Microscopic analysis of $^{11}$Li elastic scattering on protons and breakup processes within $^{9}$Li+$2n$ cluster model

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Abstract. Theoretical analysis of the elastic scattering and breakup in interactions of the $^{11}$Li nucleus with protons are presented. The hybrid model of the microscopic optical potential (OP) is applied. The OP includes the single-folding real part, while its imaginary part is derived within the high-energy approximation (HEA) theory. The spin-orbit contribution to the OP is also included. The differential cross sections of $^{11}$Li+p elastic scattering and the total reaction cross sections are calculated at energies of 62, 68.4, and 75 MeV/nucleon and are compared with the available experimental data. The breakup cross sections at 62 MeV/nucleon and the momentum distributions of the fragments using a two cluster model of the $^{11}$Li nucleus are obtained. An analysis of the single-particle density of $^{11}$Li is performed.

The experiments for the elastic scattering of $^{11}$Li+p make it possible to study the mechanism of the reactions and give tests for various theoretical methods for analysis. $^{11}$Li is a typical example of a halo nucleus with a large radius of its matter distribution, with a very large interaction radius and a small separation energy. The idea of the existence of a two-neutron halo in $^{11}$Li was experimentally verified in measurements and studies of the differential cross sections of the $^{11}$Li+p elastic scattering in the energy range 60–75 MeV/nucleon [1-3].

In the present work and in [4] we study the elastic scattering cross section for $^{11}$Li+p at three incident energies ($E < 100$ MeV/nucleon) using microscopically calculated OP’s within the hybrid model [5]. Following our previous works [5-8] we apply this model to the elastic scattering of $^{11}$Li+p.

The main expressions for the OP are

$$U_{\text{opt}} = V^F(r) + iW(r), \quad V^F(r) = V^D(r) + V^{EX}(r),$$

where the real part of the nucleon-nucleus OP is a result of a folding of the nuclear density and of the effective NN potential and involves the direct $V^D(r)$ and exchange parts $V^{EX}(r)$. The imaginary OP (ImOP) is based on the HEA. The whole OP is constructed in the form

$$U_{\text{opt}}(r) = N_R V^F(r) + iN_I W(r) + 2\lambda^2 \left\{ N^S_R V_0^F \frac{1}{r} \frac{df_R(r)}{dr} + iN^S_I W_0 \frac{1}{r} \frac{df_I(r)}{dr} \right\} (l,s),$$

where $R$ and $I$ stand for the real and imaginary parts, respectively.
Figure 1. The $^{11}$Li+$p$ elastic scattering cross section at $E = 62$, 68.4, and 75 MeV/nucleon. Solid line: without SO term; dashed line: with SO term. The experimental data are taken from [1], [2] and [3].

Table 1. Values of the $N$’s parameters, volume integrals $J_V$ and $J_W$ (in MeV fm$^3$), $\chi^2$ and total reaction cross section $\sigma_R$ (in mb) for the results at three energies $E$ (in MeV/nucleon) shown in Fig. 1.

| $E$ (MeV) | $N_R$ | $N_I$ | $N_R^{SO}$ | $N_I^{SO}$ | $J_V$ | $J_W$ | $\chi^2$ | $\sigma_R$ (mb) |
|-----------|-------|-------|------------|------------|-------|-------|----------|-----------------|
| 62        | 0.871 | 0.953 | 0.028      | 0.000      | 342.74 | 332.015| 1.415    | 456.97          |
|           | 0.851 | 0.974 | 0.028      | 0.000      | 334.610| 339.332| 1.468    | 461.21          |
| 68.4      | 0.625 | 0.186 | 0.021      | 0.000      | 232.210| 60.489 | 1.328    | 153.44          |
|           | 0.543 | 0.140 | 0.201      | 0.000      | 201.744| 45.530 | 0.316    | 122.25          |
| 75        | 0.679 | 0.370 | 0.045      | 0.000      | 238.048| 112.913| 232.62   | 232.62          |

where $\lambda_2^2 = 2$ fm$^2$ is the squared pion Compton wavelength. The ReOP and the ImOP of the SO optical potential in (2) are approximated by WS forms. The parameters of the latter are obtained by a fitting procedure to the respective calculated microscopic potentials $V^F(r)$ and $W^H(r)$. The ImOP is taken in two forms, the microscopically obtained $W^H$ within HEA ($W = W^H$) or the form of the folded real potential $V^F$ ($W = V^F$). The OP $U_{opt}(r)$ (2) is applied to calculate the elastic scattering differential cross sections using the program DWUCK4 [9]. For the protons and neutrons of $^{11}$Li we used the Large Scale Shell Model (LSSM) densities [10], that have an exponential asymptotics which is the correct one. A set of N coefficients as parameters is introduced. It can be found by fitting the experimental differential cross sections [4]. The problem of the ambiguity of the N’s parameters arises. We use a physical constraint on their choice, namely the behavior of the volume integrals $J_V$ and $J_W$ as functions of the energy [11]. The parameters N give a satisfactory agreement of our results with the experimental data. The values of the total cross sections of the elastic scattering and reaction can serve as another criterion. However the corresponding data are missing, so they are highly desirable.

The results of $^{11}$Li+$p$ elastic scattering cross sections are presented in the Fig.1. The corresponding values of the N’s parameters together with those of $J_V$, $J_W$, $\chi^2$ and $\sigma_R$ are given in Table 1.

