Relativity as classical limit in a combinatorial scenario

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Abstract

We discuss how the finiteness and universality of the speed of light arise in the theoretical framework introduced in [1], and derive generalized coordinate transformations, that allow to investigate physical systems in a non-classical regime.

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1 Introduction

In Ref. [1] we introduced an entropy-weighted sum over all possible configurations $\psi$ of a discrete space with a given amount of energy content $E \equiv N$, i.e. over all the ways of distributing $N$ energy units along a space of unit cells. This had to serve as the generating function for the (mean) values of any observable of a system that we call the “universe”:

$$Z_E = \int_E \mathcal{D}\psi \, e^{kS},$$

(1.1)

where $k$ is the Boltzmann constant, and the entropy $S$ is intended in its statistical sense, as the logarithm of the volume of occupation of a configuration $\psi(E)$ in the phase space of all the configurations, $\{\psi(E)\}$\footnote{In the following we will omit, wherever possible, namely, wherever the omission does not generate confusion, any dimensional constant.}. The space coordinates, that take only discrete values multiples of a unit, are allowed to run over any number of dimensions and extension. This unit had to be viewed as a minimal length, and, in the limit to the continuum, it has to be identified with the Planck length. In a loose sense, the entropy-weighted sum can be considered a sum over all possible “geometries”\footnote{If we consider as a geometry any possible distribution (assignment) of “energy” along discrete space coordinates, that only for large numbers and in a limit can be approximated with the usual concept of continuous, differential geometry based on the idea of dimensionless point.}, and as such contains all the information about the physics of the universe. The evolution of the universe, or in other words the time ordering, occurs through inclusion of sets. Owing to the property:

$$\{\psi\}_{E'} \subseteq \{\psi\}_E \quad \text{if} \quad E' \geq E,$$

(1.2)
the natural evolution is toward increasing total energy, that can also be identified, via appropriate conversion of units, with a time, the “age” of the universe. Expression \( \{ \psi \}_{E'} \supseteq \{ \psi \}_E \) has to be intended in the sense that \( \forall \psi_E \in \{ \psi \}_E, \exists \psi_{E'} \in \{ \psi \}_{E'} \) such that \( \psi_{E'} \subseteq \psi_E \).

From one can immediately see that the universe will be dominated by the configurations of maximal entropy. What is not at all obvious is that indeed these are the ones with three space dimensions, and with a distribution of energy corresponding to a sphere with a radius/energy relation given by the Schwarzschild black hole expression. Moreover, the fluctuations in the mean energy value due to the contribution of the “sea” of non dominant, in general not even geometrizable, configurations correspond to the Heisenberg’s time/energy Uncertainty Relation. We concluded therefore in Ref. [1] that the functional \( \mathcal{L} \) describes a quantum scenario (or, more precisely, it embeds a quantum scenario).

The quantum physical world as we experience it corresponds to the “mean value configuration” of this scenario, and it turns out that the horizon of this black hole-like universe expands at the speed of expansion of time, by definition/choice of units = \( c \). Here we will discuss how, in this universe, \( c \) is also the maximal speed of propagation of information. This leads to Relativity as a direct implication of this scenario. On the other hand, through this derivation we will also learn something more about the limitations implicit in a description of physical phenomena corresponding to the classical relativistic limit, and gaining some insight into the behaviour of physical systems beyond the domain of validity of the theory of Relativity.

We will start by showing how the speed of expansion of the radius of the average three-dimensional, black-hole-like universe is also the maximal speed of propagation of information (section 2). We say “information”, because, as also discussed in [1], in the sum \( \mathcal{L} \) are also contained configurations with faster objects (tachyonic configurations), but for these what is transferred cannot be considered something with a (geometric) structure which is conserved during transportation: by “information” we mean something that conserves its characteristics, i.e. carries, transfers, a message during its displacement.

