Dynamics of Nonhomogeneous Carbon Steel Plates

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Abstract. This paper presents finite element modeling procedure for rate-dependent elastic-plastic nonhomogeneous carbon steel plates subjected to dynamic loads. The steel plate consists of two ductile materials. We utilize a modified rule of mixture and linear strain-rate dependence to derive constitutive relations which are then implemented in ABAQUS/Explicit through user subroutine. Validation of the developed procedure is partially made with Steel 1018. Comparative numerical study is done between Steel 1018 and the nonhomogeneous plate. Displacements, stresses and relevant energy terms are calculated and compared for various sets of material configurations.

1. Introduction
A functionally graded material (FGM) is a material concept or a material itself in which the volume fractions of constituent materials vary smoothly resulting in graded macroproperties with a non-uniform microstructure [1, 2]. There has been much research done on the static and dynamic responses of linear-elastic FGMs [3-10]. There has been also some investigations of rate-independent elastic-plastic FGMs. Williamson et al. [11, 12] developed an elastic-plastic model based on a modified rule of mixture so-called Tamura–Tomota–Ozowa (TTO), Giannakopoulos et al. [13] evaluated rate-independent elastic-plastic material properties for a FGM interface layer based on the TTO model and investigated the cyclic thermal response of the layered composite. Gunes et al. [14] studied the elastic-plastic response of a circular FG plate under low velocity impact. There has been a need for the development of rate-dependent elastic-plastic constitutive relations for FG ductile steel plates. Rate-dependent elastic-plastic material models are the key to accurately analyzing FGM subjected to medium and high rate dynamic loading. Li et al. [15, 16] investigated the influence of particle volume fraction, shape, and aspect ratio on the rate-dependent elastic-plastic behavior of a particle-reinforced metal-matrix composite at high strain rates. Li et al. [17] examined the impact response of layered and graded plates. Herein we develop a rate-dependent elastic-plastic FGM model. The rate-dependent elastic-plastic material properties of the constituent materials are experimentally characterized. The modified rule of mixture is used to calculate material properties. We apply the TTO model to an FGM with two ductile constituent materials. Based on the rate-dependent elastic-plastic FGM model, an ABAQUS/VUMAT subroutine is developed and implemented to simulate the dynamic response of a hypothetic FG plate under dynamic air pressure load.
2. Constitutive Models and Material Characterization

2.1 Strain-rate Dependent Stress-Strain Laws

The flow stress of a material, with no temperature effect, is dependent on the equivalent plastic strain $\varepsilon^{pl}$ and the equivalent plastic strain rate $\dot{\varepsilon}$

$$\sigma = \sigma (\varepsilon^{pl}, \dot{\varepsilon}^p) \quad (1)$$

In commonly used rate-dependent plastic material models, such as Johnson-Cook and Cowper-Symonds model, the strain-rate dependence can be decoupled from the quasi-static stress-strain relation:

$$\sigma = \sigma_0(\varepsilon^{pl}) R(\dot{\varepsilon}^p) \quad (2)$$

where $\sigma_0(\varepsilon^{pl})$ shows the quasi-static flow stress which is a function of equivalent plastic strain $\varepsilon^{pl}$, and $R(\dot{\varepsilon}^p)$ is the strain-rate dependence. There are three models available in ABAQUS/CAE to define strain-rate dependence [18]: yield ratio model, Johnson-Cook strain-rate dependence [19] and Power-law strain-rate dependence [20]. In this work, linear strain-rate dependence is selected due to the nature of experimental data of Steel 1018 (See Figure 1(a)). The linear strain-rate dependence is defined as:

$$R = 1 + C \dot{\varepsilon}^p \quad (3)$$

where $C$ is a constant representing a linear strain-rate dependence. A bilinear hardening model is widely used in representing the hardening curve due to its descriptive mathematical form. With the bilinear hardening model used, the rate-dependent bilinear hardening model becomes

$$\sigma = (\sigma_y + E_t \varepsilon^{pl})(1 + C \dot{\varepsilon}^p) \quad (4)$$

where $\sigma_y$ is the quasi-static yield stress, $E_t$ is the tangent modulus.

