG algorithm to track generic speed profiles; and iv) energy consumption comparisons with the CPPT, Opt-CPPT and DP cases.

II. SPEED PROFILES WITH CONSTANT PEDAL POSITION TECHNIQUE

A. CASE STUDY VEHICLE

The case study EV is a prototype L7e [34] quadricycle evaluated during the H2020 European Union’s funded project STEVE [35]. The vehicle is rear-wheel drive with two in-wheel direct drive electric machines by Elapho. Table 1 shows the main EV parameters, while Figure 1 reports the combined motor and inverter efficiency map provided by
the powertrain supplier, based on an experimentally validated simulation model. The vehicle has a passive hydraulic braking system. Regenerative braking is regulated through a map based on the accelerator pedal position in the first part of the stroke, vehicle speed, and battery state of charge. The simulation results of this study consider the braking torque values within each scenario simulation. Hence, the resulting driving pattern is referred to as constant EV speed.

To ensure a driving profile that is easy-to-follow for a human driver, the accelerator pedal position is kept constant during the acceleration and braking phases, which implies fixed ATR and BTR values within each scenario simulation.

The motor torque levels in traction and regeneration, $T_m$, are functions of ATR, BTR and the maximum/minimum motor torque values, $T_{m,max}$ and $T_{m,min}$, at the current motor speed, $\omega_m (k)$:

$$T_m (k) = \begin{cases} T_{m,max} (\omega_m (k)) \cdot ATR, & \text{in traction} \\ T_{m,min} (\omega_m (k)) \cdot BTR, & \text{in regeneration} \end{cases} \quad (1)$$

where $k$ is the distance step, and the notation ( ) in (1) and the remainder indicates a function. The corresponding total tractive/regenerative force, $F$, is:

$$F (k) = T_m (k) n_{mot} \frac{G_i}{r_{wh}} \quad (2)$$

The EV acceleration, $a$, is given by:

$$a (k) = \frac{F (k) - F_v (k)}{m + m_e} \quad (3)$$

where the motion resistance force, $F_v$, includes the effects of the aerodynamic drag, $F_{ae}$, and rolling resistance force, $F_{rr}$:

$$F_v (k) = F_{ae} (k) + F_{rr} \quad (4)$$

with:

$$F_{ae} (k) = 0.5 \rho C_d A_f v (k)^2 \quad (5)$$

$$F_{rr} = mg C_r$$

In (5), $\rho$ is the air density, $g$ is the gravitational acceleration, and $v$ is the vehicle speed. The following condition ensures absence of rear wheel spinning or locking [36]:

$$C_f F_{cf} (k) + F_{ae} (k) + [m + m_{cf}] a (k) \leq \mu F_{z,r} (k) \quad (6)$$

where $m_{cf}$ is the equivalent mass of the rotating components of the front axle, and $F_{cf}$ and $F_{z,r}$ are the vertical loads on
the front and rear axles, calculated as:

\[ F_{c,f}(k) = \frac{L_{CG,r}mg - h_{CG}[F_{ae}(k) + ma(k)]}{L} \]

\[ F_{c,r}(k) = \frac{[L - L_{CG,r}]mg + h_{CG}[F_{ae}(k) + ma(k)]}{L} \]  

(7)

In the acceleration and braking phases, the relationship between the EV velocities at adjacent distance steps is:

\[ v(k + 1) = \sqrt{v(k)^2 + 2a(k)\Delta s} \]  

(8)

where \( \Delta s = 0.1 \text{ m} \) is the distance discretization. Forward and backward approaches along the defined distance are used for emulating the acceleration and braking phases, and combining them with the constant speed phase.

The powertrain losses, when considered, are calculated from the efficiency map in Figure 1. Each powertrain efficiency, \( \eta_{pwt} \), is a function of motor torque and speed:

\[ \eta_{pwt}(k) = f(T_m(k), \omega_m(k)) = f\left(\frac{F(k) r_{wh}}{G_r}, v(k) \frac{G_r}{r_{wh}}\right) \]  

(9)

where (9) neglects the longitudinal tire slip. Hence, the required battery energy, \( E_{batt} \), per unit distance, \( d \), is calculated as:

\[ \frac{E_{batt}(k)}{d} = \begin{cases} \frac{F(k)}{\eta_{pwt}(k)}, & F(k) \geq 0 \\ \frac{F(k)}{\eta_{pwt}(k)}, & F(k) < 0 \end{cases} \]  

(10)

under the hypothesis of equal torque on the two rear wheels.

C. RESULTS

Figure 2 reports \( E_{batt}/d_d \) for scenario simulations with \( d_d = 2 \text{ km} \). The combinations of \( ATR \), \( v_{con} \) and \( BTR \) lead to different driving times, \( t_d \). Consequently, the horizontal axis is expressed in terms of average speed \( v_{avg} = d_d/t_d \). Since the simulations are run at high resolution, to ensure readability, the figure shows only a selection of the results. Each data point represents a trip, with corresponding \( ATR \), \( v_{con} \), and \( BTR \). The data points outlined with a blue diamond assume 100% powertrain efficiency, while those outlined with a green ring are calculated with the powertrain efficiency map. The shades of red on the data points indicate the \( ATR \) value, i.e., the lighter the shade, the higher the \( ATR \). The shades of yellow in the center of the data points indicate the \( v_{con} \) value, i.e., the lighter the shade, the higher the \( v_{con} \). As shown in the figure, an increase in \( BTR \) allows higher \( v_{avg} \). For example, if \( BTR \) is 0, freewheeling from 90 km/h to 5 km/h takes 1.2 km, which is a substantial section of \( d_d = 2 \text{ km} \) and leads to low \( v_{avg} \).

For the 100% powertrain efficiency case, the energy consumption increases with \( v_{avg} \) due to the increase in aerodynamic drag. The horizontal line near the bottom of the figure is the asymptote of minimum \( E_{batt}/d_d \) with 100% efficiency. This refers to an EV speed that tends to zero, condition in which the rolling resistance is the only contribution to energy loss. The sweet spot for minimum energy consumption with the actual powertrain efficiency map occurs for \( v_{avg} \approx 17 \text{ km/h} \). At higher speeds, the aerodynamic drag becomes significant, while at lower speeds the powertrain is inefficient.