We discuss then how different observers can relate their time measurements (section 3). According to our approach, time has progressed as soon as something in the universe, which, we remember it, includes also the observer has changed. If nothing changes, there is, by definition, no time progress. In the framework underlying configurations are identified by their energy distribution, i.e. by their combinatorics of energy and space degrees of freedom. This means, equivalently, through their symmetry group. As we work with discrete

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3The “past” is something of which the system conserves a memory. The configuration it corresponds to belongs therefore also to the set of configurations at any subsequent time, as expressed by the inclusion relation of above.

4As discussed in [1] this non-accelerated expansion nevertheless appears to be accelerated, as the effect of a time-dependence of masses and couplings, and therefore of the frequencies in the radiation/emission spectra.

5We recall that in our theoretical approach there is no absolute universe in itself, as something independent on the relative position of its observer: when we say that the universe has a certain age and the horizon a certain extension, this is the age and the extension as they are measured by a particular observer.
groups, there are no two inequivalent configurations with the same symmetry group \( \leftrightarrow \) volume of occupation in the phase space \( \leftrightarrow \) entropy. In particular, this holds also for the dominant configuration at any time. It is therefore possible to correlate the time flow to the entropy change of the dominant configuration. Similarly, the perception of time in any subsystem of the universe must be related to the entropy of the (subset of the) dominant configuration corresponding to that subsystem. This allows us to rephrase the problem of time transformation between different frames into a problem of transformation of entropies. In this work, the Lorentz boosts are derived via entropy arguments. This opens up new perspectives, eventually allowing to determine the metric of space-time around any point in the universe directly from the formulation in terms of entropy. That is, beyond classical General Relativity, in a pure “quantum gravity” regime.

We pass then, in the following section 4, to the general coordinate transformation, and therefore also to the metric in itself. All the relations we discuss involve only the time component. Time is in fact the ground quantity in this framework, as it is directly related to the fundamental quantities of this scenario, namely energy and entropy. Space lengths are derived quantities: lengths are defined through measurements of extensions and displacements that take place during a time interval, and therefore are related to time and to the speed of the transferred information. On the other hand, this is true also for the traditional derivation of the Lorentz boosts: the Lorentz space boost is obtained by comparing measurements of length made by counting the time needed for light to travel from one edge to the other of a space segment. In a more involved way, it is in principle possible also in our framework to obtain the full coordinate transformations, something we will however not do here, for the sake of shortness and simplicity, as it would add no further fundamental ideas and deeper insight into the problem.

Raising the Einstein’s equations to entropy-dependent expressions provides us with a more general expression, valid also in a quantum scenario and for cases in which no geometry in a classical sense can be identified. At the end of the work (section 5), we briefly discuss time coordinate transformations in the case of complex quantum systems in a quantum/relativistic regime.

2 From the speed of expansion of the universe to a maximal speed for the propagation of information

As discussed in Ref. [1], the universe arising from the superposition of all possible energy distributions (= configurations) at time \( E = N \) is predominantly a three-dimensional one with an energy distribution corresponding to the geometry of a three-sphere with radius \( \sim N \). Such a three-dimensional universe, of radius \( \sim N \), at time \( N \), with total energy also \( N \), behaves like a black hole expanding at speed 1 (we can introduce a factor of conversion from time to space, \( c \), and say that, by choice of units, we set the speed of expansion to be \( c = 1 \)). Here we want to see how this scenario implies that this is also the maximal speed for propagation of information within the three-dimensional universe (i.e., inside the black hole). It is important to stress that all this refers only to the average universe, because only in this sense we can say that the universe is three-dimensional: the sum 1.1 contains in
fact also configurations in which higher speeds are allowed (we may call them “tachyonic” configurations), along with configurations in which it is not even clear what is the meaning of speed of propagating information in itself, as there is no recognizable information at all, at least in the sense we usually intend it.