2.2 Material Characterization

The Split-Hopkinson Pressure Bar (SHPB) Technique is used to obtain the constitutive relations of steel 1018 for various high strain rates at room temperature. The stress-strain curves of steel 1018 under strain rates (2000/s, 2500/s and 3100/s) obtained by Prof. Arun Shukla’s group (See Figure 1(a)). In addition, the quasi-static stress-strain curve is obtained at a rate of $10^{-5}$/s in compression (See Figure 1(a)). The strain was determined directly from the instrument, i.e., from the displacement of the crosshead. For the elastic region we use well-known material properties such as Young’s modulus=200GPa, Poisson’s ratio=0.29. The rate-dependent elastic-plastic material model of steel 1018 is obtained by the curve-fitting method using the experimental data and Eq. (4) as shown in Figure 1(a).

The FG plate is envisioned to consist of Steel 1018 and Steel A36. The stress-strain curves of steel A36 at different strain rates are shown in Figure 1(b) [21]. Steel A36 and Steel 1018 are all low-carbon steel and steel A36 is assumed to share similar strain-hardening and strain-rate dependence. The FG plate considered here has steel A36 at the front, which is more compliant and can absorb more energy by its greater deformation, and steel 1018 at the back which has a larger yield stress. The Young’s moduli of these two materials are 200GPa. Linear gradation between the front and back plates is assumed.
2.3 Strain Rate-dependent FGM model

The rate-dependent FGM model is derived based on the modified rule of mixture [12]. The TTO model relates the uniaxial stress and strain of a two-phase composite to the corresponding uniaxial stresses and strains of two constituent materials by

\[ \sigma_c = V_1 \sigma_1 + V_2 \sigma_2, \epsilon_c = V_1 \epsilon_1 + V_2 \epsilon_2 \]  

(5)

where \( \sigma_i, \epsilon_i \) and \( V_i (i=1,2) \) denote the average stresses, average strains and volume fractions of the constituent phases, respectively, and \( \sigma_c, \epsilon_c \) denote the uniaxial stress and strain of the two-phase composite, respectively. The constitutive law of an FGM can be obtained by relating the constitutive curves of the constituent materials through parameter \( Q \) at any fixed strain rate, as shown in Figure 2.

![Figure 1. Stress-strain curves of (a) steel 1018 and (b) A36 [21] at different strain rates](image)

![Figure 2. Concept of modified rule of mixture](image)

In this work, the parameter \( Q \) value is uniquely defined by the yielding points of ductile constituent materials as:

\[ Q = \frac{\sigma_y 2 - \sigma_y 1}{\epsilon_y 2 - \epsilon_y 1} \]  

(6)

The effective Young’s modulus for a FGM is obtained as [12]

\[ E = \frac{V_2 E_2 Q + E_1}{Q + E_2} + V_1 [E_1 Q + E_1] / [V_2 Q + E_2] \]  

(7)

In the plastic regime, the stress-strain relation is assumed to follow the bilinear hardening curve. For a point in material 2 \( (\epsilon_{c2}, \sigma_{c2}) \), its corresponding point in material 1 \( (\epsilon_{c1}, \sigma_{c1}) \) can be found by substituting the bilinear hardening model,

\[ Q(\epsilon_{c2} - \epsilon_{c1}) = \sigma_{c1} - \sigma_{c2} = \sigma_{c1} + E_{pl}(\epsilon_{c1} - \epsilon_{c1}) - \sigma_{c2} \]  

(8)

The corresponding strain in material 1 is

\[ \epsilon_{c1} = \frac{Q \epsilon_{c2} - \sigma_{c1} + E_{pl} \epsilon_{c1} + \sigma_{c2}}{Q + E_{pl}}, \sigma_{c1} = \sigma_{c1} + E_{pl}(\epsilon_{c1} - \epsilon_{c1}) \]  