Figure 3 isolates the data points for \( v_{con} = 55 \) and 56 km/h. The \( v_{con} = 55 \text{ km/h} \) data points have a yellow center, while the \( v_{con} = 56 \text{ km/h} \) data points have a black center.
From the full set of results, Figure 4 extracts the bottom envelopes of the data points at $d_d = 1$, 2, 5, and 10 km, for the cases with 100% powertrain efficiency (dashed lines) and the actual powertrain efficiency map (continuous lines). The envelopes represent the optimal result of the CPPT approach for each $v_{avg}$, and therefore they are referred to as Opt-CPPT.

With the realistic powertrain efficiency map, the regions with lower $v_{avg}$ than that of the sweet spot discussed in Figure 2 are discarded, since the vehicle would consume more energy while taking longer to complete the trip. Therefore, the left-most point for each $d_d$ corresponds to the sweet spot of minimum energy consumption, see Table 2. Interestingly, the inputs to minimize consumption are exactly the same for the four distances. In particular, $ATR = 0.036$ is the lowest possible $ATR$ value – among those of the brute force approach – to hit 19 km/h. The constant speed phase is at 19 km/h, which is the optimal speed of the vehicle at zero road gradient (after performing a similar analysis to that in [1]). Finally, the braking input is $BTR = 0$, which means freewheeling.

At 100% powertrain efficiency, $E_{batt}/d_d$ is a decreasing function of $d_d$ for any $v_{avg}$. In fact, to achieve a given $v_{avg}$, a longer distance requires a lower $v_{con}$, which implies reduced resistance losses. With the inclusion of the powertrain efficiency map, the trend remains the same at medium-to-high $v_{avg}$, but at low $v_{avg} (<25 \text{ km/h})$, $E_{batt}/d_d$ increases with $d_d$ (see the line crossings in the figure). In fact, for the case study EV, the powertrain operation at torque levels producing low acceleration is more efficient than at torque levels maintaining low constant speeds.

Figure 5 reports the driver inputs for the Opt-CPPT cases using the powertrain efficiency map for the different distances. For the 5 and 10 km scenarios, $ATR$ and $BTR$ are shown as clouds of data points marked with ‘x’ or ‘+’ and distributed over wide bands. Selecting $ATR$ or $BTR$ values anywhere in the band region for a specified $v_{avg}$ would lead to a consumption very close to the minimum. In fact, as distance increases, the duration and consumption of the acceleration and braking phases become less significant relative to the total trip. Hence, the 10 km scenario band is wider than the 5 km scenario band. In the $BTR$ plot, the vertical dashed lines represent the freewheeling limits for the respective distance. To the right of the dashed line, freewheeling is no longer possible while achieving the desired $v_{avg}$.

If there are no constraints on $v_{avg}$, for minimum consumption it is best to slowly accelerate to the optimal speed, maintain it, and then freewheel to a stop. In contrast, if time limits are present, the Opt-CPPT results prescribe an intensity of the brake regeneration action similar to that of the initial acceleration, which is evident from the maps of $ATR$ and $BTR$. 

**TABLE 2.** Vehicle operating conditions in the minimum energy consumption points (with consideration of powertrain efficiency map).

| $d_d$ [km] | $E_{batt}/d_d$ [Wh/km] | $ATR$ [-] | $BTR$ [-] | $v_{con}$ [km/h] | $v_{avg}$ [km/h] |
|-----------|-----------------|-----|-----|-------------|-------------|
| 1         | 26.8            | 0.036 | 0   | 19          | 15.2        |
| 2         | 28.1            | 0.036 | 0   | 19          | 16.9        |
| 5         | 28.9            | 0.036 | 0   | 19          | 18.1        |
| 10        | 29.2            | 0.036 | 0   | 19          | 18.5        |
The Opt-CPPT results can be used for the generation of rule-based algorithms for easy-to-follow energy-efficient speed profiling, by dividing a complex journey into elementary sections, each one managed through maps or expressions of the required \( v_{con} \), \( ATR \), and \( BTR \), as functions of \( v_{avg} \) and \( d_d \), or additional appropriate parameters. Eco-friendly driver assistance systems can be developed, providing real-time hints to human drivers to increase or decrease speed.

### III. COMPARISON OF THE OPT-CPPT AND DYNAMIC PROGRAMMING SOLUTIONS

The Opt-CPPT does not provide the most energy-efficient speed profile, as it is based on constant values of \( v_{con} \), \( ATR \) and \( BTR \), for ease of implementation. This section uses DP to find the optimal speed profile and energy consumption without such restrictions.

#### A. DYNAMIC PROGRAMMING ALGORITHM

In the DP implementation, the tractive force, velocity and powertrain efficiency are obtained from (2)-(9). The battery power, \( P_{batt} \), is given by:

\[
P_{batt}(k) = \begin{cases} 
F(k) v(k), & F(k) \geq 0 \\
F(k) \eta_{pwt}(k), & F(k) < 0 
\end{cases}
\]  

The objective function \( V_{opt} \) and constraints are defined as:

\[
V_{opt} = \arg\min_{v} \sum s \left[ P_{batt}(k) \frac{\Delta s}{v(k)} + \alpha \frac{\Delta s}{v(k)} \right] \\
\text{s.t.} \ T_{m,min} \leq T_m(k) \leq T_{m,max} \\
\quad P_{m,min} \leq P_m(k) \omega_m(k) \leq P_{m,max} \\
\quad 0 \leq v(k) \leq v_{max} \\
\quad v(n_{total}) = v_f \\
\quad n_{total} \Delta s = s_{total} \\
\quad C_r F_{z,f}(k) + F_{ae}(k) + \left[ m + m_e \right] a(k) \leq \mu F_r (k)
\]  

The zoomed-in subplot of Figure 6(b) shows the pattern of the oscillations: the DP accelerates the EV with \( ATR = 0.044 \), which is the lowest possible \( ATR \) (among those considered with the brute force approach) to hit a 21.5 km/h constant speed, while the DP accelerates the vehicle quicker. Once up to speed, the Opt-CPPT maintains \( v_{con} \). In contrast, with the DP, the speed profile of the intermediate section of the cycle appears “noisy”, and in the plot is represented through a sequence of lines connecting the grid points of the DP solution. Such speed oscillations are crucial to energy consumption reduction.

\[ \text{FIGURE 6. (a) Comparison of DP and Opt-CPPT at point A of Figure 4, with } v_{avg} \approx 19 \text{ km/h and } d_d = 2 \text{ km; (b) Zoomed views.} \]

### B. COMPARISON OF OPT-CPPT AND DP RESULTS

This section compares the Opt-CPPT and DP results. Figure 4 includes the Pareto front obtained through DP (marked with ‘+’), for \( d_d = 2 \) km, with \( \alpha \) in the range (10, 10^5).