Indeed, when we say we get information about, say, the motion of a particle, or a photon, we mean to speak of a non-dispersive wave packet, so that we can say we observe a particle, or photon, that remains particle, or photon, along its motion. Let’s consider the simplified case of a universe at time $N$ containing only one such a wave packet, as illustrated in figure 2.1 where it is represented by the shadowed cells, and the space is reduced to two dimensions.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.1}
\caption{A simplified case of a universe at time $N$ containing only one wave packet.}
\end{figure}

Consider now the evolution at the subsequent instant of time, namely after having progressed by a unit of time. Adding one point, $N \rightarrow N + 1$, does produce an average geometry of a three sphere of radius $N + 1$ instead of $N$. In the average, it is therefore like having added $4\pi N^2$ “points”, or unit cells. Remember that we work always with an infinite number of cells in an unspecified number of dimensions; when we talk of universe in three dimensions within a region of a certain radius, we just talk of the average geometry, in the sense explained in Ref. [1]. Let’s suppose the position of the wave packet jumps by steps (two cells) back, as illustrated in figure 2.2. Namely, as time, and consequently also the radius of the universe, progresses by one unit, the packet moves at higher speed, jumping by two units:

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.2}
\caption{The evolution of a wave packet at the subsequent instant of time.}
\end{figure}

6Like a particle, also a physical photon, or any other field, is not a pure plane wave but something localized, therefore a superposition of waves, a wave packet.

7We may think to concentrate onto only a portion of the universe, where only such a wave packet is present.
Consider now the case in which the packet jumps by just one unit, as in figure 2.3 here below:

The entropy of this latter configuration, intermediate between the first and second one, cannot be very different from the one of the second configuration, figure 2.2 in which the packet jumps by two steps, because that was supposed to be the dominant configuration at time \( N + 1 \), and therefore the one of maximal entropy. Indeed, by “continuity” it must interpolate between step 2 and the configuration at time \( N \), that was also supposed to be a configuration of maximal entropy. Therefore, the actual appearance of the universe at time \( N + 1 \) must be somehow a superposition of the configurations 2 and 3, thereby contradicting our hypothesis that the wave packet is non-dispersive. Therefore, the wave packet cannot jump by two steps, and we conclude that the maximal speed allowed is that of expansion of the radius of the universe itself, namely, \( c \).

As discussed in Refs. [1, 2], 1.1 leads to the ordinary physics and the universe as we know it, with the correct content of particles, fields and interactions. According to this theoretical framework, the reason why we have a universal bound on the speed of light is therefore that light carries what we call classical information. Information about whatever kind of event tells about a change of average entropy of the observed system, of the observer, and what surrounds and connects them too. The rate of transfer/propagation of information is therefore strictly related to the rate of variation of entropy. Variation of entropy is what gives the measure of time progress in the universe. Any vector of information that “jumps” steps of the evolution of the universe, going faster than its rate of entropy variation, becomes therefore dispersive, looses information during its propagation. Light must therefore propagate at most at the rate of expansion of space-time (i.e. of the universe itself). Namely, at the rate of the space/time conversion, \( c \).

3 The Lorentz boost

Let’s now consider systems that can be identified as “massive particles”, i.e. localizable and that exist also “at rest”, therefore travelling at speeds always lower than \( c \). Since the phase space has a multiplicative structure, and entropy is the logarithm of the volume of

\[8\] If it was dispersive, it would be something like a particle that, during its motion, “dissolves”, and therefore we cannot anymore trace as a particle. It would be just a “vacuum fluctuation” without true motion, something that does not carry any information.
occupation in this space, it is possible to separate for each such a system the entropy into the sum of an internal, “rest” entropy, and an external, “kinetic” entropy. The first one refers to the structure of the system in itself, that can be a point-like particle or an entire laboratory. The second one refers to the relation/interaction of this system with the environment, the external world: its motion, the accelerations and external forces it experiences, etc.