(9)
The corresponding stress and strain of a FGM are given by
\[
\sigma_e = V_1 \sigma_{e1} + V_2 \sigma_{e2}, \quad e_e = V_1 e_{e1} + V_2 e_{e2}
\]
from which the tangent modulus can be calculated as
\[
E'_e = \frac{\sigma_e - \sigma_y}{e_e - e_y}
\]

3. Implementation of Rate-dependent FGM model
The VUMAT subroutine in ABAQUS/Explicit and UMAT subroutine in ABAQUS/Standard provide interfaces to define the elastic-plastic material model. Different from the UMAT subroutine, the VUMAT subroutine is called for blocks of material calculation points and there is no need to update the Jacobian matrix in the explicit integration scheme [18]. A subroutine is developed based on the J2 flow theory with isotropic hardening model. The subroutine is able to read material parameters of constituent materials and calculate material properties of the FGM at each material point. At the beginning of finite element simulation, material constitutive laws of constituent materials are constructed from the VUMAT inputs. In the following steps, volume fractions at material calculation points and the constitutive models of constituent materials are exported to the hardening subroutine VHARD, and the material properties of FGM at each material calculation point are calculated. In each analysis step, the material is considered to be purely elastic and the trial elastic stresses are calculated at first. Then the von Mises stress can be calculated by trial elastic stresses and used to judge whether the material yields or not. If it doesn’t yield, the trial elastic stresses are the true stresses and the analysis continues to the next step. If it yields, the equivalent plastic strain is updated according to the von Mises stress by Newton-Raphson iteration method. Once the equivalent plastic strain is found, the stresses and strains will be updated based on the J2 flow theory.

4. Shock Tube Test of Steel 1018
The shock tube testing was carried out on a monolithic plate made of steel 1018. The shock tube facility available at the University of Rhode Island (Prof. Arun Shukla) [21-24] is used to generate the dynamic air pressure loads. The shock tube is divided into high-pressure and low-pressure driven sections, which are separated by a destructible diaphragm. The pressure difference between these two sections becomes higher when pressurizing the high-pressure driver section. When the pressure difference reaches a critical value, the diaphragm ruptures and the resulting rapid release of gas forms a one-dimensional shock wave front. When the shock wave reaches the specimen, the dynamic air pressure load is applied to the specimen [21-24]. The reflected dynamic air pressure load generated by the shock tube is measured by the sensor which is mounted 20mm away from the nozzle as shown in Figure 3(a). The measured dynamic air pressure load has an exponential decay behavior shown in Figure 3(b). Due to the lack of experimental data for functionally graded plate, the loading magnitude and history that is obtained for the monolithic plate are assumed to be not affected by material gradation.

The plate is simply supported and located with zero stand-off distance from the nozzle, as shown in Figure 3(a) and 3(c). The size of the plate is 0.203m (8in.) in length, 0.051m (2in.) in width and 0.003m (0.125in.) in thickness. The span between the two supports is around 0.152m (6in.). From the shock tube testing, the time history of the monolithic plate deformation was obtained by a high-speed camera system. During the testing, the rectangular monolithic plate bent to its largest deformation of 14.67mm at 2.5ms (milliseconds), and then oscillated until it reached its stable permanent plastic deformation state.
Figure 3. (a) Shock tube testing setup; (b) dynamic air pressure load generated by the shock tube; (c) deformation history