The first example corresponds to data point A of the DP results in Figure 4, which is characterized by \( v_{avg} = 18.8 \) km/h, and a consumption of 22.7 Wh/km. The closest available Opt-CPPT data point has \( v_{avg} = 19.0 \) km/h, and consumes 28.2 Wh/km, which is a 24.2% increase over the DP solution. Figure 6 compares the speed, torque and powertrain efficiency profiles of the DP and Opt-CPPT solutions. The Opt-CPPT accelerates the EV with \( ATR = 0.044 \), which is the lowest possible \( ATR \) (among those considered with the brute force approach) to hit a 21.5 km/h constant speed, while the DP accelerates the vehicle quicker. Once up to speed, the Opt-CPPT maintains \( v_{con} \). In contrast, with the DP, the speed profile of the intermediate section of the cycle appears “noisy”, and in the plot is represented through a sequence of lines connecting the grid points of the DP solution. Such speed oscillations are crucial to energy consumption reduction. The zoomed-in subplot of Figure 6(b) shows the pattern of the oscillations: the DP accelerates the EV from ~19.52 km/h to ~19.65 km/h over 0.5 m, then freewheels back to the initial speed over the following 1.5 m, after which the process repeats itself, with slight differences between each cycle due to the DP resolution. With this technique, in the acceleration phases each motor generates 33.9 Nm, corresponding to a powertrain efficiency of 87.9%. In contrast, with the Opt-CPPT, by holding the speed constant at 21.5 km/h, a lower torque of 9.7 Nm from each motor is used, corresponding to a powertrain efficiency of only 68.6%. By using higher torque to accelerate, the DP EV operates in a more efficient region of the powertrain map and uses the gained momentum to freewheel for a short distance before repeating the cycle, which is a form of PnG. In Figure 6, the final speed reduction phase is achieved through
freewheeling \( (BTR = 0) \) for Opt-CPPT, while the DP mostly freewheels and applies light regenerative braking towards the end.

The second comparison, reported in Figure 7, is at a higher average speed, and corresponds to data point B in Figure 4. The DP vehicle operates at \( v_{avg} = 45.1 \) km/h and consumes 46.6 Wh/km. The closest available data point for the Opt-CPPT has the same \( v_{avg} \) (\( ATR = 0.344 \), \( BTR = 0.316 \), \( v_{con} = 48 \) km/h), and consumes 47.2 Wh/km, only 1.4% more than the DP. In fact, the powertrain is more efficient at higher speed and torque values, which reduces the energy consumption difference between DP and Opt-CPPT. Because of the high average speed, the initial acceleration phase is rather swift for both the Opt-CPPT and DP. In the PnG part of the DP solution in Figure 7(b), the vehicle accelerates from 48.04 km/h to 48.14 km/h over 1 m, freewheels back to the initial speed over 1 m, and repeats the cycle. In the final phase, the DP uses a combination of freewheeling and regenerative braking, while the Opt-CPPT slows down the EV with \( BTR = 0.316 \).

The motor torque profiles for the DP are reported in Figure 8 for \( v_{avg} \approx 19 \) (point A in Figure 4), 45 (point B in Figure 4), 61, and 79 km/h. The positive torque regions slightly above zero, marked with the rings, show the PnG sections, where the torque fluctuates between the value corresponding to maximum powertrain efficiency, \( T_{m, opt} \), and zero, at a relatively constant speed. The higher positive torque regions show the acceleration torques selected by the DP, while the negative regions highlight the braking torques. In general, the higher is \( v_{avg} \), the higher are the acceleration and braking torque values.

In summary, the DP accelerates the vehicle reasonably quickly to get the EV to a “constant” speed, determined by the desired duration of the trip. In the “constant” speed section, the DP imposes cyclic acceleration and freewheeling phases, i.e., PnG, which generally leads to higher powertrain efficiency. Subsequently, if time is not a concern, the DP controlled EV freewheels; otherwise, the DP imposes a combination of freewheeling and regenerative braking.

To verify that the oscillations in Figures 6-8 are not only numerical noise of the DP results, a second DP formulation was implemented, based on the following cost function:

\[
V_{opt} = \arg \min_v \sum_k \left[ P_{batt} \left( k \right) \frac{\Delta s}{v \left( k \right)} + \alpha \frac{\Delta s}{v \left( k \right)} \right. \\
\left. + \beta \left| T_m \left( k \right) - T_m \left( k - 1 \right) \right| \right]
\]

(13)

where the constraints are the same as in (12), and the additional term \( \beta \left| T_m \left( k \right) - T_m \left( k - 1 \right) \right| \) penalizes the torque oscillations. In the modified DP, both \( v \) and the motor torque \( T_m \) are states, as the previous torque value, \( T_m \left( k - 1 \right) \), must be available to evaluate \( V_{opt} \). As a two-dimensional DP is computationally intensive, lower resolution and low distance DP runs were performed to compare the results for different values of the weight \( \beta \) and the same trip time. The main conclusions, which confirm the results in Figures 6-8, are: i) for a specified trip time, the energy consumption increases with increasing \( \beta \), i.e., the case with \( \beta = 0 \) (no torque rate penalty) consumes the least; and ii) for all \( \beta \) values, the DP solution shows PnG behavior, with decreasing PnG frequency with increasing \( \beta \). The PnG frequency will be discussed in more detail in section V-B.

