Aspects of Type I Compactifications
and Type I - Heterotic Duality

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**Abstract**

We review the construction of open descendants of the type IIB superstring on the Z-orbifold. It results in a chiral four-dimensional model with gauge group $SO(8) \otimes U(12)$ and three generations of matter in the $(8, 12^*) \oplus (1, 66)$ representations. As a test of type I - heterotic duality, that reduces to a weak/weak duality in $D = 4$, a heterotic model on the same orbifold is also presented. The massless spectrum reproduces exactly the one found in the type I case apart from additional twisted matter charged with respect to the $SO(8)$ gauge group. The puzzle is solved by noting that at generic points in the moduli space these states get masses.
1 Introduction

During the last few years, conjectures have been proposed establishing relations between apparently different string theories in various dimensions [1, 2, 3].

In string perturbation theory there are two topologically distinct classes of theories: those with only oriented closed strings (type IIA and B, heterotic $E_8 \otimes E_8$ and $SO(32)$) and those with both open and closed unoriented strings (type I). The picture that is recently taking shape suggests that these five theories describe different regions of an underlying moduli space associated to a more fundamental theory yet to be discovered. The transition functions between the different regions are realized in terms of duality transformations. In many cases these dualities require the inversion of the coupling constant, i.e. they are inherently non-perturbative. This explains why different theories look completely different in perturbation theory.

In order to check and then extract informations from dualities a special role is played by BPS states and, in particular, by type II solitons charged with respect to the Ramond-Ramond (RR) fields [5], called D-branes. A D-brane represents the manifold where open string ends are free to move, thus giving a geometrical interpretation of Chan-Paton (CP) multiplicities. A microscopic description in terms of open strings with Dirichlet boundary conditions in the directions transverse to the D-brane world-volume has brought about remarkable progress not only in the string duality context but also for what concerns (nearly extremal) black-holes thermodynamics. This is the reason why recently there has been an increasing interest in type I strings, although the initial proposal of identifying open string theories as parameter space orbifolds of left-right symmetric theories of oriented closed strings [4] was already fully and consistently systematized some time ago [3, 4]. Type I vacua have been analyzed in some detail in several dimensions [6, 8, 10, 11, 12, 13], resulting in new interesting phenomena. For instance one finds a rich pattern of CP symmetry breaking and varying numbers of tensor multiplets, including zero [6], in $D = 6$. Other puzzling phenomena occur at the boundary of the moduli space, as for example the appearance of tensionless strings [14, 15].
2  Type I - Heterotic Duality

One example of string-string duality is given by the two ten-dimensional theories with
gauge group $SO(32)$: the type I and heterotic strings \[1\]. At the massless level these
theories certainly agree. Moreover, the low energy effective action is uniquely fixed by
$N = (1, 0)$ supersymmetry. Might they in fact be equivalent? Moreover, the agreement
is such that strong coupling of one theory would turn into weak coupling of the other.
This is an essential point in any possible comparison between the two theories, since weak
coupling of one is certainly not equivalent to weak coupling of the other. Denoting by
$g_{I(H)}$ and $\phi_{I(H)}$ the ten-dimensional metric and dilaton of type I (heterotic) string, the

duality map reads:

$$g_I = e^{-\phi_H} g_H, \quad \phi_I = -\phi_H. \quad (2.1)$$

In support of this string-string duality, it can be shown that the excitations of the type I
D-string exactly coincide with the light-cone degrees of freedom of the $SO(32)$ heterotic
string \[16\]. The inverse relation seems harder to establish since the type I string is
unoriented and carries no conserved charge.

Upon toroidal compactification the map between the two theories gets much more
involved. In particular the relation between the heterotic and type I dilatons in $D$

dimensions is \[12\]

$$\phi_I^{(D)} = \frac{6 - D}{4} \phi_H^{(D)} - \frac{(D - 2)}{16} \log \det G_H^{(10-D)}, \quad (2.2)$$

where $G_H^{(10-D)}$ is the internal metric in the heterotic string frame, and there is a crucial
sign change at $D = 6$. From this relation one can deduce that there always exists a region
in the moduli space where both type I and heterotic string theories are weakly coupled,
and there we can rely on perturbation theory, which we understand.