In addition to the analysis of $^{11}$Li+$p$ elastic scattering cross section, we study other characteristics of the reaction mechanism, such as the $^{11}$Li total reaction cross section, the breakup cross section and related quantities. A simple two-cluster model in which two clusters are suggested, namely the $^9$Li core $(c)$ and the correlated pair of neutrons $h = 2n$ is considered.
The wave function of the relative motion of the two clusters $\phi_{s_0}^{(n)}(s)$ (s is the distance between the clusters) is obtained as a solution of the Schrödinger equation using WS potential for 0s or 1s states for a particle with a reduced mass of both clusters. The WS parameters are obtained by fitting the energy of a given state to the empirical separation energy value of the h-cluster $\varepsilon = 0.247$ MeV and the rms radius of the cluster function. For the latter we choose the value of 4.93 fm.

In the framework of the $^{9}$Li+$2n$ model of $^{11}$Li one can estimate the $^{11}$Li+$p$ OP as a sum of two OP’s of interactions c + $p$ ($U_c$) and h + $p$ ($U_h$) folded with the density $\rho_0^{(n)}(s)$ ($n=0, 1$ and $(\rho_0^{(n)}(s) = |\phi_{s_0}^{(n)}(s)|^2$):

$$U_r^{(b,n)}(r) = V_r^{(b,n)} + iW_r^{(b,n)} = \int ds\rho_0^{(n)}(s) \left[ U_c^{(n)}(r + (2/11)s) + U_h^{(n)}(r - (9/11)s) \right].$$

This cluster model is used to calculate the characteristics of the breakup reactions of $^{11}$Li with the proton target. The eikonal formalism can be applied to obtain the total breakup and absorption cross sections and their sum, the total reaction cross section. In this work the cross sections of the diffractive and stripping (when $h = 2n$ cluster leaves the elastic channel) $^{11}$Li+$p$ reactions at $E = 62$ MeV/nucleon are obtained. The differential and the total reaction cross sections (for elastic scattering, as well as for diffractive breakup and absorption) require calculations of the probability functions $d^2P(b, k)/dk$ that depend on the impact parameter $b$. The general expression for the probability functions can be written as [13]:

$$\frac{d^2P_{\Omega}(b, k)}{dk} = \frac{1}{(2\pi)^3} \left| \int ds\phi_k^+(s)\Omega(b, r_\perp)\phi_{s_0}^{(n)}(s) \right|^2,$$

where $\Omega(b, r_\perp)$ depends on the two profile functions $S_i(b_i)$, (i = c, h) (given in [4]), $\phi_k$ is the continuum wave functions, k is the relative momentum of both clusters in their center-of-mass frame. One can integrate over the transverse angle of the momenta $\varphi_k$ to get the double-differential probability $d^2P_{\Omega}(b, k)/dkLdk_\perp$. The diffraction breakup cross section has the form:

$$\left( \frac{d\sigma}{dkL} \right)_{diff} = \int b_{h} db_{h} \int_{0}^{2\pi} d\varphi_{h} \int_{0}^{\infty} d{k}_{L} \int_{0}^{\infty} d^2P(k,b)/dkLdk_\perp.$$  

In the case of the stripping reaction when the h - cluster leaves the elastic channel, following [13], the stripping cross section has the form:

$$\left( \frac{d\sigma}{dkL} \right)_{s} = \frac{1}{2\pi^2} \int b_{h} db_{h} d\varphi_{h} \left[ 1 - |S_h(b_h)|^2 \right] \int \rho d\rho d\varphi d|S_c(b_c)|^2 \left[ \int_{0}^{\infty} dz \cos(k_{L}z)\phi_0 \left( \sqrt{\rho^2 + z^2} \right) \right]^2.$$  

The results for the diffraction breakup and stripping $^{11}$Li+$p$ scattering are presented in Figs. 2 and 3. They give predictions because of missing experimental data for such processes accompanying the $^{11}$Li+$p$ scattering at $E \leq 100$ MeV/nucleon. For the diffractive scattering we obtain values of the widths 98 MeV/c (for n=0) and 85 MeV/c (for n=1) and for the stripping reaction 79 MeV/c (for n=0) and 72 MeV/c (for n=1), respectively.

The analysis of the breakup reaction makes it possible to understand its significant role in the formation of the OP responsible for the $^{11}$Li+$p$ elastic scattering. It turns out that the breakup channel gives the total breakup cross section $\sigma_{tot}^{b}$ that exceeds 80% from the total reaction cross section $\sigma_{tot}$, while it is around a half of $\sigma_{tot}^{b}$ in the case of $^{6}$He+$^{12}$C (as obtained in [12]).

The single-particle density distribution of $^{11}$Li can be derived in the form $\rho(r) = \int d s \rho_0^{(n)}(s) \left[ \rho_c(r_c) + \rho_h(r_h) \right]$, where $\rho_c(r_c)$ is the density for the $^{9}$Li cluster from the LSSM.
calculations with rms radius $R_c=2.31$ fm. The density of the $h$-halo $\rho_h(r_h)$ is taken in two forms: Gaussian distribution (G density) and a symmetrized Fermi distribution (SF density). The values of the parameters of G and SF densities and the results of our calculations are obtained in [4] and they are close to the phenomenological density related to the experimental data. We should mention that at the energy of the range 60–70 MeV/nucleon a distortion due to the nuclear and Coulomb forces could affect the cross sections. We have in mind also that our simplified two-cluster model could not give the correct answer and that it can be found in a more complicated three-body approach. This problem remains open and requires further analysis. Future measurements of the cross sections and momentum distribution of the $^9\text{Li}$ fragments in the $^{11}\text{Li}$ breakup reactions might provide a supplemental information on the internal spatial structure of the $^{11}\text{Li}$ nucleus.

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