IV. CRITICAL ANALYSIS OF PULSE-AND-GLIDE FOR ELECTRIC VEHICLES

In [32] and [33], PnG is only presented as optimal solution for ICE vehicles, but not for EVs, since for a given speed, the EV powertrain efficiency is simplified as a monotonically decreasing function of torque, which may not be true in general. Here this simplification is removed, and PnG will be proven optimal for EVs under certain conditions, using mathematical and graphical methods, which will also show when and how PnG should be used. The whole analysis is carried out for a single driven axle in traction.
A. MATHEMATICAL PROOF

The formulations are based on Assumptions 1 and 2.

Assumption 1: The powertrain energy consumption is negligible in the PnG glide phase, and transients do not affect the powertrain power losses.

Assumption 2: The EV speed variation is small during a PnG cycle. Therefore, the aerodynamic and rolling resistance power losses are assumed constant during PnG, and the powertrain efficiency is only a function of torque.

Theorem: For given speed and average desired motor torque $T_m,des$, the most energy-efficient solution in the range $[T_m,des, T_m,max]$ is to operate the EV at $T_m,sel$, where $T_m,sel$ is the selected torque, corresponding to the maximum powertrain efficiency in the range $[T_m,des, T_m,max]$. $T_m,des$ is achieved by switching the torque demand between $T_m,sel$ and 0 using pulse width modulation (PWM) with a duty cycle

$$D_p = \frac{t_{on}}{t_{period}} = \frac{T_{des}}{T_{sel}}$$

where $t_{on}$ and $t_{period}$ are respectively the “on time” and period. This results in PnG if $D_p < 100\%$, and constant torque operation if $D_p = 100\%$.

Proof: At a given motor speed $\omega_m$, the average battery power consumption with PnG, $P_{batt,avg,PnG}$, is:

$$P_{batt,avg,PnG} = \frac{\omega_m T_{m,sel}}{\eta_{pwt} (T_{m,sel})} = \frac{\omega_m T_{m,sel}}{\eta_{pwt} (T_{m,sel})} \frac{t_{on}}{t_{period}}$$

(14)

Let us consider an arbitrary number of torque values $T_m,i$ in the range $[T_m,des, T_m,max]$ or zero, with an average equal to $T_m,des$ along $t_{period}$:

$$T_m,des = T_{m,sel} \frac{t_{on}}{t_{period}} = \sum_i T_{m,i} \frac{t_{on,i}}{t_{period}}$$

(15)

where $t_{on,i}$ is the duration at which the torque is at the level $T_{m,i}$. The corresponding average power consumption, $P_{batt,avg,arb}$, is:

$$P_{batt,avg,arb} = \sum_i P_{batt} \frac{t_{on,i}}{t_{period}} = \sum_i \omega_m T_{m,i} \frac{t_{on,i}}{t_{period}}$$

(16)

Since, by definition, $T_{m,sel}$ provides the maximum powertrain efficiency in the range $[T_m,des, T_m,max]$, it follows that:

$$\eta_{pwt} (T_{m,i}) \leq \eta_{pwt} (T_{m,sel})$$

(17)

Therefore, by substituting (17), (15) and (14) into (16), condition (18) holds:

$$P_{batt,avg,arb} \geq \sum_i \omega_m T_{m,i} \frac{t_{on,i}}{t_{period}} = \omega_m T_{m,sel} \frac{t_{on}}{t_{period}} = P_{batt,avg,PnG}$$

(18)

which proves that the proposed PnG mode never consumes more than any other arbitrary torque profile in the solution range along $t_{period}$.

Remark 1: If $T_m,des$ corresponds to the maximum powertrain efficiency in the range $[T_m,des, T_m,max]$, then $T_m,sel = T_m,des$, and the solution implies a 100% duty cycle, i.e., constant torque operation.

Remark 2: If in the theorem, $T_m,des$ is restricted to $(0, T_{m,opt})$, $T_m,sel$ is optimal for the entire range of $T_m,i$, i.e., $[0, T_{m,max}]$, and is the torque providing maximum powertrain efficiency, i.e., $T_m,sel = T_{m,opt}$. In this case, the proof is identical to the one in (14)-(18), and valid for any $T_m,i \in [0, T_{m,max}]$.

Remark 3: Because of Remark 2, for PnG to be optimal, it must be $T_{m,opt} > T_{m,sel}$, i.e., there must be a peak in the efficiency map for the given speed at a larger torque value than $T_m,des$. For example, at 19 and 48 km/h, Figure 9 plots the efficiency characteristics with the normalized torque (the peak torque of each powertrain is used as normalization factor), for the powertrains of three different EVs: i) the L7e quadricycle presented in section II.A; ii) a production front-wheel-drive sport utility vehicle (SUV) with central motor and single-speed transmission with open differential, indicated as ‘SUV1’ in the remainder (efficiency map provided by the car maker); and iii) a production 4-wheel-drive (4WD) premium SUV with different front and rear single-speed powertrains, respectively indicated as ‘SUV2 F’ and ‘SUV2 R’ (experimentally measured efficiency characteristics). For all powertrains in Figure 9, low motor torque values correspond to low efficiency; as torque increases, the efficiency sharply rises to a peak, after which it tends to gradually decrease, with potential local maxima. Hence, based on the specific efficiency characteristics, there are torque conditions in which PnG provides energy savings.

B. PnG IMPLEMENTATION RULES

Example: For a given speed, Figure 10 shows a conceptual powertrain efficiency characteristic, $\eta_{pwt} (T_m)$, with three stationary points, i.e., two maxima, $(T_{m,opt}, \eta_{pwt,opt})$ and $(T_{m,sta,min}, \eta_{pwt,sta,min})$, and one minimum, $(T_{m,sta,max}, \eta_{pwt,sta,max})$. Moreover, $T_{m,ide}$ is defined as the torque in the interval $(T_{m,opt1}, T_{m,opt2})$, at which the powertrain has...
identical efficiency value to the next largest stationary point, i.e., \((T_{m, \text{opt} 2}, \eta_{\text{pwt, opt} 2})\).