Let us for a moment abstract from the fact that the actual configuration of the universe implied by 1.1 at any time describes a curved space. In other words, let’s neglect the so-called “cosmological term”. This approximation can make sense at large $N$, as is the case of the present-day physics, a fact that historically allowed to introduce special relativity and Lorentz boosts before addressing the problem of the cosmological constant. Let us also assume we can just focus our attention on two observers sitting on two inertial frames, $A$ and $A'$, moving at relative speed $v$, neglecting everything else. An experiment is the measurement of some event that, owing to the fact that happening of something means changing of entropy and therefore is equivalent to a time progress, gives us the perception of having taken place during a certain interval of time. Let us consider an experiment, i.e. the detection of some event, taking place in the co-moving frame of $A'$, as reported by both the observer at rest in $A$, and the one at rest in $A'$ (from now on we will indicate with $A$ and $A'$, indifferently the frame as well as the respective observer). Let’s assume we can neglect the space distance separating the two observers, or suppose there is no distance between them. For what above said, such a detection amounts in observing the increase of entropy corresponding to the occurring of the event, as seen from $A$ and from $A'$ itself. Since we are talking of the same event, the overall change of entropy will be the same for both $A$ and $A'$. One would think there is an “absolute” time interval, related to the evolution of the universe corresponding to the change of entropy due to the event under consideration. However, the story is rather different as soon as we consider time measurements of this event, as reported by the two observers, $A$ and $A'$. The reason is that the two observers will in general attribute in a different way what amount of entropy change has to be considered a change of entropy of the “internal” system, and which amount refers to an “external” change. Proper time measurements have to do with the internal change of entropy. For instance, consider the entropy of all the configurations contributing to form, say, a clock. The part of phase space describing the uniform motion of this clock will not be taken into account by an observer moving together with the clock, as it will not even be measurable. This part will however be considered by the other observer. Therefore, when reporting measurements of time intervals made by two clocks, one co-moving with $A$, and one seen by $A$ to be at rest in $A'$, owing to a different way of attributing elements within the configurations building up the system, between “internal” and “external”, we will have in general two different time measurements. Let us indicate with $\Delta S$ the change of entropy.

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9 In our approach, there does not exist a strictly point-like object. A point-like particle is an extended object of which we neglect the geometric structure.

10 In our theoretical framework, there is no “external observer”: 1.1 describes a universe “on shell”, the totality of the physical world.

11 See Ref. 1.

12 In our scenario, huge (=cosmological) distances have effect on the measurement of masses and couplings.
as it is observed by \( A \). We can write:

\[
\Delta S (\equiv \Delta S (A)) = \Delta S (\text{internal} = \text{at rest}) + \Delta S (\text{external})
\]

\[
= \Delta S (A') + \Delta S_{\text{Kinetic}} (A),
\]

with the identifications \( \Delta S (\text{internal} = \text{at rest}) \equiv \Delta S (A') \) and \( \Delta S (\text{external}) \equiv \Delta S_{\text{Kinetic}} (A) \).

In Ref. [1] we discussed how the entropy of a three sphere is proportional to \( N^2 = E^2 \). This is therefore also the entropy of the average, classical universe, that in the continuum limit, via the identification of total energy with time, can be written as:

\[
S \propto (cT)^2,
\]

where \( T \) is the age of the universe. This relation agrees with the Hawking’s expression of the entropy of a black hole of radius \( r = cT \), as indeed the universe in our theoretical framework is. It is not necessary to write explicitly the proportionality constant in \( [3.3] \), because we are eventually interested only in ratios of entropies. During the time of an event, \( \Delta t \), the age of the universe passes from \( T \) to \( T + \Delta t \), and the variation of entropy, \( \Delta S = S(T + \Delta t) - S(T) \), is:

\[
\Delta S \propto (c\Delta t)^2 + c^2 T^2 \left( \frac{2\Delta t}{T} \right).
\]