3  $D = 6$ Type I Vacua

The first consistent $D = 6$ $N = 1$ chiral open-string models \[6\] differ markedly from
perturbative heterotic K3 compactifications \[23\], since they include different numbers of
tensor multiplets that take part in a generalized Green-Schwarz mechanism \[14\]. In the last two years, additional instances of \( N = 1 \) type I \( D = 6 \) models have been constructed as irrational toroidal orbifolds \[8\]. For rational internal tori there is an elegant description of superstring propagation on manifolds of \( SU(n) \) holonomy in terms of tensor products of \( N = 2 \) superconformal minimal models \[20\]. In addition to the Virasoro generators, \( L_n \), the \( N = 2 \) superconformal algebra includes two supercurrents \( G^\pm_r \) and a \( U(1) \) current, \( J_n \). An important feature of the \( N = 2 \) superconformal algebra is the presence of an automorphism, known as spectral flow, that connects different sectors of the spectrum. The minimal models form a discrete series with central charge \( c_k = 3k/(k + 2) \). Gepner has shown how to construct \( D \)-dimensional string vacua with space-time supersymmetry tensoring \( N = 2 \) minimal models in such a way that the total internal central charge is \( c_I = \sum_i c_{k_i} = 12 - 3(D - 2)/2 \), where \( c_{k_i} \) are the central charges of the various factors \[20, 21\].

In six dimensions there are several possible types of Gepner models. For the type IIB K3 compactifications the chiral spectrum is uniquely fixed by target-space \( N = (2, 0) \) supersymmetry and anomaly cancellation, and results in a supergravity multiplet coupled to 21 tensor multiplets \[22\]. The scalar fields of the resulting low-energy supergravity parameterize the coset \( SO(5, 21)/SO(5) \otimes SO(21) \).

The general construction of perturbative open string vacuum configurations consists in a non-geometrical \( \mathbb{Z}_2 \)-orbifold, named parameter space orbifold or orientifold \[4, 18\] (see also \[17\] for more details). First of all, the conventional Polyakov perturbative series must be supplemented with the inclusion of world-sheets with boundaries and/or crosscaps. The truncation of the parent left-right symmetric closed string spectrum encoded in the torus partition function, \( T \), is implemented by the Klein bottle projection, \( K \). As a result the \( \mathbb{Z}_2 \) projection halves the number of space-time supersymmetries. These two contributions make up the “untwisted sector” of the parameter space orbifold. The role of the “twisted sector” is played by the open string spectrum encoded in the annulus partition function, \( A \), and its projection, the Möbius strip, \( M \). In standard geometrical orbifolds twisted sectors have multiplicities associated to the fixed points. Similarly, in parameter space
orbifolds, the open string states may acquire multiplicities associated to their ends through the introduction of CP factors, or multiple D-branes. Consistency requirements may be deduced transforming the above amplitudes to the transverse channel, where Klein bottle, \( \tilde{\mathcal{K}} \), annulus, \( \tilde{\mathcal{A}} \), and Möbius strip, \( \tilde{\mathcal{M}} \), amplitudes are related to tree level closed string amplitudes between boundary and/or crosscap states, and consist in the cancellation of tadpoles of unphysical massless states or, equivalently, the cancellation of total RR charge.

Then in \( D = 6 \) the open descendants have \( N = (1, 0) \) target-space supersymmetry, and the closed unoriented spectrum consists of the supergravity multiplet coupled to \( n_T^c \) tensor multiplets and \( n_H^c \) hypermultiplets. The uniqueness of the parent type IIB massless spectrum forces \( n_T^c + n_H^c = 21 \), since the Klein bottle projection simply halves the fermionic degrees of freedom. The open unoriented spectrum completes the construction of the parameter space orbifold consistently with anomaly cancellation. Actually, tadpole conditions are in one-to-one correspondence with anomaly cancellation. For details on the various models and the corresponding CP gauge groups we refer the reader to the original paper [9]. Let us just quote the model \( D_{81} \) in [9]. It is an open descendant of the type IIB superstring compactified on six copies of Gepner minimal models with \( k = 1 \). The peculiarity of this model is that the closed unoriented spectrum does not contain any tensor multiplet at all. It is very interesting because the heterotic dual model should correspond to a vacuum configuration with a frozen dilaton!

It is worth to stress that typically the rank of the CP gauge group is smaller than the one related to irrational orientifolds, and this is due to the presence of a non-vanishing background for the NS-NS antisymmetric tensor in the internal tori [19].

The relation (2.2) implies that dual (non-perturbative) heterotic vacua in \( D = 6 \) should correspond to orbifold compactifications in which the usual modular invariance constraints are violated [24].
Now let us discuss the first class of four dimensional type I chiral models, discovered in [12]. We start from the Z-orbifold of the type IIB superstring. Then, the twist is given by \((\frac{1}{3},\frac{1}{3},\frac{1}{3})\), where each factor acts on a \(T^2\) torus, whose metrics \(G_{ab} = \frac{1}{2}R^2C_{ab}\), is proportional to the \(SU(3)\) Cartan matrix \(C_{ab}\) [23], . Moreover, we choose a vanishing NS-NS antisymmetric tensor in order to get a CP gauge group of maximal size [19].