Based on the theorem in section IV-A:

- For \(T_{m, \text{des}} \in (0, T_{m, \text{opt}})\), the optimal mode is PnG, with \(T_{m, \text{sel}} = T_{m, \text{opt}}\) since \(T_{m, \text{opt}} > T_{m, \text{des}}\).
- For \(T_{m, \text{des}} \in [T_{m, \text{opt}}, T_{m, \text{ide}}]\), the efficiency decreases as the torque increases, therefore it is better to operate at the smallest possible torque, i.e., \(T_{m, \text{sel}} = T_{m, \text{des}}\), and the desired operation is at constant torque.
- For \(T_{m, \text{des}} \in [T_{m, \text{ide}}, T_{m, \text{opt} 2})\), PnG is also beneficial, with \(T_{m, \text{sel}} = T_{m, \text{opt} 2}\) since \(T_{m, \text{opt} 2} > T_{m, \text{des}}\). For the boundary condition \(T_{m, \text{des}} = T_{m, \text{ide}}\), constant torque operation at \(T_{m, \text{ide}}\) and PnG between \(T_{m, \text{opt} 2}\) and 0 are equivalent from an energy perspective.
- For \(T_{m, \text{des}} \in [T_{m, \text{opt} 2}, T_{m, \text{max}}]\), the efficiency decreases as the torque increases, and the desired operation is at constant torque.

The four regions are highlighted by the arrows at the bottom of Figure 10. The implementation of the proposed PnG rules would make the powertrain operate with the “Effective \(\eta_{\text{pwt}}\) with PnG” in Figure 10, which is a clear benefit with respect to the original powertrain efficiency characteristic as a function of torque at the specific speed. Future studies should investigate the optimal solution for \(T_{m, \text{des}} > T_{m, \text{opt}}\) to further improve the effective efficiency.

\textit{PnG Implementation Rules:} From the previous theorem and example, a set of rules is developed in the form of a pseudo-code, to identify the PnG and constant torque regions, based on the powertrain efficiency characteristic at a given speed \(\omega_m\).

1. Identify the absolute maximum, \((T_{m, \text{opt}}, \eta_{\text{pwt, opt}})\), in the efficiency characteristic at \(\omega_m\): \(T_{m, \text{opt}} = \text{arg max}_{T_m} \eta_{\text{pwt}} (T_m, \omega_m)\) \quad (19)

For example, \(T_{m, \text{opt}}\) is obtained offline from the powertrain efficiency map in Figure 1, which reports \(T_{m, \text{opt}}\) as a function of speed.

2. IF \(T_{m, \text{des}} \in (0, T_{m, \text{opt}})\) → Perform PnG switching between 0 and \(T_{m, \text{opt}}\) with \(D_{\text{batt}} = \frac{T_{m, \text{des}}}{T_{m, \text{opt}}} \times 100\) END

3. IF \(T_{m, \text{des}} \in [T_{m, \text{opt}}, T_{m, \text{max}}]\)
   IF no stationary point exists for \(T_m > T_{m, \text{opt}}\), i.e., \(T_{m, \text{opt}}\) is the right-most stationary point → Perform a constant torque operation at \(T_{m, \text{des}}\) ELSE →
   a. Find the maximum of the remaining points in the range between the next minimum stationary point \(T_{m, \text{sta}, \text{min}}\) and \(T_{m, \text{max}}\), named \((T_{m, \text{opt} 2}, \eta_{\text{pwt, opt} 2})\), defined as: \(T_{m, \text{opt} 2} = \text{arg max}_{T_m \in (T_{m, \text{sta}, \text{min}}, T_{m, \text{max}}]} \eta_{\text{pwt}} (T_m, \omega_m)\) \quad (20)
   b. Find \(T_{m, \text{ide}} \in (T_{m, \text{opt}}, T_{m, \text{opt} 2})\), with \(\eta_{\text{pwt}} (T_{m, \text{ide}}) = \eta_{\text{pwt, opt} 2}\)
   c. IF \(T_{m, \text{des}} \in [T_{m, \text{opt}}, T_{m, \text{ide}}]\) → Perform a constant torque operation at \(T_{m, \text{des}}\) END
   d. IF \(T_{m, \text{des}} \in (T_{m, \text{ide}}, T_{m, \text{opt} 2})\) → Perform PnG switching between 0 and \(T_{m, \text{opt} 2}\) with \(D_{\text{batt}} = \frac{T_{m, \text{des}}}{T_{m, \text{opt} 2}} \times 100\) END
   e. IF \(T_{m, \text{des}} \in [T_{m, \text{opt} 2}, T_{m, \text{max}}]\) → Repeat the entire routine from step 3, by replacing \(T_{m, \text{opt}}, T_{m, \text{opt} 2}, T_{m, \text{sta}, \text{min}}, T_{m, \text{ide}}\) and \(\eta_{\text{pwt, opt} 2}\) with \(T_{m, \text{opt} 2}, T_{m, \text{opt} 3}, T_{m, \text{sta}, \text{min} 2}, T_{m, \text{ide} 2}\) and \(\eta_{\text{pwt, opt} 3}\) respectively; the cycle repeats for any further stationary points END END

**C. GRAPHICAL APPROACH**

This section uses a graphical approach, which may be more intuitive to readers, to analyze and implement PnG, similar to the one outlined in [23] for ICE vehicles; here the method is systematically formulated for EVs, and generalized to the case of multiple stationary points in the powertrain efficiency characteristic.

\textit{Lemma 1:} If, for a given speed, in the graph of the battery output power as a function of powertrain torque, a line from the origin (LftO) is drawn to a generic data point \((T_{m, 1}, P_{\text{batt}} (T_{m, 1}))\), the gradient of the LftO is inversely proportional to the efficiency at \(T_{m, 1}\), i.e., \(\frac{P_{\text{batt}} (T_{m, 1})}{T_{m, 1}} \propto \frac{1}{\eta_{\text{pwt}} (T_{m, 1})}\).
the corresponding datapoints in the power train efficiency map have the same efficiency value, e.g., \( \eta_{\text{pwt}}(T_{m,1}) = \eta_{\text{pwt}}(T_{m,2}) \), then the corresponding datapoints in the \( P_{\text{batt}}(T_m) \) characteristic will share the same LfTO, i.e., \( \frac{P_{\text{batt}}(T_{m,1})}{T_{m,1}} = \frac{P_{\text{batt}}(T_{m,2})}{T_{m,2}} \). For example, in Figure 10, there are three points with the same efficiency \( \eta_{\text{pwt,opt2}} \); the corresponding points in Figure 11(a), i.e., K, R and S, share the same LfTO.