The first term corresponds to the entropy of a “small universe”, the universe which is “created”, or “opens up” around an observer during the time of the experiment, and embraces within its horizon the entire causal region about the event. The second term is a “cosmological” term, that couples the local physics to the history of the universe. The influence of this part of the universe does not manifest itself through elementary, classical causality relations within the duration of the event, but indirectly, through a (slow) time variation of physical parameters such as masses and couplings, an effect discussed in [2] and [1]. In the approximation of our abstraction to the rather ideal case of two inertial frames, we must neglect this part, concentrating the discussion to the local physics. In this case, each experiment must be considered as a “universe” in itself. Let’s indicate with \( \Delta t \) the time interval as reported by \( A \), and with \( \Delta t' \) the time interval reported by \( A' \). In units for which \( c = 1 \), and omitting the normalization constant common to all the expressions like \( [3.3] \), we can therefore write:

\[
\Delta S (A) = (\Delta t)^2,
\]

whereas

\[
\Delta S (A') = (\Delta t')^2,
\]

and

\[
\Delta S_{\text{Kinetic}} (A) = (v \Delta t)^2.
\]

These expressions have the following interpretation. As seen from \( A \), the total increase of entropy corresponds to the black hole-like entropy of a sphere of radius equivalent to the time duration of the experiment. Since \( v = c = 1 \) is the maximal “classical” speed of propagation of information, all the classical information about the system is contained within the horizon.
Figure 1: During a time $\Delta t$, the pure motion “creates” a universe with an horizon at distance $\Delta x = v\Delta t$ from the observer. As seen from the rest frame, this part of the physical system does not exist. The “classical” entropy of this region is given by the one of its dominant configuration, i.e. it corresponds to the entropy of a black hole of radius $\Delta x$.

set by the radius $c\Delta t = \Delta t$. However, when $A$ attempts to refer this time measurement to what $A'$ could observe, it knows that $A'$ perceives itself at rest, and therefore it cannot include in the computation of entropy also the change in configuration due to its own motion (here it is essential that we consider inertial systems, i.e. constant motions). “$A$” separates therefore its measurement into two parts, the “internal one”, namely the one involving changes that occur in the configuration as seen at rest by $A'$ (a typical example is for instance a muon’s decay at rest in $A'$), and a part accounting for the changes in the configuration due to the very being $A'$ in motion at speed $v$. If we subtract the internal changes, namely we think at the system at rest in $A'$ as at a point without meaningful physics apart from its motion in space\(^{13}\), the entire information about the change of entropy is contained in the “universe” given by the sphere enclosing the region of its displacement, $v^2(\Delta t)^2 = \Delta S_{\text{Kinetic}}(A)$. In other words, once subtracted the internal physics, the system behaves, from the point of view of $A$, as a universe which expands at speed $v$, because the only thing that happens is the displacement itself, of a point otherwise fixed in the local universe (see figure 1). Inserting expressions 3.5–3.7 in 3.2 we obtain:

$$\left(\Delta t\right)^2 = \frac{(\Delta t')^2}{1 - v^2},$$

that is:

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - v^2}}.$$  \(3.9\)

The time interval as measured by $A$ results to be longer by a factor $(\sqrt{1 - v^2})^{-1}$ than as measured by $A'$. We stress that, when we use expressions like “as seen from”, “it observes” and alike, we intend them in an ideal sense, not in the concrete sense of “detecting a light ray

\(^{13}\)No internal physics means that we also neglect the contribution to the energy/entropy due to the mass.
coming from the observed”. For the derivation of the Lorentz boost we did not make explicit use of the geometry of the propagation of light rays between observed and observers. The bound on the speed of information, and therefore of light, enters on the other hand when we write the variation of entropy of the “local universe” as \( \Delta S = (c\Delta t)^2 \). If \( c \to \infty \), namely, if within a finite interval of time an infinitely extended causal region opens up around the experiment, both \( A \) and \( A' \) turn out to have access to the full information, and therefore \( \Delta t = \Delta t' \). Namely, they observe the same overall variation of entropy.