5. Explicit Dynamic Finite Element Analysis

Identical finite element mesh was used for all the simulations as shown in Figure 4(a). Due to model symmetry, a quarter model is built to reduce computational time. Contact with frictionless tangential behavior and hard normal behavior is defined to simulate the simple support between the specimen and the rigid support. The model is meshed using the first-order C3D8I incompatible element, which is the first-order fully integrated C3D8 element enhanced by incompatible modes to improve its bending behavior. The incompatible elements can not only improve the bending analysis but can also reduce the computational time compared to second-order elements. Due to the high rate dynamic air pressure loading during shock tube testing, the explicit algorithm is selected. It is very challenging to determine the spatial distribution of the pressure loading at each time increment from the shock tube experiment. We observe that when the plate undergoes large deformation, the actual loading area is extended to beyond the muzzle area and the pressure loading at the extended area decays as it moves outward from the shock tube muzzle (see the paper [26] for more information). In this paper, we do not intend to perform fluid-structure analysis but approximate the actual pressure load history. The radius of the loading area is extended from 0.75in (radius of muzzle) to 1.5in to accommodate the extended loading area. The pressure load within the muzzle area is approximately the same as the measured pressure loading. And the pressure load in the extended area is assumed to have a second-order decay behavior. The simulation of Steel 1018 is carried out by the VUMAT subroutine as well as existing bilinear plastic model in ABAQUS/CAE both of which provided identical results. The dynamic behavior of an FG plate under dynamic air pressure loading is also investigated. As a comparative study, simulations of three monolithic plate models made of steel 1018, steel A36 and effective material are also carried out. The effective material property is calculated by considering the averaged material property of steel 1018 and A36.

6. Results and Discussion

6.1 Steel 1018: Test and FEM Results

The displacement history of the center point of Steel 1018 obtained from finite element simulation is compared with shock tube experimental data in Figure 4(b). The maximum deflection from the simulation is 14.33mm at 1.6ms and the maximum deformation from the shock tube test is found to be
14.67mm at 2.5ms. The finite element simulations provide reasonably good comparison with the experiment results.

![Finite element mesh](image)

**Figure 4.** (a) 3D Finite element mesh for both monolithic and FGM plates (No. of Elements: 24576; No. of Nodes: 29961); (b) Comparison of displacement histories of the center point in Steel 1018

### 6.2 Functionally Graded Plate: FEM Results

The FGM plate is simulated and compared with Steel 1018, Steel A36 and the effective plate. The displacement histories at the plate center are plotted in Figure 5. As expected, Steel A36 has the largest deformation (maximum 17.71mm), which is due to its lower strain hardening property. Steel 1018 has the smallest deformation (maximum 14.33mm). The displacements of the effective material and the FGM are slightly larger than that of steel 1018, but much smaller than that of steel A36. With the same overall volume fractions of constituent materials, the deformation of the FGM plate is smaller than that of monolithic plate with the effective material property.

![Displacement history](image)

**Figure 5.** Displacement history for the center point of four material configurations.

For Steel 1018, the energy quantities are shown in Figure 6. The external work is determined by integrating the product of load and the displacement for the entire finite element model. The external work increases until the plate bends to its maximum deformation, and then vibrates while the plate oscillates until it reaches its permanent deflection. The plastic dissipation accumulates starting from 0.6ms and the plastic dissipation increments become smaller as the dynamic air pressure load decays. When the plastic dissipation increases, the kinematic energy decreases correspondingly. For all four plates, the external work shown in Figure 6(a) increases till the plate bends to its maximum displacement and oscillates little bit when the plate bounces back and forth until its permanent deflection. The external work is largest in Steel A36, and smallest in Steel A36, and in between in the FGM and the effective plate. The external work for the FGM plate is smaller than that for the effective plate. Similar behavior is observed for the internal energy history in Figure 6(b). The kinetic energy of Steel A36 is the largest, while Steel 1018 is the smallest. The plastic dissipation increments decay as the magnitude of dynamic pressure load decays with time. The plastic dissipation of Steel A36 is the largest, while Steel 1018 is the smallest. The plastic dissipation of plate with FGM is smaller than that of a plate with an effective material.
7. Concluding Remarks

This paper addresses rate-dependent constitutive model of FGM. The FGM considered is composed with two ductile constituent materials (steel 1018 and steel A36) with linear gradation along its thickness direction. And, the linear strain-rate dependence and bilinear hardening model are used for the constituent materials. Based on the developed rate-dependent elastic-plastic model for FGM, a VUMAT subroutine is developed and implemented in the finite element simulation on the dynamic response of FGM plate subjected to dynamic air pressure load which is generated by shock tube facility. The displacement history, stress histories at critical locations and energy histories of the FGM are comparatively provided.

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