**Remark 1:** As a result of Lemma 1, if, for a given speed, two datapoints in the power train efficiency map have the same efficiency value, e.g., \( \eta_{\text{pwt}}(T_{m,1}) = \eta_{\text{pwt}}(T_{m,2}) \), then the corresponding datapoints in the \( P_{\text{batt}}(T_m) \) characteristic will share the same LfTO, i.e., \( \frac{P_{\text{batt}}(T_{m,1})}{T_{m,1}} = \frac{P_{\text{batt}}(T_{m,2})}{T_{m,2}} \). For example, in Figure 10, there are three points with the same efficiency \( \eta_{\text{pwt,opt2}} \); the corresponding points in Figure 11(a), i.e., K, R and S, share the same LfTO.

**Remark 2:** If, for a given speed, two datapoints have efficiency values such that \( \eta_{\text{pwt}}(T_{m,1}) > \eta_{\text{pwt}}(T_{m,2}) \), then in the \( P_{\text{batt}}(T_m) \) plot, the gradient of the LfTO for \( T_{m,1} \) is smaller than that for \( T_{m,2} \), i.e., \( \frac{P_{\text{batt}}(T_{m,1})}{T_{m,1}} < \frac{P_{\text{batt}}(T_{m,2})}{T_{m,2}} \). For example, in Figure 10, Q has higher efficiency than P and Q correspond to points A and G in Figure 11(b), where the LfTO OQ has a smaller gradient than the LfTO OA.

**Remark 3:** Because of Remark 2, the maximum in the \( \eta_{\text{pwt}}(T_m) \) characteristic corresponds to the datapoint on \( P_{\text{batt}}(T_m) \) with the minimum gradient LfTO. For example, in Figure 11, the LfTO OE corresponding to \( (T_{m,\text{opt}}, \eta_{\text{pwt, opt}}) \) has the lowest gradient among all possible LfTOs of \( P_{\text{batt}}(T_m) \).

**Lemma 2:** For a given speed \( \omega_m \), a stationary point in \( \eta_{\text{pwt}}(T_m) \) corresponds to a point at which the corresponding LfTO in the \( P_{\text{batt}}(T_m) \) plot is tangent to the \( P_{\text{batt}}(T_m) \) characteristic.

**Proof:** Let us define \( f(T_m) \) as:

\[
\eta_{\text{pwt}}(T_m) = \frac{P_{\text{batt}}(T_m)}{T_m} = \frac{\omega_m}{\eta_{\text{pwt}}(T_m)}
\]  

(21)

Therefore, at the datapoint \( (T_{m,1}, \eta_{\text{pwt},1}) \), it is:

\[
\frac{P_{\text{batt}}(T_{m,1})}{T_{m,1}} = \frac{\omega_m}{\eta_{\text{pwt}}(T_{m,1})}
\]  

(22)

which proves Lemma 1.

Based on (21), in Figure 11, the \( \eta_{\text{pwt}}(T_m) \) curve from Figure 10 has been transformed into the battery output power curve as a function of motor torque, \( P_{\text{batt}}(T_m) \). The point \( (T_{m,\text{opt2}}, \eta_{\text{pwt,opt2}}) \) in Figure 10 becomes \( (T_{m,\text{opt2}}, P_{\text{batt}}(T_{m,\text{opt2}})) \), or point K, in Figure 11(a). OK is the LfTO for point K; its gradient is inversely proportional to the efficiency.

**Graphical Interpretation of the Theorem of Section IV.A:** Based on Lemma 1, the theorem of section IV.A can be used via a graphical method, i.e., for a given \( T_{m,\text{des}} \), the energy consumption is reduced by selecting \( T_{m,\text{sel}} \) corresponding to the minimum gradient LfTO in the range \( [T_{m,\text{des}}, T_{m,\text{max}}] \) in the \( P_{\text{batt}}(T_m) \) plot, and the corresponding duty cycle is\( D_k = \frac{100}{T_{m,\text{des}}} \).

For example, based on Figure 11(b), if \( T_{m,\text{des}} \) is between O and F, i.e., \( T_{m,\text{des}} \in (0, T_{m,\text{opt}}) \), e.g., in B, under a conventional constant torque operation, A would be the operating point, and the powertrain efficiency would correspond to point P in Figure 10. Instead, optimal PnG would operate the pulses at point E in Figure 11(b), and the glide at the origin O. This results in the average power consumption in C, which is lower than the power consumption for: i) the condition in A; and ii) all sub-optimal PnG cases, such as the PnG between O and G, corresponding to the average consumption in H. From Lemma 2, since point E corresponds to the stationary...
point \((T_{m,\text{opt}}, \eta_{\text{pwt},\text{opt}})\). OE is tangent to \(P_{\text{batt}}(T_m)\). From Remark 3 of Lemma 1, since \((T_{m,\text{opt}}, \eta_{\text{pwt},\text{opt}})\) is the global maximum efficiency, OE is the LftO with minimum gradient intersecting \(P_{\text{batt}}(T_m)\). Therefore, if \(T_{m,\text{des}}\) is between O and F, the most energy efficient mode is a PnG that switches between the peak efficiency torque \(T_{m,\text{sel}} = T_{m,\text{opt}}\) and zero torque with a PWM duty cycle \(D_{\%}\):

\[
\frac{D_{\%}}{100} = \frac{T_{m,\text{des}}}{T_{m,\text{sel}}} = \frac{t_{\text{on}}}{t_{\text{period}}} = \frac{OB}{OF} = \frac{BC}{FE} \tag{26}
\]

In terms of average battery power consumption during a PnG cycle, from (26) and (14) it follows that:

\[
P_{\text{batt,avg,PnG}} = \frac{FE\times D_{\%}}{100} = \frac{FE\times OB}{OF} = \frac{FE\times BC}{FE} = BC
\]

\[
= \frac{\omega_m T_{m,\text{sel}} D_{\%}}{100} < \frac{\omega_m T_{m,\text{sel}} D_{\%}}{100} = \frac{\omega_m T_{m,\text{des}}}{\eta_{\text{pwt}}(T_{m,\text{des}})} = \frac{\omega_m T_{m,\text{des}}}{\eta_{\text{pwt}}(T_{m,\text{des}})} = \frac{\omega_m T_{m,\text{des}}}{\eta_{\text{pwt}}(T_{m,\text{des}})} = \frac{AB}{P_{\text{batt,avg,con}}} \tag{27}
\]

which confirms the convenience of PnG with respect to the powertrain operation at continuous torque, corresponding to the power consumption \(P_{\text{batt,avg,con}}\).