3.1 the space boost

In this framework we obtain in quite a natural way the Lorentz time boost. The reason is that, for us, time evolution is directly related to entropy change, and we identify configurations (and geometries) through their entropy. Space length is somehow a derived quantity, and we expect also the space boost to be a secondary relation. Indeed, it can be easily derived from the time boost, once lengths and their measurements are properly defined. However, these quantities are less fundamental in that they are related to the classical concept of geometry. We could produce here an argument leading to the space boost. However, this would basically be a copy of the classical derivation within the framework of special relativity. A derivation of the time boost through entropy-based arguments opens instead new perspectives, allowing to better understand where relativity ends and quantum physics starts. Or, to better say, it provides us with an embedding of this problem into a scenario that contains both these aspects, relativity and quantization, as particular cases, to be dealt with as useful approximations.

4 General time coordinate transformation

Lorentz boosts are only a particular case of a more general transformation. They are valid when systems are not accelerated; in particular, when they are not subjected to a gravitational force. Traditionally, we know that the general coordinate transformation has to be found within the context of General Relativity; in that case the measure of time lengths is given by the time-time component of the metric tensor. In the absence of mixing with space boosts, i.e., with a diagonal metric, we have:

\[
(ds)^2 = g_{00}(dt)^2.
\]

As the metric depends on the matter/energy content through the Einstein’s Equations:

\[
R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G_N T_{\mu\nu},
\]

\( g_{00} \) can be computed when we know the energy of the system. For instance, in the case of a particle of mass \( m \) moving at constant speed \( \vec{v} \) (inertial motion), the energy, the “external” energy, is the kinetic energy \( \frac{1}{2}mv^2 \), and we recover the \( v^2 \)-dependence of the Lorentz boost\(^{14}\).

\(^{14}\)In the determination of the geometry, what matters here is not the full force experienced by the particle but the field in which the latter moves. The mass \( m \) therefore drops out from the expressions (see for instance \([3]\)).
In the simple case of the previous section, we have considered the physical system of the wave packet as decomposed into a part experiencing an “internal” physics, and a “center of mass point of view” part in which the complex internal physics is dealt with as a point-like particle. The Lorentz boost has been derived as the consequence of a transformation of entropies. Indeed, our coordinate transformation is based on the same physical grounds as the usual transformation of general Relativity, based on a metric derived from the energy tensor. In the linear approximation, where one keeps only the first two terms of the expansion of the square-root \( \sqrt{1 - v^2/c^2} \), the Lorentz boost can be obtained from an effective action in which in the Lagrangian appear the rest and the kinetic energy. These terms correspond to the two terms on the r.h.s. of equation 3.2. Entropy has in fact the dimension of an energy multiplied by a time.\(^{15}\) Approximately, we can write:

\[
\Delta S \simeq \Delta E \Delta t, \tag{4.3}
\]

where \( \Delta E \) is either the kinetic, or the rest energy. The linear version of the Lorentz boost is obtained by inserting in (4.3) the expressions \( \Delta E_{\text{rest}} = m \) and \( \Delta E_{\text{kinetic}} = \frac{1}{2}mv^2 \). In this case, the linearization of entropies lies in the fact that we consider the mass a constant, instead of the full energy of the “local universe” contained in a sphere of radius \( \Delta t \), i.e. the energy (mass) of a black hole of radius \( \Delta t \): \( m = \Delta E = \Delta t/2 \). Although imprecise, this approach through the linear approximation helps to understand where things come from.