The action of the orbifold point group breaks the ten-dimensional \(SO(8)\) characters down to \(SO(2) \otimes SU(3) \otimes U(1)\) ones. The torus amplitude can then be written as

\[
T = \frac{1}{3} \Xi_{0,0}(q)\Xi_{0,0}(\bar{q}) \sum q^{\frac{1}{12}} \bar{q}^{\frac{1}{12}} + \frac{1}{3} \sum_{\epsilon=\pm} \Xi_{0,\epsilon}(q)\Xi_{0,\epsilon}(\bar{q}) + \frac{1}{3} \sum_{\eta=\pm} \sum_{\epsilon=0,\pm} \Xi_{\eta,\epsilon}(q)\Xi_{-\eta,-\epsilon}(\bar{q}),
\]

where we have introduced

\[
\Xi_{0,\epsilon}(q) = \left( \frac{A_0\chi_0 + \omega^\epsilon A_+\chi_- + \bar{\omega}^\epsilon A_-\chi_+}{H^{3}_{0,\epsilon}} \right) (q),
\]

\[
\Xi_{\pm,\epsilon}(q) = \left( \frac{A_0\chi_\pm + \omega^\epsilon A_+\chi_0 + \bar{\omega}^\epsilon A_-\chi_\mp}{H^{3}_{\pm,\epsilon}} \right) (q),
\]

and \(\{A_0, A_+, A_-\}\) are supersymmetric characters of conformal weights \(\{\frac{1}{2}, \frac{1}{6}, \frac{1}{6}\}\) respectively, \(\{\chi_0, \chi_+, \chi_-\}\) are level-one \(SU(3)\) characters of conformal weights \(\{0, \frac{1}{3}, \frac{1}{3}\}\) respectively, and

\[
H_{0,\epsilon}(q) = q^{1/12} \prod_{n=1}^{\infty} (1 - \omega^\epsilon q^n)(1 - \bar{\omega}^\epsilon q^n),
\]

\[
H_{\pm,\epsilon}(q) = H_{-\epsilon,\epsilon}(q) = \frac{1}{\sqrt{3}} q^{-1/36} \prod_{n=0}^{\infty} (1 - \omega^\epsilon q^{n+1/3})(1 - \bar{\omega}^\epsilon q^{n+2/3}),
\]

originate from the action of the twist on the bosonic coordinates. Moreover the left and right momenta are given by \((p_a)_{L,R} = m_a \pm \frac{1}{2}G_{ab}n^b\).

The massless spectrum then comprises the \(N = 2\) supergravity multiplet coupled to \(9 + 1\) hypermultiplets from the untwisted sector and \(27\) hypermultiplets from the twisted sectors, one for each fixed point.

The torus partition function (4.1) corresponds to the charge conjugation modular invariant. Together with the fact that the only real character in the Z-orbifold is the
identity, the Klein bottle amplitude gets contributions only from the untwisted sector, and its expression is given by

\[ \mathcal{K} = \frac{1}{6} \Xi_{0,0}(q^2) \sum_{m_a} q^{m_a G^{ab} m_b} + \frac{1}{6} \Xi_{0,+}(q^2) + \frac{1}{6} \Xi_{0,-}(q^2). \] (4.4)

It contains only the conventional sum over the momentum lattice since, for generic values of the internal tori volumes, the condition \( p_L = \omega p_R \) (where \( \omega \) is the \( Z_3 \) generator) does not have any non-trivial solutions. In the open sector this reflects the presence of only D9-branes. The massless states in the projected closed string spectrum comprise the \( N = 1 \) supergravity multiplet coupled to \( 1 + 9 + 27 \) chiral multiplets. The scalars in the \( 1 + 9 \) un twisted chiral multiplets parametrize the Kähler manifold \( Sp(8,\mathbb{R})/SU(4) \times U(1) \), a real slice of the coset manifold \( E_{6(\pm 2)}/SU(2) \times SU(6) \) parameterized by the un twisted scalars in the parent type IIB theory [26].

The twisted sector of the parameter space orbifold, to be identified with the open string spectrum, contains the annulus and Möbius strip amplitudes:

\[
\mathcal{A} = \frac{(N + M + \bar{M})^2}{6} \xi_{0,0}(\sqrt{q}) \sum_{m_a} q^{m_a G^{ab} m_b} + \frac{(N + \omega M + \bar{\omega} \bar{M})^2}{6} \xi_{0,+}(\sqrt{q}) + \frac{(N + \bar{\omega} M + \omega \bar{M})^2}{6} \xi_{0,-}(\sqrt{q}),
\]

\[
\mathcal{M} = -\frac{(N + M + \bar{M})}{6} \hat{\xi}_{0,0}(\sqrt{q}) \sum_{m_a} q^{m_a G^{ab} m_b} - \frac{(N + \bar{\omega} M + \omega \bar{M})}{6} \hat{\xi}_{0,+}(\sqrt{q}) - \frac{(N + \omega M + \bar{\omega} \bar{M})}{6} \hat{\xi}_{0,-}(\sqrt{q}),
\]

where \( N, M, \bar{M} \) are CP multiplicities. The Möbius amplitude presents some subtleties connected with the proper definition of a set of real “hatted” characters [6]. Tadpole cancellations in the transverse channel result in

\[
N + M + \bar{M} = 32
\]

\[
N - \frac{1}{2}(M + \bar{M}) = -4.
\] (4.7)