Conversely, if the desired torque is located between points F and J, i.e., \(T_{m,\text{des}} \in [T_{m,\text{opt}}, T_{m,\text{ide}}]\), it is better to operate with constant torque. In fact, in this region, as torque increases, efficiency decreases, and therefore the LftO gradient increases (see Remark 2 of Lemma 1). Therefore, the minimum LftO gradient is achieved at \(T_{m,\text{des}}\) with \(D_{\%} = 100\%\). For example, for \(T_{m,\text{des}}\) corresponding to point N, the line ON is the LftO with the smallest possible gradient in \([T_{m,\text{des}}, T_{m,\text{max}}]\). All other LftOs, including the tangent OK created by \((T_{m,\text{opt}2}, \eta_{\text{pwt,op2}})\), have larger gradient than ON.

Similar analyses can be carried out for the remaining two torque intervals, see Figure 11(a) for the sequence of solutions as a function of powertrain torque.

V. PRACTICAL IMPLEMENTATION ASPECTS OF PULSE-AND-GLIDE FOR ELECTRIC VEHICLES

A. RANGE OF TORQUE AND SPEED VALUES IN WHICH PnG IS BENEFICIAL

The aim of this section is to ascertain, based on real-world electric powertrain efficiency maps, whether PnG is actually beneficial for a significant range of operating conditions.

Figure 12 includes the powertrain efficiency maps for SUV1 and SUV2, while Figure 1 refers to the case study L7e quadricycle. In the graphs, the normalization is carried out with the top speed and torque of the electric machine (for SUV2, the experimental efficiency data are not available at high torque and speed). The \(T_{m,\text{opt}}\) lines indicate the torque values with the maximum powertrain efficiency for each speed, while the \(T_{m,\text{v}}\) loci show the torque required to maintain the EV at that specific speed at zero road gradient (note that the 4WD SUV2 operates with a 30:70 front-to-rear wheel torque distribution). If \(T_{m,\text{opt}} > T_{m,\text{v}}\), PnG is optimal for at least constant speed operation.

In the considered L7e quadricycle, PnG is beneficial in constant speed conditions for the whole speed range. In SUV1, PnG is optimal up to 0.7 of the EV top speed, i.e., until the intersection of the \(T_{m,\text{opt}}\) and \(T_{m,\text{v}}\) loci; beyond such speed, constant torque operation is preferred. For SUV2, PnG reduces consumption along the whole range of EV speeds for which the experimental map was available. The important conclusion is that PnG is practically useful to reduce energy consumption in a variety of operating conditions.
B. PnG FREQUENCY AND ENERGY CONSUMPTION

The PnG section of the DP solution in Figures 6-7 is characterized by jerky and uncomfortable speed profiles; future research should also analyze the potential implications of high-frequency PnG on powertrain durability. This subsection investigates the effect of lower PnG frequencies on EV consumption, which is observed when a torque rate penalty is placed on the DP (see section III.B).

In Figure 13, distance-based PnG profiles are generated with optimal PWM rectangular pulses as discussed in the theorem in section IV.A. In the example, the pulsation spatial frequency is 0.0025 pulses/m, with a duty cycle of 29.3% to maintain an average speed of 19.6 km/h. The time-based frequency is 0.014 Hz, corresponding to a 73 s period. The spatial frequency of the pulses is then varied in a sensitivity analysis, while keeping the torque amplitude constant.

Results are reported in Figure 14 for two average speeds, i.e., 19.6 and 48.1 km/h. The profile in Figure 13 corresponds to the left-most data point at 19.6 km/h in Figure 14. The right-most data points for both average speeds correspond to the frequency selected by the DP solution in Figures 6 and 7. The solid lines show the energy consumption as a function of the PnG frequency, while the horizontal dashed lines indicate the consumption for the corresponding constant speed profile. The energy consumption increases at low frequencies, because of the wide speed fluctuations, and therefore Assumption 2 of section IV.A is no longer valid. If the desired speed profile is expected to vary greatly, then a modified technique is needed, which is discussed in the next section on PnG-tracking of speed profiles.

In Figure 14(a), frequencies > 0.007 pulses/m give origin only to a marginal increase in energy consumption when compared with the DP frequency of 0.45 pulses/m. In fact, at 0.007 pulses/m the energy consumption increases only by 0.68% and 0.13% compared to the DP solution for the 19.6 and 48.1 km/h cases, which provides a useful indication for setting up a realistic rule-based PnG. At the same frequency, the EV speed fluctuations amount to ~8 km/h and ~6 km/h for the two scenarios. For the 48.1 km/h scenario, at frequencies below 0.0015 pulses/m, the solid and dashed lines cross each other, i.e., for very low frequencies constant torque operation is preferable to PnG. In the two considered speeds, the DP solution consumes 33.3% and 6.1% less than the constant speed case. This implies that the difference between high and low frequency PnG is negligible with respect to the difference between the PnG and constant speed cases. In summary, low frequency PnG is a practical alternative to DP in constant speed conditions.

C. PnG TO TRACK A REFERENCE SPEED PROFILE

This section discusses the selection of the PnG pulsating torque and duty cycle to track an average reference speed profile in traction. The same method could be extended to regenerative braking, even if this study does not include this step, which can be safety critical with respect to the front-to-rear braking force distribution.
In the first 15 m of the simulation, \( T_{m, \text{des}} > T_{m, \text{opt}} \); therefore, based on the rules defined in section IV, constant torque operation is used. In the following part of the maneuver, \( T_{m, \text{des}} < T_{m, \text{opt}} \); as a consequence, PnG is activated, which brings an overall 2.1% energy consumption reduction during the considered acceleration test, with respect to the Opt-CPPT solution.