In our theoretical framework, the general expression of the time coordinate transformation is:

\[
(\Delta t')^2 = \Delta S'(t) - \Delta S'_{\text{external}}(t). \tag{4.4}
\]

Here \( \Delta S'(t) \) is the total variation of entropy of the “primed” system as measured in the “unprimed” system of coordinates: \( \Delta S'(t) = (\Delta t)^2 \). We can therefore write expression (4.3) as:

\[
(\Delta t')^2 = [1 - \mathcal{G}(t)] (\Delta t)^2, \tag{4.5}
\]

where:

\[
\mathcal{G}(t) \overset{\text{def}}{=} \frac{\Delta S'_{\text{external}}(t)}{(\Delta t)^2}. \tag{4.6}
\]

With reference to the ordinary metric tensor \( g_{\mu\nu} \), we have:

\[
\mathcal{G}(t) = g_{00} + 1. \tag{4.7}
\]

\( \Delta S'_{\text{external}}(t) \) is the part of change of entropy of \( A' \) referred to by the observer \( A \) as something that does not belong to the rest frame of \( A' \). It can be the non accelerated motion of \( A' \), as in the previous example, or more generally the presence of an external force that produces an acceleration. Notice that the coordinate transformation (4.3) starts with a constant term, 1: this corresponds to the rest entropy term expressed in the frame of the observer. For the observer, the new time metric is always expressed in terms of a deviation from the identity.

By construction, (4.6) is the ratio between the metric in the observed system and the metric in the system of the observer. From such a coordinate transformation we can pass to

\(^{15}\)By definition, \( dS = dE/T \), where \( T \) is the temperature, and remember that in the conversion of thermodynamic formulas, the temperature is the inverse of time.
the metric of space-time itself, provided we consider the coordinate transformation between
the metric $g'$ of a point in space-time, and the metric of an observer which lies on a flat
reference frame, whose metric is expressed in flat coordinates. We have then:

$$G(t) - 1 = \frac{g(t)}{\eta_{00}} = \eta_{00} = 1.$$  \hspace{1cm} (4.8)

As soon as this has been clarified, we can drop out the denominator and we rename the
primed metric as the metric tout court.

Once the measurement of lengths is properly introduced, as derived from a measurement
of configurations along the history of the system, it is possible to extend the relations also
to the transformation of space lengths. This gives in general the components of the metric
tensor as functions of entropy and time. In classical terms, whenever this reduction is
possible, this can be rephrased into a dependence on energy (density) and time. They give
therefore a generalized, integrated version of the Einstein’s Equations. Let’s see this for
what concerns the time component of the metric. We want to show that the metric $g_{00}$ of
the effective space-time corresponds to the metric of the distribution of energy in the mean
space, i.e., in the classical limit of effective three-dimensional space as it arises from

This will mean that the geometry of the motion of a particle within this space is the geometry of
the energy distribution. In particular, if the energy is distributed according to the geometry
of a sphere, so it will be the geometry of space-time in the sense of General Relativity. To
this regard, we must remember that:

i) All this makes only sense in the “classical limit” of our scenario, namely only in an
average sense, where the universe is dominated by a configuration that can be described in
classical geometric terms. It is in this limit that the universe appears as three dimensional.
Configurations that are in general non three-dimensional, non-geometric, possibly tachyonic,
and, in any case, configurations for which General Relativity and Einstein Equations don’t
apply, are covered under the “un-sharping” relations of the Uncertainty Principle. All of
them are collectively treated as “quantum effects”;

ii) In the classical limit, nothing travels at a speed higher than $c$. As during an experiment
no information comes from outside the local horizon set by the duration of the experiment
itself, to cause some (classical) effects on it, any consideration about the entropy of the
configuration of the object under consideration can be made “local” (tachyonic effects are
taken into account by quantization). That means, when we consider the motion of an object
along space we can just consider the local entropy, depending on, and determined by, the
energy distribution around the object.