From the amplitudes (4.5), (4.6) and tadpole conditions (4.7), we can extract the CP gauge group and the massless charged matter. In particular we have

\[
G_{CP} = SO(8) \otimes SU(12) \otimes U(1),
\] (4.8)
with three generations of chiral multiplets in the $(8_v, 12^*)_{-1} \oplus (1, 66)_2$ representations. The cancellation of the twisted tadpole guarantees that this chiral spectrum is anomaly free, aside from the $U(1)$ factor. The $U(1)$ anomaly translates into a Higgs-like mechanism that gives the abelian vector a mass of the the string scale $[27]$. 

The candidate heterotic dual corresponds to a perturbative compactification on the $Z$-orbifold with non-standard embedding $[12]$. The action of the twist on the gauge degrees of freedom consists of four copies of the basic $Z_3$ twist $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ and breaks the ten-dimensional $SO(32)$ gauge group down to $SO(8) \otimes U(12)$, the CP group of the type I model. Moreover, the untwisted charged spectrum coincides with the open string spectrum of the type I model, i.e. chiral multiplets in the representations $(8_v, 12^*) \oplus (1, 66)$. A striking feature of the heterotic model is that twisted scalars are charged with respect to the gauge group; in fact, in the heterotic case, we get 27 additional chiral multiplets in the $(8_c, 1)$ representation of the surviving (non-anomalous) gauge group, $SO(8) \otimes SU(12)$. The apparent puzzle associated to the presence of these extra charged chiral multiplets can be solved if one analyses the perturbative superpotentials for the type I and heterotic string models just discussed $[28]$. 

Denoting by $\Phi^i_a$ and $\chi^k_{[rs]}$ the three generations of chiral multiplets in the $(8_v, 12^*)$ and $(1, 66)$ representations of the gauge group, the cubic superpotential of the type I model is fixed by gauge symmetry and global $SU(3)$ symmetry to be $[12]$

$$W_I = y_I \delta^{ab} \epsilon_{ijk} \Phi^{ir}_a \Phi^{js}_b \chi^k_{[rs]}.$$  

(4.9)

In the heterotic model the perturbative superpotential is more involved since the scalar fields from the twisted sector have non-trivial couplings. Denoting by $S^A$ the 27 blow-up modes and by $T^R_{\dot{\alpha}}$ the 27 twisted charged scalars in the $(8_c, 1)$ representation, the heterotic superpotential reads $[28]$

$$W_H = y_H \delta^{ab} \epsilon_{ijk} \Phi^{ir}_a \Phi^{js}_b \chi^k_{[rs]} + \Lambda_{ABC} \delta^{\dot{\alpha} \dot{\beta}} S^A T^B_{\dot{\alpha}} T^C_{\dot{\beta}}.$$  

(4.10)

Notice that the additional contribution to the superpotential is non-vanishing only for $A = B = C$ or $A \neq B \neq C \neq A$, where $A, B, C$ label the 27 fixed points. The contribution coming from states sitting at different fixed points is exponentially suppressed with respect
to the separation of the fixed points. The form of $W$ and the above considerations suggest the solution of the puzzle. After blowing-up the orbifold singularities on the heterotic model (which amounts giving a non-vanishing vev to the $S^A$ fields) the $T$ fields become massive and decouple from the spectrum [28]. As a result, the type I and heterotic vacua are perturbatively equivalent.

5 Discussion

The advent of string dualities has shed some light on non-perturbative aspects of string theories and supersymmetric Yang-Mills theories. In particular, type I - heterotic duality seems very fruitful in understanding non-perturbative effects in $D = 4$ $N = 1$ supersymmetric theories. A strong/weak coupling duality in $D = 10$, reduces after compactification to a perturbative duality in $D = 4$. Nevertheless, studying heterotic duals of $D = 4$ type I vacua with D5-branes [11, 13] could help us to learn about non-perturbative effects in the heterotic string theory (such as NS 5-brane dynamics and the generation of a non-perturbative superpotential) by mapping them onto perturbative effects on the type I side (such as D5-brane dynamics).

A deeper understanding of the relation between tadpoles and anomaly cancellations in $D = 4$ type I vacua might also shed some light on some puzzling phenomena that occur in four-dimensional orientifolds [13].

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