### D. ENERGY CONSUMPTION ASSESSMENT

To observe its energy consumption benefit, the PnG method previously discussed is applied to track the constant \( ATR \) and constant speed sections of the Opt-CPPT and a few sub-optimal CPPT results from section II. These further cases are indicated with “PnG-tracked” followed by the denomination of the original speed profile.

For \( d_d = 2 \text{ km} \) and two \( v_{\text{avg}} \) values (19 and 29 km/h), Table 3 compares the energy consumption performance along the whole mission profile of: i) the DP for the respective speed; ii) two sub-optimal CPPT cases from section II-C, called CPPT1, characterized by significant

|       | DP    | CPPT1 | Opt-CPPT | PnG-tracked CPPT1 | PnG-tracked CPPT2 | PnG-tracked Opt-CPPT |
|-------|-------|-------|----------|-------------------|-------------------|----------------------|
| \( v_{\text{avg}} \) (km/h) |       |       |          |                   |                   |                      |
| 19    | 0.076 | 0.044 | -        | -                 | -                 |                      |
| 29    | 0.112 | 0.104 | -        | -                 | -                 |                      |
| \( ATR \) (\%) | 0.924 | 0.112 | 0.010     | 0.010             | 0.010             |                      |
| \( v_{\text{avg}} \) (km/h) |       |       |          |                   |                   |                      |
| 19    | 33.5  | 31.5  | 36        | 0.056             | 0.056             |                      |
| 29    | 32.9  | 31.51 | 32.09     | 0.056             | 0.056             |                      |
| \( E_{\text{batt}}/d_d \) (Wh/km) | 28.94 | 32.09 | 31.51    | 31.41             | 30.48             | 29.04                 |
| \% increase w.r.t. DP | 15.8  | 10.9  | 9.8       | 8.5               | 5.3               | 0.3                   |

Figure 15 applies (28), (2)-(9) and (29) in a forward facing vehicle simulation model, to track the Opt-CPPT speed profile in the acceleration phase of the test in Figure 6, corresponding to \( ATR = 0.044 \) for the case study L7e vehicle. In the figure, \( D_h \) is the PnG duty cycle calculated from the theorem in section IV-A. \( D_h^{\text{avg, zoh}} \) and \( T_{m, \text{pul, avg, zoh}} \) are the zero-order hold trajectories of \( D_h^{\text{avg}} \) and \( T_{m, \text{pul, avg}} \), which are applied to the vehicle.

\[
T_{m, \text{pul, avg}}(j) = \frac{T_{m, \text{opt}}(j) + \hat{T}_{m, \text{opt}}(j + 1)}{2}
\]

\[
D_h^{\text{avg, zoh}}(j) = \frac{D_h^{\text{avg}}(j) + D_h^{\text{avg}}(j + 1)}{2}
\]

\[
T_{m, \text{pul, avg, zoh}}(j) = \frac{T_{m, \text{pul, avg}}(j) + \hat{T}_{m, \text{pul, avg}}(j + 1)}{2}
\]
initial acceleration and lower constant speed (see the ATR, BTR, and V<sub>con</sub> values in the table), and CPPT2, characterized by moderate initial acceleration and larger constant speed; iii) the Opt-CPPT cases for the respective speed (see Figure 6 for the case at 19 km/h); and iv) the PnG-tracked CPPT1, CPPT2, and Opt-CPPT.

At 19 km/h, the PnG-tracked implementation reduces energy consumption by 20.8%, 20.6%, and 19.1%, with respect to CPPT1, CPPT2, and Opt-CPPT, while at 29 km/h the energy consumption reduction with respect to the same cases amounts to 6.3%, 5.0%, and 7.8%. In addition, from Table 3, the considered CPPT and Opt-CPPT cases consume on average 20.2% additional energy with respect to the DP solution at the two speeds, while this figure reduces to 3.6% for the PnG-tracked cases.

The important conclusion is that the PnG-tracked results are consistently rather close to those of the DP, whatever is the reference speed profile. On the contrary, a significantly increased energy consumption of the CPPT and Opt-CPPT is observed with respect to the DP solution. Hence, future research should focus on practical PnG implementations for human-driven and automated vehicles, as these could bring similar benefits to the quite widely investigated solutions based on the generation of energy-efficient long-distance speed profiles.

VI. CONCLUSION

This study presented a numerical analysis of energy-efficient speed profiling for electric vehicles, including application of brute force optimization, dynamic programming (DP), and pulse-and-glide (PnG) to a case study electric L7e quadricycle.

The main conclusions are:

- In the selected test cases, the optimal results with the constant pedal position technique (Opt-CPPT) show that in absence of time constraints, it is best to slowly accelerate to the optimal speed, maintain it, and finally freewheel to a stop. If time constraints are present, the Opt-CPPT profile is a trade-off between the desired travel time and energy consumption reduction, and implies progressive increase of the initial torque demand and final regenerative braking action with decreasing travel times.

- DP consistently outperforms the Opt-CPPT, especially when the time constraints are not critical, condition in which the Opt-CPPT controlled vehicle consumes up to ∼ 24% more than the DP solution. This difference is mainly caused by the low powertrain efficiency at small torque demands, and the intrinsic absence of PnG behavior in the Opt-CPPT case.

- PnG is optimal for EVs under certain conditions. Mathematical and graphical rules are formulated to ascertain when PnG or constant torque operation is superior. The analysis of multiple real-world electric powertrain efficiency maps confirms the practical PnG applicability to EVs.

- The high frequency PnG phases typical of the DP results cannot be implemented in practice; however, the PnG frequency can be reduced to a realistic level with only marginal increases (< 1%) of the energy consumption.

- Examples of adoption of the formulated PnG rules to track the speed profiles output by the CPPT and Opt-CPPT were analyzed. Interestingly, the results show that PnG tends to reduce the energy consumption to levels that are close to those of DP, even when it tracks rather inefficient reference speed profiles originating from the CPPT cases.

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