Having these considerations in mind, let us consider the motion of a particle, or, more
precisely, a non-dispersive wave-packet, in the mean, three-dimensional, classical space. Con-
sider to perform a (generally pointwise) coordinate transformation to a frame in which the
metric of the energy distribution external to the system intrinsically building the wave packet
in itself is flat, or at least remains constant. As seen from this set of frames, along the motion
there is no change of the (local) entropy around the particle, and the right hand side of

vanishes, implying that also the metric of the motion itself remains constant (remember that
in this case gives the ratio between metrics at different points/times). If on the
other hand we keep the frame of the observer fixed, and we ask ourselves what will be the
direction chosen by the particle in order to decide the steps of its motion, the answer will
be: the particle “decides” stepwise to go in the direction that maximizes the entropy around
itself. Let us consider configurations in which the only property of particles is their mass
(no other charges), so that entropy is directly related to the “energy density” of the wave
packet. In this case, between the choice of moving toward another particle, or far away, the
system will proceed in order to increase the energy density around the particle. Namely,
moving the particle toward, rather than away from, the other particle, in order to include
in its horizon also the new system. This is how gravitational attraction originates in this
theoretical framework.

In order to deal with more complicated cases, such as those in which particles have
properties other than just their mass (electro-magnetic/weak/strong charge), we need a more
detailed description of the phase space. In principle things are the same, but the appropriate
scenario in which all these aspects are taken into account is the one discussed in Ref. [2], in
which these issues are phrased and addressed within a context of (quantum) String Theory.
In Ref. [2] it was discussed how within that framework a full world of particles with masses
and charges, with the correct interactions, arises.

5 Transforms beyond the classical limit

Lifting the issue of relating time measurements to a problem of a comparison of entropies
allows to address questions that go beyond the domain of the Theory of Relativity. For
instance, it is possible to ask how does it appear an inertially moving frame in the extremely
relativistic regime, when the speed $v$ approaches the speed of light $c$, or what happens in a
system in a strong gravitational field, or acceleration. According to the theory of Relativity,
we would say that in the limit $v \to c$ the time gets “frozen”. But, if we look at expression 3.2
we see that we are just allowed to conclude that the variation of internal entropy decreases.
Only in the classical limit this means a slowing down of the time flow, as only in this limit
we can concentrate on the configuration corresponding to a classical geometric description,
neglecting all other configurations contributing to the mean value of what we observe. As
discussed in Ref. [1], out of the classical configuration, less entropic ones contribute to what
collectively we consider the “quantum spread” of observables.

In our scenario, a quantum system, or a system considered in the quantum regime, is
precisely a superposition of less entropic configurations. According to Ref. [1], these indeed
correspond to the microscopic details of our universe. This agrees with the common picture
of what is the quantum scale of phenomena [15]. When the classical variation of entropy
becomes negligible, the system does not freeze. In the usual language, we can say it starts
showing up a quantum behaviour. What happens is that, in the computation of entropy, we
cannot anymore neglect the less entropic configurations. Moreover, we cannot go on pushing
the external entropy, for instance by going to the very edge of the limit $v \to c$, till the

\[17\] To be more precise, in our scenario there is no exact distinction between “classical” and “quantum
mechanical”, in that everything is embedded in an universal description, which allows in this way to deal
with “quantum gravity aspects” also when they manifest themselves on a large scale.
vanishing of the internal term, without loosing the meaning of what we are doing. Indeed, beyond a certain limit, the separation in two terms of the r.h.s. of expressions 3.1 and 3.2 always possible from a mathematical point of view, looses its classical sense: as we go on “taking away” terms from the internal part of entropy and attributing them to the external one, also in the external part of entropy we start to include non-classical configurations [17]. In general it happens that “peripherical” configurations, which are less localized, start to weight more on the superposition that builds up the configuration of the system in its rest frame, and this results in an overall effective increase of its de-localization.

References

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\footnote{Not for every system it is easy to distinguish between internal and external part. There may be situations in which undergoing an external field of force deeply modifies the full system. But it is also true that in these situations the system as it is characterized as free, “at rest”, does not exist anymore once in interaction. There is therefore no more “rest frame”.

13